Shot noise limited interferometry for measuring classical force

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We propose an interferometry technique, by using electromagnetically induced transparency phenomena, for measuring classical force. The classical force is estimated by measuring the phase at the output of the interferometer. The proposed measurement mechanism satisfies quantum non-demolition measurement conditions leading to back-action evasion. We further derive the sufficient condition under which the thermal noise in the interferometer is negligible. With no back-action noise and no thermal noise, the sensitivity of this technique is limited by shot noise only.

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I. INTRODUCTION

Interferometry is a vital component in precision measurement schemes. For example, the gravitational wave detector LIGO is a giant Michelson-Morley interferometer which can detect tiny changes in displacement. While LIGO requires a huge set-up, the recent advances in the field of optomechanics\cite{1} can lead to portable ultra-high precision measurement devices. The precision measurements in both the ‘huge’ LIGO and the ‘tiny’ miniaturized optomechanical cavities are limited by similar fundamental quantum aspects\cite{2,3}, like shot noise, back-action noise, and thermal noise. Back-action limited by similar fundamental quantum aspects\cite{2,3}, like shot noise, back-action noise, and thermal noise. The back-action evasion is achieved measuring classical force with out back-action noise\cite{11,22–24} and thermal noise. The back-action evasion is achieved using electromagnetically induced transparency (EIT)\cite{25–28} phenomena. We further describe the sufficient condition under which the thermal noise can be suppressed completely by using cold atoms\cite{29}. Because both thermal noise and quantum back-action noise are suppressed, shot noise is the only limiting factor in this new interferometer.

II. BACK-ACTION EVASION

Figure 1 shows the schematics of the back-action evasion interferometer. A three-level atomic medium (shown by small yellow rectangle in Fig. 1), with atomic levels $|a\rangle$, $|b\rangle$, and $|c\rangle$, is placed inside a running wave optical cavity formed by a semi-transparent mirror ‘m’ and two mirrors 1 & 2 as shown in Fig. 1. The running wave optical cavity, along with atomic medium, is placed in one of the arms of the Mach-Zehnder interferometer as shown in Fig. 1. The atomic medium is enclosed in a small volume such that no atoms enter or leave the interaction region. The energy level structure of the atoms inside the atomic medium is shown in the big yellow circle in Fig. 1. The cavity field (shown in blue color) couples $|a\rangle - |b\rangle$ transition while a strong classical field, whose area of cross-section is shown by red circle, propagating perpendicular to the YZ-plane couple $|a\rangle - |c\rangle$ transition.

The Hamiltonian\cite{34,33} $\hat{H}$ is written as

$$\hat{H} = \sum_{j} \frac{\hat{p}^2_j}{2m} + \hbar \nu (\hat{\hat{c}} + \frac{1}{2}) + \sum_{j} \frac{\hbar \omega_j}{2} \hat{a}^\dagger_j \hat{a}_j + \sum_{j} \hbar g_j \hat{a}^\dagger_j \hat{b}^\dagger_j (\hat{\hat{c}} + \hat{\hat{c}}^\dagger) + H.C.$$ \hspace{1cm} (1)

where H.C is the hermitian conjugate, the superscript $j$ indicates $j$-th atom and $N$ is the number of atoms. $\hat{p}^j$ is the momentum of the ‘$j$’ atom along Z-axis and $m$ is atom’s mass. $\hat{\hat{c}}$ and $\nu$ are the annihilation operator and frequency of the cavity field. $\hbar$ is reduced Planck constant, $\omega_j$ is the eigen frequency of the atomic level $|u\rangle$ when the atoms are at rest and $\sigma^u_{\alpha} = |u\rangle \langle u| \hat{a}^\dagger$. $q$ is the coupling constant between weak cavity field $\hat{c}$ and $|a\rangle - |b\rangle$ transition. $\widetilde{\Omega}$ is the Rabi frequency of the strong classical driving laser propagating along X-axis. Note that $|a\rangle^\dagger \langle c|\hat{c}^\dagger$ has no $\hat{\hat{c}}$-dependence in Eq. (1) as the driving field perpendicular to Z-axis. $\hat{a}^\dagger$, $\hat{\hat{c}}$, and $\hat{\hat{c}}^\dagger$ are the input and decay rate of semi-transparent mirror ‘m’, respectively. $F$ is the classical force acting on the atom, and $\hat{\hat{c}}$ is the displacement of ‘$j$’ atom.

