QUARK-LEPTON COMPLEMENTARITY; A REVIEW a

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ABSTRACT

It has been recognized recently that there is a remarkable empirical relation between lepton and quark mixing angles, $\theta_{12} + \theta_C \approx \pi/4$. If not accidental, it should testify for yet uncovered new relationship between the fundamental twin particles in nature which only differ in their ability to feel color. The nontrivial structure which is presumed to exist behind the empirical relation is named as “quark-lepton complementarity”. In this talk, I review the idea at the kind request of the organizer. Starting from pedagogical discussions of bimaximal mixing, which likely to be involved in the whole picture, I try to give a flavor of the new field which is still in rapid development. Toward the more balanced knowledges of flavor mixing in lepton and quark sectors, I describe a promising way for precision measurement of $\theta_{12}$ which utilizes solar and reactor neutrinos.

1. Introduction

In the last year, three experiments observing neutrinos originated from the atmosphere 1, the reactor 2, and the accelerator 3 all saw the oscillatory behavior, providing us with a long awaited confirmation of $\nu$ mass-induced neutrino oscillation since its discovery by Super-Kamiokande 4. Now, we can talk about neutrino masses and lepton flavor mixing 5 with confidence, and it made the by now traditional workshop series “Neutrino Telescopes” in Venice even more important to establish future direction of research in fundamental particle physics. I should note that we all owe much to Milla for her tireless great enthusiasm for having the meeting in such a scenic place.

Let me start by giving a few ward on the thus uncovered structure of lepton flavor mixing: It consists of a large and possibly maximal angle $\theta_{23}$ (atmospheric angle 4), a large but non-maximal angle $\theta_{12}$ (solar angle 6), and a known-to-be small angle $\theta_{13}$ (reactor angle 7). The rich variety in lepton mixing angles from small to nearly maximal mixing is in sharp contrast to quark mixing angles and it must be testifying something important on how nature organized the structure of, to date, the most fundamental matter. One of the key wards in understanding the structure may be the notion of lepton-quark correspondence which dates back to late fifties and early sixties 8. The contemporary theory of the fundamental matter, of course, lends supports to the relation in the form of anomaly cancellation mechanism in the

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standard model. Therefore, there are enough circumstantial evidences of the fact that the existence of leptons and quarks is mutually dependent with each others. The real problem is, however, to uncover the whole picture of how they are related. The traditional answer to this problem is, of course, grand unification in which quarks are leptons are unified into the same multiplet. Quark-lepton complementarity (QLC) is one of the approaches along the line of thought. We start from an empirical observation that the solar angle $\theta_\odot \equiv \theta_{12}$ and the Cabibbo angle $\theta_C$ add up to $\pi/4$ in a good approximation: 

$$\theta_\odot + \theta_C = 45.1^\circ \pm 2.4^\circ \quad (1\sigma).$$  

It appears that it is so close to

$$\theta_\odot + \theta_C = \pi/4,$$  
the charming relation which suggests that quarks and leptons are hiding their common roots. Many questions immediately arise; What is the interpretation of the empirical relation? Are there particle physics models that naturally embody this relation? It is the purpose of this talk to review the status of the new approach, a duty which was kindly assigned to me by the organizer. My presentation here is meant to be very pedagogical and mainly for experimentalists, or people who are trying to get to the relatively new idea. For further references of quark-lepton complementarity (whose naming is due to 10) may be found in 11, 12, 13, 14, 15, 16, 17, 18, 19.

One of the directions which will be pursued in the new era of neutrino physics will be precision determination of the lepton mixing parameters. The approach of QLC and the trend to precision measurement are “complementary” with each other. I mean, there is a real need for precision determination of $\theta_{12}$ to verify the relationship $\theta_\odot + \theta_C = \pi/4$, or find the deviation from it. Now, I would like to note that the Cabibbo angle is measured with great precision of about 1.4% in $\sin^2 \theta_C$ at 90% CL. What about the solar angle? It is about 14% in $\sin^2 \theta_{12}$ at the same CL. What a large disparity between accuracies in measurement of lepton and quark mixing angles! I am sure that nature feels sad about our unequal treatment of the twin particles she created which differ only by possessing or lacking ability of tasting color. Therefore, I will try to also cover the question of how and to what extent accuracy of determination of $\theta_{12}$ can be improved.

