Electrical and electromagnetic co-simulations of the HERA Phase I receiver system including the effects of mutual coupling, and impact on the EoR window

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ABSTRACT
The detection of the Epoch of Reionization (EoR) delay power spectrum using a "foreground avoidance method" highly depends on the instrument chromaticity. The systematic effects induced by the radio-telescope spread the foreground signal in the delay domain, which contaminates the EoR window theoretically observable. Therefore, it is essential to understand and limit these chromatic effects. This paper describes a method to simulate the frequency and time responses of an antenna, by simultaneously taking into account the analogue RF receiver, the transmission cable, and the mutual coupling caused by adjacent antennas. Applied to the Hydrogen Epoch of Reionization Array (HERA), this study reveals the presence of significant reflections at high delays caused by the 150-m cable which links the antenna to the back-end. Besides, it shows that waves can propagate from one dish to another one through large sections of the array because of mutual coupling. In this more realistic approach, the simulated system time response is attenuated by a factor 10^4 after a characteristic delay which depends on the size of the array and on the antenna position. Ultimately, the system response is attenuated by a factor 10^5 after 1400 ns because of the reflections in the cable, which corresponds to characterizable k∥-modes above 0.7 hMpc^{-1} at 150 MHz. Thus, this new study shows that the detection of the EoR signal with HERA Phase I will be more challenging than expected. On the other hand, it improves our understanding of the telescope, which is essential to mitigate the instrument chromaticity.

Key words: telescopes – instrumentation: interferometers – techniques: interferometric – methods: numerical – dark Ages, Reionization, first stars

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**1 INTRODUCTION**

The Hydrogen Epoch of Reionization Array (HERA) is a new radio-telescope under development, and dedicated to the study of the early universe. Its main goal is to characterize the period during which the first quasars, galaxies and stars started to form, by measuring the evolution of the distribution of the neutral hydrogen present in the intergalactic medium (IGM). Before the formation of these first structures, our universe was dark and mainly composed of neutral hydrogen, element that has the property to emit photons at 21.1 cm (1420.4 MHz). In the ground state, this emission occurs when the spin configuration of the nucleus and electron flips, passing from parallel to anti-parallel, in order to reach a slightly lower and so more stable energy state. During the Dark Ages of our universe, until about 300 million years after the Big Bang, the background emission was dominated by the Cosmic Microwave Background and the 21-cm signal from this hyperfine transition. Following the formation of the first stars and galaxies, the IGM started to be heated and ionized by the X-rays and UV radiations emitted. Thus, due to the decrease in the quantity of neutral hydrogen, the 21-cm signal progressively disappeared. The study of the spectral and spatial fluctuations of this signal during the Cosmic Dawn and EoR is the key to better understand the formation, distribution, and evolution of the first structures of our universe (McQuinn et al. 2007; Morales & Wyithe 2010; Pritchard & Loeb 2012; Furlanetto 2016).

The phase I of HERA has been designed to detect and characterize the power delay spectrum of the redshifted hydrogen signal between 100 and 200 MHz, in order to set some stringent constraints concerning the main parameters used to model the formation and evolution of the first galaxies (Pober et al. 2014; DeBoer et al. 2017). This is equivalent to probing a period between 325 million and 915 million years after the Big Bang, or a redshift \(z\) between 6.1 and 13.2. This radio-interferometer is being built in the Karoo desert in South Africa in an extremely radio quiet zone in order to avoid interference from telecommunication signals. Its final configuration will comprise 350 antennas, 320 forming a very dense hexagonal core with short baselines, plus 30 outriggers in order to improve the resolution (Dillon & Parsons 2016). This represents a massive collecting area of 35000 m², taking into account the aperture efficiency of the antenna which is about 65%. The antenna and configuration of HERA have been designed with the specific objective to isolate and characterize certain portions of the EoR power delay spectrum, and achieve the required sensitivity for a robust detection. Successor of the Precision Array for Probing the Epoch of Reionization (PAPER) (Parsons et al. 2010; Ali et al. 2015; Pober et al. 2015), HERA Phase I consists of a 14-m diameter parabola and re-uses the dipole feed, RF receiver, and correlator developed for this instrument. Note that this paper only focuses on the characterization of the performance of the phase I system, which has been producing data since 2015. As for the phase II, the system is being redesigned in order to mitigate the level of chromaticity, improve its sensitivity, and extend the bandwidth from 50 to 250 MHz, which corresponds to a period between 115 million and 1.3 billion years after the Big Bang. Thus, the phase II system will be able to study the Reionization period as well as the Cosmic Dawn. This new system is currently being deployed and tested in the Karoo desert. The dipoles are being replaced by new Vivaldi feeds (Fagnoni et al. 2019 – in preparation), the RF chains by new analogue receivers with radio-over-fibre links, and the correlator upgraded. In particular, thanks to these modifications, HERA may be able to correlate the power spectra that it will measure with the absorption profile observed by the Experiment to Detect the Global Epoch of Reionization Signature (EDGES) in the 21-cm global signal (Bowman et al. 2018). Centred on 78 MHz, the sky-averaged brightness temperature of the 21-cm signal measured by EDGES presents a U-shaped absorption profile with an amplitude of about -0.5 K and a full-width at half-maximum of 19 MHz, which challenges the standard astrophysical models describing the state of the early universe and the Reionization (Witte et al. 2018).

The telescope has a static beam pointing towards the zenith and collects the signal from the drifting sky during night-time for months, in order to achieve a statistical detection of the EoR signal. At a given redshift, the Reionization is expected to be an isotropic phenomenon at sufficiently large spatial scale. The signal received by the antennas is naturally the combination of the EoR signal with the foreground signal, coming from our galaxy mainly in the form of diffuse synchrotron emission, free-free emission, and from

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1 [https://reionization.org/](https://reionization.org/)
extra-galactic compact radio-sources. However, following the absorption profile previously mentioned, the order of magnitude of the global 21-cm brightness temperature during the EoR is expected to be around 10 mK (Bowman et al. 2009). In comparison, the total brightness temperature of a “cold” patch of sky smoothly decreases from about 1000 to 100 K between 100 and 200 MHz, which makes the detection very challenging.

Therefore, it is essential to get rid of the contribution from the foreground signal, either by modelling and subtracting it from the received signal, or by analysing specific portions of the EoR delay power spectrum which are not affected by the foreground (Chapman et al. 2016). The first method, difficult to achieve in practice, requires a very accurate knowledge of the sky along with the spatial and frequency response of the instrument, in order to be able to calibrate (Ewall-Wice et al. 2017) the system with the expected accuracy; for example, it is the method followed by the LOW Frequency ARray (LOFAR) (Harker et al. 2010; Chapman et al. 2012; Van Haarlem et al. 2013), or by the Murchison Widefield Array (MWA) (Tingay et al. 2013; Jacobs et al. 2016). The second option which has been preferred for HERA is more straightforward, but also more conservative and limited, since the portions of the EoR power spectrum significantly contaminated by the foreground are in principle not used to characterize the EoR signal. This foreground avoidance method developed in Parsons et al. (2012a,b) and in Liu et al. (2014) is based on the fact that the foreground is relatively smooth in the plane of the sky as well as in the frequency domain, and approximately follows a power law. On the other hand, the EoR signal varies relatively quickly in the uv-plane and along the line of sight. By translating this property into the delay domain using Fourier transform, one obtains a very compact foreground delay power spectrum, while the EoR delay power spectrum spreads. Therefore, this difference in the time delay domain can be used to separate the EoR signal from the foreground.

The effectiveness of this method highly depends on the frequency smoothness of the observed sky, and most importantly of the instrument response as detailed in Thyagarajan et al. (2016). As we will see, the received time signal is obtained by convolving the incident signal with the antenna response. Therefore, additional reflections between the different elements of the antenna, for instance between the feed and the dish, within the transmission cables, or between adjacent antennas, duplicate and distort the received signal. As a consequence, the separation between the foreground and EoR delay power spectra occurs at higher delay, which may jeopardize the characterization of the Reionization. Thus, in order to succeed in this approach and maximize the collect of data, it is crucial to limit the level of chromaticity as much as possible.

In Ewall-Wice et al. (2016) and Patra et al. (2018), the antenna response of the instrument and its impact on the EoR detection were analysed in detail for a single antenna terminated by a fixed impedance of 125 Ω using computer simulations, as well as reflectometry measurements with a vector network analyzer (VNA). The purpose of this paper is to complete these works, by presenting a method to properly simulate all the main sources of chromaticity which can affect an antenna in a large array and in reception. In the framework of the foreground avoidance method, this allows us to better understand and estimate the effects of the instrument on the EoR delay power spectrum detection. In particular, two fundamental components are added. Firstly, an accurate model of the analogue receiver, including the effects of the transmission cables, is developed and combined with the electromagnetic model of the antenna. The analogue chain has its own response which contributes to the chromaticity of the system. Secondly, the effects of mutual coupling occurring between adjacent antennas are also included. These external reflections can play a significant role in the system response. Besides, since all the antennas do not share the same electromagnetic environment, for example the antennas at the edges and the central antennas, their response differ. However, HERA relies on baseline redundancy to calibrate the system (Liu et al. 2010; DeBoer et al. 2017; Dillon & Parsons 2016; Dillon et al. 2018).

This analysis is mainly focused on the characterization of the direction-dependent voltage antenna response which links the output voltage of the system to the incident electromagnetic wave. Section 2 of this paper summarizes how the output voltage from two antennas forming a baseline can be used to calculate the delay power spectrum. We also explain how the contamination from the foreground can be avoided by analysing the delay power spectrum at high delays, and to what extent the chromatic effects caused by the instrument can affect the detection of the EoR signal. Section 3 presents a comprehensive electromagnetic and electrical co-simulation of the HERA dish along with the receiver, performed with the software CST. The propagation of the signal from the antenna up to the output of the receiver is studied in the frequency domain, in terms of reflected and transferred voltage signal as well as power. Moreover, this simulation is used to estimate the receiver temperature and the system sensitivity, when the analogue chain is connected to the frequency-dependent antenna impedance. In order to validate the consistency of these simulations, the results are compared with reflectometry and noise measurements. Thanks to the 3D farfield beams generated with the electromagnetic simulation and the scattering or S-parameters of the analogue chain, in Section 4 we demonstrate how to compute the end-to-end system response in the frequency and time domain, when the antenna is excited by a plane wave linearly polarized coming from any direction. In particular, the chromatic effects caused by the receiver and the mutual coupling in a large array are analysed. Lastly, we discuss the implications of these new results on the ability to detect the EoR delay power.

2 DETECTION OF THE EOR DELAY POWER SPECTRUM IN THE “FOREGROUND AVOIDANCE METHOD”

2.1 Antenna output voltage and delay power spectrum

HERA is a radio-interferometer which is designed to perform a per-baseline analysis of the EoR signal; each pair
of antennas is independent and samples the delay power spectrum associated with a specific spatial scale and for a given observation frequency band or redshift. In this section, we explain how this spectrum can be derived from the output voltages measured by a baseline.

