Supergravity, Supermembrane and M(atrix) model on PP-Waves

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In the first part of this paper, we study the back-reaction of large-N light cone momentum on the maximally supersymmetric anti-pp-wave background. This gives the type IIA geometry of large-N D0-branes on curved space with fluxes. By taking an appropriate decoupling limit, we conjecture a new duality between string theory on that background and dual field theory on D0-branes which we derive by calculating linear coupling terms. Agreement of decoupling quantities, $SO(3) \times SO(6)$ isometry and Higgs branch on both theories are shown. Also we find whenever dual field theory is weakly coupled, the curvature of the geometry is large. In the second part of this paper, we derive the supermembrane action on a general pp-wave background only through the properties of null Killing vector and through this, derive the Matrix model.
### 1 Introduction

At the present time, two concrete examples of the holographic principle are well known. One is AdS/CFT and the other is the M(atrix) model of M-theory [1]. The latter is conjectured to be the microscopic definition of 11 dimensional M-theory in the DLCQ description and its action is described by the large-N limit of dimensional reduction of 16 supercharges $U(N)$ gauge theory to $0 + 1$ dimensions [2]:

$$S_0 = \int d\tau \text{Tr} \left( \sum_{i=1}^{9} \frac{1}{2} \dot{X}^{i2} + \frac{1}{4} \sum_{i,j=1}^{9} [X^i, X^j]^2 + i\Psi^T \dot{\Psi} - \sum_{i=1}^{9} \Psi^T \gamma^i [X^i, \Psi] \right) \tag{1}$$

Interest in this model was revived after the paper [3] where the Matrix model on maximally supersymmetric pp-wave is derived containing extra mass terms:

$$S_{\text{mass}} = \int d\tau \text{Tr} \left( -\sum_{i=1}^{3} \frac{1}{2} \left( \frac{\mu}{3} \right)^2 X^{i2} - \sum_{i=4}^{9} \frac{1}{2} \left( \frac{\mu}{6} \right)^2 X^{i2} + \frac{i\mu}{3} \sum_{i,j,k=1}^{3} \epsilon_{ijk} X^i X^j X^k - \frac{i\mu}{4} \Psi^T \gamma_{123} \Psi \right) \tag{2}$$

This corresponds to a background that is the maximally supersymmetric pp-wave metric [4, 5],

$$ds^2 = 2dx^+ dx^- - \sum_{i=1}^{3} \left( \frac{\mu}{3} \right)^2 x^{i2} dx^{i+2} - \sum_{i=4}^{9} \left( \frac{\mu}{6} \right)^2 x^{i2} dx^{i+2} + \sum_{i=1}^{9} dx^{i2}. \tag{3}$$

$$F^{(4)} = \mu dx^+ \wedge dx^1 \wedge dx^2 \wedge dx^3$$

Several papers about analysis and generalization of this action soon followed [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. A big difference of this model from one on flat space is the non-existence of flat direction due to the mass terms which are induced by pp-wave geometry. This improves the situation, since we do not have to worry about the threshold bound state, which is a very difficult problem in the flat space case.

On the other hand, it is well-known that the M(atrix) theory action is the same action of type IIA large-N D0-branes action in flat space background. This relation is clarified by the paper [20, 21] and its claim that by infinitely boosting a large-N IIA D0 system, the small spacelike compactified radius along $x^{10}$ is infinitely boosted to a large lightlike radius along $x^-$. The reason we need to take large-N limit is because we want to take lightlike circle to infinity while keeping $p_-$ fixed. A similar story should hold in the
M(atrix) model on the pp-wave background. In [7], IIA background metric on which D0-branes action is identical to BMN action is explicitly derived. They derived it by “unboosting” 11 dimensional pp-wave metric and then compactifying this to 10 dimensions. The metric is written in string frame as,

\[ ds^2 = g_{00}dt^2 + g_{AB}dx^A dx^B. \]

\[ g_{00} = -e^{-\frac{4}{3}\phi}, g_{AB} = \delta_{AB}e^{\frac{4}{3}\phi}, \]

\[ e^{\frac{4}{3}\phi} = 1 - \frac{1}{4}F^2, \]

\[ A_0 = \frac{-F^2}{1 - \frac{F^2}{4}} \]

\[ F^2 \equiv \left( \frac{\mu}{3} \right)^2 (x_1^2 + x_2^2 + x_3^2) + \left( \frac{\mu}{6} \right)^2 (x_4^2 + \cdots + x_9^2) \]

\[ F_{0123} = \frac{\mu}{2} \frac{1 - \frac{F^2}{2}}{1 - \frac{F^2}{4}}, H_{123} = -\frac{\mu}{2} \]

The above metric makes sense only when curvature is small, that is, only when \( F^2 \ll 4 \). Actually by using this property [22], we can construct the D0-branes action in this weak background and it is the same action of BMN [7].

In [6] it was shown that the back-reaction of the large-N D0-brane charge on the geometry is important. In the flat space background case, we know large-N D0-branes action is dual to string theory on a warped \( AdS_2 \times S^8 \) background, and depending on the energy scale we consider, we have different description of this 0+1 dimensional theory. The phase diagram of this D0-branes system is shown in the beautiful paper [6].

Motivated by these developments, it would be interesting to establish a similar correspondence in the non-trivial background [1], that is, to obtain string theory on the near-horizon geometry of large-N D0-branes in the background geometry [1] that is dual to BMN action. Unfortunately we don’t know the 11-dimensional supergravity solution which corresponds to D0-branes on geometry [1] in 10-dimension. But instead, we know anti-D0-branes (=graviton which propagate along \(-x^{10}\) direction) solution on geometry [1]. Equivalently we know D0-branes on all RR fields sign flipped geometry of [1], which corresponds to “anti” pp-wave in 11-dimension. Therefore we study the back-reaction of large-N D0-brane charge on anti-pp-wave background in this paper [1].

\[ ^{1}\text{The author is grateful to Juan Maldacena for pointing out a mis-interpretation of anti-D0-branes as D0-branes in the first version of this paper.} \]
The organization of this paper is composed of two parts. In section 2, we study the effect of the back-reaction of large-N D0-brane charge on the maximally supersymmetric anti-pp-wave. By taking an appropriate decoupling limit, we derive the IIA geometry which is given by eq. (10) with flux (12). Then we consider the dual field theory whose action is derived by calculating the linear coupling terms of D0-branes with background (19). We find agreement about well-defined $O(\alpha'^2\mu)$ deformation, $SO(3) \times SO(6)$ isometry and Higgs branch on both theories. Also it is shown that whenever the dual field theory can be described by perturbation theory, the curvature of the geometry (10) is large. In section 3, this is completely independent work to section 2, we try to derive the Matrix model on a general pp-wave background. We consider the supermembrane action on a general pp-wave background up to quadratic terms of fermion fields. Then we prove that there are no higher order corrections only through the properties of null Killing vector. Finally, we obtain the Matrix model on a general pp-wave background and we study its supersymmetry which has the expected form given the background. In the appendix, we summarize the relevant facts about general pp-waves.

