TEMPORAL VARIABILITY FROM THE TWO-COMPONENT ADVECTIVE FLOW SOLUTION
AND ITS OBSERVATIONAL EVIDENCE

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ABSTRACT

In the propagating oscillatory shock model, the oscillation of the post-shock region, i.e., the Compton cloud, causes the observed low-frequency quasi-periodic oscillations (QPOs). The evolution of QPO frequency is explained by the systematic variation of the Compton cloud size, i.e., the steady radial movement of the shock front, which is triggered by the cooling of the post-shock region. Thus, analysis of the energy-dependent temporal properties in different variability timescales can diagnose the dynamics and geometry of accretion flows around black holes. We study these properties for the high-inclination black hole source XTE J1550-564 during its 1998 outburst and the low-inclination black hole source GX 339-4 during its 2006–07 outburst using RXTE/PCA data, and we find that they can satisfactorily explain the time lags associated with the QPOs from these systems. We find a smooth decrease of the time lag as a function of time in the rising phase of both sources. In the declining phase, the time lag increases with time. We find a systematic evolution of QPO frequency and hard lags in these outbursts. In XTE J1550-564, the lag changes from hard to soft (i.e., from a positive to a negative value) at a crossing frequency ($\nu_0$) of ~3.4 Hz. We present possible mechanisms to explain the lag behavior of high and low-inclination sources within the framework of a single two-component advective flow model.

Key words: accretion, accretion disks – shock waves – stars: individual (XTE J1550-564, GX 339-4)

1. INTRODUCTION

The soft X-ray transient (SXT) XTE J1550-564 is one of the most interesting Galactic black hole candidates that has been studied over a broad range of wavelengths. The typical outburst behavior of XTE J1550-564 during 1998 began and ended in a low/hard state similar to the outburst profile of other black holes (e.g., GRO J1655-40, GX 339-4, etc.). This typical outburst behavior is supposedly due to the sudden change in viscosity in the system (Hoshi 1979; Mandal & Chakrabarti 2010). In Chakrabarti et al. (2009, hereafter Paper I), a systematic study of the evolution of the quasi-periodic oscillation (QPO) frequencies was carried out during XTE J1550-564’s 1998 outburst, and it was shown that the variation of the QPO frequencies during the rising and declining phases could be understood by assuming that the centrifugal pressure supported shocks formed in the sub-Keplerian component moving in and out during the rising and declining phases, respectively, and all the while oscillating at periods comparable to the infall timescale in the post-shock region (commonly known as CENBOL or CENtrifugal barrier dominated Boundary Layer). These oscillations are primarily due to resonances occurring between the cooling timescale and the infall timescale in the CENBOL. Very recently, Chakrabarti et al. (2015) demonstrated that once the resonance sets in, it is likely to remain locked in resonance. Thus, the so-called “Propagatory Oscillatory Shock (POS)” model of QPO evolution may indeed be valid for all such outbursting candidates.

Just as the QPO frequency variation gives us a clue concerning the geometry variation of the Comptonizing electron cloud, the time- or phase-lag variation should also have independent information, not only about the variation of geometry but also about the energy-dependent physical processes which are responsible for the emission of photons from various regions of the disk. In the present paper, we extend our study of XTE J1550-564, as presented in Papers I and Dutta & Chakrabarti 2010 (hereafter Paper II), to understand the cause of the peculiarities, if any, of the time-lag behavior and its energy dependence.

In X-ray binaries (XRBs), rapid variability in the X-ray emission on timescales of milliseconds to seconds is a common and very complex phenomenon. It was proposed (Lightman & Eardley 1974; Shakura & Sunyaev 1976) long ago that instabilities in the standard accretion disk may cause the observed fast variability in XRBs. In order to match observations, the time variation could exist over a wide range of timescales that depend on the range of unstable radii (Lyubarskii 1997). Low-frequency quasi-periodic oscillations (LFQPOs) with frequencies ranging from a few mHz to ~30 Hz are observed in black hole X-ray binaries (BHBs; see, van der Klis 2004; Remillard & McClintock 2006). Another timing property is the manifestation of a time lag which represents a delay between the Fourier components of the hard and soft light curves (i.e., a time difference in the arrival times between the hard and soft photons). Although time lags are observed in a large number of astronomical sources, we did not find any satisfactory explanation in the literature as to the physics of the origin(s) of this phenomenon.

