Response characteristics of a beam-mass system with general boundary conditions under compressive axial force and accelerating masses

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Summary
In this study the problem of the dynamic characteristics of a structurally prestressed beam under compressive axial force and subjected to accelerating loads is investigated. A procedure based on Galerkin's residual method, asymptotic method of struble, and integral transformation method is developed to solve the fourth-order partial differential equation with variable and singular coefficients governing the dynamic behavior exhibited by the beam-mass system. The proposed solution procedure is very versatile and is suitable for handling moving mass beam problem for all pertinent boundary conditions. The theory and analysis proposed in this work are illustrated by various practical examples often encounter in engineering design and practice. Analytical solutions valid for all variants of classical boundary conditions are obtained for the beam-load system. The effects of the traveling velocity of the moving mass, span length, and flexural rigidity on the response of the beam are investigated. A comparative analysis of the behavior of this structural member under accelerating, decelerating, and uniform velocity type of motion is performed. Various results in plotted curves are presented.

KEYWORDS
beam-mass system, Duhamel's integral, Galerkin's method, moving load, response characteristics

1 INTRODUCTION

The study of dynamic response of beam-like structures to moving or static loads has attracted and still attracting lot of attention due to its wide range of applications in the construction and transportation industry especially when transverse by traveling masses.1-6 Moving mass system induces vibration on the structural elements, on which it travels. The effect of this vibration and complexity of structure-load interactions have become an important issue to be considered and investigated. Stress and deformation caused by moving vehicle on a bridge is a classical example of this undesirable occurrence on such system.

One of the widely studied elastic structures is a beam; among the earliest studies on the dynamic analysis of an elastic beam under moving loads are the works of Ayre et al.7 who studied the effect of the ratio of the weight of the load to the
weight of a simply supported beam for a constantly moving mass load. They obtained the exact solution for the resulting partial differential equation using the infinite series method. Wu and Gao\(^8\) who investigated the dynamic response of a simply supported viscously damped double-beam system under moving harmonic loads; the double-beam system consisted of two finite elastic homogeneous isotropic beams, which were identical, parallel, and connected continuously by a layer of elastic springs with no viscous damping.

Abu-Hilal and Zibdeh\(^9\) who presented the dynamic response of elastic homogeneous isotropic beams with different boundary conditions subjected to harmonic force traveling with three types of motion, namely, accelerating, decelerating, and constant velocity type of motion. Gbadeyan and Oni\(^10\) who investigated an analytical solution with different options on dynamic behavior of beams and rectangular plates under moving loads. Mao\(^11\) used the Adomian modified decomposition method to investigate the free vibrations of an elastically connected parallel Euler-Bernoulli beam joined by a Winkler-type elastic layer, the general solution for general boundary conditions was obtained.

Wodek\(^12\) who also studied advanced structural dynamics and active control of structures. In addition, worthy of mention is the novel research by Omolofe et al\(^13\) who investigated the flexural motions of nonuniform deep beam resting on variable elastic foundation, transverse by harmonic variable magnitude moving loads, it was found that the critical velocity of the dynamical systems increases with increase in the foundation stiffness.

In the open literature, the problem of beam or beam-like structural members under the passage of traveling load of any form has been categorized into two, namely, the moving force and moving mass beam problems. The moving force model is the simplified model, which neglects the components of inertia term in the governing equation of motion. In particular, in moving force beam-load interaction problems; effects of the support beam acceleration at the point of contact with the moving mass, the well-known Coriolis force, the centrifugal force, and the acceleration component in the vertical direction when the moving load speed is not a constant are neglected. This class of problem has received extensive attention in literature. Prominent among these studies are the works of Frybal\(^14\) who studied nonstationary response of a beam to a moving random force. Steel\(^15\) who worked on the finite beam with moving load, Timoshenko et al\(^16\) who investigated the case of a concentrated force moving with a constant velocity along a beam when neglecting damping forces, Kurihara and Shimogo\(^17\) who considered vibration of an elastic beam subjected to discrete loads, Wang and Chou\(^18\) who studied nonlinear vibration of Timoshenko beam due to moving force and the weight of the beam, Licari and Wilson\(^19\) who investigated the dynamic response of a beam subjected to moving force system, Florence\(^20\) who worked on the problem of traveling force on a Timoshenko beam, Belotserkovskiy\(^21\) who studied the oscillations of infinite periodic beams subjected to a moving concentrated force, Iwankiewicz and Sniady\(^22\) who worked on the vibration of a beam under random stream of moving forces, Omolofe and Oseni\(^23\) who scrutinized effects of damping and exponentially decaying foundation on the motions of finite thin beam subjected to traveling loads, Omolofe and Ogunyebi\(^24\) who studied dynamic characteristics of a rotating Timoshenko beam subjected to a variable magnitude load traveling at varying speed and Omolofe et al\(^25\) who considered damping influence on the critical velocity and response characteristics of structurally prestressed beam subjected to traveling harmonic load to mention a few.

When the inertia effect of the mass of the load moving on the elastic solid body is considered, the governing differential equation of motion becomes complex, intractable, and cumbersome as the coefficients become variable and singular Awodola and Omolofe.\(^26\) This scenario is referred to as the moving mass problem. A large volume of studies has been devoted to this class of problems in literatures. Among these are the works of Low et al\(^27\) who studied experimental and analytical investigations of vibration frequencies for center-loaded beams, Low and Dubey\(^28\) who considered a note on the fundamental shape function and frequency for beams under off center load, Esen\(^29\) who worked on a new finite element for transverse vibration of rectangular thin plates under a moving mass, Low\(^30\) who investigated a comparative study of the eigenvalue solutions for mass-loaded beams under classical boundary conditions, Esen\(^31\) who studied a modified FEM for transverse and lateral vibration analysis of thin beams under a mass moving with a variable acceleration, Tso et al\(^32\) who treated circular wave motions in a plate composed of transversely isotropic materials and Esen et al\(^33\) who scrutinized finite element formulation and analysis of a functionally graded Timoshenko beam subjected to an accelerating mass including inertial effects of the mass. In almost all these aforementioned studies, applications of the solution techniques and the theories proposed are limited to the cases when the velocity or the acceleration of the traveling mass is held constant throughout its motion on the structural member it traverses. However, situation arises when the mass accelerates by a forward force or decelerates, reduces speed, and come to rest at any desired position on the structure, causing the friction between the mass and the structural elements to increase considerably. Under such condition, the vibrating system exhibits dynamic behavior, which may be more complicated.\(^34\) Among few authors in recent times who made effort to tackle the problem of the elastic beam undergoing flexural vibrations under the passage of accelerating loads are, Wang\(^35\) who studied the dynamical analysis of a finite inextensible beam with an attached accelerating mass. He employed the
Galarkin procedure in conjunction with the method of numerical integration to tackle the partial differential equations, which describe the transient vibrations of the beam-mass system. He concluded that the applied forward force amplifies the speed of the mass and the displacement of the beam. Muscolino and Palmeri investigated the response of beams resting on viscoelastically damped foundation to moving oscillators. They proposed a novel state-space formulation, in which a number of internal variables is introduced with the aim of representing the frequency-dependent behavior of the viscoelastic foundation. In their study, they figured-out that as the position of the oscillator on the beam varies with time, the dynamic matrix of the governing state-space equation becomes time dependent. Abu-Hilal and Ziddeh investigated the vibration analysis of beams with general boundary conditions traversed by a single point force traveling with variable velocity. They obtained analytical solution to the beam problem and compared the results with same beam under the actions of a concentrated force traveling at constant velocity. Their method of solution is only suitable for a moving force beam problem.

