Opening the 1 Hz axion window

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Abstract

An axion-like particle (ALP) with mass $m_\phi \sim 10^{-15}$ eV oscillates with frequency $\sim 1$ Hz and it appears naturally as a consequence of grand unification (GUT) in M-theory. However, with a GUT-scale decay constant such an ALP overcloses the Universe, and cannot solve the strong CP problem. In this paper, we present a two axion model in which the 1 Hz ALP constitutes the entirety of the dark matter (DM) while the QCD axion solves the strong CP problem but contributes negligibly to the DM relic density. The mechanism to achieve the correct relic densities relies on low-scale inflation ($m_\phi \lesssim H_{\text{inf}} \lesssim 1$ MeV) and stochastic drift of both axion fields, which also highly alleviates the cosmological moduli and gravitino problems. We present two variants of the model each compatible with cosmic microwave background anisotropies and big bang nucleosynthesis, one of which in addition satisfies the trans-Planckian censorship conjecture. The 1 Hz ALP has canonical GUT-scale couplings to both photons and nucleons, and may be detectable by ABRACADABRA and CASPER. Furthermore the model predicts: (1) a value for the strong CP phase which oscillates around $\bar{\theta}_{\text{QCD}} \sim 10^{-12}$, within reach of the proton storage ring electric dipole moment experiment, (2) that any intermediate mass black holes observed in gravitational waves from binary inspiral by LISA will have low dimensionless spin and reside on Regge trajectories.
1 Introduction

String theory and M-theory compactifications predict a so-called “axiverse” [1–8]: low-energy effective theories with large numbers of axion-like particles (ALPs, or simply “axions”) with a wide range of possible masses. The axion masses can be obtained from non-perturbative effects while the typical decay constants are around the GUT scale $f_\phi \sim 10^{16}$ GeV. The QCD axion [9–12], which solves the strong CP problem, may be naturally predicted as one of the string axions (see Refs. [13–17] for reviews).

The existence of certain light axions can be excluded due to the phenomenon of “black hole superradiance” (BHSR) [18–21]. If the axion Compton wavelength is of order the BH horizon size, then spin is extracted from the BH by the Penrose process. Thus the measurement of non-zero spins (e.g. from binary inspiral gravitational waveforms or accretion disk X-ray spectroscopy) for stellar and supermassive BHs (SMBHs) set quite general exclusion limits on the axion mass, $m_\phi$, as $10^{-18} \text{eV} \lesssim m_\phi \lesssim 6 \times 10^{-13} \text{eV}$, respectively (assuming the axion quartic self-coupling is negligible, which is a good assumption for $f_\phi \sim 10^{16}$ GeV [19]). However, the non-observation of any intermediate mass BHs ($M \sim 10^4 M_\odot$), never mind the measurement of their spins, allows for the existence of the $1$ Hz axion window:

$$10^{-16} \text{eV} \lesssim m_\phi \lesssim 6 \times 10^{-13} \text{eV} \Rightarrow 0.15 \text{ Hz} \lesssim \nu_\phi \lesssim 910 \text{ Hz.} \quad (1)$$

It is, on the other hand, known that the existence of an axion may lead to the accelerated growth of the BHs [18]. Thus if there are many axions within the window, not only the null observation of the IMBH but also the observation of the SMBHs existing at the red-shift $z \gtrsim 6$ [22] (see also Ref. [23]) may be explained.

From theoretical point of view, the $1$ Hz axion is predicted if the relevant gauge coupling is around the MSSM one at the GUT scale in an M-theory realization of axiverse [5]. The axion potential is $V(\phi) = \Lambda^4 U(\phi/f_\phi)$ and the scale $\Lambda^4 \sim 3 \times 10^{-6} m_{3/2} M_{3/2}^3 e^{-2\pi V_\phi}$ is generated from membrane instanton barring $O(1)$ coefficient where $m_{3/2}$ is the gravitino mass, $M_{3/2}$ is the reduced Planck mass, $M_{3/2} = 1/\sqrt{8\pi G_N} \approx 2.4 \times 10^{18}$ GeV, and $V_\phi$ is some cycle volume. The MSSM GUT coupling $\alpha_{\text{GUT}} \approx 1/27 - 1/25$ logarithmically depending on the SUSY scale is linked to the cycle volume in string units $V_\phi \approx 1/\alpha_{\text{GUT}}$. Thus, with $f_\phi \sim 10^{16}$ GeV and $m_{3/2} = O(1 - 100)$ TeV, this realizes the $1$ Hz axion. The resulting dynamical scale is

$$\Lambda_{\text{vis}} \sim (0.03 - 0.1) \text{ MeV.} \quad (2)$$

\footnote{We take $m_{3/2}$ to be the predicted SUSY scale [24–28] in the current measurement of the top and Higgs masses. We omit the possibility that tan $\beta$ is quite large or small, e.g. Ref. [29].}
Since \( m_\phi \approx \Lambda^2_{\text{vis}}/f_\phi \), this immediately leads to

\[
m_\phi \sim (10^{-16} - 10^{-15}) \text{ eV} \left( \frac{10^{16} \text{ GeV}}{f_\phi} \right)
\]  

(3)

It was shown by Matthew J. Stott and one of the authors (DJEM) [21] in the context of the M-theory that the axion mass distributions around 1 Hz can be either very wide or narrow by taking account of the BHSR constraints. (See also Ref. [30] for the distribution without the BHSR constraints.) When it is narrow, we should have several axions with masses around 1 Hz.

