SO(10) grand unification in five dimensions: Proton decay and the $\mu$ problem

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Abstract

We construct a minimal supersymmetric SO(10) grand unified model in 5 dimensions. The extra dimension is compactified on an $S^1/(Z_2 \times Z'_2)$ orbifold which has two inequivalent fixed points. These are flat 4-dimensional Minkowski spaces: the visible and the hidden branes. By orbifolding, the gauge symmetry on the hidden brane is reduced down to the Pati–Salam gauge symmetry $SU(4) \times SU(2)_L \times SU(2)_R$. On the visible brane the SO(10) is broken by the ordinary Higgs mechanism down to $SU(5)$. The resulting 4-dimensional theory has the standard model gauge symmetry (the intersection of $SU(5)$ and $SU(4) \times SU(2)_L \times SU(2)_R$) and the massless spectrum consists of the MSSM gauge fields and two Higgs doublets. The matter fields are assumed to live on the visible brane. We discuss gauge coupling unification in our 5-dimensional model in terms of corrections to the conventional 4-dimensional unification. Supersymmetry is broken on the hidden brane (where mass terms for gauginos and a $\mu$ term are generated) and communicated to squarks and sleptons via gaugino mediation. We also discuss a possibility of linking the supersymmetry breaking on the hidden brane to the Higgs mechanism responsible for partial breaking of the gauge symmetry on the visible brane via the shining mechanism.

Finally, there are no operators of dimension 5 leading to proton decay. Proton decay through dimension 6 operators is enhanced compared to conventional GUTs and can be seen in current or next generation proton decay experiments.
1 Introduction

The quest for a unified picture of particles and gauge interactions has led physicists to consider the grand unified theories (GUTs) as serious candidates for physics at high energies. GUTs offer a simple explanation of the quark and lepton quantum numbers \( [1, 2, 3] \) and the minimal supersymmetric framework leads to a successful prediction of the weak mixing angle from gauge coupling unification at the scale \( M_G \approx 10^{16} \text{ GeV} \). Beyond the grand unification scale, one expects to see effects coming from the Planck scale physics which are hoped to be explained in the context of superstring theory. In order to get the 4-dimensional space-time from a 10-dimensional string theory, the extra dimensions need to be compactified. Therefore the ideas of supersymmetry, grand unification and extra dimensions are direct consequences of our quest for a unified picture of physics.

At the string scale, the specific compactification dynamics chosen by nature leads to the particular pattern of particles and symmetries. The nature of this dynamics is yet to be understood. We tend to follow a bottom up approach. We try to speculate about the high energy dynamics based on the low energy physics that has the advantage of being examined by experiments. The compatibility of the consistent theories at high energies with the low energy measurements is a non-trivial test for the candidates that are hoped to come out of the string theory.

The search for alternatives among theories constructed in higher dimensional space-times is motivated by the fact that conventional SUSY GUTs face many problems that remain to be answered in order to give a more complete picture of nature. Some problems like the proton decay push the conventional SUSY GUT models to the edges of viability \(^\dagger\) and cast a shadow of doubt on our understanding of nature beyond the electroweak (EW) scale. A few important other questions include the nature of SUSY breaking and mediation, the \( \mu \) problem, suppression of flavor changing neutral current effects, GUT breaking mechanism and the doublet triplet splitting problem.

The recent interest in this direction at the field theory level started after the work of Kawamura \(^7\) which provides an elegant way of an \( SU(5) \) symmetry breaking and doublet triplet splitting by an orbifold compactification of a theory formulated in 5-dimensions.\(^1\) The framework was further developed in \(^{14, 10} \) and nice examples of \( SU(5) \) models in 5-dimensions with fifth dimension compactified on the \( S^1/(Z_2 \times Z_2') \) orbifold were constructed \(^{12, 11} \). We find it interesting to see what can be achieved by an orbifold compactification in the case of \( SO(10) \) gauge symmetry in 5-dimensions.

In this paper, we assume that the physics at some high energy scale can be described by a 5-dimensional \( SO(10) \) SUSY GUT. We assume the lowest amount of supersymmetry with a minimal particle content in the 5-dimensional bulk. The extra dimension is compactified on a \( S^1/(Z_2 \times Z_2') \) orbifold which has two in-equivalent fixed points. These are flat 4-dimensional Minkowski spaces: the visible and the hidden branes. By orbifolding, the gauge symmetry on the hidden brane is reduced down to the Pati–Salam gauge sym-

\(^{1}\)We note that using orbifolds to reduce a gauge symmetry was introduced in string phenomenology \(^8\).
metry $SU(4) \times SU(2)_L \times SU(2)_R$. On the visible brane the $SO(10)$ is broken by ordinary Higgs mechanism down to $SU(5)$. The resulting 4-dimensional theory has the standard model gauge symmetry (the intersection of $SU(5)$ and $SU(4) \times SU(2)_L \times SU(2)_R$) and the massless spectrum consists of MSSM gauge fields and two Higgs doublets. The matter fields are assumed to live on the visible brane. The model is described in section 4. In section 4 we discuss gauge coupling unification in our 5-dimensional model in terms of corrections to the conventional 4-dimensional unification coming from heavy Kaluza-Klein modes of the gauge and Higgs fields.

The advantage of higher dimensional construction over the conventional SYSY GUT models is that it provides a framework with a potential to solve several challenging questions that the conventional GUTs face. In section 4 we discuss a mechanism for supersymmetry breaking. Supersymmetry is broken on the hidden brane where mass terms for gauginos are generated. Its breaking is communicated to squarks and sleptons via gaugino mediation \cite{12} which explains the suppression of flavor changing neutral current effects. We also discuss a possibility of linking the SUSY breaking on the hidden brane to the Higgs mechanism responsible for partial breaking of the gauge symmetry on the visible brane via the shining mechanism of Ref. \cite{13}. A solution to the $\mu$ problem is proposed in section 5 and suppression of proton decay is discussed in section 6. Finally we conclude in section 7.

During preparation of this article, works \cite{14} appeared considering $SO(10)$ symmetry breaking by orbifold compactification in 6-dimensions.

