Adiabatic transfer of energy fluctuations between membranes inside an optical cavity

Devender Garg, Anil K. Chauhan, and Asoka Biswas

Department of Physics, Indian Institute of Technology Ropar, Rupnagar, Punjab 140001, India

(Dated: August 1, 2017)

A scheme is presented for the adiabatic transfer of average fluctuations in the phonon number between two membranes in an optical cavity. We show that by driving the cavity modes with external time-delayed pulses, one can obtain an effect analogous to stimulated Raman adiabatic passage (STIRAP) in the atomic systems. The adiabatic transfer of fluctuations from one membrane to the other is attained through a ‘dark’ mode, that is robust against decay of the mediating cavity mode. The results are supported with analytical and numerical calculations with experimentally feasible parameters.

PACS numbers: 42.50.Wk, 03.67.Hk

I. INTRODUCTION

Controlling the quantum systems has been the main focus of research in various branches in physics and chemistry. The main goal in quantum control techniques is to develop systematic methods for the active manipulation and control of quantum systems, to obtain a deterministic output. It has witnessed many exciting applications, including coherent control of different molecular processes, e.g., photoassociation, photodissociation, and scattering, quantum computing, and control of decoherence.

Most control schemes for quantum systems rely on its interaction with light. The optomechanical system poses a suitable platform to explore such control techniques. In such systems, the light field inside an optical cavity and a mesoscopic mechanical oscillator, with a frequency far from the optical domain, interact with each other through radiation pressure force. So far, several nontrivial quantum phenomena have been realized in optomechanical systems. This includes side band and near-ground-state cooling, and squeezing of a mechanical oscillator. The strong coupling and quantum coherent coupling between cavity field mode and mechanical oscillator have also been achieved. This leads to potential applications of such systems into quantum communication and quantum information processing. For example, a mechanical oscillator can mediate high fidelity state transfer between two optical cavities. These oscillators have been used to store optical information as a mechanical excitation, as well. The state of a cavity field can even be coherently transferred into, stored-in and retrieved back from a mechanical oscillator, allowing these oscillators to pose as quantum memory. In this paper, we proceed further to exploit the quantum aspects of the oscillators. Precisely speaking, we show how the quantum fluctuations can be coherently and deterministically transferred from one mechanical oscillator to the other. This clearly opens up an avenue of quantum communication between two truly mesoscopic systems.

The main result of this paper rely on stimulated Raman adiabatic passage (STIRAP) - a quantum control technique to efficiently transfer population between two discrete quantum states (which are not dipole-coupled) of an atom, adiabatically using two resonant pulses. In a three-level Λ configuration, one applies the pulses in a so-called counter-intuitive sequence, such that the pump pulse (with Rabi frequency $\Omega_P$) follows the Stokes pulse (with Rabi frequency $\Omega_S$) so as to evoke the population transfer from the state $|1\rangle$ to $|3\rangle$, without populating the intermediate excited state $|2\rangle$. This process can be explained in terms of the eigenstates of the relevant Hamiltonian in interaction picture:

$$H = \frac{\hbar}{2} \begin{bmatrix} 0 & \Omega_P & 0 \\ \Omega_P & 2\Delta_P & \Omega_S \\ 0 & \Omega_S & 2(\Delta_P - \Delta_S) \end{bmatrix}$$

where $\Delta_P$ and $\Delta_S$ are the single-photon detunings of the pump and the Stokes pulse, respectively. In two-photon (Raman) resonance $\Delta_P = \Delta_S$, the above Hamiltonian exhibits zero eigenvalue with the corresponding eigenstate

$$|\psi_P\rangle \propto \Omega_S|1\rangle - \Omega_P|3\rangle.$$

Suitable initial condition and time-dependence of the pulses ensures the formation of the above dark state. During subsequent time-evolution of the pulses, the population remains trapped in the dark state and gets deterministically and adiabatically transferred from the state.
We consider an optomechanical cavity setup (see Fig. 2) in which two membranes are suspended inside a cavity, dividing it into three subcavities - left (L), middle (M), and right (R). If the membranes are fully reflecting at both of their surfaces, the subcavities can be considered as independent cavities, with a corresponding resonant frequency of \( \omega_{n,j} = \frac{2\pi}{L_j} \) where \( n \) is an arbitrary integer, \( L_j \) is the length of the \( j \)th sub-cavity (\( j \in L, M, R \)), and \( c \) is the velocity of light in the vacuum. In this case, the \( k \)th \((k \in 1, 2)\) membrane is partially transmitting, one can have a tunneling between the two subcavities on either side of the membrane with a rate \( J_k \). Therefore, the Hamiltonian of the system can be written as follows:

\[
H_{ac} = H_0 + H_I + H_J, \tag{3}
\]

where \( H_0 \) represents the unperturbed Hamiltonian, \( a_L \), \( a_M \), and \( a_R \) are the annihilation operators for the cavity modes of the left, middle, and the right part with corresponding frequencies \( \omega_{c,j} \) (\( j \in L, M, R \)) respectively, and \( b_1 \) and \( b_2 \) are the annihilation operators of the mechanical modes with respective frequency \( \omega_{m1} \) and \( \omega_{m2} \). Further, \( g_1 \) \((g_2)\) determines the optomechanical coupling for first (second) membrane with left (right) and middle mode, while, \( J_1 \) \((J_2)\) represents the transmission coefficient between the left (right) and middle cavity mode through first (second) membrane. Note that to obtain a coupling, that is linear in the displacement quadrature \( X \equiv (b + b^\dagger) \) in a membrane-in-the-middle set up as considered here in \( H_I \), one needs to choose \( J_k = 0.5\omega_{mk} \) \([26, 28]\).

We further drive the \( j \)th \((j \in L, M, R)\) cavity mode with the laser field of Rabi frequency \( \Omega_j \). It must be borne in mind that these cavity modes are orthogonal to each other, and therefore the laser fields that drive these modes must have orthogonal polarizations to avoid any cross talk. One possible way of doing it may be to send the pulses \( \Omega_L \) and \( \Omega_R \) (with orthogonal polarizations, and with the same angular frequency \( \omega_l \)) along the cavity axis to drive the two modes \( a_L \) and \( a_R \) (see, e.g., \([23]\)), while a linearly polarized pulse \( \Omega_M \) may be applied with a different frequency \( \omega \) \([29]\). This leads to the following Hamiltonian of the system in addition to \( H_0 \):

\[
H_p = \sum_{j \in L,R} \Omega_j \left( a_j^\dagger e^{-i\omega_l t} + \text{h.c.} \right) + \Omega_M \left( a_M^\dagger e^{-i\omega t} + \text{h.c.} \right), \tag{4}
\]
In the rotating frame of laser frequencies, the total Hamiltonian $H_{ac} + H_p$ takes the following form:

$$H = \sum_{j \in L,M,R} (\Delta_j a_j^\dagger a_j) + \sum_{k=1,2} (\omega_{mk} b_k^\dagger b_k)$$

$$- J_1 (a_L^\dagger a_M + h.c.) - J_2 (a_R^\dagger a_M + h.c.)$$

$$- g_1 (a_L^\dagger a_L - a_M^\dagger a_M) (b_1 + b_1^\dagger)$$

$$- g_2 (a_M^\dagger a_M - a_R^\dagger a_R) (b_2 + b_2^\dagger)$$

$$+ \sum_{j \in L,M,R} \Omega_j (a_j^\dagger + h.c.) ,$$

(k \in 1, 2) satisfy the following correlations \cite{30}:

$$\langle a_j^{\text{in}}(t) a_j^{\text{in}}(t') \rangle = \delta(t - t') ,$$

$$\langle a_j^{\text{in}}(t) a_j^{\text{out}}(t') \rangle = (\bar{n}_{th} + 1) \delta(t - t') ,$$

$$\langle b_k^{\text{in}}(t) b_k^{\text{in}}(t') \rangle = (\bar{n}_{th}) \delta(t - t') ,$$

where $\bar{n}_{th} = \{ \exp [\hbar \omega_{mk} / (k_b T)] - 1 \}^{-1}$ is the mean thermal excitation number in the bath, interacting with the mechanical oscillator with frequency $\omega_{mk}$ at an equilibrium temperature $T$ and $k_B$ is the Boltzmann constant.