In a more recent development, Oni and Omolofe considered flexural motions under accelerating loads of structurally prestressed beams with general boundary conditions. The method of generalized integral transform in conjunction with a modification of the asymptotic method of struble was employed to treat this dynamical beam problem. Approximate analytical solutions for both the moving force and moving mass models were obtained and illustrated by various practical examples. Much later, Oni and Omolofe studied dynamic response of prestressed Rayleigh beam resting on elastic foundation and subjected to masses traveling at varying velocity. The proposed analytical procedure is illustrated with practical examples. The effects of time-varying velocity on the vibrating system are established. These studies though impressive, illustrative examples involving only simply supported boundary condition are considered. The methods of solution in both cases are not suitable for boundary conditions other than the simple ones. In all these aforementioned studies, the authors employed the simplest mechanical foundation model developed by Winkler, which is generally referred to as a one-parameter model. The deficiency of this model is that it assumes no interaction between the springs, so it does not accurately represent the characteristics of many practical foundations. Essen studied dynamic response of a beam due to an accelerating moving mass using moving finite element approximation. The author employed a numerical procedure implemented using a MATLAB code to obtain a numerical solutions. The accelerating moving mass, that is, traveling on the beam was modeled as a moving finite element in order to include inertial effects beside gravitation force of mass. Effect of longitudinal force due to acceleration of the moving mass is incorporated into the model. Dynamic response of the beam was obtained depending on the mass ratio and the acceleration of the mass. Nevertheless, it is well known that in a study such as this, analytical procedure is desirable as solution so obtained sheds more light on some vital information about the vibrating system.

This study therefore concerns the dynamic characteristics of elastic structural member resting on non-Winkler type foundation and under compressive axial force and accelerating masses. The main objective of this study is to obtain an analytical solution valid for all variants of classical boundary conditions to this problem. Comparative analysis of the response of the beam to accelerating, decelerating, and uniform velocity type of motion will be scrutinized. Illustrative examples involving various boundary conditions of practical interest will be considered. Effects of the various vital parameters on the dynamic characteristics of the beam-mass system will be investigated.

2 | PROBLEM FORMULATION

The system to be investigated is that of an elastic beam under compressive axial force undergoing flexural motion due to accelerating masses (see Figure 1 above). The beam is assumed to be of length \( L \) and modeled as a Rayleigh beam, which has homogeneous, uniform, and isotropic material properties along its span. The traveling load, which is represented by

![Figure 1](image1.png) An elastic beam carrying moving mass
a lump-mass with constant preload, is assumed to travel along the beam with time-varying velocity. The friction force between the beam and the moving mass is considered. The mass is assumed to travel in a straight line in the horizontal direction. It is further assumed that the mass is in contact with the beam during its travel and vibration. The transverse shear and the rotatory inertia effects are considered. The equation of motion, neglecting the damping effect is given by

\[
\frac{\partial^2}{\partial x^2} \left[ EI \frac{\partial^2 \mathbf{N}(x, t)}{\partial x^2} \right] - N \frac{\partial^2 \mathbf{N}(x, t)}{\partial x^2} + \frac{\partial^2 \mathbf{N}(x, t)}{\partial t^2} + K \mathbf{N}(x, t) - G \frac{\partial^2 \mathbf{N}(x, t)}{\partial x^2} = P(x, t),
\]

(1)

where, \(x\) is the spatial coordinate, \(\mathbf{N}(x, t)\) is the transverse displacement, \(t\) is the time, \(EI\) is the flexural rigidity, \(\mu\) is the mass per unit length of the beam, \(N\) is the axial force, \(K\) is the foundation constant, \(G\) is the shear rigidity, and \(P(x, t)\) is the transverse concentrated load.

It is remarked here that the boundary condition of the above stated problem is assumed to be arbitrary and without loss of generality, the initial conditions of the beam is given as

\[
\mathbf{N}(x, 0) = 0 = \frac{\partial \mathbf{N}(x, 0)}{\partial t}.
\]

(2)

The moving load on the beam under consideration has mass commensurable with the mass of the beam. Thus, the load \(P(x, t)\) is considered to be of the form

\[
P(x, t) = P_*(x, t) \left[ 1 - \frac{1}{g} \frac{d^2 \mathbf{N}(x, t)}{dt^2} \right],
\]

(3)

where the continuous moving force \(P_*(x, t)\) acting on the beam model is given by

\[
P_*(x, t) = Mg \delta[x - f(t)],
\]

(4)

\(g\) is the acceleration due to gravity and \(\frac{d^2}{dt^2}\) is a convective acceleration operator defined as

\[
\frac{d^2}{dt^2} = \frac{\partial^2}{\partial t^2} + 2 \frac{d}{dt} f(t) \frac{\partial^2}{\partial x \partial t} + \left( \frac{d}{dt} f(t) \right)^2 \frac{\partial^2}{\partial x^2} + \frac{d^2}{dt^2} \frac{\partial}{\partial x}.
\]

(5)

The position of the load on the structure is described by

\[
f(t) = x_o + ct + \frac{1}{2}at^2,
\]

(6)

where \(x_o\) is the point of application of force \(P\) at the instance \(t = 0\), \(c\) is the initial velocity, and \(a\) is the constant acceleration of motion. Moreover, since the load is assumed to be of mass \(M\) and the time \(t\) is assumed to be limited to that interval of time within which the mass is on the beam, that is,

\[
0 \leq f(t) \leq L,
\]

(7)

and \(\delta[x - f(t)]\) is the Dirac delta function.

