Ride comfort enhancement in railway vehicle by the reduction of the car body structural flexural vibration

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Abstract. The paper approaches the issue of reduction in the vertical bending vibrations of the railway vehicle carbody and the ride comfort enhancement at high velocities, starting from the prospect of isolating the vibrations by the best possible selection of the passive suspension damping in the vehicle. To this purpose, the examination falls on the influence of the vertical suspension damping upon the vibrations regime of the vehicle at the bending resonance frequency and upon the ride comfort. The results of the numerical simulations regarding the frequency response of the carbody acceleration and the comfort index will be therefore used. A value of the secondary suspension damping can be thus identified that will provide the best ride comfort performance. Similarly, the ride comfort can be increased by raising the primary suspension damping ratio.

1. Introduction
The ride comfort stands as an imperative criterion in assessing the dynamic performance of the railway vehicle, which depends on the level of vibrations to which the vehicle is subjected during running. While the general tendency in the railway transport is to increase velocities, the issue of maintaining the level of vibrations at an enforced value, required by the provision of the ride comfort, becomes increasingly important.

A decrease in the weight is a basic principle in designing the railway vehicles with the purpose of them reaching higher velocities with lower energy consumption. This weight reduction also triggers fewer ground vibrations and construction-cost saving [1]. In spite of the fact that the vehicle carbody becomes lighter, its flexibility rises, thus allowing an easy excitation of the carbody structural vibrations. A large number of theoretical and experimental studies have confirmed that, mainly for the high velocity vehicles, the level of vibrations in the carbody can be strongly influenced by the vibration flexible modes of the carbody [2 - 6]. In some cases, this carbody structural flexibility accounts for half of the perceived vibrations, the rest being due to rigid modes [7]. The structural vibrations cause carbody fatigue which affects the dynamic performances and service life of the vehicle [8].

Even though the carbody structural vibrations are quite complex [7], the greatest influence upon ride comfort comes from the first carbody natural bending mode [3, 6], whose frequency usually ranges from 6 to 12 Hz, an interval where the human body shows a higher sensitivity to vertical vibrations [9, 10]. For instance, in a study on the railway vehicle modelling for evaluating the dynamic behaviour in terms of the vibrating comfort, Diana et al. [3] takes into account 33 modes of the
carbody structural vibrations with frequencies within 5 and 20 Hz interval, with global and local mode shapes ranging from carbody bending, torsion, and roof, floor, side and walls vibration. The numerical-experimental comparisons show that the one which most significantly influences the carbody dynamic behaviour is the first bending mode. Tomioka and Takigami [11] found that the first mode of bending vibration of the carbodies of the Japanese high speed train Shinkansen has a natural frequency of 8.5 Hz which can negatively impact the comfort of the passengers.

The reduction in the vehicle carbody structural vibrations involves the increase in the carbody stiffness, for which a series of conventional measures can be taken [12]. One is to have the shortest possible carbody but this solution contradicts the intention to maximise the number of passengers in relation to the number of bogies, as an example. Another one aims the carbody cross-section, which should be as large as possible. However, the cross-section must not interfere with the prescribed carbody gauge profile. Furthermore, crosswind stability problems may arise, which is contradictory with the requirements on lightweight carbodies for higher speeds.

More concepts regarding the reduction of the carbody structural vibrations and enhancement of the ride comfort have already been introduced. They can be classified into two complementary approaches: vibration isolation approaches, which include passive, semi-active and active concepts and play the role to stop excitation propagation through the suspension and structural damping approaches (active or passive), aiming to reduce the carbody structure vibration amplitudes [13, 14].

