FLOW-INDEPENDENT ANISOTROPIC SPACE, AND
PERTURBATION OF RESONANCES

YANNICK GUEDES BONTHONNEAU

Abstract. Given an Anosov vector field $X_0$, all sufficiently close vector fields also are of Anosov type. In this note, we check that one can choose the anistropic spaces described in [FS11], [DZ16] to be adapted to any vector field sufficiently close to $X_0$ in $C^1$ norm.

In this note, $M$ will denote a compact manifold, and $X_0$ a smooth Anosov vector field on $M$. That is to say that we have a splitting

$$TM = \mathbb{R}X_0 \oplus E_u^0 \oplus E_s^0.$$ 

This splitting is invariant under the flow of $X_0$, which is denoted $\varphi_t^0$. We also have constants $C, \beta > 0$ such that for all $t \geq 0$,

$$\|d\varphi_{-t}^0|E_u^0\| \leq Ce^{-\beta t}, \quad \text{and} \quad \|d\varphi_{t}^0|E_s^0\| \leq Ce^{-\beta t}.$$ 

(here the norm is a fixed norm a priori, and one can check that although the constants depends on that choice of norm, their existence does not).

The main theorem of this paper is the following:

**Theorem 1.** There is an $\eta > 0$ such that the following holds. For any $\alpha > 0$, we can find a Hilbert space of distributions $\mathcal{H}_\alpha$ on $M$ such that for $\|X - X_0\|_{C^1} < \eta$, the spectrum of $X$ acting on $\mathcal{H}_\alpha$ is discrete in $\{\Re s > -\alpha, |\Im s| < \alpha^2\} \cup \{\Re s > \alpha\}$. The space is build in the same fashion as the spaces in [FS11].

Starting with the original papers by Ruelle [Rue78], [Rue86] and Pollicott [Pol85], several techniques have been developped to obtain discrete spectrum from expanding and later hyperbolic dynamics. Among the works concerning Anosov flows, there is a variety of approaches. At one end of the spectrum, we have techniques heavy on dynamical systems tools, and particularly suited to low regularity problems — see [Liv04, BL07, BT07]. At the other end, one uses the least dynamical tools, and resort to semi-classical microlocal analysis to compensate; this is more adapted to $C^\infty$ systems — see [FRS08, FS11, DZ16]. This is a very small and arbitrary sample of the litterature.

To study perturbation problems, it is very convenient to have an anisotropic space that does not depend on the system. This was remarked and used by authors in the dynamical community, for example...
in the case of diffeomorphisms in [GL06], and in the case of flows in [BL07]. There seemed to lack in the litterature a proof that the same can be done with the semi-classical techniques, and this is the point of the paper. In the cited papers, one can find perturbation results for the dominating resonance, related to the SRB measure. Actually, one can follow an arbitrary finite number of resonances:

**Corollary 2.** Consider \( \lambda_1, \ldots, \lambda_N \) a finite set of resonances of \( X_0 \) and \( \epsilon \mapsto X_\epsilon \) a \( C^\infty \) family of \( C^\infty \) vector fields perturbing \( X_0 \). Then, there is a \( \epsilon_0 > 0 \), such that we can find continuous functions \( \lambda_i(\epsilon) \) on \([-\epsilon_0, \epsilon_0]\), and an open set \( \Omega \subset \mathbb{C} \) such that \( \lambda_i(\epsilon) \in \Omega \), and the intersection of the spectrum of \( X_\epsilon \) with \( \Omega \) is the set \( \{ \lambda_1(\epsilon), \ldots, \lambda_N(\epsilon) \} \). Additionally, the resonances are smooth functions of \( \epsilon \) where they do not intersect.

This would follow easily from the techniques in [BL07], for example, but it was not stated as such — or anywhere else seemingly. The same statement should also hold in finite regularity, but we are not using the best tools to tackle this type of question, so we ignore it altogether.

As the reader knowledgeable to the microlocal techniques will expect, most of the effort goes into building an escape function. In what follows, we consider \( X \) to be another vector field, assumed to be close to \( X_0 \) in \( C^1 \) norm — we will be clear when we use that assumption. We will denote by \( \varphi_t \) the corresponding flow.

