Oil price shocks and the term structure of the US yield curve: a time–frequency analysis of spillovers and risk transmission

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Abstract
This paper investigates the influence of oil demand, oil supply, and risk-driven shocks on the yield curve in the US between 1995 and 2020. The US term-structure shape is modeled by three structural factors, the level, slope, and curvature. Their empirical analysis is performed according to the Diebold-Li modified variant of the widely used Nelson-Siegel model. The technique of wavelet analysis allows investigating the interrelation of shocks in oil prices and the US yield curve along time and frequency domains, simultaneously. We report on low, medium, and high coherence zones, relative to the oil price movements and the changes in the three yield-curve factors. The low coherence intervals indicate the potential for the three latent factors to be used for creating diversification strategies capable of hedging adverse dynamics in the oil market, potentially workable through global crises. We document the variability of dynamic patterns observable for the US sovereign yield factors on per-type-of-shock basis, evidencing the potential role of the US sovereign debt investments for designing cross-asset hedge strategies for commodity and fixed-income markets.
Keywords Yield-curve structural factors · Oil-demand shocks · Oil-supply shocks · Risk-driven shocks · Wavelet coherence phase-difference · Causality · Leads and lags · Global financial crisis · Covid-19 · Diversification attributes · Hedge strategies

JEL Classification C22 · C32 · C49 · C53 · C58 · E43 · E44 · F3 · F36 · F37 · G12 · G15 · Q41

1 Introduction

Since the beginning of the twenty-first century, global financial markets have been severely hit by several deep crises, exhibiting a planetary scale of their impacts. In this context, it is worth mentioning at least the three following events: the Dot.Com crisis, the global financial crisis (GFC) of 2007 and 2008, and the current Covid-19-triggered economic meltdown. In spite of enhanced risk management techniques and increased regulatory scrutiny, the nowadays-globalized nature and interconnectedness of diverse financial markets across the most diversified geographies and asset classes makes them inherently vulnerable from macroeconomics and complexity science perspectives (Balcilar et al., 2021; Gomes & Gubareva, 2020; Naeem et al., 2020; Zaremba et al., 2021). The above-mentioned crises as well as the imminence of future crises to come serve as a motivational basis for a vast body of empirical and theoretical literature examining various aspects of cross-market, cross-assets, cross-sectorial, and cross-regional interrelations with a special focus on systemic risk factors. For advanced reading on the subject we suggest (Bernanke, 2009; Yellen, 2013. Spierdijk & Umar, 2015; Cai et al., 2018; Culp et al., 2018; Gubareva & Umar, 2020; Samitas et al., 2020; Umar et al., 2020; Umar & Gubareva, 2020, and Gubareva & Borges, 2021).

In particular, it is worth mentioning that over the two last decades, a vast body of scientific research addressing the interrelations of the US yield curve and the most relevant economic parameters has been produced. For deeper insights, we recommend consulting (Estrella, 2005; Diebold & Li, 2006; Rudebusch & Williams, 2009; Aguilar-Conraria et al., 2012; Diebold & Rudebusch, 2013; Fernandes & Vieira, 2019; Gubareva & Keddad, 2020; Gupta et al., 2021; Aharon et al., 2021) among many others. Since the yield curve prediction tool was pioneered by Harvey (1986) (see also Harvey, 1988), proposing the gap between the US Treasury short-end and long-end yields as an alert on a possible nosediving of the US economy, investors worldwide are closely monitoring the US interest rates for signs of eventual recessions (Gupta et al., 2020; Hillerbrand et al., 2018). Even though the main focus of the worldwide academy research currently remains centered on US Treasury yield curve, it is worth mentioning some studies that address the interrelationship of economic drivers and the shape of the sovereign debt curves in other geographies (Nyholm, 2015; Nymand-Andersen, 2018; Barunik & Fiser, 2019, and Umar et al., 2020). Ojo et al., (2017), study the components of the term structure in Canada, while (Vieira et al., 2017, and Caldeira et al., 2020), are focused on emerging market countries from the bloc known as BRICS (Brazil, Russia, India, China, and South Africa).

One of the macroeconomic variables of primary importance is the price of oil, which exercises a profound influence on worldwide financial markets, including the global bond market; see (Datta et al., 2017; Malik & Umar, 2019; Balcilar et al., 2020; Bouri et al., 2020; Dermier et al., 2020; Gupta et al., 2020, and Venditti & Veronese, 2020; Tiwari et al., 2021). These authors acknowledge that the role of the oil price in financial and economic analysis has considerably changed, especially during the two last decades, as the price of oil has become
seen as a gauge of global economic and financial activity. There exist a few stand-alone studies addressing the impact of crude oil market on the US fixed-income finance (Wan & Kao, 2015; Nazlioglu et al., 2020; Nguyen et al., 2020). However, the literature consistently addressing the interrelations between oil price shocks and the term structure of the US yield curve is incipient and rather scant, with just one known to us exception of Gupta et al., (2020). Our research helps to fill this void.

Out of diverse econometric approaches, potentially applicable to study the interrelation of oil price shocks and the shape of the yield curve, we point out a wavelet time–frequency analysis. Although there are a few examples, where wavelet-based techniques are used to study the influence of macroeconomic factors on sovereign yield curves of several countries (Aguiar-Conraria et al., 2012; Ojo et al., 2017), as far as we know a time–frequency analysis has never been extended to include such variables as oil price shocks. Hence, we are motivated to fill in this gap with the present research.

In line with several previous studies based on wavelet analysis (Aguiar-Conraria et al., 2012; Gubareva & Umar, 2020; Jawadi et al., 2020; Umar & Gubareva, 2020; Zaremba et al., 2019), the wavelet coherence analysis and wavelet phase-difference studies are employed, herein, to gauge the influence of shocks in oil prices on the shape of the sovereign yield-curve of the United States. The wavelet coherence and phase difference analysis allow us to investigate the movement between two series as well as the lead-lag relationship, across both time and frequency scales. Thus, accounting for the investment horizon, which is an important element of investment decisions (Spierdijk & Umar, 2014), we use the modified version (Diebold & Li, 2006) of the Nelson-Siegel, 1987, model for studying the dynamic nature of the yield curves, representing the entire term structure by level, slope, and curvature factors; see Litterman and Scheinkman, (1991). The wavelet-based approach permits obtaining time—frequency heatmaps, which contain valuable knowledge on wavelet coherence and wavelet phase-difference for selected pair-wise sets of variables. It is important to point out that the wavelet coherence technique is appropriate for analyzing the conjoint behavior of oil shocks decomposed into demand, supply, and risk components and the shape parameters of the sovereign yield curves, simultaneously along two dimensions, namely the time and frequency, the latter being representative of pretended investment maturities. This enables investigating diverse features of the interrelation of shocks in oil prices and the yield-curve parameters. To complement the wavelet coherence analysis, we undertake a phase-difference study, which allows us determining the directions of the co-movements and describing causality patterns, while searching for nexus links between the demand, supply, and risk movements in oil prices and the variations in the shape parameters of the US yield curve.

