HeteroSAg: Secure Aggregation with Heterogeneous Quantization in Federated Learning

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Abstract

Secure model aggregation across many users is a key component of federated learning systems. The state-of-the-art protocols for secure model aggregation, which are based on additive masking, require all users to quantize their model updates to the same level of quantization. This severely degrades their performance due to lack of adaptation to available communication resources, e.g., bandwidth, at different users. As the main contribution of our paper, we propose HeteroSAg, a scheme that allows secure model aggregation while using heterogeneous quantization. HeteroSAg enables the edge users to adjust their quantization proportional to their available communication resources, which can provide a substantially better trade-off between the accuracy of training and the communication time. Our proposed scheme is based on a grouping strategy by partitioning the network into groups, and partitioning the local model updates of users into segments. Instead of applying aggregation protocol to the entire local model update vector, it is applied on segments with specific coordination between users. We further demonstrate how HeteroSAg can enable Byzantine robustness while achieving secure aggregation simultaneously. Finally, we prove the convergence guarantees of HeteroSAg under heterogeneous quantization in the non-Byzantine scenario.

1 Introduction

Federated learning (FL) is gaining significant interests as it enables training machine learning models locally at the edge device, e.g., mobile phones, instead of sending raw data to a central server [1,2]. The goal in the basic FL framework is to learn a global model $\theta \in \mathbb{R}^m$ using the data stored at the edge device. This can be represented by minimizing a global objective function,

$$
\arg\min_{\theta} F(\theta) \text{ such that } F(\theta) = \sum_{i=1}^{N} \frac{n_i}{n} F_i(\theta), \text{ and } F_i(\theta) = \frac{1}{n_i} \sum_{j=1}^{n_i} f_i(\theta; x_j, y_j),
$$

where $\theta$ is the global model to be optimized. Here, $F_i$ is the local objective function of user $i$, $f_i(\theta; x_j, y_j)$ is the loss of the prediction on example $(x_j, y_j)$ for user $i$ made with global model $\theta$, $n_i$ is the data size at user $i$, and $n = \sum_i n_i$. Without loss of generality, we assume that all users have an equal-sized dataset.

To learn the global model $\theta$ that minimizes the objective in (1), stochastic gradient descent (SGD) algorithm can be easily implemented distributedly across the $N$ available devices in the presence of a central server who orchestrates the training process. The training process in FL by using the distributed SGD is illustrated in Figure 1. At iteration $t$, the server sends the current version of the global model vector, $\theta^{(t)}$, to the mobile users. User $i$ then computes its local model vector $\theta_i^{(t)}$ based on its local dataset by using the SGD, so the local model update of each user can be written as
This local model update could be a single gradient, or could result from multiple steps of the SGD taken on this user’s local dataset. User \(i\) sends the local model update \(x_i^{(t)}\) to the server. The local model updates of the \(N\) users are then aggregated by the server. The server then updates the global model \(\theta^{(t+1)}\) for the next round according to

\[
\theta^{(t+1)} = \theta^{(t)} + \frac{1}{N} \sum_{i=1}^{N} x_i^{(t)}.
\]

Although FL provides many benefits, it still suffers from key challenges such as communication bottleneck, system failures, malicious users and users’ privacy [2]. The communication bottleneck in FL is created by sending a large model from each user to the server at each iteration of the training. Researchers have proposed many approaches to provide a communication-efficient FL system. One of these approaches is to reduce the model size by performing model compression through either quantization [4–9] or sparsification [7, 10–13].

Another key challenge for FL is the Byzantine faults [14] in which some users may behave arbitrarily due to software bugs, hardware failure, or even get hacked during training, sending arbitrary or malicious values to the server, thus severely degrading the overall convergence performance. Many Byzantine robust strategies have been proposed recently for FL [15–20]. These Byzantine robust optimization algorithms combine the gradients received by all workers using robust aggregation rules, to ensure that training is not impacted by malicious users.

Preserving the privacy of the users is another main consideration for FL system. There are two approaches to achieve that. First, the training data stays on the user device, and users locally perform model updates using their individual data. Second, local models can be securely aggregated at the central server to update the global model. This is achieved through what is known as a secure aggregation (SecAg) protocol [1], where users use random masks to mask their local model updates. In this protocol, each user masks its local update through additive secret sharing using private and pairwise random keys before sending it to the server. Once the masked models are aggregated at the server, the additional randomness cancels out and the server learns the aggregate of all user models. At the end of the protocol, the server learns no information about the individual models beyond the aggregated model, as they are masked by the random keys unknown to the server. Some other algorithms for secure aggregation with additive masking have been proposed [21–24].

In general, the state-of-the-art secure aggregation protocols with additive masking have some limitations associated with:

- **(System heterogeneity)** They require all users to quantize their model updates to the same level of quantization (to guarantee correct decoding Section 3.2), even if they have different communication resources such as transmission rates. Lack of adaptation to the speed of the available network (3G, 4G, 5G, Wi-Fi) and the fluctuation of the network quality over time severely degrades the performance of these protocols. More specifically, by making all users use a low level quantizer, the communication time will be small, but the test accuracy will decrease. On the other hand, using a high level quantizer will result in increasing the test accuracy at the expense of increasing the communication time.

- **(Robustness)** Secure aggregation protocols make the adaption of the existing state-of-the-art defense strategies [15–19] against Byzantine users difficult to implement, as the server only
receive a masked model update from each user, while the success of these strategies are based on having users’ individual clear model updates.

• (Communication efficiency) The bandwidth expansion, which measures the ratio between the size of the encoded model in bits to the size of the clear model. This bandwidth expansion results from the additional $O(\log N)$ bits that should be communicated for each scalar in the model update vector, where $N$ is the total number of users, to guarantee correct decoding. Hence, this expansion makes them ineffective with aggressive quantization, specially for large $N$ [1, 2, 25].

Overcoming the aforementioned limitations, specifically the one associated with system heterogeneity, is a challenging problem as illustrated in detail in Section 3.2. Towards solving these limitations, we propose HeteroSAg.

1.1 Main contributions

HeteroSAg has the following four salient features:

1. HeteroSAg protects the privacy of the local model updates of each individual user in the strong information-theoretic sense by masking the model update of each user such that the mutual information between the masked model and the clear model is zero.

2. HeteroSAg allows using heterogeneous quantization. This enables the edge users to adjust their quantizations proportional to their available communication resources which can result in a substantial better trade-off between the accuracy of training and the communication time.

3. HeteroSAg further enables robustness against Byzantine users, by incorporating distance-based defense mechanisms such as coordinate-wise median [19].

4. HeteroSAg reduces the bandwidth expansion. For instance, we demonstrate that for the case of having $N = 2^{10}$ users using a single bit quantization, the bandwidth expansion factor when using HeteroSAg is $4 \times$, as opposite to $11 \times$ when using SecAg.

We provide the theoretical convergence guarantees of HeteroSAg under heterogeneous quantization for convex loss function in the non-Byzantine setting. Furthermore, using neural network with real-world dataset, we demonstrate the efficiency of the heterogeneous quantization given by HeteroSAg. Specifically, we show that we can achieve accuracy close to the baseline case (no-quantization) with the same communication time as the case when all users are using 1-bit quantizer. We also show that we can achieve $\sim 15\%$ higher test accuracy when compared with the setting of homogeneous quantization with 1-bit quantizer, while the communication time is the same for both settings. We then experimentally demonstrate the resiliency of HeteroSAg in the presence of Byzantine users under three different attacks and compare it to the conventional federated averaging scheme [3] by using two different datasets.

1.2 Related works

The authors in [9] provide two heterogeneous quantization algorithms for distributed ML in the absence of a central server to reduce the communication cost, but without any privacy guarantee for the model updates of the users. Our work is different from [9], since our objective is to provide a scheme that not only allows for heterogeneous quantization, but also guarantees the privacy of the users’ models by doing secure model aggregation. We also consider a network topology where there exists a parameter server. Therefore, the two setups are not comparable. We also highlight that our objective in this paper is not to design a new quantization scheme, yet to provide a general method that enables using heterogeneous quantization while doing secure aggregation.

In recent work, Byzantine-robust secure aggregation algorithms have been proposed [26, 27]. The work in [27] has been proposed for two honest (non-colluding) servers who both interact with the mobile users and communicate with each other to carry out a secure two-party protocol. Unlike this work, the authors in [26] develop BREA, a single-server Byzantine-resilient secure training framework, to facilitate robust and privacy-preserving training architectures for FL. Our work also achieves Byzantine-robust secure aggregation in a single server by a simple incorporating of some state-of-the-art Byzantine robust algorithms which have provable convergence guarantees such
as coordinate-wise median based [19] without either extra computation cost to the users or extra communication cost to the original cost of SecAg. The per-user communication cost of HeteroSAg is $O(m + N)$ as opposite to $O(N^2 + Nm)$ for BREA and $O(N + Nm)$ for the generalized BREA, where $m$ and $N$ are the model size and the number of users, respectively. Furthermore, the per-user computation cost for HeteroSAg is $O(N^2 + m \log N)$ as opposite to $O(mN^2 + Nm \log^2 N)$ for BREA and the generalized BREA. We note that the upper bound on the number of Byzantine nodes for the success of HeteroSAg is given by $B \leq \lceil 0.25G \rceil - 1$, where $G$ the number of groups given by HeteroSAg, is less than the upper bound for coordinate-wise median based in [19]. However, our proposal is initially developed for enabling secure aggregation while using heterogeneous quantization at different users, while incorporating defense technique against Byzantine nodes comes as an extra feature. Additionally, the scheme in [19] solely does not provide privacy for the local models of the users. Therefore, we conclude that HeteroSAg is the first scheme that achieves secure aggregation with heterogeneous quantization while providing Byzantine-resiliency.

2 Background

Secure aggregation (SecAg), e.g., [1], is a key component in FL that enables the distributed training process while preserving the privacy of the users. We summarize SecAg in the following five steps while considering $\mathcal{S} \triangleq \mathcal{N} \triangleq \{1, \ldots, N\}$, where $N$ is the total number of nodes. We provide this summary as this protocol is a key component in HeteroSAg.

Step 1 (Sharing keys and masks): Users first establish a secure communication channel between them by using pairwise keys through a key exchange protocol such as Diffie-Hellman key agreement [28]. All the communication is forwarded through the server. Also, each pair of users $i, j \in \mathcal{N}$ first agrees on a pairwise random seed $s_{i,j}$ by using Diffie-Hellman key agreement, such that $s_{i,j}$ is a function of the public key $s_{j}^{PK}$ of user $j$ and the private key $s_{i}^{SK}$ of user $i$. At the end, each node $i \in \mathcal{N}$ will have this set of agreement keys $\{s_{i,j}\}_{j \in \mathcal{N} / i}$. Also, according to the key generation in Diffie-Hellman key agreement [28], the public key is symmetric, i.e., $s_{i,j} = s_{j,i}$. Furthermore, the server will have all the set of public keys $s_{j}^{PK}$ for all $j \in \mathcal{N}$. In addition, user $i$ creates a private random seed $b_{i}$. The role of $b_{i}$ is to prevent the privacy breaches that may occur if user $i$ is only delayed instead of dropped (or declared as dropped by a malicious server), in which case the pairwise masks alone are not sufficient for privacy protection. Further discussion about the rule of $b_{i}$ is given in Step 5.

Step 2 (Secret sharing): User $i, i \in \mathcal{N}$, secret shares the private key $s_{i}^{SK}$ as well as $b_{i}$ with the other users in the system, via Shamir’s secret sharing [29]. To ensures that the local model is private against an adversarial server which tries to learn information about the local models of the honest users, while the mobile users are honest and do not collude with the server, the threshold of the secret share scheme should be $\lceil N/2 \rceil + 1$. For the case where users are adversaries, no matter how we set the threshold value, users on their own learn nothing about other users.