Note that Equation (6) depict a uniform accelerating or decelerating type of motion, while for the uniform velocity type of motion, we have

\[
f(t) = ct.
\]

(8)

For simplicity, in this study, a constant elastic foundation is considered. That is,

\[
K(x) = K,
\]

(9)
using Equations (3) to (6) in Equation (1) one obtains

\[
EI \frac{\partial^4 \mathbf{N}(x, t)}{\partial x^4} - N \frac{\partial^2 \mathbf{N}(x, t)}{\partial x^2} + \mu \frac{\partial^2 \mathbf{N}(x, t)}{\partial t^2} + KN(x, t) - G \frac{\partial^2 \mathbf{N}(x, t)}{\partial x^2} + M \delta \left[x - \left(x_0 + ct + \frac{1}{2}at^2\right)\right] \left[\frac{\partial^2 \mathbf{N}(x, t)}{\partial t^2} + 2(c + at) \frac{\partial^2 \mathbf{N}(x, t)}{\partial x \partial t}\right] = Mg \delta \left[x - \left(x_0 + ct + \frac{1}{2}at^2\right)\right].
\]  

(10)

Equation (10) is the equation of motion governing the response characteristics of elastic beam subjected to compressive axial force and loads traveling with accelerated or decelerated type of motion.

Setting \(a = 0\) in Equation (10) gives the equation of motion governing the response characteristics of elastic beam subjected to compressive axial force under uniform velocity type of motion.

To solve the problem above, a versatile method popularly known as Galerkin’s method often used in problems involving mechanical vibrations will be adopted.

Thus, a sequence of linearly independent functions \(U_r(x)\), which are the normalized deflection curves for the \(r\)th mode of the vibrating beam satisfying the boundary conditions are chosen as

\[
U_r(x) = \sin \frac{\lambda_r x}{L} + A_0 \cos \frac{\lambda_r x}{L} + A_1 \sinh \frac{\lambda_r x}{L} + A_2 \cosh \frac{\lambda_r x}{L},
\]

(11)

where \(\lambda_r = \pi r\) and \(A_0, A_1, A_2\) are constant whose values depend on the boundary condition. Galerkin’s technique requires that the solution of Equation (10) be of the form

\[
\mathbf{N}_n(x, t) = \sum_{r=1}^{n} Y_r(t) U_r(x).
\]

(12)

Substituting Equation (12) into Equation (10), it is straightforward to arrive at

\[
\sum_{r=1}^{n} \left\{ \dot{Y}_r(t) + \frac{\Omega(k, r)}{H_3(k, r)} Y_r(t) + \frac{\epsilon_o}{H_3(k, r)} \left[ H_3(k, r) \dot{Y}_r(t) + 2 \sum_{n=1}^{\infty} \cos \frac{n\pi}{L} (x_0 + ct) + \frac{1}{2} at^2 \right] \right\} H_4(k, r) Y_r(t) + 2(c + at) \sum_{n=1}^{\infty} \cos \frac{n\pi}{L} (x_0 + ct) \right\}
\]

\[
= \frac{P}{H_3(k, r) \mu} \left[ \sin \frac{\lambda_r}{L} \left(x_0 + ct + \frac{1}{2}at^2\right) + A_0 \cos \frac{\lambda_r}{L} \left(x_0 + ct + \frac{1}{2}at^2\right) + A_1 \sinh \frac{\lambda_r}{L} \left(x_0 + ct + \frac{1}{2}at^2\right) + A_2 \cosh \frac{\lambda_r}{L} \left(x_0 + ct + \frac{1}{2}at^2\right) \right],
\]

(13)

where,

\[
H_3(k, r) = \int_{0}^{L} U_r^{(0)}(x) U_k(x) dx,
\]

(14)

\[
H_4(k, r) = \int_{0}^{L} U_r^{(0)}(x) U_k(x) dx,
\]

(15)
\[ H_3(k, r) = \int_0^L U_r(x)U_k(x)dx, \]  
\[ H_4(k, r, n) = \int_0^L U_r(x)U_k(x) \cos \frac{n\pi x}{L}dx, \]  
\[ H_5(k, r) = \int_0^L U'_r(x)U_k(x)dx, \]  
\[ H_6(k, r, n) = \int_0^L U'_r(x)U_k(x) \cos \frac{n\pi x}{L}dx, \]  
\[ H_7(k, r, n) = \int_0^L U''_r(x)U_k(x) \cos \frac{n\pi x}{L}dx, \]  
\[ H_8(k, r, n) = \int_0^L U'_r(x)U_k(x) \cos \frac{n\pi x}{L}dx, \]

where,
\[ \Omega(k, r) = \frac{EI}{\mu} H_1(k, r) - \frac{N}{\mu} H_2(k, r) + \frac{K}{\mu} H_3(k, r) - \frac{G}{\mu} H_2(k, r), \]  
\[ \epsilon_0 = \frac{M}{\mu L}, \]  
\[ P = Mg. \]

In what follows, analytical solution to the transformed governing Equation (13) is sought. To this end, two special cases of Equation (13) will be considered, namely; the moving force and moving mass beam problems.

3 | CASE I: THE BEAM-LOAD FORCE SYSTEM

The beam-load force system is an approximate model, which assume the inertia effect of the moving load to be negligible. Setting the mass ratio \( \epsilon_0 = 0 \) in the transformed Equation (13), one obtains
\[ \sum_{r=1}^n Y_r(t) + \frac{\Omega(k, r)}{H_3(k, r)} Y_r(t) = \frac{P}{H_3(k, r)\mu} \left[ \sin \frac{\lambda_r}{L} \left( x_0 + ct + \frac{1}{2}at^2 \right) + A_0 \cos \frac{\lambda_r}{L} \left( x_0 + ct + \frac{1}{2}at^2 \right) \right. \]  
\[ + A_1 \sinh \frac{\lambda_r}{L} \left( x_0 + ct + \frac{1}{2}at^2 \right) + A_2 \cosh \frac{\lambda_r}{L} \left( x_0 + ct + \frac{1}{2}at^2 \right) \left]. \]  

To solve the above equation, the impulse response function also known as Duhamel's integral is employed. By this technique, the solution of the moving force system Equation (25) may be expressed as
\[ Y_r(t) = \int_0^t h_r(t - \tau)Q_r \tau d\tau, \]

where \( h_r(t) \) is the impulse response function defined as
\[ h_r(t) = \begin{cases} \frac{1}{\omega_o} e^{-\frac{\tau}{\omega_o}} \sin \omega_o \tau, & t \geq 0, \\ 0, & t < 0, \end{cases} \]
and

\[ \omega_{dr} = \omega_r \sqrt{1 - \varepsilon_r^2}, \]  

(28)

is the damped circular frequency of the \( r \)th mode of the beam.

