Proton-neutron pairing correlations in the nuclear shell model

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Abstract. A shell-model study of proton-neutron pairing in $f-p$ shell nuclei using a parametrized Hamiltonian that includes deformation and spin-orbit effects as well as isoscalar and isovector pairing is reported. By working in a shell-model framework we are able to assess the role of the various modes of proton-neutron pairing in the presence of nuclear deformation without violating symmetries. Results are presented for $^{44}$Ti, $^{46}$Ti and $^{48}$Cr.

1. Introduction

It is generally believed that proton−neutron (pn) pairing is important in nuclei with roughly equal numbers of neutrons and protons [1]. The standard technique for treating these correlations is through BCS or HFB approximation, generalized to include the pn pairing field in addition to the nn and pp pairing fields. Questions arise, however, as to whether these methods can adequately represent the physics of the competing modes of pair correlations, without full restoration of symmetries [2].

Important insight into this issue has been achieved recently in the context of exactly-solvable models that include these different pairing modes. Analysis of the SO(8) model, in which isoscalar and isovector pairing act in either a single active orbital or a series of degenerate orbitals, suggests that isospin restoration or equivalently quartet correlations are extremely important, especially near $N=Z$ [2]. More recent studies, carried out for models involving non-degenerate orbitals [3], reinforce earlier conclusions as to where isoscalar pairing correlations should be most important [4, 5]. Furthermore, they make possible the description of deformation, as is critical for systems with $N \approx Z$, by treating the non-degenerate orbitals as Nilsson-like. However, it is still not possible in these models to restore symmetries, either rotational or isospin.

As a consequence, there still remain many open issues concerning the role of the different possible modes of pairing in $N \approx Z$ nuclei. In this work, we report a systematic study of pairing correlations in the context of the nuclear shell model, whereby deformation can be readily included and symmetries maintained. In this way, we are able to address many of the open issues on the role of the various pairing modes in the presence of nuclear deformation.
An outline of the paper is as follows. In Section 2 we describe our model and in Section 3, we report some of the key results we have obtained, which are then summarized in Section 4.

2. Our model
To address in a systematic way the role of pairing correlations in the presence of nuclear deformation, we consider neutrons and protons restricted to the orbitals of the $1f - 2p$ shell outside a doubly-magic $^{40}$Ca core and interacting via a schematic Hamiltonian

$$ H = \chi \left( Q \cdot Q + a P^\dagger \cdot P + b S^\dagger \cdot S + \alpha \sum_i \vec{l}_i \cdot \vec{s}_i \right) $$

where $Q = Q_n + Q_p$ is the mass quadrupole operator, $P^\dagger$ creates a correlated $L = 0, S = 1$, $J = 1$, $T = 0$ pair and $S^\dagger$ creates an $L = 0, S = 0$, $J = 0$, $T = 1$ pair. The first term in the Hamiltonian produces rotational collective motion, whereas the second and third term are the isoscalar and isovector pairing interactions, respectively. The last term is the one-body part of the spin-orbit interaction, which splits the $j = l \pm 1/2$ levels with a given $l$.

We carry out calculations systematically as a function of the various strength parameters. We begin with pure SU(3) rotational motion associated with the $Q \cdot Q$ interaction and then gradually ramp up the various SU(3)-breaking terms to assess how they affect the rotational properties. This includes the isoscalar and isovector pairing interactions and the spin-orbit term.

We first consider the nucleus $^{44}$Ti, with $N = Z = 2$, and then systematically increase $N$ and $Z$ to study the role of the number of active neutrons and protons, e.g. whether there is an excess of one over the other and whether the nucleus is even-even or odd-mass. The nuclei we have treated are $^{44}$Ti ($N = Z = 2$), $^{45}$Ti ($N = 2, Z = 3$), $^{46}$Ti ($N = 2, Z = 4$), $^{46}$V ($N = 3, Z = 3$), and $^{48}$Cr ($N = 4, Z = 4$). Some of the observables we have studied are (1) the energies and associated BE(2) values of the lowest rotational band, (2) the number of $J^\pi = 6^+$, $T = 1$ pairs, (3) the number of $J^\pi = 0^+$, $T = 1$ ($S$) pairs and (4) the number of $J^\pi = 1^+$, $T = 0$ ($P$) pairs. In the following section, we present selected results for $^{44}$Ti, $^{46}$Ti and $^{48}$Cr.

3. Calculations
3.1. Optimal Hamiltonian
We first ask whether the hamiltonian (1) has sufficient flexibility to describe the nuclei under investigation. Without making an effort towards an absolute fit, we find that the choice $\chi = -0.5 \text{ MeV}$, $a = b = 12$, and $\alpha = 20$ gives an acceptable fit to the spectra of all the nuclei we have considered. This is illustrated in figure 1 for $^{44}$Ti, $^{46}$Ti and $^{48}$Cr. As can be seen, the well-known non-rotational character of $^{44}$Ti is reproduced by our calculations, as are the highly rotational patterns seen experimentally for the heavier nuclei. As we will see later, even the experimentally observed backbend in $^{48}$Cr is acceptably reproduced with this Hamiltonian. We refer to the choice $a = b$ in the optimal Hamiltonian as the SU(4) choice, from the dynamical symmetry that derives from this choice of parameters in the SO(8) model.

