Level truncation and the tachyon in open bosonic string field theory

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Abstract

The tachyonic instability of the open bosonic string is analyzed using the level truncation approach to string field theory. We have calculated all terms in the cubic action of the string field theory describing zero-momentum interactions of up to level 20 between scalars of level 10 or less. These results are used to study the tachyon effective potential and the nonperturbative stable vacuum. We find that the energy gap between the unstable and stable vacua converges much more quickly than the coefficients of the effective tachyon potential. By including fields up to level 10, 99.91% of the energy from the bosonic D-brane tension is cancelled in the nonperturbative stable vacuum. It appears that the perturbative expansion of the effective tachyon potential around the unstable vacuum has a small but finite radius of convergence. We find evidence for a critical point in the tachyon effective potential at a small negative value of the tachyon field corresponding to this radius of convergence. We study the branch structure of the effective potential in the vicinity of this point and speculate that the tachyon effective potential is globally nonnegative.
1 Introduction

The appearance of a tachyon in the spectrum of both open and closed bosonic strings has appeared to be a fundamental obstacle to a physical interpretation of these theories since the early days of the dual resonance model. Early work on the subject indicated the possible existence of a more stable nonperturbative vacuum which can be reached by a condensation of the tachyon and other string fields in the open bosonic string theory [1, 2]. At that time, however, the connection between open strings and Dirichlet branes [3] had not yet been realized, so that the significance of the nonperturbative stable vacuum was not widely appreciated.

It was recently pointed out by Sen [4] that the condensation of the tachyon in open bosonic string theory should correspond to the process of annihilation of an unstable D25-brane. The nonperturbative stable vacuum of the open string should simply be the vacuum corresponding to empty space, and the energy difference between the stable and unstable vacua should therefore be given by the mass of the D25-brane. Sen suggested that it should be possible to precisely calculate this energy gap using open string field theory. In fact, it was found over a decade ago by Kostelecky and Samuel that truncating open bosonic string field theory at low mass levels gives a systematic approximation scheme which seems to converge to a finite value for the energy difference between the unstable and stable vacua [2, 5]. Sen and Zwiebach have carried out a calculation of this type and have shown that including fields up to mass level 4 and interactions up to mass level 8 gives a mass gap of 98.6% of the D25-brane energy. Similar calculations of tachyon condensation have recently been performed in open superstring field theory [6, 7]. The level-truncation approach to string field theory has also been used to study lower-dimensional Dp-branes as solitonic lumps [8, 9]. In all these calculations, truncation of string field theory to the first few levels seems to give a sequence of successively better approximations to nonperturbative physical quantities.

In this paper we extend the level truncation approach to open bosonic string field theory to include scalar fields of mass levels > 4, following earlier work in [9]. We use the level-truncated theory as a tool for exploring various features of the tachyon and its condensation into a stable vacuum. We compute all terms in the string field theory action involving fields of level less than or equal to 10 and interactions of up to mass level 20. We perform a nonperturbative calculation of the energy gap between the stable and unstable vacua and find that 99.91% of the D25-brane tension is produced in the level (10, 20) truncated theory.

We study the structure of the tachyon effective potential in some detail using the various level-truncated theories. We compute the first 60 coefficients $c_n$ of $\phi^n$ in the effective tachyon potential in several successive level truncations up to level (10, 20). We find that the effective tachyon potential has a radius of convergence which decreases to an apparently finite asymptotic value as the level number increases. This radius of convergence is substantially smaller than the tachyon value corresponding to the stable vacuum, although the associated singularity is at a negative value of $\phi$ while the stable vacuum arises at $\phi > 0$. We investigate the branch structure of the tachyon effective potential near the singular point, and use our numerical results to speculate about the behavior of the effective potential beyond this point.
In Section 2 we review the level truncation approach to string field theory and outline our calculation of the action up to level (10, 20). In section 3 we use our results to analyze the perturbative expansion of the tachyon effective potential, and in section 4 we discuss the nonperturbative stable vacuum and the branch structure of the effective tachyon potential.

2 Level truncation of string field theory

In this section we briefly review the level truncation approach to string field theory and describe our calculation of interactions between scalar string fields of level $\leq 10$. Throughout this paper we follow the conventions and notation of [2, 9]. For more detailed reviews of open string field theory see [10, 11].

2.1 The scalar potential in open bosonic string field theory

String field theory is described in terms of a string field $\Phi$ which contains a component field for every state in the first-quantized string Fock space. For the open bosonic string, a particularly simple string field theory was suggested by Witten [12] in which the action takes the cubic form

$$S = \frac{1}{2\alpha'} \int \Phi \star Q\Phi + \frac{g}{3!} \int \Phi \star \Phi \star \Phi,$$

where $Q$ is the BRST operator and $\star$ is the string field theory star product.

In Feynman-Siegel gauge, the string field can be expanded in the form

$$\Phi = \left( \phi + A_\mu \alpha_\mu - 1 + \frac{1}{\sqrt{2}} B_{\mu\nu} \alpha_{\mu-1} \alpha_{\nu-1} + \beta b_{-1} c_{-1} + \cdots \right) |0\rangle$$

where $|0\rangle = c_1 |\Omega\rangle$ is the state in the string Hilbert space associated with the tachyon field. The expansion (2) contains an infinite series of fields. The level of each field in the expansion is defined to be the sum of the level numbers of the creation operators which act on $|0\rangle$ to produce the associated state. Thus, the tachyon is the unique field of level 0, the gauge field $A_\mu$ is the unique field of level 1, etc. The states associated with these fields are all in the subspace $H$ of the full string Hilbert space containing states of ghost number 1.

In this paper we are interested in the behavior of the tachyon field $\phi$. In particular, we wish to study questions related to the appearance of a Lorentz-invariant stable vacuum in the theory when the tachyon and other scalar fields acquire nonzero condensates. Because all the questions we will address here involve Lorentz-invariant phenomena, we can restrict attention to scalar fields in the string field expansion. We write the string field expansion in terms of scalar fields as

$$\Phi = \sum_{i=1}^{\infty} \psi_i |s_i\rangle$$

where $|s_i\rangle$ are all the scalar states in $H$. The scalar states at levels 0 and 2 are

$$|s_1\rangle = |0\rangle$$
$$|s_2\rangle = \alpha_{-1} \cdot \alpha_{-1} |0\rangle$$
$$|s_3\rangle = b_{-1} c_{-1} |0\rangle$$

The associated scalar fields can be related to the fields in the expansion (2) through

$$\phi = \psi^1$$
$$B_{\mu\nu} = \sqrt{2} \eta_{\mu\nu} \psi^2$$
$$\beta = \psi^3$$

We will often refer to the tachyon field $\psi^1$ simply as $\phi$. (Note that a superscript on $\psi$ will always indicate a field index while a superscript on $\phi$ indicates an exponent.) For the calculations of interest here, contributions from scalar fields of odd level cancel due to a twist symmetry [2, 13]. Thus, we need only consider scalar fields of even level number. All scalar states at levels 0, 2, 4 and 6 are listed in appendix A.

An explicit algorithm for computing the terms in the string field theory action (1) using the oscillator modes of the matter fields and ghost fields of the bosonic conformal field theory was given in [14, 15, 16]. The aspects of this formalism needed for the computations in this paper are reviewed in [9]. The quadratic term evaluated on a state $|s\rangle$ is simply

$$\int |s\rangle \ast Q |s\rangle = \langle s| c_0 \left( \alpha'^2 + \frac{1}{2} M^2 \right) |s\rangle$$

where $\frac{1}{2} M^2$ is the level of $s$ minus 1, $p$ is the momentum of the state $s$, $\langle s| \ast |s\rangle$ is the BPZ dual state to $|s\rangle$ produced by acting with the conformal transformation $z \to -1/z$, and the dual vacuum satisfies $\langle 0| c_0 |0\rangle = 1$. The cubic interaction terms in the string field action can be described in terms of Witten’s vertex operator $V$ through

$$\int \Phi \ast \Phi \ast \Phi = \langle V | (\Phi \otimes \Phi \otimes \Phi)$$

where $V$ is a state in the tensor product space $\mathcal{H}^* \otimes \mathcal{H}^* \otimes \mathcal{H}^*$, given in terms of oscillator modes by

$$\langle V | = \delta(p_{(1)} + p_{(2)} + p_{(3)}) \langle (0| c_0^{(1)} \otimes (0| c_0^{(2)} \otimes (0| c_0^{(3)} \exp \left( \frac{1}{2} \alpha^{(r)} \eta_{\mu\nu} \alpha^{(s)} \nu + c_n^{(r)} X_n^{rs} h^{(s)} \right).$$

The coefficients $N_{nm}^{rs}, X_{nm}^{rs}$ can be calculated from formulae in [14, 15, 16]; explicit tables of these coefficients for $n, m \leq 8$ are given in [9].