2. QLC; questions

First, let me list some immediate questions about QLC. In fact, there are bunch of them:

- Suppose that the QLC relation is correct. Then, the question is; Is there a similar relation

$$\theta_{23}^{\text{lepton}} + \theta_{23}^{\text{quark}} = \pi/4?$$
The relation is perfectly allowed by the current data; \(37^\circ \leq \theta_{23}^{\text{lepton}} \leq 53^\circ\) \(^1\), and \(2.3^\circ \leq \theta_{23}^{\text{quark}} \leq 2.5^\circ\) \(^2\), both at 90\% CL.

- What is the reason why the analogous relation
  \[
  \theta_{13}^{\text{lepton}} + \theta_{13}^{\text{quark}} = \pi/4
  \]  
  does not hold? Experimentally the sum in \(^4\) is less than \(8.5^\circ\) \(^2\), far from \(45^\circ\).

- Suppose that the relations \(^2\) and \(^3\) are approximately correct. Then, the question is; Are there relationship between the deviation from the maximal in \(^2\) and \(^3\)?

- Is the possible deviation from the maximal of \(\theta_{23}\) connected with “deviation from zero” of \(\theta_{13}\)? (For possible symmetries which lead to such connection, see \(^2\).) If so, then, what is the role played by a possible deviation from \(^2\) in the game?

- Suppose that there exist well defined models which realize the QLC relation \(^2\). Then, the question is; how the relation made stable against the changes of parameters of the model?

Unfortunately, no definitive answer is offered to any of these questions at this moment. QLC is a brand-new approach and it is in a too premature stage to answer them. Nonetheless, let me try to give some hints toward motivating the real understanding.

3. What Does It Mean?

The QLC relation \(^2\) implies the presence of maximal mixing angle in somewhere in (1-2) sector of quark-lepton mixing matrix, the MNS matrix \(^5\). Given another mixing angle close to the maximal in (2-3) sector, \(\theta_{atm} \equiv \theta_{23}\), it naturally lead us to the new bimaximal mixing hypothesis. The old version of bimaximal mixing \(^6\) was bravely invented by people who coined to the possibility that the solar neutrino problem is solved by the vacuum oscillation solution. In this viewpoint, the bimaximal mixing would have been the issue purely inside the realm of lepton flavor mixing, having nothing to do with quark mixing.

On the contrary, our new bimaximal mixing ansats requires to embed at least one of the maximal angles into the “unified” quark-lepton sectors. To me, it is one of the most interesting features of QLC ansats; QLC requires quark-lepton unification. Therefore, most probably QLC implies the existence of maximal angle which is neither
in lepton nor in quark sectors.

3.1. Pedagogical bimaximal mixing

By the bimaximal mixing I mean flavor mixing matrix

\[ U_{\text{bimax}} \equiv \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \]  

(5)

From which mass matrix does the bimaximal mixing come? Assuming that there are no other entities which come into play, it is easy to answer the question:

\[ U_{\text{bimax}} \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} U_{\text{bimax}}^\dagger = \begin{bmatrix} \frac{m_1}{2\sqrt{2}} - \frac{m_2}{2\sqrt{2}} + \frac{m_3}{4} + \frac{m_4}{4} & \frac{m_1}{2\sqrt{2}} - \frac{m_2}{2\sqrt{2}} & \frac{m_1}{2\sqrt{2}} - \frac{m_2}{2\sqrt{2}} \negthinspace \frac{m_3}{4} + \frac{m_4}{4} \negthinspace \frac{m_5}{4} + \frac{m_6}{4} - \frac{m_7}{4} \\ \frac{m_1}{2\sqrt{2}} - \frac{m_2}{2\sqrt{2}} - \frac{m_3}{4} + \frac{m_4}{4} & \frac{m_1}{2\sqrt{2}} - \frac{m_2}{2\sqrt{2}} & \frac{m_1}{2\sqrt{2}} - \frac{m_2}{2\sqrt{2}} \negthinspace \frac{m_3}{4} - \frac{m_4}{4} \negthinspace \frac{m_5}{4} + \frac{m_6}{4} + \frac{m_7}{4} \\ \frac{m_1}{2\sqrt{2}} & \frac{m_1}{2\sqrt{2}} & \frac{m_1}{2\sqrt{2}} \end{bmatrix} \]  

