Self-gravito-acoustic shock signals in astrophysical compact objects

A. A. Mamun
Department of Physics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh

The existence of self-gravito-acoustic (SGA) shock signals (SSs) associated with negative self-gravitational potential in the perturbed state of the astrophysical compact objects (ACOs) (viz. white dwarfs, neutron stars, black holes, etc.) is predicted for the first time. A modified Burgers equation (MB), which is valid for both planar and non-planar spherical geometries, by the reductive perturbation method. It is shown that the longitudinal viscous force acting in the medium of any ACO is the source of dissipation, and is responsible for the formation of these SGA SSs. The time evolution of these SGA SSs is also shown for different values (viz. 0.5, 1, and 2) of the ratio of nonlinear coefficient to dissipative coefficient in the MB equation. The theory presented here is so general that it can be applied in any ACO of planar or non-planar spherical shape.

The astrophysical compact objects (ACOs) (viz. white dwarfs, neutron stars, black holes, etc.) are significantly different from other terrestrial bodies not only because of their extra-ordinarily high density and extremely low temperature \[1^\text{–}5\], but also because they can introduce new self-gravito-acoustic mode and associated new nonlinear structures. They, in fact, contain an admixture of degenerate, non-inertial particle species (viz. electron or positron or non-zero mass quark species, or any one/two or all of them), non-degenerate or degenerate inertial light particle species (viz. proton or neutron or \(\frac{1}{2}\)He, or \(\frac{1}{2}\)C or \(\frac{1}{2}\)O species \[2\,4\]), or any one/two or all of them), and heavy particle species (viz. \(\frac{56}{26}\)Fe or \(\frac{85}{37}\)Rb or \(\frac{96}{42}\)Mo species \[7\,8\]), or any one/two or all of them).

The degeneracy of non-inertial electron or positron or quark species arises due to Heisenberg’s uncertainty principle \[\Delta p \Delta x \geq \hbar/2\], where \(\hbar\) is the reduced Planck constant, \(\Delta p\) is the uncertainty in the particle's momentum, and \(\Delta x\) is the uncertainty in particle's position. This indicates that the momentum of a highly compressed particle species is extremely uncertain, since the particle species is located in an extremely confined space. Therefore, even though this confined space is extremely cold, the particle species must move very fast on average, and give rise to a very high pressure, known as ‘degenerate pressure’, which depends only on degenerate particle number density. This means that in order to compress an object into an extremely small space, a tremendous pressure, which is the self-gravitational pressure in any ACO (viz. white dwarfs, neutron stars, black holes, etc.) is required to balance this degenerate pressure.

Recent discovery \[9\,11\] of gravitational waves \[9\,11\] (produced by merging of two black holes) has motivated space and astrophysicists to search for new gravito-acoustic modes that may exist in such ACOs (viz. white dwarfs, neutron stars, black holes, etc.). The concept of a new self-gravitational pressure, which is the self-gravitational pressure in any ACO (viz. white dwarfs, neutron stars, black holes, etc.) is predicted for the first time. A modified Burgers equation \[5, 1,\,\,\text{and} 2\] of the ratio of nonlinear coefficient to dissipative coefficient in the MB equation. The theory presented here is so general that it can be applied in any ACO of planar or non-planar spherical shape.

The existence of self-gravitational potential in the perturbed state of the astrophysical compact objects (ACOs) (viz. white dwarfs, neutron stars, black holes, etc.) is predicted for the first time. A modified Burgers equation (MB), which is valid for both planar and non-planar spherical geometries, by the reductive perturbation method. It is shown that the longitudinal viscous force acting in the medium of any ACO is the source of dissipation, and is responsible for the formation of these SGA SSs. The time evolution of these SGA SSs is also shown for different values (viz. 0.5, 1, and 2) of the ratio of nonlinear coefficient to dissipative coefficient in the MB equation. The theory presented here is so general that it can be applied in any ACO of planar or non-planar spherical shape.

The concept of a new self-gravitating (degenerate pressure brings the system back to its equilibrium shape, but during this action it is expanded (compressed) more than its equilibrium shape according to Newton’s 1st law of motion, and again the self-gravitational (degenerate pressure brings the system back to its equilibrium shape, but again during this action, it is compressed (expanded) more than its equilibrium shape according to the same reason. These compression (rarefaction) and rarefaction (compression) of the system continue, and thus, a new SGA mode is developed.

