Fractal Dimension of Grain-boundary Fracture for Characterization of High-temperature Fracture in Heat-resistant Alloys

Manabu TANAKA,1) Junji ONO,2) Manabu SAKASHITA3) and Ryuichi KATO1)

1) Department of Mechanical Engineering, Faculty of Engineering and Resource Science, Akita University, 1-1 Tegatagakuen-cho, Akita 010-8502 Japan. 2) Student, Department of Mechanical Engineering, Faculty of Engineering and Resource Science, Akita University, 1-1 Tegatagakuen-cho, Akita 010-8502 Japan. 3) Student, Department of Mechanical Engineering, Faculty of Engineering and Resource Science, Akita University, 1-1 Tegatagakuen-cho, Akita 010-8502 Japan.

(Received on September 26, 2008; accepted on March 24, 2009)

The fractal dimension of the grain-boundary fracture, \( D_f \), which represents the fracture surface pattern with grain-boundary microcracks in three-dimensional space, is proposed for characterization of high-temperature fracture in materials. The value of \( D_f \) as well as its two dimensional value, \( D_{fp} \), was estimated in the length scale range larger than about one grain-boundary length using the height data of fracture surfaces of heat-resistant alloys obtained by the stereo matching method. The value of \( D_f \) increased with increasing fractal dimension of the grain-boundary surface profile \( D_{GB} \) in the specimens of the HS-21 alloy ruptured at 1 089 K. Both rupture life and creep ductility increased with increasing value of \( D_f \) in these specimens. Similar results were obtained by the two-dimensional fractal analysis on other specimens of cobalt-base, nickel-base and iron-base heat-resistant alloys. Creep fracture process including the growth of the main creep crack was examined by the fractal analysis using the fractal dimension map (FDM, a color-coded map) on the surface notch obtained. The result of the fractal analysis was compared with that of the FRASTA (fracture surface topography analysis) in the Inconel X-750 alloy. The fractal analysis used in this study is more convenient and more advantageous than the FRASTA, and is widely applicable to the investigation of high-temperature fracture in materials.

KEY WORDS: fractal dimension; grain-boundary fracture; creep; fracture surface; fracture process; microcracks; crack growth.

1. Introduction

B. B. Mandelbrot et al.\(^1\)) first described the geometrical features of impact fracture surfaces in steels by the fractal dimension. The fractal dimension of the fracture surface represents self-similarity and complex nature of microstructures on fracture surfaces of materials, depending on the length scale range of the fractal analysis.\(^2\)) In the grain-boundary fracture of heat-resistant alloys, the fractal dimension of the grain-boundary fracture surface profile, \( D_{GB} \) \( (1<D_{GB}<2) \), is associated with patterns formed by grain-boundary microcracks on the fracture surface of ruptured specimens when estimated in the length scale range larger than about one grain-boundary length.\(^3,4\)) The value of \( D_{GB} \) is larger in the specimens of heat-resistant alloys ruptured under the lower creep stresses.\(^3,4\)) This is attributed to that the line density of grain-boundary microcracks on the fracture surface \( n_c \) increases with decreasing creep stress owing to the increased contribution of grain-boundary sliding to total creep deformation.\(^5,6\)) In the surface notch obtained specimens of the 21Cr-4Ni-9Mn steel, the value of \( n_c \) decreased with increasing distance from notch root (the nucleation site of the main creep crack) and led to the decrease in the value of \( D_{GB} \).\(^7\)) However, the creep fracture process has not been examined on the three-dimensional fracture surfaces of the specimens ruptured at high temperatures because of severe oxidation of fracture surfaces.

Computer-aided stereo matching method has been successfully applied to the reconstruction of three-dimensional images of fracture surfaces in materials.\(^8-11\)) Kobayashi and Shockey\(^9\)) examined the fracture process and the fracture toughness of thermally embrittled cast duplex stainless steels using the fracture surface topography analysis (FRASTA), in which the fracture process was simulated by matching the height data of the conjugate fracture surfaces obtained by the stereo matching method. Ueno et al.\(^12\)) applied the FRASTA using a scanning laser microscope to the simulation of creep fracture process. However, this method has some problems including precise positioning of the conjugate fracture surfaces in the acquisition of the height data and in the matching process\(^13\)) and is also time-consuming. Further, it is difficult to examine the crack growth process by the FRASTA, if the fracture surface morphology is changed by severe oxidation which results in inaccurate matching of the conjugate fracture surfaces. On the contrary, the fractal dimension of fracture surfaces is almost unaffected by undulation whose size is smaller than one grain-boundary length, if the fractal dimension is estimated in the length scale range larger than one grain-boundary length. Therefore, the fractal dimension of fracture surfaces is almost unaffected by oxidation of fracture surfaces and can characterize the morphology of the grain-boundary fracture surface containing microcracks in this case.

In this study, the fractal dimension of the grain-boundary fracture, \( D_f \) \( (2<D_f<3) \), or its two-dimensional value, \( D_{fp} \), was estimated in the length scale range larger than about one grain-boundary length on the specimens with different grain-boundary configurations (different fractal dimension...
of the grain-boundary surface profile, $D_{GB}$, $1 < D_{GB} < 2$) of cobalt-base, nickel-base and iron-base heat-resistant alloys. The relationship between the rupture properties and the value of $D_i$ or $D_p$ was examined on those specimens. Further, the creep fracture process was examined by estimating the values of $D_f$ on small areas and displaying as the fractal dimension map (FDM, a color-coded map) on the fracture surface of the surface-notched specimens. The result of the fractal analysis was compared with that of the FRASTA in the Inconel X-750 alloy.