2 Minimal SUSY SO(10) model in five dimensions

In five dimensions, $\mathcal{N} = 1$ supersymmetry is generated by 8 supercharges and is equivalent to $\mathcal{N} = 2$ supersymmetry in four dimensions. A trivial spatial compactification of $\mathcal{D} = 5$, $\mathcal{N} = 1$ results in $\mathcal{N} = 2$ supersymmetry in 4 dimensions. However, for the purposes of model building, it is desirable to have $\mathcal{N} = 1$ supersymmetry in 4 dimensions and orbifolding is an elegant way of reducing the supersymmetry. Furthermore, orbifolding can also be used to break the gauge symmetry in a grand unified theory \cite{7}.

In this section we present a minimal $\mathcal{D} = 5$, $\mathcal{N} = 1$ supersymmetric model with $SO(10)$ GUT gauge symmetry and additional structure on the orbifold fixed points. We compactify the extra dimension on an orbifold $S^1/(Z_2 \times Z'_2)$ to reduce the supersymmetry from $\mathcal{N} = 2$ to $\mathcal{N} = 1$ and also reduce the $SO(10)$ gauge symmetry. The complete breaking of the $SO(10)$ gauge symmetry to the Standard Model (SM) gauge group $SU(3) \times SU(2) \times U(1) \equiv G(SM)$ is achieved via a combination of orbifolding and Higgs mechanism. The massless sector corresponds to the usual spectrum of MSSM.

2.1 The $S^1/(Z_2 \times Z'_2)$ orbifold

We consider a 5-dimensional space-time with the 5’th dimension compactified on an orbifold. Following Ref. \cite{4, 9, 10} the orbifold is taken to be $S^1/(Z_2 \times Z'_2)$ where $S^1$ is
a circle of radius $R = 1/M_c \sim 1/M_G$ defined with a periodic coordinate $0 \leq x_5 < 2\pi R$. $S^1/Z_2$ is obtained by dividing $S^1$ with a $Z_2$ transformation $x_5 \rightarrow -x_5$. We further divide $S^1/Z^2$ orbifold by a $Z^2$ transformation $x'_5 \rightarrow -x'_5$ with $x'_5 = x_5 + \pi R/2$ to obtain $S^1/(Z_2 \times Z_2)$. $x_5 = 0$ and $x_5 = \pi R/2$ are in-equivalent orbifold fixed points. These fixed points are each a flat 4-dimensional Minkowski space and we refer to them as the visible brane and the hidden brane, respectively. (Later in section 2.3 the ordinary matter fields are assumed to be confined to the visible brane.) A generic field $\phi(x_\mu, x_5)$ in the 5-dimensional bulk is identified by its transformations under the $Z_2$ and $Z'_2$ parities $P = \pm$ and $P' = \pm$, respectively.

$$
\phi(x_\mu, x_5) \rightarrow \phi(x_\mu, -x_5) = P \phi(x_\mu, x_5), \\
\phi(x_\mu, x'_5) \rightarrow \phi(x_\mu, -x'_5) = P' \phi(x_\mu, x'_5).
$$

(1)

A field $\phi_{\pm\pm}(x_\mu, x_5)$ with a definite set of parities $(P, P') = (\pm, \pm)$ has a unique Fourier series expansion:

\[
\begin{align*}
\phi_{++}(x_\mu, x_5) &= \frac{1}{\sqrt{2\pi} R} \sum_{n=0}^{\infty} \phi_{++}^{(2n)}(x_\mu) \cos \left( \frac{2n x_5}{R} \right), \\
\phi_{+-}(x_\mu, x_5) &= \frac{1}{\sqrt{2\pi} R} \sum_{n=0}^{\infty} \phi_{+-}^{(2n+1)}(x_\mu) \cos \left( \frac{(2n + 1) x_5}{R} \right), \\
\phi_{-+}(x_\mu, x_5) &= \frac{1}{\sqrt{2\pi} R} \sum_{n=0}^{\infty} \phi_{-+}^{(2n+1)}(x_\mu) \sin \left( \frac{(2n + 1) x_5}{R} \right), \\
\phi_{--}(x_\mu, x_5) &= \frac{1}{\sqrt{2\pi} R} \sum_{n=0}^{\infty} \phi_{--}^{(2n+2)}(x_\mu) \sin \left( \frac{(2n + 2) x_5}{R} \right).
\end{align*}
\]

(2)

From the 4-dimensional perspective the Fourier component fields (Kaluza-Klein states) $\phi_{++}^{(2n)}$ acquire a mass $2n/R$, $\phi_{+-}^{(2n+1)}$ and $\phi_{-+}^{(2n+1)}$ a mass $(2n + 1)/R$, and $\phi_{--}^{(2n+2)}$ a mass $(2n + 2)/R$. Only $\phi_{++}(x_\mu, x_5)$ has a massless Kaluza-Klein (KK) mode $\phi_{++}^{(0)}(x_\mu)$ and all other KK modes have mass of order GUT scale or larger. $\phi_{++}(x_\mu, x_5)$ and $\phi_{--}(x_\mu, x_5)$ are non-vanishing on the visible brane and can directly couple to the ordinary matter living on the visible brane. $\phi_{++}(x_\mu, x_5)$ and $\phi_{--}(x_\mu, x_5)$ are non-vanishing on the hidden brane. On the other hand, the fields $\phi_{-+}(x_\mu, x_5)$ and $\phi_{-+}(x_\mu, x_5)$ (and $\phi_{-+}(x_\mu, x_5)$ and $\phi_{-+}(x_\mu, x_5)$) have non-vanishing $x_5$-derivatives on the visible (hidden) brane and their $x_5$-derivatives can couple directly to the fields on the visible (hidden) brane.

### 2.2 Gauge symmetry structure on the orbifold

$\mathcal{N} = 1$ SUSY in $D = 5$ space-time may be formulated in terms of the usual $D = 4$, $\mathcal{N} = 1$ superfield notation. A $D = 5$, $\mathcal{N} = 1$ gauge supermultiplet can be decomposed into a $D = 4$, $\mathcal{N} = 1$ gauge $(V)$ and a chiral $(\phi)$ supermultiplet. In the same way, a $D = 5$, $\mathcal{N} = 1$ hypermultiplet can be expressed as a pair of $D = 4$, $\mathcal{N} = 1$ chiral multiplets. In our model, we assume that a single $D = 5$, $\mathcal{N} = 1$ gauge supermultiplet and Higgs hypermultiplet live in the bulk. The gauge supermultiplet is in the adjoint representation (45-dimensional)
of $SO(10)$. We take the Higgs hypermultiplet to be in the 10-dimensional representation of $SO(10)$ and refer to the $\mathcal{N} = 1$ chiral superfield components as $10_H$ and $10'_H$. The action for our minimal model containing one gauge and one massless hypermultiplet in the bulk can be expressed in terms of the $\mathcal{N} = 1$ superfields as

