Near Optimal Line Segment Weak Visibility Queries in Simple Polygons

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Abstract

This paper considers the problem of computing the weak visibility polygon (WVP) of any query line segment \(pq\) (or \(WVP(pq)\)) inside a given simple polygon \(P\). We present an algorithm that preprocesses \(P\) and creates a data structure from which \(WVP(pq)\) is efficiently reported in an output sensitive manner.

Our algorithm needs \(O(n^2 \log n)\) time and \(O(n^2)\) space in the preprocessing phase to report \(WVP(pq)\) of any query line segment \(pq\) in time \(O(\log^2 n + |WVP(pq)|)\). We improve the preprocessing time and space of current results for this problem [10, 7] at the expense of more query time.

Keywords: Computational Geometry, Visibility, Line Segment Visibility

1. Introduction

Two points inside a polygon \(P\) are visible to each other if their connecting segment remains completely inside \(P\). The visibility polygon (VP) of a point \(q\) inside \(P\) (or VP(q)) is the set of vertices of \(P\) that are visible from \(q\). There have been many studies on computing VP's in simple polygons. In a simple polygon \(P\) with \(n\) vertices, VP(q) can be reported in time \(O(\log n + |VP(q)|)\) by spending \(O(n^3 \log n)\) time and \(O(n^2)\) of preprocessing space [2]. This result was later improved by [1] where the preprocessing time and space were reduced to \(O(n^2 \log n)\) and \(O(n^2)\) respectively, at the expense of more query time of \(O(\log^2 n + |VP(q)|)\).

The visibility problem has also been considered for line segments. A point \(v\) is said to be weakly visible from a line segment \(pq\) if there exists a point \(w\in pq\) such that \(w\) and \(v\) are visible to each other. The problem of computing the weak visibility polygon of \(pq\) (or \(WVP(pq)\)) inside \(P\) is to compute all points of \(P\) that are weakly visible from \(pq\). If \(P\) is simple (with no holes), Chazelle and Guibas [3] gave an \(O(n \log n)\) time algorithm for this problem. Guibas et al. [9] showed that this problem can be solved in \(O(n)\) time if a triangulation of \(P\) is given along with \(P\). Since any \(P\) can be triangulated in \(O(n)\) [4], the algorithm of Guibas et al. always runs in \(O(n)\) time [9]. Another linear time solution was obtained independently by [12].

The WV problem in the query version has been considered by few. It was shown in [2] that a simple polygon \(P\) can be preprocessed in \(O(n^3 \log n)\) time and \(O(n^3)\) space such that, given an arbitrary query line segment inside \(P\), \(O(k \log n)\) time is required to recover \(k\) weakly visible vertices. This result was later improved by [1] in which the preprocessing time and space were reduced to \(O(n^2 \log n)\) and \(O(n^2)\) respectively, at expense of more query time of \(O(k \log^2 n)\). In a recent work, we presented an algorithm to report \(WVP(pq)\) of any \(pq\) in \(O(\log n + |WVP(pq)|)\) time by spending \(O(n^3 \log n)\) time and \(O(n^2)\) space for preprocessing [10]. Later, Chen and Wang considered the same problem and, by improving the preprocessing time of the visibility algorithm of Bose et al. [2], they improved the preprocessing time to \(O(n^3)\) [7].

In this paper, we show that the WVP of a line segment \(pq\) can be reported in near optimal time of \(O(\log^2 n + |WVP(pq)|)\), after preprocessing the input polygon in time and space of \(O(n \log n)\) and \(O(n^2)\) respectively. Compared to the algorithms in [10] and [7], the storage and preprocessing time has one fewer linear factor, at expense of more query time of \(O(\log^2 n + |WVP(pq)|)\). Our approach is inspired by Aronov et al. algorithm for computing the visibility polygon of a point [1]. In Section 3, we first show how to compute the partial weak visibility polygon \(WVP(pq)\cap P\) when \(pq\) is not inside a sub-polygon \(P'\) of \(P\). Then, in Section 4, we use a balanced triangulation to compute and report the final weak visibility polygon.

2. Preliminaries

In this section, we introduce some basic terminologies used throughout the paper. For a better introduction to these terms, we refer the readers to Guibas et al. [9], Bose et al. [2], and Aronov et al. [1]. For simplicity, we assume that no three vertices of the polygon are collinear.

