Supporting Information for “A Rossby Whistle: A resonant basin mode observed in the Caribbean Sea”

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Contents of this file
1. Text S1
2. Figures S1 to S3

Additional Supporting Information (Files uploaded separately)
1. Movie S1

Introduction
The Supporting information consists of text describing in more detail the background on basin modes, a figure like Figure 2, showing the amplitude and phase of the basin mode, but in more detail over only the continental slope region, and two figures showing the actual basin-average bottom pressure time series from two ocean models, in comparison with the reconstructions based on sea level patterns. Finally, in a separate file, there is a .mpg movie showing one cycle of the amplitude and phase of the sea level and bottom pressure components of the 120-day mode, as shown in static form in Figure 2. The text and figures follow below. A more detailed description of the movie is in the caption which follows.

Movie S1. Movie showing one cycle of the least squares fit of (left) sea level and (right) bottom pressure on basin-averaged bottom pressure in the Caribbean Sea. All time series have been band-pass filtered in the 75-175 day band, and made complex using a Hilbert transform before the fit was performed. The movie corresponds to the amplitudes and phases shown in Figure 2, panels c-f. It shows the sea level and bottom pressure cycles which would be expected to accompany one cycle of basin-averaged bottom pressure, with an amplitude of 1 mbar. The peak basin-average pressure corresponds to the time at the start of the movie.

Supporting text S1
The nature of Rossby basin modes is rather subtle, and in the literature it is mostly discussed in terms of a single vertical mode in an idealized, quasigeostrophic, closed ocean basin with flat bottom and vertical sidewalls. Here we describe the main points of evolution of this theoretical literature, and describe how it generalizes and relates to the observed mode in the Caribbean Sea.

Excluding the relatively straightforward situation of a precisely nondivergent depth-integrated flow (a barotropic ocean with a rigid lid), the first correct quasigeostrophic basin mode calculation appears to have been undertaken by Larichev [1974] for a circular ocean basin, in a paper in which he introduced a subtlety in the boundary conditions for the quasigeostrophic flow in a closed basin. This work was expanded upon by McWilliams [1977] and Flierl [1977] who clarified the nature of the boundary condition. In essence, the point is that the boundary value of the quasigeostrophic stream function, while being at first order a constant in space, must be allowed to vary in time. This is required because it reflects not only the currents, but also the layer thickness, so the extra degree of freedom is required in order to allow mass to be conserved in each constant density layer of the model. The circular basin mode calculations of Flierl [1977] seemed to show that the correct boundary condition made only a small difference to the inviscid, free modes.

Physically, the need for this boundary condition is to account for the existence of boundary waves (baroclinic Kelvin waves in the cases discussed), which are assumed to travel infinitely fast in the quasigeostrophic framework. For a given interior distribution of potential vorticity, the interior streamfunction is completely defined except for a free mode with zero potential vorticity which consists of a current trapped against the boundaries, which decays exponentially away from the boundary with the scale of the Rossby radius (if the boundary is straight). In a comparison between simulations using quasigeostrophic and shallow water equations, Milliff and McWilliams [1994] showed how this boundary current is set up in the wake of the passage of a Kelvin wave. As long as the Kelvin wave speed is sufficiently fast to travel round the basin in a time short compared with the other natural timescales of the dynamics, the shallow water simulation looks very similar to the quasigeostrophic case, but only if this correct boundary condition is used. On time scales long compared with the Kelvin wave passage time, the boundary condition permits an exchange of mass between the interior and the boundary current, and the rapid transmission of signals from the western boundary across to the eastern boundary, from where they radiate out as Rossby waves.

This has an important effect on basin modes. If the wrong boundary condition is used (or in the case of modes which do not excite a boundary current), then basin modes essentially consist of a long Rossby wave propagating to the west across the basin, reflecting as a short Rossby wave (which still has westward phase speed, but eastward group speed), and then reflecting back into a long Rossby wave at the eastern boundary. The typically slow group speed and short length scale of the short Rossby wave means that it is especially prone to dissipation and interaction with other currents, making it difficult to sustain such modes in the presence of dissipation. However, the correct boundary condition allows the long Rossby wave to interact with the western boundary and excite a boundary current round the basin. This can then re-excite a long Rossby wave at the eastern boundary, allowing for basin modes without the need for the short Rossby wave.

