This study applies non-linear threshold unit-root test to assess the non-stationary properties of the real exchange rate for twenty African countries. We found that non-linear threshold unit-root test has higher power than linear method. As suggested by Caner and Hansen (2001), the true data generating process of exchange rate is a stationary non-linear process. We examine the validity of PPP from the non-linear point of view and provide robust evidence clearly indicating that purchasing power parity (PPP) holds true for six countries, namely; Egypt, Ethiopia, Gambia, Malawi, Seychelles and South Africa. Our findings point out their exchange rate adjustment is mean reversion towards PPP equilibrium values in a non-linear way.

Key words: Non-linear threshold unit-root test, purchasing power parity.

INTRODUCTION

The analysis of long-run purchasing power parity (PPP) has probably been one of the most controversial topics of the last decades within international economics. The results from validity of PPP have important implications to decision or policy makers of central banks, multinational firms and exchange rate market participants. The exchange rate policy is often regarded as the centerpiece of economic reform programmers, and the external competitiveness of any particular country is frequently gauged by the movement of its real exchange rate. The results from studies in this regard are critical for both empirical researchers and policymakers. PPP also provides an important basis for financial stabilization and structural adjustment policies and plays a role in the choice between money, inflation or exchange rate targeting in the formulation of monetary policy. In particular, the long-run PPP requires that real exchange rates must be stationary, which implies there is a long-run relationship between nominal exchange rate, domestic and foreign prices. As such, PPP cannot be used to determine the equilibrium exchange rate and invalid PPP also disqualifies the monetary approach to exchange rate determination, which requires PPP to hold true. According to Holmes (2001), the PPP is important to policy makers for two reasons.

First of all, it can be used to predict exchange rate to determine whether a currency is over or undervalued. Whether a currency is over or undervalued is particularly important for less-developed countries and also for those experiencing large difference between domestic and foreign inflation rates.

Secondly, the notion of PPP is used as the foundation on which many theories of exchange rate determination are built. Consequently, the validity is important to those policy makers in developing countries who base their adjustment on the PPP.

While empirical evidence shows that stationarity of the real exchange rate (RER) is abundant in developing countries (Bahmani-Oskooee, 1995; Breuer et al., 2001). Few studies have been conducted using date from small developing countries and, in particular, from Africa. Besides, robust researches on PPP are very important for African countries to the extent that, since the early 1990s, these have implemented numerous exchange rate policies’ modifications based on the assumption of PPP validity (Kargbo, 2003).

Recently, several studies have examined whether or not there is empirical support for long-run PPP in African countries (Odekokun, 2000; Holmes, 2000; Nagayasu,
2002; Kargbo, 2006; Hassanain, 2004; Akinboade and Makina, 2006; Bahmanee-Oskoee and Gelan, 2006). But, once again, a strong consensus could not be reached even if more results converge towards PPP validity.

For the past two decades, African countries have been implementing structural and macro-economic adjustment programmes designed to improve the external competitiveness and economic growth of these economies (Kargbo, 2004). In particular, exchange rate policy reforms were the focal point of the adjustment programmes.

Empirical research on PPP has therefore focused on the credibility of the unit root finding and on why deviations from PPP exist.

Despite a decade of multiple applications of the unit root tests in analyzing PPP, we are still unable to draw homogenous conclusions.

For previous studies, one possible explanation for the inconsistencies in the existing empirical evidence on the PPP hypothesis is that the prior studies implicitly assume that exchange rate behavior is inherently linear in nature. It is well known that if RER follows nonlinear stationary process then tests based on linear models such as the widely used augmented Dickey-Fuller (ADF) unit root models will be mis-specified (Chortareas et al., 2002).

However, Sarno (2000) and Taylor and Peel (2000) also demonstrated that the adoption of linear stationarity tests is inappropriate for the detection of mean reversion if the true process of the data generation of the exchange rate is in fact a stationary non-linear process. The presence of nonlinear mean-reverting adjustment for real exchange rates has been advanced by recent theoretical developments that emphasize the role of transaction costs. Taylor et al. (2001), Taylor and Peel (2000), Taylor and Taylor (2004), Juvenal and Taylor (2008) and Lothian and Taylor (2008) have argued that different speeds of adjustment at the disaggregated goods level average up to smooth nonlinearity at the aggregate level.

An alternative view is that non-linearity at the aggregate level is caused by other influences, such as the effects of official foreign exchange intervention (Taylor, 2004; Menkhof and Taylor, 2007; Reitz and Taylor, 2008) or heterogeneous agents (Kilian and Taylor, 2003). Additionally, the existence of structure changes in the RER might imply broken deterministic time trends and the result is a nonlinear pattern (Bierens, 1997).

