NJL with eight quark interactions: Chiral phases at finite $T$

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Abstract. The thermodynamic potential and thermal dependence of low lying mass spectra of scalars and pseudoscalars are evaluated in a generalized Nambu — Jona-Lasinio model, which incorporates eight-quark interactions. These are necessary to stabilize the scalar effective potential for the light and strange quark flavors, which would be otherwise unbounded from below. In addition it turns out that they are also crucial to i) lower the temperature of the chiral transition, in conformity with lattice calculations, ii) sharpen the temperature interval in which the crossover occurs, iii) or even allow for first order transitions to occur with realistic quark mass values, from certain critical values of the parameters. These are unprecedented results which cannot be obtained within the NJL approaches restricted to quartic and six-quark interactions.

Keywords: stable vacuum, general spin 0 eight-quark interactions, chiral and $U_A(1)$ symmetries, scalar and pseudoscalar mass spectra, finite temperature

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Nambu – Jona-Lasinio (NJL) models ([1], for reviews see e.g. [2, 3]) have the very appealing property of describing dynamical breakdown of chiral symmetry. In the present talk we show our recently obtained results [4], drawing particular attention to two distinct patterns of chiral symmetry breaking (SB) and their impact on the nature, temperature values and slopes of chiral transitions. This study has its roots in the observation [5] that extensions of the NJL model to accomodate the approximate $SU(3)$ flavor symmetry of the $u,d,s$-quarks and the $U_A(1)$ breaking instanton induced ’t Hooft interaction [6], display an unstable/ metastable vacuum in stationary phase (SPA)/ mean field approximations, and subsequent resolution of this problem by the addition of eight-quark interactions to the Lagrangian [7, 8]. The multi-quark interactions considered are the most general non-derivative chiral symmetric spin zero combinations. A set of stabilization conditions constrain the coupling strengths, from which the $N_c$ dependence of the OZI-violating eight-quark interactions is inferred. Furthermore SPA coincides then with the mean field approach. The characteristics of the low lying pseudoscalar and scalar nonets at $T = 0$ have been reevaluated in the present framework [8]. One main conclusion is that identical spectra, except for the scalar singlet-octet mixing channel (strongest effect on the $\sigma$-meson mass) can be obtained from two distinct effective potentials. They are generated by just changing the strengths of 4- and 8-quark (q) couplings keeping all other model parameters fixed. The 4q coupling regulates the curvature at the origin and thus determines either the Wigner-Weyl or broken phase (in absence of other interactions). Higher order interactions can however induce symmetry breaking on top of the Wigner-Weyl phase, i.e. a second minimum arises with a finite condensate while the origin keeps further its status as a minimum. This is in contrast with the case in which
the curvature at the origin represents a maximum of the effective potential. Then higher order interactions will not generate further minima, but simply shift the position of the existing minimum. Here their action results in a perturbative effect around the broken phase, while in the former case they are the motor for non-perturbative SB. These starting configurations with double vacua at \( T = 0 \) lead to a lowering of the critical temperature. The phenomenon of multiple vacua is known to occur within several approaches to the QCD vacuum [9]. We show below that these patterns of SB are still present for realistic values of quark masses, although the origin loses of course its significance as a reference point for the curvature. Explicit fits reveal further that it is the ’t Hooft 6q interactions, without which neither stability of the vacuum nor a "twin fit" of mass spectra is possible.

The present analysis focuses on the gap equations at finite temperature, whose solutions represent the extrema of the thermodynamic potential. The expressions were derived within a generalized heat kernel scheme [10] which takes into account quark mass differences in a symmetry preserving way at each order of the expansion at \( T = 0 \) in [7] and we will consider from now on the isospin limit, \( m_u = m_d \neq m_s \)

\[
h_u + \frac{N_c}{6\pi^2} M_u (3I_0 - \Delta_u I_1) = 0; \quad h_s + \frac{N_c}{6\pi^2} M_s (3I_0 + 2\Delta_u I_1) = 0; \quad (1)
\]

which must be solved self-consistently with the stationary phase equations

\[
G h_i + \Delta_i + \frac{\kappa}{16} h_j h_k + \frac{g_1}{4} h_i (h_j^2 + h_k^2 + h_i^2) + \frac{g_2}{2} h_i^3 = 0; \quad (2)
\]

