A Spectrum Efficient Constellation to Simultaneously Transmit Information and Synchronization Sequence

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\section*{ABSTRACT} In this paper, a new type of \textit{quadrature phase shift keying} (QPSK) modulation, named as \textit{special QPSK} (SQPSK), is proposed and optimized. For SQPSK, the quadrant of the symbol is determined by the bits in information sequence according to QPSK constellation, while the offset relative to the original QPSK constellation is controlled by the bits in synchronization sequence. Under this scheme, the simultaneous transmission of information and synchronization sequences is implemented to improve spectrum efficiency with low computational complexity. In order to achieve the best balance between the \textit{bit error rate} (BER) and the synchronization performance, we optimize the parameter design. Furthermore, a joint synchronization scheme is proposed for frame synchronization, frequency offset estimation, and frequency offset compensation to solve the performance loss caused by Doppler frequency shift in practical applications. Then based on the characteristics of the constellation map, a dynamic threshold demodulation scheme which further improves the BER performance is discussed. Finally, some simulation results are given to show that the SQPSK not only has both good error performance and acquisition performance, but also improves the spectrum utilization of channel resource significantly.

\section*{INDEX TERMS} \textit{Bit error rate} (BER), Doppler shift, frame synchronization, \textit{quadrature phase shift keying} (QPSK).

\section*{I. INTRODUCTION} The start tag of the data frame provided at the transmitter is essential for frame synchronization at the receiver. It is critical to implement frame synchronization for communication system. The basic theory of frame synchronization for a single-channel digital communication system was presented, and the design of frame markers was discussed [1]. In recent years, inserting special auxiliary bits is the key technique in most researches on frame synchronization [2]–[5]. However, the above addition of redundant bits will cause the redundancy in channel resource. How to release the channel resource occupied by frame synchronization sequence is a challenge and it has gained significant research attention.

\textit{Quadrature Phase Shift Keying}: (QPSK) is widely applied to satellite link, digital cluster and other communication services because of its good noise resistance and frequency band utilization [6]–[9]. Considering that inserting frame synchronization sequence occupies the channel resource, we propose a new modulation and demodulation scheme based on QPSK, which hides the synchronization sequence in information modulation. The proposed scheme does not occupy additional channel resource to transmit frame synchronization sequences which achieves a higher spectrum efficiency than...
traditional methods. Hence, the proposed scheme will significantly improve the efficiency of channel resource.

A. LITERATURE REVIEW
Frame synchronization refers to the process of locating the position of some periodically embedded synchronization word in the receiving data stream, and it helps establishing an agreed connection between two communication parties [10]. In the past few years, most researchers focus on the synchronization method of inserting synchronization sequences. In [11] and [12], a frame synchronization method was discussed by using the preamble sequence with good auto-correlation characteristics. In [13], based on pseudo-noise code, a frame synchronization method with low cost and low resource consumption has been proposed. Also, pseudo-noise sequence can be adopted as a guard interval or training sequence for channel estimation and synchronization which is known to transmitter and receiver [14]. In [15], a time synchronization method based on TD-LTE frame synchronization was studied. The authors in [5] and [16] considered a special short training sequence to implement symbol timing synchronization. In addition, many published works have explored the frame synchronization performance of inserting different auxiliary bit sequences. In [17], a new tight bound for the performance of synchronization word based frame synchronization algorithms was presented for the periodically embedded case. In [18] and [19], a generalized method to implement frame synchronization was introduced by using a cross-correlation detector to calculate the correlation between the local preamble sequence and the received preamble sequence. In [10], multiple-frame decision rules were considered, which were different from the single-frame decision rules. According to the hypothesis testing theory, an exact analytical performance evaluation for unknown frame lengths was provided in [20] and [21]. It is obvious that if the transmission of preamble or training sequences occupies the independent resource, it will lead to the redundancy of channel resource.

In [22]–[24], a new technique for synchronizing frames was presented. The synchronization sequence is modulated and superimposed on the modulated data sequence. In [23], the author provided the specific analysis of frame synchronization error probability. This modulation method based on superimposed pilot sequence can be further applied to channel estimation [24]. Although this method also did not occupy the channel resources of the transmission system, it wastes some useful power [25]. When SNR is 6dB, to provide good synchronization performance, the power of the synchronization sequence needs to occupy more than half of the total power per symbol which will cause the increasing of BER of data sequence.

B. OUR CONTRIBUTIONS
In order to improve the spectrum utilization of channel resources, we propose a special QPSK (SQPSK) constellation and a corresponding modulation technique in this paper. In our study, we focus on the synchronization bits hidden into information packets. In this manner, the information and synchronization sequences are transmitted simultaneously, and the synchronization sequence can achieve a zero occupancy for channel resources so that there is no loss in information rate.

In general, the key contributions of this work can be summarized as follows:
1) A novel QPSK modulation is proposed for simultaneous transmission of information and synchronization sequences. The proposed constellation is optimized by minimizing BER with the constraint of frame synchronization performance.
2) Considering the Doppler shift in practical applications, a joint synchronization scheme is proposed to implement frame synchronization, frequency offset estimation, and compensation.
3) The improvement of spectrum utilization for SQPSK leads to BER performance degradation. Nevertheless, a dynamic threshold is further optimized to improve the reliability, which helps SQPSK achieve the BER performance approaching to that of QPSK.
4) The reception complexity of the SQPSK transmission is analyzed and some simulation results are presented to demonstrate the performance improvement of the transmission.

The organization of the paper is presented as follows. In Section II, the modulation and demodulation model of SQPSK is proposed and the corresponding frame synchronization scheme is devised. The optimization model for parameter is given and derived carefully in Section III. Then, a joint synchronization scheme for power detection, frequency estimation and correlation acquisition is proposed in Section IV. In Section V, a dynamic threshold demodulation method is proposed. The computational complexity of SQPSK and the high order extension are analyzed in Section VI. Simulation results demonstrate the correctness and effectiveness of the proposed scheme in Section VII. In the end, Section VIII summarizes the paper.

II. CONSTELLATION DESIGN AND ITS DEMODULATION MODEL
In this paper, we propose a SQPSK constellation on the basis of QPSK modulation. The amplitudes of the symbol of channels I and Q are controlled by the synchronization sequence (here, it refers to m-sequence). The constellation points of SQPSK can be obtained by adding or subtracting an offset of the amplitude d from the amplitude a of QPSK. According to Fig. 1, we have

\[ d = (a_2 - a_1)/2, \quad a = (a_1 + a_2)/2, \]

where \(a_1\) and \(a_2\) are the amplitudes after the offset. In SQPSK constellation mapping, the same information is modulated to four different forms in the same quadrant at different times. As shown in Fig. 1, the constellation of SQPSK modulation is based on the constellation coordinates.
are both caused by the information sequence, and the specific position such as E₀, E₁, E₂ and E₃ in a certain quadrant is determined by the synchronization sequence. Similarly, at the receiver, the quadrants in the constellation map correspond to the transmitted information symbols, and the specific position in the quadrant corresponds to the synchronization sequence.