2.1. Visibility decomposition

Let \(P\) be a simple polygon with \(n\) vertices. Also, let \(p\) and \(q\) be two points inside \(P\). The visibility sequence of a point \(p\) is the sequence of vertices and edges of \(P\) that are
visible from \( p \). A visibility decomposition of \( P \) is to partition \( P \) into a set of visibility regions, such that any point inside each region has the same visibility sequence. This partition is induced by the critical constraint edges, which are the lines in the polygon each induced by two vertices of \( P \), such that the visibility sequences of the points on its two sides are different.

In a simple polygon, the visibility sequences of two neighboring visibility regions which are separated by an edge, differ only in one vertex. This fact is used to reduce the space complexity of maintaining the visibility sequences of the regions \([2]\). This is done by defining the sink regions. A sink is a region with the smallest visibility sequence compared to all of its adjacent regions. Therefore, it is sufficient to maintain the visibility sequences of the sinks, from which the visibility sequences of all other regions can be computed. By constructing a directed dual graph over the visibility regions (see Figure 1), one can maintain the difference between the visibility sequences of the neighboring regions \([2]\).

In a simple polygon with \( n \) vertices, the number of visibility and sink regions are \( O(n^3) \) and \( O(n^2) \), respectively \([2]\).

2.2. A linear time algorithm for computing WVP

Here, we present the \( O(n) \) time algorithm of Guibas et al. for computing WVP\((pq)\) of a line segment \( pq \) inside \( P \), as described in \([6]\). This algorithm is used in computing the partial weak visibility polygons in an output sensitive way, to be explained in Section 3.2. For simplicity, we assume that \( pq \) is a convex edge of \( P \), but we will show that this can be extended for any line segment in the polygon.

Let \( SPT(p) \) denote the shortest path tree in \( P \) rooted at \( p \). The algorithm traverses \( SPT(p) \) using a DFS and checks the turn at each vertex \( v_i \) in \( SPT(p) \). If the path makes a right turn at \( v_i \), then we find the descendant of \( v_i \) in the tree with the largest index \( j \) (see Figure 2). As there is no vertex between \( v_j \) and \( v_{j+1} \), we can compute the intersection point \( z \) of \( v_jv_{j+1} \) and \( v_kv_i \) in \( O(1) \) time, where \( v_k \) is the parent of \( v_i \) in \( SPT(p) \). Finally the counter-clockwise boundary of \( P \) is removed from \( v_i \) to \( z \) by inserting the segment \( v_iz \).

Let \( P' \) denote the remaining portion of \( P \). We follow the same procedure for \( q \). This time, the algorithm checks every vertex to see whether the path makes its first left turn. If so, we will cut the polygon at that vertex in a similar way. After finishing the procedure, the remaining portion of \( P' \) would be the WVP\((pq)\).

![Figure 1: The visibility decomposition induced by the critical constraint edges and its dual graph. The sink regions are shown in grey.](image)

![Figure 2: The two phases of the algorithm of computing WVP\((pq)\). In the left figure, the shortest path from \( p \) to \( v_j \) makes a first right turn at \( v_i \). In the right figure, the shortest path from \( q \) to \( v_j' \) makes a first left turn at \( v_j' \).](image)

3. Computing the partial WVP

Suppose that a simple polygon \( P \) is divided by a diagonal \( e \) into two parts, \( L \) and \( R \). For a query line segment \( pq \in R \), we define the partial weak visibility polygon WVP\(_L\)(\(pq\)) (or PWVP\(_L\)(\(pq\)) for clarity) to be the polygon WVP\((pq)\)\( \cap L \). In other words, WVP\(_L\)(\(pq\)) is the portion of \( P \) that is weakly visible from \( pq \) through \( e \). In this section, we will show how to compute WVP\(_L\)(\(pq\)). Later in Section 4, we will use this structure to compute WVP\((pq)\).

We will show how to use the algorithm of Guibas et al. \([9]\) to compute WVP\(_L\)(\(pq\)). To do so, we preprocess the polygon so that we can answer the visibility query in an output sensitive way. The idea is to compute the visibility decomposition of the polygon and, for each decomposition cell, compute the potential shortest path tree structures. As the number of visibility regions is \( O(n^3) \), the preprocessing cost of our approach would be high.

