Thermoelectric generator at optimal power with external and internal irreversibilities

Jasleen Kaur and Ramandeep S. Johal
Department of Physical Sciences,
Indian Institute of Science Education and Research Mohali,
Sector 81, S.A.S. Nagar,
Manauli PO 140306, Punjab, India

There are few exact results on optimal power conditions for a thermoelectric generator in the presence of both external and internal irreversibilities—modelled as non-ideal thermal contacts and Joule heating, respectively. Simplified cases, where only one kind of irreversibility is assumed, yield some well-known expressions for efficiency at maximum power (EMP), such as Curzon-Ahlborn efficiency for endoreversible model. In this work, we analyze situations under the simultaneous presence of internal and external irreversibilities. To simplify, we neglect heat leaks, and each kind of irreversibility is assumed only on the side of one of the thermal contacts. We also present the symmetric case—where each kind of irreversibility contributes with equal strengths towards the side of each thermal contact. We show the bounds satisfied by EMP in each of these regimes and compare its properties for thermal impedance matching and close to equilibrium, where we find step-wise changes in EMP.

I. INTRODUCTION

Sadi Carnot proposed an ideal heat engine exploiting the temperature difference between two heat reservoirs [1]. However, the presence of various irreversibilities makes it impractical to design such an engine. A thermoelectric generator (TEG) provides a paradigmatic model of a realistic heat engine, where both internal and external sources of irreversibilities can be considered [2–3]. Along with a sustained effort towards improving the figure of merit of a thermoelectric material (TEM) [14, 37], the characterization of optimal performance also forms a significant aspect of the study of thermoelectricity [9–15]. The framework of finite-time thermodynamics aims to characterize the performance of thermal machines with finite-rate processes [16–20].

An important quantity in this regard is the efficiency at maximum power [21–25], referred to as EMP. A simplified analysis may be done by considering either external or internal irreversibility [26–27]. Further, heat leaks can be neglected by using the strong-coupling assumption [3] [22]. In the so-called endoreversible model [28–29], an external irreversibility is caused by a finite rate of heat transfer between the working substance (TEM) and heat reservoir. Based on Newtonian heat flows, this model yields EMP

\[
\eta_{\text{ext}} = 1 - \sqrt{1 - \eta_C}, \tag{1}
\]

which was introduced in physics literature by Curzon and Ahlborn [26, 30]. Here, \(\eta_C = 1 - T_c/T_h\) is Carnot efficiency, with \(T_c\) (\(T_h\)) is the temperature of cold (hot) reservoir. This value is independent of any properties of TEM, or of thermal contacts.

On the other hand, thermal contacts between the working substance and a heat reservoir may be perfect, while Joule heating within TEM acts as the source of internal irreversibility. A different expression for EMP is then obtained

\[
\eta_{\text{int}} = \frac{\eta_C}{2 - (1 - \omega)\eta_C}, \tag{2}
\]

which can be obtained in other models too [31–33]. Here, parameter \(\omega\) is the fraction of Joule heat rejected to the cold reservoir see Eq. (1). For a homogeneous TEM, a value of \(\omega = 1/2\) is expected [14].

Apart from the above idealized cases, the exact analysis of optimal performance is not straightforward in the general scenario [2–34]. In this work, we highlight a few special cases which still lead to a tractable problem in power optimization. Thus, the external irreversibility may be considered only at one thermal contact while the other contact is assumed ideal [29]. This requires tuning the thermal conductances of contacts with reservoirs.

Secondly, we assume that Joule heat is fully transferred to one of the heat reservoirs [35–36], i.e. \(\omega = 1\), or 0. With advances in the fabrication of functionally graded thermoelectric materials [14, 37], it is possible to rectify Joule heat such that the proportion of Joule heat flowing into a reservoir can be controlled. These assumptions allow for an exact analysis of optimal power. In addition to that, we discuss the “symmetric” case, where internal and external irreversibilities contribute equally on both sides of TEM. For all these cases, we obtain exact expressions for EMP which depend on the ratio of cold to hot temperatures, as well as on the ratio of the external to internal thermal conductances. Further, EMP is limited within certain bounds that may be approached by suitably tuning the ratio of thermal conductances.

