Conditional Spontaneous Spin Polarization and Bifurcation Delay in Coupled Two-Component Bose-Einstein Condensates

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Abstract

The spontaneous spin polarization and bifurcation delay in two-component Bose-Einstein condensates coupled with Raman pulses are investigated. We find that the bifurcation and the spontaneous spin polarization depend not only on the system parameters, but also on the relative phase between two components. Through bifurcations, the system enters into the spontaneous spin polarization regime from the Rabi regime. We also find that bifurcation delay appears when the parameter is swept through the static bifurcation point. This bifurcation delay is responsible for metastability leading to hysteresis. The area enclosed in the hysteresis loop increases with the sweeping rate of the parameter.

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Electronic and nuclear-spin polarization in an atomic vapor with optical pumping have been investigated extensively [1]. Under conditions in which electronic spin exchange takes place faster than spin relaxation, spontaneous spin polarization appears. This interesting phenomenon is very similar to ferromagnetism and has been observed in wide ranges of atomic intensity, pump laser frequency and intensity. The appearance of spontaneous spin polarization means that the atomic vapor has two stable states with large spin polarization. The experimental realization of it has been applied to the field of optical bistability [2]. The atomic spin polarization exhibits striking hysteresis in switching between the bistable states [1]. This is analogous to those ferromagnetic system displays magnetic hysteresis [3].

All the previous works have only considered the case of the thermal atoms, the experimental realization of multi-component Bose-Einstein condensates (BECs) [4,5] causes our interest in considering the similar behavior of the ultracold atoms. There are many differences between thermal and cold atoms. The first one is that the collision among thermal atoms is noncoherent. However, when the temperature is close to the critical temperature realizing Bose-Einstein condensate (BEC) ($T \sim T_{BEC}$), the collision among ultracold atoms is coherent due to the path between such collision is smaller than the phase coherence length [6,7]. Does this coherent property play a role in the polarization process of the ultracold atoms? Another difference between thermal atoms and cold ones is that the interaction strength of cold atoms can be controlled easily [6,8]. In this letter, we firstly show the coherence among ultracold atoms gives rise to the conditional spontaneous spin polarization which is determined by both the system parameters and the relative phase. Then, we present the bifurcation induced by tuning the s-wave interaction between ultracold atoms. Lastly, we propose an experiment to confirm our prediction.

We consider that the same kind of bosonic atoms, which are trapped in a single-well potential, are condensed in two different hyperfine levels $|1\rangle$ and $|2\rangle$. Raman transitions between two hyperfine states are induced by the laser fields with the effective Rabi-frequency $\Omega$ and a finite detuning $\delta$. The internal Josephson effects [4,5,11,12], coherent coupling effects [13], vortices [14] and spin waves [15] in such systems have stimulated great interest.
of many theoretists and experimentalists. In the rotating frame, neglecting the damping and the finite-temperature effects, the coupled two-component BECs system can be described by a pair of coupled GPEs \[1\] \[2\] \[3\]

\[
i\hbar \frac{\partial \psi_i(r,t)}{\partial t} = (H_i^0 + H_i^{MF} - \frac{\delta}{2})\psi_i(r,t) + \frac{\Omega}{2}\psi_i(r,t),
\]

\[
i\hbar \frac{\partial \psi_1(r,t)}{\partial t} = (H_1^0 + H_1^{MF} + \frac{\delta}{2})\psi_1(r,t) + \frac{\Omega}{2}\psi_2(r,t).
\]

Here, the free evolution Hamiltonians \(H_i^0 = -\frac{\hbar^2 \nabla^2}{2m} + V_i(r)\) \((i = 1,2)\) and the mean-field interaction Hamiltonians \(H_i^{MF} = \frac{4\pi\hbar^2}{m}(a_{ii}|\Psi_i(r,t)|^2 + a_{ij}|\Psi_j(r,t)|^2)\) \((i,j = 1,2, i \neq j)\) \((a_{ij}\) is the scattering length between states \(i\) and \(j\) which satisfies \(a_{ij} = a_{ji}\)). For the cigar-shaped trap (the trap frequencies satisfying \(\omega_x = \omega_y >> \omega_z\)), when the coupling is weak enough (i.e., the Rabi frequency satisfies \(\Omega/\omega_z \ll 1\)), the macroscopic wavefunctions can be written as the variational ansatz \(\Psi_i(r,t) = \psi_i(t)|\Phi_i(r)|\) \((i = 1,2)\) with amplitudes \(\psi_i(t) = \sqrt{N_i(t)}e^{i\alpha_i(t)}\) and spatial distributions \(\Phi_i(r)\). The symbols \(N_i(t)\) and \(\alpha_i(t)\) are the atomic population and phase of the \(i\)-th component, respectively. Due to the coupling is very small, the spatial distributions vary slowly in time and are very close to the adiabatic solutions to the time-independent uncoupled case for GP equations (1), being slaved by the populations \[4\] \[5\]. Thus, the amplitudes obey the nonlinear two-mode dynamical equations

