Rainbow gravity corrections to the information flux of a black hole

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In this paper, by utilizing the rainbow functions that were proposed by Amelino-Camelia et al., the information flux of rainbow Schwarzschild black hole and the sparsity of Hawking radiation in rainbow gravity are explored. The results show that the rainbow gravity has a very significant effect on the information flux. When the mass of rainbow Schwarzschild black hole approaches to the order of Planck scale, the Bekenstein entropy loss per emitted quanta in terms of the mass of Schwarzschild black hole reduces to zero. Furthermore, we also find the sparsity of Hawking radiation in rainbow gravity is no longer a constant; instead, it monotonically decreases as the mass of black hole decreases. At the final stages of evaporation, the modified sparsity of Hawking radiation becomes infinity, which indicates the effect of quantum gravity stops Hawking radiation and leads to remnant.

I. INTRODUCTION

In the theoretical physics, one of the surprising achievements is to prove that black holes have thermodynamic properties \cite{1-3}. This discovery started out with an analogy connecting the laws of gravity and those of thermodynamics. In 1973, Bekenstein pointed out that the entropy of a black hole $S$ can be defined in terms of horizon area $A$, namely, $S = A k_B c^3 / 4 \hbar^2 G$, with the Boltzmann constant $k_B$, Planck constant $\hbar$, and Newton's gravitational constant $G$ \cite{4}. Shortly afterwards, in Refs. \cite{5, 6}, Hawking put forward the theory of black hole radiation, which is called as Hawking radiation now. The theory of Hawking radiation includes at least two novelties, one of which is showing the black holes radiate as black bodies, with characteristic temperature as $T = \kappa / 2 \pi$, where $\kappa$ is the surface gravity of the black hole, and the other indicates that the black hole is not the end of stellar evolution. Since Hawking radiation is radically affected on the thermodynamic theory, gravitational theory, and quantum mechanism, it has received wide attention\cite{7-10}.

Despite most people have focused on using the Hawking radiation to analyze the thermodynamics of black holes in the past forty years, other properties of black holes can still be obtained by Hawking radiation such as the particle emission rates and information loss of black holes. As we know, the information of black holes can be reflected by their three “hairs”, namely, the mass $M$, charge $Q$, and angular momentum $\Omega$. As the black holes radiate the particles, their emission rates and information would change \cite{11}. Page first calculated the particle emission rates from an uncharged, non-rotating hole\cite{12}. Then, this work has been extended to other spacetimes \cite{13, 14}. In Refs. \cite{15-17}, Alonso-Serrano and Visser quantified the information budget in evolution of black hole by considering that the entropy flux of black holes is compensated by hidden information. Their results showed the lifetimes of black holes are related to the particle emission rates and information loss. Nevertheless, a lot of works claimed that the previous studies are not impeccable since the classical theory of Hawking radiation has some puzzles \cite{18-21}. For example, the black holes would evaporates completely into Hawking radiation since there is no cut-offs, it makes the singularity of black holes exposed in the universe \cite{21}. If one further assumes the radiation is pure thermal, the black holes then lost all their information, which leads to “Information loss paradox”. In order to solve those puzzles, the authors in Refs. \cite{20-25} analyzed how the black holes lose their information and how to recover it. In addition, the puzzles of black holes can be also solve by combining the models of quantum gravity with theory of Hawking radiation. According to the generalized uncertainty principle (GUP), which is a quantum gravity inspired correction to the Heisenberg’s uncertainty principle at Planck scale, Adler et al. calculated the GUP corrections to the thermodynamic evolution of black holes \cite{26-31}. Those results show the GUP can stop the evaporation of black holes and leads to remnant at the late stages of evolution, which indicates the GUP have an important effect on the information loss of black holes. Therefore, the GUP corrections to the information flux of black hole and the sparsity of Hawking radiation are investigated recently \cite{32, 33}. According to those modifications, it is found that the information/entropy flux is related to the mass of black hole, and the sparsity of Hawking radiation becomes thicker and thicker when a black hole approaches the Planck scale.