Using the equations (4, 5) for the quadratic and cubic terms in the string field theory action, the potential for the zero momentum scalar fields in $\mathcal{H}$ can be written as

$$V = \sum_{i,j} d_{ij} \psi^i \psi^j + g \kappa \sum_{i,j,k} t_{ijk} \psi^i \psi^j \psi^k$$

where $g$ is the string coupling constant and

$$\kappa = \frac{3^{7/2}}{27}.$$
has been chosen so that $t_{111} = 1$. Throughout the paper we set $\alpha' = 1$. The coefficients $d_{ij}, t_{ijk}$ can be explicitly computed for any given values of the indices. The action can be truncated by only including fields up to a fixed level $L$ and interactions including terms whose total level does not exceed another fixed value $I$. In [2], the complete action with $(L, I) = (2, 6)$ was calculated and shown to be

$$V = -\frac{1}{2} \phi^2 + 26\psi_2^2 - \frac{1}{2}(\psi_3)^2$$

$$+ \kappa g \left[ \phi^3 + \phi^2 \left( -\frac{5 \cdot 26}{9} \psi_2 - \frac{11}{9} \psi_3 \right) 
+ \phi \left( \frac{4 \cdot 7 \cdot 13 \cdot 83}{3^5} \psi_2^2 + \frac{4 \cdot 5 \cdot 11 \cdot 13}{3^5} \psi_2 \psi_3 + \frac{19}{34} (\psi_3)^2 \right) 
- 2^3 \cdot 3 \cdot 7 \cdot 13 \cdot 41 \cdot 73 (\psi_2)^3 - \frac{2^2 \cdot 7 \cdot 13 \cdot 11 \cdot 83}{3^8} (\psi_2)^2 \psi_3 
- \frac{2 \cdot 5 \cdot 13 \cdot 19}{3^7} \psi_2 (\psi_3)^2 - \frac{1}{34} (\psi_3)^3 \right]$$

(Note that we have lowered indices on the fields $\psi_2, \psi_3$ for clarity.) Results using the complete action with $(L, I) = (4, 12)$ were described in [5]. In [13] calculations were performed at level $(L, I) = (4, 8)$.

With the help of the symbolic manipulation program Mathematica, we have calculated all terms in the action up to levels $(L, I) = (10, 20)$. There are 252 fields at levels $\leq 10$ and 138,202 distinct cubic interaction terms between fields whose total level is $\leq 20$, so it is clearly impractical to reproduce the full action here. It is worth mentioning, however, some points which help to simplify the calculation.

One significant simplification arises from the fact that the matter and ghost fields almost completely decouple in the action. In fact, it is clear from (6) that this decoupling is complete in the cubic terms, and from (4) that the only coupling in the quadratic terms arises from the appearance of the total level of the scalar field in question, including the levels of both the matter and ghost oscillators needed to produce the state. Because of this decoupling, we find that it is convenient to decompose the Hilbert space into the matter and ghost Hilbert spaces

$$\mathcal{H} = \mathcal{H}_{\text{mat}} \otimes \mathcal{H}_{\text{gh}} \quad (8)$$

and to separately enumerate the scalar fields in the matter and ghost sectors $|\eta_m\rangle \in \mathcal{H}_m, |\chi_g\rangle \in \mathcal{H}_g$. The first few states in each of these component Hilbert spaces are

$$|\eta_1\rangle = |\chi_1\rangle = |0\rangle$$

$$|\eta_2\rangle = \alpha_{-1} \cdot \alpha_{-1} |0\rangle$$

$$|\chi_2\rangle = b_{-1} c_{-1} |0\rangle$$

Given the decomposition (8) of the Hilbert space we can write each of the scalar fields $|s_i\rangle$ in $\mathcal{H}$ as a tensor product

$$|s_i\rangle = |\eta_{m(i)}\rangle \otimes |\chi_{g(i)}\rangle \quad (9)$$
Thus, for example, \( m(1) = 1 \) and \( g(1) = 1 \). A list of matter and ghost scalars up to level 6 is given in appendix B, and the decomposition of the scalars in \( \mathcal{H} \) into matter and ghost factors is given in appendix A for the scalars of level \( \leq 6 \).

From the decomposition into matter and ghost components, we can now write the quadratic and cubic coefficients (4,5) as

\[
d_{ij} = \frac{2(\text{level}(i) - 1)}{(\text{level}(m(i)) - 1) \cdot (\text{level}(g(i)) - 1)} d_{m(i)m(j)}^{\text{mat}} d_{g(i)g(j)}^{\text{gh}}
\]

\[
t_{ijk} = t_{m(i)m(j)m(k)}^{\text{mat}} t_{g(i)g(j)g(k)}^{\text{gh}}
\]

where \( d_{mn}^{\text{mat}}, d_{gh}^{\text{gh}}, t_{mn}^{\text{mat}} \) and \( t_{gh}^{\text{gh}} \) are the quadratic and cubic coefficients in the matter and ghost sectors, and where \( \text{level}(i) = \text{level}(g(i)) + \text{level}(m(i)) \) indicates the level of the state \(|s_i\rangle\). Quadratic and cubic coefficients of the matter and ghost scalars appearing in scalar fields up to level 6 are tabulated in Appendix C. From these coefficients and (10) one can reproduce the entire string field theory action for zero momentum scalars up to level 6 fields and level 16 interactions. We have carried out the calculation of all matter and ghost interactions up to fields of level 10 and interactions of level 20. While we do not reproduce here the complete results of this calculation, we will use these results to study questions of physical interest in the remainder of the paper.

We end this section with a brief discussion of the approach used in [13], in which a reduced Hilbert space of background-independent fields was used. It was pointed out in [4] that it is possible to truncate the Hilbert space \( \mathcal{H} \) in a consistent fashion by considering the subspace \( \mathcal{H}_1 \) obtained by considering only those states produced by acting with the ghost fields \( b, c \) and the Virasoro generators \( L_{-n}, n \geq 2 \) associated with the stress tensor of the matter fields. In the stable vacuum of the theory, only scalar fields in \( \mathcal{H}_1 \) acquire nonzero expectation values. At level 4 and above, the number of scalars in \( \mathcal{H}_1 \) is less than the number of scalars in \( \mathcal{H} \), so that particularly when the level number becomes very large the effective action for the level-truncated theory is significantly simplified by restricting attention to the truncated Hilbert space. To compare the number of scalars in \( \mathcal{H} \) at a fixed level \( n \), which we denote \( h_n \), with the number of scalars in \( \mathcal{H}_1 \), which we denote \( h_1^1 \), we can write generating functions

\[
f(x, y) = \prod_{p=2}^{\infty} \frac{1}{(1 - x^p)^{\frac{1}{p}}} \prod_{q=1}^{\infty} (1 + x^q y) (1 + \frac{x^q}{y}) = \sum_{n,m} h_{n,m} x^n y^m
\]

\[
f_1(x, y) = \prod_{p=2}^{\infty} \frac{1}{(1 - x^p)} \prod_{q=1}^{\infty} (1 + x^q y) (1 + \frac{x^q}{y}) = \sum_{n,m} h_{1,n,m}^1 x^n y^m
\]

in terms of which

\[
h_n = h_{n,0}
\]

\[
h_n^1 = h_{1,n,0}^1
\]

The number of scalars in \( \mathcal{H} \) and \( \mathcal{H}_1 \) at each even level up to \( n = 20 \) is tabulated in Table [1]. Up
Table 1: The number of scalars $h_n$ and $h_n^1$ in $\mathcal{H}$ and $\mathcal{H}_1$ at level $n$

to the level 10 fields we consider in this paper, the difference between $\mathcal{H}$ and $\mathcal{H}_1$ is less than a factor of 2, so that there is no extraordinary advantage to be gained by using the smaller Hilbert space. Clearly, however, as the level becomes much larger than 10, the reduced size of the truncated Hilbert space would make explicit calculations much easier, assuming that calculations in $\mathcal{H}_1$ could be done just as efficiently as corresponding calculations in $\mathcal{H}$.

In our calculations we have worked with the complete oscillator Hilbert space $\mathcal{H}$ and not with $\mathcal{H}_1$. The main reason for this is that at this point no method has been developed for computing cubic interactions between fields in $\mathcal{H}_1$ which is as systematic and efficient as the oscillator method described above for computing cubic interactions between fields in $\mathcal{H}$. It is possible to calculate cubic interactions in $\mathcal{H}_1$ by computing the relevant correlation functions in the open bosonic string theory \cite{10}, but this method is somewhat more complicated than the straightforward oscillator approach. Of course, the interactions in $\mathcal{H}_1$ can always be computed by rewriting each state in terms of the oscillator basis and then using (6), but this requires just as much work as the computation directly in the oscillator basis. In any case, the results in Appendix C for fields up to level 6 are given in the oscillator basis of $\mathcal{H}_1$; these interactions can easily be translated into an action for fields in $\mathcal{H}_1$ by explicitly writing the Virasoro generators $L_{-n}$ in terms of the oscillator basis.

3 Tachyon effective potential: perturbation expansion

Given the (rational) coefficients in the scalar potential (7) truncated at level $(L,I)$, we can perform a number of interesting calculations. In particular, we can study the effective potential of the tachyon field and identify the stable vacuum or vacua of the theory.

An effective potential for the tachyon field $\phi$ can be determined by starting with the complete set of terms in the cubic potential (7) truncated at level $(L,I)$, fixing a value of $\phi = \psi^1$, solving for all fields $\psi^i, i > 1$, and plugging back into the potential to rewrite it as a function of $\phi$. If there are $N$ scalar fields involved, this means that we need to solve a system of $N - 1$ simultaneous quadratic equations. In principle there are many solutions of this system of equations, but if we are interested in the branch of the solution on which all fields vanish when $\phi = 0$, it is easy to determine which branch to choose for each of the quadratic equations (an explicit example of this choice is described below). Clearly when $N$ becomes large it is impractical to find an exact analytic form for the effective tachyon potential. Various numerical methods can be used to approximate the effective potential arising from integrating out a large number of fields; for example, a numerical analysis of
the effective potential was performed in [2] using the level 2 action with the two fields $\psi^2, \psi^3$ integrated out. In section 4.2 we extend these results by analyzing the structure of the effective potential at levels 4 and 6 using numerical methods.

