Effects of viscosity variations in temporal mixing layer

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Abstract. The objective of the present investigation is to assess the effects of viscosity variations in low-speed temporally-evolving turbulent mixing layer. Direct Numerical Simulations (DNS) are performed for several viscosity ratios, \( \nu = \frac{\nu_{\text{high}}}{\nu_{\text{low}}} \), varying between 1 and 9, whereas the upper and lower streams are of equal density. The space-time evolution of Variable-Viscosity Flow (VVF) is compared with the Constant-Viscosity Flow (CVF), for which \( \nu = 1 \). The initial Reynolds number, based on the initial momentum thickness, \( \delta_{0,0} \), is \( Re_{\delta_{0,0}} = 160 \) for the considered cases. The study focuses on the first stages of the temporal evolution of the mixing-layer. It is shown that in VVF (with respect to CVF): (i) the birth of turbulent fluctuations is accelerated; (ii) large-scale quantities, i.e. mean longitudinal velocity and momentum thickness, are affected by the viscosity variations, thus dispelling the myth that viscosity is a ‘small-scale quantity that does not affect the large scales’; (iii) the velocity fluctuations are enhanced for VVF. In particular, the turbulent kinetic energy peaks earlier and is three times larger for VVF than CVF at the earliest stage of the mixing, and (iv) the transport equation for the turbulent kinetic energy is derived and favourably compared with simulations data.

1. Introduction

Most of the available turbulence theories (Kolmogorov, 1941) are underpinned by assumptions involving a single value of the kinematic viscosity. Moreover, the effect of viscosity is a priori expected to be felt only at small scales.

However, turbulence mostly occurs in heterogeneous fluids or multicomponent mixtures, therefore involving fluids with various viscosities. In such flows, strong viscosity gradients may exist at all scales, thereby modifying the mean velocity profiles. To examine the effects of viscosity variations on the mixing efficiency, Talbot et al. [1] have conducted an experimental study of a propane jet issuing into air; the latter is 3.5 more viscous than the air, whereas the densities are almost the same. These authors found that, in comparison with a classical air/air jet, the propane jet exhibits an accelerated decrease of the mean axial velocity. This behaviour was explained analytically by an increase of the magnitude of the viscous terms with respect to the inertial terms. Strong production of turbulent fluctuations was also observed in the very near-field of the propane jet, thus enhancing mixing, producing lateral fluctuations and accelerating the trend towards self-similarity and isotropy.

The objective of the present investigation is to assess the effects of viscosity variations in a canonical flow, the low-speed temporally-evolving turbulent mixing layer. For the particular purpose of our study, the temporal mixing layer (hereafter TML) involves two fluids with the same density. TML belongs to the family of turbulent shear layers, which have been studied since the pioneering work of Brown and Roshko [2], followed by a web of investigations of this configuration devoted to different issues, such as transition to turbulence, mixing efficiency, compressibility effects [3, 4, 5, 6], variable density effects [1, 7, 8]. The effect of the trailing edge thickness of the splitter plate was also investigated in detail [9, 10], as well as e.g. sound generation [11].
The paper is organized as follows. Section 2 is dedicated to the description of the numerical methods and the associated solver. Section 3 examines the one-point statistics (instantaneous fields, mean values of the longitudinal velocity, as well as the turbulent kinetic energy budget).

2. Mathematical formulation and numerical method

2.1. Governing equations and numerical schemes

The unsteady three-dimensional, incompressible Navier-Stokes equations, with two-species transport equations are solved using an inhouse high-order explicit solver (see [12] for more details about the numerical methodology). The convective terms are approximated by the fifth-order weighted, essentially non-oscillatory (WENO) schemes, while the diffusive terms are determined using fourth-order compact central difference. The discrete equations are time-integrated by means of an explicit, third order, Runge-Kutta scheme. The Wilke formula [13], which is widely used in combustion and chemical engineering applications, is employed to determine the mixture-viscosity coefficient in terms of mass fraction, namely

$$
\mu = \sum_{k=1}^{2} \mu_k \left( \frac{r_k}{w_k} \right) \left( \sum_{l=1}^{2} Y_l \phi_{kl} / W_l \right)^{-1}
$$

where \( \mu_k \) and \( W_k \) are the viscosity and the molecular mass of the k-th species and the function \( \phi_{kl} \) is defined as

$$
\phi_{kl} = \frac{1}{1 + \left( \frac{w_k}{w_l} \right)^{1/4}}.
$$

Periodic boundary conditions are applied along the streamwise x and the spanwise z directions and a non-reflecting slip condition is imposed along the cross-stream y direction.

