The Jackiw-Pi model and its symmetries

O.M. Del Cima

Universidade Federal de Viçosa (UFV), Departamento de Física - Campus Universitário, Avenida Peter Henry Rolfs s/n - 36570-000 - Viçosa - MG - Brazil.

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The non-Abelian gauge model proposed by Jackiw and Pi, which generates an even-parity mass term in three space-time dimensions, is revisited in this letter. All the symmetries of the model are collected and established by means of BRS invariance and Slavnov-Taylor identity. The path for the perturbatively quantization of the Jackiw-Pi model, through the algebraic method of renormalization, is presented.

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One of the central problems in the framework of gauge field theories is the issue of gauge field mass. Gauge symmetry is not, in principle, conflicting with the presence of a massive gauge boson. In two space-time dimensions, the well-known Schwinger model puts in evidence the presence of a massive photon without the breaking of gauge symmetry [1]. Another evidence for the compatibility between gauge symmetry and massive vector fields has been arisen in the study of three-dimensional gauge theories, when a topological mass term referred to as the Chern-Simons one, once added to the Yang-Mills term, shifts the photon mass to a non-vanishing value without breaking gauge invariance, however parity symmetry is lost [2]. In 1997, Jackiw and Pi overcame the challenge to implement both gauge and parity invariance in three space-time dimensions by breaking the Yang-Mills paradigm - non-Abelian generalizations of Abelian models. They proposed a three-dimensional non-Yang-Mills gauge model for a pair of vector fields with opposite parity transformations, which generates a mass-gap through a mixed Chern-Simons-like term preserving parity [3]. Later on, the Jackiw-Pi model has been studied in the Hamiltonian framework, where physical states consistency was demonstrated [4]. In this letter, the non-Abelian gauge model proposed by Jackiw and Pi, which generates an even-parity mass term in three space-time dimensions, is revisited. The symmetries of the model are collected and established through BRS invariance and Slavnov-Taylor identity, also the BRS approach has allowed to bypass the difficulties cited in the literature with respect to the gauge-fixing. In the Landau gauge, thanks to the antighost equations and the Slavnov-Taylor identity, two rigid symmetries are identified by means of Ward identities. The propagators computation, the spectrum consistency and the tree-level unitarity analysis are left to be presented in a forthcoming paper [5]. The Jackiw-Pi model remains unquantized up to now, however, it is presented here the key ingredients for its further perturbatively quantization through the algebraic method of renormalization [6].

I. THE MODEL AND ITS SYMETRIES

A. The model

The classical action of the Jackiw-Pi model [3] is given by:

$$\Sigma_{\text{inv}} = \text{Tr} \int d^3x \left\{ \frac{1}{2} F_{\mu\nu} F_{\mu\nu} + \frac{1}{2} (G_{\mu\nu} + g[F_{\mu\nu}, \rho])(G_{\mu\nu} + g[F_{\mu\nu}, \rho]) - m e^{\mu\nu\rho} F_{\mu\nu} \phi_{\rho} \right\} ,$$

where

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} + g[A_{\mu}, A_{\nu}] , \quad G_{\mu\nu} = D_{\mu} \phi_{\nu} - D_{\nu} \phi_{\mu} \quad \text{and} \quad D_{\mu} \bullet = \partial_{\mu} \bullet + g[A_{\mu}, \bullet] ,$$

such that, $A_{\mu} \text{ and } \phi_{\mu}$ are vector fields, $\rho$ is a scalar, $g$ is a coupling constant and $m$ a mass parameter, also, $\bullet$ means any field. Every field, $X = X_{a} \tau_{a}$, is Lie algebra valued, where the matrices $\tau$ are the generators of the group and obey

$$[\tau_{a}, \tau_{b}] = f_{abc} \tau_{c} \quad \text{and} \quad \text{Tr}(\tau_{a} \tau_{b}) = -\frac{1}{2} \delta_{ab} .$$

*Electronic address: oswaldo.delcima@ufv.br
B. Gauge symmetries

The action (1) is invariant under two sets of gauge transformations, \( \delta_\theta \) and \( \delta_\chi \):

\[
\delta_\theta A_\mu = D_\mu \theta, \quad \delta_\theta \phi_\mu = g[\phi_\mu, \theta] \quad \text{and} \quad \delta_\theta \rho = g[\rho, \theta];
\]

\[
\delta_\chi A_\mu = 0, \quad \delta_\chi \phi_\mu = D_\mu \chi \quad \text{and} \quad \delta_\chi \rho = -\chi,
\]

where \( \theta \) and \( \chi \) are Lie algebra valued infinitesimal local parameters.

