Floquet scattering theory of negative magneto-conductance of 2DEG under microwave radiation

Kang-Hun Ahn
Department of Physics, Chungnam National University, Daejeon 305-764, Republic of Korea
(Dated: January 13, 2022)

We develop a theory of magneto-transport properties of two-dimensional electron gas in cylindrical geometry where finite bias and microwave are locally applied. By investigating the Landauer conductance based on Floquet theory, we obtain the conductance without perturbation analysis. We show that the dc conductance becomes negative when dc electric field is applied inside the 2DEG with sufficient microwave power. A positive sign of the current is restored by increasing dc bias. We analyze the radiation induced non-equilibrium distribution function with respect to the frequency of the applied microwave.

PACS numbers: 73.40.-c, 73.50.-h, 78.67.-n

I. INTRODUCTION

Recently, radiation-induced vanishing magneto-resistance observed in high-mobility 2DEG has attracted a great deal of attraction.\cite{1,2,3} One of the current understanding on the phenomena is based on the possible negative resistance in certain conditions which evolves into certain "zero-resistance(conductance)" state.\cite{4,5} The idea of negative resistance(conductance) was originally invoked long time ago by Ryzhii.\cite{6} Recent work of Durst et al. proposed the negative resistance in terms of the photon assisted electron-impurity scattering.\cite{7} Subsequent theoretical works explained the negative resistance in terms of the radiation induced change of electron distribution.\cite{8}

An alternative way of explanation of the "zero resistance" exists. Based on a semiclassical approach using a phenomenological damping parameter, Iñarrea et al. directly proposed vanishingly small positive resistance.\cite{9} According to Ref.\cite{10}, the physical origin of the zero resistance is that the electric current is blocked by Pauli exclusion principle when the "Fermi level" oscillates sufficiently.

So far, most of theoretical study has been based on the perturbation analysis. The purpose of this paper is to study the radiation-induced electron transport without relying on the perturbation analysis. We will not use 'Fermi golden rule' for the transport which sometimes gives ambiguous conclusion on the validity of the theory.

By this means, we study the magneto transport of electrons in Landauer geometry. The effects of Landau level broadening are realized via finite guiding center potential and the finite bias gives rise to the shift of the density of states. We do not assume equilibrium function for a localized states defined by magnetic fields. Because when disorder in 2DEG play a role in magneto transport, (quasi-)extended states opens transport channels,\cite{11} therefore the occupation of a localized state do not follow Fermi-Dirac distribution function. We found the negative conductance in consistent with the previous calculations showing negative resistance.\cite{12,13} Our model study indicates that

(i) the negative conductance indeed exists even in purely quantum mechanical calculations without perturbation analysis.

(ii) If the magnetic orbitals open transport channels via disorder scattering, the Pauli exclusion principle do not ensure the zero conductance.

We also investigated the nonlinear effects and the breakdown of the negative conductance as a function of DC bias. We analyze the radiation induced non-equilibrium distribution functions for temperatures lower than Landau level spacing. This paper is organized as following. In section II, we introduce the model and related formalism for the calculations of transport properties. In section III, we show the linear negative conductance indeed exists in certain conditions. It will be shown that, however, if the dc bias do not change the density of states, there is no negative conductance. The nonlinear properties will be presented in section IV, and the radiation induced non-equilibrium distribution function will be presented in section V.

II. GENERAL FORMALISM

A. The Model

We consider a 2DEG confined in a cylinder with radius R subjected to a transverse magnetic field $B$. (See Fig. 1.) By doing this we study magneto-transport in periodic boundary condition as in Corbino disc geometry.\cite{14} Similar model for 2DEG with unidirectional modulation has been studied in Ref\cite{15}. We study Landauer conductance in the system where the static electric field $E$ and oscillating field are applied only restricted regime as shown in Fig. 1. In usual 2DEG subjected to a transverse magnetic field, the longitudinal transport does not arise without disorder scattering, because in that case the electron merely performs a localized cyclotron motion. To mimic the long-range smooth disorder potential which exits in high-mobility samples,\cite{16} we introduce a scattering po-
potential $V = -V_0 \cos \phi$ (Here we use cylindrical coordinates $(\rho, \phi, z)$). By doing this we could define transport channels in the 2DEG system. In the absence of applied static electric field and microwave, the Hamiltonian reads

$$H = \frac{1}{2m^*} \left( \mathbf{p} - e\mathbf{A} \right)^2 - V_0 \cos \phi,$$

