Abstract. Precast concrete lightened sandwich panels are widely used building elements. They are made by two concrete wythes separated by a layer of lightweight material: the central layer is inhomogeneous due to the presence of concrete ribs which tie the external wythe and act as thermal bridges.

Computation of thermal transmittance of sandwich panels is clearly described in European Standards, but in many cases it requires numerical simulations to determine the linear transmittance $\psi$ associated with lightweight material-concrete interfaces in the inhomogeneous layer. Although simple, these simulations represent a critical issue for many panel manufacturers and they would much rather prefer correlations to compute $\psi$.

In this work we present a correlation based on an artificial neural network (ANN) to estimate linear transmittance values for current Italian sandwich panel production. Five input parameters are considered: rib width, lightweight material conductivity, and thickness of the three panel layers. To obtain the data which are necessary to train and test the ANN, a fast and accurate Spectral Element Method is used to solve Laplace equation in the neighborhood of a rib. 5460 $\psi$ values are collected which ensure an accurate network response.

1. Introduction

Precast concrete sandwich wall panels are building elements that allow fast and economical constructions. Here we are mostly interested in panels used for one- or two-store buildings such as factories, warehouses, and malls: these panels are designed to span from the foundation to the roof and can be used as load bearing walls, or exterior claddings, or both.

Panels are produced in factories which can be far from the site of the building under construction: weight is a crucial point with regard to handling, transportation, and installation issues. The easiest way to meet both the requirements of structure robustness and weight containment, is to have a frame of concrete and fill the empty zones with lightweight materials like expanded polystyrene. In this paper this kind of panel will be referred to as LSP, precast concrete Lightened Sandwich wall Panel.

In a LSP two reinforced or prestressed concrete wythes are separated by a heterogeneous layer made by lightweight slabs and concrete ribs: so there are panel regions made of solid concrete which act as thermal bridges. Goal of this paper is to present an accurate correlation to evaluate the effect of such thermal bridges.
LSPs are not thermal efficient because the use of insulating slabs is aimed to reduce weight, reduction of the average thermal transmittance is just a by-product effect. There are other types of sandwich panels much more thermal efficient because of the presence of a continuous layer of homogeneous insulations between the concrete wythes: they are not considered here since the computation of their thermal transmittance is straightforward.

Buildings designers in order to fulfill the requirements of the European directives on the energy performance of buildings (Directives 2002/91/EC and 2010/31/EU) as implemented by the EU Member States, need reliable estimates of the thermal transmittance of building elements, including the effects of thermal bridges. In principle, computation of the thermal transmittance $U$, or $U$-value, of precast concrete sandwich panels is not a critical issue: European Standards [1–3] provide accurate methods to do that. Nonetheless, for many panel manufacturers it does represent a practical issue.

The $U$-value of LSPs has to be calculated according to Standard EN ISO 14683 which requires the evaluation of the linear transmittance associated with the conductivity discontinuity at the lightweight material-concrete interfaces in the central heterogeneous layer. Linear transmittance values can be obtained upon numerical calculations performed as described in Standard EN ISO 10211. The effort required by the computation of linear transmittances for a LSP is rather small, since simulations based on 2D model provide results of adequate accuracy. However, most panel manufacturers are SMEs (Small-Medium Enterprises) and their technical staff does not have either the know-how, nor the time to numerically compute the linear transmittances necessary to determine the $U$-value of their product range: a linear transmittance catalogue or correlations easily implemented in a spreadsheet or in an in-house code would be of relevant practical utility for them.

Other recent studies [4, 5] pursue a goal similar to ours that is to find correlations to evaluate thermal properties of building components which satisfy the following requirements: they must reproduce with good approximation the results of standardized methods; in addition, they must be suited for routine use in industry. In [4], Tenpierik et al. propose a closed analytical model for calculating linear transmittances in vacuum insulation panels (VIP) much more accurate than models based on electric circuits, but less laborious than numerical simulations. In [5], Buratti et al. develop an artificial neural network (ANN) for estimating $U$-value of wooden framed windows. Based on 278 experimental and numerical data obtained in accordance with European Standards, the ANN returns an estimate of window transmittance as a function of 10 simple parameters like window typology, wood kind, frame thickness, etc.

