Extraction of artificial boundary frequencies for damage identification

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Abstract This paper introduces some recent progress in a study which is aimed at incorporating the so-called artificial boundary condition (ABC) frequencies for damage identification. The ABC frequencies are those corresponding to the natural frequencies of the system with additional pin supports, but may be extracted from specially configured incomplete frequency response function matrix of the original structure without the need of physically imposing the additional supports. A particular focus of this paper is placed on the actual extraction of such frequencies from physical experiments and the associated data processing and analysis. Results will demonstrate that it is possible to extract the first few ABC frequencies for a variety of boundary conditions with 1-2 artificial pin supports in a beam or a slab structure.

Keywords: modal data; frequency response function; natural frequencies, anti-resonance frequencies

1. Introduction

The acquisition of high quality dynamic property data is a crucial step in the entire process of damage identification of structures. The classical modal data mainly cover the natural frequencies, mode shapes, and damping parameters. It is well recognized that modal frequencies can be measured with much higher accuracy than mode shapes, but the number of measurable natural frequencies is limited.

One way to expand the modal frequency dataset is to alter the boundary conditions of the structure, for example by adding extra pin supports at selected locations, so that different subsets of the natural frequencies for the same physical state of the structure but under different boundary conditions can be generated. However, to actually introduce extra supports on a physical structure like a building or bridge for dynamic testing is not practical.

The development of the “artificial boundary method (ABM)” (Gordis 1996, 1999) paves the way for such an idea to be considered in civil engineering applications. It was demonstrated by Gordis (1996) that, by manipulating the incomplete frequency response function matrix measured at specific points on the existing structure, it is possible to identify resonance frequencies from the resulting frequency spectrum that exactly correspond to the natural frequencies of the structure as if additional pin supports were imposed at the specific locations. Since no physical pin supports are actually involved, such altered boundary conditions are referred to as “artificial boundary conditions”, and the corresponding natural frequencies are referred to as artificial boundary condition frequencies, or in short ABC frequencies as used by the authors.
A quick overview of the ABM is as follow. For a linear system, the steady state response of can be written as

\[
\begin{bmatrix}
  k_{aa} & k_{ao} \\
  k_{oa} & k_{oo}
\end{bmatrix}
- \omega^2
\begin{bmatrix}
  m_{aa} & m_{ao} \\
  m_{oa} & m_{oo}
\end{bmatrix}
\begin{bmatrix}
  x_a \\
  x_o
\end{bmatrix}
= \begin{bmatrix}
  f_a \\
  f_o
\end{bmatrix}
\]  

(1)

where \( k \) and \( m \) are stiffness and mass matrices, \( x \) and \( f \) are vectors of generalized response and excitation amplitudes, respectively. The subscript 'a' represents measured coordinates or DOFs, and the subscript 'o' refers to the unmeasured DOFs or 'omitted coordinate set' OCS, which implies the system with the measured DOFs restrained (or pinned). Rearranging,

\[
\begin{bmatrix}
  Z_{aa} & Z_{ao} \\
  Z_{oa} & Z_{oo}
\end{bmatrix}
\begin{bmatrix}
  x_a \\
  x_o
\end{bmatrix}
= \begin{bmatrix}
  f_a \\
  f_o
\end{bmatrix}
\]  

(2)

where \( Z \) is the impedance matrix, \( Z = k - \omega^2 m \). Under the condition that no excitation force is applied on any of the unmeasured DOFs (\( f_o = 0 \)), the measured frequency response function (FRF) can be written as

\[
\begin{bmatrix}
  Z_{aa} & Z_{ao} \\
  Z_{oa} & Z_{oo}
\end{bmatrix}
^{-1}
\begin{bmatrix}
  x_a \\
  x_o
\end{bmatrix}
= \begin{bmatrix}
  f_a \\
  f_o
\end{bmatrix}
\]  

In an actual test \( \mathbf{H}_{aa} \) is the measured incomplete FRF matrix, denoted by \( \mathbf{H}^m \). At the natural frequencies of the OCS, \( Z_{oo}^{-1} \) becomes singular, and so will be \( (\mathbf{H}^m)^{-1} \). This means that the elements of \( (\mathbf{H}^m)^{-1} \) will be singular at the natural frequencies of the OCS. Hence, by identifying the resonance frequencies from the elements of \( (\mathbf{H}^m)^{-1} \), one can determine the ABC frequencies as if the structure was pinned at the measured DOFs.