Here \( \Delta_{ij} = M_i^2 - M_j^2 \), \( \Delta_i = M_i \), \( m_i \), \( i,j,k = u, d, s \) (with cyclic permutations of \( u, d, s \) for three possible equations) and \( i \neq j \neq k \), \( M_i \) denote the constituent quark masses. It is obvious that \( h_u = h_d \) for the considered case. The factors \( I_i \) are given by the average

\[
I_i = \frac{1}{3} \left[ 2J_i (M^2_u) + J_i (M^2_s) \right]; \quad J_i (M^2) = \frac{2\pi}{t^2 / t} \rho (t\Lambda^2) \exp [tM^2]; \quad (3)
\]

and represent one-quark-loop integrals with the Pauli-Villars regularization kernel [11] \( \rho (t\Lambda^2) = (1 + t\Lambda^2) \exp [t\Lambda^2] \) where \( \Lambda \) is an ultraviolet cutoff (the model is not renormalizable). For this case one needs only to know

\[
J_0 (M^2) = \frac{\Lambda^2}{M^2} \ln \left( 1 + \frac{\Lambda^2}{M^2} \right); \quad J_1 (M^2) = \ln \left( 1 + \frac{\Lambda^2}{M^2} \right) \frac{\Lambda^2}{\Lambda^2 + M^2}; \quad (4)
\]

The model parameters are the four quark coupling \( G \), \( N_c^{-1} \), the ‘t Hooft interaction coupling \( \kappa \), \( N_c^{-3} \), the eight-quark couplings \( g_1 g_2 \) (\( g_1 \) multiplies the OZI violating combination), the current quark masses \( m_i \) and the cutoff \( \Lambda \). The stability of the effective potential is guaranteed if the couplings fulfill the following inequality [7]

\[
g_1 > 0; \quad g_1 + 3g_2 > 0; \quad G > \frac{1}{g_1} \frac{\kappa}{16} \frac{2}{16}; \quad (5)
\]

from which we deduce that \( g_1 \) must scale at most as \( N_c^{-3} \) and at least as \( N_c^{-4} \) [8].

The generalization to finite temperature of these expressions occurs in the quark loop integrals \( J_0, J_1 \). After introducing the Matsubara frequencies [12]
TABLE 1: Parameters of the model at \( T = 0 \). The couplings have the following units: \( G \, (\text{GeV}^2) \), \( \kappa \, (\text{GeV}^{-5}) \), \( g_1 \); \( g_2 \, (\text{GeV}^{-8}) \), \( m_u = m_d \, m_s \), and \( \Lambda \) are given in MeV. The values of constituent quark masses \( M_u = M_d \) and \( M_s \) are shown in MeV (only the case of global minima).

| Sets | \( m_u \) | \( m_s \) | \( M_u \) | \( M_s \) | \( \Lambda \) | \( G \) | \( \kappa \) | \( g_1 \) | \( g_2 \) |
|------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| a    | 5.8     | 183     | 348     | 544     | 864     | 10.8    | 921     | 0*      | 0*      |
| b    | 5.8     | 181     | 345     | 539     | 867     | 9.19    | 902     | 3000*   | -902    |
| c    | 5.9     | 186     | 359     | 544     | 851     | 7.03    | 1001    | 8000*   | -47     |
| d    | 5.8     | 181     | 345     | 539     | 867     | 5.00    | 902     | 10000*  | -902    |

TABLE 2: The masses, weak decay constants of light pseudoscalars (in MeV), the singlet-octet mixing angle \( \theta_p \) (in degrees), and the quark condensates \( \bar{u}u \), \( \bar{s}s \) expressed as usual by positive combinations in MeV.

