Polarization of $\Lambda^0$ hyperons as a signature for the Quark–Gluon Plasma

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Abstract

The momentum distribution of $\Lambda^0$ hyperons produced from the quark–gluon plasma (QGP) in ultra–relativistic heavy–ion collisions is calculated in dependence on their polarization. The momentum distribution of $\Lambda^0$ hyperons is defined by matrix elements of relativistic quark Wigner operators, which are calculated within the Effective quark model with chiral $U(3) \times U(3)$ symmetry and the Quark–Gluon transport theory. We show that the polarization of the $\Lambda^0$ hyperon depends on the spin of the strange quark that agrees well with the DeGrand–Miettinen model. We show that $\Lambda^0$ hyperons, produced from the QGP, are fully unpolarized. This means that a detection of unpolarized $\Lambda^0$ hyperons, produced in ultra–relativistic heavy–ion collisions, should serve as one of the signatures for the existence of the QGP in intermediate states of ultra–relativistic heavy–ion collisions.

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1 Introduction

Experimentally a strong polarization of single and double strange baryons produced in high–energy nuclear reactions has been well established [1]. According to experimental studies of the polarization of the Λ⁰ hyperon in high–energy nuclear reactions, summarized by Félix [2], the polarization of the Λ⁰ hyperon depends on $p_\perp$, the momentum of the Λ⁰ hyperon transversal to the direction of the incoming beam of colliding nuclei, and $x_f = 2p_z/\sqrt{s}$, where $p_z$ is the momentum of the Λ⁰ hyperon longitudinal to the direction of the incoming beam of colliding nuclei and $s$ is the squared total center–of–mass of event energy [2]. Then, it turned out that the polarization of the Λ⁰ hyperon, produced in high–energy nucleus–nucleus reactions, is the same for different targets and different materials with different atomic numbers [2]. Different theoretical models for the description of the polarization of Λ⁰ hyperons, produced in high–energy nuclear reactions, have been discussed in details by Félix [3].

Such a high level of polarization of Λ⁰ hyperons, produced in high–energy nuclear reactions, can be treated as test for the classification of nuclear matter in the normal state. Any violation of high polarization of the Λ⁰ hyperons should testify an “anomalous” state of nuclear matter. For the first time the idea that phase transitions, peculiar for a subnuclear matter in the form of a quark–gluon plasma (QGP), can provide a symmetry breakdown leading to the complete depolarization of Λ⁰ hyperons, has been suggested by Stock [4].

A vanishing polarization of Λ⁰ hyperons as possible signature of a QGP has been analysed theoretically in Refs.[5]–[7] in the semi–classical di–quark, $s$–quark recombination model [5], Regge–type models [7]. However, as has been pointed out by Bellwied [8] that (i) recent measurements with polarized beams [9] seem to indicate that the semi–classical di–quark, $s$–quark recombination model is inconsistent with the measured forward–backward asymmetry ($A_N$) and spin transfer asymmetry ($D_{NN}$) and (ii) Regge–type models and the quark fragmentation model do not necessary lead to the complete depolarization of Λ⁰ hyperons produced from a QGP.

According to Bellwied [8] the measurements of the polarization of the Λ⁰ hyperons in ultra–relativistic heavy–ion collisions would be of interest regardless the obtained results, since nowadays the question of the depolarization of the Λ⁰ hyperon, produced from a QGP in relativistic heavy–ion collisions, is still open. The measurements of the transversal polarization of Λ⁰ hyperons produced in 11.6$A$ GeV/c Au+Au collisions at the Alternating Gradient Synchrotron (AGS) at Brookhaven National Laboratory carried out by the E896 Collaboration [8] show that for $p_\perp \geq 1.5$ GeV/c and $x_f \geq 0.5$ the Silicon Drift Detector Array (SDDA) exhibited $P_{\Lambda^0} = (-25.7 \pm 7.5)\%$, whilst the Distributed Drift Chamber (DDC) gave $P_{\Lambda^0} = (-23 \pm 11)\%$. These results show [8] that in the heaviest collision systems Λ⁰ hyperons are still polarized at freeze–out. The direction of the Λ⁰ hyperon spin is slightly affected by the rescattering phase after hadronization. These measurements are first to confirm the polarization of Λ⁰ hyperons produced in heavy–ion collisions [8].

The problem of polarization of Λ⁰ hyperons as a probe for a QGP has been recently discussed by Ayala et al. [10]. For the description of the polarization of the Λ⁰ hyperons for densities below a critical density for the formation of a QGP the authors have used the di–quark, $s$–quark recombination mechanism within the DeGrand–Miettinen model [2, 11]. Assuming that for densities higher than the critical density of the formation
of a QGP a recombination mechanism of the $Λ^0$ polarization should be changed to the coalescence of free valence quarks, the authors have found that the polarization of the $Λ^0$ hyperons should depend on the relative contribution of each process to the total number of the $Λ^0$ hyperons produced in the collision.

In this paper we suggest to analyse the problem of the polarization of $Λ^0$ hyperons, produced from a QGP in ultra-relativistic heavy–ion collisions, in terms of the momentum distribution of the number of $Λ^0$ hyperons $N_{Λ^0}(k)$ defined by [12, 13]

$$E_k \frac{d^3 N_{Λ^0}}{d^3 k} = \frac{2}{(2\pi)^3} \int_{\Sigma_f} \theta(k^0) f_{Λ^0}(x, k) k^\mu d\sigma_\mu(x), \quad (1.1)$$

where the common factor 2 is the spin degeneracy of the $Λ^0$ hyperon and $f_{Λ^0}(x, k)$ is its distribution function, and $d\sigma_\mu(x)$ is a normal vector to the freeze–out 3–dimensional surface $\Sigma_f$ in the configuration space–time [12, 13]. The distribution function $f_{Λ^0}(x, k)$ is taken in the Jüttner form [14, 15] (see also [12, 13])

$$f_{Λ^0}(x, k) = \frac{1}{e(k \cdot U(x) - \mu_{Λ^0}(x)/T(x)) + 1}, \quad (1.2)$$

where $U(x)$, $\mu_{Λ^0}(x)$ and $T(x)$ are the 4–dimensional hydrodynamical local velocity, the baryon chemical potential and the temperature, and $k^\mu = (k^0, \vec{k})$ with $k^0 = E = \sqrt{\vec{k}^2 + m_{Λ^0}^2}$ is the 4–momentum of the $Λ^0$ hyperon, $\theta(k^0)$ is the Heaviside function.

In order to take into account polarization properties of the $Λ^0$ hyperon it is convenient to determine the distribution function $f_{Λ^0}(x, k)$ in terms of the positive energy Wigner function $W_{Λ^0}^{(+)}(x, k)_{βα}$ [15]–[17]

$$\theta(k^0)f_{Λ^0}(x, k) = \sum_\alpha W_{Λ^0}^{(+)}(x, k)_{αα} = \text{tr}W_{Λ^0}^{(+)}(x, k), \quad (1.3)$$

where $W_{Λ^0}^{(+)}(x, k)_{βα}$ is defined by [15]–[17]

$$W_{Λ^0}^{(+)}(x, k)_{βα} = \int \frac{d^4 y}{(2\pi)^4} \theta(k^0) e^{-ik \cdot y} \langle \Omega : \bar{ψ}_{Λ^0} \left( x + \frac{1}{2} y \right) _β ψ_{Λ^0} \left( x - \frac{1}{2} y \right) _α : | Ω \rangle. \quad (1.4)$$

Here $| Ω \rangle$ is the vacuum wave function, $ψ_{Λ^0}(z)$ is the interpolating field for free $Λ^0$ hyperons, and $: \ldots :$ indicates the normal ordering [15].

The interpolating field $ψ_{Λ^0}(x ± y/2)$ we represent as

$$ψ_{Λ^0} \left( x ± \frac{1}{2} y \right) = \frac{1}{(2\pi)^3} \sum_{σ=±1/2} \sqrt{\frac{m_{Λ^0}}{2}} \int \frac{d^3 p}{E(\vec{p})} \left[ u(\vec{p}, σ) a_{Λ^0}(\vec{p}, σ) e^{-ip \cdot (x ± y/2)} + v(\vec{p}, σ) b_{Λ^0}^\dagger(\vec{p}, σ) e^{-ip \cdot (x ± y/2)} \right], \quad (1.5)$$

where $u(\vec{p}, σ)$ and $v(\vec{p}, σ)$ are the Dirac bispinors normalized by

$$\bar{u}(\vec{p}, σ) u(\vec{p}, σ') = -\bar{v}(\vec{p}, σ) v(\vec{p}, σ') = δ_{σσ'}. \quad (1.6)$$

The operators $a_{Λ^0}(\vec{p}, σ)$ and $a_{Λ^0}^\dagger(\vec{p}, σ)$ annihilate and create the $Λ^0$ hyperon with quantum numbers $(E, \vec{p}, σ)$, whereas the operators $b_{Λ^0}(\vec{p}, σ)$ and $b_{Λ^0}^\dagger(\vec{p}, σ)$ annihilate and create the
anti-Λ⁰ hyperon (Λ⁰-hyperon) with quantum numbers (E, p, σ). These operators satisfy canonical relativistic covariant anticommutation relations
\[
\{a_{\Lambda^0}(\vec{p}, \sigma), a_{\Lambda^0}^\dagger(\vec{q}, \lambda)\} = \{b_{\Lambda^0}(\vec{p}, \sigma), b_{\Lambda^0}^\dagger(\vec{q}, \lambda)\} = (2\pi)^3 2E(\vec{p}) \delta^{(3)}(\vec{p} - \vec{q}) \delta_{\sigma\lambda}, \\
\{a_{\Lambda^0}(\vec{p}, \sigma), a_{\Lambda^0}(\vec{q},\lambda)\} = \{b_{\Lambda^0}(\vec{p}, \sigma), b_{\Lambda^0}(\vec{q},\lambda)\} = 0.
\] (1.7)

