The Phase of the Annual Modulation: Constraining the WIMP Mass

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Abstract

The count rate of Weakly Interacting Massive Particle (WIMP) dark matter candidates in direct detection experiments experiences an annual modulation due to the Earth’s motion around the Sun. In the standard isothermal halo model, the signal peaks near June 2nd at high recoil energies; however, the signal experiences a phase reversal and peaks in December at low energy recoils. We show that this phase reversal may be used to determine the WIMP mass. If an annual modulation were observed with the usual phase (i.e., peaking on June 2nd) in the lowest accessible energy recoil bins of the DAMA, CDMS-II, CRESST-II, EDELWEISS-II, GENIUS-TF, ZEPLIN-II, XENON, or ZEPLIN-IV detectors, one could immediately place upper bounds on the WIMP mass of [103, 48, 6, 97, 10, 52, 29, 29] GeV, respectively. In addition, detectors with adequate energy resolution and sufficiently low recoil energy thresholds may determine the crossover recoil energy at which the phase reverses, thereby obtaining an independent measurement of the WIMP mass. We study the capabilities of various detectors, and find that CRESST-II, ZEPLIN-II, and GENIUS-TF should be able to observe the phase reversal in a few years of runtime, and can thus determine the mass of the WIMP if it is $\mathcal{O}(100$ GeV). Xenon based detectors with 1000 kg (XENON and ZEPLIN-IV) and with energy recoil thresholds of a few keV require 25 kg-yr exposure, which will be readily attained in upcoming experiments.

1 Introduction

Extensive gravitational evidence suggests that a dominant fraction of the matter in our Galaxy is nonluminous, or dark. Although the identity of this dark matter is currently unknown, it may consist of Weakly Interacting Massive Particles (WIMPs). Numerous
experiments aimed at the direct detection of these WIMPs are currently being developed. These experiments generically measure the nuclear recoil energy deposited in a detector when an incident WIMP interacts with a nucleus in the detector. An important signature of halo dark matter in direct-detection experiments is the annual modulation induced by the Earth’s motion with respect to the halo. In the standard model of the dark halo, in which the velocity distribution of the WIMPs is a Maxwellian distribution truncated at the escape speed of the Galaxy, the modulation of the WIMP interaction rate at high nuclear recoil energies is in phase with the motion of the Earth with respect to the halo, peaking when the relative speed is maximal (June) and reaching a minimum when the relative motion is minimal (December). For low energy recoils, however, it is well known that that the interaction rate experiences a phase reversal, peaking in December. This phase reversal occurs below a particular crossover recoil energy, $Q_c$. In this paper we examine what information about the mass of the WIMP can be obtained from this phase reversal. We emphasize that this phase reversal could constitute an important signature of a WIMP flux, and we also point out that because the crossover recoil energy is a function of the WIMP mass, the observation of the phase of annual modulation immediately places an upper limit on the allowed WIMP mass.

Furthermore, if WIMP direct detection experiments with sufficient energy resolution determine the crossover energy $Q_c$, the WIMP mass could be independently measured using only observations of the phase of the annual modulation.

This paper is organized as follows. In section 2, we briefly review the basics of WIMP direct detection experiments and the isothermal model of the dark halo. In section 3, we explore the dependence of the annual modulation on energy recoil, and revisit the phase reversal that occurs at low recoil energies. Finally, we demonstrate how this observation may be exploited to place limits on the WIMP mass and to obtain estimates of the mass directly. We emphasize that observation of this phase reversal will constitute an unambiguous signature of an extraterrestrial WIMP flux.