Herein, the issue of reducing the vibrations of the carbody vertical bending and the enhancement of ride comfort is dealt with from the perspective of the isolation of the vibrations via the best possible selection of the damping of the vehicle passive suspension. The investigation is carried out by means of a vehicle model that includes a ‘flexible carbody’ model taking into account the carbody rigid modes – pitch and bounce - and its first flexible vibration mode – the symmetrical vertical bending. The model of the secondary suspension includes elements that contribute to the excitation of the carbody vertical bending - the vertical and pitch stiffness of secondary suspension and the stiffness of the longitudinal traction system between the carbody and the bogie. To identify the possibility to reduce the carbody bending vibrations and enhance the ride comfort, the results concerning the frequency response functions of the vehicle carbody and the comfort index from the numerical simulations are being looked at.

2. Railway vehicle model

In Figure 1 is presented the mechanical model of a railway vehicle running on a track with vertical irregularities described with the functions \( \eta_j \), with \( j = 1...4 \).

The vehicle carbody is represented by a free-free equivalent beam, a constant section and uniformly distributed mass, of Euler-Bernoulli type. The beam parameters are defined in dependence on the carbody’s, namely: \( L_c \) – beam/carbody length; \( \rho_c = m_c/L_c \) – beam mass per length unit, where \( m_c \) is the carbody mass; \( \mu \) - structural damping coefficient; \( EI \) – bending modulus, where \( E \) stands for the longitudinal modulus of elasticity, and \( I \) is the inertia moment of the beam’s transversal section. The carbody inertia moment is \( J_c \).

The distances \( l_{1,2} = L_c/2 \pm a_c \) set the position of the carbody bearings on the secondary suspension, where \( 2a_c \) is the carbody wheelbase. The dimension figure \( L_u \) defines the usable carbody length, namely the length provided with seating space.

The carbody displacement \( w_c(x, t) \) is the result of overlapping between the rigid modes of vibration – bounce \( z_c \) and pitch \( \theta_\zeta \) and the first natural mode of carbody structural vibration – vertical bending (symmetrical bending). This writes as:

\[
w_c(x, t) = z_c(t) + \left( x - \frac{L_c}{2} \right) \theta_\zeta(t) + X_c(x) T_c(t),
\]

where \( T_c(t) \) is the coordinate of the carbody vertical bending and \( X_c(x) \) represents the natural function of this vibration mode, described in the equation
\[ X_c(x) = \sin \beta_c x + \sinh \beta_c x - \frac{\sin \beta_c L_c - \sinh \beta_c L_c}{\cos \beta_c L_c - \cosh \beta_c L_c} (\cos \beta_c x + \cosh \beta_c x) \]  

(2)

with

\[ \beta_c = \sqrt[4]{\frac{\omega_c^2}{\rho_c^2 EI}} \]  

(3)

and

\[ \cos \beta_c L_c \cosh \beta_c L_c - 1 = 0 , \]  

(4)

where \( \omega_c \) is the natural pulsation of the carbody bending.

Each wheelbase bogie \( 2a_b \) is represented by a rigid body of mass \( m_b \) and inertia moment \( J_b \), which is able to perform two motions in a vertical plan, namely bounce \( (z_{b_i}) \) and pitch \( (\theta_{b_i}) \), for \( i = 1 \) and \( 2 \).

The secondary suspension is modelled using two Kelvin-Voigt systems; one of them is for vertical translation, with stiffness \( 2k_{zc} \) and damping constant \( 2c_{zc} \), and another one for rotation, with the pitch angular stiffness \( 2k_{\theta c} \) and damping constant \( 2c_{\theta c} \). The model of the secondary suspension also including a Kelvin-Voigt system is placed at the distance \( h_c \) from the carbody neutral axis and at \( h_b \) from the bogie centre of gravity and models the transmission system of the longitudinal forces between the carbody and bogie. This model has an elastic constant \( 2k_{xc} \) and a damping constant \( 2c_{xc} \). The primary suspension corresponding to an axle is modelled by a Kelvin-Voigt system, with the elastic constant \( 2k_{zb} \) and the damping constant \( 2c_{zb} \).
3. **Motion equations**