2 Back-reaction of large-N D0-brane charge on anti-pp-wave

2.1 Type IIA background

The gravitational wave with N units of momentum in flat 11 dimensional space has the supergravity background

$$\begin{align*}
&ds^2 = 2dx^+ dx^- + \left( \frac{C}{|\vec{x}|^2} \right) dx^{-2} + d\vec{x}^2, \\
&F^{(4)} = 0, \\
&C \equiv 2^8 \pi^2 \Gamma \left( \frac{7}{2} \right) \alpha'^5 \left( g_{YM}^2 N \right)
\end{align*}$$

(5)

$F^{(4)}$ is field strength of 3-form gauge potential and the constant $C$ is checked from comparison of this metric when KK reduced to the 10 dimensional metric describing N D0-branes. Now because of enough supersymmetry, the superposition of “anti”-pp-wave metric

$$\begin{align*}
&ds^2 = 2dx^+ dx^- - \sum_{i=1}^{3} \left( \frac{\mu}{3} \right)^2 x^{i2} dx^{-2} - \sum_{i=4}^{9} \left( \frac{\mu}{6} \right)^2 x^{i2} dx^{-2} + \sum_{i=1}^{9} dx^{i2}
\end{align*}$$

(6)
\[ F^{(4)} = \mu dx^- \wedge dx^1 \wedge dx^2 \wedge dx^3 \]

and gravitational wave (5) is also a solution of Einstein equations. We combine them using the ansatz of plane wave,

\[ ds^2 = 2dx^+dx^- + H(\vec{x})dx^{\perp} + d\vec{x}^2 \]

\[ H \equiv \left( \frac{C}{|\vec{x}|^7} \right) - F^2 = \left( \frac{C}{|\vec{x}|^7} \right) - \left[ \left( \frac{\mu}{3} \right)^2 (x_1^2 + x_2^2 + x_3^2) + \left( \frac{\mu}{6} \right)^2 (x_4^2 + \cdots + x_9^2) \right] \]

(7)

\[ F_{-123} = \mu \]

where \( F^2 \) is defined in eq. (4). The only nontrivial component of the Ricci tensor is \( R_{--} \sim \partial_i H \sim -\mu^2 \) (except at \( \vec{x} = 0 \)). Because the curvature scalar vanishes, the superimposed solution satisfies the Einstein equation \( R_{-} - F_{-ijk}F_{+ijk} \sim -\mu^2 \) as expected. Note that the solution (7) preserves one half of the supersymmetry. Knowing the 11 dimensional metric, it is straightforward to derive the 10 dimensional metric in string frame. Taking into account \( \mu \to \frac{\mu}{\sqrt{2}} \) by unboosting, the metric is,

\[ ds^2 = g_{00}dt^2 + g_{AB}dx^Adx^B. \]

(8)

\[ g_{00} = -e^{-\frac{2}{3} \phi}, \quad g_{AB} = \delta_{AB}e^{\frac{2}{3} \phi} \]

\[ e^{\frac{4}{3} \phi} = 1 + \frac{1}{4}H \]

\[ A_0 = \frac{-H}{1 + \frac{H}{4}} \]

\[ F_{0123} = -\frac{\mu}{2} \]

This is the IIA supergravity solution called homogeneous D0-branes (HD0-branes) in [5]. Note that the above solution describing all N D0-branes sit at the center of the space, but we can put the center of each harmonic function anywhere, this is easily seen in 11-dimensional solution (7) because of supersymmetry. Physically geometry is the same as considering usual pp-wave with N anti-D0-branes. The superposition of N D0-branes and usual pp-wave is not a 11 dimensional supergravity solution unfortunately.

2.2 Decoupling limit and near horizon geometry

As usual in AdS/CFT duality, we must take the near-horizon geometry of the metric (5) so that the boundary theory decouples from the bulk theory.
The decoupling limit we take is

\[ U^2 \equiv \frac{1}{\alpha'^2} \sum_{i=1}^{3} x_i^2 = \text{fixed}, \quad V^2 \equiv \frac{1}{\alpha'^2} \sum_{i=4}^{9} x_i^2 = \text{fixed}, \quad (9) \]

\[ g_Y^2 = \frac{1}{4\pi^2 \alpha'^2} g_s \mu = \text{fixed}, \quad \mu \alpha'^2 = \text{fixed}, \quad \alpha' \to 0 \]

The type IIA supergravity solution in this decoupling limit is

\[ ds^2 = \alpha' \left[ -g_0^{-\frac{1}{2}} dt^2 + g_0^{\frac{1}{2}} \left( dU^2 + U^2 d\Omega_2^2 \right) + g_0^{\frac{1}{2}} \left( dV^2 + V^2 d\Omega_5^2 \right) \right] \quad (10) \]

where \( g_0 \) is well-defined:

\[ g_0 = \frac{1}{4} \left[ \frac{2^8 \pi^2 \Gamma \left( \frac{7}{2} \right) (g_Y^2 N)}{(U^2 + V^2)^{\frac{7}{2}}} - \left( \frac{\alpha'^2 \mu}{3} \right)^2 U^2 - \left( \frac{\alpha'^2 \mu}{6} \right)^2 V^2 \right] \quad (11) \]

In this decoupling limit, the NSNS and RR field strengths are

\[ H_{123} = -\alpha' \left( \frac{\alpha'^2 \mu}{2} \right), \quad F_{0123} = -\alpha' \left( \alpha'^2 \mu \right) \quad (12) \]

