Spin Transport Hydrodynamics of Polarized Deuterium-Tritium Fusion Plasma

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The spin transport equations for polarized deuterium-tritium (DT) fusion plasma are derived with the density matrix formulation, which are used to investigate the hydrodynamics of polarized DT-gas-filled targets during indirectly driven inertial confinement fusion implosions. The depolarization of DT ions by strong self-generated magnetic fields can be captured by the spin transport equation. The neutron yield, angular distribution and neutron beam polarization are obtained from three-dimensional spin transport hydrodynamics simulations of the target implosions. The simulation results indicate that an optimized spin alignment of the polarized target can reduce the depolarization of DT ions and the neutron beams induced by polar mode asymmetries in indirectly driven implosions.

Inertial confinement fusion (ICF) is a promising approach to produce controlled burning plasmas and high flux neutron beams in laboratory [1]. Using spin-polarized fuels in ICF can potentially enhance the neutron yield and modify the angular distribution of the neutron beam, and more importantly generate spin-polarized neutron beam in certain emission direction [2–5]. Polarized neutron scattering and polarized neutron imaging are indispensable tools to probe the structure and dynamics of magnetic systems [6]. Increasing the flux of the polarized neutron source can shorten the time required to obtain high quality scattering signals and neutron images. Both deuterium-tritium (DT) and deuterium-deuterium (DD) reactions can be used in ICF to generate polarized neutron beams [7]. However, the key nuclear physics data, such as fusion cross-section, neutron angular distribution and neutron polarization for polarized DT and DD reactions at ICF relative conditions, lack experimental measurements [8]. For DD reaction, Kulsrud et al. developed the theoretic framework to predict fusion cross-section, neutron angular distribution and neutron polarization for arbitrary DT polarizations [2]. Ab initio calculations can also be used to predict the polarized DT fusion cross-section and neutron angular distribution [9]. For DD reaction, several models give inconsistent predictions of fusion cross-sections [5]. These nuclear physics data can be measured using ICF implosions [9] [10] as long as polarized targets can be assembled. The atomic beam source can generate polarized deuterium and tritium atoms with high polarization [11,12]. The nuclear polarization of atoms can be preserved during recombination to form “hyperpolarized molecules” [13]. If the polarized gas can be filled into the ICF capsule without severe depolarization, then the most significant question remaining is whether the polarized fuel could survive in the ICF implosion and produce polarized neutron beam.

The major depolarization mechanism of polarized DT fuel during ICF implosion is magnetic field induced depolarization [14]. Hydrodynamic instabilities, like Rayleigh-Taylor instability (RTI) and Richtmyer-Meshkov instability, can generate intense magnetic fields due to the Biermann battery effect [15,17]. The periods of the Larmor precession for DT nuclei in strong magnetic fields are close to the ICF confinement time, so the depolarization can not be neglected. Due to the smaller gyromagnetic ratio, deuterons can sustain a higher polarization than tritons during the implosion. The depolarization and spin transport process can be simulated using particle-based methods [18], hydrodynamic methods [19] or hybrid methods [14]. Hydrodynamics simulations are widely used to interpret ICF experiments [20], but conventional hydrodynamics codes do not include spin transport simulation. For spin polarized fusion, the probability distributions for spin eigenstates of DT are necessary to obtain the fusion cross-section and neutron angular distribution. The previously proposed spin transport equation using the vector polarization is not enough for spin-1 particles, whose tensor polarization is also needed to obtain the probability distribution for spin eigenstates [19,21]. In this Letter, we present the unified spin transport equation for spin-$\frac{1}{2}$ (T) and spin-1 (D) particles using the density matrix formulation, and three-dimensional (3D) spin transport hydrodynamics (STHD) simulation results of spin-polarized targets in the stagnation phase of indirectly driven ICF implosions. The depolarization of DT ions by strong self-generated magnetic fields can be captured by the spin transport equation. The neutron yield, angular distribution and neutron polarization can be obtained from the STHD simulations, which solve the hydrodynamic equations, magnetic induction equation, spin transport equations and fusion rate equation self-consistently. STHD simulations can be used to interpret the polarized ICF
experiments and optimize the physics design of ICF polarized neutron source. We show as an example that an optimized spin alignment of the polarized DT fuel can reduce the neutron beam depolarization induced by polar mode asymmetries in indirectly driven implosions.

To obtain the spin transport equation for DT nuclei, we start from the single particle Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \Psi_{\alpha} = \hat{H} \Psi_{\alpha},$$

with the Hamiltonian $\hat{H}$,

$$\hat{H} = -\frac{\hbar^2}{2m} \nabla^2 - \hat{\mu} \cdot \hat{B},$$

where $\alpha$ denotes the $\alpha$-th particle, $m$ is the particle mass, $\hat{\mu} = \gamma \hat{s}$ is the magnetic moment, $\gamma$ is the gyromagnetic ratio, $\hat{s}$ is spin operator and $\hat{B}$ is the magnetic field. Here the spin-orbit and spin-spin interaction terms in the Hamiltonian are neglected because the interaction ratio, $\hat{\mu} = \gamma \hat{s}$, spin operator and $\hat{B}$ is the magnetic field. The collisional depolarization induced by polar mode asymmetries in indirectly driven implosions.