Step 3 (Quantization): SecAg and cryptographic protocols require the input vector elements to be integers, while using modular operation to transmit these vectors. By considering the case where the model update of each user takes real values, we need to do quantization first so that we can apply SecAg. Without loss of generality and for the ease of the analysis, we use the $K$-level quantizer in [8] to quantize the model update $x_{i}$, for $i \in \mathcal{S}$. We assume that the elements of each model $x_{i}$, for $i = 1, \ldots, N$, fall in the range $[r_{1}, r_{2}]$. Let $0 \leq l < K_{y}$, where $K_{y}$ is the number of quantization levels, be an integer such that when $x_{i}(k) \in [T(l), T(l + 1)]$, where $T(l) = r_{1} + l\Delta_{K_{y}}$, and $\Delta_{K_{y}} = \frac{r_{2} - r_{1}}{K_{y} - 1}$ is the quantization interval. Then

$$ Q_{K_{y}}(x_{i}(k)) = \begin{cases} T(l + 1) & \text{with probability } \frac{x_{i}(k) - T(l)}{T(l + 1) - T(l)}, \\ T(l) & \text{otherwise}. \end{cases} $$

\footnote{The dropped users are those who failed to send their masked model to the server. In other words, the server will not receive the model update of those users for the current round. On the other hand, the delayed users are those who send their model updates to the server, but their models have experienced high delay before receiving by the server. Although, these users have already sent their models to the server, the server will consider them as dropped users, because of their high delay. Therefore, the server will not include adding the received masked models from those users in the model aggregation step}
We consider a FL system that consists of a central server and a set of users. We assume that users are already clustered into different groups based on their communication resources. These groups allow training a ML model by using quantizers from a pre-assigned set of quantizers with these levels $K_g = \{0, K_1, \ldots, K_g\}$ where $K_g$ is the highest level that can be used in this system, where $K_g$ is the number of quantization levels of the quantizer $Q_{K_g}$, and $K_0 < K_1 < \ldots < K_{G-1}$. Instead of having a single quantizer as in SecAg. In this problem, we assume that users are already clustered into $G$ different groups based on their communication resources. Each user $i$ in group $g$ can quantize its model update $x_i$ by using quantizers from a pre-assigned set of quantizers with these levels $K_g = \{0, K_1, \ldots, K_g\}$ where $K_g$ is the highest level that can be used in this system.

The output of the quantizer $\bar{x}_i(k) = Q_{K_g}(x_i(k))$ takes a discrete value from this range $\{r_1, r_1 + \Delta_{K_g}, r_1 + 2\Delta_{K_g}, \ldots, r_2 - 2\Delta_{K_g}, r_2\}$.

**Step 4 (Encoding):** Following the quantization step, the set of users $S$ starts the encoding process on $\{\bar{x}_i(k)\}_{i \in S}$, for $k = 1, \ldots, |x_i|$, by first mapping the outputs of the quantizer from the $K_g$ real values that belongs to the discrete range $\{r_1, r_1 + \Delta_{K_g}, r_1 + 2\Delta_{K_g}, \ldots, r_2 - 2\Delta_{K_g}, r_2\}$ to integer values in this range $[0, K_g - 1]$. This mapping is performed such that a real value $r_1$ maps to 0 and $r_2$ maps to $K_g - 1$, etc. The encoding process is completed by allowing each pair of users in $S$ to use the pairwise random seeds to randomly generate 0-sum pairs of mask vectors to provide the privacy for individual models. The output vector of the encoder is given by

$$y_{S,i} = \bar{x}_i + \text{PRG}(b_i) + \sum_{i<j} \text{PRG}(s_{i,j}) - \sum_{j>i} \text{PRG}(s_{j,i}) \mod R,$$

where $y_{S,i}$ is a vector of $|x_i|$ elements, and $R = |U|(K_g - 1) + 1$ to ensure that all possible aggregate vectors from the $|S|$ users will be representable without overflow at the server. PRG is a pseudo random generator used to expand the different seeds to vectors in $\mathbb{Z}_R$ to mask users’ local models.

**Step 5 (Decoding):** From a subset of survived users, the server collects either the shares of private keys the belonging to dropped users, or the shares of the private seed belonging to a surviving user. The server then reconstructs the pair of users in (4) in preserving the privacy of the delayed model of user $i$. According to the SecAg protocol, the server will consider any user, user $i$, with delayed model $y_{S,i}$ as a dropped user. Hence, according to the decoding step, the server will ask the set of survived users to get the shares of the private key $s^{SK}_i$ of the delayed user $i$. The output of the quantizer $\bar{x}_i(k) = Q_{K_g}(x_i(k))$ takes a discrete value from this range $\{r_1, r_1 + \Delta_{K_g}, r_1 + 2\Delta_{K_g}, \ldots, r_2 - 2\Delta_{K_g}, r_2\}$.

$$\bar{x}_i = \sum_{b_i} (y_{S,i} - \text{PRG}(b_i)) - \sum_{b_i} \left( \sum_{j<i} \text{PRG}(s_{i,j}) - \sum_{j>i} \text{PRG}(s_{j,i}) \right) \mod R = \sum_{b_i} \bar{x}_i \mod R$$

In the following, we discuss the importance of using the private mask $b_i$ in (4) in preserving the privacy of the delayed model of user $i$. According to the SecAg protocol, the server will consider any user, user $i$, with delayed model $y_{S,i}$ as a dropped user. Hence, according to the decoding step, the server will ask the set of survived users to get the shares of the private key $s^{SK}_i$ of the delayed user $i$. The output of the quantizer $\bar{x}_i(k) = Q_{K_g}(x_i(k))$ takes a discrete value from this range $\{r_1, r_1 + \Delta_{K_g}, r_1 + 2\Delta_{K_g}, \ldots, r_2 - 2\Delta_{K_g}, r_2\}$.

3 Problem Formulation

We first describe the secure aggregation with heterogeneous quantization problem. After that, we explain why the conventional SecAg protocol can not be applied directly to our problem.

3.1 System Model

We consider a FL system that consists of a central server and a set $\mathcal{N} = \{1, \ldots, N\}$ of $N$ mobile users with heterogeneous communication resources. These $N$ users allow training a ML model locally on their local dataset, as described in the introduction. We also consider having a set $\mathcal{Q} = \{Q_{K_0}, Q_{K_1}, \ldots, Q_{K_{G-1}}\}$ of element-wise stochastic quantizers, e.g., [8], with $G$ different levels that can be used in this system, where $K_g$ is the number of quantization levels of the quantizer $Q_{K_g}$, and $K_0 < K_1 < \cdots < K_{G-1}$.
possible quantization levels that can be used by the users in group \( g \) that is suitable for his transmission rate.

**Threat model:** The server is honest, in which it honestly follows the protocol as specified, but it can be curious and try to extract any useful information about the training data of the users from their received models. On the other hand, users are curious and can only collude with each other, such that any colluding set of users only knows the models from the users in this set. Furthermore, \( B \) users out of the \( N \) available users are malicious and could share false information during protocol execution, or send malicious updates to the server.

At a high level, we want to design a scheme that achieves 1) Secure model aggregation, where the server can only decode the aggregate model from all users, while users are allowed to use different quantizers. 2) Byzantine-resilience and secure aggregation simultaneously. We will formalize the objective in Section II-C. Now, we discuss why SecAg can not be applied directly in our setting where users are using different quantizers.

### 3.2 Challenges

To describe the main challenge for applying secure aggregation protocols with additive masking (including SecAg) to the case where users are using heterogeneous quantization, we consider the following simple example. In this example, we first start by describing the case of homogeneous quantization.

**Example 1.** We consider having two users, where user \( i \) has an input \( x_i \in \mathbb{R} \), and a central server, which should only decode the sum \( x = x_1 + x_2 \). User 1 is assigned these quantization levels, \( \mathcal{K}_0 = \{2\} \), while user 2 is assigned \( \mathcal{K}_1 = \{2, 4\} \). The encoding processes for the two users are given as follows:

- **Homogeneous quantization:** As a first step, each user quantizes its input \( x_i \) by using the same \( K = 2 \) levels of quantization, where we assume without loss of generality that the output of the quantizer is denoted by \( \bar{x}_i \in \{0, 1\} \), for \( i = 1, 2 \). The encoded messages \( y_1 \) and \( y_2 \) from the two users, and the decoded message \( x \) at the server are given by

  \[
  y_1 = \bar{x}_1 + Z_{12} \mod R, \quad y_2 = \bar{x}_2 - Z_{12} \mod R, \quad x = y_1 + y_2 \mod R = \bar{x}_1 + \bar{x}_2 \mod R, \tag{6}
  \]

  where the mask \( Z_{12} \) is drawn uniformly at random from \([0, R]\). By working in the space of integers \( \mod R \) and sampling masks uniformly over \([0, R]\), this guarantees that each user’s encoded message is indistinguishable from its own input (mutual information \( I(\bar{x}_i; y_i) = 0 \)). The correct decoding in (6) is guaranteed for the following two reasons. First, the two users and the server are working in the same space of integers \( \mod R \). Second, the summation and \( \mod \) commute. Therefore, the mask pairs will be cancelled out. We note that choosing \( R = 3 \) ensures that all possible outputs will be represented without any overflow.

- **Heterogeneous quantization:** User 1 and user 2 (assuming user 2 has a higher bandwidth than user 1) quantize their inputs \( x_1 \) and \( x_2 \) by using \( K = 2 \) and \( K = 4 \) levels of quantization, respectively. We assume without loss of generality that the output of the two quantizers are \( \bar{x}_1 \in \{0, 1\} \) and \( \bar{x}_2 \in \{0, \ldots, 3\} \). By further assuming without loss of generality that user 1 and user 2 are using \( \mod 2 \) and \( \mod 4 \) in (6), respectively, instead of the same \( \mod R \), while having \( \bar{x}_1 = 1, \bar{x}_2 = 3 \) and \( Z_{12} = 1 \), the decoded output will be given by \( x = y_1 + y_2 = 2 \) instead of the true output \( x = 4 \). This confirms that having different modular at the users results in incorrect decoding. Another issue

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2 The problem of the optimal clustering of the users based on their transmission rates or the optimal assignment of the quantizers to the users is not the main scope of our paper. Instead, our focus is to provide an approach that allows for doing secure aggregation when different quantizers can be utilized at different users, which is a challenging problem as we will show in Section III-B.

3 The difficulties of applying SecAg in the presence of Byzantine users is described in bullet two in the introduction.
for using different modular at the users is that the space of integer in which each user works on to choose its masks at random will also be different. Thus, using SecAg’s approach for generating the pairwise masks does not guarantee having 0-sum pairs of masks. For instance, generating $Z_{12} = 3$ at both user 1 and 2 in Case (b) is not possible. The reason for that $Z_{12} = 3$ only belongs to the space of integers that node 2 uses to generate its masks.

One method to cancel out masks that belong to different spaces of integers is by using modulus $D$ at the server, where $D$ is an arbitrary integer, and let all users jointly choose a tuple of masks, whose sum mod $D$ equal to 0, uniformly at random from a set of possible tuples. In this tuple of masks, each mask for each user belongs to its space of integer. The main issue of this approach is that whenever wrapping around occurs for the transmitted masked model of any user, the masks will not be cancelled out at the server side and the aggregated model will be distorted. We will consider the following example for illustration.

