Numerical Studies on the In-situ Measurement of Inclusions in Liquid Steel Using the E.S.Z. or LiMCA Technique

Xiaodong WANG, Mihaiela ISAC and Roderick I. L. GUTHRIE

McGill Metals Processing Centre, McGill University, 3610 University Street, M.H.Wong Bldg., Room 2M040, Montreal, Qc, H3A 2B2, Canada. E-mail: Xiaodong.Wang@mcgill.ca, Mihaiela.Isac@mcgill.ca, Roderick.Guthrie@mcgill.ca

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The direct detection of inclusions first became possible for electronically conducting fluids following the development of the LiMCA (Liquid Metal Cleanliness Analyzer) technique. The principle of this technique is based on the R.P.T. (Resistive Pulse Technique), or E.S.Z. (Electric Sensing Zone) method, for counting, and sizing, inclusions in liquids. Its application to steel melts is now studied theoretically, in order to help in the understanding and analysis of in-situ experimental measurements of inclusion size and frequency distributions in steel plant processing operations.

In developing the theoretical model for this technique, a three-step strategy was used to explore physical events that take place within the electric sensing zone, as an inclusion passes through. First, a multiphase flow model was required, in which events that can take place during the passage of the inclusion through the sensor's ESZ were considered. Inclusion trajectories and transit times were predicted using a particle momentum equation. For this, Newton's Second Law of motion was solved, in which the mass and instantaneous acceleration of the particle was balanced against the sum of the various forces acting on the particle/inclusion, during its passage through the ESZ. The forces summed included Stokes's drag, fluid acceleration, particle acceleration with added mass, gravitational and, in particular, the external self-conducting electromagnetic force induced by passing a heavy direct current through the ESZ of an electronically conducting liquid.

In the second step of this theoretical analysis, a numerical potential-integral method was conceived in order to calculate local changes in electrical resistance within an ESZ of variable geometry, and variable location of a traversing particle. This new approach was compared to alternative analytical and numerical estimates of changes in ESZ resistivity with a second phase particle within it, which neglects the effects of radial position of the particle.

In the third and final step in the present analysis, parabolic, fluted, and cylindrical ESZ's were selected, and the influence of fluid properties, ESZ dimensions, electric currents, and the inclusion's properties (electrical conductivity, density, size, shape, etc.), were investigated to determine how these various parameters affect the resistive (or voltage) pulses generated during the passage of an inclusion through the ESZ.

KEY WORDS: inclusions; liquid steel; inclusion size distributions; particle discrimination; electromagnetic forces; orifice geometry; resistive pulse; multiphase flow; inclusion trajectories.

1. Introduction

The need to count, and size, microscopic particles entrained within a sample of liquid, is of interest for many chemical and liquid metal processing operations. Understanding that the presence of a particle, whose electrical conductivity differs from that of the conducting fluid, will change the electrical resistance of the whole, Coulter\textsuperscript{1} conceived and demonstrated the viability of the resistive pulse technique for measuring inclusion sizes and frequency distributions in aqueous, ionically conducting liquids. Its application to steel melts is now studied theoretically, in order to help in the understanding and analysis of in-situ experimental measurements of inclusion size and frequency distributions in steel plant processing operations.

The central problem of LiMCA is to determine the resistive pulse signal when an inclusion, whose electric conductivity differs from the entraining liquid metal, passes through an orifice set in the inclusion sensor probe. To reach this objective, the kinematics and dynamics of the inclusions passage through the ESZ needs to be clarified and justified. The inclusion's motion, behavior and trajectory, needs to be determined for a wide range of possible conditions. This problem is a typical multiphase flow problem. It

Figure 1 shows a schematic of the measuring portion of the probe developed by Heraeus Electro-Nite for the on-line sensing of inclusions in steels, marketed under the name ESZ-PAS. The chamber is constructed of a closed-end quartz tube provided with a push-on connector. A steel tube inside the quartz tube acts as the positive electrode and also as the gas conduit for inert-gas purging and vacuum. The aperture, drilled by laser, is positioned near the immersion end. The electrodes inside the chamber consist of two pairs of steel rods extending from the steel tube. The first pair of them acts as the anode for contacting the molten metal entering the chamber; the second pair terminate at a chill block acts as the cathode, which also as a volume regulating device. The sampling sequence begins with a system diagnostic routine that looks for pressure leaks in all connections and detects and measures the size of orifice using a pressure decay method.

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is appropriate to note that inertial effects on inclusions passing through converging shaped orifice need special attention, owing to added mass effects. Similarly, heavy DC currents are passed through the ESZ in order to be able to generate and detect resistive voltage pulses associated with the passage of inclusions in highly conducting liquids. These necessarily high currents, in turn, uniquely lead to significant self-induced electromagnetic forces which, in their turn, affect particle dynamics.

Following initial development work researching the application of the LiMCA technique to aluminum,\(^{18-20}\) zinc, lead solders, gallium,\(^{6}\) copper,\(^{10}\) and magnesium,\(^{14,15,17}\) efforts were made to extend the technique to molten iron\(^{11}\) and liquid steel.\(^{9,13,14}\) The present study was carried out in order to strengthen the successful application of this technique for the steel making industry.

