B-Meson Mixing and Lifetimes

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$B_d$ and $B_s$ meson systems play a fundamental role to test and improve our understanding of Standard Model flavor dynamics. The mixing parameters $\Delta m_{d,s}$ represent important constraints in the unitarity triangle analysis. Their theoretical estimates require non-perturbative calculations of B-meson decay constants and B-parameters; accurate results, recently obtained from the lattice, are reviewed. Other phenomenologically interesting quantities are the beauty hadron lifetime ratios and, the width differences and CP-violation parameters in $B_d$ and $B_s$ systems. We discuss their theoretical predictions which, in the last four years, have been improved thanks to accurate lattice calculations and next-to-leading order perturbative computations.

1. Introduction

The neutral $B_d$ and $B_s$ mesons mix with their antiparticles leading to oscillations between the mass eigenstates. The time evolution of the neutral meson doublet is described by a Schrödinger equation with an effective $2 \times 2$ Hamiltonian

$$\frac{d}{dt}\begin{pmatrix} B_q \\ B_{\bar{q}} \end{pmatrix} = \begin{pmatrix} M_{q}^{\bar{q}} & M_{\bar{q}}^{\bar{q}} \\ \overline{M}_{q}^{\bar{q}} & \overline{M}_{\bar{q}}^{\bar{q}} \end{pmatrix} \begin{pmatrix} B_q \\ B_{\bar{q}} \end{pmatrix},$$

The mass and width differences are defined as $\Delta m_q = m_{q}^{H} - m_{q}^{L}$ and $\Delta \Gamma_q = \Gamma_{q}^{H} - \Gamma_{q}^{L}$, where $H$ and $L$ denote the Hamiltonian eigenstates with the heaviest and lightest mass eigenvalue. These states can be written as

$$|B_q^{H,L}\rangle = \frac{1}{\sqrt{1+|q/p|_{q}^{2}}} \left(|B_q\rangle \pm (q/p)_{q}|\bar{B}_{q}\rangle\right).$$

Theoretically, the hadron lifetime is related to $\Gamma_{q}^{H}$ ($\tau_{B_q} = 1/\Gamma_{q}^{H}$), while the observables $\Delta m_q$, $\Delta \Gamma_q$ and $|q/p|_{q}$ are related to $M_{q}^{H}$ and $\Gamma_{q}$.

In $B_{d,s}$ systems, the ratio $\Gamma_{q}^{H}/M_{q}^{H}$ is of $O(m_{b}/m_{q}) \simeq 10^{-3}$. Therefore, by neglecting terms of $O(m_{b}/m_{q})$, one can write

$$\Delta m_q = 2|M_{q}^{H}|, \quad \Delta \Gamma_q = -2|M_{q}^{H}| \Re\left(\frac{\Gamma_{q}}{M_{q}^{H}}\right), \quad |q/p|_{q} = 1 + \frac{1}{2} \Im\left(\frac{\Gamma_{q}}{M_{q}^{H}}\right).$$

The matrix elements $M_{q}^{H}$ and $\Gamma_{q}$ are related, respectively, to the dispersive and the absorptive parts of the $\Delta B = 2$ transitions. In the SM, these transitions are the result of second-order charged weak interactions involving the well-known box diagrams.

The dispersive matrix element $M_{21}^{q}$ has been computed at the NLO in QCD [1]. Since the mass differences $\Delta m_q$ play a fundamental role in constraining the unitarity triangle, it is important to have precise theoretical predictions of $M_{21}^{q}$. Recently, accurate studies of the B-meson decay constants and B-parameters entering in $M_{21}^{q}$, have been performed on the lattice.

The absorptive matrix elements $\Gamma_{11}^{q}$ and $\Gamma_{21}^{q}$ can be computed by applying the heavy quark expansion (HQE) [2], with a consequent separation of the short-distance contributions from the long-distance ones. The great energy $\sim m_{b}$ released in beauty hadron decays, in fact, allows to expand the inclusive widths in powers of $1/m_{b}$. Theoretical predictions of inclusive rates are based on a non-perturbative calculation of matrix elements, widely studied in lattice QCD, and a perturbative calculation of Wilson coefficients.

