A generic two-band model for unconventional superconductivity and spin-density-wave order in electron- and hole-doped iron-based superconductors

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Abstract – Based on experimental data on the newly synthesized iron-based superconductors and the relevant band structure calculations, we propose a minimal two-band BCS-type Hamiltonian with the interband Hubbard interaction included. We illustrate that this two-band model is able to capture the essential features of unconventional superconductivity and spin-density-wave (SDW) ordering in this family of materials. It is found that bound electron-hole pairs can be condensed to reveal the SDW ordering for zero and very small doping, while the superconducting ordering emerges at small finite doping, whose pairing symmetry is qualitatively analyzed to be of nodal $d$-wave. The derived analytical formulas not only give out a nearly symmetric phase diagram for electron and hole doping, but also are likely able to account for existing main experimental results. Moreover, we also derive two important relations for a general two-band model and elaborate how to apply them to determine the band width ratio and the effective interband coupling strength from experimental data.

Since the recent discovery of a new iron-based layered superconductor [1], intensive efforts have been focused on the nature of superconductivity in this family of materials both experimentally [2–12] and theoretically [13–21]. Apart from the well-known copper oxide superconductors, these materials exhibit higher critical temperatures, 26 K in LaO$_{1-x}$F$_x$FeAs [1], 41 K in CeO$_{1-x}$F$_x$FeAs [7], 43 K in SmO$_{1-x}$F$_x$FeAs [10], and 52 K in PrO$_{0.89}$F$_{0.11}$FeAs [11], as well as 25 K in hole-doped La$_{1.2}$Sr$_2$OFeAs [6]. Very recently, a number of preliminary analyses have been made to unveil the mystery of superconducting nature, such as the multiband superconductivity, unconventional pairing symmetry, electron-doping and hole-doping effects, strong magnetic instability of the normal state. Experiments of the specific heat measurements [3], point-contact tunneling spectroscopy [4], and infrared reflectance spectroscopy [7] provided useful information. For example, according to the point-contact tunneling spectroscopy experiment [4], a remarkable zero-bias conductance peak was observed at the (110) interface, indicating the possible presence of nodal superconductivity. Angle-integrated photoemission spectroscopy measurements also provided certain support for the existence of SDW ordering and an indication of unconventional superconductivity [12].

In the theoretical aspect, the nature of unconventional superconductivity and the pairing mechanism have also been explored preliminarily by several groups based on the density functional theory (DFT) and dynamic mean-field theory [15–19]. It was pointed out that the electron-phonon interaction in this system may be too weak to lead to such high critical temperatures [20]. The possibility of spin triplet superconductivity was also suggested [21,22].

In this paper, we propose a minimal two-band BCS-type Hamiltonian with an effective interband Hubbard interaction...
interaction term included to model the system. The construction of our model Hamiltonian is based on band structure calculation results and intuitive physical pictures. Taking into account the main features of the Fermi surface for the undoped material calculated from the DFT and to capture the essential physics of the superconductivity and magnetism in the present system, we adopt a minimal version of the Fermi surface on a primary two-dimensional square lattice in the Fe-Fe plane: one hole band around $\Gamma$ and one electron band around $M$ points, both crossing the Fermi surface in the undoped case. Based on rational physical considerations, we introduce an effective interband antiferromagnetic coupling and elucidate that the effective intraband antiferromagnetic coupling could induce the superconducting pairing with a $d$-wave symmetry. Our main findings are: i) the normal state has an SDW order in the undoped case, while upon the charge carrier doping the SDW order drops rapidly and the superconducting order emerges; in particular, analytical results for both SDW and superconducting transition temperatures are explicitly presented, and their respective relations to the SDW and superconducting gaps are elaborated in details, with the former agreeing well with the existing experiment and the latter being quite useful for the future experimental verification; ii) due to the two-band (electron and hole) superconducting nature of the material, the transition temperature as a function of the effective doping density shows a nearly symmetric electron-hole doping dependence, accounting for the experimental results; iii) based on a reasonable analysis, the two-band superconducting state is expected to possess a $d$-wave pairing symmetry. Moreover, we address how to enhance superconducting $T_c$ in this family of iron-based materials and how to verify our two-band model/theory without any fitting parameter.

