HYPERACCRETING DISKS AROUND MAGNETARS FOR GAMMA-RAY BURSTS: EFFECTS OF STRONG MAGNETIC FIELDS

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ABSTRACT

Hyperaccreting neutron stars or magnetar disks cooled via neutrino emission can be candidates of gamma-ray burst (GRB) central engines. The strong field $\gtrsim 10^{15}$–$10^{16}$ G of a magnetar can play a significant role in affecting the disk properties and even lead to the funnel accretion process. In this paper, we investigate the effects of strong fields on the disks around magnetars, and discuss implications of such accreting magnetar systems for GRBs and GRB-like events. We discuss quantum effects of the strong fields on the disk thermodynamics and microphysics due to modifications of the electron distribution and energy in the strong field environment, and use the magnetohydrodynamical conservation equations to describe the behavior of the disk flow coupled with a large-scale field, which is generated by the star–disk interaction. If the disk field is open, the disk properties mainly depend on the ratio between $|B_\phi/B_z|$ and $\Omega/\Omega_K$ with $B_\phi$ and $B_z$ being the azimuthal and vertical components of the disk field, and $\Omega$ and $\Omega_K$ being the accretion flow angular velocity and Keplerian velocity, respectively. On the other hand, the disk properties also depend on the magnetar spin period if the disk field is closed. In general, stronger fields give higher disk densities, pressures, temperatures, and neutrino luminosity. Moreover, strong fields will change the electron fraction and degeneracy state significantly. A magnetized disk is always viscously stable outside the Alfvén radius, but will be thermally unstable near the Alfvén radius where the magnetic field plays a more important role in transferring the angular momentum and heating the disk than the viscous stress. The funnel accretion process will be important only for an extremely strong field, which creates a magnetosphere inside the Alfvén radius and truncates the plane disk. Because of higher temperature and more concentrated neutrino emission of a ring-like belt region on the magnetar surface covered by funnel accretion, the neutrino annihilation rate from the accreting magnetar can be much higher than that from an accreting neutron star without fields. Furthermore, the neutrino annihilation mechanism, which releases the gravitational energy of the surrounding disk, and the magnetically driven pulsar wind, which extracts the stellar rotational energy from the magnetar surface, can work together to generate and feed an ultrarelativistic jet along the stellar magnetic poles.

Key words: accretion, accretion disks – gamma-ray burst: general – magnetic fields – neutrinos – stars: neutron

1. INTRODUCTION

Hyperaccreting black hole systems formed in massive star collapses or compact object binary mergers have been considered as candidates for gamma-ray burst (GRB) central engines for two decades (e.g., Eichler et al. 1989; Narayan et al. 1992; Woosley 1993). The accreting systems can drive ultra or mildly relativistic jets with huge energy via neutrino ($\nu\bar{\nu}$) annihilation process (Popham et al. 1999) or magnetohydrodynamical (MHD) mechanism, such as the Blandford–Znajek mechanism (Blandford & Znajek 1977) or the magnetorotational instability (MRI; Balbus & Hawley 1991), and finally produce GRB explosions. The neutrino annihilation efficiency above hyperaccreting disks around black holes with elaborate considerations of disk geometry, rotation, and general relativity effects has been studied by many authors (e.g., Ruffert et al. 1997; Ruffert & Janka 1998; Popham et al. 1999; Asano & Fukuyama 2000, 2001; Miller et al. 2003; Birkl et al. 2007). Although the annihilation process can provide sufficient total energy up to $10^{40}$ erg s$^{-1}$ above neutrino-dominated flows with accretion rate $\sim 1 M_\odot$ s$^{-1}$ (Gu et al. 2006; Liu et al. 2007), it is still under debate whether such a process can successfully generate a relativistic jet from the polar region of the central black hole. For example, MacFadyen et al. (2001) discussed that the central black hole formed in the “collapsar scenario” is more frequently formed by a fallback process after a mild explosion (Type II collapsar) rather than formed promptly (Type I collapsar). The Type II collapsar can establish an accretion disk with accretion rate $\sim 0.001$–$0.01 M_\odot$ s$^{-1}$, which is not sufficient to produce a jet by neutrino annihilation. Numerical simulations showed that the MHD mechanism may be more efficient in driving an energetic magnetically dominated wind and generating a GRB explosion (Tchekhovskoy et al. 2008, 2009; Nagataki 2009), or the annihilation process and MHD winds can work together to provide the energy for GRB explosions (Harikae et al. 2009).

On the other hand, both observational and theoretical evidence show that newborn neutron stars or magnetars, rather than black holes, can form in the GRB central engines (e.g., Dai & Lu 1998a, 1998b; Zhang & Mészáros 2001; Dai 2004; Dai et al. 2006; Mazzali et al. 2006; Soderberg et al. 2006; Shibata & Taniguchi 2006; Lee & Ramirez-Ruiz 2007). Highly magnetized neutron stars formed by accretion-induced collapse, collapse, $\alpha - \Omega$ dynamo mechanism, or differential rotation can extract their rotational kinetic energy up to $10^{31}$–$10^{32}$ erg by spinning down via magnetic activities (e.g., Uskov 1992; Duncan & Thompson 1992; Kluzniak & Ruderman 1998). A thermally driven neutrino wind is dominated during the Kelvin–Helmholtz cooling epoch after the neutron star formation, lasting from a few seconds to tens of seconds based on the strengths of surface fields. After that, magnetically dominated or Poynting-dominated wind becomes significant along the polar region (Wheeler et al. 2000; Thompson 2003; Thompson et al. 2004; Metzger et al. 2007). The jets from magnetars may be accelerated to higher Lorentz factor $\sim 100$–$10^3$ at large radius several
tens of seconds after core bounce, and bring its magnetic energy to kinetic energy, which is finally dissipated in regular GRB internal shocks (Thompson et al. 2004; Metzger et al. 2007; Bucciantini et al. 2007, 2009; Lyubarsky 2009a, 2009b).

However, all the magnetized neutron star or magnetar models ignore the accretion process that occurs onto a protoneutron star for the first several seconds (Woosley & Bloom 2006). In the scenario of massive star collapse, the outgoing shock generated by a successful Type Ibc supernova explosion with a velocity of \( \sim 10^8 \text{ cm s}^{-1} \) can evacuate a cavity around the new born compact star in the center of a supernova remnant. However, whether the outgoing shock in the core collapse of a massive star can dominate the accretion process and turn the accretion around is still unknown. If the rotational core collapse can lead to the formation of a neutron star or a magnetar, it is possible that the prompt accretion or fallback process can make a hyperaccreting disk around the young formed star. On the other hand, recent simulations also showed the possibility of a debris disk around a massive neutron star as the outcome of the other hand, are much hotter, denser with much higher pressure and accretion rates compared to the normal disks. Therefore our motivation in this paper is to investigate the effects of a strong magnetic field on the hyperaccreting transient disk around a magnetar formed in the collapse of a massive star or the merger of a compact object binary, and their observational phenomena related to GRBs and associated events. The accretion process onto the magnetized neutron star has been widely studied in X-ray binaries, T Tauri stars, and cataclysmic variables since Ghosh & Lamb (1978, 1979a, 1979b), both theoretically (e.g., Königl 1991; Ostriker & Shu 1995; Lovelace et al. 1996; Long et al. 2007, 2008). The neutrino-cooled hyperaccreting disks, on the other hand, are much hotter, denser with much higher pressure and accretion rates compared to the normal disks. Therefore the structure and radiation process in these magnetized disks must be very different from the normal ones. For example, the effects of a magnetic field in the disk. Recently, Xie et al. (2007, 2009) and Lei et al. (2009) studied the structure of magnetized hyperaccreting neutrino-cooled disks around black holes based on similar models, while the magnetic fields are generated in the disks up to \( 10^{15} - 10^{16} \text{ G} \) (see also Janiuk & Yuan 2009). Xie et al. (2007, 2009) showed that the magnetic braking and viscosity can drive magnetically dominated accretion flows rather than neutrino dominated, and turn the disk temperature to be lower than that without the field. Lei et al. (2009) investigated the properties of the NDAF with the magnetic torque acting between the central black hole and the disk. The neutrino annihilation luminosity can be increased by 1 order of magnitude for accretion rate \( \sim 0.5 \text{ } \dot{M}_o \text{ s}^{-1} \), and the disk becomes thermally and viscously unstable in the inner region. However, all of their works neglected microphysics processes and the changes in equations of state in strong magnetic fields, which may significantly change the disk pressure and various neutrino cooling rates in the neutrino-cooled disks. Furthermore, if we look into the neutron star disks, we should keep in mind that the structure of magnetic fields in disks around magnetized neutron stars or magnetars is very different from their black hole counterparts. The origin of a magnetic field in this case in the central compact star constructs the initial topology of the strong field. An interaction between the star and the surrounding disk makes the stellar magnetic field partially thread the accretion disk.

\[
\begin{align*}
\mathcal{M} & = 0.01 M_{\odot} \text{ s}^{-1} \text{ is the accretion rate, } \mathcal{M} = 1.4 M_{\odot} \text{ is the central star mass, } \mu = \mu_0 10^{30} \text{ G } \text{ cm}^3 \text{ is the central magnetic flux. Figure 1 shows the Alfvén radius } r_A \text{ as a function of accretion rate with various magnetic fields. If the surface field } B_0 \text{ is less than } \\
B_0 & \leq B_{\text{eq}} = 0.89 \times 10^{15} M_{\odot}^{-1/2} \mu_3^{-1/4} \text{ G} \quad (2)
\end{align*}
\]

with \( r = r_{\text{s,6}} \times 10^6 \text{ cm } \) being the star radius, the accretion flow will continue to be confined in the disk plane without corotating with the compact object or getting funneled onto the magnetar poles. Papers I and II assumed that the strength of the stellar surface field is less than \( 10^{15} \text{ G} \) and did not consider
the size of the magnetosphere near the central star can be ignored except for a magnetic field \( \geq 10^{16} \text{ G} \). The pressure relation \( B^2/8\pi \gg P_{\text{matter}} + \rho v^2 \) will not be satisfied in most cases. More attractively, we need to study the neutrino emission and annihilation process, which are almost always the most important properties of the hyperaccreting disks but never occur in the normal magnetized ones.

This paper is organized as follows. In Section 2, we consider the quantum effects of a strong magnetic field in the microphysical scale on disk density, pressure, and various neutrino cooling rates. In Section 3, we discuss the disk conservation equations coupled with the global fields both from the central magnetar and generated via the magnetar–disk interaction. Also, we discuss the field topology of the central magnetar. Combining the equations in Sections 2 and 3, we discuss numerically the properties of the hyperaccreting disks in the strong magnetic fields in Section 4. The structure of the magnetized disks depends on the detailed field strength and configuration. In Section 5.1, we further discuss the disk outflows, which may provide the kinetic energy of the supernova associated with a GRB (MacFadyen & Woosley 1999; Kohri et al. 2005). Then in Section 5.2, we discuss the field topology of the central magnetar. Combining the equations and the central magnetar. Combining the equations in Sections 2 and 3, we discuss numerically the properties of the hyperaccreting disks in the strong magnetic fields. Moreover, whether the microphysics change in the strong magnetic fields.3 and \( P_e = u_e/3 \) is only noticeable in very opaque regions in the disk, where \( u_e \) is the energy density of neutrons. The pressures of electrons and positrons in the strong magnetic field are

\[
P_{\text{es}} = \sum_{n=0}^{\infty} g_{nL} \int_{-\infty}^{\infty} dp \frac{eB_m}{h^2c} f_{\nu} \]

and the electron pressure \( P_e = P_{\text{es}} + P_{\text{es}} \). The pressure of the magnetic field is \( P_B = B_m^2/8\pi \).

If \( B \to 0 \), the number density formulae (5) and (8) as a series of different Landau levels will switch back to their integral form without fields. However, the convergence performance of the Landau level series becomes poor for weak magnetic fields. Figure 2 shows the convergence performance of density and pressure Landau level series, and compares the summations of formulae (5) and (8) to the values without fields. Here we take temperature and chemical potential as fixed typical values in the disks. We define \( \ell_n \) as the \( n \) term in the Landau level series (5) and (8). We note that the value of \( \ell_n \) drops faster for stronger magnetic fields. So we have to take a huge number of \( \ell_n \) terms to calculate the summation for relatively weak magnetic fields. Moreover, whether the microphysics change in the strong fields is important also depends on the disk temperature. The difference between density and pressure with and without fields is more significant at lower temperatures. This conclusion can also be applied to the hyperaccreting disks around black holes, which generate the fields by MRI or different rotation. Later in this paper, we take the upper limit \( n = 10^5 \) for \( \ell_n \) summation calculation for \( B \geq B_{\text{cc}} \), as the pressure or the equations of state in a magnetized disk switch back to those without fields for \( B < B_{\text{cc}} \) and the typical temperature in the disks.

The number density of electrons and positrons with magnetic field strength \( B_m \) is

\[
n_{ei} = \sum_{n=0}^{\infty} g_{nL} \int_{-\infty}^{\infty} dp \frac{eB_m}{h^2c} f_{\nu} = 2\pi \left( \frac{m_e c}{\hbar} \right)^3 b \sum_{n=0}^{\infty} g_{nL} \int_{0}^{\infty} dx f_{ei}, (5)\]

where

\[
f_{ei} = \frac{1}{e^m c^2 \sqrt{\pi^3} \sqrt{x^3 + 1 + nL/k_B T \mp \frac{P_{\nu}}{P_{\text{es}}} \mp 1 + 1}}\]

is the Fermi–Dirac function. The total pressure in the disk is contributed by five terms, i.e., the pressure of nucleons, electrons (including positrons), radiation, neutrino, and magnetic field:

\[
P = P_{\text{nuc}} + P_{\text{rad}} + P_e + P_{\nu} + P_B. \]

We adopt the approximation that \( P_{\text{nuc}} \) and \( P_{\text{rad}} \) do not change with magnetic fields, and \( P_e = u_e/3 \) is only noticeable in very opaque regions in the disk, where \( u_e \) is the energy density of neutrons. The pressures of electrons and positrons in the strong magnetic field are

\[
P_{\text{es}} = \sum_{n=0}^{\infty} g_{nL} \int_{-\infty}^{\infty} dp \frac{eB_m}{h^2c} \sqrt{\frac{\rho^2 c^2}{\rho^2 c^2 + m_e^2 c^4 + 2nL_e B_m^2 h c}} f_{\nu} \]

and the electron pressure \( P_e = P_{\text{es}} + P_{\text{es}} \). The pressure of the magnetic field is \( P_B = B_m^2/8\pi \).

