NDM-based H-infinity robust control of parallel-connected grid-connected converters for V2G

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Abstract. Because of the unignoring line-impedance, distorted grid voltage, high penetration and the increased parallel-connected grid-connected converters (GCC) in vehicle-to-grid (V2G) system, the stability of the grid with electric vehicle (EV) faces new challenges. To deal with the stability problem, this paper proposes a numerator-denominator-model (NDM) based H-infinity controller combined with an adaptive capacitive-current-feedback active damping method with explicitly robust stability in terms of variations on filter parameters, time-delay and grid-impedance. Simulation results are investigated in two parallel-connected GCC in EV and show the validity of the proposed method.

1. Introduction
In recent years, the deployment of Electrical Vehicles (EVs) as power storage sources under emergency conditions during the plug-in period has attracted the attention of power researchers. The Vehicle to Grid (V2G) scheme is designed to inject the power back to the electrical grid from the batteries of EVs. The presence of batteries in EVs was behind the techniques of using the plug-in EVs in renewable power systems as an energy source [1]. In the V2G system, the LCL-type grid-connected converter systems (LCL-GCCs), due to their superior harmonic suppression capability, are widely used as the interface between the grid and the EV [2,3]. However, due to the increased parallel-connected LCL-GCCs in the grid, long distribution wires and the use of low-power transformers, the equivalent grid impedance of the GCCs is increased, stabilities of the LCL-GCCs face great challenges. To enhance the system stability, the current control of LCL-GCCs plays a predominant role [4].

Much research work on improving the stability of GCCs has been done [5]. In [6], Yang, etc. proposed a virtual impedance method to reshape the output impedance of the inverter, which improves the robustness of the inverter. In [7], an optimized controller design procedure against grid-impedance change is proposed to improve the robustness of grid-connected inverters with LCL filter. In addition to the stability enhancement of GCCs with capacitor-current-feedback AD and virtual impedance reshaping method, there are also some robust control methods that improve the GCCs under weak grid by optimizing the current controller design without prior knowledge of the uncertain parameter variations, such as μ-synthesis [8] and H-infinity synthesis [9],
From the literature mentioned above, we found that in the conventional method, the dynamic performance of the closed-loop system cannot be precisely designed due to the uncertainties in the synthesis procedure. Meanwhile, the margin of the allowable grid impedance variations is not elaborated, which is important in the stability analysis. In addition to grid impedance, due to the time-delay and zero-holder in the forward loop, the system stability is further degraded. The conventional method utilizes a single capacitor-current proportional coefficient feedback for resonance damping, which cannot guarantee the system stability in a wide parameter variation range. Motivated by this research gap, in this paper, a stability improvement control scheme for LCL-type GCCs with NDM-based H-infinity synthesis and an improved capacitor-current-feedback control method is proposed. The main contributions of this paper are summarized as:

1) Stability of the LCL-GCCs considering grid impedance changes is analyzed, which proves that the system will be unstable with the increase of grid impedance.

2) An NDM-based H-infinity controller is proposed, which is realized by separating the uncertainty from the numerator and denominator of the system, combined with the proposed proportional-integral capacitor-current-feedback loop control, the allowable grid impedance variation is further extended.

2. System description

Figure 1 shows the V2G system configuration for a three-phase GCC inverter with the LCL filter, where the PV modules are interfaced with the grid via three-phase GCC. In this case, a dc-dc boost converter can be employed to boost the PV voltage to the higher voltage that is optimized for the inverter. Wherein, the inductors $L_1$ and $L_2$ and the capacitor $C$ constitute the LCL filter. $U_{dc}$ is the input dc voltage; $C_{dc}$ is the dc-link capacitor. $U_{mp}$ and $i_t$ are the inverter output voltage and current, respectively. $u_g$ and $i_g$ are the grid voltage and grid-connected current; $i_{l1}$ and $i_{l2}$ are the grid impedance.

The grid-connected converter aims to regulate the grid current to be sinusoidal and keep it in phase with the voltage at PCC, thus a phase-lock-loop (PLL) is employed to get the phase angle of PCC voltage. The amplitude of the grid current reference is given as $I^*$. The capacitor current is acquired and fed back to damp the LCL filter resonance. $H_{i1}$ and $H_{i2}$ are the inductive gain of $i_c$ and $i_{l2}$, respectively.

![Figure 1. Configuration and block diagram of the proposed three-phase LCL-GCC for EV.](a) Power circuit topology. (b) V2G.