An early explanation of the time lag assumed that it was due to the Comptonization of soft seed photons by hot electrons, known as Compton reverberation (Payne 1980; Miyamoto et al. 1988), which naturally produces hard time lags. Several models have been proposed (e.g., Cui et al. 1999; Nowak et al. 1999; Poutanen 2001) to explain the hard and soft lags observed in Galactic binary sources. The soft phase lags can be explained by a model (Bottcher & Liang 1999; Lin et al. 2000) in which the perturbation is assumed to propagate from the inner disk to the outer disk. When perturbations propagate...
inward, a hard phase lag is produced. However, it is difficult to explain the transition frequency (e.g., 3.4 Hz for XTE J1550-564; 2.4 Hz for GRS 1915+105) where the lag converts from the hard type to the soft type. Qu et al. (2010) found that the frequency and phase lag are both energy dependent for 0.5–10 Hz QPOs in GRS 1915+105. In XTE J1550-564, a similar type of complex variability is also observed (Cui et al. 2000). Reig et al. (2000) proposed that hard and soft lags are due to the mechanisms of Compton up-scattering and down-scattering. Basically, these mechanisms depend on the gradient of temperature and optical depth of the plasma in the Comptonizing region. For low optical depth, only up-scattering, i.e., positive lags, is possible. For a large optical depth, diffusion plus down-scattering raise the possibility of negative lags. In the Compton up-scattering model (Lee et al. 2001), the soft lag is explained as due to Comptonization delays. Basically, these lags are due to the effects of the difference in the travel time of light and are comparable to the light crossing time of the objects. This lag does not switch sign, however. Poutanen & Fabian (1999) argue that the reflection of hard X-rays from the outer part of the accretion disk produces time delays that we may have already observed in Galactic black holes. In this case, the disk should be flared and the break in the time-lag Fourier spectra would correspond to the size of the accretion disk.

Recently, using XMM data, Uttley et al. (2011) found in GX 339-4 that the softer disk photons lead the power-law photons at frequencies below ~1 Hz. The lag behavior switches at frequencies above ~1 Hz where softer disk photons start to lag behind the harder power-law photons. This behavior was interpreted as being due to the effect of the propagation (soft photon leads) to reverberation (soft photon lags). According to Arevalo & Uttley (2006), at small disk radii, instability in the accretion rate will only cause a small variation in the disk emission inside that radius. This disk emission variability is dominated by X-ray heating effects due to the fluctuation in mass accretion at that radius.

In active galactic nuclei, both hard and soft lags are explained by reverberation lag where the reflection of Comptonized photons from the disk (Zoghbi et al. 2010; Wilkins & Fabian 2013) is considered. Indeed, the switch in lag signature (a few tens milliseconds on short timescale variability) is consistent with the light-travel lags of reprocessed hard emission (power law) from the soft (disk) emission at a few tens of gravitational radii. Since the Comptonization process could be the key reason for lags, it would be interesting to check if the size of this region, which is also responsible for QPOs, actually decides the amount of lag as the time lag is likely to be proportional to the size of the Compton cloud.

In the context of the 1998 outburst of XTE J1550-564, a study of the phase-lag evolution during the initial rising phase was carried out by Cui et al. (1999, 2000). The magnitude of the lag was found to increase with the total X-ray flux. It was found that the QPO becomes stronger at higher energies (Cui et al. 1999; Chakrabarti & Manickam 2000 for GRS 1915 +105), which further supports the view that a QPO originates due to the oscillation of the Comptonizing region. Although we consider only one high- and low-inclination source, each with similar spectral and timing properties, the results may be general enough since there are many studies in the literature which indicate that these properties do have some inclination dependence. Muñoz-Darias et al. (2013) and Heil et al. (2015) showed that in higher inclination systems, there is systematically harder X-ray power-law emission. The properties of low-frequency QPOs are also inclination dependent. On the other hand, Heil et al. (2015) found that the amplitude of the broadband noise (subtracting the low-frequency QPOs) is no longer dependent on inclination, implying its correlation with the source structure of the emitting regions. Using Monte-Carlo simulations, Ghosh et al. (2011) also find for the same disk flow properties that the spectrum changes with inclination angle.