The motion equation of the carbody vehicle has the general form

\[ EI \frac{\partial^4 w_c(x,t)}{\partial x^4} + \mu \frac{\partial^5 w_c(x,t)}{\partial x^4 \partial t} + \rho_c \frac{\partial^2 w_c(x,t)}{\partial t^2} = \sum_{i=1}^{2} F_{cci}(x-l_i) + \sum_{i=1}^{2} (M_{ci} - h_i F_{cci}) \frac{d\delta(x-l_i)}{dx}, \]  

(5)

where \( \delta(.) \) is Dirac’s delta function and \( F_{cci}, F_{cci} \) and \( M_i \) stand for the forces and moments, respectively, due to the secondary suspension of bogie \( i \)

\[ F_{cci} = -2c_{ci} \left( \frac{\partial w_c(l_i,t)}{\partial t} - \dot{z}_{bi} \right) - 2k_{ci} \left[ w_c(l_i,t) - z_{bi} \right]; \]  

(6)

\[ F_{cci} = 2c_{ci} \left( \frac{ \partial^2 w_c(l_i,t) }{ \partial x \partial t } - \dot{h}_i \dot{\theta}_{bi} \right) + 2k_{ci} \left( \frac{ \partial w_c(l_i,t) }{ \partial x } + h_i \theta_{bi} \right); \]  

(7)

\[ M_{ci} = -2c_{ik} \left( \frac{ \partial^2 w_c(l_i,t) }{ \partial \theta_{bi} } - \dot{\theta}_{bi} \right) - 2k_{ik} \left[ \frac{ \partial w_c(l_i,t) }{ \partial \theta_{bi} } - \theta_{bi} \right]. \]  

(8)

The application of the modal analysis and the orthogonality property of the eigenfunctions of the carbody bending can help infer the bounce, pitch and vertical bending equations. Similarly, the notations below can be introduced, based on the symmetry properties of the eigenfunction \( X_c(x) \),

\[ X_c(l_1) = X_c(l_2) = \varepsilon; \]  

(9)

\[ \frac{dX_c(l_1)}{dx} = -\frac{dX_c(l_2)}{dx} = \lambda. \]  

(10)

The following carbody motion equations will be therefore reached at:

\[ m_c \ddot{z}_c + 2c_{cz} [2 \dot{z}_c + 2 \dot{\theta}_c - (\dot{z}_{b1} + \dot{z}_{b2})] + 2k_{cz} [2 z_c + 2 \theta_c - (z_{b1} + z_{b2})] = 0; \]  

(11)

\[ J_c \ddot{\theta}_c + 2c_{c\theta} [2 \dot{\theta}_c - (\dot{z}_{b1} - \dot{z}_{b2})] + 2k_{c\theta} [2 \theta_c - (\theta_{b1} - \theta_{b2})] + 
+ 2c_{xh} h_i [2 \dot{h}_i \dot{\theta}_c + h_i (\dot{\theta}_{b1} + \dot{\theta}_{b2})] + 2k_{xh} h_i [2 \theta_c + h_i (\dot{\theta}_{b1} + \dot{\theta}_{b2})] + 
+ 2k_{xh} [2 \dot{\theta}_c - (\dot{\theta}_{b1} + \dot{\theta}_{b2})] + 2k_{xh} [2 \theta_c - (\theta_{b1} + \theta_{b2})] = 0; \]  

(12)

\[ m_m \ddot{\chi} + c_m \dot{\chi} + k_m \chi + 2c_{c\chi} [2 \dot{\chi} + 2 \dot{\theta}_c - (\dot{z}_{b1} + \dot{z}_{b2})] + 2k_{c\chi} [2 \chi + 2 \theta_c - (z_{b1} + z_{b2})] + 
+ 2c_{xh} h_i [2 \dot{h}_i \chi + h_i (\dot{\theta}_{b1} - \dot{\theta}_{b2})] + 2k_{xh} h_i [2 \theta_c + h_i (\dot{\theta}_{b1} - \dot{\theta}_{b2})] + 
+ 2k_{xh} [2 \dot{\theta}_c - (\dot{\theta}_{b1} - \dot{\theta}_{b2})] + 2k_{xh} [2 \theta_c - (\theta_{b1} - \theta_{b2})] = 0, \]  