(6)

You may complain that it is not very illuminating. Yes, you are quite right. So, let us examine a bit of simplified cases. Suppose that the lightest neutrino mass is much smaller than \( \sqrt{\Delta m_{\text{atm}}^2} \), the hierarchical mass pattern. Then, there are three cases, one “normal” \( (m_3 \gg m_2 \approx m_1) \) and two “inverted” \( (m_2 \approx m_1 \gg m_3) \) mass hierarchies:

Normal: \( M_{\text{atm}} \equiv U_{\text{bimax}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & m_3 \end{bmatrix} U_{\text{bimax}}^\dagger = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{m_1}{2\sqrt{2}} & -\frac{m_2}{2\sqrt{2}} \\ 0 & -\frac{m_3}{2} & \frac{m_3}{2} \end{bmatrix} \)  

(7)

Inverted I: \( M_{\text{atm}} \equiv U_{\text{bimax}} \begin{bmatrix} m_2 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} U_{\text{bimax}}^\dagger = \begin{bmatrix} m_2 & 0 & 0 \\ 0 & m_2 & \frac{m_2}{2} \\ 0 & \frac{m_2}{2} & m_2 \end{bmatrix} \)  

(8)

Inverted II: \( M_{\text{atm}} \equiv U_{\text{bimax}} \begin{bmatrix} m_2 & 0 & 0 \\ 0 & -m_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} U_{\text{bimax}}^\dagger = \begin{bmatrix} m_2 & 0 & 0 \\ \frac{m_2}{\sqrt{2}} & 0 & 0 \\ \frac{m_2}{\sqrt{2}} & 0 & 0 \end{bmatrix} \)  

(9)

where \( m_3 = \sqrt{\Delta m_{\text{atm}}^2} \) and \( m_2 = \sqrt{\Delta m_{\text{atm}}^2} \) in the normal and the inverted mass hierarchies, respectively. We note that in the above all cases the mass matrices have \( \mu \leftrightarrow \tau \) exchange symmetry \(^{[22]}\). The Inverted II case has an extra \( L_e - L_\mu - L_\tau \) symmetry widely discussed in the literature \(^{[24]}\).

3.2. Perpurbative approach

One can phenomenologically describe QLC in a perturbative way starting from the mass matrix above, corresponding to each mass pattern, as done by Ferrandis and Pakvasa \(^{[14]}\). In the case of Inverted II, which is favored by the authors, it takes
the form $M = M_{atm} + M_{sol} + M_{QLC}$, where $M_{sol}$ and $M_{QLC}$ denote the solar scale and the QLC corrections, respectively. They are

$$M_{sol} = \gamma m_2 \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}, \quad M_{QLC} = \lambda m_2 \begin{bmatrix} -1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix},$$

(10)

where $\gamma \approx (\Delta m^2_{sol}/\Delta m^2_{atm})/2 \approx 0.016$ and $\lambda \approx \sin \theta_C$. There are some interesting differences between the three cases. The ordering between $M_{sol}$ and $M_{QLC}$ differ in these three cases; $M_{QLC} < M_{sol}$ in the Inverted I and $M_{QLC} \gg M_{sol}$ in the Inverted II cases [14].

4. QLC as Indication of Quark-Lepton Unification

As I emphasized earlier the most charming features of QLC is that it strongly suggests quark-lepton unification in some forms. Let me discuss this point in more detail. While concrete models which correctly predicted the QLC relation [10] prior experimental observation are missing, the structure of embedding of the relation into GUT-like scenarios, once explicitly formulated, allows us to test the QLC embedded GUT-like scenarios experimentally. This is the topics thoroughly discussed in [10].

Let us sketch the basic points of the discussions in [10]. There are two types of scenarios, depending upon from which sector the maximal 1-2 angle comes, the lepton-origin bimaximal and the neutrino-origin bimaximal scenarios. Let us first recall, not to be confused, the definition of the MNS and the CKM matrix. They are

$$U_{MNS} = U_{\nu}^\dagger U_{\nu}, \quad V_{CKM} = V_{up}^\dagger V_{down},$$

(11)

where $U_{\nu}$ and $U_{\nu}$ denote the matrices which diagonalize neutrino and charged lepton mass matrices, respectively (and the same as in quarks). Then, the ideas behind the both scenarios can be displayed in a simple illustrative way as below.