2. Experimental Procedure

Table 1 lists the chemical composition of heat-resistant alloys used in this study. These alloys were supplied in the form of round bars of about 20 mm in diameter. Table 2 lists the heat treatment, the fractal dimension of the grain-boundary surface profile ($D_{GB}$), the grain diameter ($d$) and the hardness of specimens used for creep experiments and fractal analysis. The value of grain-boundary length, $L$, was calculated from the relationship between the value of $L$ and the grain diameter, $d$, such as $L = d/3^{1/2} \approx (0.577d)$ on the assumption that the surface profile of a grain is hexagonal. Figure 1 shows the estimation of the fractal dimension of surface profiles by the box-counting method and an example of the measurement on the fractal dimension of the grain-boundary surface profile ($D_{GB}$) in a heat-treated specimen of the Inconel X-750 alloy.

![Figure 1](image_url)

**Fig. 1.** Estimation of the fractal dimension of surface profiles by the box-counting method and an example of the measurement on the fractal dimension of the grain-boundary surface profile ($D_{GB}$) in a heat-treated specimen of the Inconel X-750 alloy. (a) Schematic illustration of the box-counting method ($r$: box size, $N$: number of boxes covering a profile and $N=15$ in this case). (b) Fractal dimension of the grain-boundary surface profile ($D_{GB}$) in the Inconel X-750 alloy (heat treatments: 1423 K, 7.2 ks → water-quenched + 1 123 K, 86.4 ks → air-cooled + 973 K, 72 ks → air-cooled).

### Table 1. Chemical composition of heat-resistant alloys used in this study (mass %).

| Alloys   | C  | N  | Cr | Ni | Co | Fe | Mn | Al | Ti | W  | Nb | Mo | Si | S  | P   |
|----------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----|
| HS-21 alloy | 0.27 | B  | 0.003 |
| Inconel X-750 alloy | 0.06 | 15.03 | Ni(Cu) | 71.80 |
| 2Cr-4Ni-9Mn steel | 0.54 | 21.10 | 4.07 | bal. |
| L-605 alloy | 0.07 | 19.92 | 9.93 | bal. |

### Table 2. The heat treatment, the fractal dimension of the grain-boundary surface profile ($D_{GB}$), the grain diameter ($d$) and the hardness of specimens used for creep experiments and fractal analysis.

| Alloys   | $D_{GB}$ | grain diameter, $d$ (grain-boundary length, $L$) (10$^{-6}$ m) | matrix hardness, Hv (load, 4.9 N) | heat treatment (grain-boundary configuration) |
|----------|----------|---------------------------------------------------------------|-----------------------------------|------------------------------------------|
| HS-21 alloy | 1.066 | 130 (75) | 315 | 1523 K, 0.6 ks → W.Q. (annealed) |
| Inconel X-750 alloy | 1.056 | 130 (75) | 418 | 1523 K, 3.6 ks → P.C. → 1323 K → W.Q. (annealed) |
| 2Cr-4Ni-9Mn steel | 1.107 | 130 (75) | 407 | 1523 K, 3.6 ks → A.C. + 1088 K, 10.8 ks → A.C. * (intermediate) |
| L-605 alloy | 1.241 | 130 (75) | 405 | 1523 K, 3.6 ks → P.C. → 1323 K → W.Q. + 1088 K, 10.8 ks → A.C. * (annealed) |

*Heat treatment of the specimens used in the previous study: $^{15,16}$ heat treatment of surface-notched specimens: W.Q.: water-quenched; A.C.: air-cooled; P.C.: furnace-cooled.

© 2009 ISIJ

1230
of the fracture surfaces was carried out using the stereo pair images by the computer-aided stereo matching method. In the resulting elevation map (height data) of the 256-gray scale levels, the relative height of the fracture surface increases with increasing color number (brightness). The fractal dimension of the fracture surface, \( D_f \), was estimated by a computer program of the box-counting method using the elevation map of the fracture surfaces. In this case, the number of boxes, \( N \), covering the three-dimensional fracture surface can be related to the box size, \( r \), through the fractal dimension, \( D_f \), by the following power law relationship:

\[
\log N = k - D_f \log r \tag{1}
\]

where \( k \) is a constant. The value of \( D_f \) can be calculated from Eq. (1) by the regression analysis using the datum sets of \( N \) and \( r \).

High-temperature fracture surfaces in heat-resistant alloys are generally characterized by the grain-boundary fracture pattern with many microcracks on grain boundaries. Therefore, the value of \( D_f \) estimated in the length scale range larger than about one grain-boundary length, which represents the fracture pattern with grain-boundary microcracks, is defined as “the fractal dimension of the grain-boundary fracture, \( D^*_f \),” in this study. The fractal dimension of the grain-boundary fracture surface profile, \( D_{gb} \) (the two-dimensional value of \( D_f \)) was estimated in the similar length scale range in the polished and etched plane of the ruptured specimen vertically sectioned along the specimen axis. Figure 2 shows the optical micrographs of the specimens solution-treated and then aged for 10.8 ks at 1089 K of the HS-21 alloy. Grain boundaries are serrated with large M23C6 type carbide particles in the specimen with the largest \( D_{gb} \) value of 1.241 (Fig. 2(d)), while they are almost straight in the specimen with the smallest \( D_{gb} \) value of 1.056 (Fig. 2(a)). Specimens between these two extreme \( D_{gb} \) values have intermediate grain boundaries (Figs. 2(b) and 2(c)). Fine M23C6 precipitates can also be observed in the grains.

