Secure quantum communication through a wormhole

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An accumulation of theoretical evidence contribute to the picture of gravity as a manifestation of quantum entanglement in a certain many-body quantum system. This is in particular expresses in the ER=EPR conjecture, which relates gravitational Einstein-Rosen (ER) bridge with the Einstein-Podolsky-Rosen (EPR) quantum entangled pairs or, more generally, with the so-called Thermofield Double State. In this letter, the ER=EPR conjecture is employed to introduce unitary quantum teleportation protocol, which recycles the entanglement forming traversable generalization of the Einstein-Rosen bridge. In consequence, the wormhole remains unaffected by the quantum teleportation. Furthermore, it is shown that the protocol guarantees the unconditional security of the quantum communication. Performance of the protocol is demonstrated in a simple setting with the use of 5-qubit Santiago IBM quantum computer, giving fidelities above the 2/3 the classical limit for a representative set of teleported states. Security of the protocol has been supported by experimental studies performed with the use of the noisy quantum processor. Possible generalization of the protocol, which may have relevance in the context of macroscopic gravitational configurations, is also considered.

I. INTRODUCTION

It is well known that quantum entanglement can be utilized to provide an unconditionally secure method of communication. This is, in particular, the case in the entanglement-based quantum key distribution (QKD) [1], complemented with the classical one-time pad (OTP) cipher. The same concerns the superdense coding [2], which allows for secure exchange of classical bits, employing quantum entanglement. Moreover, thanks to the quantum teleportation protocol [3], secure communication can be extended from bits to qubits.

The problem with the above schemes is that a constant source of maximally entangled pairs is required for practical purposes. This is because the quantum entanglement is destroyed for every exchanged bit or qubit in a measurement process. This raises a quest to look for new secure quantum communication protocols, which can recycle the quantum entanglement and minimize the number of created entangled pairs. A proposal for such a scheme in the case of the so-called port-based teleportation has been presented in Ref. [4].

Here, we introduce another entanglement-recycling quantum communication protocol, inspired by the ER=EPR conjecture [5]. The conjecture contributes to the duality between gravity and quantum entanglement, which has arisen from the studies AdS/CFT correspondence [6] and holographic principle [7]. In the ER=EPR conjecture, the Einstein-Podolsky-Rosen (EPR) pairs are considered as being equivalent to the Einstein-Rosen (ER) bridge. The conjecture finds deep support in the case of the so-called anti-de Sitter (AdS) black hole [8], which are, in fact, two back holes connected by the Einstein-Rosen bridge. It has been justified that the AdS black hole corresponds to the Thermofield Double State (TFD), which in the large temperature limit reduces to the product of the EPR pairs [9]. Further support to the ER=EPR comes, among the others, from the geometric interpretation of the entanglement entropy [10] and the relation between tensor networks and hyperbolic geometry [11].

Following the ER=EPR conjecture, we will consider the TFD state of the AdS black hole. Then, quantum teleportation between two asymptotic regions connected by the ER bridge will be considered. In the usual case, such teleportation is associated with the intermediate measurement process, which leads to the emission of thermal Hawking radiation. This is, in particular, the case in the Hayden-Preskill protocol [12] in which measurement on a photon of Hawking radiation is necessary to accomplish the teleportation [13]. In contrast, in the case considered here, no measurement on the qubit of Hawking radiation is performed, so the process remains purely unitary. Thanks to this, the protocol can be designed such that the TFD state and the associated ER bridge are preserved during the teleportation. Therefore, if a qubit is put into the black hole on one side of the bridge, it can be recovered on the other side without affecting the wormhole.

Let us finally emphasize that at the classical level of General Relativity, the Einstein-Rosen, is considered as a non-traversable wormhole, which is because of the Null Energy Condition. In consequence, a photon sent into such a wormhole from one side will never reach the other side. Therefore, from the perspective of causal communication through the wormhole, its traversable counterpart has to be considered. The necessary violation of the Null Energy Condition, while unrealistic classically, may occur when quantum gravitational effects are taken into account (see, e.g., [14, 15]). In consequence, while referring to communication through the Einstein-Rosen bridge, we have in mind its traversable generalization. In the light of the gravity-entanglement duality, such traversable worm-
holes may play the role of quantum channels [16–18].