- Lepton-origin bimaximal scenario

$$U_{\nu} = V_{CKM}^\dagger \quad \leftarrow \text{GUT} \rightarrow \quad V_{up} = V_{CKM}^\dagger$$

$$U_{\text{lepton}} = U_{\text{bimaximal}} \quad \leftarrow \text{Lopsided} \quad V_{down} = I$$

(12)

(13)

where “Lopsided” indicates that the lopsided scenario [25] may gives the relation pointed by an arrow.

- Neutrino-origin bimaximal scenario
\[ \Delta \sin^2 \theta_{12} \quad \sin^2 2 \theta_{13} \quad D_{23} \equiv \frac{1}{2} - s_{23}^2 \quad J_{lep}/\sin \delta \]

| Scenarios               | \( \Delta \sin^2 \theta_{12} \) | \( \sin^2 2 \theta_{13} \) | \( D_{23} \) | \( J_{lep}/\sin \delta \) |
|------------------------|-------------------------------|----------------------------|-------------|-----------------------------|
| neutrino bi-maximal    | 0.051                         | 0.10 \( \pm 0.032 \)     | 0.025       | 1.5 \( \times 10^{-3} \)   |
| lepton bi-maximal      | \(-6 \times 10^{-4} \)       | 2 \( \times 10^{-3} \)   | 0.035*      | 5 \( \times 10^{-5} \)     |
| hybrid bi-maximal      | 1.4 \( \times 10^{-4} \)     | 3.3 \( \times 10^{-4} \) | 0.04*       | 2.1 \( \times 10^{-3} \)   |
| neutrino max+large     | 0.057 \( \pm 0.023 \)        | 0.10 \( \pm 0.032 \)     | SK bound    | \( \leq 6.8 \times 10^{-3} \) |
| lepton max+large       | \(-6 \times 10^{-4} \)       | 2 \( \times 10^{-3} \)   | SK bound    | \( \leq 5 \times 10^{-3} \) |
| hybrid max+large       | 1.4 \( \times 10^{-4} \)     | 3.3 \( \times 10^{-4} \) | SK bound    | \( \leq 2.1 \times 10^{-3} \) |
| single maximal         | 0.015                         | 0.034                     | 0.06 - 0.16 | 9.1 \( \times 10^{-3} \)   |

Table 1: Predictions to the deviation from the QLC relation \( \Delta \sin^2 \theta_{12}, \sin^2 2 \theta_{13} \), the deviation parameter from the maximal 2-3 mixing \( D_{23} \), and the leptonic Jarlskog factor \( J_{lep} \) for different scenarios. The uncertainties indicated with \( \pm \) come from the experimental uncertainty of the atmospheric mixing angle \( \theta_{23} \). Whenever there exist uncertainty due to the CP violating phase \( \delta \) we assume that \( \cos \delta = 0 \) to obtain an “average value”. For the quantities which vanish at \( \cos \delta = 0 \) (indicated by *) the numbers are calculated by assuming \( \cos \delta = 1 \) “SK bound” implies the whole region allowed by the Super-Kamiokande: \( |D_{23}| \leq 0.16 \). The numbers for the last row (single-maximal case) are computed with the assumed values of \( \theta_{23}^l = \theta_C \) and \( \theta_{23}^\nu = 27^\circ \).

\[ U_\nu = U_{\text{bimaximal}} \quad \leftarrow \text{Seesaw enhancement} \quad V_{up} = I \quad (14) \]
\[ U_{\text{lepton}} = V_{CKM} \quad \leftarrow \text{GUT} \rightarrow \quad V_{down} = V_{CKM} \quad (15) \]

where “Seesaw enhancement” indicates that the mechanism my be responsible for neutrino-origin bimaximal (or bi-large) matrix \( [26] \). Notice that while the maximal mixing comes purely from the lepton sector in these constructions, an amalgam of quark and lepton mixing arises once the GUT constraint is imposed.

