Ultraheavy particles at the LHC?

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Abstract

If the effective cosmological constant \( \Lambda \) of the present universe is due to physical processes in the early universe operating at temperatures just above the electroweak energy scale, it is possible that new particles with multi–TeV masses exist. These ultraheavy particles may (or may not) show up at the Large Hadron Collider (LHC). If they do, they may provide new insights into the early universe and gravitational physics.

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I. INTRODUCTION

In a series of papers [1–3], we have argued that the effective cosmological constant \( \Lambda \) of the present universe, interpreted as a remnant vacuum energy density, may be due to the imprint of ultraheavy particles on the Hubble expansion of the early universe. The discussion is in the framework of the so-called \( q \)-theory which describes the evolution of the macroscopic gravitating vacuum energy density \( \rho_V[q] \) due to a microscopic conserved relativistic vacuum variable \( q \) (the original papers are [4, 5] and a brief review appears in [6]). In view of the upcoming Run 2 of CERN’s Large Hadron Collider (LHC), it may be timely to review the main assumptions of the argument and to clarify the meaning of the predictions. Natural units will be used with \( \hbar = 1 \) and \( c = 1 \).

At this point, it may already be worthwhile to present the basic equation for the remnant vacuum energy density in a flat Friedmann–Robertson–Walker (FRW) universe with cosmic time \( t \),

\[
\Lambda \equiv \lim_{t \to \infty} \rho_V(t) = r_{V,\infty} M^8 / (E_{\text{planck}})^4,
\]

(1.1)

where \( r_{V,\infty} \) is a nonnegative number, \( M \) the mass scale of the hypothetical new particles, and \( E_{\text{planck}} \) the reduced Planck energy,

\[
E_{\text{planck}} \equiv \sqrt{1/(8\pi G_N)} \approx 2.44 \times 10^{18} \text{ GeV}.
\]

(1.2)

Inverting (1.1) gives the following expression for the new mass scale \( M \):

\[
M = (r_{V,\infty})^{-1/8} \Lambda^{1/8} (E_{\text{planck}})^{1/2} \approx 5.56 \text{ TeV} \left( \frac{10^{-3}}{r_{V,\infty}} \right)^{1/8} \left( \frac{\Lambda^{1/4}}{2.25 \text{ meV}} \right)^{1/2},
\]

(1.3)

where the numerical value used for \( \Lambda \) follows from Table 2.1 in Ref. [7].

Taking Eq. (1.1) as it stands, the number \( r_{V,\infty} \) can be interpreted as an efficiency factor given the energy scales involved, \( M \) and \( E_{\text{planck}} \). Remark that the parametric dependence of (1.1) has already been discussed by Arkani-Hamed et al. [8], but without a convincing theory of how the vacuum energy density evolves (it is here that \( q \)-theory [4, 5] is supposed to take over). Still, the authors of Ref. [8] discuss rather persuasively the role of (1.1) for the so-called triple cosmic coincidence puzzle: why are the orders of magnitude of the energy densities of vacuum, matter, and radiation approximately the same in the present Universe?

Taking the point of view that the effective cosmological constant \( \Lambda \) of the present universe has been measured [7], Eq. (1.3) can be read as a prediction of the mass scale \( M \), provided the “efficiency factor” \( r_{V,\infty} \) is known.

The task, then, is to calculate the pure number \( r_{V,\infty} \) entering (1.1). This calculation is extremely difficult and, up till now, we only have an approximate phenomenological description [1–3]. In the present paper, we try to simplify the discussion as much as possible, in order to highlight the crucial assumptions of the argument. For definiteness, we assume \( q \) to come from a three-form gauge field [4], so that \( q \) has mass dimension 2.
II. SETUP

A. \( K \)-freezing model

In the framework of \( q \)-theory [4, 5], the analysis of Ref. [1] has shown that the sudden presence of ultraheavy particles (possibly created by a phase transition with decay afterwards) perturbs the Hubble expansion and kicks the vacuum energy density \( \rho_V(t) \) away from zero to a small positive value. This process occurs at a cosmic age of order

\[
t_{\text{kick}} \equiv E_{\text{planck}}/M^2 = \xi^{1/4} M^{-1},
\]

where the last expression has been written in terms of the energy-density hierarchy parameter

\[
\xi \equiv (E_{\text{planck}}/M)^4.
\]

In the same way as the authors of Ref. [8], we initially do not worry about what physics stabilizes the large hierarchy between \( M \sim \text{TeV} \) and \( E_{\text{planck}} \sim 10^{15} \text{TeV} \). In other words, we leave aside the well-known hierarchy problem and simply try to determine the mass scale \( M \) from cosmology, for the given value of \( E_{\text{planck}} \).