The 1 Hz axion window, however, is closed once we take account of the cosmological abundance of the axion, which is too much with \( f_\phi \gtrsim 10^{15} \text{ GeV} \) and an \( \mathcal{O}(1) \) initial misalignment angle, \( \theta_i \). In particular, for an axion lighter than \( 10^{-14} - 10^{-15} \text{ eV} \), the abundance can not be diluted from late time entropy production [31–35]. The onset of the oscillation occurs when \( 3H(T_{\text{osc}}) \approx m_\phi \), and the Friedmann equation \( 3H^2M_{\text{pl}}^2 = \pi^2g_*T^4/30 \Rightarrow T_{\text{osc}} \lesssim \mathcal{O}(1) \text{ MeV} \), with \( g_* \) being the relativistic degrees of freedom which we quote [36]. This is after big-bang nucleosynthesis (BBN), whose successful predictions would be spoiled by entropy production.

The QCD axion, if identified as one of the string axions, has also the overproduction problem if \( f_\phi \gtrsim 10^{12} \text{ GeV} \) and \( \theta_i \sim 1 \).

In this paper, we open the 1 Hz window in two senses: we solve the overproduction problems and show various phenomenological perspectives. First we show that if one of the would-be 1 Hz axions with string-induced dynamical scale \( \approx \Lambda_{\text{vis}} \), can be the QCD axion, i.e. it additionally couples to gluon colour anomaly and gets a QCD-induced potential (lifting the mass to the canonical value \( m_a \approx 6 \times 10^{-6}(10^{12} \text{ GeV}/f_a) \text{ eV} \)). In this case the strong CP phase is predicted to be small enough and the strong CP problem is solved. Interestingly, the strong CP phase is non-vanishing and can be searched for from the future measurement of the nucleon electric-dipole moments (EDMs) [37, 38]. In the addition of a second (or more) axion in the 1 Hz window implies the previous QCD axion and a (or more) 1 Hz axion. The 1 Hz axion can be consistent with the cosmological abundance of the dark matter if the inflationary Hubble scale, \( H_{\text{inf}} \), is low enough. The QCD axion has its relic abundance diluted to nearly zero, while a second axion in the 1 Hz window obtains the correct relic abundance. This second 1 Hz axion has a canonical coupling to electromagnetism and a shift symmetric nucleon coupling, allowing for it to be searched for by the ABRACADABRA [39–41] and CASPER-Wind [42–44]. Finally, the second 1 Hz axion can be tested if LISA observes gravitational waves from IMBH binaries during the inspiral phase [45].
2 1 Hz axion

For illustrative purpose, we will consider the model with two axions, \( \phi \) and \( a \), one of which will be identified as the QCD axion. (See also two-axion models with QCD axion and fuzzy dark matter [46], QCD axion and heavier axion dark matter [47], and QCD axion and ALP axion dark matter with level-crossing [48]. In our scenario, the level-crossing effect is not important because the oscillation of the 1 Hz axion starts much after the QCD phase transition.) As we will discuss in the last section, the inclusion of more axions in the 1 Hz axion window does not change our conclusion.

2.1 1 Hz axion and nucleon EDMs

The potential with two axions after the M-theory compactification can be given as

\[
V_{\text{Mtheory}} \approx -\Lambda_1^4 \cos \left[ c_0 \frac{\phi}{f_\phi} + c_1 \frac{a}{f_a} \right] - \Lambda_2^4 \cos \left[ c_2 \frac{\phi}{f_\phi} + c_3 \frac{a}{f_a} \right] + \cdots ,
\]

where \( \cdots \) represents the constant terms cancelling the cosmological constant, \( \Lambda_i \) are the dynamical scales of the non-perturbative effect, and \( c_i \) real numbers. In general, both \( \Lambda_i \) are non-vanishing [5]. Notice that we can eliminate the CP-phases in the cosine terms from the field redefinitions, \( \phi \rightarrow \phi + \alpha_1, a \rightarrow a + \alpha_2 \) with certain real numbers \( \alpha_1, \alpha_2 \), unless the two axions are aligned. In the following, we omit the case of alignment of axions as well as cancellations among the parameters, unless otherwise stated. Let us couple the two axions to the gluons. Since we have the freedom to redefine the field as \( a \rightarrow \cos(\Theta)a + \sin(\Theta) \phi, \phi \rightarrow -\sin(\Theta)a + \cos(\Theta) \phi \), with \( \Theta \) being real phases, gluon-axion coupling is generally given as

\[
\mathcal{L}_{\text{int}} \supset -\frac{\alpha_s}{8\pi} \left( c_s \frac{a}{f_a} + \theta \right) G\tilde{G}.
\]

where \( \alpha_s \) is the strong coupling constant, and \( G \) (\( \tilde{G} \)) is the field strength (its dual) of the gluon, and \( \theta = \mathcal{O}(1) \) is the CP phase coming from the standard model sector in the basis the quark masses are real, \( \text{arg} \det[m_u m_d] = 0 \). Due to the QCD phase transition \( a \) acquires a potential of

\[
V_{\text{QCD}}(a) \approx -\chi \cos[c_s \frac{a}{f_a} + \theta] + \cdots
\]

where \( \chi \approx (0.0756 \text{GeV})^4 \) [49]. We arrived at the potential

\[
V = V_{\text{Mtheory}}(\phi, a) + V_{\text{QCD}}(a).
\]

In the following, we take \( c_s = c_0 = 1 \) to redefine \( f_a, f_\phi \) and \( c_i \) in the basis of [5] without loss of generality.
Figure 1: The contour plot of strong CP phase, $\bar{\theta}_{QCD}$, in the two axion models by varying $\Lambda_1$ and $\Lambda_2$. We have fixed the parameters as $\theta = c_1 = 1, c_2 = 0.5, c_3 = 2$.