There are yet other effects, such as the focusing of photons due to the gravitational bending of light from regions close to the black hole horizon, which could also be important and the delay introduced would be energy dependent as the energetic photons tend to be emitted from regions close to a black hole. In the present paper, for comparison, we study the time lag of one high inclination object, namely, XTE J1550-564, and one low-inclination object, namely, GX 339-4. As we have discussed above, the result is likely to represent the whole class of objects with high and low inclinations, respectively. We explain the results of our observations by considering several effects on the time lag, namely, (i) repeated Compton scattering which introduces higher lags for higher-energy photons and for Compton clouds of larger size before they escape, (ii) reflection, and (iii) focusing due to gravitational bending. The structure of our paper is the following. In Section 2, we discuss the properties of the two black holes under consideration. In Section 3, we present observational data and our analysis procedure. Specifically, we present the lag results for XTE J1550-564 for which detailed studies of QPOs and spectral properties have already been presented in Papers I and II. We also include a similar study for GX 339-4. In Section 4, we present discussions of our results and provide an explanation of the behavior of the time lag. Finally, in Section 5, we provide our concluding remarks.

2. INTRODUCTION TO THE BLACK HOLE CANDIDATES UNDER CONSIDERATION

2.1. XTE J1550-564

The SXT XTE J1550-564 is one of the most interesting Galactic black hole candidates that has been studied over a broad range of wavelengths. The typical outburst behavior of XTE J1550-564 during 1998 began and ended in a low/hard state similar to the outburst profile of other black holes (e.g., GRO J1655-40, GX 339-4, etc.). This typical outburst behavior is supposedly due to the sudden change in viscosity in the system (Hoshi 1979; Mandal & Chakrabarti 2010). The time lag varies during the initial onset phase of the 1998 outburst (Cui et al. 1999, 2000). The magnitude of the lag increases with X-ray flux. The coherence is roughly constant and high (value is ~1). The constant value is maintained up to the first harmonics of the QPOs.

The properties of LFQPOs and the spectral variability during the 1998 outburst were reported in Papers I and II.

2.2. GX 339-4

The black hole candidate GX 339-4 is a low-mass X-ray binary with a primary of mass $\geq 6 M_e$ (Hynes et al. 2003; Muñoz-Darias et al. 2008) that was first detected by the MIT X-ray detector on board the OSO7 mission by Markert et al. (1973). Since its discovery, the source has exhibited four
outbursts at 2–3 year intervals. The evolution of the low-frequency QPOs and spectral variability during the 2010 outburst were studied in detailed by Debnath et al. (2010, 2015) with the Two Component Advective Flow model (TCAF) proposed by Chakrabarti & Titarchuk (1995; hereafter CT95) where they found that the QPO frequencies are monotonically increasing from 0.102 to 5.69 Hz over a period of ~26 days. They explained this evolution using the POS solution and found the variation of the initial and final shock locations and strengths. This behavior generally matches the values obtained from a spectral fit with TCAF model (Dutta et al. 2013; Debnath et al. 2015). Using the TCAF model, a clear physical picture of what happens in an outburst emerges.

The only major difference between these two objects is the inclination angle. GX 339-4 belongs to a class of low-inclination (<60 deg) sources (Zdziarski et al. 1998), whereas XTE J1550-564 has a high inclination angle of 74.7 deg ± 3.8 deg (Orosz et al. 2011). However, both sources exhibit similar systematic evolutionary properties during their outbursts. Here, we use this rich data set to study the evolution of lag variability during the outbursts.