You can easily see that since both the metric and the NSNS field strength are \( \mathcal{O}(\alpha') \), their contributions are at the same order and give rise to the well-defined Nambu-Goto action, that is, \( \alpha' \) cancel out in \( S_{NG} \sim \frac{1}{\alpha'} \int d^2 \sigma \sqrt{det(G + B)} \). And of course these fluxes are supported by a curvature term which is proportional to \( \alpha'^2 \mu \) in \( g_0 \). Now it is clear that this background has \( SO(9) \to SO(3) \times SO(6) \) symmetry breaking because of the \( \mathcal{O}(\alpha'^3 \mu) \) terms in the flux and the metric. Here we take parameter \( \mu \) as \( \frac{1}{9} \). If we take different limit of \( \mu \), the results are very different. For example if we take \( \mu = \text{fixed} \), the effect of \( \mu \) is negligible. The geometry is warped \( AdS_2 \times S^8 \) space and the rotational symmetry is enhanced to \( SO(9) \). If we take a limit where \( \mu \) is bigger than \( \mathcal{O}(\alpha'^{-2}) \), the IIA metric doesn’t make sense since the curvature becomes singular at \( g_0 = 0 \), i.e. from 11 dimensional point of view, the sign of metric in \( (dx^{10})^2 \) changes and becomes timelike, therefore compactification doesn’t make sense at this point.

The curvature of this metric is (for simplicity taking \( U \) and \( V \) to be the same order)

\[ \alpha' \mathcal{R} \sim g_0^{-\frac{1}{2}} U^{-2} \quad (13) \]
So the curvature small condition is satisfied for
\[
U \ll \left( g_{YM}^2 N \right)^{\frac{1}{9}} \quad \text{if} \quad \left( g_{YM}^2 N \right)^{\frac{1}{3}} \lesssim (\alpha'^2 \mu)^{-\frac{1}{3}} \tag{14}
\]
\[
U \ll \left( g_{YM}^2 N \right)^{\frac{1}{9}} (\alpha'^2 \mu)^{-\frac{2}{9}} \quad \text{if} \quad \left( g_{YM}^2 N \right)^{\frac{1}{3}} \gtrsim (\alpha'^2 \mu)^{-\frac{1}{3}} \tag{15}
\]

Also the dilaton small condition is satisfied for
\[
\frac{\left( g_{YM}^2 N \right)^{\frac{1}{3}}}{N^{\frac{1}{3}}} \ll U \lesssim \left( g_{YM}^2 N \right)^{-\frac{4}{9}} (\alpha'^2 \mu)^{-1} N^{\frac{2}{9}} \tag{16}
\]

Note that the second inequality in eq. (16) is weaker than curvature small condition in eq. (14) and (15). We claim that IIA string theory on this background (10) with fluxes (12) is well-defined as long as we consider theory at energy scale region \( U \):
\[
\frac{\left( g_{YM}^2 N \right)^{\frac{1}{3}}}{N^{\frac{1}{3}}} \ll U \ll \left( g_{YM}^2 N \right)^{\frac{1}{3}} \quad \text{if} \quad \left( g_{YM}^2 N \right)^{\frac{1}{3}} \lesssim (\alpha'^2 \mu)^{-\frac{1}{3}} \tag{17}
\]
\[
\frac{\left( g_{YM}^2 N \right)^{\frac{1}{3}}}{N^{\frac{1}{3}}} \ll U \ll \left( g_{YM}^2 N \right)^{\frac{1}{3}} (\alpha'^2 \mu)^{-\frac{2}{9}} \quad \text{if} \quad \left( g_{YM}^2 N \right)^{\frac{1}{3}} \gtrsim (\alpha'^2 \mu)^{-\frac{1}{3}} \tag{18}
\]

### 2.3 Dual field theory

The theory on D0-branes is derived by assuming the linear coupling of D0-branes with these background field:
\[
d s^2 = g_{00} d t^2 + g_{AB} d x^A d x^B \tag{19}
\]
\[
g_{00} = -e^{-\frac{2}{3} \phi}, g_{AB} = \delta_{AB} e^{\frac{4}{3} \phi}
\]
\[
e^{\phi} = 1 - \frac{1}{4} F^2
\]
\[
A_0 = \frac{F^2}{1 - \frac{F^2}{4}}
\]
\[
F^2 \equiv \left( \frac{\mu}{3} \right)^2 (x_1^2 + x_2^2 + x_3^2) + \left( \frac{\mu}{6} \right)^2 (x_4^2 + \cdots + x_9^2)
\]
\[
F_{0123} = -\frac{\mu}{2} \frac{4 - 2 F^2}{4 - F^2}, H_{123} = -\frac{\mu}{2}
\]
by following the methods of [22]. Note that this metric is derived by compactifying the “anti”-pp-wave geometry along \( x^{10} \) direction and taking into account the rescaling of \( \mu \to \frac{\mu}{\sqrt{2}} \), therefore the sign of RR-field \( A_0 \) and \( F_{0123} \)
is flipped compared with (4). By taking the weak curvature limit and neglecting the $\mathcal{O}((F^2)^2)$ terms, the above IIA background is approximated as

$$
\phi \sim -\frac{3}{16} F^2, \ h_{00} \sim -\frac{1}{8} F^2, \ h_{AB} \sim -\frac{1}{8} F^2 \delta_{AB},
$$

$$
A_0 \sim \frac{F^2}{4}, \ C_{0ij} \sim \epsilon_{ijk} \frac{\mu}{6} x^k, \ B_{ij} \sim -\epsilon_{ijk} \frac{\mu}{6} x^k
$$

The dual field theory has action\(^4\):

$$
S = S_{\text{flat}} + S_{\text{linear}}
$$

\begin{align*}
S_{\text{linear}} & = \int dt \left[ \frac{1}{4} \partial_A \partial_B h_{\mu\nu} I_{h}^{\mu\nu(AB)} + \frac{1}{2} \partial_A \partial_B \phi I^{(AB)}_{\phi} ight. \\
& \quad + \frac{1}{2} \partial_A \partial_B A_{\mu} I_{0}^{\mu(AB)} + \partial_A B_{\mu\nu} I_{s}^{\mu\nu(A)} + \partial_A C_{\mu\nu\lambda} I_{2}^{\mu\nu\lambda(A)} \bigg] \\
& = \int dt \left[ -\frac{1}{16} \partial_A \partial_B F^2 \left( T^{--(AB)} + \mathcal{O}(\alpha^2) \right) + \epsilon_{ijk} \frac{2\pi \alpha' \mu}{2} \left( J^{-ij(k)} + \mathcal{O}(\alpha^2) \right) \right]
\end{align*}

where $\partial_A = \frac{\partial}{\partial U^A}$ and $F^2 = \left( \frac{2\pi \alpha' \mu}{3} \right)^2 \left( (U^1)^2 + \cdots (U^9)^2 \right) + \left( \frac{2\pi \alpha' \mu}{6} \right)^2 \left( (U^4)^2 + \cdots (U^9)^2 \right)$.

