Kinetic, time irreversible evolution of the unstable $\pi^{\pm}$ - meson

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Abstract

In Liouville formalism the expression for density matrix, determining the time evolution of unstable $\pi^{\pm}$ - meson in the framework of unified formulation of quantum and kinetic dynamics is defined. The eigenvalues problem is investigated in the framework of Prigogine's principles of description of nonequilibrium processes at microscopic level. The problem was solved on the basis of complex spectral representation. It was shown that the approach leads to Pauli master equation for the weakly interacting system.

Key words: kinetic, irreversible, decay, dissipative, nonequilibrium

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1 Introduction

Let me examine the eigenvalues problem for the Hamiltonian $H = H_0 + gV$

$$H | \psi_\alpha > = \tilde{E}_\alpha | \psi_\alpha >,$$

where $H_0$ - free Hamiltonian, $V$ - interaction part, $g$ - coupling constant. In the conventional case Hamiltonian $H$ is a Hermitian operator, $\tilde{E}_\alpha$ is a perturbed energy of the state - a real number. It is known that the usual procedure of equation (1) solution on the basis of perturbation method can lead to the appearance of the small denominators $1/(E_\alpha - E_{\alpha'})$, where $E_\alpha, E_{\alpha'}$ are the energies corresponding to the unperturbed situation. Obviously, the divergences can arise at $E_\alpha = E_{\alpha'}$. The problem of the small denominators was determined by Poincare as "the basic problem of dynamics" [1]. According to Poincare's

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classification the systems, for which the situation can be corrected are called the "integrable" systems [2,3]. In the opposite case the systems are defined as the "non-integrable". Poincare proved that in the general case the dynamic systems are "non-integrable" ones. The basic question now is - what can we do to avoid Poincare’s divergences in case the system is "non-integrable"?

I. Prigogine and co-workers (Brussels - Austin group) noted that in the general case the satisfactory solution of this problem is impossible on the basis of the conventional formulation of quantum dynamics. The mechanism of asymmetry processes in time, which made it possible to accomplish a passage from the reversible dynamics to the irreversible time evolution was developed for the solution of this problem. The authors of the approach deny the conventional opinion that the irreversibility appears only at the macroscopic level, while the microscopic level must be described by the laws, reversed in the time. The method of description of the irreversibility at the quantum level proposed by them leads to the kinetic, time irreversible equations and determines the connection of quantum mechanics with kinetic dynamics. The approach allows to solve the problems, which could not be solved in the framework of conventional classical and quantum mechanics, for example, now we can realize the program of Heisenberg - to solve the eigenvalues problem for the Poincare’s "non-integrable" systems.

We examine the situation using the simple Friedrichs model (the model is presented closely to the text of works [2], [4] - [7]). Despite the fact that the solution of the problem for the Friedrichs model is known [8] it serves as a good example for the demonstration of the essence of situation. The model describes interaction of two level atom and electromagnetic field. In the Friedrichs model | 1 > corresponds to the atom in its bare exited level [5], | k > corresponds to the bare field mode with the atom in its ground state. The state | 1 > is coupled to the state | k >

\[
H = H_0 + gV = E_1 | 1 >< 1 | + \sum_k E_k | k >< k | + g \sum_k V_k (| k >< 1 | + | 1 >< k |),
\]

(2)

where

\[
| 1 >< 1 | + \sum_k | k >< k | = 1, \quad < \alpha | \alpha' >= \delta_{\alpha\alpha'}
\]

(3)

here \( \alpha (\alpha') = 1 \) or \( k \). The eigenvalues problem for the Hamiltonian \( H \) is formulated as follows

\[
H | \psi_1 >= \tilde{E}_1 | \psi_1 >, \quad H | \psi_k >= E_k | \psi_k >.
\]

(4)

For the eigenstate | \( \psi_1 > \) (for small \( g \)) perturbation method gives the expression

\[
| \psi_1 > \approx | 1 > - \sum_k \frac{gV_k}{E_k - E_1} | k >.
\]

(5)
If $E_1 > 0$, Poincare’s divergences appear at $E_k = E_1$.

In accordance with Brussels - Austin group approach the eigenvalues problem can be solved if the time ordering of the eigenstates will be introduced. This procedure can be realized through the introduction into the denominators imaginary terms: $-i\varepsilon$ for the relaxation processes, which are oriented into the future and $+i\varepsilon$ for the excitation processes, which are oriented into the past.

In this case the eigenvalues problem (4) is reduced to the complex eigenvalues problem

$$H |\varphi_1> = Z_1 |\varphi_1>, \quad <\bar{\varphi}_1|H = <\bar{\varphi}_1|Z_1,$$

$$H |\varphi_k> = E_k |\varphi_k>, \quad <\bar{\varphi}_k|H = <\bar{\varphi}_k|E_k,$$

where we must distinguish right - eigenstates $|\varphi_1>$, $|\varphi_k>$ and left - eigenstates $<\bar{\varphi}_1|$, $<\bar{\varphi}_k|$. $Z_1$ is a complex: $Z_1 = \bar{E}_1 - i\gamma$, $\bar{E}_1$ is a renormalized energy and $\gamma$ is a real positive value. This procedure makes it possible to avoid Poincare’s divergences and leads to the following expressions for the eigenstates $|\varphi_1>$, $<\bar{\varphi}_1|$

$$|\varphi_1> \approx |1> - \sum_k \frac{gV_k}{(E_k - \bar{E}_1 - z)^+_{-i\gamma}} |k>,$$

$$<\bar{\varphi}_1| \approx <1| - \sum_k \frac{gV_k}{(E_k - \bar{E}_1 - z)^+_{+i\gamma}} <k|,$$

In the expressions (8), (9) the designation $1/(E_k - \bar{E}_1 - z)^+_{-i\gamma}$ has been referred to as ”delayed analytic continuation” \[4,5\] and is defined through the integration with a test function $f(E_k)$

$$\int_0^\infty dE_k \frac{f(E_k)}{(E_k - \bar{E}_1 - z)^+_{-i\gamma}} \equiv \lim_{z \to -i\gamma} \left( \int_0^\infty dE_k \frac{f(E_k)}{E_k - \bar{E}_1 - z} \right)_{z \in C^+},$$

where we first have to evaluate the integration on the upper half - plane $C^+$ and then the limit of $z \to -i\gamma$ must be taken.

Now the spectral representation of the Hamiltonian has the form

$$H = \sum_\alpha Z_\alpha |\varphi_\alpha> <\bar{\varphi}_\alpha|.$$  

It was shown in ref. [4] that the Hamiltonian (11) is Hermitian operator. Since $H$ is Hermitian the corresponding eigenstates $|\varphi_1>$, $<\bar{\varphi}_1|$ are outside Hilbert space and have no Hilbert norm

$$<\varphi_1|\varphi_1> = <\bar{\varphi}_1|\bar{\varphi}_1> = 0.$$  

For the eigenstates we have relations

$$\sum_\alpha |\varphi_\alpha> <\bar{\varphi}_\alpha| = 1, \quad <\bar{\varphi}_\alpha|\varphi_{\alpha'}> = \delta_{\alpha\alpha'}.$$  


The eigenstates $|\varphi_1 \rangle$, $<\bar{\varphi}_1 |$ are called ”Gamow vectors” [9] - [12]. Thus, the Hermiticity of $H$ leads to the fact that ”usual” norms of eigenstates $|\varphi_1 \rangle$, $<\bar{\varphi}_1 |$ disappear. However, the eigenstates $|\varphi_1 \rangle$, $<\bar{\varphi}_1 |$ have a broken time symmetry. We can associate $|\varphi_1 \rangle$ with the unstable state, which vanishes for $t \to +\infty$, $|\bar{\varphi}_1 \rangle$ corresponds to the state, which vanishes for $t \to -\infty$

$$|\varphi_1(t)\rangle = \exp(-i\bar{E}_1 t - \gamma t) |\varphi_1(0)\rangle,$$  
$$|\bar{\varphi}_1(t)\rangle = \exp(-i\bar{E}_1 t + \gamma t) |\bar{\varphi}_1(0)\rangle.$$  

Obviously, at the present moment, it is very interesting and necessary to continue further development of the Brussels - Austin group approach in the framework of the more realistic models of interaction of the relativistic quantum fields. In the article, I examine $\pi^\pm$ - meson decay such as $\pi^\pm \to l^\pm + \nu_l\bar{\nu}_l$, where $l = e$ or $\mu$. In section 2 the definition of the weak interaction model is done. In section 3 I examine the complex eigenvalues problem - the complex eigenvalue and eigenstates are obtained. In section 4 on the basis of the complex representation I consider the Liouville formalism. Time evolution of the density matrix in the framework of ”subdynamics” approach is determined in section 5. Kinetic, time irreversible nature of the evolution is shown.

