Membrane Dynamics in M(atrix) Theory

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We analyze some of the kinematical and dynamical properties of flat infinite membrane solutions in the conjectured M theory proposed by Banks, Fischler, Shenker and Susskind. In particular, we compute the long range potential between membranes and anti-membranes, and between membranes and gravitons, and compare it with the supergravity results. We also discuss membranes with finite relative longitudinal velocities, providing some evidence for the eleven dimensional Lorentz invariance of the theory.
1. Introduction

Recently, Banks, Fischler, Shenker and Susskind have proposed \cite{banks1} a definition of eleven dimensional M theory\footnote{See \cite{banks2} for recent reviews of M theory.} in the infinite momentum frame as a large $N$ limit of maximally supersymmetric matrix quantum mechanics. From the string theory point of view, this description arises as the theory governing the short distance dynamics of D0-branes in type IIA string theory \cite{banks3, banks4, banks5, banks6} (which goes over to M theory in the strong coupling limit). Surprisingly, the same system arises also as a regularization of eleven dimensional supermembrane theory in the lightcone frame \cite{banks7}. This is one of the hints that the proposal of Banks et al, that naively describes only 0-branes, in fact includes all of the degrees of freedom of M theory. Another strong hint for this is provided by the recent papers \cite{banks8, banks9}, which showed that when compactified on a 3-torus to 8 dimensions, M(atrix) theory has an $SL(2, \mathbb{Z})$ invariance that corresponds to the $SL(2, \mathbb{Z})$ factor in the $SL(3, \mathbb{Z}) \times SL(2, \mathbb{Z})$ U duality group of 8 dimensional type II string theory (the $SL(3, \mathbb{Z})$ factor is trivial from the M theory point of view). Since this duality, together with the geometrical symmetries of M theory compactifications, may be used to generate all of the known string theory dualities \cite{banks11}, this suggests that M(atrix) theory indeed reproduces all the dualities of M theory (assuming, of course, that M(atrix) theory has eleven dimensional Lorentz invariance). Since this duality exchanges a membrane with a fivebrane wrapped around the 3-torus, it also suggests that M(atrix) theory includes also fivebranes. It is not yet known how to describe in M(atrix) theory fivebranes that are not wrapped around the longitudinal direction, but a consistent description of wrapped fivebranes was given in \cite{banks12}.

In this paper we analyze in detail some properties of the membrane configurations of M(atrix) theory, and check that they correspond to our expectations from membranes in M theory. We begin in section 2 by reviewing the description of membranes in M(atrix) theory, and analyzing their kinematical properties. In section 3 we compute the long range potential between a membrane and an anti-membrane, by computing the zero point energy of the corresponding configuration in the quantum mechanics. We reproduce the expected $1/r^5$ behavior of the potential, up to numerical constants. At short distances, we find that a tachyonic mode develops when the distance between the membrane and the anti-membrane is of the order of the string scale (which goes to zero in the eleven dimensional limit). In section 4 we compute the long range potential between membranes with a relative longitudinal velocity, and show that it matches our expectations from M
theory for a $v^4/r^5$ behavior (up to numerical constants). This is a direct check of eleven dimensional Lorentz invariance in M(atrix) theory. In section 5 we compute the long range potential between a membrane and a 0-brane (or a graviton). We end in section 6 with a summary of our results and a discussion of some open questions.

2. Infinite membranes in M(atrix) theory

As discussed in [1], the large $N$ limit of the matrix quantum mechanics seems to contain membrane configurations which break half of the supersymmetry. In this section we discuss some aspects of the kinematics of infinite membranes, before performing calculations with these membranes in the next sections.

The Hamiltonian of M(atrix) theory [1] is given by

$$H = R \text{tr} \left( \frac{\Pi_i \Pi_i}{2} + \frac{1}{4} [Y^i, Y^j]^2 + \theta^T \gamma_i [\theta, Y^i] \right).$$

(2.1)

Before beginning, we need to be more precise about what is the meaning of the Hamiltonian (2.1). Our conventions differ slightly from those of [1]. The Hamiltonian (2.1) is $P_+$ (which equals $P^-$), and, therefore, it equals $m^2/2P_-$ (when $P_i = 0$). $P_-$ is quantized to be $N/R$. These conventions are the natural ones for the light cone frame, and they are the only ones that give consistent kinematics for generic configurations.

An infinite membrane spanning the coordinates $Y^1$ and $Y^2$ is described by a configuration in which $[Y^1, Y^2] = 2\pi iz^2$. Since the commutator has non-zero trace, such configurations are obviously impossible for finite $N$. We will regard $Y^{1,2}$, for finite $N$, roughly as $Y^1 = z\sqrt{N}q, Y^2 = z\sqrt{N}p$ where $p, q$ are defined in [1]. The information we need about the matrices $p$ and $q$, for our purposes, is that their spectrum can be taken to go from 0 to $2\pi$. This suggests that the membrane extends from 0 to $2\pi z\sqrt{N}$ in each direction, and, thus, its area is $(2\pi)^2 z^2 N$. Plugging this solution into the Hamiltonian one

\footnote{Our conventions for indices are the following. Indices in the standard frame run from 0 to 9 and 11, and the metric is $\text{diag}(-1,1,\cdots,1)$. Light-cone coordinates are defined by $X^\pm = \frac{1}{\sqrt{2}}(X^0 \pm X^{11})$ and $\gamma^\pm = \frac{1}{\sqrt{2}}(\gamma^0 \pm \gamma^{11})$. Coordinates will generally have upper indices, and momenta will have lower indices. $\gamma^{ij}$ is defined as $\gamma^{ij} = \frac{1}{2}[\gamma^i, \gamma^j]$.}

\footnote{They are different from the conventions of [1] where $P_{11}$ was taken to be $N/R$.}

\footnote{We should caution the reader that we have not made precise our definition of the $N \to \infty$ limit in this regime. But, at this stage, we are using the finite $N$ regulator only in a mild fashion, to cut off the area of the membrane. We will return to this issue later.}
finds that \( H = P_+ = RN(2\pi z^2)^2/2 = m^2/2P_− \), and, therefore, the mass of the membrane is \( m = 2\pi z^2N \). Comparing to the area, we see that the tension of the membrane is \( 1/2\pi \). This is consistent with the calculation of the membrane tension in the appendix of [1].

