Computation Environments (1)

An Interactive Semantics for Turing Machines
(which P is not equal to NP considering it)

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Abstract

To scrutinize notions of computation and time complexity, we introduce and formally define an interactive model for computation that we call it the computation environment. A computation environment consists of two main parts: i) a universal processor and ii) a computist who uses the computability power of the universal processor to perform effective procedures. The notion of computation finds its meaning, for the computist, through his interaction with the universal processor.

We are interested in those computation environments which can be considered as alternative for the real computation environment that the human being is its computist. These computation environments must have two properties: 1- being physically plausible, and 2- being enough powerful.

Based on Copeland’s criteria for effective procedures, we define what a physically plausible computation environment is.

By being enough powerful, we mean that the universal processor of the computation environment, must be in a way that the computist can carry out (via the universal processor) all logical deductions that the human being can carry out.

We construct two physically plausible and enough powerful computation environments: 1- the Turing computation environment, denoted by $E_T$, and 2- a persistently evolutionary computation environment, denoted by $E_e$, which persistently evolve in the course of executing the computations.

We prove that the equality of complexity classes P and NP in the computation environment $E_e$ conflicts with the free will of the computist.

We provide an axiomatic system $T$ for Turing computability and prove that ignoring just one of the axiom of $T$, it would not be possible to derive P = NP from the rest of axioms.

We prove that the computist who lives inside the environment $E_T$, can never be confident that whether he lives in a static environment or a persistently evolutionary one.

1 Introduction

We distinguish between the syntax and semantics of Turing machines (following D. Goldin, and P. Wegner, see 5.2, of [16]), and scrutinize what the computation is from an interactive view. We study the computation as a result of interaction between a computist and a computer. To do this, We propose a notion called the ‘computation environment’, as an interactive model for computation which regards the computation as an interaction between

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1It is the first draft of the article. The English language is not yet perfect. This article is the first paper of some serial related articles which would be appeared later.
the human being as a computist and a processor unit called the universal processor. In this way, Turing machines (which are proposed by Turing as a model of computation) would find their meanings based on the interaction of the human being and the universal processor.

A computation environment would consist of two parts: 1- a universal processor unit and 2- a computist who uses the universal processor to perform computation.

Based on Copeland’s Criteria for effective procedures, we define physical plausibility as a property for computation environments, and attempt to construct computation environments which could be taken as alternatives for the computation environment of the real world. That is, we are interested in those computation environments that are physically plausible and if the human being lives in one of them then the computability power of the environment enables him to carry out logical deductions.

The famous problem P versus NP is a major difficult problem in computational complexity theory, and it is shown that lots of certain techniques (such as diagonalization arguments and relativization which are used to show that one complexity class differs from another one) are failed to answer to this problem (see \[27, 1, 2\]). However, for some hypercomputations (see appendix 9.1), it is proved that \( P \neq NP \).

Hypercomputation extends the capabilities of the Turing computation via using new resources such as 1) oracle information sources 2) infinite specification, 3) infinite computation and 4) the interaction (see the appendix 9.2). Among these four resources, the resources 1, 2, and 3 do not seem physically plausible as they have infinite structures. But the forth one, the interaction, seems physically plausible.

It is proved for hypercomputation models which use the resources as oracle information, and infinite computation, P is not equal to NP.

Oracle information source: an oracle machine is a Turing machine equipped with an oracle that is capable of answering questions about the membership of a specific set. It can be shown that \( P \neq NP \) for oracle Turing machines considering a specific oracle set (see page 74, of [5]).

Infinite computation: it is shown that \( P \neq NP \) for infinite time Turing machine [29]. Infinite time Turing machines are a natural extension of Turing machines to transfinite ordinal times, and the machine would be able to operate for transfinite number of steps [20].

In this paper, we aim to study the famous problem P vs NP considering the interaction resource. We introduce a new kind of Hypercomputation called persistently evolutionary Turing machine. A persistently evolutionary Turing machine is a Turing machine that its inner structure may persistently evolve during the computation. Persistently evolutionary Turing machines can be considered as a model of interactive computation [17] and should be compared with Persistent Turing machines independently introduced by Goldin and Wegner [18] and Kosub [21]. We are not going to prove that \( P \neq NP \) for persistently evolutionary Turing machines similar to what is proved about infinite time Turing machines in [29]. Instead, we take advantage of persistently evolutionary Turing machines and construct a computation environment \( E_e \) which its universal processor is a persistently evolutionary Turing machine, and in this environment P is not equal to NP. In this way, we present an interactive semantics for Turing machines and we prove P is not equal to
NP taking this semantics. Our work should be considered in the literature of interactive computation [17] which is a new paradigm of computation.

We construct a computation environment that we call it Turing computation environment $E_T$. We prove that the computationally complexity classes P and NP of this environment are exactly the same complexity classes P and NP for Turing computation. We prove that the computist who lives inside the environment $E_T$ can never be confident that whether he lives inside a static environment or a persistently evolutionary one.

The paper is organized as follows:

In Section 2 we briefly review the definition of Turing computation and time complexity, and ask some questions which their answers inspire us to distinguish between syntax and semantics of Turing machine.

In Section 3 we define formally what a computation environment is, and we introduce the Turing computation environment that the notions of computability and time complexity in this environment are exactly as the same as they are for Turing machines.

In Section 4 a kind of hypercomputations called persistently evolutionary Turing machines are introduced. Persistently evolutionary Turing machines are interactive machines, and it is shown that they are as physically plausible as Turing machines are.

In Section 5 a persistently evolutionary computation environment is introduced and proved that the equality of two complexity classes P and NP, in this environment, conflicts with the free will of the computist.

In Section 6 we provide an axiomatic system $T$ for Turing computation, and it is shown that ignoring one of the axioms of $T$, it is not possible to derive $P = NP$ from the rest of axioms.

2 Semantics vs. Syntax

In this section, we briefly review the standard definition of Turing machines and time complexity in the literature. Then we ask and answer a question about who means and executes a Turing machine.

**Definition 2.1** A Turing machine $T$ is a tuple $T = (Q, \Sigma, \Gamma, q_0, \delta)$, where $Q$ is a finite set of states containing two special states $q_0$ (initial state) and $h$ (halting state). $\Sigma$ and $\Gamma$ are two finite sets, input and work tape alphabets, respectively, with $\Sigma \subseteq \Gamma$ and $\Gamma$ has a symbol $\Delta \in \Gamma - \Sigma$, the blank symbol. $\delta : (Q - \{h\}) \times \Gamma \rightarrow Q \times \Gamma \times \{R, L\}$ is a partial function called the transition function.

- A configuration of a Turing machine $T$ is any symbolic form $(q, xay)$ where $q \in Q$, $x, y \in \Gamma^*$, and $a \in \Gamma$.

- A configuration $C$ is a successful configuration whenever $C = (h, x\Delta)$ or $C = (h, \Delta x)$. 

- A computation path of a Turing machine $T$ on a string $x$ is any finite sequence $C_0C_1...C_n$ of configuration which is started with $C_0 = (q_0, \Delta x)$ and each $C_{i+1}$ is obtained from $C_i$ by applying the transition function $\delta$.

- We say a Turing machine $T$ accepts a string $x$, if there is a computation path $C_0C_1...C_n$, for some $n \in \mathbb{N}$, where $C_0 = (q_0, \Delta x)$, and $C_n$ is a successful configuration. We define $L(T) \subseteq \Sigma^*$ to be the set of all strings accepted by $T$.

- The time complexity of computing $T$ on $x$, denoted by $\text{time}_T(x)$, is the number of configurations which appears in the computation path $C_0C_1...C_n$, for some $n \in \mathbb{N}$, where $C_0 = (q_0, \Delta x)$, and $C_n$ is a successful configuration.

**Definition 2.2** Let $f : \mathbb{N} \rightarrow \mathbb{N}$ and $L \subseteq \Sigma^*$. We say the time complexity of the computation of the language $L$ is less than $f$ whenever there exists a Turing machine $T$ such that $L(T) = L$, and for all $x \in L$, $\text{time}_T(x) < f(|x|)$.

**Definition 2.3** The computational complexity class $P \subseteq 2^{\Sigma^*}$ is defined to be the set of all languages that their time complexity is less than a polynomial function. We also define the complexity class $NP \subseteq 2^{\Sigma^*}$ as follows:

$$L \in \text{NP} \iff \text{there exists } J \in \text{P and a polynomial function } q \text{ such that for all } x \in \Sigma^*,$$

$$x \in L \iff \exists y \in \Sigma^*(|y| \leq q(|x|) \land (x, y) \in J).$$

We start by asking and answering some questions, and inspired by the questions, we propose the notion of computation environment.

**Question 2.4** Who executes and means the computation?

If we ask you who executes an algorithm written in a computer programming language, say C++, you may simply answer that the CPU (central processor unit) of the computer. Now we ask who executes the computation in the real world?

Consider the Turing machine

$$T = (Q, \Sigma, \Gamma, q_0, \delta),$$

where

$Q = \{q_0, q_1, h\}$, $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, \Delta\}$,

$$\delta = \{(q_0, \Delta) \mapsto (q_1, \Delta, R), (q_1, 0) \mapsto (q_1, 1, R), (q_1, 1) \mapsto (q_2, 1, L), (q_2, 0) \mapsto (h, 0, l)\} \cup.$$

- Who means the string of alphabets written between $\triangledown$ in above?

Consider a configuration $C = (q_1, \Delta 001101\Delta)$,

- who means and executes the transition function $\delta$ on the configuration $C$, and provides the configuration $C' = (q_2, \Delta 001101\Delta)$?

Turing describes an effective procedure as one that could be performed by an infinitely patient ideal mathematician working with an unlimited supply of paper and pencils [25]. Therefore, Turing knows a Turing machine as one that is carried out by the human being.

We may answer above questions in two different ways based on what kind of entity one may assumes a computing machine is:
1- it is a mental and subjective entity that the human being means it, and the brain of the human being as a universal processor executes it, or

2- it is a mechanical entity that the nature as a universal processor executes it and the human being is an observer of this execution.

In the first view, assuming a Turing machine $T$ and an arbitrary string $x \in \Sigma^*$, it is the brain of the human being (as a universal processor) who applies the machine $T$ on input $x$. If a Turing machine $T$ and a string $x$ are given to the human being, then he applies $T$ on $x$ having an unlimited supply of paper and pencils. He counts the number of times that he moves his pencil to left or right as the time complexity that he consumes on this computation. The human being does not have access to the brain, and ignores the amount of mental processes that happens inside it. Time complexity is defined to be the number of pencil moves not the amount of mental processes.

