Unusual temperature behavior of the entropy of the antiferromagnetic spin state in nuclear matter with an effective finite range interaction

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The unusual temperature behavior of the entropy of the antiferromagnetic (AFM) spin state in symmetric nuclear matter with the Gogny D1S interaction, being larger at low temperatures than the entropy of nonpolarized matter, is related to the dependence of the entropy on the effective masses of nucleons in a spin polarized state. The corresponding conditions for comparing the entropies of the AFM and nonpolarized states in terms of the effective masses are formulated, including the low and high temperature limits. It is shown that the unexpected temperature behavior of the entropy of the AFM spin state at low temperatures is caused by the violation of the corresponding low temperature criterion.

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The issue of spontaneous appearance of spin polarized states in nuclear matter is a topic of a great current interest due to its relevance in astrophysics. In particular, the scenarios of supernova explosion and cooling of neutron stars are essentially different, depending on whether nuclear matter is spin polarized or not. On the one hand, the models with the effective Skyrme and Gogny nucleon-nucleon (NN) interaction predict the occurrence of spin instability in nuclear matter at densities in the range from \( \tilde{\rho}_0 \) to \( 6\tilde{\rho}_0 \) for different parametrizations of the NN potential \([1,13] \) (\( \tilde{\rho}_0 = 0.16 \text{fm}^{-3} \) is the nuclear saturation density). On the other hand, for the models with the realistic NN interaction, the ferromagnetic phase transition seems to be suppressed up to densities well above \( \tilde{\rho}_0 \) \([16,22] \).

Here we continue the research of spin polarizability of nuclear matter with the use of an effective NN interaction. As was shown in Ref. \([13] \), in symmetric nuclear matter with the Gogny D1S effective interaction the antiferromagnetic (AFM) spin ordering with the oppositely directed neutron and proton spins sets in, beginning from a critical density (at zero temperature, \( \rho_c \approx 3.8\tilde{\rho}_0 \)). At finite temperature, the entropy of the AFM spin state demonstrates the unusual behavior being larger than the entropy of the nonpolarized state at low enough temperatures. The main goal of this work is to clarify this unexpected moment, utilizing the approximation of the effective mass in the single particle spectrum of nucleons. As will be shown later, this approximation is quite successful in reproducing the entropy of a spin polarized state for all relevant temperatures. The use of low and high temperature expressions for the entropy will allow us to get the corresponding conditions in terms of the effective masses for comparing the entropies of spin polarized and nonpolarized states.

Now we stop on the basic equations of the theory \([11,12] \). Given the possibility of phase transitions to the states with parallel and antiparallel ordering of neutron and proton spins, the distribution function \( f \) can be expanded in the Pauli matrices \( \sigma_i \) and \( \tau_k \) in spin and isospin spaces

\[
f(p) = f_{00}(p)\sigma_0\tau_0 + f_{30}(p)\sigma_3\tau_0 + f_{03}(p)\sigma_0\tau_3 + f_{33}(p)\sigma_3\tau_3.
\]

Expressions for the distribution functions \( f_{00}, f_{30}, f_{03}, f_{33} \) read \([11,12] \)

\[
f_{00} = \frac{1}{4}\{n(\omega_{n1}) + n(\omega_{p1}) + n(\omega_{n1} + n(\omega_{p1})\},
\]

\[
f_{30} = \frac{1}{4}\{n(\omega_{n1}) + n(\omega_{p1}) - n(\omega_{n1} - n(\omega_{p1})\},
\]

\[
f_{03} = \frac{1}{4}\{n(\omega_{n1}) - n(\omega_{p1}) + n(\omega_{n1} - n(\omega_{p1})\},
\]

\[
f_{33} = \frac{1}{4}\{n(\omega_{n1}) - n(\omega_{p1}) - n(\omega_{n1} + n(\omega_{p1})\}.
\]