Our article is organized as follows. In Section II, we describe the model of a TEG. In sections III, IV, V and VI, we discuss TEG having different combinations of external and internal irreversibilities and optimize the power

* jasleenkaur@iisermohali.ac.in
† rsjohal@iisermohali.ac.in
output. In Section VII, we discuss the TEG having symmetric contributions of internal and external dissipation. We end with a discussion of results in Section VIII and concluding remarks in Section IX.

II. TEG MODEL

Thermoelectricity is a non-equilibrium phenomenon, which can be studied within the framework of Onsager-Callen theory [38, 39]. The coupling between the gradients of temperature and electric potential gives rise to various thermoelectric effects [40, 41]. We consider TEM to be a one-dimensional substance with given values of various thermoelectric effects [40, 41]. We consider TEM dients of temperature and electric potential gives rise to Callen theory [38, 39]. The coupling between the gra-

FIG. 1: A TEG consists of two legs of TEM which are connected electrically in series and thermally in parallel. On right side, is a block diagram of TEG having external thermal conductances $K_h$ and $K_c$. $R$ is the internal resistance of TEM with electric current $I$ flowing through. $T_{h,M}$ and $T_{c,M}$ are the local temperatures of TEM towards the hot and cold side respectively. Dashed lines indicate flow of Thermal contribution of internal and external dissipation. In Section VII, we discuss the TEG having sym-

In thermoelectricity, let $\alpha$ be the Seebeck coefficient, $\alpha$ is the fraction of Joule heat received by each reservoir. Out of these, the second term has no contribution towards energy conversion [21], and so we work within the strong-coupling assumption i.e., $K = 0$ [27]. Further, we assume a Newtonian heat flow between a reservoir and TEM, whereby we have

$$\dot{Q}_h = K_h (T_h - T_{hM}),$$

$$\dot{Q}_c = K_c (T_{cM} - T_c).$$

Then, the flux-matching condition on the hot side of TEM gives

$$K_h (T_h - T_{hM}) = \alpha T_{hM} I - (1 - \omega) RI^2.$$  (7)

On solving for $T_{hM}$, and substituting in Eq. (5), we obtain

$$\dot{Q}_h = K_h \frac{\alpha T_h I - (1 - \omega) RI^2}{K_h + \alpha I}. $$  (8)

Similarly, flux-matching condition on the cold side is given as:

$$K_c (T_{cM} - T_c) = \alpha T_{cM} I + \omega RI^2.$$  (9)

So, the outgoing flux is:

$$\dot{Q}_c = K_c \frac{\alpha T_c I + \omega RI^2}{K_c - \alpha I}.$$  (10)

Now, the power output of the device is given by:

$$P = \dot{Q}_h - \dot{Q}_c,$$  (11)

with the efficiency $\eta = P/\dot{Q}_h$. It is cumbersome to optimize power of the TEG having both internal and external irreversibilities [34] i.e., with finite values of both $K_h$ and $K_c$, as well as for general values of $\omega$. In this paper, we treat a few exactly solvable cases, which are interesting in their own right:

Case 1. Finite value of thermal conductance $K_h$, while the cold contact is reversible ($K_c \to \infty$). Additionally, $\omega = 0$, implying that the Joule heat is totally transferred to the hot reservoir (see Fig. 2a).

Case 2. Finite thermal conductance $K_c$, while hot contact as reversible ($K_h \to \infty$). Additionally, $\omega = 1$, implying all the Joule heat is transferred to the cold reservoir (see Fig. 2b).

Case 3. Finite thermal conductance $K_h$, along with $\omega = 1$, implying all the Joule heat is transferred towards the cold reservoir (see Fig. 3a).

Case 4. Finite thermal conductance $K_c$. Additionally, $\omega = 0$, implying that all the Joule heat is transferred towards the hot reservoir (see Fig. 3b).

Case 5. Symmetric external and internal irreversibilities, implying $K_h = K_c$ and $\omega = 1/2$.

We show that in all the above cases, we can analytically optimize power with respect to current $I$ flowing through TEM. It is convenient to define a parameter $v$ as the ratio of external to internal thermal conductances:

$$v = \frac{K_{ext}}{K_{int}},$$  (12)
where, we have [44]

\[ K_{\text{ext}} = \frac{K_h K_c}{K_h + K_c}, \quad (13) \]

\[ K_{\text{int}} = \frac{\alpha^2}{R} \left[ \omega T_h + (1 - \omega) T_c \right]. \quad (14) \]

In the following, we are able to find compact expressions for EMP, which are functions only of \( \theta \equiv T_c/T_h \) and parameter \( v \) for each respective case.

