THREE NUMERICAL PUZZLES

AND THE TOP QUARK’S CHIRAL WEAK-MOMENT

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Abstract

Versus the standard model’s $t \to W^+ b$ decay helicity amplitudes, three numerical puzzles occur at the 0.1% level when one considers the amplitudes in the case of an additional “$(f_M + f_E)$” coupling of relative strength $\Lambda_+ \sim 53 GeV$. The puzzles are theoretical ones which involve the $t \to W^+ b$ decay helicity amplitudes in the two cases, the relative strength of the additional $(f_M + f_E)$ coupling, and the observed masses of these three particles. A deeper analytic realization is obtained for two of the puzzles. Equivalent analytic realizations are given for the remaining one. An empirical consequence of these analytic realizations is that it is important to search for effects of a large chiral weak-moment of the top-quark, $\Lambda_+ \sim 53 GeV$. A full theoretical resolution would include relating the origin of such a chiral weak-moment and the mass generation of the top-quark, the W-boson, and probably the b-quark.

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1 Introduction

Versus the standard model’s (SM) $t \to W^+ b$ decay helicity amplitudes, three numerical puzzles occur at the 0.1% level when one considers the amplitudes in the case of an additional “$(f_M + f_E)$” coupling of relative strength $\Lambda_+ \sim 53 GeV$ [1]. The puzzles are theoretical ones which involve the $t \to W^+ b$ decay helicity amplitudes in the two cases, the relative strength of the additional $(f_M + f_E)$ coupling, and the observed masses of these three particles (see discussion at end of this Section). While the observed mass-values of the top-quark, W-boson, and b-quark are involved here, it is not a matter of presently available empirical data disagreeing with a SM prediction. The puzzles arose in a search for empirical ambiguities between the standard model’s $(V - A)$ coupling and possible single additional Lorentz couplings that could occur in the ongoing [2] and forthcoming [3,4] top-quark decay experiments at hadron and $l^- l^+$ colliders.

In this paper we report analytic realizations of these three numerical puzzles. There are four types of analytical relations which are listed below as (i)-(iv) in Section 2.

For $t \to W^+ b$, the most general Lorentz coupling is $W^* \mu J_{bt}^\mu = W^* \bar{u}_b (p) \Gamma^\mu u_t (k)$ where $k_i = q_W + p_b$, and

$$\Gamma^\mu_V = g_V \gamma^\mu + \frac{f_M}{2\Lambda} \sigma^{\mu\nu} (k - p)_\nu + \frac{g_S^-}{2\Lambda} (k - p)^\mu$$

$$+ \frac{g_S}{2\Lambda} (k + p)^\mu + \frac{g_T^-}{2\Lambda} \sigma^{\mu\nu} (k + p)_\nu \tag{1}$$

$$\Gamma^\mu_A = g_A \gamma^\mu \gamma_5 + \frac{f_E}{2\Lambda} \sigma^{\mu\nu} (k - p)_\nu \gamma_5 + \frac{g_P^-}{2\Lambda} (k - p)^\mu \gamma_5$$

$$+ \frac{g_P}{2\Lambda} (k + p)^\mu \gamma_5 + \frac{g_T^+}{2\Lambda} \sigma^{\mu\nu} (k + p)_\nu \gamma_5 \tag{2}$$

For a most general treatment of additional Lorentz structures to pure $V - A$, the $g_i$ or $\Lambda_i$ must be considered as complex phenomenological parameters.
To avoid confusion, two notational details should be noted in our usage of Eqs. (1,2) in this paper: First, we will more often use chiral combinations of the couplings which are defined by

\[ g_{L,R} \equiv g_V \mp g_A, \quad g_\pm \equiv g_{fM+fE} \equiv f_M \pm f_E, \quad g_{S\pm P} = g_S \pm g_P, \ldots \]  
(for others see appendix). Second, to avoid cumbersome notation and verbiage, we find it very convenient to suppress the analogous subscript on the effective mass-scales, \( \Lambda_i \). For example, in (1) a reader might choose to write \( \frac{f_M}{2\Lambda} \) as \( \frac{f_M}{\Lambda_{2M}} \). However, a smarter reader would realize that in the analysis in this paper for each \( i \) value only the ratio of \( g_i \) and \( \Lambda_i \) ever occurs so one could just omit the \( \Lambda_i \) by completely absorbing it into its \( g_i \). Nevertheless, we proceed a little differently: we use the unsubscripted \( \Lambda \)'s in (1,2) which does keep track of the mass-dimensions and then we subscript them when we must. For instance, whenever we use “\( g_L = 1 \) units with \( g_i = 1 \)”, we do move the corresponding subscript from the \( g_i \) to its \( \Lambda_i \). In essence, this is but a slight variation on the smarter reader’s procedure but one which, we believe, is worthwhile in this paper. For present purposes, referring to a \( \Lambda_i \) scale in “\( g_L = 1 \) units with \( g_i = 1 \)” is a simple way to characterize the size and sign of the additional effective \( \frac{g_i}{2\Lambda_i} \) coupling. [ When a fundamental theory leads to a specific coupling in (1,2), both \( f_M \) and \( \Lambda_M \), and not just their ratio, would normally be independently defined and observable. In general, one would then not want to omit either such quantity nor any such subscripts! A more fundamental or more general realization of electroweak symmetry in the context of the SM might also yield deviations from \( g_L = 1, g_i = 0 \) limit but we do not consider that possibility here. ]

