How accurately can we measure the hydrogen $2S \rightarrow 1S$ transition rate from the cosmological data?

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Abstract. Recent progress in observational cosmology, and especially the forthcoming PLANCK mission data, open new directions in so-called precision cosmology. In this paper we illustrate this statement considering the accuracy of cosmological determination of the two-quanta decay rate of 2s hydrogen atom state. We show that the PLANCK data will allow us to measure this decay rate significantly better than in the laboratory experiments.

Keywords: CMBR polarization, CMBR theory, cosmological parameters from CMBR, recombination

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1 Introduction

Since the year 2000 the modern cosmology entered the stage which can be characterized as an epoch of “precision cosmology”. After Saskatoon, TOCO, BOOMERAMG and MAXIMA-1 data, and then through the WMAP, CBI, ACBAR, ACT and getting closer to the data release of the PLANCK mission, our knowledge of the Universe becomes more and more informative \[1–11\] . There is no doubt that merging the micro-physics learned on the Large Hadronic Collider (LHC) with macro-physics discovered in space missions like WMAP and PLANCK \[3, 4, 12, 13\], will make the picture of the evolving Universe more “colorful” and self-consistent.

In this paper we would like to illustrate the current status and perspectives of the "precision cosmology", considering a simple question, namely, with which accuracy one can measure the rate of the two-photons decay for $2S \rightarrow 1S$ transition in the hydrogen atom from cosmological data. Note that the process of cosmological hydrogen recombination crucially depends on this process \[14–17\].

From Quantum Electrodynamic we know the theoretical value for corresponding decay rate: $A_{2s1s} = 8.227 \text{ s}^{-1}$ (see \[18–22\]). However, although there are no doubts about this process, there is very little experimental verification of it because the corresponding experiments are very difficult \[23\]. Experimentally, 2S-1S two photon transition has been measured in \[24–26\] for the decay of K-shell vacancy in initially neutral atom using the photon-photon coincidence technique. In these experiments, the K-shell vacancy is produced by irradiating the targets by photons or radioactive isotopes, preferable decaying by nuclear electron capture. However, all these experiments mainly are devoted to investigation of the heavy ions, rather than hydrogen atom.

On the other hand for the hydrogen the two photon decay of 2S -level determines the rate of recombination in the “middle of recombination layer”, where the pattern of the CMB polarization is basically formed, we can hope that the precision cosmological data could allow us to estimate $A_{2s1s}$-constant with rather high accuracy. At least, using the CMB data in combination with HST, BAO,SDSS data sets, or CMB temperature and polarization along (like at PLANCK experiment), we can estimate the range of uncertainties of $A_{2s1s}$, as theoretical, as experimental one, which could give a significant impact to most probable value of the cosmological parameters (the baryonic density $\Omega_b$, the Cold Dark Matter density $\Omega_{DM}$, the Dark Energy density $\Omega_{DE}$, the spectral index of the adiabatic perturbations $n_s$ etc). This range of uncertainties can restrict even theoretical improvement of the kinetic of
the hydrogen recombination, which in general is very complex and requires incorporation of \( nS \rightarrow 1S \) and \( nd \rightarrow 1S \) transitions for levels \( n \gg 1 \) (see for review [27]).

We will use the WMAP 7 TT and TE observational data [3, 4] and the PLANCK mock data respectively, and show that the current cosmological data give us: \( A_{2s1s} \simeq 8^{+3.85}_{−1.8} \) (see Section 4). Given the expected sensitivity of the PLANCK data, we will show that the estimated accuracy can be further significantly improved: \( 8.086 \text{s}^{-1} < A_{2s1s} < 9.037 \text{s}^{-1} \) and \( 7.613 \text{s}^{-1} < A_{2s1s} < 9.505 \text{s}^{-1} \) at 1\( \sigma \) and 2\( \sigma \) level respectively.