### III. CASE 1: FINITE \( K_h \) WITH \( \omega = 0 \)

In this case, the internal and external irreversibilities are considered only on the hot side, while irreversibilities on the cold side are regarded null. Thus, \( \omega = 0 \), and Eq. (7) yields

\[ T_{hM} = \frac{K_h T_h + R I^2}{K_h + \alpha I}, \quad (15) \]

so that

\[ \dot{Q}_h = \frac{\alpha T_h K_h I - R K_h I^2}{K_h + \alpha I}. \quad (16) \]

Also, thermal flux on the (reversible) cold side is

\[ \dot{Q}_c = \alpha T_c I. \quad (17) \]

Optimizing \( P \) w.r.t \( I \), i.e., setting \( \partial P/\partial I = 0 \), we solve the resulting extremum condition to obtain EMP as

\[ \eta^* = \frac{v + 1 - \sqrt{v(v+1)(v\theta + 1)}}{v + v\theta + 1}, \quad (18) \]

where the parameter \( v \) (Eq. (12)) in this case is

\[ v = \frac{R K_h}{\alpha^2 T_c}, \quad (19) \]

with \( K_{\text{ext}} = K_h \) and \( K_{\text{int}} = \alpha^2 T_c/R \).

Now, we may analyze the behaviour of EMP as a function of \( v \). For a given \( \theta \), \( \eta^* \) is a monotonic increasing function of \( v \). We consider coefficient \( \alpha \) to be fixed. Consider the regime \( K_{\text{ext}} \ll K_{\text{int}} \), which implies that the external irreversibility is dominant over the internal irreversibility. In other words, we have the limit \( v \to 0 \), or, operationally it implies \( R \to 0 \). Note that if \( v \to 0 \) implies \( K_h \to 0 \), then it yields a vanishing power, and we may exclude this possibility. Thus, with a finite \( K_h \), the limit \( v \to 0 \) implies zero Joule heating, and so we approach the endoreversible model—an external irreversibility on the hot side only. The corresponding EMP is: \( \eta^* = \eta_{\text{ext}} \). On the other hand, \( K_{\text{ext}} \gg K_{\text{int}} \) implies \( v \to \infty \), or \( K_h \to \infty \) for a finite \( R \). (Here also, \( R \to \infty \) would imply a vanishing power, and we exclude this possibility). Then the efficiency reaches its upper bound, \( \eta_{\text{C}}/(2 - \eta_{\text{C}}) \). Therefore, for \( 0 < v < \infty \), \( \eta^* \) lies in the range

\[ 1 - \frac{1}{2 - \eta_{\text{C}}} \leq \eta^* \leq \frac{\eta_{\text{C}}}{2 - \eta_{\text{C}}}. \quad (20) \]

### IV. CASE 2: FINITE \( K_c \) WITH \( \omega = 1 \)

In this case, the external irreversibility is considered only on the cold side. Further, we assume \( \omega = 1 \), i.e., Joule heat flows into the cold reservoir as shown in Fig. 2b. On the cold side, the thermal flux is

\[ \dot{Q}_c = \frac{\alpha T_c K_c I + R K_c I^2}{K_c - \alpha I}, \quad (24) \]

and

\[ T_{cM} = \frac{K_c T_c + R I^2}{K_c - \alpha I}. \quad (25) \]

Thermal flux on the (reversible) hot side of TEG is

\[ \dot{Q}_h = \alpha T_h I. \quad (26) \]

Then, optimizing power w.r.t \( I \), gives EMP as

\[ \eta^* = v + 1 - \sqrt{(v + 1)(v + \theta)}, \quad (27) \]
where $v = R K_c / (\alpha^2 T_h)$, with $K_{ext} = K_c$ and $K_{int} = \alpha^2 T_h / R$. Then, it can be shown that for $0 < v < \infty$, the corresponding value of $\eta^*$ lies in the range:

$$\frac{\eta_C}{2} \leq \eta^* \leq 1 - \sqrt{1 - \frac{\eta_C}{2}}. \quad (28)$$

In this model, the dissipation due to Joule heating results in decreasing the EMP of TEG.