## 2 WIMP Direct Detection Experiments

More than twenty collaborations worldwide are presently developing detectors designed to search for WIMPs. Although the experiments employ a variety of different methods, the basic idea underlying WIMP direct detection is straightforward: the experiments seek to measure the energy deposited when a WIMP interacts with a nucleus in the detector. If a WIMP of mass $m_\chi$ scatters elastically from a nucleus of mass $m_N$, it will deposit a recoil energy of $Q = (m_\chi^2v^2/m_N)(1 - \cos \theta)$, where

$$m_r \equiv m_\chi m_N/(m_\chi + m_N)$$
is the reduced mass, \( v \) is the speed of the WIMP relative to the nucleus, and \( \theta \) is the scattering angle in the center of mass frame. We may compute (following, \( e.g. \), [17, 13, 12]) the differential detection rate, per unit detector mass (\( i.e. \), counts/day/kg detector/keV recoil energy) associated with this process,

\[
\frac{dR}{dQ} = \frac{\sigma_0 \rho_h}{2m_r^2 m_\chi} F^2(Q) T(Q, t) \tag{2}
\]

where \( \rho_h = 0.3 \text{ GeV/cm}^3/c^2 \) is the halo WIMP density, \( \sigma_0 \) is the total nucleus-WIMP interaction cross section and \( F(Q) \) is the nuclear form factor for the WIMP-nucleus interaction that describes how the effective cross section varies with WIMP-nucleus energy transfer. We consider here only spin independent interactions, wherein the target nucleus can be approximated as a sphere of uniform density smoothed by a gaussian \([14]\), and the resulting form factor is,

\[
F(Q) = \frac{3[\sin(Qr_1) - Qr_1 \cos(Qr_1) ]}{q^3r_1^3} e^{-Q^2s^2/2} \tag{3}
\]

where \( r_1 = (r^2 - 5s^2)^{1/2}, s \simeq 1 \text{ fm} \), and,

\[
r \simeq [0.91(M/\text{GeV})^{1/3} + 0.3] \times 10^{-13} \text{ cm} \tag{4}
\]

is the radius of the nucleus. For spin-dependent interactions, the form factor is somewhat different but again \( F(0) = 1 \). In general, the form factor must be evaluated for each detector nuclei. For a more extensive discussion, see [11, 1, 15, 16, 17].

For purely scalar interactions,

\[
\sigma_{0, \text{scalar}} = \frac{4m_r^2}{\pi} [Zf_p + (A - Z)f_n]^2 . \tag{5}
\]

Here \( Z \) is the number of protons, \( A - Z \) is the number of neutrons, and \( f_p \) and \( f_n \) are the WIMP couplings to nucleons. For purely spin-dependent interactions,

\[
\sigma_{0, \text{spin}} = \frac{(32/\pi)}{G_F^2 \mu^2} \Lambda^2 J(J + 1) . \tag{6}
\]

Here \( J \) is the total angular momentum of the nucleus and \( \Lambda \) is determined by the expectation value of the spin content of the nucleus (see [11, 1, 15, 16, 17]).

For the estimates necessary in this paper, we take the WIMP-nucleon cross section\(^1\) \( \sigma_p = 7.2 \times 10^{-42} \text{ cm}^2 \), and take the total WIMP-nucleus cross section to be \(^2\)

\[
\sigma_0 = \sigma_p \left( \frac{m_r}{m_{rp}} \right)^2 A^2 \tag{7}
\]

\(^1\)This is the cross section with nucleons at zero momentum transfer as discussed in Eq. (7.36) of [17].

\(^2\)In most instances, \( f_n \sim f_p \) so that the following equation results from Eq. (5).
where the $m_{\text{rp}}$ is the proton-WIMP reduced mass, and $A$ is the atomic mass of the target nucleus.

Information about halo structure is encoded into the quantity $T(Q, t)$,

$$T(Q, t) = \int_{v_{\text{min}}}^{\infty} \frac{f_d(v)}{v} dv,$$

where $f_d(v)$ is the distribution of WIMP speeds relative to the detector, and where $v_{\text{min}} \equiv (Qm_N/2m_p^2)^{1/2}$ represents the minimum velocity that can result in a recoil energy $Q$. To determine $T(Q, t)$, we must have a model for the velocity structure of the halo.

### 2.1 The isothermal halo

In what follows we will consider dark matter detection experiments without directional capabilities; such experiments are sensitive only to the WIMP flux integrated over the entire sky. The most frequently employed background velocity distribution is that of a simple isothermal sphere. In such a model, the galactic WIMP speeds with respect to the halo obey a Maxwellian distribution with a velocity dispersion $\sigma_h$,

$$f_h(v)dv = 4\pi \left(\frac{3}{2\pi \sigma_h^2}\right)^{3/2} v^2 \exp\left(-\frac{3v^2}{2\sigma_h^2}\right)$$

where $v$ is the WIMP velocity relative to the Galactic halo, and where we have performed an angular integration over the entire sky. We take the velocity dispersion of our local halo to be $\sigma_h = 270 \text{ km/s}$. This is the local velocity distribution; it is expected to vary with spatial position throughout the Galaxy.

