COSMIC RAY HEATING OF THE WARM IONIZED MEDIUM

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ABSTRACT

Observations of line ratios in the Milky Way’s warm ionized medium suggest that photoionization is not the only heating mechanism present. For the additional heating to explain the discrepancy, it would have to have a weaker dependence on the gas density than the cooling rate, $\alpha_i^2$. Reynolds et al. suggested turbulent dissipation or magnetic field reconnection as possible heating sources. We investigate here the viability of MHD-wave mediated cosmic ray heating as a supplemental heating source. This heating rate depends on the gas density only through its linear dependence on the Alfvén speed, which goes as $n_e^{-1/2}$. We show that, scaled to appropriate values of cosmic ray energy density, cosmic ray heating can be significant. Furthermore, this heating is stable to perturbations. These results should also apply to warm ionized gas in other galaxies.

Key words: cosmic rays – galaxies: halos – galaxies: ISM – H II regions – ISM: general – magnetohydrodynamics (MHD)

Online-only material: color figures

1. INTRODUCTION

Observations of [S ii]/Hα and [N ii]/Hα line intensity ratios in the warm ionized medium (WIM) show a spatial variation with distance from the galactic midplane $|z|$—larger line ratios are seen further from the disk (see Reynolds et al. 1999; Haffner et al. 2009). Such variation might be explained by variations in the ionization parameter $U$, the ratio of photon density to gas density. However, this would not explain the additional observation that the [S ii]/[N ii] ratio remains nearly constant with $|z|$. Under WIM conditions, variations in $U$ inevitably produces larger changes in sulfur (which can be either in the form of S ii or S iii) compared to nitrogen (which almost always appears as N ii), due to their different ionization potentials.

These observations may be explained by a spatial variation of the electron temperature $T_e$. An increase in $T_e$ with height above the disk could explain the enhanced [S ii]/Hα and [N ii]/Hα ratios. Also, because [S ii] and [N ii] have nearly the same excitation energy, the [S ii]/[N ii] ratio is nearly independent of $T_e$. So the near constant [S ii]/[N ii] ratio may also be explained this way.

But how can this variation in $T_e$ be explained? If only photoionization heating is important, then this increase in $T_e$ can potentially be accommodated by hardening of the spectrum away from the disk mid-plane (the possible role of hot, low mass evolved stars above the Galactic plane has recently been examined by Flores-Fajardo et al. 2011). However, a hard spectrum is inconsistent with He i λ5876 observations (Rand 1997, 1998). On the other hand, if there were a secondary heating mechanism with a weaker dependence on electron density than the $n_e^2$ dependence of photoionization heating, such heating would dominate far from the disk, where densities are low, and we would see a variation in $T_e$ that could explain the observed line ratio variations.

Many such supplementary heating mechanisms have been proposed, such as photoelectric heating from dust grains (Weingartner & Draine 2001), magnetic reconnection (Raymond 1992), and turbulent dissipation (Minter & Spangler 1997). We study here the possibility of cosmic ray (CR) heating. The process was outlined by Wentzel (1971), but not applied to the WIM (which had not been discovered at that time). If a CR population has a bulk velocity faster than the local Alfvén speed $v_A = B/\sqrt{4\pi\rho}$, magnetohydrodynamic Alfvén waves are generated and exhibit unstable growth (Wentzel 1968; Kulursd & Pearce 1969). In a steady state, these waves are damped by some other process(es), transferring energy to the gas. In this way a CR density gradient can indirectly heat the plasma. We will see that the resulting gas heating rate is proportional to $n_e^{-1/2}$, and is of the order required to explain the necessary temperature variations. Note that this process is quite different from collisional CR heating (Spitzer & Tomasko 1968). A strength of this mechanism is that the heating rate depends on only a few parameters which are either observable or can be estimated. We first consider the nature of CR trapping in the WIM in Section 2, before considering the CR heating rate and its local stability properties in Section 3.

2. ALFVÉN WAVE EQUILIBRIUM

For CR heating to be in place we must ensure that Alfvén waves are present in the WIM with enough energy to scatter the CRs. To do this we determine the wave damping, which in the WIM environment is due to ion–neutral friction and nonlinear Landau damping. We then balance this damping with the CR-induced wave growth to obtain an equilibrium condition. This condition determines the power spectrum in the waves. We can then derive a mean free path for the CRs and determine if they are well-trapped. We could also use the equilibrium damping rate to determine the heating of the gas, but as we will see the heating rate depends only on the characteristics of the CR population, provided they are well-trapped.

2.1. Ion–Neutral Damping in Nearly Ionized Gas

We follow Appendix C of Kulursd & Pearce (1969). These authors assumed the gas is nearly neutral; we assume it is almost fully ionized. We begin with the force equations for the neutral...
and charged components of the gas respectively. Assuming the transverse velocities of each component are in the form of an oscillator \( v_i = A_i e^{i(kz - \omega t)} \), we have

\[
\rho_i \omega_i^2 v_n = -i \nu_{vni} \omega_n (v_n - v_i) \tag{1}
\]

\[
\rho_i \omega_i^2 v_i = \rho_i \omega_i^2 v_i - i \nu_{vni} \omega_n (v_i - v_n). \tag{2}
\]

These equations are essentially those of two coupled, damped oscillators, one of which is driven with frequency \( \omega_k = k v_A \), the natural Alfvén wave frequency. The neutral–ion collisions are treated as a drag force parameterized by the collision frequency \( \nu_{vni} \). The movement of the ions by the Alfvén waves is impeded by the neutral particle population. This nudges the neutral component to follow behind the oscillating ions, removing energy from the Alfvén waves to do so.

