On flow reversals in Rayleigh-Bénard convection

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Abstract.
The dynamics of flow reversals are studied numerically using Fourier mode analysis. Our analysis shows that the Fourier modes represent the large-scale flows accurately. We observe that during the reversals, the amplitude of one of the large-scale modes vanishes, while another mode rises sharply, very similar to the cessation-led reversals observed earlier in experiments and numerical simulations. The Fourier coefficients of the RBC equations obey certain symmetries properties, which dictates which modes change sign in flow reversals. Based on our simulation results and symmetry properties of the Fourier modes, we provide a qualitative explanation for the flow reversals.

1. Introduction
It has been observed in many convection experiments that under turbulent conditions, the vertical velocity changes direction randomly in time (Cioni et al. (1997); Niemela et al. (2001); Brown & Ahlers (2006); Xi & Xia (2007); Sugiyama et al. (2010); Yanagisawa et al. (2010)). This phenomenon, known as “flow reversal”, has been analyzed using experiments, simulations (Benzi & Verzicco (2008); Breuer & Hansen (2009); Sugiyama et al. (2010); Mishra et al. (2011) ), and theoretical models (Sreenivasan et al. (2002); Araujo et al. (2005); Benzi & Verzicco (2008); Brown & Ahlers (2008)). This problem gains more significance due to its strong similarities with dynamo reversals. In the present paper, we restrict our study on flow reversals to an idealized convective system called the Rayleigh-Bénard convection (RBC) in which a fluid confined between two plates is heated from below and cooled at the top. Here we attempt to understand the dynamics and symmetries of the flow reversals in RBC using direct numerical simulations. We focus our investigations on two most important parameter that specify RBC flows, (a) the Rayleigh number $Ra$, a ratio of buoyancy term and the dissipative term, and (b) the Prandtl number $Pr$, a ratio of the kinematic viscosity and the thermal diffusivity.

Cioni et al. (1997) performed RBC experiment on mercury ($Pr \approx 0.02$) and measured the vertical velocity near the mid horizontal plane using eight probes near the lateral walls. They observed flow reversals in their experiment. Similar behaviour was observed by Niemela et al. (2001) in their experiments on helium gas ($Pr \approx 0.7$), and by Brown & Ahlers (2006) and Xi & Xia (2007) for water ($Pr \approx 7$). Brown & Ahlers (2006) and Xi & Xia (2007) observed two kinds of flow reversals in their RBC experiment on a cylindrical geometry: (a) “cessation-led reversals”, in which the the amplitude of the first Fourier mode vanishes abruptly, and (b) “rotation-led reversals”, in which the plane of the large-scale circulation rotates by a random angle without any significant decrease in the amplitude of the first Fourier mode. In a recent
experiment, Sugiyama et al. (2010) observed flow reversal in a quasi two-dimensional box, where the flow profile was dominated by a diagonal large-scale roll and two smaller secondary rolls at the opposite corners.

Accurate numerical simulations provide useful information about the velocity and temperature fields at any point inside the box, which may not be able to be possible in an experiment. Breuer & Hansen (2009) simulated infinite Prandtl-number fluid in a two-dimensional box with free-slip boundary conditions at all the walls, and observed flow reversals for \( R = 10^9 \). Sugiyama et al. (2010) performed simulations for the same boundary condition, but for a set of finite Prandtl numbers. Paul et al. (2010) performed similar simulations, but they applied free-slip boundary conditions at the horizontal wall, and periodic boundary conditions at the vertical wall. Flow reversals were observed in these simulations as well. Mishra et al. (2011) studied flow reversals in a cylindrical geometry, and concluded that during the cessation-led reversals, the dipolar mode decreases in amplitude, while the quadrupolar mode increases. Recently Chandra & Verma (2011) simulated two-dimensional box with no-slip boundary conditions at all the walls, and studied the dynamics and symmetries of the Fourier modes. The above numerical simulations are consistent with the experimental results.

Several models have been proposed to explain the flow reversals. Broadly they involve either stochasticity (Sreenivasan et al. (2002); Benzi & Verzicco (2008); Brown & Ahlers (2008)) or low-dimensional models with noise (Araujo et al. (2005); Brown & Ahlers (2008)). We (Mishra et al. (2011); Chandra & Verma (2011); Paul et al. (2010)) have been attempting to explain reversals in terms of interactions among the low-wavenumber modes.