The vacuum expectation values of the products of operators of creation and annihilation of Λ⁰ and Λ⁰ hyperons are equal to
\[
\langle \Omega | a_{\Lambda^0}^\dagger(\vec{q},\lambda) a_{\Lambda^0}(\vec{p},\sigma) | \Omega \rangle = (2\pi)^3 2E(\vec{p}) \delta^{(3)}(\vec{q} - \vec{p}) f_{\Lambda^0}(0, p) \delta_{\lambda\sigma}, \\
\langle \Omega | b_{\Lambda^0}^\dagger(\vec{q},\lambda) b_{\Lambda^0}(\vec{p},\sigma) | \Omega \rangle = (2\pi)^3 2E(\vec{p}) \delta^{(3)}(\vec{q} - \vec{p}) f_{\Lambda^0}(0, p) \delta_{\lambda\sigma}.
\] (1.8)

Computing the vacuum expectation value in (1.4) we define the Wigner function for the polarized Λ⁰ hyperon as
\[
W_{\Lambda^0}^{(+)}(x, k)_{\beta\alpha} = \theta(k^0) \rho(k, \zeta)_{\alpha\beta} f_{\Lambda^0}(0, k).
\] (1.9)

Here \(\rho(k, \zeta)_{\alpha\beta}\) is the spin density matrix
\[
\rho(k, \zeta)_{\alpha\beta} = \frac{\hat{k} + m_{\Lambda^0}}{4m_{\Lambda^0}} (1 + \gamma^5 \hat{\zeta}),
\] (1.10)

where \(\hat{a} = \gamma_\mu a^\mu\) and \(\zeta^\mu\) is the polarization vector given by
\[
\zeta^\mu = \left( \frac{\vec{p} \cdot \vec{\zeta}}{m_{\Lambda^0}} \vec{\zeta} + \frac{\vec{k} (\vec{k} \cdot \vec{\zeta})}{m_{\Lambda^0}(E(\vec{k}) + m_{\Lambda^0})} \right)
\] (1.11)

and \(\vec{\zeta}\) is a unit vector, \(\vec{\zeta}^2 = 1\).

In the non-relativistic limit \(m_{\Lambda^0} \gg |\vec{k}|\) the spin density matrix \(\rho(k, \zeta)\) reduces to the standard form
\[
\hat{\rho}(k, \zeta) = \frac{1}{2} (1 + \vec{\sigma} \cdot \vec{\zeta}),
\] (1.12)

where \(\vec{\sigma}\) are 2 × 2 Pauli matrices.

Substituting (1.9) in (1.3) and computing the trace over Dirac matrices we prove the relation (1.3). One can replace \(f_{\Lambda^0}(0, k)\) by \(f_{\Lambda^0}(x, k)\) for the \(\Lambda^0\)-hyperon gas in the quasi-equilibrium state \(\|\). The result should be valid for any non-equilibrium state of the \(\Lambda^0\)-hyperon gas.

Thus, we have shown that the \(\Lambda^0\) hyperon, treated as a point-like particle and produced from the QGP, should be unpolarized. It is well-known that quark degrees of freedom play an important role for production and polarization of the \(\Lambda^0\) hyperon in hadron–hadron and nucleus–nucleus collisions. Therefore, the polarization of \(\Lambda^0\) hyperons, produced from the QGP, should be investigated at the quark level.

The work is organized as follows. In Section 2 we define a momentum distribution of \(\Lambda^0\) hyperons, produced from the QGP, in terms of the matrix elements of the relativistic quark Wigner operators. In Section 3 we calculate the matrix elements of the relativistic quark Wigner operators within the Effective quark model with chiral \(U(3) \times U(3)\) symmetry and the Quark–Gluon transport theory. In Section 4 we analyse the dependence of the momentum distribution of the number of \(\Lambda^0\) hyperons on the polarization of \(\Lambda^0\) hyperons. We show that \(\Lambda^0\) hyperons produced from the QGP are unpolarized. In the Conclusion we discuss the obtained results. In Appendix A and B we calculate the momentum integrals defining the momentum distribution of \(\Lambda^0\) hyperons in our approach.
2 Momentum distribution of $\Lambda^0$ hyperons. Relativistic quark Wigner operator

At the quark level the momentum distribution of the number $N_{\Lambda^0}(\vec{k})$ of $\Lambda^0$ hyperons, produced from a QGP in ultra-relativistic heavy–ion collisions, can be defined by \[13\]

$$E_\vec{k} \frac{d^3 N_{\Lambda^0}(\vec{k})}{d^3 k} = \sum_{\lambda = \pm \frac{1}{2}} \sum_{q = u, d, s} \int d\sigma(x) k_\mu \theta(k^0) \int d^4 p \langle \Lambda^0(k, \lambda)|\hat{W}^+(x, p)|\Lambda^0(k, \lambda)\rangle,$$ \hspace{1cm} (2.1)

where $\Sigma$ is a surface of a “freeze–out” isotherm \[13\]. Then, $\hat{W}^+(q, x, p)$ is the relativistic $q$–quark Wigner operator determined by \[17\]

$$\hat{W}^+(q, x, p) = \theta(p^0) \frac{d^4 y}{(2\pi)^4} e^{-ip \cdot y} \bar{q}(x + \frac{1}{2} y) \otimes q(x - \frac{1}{2} y),$$ \hspace{1cm} (2.2)

where dots : : : indicate the normal ordering.

The vacuum expectation value of the relativistic quark Wigner operator

$$\langle \Omega|\hat{W}^+(x, p)|\Omega\rangle$$ \hspace{1cm} (2.3)

is equal to the distribution function of the $q$–quark \[15\] \[17\]. In turn, the matrix element

$$\langle \Lambda^0(k, \lambda)|\hat{W}^+(x, p)|\Lambda^0(k, \lambda)\rangle$$ \hspace{1cm} (2.4)

describes a projection of the $q$–quark, appearing in the intermediate state of ultra–relativistic heavy–ion collisions, on the physical states of $\Lambda^0$–hyperons on–mass shell.

The calculation of the matrix elements we suggest to carry out within the Effective quark model of baryons with chiral $U(3) \times U(3)$ symmetry \[19\] \[21\] and the Quark–Gluon transport theory \[17\].

3 Matrix element $\langle \Lambda^0(k, \lambda)|\hat{W}^+(x, p)|\Lambda^0(k, \lambda)\rangle$

In this section calculate the matrix element \[24\]. Using the reduction technique we get \[19\] \[21\]

$$\langle \Lambda^0(k, \lambda)|\hat{W}^+(x, p)|\Lambda^0(k, \lambda)\rangle = \lim_{k^2 \to M^{2}_{\Lambda^0}} \int d^4 x_1 d^4 x_2 e^{i k \cdot (x_1 - x_2)} \bar{u}_{\Lambda^0}(k, \lambda)$$

$$\left(\gamma^\alpha \frac{\partial}{\partial x_1^\alpha} - M_{\Lambda^0}\right)\langle \Omega|T(\psi_{\Lambda^0}(x_1)\hat{W}^+(x, p)\psi_{\Lambda^0}(x_2))|\Omega\rangle \left(\gamma^\alpha \frac{\partial}{\partial x_2^\alpha} - M_{\Lambda^0}\right) u_{\Lambda^0}(k, \lambda),$$ \hspace{1cm} (3.1)

where $\psi_{\Lambda^0}(x)$ and $u_{\Lambda^0}(k, \lambda)$ are the interpolating field operator and the Dirac bispinor of the $\Lambda^0$ hyperon, respectively.
In order to describe the r.h.s. of Eq. (3.1) at the quark level we follow \[19\] and use the equations of motion

\[
\begin{align*}
(i\gamma^\nu \frac{\partial}{\partial x_1} - M_{\Lambda^0}) \psi_{\Lambda^0}(x_1) &= \frac{g_B}{\sqrt{2}} \eta_{\Lambda^0}(x_1), \\
\bar{\psi}_{\Lambda^0}(x_2) \left(-i\gamma^\alpha \frac{\partial}{\partial x_2} - M_{\Lambda^0}\right) &= \frac{g_B}{\sqrt{2}} \bar{\eta}_{\Lambda^0}(x_2),
\end{align*}
\]  

(3.2)

where \(M_{\Lambda^0} = 1116\) MeV is a mass of the \(\Lambda^0\) hyperon. Then, \(\eta_{\Lambda^0}(x_1)\) and \(\bar{\eta}_{\Lambda^0}(x_2)\) are the three–quark current densities \[19\].

\[
\begin{align*}
\eta_{\Lambda^0}(x_1) &= -\sqrt{\frac{2}{3}} \varepsilon^{ijk} \{[\bar{u}(x_1)\gamma^\mu s_j(x_1)]\gamma^\alpha u_k(x_1) - [\bar{d}(x_1)\gamma^\mu s_j(x_1)]\gamma^\alpha u_k(x_1)\}, \\
\bar{\eta}_{\Lambda^0}(x_2) &= +\sqrt{\frac{2}{3}} \varepsilon^{ijk} \{\bar{d}(x_1)\gamma^\mu \gamma^5 s_j(x_1)\gamma^\alpha u_k(x_1) - \bar{u}(x_1)\gamma^\mu \gamma^5 s_j(x_1)\gamma^\alpha u_k(x_1)\},
\end{align*}
\]  

(3.3)

where \(i, j\) and \(k\) are colour indices and \(\bar{\psi}(x) = \psi(x)^T C\) and \(C = -C^T = -C^\dagger = -C^{-1}\) is the conjugation charge, \(T\) is a transposition, and \(g_B\) is the phenomenological coupling constant of the low–lying baryon octet \(\bar{B}_8(x)\) coupled to three–quark current \[19\].

\[
\mathcal{L}^{(B)}_{\text{int}}(x) = \frac{g_B}{\sqrt{2}} \bar{B}_8(x) \eta_8(x) + \text{h.c.},
\]

(3.4)

where the numerical value of \(g_B\) is equal to \(g_B = 1.34 \times 10^{-4}\) MeV\(^{-2}\) \[19\].

