General analysis of $B$ decays to two pseudoscalars for EWP, rescattering and color suppression effects

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Abstract: A general analysis for $B$ decays to two pseudoscalars, involving ten modes, is presented. A simple model for final state interactions and rescattering effects is proposed. We show how the data can be used to deduce important information on electroweak penguins (EWP), rescattering and color suppression effects and on the CKM parameters in a largely model independent way by using $\chi^2$-minimization. We find that the current data suggests the presence of color-suppressed tree at levels somewhat larger than simple theoretical estimates. Once the data improves the extraction of $\alpha$ and/or $\gamma$ may become feasible with this method as we illustrate with the existing data.

The recent CLEO results [1, 2] of $B$ mesons decaying into two pseudoscalars are significant in that they give strong evidence of QCD penguin processes, in particular in the instance of $K\pi$ final states where penguins are thought to dominate. Of course in the amplitude to $K\pi$ final states, the tree decay $b \to u\pi s$ will also give a contribution which will interfere with the penguin decays.

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Evidence for the influence of the tree graph may be found by considering the ratio:

\[ R_K = \frac{Br(\bar{B}^0 \to K^0\pi^0)}{Br(\bar{B}^- \to K^0\pi^-)} \]

This has the value \( R_K \approx 0.81\pm0.35 \) where the expectation if only penguin processes were involved would be \( 1/2 \). Clearly, no firm conclusion can be drawn from this central value due to the large error; however, since the color allowed tree does not contribute to either the numerator or denominator of this ratio, it does seem to suggest that the color suppressed tree (CST), rescattered tree (RST) and/or electroweak penguins (EWP) may play an important role.

These recent experimental findings are attracting considerable theoretical attention [3, 4, 5, 6]. The goal of this Letter is to propose a simple model which includes all of the features of the standard model and can be fit to current and future data. This will point to the possible interpretation of the current data and point out patterns that should be examined in future experiments. Needless to say it is very important to acquire a quantitative understanding of the role of these individual contributions such as CST, EWP and RST as they enter in an intricate manner in our ability to deduce the CKM parameters from the experimental data and to test the Standard Model (SM) through the use of the unitarity triangle for the presence of new physics—a goal of unquestionable importance.

Bearing all that in mind and also that large increases in the data sample are expected in the near future we will attempt to construct a general framework for analysis of these and related issues. We want to focus on the ten modes of \( B \) decays to two pseudoscalars, as these are the simplest, for which CLEO has already presented some data [1, 2]: 1) \( K^-\pi^0 \), 2) \( K^0\pi^- \), 3) \( K^0\pi^0 \), 4) \( K^-\pi^+ \), 5) \( \pi^-\pi^0 \), 6) \( \pi^+\pi^- \), 7) \( \pi^0\pi^0 \), 8) \( K^-K^0 \), 9) \( K^+K^- \), 10) \( K^0\bar{K}^0 \).

The amplitudes \( M_i \) for \( i = 1 \ldots 10 \) for these reactions receive contributions from four Standard Model processes: Tree (i.e. the color allowed piece), final state interactions (FSI) or rescattered tree (RST), color suppressed tree (CST), penguin and EWP. As is well known, at present no first principles method exists for calculating final state interactions or rescattering effects. We propose a simple model for that purpose. We propose that all FS rescattering effects are due to quark level reactions of the type \( u\bar{u} \to u\bar{u}, d\bar{d}, s\bar{s} \) etc. The characteristic strength of such a conversion, presumably of some
non-perturbative origin, will be denoted by a complex parameter $\kappa$ which, for simplicity, is taken to respect exact SU(3) flavor symmetry.

