Current states in superconducting films: numerical results and approximations

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We present numerical solution of equations by Aslamazov and Lempitskiy (AL) [1] for the distribution of the transport current density in thin superconducting films in the absence of external magnetic field, in both the Meissner and the vortex states. This solution describes smooth transition between the regimes of a wide film and a narrow channel and enables us to find the critical currents and current-voltage characteristics within a wide range of the film width and temperatures. We propose simple approximating formulas for the current density distributions and critical currents.

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I. INTRODUCTION

The main property of the current states in wide superconducting films, which distinguishes them from narrow channels, is an inhomogeneous distribution of the current density $j$ due to the Meissner screening of the current-induced magnetic field. It should be noted that the current state of a wide thin film is qualitatively different from that in a bulk superconductor. Whereas the transport current $I$ in the latter case flows only within a thin surface layer with the thickness of the order of the London penetration depth $\lambda$, the current in a thin film with the thickness $d \ll \lambda$ is distributed over its width $w$ according to the approximate power-law expression $j \sim [(w/2)^2 - x^2]^{-1/2}$ [2], where $x$ is the transversal coordinate with the origin in the middle of the film. Thus, the characteristic length $\lambda_\perp(T) = 2\lambda^2(T)/d$, which is commonly referred to as the penetration depth of the perpendicular magnetic field, has nothing to do with the scale of the current decay, but rather plays the role of a “cutoff factor” in the above-mentioned law of the current distribution at the distances $\lambda_\perp$ from the film edges and thereby determines the magnitude of the edge current density. The latter was estimated in [2] as $j_e \approx I/\sqrt{\pi w \lambda_\perp}$, assuming $w$ to be much larger than $\lambda_\perp(T)$ and the coherence length $\xi(T)$.

In such an inhomogeneous situation, the resistive transition of a wide film occurs [1–4] when $j_e$ reaches the value close to the critical current density $j_c^{GL}$ in the Ginzburg-Landau (GL) theory. Using this estimate, the expression $I_c \approx j_c^{GL} \sqrt{\pi w \lambda_\perp}$ for the critical current has been obtained in [2]. This equation is widely used in analysis of experimental data (see, e.g., [3, 4]) and imposes a linear temperature dependence of the critical current $I_c(T) \propto 1 - T/T_c$ near the critical temperature $T_c$. A quantitative theory by Aslamazov and Lempitskiy (AL) [1] also predicts the linear dependence $I_c(T)$ but gives its magnitude numerically larger than the above estimate. This result has been confirmed in recent experiments [7, 8].

The instability of the current state at $I = I_c$ results in the entry of vortices whose motion leads to the appearance of the electric field, i.e., to formation of the vortex part of the I-V characteristic (IVC). While the current further increases, the motion and annihilation of the vortices form a peak in the current density at the center of the film. For certain current value $I_m$, the magnitude of this peak reaches $j_c^{GL}$, which results in instability of the stationary flow of the vortices [1]. Further behavior of the film depends on the conditions of the heat removal [10] and the quality of the films. In experiments performed decades ago, an abrupt transition to the normal state has been usually observed at $I = I_m$, whereas in later researches, in which optimal heat compliance was provided, a step-like structure of the IVC is observed at $I > I_m$ (see, e.g., [6, 11, 14]). This indicates the appearance of phase-slip lines, similar to the phase-slip centers in narrow channels.

