Turbulent asymptotic suction boundary layers studied by simulation

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Abstract. The turbulent asymptotic suction boundary layer (ASBL) is studied using numerical simulations. Uniform suction is applied on the wall in order to compensate for the momentum loss inflicted by the wall friction. Four Reynolds numbers, defined as the ratio of free-stream velocity and suction rate, $Re = 333, 400$ and $500$, are considered, whereas $Re = 280$ relaminarised. In agreement with previous studies, suction causes the fluctuation intensities to decrease, and the near-wall anisotropy to increase. The shape of the mean velocity profile is considerably changed yielding a decreased slope in the overlap region. It is shown that even for moderate suction rates large values for the friction Reynolds number $Re_\tau = \delta_{59}^+$ are obtained; at $Re = 333$ a value of $Re_\tau = 1900$ is reached and $Re = 400$ yields $Re_\tau = 5700$. Artificially using smaller computational domains, limiting the size of the largest turbulent structures, gives unexpected results: The mean velocity profile starts to show a distinct wake region which only disappears for large enough domains. Moreover, the boundary layer thickness $\delta_{99}$ strongly depends on the chosen domain size. Spectral maps of the flow are analysed, showing an outer peak appearing at a spanwise size of about $0.69\delta_{99}$, albeit with considerably lower amplitude compared to cases without suction. Visualisations of the flow are also discussed.

1. Introduction

In technical as well as geophysical applications, the flow of a fluid around solid bodies is ubiquitous. Although in real cases, the geometry of the immersed object is usually complex, and the flow is dominated by pressure gradients in various directions, the study of simplified canonical flows allows for deducing important properties of the physics. Therefore, a number of canonical flow cases have emerged as standard model problems to study wall-bounded turbulence, e.g. turbulent channel or boundary-layer flows. Since turbulent channel flow is characterised by a single non-homogeneous direction, it has been a preferred case for numerical simulations (Hoyas & Jiménez, 2006). On the other hand, the simulation of open, spatially developing flows such as the zero-pressure-gradient (ZPG) turbulent boundary layer (TBL) has only recently become the focus of extensive research interest (Schlatter & Örlü, 2010), mainly because the spatially evolving character of the flow necessitates very long numerical domains, which in turn cause considerable computational cost. Nevertheless, excellent agreement with experiments could be obtained both for channel and ZPG boundary-layer flows (see e.g. Schlatter et al., 2009; Örlü & Schlatter, 2010), indicating that the numerical techniques are indeed suitable for the accurate simulation of such canonical wall-bounded flows at moderate Reynolds numbers.

To reduce the computational cost associated with spatially developing flows, but still focusing on open flows, a number of flow cases can be considered. For instance, open channel flow has
become a preferred model in the meteorological community for the atmospheric boundary layer (Nieuwstadt, 2005). However, open channel flow does not really provide true open flow conditions as the thickness of the boundary layer is fixed by the domain height and the symmetry conditions imposed. Thus, for any true open flow the growth of the boundary layer, induced through the momentum loss caused by the wall friction, needs to be opposed by an external mechanism. For instance, the Ekman layer, studied numerically by e.g. Spalart et al. (2008), relies on system rotation to cancel the growth, rendering the flow parallel in the wall-parallel directions.

Alternatively, the boundary-layer growth can also be opposed by constant, well-balanced suction at the wall. This latter flow case, commonly denoted suction boundary layer, has been the subject of many studies, mainly motivated by the potential benefit in flow control applications, as wall suction is known to reduce turbulent fluctuations and – depending on the strength – even relaminarise turbulent boundary-layer flows.

Experiments have been performed by Antonia et al. (1988), studying the influence of wall suction on the coherent structures in a boundary layer, finding a considerable effect of the suction both in the near-wall region but also further away from the wall. They also comment on the appropriate turbulent velocity scale in the presence of suction. Numerically, Piomelli et al. (1989) simulated transpired channel flow, and derived an approximate relation, based on an eddy-viscosity hypothesis, for the mean velocity profile in the presence of uniform suction. Later, Mariani et al. (1993) performed DNS of a true suction boundary layer at (low) $Re = 278$ and documented various turbulent statistics. In particular, they noted a considerable increase of anisotropy in the near-wall region, with turbulence being approaching a one-component state (Antonia et al., 1994). Also, the pressure fluctuations are considerably reduced by suction. Sumitani & Kasagi (1995) and Chung & Sung (2001) considered channel flow under the influence of uniform suction, and documented various turbulence statistics. Chung & Sung (2001) focused on the initial stages of the flow adaptation, finding a considerable inhibition of inter-component energy exchange. Chung et al. (2002) considered the effect of suction on the turbulence structures and could confirm previous findings related to increased anisotropy. More recently, Yoshioka & Alfredsson (2005) performed careful experiments on perforated plates in the range $Re = 333-500$, analysing the scaling of velocity fluctuations inside the layer.