Further, the maximum output power is

$$P_v^* = K_c T_h \left[ 1 + 2v + \theta - 2\sqrt{(v+1)(v+\theta)} \right]. \quad (29)$$

The limiting behaviour of $P_v^*$ is

$$P_{v \to 0}^* = K_c T_h (1 - \sqrt{1 - \eta_C})^2 \quad (30)$$

and $P_{v \to \infty}$ as in Eq. (23).

\section{V. CASE 3: FINITE $K_h$ WITH $\omega = 1$}

In this case, the system is simplified by considering external irreversibility on the hot side and the flow of Joule heat to the cold side of TEG, as shown in Fig. 3(a). Now, the heat fluxes on the hot and cold sides of TEM are given by

\[ Q_h = \frac{\alpha K_c T_h I}{K_h + \alpha I}, \quad (31) \]

\[ Q_c = \alpha T_c I + R I^2, \quad (32) \]

and so the power output can be written as function of $I$. Optimizing power w.r.t $I$, the EMP comes in the form

$$\eta^* = 1 - \theta - (v + \theta) A - v A^2, \quad (33)$$

where

$$A = \frac{1}{6v} \left[ x^{1/3} + \frac{(2v - \theta)^2}{x^{1/3}} - (4v + \theta) \right] \quad (34)$$

with $x = (54v^2 + (2v - \theta)^3 + 6\sqrt{3}v \sqrt{27v^2 + (2v - \theta)^3})$, which has been obtained using Mathematica software. In the above, $v = R K_h / (\alpha^2 T_h)$. Again, it is concluded that for $0 < v < \infty$, the value of $\eta^*$ lies in the same range as Eq. (28).

The maximum output power is

$$P_v^* = K_c T_h \frac{A}{A + 1} \left[ 1 - \theta - (v + \theta) A - v A^2 \right], \quad (35)$$

whose limiting behavior is identical with Eqs. (22) and (23).

\section{VI. CASE 4: FINITE $K_c$ WITH $\omega = 0$}

In this case, the external irreversibility is considered only on cold side, while internal irreversibility due to Joule heating considered to be on hot side of TEG [40], as shown in Fig. 3(b). Thermal flux on the hot side of TEG is then

$$\dot{Q}_h = \alpha T_h I - R I^2, \quad (36)$$

while the thermal flux on the cold side of TEG is given by $\dot{Q}_c = \alpha T_c M I$, where

$$T_c M = \frac{K_c T_c}{K_c - \alpha I}. \quad (37)$$

Therefore,

$$\dot{Q}_c = \frac{\alpha K_c T_c I}{K_c - \alpha I}. \quad (38)$$

Then, the EMP is given by

$$\eta^* = 1 - \frac{\theta^2}{(vB - 1)(B - \theta)}, \quad (39)$$

where

$$B = \frac{1}{6v} \left[ 4v\theta + 1 - y^{1/3} - \left( \frac{1 - 2v\theta)^2}{y^{1/3}} \right) \right]. \quad (40)$$

with

$$y = 54v^2 \theta^3 + (2v\theta - 1)^3 + 6\sqrt{3}v \theta \sqrt{(2v\theta - 1)^3 + 27v^2 \theta^3})$$

and $v = R K_c / (\alpha^2 T_c)$. Then, for $0 < v < \infty$, $\eta^*$ lies in the same range as Eq. (20).

The maximum output power is

$$P_v^* = K_c T_c \frac{B(1 - vB)}{\theta} \left[ 1 - \frac{\theta^2}{(vB - 1)(B - \theta)} \right]. \quad (41)$$

For $v \to 0$, the above expression reduces to Eq. (30) and, for $v \to \infty$, it equals Eq. (23).
VII. CASE 5: FINITE $K_h = K_c$ WITH $\omega = 1/2$

We have so far considered an external irreversibility which is towards either hot or cold reservoir. Similarly, the internal irreversibility due to Joule heating was assumed to be rectified, thus rejecting itself in one of the two reservoirs only. However, an interesting special case arises with the simultaneous presence of internal as well as external irreversibilities. This is when we treat the external irreversibilities at the hot and the cold contact in a symmetric manner, as well as consider a symmetric dumping of Joule heat at each reservoir [3]. More precisely, it means setting $K_h = K_c = K_0$, and $\omega = 1/2$. In this case, we have

$$K_{\text{ext}} = \frac{K_0}{2}, \quad K_{\text{int}} = \frac{\alpha^2 (T_h + T_c)}{2R},$$

and so parameter $v = RK_0/\alpha^2 (T_h + T_c)$. This model is also solvable for optimal power, and yields EMP of the form:

$$\eta^* = \frac{(2 - v - \theta ) - \sqrt{(v + \theta + 2)(v + \theta + 2\theta)}}{2(1 - \theta) - v(3 + 4\theta + \theta^2)} (1 - \theta).$$