Tu and Lu (2008) applied simulated ABC frequencies, in conjunction with a genetic algorithm based search procedure, for the inverse identification of structural parameters through finite element model updating. It was demonstrated that the ABC frequencies, if measured accurately, can be as effective as normal natural frequencies for the parameter identification. The main question then becomes how reliable and accurate the ABC frequencies may be obtained from a standard modal testing procedure. To answer this question, a series of experimental studies have been organised and preliminary results as reported previously (Lu et al. 2008) were promising. This paper provides a more comprehensive investigation on the extraction of the ABC frequencies for a variety of one- and two-pin combinations. The effectiveness of the associated data processing techniques, including rational fractional polynomial (RFP) and singular value decomposition (SVD), in increasing the accuracy of ABC frequency extraction is discussed.

2. Experimental programme

1) Test structures and test setup

The present experiment was conducted on a test beam and a slab. The test beam was a reduced steel beam of dimension 1000×50×6 mm. The beam was fully fixed at both ends.
The test slab was made of aluminium plate, with a length of 1m, width of 0.5m, and the slab thickness was 2mm. The slab was fixed along the two short ends, giving a one-way slab setting. Fig. 1 shows the test setup.

![Test setup](image)

Fig. 1 Test structures and setup

Standard modal testing with impact excitation was performed. The impact was applied using an instrumented hammer, while the response was measured using small size modal testing accelerometers (B&K DeltaTron accelerometers), weighing 4.9g each. The mass of the accelerometers were negligible comparing to the unit mass of the test beam and slab. The dynamic data were recorded using a data acquisition system comprising a main dynamic DAQ module (Strainbook-616) and multi-channel acceleration acquisition modules (Wavebook WBK-18 series).

2) Data acquisition: sampling frequency

In modal testing with impact hammer excitation, the measurement of the impact load is crucial in deriving the frequency response functions from the measured response and the input load. Since the duration of the impact force is very short, usually on an order of milliseconds, high sampling frequency should be used, as also noted by Reynolds and Pavic (2008).

Fig. 2 gives an example comparison of the FRFs obtained using three different sampling frequencies, all may be considered as of high sampling rate. However, difference is still visible, especially when the sampling frequency is increased from 5 kHz to 10 kHz. This indicates that a sufficiently high sampling rate is necessary and an appropriate choice should be made with necessary trial tests. This is particularly important for the present ABC frequency extraction, which requires high quality and detailed FRF results.
Fig. 2 Measured FRF curves from the test slab using different sampling frequencies

In this experimental study, a sampling rate of 20 kHz was chosen. Fig. 3 shows a pair of sample impact load and acceleration responses.

Fig. 3 Typical measured impact force and acceleration time series

3. Data processing and analysis

3.1 Initial processing of time series

The measurement time span was kept enough long so leakage was not considered a problem in the measured signals. However, a rectangular window was applied on the measured impact force to eliminate the obvious noises in the force input. It is noteworthy that due to the impact nature of the hammer excitation, there could be travelling wave effect caused by the impact force, and such wave effect may not be reflected in the measured dynamic response (lateral accelerations). From this point of view, calculating the frequency response function by relating the response (acceleration) to the impact force could become erroneous, particularly in stiff structures where the resonance frequencies are high enough with respect to the travelling wave. In the present experiment, the test beam and slab were relatively flexible; therefore the travelling wave effect was not considered as an issue.

3.2 FRF curves and relevant processing techniques

After the initial processing of the measured time series signals, the frequency response functions are obtained using the frequency spectra of the input (impact force) and output
(acceleration) signals. Some typical FRF curves are shown in Fig. 4.