There is a natural extension of \( \varphi_t^{(0)} \) to \( T^*M \), given by

\[
\Phi_t^{(0)}(x, \xi) = (\varphi_t^{(0)}(x), ((d\varphi_t^{(0)})^{-1})^T \xi)
\]

This suggests to consider the following dual splitting

\[
T^*M = E^*_{00} \oplus E^*_{u0} \oplus E^*_{s0},
\]

where

\[
E^*_{00} = (E^u_0 \oplus E^s_0)^\perp, \quad E^*_{u0} = (\mathbb{R}X_0 \oplus E^s_0)^\perp, \quad E^*_{s0} = (E^u_0 \oplus \mathbb{R}X_0)^\perp.
\]

The splitting is \( \Phi_t^{0} \)-invariant, and we get the contraction properties (seeing \( \Phi_t^{0} \) as a bundle mapping)

\[
\|\Phi_{-t|E_{s0}^*}\| \leq Ce^{-\beta t}, \quad \text{and} \quad \|\Phi_{t|E_{u0}^*}\| \leq Ce^{-\beta t}.
\]

Since the symbol of \( X^{(0)} \) is 1-homogeneous, the flow of its principal symbol is also 1-homogeneous, and we can consider its action on the sphere at infinity \( S^*M \) in the cotangent space, \( \Phi_t^{(0)}|_{S^*M} \). The main technical result is thus

**Lemma 3.** There is a function \( \mathbf{m} \in C^\infty(S^*M) \) and \( \eta_0 > 0 \), such that when \( \|X - X_0\|_{C^1} \leq \eta_0 \), \( X\mathbf{m} \geq 0 \). In a neighbourhood of \( E^*_{u0} \oplus E^*_{00} \) (resp. \( E^*_{s0} \)), \( \mathbf{m} \) is constant equal to +1 (resp. -1).
1. Building the escape function

We start as in [FS11]. We are now working in $S^*M$. Recall the following:

**Lemma 4.** The bundles $E^u$ and $E^s$ are continuous. In particular, the angle between any two bundles in $E^u$, $E^s$, $E^0 = \mathbb{R}X_0$ is bounded by below.

This is a very usual lemma, contained in theorem 3.2 in [HP70].

**Lemma 5.** Given $\epsilon > 0$, there exist $T > 0$ such that when $t \geq T$, for $\xi \in S^*M$,

$$d(\xi, E^s_{s0}) > \epsilon \Rightarrow d(\Phi^0_{\infty}(\xi), E^s_{u0} \oplus E^s_{00}) \leq \epsilon$$

and

$$d(\xi, E^s_{u0} \oplus E^s_{00}) > \epsilon \Rightarrow d(\Phi^0_{-\infty}(\xi), E^s_{s0}) \leq \epsilon$$

In the proof, the constant $\infty > C > 0$ may change at every line, but it is always controlled by the lower bound on the angles between the bundles.

**Proof.** Since the angles between the distributions $E^s_{u0,s0,00}$ are bounded by below, we have a constant $C > 0$ such that if $\xi = \xi_s + \xi_u + \xi_0$ is the decomposition with respect to $(\Pi)$,

$$d(\xi, E^s_{s0}) > \epsilon \Rightarrow |\xi_s + \xi_u| > C\epsilon.$$

Next, we also have

$$|\xi_0| = \sup_{|u|=1} \xi_0(u) = \sup \{\xi(\alpha X_0) \mid \|\alpha X_0 + u'\| = 1, \ u' \in E^u_0 \oplus E^s_0\}.$$ 

by the bounded angle property, if $\|\alpha X_0 + u'\| = 1$, we have $|\alpha| \leq C$. We deduce ($X_0$ never vanishes !)

$$\frac{1}{C}|\xi(X_0)| \leq |\xi_0| \leq C|\xi(X_0)|.$$ 

Since by definition, $\varphi^0_{t\epsilon}(X_0) = X_0$, $(\Phi^0_{t\epsilon}(\xi))(X_0(\varphi_t(x)) = \xi(X_0)$, we deduce that

$$|\Phi^0_{t\epsilon}(\xi)| \geq \frac{1}{C}(|\Phi^0_{t\epsilon}(\xi_u)| + |\Phi^0_{t\epsilon}(\xi_0)|) \geq (|\xi_u| + |\xi_0|)/C^2 \geq \epsilon/C^3.$$ 

In particular, again using the bounded angle property,

$$d(\Phi^0_{t\epsilon}(\xi), E^s_{u0} \oplus E^s_{00}) \leq C\frac{|\Phi^0_{t\epsilon}(\xi_s)|}{|\Phi^0_{t\epsilon}(\xi)|} \leq \frac{C}{\epsilon} e^{-\beta t}.$$
To obtain the first implication, taking $T \geq (\log C - 2 \log \epsilon) / \beta$ will suffice, and for the second implication, a similar reasoning will apply.