2 Definition of the weak interaction model

The Hamiltonian of weak interaction $H_{wk}$ has the form (determination of weak interaction can be found in works [13] - [15])

$$H_{wk} = \frac{G}{\sqrt{2}} \int f_\pi \bar{\psi}_l \gamma_\alpha (1 + \gamma_5) \psi_\nu \partial_\alpha \varphi_\pi dx + H.c.  \tag{16}$$

In expression (16) $G \approx 10^{-5}/m_p^2$ ($m_p$ - proton mass), $f_\pi = 0.91m_\pi$ ($m_\pi$ - $\pi^\pm$-meson mass). $H.c.$ indicates the Hermitian conjugate. For the operator of leptons field $\bar{\psi}_l$ we have the following decomposition [13]

$$\bar{\psi}_l = \bar{\psi}_l^{(+)} + \bar{\psi}_l^{(-)};$$
$$\bar{\psi}_l^{(+)} = \frac{1}{(2\pi)^{3/2}} \int \left(\frac{m_\nu}{E_{p_l}}\right)^{1/2} \bar{u}_r (p_l) \left(\begin{array}{c} e^{ip_\nu \cdot x} \cr 1 \end{array}\right) d_\nu (p_l),$$
$$\bar{\psi}_l^{(-)} = \frac{1}{(2\pi)^{3/2}} \int \left(\frac{m_\nu}{E_{p_l}}\right)^{1/2} \bar{u}_{\nu}^\dagger (p_l) \left(\begin{array}{c} e^{ip_\nu \cdot x} \cr 1 \end{array}\right) d_\nu (p_l). \tag{17}$$

For the operator of neutrino field we have

$$\psi_\nu = \frac{1}{(2\pi)^{3/2}} \int \left(\begin{array}{c} u_\nu^r (p_\nu) e^{ip_\nu \cdot x} c_{\nu_r} (p_\nu) + u_\nu^r (p_\nu) e^{-ip_\nu \cdot x} d^\dagger_{\nu_r} (p_\nu) \end{array}\right) dp_\nu. \tag{18}$$
It is assumed that the neutrino’s mass is zero. The operator of meson field is given by

\[ \varphi_\pi = \frac{1}{(2\pi)^{3/2}} \int \left( \frac{1}{2E_p}\right)^{1/2} \left( e^{-ip_p x} a^\dagger(p_\pi) + e^{ip_p x} b(p_\pi) \right) dp_\pi. \] (19)

In the decompositions (17) - (19), such value as \( c_r(p) \) \((a^\dagger(p), c^\dagger_r(p))\) is the operator of destruction (creation) of particle, \( d_r(p) \) \((b(p), d_r(p))\) is the operator of creation (destruction) of antiparticle, symbol “*” indicates the Hermitian conjugate. Spinors \( \bar{u}^r(p), \bar{u}^r(-p) \) \((u^r(p), u^r(-p))\) correspond to the states with helicity \( r = \pm 1 \), \( m_l \) - lepton mass, \( E_p = (p^2 + m_l^2)^{1/2} \) \((i = l or \pi)\). Note that we write 4 - vectors in the form \( A = (A, iA_0) \). In this case the following equalities are valid \( A^2 = A^2 + A_4^2 = A^2 - A_0^2 \) and \( px \equiv p_\alpha x_\alpha = px - p_0 x_0 \). We use units with \( h \), and the speed of light taken to be unity \((h = c = 1)\), \( \gamma_\alpha, \gamma_5 \) - Hermitian \( 4 \times 4 \) matrices \((\gamma_\alpha \gamma_\nu + \gamma_\nu \gamma_\alpha = 2\delta_\alpha\nu, \gamma_\alpha \gamma_5 + \gamma_5 \gamma_\alpha = 0, \gamma_5^2 = 1)\), \( \psi_l = \psi_l^\dagger \gamma_4 \).

3 Complex eigenvalues problem - perturbative solutions

Let examine the eigenvalues problem for the Hamiltonian \( H = H_0 + H_{wk} \), where \( H_{wk} \) - weak interaction. We will solve the problem assuming that the eigenvalue \( Z_{p_\pi} \) of Hamiltonian \( H \) is complex (the general formalism of the complex spectral representation can be found in works [117], [16] - [18]). In accordance with the approach [3], in our case, we will distinguish equation for the complex spectral representation can be found in works [117], [16] - [18]). In our case one - two - particles vector \(| \varphi_{p_\pi} > \) corresponds to the bare \( \pi^- \) - meson state with momentum \( p_\pi \). Two - particles vector \(| p_\mu, r_\mu; p_\nu, r_\nu > \), where \( p_\mu, r_\mu \) are momentum and helicity of muon and \( p_\nu, r_\nu \) are momentum and helicity of neutrino (in our case antineutrino), corresponds to the bare state consisting of muon and neutrino. In the model \(| p_\pi > \), \(| p_\mu, r_\mu; p_\nu, r_\nu > \) are eigenstates of the free

\[ H = H_0 + H_{wk}, \]

\[ Z_{p_\pi} = g \equiv G. \] (22)
Hamiltonian $H_0$. In accordance with definitions (21) (22), as will be shown in appendix, we obtain the expressions

$$Z_{p_\pi}^{(n)} = \langle p_\pi | V | \varphi_{p_\pi}^{(n-1)} > - \sum_{l=1}^{n-1} Z_{p_\pi}^{(l)} < p_\pi | \varphi_{p_\pi}^{(n-l)} >, \quad (23)$$

$$< p_\mu', r_\mu'; p_\nu, r_\nu | \varphi_{p_\pi}^{(n)} > = \frac{-1}{E_{p_\pi} + E_{p_\mu} - E_{p_\nu} + i\varepsilon_{\beta\alpha}}$$

$$\times \left( < p_\mu, r_\mu; p_\nu, r_\nu | V | \varphi_{p_\pi}^{(n-1)} > - \sum_{l=1}^{n} Z_{p_\pi}^{(l)} < p_\mu, r_\mu; p_\nu, r_\nu | \varphi_{p_\pi}^{(n-l)} > \right), \quad (24)$$

$$< \tilde{\varphi}_{p_\pi}^{(n)} | p_\mu, r_\mu; p_\nu, r_\nu > = \frac{1}{E_{p_\pi} - E_{p_\mu} - E_{p_\nu} + i\varepsilon_{\alpha\beta}}$$

$$\times \left( < \tilde{\varphi}_{p_\pi}^{(n-1)} | V | p_\mu, r_\mu; p_\nu, r_\nu > - \sum_{l=1}^{n} Z_{p_\pi}^{(l)} < \tilde{\varphi}_{p_\pi}^{(n-l)} | p_\mu, r_\mu; p_\nu, r_\nu > \right), \quad (25)$$

where the determination $H_{wk} = GV$ is used (it is necessary to note that in the paper, for simplification of the expressions, unessential normalizing volume is implied, but it is not written). In eqs. (24), (25), in accordance with the approach [4], the time ordering was introduced. Here, $\varepsilon_{\beta\alpha}$ ($\varepsilon_{\alpha\beta}$) is an infinitesimal, where $\alpha$ corresponds to the $\pi^-$ meson state, $\beta$ corresponds to the decay products $\mu^-$ and $\tilde{\nu}_\mu$.

