FAST INCOMPLETE DECOHERENCE OF NUCLEAR SPINS
IN QUANTUM HALL FERROMAGNET

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Abstract

A scenario of quantum computing process based on the manipulation of a large number of nuclear spins in Quantum Hall (QH) ferromagnet is presented. It is found that vacuum quantum fluctuations in the QH ferromagnetic ground state at filling factor $\nu = 1$, associated with the virtual excitations of spin waves, lead to fast incomplete decoherence of the nuclear spins. A fundamental upper bound on the length of the computer memory is set by this fluctuation effect.

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A growing number of models for quantum information processing (or Quantum Computing-QC) has been recently proposed\textsuperscript{1}, some of which were successfully tested experimentally in devices consisting of a few qubits. The scaling up of these toy devices to the desired large number of quantum gates seems at present a formidable challenge. Of special interest are the models based on the manipulation of nuclear spins in semiconducting
heterostructures. A scenario for the realization of QC over a large number of qubits in a model system similar to that proposed in Ref. [2] may be achieved when nuclear spins in heterojunctions are manipulated via the hyperfine interaction with the electron spins under the conditions of the odd integer Quantum Hall (QH) effect.

The main idea behind this scenario is motivated by the experimental observation in optically pumped NMR measurements on GaAS multiple quantum well structure at low temperatures of a dramatic enhancement of the very small nuclear spin lattice relaxation rate which characterizes the QH ferromagnetic state at Landau level filling factor $\nu = 1$ (i.e. corresponding to $T_1 > 250$ s), and of a sharp decrease of the Knight shift, as the filling factor is shifted slightly away from $\nu = 1$ [4]. The prevailing interpretation of these closely related effects, associates them with the creation of skyrmions (or antiskyrmions) in the electron spin distribution as the 2D electron system moves away from the QH ferromagnetic state at $\nu = 1$ [5].

It should be stressed, however, that the nuclear spin dephasing time $T_2$ in quantum well structures based on GaAS/AlGaAs, is expected to be of the order of milliseconds or shorter, namely much smaller than the shortest value of $T_1$ found in this experiments. This drawback is due to the fact that all elemental components of such structures (i.e. Ga$^{69}$, Ga$^{71}$, As$^{75}$, all with $I = 3/2$, and Al$^{27}$ with $I = 5/2$) have non-zero nuclear spins, which as a result experience large direct dipolar interactions. An alternative quantum well structure based on semiconducting host material consisting predominantly of zero nuclear spin isotopes and small amount of atoms with nonzero nuclear spins (e.g. like Si$_{1-x}$Ge$_x$ heterojunctions), may be fabricated in the future to create a system of virtually noninteracting nuclear spins in a QH ferromagnet.

In such a system the nuclear dephasing time $T_2$ is governed only by the hyperfine interactions with the electron spins. Random impurities, for example, can influence this nuclear spin dephasing only indirectly through the scattering of electrons by the impurity potential, leading, e.g., in typical GaAs quantum well structure to nuclear spin dephasing times of the order of seconds (see below) [6].
One therefore expect that after saturating the nuclear spins and tuning the filling factor at $\nu = 1$ a nearly pure nuclear spin state can be frozen for a relatively long time due to the very long nuclear spin relaxation times $T_1$ and $T_2$. The nuclear spin system can be now manipulated by varying the filling factor away from the initial value and then back to $\nu = 1$ to freeze in a new configuration. During this variation the 2D electron system is in a condensed state of a large number of spin waves (or spin excitons), which are strongly correlated over a large spatial region. It is thus expected that the proposed manipulation of the nuclear spin system can be performed in a phase coherent fashion over a spatial region with size of the order of the skyrmion radius.

In attempting to evaluate the feasibility of this hypothetical scenario several problems of various degrees of complexity arise. Perhaps, the most fundamental one concerns the mechanism in which a single nuclear spin (or qbit) looses phase coherence by the ’manipulating agent’ itself, i.e. the 2D electron system in the heterostructure. In particular, it is not at all obvious that even under the ideal conditions of the QH ferromagnetic state at $\nu = 1$, where the nuclear spin relaxation times, $T_1$ and $T_2$, are extremely long, an ensemble of large number of nuclear spins can preserve phase coherence for a sufficiently long time.