The specimens cut from the alloy bars were heat-treated to give different grain-boundary configurations, almost same grain diameter and similar initial matrix hardness. The heat-treated specimens were machined into the creep specimens of 30 mm gauge length and 5 mm diameter. Creep-rupture experiments were carried out using these specimens in the temperature range from 973 to 1311 K, where grain-boundary fracture is the dominant fracture mechanism. Some ruptured specimens of the HS-21 alloy are the same as those used in the previous study. For the surface notched specimens, a notch of about 3 mm in projected length and 0.4 mm in width (about 0.5 mm in depth) was introduced by machining on the surface of the creep specimens. Creep crack growth experiments were conducted using the surface notched specimens of the HS-21 alloy and the Inconel X-750 alloy. The density \( (N) \) or the line density \( (n) \) of grain-boundary microcracks on the fracture surface was examined on the ruptured specimens.

3. Analytical Method

Stereo pairs of scanning electron micrographs (basic image and image tilted by 10 deg.) of the fracture surfaces were taken using a scanning electron microscope and a digital camera on the ruptured specimens of heat-resistant alloys. The stereo pair images were taken into a personal computer, and the three-dimensional image reconstruction...
composing the level-sliced height images of the fracture surface obtained by the stereo matching method. The two-dimensional images used in the matching process were also produced using the fracture surface profiles of the conjugate fracture surfaces generated by the stereo matching method. The result of the fractal analysis using the FDM was compared with that of the FRLSTA.

4. Results and Discussion

4.1. Relations between Rupture Properties and Fracture Pattern

Three-dimensional image reconstruction of the fracture surface was at first made by the stereo matching method on the ruptured specimens that were the same as those used in the previous study.14 Figure 3 shows the fracture surface of the specimen with “straight” grain boundaries and the one with “serrated” grain boundaries of the HS-21 alloy ruptured under a stress of 108 MPa at 1 089 K. (a–c) Specimen with straight grain boundaries (the fractal dimension of the grain-boundary surface profile, \( D_{GB} = 1.056 \)). (d–f) Specimen with serrated grain boundaries (\( D_{GB} = 1.241 \)). (a, d) Basic image of reconstructed area (779×675 in pixel) (examples of microcracks are indicated by arrows). (b, e) Elevation map (the area for fractal analysis (660×660 in pixel) is enclosed by white line). (c, f) High magnification micrograph.

The fracture surface involves many grain-boundary microcracks (for example, indicated by arrows in Figs. 3(a) and 3(d)). The height data of the fracture surface are displayed as the elevation map (Figs. 3(b) and 3(e)). The area for fractal analysis is enclosed by a white line in the elevation map (Figs. 3(b) and 3(e)). Unfortunately, the details of grain-boundary microstructures cannot be known on these fracture surfaces even at the higher magnification because of severe oxidation in both specimens (Figs. 3(c) and 3(f)). The fracture surface is more complicated and involves more grain-boundary microcracks in the specimen with the larger value of \( D_{GB} \) (Fig. 3(d)). The complex geometry of the fracture surface can also be known from the elevation map (Fig. 3(e)).

Figure 4 shows the number of boxes \( N \) covering the fracture surface versus the box size \( r \) in the measurement of the fractal dimension of the fracture surface by the box-counting method on the specimens with different grain-boundary configurations of the HS-21 alloy ruptured under a stress of 108 MPa at 1 089 K. The fractal dimension exhibits different values in different length scale ranges of the fractal analysis smaller than and larger than about one grain-boundary length \( (7.5 \times 10^{-5} \text{ m}) \) in the same specimen. Such dependence of the fractal dimension on the length scale range of the fractal analysis has already been known in many heat-resistant alloys.19 Figure 5 shows the relationship between the fractal dimension of the fracture surface \( D_f \) and the fractal dimension of the grain-boundary surface profile \( D_{GB} \) in the specimens of the HS-21 alloy ruptured at 1 089 K. The fractal dimension of the fracture surface \( D_f \), which is estimated in the length scale range smaller than about one grain-boundary length \( (7.5 \times 10^{-5} \text{ m}) \), lies between about 2.19 and 2.24 and does not clearly depend on the fractal dimension of the grain boundary surface profile \( D_{GB} \). This result is very different from that of the two-dimensional fractal analysis, because the fractal dimension of the fracture surface profile estimated in this length scale range is generally larger in the specimens with the larger \( D_{GB} \) values.4,19 In the two-dimensional fractal analysis, the fractal dimension is estimated on the fracture surface profile, and may not be affected by oxide layer formed on the fracture surface.3,4,7,14,19 The oxide layer may mask the microstructures on grain boundaries (Figs. 3(c) and 3(f)), and may affect the results of the three-dimensional fractal analysis, because the fractal dimension of the fracture surface is estimated using the height data of fracture surfaces. The oxide layer seems to affect largely the result of the fractal analysis in the length scale range smaller than about one grain-boundary length, where grain-boundary ledges and steps are characteristic microstructures. On the other hand, the fractal dimension of the fracture surface, \( D_f \), which is estimated in the length scale range larger than about one grain-boundary length, increases with increasing value of \( D_{GB} \). The value of \( D_f \) may represent the fracture surface pattern with grain-boundary microcracks in this scale range. For example, the value of \( D_f \) increases from about 2.16 to about 2.29 with increasing value of \( D_{GB} \) from 1.056 to 1.241 under a stress of 108 MPa. The increase in the value of \( D_f \) with the value of \( D_{GB} \) is relatively smaller under the higher stress (176 MPa).