were $m^*$ is the band effective mass of the electron in the host materia and the magnetic gauge in use is $\mathbf{A} = B\mathbf{z}\phi$. For convenience, from here on we assign $e = |e|$ and $\hbar = 1$. The eigen wavefunctions for $V_0 = 0$ are $\psi_{nm}(z, \phi) = \frac{1}{\sqrt{2\pi}} e^{im\phi} \phi_n(z!z_m)$ where $\phi_n$ is $n$th eigen wavefunction of harmonic oscillator with frequency $\omega_n = eB/m^*$ and $z_m = ml^2_B/R$, and $l_B = \sqrt{1/eB}$ is the magnetic length.

We neglect the Landau level mixing caused by guiding center potential $V(\phi)$ and DC electric field. Then we get a tight binding model for the Hamiltonian

$$H_0 = \sum_{n,m} \left[ n\omega_c + \delta_m \right] c_{n,m}^\dagger c_{n,m} - \sum_{n,m} \frac{V_0}{2} \left( c_{n,m+1}^\dagger c_{n,m} + c_{n,m-1}^\dagger c_{n,m} \right),$$

where $c_{n,m}^\dagger$ ($c_{n,m}$) is creation (annihilation) operator for $\psi_{nm}(z, \phi)$. Here the DC electric field is applied in the regime of $0 < z < Ml^2_B/R$ and gives finite voltage bias $V$ which is dictated by

$$\delta_m = \begin{cases} 0 & (m \leq 0) \\ -V_0 m/M & (0 < m < M) \\ -V_0 & (M \leq m) \end{cases}$$

The transport channels are defined as eigenstates of the above Hamiltonian for $V = 0$. The eigenstates for zero bias, $V = 0$, are

$$\ket{\psi_{nm}} = \sum_{m} \exp(iqm) c_{nm} \ket{0},$$

and the eigen energies are

$$\epsilon_n(q) = n\omega_c - V_0 \cos q.$$  \hspace{1cm} (4)

Note that the DC electric field here modifies the transmission probability and the density of states (DOS) as well. As we shall see, it is crucial to consider the modification of DOS for the negative conductance. The conductance is positive definite without the change of DOS by the electric field.

We consider AC electric field $E_\omega \sin \omega t$ along z-direction localized in $0 < z < Ml^2_B/R$. By replacing $\mathbf{A}$ with a time-periodic vector potential $\mathbf{A} + \frac{ek}{c} \omega t$ in Eq. (1), we obtain the time-dependent Hamiltonian $H(t) = H_0 + h(t)$ where

$$h(t) = \frac{\pi}{M} \sum_{n,m} i \frac{eE_\omega \cos \omega t}{\sqrt{2m^*\omega}} \sqrt{n} \left( c_{n-1,m}^\dagger c_{n,m} - c_{n,m}^\dagger c_{n-1,m} \right) + \left( \frac{eE_\omega \cos \omega t}{\sqrt{2m^*\omega}} \right)^2 c_{n,m}^\dagger c_{n,m}.$$  \hspace{1cm} (5)

**B. Floquet scattering theory**

Relying on the Floquet theorem, we write the eigenstates of the time-periodic Hamiltonian $H(t)$ in terms of the coefficients $\phi_{nm}^{(l)}$:

$$\ket{\psi(t)} = e^{-iElt} \ket{\phi(t)} = e^{-iElt} \sum_l e^{-il\omega t} \ket{\phi_l} = \sum_l e^{-i(E+l\omega)t} \sum_{nm} \phi_{nm}^{(l)} \phi_{nm}^\dagger \ket{0}$$  \hspace{1cm} (6)

Then the time-dependent Schrödinger equation $\frac{d}{dt} \ket{\psi(t)} = H(t) \ket{\psi(t)}$ reads

$$(E + l\omega) \phi_{nm}^{(l)} = \sum_{m',n'} \frac{2\pi}{\omega} \int_0^{2\pi} d\tau e^{i(l-l')\omega \tau} \times \langle n'm'|H(t)|n'm'\rangle \phi_{nm'}^{(l')}.$$  \hspace{1cm} (7)