\section{A. Derivation of effective Hamiltonian}

In order to study the dynamics of the cavity mode and the membranes, we use the standard linearization procedure \cite{5}, in which one expands all the bosonic operators as a sum of the average values and the zero-mean fluctuation, as follows: $a_j \rightarrow \alpha_j + \delta \alpha_j$, $b_j \rightarrow \beta_k + \delta \beta_k$. Here $\alpha_j$ and $\beta_k$ are in general complex and denote the steady state values of the respective annihilation operators of the cavity and membrane modes. Applying this transformation to Eqs. \eqref{6} and \eqref{10}, we obtain the following equations for the average of the operators:

$$\alpha_L' = -\left( \frac{\gamma_L}{2} + i \Delta_L \right) \alpha_L + i J_1 \alpha_M - i \Omega_L$$

$$\alpha_R' = -\left( \frac{\gamma_R}{2} + i \Delta_R \right) \alpha_R + i J_2 \alpha_M - i \Omega_R$$

$$\alpha_M' = -\left( \frac{\gamma_M}{2} + i \Delta_M \right) \alpha_M + i J_1 \alpha_L + i J_2 \alpha_R - i \Omega_M$$

$$\beta_1' = -\left( \frac{\gamma_1}{2} + i \omega_m \right) b_1 + ig_1 (a_L^\dagger a_L - a_M^\dagger a_M)$$

$$\beta_2' = -\left( \frac{\gamma_2}{2} + i \omega_m \right) b_2 - ig_2 (a_R^\dagger a_R - a_M^\dagger a_M)$$

where $\gamma_j$ is the decay rate of the $j$th mode of the cavity and $\gamma_{mk}$ is the dissipation rate of the $k$th membrane. The corresponding noise operators $a_j^{\text{in}} (j \in L, M, R)$ and $b_{k}^{\text{in}}$ are (k \in 1, 2) satisfy the following correlations \cite{30}:

$$\langle a_j^{\text{in}}(t) a_j^{\text{in}}(t') \rangle = \delta(t - t') ,$$

$$\langle a_j^{\text{in}}(t) a_j^{\text{out}}(t') \rangle = (\bar{n}_{th} + 1) \delta(t - t') ,$$

$$\langle b_k^{\text{in}}(t) b_k^{\text{in}}(t') \rangle = (\bar{n}_{th}) \delta(t - t') ,$$

where $\bar{n}_{th} = \{ \exp [\hbar \omega_{mk} / (k_b T)] - 1 \}^{-1}$ is the mean thermal excitation number in the bath, interacting with the mechanical oscillator with frequency $\omega_{mk}$ at an equilibrium temperature $T$ and $k_B$ is the Boltzmann constant.

\section{A. Derivation of effective Hamiltonian}

In order to study...
Similarly, the Langevin equations for the fluctuations can be obtained using Eqs. (10) as follows:

\[ \dot{\delta a}_L = \left( \frac{\gamma_L}{2} + i\Delta_L^j \right) \delta a_L + iJ_1 \delta a_M + ig_1 \alpha_L \left( \delta b_1 + \delta b_1^\dagger \right) \]

\[ + \sqrt{\gamma_L} \delta a_{L,n}^\dagger, \]

\[ \dot{\delta a}_R = \left( \frac{\gamma_R}{2} + i\Delta_R^j \right) \delta a_R + iJ_2 \delta a_M + ig_2 \alpha_L \left( \delta b_2 + \delta b_2^\dagger \right) \]

\[ + \sqrt{\gamma_R} \delta a_{R,n}^\dagger, \]

\[ \dot{\delta a}_M = \left( \frac{\gamma_M}{2} + i\Delta_M^j \right) \delta a_M + iJ_1 \delta a_M + iJ_2 \delta a_R + \sqrt{\gamma_M} \delta a_{M,n}^\dagger \]

\[ - i g_1 \alpha_M \left( \delta b_1 + \delta b_1^\dagger \right) + ig_2 \alpha_M \left( \delta b_2 + \delta b_2^\dagger \right), \]

\[ \dot{\delta b}_1 = \left( \frac{\gamma_m^1}{2} + i\omega_{m}^1 \right) \delta b_1 + i g_1 \left( \alpha_L \delta a_{L,M} + \alpha_L^* \delta a_{L,R} \right) \]

\[ - \alpha_M \left( \delta a_M + \delta a_M^\dagger \right) + \sqrt{\gamma_m} \delta b_{1,n}^\dagger, \]

\[ \dot{\delta b}_2 = \left( \frac{\gamma_m^2}{2} + i\omega_{m}^2 \right) \delta b_2 - ig_2 \left( \alpha_R \delta a_{R,M} + \alpha_R^* \delta a_{R,L} \right) \]

\[ - \alpha_M \left( \delta a_M + \delta a_M^\dagger \right) + \sqrt{\gamma_m} \delta b_{2,n}^\dagger, \]  