For details of this calculation, see appendix. And $T^{--(AB)} \sim \mathcal{O}(\alpha^2)$ and $J^{-ij(k)} \sim \mathcal{O}(\alpha^2)$ are given by

$$
T^{--} = \left( \frac{2\pi \alpha' \mu}{4g_s l_s} \right)^4 STr \left( F_{ab} F_{bd} F_{cd} F_{da} - \frac{1}{4} F_{ab} F_{cd} F_{ca} + \Psi^2 + \Psi^4 \text{ terms} \right)
$$

$$
J^{-ij} = \left( \frac{2\pi \alpha' \mu}{6g_s l_s} \right)^3 STr \left( \hat{U}^i \hat{U}^j F_{ij} - \hat{U}^i \hat{U}^j F_{ii} - \frac{1}{2} \hat{U}^i \hat{U}^j F_{ij} + \frac{1}{4} F_{ij} F_{km} F_{ln} + F_{im} F_{ml} F_{ij} + \Psi^2 + \Psi^4 \text{ terms} \right)
$$

where $F_{oi} = \hat{U}^i$, $F_{ij} = i [U^i, U^j]$ and $a, b, \cdots$ run from 0 to 9, but $i, j, \cdots$ run 1 to 9. More details of the fermion terms are given in page 17 of [23]. The higher multipole moments of these currents are given by

$$
T^{--(AB)} = \text{Sym} \left( T^{--}; U^A U^B \right) + T_{\text{fermion}}^{--(AB)}
$$

$$
J^{-ij(k)} = \text{Sym} \left( J^{-ij}; U^k \right) + J_{\text{fermion}}^{-ij(k)}
$$

In the decoupling limit (4),

$$
S_{\text{flat}} \rightarrow \frac{4\pi^2 g_s l_s^3}{g_s} \int dt Tr \left( \sum_{i=1}^{9} \frac{1}{2} \hat{U}^{i\gamma^2} + \frac{1}{4} \sum_{i,j=1}^{9} [U^i, U^j]^2 + i \Psi^T \dot{\Psi} - \sum_{i=1}^{9} \Psi^T \gamma^i [U^i, \Psi] \right)
$$

\(^3\)We use the notation of [22]
and

\[ S_{\text{linear}} \rightarrow \int dt \left[ -\frac{1}{16} \partial_A \partial_B F^2 (T^{--(AB)}) + \epsilon_{ijk} \frac{2\pi \alpha' \mu}{2} f^{-ij(k)} \right] \]

\[ = \frac{4\pi^2 f_s^3}{g_s} \int dt \left[ \text{Tr} \left[ -\frac{1}{32} \left( \frac{4\pi^2 \alpha'^2 \mu}{3} \right) \sum_{A=1}^{3} \text{STr} \left( \text{Sym}(F_{ab} F_{bc} F_{cd} F_{da} - \frac{1}{4} F_{ab} F_{ab} F_{cd} F_{cd}); U^A U^A) \right) \right. \right. \]

\[ - \frac{1}{32} \left( \frac{4\pi^2 \alpha'^2 \mu}{6} \right) \sum_{A=1}^{9} \text{STr} \left( \text{Sym}(F_{ab} F_{bc} F_{cd} F_{da} - \frac{1}{4} F_{ab} F_{ab} F_{cd} F_{cd}); U^A U^A) \right) \]

\[ + \sum_{i,j,k=1}^{3} \epsilon_{ijk} \left( \frac{4\pi^2 \alpha'^2 \mu}{12} \right) \text{STr} \left( \text{Sym}(\dot{U}^i \dot{U}^t F_{ij} - \dot{U}^j \dot{U}^t F_{ij} \right. \}

\[ - \frac{1}{2} \dot{U}^j \dot{U}^t F_{ij} + \frac{1}{4} F_{ij} F_{lm} F_{lm} + F_{im} F_{ml} F_{ij}; U^k) \right. \}

\[ + (\Psi^2, \Psi^4 \text{ terms}) \]

(21)

We conjecture that this is the dual field theory action. Note that the exact cancellation of the mass terms (which are \(O(\mu^2 U^2)\)) in this theory is crucial because they are not well-defined in the decoupling limit (9). The first nontrivial effects of the background (19) appears as higher \(O(\alpha'^4 \mu^2 U^{10})\), \(O(\alpha'^4 \mu^2 U^2 U^6)\) and \(O(\alpha'^4 \mu^2 U^4 U^2)\) terms which are well-defined. The symmetry breaking \(SO(9) \rightarrow SO(3) \times SO(6)\) because of \(\mu \alpha'^2\) terms are easily confirmed just as in the dual geometry (11), (10). Also note that the above theory has classical vacuum where all \(U^i\) take static diagonal form. The existence of Higgs branch in dual field theory is expected, since the D0-branes can move freely in the background (19), as easily understood from 11-dimensional solution (7).

Let’s study when the perturbation theory is valid in this field theory. The dimensionless effective coupling constants of this theory are \(g_{\text{eff}}^2 = \frac{g_s^2}{U^3} (\alpha'^2 \mu U^3)^a\) where \(a\) is some constant which takes \(0 \leq a \leq 1\). Therefore for perturbation theory to be valid, it is necessary to require following two conditions:

\[ (\alpha'^2 \mu) \left( g_{YM}^2 N \right) \ll 1 \text{ and } \frac{g_{YM}^2 N}{U^3} \ll 1 \]  

(22)

Note that this is the parameter region where the curvature of the dual geometry becomes large, see (14). This is expected result of gauge/string duality as in the \(\mu = 0\) case in [1]. Here we base our the argument on the bosonic terms, but the fermion terms don’t change the above arguments at all. Finally we comment on the validity of weak approximation. From the geometry
it is reasonable to expect that weak approximation is valid as long as $U \ll (g^2_{YM}N)^{\frac{1}{3}} (\alpha'^2 \mu)^{-\frac{2}{9}}$.

