ISOSPIN EIGENSTATES OF THE PENTAQUARK MOLECULAR STATES
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Abstract

In this article, we construct the color-singlet-color-singlet type five-quark currents with the isospins \((I, I_3) = (\frac{1}{2}, \frac{1}{2})\) and \((\frac{3}{2}, \frac{1}{2})\) unambiguously to explore the \(\bar{D}\Sigma_c, D\Sigma^*_c\), \(D^*\Sigma_c\) and \(D^*\Sigma^*_c\) molecular states via the QCD sum rules for the first time. The numerical results support assigning the \(P_c(4312)\), \(P_c(4380)\), \(P_c(4440)\) and \(P_c(4457)\) as the \(\bar{D}\Sigma_c, D\Sigma^*_c, D^*\Sigma_c\) and \(D^*\Sigma^*_c\) pentaquark molecular states with the isospin \(I = \frac{1}{2}\), respectively. The corresponding \(\bar{D}\Sigma_c, D\Sigma^*_c, D^*\Sigma_c\) and \(D^*\Sigma^*_c\) pentaquark molecular states with the isospin \(I = \frac{3}{2}\) have slightly larger masses, the observations of the higher pentaquark molecule candidates in the \(J/\psi\Delta\) invariant mass spectrum would shed light on the nature of the \(P_c\) states, and make contributions in distinguishing the scenarios of pentaquark states and pentaquark molecular states.

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1 Introduction

In 1964, M. Gell-mann proposed that the multiquark states could exist [1]. Theoretically, there is no forbiddance for the existence of the exotic states which cannot be embedded into the conventional charmonium spectrum. Since the observation of the \(X(3872)\) by the Belle collaboration in 2003 [2], many exotic \(X\), \(Y\), \(Z\) particles have been observed at the Belle, BaBar, BESIII and LHCb collaborations [3]. The masses of some exotic states are close to the known two-particle thresholds, and lead to the possible hadronic molecule interpretations [4], namely, the bound states of the meson-meson, baryon-meson or baryon-baryon. In 2015, the LHCb collaboration observed two pentaquark candidates \(P_c(4380)\) and \(P_c(4450)\) via analysis of the \(\Lambda_b^0 \to J/\psi K^- p\) decays [5]. In 2019, the LHCb collaboration re-investigated the experimental data with order of magnitude larger than that previously analyzed by the LHCb collaboration, and observed a narrow pentaquark candidate \(P_c(4312)\) in the \(J/\psi\Delta\) mass spectrum [6], and proved that the \(P_c(4450)\) consists of two narrow overlapping peaks \(P_c(4440)\) and \(P_c(4457)\). The measured Breit-Wigner masses and widths of the four exotic structures are

\[
P_c(4312) : M = 4311.9 \pm 0.7^{+6.8}_{-0.6} \text{ MeV}, \; \Gamma = 9.8 \pm 2.7^{+3.7}_{-4.5} \text{ MeV}, \\
P_c(4380) : M = 4380 \pm 8 \pm 29 \text{ MeV}, \; \Gamma = 205 \pm 18 \pm 86 \text{ MeV}, \\
P_c(4440) : M = 4440 \pm 1.3 \pm 4.3 \text{ MeV}, \; \Gamma = 20.6 \pm 4.9^{+8.7}_{-10.1} \text{ MeV}, \\
P_c(4457) : M = 4457.3 \pm 0.6^{+4.1}_{-1.7} \text{ MeV}, \; \Gamma = 6.4 \pm 2.0^{+5.7}_{-1.9} \text{ MeV},
\]

respectively [5] [6].

Those resonances lie just a few MeV below the thresholds of the hidden-charm meson-baryon pairs \(\bar{D}\Sigma_c, D\Sigma^*_c\), \(D^*\Sigma_c\) and \(D^*\Sigma^*_c\), respectively. Now a typical interpretation of the \(P_c(4312)\), \(P_c(4380)\), \(P_c(4440)\) and \(P_c(4457)\) is that they are the S-wave hidden-charm meson-baryon molecules, and have definite isospin \(I\), spin \(J\) and parity \(P\). For example, in Ref. [7], it is proposed that the \(P_c(4440)\) and \(P_c(4457)\) are the \(D^*\Sigma_c\) bound states with the \(J^P = \frac{1}{2}^-\) and \(\frac{3}{2}^-\) respectively via the one-pion exchange potential between the heavy antimeson and heavy baryon, the result is consistent with the conclusion obtained in Ref. [5] via the one-boson-exchange model. Interestingly, the...
isospins are considered via the one-pion/one-boson-exchange potential model in Ref. 9 and a series of hidden-charm antimeson-baryon pentaquark molecules are predicted. In Ref. 10, the \( \tilde{D}^{(*)}\Sigma_c^{(*)} \) molecular states are studied via a coupled-channel formalism with the scattering potential involving both the one-pion exchange and short-range operators constrained by the heavy quark spin symmetry, while in Ref. 11, the \( P_c(4440) \) and \( P_c(4457) \) are interpreted as the \( \tilde{D}^*\Sigma_c \) bound states with the \( J^P = \frac{1}{2}^- \) and \( \frac{3}{2}^- \), respectively via the quasipotential Bethe-Salpeter equation approach.