The quantity $< p_\mu, r_\mu; p_\nu, r_\nu | \varphi_{p_\pi}^{(n)} > \equiv < \beta | \varphi_{p_\pi}^{(n)} >$ corresponds to the $\alpha \rightarrow \beta$ ($\pi^\rightarrow \mu^- + \tilde{\nu}_\mu$) transition, $< \tilde{\varphi}_{p_\pi}^{(n)} | p_\mu, r_\mu; p_\nu, r_\nu > \equiv < \tilde{\varphi}_{p_\pi}^{(n)} | \beta >$ corresponds to the $\beta \rightarrow \alpha$ transition. Since unstable $\pi^-$ meson disappears in the future we associate with the transition $\alpha \rightarrow \beta$ the analytic continuation $\varepsilon_{\beta\alpha} = -\varepsilon$. With the reverse $\beta \rightarrow \alpha$ transition we associate the analytic continuation oriented to the past, i.e., $\varepsilon_{\alpha\beta} = +\varepsilon$. In other words we assume that the state corresponding to muon and neutrino disappears in the past. Expressions (21) - (24) result into (A.17)

$$Z_{p_\pi} = E_{p_\pi} - \frac{G^2 f_{\pi}^2 m_\mu}{32 \pi^3 E_{p_\pi}} \sum_{r_\mu' r_\nu'} \int \frac{d^4 p_{\mu}'}{E_{p_{\mu}'}^4} \frac{d^4 p_{\nu}'}{E_{p_{\nu}'}^4} \delta (p_{\pi} - p_{\mu} + p_{\nu}')$$

$$\times \frac{p_{\alpha,\pi} \overline{u}_{\nu}' (-p_{\nu}') \gamma_\alpha (1 + \gamma_5) \delta_{\alpha} u_{\mu}' (p_{\mu}') p_{\beta,\pi} \overline{u}_{\mu}' (p_{\mu}') \gamma_\beta (1 + \gamma_5) u_{\nu}' (-p_{\nu}')} { (E_{p_{\mu}'} + E_{p_{\nu}'} - Z)^2 Z_{p_\pi}} , \quad (26)$$

where summation over internal indices $\alpha, \beta$ is implied; $\delta_{\alpha} = -1$ if $\alpha = 1, 2, 3$ and $\delta_{\alpha} = 1$ if $\alpha = 4$. Taking into account "delayed analytic continuation" (A.16)

$$\frac{1}{(E_{p_{\mu}'} + E_{p_{\nu}'} - Z)^2 Z_{p_\pi}} \equiv \frac{1}{E_{p_{\mu}'} + E_{p_{\nu}'} - Z - i\varepsilon} |_{Z = Z_{p_\pi}}, \quad (27)$$

using the formal expression $\frac{1}{w \pm i\varepsilon} \rightarrow P \frac{1}{w} \mp i\pi \delta(w)$ and being limited by order $G^2$ we present eq. (26) in the form

$$Z_{p_\pi} = E_{p_\pi} - i\gamma_{p_\pi}, \quad (28)$$

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In the expression (28)

\[ \bar{E}_{p}\pi = E_{p\pi} + \frac{G^2 f_0^2 m_\mu}{32\pi^3 E_{p\pi}} \sum_{r_\mu r'_\nu} \int d\phi'_\mu d\phi'_\nu \frac{\delta(p_\pi - \phi'_\mu - \phi'_\nu)}{E_{p\mu'} + E_{p\nu'} - E_{p\pi}} \]

\[ \times p_{\alpha,\pi,\mu,\nu}(\bar{p}_\nu)\gamma(1 + \gamma_5)u'_{\pi}(\bar{p}_\mu')\gamma_{\beta,\pi,\mu,\nu}(\bar{p}_\mu')\gamma_\beta(1 + \gamma_5)u'_{\pi}(\bar{p}_\nu') \]

is a renormalized energy of \( \pi^- \) meson, \( \mathbf{P} \) stands for the principal part and

\[ \gamma_{p\pi} = \frac{G^2 f_0^2 m_\mu}{32\pi^3 E_{p\pi}} \sum_{r_\mu r'_\nu} \int d\phi'_\mu d\phi'_\nu \frac{\delta(p_\pi - \phi'_\mu - \phi'_\nu)}{E_{p\mu'} + E_{p\nu'} - E_{p\pi}} \delta(E_{p\pi} - E_{p\mu'} - E_{p\nu'}) \]

\[ \times p_{\alpha,\pi,\mu,\nu}(\bar{p}_\nu)\gamma(1 + \gamma_5)\delta_{\alpha,\nu}u'_{\pi}(\bar{p}_\mu')\gamma_{\beta,\pi,\mu,\nu}(\bar{p}_\mu')\gamma_\beta(1 + \gamma_5)u'_{\pi}(\bar{p}_\nu'). \]

As will be shown in appendix for the eigenstates \( |\varphi_{p\pi}>, |\tilde{\varphi}_{p\pi}> \) we have

\[ |\varphi_{p\pi}> = |p_\pi> - \frac{iG}{2(2\pi)^{3/2}} \left( \frac{m_\mu}{E_{p\pi}} \right)^{1/2} \]

\[ \times \sum_{r_\mu r'_\nu} \int d\phi'_\mu d\phi'_\nu \frac{\delta(p_\pi - \phi'_\mu - \phi'_\nu)}{E_{p\mu'} + E_{p\nu'} - E_{p\pi} - z} |p_\mu', r_\mu, p_\nu, r_\nu> \]

\[ <\tilde{\varphi}_{p\pi}| = <p_\pi| + \frac{iG}{2(2\pi)^{3/2}} \left( \frac{m_\mu}{E_{p\pi}} \right)^{1/2} \]

\[ \times \sum_{r_\mu r'_\nu} \int d\phi'_\mu d\phi'_\nu \frac{\delta(p_\pi - \phi'_\mu - \phi'_\nu)}{E_{p\mu'} + E_{p\nu'} - E_{p\pi} - z} |p_\mu', r_\mu, p_\nu, r_\nu| \]

The eigenstates \( |\varphi_{p\pi}>, |\tilde{\varphi}_{p\pi}> \) have a broken time symmetry. In accordance with the expressions (20), (28)

\[ |\varphi_{p\pi}(t)> = \exp(-iHt) |\varphi_{p\pi}> = \exp(-i\bar{E}_{p\pi}t - \gamma_{p\pi}t) |\varphi_{p\pi}> \]

\[ |\tilde{\varphi}_{p\pi}(t)> = \exp(-iHt) |\tilde{\varphi}_{p\pi}> = \exp(-i\bar{E}_{p\pi}t + \gamma_{p\pi}t) |\tilde{\varphi}_{p\pi}> \]

Here \( |\varphi_{p\pi}> \) corresponds to the state which vanishes for \( t \rightarrow +\infty \), \( |\tilde{\varphi}_{p\pi}> \) corresponds to the state which vanishes for \( t \rightarrow -\infty \). The states \( |\varphi_{p\pi}>, |\tilde{\varphi}_{p\pi}> \) are "Gamow vectors" (according to classification of [5]).

We examine \( \pi^- \) meson at rest. In this case the expression (30) results into

\[ \gamma_{p\pi=0} = \frac{1}{2} \frac{G^2 f_0^2}{8\pi} m_\pi m_\mu^2 \left( 1 - \frac{m_\mu^2}{m_\pi^2} \right)^2 = \frac{1}{2} \Gamma. \]  

\[ \Gamma \]

is a well known rate for \( \pi^- \) meson decay (see for example [15]). Thus, the procedure of the time ordering of the expression (24)
leads to the complex eigenvalue of the Hamiltonian $H$ which, in turn, makes it possible to determine the rate $\Gamma$, when $1/\Gamma = 1/2\gamma_{p,\pi} = 2.6 \times 10^{-8}$ s - lifetime $\tau_0$ of $\pi^-$ - meson at rest.