(13)

where \( k_{mc}, c_{mc} \) and \( m_{mc} \) are the carbody stiffness, damping and modal mass

\[ k_{mc} = EI \int_0^L \left( \frac{d^2 X_c}{dx^2} \right)^2 \, dx; \quad c_{mc} = \mu L \int_0^L \left( \frac{d^2 X_c}{dx^2} \right)^2 \, dx; \quad m_{mc} = \rho_c L \int_0^L X_c^2 \, dx. \]  

(14)

The equations describing the bounce and pitch motions of the bogies are

\[ m_{bi} \ddot{z}_{bi} = \sum_{j=2l-1}^{2l} F_{bji} - F_{cxi}, \text{ for } i = 1,2; \]  

(15)
\begin{align*}
J_b \ddot{\theta}_b &= a_p \sum_{j=2i-1}^{2i} (-1)^{j+1} F_{jbj} - h_b F_{scl}, \text{ for } i = 1, 2; \\
\end{align*}

where $F_{jbj}$ stands for the forces due to the primary suspension, given in the relations below

\begin{align*}
F_{jb1,2} &= -2c_{jb}(\dot{z}_{b1} + a_b \dot{\theta}_b - \eta_{1,2}) - 2k_{jb}(z_{b1} + a_b \theta_b - \eta_{1,2}); \\
F_{jb3,4} &= -2c_{jb}(\dot{z}_{b2} - a_b \dot{\theta}_b - \eta_{3,4}) - 2k_{jb}(z_{b2} - a_b \theta_b - \eta_{3,4});
\end{align*}

The motion equations of the bogies write as such:

- for the front bogie:

\begin{align*}
J_b \ddot{z}_{b1} + 2c_{zc}(\dot{z}_{b1} - (\eta_1 + \eta_2)) + 2k_{zc}(z_{b1} - (\eta_1 + \eta_2)) + \\
+ 2c_{xc}(\dot{z}_{b1} - z_c + a_c \dot{\theta}_c - \varepsilon T_c) + 2k_{xc}(z_{b1} - z_c + a_c \theta_c - \varepsilon T_c) = 0;
\end{align*}

\begin{align*}
J_b \ddot{\theta}_b + 2c_{ct}(\dot{\theta}_b - (\eta_3 - \eta_4)) + 2k_{ct}(\theta_b - (\eta_3 - \eta_4)) + \\
+ 2c_{ct}(\dot{\theta}_b - z_c + a_c \dot{\theta}_c - \varepsilon T_c) + 2k_{ct}(\theta_b - z_c + a_c \theta_c - \varepsilon T_c) = 0;
\end{align*}

- for the rear bogie:

\begin{align*}
J_b \ddot{z}_{b2} + 2c_{zb}(\dot{z}_{b2} - (\eta_3 + \eta_4)) + 2k_{zb}(z_{b2} - (\eta_3 + \eta_4)) + \\
+ 2c_{zc}(\dot{z}_{b2} - z_c + a_c \dot{\theta}_c - \varepsilon T_c) + 2k_{zc}(z_{b2} - z_c + a_c \theta_c - \varepsilon T_c) = 0;
\end{align*}

\begin{align*}
J_b \ddot{\theta}_b + 2c_{zb}(\dot{\theta}_b - (\eta_3 - \eta_4)) + 2k_{zb}(\theta_b - (\eta_3 - \eta_4)) + \\
+ 2c_{ct}(\dot{\theta}_b - z_c + a_c \dot{\theta}_c - \varepsilon T_c) + 2k_{ct}(\theta_b - z_c + a_c \theta_c - \varepsilon T_c) = 0.
\end{align*}