As discussed in greater detail in the earliest paper in [1], the various couplings in (1,2) are not Lorentz independent. In particular, to obtain the identical \( \lambda_b = -\frac{1}{2} \) amplitudes as for the pure \( V-A \) coupling of the SM, there is a dominant scale of \( \Lambda_{S+P} = -\Lambda_{fM+fE} \sim \frac{m_t}{2} = 87 GeV \) and there is a negligible (for \( \lambda_b = -\frac{1}{2} \) amplitudes) scale \( \Lambda_{S-P} = -\Lambda_{fM-fE} \sim -\frac{(m_t)^2}{2m_b} = -3,400 GeV. \) These
scales are, of course, not to be confused with those for the 2 “dynamical phase-type ambiguities” [1] discussed below in this Section in association with Table 1; the “ambiguity scale” for the additional Lorentz structure is respectively \( \Lambda_{S+P} \sim -35 GeV, \Lambda_{fM+fE} \sim 53 GeV \).

These various coupling scales can be compared with the SM’s EW scale \( v_{EW} = \sqrt{-\mu^2/|\lambda|} = \sqrt{2}\langle 0|\phi|0 \rangle \sim 246 GeV \), where \( \phi \) is the Higgs field. These scales can also be compared with the “few TeV” cut-off scale needed to control the one-loop quadratic-divergent radiative corrections which arise in the Higgs mass renormalization in the SM; this “few TeV” scale thereby occurs in the SUSY and technicolor generalizations of the SM.

In the \( t \) rest frame, the matrix element for \( t \rightarrow W^+b \) is

\[
\langle \theta_1^t, \phi_1^t, \lambda_{W+}, \lambda_b | \frac{1}{2}, \lambda_1 \rangle = D^{(1/2)*}_{\lambda_1, \mu} (\phi_1^t, \theta_1^t, 0) A (\lambda_{W+}, \lambda_b) \tag{3}
\]

where \( \mu = \lambda_{W+} - \lambda_b \) in terms of the \( W^+ \) and \( b \)-quark helicities. The helicity amplitudes, \( A (\lambda_{W+}, \lambda_b) \), corresponding to the couplings in (1) and (2) are listed in the Appendix. We will denote respectively the “Standard Model’s” and the “\((V-A) + (f_M+f_E)\)” coupling’s amplitudes by an \( (SM) \) and \( (+) \) subscript.

In (3), the asterisk denotes complex conjugation. The final \( W^+ \) momentum is in the \( \theta_1^t, \phi_1^t \) direction and the \( b \)-quark momentum is in the opposite direction. \( \lambda_1 \) gives the \( t \)-quark’s spin component quantized along the \( z \) axis. \( \lambda_1 \) is also the helicity of the \( t \)-quark if one has boosted, along the “\(-z\)” direction, back to the \( t \) rest frame from the \((t\bar{t})_{cm}\) frame. It is this boost which defines the \( z \) axis in the \( t \)-quark rest frame for angular analysis [1].

To be able to quantitatively assess future measurements of competing observables in \( t \rightarrow W^+b \) decay, we considered in [1] the \( g_{V-A} \) coupling values of helicity decay parameters versus those for “ \((V-A) + \) single additional Lorentz structures.” Versus the SM’s dominant L-handed \( b-\)
quark amplitudes, there are 2 dynamical phase-type ambiguities produced respectively by an additional \((S + P)\) and by an additional \((f_M + f_E)\) coupling, see the \(A\left(0, -\frac{1}{2}\right)\) and \(A\left(-1, -\frac{1}{2}\right)\) columns of Table 1. For a reader interested in more details about this table, we note that in the earliest reference in [1], this same table is discussed at length in association with another table with the corresponding consequences for the observable helicity parameters for \(t \rightarrow W^+b\) decay in the cases that the observed top-quark is coupled respectively by \(“(V - A) + (S + P)”\) and \(“(V - A) + (f_M + f_E)”\). By tuning the effective-mass-scale associated with the additional coupling constant, the additional \((S + P)\) coupling, \((f_M + f_E)\) coupling, has respectively changed the sign of the \(A\left(0, -\frac{1}{2}\right)\), \(A\left(-1, -\frac{1}{2}\right)\) amplitude. In \(g_L = g_{S+P} = g_+ = 1\) units, the respective effective-mass scales are \(\Lambda_{S+P} \sim -35\text{GeV}\), \(\Lambda_{f_M+f_E} \sim 53\text{GeV}\). The occurrence of these two ambiguities is not surprising because these 3 chiral combinations only contribute to the L-handed \(b\)-quark amplitudes as \(m_b \rightarrow 0\). However, associated with the latter \((f_M + f_E)\) ambiguity, 3 very interesting numerical puzzles arose at the 0.1% level in the “(+)” amplitudes versus the SM’s pure “(V – A)” amplitudes, see Table 1:

