Ab initio investigation of lasing thresholds in photonic molecules

Denis Gagnon, Joey Dumont, Jean-Luc Déziel, and Louis J. Dubé*
Département de physique, de génie physique et d’optique
Faculté des Sciences et de Génie, Université Laval, Québec G1V 0A6, Canada

We investigate lasing thresholds in a representative photonic molecule composed of two coupled active cylinders of slightly different radii. Specifically, we use the recently formulated steady-state ab initio laser theory (SALT) to assess the effect of the underlying gain transition on lasing frequencies and thresholds. We find that the order in which modes lase can be modified by choosing suitable combinations of the gain center frequency and linewidth, a result that cannot be obtained using the conventional approach of quasi-bound modes. The impact of the gain transition center on the lasing frequencies, the frequency pulling effect, is also quantified.

PACS numbers: 42.55.Sa, 42.55.Ah, 42.60.Da

I. INTRODUCTION

The study of light-matter interactions in photonic molecules (PMs), formed by coupling several optically active microcavities (atoms), has been the object of much work in recent years [1]. Applications of microresonators and photonic molecules include optical communications [2], sensing [3–6], quantum computing [7] and metrology [8]. Photonic atoms and molecules are also well suited for the fabrication of microlasers owing to their high quality factor, or photon recycling rate [9]. The lasing characteristics of microresonators are often obtained from the calculation of the cold-cavity (passive) modes. An alternative approach consists in introducing the threshold material gain in the laser eigenvalue problem [10]. This approach allows assessment of the effect of the resonator geometry on lasing thresholds and emission directionality. However, to take into account the spectral properties of a given laser transition, for instance its position and linewidth, formulations such as the Maxwell-Bloch or Schrödinger-Bloch (for 2D systems) theory must be used [11, 12].

In this work, we use the steady-state ab initio laser theory (SALT) [13] to assess the effect of gain transition parameters on lasing frequencies and thresholds in a simple two-dimensional PM composed of two coupled active cylinders. The term ab initio refers to the fact that this theory involves the solution of a set of self-consistent equations that take explicitly the gain medium parameters into account. In other words, SALT allows one to determine the steady-state solutions of the Schrödinger-Bloch equations [12, 14]. The near-threshold behavior of the PM can be obtained by computing the threshold lasing modes (TLMs) as described in [13]. This choice of basis states permits to study the effect of the Lorentzian gain transition on thresholds when there are several modes competing for efficient gain extraction.

This paper is organized as follows. In Section II, we describe the theoretical background behind our computations, including the main equations of SALT and the method used to compute the lasing states for an arbitrary number of cylinders. In Section III, we compute the lasing states of a diatomic photonic molecule as a representative example. Our results show that the thresholds of closely spaced modes exhibit a non-trivial dependence on the parameters of the gain transition, specifically the gain center frequency and its linewidth. We also investigate how the lasing modes are subject to the frequency pulling effect, i.e. the effect of the gain center frequency on the spectrum of the PM laser. We summarize our findings in Section IV and mention a number of possible improvements and extensions.

II. THEORETICAL BACKGROUND

The investigation of the lasing behavior of 2D cavities usually implies the computation of the eigenstates of the passive cavity. These states are governed by the Helmholtz equation

\[ \nabla^2 + \epsilon(r) k^2 \varphi(r) = 0 \]  

where an harmonic time dependence \( \exp(-i\omega t) \) is assumed and \( \epsilon(r) \) is the passive spatially varying refractive index. Both TM (\( \varphi \equiv E_z \)) and TE (\( \varphi \equiv H_x \)) polarized waves can be considered. By applying the usual Sommerfeld radiation condition, i.e. an outgoing wave component only, we obtain a set of leaky, or quasi-bound (QB) states characterized by complex eigenfrequencies \( k_{QB} = k^\prime + ik'' \). Since the radiation condition implies \( k'' < 0 \), QB states are non-orthogonal and exhibit exponential growth towards infinity [14]. Despite this unrealistic behavior, QB states provide a useful measure of the photon lifetime in the cavity by means of the quality factor \( Q = |k'/(2k'')| \). The \( Q \)-factor gives a qualitative indication of which cavity modes will lase first. Considering a gain transition with center located at frequency

\[ k = \omega c \]  

we will refer generically to both quantities as eigenfrequencies.
with the smallest threshold defining the TLM lasing frequency and lasing threshold, axis at $k_D$ of.

