Double Charge Exchange Reactions and Double Beta Decay.

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Abstract: The subject of this presentation is at the forefront of nuclear physics, namely double beta decay. In particular one is most interested in the neutrinoless process of double beta decay, when the decay proceeds without the emission of two neutrinos. The observation of such decay would mean that the lepton conservation symmetry is violated and that the neutrinos are of Majorana type, meaning that they are their own anti-particles. The lifetime of this process has two unknowns, the mass of the neutrino and the nuclear matrix element. Determining the nuclear matrix element and knowing the cross-section well will set limits on the neutrino mass. There is a concentrated effort among the nuclear physics community to calculate this matrix element. Usually these matrix elements are a very small part of the total strength of the transition operators involved in the process. There is no simple way to “calibrate” the nuclear double beta decay matrix element. The double beta decay is a double charge exchange process, therefore it is proposed that double charge exchange reactions using ion projectiles on nuclei that are candidates for double beta decay, will provide additional necessary information about the nuclear matrix elements.

1. Introduction

One of the most important fields of study in modern nuclear physics is double beta decay in nuclei. There are a number of nuclei where such decays are energetically allowed. These are usually ground state to ground state transitions. There is presently an intense activity both in experiment and theory in the nuclear and particle physics to study double beta decays. There are two types of decays, in the first decay, the process is accompanied by two neutrino emission while in the second, more interesting one, the so called neutrinoless decay, there is no emission of neutrinos, (see for example [1].) The observation of neutrinoless decay will put serious constraints on various gauge models beyond the standard one. It would mean that the lepton number conservation symmetry is broken, that the neutrino is a Majorana particle that is, its own anti-particle. The expression of the decay rate of the neutrinoless process involves basically two unknowns, the mass of the neutrino and the nuclear transition matrix element. Having determined the nuclear matrix element one will be able to find the mass of the neutrino. In various experiments double beta decays emitting two neutrinos have been observed [2] however the neutrinoless double beta decay, as of today, was not detected. Only lower limits for the lifetime of such reaction were set. As mentioned there are many ongoing experiments, and many more are planned to study the neutrinoless double beta decay [1, 3].

The operators that are involved in the beta decay and double beta decay are the Fermi and Gamow-Teller. In the double beta-decay with neutrinos emitted, there is no r dependence and the Fermi type operator does not contribute since the ground state of the final nucleus is not the double isobaric analog of the initial state. (There might be a negligible contribution due to a very small isospin admixture). The important transition is of double Gamow-Teller type. In the case of the neutrinoless process there is essentially a 1/r dependence in the operators and the Fermi transition does contribute, but still it is small compared to the Gamow-Teller transition. The nuclear matrix elements for the double beta decays are very small, 1% or less of the total double Gamow-Teller strength (DGT) [4, 5]. In the past we wrote [4]: “The (ββ-decay) matrix element, is very small and accounts for only a 10−4 to 10−3 of the total DGT sum rule. A precise calculation of such hindered
transition is, of course, very difficult and is inherently subject of large percent uncertainties. At present there is no direct way to “calibrate” such complicated nuclear structure calculations involving miniature fractions of the DGT transitions. By studying the stronger DGT transitions and, in particular, the giant DGT states experimentally and theoretically, one may be able to “calibrate” to some extent, the calculations of ββ-decay nuclear matrix elements. In double charge exchange (DCX) reactions at least two nuclei are involved in the process. This is also the case in the double beta decays. It is therefore natural to look for DCX reactions to provide some information about the various aspects of nuclear structure involving the dynamics of two nucleons. In the past pion DCX reactions were extensively studied because of the existence of intensive pion beams produced in the so called meson factories. Experimental and theoretical studies of the pion DCX reactions were performed, not necessarily in the double beta decay context. One learned a substantial amount of information about the nature of the reaction, about the role of correlations in nuclei and about the various mechanisms that the reaction proceeds [6, 7]. However as the pion programs in the meson factories were declining these studies of DCX declined. Thoughts were given to use some other projectiles in order to perform DCX reactions. Another way to achieve probes that could produce DCX when scattered is to use light or even heavy ions as projectiles. An isospin $T \geq 1$ ion may undergo a double charge exchange reaction from a $\langle g_{18}\rangle$ state to $\langle g_{28}\rangle$ state. The use of $T = 1$ nuclei as probes is the most convenient case [4].

