Flow Equations for Yang-Mills Theories in General Axial Gauges

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Abstract

We present a formulation of non-Abelian gauge theories in general axial gauges using a Wilsonian (or 'Exact') Renormalisation Group. No 'spurious' propagator divergencies are encountered in contrast to standard perturbation theory. Modified Ward identities, compatible with the flow equation, ensure gauge invariance of physical Green functions. The axial gauge $nA = 0$ is shown to be a fixed point under the flow equation. Possible non-perturbative approximation schemes and further applications are outlined.

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1. General axial gauges [1]-[7] have enjoyed considerable attention amongst the non-covariant gauges, especially for computations in QCD at vanishing [2],[3] or non-vanishing temperature [2],[4]. The main reason for their popularity stems from the fact that the ghost sector decouples. The number of Feynman diagrams in a perturbative loop expansion is reduced, leading to an important simplification from a technical point of view. Furthermore the problem of possible Gribov copies [8], generically present in covariant gauges, is absent [2]. The price to pay is that the (perturbative) propagator receives 'spurious' poles, which have to be dealt with separately. The question about how to regularise the propagator as to allow for a consistent loop expansion stimulated extensive investigations [7]. The intricacies concerning these regularisations partly spoil the advantage of having fewer diagrams to calculate. Nevertheless it has been an appealing gauge to e.g. calculate expectation values of Wilson loops which serve as order parameters for confinement. In the strong coupling limit they are expected to fulfil Wilson’s area law which is correlated to a linear quark potential. The proper calculation of these expectation values may also necessitate the inclusion of topologically non-trivial configurations like instantons [9]. Wilson loop calculations have also been used as a testing ground for the consistency of calculations in general axial gauges.

Thus it would be interesting to see whether an alternative approach (other than standard perturbation theory) to gauge theories in general axial gauges is available which preserves the above mentioned benefits without encountering 'spurious' divergencies.

In this Letter, we shall argue that these 'spurious' poles are indeed an artifact of a perturbative loop expansion. The remedy we propose is known as the 'Exact' (or Wilsonian) Renormalisation Group [10]-[12]. This approach has already been applied to scalar [13], Abelian Higgs theories [14]-[16] and non-Abelian gauge theories [17]-[21] in covariant gauges, and is particularly useful in cases where perturbative expansion parameters tend to be large. The key difference to a perturbative loop expansion consists in the fact that flow equations integrate-out quantum fluctuations mode by mode while integrating over infinitesimal momentum shells. This connects continuously the classical with the full quantum effective action and can be interpreted as a 'coarse-graining' of the microscopic field theory. The perturbative loop expansion lacks the notion of 'coarse-graining' as all modes will be integrated-out in one step within a given loop order.

For the issues addressed in this Letter fermions will act only as spectators. Thus for the sake of simplicity we concentrate on the pure non-Abelian gauge theories.

2. Let us shortly review the appearance of 'spurious' propagator poles related to an axial gauge fixing in standard perturbation theory. We will start with the Euclidean action for a pure non-Abelian gauge theory, given in $d$ dimensions by

$$S_A[A] = \frac{1}{4} \int d^d x \, F_{\mu \nu}^a F_{\mu \nu}^a$$

with the field strength tensor

$$F_{\mu \nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{bc}^a A_\mu^b A_\nu^c$$

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and the covariant derivative
\[ D_{\mu}^{ab} (A) = \delta^{ab} \partial_{\mu} + g f^{abc} A_{\mu}^c, \quad [e^b, e^c] = f_{a}^{\, bc} e^a. \]  

(3)

A general axial gauge fixing for the (fixed) Lorentz vector \( n_{\mu} \) is given by
\[ S_{gf} = \frac{1}{2} \int d^d x \ n_{\mu} A_{\mu}^a \frac{1}{\xi n^2} n_{\nu} A_{\nu}^a. \]

(4)