C. BRS symmetry

The corresponding BRS transformations of the fields \( A_\mu, \phi_\mu \) and \( \rho \), stemming from the symmetries (4) and (5), are given by:

\[
\begin{align*}
s & A_\mu = D_\mu c, \quad s \phi_\mu = D_\mu \xi + g[\phi_\mu, c], \quad s \rho = -\xi + g[\rho, c], \\
sc & = -gc^2 \quad \text{and} \quad s\xi = -g[\xi, c],
\end{align*}
\]

where \( c \) and \( \xi \) are the Faddeev-Popov ghosts, with Faddeev-Popov charge (ghost number) one. The ghost number \((\Phi\Pi)\) of all fields and antifields are collected in Table I.

D. The gauge-fixing and the antifields action

The gauge-fixing adopted here belongs to the class of the linear covariant gauges discussed by ’t Hooft [7]. In order to implement the gauge-fixing following the BRS procedure [8, 9], we introduce two sorts of ghosts (\( c \) and \( \xi \)), antighosts (\( \bar{c} \) and \( \bar{\xi} \)) and the Lautrup-Nakanishi fields [10] (\( b \) and \( \pi \)), playing the role of Lagrange multiplier fields for the gauge condition, such that

\[
\begin{align*}
s\bar{c} &= b, \quad sb = 0; \\
s\bar{\xi} &= \pi, \quad s\pi = 0;
\end{align*}
\]

where the multiplier fields, \( b \) and \( \pi \), and the Faddeev-Popov antighosts, \( \bar{c} \) and \( \bar{\xi} \), with ghost number minus one, belong to the BRS-doublets (7) and (8).

Now, by adopting the gauge conditions

\[
\frac{\delta \Sigma_{gf}}{\delta b} = \partial^\mu A_\mu + \alpha b, \quad \frac{\delta \Sigma_{gf}}{\delta \pi} = \partial^\mu \phi_\mu + \beta \pi,
\]

it follows that the BRS-trivial gauge-fixing action compatible with the \( \Sigma_{gf} \) reads

\[
\begin{align*}
\Sigma_{gf} &= s \text{ Tr } \int d^3 x \left\{ b \partial^\mu A_\mu + \bar{c} \partial^\mu A_\mu + \bar{\xi} \partial^\mu \phi_\mu + \frac{\alpha}{2} b \bar{c} + \frac{\beta}{2} \bar{\xi} \pi \right\} \\
&= \text{ Tr } \int d^3 x \left\{ b \partial^\mu A_\mu - \bar{c} \partial^\mu \phi_\mu + c \partial^\mu \phi_\mu - \bar{\xi} \partial^\mu (D_\mu \phi_\mu + g[\phi_\mu, c]) + \frac{\alpha}{2} b^2 + \frac{\beta}{2} \pi^2 \right\}.
\end{align*}
\]

Let us now introduce the action in which the nonlinear BRS transformations are coupled to the the antifields (BRS invariant external fields), so as to control, at the quantum level, the (further) renormalization of those transformations:

\[
\Sigma_{ext} = \text{ Tr } \int d^3 x \left\{ A_\mu^* s A_\mu + \phi_\mu^* s \phi_\mu + \rho^* s \rho + c^* s c + \xi^* s \xi \right\},
\]



\[\text{The commutators among the fields are assumed to be graded, namely, } [\varphi_{g_1}^{g_1}, \varphi_{g_2}^{g_2}] \equiv \varphi_{g_1}^{g_1} \varphi_{g_2}^{g_2} - (-1)^{g_1,g_2} \varphi_{g_2}^{g_2} \varphi_{g_1}^{g_1}, \text{ where the upper indices, } g_1 \text{ and } g_2, \text{ are the Faddeev-Popov charges } (\Phi\Pi) \text{ carried by the fields } \varphi_1^{g_1} \text{ and } \varphi_2^{g_2}, \text{ respectively.}\]
where, as mentioned above, the antifields are BRS invariant, namely,
\[
 s A_\mu^* = s \phi_\mu^* = s \rho^* = s c^* = s \xi^* = 0 .
\] (13)

