The polarization signature from microlensing of circumstellar envelopes in caustic crossing events

R. Ignace,1,2⋆ J. E. Bjorkman3 and H. M. Bryce1,4

1Department of Astronomy, University of Wisconsin, 475 North Charter Street, Madison, WI 53706, USA
2Department of Physics, Astronomy, & Geology, East Tennessee State University, Johnson City, TN 37614, USA
3Ritter Observatory, Department of Physics and Astronomy, University of Toledo, Toledo, OH 43606, USA
4Department of Physics and Astronomy, University of Glasgow; Glasgow G12 8QQ

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ABSTRACT
In recent years, it has been shown that microlensing is a powerful tool for examining the atmospheres of stars in the Galactic bulge and Magellanic Clouds. The high gradient of magnification across the source during both small impact parameter events and caustic crossings offers a unique opportunity for determining the surface brightness profile of the source. Furthermore, models indicate that these events can also provide an appreciable polarization signal: arising from differential magnification across the otherwise symmetric source. Earlier work has addressed the signal from a scattering photosphere for both point mass lenses and caustic crossings. In a previous paper, polarimetric variations from point lensing of a circumstellar envelope were considered, as would be suitable for an extended envelope around a red giant. In this work, we examine the polarization in the context of caustic crossing events, the scenario that represents the most easily accessible situation for actually observing a polarization signal in Galactic microlensing. Furthermore, we present an analysis of the effectiveness of using the polarimetric data to determine the envelope properties, illustrating the potential of employing polarimetry in addition to photometry and spectroscopy with microlensing follow-up campaigns.

Key words: gravitational lensing – polarization – stars: atmospheres.

1 INTRODUCTION
Much attention has been paid to the situation in which the analytic case of the magnification of a point source by a point lens breaks down. It has been noted that, for small lens–source separations, the finite size of the source star needs to be considered (Nemiroff & Wickramasinghe 1994; Witt & Mao 1994; Witt 1995; Peng 1997). Such events not only constrain the lens properties by breaking the degeneracy in the event parameters (Gould 1994) but also provide valuable stellar atmosphere information, such as limb darkening (Hendry et al. 1998; Valls-Gabaud 1998), polarization (Simmons, Willis & Newsam 1995a; Simmons, Newsam & Willis 1995b; Newsam et al. 1998), motions in circumstellar envelopes (Ignace & Hendry 1999) and the presence of star-spots (Heyrovský & Sasselov 2000). The opportunity for studying stellar atmospheres through microlensing as described in Gould (2001) and Sackett (2001) has clear advantages over other methods such as eclipsing binaries, because the source and means of studying the source are not coupled and the flux from the source is magnified rather than diminished.

Binary lenses produce another deviation from a standard microlensing light curve, because caustics are produced (e.g. Schneider & Weiss 1986). It is the high gradient of magnification across the caustic that allows one to infer information about the source intensity profile, even though the source images are not individually resolvable with current instruments. Despite binary lens events only accounting for about 5 per cent of microlensing events (e.g. Alcock et al. 2000), it is more likely that finite source effects will be relevant for a binary lens rather than a point lens. The reason is that finite source effects are mainly discernible only when the lens transits the source itself (Gould 1994). Because the angular Einstein radius θE (see equation 3) is usually much larger than the angular source size, such transits tend to be rare. On the other hand, the caustic structures giving rise to high magnifications in binary lens events are spatially extended (of order θE in scale) and so caustic crossings by the more distant source are relatively common in events associated with binary lenses. So it is more expedient to study the resolution of stellar atmospheres due to binary lenses than point lenses. The structure of the caustics produced and the resulting magnification and light curves from binary microlenses has been discussed extensively in the literature (Maô & Paczynski 1991; Maô & Di Stefano 1995; Di Stefano & Perna 1997; Dominik 1998; Gaudi & Gould 1999; Dominik 2004b). A number of papers have reported on source

*E-mail: ignace@mail.etsu.edu
2 MICROLENSING OF EXTENDED SOURCES

The magnification of a point source by a point lens is given by

$$A_p = \frac{u^2 + 2}{u\sqrt{u^2 + 4}},$$

(1)

where $u$ is the angular separation between the lens and source normalized by the angular Einstein radius $\theta_E$, which is given by

$$\theta_E = \sqrt{\frac{4GM_L(D_S - D_L)}{c^2D_S}},$$

(2)

where $M_L$, $D_S$ and $D_L$ are the lens mass, the distance to source and the distance to lens, respectively. The lensing is a transient event relative to the Einstein radius. Concluding remarks are presented in Section 5.