Figure 6 shows the relationship between the rupture properties and the fractal dimension of the grain-boundary fracture \( D_f \) in the specimens of the HS-21 alloy at
1 089 K. Both rupture life and creep ductility increase with increasing value of $D_f$. The relative increase in rupture life or creep ductility with the value of $D_f$ is larger under the higher stress (176 MPa). In polycrystalline materials, the initiation and growth of grain-boundary microcracks are governed by grain-boundary sliding, and the proportion of the amount of grain-boundary sliding to total creep strain generally increases with decreasing stress. Then, more grain-boundary microcracks are formed under the lower stresses, leading to the larger value of $D_f$ (Fig. 5). The effect of oxidation on the initiation and growth of the microcracks may be larger at the longer rupture lives, and may decrease the toughening effect of serrated grain boundaries under the lower stress (Fig. 6). Nevertheless, the larger fractal dimension of the grain-boundary fracture ($D_f$) is indicative of the larger ductility or the longer rupture life (or the higher strength) in the specimens with different grain-boundary microstructures. Figure 7 shows the relationship between the fractal dimension of the grain-boundary fracture ($D_f$) and the density of grain-boundary microcracks ($N_N$) on the fracture surface in the specimens of the HS-21 alloy ruptured at 1 089 K. The value of $D_t$ can be correlated to the value of $N_N$ by a single curve in the HS-21 alloy, although the value of $D_t$ may represent the fracture surface pattern including both density and spatial distribution of grain-boundary microcracks.

Experimental results similar to Fig. 6 were obtained on the two-dimensional fractal analysis of the fracture surfaces in other ruptured specimens of the heat-resistant alloys. Figure 8 shows the relationship between the rupture properties and the fractal dimension of the grain-boundary fracture surface profile ($D_{GB}$) in the ruptured specimens of cobalt-base, nickel-base and iron-base heat-resistant alloys. The rupture life of the specimen with the larger value of $D_{GB}$ is longer than that of the specimen with the smaller value of $D_{GB}$ in the HS-21 alloy (Fig. 8(a)), the Inconel X-750 alloy and the 21Cr–4Ni–9Mn steel (Fig. 8(b)). The specimen with the larger value of $D_{GB}$ also exhibits the larger elongation in these alloys. However, in the L-605 alloy (Fig. 8(b)), the rupture properties of the specimen with the larger value of $D_{GB}$ (the larger value of $D_{GB}$) are not always superior to those of the specimen with the smaller value of $D_{GB}$ (the smaller value of $D_{GB}$). This may be associated with the fact that the matrix hardness of the specimens with serrated grain boundaries ($D_{GB}=1.173$) is 301 Hv and is a little smaller compared with that of the specimens with straight grain boundaries ($D_{GB}=1.117$), 326 Hv, in the L-605 alloy (Table 2).

4.2. Creep Crack Growth in Surface Notched Specimens

The result similar to Fig. 7 was obtained in the two-dimensional fractal analysis on the fracture surfaces of surface notched specimens of the other alloys. Figure 9 shows the relationship between the fractal dimension of the grain-boundary fracture surface profile ($D_f$) and the line density ($n_C$) of grain-boundary microcracks on the fracture surface at the distance of 0.3 to 1.4 mm from notch root in the surface notched specimens of the 21Cr–4Ni–9Mn steel crept at 973 K. In spite of the differences in the values of $D_f$, the gross section stresses ($\sigma_g$) and the distance from notch root, there is also a relationship between the values of $n_C$ and those of $D_f$ approximated by a single curve in these specimens, although there is a small scatter of the datum points in the figure.

Creep fracture patterns may change in the fracture process in heat-resistant alloys at high temperatures, because the stress field of the main creep crack may affect the local fracture pattern with grain-boundary microcracks. For example, Fig. 10 shows the changes in the fractal dimension of the grain-boundary fracture surface profile ($D_f$), the line density of grain-boundary microcracks ($n_C$) and the crack growth rate with the growth of the main creep crack in the surface notched specimens of the 21Cr–4Ni–9Mn steel crept at 973 K. The value of $D_f$ decreases with increasing distance from notch root ($y$) (with the growth of the main creep crack) in both the specimens with different $D_{GB}$ values (Fig. 10(a)). Similarly, the value of $n_C$ decreases with distance from notch root ($y$), and is larger in the specimens with the larger value of $D_{GB}$ under the lower gross section stresses ($\sigma_g$) (Fig. 10(b)). The value of $D_f$ also decreases with increasing gross section stresses ($\sigma_g$) in the specimen with serrated grain boundaries ($D_{GB}=1.233$),
while a good correlation is not observed between the value of $s_g$ and that of $D_{GB}$ in the specimen with straight grain boundaries ($D_{GB}/H11005 = 1.094$). This may be related with the smaller stress ($s_g$) dependence of the value of $n_C$ in the specimen with $D_{GB}/H11005 = 1.233$ compared with the specimens with $D_{GB}/H11005 = 1.094$. The value of $D_{fp}$ is larger in the specimens with the larger fractal dimension of the grain-boundary surface profile ($D_{GB}/H11005 = 1.233$). The crack growth rate is lower in the specimens with the larger value of $D_{GB}$ ($1.233$) than in those with the smaller value of $D_{GB}$ ($1.094$) under all gross section stresses ($s_g$) tested (Fig. 10(b)).