In previous works we have presented a database of about one thousand linear transmittance values of thermal bridges in LSPs, obtained using ANSYS Fluent, and proposed a set of simple correlations to evaluate linear thermal transmittance in terms of five independent variables [6, 7]. Although we were rather satisfied with the practical results obtained, we did see two main drawbacks of our work. First, the database build-up had been time-consuming. The bottleneck being the preprocessing stage that was not automated and required both creation of a new grid and problem set-up for any new combination of geometric parameters. Second, the data analysis to obtain correlations for the linear transmittance was based on a heuristic rather than a systematic approach. We were not sure that we could have extended successfully our correlations to include dependence of the linear transmittance on a sixth parameter (concrete conductivity, for example).

To overcome these limits a new approach has been taken. On one hand, we use a fast, high accurate conformal quadrilateral Spectral Element Method (SEM) to compute the linear transmittance [8, 9]. The method is implemented in an in-house MATLAB code with automated configuration set-up: thousands of different cases are solved within a single run. The solver is used to evaluate linear transmittance varying systematically five parameters within the range of values typical of current panel production.
On the other hand, the large dataset obtained numerically is used to build an artificial neural network for accurate prediction of linear thermal transmittance values of LSPs. This last tool can be supplied to LSPs manufacturers for fast computation of panel $U$-value without any need of further numerical simulations.

In literature there are only a few other studies on thermal bridges in precast concrete panels and none of them is concerned with accurate evaluation of linear transmittance. Pessiki and Lee have investigated the thermal performance of a modified type of LSP: precast concrete three-wythe sandwich wall panel, that is three concrete wythes separated by two heterogeneous layers with concrete ribs staggered in location to increase the overall thermal resistance [10, 11]. The goal of Pessiki and Lee is to compare the $U$-value of a small set of two- and three-wythe panel configurations: to do that there is no point in evaluating linear transmittances associated with the staggered ribs, therefore $U$ is directly calculated upon computation of the heat flux through the entire panel, using a FEM commercial code. Studies [12, 13] consider point transmittance of metal connectors used to tie the concrete wythes in insulated sandwich wall panels and propose either a method based on electric circuit [12] or a correlation [13] to evaluate it.

2. Evaluation of the $U$-value of a lightened sandwich panel

The heat flow rate $q$ through a wall panel is usually written as

$$q = H \Delta T = AU \Delta T$$  \hspace{1cm} (1)

where $H$ denotes the heat transfer coefficient through the wall panel, $\Delta T$ is the temperature difference between the internal and the external environments separated by the panel, $A$ is the panel area, and $U$ is the panel average thermal transmittance. In Eq. (1) the four panel edges are considered adiabatic.

$H$ can be calculated upon numerical solution of the conduction equation for the entire panel or - much more efficiently - by means of the following expression,

$$H = \sum_i A_i U_i + \sum_j l_j \psi_j + \sum_k n_k \chi_k$$  \hspace{1cm} (2)

In Eq. (2) $A_i$ and $U_i$ are area and thermal transmittance of the $i$-th section of the panel; $l_j$ and $\psi_j$ are length and linear transmittance of the $j$-th linear thermal bridge; $n_k$ and $\chi_k$ are number and point transmittance of the $k$-th point thermal bridge. Following Standard EN ISO 6946 [1, Clause 6.2.2], here section denotes a panel part made of thermally homogeneous layers. As clearly shown in figure 1, a LSP is made of two sections: the solid concrete part, corresponding to ribs (section $a$); and the three-layer lightened part, made of the concrete wythes and the lightened layer that separates them (section $b$).

The thermal transmittance of the two sections, $U_a$ and $U_b$, is easily calculated in terms of surface resistances, $R_{se}$ and $R_{st}$, and of thermal resistances of the homogeneous layers. Therefore, the first sum in Eq. (2), $\sum_i A_i U_i$ represents the heat coefficient through the panel as if the sections were thermally insulated one from the other, and the temperature field were 1D within each section.

The other two sums, $\sum_j l_j \psi_j$ and $\sum_k n_k \chi_k$, represent the corrections associated to linear and point thermal bridges, that is to the regions where the temperature field is 2D and 3D.

In LSPs, the temperature field is 2D close to the interfaces between concrete ribs and lightweight slabs (see thick lines in figure 1), whereas it is 3D in a neighborhood of the intersections of ribs (see open circles in figure 1). Each point thermal bridge can be considered as the intersection of two or more linear thermal bridges: according to Standard EN ISO 14683 [2, Clause 4.2] the influence of such a kind of point thermal bridge can be neglected and the third
term in Eq. (2) can be dropped. Therefore, for all practical purposes, an accurate evaluation of the $U$-value of an LSP is given by

$$U = \frac{A_a U_a + A_b U_b + \sum_j \psi_j l_j}{A} \quad (3)$$

where $A = A_a + A_b$. The linear thermal transmittances $\psi_j$ must be evaluated upon solving the conduction equation on proper 2D domains, in accordance with Standard EN ISO 10211.

