Exploring the $\Upsilon(4S, 5S, 6S) \to h_b(1P)\eta$ hidden-bottom hadronic transitions

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Recently, Belle Collaboration has reported the measurement of the spin-flipping transition $\Upsilon(4S) \to h_b(1P)\eta$ with an unexpectedly large branching ratio: $B(\Upsilon(4S) \to h_b(1P)\eta) = (2.18 \pm 0.11 \pm 0.18) \times 10^{-3}$. Such a large branching fraction contradicts with the anticipated suppression for the spin flip. In this work, we examine the effects induced by intermediate bottomed meson loops and point out that these effects are significantly important. Using the effective Lagrangian approach (ELA), we find the experimental data on $\Upsilon(4S) \to h_b(1P)\eta$ can be accommodated with the reasonable inputs. We then explore the decays $\Upsilon(5S, 6S) \to h_b(1P)\eta$ and find that these two channels also have sizable branching fractions. We also calculate these processes in the framework of nonrelativistic effective field theory (NREFT). For the decays $\Upsilon(4S) \to h_b(1P)\eta$, the NREFT results are at the same order of magnitude but smaller than the ELA results by a factor of 2 to 5. For the decays $\Upsilon(5S, 6S) \to h_b(1P)\eta$ the NREFT results are smaller than the ELA results by approximately one order of magnitude. We suggest future experiment Belle-II to search for the $\Upsilon(5S, 6S) \to h_b(1P)\eta$ decays which will be helpful to understand the transition mechanism.

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I. INTRODUCTION

In recent years bottomonium transitions with an $\eta$ meson or two pions in the final state have been extensively studied on the experimental side [1–7]. In 2008, the BaBar collaboration first observed an enhancement for the transition $\Upsilon(4S) \to \Upsilon(1S)\eta$ compared to the dipion transition [1]. In 2011, two charged bottomoniumlike structures $Z_b^\pm(10610)$ and $Z_b^\mp(10650)$ were observed by the Belle Collaboration in the $\pi^\pm\Upsilon(nS)$ and $\pi^\pm h_b$ invariant mass spectra of $\Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^-$ and $h_b(mP)\pi^+\pi^-$ decays [2, 3]. In 2015, the Belle Collaboration has measured for the first time the branching fraction $B(\Upsilon(4S) \to h_b(1P)\eta) = (2.18 \pm 0.11 \pm 0.18) \times 10^{-3}$ [7]. This value is anomalously large since one would expect a power suppression for the transitions with the spin flip [8, 9].

A low-lying heavy quarkonium system is expected to be compact and nonrelativistic, so the QCD multipole expansion (QCDME) [8–10] can be applied to explore the hadronic transitions. For the excited states that lie above open flavor thresholds, QCDME might be problematic due

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to the coupled channel effects. Several possible new mechanisms have been proposed in order to explain the anomalous decay widths of $\Upsilon(4S) \rightarrow h_b(1P)\eta$. For instance a nonrelativistic effective field theory (NREFT) is used in Ref. [11], where the branching ratio can reach the order of $10^{-3}$. It has been noticed for a long time that the intermediate meson loop (IML) is one prominent non-perturbative mechanism in hadronic transitions [12–14]. In recent years, this mechanism has been successfully applied to study the production and decays of ordinary and exotic states [15–45], and a global agreement with experimental data is found. This approach has also been extensively used to study the $\Upsilon(4S, 5S, 6S)$ hidden bottomonium decays [46–52]. In this work, we will investigate the process $\Upsilon(4S, 5S, 6S) \rightarrow h_b(1P)\eta$ via IML model. As we will show in the following the experimental data on $\Upsilon(4S) \rightarrow h_b(1P)\eta$ can be accommodated in this approach. We then predict the branching ratios of the decays $\Upsilon(5S, 6S) \rightarrow h_b(1P)\eta$ and find that they are measurable in future.

The rest of this paper is organized as follows. We will first introduce the effective Lagrangian for our calculation in Sec. II and calculate the IML contributions to decay widths. Then, we will present our numerical results in Sec. III. A brief summary will be given in Sec. IV.

II. RADIATIVE DECAYS

![Hadron-level diagrams](image)

FIG. 1: The hadron-level diagrams for $\Upsilon(4S, 5S, 6S) \rightarrow h_b(1P)\eta$ via charged intermediate bottomed meson loops. Similar diagrams for neutral and strange intermediate bottomed meson loops.