We select m-sequence as the synchronization sequence for its good autocorrelation and cross-correlation characteristics. In the modulation of SQPSK, the amplitude offsets of the symbol of channels I and Q are controlled by the m-sequence. Let (b₀, b₁, b₂, ..., b_N⁻₁) denote the information sequence, and (m₀, m₁, ..., m_N⁻₁) denote the m-sequence, where N denotes the length of data frame and is even. Considering a bipolar mapping in the transmission, we have bᵢ' = 2bᵢ - 1, mᵢ' = 2mᵢ - 1 ∈ {−1, 1}, i = 0, 1, ..., N - 1. The modulation of SQPSK is shown in Fig. 2. The channels I and Q of the kth SQPSK modulation symbol can be defined as sᵢ(k) = abᵢ²k + mᵢ²k d and s_Q(k) = abᵢ²k+1 + mᵢ2k+1 d, respectively, where k = 0, 1, ..., N/2 - 1 and 0 < d < a. So the transmitted symbols can be written as

\[ x(k) = A_k \exp(j \varphi(k) + j \theta_k), \]  

(2)

where \( A_k = \sqrt{(ab_k^2 + m_k^2 d)^2 + (ab_k^2+1 + m_k^2+1 d)^2} \) and \( A_k \in \left[\sqrt{2a_1}, 1, \sqrt{2a_2}\right] \), \( \theta_k = \arctan\left( \frac{ab_k^2+1 + m_k^2+1 d}{ab_k^2 + m_k^2 d} \right) - \frac{\pi}{4} \) and \( \theta_0 \in \{0, \pi/4, \pi/2, \pi\} \). In addition, the changes in \( A_k \) and \( \theta_k \) are both caused by the m-sequence and the transmitted information sequence, so we can obtain that \( P_{A_k=1} = 2P_{A_k=\sqrt{a_1}} = 2P_{A_k=\sqrt{a_2}} \) and \( P_{\theta_k=0} = 2P_{\theta_k=\pi} = 2P_{\theta_k=-\theta_0} \).

Fig. 3 shows the block diagram of the SQPSK demodulation. It is assumed that channel estimation, channel compensation and frequency offset correction are ideal. The channels I and Q of the kth received symbol can be expressed as yᵢ(k) = sᵢ(k) + nᵢ(k) and y_Q(k) = s_Q(k) + n_Q(k), respectively, where \( n_i(k) \sim \mathcal{CN}(0, \sigma^2/2) \) and \( n_Q(k) \sim \mathcal{CN}(0, \sigma^2/2) \) represent the real additive white Gaussian noise (AWGN) at the kth received symbol.

At the receiver, the SQPSK demodulation must perform the extraction of m-sequence and correlation acquisition before the sampling decision. The detailed process is shown in Fig. 4. Considering the offset controlling of amplitude and phase by m-sequence at the transmitter, the 2kth and (2k + 1)th extracted synchronization sequence can be implemented as \( \hat{m}^{2k}_k = \text{sign}(y_i(k)) \times (|y_i(k)| - a) \) and \( \hat{m}^{2k+1}_k = \text{sign}(y_Q(k)) \times (|y_Q(k)| - a) \), respectively. Here, the function, \( \text{sign}(\cdot) \), can be defined as

\[ \text{sign}(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases} \]
We then calculate the correlation between the estimated synchronization sequence \(\{\hat{m}_0', \hat{m}_1', \ldots, \hat{m}_{N-1}'\}\) and the local synchronization sequence \(\{m_0', m_1', \ldots, m_{N-1}'\}\). The frame synchronization of the system can be completed by comparing the correlation with a given acquisition threshold, which is described in Section III.

After frame synchronization, a dynamic threshold demodulation method is designed for SQPSK to improve BER performance. In this way, the 2kth information bit can be demodulated by \(\hat{b}_{2k} = (y_1(k) > V_d)\), that is, if \(y_1(k) > V_d\), \(\hat{b}_{2k}\) is judged to be 1; otherwise, \(\hat{b}_{2k}\) is judged to be 0. Similarly, the \((2k + 1)\)th information bit can be demodulated by \(\hat{b}_{2k+1} = (y_Q(k) > V_d)\). The detailed description of \(V_d\) will be given in Section V.

### III. PARAMETER OPTIMIZATION FOR THE CONSTELLATION

Since the amplitude offset \(d\) determines the off-center distance of the symbol, \(d\) directly affects the synchronization performance and the BER performance of SQPSK with the demodulation threshold 0. As the amplitude offset \(d\) increases, the synchronization performance by \(m\)-sequence is getting better, but the BER performance of effective information performs worse. Accordingly, as the amplitude offset \(d\) decreases, the BER performance of effective information is improved, but it is not so good for frame synchronization performance. When \(a_1 = a_2, d = 0\), the SQPSK is the same as QPSK and achieves the best BER performance of effective information, but it does not transmit the \(m\)-sequence and cannot implement frame synchronization. In this way, we find that the BER performance is affected by the value of \(d\) which is related to the values of \(a_1\) and \(a_2\). In other words, the most important goal of the system optimization is how to determine the values of \(a_1\) and \(a_2\).

#### A. PROBLEM FORMULATION AND PARAMETER OPTIMIZATION

Because of the randomness of the information sequence and \(m\)-sequence, the probability of the transmitted symbol appearing at each constellation point is equal. Then the average energy of the transmitted symbol is given by

\[
\bar{E} = \frac{1}{4}\left(\alpha^2 + a_1^2\right) + \frac{1}{4}\left(a_2^2 + a_1^2\right) + \frac{1}{4}\left(a_2^2 + a_2^2\right) + \frac{1}{4}\left(a_1^2 + a_2^2\right).
\]

(3)

In order to meet the requirement of average energy normalization, we have

\[
a_1^2 + a_2^2 = 1.
\]

(4)

Furthermore, we introduce the optimization factor \(\alpha\) here for simplification, and denote \(a_1 = \alpha a_2\), where \(\alpha \in [0, 1]\). Then \(a_1\) and \(a_2\) can be represented by \(\alpha\) as

\[
a_1 = \frac{\alpha}{\sqrt{1 + \alpha^2}}, \quad a_2 = \frac{1}{\sqrt{1 + \alpha^2}}.
\]

(5)

Let \(R_c\) denote the correlation peak when the extracted sequence \(\{\hat{m}_0', \hat{m}_1', \ldots, \hat{m}_{N-1}'\}\) is completely aligned with the local sequence \(\{m_0', m_1', \ldots, m_{N-1}'\}\), and \(R_c\) denote the mean of the correlation peak at other times. The synchronization performance can be quantitatively described by the ratio \(R_c/R_e\). Based on the former knowledge about correlation peak detection, the acquisition condition for frame synchronization can be expressed as

\[
E[R_c^2] \geq \lambda^2 E[R_e^2],
\]

(6)

where \(\lambda\) is the acquisition threshold and \(\lambda > 1\). With \(\lambda\) increasing, the frame synchronization performance of the system becomes more reliable.

