Economic development is typically accompanied by migration from rural to urban employment. This migration is often associated with significant urban underemployment. Both factors are important in the development process. We consider a neoclassical growth model with rural-urban migration and urban underemployment, which arises from an adverse selection problem in labor markets. We demonstrate that rural-urban migration and underemployment can be a source of development traps and can give rise to a large set of periodic equilibria displaying undamped oscillation. Many such equilibria display long periods of uninterrupted growth, punctuated by brief but severe recessions.

Two prominent aspects of actual economic development are typically omitted from conventional neoclassical growth models. First, modern economic development has invariably been accompanied by pronounced migration from rural to urban sectors of employment. Second, all economies display some unemployment, which is often quite significant in developing countries. Moreover, many developing economies have a large portion of their labor force em-

We thank Julie Anderson Schaffner, Leonardo Auernheimer, Jim Rauch, an anonymous referee, and the participants of seminars at Cornell, Instituto Tecnologico Autonomo de Mexico, and the 1996 Texas Monetary Conference for very helpful comments and conversations regarding this paper.
ployed in a relatively low-wage, “informal” urban production sector. This is often viewed as a form of underemployment.¹

There are many reasons to think that these factors play an important role in the process of economic development. One is that, at least in the early stages of development, capital formation in urban manufacturing is far more rapid than that in agriculture. As a result, urban wage rates rise relative to rural wage rates, drawing labor into the city. The consequences are often dramatic: between 1950 and 1975 the share of urban population in total third-world population rose from 16.7 to 28 percent (Williamson 1988, p. 428). This migration, in turn, permits more labor to be combined with capital, thereby promoting output growth.

Moreover, not only does urbanization change the composition of employment and output, but it is also generally accompanied by significant urban unemployment or underemployment or both. This underemployment, obviously, can be viewed as a factor that is detrimental to growth, serving as a deterrent to urban migration and also serving to reduce average income and savings. Our goal in the present paper is to produce a theoretical framework for analyzing the interaction between migration, unemployment, capital formation, and economic growth.

While the link between rural-urban migration and unemployment has attracted enormous attention, the link between these two factors and economic growth has not.² Perhaps the best-known analysis of rural-urban migration and urban unemployment originated with Todaro (1969) and Harris and Todaro (1970). That work took the view that labor migrates to wherever its expected income is highest: hence in equilibrium expected incomes—at least for relevant workers—must be equated between urban and rural employment. Since urban wage rates are invariably much higher than rural wage rates,³ the equilibration of incomes occurs through the existence of unemployed or underemployed urban labor.

What permits this unemployment to persist? In the Harris-Todaro model and in much subsequent literature, it is an “institutionally fixed” urban real wage. This is often viewed as fixed as a result of the consequences of minimum-wage legislation or the power of labor unions (Calvo 1978). However, to whatever extent that this is

¹ Sethuraman (1981) reports that the average fraction of the urban labor force employed in the informal sector in developing countries is 41 percent (see also World Bank 1995, p. 35).
² See Rauch (1993) for an important exception to this statement.
³ See, e.g., World Bank (1995, p. 76) for modern evidence or Hatton and Williamson (1992) for historical evidence.
true in modern developing countries, it is not a particularly satisfactory explanation of urban unemployment in historical economic development. Hatton and Williamson (1992) and Williamson (1988) document the existence of significant urban-rural wage differentials—and urban underemployment—in the United States and the United Kingdom of the nineteenth century: these differentials presumably are not easily attributed to the presence of minimum-wage legislation or to the power of unions. In view of Williamson’s conclusion “that Third World urbanization experience has been fairly conventional by historical standards” (1988, p. 428), it seems that some other explanation for the existence of urban unemployment is called for.

As a final point of note, Hatton and Williamson (1992) document a significant degree of fluctuation over time in urban-rural wage differentials in U.S. economic history. For example, the ratio of (nominal) rural to urban wage rates rose from slightly above 0.5 in 1890 to above 0.6 in 1915, and then fell to about 0.35 in 1940. And while one mostly observes migration from rural to urban employment in the historical development process, this migration has periodically been interrupted by brief but marked episodes of reverse net migration. This reverse migration is typically associated with sharp economic downturns: Hatton and Williamson (1992, p. 277) show that there was considerable net migration back to agriculture in the United States in the early 1930s, and their model predicts that such migration should have occurred immediately before the pronounced recession of 1920–21. Thus a reasonable view of historical economic development is that generally sustained economic growth and migration from rural to urban employment are occasionally interrupted by brief, but possibly severe, downturns accompanied by migration from urban to rural employment.

In this paper we produce a two-period, overlapping generations model that contains an urban and a rural production sector. Following conventional “dual-economy” formulations (Drazen 1982; Drazen and Eckstein 1988; Rauch 1993), we assume that production in the “formal” urban sector uses both capital and labor, whereas production in the rural sector uses labor alone. In addition, we allow for the presence of an “informal” urban sector in which production takes place using only labor.

We also assume that the labor force is heterogeneous: some types

---

1 There are many doubts that these institutional rigidities are the primary source of urban unemployment even in modern developing countries. Stiglitz (1988, p. 134), for instance, argues that “there are many instances in which wages in excess of the minimum wage are paid, and yet there are queues of unemployed.” See also Williamson (1988, p. 446).
of workers are intrinsically more skilled than others. We impose the assumption that workers’ types are private information, with the consequence that an adverse selection problem arises in urban labor markets. It is this adverse selection problem, rather than any exogenous rigidities, that permits unemployment or underemployment to exist: unemployed—or underemployed—workers operate in the informal urban sector, earning relatively low incomes.

As capital is accumulated in the urban sector, the real wage rate in formal urban manufacturing rises relative to that in agriculture. As a result, under one technical condition, labor is induced to migrate to the city, and the adverse selection problem there becomes more severe. In response, employers are forced to hire less labor than actually migrates, and the urban unemployment (underemployment) rate rises. It is this increase that equilibrates the migration process, exactly as in Harris and Todaro (1970).

Under one technical assumption, increasing capital/labor ratios in formal urban manufacturing are associated with rising urban wage rates, high average incomes, net rural-urban migration, and high savings rates. They thus lead to yet higher future aggregate capital stocks. However, in the presence of rural-urban migration, this does not necessarily imply that future capital/labor ratios in the formal urban sector will be high; indeed if too much migration is induced, the capital/labor ratio can actually fall. Thus we may not observe the monotonic relationship between the current and future capital/labor ratio that arises in so much of neoclassical growth theory.

We describe conditions under which our model possesses either one or two nontrivial steady-state equilibria. If there is a unique nontrivial steady state, it is necessarily asymptotically stable, and paths approaching it display (locally) monotone dynamics. If there are two nontrivial steady states, one has a relatively high and one a relatively low capital stock (and capital/labor ratio in formal urban manufacturing). The high-capital-stock steady state is again necessarily asymptotically stable, and dynamical equilibrium paths approaching it display locally monotone dynamics. The low-capital-stock steady state may be either asymptotically stable or unstable; in the former case dynamical equilibrium paths approaching that steady state display damped oscillation. Thus development trap phenomena and endogenous oscillation can be easily observed. Indeed, two economies with the same initial capital stocks can approach different steady states: this is obviously a reflection of an indeterminacy that is also unusual in neoclassical growth models.