As pointed out by Drukier, Freese, and Spergel [1], and studied (in the context of an isothermal halo) by Freese, Frieman, and Gould [2], Earth-bound observers of the dark halo will see a different, time dependent velocity distribution as a result of the relative motion of the Earth with respect to the Galaxy. To take this into account, we estimate the velocity of the Earth with respect to the halo as

$$v_{eh} = \sqrt{v_{es}^2 + v_{sh}^2 + 2v_{es}v_{sh}\cos(2\pi t - \phi_h)}$$

where the phase $\phi_h = 2.61 \pm 0.02$ corresponds to June 2nd $\pm$ 1.3 days (relative to $\phi = 0$ on January 1). The speed of the Earth with respect to the Sun is $v_{es} = 29.8 \text{ km/s}$, and the speed of the Sun with respect to the halo is $v_{es} = 233 \text{ km/s}$. More precise expressions for the motion of the Earth and Sun may be found in [4, 5]. Translating the distribution in Eq. 9 into the distribution as seen by an earthbound detector, $f_d(v_{eh})$, and integrating the resulting distribution in Eq. 8 one obtains
\[ T_h(Q, t) = \frac{1}{2v_{eh}(t)} \left[ \text{erf} \left( \frac{\sqrt{2}(v_{\min} + v_{eh}(t))}{\sqrt{3}\sigma_h} \right) - \text{erf} \left( \frac{\sqrt{2}(v_{\min} - v_{eh}(t))}{\sqrt{3}\sigma_h} \right) \right]. \]  

With \( T_h(Q, t) \), one can now compute the recoil energy spectrum associated with the isothermal halo, as well as the annual modulation of that spectrum that results from the motion of the Earth around the sun. For the case of an isothermal halo, one generally expects that, above a critical recoil energy, the annual modulation will be in phase with the motion of the Earth with respect to the halo, peaking near June 2nd. In the following section, we will demonstrate the dependence of the phase of the modulation on the observed recoil energy.

### 3 The Phase Reversal at Low Recoil Energies

The annual modulation of the WIMP interaction rate [11 2], and in particular the phase reversal at low recoil energies [3 5 4], has been studied in detail by several authors. Here Fig. 11 plots the expected differential event rate for a WIMP of mass 65 GeV at recoil energies of 35 keV and 10 keV. We note that there is a 180 degree phase shift between the two signals. The origin of this effect may be understood by considering a first order approximation to Eq. 11. We note that the velocity of the Earth with respect to the WIMP halo, \( v_{eh} \), given in Eq. 10 may be written, for \( v_{es} \ll v_{eh} \), as

\[ v_{eh} \sim \eta_h + \epsilon_h(t). \]

where we define,

\[ \eta_h = \left( v_{es}^2 + v_{sh}^2 \right)^{1/2}, \]

\[ \epsilon_h(t) = \frac{v_{es}v_{sh}}{\left( v_{es}^2 + v_{sh}^2 \right)^{1/2}} \cos(2\pi t - \phi_h). \]

Using this approximation, We may expand this expression about \( \epsilon(t) = 0 \), an approximation that is well justified because \( \epsilon_h(t)/\eta_h \ll 1 \) for all \( t \). With the following definitions,

\[ X_h^\pm = \text{erf} \left( \frac{\sqrt{2}(v_{\min} \pm \eta_h)}{\sqrt{3}\sigma_h} \right), \]

\[ E_h^\pm = \exp \left( -\frac{2(v_{\min} \pm \eta_h)^2}{3\sigma_h^2} \right), \]

we find that, to first order in \( \epsilon \),

\[ 5 \]
Figure 1: The annual modulation of the WIMP differential detection rate for a WIMP with a mass of 65 GeV, in a $^{73}$Ge detector, for recoil energies of 35 and 10 keV. Note the 180 degree phase shift between the two signals.