The system of the equations (11) – (13) and (19) – (22) can be matrix-like written

\begin{equation}
M \ddot{\mathbf{z}} + C \dot{\mathbf{z}} + K \mathbf{z} = \mathbf{F},
\end{equation}

where $M$, $C$ and $K$ are the inertia, damping and stiffness matrices, $\mathbf{z} = [z_c \theta_c T_c \dot{z}_{b1} \theta_{b1} \dot{z}_{b2} \theta_{b2} \dot{z}_{b3} \theta_{b3} \dot{z}_{b4} \theta_{b4}]^T$ represents the vector of the displacements’ coordinates and $\mathbf{F}$ is the vector of the heterogeneous terms.

4. The frequency response functions

To calculate the frequency response functions, the track vertical irregularities are considered to be in a harmonic form with the wavelength $\Lambda$ and amplitude $\eta_0$, being described by the functions

\begin{equation}
\eta_{1,2}(t) = \eta_0 \cos \omega \left(t + \frac{a_c \pm a_b}{V}\right); \quad \eta_{3,4}(t) = \eta_0 \cos \omega \left(t - \frac{a_c \mp a_b}{V}\right),
\end{equation}

where $\omega = 2\pi V/\Lambda$ is the pulsation coming from the track excitation.

Regarding the vehicle response, this is also supposed to be harmonic and with the same frequency as the one induced by the track excitation. Under such conditions, the coordinates describing the vehicle motions can have the general form

\begin{equation}
p_k(t) = P_k \cos(\omega t + \varphi_k), \text{ with } k = 1 \div 7,
\end{equation}

in which $P_k$ is the movement amplitude and $\varphi_k$ stands for the displacement of the coordinate $k$ in relation to the track vertical irregularities against the vehicle center.

Further on, the complex values associated with the real ones are inserted into the system of equations (23)
\[ \bar{p}_j(t) = \bar{p}_j e^{i\omega t}, \quad \text{for } j = 1 \div 4, \]  
\[ \bar{p}_k(t) = \bar{P}_k e^{i\omega t}, \quad \text{for } k = 1 \div 7. \]  

And a linear system of heterogeneous algebraic equations is thus obtained
\[ (-\omega^2 \mathbf{M} + \mathbf{A}) \mathbf{P} = \eta_0 \mathbf{B}, \]  
where:
\[ \mathbf{P} = \mathbf{P}(\omega) = [\bar{P}_1 \ \bar{P}_2 \ \ldots \ \bar{P}_7]^T; \]
\[ \mathbf{M} = \text{diag}(m_c, J_c, m_{mc}, m_b, J_b, m_b, J_b); \]
\[ \mathbf{A} = \begin{bmatrix}
2\alpha_{zc} & 0 & 2\varepsilon\alpha_{zc} & -\alpha_{zc} & 0 & -\alpha_{zc} & 0 \\
0 & A_1 & 0 & -\alpha_{zc} & A_3 & -\alpha_{zc} & A_3 \\
2\varepsilon\alpha_{zc} & A_2 & 0 & -\varepsilon\alpha_{zc} & \lambda C_3 & -\varepsilon\alpha_{zc} & -\lambda A_3 \\
-\alpha_{zc} & -\alpha_{zc} & \varepsilon\alpha_{zc} & 2\alpha_{zb} + \alpha_{zc} & 0 & 0 & 0 \\
0 & A_3 & \lambda A_3 & 0 & A_4 & 0 & 0 \\
-\alpha_{zc} & \alpha_{zc} & \varepsilon\alpha_{zc} & 0 & 0 & 2\alpha_{zb} + \alpha_{zc} & 0 \\
0 & A_3 & -\lambda A_3 & 0 & 0 & 0 & A_4
\end{bmatrix}; \]
\[ \mathbf{B} = 2\alpha_{zb} \begin{bmatrix}
\exp(i\omega t / V) \cos(\omega t / V) \\
\exp(i\omega t / V) \sin(\omega t / V) \\
\exp(-i\omega t / V) \cos(\omega t / V) \\
\exp(-i\omega t / V) \sin(\omega t / V)
\end{bmatrix}. \]

