Nonlinear Finite Element Analysis of Reinforced Concrete Beams Subjected to Torsion

Mohammed M. Handhal ¹, David A. M. Jawad ²

¹ Misan university / Engineering College, Misan, Iraq
mohammed_mahood@uomisan.edu.iq

² University of Basrah / Engineering College, Basrah, Iraq
david.jawad@uobasrah.edu.iq

ABSTRACT
This work aims to investigate the behavior of rectangular-section of reinforced concrete beams subjected to torsion by using the finite element method. An analytical study is presented in the current work on thirty-four beam specimens divided into two series. Compressive strength of concrete is the main parameter for first series which contains sixteen specimens (high strength concrete HSC and normal strength concrete NSC), the spacing of the stirrups is also investigated in this series. Second series includes eighteen beams investigated for the effect of some parameter on the torsional capacity. These parameters are amount of longitudinal and transverse reinforcement, yield strength of longitudinal and transverse steel, reinforcement stresses, and crack patterns of HSC and NSC beams. The results showed that for RC beams, the transverse reinforcement is more effective in resisting the applied torque, it is preferable to use a higher ratio of transverse reinforcement for the NSC and HSC beams, where the increase in ultimate torque was about 94% for an increase in the ratio from 6% to 16%, the effect of yield strength of steel is about 4.1% by changing the yield strength by 13.6% from 440 MPa to 500 MPa. The increasing of transverse reinforcement spacing significantly improves and increases the ultimate twisting angle. The mode of failure for the RC beams is more affected by the reinforcement ratio and the typical mode of failure for rectangular section beams is diagonal tension failure for both concrete grades.

Key words— ANSYS, Finite element analysis, High strength concrete, Nonlinear finite element, Normal strength concrete, Reinforced concrete beam, Torsion.

1. General
A moment acting about the longitudinal axis of a member is called a *twisting moment*, a *torque*, or a *torsional moment*, $T$.

Commonly subjected forces on reinforced concrete (RC) members are bending moments, shears (transverse) accompanying with those bending moments, also, axial forces often joint with bending and shear in the case of columns. In accumulation, torsional moment may act about longitudinal axis. Torsional moment occasionally acts alone and are parallel with the other major forces. The action of bending and axial loads is fairly understood. In contrast, shear and torsion, are not well understood and the empirical design methods used are very different around the world [1].

2. Behavior of Reinforced Concrete Beams Under Pure Torsion

When a pure torsion is applied on concrete beam, shearing stresses, and principal stresses progress. One or more cracks as an inclined grow when the (maximum principal tensile stress) reaches the concrete tensile strength. The concrete failure because of cracking. Furthermore [2].

Longitudinal bars at the corners and closed stirrups in the rectangular beam can resist increased load as the second stage. Fig. (1) is a torque-twist curve for such a beam., point $A$ represent the behavior at the cracking stage, which shows that the angle of twist increases without a growth in torque because some of the forces are redistributed to the reinforcement from the uncracked concrete [2].

After the cracking of a reinforced beam, failure may happen in numerous ways. The longitudinal reinforcement, or stirrups, or maybe both, may reach the yield stress for beams in torsion. When both the reinforcements yield prior to crushing of the concrete material which represent the utmost ductile behavior outcomes [2].

Tests have shown that both closed stirrups (or spirals) and longitudinal bars are essential to seize the frequent diagonal crack [3].
Figure (1): Typical torque twist curve for a reinforced concrete rectangular beam [3]

3. Details of Study

In the present study, the structural behavior of a cantilever rectangular-section plain and RC beams is simulated depending on available experimental tests.

