On the effective potential for Horava-Lifshitz-like theories

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We study the one-loop effective potential for some Horava-Lifshitz-like theories.

The Horava-Lifshitz (HL) approach has recently acquired a great scientific attention. This approach is characterized by an essential asymmetry between space and time coordinates (space-time anisotropy): the equations of motion of the theory are invariant under the rescaling $x^i \rightarrow bx^i$, $t \rightarrow b^z t$, where $z$, the critical exponent, is a number characterizing its ultraviolet behaviour. The main reason for it is that for the HL-like reformulation of the known field theory models with a nontrivial critical exponent $z > 1$ leads to an improvement of the renormalization of these models. In particular, the four-dimensional gravity becomes renormalizable at $z = 3$.

Different issues related to the HL gravity, including its cosmological aspects, exact solutions, black holes were considered in a number of papers. At the same time, the study of the impacts of the HL extension to other field theories is a very interesting problem. Some aspects of the HL generalizations for the gauge field theories were presented in [5]. Renormalizability of the scalar field theory models with space-time anisotropy has been discussed in details in [6]. The four-fermion HL-like theory has been studied in [7]. The Casimir effect for the HL-like scalar field theory has been considered in [8]. In [9], the HL modifications of the $CP^{N-1}$ were studied. The possibility of restoration of the Lorentz symmetry in the theories with the space-time anisotropy is discussed in [5, 10].

It is well known that the effective potential is a key object in the quantum field theory useful for studying many of its aspects. Some interesting results for the HL-like theories have been obtained in the papers [11, 12] where the effective potential for the $\phi^4$ and the Liouville-
Lifshitz theories have been studied. Also, some interesting results for the effective potential in scalar field theories with certain values of the critical exponent, have been obtained in [13]. In this paper, we intend to study the effective potential for a more generic class of theories including an arbitrary interaction of the scalar field with other fields. In the sequel, we will treat three cases, namely, a pure scalar model, a gauge model and a Yukawa model.

a. Scalar model. We start with the straightforward HL generalization of the usual scalar model:

\[ S = \int dtd^d x \left( \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} (-1)^z \phi \Delta^z \phi - V(\phi) \right). \]  

The renormalizability of such a model has been discussed in [6]. In general, renormalizability of such models requires a polynomial form of the potential, however, for simplicity we restrict ourselves to the form \( V(\phi) = \lambda \phi^n \). Here our aim is the study of its effective potential. To proceed with it, we, as usual, make the replacement \( \phi \rightarrow \Phi + \phi \), where \( \Phi \) is a background field, and \( \phi \) is a quantum one. For the one-loop calculations, it is sufficient to keep only the terms of the second order in the quantum field \( \phi \):

\[ S_2 = -\frac{1}{2} \int dtd^d x \phi (\partial_0^2 + (-1)^z \Delta^z + V''(\Phi)) \phi. \]

Following the standard procedure, the one-loop effective action can be cast as

\[ \Gamma^{(1)} = \frac{i}{2} \text{Tr} \ln(\partial_0^2 + (-1)^z \Delta^z + V''(\Phi)). \]

The corresponding effective potential \( U(\Phi) \) can be read off from the expression

\[ \Gamma^{(1)}|_{\Phi=const} = -\int dtd^d x U^{(1)}(\Phi). \]

To calculate \( U(\Phi) \), we must carry out the Fourier transform of (3). After the Wick rotation, we arrive at

\[ U^{(1)} = \frac{1}{2} \int \frac{dk_0 dk}{(2\pi)^{d+1}} \ln(k_0^2 + \vec{k}^2z + V''(\Phi)). \]

First, we calculate the integral over \( k_0 \). We use

\[ \frac{d}{d(A^2)} \int dk_0 \ln(k_0^2 + A^2) = \int \frac{dk_0}{k_0^2 + A^2} = \frac{\pi}{\sqrt{A^2}}, \]

so that, neglecting an irrelevant field-independent constant, we get

\[ U^{(1)} = \int \frac{dk}{(2\pi)^d} \sqrt{\vec{k}^2z + V''(\Phi)}. \]
Then, we use the identity
\[
\sqrt{B} = -\frac{1}{2\sqrt{\pi}} \int_0^\infty d\alpha \alpha^{-3/2} e^{-\alpha B}.
\] (7)