Of special interest here is the effect of vacuum quantum fluctuations in the QH ferromagnetic state on the decoherence of nuclear spins. As will be shown below, the virtual excitations of spin waves (or spin excitons), which have a large energy gap (on the scale of the nuclear Zeeman energy) above the ferromagnetic ground state energy, lead to fast incomplete decoherence in the nuclear spin system.

To study this effect we exploit an ideal model system consisting of independent nuclear spins interacting with a 2D electron gas through the Fermi contact hyperfine interaction. An external stationary magnetic field is applied perpendicular to the 2D layer with strength corresponding to filling factor $\nu = 1$. The temperature is assumed to be smaller than any electronic energy scale in the problem, and the influence of electron scattering by impurities is neglected.

The Hamiltonian of this model system is $\hat{H} = \hat{H}_0 + \hat{H}_{en}$, where
\[ \hat{H}_0 = -\gamma_n \sum_j \hat{I}_j \cdot \mathbf{B}_0 - \gamma_e \int d^2 r \hat{\mathbf{S}}(\mathbf{r}) \cdot \mathbf{B}_0 + \hat{H}_{ee} \]  

(1)

\[ \hat{H}_{en} = A \sum_j \hat{\mathbf{S}}(\mathbf{r}_j) \cdot \hat{\mathbf{I}}_j \]  

(2)

Here \( \hat{\mathbf{I}}_j \) is the nuclear spin operator located at \( \mathbf{r}_j \), \( \hat{\mathbf{S}}(\mathbf{r}) \) is the electronic spin density operator, \( \mathbf{B}_0 \) is the external magnetic field which is assumed to be oriented perpendicular to the 2D EG (\( \mathbf{B}_0 = B_0 \mathbf{z} \)), \( \hat{H}_{ee} \) is the electron-electron interaction, \( \gamma_n \) and \( \gamma_e \) the nuclear and electronic gyromagnetic ratios respectively,

\[ A = C |u(0)|^2 / ll_B^2 \]

with \( C = \frac{8\pi}{3} \gamma_n \gamma_e \) and \( u(0) \) the electron wavefunction at a nucleus. The difference between the Fermi contact hyperfine interaction parameter \( A \) in a quantum well at high magnetic field and the corresponding zero field bulk coupling constant is reflected in the appearance of the two length parameters; \( l \) -the width of the quantum well, and \( l_B = \sqrt{\frac{\hbar e B_0}{m^*}} \) - the magnetic length. The zero field bulk value,

\[ A_{Bulk} = C |u(0)|^2 / \Omega \]

where \( \Omega \) is the volume of a unit cell, is usually much larger than \( A \) given above as long as \( ll_B^2 \gg \Omega \).

The manipulation of the nuclear spins is carried out through spin flip-flop processes, associated with the 'transverse' part of the interaction Hamiltonian \( \hat{H}_{en} \) (Eq.(2)), i.e.

\[
A \sum_j \left[ \hat{I}_{j,+} \hat{\mathbf{S}}_-(\mathbf{r}_j) + \hat{I}_{j,-} \hat{\mathbf{S}}_+(\mathbf{r}_j) \right]
\]

where \( \hat{I}_{j,+} = \hat{I}_{j,x} + i\hat{I}_{j,y}, \hat{I}_{j,-} = \hat{I}_{j,x} - i\hat{I}_{j,y} \) are the transverse components of the nuclear spin operators, and \( \hat{\mathbf{S}}_+(\mathbf{r}) = \hat{\psi}_\uparrow^\dagger(\mathbf{r}) \hat{\psi}_\uparrow(\mathbf{r}) \), \( \hat{\mathbf{S}}_-(\mathbf{r}) = \hat{\psi}_\downarrow^\dagger(\mathbf{r}) \hat{\psi}_\downarrow(\mathbf{r}) \) are the corresponding components of the electron spin density operators. Here \( \hat{\psi}_\sigma(\mathbf{r}) \), \( \hat{\psi}_\sigma^\dagger(\mathbf{r}) \) are the electron field operators with spin projections \( \sigma = \uparrow, \downarrow \).
The 'longitudinal' part of $\hat{H}_{en}$, $A \sum_j \hat{I}_{j,z} \hat{S}_z (r_j)$, which commutes with the Hamiltonian $\hat{H}_0$, and so leaves the nuclear spin projections along $\mathbf{B}_0$ unchanged, can still erodes quantum coherence in the nuclear spin system.\(^2\)

To simplify the analysis we assume that the nuclei under study have spin $1/2$. In this case the transverse components $\hat{I}_{j,+}, \hat{I}_{j,-}$, are up to a proportionality constant, just the off diagonal elements (or coherences) of the density matrix of a single nuclear spin (qbit).\(^3\) The decay of these elements with time, which determines the rate of decoherence of a single qbit, can be thus found from the equations of motion for the operators $\hat{I}_{j,\pm} (t)$ in the Heisenberg representation $\hat{I}_{j,\pm} (t) = e^{i\hat{H}t/\hbar} \hat{I}_{j,\pm} e^{-i\hat{H}t/\hbar}$.