We separate the $\hat{\hat{c}}^\dagger$ dependence from $|a\rangle^\dagger \langle b|\hat{c}^\dagger$ by writing $|a\rangle^\dagger \langle b|\hat{c}^\dagger = \langle a| \langle b| e^{ik\hat{\hat{c}}^\dagger}$, where $k$ is the wave-vector of $\hat{c}$. Equations of motion for atom-cavity field interaction,
after rotating wave approximation, are given as

\[ \hat{c} = -\frac{\xi}{2} \hat{c} - i \sum_j g^* \hat{a}_{ba}(\hat{z}) + \sqrt{\xi} \hat{a}_{\text{in}} \quad (2a) \]

\[ \hat{\sigma}_{ba}^j(\hat{z}) = \Gamma^j \hat{\sigma}_{ba}^j(\hat{z}) + ig(\hat{\sigma}_{a}^j - \hat{\sigma}_{b}^j) \hat{c} - i \Omega \hat{\sigma}_{bc}^j(\hat{z}) + \hat{F}_{ba}, \quad (2b) \]

\[ \hat{\sigma}_{bc}^j(\hat{z}) = \Gamma_o \hat{\sigma}_{bc}^j(\hat{z}) + ig \hat{\sigma}_{ac}^j \hat{c} - i \Omega^* \hat{\sigma}_{ba}^j(\hat{z}) + \hat{F}_{bc}, \quad (2c) \]

\[ \hat{\sigma}_{ac}^j = -\gamma \hat{\sigma}_{ac}^j + i(\hat{\sigma}_{c}^j - \hat{\sigma}_{a}^j) \Omega^* + ig^* \hat{\sigma}_{ba}^j(\hat{z}) \hat{c}^\dagger + \hat{F}_{ac}, \quad (2d) \]

\[ \dot{\hat{p}}^j = i \hbar k (g^* \hat{c} \hat{a}_{ba}(\hat{z}) - g \hat{a}_{ab}^j(\hat{z}) \hat{c}) + F, \quad (2e) \]

where \( \Gamma^j = i(\frac{k p_i}{m} - \frac{\hbar k^2}{2m}) - \gamma \), \( \Gamma_o = \frac{k p_i}{m} - \frac{\hbar k^2}{2m} - \gamma \), \( |c\rangle \langle a| = \hat{\sigma}_{ac}^j e^{-i\omega t} \), \( |b\rangle \langle a| \langle \hat{z}| = \hat{\sigma}_{ba}^j(\hat{z}) e^{-i\omega t} \), \( |b\rangle \langle c| \langle \hat{z}| = \hat{\sigma}_{bc}^j(\hat{z}) e^{-i\omega t} \), \( \Omega = \Omega e^{-i\omega t} \), \( \hat{c} = \hat{c} e^{-i\omega t} \). We assumed that the cavity field and the driving field are on resonance with \( |a\rangle - |b\rangle \) and \( |a\rangle - |c\rangle \) transitions, respectively, when the atom is at rest. \( \hat{F}_{ac} \) is the slowly varying noise operator, \( \gamma \) is the decoherence on transitions \( |a\rangle - |b\rangle \) and \( |a\rangle - |c\rangle \). \( \hat{a}_{\text{in}} \) is the slowly varying operator of \( \hat{a}_{in} \). On the electric dipole forbidden transition \( |b\rangle - |c\rangle \), decoherence \( \gamma_o \) is added phenomenologically. The first term on the right hand side of Eq. (2e) represents the radiation pressure force \[34\] because of absorption of cavity field. From now on, we drop the recoil frequency \( \hbar k^2/2m \) term in Eq. (2) by assuming that it is much smaller than the Doppler shift term \( p^* k/m \).

