Spin polarizations in a covariant angular momentum conserved chiral transport model

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Using a covariant and total angular momentum conserved chiral transport model, which takes into account the spin-orbit interactions of chiral fermions in their scatterings via the anomalous side jump effect, we study the quark spin polarizations in quark matter. For a system of rotating and unpolarized massless quarks in an expanding box, we find that the side jump effect can dynamically polarize the quark spin with the final quark spin polarization consistent with that of thermally equilibrated massless quarks in a self-consistent vorticity field. For the quark matter produced in non-central relativistic heavy ion collisions, we find that both the quark local spin polarizations in the direction perpendicular to the reaction plane and along the longitudinal beam direction show an azimuthal angle dependence in the transverse plane similar to those observed in experiments for Lambda hyperons.

I. INTRODUCTION

Recent experimental observation of the Lambda hyperon polarization in heavy ion collisions by the STAR Collaboration at the Relativistic Heavy Ion Collider (RHIC) [1–5] has provided the evidence for the existence of a vortical field in the produced quark-gluon plasma (QGP) that is the largest ever known. Theoretical studies using the hydrodynamic approach [6–8] or the transport model [9, 10] assuming thermal equilibration of the spin degrees of freedom have been able to describe the measured Lambda hyperon global polarization [1–3]. These studies fail, however, to explain the measured azimuthal angle dependence of Lambda hyperon polarization in heavy ion collisions [4, 5]. Various suggestions have been proposed to explain the experimental results without much success [11–13] or unambiguity [14]. Also, studying the quark spin polarization from the spin-orbit interaction in quark-quark and quark-gluon scattering in QGP [15, 16] seems to cast doubts on the equilibration of the spin degrees of freedom during heavy ion collisions as a result of the long equilibration time. On the other hand, the study using the chiral kinetic approach [17, 18] for quarks in a self-consistent vorticity field can describe not only the global polarization but also partially the local polarization of Lambda hyperons measured in experiments [19]. However, this study does not address the conservation of total angular momentum in quark-quark scattering, although it is conserved during the quark propagation. Furthermore, the spin and orbital momenta in this approach is not treated in a covariant way, which is essential for the spin dynamics of chiral fermions. As known in condensed matter systems [20], to conserve the total angular momentum in a scattering process requires the inclusion of an anomalous side jump effect. The covariant formulation of this effect was developed in Refs. [21–24] for relativistic chiral fermions and shown to reproduce the chiral vortical effect in such systems [25]. Since these studies are restricted to the scattering of partons with same helicity at zero impact parameter, they are not suitable for studying the spin-orbit interaction in relativistic heavy ion collisions. In the present study, we go beyond this limitation by constructing a covariant and total angular momentum conserved chiral transport model and then use it to study the quark local spin polarizations in relativistic heavy ion collisions.

II. THE SIDE JUMP FORMALISM

Following Ref. [22], the covariant four-dimensional angular momentum tensor of a particle can be written as

\[ J^{\mu \nu} = x^\mu p^\nu - x^\nu p^\mu + S^{\mu \nu}, \]  

where \( x \) and \( p \) are, respectively, the four coordinate and momentum, and \( S^{\mu \nu} \) is the spin tensor. For massless spin 1/2 particles, \( S^{\mu \nu} \) can be chosen to be [22]

\[ S^{\mu \nu} = \lambda \frac{\epsilon^{\mu \nu \alpha \beta} p_\alpha n_\beta}{p \cdot n}. \]  

In the above, \( n \) is the four vector that specifies the frame of reference; \( \lambda = \pm 1/2 \) denotes the helicity of the particle with the plus and minus signs for positive and negative helicities, respectively. This choice ensures that the spin tensor satisfies the conditions \( p_\mu S^{\mu \nu} = 0 \) and \( n_\nu S^{\mu \nu} = 0 \). To maintain the spin of the massless particle along...
the direction of its momentum requires \( n^\mu = (1, 0) \) and thus \( S^{ij}_q = \lambda \epsilon^{ijk} p^k / |p| \). Because of parallel spin and momentum directions, the orbital motion of a particle also determines the motion of its spin, thus resulting in a strong spin-orbit coupling.

