Ad Hoc Networking With Rate-Limited Infrastructure: Generalized Capacity Scaling

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Abstract—Capacity scaling of a large hybrid network with unit node density, consisting of wireless ad hoc nodes, base stations (BSs) equipped with multiple antennas, and one remote central processor (RCP), is analyzed when wired backhaul links between the BSs and the RCP are rate-limited. We first derive the minimum backhaul link rate required to achieve the same capacity scaling law as in the infinite-capacity backhaul link case. Assuming an arbitrary rate scaling of each backhaul link, a generalized achievable throughput scaling law is then analyzed in the network based on using one of pure multihop, hierarchical cooperation, and two infrastructure-supported routing protocols, and moreover, information-theoretic operating regimes are identified. In addition, to verify the order optimality of our achievability result, a generalized cut-set upper bound under the network model is derived by cutting not only the wireless connections but also the wired connections.

I. INTRODUCTION

Gupta and Kumar’s pioneering work [1] introduced and characterized the sum throughput scaling law in a large wireless ad hoc network. For the network having \( n \) nodes randomly distributed in a unit area, it was shown in [1] that the total throughput scales as \( \Theta(\sqrt{n} \log n) \) by using the nearest-neighbor multihop (MH) routing strategy.\(^1\) In [2], [3], MH schemes were further developed and analyzed in the network. Together with the studies on MH, it was shown that a hierarchical cooperation (HC) strategy [4] achieves an almost linear throughput scaling, i.e., \( \Theta(n^{1-\epsilon}) \) for an arbitrarily small \( \epsilon > 0 \), in the dense network of unit area.

Since long delay and high cost of channel estimation are needed in ad hoc networks with only wireless connectivity, hybrid networks consisting of both wireless ad hoc nodes and infrastructure nodes, or equivalently base stations (BSs), have been introduced and analyzed in [5], [6]. In a hybrid network where each BS is equipped with a large number of antennas, the optimal capacity scaling was characterized by introducing two new routing protocols, i.e., infrastructure-supported single-hop (ISH) and infrastructure-supported multihop (IMH) protocols [6]. In hybrid networks with ideal infrastructure [5], [6], BSs have been assumed to be fully interconnected by infinite-capacity wired links. In practice, it is rather meaningful to consider a cost-effective finite-rate backhaul link. In order to deal with this issue, the throughput scaling was studied in [7] for a simplified hybrid network, where BSs are connected only to their neighboring BSs via a finite-rate link and the form of achievable schemes is limited only to MH routings. In [8], a hybrid network where BSs are directly interconnected was studied in fundamentally analyzing how much rate per BS-to-BS link is required to guarantee the optimal capacity scaling achieved for the infinite-capacity backhaul link case. More practically, packets arrived at a certain BS in a radio access network are delivered to other BSs through a core network. The cellular network operating based on a remote central processor (RCP) to which all BSs are connected is well suited to this realistic scenario [9].

In this paper, we introduce a more general hybrid network with unit node density, consisting of \( n \) wireless ad hoc nodes, multiple BSs, and one RCP, in which wired backhaul links between the BSs and the RCP are rate-limited. Specifically, we take into account a general scenario where three scaling parameters of importance including i) the number of BSs, ii) the number of antennas at each BS, and iii) each backhaul link rate can scale at arbitrary rates relative to \( n \). We first derive the minimum rate of a BS-to-RCP link required to achieve the same capacity scaling law as in the case of infinite-capacity infrastructure. Assuming an arbitrary rate scaling of each backhaul link, we then analyze a new achievable throughput scaling law. Moreover, we identify three-dimensional information-theoretic operating regimes according to the aforementioned three scaling parameters. Besides the fact that extended networks are power-limited, we are interested in further finding the case where our network is in the infrastructure-limited regime; that is, the performance is limited by the backhaul link rate. In addition, a generalized upper bound on the capacity scaling is derived for our hybrid network with finite-capacity infrastructure based on the cut-set theorem. It is shown that our upper bound matches the achievable throughput scaling for all the operating regimes. We refer to the full paper [10] for the detailed description and all the proofs.