Consider an incoming electron in the transport channel of energy $E + L\omega$ in the $N$th Landau level in Eq. (4). For $m \leq 0$, the eigenstate is the sum of the incoming wave $\ket{\phi_{L,qL<N}}$ and the reflected wave $\ket{\psi_{L,qL<N}}$ with probability amplitude $r_{ln}^{(L-1)}$

$$\ket{\psi(t)} = e^{-iELt} \ket{\phi_{L,qL<N}} + \sum_{ln} r_{ln}^{(L-1)} e^{-iElt} \ket{\psi_{L,qL<N}}.$$  \hspace{1cm} (8)

where

$$E_l = E + l\omega = n\omega_c - V_0 \cos q_l.$$  \hspace{1cm} (9)

Thus, the coefficients $\phi_{nm}^{(l)}$ for $m \leq 0$ are written as

$$\phi_{nm}^{(l)} = \exp(iqLNm) \delta_{lN} \delta_{nN} + r_{ln}^{(L-1)} e^{-iqlN}.$$  \hspace{1cm} (10)
the Fermi-Dirac distribution $f(E)$ and its corresponding coefficients $\phi^{(l)}_{nm}$ are written as
\[ |\psi(t)\rangle = \sum_{lm} t_{lm}^n e^{-i(E+\omega)t|\psi_{n,k_{in}}} \]

\[ \phi^{(l)}_{nm} = t_{lm}^n e^{ik_{in,m}}, \]

where
\[ E_l = n\omega_c - V_0 \cos k_{in} - eV. \]

By inserting Eqs. (10) and (12) into Eq. (8), we obtain equations for the coefficients of the transmission $t_{lm}^{n}$ and reflection $r_{lm}^{n}$. Detail of the calculation can be found in the Appendix. The transmission function from the energy $E$ in lead 1 to the the energy $E + \omega + eV$ in lead 2 is given by the sum of the transmission probability over all possible spatial channels in the leads 1 and 2 (See Fig. 2):

\[ \bar{T}_{21}(E + \omega + eV, E + \omega) = \sum_{nm,l} |t_{lm}^{n}|^2 \sin k_{in} \sin q_{LN} |_{k_{in},q_{LN} \text{ real}} \]

In the above equation, the sine functions are appropriate normalization factors for the electric current, $I = \frac{e}{\hbar} V_0 \sum_m c_{n,m+1}^c c_{n,m} - c_{n,m-1}^c c_{n,m}$. Assuming the both leads are in equilibrium given by the Fermi-Dirac distribution $f_{FD}(E) = 1/\exp(\beta(E - E_F)) + 1$, we arrive at the current formula:

\[ I = \frac{2e}{h} \int_{-V_0}^{\infty} dE \sum_{l=-\infty}^{\infty} [\bar{T}_{21}(E + \omega + eV, E) - \bar{T}_{12}(E + \omega - eV, E)] f_{FD}(E) \]

\[ = \frac{2e}{h} \int_{-V_0}^{\infty} dE \sum_{l=-\infty}^{\infty} \bar{T}_{21}(E + \omega + eV, E) \]

Time-reversal symmetry in the Hamiltonian gives us useful information on the transmission function:
\[ \bar{T}_{21}(E + \omega + eV, E) = \bar{T}_{12}(E, E + \omega + eV), \]

which leads to a simple form of the electric current:

\[ I = \frac{2e}{h} \int_{-V_0}^{\infty} dE \sum_{l} \bar{T}_{21}(E + \omega + eV, E) \times [f_{FD}(E) - f_{FD}(E + \omega + eV)] \]

From the above formula, one may notice that the electric current of negative sign is attributed to inelastic transition processes from occupied states of energy $E + \omega + eV$ to unoccupied states of energy $E$.

### III. LINEAR CONDUCTANCE

From the current formula in Eq. (15) we write the linear conductance as

\[ G = \frac{dI}{dV} |_{V=0} = G_{ph} + G_0 \]

\[ = \frac{2e}{h} \int dE \sum_{l} \left[ \frac{\partial}{\partial V} \bar{T}_{21}(E + \omega + eV, E) \right] |_{V=0} \times (f_{FD}(E) - f_{FD}(E + \omega)) + \frac{2e}{h} \int dE \sum_{l} \bar{T}_{21}(E + \omega, E)(-f_{FD}(E + \omega)) \]

Let us call the first part photo-conductance $G_{ph}$ and the second part $G_0$. At absolute zero temperature, $G_0$ is written as simple Landauer formula:

\[ G_0 = \frac{2e^2}{h} \sum_{l} \bar{T}_{21}(E_F, E_F + \omega) \]

\[ = \frac{2e^2}{h} \sum_{l} \bar{T}_{21}(E_F + \omega, E_F) \]

