An SQP Method Combined with Gradient Sampling for Small-Signal Stability Constrained OPF

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Abstract—Small-Signal Stability Constrained Optimal Power Flow (SSSC-OPF) can provide additional stability measures and control strategies to guarantee the system to be small-signal stable. However, due to the nonsmooth property of the spectral abscissa function, existing algorithms solving SSSC-OPF cannot guarantee convergence. To tackle this computational challenge of SSSC-OPF, we propose a Sequential Quadratic Programming (SQP) method combined with Gradient Sampling (GS) for SSSC-OPF. At each iteration of the proposed SQP, the gradient of the spectral abscissa function is randomly sampled at the current iterate and additional nearby points to make the search direction computation effective in nonsmooth regions. The method can guarantee SSSC-OPF is globally and efficiently convergent to stationary points with probability one. The effectiveness of the proposed method is tested and validated on WSCC 3-machine 9-bus system, New England 10-machine 39-bus system, and IEEE 54-machine 118-bus system.

Index Terms—small-signal stability, AC optimal power flow (ACOPF), gradient sampling, nonsmooth optimization, sequential quadratic programming, spectral abscissa.

I. INTRODUCTION

In the large interconnected power systems, the small-signal stability problem with the oscillation behavior can greatly threaten the system security [1]. The increased penetration of renewable energy sources can further deteriorate the small-signal stability [2]–[4], which is mainly driven by:

1) Converter control-based generators including variable-speed doubly fed asynchronous generators and photovoltaic (PV) plants are replacing the conventional generators. These solid-state inverters, however, usually do not contribute to the system inertia [2].

2) The fluctuating nature of the renewable resources such as wind and solar may cause rapid changes in future generation patterns, leading to rapid fluctuations of the power system’s operating point [3].

3) In some countries, such as China, due to the differences between the geographical distribution of load center and renewable energy, new Ultra-high-voltage Alternating Current (UHVAC) transmission lines are built to transfer the clean energy. The long distance power transmission over these UHVAC lines can result in oscillations [4].

Although the damping controllers can enhance small-signal stability, they cannot always guarantee the system to be small-signal stable [5]. By contrast, re-dispatch can provide additional stability measures and control strategies to make the system to be small-signal stable. Most commercial software, such as PSS®E and NEPLAN, only uses participation factors [6], [7] to perform offline re-dispatch study. Nonetheless, participation factors can neither determine whether the generation for each generator should be increased or decreased, nor tell how much generation should be dispatched [8]. On the contrary, the Small-Signal Stability Constrained Optimal Power Flow (SSSC-OPF) is a commendable model that can give the complete re-dispatch information to guarantee the small-signal stability while considering the economic objective and technical constraints. However, solving the SSSC-OPF problem can be very challenging because of the nonsmooth nature of the small-signal stability constraint. Existing methods for solving SSSC-OPF include:

- Numerical eigenvalue sensitivity based Interior Point Method (IPM): Numerical eigenvalue sensitivities have been used to improve the power transfer capability constrained by small-signal stability [5]. In [9], an SSSC-OPF is formulated to achieve an appropriate security level under stressed loading conditions, in which the small-signal stability constraint is replaced by first-order Taylor series expansion and the gradients of the real part of the critical eigenvalue are computed by numerical eigenvalue sensitivities. This method tries to make the small-signal stability constraint feasible by successively solving SSSC-OPF with IPM, but it may compromise on the economic cost due to losing the high-order terms of the small-signal constraint.

- Approximate singular value sensitivity based IPM: An SSSC-OPF is used to tune the oscillation controls in electricity markets [10], in which the minimum singular value of the modified full Jacobian matrix is proposed as a stability index. However, the gradient of this index is derived approximately by first-order Taylor series expansion and the Hessian is numerically evaluated by perturbing the gradients.

- Closed-form eigenvalue sensitivity based IPM: An expected-security-cost OPF with the small-signal stability constraint is presented in [11], in which a more computational efficient closed-form formula is used to calculate the eigenvalue sensitivities [12], [13]. Nevertheless, the
second-order eigenvalue sensitivities essential for forming the Hessian of the small-signal stability constraints for IPM have to be derived and calculated, which can be very time-consuming.

- Nonlinear semi-definite programming (NLSDP) method: A nonlinear semi-definite programming model is proposed to formulate the spectral abscissa constraint as a semi-definite constraint indirectly which is further transformed to some smooth nonlinear constraints [15]. An explicit and equivalent small-signal stability constraint is obtained based on Lyapunov theorem. However, the dense subsidiary semi-definite matrix variables may make the model computationally prohibitive for large systems.

IPM methods cannot guarantee convergence of SSC-OPF, even locally, because of the nonsmooth nature of the small-signal stability constraint function. During iterations they may suffer from oscillations between critical modes for different generator rescheduling patterns. Recently, a Sequential Quadratic Programming (SQP) method combined with Gradient Sampling (GS), called SQP-GS, is proposed for the nonsmooth constrained optimization [15]. Based on the SQP-GS, this paper proposes an optimization method to solve the SSSC-OPF problem with global convergence. The closed-form eigenvalue sensitivity is used to calculate the gradient of the spectral abscissa. Since the SQP only needs the gradients of the sum of all functions in the model, the second-order eigenvalue sensitivities are not needed. Moreover, owing to only one nonsmooth function in SSSC-OPF, GS can be performed only for this nonsmooth function, which would be beneficial to improve computation efficiency significantly.

The remainder of this paper is organized as follows. Section II describes the small-signal stability model. In Section III the property of the small-signal stability constraint and the calculation of the eigenvalue sensitivities are discussed. Section IV introduces the SQP-GS method for nonsmooth constrained optimization. Then we discuss how to solve the SSC-OPF problem by the SQP-GS method in Section V. The proposed method is tested and validated on three systems in Section VI. Finally the conclusion is drawn in Section VII.

II. SMALL-SIGNAL STABILITY MODEL

A. General Model of Small-Signal Stability

The behavior of a dynamical system can be described by differential and algebraic equations in the following form

\[
\begin{align*}
\frac{ds}{dt} &= f_d(s, u, y) \\
n0 &= f_a(s, y),
\end{align*}
\]  

(1)  

(2)

where \( s \) is the vector of state variables, \( u \) is the vector of inputs, and \( y \) is the non-state variables.