There is a general and physical CP phase $\theta$ that can not be shifted away. The strong CP phase is related with $\theta$ as

$$\bar{\theta}_{QCD} = \left\langle \frac{a}{f_a} \right\rangle + \theta,$$

where $\langle \rangle$ denotes the vacuum expectation value. By stabilizing the potential (with a quadratic approximation), at the leading order of $\Lambda_{41}$, $\Lambda_{42}$,

$$\bar{\theta}_{QCD} \approx \frac{\Lambda_{41}^2 \Lambda_{42}^2 (c_3 - c_1 c_2)^2}{\chi \left( c_2^2 \Lambda_{42}^4 + \Lambda_{41}^4 \right)} \theta.$$  \hspace{1cm} (9)

Here, the $\bar{\theta}_{QCD}$ prediction does not depend on $f_a, f_\phi$. One finds that in our model $\bar{\theta}_{QCD}$ in general is non-vanishing, but this is small if $\min[\Lambda_{41}, \Lambda_{42}] \ll \chi$ with $c_i = \mathcal{O}(1), \theta = \mathcal{O}(1)$. The strong CP phase by varying $\Lambda_1$ and $\Lambda_2$ is shown in Fig. 1 $\theta = c_1 = 1, c_2 = 0.5$, and $c_3 = 2$ are fixed. We can find that when $\min[\Lambda_1, \Lambda_2] \lesssim a \times 10^{-4}$ GeV, $\bar{\theta}_{QCD}$ can be within the experimental bound from the neutron EDM \cite{50, 51} $|d_n| \lesssim 3 \times 10^{-26}e\,cm$, where the neutron
and proton EDMs are related with $\bar{\theta}_{\text{QCD}}$ as \[52\text{,}53\]

$$d_n = (1.52 \pm 0.71) \times 10^{-16} \bar{\theta}_{\text{QCD}} e\text{ cm} \quad \text{and} \quad d_p = (-1.1 \pm 1.0) \times 10^{-16} \bar{\theta}_{\text{QCD}} e\text{ cm},$$ \hspace{1cm} \text{(10)}

respectively. The bound is represented in the figure by the gray shaded region. Also shown is the red band corresponding to $|d_p| \approx 10^{-29} e\text{ cm}$, which is the sensitivity reach of a storage ring experiment for measuring the proton EDM \[54\]. We also show the orange band for $|d_p| \approx 10^{-30} e\text{ cm}$ which should be achieved in the future. When $\text{min}[\Lambda_1, \Lambda_2] \ll 0.01\text{ MeV}$, the standard model contribution to the $\bar{\theta}_{\text{QCD}}$ term becomes important. At the limit $\text{min}[\Lambda_1, \Lambda_2] \to 0$, the CKM phase gives a tad-pole term to $a$ and changes the strong CP phase from the CP-conserving point by $\mathcal{O}(10^{-17} - 10^{-16})$ \[55\text{,}57\]. In the blue region, the standard model contribution becomes dominant and $|\bar{\theta}_{\text{QCD}}| \sim 10^{-17} - 10^{-16}$ is obtained. The estimation of the standard model contribution requires precise calculations of the QCD matrix elements as well as the quark chromo-EDMs which are beyond the scope of the paper.

One can reduce the model to a single-axion one by taking $c_1 = 0$ and $\Lambda_1^4 \to \infty$, i.e. the decoupling limit of $\phi$. At the limit, one obtains the potential of

$$V \to -\Lambda_2^4 \cos[c_3 a/f_a] - \chi \cos[a/f_a + \theta] + \cdots \quad \text{[single-axion model]} \hspace{1cm} \text{(11)}$$

with certain field and parameter redefinitions. Again, here is a CP phase which can not be shifted away, which induce a non-vanishing strong CP phase. The strong CP phase is estimated by taking $c_1 = 0$ and $\Lambda_1^4 \to \infty$ in Eq. \((9)\) as

$$\bar{\theta}_{\text{QCD}} \approx 3 \times 10^{-12} c_3^2 \left( \frac{\Lambda_2}{0.1\text{ MeV}} \right)^4 \theta \quad \text{[single-axion model]}. \hspace{1cm} \text{(12)}$$

With $\Lambda_2 \sim \Lambda_{\text{vis}}$, the axion is the QCD axion solving the strong CP problem while inducing a small non-vanishing CP-phase.

In our scenario, either with single, $\Lambda_1 \to \infty, \Lambda_2 \sim \Lambda_{\text{vis}}$, or two 1 Hz axion, $\Lambda_1, \Lambda_2 \sim \Lambda_{\text{vis}}$, the EDMs are predicted around the current bound, and significantly overlaps with the future reach. Therefore, our scenario can solve the strong CP problem, while also giving a testable prediction for small CP-violation in the QCD sector. As we will see, the contribution to $\bar{\theta}_{\text{QCD}}$ is also important for testing the scenario using spin-dependent forces. We mention that the CP-violation may also arise from the MSSM contribution although it depends on SUSY-breaking mediation. In this case, the nucleon EDM contribution from $\bar{\theta}_{\text{QCD}}$ can be identified as the minimum value unless there are cancellations between several contributions.
2.2 Mass spectrum and abundance

From now on, let us focus $\Lambda_1 \sim \Lambda_2 \sim \Lambda_{\text{vis}}$, and $c_i = \mathcal{O}(1)$. The single-axion case can be easily obtained by taking the aforementioned decoupling limit of $\phi$.