Presently due the interest in the double beta decay new attempts are being made to perform DCX reactions with heavy ions. We mention just a few: In Catania there is a large program to study DCX using ion beams for nuclei that are candidates of double beta decay studies. The large collaboration is called NUMEN. The main motivation is to help to determine the nuclear matrix elements for neutrinoless double beta decay. Reaction with ions O18 to Ne18, etc., are planned [8]. There is a DCX program using ions at RIKEN, Japan which is related to double beta decay studies but also emphasizes the exploration of new nuclear structure, as for example observing double Gamow-Telle states, suggested some years ago [4], the study of tetraneutron states [9]. Work at Osaka University [10] consists again of using light ions to study excitations of double charge exchange states, comparing the results to the pion DCX reactions [11] and exploring the possibility that these studies will help to determine better the nuclear matrix elements appearing in the double beta decays.

2. Isotensor giant resonances

A “model” giant resonance (state built on a state $|n\rangle$) is:

$$|Q_{\alpha}; n\rangle = Q_{\alpha}|n\rangle/\langle n|Q_{\alpha}^+Q_{\alpha}|n\rangle|^{-1/2} \quad (1)$$

$Q_{\alpha}$ is a one-body operator, $Q_{\alpha} = \sum_i q_{\alpha}(i)$

For $|0\rangle$, the g.s., we have the “usual” giant resonances.

The operator $Q_{\alpha}$ might depend on isospin. For example the electric dipole is an isovector and the corresponding operator is:

$$Q_{\alpha} \equiv D = \sum_i r_i Y_i(\theta_i) r_{\mu}(i), \text{ with } \mu = 0, \pm 1 \quad (2)$$

In addition to the “common” $\mu = 0$ dipole one has also the charge-exchange analogs $\mu = \pm 1$.

The action of the operator with $\mu = -1$ (or $\mu = 1$) twice will lead to states with $\Delta T_z = \pm 2$.

These states can be reached in double charge-exchange (DCX) processes.
One of the best examples, are the pion DCX reactions \((\pi^+,\pi^-), \ (\pi^-,\pi^+)\), studied extensively in the past. Because we deal with isotensor transitions, the selectivity of the DCX is large and enhances the possibilities to observe double giant resonances (states). Examples of isotensor states:

Double isobaric analog state (DIAS): The IAS is defined as:

\[
|A_1\rangle = T_+ |0\rangle / (N - Z)^{1/2}
\]

The DIAS is:

\[
|A_2\rangle = T_+^2 |0\rangle / [(N - Z)(N - Z - 1)]^{1/2}
\]

Dipole built on an analog:

\[
|D_{-1}; A_1\rangle = \sum_i \tau_i Y_{1}(\theta_i) \tau_{-1}(i) |A_1\rangle / N
\]

Yet another example, the double dipole (for the \(\Delta T_z = -2\)):

\[
|D_{-1}; D_{-1}\rangle = \sum_i \tau_i Y_{1}(\theta_i) \tau_{-1}(i) |D_{-1}\rangle / N
\]

The operators could also depend on spin, for example the Gamow-Teller operator is:

\[
G_{\pm} = \sum_i \sigma(i) \tau_{\pm}(i)
\]

The giant Gamow-Teller (GT) state:

\[
|GT\rangle = G_+ |0\rangle / N
\]

The double Gamow-Teller (DGT) state:

\[
|DGT; J\rangle = G_2^2(J) |0\rangle / N(J)
\]

J is the total spin of the DGT. For two identical phonons the wave function is symmetric in space-spin-isospin, therefore for \(\Delta T_z = 0, 2\) \(J = 0^+, 2^+ \ldots\).

Some properties of these states:

Excitation energies: \(E_{Q_1} \approx E_{Q_2} \approx E_{Q'_1} + E_{Q'_2}\)

Widths (spreading widths) \(\Gamma_{n, \lambda} = n \Gamma_{1; \lambda}\). In some theories \(n\) is replaced by \(\sqrt{n}\)

Isospin and intensities: for an isotensor excitation \(\Delta T_z = -2\) and for \((N - Z) \geq 2\) there are five isospin values \(T' = T + 2, \ T + 1, \ T, \ T - 1, \ and \ T - 2\).

For an isotensor of rank \(k\) \(F_{\mu}^{(k)}\) the intensities for the various isospin components \(T'\) are given by the corresponding Clebsh-Gordan (CG) coefficients and reduced matrix elements:

\[
S_{T'} = |\langle T, T', k, \mu | T', T + \mu \rangle|^2 \langle T' | F^{(k)} | T \rangle^2
\]

The CG coefficient for \(T \gg k\) (in our case \(T \gg 2\)) are dominated by the CG of the aligned isospin \(T' = T + \mu\). For \(-k, \ (CG)^2 = \frac{2(T-k+1)}{2T+1}\).