$$S_{N=2} = \int d^5x \left\{ \frac{2}{g^2} \text{Tr} \left[ \frac{1}{4} \int d^2\theta W^a W_a + h.c. \right. \right. \right.$$

$$+ \left. \left. \int d^4\theta \left( (\sqrt{2}\partial_5 + \phi^\dagger) e^{-V}(\sqrt{2}\partial_5 + \phi) e^V + \partial_5 e^{-V} \partial_5 e^V \right) \right] \right.$$

$$+ \left. \int d^4\theta [10'_H e^V 10'^\dagger_H + 10_H e^{-V} 10'_H] \right. \right. \right.$$

$$+ \left. \left. \left[ \int d^2\theta 10'_H(\partial_5 - \frac{1}{\sqrt{2}}\phi) 10_H + h.c. \right] \right\}. \quad (3)$$

The action is invariant under the $Z_2$ transformations:

$$V(x_\mu, x_5) \rightarrow V(x_\mu, -x_5) = P V(x_\mu, x_5) P \quad (4)$$

$$\phi(x_\mu, x_5) \rightarrow \phi(x_\mu, -x_5) = -P \phi(x_\mu, x_5) P \quad (5)$$

$$10_H(x_\mu, x_5) \rightarrow 10_H(x_\mu, -x_5) = P 10_H(x_\mu, x_5) \quad (6)$$

$$10'_H(x_\mu, x_5) \rightarrow 10'_H(x_\mu, -x_5) = -P^T 10'_H(x_\mu, x_5) \quad (7)$$

where $P$ is a $10 \times 10$ matrix acting on the gauge indexes and $P^2 = 1$. The action is also invariant under the $Z'_2$ transformations where $P'$ and $x'_5 = x_5 + \pi R/2$ replace $P$ and $x_5$. We choose $P = 1_{5\times 5} \otimes 1_{2\times 2}$ and $P' = \text{diag} (-1, -1, -1, 1, 1) \otimes 1_{2\times 2}$. The $Z_2 \times Z'_2$ charges of the superfields are listed in the table.

| $10_H$ | $Z_2 \times Z'_2$ | mass | $V$ | $Z_2 \times Z'_2$ | mass |
|--------|-----------------|------|----|-----------------|------|
| $6_H$  | (+, -)          | $(2n+1)/R$ | $V_{t_1}$ | (+, +)          | $2n/R$ |
| $4_H$  | (+, +)          | $2n/R$   | $V_{t_2}$ | (+, -)          | $(2n+1)/R$ |

| $10'_H$ | $Z_2 \times Z'_2$ | mass | $\phi$ | $Z_2 \times Z'_2$ | mass |
|---------|-----------------|------|--------|-----------------|------|
| $6'_H$  | (-, +)          | $(2n+1)/R$ | $\phi_{t_1}$ | (-, -)          | $(2n+2)/R$ |
| $4'_H$  | (-, -)          | $(2n+2)/R$ | $\phi_{t_2}$ | (-, +)          | $(2n+1)/R$ |

Table 1: The decomposition of 4d vector supermultiplet $V$, chiral supermultiplet $\phi$, and chiral multiplets $10_H$ and $10'_H$ according to their parity assignments with corresponding KK masses.

$10_H$ and $V$ have even $Z_2$ parities, while $10'_H$ and $\phi$ have odd $Z_2$ parities and vanish at $x_5 = 0$. This signals the breakdown of $\mathcal{N} = 2$ supersymmetry into $\mathcal{N} = 1$. The non-vanishing $10_H$ and $V$ on the visible brane are complete $SO(10)$ multiplets and the $SO(10)$ gauge symmetry is respected on the visible brane.

The $Z'_2$ projection breaks the $SO(10)$ gauge group into $SO(6) \times SO(4)$ on the hidden brane. The three $-1$’s in $P'$ are associated with the $SO(6)$ and the two $+1$’s are related
to the $SO(4)$. These and following observations are elaborated in detail in the appendix. Note, that $SO(6) \sim SU(4)$ and $SO(4) \sim SU(2)_L \times SU(2)_R$. In fact, $SU(4)$ contains $SU(3) \times U(1)_{B-L}$ where $SU(3)$ is the SM QCD gauge group and $U(1)_{B-L}$ is the symmetry group associated with the baryon number minus lepton number generator. $SU(2)_L$ is the weak gauge group.

The Higgs fields $10_H$ and $10'_H$ are realized under $SO(6) \times SO(4)$ subgroup of $SO(10)$ as $10 = 6 \oplus 4$. Under the SM gauge group, we further have $6 = t \oplus \bar{t}$ and $4 = d \oplus \bar{d}$ where $t$ ($\bar{t}$) is a color triplet (anti-triplet) and $d, \bar{d}$ are weak doublets.

Based on the notation used in the appendix, $V$ and $\phi$ (which are in the adjoint representation of $SO(10)$) are classified into $t_+\text{-type}$ and $t_-\text{-type}$ categories. The $t_+\text{-types}$ belong to the $SO(6) \times SO(4)$ subgroup of $SO(10)$ and commute with $P'$. The $t_-\text{-types}$ belong to $SO(10)/(SO(6) \times SO(4))$ and anti-commute with $P'$.

From the parity assignments in the table we see that only the two Higgs doublets, $d_H$ and $\bar{d}_H$ (contained in $4_H$), and the gauge fields $V_{t_+}$ of $SO(6) \times SO(4)$ gauge group have zero modes.

To summarize the orbifolding results, at the zero-mode level, we are left with $\mathcal{N} = 1$ supersymmetry and $SO(6) \times SO(4)$ gauge symmetry. $SO(10)$ gauge symmetry is respected on the visible brane while there is only $SO(6) \times SO(4)$ gauge symmetry on the hidden brane. There is $\mathcal{N} = 1$ supersymmetry on both branes.

We now need to further reduce the $SO(6) \times SO(4)$ gauge symmetry of the zero-modes to the SM gauge symmetry. Unfortunately, this is not possible to achieve by a $Z_2$ projection. In general is in not possible to break the rank of a group by an abelian orbifolding symmetry. This applies to the case of the orbifold breaking by inner automorphism. In the case of the orbifold breaking by outer automorphisms the rank reduction is possible but only in very limited ways. The $SO(10)$ symmetry cannot be reduced in this way to $SU(5)$ or $G(SM)$. These issues were recently discussed in detail in Ref. [16].