Thus,
\[
U^{(1)} = -\frac{1}{2\sqrt{\pi}} \int d\alpha \alpha^{-3/2} \int \frac{d^d k}{(2\pi)^d} e^{-\alpha (k^2 + V''(\Phi))}.
\] (8)

In spherical coordinates and after the change of variables \(k^z \to u\), so, \(k^2 = u^2\), \(k = u^{1/z}\), and \(dk = \frac{1}{z} du u^{1/z-1}\), we get
\[
U^{(1)} = -\frac{1}{2\sqrt{\pi}} \frac{1}{(2\pi)^d} \frac{1}{z} \frac{\Gamma(d/2)}{\Gamma(\frac{d}{2})} \int_0^\infty d\alpha \alpha^{-3/2} \int_0^\infty du u^{\frac{d-2}{2}} e^{-\alpha (u^2 + V''(\Phi))}.
\] (9)

After integration we arrive at
\[
U^{(1)} = -\frac{1}{2\sqrt{\pi}} \frac{1}{(2\pi)^d} \frac{1}{z} \frac{\pi^{d/2}}{\Gamma(d/2)} \frac{\Gamma(\frac{d}{2})}{\Gamma(\frac{d}{2})} \frac{\Gamma(-\frac{1}{2} - \frac{d}{2z})}{(V''(\Phi))^{1/2+d/(2z)}}.
\] (10)

It is clear that this one-loop effective potential diverges if we have \(\frac{1}{2}(1 + \frac{d}{2}) = N\), where \(N\) is a non-negative integer number, in particular, for \(z = 2\), it diverges only at \(d = 2, 6, 10, \ldots\)

For example, for \(V(\Phi) \propto \Phi^5\), with \(d = 3\) and \(z = 2\), the Green functions have a superficial degree of divergence \(\omega = 5 - \frac{E}{2}\), with \(E\) is a number of legs. For the one-loop renormalizability the model requires a counterterm \(\Phi^8\). However, the explicit calculation shows that such a correction is one-loop finite within the dimensional regularization. For \(d = 3\) that expression is, as it is well known, quadratically divergent for \(z = 1\) and linearly divergent if \(z = 3\); otherwise it is finite.

b. Gauge fields. Now, let us introduce gauge fields. For the sake of concreteness, we restrict ourselves to the case \(z = 2\). In this case, the Lagrangian of the scalar QED is
\[
L = \frac{1}{2} F_{0i} F_{0i} + \frac{1}{4} F_{ij} \Delta F_{ij} - D_0 \phi (D_0 \phi)^* + D_i D_j \phi (D_i D_j \phi)^* - m^4 \phi \phi^*,
\] (11)

where \(D_0 = \partial_0 - ie A_0\), \(D_i = \partial_i - ie A_i\) is a gauge covariant derivative, with the corresponding gauge transformations: \(\phi \to e^{ie \xi} \phi\), \(\phi^* \to e^{-ie \xi} \phi^*\), \(A_0 \to A_0 + \partial_0 \xi\), \(A_i \to A_i + \partial_i \xi\). To keep track only from the gauge-matter interaction, we suggest that there is no self-coupling of the matter field.

The propagator for the scalar field has the simplest form
\[
< \phi \phi^* > = \frac{i}{k^2_0 - \vec{k}^4 - m^4}.
\] (12)
As for the propagator of the gauge field, the situation is more complicated. Indeed, to find this propagator, we must add to the free Lagrangian of the gauge field

\[ L_2 = \frac{1}{2} F_{0i} F_{0i} + \frac{1}{4} F_{ij} \Delta F_{ij} = \frac{1}{2} \partial_i A_0 \partial_i A_0 - \partial_0 A_0 \partial_i A_i + \frac{1}{2} \partial_0 A_i \partial_0 A_i + \frac{1}{4} F_{ij} \Delta F_{ij}. \]  

(13)

the gauge-fixing term. However, since the \( L_2 \) contains a mixed term involving both \( A_0 \) and \( A_i \) (which have distinct behaviours), it would be good if the gauge-fixing term could allow for the separation of these fields.