In the limit, $v \to 0$, it reduces to the endoreversible model, yielding the $\eta_{\text{ext}}$ value. When $v \to \infty$, the internal irreversibility becomes dominant as compared to external irreversibility, and we obtain Eq. (23) with $\omega = 1/2$. Thus, for $0 < v < \infty$, $\eta^*$ lies in the range

$$1 - \sqrt{1 - \eta_C} \leq \eta^* \leq \frac{2\eta_C}{4 - \eta_C}.$$  \hspace{1cm} (43)

The maximum output power is

$$P^*_v = \frac{K_0 T_h}{2} \left[ 1 + v + \theta + v\theta - \sqrt{4\theta + v(2 + v)(1 + \theta)^2} \right].$$

Maximum power has the following limiting expressions:

$$P^*_v \to \frac{1}{2} K_0 T_h (1 - \sqrt{1 - \eta_C})^2,$$

and for $v \to \infty$, it reduces to Eq. (23).

VIII. DISCUSSION

We have analyzed the maximum power conditions of a TEG taking into account two kinds of irreversibilities, internal due to Joule heating and external due to non-ideal thermal contacts with heat exchangers. In this treatment, we have assumed tight coupling between the fluxes, and so neglected any heat leakages. Thus, we have considered special instances of these models which turn out to be exactly solvable. The main focus has been the derivation of compact expressions for EMP as well as maximum power. All expressions for EMP obey well-defined upper and lower bounds when the ratio of external to internal thermal conductances (parameter $v$) vanishes or becomes very large.

For better comparison, EMPs in all the five cases have been plotted versus $v$, in Fig. 4. The vanishing of $v$ represents so-called endoreversible limit [30, 47] and its large values represents so-called exoreversible limit [31, 33]. Thus, we are able to highlight interpolating behavior of EMP in a TEG, by studying various special cases. Similarly, we have discussed expressions for optimal power in all the cases. It is observed that the maximum power in all cases, approaches the same limiting value (Eq. (23)), as $v \to \infty$.

Apart from the exact expressions, the universal properties of EMP near equilibrium ($\theta \approx 1$) are also of interest. Thus, we find that the EMP, in all the cases, follows the linear response universality [21, 43] given by $\eta_C/2$, and is independent of parameter $v$. This is expected from the lower and upper bounds of EMP as they also obey the same feature. The coefficients of the second order terms, in general, depend on $v$. However, in the symmetric case $\omega = 1/2$, we find that the second order term is also universal and is given by $\eta_C^2/8$ [22]. Further, the second order terms in different cases show an interesting trend if we set $v = 1$, i.e. consider equal magnitudes for the external and the internal irreversibilities in each case. This is also known as the thermal impedance matching condition [31]. Note that this implies a different operating point for each case. These cases are depicted in Fig. 5.
by tuning this load. The electric power output by the TEG can also be given by

\[ P = I^2 R_{load}. \]  

(47)

Here, we are interested in the value of the load at the maximum power output. In particular, for case 1, by equating Eq. (47) with the expression of power (using Eq. (16) and (17)), we obtain the following expression for \( R_{load} \)

\[ R_{load} = \frac{\alpha K_h (T_h - T_c) - (R K_h + \alpha^2 T_c) I}{I (K_h + \alpha I)}. \]  

(48)

In order to find the \( R_{load} \) at maximum power in case 1, we substitute the following value at maximum power

\[ I^* = \frac{K_h}{\alpha} \left( \sqrt{\frac{1 + \nu \theta}{\theta (1 + \nu) \alpha}} - 1 \right). \]  

(49)

Now, plug in the value of \( I^* \) in Eq. (48). The value of \( R_{load} \) at maximum power becomes,

\[ R_{load}^* = R \left( 1 + \frac{1}{\nu} \right), \]  

(50)

where \( v = R K_h / (\alpha^2 T_c) \). Operationally, this should be the value of load resistance of the TEG works at the maximum power. In the limit, \( v \to \infty \), when thermal contacts are perfect, \( R_{load} \) tends to \( R \). On the other hand, as \( v \to 0 \), \( R_{load} \) tends to the value \( \alpha^2 T_c / K_h \). For case 2 also, we obtain the same value of optimal load as in Eq. (50). Similarly, for other cases, the value of \( R_{load} \) can be calculated. These expressions are complicated for the cases 3 and 4. We indicate the values of \( R_{load} \) in different cases for the impedance matching condition and for small temperature differences, in Table I.