In this paper, we focus on the development of a numerical model for melts of steel, so as to be able to predict the shapes and sizes of resistive pulses, based on first principles. Section 2 introduces the theories in terms of the ESZ and develops a new integral method for computing the resistive pulse. Section 3 is devoted to numerical results. Section 4 provides a discussion of the results of the present computations and general conclusions.

2. Theory and Numerical Methods

2.1. Multiphase Flow

The present two-phase flow inclusion sensor system for steel melts constitutes a “low” concentration of dispersed inclusions within a continuous phase of conducting liquid steel (\(\sigma=0.72 \times 10^5\) S/m)). This low volumetric concentration of inclusions is not expected to disturb the fluid motion of liquid steel as it flows through the small ESZ, provided no clogging or erosion of the ESZ, takes place. Consequently, in the absence of such phenomena, when a steel sample can be drawn into the sampling tube without any difficulties, we can first solve the fluid flow field through the ESZ using an Eulerian approach, and then model the inclusion’s motion as it passes through the ESZ, using a Lagrangian approach.

2.1.1. Fluid Field of Liquid Steel

We can assume the flow of liquid steel within the ESZ set in the silica steel sampling tube to be steady and laminar, owing to a low Reynolds number (\(Re \sim 3\,500\), taking the diameter of the throat of the orifice as the typical length scale, \(D=500\,\mu m\), and a typical velocity for steel flow through the ESZ of \(2\,m/s\)). As such, the flow of liquid steel can be described by the continuity and incompressible Navier–Stokes equations:

\[
\nabla \cdot \mathbf{u} = 0 \tag{1}
\]

\[
\rho_s \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \mu_s \nabla^2 \mathbf{u} + \mathbf{F}_\text{em} \tag{2}
\]

where \(\mathbf{u}(u,v)\) is the velocity vector of liquid steel; \(p\) pressure; \(\rho_s, \mu_s\) density and viscosity of liquid steel; and \(\mathbf{F}_\text{em}\) the Lorentz electromagnetic force density exerted on the liquid steel within the ESZ.

2.1.2. Electromagnetic Force

In the present study, the interplay of physical phenomena between the fluid dynamics and electromagnetic field is that of one-way coupling. In other words, the flow of a low Reynolds Number fluid conductor will have a negligible influence on the induced electromagnetic force, but the impact of the electromagnetic force on the liquid metal flow can be very significant, and must be considered. Such a force is treated as a source term and is introduced in Eq. (2). A two-dimensional axisymmetry model employing a cylindrical coordinate system \((r, z)\) was used to compute the fluid flow within the ESZ. The Lorentz electromagnetic force is defined as:

\[
\mathbf{F}_\text{em} = \mathbf{J} \times \mathbf{B} \tag{3}
\]

where \(\mathbf{J} = -\sigma_e \nabla \varphi\) (Ohm’s law), is the electric current density, \(\sigma_e\) is the electrical conductivity, which, for steel, is \(0.72 \times 10^6\) (\(\Omega\cdot m\)^{-1}), and \(\nabla \varphi\) is the electric potential. \(\mathbf{B}=\nabla \times \mathbf{A}\) is the self-induced magnetic field within the ESZ, where \(\mathbf{A}\) is the magnetic vector potential. The scalar electric potential \(\varphi\) can be solved through the Laplace equation:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \varphi}{\partial r} \right) + \frac{\partial^2 \varphi}{\partial z^2} = 0 \tag{4}
\]

The self-induced magnetic field is derived from Ampere’s law:

\[
\int_C \frac{B}{\mu_0} dl = \int_S \mathbf{J} \cdot d\mathbf{A} = I \tag{5}
\]

where \(\mu_0 (=4\pi \times 10^{-7}\,\text{H/m})\) is the magnetic permeability in free space; \(I\) is the input electric current.

2.1.3. Equation of Motion for Inclusion

The inclusion’s trajectory can be predicted using a Lagrangian approach by solving the relevant momentum equations, assuming that the inclusions entrained within the liquid steel are rigid, spherical, particles (re-oxidized alumina inclusions, silicates, aluminum-silicates, and entrained slag droplets, are often in spherical form).\(^{29}\) The momentum equation used to describe this transient process is,

\[
\frac{\pi}{6} \rho_i D^3 \frac{du_i}{dt} = \frac{\pi}{8} d^2 C_{Dp} \rho_s \left| u_i - u_p \right| (u_i - u_p) + \frac{\pi}{6} d^2 \rho_i \rho_s \frac{Du_i}{dt} + \frac{\pi}{6} d^2 C_{sp} \rho_s \left( \frac{Du_i}{dt} - \frac{du_p}{dt} \right)
\]

\[
+ \frac{\pi}{6} d^3 (\rho_i - \rho_s) g = \frac{3\pi}{8} (1 - \chi) d^3 \mathbf{F}_\text{em} \tag{6}
\]

Subject to the initial condition:

\[
u_i(t=0) = u_i(t=0) \tag{7}
\]

Note that the computational domain selected is only valid, provided the initial velocities of particle and fluid are equal.