Let \( \mathbf{e}_1(s, t) \) the electric field vector received by the first antenna of the baseline at the instant \( t \), and coming from the direction defined by the vector \( s \) perpendicular to the wavefront of the signal. We assume that the signal received from the sky is a plane wave. Moreover, the array being very compact, we can also assume that the electric field \( \mathbf{e}_2(s, t) \) coming from the same direction and received by the second antenna has the same intensity but is delayed by \( \tau_{12} \). In particular, this implies that the disturbance from the ionosphere above the array is similar for each antenna. Besides, the delay between the two antennas is purely geometric, for instance there is no additional delay line, and so it only depends on the baseline length, its orientation \( b \), the direction of arrival of the signal \( s \), and the speed of light \( c \). Therefore, \( \mathbf{e}_2(s, t) = \mathbf{e}_1(s, t - \tau_{12}) \) with:

\[
\tau_{12}(b, s) = \frac{b \cdot s}{c}.
\] (1)

The electric field received is then converted into a voltage signal by the antenna, before propagating through the RF receiver. Assuming that the properties of the antenna and the receiver do not vary during the observation time, the system can be considered as "linear and time invariant" (LTI). The generated output voltage in the time domain is obtained by convolving the input electric field \( \mathbf{e}_1(s, t) \) with the impulse voltage time response of the system \( H_1(s, t) \). More generally, the total output voltage \( V_{\text{out}}(t) \) is the sum of the electric field contributions coming from all directions and received by the beam of the antenna \( i \). Thus, the total voltages of the antennas 1 and 2 at the output of the receivers are given by the following integrals:

\[
v_{\text{out}1}(t) = \int_{4\pi} \int \left[ h_1(s, x) \cdot e_1(s, t - x) \right] d\Omega,
\]
\[
v_{\text{out}2}(t) = \int_{4\pi} \int \left[ h_2(s, x) \cdot e_1(s, t - \tau_{12} - x) \right] d\Omega,
\] (2)

with \( d\Omega \) the solid angle element used for the integration over a sphere. Note that we focus our analysis on the characterization of the received radio signal and the associated correlated voltage. Therefore, additional random noise voltage, generated for instance by the RF analogue receiver, is ignored.

Typically, since HERA is a radio-interferometer, the time voltage signals of a baseline are recorded, cross-correlated, and averaged together by the correlator, in order to compute the visibility linking the measured voltages, the sky brightness and the antenna responses in the frequency domain (Thompson et al. 2017, Ch. 2-3). In order to do this, it is easier and more computationally efficient to perform the calculations in the frequency domain. Indeed, the visibility is the Fourier transform of the cross-correlated voltage, which is equal to the product of the complex conjugate of the Fourier transform of the first time signal with the Fourier transform of the second time signal. According to the convolution theorem, Equations (2) can be transposed into the frequency domain, and the output voltage \( V_{\text{out}}(s, f) \) for a given direction is equal to the scalar product between the frequency input signal \( E_1(s, f) \) and the frequency voltage response or complex transfer function \( H_1(s, f) \) of the antenna.

\[
v_{\text{out}1}(f) = \int_{4\pi} E_1(s, f) \cdot H_1(s, f) \ d\Omega,
\]
\[
v_{\text{out}2}(f) = \int_{4\pi} E_1(s, f) \cdot H_2(s, f) e^{-2\pi f \tau_{12}} \ d\Omega.
\] (3)

We also assume that the signals coming from different radio noise sources are not spatially coherent, and therefore that the cross-correlation is null for signals coming from different directions. Moreover, in order to take into account the effects of the limited bandwidth of the correlator, the frequency cross-correlated signal is multiplied by a window function \( W(f) \). From the averaged product between \( V_{\text{out}1}(f) \) and \( V_{\text{out}2}(f) \), one obtains the visibility \( V_{12}(f) \):

\[
v_{12}(f) = \int_{4\pi} I(s, f) A(s, f) W(f) e^{-2\pi f \tau_{12}} \ d\Omega \ d f.
\] (4)

The delay spectrum \( \tilde{V}_{12}(\tau) \) associated with a baseline is then obtained by taking the inverse Fourier transform of the visibility along the frequency axis. The goal of this transformation is to separate the EoR signal from the foreground signal in the delay domain, thanks to their spectral differences.

\[
\tilde{V}_{12}(\tau) = \int \int_{4\pi} I(s, f) A(s, f) W(f) e^{2\pi i f (\tau - \tau_{12})} \ d\Omega \ d f.
\] (5)

Lastly, from this transformed measurement of the sky brightness, Parsons et al. (2012a,b) detail how to obtain the cosmological delay power spectrum associated with the redshifted 21-cm hydrogen signal. Averaged in a cylindrical volume, the delay power spectrum contains fundamental information about the spatial and time evolution of the distribution of the neutral hydrogen signal during the EoR. It is expressed in \( \Omega^2 (h^{-1} \text{Mpc})^3 \), i.e. a squared temperature times a cosmic volume, and is equal to (Thyagarajan et al. 2015a):

\[
P_d(k_\perp, k_\parallel) = \left| \tilde{V}_{12}(\tau) \right|^2 \left( \frac{\Omega}{2\pi} \right)^2 \left( \frac{D^2 \Delta D}{\Delta B} \right) \left( \frac{1}{\Omega \Delta B} \right).
\] (6)

with \( \lambda \) the wavelength associated with the centre of the observation frequency band, \( \Delta B \) its bandwidth, \( k_B \) the Boltzmann constant, \( D \) the comoving distance perpendicular to the line-of-sight, \( \Delta D \) the comoving depth along the line-of-sight associated with \( \Delta B \), and \( \Omega \Delta B \) a normalization factor equal to:

\[
\Omega \Delta B = \int_{\Delta B} \int_{4\pi} \left| A(s, f) W(f) \right|^2 d\Omega df.
\] (7)

The delay power spectrum is usually expressed as a function
of the wave vector $k$ split into two components $k_{\perp}$ and $k_{||}$:

$$k_{\perp} = \frac{2\pi b}{Dz}, \quad k_{||} = \frac{2\pi f_{21} H_0(z) \tau}{c (1+z)^2} = 2\pi \frac{\Delta B}{\Delta D} \quad (8)$$

with $f_{21}$ the emission frequency of the spin-flip transition of the neutral hydrogen equal to 1420.4 MHz, and $H_0(z)$ the Hubble parameter expressed as a function of the redshift $z$. $k_{\perp}$ corresponds to the radial or "perpendicular" component of the wave vector. It is related to the diameter of the observation cylinder, in other words the spatial scale of the observed region in the plane of sky, and is limited by the field of view of the antenna and the length of the baselines. As for $k_{||}$, it is associated with the "depth" of this cosmological volume along the line-of-sight, and is also a function of the signal delay and the bandwidth. Indeed, the frequency of the redshifted signal is directly related to its epoch of emission, and therefore the bandwidth defined around this centre frequency corresponds to the depth of this volume. The bandwidth can be adjusted thanks to the weighting function $W(f)$, and in practice, it is selected in such a way that the cosmological time evolution of the EoR is negligible. The order of magnitude of the physical size of the reionization bubbles at the end of the EoR is expected to be around 10 Mpc, which corresponds to spatial fluctuations with a characteristic angular scale of about half a degree, and typically implies a frequency bandwidth associated with the signal variation along the line of sight of 8 MHz, that is to say a variation of redshift $\Delta z = 0.5$ at 150 MHz or $z = 8.5$ (Wyithe & Loeb 2004; Mesinger et al. 2014). The delay power spectrum given in Equation (6) includes the contributions from the redshifted 21-cm hydrogen signal along with the unwanted foreground signal. However, the difference of spatial and spectral coherence scales between both signals can be used to partially differentiate them. In the next section, we outline the foreground avoidance method used to isolate the EoR delay power spectrum.

2.2 The problematic of the instrument chromaticity in the foreground avoidance method

Let us simplify Equation (5) and assume that the sky is uniform, its frequency spectrum is constant, the antenna voltage response is flat, and the window function has an infinite width and is equal to 1. Under these assumptions, for a given baseline we can calculate that the delay spectrum associated with the position of a point source in the sky is simply a Dirac delta function centred on the geometric delay $\tau_{12}$. The geometric delay is maximum for a source received by the antenna beam at its largest zenith angle, which can extend up to the horizon. This basic approach allows us to set limits on the delay power spectrum of the foreground measured by a baseline. In this "ideal" scenario, the uniform foreground emission cannot affect the delays which are higher than the maximum geometric delay $\tau_{\text{maxGeo}}$, i.e. the "horizon limit". Thus, the delay power spectra from the shortest baselines are less affected by the foreground.

This property is often illustrated by the "wedge" which is a representation of the intensity of the delay power spectrum as a function of $\|k_{\perp}\|$ (or the baseline length), and $k_{||}$ (or the delay) (Datta et al. 2010; Morales et al. 2012; Parsons et al. 2012b; Thayagarajan et al. 2013; Liu et al. 2014). In this representation, a uniform foreground is contained inside a wedge-shaped region of the $k$-space defined by:

$$\tau \leq \tau_{\text{maxGeo}} = \frac{\|b\|}{c} \quad \Rightarrow \quad k_{||} \leq \frac{H_0(z) D}{c (1+z) \|k_{\perp}\|} \quad (9)$$

On the other hand, the delay power spectrum associated with the fluctuating EoR signal spreads beyond this limit, and consequently is free from foreground contamination.

However, in practice the foreground spectrum is not exactly flat, the bandwidth of the correlator not unlimited, and the beam not achromatic. By taking a close look at Equations (4) and (5), and by applying again the convolution theorem, one can show that the delay spectrum for a specific direction is equal to the convolution between the time signals of the averaged sky intensity, the complex conjugate of the voltage response of antenna 1, the voltage response of antenna 2, the window function, and a Dirac delta function centred on $\tau_{12}$. Consequently, the delay spectrum of the foreground coming from a given direction is no longer associated with a single delay, but instead spreads, and in particular beyond the maximum geometrical delay. The spectral and time properties of each of these terms are responsible for the leakage of the foreground in the delay domain, so it is crucial to control these parameters to preserve the EoR window.

By observing diffuse emissions from cold patches of sky, it is possible to receive a foreground which is relatively spectrally smooth. This translates into a compact delay spectrum which moderately expands beyond the maximum geometric delay (Thayagarajan et al. 2015a,b). Moreover, the choice of the frequency window is also very important. Leakage associated with windowing is a classic problem in signal analysis. For instance, a simple rectangular window in the frequency domain is transformed into a cardinal sine function in the time domain. However, the convolution of the received time signal with the sidelobes of such a window contributes to spread the foreground. Ideally, the window function in the time domain should have very low sidelobes and a sharp rolloff to minimize the leakage, as well as a narrow and high main beam to keep a good resolution and sensitivity. These two aspects are usually conflicting, so a trade-off has to be made. A rectangular frequency window is a poor choice in this context since it results in high sidelobes, despite an optimal sensitivity. Parsons et al. (2012b) and Thayagarajan et al. (2013, 2016) studied this problematic and concluded that a Blackman-Harris window (Harris 1978) would be a suitable choice. As illustrated in Section 4, when a Blackman-Harris window is used, a dynamic range of more than 100 dB can be quickly obtained for signals in the time domain, whereas with a rectangular window, only 80 dB. Thus, the level of foreground leakage caused by the window can be kept below the EoR signal. However, the drawback is to significantly down-weight the frequencies at the edges of the window, causing a reduction in the spectral sensitivity of about 50%.

The third term to analyse is the frequency response of the baseline, or in a simpler way the voltage response of each antenna. As we have seen, the foreground spectrum can be relatively smooth, the spectral weighting function can be chosen to minimize the signal leakage, so the an-
Antenna response plays a very important role in this equation. The system time response is far from an ideal Dirac delta function because of signal reflections caused by the antenna structure, the array, and the receiver chain. Each additional reflection spreads the foreground still further in the delay domain. Reflections can be divided into three categories: "internal" reflections between the elements of the antenna, reflections within the RF chain, and "external" reflections between the different antennas of the array (cf. Fig. 1). Internal reflections are mainly caused by standing waves which form inside the metal cage surrounding the feed, and between the dish and the backplane of the cage. Thus, the signal is reflected multiple times between these elements, leading to oscillations in the system time response. Once received by the antenna, the signal propagates through the RF analogue chain, which consists of a "front-end module" (FEM), a 150-m dual coaxial cable, and a "post-amplification module" (PAM). Each of these elements has its own response. For instance, reflections occurring at the ends of a coaxial cable are a common issue. In addition, the array is very compact and the edges of the adjacent antennas are only 60 cm apart. Therefore, the incident signal can be reflected off the structure of one antenna and contaminate other dishes. Moreover, a fraction of the signal received by an antenna can also be re-radiated by the feed via the sidelobes of the beam towards other antennas.

