Topology and Dark Energy: Testing Gravity in Voids

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Modified gravity has garnered interest as a backstop against dark matter and dark energy (DE). As one possible modification, the graviton can become massive, which introduces a new scalar field - here with a galileon-type symmetry. The field can lead to a nontrivial equation of state (EOS) of DE which is density- and scale-dependent. Tension between Type Ia supernovae and Planck could be reduced. In voids the scalar field dramatically alters the EOS of DE, induces a soon-observable gravitational slip between the two metric potentials, and develops a topological defect (domain wall) due to a nontrivial vacuum structure for the field.

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INTRODUCTION

The non-detection of dark matter and the incomprehensibility of dark energy are challenging puzzles for theorists. Alternatives to General Relativity (GR) is one strategy that is useful to have should the current dark sector crisis persist. In modifying gravity, the one lesson that has been learnt is the difficulty of finding an alternative theory that satisfies experimental constraints on all scales where gravity can be probed. High precision measurements on laboratory and solar system scales must accommodate astrophysical constraints on the two metric potentials in modified gravity theory from galactic to cosmological scales. Growth of density fluctuations and galaxy dynamics constrain the Newtonian potential. Light propagation constrains the sum of the two potentials. See [1] for review and citations.

Modifying gravity generally introduces a scalar that complements and couples to the tensor field of general relativity. The scalar must however be screened on small scales to be reconciled with the local precision tests of gravity. Three main mechanisms have been discussed for screening the new field. Chameleon [4] and f(R) theories [5] screen via density and decouple the scalar field at high density. Symmetron fields [6] weaken the scalar coupling at high density. Galileon/Braneworld fields [7, 8] and massive gravity (focus of this paper) operate via the Vainshtein effect [9], which decouples the scalar field in high-curvature regions.

Massive gravity (by assigning the graviton a mass of order the inverse Hubble scale) naturally and transparently restores coupling on the horizon scale, where one may then hope to account for the accelerated expansion of the universe [10]. The transition scale (i.e. the Vainshtein radius where decoupling occurs) is of order tens of Mpc today. This scale corresponds to the largest self-gravitating scales in the universe, and potentially the simplest to understand. While massive gravity theories offer a rich phenomenology and resolve some theoretical issues, there are also remaining theoretical difficulties, e.g. that quantum corrections may render theories unreliable on small scales [11], or possible issues with acausality [12].

In galaxy clusters, the metric potentials are almost the same as with GR, which is not necessarily true in voids. The change in the potential is shown here to only be a few percent for over-densities (which has been found previously by [13]−[15])

\[ f(\Delta) = \frac{\dot{\delta}_{\text{void}}^2}{\rho_{\text{void}}} + \frac{1}{\rho_{\text{void}}} T_{\mu\nu} + \mathcal{L}(\pi)_{\mu\nu} \] (1)
The Einstein tensor $(\mathcal{G}_{\mu\nu} - \Gamma_{\mu\nu}^{\alpha} h_{\alpha\beta})$ comes from the Einstein–Hilbert Lagrangian $(m^{2}_{p} c^{-2} \sqrt{-g} R)$ assessed to the second order in metric fluctuations $h_{\mu\nu}$ with $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/m_{pl}$, the Newton constant $8\pi G \equiv m^{2}_{p} c^{-2}$, $\eta_{\mu\nu} = (-1,1,1,1)$, and $m_{pl}$ is the Planck mass. The second term of Eq. 1 is the standard coupling of GR to the stress-energy tensor. The third term is the Lagrangian for the scalar field $\pi$ with $\mathcal{L}(\pi, m)$ which on small scales can be deduced from the general FRW metric (for all time) which will simply be Minkowski space under- and over-densities (e.g. voids, and halos/solar system dynamics) are spherically symmetric and time-independent, $(\pi, m)$ is the Planck mass. The second term of Eq. 1 is the standard stress–energy tensor (in Fermi coordinates) is simply $\rho \Phi / \alpha \pi c^{2}$, where $\alpha$, $\beta$, and $\gamma$ are arbitrary parameters of the theory but are taken to be $O(1)$. The effective field theory for $\pi$ is valid up to the cutoff scale $\Lambda^{2} = m_{pl}^{2} (m)$ is the graviton mass. See [10, 17] for details.