The central aim of this study contributes significantly to this field of research because, first of all, we examine evidence for PPP for African countries, using the threshold autoregressive model (TAR) and the test statistics suggested by Caner and Hansen (2001), who considered a two regime TAR(k) model:

\[
\Delta r_t = \theta_1 x_{t-1} I_{Z_{t-1} \leq \lambda} + \theta_2 x_{t-1} I_{Z_{t-1} > \lambda} + \epsilon_t, \quad t = 1, \ldots, T
\]

(1)

Where, \( x_{t-1} = (r_{t-1}, v_{t-1}^r, \Delta r_{t-1}, \ldots, \Delta r_{t-k})' \) is the indicator function, \( \epsilon_t \) is an i.i.d. disturbance, \( Z_{t-1} = r_{t-1} - r_{t-m} \) for some \( m \geq 1 \) is the threshold variable, \( v_{t-1}^r \) is a vector of exogenous variables including an intercept and possibly a linear time trend, \( \lambda \) is a threshold parameter and \( k \geq 1 \) is the autoregressive unit root. The components of \( \theta_1 \) and \( \theta_2 \) can be partitioned as follows:

\[
\begin{bmatrix}
\theta_1 \\
\theta_2 
\end{bmatrix} = \begin{bmatrix}
\rho_1 \\
\rho_2 \\
\beta_1 \\
\beta_2 \\
\alpha_1 \\
\alpha_2 
\end{bmatrix}
\]

(2)

Where, \( \rho_1 \) and \( \rho_2 \) are scalar terms. \( \beta_1 \) and \( \beta_2 \) have the same dimensions as \( v_{t-1}^r \), and \( \alpha_1 \) and \( \alpha_2 \) are \( k \)-vectors. Thus \( \rho_1, \rho_2 \) are the slope coefficients on \( r_{t-1} \), \( \beta_1, \beta_2 \) are the slopes on the deterministic components, and \( \alpha_1, \alpha_2 \) are the slope coefficients on \( (\Delta r_{t-1}, \ldots, \Delta r_{t-k}) \) in the two regimes.

The threshold effect in Equation 1 has the null hypothesis

\[
H_0 : \theta_1 = \theta_2 \]

which is tested using the familiar Wald statistic:

\[
W_T = W_T(\lambda) = \sup_{\lambda \in \Lambda \cap \lambda} W_T(\lambda)
\]

The stationarity of the process \( r_t \) can be established in two ways. First, when there is a unit root in both regimes. Here the null hypothesis is of the form

\[
H_0 : \rho_1 = \rho_2 = 0
\]

which is tested against the unrestricted alternative \( \rho_1 \neq 0 \) or \( \rho_2 \neq 0 \) using the Wald statistic. The parameters \( \rho_1 \) and \( \rho_2 \) of Equation 1 control the regime-dependent unit root process of the RER. If \( \rho_1 = \rho_2 = 0 \) holds, the RER has a unit root can be described as a rejection of PPP. This statistic is:

\[
R_{2T} = \frac{t_1^2}{t_2^2} + \frac{t_2^2}{t_2^2}
\]

(2)

Where, \( t_1 \) and \( t_2 \) are the t ratios for \( \hat{\rho}_1 \) and \( \hat{\rho}_2 \) from the ordinary least squares estimation. However, Caner and Hansen(2001) claim that this two-sided Wald statistic may have less power than a one side version of the test. As a result, they propose the following one-side Wald statistic as follows:
Table 1. Univariate unit root test for real exchange rate.

| Country   | ADF  | PP    | KPSS     |
|-----------|------|-------|----------|
| Algeria   | -1.374(0) | -1.467[3] | 1.445[15]*** |
| Botswana  | -1.312(0) | -1.445[5] | 1.419[15]*** |
| Burkina Faso | -1.994(0) | -1.835[11] | 0.831[15]*** |
| Burundi   | -2.776(1)* | -3.465[8]*** | 0.302[15] |
| Cameroon  | -1.312(0) | -1.259[5] | 0.885[15]*** |
| Côte d’Ivoire | -1.168(0) | -1.145[5] | 0.959[15]*** |
| Egypt     | -2.267(0) | -2.283[4] | 0.839[15]*** |
| Ethiopia  | -1.456(0) | -1.582[4] | 0.684[15]*** |
| Gambia    | -2.174(1) | -2.062[4] | 0.364[15]*** |
| Kenya     | -0.315(1) | -0.563[5] | 1.900[15]*** |
| Madagascar | -1.697(1) | -2.329[11] | 0.395[15]*** |
| Malawi    | -3.104(0)** | -3.253[2]** | 0.310[15] |
| Mauritius | -0.765(0) | -0.771[1] | 1.811[15]*** |
| Morocco   | -0.090(0) | -0.047[2] | 1.809[15]*** |
| Niger     | -2.328(0) | -2.078[7] | 0.436[15]*** |
| Nigeria   | -1.763(0) | -1.962[6] | 0.372[15]*** |
| Senegal   | -1.679(0) | -1.656[1] | 0.596[15]*** |
| Seychelles | -2.414(0) | -2.324[8] | 1.724[15]*** |
| South Africa | -1.984(0) | -2.091[7] | 0.748[15]*** |
| Swaziland | -1.903(0) | -1.966[11] | 0.994[15]*** |