The growth of the main creep crack from notch root results in the increased net section stress of the specimens, which leads to the accelerated crack growth rate (Fig. 10(b)) and the decreased contribution of grain-boundary sliding to the total creep strain. As a result, the density of grain-boundary microcracks of which nucleation and growth are governed by grain-boundary sliding, decreases with the main crack growth. This is why the value of $D_{fp}$ decreases with the main crack growth in the creep fracture (Fig. 10(a)). The faster creep crack growth probably occurred in the area of the smaller fractal dimension where the creep crack growth was accelerated under the higher net section stress. As known from Fig. 10, the region of the largest fractal dimension is close to the nucleation site of the main creep crack (notch root), at which the crack growth rate is very low. Thus, the larger $D_{fp}$ values also correspond to the lower creep crack growth rates.

Creep crack growth and change in the fracture pattern with the crack growth were also examined on the surface notched specimens of the HS-21 alloy and the Inconel X-750 alloy. Figure 11 shows the increase of crack depth with time in the surface notched specimens during creep. The rupture lives of these specimens are shown in the figure. The main creep crack initiated at the notch root in the later stage of creep (86.4 ks) in the HS-21 alloy, while the crack was found at the notch root in the relatively early stage of creep (72 ks) in the Inconel X-750 alloy. Three-dimensional fractal analysis was made on the fracture surfaces of these surface notched specimens.

Figure 12 shows the fracture surface of the surface notched specimen of the HS-21 alloy tested under a gross...
section stress ($\sigma_g$) of 88.2 MPa at 1089 K. The reconstructed region (within the solid line in Fig. 12(a)) does not include the notched part but involves the area close to the notch root. Only grain-boundary fracture was observed in this specimen. The fracture surface has complex geometry with many grain-boundary microcracks (as indicated by arrows in Fig. 12(a)), especially in the region close to the notch root. The main creep crack has grown in the $y$-direction in Fig. 12(b). The values of the fractal dimension of the surface notched specimens during creep. (a) HS-21 alloy, (b) Inconel X-750 alloy.

Fig. 11. Increase of crack depth with time in the surface notched specimens during creep. (a) HS-21 alloy, (b) Inconel X-750 alloy.

Fig. 12. Fracture surface of the surface notched specimen of the HS-21 alloy tested under a gross section stress ($\sigma_g$) of 88.2 MPa at 1089 K. (a) Basic image (the reconstructed region (775$\times$681 in pixel) is within the solid line and the mapped area (600$\times$504 in pixel) is enclosed by the broken line). (b) Bird’s-eye view. (c) Fractal dimension map (the maximum crack depth is 1.15 mm from notch root at 91.9 ks and the hypothetical crack tip is shown by broken line) (some grain-boundary microcracks are indicated by arrows in (a)).

Fig. 13. Fracture surface of the surface notched specimen of the Inconel X-750 alloy tested under a gross section stress ($\sigma_g$) of 343 MPa at 973 K. (a, b) Basic image (the reconstructed region (775$\times$681 in pixel) is within the solid line and the mapped area (600$\times$504 in pixel) is enclosed by the broken line) (some grain-boundary microcracks are indicated by arrows in (a)); (c, d) Bird’s-eye view. (e, f) Fractal dimension map (the maximum crack depth from notch root is 1.54 mm, at 265.5 ks) (black broken line indicates the crack tip and the corners enclosed by white broken lines are the regions failed in transgranular manner).

