Elimination of the vacuum instability for finite nuclei in the relativistic $\sigma$-$\omega$ model

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Abstract

The $\sigma$-$\omega$ model of nuclei is studied at leading order in the $1/N$ expansion thereby introducing the self consistent Hartree approximation, the Dirac sea corrections and the one fermion loop meson self energies in a unified way. For simplicity, the Dirac sea is further treated within a semiclassical expansion to all orders. The well-known Landau pole vacuum instability appearing in this kind of theories is removed by means of a scheme recently proposed in this context. The effect of such removal on the low momentum effective parameters of the model, relevant to describe nuclear matter and finite nuclei, is analyzed. The one fermion loop meson self energies are found to have a sizeable contribution to these parameters. However, such contribution turns out to come mostly from the Landau poles and is thus spurious. We conclude that the fermionic loop can only be introduced consistently in the $\sigma$-$\omega$ nuclear model if the Landau pole problem is dealt with properly.
I. INTRODUCTION

The relativistic approach to nuclear physics has attracted much attention. From a theoretical point of view, it allows to implement, in principle, the important requirements of relativity, unitarity, causality and renormalizability [1]. From the phenomenological side, it has also been successful in reproducing a large body of experimental data [1–5]. In the context of finite nuclei a large amount of work has been done at the Hartree level but considering only the positive energy single particle nucleon states. The Dirac sea has also been studied since it is required to preserve the unitarity of the theory. Actually, Dirac sea corrections have been found to be non negligible using a semiclassical expansion which, if computed to fourth order, seems to be quickly convergent [6]. Therefore, it would appear that the overall theoretical and phenomenological picture suggested by the relativistic approach is rather reliable.

However, it has been known since ten years that such a description is internally inconsistent. The vacuum of the theory is unstable due to the existence of tachyonic poles in the meson propagators at high Euclidean momenta [7]. Alternatively, a translationally invariant mean field vacuum does not correspond to a minimum; the Dirac sea vacuum energy can be lowered by allowing small size mean field solutions [8]. Being a short distance instability it does not show up for finite nuclei at the one fermion loop level and within a semiclassical expansion (which is an asymptotic large size expansion). For the same reason, it does not appear either in the study of nuclear matter if translational invariance is imposed as a constraint. However, the instability sets in either in an exact mean field valence plus sea (i.e., one fermion loop) calculation for finite nuclei or in the determination of the correlation energy for nuclear matter (i.e., one fermion loop plus a boson loop). Unlike quantum electrodynamics, where the instability takes place far beyond its domain of applicability, in quantum hadrodynamics it occurs at the length scale of 0.2 fm that is comparable to the nucleon size and mass. Therefore, the existence of the instability contradicts the original motivation that lead to the introduction of the field theoretical model itself. In such a situation several possibilities arise. Firstly, one may argue that the model is defined only as an effective theory, subjected to inherent limitations regarding the Dirac sea. Namely, the sea may at best be handled semiclassically, hence reducing the scope of applicability of the model. This interpretation is intellectually unsatisfactory since the semiclassical treatment would be an approximation to an inexistent mean field description. Alternatively, and taking into account the phenomenological success of the model, one may take more seriously the spirit of the original proposal [1], namely, to use specific renormalizable Lagrangians where the basic degrees of freedom are represented by nucleon and meson fields. Such a path has been explored in a series of papers [9–11] inspired by the early work of Redmond and Bogolyubov on non asymptotically free theories [12,13]. The key feature of this kind of theories is that they are only defined in a perturbative sense. According to the latter authors, it is possible to supplement the theory with a prescription based on an exact fulfillment of the Källén-Lehmann representation of the two point Green’s functions. The interesting aspect of this proposal is that the Landau poles are removed in such a way that the perturbative content of the theory remains unchanged. In particular, this guarantees that the perturbative renormalizability is preserved. It is, however, not clear whether this result can be generalized to three and higher point Green’s functions in order to end up with
a completely well-behaved field theory. Although the prescription to eliminate the ghosts may seem to be ad hoc, it certainly agrees more with the original proposal and provides a workable calculational scheme.

The above mentioned prescription has already been used in the context of nuclear physics. In ref. [14], it was applied to ghost removal in the $\sigma$ exchange in the $NN$ potential. More recently, it has been explored to study the correlation energy in nuclear matter in the $\sigma$-$\omega$ model [11] and also in the evaluation of response functions within a local density approximation [9]. Although this model is rather simple, it embodies the essential field theoretical aspects of the problem while still providing a reasonable phenomenological description. We will use the $\sigma$-$\omega$ model in the present work, to estimate the binding energy of finite nuclei within a self-consistent mean field description, including the effects due to the Dirac sea, after explicit elimination of the ghosts. An exact mean field calculation, both for the valence and sea, does make sense in the absence of a vacuum instability but in practice it becomes a technically cumbersome problem. This is due to the presence of a considerable number of negative energy bound states in addition to the continuum states [3]. Therefore, it seems advisable to use a simpler computational scheme to obtain a numerical estimate. This will allow us to see whether or not the elimination of the ghosts induces dramatic changes in the already satisfactory description of nuclear properties. In this work we choose to keep the full Hartree equations for the valence part but employ a semiclassical approximation for the Dirac sea. This is in fact the standard procedure [3, 6]. As already mentioned, and discussed in previous work [6], this expansion converges rather quickly and therefore might be reliably used to estimate the sea energy up to possible corrections due to shell effects.

The paper is organized as follows. In section II we present the $\sigma$-$\omega$ model of nuclei in the $1/N$ leading approximation, the semiclassical treatment of the Dirac sea, the renormalization prescriptions and the different parameter fixing schemes that we will consider. In section III we discuss the vacuum instability problem of the model and Redmond’s proposal. We also study the implications of the ghost subtraction on the low momentum effective parameters. In section IV we present our numerical results for the parameters as well as binding energies and mean quadratic charge radii of some closed-shell nuclei. Our conclusions are presented in section V. Explicit expressions for the zero momentum renormalized meson self energies and related formulas are given in the appendix.

II. $\sigma$-$\omega$ MODEL OF NUCLEI

In this section we revise the $\sigma$-$\omega$ model description of finite nuclei disregarding throughout the instability problem; this will be considered in the next section. The Dirac sea corrections are included at the semiclassical level and renormalization issues as well as the various ways of fixing the parameters of the model are also discussed here.

A. Field theoretical model

Our starting point is the Lagrangian density of the $\sigma$-$\omega$ model [3–5] given by

$$L(x) = \overline{\Psi}(x) \left[ \gamma_{\mu} (i \partial^{\mu} - g_{v} V^{\mu}(x)) \right] \Psi(x) + \frac{1}{2} \left( \partial_{\mu} \phi(x) \partial^{\mu} \phi(x) - m_{s}^{2} \phi^{2}(x) \right)$$
−\frac{1}{4} F_{\mu\nu}(x)F^{\mu\nu}(x) + \frac{1}{2} m_s^2 V_\mu(x) V^{\mu}(x) + \delta\mathcal{L}(x). \quad (1)

\Psi(x) is the isospinor nucleon field, \phi(x) the scalar field, V_\mu(x) the \omega\text{-meson field and } F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu. In the former expression the necessary counterterms required by renormalization are accounted for by the extra Lagrangian term \delta\mathcal{L}(x) (including meson self-couplings).