To segregate the oil shocks and represent them by demand-, supply-, and risk-driven components, we follow Ready (2018). This approach is based on the conjoint analysis of the following three time-series: WIOGP\(^1\) index, 1-month oil future returns of the second-nearest maturity contracts at the NYMEX,\(^2\) and the VIX index. Such decomposition allows understanding how distinct drivers of moves in oil prices influence and/or are influenced by the US yield-curve shape dynamics.

The current pandemic has intellectually challenged the worldwide academic community, generating a newly consolidated knowledge domain, which is dedicated to the influence of the Covid-19 crisis on trade and economy (Akhtaruzzaman et al., 2021; Al-Awadi et al., 2020; Ali et al., 2020; Dutta et al., 2020; Gubareva, 2020; Jawadi et al., 2020; Okorie & Lin, 2020; Umar

\(^1\) WIOGP index stands for the World Integrated Oil and Gas Producer index.

\(^2\) NYMEX designates the New York Mercantile Exchange.
& Gubareva, 2021; Umar et al., 2022; Umar, Trabelsi, et al., 2021). Our research contributes to this literature too as the analyzed period covers the pandemic and as we document the reaction of the sovereign yield curve of the United States to the Covid-19 induced shocks in oil prices. Our findings are important for investors searching for arbitrage opportunities along the US sovereign curve or pursuing diversification with investments in both the US bond market and the crude oil commodity as we discuss the unique yield-curve/oil-shocks dynamics during the Covid-19 crisis.

This work studies the interrelation between the US sovereign yield curve decomposed into the latent level, slope, and curvature factors, and the demand-, supply-, and risk-related changes in oil prices. Our paper contributes to the advancement of knowledge about the yield-curves responses to oil price shocks in the three following ways. In the first place, we provide advanced findings, filling in the current void of lacking scientific analysis regarding the maps of the dynamic interrelation between oil shocks and the US yield-curve shape. To the best of our knowledge, it is the first application of the wavelet analysis to investigate the changes of the shape of the US yield curve in the wake of Covid-19 vis-a-vis the dynamics of crude oil prices. In the second turn, the present article expands the state-of-art regarding the reaction of the US bond market to the Covid-19-triggered abysmal drop in crude oil prices. The time span of our analysis spreads over the Covid-19 financial and economic turmoil. Hence, beyond the investors’ community, our findings might be potentially insightful for market regulators assessing the suitability of yield-curve control policies to promote financial stability. Third, we report on high, medium, and low coherence zones relative to the demand, supply, and risk-related components of oil price movements and the latent yield-curve factors for the US sovereign debt curve. The regions of low coherence in the time–frequency space represent remarkable manifestations, pointing out potentially capturable diversification benefits, subjacent to the three latent yield factors, which may well be used as the base of hedge strategies, suitable not only for normal market conditions, but also during global crises, such as the GFC and the current pandemic meltdown.

The remaining part of the article is structured in the following manner. Section 2 describes the wavelet econometric framework. Section 3 addresses the estimation of the yield curves and introduces the three latent yield factors. Section 4 describes the segregation of shocks in oil prices, allowing representing them by demand-, supply-, and risk-driven price movements. Section 5 focuses on the datasets and descriptive sample statistics. Section 6 provides our findings and discusses their implications. Section 7 offers conclusions.

2 Wavelet econometric framework

Our work makes use of the squared wavelet coherence (SWC) approach. We complement the SWC analysis with the wavelet coherence phase-difference (WCPD) studies. We perform our research in line with (Torrence & Compo, 1998; Torrence & Webster, 1999; Goodell & Goutte, 2020; Gubareva & Umar, 2020, and Umar & Gubareva, 2020). The data foundations of our framework are two-fold. First, we estimate the time series on a day-to-day basis of level, scope and curvature (see Sect. 3). Second, we generate the daily time series of demand-, supply-, and risk-driven movements in the price of oil (see Sect. 4). The time span of our analysis ranges from 31/12/1994 to 11/12/2020. We analyze the interdependencies of shocks in oil prices and latent yield factors’ volatility of the US sovereign curve.

For the sake of investigating an intricate interrelation of the oil price shocks and the shape-determining yield-curve parameters, we resort to the SWC analysis, based on the continuous
wavelet transformation as per (Torrence & Webster, 1999; Vacha & Barunik, 2012, and Sun & Xu, 2018). For any investment horizon, the SWC varies between zero (null correlation) and one (perfect positive correlation). Additional information regarding lead-lag patterns of oil price shocks and latent yield factors is obtained by means of the WCPD tooling in line with (Zaremba et al., 2019; Gubareva & Umar, 2020, and Umar & Gubareva, 2020).

It is worth mentioning that the SWC approach permits investigating co-movements involving a pair of time-series in two dimensions, namely, current time and investment horizon. The base of SWC approach is a continuous wavelet transform (Rua & Nunes, 2009). Two important concepts, the cross-wavelet transform (CWT) and the coherence metrics, underlie our bivariate econometric setup and are explained below.

The CWT of two historical arrays of data \( x(t) \) and \( y(t) \); see (Torrence & Compo, 1998), can be written using the respective wavelet transforms of each stand-alone time-series: \( W_x(n, u, s) \) and \( W_y(n, u, s) \):

\[
W_{xy}(n, u, s) = W_x(n, u, s) \ast W_y(n, u, s)
\]  

Here \( u \) designates the location, \( s \) denotes the scale, while an operator \( \ast \) means the complex conjugate, \( n \) is the time index. Torrence and Compo (1998) point out that the use of wavelet transforms of a series, given by \( W_x(n, u, s) \) and \( W_y(n, u, s) \), helps in analyzing time series with non-stationary power at various frequencies. Similarly, the cross-wavelet transform, given by \( W_{xy}(n, u, s) \), allows revealing regions in time–frequency space, subjacent to the comovements between the considered series of historic data. Saying it differently, the CWT provides local covariance of these historic arrays of data along each scale. E.g., a cross-wavelet transform figure in the proximity of 1 signifies an elevated level of synchronous behavior of the two time-series, while being closed to 0, a cross-wavelet transform value indicates a non-existence of synchronization.

Following (Torrence & Webster, 1999), we employ the squared wavelet coherence (SWC), capable of capturing episodes, when the analyzed series co-move:

\[
R^2(u, s) = \frac{|S(s^{-1}W_{xy}(u, s))|^2}{S(s^{-1}|W_x(u, s)|^2)S(s^{-1}|W_y(u, s)|^2)}
\]  

Here \( S \) is an operator, which smooths the individual and cross-wavelet transforms in both time and frequency dimensions. In other words, the squared wavelet coherence represents a coefficient in time–frequency space, with possible values ranging in the interval from 0 to 1.

