Effect of hygrothermal environment on the nonlinear free vibration responses of laminated composite plates: A nonlinear finite element micromechanical approach

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Abstract. The present research deals with the nonlinear free vibration responses of laminated composite flat panel under hygrothermal environment, by considering the corrugated material properties of the composite lamina through a micromechanical model. The plate has been modeled in the framework of the higher-order shear deformation theory and Green–Lagrange strain displacement relations have been used to account for the geometric nonlinearity. Moreover, the present formulation incorporates all the nonlinear higher order terms arising in the model to capture the exact flexure of the panel. Hamilton’s principle has been adopted to derive the system governing equations and suitable nonlinear finite element steps have been employed for discretization. The responses are computed using direct iterative method and compared with those available published results for validation purpose. Numerical illustrations are presented to investigate the effect of various parameters (thickness ratio, support conditions and lamination scheme) on the nonlinear frequency responses of laminated composite plate under hygrothermal environment using the present model and discussed in details.

Keywords: Laminated composite flat panel; hygrothermal free vibration; nonlinear finite element analysis; Micromechanical approach; HSDT; Green-Lagrange geometric nonlinearity

1. Introduction

Composite structures are well known for their outstanding strength/stiffness to weight ratio and tailor made properties. Owing to this they are favorably used in weight sensitive and high-performance engineering applications like aeronautical, naval, chemical, submarines, biomedical and other mechanical industries over the past few decades. However, both during the fabrication and operational life, these structures are subjected to the combined action of loading (individual and/or combined hygro-thermo-mechanical load) which not only distorts the structural geometry but also induces the residual stresses in the structural part [1]. Thus the strength and stiffness behaviour of the structural part degrades and the linear strain displacement relations are no longer possible to define the state variables [2]. Hence, it is necessary and challenging for the design engineers to evaluate numerically/analytically/experimentally the nonlinear responses of laminated structures under severe environmental conditions for optimum design of the final products. It is well known that the laminated structures fail under shear rather than tension/fatigue and the higher order shear deformation theory (HSDT) provides a more accurate approximation (parabolic distribution) of the transverse shear

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stresses/strains of the laminated structure and eliminates the need of the shear correction factor as well [3]. Moreover, the geometric nonlinearity in laminated structures due to combined hygro-thermo-mechanical load is the small strain and large (finite) deformation type of problem. Hence, the geometry matrix and the nonlinear stiffness matrices should be derived in the framework of Green-Lagrange nonlinearity (in contrast to von-Karman type) for the realistic prediction of the desired responses. It is also true that during the numerical analysis of laminated structures the numbers of unknowns to be solved in the mathematical model increase as the number of layers increase and obtaining the solution becomes tough. The finite element method (FEM) is undoubtedly the robust and versatile tool used for such analysis [4]. In addition to the above, the environmental dependent composite properties must be precisely considered for the realistic prediction of the mechanical responses under the hostile environment. Many attempts have already been made by researchers in past to study the free vibration responses of laminated structures numerically under combined hygrothermal loading by taking the hygrothermal dependent composite material properties through macromechanical approach in conjunction with FEM. However, the mathematical models in each case are based on the FSDT [5]-[6])/HSDT [7]-[8] mid-plane kinematics and von-Karman nonlinearity. Naidu and Sinha [9] first time developed a nonlinear mathematical model for the laminated composite single/doubly curved shell panel using (the FSDT kinematics and) Green-Lagrange nonlinearity to obtain the free vibration and transient response under hygrothermal load. However, they have not included all the nonlinear higher order terms in the formulation.

Few literatures are also available on the nonlinear bending [10]-[11], buckling [12], free vibration [13]-[15] and dynamic [16]-[17] responses of laminated flat panel under hygrothermal environment incorporating degraded material properties via micromechanics approach. However, all the aforementioned literature used the HSDT mid-plane kinematics and von-Karman (instead of Green-Lagrange) type geometric nonlinear strains in the formulation. To the best of the authors’ knowledge, study on the nonlinear free vibration behaviour of laminated composite plate under hygrothermal environment based on the HSDT mid-plane kinematics and Green-Lagrange nonlinearity by including the temperature and/or moisture dependent composite properties through micromechanics approach is yet to be reported. The objective of the present work is to investigate the nonlinear free vibration responses of laminated composite plate under hygrothermal environment with the aid of a general nonlinear FEM based model based on the HSDT mid-plane kinematics with Green-Lagrange type geometric nonlinearity. The composite material properties are evaluated using a micromechanical model. Moreover, all the nonlinear higher order terms are considered in the mathematical model to achieve the exact structural flexure. The governing equation is obtained using Hamilton’s principle and solved using a direct iterative method. Finally, the effect of various geometrical parameters and support conditions on the linear and nonlinear frequency responses of laminated composite plates under combined hygrothermal loading have been investigated and discussed in details.