Fig. 14. Analysis of creep fracture process by the fractal dimension map (FDM) on the surface notched specimen of the Inconel X-750 alloy tested under a gross section stress ($\sigma_g$) of 343 MPa at 973 K (the displayed area is 600$\times$504 in pixel). (a) The maximum crack depth is 0.88 mm ($t=255.8$ ks) (the lower fracture surface). (b) The maximum crack depth is 1.54 mm ($t=265.5$ ks) (the lower fracture surface). (c, d) At the stage immediately before final rupture ($t=268.50$ ks) (c) is the lower fracture surface and (d) is the upper fracture surface) ($t$ is the time corresponding to the maximum crack depth from notch root in Fig. 13(b), $t_r$ is rupture life, and the maximum crack depth is shown by black broken line in (a) and (b)) (the corners enclosed by white broken lines are the regions failed in transgranular manner).
Figure 12 shows the surface notched specimen of the Inconel X-750 alloy tested under a gross section stress of 343 MPa at 973 K. The figures of the upper fracture surface (Figs. 13(b), 13(d) and 13(f)) are almost mirror symmetry to those of the lower fracture surface (Figs. 13(a), 13(c) and 13(e)). The fracture appearance seems to be almost similar (Figs. 13(c) and 13(d)) but the details of the fracture pattern are somewhat different even in the conjugate fracture surfaces (Figs. 13(a) and 13(b)). Many grain-boundary microcracks can be seen in the region close to the notch root, although almost no cracks are observed in the lower part of the fracture surface (Figs. 13(a) and 13(b)). The main creep crack has grown in the y-direction in Figs. 13(c) and 13(d). Figures 13(e) and 13(f) show the FDMS corresponding to the areas within the broken line in Figs. 13(a) and 13(b), respectively. The displayed range of the values of \( D_f \) was chosen in order to make clear the geometry of the main creep crack in Fig. 13(f). Almost semi-circular area with color close to red (the larger value of \( D_f \)) which contains many grain-boundary microcracks, is located in the upper region (Figs. 13(a) and 13(b)). This area may correspond to the main creep crack with the maximum crack depth of 1.15 mm from notch root at 91.9 ks (the time is known from Fig. 11(a), and the hypothetical crack tip is shown by broken line). On the contrary, the region with color close to blue (the smaller value of \( D_f \)) in the lower part seems to be the area of final fracture. Only a small number of grain-boundary microcracks can be seen in this region. Further, there are some areas with orange color (relatively larger value of \( D_f \)) ahead of the main crack, which may correspond to preceding cracks (Fig. 12(c)).

Figure 13 shows the fracture surface of the surface notched specimen of the Inconel X-750 alloy tested under a gross section stress (\( \sigma_g \)) of 343 MPa at 973 K. The figures show the perspective images (775x681 in pixel) and the two-dimensional images obtained by the FRASTA on the surface notched specimen of the Inconel X-750 alloy under a gross section stress (\( \sigma_g \)) of 343 MPa at 973 K. Cracked region is shown in gray in Figs. 15(a), 15(b) and 15(c). The main creep crack is assumed to grow from the upper edge of these figures. The maximum depth of the main creep crack at a given time was known from Fig. 11(b). The hypothetical crack tip is shown by broken line in Figs. 13(e) and 13(f). There are also some areas with orange color (relatively larger \( D_f \) value) immediately ahead of the main crack, which may correspond to preceding cracks (Figs. 11(e) and 11(f)). The region with color close to blue (the smaller value of \( D_f \)) in the lower central part seems to be the area of final grain-boundary fracture. Anomalous areas with the relatively large values of \( D_f \) can also be seen in both lower sides (enclosed by white broken lines in Figs. 13(e) and 13(f)), where the number of grain-boundary cracks is very small, although the details are a little different in Figs. 13(e) and 13(f). The detailed observation using a scanning electron microscope revealed that the fracture mechanism in these areas was not grain-boundary fracture but transgranular fracture with small dimples and steps. This will be shown in the next section.

Figure 14 shows the changes in the line density of grain-boundary microcracks (\( n_c \)) and the averaged value of the fractal dimension of the grain-boundary fracture (\( D_f \)) with distance from notch root (\( y \)) in both specimens of the HS-21 alloy (Fig. 14(a)) and the Inconel X-750 alloy (Fig. 14(b)), although the value of \( n_c \) is a little larger in the specimen of the HS-21 alloy tested under the lower gross section stress (88.2 MPa) at the higher temperature (1089 K). The averaged value of \( D_f \) was obtained over 100 datum points. The averaged value of \( D_f \) also decreases with increasing distance from notch root (\( y \)), and is a little larger in the specimen of the HS-21 alloy with the larger value of \( n_c \). Both value of \( n_c \) and averaged value of \( D_f \) are slightly different between the upper fracture surface and the lower one in the Inconel X-750 alloy, because the fracture pattern is not exactly the same even in the conjugate fracture surfaces (Fig. 13). The value of \( D_f \) represents the fracture pattern including the density and spatial distribution of grain-boundary microcracks on the fracture surface, and is not a unique function of the value of \( n_c \).

4.3. Analysis of Creep Fracture Process in Surface Notched Specimen

The FRASTA was applied to the analysis of creep crack growth in a surface notched specimen.\(^{9,12,13}\) Figure 15 shows the perspective images (775x681 in pixel) and the two-dimensional images obtained by the FRASTA on the surface notched specimen of the Inconel X-750 alloy under a gross section stress (\( \sigma_g \)) of 343 MPa at 973 K. Cracked region is shown in gray in Figs. 15(a), 15(b) and 15(c). The main creep crack is assumed to grow from the upper edge of these figures. The maximum depth of the main creep crack at a given time was known from Fig. 11(b). The crack depth increases with increasing displacement (\( w \)) between the conjugate fracture surfaces (with increasing time) (Figs. 15(a), 15(b) and 15(c)). Well-developed main crack exhibits almost semi-circular shape (Fig. 15(c)), while the crack...
The growth of the main creep crack is different from the FRASTA which relies on the matching of the conjugate fracture surfaces. Therefore, this analysis is basically different from the FRASTA, was detected by the FDM, the crack growth process is examined by the change in the fracture surface pattern with locations. According to the two-dimensional images, the crack growth is delayed in the region near specimen surface (section B) (Figs. 15(g), 15(h) and 15(i)) compared with the central part (section A) (Figs. 15(d), 15(e) and 15(f)), although the two-dimensional images gives only limited information of the fracture process compared with the perspective images. The local difference in the crack growth is related to the differences in the stress state and in the fracture mechanism. The central part is almost under the plane strain state, while the stress state is close to the plane stress state in the region near specimen surface. Grain-boundary fracture was the dominant fracture mechanism in this specimen. However, transgranular ductile fracture occurred in the finally fractured region near specimen surface, whereas the FRSTA could not detect such a difference in fracture mechanism.