The wavefunction cross-sections are relatively small. The collisional depolarization induced by polar mode asymmetries in indirectly driven implosions.

The velocity of the particle can be defined as

$$\mathbf{v}_\alpha = \frac{\hat{J}_\alpha}{n_\alpha},$$

As the components of spin operator $\hat{s}$ are Hermitian and $\hat{\mu} \cdot \hat{B}$ is also Hermitian, the second term on the right-hand side of Eq. (4) is zero. The current density is defined as

$$\mathbf{J}_\alpha = \frac{\hbar}{2m} \nabla \Psi_{\alpha}^\dagger \Psi_{\alpha} + \Psi_{\alpha}^\dagger \frac{\hbar}{2m} \nabla \Psi_{\alpha},$$

and the equation of continuity can be obtained

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot \mathbf{J}_\alpha = 0.$$  

The velocity of the particle can be defined as

$$\mathbf{v}_\alpha = \frac{\hat{J}_\alpha}{n_\alpha} = \frac{\nabla S_\alpha - i\hbar \Psi_{\alpha}^\dagger \nabla \Psi_{\alpha}}{m},$$

where $[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$ is the commutator.

The total density of a particle specie is $n = \sum_\alpha n_\alpha$. The density matrix of a particle specie is defined as $\hat{n}_\alpha = \sum_\alpha n_\alpha \hat{\eta}_\alpha/n$ and the fluid velocity is $\mathbf{v} = \sum_\alpha n_\alpha \mathbf{v}_\alpha/n$. With these definitions, we can obtain the spin transport equation for a particle specie as

$$\frac{\partial \hat{n}_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{v}_\alpha) = \frac{i}{\hbar} \left[ \hat{\mu} \cdot \mathbf{B}, n_\alpha \hat{\eta}_\alpha \right] - \nabla \cdot \hat{\mathbf{K}} + \nabla \cdot \hat{\mathbf{Q}},$$

where $\hat{\mathbf{K}} = \sum_\alpha n_\alpha [\hat{\eta}_\alpha - \hat{n}(\mathbf{v}_\alpha - \mathbf{v})]$ is the thermal-spin coupling, $\hat{\mathbf{Q}} = \frac{i}{\hbar} \sum_\alpha \left[ \nabla \hat{\eta}_\alpha, n_\alpha \hat{\eta}_\alpha \right]$ is the nonlinear spin fluid contribution. The spin transport equation still contains the explicit sum over all particles, and further statistical relations are needed to close the system. If the spin distribution and thermal distribution are not correlated, the thermal-spin coupling $\hat{\mathbf{K}} = 0$. And if the typical fluid length scale $L \gg \lambda_{ik}$, where $\lambda_{ik}$ is the thermal de Broglie wavelength, the nonlinear spin fluid contribution $\hat{\mathbf{Q}}$ can be neglected.

The probability distribution for spin eigenstates can be obtained from diagonal terms of the density matrix. The trace of density matrix is conserved and unity $\text{Tr}(\hat{n}) = 1$.

For tritons, the probabilities for spin eigenstates $m_z = \{\frac{1}{2}, -\frac{1}{2}\}$ are $\eta_{00}^T$ and $\eta_{11}^T$ respectively. The triton polarization in $+z$ direction is $p_z^T = \eta_{00}^T - \eta_{11}^T$. The vector polarization of deuteron is $p_z^D = \eta_{00}^D - \eta_{22}^D$ and the tensor polarization of deuteron is $p_{zz}^D = \eta_{02}^D - 2\eta_{11}^D + \eta_{22}^D$. The fusion cross-section and neutron angular distributions can be obtained from probability distributions for spin eigenstates of DT. For simplicity, we adopt the formulas of Kuhrud et al. [2] in our simulations. The fusion cross-section of DT reaction can be calculated as

$$\sigma = \sigma_0 \left[ \frac{3}{2} \left( \eta_{00}^T \eta_{00}^D + \eta_{11}^T \eta_{22}^D \right) + \eta_{11}^D \right] + \frac{1}{2} \left( \eta_{00}^T \eta_{22}^D + \eta_{11}^T \eta_{00}^D + 2\eta_{11}^D \right) (3\cos^2 \theta + 1),$$

where $\sigma_0$ is the unpolarized cross-section. The total differential cross-section for neutrons is

$$\frac{d\sigma}{d\Omega} = \sigma_0 \left[ \frac{9}{4} \left( \eta_{00}^T \eta_{00}^D + \eta_{11}^T \eta_{22}^D \right) \sin^2 \theta \right.$$ 