Two series of RC beams based on experimental researches (Rasmussen and Baker [4], Fang and Shiau [5]) were analyzed. The major variables that characterized the beams include concrete compressive strength, and size of the plain concrete beams also concrete compressive strength, yield strength of steel reinforcement, ratios of longitudinal and transverse reinforcement, size of beam and spacing of transverse reinforcement for the RC beams.
4. Finite Element Modeling

4.1 Loads and boundary conditions

It is very important to apply the boundary conditions to get a solution, it is obvious that each boundary conditions will lead to a unique solution, also the displacement boundary condition is required to constrain the model. To ensure that the specimens are modeled correctly, boundary conditions must be applied at the right places where the supports and loadings exist in the experimental test.

The support is model as a closed steel box plates that surround the concrete beam end, one of these plates are fixed (the right end of the beam). The line of nodes on this plate along the x, y and z-direction were constraint as shown in Fig. (2).

To get torsion on the beam a couple was applied to the beam on the steel plate in the other end (the left end of the beam). Fig. (4-1) illustrates the steel plates and applied couple and boundary conditions.

![Figure (2): Loads and boundary conditions used in this work](image)

(a) beam and the end plates   (b) right side plate and the boundary condition

4.2 Modeling of geometry and material properties

The RC beams, plain concrete beams, surrounding end plates, and supports were modeled by creating nodes first on the working plane of ANSYS. Then the elements were created through nodes with auto-numbering of elements this method was helpful to control the mesh size and number of the elements and also simple.
Solid 65 element used to represent the concrete LINK 180 elements were used to create the longitudinal and transverse reinforcement. The element type number, material number, and real constant set number for the ANSYS models were set for each mesh. At one end of the beam, the displacement in all directions was held at zero, and referred to as a fixed support.

ANSYS provides several options to characterize different types of material behavior, such as bilinear isotropic (with work hardening) and multilinear isotropic hardening. The concrete is assumed to behave as a homogeneous and initially isotropic material. The multilinear isotropic representation for compressive uniaxial stress-strain relationship are used for concrete model. For reinforcement steel, the bilinear isotropic representation is assumed in FE modeling.

1) Series One

The study of this series is conducted on sixteen small RC beam specimens which were designed as over-reinforced. The primary focus of this series is the influence of the concrete strength which includes the concrete strength (Normal and High strength) and the spacing of transverse reinforcement on the ultimate torsional moment. This study is based on experimental testing conducted by Rasmussen and Baker [4], and it is analyzed by using FE and includes suggestion of two specimens with variable concrete strength in the normal strength concrete and two other specimens with variable spacing of transverse reinforcement. All the specimens have rectangular cross-section and the same length, the overall length is 3500 mm and the length of the beams between the supports is 3000 mm, heavier reinforcement used at ends of the beam to ensure that failure would occur within the 1800 mm effective span. The cover to all reinforcement was 15 mm.

Table (1) and Fig. (3) give all details of the beam's geometry and modeling.
Table (1): Details of specimens (Series 1) [4]

