Electromagnetic Structure of Light Baryons in Lattice QCD

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A method in which electromagnetic properties of hadrons are studied by direct simulation of dynamical photon effects is applied to the extraction of the isomultiplet structure of the octet baryons. Using 187 configurations at $\beta = 5.7$ with Wilson action, and up and down quark masses determined from the meson spectrum, the nucleon splitting is found to be $1.55(\pm 0.56 \text{ stat})$ MeV; the hyperon splittings are found to be $\Sigma^0 - \Sigma^+ = 2.47 \pm 0.39$, $\Sigma^- - \Sigma^0 = 4.63 \pm 0.36$, $\Xi^- - \Xi^0 = 5.68 \pm 0.24$ MeV. Estimated systematic corrections arising from finite volume and the quenched approximation are included in these results.

1. Introduction

The fact that the intrinsic up-down quark mass difference is comparable to typical hadronic electromagnetic energy shifts has made it difficult to reliably calculate the octet baryon isomultiplet splittings in the absence of a systematic treatment of virtual photon effects combined with non-perturbative QCD contributions. The problem is particularly acute in the case of the proton-neutron mass difference, where tadpole (quark mass) effects almost completely cancel the photon cloud contributions (see [1] for an early review exhibiting the massive confusion prevailing in pre-QCD days). In this talk, we extend a method recently used [2] to extract electromagnetic contributions to pseudoscalar masses to the octet baryon spectrum. The basic idea is to propagate the quarks through a $U(3) = SU(3) \times U(1)$ field including both dynamical gluon and photon effects. The calculation of baryon isomultiplet splittings can then be viewed as a two-step process:

(a) First, the bare quark masses are determined from an analysis [3] of the pseudoscalar meson spectrum (including electromagnetic contributions);
(b) Secondly, the octet baryon spectrum is computed (again including dynamical photon effects) and extrapolated to physical values of quark mass (as determined in step (a)) and electric charge.

2. Extraction and Fitting of Baryon Spectrum

The strategy of the calculation is as follows: quark propagators are generated in the presence of Coulomb gauge background $SU(3) \times U(1)$ fields. 187 gauge configurations, separated by 1000 Monte Carlo sweeps, were generated at $\beta = 5.7$ on a $12^3 \times 24$ lattice. Quark propagators are calculated for 4 electric charges and 3 light quark mass values, and with either a local or smeared source (see [2] for details). From the resulting 12 quark propagators, 936 independent octet baryon three-quark combinations can be formed.

In quenched QCD it is known [2] that baryon masses are described by a function of the bare quark masses involving nonanalytic $m_q^{3/2}$ (as well as linear) terms, and terms involving logarithms of the quark mass arising from the same hairpin diagrams familiar in the quenched meson spectrum [4,5]. The latter terms now appear to be extremely small numerically [6,7]: we neglect them throughout. However, we do include terms of order $(\text{quark mass})^{3/2}$. Thus a general octet baryon mass is written

$$m_B = A(e_{q1}, e_{q2}, e_{q3}) + \sum_i m_{q_i} B_i(e_{q1}, e_{q2}, e_{q3})$$
\[ + \sum_{i,j} (m_{q_1} + m_{q_2})^{3/2} C_{ij} (e_{q_1}, e_{q_2}, e_{q_3}) \]  \hspace{1cm} (1)

where \( e_{q_1}, e_{q_2}, e_{q_3} \) are the three quark charges, and \( m_{q_1}, m_{q_2}, m_{q_3} \) are the three bare quark masses, defined in terms of the Wilson hopping parameter by \((\kappa^{-1} - \kappa^{-1})/2a\). (Here \( a \) is the lattice spacing.) Each of the coefficients \( A, B, C \) in (1) is then expanded in powers of the quark charges \( e_{q_1}, e_{q_2}, e_{q_3} \), with terms up to fourth order for \( A \), second order for \( B \), and with no charge dependence assumed for the nonanalytic \( C_{ij} \) terms. (1) turns out to have 30 parameters once all symmetries are exploited.