With these assumptions, we have completely characterized the kinematics of the membrane, and any additional relation will test some of our assumptions. In particular, we can check the assumption that \( P_+ = H \), which in our case gives \( P_+/m = R\pi z^2 \), by comparing it to the value derived from the unbroken SUSYs. The SUSY transformation of the fermionic coordinates, for static configurations, may be written as [1]

\[
\delta \theta = \frac{1}{2} \gamma^+ \left( \frac{R}{2} [Y^i, Y^j] \gamma^{ij} + \gamma^- \right) \epsilon.
\]  

Thus, the infinite membrane configuration breaks half of the supersymmetries, and we can also use this relation to examine the kinematics of the membrane. The membrane solution defined above preserves supersymmetry generators such that \( \gamma^+(R2\pi iz^2\gamma^{12} + \gamma^-)\epsilon = 0 \). A supermembrane in the rest frame conserves only supersymmetry generators \( \epsilon \) such that \( \gamma^0 \gamma^{12} \epsilon = i\epsilon \), and in a boosted frame this becomes

\[
\frac{1}{m}(P_+ \gamma^+ + P_- \gamma^-) \gamma^{12} \epsilon = i\epsilon.
\]  

Multiplying (2.3) by \( i\gamma^+\gamma^- \), we find (using \( (\gamma^-)^2 = 0 \) and \( \{\gamma^+, \gamma^-\} = 2I \)) that \( \gamma^+(2i\frac{P_+}{m} \gamma^{12} + \gamma^-)\epsilon = 0 \), so we should identify \( P_+/m = \pi z^2 R \), which is consistent with what we found before.

For completeness, let us make the distinction between our infinite membrane configurations and the “finite” membrane configurations more precise. By finite membranes we mean membranes defined by another natural rescaling of \( Y^{1,2} \) with \( N \), in which we take \( Y^1 = R_1 q \) and \( Y^2 = R_2 p \), corresponding to \( z = \sqrt{R_1 R_2/N} \). From the spectrum of \( p \) and \( q \) we see that this apparently corresponds to a membrane of area \( (2\pi)^2 R_1 R_2 \) and mass \( 2\pi R_1 R_2 \) (which again gives a tension \( 1/2\pi \)). This type of configuration has a finite rest mass, finite area and infinite longitudinal momentum density and, like the graviton configurations discussed in [1], is one for which the infinite momentum frame Hamiltonian is the same as the light cone Hamiltonian (up to numerical factors), which is an advantage. The disadvantage is that finite size membranes are not BPS-saturated, which makes it more difficult to analyze issues such as charges of the 3-form field. We certainly do not expect to have sensible configurations that look like an \( R_1 \times R_2 \) bit of membrane, so we need work with configurations of closed membranes, and this leads to much more difficult
computations, which have not yet been performed. To summarize, the finite membranes have finite size and infinite longitudinal momentum density whereas the infinite membrane has an area that grows like $N$, giving a finite momentum density.

For infinite membranes, unlike gravitons, we can now go to the rest frame of the membrane by a finite (N independent) boost. In light cone coordinates, boosts act as $P_- \rightarrow \gamma P_-$ and $P_+ \rightarrow P_+/\gamma$. In the rest frame $P_+ = P_-$, so it is easy to see that the boost factor we need is $\gamma = \sqrt{2\pi Rz^2}$. In the lightcone we can loosely think of the system as satisfying (for any wave function $\phi$) $\phi(X^+, X^-) = \phi(X^+, X^- + 2\pi R)$. Boosting by $\gamma$ to go to the rest frame of the membrane, we find $\phi(t, x^{11}) = \phi(t + \sqrt{2\pi R} / \gamma, x^{11} + \sqrt{2\pi R} / \gamma)$. Since we are discussing a configuration that is static in this frame, its wave function satisfies $\phi(t, x^{11}) = \phi(t, x^{11} + \sqrt{2\pi R} / \gamma)$, which determines the radius of the eleventh dimension in the rest frame of the branes to be $R_{rest}^{11} = R / \sqrt{2\gamma} = 1 / 2\pi z^2$. Since we are dealing with configurations that have a definite momentum in the longitudinal direction, the wave functions of our membranes will be spread out in this direction, and we will have to average over it in our computations. Note that $R_{rest}^{11}$ that we found does not depend on $R$, so the infinite membrane system does not become decompactified as $R \rightarrow \infty$ (but rather when $z^2 \rightarrow 0$), unlike the situation for finite energy configurations.

Configurations of several gravitons are described in the matrix theory by block matrices, each of whose blocks corresponds to a single graviton (and such that all of the blocks go to infinite size in the large $N$ limit). In the same way we can describe multiple membrane configurations, by taking several “membrane blocks” with different values of the transverse coordinates. As long as the membranes are parallel (and have the same value of $z$), such a configuration still breaks only half of the supersymmetry.

3. Membrane-anti-membrane dynamics

Anti-0-branes are not present in the matrix model, since the infinite boost we performed has turned them all into 0-branes. However, “anti-membranes” in M theory are just membranes with an opposite orientation, and these have the same status as membranes in the infinite momentum frame. We can easily discuss such an anti-membrane by choosing a configuration with an opposite value of $[Y^1, Y^2]$, since this is identified with the wrapping number of the membrane. For instance, we can multiply $Y^2$ by $(-1)$ to turn a membrane configuration into an anti-membrane configuration. Note that the anti-membrane also breaks half of the supersymmetry, but that if we take a configuration
including both membranes and anti-membranes, the supersymmetry is completely broken, as expected. Configurations involving both gravitons and membranes also break all of the supersymmetry, as they should. These are discussed in section 5.

In this section we will discuss the dynamics of a system including one membrane and one anti-membrane, with some distance $r$ between them (which we will take to be along $Y^3$). Thus, our vacuum state will correspond to a configuration of the form

$$Y_0^1 = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}; \quad Y_0^2 = \begin{pmatrix} P_1 & 0 \\ 0 & -P_2 \end{pmatrix}; \quad Y_0^3 = \begin{pmatrix} 0 & 0 \\ 0 & r \end{pmatrix},$$

(3.1)

where we will now normalize $[Q_1, P_1] = [Q_2, P_2] = 2\pi iz^2$, each block is of size $N \times N$, and all the other $Y_0^i$ vanish.

Eleven dimensional supergravity predicts that the potential between a membrane and an anti-membrane, after averaging over the position in the longitudinal direction, will go like $1/r^5$. To compute this potential in M(atrix) theory, we will integrate out the off-diagonal blocks in all of the matrices, which are the remains of strings stretching from the membrane to the anti-membrane. The supersymmetry is completely broken in this vacuum, since a different half of the SUSY is broken by each block\(^5\). Thus, we would naively expect to have corrections that are much larger than $1/r^5$. The non-trivial cancelation of all higher terms in the potential, such as the $1/r$ and $1/r^3$ terms, serves as a test of the M(atrix) theory away from any obvious BPS limit.

Another reason to suspect that we might run into problems, in addition to the fact that the supersymmetry is completely broken, is the following. In string theory, the large $r$ closed string tree level potential between a membrane and an anti-membrane can not be computed in terms of the low energy excitations of the field theory on the D-branes. Instead, one needs to include the whole infinite tower of states in the open string sector. But here, when going from the full 0-brane description to the QM, we have explicitly dropped all these states.