**Example 2.** Assume having three users with the quantized model update vectors $\bar{x}_1 \in [0, 1]^m$, $\bar{x}_2 \in [0, 2]^m$, and $\bar{x}_3 \in [0, 2]^m$, with dimension $m$. Without loss of generality, we assume the masks of user 1, user 2 and user 3 take values randomly over $[0, 1]^m$, $[0, 5]^m$, and $[0, 5]^m$, respectively. The transmitted masked models and the sum of the masked models at the server are given as follows

$$\begin{align*}
y_1 &= \bar{x}_1 + Z_1 \mod 2, \\
y_2 &= \bar{x}_2 + Z_2 \mod 6, \\
y_3 &= \bar{x}_3 + Z_3 \mod 6,
\end{align*}$$

If the users choose their tuple of masks uniformly at random from the set of tuples in TABLE 1, the mask of each user becomes uniformly distributed over its mask range. This guarantees user’s model privacy in strong information-theoretic sense, i.e., $I(y_i; \bar{x}_i) = 0$, for $i = 1, 2, 3$. However, the main limitation for this approach is that no guarantee for correct decoding. In particular, once the masked models are added together, the masks will not always be canceled out, but it will only be cancelled out when there is no overflow happens for the transmitted masked model of any user. In other words, the sum of users models will be distorted whenever an overflow happens for the transmitted masked model of any user, which occurs with non-negligible probability. For example, having a tuple of masks $(Z_1(k), Z_2(k), Z_3(k)) = (1, 1, 4)$, while the $k$-th element of the model updates of the set of users $(\bar{x}_1(k), \bar{x}_2(k), \bar{x}_3(k)) = (1, 0, 0)$, an overflow will occur at user 1, $y_1 = 0$, and the sum will be $x_{\{1,2,3\}}(k) = 5$ instead of being $x_{\{1,2,3\}}(k) = 1$.

To overcome the aforementioned issues associated with incorrect decoding and 0-pairwise masks generation in SecAg when having heterogeneous quantization, SecAg can leverage multi-group structure. In multi-group structure, the set of user users in group $\mathcal{S}_g$, for $g \in \mathcal{G}$, where $\mathcal{G}$ is the number of groups, uses the same quantizer $Q_{K_g}$, which has $K_g$ levels proportional to their communication resources. After that, each group applies the SecAg protocol independently of all other groups. However, in this strategy the server would decode the aggregate of the model updates from each group which implies knowing the average gradient/model from each group. Hence, this strategy is not robust against some attacks such as membership-inference attack [30,32] and gradient inversion attack [33,35], especially when having a small group size. In gradient inversion attack, the server can reconstruct multiple images used in the training by a user (subset of users) from the averaged gradients corresponding to these images as shown in [34,35]. In membership-inference attack, the server could breach users’ privacy by inferring whether a specific data point was used in the training by a certain subset of users or not by using the average model from this targeted set of users. By letting the server observe the average model from a small group, the attack becomes much stronger and the inferred information may directly reveal the identity of the users to whom the data belongs. Our goal is to leverage the benefits of grouping strategy while limiting the threat of membership/inversion attacks. In particular, we want to address the following question “Can we design a grouping strategy that allows for secure model aggregation while using heterogeneous quantization such that the server can not unmask (decode) the entire average gradient/model from any subset of users?”.

| $Z_1$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
|-------|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $Z_2$ | 1 | 2 | 3 | 4 | 5 | 0 | 1 | 2 | 3 | 4 | 5 | 0 | 0 |
| $Z_3$ | 5 | 4 | 3 | 2 | 1 | 0 | 4 | 3 | 2 | 1 | 0 | 5 | 0 |

Table 1: Tuples of masks that users could use in Example 2.
In this paper, we propose HeteroSAg, a scheme which is based on a specific segment grouping strategy. This segment grouping strategy leverages the multi-group structure for performing secure model aggregation with heterogeneous quantization while preventing the server from unmasking (decoding) the entire average model from any subset of users. At a high level, our proposed segment grouping strategy is based on partitioning the edge users into groups, and dividing the local model updates of these users into segments. Instead of applying the secure aggregation protocol to the entire local model update vectors, it is applied on segments with specific coordination between users, allowing segments to be quantized by different quantizers. More specifically, segments from different set of users are grouped such that they are quantized by the same quantizer while being encoded and decoded together at the server independently of all other segments. This is different from SecAg where the entire local model updates from all users (or a subset of users in the multi-group structure) are quantized by the same quantizer, while being encoded and decoded together. Furthermore, unlike SecAg with multi-group structure, where the server can decode the entire average model from each group, the key objective of our segment grouping strategy is to limit the ability of a curious server from launching inference/inversion attacks on an arbitrary subset of groups. This is achieved in HeteroSAg by allowing the server to only decode a fraction of at most \( \frac{1}{2} \) segments from the average gradient/model of any set of users which approaches 0 for sufficiently large number of segments \( G \). The remaining segments from this average model interfere with segments from the average models of some other groups. We quantify the smallest fraction of segments that the server will not successfully decode from the aggregated model from any set of users \( \mathcal{S} \subseteq \mathcal{N} \) by the inference robustness \( \delta \). In the following subsection, we formally define the inference robustness and discuss its implications.

### 3.3 Performance metric

Let \( \theta^p_i = [\theta^0_i, \ldots, \theta^{G-1}_i] \) denotes the segmentation of the local model \( \theta_i \) of user \( i \), and \( \theta^p_S = \begin{bmatrix} \theta^0_S, \ldots, \theta^{G-1}_S \end{bmatrix} \) denotes the segmentation of the average model \( \theta_S \) from the set of users \( S \), where \( \theta^l_i, \theta^l_S \in \mathbb{R}^F \) for \( l = 0, \ldots, G-1, \ i \in [N], \ \text{and} \ S \subseteq N \). We define \( \mathcal{A}\{\theta^0_p, \ldots, \theta^N_p}\) to be an arbitrary segment grouping strategy that leverages the multi-group structure for doing secure model aggregation on the segment level. This strategy \( \mathcal{A} \) groups each set of segments from \( \{\theta^0_i, \ldots, \theta^N_i\} \) together such that they are encoded and decoded together independently of all other segments. We define \( \mathcal{A}\{\theta^0_i, \ldots, \theta^N_i\} \) to be feasible if it satisfies these three conditions

1) The server could only receive a masked model \( \tilde{\theta}_i \) from user \( i \in N \), where the mutual information \( I(\theta_i; \tilde{\theta}_i) = 0 \).
2) The server could only decode at most a fraction of \( \alpha_S \) segments from the average model \( \bar{\theta}_S \), where \( S \subseteq N \), while each segment in the remaining \( 1 - \alpha_S \) fraction of segments interferes with segments from the average model \( \bar{\theta}_S \), from other sets of users \( S^c \), where \( S^c \subseteq S^c \) such that \( S \cup S^c = N \).
3) The server could decode the average model \( \theta \) from all users. Now, we define our inference robustness metric.

**Definition 1.** (Inference robustness \( \delta \)) For a feasible segment grouping strategy \( \mathcal{A}\{\theta^0_p, \ldots, \theta^N_p\}\), the inference robustness \( \delta(\mathcal{A}) \), where \( \mathcal{A}(\mathcal{A}) \in [0, 1] \), is given as follows:

\[
\delta(\mathcal{A}) = \min \left\{ 1 - \alpha_S : \mathcal{S} \subseteq \{1, \ldots, N\} \right\},
\]

where \( 1 - \alpha_S \) is the fraction of segments from \( \bar{\theta}_S \) that interferes with segments from the average models \( \bar{\theta}_S \), where \( S^c \subseteq S^c \) such that \( S \cup S^c = N \).

**Remark 1** The underlying objective of a good segment grouping strategy \( \mathcal{A}\{\theta^0_p, \ldots, \theta^N_p\} \) is to limit the ability of a curious server from launching inference/inversion attacks on an arbitrary subset of users \( S \subseteq N \), by allowing the server to decode only a fraction \( \alpha_S \in [0, 1] \) of segments from the average model \( \bar{\theta}_S \). This is different from the worst case scenario where the server can decode the entire target model \( \theta_S \). The segments from the average model \( \bar{\theta}_S \) that interfere with segments from other users outside the set \( S \) can be viewed as clear segments plus random noise, where the number of noisy segments is determined by the inference robustness \( \delta(\mathcal{A}) \). The worst case inference robustness for HeteroSAg is \( \delta = \frac{G-2}{G} \), which approaches one for sufficiently large number of segments.

### 4 The proposed HeteroSAg

We first present our HeteroSAg, and then state its theoretical performance guarantees.
4.1 HeteroSAg for heterogeneous quantization

HeteroSAg starts by letting the set of $N$ users share their keys and masks according to Step 1 in Section 2. Each user $i \in N$ then uses Step 2 to secret shares its masks with all other users. For clarity and ease of analysis of the proposed scheme, we first consider the case where users are already clustered into $G$ groups based on their communication resources, each of which has the same number of users, $|S_g| = \frac{N}{G} = n$, for $g \in [G]$, where $[G] := \{0, \ldots, G-1\}$. The case of having a different number of users in each group is presented in the Appendix G. Without loss of generality, we consider users in higher groups have communication resources higher than users in lower groups.

Following the secret sharing step, each local model update vector $\{x_i\}_{i \in N}$ is equally partitioned into $G$ segments such that the segmented model update of user $i$ is given by $x_i^p = [x_i^0, x_i^1, \ldots, x_i^{G-1}]^T$, where $x_i^l \in \mathbb{R}^\pi$ for $l \in [G]$. Also, the aggregated model update $x_S$ at the server from any set of users $S \subseteq N$ can be viewed as a set of $G$ segments $x_S^p = [x_S^0, x_S^1, \ldots, x_S^{G-1}]^T$. Finally, instead of the direct implementation of SecAg protocol where (1) All the unsegmented vectors $\{x_i\}_{i \in N}$ are quantized by the same quantizer, (2) All the $N$ users jointly encode (mask) the $N$ quantized vectors together, and (3) All the $N$ encoded model updates will be decoded together at the server, we apply the segment grouping strategy $A_{\text{HeteroSAg}}(\{x_1^p, \ldots, x_N^p\})$ such that SecAg protocol is applied on the segment level where (1) Different sets of segments are quantized by using different quantizers, (2) Different sets of segments jointly encode their quantized segments together independently of all other users, and (3) The jointly encoded segments will be also jointly decoded at the server.

**Algorithm 1:** The SS Matrix $B$ for HeteroSAg

```plaintext
for $g = 0, \ldots, G-2$ do
    for $r = 0, \ldots, G - g - 2$ do
        $l = 2g + r$;
        $B(l \mod G, g) = B(l \mod G, g + r + 1) = g$;
    end
end
```

The remaining entries of $B_{\text{HeteroSAg}}$ will hold $\ast$.

Each set of segments and its corresponding set of users that jointly executes SecAg protocol together according to the segment grouping strategy $A_{\text{HeteroSAg}}$ is given by the $G \times G$ Segment Selection (SS) matrix $B$ produced by Algorithm 1. In this matrix and as illustrated in the example given in Figure 2.

$$
B = \begin{bmatrix}
0 & 1 & 2 & 3 & 4 \\
0 & 0 & 2 & \ast & 2 & 0 \\
0 & \ast & 0 & 3 & 3 & 1 \\
0 & 1 & 1 & 0 & \ast & 2 \\
0 & 1 & \ast & 1 & 0 & 3 \\
\ast & 1 & 2 & 2 & 1 & 4
\end{bmatrix}
$$

**Figure 2:** Segment selection matrix $B$ for $G = 5$ groups.

the label for each column represents the index of the group (index $g$ of the set of users $S_g$, for $g \in [G]$), where the communication resources of the set of users $S_g$ in group $g$ is smaller than those of the set of users $S_{g'}$, where $g < g'$. On the other hand, the label of each row represents the index $l$ of the segment $x_i^l$. In this matrix having an entry $B(l, g) = \ast$ means that the set of users $S = S_g$ will execute SecAg protocol on the set of segments $\{x_i^l\}_{i \in S_g}$. In other words, having $B(l, g) = \ast$ means that the set of users $S_g$ will quantize the set of segments $\{x_i^l\}_{i \in S_g}$ according to Step 3 in Section 2 by using the quantizer $Q_{K_g}$, and jointly encode the resulting quantized segments together according to Step 4. At the server, these set of segments will be decoded together. Similarly, when $B(l, g) = B(l, g') = g$, this means that the set of users $S = S_g \cup S_{g'}$ corresponding to these columns $g$ and $g'$, where $g < g'$, will quantize the set of segments $\{x_i^l\}_{i \in S_g \cup S_{g'}}$ by using the quantizer $Q_{K_g}$ and jointly encode the resulting quantized segments. At the server side, these set of segments will be decoded together. Finally, the server aggregates each set of decoded segments $\{x_S^l\}_{S \subseteq \{0, \ldots, G-1\}}$, which results from different sets of users and belongs to the same segment level $l$, together. The server concatenates these sets of aggregated segments, which belong to these levels $l \in [G]$, to get the
A system with $N$ users partitioned into $G = 5$ groups, with $n$ users in each group. Each user holds a quantized local model update $\bar{x}_i$, $i \in [N]$. The segment selection and grouping is completed by using the SS matrix $B$. To illustrate HeteroSAg and understand how its inference robustness is measured, we consider the following example.