We start from a minimal two-band model, which captures the essential physics of the multiband unconventional superconducting state and SDW ordering,

$$H = \sum_{k\sigma} \xi_{1k} c_{k\sigma}^\dagger c_{k\sigma} + \sum_{k\sigma} \xi_{2k} d_{k\sigma}^\dagger d_{k\sigma} + U_{\text{eff}} \sum_{i,\sigma} n_{i1\sigma} n_{i2\sigma}$$

$$+ \sum_{kk'} V_{11}^{kk'} c_{k\uparrow}^\dagger c_{k'\downarrow} c_{-k\downarrow} c_{-k'\uparrow} + \sum_{kk'} V_{22}^{kk'} d_{k\uparrow}^\dagger d_{k'\downarrow} d_{-k'\uparrow} d_{-k\downarrow}$$

$$+ \sum_{kk'} \left( V_{12}^{kk'} c_{k\uparrow}^\dagger c_{k'\downarrow} c_{-k'\downarrow} d_{-k\uparrow} + \text{h.c.} \right),$$

where the two band bare dispersions are respectively approximated as $\xi_{1k} = -\hbar^2 k^2 / 2m_1 + \epsilon_0^{(1)} - \mu$ and $\xi_{2k} = -\hbar^2 k^2 / 2m_2 + \epsilon_0^{(2)} - \mu$ ($h = 1$ hereafter) based on the band calculations [13,14,17,18], where $m_1 = 1/t_1$ and $m_2 = 1/t_2$ are the effective masses of the hole and electron with $t_{1/2}$ as the effective nearest-neighbor hopping integrals in the primary square lattice of sites (with each site as a unit cell consisting of two Fe ions), which is rotated by the angle of $\pi/4$ and is enlarged by a factor of $\sqrt{2} \times \sqrt{2}$ with respect to the reduced Fe-Fe lattice, $\mu$ is the chemical potential depending on filling, and $\epsilon_0^{(1)}$ stands for the band offset, where $l = 1, 2$ represents, respectively, the nearly filled valence band (hole band) around $\Gamma = (0,0)$ and the nearly empty conduction band (electron band) around $M = (\pi, \pi)$, as shown schematically in Fig. 1. $d_{k\sigma}$ and $c_{k\sigma}$ are the corresponding electron annihilation operators of bands 1 and 2. For the hole (electron) band, we have the density of states $\rho_{1,2} = 1/(4\pi t_{1,2})$ with the bandwidth $W_{h,e} = 1/\rho_{1,2}$. $\epsilon_0^{(i)}$ is set to give the carrier density in the undoped case, i.e. the hole density of band 1 $n_0^h = 2\rho_1 \epsilon_0^{(1)}$, the electron density of band 2 $n_0^e = 2\rho_2 \epsilon_0^{(2)}$, and the effective total electron number per site is $(2 + n_0^e - n_0^h)$. $V_{11}^{kk'}$ and $V_{22}^{kk'}$ are the intraband pairing potentials for the two bands, $V_{12}^{kk'}$ denotes the interband pairing interaction. $U_{\text{eff}}$ represents the effective interband Hubbard interaction term, which will be elaborated in the next paragraph.

