A Note On ADE String Compactifications

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Abstract

We address the question whether so-called $m$-invariants of the $N = 2$ super Virasoro algebra can be used for the construction of reasonable four-dimensional string models. It turns out that an infinite subset of those are pathological in the sense that – although $N = 2$ supersymmetric – the Ramond sector is not isomorphic to the Neveu-Schwarz sector. Consequently, these two properties are independent and only requiring both guarantees an $N = 1$ space-time supersymmetric string spectrum. However, the remaining 529 consistent spectra – 210 of them are mirrors of Gepner models and 76 real orbifolds – show exact mirror symmetry and are contained in a recent classification of orbifolds of Gepner models.

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1. Introduction

In 1988, Gepner [13] initiated the construction of four-dimensional $N = 1$ space-time supersymmetric string vacua with an extended gauge group $E_6 \times E_8$ using explicitly $N = 2$ supersymmetric conformal field theories (SCFT) in the internal sector [23,24,25]. In the following years his approach of tensoring unitary models of the $N = 2$ super Virasoro algebra ($VIR_{N=2}$) adding up to $c = 9$ has been extended to orbifolds [10,11,12,35] and simple current constructions [30,31]. Another, field theoretic, approach started with $N = 2$ supersymmetric Landau-Ginzburg models [27,34] which is more related to the geometric description in terms of Calabi-Yau manifolds [6]. Recently, a classification of all orbifolds including discrete torsion of so-called ADE invariants of the $VIR_{N=2}$ has been completed in [19,20]. It turned out that an earlier stochastic search using the simple current technique [30,31] was almost exhaustive.

It is known that the ADE invariants are not a complete set of modular invariant partition functions of the $VIR_{N=2}$ [8,14,28]. There also exist the so-called $m$-invariants. In this note we investigate which subset of string vacua can be obtained by these further invariants. It is clear that all these models can also be obtained by an orbifold or simple current construction [14]. However, there appear some interesting features. First, although $N = 2$ supersymmetric, a huge set of these $m$-invariants do not yield reasonable $N = 1$ space-time supersymmetric string models. The reason is that these models have an extended $W$-symmetry, the spectral flow automorphism of which does not any longer map the Neveu-Schwarz ($NS$) sector (space-time bosons) onto the Ramond ($R$) sector (space-time fermions). Thus, this example explicitly shows that $N = 2$ supersymmetry and the existence of the gravitino vertex operator are independent conditions and only both of them are sufficient for $N = 1$ space-time supersymmetry [22]. The necessity of those conditions has been derived in 1988 by Banks et al. [2]. The second feature is that these string models built up from $m$-invariants exhibit an exact mirror symmetry, which is given by a simple exchange of $m$-invariants. Since all 529 consistent string spectra we found are contained in the classification of [19,30], our calculation can be regarded as an independent check.

2. Modular invariants of the $VIR_{N=2}$

In this section we briefly review some facts about the unitary series of the $VIR_{N=2}$ [13,14,28]. By realizing $VIR_{N=2}$ as a product of parafermions times a $U(1)$ current

$$VIR_{N=2} = \frac{SU(2)_k}{U(1)} \times U(1),$$

one obtains the following grid of unitary representations:

$$c = \frac{3k}{k+2}, \quad h_{m,s}^l = \frac{l(l+2) - m^2}{4(k+2)} + \frac{s^2}{8} \mod 1, \quad q_{m,s}^l = \frac{m}{k+2} - \frac{s}{2} \mod 2, \quad (2.2)$$

$$k \in \mathbb{N}, \quad l \in \{0, \ldots, k\}, \quad m \in \{-k-1, \ldots, k+2\}, \quad s \in \{-1, \ldots, 2\}, \quad l + m + s = 0 \mod 2.$$