| Specimen | Author Beam No. | $f_{c'}$ (MPa) | Modulus of rupture (MPa) | Dimensions $b \times h \times L$ (mm) | Longitudinal reinf. $f_y$ (MPa) | Stirrups $\phi \@$spacing (mm) |
|----------|----------------|----------------|-------------------------|--------------------------------------|-----------------------------|-----------------------------|
| S1-RN1   | suggested      | 25.00          | 3.10                    | 160x275 x3500                        | 6 $\phi$18 -638             | $\phi$10@90 - 669          |
| S1-RN2   | suggested      | 30.00          | 3.40                    |                                      | 6 $\phi$18 -638             | $\phi$10@90 - 669          |
| S1-RN3   | B30.3          | 36.30          | 3.74                    |                                      | 6 $\phi$18 - 605            | $\phi$10@90 - 672          |
| S1-RN4   | B30.2          | 38.20          | 3.83                    |                                      | 6 $\phi$18 - 638            | $\phi$10@90 - 669          |
| S1-RN5   | B30.1          | 41.70          | 4.00                    |                                      | 6 $\phi$18 - 620            | $\phi$10@90 - 665          |
| S1-RH1   | B50.2          | 57.10          | 4.69                    |                                      | 6 $\phi$18 - 614            | $\phi$10@90 - 665          |
| S1-RH2   | B50.3          | 61.70          | 4.87                    |                                      | 6 $\phi$18 - 612            | $\phi$10@90 - 665          |
| S1-RH3   | B50.1          | 61.80          | 4.87                    |                                      | 6 $\phi$18 - 612            | $\phi$10@90 - 665          |
| S1-RH4   | B70.2          | 76.90          | 5.44                    |                                      | 6 $\phi$18 - 614            | $\phi$10@90 - 656          |
| S1-RH5   | B70.1          | 77.30          | 5.45                    |                                      | 6 $\phi$18 - 617            | $\phi$10@90 - 658          |
| S1-RH6   | B70.3          | 78.20          | 5.68                    |                                      | 6 $\phi$18 - 617            | $\phi$10@90 - 663          |
| S1-RH7   | B110.2         | 105.0          | 6.35                    |                                      | 6 $\phi$18 - 634            | $\phi$10@90 - 660          |
| S1-RH8   | B110.3         | 105.1          | 6.36                    |                                      | 6 $\phi$18 - 629            | $\phi$10@90 - 655          |
| S1-RH9   | suggested      | 105.1          | 6.36                    |                                      | 6 $\phi$18 - 629            | $\phi$10@90 - 655          |

Figure (3): Structural dimensions of beams of (Series 1) [4]
2) Series Two

Two parameters are established included the yield strength of longitudinal steel and transverse steel, and the ratios of longitudinal and transverse reinforcement. A total of eighteen specimens of rectangular section are studied in this series. Twelve specimens were provided from the previous experimental study that conducted by Fang and Shiau [5], and the other six specimens were suggested to investigate the effect of yield strength of longitudinal and transverse steel. The overall length of the specimens is 3100 mm and the length of the beams between the supports is 2500 mm all the specimens have rectangular cross-section and have the same length, heavier reinforcement was used at the ends of the beam to insure that failure would occur within the test region of 1600 mm span, the concrete cover was 25 mm.

Table (2) and Fig. (4) show all details of the beams geometry and modeling.

![Figure (4): Structural dimensions of beams of (Series 2) [5]](image-url)
Table (2): Details of specimens (Series 2) [5]