Among the popular theoretical tools, the QCD sum rules approach is a powerful theoretical tool in studying the exotic states, the \( P_c(4312), P_c(4380), P_c(4440) \) and \( P_c(4457) \) have been studied with the QCD sum rules, irrespective of being assigned as pentaquark states \( \text{[12, 13, 14, 15]} \) or pentaquark molecular states \( \text{[16, 17, 18, 19, 20, 21, 22]} \). In the QCD sum rules, we choose the local five-quark currents, both the pentaquark states and molecular states are compact objects, it is better to call the pentaquark molecular states as the color-singlet-color-singlet type pentaquark states.

If we prefer interpretations of the pentaquark molecular states and the theoretical approach of the QCD sum rules, we should distinguish their isospins and investigate their properties in an unambiguous way, however, in previous works, the isospins of the interpolating currents were not specified \( \text{[10, 17, 18, 19, 20, 21, 22]} \), the currents couple potentially not only to the molecular states with the isospin \( I = \frac{1}{2} \) but also to the ones with the isospin \( I = \frac{3}{2} \), there are unknown uncertainties. Since those \( P_c \) states were discovered in the \( J/\psi p \) invariant mass spectrum, their isospins should be \( I = \frac{1}{2} \) considering for conservation of the isospins in the strong interactions, and we should specify the isospins of the interpolating currents to make robust predictions, it is the key issue to solve the puzzle of those \( P_c \) states. In the present work, we explore the \( \tilde{D}^{(*)}\Sigma_c^{(*)} \) pentaquark molecular states with the \( I = \frac{1}{2} \) and \( \frac{3}{2} \) via the QCD sum rules in a systematic way.

The article is arranged as follows: we obtain the QCD sum rules for the pentaquark molecular states in Sect.2; we present the numerical results and discussions in Sect.3; Sect.4 is reserved for our conclusions.

## 2 QCD sum rules for the pentaquark molecular states

The \( u \) quark and \( d \) quark have the isospin \( I = \frac{1}{2} \), in details, \( \hat{T}u = \frac{1}{2}u \) and \( \hat{T}d = -\frac{1}{2}d \), where the \( \hat{T} \) is the isospin operator. Then the \( \tilde{D}^0, \tilde{D}^{*0}, \tilde{D}^-, \tilde{D}^{*-}, \Sigma_c^+, \Sigma_c^{*+}, \Sigma_c^{++} + \Sigma_c^{*++} \) correspond to the eigenstates \( | \frac{1}{2}, \frac{1}{2} \rangle, | \frac{1}{2}, \frac{1}{2} \rangle, | \frac{1}{2}, -\frac{1}{2} \rangle, | \frac{1}{2}, -\frac{1}{2} \rangle, | 1, 0 \rangle, | 1, 0 \rangle, | 1, 1 \rangle \) and \( | 1, 1 \rangle \) in the isospin space \( | I, I_3 \rangle \), respectively. And we can construct the following color-singlet currents to interpolate them,

\[
\begin{align*}
J^0(x) &= \bar{c}(x)i\gamma_5 u(x), \\
J^-(x) &= \bar{c}(x)i\gamma_5 d(x), \\
J^{*0}(x) &= \bar{c}(x)\gamma_\mu u(x), \\
J^{*-}(x) &= \bar{c}(x)\gamma_\mu d(x), \\
J^{++}(x) &= \varepsilon^{ijk}u^{(T)}(x)C\gamma_\mu d^j(x)\gamma_\nu \gamma_5 c^k(x), \\
J^{*+}(x) &= \varepsilon^{ijk}u^{(T)}(x)C\gamma_\mu u^j(x)\gamma_\nu \gamma_5 c^k(x), \\
J^{++}(x) &= \varepsilon^{ijk}u^{(T)}(x)C\gamma_\mu d^j(x)\gamma_\nu \gamma_5 c^k(x), \\
J^{*+}(x) &= \varepsilon^{ijk}u^{(T)}(x)C\gamma_\mu u^j(x)\gamma_\nu \gamma_5 c^k(x),
\end{align*}
\]