When there are two asymptotically stable steady states, there can also be a large set of periodic equilibria displaying undamped oscillation: in such equilibria the aggregate capital stock, the capital/labor
ratio in formal manufacturing, urban-rural wage differentials, and net rural-urban migration fluctuate over time. In order to be consistent with observation, these equilibria must have the feature that long periods of growth and net rural-urban migration are interrupted by brief, but possibly sharp, economic downturns, accompanied by reverse migration. We state conditions under which there is a large set of oscillatory equilibrium paths displaying exactly this pattern.

The remainder of the paper proceeds as follows. Section I outlines the model, and Section II states a set of equilibrium conditions that must be satisfied in factor markets. Section III derives the full set of equilibrium conditions of the model, and Sections IV and V characterize steady-state and dynamical equilibria, respectively. Section VI presents conclusions.

I. The Model

A. Environment

Our economy consists of an infinite sequence of two-period-lived, overlapping generations plus an initial old generation. Let \( t = 0, 1, 2, \ldots \) index time. At each date \( t \) a new young generation appears, containing a continuum of agents with mass \( n^t \). Thus \( n \) is the gross rate of population growth.

In each period two kinds of goods are produced, which we think of as manufactured and agricultural commodities. In addition, we divide young agents into two types indexed by \( i = 1, 2 \); types differ in terms of their intrinsic labor skills. Let \( \theta_i \) be the fraction of the population that is of type \( i \); clearly \( \theta_i \in (0, 1) \), \( i = 1, 2 \), and \( \theta_1 + \theta_2 = 1 \).

As is common in dual-economy models, we assume that the production of agricultural commodities uses only labor as an input.\(^5\) A young type \( i \) agent employed in agricultural production can produce \( \pi_i \) \((i = 1, 2)\) units of the good per unit of time. We assume that \( \pi_2 > \pi_1 > 0 \) holds, so that type 2 agents are relatively more productive than type 1 agents in agricultural employment. We also assume that agents engaged in agriculture either work for themselves or have output levels that are perfectly observed by their employers. This assumption precludes the existence of a private information problem in the agricultural sector.

Any agent can seek employment in either agriculture or manufac-

\(^{5}\) Drazen (1982), Drazen and Eckstein (1988), and Rauch (1993) make the same assumption. Ranis (1988) describes the hallmark of dual-economy models to be the assumption that agricultural production uses relatively little or no capital.
turing, but not in both. We can interpret this to mean that agents must move to the city to work in manufacturing. Thus there is a discrete choice to be made by workers as to their sector of employment.

A firm engaged in manufacturing production that employs \( K_t \) units of capital and \( L_t \) units of young type 2 labor at \( t \) can produce \( F(K_t, L_t) \) units of the manufactured good at that date. We assume that type 1 agents are totally unproductive in formal manufacturing. This assumption is not essential to the analysis and obviously represents the limiting case in which type 1 agents are merely relatively less productive than type 2 agents in urban manufacturing. We impose the limiting case largely because it delivers a standard Harris-Todaro (1970) condition that determines the urban unemployment rate.

Let \( f(k_t) = F(k_t, 1) \) denote the intensive production function, where \( k_t = K_t / L_t \) is the capital/labor ratio in the formal manufacturing sector. We assume that \( f \) has the constant elasticity of substitution form

\[
    f(k) = (ak^\rho + b)^{1/\rho}. \tag{1}
\]

We focus here on the case in which \( \rho < 0 \) holds, so the elasticity of substitution is strictly less than unity. (This is the empirically best supported case.) Bencivenga and Smith (1995) describe how the analysis must be modified when \( \rho \geq 0 \).

We shall refer to production of the manufactured good that takes place using the technology in (1) as “formal” urban manufacturing. We also assume that there is an “informal” urban sector in which young agents can produce the manufactured good using only labor as an input.\(^6\) With this technology, a young type \( i \) agent can produce \( \beta_i \) units of the manufactured good per unit of time “at home.” We shall assume that this informal sector is inefficient in the sense that

\[
    F_2(k_t, 1) > \beta_2 \tag{2}
\]

for all “relevant” values of \( k_t \). We shall also maintain the assumption that type 2 agents are more productive than type 1 agents, so that \( \beta_2 > \beta_1 \geq 0 \) holds. We shall often set \( \beta_1 = 0 \), since that specification yields a Harris-Todaro-like condition that determines the urban unemployment rate.

Finally, we assume that each agent knows his own type but that

---

\(^6\) Rauch (1993) also assumes that informal sector production uses no capital. Sethuraman (1981) estimates that, on average, informal sector production technologies have a capital/labor ratio no greater than 15 percent of that prevailing in formal manufacturing.
this is private information ex ante. This assumption leads to the existence of an adverse selection problem in urban labor markets. It is this problem that is the source of equilibrium urban unemployment.

With respect to endowments, we assume that old agents are endowed with an initial aggregate capital stock of \( K_0 > 0 \). Thereafter, no agents are endowed with either capital or final goods. All young agents are endowed with one unit of labor, which they supply inelastically in either the urban or the rural sector. Agents have no labor endowment when old.

With respect to preferences, we assume that all agents care only about second-period consumption, so that all young-period income is saved. This assumption permits us to economize on notation by ignoring any young-period consumption-savings decisions. Let \( c_{at} \) (\( c_{at} \)) denote the time \( t \) consumption of the manufactured (agricultural) good by a representative old agent. For reasons that will become apparent, it is convenient to have agents be risk neutral and at the same time to have constant aggregate expenditure shares for the two goods. We therefore assume that all agents have the utility function

\[
u(c_{at}, c_{at}; \psi) = \psi c_{at} + (1 - \psi) c_{at}, \tag{3}\]

where \( \psi \) is an independently and identically distributed (across agents) random variable with the probability distribution

\[
\psi = \begin{cases} 
0 & \text{with probability } \gamma \\
1 & \text{with probability } 1 - \gamma, 
\end{cases} \tag{4}
\]

\( \psi \) is realized at the beginning of an agent’s second period. Obviously, then, a fraction \( 1 - \gamma \) (\( \gamma \)) of old agents purchase only manufactured (agricultural) goods.\(^7\)

Finally, it remains to describe how the capital stock evolves. We assume that one unit of the manufactured good set aside at \( t \) becomes one unit of capital at \( t + 1 \). We also assume that capital is

\(^7\) The assumption that agents are risk neutral is attractive because it (a) yields a Harris-Todaro (1970)-like determination of the unemployment rate and (b) prevents employment lotteries from being useful in addressing the adverse selection problem present in labor markets. Given risk neutrality, if we want constant aggregate expenditure shares, we must either adopt the specification in the text or simply assume that a fraction \( \gamma (1 - \gamma) \) of agents care only about consumption of the agricultural (manufactured) good. If we rule out employment lotteries, we can replace the specification in the text by the assumption that agents have logarithmic utility. All our results would remain intact, and \( \gamma \) could then be interpreted as agents’ expenditure share on agricultural commodities. Parenthetically, Mas-Colell and Razin (1973) also employ the assumption of constant expenditure shares on manufactured and agricultural goods.
used in manufacturing production and then depreciates completely. The latter assumption is inessential to the analysis.