In section 3.1 we compute the masses of the relevant bosonic modes in the presence of the membrane-anti-membrane pair, and set up the general procedure for such computations. In section 3.2 we do the same for the fermionic modes. In section 3.3 we compute the long-range potential between the membrane and the anti-membrane, and check that it agrees with the known M theory result (up to numerical factors). In section 3.4 we give a short discussion of the annihilation process.

\(^5\) We will later return to some speculations regarding approximate SUSYs.
3.1. Bosonic off-diagonal quadratic terms

The masses for the off-diagonal bosonic modes arise (at tree-level) from the term \( \sum_{1 \leq i < j \leq 9} \frac{R}{2} [Y^i, Y^j]^2 \) in the matrix model Hamiltonian. We will expand this term around the vacuum \( \text{vacuum} \), and keep only the quadratic terms in the off-diagonal modes, which we will denote by \( Y^\mu \sim \begin{pmatrix} 0 & A_\mu \\ A_\mu^\dagger & 0 \end{pmatrix} \). We will then use the Born-Oppenheimer approximation to calculate the corrections to the energy of the configuration.

Expanding the potential in a straightforward manner, the terms we find for \( \mu = 4, 5, \ldots, 9 \) are

\[
R[\text{tr}(2Q_1 A_\mu Q_2 A_\mu^\dagger - A_\mu Q_2^2 A_\mu^\dagger - Q_1 A_\mu A_\mu^\dagger Q_1)] + \\
\text{tr}(-2P_1 A_\mu P_2 A_\mu^\dagger - A_\mu P_2^2 A_\mu^\dagger - P_1 A_\mu A_\mu^\dagger P_1) - r^2 \text{tr}(A_\mu A_\mu^\dagger)].
\]

(3.2)

The term \([Y^1, Y^2]^2 \) gives

\[
R[\text{tr}(A_1 A_2^\dagger [Q_1, P_1] - A_2 A_1^\dagger [Q_1, P_1] - A_2 A_1^\dagger [Q_2, P_2] + A_2 A_1^\dagger [Q_2, P_2]) + \\
\text{tr}(Q_1 A_2 A_1^\dagger P_1 - A_1 P_2 A_1^\dagger P_1 - P_1 A_1 A_1^\dagger P_1 - A_2 Q_2 A_1^\dagger P_1 + \\
Q_1 A_2 Q_2 A_1^\dagger - A_1 P_2 Q_2 A_1^\dagger - P_1 A_1 Q_2 A_1^\dagger - A_2 Q_2^2 A_1^\dagger - \\
Q_1 A_2 A_1^\dagger Q_1 + A_1 P_2 A_1^\dagger Q_1 + P_1 A_1 A_1^\dagger Q_1 + A_2 Q_2 A_1^\dagger Q_1 + \\
Q_1 A_2 P_2 A_1^\dagger - A_1 P_2^2 A_1^\dagger - P_1 A_1 P_2 A_1^\dagger - A_2 Q_2 P_2 A_1^\dagger]).
\]

(3.3)

The \([Y^1, Y^3]^2 \) term gives

\[
R \text{tr}[(Q_1 A_3 + r A_1 - A_3 Q_2)(-A_1^\dagger r + Q_2 A_3^\dagger - A_3^\dagger Q_1)],
\]

(3.4)

and the \([Y^2, Y^3]^2 \) term gives

\[
R \text{tr}[(P_1 A_3 + r A_2 + A_3 P_2)(-A_1^\dagger r - P_2 A_3^\dagger - A_3^\dagger P_1)].
\]

(3.5)

Since \([Q_i, P_i] = 2\pi i z^2 \), we can represent them as differential operators by \( Q_i = \sigma_i \) and \( P_i = -2\pi i z^2 \partial_i \), and convert the Hamiltonian into a differential operator (as in \([12]\)). In this language, the matrix \( A_\mu \) becomes now a function of \( \sigma_1 \) and \( \sigma_2 \), so we will end up with a 2+1 dimensional field theory (with space dependent couplings). Note that even though each membrane has two coordinates, each membrane contributes only one coordinate in this representation. Correspondingly, the differential operator realization of the Hamiltonian

\[6 \] Maintaining operator ordering, to which we will return momentarily.
here is quite different from the realization we get when treating commutators as Poisson brackets, as is done for the finite membrane [1]. Thus, these two objects are very different from the point of view of the light-cone formulation. One would like to believe that taking the area of the finite membrane to infinity will be equivalent to infinitely boosting the infinite membrane, but a concrete demonstration of this does not yet exist.

There are two subtleties that need to be taken into account. The first is that we will take the coordinates $\sigma_i$ to go from $-\infty$ to $\infty$. We will then use finite (large) $N$ to regulate the area, assuming that for finite $N$ we can truncate the spectrum consistently. In the best of all possible worlds, we would calculate the exact spectrum for finite $N$, and then take $N$ to infinity. One way of doing this could be to choose $\sigma_i$ to be in the line segment $[-\pi z \sqrt{N}, \pi z \sqrt{N}]$, and to also put a cut-off on the momentum of the order $P_{\text{max}} = \pi z \sqrt{N}$, and to write the matrices in an appropriate basis. However, we do not know how to perform an exact computation in this case. Instead, what we do here is take the cut-off on $\sigma$ and on the momentum to infinity, obtaining the full real line with no restrictions on momenta. We calculate the spectrum and the wave functions on the entire axis and then regulate to finite $N$ by taking wave functions that are, essentially, supported in the finite interval $[-\pi z \sqrt{N}, \pi z \sqrt{N}]$ (both in $\sigma$-space and in momentum space). This yields the correct $z$ and $N$ dependence, but might easily introduce numerical factors that we cannot determine at this level of precision.

The second subtlety is related to the first one. For finite $N$ we know exactly what is the class of matrices that we are integrating out. However, once we write them as functions it is not clear what class of functions is the correct one. For example, are we allowed to take $A_\mu$ to be singular (for example, to behave like a derivative operator)? This will make a difference in the formal manipulations that we will do shortly, which include various integrations by parts, derivatives, etc. Since at the end of the day we find that the Hamiltonian that we want to diagonalize is an harmonic oscillator, we will take the usual $L^2$ functions on the $\sigma_i$ lines, and we will use the eigenfunctions of the harmonic oscillator as a basis for the space of functions that we allow. This point seems to be more of a technical nuisance than a real issue.
The rules we will use for transforming a trace, such as the ones written above, into a field theory expression, are

\[
\text{tr}(A_m O_2 A^+_n O_1) \rightarrow \eta(O_2) \int A^*_n O_1 O_2 A_m
\]

\[
\text{tr}(A_m A^+_n O_1 O_1') \rightarrow \int A^*_n O_1 O_1' A_m
\]

\[
\text{tr}(A^+_m A_n O_2 O_2') \rightarrow \eta(O_2) \eta(O_2') \int A^*_m O_2' O_2 A_n,
\]

(3.6)

where a 1(2) subscript denotes that this is an operator of the form \( \sigma_1 \) (\( \sigma_2 \)) or \( \partial_{\sigma_1} \) (\( \partial_{\sigma_2} \)), and \( \eta(\partial_i) = -1, \eta(\sigma_i) = 1 \).