However, unlike a common Pearson coefficient describing a correlation between two analyzed arrays, possible SWC measures may reside exclusively in a positive 0–1 interval. That is why a SWC technique as stand-alone does not permit differentiating between co-movements occurring in the same versus in the opposite direction, i.e., it is not capable of identifying whether the correlation is negative or positive.

Seeking a more complete description of the correlation patterns and lead-lag interrelations of the two historic arrays of data, a WCPD approach is employed in line with Torrence and Compo (1998). This technique permits making differentiation, classifying co-movements into two respective types: in the same direction versus in the opposite directions.

Use the following expression for the phase difference:

\[
\Phi_{xy}(u, s) = \tan^{-1} \left( \frac{Im \{ S(s^{-1}W_{xy}(u, s)) \}}{Re \{ S(s^{-1}W_{xy}(u, s)) \}} \right)
\]  

Here \( Im \) represents the imaginary part of the cross-wavelet transform while \( Re \) stands for the real part.
At this point, we mention that the standard visual tools subjacent to the WCPD technique allow for graphically representing causality interrelations involving two historical arrays of data. E.g., phase-differences are depicted on the squared-wavelet-coherence heatmaps by differently oriented solid arrows. A zero value of the WCPD metrics indicates perfectly synchronized co-movements of the historical arrays. Arrowheads, oriented to the right (left) side, highlight the in-phase (anti-phase) behavior of the considered series, indicating a positive (negative) correlation. A bottom-up oriented arrow signifies that the first array of historical data exercises a leadership vis-à-vis the second time-series by \( \pi/2 \). In its turn, a top-down pointing arrow indicates that, now, on the contrary, the second dataset leads the first array of data by \( \pi/2 \). Taking into consideration these intuitive tips makes it easy to uncover an information message brought about by any arrow oriented along any direction.

In the following section, we address the process of estimating structural shape-determining yield-curve factors.

3 Shape-determining yield-curve factors: estimation methodology

The daily estimates for the structural—level, slope, and curvature—yield-curve shaping factors are obtained following Nelson and Siegel (1987), methodology. Diverse approaches could be employed for yield-curve fitting purposes; however, the above-mentioned method has become more popular amidst academic community due to three main advantages, which are explicitly outlined in Diebold and Rudebusch (2013). The first feature is that, in line with the key attributes of economics, the discount coefficients decline with maturity, eventually approaching 0 at the long-end of the term structure. The second peculiarity is that Nelson and Siegel (1987), a parsimonious methodology improves the model’s capacity to forecast shape changes in the yield curves. And finally, this model could be employed for fitting a whole range of possible yield curves, available in the market with enough empirical data for the factors’ the estimation process.

Nelson-Siegel original approach to investigate the shape variations of yield curves was further improved and fine-tuned by Diebold and Li (2006). They posited that the structural yield-curve factors behave in accordance with an order-1 vector autoregressive (VAR) process. This assumption permits modeling latent yield-curve factors using a state-space representative methodology. We specify this representation as:

\[
\begin{align*}
zt(\tau) &= \begin{pmatrix} 1 & \frac{1-e^{-\lambda_1 \tau}}{\lambda_1} & \frac{1-e^{-\lambda_1 \tau}}{\lambda_1 \tau_1} - e^{-\lambda_1 \tau} \\ 1 & \frac{1-e^{-\lambda_2 \tau}}{\lambda_2} & \frac{1-e^{-\lambda_2 \tau}}{\lambda_2 \tau_2} - e^{-\lambda_2 \tau} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda_N \tau}}{\lambda_N} & \frac{1-e^{-\lambda_N \tau}}{\lambda_N \tau_N} - e^{-\lambda_N \tau} \end{pmatrix} x_t + u_t, \quad u_t \sim N(0, R) \\
\tilde{x}_t &= \Gamma \tilde{x}_{t-1} + \eta_t, \quad \eta_t \sim N(0, G)
\end{align*}
\]

Here \( z_t(\tau) \) denotes a vector of dimension \( N \times 1 \) representing the yield of zero-coupon bonds with \( \tau \) denoting the time to maturity and \( t \) denoting the timing: \( t = 1, ..., T \); \( \lambda_\tau \) denotes the exponential decay rate at which the latent factors decay to zero with the maturity horizon; \( u_t \) stands for error-term (gaussian white noise) vector of the same dimension; \( R \) stands for a \( 3 \times 3 \) matrix of variances and covariances. \( x_t = [L_t, S_t, C_t] \) represents a

\[3\] For more detail we suggest consulting Diebold and Li (2006).
latent-factors array of $3 \times 1$ dimensionality. Intuitively, $L_t$ designates the level parameter, $C_t$ represents the curvature, with $S_t$ indicating yield-curve slope. In the subsequent equation describing the transition dynamics, $x_t = x_t - x_{t-1}$ represents the matrix, composed by the de-meaned versions of time-dependent variables, determining the shape of the yield-curve. $\Gamma$ indicates time-varying interrelation between yield-curve factors. $\eta_t$ represents an array of (gaussian white noise) errors with dimension $3 \times 1$. Herein, we use an assumption about the independency of $\eta_t$ and $u_t$. $G$ is a diagonal matrix whose dimension is $N \times N$.

The sample statistics of the structural factors of the sovereign curve of the United States along with oil shocks data are presented in Sect. 5.

4 Decomposition of movements in the price of oil: demand -, supply -, and risk—driven changes

The decomposition begins by segregating the movements in the price of oil and representing them by demand-, supply-, and risk-driven variations according to Ready (2018). There are 3 prerequisite types of data, which are necessary to disentangle innovations in crude oil prices, namely, (i) the WIOGP\footnote{WIOGP index means World Integrated Oil and Gas Producer index.} index, second, (ii) returns of oil futures relative to the second closest maturity at the NYMEX,\footnote{NYMEX stands for NY Mercantile Exchange.} and (iii) the VIX index.\footnote{Following Ready (2018), we implement an orthogonal transformation. We thank an anonymous referee for pointing this out.} The WIOGP index, possessing among its constituent members major publicly traded oil companies, allows tracking global oil producers’ stock prices. The NYMEX oil futures are employed to gauge movements in the prices of crude oil. We assess variations in VIX using an ARMA (1, 1) model for calculating the residuals terms. They are used to identify shocks associated with shifts in the risk premium that appear to be negatively correlated with equity returns; see Bollerslev et al. (2009).