### IX. CONCLUSION

We have studied optimal power conditions for a few special configurations of external and internal irreversibilities in a one-dimensional TEG model with constant properties. We find explicit expressions for EMP which are functions only of the ratio of external to internal thermal conductances, apart from the ratio of bath temperatures. The bounds on EMP are discussed for each case, whereby the model approaches either endoreversible or exoreversible limit. We have also discussed the symmetric case (case 5) and show interpolating behavior of EMP between CA efficiency and Schmiedel-Seifert efficiency [3][31]. Further, interesting trend is shown by EMP near equilibrium for the thermal impedance matching condition. This also helps to distinguish the relative magnitudes of EMP in the various configurations. From the studied cases, we conclude that higher values of EMP are obtained when both internal and external irreversibilities are taken on the hot side. Similarly, lower values

### Table I: EMP upto second order term for all cases at thermal impedance matching condition (\( v = 1 \)) along with equivalent load resistance corresponding to maximum power.

| Case | \( \omega \) | \( v = \frac{R_{case}}{R_{imp}} \) | EMP at \( v = 1 \) | \( R_{load} / R \) at \( v = 1 \) |
|------|-------------|----------------|-----------------|----------------|
| 1    | 0           | \( \frac{R K_h}{\alpha T_c} \) | \( \frac{\eta_C}{2} + \frac{3}{2} \eta_C^2 + O(\eta_C^3) \) | 2               |
| 2    | 1           | \( \frac{R K_h}{\alpha T_c} \) | \( \frac{\eta_C}{2} + \frac{3}{2} \eta_C^2 + O(\eta_C^3) \) | 2               |
| 3    | 1           | \( \frac{R K_h}{\alpha T_c} \) | \( \frac{\eta_C}{2} + \frac{3}{2} \eta_C^2 + O(\eta_C^3) \) | 2 + \( \frac{\eta_C}{2} + \frac{3}{2} \eta_C^2 \) |
| 4    | 0           | \( \frac{R K_h}{\alpha T_c} \) | \( \frac{\eta_C}{2} + \frac{3}{2} \eta_C^2 + O(\eta_C^3) \) | 2 - \( \frac{\eta_C}{2} + \frac{3}{2} \eta_C^2 \) |
| 5    | 3           | \( \frac{R K_h}{\alpha T_c} \) | \( \frac{\eta_C}{2} + \frac{3}{2} \eta_C^2 + O(\eta_C^3) \) | 2 - \( \frac{1}{2} \eta_C^2 \) |

### A. Equivalent electrical circuit of TEG

Finally, we discuss the operational meaning of optimization of power with respect to the electric current. There is an external load resistance \( R_{load} \) in the TEG circuit and the current flowing through it can be varied

\[ I = \frac{R_{load}}{R_{load} + R_{imp}} \]  

(51)

FIG. 5: Efficiency at maximum power for \( v = 1 \), as a function of \( \eta_C \) for various cases as in Fig. 4. As reference, the upper and lower bounds of EMP are marked, along with Curzon-Ahlborn (CA) value.

TABLE I: EMP upto second order term for all cases at thermal impedance matching condition (\( v = 1 \)) along with equivalent load resistance corresponding to maximum power.
of EMP are obtained if these irreversibilities are taken on the cold side. Again, dumping of Joule heat on the hot side raises EMP as compared to dumping it on the cold side. Thus our analysis provides an insight into improving the performance of a TEG. Out of five discussed cases, cases 1 and 4 are recommended for designing a TEG. If one requires TEG with higher efficiency, then one goes for a design based on case 1. If the requirement is of a device with higher output power, then one can choose a design as in case 4. In this choice, the design of specific thermoelectric materials which have an ability to rectify Joule heat, also plays an important role. Finally, a parallel analysis can be undertaken for thermoelectric refrigerators.

**ACKNOWLEDGEMENTS**

JK acknowledges financial support in the form of Senior Research Fellowship from IISER Mohali, and useful discussions with I. Iyyappan and Varinder Singh.

