Effect of trap modulation on the Raman signal of Brownian particle in optical tweezers

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Abstract: Based on the overdamped equation of motion of an optically trapped particle in a modulated harmonic potential, we estimate the fluctuation in the Raman signal due to the Brownian motion. Our results show that the Raman power is dependent on the ratio of trap-stiffness and drag coefficient, which is determined by the size of trapped particle and viscosity of the medium. Hence, it is possible to obtain information about the size and other hydrodynamic properties of the trapped particle by modulating the laser power, in addition to the molecular properties obtained from the Raman signal.

1 Introduction

Since the seminal report of optical trapping and manipulation of micron-sized particles by exploiting intensity gradient of a highly focused Gaussian beam \cite{1}, there has been numerous studies of utilizing this technique to study the properties of bio molecules and single cells. Several authors have studied the elastic properties of DNA by stretching a single DNA strand attached to two optically trapped polystyrene beads \cite{2, 3, 4}, and there have also been reports of using visible and infrared lasers to trap a single cell bacteria or biological particles \cite{5, 6}. While the technique finds diverse applications in different fields ranging from biology to physics, it has also opened door to a novel method of studying Raman spectroscopy of micron-sized trapped particles to gain insights into its properties that would otherwise be inaccessible.

The combination of optical trapping and Raman spectroscopy has been a powerful tool for the characterization and identification of micro-particles with high specificity as it can be done for a particle trapped in any viscous medium. After the report of combining the two techniques as a novel method of Raman spectroscopy \cite{7}, this method has been used in studying aerosol particles \cite{8}, gas bubbles \cite{9}, living organelles \cite{10} and cells \cite{11, 12}.

As the particle-trap interaction can be approximated as a harmonic potential with a trap stiffness, there have been numerous attempts of calibrating the trap stiffness through various methods such as by measuring the displacement produced by a known external force \cite{12, 13}, or by analysing the force of thermal fluctuations by the power spectrum analysis \cite{14}. A very practical method is by periodically modulating the trap stiffness achieved by modulating the power of the Gaussian beam \cite{15}. The effects of such modulated optical trap on Brownian motion and the position variance of the trapped particle were studied both theoretically and experimentally, and the errors it can produce in force measurements were also discussed in \cite{15}. This method has been used to perform quantitative measurement of the optical force on bio molecules by calibrating the trap-stiffness \cite{16}.

Recently, Brownian motion in an optical trap has gained considerable attention \cite{15, 17, 18, 19} since understanding the effects of modulating trapping potential on the motion of Brownian particle is very important in optical traps. While there have been studies on the dynamics of Brownian particle in the modulated harmonic trap \cite{17, 19, 20}, the effect of such modulation on the Raman signal of the trapped particle has not been explored. Since Raman spectroscopy does not distinguish materials of similar chemical composition but with different macro-level physical properties (eg. size, shape), identifying the response of trap modulation on the Raman signal would in addition allow us to investigate hydrodynamic and optical properties of the trapped particle. Based on the theoretical formulation of the position variance of optically trapped particle shown in Ref. \cite{15}, our work here will theoretically estimate the variations in Raman power of the trapped particle as a function of modulation frequency.
In section 2, we discuss the theoretical background and introduce the expression for average Raman power. In section 3, we discuss the findings of our result.

2 Theoretical formulation

An optically trapped particle in a viscous medium exhibits Brownian motion so the particle has a natural tendency to diffuse away from the equilibrium position. The average time the particle will stay in the equilibrium position, also known as dwell time, which is the inverse of roll-off frequency \( \omega_0 \); determined as the ratio of trap stiffness and drag coefficient \( \kappa/\gamma \).