On the other hand, as the basis of loop quantum gravity (LQG), non-commutative geometry, spacetime discreteness, the standard energy-momentum dispersion relation would be changed to the so-called modified dispersion relation (MDR) when it approaches the Planck scale. Using the special relativity together with MDR, the double special relativity (DSR), which takes the speed of light $c$ and the Planck scale as constants, has put forward by Amelino-Camelia \cite{34}. Subsequently, Magueijo and Smolin generalized the DSR to the curved spacetime, and arrive at the theory of rainbow gravity (or doubly general relativity) \cite{35}. In the theory of rainbow grav-
ity (RG), the authors proposed the geometry of spacetime is related to the energy of the test particles. Hence, the background of this spacetime can be represented by a family of energy dependent metrics, namely, rainbow metrics. Now, the RG is considered as other promising candidates for a quantum gravity theory, which can modify the Hawking radiation and thermodynamic evolution of black holes just like GUP [36–45].

Due to the above discussion, one can find that both the GUP and RG have a very significant effect on radiations of black holes, which can be reflected by information flux. Therefore, inspired by the work in Refs. [32, 33], we calculate the RG corrections to the information flux of Schwarzschild (SC) black hole and its sparsity in this paper. To begin with, incorporating the line element of SC black hole with the rainbow functions that were proposed by Amelino-Camelia et al., the rainbow SC black hole is constructed. Then, using the relation $E \geq 1/r_{H} = 1/2GM$, the RG corrected Hawking temperature entropy are obtained. Finally, according to these modification, the information flux of rainbow SC black hole and the sparsity of Hawking radiation are analyzed.

The paper is organized as follows. In the next section, we briefly review the thermodynamics of rainbow SC black hole. Section III is devoted to investigating the information flux of rainbow SC black hole. In Section IV, we discuss the RG corrected sparsity of Hawking radiation. Finally, the discussion and conclusion are presented in Section V.

II. A BRIEF ON THE THERMODYNAMICS OF RAINBOW SC BLACK HOLE

In this section, we briefly review the thermodynamics properties of SC black hole in the RG. For obtaining these modifications, it is necessary to constructs the rainbow functions from the general form of MDR, which is

$$E^2F^2 (E/E_p) - p^2G^2 (E/E_p) = m^2, \quad \text{(1)}$$

with the Planck energy $E_p$, and the correction terms $F (E/E_p)$ and $G (E/E_p)$ are known as rainbow functions, which are required to satisfy the relationship

$$\lim_{E/E_p \to 0} F (E/E_p) = 1 \quad \text{and} \quad \lim_{E/E_p \to 0} G (E/E_p) = 1. \quad \text{In this case, Eq. (1) goes to the standard energy-momentum dispersion relation at low energy scale, that is, } E^2 - p^2 = m^2.$$  

In the literature of RG, the forms of rainbow functions are based on different phenomenological motivations. Many forms of rainbow functions are referred to in Refs. [46–48] and references therein. In this work, we employ one of the most interesting rainbow functions that was proposed by Amelino-Camelia et al. [49, 50]. Among the $\kappa$-Minkowski non-commutative geometry and LQG, Amelino-Camelia et al. constructed a form of MDR in the high-energy regime, which takes the form as

$$E^2 - p^2 + \eta \vec{p}^2 (E/E_p)^n \approx m^2.$$  

Comparing this MDR with Eq. (1), the rainbow functions can be expressed in the following form:

$$F (E/E_p) = 1, \quad G (E/E_p) = \sqrt{1 - \eta \left(\frac{E}{E_p}\right)^n}, \quad \text{(2)}$$

where $\eta$ and $n$ are the rainbow parameter and a positive integer, respectively. In Refs. [35, 51], the authors showed that the modified metric in gravity’s rainbow can be obtained by replacing $dt \to dt/F(E/E_p)$ for time coordinates and $dx^i \to dx^i/G(E/E_p)$ for all spatial coordinates. Hence, the metric in flat spacetime is given by

$$ds^2 = -dt^2/F(E/E_p)^2 + dx^i dx^i/G(E/E_p)^2,$$

and the metric of SC black hole in RG takes the form as follows:

$$ds^2 = -\frac{A(r)}{F(E/E_p)^2} dt^2 + \frac{B(r)^{-1}}{G(E/E_p)^2} dr^2$$

$$r^2 + \frac{1}{G(E/E_p)^2} d\Omega^2, \quad \text{(3)}$$

where $A(r) = B(r) = 1 - 2GkB M/c^3 h r$ and $d\Omega^2$ is the metric of two-dimensional unit sphere, respectively. By using the null hypersurface condition $\partial F/\partial x^i(\partial F/\partial x^i) = 0$, one can easily obtain the event horizon of rainbow SC black hole is $r_H = 2GkB M/c^3 h$. For a spherically symmetric spacetime, the original Hawking temperature satisfies the following expression

$$T_H = \frac{\kappa_H}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{1}{2} \nabla \mu \xi^\nu \nabla_\mu \xi^\nu \bigg|_{r=r_H}}$$

$$= \frac{c^3 h}{8\pi GMkB}, \quad \text{(4)}$$

where $\kappa_H = 1/4GM$ and $\xi_\nu$ represent the surface gravity on the event horizon of rainbow SC black hole and the time-like Killing vectors, respectively [52, 53]. Next, according to the rainbow functions Eq. (2), the Hawking temperature of rainbow SC black hole is given by

$$T_{H}^{RG} = \frac{\kappa_{H}^{RG}}{2\pi} = \frac{G(E/E_p)}{F(E/E_p)} \frac{\sqrt{\partial_r A(r_H)} \partial_r B(r_H)}{4\pi}$$

$$= \frac{c^3 h}{8\pi GMkB} \sqrt{1 - \eta \left(\frac{E}{E_p}\right)^n}, \quad \text{(5)}$$

where the $\kappa_{H}^{RG} = \frac{1}{2} \lim_{r \to r_H} \sqrt{\frac{\partial^2 \mu (\phi_H \phi_H)}{\partial \phi_H \partial \phi_H}} = \frac{\kappa_H G(E/E_p)}{2\pi F(E/E_p)}$ is the modified surface gravity. Following the argument in Refs. [26, 36, 54], the Heisenberg uncertainty principle $\Delta x \Delta p \geq \hbar c$ can be translated to a lower bound on the energy of radiant particle $E \geq \hbar c/\Delta x$ with the uncertainty position $\Delta x$. When considering the minimum value of $\Delta x$ equals to the event horizon $r_H$, one has

$$E \geq \hbar c/\Delta x \approx \hbar c/2r_H = \hbar^2 c^3/4GMkB. \quad \text{(6)}$$
Now, Substituting Eq. (6) into Eq. (5), the rainbow temperature can be rewritten as [36]

$$T_{H}^{RG} = \frac{c^3 \hbar}{8 \pi G M k_B} \left[ 1 - \eta \left( \frac{h^2 c^4}{4 G M k_B E_p} \right)^n \right]. \tag{7}$$

Then, based on the first law of black hole thermodynamics \( c^2 dM = TdS \) and Eq. (7), the rainbow entropy is expressed as

$$S_{RG} = \int \frac{8 \pi G M k_B}{c^3 \hbar} \left[ 1 - \eta \left( \frac{h^2 c^4}{4 G M k_B E_p} \right)^n \right]^{-\frac{1}{2}} \, dM. \tag{8}$$

It should be noted that Eq. (7) and Eq. (8) vary with the value of \( n \). Meanwhile, it is clear that integral in Eq. (8) does not have a solution for general \( n \). Therefore, in order to facilitate study of thermodynamic properties of rainbow SC black holes, we set \( n = 4 \). Henceforth, the temperature and entropy of rainbow SC black hole can be rewritten as follows:

$$T_{H}^{RG} = T_H \left[ 1 - \frac{1}{2} \eta \left( \frac{2 \pi c \hbar}{E_p} T_H \right)^4 + O(\eta) \right], \tag{9}$$

and

$$S_{RG} = S_0 - \frac{\eta}{2 S_0} \left( \frac{c^3 h^3}{4 \pi G k_B E_p^2} \right)^2 + O(\eta), \tag{10}$$

where \( S_0 = 4 \pi G M^2 k_B / \hbar c \) is the entropy of original SC black hole. It finds the modifications are not only related to the original thermodynamic quantities, but also to the Planck length \( E_p \), and the rainbow parameter \( \eta \). When \( \eta \to 0 \), the modified temperature reduces to the original case. Besides, one can obtain the remnant mass of rainbow SC black hole when it approaches the final stages of evaporation. For example, by keeping the first order term of Eq. (9), the remnant mass is