II. THE PROTOCOL

Let us first remind the standard teleportation protocol, for which the corresponding quantum circuit is shown in Fig. 1. Here the operator $\hat{U}_\psi$ prepares the teleported state, i.e. $|\psi\rangle = \hat{U}_\psi |0\rangle$, belonging to let say Alice. In order to teleport the state, Alice (A) and her friend Bob (B), share the EPR pair ($\text{EPR} = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$), which is then partially entangled with the state $|\psi\rangle$. Subsequently, Alice performs measurements on her two qubits and sends the result to Bob via the classical channel. The associated two bits are used by Bob to accomplish the teleportation.

$$A |0\rangle \quad \text{U}_\psi \quad H \quad \Box$$
$$A |0\rangle \quad H \quad \Box$$
$$B |0\rangle \quad \bigcirc$$

FIG. 1. The standard quantum teleportation protocol.

The first observation leading to our protocol is that the two classical bits can be encoded in a single qubit by virtue of the superdense coding. The protocol also utilizes the EPR pair, and the corresponding quantum circuit is shown in Fig. 2.

$$|0\rangle \quad H \quad X \quad Z \quad H$$

FIG. 2. The superdense coding protocol.

The second observation is that eventually, we can get rid of the measurement process and combine the two above protocols into the one shown in Fig. 3. The resulting protocol utilizes two EPR pairs to teleport one qubit using another qubit. Appropriate controlled gates have replaced the measurement operation.

In this scheme, Alice (sender) prepares the first qubit (the top one in Fig. 3) in the state $|\psi\rangle$, which will be teleported. Furthermore, Alice shares two EPR pairs with Bob (receiver). Then, Alice applies a unitary operation to her three qubits and sends the third (from the top in Fig. 3) qubit to Bob. The unitary operation is given by the following action on the basis states:

$$\hat{V} |a\rangle |b\rangle |c\rangle = C\text{Z}_{13}(\hat{H}_1 \otimes \text{CNOT}_{23}) \text{CNOT}_{12} |a\rangle |b\rangle |c\rangle$$

$$= \frac{1}{\sqrt{2}} (-1)^{a \oplus b \oplus c} |a\rangle |a \oplus b\rangle |a \oplus b \oplus c\rangle$$

where $a, b, c \in \{0, 1\}$. Here, $\hat{H}$ is the Hadamard operator, CNOT is the Controlled-NOT operator, and $C\text{Z}$ stands for Controlled-Z operator and $\oplus$ stands for XOR operation.

Finally, by applying the same $\hat{V}$ operation to his three qubits, Bob recovers his third qubit (the bottom qubit in Fig. 3) in state $|\psi\rangle$. Moreover, the remaining qubits are two EPR pairs, as initially (up to SWAP between the first and the second qubit, to have the same order of pairs). In consequence, the EPR pairs can be reused to teleport another qubit. The action of the protocol associated with the circuit Fig. 3 is, therefore, the following:

$$|\psi\rangle_A \otimes |\text{EPR}\rangle^\otimes 2 \rightarrow |\text{EPR}\rangle^\otimes 2 \otimes |\psi\rangle_B.$$

From the viewpoint of the security of the protocol, it has to be verified if any relevant information about the teleported state $|\psi\rangle$ can be inferred from the qubit which Alice sends to Bob (the middle qubit in Fig. 3). For this purpose one can trace the density matrix $\hat{\rho}_B = |\psi\rangle \langle \psi|$, over all the qubits except the exchanged one. Here, $|\psi\rangle$ corresponds to the state between Alice and Bob perform the $\hat{V}$ operation, as depicted in Fig. 3.

The resulting reduced density matrix for the exchanged qubit is $\hat{\rho}_H = \text{tr}_{1245} \hat{\rho}_0 = \frac{1}{2} \mathbb{I}$ and the corresponding von Neumann entropy is $S(\hat{\rho}_H) = -\text{tr} \hat{\rho}_H \ln \hat{\rho}_H = \ln 2$. The calculated entropy has maximal allowed value per single qubit. Therefore, the exchanged qubit is maximally entangled with the rest of the system. In consequence, performing measurements on his qubit returns a random value of 0 and 1, with equal probability, independently on the details of the teleported state $|\psi\rangle$. This result will be supported by an experiment in the presence of noise, discussed in Sec. IV. Furthermore, let us add that the exchanged qubit is an analog of the Hawking radiation photon, being in a thermal state. This justifies why the subscript $H$ has been used.
III. GRAVITATIONAL INTERPRETATION

As we have already mentioned before, the introduced algorithm can be interpreted in terms of teleporting a quantum state through traversable wormhole. Let us now explore this connection.