To get a non-redundant set of primary fields $\Phi_{m,s}^l$, one has to take into account the reflection symmetry

$$\Phi_{m,s}^l = \Phi_{m+k+2,s+2}^{k-l}, \quad m = m \mod 2(k+2), \quad s = s \mod 4. \quad (2.3)$$
Then, the supersymmetric characters are given by
\[
\chi^l_m(z, \tau, u) = \chi^l_{m,s}(z, \tau, u) \pm \chi^l_{m,s+2}(z, \tau, u), \quad s = 0 \text{ for } NS, \tilde{NS} \text{ and } s = -1 \text{ for } R, \tilde{R} \tag{2.4}
\]
where the minus sign has to be chosen for the $\tilde{NS}$ and $\tilde{R}$ sector. The corresponding superprimary fields are denoted by $\Phi^l_m$. Under modular transformations
\[
S : (z, \tau, u) \rightarrow \left( \frac{z}{\tau}, -\frac{1}{\tau}, u + \frac{z^2}{2\tau} \right), \quad T : (z, \tau, u) \rightarrow (z, \tau + 1, u) \tag{2.5}
\]
these characters behave in a very simple way:
\[
\chi^l_{m,s}(\frac{z}{\tau}, -\frac{1}{\tau}, u + \frac{z^2}{2\tau}) = K \sum_{l', m', s'} \sin \left( \pi \frac{(l + 1)(l' + 1)}{k + 2} \right) e^{\frac{\pi i m m'}{k + 2}} e^{-\frac{\pi i s s'}{k + 2}} \chi^l_m(z, \tau, u),
\]
\[
\chi^l_{m,s}(z, \tau + 1, u) = e^{2\pi i (h^l_m - \frac{k}{2})} \chi^l_m(z, \tau, u) \tag{2.6}
\]
with some constant $K$ only depending on $k$, i.e. the level $l$ transforms like an $SU(2)_k$ character, $m$ as a $\Theta_m k$ and $s$ as a $\Theta_s 4$ function. Thus, a huge set of modular invariant partition functions is given by the product ansatz
\[
Z = \sum_{l, l', m, m', s, s'} N_{l, l'} M_{m, m'} P_{s, s'} \chi^l_{m, s} \chi^{l'}_{m', s'} \tag{2.7}
\]
with $N, M, P$ representing modular invariant combinations of the three corresponding models. The matrices $N_{l, l'}$ are classified in an ADE scheme [8] and the possible $m$-invariants $M_{m, m'}$ of the system of $\Theta_{m, k}$-functions are labeled by divisors of $k$ [14]. For any factorization $k = \alpha \cdot \beta$ there exists a modular invariant partition function
\[
M_{m, m'} = \frac{1}{2} \sum_{x \in \mathbb{Z}, y \in \mathbb{Z}_2} \delta_{m, \alpha x + \beta y} \delta_{m', \alpha x - \beta y}. \tag{2.8}
\]
All these are simple current invariants due to Schellekens and Yankielowicz [29]. The simple current is the primary field corresponding to $\Theta_{2\beta, k}$ and the length of an orbit turns out to be $\alpha$. Thus, except the three cases $E_6, E_7, E_8$ all modular invariants (2.7) can be obtained by modding out a simple current and therefore due to [20] by an orbifold construction.

### 3. Off-diagonal invariants for $k=4j-2$

For $k = 4j$ the corresponding $D_{2j+2}$ invariant of $SU(2)_k$ yields an off-diagonal partition function of $VIR_{N=2}$ which can be interpreted as a diagonal partition function of an extended superconformal algebra. In this case it is a $SW(1, j^0)$, i.e. the extension of $VIR_{N=2}$ by a primary superfield of superconformal dimension $H = j$ and $U(1)$ charge $Q = 0$. For $j \in \{2, 3\}$ these super $W$-algebras have been explicitly constructed in [3,4]. However, there are also consistent solutions for $SW(1, j^0)$ with $j \in \mathbb{Z} + \frac{1}{2}$. These are implied by $m$-invariants with $k = 4j - 2$ and the factorization $k + 2 = 4j = 2 \cdot (2j)$:
\[
Z = \sum_{m=2}^{4j} |\Theta_m 4j + \Theta_{m-4j} 4j|^2. \tag{3.1}
\]
The corresponding partition function of $VIR_{N=2}$ is

$$Z = Z_{NS} + Z_{\tilde{NS}} + Z_{R},$$

$$Z_{NS} = \frac{1}{2} \sum_{l=0}^{2j-2} \sum_{l=0 \text{ mod } 2}^{l} \sum_{m=0 \text{ mod } 2}^{m} |\chi_m^l + \chi_m^{4j-l-2}|^2 + \frac{1}{2} \sum_{l=2j}^{4j-2} \sum_{m=0 \text{ mod } 2}^{m} \sum_{m=0 \text{ mod } 2}^{m} |\chi_m^l + \chi_m^{l-4j}|^2,$$