$$M_{\text{res}} = c^2 \hbar^2 \eta^{1/4} / 4 \times 2^{1/4} G \pi k_B E_P.$$  

III. THE INFORMATION FLUX OF RAINBOW SC BLACK HOLE

According to the viewpoint in Ref. [55], it is feasible to assume an exact Planck spectrum at the Hawking temperature and the thermodynamic entropy is related to the lack of information. Therefore, in order to analyze the information flux of rainbow SC black hole, it is necessary to calculate the Bekenstein entropy loss per emitted quanta in terms of the mass and total number of emitted particles. Based on Eq. (10), one can obtain the modified Bekenstein entropy loss of rainbow SC black hole per emitted quanta [17, 56]

$$\frac{dS_{RG}}{dN} = \frac{dS_0}{dN} \left[ 1 + \frac{\eta}{2 S_0} \left( \frac{c^3 h^3}{4 \pi G k_B E_p^2} \right)^2 + O(\eta) \right], \tag{11}$$

where \( N \) is the number of particles, and Bekenstein entropy loss of original SC black hole per emitted quanta is

$$\frac{dS_0}{dN} = \frac{dS_0/\!dt}{dN/\!dt} = \frac{8 \pi k_B M}{c^2 m_p^2} \hbar \langle \omega \rangle. \tag{12}$$

With the help of the definition of Bekenstein entropy and the conservation of energy \( \langle E \rangle = \hbar \langle \omega \rangle = \pi^4 k_B T_{H}^{RG} / 30 \zeta(3) \), Eq. (12) can be rewritten as

$$\frac{dS_0}{dN} = \frac{k_B \pi^4}{30 \zeta(3)} \frac{8 \pi k_B M}{c^2 m_p^2} T_{H}^{RG}. \tag{13}$$

For a Planck spectrum of emitted particles, the Bekenstein entropy loss of original SC black hole per emitted quanta is given by

$$\frac{dS_0}{dN} = \frac{k_B \pi^4}{30 \zeta(3)} \left[ 1 - \frac{\eta}{2} \left( \frac{c^4 \hbar^2}{4 G k_B E_p} \right)^2 + O(\eta) \right]. \tag{14}$$

Then, substituting Eq. (14) into Eq. (11), the Bekenstein entropy loss of rainbow SC black hole per emitted quanta becomes

$$\frac{dS_{RG}}{dN} = \frac{k_B \pi^4}{30 \zeta(3)} \left[ 1 - \frac{\eta}{2} \left( \frac{c^2 \hbar^2}{\pi G k_B E_p} \right)^8 + O(\eta) \right]. \tag{15}$$

From abovementioned equation, one can plot the Bekenstein entropy loss per emitted quanta in terms of the mass of SC black hole for different values of rainbow parameter \( \eta \) in Fig. 1.

![Fig. 1. Bekenstein entropy loss per emitted quanta in terms of the mass of SC black hole for different \( \eta \). Here, we choose natural units \( \hbar = k_B = E_p = 1 \).](attachment:image.png)

In Fig. 1, in case \( \eta = 0 \), one can observe that the Bekenstein entropy loss of original SC black hole per emitted
quanta $dS_0/dN$ (black solid line) is a constant, which is about 2.70. The blue dashed line, red dotted line and pink dot-dashed line in this diagram illustrate the Bekenstein entropy loss of rainbow SC black hole per emitted quanta $dS_{RG}/dN$, the value of rainbow parameter $\eta$ decreases from bottom to top. When mass of the black hole is large enough, the behavior of $dS_{RG}/dN$ is similar to that of the original case, it implies the effect of RG is negligible at big scale. However, the behavior of Bekenstein entropy loss of rainbow SC black hole per emitted quanta is apart from that of the original case with the development of evolution. It is clear that $dS_{RG}/dN$ monotonically decreases in mass. Furthermore, when the mass of rainbow SC black hole approaches the Planck mass, the $dS_{RG}/dN$ reaches zero, which indicates that the effect of rainbow gravity stops Hawking radiation in the final stages of black holes’ evolution and stores the information in the remnants. This result is in line with the work in Ref. [36].