In this paper, we simulate the two-dimensional RBC flows under no-slip boundary conditions at all the walls. Using the complete flow profile, we compute the large and intermediate-scale Fourier modes accurately. These modes help us in quantitative understanding of the dynamics and symmetries of the RBC system. We contrast our results with earlier numerical and experimental results regarding reversals. The symmetry property of the RBC system is also contrasted with that of dynamo.

2. Governing Equations and the Computational Methods

The equations governing two-dimensional Rayleigh-Bénard convection under Boussinesq approximation are

\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla P + Pr \nabla^2 \mathbf{u} + Ra Pr T \hat{y}, \tag{1}
\]

\[
\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = \nabla^2 T, \tag{2}
\]

\[
\nabla \cdot \mathbf{u} = 0, \tag{3}
\]

where \( \mathbf{u} \) is the velocity field, \( T \) is the temperature field, \( P \) is the pressure, and \( \hat{y} \) is the buoyancy direction. The two nondimensional parameters are the Prandtl number \( Pr \), the ratio of the kinematic viscosity \( \nu \) and the thermal diffusivity \( \kappa \), and the Rayleigh number \( Ra = \alpha g \Delta d^3/(\nu \kappa) \), where \( \alpha \) is the thermal expansion coefficient, \( d \) is the distance between the two plates, \( \Delta \) is the temperature difference between the plates, and \( g \) is the acceleration due to gravity. The above equations have been nondimensionalized using \( d \) as the length scale, the thermal diffusive time \( d^2/\kappa \) as the time scale, and \( \Delta \) as the temperature scale.

Equations (1-3) are solved for a 2D box of aspect ratio \( \Gamma = 1 \) (denoted by \( B_1 \)) and aspect ratio \( \Gamma = 2 \) (denoted by \( B_2 \)) with no-slip walls at all the sides. The side walls are insulating, and the temperature of the top and the bottom walls are kept constant. The simulations are performed using the spectral element method with 7th order elements. We use \( 28 \times 28 \) spectral elements for \( B_1 \) and \( 48 \) spectral elements for \( B_2 \) resulting in a grid of \( 196 \times 196 \) and \( 336 \times 196 \).
respectively. The concentration of grid points is higher at the boundaries in order to resolve the boundary layers. We use the free and open source code NEK5000 (Fischer (1997)) for our simulations.

We study the dynamics of field reversals using the low wavenumber velocity and temperature Fourier modes. Note that

\[
\begin{align*}
    u(x, y) &= \sum_{m,n} \hat{u}_{m,n} \sin(mk_c x) \cos(n\pi y) \\
    v(x, y) &= \sum_{m,n} \hat{v}_{m,n} \cos(mk_c x) \sin(n\pi y) \\
    T(x, y) &= \sum_{m,n} \hat{T}_{m,n} \cos(mk_c x) \sin(n\pi y)
\end{align*}
\]

where \(k_c = \pi\) for \(B_1\) and \(k_c = \pi/2\) for \(B_2\). The projection is performed by sampling the fields on a uniform \(128 \times 128\) grid. This grid is then transformed into spectral space using fast discrete sine and cosine transforms of the FFTW (http://www.fftw.org/) library through a python interface (https://launchpad.net/pyfftw/). We will show below that the above basis functions elegantly capture the dynamics of dominant structures in the bulk despite not satisfying the no-slip boundary conditions of our simulations. The nonlinear interactions in the Fourier basis are of the form \(k = p + q\), where \(k, p, q\) are the interacting wavenumbers. This form is elegant compared to the Legendre or Chebyshev polynomials, or proper orthogonal decomposition (POD).

In the following section we will describe the our numerical results.