2.2. Initial conditions

A sketch of the simulated TML is shown in figure 1. A velocity-viscosity counter gradient is imposed; the upper stream is initialized with a low velocity, \( U_{\text{low}} \), and a high kinematic viscosity, \( \nu_{\text{high}} \), while the lower stream is initialized with a high velocity, \( U_{\text{high}} \), and a low kinematic viscosity, \( \nu_{\text{low}} \). The upper stream \( (U_{\text{low}}, \nu_{\text{high}}) \) and the lower stream \( (U_{\text{high}}, \nu_{\text{low}}) \) meet together and evolve in time. The reference velocity is \( \Delta U = U_{\text{high}} - U_{\text{low}} \) and the reference viscosity is \( \nu_{\text{ref}} = 0.5(\nu_{\text{high}} + \nu_{\text{low}}) \).

Quantities such as \( \langle U \rangle \) are averaged over homogeneous directions (x, z) for a given y, at each time t. We define the normalized time, \( \tau \), such as \( \tau = \frac{t \Delta U}{\delta_{\theta,0}} \). Therefore, averages are functions of two variables: vertical position y and time t. As an example, the longitudinal mean velocity is a function of both y and \( \tau \), viz. \( \langle U \rangle = f(y, \tau) \). The mean longitudinal velocity is initialized by a tangent hyperbolic profile \( \langle U \rangle(y, t = 0) \), while the other mean velocity components are initially set to zero. Thus,

$$
\langle U \rangle(y, t = 0) = \frac{1}{2} \left[ (U_{\text{high}} + U_{\text{low}}) - \Delta U \tanh \left( \frac{Y}{2} \right) \right].
$$

The two streams have the same initial pressure, \( P = 1 \text{ atm} \), temperature, \( T = 293 \text{ K} \), and density, \( \rho = 1.314 \text{ kgm}^{-3} \) while the velocities and the kinematic viscosities are specified as summarized in Table 1 for all investigated mixing layers cases.

To initiate turbulence, initial fluctuations are superimposed to the mean velocity field. The initial

![Figure 1. Computational domain. The middle of the domain is defined as \( (x = 0, y = 0, z = 0) \).](image-url)
prescribed values for both Reynolds stress tensor and length scales, and generates the corresponding 
fluctuating velocity field at the initial time. The initial length scales along each direction are equal to 
\( \delta_{0,0} = 4 \delta_{0,0} \), the initial vorticity thickness. The initial turbulence intensity 
is 
\( \left( \frac{\langle u' \rangle^2}{\langle u' \rangle^2} \right) = 0.20, \quad \left( \frac{\langle v' \rangle^2}{\langle u' \rangle^2} \right) = 0.013, \quad \left( \frac{\langle w' \rangle^2}{\langle u' \rangle^2} \right) = 0.012 \) for the considered DNS.

2.3. Simulation parameters
Five test cases are carried out as shown in table 1. The parameters of DNS-R2.5 to DNS-R9 are 
chosen to analyse the effects of viscosity variations on the VVF, whilst DNS-R1 provides a 
comparison basis in the constant viscosity flow (CVF). The viscosity ratio is 
\( R_v = \frac{\nu_{\text{high}}}{\nu_{\text{low}}} \geq 1 \). The 
species mixture is characterized by values for \( R_v \) of 1, 2.5, 4, 6 and 9 for DNS-R1, DNS-R2.5, DNS-
R4, DNS-R6 and DNS-R9 respectively. The molecular diffusion coefficient \( D \) between the two 
streams is kept constant for the considered cases to focus only on viscosity variations effects. The initial 
momentum thickness Reynolds number is 
\( \text{Re}_{\delta_{0,0}} = \Delta U \delta_{0,0}/v_{\text{ref}} = 160 \), the initial momentum 
thickness is \( \delta_{0,0} = 18.4 \mu \text{m} \). The initial conditions are set to have the same initial Reynolds Number 
\( \text{Re}_{\delta_{0,0}} = 160 \) rather than the same initial free stream velocities. These velocities are determined from 
the initial Reynolds number as 
\( \text{U} = \frac{\text{Re}_{\delta_{0,0}}}{\delta_{0,0}} \). Moreover, we impose that \( \text{U}_{\text{high}}/\text{U}_{\text{low}} = 11 \) for the 
test cases. For our purpose, the velocity ratio must be the same for CVF and VVF so as to assess the 
uniquely effect of viscosity variations. Indeed, several studies have showed that the mixing layer 
growth is influenced by velocity ratio.