It turns out to be that the appropriate gauge-fixing term is nonlocal:

\[ L_{gf} = \frac{1}{2} \left( \frac{1}{\sqrt{\Delta}} \partial_0 A_0 + \sqrt{\Delta} \partial_i A_i \right)^2 = \frac{1}{2} \left( \partial_0 A_0 \frac{1}{\Delta} \partial_0 A_0 + 2 \partial_0 A_0 \partial_i A_i + \partial_i A_i \Delta \partial_i A_i \right). \]  

(14)

This gauge-fixing term can be treated as the analogue of the Feynman gauge. Adding this gauge-fixing term to the \( L_g \), we arrive at the following complete Lagrangian:

\[ L_c = L_2 + L_{gf} = -\frac{1}{2} A_0 \frac{\partial_0^2 + \Delta^2}{\Delta} A_0 - \frac{1}{2} A_i (\partial_0^2 + \Delta^2) A_i. \]  

(15)

The nonlocality of this Lagrangian, however, does not give any danger for calculations. Indeed, the propagators have a reasonable form:

\[ <A_0 A_0> = \frac{i k^2}{k_0^2 - k^4}; \]

\[ <A_i A_j> = -\frac{i \delta_{ij}}{k_0^2 - k^4}. \]  

(16)

As can be checked, the model is then renormalizable for \( d \leq 4 \).

To calculate the effective potential, we must take into account that it depends only on the matter fields, thus, we treat the gauge field as a pure quantum field. Also, we must take into account that, within the one-loop approximation, only the vertices associated to two quantum fields give nontrivial contributions to the effective potential. Let us denote the background fields by \( \Phi \) and \( \Phi^* \). It is easy to see that the only relevant vertices are

\[ -e^2 A_0 A_0 \Phi \Phi^*; \quad ie A_0 (\Phi^* \partial_0 \phi - \Phi \partial_0 \phi^*), \]

\[ -ie (\partial_i A_j) \left[ \Phi \partial_i \partial_j \phi^* - \Phi^* \partial_i \partial_j \phi \right], \quad e^2 (\partial_i A_j) (\partial_i A_j) \Phi \Phi^*. \]  

(17)

To simplify the calculations, it is convenient to move within these vertices all derivatives to act on the gauge fields. So, these vertices take the form:

\[ -e^2 A_0 A_0 \Phi \Phi^*; \quad -ie (\Phi^* \phi - \Phi \phi^*) \partial_0 A_0, \]

\[ -ie [\Phi \Phi^* - \Phi^* \phi] (\partial_j \Delta A_j), \quad -e^2 A_j \Delta A_j \Phi \Phi^*. \]  

(18)
To fix the quantum corrections at the one-loop order, we must consider two types of contributions. In the first of them, all diagrams involve only the gauge field propagators in the internal lines:

\[ \ldots \]

The total result from this sector is a sum of two contributions to the effective potential – the first one, \( U_a \) is given by sum of loops of \( \langle A_0 A_0 \rangle \) propagators, and the second one, \( U_b \) – of \( \langle A_i A_j \rangle \) propagators:

\[
U_a = -\sum_{n=1}^{\infty} \frac{1}{n} \int \frac{d^d k d k_0}{(2\pi)^{d+1}} (e^2 \Phi \Phi^*)^n \left( \frac{\vec{k}^2}{k_0^2 - \vec{k}^4} \right)^n ;
\]

\[
U_b = -\sum_{n=1}^{\infty} \frac{d}{n} \int \frac{d^d k d k_0}{(2\pi)^{d+1}} (e^2 \Phi \Phi^*)^n \left( \frac{\vec{k}^2}{k_0^2 - \vec{k}^4} \right)^n .
\] (19)

The second type of diagrams involves the triple vertices as well. We should first introduce a "dressed" propagator

\[
\begin{align*}
\ldots &= \ldots + \ldots + \ldots
\end{align*}
\]