Recently, the contribution of light quarks in beauty hadron decay widths (spectator effect) has been computed at $O(\alpha_s)$ in QCD and $O(\Lambda_{QCD}/m_{b})$ in the HQE. Based on these calculations is the theoretical prediction for beauty hadron lifetimes and B-meson width differences and CP-violation parameters. Improved theoretical estimates have been obtained, to be compared...
with recent accurate experimental measurements or limits.

2. Mixing parameters

The mass difference in the $B_d - \overline{B}_d$ system ($\Delta m_d$) is proportional to $f_{B_d} \overline{B}_{B_d} |V_{td}|^2$, thus representing a constraint on the CKM element $|V_{td}|$, provided that the multiplied hadronic matrix elements are calculated. The analogous mass difference in the $B_s - \overline{B}_s$ system ($\Delta m_s$) can be used to reduce the theoretical uncertainty, by considering the ratio

$$\frac{\Delta m_s}{\Delta m_d} \propto \frac{|V_{ts}|^2 \xi^2}{|V_{td}|^2} \quad \text{with} \quad \xi = \frac{f_{B_s} \sqrt{\overline{B}_{B_s}}}{f_{B_d} \sqrt{\overline{B}_{B_d}}}. \quad (4)$$

The hadronic parameter that is better determined from lattice calculations is $f_{B_s} \overline{B}_{B_s}$, whereas $\xi$ and $f_{B_d}^2 \overline{B}_{B_d}$ are affected by larger uncertainties coming from the chiral extrapolations. These uncertainties are strongly correlated. For this reason, the best approach to constraint the unitarity triangle, recently proposed and adopted in Ref. [3], uses the following equations

$$\Delta m_d \propto f_{B_d}^2 \overline{B}_{B_d}, \quad \frac{\Delta m_s}{\Delta m_d} \propto \xi^2.$$

In lattice calculations the difficulty to treat heavy quarks has been essentially solved by introducing the HQET based lattice actions, and the results from different formulations are in good agreement, in the quenched approximation, within the systematic uncertainty of 15% [4]. Concerning $f_{B_s}$, unquenched results seem slightly higher than the quenched values ($\sim 10 - 15\%$) [4] but some systematics, as continuum scaling, are not yet investigated. In the case of the $\overline{B}_{B_s}$ parameter, instead, sea quark effects result to be negligible. Concerning the chiral extrapolation, it represents a delicate issue for $f_{B_d}$ and, nowadays, chiral log effects are finally estimated ($\sim 5\%$) [4], while chiral loops are not a problem for $\overline{B}_{B_d}$, as expected from ChPT.

In Table 2 we summarize the averages for the $B_q - \overline{B}_q$ mixing parameters, which include rough estimates of chiral logs and unquenched effects. This year, new results have been obtained by the HPQCD Collaboration [5] that confirm their previous estimates [4]. The averages in Table 2, therefore, are identical to those presented at ICHEP2004 by S. Hashimoto [4].

3. Beauty hadron lifetime ratios

The experimental values of the measured lifetime ratios of beauty hadrons are [7]

$$\frac{\tau(B^+)}{\tau(B_d)} = 1.081 \pm 0.015, \quad \frac{\tau(B_s)}{\tau(B_d)} = 0.939 \pm 0.044,$$

$$\frac{\tau(\Delta s)}{\tau(B_d)} = 0.803 \pm 0.047. \quad (6)$$

By applying the HQE, the inclusive decay width of a hadron $H_b$ can be expressed as a sum of local $\Delta B = 0$ operators of increasing dimension, as

$$\Gamma(H_b) = \sum_k \frac{\tilde{c}_k(\mu)}{m_b} \langle H_b | \tilde{O}_k^0 | H_B = 0(\mu) \rangle \langle H_b \rangle. \quad (7)$$

The HQE yields the separation of short distance effects, confined in the Wilson coefficients ($\tilde{c}_k$), from long distance physics, represented by the matrix elements of the local operators ($\tilde{O}_k^0$).

