A Matrix Aggregation Intelligent Decision Based on H-Convex Combination and Expert Preference

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Abstract: In the field of multi-dimensional intelligent decision-making, the evaluation results rely on the subjective experience of the evaluation experts. However, the evaluation information generated by personal expert preferences can have a certain impact on the decision-making results. For this problem, we present an intelligent decision model based on H-convex combination and expert preference. The H-convex combinatorial matrix algorithm is used to assemble the individual judgment matrices, which reduces the inconsistency and non-reciprocity of the group decision matrix. The expert preference quantization formula is used to obtain the preference values of the corresponding experts, which are taken into the H-convex combination as power exponents. This solves the impact of expert preference differences on decision outcomes. The feasibility and effectiveness of the proposed method are verified by some evaluation cases.

1. Introduction

With the evolution of many fields, intelligent decision-making has always been the focus of development. Especially in some complex systems, multi-dimensional decision-making often has a hierarchical structure. Different problems are similar to a hierarchical structure and are roughly divided into three levels: level layer, criterion layer and program layer, sorting and selecting the problem layer is the most important decision-making problem. A quantitative and qualitative model was founded by the famous American operations researcher T. L. Saaty in the early 1970s. This decision-making method (Analytic Hierarchy Process, AHP) has the characteristics of systemic practicability and simplicity, so it has become the first important tool for complex decision-making problems and has also been widely used in decision-making problems. When solving the actual problem through the AHP model, whether this problem is the hierarchical structure or the expert judgment matrix for the index weight construction, the subjective preference decision information of an evaluation expert has a great influence on the decision result, and the wrong judgment information may cause failure of the final decision. The essence of intelligent decision-making is to enable decision-makers to participate in decision-making with full rationality, but in real world people's subjective preferences often have a greater impact on decision-making information. In order to make the decision conclusion of AHP more objective, the group intelligent decision-making method is used to overcome the bad influence of expert preference when using the AHP model.

In recent years, there have been some research results in this field. Xinbao Liu et al. defined a matrix Hadamard convex combination operation, which proves that the Hadamard convex combination of multiple consistency judgment matrices of the same problem still meets the...
consistency under the group decision conditions. It also shows that the convex combination of judgment matrix can not only eliminate the influence of subjective bias, but also improve the consistency of judgment matrix [5]. Jianjun Zhu and so on use decision makers to judge the consistency level of preference and the distance of group comprehensive preference difference, and propose a kind the weighting model for determining the weight of decision makers [6]. Wenshuai Wu et al. proposed the EWAHP-GDM model by combining the weighted arithmetic mean method and the least squares method [7]. The analysis of Laifu Liu et al. shows that the consistency index of the geometric mean synthesis matrix is smaller than the arithmetic mean of the consistency indicators of each decision matrix [8]; Yager et al. introduce an ordered weighting operator: the aggregation (OWA) operator, which is used to solve the problem through the aggregation of multiple criteria to form a holistic decision function to make decisions [9]. Dong et al. use the classical geometric averaging algorithm to calculate small and medium-order sorting vector set problem in AHP group decision [10]; Bojan et al. extracted the optimal ordering of each stage in the aggregation group sorting, and proposed a local priority vector aggregation method [11]. The above research works show that geometric aggregation and weighted geometric mean and system clustering are still common aggregation algorithms for the assembly of expert individual judgment matrices [12]. But in the world, these methods can only get approximate group decision aggregation. If there is a large deviation of the partial judgment information, the result of the assembly is often far from the original judgment matrix. Therefore, there is still a lot of research space for the assembly of expert judgment matrices.

Combined with the actual situation of judgment matrix aggregation in multi-dimensional intelligent decision-making, this paper proposes a judgment matrix aggregation method based on H-convex combination and expert preference. The method of judging matrix convex combination is used to obtain the comprehensive decision matrix, which ensures the consistency and reciprocity of the decision matrix, and can make the actual decision rigorous. The expert preference is used to assign the power exponent in the convex combination, eliminating the difference of expert experience. The preference factor can make the decision more rational. Our method also provides a new idea for the actual decision-making evaluation.

2. Model related concept

2.1. Definition of H-convex combination

Definition 1 Set positive reciprocal matrix \( A = (a_{ij})_{nxn}, B = (b_{ij})_{nxn} \). If \( A \ast B = (c_{ij})_{nxn} \) then \( A \ast B \) is called Hadamard product of the matrices of A and B, among them \( c_{ij} = a_{ij} \ast b_{ij} \), and \( \ast \) for the Hadamard product symbol.

Definition 2 Matrix \( A = (a_{ij})_{nxn}, T \in R \), the exponential operation of the matrix is defined as \( A^T = (a_{ij}^T)_{nxn} \).