![Typical FRF curves from test beam: RFP smoothes resonance peaks but distorted anti-resonances](image)

Fig. 4 Typical FRF curves from test beam: RFP smoothes resonance peaks but distorted anti-resonances

Two possible techniques are evaluated on their adequacy for processing the FRF curves, with a view of improving the subsequent calculation of the ABC frequencies.

The first is the Rational Fraction Polynomial (RFP) technique. If the dynamic system is linear time-invariant second-order system, then the transfer function can be represented as a ratio of two polynomials within certain frequency range (Richardson and Formenti, 1982), as,

$$ H(w) = \frac{\sum_{k=0}^{m} a_k s^k}{\sum_{k=0}^{n} b_k s^k} $$  \hspace{1cm} (4) $$

In a curve fitting process, the unknown coefficients of both numerator and denominator polynomials of analytical form \((a_k, b_k)\) are determined to match the measured FRF curves.

However, there are some requirements of applying the RFP technique. Firstly, curve fitters are usually applied over a limited frequency range; moreover, prior knowledge about the number of modes within selected frequency range should be known to remove the distortion.

The effect of applying the RFP on the FRF is illustrated in Fig. 4(b). As can be seen, the curves can be smoothed and most resonances can be shown clearly; however, it should be noted that anti-resonances are distorted. Therefore, this RFP technique may only be applied to the final ABC curves, but not on the processing of FRFs for onwards calculation of the ABC frequencies.

The second technique being considered is the Singular Value Decomposition (SVD) technique. With the SVD method, the measured FRF data are firstly used to form the Hankel matrix, and then a rank of the matrix is determined and noises are removed by deleting components beyond the chosen rank. The details of this technique on the
In order to apply SVD in the present case, two factors need to be examined. Firstly the size of Hankel matrix should be determined properly. If the number of columns of the Hankel matrix is less than the rank of the system, then the Hankel matrix can not represent the system properly; on the other hand, too large a number of columns requires unnecessary computations. It is recommended that if the computation cost is not a big issue, a square or nearly square Hankel matrix is preferred (Sanliturk and Cakar, 2005). Another factor concerns the determination of the rank of the Hankel matrix so that the components beyond the rank can be deleted for best removal of noise without losing meaningful information. In practice, this may be done by plotting the singular values of the matrix and choosing the proper rank when the singular values approach an asymptote. Fig. 5 illustrates the singular value distribution of a typical measured FRF from the present experiment.

From the curve shown in Fig. 5, the rank is estimated to be 20. For a comparison, three different values of the rank are used to apply SVD on the FRF, and the corresponding results are shown in Fig. 6. It can be seen that with different ranks within a reasonable range, the FRF curves have similar shapes, indicating the convergence of this technique for the current application. It is important to note that, for the present application aiming at the subsequent calculation of the ABC frequencies, both resonance and anti-resonance frequencies must not be distorted in this process. Therefore, to be conservative the rank of Hankel matrix is selected as 30. This ensures the removal of spurious spikes while retaining all resonance and anti-resonance features as in the original curve.

Fig. 5 Singular value distribution of a typical measured FRF curve

Fig. 6 SVD-processed FRF curves with different ranks
The SVD technique with the above general guide in selecting the proper rank is applied to process the FRF data, before the FRF data are employed in the calculation of the ABC spectra.

A further issue concerns the possible existence of anti-resonance between zero (DC) to the first resonance frequency. This is checked by examining the real and imaginary parts of the FRF, respectively, in view of the fact that at a true anti-resonance frequency a zero value should register on the real part of the FRF.

4. ABC frequency curves and extraction of ABC frequencies

Following the processing of measured FRF curves with the SVD technique, ABC frequency spectral curves can be calculated by inversing the incomplete FRF matrix. In the simplest case with one artificial pin, an ABC curve is just the inverse of a single driving-point FRF. Herein we shall confine ourselves to one-pin and two-pin ABC frequencies. ABC frequencies for three-pin (and above) scenarios will involve higher order FRF matrix inversion, and hence are expected to be prone to complications due to compounded effect of errors in individual FRFs. As a matter of fact, limiting to one- and two-pin ABC already gives rise to a variety of boundary combinations, thus producing a large set of additional frequency data.

