GAUGE INVARIANCE WITH MASS:
HIGHER SPINS IN COSMOLOGICAL SPACES

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ABSTRACT
I review recent work on massive higher (s > 1) spins in constant curvature (deSitter) spaces. Some of the novel properties that emerge are: partial masslessness and new local gauge invariances, unitarily forbidden ranges of mass, correlation between fermions/bosons and negative/positive cosmological constant \( \Lambda \) and finally the consistency requirement that in the limit of infinite spin towers, \( \Lambda \) must tend to zero.

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1 Introduction

This is a report on a program being carried out in collaboration with A. Waldron [1], concerning the behavior of fields in constant curvature backgrounds. Perhaps the main novelty is that this is the first place that major and dramatic kinematical effects (as against the usual dynamical one) of gravity on matter are encountered.

Although one of our initial motivations was provided by a recent revival in the context of $\Lambda \neq 0$ of the famous vDVZ paradox, that the $m \to 0$ limit of massive spin 2 [2] as well as $s = 3/2$ [3] fields in flat space leads to a finite discontinuity (compared to $m \equiv 0$) in the source interactions they mediate, most of this report will not deal with that issue. What happens, in both $s = 2$ [4] and $s = 3/2$ [first paper of ref. [1]], is that the limits of $(m^2, \Lambda) \to (0, 0)$ can (depending on the path) yield almost any desired value. [There is also an amusing Newtonian limit aspect to this question, discussed by B. Tekin and myself [5] that I will briefly cover at the end.] To us the importance of the observations is that they are just a corner of the wider truth that the physics of massive higher spin ($s > 1$) fields (in contrast to $s \leq 1$) is in fact governed by a new phase plane coordinatized by $(m^2, \Lambda)$, instead of the one-dimensional $m^2$ line at $\Lambda = 0$. This plane displays a phase structure: it is covered by transition lines that divide it into physical and non-unitary “phases” for the spin in question, and these lines are themselves representations of entirely novel, non-Minkowskian, systems with partial masslessness, new local gauge invariances and associated truncated helicity count. They show how the confluence at $\Lambda = 0$ of the concepts of masslessness, gauge invariance (with maximal helicity only) and null propagation, is lifted (continuously this time) when $\Lambda \neq 0$. Bosons and fermions each display these properties, albeit in characteristically different regions of the $(m^2, \Lambda)$ plane: respectively in deSitter (dS) and Anti deSitter (AdS). This difference between the two types of particles actually leads to
a truly “emergent” result: the cosmological constant, being bounded by the highest particle species spins, must vanish for towers of bosons and fermions in the limit of infinite spins!

While we will not include all the details to be found in [1], we will show how the effect of (A)dS backgrounds arises in Sec. 2, explain the ensuing nonunitary domains in Sec. 3, analyze spin 2 canonically in Sec. 4 and higher spins in Sec. 5. Null propagation of the partially massive models is obtained in Sec. 6, the Newtonian limit explained in Sec. 7 and our “solution” of the cosmological constant problem is given in Sec. 8.

I end this introduction with a simple Figure that summarizes many of the above results, and should clarify much of the detailed calculations below.
Figure 1: The top/bottom halves of the half-plane represent dS/AdS (and also bosons/fermions) respectively. The $m^2 = 0$ vertical is the familiar massless helicity $\pm s$ system, while the other lines in dS represent truncated (bosonic) multiplets of partial gauge invariance: the lowest has no helicity zero, the next no helicities $(0, \pm 1)$, etc. Apart from these discrete lines, bosonic unitarity is preserved only in the region below the lowest line, namely that including flat space (the horizontal) and all of AdS. In the AdS sector, it is the topmost line that represents the pure gauge helicity $\pm s$ fermion, while the whole region below it, including the partially massless lines, is non-unitary. Thus, for fermions, only the region above the top line, including the flat space horizontal and all of dS, is allowed. Hence the overlap between permitted regions straddles the $\Lambda = 0$ horizontal and shrinks down to it as the spins in the tower of spinning particles grow; only $\Lambda = 0$ is allowed for generic ($m^2$ not growing as $s^2$) infinite towers.

2 Field Equations and Identities in Constant Curvature Spaces

The Riemann tensor in constant curvature spaces is

$$R_{\mu\nu\rho\sigma} = -\frac{2\Lambda}{3} g_{\mu[\rho} g_{\sigma]\nu} ;$$

(2.1)

the cosmological constant $\Lambda$ is positive in dS and negative in AdS. The actions of commutators of covariant derivatives are summarized by the vector-spinor example

$$[D_\mu, D_\nu] \psi_\rho = \frac{2\Lambda}{3} g_{\rho[\mu} \psi_{\nu]} + \frac{\Lambda}{6} \gamma_{\mu\nu} \psi_\rho .$$

(2.2)

The actions and field equations for massive spins $(3/2,2)$ in constant curvature backgrounds are, in an obvious notation,

$$\mathcal{L}^{(3/2)} = -\sqrt{-g} \bar{\psi} \gamma^\mu R_\mu , \quad \mathcal{L}^{(2)} = \frac{1}{2} \sqrt{-g} \phi^{\mu\nu} G_{\mu\nu} ,$$

(2.3)

^2Our metric is “mostly plus”, Dirac matrices are “mostly hermitean” and the Dirac conjugate is $\bar{\psi} \equiv \psi^\dagger i\gamma^0$. We denote (anti)symmetrization with unit weight by round (resp. square) brackets. Antisymmetrized products of Dirac matrices are given by $\gamma^{[\mu_1 \cdots \mu_n]} \equiv \gamma[\mu_1 \cdots \gamma \mu_n]$.

^3For $s = 2$ in generic gravitational backgrounds, the minimally coupled Pauli–Fierz action does not yield field equations with the correct $2s + 1 = 5$ excitation count \(^4\). Recently \(^5\), correct $s = 2$ actions have been constructed in background Einstein spaces.
\[ R_\mu \equiv \gamma_{\mu\rho} D^\rho \psi = (D^{(3/2)} - \gamma \cdot D) \psi - m \gamma_{\mu\rho} \psi^\rho. \quad (2.4) \]
\[ G_{\mu\nu} \equiv (\Delta^{(2)} - m^2 + \Lambda) (\phi_{\mu\nu} - g_{\mu\nu} \phi^\rho) + \Lambda \phi_{\mu\nu} + D_{(\mu} D_{\nu)} \phi^\rho - 2 D_{(\mu} \phi_{\nu)} + g_{\mu\nu} D D \phi. \quad (2.5) \]

The operators \( \Delta^{(2)} \) is the wave operators of \[8\]
\[ \Delta^{(2)} \phi_{\mu\nu} \equiv D^2 \phi_{\mu\nu} - \frac{8}{3} A \left( \phi_{\mu\nu} - \frac{1}{4} g_{\mu\nu} \phi^\rho \right), \quad (2.6) \]
whose introduction is justified by the following identities,
\[ \Delta^{(2)} D_{(\mu} \phi_{\nu)} = D_{(\mu} \Delta^{(1)} \phi_{\nu)}, \quad D^\mu \Delta^{(2)} \phi_{\mu\nu} = \Delta^{(1)} D \phi_{\nu}, \quad (2.7) \]
\[ \Delta^{(2)} g_{\mu\nu} \phi = g_{\mu\nu} \Delta^{(0)} \phi, \quad g^\mu\nu \Delta^{(2)} \phi_{\mu\nu} = \Delta^{(0)} \phi^\rho. \quad (2.8) \]

The fermionic version, \( D^{(3/2)} \), is given by [see first paper of ref. \[1\]]
\[ D^{(3/2)} \psi_{\mu} \equiv 2 \gamma \cdot D \psi_{\mu} - D_{\mu} \gamma \cdot \psi + \gamma_{\mu} (\gamma \cdot D \gamma \cdot \psi - D \psi) = \gamma_{\mu\rho} D^\rho \psi^\rho + \gamma \cdot D \psi_{\mu}, \quad (2.9) \]
and satisfies analogous identities
\[ D^{(3/2)} D_{\mu} \psi = D_{\mu} D^{(1/2)} \psi, \quad D^\mu D^{(3/2)} \psi_{\mu} = D^{(1/2)} D \psi, \quad (2.10) \]
\[ D^{(3/2)} \gamma_{\mu} \psi = \gamma_{\mu} D^{(1/2)} \psi, \quad \gamma^\mu D^{(3/2)} \psi_{\mu} = D^{(1/2)} \gamma \cdot \psi. \]

Notice that we have written the massive Rarita–Schwinger field equation \(2.4\) in terms of the \( s = 3/2 \) operator \( D^{(3/2)} \) with explicit mass term as well as in a more compact form involving the operator
\[ D_{\mu} \equiv D_{\mu} + \frac{m}{2} \gamma_{\mu}, \quad [D_{\mu}, D_{\nu}] = [D_{\mu}, D_{\nu}] = (m^2/2) \gamma_{\mu\nu}, \quad (2.10) \]
encountered in cosmological supergravity \[3\].