2. THERMODYNAMICS AND MICROPHYSICS IN STRONG MAGNETIC FIELDS

2.1. Density and Pressure in Magnetic Fields

The distribution and energy of charged particles will change significantly if the magnetic field is greater than the critical value \( B_c = m_e^2 c^3 / q_i \hbar \), where \( m_i \) and \( q_i \) are the mass and charge of the particle, respectively. As the critical values for electrons are \( B_{ec} \simeq 4.4 \times 10^{13} \text{ G} \) and \( B_{ec} \simeq 1.5 \times 10^{20} \text{ G} \), we have to adopt the relativistic Dirac equation for electrons in the environment near a magnetar with a field \( B > 4.4 \times 10^{13} \text{ G} \), while the protons can be still considered as classical. The main modification of electron distribution is that, the phase space factor in the absence of the magnetic field should be replaced by the summation over the possible Landau level

\[
\frac{2}{h^3} \int d^3 p \longrightarrow \sum_{n=0}^{\infty} g_{nL} \int \frac{eB_m}{h^2c} dp, (3)\]

where \( B_m \) is the field strength, and \( g_{nL} = 2 - \delta_{n0} \) with \( \delta_{n0} \) being the Kronecker delta function.

The electron energies are given by (Johnson & Lippmann 1949)

\[
E_e = \sqrt{p^2 c^2 + m_e^2 c^4 + 2nL_e B_m^2 h c} = m_e c^2 \sqrt{x^2 + 1 + nL_b}, \]

where \( n \) labels the Landau level, \( b = 2e\hbar B_m/m_e^2 c^3 \) is the dimensionless magnetic field parameter, and \( x = p/m_e c \) is the dimensionless momentum.

3 We neglect the photodisintegration process that happens far from the central star at \( r \geq 400 \text{ km} \); thus the nucleons are mainly composed of classical protons and neutrons without \( \alpha \)-particles in the disk region we focus on. The equations of state for protons and neutrons do not change with field \( B < 10^{20} \text{ G} \) as mentioned above.
2.2. Neutrino Cooling in Magnetic Fields

Neutrino cooling processes (especially, the Urca process) in the environment of compact object magnetic fields have been studied for years (e.g., Chen et al. 1974; Dorofeev et al. 1985; Dai et al. 1993; Yuan & Zhang 1998; Lai & Qian 1998; Roulet 1998; Baiko & Yakovlev 1999; Yakovlev et al. 2001; Duan & Qian 2004, 2005; Luo 2005; Riquelme et al. 2005). Similarly, strong magnetic fields also affect the neutrino emission in a hyperaccreting disk around the compact object. Here we consider the problem systematically in this section. The total neutrino cooling rates in the vertically integrated disk are taken as (Di Matteo et al. 2002; Kohri et al. 2005)

\[
Q_{ν} = \sum_{i=e,μ,τ} \frac{(7/8)σ_B T^4}{(3/4)[τ_{ν_i}/2 + 1/\sqrt{3} + 1/(3τ_{ν_i})]],
\]

where the total optical depth for three types of neutrinos \(τ_{ν_i} = τ_{ν_i} + τ_{ν_i}ν\) with \(τ_{ν_i}ν\) being the absorption depth and \(τ_{ν_i}\) the scattering depth, both of which can be affected by strong magnetic fields.

The neutrino absorption depth can be approximately given by (Popham et al. 1999) \(τ_{ν_i} = q_{ν_i}ν H/(3σ_B T^4)\), where \(q_{ν_i}ν\) are the absorption neutrino cooling rates for three types of neutrinos, and \(H\) is the half-thickness of the disk. The electron neutrino cooling rate \(q_{ν_e}ν\) in the disk can be simply taken as the summation of four terms \(q_{ν_e}ν = (q_N + q_{ν_e}ν → νēν_e + q_{brems} + q_{plasmon})H\), which are the the cooling rates due to electron–positron capture by nucleons, electron–positron pair annihilation into neutrinos, nucleon–nucleon bremsstrahlung, and plasmon decays, respectively (Kohri & Mineshige 2002). On the other hand, the muon and tau neutrino cooling rates are the summations of pair annihilation and bremsstrahlung \(q_{ν_μ}ν H = q_{ν_μ}ν H \simeq (q_{ν_μ}ν → νēν_e + q_{brems})H\).

The neutrino cooling via electron–positron capture by nucleons (or say the Urca process in the disk), which is usually the most significant cooling rate among various cooling terms in hyperaccreting disks, can be considered as the combination of three processes: \(q_N = q_{ν_μ}ν → νēν_e + q_{ν_τ}ν → νēν_e + q_{ν_μ}ν → νēν_e \). If we take the parameter \(K\) as

\[
K = \frac{G_F^2 C_V^2 (1 + 3g_A^2)}{\pi^3 h^\gamma (m_e c)^6},
\]

where \(G_F \simeq 1.436 × 10^{-49}\) erg cm\(^{-3}\), \(C_V = 1/2 + 2\sin^2 θ_W\) with \(\sin^2 θ_W = 0.23\), and \(g_A = -1.23\). Using the modification rule in strong fields (3), we can obtain the three terms of neutrino cooling rates in strong magnetic fields as

\[
q_{ν_μ}ν → νēν_e = \frac{bK}{4} \sum_{n=0}^{∞} g_{νe}L n \int_{min[q_e,\sqrt{τ_{νe}ν}]}^{∞} \frac{ε(ε - q)^3 dε}{\sqrt{ε^2 - 1 - n_L b} f_ε},
\]

\[
q_{ν_τ}ν → νēν_e = \frac{bK}{4} \sum_{n=0}^{∞} g_{νe}L n \int_{\sqrt{τ_{νe}ν}}^{∞} \frac{ε(ε + q)^3 dε}{\sqrt{ε^2 - 1 - n_L b} f_ε},
\]

Figure 2. Left: convergence performance of Landau level series of electron density and pressure with different magnetic fields \(B = 10^{16}\) G (solid line), \(10^{15}\) G (dashed line), \(10^{14}\) G (dotted line), and \(10^{13}\) G (dash-dotted line) and temperature \(T = 5 \times 10^{10}\) K. Right: comparison of density and pressure with and without magnetic fields with temperature \(T = 10^{10}\) K (solid line), \(5 \times 10^{10}\) K (dashed line), and \(2 \times 10^{11}\) K (dotted line).
rates with and without magnetic fields. Temperature \( T \) the difference is not significant for \( q_{\nu e} \rightarrow p \bar{\nu}_e \), where temperature \( T = 5 \times 10^{10} \) K and chemical potential \( \eta_e \) is adopted. Lines are as in Figure 1. Right: comparison of neutrino cooling rates with and without magnetic fields. Temperature \( T = 5 \times 10^{10} \) K and \( \eta_e = 0.1, 1, \) and 10. Also, upper is for \( q_{\nu e} \rightarrow p \bar{\nu}_e \) and lower for \( q_{\nu e} \rightarrow e^{\pm}\).

\[
\dot{q}_{n \rightarrow p e^{-} p^{+} \bar{\nu}} = \frac{\tilde{F}}{2} \sum_{n=0}^{\infty} g_{nL} n_{n} \int_{0}^{\infty} \frac{d \varepsilon}{\sqrt{\varepsilon^2 - 1} - n_{1} B} (1 - f_{e^{-}}),
\]

where \( \varepsilon = E_{\nu_{e}} / m_{e} c^2 \) is the dimensionless energy of electrons, and \( q = (m_{n} - m_{p}) / m_{e} \approx 2.531 \). Here we adopt the approximation adopted in Shapiro & Teukolsky (1983) that the total reaction on nucleons is directly proportional to the density number of nucleons, i.e., protons and neutrons. Figure 3 shows the convergence performance of the Landau level series of two dominated cooling rates (11) and (12) with the typical temperature and chemical potential in neutrino-cooled hyperaccreting disks. Similar to the density and pressure in a strong magnetic field, the convergence properties of neutrino cooling series also become poorer for a weaker field. Moreover, the total Urca neutrino cooling rate becomes lower for a higher magnetic field. Although the difference is not significant for \( T = 5 \times 10^{10} \) K, it can be greater at a lower temperature, as shown in Figure 2. We will discuss this effect later in Sections 4 and 6.

The electron–positron pair annihilation rate without magnetic fields can be calculated as (Burrows & Thompson 2004)

\[
\dot{q}_{e^{-} e^{+} \rightarrow \nu_{e} \bar{\nu}_{e}} \approx q_{e^{-} e^{+} \rightarrow \nu_{e} \bar{\nu}_{e}(B=0)} \approx 2.558 \times 10^{33} T_{11}^{3} f_{\nu_{e}} \text{ cm}^{-3} \text{ s}^{-1},
\]

where \( f_{\nu_{e}} \) is

\[
f_{\nu_{e}} = \frac{F_{1}(\nu_{e}) F_{3}(-\nu_{e}) + F_{4}(\nu_{e}) F_{3}(\nu_{e})}{2 F_{3}(0) F_{3}(0)}
\]

with the function \( F_{n}(\pm\eta_{e}) \) being

\[
F_{n}(\pm\eta_{e}) = \int_{0}^{\infty} \frac{\chi^{n}}{e^{\pm\eta_{e}} + 1} d\chi.
\]

In the case of strong magnetic fields, we can approximately take the annihilation rate as

\[
\dot{q}_{\nu_{e} \bar{\nu}_{e}} = \dot{q}_{\nu_{e} \bar{\nu}_{e}(B=0)} \times B_{m \text{eV}} m_{e} c^{5} \times \frac{28\pi^{2} T^{4} k_{B}^{4}}{A^{2}}
\]

when the field strength \( B_{m} \) satisfies \( B_{m \text{eV}} \gg T^{3} k_{B}^{2} \) (e.g., Kaminker et al. 1992; Yakovlev et al. 2001).

In most cases, the cooling processes via bremsstrahlung and plasmon decay are much less significant than \( e^{-} e^{+} \) capture and annihilation, furthermore the unchanged distribution and energy of neutrons and photons. Therefore, we still use the formulae as in Kohri et al. (2005) for these two processes.

Besides the absorption depth, three types of optical depth for neutrinos through the scattering of nucleons and electrons are given by

\[
\tau_{\nu_{p} \nu_{e}} = (\sigma_{\nu_{p} n} n_{p} + \sigma_{\nu_{e} n} n_{e} + \sigma_{\nu_{p} e} \rho) H = (\sigma_{\nu_{p} n} Y_{n} + \sigma_{\nu_{e} e} Y_{e}) \rho H / m_{B},
\]

where \( \sigma_{\nu_{p} n}, \sigma_{\nu_{e} n}, \) and \( \sigma_{\nu_{p} e} \) are the cross sections of scattering on protons, neutrons, and electrons, and \( \rho \) is the density of the disk. We take the cross sections of \( \sigma_{\nu_{p} n} \) and \( \sigma_{\nu_{e} n} \) in strong magnetic fields as

\[
\sigma_{\nu_{p} n} \approx \sigma_{\nu_{p} n}(B=0) = \frac{\sigma_{0}}{6} \left( \frac{E_{\nu}}{m_{e} c^{2}} \right)^{2} \left[ (C_{V} - 1)^{2} + 5g_{A}^{2}(C_{A} - 1)^{2} \right],
\]
which are still classical (Tubbs & Schramm 1975; Burrows & Thompson 2004). Here \( \alpha_0 \simeq 1.705 \times 10^{-44} \) cm\(^2\) is the neutrino cross section coefficient. On the other hand, neutrino–electron scattering process in a strong magnetic field has been described by many authors using various approaches (e.g., Bezchastnov & Haensel 1996; Kuznetsov & Mikheev 1997, 1999; Hardy & Thoma 2000; Mikheev & Narynskaya 2000). Although the analytic results are quite complicated and different based on various approximated treatments, all of these works show that the cross section \( \sigma_{\nu,e} \) is proportional to the field strength \( \propto e B_m \).

For simplicity, we use the factor 2 in Equation (26) to compare the importance of electron energies in strong fields and the thermal energy, and consider the cross section \( \sigma_{\nu,e} \) in strong magnetic fields as

\[
\sigma_{\nu,e} \simeq \sigma_{\nu,e(B=0)} \times [1 + 2 e B_m \hbar c / (k_B T)^2],
\]

with

\[
\sigma_{\nu,e(B=0)} = \frac{3 \alpha_0}{8} \left( \frac{k_B T}{m_e c^2} \right)^2 \left( \frac{e}{m_e c^2} \right) \left( 1 + \frac{\eta_t}{4} \right) \times [(C_V + C_A)^2 + (C_V - C_A)^2/3].
\]

3. CONSERVATION EQUATIONS IN MAGNETIZED DISKS AND FIELD TOPOLOGY

3.1. Conservation Equations

In Section 2, we discussed the microphysics and thermodynamic equations in strong magnetic fields, and compare them with the equations without fields. Now we consider the basic conservation magnetohydrodynamical (MHD) equations for a vertically integrated steady-state magnetized disk. We modify the hydrodynamical equations without fields by adding the coupling of a large-scale magnetic field. In this case we consider the accretion flow to be still constrained in the disk plane. The disk with an outflow structure will be discussed in Section 5.1, and the funnel accretion process in an extremely strong field in the stellar magnetosphere will be discussed in Section 5.2. First of all, the mass continuity equation in a vertically integrated disk does not change

\[
\dot{M} = -2 \pi r \Sigma v_r,
\]

where \( \Sigma = 2 \rho H \) is the disk surface density and \( v_r \) is the radial velocity of the accretion flow.