Let us, for example, show how to derive the first relation for \( O_i = \partial_i \). We replace the trace by a sum over a complete set of functions \( H_n(\sigma) \) (such as eigenfunctions of the harmonic oscillator), satisfying \( \sum_n H_n(\sigma) H_n(\tau) = \delta(\sigma - \tau) \). Every summation over an index corresponding to the membrane is replaced by an integral over \( \sigma_1 \), and every summation over an index corresponding to the anti-membrane is replaced by an integral over \( \sigma_2 \). Using the explicit form of the operators, we obtain

\[
\text{tr}(A_m O_2 A^+_n O_1) = \sum_n \int d\sigma_1 d\sigma_2 d\hat{\sigma}_1 H_n(\sigma_1) A_m(\sigma_1, \sigma_2) \partial_{\sigma_2} A^*_n(\hat{\sigma}_1, \sigma_2) \partial_{\hat{\sigma}_1} H^*_n(\hat{\sigma}_1) =
\]

\[
= -\sum_n \int d\sigma_1 d\sigma_2 d\hat{\sigma}_1 H_n(\sigma_1) H_n(\hat{\sigma}_1) A_m(\sigma_1, \sigma_2) \partial_{\sigma_2} \partial_{\hat{\sigma}_1} A^*_n(\hat{\sigma}_1, \sigma_2) =
\]

\[
= -\int d\sigma_1 d\sigma_2 A^*_n(\sigma_1, \sigma_2) \partial_{\sigma_1} \partial_{\sigma_2} A_m(\sigma_1, \sigma_2),
\]

(3.7)

where we have performed several integrations by parts that are well defined on our class of functions.

Using these rules, and defining \( P = \frac{1}{\sqrt{2}}(P_1 - P_2) \) and \( Q = \frac{1}{\sqrt{2}}(Q_1 - Q_2) \) (satisfying \([P, Q] = -2\pi i z^2\)), the relevant terms (equations (3.2)-(3.5)) become

\[
-R \int A^*_\mu(2P^2 + 2Q^2 + r^2) A_\mu,
\]

(3.8)

\[
-2R \int A^*_1 P^2 A_1 + A^*_2 Q^2 A_2 + A^*_3 (P^2 + Q^2) A_3 + A^*_2 (QP + 2\pi i z^2) A_1 + A^*_1 (PQ - 2\pi i z^2) A_2,
\]

(3.9)

\[
R \int \sqrt{2}(-r A^*_1 Q A_3 - r A^*_3 Q A_1) - r^2 A^*_1 A_1
\]

(3.10)
\[ R \int \sqrt{2}(-rA_2^*PA_3 - rA_3^*PA_2) - r^2A_2^*A_2. \] (3.11)

The integrations were originally over both \( \sigma_1 \) and \( \sigma_2 \), but since the Hamiltonian has no dependence on \( \sigma_1 + \sigma_2 \), we can turn them into integrations just over \( \sigma_1 - \sigma_2 \), with approximately an order \( N \) degeneracy of all modes (corresponding to multiplying the \( A_i \) by an arbitrary function of \( \sigma_1 + \sigma_2 \)). The terms we found involving \( A_1, A_2 \) and \( A_3 \) can be rewritten in the form

\[
-R \int \left( \frac{2P^2 + r^2}{\sqrt{2rQ}} \right) \left( \frac{-2(PQ - 2\pi iz^2)}{2Q^2 + r^2} \right) \left( \frac{\sqrt{2rQ}}{2rP} \right) \left( \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} \right). \] (3.12)

We can write these terms as \(- \int A^* M_1 A\) where

\[
M_1 = 2R \begin{pmatrix} \frac{P^2 + r^2}{2} & -(PQ - 2\pi iz^2) & \frac{rQ}{\sqrt{2}} \\ -(PQ + 2\pi iz^2) & \frac{Q^2 + r^2}{2} & \frac{rP}{\sqrt{2}} \\ \frac{rQ}{\sqrt{2}} & \frac{rP}{\sqrt{2}} & \frac{P^2 + Q^2}{2} \end{pmatrix}. \] (3.13)

Conjugating by the unitary matrix

\[
U = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \] (3.14)

and defining \( a = (Q + iP)/\sqrt{2(2\pi z^2)} \), \( a^\dagger = (Q - iP)/\sqrt{2(2\pi z^2)} \) (satisfying \([a, a^\dagger] = 1\)) and \( \tilde{r} = 2/\sqrt{2(2\pi z^2)} \), the Hamiltonian matrix \( M_2 = UM_1U^{-1} \) becomes

\[
M_2 = 4\pi z^2 R \begin{pmatrix} a^\dagger a + \tilde{r}^2 - 1 & (a^\dagger)^2 & \tilde{r}a^\dagger \\ a^2 & aa^\dagger + \tilde{r}^2 + 1 & -\tilde{r}a \\ \tilde{r}a & -\tilde{r}a^\dagger & aa^\dagger + a^\dagger a \end{pmatrix}. \] (3.15)

Next, let us look for eigenvectors of this matrix. It is natural to define a basis of harmonic oscillator eigenfunctions \( L_n(\sigma) \) satisfying \( aL_n(\sigma) = \sqrt{n}L_{n-1}(\sigma) \) and \( a^\dagger L_n(\sigma) = \sqrt{n+1}L_{n+1}(\sigma) \). The eigenvectors are then of the form \((\alpha L_n(\sigma), \beta L_{n-2}(\sigma), \gamma L_{n-1}(\sigma))\). Letting \( M_2 \) act on a vector of this form, we can easily see that it is transformed into a vector of the same type, with the matrix \( \tilde{M}_2 \) acting on \((\alpha, \beta, \gamma)\), where

\[
\tilde{M}_2 = 4\pi Rz^2 \begin{pmatrix} \tilde{r}^2 + n - 1 & \sqrt{n(n-1)} & \tilde{r} \sqrt{n} \\ \sqrt{n(n-1)} & \tilde{r}^2 + n & -\tilde{r} \sqrt{n-1} \\ \tilde{r} \sqrt{n} & -\tilde{r} \sqrt{n-1} & 2n - 1 \end{pmatrix}. \] (3.16)
Thus, all that is left to do is to find eigenvectors of this matrix, and fortunately this is easy to do: there is one eigenvector \(v_3 = (-\sqrt{n}, \sqrt{n-1}, \tilde{r})\) with eigenvalue 0, and two eigenvectors with eigenvalue \(4\pi z^2 R(\tilde{r}^2+2n-1) = R(r^2+4\pi z^2(2n+1))\) which we can choose to be \(v_1 = (\tilde{r}, 0, \sqrt{n})\) and \(v_2 = (\sqrt{n}(n-1), n+\tilde{r}^2, -\tilde{r}\sqrt{n-1})\). Note that the eigenfunction corresponding to \(v_1\) exists for any \(n \geq 0\), while the eigenfunction corresponding to \(v_2\) exists only for \(n \geq 2\). The zero eigenvalue corresponds to a gauge transformation, which can be ignored.