In small-signal stability analysis, the linearized form of (1)–(2) is often used:

\[
\begin{bmatrix}
\Delta s \\
0
\end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \Delta s \\
\Delta y
\end{bmatrix} + \begin{bmatrix} E_1 \\
0
\end{bmatrix} \Delta u.
\]  

(3)

Eliminating \( \Delta y \) we can get

\[
\Delta s = A\Delta s + E_1\Delta u,
\]  

(4)

where \( A = \hat{A} - \hat{B}\hat{D}^{-1}\hat{C} \) is commonly referred to as the state matrix.

B. Differential and Algebraic Equations of Power Systems

1) Generator Model: The two-axis synchronous generator model [6] that has been widely used in small-signal stability analysis is considered in this paper. This model is more realistic than the classical model used in [16], which is appropriate only for the most basic studies. The differential equations for generator \( i \in S_G \) can be written as

\[
\begin{align*}
\frac{d\delta_i}{dt} &= \omega_i - \omega_s \tag{5} \\
\frac{d\omega_i}{dt} &= \frac{1}{M_i} \left( T_{Mi} - (E_{qi} - X_{qi}I_{di})I_{di} - (E_{di} + X_{di}I_{qi})I_{qi} - D_i(\omega_i - \omega_s) \right) \tag{6} \\
\frac{dE_{qi}}{dt} &= \frac{1}{T_{dqi}} \left( -E_{qi} - (X_{di} - X_{di}^*)I_{di} + E_{fdi} \right) \tag{7} \\
\frac{dE_{di}}{dt} &= \frac{1}{T_{qdi}} \left( -E_{di} + (X_{qi} - X_{qi}^*)I_{qi} \right), \tag{8}
\end{align*}
\]

where \( S_G \) is the set of generators, \( \delta_i \) is rotor angle, \( \omega_i \) is rotor speed, \( \omega_s \) is rated rotor speed, \( E_{di} \) and \( E_{qi} \) are, respectively, the d-axis and q-axis components of the internal voltage, \( I_{di} \) and \( I_{qi} \) are d-axis and q-axis components of the internal current, \( T_{Mi} \) is mechanical power output, \( M_i \) is inertia constant, \( D_i \) is damping torque coefficient, \( E_{fdi} \) is excitation output voltage, \( X_{di} \) and \( X_{qi} \) are synchronous reactance, \( X_{di}^* \) and \( X_{qi}^* \) are transient reactance, and \( T_{dqi}^* \) and \( T_{qdi}^* \) are open-circuit time constant, respectively, at d and q axes.

Besides, the stator algebraic equations for generator \( i \in S_G \) can be written in polar form as [6]

\[
\begin{align*}
E_{di}' - V_i \sin(\delta_i - \theta_i) - R_{si}I_{di} + X_{qi}'I_{qi} &= 0 \tag{9} \\
E_{qi}' - V_i \cos(\delta_i - \theta_i) - R_{si}I_{qi} - X_{di}'I_{di} &= 0, \tag{10}
\end{align*}
\]

where \( V_i \) is bus voltage magnitude, \( \theta_i \) is bus voltage phase angle, and \( R_{si} \) is the armature resistance.

2) Exciter Model: In this paper we use the IEEE Type DC-1 exciter for each generator, which can be expressed in differential equations for generator \( i \in S_G \) as [6]

\[
\begin{align*}
\frac{dE_{fdi}}{dt} &= \frac{1}{T_{Ei}} \left( - (K_{EI} + S_E(E_{fdi}) - E_{fdi} + V_{RFi}) \right) \tag{11} \\
\frac{dV_{RFi}}{dt} &= \frac{1}{T_{AI}} \left( - V_{RI} + K_{AI}R_{FI} - \frac{K_{AI}K_{FI}}{T_{FI}} E_{fdi} \right) + K_{AI}(V_{refi} - V_{i}) \tag{12} \\
\frac{dR_{FI}}{dt} &= \frac{1}{T_{FI}} \left( - R_{FI} + \frac{K_{FI}}{T_{FI}} E_{fdi} \right), \tag{13}
\end{align*}
\]

where \( V_{RFi} \) is voltage regulator output, \( R_{FI} \) is exciter rate feedback, \( S_E(E_{fdi}) = A_{ei}e^{B_{ei}E_{fdi}} \) is the field saturation function with coefficients \( A_{ei} \) and \( B_{ei} \), \( K_{EI} \) is exciter gain, \( K_{AI} \) is voltage regulator gain, \( K_{FI} \) is rate feedback gain, \( T_{EI} \) is exciter time constant, \( T_{AI} \) is voltage regulator time constant, \( T_{FI} \) is rate feedback time constant, and \( V_{refi} \) is the reference voltage.
3) Network Model: The network equations relate the real and reactive power injections at each bus to the voltage magnitudes and phase angles at the system buses. In this paper the loads are modeled as constant power. Then for a generator bus \( i \in S_G \) we have \([6]\)

\[
I_{di} V_i \sin(\delta_i - \theta_i) + I_{q_i} V_i \cos(\delta_i - \theta_i) + P_{Li} = \sum_{j \in S_n} V_i V_j \cos(\delta_i - \delta_j - \alpha_{ij}) = 0 \tag{14}
\]

\[
I_{di} V_i \cos(\delta_i - \theta_i) - I_{q_i} V_i \sin(\delta_i - \theta_i) + Q_{Li} = \sum_{j \in S_n} V_i V_j \sin(\delta_i - \delta_j - \alpha_{ij}) = 0, \tag{15}
\]

where \( P_{Li} \) and \( Q_{Li} \) are, respectively, the active and reactive load, and \( Y_{ij} e^{i \alpha_{ij}} \) is the entry of the admittance matrix.

For a non-generator bus \( i \in S_L \), there are

\[
P_{Li} - \sum_{j \in S_n} V_i V_j \cos(\delta_i - \delta_j - \alpha_{ij}) = 0 \tag{16}
\]

\[
Q_{Li} - \sum_{j \in S_n} V_i V_j \sin(\delta_i - \delta_j - \alpha_{ij}) = 0, \tag{17}
\]

where \( S_L \) is the set of non-generator buses and \( S_n = S_G \cup S_L \) is the set of all of the buses.