Let us, for the sake of simplicity, consider a single nuclear spin and evaluate its rate of decoherence due to the coupling with a 'bath' of spin excitons. Dropping the site index $j$, the corresponding equations for the coherences $I_+, I_-$, can be written in the form:

$$\frac{\partial}{\partial t} \tilde{I}_+ (t) = -i \alpha \tilde{S}_+ (t) \tilde{I}_z (t) , \quad \frac{\partial}{\partial t} \tilde{I}_- (t) = i \alpha \tilde{S}_- (t) \tilde{I}_z (t)$$

with the supplementary equation for $\tilde{I}_z (t)$

$$\frac{\partial}{\partial t} \tilde{I}_z (t) = \frac{i}{2} \alpha \left[ \tilde{S}_+ (t) \tilde{I}_- (t) - \tilde{S}_- (t) \tilde{I}_+ (t) \right]$$

Here $\alpha \equiv A/\hbar$, and the symbols $\tilde{I}_\pm (t), \tilde{S}_\pm (t)$ stand for the corresponding spin operators in the rotating reference of frame with angular velocity $\omega = \gamma_n B_0 - \alpha S_z \[\alpha\]$, i.e.:

$$\tilde{I}_\pm (t) \equiv e^{\pm i \omega t} \hat{I}_\pm (t) ; \quad \tilde{S}_\pm (t) \equiv e^{\pm i \omega t} \hat{S}_\pm (t)$$

Assuming that initially, at time $t = 0$, the electronic system is in its ground (QH ferromagnetic) state $|0\rangle$, and neglecting the effect of the nuclear spins on the electronic (bath) states, the average of Eq.(3) over the 'bath' states reduces to the expectation value in the ground electronic state $|0\rangle$. Thus, by integrating Eq.(4) over $t$, substituting into Eq.(3), and then averaging over the 'bath' states, one finds to lowest order in the hyperfine interaction parameter $\alpha$:

\[5\]
\[
\frac{\partial}{\partial t} \tilde{I}_+ (t) = -\frac{1}{2} \alpha^2 \int_0^t d\tau e^{i\omega_\tau} \chi_{+-}^\tau (t-\tau)
\]

where \( \chi_{+-} (\tau) \equiv \langle 0 | \widehat{S}_+ (t) \widehat{S}_- (t-\tau) | 0 \rangle \).

Note that since \( \chi_{+-} (\tau) \) varies on the characteristic electronic time scale \( 1/\gamma \), which is much shorter than the nuclear time scale \( \omega^{-1} \), it is allowed to use the expansion \( \tilde{I}_+ (t-\tau) = \tilde{I}_+ (t) - \tau \frac{d}{dt} \tilde{I}_+ (t) + \ldots \) under the integral in Eq.(5) to get:

\[
\frac{\partial}{\partial t} \tilde{I}_+ (t) = -\frac{1}{2} \alpha^2 \int_0^t d\tau e^{i\omega_\tau} \chi_{+-} (\tau) \tilde{I}_+ (t) + O(A^4)
\]

The 'bath' excited states are spin excitons with the well known dispersion relation \( E_{ex} (k) \)[10], which in the long wavelength limit reduces to \( E_{ex} (k) \approx \varepsilon_{sp} + \frac{1}{4} \varepsilon_c l_B^2 k^2 \), with \( \varepsilon_c = \frac{e^2}{\pi \kappa l_B^{3/2}} \) the characteristic Coulomb energy. Here \( \kappa \) is the dielectric constant in the 2D electron gas region. The energy gap is the 'bare' Zeeman energy \( \varepsilon_{sp} = \hbar \gamma_e B_0 \).