For the 6 other bosonic variables \(A_\mu\), equation (3.8) immediately implies that the eigenfunctions are again harmonic oscillator eigenfunctions \(L_n(\sigma)\), with eigenvalues (in the same normalization as above) \(R(r^2+4\pi z^2(2n+1))\) for any \(n \geq 0\). These eigenvalues are (up to a factor \(R\)) the frequencies squared of the harmonic oscillators corresponding to the off-diagonal modes.

### 3.2. Fermionic off-diagonal quadratic terms

The term in the Hamiltonian (2.1) that gives a mass to the off-diagonal fermionic modes is

\[
R \text{tr}(\theta^T \gamma_1 |\theta, Y^r_0|).
\]

(3.17)

Denoting by \(\theta\) also the upper-right off-diagonal part of \(\theta\), we find for it a term of the form

\[
R \text{tr}(\theta^T \{\gamma_1(\theta Q_1 - Q_2) + \gamma_2(\theta P_1 + P_2) + \gamma_3(-r\theta)\}).
\]

(3.18)

Translating into the field theory this becomes

\[
R \int \theta^T \{\gamma_1(Q_1 - Q_2) + \gamma_2(-P_1 + P_2) + \gamma_3 r\} \theta.
\]

(3.19)

Now, it is easy to see that the mass matrix squared is simply (in the same notations and normalizations as we used for the bosonic sector) \(M_f^2 = R^2(2(Q^2 + P^2) + r^2 - 2\gamma_1 \gamma_2 [Q, P])\).

The eigenvalues of \(\gamma_1 \gamma_2\) are \(\pm i\), and, thus, we find that the eigenvalues of \(M_f^2\) (with the usual eigenfunctions \(L_n(\sigma)\) defined above) are \(R^2(\tilde{r}^2+4\pi z^2(2n+2))\) and \(R^2(\tilde{r}^2+4\pi z^2(2n))\) for \(n = 0, 1, \cdots\). Since half of the fermions may be viewed as creation operators and half as annihilation operators, we have 4 states with each eigenvalue.

Note that for a pair of membranes, the terms we were discussing in the Hamiltonian (both for the bosons and for the fermions) would depend only on \(Q_1 - Q_2\) and on \(P_1 + P_2\), which commute with each other. Thus, all the mass shifts arising from the non-commutativity would vanish, and the masses of all the bosonic and fermionic off-diagonal modes would be the same, as required by the unbroken supersymmetry in this case.
3.3. The membrane-anti-membrane potential

Now we have all the information we need for computing the zero-point energy of the membrane-anti-membrane configuration. The variable \( \sigma_+ \sim \sigma_1 + \sigma_2 \) decoupled from all our calculations, so that it just corresponds to an (order \( N \)) degeneracy for all the modes we found. The zero-point energy of each mode is the square root of the matrices we discussed above (which correspond to \( \omega^2 \)). Since our variables are complex, the zero-point energy is the sum of the frequencies we computed above (and not of half the frequencies). Joining all of our results, we find that the formula for the total zero-point energy is

\[
V(r) = R \sum_{n=0}^{\infty} \left[ 6 \sqrt{r^2 + 4\pi z^2(2n+1)} - 4 \sqrt{r^2 + 4\pi z^2(2n+2)} - 4 \sqrt{r^2 + 4\pi z^2(2n+2)} + 4 \sqrt{r^2 + 4\pi z^2(2n+1)} \right] +
R \sum_{n=0}^{\infty} \sqrt{r^2 + 4\pi z^2(2n-1)} + R \sum_{n=2}^{\infty} \sqrt{r^2 + 4\pi z^2(2n-1)}
\]

or

\[
V(r) = R \sum_{n=1}^{\infty} \left[ \sqrt{r^2 + 4\pi z^2(2n+1)} - 4 \sqrt{r^2 + 4\pi z^2(2n+2)} + 6 \sqrt{r^2 + 4\pi z^2(2n-1)} - 4 \sqrt{r^2 + 4\pi z^2(2n-2)} + \sqrt{r^2 + 4\pi z^2(2n-3)} \right] \equiv \sum_{n=1}^{\infty} V(r, n).
\]

Although the sum of each term separately obviously does not converge, \( V(r, n) \) behaves for large \( n \) like \( n^{-7/2} \), so the sum is well-defined. The fact that the coefficients in (3.21) are given by \((1, -4, 6, -4, 1)\), which are binomial coefficients, guarantees the vanishing of the first four terms in the expansion of \( V(r, n) \) either in large \( n \) or in large \( r^2 \). In fact, we can perform the summation in (3.21) using the Euler-Maclaurin formula

\[
\sum_{k=1}^{\infty} F(k) = \int_0^{\infty} F(k) dk - \frac{1}{2} F(0) - \frac{1}{12} F'(0) + \frac{1}{720} F'''(0) + \cdots
\]

which is valid for functions \( F \) who vanish (with all their derivatives) at infinity. Plugging in \( F(k) = V(r, k) \), and restoring the order \( N \) degeneracy from the dependence on \( x_+ \), we find that the integral behaves for large \( r \) like \((-\frac{3}{16}(4\pi z^2)^3RNr^{-5} + O(r^{-7}))\), while the other terms vanish at least as fast as \( r^{-7} \), and in fact we can show that for large \( r \) (compared to \( z \))

\[
V(r) = -\frac{3(4\pi z^2)^3RN}{16r^5} + O\left(\frac{1}{r^9}\right).
\]
This is the one-loop potential generated for $r$ in the quantum mechanics, and it should correspond to the potential for infinite membranes in eleven dimensions, after integration over the separation of the membranes in the longitudinal direction (since we are working with states of definite longitudinal momentum).