First of all, as discussed in Refs. [8, 9] the eternal AdS two sided black hole with wormhole is associated with the TFD state, which takes the following form:

$$|\text{TFD}\rangle := \frac{1}{\sqrt{Z}} \sum_n e^{-\beta E_n} |E_n\rangle_A |E_n\rangle_B,$$  \hspace{1cm} (3)

where $\beta = \frac{1}{T}$ is an inverse of the temperature, $Z$ is the normalization factor and $E_n$ are energy eigenstates of the boundaries of the AdS black hole. The $|E_n\rangle_A$ and $|E_n\rangle_B$ are the energy eigenstates corresponding to the sites of Alice and Bob respectively. The energy eigenstates $|E_n\rangle$ are considered to be the basis states of the quantum register of $N$ qubits. In consequence, the index $n$ runs from 0 to $2^N - 1$. The spacial case of large temperature limit ($T \rightarrow \infty$) will be of importance for our further discussion. In this case, the state (3) simplifies as follows:

$$|\text{TFD}\rangle \xrightarrow{T\rightarrow\infty} \sum_n |E_n\rangle_A |E_n\rangle_B = |\text{EPR}\rangle^\otimes N.$$  \hspace{1cm} (4)

For $N = 2$, the light-temperature $|\text{TFD}\rangle$ is just the state we considered in the previous section to be formed between Alice and Bob. In that case one can write $|E_0\rangle = |00\rangle, |E_1\rangle = |01\rangle, |E_2\rangle = |10\rangle$ and $|E_3\rangle = |11\rangle$.

Then, the state $|\psi\rangle$ can be viewed as a state which falls into the black hole from one boundary, travels through the wormhole, and then reaches the second boundary. The qubit which is sent from Alice to Bob can be interpreted as a photon of Hawking radiation, which allows teleporting the state $|\psi\rangle$ through the bridge. Notably, in the considered protocol, no measurement on the Hawking photon is performed.

The traced state representing one side of black hole, for example Alice part of EPR pairs, is maximally mixed: $\hat{\rho}_A = \text{tr}_B \hat{\rho}_{AB} = \frac{1}{2} \mathbb{1} = \rho_B$, where $\hat{\rho}_{AB} := |\text{TFD}\rangle \langle \text{TFD}|$. Entropy of this state is maximal and equal to $S(\rho_A) = -\text{tr} \rho_A \log \rho_A = N \ln 2$. Therefore, entropy of each black hole is equal to the number of EPR pairs. Supposing any qubit, contributing to the pair, covers a fixed area, e.g. the Planck area, the entropy is proportional to total area of the black hole horizon, in agreement with the Bekenstein-Hawking formula.

IV. EXPERIMENTAL DEMONSTRATION

Thanks to the recent developments in quantum computing technologies, the quantum protocol introduced in the previous sections can already be studied experimentally. For this purpose, we have employed superconducting Santiago 5-qubit quantum computer provided by IBM. The quantum computer is characterized by relatively high quantum volume equal 32 [19]. The topology of the quantum processor is linear. Please notice that the related Hayden-Preskill protocol has already been simulated on a 7-qubit ion trap quantum computer [20]. Furthermore, quantum simulations of the TFD states have recently been discussed in Refs. [21–23]

In our studies, the quantum teleportation protocol (3), has been performed for 6 states $\{ |00\rangle, |11\rangle, |01\rangle, |10\rangle, |+\rangle, |\rangle\rangle\}$.

The states form a representative set over the Bloch sphere.