B. Trade

Three kinds of transactions occur in this economy. First, old agents use the proceeds of their savings to purchase agricultural and manufactured goods in competitive markets. Second, young agents decide in which sector to seek employment, and urban producers decide how much capital and how much labor to employ. Third, young agents save their income; here all savings ultimately take the form of physical capital.

Throughout we let the manufactured good be the numeraire at each date. In addition, let $p_t$ denote the relative price of the agricultural good at $t$ and $w_t$ denote the real wage rate paid in urban manufacturing. The term $r_t$ denotes the time $t$ capital rental rate; since all savings take the form of capital, $r_t$ is also the gross real return on savings between $t - 1$ and $t$.

II. Factor Markets

A. Preliminaries

Each young agent has a choice between seeking employment in formal urban manufacturing—and earning the wage rate $w_t$ if employed there—and working in agriculture. A young type $i$ agent employed in agriculture at $t$ earns $p_t \pi_i$. If a type $i$ agent moves to the urban sector but fails to find employment in formal manufacturing, then he engages in informal production and earns the income $\beta_i$. Agents in the urban sector who fail to find formal employment will be called unemployed, but clearly here this term also denotes employment in the informal sector. We let $\phi_t$ denote the fraction of type 2 agents who choose to seek employment in the urban sector at $t$ and $u_t$ denote the fraction of the urban labor force that is unemployed (employed in the informal sector) at the same date. Clearly there is no unemployment in agriculture.

In equilibrium, only type 2 agents will choose to work in the urban sector. Then if $\phi_{t+1} > (\leq) \phi_t$ holds, we shall say that there is net migration into (out of) that sector.

We focus here on equilibria in which $\phi_t \in (0, 1)$ for all $t$. This focus implies that there is always some remaining potential for fur-

---

8 There is considerable evidence that it is primarily the most highly skilled workers who migrate (see, e.g., Mazumdar 1987, p. 1119; Rosenzweig 1988, p. 754; Williamson 1988, p. 432).
ther rural-urban migration. In order for \( \phi_t < 1 \) to hold, it is clearly necessary that \( p_t \pi_2 > \beta_2 \) be satisfied, as we henceforth assume.

Suppose, as we have indicated, that only type 2 agents choose to seek urban employment and that the fraction \( 1 - u_t \) of them are employed in the formal sector. Then the total labor force in formal urban manufacturing at \( t \) is given by

\[
L_t = \theta_2 \phi_t (1 - u_t) n'.
\]  

If the aggregate capital stock at \( t \) is \( K_t \), then the capital/labor ratio in formal urban manufacturing is given by

\[
k_t = \frac{K_t}{L_t} = \frac{K_t}{\theta_2 \phi_t (1 - u_t) n'}.
\]  

Obviously an essential aspect of describing an equilibrium in labor markets is the determination of who seeks employment in which sector. We now turn our attention to this issue.

B. Self-Selection

A young type 2 agent employed in agriculture at \( t \) earns \( p_t \pi_2 \). This income is saved, yielding \( r_{t+1} p_t \pi_2 \) at \( t + 1 \). In addition, at \( t + 1 \) an agent will purchase only manufactured (agricultural) goods with probability \( 1 - \gamma (\gamma) \): in the former (latter) case, \( r_{t+1} p_t \pi_2 \) \( (r_{t+1} p_t \pi_2 / p_{t+1}) \) units of manufactured (agricultural) goods can be purchased. Hence the expected utility of a young type 2 agent employed in agriculture at \( t \) is given by the expression \( [(1 - \gamma) r_{t+1} + (\gamma r_{t+1} / p_{t+1})] p_t \pi_2 \). Similarly, a young type 2 agent who seeks employment in the urban sector will find urban employment with probability \( 1 - u_t \) and will then earn the real wage \( w_t \). With probability \( u_t \), the same agent will be unemployed (employed informally): in this case the agent’s real income will be \( \beta_2 \). Thus a young type 2 agent who chooses to work in the urban sector has an expected utility level of \( [(1 - \gamma) r_{t+1} + (\gamma r_{t+1} / p_{t+1})] [((1 - u_t) w_t + u_t \beta_2] \). Clearly \( \phi_t \in (0, 1) \) can hold iff young type 2 agents are indifferent between working in the two sectors: this requires that

\[
p_t \pi_2 = (1 - u_t) w_t + u_t \beta_2, \quad t \geq 0.
\]  

In view of the assumption that \( p_t \pi_2 > \beta_2 \), obviously (7) can hold only if

\[
w_t > p_t \pi_2, \quad t \geq 0.
\]  

Hence type 2 agents prefer formal sector urban employment to rural
employment, and they prefer the latter to informal urban employment.

Similarly, a young type 1 agent employed in the rural sector at \( t \) has the real income \( p_t \pi_1 \) and an expected utility level of \( [(1 - \gamma) \gamma^{\frac{\pi_1}{p_t}} + (\gamma^{\frac{\pi_1}{p_t}}/p_t)] p_t \pi_1 \). If the same agent were to seek employment in the urban sector, he would be formally (informally) employed with probability \( 1 - u_t (u_t) \) and would earn a real income of \( w_t (\beta_t) \). Hence a search for urban sector employment yields a young type 1 agent the expected utility level \( [(1 - \gamma) \gamma^{\frac{\pi_1}{p_t}} + (\gamma^{\frac{\pi_1}{p_t}}/p_t)] [(1 - u_t) w_t + u_t \beta_t] \). Obviously, then, type 1 agents are deterred from seeking urban employment only if

\[
p_t \pi_1 \geq (1 - u_t) w_t + u_t \beta_t, \quad t \geq 0. \tag{9}
\]

When (8) holds, clearly \( u_t > 0 \) must obtain in order for (9) to be satisfied. Thus urban unemployment is required in order to deter type 1 agents from entering the urban labor force.