Since the effect of damping parameters are insignificant for analyzing dynamic responses,\(^3\) we set \( \varepsilon_r = 0 \), which implies \( \omega_{dr} = \omega_r \) and this leads to

\[
Y_r(t) = \frac{P_0}{m_r \omega_r} \int_0^t \sin \omega_{dr}(t - \tau) \left[ \sin K_r \left( x_0 + ct + \frac{1}{2} at^2 \right) + A_0 \cos K_r \left( x_0 + ct + \frac{1}{2} at^2 \right) \\
+ A_1 \sinh K_r \left( x_0 + ct + \frac{1}{2} at^2 \right) + A_2 \cosh K_r \left( x_0 + ct + \frac{1}{2} at^2 \right) \right] d\tau, 
\]

(29)

where

\[
K_r = \frac{\lambda_r}{L},
\]

(30)

\[
P_0 = \frac{P}{m_r \omega_r \mu},
\]

(31)

\[
m_r = H_3.
\]

(32)

Thus, in view of Equation (29) and taking into account Equation (12) one obtains

\[
\mathcal{N}_r(x, t) = \sum_{r=1}^{n} P^* \text{Re} \left[ \Phi_1 Y_2 \left\{ (\cos m_1 - i \sin m_1) \left[ \text{erf}(J_2 - J_3 t) - \text{erf}(J_2) \right] \\
- (\cos m_2 + i \sin m_2) \left[ \text{erf}(J_5 - J_3 t) - \text{erf}(J_3) \right] \right\} \\
+ \Phi_2 Y_2 \left\{ (\cos m_3 - i \sin m_3) \left[ \text{erf}(J_7 + J_8 t) - \text{erf}(J_7) \right] \\
- (\cos m_4 - i \sin m_4) \left[ \text{erf}(J_{10} + J_8 t) - \text{erf}(J_{10}) \right] \right\} \\
- \Phi_3 \sqrt{2} Y_1 \left\{ E_1 (\cos m_6 + i \sin m_6) \left[ \text{erf}(J_{12} + J_3 t) - \text{erf}(J_{12}) \right] \\
- E_2 (\cos m_8 - i \sin m_8) \left[ \text{erf}(J_{15} + J_3 t) - \text{erf}(J_{15}) \right] \right\} \\
+ \Phi_4 \sqrt{2} Y_1 \left\{ E_3 (i \cos m_{10} - \sin m_{10}) \left[ \text{erf}(J_{17} + J_3 t) - \text{erf}(J_{17}) \right] \\
+ E_4 (i \cos m_{12} + \sin m_{12}) \left[ \text{erf}(J_{19} + J_3 t) - \text{erf}(J_{19}) \right] \right\} \\
\times \left[ \sin \frac{\lambda_r x}{L} + A_0 \cos \frac{\lambda_r x}{L} + A_1 \sinh \frac{\lambda_r x}{L} + A_2 \cosh \frac{\lambda_r x}{L} \right],
\]

(33)

which is the expression describing the response of the structurally prestressed beam to accelerating or decelerating forces.

where,

\[
P^* = \frac{P_0}{8m_r \omega_r}
\]

(34)

\[
\Phi_1 = (1 - A_0) + i(A_0 + 1),
\]

(35)

\[
\Phi_2 = (1 - A_0) - i(A_0 + 1),
\]

(36)

\[
\Phi_3 = A_1 + A_2,
\]

(37)

\[
\Phi_4 = A_2 - A_1.
\]

(38)
\[ Y_1 = \sqrt{\frac{K_{r}a}{2}}, \quad (39) \]

\[ Y_2 = \sqrt{\frac{\pi}{K_{r}a}}, \quad (40) \]

\[ J_2 = \frac{1}{2 \sqrt{K_{r}a}} \left[ (\omega_r - K_{r}c) + i(K_{r}c - \omega_r) \right], \quad (41) \]

\[ J_3 = \frac{1 - i}{2 \sqrt{K_{r}a}}, \quad (42) \]

\[ J_4 = \frac{i}{2K_{r}a} \left[ K_{r}^2(2ax_0 - c^2) - 2K_{r}\omega_r(c + at) - \omega_r^2 \right], \quad (43) \]

\[ J_5 = \frac{1}{2 \sqrt{K_{r}a}} \left[ (-\omega_r - K_{r}c) + i(K_{r}c - \omega_r) \right], \quad (44) \]

\[ J_6 = -\frac{i}{2K_{r}a} \left[ K_{r}^2(2ax_0 - c^2) + 2K_{r}\omega_r(c + at) - \omega_r^2 \right], \quad (45) \]

\[ J_7 = \frac{1}{2 \sqrt{K_{r}a}} \left[ (\omega_r + K_{r}c) + i(K_{r}c + \omega_r) \right], \quad (46) \]

\[ J_8 = \frac{1 + i}{2 \sqrt{K_{r}a}}, \quad (47) \]

\[ J_9 = -\frac{i}{2K_{r}a} \left[ K_{r}^2(2ax_0 - c^2) + 2K_{r}\omega_r(c + at) + \omega_r^2 \right], \quad (48) \]

\[ J_{10} = -\frac{1}{2 \sqrt{K_{r}a}} \left[ (\omega_r - K_{r}c) + i(-K_{r}c + \omega_r) \right], \quad (49) \]

\[ J_{11} = \frac{1}{2K_{r}a} \left[ K_{r}^2(2ax_0 - c^2) + 2iK_{r}\omega_r(c + at) + \omega_r^2 \right], \quad (50) \]

\[ J_{12} = \frac{K_{r}cI + \omega_r}{\sqrt{2K_{m}a}}, \quad (51) \]

\[ J_{13} = \sqrt{\frac{K_{r}a}{2}}, \quad (52) \]

\[ J_{14} = \frac{1}{2K_{r}a} \left[ K_{r}^2(2ax_0 - c^2) + 2iK_{r}\omega_r(c + at) + \omega_r^2 \right], \quad (53) \]

\[ J_{15} = \frac{K_{r}cI - \omega_r}{\sqrt{2K_{r}a}}, \quad (54) \]

\[ J_{16} = \frac{1}{2K_{r}a} \left[ -K_{r}^2(2ax_0 - c^2) + 2iK_{r}\omega_r(c + at) - \omega_r^2 \right], \quad (55) \]

\[ J_{17} = \frac{K_{r}cI - \omega_r}{\sqrt{2K_{r}a}}, \quad (56) \]

\[ J_{18} = \frac{1}{2K_{r}a} \left[ -K_{r}^2(2ax_0 - c^2) - 2iK_{r}\omega_r(c + at) - \omega_r^2 \right], \quad (57) \]
and

\[
R_{19} = \frac{K_r c + \omega_r}{\sqrt{2K_r a}}
\]  