**Figure 16** shows the result of analysis of creep fracture process by the fractal dimension map (FDM) on the surface notched specimen of the Inconel X-750 alloy tested under a gross section stress ($\sigma_s$) of 343 MPa at 973 K. The FDMs were produced by changing the displayed scale of the value of $D_1$ on both lower and upper fracture surfaces. The upper area with the larger value of $D_1$ corresponds to the main creep crack (displayed in red or orange color in Figs. 16(a) and 16(b)). The time ($t$) corresponding to the maximum crack depth from notch root was known from Fig. 9(b). Joining of the main crack with some isolated microcracks can be known also in Fig. 15(b) ($t=253.0$ ks) similar to Fig. 15(b) ($t=265.5$ ks), and these are similar to Fig. 15(c) ($t=263.3$ ks), whereas the displayed area of the FDM (600×504 in pixel) is a little smaller than the perspective images (775×681 in pixel). At the stage immediately before final fracture, the region with the larger value of $D_1$ on the upper fracture surface (Fig. 16(d)) is similar to that on the lower fracture surface (Fig. 16(c)), although some parts failed in transgranular manner (the corners enclosed by white broken lines). Ultimate fracture of the specimen may have occurred in these parts, whereas the region of the final grain-boundary fracture is shown in blue or green color (Figs. 16(c) and 16(d)). The result of the present fractal analysis using the FDM is similar to that of the FRSTA.

**Figure 17** shows the different fracture mechanisms observed in the surface notched specimen of the Inconel X-750 alloy tested under a gross section stress ($\sigma_s$) of 343 MPa at 973 K. Grain-boundary fracture (for example, the region A in Fig. 17(a) and 17(b)) was observed in most of the fracture surface. However, ductile transgranular fracture with small dimples and steps also occurred in the lower region near the final fracture surface (for example, the region B in Fig. 17(a) and 17(c) and 17(d)). Steps may be formed as a result of dislocation slip in the grains. The size of steps was in the range from about 10 to 200 $\mu$m, while the size of dimples was smaller than about 10 $\mu$m. The relatively large value of $D_1$ in these areas is principally attributed to steps of sizes larger than about one grain-boundary length ($6.2\times10^{-5}$ m) on the fracture surface. As described above, the difference in the fracture mechanism, which could not be known by the FRSTA, was detected by the fractal analysis using the fractal dimension map (FDM) (Figs. 13(e) and 13(f)). The FDM can also be produced using the three-dimensional image (the height data) of the fracture surface, which is used in the FRSTA, because the three-dimensional image is basically the same in both fractal analysis using the FDM and FRSTA. In the fractal analysis using the FDM, the crack growth process is examined by the change in the fracture surface pattern with grain-boundary microcracks owing to the main crack growth in materials. Therefore, this analysis is basically different from the FRSTA which relies on the matching of the conjugate fracture surfaces.

As for the fractal analysis of the fracture surface using
the FDM, the following findings are pointed out.

(i) In the present fractal analysis, the morphological feature of fracture surfaces in a wide area or the subtle change in the fracture surface pattern with creep crack growth can be quantified by the fractal dimension of the grain-boundary fracture ($D_f$), which is automatically calculated using height data of only one side of the conjugate fracture surfaces (Figs. 13, 14 and 16). On the other hand, there is inevitable arbitrariness in the determination of tilting angle of the conjugate fracture surfaces in the FRASTA (Fig. 15). Further, manual operation and manual image processing is necessary for matching of the conjugate fracture surfaces in this method, and therefore, is time-consuming.

(ii) The principal purpose of the FRASTA is the simulation of the main crack growth and the initiation of isolated cracks in the fracture process by matching the conjugate fracture surfaces. In the fractal analysis using the fractal dimension map (FDM, a color-coded map), the main crack growth and the initiation of isolated cracks can be numerically known by the value of $D_f$ calculated on small areas and visually realized by changing the imaging condition of the FDM on one side of the conjugate fracture surfaces (Fig. 16).

(iii) In the fractal analysis, the change in the fracture mechanism during fracture process can be detected by the FDM on a given fracture surface (Fig. 14). Such a change in the fracture mechanism cannot be known in the FRASTA that relies on the matching of conjugate fracture surfaces, even if both two-dimensional (fracture surface profile) and perspective images are used.

(iv) The fractal dimension of the grain-boundary fracture ($D_f$) can be related to material properties such as creep-rupture properties. As described in the Sec. 4.1, the larger value of $D_f$ corresponds to the longer rupture life and the larger creep ductility in cobalt-base, nickel-base and iron-base heat-resistant alloys. The larger value of $D_f$ also corresponds to the lower crack growth rate. Thus, the fractal dimension of the grain-boundary fracture is a suitable parameter characterizing the high-temperature fracture of materials. On the other hand, the FRASTA has been typically applied to the measurement of fracture toughness in materials.9,10,13

(v) In the FRASTA, the precise positioning of the conjugate fracture surfaces requires a special device for rigorous matching of the fracture surfaces when the height data are acquired.12,13 On the other hand, such a precise positioning is not necessary in the fractal analysis using the FDM.

