Silent Phase Qubit Based on d-Wave Josephson Junctions

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We report on design and fabrication of a new type of flux qubit that capitalizes on intrinsic properties of submicron YBCO grain boundary junctions. The operating point is protected from the fluctuations of the external fields, already on the classical level; the effects of external perturbations are absent, to the second or third order, depending on the character of the coupling. We propose an experiment to observe quantum tunneling and Rabi oscillations in the qubit. Estimate of the decoherence due to fluctuations of the external flux is presented.

Over the last few years a series of experiments provided a conclusive evidence of quantum superposition in meso- and macroscopic superconductors. The task at hand is scaling up of the system, with two goals in mind: to probe how far quantum superposition can be pushed into macroscopic world, and to develop an element base for quantum computing, which only becomes viable on the scale 10–100 qubits.

It was suggested, that use of high-Tc cuprates in qubits would dispense of the need to apply fine-tuned external fields to keep it in the operating point, due to the time-reversal symmetry breaking in systems with DD junctions. On the level of a few qubits this is not a major advantage, compared to the relative difficulty of fabrication and threat of extra decoherence from nodal quasiparticles and zero-energy states (ZES) in cuprates. Therefore the research was concentrated on conventional superconductors, where all of the aforementioned successes were achieved.

Nevertheless, the logical progression of research from single qubit to qubit-qubit coupling and further to qubit registers is bringing us to the point where intrinsic bistability of high-Tc qubits will become a major advantage. In the last few years, we saw the development of reliable fabrication of bistable submicron high-Tc structures. The danger of nodal quasiparticles and ZES seems to be overestimated. Finally, putting aside any technological motivation, it is tempting to probe high-Tc compounds for the same kind of macroscopic quantum coherence as was observed in low-Tc superconductors, just to see how and whether these phenomena depend on, e.g., symmetry of the ground state of the system.

The “silent qubit” (for the etymology, see below) can be viewed as a SQUID formed out of high-Tc film, with two DD-grain boundary junctions forming a mesoscopic island (Fig. 1a-c). The crystal lattice and d-wave order parameter orientations on the two sides of the grain boundary are chosen in such a way that each junction has doubly degenerate ground states.

To simplify the analysis, we keep only two harmonics in the current-phase relations of the DD junctions: \( I_i = I_{0i} \sin \phi_i - I'_{0i} \sin 2\phi_i \), where \( I_{0i} \) is the Josephson current, and \( \phi_i \) is the phase difference across the \( i^{th} \) junction \((i = 1, 2)\). This approximation successfully reproduces the observed behaviour of similar devices in classical regime. The flux quantization condition binds the phases \( \phi_1 \) and \( \phi_2 \) to the total flux through the loop \( \Phi \) via \( \phi \equiv \phi_1 + \phi_2 = 2\pi \Phi/\Phi_0 \), where \( \Phi_0 = h/2e \) is the flux quantum. Introducing the superconducting phase of the island \( \theta \equiv (\phi_1 - \phi_2)/2 \), the free energy of the qubit, threaded by an external magnetic flux \( \Phi_x \), in units of the Josephson energy \( E_J \equiv E_0 \), where \( E_i = I_{ci}\Phi_0/2\pi \), is given by

\[
\mathcal{U}(\theta, \phi) = \frac{(\phi - \phi_x)^2}{2 \beta L} - \mathcal{E}_\phi \left[ \cos \theta - \frac{\alpha_\phi}{4} \cos(2\theta) \right] + \mathcal{U}(\theta, \phi).
\]

Here \( \alpha_i = 2I'_{0i}/I_{0i} \), \( \mathcal{E}_\phi = (1 + \eta) \cos(\phi/2) \), \( \tilde{\alpha}_\phi = \alpha_\phi [\alpha_\phi + \eta \alpha_\phi \cos(\phi/2) / (1 + \eta) \cos(\phi/2)] \), \( \eta = E_2/E_1 \), and \( \phi_x = 2\pi \Phi_x/\Phi_0 \). The dimensionless self-inductance of the
loop $\beta_L \equiv 2\pi LI_L / \Phi_0$ is considered negligible ($\beta_L \to 0$); then $\phi \to \phi_x$ and the first term in (1) can be dropped.

The last term in (1):

$$\tilde{U}(\theta, \phi) = - \left[ \eta - 1 + (\alpha_1 - \eta \alpha_2) \cos \frac{\phi}{2} \cos \theta \right] \sin \frac{\phi}{2} \sin \theta,$$

is zero when the two junctions are identical (i.e. $\eta = 1$ and $\alpha_1 = \alpha_2$). In this case the potential energy minima are at $\theta = \pm \arccos(1/\alpha_\phi)$ when $\alpha_\phi > 1$ and zero otherwise. The $\pm$ signs correspond to the states on the right and left sides of the potential well (i.e. $|\pm\rangle$), respectively. For $\alpha_\phi > 1$ the potential has two minima, which are degenerate at any external flux $\phi_x$. The current induced in the loop by the external flux does not depend on the state of the qubit, which justifies the moniker “silent”. The potential profile (1) is similar to the one of persistent current qubits, but here we have only one independent phase (as $\beta_L \to 0$), and the problem becomes one-dimensional. The barrier between the potential minima is flux-dependent: $W = \cos(\phi_x/2)(\alpha_\phi + \alpha_\phi^{-1} - 2)$.