In the present contribution, turbulent boundary layers under the influence of uniform wall suction are studied via numerical simulation. It is assumed that the flow is homogeneous in both the streamwise ($x$) and spanwise ($z$) directions. In the laminar case, i.e. if no Reynolds stresses appear, the solution of the Navier-Stokes equations for a constant free-stream velocity $U_\infty$ and suction rate $V_0$ (in the negative wall-normal $y$ direction) can easily be derived as (see e.g. Schlichting, 1987),

$$u = U_\infty [1 - \exp(-yV_0/\nu)] , \quad v = -V_0 ,$$

with the kinematic viscosity $\nu$. The Reynolds number based on the laminar displacement thickness $\delta^*$ is then given by $Re = U_\infty/V_0$. For increasing Reynolds number, the laminar solution will successively lose its stability, and a turbulent flow will appear. After a certain time (or streamwise distance in experimental setups), the flow will reach a statistical steady state in which the averaged boundary-layer thickness will no longer change. This time obviously depends on the initial conditions and the suction rate $V_0$. However, the final asymptotic state is solely described by the Reynolds number $Re$. A direct consequence of the integral momentum conservation in the asymptotic state is that the friction velocity $u_\tau = (\tau_w/\rho)^{1/2}$ is given by

$$\left( \frac{u_\tau}{U_\infty} \right)^2 = \frac{V_0}{U_\infty} = \frac{1}{Re} .$$

In the literature, there is no recent simulation of an asymptotic suction boundary layer at moderate Reynolds numbers, which is performed in domains large enough to enable the study
Table 1. Key parameters for the simulations of the asymptotic suction boundary layer (ASBL). Note that $Re_\tau = \frac{\delta_{99}^+}{u_\tau} = \frac{\delta_{99}}{u_\tau} u_\tau / \nu$ with $u_\tau$ given by equation (2).

| Run   | $Re$ | Domain size       | Resolution | $Re_\tau$ | $\delta_{99}$ |
|-------|------|-------------------|------------|-----------|---------------|
| LES500c | 500  | $216 \times 150 \times 108$ | $128 \times 201 \times 128$ | 4950  | 145  |
| LES500b | 500  | $108 \times 150 \times 54$ | $64 \times 201 \times 64$ | 4530  | 105  |
| LES500a | 500  | $54 \times 100 \times 27$ | $32 \times 201 \times 32$ | 2110  | 63   |
| LES400c | 400  | $864 \times 375 \times 432$ | $512 \times 401 \times 512$ | 5700  | 151  |
| LES400b | 400  | $216 \times 250 \times 216$ | $256 \times 301 \times 256$ | 5850  | 156  |
| LES400a | 400  | $108 \times 100 \times 54$ | $64 \times 201 \times 64$ | 4560  | 106  |
| LES333e | 333  | $864 \times 150 \times 432$ | $512 \times 201 \times 512$ | 1930  | 47   |
| LES333c | 333  | $432 \times 150 \times 216$ | $256 \times 201 \times 256$ | 1820  | 44   |
| LES333d | 333  | $216 \times 100 \times 108$ | $128 \times 201 \times 128$ | 1750  | 43   |
| LES333b | 333  | $108 \times 100 \times 54$ | $64 \times 201 \times 64$ | 1530  | 37   |
| LES333a | 333  | $54 \times 100 \times 27$ | $32 \times 201 \times 32$ | 1160  | 28   |
| DNS333  | 333  | $108 \times 100 \times 54$ | $256 \times 301 \times 256$ | 1830  | 45   |

of the outer-layer structures, as opposed to channels and developing boundary layers (Hoyas & Jiménez, 2006; Schlatter et al., 2010). As will be discussed in this report, suction will lead to thick boundary layers, thus requiring large domains to properly resolve all the spatial scales. In particular the wake region of the mean velocity profile is strongly affected by the choice of domain size.