The vertically integrating angular momentum conservation equation with fields reads (e.g., Lovelace et al. 1987; Shadmehri & Khajenabi 2005)

\[
\frac{d}{dr} \left( r^2 \Sigma \frac{d \Omega}{dr} \right) = -r^2 B_r B_\phi \bigg|_{r=H} + \frac{d}{dr} \left( r^2 H \frac{d}{dr} (B_r B_\phi) \right),
\]

where \( \langle \cdots \rangle = \int_0^1 dz \langle \cdots \rangle / 2h, \nu = \alpha c_s H \) is the kinematic viscosity coefficient in the disk with \( \alpha \) being the viscosity parameter. Here we still use the \( \alpha \)-prescription. The magnetic field can play an important role in transferring the angular momentum. If the radial field \( B_r \) can be neglected (as will be shown in Section 3.2), we can integrate the angular momentum equation as

\[
\frac{\dot{M}}{3\pi} = \nu \Sigma - \frac{\dot{M} N_B}{3\pi r^2 \Omega},
\]

where

\[
\dot{M} N_B(r) = - \int_r^\infty r^2 B_r B_\phi \bigg|_{z=H} dr
\]

is the integrated magnetic torque, \( f = 1 - l_0/(r^2 \Omega) \) with \( l_0 \) being the specific angular momentum constant. In the standard assumption that the viscous torque is zero at the inner boundary of the disk \( r_s + b \) with \( r_s \) being the stellar radius and \( b \ll r_s \), we can take \( f = 1 - \sqrt{r_s/r} \) (Frank et al. 2002).

The local energy conservation equation in a magnetized disk is

\[
Q_{\text{vis}}^+ + Q_{\text{joule}}^+ = Q_{\text{adv}}^- + Q_{\text{rev}}^-.
\]

where \( Q_{\text{vis}}^+ \) and \( Q_{\text{adv}}^- \) remain the same as in the case of \( B_m = 0 \) (Equations (A12) and (A13) in Appendix A), and the neutrino cooling rate \( Q_{\text{rev}}^- \) in strong magnetic fields has been discussed in Section 2.2. In particular, the Joule dissipation (or say the Joule heating) term \( Q_{\text{joule}}^+ \) is

\[
Q_{\text{joule}}^+ = \frac{4 \pi H}{c^2} \eta_t (J^2) = \frac{H}{4\pi} \eta_t \langle (\nabla \times B_m)^2 \rangle
\]

\[
= \frac{H \eta_t}{4\pi} \left\{ -2 B_r \bigg|_{z=H} \frac{\partial B_r}{\partial r} \bigg|_{z=H} + \left[ \frac{\partial B_r}{\partial r} \right]^2 \right\},
\]

where the magnetic diffusivity parameter \( \eta_t \) is adopted as \( \eta_t \sim \nu \simeq \alpha c_s H \) (Lovelace et al. 1995; Bisnovatyi-Kogan & Lovelace 1997). In Section 6.1, we will discuss the Joule dissipation term in more details.

Moreover, we need to add the charge conservation equation and the chemical equilibrium equation, which are the same as in Paper II (or see Appendix A, Equations (A7) and (A14)).

3.2. Field Topology

The magnetic field of a neutron star or a magnetar threading the disk has the vertical component

\[
B_z(r) = B_0 \left( \frac{r_s}{r} \right)^n.
\]

The actual magnetic field in the vertical component near a magnetar may be a mix of monopole \((n = 2)\) and dipole \((n = 3)\) fields (Thompson et al. 2004). The dipole field drops faster than the monopole field along the disk radius; thus the disk properties

---

4 Actually, the half-thickness of the disk \( H \) and the kinematic viscosity \( \nu = \alpha c_s H \) will change depending on the field structure. For example, the half-thickness \( H \) will increase for large \( B_m \), which \( b \ll r_s \). Also, the Shakura & Sunyaev (1973) scenario \( v = \alpha c_s^2 / \Omega_k \) should be modified as \( v = \alpha' c_s^2 / \Omega_k \).
will be quite different from the other cases. We focus on the dipolar form \( n = 3 \) in this paper.

The differential rotation between the disk and the star will generate a toroidal field (Wang 1995). Following Lai (1998), we consider two possible field configurations in a disk, i.e., the stellar magnetic field threading the accretion disk in a closed configuration (i.e., the classical configuration as in Ghosh & Lamb 1979a), or when the magnetic field becomes open (e.g., Lovelace et al. 1995). The generated azimuthal component of the magnetic field in a steady-state disk is

\[
B_\phi|_{z=H} = -\beta B_z, \quad \text{(31)}
\]

for an open magnetic field in the disk, and

\[
B_\phi|_{z=H} = -\beta \left( \frac{\Omega - \Omega_\star}{\Omega_K} \right) B_z, \quad \text{(32)}
\]

for a closed magnetic field, where \( \beta \) is a dimensionless parameter, \( \Omega_\star \) and \( \Omega_K \) are the stellar angular velocity and Kepler velocity, respectively. The radial component \( B_r \) generated in the disk reads

\[
B_r|_{z=H} \propto \left( \frac{-v_r}{\Omega K r} \right) B_z|_{z=H}. \quad \text{(33)}
\]

In most cases, we consider \( v_r \ll \Omega K r \) and \( B_z|_{z=H} \ll B_r|_{z=H} \). Thus the field strength \( B_m \) in the disk can be obtained as

\[
B_m = \sqrt{B_z^2 + B_r^2 + B_\phi^2} \approx \sqrt{B_z^2 + B_r^2}. \quad \text{(34)}
\]

According to Lovelace et al. (1995), if the angular velocities of the star and disk differ substantially, the magnetic field lines threading the star and the disk undergo a rapid inflation so that the field becomes open. As a result, the outer part of the disk will maintain an open field configuration, while the inner disk in the magnetosphere still has closed lines. Since in the hyperaccreting case the magnetosphere is very small except for an extremely strong center field, in which the disk plane will be disrupted and truncated in the magnetosphere (see discussion in Section 5), the magnetized disk where it still has a plane geometry would be more favorable to maintain an open field. However, we still consider the two cases as a broader consideration. If the magnetic field in the disk is open, the magnetic torque in Equation (26) becomes

\[
\dot{MN}_B = \beta B_z^2 \frac{r^3}{2n - 3}, \quad \text{(35)}
\]

and the integrated angular momentum Equation (26) can be written as

\[
\frac{GM M}{3\pi} f + \frac{1}{3\pi s} \frac{\beta}{\sqrt{GM}} B_z^2 \frac{r^{5/2}}{2n - 3} = 2a \frac{p^{3/2}}{\rho^{1/2}} r^3, \quad \text{(36)}
\]

where we take the ratio of the angular velocity of the disk flow and the Keplerian velocity as a constant \( s = \Omega/\Omega_K \). For the case of a closed disk field, on the other hand, using Equation (32) we have

\[
\dot{MN}_B = \beta s B_z^2 \frac{r^3}{2n - 3} - \beta \Omega_\star B_z^2 (GM)^{-1/2} \frac{r^{9/2}}{2n - 9/2}, \quad \text{(37)}
\]

and the angular momentum Equation (26) reads

\[
\dot{MN}_B = \frac{GM M}{3\pi} f + \frac{\beta}{3\pi} \frac{B_z^2 r^{5/2}}{2n - 3} - \frac{2\beta}{3s} \frac{B_z^2 r^4}{P_r} \frac{1}{2n - 9/2} = 2a \frac{p^{3/2}}{\rho^{1/2}} r^3, \quad \text{(38)}
\]

where \( P_r \) is the spin period of the magnetar.

Moreover, we obtain the energy conservation Equation (28) in the disk with the detailed field topology structure as

\[
\frac{3GM M}{8\pi r^3} f + \frac{H \eta_\star}{4\pi r^2} \left[ n^2 + (n-1)^2 \beta^2 \right] B_z^2 = \dot{Q}_{adv} + \dot{Q}_v \quad \text{(39)}
\]

for a closed magnetic field, and

\[
\frac{3GM M}{8\pi r^3} f + \frac{H \eta_\star}{4\pi r^2} \left\{ n^2 + \left[ s(n-1) - \frac{\Omega_\star}{\Omega_K} \left( n - \frac{5}{2} \right) \right] \frac{\beta^2}{2} \right\} B_z^2 = \dot{Q}_{adv} + \dot{Q}_v \quad \text{(40)}
\]

for an open magnetic field.

We give a brief summary at the end of this section. We consider the quantum effects of strong fields on the disk thermodynamics and microphysics in Section 2.1, and the MHD conservation equation with coupling of a large-scale disk field in Section 2.2. The strong magnetic field can change the electron distribution and energy significantly, and also change the cross sections of the various neutrino reactions. The large-scale field can play an important role in transferring angular momentum besides the viscous stress and heat the disk as well. In order to see the differences of disks with and without fields (or more exactly to say, with fields \( \leq 10^{14} \) G in the hyperaccreting disk) clearly, we list the basic equations without fields in Appendix A for a comparison.

4. NUMERICAL SOLUTIONS OF MAGNETIZED NEUTRINO-COOLED DISKS

We adopt the \( \alpha \) turbulent viscosity model of Shakura & Sunyaev (1973) and fix \( \alpha = 0.1 \) in all of our calculations. First we want to study the direct quantum effects of strong magnetic fields on the disk, and the macrophysical field coupling in the MHD conservation equations independently. Figures 4 and 5 give the independent results. In Figure 4, we keep the conservation equations as the normal hydrodynamical equations without fields (i.e., Equations (A8), (A10)–(A14) in Appendix A) and only adopt the equations of state (4)–(23) in Section 2 to see the quantum effects of a field. On the other hand, in Figure 5, we keep the microphysical and thermodynamical equations as in the case of no field (i.e., Equations (A1)–(A7) in Appendix A and the equations in Section 2 without fields), but only discuss the field effect on angular momentum transfer (26) and energy heating (28). As discussed by Duncan & Thompson (1992), in principle magnetic fields as strong as \( 3 \times 10^{17} (P/\text{ms})^{-1} \) G can be generated in magnetars as the differential rotation is smoothed by growing magnetic stresses. Kluzniak & Ruderman (1998, see also Ruderman et al. 2000) argued that the energy stored in the differentially rotating magnetars can be extracted by the process of winding up and amplification of toroidal magnetic fields inside the star up to \( \sim 10^{17} \) G, then the ultrastrong field will be pushed to and through the surface by buoyancy force. Based on these considerations, we adopt the magnetar surface field up to \( \sim 10^{17} \) G, and discuss the field strength from \( 10^{14} \) G to \( 10^{17} \) G with a dipolar extension in our calculations. The steady-state disk can be considered as a transient hyperaccreting disk with a fixed time, or say, with a fixed accretion rate. In contrast, for a time-dependent disk model (e.g., Janiuk et al. 2004, 2007 without fields), we have to consider the evolution of the magnetar surface field, which is beyond the purpose of this paper. Moreover, we do not consider other quantum processes in strong fields such as the generation of strong electric field and electron/
Figure 4. Quantum effects of the microphysics and thermodynamics in strong magnetic fields that affect the disk properties. In order to see the effects clearly, we still take the non-magnetized conservation equations (i.e., Equations (A8), (A10)–(A14) in Appendix A), and only change a set of equations of state in Section 2. Also, we take the magnetic field as uniform $B = 10^{14}, 10^{15}, 10^{16}, 10^{17}$ G and without field, the accretion rate $\dot{M} = 0.1 M_\odot s^{-1}$.

In Figure 4, a stronger magnetic field makes the disk thinner and cooler with lower temperature and neutrino luminosity. In contrast, Figure 5 shows that the disk will be denser with higher pressure and neutrino luminosity, and becomes hotter at least in the inner disk region near the magnetar, because the stronger field plays a more significant role in transporting the disk angular momentum and heating the disk by Joule dissipation.\footnote{In this section, we extend our calculation to the inner disk region inside the Alfvén radius $r_A$. This extension is good for us to see the disk properties of magnetized disks, although we need to use another treatment to discuss the disk structure in the magnetosphere with an extreme strong field.} As a result, the quantum effects in strong fields and the field coupling in MHD equations play two opposite roles in changing the disk properties, one to decrease pressure, density, and luminosity with increasing field strength, and the other to increase them. These two competitive factors work together to establish the actual structure of the disk.

In Figure 6, we consider the quantum and coupling effects together. This figure is for the disk structure and neutrino luminosity with accretion rate $\dot{M} = 0.1 M_\odot s^{-1}$ and an open magnetic field configuration in the disk. From Equation (35) we know that the ratio between the two parameters $\beta/s$ determines the angular momentum transfer by magnetic field. As a result from Figure 6, a higher ratio of $\beta/s$ leads to higher density and pressure in the entire disk, higher temperature and neutrino luminosity in the inner region of the disk, as well as lower electron fraction at a disk radius of $\sim 30$ km. Moreover, electrons in the disk become more degenerate around $\sim 20–40$ km for higher $\beta/s$, but quickly change to be nondegenerate at the inner edge of the disk toward the magnetar surface.

Figure 7 gives the magnetized disk for various surface vertical fields $B_0$ and open magnetic field configuration in the disk. Stronger fields give higher density, pressure, temperature, and neutrino luminosity. This figure shows that the magnetic field coupling in the MHD equations of disk flow is more important than the quantum effects of strong fields at a microphysical scale. In the case of $B_0 = 10^{17}$ G, the disk has no steady-state solution in the inner region of the disk where the flow should corotate with the central star and be further channeled onto the magnetic pole along the field lines. We will discuss the funneled flows later. On the other hand, for $B_0 = 10^{14}$ G, we stop the calculation where the disk field is too weak for us to consider the quantum effect in strong fields as discussed in Section 2. In this case, the disk properties will be very similar to the case of $B_0 = 10^{14}$ G without quantum effect even in the case of $B_0 = 0$, which has been shown in Figure 5. The electron fraction decreases inward for $B \leq 10^{16}$ G, which is similar to the results without fields (e.g., Paper I, Figure 7), but a field $\sim 10^{17}$ G will change the electron fraction distribution significantly. The distribution of electron potential $\eta_e$ along the radius also changes a lot in strong fields. A larger peak with more degenerate electron state is obtained by stronger fields, while the change in $\eta_e$ becomes less obvious for relatively weaker fields.

Above we focus on the disk accretion rate of $\sim 0.1 M_\odot s^{-1}$, which is typical for a magnetar hyperaccreting disk. Moreover generally, we can study the magnetized disks with different accretion rates. We discuss the disk properties with open disk field and different accretion rates $\dot{M} = 0.02, 0.1, \text{and } 1 M_\odot s^{-1}$.
in Figure 8. The values of density, pressure, temperature, and neutrino luminosity become higher for higher accretion rate. This result is similar to the case without the field. Also, lower electron fraction and more degenerate electron state can be obtained for higher accretion rate, except for the inner region where the Joule dissipation becomes more significant than the viscous heating. Therefore, the accretion rate plays a very similar role in changing the disk properties to that without fields. However, we should point out that, there is no steady-state solution for \( \dot{M} = 0.02 \, M_\odot \, s^{-1} \) in the inner disk region \( B_0 = 10^{16} \, \text{G} \). This is reasonable and similar to the case of \( \dot{M} = 10 \, M_\odot \, s^{-1} \) and \( B_0 = 10^{16} \, \text{G} \), in that a region is already inside the Alfvén radius and almost belongs to the magnetosphere. The extension calculation in this section cannot reach this region.