*** and ** indicate significance at the 0.01 and 0.05 levels, respectively. The number in parenthesis indicates the lag order selected based on the recursive t-statistic, as suggested by Perron (1989). The number in the brackets indicates the truncation for the Bartlett Kernel, as suggested by the Newey and West test (1987).

\[ R_{1T} = t^2 I_{\lambda<0} + t^2 I_{\lambda<0} \]

(3)

\[ R_{1T} \] tests \( H_0 \) against the one-side alternative \( \lambda < 0 \) or \( \rho_2 < 0 \). Caner and Hansen (2001) show that both tests \( R_{1T} \) and \( R_{2T} \) will have power against both alternatives.

**DATA AND EMPIRICAL RESULTS**

We use monthly data that covers from 1980 to 2008 to apply the Caner and Hansen (2001) threshold unit test in testing the validity of PPP. Since the end of the 1980s and the early 1990s, there were several waves of depreciation of domestic currencies in order to stop the extensive overvaluation of exchange rates in Africa during the 1970s and the 1980s. This empirical study covers twenty African countries. The price series are based on the consumer price index, and the nominal exchange rates are the end period spot rates relative to the U.S. dollar (domestic price of the U.S. dollar). All data is taken from the International Monetary Fund’s International Financial Statistics (IMFIS). For comparison, the univariate unit root tests are first employed to examine the null of a unit root in bilateral real exchange rates for twenty countries that we study. Based on the results from Table 1, there is no question that three univariate unit root tests - the augmented Dickey Fuller (1981, ADF), the Phillips and Perron (1988) and the Kwiatkowski et al. (1992) tests all failed to reject the null of non-stationary real exchange rates among these eighteen countries except Burundi and Malawi. Our results signified that real exchange rate is a random process. In other words, PPP was not held among these eighteen African countries under this study. Next, we used Wald test \( W_f \) to examine whether or not we can reject the linear autoregressive model in favor of a threshold model. The results of Wald test in Table 2, and also reported the bootstrap critical values generated at conventional levels of significance. The bootstrap p-value for threshold variables of the form \( Z_{r-1} = r_{r-1} - r_{r-m} \) for delay parameters \( m \) is ranged from 1 to 12. The parameters \( m \) is generally unknown; there is no reason to think the optimal delay parameter will be the same across countries. To circumvent this, Caner and Hansen (2001) suggest making \( m \) endogenous by selecting the least squares estimate of \( m \) that minimizes the residual variance. This amounts to selecting \( m \) at the value that maximizes the \( W_f \) statistic. We find that \( W_f \) statistic is maximized for Burkina Faso, Burundi, Egypt, Kenya, Madagascar, Mauritius and Seychelles when \( m = 1 \), for Morocco and Nigeria when \( m = 2 \), for Côte d’Ivoire, Senegal and Swaziland when \( m = 3 \), for Niger when \( m = 4 \), for Ethiopia and Malawi when \( m = 6 \), for
Botswana and Gambia when \( m=7 \), for Algeria when \( m=9 \), and for Cameroon and South Africa when \( m=12 \).

Taken together, these results imply strong statistical evidence against the null hypothesis of linearity at least 5% in ten African countries indicating that simple linear models are inappropriate. Next, we explore the threshold unit root properties of RER based on \( R_{1T} \) statistic for each delay parameter \( m \), ranging from 1 to 12, paying particular attention to the results obtained for our preferred model. The \( R_{1T} \) test results, together with the bootstrap critical value at the conventional levels of significance and the bootstrap p-value, are reported in Table 3.