C. Linearization of Dynamic System Model

Linearization of \([5]-[17]\) yields

\[
\Delta \dot{s} = A_1 \Delta s + B_1 \Delta I_i + B_2 \Delta V_g + E_1 \Delta u \tag{18}
\]

\[
0 = C_1 \Delta s + D_1 \Delta I_g + D_2 \Delta V_g \tag{19}
\]

\[
0 = C_2 \Delta s + D_3 \Delta I_g + D_4 \Delta V_g + D_5 \Delta V_1 \tag{20}
\]

\[
0 = D_6 \Delta V_g + D_7 \Delta V_1, \tag{21}
\]

where \([18]\) is obtained by linearizing the differential equations \([5]-[8]\) and \([11]-[13]\), \( B_1 \) comes from the stator algebraic equations \([9]-[10]\) and \([20]\), \( B_2 \) comes from the network equations \([14]-[15]\), and \([21]\) is from the network equations \([16]-[17]\); \( A_1, B_1, B_2, E_1, C_1, D_1, D_2, C_2, D_3 \) are block diagonal matrices, \( D_4, D_5, D_6, D_7 \) are full matrices, and

\[
s_i = [\delta_i \ \omega_i \ E'_{qi} \ E'_{di} \ E_{di} \ V_{Ri} \ R_{Fi}]^\top \quad i \in S_G
\]

\[
I_{gi} = [I_{di} \ I_{q_i}]^\top \quad i \in S_G
\]

\[
V_{gi} = [\theta_i \ V_i]^\top \quad i \in S_G
\]

\[
V_{li} = [\theta_i \ V_i]^\top \quad i \in S_L
\]

\[
u_i = [T_{Mi} \ V_{refi}]^\top \quad i \in S_G.
\]

We can rewrite \((18)-(21)\) in the compact form \([3]\), and

\[
y = [T_g^\top \ V_g^\top \ V_1^\top]^\top
\]

\[
\tilde{A} = A_1, \quad \tilde{B} = \begin{bmatrix} B_1 & B_2 & 0 \end{bmatrix},
\]

\[
\tilde{C} = \begin{bmatrix} C_1 & C_2 & 0 \\ 0 & D_3 & D_4 & D_5 \\ 0 & 0 & D_6 & D_7 \end{bmatrix},
\]

\[
\dot{\tilde{C}} = \begin{bmatrix} D_1 & D_2 & 0 \\ 0 & D_3 & D_4 & D_5 \\ 0 & 0 & D_6 & D_7 \end{bmatrix}.
\]

III. SMALL-SIGNAL STABILITY CONSTRAINT AND EIGENVALUE SENSITIVITIES

If the eigenvalues of the state matrix \( A \) all have negative real parts, the power system is stable in small-signal stability sense. An index \( \eta \) called spectral abscissa, which is the largest real part of the eigenvalues, is often used to describe the security margin:

\[
\eta(A) = \max \{ \Re(\lambda) : \lambda \in \lambda(A) \} = \Re(\lambda_\eta), \tag{22}
\]

where \( \lambda(A) \) represents all the eigenvalues of \( A \), \( \Re(\lambda) \) is the real parts of the eigenvalues \( \lambda \), and \( \lambda_\eta \) is the eigenvalue with the largest real part. The spectral abscissa determines the decay rate of the amplitude of the oscillation. The smaller the spectral abscissa, the more stable the system is.

The state matrix \( A \) usually has complex eigenvalues due to its unsymmetrical characteristic. Generally, the spectral abscissa function is non-smooth \([17]\). Fortunately, the spectral abscissa function has been proved to be locally Lipschitz and continuously differentiable on open dense subsets of \( \mathbb{R}^n \) \([18]\), which means that it is continuously differentiable almost everywhere and its gradient can be easily obtained where it is defined by calculating first-order spectral abscissa sensitivities.

As for computing the spectral abscissa sensitivities, the numerical differentiation method is widely used, which performs eigenvalue analysis to get the spectral abscissa \( \eta(A) \) of the state matrix at the equilibrium point and then vary one variable \( x_i \) by a small quantity \( \varepsilon \) to get the perturbed state matrix \( A_\varepsilon \) and its spectral abscissa \( \eta(A_\varepsilon) \). The spectral abscissa sensitivity with respect to \( x_i \) can be approximated by

\[
\frac{\partial \eta}{\partial x_i} \approx \frac{\eta(A_\varepsilon) - \eta(A)}{\varepsilon}. \tag{23}
\]

The numerical differentiation method is easy to implement, but for large systems its calculation burden can be heavy due to the repetitive procedure. Also, the sensitivity with respect to the power of the slack bus cannot be obtained. Alternatively, the spectral abscissa sensitivity can be obtained by closed-form formulas. Specifically, the \( j \)th eigenvalue sensitivity with respect to the \( i \)th variable \( x_i \) can be written as \([12], [13]\):

\[
\frac{\partial \lambda_j}{\partial x_i} = \frac{\phi_j^\top A \phi_j}{\psi_j^\top \phi_j}, \tag{24}
\]

where \( \psi_j \) and \( \phi_j \) are, respectively, the left and right eigenvectors of the eigenvalue \( \lambda_j \), and

\[
\frac{\partial A}{\partial x_i} = \frac{\partial A}{\partial x_i} - \frac{\partial B}{\partial x_i} \tilde{D}^{-1} \tilde{C}
\]

\[+ \tilde{B} \tilde{D}^{-1} \frac{\partial \tilde{D}}{\partial x_i} \tilde{D}^{-1} \tilde{C} - \tilde{B} \tilde{D}^{-1} \frac{\partial \tilde{C}}{\partial x_i} \] \tag{25}

Then the sensitivity of the spectral abscissa with respect to \( x_i \) can be given by

\[
\frac{\partial \eta}{\partial x_i} = \Re \left( \frac{\partial \lambda_\eta}{\partial x_i} \right). \tag{26}
\]

From \([25]\) it is seen that the derivation of the eigenvalue sensitivities for all of the variables requires considerable work since the elements of \( \hat{A}, \hat{B}, \hat{C}, \) and \( \hat{D} \) can be different functions of several variables. However, the derivation is
where the objective function formulation to compute the gradient of the spectral abscissa.

IV. A SEQUENTIAL QUADRATIC PROGRAMMING ALGORITHM COMBINED WITH GRADIENT SAMPLING

Sequential Quadratic Programming (SQP) has a long and rich history in solving smooth constrained optimization problems [19]. In each iterate of the traditional SQP algorithms, a quadratic programming (QP) subproblem is solved to obtain a search direction. However, the traditional SQP algorithms will fail for nonsmooth problems in theory and in practice. In 2005 an algorithm known as Gradient Sampling (GS) was developed [20]. More recently, a SQP-GS method that combines the techniques of SQP and GS is developed [15], which is proved to be able to globally convergent to stationary points with probability one when the objective and constraint functions are locally Lipschitz and continuously differentiable on open dense subsets of \( \mathbb{R}^n \). SQP-GS has been shown to be a reliable method for many challenging nonsmooth problems, even when the objective function is not locally Lipschitz, in which case the convergence cannot be guaranteed though [15].