| Sets | \( m_\pi \) | \( m_K \) | \( m_\eta \) | \( m_\eta^0 \) | \( f_\pi \) | \( f_K \) | \( \theta_p \) | \( \bar{u}u \) | \( \bar{s}s \) |
|------|-------------|-------------|--------------|--------------|----------|----------|--------------|----------|----------|
| a    | 138*        | 494*        | 480          | 958*         | 92*      | 118*     | -13.6        | 237      | 191      |
| b    | 138*        | 494*        | 480          | 958*         | 92*      | 118*     | -13.6        | 237      | 192      |
| c    | 138*        | 494*        | 477          | 958*         | 92*      | 117*     | -14.0        | 235      | 187      |
| d    | 138*        | 494*        | 480          | 958*         | 92*      | 118*     | -13.6        | 237      | 192      |

\[
J_0 (M^2) \quad J_0 (M^2 ; T) = 16\pi^2 T \sum_{n=-\infty}^{\infty} \frac{Z}{Z} \frac{d^3 p}{(2\pi)^3} \int_0^{\infty} ds \rho (s\Lambda^2) e^{-s(M^2 + p^2 + M^2)} ;
\]

and using the Poisson formula

\[
\sum_{n=-\infty}^{\infty} F(n) = \sum_{m=-\infty}^{\infty} \int_0^{\infty} dx F(x) e^{i2\pi nx} ;
\]

where \( F(n) = \exp \left[ s(2n + 1)\pi^2 T^2 \right] \) one integrates over the 3-momentum \( p \) leading to

\[
J_0 (M^2 ; T) = \frac{d^4}{d^4} \rho (s\Lambda^2) e^{-sM^2} \left( 1 + 2 \sum_{n=0}^{\infty} \left( 1 \right)^n \exp \frac{n^2}{4sT^2} \right) ;
\]

Similarly one gets \( J_1 (M^2 ; T) = \partial J_0 (M^2 ; T) = \partial M^2 \); One recovers at \( T = 0 \) the starting expressions (4) and verifies also that \( \lim_{T \to 0} J_0 (M^2 ; T) = 0 \).

Using these formulae in (1), we solve the system (1)-(2) numerically, assuming that the model parameters \( G \); \( \kappa \); \( g_1 \); \( g_2 \); \( m_i \); \( \Lambda \) do not depend on the temperature. As a result we obtain the temperature dependent solutions \( M_i (T) \), representing the extrema of the thermodynamic potential.

The fit of the model parameters is obtained by fixing low lying pseudoscalar and scalar meson characteristics at \( T = 0 \), (stars denote input) in Tables 1-3. As already observed in [8], \( f_0 \) (600) is the main observable responsive to changes in the OZI violating eight-quark interaction term, diminishing with increasing strength of the \( g_1 \) coupling. The \( M_i (T) \) are shown in Fig. 1 (sets c and d). There are either one or three \( \left( M_u^i = M_d^i = M_s^i \right) \), \( i = 1; 2; 3 \) couples of solutions at fixed values of \( T \). For set (c) (as well as (b)) only one branch of solutions is physical, \( i.e. \) positive valued. The other two have negative values.
for the light quark masses. One sees however (set c) that the onset of the transition occurs at a value of $T = T_a$ for which the other unphysical two branches meet and cease to exist. The rapid crossover occurs in the short temperature interval $125 < T < 140$ MeV. The crossover pattern is in contrast with the $SU(3)$ limit case with zero current quark masses, where one branch collapses to the origin $M_u = M_d = M_s = 0$ for all values of the remaining model parameters and $T$. In this case the transition is first order [13].

We observe however that below a certain critical value of $G \Lambda^2$ (accompanied by a critical value of $g_1$) one obtains, also for the case of realistic quark masses, solutions with all branches positive valued at any $T$. This is the case shown in Fig. 1 set (d). Two of the branches (starting from the stable minimum and the saddle solution at $T=0$) merge in the physical region at a certain $T_b$ and the surviving branch has a significantly lower mass value. This leads to the discontinuities in observables typical of first order transitions. The decrease in temperature observed in sets (c) and (d) is welcome in view of recent lattice calculations [14], obtained for finite values of the quark masses. In this case there is evidence that a rapid crossover occurs, as opposed to an expected first order transition for the massless case [15]-[18]. Lattice QCD data have not unambiguously settled the question about the order of the chiral transition. For physical values of the quark masses, calculations with staggered fermions favor a smooth crossover transition [16], while calculations with Wilson fermions predict the transition to be first order [19]. At zero chemical potential there is growing evidence that the transition is crossover, which would set an upper bound for the OZI-violating $8q$ coupling $g_1$.