2. Theory and FE Formulation

Typical laminated composite plate geometry has been considered for the present investigation as shown in figure 1. The laminated plate consists of N number of equally thick orthotropic layers of length a, width b and thickness h. The proposed mathematical model has been developed based on the HSDT displacement field ([3]) and any arbitrary point on the laminated flat panel with respect to the mid-plane along x, y and z directions is given by

\[ \bar{u} = u + \phi_1 z + \psi_1 z^2 + \Theta_1 z^3, \]
\[ \bar{v} = v + \phi_2 z + \psi_2 z^2 + \Theta_2 z^3, \]
\[ \bar{w} = w \]

where, \((\bar{u}, \bar{v}, \bar{w})\) represent the displacements at any arbitrary point on the plate along the \((x, y, z)\), respectively. Similarly, \((u, v, w)\) indicate the corresponding displacements of the points on the mid-plane, \(\phi_1\) and \(\phi_2\) are the rotations of normal to the mid-plane with respect to the \(y\)-axis and \(x\)-axis, respectively. This displacement field accounts for the transverse shear strains as quadratic function of the ...
thickness coordinate and also comprise the parabolic distribution of shear stress across the thickness indicated by $\psi_1, \psi_2, \theta_1$ and $\theta_2$, which are the higher order terms arising in the Taylor series expansion defined at the mid-plane.

The Green-Lagrange type nonlinear strain displacement relations used in the present analysis are given by [18]:

$$\{\varepsilon\} = \{\varepsilon_L\} + \{\varepsilon_{NL}\}$$  \hspace{1cm} (2)

Replacing Eq. (1) into Eq. (2), the linear and nonlinear strain terms can be expressed as:

$$\{\varepsilon\} = [H]_{L} \{\varepsilon_{L}\} + \frac{1}{2} [H]_{NL} \{\varepsilon_{NL}\}$$  \hspace{1cm} (3)

where, $[H]_{L}$, $[H]_{NL}$ are the functions of thickness coordinate and $\{\varepsilon_{L}\}$, $\{\varepsilon_{NL}\}$ are the functions of $x$ and $y$. The detail terms of $\{\varepsilon_{L}\}$, $\{\varepsilon_{NL}\}$, $[H]_{L}$ and $[H]_{NL}$ can be seen in [18].

For any general $k$th orthotropic composite lamina with arbitrary fibre orientation angle $\theta$, the constitutive equation of generalized stress tensor can be written as

$$\{\sigma_{ij}\}^k = \left[Q_{ij}\right]^k \{\varepsilon_{ij} - \alpha_{ij} \Delta T - \beta_{ij} \Delta C\}^k$$  \hspace{1cm} (4)

where, $\{\sigma_{ij}\}^k = \{\sigma_1, \sigma_2, \sigma_6, \sigma_5, \sigma_4\}^T$ and $\{\varepsilon_{ij}\}^k = \{\varepsilon_1, \varepsilon_2, \varepsilon_6, \varepsilon_5, \varepsilon_4\}^T$ are the stress and strain vectors respectively for the $k$th layer, $\left[Q_{ij}\right]^k$ is the transferred reduced stiffness matrix for the $k$th layer, $\{\alpha_{ij}\}^k = \{\alpha_1, \alpha_2, 2\alpha_{12}\}^T$ and $\{\beta_{ij}\}^k = \{\beta_1, \beta_2, 2\beta_{12}\}^T$ are the thermal expansion/contraction and moisture expansion/contraction coefficient vectors, respectively. Here, the temperature difference $\Delta T = T - T_0$ is the difference between the applied ($T$) and reference ($T_0$) temperatures, respectively. Similarly, the weight percentage moisture concentration difference $\Delta C = C - C_0$ is the difference between applied ($C$) and reference ($C_0$) values.