Substituting (3.2) in (3.1) we obtain

\[
\langle \Lambda^0(k, \lambda)|\hat{W}_q^{(+)}(x, p)|\Lambda^0(k, \lambda)\rangle = \frac{1}{2} g_B^2 \int d^4x_1 d^4x_2 e^{i\mathbf{k} \cdot (x_1 - x_2)} \bar{u}^{(\alpha)}(k, \lambda) \\
\times \text{tr}\{\langle \Omega|T(\eta_{\Lambda^0}^{(\alpha)}(x_1)\hat{W}_q^{(+)}(x, p)\bar{\eta}_{\Lambda^0}^{(\beta)}(x_2))|\Omega\rangle\} u^{(\beta)}(k, \lambda),
\]

(3.5)

where the trace should be calculated over colour and spinorial indices of quark fields. Recall, that the \(\Lambda^0\) hyperon should be kept on mass–shell \(k^2 = M_{\Lambda^0}^2\).

The matrix element \(\text{tr}\{\langle \Omega|T(\eta_{\Lambda^0}^{(\alpha)}(x_1)\hat{W}_q^{(+)}(x, p)\bar{\eta}_{\Lambda^0}^{(\beta)}(x_2))|\Omega\rangle\}\) is defined by

\[
\text{tr}\{\langle \Omega|T(\eta_{\Lambda^0}^{(\alpha)}(x_1)\hat{W}_q^{(+)}(x, p)\bar{\eta}_{\Lambda^0}^{(\beta)}(x_2))|\Omega\rangle\} = \int \frac{d^4y}{(2\pi)^4} \theta(p^0) e^{-i\mathbf{p} \cdot \mathbf{y}} \\
\times \text{tr}\{\langle \Omega|T(\eta_{\Lambda^0}^{(\alpha)}(x_1)) : \bar{q}(x + \frac{1}{2} y) \otimes q(x - \frac{1}{2} y) : \bar{\eta}_{\Lambda^0}^{(\beta)}(x_2))|\Omega\rangle\}.
\]

(3.6)

The calculation of the vacuum expectation value in the r.h.s. of (3.6) we carry out by the example of \(W^{(\omega)}(x, p)\). This calculation runs in the way

\[
\begin{align*}
\text{tr}\{\langle \Omega|T(\eta_{\Lambda^0}^{(\alpha)}(x_1)) : \bar{u}(x + \frac{1}{2} y) \otimes u(x - \frac{1}{2} y) : \bar{\eta}_{\Lambda^0}^{(\beta)}(x_2))|\Omega\rangle\} &= -\frac{2}{3} \varepsilon^{ijk} \varepsilon^{ij'} j' k' \\
\times \text{tr}\{\langle \Omega|T([\bar{u}(x_1)\gamma^\mu s_j(x_1)](\gamma^\mu \gamma^5 d_k(x_1))^{(\alpha)} - [\bar{d}(x_1)\gamma^\mu s_j(x_1)](\gamma^\mu \gamma^5 u_k(x_1))^{(\alpha)}\}
\times : \bar{u}_\ell(x + \frac{1}{2} y) \otimes u_\ell(x - \frac{1}{2} y) : \{(\bar{d}_\ell(x_2)\gamma^\mu \gamma^5 s_j(x_2)\gamma^\sigma u_k(x_2)) - (\bar{u}_\ell(x_2)\gamma^\mu \gamma^5)\}
\end{align*}
\]
\[
\times [\bar{s}^c_j(x_2)\gamma_\nu d^c_{k'}(x_2)]|\Omega}\rangle = -\frac{2}{3} \epsilon^{ijk}\epsilon^{i'j'k'}\text{tr}\{\langle\Omega|T([\bar{u}^c_i(x_1)\gamma^\mu s_j(x_1)](\gamma_\mu\gamma^5 d_k(x_1)))^{(a)}
\times \bar{u}^c_\ell(x + \frac{1}{2} y) \otimes u^c_\ell(x - \frac{1}{2} y)(\bar{d}_\ell(x_2)\gamma^\nu\gamma^5)^{(\beta)}[\bar{s}^c_j(x_2)\gamma_\nu u^c_{k'}(x_2)]|\Omega}\rangle
\]
\[
+ \frac{2}{3} \epsilon^{ijk}\epsilon^{i'j'k'}\text{tr}\{\langle0|T([\bar{d}^c_i(x_1)\gamma^\mu s_j(x_1)])^{(a)}\bar{u}^c_\ell(x + \frac{1}{2} y) \otimes u^c_\ell(x + \frac{1}{2} y)(\bar{u}_{i'}(x_2)\gamma^\nu\gamma^5)^{(\beta)}[\bar{s}^c_j(x_2)\gamma_\nu d^c_{k'}(x_2)]|\Omega}\rangle
\]
\[
+ \frac{2}{3} \epsilon^{ijk}\epsilon^{i'j'k'}\text{tr}\{\langle0|T([\bar{u}^c_i(x_1)\gamma^\mu s_j(x_1)](\gamma_\mu\gamma^5 d_k(x_1)))^{(a)}\bar{u}^c_\ell(x - \frac{1}{2} y) \otimes u^c_\ell(x + \frac{1}{2} y)(\bar{d}_\ell(x_2)\gamma^\nu\gamma^5)^{(\beta)}[\bar{s}^c_j(x_2)\gamma_\nu u^c_{k'}(x_2)]|\Omega}\rangle
\]}
\[
(3.7)
\]

In the r.h.s. of (3.7) the trace should be calculated over spinorial indices of quark fields only. Making necessary contractions we get

\[
\text{tr}\{\langle0|T(\eta^{(a)}_{\Lambda_0}(x_1): \bar{u}(x + \frac{1}{2} y) \otimes u(x - \frac{1}{2} y) : \eta^{(b)}_{\Lambda_0}(x_2))|\Omega}\rangle =
\]
\[
= 4\text{tr}\{S^{(u)}_F(x - x_1 + \frac{1}{2} y)\gamma^\mu S^{(s)}_F(x_1 - x_2)\gamma^\nu S^{(a)}_F(x_2 - x + \frac{1}{2} y))\langle\gamma_\mu\gamma^5 S^{(d)}_F(x_1 - x_2)\gamma_\nu\gamma^5)^{(a\beta)}
+ 4\text{tr}\{\gamma^\mu S^{(s)}_F(x_1 - x_2)\gamma^\nu S^{(d)}_F(x_2 - x_1))\langle\gamma_\mu\gamma^5 S^{(u)}_F(x_1 - x_1 - \frac{1}{2} y)S^{(u)}_F(x_2 - x_2 - \frac{1}{2} y))\gamma_\nu\gamma^5)^{(a\beta)}
- 4(\gamma_\mu\gamma^5 S^{(d)}_F(x_1 - x_2)\gamma^\nu S^{(s)}_F(x_2 - x_1)\gamma_\mu S^{(u)}_F(x_1 - x_1 - \frac{1}{2} y)S^{(u)}_F(x_2 - x_2 - \frac{1}{2} y))\gamma_\nu\gamma^5)^{(a\beta)}
- 4(\gamma_\mu\gamma^5 S^{(a)}_F(x_1 - x_1 - \frac{1}{2} y)S^{(u)}_F(x_2 - x_2 - \frac{1}{2} y))\gamma_\nu\gamma^5)^{(a\beta)}
\]
\[
= 4\text{tr}\{S^{(d)}_F(x - x_1 + \frac{1}{2} y)\gamma^\mu S^{(s)}_F(x_1 - x_2)\gamma^\nu S^{(d)}_F(x_2 - x + \frac{1}{2} y))\langle\gamma_\mu\gamma^5 S^{(u)}_F(x_1 - x_2)\gamma_\nu\gamma^5)^{(a\beta)}
+ 4\text{tr}\{\gamma^\mu S^{(s)}_F(x_1 - x_2)\gamma^\nu S^{(d)}_F(x_2 - x_1))\langle\gamma_\mu\gamma^5 S^{(d)}_F(x_1 - x_1 - \frac{1}{2} y)S^{(d)}_F(x_2 - x_2 - \frac{1}{2} y))\gamma_\nu\gamma^5)^{(a\beta)}
- 4(\gamma_\mu\gamma^5 S^{(d)}_F(x_1 - x_2)\gamma^\nu S^{(s)}_F(x_2 - x_1)\gamma_\mu S^{(d)}_F(x_1 - x_1 - \frac{1}{2} y)S^{(d)}_F(x_2 - x_2 - \frac{1}{2} y))\gamma_\nu\gamma^5)^{(a\beta)}
- 4(\gamma_\mu\gamma^5 S^{(d)}_F(x_1 - x_1 - \frac{1}{2} y)S^{(d)}_F(x_2 - x_2 - \frac{1}{2} y))\gamma_\nu\gamma^5)^{(a\beta)}
\]
\[
= 3\text{tr}\{\langle0|T(\eta^{(a)}_{\Lambda_0}(x_1): \bar{d}(x + \frac{1}{2} y) \otimes d(x - \frac{1}{2} y) : \eta^{(b)}_{\Lambda_0}(x_2))|\Omega}\rangle =
\]
\[
= 4\text{tr}\{S^{(d)}_F(x - x_1 + \frac{1}{2} y)\gamma^\mu S^{(s)}_F(x_1 - x_2)\gamma^\nu S^{(d)}_F(x_2 - x + \frac{1}{2} y))\langle\gamma_\mu\gamma^5 S^{(d)}_F(x_1 - x_2)\gamma_\nu\gamma^5)^{(a\beta)}
+ 4\text{tr}\{\gamma^\mu S^{(s)}_F(x_1 - x_2)\gamma^\nu S^{(d)}_F(x_2 - x_1))\langle\gamma_\mu\gamma^5 S^{(d)}_F(x_1 - x_1 - \frac{1}{2} y)S^{(d)}_F(x_2 - x_2 - \frac{1}{2} y))\gamma_\nu\gamma^5)^{(a\beta)}
- 4(\gamma_\mu\gamma^5 S^{(d)}_F(x_1 - x_2)\gamma^\nu S^{(s)}_F(x_2 - x_1)\gamma_\mu S^{(d)}_F(x_1 - x_1 - \frac{1}{2} y)S^{(d)}_F(x_2 - x_2 - \frac{1}{2} y))\gamma_\nu\gamma^5)^{(a\beta)}
- 4(\gamma_\mu\gamma^5 S^{(d)}_F(x_1 - x_1 - \frac{1}{2} y)S^{(d)}_F(x_2 - x_2 - \frac{1}{2} y))\gamma_\nu\gamma^5)^{(a\beta)}
\]
\[
(3.9)
\]