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\(^1\)We use the following notation: i) \( f(x) = O(g(x)) \) means that there exist constants \( C \) and \( c \) such that \( f(x) \leq C g(x) \) for all \( x > c \). ii) \( f(x) = o(g(x)) \) means that \( \lim_{x \to \infty} \frac{f(x)}{g(x)} = 0 \). iii) \( f(x) = \Omega(g(x)) \) if \( g(x) = O(f(x)) \). iv) \( f(x) = \omega(g(x)) \) if \( g(x) = o(f(x)) \), and v) \( f(x) = \Theta(g(x)) \) if \( f(x) = O(g(x)) \) and \( g(x) = O(f(x)) \).
II. SYSTEM AND CHANNEL MODELS

In an extended network, \( n \) nodes are uniformly and independently distributed on a square of area \( n \), except for the area where BSs are placed. We randomly pick source-destination pairings. Assume that the BSs are neither sources nor destinations. The network is divided into \( m \) square cells of equal area, where a BS with \( l \) antennas is located at the center of each cell. The total number of BS antennas in the network is assumed to scale at most linearly with \( n \), i.e., \( ml = O(n) \). For analytical convenience, the parameters \( n, m, \) and \( l \) are related according to \( n = m^{1/\beta} = l^{1/\gamma} \), where \( \beta, \gamma \in [0, 1) \) with a constraint \( \beta + \gamma \leq 1 \).

It is assumed that all the BSs are fully interconnected by wired links through one RCP. For simplicity, the RCP is assumed to be located at the center of the network. In practice, it is natural for each BS-to-RCP (or RCP-to-BS) link to have a finite capacity that may limit the transmission rate of infrastructure-supported routing protocols. In this paper, we assume that each BS is connected to one RCP through an errorless wired link with finite rate \( R_{BS} = n^\eta \) for \( \eta \in (-\infty, \infty) \).

The uplink channel vector between node \( i \) and BS \( b \) is denoted by \( \mathbf{h}_{bi}^{(u)} = [r_{bi,1}^{(u)}, r_{bi,2}^{(u)}, \ldots, r_{bi,n/m}^{(u)}]^T \), where \( r_{bi,t}^{(u)} \) is the distance between node \( i \) and the \( t \)th antenna of BS \( b \), \( \theta_{bi,t}^{(u)} \) is the random phase uniformly distributed over \([0, 2\pi)\), and \( \alpha > 2 \) denotes the path-loss exponent. In a similar manner, the downlink channel vector and the channel between two nodes can be modeled. For the balance between uplink and downlink, it is assumed that each BS satisfies an average transmit power constraint \( nP/m \), while each node satisfies an average transmit power constraint \( P \). Then, the total transmit power of \( m \) BSs is the same as the total transmit power consumed by \( n \) wireless nodes. By following the same antenna configuration as that of \([6]\), the antennas of a BS are placed as follows:

1) If \( l = w(\sqrt{n/m}) \) and \( l = O(n/m) \), then \( \sqrt{n/m} \) antennas are regularly placed on the BS boundary and the remaining antennas are uniformly placed inside the boundary.

2) If \( l = O(\sqrt{n/m}) \), then \( l \) antennas are regularly placed on the BS boundary.\(^2\)

The aggregate throughput \( T_n \) of the network is defined as \( T_n = nT_s \), where \( T_s \) is the average transmission rate of each source, and its scaling exponent \( e \) is given by \( \lim_{n \rightarrow \infty} \frac{\log T_n}{\log n} \).

III. ROUTING PROTOCOLS WITH AND WITHOUT INFRASTRUCTURE SUPPORT

In this section, the routing protocols supported by BSs having multiple antennas in \([6]\) are illustrated in our network.

A. Routing Protocols With Infrastructure Support

In infrastructure-supported routing protocols, the packet of a source is delivered to the corresponding destination of the source using three stages: access routing, backhaul transmission, and exit routing.

1) ISH Protocol: There are \( \Theta(n/m) \) nodes with high probability in each cell \([4, \text{Lemma 4.1}]\). For the access routing, all source nodes in each cell transmit their packets simultaneously to the home-cell BS via single-hop multiple-access. The packets of source nodes in one cell are then jointly decoded at the BS, assuming that the signals transmitted from the other cells are treated as noise. In the next stage, the decoded packets are transmitted from the BS to the RCP via BS-to-RCP link and then delivered to the corresponding BSs via RCP-to-BS links. For the exit routing, each BS in each cell transmits \( n/m \) packets received from the RCP, via single-hop broadcast to all the wireless nodes in its cell. The ISH protocol is illustrated in Fig. 1.