The three-phase LCL-type GCCs can be seen as three single-phase circuit, the transfer function of LCL filter considering grid inductance $L_g$ is

$$G_{LCL} = \frac{1}{S^2 (L_1 + L_2 + L_g) + S (L_1 + L_2 + L_g) + L_1 (L_2 + L_g) C}$$

(1)

Thus, the system has a resonant frequency $\omega_r$ given by

$$\omega_r = 2\pi f_r = \frac{L_1 + L_2 + L_g}{\sqrt{L_1 (L_2 + L_g) C}}$$

(2)
The block diagram of the closed-loop control system with active damp (AD) is shown in Figure 2. Wherein $K_{PWM}$ is the gain of the inverter, which is equal to $V_{dc}/V_{tri}$. Where, $V_{dc}$ is the dc-link voltage of the inverter, $V_{tri}$ is the peak-peak voltage of the carrier signal, the control period is equal to PWM carrier frequency, the sampling rate is the same as the PWM frequency. The loop gain of the system and the transfer function of the closed-loop system will be discussed in the followed steps.

![Figure 2. Block diagram of an LCL-GCC with AD and control delays.](image)

Considering the computation delay and the sampling delay, and assuming $T_s$ is the sampling time, $T_d$ is the computation and PWM delays, express as

$$G_d(s) = \frac{1}{T_s} \exp(-\lambda T_d)G_h(s) \approx \exp(-\lambda T_d)G_h(s) = \frac{1-e^{-sT_d}}{s} \approx T_s e^{-0.5sT_d}$$

To reduce the steady-state error, and considering that the PR controller has a very big gain at the desired frequency, assuming the conventional PR controller is employed given by

$$G_{i1}(s) = K_p + \frac{2K_r \omega_0 s}{s^2 + 2\omega_0 s + \omega_0^2}$$

where $K_p$ is the proportional gain, $K_r$ is the resonant gain, $\omega_0 = 2\pi f_0$ is the fundamental angular frequency, $\omega_i$ is the bandwidth of the resonant part to reduce the sensitivity to the variation of the fundamental frequency. The block diagram of the system after simplification can be given in Figure 3.

![Figure 3. The equivalent plant transfer function of the system. (a) Before simplification. (b) after simplification.](image)

where $G_{x1}(s)$ is the TF from $i_{ref}(s)$ to $u_{c}(s)$. $G_{x2}(s)$ is the TF from $u_{c}(s)$ to $e_{g}(s)$.

$$G_{x1}(s) = \frac{K_{PWM}G_{d}(s)}{s^2L_1C+sH_1K_{PWM}G_{d}(s)+1}G_{i1}(s)$$

$$G_{x2}(s) = \frac{s^2L_1C+sH_1K_{PWM}G_{d}(s)+1}{s^2L_1C+sH_1K_{PWM}G_{d}(s)+1}$$

According to Figure 3, the loop gain $T(s)$ is given by

$$T(s) = G_{x1}(s)G_{x2}(s)H_{i2} = \frac{H_{i2}K_{PWM}G_{d}(s)}{s^2L_1C+sH_1K_{PWM}G_{d}(s)+s(L_1+L_2)}$$

The grid current is composed of two parts. Namely, the current component caused by reference current $i_{g}(s)$, and by the disturbance voltage $u_{pcc}(s)$. As a result, the grid current can be written as

$$i_g(s) = \frac{1}{H_{i2}} \cdot \frac{T(s)}{1+T(s)} \cdot i_{g}^*(s) - \frac{G_{x2}(s)}{1+T(s)} \cdot u_{pcc}(s) = i_s(s) - \frac{u_{pcc}(s)}{Z_0(s)}$$

Where in Figure 3(b), $i_s(s)$ and $Z_0(s)$ are given by

$$i_s(s) = \frac{1}{H_{i2}} \cdot \frac{T(s)}{1+T(s)}$$

$$Z_0(s) = \frac{1+T(s)}{G_{x2}(s)}$$
3. **H-infinity current controller**

As mentioned before, the perturbations of grid parameters have been proved to have significant influences on the stability of GCCs. A basic setup for an uncertain system is depicted in Figure 4.

![Figure 4. The basic setup for the uncertain system.](image)

According to the small gain theorem, a sufficient condition for a closed-loop system to be stable is

$$\|H\|_\infty < 1, \text{ when } \|H\delta\|_\infty < 1$$  \hspace{1cm} (10)

where $\|\delta\|_\infty$ is the parameter perturbation of the system, $\|H\|_\infty$ is the norm of the system, based on the above model, a method to further design and synthesize the controller is provided, which is called Numerator-Denominator Perturbation. This method relies on the representation of the transfer function $H$ in Figure 4 in fractional form. Assuming the transfer function of a system can be represented in a fractional form as:

$$P = H = \frac{N}{D}$$  \hspace{1cm} (11)

where $N$ and $D$ are the numerator and denominator of the system, respectively, the perturbations in the numerator-denominator can be given by:

$$P_0 = \frac{N_0}{D_0} \rightarrow \frac{N_0 + M_0 N_2}{D_0 + M_0 D_1}$$  \hspace{1cm} (12)

