Mach-Zehnder atom interferometer. Quantum and Doppler corrections caused by the finite pulses’ durations

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A new approach to the theory of atoms’ interaction with chirped Raman pulses is developed. When the pulses have sufficiently close effective wave lengths, which are smaller than the atomic cloud size, equations for the family of the matrix elements of the atomic density matrix in the Wigner representation are derived. The solution, involving linear (in the pulse duration) phase corrections, is obtained for the rectangular pulse. The interferometric part of the atoms’ excitation is calculated and new Doppler and quantum terms are found in the phase of the Mach-Zehnder atom interferometer.

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Since its birth about 30 years ago [1], the field of atom interferometry (AI) has matured significantly. Experiments based on AI have been used to test the Einstein equivalence principle (EEP) [2, 3] and to measure fundamental constants [4–7], the acceleration of gravity near the Earth’s surface [8–13], the gradient of the Earth’s gravitational field [14–16], and the curvature of the gravitational field produced by the source masses [17]. Atom interferometer gyroscopes allow one to measure rotation rates; experiments have utilized optical fields [18], nanofabricated structures [19], and three or four spatially or temporally separated sets of fields that drive Raman transitions to split and recombine matter waves [20–23]. The frequency shift arising from a quadratic Zeeman effect was also measured precisely [24]. Limits have been set on a non-Newtonian Yukawa-type fifth force [25] and on dark energy [26, 27] using AI, as well as theoretical proposals for using AI to measure general relativity effects [28, 29], including gravitational waves [30].

The Mach-Zehnder atom interferometer (MZAI) is one of the main tools for precise studies. The discovery and analysis of the new types and reasons for the systematic errors are always actual for a number of the precise experiments and projects. To achieve the extremely high precision, one should increase the interrogation time $T$. The interrogation time as large as $T = 1.15s$ was achieved in the article [31]. The extended study of the errors caused by the large value of $T$ in the theory [11, 32, 33] and experiment [10, 11, 15, 33, 37] was performed. Methods of eliminating some of those errors have been proposed [38, 39]. The A. Roura technique of eliminating [38] has been observed [37, 40].

Another source of the errors is the finite pulses’ durations $\tau_n$. Different techniques including the formalism of the sensitivity function [41] have been applied to obtain the linear and nonlinear in $\tau_n$ terms, but only one expression for the MZAI phase was published [33, 42, 43]

$$\phi_g = (k \cdot g - \alpha) T \left( T + \tau \left( \frac{4}{\pi} + 2 \right) \right),$$

(1)

where $k$ is the effective wave vector of the Raman field, $\tau$ is the first pulse duration, $g$ is the gravity acceleration, we assume that one chirps the Raman field with the rate $\alpha$, and we omit the term of the relative weight $(\tau/T)^2$. Generalization of the expression (1), which includes the combination of the gravity gradient terms and terms caused by the finite pulses’ durations, was also obtained [44].

The temporal sequence of the Raman pulses is shown in Fig. 1.

The Eq. (1) was derived for 3 rectangular pulses having the pulses’ duration sequence

$$\tau_1 - \tau_2 - \tau_3 = \tau - 2\tau - \tau.$$

(2)

This choice of the pulses’ duration sequence is mentioned explicitly in the articles [3, 10, 11, 31, 45–50]. This sequence is convenient because it allows one to create $\pi/2 - \pi - \pi/2$ sequence of the Raman pulses just by doubling the second pulse duration. That is why I believe that the sequence (2) has been implicitly used in an enormous amount of experiments where the $\pi/2 - \pi - \pi/2$ sequence has been explored. This sequence will be utilized in the following international programs: NASA QTEST program [51, 52], in the ESA project “Space Atom Interferometer” [53], in the project Q-WEP and STE-QUEST: Testing the equivalence principle in Space [54] (see also [55]) and the project MIGA [56].
FIG. 1: MZAI sequence of the Raman pulses. Pulse \( n \) is triggered at the time \( T_n \) and has duration \( \tau_n \). (a) \( T \) is the time separation between neighboring pulses, we assume that time separations between 1st and 2nd pulse and 2nd and 3rd pulses are the same; (b) \( T \) is the time delay between pulses’ starting moments, we assume that time separations between the starting points of the 1st and 2nd pulse and 2nd and 3rd pulses are the same.

In this article we also calculated the MZAI phase, but in addition to the term \( \phi_1 \), we obtained 2 new corrections, see Eqs. (C33a) (C33b),

\[
\phi_D = k \frac{p_i}{M} (\tau_2 - \tau_1), \tag{3a}
\]

\[
\phi_q = \omega_k (\tau_2 - \tau_1), \tag{3b}
\]

where \( p_i \) is an atomic cloud launching momentum, \( M \) is an atomic mass, \( \omega_k = \hbar k^2/2M \) is the recoil frequency. Evidently, terms (3a) and (3b) have Doppler and quantum nature. For the parameters’ value chosen in [10, 11], \( T = 160\mu s, \tau_1 = 40\mu s, \tau_2 = 80\mu s, k \approx 1.47 \cdot 10^7 m^{-1}, M_{\gamma s} \approx 133 u \), in the case of fountain geometry [57], when \( p_i \sim MgT \), one finds \( \phi_D \sim 900\text{rad}, \phi_q \approx 2\text{rad} \). The Doppler term (3a) is 4 orders of magnitude larger than the systematic error caused by the gravity gradient term \( \phi_q \), first calculated in the article [32]. Even the smaller quantum term (3b) is still almost 20 times larger than \( \phi_q \).

It is well known that the phase of the MZAI contains no quantum contributions in the uniform gravity field. Even if one uses Raman pulses with different effective wave vectors, the quantum contribution still equals 0 [58]. It is well known, but still somewhat awkward, that the phenomenon totally caused by the quantization of the atomic center-off-mass motion has a purely classical response in free space and also in the uniform gravity field. But this result has been obtained with the assumption that one can neglect the atom motion in space during the interaction with pulses. In this article, we show that the quantum term (3b) arises even in the uniform gravity field, if one includes in consideration the atom motion during pulses’ action and if the Raman rectangular pulses have different durations.

It is also well known that for MZAI the pulses have to be equidistant from one another, but, if the pulses durations are sufficiently large to affect on the MZAI phase, the variable \( T \), distance between pulses, has to be properly chosen. In the articles [33, 42, 43] \( T \) is the time between rectangular pulses (see Fig. 1a), while in the article [49], \( T \) is the time between pulses maxima. These two choices are equivalent and in both cases the corrections (3) are not eliminated. The calculations performed in this article show that for the rectangular pulses the elimination occurs if \( T \) is the time delay between pulses triggering points (see Fig. 1b). If \( \tau_1 \neq \tau_2 \) the cases shown in Figs. 1a and b are not equivalent. Evidently, in the case 1a the triggering points are not equidistant, while in the case 1b the time intervals between 1st and 2nd pulses and between 2nd and 3rd pulses are different.

The quantum contribution to the MZAI phase arises in the rotating frame [59], or in the slightly non-uniform field, in the presence of the gravity-gradient tensor (see for example [22]) or in the presence of the gravity curvature tensor [60], or in the strongly non-uniform field of the external test mass [61]. The quantum part of the phase in the gravity-gradient field of the external test mass has been recently observed [13]. When \( k || g \), the quantum term caused by weak gravity gradient force is given by [22]

\[
\phi_{qg} = \omega_k \gamma_{zz} T^3, \tag{4}
\]

where \( \gamma_{zz} \) is the zz-component of the gravity gradient tensor \( \gamma \), z–axis is the vertical direction. For the Earth gravity, when \( \gamma_{zz} \sim 3 \cdot 10^{-6} s^{-2} \), one finds that our quantum term (3b) is an order of magnitude larger than the term (4), even for the largest value, \( T = 1.15 s \). The technique of alternating the wave vector direction [61] allows one to eliminate both quantum terms (3b) and (4).
Term (5a) should lead to the systematic error in the absolute gravity measurements. For the fountain geometry this error is given by

$$\delta \eta \sim \frac{\tau_2 - \tau_1}{T},$$  \hspace{1cm} (5)

for the gravimeter \[10, 11\] $\delta \eta \sim 2 \cdot 10^{-4}$, while for the gravimeter $\[12\]$, $\delta \eta$ could be 2 orders of magnitude smaller. The situation would be better if (instead of fountain geometry) one just drops atoms with $p_i = 0$ initial momentum.

Terms (3) do not affect the gravity-gradiometer measurements, if the atomic initial momenta $p_i$ are the same for both interferometers comprising the given gradiometer.

The Doppler term (6a) violates the EEP and leads to a systematic error of the order of (5) in Etvöss parameter $\eta$. To eliminate this error one has to launch both atomic species with the same velocity $v_i = p_i / M$. The velocity inaccuracy $\delta v$ leads to the error in the Etvös parameter $\eta$,

$$\delta \eta \sim \frac{\delta v (\tau_2 - \tau_1)}{gT^2}. \hspace{1cm} (6)$$

If the ultimate goal of the EEP test is $\delta \eta \lesssim 10^{-14}$, then one has to hold a velocity with the inaccuracy better then $3 \cdot 10^{-9} \text{m/s}$ (for $T \sim 1.15 \text{s}$, $\tau_2 - \tau_1 \sim 40 \mu \text{s}$), which could be a severe restriction.