For a given choice of exterior real frequency $k$, the remainder of this work.

We restrict ourselves to the case of near-threshold uniform pump profiles [13, 16]. For the purpose of this discussion, we suppose that every cylinder is infinite along the axial $z$ direction. The central hypothesis of 2D-GLMT is that the total field outside the cylinders can be expanded in a basis of cylindrical functions centered on each individual scatterer, that is

$$\varphi(r) = \sum_{n=1}^{N} \sum_{l=-\infty}^{\infty} a_{nl} \tilde{H}_l^{(+)}(k_0 \rho_n) e^{il\theta_n},$$

$$\varphi(r) = \sum_{l=-\infty}^{\infty} c_{nl} J_l(k_n \rho) e^{il\theta_n},$$

where $J_l$ is a Bessel function of the first kind.

In order to apply electromagnetic boundary conditions at the interface of the $n^{th}$ scatterer, one must find an expression for $\varphi(r)$ outside the scatterers containing only cylindrical harmonics centered on the $n^{th}$ scatterer, that is

$$\varphi(r) = \sum_{l=-\infty}^{\infty} \left[ a_{nl} J_l(k_0 \rho_n) + b_{nl} \tilde{H}_l^{(+)}(k_0 \rho_n) \right] e^{il\theta_n}.$$
This can be achieved via the application of Graf’s addition theorem for cylindrical functions, allowing a transformation from the frame of reference of scatterer \( n' \) to the frame of reference of scatterer \( n \) \cite{22}. The theorem states that

\[
H^{(+)}_{l}(k_0 \rho_{n'}) e^{il\theta_n'} = \sum_{l=-\infty}^{\infty} e^{i(l'-l)\phi_{nn'}} H^{(+)}_{l-l'}(k_0 R_{nn'}) J_l(k_0 \rho_n) e^{il\theta_n} \tag{8}
\]

where \( R_{nn'} \) is the center-to-center distance between scatterers \( n \) and \( n' \) and \( \phi_{nn'} \) is the angular position of scatterer \( n' \) in the frame of reference of scatterer \( n \). Substituting (8) in (5) yields

\[
\varphi(r) = \sum_{l=-\infty}^{\infty} b_{nl} H^{(+)}_{l}(k_0 \rho_n) e^{il\theta_n} + \sum_{l=-\infty}^{\infty} \sum_{n' \neq n} \sum_{l'=-\infty}^{\infty} b_{n'l'} e^{i(l'-l)\phi_{nn'}} H^{(+)}_{l-l'}(k_0 R_{nn'}) J_l(k_0 \rho_n) e^{il\theta_n}. \tag{9}
\]

The comparison of (7) with (9) then yields the following relation between the \( \{a_{nl}\} \) and \( \{b_{nl}\} \) coefficients

\[
a_{nl} = \sum_{n' \neq n} \sum_{l'=-\infty}^{\infty} e^{i(l'-l)\phi_{nn'}} H^{(+)}_{l-l'}(k_0 R_{nn'}) b_{n'l'}. \tag{10}
\]

A further relation between the \( \{a_{nl}\} \) and \( \{b_{nl}\} \) coefficients is obtained by applying electromagnetic boundary conditions to (6) and (7) at \( \rho_n = u_n \). This finally leads to the homogeneous equation for the coefficient vector \( \mathbf{b} \), \( \mathbf{T}(k_n, k_0) \mathbf{b} = 0 \), whose non-trivial solutions are given by the condition

\[
det[\mathbf{T}(k_n, k_0)] = 0 \tag{11}
\]

where \( k_n \) is the frequency inside the \( n \)th cylinder and \( k_0 \) is the exterior frequency (both can be complex). One immediately recognizes the transfer matrix \( \mathbf{T} \) as the inverse of the usual scattering matrix. \( \mathbf{T} \) has a well defined structure; it is composed of blocks containing coupling coefficients between cylindrical harmonics centered on each circular scatterer. Its elements are given by

\[
\mathbf{T}^{nn'}_{ll'}(k_n, k_0) = \delta_{nn'} \delta_{ll'} - (1 - \delta_{nn'}) e^{i(l'-l)\phi_{nn'}} H^{(+)}_{l-l'}(k_0 R_{nn'}) s_{nl}(k_n, k_0). \tag{12}
\]

