An Optimization Approach to the Low-Frequency Entire Train Formation at the Loading Area

Boliang Lin *, Fan Yang, Shuting Zuo, Chang Liu, Yinan Zhao and Mu Yang

School of Traffic and Transportation, Beijing Jiaotong University, Beijing 100044, China; 18120930@bjtu.edu.cn (F.Y.); 18120977@bjtu.edu.cn (S.Z.); 18120832@bjtu.edu.cn (C.L.); 17114216@bjtu.edu.cn (Y.Z.); 18140578@bjtu.edu.cn (M.Y.)

* Correspondence: bllin@bjtu.edu.cn; Tel.: +86-10-51682598

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Abstract: It is well known that the shift of transporting bulk cargo from roads to railways is an important measure to reduce carbon emissions of the overall transportation systems. In order to increase the attractiveness of railway transport, companies usually provide some discounts to the customers with great transport demand so that entire trains can be operated. Since the operation of entire trains can reduce the reclassification times of shipments, the expenses of railway operations can be reduced. However, when the volume of shipment is not sufficient, the door-to-door direct transportation (in the railway industry specifically, “door-to-door” means running trains from supplier’s warehouse to customer’s warehouse) of the entire train often leads to a decrease in the frequency of delivery, which increases the average stock of users, thus increasing the inventory cost of users. Therefore, how to balance the pros and cons of the two is exactly the problem to be studied. In this paper, the optimal operation plan is obtained by minimizing the total cost of the stockholding of suppliers and customers, as well as the transportation costs of an entire train and non-direct train. Based on the classic economic order quantity (EOQ) model, a 0-1 integer programming model with the constraint of the maximum stock level is proposed to solve this problem. And an innovative approach is used to calculate the actual average stock of the customer. Finally, the model is validated and its effectiveness is confirmed using a real-world case, which is carried out using data from the China rail system.

Keywords: railway transportation; inventory cost; entire train; 0-1 integer programming

1. Introduction

The transportation system is one of the main sources of carbon emissions. Therefore, the development of low-carbon transportation is of great significance to promote energy conservation and emission reduction. As a critical component of the transportation system, railway transportation has the advantages of a large capacity and low emissions in comparison with road transportation, and the commodities carried by rail wagon are mainly bulk goods, such as coal, oil, coke, iron ore, etc. In contrast to the high electrification rate of railways, the road transportation has a single energy consumption structure: the burning of oil. Despite this fact, road transport still occupies the leading position in China freight transportation. In 2017, the railway freight turnover accounted for 13.7% of the total, while the road freight turnover accounted for 33.4% of the total in China [1]. Hence, shifting bulk cargo from road to rail is an important measure to optimize the transportation structure.

In order to accelerate the shifting process of freight transportation from road to railway, the optimization of the railway transport operation plan is particularly important. Using the entire train can reduce the services of reclassification, raise the transporting efficiency, and reduce transportation costs, while not all shipments are suitable for being delivered by loading an entire train. If the traffic
volume of the shipment is large, the railroads can provide entire trains between the loading and the unloading station to achieve “door-to-door” transportation. In the railway industry specifically, “door-to-door” means running trains from the supplier’s warehouse to the customer’s warehouse; otherwise, it will first be delivered to the adjacent yard on its itinerary by local train, and then, after a series of classification stations, it can reach the terminal.

The mode proposed in this paper for low-frequency entire trains (entire trains with long departure intervals) is meant to lessen the operation cost. Since it can effectively lessen the operation cost, the railway companies can provide a discount to attract users. Generally speaking, the in-transit time of the entire train can be shortened as the train does not need to be reclassified. However, this mode requires large storage capacities of warehouses and high loading and unloading efficiencies. The long-cycle departure interval and the high-volume delivery will result in a significant increase in average stock and then lead to additional inventory cost for customers. How to balance the pros and cons of operating low-frequency entire trains constitutes the prime problem to be studied in this paper.

The paper is organized as follows: In Section 2, we first review the existing literature on the optimization of traffic organization and the logistics costs combined with transportation. Section 3 describes the problems of rail train formation corresponding with the inventory cost. Section 4 presents a 0-1 integer model to solve the problem proposed in Section 3. A real-world case is described in Section 5 to illustrate the effectiveness of our innovative model. Section 6 concludes the study and provides suggestions for future research.