3 POLARIZATION MODEL

We review now the intensity profile that will be used with equation (5) to calculate the microlensing light curves. As stated above, the formalism follows that of Simmons et al. (2002). As emphasized in that work, the model employed is well suited to evolved cool stars. This class of stars exhibits stellar winds that are significantly stronger than those of the Sun, with mass-loss rates ranging from $10^{-10} M_\odot \text{yr}^{-1}$ for typical red giants up to $10^{-5} M_\odot \text{yr}^{-1}$ for red supergiants and asymptotic giant branch stars (e.g. Lamers & Cassinelli 1999). The more extreme stellar winds are clearly dust driven (e.g. Netzer & Elitzur 1993; Habing, Tignon & Tielens 1994). Stars with milder winds are less understood, possibly driven by a Parker-style wind augmented by molecular and dust opacities (Jorgensen & Johnson 1992).
Assuming the flows are spherically symmetric, the run of the bulk gas density \( \rho \) in the wind with radius \( r \) will be given by

\[
\rho = \frac{M}{4\pi r^2 v(r)},
\]

where \( M \) is the mass-loss rate and \( v(r) \) is the radial velocity of the flow, beginning subsonically and asymptoting to a terminal speed \( v_\infty \) at large radius.

The scattering opacity responsible for producing polarization could be molecular Rayleigh scattering or dust scattering. Consequently, the density profile of the polarigenic species can differ from equation (7) by virtue of how molecules or dust are produced and destroyed as a function of radius. Particularly in the case of dust, the location of the condensation radius will be important. This latter point concerning dust is interesting, because the stars under consideration have photospheric effective temperatures of 3000–4000 K, whereas the dust condensation temperature is typically around 1500 K (Gail & Sedlmayr 1986). The implication is that dust-driven winds can have central cavities (not vacuums, but rather interior zones for which there is no significant scattering opacity). Models for dust-driven winds naturally predict the radial extent of these cavities, yet the condensation radius is largely unconstrained by observations (Bloemhof & Danen 1995). Microlensing can provide relevant observational constraints on the location of the condensation radius. For this pilot study of the polarimetric signals from caustic crossing events, we choose to parametrize the scattering number density by a simple power law with

\[
n(r) = n_0 \left( \frac{R_b}{r} \right)^\beta,
\]

instead of fretting about the details of the wind acceleration, or how the scattering opacity evolves in the flow. We additionally assume the envelope is optically thin for this application, but will explore optical depth effects of the envelope in a subsequent paper.

We adopt the Stokes vector notation \( \mathbf{I} = (I, Q, U, V) \) for the intensities and \( \mathbf{F} = (F_I, F_Q, F_U, F_V) \) for the fluxes. Following the notation of Simmons et al. (2002), the Stokes intensities for direct and scattered light will be given by

\[
I(p, \alpha) = I_0(p) + \frac{3}{16} I_s(\beta - 1) \left( \frac{R_b}{p} \right)^{\beta - 1} \left( \frac{R_b}{p} \right)^2 \\
\times \tau_{sc} g_0(p) \begin{pmatrix} G_I \\ G_P \cos(2\alpha) \\ -G_P \sin(2\alpha) \\ 0 \end{pmatrix},
\]

where \( R_b \) is the radius for any ‘hole’ of scattering opacity and the total optical depth, \( \tau_{sc} \), is defined as

\[
\tau_{sc} = n_0 \sigma R_b \left( \frac{R_b}{p} \right)^{\beta - 1},
\]

with \( \sigma \) the scattering cross-section and where the scatterers are taken to exist only for \( r \geq R_b \).

For simplicity, we shall assume a uniform surface brightness profile for the star, such that

\[
I_0(p) = \begin{cases} (I_* \cos \alpha, 0, 0) & \text{for } p < R_* \\ 0 & \text{for } p \geq R_* \end{cases},
\]

where \( I_* \) is the intensity of the star. Although we ignore limb darkening, its main effect would be to enhance the polarization from lines of sight that are close to the star (bearing in mind that we are currently ignoring scattering polarization from the stellar photosphere, an assumption that we shall justify a posteriori). The reason is that, at a fixed distance from the star, limb darkening causes the stellar radiation field impinging upon a given point at this distance to be more radially directed than is the case for a uniformly bright star (Cassinelli, Nordsieck & Murison 1987).

Also appearing in equation (9) is the stellar occultation factor, \( g_0 \), that is given by

\[
g_0(p) = \begin{cases} 1/2 & \text{for } p < R_* \\ 1 & \text{for } p \geq R_* \end{cases}.
\]

The occultation factor simply accounts for the fact that radiation scattered on the far side of the star will not reach the observer. The integral factors \( G_I \) and \( G_P \) are

\[
G_I = \int_0^{\tau_{max}} \sqrt{n^2 \pi^2} \frac{(1 - sq)}{1 - s} \left[ \left( \frac{1}{s^2 q} \right)^{1/2} - (1 - \frac{1}{4 q}) \right] ds
\]

and

\[
G_P = \int_0^{\tau_{max}} \sqrt{n^2 \pi^2} \frac{(1 - sq)}{1 - s} ds
\]

with \( s = (p/r)^2, q = (R_*/p)^2, s_{max} = (p/r_{min})^2, \) for \( r_{min} = \max(R_*, p) \).