In the general case the value $\gamma_{p,\pi}$ depends on the momentum $p$. We examine the situation, when the angle $\vartheta_{p,\pi}$ of the vector $p_{\pi}$ (in the spherical coordinates) is zero. In this case from the expression (30) we obtain

$$\gamma_{|p|_{\pi}} = \frac{G^2 f^2_{\pi} m^2_{\pi} (m^2_{\pi} - m^2_{\mu})}{16\pi E|p|_{\pi}}$$

$$\times \int_{-1}^{1} dx \frac{E'_{p_{\nu}}}{(|p|_{\pi}^2 + m^2_{\mu} + E'^2_{p_{\nu}} - 2|p|_{\pi} E'_{p_{\nu}} x)^{1/2} + E'_{p_{\nu}} - |p|_{\pi} x},$$

(36)

where the energy of neutrino depends on the momentum of $\pi^-$ - meson and is determined by the expression

$$E'_{p_{\nu}} = \frac{m^2_{\pi} - m^2_{\mu}}{2(E|p|_{\pi} - |p|_{\pi} x)}, \ x \equiv \cos \vartheta_{p_{\nu}}.$$  

(37)

Note that the obtained results (35) - (37) are valid also for $\pi^+$ - meson. As the test of the expression (35) let me examine the lifetime $\tau$ of $\pi^\pm$ - meson depending on the momentum $|p|_{\pi}$. The use of the expression (36) leads to the following approximate results for the lifetime $\tau = 1/2\gamma_{|p|_{\pi}}$: $|p|_{\pi} = 0.5$ GeV, $\tau \approx 9.8 \times 10^{-8}$ s; $|p|_{\pi} = 1.5$ GeV, $\tau \approx 2.8 \times 10^{-7}$ s; $|p|_{\pi} = 3$ GeV, $\tau \approx 5.7 \times 10^{-7}$ s - the lifetime of $\pi^\pm$ - meson increases with an increasing of the momentum $|p|_{\pi}$. Thus, Brussels - Austin group approach leads to results which are in agreement with Einstein time dilation.

In such a way, on the basis of the approach the value of the rate $\Gamma$ is obtained as the solution of the eigenvalues problem on the basis of the complex spectral representation.

4 Liouville formalism in the framework of the complex representation

In the Liouville formalism the time evolution is determined by Liouville - von Neumann equation for the density matrix $\rho$

$$i \frac{\partial \rho(t)}{\partial t} = L \rho(t),$$  

(38)

where "Liouvillian" $L$ has the form

$$L = H \times 1 - 1 \times H,$$  

(39)

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here symbol ”×” denotes the operation \((A \times B)\rho = A\rho B\) (the Liouville formalism, for example, can be found in ref. [19]). In accordance with the determination [39], \(L\) can be written down in the sum of free part \(L_0\) that depends on the free Hamiltonian \(H_0\) and interaction part \(L_I\) that depends on the interaction \(gV\): \(L = L_0 + L_I\). For the Liouville operator \(L\) we have the equation (the text is written close to the materials of works [25])

\[
L | f_\nu \rangle\rangle = w_\nu | f_\nu \rangle\rangle,
\]

(40)

where \(| f_\nu \rangle\rangle \equiv | \psi_\alpha > \psi_\beta |\), \(w_\nu = \tilde{E}_\alpha - \tilde{E}_\beta\) and here \(\nu\) is the correlation index (\(\nu\) determines the variety of combinations of the initial and the final states of the system): \(\nu = 0\) if \(\alpha = \beta\) - diagonal case and \(\nu \neq 0\) in the remaining off-diagonal case (the details of the theory of correlations can be found, for example, in works [2,20]). The eigenvalues problem (40) for Liouville operator \(L\) has the similar features as for Hamiltonian \(H\). If we expand the values \(| f_\nu \rangle\rangle\), \(w_\nu\) in the perturbation series, the problem of Poincare’s divergences will arise again. In accordance with the approach, we have to introduce the time ordering. This leads to new formulation of the eigenvalues problem for the operator \(L\)

\[
L | \Psi_j^{\nu} \rangle\rangle = Z_j^{\nu} | \Psi_j^{\nu} \rangle\rangle, \quad \langle\langle \tilde{\Psi}_j^{\nu} | L = \langle\langle \tilde{\Psi}_j^{\nu} | Z_j^{\nu},
\]

(41)

where \(Z_j^{\nu}\) are the complex values and \(j\) is a degeneracy index since one type of correlation index can correspond to the different states (the complex eigenvalues problem for the Liouville operator is examined in works [5], [20] - [22]). In eq. (41) \(L\) is Hermitian. It is possible in case corresponding eigenstates have no Hilbert norm. For the eigenstates \(| \Psi_j^{\nu} \rangle\rangle\), \(\langle\langle \tilde{\Psi}_j^{\nu} |\) we have the following biorthogonality and bicompleteness

\[
\langle\langle \tilde{\Psi}_j^{\nu} | \Psi_i^{\nu} \rangle\rangle = \delta_{\nu\nu} \delta_{ji}, \quad \sum_{\nu j} | \Psi_j^{\nu} \rangle\rangle \langle\langle \tilde{\Psi}_j^{\nu} | = 1. \quad (42)
\]

The spectral representation of the Liouville operator can be written as follows

\[
L = \sum_{\nu j} Z_j^{\nu} | \Psi_j^{\nu} \rangle\rangle \langle\langle \tilde{\Psi}_j^{\nu} | .
\]

(43)

It was shown that the eigenstates of \(L\) can be written in the terms of kinetic operators \(C^{\nu}\) and \(D^{\nu}\). Operator \(C^{\nu}\) creates correlations other than the \(\nu\) correlations, \(D^{\nu}\) is destruction operator [5,23,24]. The use of the kinetic operators allows to write down expressions for the eigenstates of Liouville operator in the following form [5]

\[
| \Psi_j^{\nu} \rangle = (N_j^{\nu})^{1/2} (P^{\nu} + C^{\nu}) | u_j^{\nu} \rangle\rangle, \quad \langle\langle \tilde{\Psi}_j^{\nu} | = \langle\langle \tilde{\Psi}_j^{\nu} | (P^{\nu} + D^{\nu}) (N_j^{\nu})^{1/2},
\]

(44)

where \(N_j^{\nu}\) - is a normalization constant. The determination of the states \(| u_j^{\nu} \rangle\rangle\), \(\langle\langle \tilde{\Psi}_j^{\nu} |\) and operators \(P^{\nu}, C^{\nu}, D^{\nu}\) can be found in works [5,6,20]. In the general
case, for example, the operators $P^\nu$ are determined by the following conditions \[20\]

$$P^\nu = \sum_j |u^\nu_j\rangle\langle\langle \tilde{u}^\nu_j|, \langle\langle \tilde{u}^\nu_j|u^\nu_{j'}\rangle = \delta^\nu\delta_{jj'}.$$  

(45)

Similarly for $P^\nu$ and $\langle\langle \tilde{v}^\nu_j| we have:

$$P^\nu = \sum_j |v^\nu_j\rangle\langle\langle \tilde{v}^\nu_j|, \langle\langle \tilde{v}^\nu_j|v^\nu_{j'}\rangle = \delta^\nu\delta_{jj'}.$$  

(46)

Substituting expression (44) in eq. (41) and multiplying $P^\nu$ from left on both sides, we obtain \[5\]

$$\theta^\nu_C |u^\nu_j\rangle\rangle = Z^\nu_j |u^\nu_j\rangle\rangle,$$

(47)

where

$$\theta^\nu_C \equiv P^\nu L(P^\nu + C^\nu) = L_0 P^\nu + P^\nu L_1 (P^\nu + C^\nu) P^\nu.$$  

(48)

In eq. (47) $\theta^\nu_C$ is the collision operator connected with the kinetic operator $C^\nu$. This is non-Hermitian dissipative operator, which plays the main role in the nonequilibrium dynamics. As was shown in ref. \[20\] operator $\theta^\nu_C$ can be reduced to the collision operator in Pauli master equation for the weakly coupled systems. Comparing eqs. (41), (47) we can see that $|u^\nu_j\rangle\rangle$ is eigenstate of collision operators $\theta^\nu_C$ with the same eigenvalues $Z^\nu_j$ as $L$. It is possible to obtain the equation for the operator $\theta^\nu_D$ analogous to eq. (47), which is associated with the destruction kinetic operator $D^\nu$.

In that way, the determination of the eigenvalues problem for the Liouville operator $L$ outside the Hilbert space leads to the connection of quantum mechanics with kinetic, time irreversible dynamics.