The GS algorithm is conceptually simple. Basically, it is a stabilized steepest descent algorithm [21]. In each iteration, a descent direction is obtained by evaluating the gradient of the objective function at the current iterate and an additional set of nearby points and computing the vector in the convex hull of the gradients with the smallest norm. A standard line search is then used to obtain a lower point. The stabilization is controlled by the gradient sampling radius. As a natural extension to constrained optimization, the GS procedure samples the gradients of the constraint functions along with the objective function, thereby ensuring that good search directions are produced in nonsmooth regions.

Generally, the SQP-GS algorithm is developed to solve optimization problems in the following form:

\[
\begin{align*}
\min_x f(x) \\
\text{s.t.} \quad h(x) &= 0 \\
g(x) &\leq 0
\end{align*}
\]

where the objective function \( f : \mathbb{R}^n \rightarrow \mathbb{R} \), the equality constraint functions \( h : \mathbb{R}^n \rightarrow \mathbb{R}^s \), and the inequality constraint functions \( g : \mathbb{R}^n \rightarrow \mathbb{R}^m \) are locally Lipschitz and continuously differentiable on open dense subsets of \( \mathbb{R}^n \).

At the heart of SQP-GS is the following QP used to compute a search direction in the 4th iteration:

\[
\begin{align*}
\min_{d, z, e, \rho} \quad & \rho z + \frac{1}{2} d^\top H_k d_k + 1 \sum_{i=1}^{m_h} e_i + \sum_{j=1}^{m_g} (\sigma^j + \tau^j) \\
\text{s.t.} \quad & f(x_k) + \nabla f(x_k)^\top d_k \leq z \\
& h_i(x_k) + \nabla h_i(x_k)^\top d_k \leq e_i \\
& -h_i(x_k) - \nabla h_i(x_k)^\top d_k \leq -e_i \\
& g^j(x_k) + \nabla g^j(x_k)^\top d_k \leq \sigma^j + \tau^j \\
& -g^j(x_k) - \nabla g^j(x_k)^\top d_k \leq \tau^j - \sigma^j \\
& (e, \sigma, \tau) \geq 0,
\end{align*}
\]

where \( \rho \) is a penalty parameter, \( d_k \) is the search direction, \( H_k \) is the approximated Hessian of the Lagrangian of (27), \( m_h \) and \( m_g \) are, respectively, the number of the equality and inequality constraints, \( z, e, \sigma, \tau \) and \( \rho \) are slack variables, and

\[
\begin{align*}
B_{e,k}^j := \{x_{k,0}^j, x_{k,1}^j, \ldots, x_{k,p}^j\}, \quad & \text{where } x_{k,0}^j := x_k \\
B_{h,k}^j := \{x_{k,0}^j, x_{k,1}^j, \ldots, x_{k,p}^j\}, \quad & \text{where } x_{k,0}^j := x_k \\
B_{g,k}^j := \{x_{k,0}^j, x_{k,1}^j, \ldots, x_{k,p}^j\}, \quad & \text{where } x_{k,0}^j := x_k
\end{align*}
\]

are sets of \( p \) (sample size) independent and identically distributed random points uniformly sampled from

\[
\mathbb{B}_\epsilon(x_k) := \|x - x_k\|_2 \leq \epsilon,
\]

where \( \epsilon \) is the sample radius. To indicate the progress of the algorithm iterations, an infeasibility vector is defined as

\[
\sigma_k(x_k) := \begin{pmatrix} |h(x_k)| \\ \max g(x_k) - \max (g - g(x_k), 0) \end{pmatrix}.
\]

The following model reduction is also defined in terms of primal and dual infeasibility, which can be zero only if \( x_k \) is \( \epsilon \)-stationary [15]:

\[
\Delta q_k := \rho f(x_k) + \sum_{k} \sigma_k(x_k) - \rho \max_{x \in B_{e,k}} \{ f(x) + \nabla f(x)^\top d_k \}
\]

\[
- \frac{1}{2} d_k^\top H_k d_k - \sum_{i=1}^{m_h} \max_{x \in B_{h,k}^i} |h_i(x_k) + \nabla h_i(x_k)^\top d_k| \\
- \sum_{j=1}^{m_g} \max_{x \in B_{g,k}^j} \{ \max g^j(x_k) + \nabla g^j(x_k)^\top d_k - \sigma^j, 0 \} \\
- \sum_{j=1}^{m_g} \max_{x \in B_{g,k}^j} \{ \max \{ -g^j(x_k) - \nabla g^j(x_k)^\top d_k + \sigma^j, 0 \} \}.
\]

Then the SQP-GS algorithm is presented in Algorithm 1.

The SQP-GS algorithm generalizes the traditional SQP method to nonsmooth constrained problems. When solving a smooth constrained problem, the sample size \( p \) can be chosen as zero, in which case the SQP-GS algorithm reduces to the traditional SQP method.

If a function is known to be continuously differentiable everywhere in \( \mathbb{R}^n \), it is unnecessary to sample its gradient at nearby points. This can significantly improve the performance of the algorithm because the evaluation of the linear inequality constraints from the quadratic programming can be largely eliminated. Besides, for those functions that depend on fewer than \( n \) variables, sampling fewer points can still yield good results. Also, since the points are sampled independently, it allows us to use parallel computing to further reduce CPU time.

V. SOLVING SSSC-OPF BY SQP-GS

Here we discuss how to apply the SQP-GS algorithm in Section IV to solve the SSSC-OPF problem.
Algorithm 1 SQP-GS Algorithm

1: Initialization: Set \( k = 1 \), \( K_{\text{max}} = 100 \); Choose a sampling radius \( \epsilon > 0 \), penalty parameter \( \rho > 0 \), constraint violation tolerance \( \tau > 0 \), sample size \( p > 0 \), line search constant \( \varpi \in (0, 1) \), backtracking constant \( \gamma \in (0, 1) \), sampling radius reduction factor \( \mu_r \in (0, 1) \), penalty parameter reduction factor \( \mu_p \in (0, 1) \), constraint violation tolerance reduction factor \( \mu_v \in (0, 1) \), infeasibility tolerance \( \nu_{\text{in}} > 0 \), and stationarity tolerance parameter \( \nu_s > 0 \). Choose an initial iterate \( \mathbf{x} \).