(58)

Similarly, no difficulty arises to show that the dynamic response of axially prestressed thin beam to forces traveling with uniform velocity type of motion is obtained as

\[
N_n(x, t) = \sum_{r=1}^{n} P^r \left[ \frac{K_r c \sin \omega_r t - \omega_r \sin K_r ct}{K_r c^2 - \omega_r^2} + A_0 \frac{\omega_r \cos \omega_r t - \omega_r \cos K_r ct}{K_r c^2 - \omega_r^2} 
+ A_1 \frac{\omega_r \sinh K_r ct + K_r c \sin \omega_r t}{K_r c^2 + \omega_r^2} + A_2 \frac{\omega_r \cosh K_r ct - \omega_r \cos \omega_r t}{K_r c^2 + \omega_r^2} \right] 
\times \left[ \sin \frac{\lambda_r x}{L} + A_3 \cos \frac{\lambda_r x}{L} + A_1 \sinh \frac{\lambda_r x}{L} + A_2 \cosh \frac{\lambda_r x}{L} \right].
\]  

(65)

4 \quad CASE II: THE BEAM-LOAD MASS SYSTEM

When \( \varepsilon_o \neq 0 \) in Equation (13), then all the inertial terms are retained and thus leading to a moving mass problem. In this case, one is required to solve the entire second-order nonhomogeneous ordinary differential equation Equation (13). To do this, Equation (13) is written to take the form

\[
\sum_{r=1}^{n} \left\{ \bar{Y}_r(t) + \omega_{m_r}^2 \bar{Y}_r(t) + \varepsilon_o \left[ Q_2(k, r) \bar{Y}_r(t) + 2 \sum_{n=1}^{\infty} \cos \frac{n \pi}{L} \left( x_0 + ct + \frac{1}{2} at^2 \right) Q_3(k, r) \bar{Y}_r(t) 
+ 2(c + at) Q_4(k, r) \bar{Y}_r(t) + 4(c + at) \sum_{n=1}^{\infty} \cos \frac{n \pi}{L} \left( x_0 + ct + \frac{1}{2} at^2 \right) Q_5(k, r, n) \bar{Y}_r(t) 
+ (c + at)^2 Q_6(k, r) \bar{Y}_r(t) + 2(c + at)^2 \sum_{n=1}^{\infty} \cos \frac{n \pi}{L} \left( x_0 + ct + \frac{1}{2} at^2 \right) Q_7(k, r, n) \bar{Y}_r(t) 
+ aQ_8(k, r) \bar{Y}_r(t) + 2a \sum_{n=1}^{\infty} \cos \frac{n \pi}{L} \left( x_0 + ct + \frac{1}{2} at^2 \right) Q_9(k, r, n) \bar{Y}_r(t) \right] \right\}
\]  

\]
Similarly, the deflection of the beam under uniform velocity type of motion is thus obtained as

$$= \frac{\varepsilon_o L g}{H_3(k, r) \mu} \left[ \sin \frac{\lambda_r}{L} \left( x_0 + ct + \frac{1}{2} at^2 \right) + A_0 \cos \frac{\lambda_r}{L} \left( x_0 + ct + \frac{1}{2} at^2 \right) + A_1 \sinh \frac{\lambda_r}{L} \left( x_0 + ct + \frac{1}{2} at^2 \right) + A_2 \cosh \frac{\lambda_r}{L} \left( x_0 + ct + \frac{1}{2} at^2 \right) \right],$$

where

$$Q_2(k, r) = \frac{H_5(k, r)}{H_3(k, r)}, \quad Q_3(k, r) = \frac{H_4(k, r)}{H_3(k, r)}, \quad Q_4(k, r) = \frac{H_5(k, r)}{H_3(k, r)},$$

$$Q_5(k, r, n) = \frac{H_6(k, r, n)}{H_3(k, r)}, \quad Q_6(k, r) = \frac{H_2(k, r)}{H_3(k, r)}, \quad Q_7(k, r, n) = \frac{H_7(k, r, n)}{H_3(k, r)}, \quad Q_8(k, r) = \frac{H_5(k, r)}{H_3(k, r)}, \quad Q_9(k, r, n) = \frac{H_8(k, r, n)}{H_3(k, r)},$$

subjecting Equation (66) to Struble’s technique in conjunction with the impulse response function after some simplification and rearrangement one obtains

$$N_n(x, t) = \sum_{r=1}^{n} \frac{\varepsilon^* L g}{\omega_{mn}} \text{Re} \left[ \Phi_1 Y_2 \left\{ (\cos m_1 - i \sin m_1) \left[ \text{erf}(J_2 - J_3 t) - \text{erf}(J_2) \right] \right. \right.$$

$$\left. + (\cos m_2 + i \sin m_2) \left[ \text{erf}(J_5 - J_3 t) - \text{erf}(J_3) \right] \right\}$$

$$\left. + \Phi_2 Y_2 \left\{ (\cos m_3 - i \sin m_3) \left[ \text{erf}(J_7 + J_3 t) - \text{erf}(J_7) \right] \right. \right.$$

$$\left. - (\cos m_4 - i \sin m_4) \left[ \text{erf}(J_{10} + J_3 t) - \text{erf}(J_{10}) \right] \right\}$$

$$\left. - \Phi_3 \sqrt{2} Y_1 \left\{ E_1(\cos m_6 + i \sin m_6) \left[ \text{erf}(J_{12} + J_3 t) - \text{erf}(J_{12}) \right] \right. \right.$$

$$\left. - E_2(\cos m_8 - i \sin m_8) \left[ \text{erf}(J_{15} + J_3 t) - \text{erf}(J_{15}) \right] \right\}$$

$$\left. + \Phi_4 \sqrt{2} Y_1 \left\{ E_3(\cos m_{10} - i \sin m_{10}) \left[ \text{erf}(J_{17} + J_3 t) - \text{erf}(J_{17}) \right] \right. \right.$$

$$\left. + E_4(\cos m_{12} + i \sin m_{12}) \left[ \text{erf}(J_{19} + J_3 t) - \text{erf}(J_{19}) \right] \right\}$$

$$\times \left[ \sin \frac{\lambda_r x}{L} + A_0 \cos \frac{\lambda_r x}{L} + A_1 \sinh \frac{\lambda_r x}{L} + A_2 \cosh \frac{\lambda_r x}{L} \right],$$

which represents the dynamic response of Euler-Bernoulli beams on non-Winkler elastic foundation and under compressive axial force and mass traveling with accelerated or decelerated motion.