2. Literature Review

Freight transportation in railway system can be classified into two modes. Firstly, some shipments with large volume will be directly delivered to the unloading area by forming a “door-to-door” entire train. Secondly, other shipments will be delivered to the destination through a shipment-block sequence, i.e., the shipments need to be reclassified on their itineraries. It is a complicated optimization decision problem to determine which mode should be employed, since it is related to the train formation at the loading areas and the structure of the block network. Related research works are as follows. Assad [2] proposed a mixed integer programming (MIP) formulation for routing and makeup in order to determine between which terminals the direct train service should be placed. In their works, technology requirements of motive power and traction as well as resource allocation were taken into account. A discussion on European (EU) rail freight transport and current single wagonload (SWL) trends was presented by Marinov et al. [3], with the aim of gaining an understanding of how SWL services, policy, and practice can benefit from the implementation of scientific methods and information technologies. Assad [4] studied the optimal classification strategy for a line network of yards targeted at minimizing reclassification cost. Ahujia et al. [5] gave an overview of railroad blocking problems, attempted to develop cutting-edge algorithms, and employed the algorithm to railroad planning and scheduling problems. Keaton [6] studied comprehensive optimal strategies of train connections, frequencies, and blocking and routing plans for freight cars in single-carload general commodity services with an all-integer linear programming model. A dual adjustment procedure which makes it possible to efficiently obtain close-to-optimal solutions to problems of a realistic size, was used by implementing Lagrangian relaxation. Martinelli and Teng [7] formulated a 0-1 programming model for a train formation plan to minimize in-transit time of cars. The formation plan of a freight train connecting service (TCS) for shipments which is not enough to be shipped by entire trains formed in the loading area, is the foundation of the operation plan. Lin et al. [8] presented a formulation and solution for the TCSs problem aimed at determining which pairs of yards are to be provided with a direct train service and which cars are consolidated into a given train service. The objective was to minimize the sum of operating costs, accumulation delay, classification delay, and assembly delay while satisfying technological constraints on train and yard capacity. Lin et al. [9] considered the total consumption of loading conditions, traffic organization, and unloading conditions of the unloading area and described the various combination schemes of the initial traffic flow. A nonlinear 0-1 planning
model was constructed, and the model was solved by a simulated annealing algorithm. Zhao and Lin [10] took the car hour cost at the loading station of stepped direct trains and non-direct trains, as well as the time delay caused by loading sequence as the optimization target, and took the train length and reclassification station capacity as constraints. In order to improve the railway rail freight services, many European companies and research institutes in the field of railway transport have focused on research and development projects. Viable wagonload production schemes (ViWaS) and the Shift2Rail Joint Undertaking (S2R JU) project have played a great role in promoting the progress of railway freight transportation.

Over the past years, many scholars have paid attention to add transportation costs into inventory management, which is a systematic optimization of transportation and the related enterprise inventory from the perspective of logistics, but little research on the influence of railway operation (for example, whether to drive a direct train) on the inventory cost and system cost has been done. Harris [11] first proposed the importance of integrating transportation and inventory systems research, proposed a model with constant demand speed for delivering single-issue single-receiving points at a continuous rate minimizing inventory and total transportation costs, and gave a classic basic-economy economic order quantity (EOQ) model. The study carried out by Swenseth and Godfrey [12] identified transportation cost functions that simulated reality and demonstrated that straightforward freight rate functions can be incorporated into inventory replenishment decisions without compromising the accuracy of the decision. Hill and Omar [13] showed that increasing the batch size by a fixed factor instead of sending batches with same size was the optimal plan in a single supplier–single buyer model. Glock and Christoph [14] studied the flow of materials between two vendors and a buyer and developed six alternative delivery structures with the intention of minimizing total system costs. The results showed that the total costs of the system can be reduced by permitting different lot sizes, shipment frequencies, and production intervals. Baller et al. [15] incorporated transportation costs approximated by classical schemes into a dynamic-demand joint replenishment problem (DJRP) and assumed that the supplier paid a fixed fee for replenishing a customer. They analyzed the data from test instances and concluded that cooperation between the two can lead to greater cost savings. Ji et al. [16] analyzed the problem using a batching and scheduling model involving both batch supply and batch delivery. In this paper, they considered four non-deterministic polynomial (NP)-hard cases that were classified based on whether the arrival and delivery of the goods were individual or not. The minimized sum of total weighted inventory cost and transport cost was calculated by the fully polynomial-time approximation scheme (FPTAS) solution. However, these articles were studied from the perspective of logistics; hence, they only considered the transportation process as a whole but not from the internal perspective of transportation.