For \( s \geq 1 \) there are more relativistic field components than degrees of freedom (DoF). As usual, the correct DoF count is obtained by studying the constraints implied by divergences and (gamma-)traces of the field equations. Less usual, for special lines in the \( (m^2, \Lambda) \) plane, these constraints are satisfied identically and become Bianchi identities associated with gauge invariances. Explicitly, the divergences of the field equations \( (2.4) - (2.5) \) read
\[ D \cdot R = -\frac{1}{2} (3m^2 + \Lambda) \gamma \cdot \psi, \quad D \cdot G_{\mu} = -m^2 (D \phi_{\mu} - D_{\nu} \phi) \quad (2.11) \]
and are constraints for generic values of the parameters $m^2$ and $\Lambda$. For $s = 2$, the value $m^2 = 0$ yields Bianchi identities and their associated (“general coordinate”) gauge invariances

$$\delta \phi_{\mu \nu} = D_{(\mu} \xi_{\nu)}, \quad (2.12)$$

the $m^2 = 0$ theories are strictly massless: they propagate with two physical helicity states. For $s = 3/2$, the sole gauge invariance

$$\delta \psi_{\mu} = D_{\mu} \varepsilon = D_{\mu} \varepsilon + \frac{1}{2} \sqrt{-\Lambda/3} \gamma_{\mu} \varepsilon, \quad (2.13)$$

is inherited from cosmological supergravity and occurs for $m^2 = -\Lambda/3$ in AdS. This (rather than the $m = 0$ model) is the strictly massless helicity $\pm 3/2$ theory.

For $s = 2$ there is a new effect: the field equation has two open indices and thereby also admits a double divergence Bianchi identity. The double divergence constraint

$$D.D.G + \frac{1}{2} m^2 G_{\rho}^{\rho} = \frac{1}{2} m^2 (3m^2 - 2\Lambda) \phi_{\rho}^{\rho}, \quad (2.14)$$

becomes a Bianchi identity not only along the strictly massless line $m^2 = 0$, but also along the dS gauge line $m^2 = 2\Lambda/3$, which actually corresponds to the theory of [10]. The gauge invariance associated with this identity,

$$\delta \phi_{\mu \nu} = \left(D_{(\mu} D_{\nu)} + \frac{\Lambda}{3} g_{\mu \nu}\right) \xi, \quad (2.15)$$

may be employed to show that this partially massless model propagates with helicities ($\pm 2, \pm 1$) on the null cone in dS. We discuss this theory further in Section 4.

To summarize, the $(m^2, \Lambda)$ half-plane offers no surprises for spins $s \leq 1$, the usual null propagating massless theories inhabit the line $m^2 = 0$ and all other points $m^2 \geq 0$ describe $2s + 1$ massive DoF. For $s = 3/2$, the massless line is $m^2 = -\Lambda/3$ and bisects the $(m^2, \Lambda)$ half-plane. For $s = 2$, the massless theory again lies on the axis $m^2 = 0$ and a new gauge invariance emerges at $m^2 = 2\Lambda/3$ which also divides the half-plane into two distinct physical regions. This pattern will be seen to continue for $s > 2$ as well.
3 (Anti)commutators and Nonunitary Regions

Our analysis is rather simple and harks back to the original inconsistency of the local field theory of charged spin 3/2 particles \([11]\). Generically, given (anti)commutation relations \(\{\psi, \psi^\dagger\} = \epsilon = [a, a^\dagger] = -i[x, \dot{x}]\) (with \(a = (x + i\dot{x})/\sqrt{2}\) and a vacuum\(^4\) \(\psi|0\rangle = 0 = a|0\rangle\), positivity of norms requires \(\epsilon > 0\). For quantum field theories, exactly the same criterion can be applied, but now in a distributional sense.

The local canonical (anti)commutators for quantum fields with \(s \leq 2\) in cosmological spaces were presented long ago \([13]\). Let us summarize the relevant results:

For \(s \geq 1\), away from the gauge invariant boundaries, the constraints (2.11) and (2.14) imply the (gamma-)traceless–transverse conditions

\[
D.\psi = 0 = \gamma.\psi, \quad D.\phi_\nu = 0 = \phi_\mu^\rho,
\]

which allow the field equations (2.4)-(2.5) to be rewritten as

\[
\left(\frac{1}{2}D^{(3/2)} + m\right)\psi_\mu = 0, \quad (\Delta^{(2)} - m^2 + 2\Lambda)\phi_{\mu\nu} = 0.
\]

We must now write (anti)commutators for fields satisfying both (3.1) and (3.2). The former is easily imposed using the (gamma-)traceless–transverse decompositions

\[
\phi^T_\mu = \phi_\mu - D_\mu \frac{1}{D^2} D.\phi,
\]

\[
D.\phi^T = 0;
\]

\[
\psi^T_{\mu} = \psi_\mu - \frac{1}{4} \gamma_\mu \gamma.\psi + D_\mu \frac{1}{3D^2 + A} (\gamma \cdot D\gamma.\psi - 4D.\psi),
\]

\[
D.\psi^T_{\mu} = 0 = \gamma.\psi^T_{\mu};
\]

\(^4\)It has recently been suggested that the non-perturbative definition of quantum gravity in dS be reexamined \([12]\); in particular the definition of the vacuum requires careful consideration. Our local quantum field theoretic computation ignores such subtleties.
\[\phi_{\mu\nu}^{TT} = \phi_{\mu\nu} - D_{(\mu} \frac{2}{D^2 + A} (D\phi_{\nu})^T - \frac{1}{4} g_{\mu\nu} \phi^\rho_{\rho} - D_{(\mu} D_{\nu)} \frac{4}{D^2(3D^2 + 4A)} [D^2 \phi - \frac{1}{4} D^2 \phi^\rho_{\rho}],\]

\[D.\phi_{\mu}^{TT} = 0 = \phi_{\rho}^{TT\rho}; \quad (3.5)\]

where a tilde over an index denotes its gamma-traceless part, i.e. \(X_{\mu} \equiv X_{\mu} - \frac{1}{4} \gamma_{\mu} \gamma_4 X\). and \(\{\cdots\}\) denotes the symmetric-traceless part of any symmetric tensor, i.e. \(X_{(\mu\nu)} \equiv X_{(\mu\nu)} - \frac{1}{4} g_{\mu\nu} X^\rho_{\rho}\).

Therefore the (anti)commutators for spins \(1 \leq s \leq 2\) are given by

\[
\{\psi_\mu(x), \overline{\psi}_\nu(x')\} = i S_{\mu\nu}^{(3/2)}(x, x'; 2m) - \frac{i}{4} \gamma^s_\mu S^{(1/2)}(x, x'; 2m) \gamma_s^\nu' + \frac{i}{3m^2 + A} D_{\mu}^{(1/2)}(x, x'; 2m) \phi_{\nu}^{x'}; \quad (3.6)
\]

\[
[D_{\mu\nu}(x), \phi_{\rho\sigma}(x')] = i D_{\mu\nu,\rho\sigma}^{(2)}(x, x'; m^2 - 2\Lambda) + \frac{2i}{m^2} D_{\mu}^{x} D_{\nu}^{y} D_{\rho\sigma}^{(1)}(x, x'; m^2 - 2\Lambda) + \frac{i}{m^2(3m^2 - 2\Lambda)} \left[2 D_{\mu}^{x} D_{\nu}^{y} D_{\rho}^{x'} D_{\sigma}^{y'} + m^2(\Lambda - m^2) g_{\mu\nu} g_{\rho\sigma} + m^2 D_{\mu}^{x} D_{\nu}^{y} g_{\rho\sigma} + m^2 g_{\mu\nu} D_{\rho}^{x'} D_{\sigma}^{y'}\right] D^{(0)}(x, x'; m^2 - 2\Lambda). \quad (3.7)
\]

For brevity, we have suppressed obvious symmetrizations \((\mu\nu)\) and \((\rho\sigma)\) on the right hand side of (3.7). The field equations (3.2) have been used throughout to eliminate any factors \(D^2\) appearing in the (gamma-)traceless–transverse decompositions (3.3)-(3.5), since the higher distributions are also onshell

\[
(\Delta^{(n)} + m^2) D_{\mu_1...\mu_n,\nu_1...\nu_n}^{(n)}(x, x'; m^2) = 0, \quad (3.8)
\]

\[
(\mathcal{D}^{(n+1/2)} + m) S_{\mu_1...\mu_n,\nu_1...\nu_n}^{(n+1/2)}(x, x'; m) = 0. \quad (3.9)
\]