In Eq. (1), the term on the left-hand side represents the force required to accelerate the inclusion. The first term on the right-hand side models viscous resistance according to Stokes’s law. \(C_D\) is the drag coefficient which can be expressed as \(C_D = (24/Re_p)(1+0.15Re_p^{0.67})\) based on the Reynolds number of the inclusion: \(Re_p = (\rho_s d |u-u_p|/\mu_s)^{0.5}\). The second term is due to the pressure gradient resulting from the acceleration of the liquid steel. Note that the derivative of fluid velocity with respect to time is in an Eulerian form: \(Du_i/ dt = du_i/ dt + \mathbf{u} \cdot \nabla \mathbf{u}_i\). The first term on the right-hand side is zero. The third term in Eq. (6) is the force required to accelerate the apparent mass of the inclusion relative to the ambient liquid steel. Note that in a previous study by Mei Li and R. Guthrie,\(^{21-23}\) the Basset history force due to unsteady effects was also considered, but shown to be negligible. In the present study, the inertial ef-
fect is also much higher than that of the drag, so the history force can again be omitted from the momentum equation. The fourth term is the buoyancy force. Again, under the set of operating conditions applying for LiMCA, the buoyancy term also has a negligible influence on the motion of the inclusion. The fifth term is the electromagnetic body force on the inclusions, which acts in the opposite direction to the electromagnetic force acting on the liquid steel within the ESZ. The term \( \frac{\partial^2 \rho \sigma}{\partial t^2} \) represents the dependence of the electromagnetic force on the inclusion’s electrical conductivity \( \sigma_e \) and that of the liquid steel \( \sigma_m \).

Finally, the presence of a solid orifice wall, in the vicinity of a moving particle, will affect the fluid’s drag force on it.\(^{25,26}\) The present numerical solution also considers this boundary effect. Further details are given in Ref. 22).

### 2.2. Computation of Resistive Pulse

#### 2.2.1. Maxwell’s Formula

The change in the effective electrical conductivity of a fluid when a particle of different electrical conductivity is present within it was of early interest to theoreticians. The circumstance of a static uniform electric field prior to the presence of a particle was investigated by Maxwell.\(^{27}\) He obtained an expression for the effective resistivity, \( \rho_{\text{eff}} \), for an insulating inclusion (i.e., zero conductivity) within a cylindrical tube, containing a liquid of resistivity \( \rho_e \) (= 1/\( \sigma_e \)). Maxwell’s approximation is,

\[
\rho_{\text{eff}} = \rho_e \left( 1 + 3f/2 + \ldots \right). 
\]

Here \( f \) represents the fractional volume of the particle within the conducting liquid. Based on this effective resistivity, an increase in resistance, \( \Delta R \), in the presence of a non-conducting inclusion is given by,\(^{19,27}\)

\[
\Delta R_{\text{non-conducting}} = \frac{4 \rho_e d^3}{\pi D^2}. 
\]

For a perfectly conducting inclusion, the change in effective resistance is,

\[
\Delta R_{\text{conducting}} = -\frac{8 \rho_e d^3}{\pi D^2}. 
\]

When the electric conductivity of an inclusion lies between non-conducting and perfectly conducting values, the change in resistance will fall between the values given by Eq. (9) and Eq. (10). Maxwell notes that Eq. (9) represents a lower bound to the resistance, and that Eq. (9) is only correct for a sphere that is small in comparison to the orifice diameter.

When the inclusion size increases towards that of the orifice, then there is a correction factor which must be applied, that will be a function of \( f(d/D) \). There are several expressions for \( f(d/D) \); Ref. 3 supplies a multiplier, \( f(d/D) = (2/3)[1 - 0.8(d/D)^3]^{-1} \).

Another expression, that is valid over a broader range of particle to orifice size ratios, requires a complete solution of Laplace’s equation for the potential, subject to boundary conditions at both the sphere and the orifice wall. De-Blois\(^{24}\) solved this problem for a sphere in a uniform cylindrical orifice. By solving Laplace’s equation that satisfied the spherical boundary conditions around the particle, he obtained the approximation,

\[
\Delta R_{d/D_m<1,D_m/L<1} = \frac{4 \rho_e d^3}{\pi D_m^2} \left[ 1 + \frac{3}{8} \left( \frac{D_m}{L} \right)^4 + \ldots \right]. (11)
\]

Here \( D_m \) represents the cross-section of a uniform tube whose diameter is equal to that of the distorted current field at its maximum bulge (see Fig. 2). This expression has the Maxwellian value as its limit for \( d \ll D \). This result is extraordinarily insensitive to the distance \( L \) over which the potential is measured, in agreement with Eq. (9).

#### 2.2.2. Ohm’s Model

There is no exact solution for this problem.\(^{24}\) Hence, one must use approximations. One of the most useful approximations for the resistance of a tube of varying cross section is,

\[
R = \rho_e \int \frac{dz}{A(z)}. 
\]

where \( A(z) \) is the cross-sectional area perpendicular to a length coordinate \( z \).