The gauge fixing parameter \( \xi \) has mass dimension \(-2\) and may also be momentum dependent. In particular, the case \( \xi = 0 \) \( (\xi p^2 = -1) \) is known as the axial (planar) gauge. The propagator \( P_{\mu\nu} \) related to \( S = S_A + S_{gf} \) is
\[ P_{\mu\nu} = \frac{\delta_{\mu\nu}}{p^2} + \frac{n^2(1 + \xi p^2)}{(np)^2} \frac{p_{\mu} p_{\nu}}{p^2} - \frac{1}{p^2} \frac{(n_{\mu} p_{\nu} + n_{\nu} p_{\mu})}{np}. \]

(5)

It displays the usual IR poles proportional to \( 1/p^2 \). In addition, we observe additional divergencies for momenta orthogonal to \( n_{\mu} \). These poles appear explicitly up to second order in \( 1/np \) and can even be of higher order for certain \( np \)-dependent choices of \( \xi \). For the planar gauge, the spurious divergencies appear only up to first order.

This artifact makes the application of perturbative techniques very cumbersome as an additional regularisation for these spurious singularities has to be introduced.

3. The key idea of the renormalisation group “à la Wilson is the step-by-step ‘integrating-out of degrees of freedom’. This program can be achieved simply by adding a scale-dependent term to the action \[14\]-\[21\],
\[ \Delta_k S[A] = \frac{1}{2} \int \frac{d^d p}{(2\pi)^d} A_{\mu}^a R_{k,\mu\nu}(p) A_{\nu}^b. \]

(6)

Here we have introduced the infra-red (momentum) scale \( k \) which will interpolate from some UV scale \( \Lambda \) to the IR limit \( k = 0 \). Eq. (6) is quadratic in the fields and leads therefore to a modification of the propagator.

The scale dependent Schwinger functional \( W_k[J] \) related to \( S_k = S_A + S_{gf} + \Delta_k S \) is given by
\[ \exp W_k[J] = \int DA \exp \left\{ -S_k[A] + \int d^d x \ A_{\mu}^a J_{\mu}^a \right\}, \]

(7)

and the scale dependent effective action \( \Gamma_k \) is defined as the Legendre transform of (6)
\[ \Gamma_k[A] = \int d^d x \ J_{\mu}^a A_{\mu}^a - W_k[J] - \Delta_k S[A], \quad A_{\mu}^a = \frac{\delta W_k[J]}{\delta J_{\mu}^a}. \]

(8)

For later convenience, we have subtracted \( \Delta_k S \) from the Legendre transform of \( W_k \). The regulator \( R_k \) enjoys the following properties:

(i) It has a non-vanishing limit for \( p^2 \to 0 \), typically \( R \to k^2 \). This precisely ensures the IR finiteness of the propagator at non-vanishing \( k \) even for vanishing momentum \( p \).
(ii) It vanishes in the limit $k \to 0$. In this limit, any dependence on $R_k$ drops out and $\Gamma_{k \to 0}$ reduces to the full quantum effective action $\Gamma$.

(iii) For $k \to \infty$ (or $k \to \Lambda$ with $\Lambda$ being some UV scale much larger than the relevant physical scales), $R_k$ diverges like $\Lambda^2$. Thus, the saddle point approximation to (7) becomes exact and $\Gamma_{k \to \Lambda}$ reduces to the gauge-fixed classical action $S_A + S_{gf}$.