The identity \(D_{\mu}^{x} S^{(1/2)}(x, x'; 2m) = D_{\mu}^{x} S^{(1/2)}(x, x'; 2m)\) is also useful. The (anti)commutators (3.6)-(3.7) are the difference between advanced and retarded propagators. The distributions \(D^{(2)}\) and \(S^{(3/2)}\) above satisfy

\[
S_{\mu\nu}^{(3/2)}(x, x'; m) D_{\nu}^{x'} = -D_{\mu}^{x} S^{(1/2)}(x, x'; m), \quad (3.10)
\]

\[
S_{\mu\nu}^{(3/2)}(x, x'; m) \gamma_{\nu}^{x'} = \gamma_{\mu}^{x} S^{(1/2)}(x, x'; m), \quad (3.11)
\]

7
\[
D_\mu^{\rho,\sigma} D^{(2)}_{\rho,\sigma}(x, x', m^2) = -D^{\mu}_{(\mu, \nu)} D^{(1)}_{(\nu, \sigma)}(x, x'; m^2), \tag{3.12}
\]

\[
g^{\rho \sigma}_{x} D^{(2)}_{\rho,\sigma}(x, x', m^2) = g^{x}_{\mu \nu} D^{(0)}(x, x', m^2), \tag{3.13}
\]

with boundary conditions

\[
\epsilon^{(3/2)}_{\mu \nu}(x, x') \bigg|_{x^0 = x'^0} = \frac{g_{\mu \nu} \gamma^0}{\sqrt{-g}} \delta^3(\vec{x} - \vec{x}'), \tag{3.14}
\]

\[
\frac{d}{dx^0} D^{(2)}_{\mu \nu, \rho \sigma}(x, x') \bigg|_{x^0 = x'^0} = \frac{g_{\mu (\rho} g_{\nu) \sigma}}{\sqrt{-g}} \delta^3(\vec{x} - \vec{x}'), \quad D^{(2)}_{\mu \nu, \rho \sigma}(x, x') \bigg|_{x^0 = x'^0} = 0.
\]

Equipped with the above tools, one can easily uncover the nonunitary regions. Starting with the (anti)commutators (3.6)-(3.7), the aim is to determine whether the distributions on their right hand sides have definite sign in the dangerous lower helicity sectors.

For concreteness we work in the simple synchronous dS metric

\[
ds^2 = -dt^2 + e^{2Mt} d\vec{x}^2, \quad M \equiv \sqrt{\Lambda/3}, \tag{3.15}
\]

and concentrate on the equal time (anti)commutators of the time components of the fields (and their time derivatives). While the metric (3.15) does not cover the entire dS space, nor is it real when continued to negative AdS values of $\Lambda$, these disadvantages are outweighed by its simplicity. Selecting the lowest helicity components by looking at time components of fields, a simple computation reveals that

\[
\{\psi_0(t, \vec{x}), \psi_0^\dagger(t, \vec{x}')\} = -\frac{\nabla^2}{3m^2 + \Lambda} \frac{1}{\sqrt{-g}} \delta^3(\vec{x} - \vec{x}'), \tag{3.16}
\]

\[
[\phi_{00}(t, \vec{x}), \dot{\phi}_{00}(t, \vec{x}')] = \frac{2 \nabla^4}{m^2 (3m^2 - 2A)} \frac{i}{\sqrt{-g}} \delta^3(\vec{x} - \vec{x}'), \tag{3.17}
\]

where $\nabla^2 \equiv g^{ij} \partial_i \partial_j = e^{-2Mt} \tilde{\nabla}^2$ is a negative operator. The final equation (3.17) agrees with the detailed massive $s = 2$ Dirac analysis presented in [13]. Our derivation only requires writing out the Laplace and Dirac operators explicitly in the metric (3.15) and using the field equations (3.2) to maximally eliminate time derivatives, after which the boundary conditions (3.14) may be applied.
The interpretation of equations (3.16)-(3.17) is as follows: Positivity of the distributions on the right hand sides is completely determined by the respective denominators $3m^2 + \Lambda$ and $3m^2 - 2\Lambda$, precisely the factors appearing in the Bianchi identities of the previous Section. For the various spins, we learn:

- **Spin 3/2**: The model is unitary in the region $m^2 > -\Lambda/3$ which includes the Minkowski background. Strictly massless, gauge invariant unitary models are found along the AdS line $m^2 = -\Lambda/3$. The region $m^2 < -\Lambda/3$ is non-unitary. In contrast to flat space, the $m^2 = 0$ theories are massive when $\Lambda \neq 0$ and even non-unitary for negative (AdS) values of $\Lambda$.

- **Spin 2**: Models with $m^2 > 2\Lambda/3$ are unitary. There are now two lines of gauge invariant theories; the usual linearized cosmological Einstein theory at $m^2 = 0$ and a partially massless theory \[10\] at $m^2 = 2\Lambda/3$. Both are unitary but the region $m^2 < 2\Lambda/3$ is not.

Finally, as promised, we address the concern that, strictly, the metric (3.15) applies only to dS. On the one hand, given that (i) the final results are a function of the real variable $\Lambda$ only and (ii) the picture presented here is backed up by the emergence of Bianchi identities, there can be no doubt of its correctness. However, for complete certainty, we repeat, as an example, the $s = 3/2$ computation in the metric \((M \equiv \sqrt{\Lambda/3})\)

$$\begin{aligned} ds^2 &= -dt^2 + \cosh^2(Mt) \left\{ dr^2 + \frac{1}{M^2} \sin^2(Mr) \left( d\theta^2 + \sin^2 \theta \: d\phi^2 \right) \right\}. \tag{3.18} \end{aligned}$$

Upon rescaling $r \to \rho/M$, the three-metric $d\Omega^2 = d\rho^2 + \sin^2 \rho \left( d\theta^2 + \sin^2 \theta \: d\phi^2 \right)$ is seen to describe a unit three-sphere. We prefer the initial parametrization, however, since for pure imaginary values of $M$, the cosmological constant $\Lambda$ is negative and the metric continues to AdS. Performing a calculation analogous to the one above we find

$$\begin{aligned} \{ \psi_0(t,r,\theta,\phi), \psi_0^\dagger(t',r',\theta',\phi') \} &= \cosh^{-2}(Mt) \left( - \frac{3}{2}D^2 - \frac{\Lambda}{4} \right) \frac{1}{3m^2 + \Lambda} \sqrt{-g} \delta(r - r') \delta(\theta - \theta') \delta(\phi - \phi'). \tag{3.19} \end{aligned}$$

\[9\]
The operator \((3)^2D^2\) is the square of the intrinsic 3-dimensional covariant derivative (Laplace–Beltrami operator) acting on a spinor. In dS the operator \(- (3)^2D^2 - \Lambda/4\) is not manifestly positive. However (in our parametrization) the eigenvalues of \((3)^2D^2\) acting on spinors are \((\Lambda/3)(-l(l + 2) + 1/2)\) with \(l \geq 1/2\) (see, e.g. [13]), and the highest eigenvalue is precisely \(-\Lambda/4\). Hence the operator \(- (3)^2D^2 - \Lambda/4\) is indeed positive and in dS we may draw precisely the conclusions given above. Now, continuing the metric (3.18) to AdS space the same result holds for the local anticommutator except the 3-space is a hyperboloid. Nonetheless, (assuming we can neglect spatial boundary terms), both \(- (3)^2D^2\) and \(- \Lambda/4\) are now separately positive, and unitarity is determined by the sign of the denominator \(3\Lambda + m^2\). This concludes our derivation of the unitarily forbidden regions for spins \(s = 3/2, 2\).

We emphasize that once one knows the gauge lines and their corresponding Bianchi identities, our unitarity results in fact follow by inspection: Whenever a coefficient in a massive constraint vanishes and then becomes negative, all the corresponding lower helicity modes are first excised by the accompanying gauge invariance and thereafter reemerge with opposite norms. Therefore, starting from the unitary Minkowski region, it is easy to map out the unitarily allowed and forbidden regions, as shown in the Figure. Furthermore, for higher spin partially massless theories to be unitary, the ordering criterion for the gauge lines, discussed in the introduction, must hold. A simple example is provided by the \(s = 2\) strictly massless \((m^2 = 0,\ \text{linearized graviton})\) theory for \(\Lambda > 0\): To reach it starting from the unitary Minkowski region, one must pass through the unitarily forbidden region \(0 < m^2 < 2\Lambda/3\). Nonetheless, the theory is unitary, since the highest helicities \(\pm 2\) are left untouched by the unitarity flip of the helicity 0 mode across the \(m^2 = 2\Lambda/3\) gauge line.