In the second view, given a Turing machine $M$ and an arbitrary string $x \in \Sigma^*$, it is the nature (as a universal processor) who executes the machine $T$ on input $x$. The human being just is a computist who knows what a Turning machine is and sees that the nature behaves well defined, that is, the computist cannot observe any differences between two execution of $T$ on $x_0$ that the nature may complete in different periods of time. Note that the nature, in order to perform one transition of a Turing machine $T$ on a string $x$, may need to do lots of mechanical processes that could be invisible for the computist, and the computist ignores them and he does not count them as the time complexity.

The reason that the human knows the Church-Turing statement to be a thesis, is that the resource which executes effective procedures is a black for him. The human being takes the resource of computation (the brain or the nature) as a black box, and does not consider the inner structure and the internal action of the resource, in formalizing the notions of computability and time complexity.

By above arguments, we do not consider Turing machines as autonomous and independent entities. They are subject to a universal processor that could be the human's brain or the nature. Nobody can build a Turing machine in the outer world, since a Turing machine has an infinite tape! However, if we consider Turing machines as instruction sets for a universal processor, then they find their meanings. In this way, we differ between semantics and syntax of Turing machines, following D. Goldin and P. Wegner (5.2 of [16]):

Statements about TM (Turing Machines) expressiveness, such as the Church-Turing Thesis, fundamentally depend on their semantics. If these semantics were defined differently, it may (or may not) produce an equivalent machine. Kugel uses the terms machinery and machines to describe the same distinction between what a machine contains and how it is used (Kugel, 2002). He points out that if used differently, the same machinery results in a different machine. An example of a model that shares TMs syntax (machinery) but has different semantics are Persistent Turing Machines; its semantics are based on dynamic streams and persistence. (page 19 of [16])

There are lots of formalization of the notion of computation, like Church $\lambda$ calculus, Turing machines, and Markov algorithms and etc (see, Chapter 8 of [15]). What all formalization for computation have in common is that they are purely syntactical. Mostowski says (see page 70 of [15]):
However much we would like to “mathematize” the definition of computability, we can never get completely rid of the semantics aspects of this concept. The process of computation is a linguistic notion (presupposing that our notion of language is sufficiently general); what we have to do is to delimit a class of those functions (considered as abstract mathematical objects) for which there exists a corresponding linguistic object (a process of computation). [Mostowski, p.35]

Therefore, it seems reasonable that we distinguish between semantics and syntax of a Turing machine, and propose different semantics for it. In this way, for a Turing machine, we face with two things:

1) the **syntax**: what does a Turing machine consist of?, and
2) the **semantics**: how is a Turing machine executed?

The syntax of a Turing machine is a finite set of basic transitions coded in a finite string, and its semantics prescribes that how a the computation starts from a configuration, transits to other configurations and finally stops.

In this paper, we propose computation environments to present an interactive semantics for Turing machines. We prove that regarding this semantics P is not equal to NP.

The computation environment would have two parts:

1- a universal processor which means and executes the syntax of computation through interaction with a computist, and
2- a computist who ignores the inner structure and the internal actions of the universal processor in formalizing the notion of computability and time complexity.

**Definition 2.5** An instruction-set of a processor is a finite set of its instructions. An instruction (a command) is a single operation of a processor that the processor interacts via it with its outer.

Note that instructions differ from the internal actions that a processor may execute to perform one of its instruction. Instructions are assumed to be atomic statements that make it possible for a user to communicate (interact) with the processor.

The universal processor (nature, or brain of the computist) executes the computation of an instruction-set $M$ on an input string $x$.

Assuming the human being as a computist of the computation environment of the real world, we may say

1. Turing machines are finite instruction-sets of the universal processor of the real world, and
2. if the human being can effectively compute a function $f$ then there exists a code of a Turing machine as an instruction-set of the universal processor such that the human being can communicate through the code with the universal processor to compute the function.
3 Computation Environments

As it is mentioned in introduction, we divide a computation environment to two main parts: 1) a universal processor, and 2) a computist. In this section, we provide a formal definitions for computation environments.

In the following of the paper, we may use two terms ‘the observer’ of an environment and ‘the god’ of an environment.

- An observer of an environment is an ideal mathematician who does not have access to the inner structure of the universal processor.

- A god of an environment is an ideal mathematician who has access to the inner structure of the universal processor.

We start some propositions and theorems by symbol $\textbf{GV}$ to declare that they are proved by the ideal mathematician who has access to the inner structure of the universal processor.

**Definition 3.1** A universal processor is a tuple $U = (T\text{BOX}, S\text{BOX}, \text{INST}, \text{CONF})$, where

1. $\text{INST}$ is a nonempty set (assumed to be the set of all instructions of the universal processor), and $\text{INST}_0 \subseteq \text{INST}$ is a nonempty subset called the set of starting instructions.

2. $\text{CONF}$ is a nonempty set called the set of configurations such that to each $x \in \Sigma^*$, a unique configuration $C_{0,x} \in \text{CONF}$ is associated as the start configuration, and to each $C \in \text{CONF}$, a unique string $y_C \in \Sigma^*$ is associated.

Two sets $\text{INST}$ and $\text{CONF}$ consist the programming language of the universal processor that via them a computist communicates with the processor.

3. $\text{TBOX}$ (the transition black box) is a total function from $\text{CONF} \times \text{INST}$ to $\text{CONF} \cup \{\bot\}$,

4. $\text{SBOX}$ (the successful black box) is a total function from $\text{CONF}$ to $\{\text{YES}, \text{NO}\}$,

The two boxes $\text{TBOX}$ and $\text{SBOX}$ are processor units of the universal processor.

**Definition 3.2**

i. A syntax-procedure is a finite set $M \subseteq \text{INST}$ (a finite set of instructions), satisfying the following condition (called the determination condition): for every $C \in \text{CONF}$ either for all $\iota \in M$, $T\text{BOX}(C, \iota) = \bot$, or at most there exists one instruction $\tau \in M$ such that $T\text{BOX}(C, \tau) \in \text{CONF}$. We refer to the set of all syntax-procedures by the symbol $\Xi$.

ii. We let $\upsilon : \Xi \times \text{CONF} \rightarrow \text{INST} \cup \{\bot\}$ be a total function such that for each syntax-procedure $M$ and $C \in \text{CONF}$, if $\upsilon(M, C) \in \text{INST}$ then $\upsilon(M, C) \in M$, and $T\text{BOX}(C, \upsilon(M, C)) \in \text{CONF}$.
Definition 3.3 A computation environment is a pair $E = (U, O)$ where $U$ is a universal processor and $O$ is a computist which satisfy the following conditions:

\begin{enumerate}
\item The computist $O$ just observes the input-output behavior of two black boxes of the universal processor, and ignores internal actions of the universal processor (similar to what the human being does in formalizing notions of computation and time complexity)
\item The computist $O$ has a free will to do the following things in any order that he wants:
\begin{itemize}
\item he can freely choose an arbitrary instruction $i \in \text{INST}$ and an arbitrary configuration $C \in \text{CONF}$ to apply the $\text{T BOX}$ on $(C, i)$, and
\item he can freely choose an arbitrary configuration $C \in \text{CONF}$ to apply the $\text{S BOX}$ on.
\end{itemize}
\item The computist $O$ effectively defines (effectively intends) a string $x \in \Sigma^*$ to be in the language of a syntax-procedure $M$, whenever he can construct a sequence $C_0C_1, \ldots, C_n$ of configurations in $\text{CONF}$ such that
\begin{itemize}
\item $C_0 = C_{0,x}$,
\item each $C_i$, $i \geq 1$, is obtained by applying $\text{T BOX}$ on $(C_{i-1}, \upsilon(M, C_{i-1}))$,
\item the $\text{S BOX}$ outputs $\text{YES}$ for $C_n$,
\item and either $\upsilon(M, C_n) = \bot$ or $\text{T BOX}(C_n, \upsilon(M, C_n)) = \bot$.
\end{itemize}

The computist $O$ calls $C_0C_1, \ldots, C_n$ the successful computation path of $M$ on $x$. The length of a computation path is the number of configurations appeared in. We refer to the language of a syntax-procedure $M$, by $L(M)$. Note that the language $L(M)$ is dependent to the computist and is the set of all strings, say $x$, that the computist $O$, in his living in the computation environment, can construct a successful computation path of $M$ for them.

\item The computist $O$ effectively defines (effectively intends) a partial function $f : \Sigma^* \to \Sigma^*$ through a syntax-procedure $M \in \Xi$, whenever for $x \in \Sigma^*$, he can construct a sequence $C_0C_1, \ldots, C_n$ of configurations in $\text{CONF}$ such that
\begin{itemize}
\item $C_0 = C_{0,x}$,
\item each $C_i$, $i \geq 1$, is obtained by applying $\text{T BOX}$ on $(C_{i-1}, \upsilon(M, C_{i-1}))$,
\item the $\text{S BOX}$ outputs $\text{YES}$ for $C_n$,
\item and either $\upsilon(M, C_n) = \bot$ or $\text{T BOX}(C_n, \upsilon(M, C_n)) = \bot$,
\item $y_{C_n} = f(x)$.
\end{itemize}
\item The semantics of a syntax-procedure $M \in \Xi$ (the way it is executed in the environment) is determined through the interaction of the computist with the universal processor.
\end{enumerate}

\footnote{By the term free will, we mean there is no syntax-procedure $M_1$ that can simulate the behavior of the computist $O$, that is, there is no syntax-procedure $M_1$ that can recognize the ordering that the computist $O$ inputs strings to the universal processor $U$. Or in other words, the order that the computist chooses strings to input to the universal processor need not to be predetermined.}
c6. The computist may start to apply the universal processor $U$ on a syntax-procedure $M$ and a string $x$, however he does not have to keep on the computation until the successful box outputs $\text{Yes}$. The computist can leave the computation $M$ on $x$ at any stage of his activity and choose freely any other syntax-procedure $M'$ and any other string $x'$ to apply $U$ on them.

c7. The computist $O$ defines the **time complexity** of computing a syntax-procedure $M$ on an input string $x$, denoted by $\text{time}_M(x)$, to be $n$, for some $n \in \mathbb{N}$, whenever he constructs a successful computation path of the syntax-procedure $M$ on $x$ with length $n$.

c8. The inner structure of the black boxes of the universal processor may evolve through their inner processes on inputs, however they have to behave well-defined through time, and the computist cannot realize the evolution, because he does not have access to the inner part of the universal processor.

c9. Let $f : \mathbb{N} \to \mathbb{N}$ and $L \subseteq \Sigma^*$. The computist says that the time complexity of the computation of the language $L$ in the computation environment $E$, is less than $f$ whenever there exists a syntax-procedure $M \in \Xi$ such that the language defined by the computist via $M$, i.e., $L(M)$, is equal to $L$, and for all $x \in L$, $\text{time}_M(x) < f(|x|)$.

c10. The computist defines the time complexity class $\mathbb{P}_E \subseteq 2^{\Sigma^*}$ to be the set of all languages that he can effectively define in polynomial time. He also defines the complexity class $\mathbb{NP}_E \subseteq 2^{\Sigma^*}$ as follows:

\[
L \in \mathbb{NP}_E \text{ iff there exists } J \in \mathbb{P}_E \text{ and a polynomial function } q \text{ such that for all } x \in \Sigma^*, \\
x \in L \iff \exists y \in \Sigma^* (|y| \leq q(|x|) \land (x, y) \in J).
\]

c11. **Logical Deduction.** We say a computation environment is enough powerful whenever the computist can carry out logical deduction using the universal processor.

