Residual strength of GFR/POM as a function of damage

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Abstract. A relation between the residual strength and the dispersed damage accumulated in a short fiber reinforced polyoximethylene (GFR/POM) samples under tension is found. For that purpose dependencies of damage and residual strength on loading percentage are used. Damage as a function of loading percentage is known for the material under study. To find the dependency of residual strength on loading percentage a subsidiary function is introduced and a method is proposed for determination of the parameters in the dependency on the basis of the experimental data. Both damage and residual strength are measured after unloading samples that have been loaded applying different loading percentages. Damage is the accumulation of new internal surfaces that arise under mechanical loading in the whole volume of the material. They are registered by a new original method of X-ray refraction. The analytical relation between the residual strength and damage accumulated is compared to the experimental results found for the residual strength under different damage degrees.

1. Introduction

Mechanical loading triggers processes of material deformation and fracture. While the process of deformation can be followed by means of various methods, recording the macrodeformation of a specific basis, the registration of fracture (its kinetics) is a far more complex problem. This is the reason why, performing mechanical investigations, one can not directly measure fracture but such integral quantities as strength and time to fracture. Meanwhile, serious progress in fracture record was marked applying direct physical methods (X-ray treatment, acoustic emission etc.). Yet, the problem of relating the measured quantities to material strength capacity still persists.

The process of fracture in our case is a process of accumulation of dispersed damage in the whole material volume. Accumulated damage determines the degree of fracture. The aim of the present study is to find a relation between the residual strength and the degree of fracture (damage).

2. Experimental

2.1. Material
A material in which mechanical loading induces accumulation of dispersed (delocalized) damage till macrofracture was studied – injection moulded short glass fiber reinforced polyoximethylene. Samples

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were fabricated by Ticona GmbH, Frankfurt a.M. according to DIN EN ISO 527. The glass fiber part (E – glass) is 26 wt.%, the mean fiber length is 320 \( \mu \text{m} \) (max. 800\( \mu \text{m} \), min. 60\( \mu \text{m} \)) and the fiber diameter – 10 ± 1\( \mu \text{m} \). Silan compounds were used for fiber adhesion dressing.

2.2. Mechanical loading
The samples were loaded in tension by an Instron testing machinery with a three constant loading rates equal to 0.22, 1.1 and 5.5 kN/min. After reaching a certain loading percentage (part of the force at fracture) the samples were unloaded instantly. The unloaded samples were loaded till fracture with loading rate 1.1 kN/min to find the residual strength. Nine loading percentages were used – 50, 70, 80, 85, 90, 93, 95, 97 and 98% of the force at fracture. Three samples were loaded at each step and loading rate. Experiments were carried out at room temperature and 50% humidity.

2.3. Measure of damage process
The damage process has a clear physical meaning – it is the accumulation of new internal surfaces in the whole volume of the material: micro and mezzo cracks (called cracks in what follows). They are oriented perpendicularly to the loading direction and to the scattering plane (the plane determined by the collimated and refracted X-ray beams) (figure 1). Damage accumulation proceeds relatively homogeneously in isolated volumes until there occurs interaction between these zones as well as damage transfer to the macrovolume leading to collapse.

![Figure 1. Damage as a result of cracking. Fiber parallel to the scattering plane.](image)

![Figure 2. Damage accumulation \( \omega \) (cracks) as a function of loading percentage \( p \). Description according to (1).](image)

The new internal surfaces in the volume which occur as a result of mechanical loading correspond to the accumulated damage. They are registered by a new original method of X-ray refraction developed in the Federal Institute of Materials Research and Testing in Berlin (Prof. M. Hentschel – [1]). A measure of damage \( \omega \) is the quantity \( \Delta c_\omega d \) – the mean difference between the refractions \( cd \) measured on loaded-unloaded and virgin samples in the scanned area (figure 1). Quantity \( c = kS/V \) denotes the refraction factor proportional to the inner surface density (ratio “surface S / volume V”), \( k \) is a constant of the instrument, \( d \) is sample thickness and \( m \) denotes a mean value.

3. Relation residual strength - damage
We are going to find the relation between the residual strength \( R_{rs} \) and damage \( \omega \) on the basis of their dependencies on the loading percentage \( p \). The dependence \( \omega - p \) is shown in figure 2. Damage values \( \omega \) are in relative units. Their analytical description based on the idea for the distribution of weak spots in the material [2] is shown there, too. Parameters in the equation are determined according to the mean values of \( \omega \) for the three load rates. It was ascertained in [2], that for the material investigated and for the applied loading rates, damage depends on the loading percentage, only, but not on the loading rate. The mean values of \( \omega \) are shown in figure 2 as circles. The dependence \( \omega - p \) reads:

\[
\omega(p) = A[\exp(Bp) -1]
\]  

(1)
with $A = 9.82 \times 10^5$ and $B = 7.69 \times 10^7$ (a correlation coefficient $r \approx 1$).