In an analogous way one can calculate the matrix elements of the operators \(\bar{d}(x + \frac{1}{2} y) \otimes d(x - \frac{1}{2} y)\) and \(\bar{s}(x + \frac{1}{2} y) \otimes s(x - \frac{1}{2} y)\). They read

\[
\text{tr}\{\langle\Omega|T(\eta^{(a)}_{\Lambda_0}(x_1): \bar{d}(x + \frac{1}{2} y) \otimes d(x - \frac{1}{2} y) : \eta^{(b)}_{\Lambda_0}(x_2))|\Omega}\rangle =
\]
\[
= 4\text{tr}\{S^{(d)}_F(x - x_1 + \frac{1}{2} y)\gamma^\mu S^{(s)}_F(x_1 - x_2)\gamma^\nu S^{(d)}_F(x_2 - x + \frac{1}{2} y))\langle\gamma_\mu\gamma^5 S^{(d)}_F(x_1 - x_2)\gamma_\nu\gamma^5)^{(a\beta)}
+ 4\text{tr}\{\gamma^\mu S^{(s)}_F(x_1 - x_2)\gamma^\nu S^{(d)}_F(x_2 - x_1))\langle\gamma_\mu\gamma^5 S^{(d)}_F(x_1 - x_1 - \frac{1}{2} y)S^{(d)}_F(x_2 - x_2 - \frac{1}{2} y))\gamma_\nu\gamma^5)^{(a\beta)}
- 4(\gamma_\mu\gamma^5 S^{(d)}_F(x_1 - x_2)\gamma^\nu S^{(s)}_F(x_2 - x_1)\gamma_\mu S^{(d)}_F(x_1 - x_1 - \frac{1}{2} y)S^{(d)}_F(x_2 - x_2 - \frac{1}{2} y))\gamma_\nu\gamma^5)^{(a\beta)}
- 4(\gamma_\mu\gamma^5 S^{(d)}_F(x_1 - x_1 - \frac{1}{2} y)S^{(d)}_F(x_2 - x_2 - \frac{1}{2} y))\gamma_\nu\gamma^5)^{(a\beta)}
\]
\[
and
\]
\[
\text{tr}\{\langle\Omega|T(\eta^{(a)}_{\Lambda_0}(x_1): \bar{s}(x + \frac{1}{2} y) \otimes s(x - \frac{1}{2} y) : \eta^{(b)}_{\Lambda_0}(x_2))|\Omega}\rangle =
\]
\[4 \text{tr} \{ S_F^{(u)}(x_2 - x_1) \gamma^\mu S_F^{(s)}(x_1 - x_1 + \frac{1}{2} y) \gamma^\nu \} (\gamma_\mu \gamma^5 S_F^{(d)}(x_1 - x_2) \gamma_\nu \gamma^5)^{\alpha \beta} \]
\[+ 4 \text{tr} \{ S_F^{(d)}(x_2 - x_1) \gamma^\mu S_F^{(s)}(x_1 - x_1 + \frac{1}{2} y) \gamma^\nu \} (\gamma_\mu \gamma^5 S_F^{(u)}(x_1 - x_2) \gamma_\nu \gamma^5)^{\alpha \beta} \]
\[+ 4 (\gamma_\mu \gamma^5 S_F^{(d)}(x_1 - x_2) \gamma_\nu \gamma^5)^{\alpha \beta} \]
\[= 4 \text{tr} \{ S_F^{(u)}(x_2 - x_1) \gamma^\mu S_F^{(s)}(x_1 - x_1 + \frac{1}{2} y) \gamma^\nu \} (\gamma_\mu \gamma^5 S_F^{(d)}(x_1 - x_2) \gamma_\nu \gamma^5)^{\alpha \beta} \]
\[+ 4 \text{tr} \{ S_F^{(d)}(x_2 - x_1) \gamma^\mu S_F^{(s)}(x_1 - x_1 + \frac{1}{2} y) \gamma^\nu \} (\gamma_\mu \gamma^5 S_F^{(u)}(x_1 - x_2) \gamma_\nu \gamma^5)^{\alpha \beta} \]
\[-4 (\gamma_\mu \gamma^5 S_F^{(d)}(x_1 - x_2) \gamma_\nu \gamma^5)^{\alpha \beta} \]
\[-4 (\gamma_\mu \gamma^5 S_F^{(u)}(x_1 - x_2) \gamma_\nu \gamma^5)^{\alpha \beta} \]
\[-4 (\gamma_\mu \gamma^5 S_F^{(d)}(x_1 - x_2) \gamma_\nu \gamma^5)^{\alpha \beta} \]
\[-4 (\gamma_\mu \gamma^5 S_F^{(u)}(x_1 - x_2) \gamma_\nu \gamma^5)^{\alpha \beta} \]
\[\text{The two-point fermion Green function has the following form} \]
\[S_F(z) = \int \frac{d^4q}{(2\pi)^4} S_F(q) e^{-iq \cdot z}, \quad (3.10)\]
\[\text{where} \ S_F(q) \ \text{is the Fourier transformation of the two-point Green function} \ S_F(z). \ \text{For finite temperature} \ T \ \text{it is equal to} \ [22] \]
\[S_F(q) = -2\pi i \theta(q^0) \delta(q^2 - m^2) \frac{(\hat{q} + m)}{e(q \cdot U - \mu)/T + 1}, \quad (3.12)\]
\[\text{where} \ U \ \text{is a hydrodynamical velocity}, \ \mu \ \text{is a quark chemical potential of light} \ u \ \text{and} \ d \ \text{quarks related to the chemical potential of the} \ \Lambda^0 \ \text{hyperon by} \ \mu_{\Lambda^0} = 2\mu. \ \text{The chemical potential of strange quarks we set zero, since in our approach they are massive with mass} \ m_s. \]
\[\text{For the non–equilibrium quark gas the quantities} \ U, \ \mu \ \text{and} \ T \ \text{should depend on a space–time point} \ x \ [12]. \ \text{The hydrodynamical velocity} \ U(x), \ \text{the quark chemical potential} \ \mu(x) \ \text{and temperature} \ T(x) \ \text{as functions of the space–time point} \ x \ \text{should be obtained by solving transport equations within the Quark–Gluon transport theory} \ [17]. \ \text{However, as we show below for the analysis of the polarization properties of} \ \Lambda^0 \ \text{hyperons produced from the QGP we do not need to know the explicit expressions and} x–\text{dependence of the parameters} \ U(x), \ \mu(x) \ \text{and} \ T(x). \]
\[\text{We would like to emphasize that our definition of the two–point quark Green function runs parallel to the ideology of the Parton Model} \ [23, 25] \ \text{and the Coalescence Quark Model} \ [26]. \]
\[\text{Using the vacuum expectation values defined above we are able to proceed to the momentum representation of the quark Green function and to calculate the quantity} \]
\[W^{(+)}(x, p; k)^{\alpha \beta} = \]
\[\theta(p^0) \int d^4x_1d^4x_2d^4ye^{ik \cdot (x_1 - x_2)} e^{-ip \cdot y} \text{tr} \{ \langle \Omega | T(\eta_{\Lambda^0}^{\alpha}(x_1) \tilde{W}^{(+)}_u(x, p) \eta_{\Lambda^0}^{(\beta)}(x_2)) | \Omega \rangle \}
\[+ \theta(p^0) \int d^4x_1d^4x_2d^4ye^{ik \cdot (x_1 - x_2)} e^{-ip \cdot y} \text{tr} \{ \langle \Omega | T(\eta_{\Lambda^0}^{\alpha}(x_1) \tilde{W}^{(+)}_d(x, p) \eta_{\Lambda^0}^{(\beta)}(x_2)) | \Omega \rangle \}
\[+ \theta(p^0) \int d^4x_1d^4x_2d^4ye^{ik \cdot (x_1 - x_2)} e^{-ip \cdot y} \text{tr} \{ \langle \Omega | T(\eta_{\Lambda^0}^{\alpha}(x_1) \tilde{W}^{(+)}_s(x, p) \eta_{\Lambda^0}^{(\beta)}(x_2)) | \Omega \rangle \}. \quad (3.13)\]
We get
\[\theta(p^0) \int d^4x d^4x_2 e^{ik \cdot (x_1 - x_2)} \text{tr}\{\langle \Omega | T(\eta^\alpha_{\Lambda^0}(x_1)\tilde{W}_d^{(+)}(x,p)\eta^\beta_{\Lambda^0}(x_2)) \rangle \Omega \} =\]
\[= 4 \theta(p^0) \int \frac{d^4q}{(2\pi)^4} \text{tr}\{\gamma^\mu S_F^{(s)}(-q)\gamma^\nu S_F^{(u)}(-p)S_F^{(u)}(-p)\} \left(\gamma_\mu \gamma_5 S_F^{(d)}(q - p - k)\gamma_\nu \gamma_5\right)^{\alpha\beta}\]
\[+ 4 \theta(p^0) \int \frac{d^4q}{(2\pi)^4} \text{tr}\{\gamma^\mu S_F^{(s)}(q)\gamma^\nu S_F^{(d)}(q + p - k)\} \left(\gamma_\mu \gamma_5 S_F^{(u)}(p)S_F^{(u)}(p)\gamma_\nu \gamma_5\right)^{\alpha\beta}\]
\[-4 \theta(p^0) \int \frac{d^4q}{(2\pi)^4} \left(\gamma_\mu \gamma_5 S_F^{(d)}(q)\gamma^\nu S_F^{(u)}(q + p - k)\right) \left(\gamma_\mu \gamma_5 S_F^{(u)}(p)S_F^{(u)}(p)\gamma_\nu \gamma_5\right)^{\alpha\beta}\]
\[-4 \theta(p^0) \int \frac{d^4q}{(2\pi)^4} \left(\gamma_\mu \gamma_5 S_F^{(u)}(p)S_F^{(u)}(p)\gamma^\nu S_F^{(s)}(q)\right) \left(\gamma_\mu \gamma_5 S_F^{(d)}(q + k - p)\gamma_\nu \gamma_5\right)^{\alpha\beta}\]
\[= 4 \theta(p^0) \int \frac{d^4q}{(2\pi)^4} \left(\gamma_\mu \gamma_5 S_F^{(d)}(q)\gamma^\nu S_F^{(u)}(q + p - k)\right) \left(\gamma_\mu \gamma_5 S_F^{(u)}(p)S_F^{(u)}(p)\gamma_\nu \gamma_5\right)^{\alpha\beta}\]
\[\text{(3.14)}\]