Now, let us consider $m_0 \in C^\infty(S^*M)$ taking values in $[0,1]$, such that

$$
\{ \xi \mid d(\xi, E^*_{s0}) \leq \epsilon \} \subset \{ m_0 = 0 \},
$$

and

$$
\{ \xi \mid d(\xi, E^*_{a0} \oplus E^*_{b0}) \leq \epsilon \} \subset \{ m_0 = 1 \}.
$$

This can be done for $\epsilon$ small enough because the bundles are continuous and always transverse. Then, let

$$
m := \int_{-T}^{T} m_0 \circ \Phi^0_{t,\infty} dt.
$$

The derivative of $m$ along the flow $\Phi^0_{t,\infty}$ is

$$
F := m_0 \circ \Phi^0_{T,\infty} - m_0 \circ \Phi^0_{-T,\infty}.
$$

(it is a smooth function). Consider a point $\xi \in S^*M$ such that $F(\xi) = 0$.

Assume that $m_0(\xi) \in ]0,1[$. Then by definition of $T$, we get that $m_0(\Phi^0_{T,\infty}(\xi)) = 1$ and $m_0(\Phi^0_{-T,\infty}(\xi)) = 0$. This contradiction implies that $m_0(\xi)$ is either 0 or 1. By symmetry, we can assume that $m_0(\xi) = 0$.

In that case, we get that $m_0(\Phi^0_{T,\infty}(\xi)) = 0$. Then, again using the definition of $T$, we get that $m_0(\Phi^0_{-T,\infty}(\xi)) = 0$ for $t \geq 0$. In particular, we have

$$
m(\xi) = \int_{0}^{T} m_0(\Phi^0_{t,\infty}(\xi)) dt < T.
$$

Conversely, if we assumed that $m_0(\xi) = 1$, we would find that

$$
m(\xi) > T.
$$

Now, we deduce the crucial lemma

**Lemma 6.** There are constants $\epsilon, \delta > 0$ such that

$$
\{ |m - T| < \epsilon \} \subset \{ F(\xi) \geq \delta \}.
$$

**Proof.** Since $\{ F(\xi) = 0 \}$ is compact, the inf of $|m - T|$ is attained and by the preceding argument, is strictly positive. Denote it by $2\delta$.

Now, consider

$$
\ell(\epsilon) := \sup\{ d(\xi, \{ F = 0 \}) \mid F(\xi) \leq \epsilon \}.
$$

By continuity of $F$ and compactness of $S^*M$, we have that $\ell(\epsilon) \to 0$ as $\epsilon \to 0$. Finally, since $m$ is continuous, we have an $\epsilon' > 0$ such that
whenever \(d(\xi, \{F = 0\}) < \epsilon', \ |m - T| > \delta\). It suffices then to take \(\epsilon\) such that \(\ell(\epsilon) < \epsilon'\). \(\square\)

Let us now come back to our perturbation problem. Consider \(X = X_0 + \lambda V\) with \(V\) a smooth vector field with \(\|V\|_{C^1} \leq 1\), and \(\lambda > 0\) small. The vector fields generating \(\Phi_t^{(0)}\) are the hamiltonian vector fields of the principal symbols of \(-iX_0\), which are \(\xi(X_0)\). In particular, they involve the first derivative of the vector fields \(X_0\), so that they are \(\mathcal{O}(\lambda)\)-\(C^0\)-close, with a constant not depending on \(V\).

Then the vector fields on \(S^*M\), \(X_0^\infty\) and \(X^\infty = X_0^\infty + \lambda V^\infty\) that generate the boundary flows \(\Phi_t^{(0),\infty}\) also are \(\mathcal{O}(\lambda)\)-\(C^0\)-close, since they are the projection on \(T(S^*M)\) of the previous hamiltonian vector fields. Observe that

\[
X^\infty m = X_0^\infty m + \lambda \int_{-T}^T V^\infty(m_0 \circ \Phi_t^{0,\infty}) dt
\]

Since \(\|V^\infty\|_{L^\infty} = \mathcal{O}(1)\), the integral in the RHS is of size \(\mathcal{O}(T\lambda)\). Now we will use our previous arguments. Let \(\chi\) be a smooth function on \(\mathbb{R}\), such that \(\chi\) is constant equal to \(-1\) in \([-\infty, -\epsilon]\) and constant equal to \(1\) in \([\epsilon, +\infty[\), and strictly increasing in \([-\epsilon, \epsilon]\). Let

\[
m := \chi(m - T).
\]

We get directly that

\[
X_0^\infty m \geq 0
\]

But we also have

\[
X^\infty m = \chi'(m - T) \left[ X_0^\infty m + \lambda \int_{-T}^T V^\infty(m_0 \circ \Phi_t^{0,\infty}) dt \right]
\]

On the support of \(\chi'(m - T)\), we have \(X_0^\infty m \geq \delta\). In particular, with \(\lambda\) smaller than \(\eta_0 = \delta/CT\) with \(C > 0\) large enough, we get that \(X^\infty m \geq 0\).