For each of the ten reactions we first decompose the amplitudes in terms of the quark level processes:

$$M_i = V_{tq}^* V_{tb} P_i + V_{uq}^* V_{ub} T_i + V_{uq}^* V_{ub} \hat{T}_i + V_{tq}^* V_{tb} E_i$$  \hspace{1cm} (2)$$

where $P_i$ is the penguin contribution to the amplitude, $T_i$ is the tree, $\hat{T}_i$ is the color suppressed tree, and $E_i$ the electro-weak penguin. In the above, $q = s$ in the case of final states which carry one unit of net strangeness (i.e. $K\pi$) and $q = d$ in the cases with no net strangeness ($\pi\pi$ and $KK$). Note also that we have used unitarity to eliminate terms proportional to $V_{cq}^* V_{cb}$. In this notation, therefore, the difference between graphs with an internal $u$-quark and an internal $c$-quark contributes to what we designate the tree. This causes no trouble since such contributions have SU(3) properties consistent with part of the tree amplitude and this notation is consistent with what is usually taken as tree-like processes [7].

Using the conventions of [8], SU(3) provides the following relationships between these amplitudes:

\begin{align*}
    P_1 &= -P_2/\sqrt{2} = -P_3 = P_4/\sqrt{2}; \quad P_8 = -P_1/\sqrt{2} \\
    P_9 &= \sqrt{2}P_1 + P_6; \quad P_5 = 0; \quad P_6 = -\sqrt{2}P_7 = -P_{10} \\
    T_3 &= T_1 + T_2/\sqrt{2} - T_4/\sqrt{2}; \quad T_5 = T_1 + T_2/\sqrt{2}; \\
    T_7 &= T_1 + T_2/\sqrt{2} - T_6/\sqrt{2}; \quad T_8 = T_2; \quad T_9 = -T_4 + T_6; \quad (3) 
\end{align*}

Note that the relations given above for $T_i$ apply also to $\hat{T}_i$ and $E_i$. To specify our model, we therefore need give the expressions only for $\{P_1, P_6\}$ and $\{T_1, T_2, T_4, T_6, T_{10}\}$ (and likewise for $\hat{T}$ and $E$) whence the other terms are determined.

Let us now introduce the quantities $p, t, \hat{t}, e$ as the basic penguin, tree, color suppressed tree and EWP amplitudes at the quark level. In our rescattering model, the meson decay amplitudes can be expressed as:

$$P_1 = (1 + 3\kappa)p/\sqrt{2}; \quad P_6 = (1 + 5\kappa)p;$$
\[ T_1 = (1 + \kappa)t/\sqrt{2}; \quad T_2 = -\kappa t; \quad T_4 = (1 + \kappa)t; \quad T_6 = (1 + 2\kappa)t; \quad T_{10} = -\kappa t; \]
\[ \hat{T}_1 = (1 + \kappa)\hat{t}/\sqrt{2}; \quad \hat{T}_2 = -\kappa \hat{t}; \quad \hat{T}_4 = \hat{T}_6 = \hat{T}_{10} = 0; \]
\[ E_1 = (1 + \kappa/3)e/\sqrt{2}; \quad 2E_2 = E_4 = E_6 = -E_{10} = -2\kappa e/3; \] 

In this model we take the quark level quantities to be pure real and attribute all rescattering phases to the quantity \( \kappa \). The model, therefore, for simplicity, does not include, for now, phases which can occur at the quark level in perturbation theory calculations \[4\].

It is easy to see that our calculational procedure, in general, involves the following eight parameters: \( t, \hat{t}, p, e, \) Real \( \kappa \equiv \kappa_R, \) Im \( \kappa \equiv \kappa_I, \) \( \rho \) and \( \eta \) where \( \rho \) and \( \eta \) are the CKM parameters \[10\] in the Wolfenstein parameterization which are rather poorly determined. Indeed a recent fit gives \[11\]:
\[ \rho = 0.10^{+0.13}_{-0.38}, \quad \eta = 0.33^{0.06}_{-0.09}. \] Of course, \( \lambda \equiv \sin \theta_c \simeq 0.22 \) and \( A \simeq 0.81 \) are the other two CKM parameters that enter these decays. We will take the results of this fit of the CKM parameters as an input to our subsequent fits.

In the model there are, in general, 20 reactions which are controlled by a maximum of eight parameters. The 20 reactions are the ten listed above for \( B^- \) and \( \overline{B^0} \) decay together with their charge conjugate decays in the case of \( B^+ \) and \( B^0 \) decay.