and
\[\theta(p^0) \int d^4x d^4x_2 e^{ik \cdot (x_1 - x_2)} \text{tr}\{\langle \Omega | T(\eta^\alpha_{\Lambda^0}(x_1)\tilde{W}_d^{(+)}(x,p)\eta^\beta_{\Lambda^0}(x_2)) \rangle \Omega \} =\]
\[= 4 \theta(p^0) \int \frac{d^4q}{(2\pi)^4} \left(\gamma_\mu \gamma_5 S_F^{(d)}(q)\gamma^\nu S_F^{(s)}(q + p - k)\gamma_\mu S_F^{(d)}(p)\gamma_\nu \gamma_5\right)^{\alpha\beta}\]
\[+ 4 \theta(p^0) \int \frac{d^4q}{(2\pi)^4} \left(\gamma_\mu \gamma_5 S_F^{(d)}(q)\gamma^\nu S_F^{(d)}(q + p - k)\right) \left(\gamma_\mu \gamma_5 S_F^{(u)}(p)S_F^{(u)}(p)\gamma_\nu \gamma_5\right)^{\alpha\beta}\]
\[-4 \theta(p^0) \int \frac{d^4q}{(2\pi)^4} \left(\gamma_\mu S_F^{(s)}(-q)\gamma^\nu S_F^{(d)}(-p)S_F^{(d)}(-p)\right) \left(\gamma_\mu \gamma_5 S_F^{(u)}(q - p - k)\gamma_\nu \gamma_5\right)^{\alpha\beta}\]
\[-4 \theta(p^0) \int \frac{d^4q}{(2\pi)^4} \left(\gamma_\mu S_F^{(s)}(q)\gamma^\nu S_F^{(u)}(q + p - k)\right) \left(\gamma_\mu \gamma_5 S_F^{(d)}(p)S_F^{(d)}(p)\gamma_\nu \gamma_5\right)^{\alpha\beta}\]
\[= 4 \theta(p^0) \int \frac{d^4q}{(2\pi)^4} \left(\gamma_\mu S_F^{(s)}(q)\gamma^\nu S_F^{(u)}(q + p - k)\right) \left(\gamma_\mu \gamma_5 S_F^{(d)}(p)S_F^{(d)}(p)\gamma_\nu \gamma_5\right)^{\alpha\beta}\]
\[\text{(3.15)}\]

and
\[\theta(p^0) \int d^4x d^4x_2 e^{ik \cdot (x_1 - x_2)} \text{tr}\{\langle \Omega | T(\eta^\alpha_{\Lambda^0}(x_1)\tilde{W}_s^{(+)}(x,p)\eta^\beta_{\Lambda^0}(x_2)) \rangle \Omega \} =\]
\[= 4 \theta(p^0) \int \frac{d^4q}{(2\pi)^4} \left(\gamma_\mu \gamma_5 S_F^{(d)}(q)\gamma^\nu S_F^{(u)}(p)S_F^{(u)}(p)\gamma_\mu S_F^{(u)}(p + k - q)\gamma_\nu \gamma_5\right)^{\alpha\beta}\]
\[+ 4 \theta(p^0) \int \frac{d^4q}{(2\pi)^4} \left(\gamma_\mu \gamma_5 S_F^{(d)}(q)\gamma^\nu S_F^{(u)}(p)S_F^{(u)}(p)\gamma_\mu S_F^{(d)}(p + k - q)\gamma_\nu \gamma_5\right)^{\alpha\beta}\]
\[-4 \theta(p^0) \int \frac{d^4q}{(2\pi)^4} \text{tr}\{S_F^{(u)}(-q)\gamma^\mu S_F^{(s)}(-p)S_F^{(s)}(-p)\gamma^\nu\} \left(\gamma_\mu \gamma_5 S_F^{(d)}(q + p + k)\gamma_\nu \gamma_5\right)^{\alpha\beta}\]
\[-4 \theta(p^0) \int \frac{d^4q}{(2\pi)^4} \text{tr}\{S_F^{(d)}(q)\gamma^\mu S_F^{(s)}(-p)S_F^{(s)}(-p)\gamma^\nu\} \left(\gamma_\mu \gamma_5 S_F^{(u)}(q + p + k)\gamma_\nu \gamma_5\right)^{\alpha\beta}\]
\[\text{(3.16)}\]

Since the quarks \(u\) and \(d\) are massless and the strange quark \(s\) is massive with mass \(m_s\), we can set
\[S_F^{(u)}(p)S_F^{(u)}(p) = S_F^{(d)}(p)S_F^{(d)}(p) = 0.\]
\[\text{(3.17)}\]
Indeed, using (3.12) we get

\[
S_F^{(u)}(p)S_F^{(u)}(p) = -(2\pi)^2 \delta(p^2) \frac{\hat{p}}{e(p \cdot U - \mu)/T + 1} \delta(p^2) \frac{\hat{p}}{e(p \cdot U - \mu)/T + 1} = \]

\[
= -(2\pi)^2 \delta(0) \frac{p^2 \delta(p^2)}{(e(p \cdot U - \mu)/T + 1)^2} = 0,
\]

(3.18)

where we have used the property of the \( \delta \)-function \( p^2 \delta(p^2) = 0 \). 

As a result the quantity \( W^{(+)}(x, p; k)^{\alpha\beta} \) is defined by the contribution of the strange \( s \)-quark only

\[
W^{(+)}(x, p; k)^{\alpha\beta} =
\]

\[
= 4 \theta(p^0) \int \frac{d^4 q}{(2\pi)^4} \text{tr} \{ S_F^{(u)}(q) \gamma^\mu S_F^{(s)}(-p) S_F^{(s)}(-p) \gamma^\nu \} (\gamma_\mu \gamma_5 S_F^{(d)}(q + p + k) \gamma_\nu \gamma_5)^{\alpha\beta}
\]

\[
+ 4 \theta(p^0) \int \frac{d^4 q}{(2\pi)^4} \text{tr} \{ S_F^{(d)}(q) \gamma^\mu S_F^{(s)}(-p) S_F^{(s)}(-p) \gamma^\nu \} (\gamma_\mu \gamma_5 S_F^{(u)}(q + p + k) \gamma_\nu \gamma_5)^{\alpha\beta}
\]

\[
- 4 \theta(p^0) \int \frac{d^4 q}{(2\pi)^4} (\gamma_\mu \gamma_5 S_F^{(d)}(q) \gamma^\nu S_F^{(s)}(p) S_F^{(s)}(p) \gamma^\mu S_F^{(u)}(p + k - q) \gamma_\nu \gamma_5)^{\alpha\beta},
\]

(3.19)

Thus, in our approach the polarization properties of the \( \Lambda^0 \) hyperon are fully defined by the strange \( s \)-quark. This agrees well with the DeGrand–Miettinen model of polarization of \( \Lambda^0 \) hyperons [11] and the experimental data on polarization of \( \Lambda^0 \) hyperons in proton–induced reactions testifying that the spin of the \( \Lambda^0 \) hyperon is caused by the strange \( s \)-quark and that the \( u \) and \( d \) quarks are combined into a diquark with zero total angular momentum and isospin [27].

