A new generator for the Drell-Yan process

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Abstract. We present the Monte Carlo event generator LePaProGen for lepton pair production at hadron colliders. LePaProGen focuses on the description of higher-order electroweak radiative corrections. The generator is implementing a new algorithm for the selection of the optimal variables for phase space parametrization.

1. Introduction
The processes of single electroweak boson production are very significant for physical studies at LHC. There are many groups working on them to provide adequate theoretical predictions. FEWZ [1], POWHEG [2–4], SANC [5], HORACE [6] et al. programs has been developed for this purpose. At energies 1-5 TeV charged and neutral current Drell-Yan processes are background to the processes of new physics. To make distinguish between different models it is important to have both fast and accurate tool for Monte-Carlo simulation. Designed for this purpose code READY [7] based on Sudakov and collinear logarithms approximation working only at extra large dilepton invariant mass. It is convenient to have a tool which works equally good at all kinematic regions and nevertheless is very fast. For this reason we introduce our event generator LePaProGen.

2. LePaProGen
LePaProGen (Lepton Pair Production Generator) is a Monte-Carlo generator for the neutral and charged current Drell-Yan process. It is intended for fast and accurate generation of events and uses independent generation algorithm. Now it implements one loop radiative corrections with the exact hard QED Bremsstrahlung contribution. But the higher order QCD or electroweak corrections could be easily added by design. LePaProGen is written in C++ using modular architecture. To overcome the C++ templates limitations we use a Python module which calculates all the constants and preprocesses the input settings. A glue layer between Python and C++ parts of the program is Mako template library which performs optimized code generation for the process chosen by user. LePaProGen can be used as a Pythia8 plug-in. It has all needed interfaces for convenient work: it generates events in the Les Houches Accord (LHA) format, the HepMC output format; it uses the LHAPDF interface for parton density functions. It supports a variety of renormalization schemes and performs the POWHEG-like matching.

In the following sections we will describe the main algorithms implemented in LePaProGen in detail.
3. Generation of phase space points
First of all we start with phase space points generation. All the variables can be divided into two groups: the “slow” and the “fast”. The “slow” variables can be generated uniformly but the “fast” ones require importance sampling. In tree level amplitude all “fast” variables are due to propagators. In order to generate a point in phase space we need to parametrize it by the invariant variables, which appear in the propagators. It can be done by effectively replacing all the propagators by \( \delta \)-functions
\[
\int dR_n \left( \frac{1}{p^2 - m^2} \right) = \int ds' \left( \frac{1}{s' - m^2} \right) \int dR_n \delta(p^2 - s') \ldots.
\] (1)
Such integrals appear in generalized unitarity methods and can be expressed by Feynman graphs (Fig. 1) with the help of the optical theorem. In fact all intermediate and final particles in such graph are on-shell. To calculate the integral we subsequently contract one loop subdiagrams starting with ones containing lesser number of propagators (Fig. 2-3). As a result we obtain a Jacobian determinant of transformation. This algorithm is a generalization of the recurrence relation for phase space volume from Byckling and Kajantie [8].

4. Particles Momenta Reconstruction
The cross-section for the process \( a + b \rightarrow 1 + \ldots + n \) is the following phase space integral:
\[
\sigma_n = \frac{(2\pi)^4}{2\lambda^{1/2}(s, m_a^2, m_b^2)} \frac{1}{(2\pi)^3 n} \int \prod_{i=1}^{n} \frac{d^3 p_i}{2E_i} |M|^2 \delta(p_a + p_b - \sum_i p_i).
\] (2)
For now lets formulate the problem of phase space reconstruction.
- We select some set of invariant variables suitable for generation.
- Then we sample these variables according to some importance density.
- We must evaluate the matrix element at the selected point of phase space. If the matrix element is in the invariant form then we need only the full set of scalar products \( p_i \cdot p_j \).
- If the matrix element is expressed in terms of noninvariant components of vectors in some reference frame, then we must express them in our invariant set of variables. This task we call “reconstruction”.

Consider reconstruction in the case of 3-propagators phase space integral.
4.1. Three-Point Reconstruction

This case corresponds to the interference between the decay and the t-channel scattering, or to the subdiagram of interference between two 3-particle decay channels. Our reconstruction algorithm is universal and works for both interpretations. Differences appear only in the integration limits.