Then we construct the optimization model below for optimizing the BER performance and ensuring the synchronization performance at the same time. The optimization model can be written as

\[
\alpha_{\text{opt}} = \arg \min_{\alpha} P_{\text{BER}},
\]

\[
\text{s.t. } E[R_c^2] \geq \lambda^2 E[R_e^2],
\]

\[
a_1^2 + a_2^2 = 1,
\]

\[
a_1 = \alpha a_2, \quad \alpha \in [0, 1],
\]

(7)

where \(P_{\text{BER}}\) denotes the BER of the SQPSK with demodulation threshold 0.

In the following, we will first deduce the expression of \(P_{\text{BER}}\). Then, according to the requirements of acquisition performance, we derive the closed-form expression for frame synchronization related parameters \(E[R_c^2]\) and \(E[R_e^2]\) of SQPSK. Finally, we solve the optimization model in (7) to obtain the optimal factor \(\alpha^\ast\).

### B. BER FOR SQPSK TRANSMISSION

Assuming that the channel estimation, channel compensation, and frequency offset correction are ideal, the in-phase and quadrature components of the \(k\)th received symbol are \(y_1(k) = s_1(k) + n_1(k)\) and \(y_Q(k) = s_Q(k) + n_Q(k)\), respectively, where \(k = 0, 1, \ldots, N/2 - 1\). Take the first quadrant for example. When the transmitted symbol is \(E_0\), the corresponding BER can be calculated as

\[
P_{E_0} = \frac{1}{2} P(y_1(k) < 0, y_Q(k) > 0) + \frac{1}{2} P(y_1(k) > 0, y_Q(k) < 0) + \frac{1}{2} P(y_1(k) < 0, y_Q(k) < 0)
\]

\[
= \frac{1}{2} P(n_1(k) < a_1, n_Q(k) > -a_1) + \frac{1}{2} P(n_1(k) > a_1, n_Q(k) < -a_1)
\]

\[
= \frac{1}{2} P_1 (1 - P_1) \times 2 + P_1^2.
\]

(8)

where \(1/2\) indicates one bit error introduced by the decision error, and \(P_1 = \frac{1}{2\sqrt{2}} \text{erfc}(\frac{a_1}{\sigma})\).
Similarly, when the system transmits $E_1$, $E_2$ or $E_3$ in the first quadrant, and let $P_f = \frac{1}{2\sigma^2} \text{erfc}\left(\frac{a}{\sigma}\right)$, the corresponding BER is

$$P_{E_1} = \frac{1}{2} P_1 (1 - P_2) + \frac{1}{2} (1 - P_1) P_2 + P_1 P_2, \quad (9)$$
$$P_{E_2} = \frac{1}{2} P_2 (1 - P_2) \times 2 + P_2^2, \quad (10)$$
and

$$P_{E_3} = \frac{1}{2} P_1 (1 - P_2) + \frac{1}{2} (1 - P_1) P_2 + P_1 P_2, \quad (11)$$
respectively.

Because of the same probability of received symbols appearing on each SQPSK constellation point, the $P_{BER}$ can be formulated as

$$P_{BER} = 4 \times \left[ \frac{1}{4} \times (P_{E_0} + P_{E_1} + P_{E_2} + P_{E_3}) \right]$$
$$= \frac{1}{4\sqrt{2}} \text{erfc}\left(\frac{a_1}{\sigma}\right) + \frac{1}{4\sqrt{2}} \text{erfc}\left(\frac{a_2}{\sigma}\right). \quad (12)$$

Considering $a_1 = \alpha a_2$ and (4), $P_{BER}$ can be rewritten as

$$P_{BER} = \frac{1}{4\sqrt{2}} \text{erfc}\left(\frac{\alpha}{\sigma\sqrt{1 + \alpha^2}}\right) + \frac{1}{4\sqrt{2}} \text{erfc}\left(\frac{1}{\sigma\sqrt{1 + \alpha^2}}\right). \quad (13)$$

### C. Correlation Acquisition

The $m$-sequence is hidden in the energy changing of each symbol by SQPSK. Therefore, we can extract the synchronization sequence from the received signal and make correlation acquisition with local $m$-sequence. Then the frame header can be found by the correlation results. If there appears a sharp correlation peak, and at the same time the peak value is larger than or equal to $\lambda$ times of the average value of the correlation at other times, the system find the frame header successfully. In this subsection, we will derive the relationship between $\alpha$, $\lambda$, and the length of $m$-sequence $N$ based on the acquisition condition.

For the length of original $m$-sequence is $N - 1$, we add one bit, 0, at the end of the $m$-sequence to make the length be $N$, an even number. In addition, it makes the numbers of 0s and 1s in the $m$-sequence equal. From the calculation in (26), the normalized autocorrelation function of the improved $m$-sequence can be represented as

$$R_{mm}(i) = \delta(i) = \begin{cases} 1, & i = 0 \\ 0, & i \neq 0 \end{cases}.$$  

At the receiver, frame synchronization is implemented by the extraction of $m$-sequence and correlation acquisition as shown in Fig. 4. Since the $m$-sequence is hidden in the transmitted signals, the process of extracting $m$-sequence is the inverse process of $m$-sequence modulation at the transmitter. Therefore, the $2k$th and $(2k + 1)$th bits in the $m$-sequence can be calculated by $\hat{m}_{2k} = \text{sign} \left[ y_1(k) \right] \left( |y_1(k)| - a \right)$ and $\hat{m}_{2k+1} = \text{sign} \left[ y_0(k) \right] \left( |y_0(k)| - a \right)$, respectively. When SNR is relatively high, both $s_1(k) \times y_1(k) > 0$ and $s_0(k) \times y_0(k) > 0$ are hold with the probability close to 1. Then we have

$$\hat{m}_{2k} = \text{sign} \left( y_1(k) \right) \left( |y_1(k)| - a \right)$$
$$= \text{sign} \left( y_1(k) \right) \left( s_1(k) \left| s_1(k) + n_1(k) \right| - a \right)$$
$$= \text{sign} \left( y_1(k) \right) \left( \frac{s_1(k) \left| s_1(k) + n_1(k) \right|}{|s_1(k)|} \right) - a$$
$$= \text{sign} \left( y_1(k) \right) \left( \frac{s_1(k) \left| s_1(k) + n_1(k) \right|}{|s_1(k)|} \right) - a + n'_1(k), \quad (14)$$

where $n'_1(k) = \text{sign} \left( y_1(k) \right) \frac{s_1(k) \left| s_1(k) + n_1(k) \right|}{|s_1(k)|} n_1(k)$.

