The mass–concentration relationship of virialized haloes and its impact on cosmological observables

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ABSTRACT

A generic property of the cuspy simulated virialized haloes in cold dark matter (CDM) cosmogonies is that their concentration is inversely correlated with their mass. This behaviour has also been confirmed in observations, although differences in the exact form and dispersion of this so-called mass–concentration relationship have been reported. Some observational studies of massive haloes suggest that they are statistically overconcentrated with respect to the expectations of ΛCDM. Here we investigate the impact that various published mass–concentration relationships, both from simulations and derived from observations, would have on other cosmological observables, in particular considering upcoming surveys. We find that an integral measure of lensing shear, such as counts of peaks from haloes, is very sensitive to the relationship between mass and concentration at fixed $\sigma_8$, and the disparity between some reported fits is much larger than the impact of uncertainty in $\sigma_8$ itself. We also briefly assess the impact of baryonic physics on cluster scale observables, using state-of-the-art simulations, concluding that it is unlikely to give rise to the high concentrations reported for some clusters.

Key words: gravitational lensing: weak – galaxies: haloes – cosmological parameters.

1 INTRODUCTION

Galaxy clusters are the most massive bound structures in the universe, some with masses in excess of $10^{15} M_{\odot}$. Many observables related to clusters are sensitive cosmological probes. The evolution of the abundance of rich clusters has a strong dependence on $\sigma_8$, the amplitude of mass fluctuations on the scale of $8 h^{-1}$ Mpc (e.g. Fan, Bahcall & Cen 1997), and the covariance of cluster counts is very sensitive to primordial non-Gaussianity (Oguri 2009; Cunha, Huterer & Dore 2010). Furthermore, our understanding of the interplay between baryonic and dark matter in clusters, and of cluster formation and evolution can be improved by comparing observations of clusters with those formed in cosmological simulations that incorporate baryonic physics (e.g. Mead et al. 2010).

Numerical simulations of structure formation in cold dark matter (CDM) cosmogonies reveal that virialized dark matter haloes, with masses spanning many orders of magnitude, possess a cusped universal mass density profile, independent of the initial density fluctuations and cosmological parameters (Navarro, Frenk & White 1996). The density profile progressively steepens from the centre to the virial radius $r_{\text{vir}}$, with an isothermal slope at the scale radius $r_s$. A measure of the mass concentration of haloes is given by $c_{\text{s}} = r_{\text{vir}} / r_s$. This concentration parameter shows a good correlation with virial mass, both in simulations and in observations, and in CDM there is a generic trend that more massive haloes are less concentrated. Wang & White (2009) have demonstrated that there is also a correlation between mass and concentration in virialized haloes formed in hot dark matter (HDM) cosmogonies, but with more massive haloes tending to be more concentrated. Given that in CDM and HDM structure formation proceeds either via hierarchical aggregation or monolithic collapse, respectively, their work indicates that mergers are not the dominant mechanism responsible for the universal profiles and distributions.

Some analyses of rich galaxy clusters, in particular using gravitational lensing data, have shown concentrations higher than expected in ΛCDM, e.g. MS 2137–03 (e.g. Gavazzi et al. 2003), Cl 0024+1654 (e.g. Kneib et al. 2003) and the very well-studied Abell 1689 (e.g. Broadhurst et al. 2005; Umetsu & Broadhurst 2008; also see Corless, King & Clowe 2009 for a compilation of estimates from various works). Generally speaking, factors that may contribute to this apparent excess in concentration include having a triaxial halo with the major axis close to the line of sight (Oguri et al. 2005; Gavazzi 2005; Corless & King 2007), where by adopting a spherical model (circularly symmetric in projection) in the analysis one can bias parameter estimates and underestimate error bars. Projection of other structures close to the line of sight can similarly result in apparently high concentration (e.g. King & Corless 2007). Observational results are often compared with dark matter simulations, but baryons – with their more complex physics – are dominant in the centres of galaxy clusters, galaxy groups and massive galaxies. One important process due to the presence of baryons, is that...
dark matter haloes may undergo contraction (Eggen, Lynden-Bell & Sandage 1962; Blumenthal et al. 1986; Gnedin et al. 2004) resulting in significant deviation from the form arising in dissipationless collapse.

Differences in the exact form of the so-called mass–concentration (hereafter M–c) relationship have been reported, both measured in CDM simulations and estimated from observations (e.g. Bullock et al. 2001; Comerford & Natarajan 2007; Duffy et al. 2008; Macciò, Dutton & van den Bosch 2008; Mandelbaum, Seljak & Hirata 2008; Oguri et al. 2009; Okabe et al. 2010; Prada et al. 2011). For example, considering only galaxy clusters whose radial profiles are well constrained from combining weak and strong lensing data, Oguri et al. (2009) note a 7σ excess of the concentration of mass compared with expectations in ΛCDM, even including an enhancement for any bias in the samples of lensing clusters (e.g. Hennawi et al. 2007). Mandelbaum et al. (2008) performed a statistical analysis to estimate the mass–concentration relation over three orders of magnitude in mass using the stacked weak lensing signals of galaxies, groups and clusters in Sloan Digital Sky Survey (SDSS). Their analysis used weak lensing data in order to avoid the central regions of haloes where baryons dominate, and to minimize errors from mis-estimation of clusters’ centres. They find a M–c relationship with a slope consistent with simulations, but an amplitude that is about 2σ below theoretical expectations.

In order to make progress in understanding observed departures in the relationship between mass and concentration – beyond that expected from intrinsic scatter, and from known observational biases – in this paper we start by considering how particular M–c fits derived from observations or measured in CDM simulations would impact on other cosmological observables in upcoming surveys. The Dark Energy Survey (DES1), for example, is expected to contain ∼30 000 000 galaxies out to z ∼ 1 and ∼20 000 clusters in excess of 2 × 1014 M⊙, thus containing a wealth of information on virialized objects. The ingredients of the halo model formalism, used in the predictions, are summarized in Section 2, in particular the M–c relationships used in the later sections. The resulting cosmic shear power spectra and the lensing selected halo counts are considered in Sections 3 and 4, respectively. In Section 5 the impact of baryons on cluster-scale mass profiles is explored, in comparison with analytic profiles of that mass and with concentrations derived from M–c relationships. We summarize and discuss our findings in Section 6. Throughout we assume a flat cosmological model, and a dark energy equation of state parameter w = −1.