As input parameters we have two 4-momenta. These are the momenta of the particles scattered in t-channel $p_1$ and $p_{13}$ (Fig. 4). Let the momenta after scattering be $p_2$ and $p_{12}$. The masses of the decay products and the intermediate t-channel propagators are known ($m_1$, $m_{12}$ and $m_{123}$). We use only the invariant squared masses $s_1 = p_1^2$, $s_{12} = p_{12}^2$ and $s_{123} = p_{123}^2$ because a contracted one-loop subdiagram can appear at the place of a particle. It is evident, that due to the 4-momentum conservation it is enough to reconstruct only the 4-momentum of one of the particles.

The first necessary condition for the decay is time-likeness of the 2-plane spanned by external momenta:

$$\pi_{13} = p_1 \wedge p_3 = p_1 \wedge p_{13}, \quad ||\pi_{13}||^2 > 0. \quad (3)$$

This condition guarantees finiteness of the phase space volume and is weaker then the corresponding one from [8]. Consequently our approach is more general and can be applicable in cases when relations from [8] does not work.

The scalar products are fixed by the masses of decay products:

$$p_1 \cdot p_2 = \frac{s_{12} - s_1 - s_2}{2}, \quad p_2 \cdot p_{13} = \frac{s_{123} - s_2 - s_{13}}{2}. \quad (4)$$

The next step is to construct reciprocal basis:

$$p_1 = p_{13} \pi_{13}^{-1}, \quad p_{13} = -p_1 \pi_{13}^{-1}. \quad (5)$$

Using this basis we can reconstruct the longitudinal component of $p_2$ and norm of the transverse component:

$$p_{2L} = (p_1 \cdot p_2)p_1 + (p_2 \cdot p_{13})p_{13}, \quad p_{2T}^2 = s_2 - p_{2L}^2. \quad (6)$$

Here is the place to apply the second necessary condition on the input parameters: it is evident that the longitudinal part is time-like ($p_{2L}^2 > 0$) so the transverse part should be space-like:

$$p_{2T}^2 < 0. \quad (7)$$
The direction of the transverse part of \( p_2 \) cannot be determined from provided data and should be generated randomly and uniformly. Let’s generate a unit 2-vector \( n \) uniformly on a circle.

In order to use this as the direction of \( p_{2T} \) we should Lorentz-transform it to the plane orthogonal to the all external momenta. We are going to construct a Lorentz transformation which maps \( \pi_{13}/||\pi_{13}|| \) to the bivector \( \gamma_0 \wedge \gamma_3 \). In Clifford algebra this transformation is determined by the rotor \( R \):

\[
R = \sqrt{S}, \quad S = \text{sgn}(\langle \pi_{13}\gamma_{03} \rangle)\pi_{13}\gamma_{03}, \quad \gamma_{03} = \gamma_0 \wedge \gamma_3. \tag{8}
\]

\[
\sqrt{S} \propto (||S|| + S) \left( (S)_+ + \sqrt{(S)_+ (S)_-} \right) (S)_-, \quad (S)_+ = ||S|| + (S) \pm (S)_4.
\]

\[
p_{2T} = \sqrt{-p_{2T}^2} R n R^{-1}, \quad n = n_x \gamma_1 + n_y \gamma_2. \tag{10}
\]

The above mentioned procedure is correct because Lorentz-transformations preserve orthogonality. Moreover, this 4-rotation has minimum norm in parametric space.

The Jacobian determinant we need is

\[
J_3 = \frac{\pi}{2(2\pi)^6 \sqrt{\lambda(s_1, s_3, s_{13})}}. \tag{11}
\]

5. The Amplitude

In this section we turn our focus on the hard Bremsstrahlung amplitudes. They are generated using helicity amplitude technique by our MetaAmp package. We use the Kleiss-Stirling convention [9]. Let’s choose two basis vectors \( k_0 \) and \( k_1 \), which satisfy relations:

\[
k_0^2 = 0, \quad k_0 \cdot k_1 = 0, \quad k_1^2 = -1. \tag{12}
\]

So then we construct basis spinors \( u_+(k_0) \) and \( u_-(k_0) \), such that

\[
u_\lambda(k_0) \bar{u}_\lambda(k_0) = \omega_\lambda k_0, \quad u_\lambda(k_0) = \lambda k_1 u_{-\lambda}(k_0), \quad \omega_\lambda = \frac{1 + \gamma_5 \gamma_3}{2}. \tag{13}
\]

If we take \( k_0 = \{1/2, 0, 0, 1/2\} \) and \( k_1 = \{0, 1, 0, 0\} \) then in the Weyl representation (up to an arbitrary phase) the basis spinors look like \( u_+(k_0) = \{1, 0, 0, 0\}^T \) and \( u_-(k_0) = \{0, 0, 0, -1\}^T \).