The teleported states have been compared with the reference states employing the quantum fidelity measure:

$$F(\hat{\rho}_1, \hat{\rho}_2) := \left( \text{tr} \sqrt{\hat{\rho}_1 \hat{\rho}_2 \hat{\rho}_1} \right)^2.$$  \hspace{1cm} (5)

The experimental density matrix has been recovered by performing quantum tomography on the teleported qubit (the bottom qubit in Fig. 3). For each of the teleported states 12 runs (each with 8192 shots) of the algorithms have been made. Furthermore, both the case with and without applying measurement error mitigation were considered [24]. The resulting fidelities are shown in Fig. 4.

![FIG. 4. Quantum fidelities of the states teleported with the use of the protocol depicted in Fig. 3. The results have been obtained with the use of 5-qubit IBM Santiago superconducting processor. The dashed horizontal line corresponds to the quantum limit 2/3.](image-url)
One can think about generalizing our protocol to cases with bigger black holes, i.e., with more EPR pairs. We aim to find a unitary operator $\hat{V}$ with bigger black holes, i.e., with more EPR pairs. We presented in Fig. 5. The ansatz can be then related to the quantum circuit given by Eq. 1 to a higher number of qubits. The generalization protocol works for any qubit state $|\psi\rangle$, so we have to check fidelity for a set of $n$ states. Furthermore, for security reason, we also want to have the intermediate qubit being in maximally entanglement with the wormhole, which results in additional entropy term to the cost function. The proposed cost function is:

$$C = 1 - \frac{1}{2} \sum_{k=1}^{n} \left( F(\hat{\rho}_{\text{final}}, \hat{\rho}) + S(\hat{\rho}_{H}) / \ln 2 \right),$$

where the $1/2$ factor ensures that the function is non-negative and is equal zero for the optimal case. Here $\hat{\rho}$ is the density matrix of the measured state and $\hat{\rho}_{\text{final}} = |\psi_{\text{final}}\rangle\langle \psi_{\text{final}}|$ is the desired final density matrix.

In what follows, we show results of the procedure for the case of $N = 3$, which is a first step beyond the case considered so-far. Furthermore, the $n = 6$ states considered in Sec. IV are used in the averaging procedure. Concerning the assumed form of the $V$ operator the so-called $R_V$ ansatz is considered, for which the quantum circuit is shown in Fig. 6.

![FIG. 6. The ansatz $R_V$ for four qubits with 2 repetitions. The boxes are $R_{\theta}(\theta)$ rotation gates.](image-url)

In this case, the parameter space is given by the 12 angles $\{\theta_0, \theta_1, \ldots, \theta_{11}\}$, which significantly reduces the number of 30 real parameters for a general case (excluding the total phase and normalization). The initial values are randomly choosen from flat distributions for each of the angles, $\theta \in [0, 2\pi)$. Evolution of the cost function (7) for exemplary five optimization runs are shown in Fig. 7.

In total, 500 optimization steps have been performed with the use of the Constrained Optimization by Linear Approximation (COBYLA) algorithm [27]. The resulting complexity of the operator goes beyond the scope of this letter and will be addressed elsewhere. Here, our main focus is on verifying whether the operator $\hat{V}$ for a higher number of qubits can be found at all.

Our strategy is to determine the form of $\hat{V}$ using variational methods, which can be applied on a quantum computer or on a classical simulator of a quantum computer. We prepare quantum circuits consist of $2N + 1$ qubits, which has first qubit in state $|\psi\rangle$ and $2N$ qubits in the $|\text{TFD}\rangle$ state. Then, we apply a parametrization ansatz for $\hat{V}$ on the first $N + 1$ qubits. Finally, the same ansatz with the same parameters is applied on the last $N + 1$ qubits.

When our ansatz with given parameters is a good approximation of $\hat{V}$, the final state is expected to be $|\psi_{\text{final}}\rangle := |\text{EPR}\rangle \otimes N \otimes |\psi\rangle$. We require that our protocol works for any qubit state $|\psi\rangle$, so we have to check fidelity for a set of $n$ states. Furthermore, for security reason, we also want to have the intermediate qubit being in maximally entanglement with the wormhole, which results in additional entropy term to the cost function. The proposed cost function is:

$$C = 1 - \frac{1}{2} \sum_{k=1}^{n} \left( F(\hat{\rho}_{\text{final}}, \hat{\rho}) + S(\hat{\rho}_{H}) / \ln 2 \right),$$

where the $1/2$ factor ensures that the function is non-negative and is equal zero for the optimal case. Here $\hat{\rho}$ is the density matrix of the measured state and $\hat{\rho}_{\text{final}} = |\psi_{\text{final}}\rangle\langle \psi_{\text{final}}|$ is the desired final density matrix.