To overcome, we only consider the critical constraint edges that cut \( e \). The number of such constraint edges is \( O(n) \) and the complexity of the decomposition is reduced to \( O(n^2) \). This decomposition can be computed in \( O(n^2) \) time. We call this decomposition the partial visibility decomposition of \( P \) with respect to \( e \). The remaining part of this section shows how to modify the linear algorithm of
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Guibas et al. [9] so that \( WVP_L(pq) \) can be computed in an output sensitive way. First, we show how to compute the shortest path trees, and then present our algorithm for computing \( WVP_L(pq) \).

3.1. Computing the partial \( SPT_L(p) \)

We define the partial shortest path tree \( SPT_L(p) \) to be the subset of \( SPT(p) \) that lead to a leaf node in \( L \). In other words, \( SPT_L(p) \) is the union of the shortest paths from \( p \) to all the vertices of \( L \). In this section, we show how to preprocess the polygon \( P \), so that for any given point \( p \in R \), any part of \( SPT_L(p) \) can be traversed in an output sensitive way. The shortest path tree \( SPT_L(p) \) is composed of two kinds of edges: the primary edges that connect the root \( p \) to its direct visible vertices, and the secondary edges that connect two vertices of \( SPT_L(p) \) (see Figure 4). We also recognize two kinds of secondary edges: The 1st type of secondary edges (1st type for short) are those edges that are connected to a primary edge, and the 2nd type are the ones that connect other vertices of the polygon. Notice that if a point \( p \) crosses a critical constraint and that constraint does not cut \( e \), then the structure of \( SPT_L(p) \) would not change.

We can compute the primary edges of \( SPT_L \) by using Aronov’s output sensitive algorithm of computing the partial visibility polygon [1]. More precisely, with a processing cost of \( O(n^2 \log n) \) time and \( O(n^2) \) space, giving a point \( p \) in query time, a pointer to the sorted list of the vertices that are visible to \( p \) can be computed in \( O(\log n) \) time.

It is also necessary to compute the list of the secondary edges of every vertex of \( SPT_L \). Each vertex \( r \) in \( SPT_L \) have \( O(n) \) possible 2nd type edges. Depending on the parent of \( r \), a sub-list of these edges would appear in \( SPT_L \). To store all the possible 2nd type edges of \( r \), we compute and store this sub-list, or to be precise, the starting and ending edges of the list, for all the possible parents of \( r \). As there are \( O(n) \) possible parents for a vertex, these calculations can be performed for all the vertices of the polygon in total time of \( O(n^2 \log n) \) and the data can be stored in \( O(n^2) \) space. Having these data, we can, in the query time, access the list of the 2nd type edges of any vertex in constant time.

We build the same structure for the 1st type edges. The parent of a 1st type edge is the root of the tree. As the root can be in any of the \( O(n^2) \) different visibility regions, computing and storing the starting and ending edges in the list of 1st type edges of a vertex cost \( O(n^3 \log n) \) time and \( O(n^3) \) space.

We can reduce the time and space needed to compute and store these structures, having this property that two adjacent regions have only \( O(1) \) differences in their 1st type edges.

Lemma 1. Consider a visibility region \( V \) in the polygon and suppose that the 1st type secondary edges are computed for a point \( p \) in this region. For a neighboring region that share a common edge with \( V \), these edges can be updated in constant time.

Proof. When a view point \( p \) crosses the border of two neighboring regions, a vertex becomes visible or invisible to \( p \) [2]. In Figure 5 for example, when \( p \) crosses the border specified by \( u \) and \( v \), a 1st type secondary edge of \( u \) becomes a primary edge of \( p \), and all the edges of \( v \) become the 1st type secondary edges. We can see that no other vertex is affected by this movement. Processing these changes can be done in constant time, since it includes the following changes: removing a secondary edge of \( u \) (\( uv \)), adding a primary edge (\( pv \)), and moving an array pointer (edges of \( v \)) from the 2nd type edges of \( uv \) to the 1st type edges of \( pv \). Note that we know the exact positions of these elements in their corresponding lists. Finally, the only edge which involves in these changes can be identified in the preprocessing time (the edge corresponding to the crossed critical constraint), so, the time we spent in the query time would be \( O(1) \).
Having this fact and using a persistent data structure, e.g. persistent red-black tree [11], we can reduce the cost of storing the 1st type edges by a linear factor. A persistent red-black tree is a red-black tree that can remember all its intermediate versions. If a set of \( n \) linearly ordered items are stored it the tree and we perform \( m \) update into it, any version \( t \), for \( 1 \leq t \leq m \), can be retrieved in time \( O(\log n) \). This structure can be constructed in \( O((m+n) \log n) \) time by using \( O(m+n) \) space.

