Dark energy, Ricci-nonflat spaces, and the Swampland

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It was recently pointed out that the existence of dark energy imposes highly restrictive constraints on effective field theories that satisfy the Swampland conjectures. We provide a critical confrontation of these constraints with the cosmological framework emerging from the Salam-Sezgin model and its string realization by Cvetic, Gibbons, and Pope. We also discuss the implication of the constraints for string model building.

Very recently, Montefalcone, Steinhardt, and Wesley (MSW) pointed out that fundamental theories which are based on compactification from extra dimensions struggle to accommodate a period of accelerated cosmological expansion [1]. More concretely, they derived constraints on the subset of “consistent looking” (3+1) dimensional effective quantum field theories coupled to gravity that satisfy the Swampland conjectures [2,19] (for reviews see [20,21]) and thereby are also consistent with string theory [22]. In a recent study, we developed a concrete realization of the cosmological string framework of dark matter [23] that can accommodate a period of accelerated expansion [24]. In this Letter, we confront the predictions of our model with the constraints derived in [1] and we demonstrate that it remains a viable framework to explain the overall data sets of the latest cosmological observations.

We begin by summarizing some desirable features of effective field theories that are inherited from properties of the overarching string theory. The Swampland conjectures closely related to our study are those germane to effective scalar field theories canonically coupled to gravity and endowed with a canonical kinetic term, which dominates the energy density of the present epoch universe. For these theories to be consistent with string theory, the following two conditions are conjectured to hold:

- Distance Swampland conjecture: If a scalar field transverses a trans-Planckian range in the moduli space, a tower of string states becomes light exponentially with increasing distance [3,7].
- de Sitter conjecture: The gradient of the potential $V$ must satisfy either the lower bound, $M_{Pl} |V| \geq c V$ or else must satisfy $M_{Pl}^2 \min(V, V) \leq -c' V$, where $c$ and $c'$ are positive order-one numbers in Planck units and $M_{Pl}$ is the reduced Planck mass [5,10].

For the purposes of this study, however, we can ignore the criterium that restricts near-zero slope because we are considering the specific application of quintessence scalar fields as models for dark energy.

A key assumption in the derivation of the MSW constraints is that the internal space should be compact and conformally Ricci flat, and hence without loss of generality the metric tensor of the 10-dimensional space can be written as

$$ds^{10} = e^{2\Omega(t,y)} g^{RF}_{\mu
u}(t) dx^\mu dx^\nu + e^{-2\Omega(t,y)} g^{RF}_{mn}(t,y) dy^m dy^n\,,$$

where $g^{RF}$ is the flat Friedman-Robertson-Walker metric with time-dependent scale factor $a(t)$, Greek subscripts ($\mu, \nu$) are the indices along the non-compact dimensions with coordinates $x_{\mu}$, Latin subscripts ($m, n$) are the indices along the 6 compact extra dimensions with coordinates $y_{m}$, and the metric of the internal space is chosen such that $g^{RF}$ has vanishing Ricci scalar curvature with warp factor $e^{-2\Omega}$. For compact spaces with this specific structure, the expansion rate can be expressed in terms of the 4-dimensional effective scale factor $a(t)$, with $e^\Omega = \int e^{2\Omega} \sqrt{g_{10}} dx^1 \cdots dx^6$, and the variation of Newton’s constant $G_4$ can be related to the Hubble parameter $H$ according to $G_4/G_4 = -H^2$, where $\kappa = H^{-1} \sqrt{e^{2\Omega} \sqrt{g_{10}}} \frac{\partial}{\partial y^m} a$, and the variation of $\kappa$ drives the local expansion of the extra dimensions [27]. Now, using limits on the instantaneous variation of $G_4$ today [28] MSW derived constraints to be imposed on the $\kappa(a)$ trajectories for quintessence scalar field dark energy $\chi$ with potential $V_\chi \propto e^{\chi/k}$, where $\kappa \sim O(1)$. It turns out that for $\kappa < 1$, the computed values of $\kappa(a = 1)$ are outside the $3\sigma$ range of the observed instantaneous value of $G_4/G_4$ today [1].

By all means, the metric of the internal manifold is not always factorable in terms of a warping factor times a Ricci flat space. A particular string framework where the internal space is not conformally Ricci flat is that of the Salam-Sezgin model [29] with its string realization by Cvetic, Gibbons, and Pope [30]. The Salam-Sezgin model is fairly simple, it describes the compactification of a 6-dimensional supergravity to four dimensions with a monopole background on a 2-sphere, allowing for time dependence of the 6-dimensional dilaton $\phi$ and the breathing mode of the sphere $f$, while tolerat-
ing a 4-dimensional metric with a Friedmann-Robertson-Walker form \([31]\). The metric tensor of the 6-dimensional spacetime is given by

\[
\begin{align*}
    ds^2_6 &= e^{2f} \left[ -dt^2 + e^{2h} dx^2 + r_c^2 (d\delta^2 + \sin^2 \delta d\phi^2) \right],
\end{align*}
\]

where \(r_c\) is the compactification radius and \(h = \ln \bar{a}\). The gauge field \(F_{\delta\phi} = -b \sin \delta\) is excited on \(S^2\) supporting the monopole configuration \([29]\).