Due to the Heaviside function \( \theta(p^0) \) the r.h.s. of (3.19) can be transcribed into the form

\[
W^{(+)}(x, p; k)^{\alpha\beta} =
\]

\[
= -8 \theta(p^0) \int \frac{d^4 q}{(2\pi)^4} (\gamma_\mu \gamma_5 S_F^{(d)}(q) \gamma^\nu S_F^{(s)}(p) S_F^{(s)}(p) \gamma^\mu S_F^{(u)}(p + k - q) \gamma_\nu \gamma_5)^{\alpha\beta}.
\]

(3.20)

Now we are able to calculate the momentum distribution of \( \Lambda^0 \) hyperons produced from the QGP in dependence on the \( \Lambda^0 \)--hyperon polarization.

4 Polarization of \( \Lambda^0 \) hyperons

Substituting (3.20) in (2.11) we obtain the momentum distribution of \( \Lambda^0 \) hyperons produced from the QGP in terms of the Green functions of quarks treated in the non–equilibrium

---

\(^1\)The infinity related to \( \delta(0) \) can be deleted by introducing the rate of the number of \( \Lambda^0 \) hyperons (see Eq. (3.12) in Section 4).
state with \( U, \mu \) and \( T \) depending on a space–time point \( x \). It reads

\[
E_k \frac{d^3 N_{\lambda^0}(\vec{k})}{d^3 k} = \frac{g_B^2}{m_{N^0}} \int d\sigma^\mu(x) k_\mu \theta(k^0) \int d^4pd^4q (2\pi)^4 (-1) \text{tr}\{\gamma_\mu \gamma^5 S_F^{(d)}(x; q)\gamma^\nu \times S_F^{(s)}(x; p) S_F^{(s)}(x; p) \gamma_\mu S_F^{(s)}(x; p + k - q) \gamma_\nu \gamma^5 (\hat{k} + m_{N^0})(1 + \gamma^5 \hat{\zeta}) \}. \tag{4.1}
\]

Using the expressions for the quark Green functions \( (3.12) \) with \( U(x), \mu(x) \) and \( T(x) \), we reduce the r.h.s. of \((4.1)\) to the form

\[
E_k \frac{d^3 N_{\lambda^0}(\vec{k})}{d^3 k} = 2 g_B^2 \frac{m_s}{m_{N^0}} \int d\sigma^\mu(x) k_\mu \theta(k^0) \int d^4pd^4q \theta(p^0) \frac{\delta(p^2 - m_s^2)}{e \cdot p \cdot U(x)/T(x) + 1} \times \frac{\delta(q^2)}{e(q \cdot U(x) - \mu(x))/T(x) + 1} \times \frac{\theta(p^0 + k^0 - q^0)\delta((p + k - q)^2)}{e((p + k - q) \cdot U(x) - \mu(x))/T(x) + 1} \times (-1) \text{tr}\{\gamma_\mu \gamma^5 \hat{q} \gamma^\nu (\hat{p} + m_s) \gamma_\mu (\hat{p} + \hat{k} - \hat{q}) \gamma_\nu \gamma^5 (\hat{k} + m_{N^0})(1 + \gamma^5 \hat{\zeta}) \}. \tag{4.2}
\]

The trace over \( \gamma \)-matrices is equal to

\[
(-1) \text{tr}\{\gamma_\mu \gamma^5 \hat{q} \gamma^\nu (\hat{p} + m_s) \gamma_\mu (\hat{p} + \hat{k} - \hat{q}) \gamma_\nu \gamma^5 (\hat{k} + m_{N^0})(1 + \gamma^5 \hat{\zeta}) \} = (16(k \cdot p) + 8m_s m_{N^0}(p + k)^2 + 16im_s \varepsilon^{\mu\nu\alpha\beta} p_\mu q_\nu k_\alpha \zeta_\beta, \tag{4.3}
\]

where we have taken into account the properties of the \( \delta \)-functions \( \delta(q^2) \) and \( \delta((p + k - q)^2) \). The r.h.s. of \((4.2)\) we suggest to rewrite as follows

\[
E_k \frac{d^3 N_{\lambda^0}(\vec{k})}{d^3 k} = \delta(0) g_B^2 \frac{m_s}{m_{N^0}} \int d\sigma^\mu(x) k_\mu \theta(k^0) \int d^4pd^4q \frac{\theta(p^0)\delta(p^2 - m_s^2)}{\sqrt{p^2 + m_s^2}} \frac{\theta(0)\delta(q^2)}{e(q \cdot U(x) - \mu(x))/T(x) + 1} \times \frac{\theta(p^0 + k^0 - q^0)\delta((p + k - q)^2)}{e((p + k - q) \cdot U(x) - \mu(x))/T(x) + 1} \times (-1) \text{tr}\{\gamma_\mu \gamma^5 \hat{q} \gamma^\nu (\hat{p} + m_s) \gamma_\mu (\hat{p} + \hat{k} - \hat{q}) \gamma_\nu \gamma^5 (\hat{k} + m_{N^0})(1 + \gamma^5 \hat{\zeta}) \}, \tag{4.4}
\]

where we have used the definition of the \( \delta \)-function

\[
\theta(p^0) \delta(p^2 - m_s^2) = \frac{\theta(p^0) \delta(p^0 - \sqrt{p^2 + m_s^2})}{2\sqrt{p^2 + m_s^2}} \tag{4.5}
\]

According to this definition the \( \delta \)-function \( \delta(0) \) can be determined as

\[
\delta(0) = \lim_{T \to \infty} \frac{\tau}{2\pi}. \tag{4.6}
\]

Substituting \((4.6)\) into \((4.4)\), dividing both sides by \( \tau \) and introducing the notation

\[
\tilde{N}_{\lambda^0}(\vec{k}) = \lim_{T \to \infty} \frac{N_{\lambda^0}(\vec{k})}{\tau} \tag{4.7}
\]
for the rate of $\Lambda^0$ hyperons produced from the QGP we get

$$E^k \frac{d^3\tilde{N}_{\Lambda^0}(\vec{k})}{d^3k} = \frac{g_B^2}{2\pi} \frac{m_s}{m_{\Lambda^0}} \int d\sigma^\mu(x)k_\mu \theta(k^0) \int \frac{d^4pdq}{\sqrt{p^2+m_s^2}} \frac{\theta(p^0)\delta(p^2-m_s^2)}{(e\cdot p\cdot U(x)/T(x)+1)^2} \times \frac{\theta(q^0)\delta(q^2)}{e(q\cdot U(x)-\mu(x))/T(x)+1} \times \frac{\theta(p^0+k^0-q^0)\delta((p+k-q)^2)}{e((p+k-q)\cdot U(x)-\mu(x))/T(x)+1} \times (-1) \text{tr}\{\gamma_\mu\gamma^5\hat{\gamma}_\nu(p+m_s)\gamma^\mu(\hat{p}+\hat{k}-\hat{q})\gamma^5(\hat{k}+m_{\Lambda^0})(1+\gamma^5\hat{\gamma})\}. \quad (4.8)$$

Substituting (4.3) in (4.8) we end up with the expression

$$E^k \frac{d^3\tilde{N}_{\Lambda^0}(\vec{k})}{d^3k} = 32 \frac{g_B^2}{2\pi} \frac{m_s}{m_{\Lambda^0}} \int d\sigma^\mu(x)k_\mu \theta(k^0) \int \frac{d^4pdq}{\sqrt{p^2+m_s^2}} \frac{\theta(p^0)\delta(p^2-m_s^2)}{(e\cdot p\cdot U(x)/T(x)+1)^2} \times \frac{\theta(q^0)\delta(q^2)}{e(q\cdot U(x)-\mu(x))/T(x)+1} \times \frac{\theta(p^0+k^0-q^0)\delta((p+k-q)^2)}{e((p+k-q)\cdot U(x)-\mu(x))/T(x)+1} \times \left[\left((k\cdot p) + \frac{1}{2}m_s m_{\Lambda^0}\right)(p+k)^2 + im_s\epsilon^{\nu\sigma\beta\delta}p_\mu q_\nu k_\sigma \zeta_\delta\right]. \quad (4.9)$$

It is obvious that integration over the momenta $p$ and $q$ leads to the vanishing of the term proportional to the polarization vector of $\Lambda^0$ hyperons $\zeta$. This means that $\Lambda^0$ hyperons produced from the QGP are unpolarized.

Now let us show that the momentum distribution (4.9) can be reduced to the form analogous to (4.2). For this aim we assume that the Fermi–Dirac gasses of quarks and $\Lambda^0$ hyperons can be described well in the Boltzmann gas approximation [14]. This allows to neglect the contributions of unities with respect to exponentials in the distribution functions of quarks and $\Lambda^0$ hyperons.

In the Boltzmann quark gas approximation the integrand of (4.9) can be transcribed into the form

$$E^k \frac{d^3\tilde{N}_{\Lambda^0}(\vec{k})}{d^3k} = 32 \frac{g_B^2}{2\pi} \frac{m_s}{m_{\Lambda^0}} \int d\sigma^\mu(x)k_\mu \theta(k^0) e - (k\cdot U(x)-\mu_{\Lambda^0}(x))/T(x) \times \int \frac{d^4p}{\sqrt{p^2+m_s^2}} \theta(p^0)\delta(p^2-m_s^2) \left((k\cdot p) + \frac{1}{2}m_s m_{\Lambda^0}\right)(p+k)^2 e^{-3p\cdot U(x)/T(x)} \times \int d^4q \theta(q^0)\delta(q^2) \theta(p^0+k^0-q^0)\delta((p+k-q)^2), \quad (4.10)$$

where we have used $\mu_{\Lambda^0}(x) = 2\mu(x)$. Then, the function

$$e - (k\cdot U(x)-\mu_{\Lambda^0}(x))/T(x) \quad (4.11)$$

coincides with the distribution function of the $\Lambda^0$ hyperon (4.2) in the Boltzmann gas approximation. This testifies the correctness of our quark level approach to the description of the momentum distribution of the number of $\Lambda^0$ hyperons produced from the QGP. The calculation of the integrals over $q$ and $p$ in (4.10) are adduced in the Appendix A and B.
Using the results obtained in Appendix A and B we get

\[ E_k \frac{d^3 \hat{N}_{\Lambda_0}(\vec{k})}{d^3 k} = \frac{2}{(2\pi)^3} \int_{\Sigma} d\sigma \nu(x) k_\mu \theta(k^0) F(U(x), T(x), k) f_{\Lambda_0}(x, k). \quad (4.12) \]

We have denoted \( F(U(x), T(x), k) = 4\pi^3 I(U(x), T(x), k) \) (see (B.7) of the Appendix B).