3. OBSERVATIONAL ANALYSIS

We analyzed RXTE public archival observations of BHB GX 339-4 during its 2007 outburst and observations covering the rising and declining phases of QPOs in the 1998 outburst of XTE J1550-564 (for details, see, Paper I), limiting our analysis to observations for which low-frequency QPOs are observed. We produced background-corrected PCU2 rates in the Standard 2 channel bands $A = 4-44$ (3.3–20.20 keV), $B = 4-10$ (3.3–6.1 keV), and $C = 11–20$ (6.1–10.2 keV) and defined the hardness ratio as $C/B$. We used the Good Xenon, Event, and Single Bit data modes, which contain high time resolution data for our timing analysis, which was performed using the GHATS software. For each observation, we produced a PDS every 16 s in the channel band 0–35 (2–15 keV). We averaged them to obtain an average PDS for each observation and subtracted the Poissonian noise contribution (Zhang et al. 1995). The PDSs were normalized and converted to a fractional squared rms. The power spectra were then fit with a combination of Lorentzians (see Nowak 2000) using XSPEC v 12.0.

We also take the cross-spectrum, which is defined as $C(j) = X_1(j) X_2^*(j)$, where $X_1$ and $X_2$ are the complex Fourier coefficients for the two energy bands at a frequency of $\nu$, and $X_2^*$ is the complex conjugate of $X_2(j)$ (van der Klis et al. 1987). The phase lag between the signals of two different energy bands at Fourier frequency $\nu$ is $\phi = \arg[C(j)]$ (i.e., $\phi$ is the position angle of $C(j)$ in the complex plane) and the corresponding time lag is $\phi/2\pi\nu$. An average cross vector $C$ is determined by averaging the complex values for every stretch of time length. In our analysis, we produced a phase-lag spectrum for each observation in our sample, dividing the data into two energy bands (2–5 keV and 5–13 keV) and extracting cross-spectra from 16 s intervals which are then averaged yielding one phase-lag spectrum per observation. A positive phase-lag indicates that the hard photons (5–13 keV) lag the soft photons (2–5 keV). Following Reig et al. (2000), we calculate the QPO phase lag as the average of the phase lags over the interval $\nu_c \pm$ FWHM, where $\nu_c$ is the centroid frequency of the QPO and FWHM is its full-width at half-maximum as measured through a fit with a Lorentzian component.

Figure 1. Evolution of time lag (filled squares) and QPO frequency (filled circles) are plotted with time (days) during the rising (MJD 51065–MJD 51076) and declining (MJD 51076–MJD 51084) phases of XTE J1550-564 in the 1998 outburst. Time lag is calculated at QPO frequency ($\nu_Q$) integrated over the width, which is equal to the FWHM ($\Delta\nu$) of the QPO itself.

3.1. XTE J1550-564

Figure 1 shows the systematic variation of the time lag (filled squares), which is QPO frequency-dependent, and the QPO frequency (filled circles) as a function of time (days) during the onset (MJD 51076–MJD 51084) and declining (MJD 51076–MJD 51084) phases of the 1998 outburst in XTE J1550-564. We consider the 0th day (i.e., MJD 51065) when the first QPO was detected. We studied the observations that cover the first three weeks of the outburst. In the onset phase, we fit the time-lag variation with a curve where the time lag $\sim t^{-0.423 \pm 0.02}$ and the reduced $\chi^2$ is close to 1.3 ($\chi^2 /10$). We find that time lag decreases with the increase of both the QPO frequency and time (day) during the onset phase. In the declining phase, we also find a systematic variation of the same time lag. The declining phase starts after observing the highest value of $\nu_Q = 13.1$ Hz on MJD 51076. The fitted curve represents the time lag $\sim t^{-0.663 \pm 0.03}$ with a reduced $\chi^2$ close to 1.6 ($\chi^2 /7$).

In the TCAF solution, the QPO frequencies are derived from the inverse of the infall time in the post-shock region. According to the POS solution (Chakrabarti et al. 2008, 2009; Debnath et al. 2010, 2013), we can get an idea about the location of the shock wave ($X_s$) from the observed QPO frequency ($\nu_Q$) as it is believed that QPOs are generated due to the oscillations of shocks. The generated QPO frequency ($\nu_Q$) is proportional to the inverse of the infall time ($t_{infall}$, i.e., the light crossing time from the shock location to the black hole), that is, $\nu_Q \sim t_{infall}^{-1}$, and $t_{infall} \sim X_s(X_s - 1)^{1/2} \sim X_s^{3/2}$ $R$ is the shock strength ($=\rho_s /\rho_m$, i.e., the ratio of the post-shock to pre-shock densities.) Thus, according to this model, the QPO frequency is $\nu_Q \sim X_s^{-3/2}$. The time-dependent shock location is given by $X_s(t) = r_s \pm \nu/\gamma v$, where $\nu$ is the velocity of the shock wave. The positive sign in the second term is to be used for an outgoing shock and the negative sign is to be used for an infalling shock. $X_s$ is the measured units of the Schwarzschild radius $r_s = 2GM/c^2$, where $M$ is the BH mass and $c$ is the velocity of light.