The identification technique, developed by Ready (2018), breaks down movements in the price of oil and represents them by three components: supply-, demand-, and risk-related variations. The decomposition model of Ready (2018), can be written as:

$$X_t = AZ_t$$

where $X_t$ is a $3 \times 1$ column vector such as $X_t = [\Delta p_t \quad R^{Prod}_t \quad \zeta_{VIX,t}]'$, with the symbol “′” signifying the transpose operation. Here $\Delta p_t$ represents variations in oil price, $R^{Prod}_t$ designates the return from WOIGP, and $\zeta_{VIX,t}$ represents the innovations to VIX. $Z_t = [S_t \quad D_t \quad V_t]'$, is a vector of supply-driven $S_t$, demand-driven $D_t$, and risk-driven $V_t$ price changes. Lastly, $A$ is a $3 \times 3$ matrix

$$A \equiv \begin{bmatrix} 1 & 1 & 1 \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$$

To ensure orthogonality, the following criterion by Ready (2018), is fulfilled

$$A^{-1} \sum X (A^{-1})^T = \begin{bmatrix} \sigma^2_s & 0 & 0 \\ 0 & \sigma^2_d & 0 \\ 0 & 0 & \sigma^2_v \end{bmatrix}$$
The matrix of covariances relative to the discernible parameter $X_t$ appears as $\sum_X$ in the above equation, whereas $\sigma_s$, $\sigma_d$, and $\sigma_v$ represent the volatilities of demand-, supply-, and risk-driven price movements. The typical orthogonalization procedure that serves to identify structural shocks is renormalized by the identifications in a VAR context. In addition, we are aggregating the shocks to reflect total oil price changes instead of standardizing the shock volatility to one.

5 Data and descriptive statistics

Our analysis covers the years 1995–2020. For the sake of assessing the influence of movements in the price of oil on the latent level, slope, and curvature dynamic parameters of the US term structure, we need first to disentangle the movements in the oil prices into three, demand, supply, and risk components. In line with what we mentioned in Sect. 4, we use the following data-series: the WIOGP index, 1-month oil future returns for the second maturing contract at NYMEX, and the CBOE VIX index.

Apart from these time series, our dataset includes the historical data arrays containing three latent dynamic factors for US yield curve. Level, slope, and curvature of this term-structure are estimated in line with the dynamics Nelson—Siegel, 1987, methodology modified by Diebold and Li (2006). To achieve our objective, the time series for a set of fifteen maturity points on the term structure are considered, namely: 3, 6, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 180, 240, and 360-month horizons. The employed data are sourced through Bloomberg terminal.

Table 1 reports the summary statistics for the daily changes in the level, slope, and curvature factors of the US sovereign yield curve along with the daily changes in demand, supply, and risk shocks in crude oil prices, for the entire analyzed period 1995–2020.

As per Table 1, the Jarque–Bera test results for all variables as well as the high kurtosis observed in all series indicate that the series are non-normal, implying significantly fatter tales
Table 2 Pair correlations: yield-curve parameters and oil shocks

|          | C_USA | L_USA | S_USA | RiskShock | SupplyShock | DemandShock |
|----------|-------|-------|-------|-----------|-------------|-------------|
| C_USA    | 1.0000|       |       |           |             |             |
| L_USA    | 0.0672| 1.0000|       |           |             |             |
| S_USA    | 0.6189| −0.2192| 1.0000|           |             |             |
| RiskShock| 0.0161| 0.0141| −0.0056| 1.0000    |             |             |
| SupplyShock| 0.0087| −0.0037| −0.0010| −0.0001| 1.0000    |             |
| DemandShock| 0.0086| 0.0306| −0.0016| −0.0001| −0.0001| 1.0000 |

The table reports Pearson correlation coefficients subjacent to the daily changes of US term-structure shape parameters along with the demand-, supply-, and risk-related moves in the price of oil. Analyzed period: 1995–2020. C_USA, L_USA, and S_USA represent, correspondingly, the curvature, level, and slope parameters, determining the shape of US yield curve. DemandShock, SupplyShock, and RiskShock designate, respectively, those previously discussed demand-, supply-, and risk-driven components in the crude oils price movements.

than those of a normal distribution. The highest kurtosis is observed for supply-driven shocks (29.18035), indicating that daily moves in SupplyShock time series are subject to leptokurtic characteristics, revealing a highly peaked asymmetric distribution. In addition, a considerably negative value of SupplyShock skewness (−0.88825) indicates that the distribution of daily movement in the supply shocks has a fat long-left tail. This is consistent with the findings of Malik and Umar (2019), stating that supply shocks are not so much market related responses, but rather specific event-driven price movements, caused by stand-alone disruptions on the supply side, relative to drastic, mostly operations-related changes in the output of a specific oil producing corporation.

Table 2 reports the correlations subjacent to daily changes in the six variables time series for the entire analyzed period 1995–2020.

The cross-shock non-diagonal elements (see the respective quadrant in Table 2 on the right at the bottom) are extremely small, being very close to zero, thus confirming that the disentangling the demand-, supply-, and risk-driven components in the crude oils price, has been performed correctly. Note that by construction (see Ready, 2018), the nullity of the off-diagonal elements of the correlation matrix (see Eq. (7)) certifies the orthogonality of the segregated oil shock components, assuring their mutually exclusive condition.

Wrapping up our three-sections-long excursus into the methodologies employed in our research, it is worth stating that carrying out our research we pursue a two-stage approach. First, we engage in generating historical daily arrays of data for the six fundamental variables underlying our investigation. Based on Ready’s, 2018, approach, we construct demand-, supply-, and risk-driven variations in the price of the crude and, in parallel, using the Diebold and Li (2006), modified version of the Nelson-Siegel, 1987, methodology, we estimate the level, slope, and curvature factors for the term-structure of the US yield rates.

Second, by means of squared-wavelet-coherence (SWC) and the respective phase-difference (WCPD) methodologies (Torrence & Compo, 1998), we analyze the relation of the decomposed oil movements and the latent dynamic factors, responsible for the shape of US yield-curve.

In the following Sect. 6, we discuss our findings and address the most relevant implications.
6 Results

At the beginning of this section, we provide a description on how to interpret the SWC and WCPD heatmaps. We use standard visual tools, commonly used in wavelet analyses, which permit us to represent the pairwise causality linkages between the two selected variables along with their lead-and-lag patterns. For instance, the arrows on the analyzed SWC panels are informative of the phase-differences between the considered time-series. In graphical terms, the → and ← arrows inform that the two chosen variables behave in phase and anti-phase, respectively, while the ↑ and ↓ arrows imply that the first time-series in the respective figure caption leads or is lagging behind the second time-series by $\pi/2$. In addition, interpreting the produced heatmaps is made quite intuitive by the respective color-wise legends on the right of each panel. The horizontal axis measures time, while the vertical one gauges the frequency or investment horizon length expressed in months. The colors, seen in the heatmaps, arrive as the results of calculations for all points of the analyzed time–frequency space. For the SWC panels, hot and warm colors such as red and yellow correspond to greater degrees of coherence, indicating a mutual dependence of the considered time series. On the contrary, chilly-fresh and deep cold colors, such as turquoise and blue, signify low levels of coherence. For the WCPD heatmaps, the cooler (hotter) the color, the more negative (positive) is the phase difference. Moreover, green tonalities represent near-zero values of the WCPD metrics, indicating fairly synchronized co-movements of the historic arrays.