Including Dirac sea corrections requires to take care of renormalization issues. The best way of doing this in the present context is to use an effective action formalism. Further we have to specify the approximation scheme. The effective action will be computed at lowest order in the \(1/N\) expansion, \(N\) being the number of nucleon species (with \(g_s\) and \(g_v\) of order \(1/\sqrt{N}\)), that is, up to one fermion loop and tree level for bosons \[11\]. This corresponds to the Hartree approximation for fermions including the Dirac sea \[15\].

In principle, the full effective action would have to be computed by introducing bosonic and fermionic sources. However, since we will consider only stationary situations, we do not need to introduce fermionic sources. Instead, we will proceed as usual by integrating out exactly the fermionic degrees of freedom. This gives directly the bosonic effective action at leading order in the \(1/N\) expansion:

\[\Gamma[\phi, V] = \Gamma_B[\phi, V] + \Gamma_F[\phi, V],\]

where

\[\Gamma_B[\phi, V] = \int \left( \frac{1}{2} \left( \partial_\mu \phi \partial^\mu \phi - m_s^2 \phi^2 \right) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_v^2 V_\mu V^{\mu} \right) d^4x,\]

and

\[\Gamma_F[\phi, V] = -i \log \text{Det} [\gamma_\mu (i\partial^\mu - g_v V^{\mu}) - (M - g_v \phi)] + \int \delta\mathcal{L}(x)d^4x. \quad (4)\]

The fermionic determinant can be computed perturbatively, by adding up the one-fermion loop amputated graphs with any number of bosonic legs, using a gradient expansion or by any other technique. The ultraviolet divergences are to be canceled with the counterterms by using any renormalization scheme; all of them give the same result after fitting to physical observables.

The effective action so obtained is uniquely defined and completely finite. However, there still remains the freedom to choose different variables to express it. We will work with fields renormalized at zero momentum. That is, the bosonic fields \(\phi(x)\) and \(V_\mu(x)\) are normalized so that their kinetic energy term is the canonical one. This is the choice shown above in \(\Gamma_B[\phi, V]\). Other usual choice is the on-shell one, namely, to rescale the fields so that the residue of the propagator at the meson pole is unity. Note that the Lagrangian mass parameters \(m_s\) and \(m_v\) do not correspond to the physical masses (which will be denoted \(m_\sigma\) and \(m_\omega\) in what follows) since the latter are defined as the position of the poles in the corresponding propagators. The difference comes from the fermion loop self energy in \(\Gamma_F[\phi, V]\) that contains terms quadratic in the boson fields with higher order gradients.

Let us turn now to the fermionic contribution, \(\Gamma_F[\phi, V]\). We will consider nuclear ground states of spherical nuclei, therefore the space-like components of the \(\omega\text{-meson field vanish}
and the remaining fields, \( \phi(x) \) and \( V_0(x) \) are stationary. As it is well-known, for stationary fields the fermionic energy, i.e., minus the action \( \Gamma_F[\phi, V] \) per unit time, can be formally written as the sum of single particle energies of the fermion moving in the bosonic background \[13\],

\[
E_F[\phi, V_0] = \sum_n E_n, \quad (5)
\]

and

\[
[-i\alpha \cdot \nabla + g_v V_0(x) + \beta(M - g_s \phi(x))] \psi_n(x) = E_n \psi_n(x). \quad (6)
\]

Note that what we have called the fermionic energy contains not only the fermionic kinetic energy, but also the potential energy coming from the interaction with the bosons.

The orbitals, and thus the fermionic energy, can be divided into valence and sea, i.e., positive and negative energy orbitals. In realistic cases there is a gap in the spectrum which makes such a separation a natural one. The valence energy is therefore given by

\[
E_{F}^{\text{val}}[\phi, V] = \sum_n E_n^{\text{val}}. \quad (7)
\]

On the other hand, the sea energy is ultraviolet divergent and requires the renormalization mentioned above \[3\]. The (at zero momentum) renormalized sea energy is known in a gradient or semiclassical expansion up to fourth order and is given by \[2\],

\[
E_{0}^{\text{sea}} = -\frac{\gamma}{16\pi^2} M^4 \int d^3 x \left\{ \left( \frac{\Phi}{M} \right)^4 \log \frac{\Phi}{M} + \frac{g_s \phi}{M} - \frac{7}{2} \left( \frac{g_s \phi}{M} \right)^2 + \frac{13}{3} \left( \frac{g_s \phi}{M} \right)^3 - \frac{25}{12} \left( \frac{g_s \phi}{M} \right)^4 \right\},
\]

\[
E_{2}^{\text{sea}} = \frac{\gamma}{16\pi^2} \int d^3 x \left\{ \frac{2}{3} \left( \frac{\Phi}{M} \right)^2 (\nabla V)^2 - \log \frac{\Phi}{M} (\nabla \Phi)^2 \right\},
\]

\[
E_{4}^{\text{sea}} = \frac{\gamma}{5760\pi^2} \int d^3 x \left\{ -11 \Phi^{-4}(\nabla \Phi)^4 - 22 \Phi^{-4}(\nabla V)^2(\nabla \Phi)^2 + 44 \Phi^{-4}((\nabla_i \Phi)(\nabla_i V))^2 \right.
\]

\[
-44 \Phi^{-3}((\nabla_i \Phi)(\nabla_i V))(\nabla^2 V) - 8 \Phi^{-4}(\nabla V)^4 + 22 \Phi^{-3}(\nabla^2 \Phi)(\nabla \Phi)^2
\]

\[
+14 \Phi^{-3}(\nabla \Phi)^2(\nabla^2 \Phi) - 18 \Phi^{-2}(\nabla^2 \Phi)^2 + 24 \Phi^{-2}(\nabla^2 V)^2 \right\}. \quad (8)
\]

Here, \( V = g_v V_0, \Phi = M - g_s \phi \) and \( \gamma \) is the spin and isospin degeneracy of the nucleon, i.e., \( 2N \) if there are \( N \) nucleon species (in the real world \( N = 2 \)). The sea energy is obtained by adding up the terms above. The fourth and higher order terms are ultraviolet finite as follows from dimensional counting. The first two terms, being renormalized at zero momentum, do not contain operators with dimension four or less, such as \( \phi^2, \phi^4, \) or \( (\nabla V)^2 \), since they are already accounted for in the bosonic term \( \Gamma_B[\phi, V] \). Note that the theory has been renormalized so that there are no three- or four-point bosonic interactions in the effective action at zero momentum \[3\].

By definition, the true value of the classical fields (i.e., the value in the absence of external sources) is to be found by minimization of the effective action or, in the stationary case, of the energy

\[
E[\phi, V] = E_B[\phi, V] + E_F^{\text{val}}[\phi, V] + E_F^{\text{sea}}[\phi, V]. \quad (9)
\]
Such minimization yields the equations of motion for the bosonic fields:

\[
(\nabla^2 - m_s^2)\phi(x) = -g_s \left[ \rho_{s}^{\text{val}}(x) + \rho_{s}^{\text{sea}}(x) \right],
\]
\[
(\nabla^2 - m_v^2)V_\mu(x) = -g_v \left[ \rho^{\text{val}}(x) + \rho^{\text{sea}}(x) \right].
\]

(10)

Here, \( \rho_s(x) = \langle \overline{\Psi}(x)\Psi(x) \rangle \) is the scalar density and \( \rho(x) = \langle \overline{\Psi}^\dagger(x)\Psi(x) \rangle \) the baryonic one:

\[
\rho_{s}^{\text{val (sea)}}(x) = -\frac{1}{g_s} \frac{\delta E^{\text{val (sea)}}_F}{\delta \phi(x)},
\]
\[
\rho^{\text{val (sea)}}(x) = +\frac{1}{g_v} \frac{\delta E^{\text{val (sea)}}_F}{\delta V_\mu(x)}.
\]

(11)

The set of bosonic and fermionic equations, eqs. (10) and (6) respectively, are to be solved self-consistently. Let us remark that treating the fermionic sea energy using a gradient or semiclassical expansion is a further approximation on top of the mean field approximation since it neglects possible shell effects in the Dirac sea. However, a direct solution of the mean field equations including renormalization of the sum of single-particle energies would not give a physically acceptable solution due to the presence of Landau ghosts. They will be considered in the next section.