To compute the realistic response of laminated structure, the corrugated hygro-thermo-elastic material properties (to be used in Eq. 4) of the composite lamina are evaluated using a micromechanical model as presented in [11].
Now, the in-plane hygrothermal forces are evaluated by following the steps in [19] and given by

$$\{N\} = \left[ \begin{array}{c} N_1 \\ \vdots \\ N_k \end{array} \right]$$

where, \(N\) is the resultant compressive in-plane hygrothermal force vector. The work done (W) due to in-plane hygrothermal force is expressed as

$$W = \frac{1}{2} \{\varepsilon\}_G^T [D_{G}] \{\varepsilon\}_G \ dA$$

where, \(\{\varepsilon\}_G\) is the geometric strain vector and \([D_{G}]\) is the material property matrix. The total strains energy of the plate expressed as the sum of linear and nonlinear strain energies as

$$U = \frac{1}{2} \int \sum_{i=1}^{N} \sum_{k=1}^{N_k} \rho_k \left( \varepsilon_i \right)^T \left[ T_{Ln} \right] \left( \varepsilon_i \right) \ dA$$

where, \(\rho\) is the density, \(\{\varepsilon\}\) is the global displacement vector, \(\left[ T_{Ln} \right]\) is the linear stiffness matrix and \(\left[ T_{Ln} \right]\) is the nonlinear stiffness matrix which depend on the displacement vector linearly and quadratically respectively. In order to obtain an eigen value solution of the nonlinear responses, Eq. (11) is modified as follows:

$$L \delta = \left[ F_{L} + F_{NL} \right]$$

Now, using the Eq. (9) in Eq. (6)-(8) the elemental form is derived as in [18] and subsequently Eq. (10) can be expressed following steps as in [19]

$$[M][\delta] + \left[ K_L \right] + \frac{1}{2} \left[ K_{NL} \right]_1 + \frac{1}{3} \left[ K_{NL} \right]_2 = \left[ F_{LT} + F_{AC} \right]$$

where, \([M]\) is the global mass matrix, \([K_L]\) is the global linear stiffness matrix, \([K_{NL}]_1\) and \([K_{NL}]_2\) are the nonlinear mixed stiffness matrices which depend on the displacement vector linearly and quadratically respectively.
where, \([K_G]\) is the global geometry stiffness matrix. The inclusion of geometry matrix in the governing equation is possible by dropping the hygrothermal force terms from Eq. (11). The above equation is solved using a direct iterative method and the detail steps can be seen in [18].

3. Results and Discussion

The desired responses are computed by using the homemade FEM code developed in MATLAB 7.10.0 environment based on the present model. Figure 2 shows the convergence and validation behaviour of the present model. Based on the convergence, a (5×5) mesh is found adequate for throughout the analysis. The nondimensional frequency parameters are obtained using the present model and presented in Table 1 along with the reference [14] values. The geometry, material property, support conditions and the nondimensional form are considered similar to as in Kumar and Patil [14].

It is clearly observed that the present model gives higher frequency values at lower amplitude ratios whereas for higher amplitude ratios it results lower frequency parameters than the references. This is due to the Green-Lagrange geometrical nonlinearity in the framework of the HSDT considered in the present model instead of the von-Karman type nonlinear kinematics as adopted in the references. Thus the inevitability of Green-Lagrange geometrical nonlinearity and inclusion of all the nonlinear higher order terms during the modelling and analysis of laminated structures under hostile environmental conditions is clearly illustrated.

Figure 2. Convergence study of linear/nonlinear frequency parameters for laminated composite flat-panels ([±45]_2, a/h=20, a/b=1, v_f=0.6) under hygrothermal environment (ΔT=100°C, ΔC=1%).
In order to show the applicability of the developed nonlinear model and to evaluate the effect of various parameters on the nonlinear frequency response of laminated composite flat panel, few new examples are solved. For computation purpose, Graphite/epoxy composite lamina is considered and the material properties corresponding to reference condition (temperature 21°C and moisture concentration 0%) are as given below [11].

\[ E_{f1} = 220 \text{ GPa}, \quad E_{f2} = 13.79 \text{ GPa}, \quad E_m = 3.45 \text{ GPa}, \quad G_{f12} = 8.97 \text{ GPa}, \quad \nu_{f12} = 0.2, \quad \nu_m = 0.35, \quad \alpha_{f1} = -0.99 \times 10^{-6} \degree C^{-1}, \quad \alpha_{f2} = 10.08 \times 10^{-6} \degree C^{-1}, \quad \alpha_m = 72 \times 10^{-6} \degree C^{-1}, \quad \beta_m = 0.33, \quad T_0 = 216 \degree C, \quad v_t = 0.6. \]

The boundary conditions used for analysis are given below:

(a) All edges simply support (SSSS):
\[ v = w = \Phi_2 = \Psi_2 = \theta_2 = 0 \at \at x = 0, \quad a \quad \text{and} \quad u = w = \Phi_1 = \Psi_1 = \theta_1 = 0 \at y = 0, \ b. \]

(b) All edges clamped (CCCC):
\[ u = v = \Phi_1 = \Phi_2 = \Psi_1 = \Psi_2 = \theta_1 = \theta_2 = 0 \at x = 0, \ a \quad \text{and} \quad u = v = \Phi_1 = \Psi_1 = \theta_1 = 0 \at y = 0, \ b. \]

(c) All edges clamped (HHHH):
\[ u = v = \Phi_1 = \Phi_2 = \Psi_1 = \Psi_2 = \theta_1 = \theta_2 = 0 \at x = 0, \quad a \quad \text{and} \quad u = v = w = \Phi_1 = \Psi_1 = \theta_1 = 0 \at y = 0, \ b. \]

The linear and nonlinear frequency responses in nondimensional form is obtained using the equation \[ \overline{\omega} = \overline{\omega}_a \overline{a}^2 / h \sqrt{(\rho / E_2)}, \] where \( E_2 \) indicate the value under reference environmental condition \( (T_0 = 210 \degree C \quad \text{and} \quad C_0 = 0\%) \).

3.1 Illustration 1. Simply supported anti-symmetric angle-ply ([±30\degree]_2) laminated square plates \( (a/b=1) \) of different thickness ratios are subjected to varying hygrothermal loads \( (\Delta T=1000 \degree C, \Delta C=1\%, \Delta T=3000 \degree C, \Delta C=3\%) \) at different amplitude ratio and presented in figure 3. It is evident that, the nonlinear frequency parameter increases with thickness ratio and decreases with hygrothermal load. However, the nonlinearity is more pronounced at higher amplitude ratio. Thin panels are more sensitive to hygrothermal load in comparison to thick panels.

3.2 Illustration 2. Figure 4 presents the nonlinear frequency parameter of anti-symmetric angle-ply ([±60\degree]_2) laminated square plates \( (a/b=1) \) for four different support conditions (SSSS, CCCC, HHHH and CSCS) under different hygrothermal loading \( (\Delta T=100 \degree C, \Delta C=1\%, \Delta T=200 \degree C, \Delta C=2\%) \). It is observed that the nondimensional frequency parameters are the least and highest for the simply support and clamped conditions, respectively irrespective of the hygrothermal conditions. This is owing to the fact that as the number of constraints increases the stiffness of the structure increases and the frequency response increases monotonically.

3.3 Illustration 3. Moderately thick \( (a/h=30) \) square plates \( (a/b=1) \) with clamped support conditions are considered under various hygrothermal load \( (\Delta T=100 \degree C, \Delta C=1\%, \Delta T=300 \degree C, \Delta C=3\%) \) and different amplitude ratios. The values of nonlinear frequency parameters for symmetric, anti-symmetric, cross-ply and angle-ply laminations are presented in figure 5. It is noted that, nonlinear
vibration responses of symmetric laminated plates are higher in comparison to the anti-symmetric laminations. It is also seen that for equal numbers of layers and particular hygrothermal load, cross-ply laminates exhibit higher nonlinear frequencies than the angle-ply laminates.

**Figure 3.** Effect of thickness ratio \((a/h)\) on the nonlinear frequency parameter of laminated composite flat panel.

**Figure 4.** Effect of support conditions on the nonlinear frequency parameter of laminated composite flat panel.

**Figure 5.** Effect of lamination schemes on the nonlinear frequency parameter of laminated composite flat panel.
4. Conclusion

The nonlinear frequency responses of laminated composite plate under elevated hygrothermal loading have been analyzed using a novel nonlinear finite element micromechanical model. The plate model is developed in the framework of the HSDT mid-plane kinematics and Green-Lagrange type geometric nonlinearity. The temperature and/or moisture dependent composite material properties are computed through a micromechanical model and all the nonlinear higher order terms are included in the formulation to capture the realistic response. The developed model is discretised using nonlinear FEM and solved using a direct iterative method. The convergence and validation of the present mathematical model has been established. The inevitability of the HSDT and Green-Lagrange type nonlinear model for accurate prediction of the free vibration responses has been demonstrated. Based on the numerical experimentation it is observed that the nonlinear frequency responses of the laminated composite plate increase with thickness ratio and number of constraints at the support. It is also seen that corresponding to high temperatures the nonlinearity due to moisture absorption is severe for symmetric angle-ply scheme as compared to other lamination schemes.

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