Definition 3 Suppose that \( A_1, A_2, \cdots, A_m \) is m judgment matrices about the same problem. If \( \lambda_L = \left[ 0, 1 \right] (L = 1, 2, \cdots, m), \sum_{L=1}^{m} \lambda_L = 1 \) is present, let \( A' = A_1^\lambda_1 \ast A_2^\lambda_2 \ast \cdots \ast A_m^{\lambda_m} \). Then \( A' \) is called a Hadamard convex combination of \( A_1, A_2, \cdots, A_m \).

Obviously, \( A' \) is also a judgment matrix for this problem.

Theorem 1 If \( A_1, A_2, \cdots, A_m \) is m judgment matrices about the same problem, and there is a sufficiently small \( R > 0 \) let \( \frac{2}{n(n-1)} \sum_{i \leq j \leq n} \log X_{ij}^L \leq R, (L = 1, 2, \cdots, m) \), \( A' \) is a Hadamard convex combination of \( A_1, A_2, \cdots, A_m \). Then

\[
\frac{2}{n(n-1)} \sum_{i \leq j \leq n} \log X_{ij} \leq R.
\]

The literature[3] has given detailed proof and will not be described in detail in this paper.
Inference 1 If $A_1, A_2, \ldots, A_m$ are the judgment matrices of the same problem, then their Hadamard convex combination $A'$ also has satisfactory consistency and reciprocity.

2.2. Expert preference definition

In the field of intelligent decision-making evaluation, different experts have different subjective preferences, professional knowledge, and experience backgrounds. In real decision-making, the complexity of real problems will also greatly influence the decision-making judgment of experts[13]. The existing expert preferences and weights are mainly determined by subjective factors such as personal prestige and authority in their respective fields. They are determined by subjective valuation method in decision-making, and their expert preference values have great subjectivity and uncertainty. Therefore, it is meaningful to define a new expert preference evaluation method.

According to the analytic hierarchy process, the consistency ratio of the $t$ positive reciprocal judgment matrix given by the $k$ expert is $CR^t_k$. In the theoretical system of AHP, the important criterion for weighing the professionalism and validity in the specific case is the size of the judgment matrix consistency ratio. The consistency ratio is $0 \leq CR < 0.1$, and the expert's judgment matrix is in conformity with the satisfactory consistency condition. At this time, the expert opinion is considered to be effective. If $CR > 0.1$, need to adjust the expert judgment matrix. On this basis, it is shown that the consistency ratio $CR$ of the judgment matrix given by the evaluation expert also reflects the importance of the expert's preference. If the consistency ratio of the judgment matrix given by the expert is $0 \leq CR < 0.1$, the satisfactory consistency condition is met, and the judgment matrix is valid at this time. When $CR=0$, the judgment matrix is completely consistent, and the expert opinion is absolutely authoritative. In the actual decision-making case, the effective consistency ratio ranges from $(0, 0.1)$ [13]. This paper presents that the consistency ratio of the judgment matrix given by the expert in the decision model and the preference importance of the decision-making expert should satisfy one-to-one correspondence, and inversely proportional.

Thus, the formula for defining the expert preference $\alpha^t_k$ is as follows:

$$\alpha^t_k = \frac{1}{1 + \beta CR^t_k} \left( \beta > 0, 1 \leq k \leq n, 1 \leq t \leq m \right)$$  \hspace{1cm} (1)

$$\alpha_k = \frac{\sum_{t=1}^{m} \alpha^t_k}{m} \left( 1 \leq k \leq n, 1 \leq t \leq m \right)$$  \hspace{1cm} (2)

Where $m$ is the total number of judgment matrices given by each expert, and parameter $\beta$ is set to $10^4$ to ensure the stability of the calculation.

Finally, the standardization weights of all experts involved in the assessment are:

$$\bar{\alpha}_k = \frac{\alpha_k}{\sum_{k=1}^{n} \alpha_k}$$  \hspace{1cm} (3)

Definition 4 If $\bar{\alpha}_k$ is the expert weight and $A^*_k$ is the judgment matrix of the corresponding expert, then the decision matrix after assembly will be:

$$A^* = A^*_{1\bar{\alpha}_1} \times A^*_{2\bar{\alpha}_2} \times \cdots \times A^*_{n\bar{\alpha}_n}$$  \hspace{1cm} (4)

Definition 5 In the matrix $A = (a_{ij})_{n \times n}$, set its maximum eigenvalue as $\lambda_{max}$, then

$$CR = \frac{\lambda_{max}}{n(n-1)} \cdot RI^{-1}$$  \hspace{1cm} (5)

is the consistency ratio of the matrix, where $RI$ is the average random consistency indicator[14].