\[
i\hbar \frac{d}{dt}\psi_2(t) = (E_2^0 - \frac{\delta}{2} + U_{22}|\psi_2(t)|^2 + U_{21}|\psi_1(t)|^2)\psi_2(t) + \frac{K}{2}\psi_1(t),
\]

\[
i\hbar \frac{d}{dt}\psi_1(t) = (E_1^0 + \frac{\delta}{2} + U_{11}|\psi_1(t)|^2 + U_{12}|\psi_2(t)|^2)\psi_1(t) + \frac{K}{2}\psi_2(t).
\]

The parameters \(E_i^0 = \int \Phi_i(r)H_0^0\Phi_i(r)dr\) \(\wedge r\), \(U_{ij} = \frac{4\pi\hbar^2a_{ij}}{m}\int |\Phi_i(r)|^2|\Phi_j(r)|^2dr\) \(\wedge r\) = \(U_{ji}\) and \(K = \Omega\int \Phi_1(r)\Phi_2(r)dr\) \(\wedge r\) \((i,j = 1,2)\). The terms in \(K\) describe the internal tunnelling between two BEC states, whereas the terms in \(U_{ij}\), which depend on the numbers of atoms in each BEC state, describe the mean-field interaction between atoms. When \(U_{21}\) and \(\delta\) equals zero, these coupled equations can also describe the BECs in a double-well potential \[6\]. Introducing the Bloch’s spin vectors

\[
u = \frac{\psi_2^*\psi_1 + \psi_2^*\psi_2}{\psi_1^*\psi_1 + \psi_2^*\psi_2}, \hspace{1cm} w = \frac{\psi_2^*\psi_2 - \psi_2^*\psi_1}{\psi_1^*\psi_1 + \psi_2^*\psi_2}.
\]

Obviously, \(u^2 + v^2 + w^2 = 1\). When the total atomic numbers \(N_T = N_1 + N_2 = \psi_1^*\psi_1 + \psi_2^*\psi_2\) is conserved, setting the Planck constant \(\hbar = 1\), the Bloch’s spin vectors satisfy
In this Bloch’s equation, the parameters satisfy
\[ \gamma = E_2^0 - E_1^0 + N_T(U_{22} - U_{11})/2 - \delta \]
and
\[ G = N_T(U_{22} + U_{11} - 2U_{12})/2. \]
Comparing the above equation with the one for the linear case \((U_{ij} = 0)\) of equation (2), one can find that the mean-field interaction induces a shift \((Gw)\) in the transition frequency and this shift is proportional to the relative population \(w\).

Taking \(|1\rangle\) as spin-up state and \(|2\rangle\) as spin-down state, the above two-component BECs system can be regarded as an ensemble of quantum spin-1/2 particles. Thus, the longitudinal component \(w\) of the pseudospin describes the relative population, and the transverse components \(u\) and \(v\) characterize the coherence. In this language, the effective Rabi frequency causes an effective transverse magnetic field \(K\) along axis-\(u\), the effective detuning induces an effective longitudinal magnetic field \(\gamma\), and the mean-field interaction brings an effective longitudinal magnetic field \(Gw\) which depends on the longitudinal spin component.

From the definition of the Bloch’s spin vectors, we know that the above system can be described with only two independent variables. If we use the longitudinal spin component \(w\) and the relative phase \(\phi = \alpha_2 - \alpha_1\) as independent variables, rescaling the time \(Kt\) to \(t\), the motion equations
\[
\frac{dw}{dt} = -\sqrt{1 - w^2} \sin \phi,
\]
\[
\frac{d\phi}{dt} = -\gamma/K - (G/K)w + w \cos \phi / \sqrt{1 - w^2}.
\]
are equivalent to the Bloch’s equation. The above equations are consistent with those derived from the secondary quantized model [4].