**Example 3.** We consider a system which consists of $N$ users, and a set of $G = 5$ quantizers $Q = \{Q_{K_0}, Q_{K_1}, Q_{K_2}, Q_{K_3}, Q_{K_4}\}$, where $K_0 < \cdots < K_4$. HeteroSAg execution starts by letting the $N$ users first share their keys and masks with each other, and then each user secret shares its masks with the other users in the system. We consider having $G = 5$ groups with $n$ users in each group, where groups are arranged in ascending order based on the communication resources of their users. The local model of each user, $x_i$ for $i \in [N]$, is equally partitioned into $G = 5$ segments $x_i = [x^0_i, x^1_i, x^2_i, x^3_i, x^4_i]^T$, where $x^l_i \in \mathbb{R}^{m_5}$, for $l \in [5]$. The SS matrix $B$ that is used for managing the execution of HeteroSAg is given in Figure. 2.

To further formalize the execution of HeteroSAg for this example, we consider Figure. 3. In this figure, each set of segments that executes the SecAg together is given the same color. In particular, the set of segments $\{x^0_i\}_{i \in S_0 \cup S_1}$ will be quantized by the quantizer $Q_{K_0}$ according to the third step in Section 2. By using the encoding step in Section 2, the output of the quantizer $\{\bar{x}^0_i\}_{i \in S}$, where $S = S_0 \cup S_1$, will be first mapped from the values that belongs to its discrete range to integer values in this range $[0, K_0 - 1]$. By generating the random 0-sum pairs of masks and the individual masks, the encoded segment for each user $i \in S$ will be given as follows

$$y_{S,i}^0 = \bar{x}^0_i + \text{PRG}(b_i) + \sum_{j:i<j} \text{PRG}(s_{i,j}) - \sum_{j:i>j} \text{PRG}(s_{j,i}) \mod R,$$

where $R = |S|(K_0 - 1) + 1$, $|S| = 2n$, and $j \in S$, while the PRG is used to expand the different seeds to segments in $\mathbb{Z}_R$. The server collects the shares and reconstructs the private seed of each surviving user, and the pairwise seeds of each dropped user. Then its uses the PRG along with the reconstructed seeds to expand them to segments in $\mathbb{Z}_R$, where $R = |S|(K_0 - 1) + 1$, $|S| = 2n$, to be removed from the aggregate of the masked segments. The server then computes this segment

$$x^0_U = \sum_{i \in U} (y^0_{S,i} - \text{PRG}(b_i)) - \sum_{i \in D} \left( \sum_{j:i<j} \text{PRG}(s_{i,j}) - \sum_{j:i>j} \text{PRG}(s_{j,i}) \right) \mod R,$$

where $U = U_0 \cup U_1$.

![Figure 3: A system with $N$ users partitioned into $G = 5$ groups, with $n$ users in each group. Each user holds a quantized local model update $\bar{x}_i$, $i \in [N]$. The segment selection and grouping is completed by using the SS matrix $B$.](image-url)
Figure 4: A system with $N$ users partitioned into $G = 5$ groups, with $n$ users in each group. Each user holds a local model update $\bar{x}_i$, $i \in [N]$. The segment selection and grouping is completed by using the SS matrix $B$. Segments with the same color will be encoded and decoded together independently of all other segments.

where the set $\mathcal{U}_i$ represents the set of survived users from $\mathcal{S}_i$, for $i = 0, 1$.

The aggregate model update from group 0 after fully unmasking its users’ models is given by Table 2, where $\mathcal{U}_g \subseteq \mathcal{S}_g$ for $g \in [4]$, represents the set of survived users from group $g$. According to Table 2, the server will decode only the last segment from the aggregated model update from group 0, while the other segments from that group interfere with segments from some other groups. In particular, the first segment from group 0 (first row in Table 2) results from the sum of the first set of segments from the survived users in group 0 and group 1, as these segments were encoded together, and hence must be decoded together. More generally, the server will decode only one clear segment from each individual group which is corresponding to the index denoted by * in the SS matrix $B$. Also, it can be easily seen that the server will not decode more than 0.2 of clear segments from the average model from any set of users $\mathcal{S} \subseteq \mathcal{N}$, and hence the inference robustness will be $\delta(\mathcal{A}_{\text{HeteroSAg}}) = \frac{3}{5} = 0.8$.

Table 2: The aggregated model update $\bar{x}_{\mathcal{S}_0}$ from group 0 after fully unmasking the model updates of its users

| $\mathcal{I}_0 \cup \mathcal{I}_1$ | $\mathcal{I}_0 \cup \mathcal{I}_2$ | $\mathcal{I}_0 \cup \mathcal{I}_3$ | $\mathcal{I}_0 \cup \mathcal{I}_4$ | $\mathcal{I}_4$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\bar{x}_{\mathcal{I}_0 \cup \mathcal{I}_1}$ | $\bar{x}_{\mathcal{I}_0 \cup \mathcal{I}_2}$ | $\bar{x}_{\mathcal{I}_0 \cup \mathcal{I}_3}$ | $\bar{x}_{\mathcal{I}_0 \cup \mathcal{I}_4}$ | $\bar{x}_{\mathcal{I}_4}$ |

4.2 HeteroSAg for Byzantine-Resilience

Now, we extend the segment grouping strategy $\mathcal{A}_{\text{HeteroSAg}}$ given by the SS matrix $B$ generated by Algorithm 1 to further provide Byzantine robustness while achieving secure model aggregation simultaneously. This can be done by integrating $\mathcal{A}_{\text{HeteroSAg}}$ with some coordinate-wise defense techniques, such as coordinate-wise median (Median) [19], which have provable convergence guarantees.

$^4$Decoding a segment $\theta_{\mathcal{S}}^l$ from the model update $\bar{x}_{\mathcal{S}}$ implies decoding the segment $\theta_{\mathcal{S}}^l$ from the average model from that $\mathcal{S}$. We note that $\theta_{\mathcal{S}}^l = \theta^l + \frac{1}{|\mathcal{S}|} x_{\mathcal{S}}^l$, where $\theta^l$ is the $l$-th segment from the global model $\theta$. 

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Integrating Median with HeteroSAg is possible thanks to the design of the SS matrix. Particularly in the SS matrix, for a given row $l$ out of the available $G$ rows (segment index $l$ out of the $G$ segments indices), the server observes a set of unmasked segments. Each unmasked segment results from averaging the segments that were encoded together. This is different than the design of the other secure aggregation protocols which make the server only receive the masked model of each user and the aggregate of all users model updates. Observing the masked model from each user makes the adaption of the state-of-the-art Byzantine-robust techniques difficult to implement, as these defense techniques are based on observing the individual clear model update of each user to compare the updates from different users with each other and remove the outliers. To further illustrate how the design of the SS matrix has solved the aforementioned limitation of the convention secure aggregation protocol, we consider the following example.

**Example 4.** We consider a system with $N$ users, where users are divided equally among $G = 5$ groups, and each group has $n$ users, as illustrated in Figure. 2. We assume that node 1 in the first group is a Byzantine node. The local model update of each node is divided equally into $G = 5$ segments. Each set of segments that is encoded and decoded together are given the same color as shown in Figure. 3. The segments are grouped and colored according to the SS matrix $B$ given in Figure. 2. As can be seen from Figure. 4, the server decodes the segments that were encoded together, e.g., $\{y_{0,1}^0 = \frac{1}{2n}\sum_{i\in N_0\cup N_3} \bar{x}_i^0, y_{2,4}^0 = \frac{1}{2n}\sum_{i\in N_2\cup N_4} \bar{x}_i^0, y_{3}^0 = \frac{1}{n}\sum_{i\in N_3} \bar{x}_i^0\}$. The segments in this set are from the sets of users in this tuple of sets $(S_0 \cup S_1, S_2 \cup S_4)$, respectively, and belong to the same segment level $l = 0$. Since we have more than one segment in this set, coordinate-wise median can be applied as demonstrated in Figure. 4.

The coordinate-wise median scheme in [19] is presented for the case where the number of Byzantine users is less than half the total number of users, i.e., the number of benign models is more than the number of faulty models. To find the number of allowed Byzantine users in our setting while using the segment grouping strategy $A_{\text{HeteroSAg}}$, we consider the worst case scenario where Byzantine users are distributed uniformly among the groups. To make sure that each set of unmasked segments, which belongs to the same level $l$, e.g., this set of unmasked segments $\{y_{0,1}^0 = \frac{1}{2n}\sum_{i\in N_0\cup N_3} \bar{x}_i^0, y_{2,4}^0 = \frac{1}{2n}\sum_{i\in N_2\cup N_4} \bar{x}_i^0, y_{3}^0 = \frac{1}{n}\sum_{i\in N_3} \bar{x}_i^0\}$, contains benign segments more than faulty segments, the number of Byzantine users, $B$, should be $B \leq \lceil 0.25G \rceil - 1$. The former result comes from the fact that having one Byzantine user in each group makes all the segments of the average model from this group faulty. Also in HeteroSAg, we can see that some segments belong to the aggregated model from two groups. In particular, we can see the faulty model from user 1 in group 0 results in the following faulty segments $\{y_{0,1}^0, y_{0,2}^0, y_{0,3}^0, y_{0,4}^0, y_{0}^0\}$. By taking this extreme case where one Byzantine node has an impact on the segments from two groups, and the number of benign segments should be more than the faulty segments within each set of decoded segments, the number of Byzantine users, $B$, should be $B \leq \lceil 0.25G \rceil - 1$.

We have included further discussion for the Byzantine robustness of HeteroSAg in Appendix II.

### 4.3 Theoretical guarantees of HeteroSAg

We state our main theoretical results. The proofs of the theorems, propositions, and lemmas are presented in the Appendix.

**Theorem 1** (Inference robustness) For a FL system with $N$ users clustered into $G$ groups, and the model update of each user is divided equally into $G$ segments, the segment grouping strategy of HeteroSAg achieves inference robustness of $\delta(A_{\text{HeteroSAg}}) = \frac{G-1}{G}$ when the number of groups is even, and $\delta(A_{\text{HeteroSAg}}) = \frac{G-1}{G}$ when the number of groups is odd.

**Remark 2** Theorem 1 shows that we can achieve full inference robustness for sufficiently large number of groups. In particular, the maximum value for inference robustness, $\delta$, is reached when $G = n$. In this case, the server can decode $\frac{G}{n}$ (or $\frac{1}{m}$ when $m$ is odd) of the average model from any set of users, which approaches zero for sufficiently large model size. In the Appendix, we show how we can further portion the users in each group to smaller sub-groups when the size of the set of quantizers $G$ is small to increase $\delta$.

We derive the convergence guarantees of HeteroSAg under the following standard assumptions.

**Assumption 1** (Unbiasedness) The stochastic gradient $\bar{x}_i^{(t)} = g_i(\theta^{(t)})$ is an unbiased estimator for the true gradient of the global loss function in (1) such that $\mathbb{E}[\bar{x}_i^{(t)}] = \nabla F(\theta^{(t)})$.
Assumption 2 (Smoothness) The objective function $F(\theta)$ in (1) is convex, and its gradient is $L$-Lipschitz that is $\|\nabla F(\theta) - \nabla F(\theta')\| \leq L\|\theta - \theta'\|$, for all $\theta, \theta' \in \mathbb{R}^n$.