the superscripts \( i, j, k \) are color indices and the \( C \) represents the charge conjugation matrix. Accordingly, we can construct the color-singlet-color-singlet type five-quark currents to interpolate the \( \tilde{D}^{(*)}\Sigma_c^{(*)} \)-type pentaquark molecular sates, where the \( \tilde{D}^{(*)} \) and \( \Sigma_c^{(*)} \) represent the color-singlet clusters having the same quantum numbers as the physical states \( \tilde{D}^{(*)} \) and \( \Sigma_c^{(*)} \), respectively, and
we write down the two-point correlation functions,
\[
\Pi(p) = i \int d^4x e^{ip\cdot x} \langle 0 | T \{ J(x) \bar{J}(0) \} | 0 \rangle,
\]
\[
\Pi_{\mu\nu}(p) = i \int d^4x e^{ip\cdot x} \langle 0 | T \{ J_\mu(x) \bar{J}_\nu(0) \} | 0 \rangle,
\]
\[
\Pi_{\mu\nu\alpha\beta}(p) = i \int d^4x e^{ip\cdot x} \langle 0 | T \{ J_{\mu\nu}(x) \bar{J}_{\alpha\beta}(0) \} | 0 \rangle,
\]
where the currents
\[
J(x) = \frac{1}{\sqrt{3}} J^{\bar{D}\Sigma_{c}}(x), \quad J^{\bar{D}^{*}\Sigma_{c}}(x),
\]
\[
J_{\mu}(x) = \frac{1}{\sqrt{3}} J_{\mu}^{\bar{D}\Sigma_{c}}(x), \quad J_{\mu}^{\bar{D}^{*}\Sigma_{c}}(x), \quad J_{\mu}^{\bar{D}^{*}\Sigma_{c}}(x), \quad J_{\mu}^{\bar{D}^{*}\Sigma_{c}}(x),
\]
\[
J_{\mu\nu}(x) = \frac{1}{\sqrt{3}} J_{\mu\nu}^{\bar{D}\Sigma_{c}}(x), \quad J_{\mu\nu}^{\bar{D}^{*}\Sigma_{c}}(x),
\]
(4)
\[
J_{\frac{3}{2}}^{\bar{D}\Sigma_{c}}(x) = \frac{1}{\sqrt{3}} J^{\bar{D}\Sigma_{c}}(x) J^{\Sigma_{c}^{+}}(x) - \sqrt{\frac{2}{3}} J^{\bar{D}^{*}}(x) J^{\Sigma_{c}^{-}}(x),
\]
\[
J_{\frac{3}{2}}^{\bar{D}^{*}\Sigma_{c}}(x) = \sqrt{\frac{2}{3}} J^{\bar{D}\Sigma_{c}}(x) J^{\Sigma_{c}^{+}}(x) + \frac{1}{\sqrt{3}} J^{\bar{D}^{*}}(x) J^{\Sigma_{c}^{-}}(x),
\]
\[
J_{\frac{3}{2}}^{\bar{D}^{*}\Sigma_{c}}(x) = \frac{1}{\sqrt{3}} J^{\bar{D}\Sigma_{c}}(x) J^{\Sigma_{c}^{-}}(x) - \sqrt{\frac{2}{3}} J^{\bar{D}^{*}}(x) J^{\Sigma_{c}^{+}}(x),
\]
\[
J_{\frac{3}{2}}^{\bar{D}^{*}\Sigma_{c}}(x) = \sqrt{\frac{2}{3}} J^{\bar{D}\Sigma_{c}}(x) J^{\Sigma_{c}^{-}}(x) + \frac{1}{\sqrt{3}} J^{\bar{D}^{*}}(x) J^{\Sigma_{c}^{+}}(x),
\]
\[
J_{\frac{3}{2}}^{\bar{D}^{*}\Sigma_{c}}(x) = \frac{1}{\sqrt{3}} J^{\bar{D}\Sigma_{c}}(x) J^{\Sigma_{c}^{+}}(x) - \sqrt{\frac{2}{3}} J^{\bar{D}^{*}}(x) J^{\Sigma_{c}^{-}}(x),
\]
\[
J_{\frac{3}{2}}^{\bar{D}^{*}\Sigma_{c}}(x) = \sqrt{\frac{2}{3}} J^{\bar{D}\Sigma_{c}}(x) J^{\Sigma_{c}^{+}}(x) + \frac{1}{\sqrt{3}} J^{\bar{D}^{*}}(x) J^{\Sigma_{c}^{-}}(x),
\]
\[
\mu \leftrightarrow \nu,
\]
(5)
the subscripts $\frac{1}{2}$ and $\frac{3}{2}$ represent the isospins $I$ \[21\]. The currents are the isospin eigenstates $|I, I_3 = \frac{1}{2}, \frac{1}{2} \rangle$ or $|I, I_3 = \frac{3}{2}, \frac{1}{2} \rangle$, respectively.