By diagonalizing the mass matrix, the mass eigenvalues are found to be

$$M_H \approx \frac{\sqrt{\chi}}{f_a}, \quad M_L \approx \frac{\sqrt{\Lambda_1^4 + c_2^2 \Lambda_2^4}}{f_\phi},$$

where $M_H$ ($M_L$) is the heavier (lighter) axion composed respectively by $a$ ($\phi$), i.e. the eigenstates are

$$A_H \approx a, \quad A_L \approx \phi$$

with decay constants, respectively,

$$f_H \approx f_a, \quad f_L \approx f_\phi.$$ 

Explicitly

$$M_L \sim (10^{-16} - 10^{-15}) \text{ eV} \left(\frac{10^{16} \text{ GeV}}{f_\phi}\right)$$

if $\Lambda_1, \Lambda_2 \sim \Lambda_{\text{vis}}$, and $c_2 = \mathcal{O}(1)$. The heavier axion gets its mass mostly from QCD instatons, and is close to the canonical value

$$M_H \approx 6 \times 10^{-10} \text{ eV} \left(\frac{10^{16} \text{ GeV}}{f_L}\right).$$

The heavier axion can successfully solve the strong-CP problem, as we have discussed.

Axion fields with non-vanishing initial amplitude start to oscillate around the potential minima when the masses become comparable to the Hubble expansion rate. The oscillation energy contributes to the matter density in the usual vacuum realignment mechanism \cite{31,58,59}. The light axion has a dynamical scale $(\Lambda_1^4 + c_2^2 \Lambda_2^4)^{1/4}$, which is temperature independent. Neglecting anharmonic effects (i.e. assuming $\theta_i \ll \pi$), the light axion abundance is

$$\Omega_L h^2 \approx 0.1 \left(\frac{g_{*,\text{osc}}}{11}\right)^{-1/4} \left(\frac{M_L}{10^{-15} \text{ eV}}\right)^{1/2} \left(\frac{\theta_i}{0.2}\right)^2 \left(\frac{f_L}{10^{16} \text{ GeV}}\right)^2.$$ 

where $\theta_i$ with $I = L, H$ is the initial misalignment angle defined by the initial amplitude normalized by $f_I$, $g_{*,\text{osc}}$ is $g_*$ at the onset of the oscillation, and the reduced Hubble rate today, $h$, defined from $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$, and we take the value $h \approx 0.68$ \cite{60}. The abundance of the QCD axion has a temperature dependent dynamical scale. The oscillation
starts at around the QCD phase transition. The abundance is given by \[61\]

\[
\Omega_H h^2 \approx 0.35 \left( \frac{\theta_H}{0.001} \right)^2 \times \left\{ \begin{array}{ll}
\left( \frac{f_H}{3 \times 10^{17} \text{GeV}} \right)^{1.17} & f_H \lesssim 3 \times 10^{17} \text{GeV} \\
\left( \frac{f_H}{3 \times 10^{17} \text{GeV}} \right)^{1.54} & f_H \gtrsim 3 \times 10^{17} \text{GeV}
\end{array} \right.
\]

(19)

The total abundance, \(\Omega_{\text{tot}} h^2 \equiv \Omega_H h^2 + \Omega_L h^2\), must be smaller than the observed dark matter abundance \(\Omega_{\text{DM}} h^2 = 0.120 \pm 0.001\) \[60\], i.e.

\[
\Omega_{\text{tot}} h^2 \lesssim \Omega_{\text{DM}} h^2.
\]

(20)

They may also explain the dominant dark matter, i.e. \(\Omega_{\text{tot}} h^2 \approx \Omega_{\text{DM}} h^2\). Unfortunately, the total abundance of the two axions with \(f_I \sim 10^{16} \text{GeV}\) is too large unless \(\theta_L, \theta_H \ll 1\), e.g. for the QCD axion abundance alone \(\theta_H \lesssim 0.01\) is required for \(f_H \approx 10^{16} \text{GeV}\). In the next section we show that small \(\theta_i\)'s can be obtained naturally from low-scale inflation in two different models, and that the lighter axion can be the dominant dark matter.

### 2.3 Axion Couplings

The axion can couple to the standard model particles in a general manner. The axion couplings to the standard model particles up to dimension five terms are as follows (at the renormalization scale below the QCD scale):

\[
\mathcal{L}_{\text{int}} \supset \frac{\alpha_e}{4\pi} \left( c_{\gamma} \phi \frac{\partial \phi}{f_{\phi}} + c_{a} a \frac{\partial a}{f_{a}} \right) F \tilde{F} + \left( c_{\gamma} \phi \frac{\partial \phi}{f_{\phi}} + c_{a} a \frac{\partial a}{f_{a}} \right) \bar{\Psi} \gamma^{\mu} \gamma_{5} \Psi.
\]

(21)

Here \(c_{\gamma}, c_{a}\) are anomaly coefficients, \(F\) and \(\tilde{F}\) the field strength and its dual of the photon, and \(\alpha_e \approx 1/137\) the gauge coupling constant. \(\Psi\) represents nucleons, neutrinos and leptons, where \(c_{\gamma}\) can be different for different fermion. The Lagrangian satisfies the shift symmetry (up to total derivatives). If the axion couplings to the gauge bosons are universal i.e. respecting the GUT relation, \(c_{\gamma} = 0.75, c_{a} = 0\) should be satisfied. However, this may not be always since the GUT breaking mechanism may induce the non-universal couplings e.g. Refs. \[62\]–\[67\]. It is also argued in Ref. \[68\] that in general string axions can be non-universally coupled. With the general couplings, \(A_H \approx a\) and \(A_L \approx \phi\) both couple to photons and fermions. Interestingly, \(A_L\) can naturally couples to the nucleon/photon without a gluon coupling, i.e. the \(A_L\)-gluon-gluon anomaly automatically vanish due to the mixing (see Eqs. (5) and (14)). The couplings give nice opportunity for the dark matter search for \(A_L \approx \phi\).
For the photon coupling we consider the sensitivity reaches of the ABRACADABRA (39-41) (see also Ref. [69] for the latest result) and DANCE (70,71) experiments for \( c_\gamma = 1 \) and 10. For ABRACADABRA, seismic noise in broadband is neglected, which also limits the extrapolation to low masses. For the nucleon coupling, we consider the CASPER-Wind experiment (42,43,72,73) (see the latest result [44]) in optimistic (spin noise only) and conservative (CASPER-ZULF) cases for \( c_\phi = 1 \) and 10. In Ref. [44], the projection is 3 orders of magnitude smaller than Ref. [73]. The \( 10^3 \) increase from the projections in Ref. [44] is obtained assuming hyper polarisation. Furthermore, for low axion masses in all direct searches, stochastic fluctuation of the galactic axion field become important (74) and affect the sensitivity reach by an additional \( \mathcal{O}(1) \) factor. However, in the 1 Hz axion window, the coherence time is shorter than a year and the effect may not be very important if a long enough measurement is made. For the CASPER projections, we consider constant sensitivity extrapolations to low masses. We also mention that the 1 Hz axion is right inside the focused mass range of the AION experiment (75).