Lemma 1 For any vector $x_i = [x_i^0, \ldots, x_i^{G-1}] \in \mathbb{R}^m$, and by letting $x_i$ be the quantization of $x_i$, we have that (i) $E[x_i] = x_i$ (unbiasedness), (ii) $E[\|x_i - x_i\|^2] \leq \frac{m}{G} \sum_{g=0}^{G-1} \frac{1}{(K_g - 1)^2}$ (bounded variance), where $\Delta_i$ is the quantization interval associated with the stochastic quantizer $Q_{K_i}$ to quantize the $l$-th segment $x_i$. (iii) $E[\|\hat{p} - p\|^2] \leq \sigma^2$ (total quantization error), where $\hat{p} = \frac{1}{N} \sum_{i=1}^{N} x_i$, $p = \frac{1}{N} \sum_{i=1}^{N} x_i$, and $\sigma^2 = \frac{(r_x - \epsilon)^2}{4N^2} \frac{m}{G} \sum_{g=0}^{G-1} \frac{1}{(K_g - 1)^2}$. Here, $G$ is the number of segments and $K_i$ is the number of levels used to quantize the $l$-th segment of $x_i$.

Theorem 2 (Convergence) Consider FL system with $N$ users, each of which has a local gradient vector $x_i(t) = g_i(\theta(t)) \in \mathbb{R}^m$, such that the elements of each local gradient $x_i(t)$, for $i = 1, \ldots, N$, fall in the range $[r_1, r_2]$. Suppose the conditions in Assumptions 1-2 are satisfied. When each local gradient vector is partitioned equally into $G$ segments such that segments are quantized by using the set $Q$ of $G$ quantizers according to the SS matrix $B$, and by using constant step size $\eta = 1/L$, HeteroSAg guarantees

$$E \left[ F \left( \frac{1}{J} \sum_{l=1}^{J} \theta(t) \right) \right] - F(\theta^*) \leq \frac{\|\theta^0 - \theta^*\|^2}{2\eta J} + \eta \sigma^2_{\text{HeteroSAg}},$$

where $\sigma_{\text{HeteroSAg}} = \frac{(r_x - \epsilon)^2}{4N^2} \frac{m}{G} \sum_{g=0}^{G-1} \frac{2(G-g)-1}{(K_g-1)^2}$, and $\theta^0$ is the initial model.

Remark 3 HeteroSAg has a convergence rate of $O(1/J)$. The term $\eta \sigma_{\text{HeteroSAg}}$ is a residual error in the training which can be reduced by using an adaptive (decreasing) learning rate and by using a set of high level quantizers.

Remark 4 According to Theorem 1 and the bound on the number of Byzantine nodes given in Section 4.2 increasing the number of groups by further partitioning each group out of the G available groups equally into L subgroups results in increasing 1) The number of Byzantine to be tolerated $B \leq \left[0.25LG\right] - 1$, 2) The inference robustness $\delta = \frac{1}{2} \frac{LG-1}{G^2}$. On the other hand, the residual error in Theorem 2 will not increase as stated in the next proposition.

Proposition 1 (Quantization error) Let $G$ be the number of quantizers to be used in the system, and users are partitioned equally into $G$ groups, by extra partitioning each group equally into $L$ subgroups while using the segment grouping strategy of HeteroSAg, the total quantization error will not be changed and will also be given by $\sigma_{\text{HeteroSAg}}^2$.

Theorem 3 (Privacy leakage and dropout) For HeteroSAg when the number of users in each subgroup is given by $\bar{n} = \frac{N}{LG}$, where $LG$ is the total number of subgroups, and the dropout probability of each user is $p$, the probability of privacy leakage, i.e., having only one survived user in any subgroup, is given by

$$P[\text{Privacy leakage}] = P(X = 1) = \bar{n}(1-p)^{\bar{n}-1},$$

where having one survived user in any group implies that the server will be able to decode one clear segment from the model of that user; i.e., $I(x_i^l, y_i^l) \neq 0$, for a given segment $l$, where $y_i^l$ is the $l$-th encoded segment from user $i$.

Remark 5 The probability in (12) approaches zero by either having a small probability of dropout $p$, or by increasing the number of users in each subgroup. Therefore, the number of users in each subgroup makes a trade-off between the benefits of extra partitioning discussed in Remark 4 and the privacy of users’ models. To further illustrate the impact of the subgroup size $\bar{n}$ on the probability in (12), we consider the following example. Let the total number of users in each subgroup to be $\bar{n} = 8$, and by considering $p = 0.1$, a typical number for the probability of dropout [36], the probability in (12) turns out to be $7.2 \times 10^{-2}$, which is negligible. We further note that by using HeteroSAg, the dropout rate of users becomes smaller. The reason for that in HeteroSAg users consider their transmission rates when they choose their quantizers. This decreases their probabilities of being delayed and hence being considered dropped out by the server.
Proposition 2 (HeteroSAg communication and computation costs) Each user has a computation cost of $O(N^2 + m n)$ as opposed to $O(N^2 + m N)$ for SecAg, where $m$, $N$ and $n$ are the model size, total number of users and number of users in each group, respectively, and the same communication cost $O(N + m)$ as SecAg. However, the bandwidth expansion, which measures the ratio between the size of the encoded model in bits to the size of the clear model, is much lower for HeteroSAg. The communication and the computation complexities at the server are the same as in SecAg.

Table 3: A Comparison between SecAg [1] and our proposed HeteroSAg. Here, $N$ is the total number of nodes, $m$ is the model size, $G$ is the number of groups, $n$ is the number of users in each group, and $K_g$ is the number of quantization levels.

|                               | SecAg        | HeteroSAg   |
|-------------------------------|--------------|-------------|
| Adaptive quantizers          | No           | Yes         |
| Communication complexity      | $O(N + m)$   | $O(N + m)$  |
| Computation complexity        | $O(N^2 + mN)$| $O(N^2 + mn)$|
| Inference robustness         | 1            | $\frac{G - 2}{G}$ |
| Byzantine Robustness         | No           | Yes         |
| Quantization error bound      | $\frac{(r_2 - r_1)^2}{4N^2 nG} \frac{1}{(K-1)^2}$ | $\frac{(r_2 - r_1)^2}{4N^2 nG} \sum_{g=0}^{G-1} \frac{2(G-g)-1}{(K_g-1)^2}$ |
| Probability of local model breach | 0           | $\rightarrow 0$ |

In Table 3, we give a comparison between HeteroSAg and SecAg [1]. As we can observe from this table that HeteroSAg is an adaptive algorithm that allows users to use different quantizers to balance their communication load to their channel quality, while achieving secure model aggregation. This is different from SecAg which requires all users to use the same quantizer to guarantee correct decoding as discussed in Section 3.2. Additionally, HeteroSAg achieves Byzantine robustness, while SecAg fails in the presence of Byzantine nodes, as we will demonstrate in Section 5. Furthermore, the communication complexity of HeteroSAg is lower than SecAg, while both algorithms have the same communication complexity. Regarding the inference robustness, SecAg achieves inference robustness of 1, meaning that the server will not decode any segment from the average model of any subset of users. On the other hand, HeteroSAg achieves lower inference robustness. However, the inference robustness of HeteroSAg approaches 1 for sufficiently large number of groups. The probability of local model breach when users dropped out in SecAg is 0, while in HeteroSAg this probability approaches 0 when increasing the number of users in each group (Theorem 3).

5 Numerical Experiments

We run two different experiments to show the performance gains achieved by HeteroSAg. Experiment 1 highlights the benefits of using heterogeneous quantization. The second experiment is to demonstrate how the secure aggregation strategy of HeteroSAg can be effective along with coordinate-wise median against Byzantine users.

5.1 Experiment 1 (Heterogeneous quantization)

We consider the setup of $N = 25$ users, where users are equally partitioned into $G = 5$ groups and each model update vector is equally partitioned into 5 segments. We consider MNIST dataset [37] and use a neural network with two fully connected layers. The details of the neural network is presented in Appendix 1.4. For the data distribution, we sort the training data as per class, partition the sorted data into $N$ subsets, and assign each node one partition. We set the number of epochs to be 5, use a batch size of 240, and constant learning rate 0.03. We consider three different scenarios for the performance comparison based on the quantization scheme. The three scenarios apply the same segment grouping strategy given by the SS matrix $B$ in Figure 2 in terms of the encoding and decoding strategy (e.g., the first segment from group 0 and group 1 will be encoded together and decoded together at the server in the same way for the three scenarios), while they are different in the quantization scheme.

Quantization. We consider three scenarios based on the quantization scheme. Heterogeneous quantization: We consider a set $Q$ of $G = 5$ quantizers with these levels of quantization $(K_0, K_1, K_2, K_3, K_4) = (2, 6, 8, 10, 12)$, where using these quantizers follows the pattern given in
Figure 5: The performance of HeteroSAg under different quantization schemes for the non-IID setting.

**Figure 2.** Homogeneous quantization: All the segments from all users are quantized by using $K = 2$ levels quantizer. No quantization: All segments are represented in floating-point numbers (32 bits).\

We consider group 0 as a straggler group which includes users with limited communication resources including low transmission rates. In particular, we let each user in group 0 to have 1Mb/s transmission rate while users in higher groups to have more than 2Mb/s, in order to have a comparison between the three cases.

We have run the same experiment with $N = 100$ users. The results lead to the same conclusion, and can be found in the Appendix J.2.1. The details of the results in Figure. 5(b) are given in Appendix J.1. Additionally, we have further evaluate the performance of HeteroSAg using CIFAR10 dataset in Appendix J.2.2.

Figure 5(a) illustrates that HeteroSAg with heterogeneous quantization achieves accuracy close to the baseline (no-quantization). Additionally, after $t = 200$ rounds of communication with the server, the total communication time when using heterogeneous is less the case with no quantization by a factor of $5.2\times$ according Figure. 6(a). Furthermore, HeteroSAg with heterogeneous quantization maintains superior performance over the case of homogeneous quantization with $K = 2$ levels with more than 15% improvement in test accuracy, while the communication time is the same for the both settings. This confirms our motivation that by adapting the quantization levels to the transmission rates of the users, we can achieve high accuracy with small training time.

Figure 6: The performance of HeteroSAg and FedAvg under three different attacks for the IID setting.

### 5.2 Experiment 2 (Byzantine robustness)

We show how the secure aggregation strategy of HeteroSAg can be effective along with the coordinate-wise median against Byzantine users. For running the experiment, we consider a setup of $N = 300$ users, in which $B = 18$ of them are Byzantines. We consider the IID setting, where we randomly

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5 Using HeteroSAg with no quantization is the same as using FedAvg with no quantization with respect to the test accuracy. The difference between the two schemes is that in HeteroSAg models are encoded unlike FedAvg where clear models are sent to the server. This just results in a model with larger size as described in details in Appendix E. We consider HeteroSAg under different quantization schemes for a fair comparison regarding the communication cost and time given in Figure. 5(b).
split the training data samples to $N = 300$ disjoint subsets, and assign each subset to a distinct user. We use a learning rate of 0.06, set the number of epochs to be 1, and use batch size 40. (model details are presented in Section J.4). The performance of HeteroSAg in the non-IID setting is presented in Appendix J.3.

**Scheme.** We consider two schemes: HeteroSAg with $G = 75$ groups and $n = 4$ users in each group along with coordinate-wise median, and FedAvg implemented with secure aggregation [3].

**Attack model.** We assume that the Byzantine users are distributed over 18 groups. We note that since the focus here is the presence of Byzantine users while doing secure aggregation, where users are sending masked model to the server, Byzantine users can send any faulty model with extreme values without being individually decoded and hence filtered out. **Gaussian Attack:** Each Byzantine user replaces its model parameters with entries drawn from a Gaussian distribution with mean 0 and standard distribution $\sigma = 5$. **Sign-flip:** Each Byzantine user multiplies its model updates by $-5$. **Label-flip:** Each Byzantine user subtract 9 from its labeled data, and then multiplies its resulting model update by 30.

