The effect of heat transfer on the stress-strain material state considering the final rate of heat spread

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Abstract. Calculation results of different wave tasks are analyzed. It is shown that deformation heating of material is intensive near the boundary surface and quickly decreases during insignificant removal from it. The conclusion is drawn that for obtaining exact solutions of wave tasks in nonisothermal statement it is necessary to consider redistribution of temperature in materials because of the heat transfer and its influence on the intense deformed condition of materials of different designs. Calculation results are given. The sensitivity of the decision to change of specific heat capacity and the thermal diffusivity entering wave equation of heat conductivity is noted.

1. Introduction

One of sections of applied mechanics is studying of dynamic loading of materials of different designs. Such loading can be shock and be followed by the advance of waves of tension in materials their heating and the subsequent redistribution of heat. Often in the works connected with heat release it is supposed that the speed of distribution of heat in metals is much less than the speed of distribution of different types of waves of tension to them. However, it is not. According to [1], the rate of heat spread in steel, \( \omega = 1800 \text{ m/s} \), on the other hand, the speed of stress spread wave in steel is: longitudinal waves in rods - \( C_0 = 5100 \text{ m/s} \) [2], radial longitudinal shearing waves, \( C_1 = 4200 \text{ m/s} \) [1], radial pressure waves, \( \alpha = 5400 \text{ m/s} \) [3]. It can be seen that in one-time interval the waves of mechanical disturbances spread 2.2 ... 3 times further into the material depth from the loaded surface than thermal disturbances. Calculation results in different wave tasks showed [1-3], that deformation heating of material is intensive near the boundary surface and quickly decreases during insignificant removal from it. An intense drop in temperature near the boundary surface leads to the subsequent heating of the material to a considerable depth, caused by heat transfer. In this case, the loaded surface can be considered how the area, where thermal disturbances are generated. At the same time not only stress-strain state parameters, but also mechanical properties and characteristics of materials are changed.

2. Problem definition

In figure 1 the phase plane \( \Delta R0t \) is represented. The straight line 1 corresponds to the line of the characteristic direction along which the wavefront of mechanical perturbations extends (for example, radial pressure waves). The straight line 2 corresponds to the wavefront of thermal perturbations. It is visible that by the time \( t \) it is warm from border \( R = R_0 \) will extend to 1/3 distances, the radial pressure which passed the wavefront of tension, and can significantly change the intense stress-strain state in zone I.
Thus, for obtaining exact solutions of wave tasks in nonisothermal statement it is necessary to consider redistribution of temperature in materials because of the heat transfer and its influence on the intense deformed condition of materials of different designs [4-5].

3. Theory
In the classical theory of heat conductivity for definition of temperature distribution in plate material with the hole when heating the hole surface, the Fourier's equation is used [1]:

\[
\frac{\partial T}{\partial t} = a \left[ \frac{\partial^2 T}{\partial R^2} + \frac{1}{R} \frac{\partial T}{\partial R} \right],
\]

where \(T\) – is the material temperature at a distance \(R\) at time \(t\); \(a\) – coefficient of thermal diffusivity. This equation is received in the assumption that the speed of distribution of heat in material is infinite therefore it is impossible to use it for determination of temperature in the point together with the solution of the wave problem of distribution of mechanical perturbations. The authors of [1] used an equation of the form (1) at the solution of the problem of temperature redistribution in material because of the heat transfer for the purpose creating of dependence \(T_{\text{max}} = T_{\text{max}}(Z)\) in the rod, on which the blow was made, after the end of wave processes and irrespective of them, influence of the heat transfer on the solution of the considered task, did not consider. For the joint solution of problems of distribution of mechanical and thermal waves in materials, it is necessary to use wave equations of heat conductivity. The most known of them is the equation [3]:

\[
\frac{\partial T}{\partial t} + \frac{a}{\omega^2} \frac{\partial^2 T}{\partial t^2} = \frac{a}{\omega^2} \left[ \frac{\partial^2 T}{\partial R^2} + \frac{1}{R} \frac{\partial T}{\partial R} \right]
\]

(2)

In the last equation member, additional in relation to the equation (1).

\[
\frac{a}{\omega^2} \frac{\partial^2 T}{\partial t^2}
\]