| Specimen | Author | Beam No. | f'c (MPa) | Modulus of rupture (MPa) | Longitudinal reinf. $\rho_l$ (%) - bar No. - $f_{yl}$ (MPa) | Stirrups $\rho_h$ (%) - D@spacing(mm) - $f_{ys}$ (MPa) |
|----------|-------|----------|-----------|------------------------|---------------------------------|---------------------------------|
| S2-RN1   | N-06-06 | 35.50    | 3.69      | 0.6 - 6$\phi$16 - 440 | 0.6 - $\phi$10@100 - 440       | 16$\phi$12 - 410                |
| S2-RN2   | N-06-12 | 35.50    | 3.69      | 1.2 - 16$\phi$12 - 410 | 12 - $\phi$10@50 - 440         | 0.6 - $\phi$10@100 - 440       |
| S2-RN3   | N-12-12 | 35.50    | 3.69      | 1.2 - 16$\phi$12 - 410 | 1.2 - $\phi$10@50 - 440        | 0.6 - $\phi$10@100 - 440       |
| S2-RN4   | N-12-16 | 35.50    | 3.69      | 1.6 - 10$\phi$20 - 520 | 1.2 - $\phi$10@50 - 440        | 0.6 - $\phi$10@100 - 440       |
| S2-RN5   | N-20-20 | 35.50    | 3.69      | 2.0 - 12$\phi$20 - 560 | 2.0 - $\phi$12@55 - 440        | 0.6 - $\phi$10@100 - 440       |
| S2-RN6   | N-07-10 | 33.50    | 3.59      | 1.0 - 6$\phi$20 - 500  | 0.7 - $\phi$10@90 - 420        | 0.6 - $\phi$10@100 - 440       |
| S2-RH1   | H-06-06 | 78.50    | 5.49      | 0.6 - 6$\phi$16 - 440  | 0.6 - $\phi$10@100 - 440       | 0.6 - $\phi$10@100 - 440       |
| S2-RH2   | H-06-12 | 78.50    | 5.49      | 1.2 - 16$\phi$12 - 410 | 0.6 - $\phi$10@100 - 440       | 0.6 - $\phi$10@100 - 440       |
| S2-RH3   | suggested | 78.50    | 5.49      | 1.6 - 10$\phi$20 - 520 | 0.6 - $\phi$10@100 - 440       | 0.6 - $\phi$10@100 - 440       |
| S2-RH4   | H-12-12 | 78.50    | 5.49      | 1.2 - 16$\phi$12 - 410 | 1.2 - $\phi$10@50 - 440        | 0.6 - $\phi$10@100 - 440       |
| S2-RH5   | suggested | 78.50    | 5.49      | 1.2 - 16$\phi$12 - 410 | 0.7 - $\phi$10@90 - 420        | 0.6 - $\phi$10@100 - 440       |
| S2-RH6   | H-12-16 | 78.50    | 5.49      | 1.6 - 10$\phi$20 - 520 | 1.2 - $\phi$10@50 - 440        | 0.6 - $\phi$10@100 - 440       |
| S2-RH7   | H-20-20 | 78.50    | 5.49      | 2.0 - 12$\phi$20 - 560 | 2.0 - $\phi$12@55 - 440        | 0.6 - $\phi$10@100 - 440       |
| S2-RH8   | H-07-10 | 68.40    | 5.13      | 1.0 - 6$\phi$20 - 500  | 0.7 - $\phi$10@90 - 420        | 0.6 - $\phi$10@100 - 440       |
| S2-RH9   | suggested | 78.50    | 5.49      | 0.6 - 6$\phi$16 - 440  | 0.6 - $\phi$10@100 - 440       | 0.6 - $\phi$10@100 - 440       |
| S2-RH10  | suggested | 78.50    | 5.49      | 0.6 - 6$\phi$16 - 440  | 0.6 - $\phi$10@100 - 440       | 0.6 - $\phi$10@100 - 440       |
| S2-RH11  | suggested | 78.50    | 5.49      | 0.6 - 6$\phi$16 - 440  | 0.6 - $\phi$10@100 - 440       | 0.6 - $\phi$10@100 - 440       |
| S2-RH12  | suggested | 78.50    | 5.49      | 0.6 - 6$\phi$16 - 440  | 0.6 - $\phi$10@100 - 440       | 0.6 - $\phi$10@100 - 440       |

5. Presentation and Discussion of Results

5.1. Series one

1. Torques at Failure

All the results obtained from ANSYS model at failure and those predicted according to space truss analogy and skew bending theory in addition to the experimental results are listed in Table (3). It shows good agreement between results of the present models and the experimental tests. A comparison between the ultimate torques of the experimental and ANSYS results is obtainable. The verification with the experimental results based on the ultimate torque are made and the ratio of the predicted (ANSYS) ultimate torques to experimental torques
ranged from 87.2% to 91.7% for the normal strength concrete, the FE failure torque underestimates the experimental results for NSC. While the result for the high strength concrete achieves ratios from 92.8% to 102.7% which is considered a good verification. This difference in results can be related to many factors that affect the verification ratios, these factors are the assumption of full bond between concrete and steel in the FE analysis but this assumption is not true for the actual beams. The difference between the compressive strength of concrete for the beams and the test cubes that are used to find the compressive strength of concrete. Microcracks are present in the concrete to some degree which may reduce the stiffness of the actual beams, while the FE models do not take this factor in consideration.