Assuming that cavity field as weak field, we treat \( \hat{c} \) up to its first order while keeping the \( \Omega \) to all orders (the superscript ‘0’ indicates zeroth order in \( \hat{c} \), while the superscript ‘1’ indicates first order in \( \hat{c} \)). Now the relevant equations of motion are given as

\[ \dot{\hat{a}} = \hat{a}_{\text{in}} - \sqrt{\xi} \hat{c}, \quad (3a) \]

\[ \dot{\hat{c}} = -\frac{\xi}{2} \hat{c} - i \sum_j g^* \hat{a}_{ba}^j(\hat{z}) + \sqrt{\xi} \hat{a}_{\text{in}} \quad (3b) \]

\[ \dot{\hat{F}}_{ba}^j(\hat{z}) = \Gamma^j \hat{F}_{ba}^j(\hat{z}) - ig \hat{c} - i \Omega \hat{F}_{bc}^j(\hat{z}) + \hat{F}_{ba}, \quad (3c) \]

\[ \dot{\hat{F}}_{bc}^j(\hat{z}) = \Gamma_o \hat{F}_{bc}^j(\hat{z}) - i \Omega^* \hat{F}_{ba}^j(\hat{z}) + \hat{F}_{bc}, \quad (3d) \]

\[ \dot{\hat{p}}^j = F, \quad (3e) \]

where \( \Gamma^j = i k \hat{p}^j(\hat{z}) / m - \gamma \), \( \Gamma_o^j = i k \hat{p}^j(\hat{z}) / m - \gamma_o \), \( \hat{a} \) is the output field \[35 \hat{36} \] from ‘m’. In writing Eq. (2), I used the EIT system properties that all the atomic population resides in |b\rangle, and hence \( \hat{\sigma}_{b}^j = 1 \) while \( \hat{\sigma}_{a}^j = \hat{\sigma}_{c}^j = \hat{\sigma}_{ac}^j = \hat{\sigma}_{bc}^j = \hat{\sigma}_{ba}^j = 0 \) \[37 \hat{41} \]. Equation (3e) implies that the time evolution of \( \hat{p}^j(\hat{z}) \) is not disturbed by the measuring device and hence momentum \( \hat{p}^j(\hat{z}) \) is a quantum non-demolition variable. Its worth noting that back-action is present in Eq. (2a) and it is the EIT system which leads to back-action evasion in Eq. (3). Application of EIT for velocity read out purpose has been studied \[42 \hat{45} \] however, the possibility of back-action evasion is not shown explicitly.

By solving Eq. (3e), we can replace \( k \hat{p}^j(\hat{z}) / m \) and \( \hat{z}(\hat{t}) \) with classical values \( Fkt_m / m \) and \( z(\hat{t}) \), respectively, where \( t_m \) is the time of measurement. Throughout this manuscript, we assume that \( t_m \) is much less than the characteristic time an atom takes to travel from one end of the atomic medium to the other, along Z-axis.

### A. Signal

Assuming that \( t_m \ll 1/\gamma \), mean value of \( \hat{c} \) can be obtained by solving Eq. (3) as

\[ \hat{c} = -\frac{\sqrt{\xi} \hat{a}_{\text{in}}}{\Lambda_n - \frac{1}{2}} \]
where
\[
\lambda_o = \frac{N|g|^2(\frac{kt_m}{m} F - \gamma)}{(\frac{kt_m}{m} F - \gamma)^2 - N|g|^2} + |\Omega|^2. \tag{5}
\]
\(\bar{c}\) and \(\bar{a}_{in1}\) are classical mean values of \(\hat{c}\) and \(\hat{a}_{in}\), respectively. Assuming that \(\Omega^2 \gg \gamma \gamma_o\), and by considering \(\lambda_o\) only up to the first order of \(F kt_m/m\), we can approximate the output from semitransparent mirror ‘m’ as
\[
\bar{a} \approx \left( \frac{\frac{kt_m}{m} N|g|^2}{|\Omega|^2} - \frac{N|g|^2 |a_{in}|^2}{|\Omega|^2} \right) \bar{a}_{in}, \tag{6}
\]
where \(\bar{a}\) is the mean value of \(\hat{a}\). Electromagnetic field in the arm-2 of Fig. (1) is represented by \(\hat{a}_1\). Hence the difference in the photo detector readings, after adding a constant \(\pi/2\) phase to \(\hat{a}_1\), is given as
\[
I_1 - I_2 = \hat{a}_1^\dagger a_1 + \hat{a}_1 a_1^\dagger, \tag{7}
\]
where \(I_1\) and \(I_2\) are the intensities at photo detectors D1 and D2, respectively. By using Eq. (6) and the relation \(a_{in} = i \bar{a}_{in}\), we can write mean value of Eq. (7) as
\[
\langle I_1 - I_2 \rangle = \frac{-N|g|^2 \xi}{|\Omega|^2} |\bar{a}_{in}|^2. \tag{8}
\]
Equation (7) has maximum value when \(N|g|^2 |\gamma_o|/|\Omega|^2 = \zeta/2\). Hence the maximum signal we can obtain is
\[
\langle I_1 - I_2 \rangle = \frac{-kt_m}{m} F \xi |\bar{a}_{in}|^2. \tag{9}
\]
**B. Noise spectrum**