The present article is aimed at identifying the SGA shock signals associated with the self-gravitational potential in ACOs (viz. white dwarfs, neutron stars, black holes, etc.) which are assumed to contain arbitrary number of non-inertial degenerate particle species \(s\) (viz. electron or/and positron or/and non-zero mass quark, etc. \[2\,5\]), and of inertial degenerate particle species \(j\) (viz. proton or/and neutron, and \(\frac{1}{2}\)He or \(\frac{1}{2}\)C or \(\frac{1}{2}\)O, or/and \(\frac{56}{26}\)Fe or/and \(\frac{85}{37}\)Rb or/and \(\frac{96}{42}\)Mo, etc. \[2\,5\]). The perturbed state of such such ACOs can be described by generalized hydrodynamic model \[12\,16\] in planar \((\nu = 0)\) or nonplanar spherical \((\nu = 2)\) geometry \[16\,17\] by

\[
\frac{\partial \psi}{\partial r} = -\frac{3}{2} \frac{\partial \rho_s}{\partial r},
\]

\[
\frac{1}{\nu} \partial_r (r^\nu \delta_r \psi) = 0,
\]

\[
(1 + \tau_m \delta_l t) \left[ \rho_j \left( \partial_r u_j + \partial_r \psi + \frac{2}{3} \beta_j \partial_r \rho_j \right) \right] = \frac{1}{\nu} \partial_r (r^\nu \partial_r \psi) + (\zeta + \eta) \partial_r \left[ \frac{1}{\nu} \partial_r (r^\nu u_j) \right],
\]

\[
\frac{1}{\nu} \partial_r (r^\nu \partial_r \psi) = \sum_j s \rho_s + \sum_j \mu_j \rho_j,
\]

where \(\partial_t = \partial/\partial t, \partial_r = \partial/\partial r, \partial_r = \partial/\partial r\) and \(\delta_l = \partial_l + u_l \partial_r\); \(\rho_s\) \((\rho_j)\) is the number density of the degenerate, non-inertial (inertial) particle species \(s\) \((j)\), and is normalized by its equilibrium value \(\rho_{e0}\); \(s\) \((j)\) is the degenerate fluid speed of the species \(s\), and is normalized by \(C_q\) in which \(C_q = (\sqrt{\pi h} \rho_{e0}^{1/3} m_e^{1/3} m_p, m_p\) \((m_e)\) is the proton (electron) mass and \(\rho_{e0}\) is the equilibrium mass density of the electron species; \(\psi\) is
the self-gravitational potential, and is normalized by

$$C_q \equiv \left( m_p/m_e \right)^2 \left( m_e/m_n \right)^{2/3} \left( \rho_0/\rho_n \right)^{2/3}$$

and

$$\beta_j = \left( m_p/m_j \right)^2 \left( m_j/m_e \right)^{2/3} \left( \rho_j/\rho_n \right)^{2/3},$$

in which $m_j$ ($m_i$) is the mass of the non-inertial (inertial) degenerate particle species $s$ ($j$); $t$ is the time variable normalized by

$$\omega^{-1}_{jp} = \left( 4\pi G \rho_0 \right)^{-1/2};$$

$\tau$ is the space variable normalized by

$$L_q = C_q / \omega_{jp};$$

and $\delta_s = \rho_s/\rho_0$ and $\mu_j = \rho_j/\rho_0$ in which $\rho_j$ is the mass density of the proton species; $\eta$ ($\zeta$) is the shear (bulk) viscosity coefficient, and is normalized by $\rho_0 C_q L_q$, and $\tau_m$ is the viscoelastic relaxation time normalized by $\omega_{jp}$. There are various approaches for calculating these transport coefficients, $\eta, \zeta, \omega, \tau_m$. These have been widely discussed in the existing literature [12 13 16 18]. We note that in [1] the non-inertial degenerate species $s$ are assumed to be non-relativistically degenerate. This has been considered by many authors during the last few years [19–25] to study the electro-acoustic or magneto-acoustic linear/nonlinear waves, but not to study any kind of self-gravit-acoustic waves/modes, which is the basis of the present work. We also note that [1] is obtained by equating the outward degenerate pressure to the inward self-gravitational pressure of the species $s$ [1]. This is, however, valid for the SGA perturbation mode whose phase speed is much smaller than $C_c$, where $C_c = \sqrt{\pi G \rho_0 / m_n^{2/3}}$.