2) IMH Protocol: Since the extended network is fundamentally power-limited, the ISH protocol may not be effective especially when the node–BS distance is quite long, which motivates us to introduce the IMH protocol illustrated in Fig. 2. Each cell is further divided into smaller square cells of area \( 2 \log n \), termed routing cells. Since \( \min\{l, \sqrt{n/m}\} \) antennas are regularly placed on the BS boundary, \( \min\{l, \sqrt{n/m}\} \) MH paths can be created in each cell. For the access routing, each antenna placed only on the BS boundary can receive its packet transmitted from one of the nodes in the nearest-neighbor routing cell. The BS-to-RCP and RCP-to-BS transmission is the same as the ISH protocol case. For the exit routing, each antenna on the BS boundary transmits the packet to one of the nodes in the nearest-neighbor routing cell.

B. Routing Protocols Without Infrastructure Support

The protocols based only on infrastructure support may not be sufficient to achieve the optimal capacity scaling especially for small \( m \) and \( l \). Using one of the MH transmission \([1]\) and the HC strategy \([4]\) may be beneficial in terms of improving the achievable throughput scaling.

\(^2\)Such an antenna deployment guarantees the nearest-neighbor transmission from/to each BS antenna, and thus enables the IMH protocol to work well.
Fig. 3. The operating regimes on the achievable throughput scaling with respect to $\beta$ and $\gamma$ for $\eta \to \infty$.

C. The Transmission Rates of Routing Protocols

As addressed earlier, the RCP is incorporated into the hybrid network model using BSs. We remark that the transmission rates in both access and exit routings are irrelevant to the rate of backhaul links and thus are essentially the same as the infinite-capacity backhaul link case [6]. The transmission rates of the ISH and IMH protocols at each cell are given by $\Omega\left(l (m/n)^{\alpha/2-1}\right)$ and $\Omega\left(\min\{l, (n/m)^{1/2-\epsilon}\}\right)$, respectively, for an arbitrarily small $\epsilon > 0$.

The total throughput scaling laws achieved by the MH and HC protocols are given by $\Omega\left(n^{1/2-\epsilon}\right)$ [1] and $\Omega\left(n^{2-\alpha/2-\epsilon}\right)$ [4], respectively.

IV. ACHIEVABILITY RESULT

We first introduce two-dimensional operating regimes with respect to scaling parameters $\beta$ and $\gamma$ for the infinite-capacity backhaul link scenario. We then derive the minimum rate of each backhaul link, required to achieve the optimal capacity scaling. Assuming that the rate of each backhaul link scales at an arbitrary rate relative to $n$, we characterize a new achievable throughput scaling. Furthermore, we scrutinize our achievability result according to the three-dimensional operating regimes identified by introducing a new scaling parameter $\eta$.

A. Two-Dimensional Operating Regimes With Infinite-Capacity Infrastructure

As illustrated in Fig. 3, when $\eta \to \infty$, the two-dimensional operating regimes with respect to $\beta$ and $\gamma$ are divided into four sub-regimes. The best scheme and its corresponding throughput scaling exponent $\epsilon$ in each regime are summarized in Table I. The four sub-regimes are described as follows.

- In Regime A, the infrastructure is not helpful to improve the capacity scaling since $\beta$ and $\gamma$ are too small.
- In Regime B, the HC and IMH protocols are used to achieve the optimal capacity scaling. As $\alpha$ increases, the IMH outperforms the HC.
- In Regime C, using the HC and IMH protocols guarantees the order optimality as in Regime B.
- In Regime D, the HC protocol has the highest throughput when $\alpha$ is small, but as $\alpha$ increases, the best scheme becomes the ISH protocol. Finally, the IMH protocol becomes dominant when $\alpha$ is very large.

### Table I

| Regime | Condition | Scheme | $\epsilon$ |
|--------|-----------|--------|------------|
| A      | $2 < \alpha < 3$ | HC | $\frac{1}{2}$ |
| B      | $2 < \alpha < 4 - 2\beta - 2\gamma$ | HC | $\frac{1}{2}$ |
| C      | $2 < \alpha < 3 - \beta$ | HC | $\frac{1}{2}$ |
| D      | $2 < \alpha < 1 + \alpha$ | HC | $\frac{1}{2}$ |

B. The Minimum Required Rate of Backhaul Links

In order to give a cost-effective backhaul solution for a large-scale network, we derive the minimum required rate $C_{BS}$ of each link between a BS and the RCP.