5 Time evolution of the density matrix

The fundamental quantum-mechanical Liouville - von Neumann equation (38) describes the reversible evolution in the time. Basic question is - how can the irreversibility arise? The response to the question found its embodiment in the works of Brussels-Austin group. It was shown that the description of the time irreversible evolution is possible in the space of $\Psi^\nu_j, \tilde{\Psi}^\nu_j$ – functions. Let introduce the ”subdynamics” approach. The ”subdynamics” approach \[25\] - \[27\] means the construction of the complete set of the spectral projectors $\Pi^\nu$

$$\Pi^\nu = \sum_j |\Psi^\nu_j\rangle\langle\langle \tilde{\Psi}^\nu_j|.$$  

(49)
The projectors $\Pi^\nu$ satisfy the following relations:

\[
\Pi^\nu L = L \Pi^\nu, \quad \text{(commutativity)}; \quad \sum_{\nu} \Pi^\nu = 1, \quad \text{(completeness)};
\]
\[
\Pi^\nu \Pi^\nu' = \Pi^\nu \delta_{\nu \nu'}, \quad \text{(orthogonality)}.
\]

Thus, the following decomposition of the density matrix is possible

\[
\rho(t) = \sum_{\nu} \Pi^\nu \rho(t) \equiv \sum_{\nu} \rho^\nu(t),
\]

where $\rho^\nu(t) \equiv \Pi^\nu \rho(t)$. In the framework of "subdynamics" approach we can reduce eq. (38) to the equation for $P^\nu \rho^\nu(t)$ - component for each $\Pi^\nu$ - subspace

\[
i \frac{\partial}{\partial t} P^\nu \rho^\nu(t) = \theta^\nu_C P^\nu \rho^\nu(t),
\]

where operator $\theta^\nu_C$ is determined by the expression (48). $P^\nu \rho^\nu(t)$ - components were called as the "privileged" components of $\rho^\nu(t)$. Projectors $\Pi^\nu$ can be associated with the introduction of the concept of "subdynamics" because the components $\rho^\nu$ satisfy separate equations of motion. Our great interest is to investigate eq. (52) for the system of the weak interacting fields. For this purpose we present the latter in the Dirac - representation

\[
i \frac{\partial}{\partial t} P^\nu \rho^\nu(t) = \vartheta^\nu(t) P^\nu \rho^\nu(t),
\]

where

\[
\vartheta^\nu(t) \equiv P^\nu L(t) C^\nu P^\nu. \quad \text{(54)}
\]

Note that in eq. (53), the previous designations of operators are preserved. The general solution of eq. (53) can be found after examining the equivalent integral equation

\[
P^\nu \rho^\nu(t) = P^\nu \rho^\nu(t_0) + (-i) \int_{t_0}^{t} dt_1 \vartheta^\nu(t_1) P^\nu \rho^\nu(t_1),
\]

where $P^\nu \rho^\nu(t_0)$ corresponds to the initial moment $t_0$. The result can be put down in the form

\[
P^\nu \rho^\nu(t) = \Omega^\nu(t, t_0) P^\nu \rho^\nu(t_0),
\]

where

\[
\Omega^\nu(t, t_0) = \sum_{n=0}^{\infty} (-i)^n \int_{t_0}^{t} \int_{t_0}^{t_1} \ldots \int_{t_0}^{t_{n-1}} dt_1 dt_2 \ldots dt_n \vartheta^\nu(t_1) \vartheta^\nu(t_2) \ldots \vartheta^\nu(t_n). \quad \text{(57)}
\]
The non-Hermitian operator $\Omega^{\nu}(t, t_0)$ determines the time evolution of the "privileged" component $P^{\nu}\rho^{\nu}$ - the time irreversible evolution of the unstable state. Determination of eq. (53) we will carry out for the matrix element of the form $\langle \langle p_{\pi} p_{\pi}|P^{\rho}\rho^{\rho}(t)\rangle\rangle \equiv < p_{\pi} | \rho^{\rho}(t) | p_{\pi}> \equiv \rho^{0}_{p_{\pi}p_{\pi}}(t)$. Using the determinations of the operators $P^{\nu}$, $C^{\nu}$, $D^{\nu}$ [56][20], for the diagonal "privileged" component ($\nu = 0$) (being limited by order $G^2$) from the expression (56) we have the result

$$\rho^{0}_{p_{\pi}p_{\pi}}(t) = e^{-2\gamma_{p_{\pi}} t}\rho^{0}_{p_{\pi}p_{\pi}}(0) + (1 - e^{-2\gamma_{p_{\pi}} t}) \sum_{\nu, r_{\nu}} \int d\rho_{\mu}d\rho_{\nu}\Gamma_{p_{\pi} p_{\mu}, r_{\mu}; p_{\nu}, r_{\nu}} (0),$$

(58)

where $t_0 = 0$, $\rho^{0}_{p_{\mu}, r_{\mu}; p_{\nu}, r_{\nu}} \equiv < p_{\mu}, r_{\mu}; p_{\nu}, r_{\nu} | \rho^{0} | p_{\mu}, r_{\mu}; p_{\nu}, r_{\nu}>$. The result (58) reflects the evolution from the unstable $\pi^-$ meson state to the decay products. As follows from expression (58) the time evaluation of $\pi^-$ meson has strictly exponential behavior. Function $\Gamma_{p_{\pi} p_{\mu}, r_{\mu}; p_{\nu}, r_{\nu}}$ in expression (58) is determined as

$$\Gamma_{p_{\pi} p_{\mu}, r_{\mu}; p_{\nu}, r_{\nu}} = \frac{G^{2} f_{p_{\pi}}^{2} m_{\pi}}{32\pi^{3} E_{p_{\pi}} E^{\nu}_{p_{\mu}}} \delta(p_{\pi} - p_{\mu} - p_{\nu})$$

$$\times p_{\alpha, \pi}^{\nu}(p_{\nu})\gamma_{\alpha}(1 + \gamma_{\nu})(-\delta_{\alpha} u^{\nu}(p_{\mu})p_{\beta, \pi}^{\mu}(p_{\mu})\gamma_{\beta}(1 + \gamma_{\nu})u^{\nu}(-p_{\nu})$$

$$\times \left(\frac{E_{p_{\mu}} + E_{p_{\nu}} - E_{p_{\pi}} - z}{E_{p_{\mu}} + E_{p_{\nu}} - E_{p_{\pi}} - z} + \frac{1}{i\gamma_{p_{\pi}}}\right),$$

(59)

where designation $1/(E_{p_{\mu}} + E_{p_{\nu}} - E_{p_{\pi}} - z)$ corresponds to the integration, which first of all is carried out in the lower half complex plane $C^-$ and, after that, the limit of $z \rightarrow +i\gamma_{p_{\pi}}$ is taken. In accordance with works [5], [6] we will determine the function $\Gamma_{p_{\pi} p_{\mu}, r_{\mu}; p_{\nu}, r_{\nu}}$ as the line shape of the $\pi^-$ meson decay products.

There is the direct connection between eq. (53) and Pauli master equation. From eq. (53) we obtain

$$\frac{\partial \rho^{0}_{p_{\pi}p_{\pi}}(t)}{\partial t} = \sum_{\nu, r_{\nu}} \int d\rho_{\mu}d\rho_{\nu} W_{p_{\pi} p_{\mu}, r_{\mu}; p_{\nu}, r_{\nu}} \left(\rho^{0}_{p_{\mu}, r_{\mu}; p_{\nu}, r_{\nu}} p_{p_{\pi}, r_{\pi}}(t) - \rho^{0}_{p_{\pi}p_{\pi}}(t)\right)$$

(60)

the analogue of Pauli master equation for $\pi^-$ meson decay, where the transition rate $W_{p_{\pi} p_{\mu}, r_{\mu}; p_{\nu}, r_{\nu}}$ is given by

$$W_{p_{\pi} p_{\mu}, r_{\mu}; p_{\nu}, r_{\nu}} = 2\gamma_{p_{\pi}} \Gamma_{p_{\pi} p_{\mu}, r_{\mu}; p_{\nu}, r_{\nu}}.$$ 

(61)

For the function $W_{p_{\pi} p_{\mu}, r_{\mu}; p_{\nu}, r_{\nu}}$ the following expression is correct

$$\sum_{\nu, r_{\nu}} \int d\rho_{\mu}d\rho_{\nu} W_{p_{\pi} p_{\mu}, r_{\mu}; p_{\nu}, r_{\nu}} = 2\gamma_{p_{\pi}}.$$ 

(62)
Obviously, results (58), (60) can be interpreted as follows: diagonal element of the density matrix gives probability to reveal $\pi^-$ meson with momentum $p_{\pi}$ at the moment of time $t$. This probability decreases due to the evolution in the time to the products consisting of muon and neutrino. Results (58), (60) correspond to the kinetic, time irreversible evolution of the unstable state in the time, which is oriented into the future. They reflect the energy transfer during the evolution from unstable $\pi^-$ meson to the decay products without appearance of the other spontaneous, unstable states. It is necessary to note, the analogous to the expressions (58), (60) results were obtained in works [5] (for the Friedrichs model) and [20] (for the potential scattering).