As the trap stiffness \( \kappa \) is directly proportional to the laser power, a sinusoidally modulated laser power results in the modulation of trap stiffness so the equation of motion given by the Langevin equation is

\[
\dot{x}(t) + \frac{\kappa}{\gamma} [1 + \epsilon \cos(\omega t)] x(t) = \sqrt{2D}\eta(t). \tag{1}
\]

Here \( x(t) \) is the trajectory of the Brownian particle, \( \omega \) is the modulation frequency, \( \epsilon \) is the modulation depth, \( -\kappa x(t) \) is the harmonic force from the trap and \( D = k_B T/\gamma \) is the Einstein’s equation relating the diffusion constant with the Boltzmann energy and drag coefficient. \( \sqrt{2D}\eta(t) \) is a stochastic Gaussian process that represents Brownian forces at absolute temperature \( T \) such that for all \( t \) and \( t' \):

\[
\langle \eta(t) \rangle = 0; \quad \langle \eta(t)\eta(t') \rangle = \delta(t - t'). \tag{2}
\]

For a spherical particle of radius \( r \), trapped in a medium of density \( \rho \), and kinematic viscosity \( \nu \), the drag coefficient approximated by the Stoke’s law is

\[
\gamma = 6\pi\rho
\nu
r. \tag{3}
\]

Following the derivation of Itô’s lemma \[21,22\], the position variance of the trapped particle satisfies the equation (see Appendix A for the derivation)

\[
\dot{\sigma}_{xx}^2(t) = 2 \left[D - \omega_0(1 + \epsilon \cos(\omega t))\sigma_{xx}^2(t) \right]. \tag{4}
\]

From the equipartition theorem the initial position variance in the \( x \) direction takes the equilibrium value of

\[
\sigma_{xx}^2(0) = \frac{k_B T}{\kappa}. \tag{5}
\]

The solution of equation (4) with the initial position variance \( \sigma_{xx}^2(0) \) is the position variance of the trapped particle. Writing the ratio of \( \sigma_{xx}^2(t)/\sigma_{xx}^2(0) \) as \( \zeta(t) \) we have

\[
\zeta(t) = 1 - 2\omega_0 \epsilon e^{-2\omega_0(t + \frac{\omega}{\omega_0 \sin(\omega t)}} \times \int_0^t e^{2\omega_0(t + \frac{\omega}{\omega_0 \sin(\omega t)}} \cos(\omega_0 t) \, dt. \tag{6}
\]

This equation can be analyzed to see the nature of position variance of the particle in the limits of high and low modulation frequencies and gain insight into how the modulation frequency affects the position variance of the trapped particle. From the simulation of equation (6) in figure [1], we see that the position variance approaches the equilibrium value \( \sigma_{xx}(0) \) in the high frequency modulation region, and the transition occurs at the roll-off frequency \( \kappa/\gamma \).

![Figure 1: Position variance as a function of modulation frequency \( \omega \) at two different modulation depths \( \epsilon = 1 \) and 0.5.](image)

Raman Power of the trapped particle

We are interested to know the nature of Raman power that can be collected from the optically trapped particle as a function of modulation frequency. For a trapped particle at position \( x(t) \), the Raman signal emitted from the particle with the Raman cross-section \( \sigma_R \) is

\[
P(x) = \sigma_R I_o e^{-\frac{x^2(t)}{\sigma_{xx}(t)}}, \tag{7}
\]

where \( I_o \) is the intensity of the laser. Assuming the same excitation and collection coefficient, the normalized Raman power is

\[
R(x) = \frac{P(x)}{\sigma_R I_o} = e^{-\frac{x^2(t)}{\sigma_{xx}(t)}}, \tag{8}
\]

where \( w(z) \) is the beam waist radius at which the field value fall to 1/e of its axial value and \( z \) is the propagation direction. Since both the excitation and collection signal is a Gaussian beam, using the Gaussian probability density function, we can evaluate the average Raman power collected to be,

\[
(R(x)) = \frac{1}{\sigma_{xx}(t)} \sqrt{\frac{\sigma_{xx}^2(t)w^2(z)}{8\sigma_{xx}^2(t) + w^2(z)}}. \tag{9}
\]