2 THE HALO MODEL DESCRIPTION OF THE MATTER POWER SPECTRUM

In the Press & Schechter model (Press & Schechter 1974), all matter in the universe is structured in virialized dark matter haloes. Under the halo model formalism, the halo mass function and halo clustering strength, as well as the distribution of mass within the haloes is used to estimate statistical properties of the density field (McClelland & Silk 1977; Scherrer & Bertschinger 1991; Peacock & Smith 2000; Seljak 2000; Scoccimarro et al. 2001; Cooray & Sheth 2002) which readily allows estimation of the impact of the properties of haloes on observables (e.g. Hamana, Yoshida & Suto 2002; Neyrinck & Szapudi 2007; Fedeli & Moscardini 2010; Pielorz et al. 2010).

1 http://www.darkenergysurvey.org

The matter power spectrum \( P(k) \), at wavenumber \( k \), is the sum of one halo, \( P^{1h}(k) \), and two-halo, \( P^{2h}(k) \), terms:

\[
P(k) = P^{1h}(k) + P^{2h}(k) .
\]

The one-halo term depends on the distribution of mass in individual haloes and dominates on small scales:

\[
P^{1h}(k) = \int \frac{dM}{dM} \frac{M}{\rho_0} (\frac{M}{\rho_0})^2 |y(k, M)|^2 ,
\]

where \( M \) denotes halo mass and \( dN/dM \) is the halo mass function, giving the number of haloes in the mass range \( M \rightarrow M + dM \) at a certain redshift, per unit volume. \( \rho_0 \) is the present mean mass density of the universe and \( y(k, M) \) is the Fourier transform of the matter density profile of haloes, normalized by their mass.

The two-halo term arises from elements in distinct haloes:

\[
P^{2h}(k) = P^{lin}(k) \int dM \frac{dN}{dM} \frac{M}{\rho_0} |y(k, M)b_h(M)|^2 ,
\]

where \( P^{lin}(k) \) is the linear matter power spectrum, and \( b_h(M) \) is the halo bias that takes account of massive haloes clustering more strongly than low-mass haloes.

2.1 Halo mass function and bias model

The halo mass function and bias are taken from Sheth & Tormen (1999). The mass function is given by

\[
\frac{dn}{dM} = \frac{\rho_0}{M} f(v) dv = \frac{\rho_0}{M} A \left[ 1 + \frac{1}{(av)^p} \right] \left( av \right)^2 \exp^{-av^2/2} dv,
\]

where \( v \equiv \left[ \frac{\delta_c(z)}{\sigma(M, z)} \right]^2 \),

where \( \delta_c(z) \) is the cosmology-dependent critical value of the overdensity required for spherical collapse at \( z \), for which we use a fitting function (see appendix of Henry 2000 and references therein)

\[
\delta_c(z) = \frac{3(12\pi^2)^{1/3}}{20} \left[ 1 - 0.0123 \log(1 + X^3) \right] ;
\]

\[
X = \left( \frac{\Omega_m - 1}{1 + z} \right)^{1/3}
\]

appropriate for \( \Omega_m = 0 \). \( \sigma(M, z) \) denotes the variance of the linear density field, spherical top-hat smoothed in spheres enclosing mass \( M \), i.e. within radius \( R(M) \equiv (3M/4\pi\rho_0)^{1/3} \). The coefficients \( a = 0.707 \) and \( p = 0.3 \) are fit from \( N \)-body simulations, and \( A \) is a normalization constant obtained by the requirement that all mass is in haloes:

\[
\rho_0 = \int dM M \frac{dN}{dM} .
\]

Haloes are biased tracers of the background dark matter field, being associated with peaks of the initial density field. Under the peak-background split ansatz (Kaiser 1984; Bardeen et al. 1986; Cole & Kaiser 1989; Mo & White 1996), the large-scale halo bias can be related to the halo mass function; for the Sheth–Tormen mass function this reads

\[
b_h(M) = 1 + \frac{av - 1}{\delta_c} + \frac{2p}{\delta_c \left[ 1 + \left( av \right)^p \right]} .
\]
2.2 Halo mass density profile

The NFW profile (Navarro et al. 1996) is taken as the fiducial halo matter density profile that enters as an ingredient in the halo model via its Fourier transform. The mass density $\rho$ as a function of radius $r$ is given by

$$\rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2},$$

where $\rho_s$ is the matter density at the scale radius $r_s$. The concentration parameter $c$ is given by $c = r_v/r_s$, where $r_v$ is the virial radius of the halo (roughly equivalent to $r_{200}$, the radius within which the mean enclosed density is 200 times the critical density of the universe at the redshift at which the halo virialized). The virial mass of a halo with the NFW profile is

$$M_{\mathrm{vir}} = \frac{4\pi\rho_s r_s^3}{c^3} \left[ \log(1 + c) - \frac{c}{1 + c} \right],$$

which, from the spherical collapse model, is equivalent to

$$M_{\mathrm{vir}} = \frac{4}{3} \pi r_s^3 \delta_{\mathrm{vir}}(z) \rho_0$$

with $\delta_{\mathrm{vir}}$ being the virial overdensity. A fitting function for the virial overdensity is (Eke, Cole & Frenk 1996; Nakamura & Suto 1997)

$$\delta_{\mathrm{vir}} = 18\pi^2 \left( 1 + 0.4093 X^{2.71572} \right),$$

where $X$ is defined as in equation (5). The mass-normalized Fourier transform of a density profile $\rho(r)$, out to the virial radius is given by

$$y(k, M) = \frac{1}{M} \int_0^{r_{\mathrm{vir}}} \frac{d r \rho(r) r^4 r^2 \sin(kr)}{kr}$$

and for the truncated NFW profile this reads (e.g. Scoccimarro et al. 2001):

$$y(k, M) = 4\pi \rho_s r_s^3 \left[ \cos(kr_s) [\mathrm{Ci}(kr_s(1+c)) - \mathrm{Ci}(kr_s)] + \sin(kr_s) [\mathrm{Si}(kr_s(1+c)) - \mathrm{Si}(kr_s)] - \frac{\sin(kr_s c)}{kr_s(1+c)} \right],$$

where $\mathrm{Ci}$ and $\mathrm{Si}$ are cosine and sine integrals, respectively,

$$\mathrm{Si}(z) \equiv \int_0^z \frac{\sin(t)}{t} \, dt; \quad \mathrm{Ci}(z) \equiv -\int_z^{\infty} \frac{\cos(t)}{t} \, dt.$$

For simplicity and ease of comparison with other work, we focus on the NFW profile here, although higher resolution simulations of halo formation show a softening of the profile with a logarithmic slope that decreases with radius, the so-called Einasto profile (Stadel et al. 2009). In Section 5 deviations from the NFW profile due to the inclusion of baryons are considered.