We now attempt to give an intuitive physical picture to understand the origin of superconducting pairing and the SDW ordering in this system. It is well known that the superexchange antiferromagnetic coupling derived from the strong coupling of the single-band Hubbard model provides a driving force for the unconventional $d$-wave superconducting pairing state in cuprate superconductors. In the present case with the two bands, the $i$-th site Hamiltonian is given by $H_i = U n_{i\uparrow} n_{i\downarrow} + U' n_{i\sigma} n_{i2\sigma} - J_{\text{H}1} \sigma_{1i} \sigma_{1i} - J_{\text{H}2} \sigma_{1i} \sigma_{2i}$ with $n_i$ and $\sigma_i$ as the $i$-site electron number and spin operators, while $U$, $U'$, and $J_{\text{H}}$ are, respectively, the on-site intraband Hubbard repulsion, the interband one, and Hund’s coupling constant, which involves also the Hund’s coupling and interband $U'$ and thus introduces more complications. To simplify this complicated and subtle issue but without loss of the key point, let us consider only the energy range above the bottom of band 2 (i.e., $\mu - \epsilon_0^{(2)}$), below which only band 1 is available for the occupation by electrons and thus adjust Hund’s coupling.
plays no role. The number of electrons in this range at zero temperature is \((1 + \rho_h / \rho_c)n_0^2 N\) with \(N\) as the number of sites. For the two empty neighboring sites, each having two levels, there will be four levels available for the occupation by four electrons in the considered energy range. In the absence of hopping, due to the on-site Hubbard repulsion, each site would have two electrons to lower the energy (see the left panel of fig. 2). In this case, there are six states, where the two lowest-energy states have two opposite spins in the two different bands (both up or down) due to Hund’s coupling, while the two relatively higher-energy degenerate states have two electrons/sites are involved), which introduces the effective antiferromagnetic interaction.

Fig. 2: Schematic representation of the origin of the antiferromagnetic interaction.

term lifts the energy \(J_H/2\) above the bare ground-state energy level, \(U_{\text{eff}}\) is \(\propto J_H/2\). Below we elaborate first that this \(U_{\text{eff}}\)-term can indeed lead to an SDW order at zero or very small electron/hole doping, where the superconducting pairing order is suppressed.

The SDW transition temperature \(T_{\text{SDW}}\) can be calculated from the following equation [23]:

\[
1 = U_{\text{eff}}\chi_0^{12}(Q),
\]

where \(\chi_0^{12}\) is the interband spin susceptibility

\[
\chi_0^{12}(Q) = -\sum_k \frac{f(\xi_{1k}) - f(\xi_{2k+Q})}{\xi_{1k} - \xi_{2k+Q}},
\]

with \(f(\varepsilon)\) as the Fermi-Dirac function \(f(\varepsilon) = 1/(1 + e^{\varepsilon/T})\). To obtain a simple analytic formula of \(T_{\text{SDW}}\), here we set \(m_1 = m_2\) and \(\epsilon_0 = \epsilon_0^{(2)} = \epsilon_0\), where the perfect nesting with vector \(Q = (\pi, \pi)\) between the two bands occurs in the undoped case \((\mu = 0)\), which leads to the SDW instability. Integrating the RHS of eq. (3), we have an equation for \(T_{\text{SDW}}\),

\[
\frac{T_{\text{SDW}}}{W} \approx \frac{2\gamma}{\pi} \frac{\epsilon_0}{W} \left(1 - \frac{\epsilon_0}{W} e^{-\left(\frac{\epsilon_0}{W}\right)}\right)^{-1} - 1.71 \left(\frac{W}{\epsilon_{\text{SDW}}}\right)^2,
\]

where \(\gamma \approx 0.577\) is the Euler constant, \(x\) is the effective small doping, and the condition \(\epsilon_0 \gg T_{\text{SDW}}\) has been used. When the electron or hole doping is zero, \(i.e., x = 0\), we have the largest \(T_{\text{SDW}}\). Remarkably, \(T_{\text{SDW}}\) drops drastically whenever more electrons or holes are doped, as seen clearly in eq. (4) and fig. 3. If \(m_1 \neq m_2\), \(T_{\text{SDW}}\) is expected to be lowered.