Table 1. Parameters of DNS configuration.

| Case     | \( R_v \) | \( \nu_{\text{high}} \) (m²s⁻¹) | \( \nu_{\text{low}} \) (m²s⁻¹) | \( \nu_{\text{ref}} \) (m²s⁻¹) | \( \text{U}_{\text{high}} \) (ms⁻¹) | \( \text{U}_{\text{low}} \) (ms⁻¹) | \( \Delta \text{U} \) (ms⁻¹) | \( \text{U}_{\text{high}}/\text{U}_{\text{low}} \) |
|----------|---------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| DNS-R1   | 1       | 6.85                | 6.85                | 6.85                | 5.95                | 65.53               | 59.58               | 11                  |
| DNS-R2.5 | 2.5     | 6.85                | 2.74                | 4.78                | 4.17                | 45.87               | 41.70               | 11                  |
| DNS-R4   | 4       | 6.85                | 1.71                | 4.28                | 3.72                | 40.96               | 37.24               | 11                  |
| DNS-R6   | 6       | 6.85                | 1.14                | 3.99                | 3.47                | 38.23               | 34.76               | 11                  |
| DNS-R9   | 9       | 6.85                | 0.76                | 3.80                | 3.31                | 36.41               | 33.10               | 11                  |

The challenge of this study is twofold. First, the grid resolution must be sufficiently high to correctly 
capture the dynamics of the TML from its birth to the later stages of the flow evolution.
Second, the computational domain must be wide enough to ensure the spatial development of the 
large-scale turbulence. The computational domain sizes, normalized by the initial momentum 
thickness, \( \delta_{0,0} \), are 345, 172, 86 in \( x, y \) and \( z \) directions, respectively. The latter matches the DNS of 
Pantano & Sarkar [14]. The spatial resolution along the direction \( y \) is taken as 1.05 \( \mu \text{m} \), to ensure good 
discretization of the initial momentum thickness (≈18 points).

3. One-point statistics
In this section, we present results for one-point statistics. The morphological behavior of VVF and 
CVF are discussed in Subsection 3.1. The mean velocity field is analyzed in Subsection 3.2, and the 
turbulent kinetic budget is discussed in Subsection 3.3.

3.1. Morphology of the flow
We provide qualitative observations of the temporal evolution of the mixing layer in both CVF and 
VVF cases. In particular, the focus is on the way the velocity and the vorticity fluctuations 
are generated. Figure 2 shows the instantaneous turbulent kinetic energy 
\( q^2 = 0.5 \langle u_i u_i \rangle / \Delta U^2 \) (summation applies for double indices) in the plane (\( x, z \)), in the middle of the numerical domain (\( y=0 \)). The 
maxima of energy (black surfaces) are higher for VVF than for CVF. Moreover, these
velocity fluctuations are enhanced in this case. A possible scenario of the increased fluctuations for VVF is that the viscous fluid (slow fluid) acts as “spots” which decelerate the less viscous fluid (rapid fluid) leading to the creation of velocity fluctuations.

Figure 2. Middle plane (y = 0) spatial distribution of turbulent kinetic energy $q^2/\Delta U$ at time $\tau = 100$ for (left) $R_v = 9$ and (right) $R_v = 1$.

Figure 3 shows evidence of the modification of the vorticity field. The spanwise vorticity $\omega_z$ is modified by the viscosity variations. For VVF, the 2D rollers of Brown and Roshko are more diffused and thus destructed in the more viscous fluid. Conversely, the 2D rollers are symmetric and of higher vorticity in CVF.

Figure 3. Middle plane (z = 0) spatial distribution of spanwise vorticity $\omega_z$ at time $\tau = 100$.

3.2. Effect of variable viscosity on the mixing-layer spreading and mean velocity

The growth rate of the mixing layer depends on the velocity ratio $U_{high}/U_{low}$ as well as the density ratio $\rho_{high}/\rho_{low}$ [4]. However, less attention was paid to the role of the viscosity variations on the temporal and spatial evolution of mixing layer thickness. The mixing layer momentum thickness is defined as

$$\delta_\theta(\tau) = \int_{\frac{L_y}{\tau}}^{L_y} \frac{[\langle U \rangle - U_{low}] [U_{high} - \langle U \rangle]}{\Delta U^2} dy. \quad (3)$$

Figure 4 illustrates the TML momentum thickness for both VVF and CVF, as functions of the dimensionless time. In the early stage of the flow evolution, the TML momentum thickness is higher for VVF than for CVF. Physically, this result signifies that the VV-TML is affected to a larger extent by the same (initial) momentum. This behaviour might be the result of both

i) an accelerated generation of momentum fluctuations, as well as
ii) an enhanced diffusion of the momentum.