In terms of linear combinations of the \(S^2\) moduli field \(f = \sqrt{G_4 (X - Y)}/4\) and the 6-dimensional dilaton \(\phi = \sqrt{G_4 (X + Y)}/2\), the 4-dimensional effective potential in the Einstein frame consists of a pure exponential function of a quintessence field \(Y\) (which is the 4-dimensional dilaton) times a quadratic polynomial in the field \(e^{-X}\). It turns out that \(X\) is a source of cold dark matter, with a mass proportional to an exponential function of the quintessence field. When making the volume of the 2-sphere large, namely for large values of \(\rho\), there appears a tower of states, which according to the infinite distance swampland conjecture becomes exponentially massless. If the standard model fields are confined on a non-compact internal manifold.

\[
\begin{align*}
    ds^2_{10} &= (\cosh 2\rho)^{1/4} e^{\phi/2} \left\{ d\phi^2 + 4 \frac{\cos^2 \rho}{\cosh 2\rho} \left[ d\alpha - \sqrt{\frac{c}{8} b \cos \delta d\varphi} \right]^2 + \sinh^2 \rho \left[ d\beta + \sqrt{\frac{c}{8} b \cos \delta d\varphi} \right]^2 \right\}, \quad (1)
\end{align*}
\]

where \(\rho, z, \alpha, \beta\) are the four extra coordinates, \(\xi\) is the rescaled gauge coupling, and the 10-dimensional dilaton (denoted by \(\phi\)) satisfies \(e^\phi = e^\phi/\sqrt{\cosh 2\rho}\) \([30]\). As can be read off by inspection of \([3]\) the 6-dimensional metric tensor of the internal space cannot be factorized to conform with \([1]\), and therefore the MSW constraint on \(G_4/G_6\) can be evaded.

A point worth noting at this juncture is that the uplifted procedure leading to \([5]\) implies a non-compact internal manifold. As a consequence, the string coupling constant, \(g_s = e^0\), goes to zero at large distances \(\rho\) in the internal directions. In addition, the ratio \(G_{10}/G_6 = 16\pi^2 \xi^{-3/2} \int dz \int_0^\infty d\rho \sinh 2\rho\), points to a vanishing \(G_6\) to accommodate the diverging \(\rho\) integration. However, the metric in \([3]\) can be interpreted within the context of a Klebanov-Strassler throat like in \([35]\), with \(0 \leq \rho \leq L, L \gg 1\) being an infrared cutoff, to obtain a compact internal space and therefore \(G_6 \neq 0\).

A second constraint discussed by MSW pertains to the equation of state for dark energy as a function of redshift, \(w_\gamma(z)\). Before proceeding, we pause to note that it is nearly impossible to constrain a general history of \(w_\gamma(z)\). This is because the dark energy density, which regulates \(H(z)\), is given by an integral over \(w_\gamma(z)\), and hence length scales and the growth factor involve a further integration over functions of \(H(z)\). Several parametrizations for \(w_\gamma(z)\) have been proposed; see e.g. \([39, 40]\). It has become conventional to phrase constraints on \(w_\gamma(z)\) in terms of a linear evolution model, \(w_\gamma(z) = w_0 + w_a z/(1 + z)\) \([37, 39]\). Indeed, MSW adopt the constraint on \(w_\gamma(z)\) derived in \([41]\) on the basis of the linear evolution model and the best fit parameters of supernovae type Ia (SNe Ia), cosmic microwave background (CMB), and baryon acoustic oscillation (BAO) measurements \([42]\). More concretely, when Planck 2015 CMB measurements are combined with data from the Pantheon SNe Ia sample and constraints from BAO the best fit parameters are \(w_0 = -1.007 \pm 0.089\) and \(w_a = -0.222 \pm 0.407\) \([42]\). Over and above, when SNe Ia and BAO datasets are combined with the most recent Planck 2018 observations the precision on the best fit parameter improves, yielding \(w_0 = -0.964 \pm 0.077\) and \(w_a = -0.25^{+0.30}_{-0.26}\) \([43]\). However, recent observations provided evidence to support the possibility that intrinsic SNe Ia luminosities could either evolve with redshift \([44, 45]\) (see however \([46]\), or else correlate with the host star formation rate or metallicity \([47, 50]\). All in all, the effect of the new SNe Ia systematic uncertainties leads to both a shift in the peak and a broadening of the marginalized posterior probability distributions from the multi-dimensional fit used to determine the