The reduction of the $SO(6) \times SO(4)$ gauge symmetry to the SM gauge group can be accomplished via the ordinary Higgs mechanism on either the visible or the hidden brane. We assume that a pair of 16, $\bar{16}$ living on the visible brane gets a vacuum expectation value of order $M_e$ in the right handed neutrino direction. This breaks $SO(10)$ down to $SU(5)$ on the visible brane and gives GUT-scale mass to those gauge fields among $V_{t_+}$ that belong to $SO(6) \times SO(4)/G(SM)$ including the orbifold zero-modes. Since $SO(10)$ is broken on the hidden brane to $SO(6) \times SO(4)$ we are left with the SM gauge symmetry in the four dimensional theory (the intersection of $SU(5)$ and $SO(6) \times SO(4)$).

The massless spectrum consists of MSSM gauge fields and two Higgs doublets.

### 2.3 MSSM matter fields

Our next task is to identify the ordinary matter and other fields living on the branes along with all possible interactions of the brane and bulk fields. The interactions should

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Another possibility is to assume that a field transforming as $(4, 2)$ under $SO(6) \times SO(4)$ lives on the hidden brane and gets a vev that breaks the gauge symmetry down to the SM gauge group.
respect the $SO(10)$ gauge symmetry on the visible brane and $SO(6) \times SO(4)$ on the hidden brane.

There are several approaches explored in Refs. [9, 10, 11] within the context of SU(5). Realistic fermion masses can be obtained by mixing brane–localized multiplets with bulk multiplets [10], or it is possible to have a complete freedom in Yukawa couplings if matter fields originate from different bulk multiplets [11]. Many of these mechanisms can be generalized to the SO(10) case. Since fermion masses are not the subject of this paper we assume here the simplest possibility that ordinary matter superfields are localized on the visible brane. Each family of ordinary matter fermions and their superpartners is represented by a chiral superfield in the 16-dimensional representation of $SO(10)$. For the purposes of this paper, we only consider the third generation and refer to it as $16_3$.

In order to construct an action for the matter fields on the visible brane, we need to know their $Z_2 \times Z_2'$ transformation properties. After all, we are interested in having an action which is invariant under the $Z_2 \times Z_2'$ symmetry. The $Z_2$ parity of the matter fields living on the visible brane must all be plus. The $Z_2'$ parity of these fields are determined by requiring that any $SO(10)$ invariant operator on the visible brane transforms covariantly under $Z_2'$. $Z_2'$ orbifold identifies the points $x_5 = 0$ and $x_5 = \pi R$. If various parts of an $SO(10)$ invariant operator at $x_5 = 0$ transform differently under $Z_2'$, the corresponding terms at $x_5 = \pi R$ cannot combine into an $SO(10)$ invariant operator. To our choice of $P'$ acting on 10 dimensional representation correspond these possible $P'$ charge assignments of fields in a spinor representation $P'(Q, U, D, L, E, \nu) = \pm (+, - ,-, +, -, -)$. The invariant action for $16_3$ coupled to the Higgs superfield non-vanishing on the visible brane ($10_H$) is

$$S_{\text{matter}} = \int d^5 x \frac{1}{2} \{ \delta(x_5) - \delta(x_5 - \pi R) \} \sqrt{2\pi R} \lambda_3 \int d^2 \theta \ 16_3 10_H 16_3 + \text{h.c.},$$

where $\lambda_3$ is the dimensionless Yukawa coupling. The coupling of the ordinary matter supermultiplet $16_3$ to the Higgs $10_H$ generates the desired MSSM couplings of the quark and lepton superfields to the massless Higgs zero-mode doublets. Integrating out the extra dimension and writing the effective 4-dimensional Lagrangian in terms of the KK modes of the bulk fields we end up with

$$\mathcal{L}_4 = \sqrt{2} \lambda_3 \sum_{n=0}^{\infty} \int d^2 \theta \left[ \frac{1}{\sqrt{2^{2n+1}}} Q Ud_\mathcal{H}^{(2n)} + \frac{1}{\sqrt{2^{2n+1}}} Q D d_\mathcal{H}^{(2n)} + \frac{1}{\sqrt{2^{2n+1}}} Ld_\mathcal{H}^{(2n)} \right. \\
\left. + \frac{1}{\sqrt{2^{2n+1}}} LE d_\mathcal{H}^{(2n)} + \frac{1}{2} QQt_\mathcal{H}^{(2n+1)} + UE t_\mathcal{H}^{(2n+1)} + QLt_\mathcal{H}^{(2n+1)} + U Dt_\mathcal{H}^{(2n+1)} \right] + \text{h.c.}.$$

The Higgs zero-modes are coupled exactly in the form of the MSSM. However, a $\mu$ term is absent in the superpotential and the appropriate mechanism to generate the $\mu$ term at the right scale will be presented in section 5.

$10_H'$ vanishes on the visible brane and therefore there is no coupling in the superpotential like $16_3 10_H' 16_3$. However there are other possible interactions on the branes.
that do not vanish either on the visible or hidden brane. Consider an interaction on the visible brane
\[ \int d^5x \frac{1}{2} \{ \delta(x_5) + \delta(x_5 - \pi R) \} \int d^2\theta \, 10_H 10_H. \] (10)

Similar interactions respecting the $SO(6) \times SO(4)$ symmetry can in principal be present on the hidden brane too,
\[ \int d^5x \frac{1}{2} \{ \delta(x_5 - \pi R/2) + \delta(x_5 + \pi R/2) \} \int d^2\theta \, (\lambda_6' 6_H' + \lambda_4 4_H). \] (11)

The Higgs triplets are coupled in such terms by a mass of order $M_c$ and seem to confront our model with the usual challenge of the conventional SUSY GUT models – proton decay. Another major complication expected from these terms is that they couple the zero-mode Higgs doublets $d^{(0)}_H$ and $\bar{d}^{(0)}_H$ in the superpotential and produce a GUT scale $\mu$ parameter that is too large. To cure this problem, we notice that the action (3) is invariant under a $U(1)_R$ symmetry and also a vector-like $U(1)_{PQ}$ Peccei-Quinn (PQ) symmetry under which $10_H$ and $10'_H$ have opposite charges. We see that interactions in Eqs. (10), (11) can be easily forbidden by requiring that any of these symmetries is respected on branes. Later in section 5 we will find convenient to assume that the full theory is invariant under $U(1)_{PQ}$ with $Q_{10H} = +1$ and $Q_{10'_H} = -1$.

