Integrating ridesharing services with automated vehicles into macroscopic travel demand models

Emely Richter  
Chair for Transport Planning and Traffic Engineering  
University of Stuttgart  
Stuttgart, Germany  
emely.richter@isv.uni-stuttgart.de

Alexander Migl  
Chair for Transport Planning and Traffic Engineering  
University of Stuttgart  
Stuttgart, Germany  
alexander.migl@isv.uni-stuttgart.de

Markus Friedrich  
Chair for Transport Planning and Traffic Engineering  
University of Stuttgart  
Stuttgart, Germany  
markus.friedrich@isv.uni-stuttgart.de

Johann Hartleb  
Chair for Transport Planning and Traffic Engineering  
University of Stuttgart  
Stuttgart, Germany  
johann.hartleb@isv.uni-stuttgart.de

Abstract—As the introduction of fully automated vehicles enhances the attractiveness of carsharing and ridesharing systems, cities and regions may want to examine the effects of this development. This paper presents a framework for how to integrate those services in traditional macroscopic travel demand models, which are commonly used to evaluate the impacts of changed transport supply. Addressed topics are (1) the implementation of direct and intermodal ridesharing into the demand modeling process, presenting two approaches for the latter, (2) the pooling of ridesharing trips and (3) the scheduling of automated and shared vehicles. The first approach for integrating intermodal ridesharing includes ridesharing as an additional transport system, which uses the road network and which is integrated in the timetable-based public transport assignment. The second approach uses direct-link connections between traffic zones and suitable public transport transfer stops for the ridesharing feeder trips instead. Using the second approach, preliminary results of a test scenario for the Stuttgart region are presented.

Keywords—automated vehicles, ridesharing, on-demand services, macroscopic travel demand model, trip pooling, vehicle scheduling

I. INTRODUCTION

Automated and connected vehicles will change transport supply and in consequence influence travel demand. Especially fully automated vehicles can have a major impact as carsharing and ridesharing systems become more attractive and therefore a relevant mode choice alternative. For evaluating impacts of changes in transport supply, many cities and regions operate macroscopic travel demand models. For them it would be helpful to supplement these models in a way that enables the testing of scenarios including fleets of shared automated vehicles.

However, current research in the field of modeling the impacts of automated and/or shared vehicles mainly concentrates on microscopic traffic flow models (e.g. [1], [2]) or on microscopic travel demand models (e.g. [3], [4]). Macroscopic approaches to the subject of shared automated vehicle fleets are scarce. Furthermore, those studies using macroscopic models, as for example [5] for the Paris region and [6] for the Stuttgart region, often skip the calculation step of mode choice. Instead they assume shares of ridesharing trips and examine the effects. Another macroscopic modeling approach can be found in [7], where the impacts of vehicle automation on mobility behavior in Germany, the United States and China are discussed. In the underlying model of this study, the mode choice is considered, but the applied model has no spatial dimension and is highly aggregated. Therefore, an assignment of the vehicles to a road network is omitted.

The aim of this paper is to present a framework for integrating ridesharing services provided by fully automated vehicles into the whole demand modeling process of a macroscopic model. For that, it has to be taken into consideration that ridesharing can be used in two ways: (1) Direct ridesharing, where ridesharing is the only means of transport for the entire trip. (2) Intermodal ridesharing, where ridesharing acts as a feeder system for the first/last mile of a combined trip with public transport. Especially in this second form, ridesharing holds system properties of public transport systems as well as private transport. As an example, it needs to meet the departure times of the timetable-based trains and busses, but uses the road network of private transport, not following a dedicated line route.

Apart from presenting a method for integrating both direct and intermodal ridesharing into macroscopic travel demand models, this paper discusses the steps of trip pooling for ridesharing as well as scheduling the fully automated ridesharing vehicles in a macroscopic approach. Finally, first results of a test scenario run with the presented approach are described.