The fluxes during the microlensing event are then computed from the integral expressions:

\[
F_I = \int_0^{\infty} \int_0^{2\pi} I(p, \alpha) A_{\text{cau}}(d) \ p \ d p \ d\alpha,
\]

\[
F_Q = \int_0^{\infty} \int_0^{2\pi} I(p, \alpha) \cos 2\alpha A_{\text{cau}}(d) \ p \ d p \ d\alpha,
\]

\[
F_U = \int_0^{\infty} \int_0^{2\pi} I(p, \alpha) \sin 2\alpha A_{\text{cau}}(d) \ p \ d p \ d\alpha,
\]

\[
F_V = 0.
\]

As is usual, the observed fractional polarization can then be calculated as

\[
P = \frac{\sqrt{F_Q^2 + F_U^2}}{F_I},
\]

where the total flux \( F_I \) is a sum of the direct stellar flux \( F_* \) and the scattered intensity \( F_{\text{sc}} \). The polarization position angle is defined as

\[
\psi = \frac{1}{2} \tan^{-1} \frac{F_U}{F_Q}.
\]

4 RESULTS

Having described the lensing approximation for a straight fold caustic and the underlying source model, we have conducted a parameter study for microlensing light curves associated with caustic crossing events as various source and lens properties are varied. In displaying results, our goal is to highlight those features that pertain to elucidating the properties of the source; therefore, instead of light curves as a function of time, we choose to plot flux observables as a
function of caustic position relative to the star. To do so, we define the coordinate $x_{\text{lens}}$ as the normal projected distance between the straight line caustic and star centre in the source plane. Then $y_{\text{lens}}$ is the coordinate along the caustic direction. Both coordinates are normalized to the stellar radius $R_\star$. The case $x_{\text{lens}} < 0$ is when the star centre lies inside the caustic; the case $x_{\text{lens}} > 0$ is when the star centre lies outside the caustic; and $x_{\text{lens}} = 0$ is the moment of transit for the star centre.

Conversion to a time coordinate is achieved with $t = (x_{\text{lens}} R_\star)/(D_S \mu_{\text{rel}}A)$, where $\mu_{\text{rel}} = \mu_{\text{tot}} \cos \gamma$, with $\mu_{\text{tot}}$ the magnitude of the relative proper motion between the source and caustic, and $\gamma$ is the trajectory orientation of the source relative to the caustic. So $\gamma = 0^\circ$ means the source is traveling in the $+x$ direction in the frame of the caustic, whereas $\gamma = \pm 90^\circ$ means the source is moving in the $\pm y$ direction, respectively. The orientation of the trajectory has no bearing on the shape of the lensing light curves in time, except as a ‘stretch’ factor for scaling purposes; for example, $\gamma$ does not affect the value of the peak polarization achieved during the event.

4.1 Test case

Fig. 1 shows model results for a test scenario. The upper panel shows the flux of the source, relative to the case of being unlensed, as a function of the caustic location. The lower panel is for the polarized flux, also normalized to the unlensed flux of the source $F_0$ and multiplied by 100 to simulate a kind of per cent polarized flux (the reason for this rather odd choice will become apparent in a moment).

The envelope parameters are fixed with an envelope optical depth of $\tau_{\text{en}} = 0.1$, density parameter $\beta = 2$ and no cavity (i.e. $R_\text{h} = R_\star$). The lens parameter $b_0$ is fixed at a value of 10, but $A_0$ is allowed to vary from 1 to 15, with larger values giving stronger peak flux magnifications.

We have argued that $A_0$ should have no bearing on the polarized emission and indeed that is seen to be the case in Fig. 1. All of the variation of the polarized flux comes from the second term in equation (6) with scaling $b_0$. The standard representation of polarization with $p = F_Q/F_I$ is used. The fractional or per cent polarization can be affected by the value $A_0$, because the total flux variation as the microlensing event evolves does depend on $A_0$. A value of $A_0 = 1$ has been adopted for the rest of the model calculations.

It is useful at this point to introduce a schematic figure that demarcates regions contributing to the polarized light as the caustic crossing evolves. Fig. 2 shows four panels, with the source moving from inside the caustic to outside in the sequence (A) to (D). The vertical dotted line is the caustic, hence the left region is interior to the caustic and the right region is outside it. The star is cross-hatched. The double-headed arrows show the sense of orientation that would result for the emergent polarized flux from right angle scattering in the plane of the sky if the source were resolved. The dashed diagonal lines are where $Q = 0$. Thus, the figure is useful in mapping how the different zones of polarized flux will contribute to the total polarized emission from the unresolved lensed source as a function of its location relative to the caustic line.

