The modified nodal analysis method applied to the modeling of the thermal circuit of an asynchronous machine

O Nedelcu¹, C I Salisteanau¹, F Popa¹, B Salisteanau², C V Oprescu² and V Dogaru³

¹Valahia University of Targoviste, Department of Electronics, Telecommunications and Energy Engineering, Targoviste, Romania
²Valahia University of Targoviste, PhD Student, Targoviste, Romania
³Valahia University of Targoviste, Doctoral School, Targoviste, Romania

E-mail: otilia.nedelcu@valahia.ro

Abstract. The complexity of electrical circuits or of equivalent thermal circuits that were considered to be analyzed and solved requires taking into account the method that is used for their solving. Choosing the method of solving determines the amount of calculation necessary for applying one of the methods.

The heating and ventilation systems of electrical machines that have to be modeled result in complex equivalent electrical circuits of large dimensions, which requires the use of the most efficient methods of solving them.

The purpose of the thermal calculation of electrical machines is to establish the heating, the overruns of temperatures or over-temperatures in some parts of the machine compared to the temperature of the ambient, in a given operating mode of the machine.

The paper presents the application of the modified nodal analysis method for the modeling of the thermal circuit of an asynchronous machine.

1. Introduction

By modeling the heating and ventilation systems of the electrical machines, equivalent complex electrical circuits that have large dimensions are achieved and they require a complex analysis. The analysis of these circuits, in stationary and dynamic regime, requires the use of effective methods of solving, [1]. Due to the simplicity and flexibility of the modified nodal analysis method, it is one of the most frequently used as it formulates equations for electrical circuits and your computer simulates, [2].

Generally, circuit analysis in a dynamic regime is reduced to the analysis of a string of resistive circuits, by replacing coils and condensers with discrete patterns resistive of circuit, associated to an implicit numerical algorithm of numerical integration. Developing methods for analyzing nonlinear resistive electrical circuits, as effective as possible, has a special importance, resulting from substituting dynamic circuit elements with discrete resistive models associated with an implicit algorithm of integration.

The two theorems of Kirchhoff, together with the constitutive equations of the sides, are the base of nodal analysis, [3], [4]. Currents on certain sides cannot always be expressed according to the parameters that are characteristic of the respective sides and the potential nodes, situation that does not allow the use of a classical nodal analysis method, for this reason its extension was appealed, to yield the modified nodal analysis method.
By doing the analogy between a network of cooling ventilation and a non-linear resistive circuit it can be determined the flow values of the cooling fluid of an electrical machine.

2. Description of modified nodal method
The modified nodal analysis method is a generalization of the classical nodal method, which allows the analyzed circuit to contain circuit elements that are incompatible with the classical method.

The structure of circuits to which they apply the nodal method consists in the following circuit elements: resistors, linear capacitors and non-coupled magnetically coils; nonlinear resistors commanded in voltage; nonlinear capacitors; independent sources of current; independent sources of power that do not form by themselves a side.

The difference between the classical nodal method and the modified nodal method is that in the modified nodal method currents of the circuit elements that are non-compatible with the classical nodal method (such as currents of non-linear coils commanded in current and currents of linear coils magnetically coupled) appear additionally, as independent variables. Linear coils that are non-coupled do not introduce additional variables, [4-7].

For the modified nodal analysis method, the reference is considered the potential of one of the nodes of the circuit for which it will be deemed \( V = 0 \), this node of reference in the matrix of incidence \( A \) in the node which is missing.

It is applied the Kirchhoff's first theorem to the other nodes, i.e. \((n-1)\) nodes, writing the currents of sides depending on the potentials at nodes. The currents of sides, having null resistance, cannot be expressed by themselves depending on the potentials at nodes, but shall be considered as independent variables found in the vector \( \mathbf{V}_{(n-1),1} \), together with potentials at nodes.

Depending on potentials, all nodes and tensions on the sides will be expressed as shown in relation (1):

\[
\begin{bmatrix}
U_P \\
U_E \\
U_J
\end{bmatrix} = \begin{bmatrix}
A_P^T \\
A_E^T \\
A_J^T
\end{bmatrix} \cdot \mathbf{V} = \begin{bmatrix}
A_P^T \cdot \mathbf{V} \\
A_E^T \cdot \mathbf{V} \\
A_J^T \cdot \mathbf{V}
\end{bmatrix}.
\]  

(1)

The first Kirchhoff theorem shall be written as in relation (2):

\[
A_P I_P + A_E I_E + A_J I_J = 0_{(n-1),1}.
\]  

(2)

The tensions of the passive sides will determine the currents of the passive sides, as shown in (3):

\[
I_P = G \cdot U_P = G \cdot A_P^T \cdot \mathbf{V}.
\]  

(3)

After they determined the currents of the passive sides, the expression obtained will replace the first Kirchhoff theorem, and then the characteristic equations of sides with null resistance will be added, affording a corresponding matrix system for the modified nodal method, relations (4) - (6):

\[
A_P \cdot G \cdot A_P^T + A_E I_E = -A_J J,
\]  