Since $\lambda_\perp(T)$ unlimitedly grows as $T \to T_c$, any film reveals the features of a narrow channel in the immediate vicinity of $T_c$: at $\lambda_\perp \gg w$, its critical current is due to the uniform pair-breaking (narrow channel regime) thus showing the temperature dependence of the GL pair-breaking current $j_c^{GL}(T) \propto (1 - T/T_c)^{3/2}$. As the temperature decreases, the film exhibits a crossover to an inhomogeneous current state, in which vortex nucleation is responsible for the resistive transition (wide film regime). In the experiment [8, 9], the linear temperature dependence of $I_c$ predicted in [1, 2] for wide films becomes pronounced only at low enough temperatures, when $\lambda_\perp(T)$ becomes smaller than the film width by the factor of 20 – 30, although the vortex state already occurs at much larger values of $\lambda_\perp \approx w/4$. Similar difficulties in the fitting of the IVC measurements with the asymptotical results of the AL theory were met in the experiment [15], because the condition $I_c \ll I_{m}$, used in [11], can be fulfilled only in extremely wide films whose width exceeds $\lambda_\perp(T)$ by several orders of magnitude. Thus, there exist a considerable intermediate region of the film widths and temperatures, where the asymptotic results of the AL theory cannot give a satisfactory description of the experimental data, although the assumptions and initial equations of this theory remain valid in this region.

In order to fill up this gap, we perform in this paper a numerical solution of the AL equations within a wide region of the ratio $w/\lambda_\perp$. The results of our computations describe smooth transition between the regimes of a wide film and a narrow channel and demonstrate evolution of the current density distribution with the increase of the transport current in both the Meissner and the vortex states. We notice that within the AL theory, the critical currents $I_c$ and $I_m$, being normalized on the GL critical current $j_c^{GL}$, as well as the specifically normalized IVC, are universal functions of the ratio $w/\lambda_\perp(T)$. We calculate the fitting constants in the asymptotic formulas of the AL
theory and propose approximating expressions for the current density distributions, which are in rather good agreement with the results of numerical computations.

II. BASIC EQUATIONS AND RESULTS OF THE AL THEORY

A starting point of the AL theory are the static GL equations for the dimensionless modulus \( F \) of the order parameter (normalized on its equilibrium value in the GL theory) and the gauge-invariant vector potential \( \mathbf{Q} = -\mathbf{\kappa} \nabla \chi \),

\[
\mathbf{\kappa}^{-2} \mathbf{\nabla}^2 \mathbf{F} + F (1 - F^2 - \mathbf{Q}^2) = 0, \quad (1)
\]

\[
\nabla \times \mathbf{Q} = -F^2 \mathbf{Q} \delta(z). \quad (2)
\]

Here \( \mathbf{A} \) is measured in units of \( \Phi_0/2\pi \xi \), \( \mathbf{\Phi}_0 \) is the magnetic flux quantum, \( \chi \) is the order parameter phase and \( \mathbf{\kappa} \equiv \lambda_\perp/\xi \) is the effective GL parameter. The axis \( z \) is directed perpendicular to the film whose thickness is assumed to be infinitely small, and all distances are measured in units of \( \lambda_\perp \).

Usually in thin films, the GL parameter is large, \( \mathbf{\kappa} \gg 1 \). Assuming the film width much larger than \( \xi(T) \), one thus can neglect the gradient term in Eq. (1) and use the local relation \( F^2 = 1 - Q^2 \) between the order parameter and the vector potential. Inside the thin film, the latter has only one component \( Q = Q_x \), and can be found from equation

\[
dQ_x = -\frac{1}{2\pi} \int_{-\bar{w}/2}^{\bar{w}/2} \frac{Q(x') [1 - Q^2(x')]}{x' - x} dx', \quad \bar{w} = w/\lambda_\perp, \quad (3)
\]

with the Biot-Savard integral which relates the magnetic field \( dQ/dx \) to the dimensionless density \( j = Q(1 - Q^2) \) of the surface current. Equations (1)–(3) determine the stability threshold of the Meissner state, when the vortices begin to penetrate into the film and the edge value of the vector potential appears to be close to its critical value \( Q^{\text{GL}} = 1/\sqrt{3} \) in the GL theory for narrow channels; this fact was used in our calculations [16]. The asymptotic value of the critical current at \( w \gg \lambda_\perp \) was calculated in [1] and then refined in [17]:

\[
I_c^{\text{GL}} = \sqrt{15/8} I_c^{\text{GL}} (\pi \lambda_\perp/w)^{1/2}. \quad (4)
\]

