Standard Model and SU(5) GUT with Local Scale Invariance and the Weylon

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Abstract. Weyl’s scale invariance is introduced as an additional local symmetry in the standard model of electroweak interactions. An inevitable consequence is the introduction of general relativity coupled to scalar fields à la Dirac and an additional vector particle we call the Weylon. Once Weyl’s scale invariance is broken, the phenomenon (a) generates Newton’s gravitational constant $G_N$ and (b) triggers the conventional spontaneous symmetry breaking mechanism that results in masses for all the fermions and bosons. The scale at which Weyl’s scale symmetry breaks is of order Planck mass. If right-handed neutrinos are also introduced, their absence at present energy scales is attributed to their mass which is tied to the scale where scale invariance breaks. Some implications of these ideas are noted in grand unification based on the gauge symmetry SU(5).

Keywords: Weyl, Scale Invariance, Standard Model, General Relativity, Extra Gauge Bosons

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INTRODUCTION

This work falls under the category of unconventional pursuits. Nevertheless the research is respectable and as I will show, leads to some very interesting and profound results.

The notion that the standard model [1] is the underlying theory of elementary particle interactions, excluding gravity, is the prevailing consensus supported by all experiments of the present time. The only missing ingredient is the elusive Higgs particle [2]. It is conceivable that the symmetry breaking mechanism is indeed spontaneous and the Higgs particle will be discovered. However, there are reasons, both aesthetic and otherwise, that necessitate the extensions of the standard model. Seeking unity of all particle interactions (grand unification) and explaining the ultimate instability of matter (proton decay) [3] are examples that fall in the former category while neutrino oscillations [4, 5] is an example that falls in the latter category.

At a much deeper level, the very notion of the origin of scales in physics is yet another fundamental issue yearning explanation. The problem reduces to comprehending the origin of just one fundamental scale, all other scales being different manifestations of this fundamental scale. To this end, either Weyl’s scale invariance symmetry [6, 7] or the much larger symmetry, the fifteen parameter group of conformal invariance [8, 9, 10], are thought to play a significant role as fundamental symmetries of Nature. A glance at the elementary particle mass spectrum attests to the fact that scale invariance and conformal invariance are badly broken symmetries of Nature. In the past, these

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symmetries were employed to gain insight on the origin of Newton’s gravitational constant $G_N$, a dimensionful quantity, as a symmetry breaking effect, induced either spontaneously or due to quantum corrections [11, 12, 13].

In this work we attempt at combining gauge and scale symmetries in an extension of the standard model in which not only gravity but also the entire particle mass spectrum of the standard model are generated in terms of just one fundamental scale associated with scale symmetry breaking. The approach is modest in that we exercise economy and consider extending the standard model with only Weyl’s local scale invariance [6, 7], the doomed symmetry that gave birth to the gauge principle and ultimately paved the way for implementing gauge invariance as we know it and practise it today. As will be shown, in the absence of fine-tuning, the scale at which the scale invariance symmetry breaks turns out to be of order Planck mass $M_p \approx 1.2 \times 10^{19}$ GeV. The extended model predicts the existence of an additional vector particle we will call the Weylon. Its mass is tied to the scale at which Weyl’s symmetry breaks and is also of order $M_p$.

Implementing scale invariance in the standard model has been previously considered [14]. The main result there was the elimination of the Higgs boson from the standard model particle spectrum. Here we present a different philosophy of the same work which has been recently considered [15]. In the present model, the standard model Higgs particle is not eliminated, and is the sought-after particle. In other words, after scale breaking our model at low energy describes the standard model of elementary particles supplemented with the Einstein-Hilbert action for gravitational interactions.

**SCALE INVARIANCE**

Under scale invariance the parallel transport of a vector around a closed loop in four dimensional space-time not only changes its direction but also its length. In such a manifold the line element $ds$ has no absolute meaning because a comparison of lengths at two different points involves the scale factor $\Lambda(x)$ where $\Lambda(x)$ is the parameter of scale transformations. The fundamental metric tensor $g_{\mu \nu}$ transforms as

$$g_{\mu \nu}(x) \rightarrow \tilde{g}_{\mu \nu}(x) = e^{2\Lambda(x)} g_{\mu \nu}(x) .$$

(1)