(4)

\[
U_E = A_E^T \cdot \mathbf{V}; \quad U_E = -E,
\]  

(5)

\[
\begin{cases}
A_P \cdot G \cdot A_P^T \cdot \mathbf{V} + A_E \cdot I_E = -A_J \cdot J \\
A_E^T \cdot \mathbf{V} = -E
\end{cases}
\]  

(6)
3. Applying the modified nodal method in modeling an asynchronous machine with rotor in cage

In Figure 1 it is presented a section of an asynchronous machine with a rotor in a cage, in which case the rotor is the thermal source and calculations must be made for the thermal resistance’s corresponding heat transmission, developed in the rotor, outwards, [8].

![Asyncronous machine with rotor in cage](image)

**Figure 1.** Asynchronous machine with rotor in cage
1 – ax; 2 – rotor yoke; 3 – rotor notch; 4 – rotor windings;
5 – air gap; 6 – stator notch; 7 – stator tooth; 8 – stator yoke;
9 – radiator; 10 – housing

Figure 2 shows the equivalent circuit diagram with heating systems (in stationary regime). Using the program PDCEN – Program of Rotting Decompose of Electrical Circuit after the Nodes, it will decompose „after the nodes” the circuit from Figure 2, [9].

The program, [10], is written in C++ and implemented on a microcomputer type PC compatible with IBM, used for the numerical integration of an ordinary differential equation generalized with the first order regressive method. Dynamic circuit elements are replaced with discreet resistive circuits associated with these numerical methods, and for the analysis of electrical circuits the program used the modified nodal method, [10], [11].

Running the program the results presented in tables 1 and 2 were obtained. The sub-circuits structure is presented in Table 1.

**Table 1.** Sub-circuits structure

| SUBCIRCUITS=8. |
|----------------|
| SUBCIRCUIT 1 of size 4 |
| 4 5 6 7 |
| END |
| SUBCIRCUIT 2 of size 1 |
| 2 |
| END |


The nodes of interconnection are: \{1,3,8,10,12,20,24,26,32\}.

Table 2 shows the distribution of 9 interconnection nodes to the 8 sub-circuits obtained.

| 9 INTERCONNECTION NODES |
|--------------------------|
| INTERCONNECTION NODE 32 IS ADJACENT TO: |
| CLUSTER 8 OF SIZE 4 (5 CONNECTIONS) |
| CLUSTER 7 OF SIZE 2 (2 CONNECTIONS) |
| CLUSTER 6 OF SIZE 3 (2 CONNECTIONS) |
| CLUSTER 5 OF SIZE 1 (1 CONNECTIONS) |
| CLUSTER 4 OF SIZE 4 (3 CONNECTIONS) |
| CLUSTER 3 OF SIZE 4 (2 CONNECTIONS) |
| BEST CLUSTER FOR NODE 32: 8 OF SIZE 4 (5 CONNECTIONS) |
| INTERCONNECTION NODE 12 IS ADJACENT TO: |
| CLUSTER 8 OF SIZE 4 (1 CONNECTIONS) |
| CLUSTER 4 OF SIZE 4 (1 CONNECTIONS) |
| BEST CLUSTER FOR NODE 12: 8 OF SIZE 4 (1 CONNECTIONS) |
| INTERCONNECTION NODE 20 IS ADJACENT TO: |
| CLUSTER 8 OF SIZE 4 (1 CONNECTIONS) |
| CLUSTER 6 OF SIZE 3 (1 CONNECTIONS) |
| CLUSTER 3 OF SIZE 4 (1 CONNECTIONS) |
| BEST CLUSTER FOR NODE 20: 6 OF SIZE 3 (1 CONNECTIONS) |
| INTERCONNECTION NODE 26 IS ADJACENT TO: |
| CLUSTER 7 OF SIZE 2 (1 CONNECTIONS) |
| CLUSTER 4 OF SIZE 4 (1 CONNECTIONS) |
| BEST CLUSTER FOR NODE 26: 7 OF SIZE 2 (1 CONNECTIONS) |
| INTERCONNECTION NODE 3 IS ADJACENT TO: |
| CLUSTER 6 OF SIZE 3 (1 CONNECTIONS) |
| CLUSTER 2 OF SIZE 1 (1 CONNECTIONS) |
| CLUSTER 1 OF SIZE 4 (1 CONNECTIONS) |
| BEST CLUSTER FOR NODE 3: 2 OF SIZE 1 (1 CONNECTIONS) |
| INTERCONNECTION NODE 24 IS ADJACENT TO: |
| CLUSTER 5 OF SIZE 1 (1 CONNECTIONS) |
| CLUSTER 4 OF SIZE 4 (1 CONNECTIONS) |
| BEST CLUSTER FOR NODE 24: 5 OF SIZE 1 (1 CONNECTIONS) |
INTERCONNECTION NODE 10 IS ADJACENT TO:
CLUSTER 4 OF SIZE 4 (2 CONNECTIONS)
CLUSTER 1 OF SIZE 4 (1 CONNECTIONS)
BEST CLUSTER FOR NODE 10: 4 OF SIZE 4 (2 CONNECTIONS)