**Ansatz**

We look for solutions which can accommodate both under- and over-densities (e.g. voids, and halos/solar system respectively) and are consistent with cosmology. The equivalence principle implies that locally we can always define a metric (for all time) which will simply be Minkowski space plus a perturbation. The perturbation component will account for the non-trivial geometry of the space-time. Such coordinates are referred to as Fermi coordinates [7, 18]. In the Newtonian gauge,

$$ds^{2} = -(1 + 2\Phi(r)) dt^{2} + (1 - 2\Psi(r)) dx^{2}.$$  

which on small scales can be deduced from the general FRW metric by using the transformations $t_{c} = t + H^{2}x^{2}/2$ and $x_{c} = x/\alpha(1 + 1/4H^{2}x^{2})$ between co-moving ($t_{c}, x_{c}$) and the local, physical Fermi coordinates ($t, x$). $H$ is the Hubble parameter, and $\alpha$ is the scale factor. We then add on local perturbations of the metric, $\Phi$ and $\Psi$ encode both the background geometry and local metric perturbations. The standard stress–energy tensor (in Fermi coordinates) is simply $T_{\mu\nu} \approx (\rho, \delta_{ij} p)$ where we have neglected off-diagonal terms ($O(H^{2}x^{2}) \sim v^{2} \sim H^{2}x^{2}x^{2}$ with velocity $v$). Similarly, $\rho$ and $p$ include local perturbations of matter and radiation relative to a background density. Finally, we make the ansatz that the field $\pi$ (as well as the local metric and matter perturbations) are spherically symmetric and time-independent, $\pi = \pi(r)$. A subset of solutions with the above ansatz has been explored in Refs. [14, 15] - we find and explore a new class of solutions.

By the least-action principle applied to Eq. 1 we obtain two non-trivial Equations of Motion (EOM) for the metric and one EOM for $\pi$ with $t = d/dr$

$$2\nabla^{2} \Psi = 8\pi G \rho / m_{pl} r + 6 \alpha (\pi')^{2} / m_{pl} \Lambda^{2} r^{2} + 6 \gamma (\pi')^{3} / m_{pl} \Lambda^{2} r^{3},$$  

$$\left( \nabla^{2} - \nabla_{i}^{2} \right) (\Phi - \Psi) = 8\pi G (\rho / m_{pl} r - 4 \alpha (\pi')^{2} / m_{pl} \Lambda^{2} r^{2} - 2 \beta (\pi')^{2} / m_{pl} \Lambda^{2} r^{2} - 2 \gamma (\pi')^{2} / m_{pl} \Lambda^{2} r^{2},$$  

$$\alpha \nabla^{2} (2\Psi - \Phi) + \frac{2\beta \pi'}{\Lambda r} \nabla^{2} (\Psi - \Phi) - \frac{3\gamma (\pi')^{2}}{\Lambda^{2} r^{2}} \nabla^{2} \Phi = 0$$

where in Eq. 4 we are not summing over $i$ but indexing over the spatial dimensions ($x, y, z$). Previous authors [14, 15] have found the above EOM, but have neglected the effect of pressure in Eq. 4. The operator $\langle \rangle$ gives the average value of a given quantity inside a radius $r$.

In linearized GR, perturbations and the background can be separated by subtracting off the background from the perturbed Einstein equations. In massive gravity, this proves impossible. Eq. 5 is non-linear in $\pi$ and in the metric perturbations, which introduces cross-terms between the perturbed solution and the background solution. At zeroth order in $\pi$ we look for solutions that accommodate both the background and local perturbations in the metric, density, and pressure.