[1] S. Carnot. *Reflections on the Motive Power of Fire*. Dover books on science. Dover Publ., 1960.
[2] J. M. Gordon. Generalized power versus efficiency characteristics of heat engines: The thermoelectric generator as an instructive illustration. *American Journal of Physics*, 59:551–555, June 1991.
[3] Y. Apertet, H. Ouerdane, C. Goupil, and P. Lecoeur. Irreversibilities and efficiency at maximum power of heat engines: The illustrative case of a thermoelectric generator. *Phys. Rev. E*, 85(3):031116, March 2012.
[4] D. Nemir and J. Beck. On the significance of the thermoelectric figure of merit $z$. *Journal of Electronic Materials*, 39(9):1897–1901, Sep 2010.
[5] H. Littman and B. Davidson. Theoretical bound on the thermoelectric figure of merit from irreversible thermodynamics. *Journal of Applied Physics*, 32(2):217–219, 1961.
[6] G. J. Snyder and E. S. Toberer. Complex thermoelectric materials, 2011.
[7] A. Majumdar. Thermoelectricity in semiconductor nanostructures. *Science*, 303(5659):777–778, 2004.
[8] A. Shakouri. Recent developments in semiconductor thermoelectric physics and materials. *Annual Review of Materials Research*, 41:399–431, 2011.
[9] S. B. Rifat and X. Ma. Thermoelectrics: a review of present and potential applications. *Applied Thermal Engineering*, 23(8):913 – 935, 2003.
[10] F. J. DiSalvo. Thermoelectric cooling and power generation. *Science*, 285(5428):703–706, 1999.
[11] Y. Pei, X. Shi, A. LaLonde, H. Wang, L. Chen, and G. J. Snyder. Convergence of electronic bands for high performance bulk thermoelectrics. *Nature*, 473:66–69, May 2011.
[12] C. Goupil, W. Seifert, K. Zabrocki, E. Miller, and G. J. Snyder. Thermodynamics of thermoelectric phenomena and applications. *Entropy*, 13(8):1481–1517, 2011.
[13] H. J. Goldsmid. *Introduction to thermoelectricity*. Springer series in materials science., Springer., New York, 2010.
[14] D. M. Rowe, editor. *CRC Handbook of Thermoelectrics*. CRC Press, Boca Raton, Florida, 1995.
[15] G. J. Snyder and T. S. Ursell. Thermoelectric efficiency and compatibility. *Phys. Rev. Lett.*, 91:148301, Oct 2003.
[16] B. Andresen, P. Salamon, and R. S. Berry. Thermodynamics in finite time. *Phys. Today*, 37:62, 1984.
[17] G. Lebon and D. Jou. *Understanding non-equilibrium thermodynamics*. Springer., Berlin, 2008.
[18] B. Andresen. Current trends in finite-time thermodynamics. *Angewandte Chemie International Edition*, 50(12):2690–2704, 2011.
[19] P. Salamon, J. D. Nulton, G. Siragusa, T. R. Andersen, and A. Limon. Principles of control thermodynamics. *Energy*, 26(3):307–319, 2001.
[20] G. Benenti, G. Casati, K. Saito, and R. S. Whitney. Fundamental aspects of steady-state conversion of heat to work at the nanoscale. *Physics Reports*, 694:1 – 124, 2017. Fundamental aspects of steady-state conversion of heat to work at the nanoscale.
[21] C. Van den Broeck. Thermodynamic efficiency at maximum power. *Phys. Rev. Lett.*, 95:190602, Nov 2005.
[22] M. Esposito, K. Lindenberg, and C. Van den Broeck. Universality of efficiency at maximum power. *Phys. Rev. Lett.*, 102:130602, Apr 2009.
[23] H. Ouerdane, Y. Apertet, C. Goupil, and Ph. Lecoeur. Continuity and boundary conditions in thermodynamics: From Carnot’s efficiency to efficiencies at maximum power. *The European Physical Journal Special Topics*, 224(5):839–864, Jul 2015.
[24] Gaveau B. Moreau, M. and L. S. Schulman. Efficiency of a thermodynamic motor at maximum power. *Phys. Rev. B*, 85:021129, 2012.
[25] Y. Wang and T. Z.-Chun. Bounds of efficiency at maximum power for normal-, sub- and super-dissipative Carnot-like heat engines. *Communications in Theoretical Physics*, 59(2):175, 2013.
[26] D. C. Agrawal and V. J. Menon. The thermoelectric generator as an endoreversible Carnot engine. *Journal of Physics D: Applied Physics*, 30(3):357, 1997.
[27] Y. Apertet, H. Ouerdane, A. Michot, C. Goupil, and Ph. Lecoeur. On the efficiency at maximum cooling power. *EPL (Europhysics Letters)*, 103(4):40001, 2013.
[28] P. Chambadal. Les centrales nucléaires. *Armard Colin, Paris, France*, 4:1–58, 1957.
[29] I. I. Novikov. The efficiency of atomic power stations. *J. Nucl. Energy II*, 7:125–128, 1958.
[30] F. L. Curzon and B. Ahlborn. Efficiency of a Carnot engine at maximum power output. *American Journal of Physics*, 43:22–24, January 1975.
[31] T. Schmiedl and U. Seifert. Efficiency of molecular motors at maximum power. *EPL (Europhysics Letters)*, 83(3):30005, 2008.
[32] L. Chen and Z. Yan. The effect of heat transfer law on performance of a twoheatsource endoreversible cycle. *J. Chem. Phys.*, 90:3740, 1989.
8