3. Linear transmittance calculation: problem description

In figure 2 it is sketched the 2D model used to determine linear thermal transmittance values: it represents a panel section around the interface between a concrete rib and a lightweight slab, where the temperature field is 2D. The rectangle $A'B'B''A''$ belongs to the solid concrete section $a$ of the panel, whereas the other part $B'C'C''B''$ belongs to the three-layer lightened section $b$.

The geometrical model coincides with a rectangular domain $\Omega$ of width $L = L_a + L_b$ and height (panel thickness) $d = d_1 + d_2 + d_3$. $d_1$ and $d_3$ denote the thickness of the external and internal wythe, respectively, whereas $d_2$ is the lightweight material thickness.

According to Italian Standard UNI 10351 that supplies thermal conductivity values of building materials, the effective thermal conductivity of the outer layer of an external wall must be increased to take into account the stronger effect of aging and moisture content [14, Note 3, p. 16]. For this reason, the external wythe concrete is considered a material different from the internal wythe and rib concrete. Their thermal conductivities will be denoted $\lambda_{ce}$ and $\lambda_{ci}$, respectively. Thermal conductivity of lightweight slabs is $\lambda_{lw}$.

On the domain boundaries the following conditions must be satisfied. On the external and internal surfaces $A'C'$ and $A''C''$ Robin (mixed) condition is imposed. The heat transfer from surface $A'C'$ to the external environment at temperature $T_e$ is characterized by a heat transfer coefficient $h_e$ and a surface resistance $R_{se} = 1/h_e$. Internal environment temperature $T_i$, heat transfer coefficient $h_i$, and surface resistance $R_{si} = 1/h_i$ characterize heat transfer to the internal surface $A''C''$.

$A'A''$ and $C'C''$ are cut-off planes as defined in Standard EN ISO 10211 [3, Clause 5.2], which coincide with adiabatic surfaces. A cut-off plane is either a symmetry plane or a plane placed in a region so far from any thermal bridge that the temperature field can be considered 1D.

Boundary $A'A''$ represents either one of the adiabatic lateral surfaces of the panel (as in domain 1 in figure 1) or the symmetry plane of an internal rib (as in domain 2 in figure 1): therefore $L_a$ is either the width of a bounding rib or the half-width of an internal rib.
Boundary C′/C′′ either belongs to the symmetry plane of a lightweight slab (as in domain 1 in figure 1), or it falls far enough from the closest thermal bridge (as in domain 2 in figure 1). With reference to figure 2 the interface D′/D′′ between the concrete and the lightweight material represents the thermal bridge: according to Standard EN ISO 10211 the distance \(L_b\) between the cut-off plane C′/C′′ and the thermal bridge must be equal to \(\max(1m, 3d)\), where \(d\) is the total thickness of the panel.

Denoting \(q_l\) the heat flux per unit length through the 2D domain \(\Omega\), from Eqs. (1), (2), and (3) it holds,

\[ q_l = (L_a U_a + L_b U_b + \psi) \Delta T, \]

therefore, upon numerical approximation of \(q_l\) (see Sec. 4), the linear thermal transmittance associated with the conductivity discontinuity at the interface D′/D′′ in figure 2, can be computed by the following equation

\[ \psi = \frac{q_l}{\Delta T} - (L_a U_a + L_b U_b) \]

4. Approximation of the heat flux by Spectral Element Methods
The conduction problem described in Sec. 3 is recasted in the following mathematical setting in order to solve it through Spectral Element Methods.

With reference to figure 2, \(\Omega\) is the union of three non-overlapping regions \(\Omega_{ce}\), \(\Omega_{ci}\), and \(\Omega_{lw}\), corresponding to the different materials forming the panel. We introduce a cartesian system of coordinates with origin in the left bottom vertex of \(\Omega\): \(\Omega = [0, L] \times [0, d]\).