However, the ratio of two infinitesimal lengths is well defined when both lengths refer to the same point. This implies that the angle $\theta$ between two infinitesimal vectors $dx$ and $\delta x$ remains unchanged since

$$\cos \theta = \frac{g_{\mu \nu} dx^\mu \delta x^\nu}{\sqrt{g_{\alpha \beta} dx^\alpha dx^\beta} \cdot \sqrt{g_{\lambda \sigma} \delta x^\lambda \delta x^\sigma}} .$$

(2)

Thus, in reality, scale transformations lead to the larger fifteen parameter conformal transformations under which the coordinates $x^\mu$ undergo the following transformations

**Translations**;

$$x^\mu \rightarrow x^\mu + a^\mu \ \ (4 \ \text{parameters}) ,$$

(3)
Lorentz Transformations;

\[ x^\mu \rightarrow L^\mu_\nu x^\nu \] (6 parameters),

Accelerations;

\[ x^\mu \rightarrow x^\mu + \frac{a^\mu x^2}{1-2a^\alpha x^\alpha + x^2} \] (4 parameters),

and Dilatations;

\[ x^\mu \rightarrow e^\Lambda x^\mu \] (1 parameter).

The generators of these transformations are

\[ M_{\mu\nu} = x^\mu P_\nu - x^\nu P_\mu \] (Lorentz Rotations),
\[ P_\mu = -i\partial_\mu \] (Translations),
\[ K_\mu = 2x^\mu x^\nu P_\nu - x^2 P_\mu \] (Accelerations),
\[ D = x^\mu P_\mu \] (Dilatation).

These satisfy the broader algebra

\[
\begin{align*}
[M_{\mu\nu},M_{\rho\sigma}] &= g_{\mu\rho}M_{\nu\sigma} - g_{\nu\sigma}M_{\mu\rho} - g_{\nu\rho}M_{\mu\sigma} - g_{\mu\sigma}M_{\nu\rho}, \\
[M_{\mu\nu},P_\sigma] &= g_{\nu\sigma}P_\mu - g_{\mu\rho}P_\nu, \\
[M_{\mu\nu},K_\lambda] &= g_{\nu\lambda}K_\mu - g_{\mu\lambda}K_\nu, \\
[M_{\mu\nu},D] &= 0, \\
[P_\mu,P_\nu] &= 0, \\
[P_\mu,K_\nu] &= 2(g_{\mu\nu}D - M_{\mu\nu}), \\
[P_\mu,D] &= P_\mu, \\
[K_\mu,K_\nu] &= 0, \\
[K_\mu,D] &= -K_\mu, \\
[D,D] &= 0.
\end{align*}
\] (8)

In what follows we will only deal with the restricted symmetry associated with the generators \( M_{\mu\nu}, P_\mu \) (the Poincaré group) and the one parameter group associated with Weyl’s scale transformations.

**SCALE INVARIANT ACTION**

Under Weyl’s scale invariance as a local symmetry the electroweak symmetry \( SU(2) \times U(1) \) is extended to

\[ G = SU(2) \times U(1) \times \tilde{U}(1), \] (9)

where \( \tilde{U}(1) \) represents the local non-compact Abelian symmetry associated with Weyl’s scale invariance. The additional particles introduced are the vector boson \( S_\mu \) associated with \( \tilde{U}(1) \) and a real scalar field \( \sigma \) [16, 17, 18, 19, 20] that transforms as a
singlet under $G$. The distinct feature of the new symmetry is that under it fields transform with a real phase whereas under the $SU(2) \times U(1)$ symmetries fields transform with complex phases.

Under $\tilde{U}(1)$ a generic field in the action is taken to transform as $e^{w\Lambda(x)}$ with a scale dimension $w$. Thus under $G = SU(2) \times U(1) \times \tilde{U}(1)$ the transformation properties of the entire particle content of the extended model are the following: The $e$-family ($g = 1$),

\[
\Psi_{Lq}^{1g} = \begin{pmatrix} u \\ d \end{pmatrix} \sim (2, \frac{1}{3}, -\frac{3}{2}) \quad ; \quad \Psi_{L}^{1l} = \begin{pmatrix} v_e \\ e \end{pmatrix} \sim (2, -1, -\frac{3}{2}) ;
\]

\[
\Psi_{1R}^{1g} = u_R \sim (1, \frac{4}{3}, -\frac{3}{2}) \quad ; \quad \Psi_{2R}^{1g} = d_R \sim (1, -\frac{2}{3}, -\frac{3}{2}) ;
\]

\[
\Psi_{2R}^{1g} = e_R \sim (1, -2, -\frac{3}{2}) ,
\]