To obtain the MZAI phase we used equations for the atomic density matrix in the Wigner representation $[62]$. Starting from the article $[63]$ the Wigner representation is widely used in the atom optics. One can find several examples of that use in the book $[64]$. But, nevertheless, I can mention only articles $[22, 58–60, 65]$, where this approach has been used in the theory of the atom interference and in those articles the atomic motion during interaction with the pulse was ignored. The feature of this article is that we applied here equations for the atomic density matrix in the Wigner representation during the atom interaction with the Raman pulse. The small parameters of the problem are the ratio of the effective wave length $\lambda = 1 / k$ to the atomic cloud size $a$ and $\Omega_E \tau$, where $\Omega_E$ is the rotation rate of the gravity source. In the Appendix A we show that for the

$$\frac{\lambda}{a} \ll 1, \quad \Omega_E \tau \ll 1,$$  \hspace{1cm} (7)

and, when the quantum terms caused by the gravity-curvature tensor are neglected $[66]$, one can describe an evolution of the density matrix in the Wigner representation in terms of the atomic-center-of-mass classical trajectory in phase space $\{X(x, p, t), P(x, p, t)\}$ both between Raman pulses and inside of the each pulse.

In the theory $[33, 42]$, the interrogation time $T$ was defined as the time interval between the end of the given pulse and the start of the next pulse (see Fig. 1a),

$$T = T_2 - T_1 - \tau_1 = T_3 - T_2 - (8)$$

In our calculations we did not follow this definition, allow arbitrary values of $T_n$ and $\tau_n$ and arrive to the Eqs. $[C20, C30]$. One sees from these equations that both Doppler and quantum terms will be eliminated if one defines $T$ as the time delay between triggering moments $T_i$ and holds those moments to be equidistant (see Fig. 1a),

$$T = T_2 - T_1 = T_3 - T_2,$$  \hspace{1cm} (9)

but in this case for the $\pi / 2 - \pi - \pi / 2$ sequence and the pulses’ durations $[2]$ one gets for the gravitational phase instead of Eqs. $[11]$

$$\phi_g = (k g - \alpha) \left(T^2 + \left[ T \left( \frac{4}{\pi} - 2 \right) - T_1 \right] \tau \right) \hspace{1cm} (10)$$

Analysis shows that the difference between Eqs. $[11]$ and $[10]$ occurs because in the case shown in Fig. 1a, the triggering points $T_n$ of the 2nd and 3rd pulses depend on $\tau_n$ [see Eq. $[3]$] and being substituted to the general expression $[C29]$ contribute to the linear in $\tau_n$ terms, while in the case shown in Fig. 1b $T_n$ are $\tau$-independent [see Eq. $[4]$].

Terms $[3]$ can also be eliminated in the choice $[3]$ if the 1st and 2nd Raman pulses have the same duration. But to still get $\pi / 2 - \pi - \pi / 2$ sequence (and get the unity contrast), one has to double the Raman-Rabi frequency of the 2nd pulse. In this case, from the general expressions $[C29, C38]$, one gets instead of Eq. $[11]$

$$\phi_g = (k \cdot g - \alpha) T \left( T + \tau \left( \frac{4}{\pi} + 1 \right) \right) \hspace{1cm} (11)$$

If one still prefers to hold the same Raman Rabi frequencies for all pulses, then the sequence of the pulses becomes $\pi / 2 - \pi / 2 - \pi / 2$. For this sequence, the contrast becomes twice smaller {see, for example, Eq. (57) in $[22]$}, while the phase is still given by Eq. (11).
Corrections caused by the pulses finite duration is sensitive to the shape of the Raman pulses \[49\]. To verify the sensitivity of the terms (3) to the pulses’ shapes, we are going in the future to perform the MZAI calculations for the Rosen-Zener pulses \[67\].

The Doppler and quantum corrections could also arise in the atomic gyroscope \[21–23\], which explores the double-loop atom interferometer, where the pulses’ sequence is $\pi/2 - \pi - \pi - \pi/2$. Recently, this sequence has been also used for the new scheme of the atomic gravity gradiometer \[10\]. If, to achieve this sequence, one uses the pulses’ durations $\tau - 2\tau - 2\tau - \tau$, then one could get the Doppler and quantum terms owing to the differences of the pulses’ durations. We are going to perform calculations for 4 Raman pulses in the future.

Conclusion. For the equal or similar effective wave vectors, sufficiently large atom cloud size (on the scale of the effective wave length), and sufficiently small pulse duration (on the scale of the inverse gravity source rotation rate) we derived the solution for the density matrix equation in the Wigner representation for the rectangular shape of the Raman pulses. The solution brought us to the expression for the Mach-Zehnder atom interferometer phase. Linear in the pulses’ durations’ contributions to the phase were calculated. We found 2 new terms proportional to the launching atomic momentum and to the recoil frequency, which we called Doppler and quantum corrections. They are caused by the uncompensated parts of the atomic coherence phase associated with the Doppler shift of the frequency and with the quantization of the atomic center-of-mass motion. For the typical pulses durations, these terms’ magnitudes are larger than corrections caused by the gradient of the gravity field.

In addition, we calculated remaining part of phase for the arbitrary triggering times, two-quantum detunings, durations, and Rabi Raman frequencies of the rectangular Raman pulses.

Two opportunities of the Doppler and quantum corrections elimination are proposed. One can assign to be equidistant the pulses’ triggering points [see Fig. 1 and Eq. 3]. In this case the correction to the MZAI phase changes the sign and becomes 4.5 times smaller in magnitude [compare Eqs. 1 and 10]. Another option here is to choose the same duration for all 3 Raman pulses. In this case one has to use Eq. 10 instead of Eq. 1. If in addition one uses the same effective Raman Rabi frequencies for all 3 Raman pulses, then the MZAI contrast becomes twice smaller.

Though the quantum and Doppler corrections are larger than other terms, and larger than the sensitivity of the precise experiments, they have never been observed. One of the reasons here is that these corrections are independent on the time separation between pulses $T$ and the chirping rate $\alpha$. To observe these terms one can use differential technique, the phase difference of the two interferometers irradiated with the Raman pulses having different pulses’ durations should contain $T$-independent parts given by Eqs. 3.

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Appendix A: Atom interaction with the Raman pulse

Let us consider an interaction of the three-level atom with the pulse of the Raman field

\[
\mathbf{E}(\mathbf{x}, t) = \left\{ \mathbf{E}_1 \exp \left[ i \left( \mathbf{q}_1 \cdot \mathbf{x} - \omega_1 t - \phi^{(1)}(t) \right) \right] + \mathbf{E}_2 \exp \left[ i \left( \mathbf{q}_2 \cdot \mathbf{x} - \omega_2 t - \phi^{(2)}(t) \right) \right] + \text{c.c.} \right\} \sqrt{f(t)} \tag{A1}
\]

where $\mathbf{E}_n$, $\mathbf{q}_n$, $\omega_n$, $\phi^{(n)}(t)$ are amplitudes, wave vectors, frequencies and phases of the traveling waves in Eq. (A1), $n = 1, 2$, $f(t)$ is the shape of the Raman pulse, which is triggered on and off at the moments $T$ and $T + \tau$,

\[
f(t) > 0, \quad \text{at } T < t < T + \tau, \quad f(t) = 0, \quad \text{at } t < T \text{ or } t > T + \tau, \tag{A2}
\]

where $\tau$ is a pulse duration. We assume that the field $\mathbf{E}_1$ is resonant to the transition $g \rightarrow 0$ and field $\mathbf{E}_2$ is resonant to the transition $e \rightarrow 0$, where $g$ and $e$ are hyperfine sublevels of the atomic ground state manifold, 0 is an excited state. The Hamiltonian of the interaction in the presence of the gravity source rotating with a permanent rate $\Omega_E$ is
given by

\[ H = H_{cm}(x, p) + \hbar \left\{ \Omega^{(1)} \exp \left[ i \left( q_1 \cdot x - \Delta_1 t - \phi^{(1)}(t) \right) \right] |0\rangle \langle g| + \Omega^{(2)} \exp \left[ i \left( q_2 \cdot x - \Delta_2 t - \phi^{(2)}(t) \right) \right] |0\rangle \langle e| + H.c. \right\}, \quad (A3a) \]

\[ H_{cm}(x, p) = \frac{p^2}{2M} + p \cdot (x \times \Omega_E) + U(x), \quad (A3b) \]

where \( \Delta_1 = \omega_1 - \omega_{0g} \) and \( \Delta_2 = \omega_2 - \omega_{0e} \) are detunings of the traveling waves frequencies from the frequencies of the atomic transitions, \( \Omega^{(1)} = -E_1 \cdot d_{0g}/\hbar \) and \( \Omega^{(2)} = -E_2 \cdot d_{0e}/\hbar \) are Rabi frequencies of the traveling waves, \( d_{0m} \) is the matrix element of the dipole moment operator, \( p, x \) and \( M \) are the atomic momentum, position and mass, \( U(x) \) is a gravitational potential. Atomic amplitudes evolve as