The \( s_{nl} \) factor results from the application of electromagnetic boundary conditions and is given by

\[
s_{nl}(k_n, k_0) = -\frac{J_{l'}'(k_0 u_n) - \Gamma_{nl} J_l(k_0 u_n)}{H^{(+)}_{l-l'}(k_0 u_n) - \Gamma_{nl} H^{(+)}_{l-l'}(k_0 u_n)} \tag{13}
\]

where

\[
\Gamma_{nl} = \xi_n \frac{k_n J_{l'}'(k_n u_n)}{k_0 J_l(k_n u_n)} \tag{14}
\]

and \( \xi_{ij} = 1 (\epsilon_j / \epsilon_i) \) for TM (TE) polarization. Prime symbols indicate differentiation with respect to the whole argument. In a typical implementation, \( \mathbf{T} \) is composed of \( N \times N \) blocks of dimension \( 2l_{\text{max}} + 1 \), where \( l_{\text{max}} \)

\[\text{is chosen sufficiently large to ensure convergence of the cylindrical function expansions. Its value is usually fixed by } l_{\text{max}} \geq 3 k_{\text{max}} \{u_n\}. \text{ In the case of a diatomic photonic molecule } (N = 2), \text{ the matrix is of the form}\]

\[
\mathbf{T} = \begin{bmatrix} T^{11} & T^{12} \\ T^{21} & T^{22} \end{bmatrix} \tag{15}
\]

with the diagonal blocks equal to identity matrices and dense off-diagonal blocks, consistent with (12). More details on the method can be found in \cite{17–20, 23}.

The QB states of a PM can be computed by substituting \( k_n \rightarrow k \sqrt{\epsilon_n} \) and \( k_0 \rightarrow k \sqrt{\epsilon_0} \) in (11) and looking for solutions in the complex \( k \)-plane. As for the CF states, the appropriate substitution is \( k_n \rightarrow K \sqrt{\epsilon_n} \) if the \( n \)th cylinder is part of the cavity region \( C \) and \( k_n \rightarrow k \sqrt{\epsilon_n} \) otherwise, with real \( k \). The solutions are in this case located in the complex \( K \)-plane. We note that a countably infinite set of CF states can be computed for each different value of the real exterior frequency \( k \).

\[\text{III. LASING STATES OF A SIMPLE PHOTONIC MOLECULE}\]

For illustrative purposes, we consider a diatomic PM composed of two coupled cylinders, and restrict the discussion to TM-polarized modes. The cylinder radii are
Figure 2. (Color online) Profile of four TM-polarized QB states of a diatomic photonic molecule composed of two cylinders of different diameters, as shown in Fig. 1. The z-coordinate is proportional to the intensity (arbitrary units).

u_1 and u_2, with u_2 = 0.8908 u_1, the center-to-center distance is R_{12} = 2.448 u_1 and the relative permittivity of the cylinders is \(\varepsilon_c = 4\) (see Fig. 1). The y axis is perpendicular to the line connecting the two cylinders, while the x axis is taken along R_{12}. This special geometry [24] has been chosen because it corresponds to an arrangement where one finds a near-coalescence of the eigenvalues of the QB states resulting from the proximity of an avoided crossing between the associated modes. The near-coalescent states located near \(k'u_1 = 5.4\) are shown in Fig. 2, and the evolution of the associated complex eigenfrequencies is shown in Fig. 3a. The states are split in two symmetry classes with respect to the x axis, odd modes (M1 and M4) and even modes (M2 and M3). They result from the coupling between whispering-gallery modes of slightly different angular momenta of the uncoupled cylinders, creating doublet states [1, 25]. As a consequence, the four QB states located near \(k'u_1 = 5.4\) have closely spaced resonance frequencies. Moreover, the \(Q\)-factors are all similar (\(Q \sim 200\)) and the four states compete for gain. The order in which the modes will lase is not obvious, especially if we consider a gain center with a frequency higher than that of M4. In other words, the conventional approach of considering only \(Q\)-factors does not allow a quantitative determination of the lowest threshold mode.

A. Influence of gain medium parameters

As stated previously, the computation of QB states cannot account for the influence of the gain center frequency and width on the lasing characteristics of a PM. However, if the exterior frequency \(k\) is chosen to be close to the frequency \(\text{Re}(k_{QB}) = k'\) of a QB state, then this state can be associated to a single CF state and their intensity profiles look similar inside the active medium [14]. This correspondence can be seen by comparing the symmetries of the amplitude profiles between Figs. 2 and 4. For other values of the exterior frequency \(k\), the intensity profiles may look somewhat different, but the one-to-one correspondence with the QB states still holds (see for instance Figs. 3 and 4).