From the modulation principle shown in Fig. 2, we can obtain $|s_1(k)| - a = b_{2k} m_{2k}^d d$. According to sign $[y_1(k)] = \text{sign} [b_{2k}]$, (14) can be simplified as

$$\hat{m}_{2k} = m_{2k}^d d + n'_1(k). \quad (15)$$

Similarly, we have $\hat{m}_{2k+1} = m_{2k+1}^d d + n'_0(k)$. After serial-to-parallel conversion as shown in Fig. 4, the noise is denoted as $[n_0, n_1, \ldots, n_{N-1}]$. When the extracted $m$-sequence $[\hat{m}_0, \hat{m}_1, \ldots, \hat{m}_{N-1}]$ is completely aligned with the local $m$-sequence $[m_0, m_1, \ldots, m_{N-1}]$, the normalization-related results can be given by

$$\frac{1}{N} \sum_{i=0}^{N-1} (\hat{m}_i \times m'_i) = \frac{1}{N} \sum_{i=0}^{N-1} \left[ (m_i d + n_i) \times m'_i \right]$$
$$= d + \frac{1}{N} \sum_{i=0}^{N-1} (n_i m'_i). \quad (16)$$

where $n_i \sim CN \left( 0, \sigma^2/2 \right)$, $i = 0, 1, \ldots, N - 1$. $b'_1, n_1$ and $m_i$ are pairwise independent. Therefore, $E \left[ R_c^2 \right]$ can be written as

$$E \left[ R_c^2 \right] = \mathbb{E} \left[ d + \frac{1}{N} \sum_{i=0}^{N-1} (n_i m'_i) \right]^2$$
$$= d^2 + 2d \times 0 + \frac{\sigma^2}{N} = d^2 + \frac{\sigma^2}{N}. \quad (17)$$

On the other hand, when the extracted sequence is not aligned with the local synchronization sequence, the mean square of the correlation results $E \left[ R_c^2 \right]$ is

$$E \left[ R_c^2 \right] = \mathbb{E} \left[ \frac{1}{N} \sum_{i=0}^{N-1} (\hat{m}_i \times m'_i) \right]^2 = \frac{\sigma^2}{N}. \quad (18)$$

On the basis of the derivation above, the constraint for successful synchronization can be formulated as

$$d^2 + \frac{\sigma^2}{N} \geq \frac{\lambda^2 \sigma^2}{N}. \quad (19)$$

Then substituting (4) into (19), we can obtain the detailed expression of (19) as

$$\frac{(1 - \alpha)^2}{4 \left(1 + \alpha^2\right)} + \frac{\sigma^2}{N} \geq \frac{\lambda^2 \sigma^2}{N}. \quad (20)$$
D. PARAMETER OPTIMIZATION

Substituting (13) and (20) into (7), we can obtain

$$\alpha_{\text{opt}} = \arg \min_{\alpha} P_{\text{BER}}.$$  

s.t.  

$$\frac{(1 - \alpha)^2}{4(1 + \alpha^2)} + \frac{\sigma^2}{N} \geq \frac{\lambda^2 \sigma^2}{N},$$  

$$\alpha \in [0, 1].$$  

(21)

where $P_{\text{BER}} = \frac{1}{4\sqrt{2}} \text{erfc} \left( \frac{a}{\sigma \sqrt{1 + \alpha^2}} \right) + \frac{1}{4\sqrt{2}} \text{erfc} \left( \frac{1}{\sigma \sqrt{1 + \alpha^2}} \right)$.  

First, we analyze the monotonicity of $P_{\text{BER}}$ by its first-order derivative as follows. Taking the first-order derivative of $P_{\text{BER}}$ with respect to $\alpha$, we can obtain

$$P'_\text{BER} = \frac{1}{2\sqrt{2\pi}(1 + \alpha^2)^{\frac{3}{2}}} \left[ ae^{-t} - e^{-a^2t} \right].$$  

(22)

where $t = \frac{1}{\sigma^2(1 + \alpha^2)} > 0, \alpha \in [0, 1]$.  

Then let $f(\alpha) = ae^{-t} - e^{-a^2t}$, $g(\alpha) = ae^{-a^2t} - e^{-a^2t}$, so we can easily get $f(\alpha) \leq g(\alpha)$ and the equality is satisfied when $\alpha = 1$. With $g(\alpha) = (\alpha - 1)e^{-a^2t}$, we easily conclude that $g(\alpha) \leq 0$ for $\alpha \in [0, 1]$, and the equality is satisfied only when $\alpha = 1$. Then, $f(\alpha) \leq 0$ is established in $\alpha \in [0, 1]$.  

Further, we can obtain $P'_\text{BER} \leq 0$, and $P'_\text{BER} = 0$ only when $\alpha = 1$. Therefore, we conclude that $P_{\text{BER}}$ is monotonically decreasing with respect to $\alpha$ when $\alpha \in [0, 1]$.  

From the above analysis, we can minimize $P_{\text{BER}}$ at the boundary for the first constraint. Therefore, the optimal factor $\alpha$ for (21) can be calculated as

$$\frac{(1 - \alpha)^2}{4(1 + \alpha^2)} + \frac{\sigma^2}{N} = \frac{\lambda^2 \sigma^2}{N}.$$  

(23)

For $\alpha \in [0, 1]$, we can obtain the optimal $\alpha$ by

$$\alpha^* = \frac{N - \sqrt{2N\psi - \psi^2}}{N - \psi},$$  

(24)

where $\psi = 4(\lambda^2 - 1)\sigma^2$ and $0 < \psi < N$. Taking the first order derivative of $\alpha^*$ with respect to $N$ and some rearrangement, we can get

$$\frac{\partial \alpha^*}{\partial N} = \frac{N\psi^3 - \psi^2}{\sqrt{2N\psi - \psi^2}}.$$  

(25)

When $0 < \psi < N$, we have $\frac{\partial \alpha^*}{\partial N} > 0$ and $\alpha^*$ is monotonically increasing with $N$. In the same way, we can obtain that $\alpha^*$ is a monotonically decreasing function of $\lambda$ when $\lambda > 1$.  

Since the $\alpha^*$ becomes larger with $N$ increasing, $d = \frac{\alpha^* - \alpha_1}{\sqrt{1 + \alpha^2}}$ is smaller and the SQPSK is closer to QPSK gradually. When the length of data frame is $N$, the acquisition threshold parameter $\lambda$ and the SNR are determined, the unique optimal factor $\alpha$ can also be obtained. In this paper, we take SNR $= 4$ dB, $N = 512$ and $\lambda = 8$ (the value of $\lambda$ is evaluated in VII-A), we can obtain $\alpha^* = 0.671$, that is

$$\alpha_1 = \frac{\alpha^*}{\sqrt{1 + \alpha^2}} = 0.5572,$$

$$\alpha_2 = \frac{1}{\sqrt{1 + \alpha^2}} = 0.8304.$$  

(25)

IV. JOINT SYNCHRONIZATION OF FRAME AND FREQUENCY

In a practical communication system, frame synchronization is a key technology. Many superior algorithms for frame synchronization in communication systems with Doppler shift have been proposed [27]–[29]. However, due to the specifications of SQPSK, conventional synchronization methods are no longer applicable when Doppler shift exists. Therefore, a joint synchronization scheme for power detection, frequency estimation and correlation acquisition is proposed as shown in Fig. 5.