2: while \( k < K_{\text{max}} \)
3: if max(\( \Delta \mathbf{q}_k \)) < \( \nu_s \) and max(\( \mathbf{s}_k \)) < \( \nu_{\text{in}} \)
4: Output solution and stop.
5: end if
6: Gradient sampling: Generate \( B^f_{e,k} \), \( B^h_{e,k} \), and \( B^g_{e,k} \) by (35)–(38).
7: Search direction calculation: Solve (28)–(34) to get \( (d\mathbf{k}, z_k, \bar{r}_k, r_k, e_k) \).
8: L-BFGS update: Limited-memory Broyden-Fletcher-Goldfarb-Shanno (L-BFGS) method [22] that is typical in smooth optimization is used to update \( H_k \).
9: if \( \Delta \mathbf{q}_k \geq \nu_s\epsilon^2 \)
10: go to step 20.
11: else
12: if max(\( \mathbf{s}_k \)) \leq \( \tau \)
13: set \( \tau \leftarrow \mu_\tau \tau \)
14: else
15: set \( \rho \leftarrow \mu_\rho \rho \)
16: end if
17: set \( \epsilon \leftarrow \mu_\epsilon \epsilon \), \( \beta_k \leftarrow 0 \)
18: go to step 21
19: end if
20: Line search: Set \( \beta_k \) as the largest value in the sequence \( \{1, \gamma, \gamma^2, \cdots\} \) such that \( \mathbf{x}_{k+1} \leftarrow \mathbf{x}_k + \beta_k d\mathbf{k} \) satisfies:
\[
\rho f(\mathbf{x}_{k+1}) + \sum \mathbf{s}_k(\mathbf{x}_{k+1}) \leq \rho f(\mathbf{x}_k) + \sum \mathbf{s}_k(\mathbf{x}_k) - \varpi \beta_k \Delta \mathbf{q}_k
\] (41)
21: Iteration increment: Set \( k \leftarrow k + 1 \)
22: end do

A. Model of SSSC-OPF

The model of SSSC-OPF is actually a ‘standard’ OPF model defined as a smooth nonlinear programming problem with an additional small-signal stability constraint. Specifically, the SSSC-OPF model can be represented as follows.

1) Minimizing the generation cost is usually considered as the objective function
\[
f = \sum_{i \in S_G} (a_{2i} P_{Gi}^2 + a_{1i} P_{Gi} + a_{0i}),
\] (42)
where \( P_{Gi} \) is the active power output of the \( i \)th generator, and \( a_{2i} \), \( a_{1i} \), and \( a_{0i} \) are the corresponding cost coefficients.

2) Power flow equations for bus \( i \in S_B \)
\[
P_{Gi} - P_{Li} - \sum_{j \in i} V_i Y_{ij} V_j \cos(\theta_i - \theta_j - \alpha_{ij}) = 0
\] (43)
\[
Q_{Gi} - Q_{Li} - \sum_{j \in i} V_i Y_{ij} V_j \sin(\theta_i - \theta_j - \alpha_{ij}) = 0
\] (44)
where \( Q_{Gi} \) is the reactive power output of the \( i \)th generator.

3) Initial condition equations for generator \( i \in S_G \) with a two-axis model.
- Stator algebraic equations
\[
E'_{di} - V_i \sin(\delta_i - \theta_i) - R_s I_{di} + X'_q I_{qi} = 0
\] (45)
\[
E'_{qi} - V_i \cos(\delta_i - \theta_i) - R_s I_{qi} - X'_d I_{di} = 0
\] (46)
- The generator terminal power can be obtained by the product of voltage and current transformed from \( d - q \) rotor reference frame to network reference frame as
\[
P_{Gi} - V_i I_{qi} \sin(\delta_i - \theta_i) - V_i I_{di} \cos(\delta_i - \theta_i) = 0
\] (47)
\[
Q_{Gi} - V_i I_{di} \cos(\delta_i - \theta_i) + V_i I_{qi} \sin(\delta_i - \theta_i) = 0
\] (48)
- In steady state, \( dE'_d/dt \) in (47) and \( dE'_q/dt \) in (48) are zero and thus
\[
E_{di} - E'_{di} - (X_{di} - X'_{di}) I_{di} = 0
\] (49)
\[
E'_{di} - (X_{di} - X'_{di}) I_{qi} = 0
\] (50)

4) Technical constraints include
\[
V_i \leq \overline{V}_i \quad \forall i \in S_B
\] (51)
\[
P_{Gi} \leq \overline{P}_{Gi} \quad \forall i \in S_G
\] (52)
\[
Q_{Gi} \leq \overline{Q}_{Gi} \quad \forall i \in S_G
\] (53)
\[
I_{ij}^2 \leq T_{ij} \quad \forall (i, j) \in S_{\text{Line}}
\] (54)
where \( I_{ij} \) is the current of line \((i, j)\), \( S_{\text{Line}} \) is the set of all lines and \((\overline{\cdot})\) and \((\underline{\cdot})\) denote the upper and lower limits.

5) Small-signal stability constraint:
\[
\eta(\mathbf{x}) \leq \overline{\eta}
\] (55)
where \( \mathbf{x} = [P_G^T \quad Q_G^T \quad \theta^T \quad \delta^T \quad E_d^T \quad E_q^T \quad I_d^T \quad I_q^T \quad E_{di}^T] \) is the vector of variables in the model. The choice of \( \overline{\eta} \) depends on the system characteristics and can be determined based on offline stability studies.