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In total, 500 optimization steps have been performed with the use of the Constrained Optimization by Linear Approximation (COBYLA) algorithm [27]. The resulting

| State | Fidelity | Entropy / ln 2 |
|-------|----------|---------------|
| $|0\rangle$ | $0.99960 \pm 0.00022$ | $0.99911 \pm 0.00061$ |
| $|1\rangle$ | $0.99950 \pm 0.00022$ | $0.99854 \pm 0.00063$ |
| $|+\rangle$ | $0.99978 \pm 0.00010$ | $0.99937 \pm 0.00030$ |
| $|-\rangle$ | $0.99977 \pm 0.00015$ | $0.99933 \pm 0.00043$ |
| $|\rangle$ | $0.99911 \pm 0.00030$ | $0.99745 \pm 0.00084$ |
| $|\rangle$ | $0.99944 \pm 0.00021$ | $0.9984 \pm 0.0006$ |

**TABLE I.** Uncorrected quantum fidelities and entropies (divided by $\ln 2$) of the intermediate (Hawking) qubit. The results were obtained on the 5-qubit IBM Santiago quantum processor.
values of the five cases are collected in Tab. II. The results confirm that the protocol under consideration can be successfully extended beyond the $N = 2$ case. Worth emphasizing is that the extension is non-trivial, e.g., is not just a result of adding “disconnected” EPR pair to the protocol shown in Fig. 3.

| Run | Cost function | Fidelity | Entropy/\ln 2 |
|-----|---------------|----------|--------------|
| 1.  | $2.8 \times 10^{-5}$ | 1        | $0.999945 \pm 7.7 \times 10^{-5}$ |
| 2.  | $2.9 \times 10^{-5}$ | 1        | $0.999942 \pm 5.1 \times 10^{-5}$ |
| 3.  | $5.0 \times 10^{-5}$ | 1        | $0.999903 \pm 7.3 \times 10^{-5}$ |
| 4.  | $2.8 \times 10^{-5}$ | 1        | $0.999944 \pm 4.6 \times 10^{-5}$ |
| 5.  | $3.1 \times 10^{-5}$ | 1        | $0.999938 \pm 5.8 \times 10^{-5}$ |

TABLE II. Values of fidelity and cost function for the final state and entropy of the “Hawking qubit” obtained in 500 iterations of the optimization. The optimization has been performed with the use of simulator of a quantum computer.

One can show that the $R_Y$ ansatz for $N = 2$ and with three repetitions covers the case of operator $\hat{V}$ defined in Eq. 1. As we have checked, the $N = 2$ case cannot be recovered with two repetitions. However, for the higher than two number of EPR pairs, the $R_Y$ ansatz is a promising candidate for the general form of the $\hat{V}$ operator. Further studies are needed to investigate properties of the general $\hat{V}$ operator in details, including its possible action as a scrambling operator.

VI. SUMMARY

In this letter, we have introduced an entanglement-recycling quantum teleportation protocol inspired by the considerations of the AdS black hole. The protocol works in analogy to sending a qubit through a traversable gravitational wormhole, which in our studies corresponds to the large temperature limit of the Thermofield Double State. The protocol has been tested on a representative set of states with the use of a 5-qubit IBM Santiago quantum computer, giving fidelities above the classical best strategy value. This confirms that the algorithm can already be implemented with the use of currently available quantum technologies. Furthermore, it has been justified both theoretically and by performing a quantum experiment that the protocol is secure with respect to performing measurements on the exchanged qubit. Finally, possible generalization of the protocol to the case with a higher number of EPR pairs has been investigated. The generalizations, which eventually may allow us to apply our results to the case of macroscopic AdS black hole, require further, detailed studies. Some of the remaining open theoretical issues are: answering whether the protocol can be generalized such that the $\hat{V}$ part acts as a scrambling operator, fate of the protocol in finite-temperature regime ($T < \infty$), and teleportation of higher dimensional quantum states (not only qubits). On the other hand, from the practical viewpoint, an open challenge is the photonic implementation of the introduced protocol.

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