When a non-conducting spherical inclusion is entrained into the ESZ, the resistive pulse generated, \( \Delta R \), can be computed according to:

\[
\Delta R = \rho_e \int \frac{A_i(z)dz}{A_i(z) - A_o(z)}. \]

where \( A_i(z) \) and \( A_o(z) \) are the surface areas on each electric potential contour (equi-potential) occupied by the liquid steel and the inclusion, respectively. In Refs. 17–20, for simplifying computations, the authors assumed that the equi-potential contours are a series of vertical slices, perpendicular to the axis, extending to the wall. With contoured wall this is not the case, and the change in resistance will be underestimated. To overcome this defect, in Ref. 22), we numerically integrated the areas of equi-potential contours occupied within the liquid metal domain. Due to the complexity of the computations within the domain of an inclusion, we also used an approximate method in computing the relative areas occupied by the fluid and particle (or curved slices) by neglecting the deformation of the electric potential due to the presence of the non-conducting inclu-

### Fig. 2.

Distortion of electric field (streamlines and equi-potential contours) in (a) absence of inclusion (uniform field); (b) presence of a non-conducting inclusion and (c) presence of a perfectly conducting inclusion (\( \sigma_e = 10^7 \) (\( \Omega \cdot m \)^{-1}). The diameter of the spherical inclusion is \( d = 100 \mu m \), placed in liquid steel, Input current \( I = 20 \) A.
sion.

2.2.3. Numerical Integral Method

Both Maxwell’s Formulas, and Ohm’s model, mathematically treat the details of potential contours which become distorted around an inclusion. But for the present LiMCA orifices of non regular geometry, the distribution of the electric field within the steel is not uniform, and a new numerical approach has been developed. As is well-known, the resistance for a segment of a regular cross-section isotropic conducting media, when a certain voltage \( U \) is imposed on the two ends, and current \( I \) is delivered, the resistance can be defined as \( R=U/I \). From the view of electromagnetism, when an inclusion, whose electrical conductivity differs from the liquid metal, enters into a uniform electric field within the liquid metal, this electric field becomes distorted. A two-dimensional electric field is modelled to illustrate this distortion. In Fig. 2(a), the electric field is seen to be uniform, and the electric streamlines are equidistant parallel lines. When a non-conducting inclusion is introduced into this uniform electric field, the electric streamlines have to distort around the inclusion, and the orthogonal equi-potential contours near the inclusion then have to contract towards it, as illustrated in Fig. 2(b); Inversely, when a perfectly conducting inclusion (i.e. the electric conductivity \( \sigma_e=10^5(\Omega \cdot m)^{-1} \) enters a uniform electric field, the electric streamlines contract towards and into, the inclusion, while the corresponding equi-voltage contours near the inclusion become concave, as illustrated in Fig. 2(c).

For the case of an inclusion present in liquid steel within the ESZ, such as that illustrated in Fig. 1, the electric field will be redistributed around it. Since the electric potential of the inclusion become concave, as illustrated in Fig. 2(c). As a consequence, the change in resistance can be computed by the following equation,

\[
\Delta R = \int \frac{(U_i-U_o)dx}{A} \tag{14}
\]

where \( U_i, U_o \) the voltage of each cell on the electric outlet boundary surface of the ESZ when the inclusion is present or absent, respectively; \( A \), the area of the electric outlet surface.

3. Results

3.1. Fluid Dynamics in the ESZ

To ensure that the liquid steel with dispersed inclusions entering the ESZ during the measurement is representative, we need to ensure that the inclusion’s entry into the orifice is successful, and that the two-phase flow is well set as it enters the ESZ. That is, the flow can approximately satisfy boundary condition Eq. (7), such that the particle enters the ESZ domain at the same speed and path-line as the entraining liquid. This will be the case, provided the Reynolds number of inclusion is low, and buoyancy forces are moderate. In the present study therefore, the initial conditions were set at: \( u_p(t=0)=u_0(t=0)=0.2 \text{ m/s} \).

3.1.1. Orifice Shape

From a hydrodynamic point of view, the orifice should ideally provide for a streamlined entry, such that the cross-sectional diameter of the entrance is larger than that of the throat. This is helpful in allowing a representative liquid element to successfully enter, and pass through, the ESZ. Based on the law of mass conservation, a narrow entrance will lead to a significant perturbation in the flow field outside the ESZ, as liquid accelerates into the ESZ axially. Alternatively, a large aperture provides for a curved surface, and this can lead to the risk of recirculating flows inside the ESZ at high current densities, leading to anomalous inclusion behavior.