B. Employing SQP-GS to Solve SSSC-OPF

Obviously, the SSSC-OPF model belongs to the type of optimization problem in (27). Before employing the SQP-GS method that relies on the gradients to construct the QP subproblem, the gradients of all functions in the model with respect to \( \mathbf{x} \), the variables of the model, should be derived. The objective function and constraint functions in (43)–(54) are smoothly nonlinear or linear, and their gradients with respect to \( \mathbf{x} \) can be easily derived. As discussed in Section III, the spectral abscissa function in (55) is implicit but the function value and its gradient can also be evaluated.
As discussed in Section [IV] it is unnecessary to sample the gradients of the smooth functions. In SSSC-OPF, the function in (55) is the only nonsmooth function, and thus only its gradient need to be sampled, which requires the following steps for the $k$ iteration:

1) **Sampling Points**: Generate $p$ points $B_{ik}^n$ by (38).
2) **Eigenvalue Analysis**: Set up state matrix $A$ for each point $x \in B_{ik}^n$ and calculate the most critical eigenvalue $\lambda_\eta$ and the corresponding left and right eigenvectors $\psi_\eta$ and $\phi_\eta$ for each $A$;
3) **Sensitivities**: Calculate the gradients $\partial A/\partial x_i$, $\partial B/\partial x_i$, $\partial C/\partial x_i$, and $\partial D/\partial x_i$ for each point with respect to the $i$th variable $x_i \in x$; obtain the gradients $\partial A_i/\partial x_i$ for each $A_i$ in terms of (25); and calculate the eigenvalue sensitivity $\partial \lambda_\eta/\partial x_i$ in (24) for $x \in B_{ik}^n$.
4) **Gradient**: Get the gradient $\partial h/\partial x_i$ with respect to the $i$th variable $x_i \in x$ referred to (26) for each point.

Note that the steps (2)–(4) can be performed in parallel.

### VI. CASE STUDIES

The proposed method is applied to the WSCC 3-machine 9-bus, New England 10-machine 39-bus, and the modified IEEE 57-machine 118-bus systems to illustrate the effectiveness in solving SSSC-OPF. For all systems, the generators are described by the two-axis model with an IEEE type-I exciter. The loads are modeled as constant power.

The SQP-GS method is implemented in MATLAB by using CPLEX 12.60 [23] as the QP solver for the subproblem. The eigenvalues and eigenvectors are computed by QR decomposition using the MATLAB function eig. Flat start is used for which all voltage angles are set to zero, all voltage magnitudes are set to be 1.0 p.u., $P_G = (\overline{P}_G + \overline{P}_G)/2$, and $Q_G = (\overline{Q}_G + \overline{Q}_G)/2$. The parameters of SQP-GS are chosen as $\rho = 0.1$, $\mu_p = 0.5$, $\epsilon = 0.1$, $\mu_q = 0.5$, $\tau = 0.1$, $\mu_r = 0.8$, $\varpi = 1$, $\gamma = 0.8$ from [15]. The tolerances $\nu_{in}$ and $\nu_s$ are set to $10^{-3}$ and $10^{-2}$, respectively.

#### A. WSCC 3-Machine 9-Bus System

The WSCC 9-bus system is often used for stability analysis [6]. The system data, including power limit and security data can be found [14]. The generator cost coefficients are listed in Table I. To analyze the effectiveness of the proposed method, we consider the following three cases:

1) **Case 0**: Base case, which is a standard OPF without any small-signal stability constraint.
2) **Case 1**: SSSC-OPF without any binding small-signal stability constraint.
3) **Case 2**: SSSC-OPF with a binding small-signal stability constraint.

- **Case 0**: We use IPM to solve the standard OPF without small-signal stability constraint and the results are listed in the first row of Table I. The eigenvalue analysis is performed and the spectral abscissa $\eta = -0.04$.
- **Case 1**: A security margin $\eta = -0.01$ is used, which is larger than the spectral abscissa in the base case. Since the small-signal stability constraint is not binding in this case, the problem can be successfully solved by the SQP method without gradient sampling and the results are the same as those in the base case.

- **Case 2**: $\eta$ is set to be three different values, all of which are less than the spectral abscissa in the base case. When $\eta$ is set to be $-0.45$ or $-0.5$, the SQP will not converge without a GS procedure. Here the sample size is chosen as 10. From Table I it is seen that the power outputs of the generators are re-dispatched to satisfy the small-signal stability constraint. Also, the more binding the small-signal stability constraint, the more expensive the generation cost is. As $\eta$ decreases, the active power generated by generator G3 which has the cheapest generation cost gradually goes down, mainly because G3 has the smallest inertial constant and generating more power from the other two generators can help improve the stability. When $\eta = -0.5$, the re-dispatch from SSSC-OPF will allow significantly more generation from G1 which has the most expensive generation cost but the largest inertial constant.

#### B. New England 10-Machine 39-Bus System

The full dynamic data of the New England 10-machine 39-bus system are extracted from [24] and the economic and technical data are from [9]. For the voltage magnitude limits, we choose $\overline{V} = 1.1$ p.u. and $\underline{V} = 0.9$ p.u. for all buses. For standard OPF, the spectral abscissa $\eta = -0.11$. To reduce the spectral abscissa, the SSSC-OPF is applied with $\eta = -0.2$.

The SSSC-OPF is calculated by SQP-GS with no samples, 30 samples, and 60 samples, respectively. As shown in Fig. 1, the infeasibility defined in (59) cannot reduce to the tolerance with 100 iterations when the sample size is 0. By contrast, when the sample size is 30 or 60, the infeasibility reduces to $\nu_{in} = 10^{-3}$ rapidly. Also, as shown in Fig. 2, the model reduction defined in (60) decreases to an acceptable tolerance with sampling gradients. In Fig. 3 we show the generation cost which stably approaches an optimal value. From Figs. 1-3, we can see that the gradient sampling plays an important role in solving the SSSC-OPF problems. Moreover, based on our tests, the GS with 30 samples are good enough to improve the search direction of SQP for the SSSC-OPF problem.

We also compare SQP-GS with the numerical eigenvalue sensitivity based IPM (IPM-NES) [9] and the results are listed in Table III where $\overline{P}_G$ is the maximum active power output of a generator. The results for the OPF without small signal stability constraint (denoted by ‘OPF’) is also listed for reference. IPM-NES can also solve the problem but needs higher cost than SQP-GS to ensure the same level of small-signal stability. Comparing the generators of IPM-NES and SQP-GS, we can see that the generations of G3 and G5 are

| Generator# | $a_2$/MW | $a_1$/MW | $a_0$/ $\$$ |
|-----------|----------|----------|-----------|
| 1         | 9.76 x 10^{-4} | 14.712 | 0.00       |
| 2         | 7.20 x 10^{-4} | 11.290 | 0.00       |
| 3         | 5.46 x 10^{-4} | 8.001  | 0.00       |
TABLE II

| Summary of SSSC-OPF Results With Different Small-Signal Constraints for WSCC 3-Machine 9-Bus System |
|---------------------------------------------------------------|
|                                                                 |
| Base Case | 2901.3 | -0.04 | 25.0 | 25.0 | 276.0 | 1.040 | 1.045 | 1.022 |
| Case 1 ($\eta \leq -0.01$) | 2901.3 | -0.04 | 25.0 | 25.0 | 276.0 | 1.040 | 1.045 | 1.022 |
| Case 2 ($\eta \leq -0.30$) | 2919.7 | -0.30 | 25.0 | 47.1 | 252.4 | 1.040 | 1.045 | 1.045 |
| Case 2 ($\eta \leq -0.45$) | 2957.5 | -0.45 | 25.0 | 59.8 | 239.3 | 1.026 | 1.045 | 1.045 |
| Case 2 ($\eta \leq -0.50$) | 3217.9 | -0.50 | 84.4 | 25.0 | 211.8 | 0.997 | 1.045 | 1.022 |