Note that $G_0$ is the conductance obtained when the applied voltage drop is located only at the contacts between 2DEG and leads. In this case there is no applied electric field in 2DEG. If there were no electric field inside the conductor, the net current $I$ could be written as

\[ I = \frac{2e}{h} \int dE \sum_{l} \bar{T}_{21}(E + \omega, E) \times (f_{FD}(E - eV) - f_{FD}(E)) \]

Then the conductance is given as in Eq. (19) for zero temperature. Although $G_0$ is close to the conductance in the case without microwave, they are not necessarily same. It is instructive to note that, since the total transmission probability $\sum \bar{T}_{21}(E_F + \omega, E_F)$ is always positive definite, the conductance can not be negative without dc electric field inside 2DEG.

In Fig. 3 we show the numerical results of the conductances as functions of the microwave frequency. The detail of the calculation on the transmission probability was introduced in the Appendix. Since we found that
the all the features discussed in this work are not significantly influenced by the size of $M$, from here on, we focus on the case when the single orbital is irradiated, $M = 2$. The range of $l$’s for all the data were chosen after convergence tests. (For instance, $|l| \leq 6$ was enough to produce Fig.3)

The dips in $G_0$ are due to so called “threshold anomaly”. The threshold anomaly arises whenever new transport channel is opening or closing. In our model, the density of states for the calculations.

For 0.5$\omega_c < V_0 < \omega_c$, $M(E)$ has maximum value of 2 when $E$ is located in $(n + 1)\omega - V_0 < E < n\omega + V_0$, where $n$ is a non-negative integer. Otherwise $M(E)$ equals to 1. In Fig. 3 where $V_0 = 0.7\omega_c$, the dip of $G_0$ at $\omega = 0.8\omega_c(1.2\omega_c)$ is due to the opening (closing) of inter-Landau-level (intra-Landau-level) transition where $M(E_F \pm \omega)$ increases (decreases) by 1. The absolute negative conductance $G = G_0 + G_{ph}$ is clearly shown in Fig.4 where the sudden sign reversal of $G_{ph}$ at $\omega = \omega_c$ is responsible for the sudden decrease of the net conductance. The associated photo-current $I_{ph}$ ($G_{ph} = dI_{ph}/dV$) reads

\[
I_{ph} = \frac{2e}{h} \int dE \sum_l \bar{T}_{21}(E + l\omega + eV, E) \times (f_{FD}(E) - f_{FD}(E + l\omega)).
\]

At zero temperature, the processes where both of $E, E + l\omega + eV$ are below or above the Fermi level do not contribute to the photo-current. This fact simplify our consideration, because when $eV < \omega$ we only have to consider photon absorption. Furthermore, the sign of photo-current due to the process of given $l$ is given by the sign of $l$. Now let us consider the most significant transition process which arise when the energies of incoming electron and outgoing electrons are all located in the regime where $M(E)$ is maximum. Since $M(E) = M(E + \omega_c)$, such an process is expected when $(E + l\omega + eV) - E = l\omega + eV$ is close to integer multiple of $\omega_c$. Therefore when $\omega \approx \omega_c$, the term with $l = +1$ becomes most significant for $eV > 0$, which yields positive sign of the photo-current. On the other hand, for $\omega \approx \omega_c$, the condition

\[
l\omega + eV \approx \omega_c
\]

is met when $l = -1$, which yields negative sign of photo-current.

**IV. NONLINEAR PROPERTIES**

In Fig. 4 we plot the differential conductance $dI/dV$ as a function of microwave power $P = \omega E_c^2$. For $\omega < \omega_c$, the conductance is enhanced by the microwave and eventually reach a maximum value and decreases. The increase of the conductance is due to the strong absorption of the single photon and the appearance of the nonlinear two-photon process reduce the conductance as the power increases. For $\omega > \omega_c$ the conductance decreases as the microwave power increases and eventually reaches the negative conductance regime. In contrast to the positive conductance showing non-monotonic behavior, the negative conductance just show monotonic decreases.

In Fig. 5 we plot the electric current as a function of the applied bias voltage. The negative absolute conductance is shown in a a region of finite bias
and it is recovered as the bias increases. The decrease of the photo current is the main reason of the recovery of the negative conductance. According to previous works by Ryzhii, there is electric field $E_0$ at which the conductivity becomes zero $\sigma(E_0) = 0$. The electric-field breakdown arises at rather high electric field $E_{0lB} > \max(\omega - \omega_c, \Gamma)$, where $\Gamma$ is the Landau level broadening. In our model, $\Gamma \sim V_0$. As shown in Fig. 4, the electric-field breakdown appears at rather small electric field where the breakdown field is roughly an order of magnitude smaller.