As in Ref. [22], to maintain the helicity of a particle in any frame \( n' \) requires the imposition of \( n' = (1, 0) \) and also the introduction of an anomalous Lorentz transformation under which \( J' = \Lambda J \Lambda^T \) and \( p' = \Lambda p \) but

\[
x'^\alpha = \Lambda^\alpha_\beta x^\beta + \Delta^\alpha_{\beta n'},
\]

where the term \( \Delta^\alpha_{\beta n'} \) describes the side jump and is given by

\[
\Delta^\alpha_{\beta n'} = \lambda \xi(n') \cdot \delta(n) (p' \cdot n')
\]

with \( \xi(n') = \Lambda^\alpha_\alpha (1, 0) \alpha = \Lambda^{\beta \gamma}_0 \).

The above anomalous Lorentz transformation makes it possible to treat in a covariant way the conservation of total angular momentum in a two-body scattering process. With the initial momenta \( p_1 = -p_2 = p \) and final momenta \( p_1' = -p_2' = p' \) of the two scattering particles in their center-of-mass (CM) frame, conservation of the spatial part of the total angular momentum tensor, \( J_1 + J_2 = J_1' + J_2' \), is reduced to

\[
J = \Delta \times p + (\lambda_1 - \lambda_2) \tilde{p} = \Delta' \times p' + (\lambda_1 - \lambda_2) \tilde{p}'
\]

with \( \Delta = x_1 - x_2 \) and \( \Delta' = x_1' - x_2' \). Since \( \tilde{p} \cdot J = \tilde{p} \cdot J = \lambda_1 - \lambda_2 \) from Eq.(5) is a constant and \(|p| = |p'|\), the momentum \( p' \) can be obtained from \( p \) by a rotation of angle \( \phi \) around the direction of the total angular momentum \( J \), that is

\[
p' = R_3(\phi)p
\]

\[
\equiv (p \cdot \hat{J}) \hat{J} + [p - (p \cdot \hat{J}) \hat{J}] \cos \phi + (\hat{J} \times p) \sin \phi.
\]

The value of the rotation angle \( \phi \) depends on the angular distribution of the scattering cross section. For an isotropic cross section as considered in the present study, it is a random number between 0 and 2\( \pi \). Similarly, the relative distance \( \Delta' \) can be obtained from \( \Delta \) via

\[
\Delta' = R_3(\phi) \Delta + \xi p',
\]

where the second term represents the “gauge” freedom to shift \( \Delta \) along the momentum direction without changing the cross product in Eq.(5). For simplicity, we choose \( \xi = 0 \) in the present study. Since \( \Delta \neq \Delta' \) in general, there is a change or jump in the relative distance between the two particles before and after scattering in their CM frame, which is absent for scattering at zero impact parameter [21, 22].

For the time component of the total angular momentum tensor, its conservation leads to the relation

\[
X' = X - (p - p')dt / \sqrt{s}
\]

between the CM coordinates of the two colliding particles \( X = (x_1 + x_2)/2 \) and \( X' = (x_1' + x_2')/2 \) in their CM frame before and after the scattering. In Eq.(8), \( dt = x_1^0 - x_2^0 \) is nonzero in the CM frame since the scattering occurs at the same time in the laboratory (LAB) frame for the two particles, and \( \sqrt{s} \) is the invariant energy of the two colliding particles in their CM frame. With the scattering in the CM frame being local in time, one has \( x_1^0 = x_1'^0 \) and \( x_2^0 = x_2'^0 \) and thus \( dt' = x_1'^0 - x_2'^0 = dt \). The coordinates of the two particles after scattering in the CM frame are then given by \( x_1' = X' + \Delta'/2 \) and \( x_2' = X' - \Delta'/2 \). Using Eqs. (3) and (4), the coordinates and momenta of the two particles in the LAB frame can be obtained by a Lorentz boost with \( \hat{n} = (p_1 + p_2) / \sqrt{s} \). Since the time of the two particles in the LAB frame changes after their scattering, there is an additional shift in their coordinates due to free propagation during this time change. The above procedure is time reversal invariant and automatically guarantees the conservation of energy-momentum and also the covariant total angular momentum tensor in the LAB frame.