To study the influence of a shaped-wall on the fluid patterns and self-induced electromagnetic forces, three typical ESZ geometries, illustrated in Fig. 3, were considered. Going from “sharp” to “smooth”, Fig. 3(a) shows a cone-cylinder ESZ, fabricated from BN for magnesium melts. The locus of the conical wall is described by \( z=ar+b \) referenced to the throat of the ESZ. The other two orifice walls are parabolic, Fig. 3(b) with \( r \)-axisymmetry \( (r=a^2z^2+c) \) and for Fig. 3(c) \( z=dr^2+ce \) where the coefficients \( a, b, c, d, e, r_0, c \) are constants. It is appropriate to note that the selection of orifice geometry and materials of construction depends on the “corrosiveness” of the liquid metal on the sensing tube, and the technique for producing the small orifice for the ESZ. For example, alumino-silicate tubes are used for water and aluminum and a Type II ESZ LiMCA probe is manufactured by glass blowing. Boron nitride tubes are chemically inert to melts of magnesium, and a type I ESZ is drilled mechanically. Silicon tubes are used for steel melts, and a type III ESZ is produced by laser ablation.

3.1.2. Electromagnetic Force Distribution

The electromagnetic force field is self-created by the input of a direct current passing through the electric sensing zone (ESZ) from the inner anode inside the tube, to an outer cathode. A typical input electric current \( I=20 \text{A} \) for a liquid steel ESZ-PAS system is used in the present study. Note that in such cases of lower magnetic Reynolds number, the electromagnetic field is little influenced by the velocity of the liquid steel flow. As shown in Fig. 4, the electromagnetic body force on the liquid steel diminishes asymptotically from its highest value at the wall of the orifice in moving toward the axis. Also, the electromagnetic force near the wall of the throat of the orifice is much stronger than that in the entrance or exit regions of the ESZ.

3.1.3. Velocity Profiles

The present flow can be considered as a rapid, smoothly converging, micro fluidic flow within a shape-varying short duct. Velocity fields for the three types of orifices are compared for a direct current of 20A, and an initial velocity 0.2 m/s (normal to the inlet control surface of the ESZ), as illustrated in Fig. 5. As the cross-sections become smaller, the streamlines contract as the flow accelerates. After the vena contract, the flow domain expands for ESZ types I and

![Fig. 3](https://via.placeholder.com/150)

**Fig. 3.** Three types of shaped orifice and the relative computing domain of the ESZ with non-orthogonal boundary-fitted grid. (a) Type I: opposing cones and cylinder \((z=ar+b)\); (b) type II: \(r\)-axisymmetry parabola \((r=a^2z^2+c)\); (c) type III: fluted entry parabola \((z=d(r-r_0)^2+c)\).
II, but flow separation takes place, leading to high velocity jet flows as the liquid leaves the ESZ. For the type III ESZ this is not an issue. Figure 5 shows that for the same entry flowrates into the ESZ, the maximum throat velocities can reach to 3.37 m/s, 3.37 m/s, and 0.93 m/s for type I, II, and III sensing zones.

Note that as the exit cross-section expands in the type I and II ESZ, there is an accompanying re-circulating flow, as illustrated in subfigures of Figs. 5(a), 5(b) where normalised velocity fields are shown. These subfigure velocity fields clearly show that a small amount of liquid metal inside the sampling tube will re-enter the ESZ. If inclusions within the sampling tube are re-entrained in such backflows, the measurements will be compromised. The type III ESZ for steel melts poses no such risks.

3.2. Influence of Electromagnetic Force Intensity on Fluid Motion

The electromagnetic force vectors act perpendicularly to the equi-potential contours, as shown in Fig. 4. As the input currents are increased for improved resistive pulses, these body forces also increase. Once the input current reaches a certain threshold value, the axial components of the electromagnetic forces opposing fluid entry, become greater that the fluid pressure differential driving the entry flow, and flow reversals can be generated. Figure 6 illustrate this evolution in the flow patterns as input currents increase from $I=20, 50, 100$, and $300$ A respectively. The advantages and disadvantages of reversed flows are discussed in Ref. 22). According to the principle of LiMCA, this situation should be avoided during normal operations in order to allow entrained inclusions to pass through the ESZ successfully. However, our research with aluminum melts has shown that such over-imposing currents can effectively remove any inclusions accumulating on the walls of the orifice. Indeed, to ensure that the orifice is clean at the start of an inclusion sampling measurement, a heavy current is purposefully used for aluminum melts (not steel), to scour out any accumulating ‘rafts’ of inclusions at the entrance to the type II ESZ orifice.

3.3. Kinematics and Dynamics of Inclusion Motion

3.3.1. Forces Acting on Inclusion

In the Sec. 1.1, we determined that there are essentially four important force terms: 1) Stokes drag force, 2) fluid acceleration force, 3) the added mass force and 4) the electromagnetic forces acting on the inclusion. In this section, we discuss the dynamic role of each of these forces, as applied to the Type III ESZ used for the steelmaking probe. By numerically solving Eq. (1), the resolved forces in the axial and radial directions are compared along with corresponding trajectories of inclusions for three entry locations at $r=0$, $r=0.4R$, and $r=R$ for the Type III orifice, where $R$ is the radius of the fluted entry orifice. These are shown in Fig. 7 and Fig. 8. Given the geometry of the type III ESZ, it is more conveniently divided into two segments, segment I: $z=0.6$ mm, segment II: $0.6 < z < 0.6$ mm, for the sake of discussion.