6 Concluding remarks

The meson decay has been investigated long ago (for example in work [28]). However it is known that the description of physical world on the microscopic level, on the basis of conventional quantum dynamics is defined by the laws of the nature, which are deterministic and time reversible. The time in the conventional method (S - matrix approach [13] - [15]) does not have the chosen direction and the future and the past are not distinguished. It is obvious that the facts given before are in the contradiction to our experience, because the world surrounding us has obvious irreversible nature. In this world the symmetry in the time is disrupted and the future, and the past play different roles. Difference between the conventional description of the nature and those processes in the nature which we observe creates the conflict situation. The alternative formulation of quantum dynamics found its realization in the works of Brussels-Austin group that was headed by I. Prigogine. The basic idea of the I. Prigogine and co-workers is to develop the precise method for the description of the nature at the macroscopic and microscopic levels, where the irreversible processes predominate. It was noted that the exact solution of this problem is impossible on the basis of the conventional method, unitary principles. Therefore one should speak about the alternative formulation of the dynamics, that makes it possible to include the irreversibility in a natural way. In this connection the studies of the irreversible processes at the microscopic level - the microscopic formulation of the irreversibility represents the special interest. The authors of the approach deny the conventional opinion that the irreversibility appears only at the macroscopic level, while the microscopic level can be described by the laws, reversed in the time. Thus, new irreversible dynamics with the disrupted symmetry in the time was formulated. In the approach of Brussels-Austin group the irreversibility is presented as the property of material itself and is not defined by the active role of the observer. This approach implies the passage from the reversible dynamics to the irreversible time evolution, where the eigenstates have a broken time
symmetry. In the case of "non-integrable" systems the approach leads to the asymmetry between the past and the future. For $\pi^\pm$ meson decay, on the basis of Brussels-Austin group approach the value of the rate $\Gamma$ is obtained as the solution of the eigenvalues problem on the basis of the complex spectral representation. Whereas the conventional method is based on the set of the well known, mnemonic rules. The approach contains the important assembling element. It leads to the unified formulation of quantum and kinetic dynamics. Let me emphasize that the main purpose of the work is to describe $\pi^\pm$ - meson decay as the irreversible process. Despite the fact that this requires a lot of rather lengthy formulas than the conventional method, where $\pi^\pm$ - meson decay is the time reversible process, it is possible to say: the approach leads to the adequate description of the evolution in the time of the relativistic, unstable state, including irreversibility. This approach clarifies the passage from reversible dynamics to irreversible time evolution at the microscopic level (one of the fundamental problems in physics), as it describes an irreversible process as a rigorous dynamical process in conservative Hamiltonian systems when the systems are non-integrable in the sense of Poincare.

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Appendix. Derivation of the relations

In appendix, we will obtain the expressions (26), (31). Let me introduce the designation $H_{wk} = GV$. In accordance with the expressions (20), (21) the eigenvalues problem for $|\varphi_{p_\pi}|$ can be rewritten in the form

$$ (H_0 + GV) \sum_{n=0}^{\infty} G^n |\varphi^{(n)}_{p_\pi}| = \sum_{n=0}^{\infty} G^n Z_{p_\pi}^{(n)} \sum_{n'=0}^{\infty} G^{n'} |\varphi^{(n')}_{p_\pi}|. \quad (A.1) $$

The multiplication of eq. (A.1) by one-particle vector $|\vec{p}_\pi>$ leads to the expression:

$$ <\vec{p}_\pi| \left( H_0 \sum_{n=0}^{\infty} G^n |\varphi^{(n)}_{p_\pi}| + GV \sum_{n=0}^{\infty} G^n |\varphi^{(n)}_{p_\pi}| \right) $$

$$ = <\vec{p}_\pi| \sum_{n=0}^{\infty} G^n Z_{p_\pi}^{(n)} \sum_{n'=0}^{\infty} G^{n'} |\varphi^{(n')}_{p_\pi}| >. \quad (A.2) $$

Since, in the model, $|\vec{p}_\pi>$ is the eigenstate of the free Hamiltonian $H_0$:
We can present eq. (A.3) in the form

\[ E_{p_\pi} \sum_{n=0}^{\infty} G^n < p_\pi \mid \varphi_{p_\pi}^{(n)} > + \sum_{n=0}^{\infty} G^{n+1} < p_\pi \mid \varphi_{p_\pi}^{(n)} > 
\]

\[ = E_{p_\pi} + E_{p_\pi} \sum_{n=1}^{\infty} G^n < p_\pi \mid \varphi_{p_\pi}^{(n)} > + \sum_{n=1}^{\infty} G^n Z_{p_\pi}^{(n)} 
\]

\[ + \sum_{n=1}^{\infty} G^n Z_{p_\pi}^{(n)} \sum_{n'=1}^{\infty} G^{n'} < p_\pi \mid \varphi_{p_\pi}^{(n')} > . \quad (A.3) \]

We can present eq. (A.3) in the form

\[ \sum_{n=1}^{\infty} G^n < p_\pi \mid V \mid \varphi_{p_\pi}^{(n+1)} > - \sum_{n=1}^{\infty} G^n Z_{p_\pi}^{(n)} \sum_{n'=0}^{\infty} G^{n'+1} < p_\pi \mid \varphi_{p_\pi}^{(n'+1)} > 
\]

\[ = \sum_{n=1}^{\infty} G^n Z_{p_\pi}^{(n)}. \quad (A.4) \]

From the expression (A.4) we obtain eq. (23).

The multiplication of eq. (A.1) by two - particles vector \( | p_\mu, r_\mu; p_\nu, r_\nu > \) gives

\[ (E_{p_\mu} + E_{p_\nu}) \sum_{n=1}^{\infty} G^n < p_\mu, r_\mu; p_\nu, r_\nu \mid \varphi_{p_\pi}^{(n)} > 
\]

\[ + \sum_{n=0}^{\infty} G^{n+1} < p_\mu, r_\mu; p_\nu, r_\nu \mid V \mid \varphi_{p_\pi}^{(n)} > = (E_{p_\pi} + \sum_{n=1}^{\infty} G^n Z_{p_\pi}^{(n)} ) \quad (A.5) \]

\[ \times \sum_{n'=1}^{\infty} G^{n'} < p_\mu, r_\mu; p_\nu, r_\nu \mid \varphi_{p_\pi}^{(n')} > , \]

where the determination \( H_0 \mid p_\mu, r_\mu; p_\nu, r_\nu >(E_{p_\mu} + E_{p_\nu}) \mid p_\mu, r_\mu; p\nu, r_\nu > \)

was used. Now we can obtain the expression

\[ \sum_{n=1}^{\infty} G^n < p_\mu, r_\mu; p_\nu, r_\nu \mid \varphi_{p_\pi}^{(n)} > 
\]

\[ = -1 \frac{1}{E_{p_\mu} + E_{p_\nu} - E_{p_\pi}} \left( \sum_{n=1}^{\infty} G^n < p_\mu, r_\mu; p_\nu, r_\nu \mid V \mid \varphi_{p_\pi}^{(n-1)} > 
\]

\[ - \sum_{n=1}^{\infty} \sum_{n'=0}^{\infty} G^n G^{n'} Z_{p_\pi}^{(n')} < p_\mu, r_\mu; p_\nu, r_\nu \mid \varphi_{p_\pi}^{(n')} > \right), \quad (A.6) \]

which leads to the equation

\[ < p_\mu, r_\mu; p_\nu, r_\nu \mid \varphi_{p_\pi}^{(n)} > 
\]

\[ = -1 \frac{1}{E_{p_\mu} + E_{p_\nu} - E_{p_\pi}} \left( < p_\mu, r_\mu; p_\nu, r_\nu \mid V \mid \varphi_{p_\pi}^{(n-1)} > 
\]

\[ - \sum_{l=1}^{n} Z_{p_\pi}^{(l)} < p_\mu, r_\mu; p_\nu, r_\nu \mid \varphi_{p_\pi}^{(n-l)} > \right). \quad (A.7) \]
In accordance with Brussels - Austin group approach the time ordering of eq. (A.7) must be introduced. This can be realized in accordance with the rules of section 3 through the introduction into the denominator imaginary term $-i\varepsilon$ with respect to one of the particle $\mu^-$ or $\nu_-^\mu$ (we assume that $\mu^-$ and $\nu_-^\mu$ appear in the future), then we obtain eq. (24).