### 4 Partially Massless Spin 2: Canonical Analysis

At the partially massless dS boundary, \(m^2 = 2\Lambda/3 = 2M^2\), we showed that the scalar constraint (2.14) is a Bianchi identity; as such it removes the fifth DoF, leaving the 4 physical DoF corresponding to helicities \(\pm 2, \pm 1\). In this Section we prove this claim via an explicit canonical analysis\(^5\). Our method is

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\(^5\)A detailed canonical analysis of massive \(s = 2\) for general \(m^2\) is given in the fourth reference in [1]; an early attempt can be found in [4].
similar to that originally used to prove the stability of massless cosmological gravity \[15\].

A possible starting point is the second order massive \( s = 2 \) action in equation (2.3). Equivalently (and much simpler) one can begin with the first order ADM form of cosmological Einstein gravity,

\[
S_\Lambda + E = \int d^4x \left[ \pi^{ij} g_{ij} + N\sqrt{g} \left( (3)R - 2\Lambda \right) + 2N_i D_j \pi^{ij} \right.
\]

\[
+ \frac{N}{\sqrt{g}} \pi^{im} \left( \frac{1}{2} g_{ij} g_{lm} - g_{il} g_{jm} \right) \Big] ,
\]

(4.1)

then linearize around a dS background and add by hand an explicit mass term. Here \( g \) is the determinant of the 3-metric \( g_{ij} \) and \( N \equiv (-g_{00})^{-1/2} \), \( N_{0i} \equiv g_{0i} \). We take the synchronous dS metric (3.15), denoted \( ds^2 = -dt^2 + \overline{g}_{ij} dx^i dx^j \) in this Section, reserving \( g_{ij} \) for the dynamical 3-metric which we linearize as

\[
g_{ij} \equiv \overline{g}_{ij} + \phi_{ij}, \quad \overline{g}_{ij} \equiv f^2(t) \delta_{ij}, \quad f(t) \equiv e^{Mt} .
\]

(4.2)

The remaining fields are linearized as

\[
\pi^{ij} \equiv \overline{\pi}^{ij} + P^{ij}, \quad \overline{\pi}^{ij} \equiv -2Mf \delta^{ij}, \quad N \equiv 1 + \tilde{n} .
\]

(4.3)

(The background metric is block diagonal so no expansion is needed for \( N_i \).)

In terms of these deviations, the mass term is

\[
S_m = -\frac{m^2}{4} \int d^4x \sqrt{g} \left( \phi_{\mu\nu} \phi_{\rho\sigma} \overline{g}^{\mu\rho} \overline{g}^{\nu\sigma} - (\phi_{\mu\nu} \overline{g}^{\mu\nu})^2 \right) ,
\]

(4.4)

here \( \phi_{0i} \equiv N_i, \overline{g} \equiv \det \overline{g}_{ij} = -\det \overline{g}_{\mu\nu} \) and \( \phi_{00} \equiv g_{00} + \overline{g}_{00} = -(1 + N^2) \). The final action is the sum \( S = S_{A+E} + S_m \), discarding any terms of higher than quadratic order in (dynamical) fields.

Notice that the only explicit time dependence of the integrand of (4.4) is through \( f^{-1} \). Indeed it proves useful to make the field redefinition

\[
\phi_{ij} \equiv f^{1/2} h_{ij}, \quad P^{ij} \equiv f^{-1/2} p^{ij} .
\]

\[ N_i \equiv f^{1/2} n_i, \quad \tilde{n} = f^{-3/2} n . \]

(4.5)

The cost is an extra contribution generated by the symplectic term of (4.1), \( P^{ij} \dot{\phi}_{ij} \rightarrow p^{ij} \dot{h}_{ij} + (M/2)p^{ij} h_{ij} \). A dividend is that the only explicit time
dependence in what follows will be through $\nabla^2 \equiv \nabla^i \partial_i \partial_j \equiv f^{-2} \nabla_0^2$. Index contractions are just with $\delta_{ij}$, all quantities are now in (3 + 1) form.

Next examine the mass term

$$S_m = \int d^4 x \left[ - \frac{m^2}{4} (h^2_{ij} - h^2_{ii}) + \frac{m^2}{2} N_i^2 + m^2 n h_{ii} \right].$$

(4.6)

Were it not for the term proportional to $N_i^2$, the field $N_i$ would be a Lagrange multiplier for 3 constraints (as is the case for the $m = 0$ strictly massless theory). Instead, when $m \neq 0$ we must integrate out $N_i$ via its algebraic equation of motion. The field $n$, however, only appears linearly and remains a Lagrange multiplier for the constraint

$$\left[ \sqrt{g} (^{(3)}R - 2\Lambda) + m^2 h_{ii} + \frac{1}{\sqrt{g}} \pi_{ij} \pi^{lm} \left( \frac{1}{2} g_{ij} g_{lm} - g_{il} g_{jm} \right) \right]_{\text{linearized}} = 0.$$

(4.7)

For generic values of $(m^2, \Lambda)$, this constraint eliminates one degree of freedom from the 6 pairs $(p_{ij}, h_{ij})$ leaving 5 physical helicities $(\pm 2, \pm 1, 0)$. Our aim now is to show that a (single) further constraint emerges at the gauge invariant value $m^2 = 2M^2$.

We decompose the fields $h_{ij}$ and $p_{ij}$ according to their helicity and drop (for now) the helicity $\pm 2$ traceless-transverse ($h_{ij}^{tt}, p_{ij}^{tt}$) and helicity $\pm 1$ transverse ($h_{ij}^{t}, p_{ij}^{t}$) modes since they manifestly decouple from the 2 remaining helicity 0 modes (to the quadratic order used here). The latter are defined by the projection

$$h_{ij} = \frac{1}{2} \left( \delta_{ij} - \frac{\partial_i \partial_j}{\nabla^2_0} \right) h_T + \frac{\partial_i \partial_j}{\nabla^2_0} h_L,$$

(4.8)

and similarly for $p_{ij}$. (Note that under the integral $\int A_{ij} B_{ij} = \frac{1}{2} \int A_T B_T + \int A_L B_L$.) In terms of these variables the linearized constraint is

$$C \equiv \nabla^2 h_T + 2M (p_T + p_L) - (m^2 - 2M^2) (h_T + h_L) = 0.$$

(4.9)

Note that the leading term comes from the linearized 3-dimensional curvature scalar and that both the cosmological $-2\Lambda \sqrt{g}$ and momentum squared terms in (4.7) contribute to the final term in (4.9), which vanishes on the critical gauge line $m^2 = 2M^2$. Henceforth we concentrate on the critical case and eliminate $m^2$ via this relation.
Next we write out the quadratic action $S = S_{E+A} + S_m$ remembering the constraint (4.9) which we solve as

$$p_T = -p_L - \frac{\nabla^2}{2M} h_T. \quad (4.10)$$

Observe that the symplectic terms become

$$p_{ij} \dot{h}_{ij} = \frac{1}{2} p_T \dot{h}_T + p_L \dot{h}_L = p_L \dot{q} - \frac{1}{4} h_T \nabla^2 h_T, \quad q \equiv h_L - \frac{1}{2} h_T. \quad (4.11)$$

(suppressing integrations throughout). Upon eliminating $h_L = q + h_T/2$ in favor of $q$ and $h_T$, the action depends only on the 3 variables $(p_L, q, h_T)$ and its most general form is

$$S(p_L, q, h_T) - p_L \dot{q} = \frac{1}{2} A h_T^2 + B h_T + C, \quad (4.12)$$

where $A$ is constant, $B$ linear and $C$ quadratic in $(p_L, q)$. If $A = 0$, we have an additional constraint $B(p_L, q) = 0$ and no zero helicity DoF remain, whereas for non-zero $A$, $h_L$ can be removed via its algebraic field equation leaving behind one zero helicity DoF. In fact, $A$ does vanish on the critical line $m^2 = 2\Lambda/3$ so this model describes helicities $(\pm 2, \pm 1)$ only. Indeed, a lengthy calculation yields

$$S(p_L, q, h_T) - p_L \dot{q} = \frac{1}{2} \left( \frac{p_L}{M} - q \right) \nabla^2 h_T + \left( \frac{p_L}{M} - q \right) \left( \frac{\nabla^2}{m^2} - \frac{3}{2} \right) p_L - \left( \nabla^2 - m^2 \right) q. \quad (4.13)$$

As claimed, $A = 0$ and even the zero helicity Hamiltonian vanishes once the Lagrange multiplier $h_T$ is integrated out.