At this point it is appropriate to make some comments on renormalization. As we have said, one can choose different normalizations for the mesonic fields and there are also several sets of mesonic masses, namely, on-shell and at zero momentum. If one were to write the mesonic equations of motion directly, by similarity with a classical treatment, there would be an ambiguity as to which set should be used. The effective action treatment makes it clear that the mesonic field and masses are those at zero momentum. On the other hand, since we have not included bosonic loops, the fermionic operators in the Lagrangian are not renormalized and there are no proper vertex corrections. Thus the nucleon mass \( M \), the nuclear densities \( \langle \overline{\Psi}\Psi \rangle \) and the combinations \( g_s\phi(x) \) and \( g_vV_\mu(x) \) are fixed unambiguously in the renormalized theory. The fermionic energy \( E_F[\phi, V] \), the potentials \( \Phi(x) \) and \( V(x) \) and the nucleon single particle orbitals are all free from renormalization ambiguities at leading order in \( 1/N \).

B. Fixing of the parameters

The \( \sigma-\omega \) and related theories are effective models of nuclear interaction, and hence their parameters are to be fixed to experimental observables within the considered approximation. Several procedures to perform the fixing can be found in the literature \cite{2,4,5}; the more sophisticated versions try to adjust, by minimizing the appropriate \( \chi^2 \) function, as many experimental values as possible through the whole nuclear table \cite{4}. These methods are useful when the theory implements enough physical elements to provide a good description of atomic nuclei. The particular model we are dealing with can reproduce the main features of nuclear force, such as saturation and the correct magic numbers; however it lacks many of the important ingredients of nuclear interaction, namely Coulomb interaction and \( \rho \) and \( \pi \) mesons. Therefore, we will use the simple fixing scheme proposed in ref. \cite{2} for this model.
Initially there are five free parameters: the nucleon mass ($M$), two boson Lagrangian masses ($m_s$ and $m_v$) and the corresponding coupling constants ($g_s$ and $g_v$). The five physical observables to be reproduced are taken to be the physical nucleon mass, the physical ω-meson mass $m_\omega$, the saturation properties of nuclear matter (binding energy per nucleon $B/A$ and Fermi momentum $k_F$) and the mean quadratic charge radius of $^{40}$Ca. In our approximation, the equation of state of nuclear matter at zero temperature, and hence its saturation properties, depends only on the nucleon mass and on $m_s,v$ and $g_s,v$ through the combinations 

$$C_s^2 = g_s^2 \frac{M^2}{m_s^2}, \quad C_v^2 = g_v^2 \frac{M^2}{m_v^2}.$$  \hspace{1cm} (12)

At this point, there still remain two parameters to be fixed, e.g., $m_v$ and $g_s$. Now we implement the physical ω-meson mass constraint. From the expression of the ω propagator at the leading $1/N$ approximation, we can obtain the value of the physical ω pole as a function of the Lagrangian parameters $M$, $g_v$ and $m_v$ or more conveniently as a function of $M$, $C_v$ and $m_v$ (see appendix). Identifying the ω pole and the physical ω mass, and given that $M$ and $C_v$ have already been fixed, we obtain the value of $m_v$. Finally, the value of $g_s$ is adjusted to fit the mean quadratic charge radius of $^{40}$Ca. We will refer to this fixing procedure as the ω-shell scheme: the name stresses the correct association between the pole of the ω-meson propagator and the physical ω mass. The above fixing procedure gives different values of $m_s$ and $g_s$ depending on the order at which the Dirac sea energy is included in the semiclassical expansion (see section IV).

Throughout the literature the standard fixing procedure when the Dirac sea is included has been to give to the Lagrangian mass $m_v$ the value of the physical ω mass \[4,5\] (see, however, refs. \[10,6\]). Of course, this yields a wrong value for the position of the ω-meson propagator pole, which is underestimated. We will refer to this procedure as the naive scheme. Note that when the Dirac sea is not included at all, the right viewpoint is to consider the theory at tree level, and the ω-shell and the naive schemes coincide.

## III. LANDAU INSTABILITY SUBTRACTION

As already mentioned, the σ-ω model, and more generally any Lagrangian which couples bosons with fermions by means of a Yukawa-like coupling, exhibits a vacuum instability \[7,8\]. This instability prevents the actual calculation of physical quantities beyond the mean field valence approximation in a systematic way. Recently, however, a proposal by Redmond \[12\] that explicitly eliminates the Landau ghost has been implemented to describe relativistic nuclear matter in a series of papers \[9–11\]. The main features of such method are contained already in the original papers and many details have also been discussed. For the sake of clarity, we outline here the method as applies to the calculation of Dirac sea effects for closed-shell finite nuclei.

### A. Landau instability

Since the Landau instability shows up already at zero nuclear density, we will begin by considering the vacuum of the σ-ω theory. On a very general basis, namely, Poincaré
invariance, unitarity, causality and uniqueness of the vacuum state, one can show that the
two point Green’s function (time ordered product) for a scalar field admits the Källén-
Lehmann representation

\[ D(x' - x) = \int d\mu^2 \rho(\mu^2) D_0(x' - x; \mu^2), \]  

(13)

where the full propagator in the vacuum is

\[ D(x' - x) = -i\langle 0|T\phi(x')\phi(x)|0 \rangle, \]

(14)

and the free propagator reads

\[ D_0(x' - x; \mu^2) = \int d^4p (2\pi)^4 \frac{1}{p^2 - \mu^2 + i\epsilon}. \]

(15)

The spectral density \( \rho(\mu^2) \) is defined as

\[ \rho(q^2) = (2\pi)^3 \sum_n \delta^4(p_n - q)|\langle 0|\phi(0)|n \rangle|^2. \]

(16)

It is non negative, Lorentz invariant and vanishes for space-like four momentum \( q \).

The Källén-Lehmann representability is a necessary condition for any acceptable theory, yet it is violated by the \( \sigma - \omega \) model when the meson propagators are approximated by their leading \( 1/N \) term. It is not clear whether this failure is tied to the theory itself or it is an artifact of the approximation—it is well-known that approximations to the full propagator do not necessarily preserve the Källén-Lehmann representability—. The former possibility would suppose a serious obstacle for the theory to be a reliable one.