$$Z_{\tilde{NS}} = \frac{1}{2} \sum_{l=0}^{2j-2} \sum_{l=0 \text{ mod } 2}^{l} \sum_{m=0 \text{ mod } 2}^{m} |\tilde{\chi}_m^l - \tilde{\chi}_m^{4j-l-2}|^2 + \frac{1}{2} \sum_{l=2j}^{4j-2} \sum_{m=0 \text{ mod } 2}^{m} \sum_{m=0 \text{ mod } 2}^{m} |\tilde{\chi}_m^l - \tilde{\chi}_m^{l-4j}|^2,$$

$$Z_{R} = \frac{1}{2} \sum_{l=1}^{2j-3} \sum_{l=1 \text{ mod } 2}^{l} \sum_{l=0 \text{ mod } 2}^{m} |\chi_m^l + \chi_m^{4j-l-2}|^2 + \frac{1}{2} \sum_{l=2j+1}^{4j-3} \sum_{l=1 \text{ mod } 2}^{l} \sum_{m=0 \text{ mod } 2}^{m} |\chi_m^l + \chi_m^{l-4j}|^2 + \sum_{m=-2j+2}^{2j} |\chi_m^{2j-1}|^2. \quad (3.2)$$

The $\tilde{R}$ sector is left invariant by the modular group:

$$Z_{\tilde{R}} = \frac{1}{2} \sum_{l=1}^{2j-3} \sum_{l=1 \text{ mod } 2}^{l} \sum_{l=0 \text{ mod } 2}^{m} |\chi_m^l - \chi_m^{4j-l-2}|^2 + \frac{1}{2} \sum_{l=2j+1}^{4j-3} \sum_{l=1 \text{ mod } 2}^{l} \sum_{m=0 \text{ mod } 2}^{m} |\chi_m^l - \chi_m^{l-4j}|^2. \quad (3.3)$$

The intriguing feature is that, since in the $R$ sector odd values of $m$ are projected out, the spectral flow operator $\Phi_0^1$ does not occur. This can be understood in the following way: The above partition function is the diagonal one for the extension of $VIR_{N=2}$ by the primary superfield $\Phi_0^{4j-2}$ of dimension $(H, Q) = (j - \frac{1}{2}, 0)$, cf. [3] for the notation:

$$\Phi(Z) = \phi_0^{j - \frac{1}{2}}(z) + \frac{1}{\sqrt{2}} \left( \theta^+ \psi_j^{-1}(z) - \theta^- \psi_j^{+1}(z) \right) + \theta^+ \theta^- \phi_0^{j + \frac{1}{2}}(z). \quad (3.4)$$

A spin structure $\left(\square | \ldots | \square\right)$ on the torus is defined by different boundary conditions of the involved fermions ($A$: antiperiodic, $P$: periodic). Besides the supercurrents $G^\pm(z)$ there are two further fermionic fields in the symmetry algebra, namely $\phi(z)$ and $\chi(z)$. In the following the first box represents the boundary conditions along the two cycles of $G^\pm(z)$ and the second one those of $\phi(z)$ and $\chi(z)$. Under modular transformations one obtains the well-known chain

$$\left( \begin{array}{c|c} A & \square \\ \hline \square & A \end{array} \right) \overset{T}{\rightarrow} \left( \begin{array}{c|c} P & \square \\ \hline \square & P \end{array} \right) \overset{S}{\rightarrow} \left( \begin{array}{c|c} A & \square \\ \hline \square & A \end{array} \right). \quad (3.5)$$

The sector $\left(\begin{array}{c|c} P & \square \\ \hline \square & P \end{array}\right)$ is invariant under the action of the modular group. In the $R$ sector of the above partition function all fermions are periodic around a circle of constant time.
or carry integer modes, equivalently. However, the spectral flow automorphism \([33]\) has to be extended to the entire \(W\)-algebra. Denoting the action of the spectral flow by \((\cdot)’\), the only way to preserve the primarity of the further generators is

\[
(L_n)' = L_n + \eta J_n + \frac{c}{6} \eta^2 \delta_{n,0}, \quad (G_r^\pm)' = G_r^\pm, \quad (J_n)' = J_n + \frac{c}{3} \eta \delta_{n,0},
\]

\[
(F_n)' = F_{n+Q(F)\eta} \quad \text{for} \quad F = \phi, \psi^\pm, \chi.
\]  \(3.6\)

