Inverse Identification of Material Properties of the Human Eye Using Optical Deformation Measurements

Stefan Muench1,2,3,*, Mike Roellig1,2, and Daniel Balzani3,**
1 Fraunhofer Institute for Ceramic Technologies and Systems (IKTS), 01109 Dresden, Germany
2 Dresden Center for Computational Materials Science (DCMS), 01062 Dresden, Germany
3 Chair of Continuum Mechanics, Ruhr University Bochum, 44801 Bochum, Germany

The aim of the study presented here is the non-invasive and in-vivo measurement of material parameters of the eye. For this purpose, we propose a method for the construction of full-field displacements from 2D deformation contours to enable a fast identification of material parameters, which could be used as advanced diagnostic tool for a broad range of diseases of the human eye. First results are presented here based on virtual test setups which should be confirmed through the use of real medical data in the future.

© 2021 The Authors
Proceedings in Applied Mathematics & Mechanics published by Wiley-VCH GmbH

1 Introduction

Severe diseases of the eye, especially if occurring during puberty or in young adulthood, often result in a considerable reduction of the quality of life if the disease is not diagnosed early enough. In many cases, an early diagnosis is believed to be achievable by tracking changes of the mechanical behavior of the eye. For instance, keratoconus is a softening of the cornea, the transparent anterior part of the eye. Since the intraocular pressure (IOP) remains constant, the cornea deforms to the outside, leading to visual impairments. Approximately 1 out of 2000 people is affected [1] and while late diagnosed cases can only be treated surgically, corneal-cross-linking (CXL) is a procedure that can stop the progression of the disease if applied early enough. Non-contact-tonometers (NCT) apply an air-puff to the eye and the resulting deformation of the cornea is recorded. So far, various attempts have been made to determine the material properties inversely from this data. Due to limited displacement information, this was only possible with a displacement-based finite element model updating method (FEMU-U). Therein, however, the solution of the inverse problem is time-consuming and unattractive for a broad application. Furthermore, a unique identification of material parameters is impossible, which is however the basis for a quantitative interpretation of the parameter values as part of medical diagnostics. For this reason, we propose a method based on the construction of full-field displacements using the deformation contours extracted from the NCT.

2 Construction of Full-Field Displacements

One representative of NCT devices is the Corvis ST (OCULUS Optikgeräte GmbH, Wetzlar, Germany). It uses a high-speed camera to record the deformation behavior of the cornea during tonometry. Thereby, 140 cross-sectional images of the cornea are created. Within the cross-sectional images, the CoST automatically identifies the outer and inner contours. The contours of the undeformed state can then be used to build a patient-specific eye model. Note that the undeformed geometry does not reflect the stress-free geometry. This must first be determined taking into account the intraocular pressure (IOP) and a simplified material model, see [2]. The contours of the deformed states reflect the changed shape of the cornea under stress. They also contain information about the axial displacement of the cornea and a possible change in thickness. However, no conclusions can be drawn regarding displacements of individual material points. This deficit can be fixed by deforming a virtual twin. As shown in Fig. 1, a finite-element model of the patient-specific cornea under IOP is generated. The cornea contours are used to create stamps. According to the information about the axial displacement, the corneal thickness and the thickness change, these stamps can be moved to the specified position. Contact elements ensure the transmission of axial forces. On the other hand, they allow a sliding between the cornea surface and the surface of the stamp to ensure an appropriate in plane displacement of the cornea. Due to the incompressibility of the corneal material, the corneal twin...

Fig. 1: Contour data from a Corvis ST measurement (image taken from [3]) is used to create stamps to morph a virtual corneal twin.

* Corresponding author: e-mail stefan.muench@ikts.fraunhofer.de, phone +49 351 88815 518, fax +49 351 88815 509

** Corresponding author: e-mail daniel.balzani@rub.de, phone +49 (0)234 32-23080
undergoes displacements that are qualitatively and quantitatively similar to the displacements in the real cornea. The thereby obtained displacements are however accessible as full-fields.