Figure 9 shows the disk structure for a closed magnetic field in the disk. We discuss it besides the open disk field configuration for completeness. In this case, the disk properties not only depend on the disk field structure (\( \beta \)) and angular velocity (\( s \)), but also depend on the spin period of the central magnetar. A normal pulsar has a spin period around 1–100 s, while the new born pulsar, as a candidate for GRB central engines, usually has a much shorter period with a timescale about tens of milliseconds or even less. Here we consider the magnetar with a period \( P_r = 5, 10, 100 \, \text{ms} \). A rapidly rotating magnetar can act as an extra torque on the disk. However, we only consider the magnetic field torques from the magnetar for simplicity. A shorter period of the central magnetar slightly decreases the disk density, pressure, and electronic chemical potential, but increases the temperature at 20–40 km and the electron fraction at \( r \geq 20 \, \text{km} \) (note that there is a peak of \( \eta_e \)) at \( r = 20 \, \text{km} \) for \( B_0 = 10^{16} \, \text{G} \). The effects of period change on the disk are only obvious for the surface vertical field \( B_0 \geq 10^{16} \, \text{G} \) in Figure 9. However, since the neutrino luminosity does not change significantly even for \( B_0 \simeq 10^{16} \, \text{G} \), we cannot expect any obvious observational events from the disk plane related directly to the effect of period on the disks.

Figure 10 shows the \( M - \Sigma \) curves for a given disk radius \( r = 40 \) and \( 80 \, \text{km} \). In order to see the stable performance clearly, we extend our calculation to \( \dot{M} = 10 \, M_\odot \, s^{-1} \), although such an accretion rate is impossible for an accreting magnetar system. The condition for viscous stability is \( d\dot{M}/d\Sigma > 0 \). Therefore, the disk is viscously stable for \( B_0 \leq 10^{16} \, \text{G} \), but becomes unstable in the inner region \( r \leq 40 \, \text{km} \) for \( B_0 \sim 10^{17} \, \text{G} \), such an entire unstable region actually interacts in the stellar magnetosphere. Therefore, we can conclude that the accretion flow in the disk plane can always be viscously stable. This conclusion is consistent with that in the case of no fields (Narayan et al. 2001; Di Matteo et al. 2002; Kawanaka & Mineshige 2007), but is somewhat different to that in Lei et al. (2009), who showed that the disk is always unstable in the inner disk region. The main difference is caused by different magnetic structures between neutron star disks and their black hole counterparts.

The black hole hyperaccreting disk without fields is thermally stable, at least in the region where the nucleon gas pressure dominates over the total pressure (Narayan et al. 2001; Di Matteo et al. 2002; Kawanaka & Mineshige 2007). However,
Figure 6. Disk structure with $M = 0.1 \, M_\odot \, s^{-1}$ for the magnetar surface vertical field $B_0 = 10^{16} \, G$. The magnetic field in the disk is open with parameter $\beta = 0.2, 0.6, 1$ and disk angular velocity $s = 0.5, 1$.

A magnetic field in the disk can make the disk thermally unstable. We roughly estimate it with an analytic method. The general condition for thermal stability can be taken as (Narayan et al. 2001)

$$|\Sigma| \left( \frac{d \ln Q^+}{d \ln H} \right)_\Sigma < \left( \frac{d \ln Q^-}{d \ln H} \right)_\Sigma.$$  

The ratio of viscous and Joule heating is

$$\frac{Q^+}{Q^\text{Joule}} \propto \frac{G M f}{r H \eta_t B_z^2} \propto \frac{\nu \Sigma}{\eta_t H} \propto H^{-1}. \quad (40)$$

As a result, we have

$$\left( \frac{d \ln Q^+}{d \ln H} \right)_\Sigma \ll \left( \frac{d \ln Q^-}{d \ln H} \right)_\Sigma \ll \left( \frac{d \ln Q^\text{vis}}{d \ln H} \right)_\Sigma, \quad (41)$$

i.e., the value of $\left( \frac{d \ln Q^+}{d \ln H} \right)_\Sigma$ increases by unity if $Q^\text{Joule}$ dominates over $Q^\text{vis}$ in the magnetized disk. Now we calculate the ratio $\left( \frac{d \ln Q^\text{vis}}{d \ln H} \right)_\Sigma$ in the magnetized disk. Taking the open field configuration as an example, the ratio of angular momentum transferred by magnetic torque and viscous stress is

$$\frac{\dot{J}_m}{\dot{J}_\text{vis}} = \frac{\beta}{s(2n - 3)} \frac{B_z^2}{\eta_t H} \propto H^{-1}. \quad (42)$$

Furthermore,

$$\left( \frac{d \ln Q^\text{vis}}{d \ln H} \right)_\Sigma = 2 \left( 1 + \frac{\dot{J}_m}{\dot{J}_\text{vis}} \right) = 2 + 24.6 \beta s^{-1} M_{1.4}^{-1/2} M_{-1}^{-1} f^{-1} r_t^{6/2} r_6^{-7/2} B_0^{2}. \quad (43)$$

We can see that in the outer part of the disk, the ratio (43) is still $\approx 2$, but it can be significantly increased in the disk's inner
Figure 7. Disk structure with $\dot{M} = 0.1 M_\odot \text{s}^{-1}$ for the magnetar surface vertical field $B_0 = 10^{14}, 10^{15}, 10^{16}, \text{and} 10^{17} \text{G}$, where the magnetic field in the disk is in an open configuration with $\beta = 1$ and $s = 1$.

part where the magnetic field is sufficiently high. If we take the radius $r \simeq r_A$ or $r_6 \simeq 2 M^{-2/7} M_{1.4}^{-1/7} B_0^{4/7} B_{0,16}^{12/7}$, Equation (43) becomes

$$\frac{d \ln Q^+}{d \ln H}|_\Sigma \simeq 2 + 2.174 \beta s^{-1} (1 - 0.707 M^{-1/7} B_{0,16}^{-2/7})^{-1}.$$ (44)

On the other hand, for the neutrino-cooled region, the neutrino cooling rate under strong fields can be approximately taken as $Q^- \alpha \Sigma T^4 \propto H^5$, or $(d \ln Q^-/d \ln H)|_\Sigma \simeq 8$. Based on the above calculation, whether the gas-dominated disk region is thermally stable or not, depends on the disk radius, magnetic field strength, disk angular velocity, and accretion rate. For typical values $\beta \sim s, M_\odot = 1, B_{0,16} = 1$, and $r$ is comparable to $r_A$, we have $(d \ln Q^+/d \ln H)|_\Sigma \simeq 10.4$, which shows a thermally unstable performance. Furthermore, The radiation-dominated region of accretion disks is expected to be thermally unstable in the $\alpha$-model without a large-scale field (Lightman & Eardley 1974; Shakura & Sunyaev 1973; Narayan et al. 2001); therefore such a region in the magnetized disk around a magnetar will be always thermally unstable. On the other hand, Hirose et al. (2009) showed the disk thermal stability based on the three-dimensional radiation MHD simulations of a vertically stratified shearing box, in which the turbulent magnetic field is generated by MRI. The main reason for that difference is based on different microphysics considerations. The stress is assumed to be proportional to the radiation pressure in the $\alpha$-prescription, while the stress fluctuations precede pressure fluctuations, or say the magnetic energy fluctuations drive pressure fluctuations in the MHD simulation. However, their MHD simulation focuses on the accretion rate near a fraction of the Eddington limit with
Figure 8. Disk structure for different disk accretion rates $\dot{M} = 0.02$ (solid line), 0.1 (dashed line), and $1 \, M_\odot \, s^{-1}$ (dotted line) with open disk field and $\beta = 0.6$ and $s = 1$, while the surface vertical field $B_0 = 10^{16}$ G.

a normal radiation process rather than neutrino cooling. In our paper we still use the $\alpha$-prescription with the large-scale field. Thus, we have a thermally unstable conclusion in the radiation-dominated regions of the magnetized disks.

5. DISK OUTFLOWS AND FUNNEL FLOWS

So far we have studied the structure and luminosity of the magnetized disks with an open or closed field configuration constrained in the disk plane. If we define the Alfvén radius $r_A$ as the point at which the magnetic pressure $P_B = B^2_m / 8\pi$ is equal to the ram pressure of the accretion material $\rho v_r^2$, and the magnetospheric radius $r_m$ as the point where the magnetic pressure is approximately equal to the total matter pressure $P_B \simeq P_{\text{matter}} + \rho v_r^2$, the disk plane will be disrupted near the magnetosphere with $r \lesssim r_A$ as shown in Equation (1) and Figure 1, and the disk flow will be prevented from accretion in the disk plane at $r_m$. Then almost the entire disk will be funneled along the magnetar field lines onto only a small fraction of the region of the stellar surface at $r < r_m$. The value of $r_m$ depends on the details of disk–magnetar interaction. In a normal case such as accretion onto magnetized neutron stars in X-ray binaries, we have $P_{\text{matter}} \ll \rho v_r^2$. Thus, $r_m$ can be calculated as $P_B \sim \rho v_r^2$. It is estimated that $r_m$ is close to or slightly less than the Alfvén radius (Lai 1998). Let us now consider the situation in the magnetized hyperaccreting disks. Differently, we have $\rho v_r^2 < P_{\text{matter}}$ or $v_r < c_s$ in the disk plane for the hyperaccreting case. Figure 11 gives the pressure ratio of magnetic stress and matter stress, and the cooling efficiency along the radius. In this figure, we confine the calculation to the disk plane without considering the funnel effect. We find that the pressure ratio remains $P_B / P_{\text{matter}} < 1$ in the disk plane; thus the accretion flow can hardly be lifted by the field pressure.
Figure 9. Disk structure with $\dot{M} = 0.1 M_\odot \, s^{-1}$ for the surface vertical field $B_0 = 10^{15}, 10^{16} \, G$, the disk magnetic field is closed with $\beta \simeq 1$, $s = 1$. The period of a central magnetar $P_t = 0.005, 0.01$, and $0.1 \, s$.

above the plane. However, since the accretion flow begins to corotate with the magnetar surface inside the Alfvén radius, the viscous heating and angular transport, which are generated by differential rotation in the disk, become ignorable. Also, the generated azimuthal component of the disk field, which is proportional to the difference between the disk and magnetar surface angular velocity, should be ignorable. As a result, the disk begins to cool and loses its angular momentum. The matter pressure $P_{\text{matter}}$ drops at the radius inside the Alfvén radius. In addition, as the magnetic pressure increases due to the accretion flow moving toward the magnetar surface, the ratio $P_B / P_{\text{matter}}$ keeps increasing and can be greater than unity as well. Thus, the magnetosphere (with its edge at $r_m$) can actually form inside the Alfvén radius, i.e., we still have the magnetosphere region with $r_s < r_m < r_A$ in the magnetized hyperaccreting disks.

Furthermore, the strong field in the Alfvén radius would cause a magnetically driven outflow from the inner disk region; the interaction between the disk and central star is favorable for launching an X-type wind from the disk–magnetosphere boundary (Shu et al. 1994), both of which also decrease the pressure of the accretion matter and increase the ratio $P_B / P_{\text{matter}}$. In contrast, it is most likely that a thermally driven outflow can form in this outer region $r > r_A$. Generally speaking, the thermal outflow can take away angular momentum and energy outside $r_A$, lower the direct flow accretion rate into the magnetosphere and magnetar surface, and give a possible energy source for a supernova explosion associated with a GRB.

In Section 5.1, we discuss the disk outflow caused by thermal heating. In Section 5.2, we discuss the funnel flow process due to the corotation and magnetic pressure near the stellar magnetosphere.
5.1. Disk Outflows

The thermal wind is induced by viscous heating (MacFadyen & Woosley 1999), or neutrino heating (Metzger et al. 2008a), depending on the dominated heating and cooling processes in the disk. In Paper II, we study the structure of the neutron star disk outflows around neutron stars without fields. As shown by Narayan & Yi (1994, 1995), if the adiabatic index $\gamma < 3/2$ in an advection-dominated flow, the Bernoulli constant of the flow is positive and a thermally driven wind can be driven from the disk. In this case, the energy carried by the outflow is expected to feed a supernova explosion, which is associated with a GRB in the collapsar scenario (MacFadyen & Woosley 1999; Kohri et al. 2005). This thermally driven wind can also exist in the magnetized disks beyond the Alfvén radius, where the matter pressure dominates over the magnetic pressure, and heating energy generated by viscous and Joule dissipation is advected inward along the disk radius. As shown in Figure 11 (right panels), the neutrino cooling efficiency increases with the increasing strength of a magnetic field for $\dot{M} \sim 10^{-1} M_\odot s^{-1}$, and the entire disk beyond the Alfvén radius becomes neutrino dominated for $\dot{M} \sim 10^{-1} M_\odot s^{-1}$. Therefore, the outflow-driven process should be significant below $\dot{M} \sim 10^{-1} M_\odot s^{-1}$. In this section, we adopt a similar treatment as in Kohri et al. (2005) and Paper II for simplicity, i.e., we only consider the viscously induced outflows and assume that the accretion rate varies as a power law in radius for the case $\dot{M} < 1.0 M_\odot s^{-1}$ as

$$M = \dot{M}_{\text{out}} \left( \frac{r}{r_{\text{out}}} \right)^{\xi},$$

where $\dot{M}_{\text{out}}$ is the initial accretion rate at the outer edge of the disk, and $\xi$ is the outflow index in the disk. Stronger outflows have a lower value of $\xi$. Using the outflow structure (45), the angular momentum Equation (46) is modified as

$$\frac{1}{1 + 2\xi} \frac{f}{3\pi} = \frac{\nu \Sigma - \dot{M} N_B}{3\pi r^2 \Omega}.$$  

Here we have assumed that the outflow has the same angular velocity as the accretion flow outside the Alfvén radius and takes away angular momentum from the disk.