Nevertheless, many uncertainties regarding automated vehicles remain. Examples for this include operating costs of ridesharing services with automated vehicles, the perception of travel time in automated vehicles and the willingness to share a vehicle or ride with other travelers. The paper does not address these uncertainties, but it suggests a modeling framework to examine assumptions and their impact on travel demand.

II. INTEGRATING DIRECT AND INTERMODAL RIDESHARING SERVICES INTO THE DEMAND MODEL

In order to replicate the decision-making process of individual travelers, macroscopic travel demand models commonly use the four-stage algorithm. Fig. 1 shows the traditional four steps of trip generation, destination choice, mode choice and route choice supplemented by the step of departure time choice. The model determines the number of trips from origin o to destination d using mode m on route r, departing at time interval t. The steps of destination, mode,
departure time and route choice are influenced by supply quality, e.g. saturation dependent travel times. An iteration process ensures an equilibrium between demand dependent supply quality and supply dependent demand. This way congestion effects in the road network are considered. For further information on the general demand modeling process see [8] or [9].

Integrating ridesharing services with automated vehicles requires some modifications and extensions of this basic algorithm. As shown in Fig. 2, this paper suggests an extension by three additional steps: (1) mobility tool choice to determine car ownership or availability of a public transport season ticket, (2) trip pooling of ridesharing demand and (3) scheduling of shared automated vehicles. While the steps of trip pooling and vehicle scheduling are further examined in this paper, mobility tool ownership is not discussed in detail (see [10] for this).

Apart from extending the demand-modeling process by new steps, modifications of already existing steps are needed. These modifications depend on the form of ridesharing, i.e. direct or intermodal ridesharing.

For direct trips, this paper proposes to add ridesharing as an independent mode to the mode choice set. By this, travelers obtain a new alternative in mode choice.

Intermodal ridesharing does not lead to an additional mode choice alternative. Instead, it offers the new option within the public transport supply to choose ridesharing as feeder for public transport instead of walking. This will alter the travel times and route choice alternatives in the public transport supply. To model this effect, the following two approaches were explored.

A. Modeling intermodal ridesharing as an additional transport system

Public transport trips always start and end with access and egress trip legs, i.e. the ways from the origin of a trip in a traffic zone to the first bus or train stop and from the last stop to the traffic zone of the destination. As these access and egress trip legs are usually covered by foot, the number of stops within reach is restricted. This leads to a low number of alternative routes connecting an origin-destination-pair (od-pair) in the public transport system. In the example network shown in Fig. 3 (top) only one route connects the origin zone to a train stop by public transport, because the direct walking distance is too far. In addition to spatial restrictions, temporal restrictions depending on the timetable of trains and busses need to be considered in the assignment as well. These restrictions can be modeled by using a timetable-based public transport assignment (see [11] or [12]).

The main idea behind this first approach of adding ridesharing as an additional public transport system is to consider ridesharing in the timetable-based assignment while maintaining system properties of private cars. That means, the departure time of a ridesharing vehicle serving as feeder is derived directly from the departure times of trains and busses provided by the timetable, but the spatial restrictions of the ridesharing trip leg are similar to privately owned cars. A similar approach can be found in [13]. Fig. 3 (middle) shows some of the routes that are possible for a trip from the origin zone to a train stop with an additional ridesharing transport system using the roads and access links of private transport. This small example illustrates that ridesharing can increase the number of alternative routes considerably. As urban and regional travel demand models usually contain a high number of stop locations, which could all be reached by the feeder ridesharing at every time, this leads to a large set of alternative routes. As routing models with such a large number of alternatives are computational demanding, a second, simplified approach is developed.

B. Modeling intermodal ridesharing trip legs as direct links

In this approach, the ridesharing feeder trip legs are modeled as direct links between the traffic zones and suitable public transport stops, similar to modeling walking feeder trip legs. Thus the set of alternative routes is limited, as visualized in the example network in Fig. 3 (bottom).