Now we can express a spinor for arbitrary external particle with momentum \( p \):

\[
u_\lambda(p) = \frac{\hat{p} \pm m}{\sqrt{2p \cdot k_0}} u_{-\lambda}(k_0). \tag{14}
\]

To generate amplitudes MetaAmp uses Fiertz identities and Dirac matrix algebra. In that way we reduce the task to calculation of the following expressions:

\[
\langle pq \ldots k \rangle = \bar{u}_+(k_0) \hat{p} \bar{q} \ldots \hat{k} \omega_+ u_+(k_0) = \text{Tr}[\hat{p} \bar{q} \ldots \hat{k} \omega_+ \hat{k_0}],
\]

\[
\langle pq \ldots k \rangle = \bar{u}_+(k_0) \hat{p} \bar{q} \ldots \hat{k} \omega_- u_-(k_0) = \text{Tr}[\hat{p} \bar{q} \ldots \hat{k} \omega_- \hat{k_1}],
\]

\[
[pq \ldots k] = \bar{u}_-(k_0) \hat{p} \bar{q} \ldots \hat{k} \omega_+ u_+(k_0) = \text{Tr}[\hat{p} \bar{q} \ldots \hat{k} \omega_+ \hat{k_0}],
\]

\[
[pq \ldots k] = \bar{u}_-(k_0) \hat{p} \bar{q} \ldots \hat{k} \omega_- u_-(k_0) = \text{Tr}[\hat{p} \bar{q} \ldots \hat{k} \omega_- \hat{k_0}]. \tag{15}
\]
Normalization factors from external wave-functions are factorized out from the whole amplitude so square roots are not presented. We have chosen \(k_0 = \{1/2, 0, 0, 1/2\}\) and \(k_1 = \{0, 1, 0, 0\}\), so operations (15) are reduced to taking the component of the resulting matrix making numerical calculation of generated amplitudes really fast.

This is an example of the single bremsstrahlung amplitudes generated by MetaAmp:

\[
A_{++--}^\gamma = 4e^3 \delta_{i_2}^{i_1} \left[ Q(l)D_{RR}(p_{12}^2) \left( \frac{1}{p_{45}^2} [p_{2p3}] [p_{4p5}] [p_{1p4}] [p_{2p3p5}] - \frac{1}{p_{35}^2} [p_{3}] [p_{1p4}] [p_{2p3p5}] \right) 
+ Q(q)D_{RR}(p_{34}^2) \left( \frac{1}{p_{25}^2} [p_{2}] [p_{1p4}] [p_{3p2p5}] - \frac{1}{p_{15}^2} [p_{2p3}] [p_{1p4}] [p_{4p15}] \right) \right],
\]

where \(D_{LR}(s) = Q(q)Q(\mu) + g_L(q)g_R(l) - M_Z^2 + i\Gamma_Z M_Z \); \(Q(q)(Q(\mu))\) – charge of quark (muon); \(g_L(q), g_R(l)\) – weak current coupling constants; \(i_1, i_2\) – color indices.

6. Plots
In order to demonstrate work of our generator we present some plots Fig. 5-7 for the neutral-current Drell-Yan process generated using LePaProGen.

Figure 5. The transverse momentum distribution of the muon. Born – green, Born + EW – guaranty finiteness red.

Figure 6. The pseudorapidity distribution of the muon. Born – green, Born + EW – red.

Figure 7. The muon pair invariant mass distribution. Born green, Born + EW – red.

7. Conclusions
In this paper a new generator LePaProGen for Drell-Yan processes is presented. A new algorithm for the selection of the optimal variables for phase space parametrization is implemented. The resulting efficiency is under investigation. A comparison with existing Monte-Carlo algorithms, like Vegas and FOAM[10], will be performed. A fast modification of helicity amplitude method is used. The detailed numerical comparison between the predictions of LePaProGen and other MC event generators for charged and neutral current Drell-Yan processes will be fulfilled in the near future.

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