As a consequence, the functional $\Gamma_k$ interpolates between the gauge-fixed classical and the full quantum effective action. The corresponding flow equation follows from (8) as

$$
\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \left\{ \left( \Gamma_k^{(2)} + R_k \right)^{-1} \frac{\partial R_k}{\partial t} \right\}
$$

(9)

where the trace sums over all momenta and indices, $t = \ln k$, and

$$
\Gamma_k^{(2)ab}(x, x') = \frac{\delta^2 \Gamma_k}{\delta A^a_{\mu}(x) \delta A^b_{\nu}(x')}.
$$

(10)

Note that for any given scale $k$, the main contributions to the running of $\Gamma_k$ in (9) come from momenta around $p^2 \approx k^2$. This is so because $\partial_t R_k$ is peaked around $p^2 \approx k^2$, and (exponentially) suppressed elsewhere. The physics behind this is that a change of $\Gamma_k$ due to a further coarse graining (i.e. the integrating-out of a thin momentum shell around $k$) is dominated by the fluctuations with momenta around $k$. Contributions from fluctuations with momenta much smaller/larger than $k$ should be negligible.

4. What have we gained with the propagator $P_{k,\mu\nu}$ related to $S_k$? Let us specify the regulator as

$$
P_{k,\mu\nu}^{ab}(p) = \delta^{ab} \left[ r(y)p^2 \delta_{\mu\nu} - \tilde{r}(y)p_{\mu}p_{\nu} \right]
$$

(11)

and $y = p^2/k^2$. A typical class of regulator functions with the above properties is given by ($m \geq 1$)

$$
r_m(y) = \frac{1}{\exp(y^m) - 1}.
$$

(12)

(The limit $m \to \infty$ corresponds to the sharp cut-off limit \[10\].) The propagator takes the form

$$
P_{k,\mu\nu} = a_1 \frac{\delta_{\mu\nu}}{p^2} + a_2 \frac{p_{\mu}p_{\nu}}{p^4} + a_3 \frac{n_{\mu}p_{\nu} + n_{\nu}p_{\mu}}{p^2(np)} + a_4 \frac{n_{\mu}n_{\nu}}{n^2p^2},
$$

(13)

with the dimensionless coefficients

$$
a_1 = 1/(1 + r), \quad a_2 = (1 + \tilde{r})(1 + \xi p^2(1 + r))/z
$$

$$
a_3 = -(1 + \tilde{r})s^2/z, \quad a_4 = -(r - \tilde{r})/z
$$

(14)

and

$$
s^2 = (np)^2/(n^2p^2)
$$

$$
z = (1 + r)[(1 + \tilde{r})s^2 + (r - \tilde{r})(1 + p^2\xi(1 + r))].
$$

(15) (16)
The most convenient choice for practical purposes is $\tilde{r} = 0$. The cut-off function $r$ obeys the limits ($y = p^2/k^2$)
\begin{align}
\lim_{y \to 0} r_m = y^{-m}, \quad \lim_{y \to \infty} r_m = 0. \tag{17}
\end{align}
For $\tilde{r} = 0$ the propagator $P_{k,\mu\nu}$ has the limits
\begin{align}
\lim_{k^2 \to 0} P_{k,\mu\nu} = P_{\mu\nu}, \quad \lim_{p^2 \to 0} P_{k,\mu\nu} = \frac{1}{k^2} \left( \delta_{\mu\nu} + \frac{n_\mu n_\nu}{n^2} \frac{1}{1 + \xi k^2} \right) \delta_{m1}, \\
\lim_{k^2 \to \infty} P_{k,\mu\nu} = 0, \quad \lim_{p^2 \to \infty} P_{k,\mu\nu} = P_{\mu\nu}, \tag{18}
\end{align}
with $P_{\mu\nu}$ given by (13). The infrared regulator does not contribute for both large momenta or $k \to 0$. For $k \to \infty$ the propagator vanishes as all quantum modes are suppressed. By construction, the propagator (13) is IR finite for any $k > 0$. The important observation is now the following: In contrast to the perturbative propagator $P_{\mu\nu}$, the limit of $P_{k,\mu\nu}$ for $np \to 0$ is finite! This holds true even for an arbitrary choice of $\xi(p, n)$ and leads to
\begin{align}
P_{k,\mu\nu} = \frac{1}{1 + r} \frac{\delta_{\mu\nu}}{p^2} + \frac{1 + \tilde{r}}{1 + r} \frac{p_\mu p_\nu}{p^4} - \frac{1}{(1 + r)(1 + p^2 \xi(1 + r) \frac{n_\mu n_\nu}{n^2 p^2}}. \tag{19}
\end{align}
Thus (19) is perfectly well-behaved and finite for all momenta $p$ (as long as the regulators $r$ and $\tilde{r}$ have not been chosen to be identical). It is noteworthy that the 'spurious' divergencies are absent as soon as the infra-red behaviour of the propagator is under control.