In terms of the spin tensor \( S^{\mu \nu} \) introduced in Eq.(2) and the phase-space distribution functions \( f_{R/L} \) of left/right handed particles, the covariant right/left current density is given by [22]

\[
j^\mu_{R/L}(x) = \int \frac{d^3p}{(2\pi)^3} (p^\mu f_{R/L} + S^{\mu \nu} \partial_\nu f_{R/L})
\]

In the above, the first term is the usual contribution \( S_\lambda = \int (2\pi)^{-3} d^3p d^3x \lambda p f_{\lambda}(p, x) \equiv \sum_\lambda \lambda \hat{p}_\lambda \) from the spin of a particle, while the second term is the magnetization contribution at \( h \) order [21, 22]. Since the latter contribution is proportional to the vorticity due to the orbital motions of particles, it can be considered as the contribution from the local orbital angular momentum. This is similar to the nontrivial “gauge freedom” in separating the spin and orbital angular momenta in relativistic classical mechanics [22]. We note that the jump current term discussed in Ref. [22] is neglected in the above since we are only interested in systems close to equilibrium in which this contribution is small.

Using the vector and axial currents given respectively by \( j^\mu = j^\mu_R + j^\mu_L = (n_1 \hat{J}) \) and \( j^\mu_a = j^\mu_R - j^\mu_L = (n_5 \hat{J}_L) \), the distribution of spin polarizations \( 2(S_R + S_L)/(N_R + N_L) \) in the system, where \( N_R \) and \( N_L \) are, respectively, the numbers of right- and left-handed particles, can then be expressed as

\[
\mathcal{P} = \int d^3x j^\mu_5(x) / \int d^3x n(x).
\]

It is worthy to point out that defining the spin polarization through the covariant currents \( j^\mu \) and \( j^\mu_a \) makes it possible to study its values in different frames of reference.
III. NUMERICAL IMPLEMENTATIONS AND RESULTS

The above theoretical framework can be implemented in the transport model by extending the usual parton scattering to include the side-jump effect. We choose an isotropic cross section of 10 mb to mimic a strongly coupled medium and use the geometric method to determine the conditions for a scattering [26, 27]. To achieve the correct equilibrium distribution, the coordinates \( x_{\perp,2} \) of the two particles in the conditions for their scattering in the LAB frame is, however, modified by the side jump effect to \( x_{\perp,2} - \Delta_{1,2} \). Using the anomalous Lorentz boost of Eq. (3), the scattering can also be treated in the CM frame using the geometric method with appropriate conditions.

To illustrate the side jump effect on the scattering of massless quarks, we consider the case of a quark matter initially uniformly distributed in a box of sizes \( 5 \times 5 \times 5 \text{ fm}^3 \). For their initial momenta, they are taken to have a Boltzmann distribution at temperature of 300 MeV and a flow profile \( \gamma v = 2 \omega \times (x, 0, 0) \), where \( v \) is the flow velocity, \( \gamma = 1/\sqrt{1 - v^2} \) is the corresponding gamma factor, and \( \omega = (0, 0.12, 0) \text{ fm}/c \) is a constant vorticity field along the \( y \)-direction. The system is then allowed to expand for 2 fm/c.

![Figure 1](image-url) **FIG. 1:** Left panel: quark spin \( S_y \), orbital \( L_y \), and total \( J_y \) angular momenta in the \( y \)-direction. Right panel: quark total spin polarization in the \( y \)-direction and its spin and orbital contributions together with the spin polarization expected from the thermal model.