Lastly, the differences between the expected delay power spectra obtained for the foreground and the EoR are illustrated in Ewall-Wice et al. (2016) and in Thyagarajan et al. (2016), for various baselines in the HERA array. In particular at very low delays and compared with the EoR signal, the foreground delay spectrum (cf. Equation (5)) is four to five orders of magnitude higher, and its delay power spectrum (cf. Equation (6)) is about nine orders of magnitude more intense. Consequently, these papers show that the voltage time response of the system should ideally be attenuated by about $10^5$ to limit the contamination of the EoR window by the foreground.

### 3 PARAMETERS OF THE ELECTROMAGNETIC AND ELECTRICAL CO-SIMULATIONS

#### 3.1 Antenna model

In this section, we present the HERA antenna model which is used in the co-simulation along with the analogue receiver. In this first version, the dipole feed previously developed for PAPER is re-used after optimization to work with a parabolic reflector (DeBoer 2015; DeBoer et al. 2017). Developed with CST (cf. Fig. 2), the antenna model can be divided into 3 main elements: the reflector, the balanced feed, and a cylindrical cage located just above the feed. The reflector is modelled by a faceted paraboloid made up of 24 segments in aluminium, has a diameter of 14 m, and a focal ratio of 0.32. At its vertex, a cylindrical concrete slab with a 91-cm diameter, called the "hub", firmly holds the PVC pipes supporting the structure of the antenna. This hub has also a 46-cm diameter hole at its centre and is surrounded by two aluminium cylinders.

The feed consists of two perpendicular 1.3-m long dipoles made of copper, as well as two aluminium discs, one below and one above, used to broaden the frequency response. The dipoles are terminated by four connectors in copper, which are directly connected to the front-end module. Thus, the losses introduced by the transmission line between the antenna and the FEM are minimized. This also limits the additional noise entering the first amplification stage and keeps the receiver noise temperature low. Besides, a very short transmission line does not significantly modify the input impedance "seen" by the analogue receiver. The presence of the FEM structure is also modelled by a brass tube terminated by two output coaxial cables, one for each polarization. The feed of HERA is surrounded by a cylindrical cage which tapers the beam radiated by the dipoles, and protects them from external signals coming from the adjacent antennas. The cage is made up of two elements in aluminium in contact: the "backplane", which is a 172-cm diameter disc, and a cylindrical "skirt" with a height of 36 cm.

The feed is suspended by three Kevlar ropes, 4.9 m above the dish, but this height can be adjusted. We also include in our model the effects of the metals by taking into account their finite conductivity, as well as the effects of the dielectric materials by defining their relative permittivity and loss tangent as a function of the frequency. Besides, the presence of the sandy soil under the antenna is also taken into account by defining a slice of dielectric with a thickness of 1.5 m. After simulation, it turns out that the ohmic and dielectric losses are very low, with a radiation efficiency above 98%. The parameters used to simulate this model with CST are detailed in Section 3.3.

#### 3.2 Analogue receiver model

In this study, the receiver model is divided into three blocks: the FEM which is directly connected to the dipole arms via four short pins, the coaxial cable, and the PAM. For a given polarization, the received electromagnetic wave is transformed into differential voltage signals by the feed, and...
the two 180° out of phase components coming from the dipole arms are directly transmitted to the FEM. Inside this active balun, a transformer is used to adjust the input impedance of the FEM and match the antenna impedance. The differential signals then pass through three amplification stages in cascade, before being combined by a 180° hybrid coupler to form an unbalanced signal. The FEM is terminated by a 3-dB attenuator in order to limit the amplitude of the reflections between the balun output, the cable and the PAM. The global gain of this active balun is about 30 dB between 100 and 200 MHz. The unbalanced signals from the two polarizations are then transmitted to the PAM via a dual RG6U coaxial cable with an impedance of 75 Ω and terminated by F-connectors. The 150-m cable attenuates the signals by about 12.5 dB at 150 MHz. The last element of the analogue system is the PAM which provides an additional gain of 66 dB. It consists of a transformer to match the impedance of the 75-Ω cables, five amplification stages separated by 3-dB attenuators for interstage isolation, and a bandpass filter. Thus, in total the gain of the whole system is about 83.5 dB. The bandpass filter has a 3-dB passband bandwidth of 74 MHz, centred on 148 MHz, and a 30-dB stopband bandwidth of 96 MHz. The main characteristics of this receiver developed for PAPER by the NRAO (Parashare & Bradley 2006a,b, 2007) are summarized in Fig. 3. In addition, Sections 3.6 and 3.7 detail the performance of each block in terms of reflected signals and generated noise.

3.3 Simulation parameters

The electromagnetic properties of the antenna are simulated with the time domain solver of CST Microwave Studio. This transient solver is based on the "finite integration technique" (FIT) proposed by T. Weiland in 1977 (Weiland 1977; Clemens & Weil 2001). The simulation space is divided into adjacent hexahedral cells, and the discretized Maxwell’s equations in their integral form are solved on the facets of each cell, guaranteeing the consistency of the solution through the mesh. This solver is stable and efficient for problems with a complex geometry which include small and large elements, and is also recommended for wideband simulations. The antenna is fully characterized in the time domain, which is more suitable for analysing the multiple reflections occurring in the array, and the frequency parameters are obtained by Fourier transforming the time results. Unlike classic frequency solvers, only one simulation run is required to obtain comprehensive results.

The boundaries of the simulation box are defined in the solver as "open" to simulate the conditions of propagation in free space. The boundaries consist of a "perfect matched layer" (PML) of material which absorbs the radiated field. The simulation box is about 20.6-m large, 6.4-m high, and is defined in such way that the distance between the PML and the structure is at least equal to a quarter of the longest wavelength, i.e. 1.5 m at 50 MHz. The simulation is divided into 33 million hexahedral cells for a detailed spatial discretization of the structure. The smallest cell is 2-mm large, which is sufficient to mesh the coaxial cables or the pins, and the biggest cell is 100-mm large to mesh the empty space far from the structure.

For a given polarization, the centre-fed dipole is excited by a discrete termination port connecting together the two dipole arms. The dipoles are designed to work with a balanced feed line at their input, and so to produce or respond to differential signals. For a given dipole, both arms are simultaneously excited by the same signal coming from the "differential" port, but 180° out of phase. This simplified representation of the balanced feed line is equivalent to a power generator delivering 1 W, and can also absorb the signal coming from the antenna. Note that according to the maximum power transfer theorem, half of the power can be effectively delivered to the antenna at most. In addition, the port has an internal impedance which in theory represents the impedance of the RF receiver seen by the antenna. However, in practice we are limited by the software which only allows us to define a constant and real impedance in the electromagnetic simulation module. By default, a differential impedance of 100 Ω is used, which is equivalent to terminate each dipole arm with a 50-Ω impedance, value typically used in single-ended measurements with a VNA. We will see in the next sections how to properly account for the frequency-dependent impedance of the receiver chain.
The excitation signal is a broadband Gaussian pulse centred on 150 MHz with a bandwidth of 200 MHz, and which lasts about 36 ns. The time step of the simulation is proportional to the size of the smallest cell, and is about 0.004 ns. Besides, the simulation is configured to stop after 1500 ns in order to record a significant number of reflections, characterize the antenna voltage response at high delays, and ensure that the electromagnetic energy correctly dissipates inside the domain. In practice, we verify that the simulated output voltage, and therefore the voltage time response, is attenuated by a factor $10^5$ at least.

In this simulation, the antenna works in “transmission mode”: a spherical wave is generated by the feed and is then reflected by the dish. In particular, this allows the software to compute the time signals at the differential ports, as well as the $S$-parameters, the antenna impedance, and the radiated farfields in the frequency domain. The antenna beams are simulated every 0.5 MHz between 50 and 250 MHz, and with an angular step of 1 degree. However, HERA is obviously designed to work in “reception mode”. An electromagnetic plane wave coming from a specific direction of the sky is first reflected by the dish, then absorbed by the feed, and potentially a part of the received signal is re-radiated. Thus, the system response or the voltage at the antenna termination may be significantly different between these two modes of operation, because the signal follow different paths. For instance, in transmission, the voltages simulated at the antenna ports do not account for the effects of the polarization and angular direction. Note that it is possible with CST to excite the system with a plane wave coming from a certain angle and with a specific polarization. Nevertheless, such a simulation has to be repeated multiple times to fully characterize the direction-dependency of the antenna response, which is time consuming. In order to avoid that, we use the fact that the farfield beams of an antenna in transmission and reception are similar. Thanks to this property, we will see in Section 4 how the radiated electric farfield simulated and reception are similar. Thanks to this property, we will see in Section 4 how the radiated electric farfield simulated and reception are similar. Nevertheless, such a simulation has to be repeated multiple times to fully characterize the direction-dependency of the antenna response, which is time consuming. In order to avoid that, we use the fact that the farfield beams of an antenna in transmission and reception are similar. Thanks to this property, we will see in Section 4 how the radiated electric farfield simulated and reception are similar. Nevertheless, such a simulation has to be repeated multiple times to fully characterize the direction-dependency of the antenna response, which is time consuming. In order to avoid that, we use the fact that the farfield beams of an antenna in transmission and reception are similar. Thanks to this property, we will see in Section 4 how the radiated electric farfield simulated and reception are similar. Nevertheless, such a simulation has to be repeated multiple times to fully characterize the direction-dependency of the antenna response, which is time consuming. In order to avoid that, we use the fact that the farfield beams of an antenna in transmission and reception are similar.

3.4 Equivalent electrical circuit

The electrical properties of the antenna along with the analogue receiver can be described using the formalism commonly used in RF network analysis. This theory offers powerful tools to describe the propagation and reflections of a signal through a linear electrical system, which can be seen as a "black box" simply characterized by the voltage and current at its ports. In this framework, after receiving an electromagnetic wave, the antenna can be considered as a generator delivering a voltage $V_{oc}$ with an internal impedance $Z_{ant} = R_{ant} + jX_{ant}$, $R_{ant}$ being the resistance and $X_{ant}$ the reactance of the antenna. As for the receiver, in order to make the analysis easier, the FEM, cable, and PAM are combined together and form a two-port network with the input connected to the antenna, and the output connected to a load with an impedance $Z_L = R_L + jX_L$. In practice, this load represents the 50-Ω coaxial cable which connects the PAM output to the correlator. Thus for a given polarization, the system can be represented by the circuit in Fig. 4.

In this circuit, the two-port network is characterized by the voltage $V_1$, current $I_1$, at the input port 1, and by the quantities $V_2$, $I_2$ at the output port 2. Besides, $Z_{in}$ denotes the input impedance of the two-port network seen by the antenna, and $Z_{out}$ the output impedance seen by the termination load (Pozar 1998, Ch. 4). The voltages $V_1$ and currents $I_1$ flowing into the ports can be related together thanks to the impedance or $Z$-matrix $Z$. For a two-port network, $Z$ is a 2x2 matrix defined such as:

$$V_1 = Z_{11}I_1 + Z_{12}I_2,$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2.$$  

(11)

In this electrical circuit, we can demonstrate that the currents $I_1$ and $I_2$ flowing into the ports 1 and 2 are equal to:

$$I_1 = \frac{V_{oc} - V_1}{Z_{ant}}, \quad I_2 = -\frac{V_2}{Z_L}.$$  

(12)

Note that in the expression of $I_2$, there is a negative sign because of the orientation of the current, which flows into the port 2 in this definition. We have defined four equations, therefore the quantities $V_1$, $V_2$, $I_1$, and $I_2$, can be expressed as a function of the $Z$-parameters of the two-port network, the antenna impedance, the load impedance, and the generator voltage. The goal is to derive the output voltage $V_2$ at the termination load in order to calculate the response of the system. Thus, solving this system of equations yields:

$$V_2 = \frac{V_{oc}Z_{10}Z_{20}}{(Z_L + Z_{22})(Z_{ant} + Z_{11}Z_{22})} - Z_{21}Z_{12}.$$  

(13)

The previous electrical circuit can be further simplified,
by expressing $V_1$ as a function of $I_1$ thanks to Equations (11) and (12). The factor linking $V_1$ and $I_1$ is actually the input impedance $Z_{in}$ seen by the antenna at the port 1. Therefore, by writing that $V_1 = Z_{in}I_1$, and after calculation we obtain:

$$Z_{in} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22} + Z_{11}}. \quad (14)$$

$Z_{in}$ accounts for the effects of the transmission of the signal through the analogue receiver, as well as the reflections within the two-port network and at the interface with the termination load. Thus, in reception mode the interface between the antenna and the electronics is simply equivalent to a voltage source $V_{oc}$ in series with the antenna impedance $Z_{ant}$ and terminated by the input impedance of the receiver $Z_{in}$. In the same way, in transmission mode, the load previously defined can be associated with a generator, and the antenna impedance plays the role of termination. In this configuration, the load sees an impedance $Z_{out}$ towards the antenna at port 2 equal to:

$$Z_{out} = Z_{22} - \frac{Z_{12}Z_{21}}{Z_{ant} + Z_{11}}. \quad (15)$$

As discussed in the next section, in practice the Z-parameters are obtained from simulated or measured reflectometry data. Moreover, in Section 4 we explain how the input voltage $V_{oc}$ from the antenna can be calculated thanks to the electric farfield and an incident plane wave.