Now we examine the expression (23). We define: $|p_\pi> = b^\dagger(p_\pi)|\Phi_0>$ and $|p_\mu, r_\mu; p_\nu, r_\nu> = c^\dagger_{r_\mu}(p_\mu)d^\dagger_{r_\nu}(p_\nu)|\Phi_0>$, where $b^\dagger(p_\pi)$, $c^\dagger_{r_\mu}(p_\mu)$, $d^\dagger_{r_\nu}(p_\nu)$ are the creation operators of $\pi^-$, $\mu^-$, $\nu_-^\mu$ particles respectively, $|\Phi_0>$ - the vacuum state. Substituting the expressions (16) - (19) into the first term of eq. (23) (where $V = H_{\omega_k}/G$) we obtain

$$Z_{p_\pi}^{(n)} = -i\frac{f_\pi}{2(2\pi)^{3/2}} \sum_{r_\mu,r_\nu} \int dp_\mu dp_\nu \sqrt{\frac{m_\mu}{E_{p_\mu} E_{p_\nu}}} \delta(p_\pi - p_\mu' - p_\nu')$$

$$\times e^{i(E_{p_\mu} + E_{p_\nu} - E_{p_\pi})/\mu} \overline{\psi}_\pi \gamma_\alpha(1 + \gamma_5)\delta_\alpha \psi_\mu' (p_\mu')$$

$$\times <p_\mu', r_\mu' ; p_\nu', r_\nu' | \varphi_{p_\pi}^{(n-1)}> - \sum_{l=1}^{n-1} Z_{p_\pi}^{(l)} <p_\pi | \varphi_{p_\pi}^{(n-l)}>,$$

where summation over internal index $\alpha$ is implied; $\delta_\alpha = -1$ if $\alpha = 1, 2, 3$ and $\delta_\alpha = 1$ if $\alpha = 4$. By multiplying eq. (A.8) by $G^n$ and summing with respect to $n$, we have

$$Z_{p_\pi} = E_{p_\pi} - i\frac{f_\pi G}{2(2\pi)^{3/2}} \sum_{r_\mu,r_\nu} \int dp_\mu dp_\nu \sqrt{\frac{m_\mu}{E_{p_\mu} E_{p_\nu}}} \delta(p_\pi - p_\mu' - p_\nu')$$

$$\times e^{i(E_{p_\mu} + E_{p_\nu} - E_{p_\pi})/\mu} \overline{\psi}_\pi \gamma_\alpha(1 + \gamma_5)\delta_\alpha \psi_\mu' (p_\mu')$$

$$\times \frac{\varphi_{p_\pi}^{(n)} - 1}{E_{p_\mu'} + E_{p_\nu'} - E_{p_\pi} - i\varepsilon}$$

$$\times e^{i(E_{p_\mu'} + E_{p_\nu'} - E_{p_\pi})/\mu} \overline{\psi}_\pi \gamma_\alpha(1 + \gamma_5)\delta_\alpha \psi_\mu' (p_\mu')$$

$$\times <p_\mu', r_\mu' ; p_\nu', r_\nu' | \varphi_{p_\pi}^{(n-1)}> - \sum_{l=1}^{n-1} Z_{p_\pi}^{(l)} <p_\mu', r_\mu' ; p_\nu', r_\nu' | \varphi_{p_\pi}^{(n-l)}>,$$

Expression for $<p_\mu', r_\mu' ; p_\nu', r_\nu' | \varphi_{p_\pi}>$ can be obtained from eq. (24). Using (16) - (19) we find

$$<p_\mu', r_\mu' ; p_\nu', r_\nu' | \varphi_{p_\pi}^{(n)}> = \frac{-1}{E_{p_\mu'} + E_{p_\nu'} - E_{p_\pi} - i\varepsilon}$$

$$\times e^{i(E_{p_\mu'} + E_{p_\nu'} - E_{p_\pi})/\mu} \overline{\psi}_\pi \gamma_\alpha(1 + \gamma_5)\delta_\alpha \psi_\mu' (p_\mu')$$

$$\times <p_\pi | \varphi_{p_\pi}^{(n-1)}> - \sum_{l=1}^{n-1} Z_{p_\pi}^{(l)} <p_\mu', r_\mu' ; p_\nu', r_\nu' | \varphi_{p_\pi}^{(n-l)}>,$$

where summation over internal index $\beta$ is implied. By multiplying $G^n$ to
eq. (A.10) and summing with respect to $n$, we get

$$
\langle \mathbf{p}_\mu', r'_\mu; \mathbf{p}_\nu', r'_\nu \mid \varphi_{p_\pi} \rangle = \frac{-1}{E_{p'_\mu} + E_{p'_\nu} - E_{p_\pi} - i\varepsilon} 
\times \frac{f_\pi G}{2(2\pi)^{3/2}} \int d\mathbf{p}'_\pi \sqrt{\frac{m_\mu}{E_{p'_\mu} E_{p'_\nu}}} \delta(\mathbf{p}'_\pi - \mathbf{p}'_\mu - \mathbf{p}'_\nu) 
\times e^{i(E_{p'_\mu} + E_{p'_\nu} - E_{p_\pi})t} \sum_{\beta,\pi} \mathcal{M}_{\mu\nu}^{\beta\pi}(p'_\mu) \gamma_\beta(1 + \gamma_5) u^{r'_\nu}(-p'_\nu) 
\times \langle \mathbf{p}'_\pi \mid \varphi_{p_\pi} \rangle + (E_{p_\mu} - Z_{p_\pi}) \langle \mathbf{p}_\mu', r'_\mu; \mathbf{p}_\nu', r'_\nu \mid \varphi_{p_\pi} \rangle.
$$

Expression (A.11) can be represented in the form

$$
\langle \mathbf{p}_\mu', r'_\mu; \mathbf{p}_\nu', r'_\nu \mid \varphi_{p_\pi} \rangle = -i \frac{f_\pi G}{2(2\pi)^{3/2}} \int d\mathbf{p}'_\pi \sqrt{\frac{m_\mu}{E_{p'_\mu} E_{p'_\nu}}} \delta(\mathbf{p}'_\pi - \mathbf{p}'_\mu - \mathbf{p}'_\nu) 
\times \delta(\mathbf{p}_\pi - \mathbf{p}_\mu - \mathbf{p}_\nu)e^{i(E_{p'_\mu} + E_{p'_\nu} - E_{p_\pi})t} \sum_{\beta,\pi} \mathcal{M}_{\mu\nu}^{\beta\pi}(p'_\mu) \gamma_\beta(1 + \gamma_5) u^{r'_\nu}(-p'_\nu) 
\times \langle \mathbf{p}'_\pi \mid \varphi_{p_\pi} \rangle + \sum_{n=0}^{\infty} \frac{(Z_{p_\pi} - E_{p_\pi})^n}{(E_{p'_\mu} + E_{p'_\nu} - E_{p_\pi} - i\varepsilon)^{n+1}}.
$$