(Table 3): Analytical and experimental ultimate torque results (Series 1)

| Specimen No. | Exp. moment (kN.m) | Theoretical torsional moment (kN.m) | T (FEM)/T(Exp.) |
|--------------|--------------------|------------------------------------|-----------------|
| S1-RN1       | -                  | 8.87                               | -               |
| S1-RN2       | -                  | 11.26                              | -               |
| S1-RN3       | 15.25              | 13.30                              | 0.872           |
| S1-RN4       | 15.29              | 14.02                              | 0.917           |
| S1-RN5       | 16.62              | 14.63                              | 0.880           |
| S1-RH1       | 18.46              | 17.67                              | 0.957           |
| S1-RH2       | 19.13              | 17.83                              | 0.932           |
| S1-RH3       | 19.95              | 18.52                              | 0.928           |
| S1-RH4       | 20.74              | 19.81                              | 0.955           |
| S1-RH5       | 20.06              | 20.54                              | 1.024           |
| S1-RH6       | 20.96              | 20.05                              | 0.957           |
| S1-RH7       | 23.62              | 24.25                              | 1.027           |
| S1-RH8       | 24.770             | 24.50                              | 0.99            |
| S1-RH9       | -                  | 22.18                              | -               |
| S1-RH10      | -                  | 19.20                              | -               |
| S1-RH11      | 24.72              | 25.25                              | 1.021           |

It has been observed that changing the compressive strength of normal strength...
concrete from 25 MPa to 41.7 MPa, leads to increase the failure torque to 64.9% changing from 8.87 kN.m to 14.63 kN.m, as shown in Fig. (5).

Figure (5): The effect of concrete compressive strength (NSC)

For the high strength concrete by increasing the compressive strength by 34.6% and 84%, the ultimate torque will increase by 12.1% and 38.6% respectively, as shown in Fig. (6).

Figure (6): The effect of concrete compressive strength (HSC)
The relationship between the compressive strength and the ultimate torque for both normal strength concrete and high strength concrete is plotted in Figs. (7) and (8) respectively.

Figure (7): Concrete compressive strength versus ultimate torsional moment (NSC)

Figure (8): Concrete compressive strength versus ultimate torsional moment (HSC)
The spacing of the transverse reinforcement has significant effect on the ultimate torsional capacity. By increasing the spacing by 200% from 90 mm to 270 mm, the ultimate torque decrease by about 21.6% as shown in Fig. (9).

![Figure (9): Spacing of the transverse reinforcement versus ultimate torsional moment](image)

2. Crack Patterns

The crack patterns for all specimens are approximately similar. As the applied torque reaches the cracking torque, the cracks first appear in the specimen at the mid span on the wider face and after that the cracks spread along all faces of the beam, the main feature for these cracks are the angle which is nearly 45°cracks and the second feature is that it is spread on the whole faces of the specimens. The crack patterns predicted by ANSYS model for all specimens at the failure torque are shown in Fig. (10).
Figure (10): Crack patterns at failure torques (S1-RN5)

3. Torque-Twist

Table (4) contains all the details of twist results.
Table (4): Analytical and experimental twist at ultimate torque (Series 1)

| Specimen | Angle of twist per length (Degree/m) (Exp.) | Angle of twist per length (Degree/m) (FEM) | Twist (FEM) / Twist (Exp.) |
|----------|-------------------------------------------|------------------------------------------|-----------------------------|
| S1-RN1   | -                                         | 1.299                                    | -                           |
| S1-RN2   | -                                         | 1.696                                    | -                           |
| S1-RN3   | 2.574                                     | 1.955                                    | 0.76                        |
| S1-RN4   | 2.906                                     | 1.982                                    | 0.68                        |
| S1-RN5   | 2.964                                     | 1.980                                    | 0.67                        |
| S1-RH1   | 2.631                                     | 2.270                                    | 0.86                        |
| S1-RH2   | 3.096                                     | 2.080                                    | 0.67                        |
| S1-RH3   | 2.551                                     | 2.189                                    | 0.86                        |
| S1-RH4   | 2.339                                     | 2.140                                    | 0.92                        |
| S1-RH5   | 2.396                                     | 2.255                                    | 0.94                        |
| S1-RH6   | 2.677                                     | 2.213                                    | 0.83                        |
| S1-RH7   | 2.946                                     | 2.501                                    | 0.85                        |
| S1-RH8   | 2.740                                     | 2.518                                    | 0.92                        |
| S1-RH9   | -                                         | 2.480                                    | -                           |
| S1-RH10  | -                                         | 2.961                                    | -                           |

The angle of twist was obtained by measuring the rotation between two cross sections and then dividing the rotation by the distance between the sections at same location for experimental beams.