The fundamental issue is the freezing of the vacuum energy density created by the kick. As realized in Ref. [3], this freezing can be modeled by a time-dependent (or, better, temperature-dependent) gravitational coupling; see also Sec. II B for further comments. Specifically, we take a Brans–Dicke-type term in the action density of the form

\[
L_{\text{grav}} = K[q, \Phi] R[g],
\]

where \( \Phi \) stands for one or more of the new matter fields. Recall that the standard Einstein theory has an action-density term \( K_0 R[g] \) with constant \( K_0 = 1/(16\pi G_N) \). In a flat FRW universe with cosmic time \( t \), we now assume the following simplified behavior:

\[
K[q, t] = q(t)/2 + \theta_K(t) \left[q_0/2 - q(t)/2\right],
\]

\[
\theta_K(t) = \theta(t - t_K),
\]

in terms of the standard stepfunction

\[
\theta(t) = \begin{cases} 
1 & \text{for } t > 0, \\
0 & \text{for } t \leq 0.
\end{cases}
\]

Observe that \( q_0 \) in (2.4a) corresponds to the constant equilibrium value of the vacuum variable \( q \) and that this \( q_0 \) is inversely proportional to Newton’s constant, \( q_0 = 1/(8\pi G_N) \).

We also let the coupling constant of the ultraheavy matter component be controlled by another stepfunction,

\[
g^2(t) = \overline{g}^2 \theta_g(t),
\]

\[
\theta_g(t) = \theta(t - t_g).
\]

This time-dependent coupling constant generates an ultraheavy matter component, even if it is not present initially \((t < t_g)\). At a later moment, the ultraheavy particles decay, which
can be modelled by letting $g^2(t)$ drop to zero again and by having a constant decay constant $\lambda^2$ instead. For the kick mechanism to operate (that is, $r_V$ first kicked away from zero and then frozen), the following inequality is required:

$$t_g < t_K.$$  \hspace{1cm} (2.7)

Hopefully, these two timescales are of the same order of magnitude (see Sec. [V] for further discussion).

Next, define dimensionless variables [3] by use of the energy scales $M$ and $E_{\text{Planck}}$, together with the auxiliary hierarchy parameter $\xi$ from (2.2). The dimensionless cosmic time, in particular, is defined by $\tau \equiv t/t_{\text{kick}}$ with $t_{\text{kick}}$ from (2.1). The relevant dynamic variables are the dimensionless Hubble parameter $h(\tau)$, the rescaled relative $q$-parameter shift $x(\tau)$, the rescaled dimensionless energy density $r_{M1}(\tau)$ of the ultraheavy particles (called type-1), and the rescaled dimensionless energy density $r_{M2}(\tau)$ of the massless particles (called type 2). The type-1 particles are, for definiteness, assumed to be bosons. In a finite temperature context without chemical potential, the type-1 bosons have an equation-of-state parameter $w_{M1}$, which, depends only on $M$ and $T$. The massless type-2 particles have the equation-of-state parameter $w_{M2} = 1/3$ and the effective number of degrees of freedom $N_{\text{eff},2} = 10^2$, in order to represent the lighter particles of the Standard Model (see Sec. V.B of Ref. [2] for further discussion). With the temperature $T$ obtained from the energy density $\rho_{M2} = N_{\text{eff},2} (\pi/30) T^4$, it is then possible to write $w_{M1}$ in terms of $M$ and $\rho_{M2}$ and the resulting expression is denoted $\overline{w}_{M1}$; see Sec. A2 of Ref. [2] for details. Finally, we define the combination

$$\overline{r}_{M1} \equiv 1 - 3 \overline{w}_{M1},$$  \hspace{1cm} (2.8)

which has been found to drive the kick of the vacuum energy density [1].