5. Now we shall discuss the implications of gauge invariance, and in particular the question about how to ensure gauge invariance of physical Green functions. To that end we consider the Schwinger functional as given in (1). Invariance of the measure $DA$ under the infinitesimal gauge transformation $A \to A + D\alpha$ leads to the so-called modified Ward-identity (mWI)
\begin{align}
0 = \left( D^a \mu \gamma^b \mu - n_\mu D^a \mu (x) \frac{1}{n^2 \xi} n_\mu A^b (x) - D^a \mu (x) R^b c k,\mu\nu A^c (x) \frac{1}{n_\mu n_\nu} \right) J. \tag{20}
\end{align}
The mWI contains quadratic powers of the gauge field, which is a consequence of the gauge fixing and the cut-off term. Usually one converts the WI into a linear form by introducing ghosts and using BRST-invariance. However, due to the cut-off term this is no longer possible. With
\begin{align}
G^{ab}_{k,\nu\mu}(x, x') = \left( \Gamma^{(2)ab}_{k,\nu\mu} + R^{ab}_{k,\nu\mu} \right)^{-1} (x, x') \tag{21}
\end{align}
the mWI can be converted into
\begin{align}
W^a_k[A] \equiv D^a \mu (x) \frac{\delta \Gamma_k[A]}{\delta A^b \mu (x)} - n_\mu D^a \mu (x) \frac{1}{n^2 \xi} n_\mu A^b (x) \\
- g \int d^d y \int f^{abc} \left( \frac{1}{n^2 \xi} n_\mu n_\nu \delta^{cd} + R^{cd}_{k,\mu\nu} \right) (x, y) G^{db}_{k,\nu\mu}(y, x) = 0 \tag{22}
\end{align}
The first term is the usual covariant functional derivative of $\Gamma_k$, the second one stems from the gauge fixing, while the third one, a consequence of the regularisation and a possible momentum dependence of $\xi$, introduces non-local contributions. Note that the $R_k$-dependent term vanishes for both $R_k \to \infty$ and $R_k \to 0$. Both (20) and (22) reduce to the usual Ward identity in the limit $k \to 0$ [3]. For $k \neq 0$, they explicitly depend on the regulator, and the relations amongst different $n$-point functions of the theory as implied by (22) will depend on the particular form of the coarse-graining.

We will now derive the important result which is that the flow equation (9) and the mWI (22) are compatible. To that end we calculate the scale dependence of (22). Using the flow equation (9) for $\Gamma_k$, one can check explicitly that

$$\partial_t W^a_k = -\frac{1}{2} Tr \left( G_k \frac{\partial R_k}{\partial t} G_k \frac{\delta}{\delta A} \times \frac{\delta}{\delta A} \right) W^a_k$$

(23)

where the trace sums over momenta and internal indices. It follows that if $\Gamma_k$ fulfils the Ward identity at some scale $k_0$, and if it evolves according to the flow equation (9), then the new action will again fulfil the corresponding mWI. Thus it is in principle sufficient to fulfil the mWI at some initial scale $\Lambda$ in order to ensure gauge invariance of the physical Green functions, that is in the limit $k \to 0$.