C. IEEE 54-Machine 118-Bus System

We also test the proposed method on a modified version of the IEEE 54-machine 118-bus benchmark system with dynamic data from [25]. This system has 54 synchronous machines with IEEE type-1 exciters, 20 of which are synchronous compensators used only for reactive power support and 15 of which are motors. For a standard OPF, the spectral abscissa $\eta = 0.35$. We set $\eta$ to be $-0.1$. The voltage magnitudes at all buses must be between 0.9 p.u. and 1.1 p.u. The reactive power limits can be found in [26].

IPM-NES fails to solve this problem. By contrast, SQP-GS only needs 13 and 12 iterations to solve the problem, respectively, when there are 30 samples and 60 samples. The generation cost of SSSC-OPF will increase to 6110.8$/$h from 5779.2$/$h for the standard OPF with $\eta = 0.35$.

Since eigenvalue analysis and the SSSC-OPF use the linearized system model, they cannot take into account the full nonlinear behavior of the power system. Therefore, in order to
Further validate the solution of SSSC-OPF from SQP-GS, we increase the load at bus 2 by 5 MW and perform time-domain simulation using the full nonlinear power system model for the operating states obtained from both the standard OPF and SSSC-OPF. The rotor frequencies of all generators for the standard OPF and SSSC-OPF are shown in Figs. 3 and 5 respectively. It is seen that under the standard OPF the system is unstable while under SSSC-OPF the oscillation can be quickly damped.

Since the sampling in SQP-GS is random, the solutions may be different for different runs. We run SQP-GS for 20 times and the standard deviation of the generation cost is 0.06 $/h$. In Fig. 6 we show the coefficient of variation, the standard deviation divided by the mean, of the generation output for each generator over 20 runs, which is very small and indicates that the difference between the solutions is small.

### D. Efficiency

We also test the efficiency of the proposed method on 39- and 118-bus systems. All simulations are carried out on a Dell Precision T5810 with a four-core 3.5 GHz processor and 64 GB RAM memory. Each iteration of the proposed method involves eigenvalue analysis, computing sensitivities, and solving QP subproblem.

| Iter. | SQP-GS | IPM-NES |
|-------|--------|---------|
| 1     | -0.193 ± j0.4060 | -0.0965 ± j0.5086 |
| 2     | -0.199 ± j0.4059 | -0.1937 ± j0.5032 |
| 3     | -0.1976 ± j0.4037 | -0.1876 ± j0.5032 |
| 4     | -0.187 ± j0.4374 | -0.1768 ± j0.4028 |
| 5     | -0.1872 ± j0.3894 | -0.1815 ± j0.4092 |
| 6     | -0.1905 ± j0.4458 | -0.1945 ± j0.6110 |
| 7     | -0.1932 ± j0.2018 | -0.1922 ± j0.3966 |
| 8     | -0.1938 ± j0.4522 | -0.2032 ± j0.5364 |
| 9     | -0.1938 ± j0.4532 | -0.2066 ± j0.6721 |
| 10    | -0.1943 ± j0.4533 | -0.2118 ± j0.5172 |
| 11    | -0.1946 ± j0.4570 | -0.2035 ± j0.6043 |
| 12    | -0.2067 ± j0.4534 | -0.2118 ± j0.5417 |

Table [V] lists the average step and the total CPU time with sample size 30 and 60. It is seen that the proposed SQP-GS method can solve the SSSC-OPF problem efficiently. Since in each iteration the gradient sampling performs eigenvalue analysis, computing sensitivities, and solving QP subproblem.

From Table [V] it is also seen that the number of iterations
for the IEEE 118-bus system is smaller than that for the New England 39-bus system. The iteration times of SQP-GS can depend on the system size, the constraints, and the objective function. In our test cases it seems that the small-signal stability constraint is the dominant factor, and the IEEE 118-bus case has a smaller number of iterations mainly because its \( \eta \) in the small-signal stability constraint is greater than that in the New England 39-bus case (\( \eta = -0.1 \) for IEEE 118-bus case and \( \eta = -0.2 \) for New England 39-bus case).

### VII. CONCLUSION

In this paper, we propose an SQP-GS method to solve the SSSC-OPF problem, which is a nonsmooth optimization problem due to the property of the spectral abscissa function. In SQP-GS, a GS technique is used to evaluate the gradients around the current iterate to make the search direction computation effective in the nonsmooth regions. The closed-form eigenvalue sensitivity is used to calculate the gradient of the spectral abscissa. Simulation results on three test systems show that the proposed method can solve the SSSC-OPF problem effectively without any convergence problem. By contrast, the existing Interior Point Method either cannot get as good solution as SQP-GS or even fail for relatively large systems.

### REFERENCES

[1] G. Rogers, *Power System Oscillation*. New York: Springer, 2000.

[2] J. Quintero, V. Vittal, G. Heydt, and H. Zhang, “The impact of increased penetration of converter control-based generators on power system modes of oscillation,” *IEEE Trans. Power Syst.*, vol. 29, no. 5, pp. 2248–2256, Sept. 2014.

[3] T. Weckesser, H. Johannsson, and J. Ostergaard, “Real-time remedial action against aperiodic small signal rotor angle instability,” *IEEE Trans. Power Syst.*, vol. 31, no. 1, pp. 387–396, Jan. 2016.

[4] C. Wang, Y. Lv, H. Huang, J. Zhang, J. Li, Y. Li, W. Sun, and Y. Chang, “Low frequency oscillation characteristics of east china power grid after commissioning of haihu ultra-high voltage alternating current project,” *J. Mod. Power Syst. Clean Energy*, vol. 3, no. 3, pp. 332–340, 2015.