### 3.5 Measurements and simulations of the reflection and transmission coefficients

At high frequencies, it becomes more complex to measure the current or voltage at the ports of an electrical circuit. In practice, a VNA is used to measure the S-parameters of the device (Orfanidis 2016, Ch. 14). The ports of the device-under-test (DUT) are connected to the VNA via coaxial cables, which injects a signal, and measures the reflected and transmitted signals between the ports of the network. The instrument also calibrates out the effects of the cables, so that the reference plane of the measurements is defined at the ports of the DUT. It is important to bear in mind that the S-parameter measurements are performed with respect to the characteristic impedance $Z_0$ of the transmission line connected to the DUT, which is assumed to be real and typically equal to 50 $\Omega$.

In this method, the S-parameters of the antenna and of each block of the RF receiver are measured, and the entire system can be represented by a two-port network as in Fig. 4. The description of the coaxial cable and PAM is straightforward since they only have one input port and one output port. The cases of the antenna which has four single-ended ports and of the FEM which is in theory a six-port network are a bit particular, nevertheless this can be simplified. For a given polarization and in reception for instance, both arms of a dipole are simultaneously excited by the electromagnetic wave, and the antenna output delivers differential signals to the input of the FEM via two pins. Therefore, it makes more sense to calculate the “mixed-mode” S-parameters, and in particular the “differential” S-parameters (Bockelman & Eisenstadt 1995). In this case, these two physical ports are represented by only one logical differential port, and the S-parameters describe the propagation and reflection of the combined differential signals. By default and without using an external balun which then will have to be de-embedded, a VNA usually performs single-ended measurements. So in practice, if one wants also to include the effects of the polarization leakage between the two dipoles, a four-port VNA will have to be used. Then, the 4x4 S-matrix can be converted into a 2x2 differential S-matrix in which each logical port corresponds to a dipole (Fan et al. 2003). Concerning the FEM, we assume that the two polarization channels have similar properties and are perfectly isolated. By neglecting the crosstalk between the channels, the FEM can be considered as a three-port device, in which the two input pins form a differential port, and the output port is unbalanced. In the same way, the 3x3 single-ended S-matrix can be transformed into a 2x2 differential S-matrix (Anaren 2005). Note that after these conversions, the reference impedance of a differential port is equal to the sum of the reference impedances of the single-ended ports which define it. Thus, the differential S-parameters of the antenna and FEM are calculated with respect to $50 + 50 = 100$ $\Omega$, but the output single-ended port of the FEM is defined with respect to 50 $\Omega$. In simulation, this approach is rather straightforward since a differential port can directly be defined to feed a dipole or the FEM input.

In order to properly describe the transmission and reflection properties at the interface between each element, the measured or simulated S-parameters must be adjusted with respect to the right termination impedances, which can be complex and frequency-dependent. First, thanks to the initial S-parameters, the Z-parameters can be calculated; they are independent from the reference impedance of the port. Then from the Z-matrix, the S-parameters can be recalculated and the reference impedances $Z_{01} = R_{01} + jX_{01}$ and $Z_{02} = R_{02} + jX_{02}$ independently chosen for each port. For instance, the complex antenna impedance can be used to terminate the FEM input. In addition, once the (differential) S-parameters of the FEM, coaxial cables, and PAM are obtained and expressed as a function of the same reference impedance, 50 $\Omega$ for instance, they can be combined into a single 2x2 S-matrix which describes the entire receiver chain. In order to do this, it is easier to calculate the "transmission
rents in the transmission lines connected to the ports of the
are associated with the propagation of the voltages and cur-
the ports, as initially developed in Youla (1961). These waves
defined from the incident and reflected "travelling waves" at
respect to a real and positive impedance, they are actually
S-parameters. When the S-parameters are expressed with
one must be careful with the interpretation of the obtained
results. Thus, in this definition $S_{11}$ (resp. $S_{22}$) represents the
voltage reflection coefficient at port 1 (resp. port 2) when
all the other ports are perfectly matched, i.e., there is no
reflection coming from these ports. As for $S_{12}$ (resp. $S_{21}$), it
corresponds to the voltage transmission coefficient from
port 1 to port 2 (resp. from port 2 to port 1). However, the
physical interpretation differs when the S-parameters are ex-
pressed with respect to complex impedances. In this case,
the system can no longer be directly described by forward
and backward travelling waves. Another type of waves called
"power waves" and initially developed by Kurokawa (1965)
is used to defined the "generalized S-parameters". In the same
way as the travelling waves, in this context the S-parameters
describe the incident and reflected power at the interfaces
of the network, which is useful to quantify the power trans-
ferred from the antenna to the receiver. Note that the de-
inition of the power waves is similar to the travelling waves
for real impedances.

In the Section 3.6, the generalized S-parameters are di-
rectly used to describe the performance of the system in
terms of reflected and exchanged power between each block.
However, since the voltage response of the instrument is es-
sential in the foreground avoidance method, the system is
also studied in terms of reflected and transmitted voltage. If
we assume that the voltage travels from a source defined by
its impedance $Z_S$ to a load defined by $Z_L$, the voltage reflec-
tion coefficient $\Gamma$ at the interface between these two elements
is given by the following expression (Rahola 2008):

$$\Gamma = \frac{Z_L - Z_S}{Z_L + Z_S}$$  \hspace{1cm} (16)

In a similar way, the power wave reflection coefficient $S_{11}$
between a source and a load can be expressed as a function
of their impedance:

$$S_{11} = \frac{Z_L - Z_S}{Z_L + Z_S}$$  \hspace{1cm} (17)

These equations illustrates the principle of impedance
mismatch which is a well-known problem in antenna design
and transmission lines (Orfanidis 2016, Ch. 13). When there
is an impedance discontinuity between two transmission me-
dia or two elements of the RF chain, a fraction of the incident
voltage is reflected back. Some of these reflections are then
radiated by the dipoles, reflected by the dish, and eventually
partially re-absorbed. This mechanism of re-radiation and
absorption occurs multiple times, which can significantly
spread the time response of the system. The reflections be-
tween the antenna and the RF receiver can be studied by
considering $Z_{ant}$ as the source impedance and the equivalent
impedance $Z_{rec}$ as the load impedance in Equation (16). In
order to avoid voltage reflections at this precise interface,
$Z_{rec}$ should be ideally equal to $Z_{ant}$. However, despite be-
ing crucial, the impedance matching between the antenna
and the RF receiver is not necessary enough to guaranty
a system response perfectly smooth. Indeed, this condition
only implies that no signal will be re-radiated by the dipole,
but standing waves and reflections can still occur within
the receiver chain. Therefore, to complete this approach, we
also need to study the voltage reflection coefficients between
each element of the receiver chain. Ideally, for a perfectly
reflectionless system, each element should have its output

![Figure 5. Block diagrams of the X-receiver a) and of the antenna b), described as two-port networks and including both polariza-
tions.](image-url)

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matrix" $T$ from the S-parameters. When the components are
cascaded, the global T-matrix is simply equal to the product
of the T-parameters of each block. The T-parameters of the
receiver can then be converted again in order to get the S-
parameters of the RF chain with respect to 50 $\Omega$, and lastly
with respect to the differential impedance of the antenna
$Z_{ant}$. Frickey (1994) describes in detail how to perform these
calculations and provides the necessary conversion tables for
a two-port network.

In this paper, the reference system is the receiver which
can be described as a two-port system as in Fig. 5 a). Let as-
sume that the receiver corresponds to the X-polarization. Its
output port sees the impedance of the termination load $Z_L$, i.e.
50 $\Omega$, and its input port the impedance of the X-dipole
along with the Y-dipole connected to its own receiver. De-
spite low, the polarization leakage from the Y to X-dipole
is responsible for an additional voltage proportional to the
mutual impedance terms $Z_{XY}$ in the 2x2 Z-matrix of the an-
tenna. However, as we saw previously, the description of a
two-port network can be simplified. First, by applying Equation
(14) to the Y-receiver terminated by the 50-$\Omega$ load, one
can calculate its equivalent input impedance $Z_{recL}$ seen by
the differential port of the Y-dipole. Then, by describing the
antenna also as a two-port network as in Fig. 5 b) termi-
nated by the equivalent input impedances of each receiver,
one can apply the same equation to calculate the equiva-

tent input impedance of the antenna $Z_{ant}$ connected to the
Y-receiver and seen by the X-receiver. Thus, such a repre-
sentation centred on the X-receiver is actually equivalent to the
electrical circuit defined in Fig. 4, and is able to entirely
describe the dual-polarization system.

As explained in Kurokawa (1965) and Rahola (2008),
one must be careful with the interpretation of the obtained
S-parameters. When the S-parameters are expressed with
respect to a real and positive impedance, they are actually
defined from the incident and reflected "travelling waves" at
the ports, as initially developed in Youla (1961). These waves
are associated with the propagation of the voltages and cur-
rents in the transmission lines connected to the ports of the
network. In particular, this is the case with VNA measure-
impedance matched with the input impedance of the following element. In practice, it can be challenging and electrical circuits are inserted between the different elements to improve the matching. For instance, Fagnoni & de Lera Aceedo (2016) studied the effects of the insertion of such a matching circuit between the HERA dipole output and the FEM input, and showed that it was possible to divide by three the level of reflections in the time domain. Lastly, note the presence of a complex conjugate in the definition of the power wave reflection coefficient. Thus, $S_{11}$ is null when the load impedance is equal to the complex conjugate of the source impedance, which does agree with the condition of maximum power transfer. Unless the impedance of the source is real, $S_{11}$ equal to zero does not imply the absence of voltage reflections at the interface. In other words, in a system with complex input and output impedances, standing waves are actually necessary for a maximum power transfer. Therefore, a trade-off has to be found between impedance matching for power and voltage reflection. In the framework of the foreground avoidance method, it is clear that reflectionless matching should be optimized, but bearing in mind that it will certainly affect the gain of the system.

### 3.6 Reflectometry results

In this section, the reflection and transmission properties of the RF receiver when connected to the antenna, are studied in terms of voltage and power. First, for each element (antenna, FEM, cable, and PAM), the input and output impedances are calculated. For a rigorous description, the impedances are given taking into account all the elements connected in the chain. For instance, the input impedance of the FEM also includes the presence of the coaxial cable, PAM and termination load connected at its output. Then, the voltage and power wave reflection coefficients are obtained thanks to Equations (16) and (17) (cf. Fig. 6 and 7). This approach allows us to quantify the quality of the impedance matching between each element, and so to find out where most of the reflections form in the system. Lastly, the transducer gains of each element are given in Fig. 8 to characterize the system in terms of power amplification, attenuation and passband. The results from the simulations are also compared with the measurements in order to verify that our models are consistent with the system that has been built.