Below, let us analyze the stationary states of the coupled two-component BECs system. The stationary states can be obtained from the stable fixed points of the system. The fixed points correspond to those solutions satisfying \(dw/dt = 0\) and \(d\phi/dt = 0\). In the region \([0, 2\pi)\) of the relative phase, we find two different modes of stationary states existing in the system: one is the equal-phase mode with zero relative phase \((\phi = 0)\), the other one is the
anti-phase mode with $\pi$ relative phase ($\phi = \pi$).

The number of the fixed points and the stationary states depend on the ratios $\gamma/K$, $G/K$ and the relative phase. For the equal-phase mode, only a fixed point exists when $G/K \leq 1$ and this fixed point is stable. When $G/K > 1$, there are two stable fixed points and an unstable one for $(G/K)^{2/3} - (\gamma/K)^{2/3} > 1$; there is only one stable fixed point for $(G/K)^{2/3} - (\gamma/K)^{2/3} < 1$; the saddle-node bifurcations occur at the points satisfying $(G/K)^{2/3} - (\gamma/K)^{2/3} = 1$. In the left column of Fig. 1, for the equal-phase mode, we show the values for the longitudinal component of the fixed points with different ratios $\gamma/K$ and $G/K$. For the anti-phase mode, the corresponding case is different. When $G/K \geq -1$, only a fixed point appears and it is stable. When $G/K < -1$, two stable fixed points and an unstable one exist for $(G/K)^{2/3} - (\gamma/K)^{2/3} > 1$; only one stable fixed point emerges for $(G/K)^{2/3} - (\gamma/K)^{2/3} < 1$; the saddle-node bifurcations occur at the points satisfying $(G/K)^{2/3} - (\gamma/K)^{2/3} = 1$. The fixed points of the anti-phase mode with different ratios $\gamma/K$ and $G/K$ are exhibited in the right column of Fig. 1. In the Fig. 1, the fixed points between a pair of corresponding bifurcation points are unstable and the values for $d(\gamma/K)/dw$ at the bifurcation points equal zero. From the previous analysis, we find bistability exists in either the equal-phase mode or the anti-phase mode when the parameters obey $(G/K)^{2/3} - (\gamma/K)^{2/3} > 1$. The appearance of bistability indicates the existence of spontaneous spin polarization ($<w>=\int_0^T w dt/T \neq 0$, $T$ is the period for the oscillation of $w$) in this coupled two-component BECs system. When $|K/G| < 1$, and $\gamma/K$ goes through the bifurcation points which satisfy $(G/K)^{2/3} - (\gamma/K)^{2/3} = 1$, the spin polarization of either the equal-phase mode or the anti-phase mode is discontinuous at the bifurcation points. This means a first-order phase transition occurs. For the zero effective detuning $\gamma$, two metastable states with negative and positive spontaneous spin polarization coexist. As $|K/G|$ is increased to 1, the spontaneous spin polarization vanishes: this corresponds to a second-order phase transition.

Similar to the thermal atoms, the spontaneous spin polarization can be induced by adjusting the coupling lasers. Additionally, the collisions among ultracold atoms can also be
controlled. Below, we will consider the bifurcation and spontaneous spin polarization in Bose condensated atoms induced by the ultracold collisions. Tuning the coupling laser with fixed intensity to a certain detuning satisfying $\gamma = 0$, the bifurcation and the spontaneous spin polarization caused by the ultracold collisions in the above system can be obtained. For the equal-phase mode, only one stable fixed point $w = 0$ exists when $G/K < 1$, two new stable fixed points $w_\pm = \pm \sqrt{1 - (G/K)^{-2}}$ appear and the original one $w = 0$ becomes unstable when $G/K > 1$. This means a Hopf bifurcation occurs at $G/K = 1$. The system goes from the Rabi regime ($G/K < 1$) into the spontaneous spin polarization regime ($G/K > 1$) through the Hopf bifurcation. However, for the anti-phase mode, the Hopf bifurcation occurs at $G/K = -1$, there is only one stable fixed point $w = 0$ for $G/K > -1$, there two stable fixed points $w_\pm = \pm \sqrt{1 - (G/K)^{-2}}$ and an unstable one $w = 0$ for $G/K < -1$. The Hopf bifurcations of both the equal-phase mode and the anti-phase mode are shown in Fig. 2. The solid lines are stable equilibria (stationary states), the dot lines are unstable equilibria.