and similarly for the $\mu$-family ($g = 2$) and the $\tau$-family ($g = 3$). All of these fermions have the same scale dimension $w = -3/2$. The scalar boson sector comprises the Higgs doublet $\Phi$ and the real scalar $\sigma$,

\[
\Phi \sim (2, -1, -1) ; \quad \sigma \sim (1, 0, -1) ,
\]

with the common scale dimension $w = -1$. We introduce $W_\mu, B_\mu$ and $S_\mu$ as the gauge potentials respectively associated with the $SU(2), U(1), \tilde{U}(1)$ symmetries. We suppress the $SU(3)$ of strong interactions as neglecting it will not affect our results and conclusions. The four dimensional volume element transforms as

\[
d^4x \sqrt{-g} \rightarrow e^{4\Lambda(x)} d^4x \sqrt{-g} .
\]

Since the vierbein $e_\mu^m$ and its inverse $e_m^\mu$ satisfy $e_\mu^m e_{vm} = g_\mu^v$ and $e_\mu^m e_{\mu \mu} = \eta_{mn}$ where $\eta_{mn} = \text{diag.} (1, -1, -1, -1)$ is the tangent space metric, it follows that the transformation properties of $e_\mu^m$ and its inverse $e_m^\mu$ under Weyl’s symmetry are

\[
e_\mu^m \rightarrow e^{\Lambda(x)} e_\mu^m , \quad e_m^\mu \rightarrow e^{-\Lambda(x)} e_m^\mu .
\]

The action $I$ of the model is [15]

\[
I = \int d^4x \sqrt{-g} \left[ -\frac{1}{4} g^{\mu \rho} g^{\nu \sigma} (W_{\mu \nu} W_{\rho \sigma} + B_{\mu \nu} B_{\rho \sigma} + U_{\mu \nu} U_{\rho \sigma}) \right. \\
+ \sum_{g,l=1,2,3} \left( \Sigma_g^L e_\mu^m \gamma^m D_\mu \Psi_l^{gf} + \Sigma_i^f e_\mu^m \gamma^m D_\mu \Psi_l^{gf} \right) + g^{\mu \nu} (D_\mu \Phi)(D_\nu \Phi) + \frac{1}{2} (D_\mu \sigma)^2 \\
+ \sum_{g, g', f=1,2,3} \left( \Sigma_{gf}^L \Phi \Psi_l^{gf} + \Sigma_{ig}^f \Phi \Psi_l^{gf} \right) + \text{h.c.} \\
- \frac{1}{2} (\beta \Phi^\dagger \Phi + \zeta \sigma^2) R + V(\Phi, \sigma) \right] .
\]
where \( \Phi \equiv i\sigma_2\Phi' \), the indices \((g, g')\) are for generations, the indices \( i = (q, l) \) refer to (quark, lepton) fields, \( Y^{lf}_{gg'} \) or \( Y^{lf}_{gg} \) are quark, lepton Yukawa couplings that define the mass matrices after symmetry breaking, the index \( i = 1, 2 \) is needed for right-handed fermions, while \( \beta \) and \( \zeta \) are dimensionless couplings. The various \( D \)'s acting on the fields represent the covariant derivatives constructed in the usual manner using the principle of minimal substitution. Explicitly,

\[
D_\mu \Psi^{gf}_L = \left( \partial_\mu + ig\tau \cdot W_\mu + \frac{i}{2}g'Y^{gf}_L B_\mu - \frac{3}{2}fS_\mu - \frac{1}{2}\tilde{\omega}_\mu^{mn}\sigma_{mn} \right) \Psi^{gf}_L, \\
D_\mu \Psi^{gf}_{iR} = \left( \partial_\mu + ig'Y^{gf}_{iR} B_\mu - \frac{3}{2}fS_\mu - \frac{1}{2}\tilde{\omega}_\mu^{mn}\sigma_{mn} \right) \Psi^{gf}_{iR}, \\
D_\mu \Phi = \left( \partial_\mu + ig\tau \cdot W_\mu - \frac{1}{2}g' B_\mu - fS_\mu \right) \Phi, \\
D_\mu \sigma = \left( \partial_\mu - fS_\mu \right) \sigma. 
\] (15)