We can rearrange these equations into convenient matrix form by using \( v_{ni} / v_{ni} = \rho_i / \rho_n \):

\[
(\omega^2 - \omega_k^2 + i \omega v_{ni}) v_i - i \omega v_{ni} v_n = 0 \tag{3}
\]

\[-i \omega v_{ni} v_i + (\omega^2 + i \omega v_{ni}) v_n = 0. \tag{4}
\]

Setting the determinant of this matrix to zero gives us the dispersion relation for \( \omega_k \):

\[
\omega^3 + i v_{ni} \left( 1 + \frac{\rho_n}{\rho_i} \right) \omega^2 - \omega_k^2 \omega - i \nu_{vni} \omega_k^2 = 0 \tag{5}
\]

We can solve this perturbatively in the two limits \( \omega_k \ll v_{ni}, \omega_k \gg v_{ni} \). Let’s further assume we are in the \( \epsilon = \rho_n / \rho_i \ll 1 \) limit, to match the WIM.\(^3\) Let’s first consider the \( \omega_k \ll v_{ni} \) case. To leading order in \( \omega_k / v_{ni} \) we have

\[
i v_{ni}(1 + \epsilon) \omega_k^2 - i v_{ni} \omega_k^2 = 0 \quad \Rightarrow \quad \omega_0 = \omega_k \sqrt{1 + \epsilon}. \tag{6}
\]

The first order equation is

\[
\omega_0^2 + 2i v_{ni}(1 + \epsilon) \omega_0 \omega_1 - \omega_k^2 \omega_0 = 0
\]

\[
\Rightarrow \omega_0 \sqrt{1 + \epsilon} + 2i v_{ni}(1 + \epsilon) \omega_k \sqrt{1 + \epsilon} \omega_1 = 0
\]

\[
\Rightarrow \omega_1 = -\frac{i \omega_k^2}{2 v_{ni} \sqrt{1 + \epsilon}}. \tag{7}
\]

So in this limit the waves are damped at a rate

\[
\Gamma_{in} = -\frac{i \omega_k^2}{2 v_{ni} \sqrt{1 + \epsilon}}. \tag{8}
\]

\( k_z \), which is the relevant limit for CR-generated waves, we have

\[
\omega_0^3 - \omega_k^2 \omega_0 = 0 \quad \Rightarrow \quad \omega_0 = \omega_k
\]

\[
3 \omega_k^2 \omega_1 + i v_{ni}(1 + \epsilon) \omega_k^2 - \omega_k^2 \omega_1 - i v_{ni} \omega_k^2 = 0
\]

\[
\Rightarrow 2 \omega_k^2 \omega_1 + i v_{ni} \omega_k^2 = 0 \quad \Rightarrow \quad \omega_1 = -\frac{i v_{ni}}{2 \epsilon} = -\frac{i v_{ni}}{2}. \tag{10}
\]

and so we have damping rate

\[
\Gamma_{in} = -\frac{v_{ni}}{2}. \tag{11}
\]

Note that this is the same damping rate derived in Kulsrud & Pearce (1969), even though we are in the opposite limit of a mostly ionized gas rather than a mostly neutral one.

The ion–neutral collision frequency is

\[
v_{in} = \frac{m_n}{m_i + m_n} n_i (\sigma v) = \frac{1}{2} n_i (\sigma v) = \frac{1}{2} \epsilon n_i (\sigma v). \tag{12}
\]

Here \( n_i \) is the density of the neutral component and \( \langle \sigma v \rangle \) is the average rate of exchange of velocity per particle for ion–neutral collisions. For temperatures around \( 10^4 \) K, we have \( \langle \sigma v \rangle \approx 10^{-3} \text{cm}^3 \text{s}^{-1} \) (see Kulsrud & Cesarsky 1971; De Pontieu et al. 2001). We assume here that hydrogen is the dominant neutral species; up to 10% of the hydrogen in the WIM is thought to be neutral (Haffner et al. 2009); He should be mostly neutral, but has a lower collision rate (De Pontieu et al. 2001). So in the short-wave, almost completely ionized limit, the ion–neutral damping rate is a function of only the density of the neutral component

\[
\Gamma_{in} = -\frac{1}{4} n_i \langle \sigma v \rangle = -\frac{1}{4} \epsilon n_i \langle \sigma v \rangle. \tag{13}
\]

2.2. Non-linear Landau Damping

In some circumstances, non-linear Landau damping may be comparable to or dominate over ion–neutral damping. Non-linear Landau damping occurs when ions ride along the envelope of a beat wave formed by two interfering Alfvén waves. Ions whose random motions are slightly slower than the speed of this envelope will take energy from the waves, damping them. Ions that are slightly faster will give energy to the waves, but for a thermal distribution we expect there to be more of the slower particles, and so the net effect is a wave damping. The strength of this damping depends on the strength of the waves as (Kulsrud 1978)

\[
\Gamma_{NL} = -\frac{\pi}{8} v_i k \left( \frac{\delta B}{B} \right)^2. \tag{14}
\]

Here, \( v_i = \sqrt{k B T / m} \) is the thermal speed of the ions.

2.3. Cosmic Ray Instability

A CR traveling along a magnetic field line with speed \( v \) and pitch angle cosine \( \mu \) will interact with an Alfvén wave with parallel wave number \( k_z \) under the resonance condition

\[
\frac{1}{\mu r_L} = \frac{\Omega_0}{v \gamma v}. \tag{15}
\]

Here, \( r_L \) is the CR’s relativistic gyroradius, \( \Omega_0 = e B_0 / mc \) is its nonrelativistic gyrofrequency, and \( \gamma \) is its Lorentz factor. In other words, a CR and an Alfvén wave are resonant if the wave’s wavelength is roughly equal to the distance the CR travels along the \( B \)-field in one gyration.