## 2 Recombination of cosmological hydrogen

After He\(^4\) recombination the ionization history characterized by the free electron fraction \( x_e \) over redshift \( z \) is described by the following equation [14–16]:

\[
\frac{dx_e}{dz} = \frac{1}{(1+z)H(z)} C [\alpha_e n x_e^2 - \beta_e (1 - x_e) \exp \left( -\frac{B_1 - B_2}{k_B T} \right)],
\]

where \( H(z) \) is the Hubble expansion rate at a redshift \( z \), \( n \) is the number density of atoms, \( B_i \) is the binding energy of hydrogen in the \( i \)th quantum state, \( T \) is the temperature of the cosmic plasma, \( \alpha_e \) and \( \beta_e \) are the effective recombination and photo-ionization rate for the states of a principal quantum number greater than one. The factor \( C \) in Eq. 2 is given by [16]:

\[
C = \frac{1 + K A_{2s1s} n_H (1 - x_e)}{1 + K A_{2s1s} n_H (1 - x_e) + K \beta_e n_H (1 - x_e)},
\]

where \( A_{2s1s} \) is the two-photon decay rate of the 2s hydrogen state and \( K = \lambda_\alpha^2 / 8\pi H(z) \) with the wavelength of Ly-\( \alpha \) photon \( \lambda_\alpha \). This equation is applicable at the range of redshifts \( 800 \leq z \leq 1100 \), providing initial condition for the next stage \((z \lesssim 800)\), when the two-photon decay is no longer significant [16]. Currently, the decay rate \( A_{2s1s} \) is theoretically calculated to be \( 8.22458 \text{s}^{-1} \), where the slight improvement in numerical accuracy is made compared to earlier results [18, 19].

In Fig. 1, we show the ionization history of cosmic plasma for various decay rate \( A_{2s1s} \), where we numerically computed it with the help of the widely used RECFAST code with a slight modification [28–30]. In the same figure, we plotted the visibility function, which shows the probability of last scattering at a redshift \( z \). As it is seen from Fig. 1, the fraction of ionization \( x_e \) in the vicinity of \( z \approx 1000 \) increases with respect to the standard one, if the two-photon decay rate \( A_{2s1s} \) is lower, and vice versa. From Fig. 1, we may see that the last scattering occurs at more recent time with slightly wider spread, as the two-photon decay rate \( A_{2s1s} \) gets smaller. The change of the recombination rate as a function of \( A_{2s1s} \) leads to the observational traces in the CMB TT, TE, EE power spectra (refer to [31–35] for details) and therefore this allows us to determine \( A_{2s1s} \) with rather good accuracy.

## 3 CMB anisotropy

The whole-sky Stokes parameters of the CMB anisotropy can be decomposed in terms of spin 0 and spin \( \pm 2 \) spherical harmonics:

\[
\Delta T(\hat{n}) = \sum_{l,m} a_{T,lm} Y_{lm}(\hat{n}),
\]

\[
Q(\hat{n}) \pm iU(\hat{n}) = \sum_{l,m} -(a_{E,lm} \pm i a_{B,lm}) \mp 2 Y_{lm}(\hat{n}).
\]
Figure 1. Left Top. Ionization history: fraction of free electrons, \( x_e \), is plotted over a range of redshift \( z \) for various values of \( A_{2 \times 10^3} \) with \( A_0 \) being the theoretical prediction \( 8.22458 \times 10^{-3} \). Right Top. Relative variations of the ionization fraction \( \Delta x = 2(x_e - \overline{x_e})/(x_e + \overline{x_e}) \). Here \( x_e \) corresponds to current value of \( A_{2 \times 10^3} \), and \( \overline{x_e} \) corresponds to theoretical value of \( A_{2 \times 10^3} \). Bottom. The visibility function, which corresponds to the probability of the last scattering at a redshift \( z \), is plotted for various \( A_{2 \times 10^3} \).

In case of the Gaussian fluctuations the decomposition coefficients satisfy the following statistical properties \([36–39]\):

\[
\langle a_{T,lm} a_{T,l'm'}^{*} \rangle = C_{l}^{TT} \delta_{ll'} \delta_{mm'},
\]

\[
\langle a_{E,lm} a_{E,l'm'}^{*} \rangle = C_{l}^{EE} \delta_{ll'} \delta_{mm'},
\]

\[
\langle a_{T,lm} a_{E,l'm'}^{*} \rangle = C_{l}^{TE} \delta_{ll'} \delta_{mm'},
\]

where \( \langle \ldots \rangle \) denotes the average over an ensemble of universes. The power spectra for the temperature fluctuations \( C_{l}^{TT} \), for the E mode of polarization \( C_{l}^{EE} \) and for the TE cross correlation \( C_{l}^{TE} \), provide us invaluable information about early Universe \([36–39]\). Since the rate of cosmic recombination during its most important stage is mainly determined by the two-photon decay rate \( A_{2 \times 10^3} \), the correlation functions above are rather sensitive to the particular numerical value of \( A_{2 \times 10^3} \).