The \( Y^{gf}_L \)'s, \( Y^{gf}_{iR} \)'s represent the hypercharge quantum numbers (e.g., \( f = q, \ g = 1, \ i = 1, \ Y^{1q}_L = 1/3, \ Y^{1q}_{iR} = 4/3, \ etc.)\), \( g, \ g', \ f \) are the respective gauge couplings of \( SU(2), \ U(1), \ U(1) \). The \( W_{\mu\nu} \) and \( B_{\mu\nu} \) are the field strengths associated with the gauge fields \( W_\mu, B_\mu \) of \( SU(2), \ U(1) \) while

\[
U_{\mu\nu} \equiv \partial_\mu S_\nu - \partial_\nu S_\mu 
\] (16)

is the field strength associated with Weyl's \( \tilde{U}(1) \). It is gauge invariant, since \( S_\mu \) transforms as

\[
S_\mu \rightarrow S_\mu - \frac{1}{f}\partial_\mu \Lambda . 
\] (17)

The gauge fields and the field strengths carry scale dimension \( w = 0 \). The spin connection \( \tilde{\omega}_\mu^{mn} \) [21] is defined in terms of the vierbein \( e_\mu^m \)

\[
\tilde{\omega}_{mrs} = \frac{1}{2} \left( \tilde{C}_{mrs} - \tilde{C}_{msr} + \tilde{C}_{rsm} \right), \\
\tilde{C}_{\mu\nu}^r = \left( \partial_\mu e_\nu^r + f S_\mu e_\nu^r \right) - \left( \partial_\nu e_\mu^r + f S_\nu e_\mu^r \right), 
\] (18)

while the affine connection \( \tilde{\Gamma}_\mu^{\rho\nu} \) is defined by

\[
\tilde{\Gamma}_\mu^{\rho\nu} = \frac{1}{2}g^{\rho\sigma} \left[ (\partial_\mu + 2fS_\mu)g_{\nu\sigma} + (\partial_\nu + 2fS_\nu)g_{\mu\sigma} - (\partial_\sigma + 2fS_\sigma)g_{\mu\nu} \right]. 
\] (19)

The Riemann curvature tensor \( \tilde{R}_\rho^{\sigma\mu\nu} \) is

\[
\tilde{R}_\rho^{\sigma\mu\nu} = \partial_\mu \tilde{\Gamma}_\nu^{\rho\sigma} - \partial_\nu \tilde{\Gamma}_\mu^{\rho\sigma} - \tilde{\Gamma}_\mu^{\lambda\rho} \tilde{\Gamma}_\nu^{\sigma\lambda} + \tilde{\Gamma}_\nu^{\lambda\rho} \tilde{\Gamma}_\mu^{\sigma\lambda}, 
\] (20)

where \( \tilde{\Gamma}_\mu^{\rho\nu}, \tilde{R}_\rho^{\sigma\mu\nu} \) and the Ricci tensor \( \tilde{R}_\mu^{\rho\nu} = \tilde{R}_{\mu\nu} \) have scale dimension \( w = 0 \), while the scalar curvature \( \tilde{R} = g^{\mu\nu} \tilde{R}_{\mu\nu} \) has the form

\[
\tilde{R} = R - 6fD_\mu S^\mu + 6f^2S_\mu S^\mu, \\
D_\kappa S^\mu = \partial_\kappa S^\mu + \tilde{\Gamma}_{\kappa\mu}^\nu S^\nu, 
\] (21)
and transforms with scale dimension \( w = -2 \). The potential \( V(\Phi, \sigma) \) is given by

\[
V(\Phi, \sigma) = \lambda (\Phi^\dagger \Phi)^2 - \mu (\Phi^\dagger \Phi) \sigma^2 + \xi \sigma^4 ,
\]

where \( \lambda, \mu, \xi \) are dimensionless couplings.

### BREAKING OF SCALE INVARIANCE AND IMPLICATIONS

The scalar potential in this model consists of quartic terms only as required by Weyl’s scale invariance. Yet the desired descent, a two stage process, of \( G \) to \( U(1)_{\text{em}} \)

\[
G = SU(2) \times U(1) \times \tilde{U}(1) \to SU(2) \times U(1) \to U(1)_{\text{em}}
\]

is possible. In the primary stage of symmetry breaking, scale invariance symmetry is broken. This occurs spontaneously and is achieved by setting

\[
\sigma(x) = \frac{1}{\sqrt{2}} \Delta ,
\]

where \( \Delta \) is a constant for the symmetry breaking scale associated with Weyl’s \( \tilde{U}(1) \). It is to be noted that this phenomenon of spontaneous scale breaking is conceptually no different from conventional spontaneous symmetry breaking. In conventional spontaneous symmetry breaking, the term quadratic in the Higgs field changes sign suddenly from positive to negative while in spontaneous scale breaking under discussion here the scalar field \( \sigma \) freezes suddenly. The primary stage of symmetry breaking also determines Newton’s gravitational constant \( G_N \),

\[
\zeta \Delta^2 = \frac{1}{4\pi G_N} .
\]