By definition of computation environments, we attempt to mathematically model the situation that the human being has with the notion of computation in the real world. In a computation environment,

a language $L$ (a function $f$) is effectively calculable if and only if there exists a syntax-procedure $M \in \Xi$ such that the computist can define the language $L$ (the function $f$) through it.

In this way, the syntax-procedures find their meaning via the interaction of the computist with the universal processor of the environment.

**Example 3.4 (The Turing Computation Environment).**

Let $Q_T = \{h\} \cup \{q_i \mid i \in \mathbb{N} \cup \{0\}\}$, $\Sigma, \Gamma$ be two finite set with $\Sigma \subseteq \Gamma$ and $\Gamma$ has a symbol $\triangle \in \Gamma - \Sigma$.

The universal processor $U_s = (TBOX_s, SBOX_s, INST_s, CONF_s)$ is defined as follows:
1) $INST_s = \{ ((q, a) \to (p, b, D)) \mid p, q \in Q_T, a, b \in \Gamma, D \in \{R, L\} \}$, 
   $(INST_s)_0 = \{ ((q, a) \to (p, b, D)) \in INST_s \mid q = q_0 \}$, and 

2) $CONF_s = \{ ((q, xa) \mid q \in Q_T, x, z \in \Gamma^*, a \in \Gamma \}$, for each $x \in \Sigma^*$, $C_{0,x} = (q_0, \Delta x)$, and for each $C = (q, xa) \in CONF_s$, $y_C = xaz$.

Note that the programming language of $U_s$ is exactly the standard syntax of configurations and transition functions of Turing machines.

3) Let $C = (q, xbx\bar{b}_2y)$ be an arbitrary configuration then

- $TBOX_s(C, ((q, a) \to (p, c, R)))$ is defined to be $C' = (p, xbxcb_2y)$,
- $TBOX_s(C, ((q, a) \to (p, c, L)))$ is defined to be $C' = (p, xbxcb_2y)$, and
- for other cases $TBOX_s$ is defined to be $\bot$.

4- Let $C \in CONF_s$ be arbitrary

- if $C = (h, \Delta x)$ then $SBOX_s(C)$ is defined to be $YES$,
- if $C = (h, x\Delta)$ then $SBOX_s(C)$ is defined to be $YES$, and
- otherwise $SBOX_s(C)$ is defined to be $NO$.

5- For each $M \in \Xi_s$, and $C = (q, xay) \in CONF_s$, if there exists $((q, a) \to (p, b, D)) \in M$ for some $p \in Q_T, b \in \Gamma, \text{ and } D \in \{R, L\}$, then $v(M, C)$ is defined to be $((q, a) \to (p, b, D))$ else it is defined to be $\bot$.

We call the computation environment $E_T = (U_s, O_f)$ (where $O_f$ is a computist with the free will) the Turing Computation Environment.

Note that each syntax-procedure $M \in \Xi_s$ of the Turing computation environment $E_T$ is exactly a transition function of a Turing machine. Let $TM = \{ T = (Q, \Sigma, \Gamma, q_0, \delta) \mid Q \subseteq Q_T \}$ be the set of all Turing machines (according to definition 2.1) that the set of their states is a finite subset of $Q_T$. Define two functions $F_1 : TM \to \Xi_s$ and $F_2 : \Xi_s \to TM$ as follows:

- for each $T \in TM$, $F_1(T) = \delta$,
- for each $M \in \Xi_s$, $F_2(M) = (Q, \Sigma, \Gamma, q_0, M)$, where $Q$ is the set of all states in $Q_T$ which appeared in the instructions of $M$.

One may easily prove that, for all $T \in TM$, if the human being computes $L(T)$ in the real world, and the computist $O_f$ computes $L(F_1(T))$ in the Turing computation environment $E_T$ then $L(T) = L(F_1(T))$.

Also, for all $M \in \Xi_s$, if the computist $O_f$ computes $L(M)$ in the Turing computation environment $E_T$, and the human being computes $F_2(M)$ in the real world then $L(M) = L(F_2(M))$.

**Theorem 3.5 (GV).**
1. For each syntax-procedure $M \in \Xi_s$, there exists a Turing machine $T = F_2(M)$ such that for every $x \in \Sigma^*$, $x \in L(M)$ and $time_M(x) = n$ iff $x \in L(T)$ and $time_T(x) = n$.

2. For each Turing machine $T$, there exists a syntax-procedure $M \in \Xi_s$, $M = F_1(T)$, such that for every $x \in \Sigma^*$, $x \in L(T)$ and $time_T(x) = n$ iff $x \in L(M)$ and $time_M(x) = n$.

**Proof.** It is straightforward.\[\]

The above theorem is true from the god’s view. The computist who lives inside the environment cannot prove the above theorem as he does not have access to the inner structure of the universal processor $U_s$. However he is always free to propose a hypothesis about its computation environment like Church and Turing did. Note that since we (the human beings) do not have access to the inner structure of our mind, we call the Church Turing statement to be a thesis. If we know that how effective procedures are executed by our mind, then we should not call the Church Turing statement to be a thesis.

**Corollary 3.6 (GV).** The time complexity classes in the Turing computation environment are exactly the same classes of Turing machines. Particularly,

1. $PE_T = P$, and
2. $NP_{E_T} = NP$.

**Definition 3.7** A universal processor $U = (T BOX, S BOX, INST, CONF)$ is called to be static, whenever the inner structure of two boxes $T BOX$ and $S BOX$ does not change due to interaction with the computist. It is called persistently evolutionary whenever the the inner structure of at least one boxes changes but persistently, i.e., in the way that the boxes work well-defined.

The universal processor of the Turing computation environment defined in the example is static, in Section We introduce a computation environment which universal processor persistently evolve.

We may construct lots of computation environments. But we are interested in those computation environments that are

- physically plausible
- enough powerful,

and thus could be considered as alternatives of the computation environment of the real world.

By enough powerful, we mean that the computability power of the universal processor must be enough high such that the the computist who lives in the environment can carry out all deductions in first order logic that the ideal mathematician can carry out in the real world.

Note that if a theory $\mathcal{T}$ in the first order logic is recursively enumerable then the set of all statements that could be logically derived from the theory is also recursively enumerable, that is, deduction is Turing computable. Hence, We consider the following definition.
Definition 3.8 A computation environment $E = (U, O)$ is enough powerful, whenever for every recursively enumerable language $L$, there exists a syntax-procedure $M$ in the environment that the computist can define $L$ through $M$, i.e., $L(M) = L$.

Proposition 3.9 The Turing computation environment $E_T$ is enough powerful.

Proof. It is straightforward. $\square$

By physical plausibility, we mean the inner structures of the black boxes have to be as physically plausible as a Turing machine is. Copeland gives four criteria that any set of instructions that makes up a procedure should satisfy in order to be characterized as an effective procedure (see the Church-Turing Thesis, Chapter 2, Page 21, [31]):

1) Each instruction is expressed by a finite string of Alphabet, that is the elements of $INST$ must be definable by finite strings.

2) The instructions are basic and produce the result in one step (or at most in a finite number of steps).

3) They can be carried out by the computist (the human being) unaided by any machinery, ignoring the internal actions of the universal processor (his mind).

4) They demand no insight on the part of the computist carrying it out (the computist just simply inputs an instruction and a configuration in the transition box, and receives the next configuration. No insight is needed).

Also a procedure has to be finite, definite (each instruction must be clear and unambiguous), and effective (every instruction (like $[(p, a) \rightarrow (q, b, R)]$) has to be sufficiently basic).

Inspired by Copeland’s criteria, we define physical plausibility for computation environments.

Definition 3.10 A computation environment $E = (U, O)$, where $U = (TBOX, SBOX, INST, CONF)$ is physically plausible whenever

a1. The inner structure of two black boxes $TBOX$ and $SBOX$ are as physically plausible as the human being knows the Turing machines are.

a2. The instructions in $INST$ can be described in finite words,

a3. The configurations in $CONF$ can be described in finite words.

Corollary 3.11 Every computation environment $E = (U = (TBOX, SBOX, INST, CONF), O)$ which

1. $INST = INST_s$,

2. $CONF = CONF_s$ (see the definition of $E_T$ in example 3.4),

See the definition of algorithm presented in page 21, [31] which is accepted by most computer scientist and engineers.
3. and the inner structure of TBOX and SBOX are physically plausible machines, is physically plausible.

**Proof.** Since the instructions in INSTs and configurations in CONFs are describable in finite strings, the items a2, and a3 of definition 3.10 is fulfilled. ⊢

**Question 3.12** Is there a physically plausible and enough powerful computation environment $E$ such that $P_E$ is not equal to $NP_E$ for the computist who uses the universal processor freely?

In the next section, we introduce persistently evolutionary Turing machines as a kind of hypercomputations. Then in a sequel, based on this hypercomputation, we construct a physically plausible computation environment that the successful box of its universal processor is a persistently evolutionary Turing machine, and $P$ is not equal to $NP$ in the environment.

## 4 Persistently Evolutionary Turing Machines

The notion of being constructive in Brouwer’s intuitionism goes beyond the Church-Turing thesis (see the appendix 9.2). In this section, inspired by Brouwer’s choice sequences, we introduce persistently evolutionary Turing machines. A persistently evolutionary Turing machine is a machine that its inner structure during its computation on any input may evolve. But this evolution is in the way that if a computist does not have access to the inner structure of the machine then he cannot recognize whether the machine evolves or not.

**Definition 4.1** Let $M_1$ and $M_2$ be two (deterministic) Turing machines, and $x \in \Sigma^*$ be arbitrary. We say $M_1$ and $M_2$ are $x$-equivalent, denoted by $M_1 \equiv_x M_2$ whenever

if one of the two machines $M_1$ and $M_2$ outputs $y$ for input $x$, then the other one also outputs the same $y$ for the same input $x$.

