A Bayesian Approach Accounting for Stochastic Fluctuations in Stellar Cluster Properties

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Abstract. The integrated spectro-photometric properties of star clusters are subject to large cluster-to-cluster variations. They are distributed in non-trivial ways around the average properties predicted by standard population synthesis models. This results from the stochastic mass distribution of the finite (small) number of luminous stars in each cluster, stars which may be either particularly blue or particularly red. The color distributions are broad and usually far from Gaussian, especially for young and intermediate age clusters, as found in interacting galaxies. When photometric measurements of clusters are used to estimate ages and masses in conjunction with standard models, biases are to be expected. We present a Bayesian approach that explicitly accounts for stochasticity when estimating ages and masses of star clusters that cannot be resolved into stars. Based on Monte-Carlo simulations, we are starting to explore the probability distributions of star cluster properties obtained given a set of multi-wavelength photometric data.

1 Introduction

Observing star clusters offers a powerful tool for studies of the formation history of galaxies. Clusters can be measured out to much further distances than individual field stars, and they provide complementary information to the integrated light emitted by galaxy fields. Star formation and cluster formation are closely related, although the interplays between clusters and the field remain to be properly understood. The age distribution of star clusters may be a record of the star formation history of galaxies. Their age and mass distributions are especially important when we wish to understand what happens during collisions or mergers of gas-rich galaxies, where the interaction is well known to trigger the formation of stars.

For clusters that are not resolved into stars, estimates of ages, masses and other intrinsic properties are based on integrated photometry or spectra. The standard approach is to compare these observed properties with synthetic spectra produced by population synthesis codes that assume a continuously populated stellar initial mass function (IMF).

However the energy distribution of clusters is dominated by the light of a finite (often small) number of massive stars, whereas the mass is mostly produced by early evolved stars with relatively faint luminous contributions. As an example, red supergiants are the predominant source of near-IR light in young clusters (6-60 Myr). These stars are intrinsically rare. As a combined result of the Initial Mass Function (IMF) and the short duration of the relevant evolu-
tionary phases, we expect on average only one red supergiant for a total cluster mass of $10^4 \, M_\odot$ for a standard IMF with a lower cut-off of $0.1 \, M_\odot$ (Lançon et al. 2008). Hence a $10^4 \, M_\odot$ star cluster may contain zero, one or a few red supergiants, which will result in multi-modal probability distributions for integrated colors and fluxes (Chiosi, Bertelli, & Bressan 1988; Lançon & Mouhcine 2000; Cerviño & Luridiana 2004; Lançon & Fouesneau 2009). Large spreads result from the above-mentioned small numbers of luminous stars, the so-called stochastic fluctuations (Figure 1). It requires clusters with several $10^6 \, M_\odot$ to narrow down the fluctuations to 5% in the K-band. In this case, colors and fluxes follow uni-modal distributions and Gaussian approximations become tolerable.

Massive clusters for which the Gaussian approximation holds exist in very small numbers, because cluster mass functions tend to fall rapidly with increasing mass (Whitmore et al. 1999; Bastian et al. 2009). On the other hand, many low and intermediate mass clusters have already been observed with space-based and large ground-based telescopes. It is thus very important to develop methods that explicitly deal with the stochastic fluctuations.

2 Estimates of Star Cluster Properties

The method we are developing follows a Bayesian approach. It is a close analog to the one introduced by Kauffmann et al. (2003) for the study of star formation histories in the Sloan Digital Sky Survey. However the variety of observable properties has completely different origins in both contexts: stochasticity at a given age, mass and metallicity, plays a predominant role here while it is negligible in most galaxies taken as a whole.

We have started a campaign of Monte-Carlo (MC) simulations (Fig. 1), that is used as a catalog for establishing the joint probability distributions of the intrinsic parameters of a cluster (age, total number of stars or total mass, metallic-
Figure 2. Example of a posteriori probability distributions of age, mass and extinction, for a synthetic cluster “observed” in U, B, V, I, J and K with 1-σ photometric errors of 0.05 magnitudes. The prior is that the cluster mass distributions in galaxies vary as $M^{-2}$, and that the cluster age distributions in galaxies are constant in logarithmic age units (only clusters that survived dynamical disruption matter here). The true properties of the “observed” cluster are indicated as triangles, the most probable values as derived from the “observations” are shown as small circles.

ity, extinction), given a set of photometric measurements and their uncertainties (Fouesneau et al., in prep). The total mass is not a simple scaling factor. It enters the problem even when only colors are available because it affects the color probability distributions of clusters of a given age and metallicity.

Among the assumptions in the method are the stellar IMF, and the adopted stellar evolutionary tracks and stellar spectral libraries. The main a priori of the Bayesian inversion are the underlying cluster formation history (number of clusters per age bin in the catalog), and the cluster mass function. In applications to real cluster populations, several a priori distributions should be tested and compared. The effects of dynamical evolution on the IMF and on the cluster mass function also need to be discussed.

We generate integrated photometric properties of individual clusters with a modified version of PÉGASE (Fioc & Rocca-Volmerange 1999) that implements a discrete IMF. The models take into account the nebular continuum and line emission of clusters that contain ionizing stars.

The assumption that errors in the observational data are Gaussian leads to a simple expression for the a posteriori probability distribution of an intrinsic property, such as the age or mass of an individual cluster. The probability for an intrinsic property $X$ to be located in an interval $[x_1, x_2]$, given the photometric measurements $Y = \{Y_k\}_{k \in [1..n]}$, is:

$$
\mathcal{P}(X \in [x_1, x_2] \mid Y) = \alpha \times 
\sum_{i, X(M_i) \in [x_1, x_2]} \prod_k \frac{1}{\sqrt{2\pi\sigma_k^2}} \exp\left(-\frac{(Y_k - Y_{k,M_i})^2}{2\sigma_k^2}\right) \times \mathcal{P}(M_i),
$$

(1)

where $\mathcal{P}(M_i)$ is the probability of the individual model $M_i$ to be chosen and $\alpha$ is the normalization constant. Through the factors $\mathcal{P}(M_i)$, this expression adapts to any prior mass or age distribution.
As an example, Figure 2 shows recovered age, mass and extinction probability distributions based on pseudo-observational data, i.e. on the synthetic magnitudes of one of the clusters in the MC-model catalog (modified by the addition of reasonable noise). Although six absolute fluxes across the spectrum are used, the possible age and mass ranges remain significantly broad, and asymmetric. We are in the process of establishing the systematic differences between the actual properties of a cluster and various estimates thereof (best $\chi^2$ model, most probable values, and values recovered using the classical approach with a continuously populated IMF). We will also reinvestigate which minimal combination of observations constrains the intrinsic properties most efficiently.

Varying the metallicity, the allowed extinction range or the $a$ priori distributions of mass and age will significantly change the results. We are still exploring the different effects, especially those of metallicity and extinction. We are also extending the MC-model catalog by adding new massive clusters, which will be necessary to test the effect of adopting flatter $a$ priori cluster mass distributions.

3 Conclusion

Although massive clusters contain important information on active episodes of star formation in galaxies, the need to study smaller clusters is increasing as the amount of available observational data grows. Intrinsically rare but luminous stars are responsible for large stochastic fluctuations in the integrated energy distribution of these objects. Hence, new tools that deal with stochastic fluctuations explicitly need to be developed. In this context we have implemented a Bayesian approach that, given a set of photometric data, provides most probable cluster parameters as well as confidence intervals. The ongoing study of systematic differences between true and estimated values should allow us to correct any significant biases in previous studies.

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