A simple calculation yields for the correlation function \( \chi_{+-} (\tau) = \sum_k e^{-\frac{1}{2} \tilde{k}^2} e^{-iE_{ex}(k)\tau/\hbar} \), where \( \tilde{k} = l_B k \). Integrating Eq.(5) over \( t \), and solving for \( \tilde{I}_+ (t) \), the time dependence of the coherence \( I_+ \) is given by

\[
I_+ (t) = I_+ (0) J (t)
\]

where \( J (t) = e^{i\Omega (t) - \Gamma (t)} \), and

\[
\Gamma (t) = \left( \frac{C |u (0)|^2}{4\pi l_B^2} \right)^2 \int_0^\infty \tilde{k} d\tilde{k} e^{-\frac{1}{2} \tilde{k}^2} \frac{1 - \cos \left[ \frac{1}{\hbar} E_{ex} (k) - \omega \right] t}{\left[ E_{ex} (k) - \hbar \omega \right]^2}
\]

This result is similar to the expression found by Palma et al.[12] in an artificial model of pure decoherence, i.e. when energy transfer between the qbit and its environment is not allowed. The remarkable feature of this expression is due to the presence of the energy gap \( \varepsilon_{sp} \) in the spin exciton spectrum, which is typically much larger than the nuclear Zeeman energy \( \hbar \omega \). This guarantees that the denominator in the integrand in Eq.(7) is always larger than or equal to \( \varepsilon_{sp}^2 \), and that for times \( t \gg \hbar / \varepsilon_{sp} \),

\[
\Gamma (t) \rightarrow \tilde{A}^2 \int_0^\infty \tilde{k} d\tilde{k} e^{-\frac{1}{2} \tilde{k}^2} \frac{1}{\left[ \tilde{E}_{ex} (k) \right]^2}
\]
with the dimensionless exciton energy \( \tilde{E}_{\text{ex}}(k) = E_{\text{ex}}(k)/\varepsilon_{sp} \geq 1 \), and hyperfine coupling constant \( \tilde{A} = \frac{C|u(0)|^2}{4\pi\hbar^2\varepsilon_{sp}} \).

Thus, we find that during a short time scale, of the order of \( \hbar/\varepsilon_{sp} \), the coherence \( I_+(t) \) of a single nuclear spin diminishes and then saturates for a very long time (i.e. of the order of the relaxation time \( T_2 \), see below) at \( I_+(0) e^{-\kappa\tilde{A}^2} \), where \( \kappa = \int_0^\infty \frac{k^2 \tilde{E}_{\text{ex}}(k)}{E_{\text{ex}}(k)} \sim \frac{2\varepsilon_{sp}}{\varepsilon_c} \).

For GaAs/Al\(_x\)Ga\(_{1-x}\)As heterostructure the coupling constant \( \tilde{A} \) is typically of the order of \( 10^{-4} \) [15], implying an extremely small deviation, i.e. \( \sim \kappa\tilde{A}^2 \sim 10^{-9} \), from a pure state of a single nuclear spin.

It is interesting to compare this effect to the decoherence caused to a nuclear spin as a result of the scattering of spin excitons by random impurities. This mechanism leads to a complete decoherence within a time scale [1]

\[
T_2 \sim \frac{1}{\tilde{A}^2} \frac{\hbar}{\varepsilon_{sp}}
\]

where \( U_2 \equiv \frac{1}{2\pi^2 l^2 \varepsilon_c} \int d^2 r \langle U_{\text{imp}}(r) U_{\text{imp}}(r') \rangle \) is a dimensionless correlator of the impurity potential \( U_{\text{imp}}(r) \) [14]. In GaAs/Al\(_x\)Ga\(_{1-x}\)As heterostructures \( \hbar/\varepsilon_{sp} \sim 10^{-12} \) sec and \( U_2 \) is typically \( \sim 0.001 \) [17], so that \( T_2 \sim 0.1 \) sec.

Thus, due to the extreme weakness of the hyperfine contact interaction with the 2D electron gas under high magnetic fields the decoherence of a single qbit arising from both impurity scattering and quantum fluctuations in such a system is extremely small. A coherent superposition of a great number of qbits can therefore survive in the computer memory during a very long time period \( t \ll T_2 \). To find an upper bound for the length of such a memory let us consider \( N \) independent nuclear spins located at various positions \( \mathbf{r}_j \) in the quantum well. A number, \( n \), stored in the memory, corresponds to the direct product of \( N \) pure nuclear spin states \( |n\rangle = |n_1\rangle \otimes |n_2\rangle \otimes \ldots \otimes |n_N\rangle \), where \( |n_j\rangle = \sum_{\sigma=\pm 1} \delta_{n_j,\sigma} |j,\sigma\rangle \), \( \delta_{n_j,\sigma} \) is the Kronecker delta, and \( |j,\sigma\rangle \) is a nuclear state with spin projection \( \sigma \) for a nucleus located at \( \mathbf{r}_j \).