In current data, the conjugate pairs have been taken together since in these modes CP violation has not been experimentally measured. If one takes the modes averaged with their conjugates, then, for now, we can regard these as ten reactions controlled by a maximum of eight parameters. We will search for self consistent solutions to the data and solve for these parameters along the way by using \( \chi^2 \)-minimization. As an illustration of how the model works we will use the currently available experimental data. Clearly as the quality of the data improves one can hope to improve the evaluation of these parameters as well. Improvements will also result when CP violating rate differences are either measured or bounded in these modes. The \( \chi^2 \) function we find is flat in the direction of changing \( \eta \) suggesting that without measurements of CP violation, the existing data is unable to provide a useful constraint on \( \eta \). In our fits, we thus hold \( \eta \) fixed at the central value of \[11\]; thus for analyzing the current data averaged over conjugate pairs we have in effect only 7 parameters for the ten modes.

The input data we use in our fits is shown in Table 1. We systematically study eight types of fits to this data (see Table 2) to search for the presence of EWP, RST and CST i.e. to solve for \( e, \kappa \) and \( \hat{t} \) (along with the other
parameters). In this table, we use the notation $t_0 = |V_{us}^* V_{ub}| t,$ $p_0 = |V_{us}^* V_{ub}| p$ and $e_0 = |V_{ts}^* V_{tb}| e$.

Solution $A$ is with $e$, $\hat{t}$ and $\kappa$ switched off (i.e. $e = \kappa = \hat{t} = 0$) and thus has the worst $\chi^2$ amongst the eight cases.

In particular the best fit under this hypothesis has low $Br(B^0 \to K^0 \pi^0)$ compared to experiment (Table 1). As we shall see, this trend continues in the fits with more free parameters. Also the ratio $R^- \equiv Br(B^- \to \pi^- \pi^0)/Br(B^0 \to \pi^+ \pi^-)$ has the experimental result $R^- \approx 1.15 \pm 0.6$ while the fit gives 0.64. Although this in itself is barely significant, again it is interesting to follow this ratio through the other cases.

Set $B$ consists of three solutions ($B_1, B_2, B_3$) which allow switching on of either $e$, or $\kappa$ or $\hat{t}$ respectively. Thus, for example, for $B_1$, $\kappa = \hat{t} = 0$; and $\chi^2$-minimization is used to solve for $e$. Similarly for $B_2$, $e = \hat{t} = 0$ and $\kappa \neq 0$ and for $B_3$, $e = \kappa = 0$ and $\hat{t} \neq 0$. Amongst these three solutions in the $B$ set, the set $B_3$, where the CST is turned on, is the best fit. Indeed it has the highest confidence level of all of the solutions. Likewise the solution $B_2$ which allows rescattering has a high confidence level. This suggests that the fit of the basic model $A$ is best improved by allowing a large CST contribution or introducing rescattering. In the case $B_2$, where $\kappa$ is introduced, we obtain a satisfactory fit which, however, has the important consequence that it gives a substantial phase to $\kappa$ which in turn could give rise to substantial CP violation.

Set $C$ (consisting of three solutions) requires two of the three types of contributions to be non-vanishing simultaneously, while the set $G$ allows CST, FSI and EWP to be all present simultaneously.

It is interesting to compare $C_3$ where the CST is turned off, in other words where we assume that the data is explained with rescattering and EWP with $B_3$ where only the CST only is turned on. It would seem that these two complimentary models are best distinguished by considering the $\pi\pi$ and $KK$ modes. First of all, in $B_3$, $Br(\pi^- \pi^0) > Br(\pi^+ \pi^-)$ while in the case of $C_3$ they are roughly the same. The $\pi^0 \pi^0$ mode is smaller in the case of $C_3$ and, by considering all the cases it is apparent that this mode is particularly sensitive to the presence of the CST. On the other hand, the $K^- K^0$ mode is larger in the case of $C_3$ since, as has been pointed out in the literature [12, 13, 14], this mode is sensitive to rescattering effects because the tree cannot contribute to this mode without rescattering. The same is
also true for the mode $K^+K^-$ where the quark content of the final state can only arise through rescattering.