Wentzel (1968), Kulsrud & Pearce (1969) showed that a population of CRs whose bulk velocity is faster than the Alfvén speed will spur unstable growth in the waves. If we have such a distribution of CRs \( f(x, p, \mu, t) \), the resulting growth rate can be written, in the wave frame (Skilling 1971):

\[
\Gamma_{\text{growth}}(k_z) = \frac{\pi m^2 \Omega_0 v_A}{2 k_z B^2} \int d^2 p (1 - \mu^2) v \frac{d f}{d \mu} \left[ \frac{1}{\mu p - m \Omega_0 (k_z)} + \frac{1}{\mu p + m \Omega_0 (k_z)} \right]. \tag{16}
\]

\footnote{Note that our \( \epsilon \) is the inverse of the one used in Kulsrud & Pearce (1969).}
The above holds for Alfvén waves propagating nearly parallel to the background magnetic field \( B \). From here on we drop the \( z \) subscripts. The delta functions encode the resonance condition for CRs traveling in both directions.

We can rewrite this expression in terms of the CR gradient along the background magnetic field \( \partial f/\partial z \). In the absence of any sources or sinks, the CR transport equation is

\[
\frac{\partial f}{\partial t} + \nu \partial f = \frac{\partial}{\partial \mu} \left[ \frac{1 - \mu^2}{2} \nu(\mu) \frac{\partial f}{\partial \mu} \right].
\]

(17)

The scattering frequency \( \nu \) is related to the energy density \( E \) of resonant Alfvén waves (Kulsrud & Pearce 1969):

\[
\nu(\mu) = \frac{2\pi^2}{B^2} k \mathcal{E}(k) = \frac{\pi}{4} \Omega \left( \frac{\delta B}{B} \right)^2, \quad k = \frac{1}{|\mu| r_L}.
\]

(18)

This collision frequency is expected to be very large compared to the CR dynamical timescale (a condition we must check later for consistency), so we can expand \( f \) in inverse powers of \( \nu \), \( f = f_0 + f_1 + f_2 + \cdots \). To lowest order, Equation (17) becomes

\[
0 = \frac{\partial}{\partial \mu} \left[ \frac{1 - \mu^2}{2} \nu(\mu) \frac{\partial f_0}{\partial \mu} \right] \Rightarrow \frac{\partial f_0}{\partial \mu} = 0.
\]

(19)

To first order we have

\[
\mu v \frac{\partial f_0}{\partial z} = \frac{\partial}{\partial \mu} \left[ \frac{1 - \mu^2}{2} \nu(\mu) \frac{\partial f_1}{\partial \mu} \right].
\]

(20)

If we integrate both sides over \( \mu \),

\[
\frac{\partial f_1}{\partial z} = -v \frac{\partial f_0}{\partial z}.
\]

(21)

We can now eliminate \( \partial f/\partial \mu \) from Equation (16):

\[
\Gamma_{\text{growth}}(k) = -\frac{2\pi^2 m^2 \Omega_0 v_A}{k \delta B^2} \int d^3 p (1 - \mu^2) \frac{v^2}{\nu} \frac{\partial f}{\partial z} \left[ \delta(\mu p - m \Omega_0) + \delta(\mu p + m \Omega_0) \right].
\]

(22)

Plugging in Equation (18) for \( \nu \) we have

\[
\Gamma_{\text{growth}}(k) = -\frac{2\pi^2 m^2 \Omega_0 v_A}{k \delta B^2} \int d^3 p (1 - \mu^2) \frac{v^2}{\nu} \frac{\partial f}{\partial z} \left[ \delta(\mu p - m \Omega_0) + \delta(\mu p + m \Omega_0) \right]
\]

\[
\times \left[ \delta(\mu p - m \Omega_0) + \delta(\mu p + m \Omega_0) \right]
\]

\[
= -\frac{2\pi^2 m \Omega_0 v_A}{k \delta B^2} \int_0^\infty 2\pi p^2 dp \int_{-1}^1 d\mu (1 - \mu^2) pv \frac{\partial f}{\partial z}
\]

\[
\times \left[ \delta(\mu p - m \Omega_0) + \delta(\mu p + m \Omega_0) \right].
\]

Integrating the delta function over \( \mu \) gives

\[
\Gamma_{\text{growth}}(k) = -\frac{8\pi^2 m \Omega_0 v_A}{k \delta B^2} \int_{p_k}^\infty dp v f(p^2 - p_k^2) \equiv \frac{1}{4\pi} c n_{\text{CR}} A(k)
\]

(23)

where we have denoted \( p_k = m \Omega_0 / k \).

To make Equation (23) look a bit simpler, let us rewrite the integral in terms of a unitless factor of order unity \( A(k) \):

\[
\int_{p_k}^\infty dp v f(p^2 - p_k^2) \equiv \frac{1}{4\pi} c n_{\text{CR}} A(k)
\]

\[
A(k) = \frac{1}{n_{\text{CR}}} \int_{p_k}^\infty dp v f(p^2 - p_k^2), \quad 0 \leq A(k) \leq 1
\]

(24)

and let’s define a CR length scale by

\[
\frac{\partial n_{\text{CR}}}{\partial z} \equiv n_{\text{CR}} A(k).
\]

(25)

The growth rate is then

\[
\Gamma_{\text{growth}}(k) = 2\pi m \Omega_0 v_A c \frac{n_{\text{CR}}}{k \delta B^2} A(k).
\]

(26)