### 3 Gauge coupling unification

In this chapter, we discuss the issue of gauge coupling unification. The massless spectrum in this model is exactly that of the MSSM. The gauge couplings run from the EW scale to the compactification scale ($M_c \sim 1/R$) with the ordinary MSSM $\beta$ functions. They almost unify at $M_c$ which is below but very close to the conventional GUT scale. Beyond $M_c$, the gauge couplings run slowly due to the heavy KK modes that do not fill degenerate GUT multiplets and unify at scale $M_*$. Such threshold corrections coming from the KK states can be easily estimated \[17, 10\]. If we assume a unified value $\alpha_*$ at a scale $M_*$, we may write a one-loop expression for the value of the gauge couplings at the EW scale

\[ \alpha_i^{-1}(M_Z) = \alpha_*^{-1}(M_*) + \frac{1}{2\pi} \left( \alpha_i \ln \frac{m_{\text{SUSY}}}{M_Z} + \beta_i \ln \frac{M_*}{M_Z} \right) \]

\[ + \gamma_i \sum_{n=0}^{N_i} \ln \frac{M_*}{(2n+2)M_c} + \delta_i \sum_{n=0}^{N_i} \ln \frac{M_*}{(2n+1)M_c}, \] (12)

where $(\alpha_1, \alpha_2, \alpha_3) = (-5/2, -25/6, -4)$, $(\beta_1, \beta_2, \beta_3) = (33/5, 1, -3)$ are usual MSSM coefficients and $(\gamma_1, \gamma_2, \gamma_3) = (4, -2, -5)$ and $(\delta_1, \delta_2, \delta_3) = (-18, -12, -9)$ correspond to odd and even KK modes of Higgs and gauge fields in our model. All KK modes below

\[ \text{In order to calculate the $\beta$ functions, we have included the Higgs and gauge hypermultiplets in the bulk along with the matter fields on the visible brane. This is a rough approximation and we are ignoring possible additional fields in the gauge symmetry and SUSY breaking sectors and also fields in the flavor sector of the theory.} \]
M_\ast$, where \((2N_l + 2)M_c \leq M_\ast\), are included in the sum on \(n\). We assume that the model is \(SO(10)\) symmetric beyond \(M_\ast\) and also that a more fundamental theory can justify the termination of the sums at \(M_\ast\).

The gauge coupling unification in the conventional GUTs with only the MSSM particle content below the GUT scale is well established within the error-bars of the experimental values of the couplings at the EW scale. However, in our model, in addition to the MSSM states, we have a tower of heavy KK modes that are GUT non-degenerate. Such KK modes alter the running of the couplings. As a result the conventional 4-d GUT unification can not coexist with the assumed unification in our model. In the following, we assume that the gauge couplings exactly unify in our model and based on that, we try to estimate the amount of the non-unification of the couplings in the conventional GUTs. We show that the amount of the non-unification is small enough to be justified within the experimental error-bars of the couplings and our approximations. We thus have a model that gives a consistent picture of the gauge coupling unification.

Define \(M_\ast = (2N_l + 2)M_c\) to be the scale at which all three gauge couplings unify. As an example, for \(\alpha_1\) and \(\alpha_2\) we can write

\[
(\alpha_1^{-1} - \alpha_2^{-1})(M_Z) = \frac{1}{2\pi} \left( \frac{5}{3} \ln \frac{m_{\text{SUSY}}}{M_Z} + \frac{28}{5} \ln \frac{(2N_l + 2)M_c}{M_Z} - 6 \sum_{n=0}^{N_l} \ln \left( \frac{2n+2}{2n+1} \right) \right). \tag{13}
\]

In order to compare this with the conventional 4-d GUTs, we define \(M_G\) to be the scale where \(\alpha_1\) and \(\alpha_2\) meet and take the value \(\alpha_G\). We assume that \(\alpha_3\) does not exactly unify with the other two couplings and we parametrize the non-unification by a small parameter \(\xi\) given by \(\alpha_3^{-1}(M_G) = \alpha_1^{-1} + \xi\). A similar formula to Eq. (13) can be written in the conventional 4-dimensional GUTs:

\[
(\alpha_1^{-1} - \alpha_2^{-1})(M_Z) = \frac{1}{2\pi} \left( \frac{5}{3} \ln \frac{m_{\text{SUSY}}}{M_Z} + \frac{28}{5} \ln \frac{M_G}{M_Z} \right). \tag{14}
\]

By comparing Eqs. (13) and (14) we can calculate the value of the compactification scale:

\[
\ln \frac{M_c}{M_G} = \frac{15}{14} \sum_{n=0}^{N_l} \ln \left( \frac{2n+2}{2n+1} \right) - \ln(2N_l + 2). \tag{15}
\]

For example, if we take \(N_l = 4\) (\(M_\ast = 10M_c\)), for the range \(1 \times 10^{16} < M_G < 3 \times 10^{16}\) the compactification scale comes to be in the range \(4.5 \times 10^{15} < M_c < 1.3 \times 10^{16}\).

In order to estimate \(\xi\), we write equations similar to Eqs. (13), (14) for \(\alpha_1\) and \(\alpha_3\) and solve for the compactification scale.

\[
\ln \frac{M_c}{M_G} = \frac{45}{48} \sum_{n=0}^{N_l} \ln \left( \frac{2n+2}{2n+1} \right) - \ln(2N_l + 2) - \frac{5}{24\pi} \xi. \tag{16}
\]

From Eqs. (15), (16) one can calculate the level of non-unification \(\xi \approx -0.3\)\(^{\dagger}\). This is a small correction, less than 1.5\% of the GUT scale value of the gauge coupling constant.

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\(^{\dagger}\)Note, \(\xi\) is an increasing function of \(N_l\), however this dependence is quite mild.
\( \alpha_G^{-1} \approx 24 \). However, we note that our estimates are at one loop and subject to a few percent correction coming from higher loops. The fact that our estimated correction is not too large gives us hope that our model is consistent with the gauge coupling unification picture and the experimental values at the EW scale. In section 3 we will show that large values of \( N_l \) (larger than \( \sim 30 \)) would be inconsistent with the current experimental limits on proton decay.