2. Sketch of proof of the discrete spectrum

Now that we have built our weight function, we have to build the global escape function. Take a cutoff \(\kappa \in C^\infty(T^*M)\), supported in \(\{\|\xi\| < 2\}\), and equal to \(1\) in \(\{\|\xi\| \leq 1\}\); let

\[
G(x, \xi) = \chi(x, \xi) m(x, \xi/\|\xi\|) \log(1 + \|\xi\|).
\]

This is a symbol of order \(\log\). (note that the weight is constant in a neighbourhood of \(E_{u0}^* \oplus E_{00}^*\) instead of the usual more refined escape
function). If \( X \) is the hamiltonian vector field associated to the symbol \( p = \xi(X) \) of \(-iX\) seen as a differential operator, we have

\[
X \star G = \left( X \star \chi \right) m(x, \xi/|\xi|) \log(1 + |\xi|) + \chi(X^\infty m) \log(1 + |\xi|) + \chi m \frac{X_s |\xi|}{1 + |\xi|}.
\]

This is bounded by below by some constant \(-C \leq 0\).

Now, consider the space

\[
\mathcal{H}_\alpha := \text{Op}_h(e^{-\alpha G}) \cdot L^2(M).
\]

for some \( h > 0 \). The action of \( X \) on \( \mathcal{H}_\alpha \) is equivalent to the action on \( L^2(M) \) of

\[
X_\alpha := X - \alpha \text{Op}_h(X_s G) + O(h\Psi^{-1+}).
\]

The principal semi-classical symbol of \(-ihX_\alpha\) is

\[
p_\alpha = \xi(X) + ih\alpha X_s G \mod (hS^0).
\]

We get that \( \Re p_\alpha \geq 0 \). In particular, we can apply the propagation of singularities theorem to \( X_\alpha \).

Since we will base ourselves on the arguments of [DZ16], we use the concepts of sources and sinks. A closed invariant conical set \( L \subset T^*M \) is a sink for \( \Phi_t \) if it has an open conical neighbourhood \( U \) such that for all \( \xi \in U \), for some constants,

\[
|\Phi_t(\xi)| \geq e^{\beta t}|\xi|/C,
\]

and

\[
\Phi_t^\infty(U \cap S^*M) \to L \cap S^*M.
\]

A source is a sink for negative times. We get

**Lemma 7.** There is an \( 0 < \eta \leq \eta_0 \) such that the following holds. Let \( X \) be a smooth vector field such that \( \|X - X_0\|_{C^1} \leq \eta \). Then the corresponding flow on \( T^*M \), \( \Phi_t \), when restricted to \( \{\xi(X) = 0\} \) has a source \( E^s_\alpha \) and a sink \( E^u_\alpha \), which are close to respectively \( E^s_{\alpha 0} \) and \( E^u_{\alpha 0} \).

We also get that all points that are not in a given neighbourhood of the source end up in finite positive time (depending only on the neighbourhood) in a small neighbourhood of the sink independent of \( X \) (and the converse statement in negative time for the source).

Finally, we have that the source is contained in \( \{m = -1\} \) (and the sink in \( \{m = 1\} \)).

**Proof.** Following the arguments in the proof of Lemma 5, we can find open cones \( V_1 \subset V_2 \) containing \( E^s_{\alpha 0} \) as small as desired, such that \( \overline{V_1} \subset V_2 \), and for some \( t > 0 \), \( \Phi^0_t(V_2) \subset V_1 \), and \( |\Phi^0_t(\xi)| > 3|\xi| \) for all \( \xi \in V_2 \).
Since $\|V\|_{C^1} \leq 1$, for $\lambda$ small enough, we find that $\Phi_t(V_2) \subset V'_1$, where $V_1 \subset V'_1 \subset V_2$, and $\overline{V}_1 \subset V_2$. We also get that $|\Phi_t(\xi)| > 2|\xi|$ for all $\xi \in V_2$. Now, we let

$$E_u^*(x) := \{\xi \in V_2(x) \mid \Phi_{-t}(\xi) \in V_2 \text{ for all } t \geq 0\}.$$ 

Since we can write this as a decreasing intersection of compacts sets (compact in $T^*_x M \cup S^*_x M$), it is non empty, closed, and it is a cone by linearity of $\Phi_t$. We deduce that $E_u^*$ is a sink for $\Phi_t$.