Since the fermionic generators \(\phi, \chi\) have even \(U(1)\) charge and the bosonic ones \(\psi^\pm(z)\) odd, the spectral flow \(O_{\eta=\frac{1}{2}}\) does not any longer connect the NS and the \(R\) sector. But it does generate new twisted sectors, where also the bosonic fields carry half-integer modes. For instance, applying the spectral flow to the \(NS\) sector yields

\[
\begin{pmatrix} 
\text{A} \\
\text{A} 
\end{pmatrix}
\begin{pmatrix} 
\text{A} \\
\text{A} 
\end{pmatrix}
\xrightarrow{O_{\frac{1}{2}}}
\begin{pmatrix} 
\text{A} \\
\text{P} 
\end{pmatrix}
\begin{pmatrix} 
\text{A} \\
\text{A} 
\end{pmatrix}
\]  \(3.7\)

Now, the question arises, whether it is possible to extend the partition function to a flow invariant one without loosing modular invariance. To this end, one has to sum over the entire orbit generated by successive application of the modular transformations \(T, S\) and the spectral flow \(O_{\frac{1}{2}}\). The result is a sum over all 16 possible spin structures. However, this is nothing but the diagonal partition function for the non-extended \(VIR_{N=2}\). Schematically, one has to take into account the relation

\[
\sum_{x,y \in \{P,A\}} \begin{pmatrix} 
\text{m} \\
\text{n} 
\end{pmatrix}
\begin{pmatrix} 
\text{x} \\
\text{y} 
\end{pmatrix}
= m \begin{pmatrix} 
\text{n} 
\end{pmatrix}
\]  \(3.8\)

where the box on the r.h.s. has to be understood as a spin structure of the non-extended \(VIR_{N=2}\), i.e. of the two supercurrents \(G^\pm(z)\) only. Summarizing, the model \((3.2)\) gives an example of an \(N = 2\) supersymmetric CFT in which the \(R\) sector is not isomorphic to the \(NS\) sector. However, although the spectral flow \(O_{\frac{1}{2}}\) is not an automorphism of the model \((3.2)\), the square of it, \(O_1\), is. It is realized by the primary field \(\Phi_0^{(2,3)} \Phi_k^k\). Thus, if one tensors such models adding up to \(c = 9\), e.g. \((k = 2)^{\otimes 6}\), and chooses the product of the partition functions of the factors, the latter flow is still present in the theory. It can be realized in the well-known way by the \(U(1)\) current

\[
X^\pm(z) = \sqrt{6} : e^{\pm i \sqrt{3} \varphi(z)} : \quad \text{with} \quad j(z) = \sqrt{3}i \partial \varphi(z).
\]  \(3.9\)

As usual, this simple current can be used to project the internal \(N = 2\) SCFT onto integer \(U(1)\) charges in the \(NS\) and half-integer charges in the \(R\) sector, respectively \([5,9,13]\). One obtains an \(N = 2\) SCFT with \(c = 9\) and (half-)integer \(U(1)\) charges which fails to contain the internal, holomorphic part of the gravitino vertex operator

\[
\Sigma^\pm(z) = : e^{\pm i \sqrt{3} \varphi(z)} :.
\]  \(3.10\)
Consequently, after combining the space-time sector and the internal sector, the next GSO projection onto odd $U(1)$ charges cannot be carried out. In order to get a space-time supersymmetric spectrum one would have to form supersymmetric orbits by successive application of the operator $S_\alpha \Sigma^+$, where $S_\alpha$ denotes the space-time spinor. Thus, this example shows that the existence of $N = 1$ space-time supersymmetry is not guaranteed by $N = 2$ world-sheet supersymmetry and integer $U(1)$ charges alone. Independently, the existence of the spectral flow operator with $\eta = 1/2$ in the spectrum has to be required. Then, Gepner’s construction inherently implies that these two conditions ensure space-time supersymmetry.