The currents $J(x)$, $J_{\mu}(x)$ and $J_{\mu\nu}(x)$ couple potentially not only to the hidden-charm pentaquark molecular states with negative-parity but also to the ones with positive parity, we separate their ground state contributions at the hadron side,
\[
\Pi(p) = \lambda \frac{\not{p}^2 + M_{-}}{M_{-}^2 - p^2} + \lambda \frac{\not{p}^2 - M_{+}}{M_{+}^2 - p^2} + \cdots,
\]
\[
\Pi_{\mu\nu}(p) = \lambda \frac{\not{p}^2 + M_{-}}{M_{-}^2 - p^2} (-g_{\mu\nu}) + \lambda \frac{\not{p}^2 - M_{+}}{M_{+}^2 - p^2} (-g_{\mu\nu}) + \cdots,
\]
\[
\Pi_{\mu\nu\alpha\beta}(p) = -\Pi_{\mu\nu\alpha\beta}(p) g_{\mu\nu} - \Pi_{\mu\nu\alpha\beta}(p) g_{\mu\nu} + \cdots,
\]
(6)
\[
\Pi_{\mu \nu \alpha \beta} (p) = \lambda_i^{-2} \hat{\rho} + \frac{M_+}{M_+^2 - p^2} (g_{\mu \alpha} g_{\nu \beta} + g_{\mu \beta} g_{\nu \alpha}) + \lambda_i^{2} \hat{\rho} - \frac{M_+}{M_+^2 - p^2} (g_{\mu \alpha} g_{\nu \beta} + g_{\mu \beta} g_{\nu \alpha}) + \cdots ,
\]

\[
= \Pi_i^1 (p^2) \hat{\rho} (g_{\mu \alpha} g_{\nu \beta} + g_{\mu \beta} g_{\nu \alpha}) + \Pi_i^2 (p^2) (g_{\mu \alpha} g_{\nu \beta} + g_{\mu \beta} g_{\nu \alpha}) + \cdots ,
\]

where the subscripts \(1\) and \(2\) are the spins of the molecular states, the subscripts/superscripts \(\pm\) denote the positive-parity and negative-parity, respectively, and we have smeared the isospin indexes. The pole residues are defined by

\[
(0|J(0)|P_{-i}^- (p)) = \lambda_i^{-2} U_-(p, s) ,
\]

\[
(0|J(0)|P_{-i}^+ (p)) = \lambda_i^2 \gamma_5 U_+ (p, s) ,
\]

\[
(0|J_{\mu} (0)|P_{-i}^- (p)) = \lambda_i^{-2} U_{\mu, -} (p, s) ,
\]

\[
(0|J_{\mu} (0)|P_{-i}^+ (p)) = \lambda_i^2 \gamma_5 U_{\mu, +} (p, s) ,
\]

\[
(0|J_{\mu \nu} (0)|P_{-i}^- (p)) = \sqrt{2} \lambda_i^{-2} U_{\mu \nu, -} (p, s) ,
\]

\[
(0|J_{\mu \nu} (0)|P_{-i}^+ (p)) = \sqrt{2} \lambda_i^2 \gamma_5 U_{\mu \nu, +} (p, s) ,
\]

where the \(U_{\pm} (p, s)\), \(U_{\mu, \mp} (p, s)\) and \(U_{\mu \nu, \mp} (p, s)\) are the Dirac and Rarita-Schwinger spinors, for all the technical details, one can consult Refs. [12, 13, 14, 15, 20, 21].

In the present work, we choose the components associated with the structures \(\hat{\rho}\), \(1\), \(\hat{\rho} g_{\mu \nu}\), \(g_{\mu \nu}\) and \(\hat{\rho} (g_{\mu \alpha} g_{\nu \beta} + g_{\mu \beta} g_{\nu \alpha})\), \(g_{\mu \alpha} g_{\nu \beta} + g_{\mu \beta} g_{\nu \alpha}\) in the correlation functions \(\Pi (p), \Pi_{\mu \nu} (p)\) and \(\Pi_{\mu \nu \alpha \beta} (p)\) respectively to investigate the pentaquark molecular states with the spin-parity \(J^P = \frac{1}{2}^-, \frac{3}{2}^+\) and \(\frac{5}{2}^+\), respectively.

We carry out the complex operator product expansion, and analyze the contributions of all kinds of vacuum condensates. Firstly, the contributions of the related vacuum condensates are tiny in the case of \(k \geq \frac{1}{2}\) for the counting-rules in terms of the strong fine-structure constant \(\mathcal{O}(\alpha_s^2)\) [23, 24, 25, 26], it is accurate enough for us to calculate the terms for \(k \leq 1\) [27]. Secondly, the highest dimension of the vacuum condensates is usually estimated from the leading order Feynman diagrams. In the present work, the correlation functions contain two heavy quark lines and three light quark lines. If each heavy quark line emits a gluon and each light quark line contributes a quark-antiquark pair, we obtain the quark-gluon operator \(g_s G_{\alpha \beta} g_s G_{\gamma \delta} \eta_{qg} \eta_{qg} \eta_{qg} \eta_{qg}\) with the dimension 13, this operator can be factorized into the vacuum condensates \((\frac{2}{3} GG) \eta_{qg}^3\) and \((\frac{2}{3} GG) \eta_{qg} \eta_{qg} \eta_{qg} \eta_{qg}\). Thirdly, the four-quark condensates \((\bar{q} q)^2 = \sum_{u, s, d} (\bar{q} q)^2\) are neglected as they come from condensations between the two heavy quark lines through equation of motion and play a tiny role [27]. Thus, in this work, there are solid reasons for us to choose the terms \((\bar{q} q), (\frac{2}{3} GG), (\bar{q} q)^2, (\frac{2}{3} GG) (\bar{q} q), (\bar{q} q) (\bar{q} q), (\bar{q} q)^3, (\frac{2}{3} GG) (\bar{q} q)^2, (\bar{q} q) (\bar{q} q)^2, (\bar{q} q)^4, (\bar{q} q) (\bar{q} q)^3, (\frac{2}{3} GG) (\bar{q} q)^3\) in the operator product expansions.

