Dimensional optimization of the crossbeam of the spectacle frame CNC polishing machine based on sensitivity analysis

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\textbf{Abstract.} Based on finite element analysis, we obtained the first six natural frequencies and corresponding vibration modes of the crossbeam of the spectacle frame CNC polishing machine, then we proposed some improvement according to the FEM analysis results. In order to determine the reasonable size parameters, we conducted the sensitivity analysis to the mass, total deformation and first frequency of the crossbeam. The results showed that three key parameters had greater impact to the structural characteristics of the crossbeam, while the other parameters had almost no effect. An optimization problem had been established to determine the three parameters (L, H and B), and the mass, total deformation and first frequency of the crossbeam were set as optimization target. With the help of the ANSYS optimization tool, we got three groups of candidate points, and later we used an evaluation function to determine the suitable parameters.

\section{1. Introduction}

The crossbeam of numerical control grinding machine is an important part to connect the equipment and polishing tool structure of machine tools. When the machine is working properly, the mass of the crossbeam will affect the rapidity and sensitivity of the response of the control system. The machine is equipped with motor and transmission parts inside the grinding machine, the vibration of the motor can be transmitted to the tool by means of the mounting plate, and it would have a bad effect on the process. Therefore, it is important to improve the machining accuracy and stability of the machine tool.

Structure optimization is an effective measure to enhance the dynamic characteristics of the machine tool. The research of structural optimization is mainly carried out in two aspects\cite{1}, one is structural topology optimization, which is based on finite element analysis, the other is to optimize the shape feature with the help of the optimization algorithm. Dimensional optimization is a mature method of detailed design, the advantage of dimensional optimization is that the performance of the structure is taken into account in the design process, so we could obtain the reasonable structure parameters that meet the requirement.

The sensitivity analysis method is a kind of optimization analysis method, which is based on the establishment of an accurate finite element model and a reasonable mathematical model\cite{2}--\cite{5}. The
shape and size parameters of different parts have different effects on the structure performance, that is to say, the sensitivity is different[6]. By analyzing the sensitivity of the structural parameters, the most sensitive parameters are found out and the optimization design is carried out.

In this paper, the finite element model of the crossbeam of the frame grinding machine was established, later we obtained the first six natural frequencies and the corresponding vibration modes of the beam based on the basic theory of modal analysis. According to the simulation results, the improvement measures were put forward, then we carried out sensitivity analysis to the original dimension of the crossbeam. Based on the sensitivity analysis, we took 3 key parameters to conduct the optimization, and the mass, total deformation and frequency were set as the optimization target. We used the direct optimization tool in ANSYS to get 3 groups of candidate points, and later we chose the best one by evaluation function.

2. The introduction of the polishing machine structure

2.1. Introduction of the polishing machine
The spectacle frame CNC polishing machine was used to burnish plastic resin frames of the spectacles. To make the polishing process more smooth, we took a four-axis scheme, in which three sliders was fixed on the holder and the sliders were orthogonal to each other. As shown in Figure 1, the three sliders could move in straight-line motion independently. The crossbeam was fixed on the slider of Z-direction, and it could move in compound space motion by following the sliders. The turntable was the fourth axis which could turn around the Y-axis of the machine tool coordinate system, the fixtures and workpiece were fixed on the turntable. Under the coordination of the four shafts, the machine tool can finish the machining of the complex surface of the workpiece.

2.2. Introduction of the crossbeam
The crossbeam plays a role of coupling and transmission, and the tool driven by the crossbeam could be able to reach any point in space. There are three groups of tool fixed on the crossbeam, so the

![Figure 1. The structure of the polishing machine](image)
stability and the static rigidity of the crossbeam could affect the working performance of the machine tool. What’s more, the weight of the crossbeam reflected the rapidity and sensitivity of the sliders.

As shown in Figure 2, the crossbeam was made of alloy steel, the dimensional parameters were shown in Table 1, and we listed the interval of these parameters, we would conduct modal analysis by finite element method to get the crossbeam.