**Definition 4.2** A Persistently evolutionary Turing machine is a couple $N = (\langle z_0, z_1, ..., z_i \rangle, f)$ where $\langle z_0, z_1, ..., z_i \rangle$ is a growing sequence of codes of deterministic Turing machines, and $f$ (called the persistently evolutionary function) is a computable partial function from $\Sigma^* \times \Sigma^*$ to $\Sigma^*$ such that for any code of a Turing machine, say $y$, and any string $x \in \Sigma^*$, if $y$ halts on $x$ then $f(y, x)$ is defined and it is a code of a new Turing machine. The function $f$ satisfies the following condition (that we call it the persistent condition):

- for every finite sequence $(x_1, x_2, ..., x_n, x_{n+1})$ of $\Sigma^*$, and every Turing machine $y_0$, if $y_1 = f(y_0, x_1), y_2 = f(y_1, x_2), ..., y_n = f(y_{n-1}, x_n)$ and $y_{n+1} = f(y_n, x_{n+1})$ are defined then we have for all $0 \leq i \leq n$, $y_{n+1} \equiv_{x_{n+1}} y_i$.

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4Similar computing machines (dynamic machines) were presented in the author’s PhD thesis [26].
Whenever an input $x$ is given to $N$, the output of the evolutionary machine $N$ is computed according to the $z_i$ ($z_1 = f(z_0, x_1)$, $z_2 = f(z_1, x_2)$, ..., $z_i = f(z_{i-1}, x_i)$, where $x_1, x_2, ..., x_i$ are strings that sequentially have given as inputs to $N$ until now, and $N$ has halted for all of them), and the machine evolves to $N = ((z_0, z_1, ..., z_i, z_{i+1} = f(z_i, x))$, $f$ (if $z_i$ halts for $x$). Note that since the evolution happens persistently, as soon as $N$ provides an output for an input $x$, if we will input the same $x$ to $N$ again in future, then $N$ provides the same output as before. It says that, the machine $N$ behaves well-defined as an input-output black box.

The persistently evolutionary Turing computation could be assumed as one of forms of Hypercomputation [24, 31]. Note that at each moment of time, a persistently evolutionary Turing machine has a finite structure, but during computations on inputs, its structure may change. So it is not possible to encode a persistently evolutionary Turing machine in a finite word. A persistently evolutionary Turing machine is an interactive machine (see Chapter 5 of [31], and [17]) that evolves according to how it interacts with its users. One may compare persistently evolutionary Turing machines with persistent Turing machines (PTM). The difference between these two kinds of machines is that PTM’s are static machines that transform input streams (infinite sequence of strings) to output streams persistently [19, 18, 21], whereas persistently evolutionary Turing machines are evolutionary machines that transform input strings to output strings.

We may start with two equal persistently evolutionary Turing machines $N_1 = (z_0, f)$ and $N_2 = (z_0', f')$ where $z_0 = z_0'$ and $f = f'$, but as we input strings to two machines in different orders the machines may evolve in different ways.

**Example 4.3** Let $z_0$ be a code of an arbitrary Turing machine, and $I : \Sigma^* \to \Sigma^*$ be the identity function, $I(x) = x$ for all $x \in \Sigma^*$. Then $N = (z_0, I)$ is a persistently evolutionary Turing machine. Thus any Turing machine can be considered as a persistently evolutionary Turing machine.

**Fact 4.4** A user who just has access to the input-output behavior of an evolutionary Turing machine $N$, cannot become conscious whether the machine $N$ evolves or not. In other words, if we put a persistently evolutionary machine in a black box, a computist can never be aware that whether it is a (static) Turing machine in the black box, or a persistently evolutionary one.

**Corollary 4.5** Let $E = (U, O)$ be a computation environment that the computist $O$ does not have access to the inner structure of the processor $U$. The computist $O$ can never be aware that whether its environment persistently evolves or not.

In the next example, we let NFA$_1$ be the class of all nondeterministic finite automata that for each $M \in$ NFA$_1$, each state $q$ of $M$, and $a \in \Sigma$, there exists at most one transition from $q$ with label $a$.

**Example 4.6** We define a function $h : \text{NFA}_1 \times \Sigma^* \to \text{NFA}_1$ as follows. Let $M \in \text{NFA}_1$, $M = (Q, q_0, \Sigma = \{0, 1\}, \delta : Q \times \Sigma \to Q, F \subseteq Q)$, and $x \in \Sigma^*$. Suppose $x = a_0a_1 \cdots a_k$ where $a_i \in \Sigma$. Applying the automata $M$ on $x$, one of the three following cases may happen:

1. The automata $M$ could read all $a_0, a_1 \cdots, a_k$ successfully and stops in an accepting state. Then we let $h(M, x) = M$. 

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2. The automata $M$ cannot read all $a_0, a_1 \ldots, a_k$ successfully and stops in a state $p$ which is not an accepting state. If the automata $M$ can transit from $p$ to an accepting state by reading one alphabet, then we define $h(M, x) = M$. If it cannot transit (to an accepting state) then we define $h(M, x)$ to be a new automata $M' = (Q, q_0', \Sigma = \{0, 1\}, \delta' : Q' \times \Sigma \rightarrow Q', F' \subseteq Q')$, where $Q' = Q$, $\delta' = \delta$, $F' = F \cup \{p\}$.

3. The automata $M$ cannot read all $a_0, a_1 \ldots, a_k$ successfully, and after reading a part of $x$, say $a_0a_1 \ldots a_i$, $0 \leq i \leq k$, it crashes in a state $q$ that $\delta(q, a_{i+1})$ is not defined. In this case, we let $h(M, x)$ be a new automata $M' = (Q, q_0', \Sigma = \{0, 1\}, \delta' : Q' \times \Sigma \rightarrow Q', F' \subseteq Q')$, where $Q' = Q \cup \{s_{i+1}, s_{i+2}, \ldots, s_k\}$ (all $s_{i+1}, s_{i+2}, \ldots, s_k$ are different states that do not belong to $Q$), $\delta' = \delta \cup \{(q, a_{i+1}, s_{i+1}), (s_{i+1}, a_{i+2}, s_{i+2}), \ldots, (s_{k-1}, a_k, s_k)\}$, and $F' = F \cup \{s_k\}$.

For each $M \in NF\!A_1$, we let $T_M$ be a Turing machine that for each input $x \in \Sigma^*$, the machine $T_M$ first constructs the automata $h(M, x)$, and if $h(M, x)$ accepts $x$, then $T_M$ outputs 1, else it outputs 0.

We define the persistently evolutionary Turing machine $PT_1 = ([T_{M_0}], f)$, where $M^0 = (Q^0 = \{q_0\}, q_0, \Sigma = \{0, 1\}, \delta^0 = \emptyset, F^0 = \emptyset)$, and $f([T_M], x) = [T_{h(M, x)}]$.

A language which is intended by a persistently evolutionary Turing machine is an unfinished object (similar to Brouwer’s choice sequences). At each stage of time, only a finite part of it, is recognized with the human being who intends the language. Persistently evolutionary Turing machines exist in time and are temporal dynamic mental constructions (see page 16 of [7]). Also a language intended by a persistently evolutionary Turing machine is user-dependent. Two users with two persistently evolutionary Turing machines $N_1$ and $N_2$ with the same initial structure may intend two different languages. A language which is defined through a persistently evolutionary Turing machine is not predetermined and is dependent to the free will of the user.

The initial structure $N = (z_0, f)$ of a persistently evolutionary Turing machine is constructed at a particular moment of time, and then evolves as the user chooses further strings to input. For persistently evolutionary Turing machines what remains invariant is the character of the machine as a evolutionary machine, an evolution that started at a particular point in time and preserves well-definedness. The machine evolves but it is the same machine that evolves and the machine is an individual unfinished object. Note that the user is not allowed to reset the machine and goes back to past. It is because that the evolution is a characteristic of persistently evolutionary Turing machines.

**Definition 4.7**

We say a persistently evolutionary Turing machine $N = (z_0, z_1, \ldots, z_n, f)$ evolves in a feasible time whenever the function $f$ can be computed in polynomial time.

We say a persistently evolutionary Turing machine $N = (z_0, z_1, \ldots, z_n, f)$ works in polynomial time, whenever there exists a polynomial function $p$, such that for every $x \in \Sigma^*$, if we input $x$ to $N$, the machine $z_i$ provides the outputs in less than $p(|x|)$ times, and the function $f$ constructs the function $z_{i+1} = f(z_i, x)$ in less than $p(|x| + |z_i|)$.

**Proposition 4.8** The persistent evolutionary Turing machine $PT_1$ works in polynomial time.
4.1 Physical Plausibility

Hypercomputation extends the capabilities of Turing computation via using new resources such as 1- infinite memory, 2- infinite specification, 3- infinite computation and 4- the interaction (see the appendix 9.2). Among these four resources, the resources 1,2, and 3 do not seem physically plausible as they have infinite structures. But the forth one, the interaction, seems physically plausible for the human being. The human being interacts with its environment and it could be possible that its environment persistently evolve because of interaction. The persistently evolutionary Turing machines use two resources to be Hypercomputations 1- evolution, and 2- interaction. Both of these two resources could be accepted by the human being as physically plausible resources (Biological structures evolve in Darwin theory). One may hesitate to accept that Persistently evolutionary Turing machines are physically plausible due to his presupposition that the real world is a Turing machine, but if he releases himself from this confinement then it seems to him that persistently evolutionary Turing machines are as physically plausible as Turing machines are. One can implement a persistently evolutionary Turing machine on his personal computer (assuming that the computer has an infinite memory. Note that the same assumption is needed for executing Turing machines on a computer). The difference between Turing machines and persistently evolutionary Turing machines is that the languages that the first category recognize are predetermined, whereas in the second category, we can recognize an unfinished and not predetermined language that depends on our interactions.

Therefore, we may list the following items for why persistently evolutionary Turing machines are physically plausible:

1- The persistently evolutionary Turing computation is a kind of interactive computation \cite{17} which today is accepted as a new paradigm of computation by some computer scientist.

2- The persistently evolutionary Turing machines are as plausible as Turing machines are. Both machines can be simulated by a personal computer (assuming that the computer has an infinite memory).

3- In Brouwer’s intuitionism, choice sequences are accepted as constructive mathematical objects. Choice sequences are growing, unfinished objects. Therefore, the persistently evolutionary Turing machines as growing unfinished objects are acceptable as constructive mathematical objects in Brouwer’s intuitionism.

4.2 Persistent Evolution

Suppose $B$ to be an input-output black box. For the observer who does not have access to the inner structure of the black box, it is not possible to get aware that whether the inner structure of the black box persistently evolves or not. We only sense a change whenever we discover that an event which has been sensed before is not going to be sensed similar to past. Persistently evolution always respects the past. As soon as, an agent experiences an
event then whenever in future he examines the same event, he will experience it similar to
past. However, persistent evolution effects the future which has not been predetermined,
and not experienced by the agent yet. Therefore,

it is not possible for an agent to distinguish between persistent evolution and
being static based on the history of his observation.