The total action at the tree level for the Jackiw-Pi model, \( \Gamma^{(0)} \), is therefore given by:
\[
 \Gamma^{(0)} = \Sigma_{\text{inv}} + \Sigma_{\text{gf}} + \Sigma_{\text{ext}} ,
\] (14)
which is invariant under the BRS transformations given by the equations (6), (7), (8) and (13). The action (14) preserves the ghost number. The values of the ghost number, the ultraviolet (UV) and the infrared (IR) dimensions (respected to the Landau gauge) are displayed in Table I. However, all subtleties concerning the determination of the UV and the IR dimensions of the fields, in the Landau gauge, shall be presented in Ref.[5]. The statistics is defined as follows: the fields of integer spin and odd ghost number as well as the fields of half integer spin and even ghost number are anticommuting; the other fields commute with the former and among themselves.

An interesting feature of the Jackiw-Pi action \( \Gamma^{(0)} \) is that it is not BRS local invariant thanks to the parity-even mass term:
\[
 s \Sigma_m = g [F_{\mu\nu}, c] ,
\] (16)
then
\[
 s \Sigma_m = -m \, \text{Tr} \int d^3 x \{ \epsilon^{\mu\nu\rho} F_{\mu\nu} \phi_\rho \} = -m \, \text{Tr} \int d^3 x \{ \epsilon^{\rho\mu\nu} \partial_\rho (F_{\mu\nu} \xi) \} ,
\] (17)
which is invariant only up to a total derivative, possibly indicating that at the quantum level the \( \beta \)-function associated to the mass parameter \( m \) vanishes [11, 12].

**E. Slavnov-Taylor identity, ghost and antighost equations and Ward identities**

This subsection is devoted to establish the Slavnov-Taylor identity, ghost and antighost equations, and two hidden rigid symmetries. The BRS invariance of the action \( \Gamma^{(0)} \) is expressed through the Slavnov-Taylor identity
\[
 S(\Gamma^{(0)}) = \text{Tr} \int d^3 x \left\{ \frac{\delta \Gamma^{(0)}}{\delta A_\mu^*} \frac{\delta \Gamma^{(0)}}{\delta A^\mu} + \frac{\delta \Gamma^{(0)}}{\delta \phi_\mu^*} \frac{\delta \Gamma^{(0)}}{\delta \phi^\mu} + \frac{\delta \Gamma^{(0)}}{\delta \rho^*} \frac{\delta \Gamma^{(0)}}{\delta \rho} + \frac{\delta \Gamma^{(0)}}{\delta c^*} \frac{\delta \Gamma^{(0)}}{\delta c} + \frac{\delta \Gamma^{(0)}}{\delta \xi^*} \frac{\delta \Gamma^{(0)}}{\delta \xi} - b \frac{\delta \Gamma^{(0)}}{\delta \bar{c}} + \pi \frac{\delta \Gamma^{(0)}}{\delta \bar{\xi}} \right\} = 0 ,
\] (18)
which translates, in a functional way, the invariance of the classical model under the BRS symmetry. It is suitable to define, for later use, the linearized Slavnov-Taylor (\( S_{\Gamma^{(0)}} \)) operator as below
\[
 S_{\Gamma^{(0)}} = \text{Tr} \int d^3 x \left\{ \frac{\delta \Gamma^{(0)}}{\delta A_\mu^*} \frac{\delta \Gamma^{(0)}}{\delta A^\mu} + \frac{\delta \Gamma^{(0)}}{\delta \phi_\mu^*} \frac{\delta \Gamma^{(0)}}{\delta \phi^\mu} + \frac{\delta \Gamma^{(0)}}{\delta \rho^*} \frac{\delta \Gamma^{(0)}}{\delta \rho} + \frac{\delta \Gamma^{(0)}}{\delta c^*} \frac{\delta \Gamma^{(0)}}{\delta c} + \frac{\delta \Gamma^{(0)}}{\delta \xi^*} \frac{\delta \Gamma^{(0)}}{\delta \xi} + \frac{\delta \Gamma^{(0)}}{\delta c} \frac{\delta \Gamma^{(0)}}{\delta \bar{c}} + \frac{\delta \Gamma^{(0)}}{\delta \xi} \frac{\delta \Gamma^{(0)}}{\delta \bar{\xi}} + b \frac{\delta \Gamma^{(0)}}{\delta \bar{c}} + \pi \frac{\delta \Gamma^{(0)}}{\delta \bar{\xi}} \right\} .
\] (19)