\[ i\hbar \partial_t \tilde{a}(0, p, t) = H_{cm} \tilde{a}(0, p, t) + \hbar \left\{ \Omega^{(1)} e^{-i[\Delta_1 t + \phi^{(1)}(t)]} \tilde{a}(g, p - hq_1, t) + \Omega^{(2)} e^{-i[\Delta_2 t + \phi^{(2)}(t)]} \tilde{a}(e, p - hq_2, t) \right\} \sqrt{f(t)}, \quad (A4a) \]

\[ i\hbar \partial_t \tilde{a}(e, p, t) = H_{cm} \tilde{a}(e, p, t) + \hbar \Omega^{(2)*} e^{i[\Delta_2 t + \phi^{(2)}(t)]} \tilde{a}(0, p + hq_2) \sqrt{f(t)}, \quad (A4b) \]

\[ i\hbar \partial_t \tilde{a}(g, p, t) = H_{cm} \tilde{a}(g, p, t) + \hbar \Omega^{(1)*} e^{i[\Delta_1 t + \phi^{(1)}(t)]} \tilde{a}(0, p + hq_1) \sqrt{f(t)}. \quad (A4c) \]

For the large detunings, \( \Delta_1 \approx \Delta_2 \approx \Delta \),

\[ |\Delta| \gg \text{Max} \left\{ |\delta|, |\tau^{-1}|, |\phi^{(n)}|, |q_n \cdot g|, |q_n \cdot \frac{p}{M}|, |\Omega^{(n)}|, |\omega_{q_n}| \right\}, \quad (A5) \]

where

\[ \tilde{\delta} = \Delta_1 - \Delta_2 \]

is Raman detuning and

\[ \omega_q = \frac{\hbar q^2}{2M} \quad (A6) \]

is a recoil frequency, one can eliminate the rapidly oscillating amplitude of the excited state as

\[ \tilde{a}(0, p, t) = \left\{ \Omega^{(1)} e^{-i[\Delta_1 t + \phi^{(1)}(t)]} \tilde{a}(g, p - hq_1, t) + \Omega^{(2)} e^{-i[\Delta_2 t + \phi^{(2)}(t)]} \tilde{a}(e, p - hq_2, t) \right\} \sqrt{f(t)}/\Delta. \quad (A7) \]

Then the slow varying amplitudes of the ground sublevels

\[ a(m, p, t) = \tilde{a}(m, p, t) \exp \left[ i\Omega^{AC} \int_T^t dt' f(t') \right], \quad (A8) \]

evolve as

\[ i\hbar \partial_t a(e, p, t) = H_{cm} a(e, p, t) + \frac{\Omega^*}{2} \exp \left[ -i\delta (t - T) - i\phi(t) \right] a(g, p - \hbar k, t) f(t), \quad (A9a) \]

\[ i\hbar \partial_t a(g, p, t) = H_{cm} a(g, p, t) + \frac{\Omega}{2} \exp \left[ i\delta (t - T) + i\phi(t) \right] a(e, p + \hbar k, t) f(t), \quad (A9b) \]

where \( m = \{e, g\} \),

\[ k = q_1 - q_2 \quad (A10) \]

is the effective wave vector,

\[ \Omega = 2\Omega^{(1)}\Omega^{(2)*}/\Delta \quad (A11) \]

is the Rabi Raman frequency,

\[ \phi(t) = \tilde{\delta}T + \phi_1(t) - \phi_2(t), \quad (A12a) \]

\[ \delta = \tilde{\delta} - \delta_{\text{AC}}, \quad (A12b) \]

\[ \delta_{\text{AC}} = \frac{\Omega^{AC}}{t - T} \int_T^t dt' f(t'), \quad (A12c) \]

\[ \Omega^{AC} = \left| \Omega^{(2)} \right|^2/\Delta, \quad \Omega^{AC} = \left| \Omega^{(1)} \right|^2/\Delta. \quad (A12d) \]
Let us consider now the atomic density matrix in the Wigner representation

\[
\rho(x, p, t) = \int \frac{d\pi}{(2\pi \hbar)^6} \exp \left( \frac{i\pi x}{\hbar} \right) \rho \left( p + \frac{\pi}{2}, p - \frac{\pi}{2}, t \right), \quad (A13a)
\]

\[
= \int \frac{d\xi}{(2\pi \hbar)^3} \exp \left( -\frac{i\pi p \cdot \xi}{\hbar} \right) \rho \left( x + \frac{\xi}{2}, x - \frac{\xi}{2}, t \right), \quad (A13b)
\]

where

\[
\rho(p, p', t) = a(p, t) a^\dagger(p', t), \quad (A14a)
\]

\[
\rho(x, x', t) = a(x, t) a^\dagger(x', t) \quad (A14b)
\]

are the atomic density matrices in the momentum and coordinate representations. The time derivative of the density matrix consists of 2 parts,

\[
\frac{\partial}{\partial t} \rho(x, p, t) = [\partial_t \rho(x, p, t)]_{cm} + [\partial_t \rho(x, p, t)]_R, \quad (A15)
\]

associated with the Hamiltonian of the center-of-mass motion \((A3b)\) and with the Raman field. To calculate the first term, it is more convenient to use Eq. \((A13b)\), while for the second term, we used Eq. \((A13a)\). The first term in the Eq. \((A15)\) is given by \([22]\)

\[
[\partial_t \rho(x, p, t)]_{cm} = - \{H_{cm}(x, p), \rho(x, p, t)\} - Q \rho(x, p, t), \quad (A16)
\]

where \(\{H_{cm}, \rho\}\) is a Poisson brackets,

\[
\{H_{cm}(x, p), \rho(x, p, t)\} = \left[ \left( \frac{p}{M} + x \times \Omega_E \right) \cdot \partial_x + (p \times \Omega_E - \partial_x U) \cdot \partial_p \right] \rho(x, p, t) \quad (A17)
\]

and for the \(Q\)-term one gets

\[
Q = -\frac{1}{i\hbar} \left[ U \left( x + \frac{1}{2} \frac{i\hbar \partial_p}{\hbar} \right) - U \left( x - \frac{1}{2} \frac{i\hbar \partial_p}{\hbar} \right) \right] + \partial_x U \cdot \partial_p. \quad (A18)
\]

The relative weight of the \(Q\)-term is of the order of \(\{\text{see Eq. (16) in [60]}\} \frac{\phi_Q}{\phi_g} \sim \omega_k \frac{\hbar T^2}{12ML^2}\), where \(L\) is the size of the gravity source. If \(L\) is the Earth radius, then the weight is sufficiently small \(\frac{\phi_Q}{\phi_g} \sim 10^{-15}\) to neglect \(Q\)-term in further calculations. Calculating the term \([\partial_t \rho(x, p, t)]_R\), one arrives at the following equations

\[
\frac{\partial}{\partial t} \rho_{eg}(x, p, t) + \{H_{cm}(x, p), \rho_{eg}(x, p, t)\} = i\frac{\Omega}{2} \exp \left[ ik \cdot x - i\delta(t - T) - i\phi(t) \right] \times \left[ \rho_{ee} \left( x, p + \frac{\hbar k}{2}, t \right) - \rho_{gg} \left( x, p - \frac{\hbar k}{2}, t \right) \right] f(t), \quad (A19a)
\]

\[
\frac{\partial}{\partial t} \rho_{mm}(x, p, t) + \{H_{cm}(x, p), \rho_{mm}(x, p, t)\} = j_m \text{Re} \left\{ i\Omega^* \exp \left[ -ik \cdot x + i\delta(t - T) + i\phi(t) \right] \rho_{eg} \left( x, p - \frac{\hbar k}{2}, t \right) \right\} f(t), \quad (A19b)
\]

\[
\Omega \cdot e_\parallel = -j_y = 1. \quad (A19c)
\]