It remains to describe the behavior of employers in the formal urban sector. Following standard conventions in models of adverse selection, we assume that firms in this sector are Nash competitors in labor markets.\(^9\) In particular, each firm announces a contract consisting of a wage \( w_t \) and an employment probability \( 1 - u_t \), with the contract announcements of other firms taken as given. Clearly there are some constraints on contract announcements. First, they must satisfy (7) and (9) unless all workers are to be drawn into the urban sector. Second, firms must earn nonnegative profits, given the workforce they attract. Then if a firm hires \( N_t \) workers, of whom a fraction \( \epsilon_t \) are of type 2, the nonnegative profit condition requires that

\[
\max_{K_t, \epsilon_t} \left\{ F(K_t, \epsilon_t, N_t) - w_t N_t - r_t K_t \right\} \geq 0. \tag{10}
\]

Standard arguments (Rothschild and Stiglitz 1976) establish that any Nash equilibrium contract announcements must maximize the expected utility of type 2 agents, subject to the constraints already stated. Thus, in particular, \( w_t, u_t, \) and \( K_t \) must be chosen to maximize \( (1 - u_t) w_t + u_t \beta_t \), subject to (9) and (10). At the same time, it is easy to show (Rothschild and Stiglitz 1976) that any Nash equilibrium contracts induce a separating equilibrium, so that only type 2 workers seek urban employment. Therefore, in any Nash equilibrium, (9) must hold and—if (8) is satisfied—it must hold as an equality. In addition, \( \epsilon_t = 0 \) holds for all \( t \). Since it is easy to see that (10)

\(^9\) At the same time, we assume that they behave competitively in product markets.
must also hold with equality in a Nash equilibrium, the standard factor pricing relationships obtain. In other words, capital and labor both earn their marginal products in formal urban manufacturing, or

\[ r_t = f'(k_t) = \alpha^{1/\rho} \left[ 1 + \left( \frac{\beta}{\alpha} \right) k_t^{-\rho} \right]^{(1-\rho)/\rho} \]  \quad (11)

and

\[ w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t) = \beta^{1/\rho} \left[ \left( \frac{\alpha}{\beta} \right) k_t^{\rho} + 1 \right]^{(1-\rho)/\rho}, \quad t \geq 0. \]  \quad (12)

In addition, if we impose \( \beta_1 = 0 \) in (9), then the equilibrium unemployment rate must satisfy \[^{10}\]

\[ p_t \pi_t = (1 - u_t) w_t, \quad t \geq 0. \]  \quad (13)

Equation (13) asserts that the unemployment rate equilibrates the expected incomes type 1 agents obtain in both urban and rural employment. It obviously closely resembles the same condition in the Harris-Todaro (1970) model, except that here the urban wage rate is endogenous.\[^11\]

III. General Equilibrium

We now characterize the determination of \( p_t, k_t, u_t, \) and \( \phi_t \) in a full general equilibrium. Doing so requires an analysis of several conditions. First, self-selection in labor markets must occur as part of a full general equilibrium, as opposed to a partial equilibrium of the type discussed in Section II. Second, we must describe the evolution of the aggregate capital stock and the capital/labor ratio in formal urban manufacturing. Third, the markets for agricultural and manu-
factured goods must clear. We now describe what is required by each of these conditions.

A. Self-Selection

Suppose we write equation (7) as

$$p_t \pi_2 = p_t \pi_1 \left( \frac{\pi_2}{\pi_1} \right) = (1 - u_t) w_t + \beta u_t, \quad t \geq 0. \quad (7')$$

If (8) holds, then $\beta_1 = 0$ implies that (13) holds as well. Substituting (13) into (7') yields

$$u_t = \frac{[(\pi_2 - \pi_1) / \beta]}{[(\pi_2 - \pi_1) / \pi_1] w(k_t) + 1}, \quad t \geq 0. \quad (14)$$

Equation (14) gives the unemployment rate in formal urban manufacturing as a function of the wage rate (and the capital/labor ratio) in the same sector. Clearly the higher the formal urban wage rate (and, since $w'(k) > 0$, the capital/labor ratio in formal manufacturing), the higher the rate of urban unemployment. This is what would be expected from the Harris-Todaro model.

B. Savings Equals Investment, and the Formal Sector Capital/Labor Ratio

Since all young-period income is invested in capital, the time $t + 1$ capital stock satisfies

$$K_{t+1} = n' \left[ \theta_1 p_t \pi_1 + \theta_2 (1 - \phi) p_t \pi_2 
+ \theta_2 (1 - u_t) w_t + \theta_2 u_t \beta_2 \right]$$

$$= n' \left[ \theta_1 \pi_1 + \theta_2 \pi_2 \right] p_t \pi_1$$

$$= n' \left[ \theta_1 + \theta_2 \pi_2 \pi_1 \right] p_t \pi_1$$

$$= n' \left[ \theta_1 + \theta_2 \pi_2 \pi_1 \right] (1 - u_t) w(k_t), \quad t \geq 0. \quad (15)$$

The first equality in (15) obtains because $\theta_1 n'$ type 1 workers work in agriculture, earning $p_t \pi_1$ each. They save all of this income. Similarly, a fraction $1 - \phi, (\phi)$ of the $\theta_2 n'$ type 2 workers are employed in agriculture (manufacturing), where their (expected) income is $p_t \pi_2 ((1 - u_t) w_t + \beta_2 u_t)$. Again, all this income is saved, yielding the first equality in (15). The second equality in (15) is then implied by (7), and the last equality is a consequence of equation (13).
We now wish to express equation (15) in terms of the capital/labor ratio. Noting that equation (6) can be written as 
\[ K_{t+1} = \theta_2 k_{t+1} n^{r+1} \phi_{t+1} (1 - u_{t+1}) \]
and using this observation in (15), we know that the capital/labor ratio evolves according to
\[ \theta_2 k_{t+1} \phi_{t+1} (1 - u_{t+1}) = \left[ \frac{\theta_1 + \theta_2 (\pi_2/\pi_1)}{n} \right] (1 - u_t) w(k), \quad t \geq 0. \] (16)

It is now clearly necessary to analyze the equilibrium determination of \( \phi_t \). To do so, we examine the implications of the requirement that the market in agricultural commodities clears.

C. Goods Market Clearing

The supply of agricultural commodities at \( t \) is given by \( [\theta_1 \pi_1 + \theta_2 (1 - \phi_t) \pi_2] n^i \), since \( \theta_1 \) type 1 workers and \( \theta_2 (1 - \phi_t) \) type 2 workers are engaged in agricultural production at that date. The demand for agricultural products at \( t \) is given by \( \gamma r K_t \), since (a) only old agents consume, (b) the income of old agents is the rate of return on savings times the previous period’s savings \( r K_t \), and (c) a fraction \( \gamma \) of this income is expended on agricultural goods. Therefore, the supply of and the demand for agricultural goods are equal when
\[ [\theta_1 \pi_1 + \theta_2 \pi_2 (1 - \phi_t)] p_t n^i = \gamma r K_t, \quad t \geq 0. \] (17)

When we substitute \( K_t = \theta_2 k_{t+1} n' \phi_t (1 - u_t) \) into (17), the agricultural commodities market clears at \( t \) iff
\[ \phi_t \gamma k_f'(k_t) \theta_2 (1 - u_t) = \left[ \theta_1 + \theta_2 \left( \frac{\pi_2}{\pi_1} \right) (1 - \phi_t) \right] p_t \pi_1, \quad t \geq 0. \] (18)

Substituting (13) into (18) and rearranging terms, we obtain the equilibrium value of \( \phi_t \):
\[ \phi_t = \frac{(\theta_1/\theta_2)(\pi_1/\pi_2) + 1}{1 + [(\pi_1/\pi_2) \gamma k_f'(k_t)] / w(k)} \]
\[ = \frac{(\theta_1/\theta_2)(\pi_1/\pi_2) + 1}{1 + \gamma (\pi_1/\pi_2)(a/b) k^p}, \quad t \geq 0, \] (19)

where we have used the fact that \( k_f'(k_t) / w(k_t) = (a/b) k^p \). Evidently, \( \phi_t < 1 \) can hold iff
D. The Equilibrium Law of Motion for the Capital/Labor Ratio