In the pion DCX reactions the various spin independent DGRs listed above were excited in several nuclei [7, 11]. These observations were confirmed recently in experiments in the double charge exchange reaction \((B^{\pm1}_2, Li^{\pm1}_2)\) [10].

3. DCX and Correlations in Nuclei.
The DCX process has held out the hope that it would be a means of probing two-body correlations in nuclei. In the initial studies it was found that uncorrelated nuclear wave functions produced qualitative agreement with experiment at higher energies (pion energies around 300 MeV.) (The cross-sections were proportional to (N-Z) (N-Z-1)). This was the situation when the double giant resonances were studied. However at low pion energies (around 50 MeV) the disagreement with experiment was very large, (sometimes a factor of 50) when uncorrelated wave functions were used to study the double isobaric analog state. It was necessary to include wave functions with correlated nucleons.

The DCX process, as determined in pion charge-exchange reactions to the DIAS involves two basic routes.

1. For uncorrelated particles in the transition to a double analog state the route from the initial state to the final state goes via the excitation of the single analog in the intermediate stage, and from there in a charge-exchange to the double analog. This is termed as the analog route or more generally as the “sequential” process. The cross section is proportional to the number of pairs one can form from the n nucleons, \( n(n-1)/2 \).

2. The second process involves correlated nucleons and the process proceeds through intermediate states that are not the analog. This we term the “non-analog route”.

An expression that represents this approach was derived [6] for a DCX transition between the J=0\(^+\) and its double analog, for a single configuration \( j^n \), where n is an even number of neutrons occupying the orbit.

The DCX cross section can be written as:

\[
\sigma_{DCX} = \frac{n(n-1)}{2} \left| A + \frac{(2j+3+2n)}{(2j-1)(n-1)}B \right|^2
\]  
(12)

This formula is remarkable in its simplicity. The DCX cross section to the DIAS has two amplitudes A and B (which are angle and energy dependent but j independent). The first represents transitions which occur in the absence of correlations in the nuclear wave function. The cross section due to this term is proportional to the number of neutron pairs to be made into pairs of protons when going from the ground state to the DIAS. This counting rule is independent of the mutual location of the two nucleons in the pair and therefore this dominates the cross section when the DCX operator is of long range. The second term represents the DCX transitions which take place when the nuclear wave function is more than just that of independent particles, that is it involves correlations. This term will contribute when the DCX transition operator is of short range. The first term corresponds to the “sequential” while the second one represents the case when the intermediate states are non-analog states [6].

The above formula is extended to the generalized seniority scheme when several orbits are involved [6, 12, 13]. Then:

\[
\sigma_{DCX} = \frac{n(n-1)}{2} \left| A + \frac{(\Omega+1-n)}{(\Omega-1)(n-1)}B \right|^2 ; \text{ with } \Omega = \sum_i (j_i + 1/2)
\]  
(13)

The characteristic behavior of the DCX amplitude to the DIAS as a function of the number of valence neutrons for a single j orbit persists also when there is rather weak configuration mixing. The change is absorbed by the altered values of the A and B coefficients in such a way that the ratio \( B/A \) is increased, meaning that the effective DCX operator is of shorter range. But even in the case of stronger mixing the effect could be similar. This may suggest why certain models in which a
limited space is employed can provide satisfactory results when effective operators are used. (An example is the IBM description of the double beta decay).

**Double beta decay and DCX**

The process of double beta decay is very similar to the nuclear structure part of DCX reactions. We will not present the specifics of double beta decay. One can find this in the extensive literature. We will concentrate on common features of these processes. In particular the similarities are close when we deal with DCX reactions involving the Fermi and Gamow-Teller strength. For example the two-neutrino amplitude can be written as:

\[
\mathcal{A}_{\alpha} = \sum |f|G_{\alpha}|n|G_{\alpha}|l\rangle \langle G_{\alpha}|n|G_{\alpha}|l\rangle
\]  

Where \(G_{\alpha}\) is the Gamow-Teller operator defined above, eq. (7). (The 0-neutrino amplitude includes also a Fermi amplitude and the operator has an \(r\) dependence.)