3 M(atrix) model on general pp-wave

In this section, we study less supersymmetric pp-wave backgrounds. One of the surprising things in pp-wave is that there are plenty of pp-wave solutions which preserve “supernumerary” or fractional number of additional supersymmetries. So far as we know pp-wave solutions preserve $\mathcal{N} = 16 + \mathcal{N}_{extra}$, where $\mathcal{N}_{extra} = 0, 2, 4, 6, 8, 10$ [23, 25, 27, 8, 28, 29]. The goal of this section is to try to derive the DLCQ action of the Matrix model on these general pp-waves. We summarize the relevant known results about supersymmetry of fractional pp-waves in the appendix.

The 11 dimensional pp-wave we consider has metric [24],

$$ds^2 = 2dx^+dx^- + H(x^i, x^+)dx^{+2} + \sum_{i=1}^{9} dx^{i2}$$

$$H(x^i, x^+) = -\sum_{i=1}^{9} \mu_i^2 x^{i2}$$

$$F^{(4)} = \sum_{i,j,k=1}^{9} \frac{1}{3!} f_{ijk} dx^+ \wedge dx^{ijk}$$

The Einstein equations require,

$$\sum_{i=1}^{9} \mu_i^2 = \frac{1}{2} \left( \frac{1}{3!} \right) \sum_{i,j,k=1}^{9} (f_{ijk})^2$$

Here pp-waves always have at least 16 supercharges, plus $2k$ “extra” supersymmetries (see eq. (31), (32), (33), (34) in the appendix).

3.1 Supermembrane on general pp-wave

The 11 dimensional supermembrane action is

$$S[Z(\zeta)] = \int d^3\zeta \left[ -\sqrt{-G(Z)} - \frac{1}{6} \epsilon^{abc} \Pi_a^A \Pi_b^B \Pi_c^C B_{CBA} (Z(\zeta)) \right]$$
where \( Z^A = (X^M(\zeta), \xi(\zeta)) \) are superspace embedding coordinates and \( B_{CBA} \) the antisymmetric tensor gauge superfield, we also take light cone gauge \( X^+ = \tau \), \( \tau \) is the world-volume time coordinate and \( G \) is given by

\[
G(X, \xi) = \det (\Pi_a^M \Pi_b^N g_{MN}) = \det (\Pi_a^i \Pi_b^i q_{rs}).
\]

Here \( \Pi_a^+ \) is the supervielbein pullback

\[
\Pi_a^+ = \partial_a Z^A E^r_A
\]

The supermembrane action on fully supersymmetric pp-wave background can be derived by using the coset space approach where we can express the supervielbein and tensor gauge superfield background in all orders of \( \xi \). In the fractional pp-wave case, we don’t know the exact supervielbein and tensor field, although it is known to order \( \xi^2 \). In the case that the gravitino has 0 vev, supervielbein pullback \( \Pi_a^+ \) is given by

\[
\Pi_a^+ = \partial_a Z^A E^r_A
\]

Also the tensor fields pullback term is given by

\[
-\frac{1}{6} \epsilon^{abc} \Pi_a^A \Pi_b^B \Pi_c^C B_{CBA} (Z(\zeta)) = \frac{1}{6} \epsilon^{abc} \partial_a X^M \partial_b X^N \partial_c X^L \left[ C_{MNL} + \frac{3}{4} \xi \Gamma_{rsl} \xi w^r_{Mst} - 3 \xi \Gamma_{MN} \Omega L \xi \right] - \epsilon^{abc} \tilde{\xi} \Gamma_{MN} \partial_\xi \left[ \frac{1}{2} \partial_a X^M (\partial_b X^N + \tilde{\xi} \Gamma^N \partial_b \xi) + \frac{1}{6} \xi \Gamma^M \partial_a \xi \tilde{\xi} \Gamma^N \partial_\xi \right] + O(\xi^4)
\]

Here the nonzero components of \( w^r_{Mst}, \Omega_M \) are given in eq. (29) in the appendix. In order to write down the explicit form of the fermion fields, we write the 11 dimensional gamma matrices in terms of 9 dimensional gamma matrices as given in eq. (30) and remove the \( \kappa \) symmetry of this action by gauge fixing

\[
\Gamma^+ \xi = 0
\]

\[
\Leftrightarrow \tilde{\xi} = \frac{1}{2^\frac{1}{4}} \begin{pmatrix} 0 & -\Psi^T \end{pmatrix}, \quad \xi \equiv \frac{1}{2^\frac{1}{4}} \begin{pmatrix} \Psi \\ 0 \end{pmatrix}
\]

Under this gauge condition, the supervielbein pullback and tensor gauge superfield pullback are written as

\[
\Pi_a^+ = \partial_a X^+, \quad \Pi_a^i = \partial_a X^i.
\]

\( ^4 \)We use notation \( A = (M, \alpha) \) for curved space-time indices, here \( M = (+, - , i (= 1, ..., 9)) \) whereas tangent space vector indices \( r = +, - , i (= 1, ..., 9) \). We are sloppy about transverse coordinates because that direction is flat. Also we use \( a = 0, 1, 2 \) for world-volume coordinate on membrane such that \( \epsilon_{012} = 1 \).

\( ^5 \)We choose world-volume coordinates \( (\tau, \sigma^1, \sigma^2) \).
\[
\Pi_a^+ = \partial_a X^- + \partial_a X^+ \left( \frac{H}{2} - i \Psi^T \theta \Psi \right) + i \Psi^T \partial_a \Psi,
\]
\[
- \frac{1}{6} \epsilon^{abc} \Pi_a \Pi_b \Pi_c B_{CBA} = \left( - \frac{1}{6} \sum_{i,j,k=1}^9 f_{ijk} \{ X^i, X^j \}_{PB} X^k - i \sum_{i=1}^9 \Psi^T \gamma^i \{ X^i, \Psi \}_{PB} \right)
\]

Here \( \{ X, \Psi \}_{PB} = \epsilon^{0ab} \partial_a X \partial_b \Psi \), and \( \theta \equiv \frac{1}{3!} f_{ijk} \gamma^{ijk} \) is the 16 \times 16 \( \text{SO}(9) \) gamma matrix. Therefore the action is given by
\[
S = \int d^3 \sigma \left( \sum_{i=1}^9 \frac{1}{2} \dot{X}^{i2} - \frac{1}{2} \mu_i^2 \dot{X}^{i2} - \frac{1}{4} \sum_{i,j=1}^9 \{ X^i, X^j \}_{PB}^2 - \frac{1}{6} \sum_{i,j,k=1}^9 f_{ijk} \{ X^i, X^j \}_{PB} X^k 
+ i \Psi^T \dot{\Psi} - \frac{i}{4} \Psi^T \theta \Psi - i \sum_{i=1}^9 \Psi^T \gamma^i \{ X^i, \Psi \}_{PB} \right) \quad (23)
\]

again up to \( \mathcal{O}(\Psi)^4 \).