The effect of this was shown by LaCasce [2000] in a paper ostensibly about Rossby wave speeds. Looking at forced, damped modes of a rectangular basin, he found that the
correct boundary condition allowed for much stronger resonances than the incorrect version. In a one-dimensional case, the resonances occur simply at an integer number of zonal wavelengths of the long Rossby wave fit into the basin. The wave is effectively absorbed at the western boundary and instantly re-emitted at the eastern boundary. The two dimensional case is more complicated, but produces almost the same result, at the same time illustrating that the boundary condition does not influence any modes which have an odd number of nodes in the meridional direction, since they do not excite the boundary current. In parallel, Cessi and Primeau [2001] found very similar results, with strong damping of all basin modes with the wrong boundary condition, but weak damping of a small subset of the modes when the correct boundary condition is used. The short Rossby wave component, although present without dissipation, was almost absent in the modes which survived with weak damping when dissipation was added.

The assumption of these studies was that the basin modes were baroclinic modes of ocean basins such as the North Atlantic or North Pacific, which would lead to characteristic time scales of years to decades. Subsequent work led to reasons to suspect that such modes might not survive in realistic conditions: Primeau [2002] showed that the resonance is significantly weakened when the Rossby wave basin transit time is latitude dependent, as it is (quite strongly) in the real ocean, though not on the beta plane used in the theory. LaCasce and Pedlosky [2004] showed that the Rossby waves themselves are unstable, and liable to break up into eddies before making a single transit of the basin except in tropical latitudes. However, Ben Jelloul and Huck [2003] showed that instabilities of a mean flow can excite the basin mode in certain circumstances. In their study, this occurred in a two-gyre case, but only in the unusual case of a westward-flowing central jet across which the relative vorticity gradient opposed the planetary vorticity gradient. It can be seen that the Caribbean Sea favours basin mode formation from all of these points of view. The basin has a narrow latitudinal extent, so that Rossby wave transit times do not vary strongly. It is also much shorter in zonal extent than a major ocean basin, so the transit time of only about 120 days is not long enough for the waves themselves to develop instability. Finally, it has a westward current flowing though it, favouring the excitation of the basin modes by instability of the current.

The Caribbean Sea has complex topography and the sidewalls are not vertical, although its shallow thermocline (most of the stratification is in the top 400 m of a basin typically 4–5 km deep) means that a shallow water approximation is probably good for the baroclinic mode in the interior of the basin. However, interaction of stratification with sloping topography means that the vertical modes are no longer separable, and we should expect the baroclinic Rossby waves to excite a mixture of barotropic and baroclinic modes on the continental slope. This matches with our understanding of coastal trapped waves. Huthnance [1978] showed that, as the coastline changes from a vertical wall to a continental slope, the boundary waves evolve from a series of Kelvin waves to a set of trapped modes with the same number of nodes in bottom pressure, as counted down the slope, with phase propagation in the same sense as the Kelvin waves. These become more barotropic as the topographic length scale increases and tend to propagate faster than the corresponding Kelvin wave speed, so a mixture of modes is inevitable as the baroclinic Rossby wave excites mixed-mode coastal trapped waves.

The essential dynamics of the basin mode appear to remain in place: the baroclinic Rossby wave excites fast, coastal trapped modes which will set up a coastal current. As the Rossby wave travels inland, the incident wave current is turned equatorward and begins to decay. An example which illustrates this is given, in quite a different context, by Marshall [2011], who terms the effect a “Rossby wormhole”. This analysis remains linear, and uses the planetary geostrophic equations, thus excluding short Rossby waves but allowing for finite topography. In this case it is found that a baroclinic Rossby wave, encountering an isolated region of closed $f/H$ contours from the east ($f$ is the Coriolis Parameter and $H$ is total ocean depth), rapidly excites a barotropic circulation round those contours. This has the effect of damping out the Rossby wave on the east, and simultaneously exciting a new one to the west of the region of isolated $f/H$ contours. The Rossby wave appears to have passed through a wormhole.