In this propagator, the summation over all quartic vertices is performed. As a result, these "dressed" propagators are equal to

\[
\begin{align*}
\langle A_0 A_0 \rangle_D &= \langle A_0 A_0 \rangle \sum_{n=0}^{\infty} [ie^2 \Phi \Phi^* \langle A_0 A_0 \rangle]^n = \frac{i\vec{k}^2}{k_0^2 - \vec{k}^4 - e^2 \vec{k}^2 \Phi \Phi^*}; \\
\langle A_i A_j \rangle_D &= -i\delta_{ij} \sum_{n=0}^{\infty} [e^2 \frac{\vec{k}^2}{k_0^2 - \vec{k}^4} \Phi \Phi^*]^n = -\frac{i\delta_{ij}}{k_0^2 - \vec{k}^4 - e^2 \vec{k}^2 \Phi \Phi^*} .
\end{align*}
\] (20)

To proceed, we follow the methodology developed in [14] and other papers. It is based on the summation over diagrams representing themselves as cycles of all possible number of links. Such diagrams look like

\[ \ldots \]

Now, it is time to take into account the derivatives in the triple vertices. Using the "rationalized" form of the vertices [13], we can find that effectively one must consider the
objects

\[ G_1 = \langle \partial_0 A_0(t_1, \vec{x}_1) \partial_0 A_0(t_2, \vec{x}_2) \rangle_D; \]
\[ G_2 = \langle \partial_i \Delta A_i(t_1, \vec{x}_1) \partial_j \Delta A_j(t_2, \vec{x}_2) \rangle_D, \]

whose Fourier transforms are

\[ G_1(k) = \frac{ik_0^2 \vec{k}_2}{k_0^2 - \vec{k}^4 - e^2 k^2 \Phi \Phi^*}; \]
\[ G_2(k) = -\frac{i\vec{k}^6}{k_0^2 - \vec{k}^4 - e^2 k^2 \Phi \Phi^*}. \]

Here we took into account that the derivatives affect different arguments of the propagator which changes the sign with respect to (20). Then, we can take into account that the effective propagators \( G_1 \) and \( G_2 \) enter the diagrams above on the same base, thus, the total contribution must be symmetric under replacement \( G_1 \leftrightarrow G_2 \). Thus, the total contribution from these graphs is

\[ U_c = -\sum_{n=1}^{\infty} \frac{1}{n} \int \frac{d^d k d k_0}{(2\pi)^{d+1}} (-e^2 \Phi \Phi^*)^n \left( (G_1 + G_2) < \phi \phi^* > \right)^n, \]

which yields

\[ U_c = -\sum_{n=1}^{\infty} \frac{1}{n} \int \frac{d^d k d k_0}{(2\pi)^{d+1}} \left( \frac{(k_0^2 - \vec{k}^4) \vec{k}_2}{k_0^2 - \vec{k}^4 - e^2 k^2 \Phi \Phi^*} \frac{1}{k_0^2 + \vec{k}^4 - m^4} \right)^n. \]

It remains to process all these expressions \( U_a, U_b \) and \( U_c \). To do it, we use the identity \( \sum_{n=1}^{\infty} \frac{a^n}{n} = -\ln(1 - a) \) and carry out the Wick rotation, thus,

\[ U_a = i \int \frac{d^d k d k_0}{(2\pi)^{d+1}} \ln \left[ 1 + \frac{e^2 \Phi \Phi^* \vec{k}_2}{k_0^2 + \vec{k}^4} \right]; \]
\[ U_b = i d \int \frac{d^d k d k_0}{(2\pi)^{d+1}} \ln \left[ 1 + \frac{e^2 \Phi \Phi^* \vec{k}_2}{k_0^2 + \vec{k}^4} \right]; \]
\[ U_c = i \int \frac{d^d k d k_0}{(2\pi)^{d+1}} \ln \left[ 1 - \frac{e^2 \Phi \Phi^* (k_0^2 + \vec{k}^4) \vec{k}_2}{k_0^2 + \vec{k}^4 + e^2 \vec{k}^4 \Phi \Phi^* k_0^2 + \vec{k}^4 + m^4} \right]. \]

In the case \( m = 0 \), \( U_c \) simplifies radically, and we have

\[ U_c = -i \int \frac{d^d k d k_0}{(2\pi)^{d+1}} \ln \left[ 1 + \frac{e^2 \Phi \Phi^* \vec{k}_2}{k_0^2 + \vec{k}^4} \right], \]
which completely cancels $U_a$. So, in this case we end just with the following contribution to the effective potential:

$$U^{(1)} = id \int \frac{d^d k dk_0}{(2\pi)^{d+1}} \ln[1 + \frac{e^2 \Phi \Phi^* k^2}{k_0^2 + \bar{k}^2}].$$