According to [1, 2, 18], the resistive vortex state of a wide film can be described within a hydrodynamic approximation for the viscous motion of the vortex fluid, by including the contribution of the vortices \( n \mathbf{\Phi}_0 \) [\( n(x) \) is the vortex density] to the net magnetic field induction. Then, using the continuity equation for the flux density \( m \) of the vortex fluid, expressing the vortex velocity \( v \) through the linear current density \( j \) and the viscosity coefficient \( \eta \) as \[19\]

\[
v = -\eta^{-1} \mathbf{\Phi}_0 j \mathbf{\ell} \mathbf{\ell}, \quad (5)
\]

and the average electric field – through the flux density as \( E = -mv \mathbf{\Phi}_0 \), the authors of [1, 2, 18] arrive at the equation

\[
4\pi \lambda_\perp \frac{d j}{w} + 2V_p \int_{-1}^{1} j(x') dx' = -\frac{\eta c^3 E}{\mathbf{\Phi}_0 j(x')} \mathbf{\ell} \mathbf{\ell}, \quad (6)
\]

(a mismatch of the coefficient in the first term with that in [1, 2] is due to the difference in the definition of \( \lambda_\perp \)). Here and below, the coordinate \( x \) is normalized on the film half-width \( w/2 \), and the expression sign \( x \) indicates the opposite direction of the vortex motion in different halves of the film.

An asymptotic analysis of Eq. (6) at \( w \gg \lambda_\perp \) shows [1] that the IVC is linear in the vicinity of \( I_c \), whereas at large currents, the voltage grows quadratically,

\[
V = E_0 L \begin{cases} (I - I_c)/I_c, & I - I_c \ll I_c; \\ C(I/I_c)^2, & I \gg I_c, \end{cases} \quad E_0 = \frac{8\Phi_0 I_c^2}{\eta w^2 c^3}. \quad (7)
\]

The current distribution in the middle of the film has the peak of the order of \( \ln^{1/2}(w/\lambda_\perp) \) which leads to the following estimate of the maximum current of existence of the vortex state,

\[
I_m = C' I_c^{\text{GL}} \ln^{1/2}(w/\lambda_\perp). \quad (8)
\]

In Eqs. (7) and (8), \( L \) is the film length, \( I_c^{\text{GL}} \) is the GL critical current formally calculated for the uniform current distribution and \( C, C' \) are fitting constants which cannot be determined within the framework of the asymptotic approach.

III. RESULTS OF NUMERICAL CALCULATIONS

In our calculations, we perform numerical solution of Eq. (6) with a certain modification. As is obvious, the left-hand side of Eq. (6) is the approximate form of Eq. (4), in which the vector potential \( Q \) in the gradient term is replaced by the current density \( j \). This corresponds to the linear London relation \( j \sim Q \) between the current and the vector potential which assumes independence of the order parameter of the vector potential. For this reason, Eq. (6) is usually referred to as a generalized London equation [1, 2]. This does not essentially affect the asymptotic results of [1] because the gradient term is small at \( w \gg \lambda_\perp \); however, in our numerical calculations, we will use the full “nonlinearized” version of Eq. (6) in a dimensionless form (see also [17]):

\[
\alpha \frac{dQ}{dx} + \frac{V_p}{4} \int_{-1}^{1} \frac{i(x') dx'}{x' - x} = -\frac{E'}{i(x)} \quad \text{sign} x, \quad (9)
\]

where the following definitions are introduced,

\[
j(x) = \frac{3\sqrt{3} I_c^{\text{GL}}}{2} i(x), \quad i = Q(1 - Q^2), \quad E = E' E_0, \quad (10)
\]

\[
E_0 = \frac{54\Phi_0}{\eta w^2 c^3} (I_c^{\text{GL}})^2, \quad \alpha = \frac{\pi \lambda_\perp}{2w}. \quad (11)
\]

