Quantum Quench Across a Zero Temperature Holographic Superfluid Transition

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Abstract

We study quantum quench in a holographic model of a zero temperature insulator-superfluid transition. The model is a modification of that of arXiv:0911.0962 and involves a self-coupled complex scalar field, Einstein gravity with a negative cosmological constant, and Maxwell field with one of the spatial directions compact. In a suitable regime of parameters, the scalar field can be treated as a probe field whose backreaction to both the metric and the gauge field can be ignored. We show that when the chemical potential of the dual field theory lies between two critical values, the equilibrium background geometry is a AdS soliton with a constant gauge field, while the complex scalar condenses leading to broken symmetry. We then turn on a time dependent source for the order parameter which interpolates between constant values and crosses the order-disorder critical point. In the critical region adiabaticity breaks down, but for a small rate of change of the source \(v\) there is a new small-\(v\) expansion in fractional powers of \(v\). The resulting critical dynamics is dominated by a zero mode of the bulk field. To lowest order in this small-\(v\) expansion, the order parameter satisfies a time dependent Landau-Ginsburg equation which has \(z = 2\), but non-dissipative. These predictions are verified by explicit numerical solutions of the bulk equations of motion.

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1 Introduction and summary

Recently there has been several efforts to understand the problem of quantum or thermal quench \cite{1,3} in strongly coupled field theories using the AdS/CFT correspondence \cite{4,7}. This approach has been used to explore two interesting issues. The first relates to the question of thermalization. In this problem one typically considers a coupling in the hamiltonian which varies appreciably with time over some finite time interval. Starting with a nice initial state (e.g. the vacuum) the question is whether the system evolves into some steady state and whether this steady state resembles a thermal state in a suitably defined sense. In the bulk description a time dependent coupling of the boundary field theory is a time dependent boundary condition. For example, with an initial AdS this leads to black hole formation under suitable conditions. This is a holographic description of thermalization, which has been widely studied over the past several years \cite{8,10} with other initial conditions as well.

Many interesting applications of AdS/CFT duality involve a subset of bulk fields whose backreaction to gravity can be ignored, so that they can be treated in a probe approximation. One set of examples concern probe branes in AdS which lead to hypermultiplet fields in the original dual field theory. Even though the background does not change in the leading order, it turns out that thermalization of the hypermultiplet sector is still visible - this manifests itself in the formation of apparent horizons on the worldvolume \cite{11,12}.

The second issue relates to quench across critical points \cite{1,3}. Consider for example starting in a gapped phase, with a parameter in the Hamiltonian varying slowly compared to the initial gap, bringing the system close to a value of the parameter where there would be an equilibrium critical point. As one comes close to this critical point, adiabaticity is inevitably broken. Kibble and Zurek \cite{1,13,14} argued that in the critical region the dynamics reflects universal features leading to scaling of various quantities. These arguments are based on rather drastic approximations, and for strongly coupled systems there is no theoretical framework analogous to renormalization group which leads to such scaling. For two-dimensional theories which are suddenly quenched to a critical point, powerful techniques of boundary conformal field theory have been used in \cite{3} to show that ratios of relaxation times of one point functions, as well as the length/time scales associated with the behavior of two point functions of different operators, are given in terms of ratios of their conformal dimensions at the critical point, and hence universal.

In \cite{15} quench dynamics in the critical region of a finite chemical potential holographic critical point was studied in a probe approximation. The “phenomenological” model used was that of \cite{16} which involves a neutral scalar field with quartic self-coupling with a
mass-squared lying in the range $-9/4 < m^2 < -3/2$ in the background of a charged $AdS_4$ black brane. The self coupling is large so that the backreaction of the scalar dynamics on the background geometry can be ignored. The background Maxwell field gives rise to a nonzero chemical potential in the boundary field theory. In [16] it was shown that for low enough temperatures, this system undergoes a critical phase transition at a mass $m_c^2$. For $m^2 < m_c^2$ the scalar field condenses, in a manner similar to holographic superfluids [17–22]. The critical point at $m^2 = m_c^2$ is a standard mean field transition at any non-zero temperature, and becomes a Berezinski-Kosterlitz-Thouless transition at zero temperature, as in several other examples of quantum critical transitions. In [15] the critical point was probed by turning on a time dependent source for the dual operator, with the mass kept exactly at the critical value, i.e. a time dependent boundary value of one of the modes of the bulk scalar. The source asymptotes to constant values at early and late times, and crosses the critical point at zero source at some intermediate time. The rate of time variation $v$ is slow compared to the initial gap. As expected, adiabaticity fails as the equilibrium critical point at vanishing source is approached. However, it was shown that for any non-zero temperature and small enough $v$, the bulk solution in the critical region can be expanded in fractional powers of $v$. To lowest order in this expansion, the dynamics is dominated by a single mode - the zero mode of the linearized bulk equation, which appears exactly at $m^2 = m_c^2$. The resulting dynamics of this zero mode is in fact a dissipative Landau-Ginsburg dynamics with a dynamical critical exponent $z = 2$, and the order parameter was shown to obey Kibble-Zurek type scaling.