Fig. 7 presents a set of four FRFs associated with a two-pin ABC configuration for the test beam, before and after processing with SVD. The processed FRFs form the incomplete 2x2 FRF matrix. The inversion of this matrix then gives rise to the ABC frequency curves, from which the ABC frequencies, or the resonance frequencies corresponding to the test structure with the additional pin supports, can be identified. One of the ABC frequency curve (i.e., an element in the inverted FRF matrix) is shown in Fig. 8. Considering the effectiveness of the RFP technique in smoothing the resonance peaks, as discussed in Section 3, this technique is finally applied on the ABC frequency. The effect is noticeable as can be seen from Fig. 8.

![Fig. 7 Four FRF curves (before and after SVD processing) for a two-pin ABC on the test beam](image-url)
Table 1 summarises the extracted ABC frequencies from the experiment on the test beam for a number of one-pin and two-pin configurations, in comparison with the predicted counterparts using finite element model where actual pin(s) are applied at the specified locations. It can be clearly observed that the experimental ABC frequencies match well with the FE predicted counterparts with actual pin supports. This indicates that such frequencies can be obtained from the procedure described above with satisfactory accuracy and reliability.

Table 1 Experimental ABC frequencies and their FE predicted counterparts (in parenthesis)

| Pin position(s) | The 1st mode | The 2nd mode | The 3rd mode |
|-----------------|--------------|--------------|--------------|
| 1               | 34.9 (35.6)  | 98.1 (98.2)  | 193.9 (192.7)|
| 3               | 55.0 (55.6)  | 151.3 (153.7)| 254 (272.5) |
| 5               | (91.7)       | 119.7 (119.5)| (276.33)    |
| 7               | 56.6 (55.6)  | 156.9 (158.8)| 251.2 (274.5)|
| 9               | 36.3 (35.6)  | 98.1 (98.2)  | 195.2 (192.7)|
| 1+2             | 45.9 (44.7)  | 123.3 (123.58)| 245.2 (242.8)|
| 3+4             | 75.0 (78.7)  | 209.7 (215.7)| 290 (302.6) |
| 6+7             | 77.2 (78.7)  | 210.4 (215.7)| 291.4 (302.6)|
| 8+9             | 45.6 (44.7)  | 123.3 (123.6)| (242.8)     |

Location indicator:

An example set of FRF and ABC curves for the test slab, with two ABC pins, is shown in Fig. 9. Generally speaking, similar level of accuracy as for the test beam can be achieved in such a relatively flexible slab. However, because of the fact that modes become highly complicated for a slab with additional pin supports within the slab, pairing of the ABC frequencies and the counterparts from an FE model with actual pins is not straightforward and further studies on this is required.
With the availability of the ABC frequencies, an inverse analysis procedure can be applied for the identification of the structural parameters, for instance using finite element model updating with a global searching engine such as using the Genetic Algorithms, as implemented by Tu and Lu (2008).

5. Concluding remarks

The expansion of the measurable frequency dataset is highly desirable in the structural identification and damage detection field. By processing the measured FRFs from a standard modal testing procedure, it is possible to acquire resonance frequencies corresponding to the structure being altered with additional pin supports, without the need of actually imposing such supports.

A successful extraction of the ABC frequencies from modal testing will require careful consideration at the experimental as well as data processing stages. In particular, an appropriate sampling rate to capture the impact force pulse, which often lasts in an order of milliseconds, is found to have a sensible influence. Application of the SVD technique can help improve the entire FRF curves, including resonance as well as anti-resonance regions, therefore is suitable for processing the measured FRFs prior to the calculation of the ABC frequency curves. The RFP technique is found to be useful in the final processing of the ABC frequency curves, especially when the curve becomes somewhat noisy around certain identifiable ABC frequencies.

The results from a test beam and a test slab, with the above processes in place, demonstrate that it is feasible to acquire the ABC frequencies for such structures under a variety of one-pin and two-pin ABC configurations. Such frequency data can be easily incorporated into an onward structural parameter identification procedure, for instance through finite element model updating.

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