To carry out a quantum computing process, however, a coherent superposition of such products, i.e. \( |\psi\rangle = \sum_{n=1}^N \alpha_n |n\rangle \) (see e.g. [3]), should be prepared at time \( t = 0 \). This
superposition may be represented more transparently for our purposes by the direct product of $N$ mixed spin up and spin down states,

$$\left| \psi (t = 0) \right\rangle = \prod_{j=1}^{N} \otimes (u_j | j, \uparrow \rangle + v_j | j, \downarrow \rangle)$$

with the normalization $|u_j|^2 + |v_j|^2 = 1$.

Let us assume that at time $t = -t_0 < 0$ the filling factor was tuned to a fixed value $\nu = \nu_0 \neq 1$ and then kept constant until $t = 0$. If $t_0 \gg T_2 (\nu_0)$ then at $t = 0$ the nuclear spin system is in the ground state corresponding to the 2D electron system at $\nu = \nu_0$. Suppose that at time $t = 0$ the filling factor is quickly switched (i.e. on a time scale much shorter than $T_2 (\nu_0)$) back to $\nu = 1$ so that the nuclear spin system is suddenly trapped in its instantaneous configuration corresponding to $\nu = \nu_0 \neq 1$. Thus the nuclear spins will find themselves for a long time $t \gg T_2 (\nu_0)$ almost frozen in the ground state corresponding to the 2D electron system at $\nu = \nu_0$, since $T_2 (\nu = 1) \gg T_2 (\nu_0)$.

The corresponding state of the nuclear spin system can be found by considering $u_j$ and $v_j$ in Eq.(8) as variational parameters, and then minimizing the energy functional $E = \langle \psi | H | \psi \rangle$,

$$H = -\gamma_n \sum_{j=1}^{N} \hat{I}_j \cdot B_0 + A \sum_{j=1}^{N} \mathbf{S} (r_j) \cdot \hat{I}_j$$

with respect to $u_j, v_j$. Note that the effective nuclear spin Hamiltonian $H$ is obtained after averaging over the electronic ('bath') states, so that $\mathbf{S} (r_j) = \langle \hat{\mathbf{S}} (r_j) \rangle \approx \langle 0 | \hat{\mathbf{S}} (r_j) | 0 \rangle$ is the expectation value of the electronic spin density at the nuclear position $r_j$. As noted above, at $\nu_0 \neq 1$, this density has nonzero transverse components, associated with the skyrmionic spin texture, smoothly varying in space.

A simple calculation shows that $E = \sum_j \varepsilon (u_j, v_j)$,

$$\varepsilon (u_j, v_j) = \frac{1}{2} \omega_j \left( |v_j|^2 - |u_j|^2 \right) + \frac{1}{2} \left[ Av_j u_j^* S_- (r_j) + c.c \right]$$

where $\omega_j = \gamma_n B_0 - A S_z (r_j)$ is the local nuclear Zeeman energy. The extremum conditions (subject to the normalization $|u_j|^2 + |v_j|^2 = 1$) $\frac{\partial E}{\partial u_j} - \epsilon_j u_j = 0$, $\frac{\partial E}{\partial v_j} - \epsilon_j v_j = 0$ are readily solved to yield:
where

\[ \epsilon_j = \frac{1}{2} \sqrt{\omega_j^2 + A^2 S_-(r_j) S_+(r_j)} \]

In this state the nuclear spin polarization \( \langle \psi | \hat{I}_j | \psi \rangle \) follows the underlying electronic spin texture; the transverse component takes the form

\[ \langle \psi | \hat{I}_{j,+} | \psi \rangle = u_j^* v_j = A S_+ (r_j) / 2\epsilon_j \]

whereas the longitudinal component is

\[ \langle \psi | \hat{I}_{j,z} | \psi \rangle = \frac{1}{2} (|u_j|^2 - |v_j|^2) = \pm \omega_j / 2\epsilon_j \]

The topological rigidity of the skyrmionic spin texture thus ensures the rigidity of the coherences \( u_j^* v_j \) over a large spatial region.