![FIGURE 5. Joint synchronization scheme for SQPSK.](image)

When the power of received signals reaches the threshold of the power detector, the frequency estimation and the frequency compensation are activated. Then, frame synchronization is achieved by calculating the correlation value of the $m$-sequence. We replace the square operation with the absolute of the signal amplitude, then the power detector is implemented by only some addition operations. Furthermore, the frame synchronization has been described in detail in Fig. 4. In this section, we focus on the derivation of the frequency estimation for SQPSK.

A. INITIAL CONSIDERATION

In space communications, the long distance and high-speed movement between the transmitter and receiver cause dynamic Doppler shift, which has a serious impact on the synchronization performance and the BER performance. Doppler shift effect is one of the major factors causing a serious degradation in system performance [30]. Hence, how to acquire the Doppler-shift value from received signals becomes an inevitable problem for signal recovery.  

In order to find a suitable frequency estimation algorithm, the characteristics of SQPSK signal are analyzed. Assuming the channel is ideal, the sampling symbol $y_0(k)$ received at the receiver can be expressed as

$$y_0(k) = s(k) = A_k \exp(j\varphi(k) + j\theta).$$  

(26)

Four times for $y_0(k)$, we can obtain

$$y_0^4(k) = A_k^4 \exp(j4\varphi(k) + j4\theta).$$  

(27)

Because of $4\varphi(k) \in \{\pi, 3\pi, 5\pi, 7\pi\}$, (27) can be expressed as

$$y_0^4(k) = -A_k^4 \exp(j4\theta).$$  

(28)

It is easy to show that $y_0^4(k)$ no longer contains phase modulation information.
The spectrum of received signals is analyzed. The average power of one symbol in the received signals is

\[
E = \frac{1}{4} \left[ (4a_1^4)^2 + (4a_2^4)^2 + 2 \times 1^2 \right] = 4a_1^8 + 4a_2^8 + \frac{1}{2}.
\]

(29)

The power of its **direct-current** (DC) signal is

\[
E_{\text{DC}} = \left[ \frac{1}{4} \left( 4a_1^4 + 4a_2^4 + 2 \cos (4\theta_0) \right) \right]^2 = a_1^8 + a_2^8 + 2a_1^4a_2^4 + \frac{1}{4} \cos^2 (4\theta_0) + a_1^4 \cos (4\theta_0) + a_2^4 \cos (4\theta_0) + a_1^2 \cos (4\theta_0).
\]

(30)

The sum of the power of all its **alternating-current** (AC) signals is

\[
E_{\text{AC}} = E - E_{\text{DC}} = 2 + 3a_1^8 + 3a_2^8 - 2a_1^4a_2^4 - \frac{1}{4} \cos^2 (4\theta_0)
\]

\[= -a_1^4 \cos (4\theta_0) - a_2^4 \cos (4\theta_0).\]

(31)

Denote the ratio of the power of AC to DC be \( r \), we have

\[
r = \frac{E_{\text{AC}}}{E_{\text{DC}}}. \quad (32)
\]

Using the optimization results obtained in III-D, we can get \( r = 0.69 \). It is easily understandable that the energy of DC is greater than the AC in the received signals. In addition, the AC signals are generated by the random offset of the phase and amplitude without fixed frequency, so its spectrum will be dispersed throughout the bandwidth. Therefore, on the spectrogram of received signals, only a very large peak will appear at zero frequency, and the magnitude of the DC signal is larger than the amplitude of the other AC signals.

**B. FREQUENCY OFFSET ESTIMATION**

Supposing the channel is AWGN channel and the transmitted signals have a frequency offset of \( \Delta f \) when they reach the receiver. Then, the \( k \)th received symbol at the receiver can be expressed as

\[
y(k) = A_k \exp \left( j \Delta f k + \varphi (k) + \theta \right) + n_0 (k), \quad (33)
\]

where \( \Delta \omega = 2\pi \Delta f \), \( n_0(k) \) is the complex AWGN which superimposed on the \( k \)th received symbol. Under high SNR, the \( k \)th symbol with four times can be simplified as

\[
y^4(k) = -A_k^4 \exp (j4\Delta \omega k + j4\theta) . \quad (34)
\]

Referring to the **maximum likelihood** (ML) estimation method for the complex signals in the literature [31], the maximum likelihood estimation function for \( \Delta \omega \) can be expressed as follows

\[
L = -\frac{1}{N} \sum_{k=1}^{N-1} \left[ (y_1(k) - Y_1(k))^2 + (y_Q(k) - Y_Q(k))^2 \right], \quad (35)
\]

where \( y_1(k) \) and \( y_Q(k) \) are the real and imaginary parts of \( y(k) \), respectively, \( Y_1(k) = -A_k^4 \cos (4\omega k + 4\theta) \), \( Y_Q(k) = -A_k^4 \sin (4\omega k + 4\theta) \). After substituting \( y_1(k), y_Q(k), Y_1(k) \) and \( Y_Q(k) \) into (35), we get

\[
L = \frac{2}{N} \sum_{k=1}^{N-1} \left[ y_1(k) Y_1(k) + y_Q(k) Y_Q(k) \right] - \frac{1}{N} \sum_{k=1}^{N-1} A_k^8.
\]

(36)

In (36), \( \frac{1}{N} \sum_{k=1}^{N-1} A_k^8 \) and \( A_k^8 \) are constant, and they will not affect the solution of likelihood function. So we omit the constant term in the formula and replace \( A_k^8 \) with \( A_k^4 \), then (36) can be simplified as

\[
L = \frac{2}{N} \sum_{k=1}^{N-1} A_k^4 \cos (4 (\Delta \omega - \omega) k)
\]

\[= \text{Re} \left( \frac{1}{N} \sum_{k=1}^{N-1} y^4(k) \exp (-j4\theta) \exp (-j4\omega k) \right), \quad (37)
\]

where \( \text{Re}(x) \) means the real part of \( x \). Hence, the ML estimation of \( \Delta \omega \) is

\[
\Delta \hat{\omega} = \frac{2\pi}{4N} \arg \max_m \left| \text{Re} \left( \sum_{k=1}^{N-1} B(k) \exp(-j4\theta) \right) \right|, \quad (38)
\]

where \( B(k) = y^4(k) \exp(-j2\pi m k) \).