In the radial direction, as seen in Fig. 7(a), the Stokes drag force along the $z$ axis of symmetry is zero because the inward radial forces are opposite and equal. As the radius...
increases, the inward acting radial drag force component increases with $z$, until the second segment of uniform cross-sectional area, is reached. This force is virtually constant for segment II, and in the order of $(10^{-5}) \text{N}$.

The radial component of the electromagnetic force is shown in Fig. 7(b). Along the centerline this force is zero. As the radius increases, this force also increases to a maximum at the wall. Note that the force exerted on the non-conducting inclusion acts towards the orifice wall. When the input current is $I=20 \text{A}$, $r>0.4R$, the order of magnitude reached is in the order of $(10^{-5}) \text{N}$.

The radial component of the fluid acceleration force is shown in Fig. 7(c). This force is numerically small except in the region of intersection of segments I and II, close to the wall. Clearly, it results from the change of cross-section.

The radial component of the added mass force is shown in Fig. 7(d). This force component pushes the lighter density inclusion towards the wall in the segment I, the magnitude can reach to the order of $(10^{-5}) \text{N}$.

In summary, in the radial direction, when the input current is about 20A, the inclusion is mainly affected by the added mass force, owing to the high density ratio between liquid steel and light density inclusions (here the inclusion is alumina).

Figure 8 shows the axial force components acting on the 100 micron diameter inclusion. The axial Stokes drag force component yields small values except in the region close the wall, where the wall effect act obviously. This force
component is negative with respect to the $z$ direction, as shown in Fig. 8(a).

The axial component of the electromagnetic force is shown in Fig. 8(b). The magnitudes are much smaller than that of radial component (see Fig. 7(b)). According to principal of the definition of Lorenz force and Laplace theorem, only in the segment I the electromagnetic force yield a small value and vanish in the segment II.

The axial component of the fluid acceleration force is shown in Fig. 8(c), the fluid acceleration force increases the relative velocity of liquid steel and inclusion. It increases significantly in segment I and drop off in the segment II.

The axial component of the added mass force is shown in Fig. 8(d), which yields the same trend as the axial component of the acceleration force. It accelerates the inclusions entrained in liquid steel.

Note that in the axial direction, the acceleration force and the added mass force play an essential role, their magnitude can reach to the order of $10^{16}$ N, they both accelerate the inclusions within the liquid steel flute flow within the ESZ.

### 3.3.2. Inclusion Trajectories

Once the fluid flow and the electromagnetic fields within the ESZ are known, the inclusions’ trajectories can next be modeled by solving Eqs. (6) and (7), using a Lagrangian approach, as illustrated in Fig. 9. As seen, 100 $\mu$m inclusions pass through each of the three ESZ orifice configurations. As seen in Fig. 9, for $I=20$ A, the inclusions successfully traverse the ESZ without impacting the sidewalls of the orifice. Note that according to the LiMCA principle, the residence time of the inclusion passing through the ESZ is very important. It is in the order of milliseconds for all three orifice types. For the type I and II orifices, the inclusions move slowly in the entrance region, and then accelerate to their maximum value at the throat of the orifice.

The important and fortunate feature, as shown in Fig. 9, is that the initial radial locations of the entering inclusions do not significantly affect net transit times. Thanks to this, we can easily convert an axially space dependant resistive pulse into a time dependant resistive pulse (or voltage pulse), and vice-versa.

At 20 amps, it is seen that the path-lines of the inclusions match the fluid streamlines, indicating that the axial force components dominate their trajectories. The dynamic aspects of the entrained alumina inclusion’s motion has been discussed in Sec. 3.3.1, where the fluid’s acceleration and the particle’s added mass force play dominant roles in the axial direction due to the major density differences between fluid and particle ($\rho_p/\rho_f=0.54$). This is even more true for microbubbles. To show this influence on inclusion trajectories, Fig. 10 compares the motion of a non-conducting $\text{Al}_2\text{O}_3$ inclusion, a gas bubble ($\rho_p=1\text{ kg/m}^3$, $d=100\mu$m) and a neutral “liquid particle” (a liquid droplet with the same physical properties of liquid steel fluid into the ESZ).

Here, we assume that the gas bubble is un-deformable, this needs further clarification. The trajectories at four moments $t=1$ ms, 1.5 ms, 2.0 ms and 2.07 ms are shown in Fig. 10. The total transit time of an $\text{Al}_2\text{O}_3$ inclusion, gas bubble and liquid particle are 2.068 ms, 1.91 ms and 2.1 ms respectively. Clearly, the bubble transit time is significantly shorter than the other two. In practice, such short transit times might be used to discriminate between bubbles and particles within the liquid metal. The lighter $\text{Al}_2\text{O}_3$ inclusion moves slightly ahead of the neutral liquid particle, and
the microbubble, much ahead ($r/r_f/H_{11005}/1.4/H_{11003}/10/H_{11002}/4$) as a result of the fluid acceleration and added mass forces (see Fig. 8).