Generally speaking, all the possible intermediate meson loops should be included in the calculation. In reality, we only pick up the leading order contributions as a reasonable approximation due to the breakdown of the local quark-hadron duality [12, 53]. In this work, we consider the IML illustrated in Fig. II as the leading order contributions of $\Upsilon(4S, 5S, 6S) \rightarrow h_b(1P)\eta$. To calculate these diagrams, we need the effective Lagrangians to derive the couplings. Based on the heavy quark symmetry and chiral symmetry [54, 55], the Lagrangian for the S- and P-wave bottomonia at leading order is given as

$$L_1 = i g_1 T_r[P^{\mu}_{b\bar{b}} H_{2,2} \gamma_{\mu} H_{1,1}] + + \text{H.c.},$$

$$L_2 = g_2 T_r[R_{b\bar{b}} H_{2,2} \partial_{\mu} \gamma^{\mu} H_{1,1}] + \text{H.c.}.$$
The S-wave bottomonium doublet and P-wave bottomonium multiplet states are expressed as

$$ R_{bb} = \frac{1 + \dot{\theta}}{2} (\Upsilon^\mu \gamma_\mu - \eta_b \gamma_5) \frac{1 - \dot{\theta}}{2}, \\

P^\mu_{bb} = \frac{1 + \dot{\theta}}{2} \left( \lambda_{bb}^\mu \gamma_\alpha + \frac{1}{\sqrt{2}} \epsilon^{\mu \nu \alpha \beta} \gamma_\alpha \gamma_\beta \chi_{b \nu} + \frac{1}{\sqrt{3}} (\gamma^\mu - v^\mu) \chi_{b 0} + h_b^\mu \gamma_5 \right) \frac{1 - \dot{\theta}}{2}, $$

where $\Upsilon$ and $\eta_b$ are the S-wave bottomonium fields. The $h_b$ and $\chi_{b J}$ ($J=0,1,2$) are the P-wave bottomonium fields. The $v^\mu$ is the 4-velocity of these bottomonium states.

The bottomed and anti-bottomed meson triplet read

$$ H_{1i} = \frac{1 + \dot{\theta}}{2} \left[ B_i^\mu \gamma_\mu - B_i \gamma_5 \right], \\

H_{2i} = [\bar{B}_i^\mu \gamma_\mu - \bar{B}_i \gamma_5] \frac{1 - \dot{\theta}}{2}, \\

\bar{H}_{1i,2i} = \gamma^0 H_{1i,2i}^\dagger \gamma^0, $$

where $B$ and $B^*$ denote the pseudoscalar and vector bottomed meson fields, respectively, i.e. $B^{(*)} = (B(+s), B^0(+s), B_s^0(+s))$. $v^\mu$ is the 4-velocity of the bottomed mesons. $\epsilon_{\mu\nu\alpha\beta}$ is the antisymmetric Levi-Civita tensor and $\epsilon_{0123} = +1$.

Consequently, the relevant effective Lagrangian for S-wave $T(nS)$ and P-wave $h_b(1P)$ read

$$ L_{T(nS)B^{(*)}B^{(*)}} = i g_{TBS} \dot{T}^{\mu} \bar{B}^\mu - \bar{B} \partial^\mu \dot{B} - g_{TBB^*} \epsilon_{\mu\nu\alpha\beta} \partial^\mu \dot{B}^{*\nu} \partial^\alpha \bar{B}^{*\beta} \\
+ \frac{i g_{TBB^*} (\Upsilon^\mu \partial_\mu B^{*\nu} \bar{B}^{*\nu} - B^{*\mu} \partial_\mu \bar{B}^{*\nu} \bar{B}^{*\nu} + \partial_\mu \Upsilon \partial_\mu B^{*\nu} \bar{B}^{*\nu} - \Upsilon \partial_\mu B^{*\nu} \partial_\mu \bar{B}^{*\nu} \bar{B}^{*\nu}) B^{*\mu} + B^{*\mu} (\Upsilon^{\nu} \partial_\nu \bar{B}^{*\nu} - \partial_\nu \Upsilon \partial_\nu \bar{B}^{*\nu} \bar{B}^{*\nu}), $$

$$ L_{h_bB^{(*)}B^{(*)}} = g_{h_bB^*B^*} \mu (\bar{B} B^*_\mu + B^*_\mu B) + i g_{h_bB^*B^*} \epsilon_{\mu\nu\alpha\beta} \partial^\mu h_b^\nu B^{*\alpha} B^{*\beta}, $$

where the coupling constants will be determined later.