The differences between direct links for ridesharing and walking feeder trip legs concern (1) the assumed speed for traveling the direct links, (2) the additional booking and boarding times for ridesharing feeder trip legs and (3) the range of stops that can be reached by the feeder system. For walking, a maximum walking time limits the distance. Ridesharing however requires a rule to limit the range. An example of such a rule could allow from each traffic zone
one ridesharing feeder trip leg to the nearest as well as to the second nearest train stop that cannot be reached by foot.

Using such an adapted network with additional direct links for ridesharing as input, the travel demand is computed using the traditional four-stage algorithm including a mode choice model that contains the four modes walking, cycling, direct ridesharing and public transport with integrated ridesharing. To replicate temporal demand patterns the travel demand model computes 96 demand matrices for each mode for every 15-minute time interval of a day. After the mode choice step, the public transport demand is assigned to the network, thus determining the demand on the direct links between stops and traffic zones. The demand on these direct links represents the demand for an intermodal ridesharing trip leg. Then all person trip legs using direct or intermodal ridesharing are pooled to vehicle trips using a trip-pooling algorithm. Finally, the vehicle trips are concatenated to vehicle tours resulting in empty trips for vehicle reallocation. The resulting vehicle matrix is then assigned to the road network, influencing the saturation dependent travel times of private transport systems and hence starting the iteration process between travel times, mode choice and the further travel-time dependent steps of demand modeling.

III. POOLING OF RIDESHARING TRIPS

Ridesharing aims at reducing vehicle kilometers by pooling trips of different persons to one vehicle trip. Person trips can be pooled if their spatial and temporal pattern is similar.

In agent-based microscopic travel demand models, it is relatively easy to match trips of individual agents with similar routes and departure times. A matching algorithm can compute the detour of a ridesharing vehicle with spare capacities for picking up another traveler and then determine, if it is reasonable to make the detour or if the loss of time is too high.

In macroscopic travel demand models, it is difficult to replicate the matching of trips because of three reasons:

1. The temporal segmentation of the demand normally uses rather long time intervals of one hour or more.
2. A macroscopic model works with probabilities. The demand between an origin and a destination is given as non-integer values, not representing the discrete trip of a specific traveler.
3. Travelers start and end their trips in centroids of traffic zones, so that pick-up and drop-off locations cannot be modeled in detail.

To solve the matching problem for macroscopic travel demand models the pooling algorithm described in [14] is implemented. This algorithm has the following characteristics:

1. It splits the travel demand in 96 time intervals and matches the trips of every time interval.
2. The algorithm does not match individual person trips but compares route sets from an all-or-nothing assignment. Each route has a specific non-integer demand and a non-integer capacity representing the vehicle size.
3. The matching process does not match routes on the level of the road network, but on the level of zones. For this, the sequence of links from a route is transformed to a sequence of zones. The sequence of zones defines a corridor along the route. The corridor approximates potential pick-up and drop-off locations.

Fig. 4 illustrates the matching process in a network with four zones and three routes, which are determined with an all-or-nothing assignment. No route is a complete part of another one, since the origins of every route do not lay on the path of the other routes. Nevertheless, route 2 can be matched to route 1, because the zone sequence of route 2 (B-C-D) is fully covered by route 1 (A-B-C-D). In the same way route 3 can be matched to route 1 or to route 2.

Fig. 3. Example network showing route alternatives for traditional public transport assignment (top), modeling approach A (middle) and modeling approach B (bottom).
Fig. 4. Identifying potential routes for a matching on the level of traffic zones.

For the actual matching of routes and the pooling of demand, all routes are sorted in descending order by their length. Then the longest route determines the first vehicle trip. The demand of following shorter routes is assigned to this vehicle trip, if their zone sequence is included in the zone sequence of the first vehicle trip as shown in Fig. 4. The matching does not necessarily need to start with the longest route, but simulation results suggest that this approach reduces vehicle kilometers compared to other sorting rules [15].