3.4. Resistance Pulses

3.4.1. Numerical Integral Method for Computing Change in Electrical Resistance

In Sec. 2.2, it was stated that there are no analytical solutions for predicting the ESZ resistance sensing problem, and that a numerical integral method is needed to compute the change of resistance when an inclusion passes through the ESZ. The outlet imaginary control surface of the ESZ is set to zero electric potential (ground), while the electric potential on the electric inlet boundary surface will be changed when an inclusion is present within the ESZ. Figure 11 shows such variations in potential on this inside control surface when a 100 $\mu$m $\text{Al}_2\text{O}_3$ inclusion is located at $z=100$ $\mu$m, ($r=0$) (Fig. 11(a)), ($r=100$ $\mu$m) (Fig. 11(b)), and ($r=200$ $\mu$m) (Fig. 11(c)), respectively. The changes of resistances are $\Delta R_{r=0}=2.89 \times 10^{-8}$ ($\Omega$), $\Delta R_{r=100 \mu m}=2.886 \times 10^{-8}$ ($\Omega$), and $\Delta R_{r=200 \mu m}=3.2 \times 10^{-8}$ ($\Omega$), respectively. This indicates that as the inclusion moves very close to the wall, the resistance becomes larger. Equation (14) was used to predict these resistive pulses in the following computations.

3.4.2. Resistive Pulse for When an Inclusion Moves along the Axis of Symmetry

First, we suppose that an inclusion moves along the axis of the ESZ, which occurs when the buoyancy force is negligibly small and other radial force components cancel (see Sec. 3.3.1 and Fig. 7). The resistive pulses generated as a function of distance along the axis are presented in Fig. 12.

Given a prescribed orifice size, i.e. $D=500$ $\mu$m, resistive pulses generated by inclusions; $d=50$ $\mu$m, $d=100$ $\mu$m and $d=150$ $\mu$m, are shown in Fig. 12(a). In segment I of the fluted orifice, the change in resistance increases as the cross-section narrows. In segment II, the curve ramps to a limit reaching its maximum value. As the size inclusion increases, the intensity of the change in resistance increases notably.

Taking a prescribed particle size, i.e. $d=100$ $\mu$m, the resistive pulses for different sizes of orifices $D=300$ $\mu$m, $D=500$ $\mu$m, and $D=700$ $\mu$m are shown in Fig. 12(b). As the orifice size increases, the electric resistance decreases. Notice that in the zone between the two segments, there appear a sharp peak for $D=300$ $\mu$m, $d=100$ $\mu$m. This is the result of the rapid variation in the walls contours in the presence of a relatively large inclusion.

3.4.3. Influence of Radial Position on Resistance

In Sec. 3.4.2, we discussed the registration of pulses when inclusions move along the axis of symmetry; this is a rare event, since they may come from any location around the entry zone to the ESZ.

![Fig. 11. Electric potential redistribute when the inclusion is introduced in different radial location within the ESZ, $z=100 \mu$m. (a) $r=0$; (b) $r=100 \mu$m; (c) $r=200 \mu$m.](image)

![Fig. 12. Resistive pulses registered when an inclusion moves along the axis of symmetry for varying size of inclusion (a) and varying sizes of orifice (b).](image)

![Fig. 13. Influence of the radial position of inclusions on the change of electric resistance ($\Omega$) using present numerical potential-integral method. $d=100 \mu$m, $D=500 \mu$m.](image)
lected $d=100\,\mu m$, and $D=500\,\mu m$, the inclusion can reach the locations in range of ($r \in (0, 200\,\mu m)$, $z \in (100, 600\,\mu m)$). The influence of radial position on the change in electric resistance using the present numerical potential-integral method is shown in Fig. 13. It can be seen that the change in resistance is largely uniform, apart from two regions, one close to the region between segment I and II of the fluted orifice, the other close to the wall, where it increases significantly close to the wall ($r \in (190, 200\,\mu m)$). However, an inclusion would not easily reach this region because the radial drag force there is very high (boundary effect, also see Ref. 22)) and centripetally directed. We can therefore neglect such effects.

3.4.4. Transition between Time- and Space-Dependent Resistive Pulses

The former section studies the space-dependent resistive pulses. In the practice of measuring the size and frequency distribution of inclusions in liquid steel, we probe time-dependent transient resistive pulses, rather than space-dependent ones. As has been shown in Fig. 9, the transit times for different radial locations are very similar, and can be considered as being only a function of the $z$-coordinate as seen in Fig. 14(b). Therefore, we can transform the resistive pulse from the $z$-dependent to time-dependent plots for the purpose of identifying of experiment signals.24

3.4.5. Comparison of Different Methods

Maxwell’s Formula provides the low bound of predicted resistances, while DeBlois made the effort to find the high bound, but his result (Eq. (11)) involves an undetermined parameter $D_m$. While DeBlois’s method derives from the Ohm’s model approach, it goes on to consider the distortion in the equi-potential contours when an inclusion is present within the ESZ. Due to the complexity in computing the equipotential areas in the liquid domain and the inclusion, the simplifying model results in numerical errors. The present method overcomes these defects but avoids computing distortions in the potential contours close to inclusion, considering the whole effect in integral potential variation on the end surface of the ESZ. Therefore, this method can then predict the resistance at any random position when the inclusion is traversing the ESZ. It needs emphasising that the accuracy of this numerical method depends on the mesh density and the cells in computing domain.