4.3 Free Will

Suppose \( N = (z_0, f) \) is a persistently evolutionary Turing machine. The machine \( N \) could
evolve in different ways due to the free will of the user who chooses freely strings to input to
\( N \). For example, consider the persistently evolutionary Turing machine \( PT_1 \). If you input
two strings 111, and 11 respectively, the machine \( PT_1 \) outputs 1 for the first input and
outputs 0 for the second one. Inputting the string 111 made the machine to evolve such
that it cannot accept 11 anymore. The time has sink back to past, and the machine \( PT_1 \)
evolved.

However, you had the free will to input first 11 and then 111, and if this case had
happened, the machine would have accepted both of them!

The user of a persistently evolutionary Turing machine cannot change the past by his free
will, but he can effect the future by his free choices. As soon as a persistently evolutionary
Turing machine evolves, it has been evolved, and it is not possible to go back to past. When
a persistently evolutionary machine evolves, it is the same machine that evolves, and the
evolution is a part of the entity of the machine.

However, since the user has the free will, he can effect the future. The future is not
necessary predetermined, and the user can make lots of different futures due to his free will.
For example, let \( L \) be the set of all strings that the evolutionary machine \( PT_1 \), during the
interaction with a user (with me), outputs 1 for them. The language \( L \) is not predetermined,
and it is a growing and an unfinished object (similar to choice sequences \([9,2]\)). Consider the
formula

\[
\phi := (\exists k \in \mathbb{N})(\forall n > k)(\exists x \in \Sigma^*)(|x| = n \land x \in L).
\]

At each stage of time, having evidence for the truth of formula \( \phi \) conflicts with the free will
of the user, and we never could have evidence for truth of \( \phi \). It is because at each stage of
time, only for a finite number of strings in \( \Sigma^* \), it is predetermined that whether they are
in \( L \) or not. Let \( m \in \mathbb{N} \) be such that \( m \) would be grater than the length of all strings that
until now are determined to be in \( L \). The user via his free will can input all strings with
length \( m + 1 \) to the machine \( PT_1 \) respectively. The machine \( PT_1 \) outputs 1 for all of them,
and evolves such that \( L \cap \{x \in \Sigma^* \mid |x| = m\} \) would be empty! Also, at each stage of time
having evidence for \( \neg \phi \) conflicts with the free will of the computist. Again, let \( m \in \mathbb{N} \) be
such that \( m \) would be grater than the length of all strings that until now are determined to
be in \( L \). The user via his free will can input a string with length \( m \) to the machine \( PT_1 \).
The machine \( PT_1 \) outputs 1 for it.

**Theorem 4.9** Let \( L \subseteq \Sigma^* \) be the growing language intended by a user through the persistently evolutionary Turing machine \( PT_1 \). let

\[
\phi := (\exists k \in \mathbb{N})(\forall n > k)(\exists x \in \Sigma^*)(|x| = n \land x \in L).
\]
We can never have evidence not for $\varphi$ and not for $\neg \varphi$, if we assume that the user has the free will.

Proof. See the above argument. $\dashv$

5 A Persistently Evolutionary Computation Environment

As we discussed in Section 2, we do not assume Turing machines as autonomous and independent entities. The meaning of a Turing machine is subject to the interaction between the computist (the human being), and the universal processor (human’s brain, or the nature). Therefore, if the universal processor changes then the meanings of Turing machines change (although their syntax remain unchanged). We may start an argument by asking a question

Question 5.1 What about if the human’s brain or the nature persistently evolve in the course that a computation is executed?

In this section, we propose a computation environment, denoted by $E_e = (U_e, O_f)$, which its universal processor $U_e$ persistently evolves, and its computist $O_f$ has the free will. The computation environment $E_e = (U_e, O_f)$ is such that $P_{E_e}$ is not equal to $NP_{E_e}$ in it, for the computist who has the free will.

Definition 5.2 The universal processor $U_e = (TBOX_e, SBOX_e, INST_e, CONF_e)$ is defined as follows.

Two sets $INST_e$ and $CONF_e$ are defined to be the same $INST_s$ and $CONF_s$ in the Turing computation environment (see example 3.4) respectively, and consequently the set of procedures of the universal processor $U_e$, i.e., $\Xi_e$ is the same $\Xi_s$.

The transition box $TBOX_e$ is also defined similar to the transition box $TBOX_s$ of the Turing computation environment $E_T$ (see example 3.4).

The successful box $SBOX_e$ is defined as follows: let $C \in CONF_e$ be arbitrary

- if $C = (h, \triangle x)$ then $SBOX_e(C) = YES$,
- if $C = (h, x\triangle)$ then the $SBOX_e$ works exactly similar to the way that the persistently evolutionary Turing machine $PT_1$ (introduced in example 4.6) performs on input $x$, and if $PT_1$ outputs 1, the successful box outputs $YES$, and
- otherwise $SBOX_e(C) = NO$.

Note that the successful box of the universal processor $U_e$ is a persistently evolutionary Turing machine. For the computist $O_f$ the set of syntax-procedures (algorithms) in the environment $E_e$ is the same set of syntax-procedures in the environment $E_T$. However the semantics of a syntax-procedure in the environment $E_e$ may be different from its semantics in the environment $E_T$.

In the computation environment $E_e = (U_e, O_f)$, the syntax-procedures in $\Xi_e$ satisfy the four criteria that Copeland gives that any set of instruction that makes up an algorithm
should satisfy to be characterized as an effective procedure (see the Church-Turing Thesis, Chapter 2, page 21, [31]). The only thing that one should pay attention is that: the meaning of procedures may persistently change in the environment for the computist $O_f$. But it does not mean that they are unambiguous for the computist. The computist does not have access to the universal processor, and is not aware of the persistent evolution. The computist using a pencil and paper, writes a configuration on the paper, then using the universal processor (that could be its brain, and the computist ignores its processes) writes the computation path on the paper, and what happens inside the universal processor is ignored by the computist. Also note that

the universal processor $U_e$ is a physically plausible engine (see [4.1]), and it could be assumed as an alternative for the universal processor of the real world.

**Theorem 5.3** (GV). Every recursively enumerable language can be recognized in the environment $E_e$. That is, for every recursively enumerable language $L$, there exists $M \in \Xi_e$ such that $L = L(M)$.

**Proof.** It is straightforward. For each Turing machine $T$, one may construct a syntax-procedure $M \in \Xi_e$ such that $L(T) = L(M)$, and it is not possible for the computist to obtain any configurations $(h, y\triangleleft)$, $y \in \Sigma^*$, using the procedure $M$. Then the universal processor $U_e$ behaves exactly similar to the universal processor $U_s$ for the procedure $M$. ⊢

**Corollary 5.4** (GV). The computation environment $E_e = (U_e, O_f)$ is enough powerful (see definition 3.8).

**Proposition 5.5** (GV). The complexity class $P$ is a subset of $P_{E_e}$.

**Proof.** It is straightforward. ⊢

The converse of Theorems 5.3, and 5.5 do not hold true. The computist can effectively define some languages in polynomial time which are unfinished and not predetermined (see the proof of theorem 5.8).

**Definition 5.6** Let $\phi$ be a statement about the syntax-procedures in $\Xi_e$. We say the truth of $\phi$ is consistent with the free will of the computist, whenever independent of any way that the computist interacts with the universal processor, the statement $\phi$ always holds true in the environment.

Any formula that forces the future to be predetermined could conflict with the free will of the computist (also see 4.3). In following, we prove that $P_{E_e} = NP_{E_e}$ conflicts with the free will of the computist.

The item c2 of definition 5.3 considers the free will as a characteristic of any computist who lives in a computation environment, therefore, if a formula conflicts with the free will, then from the god’s view, the formula would not be valid for the computation environment.

**Definition 5.7** We say a function $f : \mathbb{N} \to \mathbb{N}$ is sub-exponential, whenever there exists $t \in \mathbb{N}$ such that for all $n > t$, $f(n) < 2^n$. 

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Theorem 5.8 (GV). There exists a syntax-procedure $M \in \Xi_e$ such that

- the language $L(M)$ that the computist $O_f$ effectively defines (effectively intends) through $M$ belongs to the class $P_{E_e}$, and

- there exists no syntax-procedure $M' \in \Xi_e$, such that the language that the computist defines through $M'$, i.e., $L(M')$, is equal to $L' = \{ x \in \Sigma^* \mid \exists y(|y| = |x| \land y \in L(M)) \}$, and for some $k \in \mathbb{N}$, for all $x \in L(M')$, if $|x| > k$ then

$$\text{time}_M(x) \leq f(|x|)$$

where $f : \mathbb{N} \to \mathbb{N}$ is a sub-exponential function.

Proof. Consider the following syntax-procedure $M \in \Xi_e$

$$\Sigma = \{0, 1\}, \Gamma = \{0, 1, \triangle\},$$

$$M = \{[(q_0, \triangle) \to (h, \triangle, R)], [(h, 0) \to (h, 0, R)], [(h, 1) \to (h, 1, R)]\}.$$ 

The intension of the word $M$ is persistently evolutionary in the computation environment $(U_e, O_f)$ for the computist $O_f$. That is, as the computist chooses a string $x \in \Sigma^*$ to check whether $x$ is an element of $L(M)$, the intension of $M$ persistently changes in the environment, though since the computist does not have access to the inner structure of the universal processor $U_e$, he never gets aware of changes.

The language $L(M)$ is intended deterministically through the syntax-procedure $M$ by the computist $O_f$, but it is not a predetermined language.

It is obvious that the language that the computist $O_f$ intends by the syntax-procedure $M$, i.e., $L(M)$ belongs to $P_{E_e}$ (due to definition of time complexity in definition [3.3]).

Let $L' = \{ x \in \Sigma^* \mid \exists y(|y| = |x| \land y \in L(M)) \}$. It is again obvious that $L'$ belongs to $NP_{E_e}$.

Suppose there exists a syntax-procedure $M' \in \Xi_e$ that the computist can intends $L'$ by $M'$ in time complexity less than a sub-exponential function $f$. Then for some $k \in \mathbb{N}$, for all $x$ with length greater than $k$, $x$ belongs to $L'$ whenever

the computist constructs a successful computation path $C_{0,x}C_{1,x}, ..., C_{n,x}$ of the syntax-procedure $M'$ on $x$, for some $n \leq f(|x|)$.

Let $m_1 \in \mathbb{N}$ be the maximum length of those strings $y \in \Sigma^*$ that until now are accepted by the persistently evolutionary Turing machine $PT_1$ available in the successful box of $U_e$. Define $m = \max(m_1, k)$.