As we can see in Figure 5, HeteroSAg with coordinate-wise median is robust to the three attacks and gives performance almost the same as the case with no Byzantine users. On the other hand, in the presence of these attacks, FedAvg scheme gives very low performance. As a final remark, HeteroSAg achieves both privacy for the users’ local models and Byzantine robustness, simultaneously. This is different from the SecAg protocol that only achieves model privacy, and different from the naive coordinate-wise median [14] that solely achieves Byzantine robustness.

**6 Conclusion**

We propose HeteroSAg, a scheme that allows secure aggregation with heterogeneous quantization. This enables the users to adjust their quantization proportional to their communication resources, which can provide a substantial better trade-off between the accuracy of training and the communication time. We show that the proposed strategy used in HeteroSAg can be utilized to mitigate Byzantine users. Finally, we demonstrate that HeteroSAg can significantly reduce the bandwidth expansion of the state-of-the-art secure aggregation protocol.

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Appendices

A Illustrative example for SecAg

In this simple example, we illustrate SecAg protocol. We consider a secure aggregation problem in FL, where there are $N = 3$ nodes with drop-out resiliency $D = 1$. Node $i \in \{1, 2, 3\}$ holds a local model update vector $x_i \in \mathbb{R}^m$. In the following, we present the steps for executing the SecAg protocol.

Step 1: Sharing keys and masks. User 1 and User 2 agree on pairwise random seed $s_{1,2}$. User 1 and User 3 agree on pairwise random seed $s_{1,3}$. User 2 and User 3 agree on pairwise random seed $s_{2,3}$. Each one of these pairwise seeds is a function of the public key and the private keys of paired users (more details is given in Step 1 in Section II). In addition, user $i \in \{1, 2, 3\}$ creates a private random seed $b_i$.

Step 2: Sharing keys and masks. Each user $i \in \{1, 2, 3\}$ secret shares $b_i$ and the private key $s_{i}^{SK}$ with the other users via Shamir’s secret sharing. The threshold for secret sharing is set to 2.

Step 3: Quantizing. user $i \in \{1, 2, 3\}$ quantizes its model $x_i$ using $K$ levels of quantization according to equation (3). The output of the quantizer $\bar{x}_i(k) = Q_{K}(x_i(k))$ takes a discrete value from this range $\{r_1, r_1 + \Delta_K, r_1 + 2\Delta_K, \ldots, r_2 - \Delta_K, r_2\}$, where $\Delta_K = \frac{r_2 - r_1}{K - 1}$ is the quantization interval. The quantized model of each node will be mapped from $\{r_1, r_1 + \Delta_K, r_1 + 2\Delta_K, \ldots, r_2 - \Delta_K, r_2\}$ to the integer range $[0, K - 1]$.

Step 4: Masking.
To provide privacy for each individual model, user \( i \in \{1, 2, 3\} \), masks its model \( \bar{x}_i \) as follows:

\[
y_1 = \bar{x}_1 + n_1 + z_{1,2} + z_{1,3} \mod R, \quad y_2 = \bar{x}_2 + n_2 + z_{2,3} - z_{1,2} \mod R, \\
y_3 = \bar{x}_3 + n_3 - z_{1,3} - z_{2,3} \mod R,
\]

where \( n_i = \text{PRG}(b_i) \) and \( z_{i,j} = \text{PRG}(s_{i,j}) \) are the random masks generated by a pseudo random number generator. Here, \( R = 3(K - 1) + 1 \) to ensure that all possible aggregate vectors from the three users will be representable without any overflow. After that, user \( i \in \{1, 2, 3\} \) sends its masked model \( y_i \) to the server.

**Step 5: Decoding.** This phase for the aggregate-model recovery. Suppose that user 1 drops in the previous phase. The goal of the server is to compute the aggregate of the models \( \bar{x}_2 + \bar{x}_3 \). The aggregated model at the server from the survived users (user 2 and user 3) is given as follows

\[
x_{2,3} = \bar{x}_2 + \bar{x}_3 + (n_2 + n_3) - z_{1,2} - z_{1,3} \mod R.
\]

Hence, the server needs to reconstruct masks \( n_2, n_3, z_{1,2}, \) and \( z_{1,3} \) to recover \( \bar{x}_2 + \bar{x}_3 \). To do that, the server has to collect two shares for each of \( b_2, b_3 \) and \( s_{i}^{SK} \) from the two survived users. Therefore, the server can reconstruct the missing masks and remove them from (15). Note that, if node 1 is delayed while the server has already reconstructed \( z_{1,2}, \) and \( z_{1,3} \), the local model of node 1 is still protected by the private mask \( b_i \) in (13).

**B Proof of Theorem 1**

First, we state four main properties of the SS matrix \( B \) for HeteroSAg. These properties will be used to prove the inference robustness of HeteroSAg.

**B.1 Main properties for the SS matrix of HeteroSAg**

**Property 1.** Each column in the SS matrix contains only one ∗. This implies that each group of users independently of the other groups executes the secure aggregation on only one segment from its model update.

**Property 2.** Any two distinct columns \( g \) and \( g' \) in the SS matrix have at most one row with the same number, where we do not consider the symbol ∗ as a number. This implies that each two groups of users corresponding to these columns independently of the other groups execute the secure aggregation on only one segment from their model updates.

**Property 3.** For the case where the number of groups \( G \) is odd, each row in the SS matrix contains only one ∗. On the other hand, when the number of groups is even, only a pair of ∗’s can be found in the odd rows (the indices of the rows in the SS matrix started from 0). A pair of ∗’s for a given row belongs to one pair of groups from this set \{\((g, g + \frac{G}{2}) : g = 0, \ldots, \frac{G}{2} - 1\)\} as shown in Figure 7.

\[
B = \begin{pmatrix}
0 & 0 & 2 & 3 & 3 & 2 \\
0 & * & 0 & 3 & * & 3 \\
0 & 1 & 1 & 0 & 4 & 4 \\
0 & 1 & * & 1 & 0 & * \\
0 & 1 & 2 & 2 & 1 & 0 \\
* & 1 & 2 & * & 2 & 1
\end{pmatrix}
\]

Figure 7: Matrix \( B \) for \( G = 6 \).

**Property 4.** We say that we have a pair when having two equal numbers. In the SS matrix, if row \( i \) contains only pairs of numbers from the set of columns \( S \subset \{0, \ldots, G - 1\} \) where \( |S| = 2n \), for \( n = 2, \ldots, \frac{G - 1}{2} \), where \( i = 2 \) for even number of groups and \( i = 1 \) for odd number of groups, each
row in the remaining set of rows \( \{0, \ldots, G - 1\}/i \) corresponding to the set of columns \( S \) will have at least two unpaired numbers. For example, in the first row of the SS matrix given in Figure. 2, we have these two pairs \( (B(0, 0), B(0, 1)) \) and \( (B(0, 2), B(0, 4)) \), where \( B(0, 0) = B(0, 1) = 0 \) and \( B(0, 2) = B(0, 4) = 2 \). On the other hand, for these column indices \( S = \{0, 1, 2, 4\} \), each row in the set of remaining rows \( \{1, \ldots, 3\} \) does not contain numbers to be paired.

### B.2 Proof of Theorem 1

According to the SS matrix \( B \), we have these 4 cases:

**Case 1: Single group.** For a single group \( S \in \{0, \ldots, G - 1\} \), the server can only decode the segment denoted by * in the SS matrix from the model update \( x_S \) by using Property 1.

**Case 2: A Pair of groups.** From any pair of groups \( S \subset \{0, \ldots, G - 1\} \), where \( |S| = 2 \), the server can successfully decode at most one segment from the local model update \( x_S \) when the number of groups is odd. On the other hand, when the total number of groups is even, the server can decode at most two segments. The former results come from: First, Property 1 and Property 2 show that for any pair of groups \( S \subset \{0, \ldots, G - 1\} \), where \( |S| = 2 \), the server can decode one segment from the model update \( x_S \). This segment is the segment that is jointly encoded by the set of groups in \( S \). Second, Property 3 states that when the total number of groups \( G \) is odd, the segments denoted by *'s in any pair of groups \( S \) do not get aligned in the same row, but interfere with segments from other groups. On the other hand, when the total number of groups \( G \) is even, the segments denoted by *'s might be aligned together on the same row, and hence make the server able to decode another segment from the model update \( x_S \) according to Property 3.

**Case 3: Even number of groups.** From any even number of groups \( S \subset \{0, \ldots, G - 1\} \), where \( |S| = 2n \) for \( n = 2, \ldots, \frac{G - 1}{2} \) with \( i = 2 \) when \( G \) is even and \( i = 1 \) when \( G \) is odd, the server can only decode one segment from the model update \( x_S \) according to Property 4. This segment results from the sum of the decoded segments from each pair of groups in \( S \), while the segment denoted by * from each group in the set \( S \) interferes with a segment from another group according to Property 3.

**Case 4: Odd number of groups.** For an odd number of groups in the set \( S \subset \{0, \ldots, G - 1\} \), where \( |S| = 2n + 1 \) for \( n = 1, \ldots, \frac{G - 1}{2} \) with \( i = 4 \) when \( G \) is even and \( i = 2 \) when \( G \) is odd, the server can not decode any segment from the model update \( x_S \). The reason for that for any given row in the SS matrix there is at least two segments to not be paired. These four cases complete the proof.

### C Proof of Lemma 1

i) (Unbiasedness) One can easily prove that \( E[x_i] = x_i \).

ii) (Variance bound) \[
E[\|x_i - x_i\|^2] \leq \sum_{l=0}^{G-1} \sum_{k=1}^{\frac{G}{i}} E[(x_i'(k) - x_i'(k))^2] \leq \sum_{l=0}^{G-1} \sum_{k=1}^{\frac{G}{i}} (T(l+1) - T(l))^2 = \frac{m}{G} \sum_{l=0}^{G-1} (\Delta'_l)^2 \leq \frac{G-1}{4} \left( \frac{\sigma^2}{4} \right) \]

where \( (a) \) follows from the fact that the random quantization is IID over elements of the vector \( x_i \), (b) from the variance of the quantizer in \( \| \) \, and (c) from the bound in \( \| \), which states that having \( x \) such that \( a \leq x \leq b \), this implies \( (b-x)(x-a) \leq \frac{(b-a)^2}{4} \).

iii) (Total quantization error) \[
E[\|\hat{p} - p\|^2] = \sum_{i=1}^{N} \left( \frac{1}{N} \sum_{i=1}^{N} \bar{x}_i - \frac{1}{N} \sum_{i=1}^{N} x_i \right)^2 \leq \frac{1}{N^2} \sum_{i=1}^{N} \left( \bar{x}_i - x_i \right)^2 = \frac{1}{N^2} \sum_{i=1}^{N} \left( \bar{x}_i - x_i \right)^2 \leq \frac{\sigma^2}{4N^2} \sum_{i=1}^{N} \frac{1}{G} \sum_{l=0}^{G-1} \left( \Delta'_l - 1 \right)^2 = \sigma^2. \]

where \( (d) \) follows from the fact that the random quantization is IID over the \( N \) local gradients \( \{x_i\}_{i=1}^{N} \), and (e) from \( \| \). We note that \( \sigma^2_{\text{HeteroSag}} \) (total quantization error when using HeteroSag)
in Theorem 2 can be derived from \( E \) by counting the number of segments that is quantized by each quantizer. According to the SS matrix \( B \), each user \( i \in S_g \) in group \( g \), for \( 0 \leq g \leq G - 1 \) uses quantizer \( Q_K \) to quantize \( G - g \) segments, and the remaining \( g \) segments are quantized by the set of quantizers \( \{Q_{K_0}, Q_{K_1}, \ldots, Q_{K_{g-1}}\} \), with one segment for each quantizer. Hence, the total number of segments used quantizer \( Q_K \), where \( 0 \leq g \leq G - 1 \), is given by \((2(G - g) - 1)n\).