We have varied the baryon mass window (for each choice of Euclidean time window used to extract a mass from smeared-local correlators) until the \( \chi^2/\text{dof} \) of the fit to (1) was minimized. For example, using a Euclidean time window from \( t = 5 \) to \( t = 8 \), the mass window (lattice units) from 1.20 to 1.26 was found to contain 74 baryons. Determining the 30 parameters in (1) by fitting this set of masses gave a \( \chi^2/\text{dof} \) of 1.33. By contrast, using the mass window from 1.15 to 1.20 (122 baryons), the chi-square fit minimizes at \( \chi^2/\text{dof} = 2.16 \). For each choice of Euclidean time window, we have performed the fit to (1) using a baryon mass window which optimizes the \( \chi^2/\text{dof} \) - the overall optimal fit was found for the window \( t = 5 \) to 8. One then determines the mass of any given octet baryon by extrapolating to physical values of quark mass and charge. The propagators for different electric charge are highly correlated, so it is not surprising that the statistical error on the center of gravity of isomultiplets is considerably larger than the error on multiplet splittings. Note, in connection with the raw lattice results quoted in Table 1, the following:

1. The quark mass parameters and lattice scale assumed in generating masses for each of the fitting window choices are shown in the second row. The up and down quark masses are those obtained from the pseudoscalar spectrum (these will depend on the scale). The strange quark mass is known to fall at a higher value when determined from the baryon spectrum (due to discretization and quenched errors), so we have chosen to fix it using the center of gravity of the \( \Xi \) hyperon, which has the smallest statistical errors in our analysis. The center of gravity of the \( \Sigma \) multiplet and the \( \Lambda \) mass are then predictions of the analysis.

2. The lattice scale has been fixed in each case by requiring the nucleon center of gravity to sit at (roughly) the physical value.

| Baryon State | Window 5-8 \((\chi^2/\text{dof}=1.33)\) |
|-------------|-----------------|
| Parameters  | \( m_{u,d,s}, a^{-1}=3.57, 7.10, 15.5, 1370 \) |
| N           | 935.92 ± 42.4   |
| P           | 933.07 ± 42.9   |
| N-P         | 2.83 ± 0.56     |
| \( \Sigma^+ \) | 1171.6 ± 25.6  |
| \( \Sigma^0 \) | 1175.1 ± 25.3  |
| \( \Sigma^- \) | 1179.1 ± 25.0  |
| \( \Sigma^0 - \Sigma^+ \) | 3.43 ± 0.39   |
| \( \Sigma^- - \Sigma^0 \) | 4.04 ± 0.36   |
| \( \Sigma^+ + \Sigma^- - 2\Sigma^0 \) | 0.61 ± 0.19   |
| \( \Xi^- \) | 1312.9 ± 14.5  |
| \( \Xi^0 \) | 1308.2 ± 14.6  |
| \( \Xi^- - \Xi^0 \) | 4.72 ± 0.24   |
| \( \Lambda^0 \) | 1098 ± 52      |

3. Finite Volume and Quenched Corrections

With massless physical degrees of freedom we expect finite volume effects which fall as inverse powers of the lattice size. These corrections can be studied directly on the lattice by repeating the calculations on lattices of varying physical volume. Here we estimate them by using the known dominance of the Born contribution to the dispersive evaluation of the Cottingham formula. Single photon exchange can be written as a sum of an electric and magnetic contribution to hadronic self-energies- the electric term takes the form

\[ \delta m_{\text{el}} = 2\pi am \int \frac{E(q)}{q^3} \sum_{q \neq 0} \frac{G_E(q)^2}{|q|^2} \left( \frac{2}{q^2 + 4m^2} \right) \]
Table 2
Final results for baryon octet splittings ($\beta=5.7$, 12$^3$x24, 187 configurations)