We obtain the analytical spectral densities \(\rho_{1, QC D} (s)\) and \(\rho_{2, QC D} (s)\) at the quark-gluon level, and take the quark-hadron duality below the continuum thresholds \(s_0\) and introduce the weight functions \(\sqrt{s} \exp \left(-\frac{M_s^2}{s} \right)\) and \(\exp \left(-\frac{s}{s_0^2} \right)\) to obtain the QCD sum rules:

\[
2 M - \lambda_i^2 \exp \left(-\frac{M_s^2}{T^2} \right) = \int_{4m_s^2}^{s_0} ds \left[ \sqrt{s} \rho_{1, QC D} (s) + \rho_{2, QC D} (s) \right] \exp \left(-\frac{s}{s_0^2} \right) ,
\]

as we are only interested in the pentaquark molecular states with the negative parity, the explicit expressions of the spectral densities \(\rho_{1, QC D} (s)\) and \(\rho_{2, QC D} (s)\) at the quark level are neglected for simplicity.
We differentiate Eq. (12) with respect to \( \tau = \frac{q}{\tau_0} \), then eliminate the pole residues \( \lambda_j \) with \( j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \) to obtain the QCD sum rules for the masses of the pentaquark molecular states,

\[
M_j^2 = \frac{\int_{s_0}^{s} ds \left[ \sqrt{s} \rho_{QCD}^1(s) + \rho_{QCD}^0(s) \right] \exp(-\tau s)}{\int_{s_0}^{s} ds \left[ \sqrt{s} \rho_{QCD}^1(s) + \rho_{QCD}^0(s) \right] \exp(-\tau s)},
\]

(13)

where the spectral densities \( \rho_{QCD}^1(s) = \rho_{j, QCD}^1(s) \) and \( \rho_{QCD}^0(s) = \rho_{j, QCD}^0(s) \).

### 3 Numerical results and discussions

We apply the standard values of the vacuum condensates \( \langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3, \langle \bar{q}g_{\sigma}Gq \rangle = m_0^2(\bar{q}q) \text{ GeV}^2, m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2, \langle \bar{q}g_{\sigma}Gq \rangle = (0.33 \text{ GeV})^4 \) at the energy scale \( \mu = 1 \text{ GeV} \) \cite{28 29 30 31}, and choose the \( \bar{M}S \) mass \( m_c(m_c) = 1.275 \pm 0.025 \text{ GeV} \) from the Particle Data Group \cite{3}. We consider the energy-scale dependence of those parameters,

\[
\langle \bar{q}q \rangle(\mu) = \langle \bar{q}q \rangle(1 \text{ GeV}) \left[ \frac{\alpha_s(1 \text{ GeV})}{\alpha_s(\mu)} \right]^{33/2n_f},
\]

\[
\langle \bar{q}g_{\sigma}Gq \rangle(\mu) = \langle \bar{q}g_{\sigma}Gq \rangle(1 \text{ GeV}) \left[ \frac{\alpha_s(1 \text{ GeV})}{\alpha_s(\mu)} \right]^{33/2n_f},
\]

\[
m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{12/3},
\]

\[
\alpha_s(\mu) = \frac{1}{b_0 t} \left[ 1 - b_1 \log t + \frac{b_2}{b_0} \left( \log^2 t - \log t - 1 \right) + b_3 b_2 \right],
\]

where \( t = \log \frac{\mu^2}{\Lambda_{QCD}^2} \), \( b_0 = \frac{33 - 2n_f}{12} \), \( b_1 = \frac{153 - 19n_f}{24} \), \( b_2 = \frac{2857 - 503n_f + 32n_f^2}{128} \), and \( \Lambda_{QCD} = 213 \text{ MeV} \), \( 296 \text{ MeV} \), \( 339 \text{ MeV} \) for the flavors \( n_f = 5, 4, 3 \), respectively \cite{32}. In this paper, we choose the flavor number \( n_f = 4 \) for all the pentaquark molecular states, and apply the energy scale formula to determine the best energy scales of the QCD spectral densities \cite{12 14 20 21 29 24 25 26}.

\[
\mu = \sqrt{M_{X/Y/Z/P}^2 - 4M_0^2},
\]

(14)

where the \( M_c \) is the effective charm quark mass, we choose the updated value \( M_c = 1.85 \pm 0.01 \text{ GeV} \) \cite{21}.