The above EOM can be made dimensionless by multiplying by $m^{2} / \pi^{3}$ and then setting $\pi' / \pi = f(\Delta) \Lambda^{3}$, where $\Delta = \delta \rho / \rho_{0}$. Here, $\rho_{0}$ is the matter density today and $\delta \rho$ is the under/over-density of a void/halo with $\delta \rho_{0} = (\rho_{0}(r)) / \rho_{0}(r)$. Hence, $\delta \rho_{m} / \rho_{m0} = -1$ corresponds to an empty void as in devoid of matter but not DE. After making the appropriate substitutions, Eq. 5 becomes a quintic constraint equation for $f(\Delta)$ which depends upon the average pressure $(\rho)$ and density $(\rho)$ for a fixed radius $r$. Out of the five roots, three will typically be real.

**Classes of Solutions**

The solutions for $\pi'$ can be categorized into three separate classes. In a separate publication we will discuss in detail the various solutions. Two of the classes have been discussed previously by [14, 15]. Neither of these cases produce interesting cosmological solutions. One class of solutions degrades all mass. The second set of solutions generates an EOS of the universe which is equivalent to radiation at late times and generates negative densities.

The third class gives new cosmological solutions. The EOS of DE tracks the energy density of matter, radiation, and curvature with several different sub-cases. In the first self-accelerating case [no Cosmological Constant (CC)], we can tune $\alpha$, $\beta$, and $\gamma$ to effectively generate a CC. With a different ansatz, the authors of [10] found a similar solution which decoupled the scalar field from matter, radiation, and the metric. In a second self-accelerating solution (flat background), the EOS of DE will depend on the local density of matter. In a third self-accelerating case (with curvature), the scalar field can dynamically counteract the effect of curvature upon the expansion rate of the universe, which may appear as so-called “phantom” EOS of DE with $w < -1$.

The more general case with a CC will constitute the focus of the paper. In solving for the $\pi$’s EOM, we search for roots of a quintic equation which has no general analytic solution. We find solutions numerically, such that densities are positive, the field $\pi$ is ghost-free, and constraints on $H_{0}$ and $\Omega_{m}$ are satisfied. To be concise we will consider two cases with $\alpha = -0.45 (-0.50), \beta = -1.0 (-1.0), \gamma = -0.86 (-0.87)$ in which a topological feature appears (for other choices of parameters no domain wall exists). We set $8\pi G \rho_{0} / m^{2} = 7.65 (18.5)$ in order to have the correct matter density today where $\rho_{0}$ is the critical density. In Fig. 1, we have plotted the solution of $\pi' / \Lambda^{3} = f(\Delta)$ (for case 1) interpolating between an under-density and an over-density. We note that the qualitative behavior of the two cases (with different $\alpha, \beta, \gamma$) are the same.
**Over-Density** First, we consider a point mass $M > 0$ (as our over-density). In the limit of approaching the point mass $r \to 0$, then $\Delta \to \infty$ and $f(\Delta) \to -0.42$. The Vainshtein mechanism kicks in recovering GR ($\Phi \to \Psi \to -GM/r$). As $r \to \infty$ then we smoothly go onto the cosmological solution and $f(\Delta)$ takes on a value which gives the correct cosmological solution today with $2\Phi \to -(H^2 + \hat{H})x^2$ and $2\Psi \to H^2 x^2$ where $H$ is the Hubble parameter. From the metric EOM, we then also recover the Friedmann equations.

**Under-Density** Far outside the void $\delta \rho_m \simeq 0$ (the metric potentials match onto the cosmological solution found in the previous section). The average density $\langle \rho_m \rangle$ drops as we move to the center of the void. Eventually the derivative of $\rho$ must jump to a new branch (see Fig. 1) once $\delta \rho_m$ drops to less than half of the present mass density of the universe today. The jump forces a discontinuity in the derivative of the field $\pi'$. We still have the freedom to adjust the integration constant of $\pi'$ such that $\pi$ is continuous. Regardless, to match the derivative across the discontinuity requires the introduction of a wall.