Without specifying a CR distribution \( f \) we cannot say anything about \( A(k) \). As an example, consider a power law in momentum, \( f(x, p, t) = C(x, t) p^{-\alpha} \), with some lower momentum cutoff \( p_c \) and normalization:

\[
n_{\text{CR}}(x, t) = \int_{p_c}^{\infty} 4\pi p^2 f(x, p, t) dp = \frac{4\pi}{\alpha - 3} C p_c^{\alpha - 3}
\]

\[
\Rightarrow f(x, p, t) = n_{\text{CR}}(x, t) \frac{p}{p_c} \theta(p - p_c).
\]

(27)

Then by definition (24) we get

\[
A(k) = \left( \frac{\alpha - 3}{4\pi p_c^3} \right)^{\alpha - 3} \int_{p_c}^{\infty} dp \theta(p - p_c)
\]

\[
= \left( \alpha - 3 \right) \int_{\chi_k}^{\infty} dx \theta\left( x^{\alpha - 3} - x_k^{\alpha - 3} \right), \quad x \equiv \frac{p}{p_c}, \quad x_k \equiv \frac{p_k}{p_c}.
\]

(28)

If we take the relativistic limit \( \beta \approx 1 \) and denote \( \chi_k, 1 = \chi_k \) this becomes

\[
A(k) = (\alpha - 3) \left[ -\frac{\chi_k^{\alpha - 3}}{3 - \alpha} + \frac{\chi_k^{1 - \alpha} - 1}{1 - \alpha} \right] = \chi_k^{1 - \alpha} \left[ \frac{\chi_k^{2 - \alpha}}{\alpha - 3} - \frac{\chi_k^{3 - \alpha}}{\alpha - 1} \right]
\]

or

\[
A(k) = \left( \left( 1 - \frac{\alpha - 3}{\alpha - 1} \right) \right)^{\alpha - 3} k < k_c
\]

(29)

where \( k_c = m \Omega_0 / p_c \) is determined from the lower momentum cutoff of the spectrum.

### 2.4. Equilibrium Power Spectrum

Now that we have the total damping and growth rates of the Alfvén waves we can enforce an equilibrium condition

\[
\Gamma_{\text{growth}} + \Gamma_{\text{in}} + \Gamma_{\text{NLID}} = 0.
\]

(29)

Inserting our expressions (14) and (26) into Equation (29),

\[
\frac{2\pi m \Omega_0 v_A c}{k \delta B^2} A(k) - \Gamma_{\text{in}} - \frac{\pi}{8} v_i k \left( \frac{\delta B}{B} \right)^2 = 0
\]

or, rearranging terms,

\[
\sqrt{\frac{T}{8} v_i k} \chi^2 + \Gamma_{\text{in}} k \chi = \frac{2\pi m \Omega_0 v_A c}{k \delta B^2} A(k) = 0, \quad \chi \equiv \left( \frac{\delta B}{B} \right)^2.
\]

\footnote{In principle the quantity \( A(k) \) could vary in space, but we ignore this possibility here. Alternatively we could adjust our definition of \( L_{\text{CR}} \) to include this effect.}
We solve analytically for $\mathcal{X}$
\[
\left(\frac{\delta B}{B}\right)_k^2 = \frac{2}{\pi} \frac{\Gamma_{in}}{k v_i} \left[ -1 + \sqrt{1 + \mathcal{R}} \right],
\]
\[\mathcal{R} \equiv \frac{\pi c}{\sqrt{2}} \delta n_{CR} n_i \frac{\Omega_0^2}{\Gamma_{in}} A(k), \tag{30}\]
where $r_i \equiv v_i/\Omega_0$ is the thermal ion gyroradius.

It is informative to determine the relative importance of each damping mechanism. We can do this by looking at the quantity $\mathcal{R}$. If $\mathcal{R}$ is small, the linear term (ion–neutral damping) dominates and
\[
\left(\frac{\delta B}{B}\right)_k^2 \approx \frac{1}{2} \frac{c}{v_i} \frac{n_{CR}}{n_i} \frac{\Omega_0^2}{\Gamma_{in}} A(k), \quad \mathcal{R} \ll 1. \tag{31}\]
while if $\mathcal{R}$ is large, non-linear Landau damping dominates, and
\[
\left(\frac{\delta B}{B}\right)_k^2 \approx \left(\frac{c}{\sqrt{2 \pi} v_i} \frac{n_{CR}}{n_i} \frac{\Omega_0^2}{\Gamma_{in}} \frac{n_{CR}}{\kappa^2 r_i L_{CR}} \right)^{1/2}, \quad \mathcal{R} \gg 1. \tag{32}\]

The transition between these two limiting cases occurs at $\mathcal{R} = 1$, or
\[A(k) \approx 2.8 \times 10^{-4} \left(\frac{v_i}{10^6 \text{ cm s}^{-1}}\right)^{-1} \left(\frac{n_i}{0.01 \text{ cm}^{-3}}\right)^{5/2} \times \left(\frac{n_{CR}}{10^{-3} \text{ cm}^{-3}}\right)^{-1} \left(\frac{L_{CR}}{\text{pc}}\right) \left(\frac{\epsilon}{0.05}\right)^{1/2} \tag{33}\]
where we have used Equation (13) and taken $\langle \sigma v \rangle = 10^{-8} \text{ cm}^3 \text{s}^{-1}$. We introduce here a set of convenient fiducial values that we will use throughout this paper. For the power law spectrum with $\alpha = 4.7$ this gives us a transition wave number $k^*$ of
\[k^* \approx 0.012 k_e = 0.012 \frac{e B}{p_e c} = 3.6 \times 10^{-15} \text{ cm}^{-1} \times \left(\frac{B}{\mu G}\right) \left(\frac{p_e c}{\text{GeV}}\right)^{-1} \tag{34}\]
at the fiducial values in Equation (33).5 For $k \ll k^*$, ion–neutral damping dominates and the wave power is given by Equation (31). For $k \gg k^*$, non-linear Landau damping is dominant and the wave power is Equation (32).