3. Establish decision model

The most commonly used group decision-making methods in AHP include two methods: assembly individual ranking method and assembly individual judgment matrix method. In existing research,
they are used as independent methods, and thus their internal connections and laws would be ignored. Based on the predecessors, we present the intelligent decision-making model based on H-convex combination and expert preference, which can effectively reconcile the two classical aggregation methods of the individual ordering and individual judgment matrix ordering. This model can satisfy the consistency while satisfying the positive reciprocity, avoiding the inconsistency of the decision matrix after assembly and the time wasted in the adjustment, reducing the amount of calculation. Assigning the convex combination through expert preference, the model can eliminate the subjective factors on the decision model influence and make decision making more reasonable. The specific algorithm steps in the model are as follows:

Step 1 For each level of evaluation decision, the expert establishes a positive reciprocal matrix \( A_k = (a_{ij})_{n \times n} \), \( l \leq k \leq n \), of two pairs according to the scale of \([1/9,9]\), where \( k \) represents the number of experts.

Step 2 Find the consistency ratio \( CR_k \) of each judgment matrix from the formula (5), and calculate the expert weights \( \tilde{\alpha}_k \) by the formulas (1), (2), and (3).

Step 3 Substitute the expert weight obtained in step 2 into the formula (4) to obtain the final group decision matrix.

Step 4 Final decision ranking

When comparing the assembled group decision matrix, the sorting vector obtained by the assembled decision matrix is constructed according to

\[
A = (a_{ij})_{n \times n} = \left( \frac{w_i}{w_j} \right)_{n \times n},
\]

as a completely consistent judgment matrix \( A' \), and the square error sum \( \Delta b \) (where the size of \( \Delta b = \sum_{i,j \neq k}^n (A_{ij} - A'_{ij})^2 \)) of the corresponding elements of each element in \( A' \) and the original judgment matrix indicates how close it is to each expert opinion. When \( \Delta b \) is smaller, it indicates that the closer to the original judgment matrix, the better the aggregation effect.

In the evaluation of the final ranking results, the difference formula \( [15] \) is used for result analysis. The greater the difference is, the better the discrimination of the evaluation scheme is, which is more conducive to decision-making.

4. Case analysis

This paper adopts a practical case about investment project selection, which is evaluated by five experts from the risk and investment fields. The decision hierarchy diagram is as follows:

see Figure 1.

Step 1 Select the judgment matrix given by the 5 experts at the B1-C level as follows:

see Figure 2.

Step 2 Find the consistency ratio of all the judgment matrices in the example by the formula (5). The weight vector of the expert is obtained by the formulas (1), (2), and (3):

\[
\tilde{\alpha}_k = (0.1945, 0.2180, 0.2146, 0.1770, 0.1959)
\]

Step 3 Substitute the weight vector \( \tilde{\alpha}_k \) obtained by Step 2 into the formula (4), at this time:

\[
A' = A_1^{12} \ast A_2^{23} \ast A_3^{34} \ast A_4^{45} \ast A_5^{51}
\]

Calculated:

\[
A' = \begin{bmatrix}
1.0000 & 1.0407 & 1.0458 & 0.7468 \\
0.9609 & 1.0000 & 1.1898 & 0.7530 \\
0.9563 & 0.8404 & 1.0000 & 0.7185 \\
1.3391 & 1.3279 & 1.3917 & 1.0000 \\
\end{bmatrix}
\]

\[
CR = 0.0011 < 0.1, W_{B1-C} = (0.2351, 0.2386, 0.2159, 0.3104)^T
\]
Figure 1. Hierarchy chart

![Hierarchy chart]

Figure 2. Judgment matrix

$$A_i = \begin{bmatrix} 1 & 2 & 2 & 3 \\ 1/2 & 1 & 1 & 2 \\ 1/2 & 1 & 1 & 2 \\ 1/3 & 1/2 & 1/2 & 1 \end{bmatrix}, \ A_2 = \begin{bmatrix} 1 & 1/2 & 1 & 1/3 \\ 2 & 1 & 2 & 1/2 \\ 1 & 1/2 & 1 & 1/3 \\ 3 & 2 & 3 & 1 \end{bmatrix}, \ A_3 = \begin{bmatrix} 1 & 2 & 1/2 & 1/3 \\ 1/2 & 1 & 1/3 & 1/4 \\ 2 & 3 & 1 & 1/2 \\ 3 & 4 & 2 & 1 \end{bmatrix},$$

(2)

Figure 4. Determine the total order of the levels:

(1) Determine the group decision matrix of each level by Step 3, and calculate the corresponding weight vector by the eigenvalue method, as follows:

$$W_{A-B} = (0.4922, 0.5078)^T, W_{B-C} = (0.1655, 0.1834, 0.2553, 0.3104)^T,$$

$$W_{C1-D} = (0.2340, 0.1924, 0.2302, 0.3433)^T, W_{C2-D} = (0.2568, 0.2283, 0.1533, 0.3616)^T,$$

$$W_{C3-D} = (0.1675, 0.1710, 0.1746, 0.4870)^T, W_{C4-D} = (0.1862, 0.1069, 0.4171, 0.2898)^T,$$

$$W_{C5-D} = (0.1869, 0.1586, 0.2876, 0.3669)^T, W_{C6-D} = (0.2620, 0.2409, 0.2119, 0.2852)^T,$$

$$W_{C7-D} = (0.1706, 0.2141, 0.3042, 0.3110)^T, W_{C8-D} = (0.1973, 0.2317, 0.2682, 0.3028)^T.$$

(2) The final hierarchical ordering by the classic AHP method is:

$$W = (0.1584, 0.1530, 0.2309, 0.4577)^T.$$
5. Experimental comparison

5.1. Comparison of Group Decision Matrix after Aggregation

In the follow example in the paper, the expert judgment matrix of B1-C level is selected, and the assembled group is compared by the traditional AHP assembly method (AHP-GDM algorithm), the EWAHP-GDM algorithm in the literature[7] and the algorithm proposed in this paper. The decision matrix and the original judgment matrix correspond to the magnitude of the element error squared $\Delta b$ to illustrate the effectiveness of the proposed algorithm. The comparison results are shown in Table 1:

| Algorithm                     | $\Delta b$   | CR     |
|-------------------------------|--------------|--------|
| AHP-GDM                       | 125.9161     | -      |
| EWAHP-GDM                     | 130.2202     | 0.0293 |
| Algorithm proposed in the paper | 114.2100     | 0.0011 |

It can be seen from the table that the group decision matrix of the algorithm proposed in this paper is significantly smaller than the other two algorithms in the corresponding element error square, which indicates that the group decision algorithm after assembly is more representative of each expert opinion, which is more reasonable in practical application.

5.2. Final decision sorting comparison

In the decision evaluation process of the case above, the effectiveness of the proposed method is illustrated by the comparison between the algorithm and the above two algorithms in decision sequencing and the discrimination of adjacent schemes. The comparison results are shown in Table 2.

| Investment program | AHP-GDM | EWAHP-GDM | Algorithm proposed in the paper |
|--------------------|---------|-----------|---------------------------------|
|                    | W S D   | W S D     | W S D                           |
| Fund $D_1$         | 0.1915  3 53.00% | 0.1719  3 84.80% | 0.1584  3 45.77% |
| Bond $D_2$         | 0.1853  4 3.35%  | 0.1077  4 59.52%  | 0.1530  4 3.53%  |
| Stock $D_3$        | 0.2930  2 12.7%  | 0.3176  2 26.82%  | 0.2309  2 98.22%  |
| Estate $D_4$       | 0.3302  1 0.4028 | 0.4577  1          |

(Comment: W means Weight, S means sorting, D means difference)

It can be clearly seen from Table 2 that the ranking results obtained by the three methods are consistent, and the result is $D_4 > D_3 > D_1 > D_2$, that is, the real estate investment project is a relatively optimal investment plan among the four investment schemes, followed by the stock investment project, relatively poor are fund investment project and bond investment project. The experimental results show that the proposed algorithm is effective. And the difference degree analysis shows that the proposed algorithm is significantly different in the selection of the optimal evaluation results than the traditional AHP-GDM algorithm and EWAHP-GDM algorithm. It is easier to choose the best solution in decision-making, avoiding the blindness and uncertainty. This can further prove the feasibility of the algorithm.

6. Conclusion

In this paper, an intelligent decision-making model of judgment matrix based on H-convex combination and expert preference is proposed for some problems of individual judgment matrix aggregation in multi-dimensional decision making, which ensures the consistency and reciprocity of
decision matrices, and eliminates the difference of expert experience and preference factors. In addition, this can make decision making more rational. And in some similar evaluation cases, compared with the traditional and classic AHP-GDM algorithm and EWAHP-GDM algorithm, the relative error and relative minimum of the group decision matrix between the assembly and the original judgment matrix are combined on the basis of ensuring the final ranking result to consistence. In the same way, the difference between the optimal scheme and other schemes would be the largest, which enables decision makers to make more comprehensive judgments. This can avoid the blindness and uncertainty of decision-making, and our method is more reasonable in practical applications.

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