Quantum Field Induced Orderings in Fully Frustrated Ising Spin Systems

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Abstract

We study ordering mechanism which is induced by a quantum fluctuation in fully frustrated Ising spin systems. Since there are many degenerated states in frustrated systems, “order by thermal disorder” often takes place due to a kind of entropy effect. To consider “order by quantum disorder” in fully frustrated Ising spin systems, we apply transverse field as a quantum fluctuation. In triangular antiferromagnetic Ising spin system, there exists a ferromagnetic correlation in each sublattice. The sublattice correlation at zero temperature is enlarged due to transverse field. The quantum fluctuation enhances the solid order at zero temperature. This is an example of quantum field induced ordering in fully frustrated systems. We also study a case in which the transverse field induces a reentrant behavior as another type of order by quantum disorder, and compare correspondent cases in the classical systems.

Keywords: transverse Ising model, frustration, order by disorder, reentrant phase transition

1. Introduction

Orderings in frustrated systems are very interesting topics and have been studied by a number of researchers. Because frustrated systems, such as triangular, kagomé, and pyrochlore antiferromagnets, have many degenerated states, entropy plays an important role in ordering mechanism. In an unfrustrated model, such as ferromagnetic Ising model, ferromagnetic order destroys by increasing temperature. On the other hand, in geometrical frustrated systems, orderings due to thermal fluctuation often occurs, which is called “order by disorder” \cite{1,2,3,4,5,6}. For example, Villain et al. have studied homogeneous fully frustrated Ising spin system which is the so-called “Villain model” and concluded existence ordering owing to thermal fluctuation in this system \cite{1}. Other example is reentrant phase transition where the order parameter shows non-monotonic behavior as a function of temperature \cite{7,8,9,10,11,12}. Generally speaking, because frustration makes peculiar density of states, thermal fluctuation induced ordering can occur. Other type of fluctuation is quantum fluctuation. It is well-known that quantum fluctuation also makes orderings as well as thermal fluctuation \cite{13,14}. For example, Moessner, Sondhi, and Chandra have studied quantum dimer model \cite{14}. This model corresponds to classical dimer covering problem and Ising model with transverse field on its dual lattice by controlling the parameter of the hopping dimer and the on-site potential. In quantum dimer model, there are many ordering structure due to quantum fluctuation.

Motivation of our study is to clarify the effect of the quantum fluctuation comparing with the effect of the thermal fluctuation. We study the sublattice correlation function at the ground state as a function of transverse field on triangular Ising antiferromagnets. We also consider the quantum fluctuation induced reentrant behavior on frustrated decorated bond system.

2. Model

In this paper, we study the ground state properties of geometric fully frustrated Ising model with transverse field by exact diagonalization method for small system and by power method for relatively large system. We consider transverse Ising model on triangular antiferromagnets and on decorated bonds where thermal (\textit{i.e.} classical) reentrant phase transition occurs.

2.1. Triangular Ising Antiferromagnets

Most well-known geometric fully frustrated Ising model is triangular Ising antiferromagnets. Because there are many degenerated ground states in frustrated system, the residual entropy is very large and plays an important role in ordering phenomena. Number of researchers have studied the residual entropy of triangular Ising antiferromagnets analytically and they concluded this value is 0.323\textit{k_{B}} per spin, where the value is about 46.6\% of total entropy \cite{15,16,17,18,19}.

We consider the ground state properties of triangular antiferromagnetic Ising system with transverse field. The Hamiltonian of this system is given by

\begin{equation}
\mathcal{H} = J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - \Gamma \sum_i \sigma_i^\gamma,
\end{equation}

where \(\langle i,j \rangle\) denotes the pairs of the nearest neighbor of triangular lattice and \(\sigma_i^\gamma\) represents the \(\gamma\)-component of the Pauli matrix at \(i\)-th site:

\begin{equation}
\sigma_i^\gamma = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_i^\gamma = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.
\end{equation}