[5] C. Chung, L. Wang, F. Howell, and P. Kundur, “Generation rescheduling methods to improve power transfer capability constrained by small-signal stability,” *IEEE Trans. Power Syst.*, vol. 19, no. 1, pp. 524–530, Feb. 2004.

[6] P. W. Sauer and M. Pai, *Power system dynamics and stability*, 1st ed. Urbana-Champaign: Prentice Hall, 1998.

[7] P. Kundur, N. J. Balu, and M. G. Lauby, *Power system stability and control*. New York: McGraw-hill, 1994.

[8] Z. Yu, F. Li, L. Sun, X. Zhou, P. Zeng, and B. Li, “An on-line generation dispatching algorithm to improve small signal stability,” in *2014 IEEE PES General Meeting — Conference Exposition*, Jul. 2014, pp. 1–5.

[9] R. Zarate-Minano, F. Milano, and A. Conejo, “An OPF methodology to ensure small-signal stability,” *IEEE Trans. Power Syst.*, vol. 26, no. 3, pp. 1050–1061, Aug. 2011.

[10] S. Kodsi and C. Caizares, “Application of a stability-constrained optimal power flow to tuning of oscillation controls in competitive electricity markets,” *IEEE Trans. Power Syst.*, vol. 22, no. 4, pp. 1944–1954, Nov. 2007.

[11] F. Coudren and T. Gedra, “Expected-utility-cost optimal power flow with small-signal stability constraints,” *IEEE Trans. Power Syst.*, vol. 21, no. 4, pp. 1736–1743, Nov. 2006.

[12] F. Alvarado, C. DeMarco, I. Dobson, P. Sauer, S. Greene, H. Engdahl, and J. Zhang, “Avoiding and suppressing oscillations,” *PSERC Final Report*, 1999.

[13] H.-K. Nam, Y.-K. Kim, K.-S. Shim, and K. Lee, “A new eigen-sensitivity theory of augmented matrix and its applications to power system stability analysis,” *IEEE Trans. Power Syst.*, vol. 15, no. 1, pp. 363–369, Feb. 2000.

[14] P. Li, H. Wei, B. Li, and Y. Yang, “Eigenvalue-optimisation-based optimal power flow with small-signal stability constraints,” *IET Gener. Transm. Dis.*, vol. 7, no. 5, pp. 440–450, May 2013.

[15] F. E. Curtis and M. L. Overton, “A sequential quadratic programming algorithm for nonconvex, nonsmooth constrained optimization,” *SIAM Journal on Optimization*, vol. 22, no. 2, pp. 474–500, 2012.

[16] S. M. Deckmann and V. F. da Costa, “A power sensitivity model for electromechanical oscillation studies,” *IEEE Transactions on Power Systems*, vol. 9, no. 2, pp. 965–971, May 1994.

[17] A. S. Lewis and M. L. Overton, “Eigenvalue optimization,” *Acta numerica*, vol. 5, pp. 149–190, 1996.

[18] J. V. Burke and M. L. Overton, “Variational analysis of non-lipschitz spectral functions,” *Mathematical Programming*, vol. 90, no. 2, pp. 317–351, 2001.

[19] P. T. Boggs and J. W. Tolle, “Sequential quadratic programming,” *Acta numerica*, vol. 4, pp. 1–51, 1995.

[20] J. Nocedal and S. J. Wright, *Numerical Optimization*, 2nd ed. New York: Springer, 2006.

[21] J. Nocedal, “Updating quasi-newton matrices with limited storage,” *Mathematics of computation*, vol. 35, no. 151, pp. 773–780, 1980.

[22] IBM ILOG CPLEX*, 2013. [Online]. Available: http://www-01.ibm.com/software/commerce/optimization/cplex-optimizer/.

[23] R. H. Yeu, “Small signal analysis of power systems: Eigenvalue tracking method and eigenvalue estimation contingency screening for DSA,” Ph.D. dissertation, University of Illinois at Urbana-Champaign, 2010.

[24] “IEEE 118-bus modified test system,” KIOS Research Center for Intelligent Systems and Networks of the University of Cyprus. [Online]. Available: http://www.kios.ucy.ac.cy/testsystems/index.php/dynamic-ieee-test-systems/ieee-118-bus-modified-test-system.

[25] “Power systems test case archive,” Department of Electrical Engineering of the University of Washington. [Online]. Available: https://www.ee.washington.edu/research/pstca/.

[26] Z. Du, W. Liu, and W. Fang, “Calculation of rightmost eigenvalues in power systems using the jacobi-davidson method,” *IEEE Trans. Power Syst.*, vol. 21, no. 1, pp. 234–239, Feb. 2006.

### Table V

| Step | New England 39-bus system | Sample size = 30 | Sample size = 60 | IEEE 118-bus system | Sample size = 30 | Sample size = 60 |
|------|--------------------------|-----------------|-----------------|---------------------|-----------------|-----------------|
|      | CPU(s)/call | iter. |                  | CPU(s)/call | iter. |                  | CPU(s)/call | iter. |                  | CPU(s)/call | iter. |                  |
| Eigen. Analysis | 0.004 | 32 | 0.13 | 0.004 | 62 | 0.24 | 0.103 | 32 | 0.29 | 0.108 | 65 | 0.70 |
| Sensitivities  | 0.005 | 31 | 0.14 | 0.005 | 61 | 0.27 | 0.028 | 31 | 0.86 | 0.029 | 61 | 1.79 |
| QP       | 0.12  | 1   | 0.12 | 0.14 | 1   | 0.14 | 0.62  | 1   | 0.62 | 0.72  | 1   | 0.72 |
| Iter. Times | 68 | 13 |                  | 68 | 13 |                  | 68 | 13 |                  | 68 | 13 |                  |
| Total CPU (s) | 26.72 (9.97') | 40.60 (9.54') |                  | 60.33 (9.94') | 114.93 (10.49') |                  |

*Estimated CPU time if ideal parallel techniques are used to sample the gradients Estimated CPU time = (Eigen. Analysis CPU(s)/call + Sensitivities CPU(s)/call + QP CPU(s)/call) x Iter. Times + the other time. The other time includes L-BFGS time in Step 8 of Algorithm 1 linear search time in Step 20 of Algorithm 1 and data input and result output time.*
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