The work of [15] is at finite temperature - the dissipation in this model is of course due to the presence of a black hole horizon and is expected at any finite temperature. It is interesting to ask what happens at zero temperatures. It turns out that the model of [16] used in [15] becomes subtle at zero temperature. In this case, there is no conventional adiabatic expansion even away from the critical point (though there is a different low energy expansion, as in [23]). Furthermore, the susceptibility is finite at the transition, indicating there is no zero mode. While it should be possible to examine quantum quench in this model by numerical methods, we have not been able to get much analytic insight.

In this paper we study a different model of a quantum critical point, which is a variation of the model of insulator-superconductor transition of [24]. The model of [24] involves a charged scalar field minimally coupled to gravity with a negative cosmological constant and a Maxwell field. One of the spatial directions is compact with some radius $R$, and in addition one can have a non-zero temperature $T$ and a non-zero chemical potential $\mu$ corresponding to the boundary value of the Maxwell field. In the absence of the scalar field this model has a line of Hawking-Page type first order phase transitions in the $T-\mu$
plane which separates an (hot) AdS soliton and a (charged) black brane. Exactly on the $T = 0$ line, the two phases correspond to the AdS soliton with a constant Maxwell scalar potential, and an extremal black hole. In [24] it was shown that in the presence of a minimally coupled charged scalar, the phase diagram changes. When the charge is large the scalar and the gauge fields can be regarded as probe fields which do not affect the geometry. Now there is a phase with a trivial scalar and a phase with a scalar condensate. In the boundary theory the latter is a superfluid phase. This phase transition persists at zero temperature, where it separates an unbroken phase at low chemical potential and a broken phase - in both cases the background geometry is the AdS soliton, while the gauge field is non-trivial in the superfluid phase. The phase diagram is given in Figure 9 of [24].

The idea now is to probe the dynamics of this insulator-superfluid transition at zero temperature by turning on a time dependent source for the operator dual to the charged field. So long as the scalar is minimally coupled and the charge $q$ is large, this would involve analyzing a coupled set of equations of the scalar field and the gauge field. However, it turns out that a slight modification of the model allows us to ignore the backreaction of the scalar to the gauge field as well. This involves the introduction of a quartic self coupling of the scalar $\lambda$. Then in the regime $\lambda \gg q^2$ and $\lambda \gg \kappa^2$ (where $\kappa$ is the gravitational coupling), we can consider the dynamics of the charged scalar in isolation.

In this work we first show that in this regime of the parameters the insulator-superfluid transition persists. Concretely, for a sufficiently small negative $m^2$, there is a critical value of the background chemical potential beyond which a nontrivial static solution for the scalar becomes thermodynamically favored. Note that unlike other models of holographic superconductors the trivial solution does not become dynamically unstable. Rather the non-trivial solution has lower energy. The transition is a standard mean field critical transition. The background geometry remains an AdS soliton and the background gauge potential remains a constant, which is the chemical potential $\mu$. At the transition, the linearized equation has a zero mode solution which is regular both at the boundary and at the tip.

We then turn on a time dependent boundary condition and find that the breakdown of adiabaticity for a small rate $v$ is characterized by exponents which are appropriate for a dynamical critical exponent $z = 2$. In a way quite similar to [15] we find that in the critical region there is a new small $v$ expansion in fractional powers of $v$, and the dynamics is once again dominated by a zero mode. The real and imaginary parts of the zero mode now satisfy a coupled set of Landau-Ginsburg type equation with first order time derivatives. However the resulting system is oscillatory rather than dissipative - this
is expected since the background geometry has no horizon so that we have is a closed system. The order parameter is shown to obey a Kibble-Zurek type scaling. Finally we solve the bulk equations numerically and verify the scaling property obtained from the above small-$v$ expansion.

Thermal quench in holographic superfluids with backreaction has been recently studied in [25]. This work addresses a different issue - here the quench is applied to the system in the ordered phase away from the critical point and the resulting late time relaxation of the order parameter is studied. Our emphasis is on probing a possible Kibble-Zurek scaling when the quench crosses the critical point.

In Section 2 we define the model and discuss its equilibrium phases. In Section 3 we study quantum quench in this model by turning on a time dependent source, discuss the breakdown of adiabaticity and show that the critical region dynamics is dominated by the zero mode, leading to scaling behavior. In Section 4 we present the results of a numerical solution of the equations, verifying the scaling behavior. In an appendix we discuss a Landau-Ginsburg model similar to the critical dynamics of our holographic model.