The effective Lagrangian for a light pseudoscalar meson coupled to bottomed mesons pair can be constructed using the heavy quark symmetry and chiral symmetry [54–56]

$$ L_{B^{(*)}B^{(*)}P} = -i g_{B^*B^*P} \left( B^i \partial^\mu P_{ij} B^j_{\mu} + B^i_{\mu} \partial^\mu P_{ij} B^j_{\mu} \right) + \frac{1}{2} g_{B^*B^*P} \epsilon_{\mu\nu\alpha\beta} B^i_{\mu} \partial^\nu P_{ij} B^j_{\nu} B^{*\beta}, $$

where $P$ is a $3 \times 3$ matrix for the pseudoscalar octet. The physical states $\eta$ is the linear combinations of $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$ with the mixing scheme:

$$ |\eta\rangle = \cos \alpha_P |n\bar{n}\rangle - \sin \alpha_P |s\bar{s}\rangle. $$

The mixing angle is given as $\alpha_P \approx \theta_P + \arctan \sqrt{2}$, where the empirical value for the $\theta_P$ should be in the range $-24.6^\circ \sim -11.5^\circ$ [57]. In this work, we will take $\theta_P = -19.3^\circ$ [58].
With the above Lagrangians, we can derive the transition amplitudes for \( \Upsilon(nS)(p_1) \rightarrow [B^{(*)}(q_1)\bar{B}^{(*)}(q_3)]B^{(*)}(q_2) \rightarrow h_b(1P)(p_2)\eta(p_3) \) shown in Fig. 11.

\[
\mathcal{M}_{BB[B^*]} = \int \frac{d^4q_2}{(2\pi)^4} \left[ -2g_{TB\bar{T}\varepsilon_{12}}^\mu_q g_\eta^\rho \right] \left[ -g_B^\mu B P \varepsilon_{3\delta} \right] [g_{h_b B^* \varepsilon_{2\lambda}}] \frac{i(-g_{\rho \delta} + q_2^\mu q_2^\rho / m_2^2)}{q_2^\mu - m_2^2} \mathcal{F}(m_2, q_2^2),
\]

\[
\mathcal{M}_{BB^*[B^*]} = \int \frac{d^4q_2}{(2\pi)^4} \left[ -g_{TB\bar{T}\varepsilon_{12}}^\mu_q g_\eta^\rho \right] \left[ -g_B^\mu B P \varepsilon_{3\delta} \right] [g_{h_b B^* \varepsilon_{2\lambda}}] \frac{i(-g_{\rho \delta} + q_2^\mu q_2^\rho / m_2^2)}{q_2^\mu - m_2^2} \mathcal{F}(m_2, q_2^2),
\]

\[
\mathcal{M}_{B^*[B^*][B]} = \int \frac{d^4q_2}{(2\pi)^4} \left[ -g_{TB\bar{T}\varepsilon_{12}}^\mu_q g_\eta^\rho \right] \left[ -g_B^\mu B P \varepsilon_{3\delta} \right] [g_{h_b B^* \varepsilon_{2\lambda}}] \frac{i(-g_{\rho \delta} + q_2^\mu q_2^\rho / m_2^2)}{q_2^\mu - m_2^2} \mathcal{F}(m_2, q_2^2),
\]

\[
\mathcal{M}_{B^*[B^*][B]} = \int \frac{d^4q_2}{(2\pi)^4} \left[ -g_{TB\bar{T}\varepsilon_{12}}^\mu_q g_\eta^\rho \right] \left[ -g_B^\mu B P \varepsilon_{3\delta} \right] [g_{h_b B^* \varepsilon_{2\lambda}}] \frac{i(-g_{\rho \delta} + q_2^\mu q_2^\rho / m_2^2)}{q_2^\mu - m_2^2} \mathcal{F}(m_2, q_2^2),
\]

where \( p_1, p_2 \) and \( p_3 \) are the four momenta of the initial state \( \Upsilon(nS) \), final state \( h_b(1P) \) and \( \eta \), respectively. \( \varepsilon_1 \) and \( \varepsilon_2 \) are the polarization vector of \( \Upsilon(nS) \) and \( h_b(1P) \), respectively. \( q_1, q_3 \) and \( q_2 \) are the four momenta of the bottomed meson connecting \( \Upsilon(nS) \) and \( \eta \), the bottomed meson connecting \( \Upsilon(nS) \) and \( h_b(1P) \), and the exchanged bottomed meson, respectively.

In the triangle diagrams of Fig. 11 the exchanged bottomed mesons are off shell. To compensate the offshell effects and regularize the ultraviolet divergence \[59, 61\], we introduce the monopole form factor,

\[
\mathcal{F}(m_2, q_2^2) = \frac{\Lambda^2}{\Lambda^2 - q_2^2},
\]

where \( q_2^2 \) and \( m_2^2 \) are the momentum and mass of the exchanged bottomed meson, respectively. The parameter \( \Lambda \equiv m_2 + \alpha \Lambda_{QCD} \) and the QCD energy scale \( \Lambda_{QCD} = 220 \text{MeV} \). The dimensionless parameter \( \alpha \), which is usually of order 1, depends on the specific process.