The distribution of the vector potential is obviously symmetric with respect to the middle axis of the film, \( Q(x) = Q(-x) \), which enables us to consider Eq. (9) only in the region \( x > 0 \) and to reduce the integral in Eq. (9) to the region of positive \( x' \). After integration of the obtained equation from the film edge to a given point \( x \), we finally get

\[
\alpha [Q(x) - Q_x] = \frac{1}{4} \int_{0}^{1} i(x') \ln \left| \frac{x^2 - x'^2}{1 - x'^2} \right| dx' - E' \int_{1}^{x} \frac{dx'}{i(x')}. \quad (12)
\]
where \( Q_e \equiv Q(1) \) is the edge value of the vector potential. In these notations, the net current \( I \) is given by equation

\[
I = w \int_0^1 j(x) \, dx = \frac{3 \sqrt{3}}{2} I_{cL} \int_0^1 i(x) \, dx. \tag{13}
\]

At \( I < I_c \), the quantity \( Q_c \) increases with the current and has to be determined self-consistently from Eqs. \((12)\) and \((13)\) at zero electric field; this procedure simultaneously gives the solution for the current distribution across the film. As noted above, the resistive state of a wide film at \( I = I_c \) occurs when \( Q_c \) approaches the critical value \( Q_c = 1/\sqrt{3} \). Such a relation is also obviously valid for narrow channels, that makes it reasonable to extend it over the films of arbitrary width. In the resistive vortex state, \( I > I_c \), the quantity \( Q_c \) holds its critical value \( Q_c \), and Eqs. \((12)\) and \((13)\) determine the dependence \( E'(I) \), i.e., the IVC, \( V(I) = E'(I) E_0 L \).

A specific property of these equations is that their solutions, i.e., the normalized current density distribution \( i(x) \) and the electric field \( E' \), are universal functions of the parameters \( w/\lambda_\perp \) and \( I/I_{cL} \). This implies that the normalized critical current \( I_c/I_{cL} \) and the maximum current of existence of the vortex state, \( I_{mL}/I_{cL} \), as well as the normalized maximum electric field \( E_{cL}' = E(I_{mL})/E_0 \), are universal functions of the parameter \( w/\lambda_\perp \). Thus, the temperature dependencies of these quantities, being expressed through the variable \( w/\lambda_\perp(T) \), must coincide for the films with different widths and thicknesses, which has been demonstrated in experiments \( \text{[8,9]} \).

### A. Solution in subcritical regime \( I \leq I_c \)

Solution of Eqs. \((12)\) and \((13)\) can be found by an iteration method, using \( Q_e \) as the initial approximation for the function \( Q(x) \). Although the iteration parameter \( \alpha^{-1} \sim w/\lambda_\perp \) is large for a wide film, the convergence in this case can be nevertheless provided by introducing certain weight factors for contributions of previous and current iterations. The result of numerical calculation of the reduced critical current, shown in Fig. 1 describes transition from the uniformly distributed GL depairing current \( I_{cL}' \approx (T_c - T)^{3/2} \) in a narrow channel to the critical current \( I_{cL} \approx T_c - T \) for a wide film \( \text{[4]} \).

As seen from Fig. 1 the asymptotic dependence \( a(T) \) can be achieved with appropriate accuracy only at rather large ratio \( w/\lambda_\perp > 20 \sim 30 \).