In our approach Eq. (4.12) describes the momentum distribution of the rate of \( \Lambda_0 \) hyperons produced from the QGP. Since it does not depend on the polarization of \( \Lambda_0 \) hyperons, in our approach \( \Lambda_0 \) hyperons, produced from the QGP, are unpolarized. This agrees well with the results obtained within other theoretical approaches [4]–[7, 10].

5 Conclusion

We have analysed the polarization properties of \( \Lambda_0 \) hyperons produced from the QGP. We have described the momentum distribution of \( \Lambda_0 \) hyperons, produced from the QGP, in terms of the matrix elements of the relativistic quark Wigner operators. The matrix elements of these operators we have calculated within the Effective quark model with chiral \( U(3) \times U(3) \) symmetry and the Quark–Gluon transport theory. We have shown that using the quark distribution functions in the form of the Jüttner distribution functions with a hydrodynamical 4–velocity \( U(x) \), a quark chemical potential \( \mu(x) \) and a temperature \( T(x) \) depending on the space–time point \( x \), the momentum distribution of \( \Lambda_0 \) hyperons can be represented in the form of the integral over the freeze–out surface of the Jüttner distribution function of \( \Lambda_0 \) hyperons. We have shown that without solving the Quark–Gluon transport equations for the parameters \( U(x), \mu(x) \) and \( T(x) \) the momentum distribution of the rate of \( \Lambda_0 \) hyperons does not depend of the polarization of \( \Lambda_0 \) hyperons. This means that \( \Lambda_0 \) hyperons are unpolarized, when they are produced from the QGP.

Since it is well–known that in high–energy nuclear reactions \( \Lambda_0 \) hyperons are produced highly polarized, the obtained depolarization of the \( \Lambda_0 \) hyperons in the QGP can serve as a one more signature of the QGP.

We would like to emphasize that in our approach the momentum distribution of the rate of \( \Lambda_0 \) hyperons has turned out to be defined by the matrix elements of the relativistic strange quark Wigner operator only. Thus, in our approach the polarization properties of \( \Lambda_0 \) hyperons are fully determined by the spin of the strange \( s \)–quarks. This agrees well with the DeGrand–Miettinen model and the experimental data on the \( \Lambda_0 \) hyperon production in proton–induced nuclear reactions [27]. Such an agreement testifies the correctness of the application of the relativistic quark Wigner operators and the Effective quark model with chiral \( U(3) \times U(3) \) symmetry to the analysis of baryon production by the QGP in ultra–relativistic heavy–ion collisions.

Our result concerning the production of unpolarized \( \Lambda_0 \) hyperons from the QGP agrees well with the results obtained within other theoretical approaches [4]–[7, 10].
Appendix A. Calculation of the integral over $q$

Let us denote the integral over $q$ as $I(p, k)$. The calculation of $I(p, k)$ runs in the way

$$I(p, k) = \int d^4q \theta(q^0) \delta(q^2) \theta(p^0 + k^0 - q^0) \delta((p + k - q)^2) =$$

$$= \int d^3q \int_{-\infty}^{+\infty} dq^0 \frac{\theta(q^0)}{2|q|} \delta(q^0 - |\vec{q}|) \frac{\theta(p^0 + k^0 - q^0)}{2|\vec{p} + \vec{k} - \vec{q}|} \delta(p^0 + k^0 - q^0 - |\vec{p} + \vec{k} - \vec{q}|). \quad (A.1)$$

Integrating over $q^0$ we obtain

$$I(p, k) = \int d^3q \frac{\theta(p^0 + k^0 - |\vec{q}|)}{2|\vec{p} + \vec{k} - \vec{q}|} \delta(p^0 + k^0 - |\vec{q}| - |\vec{p} + \vec{k} - \vec{q}|). \quad (A.2)$$

In the spherical coordinates the integral over $\vec{q}$ reads

$$I(p, k) = \frac{1}{4} \int_0^{p_0 + k_0} dq |\vec{q}| \int \frac{d\Omega_{\vec{q}}}{|\vec{p} + \vec{k} - \vec{q}|} \delta(p^0 + k^0 - |\vec{q}| - |\vec{p} + \vec{k} - \vec{q}|), \quad (A.3)$$

where $d\Omega_{\vec{q}}$ is the solid angle.

The integration over the solid angle runs as follows

$$\int \frac{d\Omega_{\vec{q}}}{|\vec{p} + \vec{k} - \vec{q}|} \delta(p^0 + k^0 - |\vec{q}| - |\vec{p} + \vec{k} - \vec{q}|) = 2\pi \int_0^\pi \frac{d\theta \sin \theta}{\sqrt{(p^0 + k^0)^2 + q^2 - 2|\vec{p} + \vec{k}||\vec{q}| \cos \theta}} \times \delta(p^0 + k^0 - |\vec{q}| - \sqrt{(p^0 + k^0)^2 + q^2 - 2|\vec{p} + \vec{k}||\vec{q}| \cos \theta}). \quad (A.4)$$

Making a change of variables

$$t = \sqrt{(p^0 + k^0)^2 + q^2 - 2|\vec{p} + \vec{k}||\vec{q}| \cos \theta} \quad (A.5)$$

we get

$$\int \frac{d\Omega_{\vec{q}}}{|\vec{p} + \vec{k} - \vec{q}|} \delta(p^0 + k^0 - |\vec{q}| - |\vec{p} + \vec{k} - \vec{q}|) =$$

$$= \frac{2\pi}{|\vec{q}| |\vec{p} + \vec{k}|} \int_{||\vec{q} + \vec{p} + \vec{k}||} ||\vec{q} + \vec{p} + \vec{k}|| dt \delta(p^0 + k^0 - |\vec{q}| - t) =$$

$$= \frac{2\pi}{|\vec{q}| |\vec{p} + \vec{k}|} \left[ \theta(p^0 + k^0 - |\vec{q}| - ||\vec{p} + \vec{k}|| + |\vec{q}||) - \theta(p^0 + k^0 - |\vec{p} + \vec{k}|| - 2|\vec{q}||) \right]. \quad (A.6)$$

Thus, $I(p, k)$ is defined by the expression

$$I(p, k) = \frac{\pi}{2|\vec{p} + \vec{k}|} \int_0^{p_0 + k_0} d|\vec{q}| \left[ \theta(p^0 + k^0 - |\vec{q}| - ||\vec{p} + \vec{k}|| + |\vec{q}||) - \theta(p^0 + k^0 - |\vec{p} + \vec{k}|| - 2|\vec{q}||) \right]. \quad (A.7)$$

Integrating by parts we get

$$I(p, k) = \frac{\pi}{2|\vec{p} + \vec{k}|} \int_0^{p_0 + k_0} d|\vec{q}| ||\vec{q}|| \theta(|\vec{q}| - |\vec{p} + \vec{k}|) \delta\left(\frac{p^0 + k^0 + |\vec{p} + \vec{k}|}{2} - |\vec{q}|\right)$$

$$- \delta\left(\frac{p^0 + k^0 - |\vec{p} + \vec{k}|}{2} - |\vec{q}|\right) = \frac{\pi}{2}. \quad (A.8)$$

Hence, the integral over $p$ does not depend on the integral over $q$. 

Appendix B. Calculation of the integral over $p$

Let us denote the integrals over $q$ and $p$ as $I(U,T,k)$

$$I(U,T,k) = 8 g_B^2 \frac{m_s}{m_{\Lambda_0}} \int \frac{d^4p}{\sqrt{p^2 + m_s^2}} \theta(p^0) \delta(p^2 - m^2_s)$$

$$\times \left( (k \cdot p + \frac{1}{2} m_s m_{\Lambda_0}) (p + k)^2 e^{-3 p \cdot U(x)/T(x)} \right). \quad (B.1)$$

Due to the Heaviside function $\theta(p^0)$ we can rewrite the integral as follows

$$I(U,T,k) = 4 g_B^2 \frac{m_s}{m_{\Lambda_0}} \int d^3p \int_0^\infty \frac{dp}{p^2 + m_s^2} \delta(p^0 - \sqrt{p^2 + m_s^2}) \left( k^0 p^0 - \vec{k} \cdot \vec{p} + \frac{1}{2} m_s m_{\Lambda_0} \right)$$

$$\times (m_{\Lambda_0}^2 + m_s^2 + 2 k^0 p^0 - 2 \vec{k} \cdot \vec{p}) \exp \left\{ -3 \frac{p^0}{T(x)} + 3 \vec{p} \cdot \vec{U}(T(x)) \right\}. \quad (B.2)$$