The random offset of the phase and amplitude in the complex signal \( y^4(k) \) does not affect the maximum value in the spectrum which has been demonstrated in IV-A. Hence, we can drop from (38) that

\[
\Delta \hat{\omega} = \frac{2\pi}{4N} \arg \max_{0 \leq m < N} \left| \text{Re} \left( \sum_{k=0}^{N-1} B(k) \right) \right|, \quad (39)
\]

Recalling the **discrete Fourier transform** (DFT) and referred to [31]–[34], the estimation of \( \Delta \hat{\omega} \) can be made directly from the DFT of \( y^4(k) \). Since \( \Delta \omega \in [-\pi/4, \pi/4] \), \( \Delta \hat{\omega} \) can be re-represented as

\[
\Delta \hat{\omega} = \begin{cases} \frac{2\pi}{4N} \arg \max_{0 \leq m < N/2} \left| \sum_{k=0}^{N-1} B(k) \right|, & 0 \leq m < N/2 \\ \frac{\pi}{4} - \frac{2\pi}{4N} \arg \max_{0 \leq m < N} \left| \sum_{k=0}^{N-1} B(k) \right|, & N/2 \leq m < N. \end{cases}
\]

(40)
Referring to [35], it is easy to show that the variable $\Delta \hat{\omega}$ is closed to the CR bound.

Finally, we get the estimation of $\Delta f$ as

$$\Delta \hat{f} = \begin{cases} \frac{f_s}{4N} \max_{0 \leq m < N} \sum_{k=0}^{N-1} B(k), & 0 \leq m < N/2 \\ \frac{f_s}{8} - \frac{f_s}{4N} \max_{0 \leq m < N} \sum_{k=0}^{N-1} B(k), & N/2 \leq m < N, \end{cases}$$

where $f_s$ is the sampling frequency.

**C. JOINT SYNCHRONIZATION ALGORITHM**

In summary, the specific implementation process of the joint synchronization scheme is summarized in Algorithm 1, where the scripts in Lines 1-11, Lines 12-20, and Lines 22-28 implement power detection and frequency estimation, frequency compensation, and frame synchronization, respectively. In addition, we can discover that frequency estimation is independent of other modules on the time axis.

**V. MORE RELIABLE DEMODULATION WITH PROPOSED DYNAMIC THRESHOLD**

In this section, we propose a more reliable SQPSK demodulation method using a proposed dynamic threshold to further improve the BER performance.

At first, the optimal dynamic threshold $V_d$ will be derived in the cases of channel I and channel Q. For channel I, when $m_{2k} = 1$, the received symbol will be $E_1$ or $E_2$ in each quadrant as shown in Fig. 1, where $m_{2k}$ is the local m-sequence in channel I. And if $1$ is sent, the coordinate of channel I is $a_2 + n_1(k)$ or $-a_1 + n_1(k)$. Therefore, when a symbol is sent under $m_{2k} = 1$, the BER of channel I can be expressed as

$$P'_e = P'_{E_1} = P'_{E_2} = \mathbb{P}(a_2 | a_2) + \mathbb{P}(-a_1 | a_2) + \mathbb{P}(a_1 | -a_1) = \frac{1}{2 \sqrt{2\pi}} \int_{-\infty}^{\sqrt{6} \sigma} \exp\left(\frac{\eta^2}{\sigma^2}\right) d\eta + \frac{1}{2 \sqrt{2\pi}} \int_{\sqrt{6} \sigma}^{\infty} \exp\left(\frac{\eta^2}{\sigma^2}\right) d\eta.$$ \hspace{1cm} (42)

To obtain the minimum of $P'_e$, we should solve $\frac{\partial P'_e}{\partial V_d} = 0$. Then for channel I, the optimal dynamic threshold under $m_{2k} = 1$ can be expressed as

$$V_d = (a_2 - a_1)/2 = d.$$ \hspace{1cm} (43)

When $m_{2k} = -1$, the corresponding received symbol is $E_0$ or $E_3$. And if $1$ is sent, the corresponding coordinates of received symbol in I axis is $-a_2 + n_1(k)$ or $a_1 + n_1(k)$. The BER of channel I under $m_{2k} = -1$ can be expressed as

$$P''_e = P'_{E_0} = P'_{E_3} = \mathbb{P}(a_1 | a_2) + \mathbb{P}(-a_1 | a_2) + \mathbb{P}(-a_2 | -a_1) = \frac{1}{2 \sqrt{2\pi}} \int_{-\infty}^{\sqrt{6} \sigma} \exp\left(\frac{\eta^2}{\sigma^2}\right) d\eta + \frac{1}{2 \sqrt{2\pi}} \int_{\sqrt{6} \sigma}^{\infty} \exp\left(\frac{\eta^2}{\sigma^2}\right) d\eta.$$ \hspace{1cm} (44)

Then we have $V_d = -d$ when $m = -1$ by solving $\frac{\partial P''_e}{\partial V_d} = 0$.

In summary, the specific implementation process of the joint synchronization scheme for power detection, frequency estimation and correlation acquisition is as follows:

1. Let $P_r$ and $P_p$ be the power of one frame under the ideal channel and the practical channel. Assume $P_{th}$ be the power detection threshold with $P_{th} = 0.8P_p$.
2. If $P_r \geq P_{th}$ then $n = 0$.
3. Calculate $\Delta f$ according to (41).
4. If $n \geq \frac{N}{10}$ then $n = 0$.
5. Calculate $\Delta f'$ according to (41).
6. If $\Delta f' = \Delta f \pm 1$ then $\Delta f = \Delta f'$.
7. For $k = 1 : N$ do
8. $R_c = R_c + m_k'$.
9. End for
10. If $R_c \geq 1.6\max(R_c, 0.6R)$ then Break; // Frame acquired successfully.
11. End if
12. End while

Based on the derived $V_d$, we can calculate the expression of theoretical BER under dynamic threshold. Assuming that

**Algorithm 1 Joint Synchronization Scheme for Power Detection, Frequency Estimation and Correlation Acquisition**

1. **while** (1) **do**
2. Calculate $P_r$ from power detector
3. if $P_r \geq P_{th}$ **then**
4. $n = 0$
5. Calculate $\Delta f$ according to (41)
6. **else** if $n \geq \frac{N}{10}$ **then**
7. $n = 0$
8. Calculate $\Delta f'$ according to (41)
9. **else**
10. $n = n + 1$
11. **end if**
12. if $\Delta f' = \Delta f$ **then**
13. $\Delta f' = \Delta f$
14. for $k = 1 : N$ do
15. $R_c = R_c + m_k'$
16. End for
17. **else**
18. $y(k) = y(k) \exp(-j2\pi \Delta f')$
19. **end if**
20. $R_c = 0$
21. for $k = 1 : N$ do
22. $R_c = R_c + m_k'$
23. End for
24. if $R_c \geq 1.6\max(R_c, 0.6R)$ then Break; // Frame acquired successfully.
25. **end if**
26. **end while**
the received symbol is in the first quadrant, for \( E_0 \), there are the following three cases that errors happen
\[
P'_{E_0} = \frac{1}{2} P \left( y_1(k) < -d, y_0(k) > -d \right) \\
+ \frac{1}{2} P \left( y_1(k) > -d, y_0(k) < -d \right) \\
+ P \left( y_1(k) < -d, y_0(k) < -d \right).
\] (45)