The ordinary differential equations (ODEs) for the four dynamic variables are [3]:

\begin{align*}
(1 - \theta_K) \left[ 3h^2 - h \dot{x} + 6h^2 - x \right] + \theta_K \left[ 3h^2 - h \dot{x} + 6h^2 - x \right] = 0, \\
\dot{x} + (4 - \overline{r}_{M1}) h r_{M1} = +g^2 r_{M2} - g^2 r_{M1} - \lambda^2 r_{M1}, \\
\dot{r}_{M2} + 4 h r_{M2} = -g^2 r_{M2} + g^2 r_{M1} + \lambda^2 r_{M1}, \\
(1 - \theta_K) \left[ 3h^2 - h \dot{x}/\xi + 3h^2 x/\xi - \left( x^2/(2\xi) + r_{M1} + r_{M2} - 3h^2 \right) \right] + \theta_K \dot{x} = 0,
\end{align*}

(2.9a, 2.9b, 2.9c, 2.9d)

with $\theta_K$ given by (2.4b) and $g^2$ by (2.6), both in term of the dimensionless cosmic time $\tau$ (the overdot in these ODEs denotes differentiation with respect to $\tau$). The source terms in Eqs. (2.9b) and (2.9c) are somewhat different compared to those of (3.3b) and (3.3c) in Ref. [3]. Equations (2.9a) and (2.9d) correspond to (3.3a) and (3.3d) of Ref. [3] but are slightly rewritten (actually, these rewritten ODEs were already used for the numerics presented in Ref. [3]). Note that (2.9a) in the $\theta_K = 1$ phase (late times) gives the derivative of the standard Hubble equation $3h^2 - r_{M1} - r_{M2} - r_V = 0$ with a term $r_V = (1/2) x^2 = \text{const.}$, according to (2.9d) for $\theta_K = 1$. 

4
An exact solution of the ODEs (2.9) for \( g^2 = 0 \) is given by [1]

\[
\begin{align*}
  h(\tau) &= 1/(2\tau), \\
  x(\tau) &= 0, \\
  r_{M1}(\tau) &= 0, \\
  r_{M2}(\tau) &= 3[h(\tau)]^2 = r_{M2}(0) [a(0)/a(\tau)]^4,
\end{align*}
\]

(2.10a) (2.10b) (2.10c) (2.10d)

where the last expression uses the scale factor \( a(\tau) \), in terms of which \( h(\tau) \) is defined by \( \dot{a}(\tau)/a(\tau) \). The exact solution (2.10) holds for both phases, the early one with \( \theta_K = 0 \) and the late one with \( \theta_K = 1 \). Incidentally, the dimensionless cosmic time \( \tau = 0.27 \) corresponds to having \( T \approx M \).

The goal, now, is to study the kick of \( r_V(\tau) \) by the sudden presence of ultraheavy particles, which, in our model, results from a nonzero coupling constant \( g^2(\tau) \) for \( \tau \geq \tau_g > \tau_{\text{min}} \). For this purpose, the ODEs with \( g^2(\tau) \) from (2.6) are to be solved the following boundary conditions:

\[
\begin{align*}
  h(\tau_{\text{min}}) &= 1/(2\tau_{\text{min}}), \\
  x(\tau_{\text{min}}) &= 0, \\
  r_{M1}(\tau_{\text{min}}) &= 0, \\
  r_{M2}(\tau_{\text{min}}) &= 3[h(\tau_{\text{min}})]^2,
\end{align*}
\]

(2.11a) (2.11b) (2.11c) (2.11d)

which matches the solution (2.10) of the earlier phase \( \tau < \tau_{\text{min}} \).

For later use, we can mention that the \( \xi = \infty \) equations have been discussed in Ref. [3] and that an analytic result has been obtained for the vacuum energy density without \( K \)–freezing effects,

\[
  r_V(\tau) \bigg|_{(\xi=\infty, \text{no } K\text{–freezing})} = \frac{1}{8} \left[ \kappa_{M1}(\tau) r_{M1}(\tau) \right]^2.
\]

(2.12)

But (2.12) still needs two \( \xi = \infty \) ODEs to be solved, in order to obtain the explicit functions \( \kappa_{M1}(\tau) \) and \( r_{M1}(\tau) \).

**B. Dissipation model**

In this article, we primarily model the freezing of \( r_V(\tau) \) by taking a particular time-dependent \( K[q,t] \) and by remaining entirely within \( q \)–theory which describes reversible processes. But, as argued in Sec. IV of Ref. [1], the freezing of \( r_V(\tau) \) may very well be due to quantum-dissipative effects, that is, irreversible processes [9]. Concretely, very low-energy gravitons (and possibly neutrinos) may be responsible for the dissipation [10]. Specifically, the energies of these particles must be of the order of meV, which value traces back to (2.1).