**Theorem 1:** The minimum rate of each backhaul link required to achieve the optimal capacity scaling of hybrid networks with infinite-capacity infrastructure is given by

$$C_{BS} = \begin{cases} 0 & \text{for Regime A} \\ \Omega\left(l\right) & \text{for Regime B} \\ \Omega\left(n^{-\alpha/2-\epsilon}\right) & \text{for Regime C} \\ \Omega\left(l\left(\frac{n}{m}\right)^{\log_m\left(n/l\right)\alpha/2-1}\right) & \text{for Regime D} \end{cases}$$

(1)

for an arbitrarily small constant $\epsilon > 0$.

**Remark 1:** From Theorem 1, it is shown that $C_{BS} = O(n^\gamma)$ if $\gamma = \epsilon$ in Regimes B and D or if $\beta = 1 - \epsilon$ in Regimes C and D. This result reveals that for the case where the number of antennas at each BS is very small or the number of BSs is almost the same as the number of nodes, surprisingly, the backhaul link rate $R_{BS}$ does not need to be infinitely high.

C. Generalized Achievable Throughput Scaling With Finite-Capacity Infrastructure

If $R_{BS}$ is smaller than $C_{BS}$ in Theorem 1, then the throughput $T_n$ will be decreased accordingly depending on the operating regimes for which the infrastructure-supported routing protocols are used. In this subsection, $T_n$ is derived with an arbitrary rate scaling of $R_{BS}$.

**Theorem 2:** In the hybrid network with the backhaul link rate $R_{BS}$, the aggregate throughput $T_n$ scales as

$$\Omega\left(\max\left\{\min\left\{\max\left\{ml, m\left(\frac{n}{m}\right)^{1/2-\epsilon}\right\}, mR_{BS}\right\}, n^{1/2-\epsilon}, n^{2-\alpha/2-\epsilon}\right\}\right),$$

(2)

where $\epsilon > 0$ is an arbitrarily small constant.

**Remark 2 (Infrastructure-limited regimes):** Let us introduce the infrastructure-limited regime where the performance is limited by the backhaul link rate $R_{BS}$. Two new operating regimes B and D causing an infrastructure limitation for some
The two-dimensional operating regimes specified by $\beta$ and $\gamma$ can be extended to three-dimensional operating regimes by introducing $\eta$. We identify the three-dimensional operating regimes by illustrating five types of two-dimensional regimes, showing different characteristics, with respect to $\beta$ and $\gamma$ according to the value of $\eta$.

**Remark 3 (Three-dimensional operating regimes):** The operating regimes with respect to $\beta$ and $\gamma$ are plotted in Figs. 4–7 for $\eta = -1/2$, $-1/2 \leq \eta < 0$, $0 \leq \eta < 1/2$, and $1/2 \leq \eta < 1$, respectively. Let us scrutinize each case.

- $\eta < -1/2$: As shown in Fig. 4, the entire regimes are included in Regime A. This indicates that the infrastructure does not improve the capacity scaling if $R_{\text{BS}} = o(1/\sqrt{n})$.
- $-1/2 \leq \eta < 0$: As $\eta$ becomes greater than $-1/2$, the infrastructure can improve the capacity scaling for some cases, but the network is limited by the backhaul transmission, thereby resulting in Regime B (see Fig. 5).
- $0 \leq \eta < 1/2$: In Regimes B and D, the network using either the ISH or IMH protocol is still limited by the backhaul transmission (specifically when the IMH in Regime B or the ISH in Regime D is used). We refer to Fig. 6.
- $1/2 \leq \eta < 1$: As $\eta$ further increases beyond $1/2$, Regime B disappears and the area of Regime D gets reduced (see Fig. 7).
- $\eta \geq 1$: As long as $R_{\text{BS}} = \Omega(n)$, the network has no infrastructure limitation at all. The associated operating regimes are shown in Fig. 3.