![Figure 2. Crossbeam Model](image)

### Table 1. Structural parameters of the crossbeam

| Item                      | Initial dimension (mm) | Variation (mm) |
|---------------------------|------------------------|----------------|
| Lateral span: L           | 580                    | 560–600        |
| Beam height: H            | 80                     | 75–95          |
| Beam thickness: B         | 10                     | 5–15           |
| Groove width: W           | 120                    | 120            |
| Groove center distance: S | 220                    | 220            |
| Thickness of connecting plate: D | 5                  | 5–10          |
| Groove depth: A           | 5                      | 5              |
| Height of connecting plate: T | 150                | 150–200        |
| Board width: K            | 120                    | 120            |

### 3. FEM modal analysis of the crossbeam

#### 3.1. Modal analysis

The crossbeam model was established in ANSYS Design Module, we imported the built model into ANSYS Workbench, and we set the parameters of the simulation environment, as shown in Table 2.

### Table 2. Properties and parameters of the FEM model

| Properties               | Parameters         |
|--------------------------|--------------------|
| Material of frame        | Alloy steel        |
| Density                  | 7.9*103            |
| Young’s modulus          | 0.3                |
| Grid quantity            | 2187               |

After we set the primary parameters, we adopted the hexahedral element to generate the mesh. To avoid stress concentration, we made the boundary dense and the transition was gentle, in the non key area, we used larger mesh, so the computing time was reduced. According to the actual installation of the crossbeam, the back connecting plate of the cross beam was provided with a fixed restraint. After we set the solution condition, the first six order natural frequencies and vibration modes of the beam were calculated, as shown in Table 3 and Figure 3– Figure 8.
### Table 3. The first six frequencies and modes of the crossbeam

| Order     | Frequencies (Hz) | Description of modal shape                                           |
|-----------|------------------|-----------------------------------------------------------------------|
| First order | 147.06           | The left wing plate swung back and forth                               |
| Second order | 147.72           | The offside plate swung back and forth                                |
| Third order | 364.74           | The offside plate twisted around the horizontal center                 |
| Fourth order | 364.79           | The left side plate twisted around the horizontal center              |
| Fifth order  | 576.85           | The left side plate swung back and forth in a wave motion              |
| Sixth order  | 578.31           | The offside plate swung back and forth in a wave motion               |

3.2. Analysis of the results

From the Figure 3~8, we can see the first six frequencies of the beam were lower, and among them, the first two frequencies were close to the free rotation frequency of the motor, which was around 100 Hz. The vibration of the crossbeam would increase with the increasing of the motor rotating speed, so we need to improve the natural frequencies of the crossbeam. If the tool is subjected to greater damping after the motor starts, the speed may decrease slightly. The operating frequency is close to the low order natural frequency of the crossbeam, in this condition, the resonance occurs.

We can see from the modal shape that the wing of the crossbeam swung on either side, so we can add some stiffened plate or apply prestressing to increase the stiffness in that direction. Later we would use optimization method to change the characteristic of the crossbeam.

4. Sensitivity of the crossbeam

4.1. Dimensional sensitivity analysis

The sensitivity analysis method is a kind of optimization analysis method based on the establishment of an accurate finite element model and a reasonable mathematical model. In the process of structural
optimization, sensitivity can be used to indicate the extent to which the performance parameters of a system are affected by the size parameter. Usually the size parameter can be height, thickness, performance parameters can be the quality of the components and the natural frequency, dimensional sensitivity analysis can optimize the design parameters of the most sensitive component.

Assume a set of variables are: \( X_1, X_2 \ldots X_n \), function \( S \) is the partial derivative of function \( f(X_1,X_2\ldots X_n) \), \( S_i \) is the sensitivity of a function to variable \( X_i \), we call the sensitivity function \( S \),

\[
S_i = \frac{\partial f}{\partial X_i} \left( X_1, X_2, \ldots X_n \right)
\]  

(1)

According to function \( S \), we could analysis the sensitivity of different dimension to the quality and the nature frequency of the crossbeam.