By using \textsc{Recfast} and \textsc{Camb} code \([28–30, 40]\) with small modifications, we have computed CMB power spectra for various \( A_{2 \times 10^3} \). In Fig. 2 we show these spectra together with the WMAP data \([3, 4, 41]\). Though we show only the binned data not to clutter the plots, we used the full WMAP data likelihood in the analysis in the next section. As noticed in Fig. 2, the shape of \( C_{l}^{TT}, C_{l}^{TE} \) and \( C_{l}^{EE} \) are sensitive to the value of \( A_{2 \times 10^3} \). As shown
in Fig. 1, the last scattering surface is affected by the variation of $A_{2s1s}$. The acoustic peaks of temperature anisotropy is, in particular, sensitive to the shift of the last scattering surface, and polarization is affected by the change in the thickness of the last scattering surface. Therefore, the EE powerspectrum and TE correlation as well as TT powerspectrum are essential to provide the tight contraint on the values of $A_{2s1s}$. Additionally, we find that CMB anisotropy at high multipoles is affected more than those at low multipoles, which may be understood by the fact that the shift and thickness change of the last scattering surface is negligible in comparison with the physical scales of low multipoles.

4 Constraints from the recent observational data

As discussed in the previous sections, the CMB power spectra are sensitive to the value of the decay rate $A_{2s1s}$. Noting this, we constrained the value $A_{2s1s}$ by the WMAP CMB data [3, 4]. For a cosmological model, we assumed $\Lambda$CDM + SZ effect + weak-lensing. Since the co-moving distance to the last scattering has dependence on Hubble expansion, there exist some level of parameter degeneracy between $A_{2s1s}$ and the Hubble parameter. From Fig. 4, we may see some degeneracy with respect to $A_{2s1s}$, and Hubble parameter. Besides WMAP
Table 1. the best-fit values of cosmological parameters + $A_{2s1s}$ with 1σ interval indicated. The scalar amplitude $A_s$ is at the $k_0 = 0.05$ [Mpc$^{-1}$].

| symbol | description | value |
|--------|-------------|-------|
| $\Omega_b h^2$ | baryonic density $\times h^2$ | $0.023^{+0.0008}_{-0.0003}$ |
| $\Omega_{DM} h^2$ | cold dark matter density $\times h^2$ | $0.114^{+0.006}_{-0.005}$ |
| $\tau$ | optical depth | $0.081^{+0.021}_{-0.009}$ |
| $n_s$ | spectral index | $0.965^{+0.015}_{-0.013}$ |
| $\log[10^{10} A_s]$ | scalar amplitude | $3.08^{+0.02}_{-0.04}$ |
| $A_{sz}$ | fitting coefficient of SZ effect | $0.047^{+0.06}_{-0.04}$ |
| $H_0$ [km/s/Mpc] | Hubble constant | $69.06^{+3.14}_{-3.06}$ |
| $A_{2s1s}$ [s$^{-1}$] | two-photon decay rate | $8.04^{+3.99}_{-1.8}$ |

CMB data, we additionally used data such as the Hubble Constant measurement with the Hubble Space Telescope (HST), Baryonic Acoustic Oscillation (BAO) data from SDSS and WiggleZ, and Big Bang Nucleosynthesis constraint [42–45]. These data are not directly sensitive to the $A_{2s1s}$, but they enhance the constraint on $A_{2s1s}$ by reducing the uncertainty of Hubble parameter. We ran the CosmoMC with slight modifications on a MPI cluster with 6 chains [40, 46]. For the convergence criterion, we adopted the Gelman and Rubin’s “variance of chain means” and set the R-1 statistic to 0.03 for stopping criterion [47, 48]. Analyzing the Markov chains produced by the CosmoMC, we obtained the best-fit values of the parameters and their confidence intervals. From the analysis, we impose the following constraint on the decay rate $A_{2s1s}$: $6.24s^{-1} < A_{2s1s} < 11.89s^{-1}$ and $4.47s^{-1} < A_{2s1s} < 14.67s^{-1}$ with 1 and 2σ confidence respectively. In Table 1, we show the best-fit values of the decay rate $A_{2s1s}$ and cosmological parameters with 1σ interval indicated. In Fig. 3, we show the likelihood distribution for each parameter and in Fig. 4 the marginalized likelihoods in the plane of $A_{2s1s}$ versus other parameters. From Fig. 4, we infer some level of parameter degeneracy between Hubble constant and $A_{2s1s}$. Fig. 4 also shows slight parameter degeneracy with the spectral index $n_s$. However, the spectral index, which determines the shape of the primordial power spectrum, often have slight level of degeneracy with other cosmological parameters, since the variation of the spectral index can mimic the variation of other parameters more or less. The degeneracy with $\Omega_{DM} h^2$ seen in Fig. 4 is attributed to the factor $h^2$.