where $P_0$, $N_0$, and $D_0$ represent the nominal transfer function and its numerator and denominator, respectively. For inverters with capacitor current feedback active damping strategy, the open-loop transfer function $G_C(s)$ from the input reference current $i_{ref}(s)$ to output current $i_g(s)$ without considering $G_t(s)$ is:

$$G_C(s) = P_0 = \frac{T(s)}{G_t(s)} = \frac{H_{i2}K_{PWM}}{s^3L_1L_2C + s^2H_{i1}CL_2K_{PWM} + s(L_1 + L_2)}$$  \hspace{1cm} (13)

where $N_0 = H_{i2}K_{PWM}$, $D_0 = s^3L_1L_2C + s^2H_{i1}CL_2K_{PWM} + s(L_1 + L_2)$. It should be mentioned that the control and modulation delay are ignored for simplicity in the synthesis. The terms $M_0N_1$ and $M_0D_2$ modeling the uncertainties in the plant. In the condition that the magnitude of $\delta_N$ and $\delta_D$ is not greater than one, $MW_2$ and $MW_1$ represent the largest possible perturbations. The factor $M$ is added for flexibility in design. This model and the block diagram may be rearranged as shown in Figure 5, the block $\delta_p$ is described by $\delta_p = [-\delta_D \delta_N]$.

![Figure 5. The Numerator-Denominator Model.](image)

$$p = -\delta_p q_1 + \delta_N q_2 = [-\delta_D \delta_N] \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \delta_p \cdot q$$  \hspace{1cm} (14)

Therefore, Figure 5 can be changed to Figure 6 as

![Figure 6. The rearranged Numerator-Denominator Model.](image)
As a result, the system would be stable when \( \| \delta_p \| < 1 \) and \( \| H \|_\infty < 1 \). H block has input \( p \) and output \( q = [q_1 \ q_2] \). The plant has an output \( z \). The transfer function of \( H \) is
\[
H = \begin{bmatrix}
W_1 S_0 V \\
-W_2 U_0 V
\end{bmatrix}
\]  
(15)
wherein \( S_0 \) is the sensitivity function of the system, and \( U_0 \) is the input sensitivity of the system.
\[
S_0 = 1/(1 + P G_c), \quad U_0 = G_c/(1 + P G_c), \quad V = M/D, \quad T_0 = P U_0.
\]  
(16)
wherein \( G_c \) is the grid current controller in Figure 2, as a result, the norm \( \| H \|_\infty \) can be written as:
\[
\| H \|_\infty = \text{sup}(|W_1 S_0 V|^2 + |W_2 S_0 V|^2)
\]  
(17)
According to Eq. (15), the stability condition of the system in Figure 5 can be expressed as:
\[
|W_1 S_0 V|^2 + |W_2 S_0 V|^2 < 1
\]  
(18)
To further design its robustness, assuming that the uncertainness of the denominator and numerator can be respectively defined as:
\[
\Delta_D = \frac{(D-D_0)}{D_0} = M \delta_D W_1 = V W_1 \delta_D = \omega_1 \delta_D
\]  
(19)
\[
\Delta_N = \frac{(N-N_0)}{N_0} = M \delta_N W_2 = V W_2 \delta_N = \omega_2 \delta_N
\]  
(20)
Rewriting Eq. (11) into, defining \( \omega_1 = W_1 V \) and \( \omega_2 = W_2 V / P \), (18) can be written by:
\[
\| H \|_\infty = \text{sup}(|\omega_1 S_0|^2 + |\omega_2 T_0|^2)
\]  
(21)
\( T_0 \) is the complementary sensitivity function of the system, \( S_0 \) and \( T_0 \) must satisfy \( S_0 + T_0 = 1 \), then
\[
|\omega_1 S_0|^2 + |\omega_2 T_0|^2 < 1, \text{ when } \left(\frac{(D-D_0)/D_0}{\omega_1}\right) + \left(\frac{(N-N_0)/N_0}{\omega_2}\right) < 1
\]  
(22)
Eq. (17) can be used to design the system’s robustness. The larger the weight function is, the larger perturbation is allowed. Meanwhile, the transfer function of \( S_0 \) and \( T_0 \) can be shaped by properly selecting the weight function \( W_1(s) \) and \( W_2(s) \).
A proportional-resonant transfer function is suitable to shape the sensitivity function to be a notch. Choosing the weight function \( W_1(s) \) as
\[
W_1(s) = \frac{(s^2 + 2 \omega_c + 1)^2}{s^2 + 2 \omega_c s + \omega_c^2} \frac{W_1(s)}{s}
\]  
(23)
which provides a significant gain at line frequency to eliminate the disturbances from the grid. The term \( \frac{W_1(s)}{s} \) is reserved to add flexibility in the following design. And double zeros in \( \omega_{c1} \) is added to eliminate the influence of \( W_1(s) \) in higher frequency. In the following synthesis, \( \omega_{c1} = 1 \times 104 \text{ rad/s}, \omega_c = 1 \text{ rad/s}, \omega_0 = 314 \text{ rad/s} \) are adopted.
With the satisfied condition \( \| \delta_p \| < 1 \), the weight function \( W_1(s) \) should be greater than the relative dominator uncertainty over all frequencies. More elaborations are given on this constraint. With the aforementioned result that the system is stable when \( \| H \|_\infty < 1 \) and \( \| \delta \|_\infty < 1 \) are satisfied. And the algorithm for \( H_\infty \) synthesis problem is a numerical solution, which leads to uncertainty in the resulting \( H_\infty \) norm. So, it’s necessary to keep the weight function \( W_1(s) \) to be greater than the relative dominator uncertainty over all frequency ranges, this constraint gives margin in the resulting controller and is convenient to design the desired robustness as long as the parameter perturbation range is known.
The relative dominator uncertainty \( \Delta_D \) is large at low frequency and tends to constant at high frequency. So, the weight function \( W_1(s) \) should be chosen as a low-pass filter. To ensure \( \omega_1 \) to be a constant at high frequency, another zero at \( \omega_{c2} \) is added to \( s \), which makes \( M(s) \) as
\[
M(s) = \frac{(s^2 + 2 \zeta \omega_n + \omega_n^2)}{\omega_n^2} \frac{s}{(s + \omega_{c2})}
\]  
(24)
4. Simulation results