The 1st puzzle is that the \(A_+(0, -1/2)\) amplitude for \(g_L + g_{f_M+f_E}\) has the same value in \(g_L = 1\) units, as the \(A_{SM}(-1, -1/2)\) amplitude in the SM; see the corresponding two “220” entries in the top of Table 1. From the empirical t-quark and W-boson mass values [5], the mass ratio \(y = \frac{m_W}{m_t} = 0.461 \pm 0.014\). This can be compared with the puzzle’s associated mass relation

\[
1 - \sqrt{2}y - y^2 - \sqrt{2}y^3 = x^2\left(\frac{2}{1 - y^2} - \sqrt{2}y\right) - x^4\left(\frac{1 - 3y^2}{(1 - y^2)^2}\right) + \ldots
\]

\[
= 1.89x^2 - 0.748x^4 + \ldots
\]

which follows by expanding the \(A_+(0, -1/2)\) amplitude, given in the appendix, in the mass ratio \(x^2 = (m_b/m_t)^2\). Before expanding, to express the \(A_+(0, -1/2)\) amplitude in the mass ratios \(x\)
and \( y \), the relative strength of the additional \( "(f_M + f_E)" \) coupling is specified by substituting 
\[ \Lambda_+ = E_W/2 = \frac{m_b}{4}[1 + y^2 - x^2] \text{ in } g_L = g_+ = 1 \text{ units} \] [ see discussions following (8) and (9) below ]. Since empirically \( x^2 \simeq 7 \cdot 10^{-4} \), there is only a 4th significant-figure correction from the finite b-quark mass to the only real-valued solution \( y = 0.46006 \) \( (m_b = 0) \) of this mass relation. Note versus theoretical interpretation of this mass relation that \( x^2 \) is very small and not of order one. [ The 0.1\% level of agreement of the two “220” entries of Table 1 is due to the present central value of \( m_t \), and to the central value and 0.05\% precision of \( m_W \). The error in the empirical value of the mass ratio \( y \) is dominated by the current 3\% precision of \( m_t \). ]

The 2nd and 3rd numerical puzzles are the occurrence of the same magnitudes of the two R-handed b-quark amplitudes \( A_{New} = A_{gL=1}/\sqrt{\Gamma} \) for the SM and for the case of \( (g_L + g_{f_M+f_E}) \). This is shown in the \( A \left( 0, \frac{1}{2} \right) \) and \( A \left( 1, \frac{1}{2} \right) \) columns in the bottom half of Table 1. Except for the differing partial width, \( \Gamma_+ = 0.66 GeV \) versus \( \Gamma_{SM} = 1.55 GeV \), by tuning the magnitude of L-handed amplitude ratio to that of the SM, we found that the R-handed amplitude’s moduli became those of the SM to the 0.1\% level. [As explained below following (8), for \( \Lambda_+ = E_W/2 \) the magnitudes of these two R-handed moduli are actually exactly equal and not merely numerically equal to the 0.1\% level.]

Notice that due to the additional \( f_M + f_E \) coupling, it is the \( \mu = \lambda_{W+} - \lambda_b = -1/2 \) helicity amplitudes \( A_{New} \) which get an overall sign change.
2 Four types of analytic relations

The first three of the four analytical relations, (i)-(iv) below, are a deeper realization of the just discussed 2nd and 3rd numerical puzzles: The first type of relation is (i):

\[
\frac{A_i(0,1/2)}{A_i(-1,-1/2)} = \frac{1}{2} \frac{A_i(1,1/2)}{A_i(0,-1/2)}
\] (4)

This holds separately for \( i = (SM), (+). \) (i) relates the two amplitudes which change or "flip" sign, i.e. the amplitudes with \( \mu = \lambda_W - \lambda_b = -1/2, \) to the two amplitudes which do not in the case of the 3 numerical puzzles. The second type of relation is (ii):

\[
\frac{A_+(0,1/2)}{A_+(-1,-1/2)} = \frac{A_{SM}(0,1/2)}{A_{SM}(-1,-1/2)}
\] (5)

Notice that this relation relates the \( t \rightarrow W^+b \) amplitudes in the two cases, (SM) and (+). (ii) is for the sign-flip amplitudes, c.f. Table 1, but the analogous relation holds for the non-sign-flip amplitudes. [This non-sign-flip relation is not independent of (i) and (ii); it follows by simply substituting (4) with respectively \( i = (+), (SM) \) on each side of (5).]

These 3 equations, the two in (4) and one in (5), occur to all orders in the \( y \) and \( x \) mass ratios. (5) and the (+) one in (4) are also independent of the effective-mass scale \( \Lambda_+ \). This independence with respect to specific values of \( x, y \) and \( \Lambda_+ \) is an analytic generalization of the numerical agreement discussed in Section 1. Due to (ii), the two equations in (i) imply but are stronger than simply an independent equality of the ratios of the (SM) and (+) amplitudes which do/do-not change sign, i.e. the existence of 3 equations is also a stronger result than is apparent from only the 2nd and 3rd numerical puzzles.