\[ T_h(Q, t) \approx \frac{1}{2} \left[ \frac{X^+_h - X^-_h}{\eta_h} \right] + \epsilon_h(t) \left[ \sqrt{\frac{8}{3\pi}} \frac{E^+_h + E^-_h}{\sigma_h \eta_h} - \frac{X^+_h - X^-_h}{\eta^2_h} \right]. \]  

(16)

We see that the differential detection rate will be modulated by $\epsilon(t) \propto \cos(2\pi t - \phi_h)$, as expected. We are particularly interested in the behavior of the amplitude of the oscillation.

If we rewrite $T_h(Q, t)$ of Eq. (16) as

\[ T(Q, t) = B + A \cos(2\pi \omega t - \phi_h), \]  

(17)

we identify the amplitude of oscillation as

\[ A(Q) = \frac{v_{es}v_{sh}}{(v_{es}^2 + v_{sh}^2)^{1/2}} \left[ \sqrt{\frac{8}{3\pi}} \frac{E^+_h + E^-_h}{\sigma_h \eta_h} - \frac{X^+_h - X^-_h}{\eta^2_h} \right]. \]  

(18)

For $A(Q) > 0$, the modulation of the WIMP differential event rate is in phase with the motion of the Earth with respect to the halo, and will peak at June 2nd as usual. For $A(Q) < 0$, however, the modulation of the event rate moves 180 degrees out of phase with terrestrial motion, peaking in December rather than June. Values of $A(Q)$ are plotted in
Fig. 2 for $0 \leq Q \leq 100$ keV, for a WIMP mass of 30, 50 and 70 GeV in a $^{73}$Ge detector. In general, for a WIMP mass $m_\chi$ and nuclear mass $m_N$ there will exist a nonzero, crossover recoil energy, $Q_c$, at which $A(Q_c) = 0$, and below which $A(Q < Q_c) < 0$. That is, for $Q < Q_c$ the WIMP interaction rate will peak in December rather than June. We emphasize that these approximations, while illustrative, give rise to errors of a few percent; in practice, Eq. 11 should be used for all computations.

Figure 2: The amplitude of modulation $A(Q)$ as a function of recoil energy $Q$ for WIMP masses of 30, 50, and 70 GeV in a $^{73}$Ge based detector. For each WIMP mass there exists a particular recoil energy $Q_c$, at which the first order annual modulation of the WIMP differential event rate vanishes. For $Q < Q_c$, we find that $A(Q < Q_c) < 0$, and the differential WIMP interaction rate peaks in December, rather than June.

The crossover recoil energy, $Q_c$ is a function of the mass of the WIMP, as well as the mass of the target nuclei in the detector. We can estimate the critical recoil energy, below which we observe the phase reversal. Fig. 3 plots the crossover recoil energy as a function of WIMP mass, for NaI, $^{73}$Ge, $^{131}$Xe, CaWO$_4$ and Al$_2$O$_3$ based WIMP direct detection experiments.
| Collaboration | Material | $Q_{\text{thresh}}$ (keV) | $M_D$ (kg) |
|--------------|----------|--------------------------|-----------|
| CDMS-II      | $^{73}\text{Ge}$ | $10^3$ | 5 |
| CRESST-II    | $\text{CaWO}_4$ | $\sim 1$ | 10 |
| DAMA         | NaI      | 22 | 100 |
| EDELWEISS-II | $^{73}\text{Ge}$ | 20 | 38 |
| GENIUS-TF    | $^{73}\text{Ge}$ | 1 | 40 |
| XENON        | LXe      | 4 | 1000 |
| ZEPLIN-II    | LXe      | 10 | 40 |
| ZEPLIN-IV    | LXe      | 4 | 1000 |

Table 1: Properties of various dark matter detection experiments including: the detector material, the detector energy recoil threshold, $Q_{\text{thresh}}$, and the total detector mass, $M_D$. In the case of future experiments, the quantities are projected.