Thus \( \Delta \approx 0.3 \times M_P / \sqrt{\zeta} \) and barring any fine-tuning \( \Delta \approx \mathcal{O}(M_P) \), if we take \( \zeta \approx \mathcal{O}(1) \). At this stage the scalar field \( \sigma \) becomes the goldstone boson \([22, 23]\). The vector particle associated with \( \tilde{U}(1) \) breaking, the Weylon, absorbs the goldstone field and becomes massive with mass \( M_S \) given by

\[
M_S = \sqrt{\frac{3\zeta^2}{4\pi G_N}} \approx 0.5 \times f M_P .
\]

Thus \( M_S \approx \mathcal{O}(M_P) \) in the absence of fine-tuning \( f \approx \mathcal{O}(1) \). Weyl’s \( \tilde{U}(1) \) symmetry decouples completely and the scalar potential after the primary stage of symmetry breaking takes the form

\[
V(\Phi) = -\mu \Delta^2 (\Phi^\dagger \Phi) + \lambda (\Phi^\dagger \Phi)^2 + \frac{\xi}{4} \Delta^4 .
\]

It is to be noted that this form of the potential, apart from the vacuum energy density term contributing to the cosmological constant, is of the same form as the standard Higgs potential in the standard model. All the conventional particles are still massless at this stage. With \( G_N \) defined, it is appropriate to work in the weak field approximation.
Henceforth we set $\sqrt{g_{\mu\nu}} \approx \eta_{\mu\nu} + O(\kappa)$ where $\kappa^2 = 16\pi G_N$. The secondary stage of symmetry breaking is spontaneous in the conventional sense. This takes place when $\Phi \rightarrow \langle \Phi \rangle$ where

$$
\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta \\ 0 \end{pmatrix},
$$

$$
\eta = \sqrt{\frac{\mu \Delta^2}{\lambda}},
$$

and $\eta$ is the electroweak symmetry breaking scale of order 250 GeV. In the standard model, $\mu$ and $\lambda$ are unrelated while in this model they are related,

$$
\frac{\mu}{\lambda} = \left(\frac{\eta}{\Delta}\right)^2 \approx 2.4 \times \zeta G_N^{-1} M_p^{-2} \approx 10^{-33} \times \zeta.
$$

After spontaneous symmetry breaking (SSB), the conventional particles acquire masses as in the standard model,

$$
M_W = \frac{1}{2} g \eta, \quad M_Z = \frac{M_W}{\cos \theta_W},
$$

$$
M_{f_{gg'}} = \frac{1}{\sqrt{2}} Y_{gg'} \eta, \quad M'_{f_{gg'}} = \frac{1}{\sqrt{2}} Y'_{gg'} \eta,
$$

where $\theta_W$ is the weak angle and $M_{f_{gg'}}$, $M'_{f_{gg'}}$ are the quark ($f = q$) and the charged lepton ($f = l$) mass matrices in terms of the Yukawa couplings $Y_{gg'}$ and $Y'_{gg'}$. At this stage neutrinos are still massless. In this model there is still left over the conventional Higgs particle $h_0$ with mass given by

$$
M_{h_0} = \sqrt{\mu \Delta} \approx 0.3 \times \sqrt{\frac{\mu}{\zeta}} M_p,
$$

which is undetermined as $\mu$ and $\zeta$ are still free parameters. It is interesting to note that in this model the mass of the Higgs particle is tied to the scale associated with the breaking of Weyl’s $\tilde{U}(1)$ symmetry which is of order Planck mass. In principle, $M_{h_0}$ can be as large as $M_p$ posing problems with unitarity. However, although the standard model is a renormalizable theory [24, 25], the present model is not. This puts into doubt the validity of the unitarity constraint derived in the renormalizable standard model and extrapolated to the non-renormalizable extended model considered here. After SSB, the mass of the Weylon gets shifted,

$$
M_S \rightarrow \sqrt{\frac{3}{4\pi G_N}} \left(1 + \frac{\beta \eta^2}{\zeta \Delta^2}\right).
$$

However, the additional contribution is negligibly small as $\eta^2/\Delta^2 \approx 10^{-33}$. Apart from being superheavy, another distinct property of the Weylon is that it completely decouples from the fermions of the standard model.
NEUTRINO MASSES