Another approach to studying the effective potential, which we consider in this section, is to determine the terms in a power series expansion of the potential around the unstable vacuum $\phi = 0$. In subsection 3.1 we describe an algebraic method for efficiently summing all diagrams contributing to the $\phi^n$ term in the effective potential. In subsection 3.2 we summarize the results of our calculation of these coefficients up to $n = 60$ at level $L \leq 10$, and discuss the implications of these results. In particular, we find that the power series expansion of the tachyon effective potential seems to have a radius of convergence which approaches a finite but nonzero value as the level number is increased. As we will discuss in the next section, the stable vacuum of the theory lies outside the radius of convergence of this power series.

3.1 Summing planar diagrams

In the vicinity of the unstable vacuum $\psi^i = 0$ we can perform a power series expansion of the effective tachyon potential

$$V(\phi) = \sum_{n=2}^{\infty} c_n (g\kappa)^{n-2} \phi^n = -\frac{1}{2} \phi^2 + g\kappa \phi^3 + \cdots$$

There are a number of ways in which we might try to calculate the coefficients $c_n$ in this expansion. Given a cubic potential (7) for $N$ fields, one approach to determining the coefficients $c_n$ would be to write the quadratic equations of motion for each of the $N - 1$ fields which we wish to integrate out, expand each field $\psi^i, i > 1$ as a formal power series in $\phi$, and solve a linear system of equations at each order in $\phi$, plugging the final results back into the cubic potential. We will not directly use this method, although it is technically equivalent to the graphical approach we will use.

An alternative approach to calculating the coefficients in the effective tachyon potential is to treat (7) as the action for a 0-dimensional field theory and to use Feynman diagrams to sum all terms contributing to a given coefficient $c_n$. This method was used by Kostelecky and Samuel in [2] to calculate the level 2 and level 4 approximations to the quartic coefficient $c_4$, which they had calculated exactly in [17]. They found that at level 2, 72% of the exact coefficient is generated, and at level 4 this increases to 84%. This calculation was extended in [9], where contributions from all fields up to level 20 were included, generating 96-97% of the exact term $c_4$.

We can use the exact cubic terms we have found up to level (10, 20) as cubic interaction vertices in Feynman diagrams which we can then sum to find the contributions to each of the coefficients $c_n$. Because the combinatorics of the Feynman diagrams grows exponentially, we will find it useful to use an algebraic simplification to expedite the summation over graphs. The essence of the simplification we will use is that because we are summing over planar tree graphs, we can write a recursion relation relating the set of graphs $G_k$ with $k$ external
φ edges and single external \( \psi^i \) edge to the sets of graphs \( G_{k'} \) with \( k' < k \). This relationship is indicated schematically by

\[
\psi^i = \sum_{m,k,l} d^{ij} t_{jkl} \hat{v}_m \hat{v}_{n-m}
\]

where \( d^{ij} \) is the inverse matrix to \( d_{ij} \) and

\[
\hat{v}_n^i = \begin{cases} 0, & i = 1 \text{ and } n > 1 \\ v_n^i, & \text{otherwise} \end{cases}
\]

Algebraically, we can define an \( N \)-dimensional vector \( v_n^i \) for each \( n \), representing the summation over all graphs with \( n \) external φ edges and a single external \( \psi^i \) (including the propagator for \( \psi \)). We then have

\[
v_1^i = \delta_1^i
\]

\[
v_n^i = \frac{3}{2} \sum_{m=1}^{n-1} d^{ij} t_{jkl} \hat{v}_m \hat{v}_{n-m}
\]

where \( d^{ij} \) is the inverse matrix to \( d_{ij} \) and

\[
\hat{v}_n^i = \begin{cases} 0, & i = 1 \text{ and } n > 1 \\ v_n^i, & \text{otherwise} \end{cases}
\]

has been defined to project out internal φ edges. In terms of the vectors \( v_n^i \), the coefficients \( c_n \) are given by

\[
c_n = \frac{1}{n} v_{n-1}^1
\]

In fact, the recursion relations (14) precisely encode the relations we would find between the coefficients of the fields \( \psi^i \) expanded in powers of \( \phi \) if we used the power series approach to solving the \( N - 1 \) quadratic equations term by term in \( \phi \) as described in the first paragraph of this subsection. The terms \( v_n^i \) are precisely the coefficients of \( \phi^n \) in the expansion of the field \( \psi^i \).

In any case, using this recursive formalism, the computation of \( c_n \) becomes a polynomial-time algorithm taking time of order \( O(N^3 n^2) \), instead of an exponentially hard algorithm such as we would encounter if we tried to directly sum over all Feynman diagrams. It may be helpful to illustrate this approach with a simple example. Consider a single massive field \( \psi \) which couples to \( \phi \) only through a term of the form \( \phi^2 \psi \) in the potential

\[
V = -\frac{1}{2} \phi^2 + g \kappa \phi^3 + \frac{1}{2} \psi^2 + g \kappa \left( \frac{\gamma}{6} \psi^3 + \beta \phi^2 \psi \right).
\]

In this case we can explicitly solve for \( \psi \)

\[
\psi = \frac{1}{g \kappa \gamma} \left( -1 + \sqrt{1 - 2g^2 \kappa^2 \beta \gamma \phi^2} \right)
\]
As mentioned above, we have chosen the branch of the square root which gives \( \psi = 0 \) when \( \phi = 0 \). Substituting (18) into (17) gives us the effective potential for \( \phi \) on the branch containing the unstable vacuum

\[
V = -\frac{1}{2} \phi^2 + g \kappa \phi^3 + \frac{1}{3g^2\kappa^2\gamma^2} \left( 1 - 3g^2\kappa^2\beta\gamma\phi^2 - (1 - 2g^2\kappa^2\beta\gamma\phi^2)^{3/2} \right)
\]  

(19)

The coefficients in a power series expansion of this potential are

\[
c_{2n} = -\frac{(2n - 5)!!}{n!} \beta^n \gamma^{n-2}
\]

(20)

for \( 2n \geq 4 \).

To derive these coefficients using the recursive formalism (14) we need the coefficients

\[
d_{11} = -\frac{1}{2}, \quad t_{112} = \frac{\beta}{3},
\]

\[
d_{22} = \frac{1}{2}, \quad t_{222} = \frac{\gamma}{6},
\]

which give the recursion relations

\[
v_1^1 = 1
\]

\[
v_n^1 = -2\beta \sum_{m=1}^{n-1} \hat{v}_m^1 \hat{v}_{n-m} = -2\beta v_{n-1}^2, \quad n > 1
\]

\[
v_1^2 = 0
\]

\[
v_2^2 = \beta \hat{v}_1^1 \hat{v}_1^1 = \beta
\]

\[
v_n^2 = \frac{\gamma}{2} \sum_{m=1}^{n-1} v_m^2 v_{n-m}^2, \quad n > 2.
\]

We begin by solving for \( v_n^2 \). These terms clearly vanish unless \( n \) is even. Defining

\[
w_m = v_{2m}^2
\]

we have

\[
w_1 = \beta
\]

\[
w_m = \frac{\gamma}{2} \sum_{k=1}^{m-1} w_k w_{m-k}.
\]

Defining a generating function

\[
f(x) = \sum_{m=1}^{\infty} w_m x^{2m}
\]

(23)

The equations (22) are equivalent to the quadratic equation

\[
f = \frac{\gamma}{2} f^2 + \beta x^2
\]

(24)
with solution

\[ f = \frac{1}{\gamma} (1 - \sqrt{1 - 2\beta \gamma x^2}) = \sum_{m=1}^{\infty} \frac{(2m-3)!!}{m!} \beta^m \gamma^{m-1} x^{2m} \]  \hspace{1cm} (25)

So

\[ c_{2n} = \frac{1}{2n} (-2\beta) w_{n-1} = -\frac{(2n-5)!!}{n!} \beta^n \gamma^{n-2} \]  \hspace{1cm} (26)

in agreement with (20).