INTERCONNECTION NODE 8 IS ADJACENT TO:
CLUSTER 3 OF SIZE 4 (1 CONNECTIONS)
CLUSTER 1 OF SIZE 4 (1 CONNECTIONS)
BEST CLUSTER FOR NODE 8: 3 OF SIZE 4 (1 CONNECTIONS)

FINAL NUMBER OF INTERCONNECTION NODES: 9; 0 COULD BE
MOVED DIRECTLY TO CLUSTERS
END_INTERCONN

CLUSTERS
CLUSTER 1 OF SIZE 4
4 5 6 7
END

CLUSTER 2 OF SIZE 1
2
END

CLUSTER 3 OF SIZE 4
9 29 30 31
END

CLUSTER 4 OF SIZE 4
11 13 14 15
END

CLUSTER 5 OF SIZE 1
25
END

CLUSTER 6 OF SIZE 3
21 22 23
END

CLUSTER 7 OF SIZE 2
27 28
END

CLUSTER 8 OF SIZE 4
16 17 18 19
END

END_CLUSTERS

RENUM

4
5
6
7
2
9
29
30
31
11
13
14
15
25
21
After the decomposition circuit, the nodal matrix has the bordered block diagonal structure as in Figure 3.

Figure 2. The decomposition of thermal circuit of the asynchronous machine in subcircuits
4. Conclusions

The modified nodal method does not require restrictions of the circuit structure to be analyzed, [12]. For the analysis, in a stationary regime and also in a dynamic regime, of the equivalent complex electrical circuits with large dimensions which are obtained from modeling heating and ventilation systems of the electrical machines, the modified nodal analysis method is used.

The bordering nodal matrix consists of rows and columns corresponding with potential interconnection nodes. In the case of the modified nodal method, the bordering is formed with rows and columns which correspond to the nodes of interconnection and the command currents of interconnection.

The purpose of a thermal calculation for electrical machines is to establish heating, overruns of temperatures or over-temperature in some parts of the machine compared to the ambient temperature, at a given operating mode of the machine.

References

[1] Hănciţă F 1979 A Method for Solving Nonlinear Resistive Networks, Roum. Sci. Techn. Électrotechn. et Énerg. 24(2) 21-30
[2] Iordache M and Dumitriu L 2004 Computer-aided analysis of non-linear analogue circuits (in Romanian: Analiza asistată de calculator a circuitelor analogice neliniare), Politehnica Press Publishing House, Bucharest, Romania
[3] Iordache M, Dumitriu L and Matei I 1999 Symbolic analysis based on Modified NOdal Method, User’s Manual (in Romanian: ASINOM – Analiza Simbolică bazată pe Metoda Nodală Modificată, Manual de utilizare), Department of Electrical Engineering, University Politehnica of Bucharest
[4] Schwarz A E 1987 Computer-aided design of microelectronic circuits and systems, Academic Press, London, UK
[5] Chua L O, Desoer C A and Kuh E S 1987 *Linear and nonlinear circuits*, McGraw-Hill, N.Z.

[6] Topan D and Mandache L 2002 *Methods of analysis in complex electric circuits* (in Romanian: *Metode de analiză în circuite electrice complexe*), Universitaria Publishing House, Craiova, Romania

[7] Iordache M 2007 *Modified Nodal Method* (in Romanian: *Metoda Nodală Modificată*), Simpozionul National de Electrotehnica Teoretica SNET’07, Bucharest, Romania, October 12-14, pp 10 - 20

[8] Magureanu R and Vasile N 1990 *Brushless synchronous servomotors* (in Romanian: *Servomotoare fara perii tip sincron*), Technical Publishing House, Bucharest, Romania

[9] Iordache M and Perpelea M 1995 *Computer-aided analysis of large complex linear and non-linear electric circuits* (in Romanian: *Analiza asistată de calculator a circuitelor electrice și electronice nelineare complexe de mari dimensiuni*), Didactic and Pedagogical Publishing House, Bucharest, Romania

[10] Dumitriu L and Iordache M 1998 *Modern theory of electric circuits- Vol. I: Theoretical background, Applications, Algorithms and Computational softwares* (in Romanian: *Teoria modernă a circuitelor electrice – Vol. I: Fundamentare teoretică, Aplicații, Algoritmi și Programe de calcul*), All Educational S.A. Publishing House, Bucharest, Romania

[11] McCalla W J 1988 *Fundamentals of computer-aided circuit simulation*, Kluwer Academic Publishers, Boston, USA

[12] Nedelcu O, Iordache M and Enescu D 2006 *An efficient method for computing of the electric machine ventilation*, The Sixth WESC, Torino, Italy, July 10-12, p 251