The variation of the model polarized flux shows interesting sign changes. Here, we are assuming an observational scenario in which a second exiting caustic crossing has been predicted from an
earlier interior crossing event. Thus in Fig. 2 at time (A), the source is approaching the caustic to exit. The quadrant closest to the fold is dominated by scattered light with $Q > 0$ and so initially the polarization is oriented parallel to the fold caustic in the sky. As the event progresses, the situation of case (B) is reached. Only scattered light leftwards of the caustic will be subject to a magnification gradient, thus breaking the symmetry and leading to a net observed polarization. By time (B), the light being most strongly magnified is for $Q < 0$ and so the polarization position angle has rotated $90^\circ$ by the time the caustic is first tangent to the photosphere. By time (C), a minimum in the polarized flux has passed. Now the polarization changes rapidly, so that by the time the caustic is tangent to the far side of the photosphere, the polarized flux has changed sign again, becoming positive. Note in terms of the polarized light side of the photosphere, the polarized flux has changed sign again, a minimum in the polarized flux has passed. Now the polarization plots the standard form of polarization,

\[ \frac{F_Q}{F_I} = \frac{\tan \alpha}{\sqrt{1 - \alpha^2}} \]

Fig. 3 shows the response of the flux and polarization light curves. The different curves are for different values of positive $\beta$ and $b$ (no cavity), $A_0 = 1$ and $b_0 = 10$ are used with a density distribution described by $\beta = 2$. These values for $A_0$, $b_0$ and $\beta$ will be standard in our calculations unless noted otherwise. The envelope has little effect on the flux light curve except when $\tau_c < 1$. On the other hand, the amplitude of the polarization light curve is approximately linear in $\tau_c$.

![Figure 3](https://example.com/figure3.png)

**Figure 3.** Plots like Fig. 1, but now with fixed lens parameters and variable envelope optical depths, and with the lower panel as per cent polarization normalized by the total intensity flux $F_I$. In this case, values of $\beta = 2$, $R_b = R_*$ (no cavity), $A_0 = 1$ and $b_0 = 10$ are assumed. The different curves are for different envelope scattering optical depths, with $\tau_c = 0.001, 0.01, 0.1, 0.3$ and 1.0 (in order of increasingly stronger peak polarizations). Note the change in sign of the polarization as the stellar photosphere transits the caustic and the strong peak value that results immediately after the photosphere has completely exited the caustic.

4.2 Variable envelope optical depth

The envelope is reasonably thin and so the flux magnification is dominated by the photospheric emission. Agol (1996) has investigated the polarimetric variations from scattering polarization in stellar atmospheres. We have purposely ignored photospheric contributions to the polarized emission, because circumstellar envelopes are more efficient at producing polarized emission (albeit, this is a function of optical depth) and because we wish to investigate the effects of a circumstellar envelope for the light curves. Although $A_0$ and $b_0$ are not exactly the same, the case of the solid line in the upper panel of Fig. 3 is roughly comparable to the $r = 0.01$ case shown in the upper panel of Agol’s fig. 1.

Overall, the polarization curves are generally similar to those of Agol (1996); however, there are notable quantitative and qualitative differences. First, quantitatively, the peak polarization achieved by photosphere crossings were rarely in excess of 1 per cent and, in some cases, only a few tenths of a per cent, whereas thin scattering envelopes can achieve values in excess of 5 per cent when $\tau_c$ is large enough ($\tau_c \gtrsim 0.5$). Secondly, there are qualitative differences as well. The underlying source models are drastically different. For example, a stellar photosphere has an intrinsic polarization profile (as a function of $p$) that is maximum at the stellar limb and decreases to zero at the centre of the star. In our case, the photospheric polarization is ignored because it is small compared with the envelope polarization. The extended envelope has a polarization profile that is zero at large distance from the star and increases towards the stellar limb. The polarization peaks outside the stellar limb and decreases to zero again at the stellar centre. So the source models are quite different but, at the same time, the variable polarization from microlensing for the two cases is somewhat similar. Basically, the photospheric polarization has a discontinuous jump in moving from off the star across the stellar limb, whereas the circumstellar envelope has a more gradual peak off the stellar limb. Microlensing involves a weighted surface integral, thus ‘smoothing’ over these detailed differences, leading to somewhat similar light curves (Dominik 2004a). Still, the fact that the peak polarization appears at the stellar limb for a photosphere in a discontinuous way explains why there is a peak (sometimes cuspy) at $x_{\text{lens}} = -R_*$ in Agol’s models and not in ours. (This is $y_\star = -r$ in Agol’s notation; see his fig. 1.)