In the above mentioned approximation, eq. (13) still holds both for the \( \sigma \) and the \( \omega \) cases (in the latter case with obvious modification to account for the Lorentz structure) but the spectral density gets modified to be

\[ \rho(\mu^2) = \rho^{KL}(\mu^2) - R_G \delta(\mu^2 + M_G^2) \]

(17)

where \( \rho^{KL}(\mu^2) \) is a physically acceptable spectral density, satisfying the general requirements of a quantum field theory. On the other hand, however, the extra term spoils these general principles. The residue \( -R_G \) is negative, thus indicating the appearance of a Landau ghost state which contradicts the usual quantum mechanical probabilistic interpretation. Moreover, the delta function is located at the space-like squared four momentum \( -M_G^2 \) indicating the occurrence of a tachyonic instability. As a perturbative analysis shows, the dependence of \( R_G \) and \( M_G \) with the fermion-meson coupling constant \( g \) in the weak coupling regime is \( R_G \sim g^{-2} \) and \( M_G^2 \sim 4M^2 \exp(4\pi^2/g^2) \), with \( M \) the nucleon mass. Therefore the perturbative content of \( \rho(\mu^2) \) and \( \rho^{KL}(\mu^2) \) is the same, i.e., both quantities coincide order by order in a power series expansion of \( g \) keeping \( \mu^2 \) fixed. This can also be seen in the propagator form of the previous equation

\[ D(p) = D^{KL}(p) - \frac{R_G}{p^2 + M_G^2}. \]

(18)
For fixed four momentum, the ghost term vanishes as \( \exp(-4\pi^2/g^2) \) when the coupling constant goes to zero. As noted by Redmond [12], it is therefore possible to modify the theory by adding a suitable counterterm to the action that exactly cancels the ghost term in the meson propagator without changing the perturbative content of the theory. In this way the full meson propagator becomes \( D_{KL}^{\text{Kl}}(p) \) which is physically acceptable and free from vacuum instability at leading order in the \( 1/N \) expansion.

It is not presently known whether the stability problems of the original \( \sigma-\omega \) theory are intrinsic or due to the approximation used, thus Redmond’s procedure can be interpreted either as a fundamental change of the theory or as a modification of the approximation scheme. Although both interpretations use the perturbative expansion as a constraint, it is not possible, at the present stage, to decide between them. It should be made quite clear that in spite of the seemingly arbitrariness of the no-ghost prescription, the original theory itself was ambiguous regarding its non perturbative regime. In fact, being a non asymptotically free theory, it is not obvious how to define it beyond finite order perturbation theory. For the same reason, it is not Borel summable and hence additional prescriptions are required to reconstruct the Green’s functions from perturbation theory to all orders. As an example, if the nucleon self energy is computed at leading order in a \( 1/N \) expansion, the existence of the Landau ghost in the meson propagator gives rise to a pole ambiguity. This is unlike physical time-like poles, which can be properly handled by the customary \(+i\epsilon\) rule, and thus an additional ad hoc prescription is needed. This ambiguity reflects in turn in the Borel transform of the perturbative series; the Borel transform presents a pole, known as renormalon in the literature [17]. In recovering the sum of the perturbative series through inverse Borel transformation a prescription is then needed, and Redmond’s proposal provides a particular suitable way of fixing such ambiguity. Nevertheless, it should be noted that even if Redmond’s prescription turns out to be justified, there still remains the problem of how to extend it to the case of three- and more point Green’s functions, since the corresponding Källén-Lehmann representations has been less studied.

**B. Instability subtraction**

To implement Redmond’s prescription in detail we start with the zero-momentum renormalized propagator in terms of the proper self-energy for the scalar field (a similar construction can be carried out for the vector field as well),

\[
D_s(p^2) = (p^2 - m_s^2 - \Pi_s(p^2))^{-1},
\]

where the \( m_s \) is the zero-momentum meson mass and the corresponding renormalization conditions are \( \Pi_s(0) = \Pi_s'(0) = 0 \). The explicit formulas for the scalar and vector meson self energies are given in the appendix. Of course, \( D_s(p^2) \) is just the inverse of the quadratic part of the effective action \( K_s(p^2) \). According to the previous section, the propagator presents a tachyonic pole. Since the ghost subtraction is performed at the level of the two-point Green’s function, it is clear that the corresponding Lagrangian counterterm must involve a quadratic operator in the mesonic fields. The counterterm kernel \( \Delta K_s(p^2) \) must be such that cancels the ghost term in the propagator \( D_s(p^2) \) in eq. (18). The subtraction does not modify the position of the physical meson pole nor its residue, but it will change the zero-momentum
parameters and also the off-shell behavior. Both features are relevant to nuclear properties. This will be discussed further in the next section.

Straightforward calculation yields
\[
\Delta K_s(p^2) = -\frac{1}{D_s(p^2)} \frac{R^*_G}{R^*_G + (p^2 + M^2_s)} D_s(p^2). \tag{20}
\]

As stated, this expression vanishes as \(\exp(-4\pi^2/g^2_s)\) for small \(g_s\) at fixed momentum. Therefore it is a genuine non perturbative counterterm. It is also non local as it depends in a non polynomial way on the momentum. In any case, it does not introduce new ultraviolet divergences at the one fermion loop level. However, it is not known whether the presence of this term spoils any general principle of quantum field theory.

Proceeding in a similar way with the \(\omega\)-field \(V_\mu(x)\), the following change in the total original action is induced
\[
\Delta S = \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \phi(-p) \Delta K_s(p^2) \phi(p) - \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} V_\mu(-p) \Delta K^{\mu\nu}_s(p^2) V_\nu(p), \tag{21}
\]
where \(\phi(p)\) and \(V_\mu(p)\) are the Fourier transform of the scalar and vector fields in coordinate space, \(\phi(x)\) and \(V_\mu(x)\) respectively. Note that at tree-level for bosons, as we are considering throughout, this modification of the action is to be added directly to the effective action —in fact, this is the simplest way to derive eq. (20)—. Therefore, in the case of static fields, the total mean field energy after ghost elimination reads
\[
E = E_{\text{val}}^F + E_{\text{sea}}^F + E_B + \Delta E, \tag{22}
\]
where \(E_{\text{val}}^F\), \(E_{\text{sea}}^F\) and \(E_B\) were given in section II and
\[
\Delta E[\phi, V] = \frac{1}{2} \int d^3x \phi(x) \Delta K_s(\nabla^2) \phi(x) - \frac{1}{2} \int d^3x V_0(x) \Delta K^{\mu\nu}_s(\nabla^2) V_\nu(x). \tag{23}
\]
One can proceed by minimizing the mean field total energy as a functional of the bosonic and fermionic fields. This yields the usual set of Dirac equations for the fermions, eqs. (\text{III}) and modifies the left-hand side of the bosonic eqs. (\text{II}) by adding a linear non-local term. This will be our starting point to study the effect of eliminating the ghosts in the description of finite nuclei. We note that the instability is removed at the Lagrangian level, i.e., the non-local counterterms are taken to be new terms of the starting Lagrangian which is then used to describe the vacuum, nuclear matter and finite nuclei. Therefore no new prescriptions are needed in addition to Redmond’s to specify how the vacuum and the medium parts of the effective action are modified by the removal of the ghosts.

So far, the new counterterms, although induced through the Yukawa coupling with fermions, have been treated as purely bosonic terms. Therefore, they do not contribute directly to bilinear fermionic operators such as baryonic and scalar densities. An alternative viewpoint would be to take them rather as fermionic terms, i.e., as a (non-local and non-perturbative) redefinition of the fermionic determinant. The energy functional, and thus the mean field equations and their solutions, coincide in the bosonic and fermionic interpretations of the new term, but the baryonic densities and related observables would differ, since
they pick up a new contribution given the corresponding formulas similar to eqs. (11). Ambiguities and redefinitions are ubiquitous in quantum field theories, due to the well-known ultraviolet divergences. However, in well-behaved theories the only freedom allowed in the definition of the fermionic determinant comes from adding counterterms which are local and polynomial in the fields. Since the new counterterms induced by Redmond’s method are not of this form, we will not pursue such alternative point of view in what follows. Nevertheless, a more compelling argument would be needed to make a reliable choice between the two possibilities.