Anily and Federgruen [17] assumed that each retailer absorbed products at a constant rate, and studied deterministic demand-time continuous inventory and vehicle routing optimization issues in the case of a single product. Bertazzi and Speranza [18] dealt with the problem of minimizing the sum of the inventory and transportation costs in the multi-products logistic network with one origin, some intermediate nodes, and one destination when a set of possible shipping frequencies was given. The inventory cost was computed through the aggregation of the inventory over time or over nodes. Chen and Sarker [19] studied a buyer who received a product from multiple vendors and assumed that the products from the vendors in a milk-run were collected by a single truck. A multi-vendor optimal model was developed here for deciding the batch size of the vendor’s production and the delivery frequencies of different vendors to the manufacturer. A freight cost function shows that the freight rate is critical for the total cost of the system. Larger delivery quantities and fewer deliveries are suitable for higher freight rates.

Burns et al. [20] studied the problem of joint inventory and transportation minimum cost for a single-point/multi-point logistics network. The model was characterized by only one product and subject to deterministic requirements. Chandra and Fisher [21] studied the joint transportation and inventory models of single-product, one-to-many fleets from a single-origin transport product to
multiple demand points. The author examined the advantages of inventory control and transport operations, and through empirical analysis, the results showed the cooperation between the two can lead to greater cost savings. The above scholars built inventory models based on the minimizing of transportation cost and inventory cost. However, these models are limited to road transportation, which is quite different from railway transportation. Guglielminetti et al. [22] pointed out one of the main barriers hampering SWL development is the price-competitive position of road transport. It is rare to put the problem of railway direct transportation into the logistics system and study the problem from the perspective of system optimization. Guglielminetti et al. [23], starting from a European context, carried out a very exhaustive statistical analysis of the cost from the aspect of railway wagon load system. The authors mentioned the costs for SWL services are about twice the costs for full train load (FTL), and the exact cost is given. However, they only considered the costs within the transportation system. Ji et al. [24] built a non-linear 0-1 integer programming model of car flow organization in a loading area based on logistics cost, in which the changes of logistics costs of both sides caused by different car flow organizations was taken into account. They used a real-world network with one loading station and multiple unloading stations to illustrate the car flow organization plan.

3. Problem Description

3.1. The Classic EOQ Model

As early as 1913, Harris [10] first proposed the importance of integrating transportation and inventory systems research, and discussed models with constant demand rates. He gave the classic economic order quantity (EOQ) model to minimize inventory cost and transportation cost. The total cost in this model includes ordering cost, procurement cost, stockholding cost, transport cost and in-transit inventory cost. In order to obtain the economic order quantity, the total cost is minimized and a derivative on the equation with respect to the order quantity is performed. Then let the derivative be zero and get the optimal value of order quantity. In addition, the model can also obtain the best order cycle.

3.2. The Entire Train and Non-Direct Train

Generally speaking, the basic rail train formation patterns can be classified into two categories. The first one is that if the volume of the shipment is large (for example, more than 25 cars per day, i.e., about 500 thousand tons per year), entire trains can be operated between the loading station and the unloading station to achieve “door-to-door” transportation. There is no need to be reclassified at every intermediate yard they pass through. Otherwise, if the volume of individual commodity is not sufficient for an entire train, it is firstly delivered to the closest yard on its itinerary by local trains, and then it is grouped together with other commodities [7], which is called a non-direct train. Therefore, an entire train can generally obtain the operating time savings in three reclassification stations compared with the non-direct trains. Besides, the entire train at the same time can save the relevant expenses at the marshalling station. For railway companies, the savings in operating expenses mean an increase in economic benefits.

If the shipment volume is not sufficient, railway companies can operate daily non-direct trains, which will spend a lot of time and money on the classification, or they can operate entire trains at a lower frequency and take a part of the cost saved by using entire trains as a discount to attract cargo owners. However, the departure interval and the arrival volume will also affect the decision of the cargo owners to decide whether to choose the low-frequency entire trains.