To construct a weakly nonlinear theory for the nonlinear propagation of this perturbation mode by using the reductive perturbation method, we, first introduce the stretched co-ordinates [13 17]. $R = - (r + V_0 t)$, $T = \tau^2 t$ [where $V_0$ is normalized by $C_q$, and $R$ ($T$) is normalized by $L_q \left( \omega^{-1}_{jp} \right)$], and expand the perturbed quantities in power series of $\epsilon$: $\rho_{s,j} = 1 + \epsilon \rho_{s,j}^{(1)} + \epsilon^2 \rho_{s,j}^{(2)} + \ldots$, $u_j = \epsilon u_j^{(1)} + \epsilon^2 u_j^{(2)} + \ldots$, and $\psi = \epsilon \psi^{(1)} + \epsilon^2 \psi^{(2)} + \ldots$, where $\epsilon$ is an expansion parameter ($0 < \epsilon < 1$). We next develop equations in various powers of $\epsilon$ by using the stretched co-ordinates, and the expansions of these perturbed quantities. Now, keeping the terms containing $\epsilon^2$ from [1]–[3], and $\epsilon$ from [4], we obtain a set of linear equations. On the other hand, keeping the terms containing $\epsilon^3$ from [1]–[3], and $\epsilon^2$ from [4], we obtain a set of nonlinear equations. These linear and nonlinear sets of equations can be reduced to a modified Burgers (MB) equation in the form

$$\partial_T \psi^{(1)} + \frac{\nu \psi^{(1)}}{2T} - A \psi^{(1)} \partial_R \psi^{(1)} = C \partial^2_R \psi^{(1)},$$

where $A$ and $C$ are the nonlinear and dissipation coefficients, respectively, and are given by

$$A = \frac{3}{2V_0} \sum_j \sum_{j'} \eta_j \gamma_j \gamma_{j'} \gamma_{j''},$$

$$C = \frac{1}{2} \sum_j \sum_{j'} \mu_j \gamma_j \gamma_{j'},$$

in which $\gamma_j = \left( 1 - \beta_j / V_0^2 \right)^{-1}$, $\gamma_j' = 1 + \beta_j / 6 V_0^2$, and $\eta_j$ ($= \zeta + \eta_j / 3$) is the longitudinal viscosity coefficient.

Now, transforming $T$ to $\tau = C T$, and denoting $A/C$ by $\Gamma$ (a ratio of the nonlinear coefficient $A$ to the dissipative coefficient $C$), one can express the MB equation (5) as

$$\partial_\tau \psi^{(1)} + \frac{\nu \psi^{(1)}}{2\tau} - \Gamma \psi^{(1)} \partial_\tau \psi^{(1)} = \partial^2_\tau \psi^{(1)}. \quad (8)$$

It is obvious from this equation that the extra-term, $\nu \psi^{(1)}/2\tau$ is due to the effect of the non-planar spherical geometry [since this extra-term disappears for a planar geometry ($\nu = 0$)], and that the effect of this extra-term diminishes as $\tau$ become significantly large. This means that for a large value of $\tau$, the SGA SSs for the nonplanar spherical spherical geometry ($\nu = 2$) are identical to those for the planar geometry ($\nu = 0$). Thus, for a large value of $\tau$ or $\nu = 0$, the stationary shock signal solution of (8) becomes [12 16]

$$\psi^{(1)} = -\frac{1}{2} \psi_m^{(1)} \left[ 1 - \tanh \left( \frac{\xi}{\Delta} \right) \right]. \quad (9)$$