The linearized equations of motion for the fluctuations are
\[
\frac{\partial}{\partial t} \delta_c = -\frac{\xi}{2} \delta_c - i \sum_j g^* \delta_{b_{a}}^{(1)} + \sqrt{\xi} \delta_{b_{in}}, \tag{10a}
\]
\[
\frac{\partial}{\partial t} \delta_{b_{a}}^{(1)} = \left( \frac{\frac{kt_m}{m} F - \gamma}{|\Omega|^2} - \frac{i \Omega^{*} \delta_{b_{c}}^{(1)}}{N|g|^2} \right) \delta_{b_{a}}^{(1)} - i \xi \delta_{b_{a}}^{(1)} + \hat{F}_{b_{a}}, \tag{10b}
\]
\[
\frac{\partial}{\partial t} \delta_{b_{c}}^{(1)} = \left( \frac{\frac{kt_m}{m} F - \gamma}{|\Omega|^2} - \frac{i \Omega^{*} \delta_{b_{c}}^{(1)}}{N|g|^2} \right) \delta_{b_{c}}^{(1)} - i \xi \delta_{b_{c}}^{(1)} + \hat{F}_{b_{c}}, \tag{10c}
\]
where \(\delta_c, \delta_{b_{a}}^{(1)}, \delta_{b_{c}}^{(1)}, \delta_{b_{in}}\) are the quantum fluctuation of \(\hat{c}, \hat{b}_{a}, \hat{b}_{c}, \hat{a}_{in}\), respectively. The terms \(kt_m F/m, \delta_{b_{a}}^{(1)}, \delta_{b_{c}}^{(1)}\) are very small, hence the product terms such as \(ikt_m F \delta_{b_{a}}^{(1)}/m\), can be neglected in