The specification of the relation between the residual strength $R_{rs}$ and $p$ faces difficulties due to the character of function $R_{rs}(p)$: the residual strength varies slightly up to $p = 97\%$ and sharply (almost jump-wise) in the range $99 - 100\%$ (figure 3). The problem is to find an analytical description of $R_{rs}$ for the whole range of loading percentage $p$ from 0 till 100\%, on the basis of experimental data before the “jump”. We get a “smooth run” of $R_{rs}$ choosing a function $f(p)$ and specifying $R_{rs}$ as depending on $f(p)$, i.e. $R_{rs}[f(p)]$

$$R_{rs}[f(p)] = R_0 - \Delta R_{rs}[f(p)]$$

where $R_0$ is the initial strength of the undamaged material, $\Delta R_{rs}$ is its decrease caused by damage.

Function $f(p)$ is chosen in the form:

$$f(p) = p/(p_0 - p)$$

where $p_0$ is a parameter.

It is assumed, that function $\Delta R_{rs}$ is proportional to $f(p)$:

$$\Delta R_{rs} = k.f(p) = k[p/(p_0 - p)]$$

with a coefficient of proportionality $k = R_0.a$, and $a$ is a parameter.

Parameters $a$ and $p_0$ in equation (4) are found applying a linear regression with iteration. As a first step it is assumed that $p_0 = p_{01} = 100$. The respective parameter $a = a_1$ is found on the basis of the method of least squares:

$$a_1 = (1/R_0)[p_i\Delta R_{rsi}]/[p_i^2]$$

where $p_i = p_i/(p_{01} - p_i)$, $p_i$ and $\Delta R_{rsi}$ (mean values) correspond to the experimental data at point $i$ (nine points altogether – for the loading percentages 50, 70, 80, 85, 90, 93, 95, 97 and 98\%). Square brackets denote sums.

As a second step it is assumed that $p_0 = p_{02} = (1 + a_1).100$. Repeating the procedure in (5) with $p_{02}$, we calculate parameter $a_2$. It is assumed at the third step that $p_0 = p_{03} = (1 + a_2).100$ and according to (5) we calculate parameter $a_3$. The procedure is repeated until $a_j = a_{j-1}$, where $j$ is the j-th step. This condition is fulfilled at the 57-th step, and for the parameters $a$ and $p_0$ follows that: $a = a_5 = 0.0009656$ and $p_0 = p_{05} = 100.09656$.

So, equation (4) reads:
$$\Delta R_{rs} = aR_0[p/(p_0 - p)]$$ (6)

with a and $p_0$ given above.

The line according to eq.(6) satisfies three conditions: it crosses point $\Delta R_{rs} = 0$, $p = 0\%$, point $\Delta R_{rs} = R_0$, $p = 100\%$ and passes maximally close to the experimental points of the loading percentages.

The “smooth run” of $\Delta R_{rs}$ as a function of $f(p)$ is shown in figure 4. It is seen, that the sharp jump in figure 3 near $p = 100\%$ disappears in figure 4. This “smooth run” of the experimental data allows the determination of parameters $a$ and $p_0$ in equations (6) and (2).

The dependence $\Delta R_{rs}$ ($R_{rs}$ respectively) on $p$ is shown in figure 5.

Residual strength $R_{rs}$ as a function of the degree of fracture (damage) $\omega$ is found for known functions $\omega(p)$ and $R_{rs}[f(p)]$ according to equations (1), (2) and (6):

$$\Delta R_{rs}(\omega) = aR_0\ln(1 + \omega \alpha' A)/[Bp_0 - \ln(1 + \omega \alpha' A)]$$ (7)

The decrease of residual strength $\Delta R_{rs}$ and the residual strength $R_{rs}$, respectively, as a function of damage $\omega$ are shown in figure 6. Initial strength $R_0 = 124.26$ N/mm$^2$, $a$, $p_0$, $A$ and $B$ are given above. The comparison of the experimental data with the theoretical dependence according to the equation (7) in figure 6 shows a good coincidence.

4. Conclusions
- A relation between the residual strength and the dispersed damage at short-term loading is found – equation (7).
- The relation found is verified for short glass fiber reinforced polyoximethylene.
- A method is proposed, which allows the determination of the parameters in the equation that relates the residual strength to the loading degree.
- A proof of the above dependencies is needed for other materials that undergo dispersed damage until fracture.

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