[33] R. S. Johal. Global linear-irreversible principle for optimization in finite-time thermodynamics. *EPL (Europhysics Letters)*, 121(5):50009, 2018.

[34] Y. Apertet, H. Ouerdane, O. Glavatskaya, C. Goupil, and P. Lecoeur. Optimal working conditions for thermoelectric generators with realistic thermal coupling. *EPL (Europhysics Letters)*, 97:28001, January 2012.

[35] T. Lu, J. Zhou, N. Li, R. Yang, and B. Li. Inhomogeneous thermal conductivity enhances thermoelectric cooling. *AIP Advances*, 4(12):124501, 2014.

[36] C. W. Chang, D. Okawa, A. Majumdar, and A. Zettl. Solid-state thermal rectifier. *Science*, 314(5802):1121–1124, 2006.

[37] A. F. Ioffe. *Semiconductor thermoelements, and Thermoelectric cooling*. Infosearch, ltd., 1957.

[38] L. Onsager. Reciprocal relations in irreversible processes. *Phys. Rev.*, 38:2265–2279, Dec 1931.

[39] H. B. Callen. The application of Onsager’s reciprocal relations to thermoelectric, thermomagnetic, and galvanomagnetic effects. *Phys. Rev.*, 73:1349–1358, Jun 1948.

[40] N. Pottier. *Nonequilibrium Statistical Physics: Linear Irreversible Processes*. Oxford University Press, 2014.

[41] H. B. Callen. *Thermodynamics and an introduction to thermostatistics*. New York : Wiley, 2nd ed edition, 1985. Rev. ed. of: Thermodynamics. 1960.

[42] C. A. Domenicali. Irreversible thermodynamics of thermoelectricity. *Rev. Mod. Phys.*, 26:237–275, Apr 1954.

[43] Y. Apertet, H. Ouerdane, C. Goupil, and P. Lecoeur. From local force-flux relationships to internal dissipations and their impact on heat engine performance: The illustrative case of a thermoelectric generator. *Phys. Rev. E*, 88(2):022137, August 2013.

[44] Y. Apertet, H. Ouerdane, C. Goupil, and Ph. Lecoeur. Internal convection in thermoelectric generator models. *Journal of Physics: Conference Series*, 395(1):012103, 2012.

[45] A. Sisman and H. Yavuz. The effect of Joule losses on the total efficiency of a thermoelectric power cycle. *Energy*, 20(6):573 – 576, 1995.

[46] Y. Apertet, H. Ouerdane, C. Goupil, and Ph. Lecoeur. Revisiting feynman’s ratchet with thermoelectric transport theory. *Phys. Rev. E*, 90:012113, Jul 2014.

[47] M. Josefsson, A. Svilans, A. M. Burke, E. A. Hoffmann, S. Fahlvik, C. Thelander, M. Leijnse, and H. Linke. A quantum-dot heat engine operating close to the thermodynamic efficiency limits. *Nature Nanotechnology*, 2018.

[48] I. Iyyappan and M. Ponnurugan. Thermoelectric energy converters under a trade-off figure of merit with broken time-reversal symmetry. *Journal of Statistical Mechanics: Theory and Experiment*, 2017(9):093207, 2017.