It should be noted that in some experiments \( \text{[7,9,20]} \), the behavior of \( I_c(T) \) at the beginning of transition to the wide film regime was found to be different from the smooth dependence following from the AL theory. Namely, when the temperature decreases and the ratio \( w/\lambda_\perp \) exceeds \( 4 \div 5 \), the critical current sharply falls to the value \( I_c(T) \approx 0.8 I_{cL}'(T) \) and holds this level until \( w/\lambda_\perp \lesssim 10 \). Within this temperature interval, the film enters the vortex state at \( I > I_c \), although the temperature dependence of \( I_c \) is similar to the case of a vortex-free narrow channel. An analogous behavior of the critical current in wide films has been registered in early experiments \( \text{[21,22]} \). To explain such a specific dependence of \( I_c(T) \), it was supposed in \( \text{[9]} \) that the Pearl’s vortices in moderately wide films may overcome the edge barrier at the edge current density \( \sim (1 - T/T_c)^2 \) much smaller than the GL critical current density \( \sim (1 - T/T_c)^{3/2} \).

In Fig. 2(a) we present the results of numerical calculations of the current density distribution across the film at the resistive transition point \( I = I_c \) and different ratios \( w/\lambda_\perp \). Similar results were obtained in \( \text{[23]} \) by using London’s equation for the superconducting current, i.e., neglecting effect of the current on the order parameter, which is equivalent to usage of Eq. \((6)\). As shown in \( \text{[23]} \), a comparison of the calculated and experimentally measured distributions of the supercurrent density may be used for determination of the penetration depth \( \lambda_\perp \). Interestingly, these distributions are well approximated at arbitrary currents by the function

\[
j_1(x) = J_e \frac{a}{\sqrt{1 - (a^2 x^2)}}. \tag{14}
\]
Equation (14) represents a modification of the asymptotic function $j(x) = j(0)(1 - x^2)^{-1/2}$ in [1,2,18] with a regularization parameter $a = j_1(0)/j_e$ which provides finiteness of the approximated current density (14) at the film edges. As follows from its definition, this parameter characterizes suppression of the current in the middle of the film due to the Meissner screening. Substituting Eq. (14) with $j_e = J_e^{GL} = F_e^{GL}/w$ into Eq. (13), we obtain equation for its value $a_c = \cos \phi$ at the critical current,

$$I_c / I_e^{GL} = \phi / \tan \phi. \quad (15)$$

In the case of a wide film, $w \gg \lambda_\perp$, the coefficient $a_c$ is small, $a_c \ll 1$, and it can be estimated by using the asymptotic value of the critical current as

$$a_c = 2.74 (\lambda_\perp / \pi w)^{1/2}. \quad (16)$$

Within a framework of the generalized London’s equation [9], the coefficient $a$ is independent of the current and holds a constant value $a_c$, because in this approximation, the current distribution is determined only by geometric factors and holds its shape at arbitrary currents $I < I_c$. The edge current density in this case varies linearly with the transport current, that reproduces the result of [2],

$$j_e = (I / I_c) J_e^{GL} \quad (17)$$

[see dashed lines in Fig. 2(b)]. Numerical calculations by means of Eq. (9) demonstrate rather weak dependence $a(I)$ and nonlinearity of $j_e(I)$ shown in Fig. 2(b) by solid lines. In particular, the coefficient $a$ increases with the current and approaches a maximum value $a_c$ at $I = I_c$. Physically, this is due to suppression of the order parameter by the transport current, that weakens the screening effect while the current increases.

**B. Solution in the vortex state, $I_c < I < I_m$**

In the region of vortex resistivity, the distribution of the screening current with sharp maxima of the height $j_e^{GL}$ at the film edges is superimposed by the distribution associated with vortex motion and having a peak at the middle of the film.

![Figure 3](image3.png)

**FIG. 3:** Solid lines - distributions of the net current density (a) and the vortex contribution (b) in the resistive vortex state at $w/\lambda_\perp = 20$ numerically calculated at different values of the transport current: $1 - I = I_c$, $2 - I = 0.5(I_c + I_m)$, $3 - I = I_m$. The approximating distributions (18) are shown by dashed lines.

as shown in Fig. 3. The logarithmic feature $j_2 \sim (I / \lambda_\perp)^{1/2}$ of this peak, predicted in [1] is rather weak and remains visible only for a certain intermediate current value; at $I \to I_m$, this feature practically vanishes. In the experiment [27], an inhomogeneous current distribution with three peaks in the vortex state of wide films has been visualized by using the laser scanning microscope.