We note that the gauge group on the hidden brane is only \( SO(6) \times SO(4) \). In fact, one may write terms on the hidden brane that do not respect the full \( SO(10) \) symmetry of the bulk that is necessary for complete unification of the gauge couplings. Ref. [10] shows that the effects of these terms are small enough to be ignored in these models.

4 Breaking the \( \mathcal{N} = 1 \) supersymmetry

In the previous sections, we benefitted from orbifolding to reduce the amount of supersymmetry. However, we are still left with \( \mathcal{N} = 1 \) SUSY that survives the compactification. Since there is yet no experimental evidence of SUSY particles at current collider energies, the \( \mathcal{N} = 1 \) SUSY must be broken at scale of a TeV or higher. In this paper, we assume that SUSY is broken by the vacuum expectation value of a \( SO(6) \times SO(4) \) singlet chiral superfield \( X \) that is localized on the hidden brane.

\[ \langle X \rangle = \theta^2 F_X \] (17)

\( X \) can couple directly to the gauge fields on the hidden brane through the ultraviolet scale suppressed terms

\[ \mathcal{L}_5 = \frac{1}{2} \{ \delta(x_5 - \pi R/2) + \delta(x_5 + \pi R/2) \} \int d^2 \theta \left( \frac{\lambda'}{M_c^2} W^{i\alpha} W^i_{\alpha} + \lambda' \frac{X}{M_c^2} W^{i\alpha} W^i_{\alpha} + h.c. \right), \] (18)

where index \( i(j) \) runs over the number of gauge fields of the \( SO(6) \) (\( SO(4) \)) symmetry group. This will give universal masses to the gauginos of \( SO(6) \) and \( SO(4) \) gauge groups separately,

\[ M_6 = \frac{\lambda'_6 F_X M_c}{M_c^2}, \quad M_4 = \frac{\lambda'_4 F_X M_c}{M_c^2}. \] (19)

The factor \( M_c \) is from the wave function normalization of the 4-dimensional gaugino fields. The masses of the gauginos of the MSSM \( (M_1, M_2, M_3) \) are given as

\[ M_1 = \frac{2}{5} M_6 + \frac{3}{5} M_4, \quad M_2 = M_4, \quad M_3 = M_6. \] (20)

The special form of \( M_1 \) is related to the fact that the hypercharge operator is expressed as \( Y = \sqrt{\frac{2}{5}} (B - L) - \sqrt{\frac{2}{5}} t_{3R} \) where \( B - L \) and \( t_{3R} \) are the generators of the \( SO(6) \) and \( SO(4) \) symmetry as discussed in the appendix. The couplings of \( X \) to the fields on
the visible brane are suppressed at short distances by locality. The soft SUSY breaking scalar masses and trilinear couplings are negligible at the GUT scale. By RGE running down to the EW scale, they receive large contributions from the gaugino mass terms and acquire finite values. These contributions to the scalar masses are flavor blind and thus do not cause large flavor changing neutral currents. Such a scenario for mediating supersymmetry breaking is called gaugino mediation \[12\]. Minimal gaugino mediation is characterized by finite universal gaugino masses and negligible trilinear couplings and scalar masses at the GUT scale. Here, we have a special case of the non-universal gaugino mediation where the Bino mass \(M_1\) is completely constrained by the Wino mass \(M_2\) and the gluino mass \(M_3\). The usual universal gaugino mediation models predict the stau to be the lightest supersymmetric particle (LSP). However stau is charged and is strongly disfavored by experimental data as the LSP. The partial non-universality of the gaugino masses in our model might provide a solution to this problem.

The SUSY breaking mechanism on the hidden brane can be easily linked to the Higgs mechanism responsible for partial breaking of the gauge symmetry on the visible brane. This is achieved via the shining mechanism \[13\]. Consider a massive gauge singlet hypermultiplet in the bulk containing a pair of chiral superfields \(\Phi\) and \(\Phi^c\). The hypermultiplet couples to the field \(X\) on the hidden brane and to the \(16\), \(\bar{16}\) on the visible brane. The action can be expressed as

\[
S = \int d^5x \left( \int d^4\theta [\Phi^\dagger \Phi + \Phi'^\dagger \Phi'^c] + \int d^2\theta [\Phi^c (m + \partial_5) \Phi \\
+ \frac{1}{2} (\delta(x_5) + \delta(x_5 - \pi R)) 16 \bar{\Phi}^c + \frac{1}{2} (\delta(x_5 - \pi R/2) + \delta(x_5 + \pi R/2)) X \Phi] \right).
\]

\(\langle 16 \bar{16} \rangle\) acts as a source and \(\Phi\) develops a non-trivial profile in the bulk. The shining mechanism gives a SUSY breaking F-term vev to the field \(X\) on the hidden brane, \(F_X \approx \langle 16 \bar{16} \rangle \exp(-\frac{\pi m R}{2})\).

Note that the appropriate phenomenology with TeV scale SUSY masses in Eq. \([19]\) is possible with \(\sqrt{F_X} \approx 10^{11} - 10^{12}\) GeV. This is a mass scale that is almost \(10^5\) times smaller than the GUT scale. The SUSY breaking scale seems to have a completely independent nature from the grand unification scale. However, the power of shining mechanism is that the SUSY breaking scale is generated from the compactification scale and the undesirable large hierarchy between the compactification and SUSY scales is cured by taking \(m \sim 10 M_e\) which is close to its natural value, i.e. the ultraviolet scale \(\sim M_4\). We emphasize that in our model, in addition to the ultraviolet scale, we have only the compactification scale and the SUSY breaking is dynamically generated via the shining mechanism at the appropriate scale. As a result, the GUT breaking and SUSY breaking have common origins in our model and are linked together.
5 Generating the $\mu$ term, solving the $\mu$ problem

The $\mu$ term in the superpotential of the MSSM couples the two light Higgs doublet superfields with a mass dimension one parameter, $\mu$. Since the $\mu$ term respects supersymmetry and the gauge symmetries of the MSSM, one expects it to be of order the ultraviolet scale in the theory. However, for various phenomenological reasons, $\mu$ parameter needs to be of order the EW scale. The difficulty in generating the $\mu$ parameter at the right scale is called the $\mu$ problem. There is another parameter in the MSSM ($B_\mu$) with mass dimension two that couples the Higgs doublets in the soft SUSY breaking Lagrangian. $B_\mu$ is also expected to be of order the EW scale, squared. Our goal is to set up our model in order to have both $\mu$ and $B_\mu$ parameters at the right scale.