The lensing properties of the non-truncated and truncated NFW halo are obtained by integrating the density profile along the line of sight to obtain the surface mass density, $\Sigma$, often given in terms of a scaled projected distance from the centre $x \equiv r'/r_s$ where $r'$ is the projected distance. This yields the convergence, $\kappa(x)$, when scaled by the critical surface mass density $\Sigma_{\mathrm{crit}}$ that depends on the lens and source distances. Since the surface mass density is circularly symmetric, the shear $\gamma$ can be obtained using the relationship $\gamma(x) = \kappa(x) - \kappa(x)$, where $\kappa(x)$ is the mean convergence inside $x$.

2.3 Mass–concentration relationships

The relationship between the mass and the concentration of virialized haloes has been determined by various authors using N-body simulations or observational data. Here several representative fits from the literature are adopted, working in terms of virial concentrations and masses as noted above. Where necessary, normalizations are recast relative to a halo of $10^{14} h^{-1} M_{\odot}$ for ease of comparison. In Fig. 1 the $z = 0$ $M_{\mathrm{vir}} - \sigma_{\mathrm{vir}}$ fits are collated.

(i) Using N-body simulations carried out assuming the Wilkinson Microwave Anisotropy Probe (WMAP) 5-year cosmological parameters (Komatsu et al. 2009), Duffy et al. (2008) derived the mean concentration of their full sample of haloes (including relaxed and unrelaxed haloes) as a function of mass and redshift:

$$c_{\mathrm{DUFFY}}(M, z) = \frac{5.72}{(1 + z)^{0.71}} \left[ \frac{M_{\mathrm{vir}}}{10^{14} h^{-1} M_{\odot}} \right]^{-0.081},$$

where $\Omega_m = 0.26$, $\Omega_{\Lambda} = 0.74$, $h = 0.72$ and $\sigma_8 = 0.8$. All their fits used a pivot mass of $M_{\mathrm{pivot}} = 2 \times 10^{14} h^{-1} M_{\odot}$ in order to minimize the covariance between the normalization and the exponent of the mass dependence. The 1σ confidence intervals on normalization, exponent of mass dependence and exponent of redshift dependence are roughly 2, 7 and 6 per cent of their fit values.

(ii) Comerford & Natarajan (2007) presented an observed $M$–$c$ relation, drawing values from the literature (both lensing and X-ray determinations for mass and concentration), as well as using newly determined masses and concentrations for 10 strong lensing clusters. Where necessary they converted their compiled values to a flat ΛCDM cosmology with $\Omega_m = 0.3$, $\Omega_{\Lambda} = 0.7$ with $h = 0.7$. Using this extensive sample of 62 clusters they fit a concentration normalization, and an exponent for the mass dependence, keeping an inverse scaling with $(1 + z)$ for their fit and using a pivot mass of $M_{\mathrm{pivot}} = 1.3 \times 10^{15} h^{-1} M_{\odot}$:

$$c_{\mathrm{COM}}(M, z) = \frac{10.7}{1 + z} \left[ \frac{M_{\mathrm{vir}}}{10^{15} h^{-1} M_{\odot}} \right]^{-0.15},$$

with an error of ≈40 per cent on the normalization and ≈85 per cent on the exponent of the mass dependence.

(iii) Oguri et al. (2009) considered an observed sample of 10 lensing clusters whose profiles are well fitted by a combined weak

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Figure 1. $M$–$c$ relationships at $z = 0$ as described in the text.
and strong lensing analyses. Assuming the WMAP 5-year cosmological parameters as above, and taking the mass and redshift scaling from the fit to N-body simulations of Duffy et al. (2008):

$$c_{\text{OGURI}}(M, z) = 14.55 \left(\frac{1 + z}{4} \right) \Omega_m \int_0^{z_0} \frac{d\chi}{f_K(\chi)} \frac{P_{3D}(\hat{\ell} \cdot \hat{f}_K(\chi), \chi)}{a^2(\chi)} W^2(\chi) d\chi. \tag{19}$$

They note that there is a $7\sigma$ excess of the concentration parameter above the prediction from the $\Lambda$CDM simulations, when the 10 clusters with weak and strong lensing analysis are combined.

(iv) Okabe et al. (2010) used Subaru data to carry out weak lensing analyses of 30 X-ray selected galaxy clusters from the Local Cluster Substructure Survey. Throughout their analysis they assumed $\Omega_m = 0.27, \Omega_\Lambda = 0.73, h = 0.72$, consistent with Komatsu et al. (2010). The $M-c$ relationship was derived from the 19 spectroscopically confirmed clusters with observations in two filters, that are well fitted by the NFW profile

$$c_{\text{OKABE}}(M, z) = 8.75 \left[ \frac{M_{\text{vir}}}{10^{14} h^{-1} M_\odot} \right]^{-0.4}. \tag{18}$$

The authors note that none of the massive clusters in this sample show very high concentrations, as has been reported for some clusters. They also point out that the scaling of concentration parameter with mass is tentatively steeper than predicted in simulations, though there is an error of $\approx 50$ per cent on the mass exponent. For the purposes of Fig. 1, we include an inverse dependence of $c$ on $(1 + z)$ and rescale to $z = 0$, given the mean redshift of their clusters.