All the QCD sum rules should satisfy the pole dominance and convergence of the operator product expansion which are two basic criteria. What’s more, we should obtain Borel platforms to avoid additional uncertainties originated from the Borel parameters. The selections of the suitable energy scales, continuum threshold parameters and Borel parameters are accomplished via trial and error: we tentatively choose an energy scale \( \mu = 1 \text{ GeV} \) and the energy scale formula \( \mu = \sqrt{M_{X/Y/Z/P}^2 - 4M_0^2} \) are satisfied. If not, we choose another energy scale and another continuum threshold parameter until reach the satisfactory results. In calculations, we define the pole contributions (PC) as,

\[
\text{PC} = \frac{\int_{s_0}^{s} ds \left[ \sqrt{s} \rho_{QCD}^1(s) + \rho_{QCD}^0(s) \right] \exp(-\tau s)}{\int_{s_0}^{s} ds \left[ \sqrt{s} \rho_{QCD}^1(s) + \rho_{QCD}^0(s) \right] \exp(-\tau s)},
\]

(15)
The convergence of the operator product expansion is quantified via the contributions of the vacuum condensates of dimension $n$,

$$D(n) = \frac{\int_{4m_c^2}^{s_0} ds \left[ \sqrt{s} \rho_{QCD,n}^1(s) + \rho_{QCD,n}^0(s) \right] \exp \left( -\frac{\sqrt{s}}{F} \right)}{\int_{4m_c^2}^{s_0} ds \left[ \sqrt{s} \rho_{QCD}^1(s) + \rho_{QCD}^0(s) \right] \exp \left( -\frac{\sqrt{s}}{F} \right)}, \quad (16)$$

where the $\rho_{QCD,n}^1(s)$ and $\rho_{QCD,n}^0(s)$ represent the spectral densities with the vacuum condensates of dimension $n$ picked out from the $\rho_{QCD}^1(s)$ and $\rho_{QCD}^0(s)$, respectively, and the total contributions are normalized to be 1.

At last, we find the best energy scales, the ideal continuum threshold parameters, the Borel windows, see Table 1 the pole contributions for all the eight molecular states are around (or slightly larger than) $(40 - 60)\%$, thus, the pole dominance criterion for the QCD sum rules holds well.

The absolute values of the normalized contributions $D(n)$ from the vacuum condensates are displayed in the Fig. 1 where the highest dimensional condensate contributions $|D(12)|$ and $|D(13)|$ are approximately zero, the most important contributions are mainly from the lowest order contributions $\langle \bar{q}q \rangle$, $\langle \bar{q}q \rangle^2$ and $\langle \bar{q}q, \sigma Gq \rangle/\langle \bar{q}q \rangle$, and the gluon condensate plays a less important role since $|D(4)| < 5\%$ except for the $D^*\Sigma_c^+$ molecular state with the isospin $I = \frac{3}{2}$. All in all, the convergence of the operator expansions is very well satisfied.

We calculate the uncertainties of the masses and pole residues according to the standard error analysis formula, the numerical results of the masses and pole residues are shown in the Table 1 (also the Fig. 2).

From Table 1 we can see that the central value of the extracted mass of the $\bar{D}\Sigma_c^+$ molecular state with the quantum numbers $IJ^P = \frac{1}{2}^-$ is 4.31 GeV, it is only about 10 MeV below the $\bar{D}\Sigma_c^+$ threshold, so we can assign this state as the $P_c(4312)$ naturally. For the $\bar{D}\Sigma_c^+$ molecular state with the quantum numbers $IJ^P = \frac{1}{2}^-$, the central value of the mass is 4.33 GeV, we find it is about 10 MeV above the $\bar{D}\Sigma_c^+$ threshold, so we can assign this one as the $\bar{D}\Sigma_c^+$ resonance state, the isospin cousin of the $P_c(4312)$.