We are now in a position to check whether the CRs are self-trapped, i.e., whether their mean free path to scattering by self-generated turbulence is small compared to their scale height. This is a necessary condition for applying the heating theory derived in Sections 2.5 and 3. The mean free path $\lambda$ is related to the scattering frequency given in Equation (18) by
\[\lambda = \frac{v}{\omega}. \tag{35}\]
Using Equations (18) and (30) in Equation (35) yields a relatively compact expression for $\lambda$
\[\lambda = \sqrt{\frac{8}{\pi}} \frac{v_i}{\Gamma_{in}} \left[-1 + \sqrt{1 + \mathcal{R}}\right]^{-1}. \tag{36}\]
The mean free path in pc given by Equation (36) is plotted as a function of $p$ in units of the cutoff momentum $p_c$ in Figure 1.

5 We note here that the value of $k^*$ depends heavily on these quantities, particularly the gas density $n_i$. In fact, for some values there is no region of $k$-space where non-linear Landau damping dominates.

The figure spans the transition from Landau damping dominated at low momentum to ion–neutral friction dominated at high momentum. It appears from Figure 1 that CRs of energy even several hundred times the cutoff energy are quite well trapped ($\lambda \sim$ several pc). Most of the CR energy lies in the trans-relativistic $\sim$ GeV regime. In the interstellar medium (ISM), the spectra turn over at $p_c \sim 10$ MeV due to Coulomb cooling. Thus, the regime of interest is $p/p_c \sim 100$. At the upper end of the trapped range, where Equation (31) holds, Equation (36) can be written in the form
\[\frac{\lambda}{L_{CR}} \approx \frac{8}{\pi} \frac{v_i}{\Gamma_{in}} \frac{n_i}{\epsilon} \frac{n_{CR} A(k)}{\Omega_0^2} \rightarrow 4.0 \times 10^{-7} \left(\frac{p}{p_c}\right)^{1.7}, \tag{37}\]
where in the last expression we have used Equation (28) and set all parameters to their fiducial values.

Figure 1 can also be used to check that the waves are small amplitude and well described by linear theory. From Equations (18) and (35) was can see that $(\delta B/B)_k^2 \approx r_i/\lambda$. CRs of energy a few hundred GeV and less have gyroradii of order 10s of AU or less, showing that $\delta B/B \ll 1$ even at low momenta where the mean free path is short.

2.5. Heating Rate

We can now determine the heating rate of the WIM due to the dissipation of Alfvén waves created by CR streaming. To do this we want to integrate the time-derivative of $\mathcal{E}(k) = \delta B_i^2/8\pi k$ over all wave numbers $k$. We know that the time-dependence of $\delta B$ in an Alfvén wave is
\[\delta B \propto e^{-i\omega t} \tag{38}\]
and so
\[\omega = \omega_{\text{damp}} + i \Gamma_{\text{damp}} \tag{39}\]

\[\mathcal{E}(k) = \frac{\delta B_i^2}{8\pi k} \propto e^{2\Gamma_{\text{damp}}t} \Rightarrow \frac{\partial \mathcal{E}(k)}{\partial t} = 2\Gamma_{\text{damp}} \mathcal{E}(k) \tag{39}\]
(note that $\Gamma_{\text{damp}} < 0$).

Let us remove the assumption of a power law spectrum and go back to any general distribution
Let us also remove any assumptions about damping mechanisms, and only assume we have equilibrium for the Alfvén waves. Then from Equation (26) and $\Gamma_{\text{growth}} = \Gamma_{\text{damp}},$

$$H = \int_0^\infty dk 2\Gamma_{\text{damp}}(k) \frac{\delta B^2}{8\pi k}$$

$$= \int_0^\infty dk \Gamma_{\text{damp}}(k) \frac{m\Omega_0 v_{\text{A}}}{2\Gamma_{\text{damp}}(k)k^2 L_{\text{CR}}} A(k) \quad (40)$$

$$H = -\int_0^\infty dk m\Omega_0 v_{\text{A}} \frac{\partial}{\partial z} \left[ \int_{p_k}^\infty dp \beta f(p) 4\pi (p^2 - p_k^2) \right].$$

(41)

In the last step we have rewritten $A(k)$ and $L_{\text{CR}}$ in terms of their original definitions (24) and (25). We reformulate this double integral with a change of variable from $k$ to $p_k,$ $dp_k = -m\Omega_0/k^2dk.$ Let us also write $v_A = v_A n$ such that

$$H = -\frac{1}{2} v_A \cdot \nabla \left[ \int_0^\infty dp_k \int_{p_k}^\infty dp 4\pi f(p)v(p)(p^2 - p_k^2) \right].$$

(42)

Finally, let’s exchange the order of the integrals by recognizing that the double integral is over all $(p, p_k)$ under the constraint $0 \leq p_k \leq p.$

$$H = -\frac{1}{2} v_A \cdot \nabla \left[ \int_0^\infty dp \int_0^p dp_k 4\pi f(p)v(p)(p^2 - p_k^2) \right]$$

$$H = -\frac{1}{2} v_A \cdot \nabla \left[ \int_0^\infty dp 4\pi f(p)v(p)(p^2 - p_k^2) \right].$$

These integrals are now very simple—they correspond to the total CR pressure

$$P_{\text{CR}} = \frac{1}{3} \int_0^\infty dp 4\pi p^2 f(p)v(p).$$

(44)

We therefore obtain the very simple expression for the CR heating:

$$H = -v_A \cdot \nabla P_{\text{CR}}$$

(45)

in agreement with Wentzel (1971).