Table 2. Assumed instrumental properties of the PLANCK mock data.

| Beam (FWHM) [arcminute] | temperature noise [$\mu$K] | polarization noise [$\mu$K] |
|-------------------------|---------------------------|---------------------------|
| 9.5                     | 6.8                       | 10.9                      |
| 7.1                     | 6.0                       | 11.4                      |
| 5                       | 13.1                      | 26.7                      |

As discussed previously, CMB polarization is sensitive to the $A_{2s1s}$, and CMB anisotropy on smaller angular scales is more sensitive than those on large scales. Therefore, the upcoming Planck data will provide a very tight constraint on $A_{2s1s}$, thanks to the low noise polarization data and the temperature data of high angular resolution. In order to assess the constraining power of the PLANCK surveyor data, we made the parameter forecast, using the PLANCK mock data. The PLANCK mock data was generated up to the multipole $l = 2000$ by the publically available FUTURCMB code with the expected sensitivity of the PLANCK surveyor.
Figure 3. Likelihood of $A_{2s1s}$: a solid line denotes a marginalized likelihood and a dotted line a mean likelihood (refer to [46] for distinction between them).

[12, 49], where we assumed the WMAP concordance model and the decay rate $A_{2s1s}$ to $8.22458 \text{s}^{-1}$. For the mock data, we assumed three channels with a sky fraction 0.65. The assumed instrumental properties of the three channels are summarized in Table 2. For the mock data constraint, we did not use the lensing convergence power spectrum, but only TT, TE, EE power spectrum. From the run of the CosmoMC with the mock data, we found the estimation error on $A_{2s1s}$ is $0.486 \text{s}^{-1}$, which is less than 6% of the central value. To be specific, the constraints imposed by the PLANCK mock data are $8.086 \text{s}^{-1} < A_{2s1s} < 9.037 \text{s}^{-1}$ and $7.613 \text{s}^{-1} < A_{2s1s} < 9.505 \text{s}^{-1}$ at 1σ and 2σ level respectively. The improvement mainly comes from temperature data on small angular scales and low noise polarization data, which are sensitive to the value of $A_{2s1s}$. We may further enhance the constraint by adding non-CMB data to PLANCK mock data. However, the improvement by non-CMB data mainly arise from the tightened constraint on Hubble parameter, which is already well constrained by PLANCK mock data.

5 Discussion

We have shown that the recent WMAP TT and TE data sets in combination with the BAO and HST data allow us to constrain the range of uncertainties of the decay rate $A_{2s1s}$ within the interval $+3.85, -1.8$, which corresponds in average to $\pm 34\%$. The PLANCK
mock data up to the multipole \( l = 2000 \) with the expected sensitivity of the PLANCK surveyor can significantly reduce the level of error bars down to \( 8.086 \text{s}^{-1} < A_{2s1s} < 9.037 \text{s}^{-1} \), around the most probable value \( A_{2s1s} \simeq 8.2 \text{s}^{-1} \). This estimation clearly illustrate that the theory of recombination, based on the theoretical value of \( A_{2s1s} \), is self-consistent. Actually, our analysis confirm prediction made in [31], that any modifications of the kinetic of recombination, which could change the fraction of ionization at redshifts \( 800 \leq z \leq 1100 \) by factor \( \Delta x_e/x_e = \delta \) would lead to corresponding changes of the TT and TE power spectrum \( \delta C(l)/C(l) \simeq \delta \). Taking into account that the natural limit of uncertainties in the power spectrum is the cosmic variance, one can get ; \( \delta C(l)/C(l) \simeq (l + 0.5)^{-1/2} \sim \delta \). In the model discussed above, the uncertainties of the decay rate \( A_{2s1s} \) are in order of \( \Delta A/A \simeq 6\% \) leading to \( \delta \sim 0.5\Delta A/A \) (see Fig.1). Thus, for \( \bar{l} = 2000 \) the corresponding constraint is given by \( \Delta A/A \simeq 1/\sqrt{\bar{l}} \sim 4\% \), which is close to the constrain, given by CosmoMC approach. Thus, if the systematic effects for the forthcoming PLANCK data release would be comparable to the cosmic variance limit for the range of multipoles around \( \bar{l} = 2000 \), our prediction of uncertainties of the decay rate \( A_{2s1s} \) would have experimental confirmation.
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