3 WEAK LENSING CONVERGENCE POWER SPECTRUM

The power spectrum of the lensing convergence is obtained from a weighted integral of the 3D matter power spectrum as described in e.g. Bartelmann & Schneider (2001):

$$P_\kappa(\ell) = \frac{9}{4} \left( \frac{H_0}{c} \right)^4 \Omega_m^2 \int_0^{z_0} d\chi P_{3D} \left( \frac{\ell}{f_K(\chi)}, \chi \right) \frac{W^2(\chi)}{a^2(\chi)} d\chi. \tag{19}$$

where $\chi$ is the comoving distance, $z_0$ is the comoving distance to the horizon, and $f_K(\chi)$ is the comoving angular diameter distance.$^3$ The scale factor of the universe $a(\chi)$ is normalized to unity today. The wave vector $k$ is related to the angular wave vector $\ell$ and $f_K(\chi)$ through $k = \ell f_K(\chi)$. The 3D matter power spectrum is calculated using the halo model formalism as described in Section 2. $W(\chi)$ is the weighting function that takes into account the redshift distribution of the sources from which the weak lensing signal is being measured, along with the relative geometry of these sources and the density fluctuations responsible for lensing:

$$W(\chi) = \int_x^{z_0} d\gamma G(\chi) f_K(\chi - x) f_K(\chi). \tag{20}$$

where $G(\chi)$ is the normalized source distance distribution. Note that the lower integration limit arises since only density fluctuations at $\chi < x$ contribute to the convergence.

We consider sources distributed in redshift according to a normalized probability distribution (Brainerd, Blandford & Smail 1996):

$$p(z) = \frac{\beta z^2}{\Gamma(3/\beta) z_0^{3/\beta}} \exp \left[ - (z/z_0)^{3/\beta} \right]. \tag{21}$$

$^2$ PI: Smith; http://www.str.bham.ac.uk/focus

$^3$ Since the spatial curvature of the universe is taken to be $K = 0$ it follows that $f_K(\chi) \equiv 1$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.pdf}
\caption{The fiducial redshift distribution corresponding to equation (21) with $\xi \approx 1.05$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.pdf}
\caption{The dimensionless convergence power spectra for the Duffy et al. (lower solid line), Comerford & Natarajan (upper heavy dashed line), Oguri et al. (shorter dashed line) and Okabe et al. (dotted line) $M-c$ relationships with their respective cosmologies (and using the same $\sigma_8 = 0.8$ for the three observational relationships).

where $z_0$ is related to the depth of the observations and $\beta$ controls the fall-off at high redshift. For example, as in Lombardi, Schneider & Morales-Merino (2002) when $\beta = 1.5$ and $z_0 = 0.7$, the distribution has $\bar{\chi} \approx 1.05$; this is taken to be the fiducial distribution and is shown in Fig. 2. The mean redshift is representative of current cosmic shear surveys.

In Fig. 3 the cosmic shear power spectrum for each of the prescriptions for the $M-c$ relationships is plotted, assuming the fiducial redshift distribution. The uncertainty in the power spectrum, assuming Gaussian errors, is given by Kaiser (1998)

$$\Delta P_\kappa(\ell) = \sqrt{\frac{2}{(2\ell + 1) \Delta f_{\text{sky}}}} \left( P_\kappa(\ell) + \frac{\gamma_{\text{int}}^2}{\bar{n}} \right). \tag{22}$$

where $f_{\text{sky}}$ is the fraction of the sky covered by the survey, the mean-square intrinsic ellipticity is $\langle \gamma_{\text{int}}^2 \rangle$ and the number of galaxies per steradian for which an ellipticity can be measured is $\bar{n}$. The first term is the cosmic variance, dominant on large scales, and the second term is Poisson noise that dominates on small scales. We take $\langle \gamma_{\text{int}}^2 \rangle^{1/2} = 0.4$.

We now focus on the Duffy et al. (from simulations) and Oguri et al. (from observations) power spectra, since they assume the same

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cosmology in their analysis, and are representative of relationships reported in the literature. Fig. 4 reproduces the power spectra from Fig. 3, now with error bars corresponding to a survey of 5000 deg² and (i) a number density of 10 arcmin⁻², characteristic of the upcoming DES, and (ii) to a futuristic survey with the same area but a number density of 165 arcmin⁻² representative of deep space-based observations (e.g. the Hubble Deep Field as used in Huterer 2002). In practice, band power estimates of the power spectrum would be made, for which we have taken 13 equal logarithmic width bins between \( l = 10 \) and \( 8 \times 10^3 \) as in Schneider et al. (2002). The error estimate of equation (22) correspondingly scales as \( \sqrt{T/\Delta l} \). As noted above, this expression assumes Gaussian errors, whereas there is also a non-Gaussian contribution arising from mode-coupling due to non-linear clustering that becomes important at \( l \sim 1000 \) for a survey of the depth we consider (e.g. Scoccimarro, Zaldarriaga & Hui 1999), and from the statistical fluctuations in the number of haloes of a certain mass which are sampled in a finite survey volume (Hu & Kravtsov 2003). As noted by Scoccimarro et al. (1999), the line-of-sight projection relevant for weak lensing tends to reduce the importance of non-Gaussianity compared with what is expected in 3D. Takada & Jain (2009) estimate a factor of \(~2\) degradation in the cumulative signal-to-noise ratio (S/N) of the power spectrum amplitude. We return to this point in Section 6 when the results are discussed.

4 HALO COUNTS

Kruse & Schneider (1999) have shown that the weak lensing selected number counts of haloes is a sensitive probe of cosmology, and this was tested by Reblinsky et al. (1999) using ray-tracing through N-body simulations. Other authors have further explored various aspects of cluster detection using weak lensing (e.g. Hamana et al. 2004; Dietrich & Hartlap 2010; Mariano, Smith & Bernstein 2010; see also the overview in the introduction of Mariano et al. 2010), and searched for mass concentrations in data (e.g. Wittman et al. 2006; Gavazzi & Soucail 2007; Schirmer et al. 2007).