Of course none of the hypotheses are in any way ruled out or clearly favored by the current data; however, since the largest confidence level is for solution $B3$, it does suggest that perhaps color-suppression does not hold too well for the final states with two light pseudoscalars (i.e. $\pi\pi$, $K\pi$ and $K\bar{K}$) that are under consideration. Indeed in all the instances where the CST is allowed, $\tilde{t}/t \approx 1$ whereas naive color counting give $\tilde{t}/t = 1/3$. Alternatively, as shown by solution $B2$ rescattering provides an adequate explanation for the data although it becomes difficult to make $\pi^-\pi^0$ larger than $\pi^+\pi^-$. It is interesting to note that in all of the the fits, the trend is to consistently give a value for the branching ratio to $\bar{K}^0\pi^0$ which is about one standard deviation below the current central value. It would be interesting if the central value remained at this level. For example, if one assumes that the error is reduced to $\pm 0.15$ while holding the central value and all the rest of the data fixed, Solution $A$ and $B2$ begin to become untenable with confidence levels of 0.07 and 0.09. In this scenario $B3$ and $C3$ are the best solutions with confidence levels 0.30 and 0.49 respectively and so CST or EWP with RST would be the favored explanations.

We can also generalize the model somewhat by allowing the rescattering of the form $q_i\bar{q}_i \rightarrow q_j\bar{q}_j$ to be different from $q_i\bar{q}_i \rightarrow q_i\bar{q}_i$ where $i \neq j$. In particular, let us replace $\kappa$ with $\kappa + \delta\kappa$ in the case where $q_i\bar{q}_i \rightarrow q_i\bar{q}_i$. In the solution $g$ listed in the Table we assume that $\delta\kappa$ is purely imaginary. Clearly the values of the resulting fit are similar to the case $G$ where $\delta\kappa = 0$. And since the confidence level is only 0.33, this does not appear to be a particularly useful parameter to explain the current data.

We now briefly comment on some of the other interesting implications of these two solutions (see Table 3):

1. A primary objective of the intense experimental and theoretical studies of B decays is to deduce the CKM phases of the unitarity triangle. In this regard, as an illustration, we note from Table 3 that solutions to the current data with $CL \gtrsim 0.6$ seem to give $\gamma = 90 \pm 20$ degrees.

2. The ratio “penguin-like”/“tree-like” for the $\pi^+\pi^-$ is a useful parameter for facilitating the extraction of $\alpha$ from the measurement of the time dependent CP asymmetry in $B \rightarrow \pi^+\pi^-$ following the works of
Charles [7] and of Quinn and Grossman [15]. We see (Table 3) that for
the various solutions considered, this ratio \( \frac{p_\pi}{t_\pi} \) is \( \sim 0.3-0.6 \). Note
also that in the analysis of Ref. [7, 15] EWP contribution was assumed
negligible. We find the EWP contribution to be about 1% of the total
amplitude for the \( \pi^+\pi^- \) mode—thus quite small. So their idea for ex-
traction of \( \alpha \) has a good chance of working if an accurate determination
of the penguin/tree for the \( \pi\pi \) mode can be achieved.

3. The current fit to the model suggests that the EWP effects are small.
However, since the possible presence of the EWP in these pseudoscalars
modes can adversely affect a model independent determination of \( \alpha \)
and/or \( \gamma \), \( B \) decays to two vector modes [16] become an important
check of this. These modes (e.g. \( K^*\omega, K^*\rho \) and \( \rho\omega \)) allow a quantitative
determination of EWP in a model independent way and those in which
EWP are not prominent (e.g. are color suppressed) can then be used
for deduction of \( \alpha \) or \( \gamma \).

4. In conjunction with the assumption of flavor SU(3) for the rescattering
effects, the method used here allows one to incorporate also the inform-
ation from \( B_s \) decays, e.g. \( B_s \to K^+K^-, K^0\overline{K}^0, K^+\pi^-, K^0\pi^0 \), etc,
in the determination of the parameters of interest. Hopefully, these
measurements would become accessible at various accelerators in the
not too distant future.