Accordingly, the shock location is larger for lower QPO frequency. The opposite is also true. In Figure 2, the variation of the time lag (in seconds) with shock location ($X_s$) is plotted during the rising (MJD 51065–MJD 51076) phase of XTE...
J1550-564 in the 1998 outburst. We clearly see that purely from an observational point of view, the lag monotonically increases when the QPO goes down and the derived shock location goes up. Thus, we have a fully consistent understanding that an increase in the QPO frequencies necessarily implies a decrease in the size of the Comptonizing region. Even the “hiccup” on day 6 in the QPO frequency also shows up in the lag profile.

Because of the very nature of the physical processes by which high-energy photons are generated, the time lag must depend on the energy. First of all, repeated inverse Comptonization processes imply a higher time lag to generate high-energy photons. Similarly, the focusing of emitted photons is also energy dependent since higher-energy photons are expected to come from regions closer to a black hole. This motivates us to compute the energy dependence of the lag. Figure 3 shows the energy-dependent time lag for the QPO centroid frequencies: \( f_0 = 5.698 \, \text{Hz}, \ f_Q = 3.14 \, \text{Hz}, \ f_Q = 2.39 \, \text{Hz}, \ f_Q = 1.60 \, \text{Hz}, \) and \( f_Q = 0.81 \, \text{Hz} \). We averaged the time lag over the width equal to the FWHM (\( F_w \)) of the QPO itself. Here, we calculated the time lag with respect to the reference energy band 1.94–6.54 keV (0–17, channels). The other energy bands are 6.54–13.36 keV (18–36, channels), 13.36–21.78 keV (37–59, channels), and 21.78–38.44 keV (60–103, channels). We find that the lag could be positive or negative depending on the QPO frequency and photon energy. This behavior suggests that the contributions to the lag from different mechanisms vary with shock location. Time lag monotonically increases with energy for a QPO frequency of 0.81 Hz, but monotonically decreases from a frequency of \( \sim 3.4 \, \text{Hz} \) onward. The complex pattern of the phase lags associated with the QPO bears a remarkable resemblance to that observed for GRS 1915+105, which is also a high-inclination source (Cui et al. 1999; Dutta et al. 2015; Reig et al. 2000). We will discuss the physical cause in the final section.

### 3.2. GX 339-4

We now turn to a source which belongs to a low-inclination binary system. If focusing by gravitational photon bending is important in deciding the lag, then low-inclination sources are not likely to be influenced by this effect and the energy dependence of the lag would look significantly different. This is precisely what we see in the source GX 339-4.

Figure 4 shows a systematic variation of the frequency-dependent time lag as a function of the day during the rising (MJD 54133–MJD 54145) phases of GX 339-4 in the 2007 outburst. The time lag is calculated at the QPO frequency \( f_Q \) by integrating over the range of the FWHM (\( F_w \)) of the QPO.
channels) as a reference energy band. The other energy bands are 5.4–6.9 keV (13–16, channels), 6.9–9.4 keV (17–22, channels), and 9.4–13.1 keV (23–31, channels). We find that the hard lag is directly related to the energy of the photons for different QPO frequencies.

4. DISCUSSION

So far, we have seen that the time lag is not only a function of the average energy of the emitted photons, but it is also a function of the inclination angle of the binary system. What is clear in both high- and low-inclination systems is that the lag increases when the QPO frequency decreases, i.e., when the size of the Comptonizing region goes up. What is not obvious is the cause of the change of sign at a specific QPO frequency, i.e., at a specific size. Below, we discuss the major processes which control the lag and suggest a possible reason for why the change of sign occurs only in high-inclination systems.