The analysis of Fig. 6 shows that most of the voltage reflections occur at the interface between the antenna and the input of the RF receiver. The humps that we can observe have a periodicity of about 30 MHz, which corresponds to reflections occurring at 5 m, i.e. the distance between the cage and the vertex. On average, the voltage reflection coefficient is about -5 dB, but one should not be surprised to also see a voltage reflection coefficient with a magnitude superior to 1 at low frequencies. As explained in Vernon & Seshadri (1969), this can occur with complex termination impedances and on condition that $R_l R_S + X_{l} X_{S} < 0$. However, we can see in Fig. 7 that the magnitude of the reflected power is always inferior to the incident power, which means that there is no violation of the power conservation for a passive load. The cable is also source of chromaticity. Indeed, the voltage reflection coefficient between the cable, the output of the FEM, and the input of the PAM varies between -10 and -25 dB; thus the impedance matching could be improved. Because of that, non-negligible standing waves can form in the cable. In addition, the reflection coefficients present small and fast ripples in the frequency domain, which is typical in mismatched transmission lines, and is caused by destructive and constructive interferences. For a transmission line with a length $L$, the peaks of these ripples are reached when $L = \lambda/4 + n\lambda/2$ and the dips when $L = n\lambda/2$, with $n$ an integer. Therefore, the frequency periodicity of these ripples $\Delta f$ is equal to $\nu/(2L)$, with $\nu$ the velocity of the signal propagating through the cable equal to 0.82$c$ in this case. These ripples have a periodicity of about 0.8 MHz which does correspond to reflections occurring in a 150-m cable. Note that these results deduced from the frequency data are confirmed in Section 4.2 by the analysis of the system re-
Figure 9. Differential input impedance of the RF receiver and antenna - Real and imaginary part.

In order to illustrate the problem of impedance mismatch between the antenna and the input of the RF receiver, their respective impedance is compared in Fig. 9. The difference is obvious, but not surprising. Indeed, the RF receiver was initially developed for PAPER, and by adding the dish and the cage around this feed, the antenna impedance is significantly modified because of the multiple reflections induced. In particular, by simulating the impedance of the PAPER feed alone, we can notice that the presence of the dish along with the cage actually adds sharp resonance peaks on the initial impedance. Such profile makes the impedance matching challenging. Moreover, by looking at the imaginary part of these impedances, we can realize that the system actually provides better performance in terms of power reflection, which is to the detriment of the voltage reflection coefficient.

Lastly, we can see that the results from our measurements and simulations are consistent. In particular, the parameters of the antenna obtained with CST agree very well with those measured. Because of the size of the antenna and the array, it is important to have an electromagnetic model which is reliable, in order to characterize and improve the design.

3.7 Effects of the termination impedance on the receiver noise and antenna sensitivity

In addition to the transmission and reflection properties of the RF chain, it is also important to consider the noise it generates, because it impacts the antenna sensitivity. As previously, the co-simulation that we have developed is used to compute the noise figure (in dB) of each element of the RF receiver, and the results are compared with measurements. Fig. 10 presents the simulated and measured noise figure and temperature of the FEM, coaxial cable, and PAM with respect to a termination impedance of 100 Ω for the FEM input, and 50 Ω in the other cases. When cascading different components, the receiver noise figure is close to the noise figure of the first block, i.e. the FEM, however it is also important to include the PAM to take into account the effects of the bandpass filter.

Note that these results are given with respect to a fixed termination impedance which depends on the measurement device. However, as shown in Equation (18), the noise factor of an amplifier $F_{\text{amp}}$ actually varies as a function of the source impedance (Pozar 1998, Ch. 11).

$$F_{\text{amp}} = F_{\text{min}} + \frac{R_N}{R_S} |Y_S - Y_{\text{opt}}|^2,$$

(18)

with $F_{\text{min}}$ the minimum noise factor of the amplifier, $R_N$ its equivalent noise resistance, $R_S$ the real part of the impedance of the source, namely the antenna, $Y_S$ its admittance, and $Y_{\text{opt}}$ the optimum source admittance which minimizes the noise factor. For instance, in the case of the FEM, $F_{\text{min}}$ varies between 1.6 dB and 1.8 dB, $Z_{\text{opt}}$ between 119 + 34j Ω and 148 + 22j Ω, and $R_N$ is around 27 Ω.

Therefore, in order to properly estimate the noise generated, the receiver has to be terminated by the antenna impedance. In practice, this is something difficult to achieve with direct measurements, but doable in simulations, on condition that the noise parameters of the amplifiers are known. This information is usually provided by the component manufacturers. Since the results from our simulations and measurements do agree well in Fig. 10, we rely on our model to estimate the receiver noise when terminated by the antenna. Moreover, Equation (18) shows that the noise generated by the receiver can be minimized on condition that it is connected to an optimal impedance. However, the input impedance required to minimize the generated noise is usu-
ally different from the impedance required to minimize the signal reflection or to optimize the power transfer. Thus, a trade-off has to be made between these three types of matching. Nevertheless, in practice the effects of the receiver noise can be mitigated by increasing the integration time, whereas it is more complex to correct the systematics introduced by the reflections.

Fig. 11 shows the noise figure and temperature of the measured and simulated RF receiver when its input is connected to a differential impedance of 100 Ω, or the measured antenna impedance. When terminated by a 100-Ω impedance, the noise figure is stable and around 1.9 dB, or 160 K in terms of noise temperature. However, when the antenna impedance is taken into account, the noise figure varies between 1.8 and 2.5 dB, or 150 and 225 K. These significant variations illustrate the impedance mismatch between the optimal noise impedance of the amplifiers and the antenna impedance.

In this paper, the antenna sensitivity $S_A$ at the feed point of the antenna for a single element, and for a given polarization, is defined as the ratio of the effective antenna aperture $A_{\text{eff}}$ over the system noise temperature $T_{\text{sys}}$ (Cortes-Medellin 2007). It is a function of the spherical angles $\theta$ and $\phi$:

$$S_A(\theta, \phi) = \frac{A_{\text{eff}}(\theta, \phi)}{T_{\text{sys}}} = \frac{1}{4\pi \eta_{\text{rad}}} D(\theta, \phi) \frac{1}{T_{\text{sys}}},$$

(19)

Note that the effective antenna aperture, which accounts for the fact that not all the physical surface of the antenna collects the signal with the same efficiency, can be calculated from the simulated antenna directivity $D(\theta, \phi)$ and the antenna radiation efficiency $\eta_{\text{rad}}$. In this definition of $A_{\text{eff}}$, the radiation losses caused by the dielectric and metal elements are included, but not the losses arising from the polarization and impedance mismatches. As for the system noise temperature, it is equal to the sum of all the received noise contributions. In this paper, it is defined by:

$$T_{\text{sys}} = T_{\text{dis}} + T_{\text{rec}} + T_A,$$

(20)

with $T_{\text{dis}}$ a term associated with the losses caused by the dissipative elements of the antenna at the ambient temperature $T_0 = 290$ K and equal to $T_0 (1 - \eta_{\text{rad}})$, $T_{\text{rec}}$ the receiver temperature obtained by combining the noise temperature of each element of the RF chain, and $T_A$ the antenna temperature corresponding to the noise effectively received from the sky as well as the ground. In principle:

$$T_A = \eta_{\text{rad}} \int_{\alpha_{\delta}} T_B(\nu, \theta, \phi) P_{\text{rad}}(\nu, \theta, \phi) \sin \theta \, d\theta \, d\phi,$$

(21)

with $T_B$ the brightness temperature surrounding the antenna, and $P_{\text{rad}}$ its power radiation pattern. However, in order to simplify the study, we only consider the contribution from the sky, which is also assumed to be uniform. For a cold patch typically observed by HERA far from the galactic centre and at low frequencies, the sky temperature can be approximated by a power law (Furlanetto 2016):

$$T_{\text{sky}} = 180 \left( \frac{\nu}{180 \, \text{MHz}} \right)^{-2.6}. $$

(22)

Figure 11. Noise figure and temperature generated by the radiation losses, the sky, and the receiver when terminated by 100 Ω and the measured antenna impedance. Combined together, they define the system noise temperature.

Figure 12. Antenna sensitivity at zenith, for a given polarization, and when the receiver is terminated by 100 Ω and the measured antenna impedance.

Fig. 11 details the different noise contributions received by the system. In order to illustrate the effects of the termination impedance, the contribution from the receiver is estimated when terminated by a differential impedance of 100 Ω and by the measured antenna impedance. As expected, the sky temperature dominated by the galactic synchrotron emission is the main contribution at low frequency. However, as the frequency increases, the sky temperature progressively decreases and the effects of the RF chain start to become significant. Lastly, by combining all these terms, the antenna sensitivity is calculated (cf. Fig. 12). Because of the variations in the antenna impedance, the performance is slightly degraded with respect to 100 Ω, in particular at 150 MHz where the sensitivity is 10% lower. We can also notice the effects of the passband filter which limits the sensitivity below 105 MHz and after 195 MHz.

4 IMPACT OF THE ANALOGUE RECEIVER AND MUTUAL COUPLING ON THE SYSTEM RESPONSE

4.1 Open-circuit voltage and effective length

So far, we have analysed the effects of the analogue receiver on a signal propagating through the chain, in terms of transmission and reflections, and in terms of generated noise. This study is necessary to understand how the differ-
ent blocks interact with each other and with the antenna. However, it is not sufficient to completely characterize the system response. Previously, the antenna has been simply represented by a generator with a complex impedance $Z_{\text{ant}}$ delivering a certain voltage $V_{\text{oc}}$. The effects of the antenna beam, such as its frequency response or angular pattern, have not been included yet. If we refer to the equivalent circuit in Fig. 4, $V_{\text{oc}}$ actually represents the “open-circuit voltage” produced by the antenna after being excited by an electromagnetic wave. More precisely, it corresponds to the difference of electrical potential between the two arms of a dipole, when disconnected from the RF receiver. In this section, we explain how the open-circuit voltage can be calculated from the simulated antenna beam and an incident electromagnetic plane wave. This element combined with the properties of the RF receiver allows us to compute the direction-dependant system response in reception, in the frequency and in the time domain.

Excited by an incoming electric field $E_{\text{in}}$ (in V/m), the antenna generates an open-circuit voltage $V_{\text{oc}}$ (in V). Thus, one can define a vector function $I_{\text{eff}}$ with the dimension of a length such as:

$$V_{\text{oc}}(\theta, \varphi) = E_{\text{in}}(\theta, \varphi) \cdot I_{\text{eff}}(\theta, \varphi).$$

(23)

$I_{\text{eff}}$ represents the effective length of the antenna. Thanks to the reciprocity principle, it is similar in reception and in transmission, and can be calculated from the radiated electric farfield pattern $E_{\text{pat}}$ (cf. Equation (10)). Besides, in the farfield approximation, $I_{\text{eff}}$ is a function of the spherical angles $\theta$ and $\varphi$, and its radial component is neglected. For a given polarization (Orfanidis 2016, Ch. 16):

$$I_{\text{eff}}(\theta, \varphi) = -E_{\text{pat}}(\theta, \varphi) \frac{4\pi}{kZ_{\text{fs}}l_{\text{p}}}.$$

(24)

with $Z_{\text{fs}}$ the impedance of free space equal to $\sqrt{\mu_0/\varepsilon_0} \approx 377 \, \Omega$, and $l_{\text{p}}$ the current at the feeding point of the considered dipole.

Note that in CST, the generated E-farfield is by default normalized by the input travelling wave $\phi_{\text{in}}$ which corresponds to the signal used to excite the dipole (cf. Fig. 13). Therefore, in practice the CST E-farfield has to be multiplied by $\phi_{\text{in}}$ in Equation (24). One should also point out that the E-farfield radiated by a dipole and the current at its terminal depend on the differential termination impedance, in this case 100 $\Omega$. However, because the first term is divided by the second one, the effective length is actually independent from the termination impedance used in the simulation. By considering the antenna as a two-port network terminated by two loads $Z_0$, one can demonstrate that the current at the antenna port is equal to:

$$I_p = \frac{2\phi_{\text{in}}\sqrt{Z_0}}{Z_{\text{ant}} + Z_0}.$$

(25)

with $Z_{\text{ant}}$ the antenna impedance obtained thanks to Equation (14), and which does include the effects of the polarization leakage.