Since we restrict ourselves to the case of uniform pumping, each QB state shown in Fig. 2 is also associated to a TLM, making it possible to keep the same labels and compute the associated TLMs to assess the influence of the gain medium parameters. To achieve this goal, one can devise the following procedure for computing the TLM associated to a single QB state

1. Compute the complex eigenfrequency \(k_{QB} = k' + ik''\) of the QB state.
2. Compute the complex eigenfrequency \(K(k')\) of the corresponding CF state, using the fact that \(K(k')\) and \(k_{QB}\) are usually close [14], as seen in Figs. 3a and 3b for instance.
3. Compute the values of \(K(k)\) in the real neighborhood of \(k'\).
4. Using (4), map the values of \(K(k)\) to values of \(D_0(k)\). The TLM is characterized by the pair of values \((k_\alpha, D_0^4)\) for which \(D_0\) becomes purely real. An example of this behavior is shown in Fig. 5.

Once the values of \(K(k)\) are computed using (11), it is not necessary to repeat steps 1–3 when varying the values of \(k_\alpha\) and \(\gamma_\alpha\) in step 4 as long as the cavity geometry and pump profile are unchanged.

Using this straightforward approach, the dependence of the lasing thresholds and lasing frequencies on the gain medium parameters can be readily investigated for each of the four modes depicted in Fig. 2. We find that the lowest thresholds modes are always M1 and M4 owing to their higher quality factor. Therefore, we restrict our discussion to these two modes. The dependence of the lasing thresholds \(D_0^1\) and \(D_0^4\) on \(k_\alpha\) and \(\gamma_\alpha\) is shown in Figs. 5–7. As expected, M1 is the first lasing mode when the gain center frequency \(k_\alpha\) is smaller than the value of \(k'\) for that mode. However, as the value of \(k_\alpha\) is increased, M1 can still lase first even if \(k_\alpha\) is greater than the position of mode M4 (see Fig. 7). This is especially true for a large gain width \(\gamma_\alpha\) as mode M1 is able to extract energy more efficiently from the gain transition in that case, while for a narrow gain transition M4 is favored.

Interestingly, at the intersection of the two surfaces shown in Fig. 7, both modes have exactly the same threshold and lase concurrently. Although this lasing behavior is qualitatively consistent with the fact that M1 corresponds to the highest \(Q\) cold-cavity mode, the use of SALT is needed to obtain quantitative predictions of its dependence on \(k_\alpha\) and \(\gamma_\alpha\).
Figure 3. (Color online) (a) Map of $\log|\det[T]|$ in the complex $k$ plane for each of the four QB states of the photonic molecule. Eigenvalues correspond to the zeros of the function (dark spots) and are located at [M1: $k_{QB}u_1 = 5.3830 - 0.0122$, M2: $k_{QB}u_1 = 5.3958 - 0.01756$, M3: $k_{QB}u_1 = 5.3993 - 0.0154$, M4: $k_{QB}u_1 = 5.4078 - 0.0133$]. (b-c) Map of $\log|\det[T]|$ in the complex $K$ plane for two different values of the exterior frequency $k$ (purely real). Each QB state can be associated to a unique CF state, allowing the use of the same labels for QB and CF states.

Figure 4. (Color online) Profile of two TM-polarized CF states of a diatomic photonic molecule, counterparts to the QB states M1 and M4 shown in Fig. 2. These profiles are computed for the same values of the exterior frequency used in Figs. 3b and 3c. The $z$-coordinate is proportional to the intensity (arbitrary units).

It is also instructive to examine the evolution of lasing thresholds when the gain transition center frequency is far from the QB eigenfrequencies. The dependence of $D_0^\alpha$ on the value of $\gamma_\alpha$ for a gain transition for large values of $|k - k_\alpha|$ is shown in Fig. 6. Our numerical results show that the thresholds of modes approximately quadruple when the value of $\gamma_\alpha$ is reduced by half. Accordingly, one can derive the following expression for $\Re[D_0]$ from (4), under the conditions $\Im[D_0] = 0$ and $|k - k_\alpha| \gg \gamma_\alpha$

$$D_0 \approx -2\epsilon_\alpha \Re[K] \Im[K] \frac{(k - k_\alpha)^2}{\gamma_\alpha^2 k^2}. \quad (16)$$

This behavior ($D_0 \sim \gamma_\alpha^{-2}$) is consistent with the observation that modes extract energy more efficiently from a broad lasing transition.