The heating rate is simply the CR pressure gradient times the Alfvén speed. Even without the above calculation we know this must be the solution, since we require an equilibrium for the waves and Equation (45) is always the rate at which CRs give energy to the Alfvén waves regardless of damping (McKenzie & Voelk 1981). So, as hinted at in Section 2, we require only that the Alfvén waves are in equilibrium and the CRs are well-trapped to know that the CR heating is Equation (45).

3. APPLICATION TO OUR GALAXY

3.1. Observations

Let us carry through the dependence of $v_A$ on the magnetic field and ion density, and pick some representative values. To write the CR pressure in terms of the energy density, we use $P_{\text{CR}} = 0.45 E_{\text{CR}}$ from Ferrière (2001). Then we can estimate the heating rate in the WIM:

$$H \approx 5.2 \times 10^{-28} \frac{\text{erg}}{\text{cm}^3 \text{s}^1} \frac{E_{\text{CR}}}{\text{eV cm}^{-3}} \frac{B}{\mu \text{G}} \times \left( \frac{L_{\text{CR}}}{\text{kpc}} \right)^{-1} \left( \frac{n_i}{10^{-2} \text{cm}^{-3}} \right)^{-1/2}.$$

(46)

If we assume the CR energy density and magnetic energy density fall off with the same scale height $L,$ we can determine the dependence of this heating rate on height $z$ from the galactic plane

$$B(z) = B_0 e^{-|z|/L}, \quad E_{\text{CR}}(z) = E_{\text{CR,0}} e^{-|z|/L}.$$

(47)

$$H(z) \approx 5.2 \times 10^{-28} \frac{\text{erg}}{\text{cm}^3 \text{s}^1} \frac{E_{\text{CR,0}}}{\text{eV cm}^{-3}} \frac{B_0}{\mu \text{G}} \times \left( \frac{L}{\text{kpc}} \right)^{-1} \left( \frac{n_i}{10^{-2} \text{cm}^{-3}} \right)^{-1/2} e^{-3|z|/2L}.$$

(48)

Let’s compare this to the heating that would be necessary to explain the inferred temperature profile $T(z).$ Following the prescription in Reynolds et al. (1999) we find $T(z)$ in our model by solving a heating–cooling balance equation

$$G_0 n_e^2 + G_3 n_e^{-1/2} = \Lambda T^4,$$

(49)

where each term represents, from left to right, photoionization heating, CR heating, and the cooling rate. The temperature dependence of the electron density $n_e,$ the cooling function $\Lambda,$ and the photoionization heating $G_0$ are

$$n_e(z) = 0.125 T_4^{0.45} e^{-0.5 |z|/1 \text{kpc}} \text{ cm}^{-3}$$

(50)

$$\Lambda = 3.0 \times 10^{-24} T_4^{-1.9} \text{ erg cm}^3 \text{s}^{-1}$$

(51)

$$G_0 = 1.2 \times 10^{-24} T_4^{-0.8} \text{ erg cm}^3 \text{s}^{-1}.$$

(52)

$T_4$ denotes the temperature in units of $10^4$ K, and $f$ is a filling fraction describing the amount of ionized hydrogen, which we will set to $f(z) = \text{Min}[0.1 e^{z/1000 \text{pc}}, 1].$ Note that our assumptions imply that $\nabla P_e < \rho g$ at all $z,$ and is consistent with hydrostatic balance.

We then solve Equation (49) to obtain the model profile $T(z)$ and compare it to the profile inferred from line ratio observations. We adjust $G_3$ to fit the model curve to the data, and we find, for $L = 2 \text{kpc}:$

$$G_{3, \text{fit}} \approx 1.2 \times 10^{-27} \frac{\text{erg}}{\text{cm}^9/2 \text{s}} e^{-3|z|/4000 \text{pc}}.$$

(53)

$$H_{\text{fit}} = G_{3, \text{fit}} n_e^{-1/2} \approx 1.2 \times 10^{-26} \frac{\text{erg}}{\text{cm}^3 \text{s}} \times \left( \frac{n_i}{10^{-2} \text{cm}^{-3}} \right)^{-1/2} e^{-3|z|/4000 \text{pc}}.$$

(54)

See Figure 2 (blue triangles) for this fit.

Comparing this to Equation (48) with $L = 2 \text{kpc},$ we see that CR heating is sufficient to explain the observed line ratios if the magnetic field and CR energy density normalizations are high enough:

$$\frac{E_{\text{CR,0}}}{\text{eV cm}^{-3}} \frac{B_0}{\mu \text{G}} \approx 46.$$

(55)
Is this the case? The magnetic field and CR energy density in the solar neighborhood are about $B_0 = 5 \, \mu G$ and $E_{CR,0} = 1.8 \, \text{eV cm}^{-3}$ (Ferrière 2001). Beuermann et al. (1985) showed that synchrotron emissivity in the galactic spiral arms is about four times greater than in the interarm regions. Synchrotron emissivity depends on the field and CR density as (Pfrommer & Enßlin 2004)

$$j_v \propto E_{CR} B^{\alpha/2+1} \propto B^{\alpha/2+3}$$

(56)

for a power law CR density. We utilize our equipartition assumption in the last step. A spiral-arm enhancement of $j_v$ by a factor of four therefore implies the $B$-field increases by a factor of about 1.4 and the CR density increases by about 1.9 (for $\alpha = 4.7$). We might then expect the product of the $B$-field and the CR density in the Perseus arm to be about

$$E_{CR,0} B_0 \approx (1.8 \, \text{eV cm}^{-3})(5 \, \mu G) \times 1.4 \times 1.9$$

$$\approx 24 \, \mu G \, \text{eV cm}^{-3}.$$  

(57)

This falls short of the requirement from Equation (55) but still comes close to reproducing the inferred temperature profile, as shown in the red circles in Figure 2. We also have not incorporated the orientation of the magnetic field. We have assumed a vertical field, but the actual field in the WIM may be much more random. This might be accounted for with an effective efficiency parameter.