Next, according to Eq. (9), the original total number of emitted quanta can be expressed as follows:

$$dN = 30\zeta(3) \frac{c^2}{k_B^4 \pi^4 T_{RG}^4}$$

$$= 30\zeta(3) \frac{c^2}{k_B^4 \pi^4 T_H^4} \left[1 - \eta \left(\frac{\hbar^2 c^4}{4GM k_B E_p}\right)^2 \right]^{\frac{1}{2}}.$$  \hspace{1cm} (16)

By integrating the above equation, the total number of particles emitted from rainbow SC black hole is given as follows:

$$N = 30\zeta(3) \frac{4\pi G M^2}{ch} \left[1 - \eta \left(\frac{\hbar^2 c^4}{4GM k_B E_p}\right)^2 \right]^{\frac{1}{2}} + O(\eta),$$  \hspace{1cm} (17)

where $M$ is the initial mass of rainbow SC black hole. In Ref. [32], the entropy in nats is $\hat{S} = S_0/k_B = 4\pi GM^2 / ch$. In this case, Eq. (17) can expressed in terms of the entropy in nats, that is

$$N = 30\zeta(3) \frac{\hat{S}}{\pi^4} \left[\hat{S}^* - \frac{\eta}{(4\pi G)^2} \left(\frac{c^3 \hbar^3}{k_B^4 E_p^2}\right)^2 \frac{1}{\hat{S}} + O(\eta) \right].$$  \hspace{1cm} (18)

It should be noted that the original total number of particles emitted from a black hole $N_0 = 30\zeta(3) \hat{S}/\pi^4$. Obviously, Eq. (18) shows that the quantum gravity effect can effectively reduce total number of particles emitted from black hole. Moreover, by setting $\hbar = k_B = E_p = 1$, the total number of particles emitted from SC black hole as a function of $\eta$ is plot in Fig. 2.

As one can see from Fig. 2, the blue dashed line, red dotted line and purple dot-dashed line illustrate the RG corrected total number of particles emitted $N$, whereas original cases is represented by the black solid line. At the early stage of black hole evolution, these lines coincide together. After that, , the total numbers of particles emitted are gradually reduced via the Hawking radiation, the original total number of particles emitted $N_0$ vanishes when $M \rightarrow 0$, while the RG corrected one reduce to zero at $M_{\text{res}}$. This indicates that the quantum gravity effect can obviously affect the evolution of black holes.

IV. THE RG CORRECTED SPARSITY OF HAWKING RADIATION

Another important property of Hawking flux is its sparsity. In Ref. [55], the authors showed that the sparsity of Hawking flux can be describe by a dimensionless parameter $\chi$, which is the ratio between an average time between the emission of two consecutive quanta and the natural time scale. The result implies that the sparsity of Hawking radiation is thin during the whole evaporation process. Recently, researchers show that Hawking radiation is no longer sparse via the effect of GUP with positive parameter [32]. However, when considering the negative GUP parameter, the sparsity of Hawking radiation would be enhanced [56]. Therefore, it is interesting to analyze how does the effect of RG affect the sparsity of Hawking radiation. Now, the expression of dimensionless parameters is given by

$$\chi = C\lambda_{\text{thermal}}^2/gA_{\text{effective}},$$  \hspace{1cm} (19)

where $C$ represent a dimensionless constant that depends on the specific parameter $\chi$ we are choosing, $g$ is the spin degeneracy factor, $A$ is the effective area and $\lambda_{\text{thermal}} = 2\pi \hbar/c k_B T_0$ is the thermal wavelength of a Hawking particle, respectively. It is well known that the area and horizon radius of SC black hole satisfying relationship $1/4A_H = \pi r_H^2$. However, this relationship is
actually only applicable to some certain types of particles in the low frequency limit. For the high frequencies cases, the relationship between area and horizon radius of SC black hole becomes \(27A_H/16 = 27\pi r_H^2/4\) with the enhancement factor \(27/4\) \([55]\). Hence, the effective area of original SC black hole in the high frequency limit is defined by \(A_{\text{effective}} = 27A_H/4\). Substituting the original thermal wavelength and effective area into Eq. (19), the original relevant factor in any dimensionless parameter for massless bosons is

\[
\chi_0 = \frac{\lambda^2_{\text{thermal}}}{A_{\text{effective}}} = \frac{64\pi^3}{27} \approx 73.5.
\]