3 Low-scale inflation, axion dark matter, and experimental tests

Let us consider two scenarios for axion dark matter with low-scale inflation: short inflation and long inflation.

Recently, by appealing to quantum gravity arguments, it was conjectured (76,77) that the e-folding number, \( N_e \), during the whole of inflation (larger than the one corresponding to the thermal history of the Universe) is bounded above as

\[
N_e \lesssim \log\left[ \frac{M_{\text{pl}}}{H_{\text{inf}}} \right].
\]

This is known as the “trans-Planckian censorship conjecture” (TCC): it forbids any mode that ever had wavelength smaller than the Planck scale from being classicalized by inflationary expansion. It protects cosmological observables form sensitivity to trans-Planckian initial conditions. In the first scenario we consider, we impose the TCC and show the axion abundance can be suppressed due to axion dynamics during inflation, while maintaining a sufficiently large reheat temperature for successful BBN.

\textsuperscript{2}See also Refs. [78-92] for subsequent studies on the inflationary cosmology, and argument on the Conjecture [93].
Eternal inflation, on the other hand, is important in many cosmological models. This predicts an extremely large number of e-folds, which is in contradiction to the TCC. In the second part, we abandon the TCC, and find a successful suppression of the axion abundance in eternal inflation.

3.1 Ultra low-scale inflation and TCC

The TCC predicts inflation scale $H_{\text{inf}} \lesssim 1 \text{ GeV}$ [77], which is even smaller, $H_{\text{inf}} \lesssim 0.01 \text{ eV}$, for single field slow-roll inflation [87]. This Hubble parameter region suggests that during inflation the Gibbons-Hawking temperature $T_{\text{inf}} \approx H_{\text{inf}}/2\pi$ is smaller than the QCD scale and thus the QCD axion potential is non-negligible. In this case the eigensystem of the axion fields is the same as in the vacuum. We are interested in the lightest axion being the dark matter, and so we consider $H_{\text{inf}} \gtrsim M_L$, in order not to suppress its abundance too much [94]. During inflation the lighter axion undergoes slow-roll following the equation of motion

$$\dot{A}_L \approx \frac{M_L^2}{3H_{\text{inf}}^2} A_L.$$

The solution is

$$\theta_{i}^L \approx \theta_{i}^{L_\text{inf}} e^{-\frac{M_L^2}{3H_{\text{inf}}^2} N_e},$$

where $\theta_{i}^{L_\text{inf}}$ is the misalignment angle at the beginning of the inflation. As a result, $\theta_{i}^L \ll \theta_{i}^{L_\text{inf}} = \mathcal{O}(1)$ if $M_L^2/3H_{\text{inf}}^2 \gtrsim N_e^{-1}$. In particular if $H_{\text{inf}}$ is slightly greater than $M_L$, the lighter axion can remain to explain the dominant dark matter with Eqs. (18) and (22). The result after setting $\Omega_L h^2 = 0.12$ is shown in Fig. 2 with $M_L = 10^{-14}, 10^{-15}, 10^{-16} \text{ eV}$ from top to bottom by taking $\theta_{i}^{L_\text{inf}} = 1$ (i.e. natural realignment angle). On the right hand $y$-axis we show the corresponding upper limit to the reheating temperature, $T_{R_\text{max}}^\text{max} \equiv (g_\star \pi^2/90)^{-1/4} \sqrt{H_{\text{inf}} M_{\text{pl}}}$.

The observational bound is $T_R \gtrsim 2 - 5 \text{ MeV}$ from big-bang nucleosynthesis (BBN), depending on the inflaton couplings to the standard model particles [95–101]. The shaded region is the upper bound to the number of e-folds set by TCC. Given the TCC, the lower bound of $M_L \gtrsim 10^{-16} \text{ eV}$ is obtained from the BBN constraint on $T_{R_\text{max}}^\text{max}$. (Notice that $N_e \gtrsim N_\star$ where $N_\star = 13 + \log[T_{R_\text{max}}^\text{max}/3 \text{ MeV}]$ corresponds to the thermal history after inflation, is needed.) Consequently, there are parameter regions, where $T_{R_\text{max}}^\text{max} \approx \mathcal{O}(1 - 10) \text{ MeV}$, satisfying the BBN bound for a 1 Hz axion.