On the other hand, the two equations in (i) can be rewritten to relate the ratios of left-handed
and right-handed amplitudes for each coupling, that is

\[
\frac{A_i(0, -1/2)}{A_i(-1, -1/2)} = \frac{1}{2} \frac{A_i(1, 1/2)}{A_i(0, 1/2)}
\]

for \( i = \text{(SM)}, (+) \). Consequently, by determining the effective mass scale \( \Lambda_+ = \Lambda_+(m_W/m_t, m_b/m_t) \) so that there is an exact equality for the ratio of left-handed amplitudes (iii):

\[
\frac{A_+ (0, -1/2)}{A_+ (-1, -1/2)} = -\frac{A_{SM} (0, -1/2)}{A_{SM} (-1, -1/2)}
\]

the normalized \( A_{New} = A_{gL=1}/\sqrt{\Gamma} \) amplitudes for the SM and for the case of a \((g_L + g_{fM} + f_E)\) coupling are all exactly equal in magnitude to all orders in \( y \) and \( x \), with \( \Lambda_+ \sim 53 GeV \). From (7), in \( g_L = g_+ = 1 \) units, the analytic formula for \( \Lambda_+ \) is

\[
\Lambda_+ = \frac{m_t}{4} [1 + (m_W/m_t)^2 - (m_b/m_t)^2]
\]

or equivalently, \( \Lambda_+ = E_W/2 \) where \( E_W \) is the energy of the final W-boson in the t-rest frame. Eq.(7) is the third type of relation; it requires (8) to hold whereas (i) and (ii) do not.

Therefore, due to (i) and (ii), relation (iii) implies an S-matrix probability condition that the \( A_{New} = A_{gL=1}/\sqrt{\Gamma} \) amplitudes are exactly equal in magnitude between the SM and (+) case, to all orders in the two mass ratios. Only the actual value of this new EW scale \( \Lambda_+ \sim 53 GeV \) depends on the empirical values of \( m_W/m_t, m_t \), and the fact that the \( m_b/m_t \) ratio is small. This S-matrix “locking mechanism” for arbitrary \( x \) and \( y \), as described by relations (i) thru (iii), supports the simple interpretation that the 2nd and 3rd numerical puzzles arise due to a large chiral weak-moment of the top quark. Although a large chiral weak-moment, \( \Lambda_+ = E_W/2 \sim 53 GeV \), changes the \( t_R \) to \( b_L \) transition amplitude, it does not drastically effect the \( SU(2)_L \times U(1)_Y \) gauge structure. With present knowledge, it is also less radical to consider an unexpected intrinsic property of the
top-quark itself, i.e. an anomalous “moment”, instead of a tree-level occurrence of an additional EW coupling.

The SM is a theory, being renormalizable and unitary. In such a framework, an anomalous moment coupling implies other new physics, e.g. new particle production, at an effective mass scale of $\sim 2\Lambda_+ \sim 106\text{GeV}$. Unfortunately in regard to detailed predictions, such models are rather non-unique in regard to the particle multiplets and their associated couplings; they are also complex and complicated in regard to a satisfactory treatment of higher order effects. Here the additional R-chirality t-quark weak coupling could be problematic given (i) the successful agreements of the SM through the one-loop level versus EW observables and (ii) the SM’s self-consistency of 3 point-like fermion families; however, due to possible form-factor effects there need not be any inconsistencies. When there is additional empirical information on $t \rightarrow W^+b$ and if, for instance, a deeper exact symmetry were found which yields the tWb-transformation, see (10) below, this should expedite development of a theory with a top-quark with a large chiral weak-moment.

The final type of relation is simply the first numerical puzzle, i.e. the equality (iv):

$$A_+(0, -1/2) = A_{SM}(-1, -1/2), \quad (9)$$

It is important to note that (9) assumes that $\Lambda_+ = E_W/2$. The numerical agreement of the two sides of (9) and the empirical agreement of the associated mass relation discussed in Section 1 depend on the empirical value of the mass ratio $m_W/m_t$, on having set the relative coupling strength $\Lambda_+ = E_W/2$, and on $m_b/m_t << m_W/m_t$.

The top-quark mass is the most accurately measured quark mass. It is the measured values of the $W$-boson and top-quark masses that make this (iv) relation something that is not understood.
If the measured value of $m_W/m_t$ were different, (iv) would not hold. If $\Lambda_+^+$ is not set equal to $E_W/2 \sim 53\text{GeV}$, (iv) would not hold. Therefore, relations (i) thru (iii) lead to the possibility of (iv) and its equivalent realizations holding for the current values of $m_W/m_t$ and $m_b/m_t$ with their present precisions [5].