3.2. $^{44}$Ti
We next turn to the nucleus $^{44}$Ti, with two active neutrons and two active protons. In figure 2, we show the calculated energy splittings $E_1 - E_{1-2}$ associated with the ground-state band as a function of the strength parameters $a$ and $b$ that define the isoscalar and isovector pairing interactions, respectively. For these calculations we assumed a quadrupole strength of $\chi = -0.05 \text{ MeV}$ and no spin-orbit interaction. What we see is that the isoscalar and isovector pairing interactions have precisely the same effect on the properties of the ground state rotational band, in the absence of any spin-orbit interaction. Precisely the same conclusion derives when we consider the effect of isoscalar and isovector pairing on other observable properties.
Figure 1. Comparison of experimental spectra for $^{44}$Ti, $^{46}$Ti and $^{48}$Cr with the calculated spectra obtained using the optimal hamiltonian described in the text. All energies are in MeV.

Figure 2. Spectra of the ground state band of $^{44}$Ti as a function of the strength of the isoscalar pairing interaction (left panel) and of the isovector pairing interaction (right panel), in each case with no spin-orbit term present.

Some other results from our study of $^{44}$Ti are illustrated in figure 3. In panel (a), we show the spectrum that derives solely from turning on a strong spin-orbit force. We see that the spectrum is still highly rotational, despite the fact that the resulting single-particle energies are no longer SU(3)-like. To obtain the physical spectrum with a non-rotational character, it is thus essential to have pairing. It is usually accepted that it is the non-SU(3) order of the single-particle levels that is responsible for the non-rotational character seen [6], a conclusion not supported by our results. In panel (b), we address the issue of how isoscalar pairing is affected by inclusion of a spin-orbit force. We see that the number of isoscalar pairs - defined as the expectation value of $P^i \cdot P$ - decreases rapidly with an increasing spin-orbit interaction,
Figure 3. Results from calculations carried out for $^{44}$Ti. (a) Comparison of the experimental spectrum with spectra obtained with a pure $Q \cdot Q$ interaction and with both a $Q \cdot Q$ interaction and a spin-orbit term. (b) Number of isoscalar pairs as a function of the spin-orbit strength, for the various states in the ground band.

especially for the lowest angular momentum states of the ground band. The mechanism whereby the spin-orbit interaction suppresses isoscalar pairing has been discussed recently in ref. [7].

3.3. $^{46}$Ti
Next we turn to $^{46}$Ti with two additional neutrons present. Here too we compare the effect of the isoscalar and isovector pairing interactions on deformation, showing the results in figure 4 with no spin-orbit term present. Here the effect of isoscalar pairing is strongly suppressed relative to isovector pairing, suggesting that even without a spin-orbit term isoscalar pairing is very strongly focused on those nuclei with $N = Z$ with a slight excess being sufficient to suppress this pairing mode.

3.4. $^{48}$Cr
Lastly, we turn to $^{48}$Cr, which again has $N = Z$, but now with two quartet-like structures present. Here we assume as our starting point both the optimal quadrupole-quadrupole force and one-body spin-orbit force and then ramp up the two pairing strengths from zero to their optimal values. The results are illustrated in figure 5, for scenarios in which we separately include isoscalar pairing, isovector pairing and and SU(4) pairing with equal strengths.

As a reminder, the experimental spectrum for $^{48}$Cr shows a backbend near $I = 12$, which as noted earlier is reproduced by our optimal hamiltonian. The results of figure 5 make clear that (a) the backbend cannot be reproduced with pure isoscalar pairing, but requires isovector pairing as well and (b) there is no significant difference between the results obtained with pure isovector pairing and SU(4) pairing. The backbend in $^{48}$Cr was discussed extensively in the context of a shell-model study with a fully realistic hamiltonian in [8], where it was first shown to derive from isovector pairing. Our results are in agreement with that earlier conclusion.
Figure 4. Spectra of the ground state band of $^{46}$Ti as a function of the strength of the isoscalar pairing interaction (left panel) and of the isovector pairing interaction (right panel), in each case with no spin-orbit term present.

4. Summary and Concluding Remarks

In this work, we have reported a shell-model study of proton-neutron pairing in $f-p$ shell nuclei using a parametrized hamiltonian that includes deformation and spin-orbit effects as well as both isoscalar and isovector pairing. By working in a shell-model framework we are able to assess the role of the various modes of proton-neutron pairing in the presence of nuclear deformation without violating symmetries.

We first showed that our parametrized hamiltonian has enough flexibility to be able to provide a reasonable description of the evolution of nuclear structure properties in this region. We then probed the role of the various modes of pairing on deformation with or without a spin-orbit term. We did this as a function of the number of neutrons and protons, so as to assess the role both of a neutron excess and of the number of active particles.

Some of the conclusions that emerged are: (1) in the absence of a spin-orbit term, isoscalar and isovector pairing have identical effects at $N = Z = 2$, but that isoscalar pairing ceases to have an appreciable effect for nuclei with just two excess neutrons; (2) the non-rotational character of $^{44}$Ti cannot be explained solely in terms of spin-orbit effects but requires pairing for its understanding; (3) in the presence of a spin-orbit interaction, isoscalar pairing is suppressed even at $N = Z$, and (4) the known backbend in $^{48}$Cr has its origin in isovector pairing.

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Figure 5. Calculated splittings in $^{48}$Cr ground band, for isovector, isoscalar, and SU(4) pairing, respectively, as described in the text.

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