In the general case ($\alpha_1 \neq \alpha_2$, $\eta \neq 1$) the two minima are only degenerate when $\phi_x = 0$. Now a state-dependent persistent current flows in the loop even in zero external field; in units of $I_{c1}$, $I_{c1}^\pm = \pm [\eta \sqrt{\alpha_\phi^2 - 1} \sin \theta, \eta / (\alpha_2 - \alpha_1)](\alpha_2 - \alpha_1)$, where $\alpha_\phi \equiv \alpha_{\phi=0}$.

An intermediate regime ($\alpha_1 = \alpha_2$, $\eta \neq 1$) is most interesting. It takes place when junctions have similar current-phase dependences, but different critical currents (i.e. widths), and should be expected if the junctions are close enough to each other (Fig. 1). At $\phi_x = 0$, the equilibrium value of $\theta$ is the same for both junctions, and there is no spontaneous current: $I_{c0}^\pm = 0$. At finite $\phi_x$, the induced currents differ for the two states of the qubit, but the difference is of higher order in $\phi_x$, keeping the qubit silent.

Expanding the free energy to the third order in $\phi_x$, we find for the minima:

$$U_{\text{min}}^\pm = A_0 + A_2 \phi_x^2 \pm A_3 \phi_x^3$$

where $A_i$ are explicit, but cumbersome, functions of $\alpha_0$ and $\eta$. As expected, there is no first order dependence on $\phi_x$ ($A_1 = 0$). The second order term in (2) does not depend on the state of the qubit. The first state-dependent term in the minimum energy of the system is $O(\phi_x^2)$ (Fig. 2). Therefore small fluctuations of $\phi_x$ do not affect the degeneracy of the states. The difference between the energy minima grows as the external flux is increased until the point at which the potential barrier vanishes altogether, and the minimum with higher energy disappears (the jumps in Fig. 2).

The current in the loop is found from $I_{c0}^\pm = \partial_{\phi_x} U_{\text{min}}^\pm = 2A_2 \phi_x \pm 3A_3 \phi_x^2$. This current generates a state-dependent magnetic flux $\delta \phi \equiv \phi - \phi_x = \beta_L I_{c0}^\pm$. Note that the state-dependent contribution to the induced flux is $O(\phi_x^2)$. For finite self-inductances $\beta_L$, $\delta \phi$ has to be calculated self-consistently. Figure 2 shows the result of such a calculation. With the parameters chosen, an external flux close to $0.2\Phi_0$ generates an additional flux ($\sim 0.005\Phi_0$) through the loop, which is of the same order as the estimate based on the above expansion.

Tunneling between the potential minima occurs due to the uncertainty relation between the charge $Q$ of the island and its superconducting phase $\theta$. The tunneling matrix element is approximately given by ($h = k_B = 1$)

$$\Delta \approx \omega_p(\phi_x)e^{-\sqrt{\zeta E_{0}}/E_c}(\phi_x - \phi_x^0)$$

where $\zeta$ is a constant of the order of 1. The coefficient $\omega_p(\phi_x) = \sqrt{\omega_p^0 \omega_p^0}$ is determined by the frequencies $\omega_p^0$ of small oscillations in the right and left potential minima, respectively. This dependence follows from the expression of $\Delta$ as the matrix element between the lowest energy states in the two wells. In the case of $\alpha_0 \sim 1$, it is only valid qualitatively, but enough for the present analysis. Due to the symmetry of the potential profile when $\alpha_1 = \alpha_2$, the linear dependence on $\phi_x$ cancel and we are left with

$$\omega_p(\phi_x) = \sqrt{E_{0}E_{c}(\alpha_0 - \alpha_0^{-1} - 2)}(1 - \kappa \phi_x^2),$$

where $E_{c} = e^2 / 2C$ is the charging energy, $C$ is the effective capacitance of the junctions, and $\kappa$ is a dimensionless coefficient of $O(1)$. Fluctuations of $\phi_x$ influence $W$ and therefore $\Delta$. Expanding the Josephson potential near the origin, we obtain the tunnelling barrier $W = U_{\text{max}} - U_{\text{min}} = (B_0 - A_0) + (B_2 - A_2) \phi_x^2 + O(\phi_x^4)$, where $B_0 = - (\eta + 1)(1 - \alpha_0/4), B_2 = - [(\alpha_0 - 1)/4][\eta/(\eta + 1)]$. Again there is no dependence on $\phi_x$ in the lowest order. The Hilbert space of the qubit to the two lowest energy states, one can write the effective Hamiltonian of the system as