### V. Cut-Set Upper Bound

A generalized cut-set upper bound on the aggregate capacity scaling based on the information-theoretic approach is derived for the hybrid network with rate-limited BS-to-RCP (or RCP-to-BS) links. As illustrated in Fig. 8, to provide a tight upper bound, two cuts $L_1$ and $L_2$ are taken into account. Similarly as in [6], the cut $L_1$ divides the network area into two halves by cutting the wireless connections between wireless source

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**TABLE II**

| Regime | Condition | Scheme | $\epsilon$ |
|--------|-----------|--------|------------|
| $\tilde{B}$ | $\alpha \geq 4 - 2\beta - 2\eta$ | HC | $2 - \frac{\alpha}{\gamma}$ |
| $\tilde{D}$ | $\alpha \geq 4 - 2\beta - 2\eta$ | IMH | $\beta + \eta$ |

Fig. 4. The operating regime with respect to $\beta$, $\gamma$, and $\eta$, where $\eta < -1/2$.

Fig. 5. The operating regimes with respect to $\beta$, $\gamma$, and $\eta$, where $-1/2 \leq \eta < 0$.

Fig. 6. The operating regimes with respect to $\beta$, $\gamma$, and $\eta$, where $0 \leq \eta < 1/2$.

\[ R_{\text{BS}} = \Omega(n) \]
Fig. 7. The operating regimes with respect to $\beta$, $\gamma$, and $\eta$, where $\frac{1}{2} \leq \eta < 1$.

Fig. 8. The cuts $L_1$ and $L_2$ in the network. The BS-to-RCP or RCP-to-BS links are not shown in (a) since they are not in effect under $L_1$.

nodes on the left of the network and the other nodes, including all BS antennas and one RCP. In addition, to fully utilize the main characteristics of the network with finite-capacity infrastructure, we consider another cut $L_2$, which divides the network area into another two halves by cutting the wired connections between BSs and the RCP as well as the wireless connections between all nodes (including BS antennas) located on the left of the network and all nodes (including BS antennas and the RCP) on the right.

Upper bounds obtained under the cuts $L_1$ and $L_2$ are denoted by $T_n^{(1)}$ and $T_n^{(2)}$, respectively. By the cut-set theorem, the total capacity is upper-bounded by $T_n \leq \min \left\{ T_n^{(1)}, T_n^{(2)} \right\}$.

Under $L_1$, the total throughput $T_n^{(1)}$ for sources on the left half is bounded by the capacity of the multiple-input multiple-output channel between the sets of sources and destinations.

**Lemma 1:** Under the cut $L_1$ in Fig. 8(a), an upper bound on the aggregate capacity, $T_n^{(1)}$, of the hybrid network with rate-limited infrastructure is given by

$$T_n^{(1)} = O \left( n^\epsilon \max \left\{ ml \left( \frac{m}{n} \right)^{\alpha/2-1}, m \min \left\{ l, \sqrt{\frac{m}{n}} \right\} \right\} \right),$$

where $\epsilon > 0$ is an arbitrarily small constant.

The upper bound $T_n^{(1)}$ matches the achievable throughput scaling within a factor of $n^\epsilon$ in the network with infinite-capacity infrastructure ($\eta \to \infty$). Now, let us turn to the cut $L_2$ in Fig. 8(b), which deals with information flows over the wired connections as well as the wireless connections. Under $L_2$, an upper bound on $T_n^{(2)}$ is shown in the following lemma.

**Lemma 2:** Under the cut $L_2$ in Fig. 8(b), an upper bound on the aggregate capacity, $T_n^{(2)}$, of the hybrid network with rate-limited infrastructure is given by

$$T_n^{(2)} = O \left( \max \left\{ mR_{BS}, n^{1/2+\epsilon} m^{\alpha-2/\alpha+\epsilon} \right\} \right),$$

where $\epsilon > 0$ is an arbitrarily small constant.

In consequence, an upper bound on the aggregate capacity is established based on using the min-cut of the network, and is presented in the following theorem.

**Theorem 3:** In the hybrid network with the backhaul link rate $R_{BS}$, the aggregate throughput $T_n$ is upper-bounded by

$$O \left( \max \left\{ mR_{BS}, n^{1/2+\epsilon} m^{\alpha-2/\alpha+\epsilon} \right\} \right),$$

where $\epsilon > 0$ is an arbitrarily small constant.

**Remark 4:** The upper bound in (5) matches the achievable throughput scaling in Theorem 2 within $n^\epsilon$ in the hybrid extended network with the finite backhaul link rate $R_{BS}$. In other words, choosing the best of the four achievable schemes ISH, IMH, MH, and HC is order-optimal for all the operating regimes (even if the rate of each backhaul link is finite).

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