The above equations can be derived from the following linearized Hamiltonian:

\[ H = \sum_{j \in L,M,R} \left( \Delta_j^j \delta a_j^\dagger \delta a_j \right) + \sum_{k \in 1,2} \left( \omega_{mk} \delta b_k^\dagger \delta b_k \right) \]

\[ - J_1 \left( \delta a_{L,M}^\dagger \delta a_{M,L} + \text{h.c.} \right) - J_2 \left( \delta a_{R,M}^\dagger \delta a_{M,R} + \text{h.c.} \right) \]

\[ - g_1 \left( \alpha_L^* \delta a_M + \alpha_L \delta a_M^\dagger \right) \left( \delta b_1 + \delta b_1^\dagger \right) \]

\[ + g_2 \left( \alpha_R^* \delta a_R + \alpha_R \delta a_R^\dagger \right) \left( \delta b_2 + \delta b_2^\dagger \right) \]

\[ + g_1 \left( \alpha_L^* \delta a_M + \alpha_L \delta a_M^\dagger \right) \left( \delta b_1 + \delta b_1^\dagger \right) \]

\[ - g_2 \left( \alpha_R^* \delta a_R + \alpha_R \delta a_R^\dagger \right) \left( \delta b_2 + \delta b_2^\dagger \right). \]  

We choose the laser frequencies in the red sideband region so that

\[ \Delta_j = \omega_{mk} : j \in L,M,R ; k \in 1,2. \]  

This corresponds to the following relation between the cavity mode frequencies:

\[ \omega_{L,M} - \omega_{C,M} = \left( \omega_l - \omega_l^j \right) - 3 g_2 \left( \beta_2 + \beta_2^* \right), \]  

while \( g_1 \left( \beta_1 + \beta_1^* \right) = - g_2 \left( \beta_2 + \beta_2^* \right). \) This further requires \( g_1 \) and \( g_2 \) to be out-of-phase, for positive real values of \( \beta_{1,2}, \) and this is achievable in the present configuration [see Eq. (56)]. The above choice of sideband clearly allows us to drive three modes of the cavity with three different pulses of suitable polarization, frequencies, and time-dependences, without any possibility of the cross-talk. The separation between the frequencies of the two sub-cavity modes are chosen to be much larger than their respective line-widths \( \gamma_{mk}. \)

In the interaction picture with respect to the unperturbed part of the above Hamiltonian

\[ H_0 = \sum_{j \in L,M,R} \left( \Delta_j^j \delta a_j^\dagger \delta a_j \right) + \sum_{k \in 1,2} \left( \omega_{mk} \delta b_k^\dagger \delta b_k \right), \]

the condition (26) and the weak-coupling condition (24) \( \omega_{mk} \gg g_1 \alpha_L, g_2 \alpha_R, |g_1 \alpha_M|, |g_2 \alpha_M| \) allow us to take the rotating wave approximation and to obtain the following final form of the effective Hamiltonian:

\[ H = - J_1 \left( \delta a_{L,M}^\dagger \delta a_M + \text{h.c.} \right) - J_2 \left( \delta a_{R,M}^\dagger \delta a_M + \text{h.c.} \right) \]

\[ - g_1 \left( \alpha_L^* \delta a_M + \alpha_L \delta a_M^\dagger \right) \left( \delta b_1 + \delta b_1^\dagger \right) \]

\[ + g_2 \left( \alpha_R^* \delta a_R + \alpha_R \delta a_R^\dagger \right) \left( \delta b_2 + \delta b_2^\dagger \right) \]

\[ \left( \delta a_{L,R}^\dagger \delta a_M + \delta a_{L,M}^\dagger \delta a_R + \sqrt{\gamma_M} \delta a_{M,n}^\dagger \right) \]