where $\xi = R - U_0 T$ (with $U_0$ being the speed of the frame of reference), and $\psi_m^{(1)} = 2U_0 / (\Delta = 2 / U_0)$ is the height (thickness) of the monotonic SGA SSs. This equation implies that the monotonic SGA SSs are formed with $\psi < 0$, since $\Gamma$ is numerical found to be positive for all possible values of the parameters corresponding to the ACOs like white dwarfs and neutron stars. To show how the SGA SSs evolve with time, and how they are significantly modified by $\Gamma$, the transformed MB equation (8) is now numerically solved using an initial pulse represented by (3). The numerical results are displayed in figure [1] which indicates that (i) for a large value of $\tau$ (e. g. -20) planar and nonplanar spherical SSs are identical for a fixed value of $\Gamma$ since for a large value of $\tau$ the extra term ($\psi^{(1)}/\tau$ for spherical geometry) in (8) becomes insignificant; (ii) as the value of $\tau$ decreases, their height and thickness of the SGA SSs increase. This is due to the effect of spherical geometry since for lower values of $\tau$ the extra term ($\psi^{(1)}/\tau$) for spherical geometry in (8) becomes significant; and (iii) their height and thickness increase with the decrease of $\Gamma$.

To summarize, a generalized hydrodynamic model has been used to treat the nonlinear dynamics of different species of the SGDP systems like ACOs, which, in general, contains arbitrary number of degenerate, non-inertial particle (viz. electron or/and positron or/and quarks) species, and arbitrary number of degenerate, inertial particle (viz. proton or/and neutron or/and $^4\text{He}$ or/and $^6\text{C}$ or/and $^{26}\text{Fe}$ or/and $^{85}\text{Rb}$ or/and $^{92}\text{Mo}$). The existence of the SGA SSs with $\psi < 0$ in such a SGDP system is predicted for first time. It is found here that for a large value of $\tau$ planar and nonplanar (spherical) SSs are identical, but they evolve with time significantly, i. e. the height as well as the thickness of the SGA SSs increase as we observe them from an earlier time (viz. $\tau = 20$) to present time (viz. $\tau = -1$, since we cannot observe them at $\tau = 0$, where (8) has a pole). It is also observed that the coefficient of longitudinal viscosity ($\eta_j$)
acts as a source of dissipation, and is responsible for the formation of the SGA SSs in the dissipative SGDP systems like ACOs, and that their height as well thickness increases with the increase in the dissipative coefficient \( C \), which is directly proportional to \( \eta \).

The SGA SSs are associated with a new SGA mode in which if a disturbed ACO is compressed (expanded), the degenerate pressure brings it back to its equilibrium shape, but during this action it is expanded (compressed) more than its equilibrium shape according to Newton’s 1st law of motion, and again the self-gravitational pressure brings the system back to its equilibrium shape, but again during this action, it is compressed (expanded) more than its equilibrium shape according to the same reason, and so on.

The dissipative SGDP system considered here is generalized to arbitrary number of non-inertial and inertial degenerate particle species with their arbitrary mass densities. This theory is also general from the point of view that it can be applied in any ACO, where the effect of the nonlinearity is comparable to or much less/more than that of the dissipation. The investigation presented here can therefore be applied in any ACO.

It should be noted here that \( n_s, n_j, \psi_0 \) cannot be applied to describe the equilibrium state of the SGDP system under consideration. But this does not affect our investigation on the SGA SSs in any SGDQP under consideration. However, to explain the equilibrium state of the SGDQP system under consideration, one has to rewrite the basic equations in such a way that \( n_s, n_j, \psi_0 \) (self-gravitational potential at equilibrium) are not equal to zero or constant, but function of \( x \). This means that \( n_s, n_j \) and \( \psi_0 \) are not constant, but are the function of \( x \) so that the force associated with degenerate pressures is balanced by the self-gravitational force at equilibrium. This is the common scenario for many ACOs like white dwarfs and neutron stars [1]. To know the exact variation of \( \psi_0(x) \) with \( x \), one has to numerically solve Poisson’s equation for \( \psi_0(x) \) by choosing the appropriate variation of \( n_s(x) \) and \( n_j(x) \) with \( x \). However, the re-
The results presented here (particularly, the concept of new SGA mode, existence of new SGA SSs with new basic features (polarity, height, thickness, etc.) are correct from both analytical and numerical points of view.

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