3.3. One Supplier to One Customer

We list three types of supplier and customer networks. In the first type, the transportation system only has one supplier and one customer. The average daily demand is \(d\) and the volume of the entire train is \(Q_{\text{Train}}\). Assume that the production time of the supplier and the consumption time of customer
are continuous, and the loading and unloading time is very short. The actual stock level and the average stock level of supplier and customer in different rail train formation patterns are given in Figure 1. The actual stock levels of the supplier are shown in Figure 1a, c. The actual stock levels of customer are shown in Figure 1b, d. It can be seen that when the daily non-direct train is operated, the average stock levels of supplier and customer are both \( \frac{1}{2}d \). When an entire train is operated, the average stock levels are both \( \frac{1}{2}Q_{\text{Train}} \).

Figure 1. The stock level in two rail train formation patterns.

Since the stock-time curve consists of a series of right triangles, as shown in Figure 2, the integral calculation process can be simplified to calculate the area of the triangles. In the case of the full cycle, the average stock is half of a shipment. Let \( T_{\text{delivery}} \) be the delivery cycle of dispatching an entire train. As shown by Figure 2, the average stock level of the entire train scenario is \( \frac{1}{2}Q_{\text{Train}} \), and that of the non-direct train scenario is \( \frac{1}{2}d \). \( T_{\text{delivery}} \) is equal to \( \frac{D}{T} \). The smaller the daily consumption, the larger the delivery cycle, thus, the larger the difference between the average stock cost of the two modes of transportation. For example, if the average daily consumption is 240 t and the volume of an entire train is 3600 t, \( T_{\text{delivery}} \) is 15 days, which makes the average stock level of the non-direct train scenario become 120 t, while for entire train it is 1800 t, which is 15 times the amount of the non-direct train. It can be seen from the Figure 2 that when a single supplier supplies a single customer separately, the stockholding cost of operating a low-frequency entire train is much higher than operating non-direct trains delivered every day.

The cost of this network consists of the stockholding cost of the supplier and the customer, transportation cost, and in-transit inventory cost, while each of them is different in diverse operation patterns. For the supplier and the customer, if the entire train is operated, they can get a discount provided by the railway companies.
3.4. Multiple Suppliers to One Customer

This paper focuses on the second type of supplier and customer network, as shown in Figure 3, which is multiple suppliers serving one single customer. Assuming that multiple suppliers supply commodities to a customer simultaneously, the daily consumption of the customer is the quantity of the daily shipments and also the sum of the daily output of each supplier. Ideally, if each supplier delivers all commodities it produced to a customer on a daily basis, both the suppliers and the customer will only have to burden the inventory cost caused by the daily average stock. However, this will increase the transportation cost. If each supplier delivers shipment by entire trains, it will extend the delivery cycle, increase the stockholding cost, but the corresponding transportation cost can be reduced. Suppliers with fewer shipments, which are the research objects in this paper, can consider operating low-frequency entire trains.

The stock level of the customer is shown in Figure 4 when three suppliers supply at the same time. When the time coordinate is zero, the stockholding level does not mean the original stock level of the sub-warehouse, but it means the volume of the first shipments from the suppliers which arrive at the same time. Each supplier delivers shipments according to different delivery cycles, and the average stock of the customer depends on the delivery cycle and shipments volume of the supplier.
The average stock of customer is the integral of the customer’s consumption curve within one order cycle divided by the order cycle length. We assume that all the suppliers arrive simultaneously at the first time. So according to the integral principle, the integral of the customer’s consumption curve is the sum of the corresponding integral of the consumption curve of each supplier. To make it easier to understand, we assume that the customer’s warehouse is virtually divided into three sub-warehouses $S_1$, $S_2$, and $S_3$, and each house stores the goods from the corresponding supplier 1, 2, and 3. The daily consumption of $S_1$ is the daily output of supplier 1, so do the other two sub-warehouses. Thus, the total daily consumption of the customer is the sum of the daily consumption of each sub-warehouse, the total quantities of goods consumed by the customer in one order cycle are the sum consumption of all sub-warehouses.

\[ \text{Figure 4. Customer’s stock quantities in the case of three suppliers.} \]

The cost composition of this network is the same as that in the 1-1 mode. The difference is that the calculation of the average inventory of customers in this network is more complicated, and the specific calculation will be explained in Section 4. Note that normally the stock level of each sub-warehouse is not zero at the end of the order cycle. Therefore, incomplete delivery cycle stock needs to be calculated separately.