Spectator contributions, which distinguish different beauty hadrons, appear at $O(1/m_b^2)$ in the HQE. These effects, although suppressed by an additional power of $1/m_b$, are enhanced with respect to leading contributions by a phase-space factor of $16\pi^2$, being $2 \rightarrow 2$ processes instead of $1 \rightarrow 3$ decays [5-6]. In order to evaluate the spectator effects one has to calculate the matrix elements of dimension-six current-current and penguin operators, non-perturbatively, and their Wilson coefficients, perturbatively.

Concerning the perturbative part, the NLO QCD corrections to the coefficient functions of the current-current operators have been computed [10-12].
Concerning the non-perturbative part, the non-valence contributions, corresponding to contractions of two light quarks in the same point, have not been computed. Their non-perturbative lattice calculation would be possible, in principle, however it requires to deal with the difficult problem of power-divergence subtractions. On the other hand, the valence contributions, which exist when the light quark of the operator enters as a valence quark in the external hadronic state, have been evaluated. For $B$–mesons, QCD and HQET lattice results have been recently combined to extrapolate to the physical $b$ quark mass [13], while for the $Λ_b$ baryon, lattice-HQET has been used [14]. These accurate results are in agreement with the values obtained in previous lattice studies [15]–[17].

Last year, the sub-leading spectator effects which appear at $O(1/m_b^4)$ in the HQE, have been included in the analysis of lifetime ratios. The relevant operator matrix elements have been estimated in the vacuum saturation approximation (VSA) for $B$–mesons and in the quark-diquark model for the $Λ_b$ baryon, while the corresponding Wilson coefficients have been calculated at the leading order (LO) in QCD [18].

Updated theoretical predictions for the lifetime ratios are [12]

$$\frac{\tau(B^+)/\tau(B_d)}{\tau(B_d)/\tau(B)} = 1.06 \pm 0.02, \quad \frac{\tau(B_s)/\tau(B_d)}{\tau(B_d)/\tau(B)} = 1.00 \pm 0.01,$$

$$\frac{\tau(Λ_b)/\tau(B_d)}{\tau(B_d)/\tau(B)} = 0.88 \pm 0.05.$$  \(\text{(8)}\)

They turn out to be in good agreement with the experimental measurements of Eq. (6).

It is worth noting that the agreement at 1.2σ between the theoretical prediction for the ratio $\tau(Λ_b)/\tau(B_d)$ and its experimental value is achieved thanks to the inclusion of NLO (see Fig. 1) and the $1/m_b$ corrections to spectator effects. They both decrease the central value of $\tau(Λ_b)/\tau(B_d)$ by 8% and 2% respectively.

Further improvement of the $\tau(Λ_b)/\tau(B_d)$ theoretical prediction would require the calculation of the current-current operator non-valence B-parameters and of the perturbative and non-perturbative contribution of the penguin operator, which appears at the NLO and whose matrix elements present the same problem of power-divergence subtraction. These contributions are missing also in the theoretical predictions of $\tau(B^+)/\tau(B_d)$ and $\tau(B_s)/\tau(B_d)$, but in these cases they represent an effect of $SU(2)$ and $SU(3)$ breaking respectively, and are expected to be small.

4. Neutral $B_q$-meson width differences

The width difference between the “light” and “heavy” neutral $B_q$-meson ($q = d, s$) is defined in terms of the off-diagonal matrix element $Γ_{21}^q$ (see Eq. (3)).

In the HQE of $Γ_{21}^q$, the leading contribution comes at $O(1/m_b^4)$ and is given by dimension-six $ΔB = 2$ operators. Up to and including $O(1/m_b^4)$ contribution, one can write

$$Γ_{21}^q = -\frac{G_F^2 m_b^2}{24π M_{B_q}} \left[ c_2^q(μ_2) (B_q) O_1^q(μ_2) |B_q⟩ + \right.$$

$$\left. c_2^q(μ_2) (B_q) O_2^q(μ_2) |B_q⟩ + δ_{1/m}^q \right], \quad \text{(9)}$$

where $(B_q) O_1^q(μ_2) |B_q⟩$ are the matrix elements.
of the two independent dimension-six operators, \(c_7^q(\mu_2)\) their Wilson coefficients, known at the NLO in QCD [19]-[21], while \(\hat{\delta}_{1/m_b}\) represents the contribution of the dimension-seven operators [22].