At the present time, one fundamental issue is that of neutrino masses and their lightness as compared to the masses of other particles. In the standard model and the model under consideration, neutrinos are strictly massless as neither right-handed neutral lepton fields nor unconventional scalar fields are present. A popular extension of the standard model that addresses the issue of neutrino masses and mixings in an aesthetically appealing way introduces right-handed neutrinos

\[
\Psi_{1R}^l = \nu_{eR}, \quad \Psi_{2R}^l = \nu_{\mu R}, \quad \Psi_{3R}^l = \nu_{\tau R}
\]

that lead to seesaw masses \[26\] for the conventional neutrinos. This scenario is usually entertained in the \(SO(10)\) grand unified theory, where the right-handed neutrinos acquire super heavy masses. The super heavy scale is determined by the stage at which the internal symmetry \(SO(10)\) breaks, and has nothing to do with gravitational interactions. If right-handed neutrino fields are also introduced in the present model, the seesaw mechanism can naturally be accommodated due to the presence of the singlet field \(\sigma\).

The relevant interaction Lagrangian is

\[
L_\nu = \sum_{g,g'=1,2,3} \left( Y_{gg'}^l \Phi \Psi_{g'}^l + h.c. + \frac{1}{2} Y_{gg'}^{RR} \sigma_{1R}^l C \sigma \Psi_{g'}^l \right). \tag{34}
\]

Lepton number is explicitly broken by the last term. Scale breaking gives superheavy Majorana masses to the right-handed neutrinos and SSB subsequently gives Dirac masses that connects the left- and right-handed neutrinos leading to the following familiar \(6 \times 6\) mass matrix

\[
M_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & Y_l^l & \eta \\ Y_{gg'}^l \eta & Y_{gg'}^{RR} \Delta \end{pmatrix}, \tag{35}
\]

the eigenvalues of which are three seesaw masses for the light neutrinos and three heavy neutrinos with enough parameters to fit the observed solar and atmospheric neutrino oscillation phenomena. In the present model, the scale of right-handed neutrino masses is tied to the scale \(\Delta\) associated with Weyl’s \(\tilde{U}(1)\) breaking which in turn is tied to Newton’s constant \(G_N\). This is unlike the see-saw GUT scenario where right-handed neutrino masses are tied to the GUT scale at which the grand unification internal symmetry breaks. Thus in our scale invariant model the absence of right-handed neutrinos from the low energy scales is attributed to their superheavy masses which are naturally of \(\mathcal{O}(M_P)\). Perhaps this is an indication that right-handed neutrinos (and also gauge-mediated right-handed currents) and gravitational interactions are ultimately related.

We stress that our model needs only quartic potential for the scalar fields \(\Phi\) and \(\sigma\) only with dimensionless couplings as its foundation. The scale-breaking parameter \(\Delta\) then induces the quadratic terms in the resulting potential \(20\). Whereas in the standard model \(\mu\) and \(\lambda\) are not related, our model relates them in terms of \(\Delta\) via \(30\).

We note that the symmetry breaking scheme depicted in the model under consideration would apply universally to theories that accommodate local scale invariance and generate Newton’s constant \(G_N\) as a symmetry breaking effect. In the conventional
SSB mechanism the scalar potential contains terms that are quadratic in scalar fields. Such terms are either added explicitly by hand or generated via quantum corrections.

Our contention is that the present model presents a viable scheme in which gravity is unified, albeit in a semi-satisfactory way, with the other interactions. In the standard model physical fields and the couplings like electric charge $e = 1/\sqrt{g^{-2} + g'^{-2}}$ and Fermi constant $G_F = g^2/(8M_W^2)$ get defined after SSB. Similarly, in the present model, not only $e$ and $G_F$, but also $G_N$ gets defined after symmetry breaking, thus conforming to the main theme in physics that all phenomena observed in Nature are symmetry breaking effects. In the complete theory of all interactions, the model described here will emerge as an effective theory representing the four fundamental interactions in the low energy limit.

**SCALE INVARIANT SU(5) GUT**

In theories unifying all the elementary particle interactions and possessing both local scale invariance and internal symmetry invariance, it is a scale invariance breaking that would precede spontaneous symmetry breaking. This is because since all such theories would contain the scalar curvature $R$, Newton’s constant $G_N$ would be generated as the primary symmetry breaking effect. After scale breaking, the resulting potential would contain the necessary terms quadratic in scalar fields to effect SSB, similar to the discussion in the text, resulting in the GUT scale $M_G \approx M_P$, intermediate scale(s) $M_I$ ($M_I, M_{II}, M_{III}, \cdots$) and the electroweak scale $M_W \approx \sqrt{1/G_F}$ with the hierarchy $M_G > M_I > M_{II} > M_{III} > \cdots > M_W$.