For moderately wide films, in which the above-mentioned logarithmic factor is of the order of unity, the vortex contribution shown in Fig. 3(b) can be approximated by a piecewise-linear function

$$j_2(x) = J_e^{GL} b(1 - |x|) \quad (18)$$

depicted in Fig. 3(b) by dashed lines. As follows from Eq. (18), the parameter $b = J_2(0)/J_e^{GL}$ represents the relative (in units of $J_e^{GL}$) current density created by vortices in the middle of the film. Within such an approximation, this parameter, similar to the edge current density (17) at $I < I_c$, linearly depends on the transport current,

$$b(I) = 2(I - I_c)/I_e^{GL}. \quad (19)$$

According to [1], the vortex state becomes unstable when the height of the central peak of the current distribution approaches the GL depairing current density. Using this condition and solving Eqs. (2) and (12) at the critical edge value of the vector potential, $Q_e = Q_c$, we determine the maximum current of existence of the vortex state $I_m$ and the normalized maximum electric field $E_m = E'(I_m)$. The results of numerical calculation, together with the asymptotical results of the AL theory, are presented in Fig. 4 in which the difference between the definitions of the quantity $E_0$ introduced in our paper and in [1], following from the difference between Eqs. (6) and (9), has been taken into account.
FIG. 5: Numerically calculated IVC of the superconducting film in the vortex state at $w/\lambda_\perp = 20$ (solid line 1). Dashed straight line 2 is the linear AL asymptotics for $I - I_c \ll I_c$ [22], and the parabola 3 is the shifted AL asymptotics for $I \gg I_c$ [23].

At large enough values of $w/\lambda_\perp \gtrsim 20 - 30$, the asymptotic dependencies [1] shown in Fig. 4 by dashed lines,

$$I_m/I_c^{GL} = C_1 \ln^{-1/2}(C_2 w/\lambda_\perp), \quad E_m' = C_3 \left(I_m/I_c^{GL}\right)^2,$$  

(20)

can be fitted to the numerical results by an appropriate choice of the fitting constants of the AL theory (shown in the caption of Fig. 4) which cannot be evaluated within the framework of the asymptotic analysis. Note that in order to obtain a satisfactory agreement, one has to introduce an additional constant $C_2$ into the argument of the logarithm, since the formulas (20) were derived in [1] within a logarithmic accuracy. At smaller $w/\lambda_\perp \lesssim 20$, the asymptotic results (20) considerably overestimate the values of $I_m$ and $E_m'$.

Another useful expression for $I_m$ suitable for a rather wide range of film widths can be obtained from the approximating current distributions (14) and (15). At the stability threshold of the vortex state, where $j(0) = j_1(0) + j_2(0) = j_c^{GL}$, the relation $b = 1 - a_c$ is fulfilled which leads to equation

$$I_m = I_c + 0.5I_c^{GL}(1 - a_c).$$

(21)

As seen from Fig. 4 this approximation (dotted line) rather well reproduces the result of numerical calculations of $I_m$, up to the point of nucleation of the vortex resistivity at $w/\lambda_\perp \approx 4$. Of course, for extremely wide films, in which $I_m \gg I_c$ and the logarithmic peak of the current is well pronounced, the AL asymptotic expression (20) for $I_m$ with numerically calculated fitting constants is more preferable.