3.1 Placing an Upper Bound on the WIMP Mass

The observation of an annual modulation signal can be used to constrain the WIMP mass. For example, if a direct-detection experiment observes an annually modulated signal peaking in June at an energy recoil bin $Q_0$, we may place an upper limit on the WIMP mass because we know that the crossover energy $Q_c$ must have been less than $Q_0$, or a phase reversal would have been observed. Because the crossover energy is a monotonically increasing function of the WIMP mass, this places an upper limit on the WIMP mass.

We will describe the upper bounds one can obtain in a variety of dark matter detectors. Of course, the experiments must have sufficient exposure to see the annual modulation in order to obtain these bounds.

As a concrete example, consider the DAMA experiment, which reports an annual modulation in the event rate in its NaI detector in the 22-66 keV recoil energy bins [24]. Although other experiments rule out much of this region of parameter space [27], if one were to believe the DAMA results, the fact that annual modulation in DAMA peaks in June in this energy bin implies, from Fig. 3, that the maximum WIMP mass consistent with the DAMA data is $m \leq 103$ GeV.

This same logic may be applied to other dark matter experimental collaborations that may be sensitive to the phase reversal. In particular, in this paper we will consider the various versions of the CRESST [25], CDMS [26], EDELWEISS [27], XENON [29], GENIUS [30], and ZEPLIN [31] experiments. Table I collects the detector target materials, mass, and recoil energy thresholds associated with these experiments. In Table II, we present the maximum WIMP mass consistent with an observation of a June-peaking annual modulation in the lowest accessible recoil energy bins of the respective experiments, as determined using the arguments in the preceding paragraphs.
Table 2: The maximum WIMP masses consistent with observing a June-peaking annual modulation signal in the lowest accessible energy recoil bins of the respective dark matter experiments.

### 3.2 Detecting the Phase Reversal

For WIMP direct detection experiments with sufficient energy resolution and sufficient exposure, it may be possible to determine the crossover recoil energy $Q_c$ itself, thereby making an independent measurement of the WIMP mass. If one determines the recoil energy at which the modulation phase changes sign, Fig. 3 may be used to estimate the WIMP mass.

We now explore whether or not the phase reversal itself could be practically observed in the context of current and near-future direct detection experiments. In order to extract a small annual signal from the background, we require a large signal to noise ratio so that the fluctuations are small compared to the desired signal. This problem, in the context of WIMP direct detection experiments, has been discussed by Hasenbalg [3]. Because the count rate for direct detection experiments is so low, extended exposure times will be required to reliably detect a small modulation.

We write the total signal as a function of time as,

$$S(t) = \int_{Q_i}^{Q_f} \frac{dR}{dQ} dQ$$

$$= S_0(Q_i, Q_f) + S_m(Q_i, Q_f) \cos(\omega t) + \mathcal{O}(S_m^2)$$

where $S_m$ is the amplitude of modulation, $S_0$ is the unmodulated count rate, and $dR/dQ$ is defined as in Eq. 2. These quantities will depend on the limits of integration, $Q_i$ and $Q_f$, and may be be written in terms of the differential event rate evaluated at its maximum in June, $S_J$, and the rate at its minimum, in December, $S_D$. 

Figure 3: The crossover recoil energy $Q_c$ as a function of WIMP mass $m_\chi$ for NaI, $^{73}\text{Ge}$, $^{131}\text{Xe}$, $\text{CaWO}_4$, and $\text{Al}_2\text{O}_3$ detectors, assuming a halo velocity dispersion of 270 km/s. The observation of an annual modulation signal peaking in June at a given recoil energy necessarily places an upper limit on the allowed WIMP mass. For example, the observation of a June-peaking annual modulation in the 22 keV energy recoil bin implies an upper limit on the WIMP mass of about 103 GeV for a NaI based experiment.

$$S_m = \frac{1}{2}[S_J - S_D]$$
$$S_0 = \frac{1}{2}[S_J + S_D].$$  \hspace{1em} (21)

If $S_m \ll S_0$, the theoretical signal to noise ratio may be written as,

$$\frac{\langle s/n \rangle}{\sqrt{MT}} = \frac{S_m(Q_i, Q_f)}{\sqrt{S_0(Q_i, Q_f)}}\sqrt{MT}. \hspace{1em} (22)$$

where $M$ is the total detector mass and $T$ is the total exposure time. As a reasonable criterion for distinguishing the modulation signal from the noise, we require that the $s$ be at least $2\sigma$ greater than the statistical uncertainty. This amounts to requiring $\langle s/n \rangle = 2$. For low count rates, large $MT$ will be required to achieve this minimum signal to noise ratio.