In general, the twisting angle at failure for the Normal strength concrete indicates variation from 67% to 76% compared to the experimental test beams.

For High strength concrete the angle of twist at failure shows a good agreement between test and numerical results with a variation between 83% to 99% except for specimen S1-RH2 which has 67% difference.

The predicted torque-twist behavior of NSC and HSC are shown in Fig. (11) and Fig. (12) respectively.
The effect of transverse reinforcement spacing on the overall behavior of the beams can be observed on the torque-twist curves, as the spacing increases from 90 mm to 180 mm and 270 mm, the beams seem to have more ductile behavior as shown in Fig. (13)
Figure (13): Effect of spacing of the transverse reinforcement on analytical torque-twist curves (Series 1)

5.2. Series two

1. Torques at Failure

All the details of the results: experimental, FE torsional strength, and the values predicted based on space truss analogy and skew bending theory are included in Table (5). A slight difference is observed between the experimental and numerical results where the ratio of ultimate torque predicted by FEA to the experimental ultimate torque ranged between 88% to 108%.

According to the results, the ultimate torsional capacity tends to be larger with increasing the longitudinal steel ratio and transverse steel ratio, until the beam becomes an over-reinforced beam. However, the effect of increasing the transverse reinforcement ratio is higher than the longitudinal reinforcement, a (21.3 and 43.2%) were the increment ratio in the ultimate torque due to the effect of changing the transverse reinforcement ratio from 6% to 12% and 14% respectively as shown in Fig. (14).
By increasing the longitudinal reinforcement ratio from 6% to 16% the ultimate torque will increase by about 25.2% as shown in Fig. (15).

Table (5): Analytical and experimental ultimate torque results (Series 2)

| Specimen No. | Exp. (kN.m) | Analytical torsional moment (kN.m) | T(FEM)/T(Exp.) |
|--------------|-------------|-----------------------------------|----------------|
| 2-RN1        | 79.7        | 77.3                              | 0.97           |
| 2-RN2        | 95.2        | 89.5                              | 0.94           |
| 2-RN3        | 116.8       | 106.3                             | 0.91           |
| 2-RN4        | 138.0       | 124.2                             | 0.90           |
| 2-RN5        | 158.0       | 139.0                             | 0.88           |
| 2-RN6        | 111.7       | 100.5                             | 0.90           |
| 2-RH1        | 92.0        | 99.1                              | 1.08           |
| 2-RH2        | 115.1       | 117.7                             | 1.02           |
| 2-RH3        | -           | 124.1                             | -              |
| 2-RH4        | 155.3       | 142.3                             | 0.92           |
| 2-RH5        | -           | 167.5                             | -              |
2. Crack Patterns

The crack patterns evolution for all specimens are nearly similar at failure. Also, they are similar to the crack patterns for the beams in series three, the 45° cracks on the whole faces of the specimens are the main feature for these cracks.
Fig. (16) shows the crack patterns predicted by ANSYS for the specimens at the failure torque.

Figure (16): The crack pattern at failure (Series 2)

3. reinforcement stresses

Fig. (17) shows the reinforcement stresses at failure, Table (6) contains the experimental and FEM torque value as the first reinforcement bar yields.

For this series the reinforcement stresses vary as the reinforcement ratios, reinforcement yield strength, bar diameters, numbers of longitudinal bars and the spacing of the transverse reinforcement are different for each beam.