So far we have derived this action by truncating the full action, and in principle we should include terms of higher order in \( \Psi \). But even though we derived this action by the truncation of \( \mathcal{O}(\Psi)^4 \) terms, we can argue that this is the exact action on general pp-wave background. The proof is as follows.

The truncated terms have two parts, one being higher order terms in the supervielbein pullback, the other higher order terms in the tensor gauge superfield pullback. Let’s consider the supervielbein first. The supervielbein is defined as \( \Pi_i^r = \partial_r Z^A E^r_A \), and \( \partial_i Z^A \) is proportional to \( \partial_i X^M \) or \( \partial_i \xi \), while \( E^r_A \) is superfield of elfbein which is made by some complicated function of the quantities \( \xi, \Gamma^r, C_{ijk}, F_{+i,j,k}, \partial_M, w_{MNL} \equiv \epsilon_{rN} \epsilon_{sL} w_{rM}^s, \Gamma_{NL}^M \) and all other geometrical functions with indices of curved space \( M, N, \ldots \), i.e. (some derivative of \( \epsilon_{rM}^N \)) \& \( \epsilon_{rN} \). But note that \( C_{ijk}, F_{+i,j,k}, \partial_M, w_{MNL} \equiv \epsilon_{rN} \epsilon_{sL} w_{rM}^s, \Gamma_{NL}^M \) and whatever function we obtain from the derivative of elfbein, they cannot have curved space-time lower index \( M = - \). This is because the pp-wave has null Killing vector \( k^M \), which has only one nonzero component along upper index \( M = - \) and lower index \( M = + \), and the field strength is proportional to \( k^M = k^+ \). In the same way all upper curved space indices can’t have \( M = + \). On the other hand, because of the gauge condition \( \Gamma^+ \xi = 0 \), \( \xi \Gamma^r \ldots \Gamma^s \xi \) can be nonzero if and only if we have one tangent space upper index \( r = - \) and no upper index \( r = + \) for the Gamma matrices between \( \xi \) and \( \xi \). Since \( \epsilon_{rN} M \neq 0 \) if and only if \( M = + \), in order to contract the curved space-time index \( M \), we need upper \( M = + \) for each \( \xi \Gamma^r \ldots \Gamma^s \xi \) term, and the only component which can have upper \( M = + \) to contract is \( \partial_a X^{(M=+)} = \delta^0_a \). Since the supervielbein pullback is at most linear in \( \partial_a X^M \), we conclude that \( \xi \) is

\[\text{Similar argument about Green-Schwarz action on general pp-wave background which doesn’t have supernumerary supersymmetry is found in } \]

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at most bilinear in $\xi$, therefore that there are no higher order corrections in $\xi$.

In the same way we can show that there are no higher order corrections to the $\epsilon^{abc} \Pi^A a \Pi^B \Pi^C B_{CBA} (Z(\xi))$ term in the action. Since each bilinear term $\bar{\xi} \Gamma^r \ldots \Gamma^s \xi$ needs one upper index $r = -$ and $e_r = -M \neq 0$ iff $M = +$, we need $\partial_a X^{(M=+)} = \delta_a^0$ from $\partial_a Z^A$. But because $\epsilon^{abc} \partial_a X^{M=+} \partial_b X^{N=+} = 0$, we can have at most linear terms in $\partial_a X^{M=+}$. Therefore these terms are at most bilinear in $\xi$ so the action (23) which we obtained is exact.

3.2 Matrix model and its supersymmetry

Once we obtain the supermembrane action exactly, it is straightforward to derive the Matrix model action by regularizing according to the usual prescription [32].

$$X^M (\zeta) \leftrightarrow X^M_{N \times N}$$
$$\Psi (\zeta) \leftrightarrow \Psi_{N \times N}$$
$$\int d^2 \sigma \leftrightarrow Tr$$
$$\{, \}_{PB} \leftrightarrow -i [,]$$

$$S = \int d\tau Tr \left( \sum_{i=1}^{9} \frac{1}{2} \dot{X}^i \dot{x}^2 - \sum_{i=1}^{9} \frac{1}{2} \mu_i^2 X^i \dot{x}^2 + \frac{1}{4} \sum_{i,j=1}^{9} [X^i, X^j]^2 + \frac{i}{3} \sum_{i,j,k=1}^{9} f_{ijk} X^i X^j X^k + i \Psi^{T} \dot{\Psi} - \frac{i}{4} \Psi^{T} \theta \Psi - \sum_{i=1}^{9} \Psi^{T} \gamma^i [X^i, \Psi] \right)$$

(Note that on general pp-wave, the mass term is time dependent. Remember that in BFSS Matrix model description of M-theory, we have 16 dynamical supersymmetries of the action, and that the supersymmetry transformation spinor is the one for the background which survives after we introduce momentum. That is, the 16 components of the background Killing spinor which satisfy $\Gamma_0 \epsilon = \epsilon (\Leftrightarrow \Gamma^- \epsilon = 0)$ are realized as the dynamical supersymmetries of the flat space Matrix theory. The same story should hold for a Matrix model on a pp-wave background. Actually one can see that the dynamical supersymmetry transformation spinor of BMN action has exactly this form as given in eq. (31) $\chi_-$ in the appendix. (Note that because $\Gamma^+ \epsilon = \Gamma^+ \chi = 0 \Leftrightarrow \chi_- = 0$, the $\chi_+$ components are not realized as dynamical supersymmetry, but rather realized as kinematical supersymmetry of the Matrix model on pp-wave.) In the same way we expect the above action has $2k$
supersymmetries as expected from the background pp-wave geometry. Now it is straightforward to check the condition of supersymmetry of this action.