4.3 The influence of a cavity

Fig. 4 shows how a cavity of scatterers at the inner envelope impacts the polarization light curves. The different curves are for different ‘hole’ radii of $R_b = 1.0, 1.5, 3.0, 5.0$ and $8.0 R_*$. Clearly, as the extent of the cavity increases, the polarimetric variation occurs over a longer time-scale, with the peak polarization shifting towards larger values of positive $x_{\text{lens}}$ and the negative ‘trough’ growing in extent towards negative $x_{\text{lens}}$ as a precursor to the transit of the star.

In each case, the envelope optical depth is maintained at $\tau_c = 0.1$. First, this is rather optically thin, as evidenced by the total flux light curve (upper panel), which does not vary much between the different cases and is dominated largely by the photosphere. Secondly, the scale of the polarization is to zeroth order determined by the value of $\tau_c$, which explains why all of the curves have similar peak polarization values. Thirdly, holding $\tau_c$ constant implies conserving the total number of scatterers; hence, although the different cases shown have the same density distribution at $\beta = 2$, these require different density scales $n_b$, so as to maintain fixed $\tau_c$. Using equation (10), the density scale for fixed envelope optical depth is given by

\[ n_b = (\beta - 1) \frac{\tau_c}{\sigma R_b} \]
Consequently a cavity is present (see Fig. 5). Some notable qualitative differences in the variation of polarization when a cavity is present (see Fig. 5).

So our approach for inserting a cavity is not to delete scatterers, but to redistribute them outwards. The goal of considering cavities is to illustrate how microlensing can neatly trace the cavity extent through the polarization light curve, which is relevant for the case of red giants that can form dust in their winds at a condensation radius that is offset from the stellar photosphere.

Fig. 5 shows the same curves as Fig. 4, but with $x_{\text{lim}}$ normalized to $R_h$ instead of $R_\ast$, which nicely shows how the polarimetric variations are set by the crossing of the cavity. Clearly, the peak polarization is affected by the cavity (generally smaller), and substructure is seen around the passage of the photosphere from inside the caustic to outside. The peak polarization consistently occurs just before the cavity transits entirely out of the caustic and so becomes an excellent tracer of the cavity extent relative to the stellar radius, which can be determined from the total flux variations.

In other words, the total flux variations show a peak at a time when the stellar limb just begins to transit the caustic. As the event proceeds, the total flux shows a precipitous drop and then goes flat. The extended envelope can produce a tail of enhanced brightness after the star has transited out of the caustic, but the drop is dominated by the star. So that time-scale is $2t_s = 2\theta_s/\mu_{\text{rel}}$, where $\theta_s$ is the angular radius of the star. On the other hand, from this time until the cavity is completely out of the caustic, the negative ‘trough’ has approximately constant width. The solid line is for no cavity (i.e. $R_h = R_\ast$); notably, the presence of a cavity changes the qualitative shape of the trough, with some recovery towards net zero polarization followed by a sharp drop towards more negative values.

The different cases are all for a fixed value of $\tau_\ast = 0.1$. As noted before, this is in essence achieved via redistribution of scatterers. In this case, there is a fixed hole. Changing $\beta$ makes the density distribution more or less steep. As $\beta$ is made to increase, keeping $\tau_\ast$ fixed results in increased values of $n_0$ and so the peak polarization that is dominated by the number density of scatterers at the limb of the cavity increases as well.

So $\beta$ does not necessarily lead to larger polarizations; here, it is an artefact of maintaining a constant value of $\tau_\ast$. Fundamentally, what $\beta$ does is to alter the slope of the polarization curves after the cavity has completely passed out of the caustic, making the slopes steeper with increasing $\beta$. In fact, in the limit that the star can be treated as a point source, the asymptotic slope of the polarization for relatively large values of $x_{\text{lim}}$ can be derived analytically. The derivation is found in the Appendix; here, just the result is quoted. Asymptotically, the polarized flux ($\text{not the per cent polarization}$) will be given by

$$F_Q \propto \left( \frac{R_h}{x_{\text{lim}}} \right)^{(2\beta - 1)/2} \propto \tau^{-(2\beta - 1)/2}. \quad (23)$$