Below \(T_{\text{SDW}}\), the SDW ordering emerges, whose order parameter may be defined as

\[
\Delta_{\text{SDW}} = \frac{U_{\text{eff}}}{2} \sum_{k \sigma} \langle c_{k \sigma} d_{k+Q \sigma}^\dagger \rangle,
\]

where \(\langle \cdots \rangle\) denotes thermodynamic average. \(\Delta_{\text{SDW}}\) satisfies the following equation:

\[
1 = -U_{\text{eff}} \sum_k \frac{f(\eta_{2k} + \Omega_k) - f(\eta_{2k} - \Omega_k)}{2\Omega_k},
\]

where \(\Omega_k = \sqrt{\eta_k^2 + \Delta_{\text{SDW}}^2}\), \(\eta_k = (\xi_{1k} - \xi_{2k+Q})/2\) and \(\eta_{2k} = (\xi_{1k} + \xi_{2k+Q})/2\). Here the effective repulsive interaction \(U_{\text{eff}}\) between interband electrons may also be viewed as an attractive pairing interaction between electron and hole. Physically, it is noted that the pairing leads to form a “condensate” of bound electron-hole pairs in the triplet state or “excitons” [24], which exhibits the SDW ordering. The condensate of electron-hole pairs is actually a counterpart of Cooper electron-electron pairs. In this sense, it is straightforward to obtain the famous relation \(2\Delta_{\text{SDW}}(0) \approx 3.5T_{\text{SDW}}\), as in the case of the conventional weak-coupling superconductivity. Here we pinpoint out that this relation can be quantitatively verified by independent experimental methods, \(e.g.,\) the...
optical conductivity spectra [8] and resistivity or specific heat measurements. As seen from ref. [8], it may be estimated that $2\Delta_{\text{SDW}}(8\text{K}) \approx 350\text{ cm}^{-1} = 504\text{ K}$ and $T_{\text{SDW}} \approx 150\text{ K}$, leading to $2\Delta_{\text{SDW}}(8\text{K})/T_{\text{SDW}} \approx 3.4$, which agrees well with the present theory. It is worthwhile noting that, since the effective hopping integrals $t_{1,2}$ in the present model correspond to those between nearest-neighboring sites (unit cells) in the primary lattice, the above SDW ordering pattern is stripe-like antiferromagnetic on the reduced Fe-Fe square lattice, namely, the spin ordering pattern is ferromagnetic in each stripe along the $x$’(or $y$’)-direction of the reduced Fe-Fe lattice, while it is antiferromagnetic between stripes in the $y$’(or $x$’)-direction. This kind of SDW ordering was likely observed in a very recent neutron scattering experiment [25]. Remarkably, we estimate from eq. (5) that the antiferromagnetic moment per Fe atom (in unit of the Bohr magneton $\mu_B$): $m_\alpha \approx (4/2) \times \frac{\mu_B}{10^{-19}} = 2 \sum_{k\sigma} \langle c_{k\sigma} d_{k\sigma} \rangle = \frac{4\Delta_{\text{SDW}}}{U_{\text{eff}}} = 2 \times 3.52 \times \frac{10^{-19}}{(T_c/W)/(U_{\text{eff}}/W)} \approx 0.31$, where $m_i = (-1)^i \sum \langle c_{i\sigma} d_{i\sigma} \rangle$ is the $i$-th site moment. This estimation is also in agreement with the data reported in ref. [25]. In addition, we may have one more expectation from eq. (4) that $T_{\text{SDW}}$ is normally decreased with the increase of pressure because the effective band width $W$ (or the effective hopping integral $t$) that dominates $T_{\text{SDW}}$ in the exponential term is increased, as seen in ref. [26].

At this stage, we turn to address the superconducting ordering. In view of the fact that the SDW order drops very sharply to zero at a very small critical doping, it is reasonable and convenient to ignore the effect of the SDW order on the superconducting ordering above the critical doping level. In the following calculations, the pairing potentials involving two bands are expressed as:

$$V_{hh'}^{11} = J_{hh}\gamma_{hh'}\gamma_{hh'}', V_{hh'}^{22} = J_{ee}\gamma_{ee}\gamma_{ee'}', \quad V_{hh'}^{12,21} = J_{he,ee}\gamma_{hh'}\gamma_{ee}',$$