3.5. Multiple Suppliers to Multiple Customers

In this type, the transportation system has multiple suppliers and multiple customers, as shown in Figure 5. In China, the main shipping object of railway transportation is bulk cargo with a large volume. A supplier with such goods often supplies multiple customers, and a customer also has multiple suppliers. The multiple suppliers and customers can form a network. However, there are several combinations of suppliers’ and customers’ stock levels in this case. The complexity of these combinations is not conducive to this research, thereby this type of network is not considered in this paper.
When selecting which kind of transportation to deliver, the customer not only takes the transportation cost into consideration, but also considers inventory cost. The cost of the inventory has a certain relationship with the reasonable frequency of delivery within one order cycle. The intention of this paper is to reasonably arrange a railway transport operation plan within one order cycle and increase the competitiveness of the railway companies under the relevant constraints.

4. Mathematical Models

4.1. Notations

Set:
- $S$: The set of all suppliers in a railway network.

Parameters:
- $T_{\text{order}}$: The order cycle length for the customer (days)
- $T_{\text{delivery}}$: The delivery cycle length for the supplier $i$ (days)
- $Q_{\text{Train}}$: The volume of an entire train (tons)
- $V_{\text{max.stock}}$: The capacity of customer stock level (tons)
- $D_i$: The order quantity of the customer from the supplier $i$ (tons). Let $d_i$ be the quantity of daily production, which is equal to the customer’s consumption of the goods from the supplier $i$, $d_i = \frac{D_i}{365}$ (tons per day)
- $h_i^{\text{Stock}}$: The stockholding cost per ton for the supplier $i$ (yuan per ton)
- $p_i$: The unit purchasing price for the supplier $i$ (yuan)
- $b_i$: The unit transportation cost from the supplier $i$ to the customer (yuan per ton)
- $w_i^{\text{Stock}}$: The customer’s stockholding cost per ton (yuan per ton)
- $t_i^e$: The transportation time of an entire train from the supplier $i$ (days)
- $t_i^m$: The transportation time of a non-direct train from the supplier $i$ (days)
- $\gamma$: The interest rate per year
- $\beta$: The discount rate for dispatching an entire train.

Decision variables:
- $y_i = \begin{cases} 
1 & \text{If the shipment from supplier } i \text{ to customer is delivered by entire train, } \\
0 & \text{Otherwise } 
\end{cases}$ for $i \in S$.

4.2. Formulations

The goal of the classic EOQ model is to minimize the sum of ordering cost and stockholding cost from the customer’s perspective. The model proposed in this paper takes the inventory costs of both suppliers and customers into account, in which operating a low-frequency entire train in a loading area where the shipment is insufficient for entire train is considered. Meanwhile, a discount is given by the railway companies. Unlike replenishment when customer stock levels are reduced to a minimum in the classic model, the replenishment in this model depends on the supplier’s fixed delivery cycle.
In order to calculate the actual average stock of the customer, an innovative method of calculating average stock is proposed. The following five parts are considered in this model.

4.2.1. The Stockholding Cost of Suppliers

For the supplier $i$, if low-frequency entire trains are operated to deliver goods, the average stock will be $\frac{1}{2}Q_{\text{Train}}$. If daily non-direct trains are operated, it will be $\frac{1}{2}Q_{\text{Train}}$. The unit stockholding cost is related to unit stock cost, unit selling price $c_i$, and annual interest rate $\gamma$. Thus, the total stockholding cost of all suppliers is calculated as shown in Equation (1).

$$Z_{\text{Supplier}}(y) = \sum_{i \in S} \frac{1}{2} h_i \text{Stock} [Q_{\text{Train}} y_i + d_i (1 - y_i)] \quad (1)$$

4.2.2. The Stockholding Cost of a Customer

For the customer, based on the algorithm of the customer average stock presented in the problem description, the stock cost of a customer can be concluded by Equation (2). Equation (3) gives a method to calculate the consumption of a customer containing an incomplete period, which is obtained by subtracting the area of the shadow from the area of the rectangle (Figure 6). Note that the unit stock cost is related to the unit purchasing (selling) price and transport cost.

$$Z_{\text{Customer}}(y) = u^{\text{stock}} \sum_{i \in S} [A_{\text{Customer}} y_i + \frac{1}{2} d_i (1 - y_i)] \quad (2)$$

$$A_{\text{Customer}} = \frac{Q_{\text{Train}} T_{\text{order}} - \frac{1}{2} Q_{\text{Train}} T_{\text{delivery}} \left[ \frac{T_{\text{order}}}{T_{\text{delivery}}} \right] - (T_{\text{order}} - T_{\text{delivery}}) \left[ \frac{T_{\text{order}}}{T_{\text{delivery}}} \right]^2 d_i}{T_{\text{order}}} \quad (3)$$