It is not obvious whether (iv) is an exact or approximate relation and, unfortunately, the empirical masses will not be better known for some time. The equality (iv) is equivalent to

$$\sqrt{2} = \frac{q_W}{E_W} \left( \frac{E_W + q_W}{m_W} \right).$$

Thus, (iv) is also equivalent to the velocity formula

$$\sqrt{2} = \gamma (1 + v) = \frac{\sqrt{1 + v}}{1 - v},$$

where $v$ is the velocity of the W-boson in the $t$-rest frame, so by this cubic equation $v = 0.6506\ldots$ without input of a specific value for $m_b$. Other $Q_{em} = -\frac{1}{3}$ quark decay modes are predicted by the standard model and the observed W-boson velocity must vary with different flavor $Q_{em} = -\frac{1}{3}$ quark mass values, so $v = 0.6506\ldots$ is presumably modified by higher order corrections depending on the $Q_{em} = -\frac{1}{3}$ quark mass. In the context of these four analytic relations, we think that the relation of $v$ and $m_{Q_{em} = -\frac{1}{3}}$ is a very important, fundamental open issue that has become manifest by this equivalent realization of (iv). To empirically investigate (iv), a measurement of $v$ at a $l^-l^+$ collider near the $t\bar{t}$-threshold might eventually be better than using the empirical mass ratio

$$y = \frac{m_W}{m_t}. [\text{In the } m_b = 0 \text{ limit, the formula } v = \sqrt{2} \frac{m_W}{m_t} c \text{ transforms this cubic equation into the mass relation listed in Sec. 1 and inversely; in this formula the speed of light has not been set equal to 1. }]

In summary, the four analytic-type relations can be characterized by a $tWb$-transformation $A_+ = M A_{SM}$ where $M = v \ diag(1, -1, -1, 1)$ due to (i) thru (iii), and where

$$A_{SM} = [A(0, -1/2), A(-1, -1/2), A(0, 1/2), A(1, 1/2)]
= \sqrt{m_t (E_b + q_W)} \left[ \frac{\sqrt{2}}{v} - \frac{v}{\sqrt{2} (E_b + q_W)}, -\sqrt{2} (\frac{m_b}{E_b + q_W}) \right]$$

(10)
with \( v \) the real root of \( \sqrt{2} = v \sqrt{\frac{1 + u}{1 - u}} \) due to relation (iv). In (10), (iv) has been used for the two components involving the longitudinal component of the W. It is not clear whether there is a dynamical mechanism and/or a mathematical-symmetry origin for such a tWb-transformation.

3 Remarks

Unfortunately, model-dependent interpretations and assumptions are needed to relate the above analytic realizations to the observed \( t\bar{t} \) production [2]. Experimental tests and measurements at the Tevatron [2] and LHC [3] should be able to clarify situation, for instance by excluding simple phenomenological possibilities. Here in (1) and (2) we discuss two simple models:

(1) As the simplest assumption, the top-quarks \( t \) described by the SM weak decay amplitude and those \( T \) described by the \((+)\) weak decay amplitude are presumed identical except for their differing chiral weak-moments. Since (i) nothing sufficiently fundamental appears to forbid that the observed top is more than one top-like object and since (ii) the explanation of the first numerical puzzle (9) might be dynamical, it is important to perform simple tests so as to empirically constrain such a possibility from \( t\bar{t} \) hadroproduction. In regard to the model we now construct, these are our motivations; it is not a matter of any presently available empirical data on \( t\bar{t} \) hadroproduction disagreeing with a SM prediction. The \( t\bar{t} \) cross-section has a 25% precision from Run I at the Tevatron which is expected to be reduced to 10% in Run IIa [5]. Our limited objective is to make a simple model to investigate how one can empirically restrict the possibility that in hadroproduction the observed top-quark is a linear combination of \( t \) and \( T \), specifically

\[
t_{\text{obs}} = c_t t + c_T T
\]

where \( c_{t,T} \) are unknown complex coefficients. The observed anti-top-quark is

\[
\bar{t}_{\text{obs}} = c_t^* \bar{t} + c_T^* \bar{T}
\]

and the asterisk denotes complex conjugation. We also assume that produc-
tion of \(t\bar{t}\) and \(T\bar{T}\) combinations occur and quantum-mechanically interfere in hadroproduction, but that the production of \(t\bar{T}\) and \(T\bar{t}\) combinations is forbidden. Note that for this model to be viable, simultaneous non-zero values of both \(|c_{t,T}|^2\) are required. Given continued agreement of QCD with more precise hadroproduction data, the viability of this model will require that neither \(t\) nor \(T\) couple with full QCD strength. This might occur due to possible form-factor effects. Alternative and/or more detailed models of this type could, of course, be constructed.

Tests of this model are possible because the (+) types, \(T\), would occur with a longer lifetime and because there are simple differences in stage-two spin-correlation functions: If for simplicity only \(\lambda_b = -\frac{1}{2}\) amplitudes are considered, then for the general angular distribution in the \((t\bar{t})_{cm}\) frame, i.e. in Eq.(62) of [6], the product of decay density matrices \(R_{\lambda_1'\lambda_1} (t \to W^+b \to \ldots) \overline{R_{\lambda_2'\lambda_2} (\bar{t} \to W^-\bar{b} \to \ldots)}\) for the top and anti-top is to be replaced according to

\[
R_{\lambda_1'\lambda_1} \overline{R_{\lambda_2'\lambda_2}} \to [\Gamma_L \Gamma_L + \Gamma_T \Gamma_T + I \cos \theta \cos \bar{\theta}] (|c_t|^2 + v^2|c_T|^2)^2
\]

\[
+ [\Gamma_L \Gamma_T + \Gamma_T \Gamma_L] (|c_t|^2 - v^2|c_T|^2)^2
\]

\[
+ [I \cos \theta (\Gamma_L + \Gamma_T) + (\Gamma_L + \Gamma_T) I \cos \bar{\theta}] (|c_t|^4 - v^4|c_T|^4)
\]  