In order to observe a phase reversal in a particular WIMP detector with a given WIMP mass, an annual modulation peaking in December must be detected in the energy range $Q_{\text{thresh}}$ to $Q_c$, and a modulation peaking in June must be observed in the energy range $Q_c$ to
$Q_{\text{max}}$, where $Q_{\text{thresh}}$ is the recoil energy threshold of the detector, and $Q_{\text{max}}$ is the maximum recoil energy accessible to the detector. In Fig. 4 we plot the minimum exposure $(MT)_{\text{min}}$ (in kg-yrs) necessary to observe a phase reversal for several current and future experiments, requiring a minimum signal to noise ratio $(s/n) = 2$. We should note that we are assuming perfect energy detector resolution, and that these exposure estimates therefore represent lower limits to the required exposure in actual experiments.

Figure 4: The minimum exposure (in kg-yrs) required to observe the phase reversal in the WIMP-nucleus differential event rate at the $2\sigma$ level, as a function of WIMP mass for various detectors.

We can determine the viability of observing the phase reversal and hence obtaining an estimate of the WIMP mass in various detectors. We note that, using this phase reversal technique, the experiments are most sensitive to WIMP masses between 80 GeV to a few hundred GeV, which is the most interesting mass range. Accelerators (LEP-200) place lower bounds on the neutralino mass of roughly 45 GeV. The upper limit for sensible supersymmetric dark matter particles is roughly 1.5 TeV. Recent upper bounds on the Constrained Minimal Supersymmetric Model are tighter \[18, 19, 20\]. Hence the most sensitive mass range in the detectors covers a compelling slice of parameter space.
3.3 Summary of Results

As emphasized above, Fig. 4 presents the total exposure time required to observe the phase reversal as a function of the WIMP mass, for various detectors.

The CRESST-II experiment, which employs calcium tungstate (CaWO$_4$) crystals, should be able to detect the phase reversal. In particular, for WIMP masses near 80 GeV, only two years of total exposure would be required to observe the phase reversal.

For the $^{73}$Ge based detectors of CDMS and CDMS-II, the phase reversal is not likely to be observed. CDMS-II has projected exposures of roughly 2,500 kg-days of data [28]. For the most favorable WIMP masses, nearly 30,000 kg-days of data are required to observe the phase reversal using germanium detectors with a recoil energy threshold of 10 keV.

The EDELWEISS-II experiment, which also employs $^{73}$Ge, the relatively high energy recoil threshold of the experiment renders it unlikely to observe the phase reversal. At best, for WIMP masses near 400 GeV, the experiment requires $\sim$ 340 kg-yrs of exposure. With its current mass, this will necessitate nearly 9 years of continuous running time.

The GENIUS-TF experiment, a test facility for the projected GENIUS experiment, is currently the best germanium-based platform for observing the phase reversal [30]. Requiring only 10 kg-yrs of exposure for WIMP masses $\sim$ 80 – 120, the experiment could observe the phase reversal with as little as a year of runtime. The germanium based experiments have the advantage over other experiments in that they are sensitive to a larger range of WIMP masses due to the relative flatness of the $(MT)_{\text{min}}$ curves in Fig. 4. Other germanium based experiments not reflected in the plot, such as the 0.2 kg HDMS experiment [32], do not have sufficient exposure to observe the reversal.

The DAMA experiment, which is not represented in Fig. 4, utilizes low mass NaI detectors with a high energy recoil threshold of 20 keV. The DAMA collaboration will not be able to see the phase reversal, despite its already extensive, 60,000 kg-day exposure. For the most optimistic WIMP masses, DAMA would require nearly 1000 kg-yrs of exposure to observe the WIMP phase reversal.