One can notice that Eqs. \((A19)\) comprise an inclosed system of 3 equations for 3 variables \(\{\rho_{eg}(x, p, t), \rho_{ee}(x, p + \frac{\hbar k}{2}, t), \rho_{gg}(x, p - \frac{\hbar k}{2}, t)\}\). Introducing "population difference" and "population sum"

\[
\{n(x, p, t), R(x, p, t)\} = \rho_{ee} \left( x, p + \frac{\hbar k}{2}, t \right) \mp \rho_{gg} \left( x, p - \frac{\hbar k}{2}, t \right), \quad (A20)
\]

one obtains, using Eq. \((A17)\),

\[
\frac{\partial}{\partial t} \rho_{eg}(x, p, t) + \{H_{cm}(x, p), \rho_{eg}(x, p, t)\} = i\frac{\Omega}{2} \exp \left[ ik \cdot x - i\delta(t - T) - i\phi(t) \right] f(t) n(x, p, t), \quad (A21a)
\]

\[
\frac{\partial}{\partial t} n(x, p, t) + \{H_{cm}(x, p), n(x, p, t)\} = 2 \text{Re} \left\{ i\Omega^* \exp \left[ -ik \cdot x + i\delta(t - T) + i\phi(t) \right] f^2(t) \rho_{eg}(x, p, t) \right\}, \quad (A21b)
\]

\[
\frac{\partial}{\partial t} R(x, p, t) + \{H_{cm}(x, p), R(x, p, t)\} = -\frac{\hbar}{2} \left[ \frac{k}{M} \partial_x + (k \times \Omega_E) \cdot \partial_p \right] R(x, p, t), \quad (A21c)
\]
If the effective wave vectors of the different Raman pulses are the same or sufficiently closed to each other, then atomic level populations have no terms rapidly oscillating in the phase space, and one can estimate gradients as \( \partial_x \sim 1/a \), \( \partial_p \sim 1/\bar{p} \), where \( a \) and \( \bar{p} \) are the atomic cloud size and the thermal momentum. Since \( \partial_t \sim 1/\tau \), the relative weights of the last term in the Eqs. (A21) are given by

\[
\varepsilon_x \sim \frac{\omega_k \tau}{k \hbar}, \quad \varepsilon_p \sim \frac{\hbar k}{\bar{p}} \Omega E \tau.
\]  

(A22)

Small parameters of the problem

\[
\lambda/a \ll 1, \quad \Omega E \tau \ll 1,
\]  

(A23)

where \( \lambda = 1/k \), allow us to neglect the last terms in the Eqs. (A21) even outside of the Raman-Nath approximation, \( \omega_k \tau \gtrsim 1 \), and even for the sub-recoil cloud temperature, \( \bar{p} \lesssim \hbar k \). After omitting those terms, one can look for the solution of the Eqs. (A21) in the atomic rest frame, i.e., consider the density matrix as a function of the atomic position and momentum \( \{x_r, p_r\} \) at the time \( t' \) preceding to the Raman pulse,

\[
\{x_r, p_r\} = \{X(x, p, t - t'), P(x, p, t' - t')\},
\]  

(A24)

where \( \{X(x, p, t), P(x, p, t)\} \) is an atom classical trajectory in the phase space subject to the initial condition \( \{X(x, p, 0), P(x, p, 0)\} = \{x, p\} \). The trajectory satisfies the multiplication laws

\[
X(X(x, p, T_1), P(x, p, T_1), T_2) = X(x, p, T_1 + T_2),
\]  

(A25a)

\[
P(X(x, p, T_1), P(x, p, T_1), T_2) = P(x, p, T_1 + T_2).
\]  

(A25b)

Consider the left-hand-side (lhs) of the Eqs. (A21),

\[
\left( \frac{\partial \zeta(x_r, p_r, t)}{\partial t} \right)_{\{x, p\}} + \{H_{cm}(x, p), \zeta(x, p, t)\},
\]  

(A26)

where \( \zeta \) is \( \rho_{eg} \), \( n \), or \( R \), and \( \frac{\partial \zeta(x_r, p_r, t)}{\partial t} \) is time derivative at \( \{x, p\} = \text{const.} \) Since

\[
\left( \frac{\partial \zeta(x_r, p_r, t)}{\partial t} \right)_{\{x, p\}} = \left( \frac{\partial \zeta(x_r, p_r, t)}{\partial t} \right)_{\{x_r, p_r\}} + \frac{\partial \zeta}{\partial x_{rn}} \frac{dX_n(x, p, t' - t)}{dt} + \frac{\partial \zeta}{\partial p_{rn}} \frac{dP_n(x, p, t' - t)}{dt}
\]  

(A27)

A summation convention implicit in Eq. (A27) will be used in all subsequent equations. Repeated indices and symbols appearing on the right-hand-side (rhs) of an equation are to be summed over, unless they also appear on the lhs of that equation. Since

\[
\frac{dX_n(x, p, t' - t)}{dt} = - \frac{dX_n(x, p, t' - t)}{dt'} = - \frac{\partial H(x_r, p_r)}{\partial p_{rn}}
\]  

(A28)

and

\[
\frac{dP_n(x, p, t' - t)}{dt} = - \frac{dP_n(x, p, t' - t)}{dt'} = \frac{\partial H(x_r, p_r)}{\partial x_{rn}},
\]  

(A29)

one gets

\[
\left( \frac{\partial \zeta(x_r, p_r, t)}{\partial t} \right)_{\{x, p\}} = \left( \frac{\partial \zeta(x_r, p_r, t)}{\partial t} \right)_{\{x_r, p_r\}} - \{H_{cm}(x_r, p_r), \zeta(x_r, p_r, t)\}.
\]  

(A30)

Substituting this result into Eq. (A26) and using an invariance of the Poisson brackets along an atom trajectory,

\[
\{H_{cm}(x, p), \zeta(x, p, t)\} = \{H_{cm}(x_r, p_r), \zeta(x_r, p_r, t)\},
\]  

(A31)

one concludes that

\[
\left( \frac{\partial \zeta(x_r, p_r, t)}{\partial t} \right)_{\{x, p\}} + \{H_{cm}(x, p), \zeta(x, p, t)\} = \left( \frac{\partial \zeta(x_r, p_r, t)}{\partial t} \right)_{\{x_r, p_r\}}.
\]  

(A32)
Since in the rest frame lhs of the Eqs. (A21) contains no derivatives over position \( \mathbf{x} \), or momentum \( \mathbf{p} \), one can consider \( \{ \mathbf{x}, \mathbf{p} \} \) simply as parameters and put \( (\partial \zeta (\mathbf{x}, \mathbf{p}, t) / \partial t) \{ \mathbf{x}, \mathbf{p} \} = \dot{\zeta} (\mathbf{x}, \mathbf{p}, t) \). It is convenient now to put \( t' \) to the moment of the cloud launching, \( t' = 0 \), so that

\[
\{ \mathbf{x}, \mathbf{p} \} = \{ \mathbf{X} (\mathbf{x}_r, \mathbf{p}_r, t), \mathbf{P} (\mathbf{x}_r, \mathbf{p}_r, t) \}
\]

(A33)

and the density matrix evolves as

\[
\dot{n} (\mathbf{x}_r, \mathbf{p}_r, t) = 2 \text{Re} \left\{ i \Omega^r e^{i \psi} \rho_{eg} (\mathbf{x}_r, \mathbf{p}_r, t) \right\} f (t),
\]

(A34a)

\[
\dot{\rho}_{eg} (\mathbf{x}_r, \mathbf{p}_r, t) = i \frac{\Omega}{2} e^{-i \psi} f (t) n (\mathbf{x}_r, \mathbf{p}_r, t),
\]

(A34b)

\[
\psi = \phi (t) + \delta (t - T) - \mathbf{k} \cdot \mathbf{X} (\mathbf{x}_r, \mathbf{p}_r, t),
\]

(A34c)

\[
\dot{\mathbf{R}} (\mathbf{x}_r, \mathbf{p}_r, t) = 0.
\]

(A34d)

One can seek the solution of these equations in the shape

\[
n (\mathbf{x}_r, \mathbf{p}_r, t) = |x|^2 - |y|^2,
\]

(A35a)

\[
\rho_{eg} (\mathbf{x}_r, \mathbf{p}_r, t) = xy',
\]

(A35b)

where \( x \) and \( y \) evolve as the amplitudes of two-level atom

\[
\left( \begin{array}{c}
x \\
y
\end{array} \right) = -i \frac{1}{2} \left( \begin{array}{cc}
0 & \Omega e^{-i \psi} \\
\Omega e^{i \psi} & 0
\end{array} \right) f (t) \left( \begin{array}{c}
x \\
y
\end{array} \right)
\]

(A36)

Let’s assume now that the pulse duration is much smaller than interrogation time \( T \),

\[
t - T \lesssim \tau \ll T.
\]

(A37)

For the atomic trajectory one finds expansion

\[
\mathbf{X} (\mathbf{x}_r, \mathbf{p}_r, t) \approx \mathbf{X} (\mathbf{x}_r, \mathbf{p}_r, T) + \left[ \frac{\mathbf{P}}{M} + \mathbf{X} \times \Omega_E \right] (t - T) + \frac{1}{2} \left[ -\frac{1}{M} \frac{\partial U (\mathbf{X})}{\partial \mathbf{X}} + 2 \frac{\mathbf{P}}{M} \times \Omega_E + \Omega_E \times (\Omega_E \times \mathbf{X}) \right] (t - T)^2
\]

(A38)

For the arbitrary pulse shape, one could not consider \( \delta \), defined in Eq. (A12b) as a permanent or slow varying inside Raman pulse. There are two exclusions, the rectangular pulse

\[
f (t) = \begin{cases}
1, & \text{at } T < t < T + \tau \\
0, & \text{at } t < T \text{ or } t > T + \tau
\end{cases}
\]

(A39)

and when the hyperfine sub-levels AC-Stark shifts (A12a) coincide.