It was noted earlier in this paper that by assigning PQ charges to the 10 dimensional Higgs multiplets in the bulk, we avoided terms like $M_{10}H_{10}H$ on the branes that potentially give rise to a very large $\mu$ parameter. Our model presents a solution to the $\mu$ problem while keeping $B_\mu$ under control. One can imagine an $SO(6) \times SO(4)$ singlet chiral superfield $Y$ on the hidden brane with PQ charge $Q_Y = +2$. An ultraviolet suppressed term of the form

$$\int d^5x \frac{1}{2} \left\{ \delta(x_5 - \pi R/2) + \delta(x_5 + \pi R/2) \right\} \int d^4\theta (\frac{Y^\dagger M_2^*}{M_s^2} 4_H^4_H + h.c.)$$

is allowed on the hidden brane and can result in a $\mu$ term

$$\mu \sim \frac{F_Y M_c}{M_s^2}$$

if

$$\langle Y \rangle = \theta^2 F_Y.$$  \hspace{1cm} (25)

Comparing this with the result in Eq. (19), one notices that the value of $\mu$ is at the right scale provided that the SUSY breaking vevs of $Y$ and $X$ are comparable.\footnote{We proposed that $X$ gets a SUSY breaking vev through the shining mechanism. However, we assume that there exists a potential on the hidden brane for $X$ and $Y$ that relates the vev of $Y$ to the vev of $X$ at the same scale.} It is important to note that $B_\mu$ is also generated at this order since

$$\int d^4\theta (\frac{X Y^\dagger}{M_s^2} 4_H^4_H + h.c.)$$

is allowed by the PQ symmetry ($Q_X = 0$).

$$B_\mu \sim \frac{F_Y^2 M_c}{M_s^2}.$$ \hspace{1cm} (27)

It is now evident from $\sqrt{B_\mu/\mu} \sim \sqrt{M_s/M_c} \sim 3$ that $B_\mu$ is also generated at the right scale.
6 Proton decay

Proton decay in grand unified theories can happen through dimension six operators coming from the exchange of the X gauge bosons. The mass of the X gauge bosons must be large enough to bring the predicted proton decay rate below the current experimental bounds. Note that the mass of the X gauge bosons in our model is the compactification scale and so the experimental bounds on the proton decay set limits on the compactification scale.

Super-Kamiokande puts a bound on $\tau_{p\to e^+\pi^0} > 4.4 \times 10^{33}$ (90% CL) \footnote{Ref. 19} and this translates into a limit on the compactification scale $M_c > 6 \times 10^{15}$ GeV. In our model, the size of the compactification scale from Eq. (13) is a decreasing function of $N_l$. Therefore, the current experimental bound on $p \to e^+\pi^0$ gives an upper bound on $N_l \sim 30$. However, it should be noted that this is a quite conservative limit. Improving the experimental bounds will result in a lower upper limit on the value of $N_l$. Thus the predictions of our model can be tested in the current or the next generation proton decay experiments.

There are other possible dangerous sources for proton decay in the $SO(10)$ grand unified theories. Large contributions to proton decay are expected from the Higgs triplet exchanges. We show that unlike the conventional $SO(10)$ GUTs, the Higgs triplet contribution vanishes in our 5-dimensional model. Consider the relevant terms coming from Eq. (9) for the proton decay through the Higgs triplets

$$\frac{1}{2}QQt_H^{(2n+1)} + UEt_H^{(2n+1)} +QLt_H^{(2n+1)} + UD\bar{t}_H^{(2n+1)}.$$  \hspace{1cm} (28)

The only mass terms consistent with the PQ symmetry in our model are coming from the bulk contribution Eq. (3) of the form

$$M_c(t_H^{(2n+1)}\bar{t}_H^{(2n+1)} + t_H^{(2n+1)}t_H^{(2n+1)}).$$  \hspace{1cm} (29)

After integrating out the GUT-scale heavy fields $t_H, \bar{t}_H, t_H^{(2n+1)}, \bar{t}_H^{(2n+1)}$ we observe that we get no terms of the type $QQQL$ or $UD\bar{U}E$ and thus no proton decay from dimension five operators. It is worth to note that even an additional mass term for the Higgs triplets of the form

$$M_c\bar{t}_H^{(2n+1)}t_H^{(2n+1)}$$  \hspace{1cm} (30)

does not affect the proton decay result that we just mentioned. Another possible source for proton decay is the operator $16 16 16 16$ on the visible brane where $16$s represent generations of matter fields. This term is also forbidden by the PQ symmetry.

The conventional SUSY GUTs are pushed to their limits by the updated experimental bounds on the proton decay. We showed that our model survives the current experimental bounds yet may be tested in the near future.

\footnote{The discussion of proton decay in our $SO(10)$ model is similar to the case of $SU(5)$ discussed in Ref. 10.}
7 Conclusions

We have constructed an $SO(10)$ supersymmetric grand unified model in 5 dimensions. A gauge supermultiplet and a single Higgs hypermultiplet live in the 5-dimensional bulk. The extra dimension is compactified on an orbifold $S^1/(Z_2 \times Z'_2)$ which has two fixed points: the visible and the hidden branes. The matter supermultiplets are confined on the visible brane. By orbifolding $\mathcal{N} = 2$ supersymmetry is reduced to $\mathcal{N} = 1$, and the $SO(10)$ gauge symmetry is reduced to $SO(6) \times SO(4)$. Unfortunately it is not possible to reduce $SO(10)$ down to the SM gauge symmetry by an abelian orbifolding and the further reduction of the gauge symmetry to that of SM is achieved by ordinary Higgs mechanism. However, we argue that this does not have to be viewed as a downside of the model. The Higgs mechanism responsible for partial breaking of the gauge symmetry can be used to trigger the breaking of the remaining $\mathcal{N} = 1$ supersymmetry via the shining mechanism. The GUT breaking vev of the 16, $\bar{16}$ pair living on the visible brane acts as a source for the shining mechanism which gives a SUSY breaking F-term vev to the field $X$ living on the hidden brane. The large hierarchy between the SUSY breaking scale and the GUT breaking scale is achieved by the exponential suppression of the SUSY breaking vev with respect to the source. The F-term vev of $X$ gives soft SUSY breaking masses to gauginos. The breaking of supersymmetry is communicated to squarks and sleptons through gaugino mediation and so the flavor changing neutral currents are suppressed. The specific feature of this model is the non-universality of gaugino masses. The Bino mass $M_1$ is completely constrained by the Wino mass $M_2$ and the gluino mass $M_3$ which is dictated by the $SO(6) \times SO(4)$ gauge symmetry on the hidden brane. This might provide a solution to the usual problem of universal gaugino mediation models which typically predict stau to be the LSP. The $\mu$ and $B_\mu$ terms are also generated on the hidden brane at the scale that can also be linked to the GUT symmetry breaking vev. Operators of dimension 5 which could lead to proton decay are forbidden by PQ symmetry. Proton decay through dimension 6 operators is enhanced compared to conventional GUTs and can be seen in current or next generation proton decay experiments.