In the left window of Fig. 1, we show the time evolution of the spin \( S_y \) (solid), orbital \( L_y \) (dashed line) and total \( J_y \) (dash-dotted line) angular momenta of the system along the \( y \)-direction with the orbital and total angular momenta subtracted by the constant value of \( 5,425 \hbar \). It is seen that the spin angular momentum increases from zero and quickly reaches 12\( \hbar \) within 0.5 fm/c. The orbital angular momentum decreases by the same amount, and the total angular momentum of the system is conserved during the evolution. The total polarization in the \( y \)-direction (dash-dotted line) given by the sum of the spin (solid line) and orbital (dashed line) terms in Eq. (9) is shown in the right window of Fig. 1 and compared to the thermally equilibrated value \( \omega/(2T) \) (dotted line), where both \( \omega \) and \( T \) are calculated assuming local equilibrium. In calculating the average polarization, only the volume \( 3 \times 3 \times 3 \text{ fm}^3 \) around the center of the system are used to avoid numerical complications near the boundary of the expanding box. We see that the contribution to the total polarization from the spin term in Eq. (9) increases from zero and also quickly reaches a plateau as in the behavior of the total spin. For the contribution from the orbital term in Eq. (9), it is finite even at \( t = 0 \) due to local orbital motions as a result of nonzero vorticity. As to the total spin polarization, its final value is close to that expected from the thermal model result.

We further study the quark spin polarizations in heavy ion collisions by considering Au+Au collisions at \( \sqrt{s} = 200 \text{ GeV} \) and at impact parameter \( b = 8.87 \text{ fm} \), corresponding to the collision centrality of 30-40%. For the initial quark and antiquark phase-space distributions, we take them from a multi-phase transport (AMPT) model with string meltings [28]. The resulting partonic matter is then evolved according to the above described covariant and angular momentum conserved transport model. Because the quark polarization is related to the axial current \( j^a_5 \), which is a Lorentz four-vector, its value depends on the frame of reference it is evaluated. With the flow velocity given by \( v(x) = j(x)/n(x) \) as well as the relations \( \int d^3x' n'(x) = \int d^3x n(x) \) and \( \int d^3x' j_5'(x) = \int \gamma d^3x A_j(x) \) between quantities in the local medium rest frame and the LAB frame, the quark spin polarizations in the medium rest frame can then be evaluated from the spin polarization in the LAB frame using Eq.(10). As in the experimental measurement of Lambda hyperon polarization [2, 5], we use the event plane determined from the elliptic flow of partons to define the azimuthal angle \( \phi \) in the transverse plane of a heavy ion collision. To mimic the pseudorapidity cut \(-1 < \eta < 1\) in experiments, we only integrate the spatial region where the flow velocity is in the pseudo-rapidity range \(-1 < 1/2 \log[(|v(x)| + v_z(x))/(|v(x)| - v_z(x))] < 1\).

Figure 2 shows the azimuthal angle distribution in the transverse plane for local quark spin polarizations \( P_y \) along the \( y \)-direction perpendicular to the event plane (left windows) and \( P_z \) along the longitudinal \( z \)-direction (right windows) in the LAB frame (upper windows) and the medium rest frame (lower windows). For the total spin polarization \( P_y \) (dash-dotted line) along the \( y \)-direction, both the spin (solid line) and orbital (dashed line) contributions have the similar azimuthal angle dependence \(-a + b \cos 2\phi \) in the LAB frame with the former having a much larger magnitude. After transforming to the medium rest frame, the azimuthal angle dependence of the orbital contribution remains the same except having a larger magnitude but the spin contribution changes its azimuthal angle dependence to \(-a - b \cos 2\phi \) in the LAB frame. For the spin polarization \( P_z \) along the \( z \)-direction, its spin and orbital contributions have the azimuthal angle dependence \( \sin 2\phi \) and \( -\sin 2\phi \), respectively, in the LAB frame. The magnitude of both spin and orbital contribution are enhanced after transforming to the medium rest frame. As to the relative strength of the orbital vs spin contribution, it changes...
FIG. 2: Azimuthal angle $\phi$ dependence of spin polarizations along the $y$-direction and $z$-direction in the LAB and medium rest (REST) frame at times $t = 3$ and $8 \text{ fm}/c$.