6.1 US yield-curve level factor (L_USA) and the demand -, supply -, and risk—driven moves in oil price

We initiate our study assessing the SWC and WCPD involving the demand, supply, and risk shocks in oil prices and the latent level parameter for the US term structure (L_USA).

Figure 1 provides the SWC panel and WCPD heatmap, revealing, among others, the lead-lag patterns of interrelation of the demand shocks in the price of oil and the US term-structure level factor L_USA.

For the pair L_USA-DemandShock, coherence varies between low and high along the analyzed 1995–2020 period. For the below-1-month relatively high frequency band, the
left panel of Fig. 1 is mostly blue, implying low coherence for the entire time span of our analysis. Still, the 6-months-plus frequency band is characterized by the predominance of red tonalities, implying that coherence augments with the investment horizon. Numerous blue spots in the reddish domain of low frequencies reveal the respective low coherence zones, indicating potentially attractive diversification attributes.

As per Fig. 1 left panel, within the 1-to-4-days frequency band, we observe several spaced-in-time piles of diversely oriented arrows. The most pronounced are the two piles of the ↗ arrows, corresponding to the GFC and Covid-19 meltdown periods, indicating that the movements of the two time-series are mostly in-phase although not completely, as the innovations in the level factor of the US yield curve lead the variations in the demand risk shock in oil prices. For the same frequency band, we also observe a pile of the up-and-left-directed arrows, corresponding to the oil plunge in 2014, indicating that the two time-series are largely in anti-phase with the variations in the level factor of the US yield curve lagging behind the innovations in the demand risk shock in oil prices. It indicates that the investors’ expectations of the long-term interest rates follow the drastic changes occurring in the crude oil market.

Figure 1, right panel, depicts the patterns of leads and lags relative to the interrelation of the L_USA and DemandShock time series. Analyzing this heatmap, we observe four relevant aspects. For the 9-to-18-months frequency band, we observe the two predominantly blue anti-phase regions: from 1995 to 2003 and from 2009 onwards, clearly evidencing a lead by the DemandShock over the L_USA factor. This implies that the aggregate demand, revealing itself through the crude oil demand, drives alterations of the US long-term yields’ level, with the exception of the 2004–2008 period, which covers the final stage of the “irrational exuberance of markets” and the subsequent GFC.

Figure 2 provides the SWC panel and WCPD heatmap, revealing, among others, the lead-lag patterns of interrelation of the supply shocks in the price of oil and the US term-structure level factor L_USA.

For the L_USA-SupplyShock pair, similarly to the L_USA-DemandShock case relation, the ever-present variations in coherence are observed for the whole time–frequency space. For the below-1-month frequency band, the SWC panel is mostly blue, implying a low coherence, while the 6-months-plus frequency band is characterized by the predominance of red tonalities, which represent high coherence levels. As in the previous case, we also observe a recurrence of blue low-coherence spots within the high coherence zone.

![Wavelet Coherence between L_USA and SupplyShock](image)

**Fig. 2** Wavelet analysis: US curve level factor (L_USA) and supply oil shocks (SupplyShock)
However, now we do not register any causality relation between the two time-series around the 2014 oil plunge. This is expectable as the origins of this oil price collapse were not linked to specific supply problems of operation origin. Therefore, while oil price shocks are disentangled, the supply shocks do not represent any relevance for oil price movements, nor they are able to cause any alterations of the US level factor.

The right panel in Fig. 2 depicts the patterns of leads and lags relative to the interrelation of the L_USA and SupplyShock time series. The overall pattern of this panel resembles the pattern of the WCPD L_USA-DemandShock heatmap of Fig. 1. However, now we observe the two distinctive features. First, the red region at around 1-year-long investment horizons along the GFC and the recovery in 2008–2009 (see Fig. 1), indicating a lead of the level factor L_USA over the DemandShock is now absent, giving place to the dark blue region, which evidences that the moves in the L_USA factor lag behind the SupplyShock movements. These observations highlight interesting diversification features potentially useful for designing hedge strategies workable through global crises.

Second, during the Covid-19 pandemic year of 2020, we observe, at around 1-year long investment horizon, a pronounced red zone, indicating the lead of the level factor L_USA over SupplyShock time series. Considered jointly with the first observation, this finding evidences that the lead-lag relations between the L_USA-SupplyShock, have changed from lead by the SupplyShock during the GFC to L_USA lead during the Covid-19 economic and financial meltdown. We ascribe the level-factor lead through the pandemic year to the fact that the Covid-19 contingency measures put oil prices at historical lows in April, 2020, making the whole oil industry seem way too vulnerable, both demand and supply side, so that both, demand and supply shocks were led by the long-term level of interest rate expectations.

Figure 3 provides the SWC panel and WCPD heatmap, revealing, among others, the lead-lag patterns of interrelation of the risk shocks in the price of oil and the US term-structure level factor L_USA.

The overall L_USA-RiskShock patterns exhibited in both panels of Fig. 3 are similar to those of the heatmaps of Figs. 1 and 2. Herein, once again, we report on differing degrees of coherence, changing between low and high levels for the whole timespan of our study. For the below-1-month frequency band, the SWC panel exhibits a mostly blue low-coherence tonality. However, even within the red 6-months-plus frequency band we observe a recurrence
of blue spots corresponding to low coherence zones, indicating attractive and potentially explorable diversification attributes.

We also undertake an identification of phase-differences and causality patterns subjacent to the L_USA and RiskShock time series. In the left panel, see Fig. 3, for the 1-to-4-days frequency band, we observe several spaced-in-time piles of diversely oriented arrows, indicating regions of statistically significant causation patterns. This means that risk shocks, orthogonal to both demand and supply innovations, might result in observable causation relationships with the US yield-curve level factor, linked to long-term investors’ expectations that cause changes in the current levels of uncertainty in financial and oil markets, which reveal themselves as uncertainty-driven, i.e., risk shocks.