Now we briefly review these scenarios and their consequences in a minimal way; See \[10\] for more detailed discussions. In the lepton-origin bimaximal scenario, the MNS matrix can be written as

\[ U_{MNS} = R^{m}_{12 \gamma} \Gamma_\delta R^{m}_{12} V^{CKM \dagger} = R^{m}_{23 \gamma} R^{m}_{12} (\pi/4 - \theta^{CKM}_{12}) R^{CKM \dagger}_{13} R^{CKM \dagger}_{23} \quad (16) \]

where \( \Gamma_\delta = \text{diag}[1, 1, e^{i \delta}] \). The lepton-origin bimaximal scenario is also discussed in \[11\]. Whereas in the neutrino-origin bimaximal scenario, it takes the form

\[ U_{MNS} = V^{CKM \dagger} \Gamma_\delta R^{m}_{12} R^{m}_{12} = R^{CKM \dagger}_{12} R^{CKM \dagger}_{13} R^{CKM \dagger}_{23} \Gamma_\delta R^{m}_{23} R^{m}_{12} \quad (17) \]

It is worth to note that the order of rotations and the location where the maximal angle is inserted deserve careful attention \[11\].

Having specified the MNS matrix it is straightforward to work out the phenomenological consequences. Instead of repeating the discussion given in \[10\], we give a summary Table \[1\]. We define the parameter which describes deviation from the QLC
relation (2) as

$$\Delta \sin^2 \theta_{12} \equiv \sin^2 \theta_\odot - \sin^2 \left( \frac{\pi}{4} - \theta_C \right).$$

(18)

At the moment, $\Delta \sin^2 \theta_{12} = 0.002 \pm 0.040$ experimentally. Let us focus on the neutrino- and the lepton-origin bimaximal scenarios, the first and the second rows in Table 1. It should be noticed that there are the characteristic differences between them; In the lepton-origin bimaximal scenario, the deviation from the QLC relation (2) is extremely small so that it is very difficult, if not impossible, to verify it experimentally. In the neutrino-origin bimaximal scenario, on the other hand, the deviation is sizable and may be in reach in the future solar and the reactor neutrino experiments. We will discuss in Sec. 7 how the accuracy of testing the QLC relation can be improved.

The readers might be surprised by proliferation of scenarios in Table 1. In addition to the first and the second rows that are discussed above there exist five more scenarios. It is because the QLC relation (2) is satisfied at least approximately by scenarios with a single maximal angle in 1-2 sector. Therefore, there exist much wider possibilities, as given in Table 1. I stop here, leaving examination of these scenarios for interested readers.

5. Renormalization Stability

Suppose that there exists a GUT model which embodies the QLC relation. It is not quite sufficient to guarantee the QLC relation (1) at low energies, because the renormalization flow could destroy the relationship. It is known that the running of the Cabibbo angle is negligibly small in the SM and in the MSSM. For instance, in MSSM with $\tan \beta = 50$ the parameter $\sin \theta_C$ decreases from 0.2225 at the $m_Z$ down to 0.2224 at the $10^{16}$ GeV.

The issue is, therefore, located in the running of leptonic angle $\theta_{12}$. The renormalization effect on the leptonic $\theta_{12}$ has been investigated by many people. It depends on the type of mass spectrum of light neutrinos. For the spectrum with normal mass hierarchy, $m_1 < m_2 \ll m_3$, the effect is negligible. In contrast, in the case of quasi-degenerate spectrum, $m_1 \approx m_2 \approx m_3 = m_0$, or the spectrum with inverted mass hierarchy the effects can be large. The most recent analysis of the renormalization effects in mixing parameters reassures that in most of the parameter space the QLC relation (2) is stable under the renormalization flow.

6. Quark-Lepton Mass Models with QLC Relation

It is important to construct concrete models to which the QLC relation (2) is embedded in a natural way. Let me describe a possible idea toward this direction by abstracting an essence from the detailed discussion given in [15]. Suppose that one
can prepare a zeroth-order model in which the lepton and quark mixing matrix have the following form

\[ U_{MNS} = U_{bimax}, \quad V_{CKM} = 1. \]  

(19)

Then, one envisage the mechanism that generates the first-order correction to the leading-order formula such that it modifies (19) by the same amount given by the Cabibbo rotation,

\[ U_{MNS} = U_{bimax} \times V_{\text{Cabibbo-like}}, \quad V_{CKM} = 1 \times V_{\text{Cabibbo-like}}, \]  

(20)

where \( V_{\text{Cabibbo-like}} \) denotes the rotation only in 1-2 subspace by the amount of \( \simeq \theta_C \) the Cabibbo angle. We note that it belongs to the neutrino-origin bimaximal scenario in the classification above.