\[ - i g_1 \alpha_M \left( \delta b_1 + \delta b_1^\dagger \right) + ig_2 \alpha_M \left( \delta b_2 + \delta b_2^\dagger \right), \]  

\[ \lambda = \left[ \begin{array}{cccc} J_1 & J_2 & 0 & -g_1 \alpha_L \\
 -J_1 & -J_2 & \frac{g_1}{2} & \frac{g_2}{2} \\
 0 & -J_2 & 0 & g_2 \alpha_R \\
 -g_1 \alpha_L & g_2 \alpha_R & 0 & \frac{g_1}{2} \end{array} \right]. \]  

\[ \lambda = \left[ \begin{array}{cccc} \frac{-i g_1}{2} & J_1 & 0 & -g_1 \alpha_L \\
 -J_1 & \frac{-i g_1}{2} & -J_2 & \frac{g_1}{2} \frac{g_2}{2} \\
 0 & -J_2 & 0 & \frac{g_2}{2} \frac{g_2}{2} \\
 -g_1 \alpha_L & g_2 \alpha_R & 0 & \frac{-i g_1}{2} \frac{g_2}{2} \frac{g_2}{2} \end{array} \right]. \]  

\[ \psi_D = \left[ \begin{array}{c} 2 g_1 g_2 \alpha_L \alpha_R \delta a_M - \left( g_2 \alpha_R \right) \delta b_1 + \left( g_1 \alpha_L \right) \delta b_2 \end{array} \right]. \]

We find that the above matrix \( M \) exhibits a zero eigenvalue, in absence of the decay terms, with the corresponding eigenmode.

\[ \psi_D = \left[ \begin{array}{c} 2 g_1 g_2 \alpha_L g_2 \alpha_R \delta a_M - \left( g_2 \alpha_R \right) \delta b_1 + \left( g_1 \alpha_L \right) \delta b_2 \end{array} \right]. \]
suggests that the time-evolution should be fast enough to avoid the decay of the cavity mode \(\alpha_M\). We emphasize that, to have \(\alpha_M = 0\), the middle mode also needs to be driven by another field \(\Omega_M\), given by

\[
\Omega_M = J_1 \alpha_L + J_2 \alpha_R ,
\]

as clear from the Eq. (19).

From the steady state expressions (17, 18), it is obvious that the time-dependence of \(\alpha_L\) and \(\alpha_R\) are effectively governed by the driving fields \(\Omega_L\) and \(\Omega_R\). We choose these fields with the following Gaussian envelope in time-domain:

\[
\Omega_L(t) = A \exp \left[ -\left( t - \tau \right)^2 / T^2 \right] \tag{39} \\
\Omega_R(t) = A \exp \left[ -\left( t + \tau \right)^2 / T^2 \right] \tag{40} \\
\Omega_M(t) = J_1 \frac{\Omega_L}{-\Delta_L} + J_2 \frac{\Omega_R}{-\Delta_R} . \tag{41}
\]

Here \(A\) represents the amplitude of the Gaussian pulses, \(T\) is the width of the pulses, and \(2\tau\) represents the pulse delay between the pulses. Note that such a time-delay between the pulses can be obtained by introducing path-difference, as routinely done in standard optical experiments. The time-dependent optomechanical coupling can then be written, using Eqs. (17, 18) and for negligible decay rates, as

\[
g_1 \alpha_L = \frac{g_1 A}{-\Delta_L} \exp \left[ -\left( t - \tau \right)^2 / T^2 \right] \tag{42} \\
g_2 \alpha_R = \frac{g_2 A}{-\Delta_R} \exp \left[ -\left( t + \tau \right)^2 / T^2 \right] . \tag{43}
\]

This represents a delayed and counter-intuitive pulse sequence to transfer the fluctuation excitation from the \(b_1\) mode to the \(b_2\) mode, through the evolution of the eigenmode \(\psi_D\), akin to STIRAP.