Since these somehow pathological models still provide one with an $N = 2$ SCFT with $c = 9$ and (half-)integer $U(1)$ charges, the question arises, whether they can be interpreted as some nonlinear $\sigma$ model partition functions of not Calabi-Yau type.

4. String spectra and the mirror map

Of course, not in all $m$-invariants the spectral flow is projected out. For all 168 combinations of unitary models adding up to $c = 9$ [25] with different ADE invariants we have used all possible $m$-invariants in all factors for the construction of a GSO projected, space-time supersymmetric string compactification. However, the following two conditions put severe constraints on the allowed combinations of invariants:

(a) The spectral flow operator $(H, Q; \overline{H}, \overline{Q}) = \left(\frac{3}{8}, \frac{3}{2}; \frac{3}{8}, \frac{3}{2}\right)$ must be contained in the $R$ sector of the $c = 9$ internal SCFT.

(b) The field theoretic analogues of the $(3, 0)$ and $(0, 3)$ form on the Calabi-Yau manifold have to survive the GSO projection. These are exactly the (anti-)holomorphic, chiral fields with $(H, Q; \overline{H}, \overline{Q}) = \left(\frac{3}{2}, 3; 0, 0\right)$ and $(H, Q; \overline{H}, \overline{Q}) = \left(0, 0; \frac{3}{2}, 3\right)$ in the $c = 9$ internal SCFT which correspond to spectral flow with $\eta = 1$.

One can show that the condition (b) is equivalent to the requirement that the state $(H, Q; \overline{H}, \overline{Q}) = \left(\frac{3}{2}, 3; \frac{3}{2}, 3\right)$ survives the projection. Since applying two times the spectral flow with $\eta = 1/2$ gives the flow with $\eta = 1$, (b) follows from (a). However, the model (3.2) shows that both conditions are not equivalent meaning that there is not necessarily a kind of ‘squareroot’ of the flow with $\eta = 1$. Furthermore, unlike the orbits generating generations and antigenerations the vacuum orbit is invariant under a $U(1)$ flip, i.e. $Q \to -Q$. Therefore, the fields with $(H, Q; \overline{H}, \overline{Q}) = \left(\frac{3}{2}, -3; 0, 0\right)$ and $(H, Q; \overline{H}, \overline{Q}) = \left(0, 0; \frac{3}{2}, -3\right)$ are automatically contained in the spectrum.

Surprisingly, besides the ordinary Gepner models and their mirrors which merely contain $m$-invariants $k_i + 2 = 1 \cdot (k_i + 2)$ and $k_i + 2 = (k_i + 2) \cdot 1$, respectively, only 76 further spectra are consistent. They are listed in Table 1 which can be found in the appendix including the used combination of invariants and the massless string spectrum. The relation

$$M_{m, m'}^{k+2=\alpha \cdot \beta} = M_{m', -m'}^{k+2=\beta \cdot \alpha}$$

(4.1)

for the invariants of the $\Theta$ functions (2.8) implies that all consistent models occur in mirror pairs. To get the mirror partner one only has to replace the $k_i + 2 = \alpha_i \cdot \beta_i$ $m$-invariant by the $k_i + 2 = \beta_i \cdot \alpha_i$ invariant in each factor. This couples the left and right sector in such a way that a state $(H_L, Q_L; H_R, Q_R)$ is substituted by $(H_L, Q_L; H_R, -Q_R)$. Especially, the mirror
partner of an ordinary Gepner model can be obtained by the $k_i + 2 = (k_i + 2) \cdot 1$ invariants. Note, that this simple map yields the mirror partition function without explicitly carrying out an orbifold construction [1,15,26]. There are further interesting properties:

(i) All Euler numbers of the 76 orbifold spectra are divisible by 12.
(ii) There are nontrivial mirrors, i.e. one obtains the mirror spectrum by an $m$-invariant different from the flip (4.1) in all factors. This phenomenon occurs at almost all tensor products. In a few cases one needs no flip (4.1) in any factor at all.
(iii) Some $m$-invariants produce the same spectrum as different $l$-invariants. This generalizes an observation made in [25]. It does not happen as often as (ii).