Similarly, the deflection of the beam under uniform velocity type of motion is thus obtained as

$$N_n(x, t) = \sum_{r=1}^{n} \frac{\varepsilon^* L g}{\omega_{mn}} \left[ K_c \sin \omega_r t - \omega_r \sin K_c \omega_r t + A_0 \frac{\omega_r \cos \omega_r t - \omega_r \cos K_c \omega_r t}{K_c \omega_r ^2 - \omega_r ^2} \right.$$  

$$\left. + A_1 \frac{\omega_r \sinh K_c \omega_r t + K_c \sin \omega_r t}{K_c \omega_r ^2 + \omega_r ^2} + A_2 \frac{\omega_r \cosh K_c \omega_r t - \omega_r \cos \omega_r t}{K_c \omega_r ^2 + \omega_r ^2} \right]$$

$$\times \left[ \sin \frac{\lambda_r x}{L} + A_0 \cos \frac{\lambda_r x}{L} + A_1 \sinh \frac{\lambda_r x}{L} + A_2 \cosh \frac{\lambda_r x}{L} \right],$$

where

$$\varepsilon^* = \frac{\varepsilon_o}{1 + \varepsilon_o},$$

$$\varepsilon_o = \frac{M}{\mu L}.$$
and

\[
\omega_{nm} = \omega_{mf} \left\{ 1 - \frac{\xi}{2} \left[ Q_2(k, r) - \left\{ \frac{c^2 Q_6(k, r) + a Q_8(k, r)}{\omega_{mf}^2} \right\} \right] \right\},
\]

(72)
is called the modified natural frequency representing the frequency of the free system due to the presence of the moving mass.

5 | RESULTS AND DISCUSSION

In order to study the nature of the dynamic and response characteristics of a beam under compressive axial force and subjected to accelerating masses, homogenous beam whose material, and geometric properties are given below is considered. The modulus of elasticity \( E = 3.9012 \times 10^8 \text{N/m}^2 \), the moment of inertia \( I = 2.87698 \times 10^{-3} \text{m}^4 \), the beam span \( L = 30 \text{m} \), and the mass per unit length of the beam \( \mu = 2758.291 \text{kg/m} \). The values of foundation moduli are varied between 0 and 400000 N/m³, the values of axial force \( N \) are varied between 0 and 2.0 × 10^8 N. The mass \( M \) is assumed to strike the beam at the point \( x = 0 \) and time \( t = 0 \) and travels across the beam with uniformly accelerated, decelerated, or uniform velocity type of motion. The dimensionless time \( s \) is given as \( \frac{t}{t_0} \), with \( t_0 = \frac{L}{c} \), where \( c \) is the velocity of the load.

The analysis in this study is applied to homogeneous beam subjected to concentrated loads moving with uniformly accelerated, decelerated, or uniform velocity type of motion under pinned-pinned, fixed-fixed, fixed-free, fixed-pinned, fixed-guided, and pinned-fixed end conditions as shown in Figures 2 to 7, respectively.

5.1 | Pinned-pinned

Figure 8A-C show the dimensionless mid-span dynamic displacement of a pinned-pinned uniform beam under the actions of accelerating, decelerating, or uniform velocity type of motion for various values of axial force \( N \) and for fixed values of subgrade moduli \( K = 40000 \) and shear modulus \( G = 30000 \). The figures show that as \( N \) increases, the response amplitude of the beam decreases for all the three type of motions. It is observed that the maximum deflection occurs under the decelerated type of motion.
Figure 9A-C depicts the mid-span response of a pinned-pinned uniform beam under the actions of accelerating, decelerating, or uniform velocity type of motion for various values of span length $L$. The figures show that as $L$ increases, the response amplitude of the beam decreases for all the three type of motions considered.

Figure 10A-C display the mid-span deflection profile of the pinned-pinned uniform beam under the actions of accelerating, decelerating, or uniform velocity type of motion of the concentrated mass for various values of velocity $V$ and for fixed values of axial force $N = 20\,000$, foundation modulus $K = 40\,000$, and shear modulus $G = 30\,000$. These figures clearly show that as the velocity of the moving mass system increases the deflection also increases. It is observed that under uniform velocity type of motion the deflection increases as the velocity increases until it get to a certain point and start decreasing.
FIGURE 9  Dimensionless dynamic deflection vs the dimensionless time for a pinned-pinned beam; (A) accelerated motion; (B) decelerated motion; (C) uniform motion; for moving mass; ( ) $L = 20$ m; ( ) $L = 30$ m; ( ) $L = 40$ m

FIGURE 10  Dimensionless dynamic deflection vs the dimensionless time for a pinned-pinned beam; (A) accelerated motion; (B) decelerated motion; (C) uniform motion; for moving mass; ( ) $V = 10$ m/s; ( ) $V = 20$ m/s; ( ) $V = 30$ m/s

5.2  Fixed-fixed

Figure 11A-C show the dimensionless mid-span dynamic displacement of a fixed-fixed uniform beam under the action of accelerating, decelerating, or uniform velocity type of motion of a traveling concentrated mass for various values of axial force $N$ and for fixed values of subgrade moduli $K = 40000$ and shear modulus $G = 30000$. The figures show that as $N$ increases, the response amplitude of the beam decreases for all the three type of motions. It is observed that the maximum deflection occurs under the decelerating type of motion.

Figure 12A-C display the dimensionless mid-span displacement of a fixed-fixed uniform beam under the actions of accelerating, decelerating, or uniform velocity type of motion of a traveling concentrated mass for various values of span length $L$. The figures show that as the span length $L$ increases, the response amplitude of the beam decreases for all the type of motions considered.