### 3.2 Coefficients of effective tachyon potential

We have used the method described in the previous subsection to calculate the coefficients \( c_n \) in the perturbative expansion of the effective tachyon potential around the unstable vacuum for \( n \leq 60 \) in each of the level-truncated theories up to (10, 20). In Table 2 we have tabulated the successive approximations to \( c_n \) for a representative set of values of \( n \) at various levels. There are several observations we can make based on these results. In [4] successive approximations to the coefficient \( c_4 \) were computed using fields at up to level 20. It was found that while the level truncation method gives a monotonic sequence of approximations which seem to converge to the known exact value for this coefficient \( c_4 \approx -1.75 \), the convergence to the asymptotic value is fairly slow. For \( c_4 \), the contribution from the set of graphs including at least one field of level \( k \) but no fields of level \( k+1 \) decreases as \( k \) increases. The same is true for successive approximations to \( c_n \) for small \( n \), but as \( n \) increases we find that at \( n = 8 \), the contribution from graphs with some fields of level 4 exceeds that of graphs with only fields of level 2. At \( n = 15 \), the contribution from graphs with some fields of level 6 exceeds that of graphs with fields of level at most 4. The corresponding thresholds where fields of levels 8 and 10 dominate the contributions from lower levels are \( n = 26 \) and \( n = 43 \) respectively.

| \( n \) | (2, 6) | (4, 12) | (6, 18) | (8, 20) | (10, 20) |
|---|---|---|---|---|---|
| 3 | 1 | 1 | 1 | 1 | 1 |
| 4 | -1.2592 | -1.4724 | -1.5562 | -1.6004 | -1.6276 |
| 5 | 5.9478 | 7.8370 | 8.6398 | 9.0756 | 9.3479 |
| 6 | -27.909 | -43.529 | -50.768 | -54.816 | -57.353 |
| 7 | 143.67 | 269.25 | 333.83 | 371.36 | 395.12 |
| 8 | -786.81 | -1777.5 | -2346.7 | -2691.9 | -2913.4 |
| 9 | 4513.9 | 12323 | 17345 | 20529 | 22606 |
| 10 | -26845 | -88690 | -133179 | -162693 | -182299 |
| 20 | -4.0710 \cdot 10^{12} | -9.2769 \cdot 10^{14} | -2.6957 \cdot 10^{14} | -4.5477 \cdot 10^{14} | -6.0780 \cdot 10^{14} |
| 30 | -1.3415 \cdot 10^{21} | -2.1308 \cdot 10^{23} | -1.2050 \cdot 10^{24} | -2.8154 \cdot 10^{24} | -4.4920 \cdot 10^{24} |
| 40 | -6.017310^{29} | -6.6758 \cdot 10^{32} | -7.3563 \cdot 10^{33} | -2.3817 \cdot 10^{34} | -4.5372 \cdot 10^{34} |
| 50 | -3.1889 \cdot 10^{38} | -2.4732 \cdot 10^{42} | -5.3126 \cdot 10^{43} | -2.3840 \cdot 10^{44} | -5.4231 \cdot 10^{44} |

Table 2: Level truncation approximations to coefficients \( c_n \) in effective tachyon potential
The fact that higher level fields become more relevant for higher order terms in the effective potential is a natural consequence of the tradeoff between the exponential growth in the number of diagrams contributing to $c_n$ and the suppression of higher order diagrams by $(g\kappa)^{n-2}$. From this behavior, however, we see that for large $n$ it will be necessary to include fields of increasingly high level in order to have a good approximation to the coefficients $c_n$.

One interesting question which we can explore using our results for the coefficients $c_n$ is the radius of convergence of the power series expansion of the effective potential after truncating at a fixed level. Because of the square root branch cuts which arise from the quadratic equations of motion for the fields, at some finite value of $\phi$ the effective potential becomes singular for any given level truncation. For example, the effective potential arising from (17) has a radius of convergence $r_c = 1/(g\kappa\sqrt{2\beta\gamma})$, which is manifest in (19) and which can be seen from the asymptotic form of the coefficients $c_{2n} \sim \alpha n^\eta (\sqrt{2\beta\gamma})^{2n}$ where $\alpha, \eta$ are numerical constants. It was found in [2] that after truncation at level 2, there is a singularity at $-\phi = r_c \approx 0.35/g$. This radius of convergence can be seen from the coefficients of the effective potential in the level 2 truncated theory, which for large $n$ go as

$$|c_n| \sim \alpha n^\eta \frac{1}{(g\kappa r_c)^n}$$

A graph of $(\ln |c_n|)/n$ as a function of $n$ is shown for each level truncation in Figure 1. The asymptotic value of $(\ln |c_n|)/n$ gives the (logarithm of the inverse of the) radius of convergence of the effective potential at each level. It is clear from the graph that this radius of convergence is decreasing and seems to be approaching a nonzero limiting value. By matching the $c_n$ trajectories to a 3-parameter family of functions of $n$ of the form (27)

Figure 1: $(\ln |c_n|)/n$ for coefficients $c_n$ in effective tachyon potential in different level truncations

convergence of the effective potential at each level. It is clear from the graph that this radius of convergence is decreasing and seems to be approaching a nonzero limiting value. By matching the $c_n$ trajectories to a 3-parameter family of functions of $n$ of the form (27)
we can find a close approximation to the radius of convergence at each of the levels we have
computed. Table 3 shows the approximate radius of convergence at each level; we see that
in the limiting theory the radius of convergence approaches something like

\[ r_c \approx 0.25/g. \] (28)

Because the signs of the coefficients \( c_n \) are alternating, this corresponds to a singularity in
the effective potential at \( g\phi \sim -0.25 \). We study the behavior of the effective potential in
the vicinity of this singularity in more detail in section 4.2.

| level  | (2, 6)   | (4, 12)  | (6, 18)  | (8, 20)  | (10, 20) |
|--------|----------|----------|----------|----------|----------|
| \( gr_c \) | \( \approx 0.345 \) | \( \approx 0.283 \) | \( \approx 0.265 \) | \( \approx 0.256 \) | \( \approx 0.252 \) |

Table 3: Approximate radius of convergence of effective potential in different level truncations

From this analysis of the perturbative effective potential using string field theory we have
found several things. First, we find that we can in principle determine any coefficient in the
perturbative expansion of the effective tachyon potential to an arbitrary degree of accuracy
with a finite calculation in string field theory, but that the size of the calculation needed to
determine the coefficients \( c_n \) increases significantly as \( n \) increases. Second, we have found
that the resulting effective potential has a radius of convergence on the order of (28). As we
will discuss in the next section, the stable vacuum of the theory lies well outside this radius,
near \( \phi \approx 1.1/g \). It is tempting to conclude from this that, even in principle, the existence
and structure of the stable vacuum of the theory are fundamentally nonperturbative phenomena.

4 The true vacuum and other nonperturbative features

The appearance of a stable vacuum in the open bosonic string field theory was first shown by
Kostelecky and Samuel in [2]. They found the vacuum in the (2, 6) level-truncated theory.
In later work, Kostelecky and Potting extended this analysis to the (4, 12) level-truncated
theory and reported that the energy density and the values of the scalar fields in the new
vacuum both seemed to be rapidly converging as the level number was increased. In [4],
Sen argued that the energy difference between the false and true vacua should precisely
match to the tension of the bosonic D25-brane. Strong evidence for this conclusion was
given by Sen and Zwiebach in [13], where they showed that at level (4, 8) the energy gap
between the stable and unstable vacua was 98.64% of the D-brane energy.

In this section we use our results for the level-truncated string field action to compute
some nonperturbative features of the open bosonic string. In subsection (4.1) we determine
the values of the scalar fields and the total energy of the system in the stable vacuum
including fields of up to level 10. At level (10, 20) we find that the energy gap between the
stable and unstable vacua corresponds to 99.91% of the D25-brane energy. In subsection
4.2 we study the effective potential of the tachyon, focusing on the branch structure of the effective potential in the region $\phi < 0$.

4.1 The stable vacuum

In order to find the stable vacuum of the level-truncated string field theory action in the zero momentum sector it is necessary to solve a system of $N$ coupled quadratic equations, where $N$ is the number of scalar string fields involved. In general, as $N$ becomes large it is difficult to rapidly solve such a system of equations to a high degree of precision. Because we know which branch of each quadratic equation the physical solution lies on, however, we can use an iterative approximation algorithm to rapidly converge on the true vacuum.

As discussed in section 3, the branch of the solution for each of the $N$ quadratic equations arising from the level-truncated string field theory action is determined by the condition that the stable vacuum lies on the same branch as the unstable vacuum which has all fields vanishing. Thus, the choice of branch for each field is dictated by the sign of the quadratic term in the string field action. For most fields $\psi^i$, therefore, the value of the field can be expressed in terms of the remaining fields $\psi^j, j \neq i$ through

$$\psi^i = \frac{-b + \text{sign}(d_{ii}) \sqrt{b^2 - 4ac}}{2a} \quad (29)$$

where

$$a = 3t_{iii}, \quad b = \sum_{j \neq i} 6t_{iji} \psi^j + 2d_{ii}, \quad c = \sum_{j,k \neq i} 3t_{ijk} \psi^j \psi^k + \sum_{j \neq i} 2d_{ij} \psi^j \quad (30)$$

There are some exceptions to this general rule, however. For the tachyon $\phi = \psi$ we choose the $+$ branch of the square root even though the kinetic term is negative. In addition, for most fields with ghost excitations the quadratic term in the action is off-diagonal. For example, the level 4 fields $\psi^8, \psi^{10}$ are coupled through the quadratic term $d_{810} = -3/2$. For fields such as this we use the equation of motion for field $\psi^i$ to solve for the dual field $\tilde{\psi}^i$ which has ghost and antighost modes exchanged through $c_{-n} \leftrightarrow b_{-n}$.

We have used (29, 30) to solve iteratively for the stable vacuum. We begin by solving (29) for all fields $\psi^i$, using zero for all fields appearing on the RHS. We then insert the first-round solutions for the fields back on the RHS to solve again for all the $\psi^i$, and repeat this process many times. This algorithm is numerically very stable near the nonperturbative vacuum and converges quite rapidly to a simultaneous solution of all $N$ equations. At level 6, for example, including all interactions up to level 18, after 30 rounds of this procedure the energy stabilizes to 10 digits, and after 50 rounds the values of all fields stabilize to 10 digits.
In the units we are using here, the tension of the bosonic D25-brane is \[ T_{25} = \frac{1}{2\pi^2 g^2} \] (31)

In Table 4 we have tabulated the results of our calculation of the exact vacuum in the theory truncated at various levels. The value of the tachyon is given in units of \( 1/g \) (with \( \alpha' = 1 \)),

| level | \( g\langle \phi \rangle \) | \( V/T_{25} \) |
|-------|----------------|----------------|
| (0, 0) | 0.91236 | -0.68462 |
| (2, 4) | 1.08318 | -0.94855 |
| (2, 6) | 1.08841 | -0.95938 |
| (4, 8) | 1.09633 | -0.98640 |
| (4, 12) | 1.09680 | -0.98782 |
| (6, 12) | 1.09602 | -0.99514 |
| (6, 18) | 1.09586 | -0.99518 |
| (8, 16) | 1.09424 | -0.99777 |
| (8, 20) | 1.09412 | -0.99793 |
| (10, 20) | 1.09259 | -0.99912 |

Table 4: Tachyon VEV and vacuum energy in stable vacua of level-truncated theory

and the vacuum energy difference from the unstable vacuum is given as a proportion of the D25-brane tension. The results at levels (2, 4) and (2, 6) agree with those of [2], and the results at level (4, 8) agree with [13].