The equation (2) is second kind equation in partial derivatives of hyperbolic type and can be solved by method of characteristics. Let's enter the following support functions:

\[
Y = \frac{\partial T}{\partial t} \quad \text{and} \quad U = \frac{\partial T}{\partial R}
\]

Then the system of two differential equations of first order in partial derivatives is equivalent to the equation (2):

\[
\begin{align*}
\frac{1}{\omega^2} \frac{\partial Y}{\partial t} &= \frac{\partial U}{\partial R} + \frac{U}{R} \\
\frac{\partial U}{\partial t} &= \frac{\partial Y}{\partial R} - \frac{\omega^2}{a} U
\end{align*}
\]

(3)

**Figure 1.** Phase plane \(\Delta R0t\)
Supplementing the system (3) equations of total differentials of the unknown Y and U functions and equating zero system determinants, received the equations of characteristics and the ratio between required functions along them which register as:

\[ dR = \pm \omega dt \]

\[ \pm dU - \frac{1}{\omega} dY \pm \frac{\omega^2}{a} U dt + \omega \frac{U}{R} \]

Let's consider process of loading of the plate the waves of tension of radial pressure arising, for example, at the penetration of openings of radius \( R_0 \) [7]. Motion equation for the elemental annulus of mass of the plate registers as [8]:

\[ \rho \frac{\partial V_R}{\partial t} = \frac{\partial \sigma_R}{\partial R} + \frac{\sigma_R - \sigma_v}{R} \]

where \( \rho \) – is pipe material density; \( V_R \) – is the radial speed of particles of material of the plate. Deformations \( \varepsilon_R \) and \( \varepsilon_v \) and the speed \( V_R \) are connected among themselves by consistency relations:

\[ \frac{\partial \varepsilon_R}{\partial t} = \frac{\partial V_R}{\partial R} \]

\[ \frac{\partial \varepsilon_v}{\partial t} = \frac{V_R}{R} \]

As indicial equations for material of the pipe the generalized Hooke's law which in the conditions of the considered task registers as is used:

\[ \frac{\partial \varepsilon_R}{\partial t} = \frac{1}{E(T)} \left( \frac{\partial \sigma_R}{\partial t} - \mu \frac{\partial \sigma_v}{\partial t} \right) \]

\[ \frac{\partial \varepsilon_v}{\partial t} = \frac{1}{E(T)} \left( \frac{\partial \sigma_v}{\partial t} - \mu \frac{\partial \sigma_R}{\partial t} \right) \]

\[ \frac{\partial \varepsilon_z}{\partial t} = -\mu \frac{1}{E(T)} \left( \frac{\partial \sigma_R}{\partial t} + \frac{\partial \sigma_v}{\partial t} \right) \]

where \( E(T) \) – is the variable elastic modulus depending on temperature: \( E(T) = E^* - k_1 T \); \( E^*, k_1 \) – are the material constants defined experimentally. Boundary conditions of the task are written down in the look:

\[ \sigma_R(R_0, t) = \sigma_{R_0} \]

where in quality \( \sigma_{R_0} \) is taken as a predetermined number.

The given initial conditions correspond not to stress of material of the plate.

The equations (5)–(7) also represent the system of quasilinear differential equations in partial derivatives of first order of hyperbolic type which together with initial and boundary conditions completely and unambiguously describe the stress-strain state of the plate material and distribution process in it radial pressure waves.

The decision of the system of equations was passed by method of characteristics [5]. Expressions of communication between required functions on the characteristic directions are received:

- along characteristic \( dR = 0 \):

\[ d\varepsilon_v = \frac{V_R dt}{R} \]
\[
d\varepsilon_v = \frac{\sigma_v - \mu \sigma_R}{E} \, dt
\]
\[
d\varepsilon_R = \frac{\sigma_R - \mu \sigma_v}{E} \, dt
\]
\[
d\varepsilon_Z = -\mu \frac{\sigma_R + \sigma_v}{E} \, dt
\]
\[
d\Delta = \pm \frac{E(T)}{\rho(1-\mu^2)} \, dt = \pm \alpha dt
\]

Figure 2 shows in the phase plane the combined grid of characteristics for the conjugate solution of systems of equations (5 - 7) from the problem of radial pressure waves, and equation (4) – from the solution of the wave equation (2) of heat conduction.