We now estimate the number counts due to the most massive haloes, adopting the Duffy et al. and Oguri et al. \( M-c \) relationships. Again we assume that haloes can be described by NFW profiles, truncated at their virial radii. Selecting haloes by integral measures of their gravitational lensing shear is sensitive not only to their mass function, but to how their mass is distributed, so one expects a difference in halo counts that depends on how concentrated a halo is at fixed mass. We consider a Gaussian smoothed convergence map, that results in haloes with peak convergence \( \kappa_\text{p} \) after smoothing with a filter of scale \( \theta_\text{f} \). We ask how many peaks would have a S/N \( v \) above a threshold \( v_\text{th} \), where \( v \) is defined as \( \kappa_\text{p}/\sigma_\text{th} \), and \( \sigma_\text{th} \) is the noise inside the same filter arising from having a finite number density of galaxies (\( \bar{n} \)) from which to measure the lensing signal, and from their intrinsic ellipticities (Kaiser & Squires 1993)

\[
\sigma_\text{th}^2 = \frac{\bar{n}_\text{int}^2}{4\pi \bar{\theta}^2 \bar{\theta}_\text{th}^2}.
\]

For sources at redshift \( z \), haloes with and an assumed \( M-c \) relationship, smoothing on a scale of \( \theta_\text{f} \) and a threshold of \( v_\text{th} \)

\[
N(v > v_\text{th}) = \frac{1}{4\pi} \int d\chi \frac{dV}{d\chi} \int dM \frac{dn}{dM} \mathcal{H}(v(M, c, z) - v_\text{th}),
\]

where \( dV/d\chi \) is the comoving volume element, \( \mathcal{H}(x) \) is the Heaviside step function (\( \mathcal{H}(v(M, c, z) - v_\text{th}) = 1 \) if \( x > 0 \)) through which the dependence on the \( M-c \) relation enters. We consider a survey of 5000 deg² and a number density of 10 galaxies per arcmin² usable for shear measurements, with the data Gaussian smoothed on a scale of 1 arcmin, appropriate to the scale of massive haloes. In practice a filter that is better matched to the profile expected could also be employed as in e.g. Bartelmann, King & Schneider (2001), or as discussed in Section 6.2 an optimal filter tailored to minimize confusion of real haloes with large scale structure, e.g. Maturi et al. (2005), but for the purposes of illustrating the differences between \( M-c \) relationships, Gaussian smoothing suffices. We first assume that sources are at each of \( z = 0.5 \) and 1.0 and fix the cosmology at the WMAP 5-year parameters (\( \sigma_8 = 0.8 \)) that were used in the Duffy et al. simulations, and adopted in the Oguri et al. analysis. The results are shown in Fig. 5.

Assuming a scatter in the \( M-c \) relations of \( \sigma(\log M) = 0.15 \) as estimated in Duffy et al. (2008), Fig. 6 shows predictions for \( \pm \sigma \) offsets in the mean relations, now focusing on sources at \( z = 1 \). Finally, the comparative sensitivity to \( \sigma_8 \) is also illustrated in Fig. 6 with predictions for the Duffy et al. \( M-c \) relationship, and now taking \( \sigma_8 = 0.77, 0.83 \), an offset roughly consistent with the current \( 2\sigma \) bounds from Komatsu et al. (2010).

We keep sources at \( z = 1 \), and for \( \sigma_8 = 0.8, \Omega_m = 0.26 \) consider the dependence of counts on various of the fit parameters, for a
Figure 5. The number of peaks detected above a threshold $S/N = \nu$, for the Duffy et al. and Oguri et al. $M-c$ relationships for a source number density of 10 arcmin$^{-2}$ and sky coverage of 5000 deg$^2$, assuming sources at $z = 1$ and 0.5; the key shows the $M-c$ relationship and source redshift, with the middle line of each set corresponding to the estimated number of detections and Poisson errors indicated by the error bars.

Figure 6. The number of peaks detected above a threshold $S/N = \nu$, for the Duffy et al. and Oguri et al. $M-c$ relationships for a source number density of 10 arcmin$^{-2}$ and sky coverage of 5000 deg$^2$, assuming sources at $z = 1$. For the WMAP 5-year parameters ($\sigma_8 = 0.8$) the lower (upper) shaded regions indicate $\pm 1\sigma$ offsets in concentration normalization (from intrinsic scatter) in the Duffy et al. (Oguri et al.) $M-c$ relationships. For the Duffy et al. relation, the dotted lines indicate the impact of offsets in $\sigma_8$ approximately corresponding to the $2\sigma$ bounds ($\sigma_8 = 0.77, 0.83$) from Komatsu et al. (2010).

The general $M-c$ relationship

$$c(M,z) = \text{cnorm} \left(1 + z\text{power} \left(\frac{M}{10^{14} M_\odot} \right) \text{mpower} \right).$$

Note that this is for the purposes of illustrating how the counts would vary for changes in two of the parameters, and that some of the combinations are rather extreme. For a threshold $\nu_{th} = 4$ the counts are shown in Fig. 7.

5 BARYONIC PHYSICS

The presence of baryons will cause differences in the distributions of concentrations measured from dark matter only simulations, and derived from observational data. One important process is baryonic cooling that can lead to adiabatic contraction of dark matter haloes, significantly modifying their mass distributions (Eggen et al. 1962; Blumenthal et al. 1986; Gnedin et al. 2004; White 2004).

Baryonic physics is likely to be most important for lower mass objects ($\lesssim 10^{14} M_\odot$), due to their shorter cooling time-scale (Rees & Ostriker 1977; Silk 1977; White & Rees 1978). A stark illustration of this is provided by the failure to reproduce the image separation distribution of galaxy-scale gravitational lens systems, when the mass function is estimated using Press–Schechter theory (or its extensions), normalized to match the local abundance of massive clusters (see Kochanek & White 2001 and references therein). Based on gravitational lensing and stellar dynamics constraints from the Sloan Lens ACS Survey (SLACS; e.g. Bolton et al. 2006) massive ($\sim L_*$) early-type galaxies – that tend to be central galaxies in...
a group or cluster, or lack more luminous neighbours – on average have isothermal density profiles on scales of a few kpc, with a small scatter of \( \lesssim 10 \) per cent (e.g. Koopmans et al. 2009. This is well inside the scale radius of a typical object in their sample, where in the absence of baryons the profile is expected to be flatter than isothermal. Satellite galaxies – with more luminous companion(s) – tend to have marginally steeper mass–density profiles (e.g. Auger 2008; Treu et al. 2009), which from N-body simulations is consistent with tidal truncation by an external potential (Dobke, King & Fellhauer 2007). Schulz, Mandelbaum & Padmanabhan (2009) fit NFW profiles to stacked weak lensing measurements in the outskirts of SDSS early-type galaxies, and find that the mass inside the half light radius directly obtained from stellar velocity dispersion observations far exceeds what is expected from the extrapolation of the weak lensing-derived profile, even when the contribution of stars is also accounted for. This is consistent with modification of dark matter profiles by baryons.