Here \( n(\omega) = \{\exp(\omega/T) + 1\}^{-1} \) and

\[
\begin{align*}
\omega_{n1} &= \varepsilon_0 + \tilde{\varepsilon}_0 + \tilde{\varepsilon}_{30} + \tilde{\varepsilon}_{03} + \tilde{\varepsilon}_{33} - \mu_{n1}, \\
\omega_{p1} &= \varepsilon_0 + \tilde{\varepsilon}_0 + \tilde{\varepsilon}_{30} - \tilde{\varepsilon}_{03} - \tilde{\varepsilon}_{33} - \mu_{p1}, \\
\omega_{n1} &= \varepsilon_0 + \tilde{\varepsilon}_0 - \tilde{\varepsilon}_{30} + \tilde{\varepsilon}_{03} - \tilde{\varepsilon}_{33} - \mu_{n1}, \\
\omega_{p1} &= \varepsilon_0 + \tilde{\varepsilon}_0 - \tilde{\varepsilon}_{30} - \tilde{\varepsilon}_{03} + \tilde{\varepsilon}_{33} - \mu_{p1},
\end{align*}
\]

are the branches of the quasiparticle spectrum corresponding to neutrons and protons with spin up and spin down, and \( \mu_{\tau\sigma} \) are their respective chemical potentials \( (\tau = n, p; \sigma = \uparrow, \downarrow) \). Under derivation of Eqs. \((2), (3)\), it is assumed that the populations of neutrons and protons with spin up and spin down are held fixed. In Eq. \((3)\), \( \varepsilon_{00}(p) \) is the free single particle spectrum, and \( \tilde{\varepsilon}_{00}, \tilde{\varepsilon}_{30}, \tilde{\varepsilon}_{03}, \tilde{\varepsilon}_{33} \) are the Fermi liquid (FL) corrections to the free single particle spectrum, related to the FL am-
plitudes \( U_0(k), ..., U_3(k) \) by formulas

\[
\begin{align*}
\tilde{\varepsilon}_{00}(p) &= \frac{1}{2V} \sum_q U_0(k)f_{00}(q), \quad k = \frac{p - q}{2}, \\
\tilde{\varepsilon}_{30}(p) &= \frac{1}{2V} \sum_q U_1(k)f_{30}(q), \\
\tilde{\varepsilon}_{03}(p) &= \frac{1}{2V} \sum_q U_2(k)f_{03}(q), \\
\tilde{\varepsilon}_{33}(p) &= \frac{1}{2V} \sum_q U_3(k)f_{33}(q).
\end{align*}
\]

The distribution functions \( f_{00}, f_{03}, f_{30}, f_{33} \), in turn, should satisfy the normalization conditions for the total density \( \nu_n + \nu_p = \rho \), excess of neutrons over protons \( \nu_n - \nu_p = \alpha \rho \), ferromagnetic (FM) \( \nu_{\uparrow} - \nu_{\downarrow} = \Delta \nu_{\uparrow} \) and antiferromagnetic (AFM) \( (\nu_{\uparrow} + \nu_{\downarrow}) - (\nu_{\uparrow} + \nu_{\downarrow}) \parallel \Delta \nu_{\uparrow} \) spin order parameters, respectively (\( \tau \) being the isospin asymmetry parameter, \( \nu_{\uparrow} = \nu_{\uparrow} + \nu_{\downarrow} \) and \( \nu_{\downarrow} = \nu_{\uparrow} + \nu_{\downarrow} \) with \( \nu_{\uparrow}, \nu_{\downarrow} \) and \( \nu_{\uparrow}, \nu_{\downarrow} \) being the neutron and proton number densities with spin up and spin down, respectively). To check the thermodynamic stability of different solutions of the self-consistent equations (2)–(4), it is necessary to compare the corresponding free energies \( F = E - TS \), where \( E \) is the energy functional and the entropy reads

\[
S = -\sum_p \sum_{\tau,\mu} \sum_{\sigma = \uparrow, \downarrow} \{n(\omega_{\tau\sigma}) \ln n(\omega_{\tau\sigma}) \\
+ \bar{n}(\omega_{\tau\sigma}) \ln \bar{n}(\omega_{\tau\sigma}) \}, \quad n(\omega) = 1 - n(\omega).
\]