Likewise, we can find similar cones around $E_{s_0}^*$ for negative times, and obtain that the corresponding $E_s^*$ is a source.

For the points that are neither in the source nor in the sink, we can directly use Lemma 5. Finally, since we could choose the neighbourhood $V_2$ as small as desired, since $E_u^* \subset V_2$, and since $m = +1$ in a neighbourhood of $E_{s_0}^*$, the proof is complete. \hfill $\square$

One can check that the arguments in [DZ16] apply in our context, and we will no go into further details about the proof of theorem 1. However, we will need the following lemma. First, define

$$H^n_\alpha := \text{Op}_h(e^{-\alpha G})H^n(M).$$

Then, by the methods in [DZ16],

Lemma 8. Consider $X_\epsilon$ a smooth family of vector fields perturbing $X_0$. Fix $Q \in \Psi^{-\infty}$ compactly microsupported around the 0 section, and elliptic there. Then, there is $\epsilon_0 > 0$ such that the following holds. Given any $n$, there are $\alpha_n, h_n > 0$ small enough such that $(X_\epsilon - h^{-1}Q - s)^{-1}$ is invertible on $H^n_\alpha$ for $h < h_n$, $\alpha > \alpha_n$, $|\Re s| > -\alpha/2$, $|\Im s| < h^{-1/2}$ and $|\epsilon| < \epsilon_0$. The inverse is then bounded as $O(1)$ independently of $\epsilon$.

3. Perturbation of resonances

We observe that

$$(X - \epsilon)(X - h^{-1}Q - s)^{-1} = 1 + h^{-1}Q(X - h^{-1}Q - s)^{-1}.$$ 

Let us denote $D(X, s) = h^{-1}Q(X - h^{-1}Q - s)^{-1}$. In particular, the resonances of $X$ in $\Omega_{h, \alpha} = \{\Re s > -\alpha/2, |\Im s| < h^{-1/2}\}$ are the $s$’s such that the Fredholm determinant

$$F(X, s) := \det(1 + D(X, s)),$$

vanishes. For fixed $X C^1$-close to $X_0$, $F(X, \cdot)$ is a holomorphic function in $\Omega_\alpha$. Now, we consider a smooth family $X_\epsilon$. We observe that

$$\partial_\epsilon F(X_\epsilon, s) = F(X_\epsilon, s) \text{ Tr } [(1 + D(X_\epsilon, s))^{-1} \partial_\epsilon D(X_\epsilon, s)],$$

and

$$\partial_\epsilon D(X_\epsilon, s) = h^{-1}Q(X_\epsilon - h^{-1}Q - s)^{-1} \partial_\epsilon X_\epsilon(X_\epsilon - h^{-1}Q - s)^{-1}.$$
Since in the Tr in the formula for $\partial_\epsilon F$, the operators are smoothing, this trace does not depend on the Sobolev space with respect to which we are taking the trace. We will denote by $\|\cdot\|\text{Tr}$ the norm on the space $L^1(\mathcal{H}_\alpha, \mathcal{H}_\alpha)$. Additionally,

$$\|F(X_\epsilon, s)(1 + D(X_\epsilon, s))^{-1}\| \leq \exp 2\|D\|\text{Tr} + 1,$$

is uniformly bounded (see equation B.5.15 in [DZ]). From the formula, we deduce that $\partial_\epsilon F(X_\epsilon, s)$ defines a holomorphic function in the $s$ parameter. In particular, to obtain estimates on its derivatives in $s$, it suffices to estimate

$$\|\partial_\epsilon D(X_\epsilon, s)\|\text{Tr} \leq Ch^{-1}\|Q(X_\epsilon - h^{-1}Q - s)^{-1}\partial_\epsilon X_\epsilon\|\text{Tr}$$

$$\leq Ch^{-1}\|\partial_\epsilon X_\epsilon\|_{\mathcal{H}_\alpha \to \mathcal{H}_\alpha} + \|Q\|_{L^1(\mathcal{H}_\alpha^{-1}, \mathcal{H}_\alpha)}$$

$$= O(h^{-2-n}).$$

(with a constant $C$ changing at every line). Here, we needed $(X - h^{-1}Q - s)^{-1}$ to be bounded on $\mathcal{H}_\alpha^{-1}$, so that the computation is valid for $h < h_1$, and $\alpha > \alpha_1$.