**Figure 1.** Top panel: Convective flow profile computed from a spectral element simulation for a box of aspect ratio \(\Gamma = 1\) \((B_1)\), Prandtl number \(Pr = 1\), Rayleigh number \(Ra = 2 \times 10^7\): (a) before the reversal \((t = 10.52\) thermal diffusive time units\)), (b) during the reversal \((t = 10.68)\), (c) after the reversal \((t = 10.79)\). Bottom panel: Velocity field corresponding to (a) \(\hat{u}_{1,1}\), (b) \(\hat{u}_{2,2}\), (c) \(-\hat{u}_{1,1}\).
Figure 2. For $\Gamma = 1$ ($B_1$), $Pr = 1$, and $Ra = 2 \times 10^7$, time series of (a) $y$ component of the real velocity field at the point $(0.25, 0.25)$, (b) the Fourier amplitudes $\hat{v}_{1,1}$ (blue) and $\hat{v}_{2,2}$ (red), (c) the ratio $|\hat{v}_{2,2}/\hat{v}_{1,1}|$, and (d) Nusselt number ($Nu$). During the reversal, the sign of mode $\hat{v}_{1,1}$ changes sign after vanishing, while the mode $\hat{v}_{2,2}$ rises sharply. The Nusselt number shows large fluctuations during the reversals.

3. Numerical Results

All the simulations presented in this paper were performed for the unit Prandtl number. We take $Ra = 2 \times 10^7$, $10^8$, and $10^9$ for $B_1$, and $Ra = 10^7$, $2 \times 10^7$, and $10^8$ for $B_2$. We observe flow reversals for $Ra = 2 \times 10^7$ in the $\Gamma = 1$ box, and for $Ra = 10^7$ in the $\Gamma = 2$ box.

For one of the reversing case ($Ra = 2 \times 10^7, \Gamma = 1$), we display three frames of the velocity and temperature fields in Fig. 1(a,b,c): (a) before the reversal at $t = 10.52$ thermal diffusive time units, (b) during the reversal at $t = 10.68$, and (c) after the reversal at $t = 10.79$ (also see videos at http://turbulence.phy.iitk.ac.in/animations:convection). These figures are very similar to those presented by Sugiyama et al. (2010).

We can capture the convective flow profiles quite nicely using the low wavenumber Fourier modes. In Fig. 1(d,e) we display the velocity profiles of the primary modes $(1,1)$, or, $(m=1, n=1)$ and the mode $(2,2)$. The mode $(1,1)$ corresponds to a single roll, while the mode $(2,2)$ corresponds to four rolls. These are the most dominant modes of our $\Gamma = 1$ box simulations, and they represent dipolar and quadrupolar configurations respectively. The sign of the Fourier mode of Fig. 1(f) is reversed compared to that of Fig. 1(d). Fig. 1(a) is a superposition of (d) and (e) modes with appropriate amplitudes, while Fig. 1(c) is a superposition of (f) and (e) modes. We will show later that during the reversal, the dipolar mode (subfigures (d,f)) become weak, and the quadrupolar mode (subfigure (e)) becomes most dominant. As a result, the velocity profile during the reversal is as shown in subfigure (b).

Now we perform a quantitative study of the field reversal using real space probes and Fourier space amplitudes. In Fig. 2(a) we illustrate the time series of $V_y$ (measured at the point $(0.25, 0.25)$) that clearly shows field reversals. Subfigure 2(b) shows that the Fourier modes $(1,1)$ and $(2,2)$ change abruptly during the reversal. The mode $(1,1)$ dominates the mode $(2,2)$ between the reversal, and it maintains almost a constant absolute value. However during the reversal, the mode $(1,1)$ decreases and becomes zero, and while the mode $(2,2)$ mode shows a
spike. This feature is clearly evident in the time series of the ratio $\hat{v}_{2,2}/\hat{v}_{1,1}$ depicted in Fig. 2(c). The appearance of four rolls during the reversal is due to the dominance of (2,2) Fourier mode during the reversal. We also observe an offshoot of about 40% for the (1,1) mode before it settles into its new steady state (reversed in sign). Due to the highly fluctuating nature of the Fourier modes during the reversal, the Nusselt number too shows a spiky behaviour as shown Fig. 2(d). Surprisingly, the Nusselt number becomes negative for a brief period during the reversal; here, the heat energy is transferred from the cold plate to the hot plate.

Similar features are observed for the aspect ratio two box ($B_2$). Two snap-shots of the dominant flow profiles (during nonreversing phase) for Rayleigh numbers $Ra = 10^7$ and $Ra = 10^8$ are shown in Fig. 3. Here the first and the second most-dominant modes are (2,1) and (2,2) respectively. The corner rolls are due to the presence of the (2,2) mode. A time series of the $V_0(1,0.5)$, $\langle |\hat{v}_{2,1}/\hat{v}_{2,2}| \rangle$, and the Nusselt number $Nu$ for $Ra = 2 \times 10^7$ are shown in Fig. 4. We observe a sharp peak for $|\hat{v}_{2,1}/\hat{v}_{2,2}|$ during the reversal, very similar to that observed for the aspect ratio one box during reversals.