In order to analyze the supersymmetry of this action, it is enough to analyze just the Abelian case. We therefore consider the Abelian action

$$S = \int d\tau \left( \sum_{i=1}^{9} \frac{1}{2} \dot{x}^{i2} - \sum_{i=1}^{9} \mu_{i}^{2} x^{i2} + \Psi^{T} \partial_{\tau} \Psi - \frac{a}{4} \Psi^{T} \theta \Psi \right). \quad (25)$$

Even though we expect $a = 1$ from an derivation, here we take $a$ to be an unknown constant. Taking the ansatz of supersymmetry transformation as,

$$\delta X^{i} = \Psi^{T} \gamma^{i} N_{i} \varepsilon \text{ (no sum over } i)$$

$$\delta \Psi = \sum_{i=1}^{9} \left( \dot{X}^{i} \gamma^{i} N_{i} \varepsilon + \mu X^{i} \gamma^{i} M'_{i} \varepsilon \right)$$

$$\varepsilon = \varepsilon (\tau) = e^{\mu M \tau} \varepsilon_{0}$$

We wish to determine the matrices $N_{i}, M'_{i}, M$ as well as $a$. Terms of $O(\mu^{0})$ automatically cancel. Terms of $O(\mu^{1})$ give

$$\mu M'_{i} = -\mu N_{i} M + \frac{a}{4} \theta^{i} N_{i} \quad (26)$$

and

$$\mu = 0.$$

Here $\theta^{i} \equiv \gamma^{i} \theta^{\gamma^{i}}$. Finally the $O(\mu^{2})$ terms give

$$\left( \mu^{2} N_{i} M^{2} - \frac{a \mu}{2} \theta^{i} N_{i} M + \left( \frac{a}{4} \right)^{2} \theta^{i2} + \mu_{i}^{2} \right) N_{i} \varepsilon = 0 \quad (27)$$

for all $i = 1...9$. Let’s consider some solutions of this equation. Setting the matrices $N_{i} = 1$ for all $i = 1...9$, equation (27) becomes

$$\left( \mu^{2} M^{2} - \frac{a \mu}{2} \theta^{i} M + \left( \frac{a}{4} \right)^{2} \theta^{i2} + \mu_{i}^{2} \right) \varepsilon = 0.$$

By setting $M \equiv b \frac{\theta}{\mu}$, with $b$ some unknown constant, this equation reduces to

$$\left( b^{2} \theta^{2} - \frac{ab}{2} \theta^{i} \theta + \left( \frac{a}{4} \right)^{2} \theta^{i2} + \mu_{i}^{2} \right) \varepsilon = 0.$$

You can see that the solution $a = \pm 1$, $b = \mp \frac{1}{12}$ exactly coincides with eq. (32). Also the form of the spinor is given by $\varepsilon = e^{\pm \frac{i}{12} \theta}$ and again this
is exactly the expected form of the $2k$ Killing spinor in (31). So far we have found that some supersymmetry transformations of the Matrix action are inherited from the background action.

Note also that this action has kinematical $\mathcal{N} = 16$ supersymmetry under which fields transform as:

$$\delta X^i = 0$$
$$\delta \Psi = \exp\left(\frac{1}{4} \int dx^+ \theta (x^+)\right) \eta$$

for some constant spinor $\eta$. This is again expected from the supersymmetry of the background.

## 4 Conclusions

In the first part, we have studied the back-reaction of the large-N momentum on the maximally supersymmetric anti-pp-wave geometry. By taking an appropriate decoupling limit, we derived the type IIA geometry on which string theory is well-defined. The field theory action which we conjecture to be dual to the geometry is derived through the linear coupling between D0-branes and background fields. This gives a deformed version of 0+1-dimensional SYM with 16 supercharges, dual to warped $AdS_2 \times S^8$ geometry deformed by $O(\mu \alpha'^2)$ terms. Both theories possess $SO(3) \times SO(6)$ symmetry and Higgs branch. Also similarly perturbation theory and dual geometry don’t make sense at the same time. In the second part, we succeed in deriving the supermembrane action and the Matrix model on a general pp-wave background only through the properties of null Killing vector and checked that the supersymmetry of this action is as expected from the background. There are several points which are not fully understood. It would be nice to understand the amount of supersymmetry in the geometry (10), and also it would be nice to understand whether there are any other vacua in the dual field theory. And of course it is very nice to derive the 11-dimensional supergravity solution of D0-branes on usual pp-wave, if it exists. Finally it would be interesting to analyze the light cone time-dependent Matrix theory which is dual to M-theory on a time dependent pp-wave.

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A Fractionally supersymmetric pp-waves

Here we summarize the known results of 11 dimensional general pp-waves which posses $16 \leq \mathcal{N} \leq 32$ SUSY.

The 11 dimensional pp-wave has metric

$$ds^2 = 2dx^+dx^- + H (x^i, x^+) dx^+^2 + \sum_{i=1}^{9} dx^{i2}$$

$$H (x^i, x^+) = - \sum_{i=1}^{9} \mu_i^2 x^{i2}$$

$$F^{(4)} = \sum_{i,j,k=1}^{9} \frac{1}{3!} f_{ijk} dx^+ \wedge dx^{ijk}$$

$$\sum_{i=1}^{9} \mu_i^2 = \frac{1}{2 \cdot 3!} \sum_{i,j,k=1}^{9} (f_{ijk})^2$$

Generally $\mu_i$ and $f_{ijk}$ are functions of $x^+$. The Killing spinor $\epsilon$ should satisfy

$$D_M \epsilon = \nabla_M \epsilon + \Omega_M \epsilon = 0.$$  \hspace{1cm} (28)

$$\Omega_M = \frac{1}{288} \left( \Gamma_M^{PQRS} - 8 \delta_M^P \Gamma^{QRS} \right) F_{PQRS}$$

for $M = 0, 1, ..., 11$. In a pp wave background, the non-zero components of the elfbein $e_{rM}$, spin connection $w_{rM}^i$, and $\Omega_M$ are

$$e_{r=+,M=+} = \frac{H}{2}, \quad e_{r=+,M=-} = 1, \quad e_{r=-,M=+} = 1, \quad e_{r=i,M=j} = \delta_{ij}$$  \hspace{1cm} (29)

$$w_{+i} = \frac{1}{2} \partial_i H = \mu_i^2 x^i.$$  

$$\Omega_+ = -\frac{1}{12} \Theta (\Gamma_+ \Gamma_+ + 1)$$  

$$\Omega_i = \frac{1}{24} (3\Theta \Gamma_i + \Gamma_i \Theta) \Gamma_-$$
where $\Theta$ is defined as
\[
\Theta \equiv \sum_{ijk=1}^{9} \frac{1}{3!} f_{ijk} \Gamma^{ijk}.
\]

In order to discuss the supersymmetry of these pp-wave backgrounds, it is convenient to decompose the 32-component Killing spinor $\epsilon$ in terms of two 16-component $SO(9)$ spinors as
\[
\epsilon = \sum_{i=1}^{9} (1 - x^i \Omega_i) \chi, \quad \chi \equiv \begin{pmatrix} \chi^+ \\ \chi^- \end{pmatrix}
\]
Here we extract $x^i$ dependence from $\epsilon$ explicitly, therefore $\chi \equiv \begin{pmatrix} \chi^+ \\ \chi^- \end{pmatrix}$ depends only on $x^+$. One can check that this $x^i$ dependence satisfies (28).