The mechanism for this lies in the fact that the interaction between baroclinic signals and topography leads to a continuously accelerating barotropic circulation around closed $f/H$ contours unless there is a certain symmetry in the stratification across these closed contours. As the Rossby wave approaches the regions of closed contours, it disturbs that symmetry so that a barotropic current begins to flow. Where the flow is poleward, potential vorticity conservation means that there is a downward vertical velocity at the bottom, decreasing to zero at the surface. This has the effect of pulling the thermocline down, damping out the Rossby wave. However, the opposite happens where the flow is equatorward, causing the thermocline to rise on the western side of the region of closed $f/H$ contours, and this rise in the thermocline then radiates to the west as a Rossby wave.

The only difference in the Caribbean Sea is that the Rossby wave is initially in the centre of the region of closed $f/H$ contours, so when it encounters the western boundary it excites an equatorward flow there and therefore a poleward flow at the eastern boundary of the basin. The wormhole transmits the wave back to the east, just as described by LaCasce [2000] for the quasigeostrophic case. The main difference is that, by neglecting relative vorticity and allowing finite topography, the implicit boundary waves are now barotropic continental shelf waves rather than baroclinic Kelvin waves. The waves remain implicit because, as in the quasigeostrophic case, they are assumed to propagate infinitely fast in the planetary geostrophic limit.

If we allow for a lag associated with the boundary propagation, we should expect this to be around 10 days for a baroclinic Kelvin wave in the Caribbean Sea (as discussed in the main text), or less if the coastal trapped wave is more barotropic. The combined Venezuela and Colombia basins, up to the Central American Rise, have a zonal extent very close to 2000 km and a typical meridional extent of 600 km. Together with a Rossby radius of 80 km, this allows us to calculate the expected resonances for a particular boundary lag. This is most straightforward if, instead of choosing a boundary lag, we choose a zonal wavelength which for which $n$ wavelengths is equal to the 2000 km basin extent plus a small excess, $\delta x$. The boundary lag $\delta t$ required for resonance can then be calculated from $\delta t = \delta x/c$ where $c$ is the Rossby wave phase propagation speed for that wavelength.

Applying this formula with the above parameters, we find 4 possible resonances for Rossby waves with westward group velocities. In order of decreasing wavelength, and for boundary lags of $\delta t = 0$ to 10 days, these have periods of 203.7–212.3, 117.5–120.7, 95.6–96.8 and 89.9–90.1 days, and the shortest possible period for Rossby waves is at 89.6 days.

There is no sign of the wavenumber 1 mode, perhaps because of the partial division of the basin into two sub-basins. This would also be expected to interfere with the wavenumber 3 mode. However, the wavenumber 2 mode is observed in the expected period range, and a secondary peak is also seen (black line in Figure 1c) at a period of 92 days. It is
possible that this is indicative of one of the other resonant modes, although it could also simply represent energy accumulating near to the minimum possible Rossby wave period, at which the Rossby wave group velocity is zero. We have not attempted to alter the assumed geometry or Rossby radius to obtain better agreement, these numbers are based on a first estimate of the relevant parameters. If we are to understand the secondary peak, a more comprehensive mode calculation is needed, with realistic geometry and stratification including a Caribbean current. Such a calculation is beyond the scope of the present work.

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Figure S1. A repeat of Figure 2, but focusing on the southern boundary. Amplitude of (a,b) the complex correlation, and (c,d) the complex fit of (a,c) sea level and (b,d) bottom pressure on basin-averaged bottom pressure in the NEMO12 model. The corresponding phase is shown in (e,f). All time series are band-pass filtered in the 75-175 day band, and made complex using a Hilbert transform. Phase is zero for a time series which peaks at the same time as the basin-averaged time series, and small positive phase means the peak follows the basin average. Black contours show depths of 200, 400, 800 and 1600 m.
Figure S2. Time series of basin-average bottom pressure from the NEMO12 model as described in the main text. Black is the actual bottom pressure, and blue shows the prediction based on sea level, using the relationship derived from this model. The upper panel is unfiltered, and the lower panel is band pass filtered to show periods between 75 and 175 days.
Figure S3. Time series of basin-average bottom pressure from the ECCO2 model as described in the main text. Black is the actual bottom pressure, and blue shows the prediction based on sea level, using the relationship derived from the NEMO12 model. The upper panel is unfiltered, and the lower panel is band pass filtered to show periods between 75 and 175 days.