4.1. Evolution of Time Lags and Quasi-periodic Oscillations

The typical evolution of QPOs in transient black hole sources during outburst has been established for a long time (Chakrabarti et al. 2005, 2008, 2009; Debnath et al. 2008, 2010, 2013; Dutta & Chakrabarti 2010). This evolution suggests that certain specific physical mechanisms are in place which are responsible for the generation of QPOs day after day (Chakrabarti et al. 2005, 2008, 2009; Debnath et al. 2008, 2010, 2013). These authors found that the Comptonization region gradually shrinks in the rising phase of an outburst when QPO frequencies go up and the reverse is true in the declining phase. We would also expect that, in general, there would be a monotonically decreasing lag with increasing QPO frequency and a monotonically increasing lag with shock location. This is precisely what happens in both objects: the net time lag due to Comptonization is the sum over light crossing times of the mean free paths $l_{\text{Comp}}$ between two successive Compton scatterings, i.e., $t_{\text{lag}}^c = \sum l_{\text{Comp}}/c \propto X_s/c$. However, when one considers the energy dependence, the picture is more complex. Since energy is expected to rise after each successive scattering, higher energies are expected to have higher lags. As pointed out by earlier workers (Cui et al. 1999, 2000; Reig et al. 2000), we also find that the energy dependence of the time lag is not straightforward to understand.

In XTE J1550-564 (Cui et al. 2000), the lag first increases with frequency, peaks at some characteristic frequency, and then decreases and moves toward negative (soft) lag at frequency ∼3.4 Hz. This pattern of phase lag associated with the QPO bears a remarkable resemblance to that observed in GRS 1915 +105 (Cui et al. 1999; Reig et al. 2000), albeit with “zero-lag” occurring at a different QPO frequencies. This is due to the very nature of the emission process in both the Keplerian and sub-Keplerian components.

The physical processes contributing to lags may be understood from Figure 7 where we show how the radiation from an accretion disk and Comptonizing cloud (CENBOL) may reach observers located at angles of $\theta \lesssim 60$ deg and $\theta \gtrsim 60$ deg through various paths. As has already been noted, the time of travel is proportional to the amount of scattering at the CENBOL, and one expects harder X-rays (HXR) to arrive at later times. However, radiation emitted from the inner CENBOL (CENBOL-B) are also focussed due to the gravitational bending of light and may be further delayed. Hard photons reflected from the Keplerian disk as softer radiation (RSXR) also take a longer route, especially for high-inclination sources. Thus, while each soft photon may have a range of time lags (high and low) depending on whether it is focussed and/or reflected before reaching the observer, hard X-rays are always expected to be delayed. This causes the non-monotonicity of the lag with energy when the inclination angle is high. The qualitative behavior could be understood from Figure 8 where we schematically plot the time lag ($t_{\text{lag}}^c$) as a function of the size of the Comptonizing region ($R_s$), which is believed to be located in the inner region (CTR95). We plot this for $\theta \lesssim 60$ deg on the left and for $\theta \gtrsim 60$ deg on the right. The
boundary of this region (CENBOL) is the shock location $X_s$. For convenience, we assume that CENBOL is divided into two gross parts: CENBOL-A emits relatively softer photons than CENBOL-B. The lag due to the Comptonization (superscript “c”) of hard X-rays (HXR) would be proportional to the amount of scattering which took place. Therefore, the energy would roughly monotonically increase linearly with the size and its lag will also increase ($dt_{cHXR}$ in Figure 8). Soft X-rays from the Keplerian disk will not be strongly affected by the size of the CENBOL. Hence, in Figure 8, $dt_{cSXR} \sim 0$ always. When $\theta \lesssim 60$ deg, the focusing and reflection effects are also negligible. So, we expect the lag $dt_{fHXR} - dt_{cSXR}$ to be always positive in this case (left panel of Figure 8). However, when the inclination is high ($\theta \gtrsim 60$ deg), apart from the above two lags, we have monotonically decreasing effects of the soft ($dt_{fSXR}^R$-big circled curve; from CENBOL-A) and hard X-ray ($dt_{HXR}^C$-squared-curve; from CENBOL-B) lags due to focusing by gravitational bending effects. These are shown in the right panel. This is because bending is important only when the emission is closer to the black hole. Focusing is expected to cause a delay of the order of $1 - 2r_s/c$ for hard X-rays (from CENBOL-B) and $5 - 10r_s/c$ for soft X-rays (CENBOL-A) since they must be proportional to the size of the emitting region of the corresponding radiation. Thus, the lag due to the focusing of soft X-rays is higher on an average. The reflection component is reprocessed CENBOL emission from the Keplerian disk and is relatively softer radiation. The delay increases linearly with the size of CENBOL ($dt_{fSXR}^R$, small circled curve in right panel of Figure 8) and the inclination angle. The combination of these four effects could result in interesting patterns. In the right panel of Figure 8, we draw the total delay of hard X-rays (solid curve $dt_{HXR}^C$ which is the sum of $dt_{fHXR}^C$ due to Comptonization and $dt_{HXR}^C$ due to focusing) and the total delay of soft X-rays (solid curve $dt_{SXR}^R$ is the sum of $dt_{fSXR}^R$ from the Keplerian disk, $dt_{SXR}^C$ due to Comptonization, $dt_{SXR}^C$ due to focusing and $dt_{fSXR}^R$ due to reflection); here, we have assumed that no flare-like situation (except for flashes like solar flares, which are rare in persistent sources) occurs where hard X-rays could perhaps be expected earlier than the down-scattered soft X-rays. Therefore, we did not address this issue at all. These two solid curves intercept at $R_c = R_{tr}$, below which the lag ($dt_{fHXR}^C - dt_{SXR}^C$) is negative. This means that according