After substituting (1) into (45) and some rearrangement, we can obtain
\[
P'_{E_0} = \frac{1}{2\sqrt{2}} \text{erfc} \left( \frac{a}{\sigma} \right).
\] (46)

Similarly, we can obtain
\[
P'_{E_1} = P'_{E_2} = P'_{E_3} = \frac{1}{2\sqrt{2}} \text{erfc} \left( \frac{a}{\sigma} \right).
\] (47)

For the other three quadrants, the results of BER are the same. Therefore, \( P'_\text{BER} \) can be written as
\[
P'_\text{BER} = 4 \times \frac{1}{4} \times \left[ \frac{1}{4} \times \left( P'_{E_0} + P'_{E_1} + P'_{E_2} + P'_{E_3} \right) \right]
\]
\[
= \frac{1}{2\sqrt{2}} \text{erfc} \left( \frac{a}{\sigma} \right).
\] (48)

Compared the BER performance of SQPSK with dynamic threshold and traditional method. The BER of SQPSK with traditional method (\( V_d = 0 \)) is given in (12). Due to \( \text{erfc}(x) \) is a convex function and \( a = \frac{\sqrt{2}}{2} \sigma \), it is easy to learn that
\[
\frac{1}{2\sqrt{2}} \text{erfc} \left( \frac{a}{\sigma} \right) < \frac{1}{4\sqrt{2}} \text{erfc} \left( \frac{a_1}{\sigma} \right) + \frac{1}{4\sqrt{2}} \text{erfc} \left( \frac{a_2}{\sigma} \right).\]
Hence, we can conclude that the BER performance with dynamic threshold is better than traditional method.

VI. COMPLEXITY ANALYSIS, EXTENSION AND COMPARISON

In this section, we first analyze the complexity for each module at the transmitter and receiver. Then we extend our scheme to high order modulation schemes. Finally, we make a comparison with 16-QAM when considering simultaneous transmission of information and synchronization sequences.

A. COMPLEXITY ANALYSIS

Here, assuming that the length of a frame is \( N \), we analyze the complexity of SQPSK modulation and demodulation system.

At the transmitter, compared to QPSK, we only need to preprocess the modulation information to achieve SQPSK modulation. In terms of computational complexity, two multiplications and one addition are added for one bit. Using the pipeline parallel mode, only delays the time of one multiplication and one addition. In other respects, SQPSK is completely consistent with QPSK.

At the receiver, the performance analysis is focused on the joint synchronization scheme in Fig. 5 and the dynamic threshold demodulation in Fig. 4. Assume that a new downsampled symbol is received. The complexity of frame synchronization and dynamic demodulation at the receiver is analyzed as follow.

a) Power detection. As shown in Fig. 5, the power detector implemented in first input first output (FIFO) requires only one addition and one subtraction, where the absolute value of the symbol is used instead of the square operation.

b) Frequency estimation. Frequency estimation mainly requires \( \frac{N}{2} \log_2 N \) complex multiplications and \( \frac{N}{2} \log_2 N \) complex additions, but it is an intermittent operation and independent of the frame synchronization process.

c) Frequency compensation. The frequency compensation only needs one complex multiplications with the last value of frequency estimation.

d) Frame synchronization. As shown in Fig. 4, the received symbols are reversed according to the sign of amplitude and then the correlation value is calculated. Hence, the frame synchronization mainly includes \( 2N \) multiplications and \( 2N \) additions.

e) Dynamic demodulation. The implementation of the dynamic threshold only needs to add an operation to reverse the number of \( d \) according to the \( m \)-sequence.

In summary, the computational complexity of each module at the receiver is shown in TABLE 1. We can deduce that the SQPSK can be implemented with low computational complexity. Note that we here list the complexity of each module in SQPSK transmission to the hardware implementation of the proposed SQPSK with low complexity. Moreover, the complexities of power detection, frequency estimation and frequency compensation are shared with the transmission based on SQPSK and other modulation.

|               | Multiplications | Additions |
|---------------|-----------------|-----------|
| Power detector | 0               | \( N \)   |
| Frequency estimation | \( \frac{N}{2} \log_2 N \) | \( \frac{N}{2} \log_2 N \) |
| Frequency compensation | \( N \)  | 0         |
| Frame synchronization | \( 2N \) | \( 2N \) |
| Dynamic demodulation | 0           | \( N \)   |

B. THE EXTENSION TO HIGH ORDER MODULATION

Besides QPSK, the proposed constellation scheme in this paper is not constrained to QPSK modulation and can be also extended to higher order \( M \)-PSK or \( M \)-QAM by following the same procedure of SQPSK. Fig. 6 gives an extended example of 8-PSK (denoted as 8-PSK extension), for which each initial constellation point of 8-PSK can be extended to a QPSK constellation. More specifically, in Fig. 6, the red numbers represent the transmitted information sequence, while the black numbers represent the transmitted synchronization sequence. For 8-PSK extension, each constellation point
involves the transmission of 3-bit information sequence and 2-bit synchronization sequence.

Similarly, an extension of 16-QAM in the first quadrant is given in Fig. 7, for which each initial constellation point of 16-QAM (denoted as empty circle) can be extended to a QPSK constellation (denoted as full circle).

As shown in Figs. 6 and 7, the constellation points are more dense in the extension to high order modulation. The BER performance indeed deteriorates due to the smaller distances. However, the proposed dynamic threshold of SQPSK demodulation is still carefully constructed in distance or angular domain.

C. COMPARISON WITH 16-QAM

Recalling the constellation of SQPSK in Fig. 1, SQPSK has 16 constellation points and square shape, which shares the same distribution with the conventional 16-QAM. Comparing the constellations of these two modulation, we note that the coordinates of the points are totally different owing to the optimization design of SQPSK for BER performance. Specifically, the normalized coordinates of 16-QAM are $1/\sqrt{10}$ and $3/\sqrt{10}$ while the normalized coordinates of SQPSK are 0.5572 and 0.8304.

Motivated by the analogy between SQPSK and 16-QAM, we can also design the scheme of simultaneous transmission of information and synchronization sequence based on 16-QAM, which is named as special 16-QAM. For special 16-QAM, the first and third bits from left represent information sequence, while the second and fourth bits represent synchronization sequence. The constellation map of special 16-QAM is shown in Fig. 8, where the red numbers denote the information sequence of the constellation points in their quadrant, and the black numbers denote the synchronous sequence corresponding to each constellation point. Again owing to the optimization of BER performance, the proposed SQPSK is expected to outperform special 16-QAM in terms of BER indicator. However, this superiority is at sacrifice of acquisition performance compared to 16-QAM. The acquisition is implemented over a long received sequence, which results in small or even negligible sacrifice for acquisition performance of SQPSK. The performance of SQPSK and special 16-QAM are detailedly compared in Subsection VII-A.