Further calculations are performed for the rectangular pulse (A39). If one starts to chirp the Raman field at the moment of atoms’ launching, which means that

\[
\phi (t) = \phi + \delta T + \alpha t^2 / 2,
\]

(A40)

where \( \alpha \) is the chirping rate, and if quadratic in \( (t - T) \) terms in the rhs of Eq. (A38) are negligibly small,

\[
\left| \alpha - \mathbf{k} \left[ -\frac{1}{M} \frac{\partial U (\mathbf{X})}{\partial \mathbf{X}} + 2 \frac{\mathbf{P}}{M} \times \Omega_E + \Omega_E \times (\Omega_E \times \mathbf{X}) \right] \right| \tau^2 \ll 1,
\]

(A41)

then

\[
\psi = \phi (\mathbf{x}_r, \mathbf{p}_r, T) + \delta (\mathbf{x}_r, \mathbf{p}_r, T) (t - T),
\]

(A42a)

\[
\phi (\mathbf{x}_r, \mathbf{p}_r, T) = \phi + \delta T + \frac{\alpha}{2} T^2 - \mathbf{k} \cdot \mathbf{X} (\mathbf{x}_r, \mathbf{p}_r, T),
\]

(A42b)

\[
\delta (\mathbf{x}_r, \mathbf{p}_r, T) = \delta + \alpha T - \mathbf{k} \cdot \left( \frac{\mathbf{P} (\mathbf{x}_r, \mathbf{p}_r, T)}{M} + \mathbf{X} (\mathbf{x}_r, \mathbf{p}_r, T) \times \Omega_E \right)
\]

(A42c)
and Eq. (A30) becomes the equation for the two-level atom interacting with the resonant pulse. The solution of Eq. (A30) is well-known and given by

\[
\begin{pmatrix}
  x(T + \tau) \\
y(T + \tau)
\end{pmatrix} = F(x_r, p_r, T) \begin{pmatrix} x(T) \\
y(T) \end{pmatrix},
\]

(A43a)

\[
F(x_r, p_r, T) = \begin{pmatrix}
f_{ee}(x_r, p_r, T) \\
f_{eg}(x_r, p_r, T) \exp[-i\phi(x_r, p_r, T)] f_{gg}(x_r, p_r, T)
\end{pmatrix},
\]

(A43b)

\[
f(x_r, p_r, T) = \begin{pmatrix}
\exp[-i\delta(x_r, p_r, T) \tau / 2] f_a(\Omega \delta(x_r, p_r, T)) \\
-i \exp[i\delta(x_r, p_r, T) \tau / 2] f_a(\Omega \delta(x_r, p_r, T))
\end{pmatrix},
\]

(A43c)

\[
f_d(\Omega, \delta) = \cos \frac{\Omega r (\Omega, \delta) \tau}{2} + i \frac{\delta}{\Omega r (\Omega, \delta)} \sin \frac{\Omega r (\Omega, \delta) \tau}{2},
\]

(A43d)

\[
f_a(\Omega, \delta) = \frac{\Omega}{\Omega r (\Omega, \delta)} \sin \frac{\Omega r (\Omega, \delta) \tau}{2},
\]

(A43e)

\[
\Omega r (\Omega, \delta) = (\Omega^2 + \delta^2)^{1/2}.
\]

(A43f)

Using this solution one finds from Eqs. (A34d, A35)

\[
n(x_r, p_r, T + \tau) = \delta^2(x_r, p_r, T) + |\Omega|^2 \cos \Omega r [\Omega, \delta(x_r, p_r, T)] \tau n(x_r, p_r, T)
\]

(A44a)

\[
+ 4 \text{Re} \{i \exp[i\phi(x_r, p_r, T)] f_d(\Omega, \delta(x_r, p_r, T)) f_a(\Omega, \delta(x_r, p_r, T)) \rho_{eg}(x_r, p_r, T)\},
\]

(A44b)

\[
\rho_{eg}(x_r, p_r, T + \tau) = \exp[-i\delta(x_r, p_r, T) \tau / 2] \{i \exp[-i\delta(x_r, p_r, T) \tau / 2] f_a(\Omega, \delta(x_r, p_r, T)) n(x_r, p_r, T)
\]

(A44c)

\[
+ f_d^2(\Omega, \delta(x_r, p_r, T)) \rho_{eg}(x_r, p_r, T) + \exp[-2i\delta(x_r, p_r, T) \tau / 2] f_a(\Omega, \delta(x_r, p_r, T)) \rho_{eg}^* (x_r, p_r, T)\}
\]

Let us now return back to the lab-frame \(\{x, p\}\),

\[
\{x_r, p_r\} = \{x_{T+\tau}, p_{T+\tau}\},
\]

(A45a)

\[
\{x_t, p_t\} = \{x(x, p, -t), p(x, p, -t)\}.
\]

(A45b)

One should notice that replacing \(\zeta(x_r, p_r, T + \tau)\) with \(\xi(x, p, T + \tau)\), where \(\zeta\) is \(n\) or \(\rho_{eg}\), correct only at time \(T + \tau\). The \(\zeta(x_r, p_r, T)\) in the rhs of Eqs. (A44) should be replaced in the lab-frame with \(\xi(x', p', T)\), where

\[
\{x', p'\} = \{X(x_r, p_r, T), P(x_r, p_r, T)\}
\]

(A46)

\[
= \{X(x, p, -T - \tau), P(x, p, -T - \tau), T), P(x, p, -T - \tau), T)\}
\]

Using multiplication law (A25), one finds

\[
\{x', p'\} = \{x_r, p_r\},
\]

(A47)

where \(\{x_t, p_t\}\) is given by Eq. (A45b). This result has evident meaning, we are expressing the density matrix in the point \(\{x, p\}\) after the Raman pulse action (at the moment \(T + \tau\)) in terms of the density matrix before the Raman pulse (at the time \(T\)) in the point \(\{x', p'\}\), where the atom was an interval \(\tau\) before. Keeping this in mind, one gets

\[
n(x, p, T + \tau) = \frac{\delta^2(x_{T+\tau}, p_{T+\tau}) + |\Omega|^2 \cos \Omega r [\Omega, \delta(x_{T+\tau}, p_{T+\tau})] \tau n(x_r, p_r, T)
\]

(A47a)

\[
+ 4 \text{Re} \{i \exp[i\phi(x_{T+\tau}, p_{T+\tau})] f_d(\Omega, \delta(x_{T+\tau}, p_{T+\tau})) f_a(\Omega, \delta(x_{T+\tau}, p_{T+\tau})) \rho_{eg}(x_r, p_r, T)\},
\]

(A47b)

\[
\rho_{eg}(x, p, T + \tau) = \exp[-i\delta(x_{T+\tau}, p_{T+\tau}) + \delta_{AC} \tau] \{i \exp[-i\delta(x_{T+\tau}, p_{T+\tau}) + \delta_{AC} \tau] f_d(\Omega, \delta(x_{T+\tau}, p_{T+\tau})) n(x_r, p_r, T)
\]

(A47c)

\[
+ f_d^2(\Omega, \delta(x_{T+\tau}, p_{T+\tau})) \rho_{eg}(x_r, p_r, T) + \exp[-2i\delta(x_{T+\tau}, p_{T+\tau}) + \delta_{AC} \tau] f_a(\Omega, \delta(x_{T+\tau}, p_{T+\tau})) \rho_{eg}^* (x_r, p_r, T)\}
\]

where we took into account that if in the expression for the density matrix (A44a) one uses amplitudes \(\tilde{a}(m, p, t)\) instead of the amplitudes \(a(m, p, t)\), defined by Eq. (A3), then it leads only to the phase factor \(\exp(-i\delta_{AC} \tau)\) in the atomic coherence \(\rho_{eg}\), which we inserted in the Eq. (A47b) rhs.
Using the definitions of the "population difference and sum" Eq. (A20), one gets for the density matrix jump during an interaction with the Raman pulse

$$\rho_{mm}(x_+, p_+, T + \tau) = \sum_{m' = -g} \rho_{mm', \tau m}(x_+, p_+, T + \tau) + \rho_{eg mm}(x_+, p_+, T + \tau), \quad (A48a)$$

$$\rho_{eg}(x_+, p_+, T + \tau) = \sum_{m' = -g} \rho_{m'm', eg}(x_+, p_+, T + \tau) + \rho_{eg mm}(x_+, p_+, T + \tau) + \rho_{ge mm}(x_+, p_+, T + \tau), \quad (A48b)$$