We see from Table 4 that at level (10, 20) the energy gap between the unstable and stable vacua is 99.91% of the D25-brane tension. This seems to affirm the prediction of Sen in [4] beyond any reasonable doubt. It is interesting to note that the error in the vacuum energy \( (1 + V/T_{25}) \) is multiplied by approximately \( 1/3 \approx \kappa \) as each new level is added. It would be nice to have a theoretical explanation for this rate of convergence.

In [2] it was found that the vacuum energy and tachyon expectation values change very little between the (2, 4) and (2, 6) level truncations. In [13] a calculation was performed at level (4, 8) giving 98.6% of the D-brane tension. The improvement on this result given by including level 10 and 12 interactions between level 4 fields is fairly small. We have found that this pattern persists at higher level. The results at level (6, 18) are not very different from those found at level (6, 12), and the results at level (8, 20) are very close to those at level (8, 16). If, however, we drop cubic interactions at the same level as the quadratic interactions at that level, for example by truncating all cubic interactions to level (6, 6), the energy of the stable vacuum varies much more wildly and can even decrease below \(-T_{25}\). For this reason, we do not think that it would be useful to consider including higher level fields unless the level of cubic interactions calculated could be increased to at least twice the level number. Using our rather inefficient program it would take a significant amount of computer time to go to level (12, 24) on a standard desktop machine. We discuss in the last
section the prospects for continuing these numerical studies to higher levels of truncation of the full theory.

As we can see from Table 4, the expectation value of the tachyon in the stable vacuum converges fairly quickly as the level number is increased. The same is true of all the scalar fields; in Appendix A the values of the scalar fields at levels \( \leq 6 \) are given for the level \((4, 12)\), \((6, 18)\) and \((10, 20)\) truncations. The tachyon field itself converges to a value near

\[
g\langle \phi \rangle \approx 1.09 \quad (32)
\]

In [13] Sen and Zwiebach observed that the level 2 fields with canonical normalization

\[
\begin{align*}
  u &= -\psi^3 \\
  v &= \frac{1}{2\sqrt{13}}\psi^2
\end{align*}
\]

take almost identical expectation values in the level \((4, 8)\) truncation. This led these authors to conjecture that in the exact theory these fields would take identical expectation values, possibly due to some hidden symmetry in the theory. Checking this relationship in the vacua of the higher level truncations, we find that this conjecture is not supported. While at level \((4, 8)\) these fields differ by about 0.1%, at level \((10, 20)\) the difference has grown to over 5%.

The values of these fields in the different level-truncated vacua are graphed in Figure 2, using the value \( g = 2 \) to conform with the conventions of [13]. The failure of this equality to hold in the asymptotic theory indicates that it may be more difficult than previously thought to implement the suggestion made in [13] of finding an exact solution for the nonperturbative vacuum in the full theory using a hidden symmetry relating fields of this type.
As a check on our results, we can verify that the vacuum expectation values of the fields $\psi^i$ are such that the nonperturbative vacuum lies in the truncated Hilbert space $H_1$ described in [4]. For example, the level 4 fields should obey the linear relation

$$\langle \psi^4 \rangle - 2\langle \psi^5 \rangle - 4\langle \psi^6 \rangle = 0 \quad (33)$$

in the vacuum. We have checked this relation and other analogous relations for higher level fields and find that they are satisfied up to the level of accuracy (10 significant digits) to which we have calculated the vacuum expectation values.

Now that we have identified the stable vacuum of the theory after truncating at levels up to (10, 20), a natural next step is to investigate the structure of the theory around this stable vacuum. The stable vacuum should correspond to the empty vacuum of the closed bosonic string theory, with modes corresponding to the open string fields on the D25-brane decoupling from the theory. Of particular interest in this regard is the fate of the U(1) vector field $A^\mu$ on the brane [18, 19, 20]. Some progress in understanding the structure of the theory around the stable vacuum was made in [2]. It would be interesting to study this question further and to investigate the spectrum of modes around the stable vacuum using the more detailed picture we have given of the expectation values of the scalar string fields in this vacuum. We leave these questions to further work.

### 4.2 Effective tachyon potential

In section 3.2 we studied the effective potential of the tachyon through its power series expansion around the unstable vacuum $\psi^i = 0$. We found that the perturbation series had a finite radius of convergence in each of the level-truncated theories, indicating a possible singularity near $\phi \approx -0.25/g$. In this section we study the nonperturbative effective potential, and investigate the branch structure of the potential near the singularity.

For a fixed value of the tachyon field, there are in general many solutions of the equations for the remaining fields $\psi^i, i > 1$ which correspond to different branches of the effective potential. The perturbative expansion describes only one branch of the effective potential, the one which is connected to the unstable vacuum. We will refer to this as branch 1. For levels higher than (2,4), we cannot exactly integrate out all the non-tachyonic fields, so we are forced to use numerical methods to study the effective potential outside its radius of convergence or on other branches. We have used Newton’s method to find the zeros of the partial derivatives of the potential. The choice of initial values for the fields determines on which branch the algorithm will converge. At mass level 2, we can find all the solutions, and thus determine all the branches. We can then use the field values from these branches as initial values for the algorithm at higher levels in order to stay on the same branch.

In Figure 3, we have plotted branch 1 of the effective potential at levels (2,6), (4,12) and (6,18). At level (4,12), our algorithm stops converging after $g\phi \approx 1.8$. At level (6,18), the algorithm becomes unstable after $g\phi \approx 1.6$. This may indicate either that the branch ends at that point or that it meets one or more other branches which play the role of attractors,
making it difficult to converge to the chosen branch. A similar breakdown of convergence also happens at level \((2, 6)\) around \(g\phi \approx 6\), where a new pair of branches appear \([2]\).

A particularly interesting physical question is to determine the structure of the tachyon effective potential for negative values of \(\phi\). In the level 0 truncated theory, this potential decreases to \(-\infty\) as \(\phi \to -\infty\). If this behavior is not modified by higher level corrections, it poses the natural question of why the D25-brane would choose to condense to the stable vacuum rather than the runaway solution at negative \(\phi\).

When we attempt to continue branch 1 of the effective potential to include arbitrary negative values of the tachyon field, we find that at each level our algorithm stops converging very near the radius of convergence given in Table 3, which we calculated using the perturbative expansion coefficients of the effective potential. In \([2]\), Kostelecky and Samuel studied the effective potential at mass level 2 near this point. They found that the branch connected to the unstable and stable vacua (branch 1) ends at a singularity at \(g\phi \approx -0.35\) and, at that point, meets another branch (branch 2) which continues upward for increasing values of \(\phi\). This branch then meets a third branch (branch 3) which continues downward for decreasing values of \(\phi\). We have studied the branch structure of the effective potential in level truncations \((4, 12)\) and \((6, 18)\) and we find precisely the same branch structure, although the shape of the branches changes noticeably at each level. In Figure 4, a graph is given of the branch structure of the effective potential at each of these levels. We observe that branch 3 becomes less steep as we go at higher levels, and that it begins to approach the singular point where branch 1 meets branch 2. While branch 3 does not seem to approach the singular point of branch 1 exponentially quickly, its rate of approach seems compatible with the rate of convergence of the coefficients in the perturbative expansion of the tachyon.
effective potential. Although we only have limited evidence for this suggestion at this time, we conjecture that in the limit of the full string field theory, branch 3 precisely intersects the singular point connecting branches 1 and 2. The physical effect of this would be to create a critical point near \( g\phi \approx -0.25 \) at which there would be a higher-order phase transition in the tachyon effective potential. Unlike the effective potential on branch 1, the structure of the effective potential on branch 3 changes dramatically with each additional level of string fields which are included. Thus, we cannot trust our low-level truncation of the theory to accurately describe the effective potential beyond the critical point. It is natural to speculate that the effective potential becomes nonnegative (relative to the energy of the stable vacuum) for all values of \( \phi \) in the full string field theory.

It would be very nice to have some better understanding of the physics of open bosonic string theory beyond the critical point; there must also be a natural physical explanation for the existence of the critical point, which might be understood from the point of view of bosonic D-branes. We leave these questions for further research, but we conclude this section by pointing out one possible amusing scenario: it is quite possible that the numerical difficulties we have encountered in extending branch 1 of the effective potential near \( g\phi \approx 1.6 \) indicate a second critical point at approximately the same energy as that we have found at \( g\phi \approx -0.25 \). If this is indeed the case, it may be that these two critical points should be physically identified in the full theory, so that the tachyon effective potential will essentially become (in some coordinates) periodic, and perhaps even smooth. This would provide a satisfying resolution to the question of what happens when the tachyon rolls in the negative direction: it would simply bring the system to the same stable vacuum as if it rolled in the positive direction. More evidence is needed before this possibility can be taken seriously, but
it would provide a nice scenario in which the effective potential for the tachyon has a unique global minimum corresponding to the stable vacuum, just as occurs for the superstring \[\text{[7]}\]. One potentially serious drawback to this scenario, however, is that it seems to predict the existence of a stable 24-brane kink for whose existence there is no evidence in the bosonic string theory.