Another identities, the ghost equations
\[
 \mathcal{G}_I \Gamma^{(0)} = \frac{\delta \Gamma^{(0)}}{\delta \bar{c}} + \partial_\mu \frac{\delta \Gamma^{(0)}}{\delta A_{\mu}^*} = 0 ,
\] (20)
\[
 \mathcal{G}_II \Gamma^{(0)} = \frac{\delta \Gamma^{(0)}}{\delta \bar{\xi}} + \partial_\mu \frac{\delta \Gamma^{(0)}}{\delta \phi_{\mu}^*} = 0 ,
\] (21)
follow from the gauge-fixing conditions, (9) and (10), and from Slavnov-Taylor identity (18), meaning that \( \Gamma^{(0)} \) depends on the antighosts, \( \bar{c} \) and \( \xi \), and the antifields, \( A^\mu \) and \( \phi^{\ast \mu} \), through the combinations

\[
\bar{A}^\mu = A^\mu + \partial_\mu \bar{c} \quad \text{and} \quad \phi^{\ast \mu} = \phi^{\ast \mu} + \partial_\mu \xi .
\]

The Jackiw-Pi model presents two antighost equations, they are listed as below:

\[
\bar{\psi}_I \Gamma^{(0)} = \int d^3x \left\{ \frac{\delta \Gamma^{(0)}}{\delta c} - g \left[ \bar{\xi}, \frac{\delta \Gamma^{(0)}}{\delta b} \right] - g \left[ \xi, \frac{\delta \Gamma^{(0)}}{\delta \pi} \right] \right\} = \bar{\Sigma}_I ,
\]

(23)

where \( \bar{\Sigma}_I = -g \int d^3x \left\{ [A^\mu, A^\nu] + [\phi^{\ast \mu}, \phi^{\ast \nu}] + [\rho^{\ast}, \rho] - [c^{\ast}, c] - [\xi^{\ast}, \xi] + \alpha \bar{c} b + \beta [\xi, \pi] \right\} ;

\[
\bar{\psi}_{II} \Gamma^{(0)} = \int d^3x \left\{ \frac{\delta \Gamma^{(0)}}{\delta \xi} - g \left[ \bar{\xi}, \frac{\delta \Gamma^{(0)}}{\delta b} \right] \right\} = \bar{\Sigma}_{II} ,
\]

(25)

where \( \bar{\Sigma}_{II} = -g \int d^3x \left\{ [\phi^{\ast \mu}, A^\mu] - [\xi^{\ast}, c] - \frac{\rho^{\ast}}{\rho} + \alpha \bar{c} b \right\} .

(26)