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**Figure 7.** Cartoon diagram of two-component advective flows where soft photons (SXR) from Keplerian disks are inverse Comptonized by the post-shock region, namely, Centrifugal pressure supported Boundary Layer (CENBOL) and hard X-rays (HXR) are produced. Radiation could directly reach the observer or through focusing effects (FSXR, FHXR). Harder radiation from the inner regions of CENBOL (CENBOL-B) would be more focussed than the Comptonized photons which had undergone less scattering from the outer CENBOL (CENBOL-A). The reflected component (RSXR) from a Keplerian disk would primarily affect softer X-rays and may delay soft X-ray travel time. Observers with $\theta \lesssim 60$ deg will see fewer focusing effects.

**Figure 8.** Schematic diagram explaining the inclination angle and energy dependence of the time lag. We consider two cases: $\theta \lesssim 60$ deg (left) and $\theta \gtrsim 60$ deg (right). In the former case, only Comptonization is important. In latter case, Comptonization, reflection and focussing are important. As a result, the lag may change sign at a given frequency of the power density spectrum. See the text for details.
to TCAF, there is a cross-over QPO frequency (the inverse of the infall time of matter from $R_g$ to the horizon) above which the hard lag would be negative. This cross-over frequency clearly depends on the mass of the black hole, which determines the length scales and timescales of the disk components and the CENBOL.

Heil et al. (2015) analyzed a large number of black hole candidates and found that higher-inclination sources with the same power spectral shape (in PDS) exhibit systematically harder X-ray power-law emission. Although they found that the broadband noise in the power density spectrum is independent of the inclination angle, Type-C QPOs located at different regions of the spectrum are found to be inclination dependent. They conclude that this property strongly depends on the inclination angle. In soft states, there are no low-frequency QPOs and there is no distinction in inclination angle. Muñoz-Darias et al. (2013) found that marginally soft states will be marginally harder in high-inclination systems. In other words, the hardening of the system is due to the higher abundance of harder radiation in relation to soft radiation. This is precisely what happens in Figure 7. Ghosh et al. (2011) found that the same outgoing photons from Monte-Carlo simulations, when binned with respect to inclination angles, showed distinctly harder spectra at higher inclinations.