Of course, the real question is if one can construct such a model as that it possesses the desirable zeroth-order structure and is able to generate the first-order corrections of the required form. The authors of 15 presented a model based on the Pati-Salam gauge group \( SU(2)_L \times SU(2)_R \times SU(4)_C \), and presented arguments that it satisfies the above requirements. Since they are quite involved, I urge the interested readers to go to their paper 15.

We note, in passing, that once the MNS matrix is written in the form as in (20) it is identical to the parametrization of lepton mixing matrix which is examined by many authors in much more generic context than the QLC relation 16,30.

One may ask: “To which point we have reached and where to go?”, which may be too premature question to ask. I feel at this point that we still lack simple models in which the QLC relation is naturally implemented. Or, there might be a mechanism that can be called as “built-in stability” which remains to be understood. After the Neutrino Telescope workshop several papers related to QLC were submitted on the Archiv. The authors of 18 attempt a systematic search for higher dimensional operators which lead to the QLC relation within the framework of inverted mass hierarchy. Whereas in 19 a mechanism called “screening” is proposed to prevent Dirac flavor structure from contaminating to the lepton mixing.

7. Experimental Test of the QLC Relation

We now discuss how the QLC relation \(^2\) can be tested experimentally. Since the Cabibbo angle is measured in an enormous precision as emphasized earlier, the real problem is to what accuracy the solar angle \( \theta_{12} \) can be measured experimentally. At this moment there exist two approaches to measure \( \theta_{12} \) accurately. The first one is a natural extension of the method by which \( \theta_{12} \) is determined with the highest precision today, namely, combining the solar and the KamLAND experiments. The other one is to create a dedicated new reactor experiment with detector at around
the first oscillation maximum of reactor neutrino oscillation. Let me briefly explain about the basic ideas behind them one by one.

7.1. Solar-KamLAND method

Combining the solar and the KamLAND experiments is powerful because solar neutrino measurement is good at constraining $\theta_{12}$ and KamLAND determines with high precision the other parameter $\Delta m^2_{21}$, which makes the solar neutrino analysis essentially 1-dimensional. The former characteristics is particularly clear from the fact that the ratio of CC to NC rates in SNO directly determines $\sin^2 \theta_{12}$ in the LMA solution. The current data allows accuracy of determination of $\sin^2 \theta_{12}$ of about $\sim 15\%$ (2 DOF)\(^2\). Further progress in measurement in SNO and KamLAND may improve the accuracy by a factor of $\sim 2$ but not too much beyond that.

However, if one want to improve substantially the accuracy of $\theta_{12}$ determination, the existing solar neutrino experiments are not quite enough. Measurement of low-energy $^7$Be and the pp neutrinos is particularly useful by exploring vacuum oscillation regime. The improvement made possible by these additional measurement is thoroughly discussed by Bahcall and Peña-Garay\(^3\). Since the vacuum oscillation is the dominant mechanism at low energies measuring pp neutrino rate gives nothing but measurement of $\sin^2 2\theta_{12}$. On the other hand, $^7$Be neutrino may carry unique informations of oscillation parameters due to its characteristic feature of monochromatic energy. The solar-KamLAND method will allow us to determine $\sin^2 \theta_{12}$ to 4% level at $1\sigma$ CL\(^3\). In the upper panels of Table 2 we tabulate the sensitivities (1 DOF) currently obtained and expected by the future measurement. We show in Fig. 1 the contour of sensitivity expected by the method in the two-dimensional space spanned by $\tan^2 \theta_{12}$ and $\Delta m^2_{21}$.

Fortunately, varying proposal for such low energy solar neutrino measurement are available in the world\(^4\). Measurement of $^7$Be neutrinos is attempted in Borexino\(^5\) and in KamLAND\(^6\).