We are now prepared to describe the evolution of the capital/labor ratio in formal manufacturing. Define the function $H(k)$ by

$$H(k) = \frac{k(\pi_1/\pi_2)}{1 + [(\pi_2 - \pi_1)/\beta_2\pi_1]w(k)[1 + [\gamma(\pi_1/\pi_2)kf'(k)/w(k)]]}. \quad (21)$$

Then if we substitute equations (14) and (19) into (16), we obtain the following equilibrium law of motion for $k_t$:

$$H(k_{t+1}) = \frac{w(k_t)}{n} \left(\frac{\pi_2 - \pi_1}{\beta_2\pi_1}w(k_t) + 1\right)^{-1}, \quad t \geq 0. \quad (22)$$

Given an initial value $k_0$,\textsuperscript{13} equation (22) describes the subsequent possible evolutions of the equilibrium sequence \{$k_t$\}. Once we have obtained this sequence, \{$\phi_t$\} can be computed from (19), and the sequence of unemployment rates \{$u_t$\} can be calculated from (14). The equilibrium values \{$p_t$\} can then be derived from (13).\textsuperscript{14}

\textsuperscript{12}More generally, $\phi_t$ increases with the capital/labor ratio in formal manufacturing iff the elasticity of substitution is less than one. Most empirical estimates suggest that this is the natural case to consider.

\textsuperscript{13}The value $k_0$ is not given as an initial condition, although $K_0$ is. We describe the determination of $k_0$ in Sec. V.

\textsuperscript{14}In order to illustrate the consequences of the informational asymmetry and of the underemployment that results from it, it is instructive to compare (22) with the equilibrium law of motion that would obtain under full information. That is given by $k_{t+1}/[1 + \gamma(k_{t+1}f'(k_{t+1})/w(k_{t+1})] = w(k_t)/n$. Under the assumption of an elasticity of substitution no greater than one, the resulting law of motion for $k_t$ would necessarily be monotone, as in the standard Diamond (1965) model. Indeed, setting $\gamma = 0$ delivers exactly the Diamond model with a unitary savings rate.

We also note that our derivation of equilibrium has been predicated on the maintained hypotheses that $w(k_t) > \beta_2$ and $\phi_t < 1$ hold at each date. Hence legitimate equilibrium sequences must also satisfy $k_t > w^{-1}(\beta_2)$ and (20) for all $t$. In addition, they must satisfy the conditions stated in n. 11.
IV. Steady-State Equilibria

Imposing \( k_t = k_{t+1} \) in (22), using the definition of \( H(k) \), and utilizing the constant elasticity of substitution specification in equation (1), we obtain the following condition determining the steady-state value(s) of \( k \):

\[
n \left( \frac{\pi_1}{\pi_2} \right)^{1-\rho} = \left[ \gamma \left( \frac{\pi_1}{\pi_2} \right) + \left( \frac{b}{ak^\rho} \right) \right] \left[ 1 + \left( \frac{b}{ak^\rho} \right) \right]^{(1-\rho)/\rho}.
\]  

(23)

Equation (23) gives the steady-state capital/labor ratio in formal manufacturing as a function of the population growth rate \( n \), the relative productivity of type 1 and 2 workers in agriculture \( (\pi_1/\pi_2) \), the share of agricultural goods in total expenditure \( (\gamma) \), and technological parameters.

It will now be useful to make the following transformation. Define

\[
x_t \equiv \frac{b}{ak_t^\rho} = \frac{w(k_t)}{k_t f''(k_t)}
\]  

(24)

to be the ratio of labor’s share in formal manufacturing to capital’s share; \( x_t \) is increasing in \( k_t \) when \( \rho < 0 \). If we also define the function \( Q(x) \) by

\[
Q(x) \equiv \left[ \gamma \left( \frac{\pi_1}{\pi_2} \right) + x \right] \left[ 1 + x \right]^{(1-\rho)/\rho},
\]  

(25)

then we can rewrite the steady-state equilibrium condition (23) in the form

\[
Q(x) = n \left( \frac{\pi_1}{\pi_2} \right)^{a^{-1/\rho}}.
\]  

(26)

Equation (26) determines the steady-state value of \( x_t \); the steady-state value of \( k_t \) can be recovered from (24).

Evidently, it will be important to know some properties of the function \( Q \). They are summarized in the following lemma (see Benaviega and Smith [1995] for a proof).

**Lemma 1.** (a) \( Q(0) = \gamma (\pi_1/\pi_2) < 1 \) holds. (b) \( Q'(x) \) satisfies

\[
\frac{Q'(x)}{Q(x)} = \frac{(1 - \rho)/\rho}{1 + x} + \frac{1}{\gamma (\pi_1/\pi_2) + x}.
\]  

(27)
Thus $Q'(x) \geq 0$ is satisfied iff $x \leq -\rho - (1 - \rho) \gamma(\pi_1/\pi_2)$. (c) $\lim_{x \to \infty} Q(x) = 0$.

Lemma 1 implies that the function $Q$ has the configuration depicted in figure 1. There are now three possibilities regarding the existence of a steady-state equilibrium. They are summarized in the following proposition. Its proof appears in Bencivenga and Smith (1995).

**Proposition 1.** (a) Suppose that

$$-\rho > (1 - \rho) \gamma\left(\frac{\pi_1}{\pi_2}\right)$$

(28)
are both satisfied. Then there are two values of $x$—denoted by $\bar{x}$ and $\tilde{x}$—satisfying (26). (b) Suppose that either (28) or the first inequality in (29) is violated but the second inequality in (29) holds. Then there is a unique solution to (26), which we denote by $\bar{x}$. (c) Suppose that the second inequality in (29) is violated. Then there is no steady-state equilibrium.15

When (28) and (29) both hold, two nontrivial steady-state equilibria exist. The potential existence of multiple steady states derives from the possibility of rural-urban migration. When $x = \bar{x}$ in the steady state, the capital/labor ratio is low in formal manufacturing. As a result, the real wage in that sector is also low, the urban (rural) labor force is relatively small (large), and the supply of the agricultural good is correspondingly high. This tends to depress the relative price of the agricultural good, with the consequence that aggregate income is low, as is the aggregate capital stock. Thus the economy is stuck in a development trap in which both the urban labor force and the capital stock are relatively small. When $x = \tilde{x}$ in the steady state, on the other hand, there is a relatively high wage rate in formal manufacturing, the urban labor force is correspondingly large, and average incomes are high. The induced savings then generate a relatively large aggregate capital stock. In this steady state, of course, the high wage rate leads to more unemployment than is observed in the low-capital-stock steady state.

When is there a potential for multiple steady-state equilibria to arise? As is apparent from proposition 1, there will be a unique steady-state equilibrium if the rate of population growth is sufficiently low. For intermediate levels of the population growth rate there can be multiple steady states, whereas excessive rates of population growth preclude the existence of any steady-state equilibria, as is potentially true in the Diamond (1965) model. Thus intermediate rates of population growth are conducive to the existence of more than one steady state.