Since a vehicle has only a limited capacity $v_{cap}$ for transporting passengers, determining the capacity of a route $r_{cap}$ is an important part of the pooling algorithm. In a microscopic model, the first traveler is assigned to an empty vehicle, which then has a remaining capacity of $v_{cap} - 1$. In a macroscopic model, demand is non-integer and often below 1. To set the capacity $r_{cap}$ of a route it is assumed that the demand $r_{demand}$ of the route determines the capacity of the route. If the demand is for example $r_{demand} = 0.1$ and the vehicle capacity is $v_{cap} = 6$, it is assumed that the capacity of the route equals $0.1 \cdot 6 = 0.6$, i.e. it offers a spare capacity of $r_{cap} = 0.6 - 0.1 = 0.5$, which can be used to transport the demand of other matching routes. If the demand $r_{demand}$ exceeds the capacity $v_{cap}$ of one or more vehicles, only the remaining demand is considered to calculate the spare capacity $r_{cap}$ of a route, since it is not possible to match another trip to a fully occupied vehicle. This leads to the following equation (1):

$$
\text{if } (r_{demand} \mod v_{cap}) < 1:
\quad r_{cap} = (r_{demand} \mod v_{cap}) \cdot v_{cap} - (r_{demand} \mod v_{cap})
$$

else:
\quad $$r_{cap} = v_{cap} - (r_{demand} \mod v_{cap})$$

The processing of a route terminates, if the matching process does not find suitable matches or if the capacity of a route is exhausted. Then the process continues with the next route.

IV. SCHEDULING OF AUTOMATED RIDEHARING VEHICLES

The pooling algorithm determines the number of shared vehicle trips required to transport all travelers. However, these trips cannot be interpreted as a quality measure for evaluating ridesharing services, as neither the number of required vehicles nor the empty vehicle kilometer traveled are known. Therefore, these vehicle trips are concatenated to vehicle tours. A vehicle tour is a sequence of consecutive vehicle trips such that the destination and end time of one trip coincide with the origin and start time of the subsequent trip. Thus, the aim is to concatenate all vehicle trips to vehicle tours while keeping the number of used vehicles as low as possible.

Fig. 5 shows an example network with three zones and three time intervals. Each arc represents a vehicle trip, which can be non-integer. Solid lines indicate demanded vehicle trips and their volume is written next to the arcs. For example, starting in the first time interval, there are 1.9 vehicle trips required from zone B to A, and 0.6 from A to C. Since zones A and C are far apart from each other, the trip spans over two time intervals.
The task of designing vehicle tours can be formulated as a mathematical optimization problem. The structure of the problem even allows a formulation as a linear continuous program, which generally can be solved to optimality efficiently. However, for large travel demand models the resulting number of variables requires excessive computational power and implicates impractically long running times. An optimization approach is therefore not suited for this application of the problem.

Hence, the vehicle tours are determined by a heuristic approach. The basic idea is a chronological processing of all vehicle trips with the objective to reuse as many vehicles as possible. That means, each step of the algorithm corresponds to a time interval and processes all vehicle trips originating in that time interval. These vehicle trips are assigned to vehicles, which extends their tours. The algorithm uses vehicles from three different sources in the following priority order.

First, preferably vehicles that are available in the zone are used. Vehicles might be available in a zone because the previous vehicle trip ended in that zone. Since using these vehicles does not cause any acquisition or routing cost, this is the preferred option. In the example network in Fig. 5, there are 1.2 vehicle trips demanded which start in zone A in the second time interval. Since there are 1.9 incoming vehicle trips from zone B, these vehicles can be assigned to the 1.2 demanded vehicle trips in the second interval. A solution to this example network is depicted in Fig. 6. The resulting vehicle tours are represented by solid lines and the numbers in brackets state the number of vehicles driving.