The below notations have been introduced in the earlier equations
\[ \alpha_{zc} = 2(i\omega_{zc} + k_{zc}); \quad \alpha_{xc} = 2(i\omega_{xc} + k_{xc}); \quad \alpha_{th} = 2(i\omega_{th} + k_{th}); \]
\[ \alpha_{mc} = i\omega_{mc} + k_{mc} ; \quad \alpha_{j} = 2(i\omega_{j} + k_{j}); \]
\[ A_1 = 2\alpha_{zc}^2 + 2h_{c}^2\alpha_{xc} + 2\alpha_{dk}; \quad A_2 = \alpha_{mc} + 2e^2\alpha_{zc} + 2h_{c}^2\alpha_{xc} + 2\lambda^2\alpha_{dk}; \]
\[ A_3 = h_{d}h_{b}\alpha_{xc} - \alpha_{dk}; \quad A_4 = 2a_{zb}^2\alpha_{j} + h_{c}^2\alpha_{xc} + \alpha_{dk}. \]

The answer to the system of equations (28) allows the calculation of the vehicle frequency response functions. The response function of the carbody movement in a random point \( x \) located on the longitudinal axis of the carbody going through its centre of gravity is thus
\[ H_c(x, \omega) = H_{zc}(\omega) + \left( \frac{L}{2} - x \right) H_{\theta c}(\omega) + X_c(x) H_{T c}(\omega), \]
\[ H_{zc}(\omega) = \frac{\bar{P}_c(\omega)}{\bar{n}_0}; \quad H_{\theta c}(\omega) = \frac{\bar{P}_c(\omega)}{\bar{n}_0}; \quad H_{T c}(\omega) = \frac{\bar{P}_3(\omega)}{\bar{n}_0}. \]  

are the response functions pertinent to the vibration modes of the carbody.

Similarly, the response function of the carbody acceleration can be calculated via the relation
\[
\bar{H}_{ac}(x, \omega) = \omega^2 \bar{H}_c(x, \omega).
\]  

Based on the frequency response functions in a permanent regime of vibrations, the power spectral density of the carbody acceleration can be further calculated

\[
G_{ac}(x, \omega) = G(\omega) \left| \bar{H}_{ac}(x, \omega) \right|^2,
\]

where \(G(\omega)\) represent the power spectral density of the track vertical irregularities [16].

5. Ride comfort evaluation

Ride comfort in the vertical direction is evaluated by the partial comfort index [9]

\[
N_{MV} = 6a^{W_{ab}},
\]

where

\[
a(x) = \frac{1}{\pi} \int_0^\infty G_{ac}(x, \omega) d\omega,
\]

is the root mean square of the vertical acceleration and \(W_{ab} = W_a W_b\) represents the weight filter of the accelerations in the vertical direction [10].

When adopting the hypothesis that the vertical accelerations have a Gaussian distribution with the null mean value, we will have the following relation to calculate the partial comfort index

\[
N_{MV}(x) = \sqrt{\frac{1}{\pi} \int_0^\infty G_{ac}(x, \omega) |H_{ab}(\omega)|^2 d\omega},
\]

where \(H_{ab}(\omega) = H_a(\omega) H_b(\omega)\), where \(H_a(\omega)\) and \(H_b(\omega)\) are the transfer functions corresponding to the filters \(W_a\) and \(W_b\) [10].