5 Conclusions

In this paper we have used the level-truncation approach to string field theory to perform a detailed study of the tachyonic instability of the open bosonic string. We have calculated the cubic string field theory potential up to terms of mass level 20, including fields up to level 10. We have used these results to analyze the effective tachyon potential and the nonperturbative stable vacuum of the system. Our results for the perturbative form of the effective potential indicate that the radius of convergence of this potential decreases to an apparently finite value as higher level fields are included, indicating a critical point in the effective potential near \(\phi \approx -0.25/g\). The stable vacuum lies well outside this radius of convergence, although the critical point and the stable vacuum lie on opposite sides of the unstable vacuum so there is no phase transition between the two vacua. We have found that while the coefficients in the effective potential converge relatively slowly using the level truncation method, the energy of the stable vacuum solution and the values of the fields in this vacuum converge much more quickly. We found that the discrepancy between the exact D-brane tension and the vacuum energy calculated in the truncated string field theory decreases by approximately a factor of \(1/3\) when string fields at each additional mass level are included, although we do not have a theoretical explanation for this rate of convergence.

It is perhaps somewhat surprising that this method converges more rapidly for the vacuum energy calculation, which is a truly nonperturbative feature of the system, than for the coefficients of the effective potential, which are in principle computable using perturbative methods. This finding seems to indicate that string field theory has the potential to be an extremely useful tool in studying detailed aspects of nonperturbative string physics. Even if it is not possible to find an exact analytic solution of string field theory describing the nonperturbative vacuum, it would be of great interest to carry out a more detailed analysis of the asymptotic properties of the level truncation approach.

The methods used in this paper give us a fairly complete picture of the behavior of the effective tachyon potential when \(-0.25 < g\phi < 1.5\). We have found that a critical point appears in the tachyon potential near \(g\phi \approx -0.25\), beyond which it is difficult to precisely determine the physics. We have speculated that the effective potential stays above the stable vacuum energy for negative \(\phi\) beyond the critical point, but we do not yet have conclusive evidence for this conclusion. It would be very nice to have a better understanding of the behavior of the theory in the regime where the tachyon is large and negative. We have outlined one possible scenario, in which the tachyon effective potential becomes periodic and the stable vacuum appears at the unique minimum of this potential. If this scenario is
realized it would provide a simple answer to the question of how the theory behaves if the
tachyon chooses to roll in the negative direction. This would also provide a picture in which
unstable bosonic D-branes could naturally be interpreted as sphalerons, as advocated in the
supersymmetric case in [21].

There are many directions in which it would be interesting to extend the work in this
paper. One obvious question is to ask how far it is possible to extend the level truncation
method in the open bosonic string field theory. For the results in this paper where we included
fields at level 10 we have computed 138,202 distinct cubic vertices. Because many of these
vertices involve hundreds of possible index contraction combinations, this computation is
already rather time-consuming using a higher-level symbolic manipulation program such as
Mathematica on a desktop PC. To continue to level (12, 24), over a million vertices (1,381,097)
would be needed, each involving hundreds or thousands of index contractions. With a more
efficient program and a powerful computer, it might be possible to push as far as level (16,
32) or higher in the foreseeable future if there are physical questions of sufficient interest to
motivate further development in this direction (at this level there are over 10^8 cubic vertices).
Although the number of vertices grows as the cube of the number of fields at the level at
which the theory is truncated, both the number of fields and the complexity of computing
each vertex grows exponentially, so that unless a better theoretical understanding of the
structure of the theory is developed it will probably never be feasible to perform calculations
in this theory beyond interactions of total level 40 or so.

There are a number of other questions of significant interest which could be investigated
using truncations of bosonic string field theory at levels considered in this paper. As discussed
in section [1] it is clearly important to attain a better understanding of the structure of the
theory around the nonperturbative vacuum. While some progress was made in this
direction in [2], we do not yet have a very clear picture of how the open string fields such
as the U(1) gauge field decouple from the theory in the stable vacuum. Another question of
interest which can be addressed in the bosonic theory is the existence of vacua which break
Lorentz symmetry. Such vacua have been shown to exist and to converge up to level (6, 18)
truncations of the theory [2, 3]. It would be nice to have a better theoretical understanding
of the significance of these vacua.

A related question which can be addressed using open bosonic string field theory is that
of describing the structure of Dp-branes with p < 25 as unstable configurations of open
string fields. Solutions of this type have been found in conformal field theory [22, 23, 24, 25].
The possibility of realizing a bosonic D(p − 1)-brane as a tachyonic lump on a Dp-brane
was discussed in [26]. The use of the level truncation approach to string field theory in this
context was suggested in [13], and was implemented by Harvey and Kraus [8], who found
numerical evidence for Dp-branes with p > 18 as lumps on the D25-brane after including
effects of some level 2 fields. It would be very interesting to extend this work to higher level
truncations and to see whether the physics of all Dp-branes is indeed accurately described
in level-truncated string field theory.

The open bosonic string is an interesting model in which string field theory is particu-
larly simple. Studying this theory further may allow us to refine our understanding of certain qualitative aspects of D-brane physics and tachyonic instabilities. Nonetheless, if string theory is ever to make concrete contact with observable phenomenology it will almost certainly be in the context of a supersymmetric theory. Given that the open string field theory seems to be able to access interesting nonperturbative structure in string theory, it is clearly of substantial interest to develop an equally systematic approach to performing nonperturbative calculations using superstring field theory. While Witten’s formulation of open string field theory can be extended to the supersymmetric theory \cite{27, 28}, this formalism may be problematic due to the necessity of considering higher order contact interactions \cite{29, 30}. Several alternative formulations of open superstring field theory have been suggested \cite{31, 32, 33}. Recently Berkovits has used his alternative formulation of the supersymmetric open string field theory \cite{33} to carry out a first approximation of the energy gap in a tachyonic brane system, and has found that 60% of the brane energy is reproduced even in the first truncation of the theory \cite{33}. This formalism has been more explicitly developed and the calculation of the energy gap has been extended by Berkovits, Sen and Zwiebach \cite{7}, who showed that including the three scalar fields at the next relevant level gives 85% of the brane energy. If explicit calculations in open superstring field theory can indeed be systematically carried out in high-level truncations as we have done for the bosonic theory, this promises to provide an exciting new tool for investigating in detail nonperturbative issues in string theory such as the description of non-BPS D-branes as brane-antibrane bound states \cite{34, 35, 36} and the associated connection between D-brane charges and K-theory \cite{19, 37}.

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The following table describes the scalar states at levels \( \leq 6 \). The SZ column relates these fields to the background-independent fields used in \([13]\). \( m \) and \( g \) are the indices of the matter and ghost states into which each scalar decomposes through \((4)\). \( g(\psi^i)_L \) denote expectation values of the scalar fields in level truncations \((4, 12), (6, 18)\) and \((10, 20)\).