Figure 12 gives the disk structure with a viscous and Joule heating induced thermally driven outflow for various outflow indices $\xi = 0.2, 0.6, 0.9$, different magnetar surface fields $B_0 = 10^{16}, 10^{17}$ G, and a fixed accretion rate $\dot{M} = 0.5 M_\odot s^{-1}$ in the radius $r = 124$ km (i.e., $30R_\text{sh}$ of a 1.4 $M_\odot$ star). Similar to the results in Paper II, the density, pressure, and neutrino luminosity decrease with increasing outflow strength. On the other hand, the ratio of $P_B/P_{\text{matter}}$ and the thickness of the disk become larger for stronger outflows. Therefore, a disk with stronger thermally driven outflow outside the Alfvén radius carries more energy away into the stellar envelope at a fixed radius. However, the total outflow energy strongly depends on the size of the region outside the Alfvén radius, where it generates viscous and Joule heating energy and induces outflows. A strong central stellar field makes the Alfvén radius larger and decreases the total heating and the potential outflow injection energy. Table 1 gives our estimate of the maximum energy carried by the outflow, which can be calculated as the
The maximum energy injection rate decreases significantly for stronger stellar fields, which make the magnetically driven wind and funnel effect more important than the thermal outflow for strong fields. Besides the thermally driven outflow in the disk outer region, a strong field inside the Alfvén radius causes the accretion flow to corotate with the stellar field, and it is possible to launch MHD wind along the field lines (Metzger et al. 2008b). Moreover, it is also clear that an X-type field configuration near the central star is favorable for launching magnetically driven wind from the disk–magnetosphere boundary (Shu et al. 1994). As shown by simulations (e.g., Romanova et al. 2008), the properties of these MHD outflows depend on many factors such as the field structure, the accretion rate, the related viscosity, and magnetic diffusivity, etc. The magnetically driven winds in the disk inner region will decrease the rate of accretion onto the magnetar surface. Also, the magnetically driven winds can provide energy to a supernova explosion together with the thermal outflow. But this issue is beyond the scope of this paper.

Figure 11. Ratio of magnetic pressure $P_B$ to the accretion matter $P_{\text{matter}}$, and ratio of neutrino emission $Q_{\nu}^{-}$ to the local heating rate $Q^{\text{heat}}(\xi = 0)$ as functions of disk radius. Upper two panels for the accretion rate $M = 0.1 M_\odot \text{s}^{-1}$. Open disk field with $B_0 = 10^{15}$ G, $\beta = 0.2$, $s = 1$ (thick solid line), $\beta = 0.6$, $s = 1$ (thick dashed line), $\beta = 1$, $s = 1$ (thick dotted line) and closed field with $\beta = 1$, $s = 1$, $P_\nu = 0.005$ s (thin solid line). Bottom two panels for the accretion rate $M = 1 M_\odot \text{s}^{-1}$, and the lines are the same as the upper panels.

### Table 1

| Disk Heating Energy Rate ($10^{51}$ erg s$^{-1}$) | Max Outflow Energy Rate ($10^{51}$ erg s$^{-1}$) |
|-----------------------------------------------|-----------------------------------------------|
| Outflow Index | $B_0 = 10^{15}$ G | $B_0 = 10^{16}$ G | $B_0 = 10^{17}$ G | $B_0 = 10^{15}$ G | $B_0 = 10^{16}$ G | $B_0 = 10^{17}$ G |
| $\xi = 0.2$ | 29.3 | 25.9 | 8.08 | 8.60 | 8.63 | 1.45 |
| $\xi = 0.6$ | 18.3 | 15.7 | 6.47 | 19.5 | 18.8 | 3.07 |
| $\xi = 0.9$ | 13.5 | 11.7 | 5.53 | 24.4 | 22.9 | 4.01 |

**Notes.** The disk heating rate is calculated in the region outside the Alfvén radius, where differential rotation and viscosity are significant: $E_{\text{heat}} = \int r_{\text{out}}^{r_{\text{in}}} 2\pi rdr$. Here the outflow index $\xi = 0.2, 0.6, 0.9$, and the accretion rate $M = 0.5 M_\odot \text{s}^{-1}$. The maximum thermally driven energy rate is estimated using Equation (47). The total thermal outflow energy can be considered as 0.1–1 fraction of the maximum energy, as discussed in Paper II.
Figure 12. Disk structure and luminosity for three values of outflow index $\xi = 0.2, 0.6, 0.9$ with the open disk field with $\beta = 0.6, s = 1, B_0 = 10^{16}, 10^{17}$ G. We give an initial accretion rate $\dot{M} = 0.5 M_\odot \text{s}^{-1}$ at the outer radius $r = 124$ km.

5.2. Funnel Flows from Disks to Magnetars

When the magnetic pressure becomes equal to or larger than the matter pressure inside the Alfvén radius $P_B \geq P_{\text{matter}}$, the funnel accretion process becomes significant. For the case of hyperaccreting disks, this process can be important only for an extremely strong field that depends on the accretion rate. For simplicity, we consider the funnel process to take in the region between the magnetosphere and the Alfvén radius, and the accretion in the disk plane will be truncated at the radius $r_m$.

The scenario of the funneled process is similar to many previous works (e.g., Lovelace et al. 1995, Figures 2 and 3; Koldoba et al. 2002, Figure 1; Frank et al. 2002, Figure 6.4). Different from the disk with cylindrical coordinates, in this section we use the spherical coordinates $(r_l, \theta, \phi)$ with the origin at the stellar center to describe the funneled flow, where $\theta$ is the angle between the axis of the disk plane and a given polar radius $r$, $\phi$ is the angle in the disk plane. Thus, the equation for the dipole field geometry line is (Frank et al. 2002)

$$r_l = r_d \frac{\sin^2 \theta}{\sin^2 \Theta},$$

where $r_d$ is the radius at which the magnetic field line passes through the disk, and $\Theta$ is the angle between the magnetic pole and disk axis. We can approximately take $r_m < r_d < r_A$.

The magnetic field along the field line reads

$$B_p(r_l) = B_0 \left( \frac{r_d}{r_l} \right)^3 \left( 4 - 3 \sin^2 \theta \right) \left( 4 - \frac{3r_l}{r_d} \frac{\sin^2 \theta}{\sin^2 \Theta} \right)^{1/2}.$$
Note that the dipole field is somewhat different from the vertical component $B_r(r) = B_0(r_a/r)^3$ in Section 3, in which $r$ is the radius of the disk in the disk plane. The mass conservation along the magnetic flux line requires the poloidal part $(r, \theta)$ of the velocity $v_p$ and the density $\rho$ to satisfy the relation
\[
\frac{4\pi v_p\rho}{B_p} = K, \tag{50}
\]
where $K$ is the constant along the flux line. Near the magnetar surface, the accretion rate satisfies
\[
4\pi r_a^2 v_s \rho_s \left( \frac{\Delta \Omega}{2\pi} \right) = \dot{M}, \tag{51}
\]
where $\Delta \Omega$ is the opening solid angle of the funnel flows, $v_s$ and $\rho_s$ are the average velocity and density at this angle $\Delta \Omega$ onto the stellar surface. We use $2\pi$ as the total solid angle because the funnel flow can be accreted toward two poles. The difference between the final accretion rate $\dot{M}$ onto the magnetar surface and the initial accretion rate $\dot{M}_{\text{ini}}$ in the disk strongly depends on the strength of the magnetically and thermally driven winds. Different from the case of Bondi accretion onto a magnetized compact star, which forms an accretion column to cover the magnetic pole, the funnel accretion from a half-thickness disk forms a ring-like belt at a latitude of $\theta$ between $\arcsin \sqrt{r_a/r}$ and $\arcsin \sqrt{r_a/r_A}$ to cover a part of the stellar surface. Recent simulations show that the geometric shape and physical properties of this belt (or say the “hot spot”) are very complicated, depending on the detailed field topology and structure of the funnel flows (Long et al. 2008 or see Romanova et al. 2008 for a review). In this paper, we take this ring-like belt to axisymmetric around the magnetic pole axis for simplicity. Appendix B gives our estimate of the ratio of $\Delta \Omega/2\pi$ in the case of hyperaccreting and neutrino-cooled disks. Combining Equations (49)–(51), we have
\[
\rho v = \frac{M(2\pi/\Delta \Omega)}{4\pi r_a^2} \left( \frac{B_p}{B_{ps}} \right), \tag{52}
\]
with $B_{ps}$ being the strength of the dipole field on the stellar surface.

In the normal cases, the infalling funneled flow from the disk plane onto a magnetized compact star will usually go through a strong shock before reaching the stellar surface (e.g., Ferrari et al. 1985; Ryu et al. 1995; Li et al. 1996; Frank et al. 2002). However, such a shock is probably unlikely to develop in the hyperaccreting disks. We now discuss it in more detail. The Mach number $\mathcal{M}$ in the funnel flow along the magnetic field pole can be calculated as
\[
\mathcal{M}^2 = \frac{v^2}{a^2} = \frac{(v \rho)^2}{\gamma \rho P_{\text{matter}}} \left( \frac{2\pi}{\Delta \Omega} \right)^2 \left( \frac{B_p}{B_{ps}} \right)^2 \frac{1}{\gamma \rho P_{\text{matter}}}, \tag{53}
\]
with $\gamma$ being the adiabatic index of the disk matter. Thus,
\[
\mathcal{M}^2 \sim 8\pi \left( \frac{M}{4\pi r_a^2} \right)^2 \left( \frac{2\pi}{\Delta \Omega} \right)^2 \left( \frac{B_p}{B_{ps}} \right)^2 \frac{1}{P_{\text{matter}}} \gamma B_p^2 \rho
\]
\[
= 6.30 \times 10^{-5} \rho^{-1} M_{12}^2 r_a^{-4} B_{ps, 16}^2 (2\pi/\Delta \Omega)^2 (P_B/P_{\text{matter}}), \tag{54}
\]
where $\rho_{12} = \rho/10^{12} \text{ g cm}^{-3}$ and $B_{ps, 16} = B_{ps}/10^{16} \text{ G}$. For a typical hyperaccreting disk with a stellar field $\sim 10^{16} \text{ G}$ and an accretion rate $M = 0.1 M_\odot \text{ s}^{-1}$, we have $P_B \sim P_{\text{matter}}$, $\rho \sim 10^{12} \text{ g cm}^{-3}$, and $2\pi/\Delta \Omega \sim 10^2$ (from Appendix B); therefore the Mach number is $\mathcal{M} \sim 1$, which shows that there is no shock wave existing in the funnel flow. A strong magnetically driven wind will decrease the density and pressure in the funnel flow, but the accretion rate $\dot{M}$ onto the star will also decrease. Although $2\pi/\Delta \Omega$ can reach up to $10^3$ and the factor $P_B/P_{\text{matter}}$ increases for a strong field $\sim 10^{17} \text{ G}$ with $M = 0.1 M_\odot \text{ s}^{-1}$, the decreasing factor $\rho_{12}^{-1} B_{ps, 16}$ makes the Mach number hardly exceed unity.

In the funnel channel, the flow is accelerated via the stellar gravitational force. The gravitational binding energy is converted to the kinematic energy of the flow; then the kinematic energy will be converted to the heating energy near the magnetar surface, which is cooled via thermal neutrino emission. The total energy release rate in the funnel process can be approximately taken as
\[
\dot{E}_{\text{funnel}} \sim \frac{GM \dot{M}}{4} \left( \frac{1}{r_s} - \frac{1}{r_A} \right), \tag{55}
\]
and the energy equation in the magnetar surface boundary is
\[
\frac{7}{8} \frac{\sigma_B T^4}{\tau_\nu} \times S = \frac{GM \dot{M}}{4r_s} \epsilon, \tag{56}
\]
where the parameter $\epsilon$ is introduced to show the combined efficiency of acceleration and cooling. The accretion rate here is adopted as the final accretion rate onto the magnetar. The area of the “hot spot” is $S = \Delta \Omega r_s^2$. Here we consider the new born magnetar with a lifetime $\geq 100$ ms (Dessart et al. 2009); thus the temperature of the “hot spot” can be significantly higher than the other region of the magnetar surface. The temperature in the “hot spot” can be calculated as
\[
T = 7.4 \times 10^{10} (2\pi M_{1,4} \dot{M}_{-1} \epsilon/\Delta \Omega)^{1/4} r_s^{-3/4} \text{ K}, \tag{57}
\]
where we take $\epsilon$ as 0.5. Since the temperature is insensitive to the neutrino optical depth as $\tau_{1/4}$, we take $\tau_\nu \sim 1$ in our calculation. The temperature in this surface region increases slightly with increasing ratio of $2\pi/\Delta \Omega$: $T = 1.3 \times 10^{11}$ K (10 MeV) for $2\pi/\Delta \Omega = 10$, or $T = 2.3 \times 10^{11}$ K (20 MeV) for $2\pi/\Delta \Omega = 100$.

The neutrino pair annihilation process $\nu_e + \bar{\nu}_e \rightarrow e^- + e^+$ is the most important mechanism for providing the energy for a relativistic jet formed from a neutron star disk with weak stellar magnetic field. According to Paper II, a neutron star disk with a hot stellar surface layer could increase the neutrino annihilation luminosity by about 1 order of magnitude higher compared with the black hole disk. If the stellar field is strong up to $\geq 10^{15}$ G, as discussed in Section 3, the disk with strong magnetic field will have a higher density, pressure, and neutrino luminosity; therefore the annihilation rate above the disk plane will also be higher than that emitted from a disk with the same radius range without field or with weak field. This conclusion is similar to that in Lei et al. (2009, their Figure 2). However, the strong field truncates the disk plane accreting in the inner region and will decrease the total annihilation luminosity from the disk. In this case, the annihilation mechanism will be more significant from the magnetar surface where it accretes the funnel flow. Similar to the case without field, the total neutrino annihilation rate is contributed by three components:
The annihilation energy along the pole is 

\[
\frac{\sin \theta}{\Delta \Omega} \lesssim \frac{10^{-2}}{2 \pi} \text{ s}^{-1} \text{ cm}^{-2}
\]

for the value of \( \sin \theta = 0.2, 0.4, 0.6, 0.8, 1.0 \), \( \Delta \Omega/2\pi = 10^{-2} \), accretion rate \( M = 0.5 M_\odot \text{ s}^{-1} \), and the energy release efficiency \( \epsilon = 0.5 \). The integrated annihilation energy along the pole is

\[
\int_{r_\nu}^{r_\nu} \nu dz = 4.95 \times 10^{33}, 3.31 \times 10^{33}, 5.92 \times 10^{32}, 1.31 \times 10^{32}, \text{ and } 9.21 \times 10^{31} \text{ erg s}^{-1} \text{ cm}^{-2}
\]

for the value of \( \sin \theta = 0.2, 0.4, 0.6, 0.8, \text{ and } 1.0 \), respectively.

the annihilation between the neutrinos both from the disk, both from the stellar surface, and one from the disk and the other from the stellar surface, respectively. If the stellar surface is bright to the neutrino emission, then it will contribute to the main annihilation luminosity in the accreting system. This is why the neutron star accretion system will have a brighter annihilation luminosity than its black hole counterpart. In the funnel process, on the other hand, for the stellar surface area where there is a solid angle \( \Delta \Omega \ll 2 \pi \) and higher latitude \( \theta_{\nu} \approx \arcsin \sqrt{r_{\nu}/r_A} \) to emit a thermal neutrino will be more geometrically concentrative and hotter than that without field, so the annihilation rate will be more efficient (Birkl et al. 2007). Figure 13 shows the neutrino annihilation rate as a function of height along the magnetic poles with different latitudes of the emitting “hot spot” area.