Writing \( w(z) \) in terms of the position variance at the equilibrium \( \sigma_{xx}^2(0) \)

\[
w^2(z) = \xi \sigma_{xx}^2(0), \tag{10}
\]

the average Raman power of modulated Raman signal will be (see Appendix B for alternate derivation)

\[
R_{\text{mod}} = (R(x)) = \frac{1}{8\sigma_{xx}(t)} \sqrt{\frac{\xi}{\zeta(t) + \xi}}. \tag{11}
\]
For unmodulated case, $\zeta(t) = 1$

$$R_{\text{unmod}} = \sqrt{\frac{\xi}{8 + \xi}},$$  \hspace{1cm} (12)

which is the change in Raman signal due to diffusion. Therefore,

$$\frac{R_{\text{mod}}}{R_{\text{unmod}}} = \sqrt{\frac{8 + \xi}{8\zeta(t) + \xi}}.$$  \hspace{1cm} (13)

Using equation (6) and choosing the value of $\xi$ according to the source Gaussian beam, we can analyse equation (13) to gain some insights into the nature of Raman signal with respect to the modulating frequency.

3 Results and discussion

Using the relation of the Rayleigh range of a laser source of wavelength $\lambda$ relating to the Gaussian beam waist radius at the focus $w(0)$, Numerical Aperture of the optical system to focus the beam and the refractive index of the medium, we have the relation between $w(0)$ and $\lambda$ as

$$w(0) = \frac{\lambda}{\pi NA}.$$  \hspace{1cm} (14)

For an optical trap with a source wavelength $\lambda = 785nm$, a high Numerical Aperture objective lens of $NA = 1$, the beam waist radius at the focus is $w(0) = 250nm$. Since from equation (5) the estimated equilibrium value of the position variance at the room temperature is $\sigma^2_{xx}(0) \sim 4.14 \times 10^{-16} m^2$, using $w(0) = 250nm$, we get $\xi \sim 151$ from equation (10). Using this value for $\xi$ the plot of equation (13) with respect to the ratio of modulation frequency and roll-off frequency $\omega/\omega_0$ is shown in figure (2). From figure (2) we see that in the low frequency regime i.e. when the modulation frequency is less than the roll-off frequency ($\omega < \omega_0$), there is significant fluctuations in the average Raman signal from the trapped particle. This is due to the increased position variance of the particle in the low frequency region as seen in figure (1). However, in the high frequency regime i.e. when the modulation frequency is greater than the roll-off frequency ($\omega > \omega_0$), the Raman power of the trapped particle approaches to the value of the Raman power for an unmodulated case given by equation (12). This is because the position variance of the trapped particle approaches the equilibrium value $\sigma^2_{xx}(0)$ in the high frequency region, as seen in figure (1).

Based on the arguments presented in Appendix A, we estimated the fluctuations in the Raman signal as a function of modulation frequency using equation (13) for an optical trap system with $\xi = 151$ and by setting the roll-off frequency as unity. The maximum fluctuation in the Raman power seen in the low frequency regime is approximately 38%, while in the high frequency region, the Raman power is that of the power for unmodulated case.

It can be inferred that since the Raman signal of an optically trapped particle exhibiting Brownian motion in a viscous medium, depends entirely in the particle’s roll-off frequency, which is determined by the size of the particle and viscosity of the medium, it is in principle possible to extract useful information such as the size of the particle and its hydrodynamic properties, by studying its Raman signal.