The main results of our RBC simulations are as follows:

- During the reversal, the magnitude of the most dominant Fourier mode drops sharply, and the second-most important mode increases in magnitude. After some transient fluctuations, these modes settle down to their new steady-state values.
- The time interval between the two consecutive reversals is random, similar to those observed in all the RBC experiments. The average intervals between consecutive reversals for $\Gamma = 1, Ra = 2 \times 10^7$ and $\Gamma = 2, Ra = 10^8$ are approximately 0.6 and 0.05 thermal diffusive time units, respectively. The duration of the reversals is approximately 0.03 time units for both the cases. Geometry appears to play an important role in reversal dynamics.
- We do not observe flow reversals for some cases till several thermal diffusive time units. The maximum simulation time of our non-reversing cases is 10 thermal diffusive time units. Some
Figure 4. For $\Gamma = 2 \ (B_2)$, $Pr = 1$, and $Ra = 10^7$, time series of (a) $y$ component of the real velocity field at the point $(1,0.5)$, (b) the Fourier amplitudes $\hat{v}_{2,1}$ (blue) and $\hat{v}_{2,2}$ (red), (c) the ratio $|\hat{v}_{2,2}/\hat{v}_{2,1}|$, and (d) Nusselt number ($Nu$). During the reversal, the sign of mode $\hat{v}_{2,1}$ changes sign after vanishing, while the mode $\hat{v}_{2,2}$ rises. The Nusselt number shows large fluctuations during the reversals.

of the non-reversal cases are: for $\Gamma = 1$, $Ra = 10^8, 10^9$, and for $\Gamma = 2$, $Ra = 2 \times 10^7, 10^8$. Whether reversal will occur or not at a later time for these cases is not certain.

• The ratio $|\hat{v}_{2,2}/\hat{v}_{2,1}|$ appears to be smaller for the non-reversing cases compared to the reversing cases. Note that this ratio quantifies the importance of the corner rolls ((2,2) mode) compared to the dominant roll structure. As the Rayleigh number is increased, this ratio tends to decrease, thus making the reversal somewhat more difficult. This result is consistent with those of Sugiyama et al. (2010).

• Buoyancy and Nusselt number exhibit spiky behaviour during the reversal. This feature is illustrated in the plot of the energy feed into the (1,1) and (2,2) modes due to buoyancy for $\Gamma = 1, Ra = 2 \times 10^7$ case (see Fig. 5).

• The properties of the dominant Fourier modes are governed by symmetry properties, which will be described below.

In the next section we will describe the symmetry properties of the the Fourier modes of the RBC equations.

4. Symmetries of the RBC equations and the Flow Reversals

We observe in our simulations that some modes switch sign during a reversal, while others do not. This feature can be explained using the underlying symmetries of the governing equations in Fourier space. First we will illustrate these symmetries for two-dimensional RBC, and then we will extend them to three dimensions. The RBC equations in the Fourier space are

$$\frac{\partial \hat{u}_i(k)}{\partial t} = -ik_j \sum_{k=p+q} \hat{u}_j(p)\hat{u}_i(q) - ik_i P(k) + RaPr\hat{\theta}(k)\delta_{i,2} - Prk^2\hat{u}_i(k)$$

(7)

$$\frac{\partial \hat{\theta}(k)}{\partial t} = -ik_j \sum_{k=p+q} \hat{u}_j(p)\hat{\theta}(q) + \hat{u}_2(k) - k^2\hat{\theta}(k)$$

(8)

where $\theta$ is the perturbation of the temperature about the conduction state. For a two-dimensional box, the Fourier modes (coefficients of the basis functions) of the fields are of
Figure 5. (a) The energy flow due to buoyancy \((RaPr\hat{T}(k)\hat{v}(k))\) into the \(k = (1,1)\) and \((2,2)\) modes for the \(\Gamma = 1 (B_1)\) box for \(Pr = 1\) and \(Ra = 10^7\). The buoyancy contribution into these modes shows a large increase before a reversal. The buoyancy of the \((2,2)\) mode is always positive but is negative during the reversal process.