We adopt the formula of neutrino annihilation as in Popham et al. (1999) and Rosswog et al. (2003). The difference here is that the neutrinosphere is a ring-like belt around the magnetar not a plane disk. The neutrino annihilation rate \( l_{\nu} \) from a higher latitude ring-like belt area is larger near the stellar surface, and one from the disk and the other from the stellar surface.

6. DISCUSSIONS

\subsection{6.1. Joule Dissipation}

In Section 3, we discuss two competitive sets of effects to affect the disk structure and neutrino emission, i.e., the microphysical quantum effects and the macrophysics of magnetic field coupling in the disk MHD equations. The quantum effects decrease the pressure, and luminosity with increasing strength of field, but magnetic coupling makes these quantities higher. However, in most cases, the magnetic field coupling is more important than the quantum effects at a microphysical scale. We discuss this conclusion in detail in this section. In fact, in order to see the quantum effects clearly in Figure 4, we adopt a unified field (i.e., without a dipolar form) in the disk, and find that this microphysical effect becomes more significant in the disk region at a relatively larger radius. However, such a situation is unclear if we adopt the dipolar field from the stellar surface; thus the microphysical quantum effects will be less important as shown in Figure 4. On the other hand, as shown in the angular momentum equations (35) and (37) and the local energy equations (38) and (39), stronger magnetic fields will be more important in transferring angular momentum in the disk as well as to heat the disk by Joule dissipation. Therefore, it is not difficult to understand why a disk with stronger field will be hotter and denser with higher pressure and brighter neutrino luminosity.

In fact, the parameter \( \beta \), which gives the relation between the vertical and azimuthal components of the disk magnetic field as in Equation (32) (i.e., the closed field configuration), can be calculated using Ohm’s law \( j = \sigma_m (E + v \times B)/c \) and Ampere’s law. 

\begin{table}[h]
\centering
\begin{tabular}{|l|l|}
\hline
Field Strength & Disk Properties \\
\hline
\hline
\( B_0 \sim 10^{14} \text{ G} \) & No funnel accretion, disk is similar to that without fields \\
\hline
\( B_0 \sim 10^{15} \text{ G} \) & No funnel accretion, MHD coupling is important in the disk inner region, thermal outflow \\
\hline
\( B_0 \sim 10^{16} \text{ G} \) & Weak funnel accretion, disk is denser, hotter with higher pressure, brighter \( l_{\nu} \) from the disk, thermal outflows from \( r > r_A \), magnetic winds inside \\
\hline
\( B_0 \sim 10^{17} \text{ G} \) & Strong funnel accretion, but no shock, much brighter \( l_{\nu} \) from the stellar “hot spot,” significantly magnetically dominated wind \\
\hline
\end{tabular}
\end{table}

\textbf{Notes.} The accretion rate near the Alfvén radius \( r_A \) or the stellar surface \( r_* \) (depending on \( r_A > r_* \) or \( r_A < r_* \), respectively) is \( M = 0.1 M_\odot \text{ s}^{-1} \). The neutrino cooling emission is efficient in this case, and the accretion flow is an NDAF. The inner region satisfying the self-similar structure (Papers I and II) can be ignored even when the disk is similar to that without fields.
Figure 14. Neutrino luminosity for accretion rate $\dot{M} = 0.01, 0.1, 1 \, M_\odot \, s^{-1}$ and magnetar surface field $B_0 = 10^{15}, 10^{16},$ and $10^{17} \, G$ without Joule dissipation in the disk. The open field model is adopted with $\beta = 0.6$ and $s = 1$.

law $\nabla \times \mathbf{B} = 4\pi \mathbf{j}/c$. We have (Lee 1999)

$$\beta = \frac{4\pi \sigma_m}{c^2} H \Omega_K = \frac{H \Omega_K}{\eta_m}, \tag{58}$$

where $\sigma_m$ and $\eta_m$ are the microscopic electrical conductivity and magnetic diffusivity. However, as argued by some authors (e.g., Bisnovatyi-Kogan & Ruzmaikin 1976; Lovelace et al. 1995; Bisnovatyi-Kogan & Lovelace 1997), since the accretion flow is in general turbulent, the microscopic $\eta_m$ should be replaced by a turbulent transport parameter $\eta_t$, which can be considered as being comparable to the turbulent $\alpha$ viscosity $\eta_t \sim \nu \sim \alpha c_s H$. In this case, Equation (58) gives the relation $\beta \sim \nu \Omega_K / \alpha c_s H$. However, this new relation has its limitations. As discussed by Lovelace et al. (1995), the twist $|B_o / B_z|$ can never be much larger than unity. Therefore, in this paper, we take $\beta$ as a parameter rather than an obtained variable. Similar to Lai (1998), we consider that $\beta \sim 1$ as the maximum twist of the original magnetic field from the central stellar surface. On the other hand, we take the magnetic diffusivity $\eta_t$ in the local energy equation to be of the order of the turbulent viscosity. As a result, the magnetic diffusivity can be smaller than the classical diffusivity shown by Ghosh & Lamb (1979a, 1979b; see Lovelace et al. 1995 for further discussion).

The Joule heating plays a key role in affecting the neutrino luminosity from the hyperaccreting disks with small radius, because the ratio of the viscous heating and Joule dissipation will decrease with small radius, and the Joule dissipation will quickly dominate over the viscosity as the main heating source in the disk. Of course all of these results only occur in the region outside the Alfvén radius $r_A$, although we usually extend our calculations to the inner region. Figure 14 shows the neutrino cooling luminosity without Joule dissipation. Compared with the neutrino luminosity from the disk with Joule dissipation in Figures 5, 7, and 9, the main difference is that the neutrino luminosity drops quickly in relatively strong fields at small radius without fields, except for the case when the disk has a relatively low accretion rate and is advection dominated. This result is consistent with the above discussion that Joule dissipation plays a key role in heating the disk’s inner region near the Alfvén radius.

6.2. Application to GRBs

As discussed in Section 5.2, the funnel accretion makes the ring-like belt of the “hot spot” more concentrative with higher latitude than the equatorial accretion without fields or with moderate fields. Also, the neutrino luminosity outside the Alfvén radius is brighter for stronger stellar fields. As a result, the neutrino annihilation efficiency will be significantly increased in the hyperaccreting disks around magnetars compared to the normal neutron star disks, and definitely more efficient than the black hole disks. However, a higher concentration neutrino emission region means higher temperature on the magnetar surface with more massive neutrino-driven winds (Qian & Woosley 1996; Dessart et al. 2009), which will make the jet heavily baryon loaded. The problem is whether the more powerful annihilation and massive mass loss rate driven by neutrino absorption can work together to produce a relativistic jet required for GRB explosion? This problem is similar to that in the neutron star disks without fields, which has been discussed in Paper II. We consider that a GRB-related jet forms along the stellar poles (i.e., the disk axis if there is no field), and only the wind materials along this axis can feed the jet and
affect the bulk Lorentz factor of the jet. The wind ejected in an off-axis direction and that evaporated from the disk will not affect the jet, but only affect the potential supernova explosion associated with the GRB. If there is no disk around the central compact star, the neutrino-driven wind from the star surface can be reasonably approximated as a quasi-steady spherical outflow (Qian & Woosley 1996). However, the hyperaccreting accretion disk around the star can significantly change the distribution of the neutrino-driven wind. The entire stellar surface will be very hot and will inject a heavily mass-loading wind along the axis after its formation in a timescale of $\sim 100$ ms, and will then cool down with the power law $M_{\text{wind}} \propto t^{-5/3}$. In most cases, the surface ring-like belt region where it directly accretes the disk flow will be the hottest region in the disk for time $\gg 100$ ms. However, most parts of the wind from this ring-like belt should be off-axis. The polar region that directly drives a wind along the axis is cooler than the equatorial ring-like belt. Even then, we use the temperature of the hot ring-like belt to estimate the maximum strength of the neutrino-driven wind along the poles and calculate the bulk Lorentz factor of the wind along the pole with field $\lesssim 10^{15} \text{G}$ as in Paper II. A moderate or ultrarelativistic jet can be produced in the hyperaccreting neutron star system with sufficiently high disk accretion rate and bright boundary emission.

However, the situation in a strong field will be different in some aspects. First of all, the neutrino-to-nucleon absorption reaction rate, which is dominated by

$$\nu_e + n = p + e^-$$ \hspace{1cm} (59)

will be significantly reduced in strong fields $B_0 \gtrsim 10^{16} \text{G}$ (Duan & Qian 2004, 2005) because of the quantum effects. As a result, the mass loss rate from the stellar surface, which is proportional to the absorption rates, will decrease as well. Moreover, we have to note that part of the newly generated $e^-e^+$ pairs in the neutrino absorption reaction will move along the closed field lines from the stellar surface rather than be injected far away from the central star. There are some open field lines in the surface region near the stellar magnetic poles (Lovelace et al. 1995), but many of them are more likely to induce winds off-axis rather than the neutrino-driven wind. As a result, the total mass loss rate $M_{\text{polar}}$ along the magnetic polar region will only be a fraction of the total mass loss rate $M_{\text{wind}}$. On the other hand, the field structure around the stellar surface will also affect the neutrino annihilation process, because part of the generated $e^-e^+$ pairs in the reaction $\nu_e + \bar{\nu}_e \leftrightarrow e^- + e^+$ will also move along the closed field lines. Therefore, part of the neutrino annihilation energy cannot be loaded into the jet, which propagates along the magnetic poles. The precise fraction of the total annihilation energy feeding the polar jet depends on the energy–momentum distribution of the $e^-e^+$ pairs above the central star and the entire disk, as well as the ratio of $e^-e^+$ plasma pressure to the magnetic pressure. The reliable estimate of these considerations requires further MHD simulations. However, since the annihilation process happens in the extended space above the accreting system, while the neutrino absorption reaction mainly occurs around the stellar surface with a sufficiently high density of nucleons, the strong magnetic fields will play a more significant role in changing the properties of the neutrino-driven wind rather than the annihilation process. Therefore, the mass loaded in the jet will decrease more significantly than that of the annihilation energy. Based on the above considerations, that is, the neutrino annihilation efficiency is larger for a magnetized disk, such an efficiency can be even higher for the funnel accretion process, and the mass loss along the magnetic poles $M_{\text{wind}}$ decreases in strong fields, we can conclude that the jet along the magnetic poles can be accelerated to a larger bulk Lorentz factor compared with its neutron star counterparts without fields or with weak fields. In addition, if the magnetic pole and the disk axis do not overlap ($\Theta \neq 90^\circ$), the direction of the relativistic jet and the disk rotation axis will also not overlap. This is interesting because if the annihilation rate from the disk is much less than that from the star, the jet will precess along the disk axis with the period of the magnetar.

Furthermore, another significant difference between a magnetar and a non-magnetized neutron star in the hyperaccreting systems is that a thermally dominated neutrino-driven wind from the stellar surface may switch to be magnetically dominated instead after seconds of the compact star formation (Thompson et al. 2004; Metzger et al. 2007). The condition of the magnetic wind can be parameterized using the term of magnetization

$$\sigma = \frac{\Phi_B^2 \Omega^2}{M_{\text{wind}} c^3},$$ \hspace{1cm} (61)

where $\Phi_B$ is the magnetic flux. The parameter $\sigma$, which strongly depends on the rotational period of the central star, is the maximum Lorentz factor the wind can achieve if all the magnetic energy is converted into kinetic energy, either via magnetic reconnection (Spruit et al. 2001) or collimation by the interaction between the wind and the stellar envelope (Bucciantini et al. 2006, 2007, 2009; Tchekhovskoy et al. 2008, 2009). The critical period for $\sigma \gtrsim 1$, i.e., the wind to be magnetically dominated and relativistic is

$$P_{r c} = 85 B_0^{-2/5}_{\text{gy}} M_{\text{wind}}^{-1/2} 5 \text{ms}.$$ \hspace{1cm} (62)

The critical period $P_{r c}$ increases with increasing stellar field, and decreases with increasing mass loss rate $\dot{M}$. The properties of the magnetically dominated wind will be different with different ranges of $\sigma$ for the stellar period $P_{\ast} \leq P_{r c}$. The wind is collimated along the magnetic poles for $\sigma < 5$, but will be distributed around the direction at low latitudes and the equatorial plane for $\sigma > 30$. For example, Bucciantini et al. (2006, 2007, 2009) showed that, the wind–stellar envelope interaction may provide a viable mechanism for collimating the jet along the polar region. Their works are based on the consideration that a cavity with the radius $> 10^8 \text{cm}$ has been evacuated by the outgoing supernova shock before the compact star formation. Since the wind with high $\sigma$ is ejected around the equatorial plane initially, whether or not the hyperaccreting disk with a small size $10^7-10^8 \text{cm}$ can help or prevent the jet collimation needs to be further studied.