Let us now examine the remaining helicities $(\pm 2, \pm 1)$. A series of canonical transformations yields a simple action

$$S_{(\pm 2, \pm 1)} = \sum_{\varepsilon=(\pm 2, \pm 1)} \left\{ p_{\varepsilon} \dot{q}_{\varepsilon} - \frac{1}{2} \left[ p^2_{\varepsilon} + q_{\varepsilon} \left( -\nabla^2 - \frac{M^2}{4} \right) q_{\varepsilon} \right] \right\}. \quad (4.14)$$

Notice again, all time dependence is through $\nabla^2$ in the Hamiltonian. The field equations are

$$p_{\varepsilon} = \dot{q}_{\varepsilon}, \quad \left( -\frac{d^2}{dt^2} + \nabla^2 - \frac{M^2}{4} \right) q_{\varepsilon} = 0. \quad (4.15)$$
The covariant field equation (3.2) evaluated at $m^2 = 2M^2$ is $(D^2 - 4M^2) \phi_{\mu\nu} = 0$. Consider, for example, helicities $\pm 2$, for which $\partial^i \phi_{ij} = 0 = \phi_{ii}$. In this frame the transverse-traceless part of the covariant field equation reads

$$( - \frac{d^2}{dt^2} + M \frac{d}{dt} + \nabla^2 ) \phi_{ij} = 0 .$$

(4.16)

The action (4.14) was obtained by the same rescaling as in (4.5), namely, $\phi_{ij} = f_{1/2} q_{ij}$. The factor $f_{1/2}$ is precisely the integrating factor which removes the single time derivative from equation (4.16) at the cost of a term $-M^2/4$, i.e. equations (4.16) and (4.15) are identical (helicities $\pm 1$ agree via a similar calculation).

Stability of the partially massless theory requires that it possess a conserved, positive, energy function. The latter can be obtained by an argument similar to that given in [15] for the strictly massless $s = 2$ theory: The Hamiltonian in (4.14) is not conserved because of the explicit time dependence of $\nabla^2$. However, inside the intrinsic dS horizon at $(fM x^i)^2 = 1$, the background metric (3.15) possesses a timelike Killing vector $\xi^\mu = (-1, M x^i) = \Rightarrow \xi^2 = -1 + (fM x^i)^2$.

(4.17)

Therefore, the energy associated with time evolution in this Killing direction

$$E = T^0_\mu \xi^\mu = H - M x^i \left[ p_\epsilon \partial_i q_\epsilon - \frac{1}{2} \partial_i (p_\epsilon q_\epsilon) \right]$$

(4.18)

satisfies $\dot{E} = 0$ ($H$ is the Hamiltonian in (4.14) and we have suppressed the sum over helicities $\epsilon$). Furthermore, writing out $E$ explicitly and relabeling the variable $p_\epsilon \rightarrow p_\epsilon + (3M/2)q_\epsilon$ gives

$$E = \frac{1}{2} \left( \tilde{x}^i p_\epsilon \right)^2 + \frac{1}{2} \left( f^{-1} \partial_i q_\epsilon \right)^2 - fM|x| \left( \tilde{x}^i p_\epsilon \right) \left( f^{-1} \partial_i q_\epsilon \right) + \frac{1}{2} (2M^2) q_\epsilon^2 ,$$

(4.19)

with $x^i \equiv |x| \tilde{x}^i$. The last (mass) term is manifestly positive and the first three terms are positive by the triangle equality whenever

$$fM |x| < 1 ,$$

(4.20)

that is, inside the physically accessible region.

A final interesting feature of the partially massless $s = 2$ theory is null propagation. The dS metric is conformally flat and it can be shown that the $m^2 = 2\Lambda/3$ theory propagates on its null cone [10]. We will later see that this property is shared by all partially massive systems of higher spins.
5 Higher Spins

Having seen that the $s = 2$ field equation $\mathcal{G}_{\mu\nu}$ yields both single divergence, $D.G_{\nu}$, and double divergence, $D.D.G$, Bianchi identities we are led to inquire whether even higher divergence Bianchi identities occur for $s > 2$. The answer (see the third paper of [1]) is yes: In addition to the usual massive and strictly massless possibilities, a spin $s$ field in (A)dS can be partially massless with propagating helicities $(\pm s, \pm(s-1), \ldots, \pm(s-t)) \ (t < s)$. In this section, we concentrate on the explicit analysis of $s = 5/2$ and $3$ as examples of the generic cases.

Spin 5/2

The $s = 5/2$ spinorial field equation has two open indices, so as for $s = 2$, there are two possible Bianchi identities; they appear along the AdS gauge lines $m^2 = -4\Lambda/3$ and $m^2 = -\Lambda/3$. The former is the strictly massless theory with helicities $\pm 5/2$ whereas the novel gauge invariance of the latter removes only the lowest $\pm 1/2$ leaving helicities $(\pm 5/2, \pm 3/2)$. Since the massless gauge lines all lie in AdS (just as for their $s = 3/2$ counterpart), the $(m^2, \Lambda)$ half-plane is divided into 3 regions. Only the $m^2 > -4\Lambda/3$ one including Minkowski space, is unitary.

The $s = 5/2$ action and field equations are
\begin{equation}
\mathcal{L} = -\sqrt{-g} \overline{\psi}^{\mu\nu} \mathcal{R}_{\mu\nu} - \sqrt{-g} \chi \mathcal{R}_5 ,
\end{equation}
\begin{equation}
\mathcal{R}_{\mu\nu} = (D^{(5/2)} - 2\gamma \cdot D) \psi_{\mu\nu} + g_{\mu\nu} \gamma.D.\psi + (D_{(\mu} \gamma_{\nu)} - \frac{1}{2} g_{\mu\nu} \gamma.D) \psi_{\rho}^\rho \\
+ m \left( \psi_{\mu\nu} - 2 \gamma(\mu,\psi_{\nu}) - \frac{1}{2} g_{\mu\nu} \psi_{\rho}^\rho \right) - \frac{5}{12} \mu g_{\mu\nu} \chi = 0 ,
\end{equation}
\begin{equation}
\mathcal{R}_5 = -\alpha (\gamma \cdot D - 3m) \chi - \frac{5}{12} \mu \psi_{\rho}^\rho = 0 .
\end{equation}

Minimal coupling alone does not provide equations of motion describing the $6 = 2s + 1$ massive DoF: An additional non-minimal coupling contained by the term $\mu \chi \psi_{\rho}^\rho$ is necessary. In fact, to achieve a proper set of constraints requires fixing the auxiliary coupling to
\begin{equation}
\mu^2 = \frac{12\alpha}{5} (m^2 + 4\Lambda/3) .
\end{equation}
Here we encounter a new subtlety. Implicitly we have so far assumed that physical models live in the half plane $m^2 > 0$, since for fermions negative $m^2$ implies a non-hermitean mass term, and for bosons, one that is unbounded below. While there are regions with both $m^2 < 0$ and the correct sign for anticommutators, the dynamics is non-unitary there. Therefore we continue to require $m^2 > 0$ and examine the relation (5.4). Since hermiticity of the action (5.1) demands $\mu^2 > 0$, we find two regions: (i) $m^2 > -4A/3 > 0$, (ii) $0 < m^2 < -4A/3$. Up until now, $\alpha$ was a free parameter which we could set to $\pm 1$. In the region (i), $m^2 + 4A/3 > 0$ so we must take $\alpha = +1$. In region (ii), hermiticity of the action can be maintained at the cost of changing the sign of $\alpha$ to $\alpha = -1$ (the actions are then different in each region). In either case, we will find that region (ii) is unitarily forbidden.

Before continuing, it is interesting to compare these difficulties to $s = 3/2$ and the problem of constructing dS supergravities. As we have shown, $s = 3/2$ is unitary for $m^2 \geq -A/3 \geq 0$ and the boundary is the strictly massless AdS theory corresponding to cosmological supergravity. As one follows the massive theory into dS, the canonical anticommutators all have the correct sign for unitary representations. In fact, keeping $A > 0$, there is no obstruction at the level of anticommutators to continuing to $m^2 < 0$ until the branch of the line $m^2 = -A/3$ with $m^2 < 0 < A$ is reached. The theory there is formally supersymmetric (i.e., strictly massless) but the action is no longer hermitean, which is an example of the general statement that dS supergravities do not exist [16]. One might speculate that this clash is generic to higher spin fermions.