Neveu-Schwarz 5-branes \([32]\) the 6-dimensional gauge couplings are independent of the string dilaton in the string frame, and upon compactification to four dimensions the 4-dimensional gauge couplings depend on \(X\) (rather than the dilaton \(Y\)) which is fixed at the minimum of the potential \([24]\). This avoids direct couplings of the dilaton to matter suppressing extra forces competing with gravity. The asymptotic behavior of the Hubble parameter, \(h \sim t\), leads to a conformally flat Friedmann-Robertson-Walker metric for large times. The de Sitter (vacuum) potential energy density is characterized by an exponential behavior \(V_\gamma \propto e^{-\sqrt{2}Y}\). Asymptotically, this represents the crossover situation with the equation of state for the quintessence field \(w_\gamma = -1/3\), implying expansion at constant velocity with \(Y\) varying logarithmically \(Y \sim -\ln t\) \([33, 34]\). The deviation from constant velocity expansion into a brief accelerated phase encompassing the recent past (redshift \(z \leq 6\)) makes the model phenomenologically viable \([24]\).

The Salam-Sezgin model can be uplifted to obtain a full Type I string configuration, where the metric tensor takes the form

\[
\begin{align*}
    ds^2_{10} &= (\cosh 2\rho)^{1/4} e^{\phi/2} \left\{ d\phi^2 + 4 \frac{\cos^2 \rho}{\cosh 2\rho} \left[ d\alpha - \sqrt{\frac{c}{8} b \cos \delta d\varphi} \right]^2 + \sinh^2 \rho \left[ d\beta + \sqrt{\frac{c}{8} b \cos \delta d\varphi} \right]^2 \right\}, \quad (1)
\end{align*}
\]
FIG. 1: Left. The 95%CL upper limit on \( w_Y(z) = w_0 + w_a z/(1+z) \) based on SNe Ia, CMB and BAO data. Following [41], the limit is determined from Fig. 5 in [43] by finding the values of \((w_0, w_a)\) all along the 95%CL contour, plotting all \(w_Y(z)\), and finding the upper hull. Right. A comparison between the 95% CL upper limit derived in the left panel and various predictions for the Salam-Sezgin-Cvetič-Gibbons-Pope model.

dark energy parameters: when Pantheon SNe Ia, BAO, and Planck 2018 datasets are combined, \( w_0 = -0.85^{+0.15}_{-0.21} \) and \( w_a = -0.52^{+0.57}_{-0.49} \), whereas when JLA SNe Ia, BAO, and Planck 2018 datasets are combined, \( w_0 = -0.70 \pm 0.19 \) and \( w_a = -0.91 \pm 0.52 \). In Fig. 1 we show a comparison between the predictions for \( w_Y(z) \) of the models studied in [24] and the 95%CL upper limit on \( w_Y(z) \) derived in [43], taking into account SNe Ia systematics. The predictions of the models are partially consistent with the upper limit. Given the large theoretical uncertainties in the determination of the functional form of \( w_Y(z) \) [51, 52], we conclude that our cosmological framework remains phenomenologically viable.

In summary, we have shown that the Friedmann-Robertson-Walker-Salam-Sezgin model and its string realization by Cvetič, Gibbons, and Pope remains a well equipped framework to describe cosmological observations. Besides, for the sake of completeness, it is important to stress that in [1] there is an implicit assumption of critical string theory which does not hold for time dependent solutions. Consider for instance the simplest time-dependent exact solution of string theory described by the linear dilaton background in string frame, corresponding to a linearly expanding universe and logarithmic dilaton in the Einstein frame [33, 34]. The underline (super-)conformal field theory (CFT) in the world-sheet is a free coordinate with a background charge, implying a positive central charge deficit for the internal CFT. Using a 6-dimensional \( \sigma \)-model, this implies a negatively curved internal manifold violating the Ricci-flatness assumption of the metric \( g_{\text{RF}}^{mn} \), such as in the model we described above. Alternatively, one may use flat compact coordinates in a higher dimensional space, since positive central charge deficit increases effectively the critical dimension of string theory. Another property shared by the model we studied here is the non-uniform time dependence of the internal space (i.e., internal dimensions may have different time dependence). Allowing in general different directions/cycles to have different time dependence, leaves plenty of room still available for model builders.

We end with an observation: the fading dark matter hypothesis relieves tensions in \( H_0 \) measurements but it does not fully resolve them. String theory provides a plethora of candidates for long-lived relics that can modify the expansion rate at recombination and thus affect the evolution of \( H \) and \( w_Y \) [24, 53, 54]. A comprehensive study of the full parameter space is beyond the scope of this Letter and will be presented elsewhere.

The work of L.A.A. and J.F.S. is supported by the U.S. National Science Foundation (NSF Grant PHY-1620661) and the National Aeronautics and Space Administration (NASA Grant 80NSSC18K0464). The research of I.A. is funded in part by the “Institute Lagrange de Paris”, and in part by a CNRS PICS grant. The work of D.L. is supported by the Origins Excellence Cluster. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the NSF or NASA.

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