As a concrete example we illustrate this scenario in a scale invariant $SU(5)$ model. The $SU(5)$ GUT consists of the usual gauge bosons in the $24$, the fermions in the $5$ and the $\overline{10}$, and the scalar fields in the $\overline{5}$ ($\equiv H$) and the $24$ ($\equiv \Phi$) representations of $SU(5)$. To make scale invariant $SU(5)$ GUT, we extend the gauge symmetry from $SU(5)$ to

$$G = SU(5) \times \overline{U}(1)$$

and add a real scalar $\sigma$ that is a singlet of $SU(5)$. The scale invariant Lagrangian is straightforward to write down along the lines discussed in the text. The most important term is the scalar potential $V(H, \Phi, \sigma)$ where

$$V(H, \Phi, \sigma) = \lambda_H (H^\dagger H)^2 + \lambda_\Phi (\text{Tr} \Phi^2)^2 + \lambda_\Phi' \text{Tr} (\Phi^4) + \lambda_\sigma \sigma^4 + \lambda_{HI} H^\dagger H \text{Tr} \Phi^2 + \lambda_{HI} \sigma H^\dagger H \sigma^2 + \lambda_{\Phi\sigma} (\text{Tr} \Phi^2) \sigma^2 + \lambda_{HI} \Phi (H^\dagger \Phi^2 H) + \lambda_{\sigma HI} \sigma H^\dagger \Phi H + \lambda_{\sigma HI} \Phi \sigma \text{Tr} \Phi^3 .$$

This is the most general potential consistent with the symmetries of the theory. Notice the important fact that all terms are quartic in the scalar fields. The primary descent occurs when the singlet $\sigma$ acquires a VEV i.e., $\langle \sigma(x) \rangle = \Delta/\sqrt{2}$. In this stage scale invariance is spontaneously broken and

$$G = SU(5) \times \overline{U}(1) \overset{\langle \sigma \rangle \equiv M_P}{\longrightarrow} SU(5)$$
After this stage of symmetry breaking the potential is the usual one of the $SU(5)$ GUT and consists of the usual fields $H$ and $\Phi$. Dimensionful couplings linear and quadratic in the mass dimension appear. The potential, after rescaling, now contains terms quadratic, cubic and quartic in $H$ and $\Phi$ and has the required rich structure to trigger spontaneous symmetry breaking in the conventional sense with the secondary stage and the ternary stage characterized by the vacuum expectation values of $\Phi$ and $H$,

$$SU(5) \xrightarrow{\langle \Phi \rangle \equiv M_I} SU(3) \times SU(2) \times U(1) \xrightarrow{\langle H \rangle \equiv M_W} SU(3) \times U(1)_{em} \quad (39)$$

This model is now the usual $SU(5)$ model with an additional gauge boson, the Weyl-ron. Conceptually, there are marked differences. The standard $SU(5)$ theory fell out of repute because it predicted low weak angle $\sin^2 \theta_W$ and rapid Proton decay, predictions that turned out to be contrary to empirical observations. The present model may not suffer from such defects. The main reason is that the scale invariant $SU(5)$ model described here is semi-renormalizable. It is an effective theory that will eventually emerge from a unified scheme of all interactions that successfully incorporates quantum gravity. Thus the renormalization effects that sent the standard $SU(5)$ theory to disrepute do not apply to the scale invariant $SU(5)$ model discussed here. The additional renormalization effects due to gravitational interactions may easily provide the patch necessary to restore the standard $SU(5)$ model back to its full glory. Donohue [27] has argued that treating conventional field theory models with quantum gravity included (such as the one described here) leads to viable effective theories with quantum corrections due to gravitational interactions as legitimate contributions to the part of the theory that has conventional renormalizable interactions. Consider the one loop renormalized gauge couplings in the $SU(5)$ model with additional contributions $\delta_x = \delta_x(M_P,M_I,M_W), x = 1,2,3$ resulting from the complete theory,