$$\left. + \frac{1}{4} \left( \eta_{00}^T \eta_{22}^D + \eta_{11}^T \eta_{00}^D + 2\eta_{11}^D \right) (3\cos^2 \theta + 1) \right],$$

where $\theta$ is the polar angle. The differential cross-sections for neutrons with $m_z = \{\frac{1}{2}, -\frac{1}{2}\}$ are $d\sigma^+/(d\Omega)$ and $d\sigma^-/(d\Omega)$ respec-
FIG. 1. 3D STHD simulation results of a polarized DT-gas-filled capsule at bang time, (a) fuel density, (b) magnetic field strength, (c) fraction of depolarized tritons ($\eta_{T11}$), (d) fraction of depolarized deuterons ($\eta_{D11} + \eta_{D22}$). The initial fuel polarization is 0.9, the initial spins of DT fuel are perpendicular to the axis of hohlraum. The data in $x,y > 0$ region are set to be transparent in (a), (c) and (d) for better visibility.

The cross-section and differential cross-sections (9)-(11) are used in the fusion rate equations to obtain the neutron yield, neutron angular distribution and neutron polarization of polarized DT fusion.

As the spin transport equation contains fluid quantities $n$, $v$ and magnetic field $B$, it must be solved in combination with hydrodynamic equations and magnetic induction equation. A numerical scheme to solve the spin transport equation is developed and implemented in a 3D STHD simulation code SPINSIM [23], which numerically guarantees that the diagonal terms of the density matrix are bounded in [0,1] and $\text{Tr}(\hat{\eta}) = 1$. The capsule-only STHD simulation results of a polarized DT-gas-filled capsule are shown in Fig. 1. The capsule is made of a high density carbon (HDC) shell filled with highly polarized DT gas ($p_T^z = p_D^z = p_D^{zz} = 0.9$). The outer radius and thickness of the HDC shell are 1040 $\mu$m and 40 $\mu$m respectively. The density of HDC is 3.52 g/cm$^3$ and the density of the DT gas is 4 mg/cm$^3$. The initial temperature of the capsule is 65.65 K. Because the hydrodynamic instabilities and magnetic fields are amplified during the stagnation phase of the implosion [17], only the stagnation phase is simulated with SPINSIM. The radiation hydrodynamics code MULTI-IFE [24] is used to provide the fluid quantities as input data for STHD simulations. The capsule is ablated by radiations with peak temperature of 250 eV and reaches maximum fusion rate after 12.85 ns (bang time) [23]. The polar mode-2 ($P_2$) perturbation which forms from low-mode radiation drive asymmetries [25, 26] is added to the implosion velocity of STHD simulation. The asymmetry amplitude of shell radius measured at bang time is $P_2/P_0 = 0.21$. High-mode perturbations, which rise from the defects of the target, are also added with 64 random RTI spikes and bubbles [23]. The density distribution of the fuel at bang time is shown in Fig. 1(a). The axis of polar asymmetry, which
alignment cases have larger DT polarizations than the parallel alignments are smaller than the deuteron polarizations as shown in Figs. 2(a) and 2(b). The deuteron vector polarization $p^D_2$ measured at $\theta = 90^\circ$ for different $P_2$ amplitudes with various conditions of initial fuel polarization, spin alignment and high-mode perturbation shown in Figs. 2(b) and 2(c). The deuteron yields and absolute values of neutron polarization decrease with the increasing of absolute values of $P_2$ perturbation amplitudes. Reduction of initial fuel polarization and increment of high-mode perturbations can cause the reduction of neutron yields and absolute values of neutron polarization. Reductions of neutron beam depolarization by $P_2$ perturbations with perpendicular alignment are significant under all conditions as depicted in Fig. 3(c).

In summary, we have derived the spin transport equation using the density matrix formulation to model the spin transport hydrodynamics of spin-$\frac{1}{2}$ and spin-1 particles. The spin transport equations can be solved in combination with hydrodynamic equations and magnetic induction equations. The solutions of spin transport equations can be used in fusion rate equations to obtain the neutron yield, neutron angular distribution and neutron polarization of polarized DT fusion. The STHD simulation results show that optimized spin alignment of the polarized DT-gas-filled target can reduce the neutron beam depolarization induced by polar mode asymmetries in indirectly driven implosions. The polarized DT-gas-filled target investigated in this Letter is different from the ignition target which contains a high density DT ice layer [1]. Polarized DT ice are more difficult to produce than polarized DT gas [3]. The polarized DT-gas-filled targets can be useful in experiments of nuclear data measurement [9, 10] and neutron beam production [27]. It is promising to obtain highly polarized neutron beams using polarized DT-gas-filled target implosions, which will expand the range of applications of fusion based neutron sources.

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