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5 Appendix

Appendix A

The overdamped equation of motion of the trapped particle is

$$\dot{x}(t) + \frac{\kappa}{\gamma} [1 + \epsilon \cos(\omega t)] x(t) = \sqrt{2D} \eta(t),$$  \hspace{1cm} (15)

where $\kappa/\gamma = \omega_0$ is the particle’s roll-off frequency. Comparing equation (15) with the stochastic differential equation of the form

$$dX_t = \mu_t dt + \sigma_t dB_t,$$  \hspace{1cm} (16)

we have $\mu = -\omega_0 (1 + \epsilon \cos \omega t)x_t$ and $\sigma = \sqrt{2D}$. Here $B_t$ is a Wiener process [21]. To calculate the position variance of the Brownian motion described by equation (15) we apply Itô’s lemma [22] to calculate the mean of
Taking $f = x^2$, $df$ is calculated as

$$\frac{\partial f}{\partial t} \mu_t \frac{\partial f}{\partial x} + \sigma_t^{2} \frac{\partial^2 f}{2 \partial x^2} \right) \right) dt + \sigma_t \frac{\partial f}{\partial x} dB_t. \tag{17}$$

Here $\langle \frac{\partial f}{\partial t} \rangle = 0$, $\langle \mu_t \frac{\partial f}{\partial x} \rangle = -2\omega_o (1 + \cos \omega t)\sigma_{xx}^2(t)$ and $\langle \sigma_t^{2} \frac{\partial^2 f}{\partial x^2} \rangle = 2D$ and $\langle \sigma_t \frac{\partial f}{\partial x} dB_t \rangle = 0$ as $\langle dB_t \rangle = 0$. So the position variance is

$$d\langle x^2 \rangle = d\sigma_{xx}^2(t) = -2\omega_o (1 + \cos \omega t)\sigma_{xx}^2(t) dt + 2D Dt.
\sigma_{xx}^2(t) = 2 \left[D - \omega_o (1 + \cos \omega t)\sigma_{xx}^2(t) \right]. \tag{18}$$

### Appendix B

Here we present an alternate derivation of the Raman signal for a modulated Gaussian beam using the stochastic differential equation.

The Taylor’s expansion of the normalized Raman power (equation 5) is:

$$R(x) = 1 - \frac{4 x^2}{w^2(z)} \frac{192 w^4(z) x^4}{4! \times w^{10}} - \ldots \tag{19}$$

Taking the first two terms, we write $f = 1 - \frac{4 x^2}{w^2(z)}$. Since $\mu_t = -\omega_o (1 + \cos \omega t)x_t$ and $\sigma_t = \sqrt{2D}$, $\langle \mu_t \frac{\partial f}{\partial x} \rangle = 8\omega_o (1 + \cos \omega t)\frac{\sigma_{xx}^2(t)}{w^2(z)}$ and $\langle \sigma_t \frac{\partial f}{\partial x} dB_t \rangle = 0$ as $\langle dB_t \rangle = 0$. Using equation (17), we have

$$d \left(1 - \frac{4 x^2}{w^2(z)} \right) = \frac{8 \omega_o \sigma_{xx}^2(t) (1 + \cos \omega t) - D}{w^2(z)} dt. \tag{20}$$

Using equations (6) and (10) to express in terms of $\zeta(t)$ and $\xi$ we have,

$$\frac{d}{dt} \langle R'(x) \rangle = \frac{8 \omega_o}{\xi} [(1 + \cos \omega t)\zeta(t) - 1]. \tag{21}$$

Now to compare this expression with our equation of the normalized Raman power for the modulated case, we take the time derivative of equation (11).

$$\frac{d}{dt} \langle R(x) \rangle = \frac{8 \omega_o}{\xi} [(1 + \cos \omega t)\zeta(t) - 1] \left(\frac{\xi}{\xi + 8\zeta(t)}\right)^\frac{3}{2}. \tag{22}$$

Since equations (21) and (22) differs by a factor $\left(\frac{\xi}{\xi + 8\zeta(t)}\right)^\frac{3}{2}$, for an optical trap system with the $\xi$ value such that, the factor $\left(\frac{\xi}{\xi + 8\zeta(t)}\right)^\frac{3}{2} \sim 1$, equation (13) would be a good approximation to estimate the fluctuations of the average Raman signal for the modulated case.

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