(27)

Adding and subtracting the constant $id \int \frac{d^d k dk_0}{(2\pi)^d} \ln[1 + \frac{\bar{k}^4}{k_0^2}]$, we find that the effective potential, up to an additive constant, looks like

$$U^{(1)} = id \int \frac{d^d k dk_0}{(2\pi)^{d+1}} \ln[1 + \frac{k^4 + e^2 \Phi \Phi^* k^2}{k_0^2}].$$

(28)

Then, we use the integral $\int_0^\infty dk_0 \ln(k_0^2 + A^2) = \pi \sqrt{A^2}$, so,

$$U^{(1)} = id \int \frac{d^d k}{(2\pi)^d} \sqrt{\bar{k}^2 + e^2 \Phi \Phi^*} = id I.$$

(29)

Following the same steps as before, in the case of the scalar model, we arrive at

$$I = \frac{\pi^{d/2}}{2(2\pi)^d} (e^2 \Phi \Phi^*)^{d/2+1} \frac{\Gamma(-1 - \frac{d}{2}) \Gamma(\frac{d}{2} + \frac{1}{2})}{\Gamma(\frac{d}{2}) \Gamma(-\frac{d}{2})}.$$

(30)

We see that for odd spatial dimension $d$, this expression is finite, while for even $d$ it diverges, and, we would need to add the corresponding counterterms (in particular, for $d = 2$, one will need the quartic interaction to achieve a multiplicative renormalizability).

For completeness, we note that sometimes, the Coulomb gauge $\partial_i A_i = 0$ maybe convenient. It is considered in Appendix.

### c. Yukawa theory.

Then, let us formulate the Yukawa theory. It is natural to consider now the $z = 2$ version of the spinor field theory, so, the $(d+1)$-dimensional Lagrangian for the theory looks like

$$L = \bar{\psi}(i\gamma^0 \partial_0 + \Delta - m^2 - h\Phi)\psi.$$

(31)

To keep track only from the Yukawa coupling, we treat the scalar field as a purely external one. The generalization of this study for the case of the self-interacting scalar field is straightforward. The one-loop effective potential corresponding to this Lagrangian, looks like

$$\Gamma^{(1)} = i\text{Tr} \ln(i\gamma^0 \partial_0 + \Delta - m^2 - h\Phi).$$

(32)

We can present this expression as

$$\Gamma^{(1)} = i\text{Tr} \ln(i\gamma^0 \partial_0) + i\text{Tr} \ln(1 - i\frac{(\Delta - m^2 - h\Phi) \gamma^0 \partial_0}{\partial_0^2}).$$

(33)
Disregarding an irrelevant additive constant, expanding the logarithm in power series, calculating the matrix trace and doing the sum, we arrive at

$$\Gamma^{(1)} = i\frac{\delta}{2} \text{Tr} \ln \left[ 1 - \frac{(\Delta - m^2 - h\Phi)^2}{\delta_0^2} \right].$$  \hspace{1cm} (34)$$

Here $\delta$ is a dimension of the Dirac matrices in the corresponding representation. After Fourier transform by the rule $i\partial_{0,i} \rightarrow k_{0,i}$, this expression yields the following effective potential

$$U^{(1)} = -i\frac{\delta}{2} \int \frac{d^dk_0}{(2\pi)^d+1} \ln \frac{k_0^2 - (\vec{k}^2 + m^2 + h\Phi)^2}{k_0^2}. \hspace{1cm} (35)$$

Doing the Wick rotation and integrating over $k_0$, we arrive at

$$U^{(1)} = -\frac{\delta}{2} \int \frac{d^dk}{(2\pi)^d} \left( \vec{k}^2 + m^2 + h\Phi \right)^{1+\epsilon}. \hspace{1cm} (36)$$

This integral, for any positive $d$, vanishes within the dimensional regularization being proportional to $\frac{1}{\Gamma(-1-\epsilon)}$ which is zero as $\epsilon \rightarrow 0$.