6. Numerical analysis

This section investigates the possibilities of reducing the carbody vertical bending vibrations and improving the ride comfort by the best selection vertical suspension damping of the vehicle. To this end, the results about the frequency response functions of the vehicle and the comfort index derived from numerical simulations are being used.

| Table 1. The parameters of the vehicle’s numerical model. |
|----------------------------------------------------------|
| Parameter       | Value                              |
|-----------------|------------------------------------|
| \(m_c\)         | 34.000 kg                          |
| \(m_b\)         | 3.200 kg                           |
| \(J_c\)         | 1.963.840 kg·m²                   |
| \(J_b\)         | 2.048 kg·m²                       |
| \(EI\)          | 3.158·10^9 Nm²                    |
| \(m_{a2}\)      | 35.224 kg                          |
| \(m_{a3}\)      | 33.950 kg                          |
| \(L_c\)         | 26.4 m; \(L_a\) = 21 m            |
| \(h_c\)         | 1.3 m; \(h_b\) = 0.2 m            |
| \(k_{ac}\)      | 1.2 MN/m                           |
| \(k_{bc}\)      | 4 MN/m                            |
| \(k_{ac}\)      | 256 kN/m                           |
| \(k_{bc}\)      | 34.28 kN/m                        |
| \(k_{ac}\)      | 50 kN/m                            |
| \(k_{ab}\)      | 2 kN/m                            |
| \(k_{na2}\)     | 88.998 MN/m                       |
| \(c_{na2}\)     | 53.117 kN/m/s                     |
| \(4k_{z}\)      | 4.4 MN/m                          |
| \(4k_{zb}\)     | 52.21 kN/m                        |
For a simpler analysis, the damping ratios corresponding to the two levels of suspension will be brought in, as below

\[
\zeta_c = \frac{4c_{zc}}{2\sqrt{4k_{zc}m_c}}, \quad \zeta_b = \frac{4c_{zb}}{2\sqrt{4k_{zb}m_b}}.
\]

(40)

The reference parameters of the numerical model are featured in table 1, according to which the eigenfrequency of the carbody vertical bending is 8 Hz and the damping ratios of the suspension have the values of \(\zeta_c = 0.12\) and \(\zeta_b = 0.22\).

The usable carbody length \((L_u)\) will be taken into account for the analysis of the carbody behavior of vibrations and the evaluation of comfort for the vertical vibrations.

Figure 2 shows the acceleration response function calculated at the carbody bending frequency for its entire usable length, for velocities of up to 300 km/h. The level of vibrations coming from bending can be noticed being non-uniform along the carbody. The acceleration response function rises at the carbody ends (above the bogies) and at its centre. On the other hand, the carbody regime of vibrations does not have a constant intensification along with a higher speed, due to the geometric filtering effect and to the bogie wheelbase. This effect has a selective nature, depending on velocity – the higher the effect, the lower the speed. The interval between two consecutive filtering speeds is larger [6] when velocity increases.

![Figure 2. The response function of the carbody acceleration.](image1)

![Figure 3. Acceleration power spectral density.](image2)
Figure 3 features the power spectral density of the carbody acceleration calculated at the carbody bending frequency. The power spectral density of acceleration becomes significant at speeds higher than 200 km/h, unlike the response function of the carbody acceleration (see figure 1) with important values at low speeds, as well. The explanation lies in the fact that the excitation power of the track irregularities increases along with the velocity [17]. In spite of that, the power spectral density of acceleration maintains a similar tendency with the acceleration response function, namely a higher value above the bogies and in its centre.

Figure 4 (a) shows the power spectral density of the carbody acceleration calculated at speed of 300 km/h and the carbody bending frequency for various values of the secondary suspension damping ratio within 0.01 and 0.5. For the usable carbody length, the power spectral density of acceleration is noticed to decrease along with the increase of the secondary suspension damping ratio up to a certain value. Beyond that value, the power spectral density of acceleration starts going up again. Consequently, a value of the secondary suspension damping ratio can be identified and it minimizes the level of carbody vibrations at the bending resonance frequency. Figure 4 (b) shows the power spectral density of carbody acceleration calculated at the same speed of 300 km/h and the carbody bending frequency, while taking into account different values of the primary suspension damping ratio, from 0.01 to 0.5. In this case, the power spectral density of acceleration constantly decreases with the raise of the primary suspension damping ratio. The decrease is more visible for the small dampings and less sensitive should the primary suspension damping ratio rise over 0.2.