Returning to the $s = 5/2$ field equations, for generic $(m^2, A)$ the constraints

$$C_\nu \equiv D_\nu R_{\nu} + \frac{1}{4} m \gamma R_{\nu}$$

$$= -\frac{5}{4} \left( m^2 + 4A/3 \right) \gamma \psi_{\nu} - \frac{5}{12} \mu D_{\nu} \chi , \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 

$$m^2 = -4A/3 \quad m^2 = -A/3 \quad (5.9)
the constraints (5.6) and (5.8) transmute to Bianchi identities with respective (distinct) gauge invariances

\[
\delta \psi_{\mu\nu} = D_{(\mu} \varepsilon_{\nu)} + \frac{1}{2} \sqrt{-\Lambda} \gamma_{(\mu} \varepsilon_{\nu)}, \quad \delta \chi = 0, \quad (5.10)
\]

\[
\delta \psi_{\mu\nu} = D_{(\mu} D_{\nu)} \varepsilon + \frac{5 \Lambda}{16} g_{\mu\nu} \varepsilon, \quad \delta \chi = -\frac{1}{8\alpha} \sqrt{15\alpha \Lambda} (\gamma \cdot D + \sqrt{-3\Lambda}) \varepsilon. \quad (5.11)
\]

The vector-spinor Bianchi identity (5.5) at \( m^2 = -4\Lambda/3 \) implies strict masslessness (propagating helicities \( \pm 5/2 \)) since its invariance removes helicities \( \pm 3/2, \pm 1/2 \). Notice also that \( \mu = 0 \) on the strictly massless line so, as claimed above, the spinor auxiliary \( \chi \) decouples there. The novel spinor Bianchi identity (5.8) at \( m^2 = -\Lambda/3 \) and invariance (5.11) removes helicities \( \pm 1/2 \) leaving a partially massless theory of helicities \( \pm 5/2, \pm 3/2 \).

Once again, the coefficients \( (m^2 + 4\Lambda/3) \) and \( (m^2 + \Lambda/3) \) appearing in the constraints (5.5) and (5.8) control the positivity of equal time anticommutators. Therefore, since the gauge lines all lie in AdS, the \( (m^2, \Lambda) \)-plane is divided into 3 regions; only the one including Minkowski space \( m^2 > -4\Lambda/3 \) is unitary. Although the strictly massless, AdS, \( m^2 = -4\Lambda/3 \) theory is unitary, the partially massless one is not, as it fails the line ordering requirement: Starting from the unitary Minkowski region where all norms are positive, one would like first to traverse the line \( m^2 = -\Lambda/3 \), but that is only possible in dS with negative \( m^2 \) (imaginary values of \( m \) violate hermiticity of the action and unitary evolution). Crossing the AdS strictly massless line \( m^2 = -4\Lambda/3 \) first flips the norm of both lower helicities \( \pm 3/2, \pm 1/2 \) so the partially massless AdS theory cannot be unitary. Ironically, were negative values of \( m^2 \) not prohibited, we could traverse the lines in the correct order in dS. This observation lends weight to the speculation that the unitarity difficulties of partially massless theories are peculiar to half integer spins. Indeed, in the next Section we exhibit the bosonic \( s = 3 \) example, which enjoys two partially massless unitary dS lines.

**Spin 3**

Spin 3 is the first example of a system with two new Bianchi identities over and above the usual one at \( m^2 = 0 \). The action and field equations are

\[
\mathcal{L} = \frac{1}{2} \sqrt{-g} \phi^{\mu\nu\rho} g_{\mu\nu\rho} - \frac{3}{8} \sqrt{-g} \chi G_5, \quad (5.12)
\]
\[
\mathcal{G}_{\mu\nu\rho} = (\Delta^{(3)} - m^2 + 16A/3) \phi_{\mu\nu\rho} - 3D_{(\mu}D.\phi_{\nu\rho)} + 3D_{(\mu}D_{\nu}\phi_{\rho)\sigma}^\sigma \\
- 3g_{(\mu\nu} \left( (\Delta^{(1)} - m^2 + 11A/3) \phi_{\rho)\sigma}^\sigma - D.D.\phi_{\rho)\sigma}^\sigma + \frac{1}{2} D_{\rho)}D.\phi_{\sigma}^\sigma \right) \\
+ \frac{3m}{4} g_{(\mu\nu} D_{\rho)} \chi = 0 ,
\]
(5.13)

\[
\mathcal{G}_5 = \frac{3}{2} (\Delta^{(0)} - 4m^2 + 8A) \chi + m D.\phi_{\sigma}^\sigma = 0 .
\]
(5.14)

The field \( \phi_{\mu\nu\rho} \) is a symmetric 3-tensor and the auxiliary field \( \chi \) decouples at \( m = 0 \) (the strictly massless theory). Fixing the ordering of covariant derivatives as shown and requiring constraints to remove all but the physical \( 7 = 2s + 1 \) degrees of freedom, uniquely specifies all terms with an explicit \( \Lambda \)-dependence. Indeed, we find the following constraints

\[
\mathcal{B}_{\{\nu\rho\}} \equiv D.\mathcal{G}_{\{\nu\rho\}} = -\frac{1}{2} m \left( D_{(\nu}D_{\rho)} \chi + 2m D_{(\nu} \phi_{\rho)\sigma}^\sigma - 4m D_{(\nu} \phi_{\rho)\sigma}^\sigma \right) ,
\]
(5.15)

\[
\mathcal{B}_{\rho} \equiv D.\mathcal{B}_{\rho} - \frac{m}{4} D_{\rho} \mathcal{G}_5 + \frac{m^4}{4} \mathcal{G}_{\rho\sigma}^\sigma = \frac{5}{8} m (3m^2 - 4A) (D_{\rho} \chi + \frac{2}{3} m \phi_{\rho\sigma}^\sigma) ,
\]
(5.16)

\[
\mathcal{B} \equiv D.\mathcal{B} - \frac{5}{12} m (3m^2 - 4A) \mathcal{G}_5 = \frac{5}{2} m (3m^2 - 4A) (m^2 - 2A) \chi .
\]
(5.17)

The explicit tensorial structures on the right hand sides of (5.15)-(5.17) exhibit the successive splitting of the prefactors to \( m \), namely \( (3m^2 - 4A) \) and \( (m^2 - 2A) \), thanks to the additional parameter \( \Lambda \). Therefore, in addition to the usual massless theory at \( m = 0 \) there are new gauge invariant systems at \( m^2 = 4A/3 \) and \( m^2 = 2A \), since whenever these prefactors vanish, the corresponding constraints in (5.15)-(5.17) become Bianchi identities with accompanying, respective, gauge invariances

\[
\delta \phi_{\mu\nu\rho} = D_{(\mu} \xi_{(\nu\rho)}), \quad \delta \chi = 0 ;
\]
(5.18)

\[
\delta \phi_{\mu\nu\rho} = D_{(\mu}D_{(\nu} \xi_{\rho)}) + \frac{A}{3} g_{(\mu\nu} \xi_{\rho)} , \quad \delta \chi = -\frac{2}{3} \sqrt{\frac{A}{3}} D.\xi ;
\]
(5.19)

\[
\delta \phi_{\mu\nu\rho} = D_{(\mu}D_{(\nu}D_{\rho)}) \xi + \frac{A}{2} g_{(\mu\nu} D_{\rho)} \xi , \quad \delta \chi = -\frac{2}{3} \sqrt{\frac{A}{2}} (D^2 + \frac{10A}{3}) \xi .
\]
(5.20)

The new gauge invariant lines bound regions in the \( (m^2, \Lambda) \) half-plane whose unitarity properties are determined by the signs of the prefactors \( (3m^2 - 4A) \)
and $(m^2 - 2\Lambda)$. To analyze these new properties, decompose the $7 = 2s + 1$ physical DoF into helicities $(\pm 3, \pm 2, \pm 1, 0)$. We find the following “phase” structure of the $(m^2, \Lambda)$ half-plane

- **$m^2 > 2\Lambda > 0$**: This region includes Minkowski space and is clearly unitary. All helicities $(\pm 3, \pm 2, \pm 1, 0)$, propagate with positive norm.

- **$m^2 = 2\Lambda$**: A partially massless theory appears since the scalar constraint $\mathcal{B} = 0$ is now a Bianchi identity whose associated gauge invariance removes the scalar helicity 0 excitation. The remaining 6 DoF, $(\pm 3, \pm 2, \pm 1)$, propagate with positive norm since they are unaffected by the scalar gauge invariance.

- **$4\Lambda/3 < m^2 < 2\Lambda$**: Although all 7 DoF are now again propagating, the scalar helicity 0 mode reemerges from the gauge boundary $m^2 = 2\Lambda$ with negative norm (since the factor $(3m^2 - 4\Lambda)(m^2 - 2\Lambda)$ appears in canonical commutators as a negative denominator). This is a unitarily forbidden region.

- **$m^2 = 4\Lambda/3$**: This partially massless theory has Bianchi identities $\mathcal{B} = 0 = \mathcal{B}_\rho$ whose gauge invariances excise the helicities $(\pm 1, 0)$. The remaining 4 DoF, helicities $(\pm 3, \pm 2)$ propagate with positive norm.