For every $y \in \Sigma^*$, let $\text{path}(y) := C_{0,y}C_{1,y}, ..., C_{f(|y|),y}$ be the computational path of syntax-procedure $M'$ on the string $y$. The $\text{path}(y)$ can be generated by the transition box of $U_e$. Note that $TBOX_e$ is a static machine and does not evolve, thus $\text{path}(y)$ is independent of the behavior of the computist. Let

$$S(w) = \{ C_{j,y} \mid C_{j,y} \in \text{path}(y) \land \exists x \in \Sigma^* (C_{i,y} = (h, x\triangle)) \}$$

and
\[ H(y) = \{ x \in \Sigma^* \mid \exists C_{j,y} \in \text{path}(y)(C_{j,y} = (h, x\triangle)) \} \]

We refer by \(|H(y)|\) to the number of elements of \(H(y)\), we have \(|H(y)| \leq f(|y|)\) if \(|y| > k\).
Also let \(E(y) = H(y) \cap \{ x \in \Sigma^* \mid |x| = |y| \}\), and \(D(y) = H(y) \cap \{ x \in \Sigma^* \mid |x| = |y| + 2 \}\).

Consider \(w \in \Sigma^*\) with \(|w| > m\). Two cases are possible: either \(S(w) = \emptyset\) or it is inhabited.

Consider the first case, \(S(w) = \emptyset\).

The computist chooses a string \(w \in \Sigma^*\) with length greater than \(m\) and applies the universal processor \(U_e\) on the syntax-procedure \(M'\) and the string \(w\), and recognizes whether \(w\) is in \(L(M')\) or not. Since the set \(S(w)\) is empty, the execution of \(M'\) on \(w\) does not make the \(SBOX_e\) to evolve, and it remains unchanged.

If the computist realizes that \(w \in L(M')\) then it means that there exists a string \(v \in \Sigma^*\) such that \(|v| = |w|\) and \(v \in L(M)\). But length \(v\) is greater than \(m\), and it contradicts with the free will of the computist that he can apply the universal processor in any arbitrary orders on instructions and configurations (see c2 of definition 3.3). Since the computist has the free will, he can apply the universal processor \(U_e\) on the syntax-procedure \(M\) and all strings in \(\Sigma^*\) with length \(|v| + 1\) sequentially. As the length of \(v\) is greater than \(m\), all strings with length \(|v| + 1\) are accepted by the persistently evolutionary Turing machine \(PT_1\) (see item-3 of example 4.6) available in \(SBOX_e\), and the universal processor evolves. Now if the computist applies the universal processor on the syntax-procedure \(M\) and the string \(v\), then \(v\) cannot be an element of \(L(M)\) (see the item-2 of example 4.6) and \(v \notin L(M)\), contradiction.

If the computist realizes that \(w \notin L(M')\) then it means that for all strings \(v \in \Sigma^*\), \(|v| = |w|\), we have \(v \notin L(M)\). But it contradicts with the free will of the computist again. As the length \(w\) is greater than \(m\), the computist may choose a string \(z\) with \(|z| = |w|\) and by the item-3 of example 4.6 we have \(z \in L(M)\), contradiction.

Consider the second case, \(S(w) \neq \emptyset\).

Suppose that before computing \(M'\) on \(w\), the computist applies the universal processor on the syntax-procedure \(M\) and all strings \(v0's\), for \(v \in E(w)\), and then applies the universal processor on the syntax-procedure \(M\) and strings \(v0's\), for \(v \in D(w)\) respectively. Since \(|w| > m\), the computist would have \(v0 \in L(M)\) for all \(v \in E(w) \cup D(w)\), and the successful box of the universal processor \(U_e\) evolves through computing \(M\) on \(v0's\).

After that, the computist applies the universal processor on \(M'\) and \(w\). The universal processor already evolved in the way that the successful box outputs \(No\) for all configuration in \(\{ C_{i,w} \in S(w) \mid \exists x \in E(w) \cup D(w)(C_{i,w} = (h, x\triangle)) \}\). Either the computist finds \(w \in L(M')\) or \(w \notin L(M')\). Suppose the first case happens and \(w \in L(M')\). It contradicts with the free will of the computist. The computist can apply the universal processor on \(M\) and strings \(v0\), \(|v| = |w|\) sequentially, and would make \(\{ v0 \in \Sigma^* \mid |v| = |w| \} \subseteq L(M)\). Then the successful box evolves in the way that, it will output \(No\) for all configurations \((h, v\triangle), |v| = |w|\), and thus there would exist no \(v \in L(M) \cap \{ x \in \Sigma^* \mid |x| = |w| \}\).

Suppose the second case happens and \(w \notin L(M')\). Since \(|H(w)| < f(|w|) < 2^{|w|}\), during the computation of \(M'\) on \(w\), only \(f(|w|)\) numbers of configurations of the form \((h, x\triangle), x \in \{ v0 \mid |v| = |w| \} \cup \{ v1 \mid |v| = |w| \}\) are given as input to the successful box. Therefore there exists a string \(z \in \{ x \in \Sigma^* \mid |x| = |w| \}\) such that none of its successor has been input to the persistently evolutionary Turing machine \(PT_1\), and if the computist chooses \(z\) and computes \(M\) on it, then \(z \in L(M)\). Contradiction. \(\dag\)
Note that the proof of Theorem 5.8 cannot be carried by the computist \( O_f \) who lives in the environment. The above proof is done by the god, the ideal mathematician who has access to the inner structure of the universal processor.

**Theorem 5.9 (GV).** Free Will\(^5\).

\[ \text{NP}_{E_e} \neq \text{P}_{E_e}. \]

**Proof.** It is a consequence of theorem 5.8. ⊢

The above theorem says that there exists a syntax-procedure \( M \in \Xi_e \) that the computist \( O_f \) can effectively define the language \( L(M) \) in polynomial time through the syntax-procedure \( M \), such that for \( L' = \{ x \in \Sigma^* \mid \exists y(|y| = |x| \land y \in L(M)) \} \), if \( L' \) can be effectively defined through a syntax-procedure \( M' \in \Xi_e \) then it forces the computist to use the universal processor in some certain orders, which conflicts with the free will of the computist\(^6\).

**Corollary 5.10** The computation environment \( E_e = (U_e, O_f) \) is a mathematical model for the Copeland’s criteria of effective procedures. Thus, one cannot derive \( \text{P} = \text{NP} \) from the criteria introduced by Copeland for effective procedures.

**Corollary 5.11** There exists a physically plausible and enough powerful computation environment which \( \text{P} \) is not equal to \( \text{NP} \) in.

**Proof.** Persistently evolutionary Turing machines seem as physically plausible as Turing machines are. The structure of the transition box of the universal processor \( U_e \) is a Turing machine and the structure of its successful box is a Persistently evolutionary Turing machine. Therefore \( E_e = (U_e, O_f) \) is a physically plausible computation environment. Also \( E_e \) is enough powerful by Theorem 5.3. ⊢

### 5.1 Complexity Equivalence

In this part, we define an equivalency between universal processors from time complexity view. We prove that the universal processor \( U_s \) of Turing computation environment \( E_T \) is polynomial time equivalent to the universal processor \( U_e \) of the persistently evolutionary computation environment \( E_e \).

**Definition 5.12** Let \( U_1 = (TBOX_1, SBOX_1, INST_s, CONF_s) \), and \( U_2 = (TBOX_2, SBOX_2, INST_s, CONF_s) \) be two universal processors where their instructions and configurations are the same instructions and configurations of the Turing computation environment \( E_T \). Also let \( f : \mathbb{N} \to \mathbb{N} \) be a function. We say two universal processor \( U_1 \) and \( U_2 \) are \( f \)-complexity equivalent whenever:

\(^5\)Readers may see [4], where the notion of the free will is used to settle the well-known Surprise Exam Paradox. The notion of the free will is also discussed in author’s PhD thesis [26], and in [3].

\(^6\)Note that the above argument says that if the language of a syntax-procedure \( M \) is not predetermined and the computist who computes the language through \( M \) has free will, then future is always undetermined and any assumption that forces the future to be determined conflicts with the free will. The statement \( \text{P} = \text{NP} \) is one of those assumptions that forces the future behavior of the computist to be determined.
1- for each $C = (q, xay) \in \text{CONF}_s$, and each $\iota \in \text{INST}_s$, if $\text{TBOX}_1(C, \iota) = C'$ for some $C' \in \text{CONF}_s$, then there exist $\tau_1, \tau_2, ..., \tau_k \in \text{INST}_s$, for some $k \leq f(|xay|)$, such that for some $C_1, C_2, ..., C_k \in \text{CONF}_s$, $\text{TBOX}_2(C, \iota_1) = C_1$, for all $2 \leq i \leq k$, $\text{TBOX}_2(C_{i-1}, \iota_{i-1}) = C_i$ and $C_k = C'$.

2- for each $C = (q, xay) \in \text{CONF}_s$, and each $\iota \in \text{INST}_s$, if $\text{TBOX}_2(C, \iota) = C'$ for some $C' \in \text{CONF}_s$, then there exist $\tau_1, \tau_2, ..., \tau_k \in \text{INST}_s$, for some $k \leq f(|xay|)$, such that for some $C_1, C_2, ..., C_k \in \text{CONF}_s$, $\text{TBOX}_1(C, \iota_1) = C_1$, for all $2 \leq i \leq k$, $\text{TBOX}_1(C_{i-1}, \iota_{i-1}) = C_i$ and $C_k = C'$.

3- for each $C = (q, xay) \in \text{CONF}_s$, if $\text{SBOX}_1(C) = \text{YES}$, then there exist $\tau_1, \tau_2, ..., \tau_k \in \text{INST}_s$, for some $k \leq f(|xay|)$ such that for some $C_1, C_2, ..., C_k \in \text{CONF}_s$, $\text{TBOX}_2(C, \iota_1) = C_1$, for all $2 \leq i \leq k$, $\text{TBOX}_2(C_{i-1}, \iota_{i-1}) = C_i$ and $\text{SBOX}_2(C_k) = \text{YES}$.

4- for each $C = (q, xay) \in \text{CONF}_s$, if $\text{SBOX}_2(C) = \text{YES}$, then there exist $\tau_1, \tau_2, ..., \tau_k \in \text{INST}_s$, for some $k \leq f(|xay|)$, such that for some $C_1, C_2, ..., C_k \in \text{CONF}_s$, $\text{TBOX}_1(C, \iota_1) = C_1$, for all $2 \leq i \leq k$, $\text{TBOX}_1(C_{i-1}, \iota_{i-1}) = C_i$ and $\text{SBOX}_1(C_k) = \text{YES}$.