The choice of the step for the solution of the equations (4) is secondary and is carried out from the condition that lines of characteristics (4) passed through nodal points of characteristics of the equations (8). When using as plate material from steel 1017 (USA) \((\omega = 1800 \, \text{m/s}, \, a = 5400 \, \text{m/s})\), the front of the thermal wave occurs through points of phase plane (1.1), (4.2), (7.3)..., \((j+3, i+1)\). It should be noted that similar decisions can be constructed also for other wave tasks.

4. Discussion of results

Figure 3 shows snapshots of the temperature distribution along the radius of the plate at a fixed point in time \(t = 3.3 \times 10^{-6} \, \text{s}\). The problem was solved at \(T_0 = 180^\circ \text{C}, \, \sigma_{R0} = 400 \, \text{MPa}, \) radius of the hole \(R_0 = 0.005 \, \text{m}\). The analysis of the effect of heat transfer during loading on the temperature distribution at the specific heat capacity of the plate material, \(C = 250 \, \text{J/(kg} \cdot \text{K)}\) is carried out for different thermal diffusivities \(\alpha\) :

- the curve 1 is constructed without the heat transfer \((\alpha = 0)\),
- the curve 2 – is constructed with \(\alpha = 0.024 \, \text{m}^2/\text{s}\),
- the curve 3 – is constructed with \(\alpha = 0.033 \, \text{m}^2/\text{s}\).
It is visible that deformation heating of material is intensive near the boundary surface and quickly decreases during insignificant removal from it (the curve 1), intensive falling leads to the subsequent warming up of material caused by the heat transfer on considerable depth (curves 2 and 3). And increase in the thermal diffusivity $\alpha$ leads to increase in heat extraction from the boundary surface.

![Figure 3. Influence of the heat transfer on distribution of temperature in the plate in the fixed instants](image)

In figure 4 moment pictures of profiles of waves of radial tension at $t = 15 \cdot 10^{-6}$ s with are represented at $R_0 = 0.01$ m. Shaped lines correspond to the solution of the task in isothermal statement, and continuous - to the decision in nonisothermal statement. It is visible that not accounting of heat production distorts the solution of the task, and it collects in time.

![Figure 4. Profile snapshots of radial stress waves](image)

In addition, it can be concluded that underestimation of heat transfer significantly overestimates the level of real stresses. In this case, the difference reaches 20%.

5. Conclusions
Numerical simulation of wave deformation of the plate material with allowance for heating is carried out. It is shown that it is necessary to take into account the redistribution of temperature during heat transfer to obtain a refined decision. The nonlinear influence of the specific heat and the thermal diffusivity, included in the wave equation of thermal conductivity, on the temperature distribution and the stress-strain state of the material is noted.
6. References
[1] Baranov V L and Lopa I V 1989 Radial waves of torsion and longitudinal shift in a thick elastic and visco-plastic plate in no isothermal statement News of higher educational institutions. Mechanical engineering 7 pp 27–30
[2] Baranov V L and Lopa I V 1993 Longitudinal it is elastic visco-plastic waves in rod stock of final length // News of higher educational institutions. Mechanical engineering 1 pp 54
[3] Baranov V L and Lopa I V 1990 Radial waves of pressure in thermo - a resilient and visco-plastic plate News of higher educational institutions. Mechanical engineering 2 pp 16–19
[4] Baranov V L and Lopa I V 1995 Instability of with great dispatch loaded rod stock News of higher educational institutions. Mechanical engineering 1-3 pp 45
[5] Proskuriakov N E and Lopa I V 2017 The monitoring of pipeline strength in dynamic loading AIP Conference Proceedings 020095
[6] Proskuriakov N E and Lopa I V 2016 Calculation of Spindle of Pipeline Fittings on the Longitudinal Stability Procedia Engineering 152C pp 265–269
[7] Avraamov G V, Baranov V L, Gevorkyan A M and Lopa I V 1996 Multi-layered armour Pat. 2064647 C1 RU
[8] Baranov V L, Zubachev V I, Lopa I V and Schitov V N 1996 Some problems of design of bullets for small arms Tula State University Publisher, 192 p.