Let us compare the potential we found with the supergravity result for the long-range potential between a membrane and an anti-membrane. We wish to find the correction to the rest mass of the system per unit area. In supergravity, the long-range gravitational force between two membranes exactly cancels the long-range force arising from the 3-form field. Thus, for a membrane-anti-membrane pair, the long-range potential is exactly twice the gravitational potential. The metric corresponding to a supermembrane solution of 11D supergravity \cite{13} is (see, e.g., \cite{3})

$$g_{00} = (1 + \frac{2\kappa_{11}^2 T_M}{\Omega_7 r^6})^{-2/3}$$

where $\kappa_{11}$ is the 11D gravitational constant, $T_M$ is the membrane tension, and $\Omega_7 = \pi^4/3$ is the volume of a 7-sphere of unit radius. $\kappa_{11}$ is related to the membrane tension by $\kappa_{11}^2 T_M^3 = \frac{2\pi}{\sqrt{3}}$, and in the Newtonian limit $g_{00} \sim 1 - 2V_{11}^g(r)$ where $V_{11}^g(r)$ is the gravitational potential, so that we find in eleven dimensions $V_{11}^g(r) = -4/\pi^2 T_M r^6$. The membrane-anti-membrane potential per unit area is this potential multiplied by the membrane tension, and restoring the factor of 2 we find that the total 11D potential is $V_{11}(r) = -8/\pi^2 T_M r^6$. As we mentioned earlier, in their rest frame the membranes live on a compact circle of radius $R_{11}^{\text{rest}} = 1/2\pi z^2$. To get the M(atrix) theory potential which is in the lightcone frame, we should average over the longitudinal direction, while adding images of the membranes due to the periodicity requirement:

$$V_{LC}^{\text{rest}}(r) = \frac{1}{2\pi R_{11}^{\text{rest}}} \int_{-\infty}^{\infty} dx_{11} V_{11}(\sqrt{r^2 + x_{11}^2}) = -\frac{3}{\pi(2\pi R_{11}^{\text{rest}}) T_M r^5}.$$ (3.25)

As discussed in section 2, in the M(atrix) theory the membrane tension is given by $T_M = 1/2\pi$, and plugging in the value of $R_{11}^{\text{rest}}$ we find $V_{LC}^{\text{rest}}(r) = -6z^2/r^5$. Since the area of the membrane is $(2\pi)^2 N z^2$, we find that the rest energy of this configuration behaves as

$$m \sim (2\pi)^2 N z^2 (2 \cdot \frac{1}{2\pi} - \frac{6z^2}{r^5}).$$ (3.26)

Thus, we expect the leading order (in $1/r$) correction to the M(atrix) theory calculation of $m^2/2P_-$ to be:

$$2 \cdot ((2\pi)^2 N z^2)^2 \cdot 2 \cdot \frac{1}{2\pi} \cdot \frac{-6z^2}{r^5} / (2 \cdot 2N/R) = -\frac{48\pi^3 N R z^6}{r^5}.$$ (3.27)
This is identical to what we found in \((3.23)\), up to a factor of 4. We are not sure if this factor arises from a difference between our conventions and the supergravity conventions, or if it arises from the assumptions we made as to the degeneracy of each state in the large \(N\) limit. To the level of precision we have been working in we do not have good control over numerical factors arising from this degeneracy.

The decay of the potential as \(r^{-5}\), necessary for locality in eleven dimensions, depended on many cancelations from the quantum mechanics point of view. Naively, we would expect \(V(r)\) in a theory which has the same number of bosons and fermions to behave as \(r\) for large \(r\). However, the fact that for theories with spontaneously broken SUSY (as is the case in our computation) the sum of the masses squared of the bosons still equals that of the fermions, ensures that \(V(r)\) should decay at least as \(1/r\). And indeed, this is the behavior we would have found for the same computation in a different number of transverse dimensions, since our cancelations of the \(1/r\) and \(1/r^3\) terms depended on \((1, 4, 6, 4, 1)\) being binomial coefficients. For instance, in the analogous seven dimensional theory, the coefficients in \((3.21)\) would have been \((1, 2, 2, 2, 1)\), and we would have found a potential behaving like \(1/r\) (which is, by the way, the expected potential for membranes in this theory). Presumably, the fact that in our case we get a decay like \(1/r^5\) is related to the larger amount of SUSY that exists in the eleven dimensional theory, but the exact way in which this works is still unclear to us. For instance if, in theories with 16 supersymmetry generators, there are also sum rules on the fourth and sixth powers of the masses, that would explain our results, but we do not know of the existence of such sum rules.

Another way in which one may try to understand the cancelation to this order, which was suggested to us by M. Douglas and S. Shenker, is the following. The leading correction to the 0-brane action\([4]\), which gives the desired graviton-graviton scattering, is given by a term of order \(\nu^4/r^7\), whose \(D = 10\) SYM origin is an abelian \(F_{\mu\nu}^4\) term. The non-abelian completion of this term, if such a term exists, has the commutator in it. If we can treat the membrane-anti-membrane as soft breaking of SUSY, the term \(\nu^4/r^7\) will be replaced by \(z^8/r^7\). Integrating over two coordinates of the membrane this becomes \(z^6/r^5\), as we found above.

### 3.4. Membrane-anti-membrane annihilation

For \(r^2 = 4\pi z^2\), one of the frequencies we computed vanishes, while for \(r^2 < 4\pi z^2\) the field theory develops a tachyon, signaling an instability in the configuration. Note that since \(R_{\text{rest}}^{11} = 1/2\pi z^2\), this distance is exactly proportional to the string scale in our formalism,
where a similar instability may be found for a D-brane-anti-D-brane configuration. Of course, for small values of the frequencies, our one-loop approximation is no longer valid, and we can no longer neglect the excitations of the off-diagonal modes. However, when the string scale is large compared to the Planck scale (i.e. the radius of the eleventh dimension is small), the other off-diagonal modes will still have high frequencies when the tachyon develops, and it is legitimate to ignore them and assume that only the tachyonic mode is excited.

The wave function corresponding to the ground state of this tachyonic mode is of the form $A_1 \sim e^{-\frac{1}{2}(\sigma_1 - \sigma_2)^2}$, $A_2 \sim ie^{-\frac{1}{2}(\sigma_1 - \sigma_2)^2}$, and this can be multiplied by any function of $\sigma_1 + \sigma_2$. Since we interpreted the $\sigma_i$ as positions along one of the coordinates of the membrane (recall that they came from the $Q_i$), such an excitation is localized at the same place in both membranes. Note that in momentum space, which we identify with the second coordinate of the membrane, the tachyonic mode is also proportional to $e^{-\frac{1}{2}(\partial_1 - \partial_2)^2}$, so the excitation is in fact localized in both membrane coordinates. Adding an arbitrary function of $\sigma_1 + \sigma_2$ just corresponds to a superposition of many such local excitations along the surface of the membranes. When the tachyon condenses, the fields corresponding to the distance between the two membranes become massive, so that the membranes tend to join together, and presumably eventually annihilate into gravitons. Presumably, this may be interpreted as the condensation of a string between the two membranes, which in M theory is identified with a “wormhole” configuration connecting the membrane and the anti-membrane [14]. The condensation of such strings “eats up” the surface of the membranes, and they annihilate into gravitons. It is less clear what happens in the eleven dimensional limit, when the string scale (where the tachyonic mode arises) is much smaller than the Planck scale. In this case we cannot trust our approximations, and different methods should apparently be used to analyze the annihilation.