Focusing on cluster scales, since these objects contribute to the halo counts discussed in Section 4, we now illustrate the differences between having dark matter only and baryonic physics [with cooling and star formation, and with and without active galactic nucleus (AGN) feedback]. We use the total mass profiles of a \( \sim 3 \times 10^{14} \, M_\odot \) galaxy cluster from the simulations of Puchwein, Sijacki & Springel (2008) simulated with different types of physics, using the numerical prescription described in Sijacki et al. (2007). The stellar mass fraction of the run with AGN feedback is consistent with observations (Puchwein et al. 2010). Without AGN feedback the stellar mass fraction in the cluster’s centre is too high, and the contraction of the dark matter is greater than seen in the simulations with feedback. Other authors have also recently noted that when feedback is included at a level that yields stellar mass fractions consistent with observations, the central density is less enhanced than previously thought. This has been confirmed on group scales by Duffy et al. (2010), where in fact the concentration parameters in the runs with baryons can actually be less than in the dark matter runs.

In Fig. 8 we plot the the mass-normalized fast Fourier transform (FFT), \( \gamma \), of the spherically averaged density profile for each of the dark matter and baryonic physics runs without and with AGN feedback. We also plot the FFTs of analytic NFW profiles with the same virial radius, but with concentrations calculated using the mean for clusters of that mass from the Duffy et al. and Oguri et al. \( M-c \) relationships.

6 DISCUSSION AND CONCLUSIONS

Haloes formed in CDM simulations have concentrations that are anticorrelated with mass. This \( M-c \) relationship has been reported for both simulations and observations, with some differences in the normalization and in the evolution with redshift. We compiled several \( M-c \) relationships from the literature, and then focused on two representative fits from simulations (Duffy et al. 2008) and observations (Oguri et al. 2009) that have the same assumed cosmology. Our aim was to examine how, if objects in the universe are described by one of these forms, this would manifest in other cosmological observables measured in future surveys. The predictions have been undertaken in the framework of the halo model.

6.1 Cosmic shear power spectrum

In Section 3, the cosmic shear power spectra were derived for a near-future survey (with approximately the characteristics of DES) and a more futuristic space-based survey, for haloes following the Duffy et al. and Oguri et al. \( M-c \) relationships (see Fig. 4). Since the assumed cosmology was the same for each of these, on large scales the signals are identical. Differences in halo properties are manifest above \( l \sim 1000 \) where, as noted earlier, non-Gaussian errors are likely to be important. With the typical \( ~2 \), or more pessimistic \( ~5 \) degradation in the cumulative S/N expected (Takada & Jain 2009), the normalization of the \( M-c \) relationship would be difficult to directly constrain, until deep space-based data become available (lower panel of Fig. 4). However, as proposed by Zentner, Rudd & Hu (2008), self-calibration of cosmic shear tomographic data could be used to simultaneously extract information on halo structure as well as on cosmological parameters, in the regime where \( l < 3000 \), until non-Gaussian errors dominate. Given that the differences in the normalizations of the two relationships considered here are apparent at \( l \sim 1000 \), it could well be that combining tomography with pushing to higher \( l \) would allow useful constraints on the \( M-c \) relationships for even the shallow survey. Zentner et al. note that the concentrations of \( 10^{14} \, h^{-1} \, M_\odot \) haloes at \( z \sim 0.2 \) could be constrained to better than 10 per cent.

6.2 Halo counts

The numbers of lensing shear-selected mass peaks detected above a certain threshold were estimated in Section 4. These turn out to be rather disparate for the two \( M-c \) relations considered, already in the near-future survey as shown in Fig. 5. In Fig. 6 we show the results for the peak counts assuming that the normalizations of the mean relations are offset by \( \pm 1 \sigma \), based on the intrinsic dispersion that likely reflects formation history. For the Duffy et al. relation, we also considered changes in \( \sigma_8 \) from 0.8 to 0.77 and 0.83, at a level consistent with the 2\( \sigma \) bound from Komatsu et al. (2010), so changing the halo mass function. The impact on number counts of this departure in \( \sigma_8 \) is much less than the disparity between the different \( M-c \) relations. Our fiducial cosmological parameters are fixed at WMAP 5-year values (for consistency with the \( M-c \) relations), with errors on \( \sigma_8 \) consistent with the current 7-year
values of Komatsu et al. (2010); the errors on parameters will be substantially smaller after the Planck mission.4

In this work we have employed a Gaussian filter, applied to the convergence field. In practice, one would filter the reduced shear, the direct observable from the ellipticities of weakly lensed galaxies. Various filters have been developed and applied to simulations and to real data. Schneider (1996) and Schneider et al. (1998) proposed the aperture mass statistic, with a compensated weight function that gives an optimized S/N for approximately isothermal mass profiles. Padmanabhan, Seljak & Pen (2003), Schirmer et al. (2004) and Schirmer (2004) suggested filter functions that more closely follow the NFW profile. Hennawi & Spergel (2005) developed a tomographic matched filtering scheme for cluster detection, that in addition uses redshift information for the source galaxies. With and without tomographic information, they consider the performance of the Gaussian filter, of the aperture mass statistic – with a commonly used polynomial filter from the family in Schneider et al. (1998), of the Padmanabhan et al. (2003) filter approximately tuned to the NFW profile, and of the latter modified by an exponential cut-off to minimize the impact of large-scale structure. Using cosmological simulations to generate weak lensing data, for the case without tomographic information, Hennawi & Spergel (2005) find that at moderate detection significance (S/N ~ 3.5–4.5) the truncated NFW filter is most effective at cluster detection, followed by the Gaussian filter. For the highest detection significance, the aperture mass statistic performs best. For a small range of detection S/N, the Gaussian filter performs best.