where $J_{hh}$, $J_{ee}$, and $J_{he,ee}$ are the corresponding coupling constants, $\gamma_k = \cos(L\theta_k)$ with $L = 2, 1, 0$ denoting, respectively, the nodal $d$-wave, nodal $p$-wave, and isotropic $s$-wave pairing functions, and $\theta_k$ as the angle between the vector $\mathbf{k}$ and the $x$-axis. Taking the BCS mean-field approximation, the quasiparticle eigenspectrum of the $l$-th band is then given by

$$E_{lh} = \sqrt{\Delta_0^2 + |\Delta_l|^2},$$

where $|\Delta_l|$ is the gap amplitude of the $l$-th band ($l = h, e$). They are determined from the following coupled gap equations [27]:

$$\Delta_h = \sum_k \gamma_k (J_{hh} \langle c_{-k\downarrow} c_{k\uparrow} \rangle + J_{he,ee} \langle d_{-k\downarrow} d_{k\uparrow} \rangle),$$

$$\Delta_e = \sum_k \gamma_k (J_{ee} \langle c_{-k\downarrow} c_{k\uparrow} \rangle + J_{he,ee} \langle e_{-k\downarrow} e_{k\uparrow} \rangle).$$

Then the self-consistent gap equations for $\Delta_h$ and $\Delta_e$ read

$$\begin{pmatrix} J_{hh} K_1 & J_{hh} K_2 \\ J_{ee} K_1 & J_{ee} K_2 \end{pmatrix} \begin{pmatrix} \Delta_h \\ \Delta_e \end{pmatrix} = \frac{\Delta_h}{\Delta_e},$$

where $K_{1,2} = \sum_k \gamma_k^2 \tanh(E_{1,2k}/2T)/E_{1,2k}$ satisfy

$$\det \begin{pmatrix} J_{hh} K_1 - 1 & J_{hh} K_2 \\ J_{ee} K_1 & J_{ee} K_2 - 1 \end{pmatrix} = 0.$$  

The superconducting transition temperature $T_c$ is actually determined from eq. (9) with $\Delta_0 \to 0$.

Let us first address a more general case: $J_{ee}, J_{hh} > 0$, and $J_{ee} J_{hh} \neq J_{ee} J_{he} > 0$ ($J_{ee} = J_{he}$). By the introduction of dimensionless couplings: $J_{hh,ee} = J_{hh,ee}/W_h, J_{he,ee} = J_{he,ee}/W_h, J_{he,ee}/W_h$, and $J_{he} = J_{he} J_{ee} = J_{he} J_{hh}$, the $T_c$ formula is derived as

$$\frac{T_c}{\sqrt{W_h W_h}} = e^{C_{T_c}} \left[ n_e n_h (2 - n_e) (2 - n_h) \right]^{1/4} e^{-\pi_0 \frac{m_i}{W_h}},$$

with $C_{T_c} = \ln(e^\gamma/\pi) \approx -0.568$ (for $L = 2, 1, 0$ cases) and

\(^3\)Considering that the almost degenerated two hole bands and two electron bands connected to two Fe atoms per site and the $m_i$ counts merely the contribution per site from one nesting channel between one hole and one electron bands, while there are four such independent channels, we have approximately the factor $(4/2)$ here.
\( \lambda_{\text{red}} \) as the reduced pairing strength being given by

\[
\lambda_{\text{red}}^{-1} = \left\{ \left[ \frac{1}{4} J J \ln \frac{n_e (2 - n_e) W_e^2}{n_h (2 - n_h) W_h^2} + \frac{\tilde{J}_{hh} - \tilde{J}_{ee}}{2} \right]^2 + \tilde{J}_{eh} \tilde{J}_{he} \right\}^{1/2} / JJ,
\]

where \( n_e = n_0^e + [W_h/(W_e + W_h)] x \) and \( n_h = n_0^h - [W_e/(W_e + W_h)] x \), and the condition \( \epsilon_0 (1,2) \gg T_c \) has been used. For a special case: \( \tilde{J} J = 0 \), by taking the limit of \( \tilde{J} J \to 0 \) in the above equation, the \( T_c \) formula is reduced to

\[
T_c = \frac{e^2}{\pi} \left( \frac{W_e}{W_h} \right) \left[ \sqrt{n_e (2 - n_e)} \right]^{J_{ee}}_{J_{eh} + J_{he}} \times \left[ \sqrt{n_h (2 - n_h)} \right]^{J_{hh}}_{J_{he} + J_{eh}} e^{-\frac{1}{J_{ee} + J_{hh}}}.
\]