### III. NUMERICAL RESULTS

With the experimental data on the decay width of \( \Upsilon(4S) \rightarrow B\bar{B} \)[57], the coupling constant \( g_{\Upsilon(4S)BB} \) is determined as \( g_{\Upsilon(4S)BB} = 24.2 \) which is comparable to the estimation in the vector
meson dominance model. Since the mass of $\Upsilon(4S)$ is only above the $B\bar{B}$ threshold, the coupling constants $g_{\Upsilon(4S)B^*\bar{B}^*}$ are determined as follows

\[ g_{\Upsilon(4S)B^*\bar{B}^*} = \frac{g_{\Upsilon(4S)BB^*}}{\sqrt{m_{B^*}m_B}} , \quad g_{\Upsilon(4S)B^*B^*} = g_{\Upsilon(4S)B^*B^*} \frac{m_{B^*}}{m_B} \sqrt{m_{B^*}m_B} . \]  

(14)

For the coupling constants between $\Upsilon(5S)$ and $B^{(*)}\bar{B}^{(*)}$, we use the experimental data on the decay width of $\Upsilon(5S) \to B^{(*)}\bar{B}^{(*)}$ [57]. The measured branching ratios and the corresponding coupling constants are given in Table I. One can see that the values determined from the $\Upsilon(5S)$ data in Table I are very small. This is partly due to the fact that as a high-excited $b\bar{b}$ state, the wave function of $\Upsilon(5S)$ has a complicated node structure, and the coupling constants will be small if the $p$ values of $B^{(*)}\bar{B}^{(*)}$ channels (1060 – 1270 MeV) are to those corresponding to the zeros in the amplitude [48]. Since there is no experimental information on $\Upsilon(6S) \to B^{(*)}\bar{B}^{(*)}$ [57], we choose the same values as the $\Upsilon(5S)$ ones.

The coupling constants between $h_b(1P)$ and $B^{(*)}\bar{B}^*$ in Eq. (9) are determined as

\[ g_{h_bBB^*} = -2g_1\sqrt{m_{h_b}m_Bm_{B^*}} , \quad g_{h_bB^*B^*} = 2g_1\frac{m_{B^*}}{\sqrt{m_{h_b}}} , \]  

(15)

where $g_1 = -\sqrt{m_{\chi_{b0}}/3}/f_{\chi_{b0}}$. $m_{\chi_{b0}}$ and $f_{\chi_{b0}}$ are the mass and decay constant of $\chi_{b0}(1P)$, respectively [62], i.e. $f_{\chi_{b0}} = 175 \pm 55$ MeV [63].

In the chiral and heavy quark limits, the couplings between bottomed meson pair and light pseudoscalar mesons have the following relationships [55],

\[ gb^{*}\bar{B}^*P = \frac{gb^{*}\bar{B}P}{\sqrt{m_Bm_{B^*}}} = \frac{2}{f_{\pi}}g , \]  

(16)

where $f_{\pi} = 132$ MeV is the pion decay constant, and $g = 0.59$ [64].

For the tree-level contributions to $\Upsilon(nS) \to h_b(1P)\eta$, the amplitude scales as the quark mass difference

\[ \mathcal{M}_{\text{tree}} \sim \delta \]  

(17)

with $\delta = m_s - (m_u + m_d)/2$. 

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| Final state $B(\%)$ Coupling | Final state $B(\%)$ Coupling | Final state $B(\%)$ Coupling |
|-------------------------------|-------------------------------|-------------------------------|
| $B\bar{B}$ | 5.5 | 1.76 | $BB^* + c.c.$ | 13.7 | 0.14 GeV$^{-1}$ | $B^*\bar{B}^*$ | 38.1 | 2.22 |
| $B_s\bar{B}_s$ | 0.5 | 0.96 | $B_sB_s^* + c.c.$ | 1.35 | 0.10 GeV$^{-1}$ | $B_s^*\bar{B}_s^*$ | 17.6 | 5.07 |
FIG. 2: (a) The $\alpha$ dependence of the branching ratios of $\Upsilon(4S) \to h_b(1P)\eta$. (b) The $\alpha$ dependence of the branching ratios of $\Upsilon(5S) \to h_b(1P)\eta$. (c) The $\alpha$ dependence of the branching ratios of $\Upsilon(6S) \to h_b(1P)\eta$.