Liquid xenon experiments that are currently underway, such as the 40 kg ZEPLIN-II, will be sensitive to the phase reversal [31]. With a recoil energy threshold of 10 keV, the ZEPLIN-II experiment will require only 40 kg-yrs of exposure for WIMP masses near 150 GeV. For these WIMP masses, it may observe the phase reversal with only a year of continuous running time.

The upcoming, large scale LXe detectors such as ZEPLIN-IV and XENON with masses on the order of 1000 kg and energy recoil thresholds of a few keV are in a excellent position to observe the phase reversal. LXe detectors with energy recoil thresholds on the order of a few keV require 25 kg-yrs exposure for WIMP masses in the range 100-150 GeV. Hence, such experiments could observe a phase reversal with as little as a year of continuous runtime.

As seen in Fig. 4, a disadvantage of xenon based detectors is that the minimum required
exposure is low only for a narrow span of WIMP masses. Outside of the 100-200 GeV mass range, the required exposure rapidly becomes larger. The other detectors, particularly CRESST-II and GENIUS-TF, have flatter curves, i.e., the minimum required exposure does not change rapidly as a function of WIMP mass. However, future 1 tonne xenon detectors, such as the XENON and ZEPLIN-IV experiments, will have the advantage of sheer size, so that they may reach the required exposures on reasonable timescales.

4 Conclusions

The reversal of phase at low recoil energy constitutes an unambiguous signature of an incident WIMP flux. The usual annual modulation, while an important indicator of a WIMP halo, could conceivably be mimicked by Earth bound physics (such as annually modulated temperature fluctuations in the detector). It is difficult, however, to imagine non-WIMP scenarios that could engender a phase reversal only at low energy recoils. In addition, the critical recoil energy depends in a well understood way on the WIMP mass, so that an independent estimation of the WIMP mass can be obtained by locating the critical recoil energy, \( Q_c \), below which the phase reversal is observed. Furthermore, because the crossover recoil energy is a monotonically increasing function of the WIMP mass, the observation of an annual modulation in the WIMP signal will necessarily imply an upper limit to the WIMP mass.

If a June-peaking annual modulation were observed in the lowest accessible energy recoil bins of the DAMA, CDMS-II, CRESST-II, EDELWEISS-II, GENIUS-TF, ZEPLIN-II or XENON/ZEPLIN-IV detectors, one could immediately place upper bounds on the WIMP mass of [103, 48, 6, 97, 10, 52, 29] GeV, respectively, provided the experiments have sufficient exposure to observe the annual modulation at all.

The observation of a phase reversal would provide independent confirmation of the WIMP mass. The CaWO\(_4\) based detectors of CRESST-II have the lowest required exposure, on the order of 10 kg-yrs, and would require only 2 years of running time to observe the phase reversal. With 40 kg of detector, GENIUS-TF and ZEPLIN-II could observe the phase reversal with as little as a year of runtime. We note that the 1000 kg Xenon based detectors, ZEPLIN-IV and XENON, operating with recoil energy thresholds on the order of a few keV provide an excellent means of detecting the phase reversal in the differential rate for WIMP masses on the order of 100 GeV, requiring only 1 year of running time. Hence, should the annual modulation be observed, its phase reversal will provide a means to obtain the WIMP mass in these detectors.

The analysis of this paper has assumed an isothermal halo model. Galaxies, however, are thought to form hierarchically, with small substructures condensing first and subsequently merging into progressively larger structures. It is therefore reasonable to think that the dark
halo of our galaxy may be populated with clumps or streams of dark matter. Indeed, work by Stiff, Widrow, and Frieman [33] suggests that there is a high ($\mathcal{O}(1)$) probability that residual substructure exists at the solar radius, with a contribution that is an additional few percent of the local Halo density. In fact, as discussed by [34, 35], the Sagittarius dwarf galaxy, which is being tidally disrupted by the Milky Way, is pouring dark matter down upon the solar neighborhood, at the level of an additional (0.3-25)% of the local Halo density. The peak of the annual modulation and value of the crossover recoil energy will differ in halo models containing substantial clumps or streams of dark matter, or characterized by anisotropic velocity distributions [36]. In these alternate halo models, the phase and amplitude of the annual modulation are generically altered, and the impact on the crossover recoil energy must therefore be computed on a case by case basis. Such a study will be the subject of future efforts.

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