Suppose you, as a computist, have two personal computer on your desk. The CPU of one of them is the universal processor $U_1$, and the CPU of the other one is the universal processor $U_2$, the programming language that you can communicate with the processor are the same, and you know that

- if you can transit from a configuration $C = (q, xay)$ and an instruction $\iota$ to a configuration $C'$ using the transition box $\text{TBOX}_1$ ($\text{TBOX}_2$) only one time, then you can transit from the same configuration $C$ to the same configuration $C'$ using the transition box $\text{TBOX}_2$ ($\text{TBOX}_1$) at most $f(|xay|)$ times, and

- if you input a configuration $C = (q, xay)$ to the $\text{SBOX}_1$ ($\text{SBOX}_2$), and receives $\text{YES}$, then you can transit from the configuration $C$ to a configuration $C'$ using the transition box $\text{TBOX}_2$ ($\text{TBOX}_1$) at most $f(|xay|)$ times, such that the successful box $\text{SBOX}_2$ ($\text{SBOX}_1$) outputs $\text{YES}$ for $C'$.

Then, for you as a computist, it seems (wrongly) that if you can do some work using the processor $U_1$ in $m$ times, then you may do the same work using the processor $U_2$ for at most $2m \times f(m)$ times, and vice versa.

**Proposition 5.13** Two universal processors $U_s$ and $U_e$ are $f$-complexity equivalent where $f(n) = 2n$ for all $n \in \mathbb{N}$.

**Proof.** It is straightforward. \(\neg\)

### 6 An Axiomatic System for Turing Computability

In this section, we provide an axiomatic system for Turing computation. Let $\text{CONF} = \text{CONF}_s$, and $\text{INST} = \text{INST}_s$ (where $\text{CONF}_s, \text{INST}_s$ are defined in example 3.4). Also let $\Xi$ be the set of all finite subsets of $\text{INST}$ such that for every $M \in \Xi$, for every two instructions $\iota_1 = [(q_1, a_1) \to (p_1, b_1, D_1)], \iota_2 = [(q_2, a_2) \to (p_2, b_2, D_2)]$, if $\iota_1, \iota_2 \in M$, $q_1 = q_2$, and $a_1 = a_2$ then $p_1 = p_2$, $b_1 = b_2$, and $D_1 = D_2$. Consider
a function symbol $TB : CONF \times INST \rightarrow CONF \cup \{\bot\}$, and
a predicate symbol $SB : CONF \rightarrow \{YES, NO\}$.

For each $x \in \Sigma^*$, define $C_{0,x} = (q_0, \Delta x)$.

**Definition 6.1** For each $x \in \Sigma^*$, for each $M \in \Xi$, we define

$x \in L(M) \equiv \text{there exists } n \in \mathbb{N}, \text{there exists } \tau_1, \tau_2, \ldots, \tau_n \in M, \text{there exists } C_1, C_2, \ldots, C_n \in CONF, \text{ such that } C_1 = TB(C_{0,x}, \tau_1), \text{ and } C_i = TB(C_{i-1}, \tau_i) \text{ for } 1 < i \leq n, \text{ and }$ 

$SB(C_n) = YES, \text{ and for all } i \in M, TB(C_n, i) = \bot.\)

$time_M(x) = n \text{ whenever there exists } \tau_1, \tau_2, \ldots, \tau_n \in M, \text{ there exists } C_1, C_2, \ldots, C_n \in CONF, \text{ such that } C_1 = TB(C_{0,x}, \tau_1), \text{ and } C_i = TB(C_{i-1}, \tau_i) \text{ for } 1 < i \leq n, \text{ and }$ 

$SB(C_n) = YES, \text{ and for all } i \in M, TB(C_n, i) = \bot.\)

An axiomatic system $\mathcal{T}$ for Turing computation consists of following axioms A1-A4.

**A1.** for all $x, y \in \Sigma^*$, for all $a, b_1, b_2, c \in \Sigma$, for all $q, p \in Q,$

\[ TB((q, xb_1a_2y), [(q, a) \rightarrow (p, c, R)]) = (p, xb_1c_2y), \text{ and } \]
\[ TB((q, xb_1a_2y), [(q, a) \rightarrow (p, c, L)]) = (p, xb_1c_2y). \]

**A2.** If $SB(C) = YES$ then for some $x \in \Sigma^*$, either $C = (h, \Delta x)$ or $C = (h, x\Delta)$.

**A3.** For all $x \in \Sigma^*$, if $C = (h, \Delta x)$ then $SB(C) = YES$.

**A4.** For all $x \in \Sigma^*$, if $C = (h, x\Delta)$ then $SB(C) = YES$.

Let $TM = \{T = (Q, \Sigma, \Gamma, q_0, \delta) \mid Q \subseteq QT\}$ be the set of all Turing machines (see definition 2.7). Define two functions $F_1 : TM \rightarrow \Xi$ and $F_2 : \Xi \rightarrow TM$ as follows:

- for each Turing machine $T \in TM$, $F_1(T) = \delta$,
- for each $M \in \Xi$, $F_2(M) = (Q, \Sigma, \Gamma, q_0, M)$, where $Q$ is the set of all states appeared in the instructions of $M$.

**Theorem 6.2** For each $x \in \Sigma^*$, for every $T \in TM$, for every $M \in \Xi$,

- $x \in L(T)$ iff $\mathcal{T} \vdash x \in L(F_1(T))$.
- $time_T(x) = n$ iff $\mathcal{T} \vdash time_{F_1(T)}(x) = n$.
- $\mathcal{T} \vdash x \in L(M)$ iff $x \in L(F_2(M))$.
- $\mathcal{T} \vdash time_M(x) = n$ iff $time_{F_2(M)}(x) = n$.

**Proof.** It is straightforward. $\square$

The above theorem declares that the axiomatic system $\mathcal{T}$ is an appropriate axiomatization of Turing computation.
Theorem 6.3 Let $T'$ be the axiomatic system $T - \{A_4\}$.

$$T' \not\vdash P = NP.$$  

Proof. We need to provide a model for the theory $T'$ such that $P$ is not equal to NP in the model. Interpret the predicate symbol $SB$ to be the successful box $SBOX_e$ of computation environment $E_e$, and the function symbol $TB$ to be the transition box $TBOX_e$. All axioms in $T'$ are satisfied by this interpretation. On the other hand, by Theorem 5.9 in this model, we have $P \neq NP$. $\Box$ |  

7 P vs NP for the Computist

We claim that the computist who lives inside the Turing computation environment $E_T$ can never be confident that whether he lives inside a static environment or a persistently evolutionary one. It is simply because of the three following facts:

1. The language (syntax) of two computation environments $E_T$ and $E_e$ are the same. That is, $INST_e = INST_s$, $CONF_e = CONF_s$, and $\Xi_e = \Xi_s$.

2. The universal processor of the Turing computation environment is a black box for the computist. The computist can never know that whether the universal processor of its environment persistently evolves or not. It is because an observer who does not have access to the inner structure of a black box, can never distinguish that whether the black box persistently evolves or not.

3. If the universal processor of an environment persistently evolves, then it is possible that there are syntax-procedures in $\Xi$ which their languages are not predetermined and $P = NP$ conflicts with the free will of the computist. For example, $P$ is not equal to NP in the computation environment $E_e$.

In follows, we show that for the computist who lives in one of the environments $E_T$ or $E_e$, it is not possible to get aware that actually he lives in a static environment or a persistently evolutionary one.

Definition 7.1 (BLACK BOX). Let $X$ and $Y$ be two sets,

- an input-output black box $B$, for an observer $O$, is a box that
  - The observer $O$ does not see the inner instruction of the box, and
  - the observer $O$ chooses elements in $X$, and input them to the box, and receives elements in $Y$ as output.

$\Omega = \{A_1, A_2\}$ is a theory for Turing computability, and for every $M \in \Xi$, for every $x \in \Sigma^*$, $\Gamma' \vdash x \in L(M)$ iff $\Omega \vdash x \in L(M)$, and for every $M \in \Xi$, for every $x \in \Sigma^*$, for every $n \in N$, $\Gamma' \vdash time_M(x) = n$ iff $\Omega \vdash time_M(x) = n$.  

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$A_2$. $SB(C) = YES$ if and only if for some $x \in \Sigma^*$ $C = (h, \Delta x)$. One may check that $\Omega = \{A_1, A_2\}$ is a theory for Turing computability, and for every $M \in \Xi$, for every $x \in \Sigma^*$, $\Gamma' \vdash x \in L(M)$ iff $\Omega \vdash x \in L(M)$, and for every $M \in \Xi$, for every $x \in \Sigma^*$, for every $n \in N$, $\Gamma' \vdash time_M(x) = n$ iff $\Omega \vdash time_M(x) = n$.  

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• We say an input-output black box $B$ behaves well-defined for an observer $O$, whenever if the observer $O$ inputs $x_0$ to the black box, and the black box outputs $y_0$ at a stage of time, then whenever in future if the observer $O$ inputs the same $x_0$ again, the black box outputs the same $y_0$.

• We say a well-defined black box is static (or is not order-sensitive) whenever for all $n \in \mathbb{N}$, for every $x_1, x_2, ..., x_n \in X$, for every permutation $\sigma$ on $\{1, 2, ..., n\}$, if the observer inputs $x_1, x_2, ..., x_n$ respectively to $B$ once, and receives $y_1 = Bx_1, y_2 = Bx_2, ..., y_n = Bx_n$, and then the god (the one who has access to the inner structure of the black box) resets the inner structure of the black box $B$, and after reset, the observer inputs $x_{\sigma(1)}, x_{\sigma(2)}, ..., x_{\sigma(n)}$ respectively to $B$, then the outputs of $B$ for each $x_i$ would be the same already output $y_i$.

• An observer who does not have access to the inner structure of a well-defined black box $B$, and cannot reset the inner structure black box $B$, can never get aware whether the black box is static or order-sensitive.

Example 7.2 Let $X = Y = \mathbb{N}$, and $B$ be the black box that works as follows. I hide in the black box, and do the following strategy. I plan to output 1 for each $n$ before the observer gives the black box 5 or 13 as inputs. If the observer gives 5 as input (and has not yet given 13 before) then after that time, I output 2 for all future inputs that have not been already given to the black box. For those natural numbers that the observer have already given them as input, I still output the same 1. If the observer gives 13 as input (and has not yet given 5 before) then after that time, I output 3 for all future inputs that have not been already given to the black box. For those natural numbers that the observer has already given them as input, I still output the same 1. It is easy to verify that

1- The black box $B$ works well-defined.

2- The black box is order-sensitive.

3- For the observer $O$, it is not possible to be aware that the black box is order sensitive. He always could assume that actually a machine Turing is in the black box.

4- The function that the observer computes via the black box $B$ is not a predetermined function, but the observer can never find out that whether the function is predetermined or not!

Proposition 7.3

• For every finite sets of pairs $S = \{(i_k, o_k) \mid 1 \leq k \leq n, n \in \mathbb{N}, i_k, o_k \in \Sigma^*\}$, there exists a Turing machine $T$ such that for all $(i_k, o_k) \in S$, if we give $i_k$ as an input to $T$, the Turing machine $T$ outputs $o_k$.