C. Application to finite nuclei

In this section we will take advantage of the smooth behavior of the mesonic mean fields in coordinate space which allows us to apply a derivative or low momentum expansion. The quality of the gradient expansion can be tested a posteriori by a direct computation. The practical implementation of this idea consists of treating the term $\Delta S$ by expanding each of the kernels $\Delta K(p^2)$ in a power series of the momentum squared around zero

$$\Delta K(p^2) = \sum_{n \geq 0} \Delta K_{2n} p^{2n}. \quad (24)$$

The first two terms are given explicitly by

$$\Delta K_0 = -\frac{m^4 R_G}{M_G^2 - m^2 R_G},$$

$$\Delta K_2 = \frac{m^2 R_G (m^2 - m^2 R_G + 2M_G^2)}{(M_G^2 - m^2 R_G)^2}. \quad (25)$$

The explicit expressions of the tachyonic pole parameters $M_G$ and $R_G$ for each meson can be found below.

Numerically, we have found that the fourth and higher orders in this gradient expansion are negligible as compared to zeroth- and second orders. In fact, in ref. [6] the same behavior was found for the correction to the Dirac sea contribution to the binding energy of a nucleus. As a result, even for light nuclei, $E_{\text{sea}}^4$ in eq. (8) can be safely neglected. Furthermore, it has been shown [18] that the fourth order term in the gradient expansion of the valence energy, if treated semiclassically, is less important than shell effects. So, it seems to be a general rule that, for the purpose of describing static nuclear properties, only the two lowest order terms of a gradient expansion need to be considered. We warn, however, that the convergence of the gradient or semiclassical expansion is not the same as converging to the exact mean field result, since there could be shell effects not accounted for by this expansion at any finite order. Such effects, certainly exist in the valence part [18]. Even in a seemingly safe case as infinite nuclear matter, where only the zeroth order has a non vanishing contribution, something is left out by the gradient expansion since the exact mean field solution does not exist due to the Landau ghost instability (of course, the situation may change if the Landau pole is removed). In other words, although a gradient expansion might appear to be exact in the nuclear matter case, it hides the very existence of the vacuum instability.

From the previous discussion it follows that the whole effect of the ghost subtraction is represented by adding a term $\Delta S$ to the effective action with same form as the bosonic
part of the original theory, $\Gamma_B[\phi, V]$ in eq. (3). This amounts to a modification of the zero-momentum parameters of the effective action. The new zero-momentum scalar field (i.e., with canonical kinetic energy), mass and coupling constant in terms of those of the original theory are given by

$$
\tilde{\phi}(x) = (1 + \Delta K^s_2)^{1/2}\phi(x),
\tilde{m}_s = \left(\frac{m^2_s - \Delta K^s_0}{1 + \Delta K^s_2}\right)^{1/2},
\tilde{g}_s = (1 + \Delta K^s_2)^{-1/2}g_s.
$$

(26)

The new coupling constant is obtained recalling that $g_s\phi(x)$ should be invariant. Similar formulas hold for the vector meson. With these definitions (and keeping only $\Delta K_{s,v}(p^2)$ till second order in $p^2$) one finds

$$
E_B[\tilde{\phi}, \tilde{V}; \tilde{m}_s, \tilde{m}_v] = E_B[\phi, V; m_s, m_v] + \Delta E[\phi, V; m_s, m_v],
$$

$$
E_F[\tilde{\phi}, \tilde{V}; \tilde{g}_s, \tilde{g}_v] = E_F[\phi, V; g_s, g_v].
$$

(27)

The bosonic equations for the new meson fields after ghost removal are hence identical to those of the original theory using

$$
\tilde{m}^2 = m^2 M^2_G \frac{M^2_G - m^2 R_G}{M^4_G + m^4 R_G},
\tilde{g}^2 = g^2 \frac{(M^2_G - m^2 R_G)^2}{M^4_G + m^4 R_G},
$$

(28)

as zero-momentum masses and coupling constants respectively. In the limit of large ghost masses or vanishing ghost residues, the reparameterization becomes trivial, as it should be. Let us note that although the zero-momentum parameters of the effective action $\tilde{m}_{s,v}$ and $\tilde{g}_{s,v}$ are the relevant ones for nuclear structure properties, the parameters $m_{s,v}$ and $g_{s,v}$ are the (zero-momentum renormalized) Lagrangian parameters and they are also needed, since they are those appearing in the Feynman rules in a perturbative treatment of the model. Of course, both sets of parameters coincide when the ghosts are not removed or if there were no ghosts in the theory.

To finish this section we give explicitly the fourth order coefficient in the gradient expansion of $\Delta E$, taking into account the rescaling of the mesonic fields, namely,

$$
\frac{\Delta K_4}{1 + \Delta K^s_2} = -\frac{R_G(M^2_G + m^2)^2}{(M^4_G + m^4 R_G)(M^2_G - m^2 R_G)} - \frac{\gamma g^2}{\alpha \pi^2} \frac{m^4 R_G^2 - 2 m^2 M^2_G R_G}{M^4_G + m^4 R_G},
$$

(29)

where $\alpha$ is 160 for the scalar meson and 120 for the vector meson. As already stated, for typical mesonic profiles the contribution of these fourth order terms are found to be numerically negligible. Simple order of magnitude estimates show that squared gradients

---

1Note that $E_{B,F}[\cdot]$ refer to the functionals (the same at both sides of the equations) and not to their value as is also usual in physics literature.
are suppressed by a factor \((RM_G)^{-2}\), \(R\) being the nuclear radius, and therefore higher orders can also be neglected. That the low momentum region is the one relevant to nuclear physics can also be seen from the kernel \(K_s(p^2)\), shown in fig. [1]. From eq. (21), this kernel is to be compared with the function \(\phi(p)\) that has a width of the order of \(R^{-1}\). It is clear from the figure that at this scale all the structure of the kernel at higher momenta is irrelevant to \(\Delta E\).

D. Fixing of the parameters after ghost subtraction

As noted in section [1], the equation of state at zero temperature for nuclear matter depends only on the dimensionless quantities \(C_s^2\) and \(C_v^2\), that now become

\[
C_s^2 = \hat{g}_s^2 \frac{M^2}{\hat{m}_s^2}, \quad C_v^2 = \hat{g}_v^2 \frac{M^2}{\hat{m}_v^2}.
\]

(30)

Fixing the saturation density and binding energy to their observed values yields, of course, the same numerical values for \(C_s^2\) and \(C_v^2\) as in the original theory. After this is done, all static properties of nuclear matter are determined and thus they are insensitive to the ghost subtraction. Therefore, at leading order in the \(1/N\) expansion, to see any effect one should study either dynamical nuclear matter properties as done in ref. [10] or finite nuclei as we do here.

It is remarkable that if all the parameters of the model were to be fixed exclusively by a set of nuclear structure properties, the ghost subtracted and the original theories would be indistinguishable regarding any other static nuclear prediction, because bosonic and fermionic equations of motion have the same form in both theories. They would differ however far from the zero four momentum region where the truncation of the ghost kernels \(\Delta K(p^2)\) at order \(p^2\) is no longer justified. In practice, the predictions will change after ghost removal because the \(\omega\)-meson mass is quite large and is one of the observables to be used in the fixing of the parameters.