In a similar way, according to the numerical results of the extracted masses, we have very good reasons to assign the $P_c(4308)$, $P_c(4440)$ and $P_c(4457)$ as the $\bar{D}\Sigma_c$, $\bar{D}\Sigma_c^*$, and $\bar{D}\Sigma_c^*$ molecular states with the quantum numbers $IJ^P = \frac{1}{2}^-$, $\frac{1}{2}^-$ and $\frac{1}{2}^-$, respectively. For the pentaquark molecular states (resonances) $\bar{D}\Sigma_c^+$ with $IJ^P = \frac{3}{2}^-$, $\bar{D}\Sigma_c^+$ with $IJ^P = \frac{3}{2}^-$ and $\bar{D}\Sigma_c^*$ with $IJ^P = \frac{3}{2}^-$, the central values of the extracted masses are about 20 MeV, 10 MeV and 90 MeV above the corresponding meson-baryon thresholds, respectively.
Figure 2: The $M - T^2$ curves, where $M_i (i = 1, 2, \cdots, 8)$ denote the masses of the $\bar{D}\Sigma_c$ with $I = \frac{1}{2}$, $D\Sigma_c$ with $I = \frac{3}{2}$, $D\Sigma_c^*$ with $I = \frac{1}{2}$, $D^*\Sigma_c$ with $I = \frac{3}{2}$, $D^*\Sigma_c^*$ with $I = \frac{1}{2}$ and $D^*\Sigma_c^*$ with $I = \frac{3}{2}$, respectively.
If we choose the same input parameters, the pentaquark molecular states with the isospin \( I = \frac{3}{2} \) have slightly larger masses than the corresponding molecules with the isospin \( I = \frac{1}{2} \). In calculations, we observe that the masses and pole residues increase monotonously with the increase of the continuum threshold parameters, we determine the continuum threshold parameters \( s_0 \) by adopting the uniform constraints, such as the continuum thresholds \( \sqrt{s_0} = M_\pi + 0.65 \pm 0.1 \text{GeV} \), pole contributions \( (40 \sim 65\%) \) and intervals \( T_{\text{max}} - T_{\text{min}}^2 = 0.6 \text{GeV}^2 \) to acquire reliable predictions, where the \( T_{\text{max}}^2 \) and \( T_{\text{min}}^2 \) stand for the maximum and minimum values of the Borel parameters, respectively.

It is clearly that the \( P_c(4312) \), \( P_c(4380) \), \( P_c(4440) \) and \( P_c(4457) \) can be assigned to be the \( \bar{D}\Sigma_c \), \( \bar{D}'\Sigma_c \), \( D^*\Sigma_c \) and \( \bar{D}^*\Sigma_c \) pentaquark molecular states with the isospin \( I = \frac{1}{2} \), since the two-body strong decays \( P_c \rightarrow J/\psi p \) conserve isospin. If the assignments are robust, there exist four slightly higher pentaquark molecular states \( \bar{D}\Sigma_c \), \( \bar{D}'\Sigma_c \), \( D^*\Sigma_c \) and \( \bar{D}^*\Sigma_c \) with the isospin \( I = \frac{3}{2} \), we can search for the four resonances in the \( J/\psi \Delta \) invariant mass spectrum, as the two-body strong decays \( P_c \rightarrow J/\psi \Delta \) also conserve isospin, the \( J/\psi \), \( p \) and \( \Delta \) have the isospins \( I = 0, \frac{1}{2} \) and \( \frac{3}{2} \), respectively. If the four resonances are observed one day, we can obtain additional proofs for the molecule assignments, and shed light on the nature of the \( P_c \) states and dynamics of the low energy QCD.

In this work, we construct the local color-singlet-color-singlet type five-quark currents with the definite isospins, which couple potentially to the hidden-charm molecular states (more precisely, the color-singlet-color-singlet type compact pentaquark states) rather than to the meson-baryon scattering states or thresholds, the thresholds in Table 1 are taken from Particle Data Group \[3\], as the color-singlet-color-singlet type compact pentaquark states rather than to the meson-baryon scattering states or thresholds, the \( \bar{D}\Sigma_c \), \( \bar{D}'\Sigma_c \), \( D^*\Sigma_c \) and \( \bar{D}^*\Sigma_c \) pentaquark molecular states with the isospin \( I = \frac{1}{2} \), the two-body strong decays \( P_c \rightarrow J/\psi p \) conserve isospin. If the assignments are robust, there exist four slightly higher pentaquark molecular states \( \bar{D}\Sigma_c \), \( \bar{D}'\Sigma_c \), \( D^*\Sigma_c \) and \( \bar{D}^*\Sigma_c \) with the isospin \( I = \frac{3}{2} \), we can search for the four resonances in the \( J/\psi \Delta \) invariant mass spectrum, as the two-body strong decays \( P_c \rightarrow J/\psi \Delta \) also conserve isospin, the \( J/\psi \), \( p \) and \( \Delta \) have the isospins \( I = 0, \frac{1}{2} \) and \( \frac{3}{2} \), respectively. If the four resonances are observed one day, we can obtain additional proofs for the molecule assignments, and shed light on the nature of the \( P_c \) states and dynamics of the low energy QCD.