$$H = \frac{1}{2} \sum_{x} \phi_x^2 \sigma_x + \frac{1}{2} (\phi_x^2) \sigma_z,$$

where $\epsilon(\phi_x) \approx E_{0}A_3 \phi_x^3$. All single qubit operations can be realized by applying controlled flux $\phi_x$. Note that the qubit only leaves the operating point when a finite external flux is applied. Unlike the earlier qubit design [23] this point is protected from external flux fluctuations, already on the classical level [cf. Eq. (2)].

Readout of the quantum state can in principle be achieved using a dc-SQUID to measure the magnetic flux generated by the qubit when it is biased by an external magnetic field. In this work the readout SQUID was fabricated onchip using a high-$T_c$ grain boundary SQUID (cf. Fig. 1); alternatively it can be made of aluminum.
Although coherent oscillations in an all-aluminum persistent current qubit have previously been observed using a similar readout design, the measured decoherence time appeared to be rather short. Subsequent results of Il'ichev et al.\textsuperscript{12} have demonstrated a longer decoherence time in a measurement scheme which couples the qubit to a high quality tank circuit. Here, we calculate the decoherence time of our silent qubit in the same setup (see Fig. 1b). The circuit is assumed to have a resonance inductance $L$ and tank respectively.

Without limiting the generality of our approach, we can ascribe all the dephasing and dissipation in the qubit to its interaction with the fluctuating flux $\phi_k(t)$, created by the tank current $I_T(t)$. These fluctuations are characterized by correlator $K(t, t') = \langle I_T(t), I_T(t') \rangle$, spectral density $K(\omega) = \frac{\omega(\gamma T \omega^2 / L_T \coth(\omega/2T) + 1)}{(\omega^2 - \omega_T^2)^2 + \gamma_T^2 \omega^2}$, and dispersion $\langle I_T^4 \rangle = K(t, t) = (\omega T / 2L_T \coth(\omega_T/2T)).$ At $\phi_k = 0$, the qubit Hamiltonian becomes $H = (\Delta/2)\sigma_x - \lambda_1 I_T^2 \sigma_z - \lambda_2 I_T^2 \sigma_z$

where $\lambda_1$ and $\lambda_2$ are coupling coefficients depending on the qubit parameters. In the Bloch-Redfield approximation\textsuperscript{13} we calculate the energy relaxation rate $\Gamma = 30\Delta^2 L_T^2 \gamma_0 / (\Delta^2/2T)$, together with a dephasing time of the qubit $\gamma^{-1}$, where $\gamma = 1/2 + \gamma_0$, $\gamma_0 = (16\pi/3)\lambda_2^2 (T^2 / L_T^2 \omega_T^4)$, if $T \ll \omega_T$, and $\gamma_0 = 3Q_T (\omega_T / L_T^2) [\coth^2 (\omega_T / 2T) - 1]$ at temperatures $T \gg \omega_T$.

Using the experimental data of Ref.\textsuperscript{12} ($I_c = 0.5 \mu A$, $I_0 = 0.6 \mu A$, $C \approx 10$ fF), we find $E_c \approx 2$ GHz, $\omega_c / 2\pi \approx 40$ GHz, and $\Delta / 2\pi \approx 1.6$ GHz. For $\beta_L \approx 0.01$, which is the value used in Fig. 3, the inductance of the loop is of the order of 10$^3 \mu H$. To estimate the contributions of the cubic and quadratic terms to qubit dephasing and dissipation, we chose the following parameters: $\gamma = 2$, $\alpha_1 = \alpha_2 = 2.4$, and $E_J = 1.66 \times 10^{-22} J$. If the tank frequency $\omega_T / 2\pi = 10$ MHz, its quality factor $Q_T = 2000$, and the coupling coefficient $k \sim 1/33$, then contribution of quadratic flux fluctuations to dephasing rate is small, so that the dephasing time due to qubit coupling to the tank is $\gamma_0^{-1} \approx 20$ ns at temperatures of order 10 mK, while the contribution of the cubic fluctuations to the dephasing and relaxation rates is totally negligible. It means that at the operating point the silent qubit is practically decoupled from the fluctuations caused by the controlling circuits. The dominant source of decoherence is from the nodal quasiparticles at the junction, considered in Ref.\textsuperscript{12} which may reduce the decoherence time to about 1–100 ns.

In conclusion, a new type of flux qubit, based on specific properties of submicron YBCO grain boundary junctions, is proposed and fabricated. The symmetry of the device provides an operating point, which is stable and protected against the external field fluctuations.

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