5. We have examined partial rate asymmetry (PRA) ( as usual defined to
be \( \frac{\Gamma(B \to X) - \Gamma(B \to \overline{X})}{\Gamma(B \to X) + \Gamma(B \to \overline{X})} \)) in these ten
modes for the solutions which allow \( \text{Im}(\kappa) \) to be non-zero where we are
taking the needed strong phase originating only from \( \text{Im}(\kappa) \) charac-
teristic of the RST. Since the likelihood distribution for these quantities
is not well described by a Gaussian, the number we give indicates the
range of values for the magnitude of the PRA which contains 68% of
the likelihood distribution with the remainder split evenly above and
below the range. The absolute sign of the PRA, of course, is propor-
tional to the sign of \( \text{Im}(\kappa) \) which cannot be determined without some
CP odd experimental data.

We find that the PRA in the \( K\pi \) modes tend to be \( \lesssim 15\% \) whereas for
the \( \pi\pi \) modes appreciably larger asymmetries \( \sim 10-80\% \) seem possible
if indeed large $Im(\kappa)$ is possible.

Measurements of direct CP violation in the $\pi\pi$ final states will, of course, be useful in clarifying whether there is large rescattering present in $B$ decays to two pseudoscalars. The same is also true for $K\pi$ final states, although the CP violation is smaller, it should be easier to measure.

6. Perhaps the most interesting finding suggested by this study is that in contrast to $B \to D$ decays, the best fit to the data in our model seems to suggest that there is little or no color suppression operative in $B$ decays to two pseudoscalars. It will be extremely interesting if this assessment persists with improvements in the data.

7. In all the solutions the central value for the EWP is small, comparable with theoretical estimates [7, 8]. Clearly there is considerable uncertainty and since the impact of EWP is larger in the $K\pi$ modes, improved data on those modes should clarify the situation. For example, if we perform the same fit with the branching ratio for $\pi^0K^0$ increased to $2 \times 10^{-5}$ (i.e. 1-sigma) and all the other data and error bars fixed, then fit G produces a value for $e_0$ about 1/2 that of $p_0$ with a confidence level of 0.30. In this case, the best solution is $C3$ (EWP and rescattering turned on) with a confidence level of 0.49 with a similar value for $e/p$.

8. Finally, since $\pi\pi$ and $KK$ states receive a larger influence from the tree graph, one would expect more data on these processes to be helpful in distinguishing between CST and RST explanations. Indeed the results in Table 1 suggest how to interpret experimental results to determine whether CST or rescattering are present. As discussed above the presence of the CST is associated with larger branching ratios to $\pi^0\pi^0$ while the presence of rescattering is associated with larger branching ratios to $K^-K^0$ and $K^+K^-$. We are grateful to George Hou for discussions. This research was supported in part by US DOE Contract Nos. DE-FG01-94ER40817 (ISU) and DE-AC02-98CH10886 (BNL).
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Table 1: Summary of the experimental data that we use in constructing the fits to our model. The branching ratios are in units of $10^{-5}$ and are the average for each given mode and the conjugate. The results in the case of $K^-\pi^+$, $K^-\pi^0$ and $\bar{K}^0\pi^+$ are the experimental results from [1, 2]. In the other cases, we infer the branching ratios with the stated errors from the yields together with the efficiencies given in [2].

| Mode # | Mode   | $Br$   |
|--------|--------|--------|
| 1      | $K^-\pi^0$   | 1.21$^{+0.30}_{-0.28}$ |
| 2      | $\bar{K}^0\pi^-$ | 1.82$^{+0.46}_{-0.40}$ |
| 3      | $\bar{K}^0\pi^0$ | 1.48$^{+0.59}_{-0.51}$ |
| 4      | $K^-\pi^+$   | 1.88$^{+0.28}_{-0.26}$ |
| 5      | $\pi^-\pi^0$ | 0.54$^{+0.25}_{-0.26}$ |
| 6      | $\pi^+\pi^-$ | 0.47$^{+0.15}_{-0.18}$ |
| 7      | $\pi^0\pi^0$ | 0.16$^{+0.16}_{-0.10}$ |
| 8      | $K^-K^0$     | 0.20$^{+0.24}_{-0.16}$ |
| 9      | $K^+K^-$     | 0$^{\pm}$.5 |
| 10     | $K^0\bar{K}^0$ | 0$^{\pm}$.7 |
Table 2: Summary of the solutions under the various hypothesis. $N$ means the corresponding entity is switched off (i.e. set to zero) whereas $Y$ means it is on and being determined. The branching ratio for the ten modes are given in units of $10^{-5}$; the individual amplitudes $t_0$, $p_0$, etc. are expressed accordingly.