In order to approximate the heat flux \(q_l\) we compute the temperature \(T = T(x)\) inside the panel. It solves a homogeneous elliptic equation with discontinuous thermal conductivity \(\lambda\) such that \(\lambda|_{\Omega_{ce}} = \lambda_{ce}\), \(\lambda|_{\Omega_{ci}} = \lambda_{ci}\), and \(\lambda|_{\Omega_{lw}} = \lambda_{lw}\), i.e., \(T\) is the solution of

\[
\begin{align*}
-\nabla \cdot (\lambda \nabla T) &= 0 & \text{in } \Omega, \\
\lambda \frac{\partial T}{\partial n} + h_e T &= h_e T_e & \text{on } \partial \Omega_e, \\
\lambda \frac{\partial T}{\partial n} + h_i T &= h_i T_i & \text{on } \partial \Omega_i, \\
\lambda \frac{\partial T}{\partial n} &= 0 & \text{on } \partial \Omega_0.
\end{align*}
\]
For simplicity the bottom side of $\Omega$ is denoted by $\partial\Omega_e$ (external surface), the top side by $\partial\Omega_i$ (internal surface), whereas $\partial\Omega_i$ denotes the union of the adiabatic vertical sides.

Problem (6) can be reformulated in a weak sense, and it can be proved that it admits a unique weak solution $T \in H^1(\Omega)$, being $H^1(\Omega) = \{ v \in L^2(\Omega) : \nabla v \in [L^2(\Omega)]^2 \}$ the Sobolev space of order one (see, e.g. [15]). Notice that the weak form of (6) is well posed even if $\lambda$ is a discontinuous bounded function.

To approximate the temperature field inside the panel we consider conformal quadrilateral Spectral Element Methods (SEM) (see, e.g. [8, 9]). SEM, sometimes known as $hp$—Finite Element Methods ($hp$—FEM), are high accurate methods (more largely accurate than classical FEM or Finite Volume Methods) designed to discretize partial differential equations. They do their best when the differential problem is set on cartesian geometries, exactly as in the problem we are dealing with, and when the data are regular.

To this aim the computational domain $\Omega$ is partitioned in $N_e$ non-overlapping rectangles $Q_k$ (also named elements) of size $h$ (typically $h$ denotes the diagonal), such that two adjacent elements share a common vertex or a complete edge, and we denote by $Q_h = \bigcup_{k=1}^{N_e} Q_k$ such a partition. We accept that the elements $Q_k$ can have different size $h_k$, in such a case we set $h = \max_k h_k$. Finally, let us denote by $Q_p$ the space of polynomials defined on $\mathbb{R}^2$ that are of degree less than or equal to $p \geq 1$ with respect to each variable. Given a partition $Q_h$ of $\Omega$ and fixed the polynomial degree $p$, we look for an approximation $T_h$ of $T$ that is globally continuous on $\overline{\Omega}$ and locally (that is in each element $Q_k$) a polynomial of degree $p$. By introducing the finite dimensional space

$$ X_h = \{ curb \in C^0(\overline{\Omega}) : v|_{Q_k} \in Q_p, \forall Q_k \in Q_h \} \subset H^1(\Omega), $$

the weak discrete counterpart of (6) reads:

$$ \text{find } T_h \in X_h : \int_{\Omega} \lambda \nabla T_h \cdot \nabla v_h \, d\Omega + \int_{\partial\Omega_e} h_e T_h v_h \, d\partial\Omega + \int_{\partial\Omega_i} h_i T_h v_h \, d\partial\Omega = 0, \quad \forall v_h \in X_h. \quad (7) $$

It can be proved that the discrete solution $T_h$ converges in a suitable norm to the weak solution $T$ of (6) when the mesh size $h$ tends to zero and/or the polynomial degree $p$ grows to infinity.

As for classical Lagrangian FEM, the degrees of freedom of the discrete problem are the values of the function $T_h$ at suitable points $\{ x_j \}_{j=1}^{N_h}$ in $\Omega$, and, when SEM are adopted, these points are the Legendre-Gauss-Lobatto (LGL) nodes, that are defined on $[-1, 1]^2$ and mapped on each element $Q_k$ of the partition ([16]). If $p$ is the polynomial degree chosen in each element $Q_k$, there are $(p+1)^2$ LGL points in $Q_k$ matching, on the edges of $Q_k$, with those of the adjacent elements.

The choice of LGL nodes guarantees the high accuracy of the method, but also a low computational effort since LGL nodes are used to both represent and integrate the approximating function, thus avoiding time-consuming interpolation processes typical of FEM.