All 529 (i.e. also including the ordinary Gepner models and their mirrors) resulting from combining $l$- and $m$-invariants are plotted in Figure 1 in the usual manner. The dots ‘.’ denote Gepner spectra and their mirrors, whereas ‘.’ stand for the spectra also listed in Table 1. The plot shows exact mirror symmetry, in distinction to the results in more general constructions [7,17,18,21].

![Figure 1 mirror plot](image-url)
Additionally, the GSO projection admits even more redundancies than (ii) and (iii) like the well-known identities

\[ 10_E \cong 1_A \otimes 2_A, \quad 28_E \cong 1_A \otimes 3_A, \quad 4_D \cong 1_A \otimes 1_A \]

which can be easily checked in the Landau-Ginzburg formulation [32]. These identifications are valid for the diagonal \( m \)-invariant \( k+2 = 1 \cdot (k+2) \) and for its mirror \( k+2 = (k+2) \cdot 1 \). Further identifications can be read off from Table 1 whenever different tensor products produce equal massless spectra:

\[ 10_{E,3-4} \cong 1_{A,3-1} \otimes 2_{A,1-4}, \quad 4_{D,2-3} \cong 1_{A,1-3} \otimes 1_{A,1-3} \cong 4_{D,1-6} \]

and the corresponding mirror relations. For \( 28_E \) similar identities will hold.

5. Summary

In this note we have investigated to which extent one can use \( m \)-invariants for the construction of \( N = 1 \) space-time string vacua. It turned out that some conditions derived from string theory limited the number of models drastically. We gave an example of a class of modular invariant \( N = 2 \) supersymmetric partition functions which failed to allow the spectral flow automorphism to map the \( NS \) sector onto the \( R \) sector. For the surviving models we calculated the massless string spectrum showing that almost half of all orbifold models can also be obtained using \( m \)-invariants. Furthermore, this subclass of theories exhibits exact mirror symmetry.

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Appendix

The 76 orbifold spectra are listed in Table 1. \( n_{27} \) denotes the number of generations, \( n_{27} \) of antigenerations, \( n_1 \) of singlets and \( n_g \) of gauge bosons. \( \chi = 2(n_{27} - n_{27}) \) means the Euler number of the underlying Calabi-Yau manifold. The \( m \)-invariants for a model \( k_1, \ldots, k_r \) are labeled by divisors of \( k_i + 2 \) for each \( i \). For brevity, the spectra with \( \chi > 0 \) are omitted. These missing spectra can be obtained by the flip (4.1).