Equation (3) suggests that this behaviour is most likely attributable to differences in the mean stream-
wise velocity \( U \), all the other parameters being the same. It is then clear that the longitudinal mean velocity \( U \) is modified by viscosity effects. This is an a priori surprising result, which may dispel the myth that ‘viscosity is a small-scale quantity with small-scale effects’. Figure 5 shows the spatial distribution along the inhomogeneous direction, \( y \), of the mean velocity \( U \) and mean velocity gradient \( \frac{\partial U}{\partial y} \) profiles for different viscosity ratio at the dimensionless time \( \tau = 40 \). It is clear that the viscosity variations act on the mean velocity field, leading for VVF to

i) a decrease of velocity at the centre of mixing layer at \( \tau = 40 \) (figure 1.a) where the viscosity gradients are important;

ii) an increase of velocity gradient \( \frac{\partial U}{\partial y} \) (figure 5.b); the peaks are located in the high speed stream side (less viscous).

Figure 5. Spatial distribution of (a) mean dimensionless longitudinal velocity and (b) mean dimensionless longitudinal velocity gradient profiles, for different viscosity ratios \( R_\nu \) at \( \tau = 100 \).

3.3. Effect of variable viscosity on turbulent kinetic energy budget

It is found that the quantity \( q^2 = 0.5 \langle u'_i u'_j \rangle / \Delta U^2 \) (summation applies for double indices) is stronger for VVF, indicating that the fluctuating velocity are increased due to the viscosity variations. However, the decay of kinetic energy is more enhanced VVF at time larger \( \tau \geq 200 \).

To unravel the issues associated with variations of viscosity, the one-point kinetic energy budget is derived, in its general expression, to variable viscosity flows first by Talbot and al [1]. The spatial averaging over the homogeneous directions (x and z) allows some simplifications for the particular case of TML.

The aim of this section is to provide a direct evaluation of different terms in equation (5) and to investigate the effects of viscosity variations on the classical expression of the mean energy dissipation
Here, double indices indicate summation and \( s_{ij} \) is the strain tensor, viz.
\[
s_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right).
\]

The transport equation of the turbulent kinetic energy is derived by multiplying equation (4) with the velocity fluctuations \( u'_i \) and space-averaging over the two homogeneous directions (x and z). Finally, the one-point kinetic energy budget is expressed in equation (5).

Terms \( Tt, D_T, P_k \) and \( D_p \) represent the temporal variation of the kinetic energy, the turbulent diffusion, the energy production and the pressure diffusion, respectively.

\[
\frac{\partial}{\partial t} \left( \frac{\langle u'_i^2 \rangle}{2} \right) = -\langle u'_i \frac{\partial}{\partial x_j} \left( \frac{u'_j^2}{2} \right) \rangle - \langle u'_i u'_j \rangle \frac{\partial \langle U \rangle}{\partial y} - \frac{1}{\rho} \frac{\partial}{\partial y} \left( \langle u'_i u'_j \rangle \frac{\partial^2 \langle U \rangle}{\partial y^2} \langle u'_i u'_j \rangle \frac{\partial}{\partial y} \langle u'_i u'_j \rangle \right)
\]

\[
D_T = -\langle u'_i \frac{\partial}{\partial x_j} \left( \frac{u'_j^2}{2} \right) \rangle - \langle u'_i u'_j \rangle \frac{\partial \langle U \rangle}{\partial y} \]

\[
P_k = \frac{\partial v}{\partial x} \left( \frac{1}{2} \frac{\partial u_i^2}{\partial x_i} \right) + \frac{1}{2} \langle v \frac{\partial^2 u_i^2}{\partial x_j^2} \rangle
\]

\[
D_p = \frac{\partial v}{\partial x} \left( \frac{1}{2} \frac{\partial u_i^2}{\partial x_i} \right)
\]

Terms \( P_v \) (line 5.b) represent correlations between the mean velocity gradients and velocity fluctuations. Terms \( \langle \epsilon \rangle_{VG} \) regroup expressions in line (5.c), where 'VG' stands for 'Viscosity Gradients'. The last term represents the homogeneous form of \( \langle \epsilon \rangle_{VV} \). Note that the viscosity is inside the brackets (spatial averages).

Different terms of equation (5) are assessed through our DNS. We now make a distinction between terms that explicitly depend on the kinematic viscosity \( \nu \) (5.b) and terms that do not (5.c). Figure 6 shows the one-point TKE budget for both CVF and VVF cases, plotted at time \( \tau = 100 \). In the caption of figure 6, 'Budget' stands as the RHS of equation (5), which collapses with the LHS for well-balanced TKE budget.