To fix the parameters of the theory we choose the same observables as in section [1]. Let us consider first the vector meson parameters \(\hat{m}_v\) and \(\hat{g}_v\). We proceed as follows:

1. We choose a trial value for \(g_v\) (the zero-momentum coupling constant of the original theory). This value and the known physical values of the \(\omega\)-meson and nucleon masses, \(m_\omega\) and \(M\) respectively, determines \(m_v\) (the zero-momentum mass of the original theory), namely

\[
m_v^2 = m_\omega^2 + \frac{\gamma g_v^2}{8 \pi^2} M^2 \left\{ \frac{4}{3} + \frac{5}{9} \frac{m_\omega^2}{M^2} - \frac{2}{3} \left(2 + \frac{m_\omega^2}{M^2}\right) \sqrt{\frac{4 M^2 m_\omega^2}{m_\omega^2 - 1}} \arcsin\left(\frac{m_\omega}{2 M}\right) \right\}.
\]

(31)

(This, as well as the formulas given below, can be deduced from those in the appendix.)

2. \(g_v\) and \(m_v\) provide the values of the tachyonic parameters \(R_v^G\) and \(M_v^G\). They are given by

\[
R_v^G = \frac{2 M}{\sqrt{\kappa_v^2 - 1}}
\]

\[
\frac{1}{R_v^G} = -1 + \frac{\gamma g_v^2}{24 \pi^2} \left\{ \left(\frac{\kappa_v^3}{4} + 3 \frac{\kappa_v}{4}\right) \log \frac{\kappa_v + 1}{\kappa_v - 1} - \frac{\kappa_v^2}{2} - \frac{1}{6} \right\},
\]

(32)
where the quantity $\kappa_v$ is the real solution of the following equation (there is an imaginary solution which corresponds to the $\omega$-meson pole)

$$1 + \frac{m_\omega^2}{4M^2} (\kappa_v^2 - 1) + \frac{\gamma g_v^2}{24\pi^2} \left\{ \left( \frac{\kappa_v^3}{2} - \frac{3\kappa_v}{2} \right) \log \frac{\kappa_v + 1}{\kappa_v - 1} - \kappa_v^2 + \frac{8}{3} \right\} = 0. \quad (33)$$

3. Known $g_v$, $m_v$, $M_v^G$ and $R_v^G$, the values of $\hat{m}_v$ and $\hat{g}_v$ are obtained from eqs. (28). They are then inserted in eqs. (30) to yield $C_v^2$. If necessary, the initial trial value of $g_v$ should be readjusted so that the value of $C_v^2$ so obtained coincides with that determined by the saturation properties of nuclear matter.

The procedure to fix the parameters $m_s$ and $g_s$ is similar but slightly simpler since the physical mass of the scalar meson $m_\sigma$ is not used in the fit. Some trial values for $m_s$ and $g_s$ are proposed. This allows to compute $M_s^G$ and $R_s^G$ by means of the formulas

$$M_s^G = \frac{2M_s}{\sqrt{\kappa_s^2 - 1}}$$
$$\frac{1}{R_s^G} = -1 - \frac{\gamma g_s^2}{16\pi^2} \left\{ \left( \frac{\kappa_s^3}{2} - \frac{3\kappa_s}{2} \right) \log \frac{\kappa_s + 1}{\kappa_s - 1} - \kappa_s^2 + \frac{8}{3} \right\}, \quad (34)$$

where $\kappa_s$ is the real solution of

$$1 + \frac{m_s^2}{4M^2} (\kappa_s^2 - 1) - \frac{\gamma g_s^2}{16\pi^2} \left\{ \kappa_s^3 \log \frac{\kappa_s + 1}{\kappa_s - 1} - 2\kappa_s^2 + \frac{2}{3} \right\} = 0. \quad (35)$$

One can then compute $\hat{m}_s$ and $\hat{g}_s$ and thus $C_s^2$ and the mean quadratic charge radius of $^{40}\text{Ca}$. The initial values of $m_s$ and $g_s$ should be adjusted to reproduce these two quantities. We will refer to the set of masses and coupling constants so obtained as the no-ghost scheme parameters.

**IV. NUMERICAL RESULTS AND DISCUSSION**

As explained in section II, the parameters of the theory are fitted to five observables. For the latter we take the following numerical values: $M = 939$ MeV, $m_\omega = 783$ MeV, $B/A = 15.75$ MeV, $k_F = 1.3$ fm$^{-1}$ and 3.82 fm for the mean quadratic charge radius of $^{40}\text{Ca}$.

If the Dirac sea is not included at all, the numerical values that we find for the nuclear matter combinations $C_s^2$ and $C_v^2$ are

$$C_s^2 = 357.7, \quad C_v^2 = 274.1 \quad (36)$$

The corresponding Lagrangian parameters are shown in table I. There we also show $m_\sigma$ and $m_\omega$ that correspond to the position of the poles in the propagators after including the one-loop meson self energy. They are an output of the calculation and are given for illustration purposes.

When the Dirac sea is included, nuclear matter properties fix the following values

$$C_s^2 = 227.8, \quad C_v^2 = 147.5 \quad (37)$$
Note that in nuclear matter only the zeroth order $E_0^{\text{sea}}$ is needed in the gradient expansion of the sea energy, since the meson fields are constant. The (zero momentum renormalized) Lagrangian meson masses $m_{s,v}$ and coupling constants $g_{s,v}$ are shown in table I in various schemes, namely, $\omega$-shell, no-ghost and naive schemes, previously defined. The scalar meson parameters differ if the Dirac sea energy is included at zeroth order or at all orders (in practice zeroth plus second order) in the gradient expansion. For the sake of completeness, both possibilities are shown in the table. The numbers in brackets in the no-ghost scheme are the zero-momentum parameters of the effective action, $\hat{m}_{s,v}$ and $\hat{g}_{s,v}$ (in the other schemes they coincide with the Lagrangian parameters). Again $m_\sigma$ and $m_\omega$ refer to the scalar and vector propagator-pole masses after including the one fermion loop self energy for each set of Lagrangian parameters. Table II shows the ghost masses and residues corresponding to the zero-momentum renormalized propagators. The no-ghost scheme parameters have been used.

The binding energies per nucleon (without center of mass corrections) and mean quadratic charge radii (without convolution with the nucleon form factor) of several closed-shell nuclei are shown in tables III and IV for the $\omega$-shell and for the naive and no-ghost schemes (these two schemes give the same numbers), as well as for the case of not including the Dirac sea. The experimental data are taken from refs. [19–21].

From table I it follows that the zero-momentum vector meson mass $m_v$ in the $\omega$-shell scheme is considerably larger than the physical mass. This is somewhat unexpected. Let us recall that the naive treatment, which neglects the meson self energy, is the most used in practice. It has been known for a long time [22,23] that the $\omega$-shell scheme is, as a matter of principle, the correct procedure but on the basis of rough estimates it was assumed that neglecting the meson self energy would be a good approximation for the meson mass. We find here that this is not so.

Regarding the consequences of removing the ghost, we find in table I that the effective parameters $\hat{m}_{s,v}$ and $\hat{g}_{s,v}$ in the no-ghost scheme are similar, within a few per thousand, to those of the naive scheme. This similarity reflects in turn on the predicted nuclear properties: the results shown in tables III and IV for the no-ghost scheme coincide, within the indicated precision, with those of the naive scheme (not shown in the table). It is amazing that the outcome parameters from such a sophisticated fitting procedure, namely the no-ghost scheme, resemble so much the parameters corresponding to the naive treatment. We believe this result to be rather remarkable for it justifies a posteriori the nowadays traditional calculations made with the naive scheme.