We remark that solving problem (7) guarantees that the discrete solution is continuous on the whole domain $\Omega$ and no additional constraints have to be imposed where the thermal conductivity jumps, since we are dealing with a unique global well-posed problem.

Once the discrete temperature $T_h$ is available, the outgoing heat flux per unit length from horizontal edges $\partial\Omega_e$ and $\partial\Omega_i$ can be computed by the following formulas:

$$ \phi_{i,h} = \int_{\partial\Omega_i} \lambda \frac{\partial T_h(x)}{\partial n} \, d\partial\Omega = \lambda c_i \int_0^L \frac{\partial T_h(x)}{\partial x_2} \bigg|_{x_2=d} \, dx_1, \quad (8) $$

$$ \phi_{e,h} = \int_{\partial\Omega_e} \lambda \frac{\partial T_h(x)}{\partial n} \, d\partial\Omega = -\lambda c_e \int_0^L \frac{\partial T_h(x)}{\partial x_2} \bigg|_{x_2=0} \, dx_1. \quad (9) $$
The computation of derivatives in (8) and (9) is simple and affordable, and the integrals are evaluated by Legendre-Gauss-Lobatto quadrature formulas [16].

The moduli of $\phi_{i,h}$ and $\phi_{e,h}$ are the discrete approximation of the heat flux per unit length through $\Omega$, $q_i$, that is required to calculate the linear transmittance $\psi$ by Eq. (5). In particular, for $T_i > T_e$ we obtain $-\phi_{i,h} \rightarrow q_i$ and $+\phi_{e,h} \rightarrow q_i$ when $h \rightarrow 0$ and/or $p \rightarrow \infty$, i.e. when the discretization errors tend to zero.

For a more in-depth description of SEM or $hp$–FEM methods and in their use to approximate partial differential equations, we refer, e.g., to [9, 15, 17].

5. Numerical results

The SEM described in Sec. 4 has been implemented in MATLAB together with an automated procedure to generate partition $Q_h$ depending on the geometrical parameters ($L_a, L_b, d_1, d_2, d_3$).

In order to get a good trade-off between accuracy and computational time one has to choose properly the partition $Q_h$ and the polynomial degree $p$ valid for all elements $Q_k$.

Accuracy of SEM depends on both size $h$ of elements $Q_k$ and polynomial degree $p$. Error tends to zero as $h^p p^{-s}$, being $s \geq 1$ the regularity order of the solution. Error decreases rather quickly for increasing $p$, but only algebraically w.r.t. $h$, as $h$ tends to zero. During the initial simulation trials, we have found that for $p \geq 12$ computational time required to solve a single case was too large, so we decided to set $p = 8$, a value that guaranteed the requested accuracy on the partition chosen with computational time ten times shorter: as a matter of fact in most applications of SEM $p$ is equal to 6 [16].

With regard to partition $Q_h$, elements $Q_k$ of non-uniform size have been taken in order to improve resolution where temperature variations are larger. The element width (along direction $x_1$) increases moving from the concrete-lightweight material interface at $x_1 = L_a$, towards the adiabatic walls. This is justified by the fact that far from the interface $x_1 = L_a$ the temperature field tends to become independent of $x_1$. As an example of partition along $x_1$ consider the case $L_a = L_b = 1$ m. The element width sequence on both sides of the interface is: $[0.5, 0.5, 1, 1.5, 1.5, 2, 2, 5, 1, 20, 20, 20, 20]$ cm.

The element thickness (along direction $x_2$) is set uniform and equal to 2 cm, except for the rows of elements close to the material interfaces at $x_2 = d_1$ and $x_2 = d_1 + d_2$. On both sides of lines $x_2 = d_1$ and $x_2 = d_1 + d_2$ we place two rows of 0.5 cm thick elements, followed by one row of 1 cm thick elements: in this way interfaces between different materials always coincide with the common boundary of adjacent elements. In principle, the discrete temperature field $T_h$ converges to $T$ even if the conductivity discontinuities fall inside elements of the partition, in practice, however, for large $h$ and small $p$, one obtains more accurate results when discontinuities fall on element boundaries.

In our simulation the total number of elements $Q_k$ ranges from $18 \times 14$ for the smallest computational domain ($L = 1.025$ m, $d = 0.12$ m), to $28 \times 20$ for the largest one ($L = 2$ m, $d = 0.24$ m).