Eq. (10c). By using the Fourier transform function \(\hat{F}(x(t)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(t) e^{i\omega t} dt\), the solution to the above equations in the Fourier frequency space is given as
\[
\mathcal{F}_{\omega} = -\frac{\hat{F}(\omega) + \sqrt{\xi} \delta_{b_{in}}(\omega)}{i\omega - \frac{\zeta}{2} + \frac{N|g|^2 (\omega - \gamma)}{|\Omega|^2}}, \tag{11}
\]
where
\[
\hat{F}(\omega) = \sum_j g^* \Omega_j \delta_{b_{c}}^{(1)} + i (i\omega - \gamma) \delta_{b_{a}}^{(1)} + \sqrt{\xi} \hat{F}(\omega), \tag{12}
\]
Fluctuation in output field is given as
\[
\delta_{\omega}^{(1)}(\omega) = \frac{|i\omega + \Lambda(\omega) + \zeta/2|^{2}}{|i\omega + \Lambda(\omega) - \zeta/2|^{2}}. \tag{13}
\]
Hence, we can write
\[
\langle \delta_{\omega}^{(1)}(\omega) \rangle = \frac{|i\omega + \Lambda(\omega) + \zeta/2|^{2}}{|i\omega + \Lambda(\omega) - \zeta/2|^{2}}. \tag{14}
\]
Fluctuation in Eq. (7) is given as
\[
\Delta = \hat{I}_1 - \hat{I}_2 - \langle \hat{I}_1 - \hat{I}_2 \rangle = \delta_1^\dagger \delta_1 + \delta_2 \delta_1^\dagger + \delta_1 \delta_2^\dagger + \delta_1^\dagger \delta_2. \tag{15}
\]
Variance of \(\Delta(\omega)\), where \(\Delta(\omega) = \mathcal{F}(\Delta)\), gives the power spectral density \(V^2\) as
\[
\frac{\Delta^2(\omega)}{\Delta(\omega)} = \frac{\hat{a}_1^2 \langle \delta_{\omega}^{(1)}(\omega) \rangle + |a_{in}|^2 \langle \delta_{\omega}^{(1)}(\omega) \rangle}{\hat{a}_1^2 + \hat{a}_1^\dagger(\omega) \hat{a}_1 + \hat{a}_1 \hat{a}_1^\dagger + \hat{a}_1 \hat{a}_1^\dagger}, \tag{16}
\]
where \(V^2\) can be computed by using the correlation functions
\[
\sum_{j,j'} \langle \hat{F}_{b_{c}}^{(1)}(t_1) \hat{F}_{b_{c}}^{(1)}(t_2) \rangle = 2 N \gamma \delta(t_1 - t_2), \tag{16a}
\]
\[
\sum_{j,j'} \langle \hat{F}_{b_{a}}^{(1)}(t_1) \hat{F}_{b_{a}}^{(1)}(t_2) \rangle = 2 N \gamma \delta(t_1 - t_2), \tag{16b}
\]
\[
\langle \delta_{b_{in}}(t_1) \delta_{b_{in}}^{(1)}(t_2) \rangle = \delta(t_1 - t_2). \tag{16c}
\]
At \(\omega = 0\), by substituting Eq. (13), Eq. (4), and \(\hat{a}_1 = i \bar{a}_{in}\) in Eq. (15), we evaluate
random thermal motion leads to Doppler broadening \[46\] which can be estimated by using Maxwell-Boltzmann distribution. Since the probe and drive lasers are in \(z\) and \(x\) directions, respectively, we only need to consider the thermal Doppler detuning because of \(v_z\) and \(v_x\). Where \(v_z\) and \(v_x\) are the thermal velocities of the atom with respect to probe and drive laser fields, respectively. So the effect of temperature can be accounted by using the Maxwellian velocity distribution as

\[
\frac{m}{2\pi k_B T} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-m(v_x^2 + v_z^2)/2k_B T} \sum_j \delta^{(1)}_{ba}(v_x, v_z) dv_x dv_z = \frac{m}{2\pi k_B T} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-m(v_x^2 + v_z^2)/2k_B T} \left( N \delta^{(1)}_{baD} + \sum_j \delta^{(1)}_{baD}(\omega) \right) dv_x dv_z,
\]

where

\[
\delta^{(1)}_{baD}(\omega) = \frac{iN \delta_{\lambda}(\omega)}{(i\omega + \frac{i\omega v_{bz} - \omega v_{ax} v_z}{c} \gamma/c) - \delta_{ba}(\omega) - \frac{[\gamma/c]}{\omega + \frac{i\omega v_{bx} - \omega v_{az} v_x}{c} \gamma/c - \delta_{ba}(\omega)} - \frac{[\gamma/c]}{\omega + \frac{i\omega v_{bx} - \omega v_{az} v_x}{c} \gamma/c - \delta_{ba}(\omega)} = \frac{N}{2\pi k_B T} \sum_j \left( \frac{i\omega v_{bx} - \omega v_{ax} v_x}{c} \gamma/c - \delta_{ba}(\omega) + \frac{[\gamma/c]}{\omega + \frac{i\omega v_{bx} - \omega v_{ax} v_x}{c} \gamma/c - \delta_{ba}(\omega)} \right).
\]

where \(N/V\) represents the density of atoms in the running wave cavity.