In any case, a phenomenological description of this quantum dissipation has been given in Ref. [1], which we will now briefly review.

The following model equation [1] can be used:

\[
  \dot{r}_V^{\text{diss}}(\tau) = -\Gamma(\tau) \left[ r_V^{\text{diss}}(\tau) - r_{V,0}(\tau) \right],
\]

(2.13)
which recalls the standard description of bulk-viscosity effects in fluid mechanics (see, in particular, Eq. (78.1) of Ref. [9]). The interpretation of (2.13) is that $r_{V,0}(\tau)$ is the “bare” vacuum energy density driven by the kick from the ultraheavy particles and that $\Gamma(\tau) \geq 0$ is the rate at which the “excess” vacuum energy density is dissipated into particles. Equation (2.13) has the following exact solution [1]:

$$r_{V}^{\text{diss}}(\tau) = \int_{0}^{\tau} d\tau' \, \Gamma(\tau') \, r_{V,0}(\tau') \exp \left[ -\int_{\tau'}^{\tau} d\tau'' \, \Gamma(\tau'') \right], \quad (2.14)$$

for boundary condition $\rho_{V}(0) = 0$, which holds for times $\tau$ well after the Planck era.

In the next section, we will obtain a numerical estimate of the asymptotic value of $r_{V}^{\text{diss}}$ based on (2.13), by use of the following approximations:

$$r_{V,0}(\tau) = \frac{1}{8} \left[ \kappa M_{1}(\tau) \, r M_{1}(\tau) \right]^{2}, \quad (2.15a)$$

$$\Gamma(\tau) = \frac{\gamma}{\tau^{2}}, \quad (2.15b)$$

where the first approximation becomes exact in the limit $\xi \to \infty$, according to (2.12), and the second approximation is purely illustrative. Taking a relatively large value for $\gamma$ forces $r_{V}^{\text{diss}}(\tau)$ to follow $r_{V,0}(\tau)$, according to (2.13) or (2.14). A smaller value of $\gamma$ allows $r_{V}^{\text{diss}}(\tau)$ to reach a nonzero asymptotic value.

Remark that (2.13) is only part of the whole story, as it does not specify the detailed changes in the matter energy densities $\rho_{M,n}(\tau)$ corresponding to the change of $\rho_{V}^{\text{diss}}(\tau)$. But this partial description is perfectly valid during the kick phase, as the amount of energy carried by the vacuum then is negligible compared to that of the ponderable matter by a factor of order $\xi \sim 10^{57}$, according to (2.2) for $M \sim 10 \text{ TeV}$.

### III. NUMERICAL RESULTS

The numerical solutions of the ODEs (2.9) with boundary conditions (2.11) are readily obtained. We present numerical solutions in Figs. 1 and 2 for two choices of coupling constants ($g$, $\lambda$), which give a ratio $r_{M1}(0.3)/r_{M2}(0.3)$ of order 1/100 and 1, respectively. In an equilibrium context, these energy-density ratios would translate into a degrees-of-freedom ratio $N_{\text{eff,1}}/N_{\text{eff,2}} = 1/100$ for case 1 and a ratio $N_{\text{eff,1}}/N_{\text{eff,2}} = 100/100$ for case 2. Note that the small oscillations on $r_{V} = (1/2) \, x^{2}$ in the top-right panels of the figures disappear for even larger values of the hierarchy parameter, $\xi \gg 10^{7}$. At this moment, it may be useful to recall the analytic $\xi = \infty$ result (2.12). This analytic result suggest to consider the quantity $1/8 \left( \kappa M_{1} \, r M_{1} \right)^{2}$ shown in the third bottom-row panels of the figures, which is indeed smoother than the numerical result $r_{V}(\tau)$ shown in the top-right panels.