V. RADIATION-INDUCED NON-EQUILIBRIUM DISTRIBUTION FUNCTION

The electron distribution becomes highly non-equilibrium due to the electron scattering with the microwave and static electric field. The distribution function of outcoming electron $f_{\text{out}}^{(1,2)}$ in the lead 1 and 2 can be written as

$$f_{\text{out}}^{(1)}(E) = \frac{1}{M(E)} \sum_l \tilde{T}_{12}(E, E_l + eV) f_{FD}(E_l + eV)$$

$$f_{\text{out}}^{(2)}(E) = \frac{1}{M(E)} \sum_l \tilde{T}_{21}(E, E_l - eV) f_{FD}(E_l - eV)$$

In Fig. 6 we show an example of the distribution function $f_{\text{out}}^{(2)}(E)$. It is shown that the distribution function has signature of 'periodic' modulation by the microwave irradiation only when $\omega$ is close to $\omega_c$.

In our Landauer approach where we assumed reflectionless contacts, the non-equilibrium distribution function do not directly affect the conductance. The non-equilibrium distribution function is a result of the electron scattering and the scattered electron just go away from conductors. For more realistic distribution function where many scattering events are considered, the reader may refer to Ref. 5. Our work might be useful for getting hint on the zero temperature case which are not accessible within semiclassical approximation.

VI. CONCLUSION

We have developed the Floquet scattering theory of the negative conductance of 2DEG in cylindrical geometry under microwave radiation. Not only the quantum Hall system but also other multi-channel conductors can have microwave induced negative conductance. The necessary condition of the negative conductance is that multi-channel conductors have dc electric field inside and microwaves are irradiated. We demonstrated that the conductance is always positive definite when the DC electric field do not affect the density of states in conductors. The chemical potential difference in contacts without modification of DOS by electric field do not assure negative conductance even in the presence of microwave.

The reason of the absence of the Pauli blocking in our work is that the magnetic orbitals are not merely localized states but form transport channels due to disorder scattering. Since the outgoing states and incoming states are different states, those states do not have a relation of Pauli exclusion principle. Situation will be different if the transport arises due to hopping between localized states. In this case, Pauli exclusion principle may play important role due to significant back scattering events. Further theoretical works on the hopping conduction with microwave are necessary to clarify the issue on the Pauli blocking effect.
VII. APPENDIX

In this appendix we develop the calculational method for the Floquet scattering matrix in use in this work. We are going to calculate the transmission(reflection) probability amplitude $t_{nN}$ for the incoming electron with $l = 0$ in $N$th Landau level to transmit(reflect) to the outcome state with $l$th channel in $n$th Landau level.

To save the notations, we introduce abstract vectors $\vec{\phi}_m$ which are defined as the linear combinations of the spinor $\chi_{nl}$, the $(n,l)$ component of $\phi_m$ denotes the coefficients $\phi_{nm}^{(l)}$ for the wave-function in Eq.(6):

$$\vec{\phi}_m = \sum_{nl} \phi_{nm}^{(l)} \chi_{nl}. \quad (27)$$

Here $\chi_{NL}$ means a spinor of which $(n,l)$ component is 1 for $(n,l) = (N,L)$ and zero for $(n,l) \neq (N,L)$.

We also introduce operators acting on the vectors:

$$L_0 \vec{\chi}_{nl} = \kappa \vec{\chi}_{nl} \quad (28)$$
$$L^+ \vec{\chi}_{nl} = \vec{\chi}_{nl\pm 1} \quad (29)$$
$$H_m \vec{\chi}_{nl} = (n\omega_c + \delta_m) \vec{\chi}_{nl} \quad (30)$$
$$N^+ \vec{\chi}_{nl} = \sqrt{n+1} \vec{\chi}_{n+1l} \quad (31)$$
$$N^- \vec{\chi}_{nl} = \sqrt{n} \vec{\chi}_{n-1l} \quad (32)$$

The Schrödinger equation in Floquet basis in Eq.(7) can be written as

$$(E + \omega L_0) \vec{\phi}_m = -\frac{V_0}{2}(\vec{\phi}_{m-1} + \vec{\phi}_{m+1}) + H_m \vec{\phi}_m + i\lambda (N^- - N^+)(L^- + L^+)\vec{\phi}_m + \lambda^2 (L^- + L^+)^2 \vec{\phi}_m, \quad (1 \leq m \leq M) \quad (33)$$

where $\lambda = \frac{eE}{2\hbar c\omega_c}$. For $m \leq 0$ and $m \geq M$, the above equation is simply modified by setting $\lambda = 0$.