The right panel in Fig. 3 depicts the patterns of leads and lags relative to the interrelation of the L_USA and RiskShock time series. Apart from the already familiar overall pattern, in this panel we observe the two important features, different from the L_USA-DemandShock and L_USA-SupplyShock panels. First, for the 6-to-18-months frequency band for the entire studied period, the predominant color of the WCPD heatmap is green, evidencing an in-phase behavior of the two time-series, and indicating that the individual innovations are likely driven by a common source, being an uncertainty regarding the long-term expectations. Second, in 2019 and 2020, for the frequency band between 18 and 24 months, we observe the red-colored region evidencing a lead of the US level factor over the risk shocks in the crude oil prices. This observation shows up simultaneously with the prolonged nosediving of the price of the crude since the second half of 2018 until the bottom reached in the middle of Covid-19 meltdown in April 2020.

6.2 The US yield-curve slope factor (S_USA) and the demand, supply, and risk shocks in oil prices

In this sub-section we present our findings regarding the interrelation of the latent US yield-curve slope factor (S_USA) and the three orthogonal categories of oil price shocks.

Figure 4 provides the SWC panel and WCPD heatmap, revealing the lead-lag patterns of interrelation of the demand shocks in the price of oil and the US term-structure slope factor S_USA.

![Wavelet analysis: US curve slope factor (S_USA) and demand oil shocks (DemandShock)](image)
For the pair S_USA-DemandShock we observe, in the left panel of Fig. 4, never-ceasing variations in a degree of coherence. However, the blue, green, and red colors dominating, respectively, the bottom, middle, and the top of the panel indicate that, in general, coherence augments with the investment horizon. Moreover, the alternating coherence patterns reveal potentially attractive diversification attributes of this pair of variables. Interpreting further the left panel, see Fig. 4, for the 1-to-4-days frequency band, we observe several spaced-in-time piles of diversely oriented arrows, indicating regions of statistically significant causation patterns. This means that demand innovations, which might result in an observable causation relationship with the slope yield factor, are linked to the steepness of the US term structure, being but the contrasting gap between the short-term and medium-term investors’ expectations. This seems to be plausible as demand is expected to be influenced by the short-term perspectives relative to financial and commodity markets.

The right panel of Fig. 4 depicts the patterns of leads and lags relative to the interrelation of the S_USA and DemandShock time series. For the 6-to-15-months frequency band, in the middle of the panel, one could observe a blue anti-phase zone, clearly evidencing a lead by the DemandShock over the S_USA factor through the GFC. This is an interesting finding, this implies that the aggregate demand revealing itself through the crude oil demand drives changes not only in the long-term level of the US interest rates (L_USA; see WCPD panel of Fig. 1), but also governs variations in the short-term expectations of yield rates as captured by the WCPD panel, presented in Fig. 4.

Figure 5 presents the SWC panel and WCPD heatmap, revealing the lead-lag patterns of interrelation of the supply shocks in the price of oil and the US term-structure slope factor S_USA.

For the S_USA-SupplyShock pair in the SWC heatmap, we newly observe persistent variations in the degree of coherence, which increases with the investment horizon. However, differently from the previous cases, for the 1,5-year-plus range of frequencies, in the middle of the SWC heatmap, see Fig. 5, we observe a cloud of mostly upward-directed arrows, implying a statistically significant lead by the US slope factor over the SupplyShock throughout the GFC. It means that during the periods of higher uncertainty, the changes of steepness of the US yield curve influence the supply side of the crude oil industry.
We come to the same conclusion by visualizing the respective white contour in the right heatmap of Fig. 5, which identifies the patterns of leads and lags for the S_USA and SupplyShock time series. In general traits, the overall color pattern of this panel is green as multiple intermittent red and blue regions are self-cancelled out, giving place for a rather synchronous behavior of these two time-series of data. Such in-phase dynamics does not classify this pair of factors as a prominent base for hedge strategy design.

Figure 6 provides the SWC panel and WCPD heatmap, revealing the lead-lag patterns of interrelation of the risk shocks in the price of oil and the US term-structure slope factor S_USA.

The left heatmap of Fig. 6 for the 1-to-4-days frequency band, exhibits several spaced-in-time piles of arrows always pointing to the right and, thus, meaning that when there exists a statistically significant relationship between the two time-series, in this case, they are always synchronized, revealing an in-phase behavior, especially since the 2011 onwards and throughout the Covid-19 pandemics. It is plausible that moves in the slope factor, a sensitive measure of the steepness of the term structure, are aligned with the moves in risk shocks on occasions of major changes in the oil market.

The right panel of Fig. 6 depicts the patterns of leads and lags relative to the interrelation of the S_USA and RiskShock time series. Generally speaking, the overall pattern of this panel resembles the pattern of the corresponding WCPD L_USA-RiskShock heatmap. We observe fractal-like structures, resembling from-top-to-bottom trees, which reveal complex patterns of phase-difference metrics subjacent to the considered pair of variables. Our observations highlight interesting diversification features potentially useful for designing hedge strategies workable through both normal market conditions and global crises.

6.3 The US yield-curve curvature factor (C_USA) and the demand, supply, and risk shocks in oil prices

Now we present our findings regarding the interrelation of the latent US yield-curve curvature factor (C_USA) and the three orthogonal categories of oil price shocks.

Figure 7 presents the SWC panel and WCPD heatmap, revealing the lead-lag patterns of interrelation of the demand shocks in the price of oil and the US term-structure curvature factor C_USA.
For the pair of variables C_USA-DemandShock, we once again observe ever-present variations in a degree of coherence, whose general trend is, however, an increase with the investment horizon. The left heatmap of Fig. 7 for the 1-to-4-days frequency band, exhibits the two piles of the ↗ and ↘ arrows, indicating that the factor C_USA leads the demand shocks during the escalation of the GFC, whereas along the early phase of recovery out of the GFC low, the lead-lag relationship inverts, when demand shocks lead the latent curvature factor of the US yield curve.

The right panel of Fig. 7 depicts the patterns of leads and lags relative to the interrelation of the C_USA and DemandShock time series. For the 1-year-plus frequency band, since the GFC onwards, we observe a blue anti-phase zone, clearly evidencing a lead by the DemandShock over the C_USA factor. This implies that the aggregate demand, revealing itself through the crude oil demand, drives changes in all three latent shape-defining parameters of the US term structure.

Figure 8 exhibits the SWC panel and WCPD heatmap, revealing the lead-lag patterns of interrelation of the supply shocks in the price of oil and the US term-structure curvature factor C_USA.
At this point, we focus on identifying phase-differences and causality patterns subjacent to the C_USA and SupplyShock time series. The left heatmap of Fig. 8 for the 1-to-4-days frequency band exhibits several spaced-in-time piles predominantly of the ↗ arrows, meaning that there exists an in-phase relationship, while the supply shocks lead the latent curvature factor time series.