4. Membranes at finite longitudinal velocities

From the point of view of supergravity, gravitons are considerably simpler then membranes. In M(atrix) theory, the situation is reversed, in the sense that we have a full description of the infinite membranes whereas the graviton wave functions remain elusive. Realizing this we can now re-examine the issue of Lorentz invariance, which we could not have done in the supergravity multiplet sector due to lack of knowledge of the bound state wave function. In the following we will take the first step towards showing Lorentz
invariance in the membrane sector. We will show that the M(atrix) model computation reproduces the supergravity potential between a pair of infinite membranes (in the kinematical setup discussed above) when they are moving with a small relative longitudinal velocity. This implies that the M(atrix) model will reproduce processes with longitudinal momentum transfer to the first sub-leading contribution to the moduli space approximation.

Let us first discuss the kinematical setup and the supergravity predictions. We will now take a pair of membranes satisfying \(|P_i, Q_i| = -2\pi iz_i^2, i = 1, 2\). Transforming to the rest frame of the first membrane, the velocity of the second membrane is proportional to \((z_2^2 - z_1^2)/z^2\) (to leading order in \(z_1 - z_2\)). We work only to leading order in \(z_1 - z_2\), and in quantities that do not depend on the difference we will denote both \(z_1\) and \(z_2\) by \(z\). In this section we will not put in all of the numerical factors, and will check only the dependence of the results on \(N, R, r\) and the \(z_i\)’s.

The supergravity result for the potential in the approximate rest frame (where the velocities are small), in the eleven dimensional theory, is \(Vol \times v^4/r^6\), where \(Vol\) is the area of the two membranes. Since we are dealing with a configuration that in the rest frame is compactified on a circle of radius \(1/z^2\), this is modified (upon averaging over the longitudinal direction) to \(Vol \times z^2v^4/r^5\). Recalling the \(z\) dependence of the area, we obtain that the potential in the rest frame is \(Nz^4v^4/r^5\). In the Hamiltonian we therefore expect a correction \(1/N/R \cdot NZ^4v^4/r^5\).

Let us perform now the M(atrix) model calculation. We implement this configuration by taking \(Q_i = z_i\sigma_i, P_i = -iz_i\partial\sigma_i\), and we use the method described above to write the Hamiltonian of the off-diagonal terms as a simple 2+1 field theory. As we will see shortly, the numerical details of the phase-space matching are more subtle than in the membrane anti-membrane case (since the field theory changes qualitatively when \(z_1 = z_2\)), but it is still easy to extract the dependence of the answer on \(z, N, R\) and \(r\).

Let us take, for example, the Hamiltonian for coordinates transverse to the brane. We obtain that the potential term is

\[
\int d\sigma_1d\sigma_2 A^*_\mu(r^2 + (Q_1 - Q_2)^2 + (P_1 + P_2)^2)A_\mu.
\]

Changing to variables \(\sigma_+ = \frac{z_1\sigma_1 + z_2\sigma_2}{\sqrt{z_1^2 + z_2^2}}\) and \(\sigma_- = \frac{z_2\sigma_1 - z_1\sigma_2}{\sqrt{z_1^2 + z_2^2}}\), we obtain that the Hamiltonian is (to leading order in \(z_1^2 - z_2^2\))

\[
\int d\sigma_+ d\sigma_- A^*_\mu(r^2 + \frac{(z_1^2 - z_2^2)^2}{z^2}(\sigma_+ + \frac{2z_1z_2}{z_1^2 - z_2^2}\sigma_-)^2 - z^2\partial_+^2)A_\mu.
\]
The energy levels of this field theory, due to the harmonic oscillator in the $\sigma_+$ direction, are of the form $r^2 + (z_1^2 - z_2^2)(2n + 1)$. Similar results pertain to all the other bosonic and fermionic modes, and we find that the sum over frequencies (performed in the same way as in the previous section) is proportional to $(z_1^2 - z_2^2)^3/r^5$.

However, in this case we need to be more careful in going to the field theory limit. We should check more carefully what is the upper value of $n$ that we should take (for finite $N$), and, correspondingly, what is the degeneracy that arises from the integration over $\sigma_-$ (since the total number of modes is $N^2$). We can determine the highest allowed value of $n$ (for finite $N$) as follows. The size of the membranes is $[-z\sqrt{N}, z\sqrt{N}]$ in each direction, so the most massive state should have a mass of order $r^2 + z^2 N$. On the other hand, the states we find for each $n$ have energies of order $r^2 + (z_1^2 - z_2^2)(2n + 1)$. Equating these two energies, we conclude that the highest allowed value of $n$ should be $N \frac{z_1^2}{z_2}$, and, correspondingly, the degeneracy of each level is of order $N \frac{z_1^2}{z_2}$. Thus, we find that the M(atrix) theory result is $\frac{1}{N^2} N \frac{z_1^2}{z_2} \frac{1}{r^2}$, which is the same as the supergravity result (up to numerical factors).

Note that if we (not necessarily justifiably) extend our calculations to small values of $r$, we find now a tachyon appearing for $r \sim \sqrt{z_1^2 - z_2^2} \sim \sqrt{v} z$ and not at the string scale $r \sim z$. The scale $r \sim \sqrt{v}$ (in string units) is known to be the relevant “interaction” scale for moving D-branes in string theory [15,16], and we see that it appears also in M(atrix) theory. We can easily generalize the calculation above to the case of a membrane and an anti-membrane moving at a relative longitudinal velocity, and we find the expected $v^2 / r^5$ correction in this case.