(11)

This equation is a compact, schematic display of the terms arising from the, now four, composite decay density matrices \(R_{\lambda_1'\lambda_1} \ldots\) per

\[
R_{\lambda_1'\lambda_1} = \Gamma_{\lambda_1'\lambda_1} + I_{\lambda_1'\lambda_1} \cos \theta, \ldots
\]  

(12)

where \(\Gamma\) does not depend on \(W\) longitudinal/transverse interference (the helicity parameters \(\xi\) and \(\sigma = \zeta\) in the case of non-coexistence) and \(I\) depends on \(W\) longitudinal/transverse interference (the \(\eta_L\) helicity parameter in the case of non-coexistence). The term “schematic” means, for instance, that the angle \(\theta\) in (12) represents the angles/momenta specifying \(W^+b \to \ldots\) which
can be used for separating the $I$ versus $\Gamma$ terms in $R_{\lambda_1\lambda_1'}$, c.f. [6]. The helicity parameters of [6] were normalized by the partial width assuming a single kind of t-quark whereas in (11) the overall normalization includes a factor $(|c_t|^2 + v^2|c_T|^2)^2$. From (11), measurement of both this factor and, e.g., the ratio $(|c_t|^2 - v^2|c_T|^2)^2 / (|c_t|^2 + v^2|c_T|^2)^2$ can be used to constrain the possibility of simultaneous non-zero values of $|c_{t,T}|^2$. By (11), it is evident that a W longitudinal/interference measurement is not needed, i.e. the $\Gamma_i\Gamma_j$ terms are sufficient. If indirect signatures for such a model were to be found, it would obviously then be desirable to perform a direct lifetime experiment to confirm and better investigate the empirical phenomena. It is to be emphasized that (11) is model dependent regarding non-$W^+b$-channel interactions of the $t, T$. In single top-production [7] by W-gluon fusion, the unknown $|c_{t,T}|^2$ factors do not occur, and both $t$ and $T$ would be produced by their respective weak current couplings, per (1) and (2).

(2) Fortunately, in the simpler non-coexistence case where either $t$ or $T$ is the observed top-quark, a sufficiently precise measurement of the sign of the $\eta_L \equiv \frac{1}{4}|A(-1, -\frac{1}{2})||A(0, -\frac{1}{2})|\cos \beta_L$ helicity parameter will determine the sign of $\cos \beta_L$ where $\beta_L = \phi^{L}_{-\frac{1}{2}} - \phi^{L}_{0}$ is the relative phase of the two $\lambda_b = -\frac{1}{2}$-amplitudes, $A(\lambda_{W^+}, \lambda_b) = |A|\exp(i\phi^{L,R}_{\lambda_{W^+}})$. Measurement of the sign of $\eta_L = \pm 0.46(\text{SM/+})$ due to the large interference between the W longitudinal/transverse amplitudes could exclude such a large chiral weak-moment. Second, measurement of the closely associated $\eta_L' \equiv \frac{1}{4}|A(-1, -\frac{1}{2})||A(0, -\frac{1}{2})|\sin \beta_L$ helicity parameter would provide useful complementary information, c.f. second paper in [1]. In the absence of $T_{FS}$-violation, $\eta_L' = 0$. The possibility of $CP$ violation effects in top-quark physics is reviewed in [8]. In the case of the SM’s top-quark, next-to-leading order QCD corrections to top-quark spin correlations at hadron colliders have been recently calculated in [9].
By single top-quark production [7] at a hadron collider the partial width for $t \to W^+b$ can be measured and so the $v^2$ factor-difference in their associated partial widths can also be used to determine whether the observed top-quark is the SM’s $t$ (with $\Gamma_{SM} = 1.55\text{GeV}$), or a $T$ (with $\Gamma_+ = 0.66\text{GeV}$). More generally, single top-production at hadron and at linear colliders can provide important information on the Lorentz-structure of the $tWb$-vertex [10,11]. For hadron colliders, the effects of a finite top-quark width in a variety of top-quark production processes have been investigated in [12].

(3) The observed top-quark may, indeed, turn out as predicted by the standard model. In this case, the empirical significance of the four types of analytic relations is not clear. They might dynamically arise due to a weak-interaction phase-interface, or some other mixed-component quantum phenomena between top-quarks $t$ without the large chiral weak-moment and others $T$ with such a moment. Such a possibility is indirectly indicated by the fact that all the analytic relations involve both the (SM) and (+) amplitudes and by the appearance in the above analytic realizations of the kinematic variables $E_W$ (as the $2\Lambda_+$ value) and $v$, the W-boson velocity in the top rest-frame (e.g. in the tWb-transformation). For the (+) amplitudes, the Lorentz structure of the effective coupling is

$$\gamma^\mu P_L + \epsilon^{\mu\nu} v_\nu P_R$$

where $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ and $v_\nu$ is the W-boson’s relativistic four-velocity. From this perspective, besides higher precision accelerator-experiment mass measurements, theoretical investigations of mechanisms to produce anomalous weak and/or electromagnetic interaction moments in quantum field theoretic systems and in quantum statistical mechanical systems can be constructive. Such studies could be useful towards (i) unraveling the physical significance of these analytical rela-
tions, (ii) indicating relevant non-accelerator and/or accelerator tests, and (iii) relating the mass
generation of the top-quark, the W-boson, and the different flavor $Q_{em} = -\frac{1}{3}$ quarks.