2. Numerical method and simulated cases

In the present study, asymptotic suction boundary layers (ASBL) are studied numerically using a fully spectral method (Chevalier et al., 2007) based on Fourier decomposition in the stream- and spanwise directions $x$ and $z$, and Chebyshev expansions in the wall-normal direction $y$. Thus, periodic boundary conditions are chosen in the wall-parallel directions, together with Dirichlet conditions at the wall, specifying no-slip $u = w = 0$ and uniform suction $v = -V_0$. In the free-stream, a similar Dirichlet condition is imposed far away from the wall. Due to the computational cost of the simulations, most cases were performed using an active subgrid-scale (SGS) model. We chose the ADM-RT model (Schlatter et al., 2004) due to its simplicity and reliable performance in conjunction with spectral discretisation (Schlatter et al., 2010). The resolution in inner units for the DNS case (DNS333) is $\Delta x^+ = 15$ and $\Delta z^+ = 7$, whereas all the LES were run using $\Delta x^+ \approx 60$ and $\Delta z^+ \approx 30$, depending on the Reynolds number.

Four different suction rates are considered, $Re = 280$ (relaminarising), $Re = 333$, $Re = 400$ and $Re = 500$. Key parameters of the various simulations are given in Table 1. The simulations were all started from localised perturbations within the boundary layer. Long transients had to be considered for the higher suction rates, as will be discussed below. The averaging of the statistics only started when a statistically stationary state had been reached.

3. Simulation results

3.1. Validation of LES method

In a first step, the employed large-eddy simulation (LES) method is validated by comparing to fully resolved direct numerical simulation (DNS) for one specific parameter set, i.e. $Re = 333$
and a domain size 108 × 100 × 54 in the streamwise, wall-normal and spanwise direction. In these length units, the viscous unit will be about 0.0246. The comparison of the mean velocity profile and the root-mean-square (rms) of the fluctuations is shown in Fig. 1. The results are reasonably close to each other; the LES slightly overpredicts the velocity and fluctuations throughout the overlap region, however the turbulence attenuation due to the suction is accurately captured. The boundary-layer thickness, listed in Table 1 is underpredicted by the LES, which is a direct consequence of the differences in the mean profile. However, given the comparably coarse LES resolution employed, and the focus of the present contribution on assessing the influence of the domain size on the large turbulent structures, the accuracy of the LES is certainly sufficient.

3.2. Mean turbulence statistics
In Fig. 2 the mean velocity and the corresponding fluctuations are shown for the various Reynolds numbers listed in Table 1. Note that an additional case with Re = 280, similar as in Mariani et al. (1993) has been run, which however did relaminarise. Therefore, only results for Re = 333, 400 and 500 are further discussed. Compared to channel and spatially evolving boundary layers, a clearly lower slope in the overlap region can be observed. All components of the fluctuations and stresses are considerably reduced by the presence of increasing suction, see also Chung et al. (2002) and Mariani et al. (1993). This reduction is lowest for the streamwise fluctuations, which increases the near-wall anisotropy as discussed by Antonia et al. (1994).

The mean velocity profile shows a reduce slope in the overlap region. Based on an eddy-viscosity hypothesis, Piomelli et al. (1989) derived a modified law of the wall, which is shown for comparison in Figs. 1a) and 4-6. The present LES results however can quite accurately be described by an empiric log law with modified coefficients κ = 0.82, B = 9.2, irrespective of Re for the studied range. This modified law approaches the form by Piomelli et al. (1989) for higher Re which indicates that a correction of the latter for lower Re might be necessary.

As discussed above, due to the imposed wall suction, the wall shear stress and thus the friction velocity is fixed by the Reynolds number. In Fig. 3a) the transient approach to the stationary friction value is shown on the example of case LES400e. In particular for these large domains, a sufficient adjustment time needs to be maintained to allow the boundary layer to settle in its final state. Another characteristic statistics of the suction layer is shown in Fig. 3b), namely the decomposition of the total shear stress into contributions from turbulence and viscosity. As already discussed by Mariani et al. (1993), to total stress drops much quicker than in boundary layers without suction, mainly because the uv contribution is reduced to about 25% of its value.

Figure 1. Comparison of case DNS333 (red solid) and LES333c (blue solid) at Re = 333. a) Mean velocity profile. - - - - free-stream values for the suction layers (U∞ = √Re) and empiric log law with κ = 0.82, B = 9.2. - - - - modified law of the wall Piomelli et al. (1989). b) Velocity fluctuations u rms, v rms, w rms and Reynolds stresses uv.
in the non-transpired case.