It is said that the energy of the magnetically driven wind is extracted by the central star spin-down process. However, in the magnetar–disk system, if the magnetically driven wind can be actually collimated along the magnetic polar region, then the stellar spin-down mechanism and neutrino annihilation from the “hot spot” and the entire disk can work together to provide the energy to the polar jet and accelerate the jet to an ultrarelativistic speed. In this general case, the bulk Lorentz factor can be estimated as

$$\Gamma = \frac{\dot{E}}{M_{\text{polar}} c^2},$$ \hspace{1cm} (63)
where the energy feed rate is the summation of neutrino annihilation and the magnetic energy from rotational extraction,

\[
\dot{E} = \dot{E}_{\text{anni}} + \dot{E}_{\text{mag}} = L_{\nu\nu} f_k + \dot{E}_{\text{rot}} = L_{\nu} (f_{\nu\nu} f_k) + \dot{E}_{\text{rot}} = \frac{GM^2}{4r_s} \left( f_{\nu\nu} f_k \right) + \frac{2}{5} \Omega^2 \dot{\Omega} f_{\text{rot}},
\]

(64)

with \(f_{\nu\nu}, f_k, f_{\text{rot}}\) being the neutrino annihilation efficiency \(L_{\nu\nu}/L_{\nu}\), the fraction of the deposited annihilation energy to provide the kinetic energy of the stellar wind, and the ratio of magnetic to total spin-down energy, respectively. \(M_{\text{pol}}\) as a fraction of \(M_{\text{wind}}\) is the wind mass loss rate along the polar region (see Paper II for more discussion about the mass loss rate). The ratio of \(E_{\text{anni}}\) to \(E_{\text{mag}}\) is

\[
\frac{\dot{E}_{\text{anni}}}{\dot{E}_{\text{mag}}} = 2.1 \times 10^5 M_{-1} \tau_j P_{r,3} f_{\nu\nu} \left( f_{\nu\nu} f_k \right),
\]

(65)

where \(\tau_j\) is the typical spin-down timescale. Based on the analysis in this paper, we take \(\tau_j \sim 0.5, f_{\nu\nu} \sim 0.005–0.01\) for \(M_{-1} \sim 1, f_k \sim f_{\text{rot}},\) and \(\tau_j \sim 7.6 B_{0,16}^2 (P_{r} / 1 \text{ ms})^2\) s \(^6\), and then the critical value of the period \(P\) for the ratio \(\dot{E}_{\text{anni}}/\dot{E}_{\text{mag}}\) being less than unity is roughly \(P_{r} \lesssim 4\) ms. The critical value \(\sim 4\) ms will decrease even if the disk accretion rate is higher with a higher annihilation efficiency \(f_{\nu\nu},\) or the spin-down timescale is longer. Keep in mind that this critical value is less than \(P_{r}\) in Equation (62), which shows the wind to be magnetically dominated and relativistic without neutrino annihilation. Therefore, in the extreme case of a millisecond magnetar, the magnetically dominated jet extracts the rotation energy as the main energy source. If the stellar spin period is around tens to hundreds of milliseconds, the polar jet can be fed by the neutrino annihilation and magnetic energy together, and has a bulk Lorentz factor even higher than \(\tau_j.\) On the other hand, for the magnetar period \(P_r \geq P_{r}\) in Equation (62),\(^7\) the jet formation process goes back to being thermally driven and fed by the annihilation process as discussed in Paper II.\(^8\)

6.3. Disk Nucleosynthesis and r-process Nucleosynthesis

Besides the GRB or GRB-like (e.g., X-ray flashes) phenomena with their associated supernovae, the hyperaccreting magnetar system can also generate enough \(^{56}\)Ni and other elements from the disk for the supernova (MacFadyen & Woosley 1999; Kohri et al. 2005; Nagataki et al. 2006, 2007), and produce heavier elements in the neutrino-driven wind via r-process nucleosynthesis.

Simulation works based on the collapsar scenario (MacFadyen & Woosley 1999; Nagataki et al. 2006, 2007) showed that \(^{56}\)Ni produced in the jet of a collapsar is not sufficient to explain the observed amount in a supernova with a duration about \(\sim 10\) s (e.g., Mazzali et al. 2006; Soderberg et al. 2006). As a result, the majority of \(^{56}\)Ni with mass \(\sim 10^{-2} M_{\odot}\) might be synthesized in the accretion disk or in the disk outflows. Nagataki et al. (2007) calculated that \(^{56}\)Ni synthesized in the disk can reach \(\sim 10^{-3} M_{\odot}\), but it should be carried out by a later phase outflow or the viscous induced outflow. Also, Kohri et al. (2005) discussed that the recombination process of nucleons \((n, p)\) into nuclei can happen in the outflow and release an energy of about \(8\) MeV per nucleon. In the hyperaccreting magnetar scenario, on the other hand, since the disk becomes hotter and denser with higher pressure due to a stronger field, the ejected \(^{56}\)Ni via disk outflow should be more than that estimated in the same region of the collapsar without fields. However, if the magnetar field strength is high enough and the funnel accretion is obvious, the disk region outside the Alfvén radius \(r_A\) will be relatively small and will eject even less \(^{56}\)Ni. Also, in the corotation region without viscous heating at radius less than \(r_A\), the disk will be cooled down and will produce much less \(^{56}\)Ni.

The main properties for determining the r-process production are the asymptotic wind entropy \(S^a\), asymptotic electron fraction \(Y_e\), and dynamical timescale \(t_{\text{dyn}}.\) In general, higher wind entropy, lower electron fraction, and short dynamical timescale are more favorable for producing heavier nuclei with higher maximum A number (Meyer & Brown 1997; see also Thompson 2003). Although the winds are thought to be candidates for the r-process, recent models without fields showed that it is difficult for the winds to produce robust r-process nucleosynthesis for the “classical” neutron star with a 1.4 \(M_{\odot}\) mass and 10 km radius (e.g., Qian & Woosley 1996; Thompson et al. 2001). Thompson (2003) roughly estimated the r-process in the strong field environment with \(B_0 \geq 6 \times 10^{14}\) G. Since the strong field can trap the wind in the neutrino heating region, the amplification of the trapping timescale can lead to the amplification of the entropy. This amplification may be sufficient to yield a robust third-peak r-process nucleosynthesis. In addition, Metzger et al. (2007) showed that the presence of a magnetar field \(\sim 2 \times 10^{14}\) G and mildly rapid rotation \(\sim 10\) ms moves the ratio \((S^a)^3/t_{\text{dyn}},\) which is the critical wind parameter to determine the condition for an r-process, to be an order of magnitude more favorable for the third-peak r-process nucleosynthesis. Therefore, the magnetic fields from the central magnetar may play a significant role in inducing a successful strong r-process nucleosynthesis. However, the problem is that all works considering magnetic fields do not include the strong field quantum effects on the state of equations of the wind. In other words, they do not consider the Landau level effect and the neutrino absorption reaction rate as modified by the strong fields. As discussed in Section 6.2, the strong field can decrease the neutrino absorption reaction rate and also decrease the mass loss rate of the neutrino-driven wind as well. Also, the strong field can affect the density and pressure distribution in the wind. Therefore, it will be interesting to consider whether the microphysical change in the strong field can affect the final results of the r-process nucleosynthesis. Furthermore, if we look into the scenario of hyperaccreting disks, we should keep in mind that the disk will increase the wind entropy and affect its
distribution in the polar region where it propagates the stellar neutrino-driven wind (e.g., Nagataki et al. 2006, 2007). Another possibility is that the disk outflows and X-type winds, rather than the stellar neutrino-driven winds, will also produce the r-process elements with $A > 130$, because the neutrino-to-proton ratio in the disk and the outflows are sufficiently high, and the mass loss rate from the disk is much higher than that from the stellar surface (Kohri et al. 2005). However, whether the disk outflows can be the site for r-process nucleosynthesis is still under debate (Metzger et al. 2008b). Since the pressure of the outflow will drop quickly above the disk and be less than the magnetic pressure, the magnetic field above the disk may also play a significant role in increasing the outflow entropy as discussed in Thompson (2003). As a result, more work should be done to understand the influence of a strong field on the r-process in hyperaccreting magnetar systems.

6.4. A Unified Scenario of Collapse-related-GRB Models

Let us compare the hyperaccreting neutron star/magnetar model with the isolated magnetar model, both of which are proposed to be able to produce GRB and GRB-like events. The hyperaccreting neutron star/magnetar disks can form in collapsars or compact binary mergers. In the collapsar scenario, rotation and magnetic fields make it possible for the core to collapse into a massive neutron star/magnetar rather than a black hole. On the other hand, isolated magnetars are proposed to form via the rotating Type-Ib/c supernovae, the mergers of compact binaries, or the accretion-induced collapse of white dwarfs. Therefore, the progenitors of the hyperaccreting compact star model and the isolated magnetar model are actually very similar to each other. The main difference is the environment: the disk around the neutron star/magnetar will produce significant phenomena that will not be produced by the isolated magnetar. Let us see the case of massive star collapse for example. The isolated magnetar forms after a successful supernova explosion, and the supernova shock has created a cavity around the magnetar before it drives magnetically dominated and Poynting-dominated winds. In contrast, the unsuccessful or weak ongoing shock leads to the hyperaccreting system. The disk material around the neutron star/magnetar comes from the continuous infalling stellar envelope, or the fallback of stellar material that has been ejected by the shock but cannot reach sufficiently high velocity to escape the gravitational potential of the core. Kumar et al. (2008) discussed the accretion rate of different accretion stages, in which the accretion rate is different depending on which stellar zone is accreted.

We try to use a unified point of view to consider the outcomes of massive star collapse. If the core collapse can initiate the formation of a successful supernova, then an isolated black hole or a rotating magnetar can form in the supernova remnant. Vietri & Stella (1998, 1999) discussed the possibility of a neutron star further losing its angular momentum by magnetic dipole or gravitational wave radiation over a long time and collapsing into an accreting black hole. In addition, the magnetar model shows the possibility that a variety of magnetic activities from the isolated magnetar can produce a GRB explosion after the supernova formation within a short time. On the other hand, if the outgoing shock formed during a core collapse cannot compete with the continuous accretion from the stellar envelope, the collapse leads to the formation of a collapsar system, in which the type of the central compact object depends on many factors such as rotation, equations of state, and so on. The black hole disk may produce a GRB explosion via neutrino annihilation or MHD processes, but the annihilation mechanism may not produce sufficient energy for energetic GRBs. The neutron star disk will increase the annihilation efficiency compared to its black hole counterpart, and the increased efficiency will be even higher if the central star is a magnetar. In the hyperaccreting magnetar system, the increased annihilation process and the magnetically driven pulsar wind can work together to generate a more powerful jet than that generated by a single mechanism. The outflows and magnetically dominated winds from the disk can possibly feed a late-disk-induced supernova associated with the GRB. In other words, the unified scenario shows that the outcomes have a close relation with the initial stellar properties and the core-collapse process itself.

However, such a unified scenario as the GRB central engine candidate cannot be confirmed directly. What we can observe are the GRB-related properties such as photon light curve, spectrum, neutrino emission, different bands of afterglows, which can be traced back to the central engine and show evidence for the existence of a central neutron star. We do not want to show further evidence here (see Dai 2004; Dai et al. 2006; Fan & Xu 2006; Yu & Dai 2007 for more details). Finally, we want to mention two points. One is that, the intermediate case, i.e., the fallback material forms a normal disk around a magnetar, has been discovered (Wang et al. 2006). Although the disk-radiated IR emission is very different from the hyperaccreting disk, this discovery shows a link between the isolated magnetar and hyperaccreting magnetar in the core collapses. Another point is that, since some stars that are less massive than Wolf–Rayet stars are more likely to form neutron stars and magnetars, we cannot preclude the possibility that the collapse of stars with an intermediate mass can also lead to the hyperaccreting neutron star/magnetar systems and produce the GRB-like phenomena. In fact, some evidence show that GRBs may be associated with Type II supernova (Germany et al. 2000; Rigon et al. 2003), although the evidence is still controversial today.

7. CONCLUSIONS

Hyperaccreting neutron stars or magnetar disk systems cooled via neutrino emission can form by the mergers of compact star binaries or collapses of rotational massive stars. Strong fields of the magnetar can play a significant role in affecting the disk properties and even changing the accretion process. Our motivation in this paper is to investigate the influence of such strong magnetic fields on the disks, and discuss implications of the magnetar disk systems for the GRB and GRB-like events. Our conclusions are as follows.

1. We consider that the magnetar field has a dipolar vertical component $B_z$. The differential rotation between the disk and the magnetar will generate a toroidal field component $B_\phi$, as well as a relatively weak radial component $B_r$. The generated field can have an open or a closed configuration, depending on the disk’s viscous turbulence, magnetic diffusivity, disk angular velocity, and twist limitation. Similar to previous works, we use the parameter $\beta$ to measure the strength of the toroidal field (Equations (31) and (32)). The generated large-scale disk field coupled with the accretion flows will transfer the angular momentum in the radial direction and heat the disk via Joule dissipation together with viscous stress outside the Alfvén radius $r_A$. On the other hand, since the distribution and energy of electrons change significantly in the strong field environment, the disk pressure and a variety of neutrino cooling processes will
be different compared to the case without fields. Therefore, we discuss the quantum effects of the strong fields on the disk thermodynamic and microphysical processes in Section 2, and list the MHD conservation equations to describe the behavior of the large-scale magnetic field coupling in the disk in Section 3.

2. The quantum effects and field coupling in MHD equations play competitive roles in changing the disk properties, the former to decrease the pressure, density, and neutrino luminosity with increasing field strength (Figure 4), while the latter to increase them (Figure 5). However, in most cases the large-scale field coupling is more significant than the microphysical quantum effect (Figure 7). Moreover, strong fields will change the electron fraction distribution \( Y_e \) in the disk significantly. A larger peak of electron chemical potential \( \eta_e \) with more degenerate electron states is obtained by stronger fields, while the change of \( \eta_e \) becomes more obvious for stronger fields.

3. Similar to the neutron star disk without fields, the values of density, pressure, temperature, neutrino luminosity, and electron chemical potential become higher for higher accretion rate, but the electron fraction \( Y_e \) decreases with increasing accretion rate.