|    | $g$ | $G$ | $C3$ | $C1$ | $C2$ | $B3$ | $B1$ | $B2$ | $A$ |
|----|-----|-----|------|------|------|------|------|------|-----|
| $\hat{t}$ | $Y$ | $Y$ | $N$  | $Y$  | $Y$  | $N$  | $N$  | $N$  | $N$ |
| $e$ | $Y$ | $Y$ | $N$  | $Y$  | $N$  | $Y$  | $N$  | $N$  | $N$ |
| $\kappa$ | $Y$ | $Y$ | $Y$  | $Y$  | $N$  | $N$  | $N$  | $Y$  | $N$ |
| $\delta \kappa$ | $Y$ | $N$ | $N$  | $N$  | $N$  | $N$  | $N$  | $N$  | $N$ |
| $t_0$ | -0.01 | 0.11 | -0.28 | -0.11 | 0.14 | -0.15 | 0.23 | -0.28 | 0.23 |
| $p_0$ | -0.19 | 0.80 | -2.21 | 0.87 | -1.42 | 1.41 | 1.37 | -2.21 | 1.37 |
| $t_0$ | 0.07 | 0.15 | 0.00 | -0.16 | 0.13 | -0.13 | 0.00 | 0.00 | 0.00 |
| $e_0$ | 0.04 | 0.06 | -0.02 | 0.00 | -0.04 | 0.00 | -0.03 | 0.00 | 0.00 |
| $Re(\kappa)$ | -0.15 | 0.27 | -0.16 | 0.20 | 0.00 | 0.00 | 0.00 | -0.16 | 0.00 |
| $Im(\kappa)$ | -1.08 | 0.00 | -0.12 | 0.00 | 0.00 | 0.00 | 0.00 | -0.12 | 0.00 |
| $\rho$ | 0.13 | 0.08 | -0.02 | 0.04 | -0.04 | -0.01 | -0.08 | -0.02 | -0.07 |
| $\eta$ | 0.32 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 |
| $K^-\pi^0$ | 1.05 | 1.06 | 1.02 | 1.06 | 1.02 | 0.99 | 1.01 | 1.03 |
| $K^0\pi^+$ | 1.97 | 2.03 | 1.92 | 1.95 | 2.01 | 2.00 | 1.87 | 1.92 | 1.88 |
| $K^0\pi^0$ | 1.18 | 0.98 | 0.94 | 0.95 | 0.98 | 1.01 | 0.98 | 0.96 | 0.94 |
| $K^-\pi^+$ | 1.96 | 1.95 | 2.02 | 1.99 | 1.98 | 2.00 | 2.08 | 2.02 | 2.06 |
| $\pi^-\pi^+$ | 0.57 | 0.57 | 0.50 | 0.57 | 0.60 | 0.60 | 0.34 | 0.50 | 0.34 |
| $\pi^-\pi^-$ | 0.47 | 0.46 | 0.50 | 0.46 | 0.45 | 0.45 | 0.45 | 0.50 | 0.53 |
| $\pi^0\pi^0$ | 0.16 | 0.16 | 0.10 | 0.15 | 0.14 | 0.14 | 0.04 | 0.10 | 0.04 |
| $K^-K^0$ | 0.20 | 0.20 | 0.19 | 0.20 | 0.12 | 0.11 | 0.12 | 0.19 | 0.12 |
| $K^+K^-$ | 0.01 | 0.02 | 0.07 | 0.02 | 0.00 | 0.00 | 0.07 | 0.00 |
| $K^0K^0$ | 0.29 | 0.17 | 0.15 | 0.18 | 0.12 | 0.11 | 0.12 | 0.15 | 0.12 |
| $\chi^2$ | 0.96 | 1.62 | 2.77 | 1.65 | 2.10 | 2.13 | 4.68 | 2.78 | 4.71 |
| $df$ | 1 | 2 | 3 | 3 | 3 | 4 | 4 | 4 | 5 |
| $\chi^2/df$ | 0.96 | 0.81 | 0.92 | 0.55 | 0.70 | 0.53 | 1.17 | 0.69 | 0.94 |
| CL | 0.33 | 0.45 | 0.43 | 0.65 | 0.55 | 0.71 | 0.32 | 0.60 | 0.45 |
Table 3: Properties of the solutions that appear most relevant to the CLEO data. Numbers after $K^-\pi^0$, $K^0\pi^+ \ldots \pi^0\pi^0$ are the magnitudes of the partial rate asymmetry for $B^-$ or $B^0$ to the given mode\textsuperscript{[3]}. For each mode, the range quoted encompasses 68% of the likelihood function with the rest split above and below the range. (See also caption to Table \textsuperscript{2}).