The nature of the $\pi$ wall is categorically different from a wall formed for a typical scalar field. The wall is not formed primordially but forms dynamically at late times. Typically, a domain wall corresponds to a kink solution interpolating between two different vacuum states on different sides of the wall. The wall tension is due to the potential separating the two vacuum states in field space. The $\pi$ field has no potential, $V(\pi) = 0$, but upon removing the static condition, the EOM for $\pi$ becomes highly non-linear. As a parallel in fluid mechanics, non-linearities of the Navier–Stokes equation can lead to a discontinuity in density and pressure gradients (a shockwave). The fluid mechanic description breaks down and microphysics kicks in which removes the discontinuity. Similarly, we conjecture that the full non-linear EOM for $\pi$ (in the presence of matter [19]) will generate a “shockwave.” At the discontinuity, the effective field theory for $\pi$ breaks down and new physics kicks in which resolves the discontinuity.

From Fig. 1, there exists a mapping of $\pi' / r$ and $\pi$ into real space, since there is a one-to-one mapping between $\pi$ and $\pi'$. The mapping of $\pi'$ is not one-to-one into real space. If we allow a complex extension of $\Delta$ and $\pi'$, the quintic equation corresponds to a hyper-elliptic surfaces (a compact Riemann surfaces with genus greater than one), with multiple branch cuts. $f$ loops around two branch points producing the multivalued function in real space in Fig. 1 but $f$ is single valued on the Reimann surface! The $\pi$ field appears to be a NGB (see [20]). Frequently, the vacuum structure of a Nambu–Goldstone Boson (NGB) is mapped into real space. By analogy, we identify Riemann surface as the vacuum of $\pi$.

We note that the topologically non-trivial vacuum of a NGB when mapped into real space leads to the appearance of topological structures. Similarly, our function $f$ due to its wrapping around branch points is equivalent to a non-trivial wrapping around the hyper-elliptic surface. Finally, if our solutions for $\pi$ are ‘onto’, then the non-renormalization theorem for $\alpha, \beta, \gamma$ may be a consequence of the compact vacuum of $\pi$, since the terms in $\mathcal{L}(\pi)$ are in fact Wess–Zumino–Witten (WZW) terms [20]. Hence further properties of the field may potentially be inferred from the Riemann surface.

**Dynamical Dark Energy** Density goes like $\langle \rho(r) \rangle = \langle \rho_m(r) \rangle + \rho_{cc} + \rho_{mg}$ where $\rho_m(r)$ is local matter density, $\rho_{cc}$ is the energy density from the CC. We identify $\rho_{mg}$ as the energy density from massive gravity via the last terms of Eq. 3. Upon neglecting radiation, $\langle \rho(r) \rangle \approx -\rho_{cc} + \rho_{mg}(r)$ where $\rho_{mg}$ is an effective pressure from massive gravity due to the last terms in Eq. 3.

Massive gravity naturally leads to inhomogeneous DE. In Fig. 1, the $\pi$ field responds to changes in the local density. As $\pi$ evolves, so does $\rho_{mg}$ and $\rho_{cc}$. The EOS of DE is $w_{DE} = \frac{\rho_{cc} + \rho_{mg}(r)}{\rho_{cc} + \rho_{mg}(r)}$. See Fig. 2 for two different cases. Outside of halos and voids, $w = -0.98$ to $-0.95$. As we move into a halo, $w$ asymptotes towards $-1$. At the center of a void, $w_{DE}$ jumps to $-0.69$ to $-0.58$. Hence the value of $w_{DE}$ inside a void depends upon the mass of the graviton $\alpha, \beta, \gamma$, etc.

We can also read Fig. 2 as a measure of how the EOS of DE changes with redshift. At high redshift, the average density of the universe becomes large compared to the mean density today. The scalar field becomes unimportant which is merely a manifestation of the Vainshtein mechanism. DE is then unimportant for the evolution of the universe.