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**Appendix: Helicity amplitudes for** $t \rightarrow W^+ b$

In terms of all the coupling constants in (1) and (2), from [6] the helicity amplitudes defined
by (3) and in the Jacob-Wick phase convention are:

For both $(V \mp A)$ couplings and for $\lambda_b = -\frac{1}{2}$,

$$
A \left(0, -\frac{1}{2}\right) = g_L \frac{E_W + q_W}{m_W} \sqrt{m_t (E_b + q_W)} - g_R \frac{E_W - q_W}{m_W} \sqrt{m_t (E_b - q_W)}
$$

$$
A \left(-1, -\frac{1}{2}\right) = g_L \sqrt{2m_t (E_b + q_W)} - g_R \sqrt{2m_t (E_b - q_W)}.
$$

and for $\lambda_b = \frac{1}{2}$,

$$
A \left(0, \frac{1}{2}\right) = -g_L \frac{E_W - q_W}{m_W} \sqrt{m_t (E_b - q_W)} + g_R \frac{E_W + q_W}{m_W} \sqrt{m_t (E_b + q_W)}
$$

$$
A \left(1, \frac{1}{2}\right) = -g_L \sqrt{2m_t (E_b - q_W)} + g_R \sqrt{2m_t (E_b + q_W)}
$$

where $g_L = g_V - g_A$, $g_R = g_V + g_A$.

For $(S \pm P)$ couplings, $g_{S\pm P} = g_S \pm g_P$ the additional contributions are

$$
A(0, -\frac{1}{2}) = g_{S+P}(\frac{m_t}{2\Lambda}) \frac{2q_W}{m_W} \sqrt{m_t (E_b + q_W)} + g_{S-P}(\frac{m_t}{2\Lambda}) \frac{2q_W}{m_W} \sqrt{m_t (E_b - q_W)}, \quad A(-1, -\frac{1}{2}) = 0
$$

$$
A(0, \frac{1}{2}) = g_{S+P}(\frac{m_t}{2\Lambda}) \frac{2q_W}{m_W} \sqrt{m_t (E_b - q_W)} + g_{S-P}(\frac{m_t}{2\Lambda}) \frac{2q_W}{m_W} \sqrt{m_t (E_b + q_W)}, \quad A(1, \frac{1}{2}) = 0
$$

The two types of tensorial couplings, $g_\pm = f_M \pm f_E$ and $\tilde{g}_\pm = g_{T^+} \pm g_{T^+_5}$, give the additional
contributions

$$
A \left(0, \pm\frac{1}{2}\right) = \mp g_\pm (\frac{m_t}{2\Lambda}) \left[ \frac{E_W \mp q_W}{m_W} \sqrt{m_t (E_b \pm q_W)} - \frac{m_b E_W \mp q_W}{m_t m_W} \sqrt{m_t (E_b \pm q_W)} \right]
$$
\[
\pm g_- \left( \frac{m_t}{2\Lambda} \right) \left[ -\frac{m_b E_W \pm q_W}{m_t} \sqrt{m_t (E_b \pm q_W)} + \frac{E_W \pm q_W}{m_W} \sqrt{m_t (E_b \mp q_W)} \right]
\]
\[
\mp \tilde{g}_+ \left( \frac{m_t}{2\Lambda} \right) \left[ \frac{E_W \pm q_W}{m_W} \sqrt{m_t (E_b \pm q_W)} + \frac{m_b E_W \pm q_W}{m_t} \sqrt{m_t (E_b \mp q_W)} \right]
\]
\[
\pm \tilde{g}_- \left( \frac{m_t}{2\Lambda} \right) \left[ \frac{m_b E_W \pm q_W}{m_t} \sqrt{m_t (E_b \pm q_W)} + \frac{E_W \pm q_W}{m_W} \sqrt{m_t (E_b \mp q_W)} \right]
\]

\[
A \left( \mp 1, \pm \frac{1}{2} \right) = \mp \sqrt{2} g_+ \left( \frac{m_t}{2\Lambda} \right) \left[ \sqrt{m_t (E_b \mp q_W)} - \frac{m_b}{m_t} \sqrt{m_t (E_b \pm q_W)} \right]
\]
\[
\pm \sqrt{2} g_- \left( \frac{m_t}{2\Lambda} \right) \left[ -\frac{m_b}{m_t} \sqrt{m_t (E_b \pm q_W)} + \sqrt{m_t (E_b \mp q_W)} \right]
\]
\[
\mp \sqrt{2} \tilde{g}_+ \left( \frac{m_t}{2\Lambda} \right) \left[ \sqrt{m_t (E_b \pm q_W)} + \frac{m_b}{m_t} \sqrt{m_t (E_b \mp q_W)} \right]
\]
\[
\pm \sqrt{2} \tilde{g}_- \left( \frac{m_t}{2\Lambda} \right) \left[ \frac{m_b}{m_t} \sqrt{m_t (E_b \pm q_W)} + \sqrt{m_t (E_b \mp q_W)} \right]
\]

