Phase transition of spacetime

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Abstract

In this work, we find both black holes and particle can be described by the complex Kerr-Newman metric and converted to each other through a phase transition. After the phase transition point at Planck energy, the particle’s imaginary radii are realized and converted into a black hole. In 4-D spacetime, the 3-D imaginary space is compacted to its 1-D temporal dimension. In the hidden 3-D imaginary space, a Dirac electron appears as an equivalent Schwarzschild black hole with a mass of $J/m$ (spin angular momentum per unit mass), which reveals the geometric origin of wave nature and spin in quantum mechanics. This work provides strong evidence for ER=EPR.

Keywords: black hole; Dirac electron; complex metric; phase transition; ER=EPR

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1. Introduction

The concern over the potential link between the black hole and the particle has a long and continuous history because it may provide us information about the connection between general relativity and quantum mechanics. In 1935, trying to in search of a geometric model for elementary particles, Einstein and Rosen [1] rediscovered the wormhole. In 1968, Carter [2] found that the newly discovered Kerr–Newman solution [3] has a gyromagnetic ratio \( g=2 \) like the Dirac electron. Then, the Kerr–Newman electron has received constant attention [4–17] and obtained supports from string theory [18–23]. What’s more, there also have been suggestions that black holes should be treated as elementary particles [24–36].

Complex Kerr-Newman metric, initiated by Newman and and his co-workers in their derivation of the Kerr-Newman metric [3], has been found to be useful in various problems [37–43]. In a previous work [44], we found that the gravitons of a Dvali-Gomez BEC black hole [45] can be described by the complex Kerr-Newman metric, resulting in a slight-naked firewall, which only allows the information of a black hole to penetrate through the horizon at a limited rate rather than the dangerous naked firewall [46]. Based on [44], we further find that both black holes and elementary particles can be described by complex Kerr-Newman metric. In fact, they are two special cases of the complex black holes and can turn into each other through a phase transition.

2. Phase transition of complex black hole

The Kerr-Newman metric describes a general black hole with both charge and spin [3]. The radius of its two horizons \((r_\pm)\) are

\[
r_\pm = m \pm \sqrt{m^2 - a^2 - Q^2}
\]

where \(m\) is its mass, \(a\) is its angular momentum per unit mass, and \(Q\) is its charge, \(c = \hbar = G = k_B = 1\) is used in this work. Equation (1) seems to lose its physical meaning when \(m^2 < a^2 + Q^2\). However, if a horizon can have complex radius, the physical meaning
of this equation can be further expanded. Re-writing equation (1), we can obtain

\[ r = m \pm \sqrt{m^2 - a^2 - Q^2} = m \pm i \sqrt{a^2 + Q^2 - m^2} \]  

(2)

The real radius of the complex horizon \( r_k \) is,

\[ r_k = m \]  

(3)

while the imaginary radii of the complex horizon \( r_i \) are,

\[ r_i = \pm i \sqrt{a^2 + Q^2 - m^2} \]  

(4)

In a previous work [44], we found that the gravitons of a Dvali-Gomez BEC black hole [45] can be described by the complex Kerr-Newman metric, resulting in a slight-naked firewall. Further, we find that both particles and black holes are also special cases of complex Kerr-Newman metric.

In the 3-D real space, an elementary particle appears as a 0-D particle in low energy because its \( r_k \) is too small to be measured. However, with the increase of its energy, the particle as a complex black hole can turn into a real black hole through a phase transition. \( r_1 \) of a particle can be described as

\[ r_1 = \begin{cases} 
\pm i \sqrt{a^2 + Q^2 - m^2} & (a^2 + Q^2 - m^2 > 0) \\
0 + 0i & (a^2 + Q^2 - m^2 = 0) \\
\pm \sqrt{m^2 - a^2 - Q^2} & (a^2 + Q^2 - m^2 < 0) 
\end{cases} \]  

(8)

which continuously reduce to \( 0i \) and then be realized. The point of phase transition of the complex black hole is an extreme black hole, \( r_f = 0 \).

The module of the graviton’s \( r \) is found to be exactly the radius of the BEC black hole [44]. Since the radial coordinate inside the horizon is time-like, the imaginary radius of the complex horizon of a particle should be its size in temporal dimensions, while the real radius is its size in spatial dimensions. In this way, the 1-D temporal dimension of 4-D spacetime should be folded from the 3-D imaginary space of particle. After the phase transition, the folded 3-D imaginary space is unfolded: a real Kerr-Newman black hole’s imaginary space, is embedded in the 3-D real space by its two horizons. A real Kerr-Newman black hole’s \( r_k \) determines the position of the origin of the realized imaginary space, appearing as a 2-D origin spherical surface, while its
realized $r_I$ determines the boundary of the realized imaginary space, appearing as its two horizons.