Evidently $\rho_{\alpha\beta\gamma\delta}$ is a transformation of the matrix element $\rho_{\alpha\beta}$ in the input of the Raman pulse to the matrix element $\rho_{\gamma\delta}$ at the output of this pulse. For the transformations from the population $m'$ to population $m$ one finds

$$\rho_{m'm', \tau mm}(x_+, p_+, T + \tau) = \frac{1}{2} \left[ 1 + j_m j_{m'} \frac{\delta^2(x_{T+\tau}, p_{T+\tau}, T) + |\Omega|^2 \cos \Omega, [\Omega, \delta(x_{T+\tau}, p_{T+\tau}, T)] \tau}{\Omega^2 \delta(x_{T+\tau}, p_{T+\tau}, T)} \right] \times \rho_{m'm'}(x_-, p_-, T), \quad (A49a)$$

$$\{x_-, p_\} = \left\{ x_+, p + \frac{\hbar k}{2} \right\}. \quad (A49b)$$

The "coherence to population" transformation is given by

$$\rho_{eg mm}(x, p, T + \tau) = 2 j_m \text{Re} \{ i \exp[i \phi(x_{T+\tau}, p_{T+\tau}, T)] f_d[\Omega, \delta(x_{T+\tau}, p_{T+\tau}, T)] f^*_a[\Omega, \delta(x_{T+\tau}, p_{T+\tau}, T)] \times \rho_{eg}(x_-, p_-, T), \quad (A50a)$$

$$\{x_-, p_\} = \{ x_+, p \}. \quad (A50b)$$

The phase point $\{x_+, p_\}$ in the Eqs. (A49-A50) is defined as

$$\{x_+, p\} \equiv \left\{ X(x_+, p_+ - j_m \frac{\hbar k}{2}, -t) , P(x_+, p_+ - j_m \frac{\hbar k}{2}, -t) \right\}. \quad (A51)$$

For the transformation "population to coherence" one finds

$$\rho_{mm', eg}(x_+, p_+, T + \tau) = ij_m \exp\{ -i \phi(x_{T+\tau}, p_{T+\tau}, T) - i [\delta(x_{T+\tau}, p_{T+\tau}, T) + \delta_{AC}] \tau \} f_d[\Omega, \delta(x_{T+\tau}, p_{T+\tau}, T)] \times f^*_a[\Omega, \delta(x_{T+\tau}, p_{T+\tau}, T)] \rho_{mm}(x_-, p_-, T), \quad (A52a)$$

$$\{x_-, p_\} = \left\{ x_+, p + j_m \frac{\hbar k}{2} \right\}. \quad (A52b)$$

while "coherence to coherence" is equal to

$$\rho_{ge mm}(x_+, p_+, T + \tau) = \exp\{ -i [\delta(x_{T+\tau}, p_{T+\tau}, T) + \delta_{AC}] \tau \} f^2_d[\Omega, \delta(x_{T+\tau}, p_{T+\tau}, T)] \rho_{eg}(x_-, p_-, T). \quad (A53)$$

Finally the mirror transformation is given by

$$\rho_{ge mm}(x_+, p_+, T + \tau) = \exp\{ -2i \phi(x_{T+\tau}, p_{T+\tau}, T) - i [\delta(x_{T+\tau}, p_{T+\tau}, T) + \delta_{AC}] \tau \} f^2_d[\Omega, \delta(x_{T+\tau}, p_{T+\tau}, T)] \times f^*_{a}[\Omega, \delta(x_{T+\tau}, p_{T+\tau}, T)], \quad (A54a)$$

$$\{x_-, p_\} = \{ x_+, p \}. \quad (A54b)$$

The phase point $\{x_+, p\}$ in Eqs. (A52, A54) is differently defined,

$$\{x_+, p\} \equiv \left\{ X(x_+, p_+ - t) , P(x, p, -t) \right\}, \quad (A55)$$

and $j_m$ in the Eqs. (A49-A52) is given by Eq. (A19c).

**Appendix B: MZAI interference term**

Consider now an interaction of the atomic cloud with three Raman pulses shown in Fig. 1. Let's assume that only sublevel $g$ is initially populated.

$$\rho_{gg}(x_i, p_i, 0) = f(x_i, p_i), \quad \rho_{ee}(x_i, p_i, 0) = \rho_{eg}(x_i, p_i, 0) = 0, \quad (B1)$$
where \( f(x_i, p_i) \) is an atomic distribution over the initial position and momentum. The purpose of our calculations is the atomic population of the sublevel \( e \) at the exit of the third Raman pulse \( \rho_{ee}(x_{3+}, p_{3+}, T_3 + \tau_3) \), where \( \{x_{3+}, p_{3+}\} \) is atomic position and momentum after the third pulse. Density matrix evolution between the neighboring Raman pulses \( n + 1 \) and \( n \) is given by \[22\]

\[
\rho_{\alpha\beta} T_{n,n+1} \rho_{\alpha\beta} (x_{n-}, p_{n-}, T_n) = \rho_{\alpha\beta} (x_{(n-1)+}, p_{(n-1)+}, T_{n-1}) ,
\]

(B2a)

\[
\{x_{(n-1)+}, p_{(n-1)+}\} = \{X (x_{n-}, p_{n-}, -T_{n-1}), P (x_{n-}, p_{n-}, -T_{n-1})\}
\]

(B2b)

where

\[
T_{n,n+1} = T_n - T_{n-1} + \tau_{n-1}
\]

(B3)

\( \{x_{n\pm}, p_{n\pm}\} \) is an atomic point in the phase space before and after the pulse \( n \) action. Evidently, \( T_0 = \tau_0 = 0 \), \( \{x_0+, p_0+\} = \{x_i, p_i\} \).

There are three types of the processes contributing to an atom excitation, i.e. to the matrix element \( \rho_{ee}(x_{3+}, p_{3+}, T_3 + \tau_3) \):

1. when only atomic populations are transferred between pulses. These processes contribute to the signal background.

2. when the coherence is transferred only between 2 pulses. These processes are responsible for the Ramsey fringes \[68\]. They are used in atomic velocimetry \[13\] for time delay between pulses smaller than inverse Doppler width \( \omega_D = k\bar{p}/M \lesssim T^{-1} \), but for the large Doppler broadening (which we consider here)

\[
\omega_D T \gg 1
\]

(B5) these processes are washed out \[69\].

3. when one transfers the coherence \( \rho_{gg} \) between first and second pulses, mirrors this coherence into \( \rho_{eg} \) using second pulse, transfers the coherence \( \rho_{eg} \) between second and third pulse and probes it with third pulse. This process is responsible for the set of transients having a common nature, spin echo \[70\], photon echo \[71, 72\], optical Ramsey fringes \[73, 74\], atom interference \[1\].

The following four terms contribute to the background (type 1 processes)

\[
\rho_1 (x_{3+}, p_{3+}, T_3 + \tau_3) = \rho_{ggT_{1}ggT_{2}ggT_{3}ee} (x_{3+}, p_{3+}, T_3 + \tau_3),
\]

(B6a)

\[
\rho_2 (x_{3+}, p_{3+}, T_3 + \tau_3) = \rho_{ggT_{1}ggT_{2}ggT_{1}eeT_{3}ee} (x_{3+}, p_{3+}, T_3 + \tau_3),
\]

(B6b)

\[
\rho_3 (x_{3+}, p_{3+}, T_3 + \tau_3) = \rho_{ggT_{1}eeT_{2}eeT_{1}ggT_{3}ee} (x_{3+}, p_{3+}, T_3 + \tau_3),
\]

(B6c)

\[
\rho_4 (x_{3+}, p_{3+}, T_3 + \tau_3) = \rho_{ggT_{1}eeT_{1}eeT_{2}eeT_{3}ee} (x_{3+}, p_{3+}, T_3 + \tau_3),
\]

(B6d)

We are going to calculate terms \[B6\] elsewhere. But only one process corresponds to the atom interference (type 3 process)

\[
\rho_I (x_{3+}, p_{3+}, T_3 + \tau_3) = \rho_{ggT_{1}ggT_{1}ggT_{2}ggT_{3}ee} (x_{3+}, p_{3+}, T_3 + \tau_3),
\]

(B7)

which we calculate here. Let us build up this process step-by-step. From the Eqs. \[B2 - B4\] for \( n = 1 \), an atomic distribution on the sublevel \( g \) at the moment \( T_1 \) is given by

\[
\rho_{ggT_{1}gg} (x_{1-}, p_{1-}, T_1) = f (x_i, p_i),
\]

(B8a)

\[
\{x_i, p_i\} = \{X (x_{1-}, p_{1-}, -T_1), P (x_{1-}, p_{1-}, -T_1)\}.
\]

(B8b)

According to the Eqs. \[A48 - A55\], the first pulse transforms this term into the coherence

\[
\rho_{ggT_{1}ggT_{1}ee} (x_{1+}, p_{1+}, T_1 + \tau_1) = e^{i\phi_1} \left\{ x_i^{(1)} T_{1+\tau_1} P_{1+\tau_1}, f^*_{(1)} \left( \Omega_1, \delta_1 \left( x_i^{(1)} T_{1+\tau_1} P_{1+\tau_1}, T_1 \right) \right) \right\}
\]

\[
\times f^*_{(1)} \left[ \Omega_1, \delta_1 \left( x_i^{(1)} T_{1+\tau_1} P_{1+\tau_1}, T_1 \right) \right] f_{(1)} (x_i, p_i),
\]

(B9a)

\[
\{x_i^{(1)}, p_i^{(1)}\} = \{X (x_{1+}, p_{1+}, -t), P (x_{1+}, p_{1+}, -t)\},
\]

(B9b)

\[
\{x_{1-}, p_{1-}\} = \left\{ x_i^{(1)}, P_{1-}, T_1 = \frac{\hbar k}{2} \right\},
\]