In this special case, more intriguingly, it is found from eq. (8) that the ratio between two gaps \( r(T) = \Delta_h / \Delta_e = \tilde{J}_{he} / J_{ee} = \tilde{J}_{hh} / J_{eh} \) being independent of temperature and other variables, therefore, if the ratio value \( |r| \) is experimentally found to be independent of temperature and other variables, it is implied that the system is just in this special case. Note that this result is valid for any two-band superconductivity model described by eq. (7).

For a general case, \( i.e. \), \( \tilde{J} J \neq 0 \), the phase diagram of \( T_c - x \) calculated from above formula is plotted in fig. 3, where we choose the parameters as \( W_e = W_h = W \) (for simplicity but without loss of generality), \( J_{ee}/W = 0.15 \), and \( \epsilon_0 (1,2) = 0.05 W \), \( U_{\text{eff}}/W = 0.3 \) (since \( \tilde{J}_H \approx 0.9 eV \), \( i.e. \), \( U_{\text{eff}} \approx 0.45 eV \), such choice leads to the effective band width \( W \sim 1.5 eV \), which is not unreasonable in the present system). From fig. 3, we note that the superconducting order emerges from a very small effective doping and then decays rather slowly to zero at \( x = x_c (e,h) = \pm 2 n_0 (e,h) \). When \( J_{ee} = J_{hh} \) and \( n_0^e = n_0^h \), the \( T_c \)-\( x \) dependence is symmetric for the effective electron and hole doping, while the symmetric feature changes slightly if \( J_{ee} \) (or \( n_0^e \)) is not too much different from \( J_{hh} \) (or \( n_0^h \)) (as estimated from the band calculations), in agreement with the experimental result [6]. In particular, if one intraband pairing strength (or the interband coupling) with the largest value is fixed, both the interband coupling (regardless of its sign) and other intraband pairing strength(s) enhance \( T_c \) significantly, reaching the maximum as \( [J_{hh} \to J_{hh} \to J_{ee}] \), whose value is explicitly given by eq. (11). This feature may be helpful for searching even higher \( T_c \) superconductors. Based on our results, another possible way to increase \( T_c \) in this family of superconductors is to increase the effective band width \( W \) (or the effective hopping integral \( t \)) since we have approximately \( T_c \propto W e^{-\alpha / W} \) with \( \alpha \) being a \( W \)-independent coefficient by noting that \( J_{ee,hh} / W \propto W \).

In this sense, a higher pressure may enhance \( T_c \), in contrast to that of \( T_{\text{SDW}} \). This expectation was seen in a very recent high-pressure experiment that showed clearly the enhancement of \( T_c \) with the shrinkage of the lattice [28].

The normalized tunneling conductances along the directions (110) and (001) of the primary lattice vs. the bias voltage are calculated self-consistently from the gap equations (8) with the nodal d-wave (cos(2\( \theta_h \)) pairing) at zero temperature and electron doping at \( x = 0.1 \), as shown in fig. 4 for a set of parameters used in plotting fig. 3 (except for the upper panel where \( J_{hh} = 0.5 J_{ee} \)). A sharp zero-bias conductance peak (ZBCP) along (110) and two coherence peaks corresponding to the two gaps (a larger \( \Delta_0^h \) and a smaller \( \Delta_0^e \)) are clearly seen, as expected. In addition, the two weak kinks in the ZBCP curve can also be seen, which correspond to the two gaps as well. The present ZBCP results are in agreement with the experimental observation reported in ref. [4].