We now consider two issues: (i) Will any steady-state equilibrium

15 Let $x$ satisfy (26). Then the associated steady-state value of $k$ given by (24) satisfies $w(k) > \beta_1$ iff $b^\gamma[(1 + x)/x]^{1-\alpha/\rho} > \beta_2$ holds. This condition, in turn, will obtain when (28) and (29) are satisfied iff $Q[(b/a)w^{-1}(\beta_2)^{\gamma}] < n(\pi_1/\pi_2)a^{1/\rho}$. It will obtain when either (28) or the first inequality in (29) is violated iff $Q[(b/a)w^{-1}(\beta_2)^{\gamma}] > n(\pi_1/\pi_2)a^{1/\rho}$.
necessarily be approached, and, if so, (ii) what is the nature of the equilibrium paths approaching that steady state?

V. Dynamical Equilibria

Dynamical equilibria consist of sequences \{k_t\} that satisfy equation (22), \(w(k) > \beta_2\), (20), and the conditions of note 11 for all \(t\). We now undertake an analysis of the equilibrium law of motion for \(k_t\) embodied in equation (22). We then produce examples in which all candidate equilibrium sequences obeying (22) also satisfy the remaining conditions at all dates. To do this, it will prove useful to work with a transformed version of equation (22). In particular, that equation can be written in the following form:

\[ w(k_t) = \frac{nH(k_{t+1})}{1 - n[\pi_2 - \pi_1]/[\beta_2 \pi_1]} H(k_{t+1}), \quad t \geq 0. \]  

Equation (30) gives \(k_t\) as a function of \(k_{t+1}\), whereas (22) is typically a correspondence. It will therefore be most convenient to analyze (30), and from that analysis to draw inferences about the natural “backward” dynamics in this economy.

Since \(w'(k) > 0\) holds, the properties of the equilibrium law of motion (30) are primarily dictated by the properties of the function \(H(k)\). Some of these properties are stated in the following lemma.

**Lemma 2.** The function \(H\) satisfies the following conditions: (a) \(H(0) = 0\); (b) \(\lim_{k \to \infty} H(k) = \infty\); and (c) \(H'(k) > 0\) holds for all \(k\) satisfying \((b/a)k^{-\rho} \geq -\rho - (1 - \rho)\gamma(\pi_1/\pi_2)\). In addition, \(H'(0) > 0\) holds.

Lemma 2 is proved in Bencivenga and Smith (1995). The lemma has two immediate corollaries. First, part (b) of lemma 1 and part (c) of lemma 2 imply that \(H'(k) > 0\) holds whenever \(Q'[(b/a)k^{-\rho}] \leq 0\) is satisfied. Thus, if there is a unique nontrivial steady state, that steady state must have \(H'(k) > 0\). Second, if there are two nontrivial steady states, \(H'(k) > 0\) must hold at the high-capital-stock steady state. In this case the low-capital-stock steady state can have \(H'(k) \equiv 0\).

Before we proceed to a characterization of equation (30), it will be useful to have a preliminary result. When \(\rho < 0\) holds, \(w(k) \leq b^{1/\rho}\) is satisfied for all \(k\). As a consequence, it is easy to verify that (30) has a solution for \(k_t\) iff \(H(k_t) < \beta_2 \pi_1/[n(\pi_2 - \pi_1) + n\beta_2 \pi_1 b^{-1/\rho}] \equiv \eta\) holds. We then have the following result (see Bencivenga and Smith [1995] for a proof).

**Lemma 3.** Suppose that (a) \(H(k) > H[w^{-1}(\beta_2)]\) holds for all \(k > w^{-1}(\beta_2)\), and (b) \(H[w^{-1}(\beta_2)] > \beta_2/n(\pi_2/\pi_1)\) holds. Then equation
(30) defines a differentiable function $G: (w^{-1}(\bar{\beta}_2), H^{-1}(\eta)) \rightarrow (w^{-1}(\bar{\beta}_2), \infty)$ such that $k_t = G(k_{t+1})$.

We computed a large number of numerical examples (a representative one is presented below) that satisfy conditions $a$ and $b$ of the lemma.

We are now prepared to characterize the equilibrium law of motion $k_t = G(k_{t+1})$. This law of motion has several possible configurations; they are described in detail by Bencivenga and Smith (1995). Here we focus on what we regard as the most interesting of these configurations, and the one that arose most frequently in our numerical examples.

Figure 2 depicts the equilibrium law of motion (30), along with the function $H(k)$, when $H^{-1}((0, \eta))$ consists of two disjoint inter-
vals. Evidently the function $G(k_{i+1})$ consists of two “pieces” as well: an upward-sloping piece and a “U-shaped” piece as shown in the figure. Apparently, if there are any steady-state equilibria, there are generically two of them: these steady states are denoted $k$ and $\bar{k}$ in figure 2.\textsuperscript{16} If we let $\bar{k}$ denote the high-capital-stock steady state (or the unique nontrivial steady state, if there is only one), the following result is easily established.

**Proposition 2.** $G'(\bar{k}) > 1$ holds.

Proposition 2 asserts that the steady state with $k_i = \bar{k}$ is unstable in the “forward” dynamics of equation (30) and therefore that it is asymptotically stable in the natural “backward” dynamics of equation (22). Hence this steady state can be approached from some initial values of $k_0$.

However, when two nontrivial steady states exist and when $G'(k) < 0$ holds—as is the case in figure 2—the low-capital-stock steady state may be either unstable or asymptotically stable in the forward dynamics of equation (30). In the former case a number of interesting dynamical phenomena can be observed. We now consider this possibility.

\textbf{A. Oscillation and Development Traps}

Figure 3 reproduces the lower panel of figure 2, and it also depicts the “negative 45-degree line” passing through the point $(k, k)$. As drawn, clearly $G'(k) < -1$ holds, so that the low-capital-stock steady state is unstable (asymptotically stable) in the forward (backward) dynamics of equation (30) ([22]). Thus this steady state can be approached from some initial values $k_0$ in the natural backward dynamics. Obviously paths approaching the low-capital-stock steady state will display damped oscillation as they do so.

Moreover, when $G'(k) < -1$ holds, both steady-state equilibria are asymptotically stable (in the backward dynamics), and hence development trap phenomena can be observed. In particular, two otherwise identical economies with different initial values for $k_0$ can approach different long-run capital/labor ratios and hence can fail to converge for purely endogenous reasons. Economies that are “trapped” in low-level equilibria will also pay an additional price: they will experience oscillation in all endogenous variables en route to the steady state.

The reason that both oscillations and development trap phenomena can be observed derives from the role of rural-urban migration. In particular, suppose that $k_i$ is relatively high, which means that,

\textsuperscript{16} Clearly $(b/a)\bar{k}^{\gamma} = \bar{x}$ and $(b/a)\bar{\bar{k}}^{\gamma} = \bar{x}$. 

on average, young agents have high incomes at $t$. The result is large savings, so that $K_{t+1}$ is correspondingly high. However, if enough labor is drawn from the rural to the urban sector as a result, $k_{t+1} \equiv K_{t+1}/L_{t+1}$ can actually fall, producing low incomes at $t + 1$. The result is a low value of $K_{t+2}$, out-migration from the city, and a potential increase in $k_{t+2}$. Thus, when rural-urban migration is a strong enough force, the capital/labor ratio in urban manufacturing can fluctuate along perfect-foresight equilibrium paths. Along such paths that approach the low-capital-stock steady state, these fluctuations dampen over time. However, it is also very easy to observe equilibria that display undamped oscillation. We now turn our attention to this possibility.