For the bottom meson loop contributions in Fig. 1, the decay amplitude scales as follows,

$$M^{\text{loop}} \sim N \frac{q^2}{\bar{v}^3 M_B^2} \Delta,$$

where $N = 1/(2\sqrt{3}\pi v_b^4)$, $q$ is the final $\eta$ momentum, $\bar{v}$ is understood as the average velocity of the intermediate bottomed mesons. The meson mass difference $\Delta$ denotes the violation of the $SU(3)$ symmetry, which has similar size as $\delta$. $v_b$ denotes the bottom quark velocity inside the bottomonia and we take $v_b = \sqrt{0.1}$ here.

For $\Upsilon(4S) \to h_b(1P)\eta$ decay, the momentum of the emitted $\eta$ is $q \simeq 388$ MeV and the velocity $v$ is about $\sqrt{[2m_B - (m_{\Upsilon(4S)} + m_{h_b})]/m_B} \simeq 0.28$. As a result, the factor $N q^2/\bar{v}^3 M_B^2$ is about 2.17, which gives an enhancement compared with the tree-level contributions. For $\Upsilon(5S) \to h_b(1P)\eta$, the velocity $\bar{v} \simeq 0.23$ and $q = 750$ MeV, so the factor $N q^2/\bar{v}^3 M_B^2$ is about 15. For $\Upsilon(6S) \to h_b(1P)\eta$, the velocity $\bar{v} \simeq 0.19$ and $q = 930$ MeV, so the factor $N q^2/\bar{v}^3 M_B^2$ is about 37. According our power counting analysis, the transitions $\Upsilon(4S,5S,6S) \to h_b(1P)\eta$ are dominated by the meson loops.

In Fig. 2(a), we plot the branching ratios for $\Upsilon(4S) \to h_b(1P)\eta$ in terms of the cutoff parameter $\alpha$ with the monopole form factor. We also zoom into details of the figure with a narrow range $\alpha = 0.1 - 0.2$ in order to show the best fit of the $\alpha$ parameter. As shown in Fig. 2(a), the branching
FIG. 3: (a). The dependence of branching ratios of $\Upsilon(4S) \to h_b(1P)\eta$ on the $\eta-\eta'$ mixing angle with the cut-off parameter $\alpha = 0.15$ (solid line) and $0.25$ (dashed line), respectively. The calculated branching ratios in NREFT approach are presented with dotted line. (b). The branching ratios of $\Upsilon(5S) \to h_b(1P)\eta$ in terms of the $\eta-\eta'$ mixing angle with $\alpha = 0.15$ (solid line) and $0.25$ (dashed line), respectively. The calculated branching ratios in NREFT approach are presented with dotted line. (c). The branching ratios of $\Upsilon(6S) \to h_b(1P)\eta$ in terms of the $\eta-\eta'$ mixing angle with $\alpha = 0.15$ (solid line) and $0.25$ (dashed line), respectively. The calculated branching ratios in NREFT approach are presented with dotted line.

ratios are not drastically sensitive to the cutoff parameter $\alpha$. Our calculated branching ratios can reproduce the experimental data at about $\alpha = 0.12$. In Fig. 2(b) and (c), we plot the predicted branching ratios for $\Upsilon(5S) \to h_b(1P)\eta$ and $\Upsilon(6S) \to h_b(1P)\eta$ in terms of the cutoff parameter $\alpha$ with the monopole form factor. The behavior is similar to that of $\Upsilon(4S) \to h_b(1P)\eta$ in Fig. 2(a). The predicted branching ratios of $\Upsilon(5S) \to h_b(1P)\eta$ are about $10^{-3} \sim 10^{-2}$ with commonly accepted $\alpha$ range. For $\Upsilon(6S) \to h_b(1P)\eta$, the results are much small, which are about $10^{-4} \sim 10^{-2}$. At the same $\alpha$, the predicted branching ratios of $\Upsilon(5S) \to h_b(1P)\eta$ are about 1 order of magnitude smaller than that of $\Upsilon(4S) \to h_b(1P)\eta$. For $\Upsilon(6S) \to h_b(1P)\eta$, the predicted branching ratios are about 2 orders smaller than that of $\Upsilon(4S) \to h_b(1P)\eta$. We suggest future experiment BelleII to carry out the search for the spin-flipping transitions $\Upsilon(5S,6S) \to h_b(1P)\eta$ which will help us understanding the decay mechanism. Here, we should notice several uncertainties may influence our numerical results, such as the coupling constants and off-shell effects arising from the exchanged...
particles of the loops, and the cutoff parameter can also be different in decay channels.