5. Membrane-0-brane dynamics

The computation for a membrane and a 0-brane (or a graviton state) is exactly the same as that of the membrane-anti-membrane case, if we take the second block to be of size $1 \times 1$ (instead of $N \times N$), and set $Q_2 = P_2 = 0$. Thus, our field theory will just be $1 + 1$ dimensional, and we will find simple mass matrices for all the fields. For the bosonic transverse modes the mass matrix squared is $R^2 (r^2 + Q^2 + P^2) \to R^2 (r^2 + 2\pi z^2(2n + 1))$, for the fermions it is $R^2 (r^2 + Q^2 + P^2 + \gamma_1 \gamma_2 [Q, P]) \to R^2 (r^2 + 2\pi z^2(2n + 1 \pm 1))$, and for the longitudinal bosons we again find the same split as before. For $N_1$ 0-branes each mode has an additional degeneracy of $N_1$ (we assume $N_1 \ll N$, since we take the $N \to \infty$ limit).
limit first). The total potential thus comes out to be, by a similar computation to the membrane-anti-membrane case,

\[ V(r) = -3(2\pi z^2)^3 N_1 R \frac{1}{16r^5} + O(\frac{1}{r^9}). \quad (5.1) \]

Again, we should compare this with the expected supergravity result. For this we again transform to the rest frame of the membrane. In the light cone frame, the 0-branes had \( P^+ = N_1/R \). In the rest frame, this is transformed to \( P^+ = N_1 \sqrt{2}\pi z^2 \), and this is also (up to a \( \sqrt{2} \) factor) the energy of the 0-branes in this frame. The result we expect to find is the same as the result in the membrane-anti-membrane case, but with the energy of the 0-brane replacing the mass of the membrane (up to a numerical constant). This is because from the 10D perspective the energy of the 0-brane is simply its mass, and it is also at rest, and both the 0-brane-2-brane and the 2-brane-anti-2-brane potentials are proportional to the corresponding gravitational potentials (but with a different constant of proportionality). The ratio of the 10D masses is \( N_1 \pi z^2 / 2\pi z^2 N = N_1 / 2N \), so we expect the 0-brane result to be \( N_1 / N \) times the membrane-anti-membrane result \( (3.23) \), up to numerical factors, and this is indeed what we find in (5.1).

As in the membrane-anti-membrane case, we find a tachyonic mode at the string scale. A similar mode exists for a D-0-brane D-2-brane configuration in type IIA string theory. The wave function of the tachyon now looks like \( e^{-\frac{1}{2}x^2} \) where \( x \) is the position of the 0-brane, so it is localized (in the membrane worldvolume) near the position of the 0-brane. The condensation of this tachyon makes the modes corresponding to the separation between the 0-brane and the membrane massive, so that they would tend to join together. Presumably, the 0-brane is swallowed into the membrane worldvolume, corresponding to a boost of the membrane from the eleven dimensional point of view, or to a gauge field inside the D-2-brane from the ten dimensional point of view. The corresponding bound states of the D-0-brane and the D-2-brane in type IIA string theory were recently discussed in [16]. As in section 3, it is less clear what happens in the eleven dimensional limit, since our approximations are not necessarily valid then.

6. Conclusions and open questions

In this paper we computed the long range potentials associated with various configurations involving infinite membrane in M(atrix) theory, and compared the results with
the known supergravity potentials. In all three cases we discussed (the membrane-anti-
membrane potential, the potential between membranes at relative longitudinal velocities
and the potential between a membrane and a graviton), we found that the M(atrix) the-
ory computation agrees with the supergravity result, up to numerical factors. We believe
this provides more evidence that M(atrix) theory indeed describes M theory membranes
correctly, and that it is Lorentz invariant (though, obviously, more tests are needed to
establish exact Lorentz invariance).

Our computations in the quantum mechanics involved only the zero point energies
of the off-diagonal matrices, ignoring their fluctuations. This is valid when the distance
between the membranes (or gravitons) in our computations is large, so this is the only
regime where we can trust our calculations. When the distance is of the order of the
string scale, we find that in the membrane-anti-membrane and graviton-membrane con-
figurations a tachyon (a mode with negative $\omega^2$) appears. We can trust this result only
if the string scale is larger than the eleven dimensional Planck scale, where we expect
additional corrections could arise, and then our tachyons presumably correspond exactly
to the corresponding tachyons arising from open strings between D-branes in the type IIA
theory (which is then weakly coupled). It is less clear what we can say about the opposite
limit, in which the eleventh dimension is large, and the string scale is much smaller than
the Planck scale. This is an issue that deserves further investigation.

In our computations we worked only with infinite flat membranes. It would be in-
teresting to find a direct connection between these configurations and the finite, closed
membranes which also exist (as long-lived semi-classical configurations) in M(atrix) the-
ory. Upon compactification, the infinite membrane solutions, if wrapped around compact
directions, become finite energy solutions, and the problems we encounter due to the infi-
nite energy and size of the membranes should disappear. Thus, it should be interesting to
perform similar computations to ours with wrapped membranes in compactified M(atrix)
theory.

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References

[1] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, “M Theory As A Matrix Model: A Conjecture”, hep-th/9610043.
[2] J. H. Schwarz, “Lectures on Superstring and M Theory Dualities”, hep-th/9607201.
[3] M. J. Duff, “M Theory (the Theory Formerly Known as Strings)”, hep-th/9608117.
[4] E. Witten, “Bound States of Strings and P-branes”, Nucl. Phys. B460 (1996) 335, hep-th/9510135.
[5] U. H. Danielsson, G. Ferretti and B. Sundborg, “D-particle Dynamics and Bound States”, hep-th/9603081.
[6] D. Kabat and P. Pouliot, “A Comment on Zero-Brane Quantum Mechanics”, hep-th/9603127.
[7] M. R. Douglas, D. Kabat, P. Pouliot and S. H. Shenker, “D-branes and Short Distances in String Theory”, hep-th/9608024.
[8] B. de Wit, J. Hoppe and H. Nicolai, “On the Quantum Mechanics of Supermembranes”, Nucl. Phys. B305 (1988) 545 ;
B. de Wit, M. Luscher and H. Nicolai, “The Supermembrane is Unstable”, Nucl. Phys. B320 (1989) 135.
[9] L. Susskind, “T Duality in M(atrix) Theory and S Duality in Field Theory”, hep-th/9611164.
[10] O. J. Ganor, S. Ramgoolam and W. Taylor IV, “Branes, Fluxes and Duality in M(atrix) Theory”, hep-th/9611202.
[11] O. Aharony, “String Theory Dualities from M Theory”, Nucl. Phys. B476 (1996) 470, hep-th/9604103.
[12] M. Berkooz and M. R. Douglas, “Five-branes in M(atrix) Theory”, hep-th/9610236.
[13] M. J. Duff and K. S. Stelle, “Multimembrane solutions of $D = 11$ supergravity”, Phys. Lett. 253B (1991) 113.
[14] O. Aharony, J. Sonnenschein and S. Yankielowicz, “Interactions of Strings and D-branes from M Theory”, Nucl. Phys. B474 (1996) 309, hep-th/9603009.
[15] C. Bachas, “D-brane Dynamics”, hep-th/9511043.
[16] J. G. Russo and A. A. Tseytlin, “Waves, Boosted Branes and BPS States in M Theory”, hep-th/9611047 ;
J. C. Breckenridge, G. Michaud and R. C. Myers, “More D-brane Bound States”, hep-th/9611174.