The main parameters are the reinforcement ratios and reinforcement yield strength, in this reinforcement ratios, reinforcement yield strength the beams are under-reinforced beams, except the specimens S2-RN5 and S2-RH7 that contain
a large amount of reinforcement, the reinforcement reaches the yield strength. reinforcement bar yields,

The verification for these torques shows good agreement when compared with the experimental values, for the NSC beams these ratios are between 91% to 113%, for the HSC the ratios changes from 95% to 126%

For the normal strength and high strength concrete beams the transverse reinforcement yields first, while the longitudinal seldom reaches the yield point, the location of the bars that yields are localized in the mid height of the mid span, changing the yield strength of the transverse reinforcement affect the overall behavior, the ultimate torque, the torque at yield, in Fig. (4-43) the reinforcement stresses at yield and failure are shown.

Table (6): Analytical and experimental reinforcement stresses results (Series 2)

| Specimen | Torque at yield of reinforcement (MPa) (Exp.) | Torque at yield of reinforcement. (MPa) (FEM) | Ty (FEM) / Ty (Exp.) |
|----------|-------------------------------------------|-------------------------------------------|---------------------|
| S2-RN1   | 71.5                                       | 65.4                                       | 0.91                |
| S2-RN2   | 80.9                                       | 91.3                                       | 1.13                |
| S2-RN3   | 113.0                                      | 121.4                                      | 1.07                |
| S2-RN4   | 125.0                                      | 132.1                                      | 1.06                |
| S2-RN5   | -                                          | -                                          | -                   |
| S2-RN6   | 93.8                                       | 90.8                                       | 0.97                |
| S2-RH1   | 79.7                                       | 76.2                                       | 0.96                |
| S2-RH2   | 83.5                                       | 105.0                                      | 1.26                |
| S2-RH3   | -                                          | 113.0                                      | -                   |
| S2-RH4   | 116.8                                      | 123.1                                      | 1.05                |
| S2-RH5   | -                                          | 140.3                                      | -                   |
| S2-RH6   | 157.0                                      | 153.4                                      | 0.98                |
| S2-RH7   | -                                          | -                                          | -                   |
| S2-RH8   | 91.1                                       | 86.7                                       | 0.95                |
| S2-RH9   | -                                          | 63.8                                       | -                   |
| S2-RH10  | -                                          | 86.1                                       | -                   |
Figure (17): The reinforcement stresses at failure (MPa) (Series 2)
4. Torque-Twist

Table (7) contains all the details of twisting angle results.

Table (7): Analytical and experimental twist at ultimate torque (Series 2)

| Specimen | Angle of twist per length at $T_{cr}$ (Degree/m) (Exp.) | Angle of twist per length at $T_{cr}$ (Degree/m) (FEM) | Twist (FEM) / Twist (Exp.) | Angle of twist per length at $T_u$ (Degree/m) (Exp.) | Angle of twist per length at $T_u$ (Degree/m) (FEM) | Twist (FEM) / Twist (Exp.) |
|----------|-----------------------------------------------------|-----------------------------------------------------|-----------------------------|-----------------------------------------------------|-----------------------------------------------------|-----------------------------|
| 2-RN1    | 0.080                                               | 0.130                                               | 1.62                        | 3.245                                               | 4.350                                               | 1.34                        |
| 2-RN2    | 0.115                                               | 0.195                                               | 1.70                        | 2.310                                               | 3.985                                               | 1.72                        |
| 2-RN3    | 0.092                                               | 0.191                                               | 2.08                        | 2.717                                               | 4.215                                               | 1.55                        |
| 2-RN4    | 0.120                                               | 0.250                                               | 2.08                        | 2.304                                               | 4.165                                               | 1.81                        |
| 2-RN5    | 0.126                                               | 0.265                                               | 2.10                        | 2.476                                               | 4.670                                               | 1.89                        |
| 2-RN6    | 0.109                                               | 0.201                                               | 1.85                        | 3.118                                               | 5.090                                               | 1.63                        |
| 2-RH1    | 0.120                                               | 0.186                                               | 1.55                        | 1.496                                               | 3.444                                               | 2.30                        |
| 2-RH2    | 0.086                                               | 0.189                                               | 2.20                        | 1.456                                               | 3.756                                               | 2.58                        |
| 2-RH3    | -                                                   | 0.193                                               | -                           | -                                                   | 3.981                                               | -                           |
| 2-RH4    | 0.075                                               | 0.145                                               | 1.95                        | 2.167                                               | 4.672                                               | 2.16                        |
| 2-RH5    | -                                                   | 0.165                                               | -                           | -                                                   | 4.825                                               | -                           |
| 2-RH6    | 0.092                                               | 0.345                                               | 3.77                        | 2.253                                               | 7.377                                               | 3.27                        |
| 2-RH7    | 0.092                                               | 0.343                                               | 3.74                        | 2.666                                               | 5.561                                               | 2.09                        |
| 2-RH8    | 0.092                                               | 0.307                                               | 3.30                        | 1.903                                               | 6.884                                               | 3.62                        |