As described above, the fractal analysis using the FDM is more convenient and more advantageous in several aspects than the FRASTA. The fractal analysis can be carried out without any special devices. Therefore, the fractal analysis used in this study is widely applicable to the analysis of fracture surfaces in materials generated at high temperatures.

5. Conclusions

The fractal dimension of the grain-boundary fracture, $D_f$ (2 $< D_f < 3$) or the fractal dimension of the grain-boundary fracture surface profile, $D_{fp}$ (1 $< D_{fp} < 2$), was estimated for characterization of high-temperature fracture in heat-resistant alloys in this study. The value of $D_f$ or $D_{fp}$ was evaluated in the length scale range more than about one grain-boundary length on the ruptured specimens of cobalt-base, nickel-base and iron-base heat-resistant alloys. The results obtained were summarized as follows.

(1) The value of $D_f$ increased with increasing fractal dimension of the grain-boundary surface profile ($D_{GB}$) in the HS-21 alloy. Both rupture life and creep ductility increased with increasing fractal dimension of the grain-boundary fracture ($D_f$) in these specimens. Similar results were obtained between the rupture properties and the value of $D_{fp}$ in other specimens of the heat-resistant alloys. The value of $D_f$ or $D_{fp}$ represents the fracture surface pattern including grain-boundary microcracks.

(2) A relationship was found between the value of $D_f$ and the density of grain-boundary microcracks ($N_{gb}$) on the fracture surface in the specimens of the HS-21 alloy, although the value of $D_f$ represents the fracture surface pattern including both density and spatial distribution of grain-boundary microcracks. A similar relationship was found between the fractal dimension of the grain-boundary fracture surface profile ($D_{fp}$) and the line density of grain-boundary microcracks ($N_{gb}$) on the fracture surface in the surface notched specimens of the 21Cr–4Ni–9Mn steel ruptured at 973 K.

(3) The values of $D_f$ calculated on small areas of the fracture surface or the value of $D_{fp}$ as well as the line density of grain-boundary microcracks ($N_{gb}$) decreased with increasing distance from notch root (with the main crack growth) in the surface notched specimens. The creep fracture process could be examined using the fractal dimension map (FDM, a color-coded map) on the surface notched specimens of the HS-21 alloy and the Inconel X-750 alloy.

(4) The result of the fractal analysis using the FDM was similar to that of the FRASTA in the surface notched specimen of the Inconel X-750 alloy. The present fractal analysis is more convenient and more advantageous in several aspects than the FRASTA. One can conduct the analysis of fracture process on a given fracture surface without any special devices. Therefore, the fractal analysis used in this study is widely applicable to the investigation of high-temperature fracture process in materials.

Acknowledgements

The authors thank The Iron and Steel Institute of Japan (Tekkou-Kenkyu-Shinkou-Josei) for financial support.

REFERENCES

1) B. B. Mandelbrot, D. E. Passoja and A. J. Paullay: Nature, 308 (1984), 721.
2) R. H. Dauskardt, F. Haubensak and R. O. Ritchie: Acta Metall., 38 (1990), 142.
3) M. Tanaka: Z. Metallkd., 84 (1993), 697.
4) M. Tanaka: J. Mater. Sci., 31 (1996), 3513.
5) T. G. Langdon and R. B. Vastava: Mechanical Testing for Deformation Model Development, ASTM STP 765, ed. by R. W. Rohde and J. C. Swearengen, American Society for Testing and Materials, Philadelphia, (1982), 435.
6) H. E. Evans: Mechanisms of Creep Fracture, Elsevier Applied Science Publishers, New York, (1984), 7.
7) M. Tanaka, R. Kato, Y. Kimura and A. Kayama: ISIJ Int., 42 (2002), 1412.
8) K. Komai and J. Kikuchi: J. Soc. Mater. Sci., Jpn., 34 (1985), 648.
9) T. Kobayashi and D. A. Shockey: Metalli. Trans., 18A (1987), 1941.
10) J. Stampf, S. Scherer, M. Berchthaler, M. Gruber and O. Koledkin: Int. J. Frac., 78 (1996), 193.
11) M. Tanaka, Y. Kimura, L. Chouanine, J. Taguchi and R. Kato: ISIJ Int., 43 (2003), 1453.
12) A. Ueno, H. Kishimoto, K. Kino and Y. Ishii: Trans. JSMSE, A63 (1997), 2393.
13) M. Murata (ed.): Fractography, Maruzen Co. Ltd., Tokyo, (2000), 63.
14) M. Tanaka: J. Mater. Sci., 32 (1997), 1781.
15) B. B. Mandelbrot: The Fractal Geometry of Nature, translated by H. Hirnoka, Nikkei Science, Tokyo, (1985), 227.
16) H. Takayasu: Fractals in the Physical Sciences, Manchester University Press, Manchester, New York, (1990), 11.
17) M. Tanaka, Y. Kimura, L. Chouanine, R. Kato and J. Taguchi: J. Mater. Sci. Lett., 22 (2003), 1279.
18) M. Tanaka, Y. Kimura, A. Kayama, J. Taguchi and R. Kato: J. Mater. Sci., 40 (2005), 6291.
19) M. Tanaka: J. Mater. Sci., 27 (1992), 4717.