One complication these estimates do not fully take into account is the multiphase nature of the ISM: the WIM has a low ($\sim 20\%$) filling factor, and above the disk is interspersed with hot diffuse coronal gas. This can cause local variations in Alfvén speed, and thus CR pressure. As long as $B$ increases more slowly than $n_i^{1/2}$ (note that in the cooler diffuse ISM sampled by H I lines, $B$ as measured by Zeeman splitting does not scale with density Crutcher et al. 2010), $v_A$ will be reduced in the WIM relative to coronal gas, and thus $P_e$ is higher. Thus, the relevant length scale for CR pressure gradients could be the cloud size, rather than the global scale height we have adopted; this leads to larger heating rates.\footnote{It also implies most heating occurs when the cosmic rays exit the cloud, when $v_A \cdot \nabla P_e < 0$.}

On the other hand, if clouds are sufficiently small ($L_{\text{cloud}} \approx \lambda \sim 10 \, \text{pc}$), then the CRs smooth over these inhomogeneities and the global gradients are appropriate. No observations and photoionization modeling indicate that the WIM is likely to have both a smooth and a clumpy component which fluctuates on a wide range of length scales, but there is no consensus picture (Haffner et al. 2009). We regard these issues as beyond the scope of this paper, but such considerations are illustrative of possible variations in the CR heating rate.

3.2. Stability

We must check that the heating from the CR pressure gradient is stable under perturbations. If the heating increases compared to the cooling for a small element of gas perturbed to a higher temperature, the heating is unstable and we would get thermal runaway. To determine if this is the case, we need the change in the heating rate for a given perturbation.

$$\delta(\Delta n^2 - H) = \delta(L_0 n^2 T^{1.9} - H_0 n^2 T^{-0.8} - H_3 B E_{CR} n^{-1/2}).$$

(58)

In terms of perturbed quantities this becomes

$$= L_0 n^2 T^2 \left(2 \frac{\delta n}{n} + 1.9 \frac{\delta T}{T} - H_0 n^2 T^{-0.8} \left(2 \frac{\delta n}{n} - 0.8 \frac{\delta T}{T}\right)\right)$$

$$- H_3 B E_{CR} n^{-1/2} \left(\frac{\delta B}{B} + \frac{\delta E_{CR}}{E_{CR}} - \frac{1}{2} \frac{\delta n}{n}\right).$$

(59)

Making use of the fact that the initial state was in thermal equilibrium, $L_0 n^2 T^2 = H_0 n^2 T^{-0.8} + H_3 B E_{CR} n^{-1/2}$, we can write this most generally as

$$\delta(\Delta n^2 - H) = \frac{\delta T}{T} [1.9L_0 n^2 T^{1.9} + 0.8 H_0 n^2 T^{-0.8}]$$

$$+ \frac{\delta n}{n} \left[2L_0 n^2 T^{1.9} - 2H_0 n^2 T^{-0.8} - \frac{1}{2} H_3 B E_{CR} n^{-1/2}\right]$$

$$- H_3 B E_{CR} n^{-1/2} \left(\frac{\delta B}{B} + \frac{\delta E_{CR}}{E_{CR}}\right)$$

$$= \frac{\delta T}{T} [1.9 L_0 n^2 T^{1.9} + 0.8 H_0 n^2 T^{-0.8}]$$

$$- H_3 B E_{CR} n^{-1/2} \left(\frac{3}{2} \frac{\delta n}{n} + \frac{\delta B}{B} + \frac{\delta E_{CR}}{E_{CR}}\right).$$

(60)
Without specifying the perturbation this is as far as we can go. Once we relate the perturbed quantities with \( \delta T \), we can determine whether the heating is stable or unstable.

Let us consider an isobaric perturbation perpendicular to the magnetic field lines. The total pressure remains constant

\[
P_{\text{tot}} = P_g + P_B + P_{\text{CR}} = \text{const}.
\]

The above terms represent the gas pressure, magnetic pressure, and CR pressure respectively. The field lines are compressed along with the gas, so

\[
\frac{\delta B}{B} = \frac{\delta n}{n}.
\]

Let us further assume that the CRs respond adiabatically:

\[
\frac{\delta E_{\text{CR}}}{E_{\text{CR}}} = \frac{\delta P_{\text{CR}}}{P_{\text{CR}}} = \gamma_{\text{CR}} \frac{\delta n}{n}.
\]

Then we can use the constant pressure condition to relate \( \delta n \) and \( \delta T \). From \( P_g \propto nT \) and \( P_B \propto B^2 \) we get

\[
\delta P_{\text{tot}} = \delta P_g + \delta P_B + \delta P_{\text{CR}} = 0
\]

\[
P_g \left( \frac{\delta n}{n} + \frac{\delta T}{T} + 2 \frac{\delta B}{B} + \gamma_{\text{CR}} \frac{\delta P_{\text{CR}}}{P_{\text{CR}}} \right) = 0
\]

\[
\frac{\delta n}{n} = - \frac{P_g}{P_g + 2P_B + \gamma_{\text{CR}} P_{\text{CR}}} \frac{\delta T}{T}.
\]