VII. SIMULATION RESULTS & ANALYSIS

In this section, we present some simulation results to demonstrate the performance of the proposed scheme and optimization. In the simulation, we set $\text{SNR} = 4 \text{ dB}, N = 512$ as an example and take the average result of 100,000 simulations.

A. PARAMETER OPTIMIZATION SIMULATION

The value of $\lambda$ affects both synchronization performance and BER performance, and it is discussed over AWGN channel. When the $\text{SNR} = 4 \text{ dB}$ and $N = 512$, the optimal factor $\alpha^*$ in (24) can be only determined by $\lambda$. According to the practical experience of frame synchronization, the decision condition of frame synchronization is set as $R_c \geq \max (1.6 \max (R_e), 0.6R)$.
The synchronization performances with different $\lambda$’s are shown in Fig. 9. It is easy to see that the synchronization performance of our proposed SQPSK scheme becomes better with increased values of $\lambda$. When $\lambda = 8$, the system successful synchronization probability is 0.991 at SNR = 4 dB, and it is very close to 1 when SNR = 5 dB. When $\lambda \geq 8$, the synchronization performance upgrade is negligible with larger $\lambda$. Specifically, when $\lambda \geq 10$, a decrease of synchronization probability occurs at low SNRs. This degradation attributes to the decrease of $\alpha^*$ and hence in $\alpha_1$, causes an increase in the error rate of the extracted $m$-sequence $\{\hat{m}_0, \hat{m}_1, \ldots, \hat{m}_{N-1}\}$. Since the system has good enough synchronization performance for $\lambda = 8$, we configure $\lambda = 8$ in the subsequent simulations. Further, we select $\lambda$ according to the requirement of the successful synchronization probability and the length of the frame in practical applications. Due to the long length of the sequence we selected, the false-alarm probability is about 0, so we do not discuss it too much here.

We plot the BER performance at different SNRs with different frame lengths to compare QPSK, SQPSK. The BER performance at $N = 256$, $N = 512$, and $N = 1024$ is shown in Figs. 10(a), 10(b), and 10(c), respectively. It is observed that QPSK has the best BER performance. After using dynamic threshold demodulation, the BER performance of SQPSK has been significantly improved and is almost equal to QPSK. The differences of SNR between QPSK and SQPSK with dynamic threshold at $N = 256$, $N = 512$, and $N = 1024$ to achieve a BER of $10^{-6}$ are 0.26 dB, 0.18 dB, and 0.08 dB, respectively. The curves of SQPSK with dynamic threshold and QPSK almost coincide as the frame length $N$ increases.

Meanwhile, the BER performance of SQPSK compared with special 16-QAM is illustrated in Fig. 10(b). It is clearly observed that the performance of SQPSK with dynamic threshold surpasses special 16-QAM by about 3.5 dB SNR gain when BER is $10^{-4}$. As we stated previously, this is because compared with the special 16-QAM, the SQPSK scheme attains the optimal BER performance under the constraint of acquisition performance.

The BER performance of SQPSK is plotted in Fig. 11 for Rayleigh flat fading and AWGN channels. Assuming that channel remains constant and time-invariant during the block transmission and channel estimation is ideal, the curves reveal that the BER performance of SQPSK with dynamic threshold is almost identical to that of QPSK.
B. SIMULATION FOR JOINT SYNCHRONIZATION

Fig. 12 depicts the performance of SQPSK with dynamic threshold under different normalized Doppler frequency offsets, and $N = 512$. It is obvious that the BER performance after using compensation by the proposed joint scheme is much better than that without compensation. Moreover, the influence of different normalized Doppler frequency offsets on the performance is negligible, which shows the superiority of our algorithm.

In Fig. 13, the performance of BER vs normalized Doppler frequency offset at different SNRs for frame length $N = 512$ is shown. It is noticed that the normalized Doppler frequency offset does not affect the BER performance of SQPSK with dynamic threshold. Hence, our proposed algorithm of frequency offset estimation and compensation can perform well under various normalized Doppler frequency offsets. In addition, as the SNR increases, the BER performance is improved.

In Fig. 14, we compare the proposed method with QPSK and special 16-QAM with Doppler frequency offset 0.1. Considering the same information transmission capacity, the SNR gap of QPSK and SQPSK with the same BER does not exceed 0.3 dB. Thus we can conclude that their performance are very close. Considering similar constellations, SQPSK outperforms the special 16-QAM by about 6 dB at BER equal to $10^{-2}$.

Under the normalized Doppler frequency offset is 0.1 and SNR = 4 dB, the simulation for frame synchronization is shown in Fig. 15. It can be seen that based on the above optimization results, the m-sequence extracted from the received signal has excellent correlation characteristics with the local m-sequence. When the extracted sequence is perfectly aligned with the local m-sequence, the peak of it is greater than other peaks specifically as shown so that we can find the frame header easily.

C. ANALYSIS ON THE IMPROVEMENT OF THE SPECTRUM EFFICIENCY

Since the SQPSK is a modified version of QPSK, they have the same spectral efficiency, 1bps/Hz. However, the SQPSK incorporates the frame synchronization sequence in the information transmission. As a result, the frame synchronization
sequence does not occupy any extra channel resource, and thus it can effectively improve the efficient channel utilization, which specifies the proportion of channel resource utilized for the data transmission rather than pilot transmission.

Assuming that a QPSK data frame consists of \( l \) pilot bits and \( N \) information bits, the efficient channel utilization of QPSK can be calculated as \( N/(N + l) \) while that of SQPSK would be 100%. The efficient channel utilizations for QPSK and SQPSK are compared with different \( l/N \)’s in Fig. 16. It can be easily deduced that the efficient channel utilization of SQPSK is better than that of QPSK. Additionally, with the increase of \( l/N \) values, the improvement of efficient channel utilization for our proposed scheme is more obviously. For example, \( l/N \approx 1/4 \) is usually used in typical single-carrier communication systems [36], [37]. In this case, the efficient channel utilization of SQPSK is increased by about 20% compared with QPSK.

FIGURE 16. The comparison of efficient channel utilizations for SQPSK and QPSK.

VIII. CONCLUSION

In this paper, a spectrum efficient constellation named SQPSK has been proposed to simultaneously transmit information and synchronization sequence. Then an optimization model for constellation was constructed and the optimal amplitude values of \( \alpha_1 \) and \( \alpha_2 \) were obtained. Afterwards, the proposed joint synchronization scheme has performed quite well after Doppler compensation. Further, we proposed a dynamic threshold demodulation method to improve BER performance and analyzed the reception complexity of the SQPSK transmission. Our simulation results have demonstrated that the proposed transmission method attained good synchronization and BER performance. Compared with QPSK, the SQPSK scheme can release the resources occupied by the frame synchronization codes and improve the efficient channel utilization while maintaining good BER performance.

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