![Figure 6. The consumption curve containing an incomplete period.](image)

4.2.3. Transport Cost

The transport cost from supplier $i$ to a customer is equal to $b_i D_i$, so the total transport cost is given by Equation (4).

$$Z^D = \sum_{i \in S} b_i D_i \quad (4)$$
4.2.4. The Discount of Dispatching an Entire Train

If the supplier \( i \) operates low-frequency entire trains to deliver goods, the railway companies will offer a discount, which is equal to the transport cost times the discount rate. Therefore, the total discount is given by Equation (5).

\[
Z_D(y) = \sum_{i \in S} b_i D_i \beta y_i
\]

(5)

4.2.5. The In-Transit Inventory Cost

The in-transit inventory cost from supplier \( i \) to customer is the product of the total demand of goods, time in transit, and daily interest rate, where in-transit time of dispatching an entire train \( t^e_i \) is the transportation distance divided by the operating rate. The in-transit time of dispatching a non-direct train \( t^{m^i} \) is the sum of \( t^e_i \), the pick-up time by a local train, the waiting time for the accumulation progress of goods and the reclassification time. Thereby, the total in-transit inventory cost is given by Equation (6).

\[
Z_{in}(y) = \sum_{i \in S} \frac{y}{365} p_i D_i [t^e_i y_i + t^{m^i}(1 - y_i)]
\]

(6)

The objective of Function (7) is to minimize the total inventory cost which is the sum of stockholding costs of supplier \( Z_{Supplier} \), the stockholding costs of customer \( Z_{Customer} \), transportation costs \( Z^T \), the discount of dispatching entire train \( Z_D \), and in-transit inventory cost \( Z_{in} \).

\[
\min Z(y) = Z_{Supplier}(y) + Z_{Customer}(y) + Z^T - Z_D(y) + Z_{in}(y)
\]

(7)

\[
s.t. \sum_{i \in S} Q_{Train} y_i + \frac{D_i}{T} (1 - y_i) \leq V_{max,stock}
\]

(8)

\[
y_i \in \{0, 1\}, i \in S
\]

(9)

Equation (8) ensures that the inventory level is within the maximum limit.

This model is a linear 0-1 integer programming model, which can be solved by using the optimization software.

5. Numerical Example

5.1. Background

To illustrate the effectiveness of our innovative model proposed in Section 4, we implement our model on a real freight rail network in China. Considering the overall transportation trend of China’s bulk cargo is from west to east and from north to south, we take \( S_1, S_2, \) and \( S_3 \) with relatively low coke production as suppliers in north-west China and one customer \( C \) in south-central China as an example. The loading stations of three suppliers are respectively the Fengxiang (FX), Xiayukou (XYK), and Meijiaping (MJP) stations of China Railway Xi’an Group Co., Ltd., and the unloading station of the customer is Lengshuijiang East (LSJE) station of China Railway Guangzhou Group Co., Ltd. The location of the suppliers and customer on the map is shown in Figure 7. There were two primary problems that needed to be solved: whether an entire train should be provided or not and which suppliers should operate the entire train.
Figure 7. The locations and the route.

5.2. Data

5.2.1. Demand Analysis

To illustrate the feasibility of selected suppliers and customers, each supplier’s annual shipment to LSJE is shown in Table 1.

Table 1. Annual shipment data for suppliers and customer.

| Loading Station | Unloading Station | Demand/Year (Thousand Tons) | Category | The Length (km) |
|-----------------|-------------------|------------------------------|----------|-----------------|
| Baicun          | LSJE              | 1.41                         | coal     | 1480            |
| FX              | LSJE              | 9.18                         | coal     | 1469            |
| Huangling       | LSJE              | 27.62                        | coal     | 1464            |
| MJP             | LSJE              | 13.68                        | coal     | 1363            |
| XYK             | LSJE              | 5.61                         | coal     | 1487            |

The definitions of abbreviations are given in the Section 5.1.

Note that there are two shipment flows, i.e., Baicun to LSJE and Huangling to LSJE, that are removed from the numerical case. Because the shipment is too small to be operated on low-frequency entire trains for Baicun to LSJE, and the shipment is too large to operate entire trains for Huangling to LSJE.