Although long-range order does not exist in antiferromagnetic Ising spin system on triangular lattice at zero temperature...
and at zero quantum fluctuation, the correlation function shows power law decay [20]:
\[
\langle \sigma^z_i \sigma^z_j \rangle \propto |r_i - r_j|^{-1/2},
\]
where \( r_i \) represents the position of the \( i \)-th site. From this result, we expect that the sublattice order can develop by additional perturbation such as transverse field. We consider the correlation function at the ground state of \( N = 18 \) triangular antiferromagnet with periodic boundary condition as shown in Figure 1. Three sublattices are defined as shown in Figure 1 and the sublattice magnetization \( m^z_a \) and \( z \)-component of total magnetization \( m^z \) are given by
\[
m^z_a = \frac{3}{N} \sum_{i} \sigma^z_i, \quad m^z = \frac{1}{N} \sum_i \sigma^z_i
\]
where \( \sigma = A, B, \) and \( C \) indicates the label of sublattice. Figure 2 shows the sublattice magnetization and the correlation of sublattice magnetization as a function of transverse field. \( (m^z_A)^2 = (m^z_B)^2 = (m^z_C)^2 \) and \( m^z_A m^z_B = m^z_B m^z_C = m^z_C m^z_A \) are satisfied from the symmetry. From Figure 2 we can observe the enhancement of the three-sublattice magnetization at small transverse field. This result is the example of quantum fluctuation induced ordering [14, 21].

2.2. Decorated Bond System

We study the correlation function of decorated bond system which is depicted in Figure 3. The circles and the triangles in Figure 3 denote system spins, \( \sigma_1, \sigma_2, \) and decoration spins, \( s_i \), respectively. The system spins connect many decoration spins with ferromagnetic coupling \(-J\) depicted solid lines and the system spins connect directly with antiferromagnetic coupling \( \frac{N_d}{2} J \) depicted dotted line in Figure 3. The Hamiltonian of this system is given by
\[
\mathcal{H} = -J \sum_{i} (\sigma^z_1 + \sigma^z_2) \sigma^z_i + \frac{N_d J}{2} \sigma^z_1 \sigma^z_2
\]
\[
-\Gamma \left( \sigma^z_1 + \sigma^z_2 + \sum_{i} s_i \right),
\]
where \( N_d \) is the number of the decoration spins.

First we consider the case of zero transverse field. The correlation function between the system spins behaves non-monotonic as a function of temperature due to entropy effect of decorated spins [7, 8, 9]. At \( T = 0 \), all the spins align the same direction, because it is the most favorable state energetically. At finite temperature the decoration spins can flip due to the thermal fluctuation. When each decoration spins have \( \pm 1 \) values randomly, the ferromagnetic paths depicted by the solid line in Fig. 3 are weakened, and the direct antiferromagnetic interaction becomes dominant. The states, in which the decoration spins align randomly, are entropically favorable. We can calculate exactly the correlation function between the system spins by tracing out the degree of the freedom of the decoration spins. The correlation function is given by
\[
\langle \sigma^z_1 \sigma^z_2 \rangle = \tanh K_{\text{eff}},
\]
\[
K_{\text{eff}} = \frac{N_d}{2} \log (\cosh 2\beta J) - \frac{\beta N_d J}{2},
\]
The upper panel of Figure 4 shows the correlation function as a function of temperature. This is thermal fluctuation induced reentrant behavior [7, 9].

Next we consider the case of zero temperature with finite transverse field. The correlation function between system spins also behaves non-monotonic as a function of transverse field.
Figure 4: (Upper panel) The correlation function of the system spins as a function of temperature. (lower panel) The correlation function of the system spins at the ground state as a function of transverse field. The correlation functions as a function of temperature and transverse field show non-monotonic behavior. The larger the the number of the decoration spins, the larger the absolute value of correlation function.

as shown in the lower panel of Figure 4. This is the similarity between the thermal fluctuation and the quantum fluctuation. From this result, we expect that the lattice systems with decorated bonds show quantum reentrant phase transition as well as thermal reentrant phase transition [22].

3. Conclusion and Future Works

In this paper, we considered the ordering in homogeneous frustrated systems due to quantum fluctuation comparing with thermal fluctuation. In antiferromagnetic Ising spin system on triangular lattice, we observed enhancement of three-sublattice spin structure at finite transverse field. We also studied the quantum reentrant behavior of the correlation function of the decorated bond systems as well as thermal fluctuation. In this study, we focus on only the ground state static properties. It is future work to clarify the dynamical ordering process, which is related to the quantum annealing [21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34].

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