with time. For $P_y$, it is dominated by the orbital contribution at earlier times and by the spin contribution at later times. This behavior is reversed for $P_z$ as it is dominated by the spin contribution at earlier times and by the orbital contribution at later times. We note that our results at earlier times are close to those predicted by the thermal model [6]. The azimuthal angle dependence of the quark spin polarizations along the $y$- and $z$-directions in the medium rest frame obtained in the above appears to be similar to that of Lambda hyperons in their rest frames measured in experiments [4, 5]. Since the spin of Lambda hyperon in the constituent model is given by its strange quark, the azimuthal angle dependence of the local spin polarizations of Lambda hyperons and strange quarks in relativistic heavy ion collisions are expected to be closely related.

The above results are related to the evolution of the axial charge distribution. In the chiral kinetic theory, the axial charge is the time component of the four-dimensional spin current $j_5^\mu$ and plays an important role in the relativistic spin transport. It was shown in Ref. [19] that the redistribution of axial charges in the transverse plane of heavy ion collisions is essential in determining the azimuthal angle dependence of the quark spin polarizations. In Fig. 3, we show the results obtained in the present study for the redistribution of final integrated axial charge density $\int dz n_5(x, y, z)$ to total particle density $\int dz n(x, y, z)$ ratio calculated in the LAB frame for the regions $z > 0$ and $z < 0$. At the early time of $3 \text{ fm}/c$, small net axial charges of opposite signs are seen to appear in the positive and negative $y$-direction. The above axial charge dipole moment in the transverse plane changes to a quadrupole distribution at the later time of $8 \text{ fm}/c$. Since the axial charge current transformed under a Lorentz boost according to $j_5' = j_5 + (\gamma - 1)(\vec{v} \cdot \vec{r})\hat{v} - \gamma n_5 v$, the quark spin polarization can have different values at different frames of reference due to the redistribution of $n_5$. For example, the flow velocity $v$ at the azimuthal angle $\phi = \pi/2$ has a positive $y$ component where the axial charge $n_5$ is negative. Thus, the Lorentz boost can increase the spin part of $P_y$ in the medium rest frame and changes its azimuthal angle dependence.

IV. CONCLUSIONS

We have included in the present study the side jump effect in the transport model to conserve the total angular momentum during the time evolution of quark matter. In the case of an expanding quark matter initially in a box with a given vorticity field, the final quark spin polarization is consistent with that expected from the thermal model of assuming that the spin degrees of freedom are in thermal equilibrium with the local vorticity field. Applying this angular momentum conserved transport model to relativistic heavy ion collisions, we have obtained the azimuthal angle dependence of the local polarizations along the transverse and longitudinal directions in the medium rest frame that are consistent with those of Lambda hyperons observed in experiments. As in studies based on the chiral kinetic equations, the redistribution of axial charges in the transverse plane induced by the side jump effect in the present study is essential for understanding the final azimuthal angle dependence.
of local spin polarizations. Besides, we have found that the anomalous magnetization or orbital contribution to the polarization also plays an important role. However, there are still theoretical uncertainties that need to be addressed in future studies. First, for strange quarks, an improved approach needs to be developed to include their finite masses in the side jump effect during their scatterings. Also, this massive side jump effect should be incorporated in a quasi-particle transport model to properly take into account of the equation of state of QGP. Furthermore, it is important to understand how to form Λ hyperon in an angular momentum conserved coalescence approach that includes the anomalous contribution to its spin and polarization. Quantitative studies in the future based on such an approach can then provide a more reliable study of anomalous transport phenomena in QCD matter from the azimuthal angle dependence of Lambda hyperon polarizations measured in experiments.

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