![Figure 4](image1.png)

**Figure 4.** Influence of vertical suspension damping upon the carbody regime of vibrations: (a) influence of secondary suspension damping; (b) influence of primary suspension damping.

![Figure 5](image2.png)

**Figure 5.** Velocity influence upon comfort index.
Figure 5 features the comfort index for the usable carbody length for velocities of up to 300 km/h. The comfort index increases along with velocity and the highest index is to be found above the bogies, irrespective of velocity. The higher the speed, the lower the comfort at also the carbody centre.

The diagram (a) in figure 6 shows the influence of the secondary suspension damping upon the ride comfort, whereas diagram (b) is about the influence of the primary secondary suspension upon the ride comfort, both for speed of 300 km/h. A function of the damping ratio of the two suspension levels, the comfort index shares characteristics with the power spectral density of the carbody acceleration. The comfort index can be lowered by increasing the secondary suspension damping to a certain value and the comfort index has a constant decrease with the raise of the primary suspension damping ratio.

![Figure 6](image_url)

**Figure 6.** Influence of vertical suspension damping upon comfort index:
(a) influence of damping upon secondary suspension;
(b) influence of damping upon primary suspension.

Figure 7 shows the minim values of the comfort index and the secondary suspension damping ratio that helps with deriving these values, in different points along the carbody, at velocity of 300 km/h. Due to the non-uniform regime of vibrations along the carbody, the damping ratio required to minimize the comfort index varies from 0.08 to 0.23. To select the best value for the secondary suspension damping, there will be introduced the concept of critical point in the carbody level of vibrations as being the point where the comfort index reaches its peak [16]. Consequently, the requirement of minimizing the comfort index in the carbody critical point will be the starting point in calculating the best value of the secondary suspension damping. For instance, the highest level of vibrations is recorded above the rear bogie of the carbody and the damping ratio that minimizes the comfort index here is 0.14.

![Figure 7](image_url)

**Figure 7.** Minim comfort index.
It should be mentioned that the selection of the best damping involves an exact knowledge of the change in the critical point position, due to the high sensitivity of the comfort index compared to the suspension damping and velocity [16].

7. Conclusions
The paper herein investigates the possibility to reduce the vertical bending vibrations of the carbody and improve the ride comfort by the best possible selection of the passive suspension damping of the railway vehicle. The analysis is based on the results from the numerical simulations developed on a vehicle model that includes a ‘flexible carbody’ model, as well as on the elements contributing to the excitation of the symmetrical mode of the carbody vertical bending. It is about the fact that the model of the secondary suspension has been included into the vertical and pitch stiffness and the stiffness of the longitudinal traction system between the carbody and the bogie.

The influence of the vertical suspension damping upon the vibrations regime of the vehicle carbody at the resonance frequency of the carbody bending has been examined, by using the frequency response functions in a permanent harmonic and random behaviour of vibration. As a principle, the vehicle regime of vibrations intensifies along with the speed increase and the level of vibrations is much higher above the bogies and at the carbody centre. As for the influence of the vertical suspension damping, a value of the secondary suspension damping ratio can be identified that will minimizes the level of carbody vibrations at the bending resonance frequency. On the other hand, it has been shown that the level of carbody vibrations can be reduced by increasing the primary suspension damping ratio. The analysis concerning the influence of the vertical suspension damping upon the ride comfort has also proved the same trend for the comfort index. Hence, the comfort index can be minimized in any point along the carbody for a certain value of the secondary suspension ratio or it can be lowered by increasing the primary suspension damping ratio. Nevertheless, due to the non-uniform behaviour of vibrations along the carbody, there will be issues related to the adopting the secondary suspension damping ratio required for the minimizing of the comfort index.

To this purpose, it is recommended to introduce the concept of the critical point of the carbody level of vibrations and to establish the best secondary suspension damping ratio in the requirement of minimizing the comfort index in the carbody critical point.

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