To assess accuracy requirements we referred to criteria A.2.d and A.2.e of Standard EN ISO 10211 and to comparison with data in the database presented in [6, 7]. In all our computations the relative difference between the two fluxes $|\phi_{i,h}|$ and $\phi_{e,h}$ is less than $10^{-4}$ as required by Clause A.2.e of [3] – actually it is always less then $10^{-9}$.

With regard to the number of nodes to be used in the computational model, Clause A.2.d of [3] requires that upon doubling the number of nodes, the computed heat flux through the building element must not vary more than 1%. The total number of LGL nodes in $\Omega$ increases of a factor slightly larger than 2 when the polynomial degree changes from $p = 8$ to $p = 12$. In a few test cases we have checked how $q_i$ and $\psi$ vary and we have verified that the requirement was satisfied: the relative change of $q_i$ is of order $10^{-4}$ or smaller, whereas the relative change of $\psi$ is of order $10^{-3}$ or smaller. For these test cases the value of $\psi$ is available in the database.
presented in [6, 7]. The relative difference with respect to the value obtained with SEM is close to \(8 \times 10^{-3}\).

The linear transmittance \(\psi\) as defined by Eq. (5) in Sec. 3 depends on several parameters: geometrical, \(L_a, L_b, d_1, d_2, d_3\); and thermophysical, \(\lambda_{ce}, \lambda_{ci}, \lambda_{lw}, R_{se}, R_{si}\).

In this study \(L_b, \lambda_{ce}, \lambda_{ci}, R_{se}, \) and \(R_{si}\) are fixed, and \(\psi\) is computed for varying values of the other five variables:

\[
\psi = f(L_a, d_1, d_2, d_3, \lambda_{lw})
\]  

(10)

The surface resistances are set equal to the conventional values prescribed by Standard EN ISO 6946 [1, Clause 5.2]: \(R_{se} = 0.04\ m^2K/W\) and \(R_{sc} = 0.13\ m^2K/W\). Concrete conductivity is set equal to the values used by most Italian panel manufacturers to estimate thermal performances of their products [6, 7]. They correspond to a concrete density of \(2400\ kg/m^3\): \(\lambda_{ci} = 1.909\ W/mK\) for external walls, and \(\lambda_{ce} = 2.075\ W/mK\) for the outer layer of external walls [14, p. 5].

In figure 3 the dependence of \(\psi\) on the section width \(L_b\) is shown – for \(L_a = 1\ m\), data are taken from [6, 7]. When \(L_b = 0\), the domain \(\Omega\) is made only of homogeneous layers, temperature field is 1D and \(\psi\) vanishes. As \(L_b\) increases \(\psi\) tends asymptotically to a constant value: the growth is fast, for \(L_b\) larger than 25 cm the value of \(\psi\) is hardly distinguishable from the asymptotic value. Since in real panels the lightweight slab width is rarely smaller than 50 cm, the asymptotic value only is needed for all practical purposes: following EN ISO 10211 in all computations we set \(L_b = 1\ m\).

The dependence of \(\psi\) on solid concrete section width \(L_a\) is similar to the one on \(L_b\) (see figure 3): \(\psi\) attains its asymptotic value for \(L_a\) larger than 20 cm, approximately. Since in current LSP production concrete rib width ranges between 10 and 25 cm, that is in the range of significant variation of \(\psi\), numerical computations have been performed for \(L_a = 2.5, 5, 7.5, 10, 12.5, 15, 20, 25, 30, 40, 60, 80, 100\) cm: this choice includes the most frequent values of rib width or half-width.

Thermal conductivity of expanded polystyrene used in most LSPs ranges between 0.032 and 0.056 W/mK, but smaller or larger values are not uncommon. \(\psi\) has been calculated for \(\lambda_{lw} = 0.02, 0.03, 0.04, 0.05, 0.06\ W/mK\). \(\psi\) decreases monotonically with \(\lambda_{lw}\).

With regard to the thickness of the two wythes, \(d_1\) and \(d_3\), and to that of the lightened layer, \(d_2\), each one of them has been varied from 4 up to 16 cm by 2 cm steps. However, the following constrain on panel total thickness has been imposed: \(d \leq 24\) cm. Only 84 combinations of values of \(d_1\), \(d_2\) and \(d_3\) satisfy condition above. They span a layer thickness parameter space slightly larger than current production of LSPs.