In Fig. 5 the calculated normalized IVC per unit length of a wide film ($w/\lambda_\perp = 20$) is shown by the curve 1 in the region of existence of the stable vortex state $I_c < I < I_m \approx 1.7I_c$. Its initial part coincides with the linear AL asymptotics (line 2) at $I - I_c \ll I_c$, which is described by the formula

$$E'(I) = \frac{4}{27} \left(\frac{I_c}{f_c^{GL}}\right)^2 \left(\frac{I}{I_c} - 1\right) \approx 0.873 \frac{\lambda_\perp}{w} \left(\frac{I}{I_c} - 1\right),$$

(22)

obtained by using approximate Eq. (4) for $I_c/f_c^{GL}$. At $I > 1.4I_c$, the calculated characteristic is well described by the modified AL asymptotics for $I \gg I_c$:

$$E'(I) = C_1 \frac{4}{27} \left(\frac{I_c}{f_c^{GL}}\right)^2 \left(\frac{I}{I_c} - C_2\right)^2 \approx 0.873C_1 \frac{\lambda_\perp}{w} \left(\frac{I}{I_c} - C_2\right)^2,$$

(23)

with the fitting constants $C_1 = 0.97$ and $C_2 = 0.7$. Introduction of an additional constant $C_2$, which shifts the original AL parabola, enables us to extrapolate the result obtained in [1] for the case of large supercriticality, $I > I_c$, to the region of currents comparable with $I_c$. Such a modification of the AL asymptotic formulas has been successfully used for fitting of the parabolic part of the IVC in [15]. In experiments with relatively narrow films (in which the vortex state nevertheless exists), only a linear part of the IVC is observed, because the region of the vortex resistivity is rather narrow in this case.

In conclusion to this Section, we note that a numerical simulation of the vortex motion in an infinitely long and thick superconducting slab [24] gives similar results for the current distribution and the IVCs, although these results cannot be directly applied to the thin film because of essential difference between strongly localized Abrikosov vortices in a bulk slab and Pearl vortices in a thin film which interact mostly via the fields in the surrounding space [25 26].

IV. SUMMARY

We studied the distributions of the transport current in thin superconducting films in zero external magnetic field within a wide range of the film widths $w$ and temperatures, using numerical solutions of the integro-differential equations for the gauge-invariant vector potential. These distributions can be effectively approximated by rather simple analytical formulas, the parameters of which have a clear physical sense and can be relatively easily calculated.

We calculated universal dependencies of the critical current $I_c$ and the maximum current of existence of the vortex state $I_m$ (normalized on the Ginzburg-Landau critical current in a uniform current state), as well as the dependencies of the reduced maximum electric field in the vortex state on the parameter $w/\lambda_\perp$. For wide enough films, $w/\lambda_\perp \gtrsim 20 - 30$, our numerical results coincide with the asymptotical dependencies found in [1 7]. We study numerically the current-voltage characteristic of a wide film in the vortex state and propose a modification of the asymptotical results of [1] which provides much better fitting with the experimental data. The pinning of the vortices can be also taken into account [1] by a certain modification of Eq. (5) and will be considered elsewhere.

In conclusion, we note that the validity of our results is confined by the boundaries of applicability of the static Ginzburg-Landau equations to the solution of the problem under consideration. Within this approach, the order parameter relaxation time $\tau_A$ is assumed to be much smaller than other characteristic times of the system. In a general case, the finiteness of $\tau_A$ results in deformation of the vortex core and in occurrence of a wake with the suppressed order parameter behind the moving vortex. As shown by numerical simulation [24], this may...
anomalously enhance the vortex velocity and lead to creation of fast moving chains of vortices treated in [24] as the nuclei of the phase-slip lines. The instability of the vortex motion due to the nonequilibrium state of quasiparticles in the vortex core [28] is also neglected, although it can be taken into account phenomenologically by introducing the dependence of the viscosity coefficient $\eta$ in (5) on the vortex velocity. Finally, the AL model assumes rather weak pinning of the vortices, the penetration of which into the film is followed by their continuous viscous motion leading to the current dissipation. The opposite case of strong pinning corresponds to the model of the critical state with unmovable vortices, which results in quite different distributions of the current and magnetic field (see, e.g., [29]).

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