$$\frac{1}{g_1^2(M_W)} = \frac{1}{g^2} + b_1 \ln \frac{M_I}{M_W} + \delta_1, \quad \frac{1}{g_2^2(M_W)} = \frac{1}{g^2} + b_2 \ln \frac{M_I}{M_W} + \delta_2, \quad \frac{1}{g_3^2(M_W)} = \frac{1}{g^2} + b_3 \ln \frac{M_I}{M_W} + \delta_3 \quad (40)$$

where the $b_i$'s are the usual one loop $\beta$-function coefficients, $g_i$'s are the renormalized gauge couplings of $SU(3),SU(2),U(1)$ at the weak scale $M_W$ and the $\delta_x,x = 1,2,3$ are the additional contributions satisfying the constraint $\delta_1 = \delta_2 = \delta_3 = \delta$ at the renormalization point $\mu = M_P$. Also,

$$\frac{1}{e^2(M_W)} = \frac{1}{g_2^2(M_W)} + \frac{5}{3g_1^2(M_W)} \quad \text{and} \quad \sin^2 \theta_W(M_W) = \frac{e^2(M_W)}{g_2^2(M_W)} \quad (41)$$

Since gravitational interactions do not contribute to electric charge, the definition of $e$ remains defined in terms of $g_1$ and $g_2$. With these modifications the predictions for the weak angle and the intermediate GUT scale are

$$\sin^2 \theta_W(M_W) = \sin^2 \theta_W(M_W) \big|_{SU(5)} + \kappa_1 [(b_2-b_3)\delta_1 + (b_3-b_1)d_2 + (b_1-b_2)d_3] \quad ,$$
\[
\ln \frac{M_I}{M_W} = \ln \frac{M_G}{M_W} |_{SU(5)} + \kappa_2 (5\delta_1 + 3d_2 - 8d_3),
\]

where \( |_{SU(5)} \) are the expressions as in the conventional \( SU(5) \) GUT, \( \kappa_1 = 20\pi\alpha_{em}/(8b_3 - 3b_2 - 5b_1) \), \( \kappa_2 = 8\pi^2/3(8b_3 - 3b_2 - 5b_1) \) and the \( \delta_i \)'s are the additional contributions. As input we take the weak mixing angle to be equal to the experimental value, \( \sin^2 \theta_W(M_W) = 0.23 \), \( \alpha_{em} \approx 1/128 \), \( \alpha_s \approx 0.11 \) and the intermediate scale to be the value \( M_G/M_W = 10^{15} \) that meets the present limit on the lifetime of the Proton. With this, the constraints on the various \( \delta_i \)'s are

\[
\begin{align*}
0.16\delta_1 - 0.40\delta_2 + 0.26\delta_3 &= 1, \\
0.40\delta_1 + 0.24\delta_2 - 0.63\delta_3 &= 1.
\end{align*}
\]

Tiny effects due to gravitational interactions can easily amplify the various \( \delta_i \)'s at the renormalization point \( \mu = M_W \) to provide the required patch such that the scale invariant \( SU(5) \) model fares better than the conventional \( SU(5) \) GUT. That this is indeed the case has been recently demonstrated by Robinson and Wilczek [28] who, working in the philosophy advocated by Donoghue [27], compute the one loop contributions due to graviton exchange to the renormalization of the gauge couplings and show that the graviton contributions work in the right direction as implied in this work.

To conclude, we have accommodated Weyl’s scale invariance as a local symmetry in the standard electroweak model. This inevitably leads to the introduction of general relativity. The additional particles are one vector particle we call the Weylon and a real scalar singlet that couples to the scalar curvature \( \tilde{R} \) à la Dirac [16]. The scale at which Weyl’s scale invariance breaks defines Newton’s gravitational constant \( G_N \). Weyl’s vector particle, i.e., the Weylon absorbs the scalar singlet \( \sigma \) and acquires mass \( \mathcal{O}(M_P) \) in the absence of fine tuning. The scalar potential is unique in the sense that it consists of terms only quartic in the scalar fields and dimensionless couplings. Yet, as we have demonstrated, symmetry breaking is possible such that the left-over symmetry is \( U(1)_{em} \) and all particle masses are consistent with present day phenomenology. If right-handed neutrinos are also introduced, the light neutrinos acquire seesaw masses and the suppression factor in the neutrino masses is of \( \mathcal{O}(M_P) \). As a concrete example, \( SU(5) \) GUT with local scale invariance is presented and the implications noted.

I don’t know about you, but

“Herman Weyl would have been very happy”

to see his work revived in the light of our present understanding of elementary particle interactions. After all, his gauge idea may turn be out not as futile as once perceived.

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