3 Evaluation of Equilibrium Equation

To set up the equilibrium equation, further inputs are required. A main component is the strain energy function to describe the corneal behavior. Here the description according to [4] with the volumetric, isotropic and transverse isotropic components:

\[ \psi_{\text{volumetric}}(J) = \frac{1}{2} K \left[ J^2 - 1 - 2 \ln(J) \right], \quad \psi_{\text{isotropic}}(\tilde{I}_1) = \frac{1}{4} \mu (\tilde{I}_1 - 3) \]

and

\[ \tilde{\psi}_{i,j}(\tilde{I}_{4,i,j}) = -\frac{k_{1,i,j}}{k_{2,i,j}} + \frac{k_{1,i,j}}{2k_{2,i,j}} \exp \left[ k_{2,i,j} \left( \tilde{I}_{4,i,j} - 1 \right)^2 \right] \left( 1 + K_i \sigma_{i,j}^2 \right) \]

is used. Therein, the Mooney-Rivlin model preferred by [4] was simplified to the Neo-Hookean. The variables \( K, \mu, k_1 \) and \( k_2 \) are the actual material parameters. Note that \( K, \mu \) and \( k_1 \) contribute linearly to the strain energy function and only \( k_2 \) acts non-linearly. The internal forces based on the full-field displacement and the strain energy function are counteracted by the external forces caused by boundary conditions. These are on one hand the intraocular pressure acting on the inner surface of the cornea. It is important to note that this pressure increases with increasing deformation of the cornea. On the other hand, the forces of the air pulse act on the outside of the cornea. These are also dependent on the corneal deformation. The deformation-dependent descriptions can be taken from [5]. Then the square of differences between discrete internal and external forces (obtained by applying a finite element discretization) is minimized to obtain the material parameters. By fixating the nonlinear parameter \( k_2 \), the strain energy function is reduced to a linear function of the remaining parameters. As shown in [6], the optimization therefore results in a quadratic problem, which can be solved directly and uniquely. The nonlinear parameter \( k_2 \) is computed as proposed by [7]. For this purpose the linear material parameters are determined for at least three different cornea deformation states. Then the standard deviation of the linear material parameters between these states is minimized to identify \( k_2 \). Fig. 2 compares the identified parameters with the parameters in the virtual twin.

![Fig. 2: Given and identified material parameters for the assumed test cases, where the parameter \( k_1 \) was varied.](image-url)

4 Conclusion

We proposed a method for the construction of full-field displacements based on the deformation contours extracted from the NCT. The derived full-field kinematics were directly inserted into the discretized equilibrium equation suitably representing the NCT procedure. Since the equilibrium equation is only evaluated and not solved, the optimization problem becomes highly efficient. By using the procedure described in [7], a unique identification of the material parameters can be ensured. Further tests of the presented method in cooperation with clinicians are necessary to prove the function on real measured data.

Acknowledgements The authors appreciate financial funding from the European Research Fund (ESF) through the training research group “Cosima” (ESF Project 100231947). Furthermore, the author Daniel Balzani thanks the Institutional Strategy “The Synergetic University” at Technische Universität Dresden funded by the DFG.

Open access funding enabled and organized by Projekt DEAL.

Correction added on 19 February 2021, after first online publication: Daniel Balzani was designated as corresponding author.

References

[1] Y. S. Rabinowitz, Surv. Ophthalmol. 42, 297-319 (1998).
[2] A. Pandolfi and G. A. Holzapfel, J. Biomech. Eng. 130, 061006 (2008).
[3] Q. Long et al., J. Ophthalmol. 2015, (2015).
[4] P. Sánchez, K. Moutsouris and A. Pandolfi, J. Cataract Refract. Surg. 40, 905–917 (2014).
[5] S. Muench et al., Exp. Mech. 59, 1285–1297 (2019).
[6] N. Cottin, H. P. Felgenhauer and H. G. Natke, Ingenieur-Archiv 54, 378-387 (1984).
[7] L. E. Perotti et al., Int. J. Numer. Method. Biomed. Eng. 33, e2866 (2017).