Recent CMB results from the Planck satellite [21] indicate (with 2.5σ significance) a nearly 10% lower value of the Hubble constant ($H_0$) compared to values inferred from the local-Universe and supernova, e.g. [22]. SNIa systematics and concordance-model uncertainties could account for this difference. Our model could also explain the effect with a significant fraction of low-density void in the local Universe or along the line of sight. With a difference of $\Delta w_{DE}$ between $w_{DE}^{\text{void}} \approx -0.7$ in voids and $w_{DE}^{\text{global}} \approx -1$ elsewhere, a higher value of $H_0$ would be inferred than from the cosmic microwave background since $\Delta H_0 / H_0 \approx 3z\Delta w_{DE} / 2w_{DE}^{\text{global}}$ between the local and global values. The required local under-density of $\rho_{ VOID}/2$ appears consistent with recent observational studies, e.g. [23, 24]. This could potentially relieve the
tension between the different Hubble constant estimates up to
the observed 10\% level.

CONCLUSION Massive gravity can naturally give rise to
an inhomogenous EOS of DE, which will be dependent upon
local matter density and scale. The scale-dependent EOS of
dark energy may reduce the tension between relatively local
measurements of dark energy from SNIa and global measure-
ments from the CMB. Detailed analysis of voids may be the
most direct way to infer the existence of a scale-dependent
and density-dependent EOS of DE since perturbations in mat-
ter will lead to perturbations in DE. We can infer the exis-
tence of inhomogeneous DE via dynamics and lensing. The
motion of galaxies from the geodesic equation constrains Φ
(in the weak-field limit). However, for massless particles,
the geodesic equation (lensing) depends upon the lensing poten-
tial ΦL = 1/2(Φ + Ψ). We can infer the amount of material in
and around a void by simply counting galaxies. It is then pos-
sible to infer the EOS of DE in a void and test the predicted
large gravitational slip between the metric potentials, which
should be achievable in future surveys [25, 26].

The Euclid satellite [27] will measure the stacked void lens-
ing (shear and magnification) signal with S/N ∼ 15, to de-
rive radial lensing-potential profiles [23]. This, combined
with the galaxy-density map, should allow for a strong detec-
tion/exclusion of the ∼ 50\% weaker lensing potential relative
to GR, due to massive gravity. See further also Refs. [25, 26]
on e.g. the Baryon Oscillation Spectroscopic Survey (BOSS)
and the Large Synoptic Survey Telescope (LSST).

The EOS of DE will change the growth rate of voids. By
measuring the shape of voids in redshift space, the Alcock–
Paczynski test can reveal local deviations in the expansion rate
due to a different equation of state [28]. For massive grav-
ity, the average equation of state in the universe until today
wDE global ≈ −1. In voids, it is wDE voids ∼ − 0.7, as empiri-
cally roughly 50-75\% of void volumes have ρ < ρm/2 [29].

The Euclid mission could detect a deviation |ΔwDE| > 0.1
at the 95\% level or better [28]. A deviation ΔwDE = 0.3
should be highly detectable. For BOSS [30], the equiva-
 lent limit is |ΔwDE| > 0.25, allowing for marginal detection.
Further, the large-scale-structure power spectrum will be im-
printed with a particular length scale derived from the EOS
transition, as well as an overall effect on the growth rate. Im-
portantly, galaxy surveys also probe the power spectrum and
redshift-space distortions, and could potentially detect local
deviations of DE. Finally, lensing and polarization of the cos-
mic microwave background by voids/halos could also be good
probes for constraining the model, as well as the integrated
Sachs–Wolfe (ISW) effect due to voids/halos [31].

The domain wall merits further study by potentially affect-
ing lensing and dynamics of galaxies. With the presence a
magnetic field, charged particles could be accelerated by in-
teracting with the π field. As with axions, distinct photon
conversion/polarization could take place. Finally, the tension
in π goes like τ ∼ A2 by dimensional analysis. Matching the
metric potentials across the discontinuity in π′ will give a pre-
cise solution for the tension in the domain wall. We should
also be able to calculate the wall tension via non-linear field
dynamics and via an analysis of the vacuum structure of τ.
GR implies that they should all give the same solution, which
is a curious mathematical connection.

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