| \( n_{27} \) | \( n_{27} \) | \( n_1 \) | \( n_g \) | \( \chi \) | tensor product | \( l \)-invariant | \( m \)-invariant |
|---|---|---|---|---|---|---|---|
| 37 | 7 | 200 | 4 | -60 | 1 4 4 4 4 | A A A A | 1 2 2 2 2 |
| 67 | 7 | 267 | 3 | -120 | 3 8 8 8 | A A A A | 1 2 2 2 |
| 27 | 9 | 193 | 3 | -36 | 2 10 10 10 | A D D D | 1 3 3 3 |
| 63 | 9 | 265 | 3 | -108 | 2 10 10 10 | A A A A | 1 3 3 3 |
| 40 | 10 | 219 | 3 | -60 | 4 4 10 10 | A A A D | 2 2 4 4 |
| 23 | 11 | 212 | 4 | -24 | 1 2 2 10 10 | A A A D D | 1 4 4 4 4 |
| 47 | 11 | 244 | 4 | -72 | 1 2 2 10 10 | A A A A A | 3 1 1 3 3 |
| 101 | 11 | 401 | 3 | -180 | 2 10 10 10 | A E A A | 1 3 3 3 |
| 30 | 12 | 215 | 5 | -36 | 2 2 4 4 4 | A A A A A | 1 1 3 3 3 |
| 25 | 13 | 188 | 4 | -24 | 1 2 2 10 10 | A A A D A | 3 1 1 3 3 |
|    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|
| 97 | 13 | 405| 3  | -168 | 1 10 16 34 | A D A A, A A A D | 1 3 3 3 |
| 27 | 15 | 212| 4  | -24  | 1 2 4 4 10  | A A A A A, A A A A D | 1 4 2 4 |
|    |    |    |    |    |    |    |    |
| 23 | 17 | 205| 3  | -12  | 2 10 10 10 | A D D A            | 1 3 3 3 |
| 35 | 17 | 229| 3  | -36  | 2 10 10 10 | A D A A            | 1 3 3 3 |
| 36 | 18 | 223| 3  | -36  | 4 4 10 10  | A A A A, A A D D   | 2 2 4 4 |
| 90 | 18 | 415| 3  | -144 | 1 8 18 58  | A A D A, A A A D   | 1 2 4 4, 3 2 4 12 |
| 19 | 19 | 204| 6  | 0    | 1 1 2 4 4  | A A A A A A        | 1 1 4 4 2 2, 3 3 1 1 3 3 |
|    |    |    |    |    |    |    |    |
| 46 | 22 | 309| 3  | -48  | 2 4 16 34  | A A D A, A A D D   | 4 2 1 4, 4 2 2 4 |
| 29 | 23 | 223| 3  | -12  | 4 4 8 13   | A A A A A          | 2 2 2 1, 2 2 1 0 5 |
| 30 | 24 | 247| 3  | -12  | 2 4 16 34  | A A E A, A A E D   | 4 2 1 4, 4 2 2 4 |
| 60 | 24 | 325| 3  | -72  | 2 4 22 22  | A A A A, A A D D   | 1 3 3 3 |
| 25 | 25 | 221| 3  | 0    | 4 4 6 22   | A A D A, A A A D   | 2 2 8 8, 3 3 1 3 |
| 39 | 27 | 277| 3  | -24  | 3 4 8 28   | A A A A A          | 1 2 2 2, 1 6 2 6 |
| 75 | 27 | 425| 3  | -96  | 1 10 14 46 | A A A A, A A D D   | 3 3 1 3 |
| 34 | 28 | 257| 3  | -12  | 2 8 8 18   | A A A A, A A A D   | 4 2 2 4 |
| 65 | 29 | 355| 3  | -72  | 1 10 16 34 | A A A A, A D A D   | 1 4 2 4 |
| 60 | 30 | 345| 3  | -60  | 1 12 12 40 | A A A A A          | 1 2 2 2, 3 2 2 6 |
| 38 | 32 | 299| 3  | -12  | 4 4 5 40   | A A A A A          | 2 2 1 2, 2 2 7 1 4 |
| 44 | 32 | 301| 3  | -24  | 1 13 13 28 | A A A A, A A A D   | 1 5 5 5, 1 5 5 1 0 |
| 47 | 35 | 337| 3  | -24  | 1 10 18 28 | A A A A, A D D A   | 3 3 1 3, 3 3 5 1 5 |
| 179| 35 | 791| 3  | -288 | 1 5 82 82  | A A A A, A A D D   | 3 1 3 3 |
|    |    |    |    |    |    |    |    |
| 61 | 37 | 413| 3  | -48  | 2 4 13 58  | A A A A, A A A D   | 4 2 1 4, 4 2 5 2 0 |
| 44 | 38 | 333| 3  | -12  | 2 4 16 34  | A A A A, A A A D   | 1 3 9 9 |
| 62 | 38 | 377| 3  | -48  | 1 8 18 58  | A A A A, A A D D   | 1 2 4 4, 3 2 4 1 2 |
| 91 | 43 | 505| 3  | -96  | 1 6 34 70  | A A A A, A D A D   | 3 1 9 9 |
| 63 | 51 | 457| 3  | -24  | 1 10 12 82 | A A A A, A D A D   | 3 3 1 3, 3 3 7 2 1 |
| 101| 53 | 565| 3  | -96  | 1 6 28 118 | A D A A, A A A D   | 1 8 2 8, 1 8 10 4 0 |
| 61 | 55 | 447| 3  | -12  | 1 8 16 88  | A A A A A          | 3 1 9 9, 3 5 9 4 5 |
| 63 | 63 | 473| 3  | 0    | 1 6 34 70  | A D A A, A A A D   | 1 8 4 8, 3 1 9 9 |
| 75 | 75 | 565| 3  | 0    | 1 6 28 118 | A A A A, A D A D   | 1 8 2 8, 3 1 5 1 5 |

Table 1  non-Gepner spectra
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