- **$0 < m^2 < 4\Lambda/3$**: Again all 7 DoF propagate but now the scalar helicity has again positive norm since its denominator $(3m^2 - 4\Lambda)(m^2 - 2\Lambda)$ is again positive. However, the region is still unitarily forbidden because now the vector helicities $\pm 1$ suffer a negative denominator $(3m^2 - 4\Lambda)$.

- **$m^2 = 0$**: This is the unitary strictly massless model with tensor Bianchi identity $\mathcal{B}_{(\nu\rho)} = 0 = \mathcal{B}_\rho = \mathcal{B}$. Only the uppermost helicities $\pm 3$ remain. An added subtlety is the remnant decoupled auxiliary field $\chi$.

Notice how the uppermost helicity $\pm 3$ always emerges unscathed as a pair of positive norm states but, unlike Minkowski space, splittings into theories with intermediate lower helicities, rather than only the full complement of $2s + 1$ states of the massive theory, are possible. Furthermore, the ordering of gauge lines is the same as for the $s = 5/2$ example. But, unlike for $s = 5/2$, the lines can be traversed in the order required for unitarity of the partially massless dS theories without recourse to unphysical, negative, values of $m^2$. 

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The general pattern for $s > 3$ should be clear; although the complexity of auxiliary fields will make the details arduous to follow, the counting is not.

6 Null Propagation of Partially Massive Systems

Here we show that the partial massless (i.e., gauge invariant) truncated multiplets propagate on the null cone, and give a general spin formula for the $m^2 : A$ tunings that govern these systems.

Bosons

Our method is direct: We solve the helicity $\pm s$ field equations for all higher spins in the dS background (3.15). When they are present in the spectrum, all lower helicities propagate in exactly the same manner, so we need not treat them separately. In our frame spatial slices are flat, which allows the usual definition of helicity. Solutions to the field equations are of Bessel type, and take the form

$$\left(\text{slowly varying}\right) \times \exp(\imath \omega u + \imath \vec{k} \cdot \vec{x}), \quad \omega^2 = \vec{k}^2,$$

whenever the index $\nu$ of the Bessel function is half integer. Here

$$u \equiv - \frac{f^{-1}(t)}{M}$$

(6.2)

is the conformal time coordinate in terms of which

$$ds^2 = \frac{1}{M^2 u^2} \left( - du^2 + d\vec{x}^2 \right).$$

(6.3)

Therefore we can read off the theories propagating on the null cone directly from the index $\nu$. We will explicitly find null propagation for all the $s \leq 3$ partially massless theories discussed previously.

The onshell conditions for a massive spin $s$ field in (A)dS are

$$\left(D^2 - m^2 - (2 + 2s - s^2) M^2\right) \phi_{\mu_1...\mu_s} = 0, \quad D_\rho \phi_{\rho\mu_2...\mu_s} = 0 = \phi_{\rho\mu_3...\mu_s}.$$ (6.4)

The parameter $m^2$ has been chosen so that $m^2 = 0$ corresponds to the strictly massless (i.e., helicity $\pm s$ only) theory. [For $s = 0$, equation (6.4) describes a conformally improved scalar.]
The traceless-transverse part of a spatial tensor is denoted by the superscript $tt$. We project out helicity $\pm s$ by computing only the $tt$ part of (6.4), which in the frame (3.15) reads,

\[
\left\{-\frac{d^2}{dt^2} + (2s - 3) M \frac{d}{dt} + f^{-2} \vec{\partial}^2 - m^2 + 2(s - 1) M^2 \right\} \phi^{tt}_{i_1...i_s} = 0 . \tag{6.5}
\]

Fourier transforming $\vec{\partial} \to i\vec{k}$, changing coordinates

\[
z \equiv -\frac{kf^{-1}(t)}{M} = ku , \quad (k \equiv |\vec{k}|) \tag{6.6}
\]

and the field redefinition (suppressing indices)

\[
\phi^{tt} \equiv z^{3/2-s}q , \tag{6.7}
\]

yield Bessel’s equation

\[
\frac{d^2q}{dz^2} + \frac{1}{z} \frac{dq}{dz} + \left(1 - \frac{\nu^2}{z^2}\right)q = 0 , \tag{6.8}
\]

with index

\[
\nu^2 = \frac{1}{4} + s(s - 1) - \frac{m^2}{M^2} . \tag{6.9}
\]

We may now read off the null propagating theories.
Examples:

1. **Conformal Scalar**: At $s = 0 = m$ we obtain $\nu = 1/2$ and $q(z) = z^{-1/2} \exp(iz)$ which implies a solution of the form (6.1). The value $\nu = 1/2$ also characterizes all the higher spin “conformal” theories.

2. **Maxwell**: In $d = 4$ the $s = 1, m = 0$ vector theory is conformal and here $\nu = 1/2$.

3. **Spin 2**: Spin 2 can be either strictly massless at $m = 0$ or partially massless when $m^2 = 2M^2$. The latter model, with its accompanying scalar gauge invariance, takes the conformal value $\nu = 1/2$. Of course, the $m^2 = 0$ linearized cosmological Einstein theory also propagates on the cone, but this is achieved by the solution $\nu = 3/2$ for which

\[ q(z) = (z + i) z^{-3/2} \exp(iz). \]

4. **Spin 3**: Here there are 3 possibilities; the strictly massless theory at $m = 0$ for which $\nu = 5/2$ \((\Rightarrow q(z) = (z^2 + 3iz - 3) z^{-5/2} \exp(iz))\), a partially massless one with helicities ($\pm 3, \pm 2$) at $m^2 = 4M^2$ with $\nu = 3/2$ and finally, a theory with a scalar gauge invariance and helicities ($\pm 3, \pm 2, \pm 1$) with $m^2 = 6M^2$ and the conformal value $\nu = 1/2$. Clearly all these theories have null propagation.

Noting that the value of $\nu$ can be associated with the type of gauge invariance (for example, the conformal value $\nu = 1/2$ always belongs to the scalar invariance) makes it likely that all partially massless higher spin bosons propagate on the null cone. The spin $s$ theory with helicities ($\pm n, \ldots, 0$) removed appears when

\[ m^2 = M^2 \left( s(s - 1) - n(n + 1) \right), \quad (6.10) \]

and has Bessel index $\nu = n + 1/2$. All these gauge theories are unitary, and (6.11) has indeed recently proved in [17].

**Fermions**

Partially massless fermionic theories are found in AdS. However, we continue to work in dS because of the simplicity of the metric (3.15). The results for the partially massless lines depend on $m^2$ and $A \equiv 3M^2$ only and continue to AdS. Just as for the lowest “multiline” $s = 5/2$ case, only the strictly massless helicity $\pm s$ gauge theory is unitary in AdS.
A massive spin $s \equiv \sigma + 1/2$ fermionic field satisfies the onshell conditions

$$ (\gamma \cdot D + m) \psi_{\mu_1...\mu_\sigma} = 0 \quad D_\nu \psi_{\mu_2...\mu_\sigma} = 0 = \gamma_\nu \psi_{\mu_2...\mu_\sigma}. \quad (6.11) $$

We choose the local Lorentz gauge

$$ e_0^0 = 1, \quad e_\mu^\lambda = f(t) \delta_\mu^\lambda, \quad (6.12) $$

where underlined indices are flattened. The Dirac equation for the (spatially transverse, gamma-traceless “tt”) helicities $\pm s$ reads

$$ \left( \frac{d}{dt} + (2 - s) M - f^{-1} \gamma_0^0 \partial_0 - \gamma_0^0 m \right) \psi_{tt}^{i_1...i_\sigma} = 0. \quad (6.13) $$

In the usual large/small component basis (suppressing indices again)

$$ \gamma^0 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \quad \gamma^j = \begin{pmatrix} 0 & i \sigma^j \\ -i \sigma^j & 0 \end{pmatrix}, \quad \psi_{tt} = \begin{pmatrix} \chi \\ \phi \end{pmatrix}, \quad (6.14) $$

we can eliminate the small component $\phi$ and obtain the second order equation for $\chi$

$$ \left( -\frac{d^2}{dt^2} + (2s - 5) M \frac{d}{dt} + f^{-2} \vec{\partial}^2 - (s - 2)(s - 3) M^2 - imM - m^2 \right) \chi = 0. \quad (6.15) $$

The coordinate transformation (6.6) and field redefinition

$$ \chi \equiv z^{5/2 - s} q, \quad (6.16) $$

yield Bessel’s equation (6.8) with index

$$ \nu^2 = \frac{1}{4} - \frac{im}{M} - \frac{m^2}{M^2} = \left(\frac{1}{2} - \frac{im}{M}\right)^2. \quad (6.17) $$

Note that $m$ itself will be imaginary for the partially massless lines when they are in dS, so the appearance of an explicit $im$ here is appropriate.