Table 1. Symmetry properties of RBC systems.

| Box Geometry | Dominant modes | Generated modes | Transformations during reversal | Symmetry |
|--------------|----------------|-----------------|--------------------------------|----------|
| Present paper: \(\Gamma = 1\) | \((1,1), (2,2)\) \(O, E\) | \(M_{eo}, M_{oe}\) | \(E\rightarrow E, O\rightarrow -O, M_{eo} \rightarrow M_{eo}, M_{oe} \rightarrow -M_{oe}\) | \(i, ii\) |
| Present paper: \(\Gamma = 2\) | \((2,1), (2,2)\) \(M_{eo}, E\) | \(\hat{v}_{1,1} \rightarrow -\hat{v}_{1,1}, \hat{v}_{2,2} \rightarrow \hat{v}_{2,2}\) | \(ii\) |
| Bruer & Hansen: \(\Gamma = 2\) | \((1,1), (2,1)\) \(O, M_{eo}\) | \(\hat{v}_{1,1} \rightarrow -\hat{v}_{1,1}, \hat{v}_{2,1} \rightarrow -\hat{v}_{2,1}\) | \(ii\) |
| Prediction: \(\Gamma = 1/2\) | \((1,2), (2,2)\) \(M_{oe}, E\) | \(\hat{v}_{1,2} \rightarrow -\hat{v}_{1,2}, \hat{v}_{2,2} \rightarrow \hat{v}_{2,2}\) | \(i\) |

In Table 1 we list the symmetry properties of the system simulated by us, as well as that of Breuer & Hansen (2009). Our simulations tend generate new modes as a result of the nonlinear processes. However, to a large extent, the primary and the generated modes tend to belong to

the type \(E = (e,e), O = (o,o), M_{eo} = (e,o), \) and \(M_{oe} = (o,e),\) where \(e,o\) are the even and odd integers respectively, and \(E, O,\) and \(M\) are even, odd and mixed modes respectively. The nonlinear interactions generate new modes with \(k = p + q.\) Consequently, in the \(\sin-cos\) basis, the modes \((m_1, n_1)\) and \((m_2, n_2)\) generate modes \((|m_1\pm m_2|, |n_1\pm n_2|)). The nonlinear interactions satisfy the following properties: \(O \times O = E; O \times E = O; E \times E = E; E \times M_{eo,oe} = M_{eo,oe}; \)
\(O \times M_{eo,oe} = M_{oe, eo}; M_{eo, eo} \times M_{eo, oe} = E; \) and \(M_{eo, oe} \times M_{oe, eo} = O.\) Here \(O \times O = E\) means that two \(O\)-modes interact to yield an \(E\)-mode. As a consequence of the above constitutive rules, the two symmetry operations that keep Eqs. (7, 8) invariant are:

(i) \((E \rightarrow E, O \rightarrow -O, M_{eo} \rightarrow M_{eo}, M_{oe} \rightarrow -M_{oe})\)

(ii) \((E \rightarrow E, O \rightarrow -O, M_{eo} \rightarrow -M_{eo}, M_{oe} \rightarrow M_{oe})\)

Thus, if \(\{E, O, M_{eo, M_{oe}}\}\) is a solution of Eqs. (7, 8), then \(\{E, -O, \pm M_{eo}, \mp M_{oe}\}\) is also a solution of these equations (cases (i, ii) respectively). Note that the above properties are universal, that is, they are independent of the box geometry, Prandtl number, etc.

In Table 1 we list the symmetry properties of the system simulated by us, as well as that of Breuer & Hansen (2009). Our simulations tend generate new modes as a result of the nonlinear processes. However, to a large extent, the primary and the generated modes tend to belong to
a class or several classes. For example, the modes generated in $\Gamma = 1$ simulation belong to $E$ and $O$ classes, and the mixed modes have negligible magnitude even in high Rayleigh number regime. Thus, the symmetry properties help us understand the reversal dynamics in terms of Fourier modes.