It should be noticed, for the sake of later quantization [6], that the breakings, \( \bar{\Sigma}_I \) and \( \bar{\Sigma}_{II} \), being nonlinear in the quantum fields will be subjected to renormalization. An interesting issue in Yang-Mills theories is that the Landau gauge [13] has very special features as compared to a generic linear gauge. This is due to the existence, besides the Slavnov-Taylor identity, of another identity, the antighost equation [14], which controls the dependence of the quantum fields on the ghost \( c \). In particular, this equation implies that the ghost field \( c \) and the composite \( c \)-field cocycles in the descent equations have vanishing anomalous dimension, allowing the algebraic proof [9] of the Adler-Bardeen nonrenormalization theorem [15] for the gauge anomaly. Back to the Jackiw-Pi model we are considering here, in the case of the general linear covariant gauges, \( (9) \) and \( (10) \), the right-hand sides of the equations, \( (23) \) and \( (25) \), are nonlinear in the quantum fields due to the presence of the terms, \( \int d^3x \alpha \bar{c} b \) and \( \int d^3x \beta [\xi, \pi] \), and \( \int d^3x \alpha [\xi, b] \), respectively. Therefore, the breakings, \( \bar{\Sigma}_I \) (24) and \( \bar{\Sigma}_{II} \) (26), in the quantized theory, have to be renormalized, which could spoil the usefulness of the antighost equations, by this reason, bearing in mind the (further) renormalization of the model [6], we adopt from now on the Landau gauge \( \alpha = \beta = 0 \).

As another feature of the Landau gauge, the following Ward identities for the rigid symmetries stem from the Slavnov-Taylor identity (18) and the antighost equations (23) and (25) with \( \alpha = \beta = 0 \):

\[
\mathcal{W}_I^{\text{rig}} \Gamma^{(0)} = 0 , \quad \text{where}
\]

\[
\mathcal{W}_I^{\text{rig}} \equiv -g \int d^3x \left\{ [A^\mu, \delta A^\mu] + [\phi^{\ast \mu}, \delta \phi^{\ast \mu}] + [\rho^{\ast}, \delta \rho] + [b, \delta b] + [\pi, \delta \pi] + [c, \delta c] + [\xi, \delta \xi] + \bar{c}, \frac{\delta \xi}{\delta c} + \bar{\xi}, \frac{\delta \xi}{\delta \xi} \right\} ;
\]

(27)

\[
\mathcal{W}_{II}^{\text{rig}} \Gamma^{(0)} = 0 , \quad \text{where}
\]

\[
\mathcal{W}_{II}^{\text{rig}} \equiv -g \int d^3x \left\{ [A^\mu, \delta \phi^{\ast \mu}] + [\pi, \delta b] + [c, \delta \xi] + [\xi, \delta \xi] + [\phi^{\ast \mu}, \delta A^\mu] + [\xi, \delta \xi] + [\xi, \delta \xi] + \frac{1}{\rho} \delta \rho \right\} ,
\]

(28)

\[\text{TABLE I: Ultraviolet dimension (d), infrared dimension (r) and ghost number (ΦΠ).}\]

| \( A_\mu \) | \( \phi_\mu \) | \( \rho \) | \( b \) | \( \pi \) | \( c \) | \( \xi \) | \( \bar{c} \) | \( \xi \) | \( A^{\ast \mu} \) | \( \phi^{\ast \mu} \) | \( \rho^* \) | \( c^* \) | \( \xi^* \) | \( g \) | \( m \) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| \( d \) | 1/2 | 1/2 | -1/2 | 3/2 | -1/2 | 1/2 | 3/2 | 1/2 | 3/2 | 5/2 | 5/2 | 7/2 | 7/2 | 7/2 |
| \( r \) | 1/2 | 1/2 | -1/2 | 3/2 | -1/2 | 1/2 | 3/2 | 1/2 | 3/2 | 5/2 | 5/2 | 7/2 | 7/2 | 7/2 |

II. CONCLUSIONS

The Jackiw-Pi model [3] which generates a mass gap preserving parity in three space-time dimensions was presented here. The BRS symmetry of the model was established and the difficulties cited in the literature concerning the gauge-fixing were bypassed. Also, BRS invariance and Slavnov-Taylor identity together with the antighost equations, in the
Landau gauge, have allowed to find out two rigid symmetries. In spite of being out of the scope of this letter, an important issue to be noticed is that, as we have shown in (17), the Jackiw-Pi even-parity mass term is BRS invariant up to a total derivative, i.e., it is not local BRS invariant. Therefore, it could be conjectured that, at the quantum level, the $\beta$-function associated to the mass parameter $m$, $\beta_m$, should be zero, $\beta_m = 0$ [11, 12]. Moreover, this fact would indicate the perturbatively ultraviolet finiteness of the Jackiw-Pi model, which is now under investigation [6] in the framework of the algebraic renormalization scheme.

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