Now we construct transfer matrix for $(\vec{\phi}_m, \vec{\phi}_{m-1})$. This task can be done similar to the well-known transfer matrix calculation for Anderson localization problem:

$$\begin{pmatrix} \vec{\phi}_{m+1} \\ \vec{\phi}_m \end{pmatrix} = \begin{pmatrix} K_m + F & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \vec{\phi}_m \\ \vec{\phi}_{m-1} \end{pmatrix} \quad (34)$$

where $m = 1, 2, ..., M-1$ and

$$K_m = \frac{2}{V_0} [-(E + \omega L_0) + H_m] \quad (35)$$

$$F = \frac{2\omega_c}{V_0} [i\lambda (N^- - N^+)(L^- + L^+) + \lambda^2 (L^- + L^+)] \quad (36)$$

Successive multiplication of the transfer matrix give us the following relation.

$$\begin{pmatrix} \vec{\phi}_{M+1} \\ \vec{\phi}_M \end{pmatrix} = T \begin{pmatrix} \vec{\phi}_0 \\ \vec{\phi}_{-1} \end{pmatrix}, \quad (37)$$

where

$$T = \begin{pmatrix} K_M -1 \end{pmatrix} \prod_{m=1}^{M-1} \begin{pmatrix} K_m + F & -1 \\ 1 & 0 \end{pmatrix} \times \begin{pmatrix} K_0 & -1 \\ 1 & 0 \end{pmatrix}. \quad (38)$$

From Eqs.(10) and (12), the transmission and reflection amplitudes are written as

$$\vec{\chi}_{nl} \cdot \vec{\phi}_{M+1} = t_{nN}^\text{in} \exp(i(M+1)k_{ln}) \quad (39)$$
$$\vec{\chi}_{nl} \cdot \vec{\phi}_M = t_{nN}^\text{in} \exp(iMk_{ln}) \quad (40)$$
$$\vec{\chi}_{nl} \cdot \vec{\phi}_0 = \delta_{00} \delta_{nN} + r_{nN} \exp(ik_{ln}) \quad (41)$$

By solving the above coupled linear equations with Eqs.(37), we obtain $t_{nN}^\text{in}$ and $r_{nN}^\text{in}$.

Acknowledgments

This work was supported by Chungnam National University through Internal Research Program. I thank Prof. P. Fulde for helpful discussion and Max Planck Institute for Physics of Complex Systems where part of this work has been done.
1 R.G. Mani, J.H. Smet, K. von Klitzing, V. narayanamurti, W.B. Johnson, and V. Umansky, Nature (London) 420, 646 (2002).
2 M.A.Zudov, R.R. Du, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. 90, 046807 (2003).
3 C.L.Yang, M.A.Zudov, T.A.Knuuttila, R.R.Du, L.N.Pfeiffer, and K.W.West, Phys. Rev. Lett. 91, 096803 (2003).
4 A.V. Andreev et al., Phys. Rev. Lett. 91, 056803 (2003).
5 Juren Shi and X.C. Xie, Phys. Rev. Lett. 91, 086801 (2003).
6 V.I.Ryzhii, Sov. Phys. Solid State 11, 2078 (1970).
7 V.I.Ryzhii, R.A.Suris, and B.S.Shchamkhalova, Sov. Phys. Semicond. 20, 1299 (1986).
8 A. C. Durst et al., Phys. Rev. Lett. 91, 086803 (2003).
9 I. A. Dmitriev et al., Phys. Rev. Lett. 91, 226802 (2003).
10 J. Ikarrea and G. Platero, Phys. Rev. Lett. 94, 016806 (2005).
11 J. Dietel et al., Phys. Rev. B 71, 045329 (2005).
12 J.T. Chalker and P.D. Coddington, J. Phys. C 21, 2665 (1988).
13 R.G. Newton, in Scattering Theory of Waves and Particles (McGraw-Hill, New York, 1966).
14 D. F. Martinez and L. E. Reichl, Phys. Rev. B 64, 245315 (2001).
15 V. Ryzhii, A. Satou, cond-mat/0306051 and references therein.