Finally, we wish to address how to verify unambiguously the present two-band model/theory experimentally, without any fitting parameter. From the definitions of \( K_{1,2} \)
below eq. (7), we have
\[ K_{1,2}(T_c) = \frac{\ln \sqrt{n_{h,c}(2-n_{h,c})} - \ln (T_c/W_{h,c}) + C_T}{W_{h,c}}, \]
\[ K_{1,2}(0) = \frac{\ln \sqrt{n_{h,c}(2-n_{h,c})} - \ln(\Delta^0_{h,c}/W_{h,c}) + C_0}{W_{h,c}}, \] (12)
\[ \text{where } C_0 = (\ln 2 - 1/2) \approx 0.193 \text{ for the nodal } d\text{-wave and } p\text{-wave cases.} \]
\[ \text{and } C_0 \text{ is zero for the } s\text{-wave and } p\text{-wave cases.} \]
\[ \text{On the other hand, from eq. (8), we have } W_{h,e} K_{1,2}(T) = [-\tilde{J}_{e,h} + r^{-1}(T)\tilde{J}_{h,e,h}] / JJ \text{ for any} \]
\[ \text{temperature } T \leq T_c. \]
\[ \text{Combining them, we can find an important relation:} \]
\[ \frac{|-\ln(\Delta^0_{c,h}/T_c) + C_0 - C_{T,c}|}{|\ln(\Delta^0_{c,h}/T_c) + C_0 - C_{T,c}|} \times |r_0r_c| = \frac{W_h}{W_e}, \] (14)
\[ \text{where } r_{0,c} = r(0), \text{ and } r(T_c) \text{ are, respectively, the above introduced gap ratio } r(T) \text{ below eq. (11) at zero and transition} \]
\[ \text{temperatures. Since the RHS of eq. (14) depends only on the ratio of the two band widths, this relation can be verified} \]
\[ \text{by experimentally measured data for } \Delta^0_{c,h} \text{ and } r_{0,c} \text{ at various doping levels and then be used to determine the ratio of } W_h/W_e. \]
\[ \text{Similarly, we can obtain another useful relation:} \]
\[ \frac{\ln |r_0|}{1 + (W_h/W_e)r_0r_c} \times |r_c - |r_0| | = |\tilde{J}_{e,h}| / J J / J \] (15)
\[ \text{Once } (W_h/W_e) \text{ is determined from eq. (14), this relation can not only be checked by experimental data of } r_{0,c} \text{ but also be used to determine the effective interband coupling strength } |\tilde{J}_{e,h}| / J J. \]
\[ \text{It is also interesting to see from eq. (15) that i) as long as } |r_0| \approx |r_0| \text{ is observed experimentally, the} \]
\[ \text{system is approximately in the mentioned special case; ii) if } |r_0| \approx 1 \text{ is seen experimentally, except for case i),} \]
\[ \text{the interband coupling is negligible; iii) if two bands} \]
\[ \text{superconduct independently with two different } T_c \text{'s, the} \]
\[ \text{system } T_c \text{ is determined by the band having a higher one, and} \]
\[ \text{the LHS of eq. (15) approaches to zero because } r_c \text{ is either divergent or zero at the lower } T_c. \]
\[ \text{It is notable that the above results are also valid for the two-band} \]
\[ \text{and gapped } p\text{-wave superconductors similar to} \]
\[ \text{the present system, except for a different constant term} \]
\[ (C_0 - C_{T,c})_g \approx 0.568. \]
\[ \text{In summary, we have proposed for the first time a minimal two-band (hole and electron) model, with the} \]
\[ \text{interband Hubbard interaction being also included. It has been shown that this simplified model is able to} \]
\[ \text{capture the essential physics of unconventional superconductivity and spin-density-wave ordering in the addressed} \]
\[ \text{new family of materials. The present theory not only accounts for very recent experimental results, but also} \]
\[ \text{demonstrates/elaborates a key role of the interband pairing coupling played in the significant enhancement of } T_c \]
\[ \text{(in comparison with that of each band) for a generic two-band system, regardless of the pairing symmetry.} \]

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