The effective length can be seen as the transfer function in reception when the antenna is not connected to the RF receiver. Remarkably, this intrinsic quantity is obtained from a beam simulated in transmission with respect to an arbitrary termination impedance. Fig. 14 represents the frequency spectrum and time signal of the Gaussian pulse used to excite the dipole in transmission mode and the plane wave in reception mode, between 50 and 250 MHz.

**Figure 13.** Frequency spectrum and time signal of the Gaussian pulse used to excite the dipole in transmission mode and the plane wave in reception mode, between 50 and 250 MHz.

RF receiver. The effective length beam, the open-circuit voltage induced by a plane wave can be immediately calculated for any angle of incidence and polarization (cf. Equation (23)). The signal defined in Fig. 13 is also used as an excitation signal for the plane wave.

4.2 Frequency and time responses of the system without mutual coupling

Once $V_{\text{oc}}$ is obtained, the antenna can be combined with the parameters of the RF receiver previously simulated or measured. The calculations are performed in the frequency domain, before being Fourier transformed in the time domain. The voltage at the output of the RF chain is calculated by applying Equation (13) to the RF receiver terminated by $Z_{\text{ant}}$ and $Z_L = 50 \, \Omega$. The output voltage is given in
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Figure 14. $\theta$ and $\phi$-components of the antenna effective length in the E and H-planes, in the frequency domain.

Figure 15. Receiver output voltage $V_2$ at zenith transformed into the time domain, with the measured and simulated RF receiver.

This is caused by the reflection of the signal at the end of the 150-m coaxial cable. The results between the simulated and measured RF receivers are consistent, and the difference in the reflection delay can probably be explained by a slight variation in the velocity factor, which is directly related to the square root of the dielectric relative permittivity of the cable.

Similarly to Equation (3), we define the voltage response or transfer function of the system $H(\theta, \phi)$ which links the output voltage of the system previously described to the incident electric field, in the frequency domain:

$$V_2(\theta, \phi) = H(\theta, \phi) \cdot E_{in}(\theta, \phi).$$

Then, thanks to Equations (23) and (24), we can derive the expression of the frequency response of the system in reception, as a function of the effective length and of the $Z$-parameters of the antenna along with the RF receiver. Thus:

$$H(\theta, \phi) = \frac{-E_{pat}(\theta, \phi)}{\sqrt{Z_0}} \left[ \frac{Z_{ant} + Z_0}{2a_n V_{\phi m}} \right] \left[ \frac{Z_1 Z_{21}}{(Z_1 + Z_{22})(Z_{ant} + Z_{11}) - Z_{21} Z_{12}} \right].$$

The frequency response at zenith with the measured and simulated RF receiver is given in Fig. 16. Compared with the effective length, we can notice the effects of the bandpass filter between 110 and 190 MHz. By zooming in on the frequency spectra, the 0.8-MHz ripples caused by the reflections in the cable are also visible. Then, by Fourier transforming this fundamental equation, it is now possible to properly quantify all the chromatic effects affecting the antenna along with the receiver, in the time domain and for any direction.

As mentioned in Section 2.2, the received signal is also windowed in the frequency domain by a Blackman-Harris function in order to mitigate the effects of the leakage associated with the 100–200 MHz passband of the correlator. Thanks to this window, the noise floor of the voltage system time response in this simulation is about $10^{-5}$ or -100 dB with a fixed termination (cf. Fig. 17). Note that this type of window tends to downweight the frequencies at its edges. For this reason and because in practice the power delay spectrum is estimated over small frequency bands, the frequency response is also divided into 20-MHz sub-bands which overlap.
In order to illustrate the differences caused by the choice of the termination, in Fig. 17 the system time response is computed when the antenna is terminated by a 125-Ω load as in Ewall-Wice et al. (2016), the equivalent impedance of the receiver, and the full reflection and transmission parameters of the RF chain. In theory, only the last approach is able to properly account for the chromatic effects caused by the propagation of the signal though the receiver, and in particular the effects of the filter and the reflections between the output of the FEM, the cable, and the PAM. Note that the response is normalized and the time axis is defined such as its origin coincides with the maximum of the response. As one can expect, the system is more chromatic when the entire receiver chain is taken into account. We can notice that at low delays the main signal is reflected every 30 ns, which corresponds to the round-trip delay time between the cage and the vertex of the dish. Moreover, after 250 ns, the response level is about 5 to 10 times higher with the receiver than with a fixed 125-Ω termination. As illustrated in Fig. 6 and 9, the variations in the antenna impedance degrade the quality of the matching. Thus, in principle the attenuation of the response at low delays mainly depends on the impedance matching between the antenna and the input of the FEM. But in practice, this difference of matching is not critical in terms of performance. For instance, the response is attenuated by a factor $10^4$ after 170 ns with the simulated receiver, and after 140 ns with the 125-Ω termination. However, when the measured transmission parameters are considered, we can see that the level of the response is still about 5 to 10 times more intense at high delays. This is due to the chromaticity of the cable. Contrary to an ideal cable used in the simulated model, micro-reflections caused by small variations in the characteristic impedance can occur all along a real cable. These variations are due to differences in the physical properties of the cable, in particular the diameter of the conductors and the dielectric permittivity. These properties may vary when the cable is curved, bent, crushed, or simply with time. Thus, with the measured data, the system time response is attenuated by a factor $10^4$ after 260 ns, but remains above $10^{-5}$. Lastly, because of the reflection of the signal at the end of the cable, the response increases by a factor $10^2$, from $10^{-5}$ to $10^{-3}$, at about 1200 – 1300 ns.

Then, in order to refine the analysis, the system time response is studied in 20-MHz sub-bands. This allows us to identify the portions of the spectrum that are the most impacted by the chromatic effects. Fig. 17 shows that in the case of a fixed termination impedance, the responses in the sub-bands are rather similar, controlled by the window, and quickly decays to a level of about $10^{-4}$ after 200 ns, and $10^{-5}$ after 300 – 400 ns. However, the situation is more complex when the receiver is connected. This analysis does confirm the presence of micro-reflections in all the sub-bands caused by the chromatic cable, which affects the response after 200 ns. The noise floor of the response in this case is a bit below $10^{-4}$. This is about one order of magnitude higher than with an ideal cable or the 125-Ω termination. Interestingly, except for the reflection at 1200 ns, these two configurations have a similar profile and noise floor between 120 and 180 MHz, which does illustrate the importance of having high quality cables and as straight as possible. Thus, in this frequency band the FEM and PAM are actually rather achronomic. Nevertheless, the system response in the lowest and highest bands is significantly higher at low delays with the receiver; this is due to the effects of the bandpass filter. Lastly, the windowed system time response is computed as a function of the angle of incidence in the E and H-plane in order to estimate the contribution from the sidelobes. The results obtained in the E-plane with the simulated and measured receiver are given as an example in Fig. 18. Note that the angular responses are normalized by the maximum amplitude of the response at zenith occurring at $t = 0$.

Lastly, the final goal of this study is to quantify the attenuation of the direction-dependent time response as a function of the delay. Thus, Fig. 19 represents the maximum delay after which the system time response is attenuated by a certain factor, as a function of the angle of incidence. In these plots, the thresholds are calculated for the first 1000 ns, in order to disregard the substantial contribution from the signal reflection at the end of the cable. In this case, it

![Figure 16](image1). Amplitude of the system frequency response $H$ at zenith, with the measured and simulated RF receiver.

![Figure 17](image2). System time response $H$ at zenith transformed into the time domain, normalized, and windowed by a Blackman-Harris function between 100-200 MHz and in 20-MHz sub-bands.
is assumed that the EoR signal will be studied for $k_{\parallel}$-modes associated with delays inferior to this limit or that it will be possible to calibrate out the effects of the cable. Thanks to this approach, it is possible to transpose the impact of the frequency structure of the beams, for instance the variations in the gain at zenith or the position and intensity of the sidelobes, into the delay domain. Considering excitation signals with similar spectra, the system time response generated by the sidelobes is about 10 times lower than the response at zenith at high delays. In other words, for one antenna, if we compare two sources, one at zenith and the other one in a sidelobe, the system response associated with the source in the sidelobe is more chromatic than the response at zenith if the intensity of the source in the sidelobe is at least one order of magnitude higher. The results are given for the E-plane only, because the performance in the H-plane is quite similar due to the symmetry of the beam. Lastly, small asymmetries can be noticed in the delay response, for instance with the results in the E-plane, although the electromagnetic model is perfectly symmetric in CST. This anomaly can be explained by the fact that the structure is excited from a single cell with a finite size which is not perfectly at the centre of the simulation domain.

4.3 System response in a mutual coupling environment

HERA is a dense array in which the edges of the adjacent antennas are 60 cm apart. Therefore, one can suspect that mutual coupling, which encompasses all the interactions between antennas, plays a significant role in the chromaticity of the telescope, as described in Craeye & González-Ovejero (2011). For instance, multipath propagation can occur when the incident electromagnetic wave follows different paths before reaching the considered receiver. In particular, the incident signal can be reflected by the structure of the adjacent dishes. In addition, the signal absorbed by a dipole may also be re-radiated, because of impedance mismatch, towards other antennas via the sidelobes. Consequently, these phenomena can be sources of additional contributions which spread the received time signal. In this last section, the system response associated with an antenna in a large array...
is analysed for three different configurations described in Fig. 20. The main challenge is the size of the models which quickly becomes very computationally intense to simulate. Typically, these simulations require about 250 GB of RAM.

In order to do this, the method previously described is used but needs to be adjusted. An array of N antennas can be described as a 2N-port network, taking into account the X and Y-polarizations. In practice, the port associated with a given antenna and polarization is excited, while the other antennas are passive. Thus, only the considered antenna is initially transmitting, and the reflected signals coming from the other antennas are simulated with CST. From these simulations, the E-farfield radiated by the reference antenna in a mutual coupling environment as well as the S-parameters with the rest of the antennas are exported. For a fixed and real reference impedance, \( S_{ij} \) represents the voltage transmission coefficient between the antennas \( i \) and \( j \), when the other ports are perfectly matched, i.e. they do not transmit any signal.

The first configuration simulated is a hexagonal array with 19 elements, HERA 19, which corresponds to a small subset of the final 320-element core. As an example, Fig. 21 and 22 show how the antenna gain pattern at 150 MHz is modified. With respect to a beam without mutual coupling, the adjacent antennas are responsible for additional ripples on the sidelobes. We can also notice that without coupling and because the antenna is symmetric, the beam of the X-polarization in the E-plane is equivalent to the beam of the Y-polarization in the H-plane, but with coupling the structure of the sidelobes is modified. The configuration of the array is different along the X and Y-axis, and consequently the two polarizations are not strictly equivalent. Moreover, by considering the antennas at the edges of the array, as one could expect the sidelobes are also asymmetric. On average, without mutual coupling the maximum sidelobe level is about -25 dB in the E-plane, and -23 dB in the H-plane across the frequency band. However, the mutual coupling increases this level by 2 to 4 dB. The main lobe is also affected. Depending on the polarization and position of the antenna, the gain at zenith can fluctuate by \( \pm 0.3 \) dB with respect to the beam without coupling. The central antenna (ports 21 and 22) is the most affected. In terms of percentage, this can represent a difference of up to 6% (cf. Fig. 23). Besides, this value oscillates with a periodicity of about 20 MHz, which corresponds to reflections occurring at 15 m, about the distance between the centres of two dishes. This result is confirmed by the time analysis hereinafter. In addition to this modification in the maximum gain, the 3-dB beamwidth can also vary up to 1°. In the case of redundant-baseline calibration, all these elements matter. For instance, Orooz et al. (2019) simulated the impact of antenna-to-antenna variations on the estimation of the delay power spectrum measured by HERA, by modelling the beam with an Airy function. In particular, modifications of the beam shape, for example caused by a misalignment between the feeds, variations in the dish construction, or by mutual coupling, can significantly contaminate the EoR window. A variation of only 1 degree in the 3-dB beamwidth or in the beam pointing has prejudicial effects. Consequently, some antennas may have to be excluded from the calibration process, for instance the ones at the edges.