A critical comparison between the TKE budget for VVF \( (R_v = 9) \) and CVF \( (R_v = 1) \) cases is made in order to highlight the viscosity variations effect on the second-order, one-point statistics. Observations can be made on the peak of the production term \( P_k \) that is higher for VVF than for CVF; turbulence production is enhanced and accelerated by viscosity variations. This peak is located within the rapid
Figure 6. Turbulent kinetic energy budget for different viscosity ratios (a) $R_\nu = 1$ and (b) $R_\nu = 9$ at time $\tau = 100$.

The supplementary terms $P_v$ and $\langle \epsilon \rangle_{\nu G}$, related directly to the viscosity variations, have respectively positive and negative contributions in the TKE budget at the early stages of the flow (not shown here). Their role, however, is negligible at time $\tau > 100$, as shown in figure 6 (black points and diamonds, respectively). The velocity fluctuations are stronger for VVF because their production is enhanced by the viscosity variations. It is shown in figure 6 (square symbols), that the VVF flow makes stronger turbulent diffusion (term $D_T$) than CVF. It is obvious that viscosity variations lead to an enhanced entrainment of the irrotational fluid inwards the central zone of the mixing layer. The pressure diffusion term $D_P$, that contributes to redistribute the kinetic energy from the centre of the TML to its edges, has a significant contribution in the TKE budget at the earlier stage of the temporal evolution of the mixing layer. Similarly to the production and turbulent diffusion terms ($P_T$ and $D_T$), TKE is stronger for VVF. We recall that, the instantaneous kinematic viscosity $\nu$ is, for VVF, included in the averages of the mean dissipation rate of the kinetic energy $\langle \epsilon \rangle_{\nu V}$. The latter may be decomposed into contributions due to both the mean viscosity and viscosity fluctuations, as follows

$$
\langle \nu \left( \frac{\partial u'_i}{\partial x_j} \right)^2 \rangle = \langle \nu \rangle \langle \left( \frac{\partial u'_i}{\partial x_j} \right)^2 \rangle + \langle \nu' \frac{\partial u'_i}{\partial x_j} \rangle^2.
$$

(6)

Figure 7. (a) Temporal evolution of the mean kinetic energy dissipation rate $\langle \epsilon \rangle_{\nu V}$ for VVF ($R_\nu = 9$) and CVF ($R_\nu = 1$). (b) Mean and fluctuating components of $\langle \epsilon \rangle_{\nu V}$, for VVF ($R_\nu = 9$), at the dimensionless time $\tau = 400$.

It is obvious that the term $\langle \epsilon \rangle_{\nu V}{\text{fluc}}$ is exactly equal to zero for CVF. Hence, as shown in figure 7.a, the kinetic energy is produced and dissipated in stronger way for different VVF flows, despite the fact that for CVF the kinematic viscosity coefficient is higher, or at least equal to those of VVF. Figure 7.a shows the temporal evolution of the dimensionless mean dissipation rate of the kinetic energy $\langle \epsilon \rangle_{\nu V}$.
It can be seen that \( \epsilon_{VV} \) achieved stronger values for CVF to decrease at \( \tau = 220 \). The location of the maximum mean energy dissipation rate can be related to the time-location to the transition to turbulence. This statement suggests that the time-life of the flow is smaller for VVF, but involves more intense velocity fluctuations and a faster transition to turbulence. Furthermore, it is shown in figure 7.b that the contribution of the viscosity-fluctuating part \( \epsilon_{Vf} \) is far from being negligible, even at \( \tau = 400 \), corresponding to a fully turbulent mixing regime. Therefore, term \( \epsilon_{VV} \) does not reduce to the term proportional to the mean viscosity. Thus, the problem of viscosity variations is far from being trivial.

4. Conclusions
The paper aims at providing rigorous evidence of qualitative and quantitative differences between Variable-Viscosity Flow (VVF) and Constant-Viscosity-CVF, for the same initial flow conditions (Reynolds number \( Re_{\theta a} \) and turbulent intensities), but with drastic differences between the viscosity of the involved fluids. For the CVF, both low and high-stream flows are characterized by the same viscosity, whereas for VVF cases the low-stream is \( Re = 9, 6.4 \) and 2.5 more viscous than the rapid part of the flow. Our DNS showed some new flow features, which are summarized as follows:

i) The morphology of the VVF is significantly affected by the viscosity gradients.

ii) The birth of turbulence is accelerated for VVF compared to CVF.

iii) Turbulent kinetic energy is (up to 2.36 times) more important in VVF at the same time.

iv) The time-life of the flow is smaller for VVF, but involves more intense velocity fluctuations and a faster transition to turbulence.

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