A slight increase in the twisting angle when the yield strength of reinforcement increases. Figs. (18) and (19) shows the Effect of yield strength of the transverse reinforcement and longitudinal reinforcement respectively.
Figure (18): Effect of yield strength of the transverse reinforcement ($f_{ys}$) on the angle of twist (Series 2)

Figure (19): Effect of yield strength of the longitudinal reinforcement ($f_{yl}$) on the angle of twist (Series 2)
By increasing the reinforcement ratio, the twisting angle also increases, as shown in Figs. (20) and (21). By increasing the transverse reinforcement ratio from 6% to 14% the twisting angle will increase by about 15.5%.

Figure (20): Effect of the transverse reinforcement ratio on the angle of twist (Series 2)
6. Conclusions

1) The nonlinear FE model, which adopted in the current study, is appropriate for simulation of the behavior of RC beams subjected to torsion and capable of predicting the ultimate torque, ultimate angle of twist and the overall behavior with good accuracy.

2) The failure torques from the FEA of the RC beams are close to the experimental failure loads. The ratios of the numerical to experimental values of ultimate torque range between 85% and 107%.

3) The transverse reinforcement is more effective in resisting the applied torque, it is preferable to use higher ratio of transverse reinforcement for the NSC and HSC beams, where the increase in ultimate torque was about 94% when the ratio increased from 6% to 16% , and using high yield strength steel will increase in ultimate torque by about 4.1% by changing the yield strength from 440 MPa to 500 MPa.

4) The mode of failure for the RC beams is significantly affected by the reinforcement ratio, and the typical mode of failure for under-reinforced beams, was diagonal tension failure for both concrete types, also it agrees with the type of failure in the available experimental results.

5) The crack patterns obtained from the FE models for RC are similar to the crack patterns available in the experimental results especially in the early stages of loading.
REFERENCES

[1] Al-Radi, S.N.A, "Three Dimensional Nonlinear Analysis of Reinforced Concrete Beams Subjected to Combined Shear and Torsion Using Finite Element Method", M.Sc. Thesis, College of Engineering, University of Basrah, 2002.

[2] Wight, J. K., and Macgregor, J. G., "Reinforced Concrete Mechanics and Design", Pearson Education, Inc., 2012.

[3] McCormac, J. C., and Brown, R. H., "Design of Reinforced Concrete", John Wiley & Sons, Inc., 2014.

[4] Rasmussen, L.J., and Baker, G., "Torsion in Reinforced Normal and High-Strength Concrete Beams-Part 1: Experimental Test Series", ACI Structural Journal, Vol.92, No.1, pp.56-62, January-February 1995.

[5] Fang, I. K., and Shiau, J. K., " Torsional Behavior of Normal- and High-Strength Concrete Beams", ACI Structural Journal, V. 101, No. 3, pp.304-313, May-June 2004.