D Proof of Theorem 2

From the \( L \)-Lipschitz continuity of \( \nabla F(\theta) \), we have

\[
F(\theta^{(t+1)}) \leq F(\theta^{(t)}) + \langle \nabla F(\theta^{(t)}), \theta^{(t+1)} - \theta^{(t)} \rangle + \frac{L}{2}||\theta^{(t+1)} - \theta^{(t)}||^2 \geq F(\theta^{(t)}) - \eta \langle \nabla F(\theta^{(t)}), \bar{p}^{(t)} \rangle + \frac{L\eta^2}{2}||\bar{p}^{(t)}||^2,
\]

where \( \bar{p}^{(t)} = \frac{1}{N} \sum_{i=1}^{N} \bar{x}_i \), and \( \bar{x}_i = g_i(\theta^{(t)}) \) is the quantized local gradient at node \( i \). We used this relation \( \theta^{(t+1)} = \theta^{(t)} - \eta \bar{p}^{(t)} \) to get (a). By taking the expectation with respect to the quantization noise and data sampling randomness,

\[
E\left[F(\theta^{(t+1)})\right] \leq F(\theta^{(t)}) - \eta \langle \nabla F(\theta^{(t)}), \theta^{(t)} \rangle - \frac{\eta}{2}||\nabla F(\theta^{(t)})||^2 + \frac{\eta^2}{2}\sigma^2_{\text{HeteroSAg}}
\]

where (a) follows from that \( E||\bar{p}^{(t)}||^2 = E||p^{(t)} - E[p^{(t)}]||^2 + E||E[p^{(t)}]||^2 \), \( E[\bar{x}_i^{(t)}] = \nabla F(\theta^{(t)}) \) according to Lemma 1-(i) and Assumption 1, and \( E||p^{(t)} - E[p^{(t)}]||^2 \leq \sigma^2_{\text{HeteroSAg}} \) according to Lemma 1-(iii), where \( E[\bar{p}^{(t)}] = p^{(t)} \) with respect to the quantization error. Furthermore, (b) follows from using \( \eta \leq \frac{1}{L} \), and (c) from the convexity of \( F(.) \). By summing the above equations for \( t = 0, \ldots, J - 1 \)

\[
\sum_{t=0}^{J-1} \left( E\left[F(\theta^{(t+1)})\right] - F(\theta^{*}) \right) \leq \frac{1}{2\eta}(E||\theta^0 - \theta^*||^2 - E||\theta^J - \theta^*||^2) + \eta J\sigma^2_{\text{HeteroSAg}} \leq \frac{||\theta^0 - \theta^*||^2}{2\eta} + \eta J\sigma^2_{\text{HeteroSAg}},
\]

By using the convexity of \( F(.) \),

\[
E\left[F\left(\frac{1}{J} \sum_{t=1}^{J} \theta^{(t)}\right)\right] - F(\theta^{*}) \leq \frac{1}{J} \sum_{t=0}^{J-1} \left( E\left[F(\theta^{(t+1)})\right] - F(\theta^{*}) \right) \leq \frac{||\theta^0 - \theta^*||^2}{2\eta J} + \eta^2\sigma^2_{\text{HeteroSAg}}.
\]

E Proof of Theorem 3

We recall that the model update \( x_i \) of user \( i \), is partitioned into \( Z \) segments when using HeteroSAg, where \( Z \) is the total number of subgroups. The partitioned model is denoted by \( x_i = [x_i^0, x_i^1, \ldots, x_i^{Z-1}]^T \), where \( x_i^l \in \mathbb{R}^\mathbb{X} \). To guarantee information theoretic privacy for the model update \( x_i \), we should have \( I(x_i^l; y_i^l) = 0 \), for \( l = 0, \ldots, Z - 1 \), where \( y_i^l \) is the \( l \)-th encoded segment from user \( i \). To achieve this information theoretic privacy for each segment in the model update, the server should not be able to decode any individual segment \( x_i^l \) when recovering all pairwise keys of dropped users and the private keys of the survived users. Each segment from user
We consider a more general scenario, where instead of assuming that the set of \( N \) users execute the secure aggregation protocol together on the same subgroup, or 2\( \bar{n} \) users from its subgroup and from an additional subgroup, where \( \bar{n} \) is the total number the number of users in each subgroup. Therefore, the number of survived users in each subgroup can be lower. In particular, if \( \bar{n} = 8 \) users execute the secure aggregation protocol together, the majority of the bandwidth expansion for the additive masking in SecAg comes from the number of users that execute the protocol together. In our proposed segment grouping strategy, all segments in clear without any encoding result in \( O(\bar{n}) \) time in total. The former result comes from the fact that each element in the model update of any user in HeteroSAg is masked by either \( \bar{n} - 1 \) masks or \( 2\bar{n} - 1 \) masks, unlike SecAg where the whole vector is masked by \( N - 1 \) 0-sum pairwise masks. Therefore, for the case where the number of users in each group \( \bar{n} = \log N \), the computation cost becomes \( O(2^{\bar{n}} + m \log N) \), as opposite to \( O(2^{\bar{n}m}) \).

User communication complexity: \( O(N + m) \) The communication complexity is the same as the secure aggregation protocol; however, the actual number of transmitted bits per user in HeteroSAg is lower. In particular, HeteroSAg gives lower per user communication cost compared to SecAg. Specifically, having a set of \(|S|\) users executes the secure aggregation protocol together on the set of segments \( \{x_i\}_{i \in S} \), the actual number of transmitted bits from each user \( i \in S \) is given by \( R = |x_i| \log(|S| (K_g - 1) + 1) \) according to [4], where \( |x_i| \) gives us the number of elements in this segment, and \( K_g \) is the number of quantizer levels. On the other hand, just sending the quantized segments in clear without any encoding results in \( |x_i| \log(K_g) \) bits. This gives us an expression for what is called the bandwidth expansion factor with respect to segments \( \frac{\log(|S| (K_g - 1) + 1)}{\log(K_g)} \), while ignoring the cost of sharing keys and masks and other cryptographic aspects of the protocol [4]. In fact, the majority of the bandwidth expansion for the additive masking in SecAg comes from the number of users that execute the protocol together. In our proposed segment grouping strategy, all segments are executed by either \( \bar{n} \) or \( 2\bar{n} \) users. On the other hand, SecAg besides the fact that the local model of all users are quantized by using the same quantizer, even if they have different communication resources, all the \( N \) users execute the secure aggregation protocol together. This implies much larger bandwidth expansion factor than our HeteroSAg.

In order to further illustrate how HeteroSAg reduces the bandwidth expansion of the SecAg protocol, we give the following numerical explanations. We assume having \( N = 2^{10} \) users, a number used to evaluate this metric in [1][25], and assume without loss of generality that only one quantizer to be used by the users. When the partitioning step results in \( \bar{n} = 8 \) users in each subgroup, and when using \( K = 2^{16} \) quantization levels, the bandwidth expansion factor becomes \( 1.25 \times \) instead of being \( 1.625 \times \) for SecAg. For a single bit quantization, the expansion factor is significantly reduced from \( 11 \times \) to \( 4 \times \).

### G HeteroSAg for Heterogeneous group Size

We have considered the case of uniform group sizes for HeteroSAg, where clustering users results in the same number of users in each group, in Section 4 in the main submission. In this section, we consider a more general scenario, where instead of assuming that the set of \( N \) users are divided equally on the \( G \) groups, where each group has \( n \) users, we assume the case where the number of users in each group is different.

---

\(^6\) The costs of sharing keys and masks in HeteroSAg are the same as SecAg, so we do not consider them in the evaluation.
G.1 Execution of HeteroSAg for heterogeneous group size

Similar to HeteroSAg in Section 4.1, key agreement and secret sharing are executed according to Step 1 and Step 2 in Section 2. Here, the size of the set of users in group $g$ is denoted by $|S_g| = n_g$, for $g \in [G]$, where $\sum_{g=1}^{G-1} n_g = N$, and $G$ is the number of possible quantizers given in the set $Q$. The second extension for HeteroSAg is that we allow further partitioning of the groups into smaller subgroups when the number of users in each group is large. Extra partitioning results in the benefits given in Remark 4, and in decreasing the expansion factor discussed in Section F, which measures the ratio between the size of the masked model in bits to the size of the clear model. Extra partitioning is achieved by dividing each set of users $S_g$, for $g \in [G]$, into $L_g$ subsets (subgroups), $S^d_g$, for $d = 0, \ldots, L_g - 1$, such that each subgroup has the same number of users $\bar{n}$. Following the clustering step and the extra partition of the groups, each model update vector $\{x_i^l\}_{i \in [N]}$ is equally partitioned into $Z$ segments $x_i^l = [x_i^1, x_i^2, \ldots, x_i^{Z-1}]^T$, where $x_i^l \in \mathbb{R}^Z$ and $Z = \sum_{g=0}^{G-1} L_g$, for $l \in [Z]$. Also, we should have $Z \leq m$, and for sufficiently large $N > m$, we might restrict the number of subgroups to equal the size of the model parameter $Z = m$, which means that each segment of the local model update is just one element.

The segment grouping strategy $A_{\text{HeteroSAg}}$ is given by the SS matrix $B$ with dimensions $Z \times Z$ according to Algorithm 2. As shown in the example SS matrix given in Figure 8:

$$B^e = \begin{pmatrix}
(0,0) & (1,0) & (1,1) & (2,0) & (2,1) \\
(0,0) & (0,0) & (1,1) & (*) & (1,1) \\
(0,0) & (*) & (0,0) & (2,0) & (2,0) \\
(0,0) & (1,0) & (1,0) & (0,0) & (*) \\
(0,0) & (1,0) & (*) & (1,0) & (0,0) \\
(*) & (1,0) & (1,1) & (1,1) & (1,0) \\
\end{pmatrix}$$

Figure 8: Matrix $B^e$ for $G = 3$ groups with number of subgroups in each group $L_0 = 1$, $L_1 = 2$, and $L_2 = 2$, respectively.

Each column is indexed by two indices $(g, d)$ representing the set of users $S^d_g$, for $g \in [G]$ and $d = 0, \ldots, L_g - 1$, while each row $l$, for $l = 0, \ldots, Z - 1$, represents the index of the segment. Similar to the description in Section 4.1, having an entry $B^e(l, (g, d)) = *$ means that the set of users $S = S^d_g$ will quantize the set of segments $\{x_i^l\}_{i \in S^d_g}$ by the quantizer $Q_{K_g}$ and encode them together, while at the server side these set of segment will be decoded together. When $B^e(l, (g, d)) = B^e(l, (g', d')) = (g, d)$, where $g \leq g'$, this means that the set of users $S = S^d_g \cup S^d_{g'}$, corresponding to these columns $(g, d)$ and $(g', d')$, will quantize the set of segments $\{x_i^l\}_{i \in S^d_g \cup S^d_{g'}}$. 

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We further discuss the intuition behind the success of HeteroSAg in mitigating the Byzantine nodes. The reason for the success of coordinate-wise median (Median) algorithm in mitigating the Byzantine nodes in the IID setting is the same reason behind the success of HeteroSAg when it is integrated with coordinate-wise median. In particular, the success of Median is guaranteed since the model updates from all benign

Algorithm 2: Segment Selection matrix $B^e$ for HeteroSAg

Define: $Z_{g-1} = \sum_{l=0}^{g-1} L_l$ and $Z_{g-1} = 0$ when $g = 0$. For $g = 0, \ldots, G-1$

\begin{algorithm}
\begin{algorithmic}
   \State \textbf{for} $d = 0, \ldots, L_g - 1 - \mathbbm{1}_{g=G-1}$ \textbf{do}
   \State \hspace{1em} $i = 0$ and $s = 0$;
   \State \hspace{1em} \textbf{for} $r = 0, \ldots, Z - Z_{g-1} - d - 2$ \textbf{do}
   \State \hspace{2em} $m = 2(Z_{g-1} + d) + r$;
   \State \hspace{2em} \textbf{if} $(d + r + 1) \mod Z_{g} = 0$ \textbf{then}
   \State \hspace{3em} $i = i + 1$ and $s = 0$;
   \State \hspace{3em} $B^e((m \mod Z_{g}, (g, d)) = B^e((m \mod Z_{g}, (g + i), s)) = (g, d)$
   \State \hspace{2em} \textbf{else}
   \State \hspace{3em} $s = s + 1$;
   \State \hspace{3em} $B^e((m \mod Z_{g}, (g, d)) = B^e((m \mod Z_{g}, (g + i), s)) = (g, d)$
   \State \hspace{1em} \textbf{end}
   \State \textbf{end}
\end{algorithmic}
\end{algorithm}

The remaining entries of Matrix $B^e$ will hold $*$

Now, we give the theoretical guarantees of HeteroSAg for heterogeneous group size.