With reference to figure 3, define $\hat{k}$ by $G'(\hat{k}) = 0$. In addition, define $k^*$ by $G(k^*) = G(\hat{k})$ and $G'(k^*) < 0$. We can then state the following result.

**Proposition 3.** Suppose that $k^* > G(\hat{k})$ holds. Then in every neighborhood of $\hat{k}$ there are infinitely many distinct periodic points. Indeed, there exist equilibrium cycles of periodicity $2^j$, for all $j \geq 0$.

The condition $k^* > G(\hat{k})$ implies the existence of a homoclinic orbit (Devaney 1989, p. 122); one such orbit is depicted in figure 3.
The existence of a homoclinic orbit, in turn, implies that there are infinitely many distinct periodic points (Devaney 1989, p. 125).

Proposition 3 asserts that, whenever \( k^* > G(\bar{k}) \) holds, as it does in figure 3, a large number of equilibria exist in which \( \{k_t\} \) displays cycles of period 2 or higher. Moreover, while such cycles arise in the forward dynamics of equation (30), if \( \{k_1, k_2, \ldots, k_j\} \) is a \( j \)-period cycle in the forward dynamics, \( \{k_j, k_{j-1}, \ldots, k_1\} \) is a \( j \)-period cycle in the normal backward dynamics. Thus such cycles also emerge in solutions to (22). Parenthetically, the existence of cyclical perfect-foresight paths creates the potential for an additional type of non-convergence phenomenon. In particular, there can exist two intrinsically identical economies, one of which has \( k_t \uparrow \bar{k} \). At the same time, the other may have \( \{k_t\} \) evolving according to a \( J \)-period cycle. The economy experiencing fluctuations may have \( k_t \) arbitrarily close to \( \bar{k} \) at some dates but will not sustain this high level of real activity.

We now display an example in which \( G(k_{t+1}) \) has the configuration depicted in figures 2 and 3 and in particular satisfies \( k^* > G(k) \). In addition, the conditions of note 11 are satisfied for all \( k \in (w^-(\beta_2), \bar{k}] \), as is the condition of equation (20). Finally, the example satisfies conditions a and b of lemma 3; hence all potential perfect-foresight paths have \( w(k_t) > \beta_2 \) for all \( t \).

Example 1.—Let \( \pi_1 = 1, \pi_2 = 16, a = b = 11.03175, \) and \( \rho = -1.5 \) be the technological parameters; \( \beta_2 = 0.02 \) and \( \beta_1 = 0 \) also hold. In addition, set \( n = 1, \gamma = 0.8, \) and \( \theta_1 = 0.2 \). This economy has the equilibrium law of motion depicted in figure 4. There are two steady states, with \( \bar{k} = 0.788357 \) and \( k = 1.95611 \). Equation (20) is satisfied for all \( k \leq 2.17153 \); hence it is satisfied at each steady state. Finally, \( G'(\bar{k}) < -1 \) and \( k^* > G(k) \) both hold, so that this economy possesses a homoclinic orbit.

In addition, numerical calculations indicate that this economy generates an equilibrium displaying a three-period cycle; such a cycle is depicted in figure 4. It is then straightforward to show that equilibrium cycles of all periods exist (Devaney 1989, p. 60).\(^{18}\)

B. The Nature of Cycles

Dynamical equilibria displaying either damped or undamped oscillation have the feature that \( k_t \) and \( w(k_t) \) fluctuate over time. When \( w(k_t) \) is relatively high (or, more generally, increasing), net migration from rural to urban employment will occur. When \( w(k_t) \) is rela-

\(^{18}\) We should note that this example is representative of a large number of examples that we computed. In particular, it is relatively easy to find examples in which \( G(k) \) has the configuration depicted in figs. 3 and 4.
Fig. 4.—Cyclical equilibria

tively low (or, more generally, falling), net migration from the urban to the rural sector will occur.

In general, the history of economic development suggests that sustained growth is accompanied by sustained net migration from rural to urban sectors, possibly punctuated by relatively brief episodes of reverse migration (see below). Thus, if equilibrium cycles are to be consistent with observation, they must have the feature that periods of out-migration from urban employment are relatively infrequent.

Figure 4 depicts the economy of example 1, which has a three-period cycle. It also depicts the natural backward dynamics associated with this cycle. Evidently, in such an equilibrium, the economy will experience two periods of sustained growth, interrupted by one period of contraction. During this “recession,” there will be net migration from urban to rural employment.

Some reflection on the homoclinic orbit depicted in figure 3 will suggest—quite correctly—that it is straightforward to construct cyclical equilibria of periodicity $j$ that have the feature that $j - 1$ periods of sustained growth are followed by one period of pronounced recession. Only during this single period does net out-migration from urban areas occur.

Hatton and Williamson (1992), using U.S. data from 1920–40, show that there has been consistent net migration from rural to urban employment, with one exception. The early 1930s experienced a pronounced net migration in the opposite direction. Thus signifi-
cantly declines in output have historically been accompanied by the reverse migration that our model predicts. And, while Hatton and Williamson do not have migration data before 1920, their statistical model—which fits quite well in sample—predicts that there was net out-migration from urban areas in 1919, immediately preceding the pronounced recession of 1920–21. Thus there appears to be a significant empirical basis for the notion that cyclical equilibria displaying long expansions—followed by relatively brief but severe recessions—can occur and are associated with net rural-urban migration, which reverses during sharp downturns.

C. Indeterminacy

There are two potential sources of indeterminacy in this economy. One derives from the fact that, in the normal backward dynamics, equation (22) is a correspondence. Hence, for any initial value \( k_0 \in \{G(k), \bar{k}\} \) in figure 3, there are multiple possible perfect-foresight equilibrium paths. For example, in figure 4, if \( k_0 \) is as shown, there is a perfect-foresight equilibrium path corresponding to the three-period cycle, and at the same time there is a monotonic perfect-foresight path with \( k_t \uparrow \bar{k} \). There are also obviously other equilibrium paths in which a period of oscillation is observed, followed by the convergence of \( \{k_t\} \) to one of its steady-state values.\(^{19}\)

There is also another source of indeterminacy here, which is a consequence of the fact that, while \( K_0 \) is given, \( k_0 \) is not. We now describe the determination of the initial value \( k_0 \).

As already noted, \( K_t = \theta_2 k_t \phi_t (1 - u_t) n' \) holds at all dates, including \( t = 0 \). Thus \( K_0 = \theta_2 k_0 \phi_0 (1 - u_0) \) holds. Substituting (14) and (19) into this relation, we obtain

\[
H(k_0) = \frac{K_0}{\theta_1 + \theta_2 (\pi_2/\pi_1)}.
\]

When the function \( H \) has the configuration depicted in figure 2, there will be an interval of values of \( K_0 \) for which (31) delivers more than one possible value of \( k_0 \). Thus, for any given initial condition \( K_0 > 0 \), there may be more than one equilibrium value of \( k_0 \), which is an additional source of indeterminacy.