An observation is in order: if we consider a model composed of the Lagrangian (11) plus an extension of (31) in which the fermions are also minimally coupled to the electromagnetic field, up to the one-loop order, no additional contribution to the effective potential given by the expressions (29,30) arises.

We studied the effective action for some scalar HL-like theories: first, the self-coupled scalar model whose one-loop effective potential was found for arbitrary values of the space dimension, critical exponent and coupling; second, the scalar QED, whose effective potential was successfully obtained in the $z = 2$ case, and third, the Yukawa theory, where the one-loop effective potential was shown to vanish. In principle, we can also introduce the coupling between the gauge and spinor fields. Nevertheless, these additional interactions will start to contribute to the effective potential only at the two-loop order. We found that the methodology for calculating the effective potential does not essentially differ from that in the usual, Lorentz invariant field theories.

### Appendix

Let us briefly describe the difference of the results for the case of gauge fields in the Coulomb gauge. After imposing this gauge, the ”mixed” term immediately vanishes in the action (13), but there is no modification of the quadratic term in $A_0$, so, the propagator $<A_0 A_0>$ in this case differs from that one in (16) being equal to

$$<A_0 A_0> = -\frac{i}{\vec{k}^2}, \hspace{1cm} (37)$$
whereas the propagator \( < A_i A_j > \) stays the same as in (16). As a result, the contribution \( U_b \) from (25) stays unchanged, while for \( U_a \) now we have

\[
U_a = i \int \frac{d^d k d k_0 E}{(2 \pi)^{d+1}} \ln [1 + \frac{e^2 \Phi \Phi^*}{k^2}].
\]  

(38)

However, the result for \( \Gamma_c \) in the Coulomb gauge is much more complicated. Let us consider it in details. The ”effective propagator” \( G_1 \) introduced in (22) in the case of the Coulomb gauge takes the form

\[
G_1 = - \frac{i k_0^2}{k^4 + e^2 \vec{k}^2 \Phi \Phi^*},
\]  

(39)

while the \( G_2 \) does not suffer any modification. After doing the sum indicated in (23) and some simple transformations, we have

\[
U_c = i \int \frac{d^d k d k_0}{(2 \pi)^{d+1}} \left[ \ln \left( \frac{k_0^2 + (k^2 + e^2 \Phi \Phi^*)(\vec{k}^2 + m^4)}{(k_0^2 + \vec{k}^2 + e^2 \vec{k}^2 \Phi \Phi^*)} \right) + \right.
\]

\[
\left. + \frac{e^2 \Phi \Phi^* \vec{k}^2}{k_0^2 k^2} \right) - \ln \frac{k_0^2 + \vec{k}^2 + e^2 \vec{k}^2 \Phi \Phi^*}{k_0^2} - \ln \frac{\vec{k}^2 + e^2 \Phi \Phi^*}{k^2} + \ln \frac{k_0^2}{k^4}. \]  

(40)

It is easy to see that the second term in the r.h.s. of this expression differs from (27) only by a constant factor, \(-d\), multiplying the last one. The third term exactly cancels with \( U_a \) (38), and the last term is a pure irrelevant constant, since it does not depend on the background fields. Thus, the complete effective potential is

\[
U = U_a + U_b + U_c = -i \frac{\pi^{d/2}}{2 (2 \pi)^d} (e^2 \Phi \Phi^*)^{d/2+1} (1 - d) \Gamma(-1 - \frac{d}{2}) \Gamma(\frac{d}{2} + 1) + \]

\[
+ i \int \frac{d^d k d k_0}{(2 \pi)^{d+1}} \ln \left( \frac{k_0^2 + (k^2 + e^2 \Phi \Phi^*)(\vec{k}^2 + m^4)}{(k_0^2 + \vec{k}^2 + e^2 \vec{k}^2 \Phi \Phi^*)} \right) + \]

\[
+ \frac{e^2 \Phi \Phi^* \vec{k}^2}{k_0^2 k^2} \right]. \]  

(41)

Notice that the first term in this expression is very similar to the result (38). The above expression differs from the results in (25–30) just because the contribution involving \( < A_0 A_0 > \) propagators was cancelled. Unfortunately, the last term is highly cumbersome.

The discrepancy between the results for the effective potential in the gauges we considered is expected because it is a gauge dependent quantity.

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