Figure 13A-C show the mid-span deflection profile of the fixed-fixed uniform beam under the actions of accelerating, decelerating, or uniform velocity type of motion of the moving concentrated mass for various values of velocity $V$ and for fixed values of axial force $N = 20000$, foundation modulus $K = 40000$, and shear modulus $G = 30000$. These figures clearly show that as the velocity of the moving mass system increases the deflection also increases. It is shown from these figures that under uniform velocity type of motion the deflection increases as the velocity increases until it get to a certain point and start decreasing.
5.3 Fixed-free

Figure 14A-C show the dimensionless dynamic displacement response of a fixed-free uniform beam under the actions of accelerating, decelerating, or uniform velocity type of motion of a traveling concentrated mass for various values of axial force $N$ and for fixed values of subgrade moduli $K = 40000$ and shear modulus $G = 30000$. The figures show that as $N$ increases, the response amplitude of the beam decreases for all the three type of motions. It is observed that the maximum deflection occurs under the decelerating type of motion.

Figure 15A-C depict the dimensionless dynamic displacement of a fixed-free uniform beam under the actions of accelerating, decelerating, or uniform velocity type of motion of moving concentrated masses for various values of span length $L$. The figures show that as the span length $L$ increases, the response amplitude of the beam decreases for all the type of motions considered.

Figure 16A-C show the deflection profile of the fixed-free uniform beam under the actions of accelerating, decelerating, or uniform velocity type of motion of concentrated masses for various values of velocity $V$ and for fixed values of axial force $N = 200000$, foundation modulus $K = 40000$ and shear modulus $G = 30000$. These figures clearly show that as the velocity of the moving mass system increases the deflection also increases. It is observed that under uniform velocity type of motion the deflection increases as the velocity increases until it get to a certain point and start decreasing.
Figure 13  Dimensionless dynamic deflection vs the dimensionless time for a fixed-fixed beam; (A) accelerated motion; (B) decelerated motion; (C) uniform motion; for moving mass; (—) $V = 10 \text{ m/s}$; (—) $V = 20 \text{ m/s}$; (—) $V = 30 \text{ m/s}$

Figure 14  Dimensionless dynamic deflection vs the dimensionless time for a fixed-free beam; (A) accelerated motion; (B) decelerated motion; (C) uniform motion; for moving mass; (—) $N = 0$; (—) $N = 200000$; (—) $N = 2000000$

Figure 15  Dimensionless dynamic deflection vs the dimensionless time for a fixed-free beam; (A) accelerated motion; (B) decelerated motion; (C) uniform motion; for moving mass; (—) $L = 20 \text{ m}$; (—) $L = 30 \text{ m}$; (—) $L = 40 \text{ m}$
**FIGURE 16** Dimensionless dynamic deflection vs the dimensionless time for a fixed-free beam; (A) accelerated motion; (B) decelerated motion; (C) uniform motion; for moving mass; ( ) $V = 10 \text{ m/s}$; ( ) $V = 20 \text{ m/s}$; ( ) $V = 30 \text{ m/s}$

**FIGURE 17** Dimensionless dynamic deflection vs the dimensionless time for a fixed-pinned beam; (A) accelerated motion; (B) decelerated motion; (C) uniform motion; for moving mass; ( ) $N = 0$; ( ) $N = 200000$; ( ) $N = 2000000$

**FIGURE 18** Dimensionless dynamic deflection vs the dimensionless time for a fixed-pinned beam; (A) accelerated motion; (B) decelerated motion; (C) uniform motion; for moving mass; ( ) $L = 20 \text{ m}$; ( ) $L = 30 \text{ m}$; ( ) $L = 40 \text{ m}$
Figure 19  Dimensionless dynamic deflection vs the dimensionless time for a fixed-pinned beam; (A) accelerated motion; (B) decelerated motion; (C) uniform motion; for moving mass; ( ) $V = 10\ m/s$; ( ) $V = 20\ m/s$; ( ) $V = 30\ m/s$

### 5.4  Fixed-pinned

Figure 17A-C show the dimensionless dynamic displacement of a fixed-pinned uniform beam under the actions of accelerating, decelerating, or uniform velocity type of motion of concentrated masses for various values of axial force $N$ and for fixed values of subgrade moduli $K = 40 000$ and shear modulus $G = 30 000$. The figures show that as $N$ increases, the response amplitude of the beam decreases for all the three type of motions. It is observed that the maximum deflection occurs when the motion is of the decelerating type.

Figure 18A-C display the dimensionless dynamic displacement response of a fixed-pinned uniform beam under the action of accelerating, decelerating, or uniform velocity type of motion concentrated masses of various values of span length $L$. The figures show that as the span length $L$ increases, the response amplitude of the beam decreases for all the three type of motions considered.

Figure 19A-C depict the deflection profile of the fixed-pinned uniform beam under the action of accelerating, decelerating, or uniform velocity type of motion of concentrated masses for various values of velocity $V$ and for fixed values of axial force $N = 200 000$, foundation modulus $K = 40 000$, and shear modulus $G = 30 000$. These figures clearly show that as the velocity of the moving mass system increases the deflection of the vibrating system also increases. It is observed that under uniform velocity type of motion the deflection increases as the velocity increases until it get to a certain point and start decreasing.

### 5.5  Fixed-guided

Figure 20A-C show the dimensionless dynamic displacement response of a fixed-guided uniform beam under the actions of accelerating, decelerating, or uniform velocity type of motion of concentrated masses for various values of axial force $N$ and for fixed values of subgrade moduli $K = 40 000$ and shear modulus $G = 30 000$. The figures show that as $N$ increases, the response amplitude of the beam decreases for all the three type of motions. It is observed that the maximum deflection occurs under the decelerating type of motion.

Figure 21A-C display the dimensionless displacement response of a fixed-guided uniform beam under the actions of accelerating, decelerating, or uniform velocity type of motion of concentrated mass for the various values of span length $L$. The figures show that as the span length $L$ increases, the response amplitude of the beam decreases for all the type of motions considered.