4. The magnetized disk maintaining a plane geometry would be more favorable for an open field configuration rather than a closed one. However, we still consider the two cases for completeness (Figures 6 and 9). For the disk with an open field and \( M = 0.1 M_\odot \text{ s}^{-1} \), a higher ratio of \( \beta/s \) leads to higher density and pressure in the entire disk plane, higher temperature and neutrino luminosity in the inner part of the disk, as well as lower \( Y_e \) in the outer part. Here \( s \) is the ratio of disk angular velocity and Keplerian velocity. Also, electrons have more degeneracy at \( \sim 20-40 \) km for higher \( \beta/s \). On the other hand, the disk properties with a closed field depend not only on the values of \( \beta \) and \( s \), but also on the spin period of the central magnetar. The shorter period of the central star decreases the disk density, pressure, and \( \eta_e \), but increases \( Y_e \) in the outer part of the disk. The effects of spin period are obvious only for sufficient field strengths (e.g., \( \sim 10^{16} \text{ G} \) for the accretion rate \( 0.1 M_\odot \text{ s}^{-1} \)) from the central star.

5. The accretion flow in the disk plane outside the Alfvén radius is viscously stable. However, whether the disk is thermally stable depends on many factors such as the disk region, magnetic field strength, disk angular velocity, and accretion rate. Generally speaking, the disk will be definitely thermally unstable if its non-field disk counterpart with the same accretion rate is unstable. The disk region can also be thermally unstable even if its non-field counterpart is stable, if the region is near the Alfvén radius where the magnetic field plays a more important role in transferring the angular momentum and heating the disk than the viscous stress.

6. The thermally driven outflows can also exist in the magnetar disk beyond the Alfvén radius for the case of \( M < 1.0 M_\odot \text{ s}^{-1} \). We assume the accretion rate as a power law in radius for simplicity (Equation (45)). The outflow will take away the disk angular momentum (Equation (46)) and may provide energy for a supernova associated with a GRB explosion. The disk density, pressure, and neutrino luminosity decrease with increasing outflow strength. Also, the ratio of magnetic and matter pressure \( P_B/P_{\text{matter}} \) and the thickness of the disk become larger for stronger outflows. However, since the total energy taken by the thermal outflow depends on the size of the disk plane outside the Alfvén radius, the total energy injection rate from the outflow decreases significantly for stronger stellar fields (Table 1). Besides the thermally driven outflow, strong fields inside the Alfvén radius lead the accretion flow to corotate with the stellar field, which makes it possible to launch MHD winds along the field lines and generate the X-type wind near the disk–magnetosphere boundary. These magnetically dominated winds are nonrelativistic and may provide energy for a supernova explosion.

7. In hyperaccreting disks, the funnel accretion can only be important for extremely strong fields, which depend on the accretion rate. The accretion process along the disk plane will be truncated in the stellar magnetosphere, so most of the accretion flow can be approximately considered as being lifted along the closed field lines onto the magnetar surface. In most cases, the hyperaccreting infalling funnel flow is unlikely to develop a shock with Mach number greater than unity (Equation (54)). The flow is accelerated by the gravitational force and transfers the gravitational binding energy to the kinematic energy and next to the heating energy near the magnetar surface. The funnel flow will cover a ring-like belt of “hot spot” around the magnetar surface and emit thermal neutrinos. Because the temperature is higher and the neutrino emission region is more concentrated than the hyperaccreting neutron star without fields, funnel accretion can accumulate more powerful neutrino annihilation luminosity (Figure 13).

8. The neutrino annihilation process both from the magnetar surface and from the disk plane will be higher than that without fields. Moreover, the neutrino annihilation mechanism and the magnetic activity from the stellar surface (i.e., the pulsar wind mechanism) can work together to generate and feed an ultrarelativistic jet along the stellar magnetic poles. If the stellar spin period is sufficiently short (e.g., \( \sim 4 \) ms for the field \( \sim 10^{16} \text{ G} \) and \( M = 0.1 M_\odot \text{ s}^{-1} \)), the jet from the magnetar will be magnetically dominated and mainly fed by extraction of the stellar rotational energy. If the magnetar spin period is long (i.e., longer than the critical value \( P_\pi \) in Equation (62)), the jet is thermally driven and fed by the annihilation process. In the intermediate case, on the other hand, the relativistic jet can be launched by the pulsar-wind-like process and neutrino annihilation together.

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APPENDIX A

DISK EQUATIONS WITHOUT FIELD

The thermodynamical, microphysical, and conservation equations in a hyperaccreting disk without fields can be seen in many previous works. In order to compare them with the case with strong fields, we list them systematically in this appendix.
Here we do not consider equations of the neutron star inner disk with a self-similar structure as discussed in Papers I and II.

The electron/positron density number reads

\[ n_{e^\pm} = \frac{(m_e c)^3}{\pi^2 \hbar^3} \int_0^\infty \frac{x^2 \, dx}{e^{(m_e c^2 \sqrt{x^2 + m_e^2})/k_B T} + 1}, \] (A1)

and the electron/positron pressure is

\[ P_{e^\pm} = \frac{1}{3} \frac{m_e^2 c^5}{\pi^2 \hbar^3} \int_0^\infty \frac{x^4 \, dx}{\sqrt{x^2 + 1} \, e^{(m_e c^2 \sqrt{x^2 + m_e^2})/k_B T} + 1}. \] (A2)

The total pressure is the summation of the pressure of electrons, nucleons, radiation, and neutrinos (without magnetic pressure):

\[ P = P_{e^-} + P_{e^+} + P_{\text{nuc}} + P_{\text{rad}} + P_v, \] (A3)

where \( P_{\text{nuc}} = \rho RT \) and \( P_{\text{rad}} = a_B T^4/3 \) with \( R \) being the gas constant and \( a_B \) being the radiation constant.

The total neutrino cooling rate is the same as Equation (9) with various absorption and scattering depths as shown in the beginning of Section 2.2.

The neutrino cooling by electron–positron captures by nucleons are contributed by three terms as follows:

\[ \dot{q}_{\nu e^-\rightarrow e^+\nu_e} = \tilde{K} n_p \int_q^\infty (\varepsilon^2 - 1)^{1/2} (\varepsilon - q)^3 f_e \, d\varepsilon, \] (A4)

\[ \dot{q}_{\nu e^+\rightarrow e^-\nu_e} = \tilde{K} n_p \int_1^q (\varepsilon^2 - 1)^{1/2} (\varepsilon + q)^3 f_e \, d\varepsilon, \] (A5)

\[ \dot{q}_{\nu e^-\rightarrow e^+\nu_e} = \tilde{K} n_p \int_1^q (\varepsilon - 1)^{1/2} (\varepsilon - q)^3 (1 - f_e^-) \, d\varepsilon, \] (A6)

when the field is absent or can be ignored. Here \( \tilde{K}, \varepsilon, q, \) and \( f_e \) are the same as in Equations (11)–(13) in Section 2.2. Other cooling rates without fields have been shown in Section 2.2.

The chemical equilibrium is

\[ \ln \left( \frac{n_n}{n_p} \right) = f(\tau_e) \frac{2\mu_e - Q}{k_B T_0} + [1 - f(\tau_e)] \frac{\mu_e - Q}{k_B T}, \] (A7)

where the weight factor \( f(\tau_e) = \exp(-\tau_e) \) combines the formula from the neutrino-transparent limit case with the neutrino-opaque limit case of the \( \beta \)-equilibrium distribution, and \( Q = q m_e c^2 \).

A set of conservation equations without fields are as follows: Mass conservation (continuity) equation reads

\[ \dot{M} = -2\pi r \Sigma v_r, \] (A8)

this equation remains the same with and without fields in the vertically integrated disks. However, we should consider another continuity equation for outflows and winds as

\[ \dot{M} = -4\pi r^2 \rho v_r (\Delta \Omega/\Omega), \] (A9)

where \( \Delta \Omega \) is the opening solid angle of the wind.

The integrated angular momentum conservation equation is

\[ \frac{\dot{M}}{3\pi} f = v \Sigma. \] (A10)

The local energy conservation without fields is

\[ Q_{\nu e} = Q_{\text{adv}} + Q_{\nu e} \] (A11)

where the viscosity heating term is

\[ Q^+ = \frac{3GMM}{8\pi r^3} f \] (A12)

and the advection term is

\[ Q_{\text{adv}} = v_r T \left\{ \frac{R}{2} \left[ (1 + Y_e) + \frac{4\gamma a T^3}{3} \right] \right\}, \] (A13)

with \( g_e = 2 \) for photons and \( g_e = 11/2 \) for a plasma of photons and relativistic \( e^- e^+ \) pairs.

The charge conservation equation is

\[ \frac{\rho Y_e}{m_B} = n_p = n_{e^-} - n_{e^+}. \] (A14)

which will not be changed in strong fields if we do not consider other charged particles created in the strong field environment.

**APPENDIX B**

**SOLID ANGLE OF FUNNEL FLOW**

We adopt two methods to estimate the latitude and area of the ring-like belt of the “hot spot” on the magnetar surface. The first one is just to follow the approximation made in Section 5.2. We assume that the funnel accretion flow is lifted in the region with radius between \( r_m \) and \( r_A \), and the disk plane is truncated at the magnetosphere \( r_m \). Taking \( r_m = r_A - \delta \), we have the field line equation from the two disk radii \( r_A \) and \( r_m \) as

\[ r_t = \frac{r_A}{r_m} \frac{\sin^2 \theta}{\sin^2 \Theta}, \] (B1)

\[ r_r = (r_A - \delta) \frac{\sin^2 \theta}{\sin^2 \Theta}, \] (B2)

or we obtain

\[ \sin \theta_1 = \left( \frac{r_r}{r_A} \right)^{1/2} \sin \Theta, \] (B3)

\[ \sin \theta_2 = \sin(\theta_1 + \Delta \phi) = \left( \frac{r_r}{r_A - \delta} \right)^{1/2} \sin \Theta. \] (B4)

If \( \delta \ll r_A \), \( \sin \Theta \approx 1 \), we can obtain an analytic solution of \( \Delta \phi \) and \( \Delta \Omega/2\pi \). We have

\[ \sin \theta_2 = \sin \theta_1 + \cos \theta_1 \Delta \phi = \left( \frac{r_r}{r_A} \right)^{1/2} \left( 1 + \frac{\delta}{2r_A} \right), \] (B5)

or

\[ \Delta \phi = \frac{\delta}{2 \sqrt{r_A (r_A - r_r)}}. \] (B6)

The solid angle spanned by the hot spot ring-like belt can be calculated as

\[ \Delta \Omega = \int_0^{2\pi} d\phi \int_0^{\theta_1} \sin \theta d\theta \] (B7)

and we obtain an analytic result of the ratio \( \Delta \Omega/2\pi \) as a function of \( r_A \) and \( \delta \):

\[ \frac{\Delta \Omega}{2\pi} = \frac{2\pi \sin \theta_1 \Delta \phi}{2\pi} = \frac{\delta}{2r_A} \sqrt{r_r/r_A - r_r}. \] (B8)
We do not want to solve detailed conservation equations or a set of MHD differential equations as in Ustyugova et al. (1999). Table 3 gives the results with different parameters of $r_A/r_{\delta}$, $\delta/r_{\delta}$, and $\Theta$ as in the case of hyperaccreting disks with strong fields. The typical value for $\Delta \Omega/2\pi$ is around $10^{-2}$, except for the case when $r_A$ is small and $\delta \sim r_A$, which can still be considered as disk plane accretion. A stronger field makes the funnel flow more significant and the value of $\Delta \Omega$ smaller.

Next we give another scenario based on the consideration that the disk has a thickness near the magnetosphere $r_m$. We try to give a simple mathematical model: the ring-like belt is formed by the accretion matter at $r_m$ with different heights $z$ in the vertical direction. In other words, the ring-like belt area on the magnetar surface can be traced back to the disk plane vertical section at $r_m$:

$$r_{11} = r_m \frac{\sin^2 \Theta}{\sin^2 \Theta},$$

$$r_{12} = r_m \frac{\sin^2 \Theta}{\sin^2 \Theta},$$

where $r'$ satisfies

$$r' = \frac{r_m(1 + H^2/r_m^2)^{1/2}}{\cos(H/r_m)} \sin^2 \Theta.$$

Similarly, if $\sin \Theta \ll 1$ and $H^2/r_m^2 \ll 1$, we have

$$\sin \theta_1 = \left( \frac{r_m}{r_m} \right)^{1/2} \sin \Theta,$$

$$\sin \theta_2 = \left( \frac{r_m}{r_m} \right)^{1/2} \frac{\cos(H/r_m)}{1 + H^2/2} \sin \Theta,$$

$$\Delta \theta = \frac{3}{4} \frac{r_m}{r_m - r_*} \frac{H}{r_m^2},$$

$$\Delta \Omega = \frac{2 \pi \sin \theta_1 \Delta \theta}{2 \pi} = \frac{3}{4} \frac{r_m}{r_m - r_*} \frac{H}{r_m^2}.$$

We can take $r_m$ as a fraction of $r_A$. Table 4 gives the numerical results for this case. The typical value of $\Delta \Omega/2\pi$ is $\sim 10^{-3}$ to $10^{-2}$, which is slightly smaller than that obtained using the first estimation.

On the other hand, the accretion rate onto the neutron star without fields, as discussed in Paper II, can be estimated as

| $\delta/r_{\delta}$ | $r_m/r_{\delta}(\Theta = 90^\circ)$ | $r_m/r_{\delta}(\Theta = 75^\circ)$ |
|---------------------|---------------------------------|---------------------------------|
| 0.1                 | 1.84e-2                         | 6.38e-3                         |
| 0.5                 | 0.110                           | 3.46e-2                         |
| 1                   | 0.292                           | 7.74e-2                         |
| 2                   | ...                             | 0.208                           |

| $H/r_m$ | $\theta = 90^\circ (r_A/r_m)$ | $\theta = 75^\circ (r_A/r_m)$ |
|---------|-------------------------------|-------------------------------|
| 0.1     | 5.28e-3                        | 3.74e-3                        |
| 0.3     | 4.60e-2                        | 2.64e-2                        |
| 0.5     | 0.120                          | 8.50e-2                        |

Notes. The ratio of the solid angle of the “hot spot” ring-like belt $\Delta \Omega$ formed by funnel accretion to the half-total solid angle $2\pi$ based on the first scenario in Appendix B, in which the accreted matter onto the ring-like belt of “hot spot” is from the region between the magnetosphere edge $r_m$ and the Alfvén radius $r_A$. We consider $\delta/r_{\delta}$ from 0.1 to 2, and $r_m/r_{\delta}$ from 2 to 7. Here the left three columns are for $\Theta = 90^\circ$ and the right three ones are for $\Theta = 75^\circ$. The typical value of $\Delta \Omega$ is around $10^{-2}$.

We can take $r_m$ as a fraction of $r_A$.