We are able to reject the unit root null hypothesis for Ethiopia and South Africa at the 1% level, for Egypt, Gambia, Malawi and Seychelles at the 5%. However, we are unable to reject the threshold unit root hypothesis for the other fourteen African countries. Taken together, our results provide strong support for PPP for six of twenty African countries and point that these countries are nonlinear stationary, implying that deviations of exchange rate is mean reverting towards the PPP equilibrium. As mentioned earlier, trade barriers, as well as interventions in the exchange markets, could be behind this nonlinear behavior. The validity of PPP is important to policy makers in six African countries who base their determination on exchange rate adjustments. The result means that the unbounded gains from arbitrage in traded goods are impossible among these three countries. Figure 1 shows the estimated division of twenty Africa countries’ RER into two threshold regimes. The threshold unit root tests of the real exchange rate employed in this study provides some evidence favoring the long-run validity of PPP for the Africa countries being studied. Many of African countries experience double-digit inflation in the last two decades, and the high inflation countries tend to favor the PPP hypothesis. These results clearly qualify the earlier findings of Liu (1992), Mahdavi and Zhou (1994), and others that PPP is most likely to hold in the case of high inflation countries. The major policy implication that emerges from this study is that PPP can be used to determine the equilibrium exchange rate for these six African countries. Our findings are consistent with Holmes (2001) that we can use PPP to predict exchange rate that determine whether a currency is over or undervalued and experiencing difference between domestic and foreign inflation rates. Other countries experienced real shocks, such as droughts, reductions in the terms of trade, oil price shocks, civil wars and other forms of political instability during the past three decades. These problems could trigger destabilizing effects on the PPP relationship in African countries. Periods of political instability are clearly associated with rapid rates of inflation in Africa.

Finally, six of African countries RER rates are nonlinearly stationary with their relative price as well as aggregate price levels by the Caner and Hansen (2001) threshold unit root tests that reinforce the validations of
Table 3. Threshold unit root test.

| Country       | $R_{1T}$ | Bootstrap critical values (%) | Bootstrap $p$-value |
|---------------|----------|-------------------------------|---------------------|
| Algeria       | 5.182    | 9.292 11.087 22.477           | 0.323               |
| Botswana      | 6.861    | 8.409 10.805 16.667           | 0.170               |
| Burkina Faso  | 6.421    | 12.524 18.713 33.213           | 0.303               |
| Burundi       | 7.494    | 12.146 13.404 16.300           | 0.220               |
| Cameroon      | 3.451    | 10.537 13.270 32.688           | 0.530               |
| Côte d'Ivoire | 4.953    | 11.416 21.165 31.784           | 0.410               |
| Egypt         | 20.519   | 14.138 16.974 36.958           | 0.050               |
| Ethiopia      | 39.799   | 16.311 19.496 27.045           | 0.010               |
| Gambia        | 17.298   | 10.912 15.235 26.556           | 0.040               |
| Kenya         | 4.118    | 8.486 11.305 15.418           | 0.430               |
| Madagascar    | 7.292    | 11.175 17.484 33.718           | 0.240               |
| Malawi        | 15.542   | 9.512 11.345 16.217           | 0.020               |
| Mauritius     | 2.701    | 8.628 10.074 14.324           | 0.530               |
| Morocco       | 3.310    | 8.859 9.777 10.768           | 0.610               |
| Niger         | 3.044    | 13.467 16.450 34.352           | 0.560               |
| Nigeria       | 4.948    | 17.434 20.490 30.883           | 0.500               |
| Senegal       | 11.386   | 12.068 16.966 22.847           | 0.140               |
| Seychelles    | 21.742   | 13.494 16.096 24.467           | 0.020               |
| South Africa  | 14.202   | 7.295 8.622 10.142           | 0.010               |
| Swaziland     | 5.016    | 9.658 12.461 15.183           | 0.310               |
In this empirical study, we applied non-linear threshold unit-root test to assess the non-stationary properties of the real exchange rate for twenty African countries. The test has higher power than linear method if the true data generating process of exchange rate is in fact a stationary non-linear process. This study examined the validity of PPP from the non-linear point of view and the findings from provide robust empirical evidence supporting the validity of the long-run PPP, suggesting that Egypt, Ethiopia, Gambia, Malawi, Seychelles and South Africa that their real exchange rate adjustment is mean reversion towards PPP equilibrium values in a non-linear way. Because of the wide variability revealed by our results, the policy formulation process in each African country must reflect the prevailing economic, social and political environment in that country. The other major policy implication of our study is that the validity of using PPP to equilibrium exchange rate and reaping unbounded gains from arbitrage in traded goods is not possible in these six countries.

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