**Theorem 2.** A simple polygon \( P \) can be processed into a data structure with \( O(n^2) \) space and in \( O(n^2 \log n) \) time so that for any query point \( p \), the shortest path tree from \( p \) can be reported in \( O(\log n + k) \), where \( k \) is the size of the tree that is to be reported.

**Proof.** First, we use Aronov’s algorithm for computing the partial visibility polygon of \( p \). For this, \( O(n^2) \) space and \( O(n^2 \log n) \) time is needed in the preprocessing phase. For the secondary edges, \( O(n^2 \log n) \) time and \( O(n^2) \) space is needed to compute and store these edges. Also, a point location structure is built on top of the arrangement.

In the query time, the partial visibility region of \( p \) can be located in \( O(\log n) \) to have the sorted list of the visible vertices from \( p \). As the visible vertices from \( p \) correspond to the primary edges of \( SPT_L \), we also have the primary edges of \( SPT_L(p) \).

For the 1st type edges, a tour is formed to visit all the cells of the partial visibility decomposition. From Lemma 1, we can start from an arbitrary cell, walk along the tour, and construct a persistent red-black tree on the 1st type edges of \( SPT_L \) of a point in each cell. As there are \( O(n^2) \) cells and, each cell has \( O(n) \) 1st type edges, the structure takes \( O(n^2) \) storage and can be built in \( O(n^2 \log n) \) preprocessing time. Having this structure, the 1st type edges of the cell containing \( p \) can be retrieved from the persistent data structure in \( O(\log n) \) time.

Finally, at each node of the tree, we have the list of 2nd type edges from that node. Therefore, the cost of traversing \( SPT_L \) is the number of visited nodes of the tree, plus the initial \( O(\log n) \) time. In other words, the query time is \( O(\log n + k) \), where \( k \) is the number of the traversed edges of the \( SPT_L \).

### 3.2. Computing \( WVP_L(pq) \)

Now that we show how to compute \( SPT_L(p) \) for any point \( p \in R \) in the query time, we can use the linear algorithm presented in Section 2.2 for computing \( WVP \) of a simple polygon and modify it to compute \( WVP_L(pq) \) in an output sensitive way. As we can see in Figure 6, the algorithm can be extended to the cases that \( pq \) is not a polygonal edge.

![Figure 5: When \( p \) enters a new visibility region, the combinatorial structure of \( SPT_L(p) \) can be maintained in constant time.](image1)

![Figure 6: In computing \( WVP_L(pq) \) we can assume \( pq \) to be a polygonal edge.](image2)
In the preprocessing phase, we compute the critical information of a point inside each region, and assign this information to that region. In the query time and upon receiving a line segment \(pq\), we find the regions of \(p\) and \(q\). Using the critical information of these two regions, the above algorithm can be applied to compute \(WVP_L(pq)\).

As there are \(O(n^2)\) visibility regions in the partial visibility decomposition, \(O(n^3)\) space is needed to store the critical information of all the vertices. For each region, we compute \(SPT_L\) of a point, and by traversing the tree, we update the critical information of each vertex with respect to this region. An array of size \(O(n)\) is assigned to each region to store these information. We also build the structure described in Section 3.1 to compute \(SPT\) in \(O(n^3 \log n)\) time and \(O(n^3)\) space. In the query time, we locate the visibility regions of \(p\) and \(q\) in \(O(\log n)\) time. By Remark 1, when we proceed the algorithm in \(SPT_L\), of \(p\) and \(q\), we only traverse the vertices of \(WVP_L(pq)\). Finally, as the processing time spent in each vertex is \(O(1)\), the total query time is \(O(\log n + |WVP_L(pq)|)\).

To improve this result, we use the fact that any two adjacent regions have \(O(1)\) differences in their critical information.

**Lemma 3.** In the path between neighboring visibility regions, the changes of the critical information can be handled in constant time.