Let us comment on the possible momentum dependence of $\xi$. As $\xi$ is dimensionful, it is natural to consider it as an operator. In perturbation theory, for example, the planar gauge $\xi(p) = -1/p^2$ introduces a momentum dependence in order to reduce the degree of the spurious divergencies. This is no longer necessary as the divergences are absent in the present approach. Therefore we may restrict ourselves to a momentum-independent gauge parameter. This is an important simplification, as the mWI (22) reduces to

$$D^{ab}_\mu(x) \frac{\delta \Gamma_k[A]}{\delta A^b_\mu(x)} - \frac{1}{n^2 \xi} \sum_{\nu=1}^{n^2} n_\mu \partial_\mu n_\nu A^a_\nu(x) - g \int d^4y f^{abc}_{\mu} R^{d}_{k,\mu}(x, y) G^{cb}_{d,\nu}(y, x) = 0.$$ 

(24)

As a consequence the possible tensor structure of vertices is considerably simplified. However, one may ask whether the limit $\xi \to 0$ defines the axial gauge also for momentum dependent $\xi(p)$. In perturbation theory, the answer is yes [5]. In the present context, it suffices to show that the term

$$\int d^4y f^{abc}_{\mu} \frac{n_\mu n_\nu}{n^2 \xi(x, y)} G^{cb}_{k,\nu}(y, x)$$

(25)

in (22) vanishes in the limit $\xi(p) \to 0$. That this is indeed the case can be seen as follows: Expanding (23) in powers of the gauge field yields expressions containing the effective propagator $G_k$ and $n$-point vertices $\Gamma^{(n>2)}_k$. Both $G^0_k$ and $\Gamma^{(n>2)}_k$ are completely determined via the (finite) flow equation. Thus they cannot exhibit singular terms proportional to

\[^3\text{It can be shown that (23) is valid for general linear gauges. For covariant linear gauges, a similar result was obtained in [13].}\]
1/ξ. Using in addition \( n_\mu G_{k,\mu\nu} = \mathcal{O}(\xi) \) (due to the gauge fixing) we find that (23) is at least of order \( \mathcal{O}(\xi) \) for \( \xi \to 0 \), which establishes the axial gauge even for a momentum dependent gauge fixing.

6. As a first application of this formalism we shall argue that the choice \( \xi = 0 \) corresponds to a fixed point w.r.t. the flow equation. In principle, this can be shown through an explicit computation of the flow equation for \( \xi(k) \). However, this would necessitate an explicit Ansatz for \( \Gamma_k \) (and would therefore be only an approximation). In contrast, we present an argument which makes use only of the mWI. With the preceding result in mind we can safely restrict ourselves to the case of \( \xi \) being momentum independent, although the result will hold true for general \( \xi(p) \). First note that \( \xi \) enters the mWI both explicitly and implicitly. The \( \xi \) appearing explicitly corresponds to the choice of \( \xi \) at some initial scale \( \Lambda \), \( \xi \equiv \xi(\Lambda) \). An implicit dependence occurs through the dependence of \( \Gamma_k \) on the scale dependent \( \xi(k) \). Let us choose \( \xi(\Lambda) = 0 \) with \( \Gamma_\Lambda \) solving (22) and assume that \( \xi(k) \neq 0 \) for some \( k < \Lambda \). This means in particular that \( \Gamma_k \) will no longer contain a singular term \( \sim 1/\xi \). Thus the only singular term appearing in the mWI is the term explicitly proportional to \( 1/\xi \). One can always find an \( A^a \) such that \( n^a n A^a \) does not vanish. Therefore \( \Gamma_k \) with \( \xi(k) \neq 0 \) can not be a solution of (22) for \( \xi(\Lambda) = 0 \). But this cannot be true as the compatibility of the flow equation and the mWI (23) implies that \( \Gamma_k \) solves the mWI. It follows that \( \xi(k) = 0 \) for \( \xi(\Lambda) = 0 \). Hence the axial gauge is indeed a fixed point of the flow equation. Note that this argument does not involve any approximations regarding \( \Gamma_k \).