In the local color-singlet-color-singlet type five-quark currents with the definite isospins, which couple potentially to the hidden-charm molecular states (more precisely, the color-singlet-color-singlet type compact pentaquark states) rather than to the meson-baryon scattering states or thresholds, the thresholds in Table 1 are taken from Particle Data Group \[3\], as the traditional charmed mesons and baryons are spatial extended objects and have average spatial sizes \( \langle r^2 \rangle \approx 0.5 \text{fm} \) and \( 0.5 \sim 0.8 \text{fm} \), respectively \[21, 33\]. Therefore, the loosely bound molecular states, meson-baryon scattering states or thresholds have spatial extensions larger than \( 1 \text{fm} \), which is too large to be interpolated by the local currents. In the local limit \( r \rightarrow 0 \), in such small spatial separations, the \( \bar{c}q \) meson and \( cq'q'' \) baryon lose themselves and merge into color-singlet-color-singlet type pentaquark states. The scenario of the pentaquark molecular states in the QCD sum rules is quite different from other theoretical methods. In the QCD sum rules, there are two color-singlet clusters, which have the same quantum numbers as the physical states \( \bar{D}^{(*)} \) and \( \Sigma_c^{(*)} \), respectively, but they are not the physical states, and we carry out the operator product expansion at the quark-gluon level at the QCD side, and can only distinguish the short distance and long distance contributions, no hadronic degrees of freedoms are needed. In fact, we can abandon the conception "molecular states" in the QCD sum rules, we just investigate the color-singlet-color-singlet type pentaquark states, which have masses near the meson-baryon thresholds.

While in the one-pion exchange potential model \[34\] and heavy-quark spin symmetry model \[35\], there are physical charmed meson and baryons states. In the one-pion exchange potential model, the short range interaction by the coupling to the 5-quark-core states plays a major role in determining of the ordering of the multiplet states, while the long range force of the pion tensor force does in producing the decay widths \[34\]. In the heavy-quark spin symmetry model, the pentaquark-like resonances can be naturally accommodated in a contact-range effective field theory description that incorporates the heavy-quark spin symmetry \[35\].

The \( P_c(4380) \) observed in the six-dimensional amplitude analysis obsolete in the updated analysis \[6\], which weakens the previously reported evidence for the \( P_c(4380) \), but does not contradict its existence, as the one-dimensional analysis is not sensitive to wide \( P_c \) states. Whether or not there exist \( P_c(4380) \)-like wide pentaquark candidates, a six-dimensional amplitude analysis of the \( \Lambda_c^0 \rightarrow J/\psi pK^- \) decays in the future could be able to answer the question. Our calculations just indicate that there exist a pentaquark molecule candidate with the mass about \( 4.38 \text{GeV} \), and it is not necessary to be the \( P_c(4380) \).

In Ref. \[36\], we assign the \( Z_c^{\pm}(3900) \) as the diquark-antidiquark type tetraquark state with the quantum numbers \( J^{PC} = 1^{+-} \), and study the hadronic coupling constants in its two-body strong decays with the QCD sum rules based on the rigorous current-hadron duality, and obtain satisfactory total width to match to the experimental data. We can explore the two-body strong decays of the pentaquark molecular states based on the rigorous current-hadron duality, and get...
the branching fractions, which can be confronted with the experimental data in the future to assign the pentaquark molecular states in more reasonable foundations.

4 Conclusions

In the present work, we distinguish the isospins of the pentaquark molecular states and construct the color-singlet-color-singlet type five-quark currents with the isospins \((I, I_3) = (\frac{1}{2}, \frac{1}{2})\) and \((\frac{3}{2}, \frac{1}{2})\) unambiguously to explore their properties with the QCD sum rules for the first time. In order to obtain accurate numerical results, we consider the vacuum condensates up to dimension 13 in a consistent way. Based on the extracted molecule masses from the Borel windows, we assign the \(\bar{D}\Sigma_c, \bar{D}\Sigma_c^*, \bar{D}\Sigma_c^*\) pentaquark molecular states with the isospin \(I = \frac{1}{2}\) to be the \(P_c(4312), P_c(4380), P_c(4440)\) and \(P_c(4457)\), respectively, see Table II. Furthermore, the present calculations indicate that there also exist four slightly higher pentaquark molecular states \(\bar{D}\Sigma_c, \bar{D}\Sigma_c^*, \bar{D}\Sigma_c^*\) with the isospin \(I = \frac{3}{2}\), which lie slightly above the thresholds of the corresponding meson-baryon pairs \(\bar{D}\Sigma_c, \bar{D}\Sigma_c^*, \bar{D}\Sigma_c^*\), respectively. We can search for the four resonances in the \(J/\psi\Delta\) invariant mass spectrum, which can lead to additional proofs for the molecule assignments, and shed light on the nature of the \(P_c\) states and dynamics of the low energy QCD.

Data Availability

All data included in this manuscript are available upon request by contacting with the corresponding authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Table 1: The Borel parameters, continuum threshold parameters, energy scales, pole contributions, masses, pole residues and assignments for the eight pentaquark molecular states, where the thresholds denote the corresponding thresholds of the meson-baryon scattering states.
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