So larger values of $\beta$ lead to steeper declines in the polarized flux as the microlensing event progresses. Two points should be mentioned. First, although real sources may not follow a power-law distribution for the density of scatterers in portions of their extended envelopes, a value of $\beta = 2$ is reasonable to expect at large scale for a spherical wind flow, for which case $F_Q \propto \tau^{-3/2}$. Secondly, the preceding equation is only valid both for when the star can be treated as a nearly point source of illumination with respect to the scattering envelope and when the caustic can be approximated as a straight line. Even if the asymptotic trend of equation (22) is not achieved in a real

\[\text{Figure 4. Illustration of how a cavity of scattering opacity impacts the polarization variation. All of the curves are for } \tau_\ast = 0.1. \text{ Each is distinguished by the extent of the cavity, with } R_h = 1.0, 1.5, 3.0, 5.0 \text{ and } 8.0 R_\ast, \text{ with larger cavities yielding peak polarizations at larger values of } x_{\text{lim}}. \text{ Note that the total flux curve (upper panel) is little influenced by the cavity extent because the scattering envelope is optically thin. Although basically similar, there are some notable qualitative differences in the variation of polarization when a cavity is present (see Fig. 5).}\]

\[\text{Figure 5. The results shown in Fig. 4, except now the lower axis for the position of the fold caustic is normalized to the size of the cavity } R_h. \text{ From Fig. 4, the total flux variation was set by the passage of the photosphere across the caustic. The polarimetric variations, on the other hand, are determined by the size of the cavity. Although the positive peak polarization has variable width, the negative ‘trough’ has approximately constant width. The solid line is for no cavity (i.e. } R_h = R_\ast); \text{ notably, the presence of a cavity changes the qualitative shape of the trough, with some recovery towards net zero polarization followed by a sharp drop towards more negative values.}\]
event, it does provide useful insight and limiting behaviour for the modelling effort.

4.5 Variation of the magnification gradient

Fig. 7 shows model results as the value of \( b_0 \) is varied, with values of 3, 5, 8, 12 and 17 (with \( A_0 = 1 \) fixed). With \( b_0 \) relatively large compared with \( A_0 \), the polarization varies little for \( x_{\text{lens}} < 1 \), prior to when the star has passed out of the caustic. The reason is that the polarization is a ratio of the polarized light to the total flux, and both scale as \( b_0 \). Clearly, the total flux shown in the upper panel is strongly dependent on \( b_0 \), increasing essentially linearly with \( b_0 \). Similarly, the polarization after the star has passed out of the caustic is affected by \( b_0 \), because now \( F_I \) is a constant that depends primarily on the photospheric flux multiplied by \( A_0 \), whereas \( F_Q \) still depends on the value of \( b_0 \).

5 DISCUSSION

This study has demonstrated that polarimetric observations of caustic crossing events could be used to probe circumstellar envelopes. Such data could constrain the number density of the scatterers within the envelope, detect the presence and trace the extent of a central cavity around the source photosphere, and provide information about the density distribution of scatterers.

To summarize, a comparison of the time-scale for the total flux variations against that of the polarimetric variations yields the extent of the cavity relative to the stellar radius. In the case of dust producing cool star winds, that information is sufficient to test models that predict the location of the dust condensation radius. The late time evolution of the polarized flux is set by the density distribution (our \( \beta \) value for this work). Lens parameters \( A_0 \) and \( b_0 \) can be derived from fitting the variation of the photospheric flux during the caustic crossing, in which case model fits to the polarization level will give the value of \( \tau_{\text{sc}} \) for the circumstellar envelope.

To connect our model results with applications to observed events, we must relate the envelope properties for the model to those of real sources. Some help towards this end is provided by Netzer & Elitzur (1993), who describe model results for dust-driven winds. In their equation (10), they provide an expression relating stellar parameters \( M \), \( v_{\infty} \) and \( L_* \) to the flux mean optical depth of the envelope \( \tau_F \):

\[
\frac{M}{2 \times 10^{-5} M_\odot \, \text{yr}^{-1}} = \tau_F \frac{L_* / 10^4 L_\odot}{v_{\infty} / 10 \, \text{km s}^{-1}},
\]

here shown in slightly modified form from their paper. The value of \( \tau_F \) will not equal the value of \( \tau_{\text{sc}} \) that we use to characterize our models; however, the flux mean opacity does give an overall scale related to the optical depth of the envelope. Although relating \( \tau_{\text{sc}} \) to \( \tau_F \) will depend on the particular opacities involved, one might generally expect that the two will scale together. The models of Netzer & Elitzur show that the minimum mass-loss for dust driving to be dominant is around \( 10^{-7} M_\odot \, \text{yr}^{-1} \). At this value for a star with \( L_* = 10^4 L_\odot \) and \( v_{\infty} = 10 \, \text{km s}^{-1} \), the flux mean optical depth will be 0.02 (of course, the optical depth at a wavelength of interest can be higher or smaller).