We conclude that out of all general axial gauges (even momentum dependent ones) the axial (\( \xi = 0 \)) gauge is singled out and appears to be the natural choice. Furthermore, \( \xi = 0 \) is well-suited for actual computations as both the flow equation and the mWI are rather simple in that case.

7. A second application concerns the question about how to control gauge invariance for an approximate solution of the flow equation. Generally speaking, the task of solving the flow equation for gauge theories faces two main problems. The first one is to find a solution of (22) at some initial scale \( k = \Lambda \) in order to ensure gauge invariance of the physical Green function in the limit \( k \to 0 \). The second one concerns an Ansatz for the functional form of \( \Gamma_k[A] \). As not all possible operators can be taken into account, the validity of (22) for \( k < \Lambda \) (automatically ensured only for the full effective action \( \Gamma_k[A] \)) can no longer be taken for granted and has to be checked independently.

As an example, consider the action \( \Gamma_0 = S_A + S_{gf} \) for \( \xi = 0 \) at the scale \( k = 0 \). Obviously, \( \Gamma_0 \) is a solution of the mWI (24). The flow equation can be integrated analytically in leading order of perturbation theory, replacing \( \Gamma_k \) through \( \Gamma_0 \) on the r.h.s. of (9), to give

\[
\Gamma_k = \Gamma_0 + \frac{1}{2} \text{Tr} \ln \left( \Gamma_0^{(2)} + R_k \right) - \frac{1}{2} \text{Tr} \ln \Gamma_0^{(2)}.
\]

The mWI without the \( R_k \)-dependent term is solved by \( \Gamma_0 \). For the remaining terms in
we obtain in leading order

\[
\frac{1}{2} D_{ab}^{\mu}(x) \frac{\delta (\text{Tr} \ln (\Gamma_0^{(2)} + R_k) - \text{Tr} \ln \Gamma_0^{(2)})}{\delta A_{\mu}^b(x)} - g \int d^4 y \ f^{abc} R_{k,\mu\nu}^{cd}(x, y) C_{k,\nu\mu}^{db}(y, x) = 0. \tag{27}
\]

\(\Gamma_0^{(2)}\) is the second derivative of \(\Gamma_0\) with respect to the gauge field \((10)\). We use the commutator \([D, \frac{\delta}{\delta A}, \frac{\delta^2}{\delta A}^2]\) and \((24)\) for \(\Gamma_0\) to obtain

\[
\frac{1}{2} D_{ab}^{\mu}(x) \frac{\delta \text{Tr} \ln (\Gamma_0^{(2)} + R_k)}{\delta A_{\mu}^b(x)} = -g \int d^4 y \ f^{abc} \Gamma_0^{(2)c\mu\nu}(x, y) \left( \frac{1}{\Gamma_0^{(2)} + R_k} \right)^{db}_{\nu\mu}(y, x). \tag{28}
\]

For \(k = 0\), \((28)\) is simply zero. Using \((\Gamma_0^{(2)} + R_k)^{-1} = G_k + \mathcal{O}(g)\) and inserting \((28)\) into \((27)\) results in

\[
- g \int d^4 y \ f^{abc} \left( \Gamma_0^{(2)c\mu\nu} + R_{k,\mu\nu}^{cd} \right) (x, y) G_{k,\nu\mu}^{db}(y, x) \sim -g^2 f^{abc} \delta^{bc} = 0. \tag{29}
\]

Thus we have shown that the compatibility of the flow and the mWI can be maintained even within an approximate solution. It is straightforward to show that this holds true systematically even for higher orders within a perturbative loop expansion \([21]\).

Note that in \([18]\) the 1-loop perturbative compatibility of the flow equation was checked explicitly (for covariant gauges) for the scale dependent gluon mass parameter using a BRST-formulation. This is a rather non-trivial task since the flow equation as computed directly from \((9)\) or from the corresponding Slavnov-Taylor identity receives contributions from quite different diagrams (involving ghosts and gauge fields). However, in our formulation (without ghosts) the consistency check is rather simple and is done without problems for the entire effective action.