The velocity profiles further show that the friction Reynolds number, defined as \( Re_f = \frac{\delta_{99} u_f}{\nu} \) based on 99\% boundary-layer thickness, assumes comparably large values even for moderate suction rates: The present results yield \( Re_f = 1900 \) for \( Re = 333 \) (note that the case \( Re = 280 \) is relaminarising) and \( Re_f = 5700 \) for \( Re = 400 \)! Such high values directly imply that the separation between largest and smallest turbulent scales must indeed be very large. This in turns means that large computational boxes need to be considered to capture these scales. In order to study the influence of these large scales onto the flow statistics, we systematically change the box dimension and thus the maximum possible size of these large structures. As an example, consider \( Re = 400 \): In Fig. 5 the mean profiles and fluctuations for four different domain sizes (LES400a to LES400e) are shown. The computational resolution is unchanged between the cases. Indeed, the velocity profiles show remarkable differences in the outer region between the different cases: When the domain is increased, the wake region is gradually reduced, whereas the inner
Figure 4. Turbulent asymptotic suction boundary layer at $Re = 333 \ (V_0/U_\infty = 0.003)$ for increasing domain size (direction of arrow). a) Mean velocity profile; --- modified law of the wall Piomelli et al. (1989), ....... empiric log law with $\kappa = 0.82, B = 9.2$. b) Turbulent fluctuations and Reynolds stresses.

Figure 5. Turbulent asymptotic suction boundary layer with $Re = 400 \ (V_0/U_\infty = 0.0025)$ for increasing domain size (direction of arrow). a) Mean velocity profile; ---- modified law of the wall, .......... empiric log law with $\kappa = 0.82, B = 9.2$. b) Turbulent fluctuations and Reynolds stresses.

Figure 6. Turbulent asymptotic suction boundary layer with $Re = 500 \ (V_0/U_\infty = 0.002)$ for increasing domain size (direction of arrow). a) Mean velocity profile; ---- modified law of the wall, .......... empiric log law with $\kappa = 0.82, B = 9.2$. b) Turbulent fluctuations and Reynolds stresses.
region (up to $y^+ \approx 300$) perfectly collapses. The turbulent stresses are equally unchanged in the near-wall region up to $y^+ \approx 150$, but clearly show a trend towards a thicker boundary layer for larger domains. This is consistent with spatially evolving boundary layers (Örlü & Schlatter, 2010). Measured in inner units, the smallest domain (LES400a, size $L_x^+ \times L_z^+ = 2000 \times 1000$) gives a boundary-layer thickness of $Re^+_\tau \approx 1650$, whereas the results become independent of domain size above $L_z^+ > 6000$ reaching $Re^+_\tau = 5700$ (cases LES400d and e). Note that the asymptotic value $U_{\infty}^+ = \sqrt{Re} = 20$ is fixed through the suction rate. Similar trends are also observed for the two other Reynolds numbers, see Figs. 4 and 6. In the latter case, due to the expected very large values for $Re^+_\tau$ independence of the domain size could not be achieved yet.

The larger the computational domain is chosen, the larger the turbulent structures are allowed to be. Indeed, such structures seem to exist, and they grow in size until they reach a given maximum size depending on the Reynolds number, which then defines the final boundary layer thickness $\delta_{99}$ for a given $Re$. Based on the development of the mean velocity profiles in the previous figures, it becomes clear that the profiles successively approach a law of the wall without distinct wake region, as opposed to spatially developing boundary layers. It can thus be expected that a formulation of the mean profile similar to the one proposed by Piomelli et al. (1989) can be used up to the edge of the boundary layer.

3.3. Spectra and visualisation

It is thus clear that the size of the largest structures plays in integral part in the development of the asymptotic suction boundary layer. In Fig. 7 premultiplied spectral maps are shown for $Re = 400$, again for increasing domain size, cases LES400a to d. It can be seen that the dominant

![Figure 7. Premultiplied spanwise energy spectrum $\Phi_{uu}^+(\lambda_z)$ for the turbulent asymptotic suction boundary layer at $Re = 400$ for increasing domain size $L_x \times L_z$ with $L_x = 2L_z$. From top left to bottom right: cases 400a-400d, $L_z^+ = 1000, 2000, 4000, 8000$. Horizontal dashed lines: Boundary-layer thickness $\delta_{99}^+ = Re^+_\tau$ in inner units: $Re^+_\tau = 1650, 2650, 3990, 5850$. Vertical dashed lines: $\lambda_z = 0.6\delta_{99}$. Contour lines start at 0.2 with spacing 0.4.](image-url)
peak in the near-wall region (at $\lambda_z \approx 150$ and $y^+ \approx 12$, i.e. slightly wider than for cases without suction) is invariant, whereas the largest domain shows the appearance of a secondary energy peak in the outer region with $\lambda_z \approx 0.6\delta_99$ and $y \approx 0.1\delta_99$. Similar spectra were also computed in the streamwise direction (not shown), and are consistent with the spanwise spectra in the sense that energy is contained in very long structures.