Fig.1 Phase transition of complex black hole. After the point of phase transition, $r_I=0$, the imaginary radii are realized. The objects in $a$ are corresponding to the objects in the same color in $b$: 0-D origin point vs 2-D origin surface, 1-D temporal dimension vs 3-D imaginary space, light cones vs horizons.

3. Hawking temperature and wave nature of Dirac electron

What kind of geometry does an electron have in the hidden 3-D imaginary space? The electron’s imaginary radius has both positive and negative values. In order to make the negative one have a physical meaning, the origin of the imaginary space of the electron has to be a 2-D origin surface like a real black hole shown in Fig.1. In this way, the electron will appear as an equivalent Schwarzschild black hole with a radius of $R_i=2r_i$ in the imaginary space.

$$R_i = 2r_i = 2i\sqrt{a^2+Q^2-m^2} \quad (9)$$

This imaginary Schwarzschild black hole has an equivalent mass of

$$M_i = r_i = i\sqrt{a^2+Q^2-m^2} \quad (10)$$

and a Hawking temperature of
\[ T_i = \frac{1}{4i\sqrt{a^2 + Q^2 - m^2}} \]  

A black hole can harvest energy from its environment and lose energy through Hawking radiation. Energy balance is a necessary condition for a stable black hole. As a stable elementary particle, the Hawking temperature of the electron is therefore a good mark of the energy level of the imaginary space.

In 4-D spacetime, the 0-D particle moves forward at the speed of light along the time dimension. In the hidden 3-D imaginary space, this motion appears in a different way: the energy moves on the horizon of the imaginary Schwarzschild black hole at the speed of light (as shown in Fig.2).

\[ \theta = \frac{ct}{|R_i|} \approx \frac{ct}{2a} = mt \]  

For an electron in low energy, equation (12) can be approximate to

\[ \theta = \frac{ct}{|R_i|} \approx \frac{ct}{2a} = mt \]  

\( \theta \) is found to be the phase angle of the wave function of the electron described by Dirac equation.

The blue ball in Fig.2 represents a component carrying a small fraction of the energy of the particle, \( dm_j \), while the red ball represents its origin component, which moves on the origin surface at half of the speed of light and therefore maintains the same phase angle. In this way, the energy of the particle makes an equivalent uniform circular motion around the origin with a radius of \(|R_i|/2\). The spin of a particle in quantum mechanics, appearing as an intrinsic concept in the 4-D spacetime, in fact has a dynamic origin in the hidden imaginary space.

\[ L = \sum_{j=1} |R_j| \frac{|R_i|}{2} \]  

In this picture, the electron has a gyromagnetic ratio of 2 as given by the Dirac equation.
Fig. 2 Geometric origin of wave nature and spin. In the hidden 3-D imaginary space, a Dirac electron appears as an equivalent Schwarzschild black hole with a radius of $|R|$. A component of the particle (blue ball) maintains the same phase angle with its origin component (red ball) and doing an equivalent uniform circular motion around the origin with a radius of $|R|/2$.

4. Conclusion and discussion

In this work, both black hole and particle are re-examined from the view of complex metric. A conjugate symmetry between them in complex space is found. This work provides strong evidence for ER=EPR [23]. In fact, in a complex space, there is no essential difference between the pair production of an elementary particle and its antiparticle and the pair production of two entangled black hole in ER=EPR picture.

Supplemental materials

1. Imaginary space before and after the phase transition of a particle

|                | $r^2_i$ | Origin | 3-D Imaginary space                  | Non-superluminal information boundary |
|----------------|---------|--------|--------------------------------------|---------------------------------------|
| Particle       | $r^2_i < 0$ | 0-D    | Folded to 1-D temporal dimension      | Future light cone                      |
| Black hole     | $r^2_i > 0$ | 2-D    | Realized to the time-like space in the horizon | Outer horizon                          |
|                |          |        |                                      | Inner horizon                          |

Table.s1 One-to-one correspondence of the imaginary space before and after the phase
2. Imaginary radius the complex horizon

Fig.s1 The imaginary radius the complex horizon of a particle (graviton), $r_I$, as a function of its mass, $m$. An extreme black hole, whose $r_I$ is 0, acts as the point of phase transition of the complex black hole: after it, $r_I$ is realized.

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