The single particle energies \( \omega \) have the following general structure

\[
\omega_{\tau\sigma}(k) = \omega_0(\sigma) + U_{\tau\sigma}(k), \quad \omega_0(\sigma) = \frac{h^2k^2}{2m_0} - \mu_{\tau\sigma},
\]

where \( m_0 \) is the bare nucleon mass, \( U_{\tau\sigma} \) is the single particle potential. Its momentum dependence can be characterized by the effective mass \( m_{\tau\sigma}(k) \), defined as

\[
\frac{m_0}{m_{\tau\sigma}(k)} = 1 + \frac{m_0}{\hbar^2} \frac{dU_{\tau\sigma}(k)}{dk}.
\]

If to use the quadratic approximation for the single particle potential

\[
U_{\tau\sigma}(k) \approx U_{\tau\sigma}(0) + \left( \frac{1}{2k} \frac{dU_{\tau\sigma}(k)}{dk} \right)_{k=k_F,\tau\sigma} \cdot k^2,
\]

where \( k_{F,\tau\sigma} \) is the Fermi momentum of nucleons in the state \( (\tau, \sigma) \), then the single particle energy can be represented in the form

\[
\omega_{\tau\sigma}(k) = \frac{h^2k^2}{2m_{\tau\sigma}} + U_{\tau\sigma}(0) - \mu_{\tau\sigma}, \quad m_{\tau\sigma} \equiv m_{\tau\sigma}(k_{F,\tau\sigma}).
\]

Within this approximation, all thermodynamic quantities can be easily calculated, analogously to the case of a free Fermi gas. Note that in order to get the effective mass \( m_{\tau\sigma} \), it is necessary to find the explicit single particle potential as a result of the solution of the self-consistent equations (2)–(4).

Further we will consider symmetric nuclear matter with the Gogny D1S interaction as a potential of NN interaction. It was shown in Ref. \[13\] that in this case the AFM spin ordering can be realized only among the states with the collinear spin ordering. In the AFM spin state, \( \nu_{\uparrow} = \nu_{\uparrow}, \nu_{\downarrow} = \nu_{\downarrow}, \) and, hence, \( \mu_{\uparrow} = \mu_{\uparrow, \downarrow} \). Besides, we have only two different branches in the quasiparticle spectrum, \( \omega_{\uparrow} = \omega_{\uparrow}, \omega_{\downarrow} = \omega_{\downarrow}, \) and, as a consequence, only two different effective massess, \( m_{\uparrow} = m_{\uparrow, \downarrow}, m_{\downarrow} = m_{\downarrow, \uparrow} \). The neutron \( \Pi_n = \frac{\omega_{\uparrow} - \omega_{\downarrow}}{\omega_{\uparrow}} \) and proton \( \Pi_p = \frac{\omega_{\uparrow, \downarrow} - \omega_{\downarrow}}{\omega_{\uparrow}} \) spin polarization parameters for the AFM spin state are opposite in sign and equal to

\[
\Pi_n = -\Pi_p = \frac{\Delta \nu_{\uparrow}}{\theta} \equiv \Pi.
\]

In Fig. \[1\] the difference between the entropies per nucleon of the AFM and nonpolarized states is shown as a function of temperature at different polarizations. The value \( \Pi = 1 \) corresponds to the totally AFM polarized nuclear matter. One can see that for low temperatures the entropy of the AFM state is larger than the entropy of the normal state. It looks like the AFM state at low finite temperatures is less ordered than the nonpolarized state. Under a further increase of temperature the difference between the entropies changes the sign and becomes negative, that corresponds to the intuitively expected behavior.