(B9c)
where \( \{ x_i, p_i \} \) is given by Eq. (B81). According to Eqs. (B12), this coherence evolves between 1st and 2nd pulses to the term \( \rho_{ggT_2} \), coinciding with rhs of Eq. (B9a), but taken at the point

\[
\{ x_{1+}, p_{1+} \} = \{ X(x_{2-}, p_{2-}, -T_{21}), P(x_{2-}, p_{2-}, -T_{21}) \}.
\]

The 2nd pulse transforms into the coherence [see Eqs. (A54) A55]

\[
\rho_{ggT_1} = \exp \left\{ i \phi_1 \left( x_{1+}, p_{1+}, T_{1+} \right) + i \left[ \delta_1 \left( x_{1+}, p_{1+}, T_{1+} \right) + \delta_{AC1} \right] \tau_1 \right\}
\]

\[
-2i \phi_2 \left( x_{2+}, p_{2+}, T_{2+} \right) - i \left[ \delta_2 \left( x_{2+}, p_{2+}, T_{2+} \right) + \delta_{AC2} \right] \tau_2 \]

\[
f_a \left[ \Omega_1, \delta_1 \left( x_{1+}, p_{1+}, T_{1+} \right) \right] f_d \left[ \Omega_2, \delta_2 \left( x_{2+}, p_{2+}, T_{2+} \right) \right] f_c \left( x_i, p_i \right),
\]

\[
\{ x_{1-}, p_{1-} \} = \{ X(x_{3+}, p_{3+}, -T_{32}), P(x_{3+}, p_{3+}, -T_{32}) \}.
\]

Finally the 3rd pulse transforms coherence into the sublevel \( e \) population [see Eqs. (A45) A50] for \( m = e \) and definition (A19a)

\[
\rho_f \left( x_{3+}, p_{3+}, T_3 + \tau_3 \right) = -2 \text{Re} \left\{ \exp \left[ i \phi_1 \left( x_{1+}, p_{1+}, T_{1+} \right) \right] \right\}
\]

\[
\left[ \delta_1 \left( x_{1+}, p_{1+}, T_{1+} \right) + \delta_{AC1} \right] \tau_1 \right\}
\]

\[
-2i \phi_2 \left( x_{2+}, p_{2+}, T_{2+} \right) - i \left[ \delta_2 \left( x_{2+}, p_{2+}, T_{2+} \right) + \delta_{AC2} \right] \tau_2 \]

\[
f_a \left[ \Omega_1, \delta_1 \left( x_{1+}, p_{1+}, T_{1+} \right) \right] f_d \left[ \Omega_2, \delta_2 \left( x_{2+}, p_{2+}, T_{2+} \right) \right] f_c \left( x_i, p_i \right),
\]

\[
\{ x_{3-}, p_{3-} \} = \{ x_{3+}, p_{3+} \}.
\]

In the Eqs. (B13) the population is considered as a function of the finite atomic position and momentum \( \{ x_{3+}, p_{3+} \} \).

One can simplify the system of equations (B13 B12 B11 B10 B9b B9c B8c) considering the response as a function of the initial atomic position and momentum \( \{ x_i, p_i \} \). Resolving Eq. (B8) in respect to \( \{ x_{1-}, p_{1-} \} \), one gets

\[
\{ x_{1-}, p_{1-} \} = \{ X(x_i, p_i, T_1), P(x_i, p_i, T_1) \}.
\]

Since from Eqs. (B9c B9d)

\[
\{ x_{1-}, p_{1-} + \frac{\hbar k_1}{2} \} = \{ x_{3+}, p_{3+} \} = \{ X(x_{1+}, p_{1+}, -T_1), P(x_{1+}, p_{1+}, -T_1) \},
\]

one finds

\[
\{ x_{1+}, p_{1+} \} = \{ X(x_{1+}, p_{1+}, -T_1), P(x_{1+}, p_{1+}, -T_1) \}.
\]

In the same manner, applying the multiplication laws (A25), one gets consequently

\[
\{ x_{2-}, p_{2-} \} = \{ X(x_{1-}, p_{1-} + \frac{\hbar k_1}{2}, T_2 - T_1), P(x_{1-}, p_{1-} + \frac{\hbar k_1}{2}, T_2 - T_1) \},
\]

\[
\{ x_{2+}, p_{2+} \} = \{ X(x_{1+}, p_{1+} + \frac{\hbar k_1}{2}, T_2 + T_2 - T_1), P(x_{1+}, p_{1+} + \frac{\hbar k_1}{2}, T_2 + T_2 - T_1) \},
\]

\[
\{ x_{3-}, p_{3-} \} = \{ X(x_{1-}, p_{1-} + \frac{\hbar k_1}{2}, T_3 - T_1), P(x_{1-}, p_{1-} + \frac{\hbar k_1}{2}, T_3 - T_1) \},
\]

\[
\{ x_{3+}, p_{3+} \} = \{ X(x_{1+}, p_{1+} + \frac{\hbar k_1}{2}, T_3 + T_3 - T_1), P(x_{1+}, p_{1+} + \frac{\hbar k_1}{2}, T_3 + T_3 - T_1) \}.
\]
One can check that the points \(\{\xi, \pi\} = \{x_{1-1+1+1}^{n}, p_{1-1}^{(n)}\} \) [defined in Eqs. (B11b, B13b)] are the same for all three pulses \((n = 1, 2, 3)\), and therefore the interference term is given by

\[
\rho_{1}(x_{1}, p_{1}, T_{3} + \tau_{3}) = -2 \text{Re} \{\exp[\text{i}\phi_{1}(\xi, \pi, T_{1}) + \text{i}(\delta_{1}(\xi, \pi, T_{1}) + \delta_{AC1})] \tau_{1}
-2\text{i}\phi_{2}(\xi, \pi, T_{2}) - \text{i}(\delta_{2}(\xi, \pi, T_{2}) + \delta_{AC2}) \tau_{2} + \text{i}\phi_{3}(\xi, \pi, T_{3})] f_{a}[\Omega_{1}, \delta_{1}(\xi, \pi, T_{1})] f_{a}^{\ast}[\Omega_{2}, \delta_{2}(\xi, \pi, T_{2})] f_{a}^{\ast}[\Omega_{3}, \delta_{3}(\xi, \pi, T_{3})] f_{a}(x_{1}, p_{1})\},
\]

where \(\{x_{1-1, p_{1-1}}\}, \phi_{n}(\xi, \pi, T_{n}), \delta_{n}(\xi, \pi, T_{n}), f_{d}[\Omega, \delta], f_{a}[\Omega, \delta]\) are consequently given by Eqs. (A14, A12b, A12c, A43d, A43e).

**Appendix C: Uniform gravity field**

Expressions (B17, B14) one can use for any close to each other effective wave vectors and any atomic trajectories, including the exact expressions for the trajectories in the presence of the gravity, gravity-gradient, centrifugal and Coriolis forces on the Earth surface or in the moving platform, derived in [35]. Let us apply them for the simplest case of the uniform gravity field \(U(x) = -Mg \cdot x\), where \(g\) is the gravity acceleration, in the absence of rotation, when

\[
X(x, p, t) = x + \frac{p}{M}t + \frac{1}{2}gt^{2},
\]

\[
P(p, t) = p + Mgt;
\]

and for \(\{\xi, \pi\}\) using Eqs. (B17b, B14) one obtains

\[
\{\xi, \pi\} = \left\{x_{1} - \frac{h\xi_{1}}{2M}T_{1}, p_{1} + \frac{h\xi_{1}}{2}\right\}.
\]

Let us also assume that three Raman pulses have the same effective wave vectors \(k_{n} = k\). Then from the definitions (A42b, A42c) one concludes that the phases \(\phi_{n}(\xi, \pi, T_{n})\) and detunings \(\delta_{n}(\xi, \pi, T_{n})\) are given by

\[
\phi_{n}(\xi, \pi, T_{n}) = \delta_{n}T_{n} + \phi_{0} - \frac{1}{2}(k \cdot g - \alpha)T_{n}^{2} - k \cdot \left(x_{1} + \frac{p_{1}}{M}T_{n}\right) - \omega_{k}(T_{n} - T_{1}),
\]

\[
\delta_{n}(\xi, \pi, T_{n}) = \delta_{n}^{(0)} - (k \cdot g - \alpha)T_{n},
\]

\[
\delta_{n}^{(0)} = \delta_{n} - \frac{k \cdot p_{1}}{M} - \omega_{k},
\]

where \(\omega_{k} = h\kappa^{2}/2M\) is the recoil frequency.