**Proof.** Suppose that we want to maintain the critical information of \(p\) and \(p\) is crossing the critical constraint defined by \(uv\), where \(u\) and \(v\) are two reflex vertices of \(P\). The only vertices that affect directly by this change are \(u\) and \(v\). Depending on the critical states of \(u\) and \(v\) w.r.t. \(p\), four situations may occur (see Figure 7). In the first three cases, the critical state of \(v\) will not change. In the forth case, however, the critical state of \(v\) will change. Before the cross, the shortest path makes a left turn at \(u\), therefore, both \(u\) and \(v\) are LC w.r.t. \(p\). However, after the cross, \(v\) is no longer LC. This means that the critical state of all the children of \(v\) in the \(SPT_L(p)\) may change as well.

To handle these cases, we modify the way the critical information of each vertex w.r.t. \(p\) is stored. At each vertex \(v\), we store two additional values: the number of LC vertices we met in the path \(SP(p, v)\) from \(p\), or its critical number, and a debit number, which is the critical number that is to be propagated in the vertex subtree. It is clear that if a vertex is LC, it means that its critical number is greater than zero (see Figure 8). Also, if a vertex has a debit number, the critical numbers of all its children must be added by this debit number. Notice that computing and storing these numbers along the critical information will not change the time and space requirements.

Now let us consider the forth case in Figure 7. When \(v\) becomes visible to \(p\), it is no longer LC w.r.t. \(p\). Therefore, the critical number of \(v\) is changed to \(0\). However, instead of changing the critical numbers of all the children of \(v\),

![Figure 7](image.png)
Theorem 4. Given a polygon $P$ and a diagonal $e$ which cuts $P$ into two parts, $L$ and $R$, and using $O(n^2 \log n)$ time, we can construct a data structure of size $O(n^2)$ so that, for any query line segment $pq \in R$, the partial weak visibility polygon $\text{WVP}_L(pq)$ can be reported in $O(|\text{WVP}_L(pq)|)$ time.

4. Computing WVP by balanced triangulation

There is always a diagonal $e$ of a simple polygon that cuts $P$ into two pieces, each having at most $2n/3$ vertices [5]. We can recursively subdivide and build a balanced binary tree where the leaves are triangles and each interior node $i$ corresponds to a subpolygon $P_i$ and a diagonal $e_i$. Each diagonal $e_i$ divides $P_i$ into two subpolygons, $L_i$ and $R_i$, which respectively correspond to the left and right subtrees of $i$ (see Figure 9). We build the data structures described in Section 3 for $L_i$ and $R_i$ with respect to $e_i$.

To compute $\text{WVP}(pq)$, $p$ and $q$ will be located among the leaf triangles. In the simplest case, both $p$ and $q$ belong to the same triangle. First we explain this situation. We construct $\text{PWVP}_i(pq)$ for each $i$ from the leaf to the root.

![Figure 8: The critical number represents the number of the left critical vertices met from $p$ in $\text{SPT}_e(p)$.

Figure 9: A balanced binary triangulation of the polygon is built so that the the weak visibility polygon can be computed recursively.

Figure 10: The specified nodes correspond to the computed partial WVPs in Figure 9

Here, $\text{PWVP}_i(pq)$ is the partial weak visibility polygon of $pq$ in $P_i$ with respect to $e_i$. For the leaf node, it is the corresponding triangle, and for other nodes, it can be computed inductively. In this case, the inductive step is similar to that of [1]. In each step, the merging of the computed polygons can be done in $O(\log n)$.

The time and space needed for building an exterior visibility decomposition of a simple polygon with $m$ vertices are $O(m^2)$ and $O(m^2 \log m)$, respectively. Thus, the space and time of the above inductive procedure can be expressed as the following equations:

$$S(n) = \max_{n/3 \leq m \leq 2n/3} (S(m) + S(n - m)) + \Theta(n^2),$$

$$T(n) = \max_{n/3 \leq m \leq 2n/3} (S(m) + S(n - m)) + \Theta(n^2 \log n)$$

Therefore, $S(n) = \Theta(n^2)$, and $T(n) = \Theta(n^2 \log n)$. With the same analysis as in [1], we can calculate the query time. Two point locations can be done in $O(\log n)$ time. As the triangulation is balanced, we path from the root to any node has $O(\log n)$ length. As we showed in Theorem 4, the time needed to query $\text{PWVP}_i(pq)$ at step $i$ is $O(|\text{PWVP}_i(pq)|)$. Also, the merging at each step can be done in $O(\log n)$ time. Therefore, the total query time is $O(\log n + \sum_i (\log n + \text{PWVP}_i(pq)))$, or $O(\log^2 n + |\text{WVP}(pq)|)$.
The tricky part is when \( p \) and \( q \) are on different triangles. Assume that at step \( i \), the query line segment is \( p,q,i \) and it is in sub-polygon \( P_i \). The sub-polygon \( P_i \) is divided by diagonal \( e_i \) to two sub-polygons \( L_i \) and \( R_i \). If \( p,q,i \) does not intersect \( e_i \), without loss of generality, assume that \( p,q,i \) is located in \( R_i \) (see Figure 11a). We do the normal procedure of the algorithm and compute \( WVP_{L_i}(p,q,i) \). We continue to recursively compute weak visibility polygon on \( R_i \). In this case, the time needed by this step can be expressed as \( T(n_i,p_i,q_i) = T(n_i/2,p_i,q_i) + O(\log n_i) + |PWVP_{L_i}(p_i,q_i)| \).

On the other hand, if \( p,q,i \) and \( e_i \) intersect at point \( r_i \), without loss of generality, assume that \( p_i \) is in \( R_i \) and \( q_i \) is in \( L_i \) (see Figure 11b). We can express \( WVP(p_i,q_i) \) as the union of four weak visibility polygons:

i) the partial weak visibility polygon of \( p_ir_i \) on \( L_i \),
ii) the weak visibility polygon of \( p_ir_i \) in \( R_i \),
iii) the partial weak visibility polygon of \( r_iq_i \) on \( R_i \),
iv) the weak visibility polygon of \( r_iq_i \) in \( L_i \).

In other words, we must compute two partial weak visibility polygons, and two weak visibility sub-problems. Having these four visibility polygons, the union of them can be merged in time \( O(|WVP_{L_i}(p_i,q_i)|) \). According to the Theorem 4, the query time spent at step \( i \) can be expressed as:

\[
T(n,pq) = O(\log n) + |PWVP_e(pq)|
\]

or,

\[
T(n,pq) = T(n/2,pr) + T(n/2,rq) + O(\log n)
+ |PWVP_e(pr)| + |PWVP_e(rq)|
\]

Here, \( PWVP_e(pq) \) is the partial weak visibility polygon of \( pq \) w.r.t. the cut \( e \). In the first case, we have

\[
T(n,pq) = O(\log^2 n + \log n + |WVP_{L_i}(pq)|)
= O(\log^2 n + |WVP(pq)|)
\]

In the second case, we have

\[
T(n,pq) = O(\log n + \log^2 n + |WVP_{L_i}(pq)|)
+ |WVP_{L_i}(pr)| + |PWVP_{L_i}(pr)| + |PWVP_{L_i}(rq)|
= O(\log^2 n + |WVP(pq)|)
\]

In these equations, \( WVP_{L_i}(pq) \) is the weak visibility polygon of \( pq \) in a polygon of size \( \frac{n}{2} \).
As for the base case, we showed that if \( pq \) is located in a single triangle, the time for computing \( WVP(pq) \) would be \( O(\log^2 n + |WVP(pq)|) \). In summary, we have the following theorem:

**Theorem 5.** A simple polygon \( P \) of size \( n \) can be processed in \( O(n^2 \log n) \) time into a data structure of size \( O(n^2) \) so that, for any query line segment \( pq \), \( WVP(pq) \) can be reported in time \( O(\log^2 n + |WVP(pq)|) \).

5. Conclusion

In this paper, we showed how to answer the weak visibility queries in a simple polygon with \( n \) vertices in an efficient way. In the first part of the paper, we defined the partial weak visibility polygon \( WVP_e(pq) \) of a line segment \( pq \) with respect to a diagonal \( e \) and presented an algorithm to report it in time \( O(\log n + |WVP_e(pq)|) \), by spending \( O(n^2 \log n) \) time to preprocess the polygon and maintaining a data structure of size \( O(n^2) \).

In the second part, we presented a data structure of size \( O(n^2) \) which can be computed in time \( O(n^2 \log n) \) so that the weak visibility polygon \( WVP(pq) \) from any query line segment \( pq \in P \) can be reported in time \( O(\log^2 n + |WVP(pq)|) \).

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