The total coherent DGT strength is:

\[
S_{DGT} = |\sum f|G_{\alpha}|n|G_{\alpha}|l\rangle \langle G_{\alpha}|n|G_{\alpha}|l\rangle|^2
\]  

\[
S_{2\beta} = |\sum f|G_{\alpha}|n|G_{\alpha}|l\rangle \langle G_{\alpha}|n|G_{\alpha}|l\rangle|^2
\]  

is the double beta decay strength (in the closure approximation) and is a coherent sum also. Thus:

\[
S_{2\beta} \sim |\langle G_{\alpha}\rangle|G_{\alpha}\rangle|G_{\alpha}\rangle|G_{\alpha}\rangle|^2
\]  

One can approach this problem of the distribution of DGT strength in a straightforward way [14, 15, 16]. One calculates the shell-model states in an extended model space in the parent nucleus, (\(N, Z\)), in the intermediate nucleus (\(N - 1, Z + 1\)), and in the final nucleus (\(N - 2, Z + 2\)). Having determined the nuclear wave functions one then evaluate directly the incoherent sum and coherent sum. In figure 1 we show the coherent running sum of the two neutrino beta decay of \(^{48}_{22}Ca\) as a function of the intermediate states. We note that this is not a monotonic function and some of the contributions are positive and some negative. This happens for all nuclei we calculated and all the different interactions used.

Some (n, p) studies were performed in which one chooses as the target states the final states of the relevant double beta decay process. In these (n, p) experiments one measures the GT strength to the various intermediate single charge exchange states. One can then take and multiply the two GT strengths obtained in (p,n) and (n,p) to the same intermediate states obtained in each of the two reactions and try to sum over the intermediate states observed. For example, let us take the \(A = 48\) nuclei. The (p, n) and (n, p) reactions are performed correspondingly on the \(^{48}_{20}Ca\) and \(^{48}_{22}Ti\) targets. The intermediate states measured are in both reactions are located in the \(^{48}_{20}Sc\) nucleus. Actual experiments were performed at the initial nucleon energies of 200 MeV. At these energies the spin states and in the forward direction GT states in particular are excited predominantly. Using the DWIA analysis, one was able to extract the GT strength for the various intermediate states observed in these two reactions. Note that only the squares of the matrix elements of the type \(\langle j_f|\sigma\tau|n\rangle\) and \(\langle n|\sigma\tau|j_f\rangle\) can be deduced from these two one-body experiments. The relative signs are not determined in such processes. By measuring transitions to a great number of intermediate states, one in principle can determine the incoherent sum for the g.s. to g.s. transition. There are several difficulties in this procedure even when one tries to determine the incoherent sum. First, because of final experimental resolution on the one hand and large density of intermediate states on the other, one is not always sure that it is the same matching intermediate state that is excited when the (p,n) and (n, p) experiments are compared. Second, and related to it is the fact that when
a strong GT transition in (p, n) is observed to a given intermediate state, the (n, p) transition to the same intermediate state might be (and often is) weak, and vice versa. This of course makes the experimental exploration of the double beta decay amplitude difficult. The ground states in the two target nuclei are not related by any simple transformation and therefore the action of the $\sigma\tau_+\tau_-$ operators leads to mismatched distributions of strength in the intermediate nucleus. The combined studies of (p, n) and (n, p) may provide only limited information concerning the nuclear structure aspects of the $2\beta$ decay matrix elements. In view of the things discussed above it is clear that because of the coherence of the $2\beta$ decay matrix elements, the use of (n, p) and (p, n) strength cannot determine the value of such matrix elements. Also, quite obviously in such experiments, one cannot excite all DGT strengths. In order to be able experimentally to study the DGT strength (including the $2\beta$ decay g.s. to g.s. transition), one must employ processes in which the leading terms are genuine two-body transitions. Double charge-exchange (DCX) reactions are the natural candidates for such a study.

We should point out that one of the difficult problems in the theoretical studies of double beta decay is the “universal” quenching of the single Gamow-Teller strength. Experiments show that 30-40% of GT strength is missing in the main GT peaks. This affects the double beta decay transition matrix elements. The source of the above quenching is not certain. There are several ideas. One is that the missing strength is due to the $\Delta-h$ configurations interacting with the GT states and the missing strength is 300 MeV above the GT peak [17]. Another theory is that the GT strength is fragmented and the missing strength is several tens of MeV above the main GT peak [18, 19, 20]. These two different theories will affect the double beta decay matrix element differently. The uncertainty, (because of this quenching), could be as large as a factor of 2 for double beta decay matrix elements.

Acknowledgement

We wish to thank the Bulgarian Academy of Science for the hospitality during authors visit in Bulgaria.
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