In more general situations, and especially within non-perturbative regions, it is not obvious how the compatibility between flow and mWI of a given truncation can be maintained. However, it is still possible to exploit the compatibility condition and to use it as a control mechanism for the Ansatz itself. A natural implementation would be to use the mWI as a flow equation for some of the relevant couplings (like mass terms) of the theory. Comparing the flow of these operators with the flow as derived directly from \((9)\) allows one to control the domain of validity of a given truncation.

For approximations beyond perturbation theory the flow equation \((9)\) and the mWI \((24)\) can also be used to control the dependence on \(n_\mu\) of the effective action and thus generalise the observations of \([2]\) to the case with a cut-off term.

8. Let us finally comment on the computation of the 1-loop \(\beta\)-function. This is another crucial test for the viability of this method. In standard perturbation theory, the 'spurious' singularities seem to play an important rôle and do already contribute on

\(^4\)See \([18], [19]\) where a similar line of reasoning has been employed on QCD in covariant gauges.

\(^5\)All the details of the computation shall be presented elsewhere \([24]\).
the 1-loop level [2]. In the present context they are absent throughout and it would be interesting to check that the correct 1-loop running is still coming out.

The running gauge coupling $g_k^2$ is related to the scale dependent wave function renormalisation of the field strength $Z_{F,k}$ via $g_k^2 = k^{d-4}g^2/Z_{F,k}$. Using the Ansatz

$$\Gamma_k = \frac{1}{4} \int d^d x \, Z_{F,k} F_{\mu\nu}^a F_{\mu\nu}^a + S_{gf}[Z_{F,k}^{1/2}A]$$

(30)

and (8) one can deduce the 1-loop flow for $Z_{F,k}$ by projecting on the $F^2$-terms in the flow equation. This is self-consistent at the 1-loop level. The traces involved in (9) can be calculated using heat kernel methods and one obtains

$$\partial_t \ln Z_{F,k} = \frac{11N}{24\pi^2}g^2 + \mathcal{O}(g^4).$$

(31)

The $\beta$-function follows immediately from (31) as

$$\beta_{g^2} \equiv \partial_t g_k^2 = -\frac{11N}{24\pi^2}g_k^4 + \mathcal{O}(g_k^6),$$

(32)

i.e. the well-known 1-loop result. The above (universal) result can be shown to hold for any regulator with the properties (i) – (iii).

9. To sum up, we have developed a self-contained and self-consistent formulation of QCD in general axial gauges which allows full control over the IR behaviour of the theory. In contrast to standard perturbation theory the 'spurious’ propagator singularities are naturally absent. Gauge invariance of physical Green functions is controlled via the mWI which is shown to be compatible with the flow equation. The absence of ghosts results in a rather simple expression for the mWI. We have shown that of all general axial gauges the axial gauge $nA = 0$ is singled out as it corresponds to a fixed point under the Wilsonian flow. Possible ways of finding approximate solutions even beyond perturbation theory have been briefly indicated. The calculation of the 1-loop running of the gauge coupling has been outlined and the $\beta$-function agrees with the known perturbative result.

This formulation seems to be a good starting point for theories where Lorentz covariance is naturally broken. This is the case for QCD at finite temperature where the heat bath singles out a rest frame. It is straightforward to apply the present approach within the imaginary time formalism. The propagator would still come out without any ‘spurious’ divergencies, in contrast to the recent proposal [22] based on a renormalisation group for the temperature fluctuations only. Another possible application concerns quantum field theories with both electrically and magnetically charged U(1)-fields, as this necessitates the introduction of a fixed Lorentz vector [23]. In either case, the Lorentz vector $n_\mu$ should be used for the axial gauge fixing as described above. It may also necessitate the inclusion of topologically non-trivial configurations into the effective average action [24]. We hope to report on these matters in the future.

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6Here, the heat kernel is not used as a regularisation since (8) is finite anyhow.
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