| \( \psi^i \) | SZ | state | \( m \) | \( g \) | \( g(\psi^i)_4 \) | \( g(\psi^i)_6 \) | \( g(\psi^i)_{10} \) |
|---|---|---|---|---|---|---|---|
| \( \psi^1 \) | \( \phi \) | \( \langle 0 \rangle \) | 1 | 1 | 1.09680 | 1.09586 | 1.09259 |
| \( \psi^2 \) | \( v/\sqrt{52} \) | \( (\alpha_{-1} \cdot \alpha_{-1}) \langle 0 \rangle \) | 2 | 1 | 0.05692 | 0.05714 | 0.05723 |
| \( \psi^3 \) | \( -u \) | \( b_{-1} c_{-1} \langle 0 \rangle \) | 1 | 2 | -0.41135 | -0.42363 | -0.43372 |
| \( \psi^4 \) | \( A + B \) | \( (\alpha_{-1} \cdot \alpha_{-3}) \langle 0 \rangle \) | 4 | 1 | -0.01142 | -0.01146 | -0.01148 |
| \( \psi^5 \) | \( A/2 \) | \( (\alpha_{-2} \cdot \alpha_{-2}) \langle 0 \rangle \) | 5 | 1 | -0.00512 | -0.00511 | -0.00509 |
| \( \psi^6 \) | \( B/4 \) | \( (\alpha_{-1} \cdot \alpha_{-1})(\alpha_{-1} \cdot \alpha_{-1}) \langle 0 \rangle \) | 6 | 1 | -0.00029 | -0.00031 | -0.00032 |
| \( \psi^7 \) | \( -F \) | \( (\alpha_{-1} \cdot \alpha_{-1}) b_{-1} c_{-1} \langle 0 \rangle \) | 2 | 2 | 0.00686 | 0.00740 | 0.00786 |
| \( \psi^8 \) | \( -C \) | \( b_{-1} c_{-3} \langle 0 \rangle \) | 1 | 5 | 0.11242 | 0.11478 | 0.11654 |
| \( \psi^9 \) | \( E \) | \( b_{-2} c_{-2} \langle 0 \rangle \) | 1 | 6 | 0.06621 | 0.06813 | 0.06990 |
| \( \psi^{10} \) | \( D \) | \( b_{-3} c_{-1} \langle 0 \rangle \) | 1 | 7 | 0.03747 | 0.03826 | 0.03885 |
| \( \psi^{11} \) | \( (\alpha_{-1} \cdot \alpha_{-3}) \langle 0 \rangle \) | 10 | 1 | | 0.00349 | 0.00350 |
| \( \psi^{12} \) | \( (\alpha_{-2} \cdot \alpha_{-4}) \langle 0 \rangle \) | 11 | 1 | | 0.00293 | 0.00291 |
| \( \psi^{13} \) | \( (\alpha_{-3} \cdot \alpha_{-3}) \langle 0 \rangle \) | 12 | 1 | | 0.00144 | 0.00143 |
| \( \psi^{14} \) | \( (\alpha_{-1} \cdot \alpha_{-3})(\alpha_{-1} \cdot \alpha_{-1}) \langle 0 \rangle \) | 13 | 1 | | 0.00028 | 0.00028 |
| \( \psi^{15} \) | \( (\alpha_{-2} \cdot \alpha_{-2})(\alpha_{-1} \cdot \alpha_{-1}) \langle 0 \rangle \) | 14 | 1 | | 0.00015 | 0.00015 |
| \( \psi^{16} \) | \( (\alpha_{-1} \cdot \alpha_{-2})(\alpha_{-1} \cdot \alpha_{-2}) \langle 0 \rangle \) | 15 | 1 | | 0.00001 | 0.00001 |
| \( \psi^{17} \) | \( (\alpha_{-1} \cdot \alpha_{-1})(\alpha_{-1} \cdot \alpha_{-1}) \langle 0 \rangle \) | 16 | 1 | | -0.00000 | -0.00000 |
| \( \psi^{18} \) | \( (\alpha_{-1} \cdot \alpha_{-3}) b_{-1} c_{-1} \langle 0 \rangle \) | 4 | 2 | | -0.00263 | -0.00280 |
| \( \psi^{19} \) | \( (\alpha_{-2} \cdot \alpha_{-2}) b_{-1} c_{-1} \langle 0 \rangle \) | 5 | 2 | | -0.00116 | -0.00121 |
| \( \psi^{20} \) | \( (\alpha_{-1} \cdot \alpha_{-1}) b_{-1} c_{-1} \langle 0 \rangle \) | 6 | 2 | | -0.00008 | -0.00009 |
| \( \psi^{21} \) | \( (\alpha_{-1} \cdot \alpha_{-2}) b_{-1} c_{-2} \langle 0 \rangle \) | 3 | 3 | | -0.00013 | -0.00015 |
| \( \psi^{22} \) | \( (\alpha_{-1} \cdot \alpha_{-2}) b_{-2} c_{-1} \langle 0 \rangle \) | 3 | 4 | | -0.00006 | -0.00007 |
| \( \psi^{23} \) | \( (\alpha_{-1} \cdot \alpha_{-1}) b_{-1} c_{-3} \langle 0 \rangle \) | 2 | 5 | | -0.00336 | -0.00351 |
| \( \psi^{24} \) | \( (\alpha_{-1} \cdot \alpha_{-1}) b_{-2} c_{-2} \langle 0 \rangle \) | 2 | 6 | | -0.00283 | -0.00295 |
| \( \psi^{25} \) | \( (\alpha_{-1} \cdot \alpha_{-1}) b_{-3} c_{-1} \langle 0 \rangle \) | 2 | 7 | | -0.00112 | -0.00117 |
| \( \psi^{26} \) | \( b_{-1} c_{-5} \langle 0 \rangle \) | 1 | 12 | | -0.06003 | -0.06094 |
| \( \psi^{27} \) | \( b_{-2} c_{-4} \langle 0 \rangle \) | 1 | 13 | | -0.03750 | -0.03816 |
| \( \psi^{28} \) | \( b_{-3} c_{-3} \langle 0 \rangle \) | 1 | 14 | | -0.02283 | -0.02291 |
| \( \psi^{29} \) | \( b_{-4} c_{-2} \langle 0 \rangle \) | 1 | 15 | | -0.01875 | -0.01908 |
| \( \psi^{30} \) | \( b_{-5} c_{-1} \langle 0 \rangle \) | 1 | 16 | | -0.01201 | -0.01219 |
| \( \psi^{31} \) | \( b_{-2} b_{-1} c_{-2} c_{-1} \langle 0 \rangle \) | 1 | 17 | | 0.01547 | 0.01612 |
## Table of matter and ghost states at levels $\leq 6$

The following tables list the matter and ghost states which contribute to scalar fields at levels $\leq 6$. The quadratic terms involving each state are listed in the last column.

### Matter states

| $\eta_m \rangle$ | state | quadratic terms |
|-----------------|-------|-----------------|
| $|\eta_1\rangle$ | $|0\rangle$ | $d_{11}^{\text{mat}} = -\frac{1}{2}$ |
| $|\eta_2\rangle$ | $(\alpha_{-1} \cdot \alpha_{-1}) |0\rangle$ | $d_{22}^{\text{mat}} = 2 \cdot 13$ |
| $|\eta_3\rangle$ | $(\alpha_{-1} \cdot \alpha_{-2}) |0\rangle$ | $d_{33}^{\text{mat}} = 2^2 \cdot 13$ |
| $|\eta_4\rangle$ | $(\alpha_{-1} \cdot \alpha_{-3}) |0\rangle$ | $d_{44}^{\text{mat}} = 3^2 \cdot 13$ |
| $|\eta_5\rangle$ | $(\alpha_{-2} \cdot \alpha_{-2}) |0\rangle$ | $d_{55}^{\text{mat}} = 2^3 \cdot 3 \cdot 13$ |
| $|\eta_6\rangle$ | $(\alpha_{-1} \cdot \alpha_{-1})(\alpha_{-1} \cdot \alpha_{-1}) |0\rangle$ | $d_{66}^{\text{mat}} = 2^5 \cdot 3 \cdot 7 \cdot 13$ |
| $|\eta_{10}\rangle$ | $(\alpha_{-1} \cdot \alpha_{-5}) |0\rangle$ | $d_{10\ 10}^{\text{mat}} = 5^2 \cdot 13$ |
| $|\eta_{11}\rangle$ | $(\alpha_{-2} \cdot \alpha_{-4}) |0\rangle$ | $d_{11\ 11}^{\text{mat}} = 2^3 \cdot 5 \cdot 13$ |
| $|\eta_{12}\rangle$ | $(\alpha_{-3} \cdot \alpha_{-3}) |0\rangle$ | $d_{12\ 12}^{\text{mat}} = 2 \cdot 3^2 \cdot 5 \cdot 13$ |
| $|\eta_{13}\rangle$ | $(\alpha_{-1} \cdot \alpha_{-3})(\alpha_{-1} \cdot \alpha_{-1}) |0\rangle$ | $d_{13\ 13}^{\text{mat}} = 2^3 \cdot 3 \cdot 5 \cdot 7 \cdot 13$ |
| $|\eta_{14}\rangle$ | $(\alpha_{-2} \cdot \alpha_{-2})(\alpha_{-1} \cdot \alpha_{-1}) |0\rangle$ | $d_{14\ 14}^{\text{mat}} = 2^5 \cdot 5 \cdot 13^2 \cdot d_{14\ 15}^{\text{mat}} = 2^4 \cdot 5 \cdot 13$ |
| $|\eta_{15}\rangle$ | $(\alpha_{-1} \cdot \alpha_{-2})(\alpha_{-1} \cdot \alpha_{-2}) |0\rangle$ | $d_{15\ 14}^{\text{mat}} = 2^4 \cdot 5 \cdot 13 \cdot d_{15\ 15}^{\text{mat}} = 2^3 \cdot 3 \cdot 5 \cdot 13$ |
| $|\eta_{16}\rangle$ | $(\alpha_{-1} \cdot \alpha_{-1})(\alpha_{-1} \cdot \alpha_{-1})(\alpha_{-1} \cdot \alpha_{-1}) |0\rangle$ | $d_{16\ 16}^{\text{mat}} = 2^7 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 13$ |

### Ghost states

| $|\chi_g\rangle$ | state | quadratic terms |
|-----------------|-------|-----------------|
| $|\chi_1\rangle$ | $|0\rangle$ | $d_{11}^{\text{gh}} = -\frac{1}{2}$ |
| $|\chi_2\rangle$ | $b_{-1}c_{-1} |0\rangle$ | $d_{22}^{\text{gh}} = -\frac{1}{2}$ |
| $|\chi_3\rangle$ | $b_{-1}c_{-2} |0\rangle$ | $d_{34}^{\text{gh}} = -1$ |
| $|\chi_4\rangle$ | $b_{-2}c_{-1} |0\rangle$ | $d_{43}^{\text{gh}} = -1$ |
| $|\chi_5\rangle$ | $b_{-1}c_{-3} |0\rangle$ | $d_{57}^{\text{gh}} = -\frac{3}{2}$ |
| $|\chi_6\rangle$ | $b_{-2}c_{-2} |0\rangle$ | $d_{66}^{\text{gh}} = -\frac{3}{2}$ |
| $|\chi_7\rangle$ | $b_{-3}c_{-1} |0\rangle$ | $d_{75}^{\text{gh}} = -\frac{3}{2}$ |
| $|\chi_{12}\rangle$ | $b_{-1}c_{-5} |0\rangle$ | $d_{12\ 16}^{\text{gh}} = -\frac{5}{2}$ |
| $|\chi_{13}\rangle$ | $b_{-2}c_{-4} |0\rangle$ | $d_{13\ 15}^{\text{gh}} = -\frac{5}{2}$ |
| $|\chi_{14}\rangle$ | $b_{-3}c_{-3} |0\rangle$ | $d_{14\ 14}^{\text{gh}} = -\frac{5}{2}$ |
| $|\chi_{15}\rangle$ | $b_{-4}c_{-2} |0\rangle$ | $d_{15\ 13}^{\text{gh}} = -\frac{5}{2}$ |
| $|\chi_{16}\rangle$ | $b_{-5}c_{-1} |0\rangle$ | $d_{16\ 12}^{\text{gh}} = -\frac{5}{2}$ |
| $|\chi_{17}\rangle$ | $b_{-2}b_{-1}c_{-2}c_{-1} |0\rangle$ | $d_{17\ 17}^{\text{gh}} = \frac{5}{2}$ |
## C Cubic interactions at levels (6, 16)

In the following pages we give tables of all cubic interactions between the pure matter and ghost scalar fields listed in Appendix B which contribute to cubic interactions of total dimension $\leq 16$ between the scalar fields listed in Appendix A. All cubic interactions of level less or equal to (6, 16) between the scalar fields listed in Appendix A can be reproduced using these tables and \([10]\).