Unless the S/N threshold is chosen to be high (~4–5 depending on the specifications of the survey), or equivalently for high-mass clusters, the contamination from other structure close to the line of sight leads to the detection of significant spurious peaks in weak lensing data (e.g. Reb .insky & Bartelmann 1999; Metzler, White & Loken 2001; White, van Waerbeke & Mackey 2002; Hamana, Takada & Yoshida 2004; Hennawi & Spergel 2005). Optimal filtering for the detection of clusters, as proposed by Maturi et al. (2005), dramatically reduces the contribution from spurious peaks arising from the large scale structure and from other noise due to the intrinsic ellipticity dispersion and positions of galaxies. More recently, Maturi et al. (2010) developed an analytic method suitable for quantifying the degree to which spurious detections are present in a weak lensing survey, for various types of filters. Crucially, by using N-body simulations, they also confirm that using optimal filters renders the contribution from spurious peaks due to large scale structure low on all filter scales, increasing the number of real detections by up to an order of magnitude. For the analysis of data from a particular survey, an estimate of spurious peaks could be made using the formalism of (Maturi et al. 2010, table A.1), or by the more time consuming optimal filtering of synthetic weak lensing data sets derived from ray-tracing through cosmological N-body simulations, with appropriate observational factors for the survey such as the number density of galaxies and their redshift distribution from which the shear signal can be estimated. What one should also keep in mind is that, as discussed by Hamana et al. (2004) and Fan, Shan & Liu (2010), peaks arising due to noise are also more likely to appear in the vicinity of real clusters. Further, in the context of cosmological parameter constraint, which is not the subject of this paper, it has been shown by Dietrich & Hartlap (2010) and Wang, Haiman & May (2009) that the number and properties of all the peaks detected in a weak lensing survey, including spurious peaks due to large-scale structure, also contain important information.

Now to illustrate why differences in concentration would impact on the shear-selected peak counts, we consider the optimal S/N of haloes as a function of their concentration, at fixed mass. In the limit of a filter that is exactly matched to the halo profile, and neglecting large-scale structure noise (so concentrating on the most significant cluster mass peaks), the S/N is given by, e.g. Bergé, Amara & Réfrégier (2010),

\[ \frac{\nu}{\nu_*} = \frac{\sqrt{\ln(1+c)} - \frac{c}{4+c}}{\ln(1+c) + \frac{c}{4+c}} \sqrt{G(c)} \]

which they show for an NFW profile is

\[ \nu \propto \rho_{\nu} \sqrt{G(c)} \]

for a given lens redshift and source redshift (or redshift distribution), and number density of source galaxies, with

\[ G(c) = \frac{1}{c^2} \left( \frac{1 - \frac{3}{c^2} + \frac{3}{c^2} \ln(1+c)}{\frac{3}{c^2} + \frac{3}{c^2} \ln(1+c) - \frac{3}{c^2}} \right) \]

\[ \nu^2 \propto \frac{1}{c} \left( \frac{1 - \frac{3}{c^2} + \frac{3}{c^2} \ln(1+c)}{\frac{3}{c^2} + \frac{3}{c^2} \ln(1+c) - \frac{3}{c^2}} \right) \]

In a given cosmology (fixed critical density, and overdensity required for collapse) since

\[ \nu \propto \rho_{\nu} \sqrt{G(c)} \]

and comparing haloes with the same mass (and \( r_{200} \)), the ratio of the S/N of haloes with different concentrations is

\[ \frac{\nu}{\nu_*} = \frac{\sqrt{\ln(1+c) - \frac{c}{4+c}}}{\ln(1+c) + \frac{c}{4+c}} \sqrt{G(c)} \]

which gives the S/N of a halo, \( \nu_* \), compared with a fiducial halo, \( \nu_\star \). In Fig. 9 this is plotted with respect to the concentration of the Oguri et al. 10^{14} h^{-1} M_\odot halo; note that the Duffy et al. halo of the same mass would yield \( \approx 65 \) per cent of the S/N due to lower concentration.

With what accuracy can the parameters of the mass–concentration relation potentially be constrained using the peak counts, for a DES-like survey? Let us assume that the Duffy et al. relation is the true, fiducial, description. Considering sources to be at \( z = 1 \), we estimate the range of parameters that would be consistent with the ‘observed’ peak counts, by running different grids of parameters and considering the most significant peaks with \( \nu > 4.5 \). To approximately account for cosmic variance, the error bars in Fig. 5 are doubled, although the statistics of highly significant peaks is dominated by

\[ \text{Figure 9. The S/N with which a NFW halo of concentration } c \text{ detected, } \nu, \text{ relative to a halo of concentration } c_\star = 14.55, \nu_\star, \text{ as a function of } c. \]
shot noise (Hu & Kravtsov 2003). From Fig. 5, it is already clear that the halo counts from the fiducial model would be inconsistent with an Oguri et al. $M$–$c$ relation. Given these observational error bars, and keeping the mass and redshift dependence of the $M$–$c$ relation fixed, the concentration normalization can be determined to within ~7 per cent. Fixing the normalization and redshift dependence, the data would be inconsistent with a flat dependence on halo mass, but would accommodate a relation that is ~30 per cent flatter or steeper in $M_{\text{vir}}$. Finally, for the evolution with redshift, fixing the normalization and mass dependence, the observed counts are inconsistent with zero evolution of the $M$–$c$ relation with redshift, but consistent with an evolution that is ~30 per cent flatter or steeper in $(1+z)$.

In practice one would explore which method of peak detection was best suited to the data set, either by application to simulations, or by testing the recovery of known lower mass clusters. Peak counts look promising as a technique that complements the determination of the $M$–$c$ relation from detailed studies of individual clusters (e.g. Comerford & Natarajan 2007; Oguri et al. 2009; Okabe et al. 2010), or from statistical studies (e.g. Mandelbaum et al. 2008), in particular when the uncertainty on cosmological parameters would be further reduced by Planck and supporting data.