| Level Splitting | Raw Lattice  | Finite Volume | Meson Cloud | Total Lattice | Physical   |
|-----------------|--------------|---------------|-------------|---------------|------------|
| N - P           | 2.83 ± 0.56  | -0.75         | -0.53       | 1.55 ± 0.56   | 1.293      |
| $\Sigma^0 - \Sigma^+$ | 3.43 ± 0.39  | -0.80         | -0.16       | 2.47 ± 0.39   | 3.18 ± 0.1 |
| $\Sigma^- - \Sigma^0$ | 4.04 ± 0.36  | +0.86         | -0.27       | 4.63 ± 0.36   | 4.88 ± 0.1 |
| $\Sigma^+ + \Sigma^- - 2\Sigma^0$ | 0.61 ± 0.19  | +1.66         | -0.11       | 2.16 ± 0.19   | 1.70 ± 0.15|
| $\Xi^- - \Xi^0$ | 4.72 ± 0.24  | +0.86         | +0.10       | 5.68 ± 0.24   | 6.4 ± 0.6  |

where the momentum vectors $\vec{q}$ are the discretized bosonic photon momenta for the finite LxLxL lattice. Using (2), one can estimate the finite volume corrections to baryon masses on our L=12 lattice- they are indicated in column 3 of Table 2, together with our final estimate (including the finite volume correction as well as quenched error estimate- see below) for the baryon mass in column 5.

Processes in which mesons are emitted and reabsorbed from a baryon include graphs with internal quark loops and are known [8] to result in a small but nonnegligible shift in isospin splittings. For example, in the static limit where the nucleon mass is infinite, the pion cloud decreases the neutron-proton splitting by an amount (in the infinite volume limit) 0.43$\Delta M_0$, where $\Delta M_0$ is the nucleon splitting in the absence of a virtual pion cloud (a fully relativistic evaluation gives 0.41$\Delta M_0$). We shall use a static approximation but include the effects of all octet pseudoscalar mesons (assuming SU(3) symmetry with a $d : (f + d)$ ratio of 0.62). Discretizing the second order shift formula (see [8]) on a LxLxL lattice, one may estimate the meson cloud shift for the particular lattices used. Since the meson cloud shift includes contributions from quenched nonplanar graphs in the cases where the emitted meson only contains valence quarks of the external baryon, these estimates are only a rough indication of the magnitude and sign (probably, an overestimate), of the quenched correction. Setting $L=12$ and using a lattice scale $a^{-1}=1370$ MeV, together with the quenched masses from column 2 of Table 1, we obtain the meson cloud shifts given in column 4 of Table 2. The lattice results, corrected for finite volume and meson cloud effects, are given in column 5, and the physical values in column 6.

The results in Table 2 (which must still be corrected for finite lattice spacing effects) suggest that this first evaluation, on a fairly coarse lattice, already reproduces - almost quantitatively- the isomultiplet pattern of the octet baryons. Of course, the extremely delicate level of baryon fine structure being considered here requires a detailed study of all systematic effects, with improved statistics on larger lattices. An upcoming run will work with lattices of varying physical volume, using improved (to O(a)) action to minimize lattice discretization errors- an important check given the known strong a-dependence in off-shell defined continuum quark mass parameters [9].

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REFERENCES

1. A. Zee, Phys. Reports 3C (1972) 129.
2. See talk of E. Eichten, this conference; also A. Duncan, E. Eichten, and H. Thacker, Phys. Rev. Lett. 76, (1996) 3894.
3. J.N. Labrenz and S.R. Sharpe, hep-lat 9605034.
4. C. Bernard and M.F.L. Golterman, Phys. Rev. D46, (1992) 853.
5. S. Sharpe, Phys. Rev. D41, (1990) 3233.
6. See talk of H. Thacker, this conference.
7. R. D. Mawhinney, Nucl. Phys. B (Proc. Suppl.) 47 (1996) 557.
8. J. Gasser and H. Leutwyler, Phys. Rep. 87 (1982) 77.
9. See talk of P. Mackenzie, this conference.