These results are interesting and perhaps unexpected as it is clearly demonstrated that the large scales in the flow are actively influencing the integral development of the boundary layer, leaving their strong footprint even in the mean velocity profile. This is in contrast to e.g. (internal) channel flows in which the mean profile is fairly insensitive to domain size and thus the largest structures. On the other hand, the near-wall peak (and the corresponding statistics) are unaffected by the outer peak as clearly demonstrated in the previous figures.

To put these spectra into perspective, in Fig. 8 the spanwise energy spectrum for the largest case, LES400e, is compared to a similar spectrum obtained in a spatially developing turbulent boundary layer (Schlatter & Örlü, 2010), using the same scaling with $u'^2$. In agreement with Fig. 2, the fluctuation intensity is lower for the ASBL, both for the inner peak ($y^+ \approx 12$) and, much more pronounced, further away from the wall.

Finally, in Fig. 9 a three-dimensional visualisation of the flow is presented. Looking onto the boundary layer from an elevated position, Fig. 9a), the larger outer-scale structures are clearly visible. Compared to channels and spatially developing boundary layers, these structures are less dominant, as also evidenced by the spectra discussed above. Note that this domain size is only slightly larger than the smallest one that gives a wake region approximately independent of box size. The view from below, Fig. 9b), reveals the smaller-scale turbulent streaks close to the wall. A modulation of their amplitude by the action of the outer structures (Schlatter et al., 2009) can be seen, albeit not as clearly as in cases without suction.

4. Conclusions

The turbulent asymptotic suction boundary layer (ASBL) is studied using numerical simulations. Uniform suction is applied on the wall in order to compensate for the momentum loss inflicted by the wall friction. Compared to spatially developing boundary layers, periodic domains in both the streamwise and spanwise direction can be considered, which is advantageous for the
Figure 9. Three-dimensional visualisation of the flow domain pertaining to case LES400e. Red and blue isocontour correspond to positive and negative disturbance velocity $u' = \pm 0.05U_\infty$, respectively. In grey-scale contours of streamwise velocity $u$ projected on the sides. a) View from top, flow from lower left to upper right. b) View from below through the solid wall (assumed fully transparent). The full simulation domain is shown.

numerical treatment. Here, four Reynolds numbers, $Re = 333, 400$ and $500$, are considered, whereas $Re = 280$ relaminarised. The Reynolds number is defined based on the ratio of the suction rate to the free-stream velocity, $Re = U_\infty/V_0$.

In agreement with previous studies, suction causes the fluctuation intensities, scaled in viscous units, to decrease, and the near-wall anisotropy to increase. Moreover, the shape of the mean velocity profile is considerably changed yielding a decreased slope in the overlap region.

Performing simulations in large computational domains shows that even for moderate suction rates close to relaminarisation, comparably large values for the friction Reynolds number $Re_\tau = \delta_{99}^+$ were reached. For instance, at $Re = 333$ a value of $Re_\tau = 1900$ is obtained; $Re = 400$ yields $Re_\tau = 5700$. Artificially using smaller computational domains thus limiting the size of the largest turbulent structures gives unexpected results: The mean velocity profile starts to show a distinct wake region which only disappears for large enough domains. Moreover, the integral boundary layer thickness $\delta_{99}$ (and consequently also $Re_\tau$) strongly depends on the domain size. This is in disagreement with channel or boundary-layer flow without suction, in which the domain size has only marginal influence on the mean profile. It is thus of uttermost importance to perform simulations (and experiments for that matter) of turbulent asymptotic suction boundary layers in domains of sufficient size; giving the flow also sufficient time (or development length in a spatial setup) to overcome its long transients.

Spectral maps are also analysed, showing an outer peak appearing at a spanwise size of about $0.6\delta_{99}$. Compared to cases without suction, the spectral energy in this peak is considerably lower, which is surprising given the significant influence of exactly these structures on the mean profile. Visualisations of the flow field essentially confirm these findings.

Given these results, the asymptotic suction boundary layer is shown to be an interesting canonical flow case which deserves further analysis both numerically and experimentally, in particular given its relevance in control applications. Additionally, aspects such as regeneration of near-wall turbulence, and turbulent structures (e.g. the relevance of hairpin vortices) could be conveniently studied in this case which does not require expensive spatial simulation setups.

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