As second choice, available vehicles located in other zones are used. Since these vehicles need to be available in the current time interval, they have to be relocated beforehand. Thus, the algorithm ensures that only vehicles are relocated, which are available in past time intervals and cannot be used better in their own zone in the current time interval. A relocation is modeled as an empty vehicle trip from the zone they are located at to the zone where their next trip starts. These empty vehicle trips are accountable for vehicle routing costs. To keep the routing costs low, only the closest available vehicles are selected for relocation. In addition to this an adjustable maximum distance for relocation is implemented to prevent empty long distance trips. If desired, a trade-off between vehicle acquisition costs and vehicle routing costs can be simulated with this parameter. The 0.7 vehicle trips demanded in zone B in the third time interval exemplify this. No vehicles are available in zone B, but exactly 0.7 vehicles are available in zone A during the second interval and not needed during the third time interval. Hence, the solution in Fig. 6 shows 0.7 empty vehicle trips from zone A to B to be used in the third time interval.

Thirdly, for all vehicle trips that cannot be assigned to available vehicles from the same or nearby zones, new vehicles have to be acquired. This is the last source as the algorithm is designed to minimize the number of vehicles needed to accommodate all vehicle trips. For example, zone C has 0.6 incoming vehicles from zone A, which is not enough to transport the demand in the third time interval. Further, it is not possible to relocate available vehicles from other zones, therefore it is necessary to acquire additional 1.1 vehicles.

To receive feasible vehicle flows, vehicle acquisition is modeled as empty trips within the zone, which is equivalent to vehicles waiting.

To summarize, the algorithm constructs all vehicle tours simultaneously extending them in each time interval with new vehicle trips. If no vehicle is available at the right time in the right zone, vehicles from other zones are requested which results in empty trips for vehicle relocation. The relocation is always performed in such a way as to minimize vehicle routing costs. Only if no relocation is possible, additional vehicles are acquired. This is a deterministic approach creating feasible vehicle tours containing all requested vehicle trips while using a preferably low number of vehicles and short relocation distances.

Obvious disadvantages of this approach compared to a mathematical model are a missing optimality guarantee and an insufficient treatment of routing costs. These disadvantages occur due to the chronological approach of the heuristic in contrast to a holistic formulation of an optimization model. In fact, in constructed situations the heuristic can yield vehicle tours requiring up to 50% more vehicles than in the optimal case.

In return, the heuristic approach overcomes the problem of scalability. While an optimization problem is expensive to solve, the heuristic can be solved on standard machines without exceptionally large memory. Tests with the macroscopic travel demand model of the Stuttgart region (1,100 traffic zones) have shown that the heuristic approach is approximately 300 times faster than solving the corresponding linear continuous program with a multipurpose solver. At the same time, the gap to the optimal solution was only 1%. In addition to that, the algorithm is independent of the travel demand. Even with high demand settings, the algorithm does not slow down.

V. TESTED SCENARIO AND FIRST RESULTS

In order to ensure the practicability of the methods described in the previous chapters, they were implemented in the macroscopic travel demand model of the Stuttgart region and a test scenario run was carried out. The base scenario distinguishes five mode choice alternatives: car driver, car passenger, public transport, bicycle and walking. Choice relevant input values such as mode specific travel times are calculated as 24-hour values of an average workday.

The tested scenario “ridesharing and rail” prohibits private car ownership and offers collective ridesharing instead. This means that travelers have to share a vehicle, taking into account certain limits for maximum vehicle
detours and extensions of travel time. Furthermore, the current bus system is assumed to be replaced by ridesharing, while rail-bound public transport systems (Suburban trains and LRT) remain. The assumptions and parameters used in the tested scenario are described in the following, according to the structure of this paper.