Putting this all together into Equation (60) gives

\[
\delta (\Lambda n^2 - H) = \frac{\delta T}{T} \left[ 1.9 L_{0.n}^2 T^{1.9} + 0.8 H_0 n^2 T^{-0.8} - H_3 B E_{\text{CR}} n^{-1/2} \left( \frac{3}{2} - \gamma_{\text{CR}} \right) \left( \frac{P_g}{P_g + 2P_B + \gamma_{\text{CR}} P_{\text{CR}}} \right) \right].
\]

If the term in the brackets is positive, a small increase in temperature causes the change in cooling to outweigh the change in heating and the perturbation is stable. If the term in brackets is negative, it is unstable. But note that both parenthesized factors in the third term must each be less than one. Also, by the thermal equilibrium condition, \( H_3 B E_{\text{CR}} n^{-1/2} \) must be less than \( L_{0.n}^2 T^{1.9} \). The first term must therefore be of higher magnitude than the third term, and so the expression in the brackets is positive and the heating is stable.

This is perhaps easier seen if we denote the total cooling by \( C \), and the fraction of the total heating due to photoelectric heating by \( 0 \leq x \leq 1 \). Then,

\[
\delta (\Lambda n^2 - H) = C \frac{\delta T}{T} \left[ 1.9 + 0.8 x - (1-x) \left( \frac{3}{2} - \gamma_{\text{CR}} \right) \right] \frac{P_g}{P_g + 2P_B + \gamma_{\text{CR}} P_{\text{CR}}}.
\]

Now let us consider an acoustic perturbation. Equations (60), (62), and (63) still hold. Pressure is no longer fixed, but the gas responds almost adiabatically. As such

\[
\frac{P_g}{\rho \gamma_{\text{m}}} = \text{const} \rightarrow \delta \left( \frac{P_g}{\rho \gamma_{\text{m}}} \right) = 0
\]
conditions produces a reasonably good fit to the observations (Figure 2). Although the heating rate coefficient is about a factor of two too small (Equation (57)), the height dependence—which follows from the height dependence of the magnetic field, gas density, and CR pressure—leads to a temperature versus height relation of the correct shape. We regard this, and matching the inferred size of the supplemental heating rate to within a factor of two—as confirmation that CR heating is a viable supplementary heat source for the WIM. CR heating also seems to be a thermally stable mechanism (Section 3.2), at least under the assumptions we considered.

We estimated the CR pressure gradient from the observed Galactic vertical gradient. While this is a reasonable average value, local propagation effects can modify the gradient considerably due to changes in density, magnetic field strength, and degree of ionization. For example, Everett & Zweibel (2011) found thin strongly heated skins at the boundaries of cold clouds. This will be the topic of future work.

The results in this paper should be generally applicable to warm ionized gas in other galaxies. In cases where synchrotron emission is detected or other estimates of the CR and magnetic field energy densities are available, it should be possible to estimate the magnitude of CR heating. It is important that the gas be diffuse and that the ionization fraction be high; in weakly ionized clouds, for example, ion–neutral friction is so strong that the CRs are not well coupled to the medium (Everett & Zweibel 2011). And, as long as the CRs are well scattered, their pressure gradient along the ambient magnetic field exerts a force which may be important in determining the scale height of the gas and in driving an outflow even when the thermal speed of the gas is well below what is needed for escape.

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REFERENCES

Beuermann, K., Kanbach, G., & Berkhuijzen, E. M. 1985, A&A, 153, 17
Breitschwerdt, D., McKenzie, J. F., & Voelk, H. J. 1991, A&A, 245, 79
Crutcher, R. M., Wandelt, B., Heiles, C., Falgarone, E., & Troland, T. H. 2010, ApJ, 725, 466
De Pontieu, B., Martens, P. C. H., & Hudson, H. S. 2001, ApJ, 558, 859
Everett, J. E., & Zweibel, E. G. 2011, ApJ, 739, 60
Everett, J. E., Zweibel, E. G., Benjamin, R. A., et al. 2008, ApJ, 674, 258
Ferrière, K. M. 2001, RvMP, 73, 1031
Flores-Fajardo, N., Morisset, C., Stasińska, G., & Binette, L. 2011, MNRAS, 415, 2182
Haffner, L. M., Dettmar, R.-J., Beckman, J. E., et al. 2009, RvMP, 81, 969
Kulsrud, R. M. 1978, in Astronomical Papers Dedicated to Bengt Stromgren, ed. A. Reiz & T. Andersen (Copenhagen: Copenhagen Univ. Obs.), 317
Kulsrud, R. M., & Cesarsky, C. J. 1971, ApJ, 189
Kulsrud, R., & Pearce, W. P. 1969, ApJ, 156, 445
McKenzie, J. F., & Voelk, H. J. 1981, in 17th International Cosmic Ray Conference, Vol. 9, (Paris: CEN-Saclay), 242
Minter, A. H., & Spangler, S. R. 1997, ApJ, 485, 182
Pfrommer, C., & Enßlin, T. A. 2004, A&A, 413, 17
Rand, R. J. 1997, ApJ, 474, 129
Rand, R. J. 1998, ApJ, 501, 137
Raymond, J. C. 1992, ApJ, 384, 502
Reynolds, R. J., Haffner, L. M., & Tufte, S. L. 1999, ApJL, 525, L21
Skilling, J. 1971, ApJ, 170, 265
Spitzer, L., Jr., & Tomasko, M. G. 1968, ApJ, 152, 971
Voelk, H. J., Drury, L. O., & McKenzie, J. F. 1984, A&A, 130, 19
Weingartner, J. C., & Draine, B. T. 2001, ApJ, 563, 842
Wentzel, D. G. 1968, ApJ, 152, 987
Wentzel, D. G. 1971, ApJ, 163, 503