We finally remark that the parameter set (a), without eight-quark interactions, evolves as function of $T$ qualitatively as in Fig. 1, however the crossover takes place at much larger temperatures, $T' \sim 210$ MeV. Also the transition is much smoother than for set (c).

The masses of scalar and pseudoscalar mesons at finite temperature obtained for the set (c) are shown in Fig. 2. As can be seen there is a rapid crossover for all meson masses in the same temperature interval as in Fig. 1. However, neither this rapid crossover nor the first order transition case (d) do imply restoration of chiral or $U_A(1)$ symmetry, but only the recovery of a distorted Wigner – Weyl phase, with the minimum of the
thermodynamic potential shifted to finite quark mass values due to flavor breaking effects.

The role played by the different multi-quark interactions can be further understood by analyzing the following two limits, with the parameter set (c) as starting condition.

Case 1: We set \( g_1 = g_2 = \kappa = 0 \) and remaining parameters as in (c). In this limit the gap equation has only one solution for the considered parameter set, thus the system is in a distorted Wigner – Weyl phase.

Case 2: We set \( \kappa = 0 \) and all other parameters fixed as in (c). In this case there is no \( U_A(1) \) breaking, but OZI violating effects are present. We verify that in this limit the gap equation has also only one solution, being again in a distorted Wigner – Weyl phase.

Thus the spontaneous symmetry breakdown seen in the full set (c) at \( T = 0 \) (and also in sets b and d) is driven exclusively by the 't Hooft interaction strength \( \kappa \). We wish not to include case (a) in the present discussion, as it violates the stability conditions of (5).

TABLE 3: The masses of the scalar nonet (in MeV) at \( T = 0 \), and the corresponding singlet-octet mixing angle \( \theta_\xi \) (in degrees).

| Sets | \( m_{a_0(980)} \) | \( m_{K_0(800)} \) | \( m_{f_0(600)} \) | \( m_{f_0(980)} \) | \( \theta_\xi \) |
|------|-------------------|-------------------|-------------------|-------------------|-------|
| a    | 963.5             | 1181              | 707               | 1353              | 24    |
| b    | 1024*             | 1232              | 605               | 1378              | 20    |
| c    | 980*              | 1201              | 463               | 1350              | 24    |
| d    | 1024*             | 1232              | 353               | 1363              | 16    |

In conclusion, the present study indicates that chiral eight-quark interactions have a strong effect on the temperature dependence of observables described by NJL models, offering a plethora of solutions deeply rooted in the nature of chiral transitions. Within the model its origins can be traced back to the pattern of dynamical chiral symmetry breaking. We have discussed at length how the different patterns emerge. Mesonic spectra built on the spontaneously broken vacuum induced by the 't Hooft interaction strength, as opposed to the commonly considered case driven by the four-quark coupling, undergo a rapid crossover to the unbroken phase, with a slope and at a temperature which is regulated by the strength of the OZI violating eight-quark interactions. This
strength can be adjusted in consonance with the four-quark coupling and leaves the spectra unchanged, except for the sigma meson mass, which decreases. This effect also explains why in the crossover region the sigma meson mass drops slightly below the pion mass. A first order transition behavior is also a possible solution within the present approach. Additional information from lattice calculations and phenomenology is necessary to fix finally the strength of interactions. We expect that the role of eight-quark interactions are of equal importance in studies involving a dense medium and extensions of the model with the Polyakov loop [20]. The latter is known to increase the transition temperature by 25 MeV [14]. In relation with the NJL model the role of the Polyakov loop has been investigated in several papers, see e.g [21]. The present study can be extended likewise. In the two flavor NJL the inclusion of eight quark interactions has been analyzed in connection with finite T, chemical potential and Polyakov loop [22], where the relevance of eight-quark interactions has been reported. Eight-quark physics has further been explored in presence of a constant magnetic field [23], and also in that case it provides for a rich structure of the effective potential.

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