A. Assumptions in the demand modeling process

For modeling intermodal ridesharing, direct links are used (approach B). Each traffic zone is directly connected to at least the nearest rail-station. If the direct feeder link length is longer than 0.5 km, it is assumed that travelers prefer ridesharing to walking. In this case, a faster travel speed and additional time for booking and boarding are assumed.

To determine the distribution between direct ridesharing and intermodal ridesharing, the existing model structures are used. Direct ridesharing is assigned to the existing mode car driver; intermodal ridesharing is integrated into the existing mode public transport. The distribution between the two modes is influenced in the following ways: (1) Travel time for direct ridesharing is assumed to be similar to car travel time with a small increase for detours and boarding waiting time. (2) Travel time for public transport is reduced for od-pairs where intermodal ridesharing replaces a feeder bus. (3) Direct ridesharing is not provided, if public transport is faster. (4) Public transport is not considered a suitable alternative, if travel time is more than 1.5 times longer than direct ridesharing.

B. Assumptions for pooling ridesharing trips

Not all person trips in the Stuttgart region are suitable for ridesharing. Trips with commercial purposes and trips leaving the Stuttgart region are not pooled to ridesharing trips. This demand is assigned to the network as a separate user class “no sharing” as in the base scenario.

The test scenario assumes automated vehicles with six seats for passengers.

C. Assumptions for vehicle scheduling

The vehicle-scheduling algorithm allows the implementation of an upper limit for the distance of relocation of vehicles. In order to minimize the number of vehicles in the tested scenario, no such upper limit is chosen. That means that long empty vehicle trips are preferred to the acquisition of additional vehicles.

D. Results

Fig. 7 and Fig. 8 compare the travel demand on the level of vehicle kilometers in the entire region and within the city boundaries. The total vehicle kilometers traveled by no sharing and ridesharing vehicles decrease slightly in the Stuttgart region (Fig. 9), but increases within the city boundaries of Stuttgart (Fig. 10). This can be explained as follows: (1) Current car trips have only a limited potential to be pooled to ridesharing trips – this is especially the case for trips in the rural areas of the region. (2) Bus trips are replaced by shared vehicle trips and (3) empty trips are necessary to reallocate the ridesharing vehicles. (4) Reductions in vehicle kilometers achieved by ridesharing reduce travel times in the road network. This makes direct ridesharing more attractive, thus reducing the demand for public transport and increasing the demand for direct ridesharing. After all, an equilibrium is obtained at a state, where the vehicle kilometers traveled by direct ridesharing and no sharing are similar to those in the base scenario.
Concerning the number of vehicles needed to serve the demand, the test scenario shows that this number can be reduced to less than 25% of the number of cars needed today.

VI. CONCLUSION AND OUTLOOK

Macroscopic travel demand models are used to evaluate changes in transport supply. The introduction of fully automated vehicles is likely to change transport supply considerably, especially by enhancing the attraction of ridesharing and carsharing services. The paper suggests a framework for integrating ridesharing services provided by automated vehicles into macroscopic travel demand models and describes the results of a test run with such an enhanced model. These first results show that the potential for trip pooling and therefore reduction of vehicle kilometers traveled is limited. The potential to reduce the number of vehicles needed, in contrast, is high.

While the focus of this paper lies on ridesharing services, the implementation process for carsharing services can be easily derived: The steps are all the same, but without the trip-pooling step, which makes the procedure less elaborate.

Even though this paper presents and successfully tests a framework for integrating automated sharing systems into macroscopic travel demand models, there are still many open issues concerning modeling demand effects of ridesharing and carsharing services. New services can increase the set of modal choices or they can enhance the existing public transport mode. Modeling this requires information on the parameters and variables in the utility functions, especially those components describing the pricing of the new services and the willingness to share. Finally, the important issue of choosing suitable mobility tools needs to be addressed. This requires an extended model structure and assumptions on future choices, e.g. if car- and ridesharing operators will provide flat rate season tickets like in public transport or if travelers will pay per ride.

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