(20)

It is clear that Eq. (20) is related to the properties of a black hole, which leads to the final result being a constant, it means the sparsity never change during the whole evaporation process. However, when considering the effect of RG, the modified effective area can be expressed as following:

\[
\lambda_{\text{RG}}^{\text{effective}} = 27 \left[ \lambda_{H} - \eta \frac{h^8}{2\pi^2k_B^4E_pA_H} + \mathcal{O}(\eta) \right],
\]

(21)

and the modified thermal wavelength becomes

\[
\lambda_{\text{thermal}}^{\text{RG}} = \frac{2\pi\hbar c}{k_B T_H} = \frac{2\pi\hbar c}{k_B T_0} \left[ 1 - \frac{1}{2\eta} \left( \frac{2\pi\hbar c}{E_p T_0} \right)^4 + \mathcal{O}(\eta^2) \right]^{-1}.
\]

(22)

According Eq. (21) and Eq. (22), the RG corrected dimensionless parameter is given by

\[
\chi^{\text{RG}} = \frac{\lambda_{\text{RG}}^{\text{thermal}}}{\lambda_{\text{RG}}^{\text{effective}}} = \frac{64\pi^3}{27} \left( \frac{\pi GMk_B E_p}{\lambda_{\text{RG}}^{\text{effective}}} \right)^{12} = \frac{64\pi^3}{27} \left( \frac{\pi GMk_B E_p}{\lambda_{\text{RG}}^{\text{effective}}} \right)^{12}.
\]

(23)

Different from the original case \(\chi_0\), Eq. (23) shows that the modified dimensionless parameter is not only dependent on the ratio \(64\pi^3/27\), but also determined by the mass of black hole \(M\), Planck energy \(E_p\) and RG parameter \(\eta\). In order to discuss the behaviors of dimensionless parameter \(\chi^{\text{RG}}\), we plot Fig. 3.

In Fig. 3, the blue dashed line, red dotted line and purple dot-dashed line for modified dimensionless parameter \(\chi^{\text{RG}}\) diverges when the mass approaches \(M_{\text{res}}\), which is different from the black solid line for original dimensionless parameter \(\chi_0\) that keeps a constant value during the whole evaporation process. This implies that, due to the effect of RG, the pause time between in Hawking radiation becomes longer and longer. The rainbow SC black hole takes infinite time to radiate a particle when the black hole at the final stages of evaporation. In other words, the black hole does not radiate any particle or lose its information at that time. Furthermore, one may find that our result is different those modified by GUP. In Ref. [32], for a positive GUP parameter, namely \(\alpha > 0\), the radiation is no longer quite sparse when the mass approaches the Planck scale. However, if the GUP parameter becomes negative, the radiation becomes infinitely sparse when \(M \to 0\) \([56]\). Those indicate that different models of quantum gravity would cause the different results.

V. DISCUSSION

In the present work we have investigated the quantum gravity corrections to information flux of SC black hole and its sparsity via the rainbow functions that have been proposed by Amelino-Camelia, et al. First, by using the thermodynamics quantities of rainbow SC black hole, we found a new relationship between the mass and Bekenstein entropy loss per emitted quanta, which implies that information flux of rainbow SC black hole varies with its mass. When rainbow SC black hole approaches the Planck scale, information flux would reduce to zero. Accordingly, the effect of RG can stop the evaporation of black hole and leads to a remnant. Hence, one can study the lifetime of rainbow SC black hole via its information flux. Subsequently, according to RG corrected information flux, sparsity of Hawking radiation has also been analyzed. The results showed that the sparsity of Hawking radiation is no longer a constant, instead, it monotonically decreases as the mass of black hole decrease. From Fig. 3, one can see that the modified sparsity diverges as \(M \to M_{\text{res}}\), which indicates that the pause time between in Hawking radiation becomes longer and longer. Finally, it is also found that the modified dimensionless param-
eter $\chi$ in this work is different from the GUP corrected dimensionless parameter. That difference may cases the different models of quantum gravity. Actually, there are a lot of works try to investigate the relationship between GR and GUP since they are able to influence the evaporation process of a black hole, and our work just showed the similarities and differences between the RG and GUP from the perspective of information loss.

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