\(^{19}\) The situation in which the equilibrium law of motion is a correspondence rather than a function clearly arises often in pure exchange overlapping generations models of money (Azariadis 1981, 1993; Benhabib and Day 1982; Grandmont 1985) when income effects are sufficiently strong. This situation does not arise in any overlapping generations models with production that we are familiar with.
D. The Effects of an Increase in the Rate of Population Growth

It is easy to verify that the effect of an increase in $n$ is to reduce the (urban) capital/labor ratio at the high-capital-stock steady state and to increase it at the low-capital-stock steady state. Thus, for economies experiencing development traps, higher rates of population growth can actually increase the per capita capital stock and lead to higher steady-state (urban and rural) wage rates.

In addition, as we have already noted, two steady-state equilibria exist only when the rate of population growth is neither too small nor too large. Finally, it is easy to show that high enough rates of population growth render the low-capital-stock steady state unstable and hence preclude development trap phenomena, as well as oscillatory equilibria. On the other hand, high population growth rates limit the size of $k$. Thus there is a “tension” regarding high rates of population growth: they preclude development traps and equilibria displaying endogenous fluctuations. However, they also imply that, in per capita terms, the economy will be fairly poor in the high-capital-stock steady state. It is not transparent which situation might be preferable.

VI. Conclusions

We have developed a model in which rural-urban migration and the existence of urban underemployment are crucial aspects of the development process. The result is that development trap phenomena can easily be observed, as can an indeterminacy of equilibrium and the existence of endogenous, undamped economic fluctuations. It is also easy to generate equilibria in which long periods of sustained growth and sustained rural-urban migration are punctuated by brief—but possibly severe—downturns, as well as reverse net migration. Such equilibria seem broadly consistent with historical experience.

The underemployment that arises in our model is not a consequence of any exogenous rigidities, but instead arises endogenously in response to an adverse selection problem in labor markets. The presence of this underemployment allows urban-rural wage differentials to vary over time and with the level of economic development. Thus, as in Harris and Todaro (1970), the process of rural-urban migration is equilibrated by the rate of urban underemployment. And, it is exactly this migration that underlies the most interesting aspects of our analysis.

To be more specific, high current capital/labor ratios in formal
urban manufacturing are associated with high average income levels and hence with high levels of saving and of the future aggregate capital stock. However, if enough labor is drawn into the city, next period’s urban capital/labor ratio may actually fall. By implication, there need not be a monotone relationship between the current and future capital/labor ratio in formal manufacturing. Such nonmonotonicities are the source of multiple asymptotically stable steady states, indeterminacies, and endogenous fluctuations.

There are obviously a number of dimensions along which the current analysis could be extended. One would be a consideration of how various policies affect the process of rural-urban migration, underemployment, and capital formation. Some natural policies to examine would include unemployment insurance, agricultural subsidies, or the provision of urban public services. A second issue of great importance concerns why urban unemployment/underemployment is such a significant phenomenon in developing countries but is not in more developed economies. A natural conjecture is that a basic premise of all dual-economy models is violated in the most developed economies; agriculture is actually highly capital intensive rather than labor intensive. Clearly an analysis of the technological change that occurs in agricultural production as development proceeds is beyond the scope of the present paper; integrating it into a model of rural-urban migration is an important topic for future investigation.

References
Azariadis, Costas. “Self-Fulfilling Prophecies.” J. Econ. Theory 25 (December 1981): 308–96.
———. Intertemporal Macroeconomics. Cambridge: Blackwell, 1993.
Bencivenga, Valerie R., and Smith, Bruce D. “Unemployment, Migration and Growth.” Manuscript. Austin: Univ. Texas, Dept. Econ., 1995.
Benhabib, Jess, and Day, Richard H. “A Characterization of Erratic Dynamics in the Overlapping Generations Model.” J. Econ. Dynamics and Control 4 (February 1982): 37–55.
Calvo, Guillermo A. “Urban Employment and Wage Determination in LDC’s: Trade Unions in the Harris-Todaro Model.” Internat. Econ. Rev. 19 (February 1978): 65–81.
Devaney, Robert L. An Introduction to Chaotic Dynamical Systems. 2d ed. New York: Addison-Wesley, 1989.
Diamond, Peter A. “National Debt in a Neoclassical Growth Model.” A.E.R. 55 (December 1965): 1126–50.
Drazen, Allan. “Unemployment in LDCs: Worker Heterogeneity, Screening, and Quantity Constraints.” World Development 10 (December 1982): 1039–47.
Drazen, Allan, and Eckstein, Zvi. “On the Organization of Rural Markets
and the Process of Economic Development.” *A.E.R.* 78 (June 1988): 431–43.

Grandmont, Jean-Michel. “On Endogenous Competitive Business Cycles.” *Econometrica* 53 (September 1985): 995–1045.

Harris, John R., and Todaro, Michael P. “Migration, Unemployment and Development: A Two-Sector Analysis.” *A.E.R.* 60 (March 1970): 126–42.

Hatton, Timothy J., and Williamson, Jeffrey G. “What Explains Wage Gaps between Farm and City? Exploring the Todaro Model with American Evidence, 1890–1941.” *Econ. Development and Cultural Change* 40 (January 1992): 267–94.

Mas-Colell, Andreu, and Razin, Assaf. “A Model of Intersectoral Migration and Growth.” *Oxford Econ. Papers* 25 (March 1973): 72–79.

Mazumdar, Dipak. “Rural-Urban Migration in Developing Countries.” In *Handbook of Regional and Urban Economics*, vol. 2, *Urban Economics*, edited by Edwin S. Mills. New York: Elsevier Sci., 1987.

Ranis, Gustav. “Analytics of Development: Dualism.” In *Handbook of Development Economics*, vol. 1, edited by Hollis Chenery and T. N. Srinivasan. New York: Elsevier Sci., 1988.

Rothschild, Michael, and Stiglitz, Joseph E. “Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information.” *Q.J.E.* 90 (November 1976): 629–49.

Sethuraman, S. V. *The Urban Informal Sector in Developing Countries: Employment, Poverty, and Environment*. Geneva: Internat. Labor Office, 1981.

Stiglitz, Joseph E. “Economic Organization, Information, and Development.” In *Handbook of Development Economics*, vol. 1, edited by Hollis Chenery and T. N. Srinivasan. New York: Elsevier Sci., 1988.

Todaro, Michael P. “A Model for Labor Migration and Urban Unemployment in Less Developed Countries.” *A.E.R.* 59 (March 1969): 138–48.

Williamson, Jeffrey G. “Migration and Urbanization.” In *Handbook of Development Economics*, vol. 1, edited by Hollis Chenery and T. N. Srinivasan. New York: Elsevier Sci., 1988.

World Bank. *World Development Report*. New York: Oxford Univ. Press (for World Bank), 1995.