In order to illustrate the impact of the $\eta$-$\eta'$ mixing angle, in Fig. 3 we present the branching ratios in terms of the $\eta$-$\eta'$ mixing angle with $\alpha = 0.15$ (solid line) and 0.25 (dashed line), respectively. As shown in this figure, when the $\eta$-$\eta'$ mixing angle $\alpha_P$ increases, the branching ratios of $\Upsilon(4S) \to h_b(1P)\eta$ decrease, while the branching ratios of $\Upsilon(5S,6S) \to h_b(1P)\eta$ increase. This behaviour suggests how the $\eta$-$\eta'$ mixing angle influences our calculated results to some extent.

As a comparison, in Fig. 3 we also give the results using the NREFT approach denoted as dotted lines. The NREFT approach provides a systematic tools to control the uncertainties [11, 34, 65]. From the figure, one can see that for the decays $\Upsilon(4S) \to h_b(1P)\eta$, the NREFT results are at the same order of magnitude but smaller than the ELA results by a factor of 2 to 5. These differences may give some sense of the theoretical uncertainties for the calculated rates and indicates the viability of our model to some extent. However, for transitions where the mass difference between the initial and final state becomes large, the NREFT may be not applicable. From Fig. 3 one can see that for the decays $\Upsilon(5S,6S) \to h_b(1P)\eta$ the NREFT results are smaller than the ELA results by approximately one order of magnitude. We suggest future experiment BelleII to carry out the search for this anomalous $\Upsilon(5S,6S) \to h_b(1P)\eta$ transitions which will help us testing this point.