Integrating over $p^0$ we get

$$I(U,T,k) = 4 g_B^2 \frac{m_s}{m_{\Lambda_0}} \int \frac{d^3p}{p^2 + m_s^2} \left( k^0 \sqrt{p^2 + m_s^2} - \vec{k} \cdot \vec{p} + \frac{1}{2} m_s m_{\Lambda_0} \right)$$

$$\times (m_{\Lambda_0}^2 + m_s^2 + 2 k^0 \sqrt{p^2 + m_s^2} - 2 \vec{k} \cdot \vec{p}) \exp \left\{ -3 \sqrt{p^2 + m_s^2} \frac{U^0(x)}{T(x)} + 3 \vec{p} \cdot \vec{U}(T(x)) \right\}. \quad (B.3)$$

Introducing the notations $\rho^0 = U^0(x)/T(x)$ and $\vec{\rho} = \vec{U}(x)/T(x)$ we can transcribe the r.h.s. of (B.3) as follows

$$I(U,T,k) = g_B^2 \frac{4 m_s}{9 m_{\Lambda_0}} \left( - k^0 \frac{\partial}{\partial \rho^0} - \vec{k} \cdot \frac{\partial}{\partial \vec{\rho}} + \frac{1}{2} m_s m_{\Lambda_0} \right)$$

$$\times \left( m_{\Lambda_0}^2 + m_s^2 - 2 k^0 \frac{\partial}{\partial \rho^0} - 2 \vec{k} \cdot \frac{\partial}{\partial \vec{\rho}} \right) \int \frac{d^3p}{p^2 + m_s^2} e^{-3 \sqrt{p^2 + m_s^2} \rho^0 + 3 \vec{p} \cdot \vec{\rho}}. \quad (B.4)$$

The integral over $\vec{\rho}$ can be calculated as follows

$$\int \frac{d^3p}{\vec{\rho}^2 + m_s^2} e^{-3 \sqrt{\vec{\rho}^2 + m_s^2} \rho^0 + 3 \vec{p} \cdot \vec{\rho}} = \frac{4 \pi}{3 |\vec{\rho}|} \int_0^\infty dp p \sinh(3p |\vec{\rho}|) e^{-\sqrt{p^2 + m_s^2} \rho^0}, \quad (B.5)$$

where we have denoted $p = |\vec{\rho}|$. The integral over $p$ is convergent. However, it is rather complicated to be expressed in terms of special functions. Therefore, we leave this integral in the form given by (B.5).

Thus, the result of the integration over $q$ and $p$ can be written as

$$I(U,T,k) = g_B^2 \frac{16 \pi}{27} \frac{m_s}{m_{\Lambda_0}} \left( - k^0 \frac{\partial}{\partial \rho^0} - \vec{k} \cdot \frac{\partial}{\partial \vec{\rho}} + \frac{1}{2} m_s m_{\Lambda_0} \right)$$

$$\times \left( m_{\Lambda_0}^2 + m_s^2 - 2 k^0 \frac{\partial}{\partial \rho^0} - 2 \vec{k} \cdot \frac{\partial}{\partial \vec{\rho}} \right) \frac{1}{|\vec{\rho}|} \int_0^\infty dp p \sinh(p |\vec{\rho}|) e^{-\sqrt{p^2 + 9 m_s^2} \rho^0}, \quad (B.7)$$

where we have changed the scale of the momentum $p \rightarrow p/3$. 

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References

[1] G. Bunce et al., Phys. Rev. Lett. 36, 1113 (1976); K. Heller et al., Phys. Lett. B 68, 480 (1977); K. Heller et al., Phys. Rev. Lett. 41, 607 (1978); S. Erhan et al., Phys. Lett. B 82, 301 (1979); F. Lomanno et al., 43, 1905 (1979); Phys. Lett. B 96, 223 (1980); K. Raychaudhuri et al., Phys. Lett. B 90, 319 (1980); C. Wilkinson et al., Phys. Rev. Lett. 46, 803 (1981); J. Bensinger et al., Phys. Rev. Lett. 50, 313 (1983); F. Abe et al., Phys. Rev. Lett. 50, 1102 (1983); L. Deck et al., Phys. Rev. D 28, 803 (1983); B. E. Bonner et al., Phys. Rev. D 38, 729 (1988); B. Lundberg et al., Phys. Rev. D 40, 3557 (1989); J. Félix et al., Phys. Rev. Lett. 82, 22 (1999); Nucl. Phys. A 721, 805c (2003).

[2] J. Félix, Mod. Phys. Letters A 12, 363 (1997).

[3] J. Félix, Mod. Phys. Lett. A 14, 827 (1999).

[4] R. Stock et al., Proceedings of the Conference on Quark Matter Formation and Heavy Ion Collisions, edited by M. Jacob and H. Satz, World Scientific Singapore, 1982, pp. 557–582.

[5] A. D. Panagiotou, Phys. Rev. C 33, 1999 (1986); Int. J. Mod. Phys. A 5, 1197 (1990).

[6] R. Barni, Phys. Lett. B 296, 251 (1992).

[7] M. Anselmino et al., Phys. Rev. D 63, 054029 (2001).

[8] R. Bellwied (E896 Collaboration), Nucl. Phys. A 698, 499c (2002).

[9] A. Bravar et al., Phys. Rev. Lett. 78, 4003 (1997).

[10] A. Ayala, E. Cuautle, G. Herrera, and L. M. Montaño, Phys. Rev. C 65, 024902 (2002).

[11] T. A. DeGrand and H. I. Miettinen, Phys. Rev. D 23, 1227 (1981); T. Fujita and T. Matsuyama, Phys. Rev. D 38, 401 (1988); T. A. DeGrand, Phys. Rev. D 38, 403 (1988).

[12] F. Cooper and G. Frye, Phys. Rev. D 10, 186 (1974).

[13] U. Heinz, K. S. Lee, and E. Schnedermann, HADRONIZATION OF A QUARK–GLUON PLASMA, in QUARK–GLUON PLASMA – ADVANCED SERIES ON DIRECTIONS IN HIGH–ENERGY PHYSICS, edited by R. C. Hwa, World Scientific Publishing Co., Vol.6, 471 (1990) and references therein.

[14] F. Jüttner, Z. Phys. 47, 542 (1928).

[15] S. R. de Groot, W. A. van Leeuwen, and Ch. G. van Weert, in RELATIVISTIC KINETIC THEORY, Principles and Applications, North–Holland Publishing Company, Amsterdam–New York–Oxford, 1980.
[16] E. P. Wigner, Phys. Rev. 40, 749 (1932); W. E. Brittin and W. R. Chappell, Rev. Mod. Phys. 34, 620 (1962); F. Cooper and D. Sharp, Phys. Rev. D 8, 194 (1975); P. Carruthers and F. Zachariasen, Phys. Rev. D 13, 950 (1976); T. R. Dominguez and R. Hakim, Phys. Rev. D 15, 1435 (1977); J. Phys. A 10, 1525 (1977); R. Hakim, Rev. Nuovo Cim. 1, 1 (1978); Ch. G. van Weert and W. P. H. de Boer, Physica A 81, 597 (1975).

[17] H.–T. Elze and U. Heinz, QUARK–GLUON TRANSPORT THEORY, in QUARK–GLUON PLASMA – ADVANCED SERIES ON DIRECTIONS IN HIGH–ENERGY PHYSICS, edited by R. C. Hwa, World Scientific Publishing Co., Vol.6, 117 (1990) and references therein.

[18] D. J. Thouless, Phys. Rev. 107, 1162 (1957); C. Bloch and C. De Dominicis, Nucl. Phys. 7, 459 (1958); M. Gordin, Nucl. Phys. 15, 89 (1960).

[19] A. N. Ivanov, M. Nagy, and N. I. Troitskaya, Phys. Rev. C 59, 451 (1999).

[20] A. Ya. Berdnikov, Ya. A. Berdnikov, A. N. Ivanov, V. F. Kosmach, M. D. Scadron, and N. I. Troitskaya, Eur. Phys. J. A 12, 87 (2001); Phys. Rev. D 64, 014027 (2001).

[21] Ya. A. Berdnikov, A. N. Ivanov, V. F. Kosmach, and N. I. Troitskaya, Phys. Rev. C 60, 015201 (1999).

[22] L. Dolan and R. Jackiw, Phys. Rev. D 9, 3320 (1974).

[23] R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969; R. P. Feynman, in PHOTON HADRON INTERACTIONS, Benjamin, New York, 1972.

[24] J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).

[25] F. E. Close, in AN INTRODUCTION TO QUARKS AND PARTONS, Academic Press, New York, 1979.

[26] a) T. S. Biró and J. Zimányi, Nucl. Phys. A 395, 525 (1983); b) P. Koch, B. Müller, and J. Rafelski, Phys. Rep. 142, 167 (1986); J. Rafelski and B. Müller, Phys. Rev. Lett. 48, 1066 (1982); J. Rafelski, Nucl. Phys. A 418, 215 (1984); A. Shor, Phys. Rev. Lett. 54, 1122 (1985); J. Rafelski, J. of Phys. G 25, 451 (1999); T. S. Biró, P. Lévai, and J. Zimányi, Phys. Lett. B 347, 6 (1995); J. Zimányi, T. S. Biró, and P. Lévai, The mischievous linear coalescence model and the correct counting in heavy ion collisions, hep-ph/9904501; A. Ya. Berdnikov, Ya. A. Berdnikov, A. N. Ivanov, V. A. Ivanova, V. F. Kosmach, V. M. Samsonov, and N. I. Troitskaya, On hadron production from the quark–gluon plasma phase in ultra–relativistic heavy–ion collisions, hep–ph/0005203; T.S. Biró, P. Lévai, and J. Zimányi, J. Phys. G 28, 1561 (2002).

[27] (see [1] and B. E. Bonner et al., Phys. Rev. D 38, 729 (1988)).