The S-parameters of the central antenna for the X and Y-polarizations are given in Fig. 24, in order to identify the antennas between which the coupling is the strongest. We can see that the level of coupling is moderate, below -35 dB, and that the central antenna is mainly coupled with the ports 31, 32, 37, and 39 (plus the other symmetric configurations) for the X-polarization (port 21), and 24, 26, 31, and 34 for the Y-polarization (port 22). Overall, the coupling between two antennas is the most intense when their dipoles
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Figure 22. Antenna 2D gain pattern without and with mutual coupling, for different antennas and polarizations in HERA 19, at 150 MHz, in the E and H-planes. Without mutual coupling, these patterns should be similar.

are parallel. However, these plots also reveal that the cross-polarization can also be affected in a non-negligible way. For instance, the level of coupling between the port 21 (X-polarization) and the port 32 (Y-polarization), and the level between the port 22 (Y-polarization) and the port 31 (X-polarization), varies between -45 dB and -55 dB. Thus, the strongest interactions occur between two adjacent antennas forming an angle of 60° with the X-axis. Further studies with the antennas at the edges and with the ports 5, 6, 25, and 26, allow us to isolate a pattern of antennas between which the coupling is the most significant. By setting a limit of -50 dB, these simulations show that the X-polarization of an antenna is mainly coupled with the antennas aligned in the H-plane as well as with the antennas in the columns directly positioned at its right and left. The Y-polarization is more affected by the antennas in the E-plane, and the antennas of the rows just above and below. Therefore, by isolating the antennas which are responsible for most of the mutual coupling as illustrated in Fig. 20, it is possible to expand the simulated array and verify if the coupling with farther antennas is important.

In order to study the effects on the X-polarization, the array is expanded along the Y-axis and includes 11 rows. As for the Y-polarization, the array is expanded along the X-axis and contains 11 columns. These two configurations correspond to half of the final core respectively in the H-plane and E-plane. The results of this analysis show that the S-parameters associated with mutual coupling go below -50 dB only after the 5th rows for the X-polarization, and after the 5th column as well for the Y-polarization. Therefore, it is not a phenomenon occurring only between nearby antennas.

So far, the properties of mutual coupling have been inferred from the results of simulations performed in transmission, when only one antenna is initially transmitting. The next step is to excite the structure of the array with a plane wave coming with a certain angle of incidence, in order to fully describe the system in reception (Lui et al.)
environment, the X and Y-polarisations for a given antenna can consequently, due to the difference in the electromagnetic environment, the X and Y-polarisations for a given antenna can be expressed as a function of the $Z$-parameters by solving the following system of linear equations:

$$V_1 = Z_{11}I_1 + \ldots + Z_{1k}I_k + \ldots + Z_{12N}I_{2N} = -Z_{\text{recL}}I_1$$

$$\vdots$$

$$V_k = Z_{1k}I_1 + \ldots + Z_{kk}I_k + \ldots + Z_{12N}I_{2N} = Z_{\text{ant}}I_k$$

$$\vdots$$

$$V_{2N} = Z_{2N1}I_1 + \ldots + Z_{2Nk}I_k + \ldots + Z_{2N2N}I_{2N} = -Z_{\text{recL}}I_{2N}.$$

(28)

In practice, the symmetries in the array can be used to simplify this system. Note that in the case of a 2-port system, we do find a result equivalent to Equation (14). Thus, Equation (27) is still valid.

Thanks to this method, the chromatic effects caused by the dish, the receiver, and the mutual coupling can be isolated or combined in order to estimate their respective contributions. Fig. 25 shows the system response between 100 and 200 MHz, for different array configurations, antennas, and polarizations, with respect to a fixed 125-Ω termination. This plot reveals that the mutual coupling significantly affects the system performance and is actually the dominant effect once the response is attenuated by a factor $10^5$.

Then, the signal slowly decays and eventually drops after a certain delay which corresponds to the propagation time between the considered antenna and the end of the simulated array. In order to know whether the mutual coupling is mainly caused by the interactions between the feeds via a re-radiation process of the received signal or by reflections between the dishes, the array in its horizontal configuration is also simulated with only one feed, still defined by the ports 1 and 2, and with the 33 dishes. In this configuration, the antenna time response at the port 2 (dark green curve) is almost similar to the response with all the feeds present (dark blue curve). Consequently, the mutual coupling in reception is mainly caused by the direct interactions between a given feed and the other dishes. Moreover, the comparison between the response of the ports 1 (red curve) and 2 (dark blue curve) shows that the receiving dipole is mostly affected by the dishes which are located in a perpendicular direction. Unsurprisingly, in this configuration the sidelobes are relatively high near the horizon as illustrated in Fig. 21, at a level of about 0 dB. This is confirmed by the analysis of the responses in HERA 19. Thus, the port 26 (light blue curve) is more chromatic than the port 25 (light green curve) which corresponds to the orthogonal polarization. Consequently, due to the difference in the electromagnetic environment, the X and Y-polarisations for a given antenna can have different responses. Besides, when the mutual coupling dominates the time response, reflections with a periodicity of about 50 ns can be observed. This delay corresponds to a propagation distance of 15 m, which is coherent with the distance between two dishes. Every 50 ns the contribution from one of the aligned dishes is received by the reference antenna. Extrapolating the slope of the system response in a strip configuration suggests that it would be attenuated by a factor $10^5$ between 600 and 700 ns, and by a factor $10^3$ after about 1000 ns. By comparison, the core of the array is about 300-m large along the X-axis and 250-m large along the Y-axis. In terms of propagation time, this is respectively equivalent to 1000 ns and 833 ns. Therefore, this study indicates that significant standing waves could form across the entire core, and that any antenna could interact with the dishes at the edges of the core. The level of the system response would remain above $10^{-3}$ only after a certain delay which mainly depends on the maximal distance between the antenna and the edges.

This analysis is confirmed by the study of the electric field propagating through an antenna array, and in the time domain. Fig. 26 presents several snapshots obtained with CST when the array is excited by a plane wave coming from the zenith and with a Y-polarization. The incident electromagnetic wave reaches the antenna at $t = 25$ ns, before being reflected by the dish at about $t = 50$ ns towards the zenith. However, a significant fraction of the wave is also reflected at 90° towards the edges of the dishes, as we can see at $t = 75$ ns. Then, the next snapshots illustrate how the reflected waves are able to propagate from one dish to another one, with little attenuation. Eventually, the waves coming from the antennas at the far left (or far right) reach the opposite antennas after 500 – 600 ns. Interestingly, the central dish is less affected by the mutual coupling, since the propagation time between this antenna and those at the edges is shorter. This explains why the port 22 (brown curve) in HERA 19 is less chromatic than the port 26 (light blue curve), despite having similar X-polarisation. Note that in this simulation, all the feeds have been removed except one at the far right, in order to analyse the propagation mechanisms between the dishes. Consequently, the interactions between the dishes are a major source of mutual coupling in the array. In terms of amplitude, the electric field which reaches the opposite antenna after propagating for 500 ns is between -40 and -50 dBV/m, i.e. between 0.01 and 0.003 V/m. As a comparison,
Figure 26. Snapshots of the electric field propagating through the array, when the antennas are excited by a plane wave coming from the zenith, at $t = 0$ ns, 25 ns, 50 ns, 75 ns, 100 ns, 150 ns, 200 ns, 300 ns, 400 ns, 500 ns, 600 ns, and 700 ns. The intensity of the E-field is expressed in dBV/m. Note how a part of the incident electromagnetic wave is reflected upwards and the other part towards the edges of the dishes. Then, the E-field propagates from one dish to the next one up to the end of the array, and with little attenuation.

the amplitude of the incident electric field is 1 V/m. Thus, the feeds receive a reflected field which is only about two to three orders of magnitude less intense than the incident field.

Lastly, all the sources of chromaticity are combined together in a final simulation. Fig. 27, 28, and 29 present the results for the X and Y-polarizations with the ports 41 and 2, and the measured receiver with a chromatic cable. Fig. 27 shows that the system time response can be divided into four distinct parts, each one being dominated by a particular type of chromatic effect: first the dish-feed reflections, then the mutual coupling, the micro-reflections within an imperfect cable, and eventually the reflection of the received signal at the end of the cable. Overall, in a large array the mutual coupling becomes the dominant effect a high delays. Fig. 28 and 29 illustrate the effects of the plane and angle of incidence on the system response, for an antenna at the edge of the array. Compared with Fig. 18 and 19, these plots provide an insight into the complexity of the interactions between the antennas, and reveal a significant level of chromaticity for a source near the horizon. Thus, if the studied antenna is at the edge of the array and an electromagnetic wave comes from the horizon, the system response of an antenna at the edge can spread up to a delay equal to twice the propagation time associated with the size of the array. In the same way, if the studied antenna is at the edge of the array and the wave comes from the most populated side of the array, the reference antenna receives the main signal more or less at the same time as the contributions from the other antennas. That is why the system response in the E-plane (resp. H-plane) of port 2 (resp. port 41) is significantly less chromatic for negative (resp. positive) angles of incidence. Moreover, it is also interesting to notice that even if the wave comes perpendicularly to the strip, which is the case when it comes from the H-plane (resp. E-plane) of port 2 (resp. port 41), the level of reflection is also intense. It implies that all the dishes reflect the incoming wave at the same time and in particular at $90^\circ$. In this case, the characteristic delay associated with the mutual coupling is equal to the propagation time between the studied antenna and the farthest antenna. Consequently, the system response of an antenna depends on its position in the array, the considered polarization, and the angle of incidence of the signal.

Mathematically, it is equal to:

$$\tau_{\text{maxCoup}} = \frac{D_{\text{max}}}{c} (\sin(\theta) + 1).$$

with $D_{\text{max}}$ the distance between the reference antenna and the farthest aligned antenna. Thus, for a signal coming from the horizon, the system response of an antenna at the edge can spread up to a delay equal to twice the propagation time associated with the size of the array. In the same way, if the studied antenna is at the edge of the array and the wave comes from the most populated side of the array, the reference antenna receives the main signal more or less at the same time as the contributions from the other antennas.
5 CONCLUSIONS AND FUTURE WORK

In this paper, we have presented a method to model and estimate the impact of the chromatic effects on the performance of an antenna in an array, including the effects of the cross-polarization leakage, RF receiver with cable, and mutual coupling. In particular, we have emphasized the importance of bringing together all these elements to properly characterize the receiver noise temperature, the antenna sensitivity, as well as the direction-dependent system response in the frequency and time domain, when it is excited by a plane wave linearly polarized. This method combines the formalism commonly used in RF network analysis with the electromagnetic properties of the antenna, and in particular its effective length. This requires to know the transmission, reflection and noise parameters of each block of the analogue receiver (FEM, coaxial cable, and PAM), the E-field radiated by the antenna in an array, and its differential impedance. In this work, the parameters of the RF receiver are either simulated with the microwave circuit simulation software Genesys or independently measured in our laboratory. As for the antenna, the beams are entirely simulated with CST, and its impedance is also simulated then validated with reflectometry measurements using a VNA. This method has the advantage to be flexible and each element can be independently replaced in order to try different scenarios and estimate their impact. Thus, these models can play an important role in the data analysis pipeline of HERA to understand and mitigate the chromatic effects. Note that in this paper, a Blackman-Harris function is applied to the system response in order to limit the leakage of the signal in the time domain associated with windowing.