|       | $G$ | $C_3$ | $C_1$ | $B_3$ | $B_2$ |
|-------|-----|-------|-------|-------|-------|
| $t$   | Y   | N     | Y     | Y     | N     |
| $e$   | Y   | Y     | N     | N     | Y     |
| $\kappa$ | Y | Y | N | N | Y |
| $\kappa_D$ | N | N | N | N | N |
| $t_0$ | 0.11±0.03 | -0.28±0.18 | -0.11±0.03 | -0.15±0.03 | -0.28±0.11 |
| $p_0$ | 0.80±0.26 | -2.21±5.47 | 0.87±0.38 | 1.41±0.07 | -2.21±3.17 |
| $t_0$ | 0.15±0.03 | 0.00±0.00 | -0.16±0.03 | -0.13±0.03 | 0.00±0.00 |
| $v_0$ | 0.06±0.14 | -0.02±0.19 | 0.00±0.00 | 0.00±0.00 | 0.00±0.00 |
| $Re(\kappa)$ | 0.27±0.18 | -0.16±0.23 | 0.20±0.21 | 0.00±0.00 | -0.16±0.14 |
| $Im(\kappa)$ | 0.00±1.00 | 0.12±0.59 | 0.00±1.12 | 0.00±0.00 | -0.12±0.37 |
| $\rho$ | 0.08±0.16 | -0.02±0.26 | 0.04±0.15 | 0.00±0.12 | -0.02±0.18 |
| $\gamma$ | 76±25 | 93±45 | 83±25 | 92±20 | 94±31 |
| $p_{\pi}/t_{\pi}$ | 0.51±0.09 | 0.37±0.88 | 0.59±0.23 | 0.51±0.18 | 0.37±0.60 |
| $e_{\pi}/A_{\pi}$ | 0.01±0.02 | 0.00±0.04 | 0.00±0.00 | 0.00±0.00 | 0.00±0.00 |
| $K^-\pi^0$ | 0.22—157 | 0.31—176 | 0.21—156 | 0 | 0.027—169 |
| $K^0\pi^+$ | 0.12—0.81 | 0.16—0.90 | 0.11—0.79 | 0 | 0.014—0.88 |
| $K^0\pi^0$ | 0.19—1.33 | 0.16—0.91 | 0.18—1.29 | 0 | 0.014—0.88 |
| $K^-\pi^+$ | 0.15—1.25 | 0.31—1.73 | 0.15—1.27 | 0 | 0.028—1.70 |
| $\pi^-\pi^+$ | 0 | 0 | 0 | 0 | 0 |
| $\pi^-\pi^0$ | 0.073—0.594 | 0.127—0.749 | 0.070—0.591 | 0 | 0.110—0.719 |
| $\pi^0\pi^0$ | 0.198—0.849 | 0.324—0.900 | 0.192—0.854 | 0 | 0.291—0.902 |