In the viable parameter regions, the heavier ($\approx \text{QCD}$) axion energy density is diluted as $\propto a^{-3}$ during the inflation since it starts to oscillate before the inflation with $H_{\text{inf}} \ll m_a \sim 10^{-10} \text{ eV}$. By assuming an instantaneous reheating, $T_R = T_{R_\text{max}}^\text{max}$ one obtains the abundance
of the heavier axion as
\[ \Omega_{TCC}^{h^2} = \frac{\rho_H}{s} \bigg|_{T=T_R} \times \frac{s_0}{\rho_c} \] (25)
where \( s_0 \approx 2900 \text{ cm}^3, \rho_c \approx 10^5 \text{ GeV} h^{-2} \text{ cm}^3, s = g_{*s} 2 \pi^2 T^3/45 \) with \( g_{*s} \) being the relativistic degrees of freedom for the entropy density, and \( \rho_H \) is the energy density of the heavier axion. Since at the beginning of the inflation, \( \rho_H \lesssim \chi \) should be satisfied (a more stringent constraint may come from \( \rho_H \lesssim H_{\text{inf}}^2 M_{\text{pl}}^2 \) i.e. the axion potential does not contribute to the inflation dynamics.), the dilution works as \( \rho_H|_{T=T_R} \lesssim \chi \exp[-3N_e] \). Total e-folds satisfies \( N_e > N_* = 13 + \log[T_R^{\text{max}}/3 \text{ MeV}] \), and thus one obtains the inequality
\[ \Omega_{TCC}^{h^2} \lesssim \Omega_{H}^{\text{max}} h^2 \] (26)
where
\[ \Omega_{H}^{\text{max}} h^2 \equiv \Omega_{TCC}^{h^2}|_{\rho_H=\chi \exp[-3N_e]} \sim 10^{-6} \left( \frac{3 \text{ MeV}}{T_R^{\text{max}}} \right)^6. \] (27)
Therefore, the QCD axion abundance is highly suppressed with \( T_R^{\text{max}} > \mathcal{O}(1) \text{ MeV} \), and \( m_a > H_{\text{inf}} \), which is the parameter region of \( \Omega_L h^2 = 0.12 \) and successful BBN. The QCD axion dark matter abundance is diluted to almost zero as \( 26 \), and thus cannot be observed in axion haloscopes \([39,42,70]\) (See also e.g. Refs. \([102–110]\) for heavier mass range \( M_H \gg 10^{-10} \text{ eV} \)).

The signal is even enhanced compared to the standard model case due to the larger value of \( \theta_{\text{QCD}} \). If the decay constant is small or the photon coupling is enhanced by e.g. a clockwork mechanism \([112,113]\), it may be searched for in the ARIADNE force experiment \([57,111]\).

In Fig. 3, we show the parameter space of the TCC scenario with \( \Omega_{\text{tot}} h^2 \approx \Omega_L h^2 = 0.12 \) maximising \( N_e = \log[M_{\phi}/H_{\text{inf}}] \) (red solid lines for \( H_{\text{inf}} = 10^{-16}, 10^{-14}, 10^{-12} \text{ eV} \) from left to right). The purple bands are excluded by BHSR, where we impose lower limits \( f_L < 10^{14} \text{ GeV} \) and \( f_L < 10^{16} \text{ GeV} \) for the stellar and supermassive regions respectively due to the Bosenova effect \([118]\) (see also Ref. \([19]\) for the single cosine term case). Experimental reaches are included as discussed above. We conclude that dark matter with \( f_L \approx 10^{16} \text{ GeV} \) can be mostly tested by ABRACADABRA if \( c_\phi^2 \) is large enough. Furthermore, if the CASPER experiment can reach the optimistic sensitivity, then most of the parameter space can also be tested via the nucleon coupling if \( c_n^2 = \mathcal{O}(1) \). The dotted line represents \( \theta_3^L = 1 \), below which we need to set the lighter axion close to the hilltop to increase the abundance via anharmonic corrections.\(^3\) Thus in this large misalignment region, one again requires tuning

\(^3\)In this region, one can also consider the QCD axion as the dominant dark matter instead. In the case, the 1 Hz axion is not the dominant dark matter and the sensitivity reaches are not applied.
of $\theta_i^L$ to obtain the correct dark matter abundance, and so we no longer consider it. In the last section, we will discuss the hilltop 1 Hz axion realized by a mixing between the axion and the inflaton.

Lastly we comment on the model-building in the inflaton sector. The inflation scale of $H_{\text{inf}} \sim 10^{-16} - 10^{-12}$ eV is very small. Recently, $H_{\text{inf}} \gtrsim \mathcal{O}(10^{-6})$ eV was achieved with ALP inflation with two cosine terms composing the inflaton potential [119–121] (see also Refs. [122, 123] for multi-natural inflation). The lower limit is set by constraints from the cosmic microwave background (CMB) on the primordial power spectrum. This is because the slow-roll period of inflation from horizon exit of the CMB scales is $N_* \lesssim 20$ for $H_{\text{inf}} \lesssim 10^{-6}$ eV. The curvature of the inflaton potential has to change fast enough to terminate slow-roll. It turns out that to satisfy this in this simple potential at horizon exit of the CMB scales,
Figure 3: The \((M_L - f_L)\) 1 Hz axion parameter space fixing with \(\Omega_{\text{tot}}h^2 \approx \Omega_L h^2 = 0.12\) with TCC-satisfying ultra-low-scale inflation. Rolling of the axion fields fixes \(\theta_i^I = (H_{\text{inf}}/M_{pl})^{M_i^I/3H_{\text{inf}}^2}\), and we consider \(H_{\text{inf}} = 10^{-16}, 10^{-14}, 10^{-12}\) eV from left to right for the red solid lines. The purple regions are excluded by BHSR. 

**Left Panel** Axion nucleon coupling. The lower green (blue) line is the sensitivity reach of the CASPEr-ZULF experiment with \(c_\phi^I = 1\) \((c_\phi^I = 10)\) taken from [43]. Also shown is the optimistic projection (upper green (blue) line) taken from Ref. [42] assuming only spin noise. Dashed lines are our extrapolations.