Consider now the factor \(f_{d}[\Omega_{n}, \delta_{n}(\xi, \pi, T_{n})]\) given by Eq. (A43d). Expanding this factor over the small term \(|(k \cdot g - \alpha)T_{n}| \ll \max\left\{|\Omega_{n}|, |\delta_{n}^{(0)}|\right\}\) in the detuning (C20b) one gets

\[
f_{d}[\Omega_{n}, \delta_{n}(\xi, \pi, T_{n})] \approx f_{d}[\Omega_{n}, \delta_{n}^{(0)}] - f_{d}^{\prime}[\Omega_{n}, \delta_{n}^{(0)}](k \cdot g - \alpha)T_{n} + \frac{1}{2}f_{d}^{\prime\prime}[\Omega_{n}, \delta_{n}^{(0)}](k \cdot g - \alpha)^{2}T_{n}^{2},
\]

where

\[
f_{d}^{\prime}[\Omega, \delta] = -\frac{\delta_{d}[\Omega, \delta]}{2\Omega_{d}(\Omega, \delta)} \sqrt{\frac{2}{\Omega_{d}(\Omega, \delta)}} \left[\frac{\tau}{\Omega_{d}(\Omega, \delta)} \cos \frac{\Omega_{d}(\Omega, \delta) \tau}{2} - \frac{1}{\Omega_{d}^{2}(\Omega, \delta)} \sin \frac{\Omega_{d}(\Omega, \delta) \tau}{2}\right],
\]

and \(f_{d}^{\prime}\) are the 1st and 2nd derivatives of the factor \(f_{d}\) over detuning. It is well-known, and we will see it below, that the part of the MZAI phase associated with the gravitational field is of the order of \(\phi_{g} \sim (k \cdot g - \alpha)T_{n}^{2}\). Since \(f_{d}^{\prime} \sim \tau^{2}\), the last term in the expansion (C21) is of the order of \(|k \cdot g - \alpha| \tau^{2} \phi_{g}\) and we have to neglect it owing to the condition (A41). Thus for

\[
|k \cdot g - \alpha| \tau^{2} \ll 1
\]
one obtains

\[ f_d [\Omega_n, \delta_n (\xi, \pi, T_n)] \approx \left| f_d \left(\Omega_n, \delta_n^{(0)}\right) \right| \exp \left\{ i \arctan \left[ \frac{\delta_n^{(0)}}{\Omega_r \left(\Omega_n, \delta_n^{(0)}\right)} \tan \frac{\Omega_r \left(\Omega_n, \delta_n^{(0)}\right) \tau_n}{2} \right] \right\} \\
- i (k \cdot g - \alpha) T_n \text{ Im} \left[ \frac{f_d' \left(\Omega_n, \delta_n^{(0)}\right)}{f_d \left(\Omega_n, \delta_n^{(0)}\right)} \right], \quad (C24) \]

where we neglected the change of the magnitude of the factor \( f_d \) caused by the finite duration of the pulse. It is evident from the Eq. (A43a) that the factor \( f_a [\Omega_n, \delta_n (\xi, \pi, T_n)] \) does not contain any additional phase corrections and one can put

\[ f_a [\Omega_n, \delta_n (\xi, \pi, T_n)] \approx f_a \left[|\Omega_n|, \delta_n^{(0)}\right] \exp (i \arg \Omega_n). \quad (C25) \]

Substituting Eq. (C20a) for \( n = 1, 2, 3 \), Eq. (C20b) for \( n = 1, 2 \), Eq. (C24) for \( n = 1, 3 \), Eq. (C25) for \( n = 1, 2, 3 \), into the expression for interference term, Eq. (B17a) one arrives at the following general expression

\[ \rho_I (\mathbf{x}_i, \mathbf{p}_i, T_3 + \tau_3) = A \cos (\phi) f (\mathbf{x}_i, \mathbf{p}_i), \quad (C26a) \]
\[ A = -2 \left| f_d \left(\Omega_1, \delta_1^{(0)}\right) f_d \left(\Omega_3, \delta_3^{(0)}\right) \right| f_a \left[|\Omega_1|, \delta_1^{(0)}\right] f_a \left[|\Omega_2|, \delta_2^{(0)}\right] f_a \left[|\Omega_3|, \delta_3^{(0)}\right], \quad (C26b) \]

where the phase \( \phi \) consists of 4 different contributions

\[ \phi = \phi_D + \phi_q + \phi_g + \tilde{\phi}, \quad (C27) \]

the quantum and Doppler parts are given by

\[ \phi_D = \frac{k \cdot p_r}{M} (T_1 - 2T_2 + T_3), \quad (C28a) \]
\[ \phi_q = \omega_k (T_1 - 2T_2 + T_3), \quad (C28b) \]

the gravitational phase is given by

\[ \phi_g = (kg - \alpha) \left[ \frac{1}{2} \left( T_1^2 - 2T_2^2 + T_3^2 \right) + T_1 \tau_1 - T_2 \tau_2 + \varepsilon_d \right], \quad (C29a) \]
\[ \varepsilon_d = -T_1 \text{ Im} \left[ \frac{f_d' \left(\Omega_1, \delta_1^{(0)}\right)}{f_d \left(\Omega_1, \delta_1^{(0)}\right)} \right] + T_3 \text{ Im} \left[ \frac{f_d' \left(\Omega_3, \delta_3^{(0)}\right)}{f_d \left(\Omega_3, \delta_3^{(0)}\right)} \right], \quad (C29b) \]

while for the remaining part of the phase one finds

\[ \tilde{\phi} = -\phi_1 + \arg \Omega_1 + 2 (\phi_2 - \arg \Omega_2) - \phi_3 + \arg \Omega_3 - \tilde{\delta}_1 T_1 + 2 \tilde{\delta}_2 T_2 - \tilde{\delta}_3 T_3 - \left( \delta_1^{(0)} + \delta_{AC1} \right) T_1 + \left( \delta_2^{(0)} + \delta_{AC2} \right) T_2 \]
\[ + \text{arctan} \left[ \frac{\delta_1^{(0)}}{\Omega_r \left(\Omega_1, \delta_1^{(0)}\right)} \tan \frac{\Omega_r \left(\Omega_1, \delta_1^{(0)}\right) \tau_1}{2} \right] - \text{arctan} \left[ \frac{\delta_3^{(0)}}{\Omega_r \left(\Omega_3, \delta_3^{(0)}\right)} \tan \frac{\Omega_r \left(\Omega_3, \delta_3^{(0)}\right) \tau_3}{2} \right]. \quad (C30) \]

Terms (C28a) and (C28b) are new Doppler and quantum corrections to the MZAI phase caused by the pulses’ finite durations; Eq. (C29) is a generalization of the Eq. (1) \{derived in [33, 42, 43]\} for the arbitrary pulses durations, Rabi Raman frequencies and detunings. If following the articles [33, 42, 43] one defines the time delay between pulses as

\[ T = T_{21} = T_{32} \quad (C31) \]

or (see Fig. IIa)

\[ T_2 = T_1 + T + \tau_1, \quad (C32a) \]
\[ T_3 = T_1 + 2T + \tau_1 + \tau_2, \quad (C32b) \]
\[
\phi_D = k \frac{P_i}{M} (\tau_2 - \tau_1),
\]
\[
\phi_q = \omega k (\tau_2 - \tau_1),
\]
\[
\phi_g = (kg - \alpha) (T^2 + T\tau_2 + \varepsilon_d)
\]

(C33a)

(C33b)

(C33c)

In the case of the rectangular pulses, one can eliminate the Doppler and quantum phases just holding permanent the times between pulses triggering moments (see Fig. II)

\[
T = T_2 - T_1 = T_3 - T_2
\]

(C34)

When all three Raman pulses tuned on the exact two-quantum resonance \( \delta^{(0)}_n = 0 \), where \( \delta^{(0)}_n \) is given by Eq. (C20c) and according to the Eq. (A12b) \( \delta_n \) consists of Raman detuning \( \delta_n \) and AC-Stark shift \( \delta_{ACn} \), so that to reach exact resonance one has to choose

\[
\delta^{(0)}_n = k \frac{P_i}{M} + \omega k + \delta_{ACn}.
\]

(C35)

In this case one arrives to the well-known expression for the interference magnitude

\[
A = -\frac{1}{2} \sin \theta_1 \sin^2 \frac{\theta_2}{2} \sin \theta_3,
\]

(C36)

where

\[
\theta_n = |\Omega_n| \tau_n
\]

(C37)

is the pulse \( n \) area. Using that \( f_d(\Omega_n, 0) = \cos \theta_n / 2 \), \( f_d'(\Omega_n, 0) = i (\tau_n / \theta_n) \sin \theta_n / 2 \) [see Eqs. (A43d) (C22) for \( \delta = 0 \)] one gets from Eq. (C29b)

\[
\varepsilon_d = -T_1 \tau_1 \tan \frac{\theta_1}{2} + T_3 \tau_3 \tan \frac{\theta_3}{2}
\]

(C38)

One should consider separately cases (C31) and (C34). For the usual \( \pi / 2 - \pi - \pi / 2 \) pulses’ sequence and \( \tau - 2\tau - \tau \) pulses durations, in the 1st case one returns to the Eq. (1) derived in [33, 42, 43], while in the 2nd case, one arrives to the Eq. (10). For the equal pulses’ durations, \( \tau_1 = \tau_2 = \tau_3 = \tau \), and \( \theta_1 = \theta_3 = \pi / 2 \), in the case (C31), one arrives to the Eq. (11).

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