5460 accurate numerical estimates of \(\psi\) have been obtained upon varying \(L_a, \lambda_{lw}, d_1, d_2, d_3\) as specified above. This large dataset has been used, in turn, to build an artificial neural network (ANN) for computation of \(\psi\).

6. Artificial neural network for \(\psi\) computation

In recent years several studies have appeared in the heat transfer literature describing the use of ANNs to correlate experimental and/or numerical data [5, 18-20]. The work by Buratti et al. on \(U\)-value of wooden windows has already been quoted in Sec. 1 [5].

In [18, 19] ANNs are successfully tested to predict Nusselt number and pressure drop for either compact or fin-and-tube heat exchangers. For ANN training and testing 170 (experimental) and 277 (experimental and numerical) data were used in [18] and [19], respectively.

In [20] Karadag and Akgöbek applied an ANN to a floor-heated room to correlate Nusselt and Rayleigh numbers to wall, ceiling and floor temperatures and room size. 235 data obtained through CFD simulations were used for training and testing.

Fitting of function (10) has been obtained through applications provided by the Neural Network Toolbox in MATLAB (NNT). In particular, we have chosen a standard architecture consisting in a two-layer feed-forward network with tan-sigmoid transfer function in the hidden
layer and linear transfer function in the output layer. As noted in [21, p. 5-11] this kind of network allows approximation of any function for a sufficient number of neurons in the hidden layer: after a few trials, this number was set equal to 10.

The default Levenberg-Marquardt backpropagation algorithm has been adopted for training. It divides the 5460 data in three sets, one for training, one for validation and one for testing. It is possible to decide the percentage of data to store for the three steps: here we have used the default values, 60% for training, 20% for validation, and 20% for testing. The training data are used by the network to estimate the weights and biases associated with each neuron layer, the validation data are used to measure the network capability of generalization to new data and avoid overfitting, and the testing data provides a measure of the performance of the network.

To assess the accuracy of the network, a regression analysis between the network response and the original data can be performed. The resulting correlation coefficient, $R$-value, is $R = 0.99930$ for the test data set (1092 data). More than 97% of predicted data fall within 0.5% from computed results: the maximum relative difference is smaller than 1.3%. These results show that the fit is very good.

Finally, values obtained with the ANN have been compared with data presented in [6, 7]. In the database built up using ANSYS Fluent simulations, 236 records have values of $(L_a, d_1, d_2, d_3, \lambda_{lw})$ falling in the range of validity of the ANN. For these configurations the ANN response, $\psi_{ANN}$, has been evaluated and compared with $\psi_{AF}$ from the database. Results are shown in figure 4: the relative difference is within 1% for 91% of data, as expected from comparison between results obtained with SEM and ANSYS Fluent (see Sec. 5).

7. Conclusions

This paper deals with the problem of finding an accurate correlation for computing the linear thermal transmittance associated with the interface between a concrete rib and a lightweight slab in precast concrete lightened sandwich panels. If such a correlation is available, evaluation of the $U$-value of the panel is straightforward and easily implemented in a spreadsheet.

To reach this goal a two-step approach has been followed. A conformal quadrilateral Spectral Element Method was developed in MATLAB to obtain accurate and fast numerical computation
of the linear transmittance. The numerical procedure is in accordance with Standard EN ISO 10211. All the preprocessing operations (domain partition generation, node mapping on the partition elements, problem set-up) are automated, thus in a single run thousands of $\psi$ values are collected.

A data set of 5460 configurations has been obtained as a function of five parameters: rib width, lightweight material conductivity, thickness of the two concrete wythes and of the internal lightened layer. The parameters span a range of values slightly larger than those of current Italian panel production.

The data set has been used for training and testing an ANN developed within the Neural Network Toolbox environment. The ANN is a two-layer feed-forward network with tan-sigmoid and linear transfer functions which is a standard choice for function fitting: 10 neurons are placed in the hidden layer. The network response is rather good. A comparison with an existing database of $\psi$ values built by numerical simulations with a finite volume code shows relative differences smaller than 2%.

The ANN can be distributed to panel manufacturers as a standalone executable, although the issue of its dissemination has not been fully addressed yet.

Finally, the results presented here can be quickly extended to include other parameters such as concrete conductivity, or to change the variability range of the input parameters to fit other countries production. Beside, the procedure that has been discussed could be successfully applied to other similar problems like the determination of correlations for point transmittances associated with metal fasteners used to connect the external wythes in insulated sandwich panels.

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