**Right Panel** Axion-photon coupling. Below the green (blue) band, the region can be tested in the ABRACADABRA experiment with \(c_\phi^\gamma = 1\) \((c_\phi^\gamma = 10)\) adopted from [117]. The lower green (blue) line shows the projected reach of recently-proposed DANCE experiment [70,71].

The derivatives of the curvature should be sizeable. The first and second derivatives induce the running or running of running of the scalar spectral index, which are both constrained from the CMB data. However the experimental bound can be alleviated by introducing more cosine-terms or non-cosine terms for the potentials to suppress the first and second derivatives at the horizon exist. (See e.g. Ref [124].)

As pointed out in Ref [121], in the regime of inflection point inflation, the inflationary period has an upper bound. It thus should be possible to satisfy the TCC. However, the inflaton mass is too small and reheating is problematic. This reheating problem could be also
Figure 4: The \( (M_L - f_L) \) 1 Hz axion parameter space fixing with \( \Omega_{\text{tot}} h^2 \approx \Omega_L h^2 = 0.12 \) with low-scale eternal inflation. The Bunch-Davies distribution fixes \( \theta_i^l \) according to Eq. (28), and we consider \( H_{\text{inf}} = 10^{-5}, 10^{-4}, 10^{-3} \) GeV from left to right for the red solid lines. The purple regions are excluded by BHSR. Left Panel Axion nucleon coupling. The lower green (blue) line is the sensitivity reach of the CASPEr-ZULF experiment with \( c_n^\phi = 1 \) (\( c_n^\phi = 10 \)) taken from \cite{43}. Also shown is the optimistic projection (upper green (blue) line) taken from Ref. \cite{42} assuming only spin noise. Dashed lines are our extrapolations. Right Panel Axion-photon coupling. Below the green (blue) band, the region can be tested in the ABRA-CADABRA experiment with \( c_\gamma^\phi = 1 \) (\( c_\gamma^\phi = 10 \)) adapted from \cite{117}. The lower green (blue) line shows the projected reach of recently-proposed DANCE experiment \cite{70,71}.

cured since we have introduced modifications of the potential, which can make the inflaton mass at the vacuum larger. With very low reheat temperatures such as in this model, low-scale baryogenesis from inflaton decays is necessary \cite{125,128} (one can also transfer the baryon asymmetry to baryonic dark matter sector from the inflaton decays e.g. Refs \cite{129,133}).

3.2 Low scale eternal inflation

In general, there are fine-tunings on the inflaton field over certain spacetime volume to start inflation. Such tuning may be explained by eternal inflation \cite{134,136}, which predicts
extremely large e-folding number, \( N_e \). Eternal inflation is obviously in contradiction with the TCC and in the following we abandon it. In this case, we can go further into the shaded region of Fig. 2 where higher \( H_{\text{inf}} \) can be allowed with larger \( N_e \).

In fact, there is an asymptotically allowed value once we take \( N_e \to \infty \) eternal inflation limit. This is because if we take \( N_e \) extremely large with a given \( H_{\text{inf}} \gg M_L \), the abundance of \( A_L \) saturates due to the quantum diffusion effect. It was pointed out that if the inflation scale is sufficiently low and the period is long enough, the initial misalignment angle of the QCD axion \[ ^{137,138} \] and string axions \[ ^{139} \] reach equilibrium between the classical motion and quantum diffusion for the time scale \( N_e \gtrsim H_{\text{inf}}^2/M_I^2 \). In this case it is the Bunch-Davies (BD) distribution that determines the misalignment angle, which is independent from \( \theta_{\text{inf}}^I \).

The typical misalignment angles in this “natural” region are given by

\[
\theta_{\text{i}}^I \approx \sqrt{\frac{3}{8\pi^2}} \frac{H_{\text{inf}}^2}{M_I f_I} \quad (N_e \gtrsim \frac{H_{\text{inf}}^2}{M_I^2}) \tag{28}
\]

where we take the variance of the BD distribution. The probability that \( \theta_{\text{i}}^I \) is much greater than the variance is exponentially suppressed since the BD distribution is a normal distribution. In this model, and taking account of the BD distribution, we show the 1 Hz axion parameter space in Fig. 4. The red lines from left to right correspond to \( \Omega_L h^2 = 0.12 \) with \( H_{\text{inf}} = 10^{-5}, 10^{-4}, 10^{-3} \) GeV. On the dotted black line \( \theta_{\text{i}}^I = 1 \); below this the distribution is almost uniform and thus one can not simply apply (28) as the typical value. We notice again that the dominant dark matter is \( A_L \approx \phi \), and the QCD axion \( A_H \approx a \) has negligible abundance with \( H_{\text{inf}} \lesssim 1 \) MeV \[ ^{137,138} \]. The predictions for direct detection are thus the same as the short inflation case.

In this model variant, the required \( H_{\text{inf}} \) can be much larger than the TCC scenario, and thus higher reheating temperature can be obtained. Model-buildings of inflation and baryogenesis, e.g. resonant leptogenesis \[ ^{140,141} \], the ARS mechanism \[ ^{142,143} \], non-thermal leptogenesis \[ ^{144,146} \], and leptogenesis via active neutrino oscillation \[ ^{133,147} \] is much easier than in the TCC model.

Finally, let us connect the two parameter regions of \( N_e \). Given a \( N_e \) in the range of

\[
\log\left[\frac{M_{L}}{H_{\text{inf}}}\right] \lesssim N_e,
\]

\[
M_L \lesssim H_{\text{inf}} \lesssim \mathcal{O}(\text{MeV}) \tag{29}
\]

to explain the abundance of dark matter in general. The last inequality becomes equality if

\[
N_e \gtrsim H_{\text{inf}}^2/M_L^2.
\]
4 Discussion and conclusions

We have studied a two axion model inspired by M-theory with two would-be 1 Hz axions. By turning on the gluon coupling, one of the axions becomes massive and becomes the QCD axion, while the other remains light. Introducing more axions in the 1 Hz window does not affect our conclusions.