• For every finite sets of pairs $S = \{(i_k, o_k) \mid 1 \leq k \leq n, n \in \mathbb{N}, i_k, o_k \in \Sigma^*\}$, there exists a Persistent evolutionary Turing machine $N$ such that for all $(i_k, o_k) \in S$, if we give $i_k$ as an input to $N$, the Persistent evolutionary machine $N$ outputs $o_k$.

Proof. It is straightforward. ⊣
Corollary 7.4 Let \( B \) be an input-output black box for an observer. At each stage of time, the observer has observed only a finite set of input-output pairs. By the previous proposition, at each stage of time, the observer knows both the following cases to be possible:

1. There exists a Turing machine inside the black box \( B \).
2. There exists a Persistent evolutionary Turing machine inside the black box \( B \).

Fact 7.5 In any computation environment \( E = (U, O) \), since the computist \( O \) cannot reset the universal processor and goes back to past, he can never realize that actually he lives in a static environment or in a persistently evolutionary one.

Therefore, the computist who lives in the Turing computation environment can never have evidence from his observation that the universal processor which means to syntax-procedures in \( \Xi_s \) is static or persistently evolutionary.

The following argument can be carried out in Epistemic logic [14].

- There are two possible worlds (two computation environments) \( E_T \) and \( E_e \) that the computist can never distinguish between them.
- In Epistemic logic an agent knows a formula \( \varphi \) whenever the formula \( \varphi \) is true in all possible worlds which are not indistinguishable from the actual world for the agent.
- In the possible world \( E_e \), \( P \neq NP \).

thus,

The computist who lives inside the Turing computation environment \( E_T \) does not know \( P = NP \). \(^8\)

8 Concluding Remarks

P versus NP is a major difficult problem in computational complexity theory, and it is shown that lots of certain techniques of complexity theory are failed to answer to this problem (see [27, 1, 2]). We constructed a computational environments \( E_e \) which

1. is physically plausible,
2. is enough powerful, and the computist who lives inside the proposed environment can carry out logical deductions,
3. similar to the human being who ignores his brain in formalizing notions of computation and time complexity, the computist of the proposed environment does not have access to the inner structure and internal actions of the universal processor,
4. the universal processor of the computation persistently evolve in a feasible time, and
5. \( P \) is not equal to NP in the proposed environment.

\(^8\)A formal version of this argument is presented in the next paper “Computation Environment (2)”.
It seems that the above five items make it possible to know the proposed computation environment as an alternative for the computation environment which surrounded the human being.

This paper is a first manuscript of some serial articles which would be appeared sequentially:

1- A Persistent Evolutionary Semantics for Predicate Logic. In (classic) model theory, the mathematician who studies a structure is assumed as a god who lives out of the structure, and the study of the mathematician does not effect the structure. Against of this, we will propose a new semantics, called persistently evolutionary semantics, for predicate logic that the meaning of functions and predicates symbols are not already predetermined, and predicate and function symbols find their meaning through the interaction of the subject with the language. Then, we will prove formally that the computist who lives inside the Turing computation environment can never know $P = NP$.

2- We will discuss the human being as a computist of its computation environment. Lots of people do not agree with us that one is allowed to distinguish between syntax and semantics of Turing machines. Although, we proposed an interactive semantics for computation and prove that $P$ is not equal to $NP$ in this semantics, but one may note that the computist who lives inside the computation environment $E_e$ does not distinguish between the syntax and semantics of procedures in $Ξ_e$ (since the computist ignores the inner structure of the universal processor, and cannot be aware that whether he lives in a static environment or a persistently evolutionary one). Therefore, we will discuss that

- it is possible that the human being lives in an interactive environment (like the environment $E_e$),
- the human being does not distinguish between the syntax and semantics of Turing machines (similar to the computist who lives inside $E_e$), and
- the human being, even by considering that the semantics and syntax to be the same, cannot prove $P = NP$.

3- There are lots of open problems in sciences. One of the main reason that why $P$ vs $NP$ is so famous, is because of the foundation of cryptography. We proved that $P \neq NP$ for interactive computation. Interactive computation is physically plausible and could be considered as a new paradigm in computer science. We will propose an interactive cryptosystem and to show that it is safe, we reduce our interactive $P \neq NP$ statement to it.

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back to the notion of dynamic computation in the author’s PhD thesis. The author would like to thank the supervisor of his PhD thesis, Mohammad Arde shir, for all of his warm kindness, supports, and counsels.

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9 Appendix A.

In the appendix, a very short review of hypercomputations and an introduction of Brouwer’s intuitionism is presented. To review the Hypercomputation resources, the section 4 of [25] is used, and the review of Brouwer’s choice sequences is derived from [9].

9.1 Hypercomputation

In what follows, we briefly review a collection of hypercomputational resources and those hypermachines that use these resources.

Non-recursive Information Sources.

The paradigm hypercomputation starts with the o-machine, proposed by Turing in 1939 [33]. O-machine is a Turing machine equipped with an oracle that is capable of answering questions about the membership of a specific set of natural numbers. If the oracle set is recursive then the o-machine gains no new power, but if the oracle set is not itself computable by Turing machines, the o-machine may compute an infinite number of non-recursive functions. The o-machines use non-recursive information sources [25] as hypercomputational resources [25].

Coupled Turing machine, introduced by Copeland and Sylvan [13], is a Turing machine with one or more input channels, providing input to the machine while the computation is in progress. The specific sequence of input determines the functions that the coupled Turing machine can perform. It exceeds a Turing machine if the sequence of input is non-recursive. Like o-machines, coupled Turing machines use non-recursive information sources. Besides the above discussed hypermachines, Asynchronous network of Turing machines [13], Error prone Turing machines [25], and Probabilistic Turing machines [22] are well-known hypermachines that all use non-recursive information sources as their hypercomputation resources.

Infinite Memory.

Another way to expand the capabilities of a Turing machine is to allow it to begin with infinite number of symbols initially inscribed on its tape. Turing machines with initial inscription which have an explicitly infinite amount of storage space are not physically plausible.

Infinite Specification.

An infinite state Turing machine is a Turing machine where the set of states is allowed to be infinite. This type of machine has an infinite amount of transitions, with only a finite number of transition from a given state. This gives the Turing machine an infinite program of which only a finite (but unbounded) amount of transitions is used in any given computation. The infinite state Turing machines require infinite specification which does not seems physically plausible.

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Infinite Computation.

In the last century, Bertrand Russell [28], Ralph Blake [10], and Hermann Weyl [34] independently proposed the idea of a process that performs its step in one unit of time and each subsequent step in half the time of the step before [25]. Therefore, such a process could complete an infinity of steps in two time unit. The application of this temporal patterning to Turing machines has been discussed briefly by Ian Stewart [30] and in much more depth by Copeland [12] under the name of accelerated Turing machines. The hypercomputation resource is infinite computation. To achieve infinite computation through acceleration, we rapidly run into conflict with physics. For the tape head to get faster and faster, its speed converges to infinite.

Joel Hamkins and Andy Lewis presented another kind of hypermachines that use infinite computation [20], named infinite time Turing machines. The infinite time Turing machine is a natural extension of Turing machine to transfinite ordinal times, the machine would be able to operate for transfinite numbers of steps. An interesting case about infinite time Turing machines is that it has been proved $P \neq NP$ for this model of computation [29].

Interaction.

Among other resources: non-recursive information sources, infinite memory, infinite specification, and infinite computation, the interaction seems to be possible with our current physics. Persistently evolutionary Turing machines which are introduced in this paper, use the interaction as a Hypercomputation resource.

A kind of hypermachines that use the interaction as resource is the class of persistent Turing machines (PTMs), multiple machines with a persistent worktape preserved between interactions, independently introduced by Goldin and Wegner [18] and Kosub [21]. Consistent PTMs, a subclass of PTMs, produce the same output string for a given input string everywhere within a single interaction stream; different interaction streams may have different outputs for the same input. PTMs are a minimal extension of Turing machines that express interactive behavior. The behavior of a PTM is characterized by input-output streams instead of input-output strings. Interaction streams have the form $(i_1, o_1), (i_2, o_2), \ldots$, where $i$’s are input strings and $o$’s are output strings by PTM. For all $k$, $o_k$ is computed from $i_k$ but preceded and can influence $i_{k+1}$. The set of all interaction streams for a PTM $M$ consists its language, $L(M)$, and two PTMs $M_1$ and $M_2$ are equivalent if and only if $L(M_1) = L(M_2)$. Actually, PTMs extend computing to computable nonfunctions over histories rather than noncomputable functions over strings, whereas persistently evolutionary Turing machines (introduced in this paper) extends computing to interactive computable functions which are not predetermined.

9.2 Intuitionism

Over the last hundred years, certain mathematicians have tried to rebuilt mathematics on constructivist principles [32]. However, there are considerable differences between various representatives of constructivist, and there exists no explicit unique answer to what a constructive object or a constructive method is. In contrast to other constructivist schools as Bishop’s and Markov’s [11], for Brouwer, the notion of “constructive object” is not restricted to have a numerical meaning, or to be presented as ‘words’ in some finite alphabet of symbols [32]. According to Brouwer, mathematical objects are mental constructions, a
languageless activity and independent of logic. Brouwer recognized the choice sequences as legitimate mathematical objects [32].

Imagine you have a collection of objects at your disposal, let’s say the natural numbers. Pick out one of them, and note the result. Put it back into the collection, and choose again. Since you have the ability to choose freely, you may choose a different one, or the same again. Record the result, and put it back. You may make further choices and keeping on. A choice sequence is what you get if you think of the sequence you are making as potentially infinite [9]. Initial segments are always finite. We cannot make an actually infinite number of choices, but we can always extend an initial segment by making a further choice. The following characteristic of the choice sequences is crucial in our consideration:

The subject successively chooses objects, restriction on future choices, restriction on restriction of future choices, etc. ([8] page 6)

The object of classical mathematics have their properties independently from us and are static. Choice sequences, in contrast, depends on the subject (who has to make the choice), and they change through time. They are individual dynamic objects that come into being, at the moment that the subject decides to intend them, and with each further choice, they grow, and they are not necessarily predetermined by some law.

static-dynamic An object is static exactly if at no moments parts are added to it, or removed from it. It is dynamic exactly if at some moment parts are added to it, or removed from it. ([9], page 12)

Since choice sequences are dynamic objects and are accepted as intuitionistic mathematical objects, the notion of “constructive method” cannot be captured by Turing computability. In addition, in intuitionism, the notion of decidability differs from the notion of recursiveness. Although any recursive set is decidable, the converse is not true. From intuitionistic view, a subset $A$ of $\mathbb{N}$, is decidable if and only if there exists a sequence $\alpha \in 2^{\mathbb{N}}$, such that, for every $n$, $n \in A$ if and only if $\alpha(n) = 1$ [3]. It is not required that the sequence $\alpha$ is given by a finite algorithm, it can be a choice sequence.