Now we turn to the analysis of the WCPD diagram of Fig. 8. In general traits, the overall color pattern of this panel is green as several intermittent red and blue regions are self-cancelled out giving place for a rather synchronous behavior of these two time-series of data. However, interestingly enough, in the top right-hand quadrant we observe the neat borders between the red in-phase and blue out-of-phase zones, clearly indicating the presence of lead-lag regime switching behavior, especially for 4-months-plus investment horizons. Such in-phase dynamics does classify this pair of factors as a prominent base for hedge strategy design.

Figure 9 provides the SWC panel and WCPD heatmap, revealing the lead-lag patterns of interrelation of the risk shocks in the price of oil and the US term-structure curvature factor C_USA.

The panels of Fig. 9 allow identifying the phase-difference and causality patterns subjacent to the C_USA and RiskShock time series. The left heatmap for the 1-to-4-days frequency band, presents several spaced-in-time piles of arrows always pointing to the left and, thus, meaning that when there exists a statistically significant relationship between the two time-series, in this case, they behave in anti-phase, especially since the GFC onwards and throughout the Covid-19 pandemics. It is plausible that moves in the curvature factor, representative of the middle-term shape of the term structure, are aligned, although in an opposite mode, with the moves in risk shocks on occasions of major changes in the market conjecture. The stronger the risk shocks, the lower the curvature, and vice versa.

The right panel of Fig. 9 depicts the patterns of leads and lags relative to the interrelation of the C_USA and RiskShock time series. In this case, we also observe fractal-like structures, resembling from-top-to-bottom trees, which reveal complex patterns of phase difference between the pair of the analyzed time-series. Our observations highlight interesting diversification features potentially useful for designing hedge strategies workable through both normal market conditions and global crises. It is especially worth noting that in 2002–2004
for investment horizons above 9 months, we observe a pronounced red in-phase zone, indicating that the curvature factor C_USA leads the time series of risk shocks. Perhaps, post 2001 recession, the prices of oil were quite depressed, below 60 USD per barrel, and, apparently, under such conditions the riskiness of oil markets comes from the riskiness of the financial markets, in general, and form the US bond market in particular. Further research into this topic is clearly desirable.

6.4 Comprehensive remarks

In this subsection, we provide a synthesis of our findings and discuss their implications for portfolio management and regulatory oversight professionals.

For all considered pairs of orthogonal oil price shocks and the yield-curve shaping factors, we document a general trait of coherence augmenting with the investment horizon. However, even within the time–frequency spaces of high coherence, we observe recurrent blue spots with green-to-yellow aureoles, signaling the existence of low coherence zones in the domain of long investment horizons. Further research of these patterns seems to be desirable in order to understand the subjacent diversification attributes and mechanisms at play, including those of behavioral nature.

Among the oil price shocks, the most persistent causality relations with all three yield-curve shaping factors we identify for the demand-driven shocks, which are found to lead all shaping factors from the GFC onwards. This implies that the aggregate demand, revealing itself through the crude oil demand, drives changes in all three latent shape-defining parameters of the US term structure. This finding is potentially useful for market players and portfolio managers gauging the appeal to invest in sovereign debt, distributing their debt holdings along the US sovereign yield curve. In particular, we find that the latent level factor seemingly possesses salient attractive hedging attributes, especially when combined with the demand side investments in crude oil. Hence, we emphasize the potential of this financial-plus-oil factors tandem for downside risk hedge strategies, which is based on their predominant anti-phase relationship, which has been observed to remain through the Covid-19 pandemic fueled meltdown, especially for 1-to-1.5-year-long investment horizons. Moreover, several observations outlined in the previous sub-sections highlight interesting diversification features potentially useful for designing hedge strategies workable through global crises.

We also demonstrate that the latent slope factor also reveals some attractive hedging attributes, especially when combined with the demand side investments in crude oil. The potential of this financial-plus-oil factors pair for downside risk hedge strategies is also based on their predominant anti-phase relationship, especially noticeable during the GFC. However, the slope factor and the demand-driven oil shocks are synchronized throughout the Covid-19 pandemic, as evidenced by the respective green zone in the corresponding phase-difference heatmap (Fig. 4). This aspect alerts that diversification attributes of this pair should be studied more deeply in order to serve as a base for workable hedge strategies throughout crises, depending on their endogenous (sub-prime problems) or exogenous (coronavirus) cause vis-à-vis the global financial system. In addition, we observe an interesting interplay of patterns between the slope factor and the supply- and risk-driven shocks. Therefore, based on our findings, we consider the slope factor of the US yield curve to be an eligible candidate for hedge strategies design, especially when considered jointly with all three types of innovations in the oil price.

In respect to the yield-curve curvature factor, we document that the demand- and supply-driven shocks influence the intertemporary dynamics of interest rate expectations, revealed
through the change of the shape of the term structure, i.e., curvature, which is duly captured in the respective phase-difference heatmap (Figs. 7, 8). The latent curvature factor seemingly possesses certain attractive hedging attributes, especially when combined with the demand side and/or supply side investments in crude oil. The potential of these two tandems composed of financial-plus-oil factors for downside risk hedge strategies are based on their pairwise predominantly anti-phase relationship, especially noticeable since the GFC onwards and also present through the Covid-19 pandemic year. Diversification attributes of these pairs should be studied more deeply in order to serve as a base for workable hedge strategies in the periods of global crises. In particular, the exact nature of the subjacent causality relationship should be studied further in order to result in workable insights on this subject.

6.5 Hedging and portfolio implications

For the sake of highlighting the economic significance of our findings, we assess the hedging and portfolio implications of our analysis for an investor who can invest in oil and 10-year US treasuries. We use the S&PGSCI oil futures index as a proxy for oil and the yield on 10-year government bonds as proxy for US treasuries. We compute the optimal hedge ratios for an investor who takes a long position in one asset and a short position in the other. Following (Kroner & Sultan, 1993, and Kroner & Ng, 1998), we construct hedge ratios as

\[ \beta_{ob,t} = \frac{h_{ob,t}}{h_{bb,t}} \]  

where \( h_{ob,t} \) denotes the conditional covariance of oil futures and 10 years treasury bonds and \( h_{bb,t} \) is the conditional variance of the 10-year treasury bonds. Figure 10 shows the time-varying hedge ratios for an investor who is long/short in oil/bond (first panel) and bond/oil (second panel). We observe that the hedge ratios exhibit a lot of volatility during the sample period. In addition, we notice that the hedge ratios are sizable, implying the hedging capacity of each asset for the other asset under consideration. Another interesting feature is the negative hedge ratios for the most part of the sample period, implying that an investor should take long positions in both assets rather than taking a short position in one asset and a long position in the other asset. The negative hedge ratios signify the diversification potential of these assets against each other. The hedge ratios obtained in this section underscore our findings of the wavelet analysis, discussed above.