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Appendix

In order to better understand the breaking of the gauge group and the notation used in this paper, we present a short review of $SO(10)$ group theory in this appendix. A generator of $SO(10)$ group is an imaginary $10 \times 10$ antisymmetric matrix and may be
written as

\[ t = A_{5 \times 5} \otimes 1_{2 \times 2} + B_{5 \times 5} \otimes \sigma^2 + C_{5 \times 5} \otimes \sigma^1 + D_{5 \times 5} \otimes \sigma^3, \] (31)

where \( \sigma^a \) are the Pauli matrices. \( A_{5 \times 5}, C_{5 \times 5} \) and \( D_{5 \times 5} \) are \( 5 \times 5 \) imaginary antisymmetric matrices while \( B_{5 \times 5} \) is a \( 5 \times 5 \) real symmetric matrix. It is possible to classify the generators of \( SO(10) \) algebra based on whether they commute (\( t_+ \)) or anti-commute (\( t_- \)) with \( P' = \text{diag} (-1, -1, -1, +1, +1) \otimes 1_{2 \times 2} \). The \( SO(10) \) generators that commute with \( P' \) can always be written as

\[
  t_+ = \left( \begin{array}{cc} A_{3 \times 3} & 0 \\ 0 & A_{2 \times 2} \end{array} \right) \otimes 1_{2 \times 2} + \left( \begin{array}{cc} B_{3 \times 3} & 0 \\ 0 & B_{2 \times 2} \end{array} \right) \otimes \sigma^2 \\
  + \left( \begin{array}{cc} C_{3 \times 3} & 0 \\ 0 & C_{2 \times 2} \end{array} \right) \otimes \sigma^1 + \left( \begin{array}{cc} D_{3 \times 3} & 0 \\ 0 & D_{2 \times 2} \end{array} \right) \otimes \sigma^3, \tag{32}
\]

where \( A_{3 \times 3}, C_{3 \times 3}, D_{3 \times 3}(A_{2 \times 2}, C_{2 \times 2}, D_{2 \times 2}) \) are \( 3 \times 3(2 \times 2) \) imaginary antisymmetric matrices and \( B_{3 \times 3}(B_{2 \times 2}) \) is a \( 3 \times 3(2 \times 2) \) real symmetric matrix. The rest of the \( SO(10) \) generators in Eq. (31) anticommute with \( P' \).

The \( t_+ \) generators of \( SO(10) \) in Eq. (32) are in fact the elements of the \( SO(6) \times SO(4) \) subalgebra of \( SO(10) \). Eq. (31) is a representation of the \( SO(10) \) generators based on 5 \( \times \) 5 matrices times Pauli sigma matrices. In the same way, Eq. (32) represents the \( SO(6) \) (\( SO(4) \)) generators based on \( 3 \times 3(2 \times 2) \) matrices times Pauli sigma matrices. Note, that \( SO(6) \sim SU(4) \) and \( SO(4) \sim SU(2)_L \times SU(2)_R \). In fact, \( SU(4) \) contains \( SU(3) \times U(1)_{B-L} \) where \( SU(3) \) is the SM QCD gauge group and \( U(1)_{B-L} \) is associated with the baryon number minus lepton number generator.

The \( SU(3) \times U(1)_{B-L} \) subalgebra consists of \( t_+ \) generators in Eq. (32) with all entries zero except \( A_{3 \times 3} \) and \( B_{3 \times 3} \). In fact, \( U(1)_{B-L} \) is generated by \( A_{3 \times 3} = 0 \) and \( B_{3 \times 3} = 1_{3 \times 3} \). \( SU(2)_L \times SU(2)_R \) contains the gauge group \( SU(2)_L \times U(1)_{t_3R} \). In our notation, \( U(1)_{t_3R} \) is generated by \( A_{2 \times 2} = 0 \) and \( B_{2 \times 2} = 1_{2 \times 2} \). Hypercharge symmetry is a combination of \( U(1)_{B-L} \) and \( U(1)_{t_3R} \) gauge symmetries. Note that one needs to define a proper normalization of the \( SO(10) \) generators. In this paper we use the usual convention \( Tr[t^a t^b] = \delta^{ab}/2 \). The properly normalized \( B - L = \frac{1}{\sqrt{12}} \text{diag} (1, 1, 1, 0, 0) \otimes \sigma^2 \) and \( t_{3R} = \frac{1}{\sqrt{8}} \text{diag} (0, 0, 0, 1, 1) \otimes \sigma^2 \) combine into the hypercharge generator \( Y = \sqrt{\frac{2}{10}} \text{diag} (2/3, 2/3, 2/3, -1, -1) \otimes \sigma^2 \) where \( Y = \sqrt{\frac{2}{5}} (B - L) - \sqrt{\frac{4}{5}} t_{3R} \).

The 10-dimensional Higgs representation of \( SO(10) \) is realized under \( SO(6) \times SO(4) \) as \( 10 = 6 \oplus 4 \). Under the standard model gauge group however, we have \( 6 = t \oplus \bar{t} \) and \( 4 = d \oplus ar{d} \) where \( t (\bar{t}) \) is a color triplet (anti-triplet) and \( d, \bar{d} \) are weak doublets. It is essential to note that under \( P' = \text{diag} (-1, -1, -1, +1, +1) \otimes 1_{2 \times 2} \) doublets \( d \) and \( \bar{d} \) have positive parities while triplets \( t \) and \( \bar{t} \) have negative parities. This is in fact the essence of the doublet-triplet splitting mechanism used in this paper.
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