Note that the other eigenvalues of \(M\) are given by

\[
\lambda_2 = -\lambda_3 = -\frac{1}{2} \sqrt{\left( \alpha_0 - \sqrt{\beta_0} \right) } , \\
\lambda_4 = -\lambda_5 = -\frac{1}{2} \sqrt{\left( \alpha_0 + \sqrt{\beta_0} \right) } \tag{44}
\]

where

\[
\alpha_0 = 1 + 2 \alpha_L^2 g_1^2 + 2 \alpha_L^2 g_2^2 \tag{45} \\
\beta_0 = 1 + 4 \alpha_L^4 g_1^4 - 8 \alpha_L^2 \alpha_R^2 g_1^2 g_2^2 + 4 \alpha_R^4 g_2^4 . \tag{46}
\]

We show in Fig. 3 how these eigenvalues \(\lambda_i\) \((i = 1, \ldots, 5)\) vary with time under the action of these pulses. We find that the gap between the two eigenvalues become larger during the maximum overlap of the two pulses. This gap reflects that the system would remain confined in the zero-eigenvalue eigenstate, as there is no level-crossing during evolution, ensuring the adiabaticity of the process.

We choose the pulse sequence (39)-(41) to drive the cavity modes. The pulse \(\Omega_R(t)\) is applied first on the right cavity \(R\), thereby increasing the average photon number in that mode and effectively the radiation pressure on the second membrane. This leads to a driven oscillation of the right membrane, with an enhanced phonon excitation. Next, the left cavity \(L\) is driven by a pulse \(\Omega_L(t)\), partially overlapping with the \(\Omega_R(t)\). This causes the phonon population sweep at resonance, in analogy of STIRAP. In this way, the excitation fluctuation of the left membrane is adiabatically transferred to the other mediated by the middle sub-cavity mode \(\alpha_M\).

To verify the adiabatic transfer of the excitation, as discussed above, we next solve numerically the Langevin...
equations [30][34] and investigate the time-evolution of the average phonon number fluctuation \( \langle \delta b_1^\dagger \delta b_1 \rangle \) and \( \langle \delta b_2^\dagger \delta b_2 \rangle \). For an initial average phonon number fluctuation \( \langle \delta b_1^\dagger \delta b_1 \rangle = 1 \) in mechanical mode \( b_1 \), we display in Fig. 4 the transfer of excitation to the \( b_2 \) mode. It is observed that for a weak optomechanical coupling \( (g_1 = g_2 = 0.001) \), complete population transfer would require large amplitude pulses. Larger coupling strength \( (e.g., g_1 = g_2 = 0.01) \) would relax the requirement of such large amplitudes for complete excitation transfer.

So the first-order correction of the eigenvalue can be obtained as

\[
\lambda_1' = \psi_D^\dagger M_{\text{decay}} \psi_D
\]

\[
= -2i \gamma_M (g_1 g_2 \alpha_L \alpha_R)^2 - i \frac{\gamma m_1}{2} (g_2 \alpha_R)^2 - i \frac{\gamma m_2}{2} (g_1 \alpha_L)^2
\]

where the zero-eigenvalue eigenmode \( \psi_D \) is considered in its matrix form. We display the temporal variation of this eigenvalue \( |\lambda_1'| \) in the inset of the Fig. 3. Clearly this does not deviate much from the value zero. This suggests that the moderate values of decay rates of the sub-cavity modes and the membranes do not affect much the adiabatic transfer. This is further verified in the Fig. 4, which shows that the transfer is nearly complete in presence of the decay of all the modes. In fact, negative imaginary shift of the eigenvalue would otherwise lead to the decay of the eigenmode \( \psi_D \); however, the magnitude of this decay rate \( |\lambda_1'| \) remains negligibly small during the adiabatic transfer.

**IV. CONCLUSION**

In summary, we have described a scheme in an optomechanical set up to adiabatically transfer the average fluctuation of excitation from one membrane to the other, both being suspended inside an optical cavity. These membrane, while suitably placed, divide the entire cavity into three sub-cavities - L, M, and R. The corresponding optical modes couple with each other via tunnelling through the membranes and to the membranes through radiation pressure force. We propose to drive the left and right modes with time-dependent pulses in counter-intuitive sequences such that a zero-eigenvalue adiabatic mode is obtained, analogous to that obtained for STIRAP. Thereby, the effective couplings between the membranes and the optical modes become time-dependent and facilitate transfer of excitation fluctuation through this adiabatic mode. In addition, a third pulse, as a suitable superposition of these two delayed pulses, is employed to drive the middle cavity mode so as to avoid its excitation. We have analyzed the adiabatic features of the transfer, through the time-evolution of the relevant eigenvalues. We have further investigated, both analytically and numerically, the robustness of this evolution against the decays of the membranes and the cavity modes. We emphasize that the exchange of the energy fluctuations between two mechanical systems may pose as a possible quantum communication protocol. The slow decay of the mechanical oscillator may facilitate the information storage as well.