This study reveals that the system is noisier than expected because of the mismatch between the antenna impedance and the amplifier optimum impedance. Thus, the receiver noise temperature presents some peaks up to 225 K, instead of 160 K with respect to a reference impedance of 100 Ω. In terms of antenna sensitivity, this represents a loss of 10%. Moreover, the system is also much more chromatic. Previous works based on simulations and measurements (Ewall-Wice et al. 2016; Thyagarajan et al. 2016; Patra et al. 2018) but performed with respect to a fixed termination impedance and without including mutual coupling suggested that the system response would be relatively quickly attenuated by a factor $10^4$, and therefore $k$-modes with a $k_\parallel$-component above $0.2 \, h\, \text{Mpc}^{-1}$ should be detectable, which is in agreement with our simulations. However, adding the effects of the receiver chain and mutual coupling significantly changes the performance of the system. To summarise, up to 200 ns, the chromaticity of the instrument is dominated by the reflections occurring between the feed and the dish, and mainly depends on the quality of the impedance matching. After having been attenuated by a factor $10^4$, the response very slowly decays because of the mutual coupling. The simulations suggest that any antenna can significantly interact with the antennas aligned along the X or Y-axis up to the edges of the core. Thus, the attenuation of the time response is limited by the position of the considered antenna and the size of the array. This could be problematic to calibrate the system, because this affects the number of truly redundant baselines. Once the propagation time associated with the size of the array is reached, the system response quickly drops to about $10^{-5}$, which was the initial goal. However, imperfections in the transmission cables can cause micro-reflections. In this case, the system response varies between $10^{-4}$ and $10^{-5}$. Lastly, the received signal is significantly reflected at about 1200 – 1300 ns because of impedance mismatch at the ends of the 150-m cable. Moreover, the study of the direction-dependant system response allows us to estimate the impact of the sidelobes and of the angle of incidence on the received signal. In particular, emissions from the horizon and coming from the less populated side of the array considerably spread the system time response, because of reflections with the farthest antennas.

The analysis of the data obtained in December 2017 with 46 HERA antennas closely packed are consistent with these simulations. These results are detailed in Kern et al. (2019 – in preparation). In particular, the study of the auto-correlated signals does show the presence of reflections caused by the cable at a similar level at 1200 ns. Moreover, up to 500 ns these data remarkably agree with our results, and present a level of chromaticity which decreases between $10^{-4}$ and $10^{-3}$ in the same way as our simulations. As expected, the mutual coupling does impact the antenna response in the array. However, after 500 ns the measured auto-correlated signals reach a noise floor at about $10^{-4}$. This could indicate either more complex “second order” interactions in a large two-dimensional array, or more chromatic cables deteriorated by the harsh environment. This difference could also be explained if more dishes were actually built but not connected when the data were recorded. Indeed, the mutual coupling is mainly caused by the reflections of the incident signal from one dish to another one.

By applying a strict foreground avoidance method without using an inverse covariance weighting technique to mitigate the foreground contribution, such as the “optimal quadratic estimator” (OQE) formalism described in Liu et al. (2010); Dillon et al. (2014); Liu et al. (2014); Ali et al. (2015), this analysis indicates that the detection of the EoR signal will be more challenging than expected with HERA Phase I. The response is attenuated by a factor $10^5$ only after 1400 ns, that is to say $k_\parallel$-modes above $0.7 \, h\, \text{Mpc}^{-1}$ at 150 MHz. However, the receiver system has been re-designed for the phase II of HERA, by using again the method developed in this paper to predict its performance (Fagnoni et al. 2019 – in preparation). New Vivaldi feeds are replacing the current dipoles, and new analogue receivers are being deployed in the Karoo desert. In addition to a wider bandwidth from 50 to 250 MHz, the new Vivaldi feeds should also be slightly less chromatic since they do not require a cage to properly radiate towards the dish. Besides, the coaxial cables are being replaced by 500-m optical fibres, which should resolve the problems associated with the reflections in the cables, and unlock lower $k_\parallel$-modes. In this context, mutual coupling will become the limiting factor. The size of the array is an advantage to reach the sensitivity required to detect the EoR signal, but it also increases the level of chromaticity. Therefore, it will be interesting to study new solutions to mitigate this phenomenon, for instance, by installing serrated or rolled edges on the dishes (Burnside et al. 1987; Gupta et al. 1990; Lee & Burnside 1996), or by adding conical shrouds on the rims (Landecker et al. 1997). Thanks to
these elements, it may be possible to limit the diffraction of the E-field on the edges of the dishes or to partially redirect it upwards, thus limiting the propagation of the signal from one dish to another one.

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REFERENCES

Ali Z. S., et al., 2015, ApJ, 809, 61
Anaren 2005, Measurement techniques for Baluns, https://cdn.anaren.com/multimedia/PDF/MeasurementTechniquesForBaluns.pdf
Bockelman D. E., Eisenstadt W. R., 1995, IEEE Trans. Microw. Theory Techn., 43, 1530
Bowman J. D., Rogers A. E. E., Monsalve R. A., Mozdzen T. J., Mort B., 2016, MNRAS, 458, 2028
Clemens M., Weil T., 2001, PIER, 32, 65
Chapman E., Mort B., 2016, MNRAS, 458, 2928
Chapman E., Zaroubi S., Abdalla F. B., Dulwich F., Jelić V., Mahesh N., 2018, Nature, 555, 67
Burnside W., Gilreath M., Kent B., Clerici G., 1987, IEEE Trans. Antennas Propag., 35, 176
Chapman E., et al., 2012, MNRAS, 423, 2518
Chapman E., Zaroudi S., Abdalla F. B., Dulwich F., Jelić V., Mort B., 2016, MNRAS, 458, 2028
Cortes-Medellín G., 2007, Antenna Noise Temperature Calculation, https://www.skatelescope.org/uploaded/6967_Memo_95.pdf
Cräeye C., González-Ovejero D., 2011, Radio Sci., 46
Datta A., Bowman J. D., Carilli C. L., 2010, ApJ, 724, 526
DeBoer D. R., 2015, HERA Phase 1 Feed Design, http://reionization.org/wp-content/uploads/2015/01/feedP1.pdf
DeBoer D. R., et al., 2017, PASP, 129, 045001
Dillon J. S., Parsons A. R., 2016, ApJ, 826, 181
Dillon J. S., et al., 2014, Phys. Rev. D, 89, 023002
Dillon J. S., et al., 2018, MNRAS, 12, 1
Ewall-Wice A., et al., 2016, ApJ, 831, 196
Ewall-Wice A., Dillon J. S., Liu A., Hewitt J., 2017, Monthly Notices of the Royal Astronomical Society, 470, 1849
Fagnoni N., de Lera Acedo E., 2016, in Int. Conf. Electromagn. Adv. Appl. (ICEAA). IEEE, Cairns, Australia, pp 629–632, doi:10.1109/ICEAA.2016.7731474
Fan W., Lu A., Wai L. L., Lok B. K., 2003, in Proc. of 5th Electrom. Packag. Technol. Conf. (EPTC). IEEE, Singapore, pp 533–537, doi:10.1109/EPTC.2003.1271579
Frickey D., 1994, IEEE Trans Microw. Theory Techn., 42, 205
Furlanetto S. R., 2016, in A. M., ed., Understanding the Epoch of Cosmic Reionization: Challenges and Progress. Springer International Publishing, Chapt. The 21-cm Line as a Probe of Reionization, pp 247–280, doi:10.1007/978-3-319-21957-8_9
Gupta I. J., Ericksen K. P., Burnside W. D., 1990, IEEE Trans. Antennas Propag., 38, 853
Harker G., et al., 2010, MNRAS, 405, 2492
Harris F. J., 1978, Proc. IEEE, 66, 51
Jacobs D. C., et al., 2016, ApJ, 825, 114
Kurokawa K., 1965, IEEE Trans. Microw. Theory Techn., 13, 194
Landecker T. L., Smeal R. J., McKinley J. M., 1997, Radio Sci., 32, 2139
Lee T.-H., Burnside W. D., 1996, IEEE Trans. Antennas Propag., 44, 87
Liu A., Teagmark M., Morrison S., Lutominiski A., Zaldarriaga M., 2010, MNRAS, 408, 1029
Liu A., Parsons A. R., Trott C. M., 2014, Phys. Rev. D, 90, 023018
Lui H.-S., Hui H. T., Leong M. S., 2009, IEEE Antennas Propag. Mag., 51, 171
McQuinn M., Lidz A., Zahn O., Dutta S., Hernquist L., Zaldarriaga M., 2007, MNRAS, 377, 1043
Mesinger A., Ewall-Wice A., Hewitt J., 2014, MNRAS, 439, 3262
Moraes M. F., Wyithe J. S. B., 2010, Annu. Rev. Astron. Astrophys., 48, 127
Moraes M. F., Hazelton B., Sullivan I., Beardsley A., 2012, ApJ, 752, 137
Orfanidis S. J., 2016, Electromagnetic Waves and Antennas. Orfanidis S. J.
Oroz N., Dillon J. S., Ewall-Wice A., Parsons A. R., Thyagarajan N., 2019, MNRAS, 487, 537
Parashare C. R., Bradley R. F., 2006a, Memo P011: 75 Ω Transmission System, http://eor.berkeley.edu/wp-content/uploads/2011/09/p011.cparashare.pdf
Parashare C. R., Bradley R. F., 2006b, Memo P012: 120-205 MHz Receiver for PAPER: Precision Array to Probe the Epoch of Reionization, http://eor.berkeley.edu/wp-content/uploads/2011/09/p012.cparashare.pdf
Parashare C. R., Bradley R. F., 2007, Memo P013: Instrument Development for PAPER: A Precision Array to Probe the Epoch of Reionization, http://eor.berkeley.edu/wp-content/uploads/2011/09/p013.cparashare.pdf
Parsons A., et al., 2010, AJ, 139, 1468
Parsons A., Pojer J., McQuinn M., Jacobs D., Aguirre J., 2012a, ApJ, 753, 81
Parsons A. R., Pojer J. C., Aguirre J. C., Carilli C. L., Jacobs D. C., Moore D. F., 2012b, ApJ, 756, 165
Patra N., et al., 2018, Exp. Astron., 45, 177
Pober J. C., et al., 2014, ApJ, 782, 66
Pober J. C., et al., 2015, ApJ, 809, 62
Pozar D. M., 1998, Microwave Engineering, 2nd edn. Wiley
Pritchard J. R., Loeb A., 2012, Rep. Prog. Phys., 75, 086901
Rahola J., 2008, IEEE Trans. Circuits Syst., II, Exp. Briefs, 55, 92
Thompson A. R., Moran J. M., Swenson G. W., 2017, Interferometry and Synthesis in Radio Astronomy, 3rd edn. Springer International Publishing, doi:10.1007/978-3-319-44341-4
Thyagarajan N., et al., 2013, ApJ, 776, 6
Thyagarajan N., et al., 2015a, ApJ, 804, 14
Thyagarajan N., et al., 2015b, ApJ, 807, L28
Thyagarajan N., Parsons A. R., DeBoer D. R., Bowman J. D., Ewall-Wice A. M., Neben A. R., Patra N., 2016, ApJ, 825, 9
Tingay S. J., et al., 2013, Publ. Astron. Soc. Aust., 30, e007
Van Haarlem M. P., et al., 2013, A&A, 556, A2
Vernon R. J., Seshadri S. R., 1969, Proc. IEEE, 57, 101
Weiland T., 1977, Electronics and Communications AEU, 31, 116
Wyithe J. S. B., Loeb A., 2004, Nature, 432, 194
Zaldarriaga M., 2007, MNRAS, 377, 1043
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Figure 28. Time response of the system terminated by the measured RF receiver and including mutual coupling for the ports 41 (X-polarization) and 2 (Y-polarization), in the E and H-planes.
Figure 29. Maximum delays before the system response is attenuated by a certain factor, with the measured receiver, including mutual coupling, for the ports 41 (X-polarization) and 2 (Y-polarization), and for the E and H-planes.