4.2. Sensitivity of dimensional parameters

In this sensitivity analysis, the structure parameters of the crossbeam were taken as the design variables. We used finite element analysis software ANSYS Workbench to get the sensitivity of mass, total deformation and frequency, as shown in Figure 9. In the left part of Figure 9, we could see that the first three variables (L, H and B) had significant influence on the mass of the crossbeam, while the other variables almost had no effect, the thickness of the crossbeam affected the most especially. In the middle part of Figure 9, we can see that only the thickness of the crossbeam affected the total deformation, and the other variables almost had no effect. In the left part of Figure 9, the length was the only influence factor of the first frequency.

![Figure 9. Sensitivities of the variables](image)

In the subsequent optimization process, to simplify the optimization model, we focus on the first three variables (L, H and B) and ignored the other variables.

5. Dimensional optimization and analysis

5.1. Establish the optimization problem
The main content of the optimization design is to guarantee the constraint condition of the model, and to achieve small deformation, light weight, small size, reasonable shape, the lowest cost, and the best mechanical properties as far as possible. We set the range of the three parameters and defined optimization goal, as shown in Table 4.

### Table 4. The optimization parameters

| Parameter    | Type   | Lower bound | Upper bound | Requirement |
|--------------|--------|-------------|-------------|-------------|
| L            | Variable | 560 mm      | 600 mm      | none        |
| B            | Variable | 5 mm        | 15 mm       | none        |
| H            | Variable | 75 mm       | 95 mm       | none        |
| Mass         | Target  | none        | none        | Minimum     |
| Total deformation | Target | none        | none        | Minimum     |
| First frequency | Target | none        | none        | Maximum     |

5.2. Solve the optimization problem

We used direct optimization tool in ANSYS to solve the optimization problem, 100 groups of points were generated by the suitable algorithm, after the computer finished computing, three groups of candidate points were selected, the candidate points and corresponding results were shown in Table 5.

### Table 5. The candidate points and corresponding results

| Parameter                  | Candidate Point 1 | Candidate Point 2 | Candidate Point 3 |
|----------------------------|-------------------|-------------------|-------------------|
| L (mm)                     | 596.2             | 592.6             | 599.8             |
| H (mm)                     | 82.13             | 85.88             | 90.56             |
| B (mm)                     | 5.46              | 5.09              | 5.83              |
| Mass (Kg)                  | 1.65              | 1.52              | 1.91              |
| Total deformation maximum (mm) | 3701.6       | 4138.4            | 3253.3            |
| First frequency (Hz)       | 54.93             | 50.62             | 52.54             |

5.3. Determine the three parameters

From the Table 5, we could see that the three candidate points behaved differently on different objective functions, the mass of candidate point 2 was the minimum of the three points, and the total deformation is the maximum of them. The first frequencies of the three candidate points were very close, we needed to take into account of both the mass and the total deformation maximum. We established an evaluation function to determine the best value of the three candidate points:

$$\min \left\{ E=0.7 \times M + 2 \times D / 10000 - F / 100 \right\}$$  \hspace{1cm} (2)

Where M is the mass, D is the total deformation maximum, F is the first frequency. We gave different weights to the two variables according to the influence degree of the performance of the polishing machine tool, the mass of the crossbeam was more important so we set the weight of the mass to 0.7, and the weight of total deformation maximum was set to 2/10000, the weight of the first frequency was set to 1/100. We calculated the value of the evaluation function, and chose the minimum. The three evaluation function values respectively were 1.35, 1.39 and 1.46, we chose the candidate point 1 as the design value according to the evaluation function, after rounding, we took the L as 596 mm, H as 82 mm and B as 5.5 mm.

6. Conclusion

In this paper, we obtained the vibration mode and natural frequency of the first 6 steps of the spectacle frame CNC polishing machine tool through the finite element analysis, and we found the weak link of the bed, which provided the basis for the structural optimization design. Later we conducted sensitivity
analysis of the crossbeam, and we found the three key parameters which had greater impact to the crossbeam. Focused on the mass, total deformation and first frequency, we set an optimization problem and solved it with the help of the ANSYS software. Eventually, we got the best group of parameters of the crossbeam.

**Acknowledgments**

This paper is supported by the Shenzhen-Hong Kong Innovative Circle Project (SGLH20150213143207909). The authors would like to thank OKIA Optical limited company for their support.

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