**Examples:**

1. **Spin 1/2:** The $m = 0$ spin 1/2 theory is well known to be Weyl invariant, and indeed we find $\nu = 1/2$, the conformal value.
2. **Spin 3/2**: As follows from linearizing cosmological supergravity, spin 3/2 is strictly massless at \( m^2 = -M^2 \). The choice of branch \( m = iM \), justified by the results, yields \( \nu = 3/2 \). This is the same value we found above for its strictly massless spin 2 superpartners.

3. **Spin 5/2**: The strictly massless theory is at \( m^2 = -4M^2 \); the choice \( m = 2iM \) yields \( \nu = 5/2 \) and null propagation. The model with a spinor gauge invariance at \( m^2 = -M^2 \) has \( \nu = 3/2 \), just as for linearized cosmological supergravity.

All the above fermionic partially massless theories propagate on the null cone. It is then reasonable here also to assume that all partially massless fermions propagate on the null cone. The spin \( s \) theory with helicities \( \pm(n + 1/2), \ldots, \pm1/2 \) removed appears at

\[
m^2 = -M^2 (n + 1)^2
\]

with Bessel index \( \nu = 3/2 + n \). This has also been established in [17]. The Hilbert space of these theories is unitary in dS, where their actions are not hermitean. In particular, the strictly massless theory with \( n = s - 3/2 \) occurs at \( m^2 = -M^2 (s - 1/2)^2 \) for \( s > 1/2 \): Its Hilbert space is unitary for any \( \Lambda \), but hermiticity of its action is lost in dS. The conformal value \( \nu = 1/2 \) is only attained by the \( m = 0 \) spin 1/2 theory.

7 **Massless (Dis)continuities in the Newtonian Limit**

As promised in the Introduction, we outline (following [5]) the effect of the vDVZ discontinuity in the Newtonian limit in the source-source interactions induced by massive (as \( m \to 0 \)) vs. strictly massless tensor fields.

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\( ^6 \)The argument of [10] shows that spin 3/2 cannot take the conformal value \( \nu = 1/2 \). This does not imply that strictly massless spin 3/2 propagates off-cone, since null propagation is achieved there by \( \nu = 3/2 \). This null propagation was already proven in the second paper of [9].
The presence of a second dimensional constant provides alternative paths, and outcomes, for the limit. In particular, the spin 2 case with, say, two (background covariantly conserved) sources \((T_{\mu\nu}, t_{\mu
u})\) leads to the Born exchange interaction,

\[
I = G_{\Lambda,m} \int d^4x \left\{ T_{\mu\nu} D t^{\mu\nu} - \frac{m^2 - \Lambda}{3m^2 - 2\Lambda} T_\mu T^\mu D t_\nu \right\},
\]

(7.1)

where \(D\) is the usual massive \((A)dS\) scalar propagator whose \(m = 0\) and \(\Lambda = 0\) limits are smooth and \(G_{\Lambda,m}\) is the gravitational constant for the particular \((\Lambda, m)\) model. The old discontinuity\(^7\) at \(\Lambda = 0\) led to a relative coefficient \(1/3\) in the second term versus \(1/2\) if \(m^2\) is identically zero. When \(\Lambda \neq 0\), there is an infinite number of limits available; in particular \(m^2 \to 0\) followed by \(\Lambda \to 0\) reproduces the \(1/2\) factor.

Before considering the details, we argue physically that the Newtonian limit of (7.1) must be immune to discontinuities because, by its very definition, it is only valid for \(c \to \infty\). Thus only \((T_0^0 = \rho, t_0^0 = \sigma)\) fail to vanish: we have an effective scalar theory with only slow sources and one “experiment” to fit with one coupling constant. There is no “light-bending” to fit, as there is no light.

If \(\Lambda = 0\), the interaction is

\[
I_{0, m} \sim \frac{2}{3} G_{0, m} \int d^3x \rho Y \sigma,
\]

(7.2)

where \(Y\) is the Yukawa potential and \(2G_{0, m}/3\) is tuned to the observed Newtonian constant. Since the Yukawa potential reduces continuously to \(1/r\), the \(m \to 0\) process is perfectly smooth.

If, on the other hand, \(\Lambda \neq 0\), the effective interaction becomes

\[
I_{\Lambda, m} \sim G_{\Lambda, m} (1 - \frac{m^2 - \Lambda}{3m^2 - 2\Lambda}) \int d^3x \rho Y_\Lambda \sigma,
\]

(7.3)

\(^7\)The effect of \(1/3\) versus \(1/2\) was a finite discrepancy between predictions for experiments involving only slow \((t_{\mu\nu} \to t_{00}\) only) and those involving light-like (e.g. \(t_{\mu\nu} = 0\)) sources. For, and only for, the value \(1/2\) could both light bending and Newtonian gravity agree with observation since the coupling constant \(G_{\Lambda, m}\) is used up to fix the latter’s strength.
where $Y_\Lambda$ is the generalized static Yukawa potential when $\Lambda \neq 0$. Thus “Newton’s constant” is

$$G_N = G_{\Lambda,m} \frac{2m^2 - \Lambda}{3m^2 - 2\Lambda}. \quad (7.4)$$

This $(m^2, \Lambda)$ dependence of $G_N$ would seem to involve some dangerous ranges and points. However, in the original Lorentz invariant domain whose limit this is, all models with $0 < 3m^2 < 2\Lambda$ are, as we saw earlier, non-unitary and so unphysical. This excludes the region where the fraction in (7.4) would turn negative, as well as the point $2m^2 = \Lambda$ where the numerator vanishes.

The $3m^2 = 2\Lambda$ model \textsuperscript{10} is unitary but has a gauge invariance that requires its conserved sources to be traceless as well, so it has no Newtonian limit at all. The physical region relevant to (7.4) thus consists of the usual gauge point $m^2 = 0$, together with that part of the $(m^2, \Lambda)$ plane for which $m^2 > 2\Lambda/3$, including of course $AdS$ space where $\Lambda < 0$. Any limit of $(m^2, \Lambda) \rightarrow 0$ in this region is perfectly smooth, with a well-defined positive $G_N$.

\section{8 A Cosmological Speculation}

We conclude with a cosmological consequence of having infinite towers of higher spin systems. Previous attempts to render $\Lambda$ small or vanishing have been of two types: (i) Those based on (unbroken) symmetries, such as the cancellation of supersymmetric zero point energies \textsuperscript{13} or the necessary absence of a cosmological terms in $d = 11$ supergravity \textsuperscript{19}. (ii) A dynamical solution based on quantum gravity loop corrections driving $\Lambda$ to zero \textsuperscript{20}. Our idea is rather different: it depends only on the kinematics of a tower of free fields. Such a leap of faith should not be foreign to string theorists, since massive string states couple an infinite tower of higher spins.

The argument is that the region of the $(m^2, \Lambda)$ plane where the entire tower has unitary content only is squeezed onto the $\Lambda = 0$ axis: The unitary

\footnote{We emphasize that, at the Newtonian level, this vanishing is a simple case of cancellation between Newtonian attraction and the non-unitary helicity zero ghost’s repulsive contribution. It corresponds to the covariant interaction $(T_\mu^{\nu}T_\nu^{\mu} - T_\mu^{\mu}T_\nu^{\nu})$ in which there is manifestly no $T_{00}^{\mu\nu}$ term.}
region for massive higher spins in the \((m^2, \Lambda)\) plane is bounded below in AdS by the (highest) strictly massless fermionic line of (6.19)

\[
m^2 = \frac{A (s - 1/2)^2}{3},
\]

and above by the (lowest) partially massless bosonic gauge line (the one excluding helicity 0), of (6.11)

\[
m^2 = \frac{A s(s - 1)}{3}.
\]

Therefore the unitarily allowed region is pinched around \(\Lambda = 0\) for large values of \(s\) (as is manifest in our Figure). Of course the robustness of this mechanism will be challenged by the usual array of difficulties introduced by interactions.

9 Conclusions

I have discussed in some detail the kinematical effects of the simplest nontrivial – constant curvature – gravitational backgrounds on matter. The initial surprise that having \(\Lambda \neq 0\) can so greatly alter the most elementary and fundamental properties of free fields, such as gauge invariance, masslessness, and helicity content. Upon further reflection, one might rather wonder that any close correspondence between the two worlds of flat and constant curvatures should remain at all, given for example the transmutation of Poincaré to homogeneous (A)dS algebras. Indeed, there are bound to be more such surprises, and of course global problems are guaranteed to exist! Still, this complex of ideas will certainly be of relevance to such realms as string theory and its slope expansion, and perhaps even to the cosmological problem’s resolution.

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