One of the challenges in detecting polarized signals in real events, such as this hypothetical bulge star, is that the crossing of the caustic by the photosphere will typically take only a few hours (e.g. Alcock et al. 2000). We can estimate the detectability of polarizations predicted by our models. Using a Kurucz model for a cool subgiant of \( g = 3.5 \) and \( T = 4500 \, \text{K} \) (parameters similar to OGLE-1999-BUL-23 from Albrow et al. 2001a), the \( I \)-band flux at a distance of 8 kpc is estimated to be \( 3 \times 10^{-11} \, \text{erg s}^{-1} \, \text{cm}^{-2} \), with a corresponding magnitude of about \( m_I \approx 18 \). Of course, during the lensing event, the source brightens and magnifications by an order of magnitude are typically achieved, at which point \( m_I \approx 15.5 \). The time \( t \) required to achieve a given signal-to-noise ratio \( S/N \) for a telescope of diameter...
D in centimeters, allowing for Poisson noise only, will be 
\[ t \approx \frac{9(S/N)^2}{D^2}. \]  
(25)

Our models indicate that peak polarizations of about 1 per cent will be achievable during caustic crossings. A 5σ detection at this polarization level requires \( S/N = 500 \). Additionally, exposures are needed at eight position angles in order to construct the Stokes I, Q and U fluxes (eight to eliminate systematics). Consequently, the required exposure time in total for this detection level, not counting overhead, will be about 30 min using a 1-m telescope. Although this exposure estimate is a lower limit (owing to neglect of background, inefficiencies and telescope overheads), the required exposure is about 10 per cent of the duration of the photosphere crossing, even smaller for the bulk of the circumstellar scattering envelope, and can be reasonably obtained with modest facilities equipped with polarimetric instrumentation.

Although the original goal of microlensing surveys was to deduce the properties of dark matter in the Milky Way, it is clear that a vast range important of by-products have resulted from the survey effort, from catalogues of variable stars to observations of finite source effects (as described in the Introduction). Our contribution to the topic of finite source effects has been to point out how novel and valuable information about circumstellar envelopes might be obtained through polarimetric monitoring of events involving binary lenses and sources that may have substantial winds.

Certainly, our models include some simplifying assumptions, such as ignoring polarization from the photosphere and the effect of limb darkening. Neither of these are severe; indeed, both will tend to increase the peak polarizations above those predicted by our models. Photospheric polarization is expected to be smaller than the circumstellar contribution for stars with significant winds, but its contribution should add to the Q and U fluxes constructively and not destructively. Limb darkening will tend to mollify the effects of the finite depolarization factor, thereby increasing the peak polarization from the inner wind where the density of scatterers is larger (although limb darkening will have little or no impact in the case of significant central cavities). We have also not allowed for optical depth effects, by way of multiple scattering effects and extinction of the photospheric emission. We intend to consider these effects in a separate paper. However, substantial scattering optical depths will be more important for quite dense winds, like those of red supergiants and asymptotic giant branch stars. For the more common red giant stars, the optically thin assumption will be a good assumption.

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REFERENCES

Abe F. et al., 2003, A&A, 411, L493
Afonso C. et al., 2000, ApJ, 532, 340
Agol E., 1996, MNRAS, 279, 571
Albrow M. et al., 1999, ApJ, 522, 1011
Albrow M. et al., 2000, ApJ, 534, 894
Albrow M. et al., 2001a, ApJ, 549, 759
Albrow M. et al., 2001b, ApJ, 550, L173
Alcock C. et al., 2000, ApJ, 541, 270
Blohmof E. E., Danen R. M., 1995, ApJ, 440, L93
Cassan A. et al., 2004, A&A, 419, L1
Cassinelli J. P., Nordsieck K. H., Murison M. A., 1987, ApJ, 317, 290
Castro S., Pogge R. W., Rich R. M., DePoy D. L., Gould A., 2001, ApJ, 548, L197
Di Stefano R., Perna R., 1997, ApJ, 488, 55
Dominik M., 1998, A&A, 349, 108
Dominik M., 2004a, MNRAS, 352, 1315
Dominik M., 2004b, MNRAS, 353, 118
Gail H.-P., Sedlmayr E., 1986, A&A, 166, 225
Gaudi B. S., Gould A., 1999, ApJ, 513, 619
Gould A., 1994, ApJ, 421, 71
Gould A., 2001, PASP, 113, 903
Habing H. J., Tignon J., Tieless A. G. M., 1994, A&A, 286, 523
Hendry M. A., Coleman I. J., Gray N., Newsam A. M., Simmons J. F. L., 1998, New Astron. Rev., 42, 125
Heyrovský D., Sasselov D., 2000, ApJ, 529, 69
Ignace R., Hendry M. A., 1999, A&A, 341, 201
Jorgensen U. G., Johnson H. R., 1992, A&A, 265, 168
Lamers H. J. G. L. M., Cassinelli J. F., 1999, Introduction to Stellar Winds. Cambridge Univ. Press, Cambridge
Mao S., Di Stefano R., 1995, ApJ, 440, 22
Mao S., Paczynski B., 1991, ApJ, 374, L37
Maoz D., Gould A., 1994, ApJ, 425, 67
Nemiroff R. J., Wickramasinghe H., 1994, ApJ, 424, 21
Netzer N., Elitzur M., 1993, ApJ, 410, 701
Newsam A. M. Simmons J. F. L., Hendry M. A., Coleman I. J., 1998, New Astron. Rev., 42, 121
Peng E. W., 1997, ApJ, 475, 43
Sackett P. D., 2001, in Menzies J. W., Sackett P. D., eds, ASP Conf. Ser. Vol. 239, Microlensing 2000: A New Era of Microlensing Astrophysics. Astron. Soc. Pac., San Francisco, p. 213
Schneider P., Wagoner R. V., 1987, ApJ, 314, 154
Schneider P., Weiss A., 1986, A&A, 164, 237
Simmons J. F. L., Willis J. P., Newsam A. M., 1995a, A&A, 293, L46
Simmons J. F. L., Newsam A. M., Willis J. P., 1995b, MNRAS, 276, 182
Simmons J. F. L., Bjorkman J. E., Ignace R., Coleman I. J., 2002, MNRAS, 336, 501
Valls-Gabaud D., 1998, MNRAS, 294, 747
Witt H. J., 1995, ApJ, 449, 42
Witt H. J., Mao S., 1994, ApJ, 430, 505