The substitution of result (A.12) into (A.9) gives

$$
Z_{p_\pi} = E_{p_\pi} - \frac{G^2 f_\pi^2 m_\mu}{32\pi^3 E_{p_\pi}} \sum_{r'_\mu, r'_\nu} \frac{d\mathbf{p}'_\mu d\mathbf{p}'_\nu}{E_{p'_\mu} E_{p'_\nu}} \delta(\mathbf{p}_\pi - \mathbf{p}'_\mu - \mathbf{p}'_\nu) 
\times \sum_{\alpha,\pi} \mathcal{M}_{\mu\nu}^{\alpha\pi}(-p'_\nu) \gamma_\alpha(1 + \gamma_5) E_{p_\pi} \gamma_\beta(1 + \gamma_5) u^{r'_\nu}(-p'_\nu) 
\times \sum_{n=0}^{\infty} \frac{(Z_{p_\pi} - E_{p_\pi})^n}{(E_{p'_\mu} + E_{p'_\nu} - E_{p_\pi} - i\varepsilon)^{n+1}},
$$

where the order $G^2$ is preserved (for this purpose the relationship $\langle \mathbf{p}'_\pi \mid \varphi_{p_\pi} \rangle = \delta(\mathbf{p}'_\pi - \mathbf{p}_\pi)$ was used). Here, for the simplification of the intermediate expressions unessential normalizing volume was not written, but it was implied. Now we examine the sum

$$
\sum_{n=0}^{\infty} \frac{(Z_{p_\pi} - E_{p_\pi})^n}{(E_{p'_\mu} + E_{p'_\nu} - E_{p_\pi} - i\varepsilon)^{n+1}}.
$$

If we sum up the series without paying attention to $i\varepsilon$, we have

$$
\sum_{n=0}^{\infty} \frac{(Z_{p_\pi} - E_{p_\pi})^n}{(E_{p'_\mu} + E_{p'_\nu} - E_{p_\pi})^{n+1}} = \frac{1}{E_{p'_\mu} + E_{p'_\nu} - Z_{p_\pi}}.
$$

Expression (A.15) has a pole in the lower half plane because, as it can be shown, $Z_{p_\pi}$ is in the lower half plane. However each term of the sum eq. (A.14) has a pole at $E_{p'_\mu} = E_{p_\pi} - E_{p'_\nu} + i\varepsilon$ or $E_{p'_\nu} = E_{p_\pi} - E_{p'_\mu} + i\varepsilon$ in the upper half plane. (the last expressions are equivalent since, both $\mu$ and $\tilde{\nu}$ particles are located in the future with respect to the $\pi^-$ meson). This implies that the
summation introduces a discontinuity. To avoid this difficulty we will adhere to the rule, which was proposed in work \[4\]

\[
\sum_{n=0}^{\infty} \frac{(Z_{p_{\pi}} - E_{p_{\mu}})^n}{(E_{p_{\pi}'} + E_{p_{\nu}} - E_{p_{\mu}} - i\varepsilon)^{n+1}} = \frac{1}{(E_{p_{\pi}'} + E_{p_{\nu}} - Z)^{\frac{1}{2}}_{Z_{p_{\pi}}}}. \tag{A.16}
\]

In work \[4\] this procedure was named as ”delayed analytic continuation”, where we first have to evaluate the integration on the upper half-plane of \(Z\), designated as ”+” (with respect to \(\mu^-\) or \(\bar{\nu}\) particle), and then substitute \(Z = Z_{p_{\pi}}\) Using (A.13), (A.16) we find eq. (26)

\[
Z_{p_{\pi}} = E_{p_{\pi}} - \frac{G^2 f_{\pi}^2 m_{\mu}}{32\pi^3 E_{p_{\pi}}} \sum_{r_{\mu}, r_{\nu}} \int \frac{d\rho_{\mu}'d\rho_{\nu}'}{E_{p_{\pi}'}^2} \delta(p_{\pi} - p_{\mu}' - p_{\nu}')
\times p_{\alpha,\pi}^\pi r_{\nu}'(-p_{\nu}') \gamma_\alpha(1 + \gamma_5) \delta(\alpha) u_{\rho_{\mu}'}(p_{\mu}') p_{\beta,\pi}^\pi r_{\mu}'(p_{\mu}') \gamma_\beta(1 + \gamma_5) u_{\rho_{\nu}'}(-p_{\nu}') \tag{A.17}
\]

\[
\frac{1}{(E_{p_{\pi}'} + E_{p_{\nu}} - Z)^{\frac{1}{2}}_{Z_{p_{\pi}}}}.
\]

Now we obtain the right-eigenstate (31). We expand eigenvector \(|\varphi_{p_{\pi}}\rangle\) in the terms of the set of eigenvectors \(|p_{\pi}\rangle\), \(|p_{\mu}', r_{\mu}'; p_{\nu}', r_{\nu}'\rangle\) of the Hamiltonian \(H_0\)

\[
|\varphi_{p_{\pi}}\rangle = \int |p_{\pi}'\rangle <p_{\pi}'| \varphi_{p_{\pi}}\rangle dp_{\pi}'
\]

\[
+ \sum_{r_{\mu}', r_{\nu}'} \int |p_{\mu}', r_{\mu}', p_{\nu}', r_{\nu}'\rangle <p_{\mu}', r_{\mu}', p_{\nu}', r_{\nu}'| \varphi_{p_{\pi}}\rangle dp_{\mu}' dp_{\nu}'. \tag{A.18}
\]

Using the energy conservation law for \(\pi^-\) meson decay, \(E_{p_{\pi}} = E_{p_{\mu}} + E_{p_{\nu}}\), with the help of (A.12), (A.16) we obtain

\[
|\varphi_{p_{\pi}}\rangle = \int <p_{\pi}'| \varphi_{p_{\pi}}\rangle (|p_{\pi}'\rangle - 
\sum_{r_{\mu}', r_{\nu}'} \int d\rho_{\mu}' d\rho_{\nu}' \frac{iG_{\pi}}{2(2\pi)^3} \sqrt{\frac{m_{\mu}}{E_{p_{\mu}'}}} \delta(p_{\rho} - p_{\mu}' - p_{\nu}')
\times p_{\beta,\pi}^\pi r_{\nu}'(p_{\mu}') \gamma_\beta(1 + \gamma_5) u_{\rho_{\mu}'}(-p_{\nu}') \tag{A.19}
\]

\[
\frac{1}{(E_{p_{\pi}'} + E_{p_{\nu}} - Z)^{\frac{1}{2}}_{Z_{p_{\pi}}}} |p_{\mu}', r_{\mu}', p_{\nu}', r_{\nu}'\rangle dp_{\mu}'.
\]

The result (28) makes it possible to rewrite expression (A.16) in the form

\[
\frac{1}{(E_{p_{\pi}'} + E_{p_{\nu}} - Z)^{\frac{1}{2}}_{Z_{p_{\pi}}}} \equiv \frac{1}{(E_{p_{\pi}'} + E_{p_{\nu}} - E_{p_{\mu}} - \varepsilon)^{\frac{1}{2}}_{Z_{p_{\pi}}}}. \tag{A.20}
\]

18
The function (A.20) is defined through the integration over $E_{p_{\mu}}$ or $E_{p_{\nu}}$

$$
\int_0^{\infty} dE'_{p_{\mu},(\nu)} \frac{f(E'_{p_{\mu},(\nu)})}{(E'_{p_{\mu}} + E'_{p_{\nu}} - E_{p_{\pi}} - z)^+_{-i\gamma_{p_{\pi}}}}
$$

$$
\equiv \lim_{z \to -i\gamma_{p_{\pi}}} \left( \int_0^{\infty} dE'_{p_{\nu},(\mu)} \frac{f(E'_{p_{\nu},(\mu)})}{(E'_{p_{\mu}} + E'_{p_{\nu}} - E_{p_{\pi}} - z)_{z \in C^+}} \right),
$$

(A.21)

where we first have to evaluate the integration on the upper half-plane $C^+$ (with respect to $E_{p_{\mu}}$ or $E_{p_{\nu}}$) and then the limit of $z \to -i\gamma_{p_{\pi}}$ must be taken, $f(E'_{p_{\mu},(\nu)})$ is a test function. The expressions (A.19) - (A.21) give the result (31).

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