Figure 15 shows the resistive results predicted by the above methods. In Maxwell’s Formula, we approximately use the $z$-dependent cross-section diameter to stand for the diameter of the regular tube. As explained in Sec. 2.2.2, Mei Li’s method underestimated the resistance for the entire ratio $d/D$. When the inclusion size is much smaller than the orifice size as seen in Fig. 15(a), $d=100\,\mu m$, $D=500\,\mu m$, the predicted of resistive pulse by Maxwell’s Formula and the present method have very similar shapes and maxima. As the inclusion size increases e.g., $d=300\,\mu m$, $D=500\,\mu m$, the resistance predicted by Maxwell’s method is underestimated, particularly in segment II, as seen in Fig. 15(b). In such a case, the inclusion’s size will be much overestimated.

3.4.6. Influence of Electric Conductivity of Inclusion

As Eqs. (6) and (7) and Fig. 1 show, the electric conductivity significantly influences the change in resistance. Figure 16 presents predicted pulses generated by a non-conducting inclusion ($Al_2O_3$), and a highly conducting inclusion ($TiB_2$), in liquid steel. The non-conducting inclusion registers a positive pulse, whereas the highly conducting inclusion registers a positive pulse, whereas the highly conducting in-
clusion (σi > σf) registers a negative pulse.

3.4.7. Discussion of Inclusion Size Ranges

Based on the above analysis, and considering a type III fluted orifice D = 500 μm in steel, we use the present numerical integral method to investigate the influence of inclusion sizes ranging between 20 to 300 μm on resistive pulses. Predicted results, shown in Fig. 17, reveal that the resistive pulses for the smaller inclusions ~ 20 μm, are very weak, and in the order of only 10^-6 Ω. Depending on the sensitivity of the measuring equipment, and electrical noise, these inclusions may be too small to detect in practice. Typically, the Heraeus probe can detect 30–50 μm inclusions passing through a 500 μm orifice, but noise is a problem for detecting smaller inclusions. To resolve the problem of detecting inclusions in the 2–10 μm size range, one would need an orifice size of about 100 μm. Depending on liquid steel quality, this could become blocked, should larger inclusions be present within the steel melt. If not, the size distribution to be targeted for measurement, can be chosen through appropriate selection of electrical currents and orifice size.

4. Concluding Remarks

This article has investigated some of the theoretical aspects for the on-line, in-situ, detection of inclusions using the Resistive Pulse Technique. The results provide the theoretical basis for the practical use of steel LiMCA sensors for the steelmaking industry.

These theoretical considerations involved the development of a two-phase flow model. This model was used to predict the electromagnetic fields and forces, and their influence on flow fields and particle motions within the electric sensing zone. It was demonstrated that heavy electrical currents can combine with specific geometries of typical electric sensing zones (i.e. fluted and parabolic shaped orifices) such as to cause flow reversals, and low pass-through fractions of inclusions.

A new numerical integral method for computing the resistive pulse was developed and compared with the Maxwell and Ohm models for predicting resistive pulses. The present method is superior, in that it can estimate the changes in electrical resistance when insulating inclusions are randomly located within any part of the electric sensing zone.

The three types of wall-shaped orifices were studied from four aspects: fluid dynamics, electromagnetic body force distribution, inclusion trajectory and the change of resistance within the ESZ. The kinematics and hydrodynamics of inclusion motion has been researched, Stokes’s drag force, fluid acceleration force, added mass force, and electromagnetic force acting on the inclusion in radial and axial directions have been numerically analyzed. The force effect of axial components forces is much higher than that of the radial. For lighter density inclusions in liquid steel, fluid acceleration and added mass force improve inclusion motion in the heavier liquid steel flow direction, they move rapidly and yield relatively short transient time. It is critical to optimize the shape of the orifice. The relatively lower ratio of entrance size and throat size is favourable; the space gently variation parabolic curve of the sidewall is appropriate; the vertical symmetrical geometry has a disadvantage, the backflow accompany with the jet flow near the exiting region increases the risk to re-entraps inclusions. The input current should not too low and not too high. The register resistive pulse is too weak if the input current is too low, while the current is not too high to avoid forming re-circular flow near the entrance region. In summary, the Type III orifice appears to be most appropriate for liquid steel melts.

The transient time is another critical value for LiMCA problem. Whatever the type of the orifice, the displacement in the radial direction of an inclusion has little influence on the electrical resistance. This allows us to correlate temporal and space dependent resistive pulses the presence of micro-bubbles and lighter inclusions yield shorter traversing transient times, and this phenomenon may provide the evidence needed for inclusion discrimination on the basis of density differences.

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