Fig. \[2\] shows the difference between the free energies per nucleon of the AFM and nonpolarized states as a function of temperature for the same polarizations as
in Fig. 1. In contrast to the difference between the entropies, the difference between the free energies preserves its negative sign for all temperatures below the corresponding critical temperature, and, hence, the AFM spin state is thermodynamically preferable as compared to the nonpolarized state for the whole corresponding temperature domain. Therefore, the unusual temperature behavior of the entropy of the AFM spin state doesn’t lead to the instability of the polarized state at low temperatures. Note that if to assume the quadratic approximation for the dependence of the free energy per nucleon of the polarized state on the spin polarization parameter \( \Pi \),

\[
\frac{F(\rho, T, \Pi)}{A} = \frac{F(\rho, T, \Pi = 0)}{A} + \gamma(\rho, T)\Pi^2,
\]

then the curve in Fig. 2 corresponding to \( \Pi = 1 \), in fact, shows the dependence of the spin-isospin symmetry parameter \( \gamma \) on temperature. Its negative value proves the stability of the AFM spin state at temperatures below the critical temperature, as clarified above.

To understand the unusual behavior of the entropy of the AFM spin state at low temperatures, we utilize the quadratic approximation for the single particle spectrum of nucleons, Eq. (8). Note that, adopting this approximation, we self-consistently determine also the chemical potentials to guarantee the fulfillment of the normalization conditions for the distribution functions. The results of the numerical determination of the entropy density, based on the exact and approximated forms of the single particle energies, are compared in Fig. 3. It is seen that the approximation turns out to be quite satisfactory, especially in the region of low temperatures, and the disagreement does not exceed 7% even for the less favorable case of totally polarized matter in the entire temperature domain under consideration. Therefore, we can provide the low temperature expansion for the entropy using the approximation of the effective mass in the single particle energies, as described in detail, e.g., in Ref. [23]. Then, requiring for the difference between the entropies per nucleon of the AFM and nonpolarized states to be negative, one can get the condition

\[
D_1 \equiv \frac{m_{\uparrow}}{m^*}(1 + \Pi) + \frac{m_{\downarrow}}{m^*}(1 - \Pi)^2 - 2 < 0. \tag{9}
\]

Here \( m^* \) is the effective mass of a nucleon in nonpolarized nuclear matter at the corresponding temperature and density. The low temperature condition (9) is valid until \( T/\varepsilon_{F_{n,\uparrow}} \ll 1 \), \( \varepsilon_{F_{n,\uparrow}} = \frac{\hbar^2 k_{F_{n,\uparrow}}^2}{2m_{\uparrow}} \) being the Fermi energy of neutrons with spin up \( (\sigma = \uparrow) \) and spin down \( (\sigma = \downarrow) \). The calculations show that at the given density \( (\rho = 0.64 \text{fm}^{-3}) \) and polarizations, the corresponding temperature interval extends approximately up to \( T = 10 \text{MeV} \). Besides, under derivation of the condition (9) it is assumed that the effective masses are temperature independent. Fig. 4 shows the dependence of the effective masses \( m_{\uparrow}, m_{\downarrow} \) on temperature in the temperature domain, where the low temperature expansion holds true. It is seen that the effective masses for partially AFM polarized nuclear matter at the given polarizations are practically independent on temperature, and for totally polarized matter the change in the effective mass is about 5% in this temperature domain. Therefore, the use of Ineq. (9) is quite justified for comparing the entropies of the AFM and nonpolarized states at low temperatures.

Fig. 5 shows the dependence of the l.h.s. \( D_1 \) of Ineq. (9) on temperature in the low temperature domain under

FIG. 2: Same as in Fig. 1 but for the free energy per nucleon, measured from its value in the nonpolarized state.

FIG. 3: (Color online) The density of entropy for the AFM spin state as a function of temperature, calculated with the exact (solid line) and approximated (dashed line) single particle energies at the spin polarization parameter \( \Pi = 0.6 \) (left) and \( \Pi = 1 \) (right).
 consideration at different polarizations. It is seen that the quantity $D_1$ is positive and increases with the spin polarization. This explains the unexpected behavior of the entropy of the AFM spin state, being larger than that of the nonpolarized state at low temperatures.