Next, we assess the influence of the gain center frequency $k_\alpha$ on the lasing frequencies of individual modes. The exact lasing frequency of a mode is always shifted by a small amount from the cold-cavity resonance frequency towards the gain center frequency, an effect known as frequency pulling [13, 15]. Since the studied PM geometry exhibits closely spaced lasing modes, this small shift may be of the order of magnitude of the mode spacing. As shown in Fig. 8a, the order of the lasing frequencies
Figure 6. (Color online) Evolution of complex $D_0$ values of the four CF states shown in Fig. 3 for different values of the gain transition width $\gamma_a$. The threshold for each mode is given by $D^\mu_0$ when $\text{Im}[D_0] = 0$, indicated by circles on the curves.

Figure 7. (Color online) Evolution of lasing thresholds of modes M1 and M4 as a function of the gain center frequency $k_a$ and gain width $\gamma_a$. The thresholds of modes M2 and M3 are higher for this range of parameters (not shown).

$k_\mu$ can be altered by changing the gain center frequency. For instance, for $k_{a}u_1 \approx 5.4$, the lasing frequencies are in the order of the QB eigenfrequencies (1, 2, 3, 4), whereas for $k_{a}u_1 \approx 6.0$, the order is (4, 3, 1, 2). This can be explained by the fact that lower $Q$ modes are pulled more strongly. As seen in Fig. 8b, mode M2 is generally the most strongly pulled mode. However, for large values of $k_{a}$, mode M1 is also subject to strong pulling since it is the mode located further away from the gain center frequency. This result shows that in the case where there are closely spaced cold cavity modes of similar $Q$-factor, the spectral characteristics of the PM laser may be strongly affected by the gain transition parameters.

IV. SUMMARY AND OUTLOOK

In summary, we have used the threshold lasing modes of SALT to obtain accurate quantitative predictions of the lasing threshold and frequencies of a simple diatomic PM composed of two coupled cylinders. These predictions were obtained from the computation of threshold lasing modes using 2D-GLMT. This combination of SALT and 2D-GLMT is general and not limited to diatomic photonic molecules. For instance, it can readily be applied to the computation of modes of random lasers for an arbitrary number of active scattering centers [17].

We found that the lasing thresholds of closely spaced modes of the diatomic PM are strongly influenced by the gain center frequency $k_{a}$ and its linewidth $\gamma_{a}$. More specifically the order in which modes lase can be changed by a suitable combination of those gain medium parameters. We also highlighted the frequency pulling effect, and found that lower $Q$ modes are usually subject to stronger pulling. These results show the importance of using ab initio theories to take the gain medium characteristics into account in microcavities research. Future work includes an extension to non-uniformly pumped
single-mode emission. Together with SALT, the path is laid out to investigate, engineer and harness the lasing properties of PMs for ultra-low threshold and directional single-mode emission.

The authors acknowledge financial support from the Natural Sciences and Engineering Research Council of Canada (NSERC). D.G. is supported by a NSERC Postgraduate Scholarship. J.D. and J.L.D. are grateful for a research fellowship from the Canada Excellence Research Chair in Photonic Innovations of Y. Messaddeq. We also acknowledge the free software projects Armadillo [27] and Mayavi [28]. Colorblind compliant colormaps are taken from [29].

[1] S. V. Boriskina, *Photonic Molecules and Spectral Engineering* (Springer, 2010), vol. 156 of *Springer Series in Optical Sciences*, chap. 16, pp. 393–421, ISBN 978-1-4419-1743-0.

[2] G. Griffel and S. Arnold, in *LEOS Conf. Proc.* (1997), pp. 165+.

[3] S. V. Boriskina, J. Opt. Soc. Am. B 23, 1565 (2006).

[4] C. Wang and C. P. Search, Opt. Lett.

[5] A. Imamoglu, *Quantum Computation Using Quantum Dot Spins and Microcavities* (Wiley, 2005), chap. 14, pp. 217–227.

[6] C. Wang and C. P. Search, Opt. Lett. 39, 26 (2014).