Lattice results of the dimension-six operator matrix elements [23]-[27] have been confirmed and improved, by combining QCD and HQET results in the heavy quark extrapolation [28]. Moreover, the effect of the inclusion of the dynamical quarks has been examined, within the NRQCD approach, finding that these matrix elements are essentially insensitive to switching from \(n_f = 0\) to \(n_f = 2\) [29,30].

Concerning the dimension-seven operators, their matrix elements have never been estimated out of the VSA. Two of these four matrix elements, however, can be related through Fierz identities to the complete set of operators studied in Ref. [29].

The updated theoretical predictions are [20]

\[
\begin{align*}
\Delta \Gamma_d/\Gamma_d & = (2.42 \pm 0.59)10^{-3}, \\
\Delta \Gamma_s/\Gamma_s & = (7.4 \pm 2.4)10^{-2}.
\end{align*}
\]

The corresponding theoretical distributions are shown in Fig. 2 where the effect of NLO corrections can be seen to be quite relevant.

The interest in \(\Delta \Gamma_s\) has largely increased due to recent experimental measurements from the CDF [31] and D0 [32] Collaborations. Previous experimental limits [7]

\[
\begin{align*}
\Delta \Gamma_d/\Gamma_d & = 0.008 \pm 0.037 \pm 0.019, \\
\Delta \Gamma_s/\Gamma_s & = 0.07^{+0.09}_{-0.07},
\end{align*}
\]

were in agreement with theoretical predictions, within the large experimental uncertainties. Last year CDF and D0 presented their results for

\[
\begin{align*}
\Delta \Gamma_s/\Gamma_s & = 0.65^{+0.25}_{-0.33} \pm 0.01 (\text{CDF}), \\
\Delta \Gamma_s/\Gamma_s & = 0.21^{+0.27}_{-0.40} (\text{D0}).
\end{align*}
\]

The CDF’s result is surprisingly large, 2\(\sigma\) away from the theoretical prediction. Updated experimental results with higher statistics are therefore needed for a significant comparison. The theoretical value, instead, is under control and comes from cancellations occurring at the NLO and \(\mathcal{O}(1/m_b^2)\), which successively reduce the LO central value from 0.26 to 0.18 and to the final 0.074 given in Eq. [11].

Our theoretical prediction is slightly smaller than the value calculated in Ref. [19] \((\Delta \Gamma_s/\Gamma_s = 0.12(5))\). This difference is mainly due to the contribution of \(\mathcal{O}(1/m_b^2)\), which, in Ref. [19] is wholly estimated in the VSA, while we express the matrix elements of two dimension-seven operators in terms of those calculated on the lattice.

5. CP Violation parameters: |\((q/p)_d\)| and |\((q/p)_s\)|

The experimental observable |\((q/p)_q|\), whose deviation from unity describes CP-violation due to mixing, is related to \(M_{21}^2\) and \(\Gamma_{21}^s\), through Eq. [13]. The theoretical prediction of |\((q/p)_q|\) is therefore based on the same perturbative and non-perturbative calculation discussed in Sec. 4 while the \(V_{CKM}\) contribution to |\((q/p)_q|\) is different from that in \(\Delta \Gamma_q/\Gamma_q\).

The updated theoretical predictions [20] are

\[
\begin{align*}
|\langle q/p\rangle_d| - 1 & = (2.96 \pm 0.67)10^{-4}, \\
|\langle q/p\rangle_s| - 1 & = -(1.28 \pm 0.28)10^{-5}.
\end{align*}
\]

The corresponding theoretical distributions are shown in Fig. 3 with an evident effect of NLO corrections.

A preliminary measurement for |\((q/p)_d| - 1\) is now available from the BABAR Collaboration [7]

\[
|\langle q/p\rangle_d| - 1 = 0.029 \pm 0.013 \pm 0.011
\]

Improved measurements are certainly needed to make this comparison significant for the unitarity triangle analysis.

Figure 2. Theoretical distributions for \(B_d\) and \(B_s\) width differences, at the LO (light/red) and NLO (dark/blue).
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Figure 3. Theoretical distributions for $|q/p| - 1$ in $B_d$ and $B_s$ systems, at the LO (light/red) and NLO (dark/blue).

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