IV. SUMMARY

Recent experiments on the $\Upsilon(4S) \to h_b(1P)\eta$ transition show anomalously large decay rates. This seems to contradict the naive expectation that hadronic transitions with spin flipping terms should be suppressed with respect those that do not have these terms. In this work, we have studied the spin-flipping transitions of $\Upsilon(4S,5S,6S) \to h_b(1P)\eta$ via intermediate bottomed meson loops in an effective Lagrangian approach. Our results have shown that the intermediate bottomed meson loops can play an important role in these process, especially when the initial states are close to the two particle thresholds. For $\Upsilon(4S) \to h_b(1P)\eta$, the experimental data can be reproduced in this approach with a commonly accepted range of values for the form factor cutoff parameter $\alpha$. We also predict the branching ratios of $\Upsilon(5S) \to h_b(1P)\eta$, which is about orders of $10^{-3} \sim 10^{-2}$. For $\Upsilon(6S) \to h_b(1P)\eta$, the results are much small, which are about $10^{-4} \sim 10^{-2}$. As a cross-check, we also calculated the branching ratios of the decays in the framework of NREFT. We suggest future experiment BelleII to carry out the search for the spin-flipping transitions $\Upsilon(5S,6S) \to h_b(1P)\eta$ which will help us understanding the decay mechanism.
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[1] B. Aubert et al. [BaBar Collaboration], Phys. Rev. D 78, 112002 (2008) [arXiv:0807.2014 [hep-ex]].
[2] I. Adachi [Belle Collaboration], arXiv:1105.4583 [hep-ex].
[3] A. Bondar et al. [Belle Collaboration], Phys. Rev. Lett. 108, 122001 (2012) [arXiv:1110.2251 [hep-ex]].
[4] K. F. Chen et al. [Belle Collaboration], Phys. Rev. Lett. 100, 112001 (2008) [arXiv:0710.2577 [hep-ex]].
[5] A. Sokolov et al. [Belle Collaboration], Phys. Rev. D 79, 051103 (2009) [arXiv:0901.1431 [hep-ex]].
[6] J. P. Lees et al. [BaBar Collaboration], Phys. Rev. D 84, 092003 (2011) [arXiv:1108.5874 [hep-ex]].
[7] U. Tamponi et al. [Belle Collaboration], Phys. Rev. Lett. 115, no. 14, 142001 (2015) [arXiv:1506.08914 [hep-ex]].
[8] Y. P. Kuang, Front. Phys. China 1, 19 (2006) [hep-ph/0601044].
[9] M. B. Voloshin, Prog. Part. Nucl. Phys. 61, 455 (2008) [arXiv:0711.4556 [hep-ph]].
[10] E. Eichten, S. Godfrey, H. Mahlke and J. L. Rosner, Rev. Mod. Phys. 80, 1161 (2008) [hep-ph/0701208].
[11] F. K. Guo, C. Hanhart and U. G. Meissner, Phys. Rev. Lett. 105, 162001 (2010) [arXiv:1007.4682 [hep-ph]].
[12] H. J. Lipkin, Nucl. Phys. B 291, 720 (1987).
[13] H. J. Lipkin and S. F. Tuan, Phys. Lett. B 206, 349 (1988).
[14] P. Moxhay, Phys. Rev. D 39, 3497 (1989).
[15] Q. Wang, C. Hanhart and Q. Zhao, Phys. Lett. B 725, no. 1-3, 106 (2013) [arXiv:1305.1997 [hep-ph]].
[16] M. Cleven, Q. Wang, F. -K. Guo, C. Hanhart, U. -G. Meißner and Q. Zhao, Phys. Rev. D 87, no. 7, 074006 (2013) [arXiv:1301.6461 [hep-ph]].
[17] X. -H. Liu and G. Li, Phys. Rev. D 88, 014013 (2013) [arXiv:1306.1384 [hep-ph]].
[18] F. -K. Guo, C. Hanhart, U. -G. Meißner, Q. Wang and Q. Zhao, Phys. Lett. B 725, 127 (2013) [arXiv:1306.3096 [hep-ph]].
[19] M. B. Voloshin, Phys. Rev. D 87, no. 7, 074011 (2013) [arXiv:1301.5068 [hep-ph]].
[20] M. B. Voloshin, Phys. Rev. D 84, 031502 (2011) [arXiv:1105.5829 [hep-ph]].
[21] G. Li, X. H. Liu and Q. Zhao, Eur. Phys. J. C 73, 2576 (2013).
[22] G. Li, X. h. Liu, Q. Wang and Q. Zhao, Phys. Rev. D 88, no. 1, 014010 (2013) [arXiv:1302.1745 [hep-ph]].
[23] G. Li and Q. Zhao, Phys. Rev. D 84, 074005 (2011) [arXiv:1107.2037 [hep-ph]].
[24] D. -Y. Chen and X. Liu, Phys. Rev. D 84, 094003 (2011) [arXiv:1106.3798 [hep-ph]].
[25] G. Li and X. -H. Liu, Phys. Rev. D 88, 094008 (2013) [arXiv:1307.2022 [hep-ph]].
[26] D. -Y. Chen, X. Liu and T. Matsuki, Phys. Rev. D 84, 074032 (2011) [arXiv:1108.4458 [hep-ph]].
[27] D. -Y. Chen, X. Liu and T. Matsuki, arXiv:1208.2411 [hep-ph].
[28] A. E. Bondar, A. Garmash, A. I. Milstein, R. Mizuk and M. B. Voloshin, Phys. Rev. D 84, 054010 (2011) [arXiv:1105.4473 [hep-ph]].