The key parameter here is the mixing parameter \( \eta_j \equiv A^2 |S_+(r_j)|^2 / \omega_j^2 \), which becomes significant when the transverse component \( S_+ (r_j) \) does not vanish over a large spatial region, as is the case for skyrmion spin texture. For very small mixing parameter \( \eta_j \) one finds a pure nuclear ferromagnetic state, namely \( |u_j|^2 = 1, |v_j|^2 = 0 \), corresponding to the ground state, or \( |u_j|^2 = 0, |v_j|^2 = 1 \), corresponding to the fully saturated nuclear spin system. In the opposite limit of very strong mixing (\( \eta_j \gg 1 \)) \( |u_j|^2 = |v_j|^2 = \frac{1}{2} \), which is the desired state for quantum computing\[17\], all the numbers \( n \) are stored in the memory with equal probability. It should be noted that, since \( |S_+| \leq 1 \), large values of \( \eta_j \) can be obtained only when the nuclear Zeeman energy \( \omega_j \) is much smaller than the hyperfine coupling constant \( A \).

Now, during the long time following \( t = 0 \), when the electronic system is set at filling factor \( \nu = 1 \) so that its ground state is a uniform quantum ferromagnet and the elementary spin excitations are the spin waves discussed above, the nuclear state \( |\psi (t)\rangle \) evolving from \( |\psi (0)\rangle \) after time \( t \) can be readily calculated in terms of the operators \( \hat{I}_{j,\pm} (t), \hat{I}_{j,z} (t) \). Using
the solutions $\hat{I}_{j,+}(t) = J(t)\hat{I}_{j,+}(0), \hat{I}_{j,z}(t) = I(t)\hat{I}_{j,z}(0)$, derived above (Eq.(7)), it is easy to show that the probability that after time $t$ the memory remains in the coherent state $\psi(0)$:

$$P_\psi = |\langle \psi(0) | \psi(t) \rangle|^2$$

$$= \frac{1}{2^N} \prod_{j=1}^{N} \{ [1 + I(t)] + 4 [I(t) - \text{Re} J(t)] \left[ |u_j|^4 - |u_j|^2 \right] \} \tag{10}$$

Let us further assume that the mixing parameter $\eta_j$ are large so that in the initial state $\psi(0)$, $|u_j|^2 = 1/2$ as required. Under these circumstances we easily find that:

$$P_\psi(t) = \left[ \frac{1 + \text{Re} J(t)}{2} \right]^N \approx \exp \left\{ -\frac{1}{2} N [1 - \text{Re} J(t)] \right\} \approx e^{-\frac{1}{2} N \Gamma(t)}$$

It is evident that due to the even distribution of nuclear spins in the initial state $\psi(0)$ the survival probability $P_\psi(t)$ depends only on the decoherence factor $J(t)$. The decay of $P_\psi(t)$ therefore follows $e^{-\frac{1}{2} N \Gamma(t)}$, saturating at $e^{-\frac{1}{2} N \kappa}$ for $t \gg \hbar/\varepsilon_{sp}$. Note that, despite the much larger drop in the level of coherence in the many qubit system, the time scale over which the coherence diminishes is the same as for a single qubit.

An upper bound on the length of possible memories in future quantum computers based on the proposed model can be now estimated by the requirement $P_\psi(t \gg \hbar/\varepsilon_{sp}) \sim 1/e$, which yields

$$N_{\text{max}} \sim 2/\tilde{A}^2$$

The restriction imposed by this condition is inherent to the manipulation mechanism of qubits via the hyperfine interaction with the electron spins, and so can not be removed or even relaxed by any technical improvement.

In conclusion we have found that a system of many nuclear spins, coupled to the electronic spins in the 2D electron gas through the Fermi contact hyperfine interaction, partially looses its phase coherence during the short (electronic) time $\hbar/\varepsilon_{sp}$, even under the ideal conditions of the QHE, where both $T_1$ and $T_2$ are infinitely long. The effect arises as a result of vacuum
quantum fluctuations associated with virtual excitations of spin waves (or spin excitons) by the nuclear spins. The incompleteness of the resulting decoherence is due to the large energy gap (on the scale of the nuclear Zeeman energy) of these excitations whereas the extreme weakness of the hyperfine interaction with the 2D electron gas under high magnetic fields guarantees that the loss of coherence of a single nuclear spin is extremely small. The memory of a quantum computer to be constructed in such a system is therefore limited in principle to lengths of the order of $N_{\text{max}}$, which is found to be about $10^9$ for GaAS multiple quantum well structure.

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