[1] W. S. Warren, H. Rabitz, and M. Dahleh, Science 259, 1581 (1993); H. Rabitz, R. de Vivie-Riedle, M. Motzkus, and K. Kompa, Science 288, 824 (2000).

[2] M. Shapiro and P. Brumer, *Principles of the Quantum Control of Molecular Processes* (Wiley-Interscience, New Jersey, 2003).

[3] M. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge Univ. Press, Cambridge, 2011).

[4] L. Viola and S. Lloyd, Phys. Rev. A 58, 2733 (1998); L. Viola, S. Lloyd, and E. Knill, Phys. Rev. Lett. 82, 2417 (1999).

[5] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Rev. Mod. Phys. 86, 1391 (2014).

[6] J. D. Teufel et al., Nature 475, 359 (2011).

[7] F. Marquardt, J. P. Chen, A. A. Clerk, and S. M. Girvin, Phys. Rev. Lett. 99, 093902 (2007).

[8] A. D. O’Connell et al., Nature 464, 697 (2010); J. Chan et al., Nature 478, 89 (2011).

[9] K. Jahne et al., Phys. Rev. A 79, 63819 (2009); A. Szorkovszky, A. C. Doherty, G. I. Harris, and W. P. Bowen, Phys. Rev. Lett. 107, 213603 (2011); A. Nunnenkamp, K. Borkje, and J. G. E. Harris, and S. M. Girvin, Phys. Rev. A 82, 021806 (2010); M. Asjad et al.
[10] S. Groblacher, K. Hammerer, M. R. Vanner, and M. Aspelmeyer, Nature 460, 724 (2009).
[11] J. D. Teufel, et al., Nature 471, 204 (2011).
[12] E. Verhagen, S. Deleglise, S. Weis, A. Schliesser, and T. J. Kippenberg, Nature 482, 63 (2012).
[13] K. Stannigel et al., Phys. Rev. Lett. 109, 013603 (2012).
[14] A. H. Safavi-Naeini and O. Painter, New J. Phys. 13, 013017 (2011).
[15] B. Rogers et al., Quantum Meas. Quantum Metrol. 2, 11 (2014).
[16] Satya Sainadh U. and A. Narayanan, Phys. Rev. A 88, 033802 (2013).
[17] V. Fiore, Y. Yang, M. C. Kuzyk, R. Barbour, L. Tian, and H. Wang, Phys. Rev. Lett. 107, 133601 (2011).
[18] T. A. Palomaki, J. W. Harlow, J. D. Teufel, R. W. Simmonds, and K. W. Lehnert, Nature 495, 210 (2013).
[19] J. Oreg, F. T. Hioe, and J. H. Eberly, Phys. Rev. A 29, 690 (1984).
[20] K. Bergmann, H. Theuer, and B. W. Shore, Rev. Mod. Phys. 70, 1003 (1998).
[21] L. Tian, Phys. Rev. Lett. 108, 153604 (2012).
[22] Y.-D. Wang and A. A. Clerk, Phys. Rev. Lett. 108, 153603 (2012).
[23] C. Dong, V. Fiore, M. C. Kuzyk, and H. Wang, Science 338, 1600 (2012).
[24] J. T. Hill et al., Nat. Commun. 3, 1196 (2012).
[25] L. Tian and H. Wang, Phys. Rev. A 82, 053806 (2010).
[26] H. K. Cheung and C. K. Law, Phys. Rev. A 84, 023812 (2011).
[27] M. Ludwig, A. H. Safavi-Naeini, O. Painter, and F. Marquardt, Phys. Rev. Lett. 109, 063601 (2012).
[28] P. Komar et al., Phys. Rev. A 87, 013839 (2013).
[29] M. J. Woolley and A. A. Clerk, Phys. Rev. A 89, 063805 (2014).
[30] D. F. Walls and G. J. Milburn, Quantum Optics (Springer, Berlin, 2008).