6.3 The recent extension to the halo model by Giocoli et al.

Recently, Giocoli et al. (2010) extended the halo model formalism to include realistic substructure within individual haloes, and to include scatter in halo concentration at fixed mass. With their prescription for substructure, as motivated by the findings of $N$-body simulations (e.g. Gao et al. 2004), the predicted power on very small scales is increased, bringing the matter power spectrum to include scatter in halo concentration at fixed mass. With their model, the extension to the halo model formalism of Giocoli et al. (2010) may be especially pertinent to properly accounting for the weak lensing signal due to low-mass haloes, as they note, and predicting the counts of low-mass haloes. For our comparison of the difference between peak counts for the Duffy et al. and Oguri et al. $M$–$c$ relations, any linear structure would be the same in both cases, and the higher normalization $M$–$c$ relation yields more peak detections. This could then be further enhanced by the inclusion of substructure. For the detection of the highest significance peaks, where the weak lensing signal is dominated by massive cluster haloes, the contrast against the large-scale structure is less of an issue. In addition, simulations show that substructure itself changes the weak lensing signal of these massive haloes by only a few per cent (King, Schneider & Springel 2001). Assessing the impact of this extended halo model formalism on weak lensing peak counts is beyond the scope of this paper, and will be the subject of future work. At that point, estimating the accuracy with which the parameters of the mass–concentration relationship can be estimated could be undertaken with a Fisher matrix analysis, also incorporating the (photometric) redshift distribution of sources from which the weak lensing signal is being measured, and combining information from the measurement of the cosmic shear power spectrum for example.

As discussed further by Giocoli (2010) their extended halo model with the inclusion of substructure is of much importance in predicting the flexion signal of haloes, and in the investigation of strong-lensing properties such as universal magnification invariants. Giocoli (2010) also note that their formalism could be further refined, for example to include the impact of non-linear bias and to include halo shapes.

6.4 Non-Gaussianity

Some degree of non-Gaussianity in the primordial density field can arise in even the simplest inflationary models as discussed in e.g. Liddle & Lyth (2000), and we now briefly compare this with the impact of $\sigma_8$. Commonly non-Gaussianity is quantified in terms of the parameter $f_{\text{NL}}$ such that the Bardeen potential $\Phi$ can be expressed as the sum of a Gaussian random field $\Phi_\text{G}$, and a quadratic departure term: $\Phi = \Phi_\text{G} + f_{\text{NL}}(\Phi_\text{G}^2 - \langle \Phi_\text{G}^2 \rangle)$. The tightest constraints currently come from CMB data – using WMAP 7-year data, Komatsu et al. (2010) find $13 < f_{\text{NL}} < 97$. Grossi et al. (2007) carried out dark matter simulations with various values of $f_{\text{NL}}$, and found that only modest departures from the halo mass function arise even for large positive or negative values of $f_{\text{NL}}$. In the context of weak lensing statistics, and using the same set of simulations, Pace et al. (2011) show that non-Gaussianity also has a minimal impact: for values of $f_{\text{NL}}$ within the current bounds, the impact on the cosmic shear power spectrum is well within the uncertainty due to the current bounds on $\sigma_8$. Pace et al. (2011) also remark that since non-Gaussianity is comparatively much more effective at small scales, the scale-dependence of observables can be used to aid parameter constraint in future surveys. Indeed, as noted earlier, it has recently been shown that the covariance of cluster counts is very sensitive to primordial non-Gaussianity (Oguri 2009; Cunha et al. 2010). Although beyond the scope of this paper, it would be interesting to examine the extent to which the uncertainty on the $M$–$c$ relation could be reduced by self-calibration of shear-selected peak counts (similar to the scheme of Zentner et al. 2008) binned in redshift, and allowing for non-Gaussianity at a level consistent with forthcoming Planck data.

6.5 Baryonic physics

The impact of baryons on dark matter haloes as seen in simulations, compared with the change in profiles due to changing the halo concentration to be consistent with $M$–$c$ relationships at the same mass, was considered in Section 5. In Mead et al. (2010) we demonstrated that simulated clusters with AGN feedback are less efficient strong lenses (specifically in their production of giant arcs) than clusters simulated without AGN feedback, and are in fact similar to dark matter only haloes. Here we considered the $\sim 3 \times 10^{14} M_\odot$ cluster from the simulations of Puchwein et al. (2008), also used in Mead et al. (2010) where the stellar mass fraction is in very good agreement with observations, in the case where AGN feedback is accounted for alongside other baryonic processes. As shown in Fig. 8, the corresponding FFTs of these density profiles that enter into the halo model predictions illustrate that the simulated model with AGN feedback has much less power on small scales than either the simulated model without AGN feedback (i.e. where there is overcooling), or of the analytic model with a concentration at the mean of the Oguri et al. $M$–$c$ relationship. From this illustration, and from the general findings of Mead et al. (2010) and Puchwein et al. (2010) we emphasize that for the observational measurements of the $M$–$c$ relation made on cluster scales, or correspondingly for individual clusters with very high concentrations, baryons are unlikely to be a significant culprit in yielding results that greatly differ from the expectations of $\Lambda$CDM. Note that the number of simulated clusters, groups and galaxies currently available is too small to specify a global $y(k, M)$ form, but an interim
possibility that we defer to later work would be to calibrate an analytic form, such as that used in Giulet, Teyssier & Colombi (2010), using a fairly large set of high-resolution simulations that reproduce the density profiles and stellar mass fractions from observations.

6.6 Conclusions
Understanding the physical processes, or the observational selection effects, that give rise to some clusters having seemingly high concentrations, and to tension in $M-c$ relations derived from various types of observations, will be possible in the near future. In particular, we have shown that the counts of haloes obtained using weak lensing observations, and the cosmic shear power spectrum are sensitive to the $M-c$ relation. Alongside the exciting preparations for observational surveys such as DES, much progress is being made in developing tools that are very useful in better understanding the implications of observations. For example, during the work for this paper, Giocoli et al. (2010) presented an extension to the halo model formalism that incorporates substructure in haloes, as well as allowing for scatter in halo concentration at fixed mass.

It will be very interesting to compare constraints on mass profiles and their concentrations from measurements of both weak and strong lensing by individual selected clusters from upcoming surveys, from carrying out statistical studies in DES such as that in SDSS of Mandelbaum et al. (2008), to explore using tomographic cosmic shear with self-calibration (Zentner et al. 2008) and from strong lensing by individual selected clusters from upcoming surveys, from carrying out statistical studies in DES such as that in SDSS of Mandelbaum et al. (2008), to explore using tomographic cosmic shear with self-calibration (Zentner et al. 2008) and from measurements of weak lensing peak counts.

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