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7 We thank an anonymous referee for useful comment.
Next, we can quantify the portfolio weights as

\[ \omega_{ob,t} = \frac{h_{bb,t} - h_{oj,t}}{h_{oo,t} - 2h_{ob,t} + h_{bb,t}} \]  

(9)

Such that

\[ \omega_{ob,t} = \begin{cases} 
0, & \text{if } \omega_{ob,t} < 1 \\
\omega_{ob,t}, & \text{if } 0 \leq \omega_{ob,t} \leq 1 \\
1, & \text{if } \omega_{ob,t} > 1 
\end{cases} \]  

(10)

\( \omega_{ob,t} \) is the weight of 10-year treasury bonds in one dollar portfolio of oil futures and the treasury bonds. Thus, \( 1 - \omega_{ob,t} \) is the weight of the oil futures. The conditional variance and covariance are calculated by employing a DCC model.

Figure 11 reports the time-varying optimal portfolio weights for the oil and bond for the investor. The first panel shows the weight of the bonds and the second panel shows the weight of oil futures (obtained by subtracting the weight of the bonds in the first panel from 1).\(^8\)

We notice that a sizable optimal weight for bonds in Fig. 11, thus underscoring the hedging attributes of bonds for oil.

In order to document the effectiveness of the above hedging strategy, we compute the hedging effectiveness as follows:

\[ HE = \frac{h_{oo,bb} - h_{\beta,w}}{h_{oo,bb}} \]  

(11)

where \( h_{ee,aa} \) represents the variance of the unhedged position for an asset and \( h_{\beta,w} \) represents the hedged position variance. Table 3 reports the summary statistics of our strategy. The last column of Table 3 reports the hedging effective. As expected, we notice that US bonds exhibit very higher effectives (0.98) for hedging oil position, whereas the hedging effective of oil for US bonds is minimal.

Wrapping up, based on our findings, we consider US yield curve and its curvature factor to be an eligible candidate for hedge strategy design, especially when considered jointly with three types of shocks in crude oil prices as well as the crude oil prices.

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\(^8\) A weight of 1 for one asset (zero for other asset) implies minimal hedging benefits.
Table 3 Statistics of the optimal portfolio

|          | Mean | Std. Dev. | 5%  | 95%  | HE   |
|----------|------|-----------|-----|------|------|
| Oil/Bond | 0.05 | 0.04      | 0.01| 0.1  | 0.98 |
| Bond/Oil | 0.95 | 0.04      | 0.9 | 0.99 | 0.07 |

This table reports the mean, standard deviation (Std. Dev.), 5th and 95th percentiles of the optimal portfolio weights for an investor who is long in the first asset and short in the second asset (long/short). We also report the hedging effectiveness (HE).

7 Conclusion

This paper investigates the interrelation of the oil demand, oil supply, and uncertainty-driven risk-related innovations in the price of crude and the yield curve in the US between 1995 and 2020. We use squared-wavelet-coherence and the respective phase-difference methodologies. We empirically evidence an inverse relation between coherence and investment horizon: low coherence for high frequencies below 1 month and high coherence for low frequencies above 6 months. However, even within the high coherence range, we observe regions of low coherence for diverse readings on the time and frequency axes for the considered time series of the latent shape-defining parameters of the US term structure and for the three structured types of shocks in oil prices. The low coherence intervals potentially allow for benefitting from and may constitute basilar pillars for the creation of hedge strategies, inclusively those workable during global catastrophes, e.g., the GFC and the current pandemic. In addition, we clearly notice important divergences in coherence patterns, subjacent to the yield factors for the three considered herein types of oil price shocks, namely demand-, supply-, and risk-driven.

Our research contributes to the advancement of knowledge on the subject in the three following ways. First, we bridge a considerable void, consisting in insufficiency and absence of scientific studies, related to the analyses of dynamic maps focused on the interrelation of oil shocks and the shape of the US yield curve. It is the first time when the wavelet analysis is applied to address the changes of the shape of the US yield curve in the wake of the GFC and the Covid-19 vis-a-vis the dynamics of crude oil prices. Second, we contribute to the literature on the response of the US bond market to the Covid-19, which triggered an abysmal drop in crude oil prices. It is worth noting that our historical data sample spans over the pandemic-triggered crisis of 2020. Therefore, beyond the investors’ community, our findings may be potentially insightful for financial markets’ regulators assessing the suitability of yield-curve control policies to promote financial stability. Third, we evidence the high, medium, and low levels of coherence between the demand, supply, and risk-related components of oil price movements and the latent yield-curve factors for the US sovereign debt curve.

In particular, we observe and document differences in the coherence patterns on a per-type-of-shock basis. For all yield factors, we observe the lead-lag regime switching revealing their attractiveness for hedging. This finding serves as an indication that our results may be useful for cross-asset hedge strategies for commodity and fixed-income markets, especially the US bond market, and supports the usage of investments in the governmental debt of the United States by investors pursuing diversification on a cross-section of crude oil and bond markets, aimed at mitigation of downside risks. The findings of our research provide evidence-based implications for the investment communities, portfolio managers, market regulators, and subsequent investigation projects. For instance, portfolio managers and professional
investors could employ our findings in the designing of cross-yield-curve-factors and cross-asset hedge strategies, potentially resilient and capable of withstanding the effects of global calamities, as evidenced by both, Global Financial Crisis and the global pandemic turmoil. In this way, financial institutions of a diverse nature may benefit from our results, potentially allowing to delineate their risk profiles in a more accurate manner, especially in what concerns their crude oil and US Treasuries portfolios. Viewed from the perspective of the regulatory oversight of the considered government bonds and oil markets, our findings constitute a relevant effort to assist policy-makers in defining the content and implementation timing of crisis-contingent policy solutions aimed at the reduction of volatility in financial markets crossing highly uncertain periods of global turmoil.

Additionally, we highlight the wavelet approach underlying our results, which allows for investigating the pairwise interrelations within the chosen tandems of the variables across diverse investment horizons by making use of the time and frequency dimensions, and enriching in this manner the informational output, potentially available to concerned regulators and policy makers. While the heterogeneous risk perceptions underlie distinctive decisions of traders and investors within their investment activities, the dimension of investment horizon or frequency permits the regulatory bodies to better assess the possible reactions of market players to proposed restrictions and contingency policies. The wavelet methodology makes it possible to track probable, hypothetical, and then real-world responses simultaneously at time and frequency scales, potentially offering relevant insights, which take into account intricate time-varying and investment-period-dependent trends, patterns, and interactions. And last but not least, differently from common time-series analyses, the wavelet approach is efficient in determining the existing lead and lag pairwise interrelations of non-linear arrays of historical data.

And finally, a possible continuation of our studies may be focused on the extending their coverage, on applying alternative techniques, and on measuring the portfolio implications of including investments in US Treasuries and in crude oil in a framework of portfolio optimization.

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