[7] A. Imamoglu, *Quantum Computation Using Quantum Dot Spins and Microcavities* (Wiley, 2005), chap. 14, pp. 217–227.

[8] P. Del’Haye, A. Schliesser, O. Arcizet, T. Wilken, A. Imamoglu, *Quantum Computation Using Quantum Dot Spins and Microcavities* (Wiley, 2005), chap. 14, pp. 217–227.

[9] F. Vollmer and S. Arnold, Nat. Methods 5, 591 (2008), ISSN 1548-7105, URL http://dx.doi.org/10.1038/nmeth.1221.

[10] F. Vollmer and S. Arnold, Nat. Methods 5, 591 (2008), ISSN 1548-7105, URL http://dx.doi.org/10.1038/nmeth.1221.

[11] S. Sunada, T. Harayama, and K. S. Ikeda, Phys. Rev. E. I. Smotrova, A. I. Nosich, T. M. Benson, and P. Sewell, IEEE J. Sel. Top. Quant. 12, 78 (2006), ISSN 1077-260X, URL http://dx.doi.org/10.1109/jstqe.2005.862940.

[12] T. Harayama and S. Shinohara, Laser Photon. Rev. 5, 247 (2011), URL http://dx.doi.org/10.1002/lpor. 200900057.

[13] L. Ge, Y. D. Chong, and A. D. Stone, Phys. Rev. A 82, 063824+ (2010), URL http://dx.doi.org/10.1103/physreva.82.063824.

[14] L. Ge, Ph.D. thesis, Yale University (2010).

[15] A. E. Siegman, *Lasers* (University Science Books, 1986).

[16] J. Andersen, A. A. Asatryan, L. C. Botten, M. A. Byrne, H. Cao, L. Ge, L. Labonté, P. Sebbah, A. D. Stone, H. E. Türeci, et al., Adv. Opt. Photon. 3, 88 (2011), ISSN 1943-8206, URL http://dx.doi.org/10.1364/apoph.3.000088.

[17] J. Andreasen, A. A. Asatryan, L. C. Botten, M. A. Byrne, H. Cao, L. Ge, L. Labonté, P. Sebbah, A. D. Stone, H. E. Türeci, et al., Adv. Opt. Photon. 3, 88 (2011), ISSN 1943-8206, URL http://dx.doi.org/10.1364/apoph.3.000088.

[18] G. Gouesbet and J. A. Lock, Appl. Opt. 52, 897 (2013), URL http://dx.doi.org/10.1364/ao.52.00897.

[19] A. A. Asatryan, L. C. Botten, M. A. Byrne, H. Cao, L. Ge, L. Labonté, P. Sebbah, A. D. Stone, H. E. Türeci, et al., Adv. Opt. Photon. 3, 88 (2011), ISSN 1943-8206, URL http://dx.doi.org/10.1364/apoph.3.000088.

[20] D. Gagnon, J. Dumont, and L. J. Dubé, J. Opt. Soc. Am. A 29, 2673 (2012).

[21] D. Gagnon, J. Dumont, and L. J. Dubé, J. Opt. Soc. Am. A 29, 2673 (2012).

[22] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover Publications, 1970).

[23] P. Del’Haye, A. Schliesser, O. Arcizet, T. Wilken, R. Holzwarth, and T. J. Kippenberg, Nature 450, 1214 (2007), ISSN 0028-0836, URL http://dx.doi.org/10.1038/nature06401.

[24] S. Nojima, J. Appl. Phys. 98, 043102+ (2005), URL http://dx.doi.org/10.1063/1.2001150.

[25] J. W. Ryu, S. Y. Lee, and S. W. Kim, Phys. Rev. A 79, 053858+ (2009), URL http://dx.doi.org/10.1103/PhysRevA.79.053858.

[26] J.-B. Shim and J. Wiersig, Opt. Express 21, 24240 (2013), URL http://dx.doi.org/10.1364/oe.21.024240.

[27] G. Painchaud-April, J. Dumont, D. Gagnon, and L. J. Dubé, in *Transparent Optical Networks (ICTON), 2013 15th International Conference on* (IEEE, 2013), pp. 1–4, ISSN 2161-2056, URL http://dx.doi.org/10.1109/ icton.2013.6602811.

[28] C. Sanderson, Tech. Rep., NICTA (2010).

[29] M. Geissbühler and T. Lasser, Opt. Express 21, 9862 (2013), URL http://dx.doi.org/10.1364/oe.21.009862.