Decomposing the 11 dimensional gamma matrices in terms of $SO(9)$ gamma matrices,
\[
\Gamma^i = \gamma^i \times \sigma_3 (i = 1...9), \quad \Gamma^0 = 1 \times i\sigma_1, \quad \Gamma^{11} = -1 \times \sigma_2
\]
and define the 9 dimensional gamma matrix $\theta$
\[
\Theta \equiv \sum_{ijk=1}^{9} \frac{1}{3!} f_{ijk} \Gamma^{ijk} = \sum_{ijk=1}^{9} \frac{1}{3!} f_{ijk} \gamma^{ijk} \times \sigma_3 \equiv \theta \times \sigma_3.
\]
Then the Killing spinor which satisfies (28) is expressed as
\[
\chi_+ = \exp \left( \frac{1}{4} \int^{x^+} dx^{i+} \theta \right) \psi_+, \quad \chi_- = \exp \left( -\frac{1}{12} \int^{x^+} dx^{i+} \theta \right) \psi_-.
\]
Here $\psi_{\pm}$ is a constant spinor. Equation (28) gives no constraint on $\psi_+$ but gives a constraint on $\psi_-$, that
\[
\left[ 144 \mu_i^2 + \left( 3 \gamma^i \theta \gamma^i + \theta \right)^2 - 12 \left( 3 \gamma^i \partial_+ \theta \gamma^i + \partial_+ \theta \right) \right] \psi_- = 0
\]
for all $i = 1...9$. Here $\partial_+ = \frac{\partial}{\partial x^+}$. The condition that there exist supernumerary supersymmetry is that there exists non-trivial $\psi_-$ which satisfy this equation, that is, the determinant must be zero, or
\[
\Pi_{i=1}^{8} \left\{ \left( 12 \mu_i^2 - \rho^2_{i,I} \right)^2 + (12 \partial_+ \rho_{i,I})^2 \right\} = 0
\]
for all $i = 1...9$. Here we choose the basis so that the $16 \times 16$ matrix $3\gamma^i\theta\gamma^i + \theta$ is skew diagonal and its skew eigenvalues are $\rho_{i,I}$ where $I = 1,...,8$.

This shows that $f_{ijk}$ should be $x^+$-independent in order to have supernumerary supersymmetry. Then (33) can be zero if

$$144\mu_i^2 = \rho_{i,i}^2 = \cdots = \rho_{k,i}^2$$

is satisfied for all $i = 1...9$. Therefore pp-wave backgrounds always possess 16 supersymmetries from $\psi_+$, and $2k$ supersymmetries from $\psi_-$ when the background has no $x^+$-dependence. $\psi_-$ has $(16-2k)$ zero components because of the $16 - 2k$ nontrivial constraints (32).

## B Dual field theory action in detail

The dual field theory has action:

$$S = S_{\text{flat}} + S_{\text{linear}}$$

$$S_{\text{linear}} = \int dt \left[ \frac{1}{4} \partial_A \partial_B h_{\mu\nu} I^{\mu\nu(AB)}_h + \frac{1}{2} \partial_A \partial_B \phi I^{(AB)}_\phi ight]$$

$$+ \frac{1}{2} \partial_A \partial_B A_{I} I^{(AB)}_0 + \partial_A B_{\mu\nu} I^{\mu\nu(A)}_s + \partial_A C_{\mu\nu\lambda} I^{\mu\nu\lambda(A)}_2$$

$$= \int dt \left[ \frac{1}{32} \partial_A \partial_B F^2 \left( I^{00(AB)}_h + I^{ii(AB)}_h \right) - \frac{3}{32} \partial_A \partial_B F^2 I^{(AB)}_\phi ight]$$

$$+ \frac{1}{8} \partial_A \partial_B F^2 I^{(0)}_0 - \epsilon_{ijk} \frac{2\pi\alpha'}{6} I^{ij(k)}_s + 3\epsilon_{ijk} \frac{2\pi\alpha'}{6} I^{0ij(k)}_2$$

$$= \int dt \left[ \frac{1}{32} \partial_A \partial_B F^2 \left( T^{++(AB)} + T^{+- (AB)} + (I^{00(AB)}_h)_s + T^{ii(AB)} + (I^{ii(AB)}_h)_s + \mathcal{O}(\alpha'^{11}) \right) ight]$$

$$- \frac{3}{32} \partial_A \partial_B F^2 \left( T^{++(AB)} - \frac{1}{3} T^{+- (AB)} - \frac{1}{3} T^{ii(AB)} + (I^{AB}_\phi)_s + \mathcal{O}(\alpha'^{11}) \right)$$

$$+ \frac{1}{8} \partial_A \partial_B F^2 \left( T^{++(AB)} \right) - \epsilon_{ijk} \frac{2\pi\alpha'}{6} \left( 3J^{++(k)} - 3J^{ij(k)} + \mathcal{O}(\alpha'^{2}) \right)$$

$$+ 3\epsilon_{ijk} \frac{2\pi\alpha'}{6} \left( J^{ij(k)} + \mathcal{O}(\alpha'^{2}) \right)$$

$$= \int dt \left[ \frac{1}{16} \partial_A \partial_B F^2 \left( T^{++(AB)} + \mathcal{O}(\alpha'^{2}) \right) + \epsilon_{ijk} \frac{2\pi\alpha'}{2} \left( J^{ij(k)} + \mathcal{O}(\alpha'^{2}) \right) \right]$$

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where \( \partial_A = \frac{\partial}{\partial U^A} \) and 
\[
F^2 = \left( \frac{2\pi \alpha' \mu}{3} \right)^2 \left( (U^1)^2 + \cdots (U^3)^2 \right) + \left( \frac{2\pi \alpha' \mu}{6} \right)^2 \left( (U^4)^2 + \cdots (U^9)^2 \right) .
\]
Note that terms involving \( T^{++(AB)} \) and \( J^{+ij(k)} \), which are origin of BMN mass term in pp-wave case, are exactly canceled out in this anti-pp-wave case.

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