**Theorem 4.** (Inference robustness) For a FL system with $N$ users clustered into $Z$ subgroups, and the model update of each node is divided equally into $Z$ segments, HeteroSAg achieves an inference robustness $\delta(A_{\text{HeteroSAg}}) = \frac{Z-2}{Z}$, when the number of subgroups is even, and $\delta(A_{\text{HeteroSAg}}) = \frac{Z-1}{Z}$, when the number of subgroups is odd, where $Z = \sum_{g=0}^{G-1} L_g$, for $l = 0, \ldots, Z - 1$, and $L_g$ is the number of subgroups in group $g$.

**Lemma 2** (Quantization error bound) Let $L_g$ be the number of subgroups in group $g$, and $Z_{g-1} = \sum_{l=0}^{g-1} L_l$ represent the sum of subgroups of group 0 to group $g - 1$, such that $Z_{g-1} = 0$ when $g = 0$. Additionally, let $Z = \sum_{g=0}^{G-1} L_g$ to be the total number of subgroups. For a set of vector $\{x_i \in \mathbb{R}^m\}_{i=1}^N$, such that the elements of each vector $x_i$, for $i = 1, \ldots, N$, take value from this interval $[r_1, r_2]$, and each vector is partitioned into $Z$ equal segments, the quantization error bound $\sigma_{\text{HeteroSAg}}$ when using the quantizers in $Q$ along with the SS matrix $B^e$ is given by

$$\sigma_{\text{HeteroSAg}} = \frac{(r_2 - r_1)^2}{4N^2} \frac{m}{Z} \sum_{g=0}^{G-1} \sum_{i=0}^{Z_g-1} (2(Z_{g-1} - i) - 1).$$

The proofs of Theorem 4 and Lemma 2 can be derived similarly to the proofs of Theorem 1 and Lemma 1, respectively. The convergence rate is the same as in Theorem 2 with replacing $\sigma_{\text{HeteroSAg}}$ with $\sigma_{\text{HeteroSAg}}^{+}$.

**H Proof of Proposition 1**

From Lemma 2, when having $G$ groups and each group is partitioned equally into $L$ subgroups each of which has a size of $\bar{n} = \frac{N}{G}$ users and using these two results $Z_{g-1} = gL$ and $Z = GL$, the quantization error bound will be the same as $\sigma_{\text{HeteroSAg}}$ given in Theorem 2. This means that extra partitioning of each group does not change the quantization error.

**I Byzantine robustness of HeteroSAg**

We further discuss the intuition behind the success of HeteroSAg in mitigating the Byzantine nodes in the following remark.

**Remark 6 (Byzantine robustness of HeteroSAg)** In this remark, we further motivate the reason behind the success of HeteroSAg in mitigating Byzantine nodes. The reason for the success of coordinate-wise median (Median) algorithm in mitigating the Byzantine nodes in the IID setting is the same reason behind the success of HeteroSAg when it is integrated with coordinate-wise median. In particular, the success of Median is guaranteed since the model updates from all benign

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users are similar to each other [19], where the similarity increases as the data at the users become more IID. Therefore, taking the median over each coordinate across the model update of all users ensures that we get a representative model for all the benign models while ignoring the outliers from each coordinate. For the same reason, integrating median with HeteroSAg can provide Byzantine robustness. In particular, unlike the case where each coordinate represents one element from the local model of each user (e.g., the k-th element $\bar{x}_i(k)$ of the local model of node $i$) in the naive coordinate-wise median algorithm. In HeteroSAg, each coordinate becomes representing the average of a set of elements from the local models of some users (e.g., the k-th element $\bar{y}_{0,1}^0(k)$ of the segment $\bar{y}_{0,1}^0$, given in Example 3, where $\bar{y}_{0,1}^0(k) = \frac{1}{|S|} \sum_{i \in N_0 \cup N_1} \bar{x}_i(k)$). Similarly, the average of a set of elements from the local models of some benign nodes is a reasonably good representative of those elements. Therefore, applying the median along the new coordinates will guarantee that we get a representative model of the benign models while ignoring the outliers from each coordinate. The outliers of each coordinate appear when having at least one faulty model contribute to the average element in that coordinate. For instance, $\bar{y}_{0,1}^0(k)$ will be faulty if at least one of these elements $\{\bar{x}_i(k), i \in N_0 \cup N_1\}$ is faulty.

J Complete Experimental Results for Section 5

J.1 Evaluating the results in Figure 5(b)

Table 4: User communication cost and the total communication time in Experiment 1.

| Quantization | Group | User communication cost (Mb) | Communication time (s) |
|--------------|-------|------------------------------|-----------------------|
| Heterogeneous $(K_0, K_1, K_2, K_3, K_4) = (2, 6, 8, 10, 12)$ | 0     | 53                           | 53                    |
|              | 1     | 87                           |                       |
|              | 2     | 90                           |                       |
|              | 3     | 97                           |                       |
|              | 4     | 101                          |                       |
| Homogeneous $K = 2$ | -     | 53                           | 53                    |
| No-quantization | -     | 279                          | 279                   |

The total communication time of the three heterogeneous scheme given in Figure 5(b) can be derived from the results in Table 4. The transmission rate of the users in group 0 is 1Mb/s, while users in higher groups have transmission rate more than 2Mb/s, as given in Section 5. The communication cost in (Mb) per user $i$ in group $g$ is given by summing the size of the masked model sent by node $i$ and the size of the global model received from the server (in Mb). The model size for the fully connected neural network considered for Experiment 1 is 79510 elements. In HeteroSAg, the encoding is done on the segment level, and the size of the encoded segment $x_l^0$ from node $i$ is given by $|x_l^0| \lceil \log(|S|(K_l^0 - 1) + 1) \rceil$, where $|S|$ is the number of users who jointly encode this segment, $K_l^0$ is the number of quantization levels used for quantizing $x_l^0$. The former result is given according to the encoding step in 4. By using the previous formula along with the segment grouping given in Figure 2 and the quantizers from the three different scenarios that we are considering (heterogeneous quantization with levels $(K_0, K_1, K_2, K_3, K_4) = (2, 6, 8, 10, 12)$), homogeneous quantization $K = 2$, and no quantization, i.e., $K = 2^{32}$, the per-user communication cost after $t = 200$ rounds can be evaluated. The cost of sharing keys and masks is the same for the three scenarios, therefore, we do not consider that in the calculations of the communication cost. The communication time can simply be computed by dividing the per-user communication cost by the corresponding transmission rate.

J.2 Additional experiment (Heterogeneous quantization)

J.2.1 MNIST dataset

We consider the same setup given for Experiment 1 in Section 5.1 while setting $N = 100$ users with $n = 20$ in each group of the $G = 5$ groups for running the experiment in Figure 5. In this experiment, we use a batch size of 60.
J.2.2 CIFAR10 dataset

Data distribution and Hyperparameters We set the total number of users to \( N = 100 \). We use a fixed learning rate of 0.02 for the first 150 rounds, \( t \leq 150 \), and then gradually decrease the learning rate according to
\[
\frac{0.02}{1 + \frac{0.2}{t}}, \quad \text{for } t > 150.
\]
We set the batch size for each user to be 20\% of its local data. We consider epoch training, where the number of epochs is 5. We use CIFAR10 dataset with non-IID data distribution. In particular, we use the generic non-IID synthesis method based on the Dirichlet distribution with parameter \( \alpha \) proposed in [38]. In this method, increasing \( \alpha \) makes the data more IID at the users. On the other hand, decreasing \( \alpha \) makes each user have very few samples from some random classes. We implement this method using FedML library [39].

![Label distribution with different \( \alpha \) parameters](image)

Figure 10: The label distribution over the \( N = 100 \) users using Dirichlet distribution with three different \( \alpha \) parameters.

Results Figure [10] illustrates the label distribution for \( N = 100 \) users with Dirichlet distribution with different \( \alpha \) parameter. Using this set of distributions given in Figure [10], we evaluate the performance of HeteroSAg using the same quantization schemes given in Section 5.1. Similar to the performance of HeteroSAg presented in Figure [5] and Figure [9] for the MNIST dataset, Figure [11] demonstrates that HeteroSAg for CIFAR10 dataset still achieves higher accuracy than the Homogeneous quantization with \( K = 2 \) over the three different data distribution settings in Figure [10]. Additionally, HeteroSAg gives a comparable test accuracy to the baseline (no-quantization). The high test accuracy of HeteroSAg over the case of homogeneous quantization with \( K = 2 \) is achieved at no extra communication time as illustrated in Figure [11](d). On the other hand, after \( t = 250 \) rounds of communication with the server, the total communication time when using heterogeneous is less the baseline case with no quantization by a factor of 5.4 ×.

J.3 Additional experiment (Byzantine robustness)

In the following set of experiments, we further demonstrate the performance of HeteroSAg under Byzantine attacks using non-IID data setting.
Dataset and Hyperparameters
Similar to the setting given in Section 5.2, we set the total number of users to \( N = 300 \), in which \( B = 18 \) of them are Byzantines. We use a fixed learning rate of 0.02, and setting the batch size for each user to be 20\% of its local data. We consider epoch training, where the number of epochs is 4. We use CIFAR10 dataset with non-IID data distribution. Here, we also generate the non-IID data distribution using Dirichlet distribution with parameter \( \alpha \).

Results
Figure 12 gives the label distribution for \( N = 300 \) users with Dirichlet distribution with three different \( \alpha \) parameters. Using this set of distributions, we evaluate the performance of HeteroSAg. As we can see in Figure 13, HeteroSAg with coordinate-wise median is robust to Gaussian attack and the sign flip attack while giving performance almost the same as the case with no Byzantine users. On the other hand, in the presence of these attacks, FedAvg scheme gives very low performance.

J.4 Models
We provide the details of the neural network architectures used in our experiments. For MNIST, we use a model with two fully connected layers, and the details are provided in Table 5. The first fully connected layers is followed by ReLU, while softmax is used at the output of the last layer.

Table 5: Details of the parameters in the architecture of the neural network used in our MNIST experiments.

| Parameter | Shape |
|-----------|-------|
| fc1       | \( 784 \times 100 \) |
| fc2       | \( 100 \times 10 \) |

For CIFAR10, we consider a neural network with two convolutional layers, and three fully connected layers, and the specific details of these layers are provided in Table 6. ReLU and maxpool is applied on the convolutional layers. The first maxpool has a kernel size \( 3 \times 3 \) and a stride of 3 and the second maxpool has a kernel size of \( 4 \times 4 \) and a stride of 4. Each of the first two fully connected layers is followed by ReLU, while softmax is used at the output of the third one fully connected layer.
Figure 12: The label distribution among $N = 300$ users using Dirichlet distribution with different $\alpha$ parameters.

Table 6: Details of the parameters in the architecture of the neural network used in our CIFAR10 experiments.

| Parameter | Shape          |
|-----------|----------------|
| conv1     | $3 \times 16 \times 3 \times 3$ |
| conv2     | $16 \times 64 \times 4 \times 4$ |
| fc1       | $64 \times 384$ |
| fc2       | $384 \times 192$ |
| fc3       | $192 \times 10$ |

We initialize all biases to 0. Furthermore, for weights in convolutional layers, we use Glorot uniform initializer, while for weights in fully connected layers, we use the default Pytorch initialization.
Figure 13: The performance of HeteroSAg and Fedavg under Gaussian and sign-flip attacks with three different data distributions.