The ODEs (2.9) result from the $K$–freezing model and the main result of the numerical calculation is the asymptotic plateau of the vacuum energy density $r_{V}(\tau)$ as shown in the top-right panels of Figs. 1 and 2. For comparison, we also show, in the bottom-right panels, the vacuum energy density resulting from the dissipation model (2.13) with approximations (2.15). Similar results are obtained with $\Gamma(\tau)$ Ansätze that drop to zero faster than $1/\tau^{2}$. An exponential tail, for example, has been used in a previous version of the present paper [arXiv:1503.03858v1].
Figure 1. Numerical solution of the dimensionless ODEs (2.9) with an equation-of-state function $\bar{\kappa}_{M1}(\tau) \equiv 1 - 3\bar{w}_{M1}(\tau)$ as defined in Sec. A2 of Ref. [2]. The panels are organized as follows: the four basic dynamic variables $h(\tau)$, $r_{M1}(\tau)$, $r_{M2}(\tau)$, and $x(\tau)$ are shown on the top row and secondary or derived quantities on the bottom row. The dashes lines in certain panels refer to the second quantity listed in the respective panel label, for example, the dashed line in the bottom-left panel corresponds to $\theta_K$. The main result is the nonzero remnant value of the dimensionless gravitating vacuum energy density $r_V \equiv x^2/2$ shown in the top-right panel. The bottom-right panel shows, for comparison, the dimensionless vacuum energy density from quantum-dissipative effects, as modeled by (2.13) with approximations (2.15). The model parameters are \{\xi, N_{\text{eff,2}}, y^2, \lambda^2, \tau_g, \tau_K, \gamma\} = \{10^7, 10^2, 1, 10^2, 0.2, 0.4, 1/5\} and the ultraheavy type-1 particles are assumed to be bosons (similar results are obtained for type-1 fermions, with somewhat lower values for $r_V$ and $r_V^{\text{diss}}$ by approximately 20\%). The ODEs are solved over the interval $[\tau_{\text{min}}, \tau_{\text{max}}] = [0.02, 0.8]$ with the following boundary conditions at $\tau = \tau_{\text{min}} = 0.02$: $x$, $h$, $a$, $r_{M1}$, $r_{M2}$ = \{0, 25, 1/10, 0, 1875\}.

Figure 2. Same as Fig. 1 but with model parameters \{\overline{y}^2, \lambda^2\} = \{64, 1\}. These coupling constants make for a ratio $r_{M1}/r_{M2} \sim 1$ at $\tau = (\tau_g + \tau_K)/2 = 0.3$, whereas the coupling constant of Fig. 1 give a ratio of order $1/100$. 
For the case of Fig. 1 ("$N_{\text{eff,1}} = 1$") with an $r_V$ peak of order $2 \times 10^{-5}$, the frozen asymptotic value $r_{V,\infty}$ obeys the bound

$$r_{V,\infty}^{(\text{case-1})} \leq \max \left[ r_V(\tau) \right]^{(\text{case-1})} \sim 10^{-5},$$

which translates into the following lower-bound on $M$ from (3.3):

$$M^{(\text{case-1})} \gtrsim 10 \text{ TeV}. (3.2)$$

For the case of Fig. 2 ("$N_{\text{eff,1}} = 10^2$") with an $r_V$ peak of order $7 \times 10^{-2}$, the asymptotic value $r_{V,\infty}$ obeys the bound

$$r_{V,\infty}^{(\text{case-2})} \leq \max \left[ r_V(\tau) \right]^{(\text{case-2})} \sim 10^{-1},$$

which gives

$$M^{(\text{case-2})} \gtrsim 3 \text{ TeV}. (3.4)$$

**IV. CONCLUSION**

The results (3.2) and (3.4) from the $K$–freezing model of Sec. II A set the mass scale of the hypothetical new particles. If the underlying physics is able to relate the time scale $t_g$ of ultraheavy (type–1) particle creation and the time scale $t_K$ of the change in the effective gravitational coupling, these inequalities could be replaced by rough equalities and be all the more convincing. Perhaps such a single physical process is similar to the one of slow-roll particle production discussed in the context of inflation models [11]. In our case, the slow-roll phase must be relatively short. More importantly, such a process must not reinstate the cosmological constant problem (with a new scale of order $M^4$) and must, therefore, include the evolution of $q$. Particle creation [10] is also crucial for quantum-dissipative effects of the vacuum energy density, as illustrated by the alternative model of Sec. II B.

As (3.2) and (3.4) are only lower bounds, we cannot predict that the mass scale $M$ must necessarily be in reach of the LHC with a 13 TeV center-of-mass energy (or HiLumi LHC [12]) and we can only adopt a “wait–and–see” attitude.1 If multi–TeV particles are discovered at the LHC (and, admittedly, this is a big ‘if’), an exciting consequence may be that they provide a new window on the early universe and gravitational physics.

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1 As to the properties of the hypothetical new particles, we remain entirely agnostic. One possibility involves ultraheavy composite scalars from top-condensation models for the light Higgs [13–15], but many other models exist.
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