[29] G. Li and Z. Zhou, Phys. Rev. D 91, no. 3, 034020 (2015) [arXiv:1502.02936 [hep-ph]].
[30] G. Li, C. S. An, P. Y. Li, D. Liu, X. Zhang and Z. Zhou, Chin. Phys. C 39, no. 6, 063102 (2015) [arXiv:1412.3221 [hep-ph]].
[31] D.-Y. Chen, X. Liu and T. Matsuki, Phys. Rev. D 88, 014034 (2013) [arXiv:1306.2080 [hep-ph]].
[32] G. Li, F. L. Shao, C. W. Zhao and Q. Zhao, Phys. Rev. D 87, no. 3, 034020 (2013) [arXiv:1212.3784 [hep-ph]].
[33] G. Li and W. Wang, Phys. Lett. B 733, 100 (2014) [arXiv:1402.6463 [hep-ph]].
[34] F. K. Guo, C. Hanhart, G. Li, U. G. Meissner and Q. Zhao, Phys. Rev. D 83, 034013 (2011) [arXiv:1008.3632 [hep-ph]].
[35] Q. Wu, G. Li, F. Shao and R. Wang, Phys. Rev. D 94, no. 1, 014015 (2016).
[36] Q. Wu, G. Li, F. Shao, Q. Wang, R. Wang, Y. Zhang and Y. Zheng, Adv. High Energy Phys. 2016, 3729050 (2016) doi:10.1155/2016/3729050 [arXiv:1606.05118 [hep-ph]].
[37] X. H. Liu and G. Li, Eur. Phys. J. C 76, no. 8, 455 (2016) doi:10.1140/epjc/s10052-016-4308-1 [arXiv:1603.00708 [hep-ph]].
[38] G. Li, X. H. Liu and Z. Zhou, Phys. Rev. D 90, no. 5, 054006 (2014) doi:10.1103/PhysRevD.90.054006 [arXiv:1409.0754 [hep-ph]].
[39] Y. J. Zhang, G. Li and Q. Zhao, Chin. Phys. C 34, no. 9, 1181 (2010). doi:10.1088/1674-1137/34/9/006
[40] C. W. Zhao, G. Li, X. H. Liu and F. L. Shao, Eur. Phys. J. C 73, 2482 (2013). doi:10.1140/epjc/s10052-013-2482-y
[41] G. Li, Eur. Phys. J. C 73, no. 11, 2621 (2013) doi:10.1140/epjc/s10052-013-2621-5 [arXiv:1304.4458 [hep-ph]].
[42] Q. Wang, G. Li and Q. Zhao, Phys. Rev. D 85, 074015 (2012) doi:10.1103/PhysRevD.85.074015 [arXiv:1201.1681 [hep-ph]].
[43] Y. J. Zhang, G. Li and Q. Zhao, Phys. Rev. Lett. 102, 172001 (2009) [arXiv:0902.1300 [hep-ph]].
[44] G. Li and Q. Zhao, Phys. Lett. B 670, 55 (2008) doi:10.1016/j.physletb.2008.10.033 [arXiv:0709.4639 [hep-ph]].
[45] G. Li, Q. Zhao and C. H. Chang, J. Phys. G 35, 055002 (2008) doi:10.1088/0954-3899/35/5/055002 [hep-ph/0701020].
[46] C. Meng and K. T. Chao, Phys. Rev. D 77, 074003 (2008) doi:10.1103/PhysRevD.77.074003 [arXiv:0712.3595 [hep-ph]].
[47] C. Meng and K. T. Chao, Phys. Rev. D 78, 034022 (2008) doi:10.1103/PhysRevD.78.034022 [arXiv:0805.0143 [hep-ph]].
[48] C. Meng and K. T. Chao, Phys. Rev. D 78, 074001 (2008) doi:10.1103/PhysRevD.78.074001 [arXiv:0806.3259 [hep-ph]].
[49] H. W. Ke, X. Q. Li and X. Liu, Phys. Rev. D 82, 054030 (2010) doi:10.1103/PhysRevD.82.054030 [arXiv:1006.1437 [hep-ph]].
[50] D. Y. Chen, X. Liu and X. Q. Li, Eur. Phys. J. C 71, 1808 (2011) [arXiv:1109.1406 [hep-ph]].
[51] D. Y. Chen, X. Liu and T. Matsuki, Phys. Rev. D 90, no. 3, 034019 (2014) [arXiv:1406.6763 [hep-ph]].
[52] B. Wang, X. Liu and D. Y. Chen, Phys. Rev. D 94, no. 9, 094039 (2016) [arXiv:1611.02369 [hep-ph]].
[53] H. J. Lipkin, Phys. Lett. B 179, 278 (1986).
[54] P. Colangelo, F. De Fazio and T. N. Pham, Phys. Rev. D 69, 054023 (2004) [hep-ph/0310084].
[55] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Rept. 281, 145 (1997) [hep-ph/9605342].
[56] H. Y. Cheng, C. K. Chua and A. Soni, Phys. Rev. D 71, 014030 (2005) [hep-ph/0409317].
[57] C. Patrignani et al. [Particle Data Group], Chin. Phys. C 40, no. 10, 100001 (2016). doi:10.1088/1674-1137/40/10/100001
[58] X. Liu, X. Q. Zeng and X. Q. Li, Phys. Rev. D 74, 074003 (2006) [hep-ph/0606191].
[59] X. -Q. Li, D. V. Bugg and B. -S. Zou, Phys. Rev. D 55, 1421 (1997).
[60] M. P. Locher, Y. Lu and B. S. Zou, Z. Phys. A 347, 281 (1994) [nucl-th/9311021].
[61] X. -Q. Li and B. -S. Zou, Phys. Lett. B 399, 297 (1997) [hep-ph/9611223].
[62] P. Colangelo, F. De Fazio and T. N. Pham, Phys. Lett. B 542, 71 (2002) [hep-ph/0207061].
[63] E. V. Veliev, H. Sundu, K. Azizi and M. Bayar, Phys. Rev. D 82, 056012 (2010) [arXiv:1003.0119 [hep-ph]].
[64] C. Isola, M. Ladisa, G. Nardulli and P. Santorelli, Phys. Rev. D 68, 114001 (2003) [arXiv:hep-ph/0307367].
[65] F. -K. Guo, C. Hanhart and U. -G. Meißner, Phys. Rev. Lett. 103, 082003 (2009) [Erratum-ibid. 104, 109901 (2010)] [arXiv:0907.0521 [hep-ph]].