Figure 22A-C depict the deflection profile of the fixed-guided uniform beam under the action of accelerating, decelerating, or uniform velocity type of motion of concentrated masses for various values of velocity $V$ and for fixed values of axial force $N = 200000$, foundation modulus $K = 40000$, and shear modulus $G = 30000$. These figures clearly show that as the velocity of the moving mass system increases the deflection of the vibrating system also increases.
**Figure 20** Dimensionless dynamic deflection vs the dimensionless time for a fixed-guided beam; (A) accelerated motion; (B) decelerated motion; (C) uniform motion; for moving mass; (–) \( N = 0 \); (––) \( N = 200 \, 000 \); (–––) \( N = 2 \, 000 \, 000 \)

**Figure 21** Dimensionless dynamic deflection vs the dimensionless time for a fixed-guided beam; (A) accelerated motion; (B) decelerated motion; (C) uniform motion; for moving mass; (–) \( L = 20 \) m; (––) \( L = 30 \) m; (–––) \( L = 40 \) m

### 5.6 Pinned-fixed

Figure 23A-C display the dynamic displacement of a pinned-fixed uniform beam under the actions of accelerating, decelerating, or uniform velocity type of motion of concentrated masses for various values of axial force \( N \) and for fixed values of subgrade moduli \( K = 40 \, 000 \) and shear modulus \( G = 30 \, 000 \). The figures show that as \( N \) increases, the response amplitude of the beam decreases for all the type of motions considered. It is observed that the maximum deflection occurs under the decelerating type of motion.

Figure 24A-C display the dimensionless displacement response of a pinned-fixed uniform beam under the action of accelerating, decelerating, or uniform velocity type of motion of concentrated masses for various values of span length \( L \). The figures show that as the span length \( L \) increases, the response amplitude of the beam decreases for all the three type of motions considered.

Figure 25A-C depict the dynamic deflection profile of the pinned-fixed uniform beam under the action of accelerating, decelerating, or uniform velocity type of motion of concentrated masses for various values of velocity \( V \) and for fixed values of axial force \( N = 200 \, 000 \), foundation modulus \( K = 40 \, 000 \), and shear modulus \( G = 30 \, 000 \). These figures clearly show that as the velocity of the moving mass system increases the deflection also increases. It is observed that under
FIGURE 22  Dimensionless dynamic deflection vs the dimensionless time for a fixed-guided beam; (A) accelerated motion; (B) decelerated motion; (C) uniform motion; for moving mass; ( ) $V = 10\; \text{m/s}$; ( ) $V = 20\; \text{m/s}$; ( ) $V = 30\; \text{m/s}$

FIGURE 23  Dimensionless dynamic deflection vs the dimensionless time for a pinned-fixed beam; (A) accelerated motion; (B) decelerated motion; (C) uniform motion; for moving mass; ( ) $N = 0$; ( ) $N = 200\; 000$; ( ) $N = 2\; 000\; 000$

FIGURE 24  Dimensionless dynamic deflection vs the dimensionless time for a pinned-fixed beam; (A) accelerated motion; (B) decelerated motion; (C) uniform motion; for moving mass; ( ) $L = 20\; \text{m}$; ( ) $L = 30\; \text{m}$; ( ) $L = 40\; \text{m}$
uniform velocity type of motion the deflection increases as the velocity increases until it get to a certain point and start decreasing.

Figure 26A-C show the comparison of the dimensionless deflection of the pinned-pinned beam for the moving force and moving mass system. It is observed from the figures that for the accelerated and uniform velocity type of motion, the deflection is higher in case of moving force than that of the moving mass system. For the decelerated motion, the deflection is higher in the case of moving mass system than that of the moving force system.

Figure 27A-C show the comparison of the dimensionless deflection of the fixed-fixed beam for the moving force and moving mass system. It is observed from the figures that, the deflection is higher in the case of moving force system than that of the moving mass system in all the three types of motion considered. Similar result is obtained for the fixed-free boundary condition as shown in Figure 28A-C.

Figure 29A-C show the comparison of the dimensionless deflection of the fixed-pinned beam for the moving force and moving mass system. It is observed from the figures that for the accelerated and decelerated type of motion, the deflection is higher in case of the moving mass system than that of the moving force system. For the uniform velocity type of motion, the deflection is higher in the case of moving force system than that of the moving mass system. Similar result is obtained for the fixed-guided and pinned-fixed boundary condition as shown in Figures 30A-C and 31A-C, respectively.
FIGURE 27  Dimensionless dynamic deflection vs the dimensionless time for a fixed-fixed beam; (A) accelerated motion; (B) decelerated motion; (C) uniform motion; for moving force and moving mass

FIGURE 28  Dimensionless dynamic deflection vs the dimensionless time for a fixed-free beam; (A) accelerated motion; (B) decelerated motion; (C) uniform motion; for moving force and moving mass

FIGURE 29  Dimensionless dynamic deflection vs the dimensionless time for a fixed-pinned beam; (A) accelerated motion; (B) decelerated motion; (C) uniform motion; for moving force and moving mass
**FIGURE 30** Dimensionless dynamic deflection vs the dimensionless time for a fixed-guided beam; (A) accelerated motion; (B) decelerated motion; (C) uniform motion; for moving force and moving mass

**FIGURE 31** Dimensionless dynamic deflection vs the dimensionless time for a pinned-fixed beam; (A) accelerated motion; (B) decelerated motion; (C) uniform motion; for moving force and moving mass

### 6 CONCLUDING REMARKS

This study assesses the response characteristics of a beam subjected to compressive axial force and accelerating masses. The fourth-order partial differential equation governing the flexural motion of the structural member is first treated using Galerkin’s residual method to reduce the governing equation to a system of coupled second-order ordinary differential equations. This sequence of second-order ordinary differential equations is further simplified by employing the Struble’s asymptotic technique. The Duhamel’s integral transform also known as the unit impulse response function is then used to obtain a closed form solution of this dynamical problem, which is valid for all variants of pertinent boundary conditions. The theory and analysis proposed in this work is illustrated by various practical examples often encounter in engineering design and practice. Results and analysis of the illustrative examples for different boundary conditions are presented. Influence of parameters like traveling load velocity, span length, axial force, subgrade moduli, and shear modulus on the dynamic characteristics exhibited by the beam-load system is established. Dynamic behaviors of this structurally prestressed beam under accelerated, decelerated, and uniform velocity type of motion is studied and presented. This study has proposed a valuable and versatile method of solution for this category of problems for all variant of classical boundary conditions. Various results and analysis presented in this study are consistent with existing results in literatures. However, in the present study, elastic beam of uniform cross section is considered and the traversing mass is assumed to
be a lump mass. An undamped system is studied, leaving a damped system for future research. The static load effects on the beam-mass system are not considered in this work, studies are ongoing to address some of these limitations.

CONFLICT OF INTEREST

The authors declare no potential conflict of interests.

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