The present paper uses the Standard Model’s (SM) and the \((f_M + f_E)\)’s amplitudes: In \(g_L = g_+ = 1\) units and suppressing a common overall factor of \(\sqrt{m_t (E_b + q_W)}\), from the above expressions these helicity amplitudes are with \(y = m_W/m_t\):

For only the \((V - A)\) coupling

\[
A_{SM} \left( 0, -\frac{1}{2} \right) = \frac{1}{y} \frac{E_W + q_W}{m_t}
\]
\[
A_{SM} \left( -1, -\frac{1}{2} \right) = \sqrt{2}
\]
\[
A_{SM} \left( 0, \frac{1}{2} \right) = -\frac{1}{y} \frac{E_W - q_W}{m_t} \left( \frac{m_b}{m_t} \right)
\]
\[
A_{SM} \left( 1, \frac{1}{2} \right) = -\sqrt{2} \left( \frac{m_b}{m_t} \right)
\]

and for only the \((f_M + f_E)\) coupling

\[
A_{f_M + f_E} \left( 0, -\frac{1}{2} \right) = -\left( \frac{m_t}{2\Lambda_+} \right) y
\]
\[
A_{f_M + f_E} \left( -1, -\frac{1}{2} \right) = -\left( \frac{m_t}{2\Lambda_+} \right) \sqrt{2} \frac{E_W + q_W}{m_t}
\]
\[
A_{f_M + f_E} \left( 0, \frac{1}{2} \right) = \left( \frac{m_t}{2\Lambda_+} \right) y \left( \frac{m_b}{m_t} \right)
\]
\[
A_{f_M + f_E} \left( 1, \frac{1}{2} \right) = \left( \frac{m_t}{2\Lambda_+} \right) \sqrt{2} \frac{E_W - q_W}{m_t} \left( \frac{m_b}{m_t} \right)
\]
From these, the \[ (V - A) + (f_M + f_E) \] coupling’s amplitudes are obtained by
\[ A_+ (\lambda_W, \lambda_b) = A_{SM} (\lambda_W, \lambda_b) + A_{f_M+f_E} (\lambda_W, \lambda_b). \]

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\[ m_W = 80.434 \pm 0.037 GeV \] from LEP and \( p\bar{p} \) experiments, G. Bella at TAU2000 workshop; \( m_t = 174.3 \pm 5.1 GeV \), PDG2000; and the pole mass \( m_b \sim 4.6 \pm 0.2 \), e.g. see talks at ICHEP2000. Expected precisions for \( m_t \) and \( m_W \) measurements from future experiments were reported at the 2001 Intl. Europhysics Conference on High Energy Physics by D. Charlton, G. Chiarelli, and D. Cavalli and for the Tevatron’s Run II at the 20th Lepton-photon Symposium by Y.-K. Kim: \( \delta m_t = \pm 3 GeV \) per experiment from run IIa at the Tevatron, and \( (\delta m_t, \delta m_W) = (\pm 1.5 GeV, \pm 0.030 GeV) \) per experiment from runs IIa and IIb combined; \( (\pm 1.5 GeV, \pm 0.015 GeV) \) per experiment from the LHC, and \( (\pm [0.1 - 0.2] GeV, \pm 0.006 GeV) \) from a NLC.

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Table Captions

Table 1: Numerical values of the helicity amplitudes $A(\lambda_{W^+}, \lambda_b)$ for the standard model and for the 2 dynamical phase-type ambiguities (with respect to the SM’s dominant $\lambda_b = -1/2$ amplitudes). The values are listed first in $g_L = g_{S+P} = g_+ = 1$ units, and second as $A_{\text{new}} = A_{g_L=1}/\sqrt{\Gamma}$. The latter removes the effect of the differing partial width in the $(f_M + f_E)$ case. The respective effective-mass scales for these 2 dynamical phase-type ambiguities are $\Lambda_{S+P} \sim -35 \text{GeV}$, $\Lambda_{f_M+f_E} \sim 53 \text{GeV}$. [$m_t = 175 \text{GeV}, m_W = 80.35 \text{GeV}, m_b = 4.5 \text{GeV}$].
|             | $A_{0, -\frac{1}{2}}$ | $A(-1, -\frac{1}{2})$ | $A(0, \frac{1}{2})$ | $A(1, \frac{1}{2})$ |
|-------------|-----------------------|------------------------|---------------------|---------------------|
| $A_{ge=1}$ in $g_L = 1$ units |                       |                        |                     |                     |
| $V - A$     | 338                   | 220                    | -2.33               | -7.16               |
| $S + P$     | -338                  | 220                    | -24.4               | -7.16               |
| $f_M + f_E$ | 220                   | -143                   | 1.52                | -4.67               |
| $A_{New} = A_{ge=1}/\sqrt{T}$ |                       |                        |                     |                     |
| $V - A$     | 0.84                  | 0.54                   | -0.0058             | -0.018              |
| $S + P$     | -0.84                 | 0.54                   | -0.060              | -0.018              |
| $f_M + f_E$ | 0.84                  | -0.54                  | 0.0058              | -0.018              |