None of the stations FX, MJP, and XYK are sufficient for entire train deliveries every few (less than three) days. Therefore, there are two operation plans for each shipment flow, i.e., operating low-frequency entire trains and a daily non-direct train. Under this situation, there are eight different combinations of customer’s stock that can result in different costs.

5.2.2. Transportation Path

The paths of the loading stations to the unloading station are shown in Figure 7. Here we show the passing stations from the suppliers to the customer.

- \( S_1 \rightarrow \) FX \( \rightarrow \) Xinfengzhen \( \rightarrow \) Shangnan \( \rightarrow \) Nanyang \( \rightarrow \) Goulin \( \rightarrow \) Xiangyang North \( \rightarrow \) Jinmen \( \rightarrow \) Yaqueling \( \rightarrow \) Xizhai \( \rightarrow \) Shimenxia North \( \rightarrow \) Yiyang East \( \rightarrow \) LSJE \( \rightarrow \) C
- \( S_2 \rightarrow \) MJP \( \rightarrow \) Xinfengzhen \( \rightarrow \) Shangnan \( \rightarrow \) Nanyang \( \rightarrow \) Goulin \( \rightarrow \) Xiangyang North \( \rightarrow \) Jinmen \( \rightarrow \) Yaqueling \( \rightarrow \) Xizhai \( \rightarrow \) Shimenxia North \( \rightarrow \) Yiyang East \( \rightarrow \) LSJE \( \rightarrow \) C
The first column represents the three suppliers in the case. \( t^s_i \) and \( t^m_i \) represent the transportation time of two delivery modes from three suppliers to the customer, respectively. \( p_i \), \( b_i \), and \( h_i^{Stock} \) represent the unit selling price, the unit transportation cost, and the unit stockholding cost of three suppliers, respectively. \( T_i^{delivery} \) represents the delivery cycle from suppliers to the customer.

### 5.3. Results and Analysis

This model is solved by the commercial software Gurobi 8.0.1. (Gurobi optimization, Beaverton, Oregon, USA) on a 3.10 GHz Intel (R) Core (TM) i5-7276U central processing unit (CPU) with 4.0 GB of random-access memory (RAM). The original value is \( Y = (0,0,0) \), that is, in the original operation plan, all three stations are dispatched by daily non-direct train. The stockholding level of this plan is...
406.83 t, and the total cost is ¥15,941,924.27. The best objective value of this model is ¥15,337,549.67, where \( Y = (1, 0, 1) \), which means supplier 1 and supplier 3 dispatch the low-frequency entire trains, and supplier 2 dispatches the daily non-direct trains. Therefore, the optimal operation plan in this network is that the low-frequency entire trains are operated from FX and MJP station and the daily non-direct trains are operated from XYK station. The actual stock level of the customer is shown in Figure 9, and according to the Equation (3), the stockholding level of the customer is 3746.83 tons. It can be seen that the delivery cycle length of the suppliers 1, 2, and 3 are 9 days, 1 day, and 14 days, respectively. The customer’s stock level reached a maximum of 7200 tons on the 45th day and a minimum of 800 tons on the 70th day. The customer’s stock is neither out of stock nor exceeds the maximum. This mode optimizes the total cost of \( \frac{15941924.27 - 15337549.67}{15941924.27} = 3.79\% \) in one quarter compared to organizing the daily non-direct train of all suppliers.

6. Conclusions

Based on the railway bulk cargo, this paper considers the possibility of operating low-frequency entire trains in areas that are insufficient for dispatching an entire train. In this paper, the optimization of the total cost for the supplier and customer is formulated as a linear 0-1 integer programming model, with the prime task of minimizing the total inventory costs, which include the stockholding costs of supplier, the stockholding costs of customer, transportation costs, and in-transit inventory costs, while satisfying the constraint of the maximum stock level of the customer. Different from the classic EOQ model, replenishment is made when the customer stock level is reduced to a minimum level. The model presented in this paper considers the situation that the supplier delivers the goods according to the delivery cycle. Using the principle of integration, a simplified calculation of the average inventory of customers was done. A real-world case was carried out including three suppliers.
and one customer. According to the results, two suppliers should operate a low-frequency entire train. The results indicate that the mathematical model proposed can be used to solve the real-world inventory cost problem. In future research, the classification time of the train in classification yards should be discussed in detail, which means the benefits of the railway companies should be included to make the discount rate more practical, and the willingness of the customer under the situation of road competition should be considered as well.

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