7.2. SADO; Several-tens of km Antineutrino DetectOr

Though natural and profitable as a dual-purpose experiment for both $\theta_{12}$ and solar flux measurement the solar-KamLAND method is not the unique possibility for reaching the region of the highest sensitivity for $\theta_{12}$. The most traditional way of measuring mixing angles at the highest possible sensitivities is either to tune beam energy to the oscillation maximum (for example\(^7\) which is for $\sin^2 2\theta_{23}$), or to set up a detector at baseline corresponding to it as employed by various reactor experiments to measure $\theta_{13}$\(^8\). It is also notable that the first proposal of prototype superbeam experiment for detecting CP violation\(^9\) entailed in a setup at around the first oscillation maximum.
Table 2: Comparisons of fractional errors of the experimentally determined mixing angle, $\delta s_{12}^2/s_{12}^2 = \delta(\sin^2 \theta_{12})/\sin^2 \theta_{12}$, by current and future solar neutrino experiments and KamLAND (KL), obtained from Tables 3 and 8 of Ref. 31, versus that by SADO single, which means to ignore all the other reactors than Kashiwazaki-Kariwa, obtained at 68.27% and 99.73% CL for 1 DOF in 38. The numbers in parentheses are for SADO multi, which takes into account all 16 reactors all over Japan.

For $\theta_{12}$ the latter method should apply to reactor neutrinos and in fact a concrete idea for a experimental proposal of dedicated reactor $\theta_{12}$ is worked out in detail 38. See also 39 for a similar but different proposal. The type of experiment is dubbed as “SADO”, an acronym of *Several-tens of km Antineutrino DetectOr* because of the range of baseline distance appropriate for the experiments 38. It is a very feasible experiment because it does not require extreme reduction of the systematic error to 1% level, as required in the $\theta_{13}$ measurement mentioned above. As is demonstrated in 38, reduction of the systematic error to 4% level would be sufficient if no energy spectrum cut at $E_{\text{prompt}} = 2.6$ MeV is performed. It should be within reach in view of the current KamLAND error of 6.5% 2. The effect of geo-neutrino background, which then has to be worried about without spectrum cut, is shown to be tolerable even for most conservative choice of geo-neutrino model, the Fully Radiogenic model 38.

The accuracy achievable by the dedicated reactor $\theta_{12}$ measurement is quite remarkable. It will reach to 2% level at 1$\sigma$ CL for 60 GWth·kt·yr exposure as shown in Table 2. With Kashiwazaki-Kariwa nuclear reactor complex, it corresponds to about 6 years operation for KamLAND size detector. It is notable that possible uncertainty that may arise from the surrounding reactors besides the principle one is also modest, as one can see in Table 2.

Notice that the measurement is not yet systematics dominated and therefore further improvement of the sensitivity is possible by gaining more statistics. In Fig. 1 we make a comparison between the extended solar-KamLAND method and SADO single setup. If SADO can run long enough it can go beyond the solar-KamLAND...
8. Conclusion

In this review talk, I introduced a new approach called “quark-lepton complementarity” which has initiated on the impact of the fascinating empirical relationship \( (1) \) obtained as a result of numerous experiments supported by uncountable numbers of people. I tried to sketch the ideas currently at hand which has been suggested in seeking deeper structure of quark-lepton relation through the the QLC relation. Most of the approach so far involves the maximal mixing in the 1-2 sector. It is one of the most important step to understand the nature of the 1-2 maximal angle.

I have also discussed possible ways of testing the QLC relation \( (2) \) to uncover deviation from it. As I have discussed it may testify from which sector, neutrinos or charged lepton, the 1-2 maximal angle originates. It is good to know that measurement of a few % level accuracies in \( \sin^2 \theta_{12} \) is certainly possible either by an extended solar-KamLAND method, or the dedicated reactor \( \theta_{12} \) experiment, SADO.

I have not covered in my talk possible role of the other mixing angles, \( \theta_{23} \) and \( \theta_{13} \). It is not understood if they are the registered members of the QLC fraternity. Yet,
since most approach to QLC anticipate nearly maximal $\theta_{23}$, possible deviation from its maximality would be of great interests. It should be noticed that the 1% level measurement of $\sin^2 2\theta_{23}$ does not translate into the similar sensitivity in $\sin^2 \theta_{23}$. It is due to a large Jacobian involved in the transformation near the maximal angle, and the $\theta_{23}$ octant degeneracy. For detailed discussions of this point and for possible ways out, see [10] and the references cited therein.

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