The Hopf bifurcations obtained from analyzing the equilibria in the previous are static bifurcations. Imagine now that the parameters are swept through the static bifurcation points. An interesting phenomenon emerges: the system starting close to the initially stable equilibrium does not immediately react to the bifurcation. Furthermore, it remains for some time close to the unstable equilibrium, then fast falls into one of the newly formed stable equilibria. This has been named as bifurcation delay which has been found in a variety of physical systems [17]. The bifurcation delay, which might lead to hysteresis, is the response to the bistability. For the equal-phase mode, fixed the effective detuning $\gamma = 0$, slowly sweeping up the ratio $G/K$ from $R_0$ with sweeping rate $r$ (i.e., $G/K = R_0 + rt$, $1 \gg r > 0$), choosing $R_0 < 1$ and the initial state close to the equilibrium, the system evolves along the unstable equilibrium for a period of time after the ratio sweeping through the static bifurcation point ($G/K = 1$), then it quickly goes into a small oscillations around one of two new stable equilibria. The equilibrium, which the system evolves around lastly, determines by the state at the static bifurcation point. The system evolves around the up branch lastly when this state is close to the up branch; otherwise, the system evolves around the down
branch. When $R_0 > 1$, slowly sweeping down the ratio through the static bifurcation point with initial state close to one of two stable equilibria, the system evolves near the stable equilibrium before it sweeps through the static bifurcation point, then it goes into a small Rabi oscillation around the ordinary equilibrium ($w = 0$). For the same sweeping rate, averaging the small oscillations, the process of sweeping up and down generates a hysteresis loop in the plane extended by $G/K$ and $w$. The area enclosed in the hysteresis loop increases with the sweeping rate. This means that the energy exchanged between the atoms and the environments increases with the sweeping rate. The bifurcation delay in the equal-phase mode with different sweeping rate is shown in Fig. 3. For the anti-phase mode, a similar behavior can be observed near the static bifurcation point $G/K = -1$.

To observe the spontaneous spin polarization and the bifurcation delay predicted in the above, we suggest a experiment based upon the works of JILA [1]. Two BECs in the $|F = 1, m_F = -1 >$ and $|2, 1 >$ spin states of $^{87}Rb$ are coupled by a two-photon pulse with the two-photon Rabi-frequency $\Omega$ and a finite detuning $\delta$. Thus, the control of the parameters $K$ and $\gamma$ can be realized by adjusting the Rabi frequency and the detuning of the coupling lasers, respectively. The tuning of the parameter $G$ can be accomplished with the Feshbach resonance [8]. The longitudinal and transverse spin components can be measured with the state-selective absorption imagining and the Ramsey interference, respectively [15].

Summary, due to the coherent ultracold collision among condensed bosonic atoms, the bifurcation and the spontaneous spin polarization in coupled two-component BECs are determined by both the relative phase and the parameters. These phenomena are different from those only determined by the parameters, we name them as conditional bifurcation and conditional spontaneous spin polarization, respectively. For the zero effective detuning $\gamma$, the Hopf bifurcation and bifurcation delay can be induced by the Feshbach resonance in either the equal-phase mode or the anti-phase mode. The system falls into the spontaneous spin polarization regime from the Rabi regime after the bifurcations occur. The appearance of bifurcation delay indicates the existence of metastability and hysteresis. Because of the inherently quantum coherence and superposition of two components, this quantum
hysteresis might open the door to storage quantum data with Bose condensed atoms \cite{18}.

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Figure caption

Fig. 1 The fixed points for the system with different ratios \(\gamma/K\) and \(G/K\). The numbers labelled on the lines are values for \(G/K\).

Fig. 2 The static Hopf bifurcation and the spontaneous spin polarization.

Fig. 3 The bifurcation delay in the equal-phase mode for different values of sweeping rate which are labelled on the lines.
Fig. 1

Equal-phase mode

Anti-phase mode
Fig. 2
Fig. 3
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