Our present work establishes this property more firmly using timing analysis. Here, we considered two sources which show very similar systematic evolution in QPOs and spectral states. In XTE J1550-564, the QPO frequency rises from $0.01$ to $14.0$ Hz and starts to fall in the declining state. During this time, the shock location also varies from $800$ to $100$. Also, based on spectral studies with TCAF, we find similar changes in the Compton cloud size (CENBOL). In GX 339-4, the QPO frequencies vary from $0.10$ to $5.69$ Hz and then monotonically decrease, whereas the shock location changes from $600$ to $200$ (Debnath et al. 2010; Nandi et al. 2012). The complex outburst profiles of both sources exhibit similar spectral evolution: hard $\rightarrow$ hard-intermediate $\rightarrow$ soft-intermediate $\rightarrow$ soft $\rightarrow$ in the rising state and in reverse order in the declining state. Moreover, they have comparable masses, differing only in inclination which affected the QPOs of both sources exhibit similar spectral properties. In XTE J1550-564, the QPO frequency rises from $0.01$ to $2.2$ Hz and then monotonically decreases, while in GX 339-4, the QPO frequency generally rises as the frequency decreases. In fact, exactly the opposite result is found in the declining phase. In a TCAF solution, this implies that the lag would increase as the size of the Comptonizing region increases. This is precisely what we see. However, the dependence of lag on photon energy is more intriguing. We find that GX 330-4 exhibits only positive lag for all QPO frequencies, while in XTE J1550-564, the lag switches sign and becomes negative above a certain frequency. We discussed major effects which could be controlling the property of the lag. Specifically, we showed that if we add up the qualitative variations of the lag components, then the high-inclination objects could have negative time lags, i.e., soft photons appearing after hard photons due to the reflection and focussing effects. We see this effect in XTE J1550-564 at frequencies higher than $3.4$ Hz. This frequency gives rise to a characteristic lengthscale ($R_g$) where the lag changes its sign. This frequency is certainly not universal as the cancellation of lags would depend on inclination angles and the mass of the black holes which determine the length scales. Indeed, in another object, GRS 1915+105, with a mass at least twice as high, the transition frequency happens to be lower (2.2 Hz). Detailed work is in progress and will be presented elsewhere.

This research has made use of RXTE data obtained through the HEASARC Online Service, provided by NASA/GSFC, in support of NASA High Energy Astrophysics Programs. We thank T. Belloni for providing the timing analysis software GHATS at X-ray Astronomy School (IUCAA, Pune). Also, B.G.D. acknowledges the support of UGC (Minor Research Project) in carrying forward his research works.

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5. CONCLUSIONS

In the present paper, we extended the earlier studies in Papers I and II to understand the behavior of the time/phase-lag properties of the black hole candidate XTE J1550-564. The properties of this high-inclination object were compared to those of a low-inclination black hole candidate, namely, GX 339-4. We concentrated on the lag in the QPO frequencies as the oscillation is more coherent and lag measurement is less erroneous. In this paper, we have used the TCAF paradigm to explain the time-lag properties primarily because only in this paradigm is there a distinct prediction for the time lag, the size of the Compton cloud, QPO frequency, and spectral states. To our knowledge, no other models in the literature, such as the disk corona model (Haardt & Maraschi 1993; Zdziarski et al. 2003) and the lamp-post models (Martocchia & Matt 1996; Miniutti & Fabian 2004), actually bring out such relations naturally. In both cases, we found that the lag in QPO frequency generally rises as the frequency decreases. In fact, exactly the opposite result is found in the declining phase. In a TCAF solution, this implies that the lag would increase as the size of the Comptonizing region increases. This is precisely what we see. However, the dependence of lag on photon energy is more intriguing. We find that GX 330-4 exhibits only positive lag for all QPO frequencies, while in XTE J1550-564, the lag switches sign and becomes negative above a certain frequency. We discussed major effects which could be controlling the property of the lag. Specifically, we showed that if we add up the qualitative variations of the lag components, then the high-inclination objects could have negative time lags, i.e., soft photons appearing after hard photons due to the reflection and focussing effects. We see this effect in XTE J1550-564 at frequencies higher than $3.4$ Hz. This frequency gives rise to a characteristic lengthscale ($R_g$) where the lag changes its sign. This frequency is certainly not universal as the cancellation of lags would depend on inclination angles and the mass of the black holes which determine the length scales. Indeed, in another object, GRS 1915+105, with a mass at least twice as high, the transition frequency happens to be lower (2.2 Hz). Detailed work is in progress and will be presented elsewhere.

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