The above (discrete) symmetry relations hold when the sin-cos basis (4-6) are good basis functions for the geometry of the system. Here the sign of the most dominant Fourier coefficient gets changed. This symmetry correspond to the “cessation-led” reversals described earlier by Brown & Ahlers (2006), Xi & Xia (2007), and Mishra et al. (2011). For the cylindrical geometry or periodic boundary conditions, the basis functions involve complex Fourier functions and the symmetry is continuous, corresponding to a change in phase of the mode. For a two dimensional periodic box (free-slip horizontal walls), Paul et al. (2010) showed that the flow reversals are due to the continuous change of phases.

The symmetry properties for the reversals in dynamo is quite different. It is dictated by the $\{u \rightarrow u, b \rightarrow -b\}$ symmetry of the governing magnetohydrodynamics (MHD) equations. As a consequence, all the Fourier modes of the magnetic field $b$ would change sign after the reversal. The dynamical equations of RBC do not have such global symmetry. This is one of the major differences between flow reversals of RBC and magnetic field reversals of dynamo.

5. Connection with Earlier Work and Reversal Mechanism

Our numerical results are same as that of Sugiyama et al. (2010). However, we attempt to explain the reversal phenomenon using the low wavenumber modes, while Sugiyama et al. invoke the dynamics of plumes and corner rolls near the wall for their explanation. Also, the dynamics of the dominant modes (e.g., (1,1) and (2,2) for the $\Gamma = 1$ box) described for the two-dimensional no-slip box is very similar to the cessation-led reversal described by Brown & Ahlers (2006), Xi & Xia (2007), and Mishra et al. (2011). The “rotation-led reversals” observed by the above researchers for the cylindrical geometry is related to the continuous symmetry along the azimuthal direction.

An intriguing question is why do the Fourier modes change so suddenly from their steady-state values during the reversals. There have been several attempts in the past to explain it using “stochasticity” (Sreenivasan et al. (2002); Benzi & Verzicco (2008); Brown & Ahlers (2008)) and/or low-dimensional models with noise (Araujo et al. (2005); Brown & Ahlers (2008)). At present there are many more questions than answers. Yet, based on the above results and the previous work (Mishra et al. (2011); Paul et al. (2010)), we provide the following qualitative arguments for the mechanism for the flow reversal.

- The RBC system is a driven nonequilibrium system that maintains its steady-state, with the energy supply to the large-scale (small wavenumber) modes by buoyancy being cascaded to the intermediate and small-scale modes. Under the steady-state, the system attains a quasi-equilibrium state with the large-scale modes maintaining somewhat steady-state values. This is evident from the steady value of the (1,1) and (2,2) Fourier modes for the $\Gamma = 1$ box, and (2,1) and (2,2) modes for the $\Gamma = 2$ box.
- The quasi-equilibrium state however is unstable. At some random time, the balance of energy supply at various scales gets disturbed due to nonlinearity, and the unstable RBC system attains a new steady-state in which the large-scale structure is reversed. Sometimes the reversal process fails and the system comes back to its original state after a brief excursion.
- The new steady state is not arbitrary or random. It is dictated by the symmetry property of the system that leads to the change of sign of some of the modes. The constant drive also plays a major role during the reversal. During the reversal, when some of the modes decreases in magnitude, the other modes increase to maintain the average energy supply of the system. This feature however is applicable to the rectangular box geometry that
respects discrete symmetry. Continuous symmetry of cylinder or periodic box would lead to different behaviour.

- The interval distribution between two consecutive reversals is being studied by various research groups. Unfortunately simulations do not provide enough number of reversals for computing statistics. It is however believed that the system loses its memory during the cessation-led reversals, hence the interval distribution for such reversals tend to follow a Poisson distribution. On the contrary, the rotation-led events may be temporally correlated without any specific time-scales, and they may follow a power-law distribution (Verma et al. (2006), Brown & Ahlers (2006)).

6. Conclusions

In conclusion, we studied the dynamics of reversals in a two-dimensional box of aspect ratio 1 and 2 using Fourier mode analysis. Our analysis shows that Fourier modes represent the large-scale flows quite accurately, both during the steady-state as well as during the reversals. We observe that during the reversal, the most-dominant mode decreases in amplitude, while the second-largest mode increases. We also derived symmetry properties of Fourier modes of the RBC equations. These properties help us in classifying the reversing and nonreversing modes of the RBC system. There observations indicate the usefulness of the Fourier modes in understanding the reversal dynamics.

Extensive numerical simulations and experiments are required for further investigation the symmetry properties and reversal dynamics of RBC.

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