The above observation is equivalent to the fact that the zero-momentum masses, $\hat{m}_{s,v}$, and the propagator-pole masses $m_{s,\omega}$ are very similar in the no-ghost scheme. This implies that the effect of removing the ghosts cancels to a large extent with that introduced by the meson self energies. Note that separately the two effects are not small; as was noted above $m_v$ is much larger than $m_\omega$ in the $\omega$-shell scheme. To interpret this result, it will be convenient to recall the structure of the meson propagators. In the leading $1/N$ approximation, there are three kinds of states that can be created on the vacuum by the meson fields. Correspondingly, the spectral density functions $\rho(q^2)$ have support in three clearly separated regions, namely, at the ghost mass squared (in the Euclidean region), at the physical meson mass squared, and above the $N\overline{N}$ pair production threshold $(2M)^2$ (in the time-like region). The full meson propagator is obtained by convolution of the spectral density function with
the massless propagator \((q^2 + i\epsilon)^{-1}\) as follows from the Källén-Lehmann representation, eq. (13). The large cancelation found after removing the ghosts leads to the conclusion that, in the zero-momentum region, most of the correction induced by the fermion loop on the meson propagators, and thereby on the quadratic kernels \(K(p^2)\), is spurious since it is due to unphysical ghost states rather than to virtual \(NN\) pairs. This can also be seen from figs. 1 and 2. There, we represent the real and imaginary parts of \(K_s(p^2)\) respectively, in three cases, namely, before ghost elimination, after ghost elimination and the free inverse propagator. In all three cases the slope of the real part at zero momentum is equal to one and the no-ghost (sea 2nd) set of parameters from table I has been used. We note the strong resemblance of the free propagator and the ghost-free propagator below threshold. A similar result is obtained for the vector meson.

One may wonder how these conclusions reflect on the sea energy. Given that we have found that most of the fermion loop is spurious in the meson self energy it seems necessary to revise the sea energy as well since it has the same origin. Technically, no such problem appears in our treatment. Indeed the ghost is found in the fermion loop attached to two meson external legs, i.e., terms quadratic in the fields. However, the sea energy used, namely, \(E^\text{sea}_0 + E^\text{sea}_2\), does not contain such terms. Quadratic terms would correspond to a mass term in \(E^\text{sea}_0\) and a kinetic energy term in \(E^\text{sea}_2\), but they are absent from the sea energy due to the zero-momentum renormalization prescription used. On the other hand, terms with more than two gradients were found to be negligible [6]. Nevertheless, there still exists the possibility of ghost-like contributions in vertex functions corresponding to three or more mesons, similar to the spurious contributions existing in the two-point function. In this case the total sea energy would have to be reconsidered. The physically acceptable dispersion relations for three or more fields have been much less studied in the literature hence no answer can be given to this possibility at present.

V. SUMMARY AND CONCLUSIONS

We summarize our points. In the present paper, we have studied the consequences of eliminating the vacuum instabilities which take place in the \(\sigma-\omega\) model. This has been done using Redmond’s prescription which imposes the validity of the Källén-Lehmann representation for the two-point Green’s functions. We have discussed possible interpretations to such method and have given plausibility arguments to regard Redmond’s method as a non-perturbative and nonlocal modification of the starting Lagrangian.

Numerically we have found that, contrary to the naive expectation, the effect of including fermionic loop corrections to the mesonic propagators (\(\omega\)-shell scheme) is not small. However, it largely cancels with that of removing the unphysical Landau poles. A priori, this is a rather unexpected result which in fact seems to justify previous calculations carried out in the literature using a naive scheme. Actually, as compared to that scheme and after proper readjustment of the parameters to selected nuclear matter and finite nuclei properties, the numerical effect becomes rather moderate on nuclear observables. The two schemes, naive and no-ghost, are completely different beyond the zero four momentum region, however, and for instance predict different values for the vector meson mass.

Therefore it seems that in this model most of the fermionic loop contribution to the meson self energy is spurious. The inclusion of the fermionic loop in the meson propagator can only
be regarded as an improvement if the Landau ghost problem is dealt with simultaneously. We have seen that the presence of Landau ghosts does not reflect on the sea energy but it is not known whether there are other spurious ghost-like contributions coming from three or higher point vertex functions induced by the fermionic loop.

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APPENDIX A:

1. Meson self energies in the leading \(1/N\) expansion

As stated in the main text, the leading order in the \(1/N\) expansion (\(N\) being the number of nucleon species) is achieved by considering 1-fermion loop and zero-boson loop Feynman graphs in the effective action. This corresponds to compute the meson self energies at 1-loop approximation.

For the \(\sigma\)-meson, the bare self energy in terms of the Lagrangian coupling constant is obtained as

\[
\Pi_{B,s}(p^2; M, \xi, \varepsilon) = -i \xi^{2\varepsilon} \int \frac{d^{4-2\varepsilon} k}{(2\pi)^{4-2\varepsilon}} \text{Tr} \left\{ \frac{i}{p \cdot k - M + i\varepsilon} i g_s \times \right. \\
\left. \times \frac{i}{k - M + i\varepsilon} i g_s \right\}.
\]  

(A1)

Imposing zero-momentum renormalization we get

\[
\Pi_s(p^2) = \left[ \Pi_{B,s}(p^2) - \Pi_{B,s}(0) - \Pi'_{B,s}(0) p^2 \right] \\
= -\frac{g_s^2 N}{4\pi^2} \left\{ \left(2M^2 - \frac{1}{2}p^2\right) I_a \left(\frac{p^2}{M^2}\right) + \frac{p^2}{3} \right\},
\]  

(A2)

where the function \(I_a(y)\) is defined as

\[
I_a(y) = \int_0^1 dx \log \left[1 - yx(1 - x) - i\varepsilon\right] \\
= \begin{cases} \\
\sqrt{1 - \frac{4}{y}} \log \frac{\sqrt{1 - \frac{4}{y} + 1}}{\sqrt{1 - \frac{4}{y} - 1}} - 2 & y < 0 \\
2 \sqrt{\frac{4}{y} - 1} \arcsin \left(\frac{\sqrt{y}}{2}\right) - 2 & 0 < y < 4 \\
\sqrt{1 - \frac{4}{y}} \log \frac{1 + \sqrt{1 - \frac{4}{y}}}{1 - \sqrt{1 - \frac{4}{y}}} - 2 - i \sqrt{1 - \frac{4}{y}} \pi & 4 < y.
\end{cases}
\]
The $\omega$-meson self energy is obtained in a similar way but taking care of its Lorentz structure,

$$\Pi_{B,v}^{\mu\nu}(p^2; M, \varepsilon, \xi) = -i \xi^2 \int \frac{d^4 k}{(2\pi)^{1-2\varepsilon}} \text{Tr} \left\{ \frac{i}{\slashed{p} + \slashed{k} - M + i\varepsilon} (-ig_v)\gamma^\mu \times \right.$$

$$\left. \times \frac{i}{\slashed{k} - M + i\varepsilon} (-ig_v)\gamma^\nu \right\}, \quad (A3)$$

which is highly simplified by baryonic current conservation,

$$\Pi_{B,v}^{\mu\nu}(p^2) = \left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2}\right) \Pi_{B,v}(p^2). \quad (A4)$$

The explicit expression of the $\omega$-meson self energy renormalized at zero-momentum is

$$\Pi_v(p^2) = \Pi_{B,v}(p^2) - \Pi_{B,v}(0) - \Pi'_{B,v}(0) p^2$$

$$= -\frac{Ng^2}{12\pi^2} \left( (2M^2 + p^2) I_0 \left( \frac{p^2}{M^2} \right) + \frac{p^2}{3} \right). \quad (A5)$$