APPENDIX A: OBTAINING \( \beta \) FROM THE ASYMPTOTIC DECLINE OF THE STOKES Q FLUX

Consider a source that lies inside the caustic and is passing out of it. (The arguments that follow also hold true when the source first approaches the caustic.) After the photosphere of the star has completely passed out of the caustic, the polarization achieves a strong positive (by our convention) peak value. This occurs because only the scattering envelope lies interior to the caustic and so is subject to the selective amplification, whereas the photosphere is amplified by an approximately constant value. As the event evolves, more of the envelope transits the fold caustic and the polarized signal at a given moment is given primarily by the highest value of the polarized flux along the caustic itself. The scale of this polarization is set by the scattering optical depth of the envelope, but the slope of the polarized light curve is determined by the density distribution of scatterers. Here, we derive this relation between the lens position and the dependence of the polarization on \( \beta \).

As the star moves farther along from the caustic, the scattered light is accurately described by a point source. Equation (10) indicates that, far from the star, \( Q_0(p, \alpha) \approx Q_0(p) \left( R_h/p \right)^{\delta+1} \cos 2\alpha \). The
polarized flux will be given by
\[ F_Q = \frac{R^2}{D^2} \int_0^\infty dx \, A(x) \int_{-\infty}^\infty dy \, Q(p, \alpha) \]
\[ = Q_0 \frac{R^2}{D^2} \int_0^\infty dx \, \frac{b}{\sqrt{x}} \int_{-\infty}^\infty dy \, \frac{(x + x_l)^2 - y^2}{(x + x_l)^2 + y^2} \]
\[ \times \left[ \frac{R^2}{(x + x_l)^2 + y^2} \right]^{(\beta+1)/2}. \] (A1)

Factoring out \((x + x_l)\) and making a suitable change of variable, the integration in \(y\) can be evaluated numerically for any particular value of \(\beta\). When \(\beta\) is an integer, analytic integration formulae will apply. Because the result is not critical for our concerns, we simply denote the result of the \(y\) integral as \(Y(\beta)\), giving
\[ F_Q = Q_0 b R^2 \frac{h}{\sqrt{x_l}} Y(\beta) \int_0^\infty dx / \sqrt{x} \frac{(x + x_l)^{\beta+1}}{(x + x_l)^{2\beta}}. \] (A2)

Using a substitution with \(z = \sqrt{x/x_l}\), the integral will have a similar form to the one for \(y\), but with a different dependence on \(\beta\). We define the result of this integration to be \(X(\beta)\), leading to
\[ F_Q = Q_0 b R^2 \left( \frac{R_h}{x_l} \right)^{(2\beta-1)/2} \frac{h}{D^2} X(\beta) Y(\beta). \] (A3)

Observationally, the lens position is linear with time \(t\) and so
\[ \log F_Q = -\frac{2\beta+1}{2} \log t + \log t_0, \]
where the other factors have been collected into the variable \(t_0\). Of course, this is a power law and its slope is directly related to the value of \(\beta\) for the density distribution of scatterers in the envelope. For example, many of our models employ \(\beta = 2\), in which case the asymptotic polarized flux varies as \(F_Q \propto t^{3/2}\). It should be pointed out that this limiting behaviour will only be achieved in the tail of the polarized light curve, somewhat following the polarimetric peak.

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