It has been indicated that large grid impedance can push the inverters into unstable states. To demonstrate the validity of the proposed method, comparisons between the conventional method and the proposed method with the change of grid impedance under steady-state will be discussed in the first place. Then the dynamic response comparisons between the conventional and proposed method with large grid impedance under a step-up current reference will be given. The grid impedance can be changed to mimic the behavior of a weak grid. Parameter configurations are shown in Table 1.

**Table 1.** Inverter parameter specifications.

| Parameter                        | Value       |
|----------------------------------|-------------|
| Grid Line-line Voltage $v_g$     | 380 VRMS    |
| DC-link Voltage $v_{dc}$         | 700 V       |
| Line Frequency $f$               | 50 Hz       |
| Rated Power $P_e$                | 150 VA      |
| Switching Frequency $f_{sw}$     | 50 kHz      |
| Sampling Frequency $T_s$         | 25 kHz      |
| Control Updating Frequency       | 25 kHz      |
| Inverter Side Inductance $L_1$   | 2 mH        |
| Grid Side Inductance $L_2$       | 1 mH        |
| Filter Capacitance $C$           | 2.2 µF      |

The simulations have been carried out under the continuous domain. Two switching periods of control delay is added to simulate the computation delay. Simulations have been developed under different grid conditions to verify the dynamic tracking and stability performance of the proposed controller. Figure 7(a) shows the simulation result of the conventional PR controller with $K_p = 0.633$ and $K_R = 500$. The grid impedance changes from 100 to 600 µH at 0.1s, which makes the system unstable. To maintain its stability, the gain of the PR controller must decrease. However, decreasing the loop gain will significantly worsen the tracking performance and disturbance rejection capability. Figure 7(b) shows the simulation result of the proposed $H_{\infty}$ controller with grid impedance changing from 100 µH to 1.5 mH. This result shows that the proposed $H_{\infty}$ controller can achieve good robust performance with the grid impedance variations in a wide range.

![Figure 7](image_url)

**Figure 7.** Grid current when $L_g$ changes from 100 to 600 µH and from 100 µH to 1.5 mH with different controllers at 0.1s. (a) $QPR$ controller (b) $H_{\infty}$ controller.

Figure 8 shows the dynamic response of the proposed $H_{\infty}$ controller. The reference current changes from 5 to 10 A. The result shows that the proposed controller can achieve a fast response. When the reference changes, the output current follows the command after a short regulation time. Figure 9 shows the grid current with $H_{\infty}$ controller when grid inductance changing to 1 to 12 mH at 0.1s, which shows that the maximum allowable grid inductance to ensure system stability is approximately 12 mH, this proves the aforementioned conclusions.
Figure 8. Waveform of the grid current with the proposed controller when a step-up grid-current reference is given.

Figure 9. Waveform of the grid current with the proposed controller when $L_g$ changes from 1 to 12 mH at 0.1s.

5. Conclusions
This paper investigates a current robust controller to guarantee the system stability under wide grid impedance changes for V2G system. The proposed method seeks to reshape the output impedance of the converter by introducing an NDM-based $H_\infty$ controller in the current loop. By properly selecting the weight function, the synthesized proposed controller exhibits high gain at line frequency to eliminate the influences generated by grid disturbance. The results show that strong robustness of the system can be achieved under uncertain parameters when the proposed method is used.

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