### 2. Poles and residues

The relation between the Lagrangian mass of a boson, $m$, and its physical mass, $m_{sh}$, is given by

$$m_{sh}^2 - m^2 - \Pi(m_{sh}^2) = 0, \quad (A6)$$

where the self energy, $\Pi(p^2)$, is assumed to be renormalized at zero momentum. From the expression of its self energy given above we find that the $\sigma$-meson physical pole, $m_\sigma$, can be obtained in terms of the Lagrangian parameters $g_s$ and $m_s$ by solving the transcendental equation

$$m_s^2 = m_\sigma^2 - \frac{Ng_s^2}{4\pi^2} M^2 \left[ 4 - 4 \frac{m_s^2}{3 M^2} + \left( -4 + \frac{m_s^2}{M^2} \right) \sqrt{\frac{4M^2}{m_s^2} - 1 \arcsin \frac{m_s}{2M}} \right]. \quad (A7)$$

Similarly, the equation to solve for the $\omega$ particle is

$$m_\omega^2 = m_\omega^2 + \frac{Ng_\omega^2}{4\pi^2} M^2 \left[ 4 + 5 \frac{m_\omega^2}{9 M^2} - \frac{2}{3} \left( 2 + \frac{m_\omega^2}{M^2} \right) \sqrt{\frac{4M^2}{m_\omega^2} - 1 \arcsin \frac{m_\omega}{2M}} \right]. \quad (A8)$$

It is interesting to note that sometimes the combination $C^2 = M^2 g^2/m^2$ is taken to be fixed by nuclear matter properties. This allows one to write the Lagrangian coupling constant, $g$, as a function of $C$ and the Lagrangian mass, $m$. Inserting the omega version of this expression into the previous equation permits to solve the Lagrangian mass in terms of $C_\omega$ and the physical $\omega$ mass, $m_\omega$. If the eqs. (A7-A8) are conveniently extended to the $m_{sh}$-complex plane they can be used to obtain the Landau ghost masses as well (better expressions for numerical calculation are found in the main text in eqs. (33) and (33)).
Once a Landau pole has been computed, the value of its zero-momentum residue, $-R_G$, is easily obtained as

$$- R_G = 1 - \Pi'(-M_G^2).$$  \hspace{1cm} (A9)

The particular expressions of this equation for the $\sigma$ and $\omega$ meson are given in eqs. (34) and (32) respectively.
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**TABLE I.** Zero momentum renormalized Lagrangian parameters in several schemes. Masses are in MeV. The meaning of the labels no sea, sea 0th, sea 2nd, ω-shell and no-ghost are as in table III. The naive scheme corresponds to not including the meson self energy. The numbers in brackets are the zero-momentum parameters of the effective action for the no-ghost scheme, $\hat{m}_{s,v}$ and $\hat{g}_{s,v}$. In all cases, $m_\sigma$ and $m_\omega$ stand for the poles in the meson propagators after including the one fermion loop self energy and using the corresponding Lagrangian parameters. Note that by construction the vector meson parameters coincide in the sea 0th and sea 2nd cases.

|        | $g_s$  | $m_s\,\text{MeV}$ | $m_\sigma\,\text{MeV}$ | $g_v$  | $m_v\,\text{MeV}$ | $m_\omega\,\text{MeV}$ |
|--------|--------|-------------------|-------------------------|--------|-------------------|-------------------------|
| no sea | 9.062  | 449.9             | 439.8                   | 13.81  | 783               | 673.6                   |
| sea 0th|        |                   |                         |        |                   |                         |
| ω-shell| 6.153  | 382.8             | 379.8                   | 11.78  | 910.8             | 783                     |
| no-ghost| 5.996  | 370.9             | 368.3                   | 14.86  | 978.6             | 783                     |
| naive  | (5.928)| (368.8)           | (10.17)                 | (786.1)|                   |                         |
| sea 2nd|        |                   |                         |        |                   |                         |
| ω-shell| 6.846  | 425.9             | 420.3                   | 11.78  | 910.8             | 783                     |
| no-ghost| 6.664  | 410.8             | 406.5                   | 14.86  | 978.6             | 783                     |
| naive  | (6.544)| (407.1)           | (10.17)                 | (786.1)|                   |                         |

**TABLE II.** Residue (up to a sign) and mass (in MeV) of the ghosts in the zero-momentum renormalized meson propagators using the no-ghost sets of Lagrangian parameters in table I.

|        | $R_{G_0}^g$ | $M_{G_0}^g\,\text{MeV}$ | $R_{G_0}^\omega$ | $M_{G_0}^\omega\,\text{MeV}$ |
|--------|--------------|--------------------------|-------------------|-------------------------------|
| sea 0th| 1.748        | 4605                     | 0.6090            | 1457                          |
| sea 2nd| 1.584        | 3863                     | 0.6090            | 1457                          |
| \(^{A}_{Z}X\) | B/A (MeV) | no sea | sea 0th | sea 2nd | exp. |
|---|---|---|---|---|---|
| \(^{40}\text{Ca}\) | 6.28 | 6.00 | 6.10 | 6.33 | 6.43 | 8.55 |
| \(^{56}\text{Ni}\) | 7.24 | 6.51 | 6.60 | 6.80 | 6.90 | 8.64 |
| \(^{90}\text{Zr}\) | 8.81 | 7.99 | 8.07 | 8.22 | 8.30 | 8.71 |
| \(^{132}\text{Sn}\) | 9.84 | 9.55 | 9.61 | 9.70 | 9.76 | 7.87 |

**TABLE III.** Binding energy per nucleon of some closed-shell nuclei computed in several ways: not including the Dirac sea in the parameter fixing (no-sea), including the Dirac sea at lowest order in a gradient expansion (sea 0th), including the Dirac sea at all orders (sea 2nd) and the experimental values (exp.). The entry \(\omega\)-shell corresponds to use the set of parameters that reproduce the \(\omega\)-meson mass after including the meson self energy. The entry no-ghost corresponds to the parameters obtained by applying Redmond’s prescription.

| \(^{A}_{Z}X\) | m.q.c.r. (fm) | no sea | sea 0th | sea 2nd | exp. |
|---|---|---|---|---|---|
| \(^{40}\text{Ca}\) | 3.48 | 3.48 | 3.48 | 3.48 | 3.48 |
| \(^{56}\text{Ni}\) | 3.72 | 3.79 | 3.79 | 3.79 | 6.80 |
| \(^{90}\text{Zr}\) | 4.22 | 4.23 | 4.24 | 4.25 | 4.25 | 4.27 |
| \(^{132}\text{Sn}\) | 4.60 | 4.66 | 4.66 | 4.68 | 4.68 |
| \(^{208}\text{Pb}\) | 5.35 | 5.39 | 5.39 | 5.41 | 5.41 | 5.50 |

**TABLE IV.** Mean quadratic charge radii of several closed-shell nuclei. Meaning of the labels and experimental values as in table III.
FIGURES

FIG. 1. Real part of the inverse scalar meson propagator $K_s(p^2)$ as a function of the squared four momentum using the no-ghost (sea 2nd) set of parameters of table I. The dashed line represents the one-loop result without ghost subtraction. The solid line is the result after ghost elimination. The dotted line shows the free inverse propagator. In all cases the slope at zero momentum is unity. Units are in nucleon mass.

FIG. 2. Imaginary part of the inverse scalar meson propagator $K_s(p^2)$. Units and meaning of the lines as in fig. 1.
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