With the expected GUT scale decay constants, this model has a relic abundance problem, and we introduced two models to suppress it. In the ultra-low scale TCC scenario, from Eq. (24) the abundance is suppressed exponentially for heavier axions with $\theta_{inf}^I$, $H_{inf}$, and $N_e$ fixed. Thus the lightest axion tends to contribute dominantly to the dark matter and the heavier ones almost do not contribute (an exception is that the axions, including the lightest one, have almost degenerate masses). Thus one can replace $M_L$ in the discussions (e.g. the horizontal axis of Figs. 2 and 3) to be the lightest axion mass as a good approximation. In this model, the ultra-low values of $H_{inf}$ lead to constraints after imposing $T_{R}^{max} \gtrsim 1 \text{MeV}$ for successful BBN. On the other hand, for the low-scale eternal inflation scenario, from Eqs. (28) and (18), the lightest axion also contributes most to the total abundance in the Bunch-Davies distribution. Since the dependence of the abundance on the axion mass is not exponential, the inclusion of the additional axions may decrease the required $H_{inf}$ slightly. However, this does not give serious problem since $H_{inf}$ is not very low and reheating does not present a problem in this model. Conversely, in the case with a single axion with (11), as we showed, it becomes the QCD axion but with a non-vanishing strong CP phase. The QCD axion can be the dominant dark matter in either the ultra-low scale inflation, with $m_a \approx H_{inf}$ or the low-scale eternal inflation scenario [137, 138]: $H_{inf} \sim m_a - 10^2 \text{MeV}$. The predictions are the signal might be searched by ABRACADABRA, via nucleon EDMs, and in the most optimistic scenarios also by CASPER. The photon coupling may also be tested by future measurements of CMB spectral distortions, if primordial magnetic fields on Mpc scales are nG or stronger [148].

Notice that once either axion coupling as well as the mass of $A_L$ is measured, the existence of the QCD axion is anticipated. This is because if there were no other axion composing the QCD axion, $A_L$ would acquire $\mathcal{O}(10^{-10})$ eV mass and become the QCD axion unless the coupling to gluons is highly suppressed. In the case of the $A_L$ discovery, one may make an effort to measure the QCD-axion mediated force in the aforementioned ARIADNE axion experiment or measure the nucleon EDM to confirm our scenario.

Let us mention the alleviation of the other cosmological problems in the UV model due to the low inflationary scale. The moduli, if stabilized by SUSY breaking, have masses
around $m_{3/2}$. If such moduli are displaced from the potential minima during inflation, they later dominate the Universe via coherent oscillations and cause the notorious cosmological moduli problem \cite{149,150}. In our scenario, obviously $H_{\text{inf}} \ll m_{3/2}$ is satisfied. The moduli oscillations start during inflation and the abundances are exponentially diluted. Therefore the moduli problem for the mass $\sim m_{3/2}$ is solved. On the other hand there may be a "moduli" problem induced by lighter axions if some axions happen to have masses in the range $10^{-22} - 10^{-18}$ eV and the decay constants $\gtrsim 10^{17}$ GeV. The abundance of these lighter axions is overproduced with the values of $H_{\text{inf}}$ required to explain the 1 Hz axion dark matter abundance (see Ref. \cite{139}). However, such axions may be absent if the mass distributions take particular shapes or there is a small total number of axions \cite{21}. Another implicit cosmological problem is the gravitino problem. For $O(1)$ TeV $\lesssim m_{3/2} \lesssim 50$ TeV, gravitino decays may spoil BBN. Thus the gravitino abundance, which is produced most efficiently at high temperature, should be small enough at its decay. This sets an upper bound on the reheating temperature $T_R \lesssim 10^{5-9}$ GeV \cite{151}. This is easily (absolutely) consistent with our (ultra) low-scale inflation scenario. These facts imply that the moduli and SUSY breaking scales can be lower than the traditional 10-100 TeV bound from the moduli and gravitino problems. (There are collider bounds setting SUSY scale higher than $O(1)$ TeV.)

It is also possible to set the 1 Hz axion around the hilltop of the potential due to the mixing between the inflaton and the axion \cite{152}, which flips the potential of the axion after inflation. The condition that the axion field is frozen at the flip can be consistent with low scale eternal inflation, which suppresses the QCD axion abundance. In this large-realignment scenario, 1 Hz axion dark matter can form dense structures which can be tested from the gravitational lensing effect or the axion implosion can cause detectable stochastic gravitational waves \cite{153}. A sub-GUT scale decay constant is required for a correct dark matter abundance. A smaller decay constant not only is accessible in the CASPER-ZULF and DANCE experiments, but also could lead to novae where the axionic clouds of the BH collapse due to self-interactions. The axion novae may lead to testable gravitational wave bursts, as well as the gamma-ray bursts with the axion-photon conversion given cosmic magnetic fields.

In conclusion, we have opened the window of 1 Hz axions which it has been suggested are a natural prediction of the M-theory axiverse \cite{5,30}. We showed that a would-be 1 Hz axion can be the QCD axion and solve the strong CP problem while at the same time inducing a testable non-vanishing strong CP phase. The abundance of the axions can be consistent with the observed one, while maintaining GUT-scale decay constants with sufficiently low-

\footnote{A smaller $\Lambda$ for the 1 Hz axion window can be realized in High scale SUSY scenarios \cite{154,161} which lead to smaller $\alpha_{\text{GUT}}$ due to renormalization group effects.}
scale inflation. Since the 1 Hz axion dark matter is dominant, the scenario predicts the non-observation of the QCD axion dark matter but many phenomenological observables in the 1 Hz window from the lighter field.

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