By an induction argument, we obtain that for $h < h_k$ and $\alpha > \alpha_k$,

$$|\partial_\epsilon^k F(X_\epsilon, s)| \leq C_k h^{-2+2^k},$$

so that $\epsilon, s \mapsto F(X_\epsilon, s)$ is valued in

$$C^k([-\epsilon_0, \epsilon_0], \mathcal{C}(\Re s > -\alpha/2, |\Im s| \leq h^{-1/2})).$$

Since the resonances do not depend on the choice of space, it does not matter if we were using $\mathcal{H}_\alpha$ or $\mathcal{H}_\alpha^\alpha$.

We consider now a finite sequence $\lambda_1, ..., \lambda_N$ of resonances of $X_0$. For some $\delta > 0$, for $h$ small enough, and $\alpha$ large enough, $|F(X_\epsilon, s)| > \delta$ for $s \in \Omega_\delta := \{s \mid d(s, \{\lambda_1, \ldots, \lambda_N\}) = \delta\}$. In particular, there is $\epsilon'_0 < \epsilon_0$ such that $|F(X_\epsilon, s)| > \delta/2$ for $d(s, \{\lambda_1, \ldots, \lambda_N\}) = \delta$ and $|\epsilon| < \epsilon'_0$.

By the Rouché theorem, we deduce that the zeroes of $F(X_\epsilon, s)$ in $\Omega_\delta$ can be parametrized by continuous functions, which are $C^\infty$ when the resonances are simple. Additionally, we obtain that for any smooth closed curve $\gamma$ inside $\Omega_\delta$, the spectral projector

$$\Pi_\gamma(\epsilon, s) := \int_\gamma (X_\epsilon - s)^{-1}ds,$$

depends on $\epsilon$ in a $C^\infty$ fashion on the open sets where no resonance hits $\gamma$. 


REFERENCES

[BL07] Oliver Butterley and Carlangelo Liverani. Smooth Anosov flows: correlation spectra and stability. J. Mod. Dyn., 1(2):301–322, 2007.

[BT07] V. Baladi and M. Tsujii. Anisotropic Hölder and Sobolev spaces for hyperbolic diffeomorphisms. Ann. Inst. Fourier, 57(1):127–154, 2007.

[DZ] Semyon Dyatlov and Maciej Zworski. Mathematical Theory of Resonances. Version préliminaire.

[DZ16] Semyon Dyatlov and Maciej Zworski. Dynamical zeta functions for Anosov flows via microlocal analysis. Ann. Sci. Éc. Norm. Supér. (4), 49(3):543–577, 2016.

[FRS08] Frédéric Faure, Nicolas Roy, and Johannes Sjöstrand. Semi-classical approach for anosov diffeomorphisms and ruelle resonances. The Open Mathematics Journal, (1):35–81, 2008.

[FS11] Frédéric Faure and Johannes Sjöstrand. Upper bound on the density of Ruelle resonances for Anosov flows. Comm. Math. Phys., 308(2):325–364, 2011.

[GL06] S. Gouëzel and C. Liverani. Banach spaces adapted to Anosov systems. Ergodic Theory Dyn. Syst., 26(1):189–217, 2006.

[HP70] Morris W. Hirsch and Charles C. Pugh. Stable manifolds and hyperbolic sets. In Global Analysis (Proc. Sympos. Pure Math., Vol. XIV, Berkeley, Calif., 1968), pages 133–163. Amer. Math. Soc., Providence, R.I., 1970.

[Liv04] Carlangelo Liverani. On contact Anosov flows. Ann. of Math. (2), 159(3):1275–1312, 2004.

[Pol85] Mark Pollicott. On the rate of mixing of Axiom A flows. Invent. Math., 81(3):413–426, 1985.

[Rue78] David Ruelle. An inequality for the entropy of differentiable maps. Bol. Soc. Brasil. Mat., 9(1):83–87, 1978.

[Rue86] David Ruelle. Resonances of chaotic dynamical systems. Physical review letters, 56(5):405, 1986.

Université de Rennes 1, CNRS, IRMAR - UMR 6625, F-35000 Rennes, France