Cubic matter field coefficients $t_{ijk}^{\text{mat}}$

| $ijk$ | 1 | 2 | 3 | 4 | 5 | 6 |
|-------|---|---|---|---|---|---|
| 1-1   | 1 | $2\sqrt{3}$ | 4 | $2\sqrt{3}$ | 4 | $2\sqrt{3}$ |
| 1-2   | $2\sqrt{3}$ | $2\sqrt{3}$ | $2\sqrt{3}$ | $2\sqrt{3}$ | $2\sqrt{3}$ | $2\sqrt{3}$ |
| 1-3   | 0 | 4 | 4 | 4 | 4 | 4 |
| 1-4   | $2\sqrt{3}$ | 4 | 4 | 4 | 4 | 4 |
| 1-5   | $2\sqrt{3}$ | 4 | 4 | 4 | 4 | 4 |
| 1-6   | $2\sqrt{3}$ | 4 | 4 | 4 | 4 | 4 |
| 2-1   | $2\sqrt{3}$ | 4 | 4 | 4 | 4 | 4 |
| 2-2   | 0 | 4 | 4 | 4 | 4 | 4 |
| 2-3   | $2\sqrt{3}$ | 4 | 4 | 4 | 4 | 4 |
| 2-4   | $2\sqrt{3}$ | 4 | 4 | 4 | 4 | 4 |
| 2-5   | $2\sqrt{3}$ | 4 | 4 | 4 | 4 | 4 |
| 2-6   | $2\sqrt{3}$ | 4 | 4 | 4 | 4 | 4 |
| 3-1   | $2\sqrt{3}$ | 4 | 4 | 4 | 4 | 4 |
| 3-2   | $2\sqrt{3}$ | 4 | 4 | 4 | 4 | 4 |
| 3-3   | 0 | 4 | 4 | 4 | 4 | 4 |
| 3-4   | $2\sqrt{3}$ | 4 | 4 | 4 | 4 | 4 |
| 3-5   | $2\sqrt{3}$ | 4 | 4 | 4 | 4 | 4 |
| 3-6   | $2\sqrt{3}$ | 4 | 4 | 4 | 4 | 4 |

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| $j$ | $k$ | 1  | 2  | 3  | 4  | 5  | 6  |
|-----|-----|----|----|----|----|----|----|
| 3  | 10 | $-2^{15} \cdot 1.3^{16} \cdot 1.5^{18}$ | $-2^{15} \cdot 1.3^{17}$ | $-2^{15} \cdot 1.3^{18}$ | $-2^{15} \cdot 1.3^{19}$ | $-2^{15} \cdot 1.3^{20}$ | $-2^{15} \cdot 1.3^{21}$ |
| 3  | 11 | $-2^{15} \cdot 1.3^{17}$ | $-2^{15} \cdot 1.3^{18}$ | $-2^{15} \cdot 1.3^{19}$ | $-2^{15} \cdot 1.3^{20}$ | $-2^{15} \cdot 1.3^{21}$ |
| 3  | 12 | $-2^{15} \cdot 1.3^{18}$ | $-2^{15} \cdot 1.3^{19}$ | $-2^{15} \cdot 1.3^{20}$ | $-2^{15} \cdot 1.3^{21}$ |
| 3  | 13 | $-2^{15} \cdot 1.3^{19}$ | $-2^{15} \cdot 1.3^{20}$ | $-2^{15} \cdot 1.3^{21}$ |
| 4  | 10 | $-2^{15} \cdot 1.3^{20}$ | $-2^{15} \cdot 1.3^{21}$ |
| 4  | 11 | $-2^{15} \cdot 1.3^{21}$ |
| 4  | 12 | $-2^{15} \cdot 1.3^{22}$ |
| 4  | 13 | $-2^{15} \cdot 1.3^{23}$ |
| 5  | 10 | $-2^{15} \cdot 1.3^{24}$ |
| 5  | 11 | $-2^{15} \cdot 1.3^{25}$ |
| 5  | 12 | $-2^{15} \cdot 1.3^{26}$ |
| 6  | 10 | $-2^{15} \cdot 1.3^{27}$ |
| 6  | 11 | $-2^{15} \cdot 1.3^{28}$ |
| 6  | 12 | $-2^{15} \cdot 1.3^{29}$ |

Table of $i_{mat}^{ijk}$ (continued)

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Table of $i_{ijk}^{\text{mat}}$ (continued)

|   | 1          | 2          | 3          | 4          | 5          | 6          |
|---|------------|------------|------------|------------|------------|------------|
| 10 | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} |
| 11 | 2^{11} \cdot 5^{11} \cdot 154601^{6} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} |
| 12 | 2^{11} \cdot 5^{11} \cdot 147^{67} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} |
| 13 | 2^{11} \cdot 5^{11} \cdot 13^{111} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} |
| 14 | 2^{11} \cdot 5^{11} \cdot 11^{111} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} |
| 15 | 2^{11} \cdot 5^{11} \cdot 10^{111} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} |
| 16 | 2^{11} \cdot 5^{11} \cdot 9^{111} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} |
| 17 | 2^{11} \cdot 5^{11} \cdot 8^{111} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} |
| 18 | 2^{11} \cdot 5^{11} \cdot 7^{111} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} |
| 19 | 2^{11} \cdot 5^{11} \cdot 6^{111} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} |
| 20 | 2^{11} \cdot 5^{11} \cdot 5^{111} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} | 2^{10} \cdot 5^{10} \cdot 109^{10} \cdot 839^{3} |

Note: The table continues with similar entries.
| 14 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|----|---|---|---|---|---|---|---|
| 1  | 1 | 1 1/3 | 0 | 0 | 21/3 | 19 | 25/3 |
| 2  | 11/3 | 19/3 | 25/3 | 27/3 | 27/3 | 19/3 | 25/3 |
| 3  | 0 | 25/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 |
| 4  | 0 | 25/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 |
| 5  | 11/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 |
| 6  | 19/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 |
| 7  | 19/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 |
| 8  | 25/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 |
| 9  | 25/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 |
| 10 | 25/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 |
| 11 | 25/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 |
| 12 | 25/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 |
| 13 | 25/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 |
| 14 | 25/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 |
| 15 | 25/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 |
| 16 | 25/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 |
| 17 | 25/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 | 27/3 |

Cubic ghost field coefficients $f_{ijk}^{gh}$.
Table of $i^\text{th}_{ij,k}$ (continued)

| $i$ | 1  | 2  | 3  | 4  | 5  | 6  | 7  |
|-----|----|----|----|----|----|----|----|
| 3.12| $-2^{5.71}.$ | $-2^{5.71}.$ | $-2^{5.71}.$ | $-2^{5.71}.$ | $-2^{5.71}.$ | $-2^{5.71}.$ | $-2^{5.71}.$ |
| 3.13| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.14| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.15| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.16| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.17| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.18| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.19| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.20| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.21| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.22| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.23| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.24| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.25| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.26| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.27| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.28| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.29| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.30| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.31| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.32| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.33| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.34| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.35| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.36| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.37| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.38| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.39| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.40| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
| 3.41| $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ | $2^{5.19}.$ |
Table of $f_{ijk}^{\text{th}}$ (continued)

| i  | 1 | 2 | 5 | 6 | 7 |
|----|---|---|---|---|---|
| 12 | $2^6 j^2$ | $-2^6 j^2$ | $2^7 71^2$ | $-2^6 71^2$ | $2^6 71^2$ |
| 13 | $2^6 j^2$ | $-2^6 j^2$ | $2^7 71^2$ | $-2^6 71^2$ | $2^6 71^2$ |
| 14 | $2^6 j^2$ | $-2^6 j^2$ | $2^7 71^2$ | $-2^6 71^2$ | $2^6 71^2$ |
| 15 | $2^6 j^2$ | $-2^6 j^2$ | $2^7 71^2$ | $-2^6 71^2$ | $2^6 71^2$ |
| 16 | $2^6 j^2$ | $-2^6 j^2$ | $2^7 71^2$ | $-2^6 71^2$ | $2^6 71^2$ |
| 17 | $2^6 j^2$ | $-2^6 j^2$ | $2^7 71^2$ | $-2^6 71^2$ | $2^6 71^2$ |
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