Note that at higher temperatures the entropy of the AFM spin state becomes smaller than the entropy of the nonpolarized state. To explain this, we can again use the approximation of the effective mass in the single particle energies for getting the high temperature expression for the entropy. If $\varrho_{n\sigma}\lambda_{n\sigma}^3 \ll 1$ ($\lambda_{n\sigma} = \sqrt{2\pi \hbar^2/m_{n\sigma}^*} = \sqrt{2\pi \hbar^2/m_{n\sigma}^*}$ is the thermal wavelength of neutrons with spin up and spin down), then the condition for the difference between the entropies per nucleon of the AFM and nonpolarized states to be negative is

$$D_2 \equiv \left( \frac{m_{n\sigma}^*}{1 + \Pi} \right)^{\frac{1+\Pi}{2}} \left( \frac{m_{n\sigma}^*}{1 - \Pi} \right)^{\frac{1-\Pi}{2}} \frac{1}{m_{n\sigma}^*} - 1 < 0.$$  \hspace{1cm} (10)

Fig. 4 shows the dependence of the quantity $D_2$ on temperature in the high temperature region at the given density and polarizations. One can see that the condition (10) is fulfilled, and, hence, the entropy of the AFM spin state turns out to be smaller than that of the nonpolarized state. Note that from the derivation procedure of the high temperature expression for the entropy it follows that the effective masses in Ineq. (10) can be temperature dependent.

Thus, using the approximation of the effective mass, it is possible to explain both the low temperature and high temperature peculiarities of the entropy of spin polarized nuclear matter with the D1S Gogny interaction. In Ref. [14], it was found that the entropy of spin polarized neutron matter with the Skyrme effective interaction above some critical density $\varrho_S$ is larger than that of nonpolarized matter. As a consequence, the critical density for the appearance of a spin polarized state decreases with temperature, contrary to the intuitive suggestion. However, there is an important difference between the cases with the finite range Gogny and zero range Skyrme forces: While for the Skyrme interaction the effective masses in Eq. (7) are momentum and temperature independent, for the Gogny interaction they do depend on momentum and temperature. By this reason, the difference between the entropies of polarized and nonpolarized states is positive for all temperatures at densities above $\varrho_S$ for the Skyrme interaction. For the Gogny interaction, this difference changes the sign from the positive one at low temperatures to the negative one at high temperatures. The critical density $\varrho_S$, above which the difference of the entropies becomes positive at low temperatures, depends on polarization, for example, $\varrho_S(\Pi = 0.3) \approx 0.07 \, \text{fm}^{-3}$, $\varrho_S(\Pi = 0.6) \approx 0.08 \, \text{fm}^{-3}$, and $\varrho_S(\Pi = 1) \approx 0.17 \, \text{fm}^{-3}$. For comparison, for totally polarized neutron matter with the SLy4 Skyrme interaction $\varrho_S \approx 0.15 \, \text{fm}^{-3}$ [15]. Since the entropy of the AFM spin state is smaller than that of nonpolarized matter at high temperatures, the critical density for the appearance of the AFM state increases with temperature [14], in agreement with the intuitive considerations and contrary to the scenario with the Skyrme interaction [15].

In summary, it has been shown that the entropy of the AFM spin state in symmetric nuclear matter with the Gogny D1S interaction demonstrates the unusual behavior, being larger at low temperatures than the entropy of nonpolarized matter. By comparing the free energies of polarized and nonpolarized states, it has been clarified that this unconventional temperature behavior...
doesn’t lead to the instability of the AFM state. This entropy peculiarity has been related to the dependence of the entropy on the effective masses of nucleons in a spin polarized state, which for the finite range D1S Gogny interaction do depend on temperature. The corresponding conditions for comparing the entropies of the AFM and nonpolarized states in terms of the effective masses have been formulated, including the low and high temperature limits. It has been shown that the unexpected temperature behavior of the entropy at low temperatures is caused by the violation of the corresponding low temperature criterion.

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