By considering realistic parameters: \(N/V \approx 10^{18} \text{ m}^{-3}\), \(c = 3 \times 10^8 \text{m/s}\), \(\gamma = 10^9 \text{ Hz}\), \(\omega_{ab} = 5 \times 10^{15} \text{ Hz}\), \(\gamma_{a0} = 10^3 \text{ Hz}\), \(\zeta = 10^9 \text{ Hz}\), we estimate that \(|\Omega|^2 \approx 3 \times 10^{16} \text{ Hz}^2\). By considering that \(\omega_{cb} = 10^{-6} \omega_{ab}\), \(m \approx 1.4 \times 10^{-25} \text{J/K}\), and \(T = 1 \text{K}\), the Doppler width of \(|a\rangle - |b\rangle\) transition is \(\omega_{ab} \sqrt{2\ln 2k_B T/mc^2} \approx 2.8 \times 10^8 \text{Hz}\) and the Doppler width of \(|c\rangle - |b\rangle\) transition is \(\omega_{cb} \sqrt{2\ln 2k_B T/mc^2} \approx 264 \text{Hz}\). Hence Eq. (21) is practically fulfilled \[47\] when the temperature of the atomic medium is sufficiently low and then we can approximate Eq. (20) as

\[
\delta^{(1)}_{baD} = iN \delta_{\lambda}(\omega)(i\omega + i\frac{(\omega v_{bx} - \omega v_{ax} v_x}{c} \gamma/c - \omega}{i\omega - \gamma}) \left( i\omega + i\frac{(\omega v_{bx} - \omega v_{ax} v_x}{c} \gamma/c - \omega}{i\omega - \gamma} + \frac{[\gamma/c]}{\omega + \frac{i\omega v_{bx} - \omega v_{ax} v_x}{c} \gamma/c - \omega} + \frac{[\gamma/c]}{\omega + \frac{i\omega v_{bx} - \omega v_{ax} v_x}{c} \gamma/c - \omega} \right),
\]

where

\[
\delta^{(1)}_{baD} = iN \delta_{\lambda}(\omega) \left( i\omega + i\frac{(\omega v_{bx} - \omega v_{ax} v_x}{c} \gamma/c - \omega}{i\omega - \gamma} \right) \left( i\omega + i\frac{(\omega v_{bx} - \omega v_{ax} v_x}{c} \gamma/c - \omega}{i\omega - \gamma} + \frac{[\gamma/c]}{\omega + \frac{i\omega v_{bx} - \omega v_{ax} v_x}{c} \gamma/c - \omega} + \frac{[\gamma/c]}{\omega + \frac{i\omega v_{bx} - \omega v_{ax} v_x}{c} \gamma/c - \omega} \right).
\]
By substituting Eq. \((23)\) in Eq. \((19)\), we will see that

\[
\frac{m}{2\pi k_B T} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-m(v_x^2+v_y^2)/2k_B T} \sum_j \sigma_{pa}^{j(1)}(v_x, v_y) dv_x dv_y = \sum_j \sigma_{pa}^{j(1)}.
\]  

(24)

Hence the Doppler broadening or the thermal noise effect is practically zero when the condition given in Eq. \((21)\) is fulfilled.

IV. DISCUSSION

By comparing the noise in Eq. \((18)\) with signal in Eq. \((9)\), we estimate the force sensitivity \(F_s\) as

\[
F_s = \frac{V}{\langle I_1 - I_2 \rangle} \frac{m \gamma_o}{k_B T \sqrt{\langle \sigma_{ba} \rangle}}
\]

(25)

Recently, optomechanical systems \([48]\) has gained a lot of attention as ultra-sensitive force detectors \([17, 49, 51]\). Generally, classical force is estimated in optomechanical systems by measuring the position \([52]\) of the object on which force is acting. However, accuracy of position measurement intrinsically leads to quantum back-action noise \([53]\) which limits the best precision achievable. Application of squeezed states \([54, 55]\) and back-action evasion \([50, 56–59]\) methods are proposed to overcome SQL, but still, thermal noise \([52, 60–62]\) limits the sensitivity of optomechanical systems. Moreover, preparing optimum squeezed states to overcome the SQL is experimentally challenging. In this manuscript we proposed an interferometry technique in which both the back-action noise and the thermal noise are suppressed.

V. CONCLUSION

A technique to measure classical force without measurement back-action noise and thermal noise is described. Back-action evasion is achieved by using EIT phenomena. Role of temperature is studied and derived the sufficient conditions for nullifying the thermal noise. With no thermal noise and no back-action noise, shot noise is the only limitation in this measurement scheme.

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