## 2 The model and equilibrium phases

The “phenomenological” holographic model we consider is a slight variation of the model of [24]. The bulk action in $(d+2)$-dimensions is

$$S = \int d^{d+2}x \sqrt{g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{d(d+1)}{L^2} \right) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{\lambda} \left( \left| \nabla_\mu \Phi - iq A_\mu \Phi \right|^2 - m^2 |\Phi|^2 - \frac{1}{2} |\Phi|^4 \right) \right],$$

where $\Phi$ is a complex scalar field and $A_\mu$ is an abelian gauge field, and the other notations are standard. Henceforth we will use $L = 1$ units.

One of the spatial directions, which we will denote by $\theta$ will be considered to be compact. We will consider the regime

$$\lambda \gg q^2, \quad \lambda \gg \kappa^2.$$  \hspace{1cm} (2.2)

In this regime the scalar field is a probe field, and its backreaction to both the metric and the gauge field can be ignored.

### 2.1 The background

The background metric and the gauge field can be then obtained by solving the Einstein-Maxwell equations with the appropriate periodicity condition on $\theta$. It is well known that
there are two possible solutions. The first is the $AdS_{d+2}$ soliton,

\begin{align}
    ds^2 &= \frac{dr^2}{r^2 f_{sl}(r)} + r^2 \left( -dt^2 + \sum_{i=1}^{d-1} dx_i^2 \right) + r^2 f_{sl}(r) d\theta^2, \\
    f_{sl}(r) &= 1 - \left( \frac{r_0}{r} \right)^{d+1}, \\
    A_t &= \mu, \tag{2.3}
\end{align}

with constant parameters $\mu$ and $r_0$. The periodicity of $\theta$ in this solution is

\begin{equation}
    \theta \sim \theta + \frac{4\pi}{(d+1)r_0}, \tag{2.4}
\end{equation}

while the temperature can be arbitrary. The second solution is a $AdS_{d+2}$ charged black hole

\begin{align}
    ds^2 &= -r^2 f_{bh}(r) dt^2 + \frac{dr^2}{r^2 f_{bh}(r)} + r^2 \left( \sum_{i=1}^{d-1} dx_i^2 + d\theta^2 \right), \\
    f_{bh}(r) &= 1 - \left[ 1 + \frac{d-1}{2d} \left( \frac{\mu}{r_+} \right)^2 \right] \left( \frac{r}{r_+} \right)^{d+1} + \frac{d-1}{2d} \left( \frac{\mu}{r_+} \right)^2 \left( \frac{r}{r_+} \right)^{2d}, \\
    A_t &= \mu \left[ 1 - \left( \frac{r_+}{r} \right)^{d-1} \right]. \tag{2.5}
\end{align}

The temperature of this black brane is

\begin{equation}
    T = \frac{r_+}{4\pi} \left[ d + 1 - \frac{(d-1)^2}{2d} \left( \frac{\mu}{r_+} \right)^2 \right], \tag{2.6}
\end{equation}

while the period of $\theta$ is arbitrary. As shown in [24], this system undergoes a phase transition between these two solutions when

\begin{equation}
    r_0^{d+1} = r_+^{d+1} \left[ 1 + \frac{d-1}{2d} \left( \frac{\mu}{r_+} \right)^2 \right]. \tag{2.7}
\end{equation}

The AdS soliton is stable when the temperature and the chemical potential are small. At $T = 0$ the transition happens at a critical chemical potential $\mu_{c2}$ given by

\begin{equation}
    \mu_{c2} = \frac{r_0 (d+1)(2d) \frac{d-1}{2(d+1)}}{(d-1) \pi^2 (d+1)^{1/2}}. \tag{2.8}
\end{equation}
2.2 Scalar condensate

Consider now the scalar wave equation in the AdS soliton background \([2.3]\). We first rescale
\[
    r \rightarrow \frac{r}{r_0}, \quad t \rightarrow t r_0, \quad \mu \rightarrow \frac{\mu}{r_0}.
\]
In the rest of the paper we will use these rescaled coordinates (i.e., \(r_0 = 1\) and chemical potentials.

For fields which depend only on \(t\) and \(r\), the equation of motion is given by
\[
    \left[ -\frac{1}{r^2} (\partial_t - i \mu)^2 + \frac{1}{r^d} \partial_r \left( r^{d+2} f_{sl}(r) \partial_r \right) \right] \Phi - m^2 \Phi - \Phi |\Phi|^2 = 0 .
\]
In this paper we will consider \(-\frac{(d+1)^2}{4} < m^2 < -\frac{d(d-1)}{4}\). The asymptotic behavior of the solution at the AdS boundary \(r \to \infty\) is of the standard form
\[
    \Phi(r, t) = J(t) r^{-\Delta_-} \left[ 1 + O(1/r^2) \right] + A(t) r^{-\Delta_+} \left[ 1 + O(1/r^2) \right] + \cdots,
\]
where
\[
    \Delta_{\pm} = \frac{d+1}{2} \pm \sqrt{m^2 + \frac{(d+1)^2}{4}}.
\]
In “standard quantization” \(J(t)\) is the source, while the expectation value of the dual operator is given by
\[
    \langle \mathcal{O} \rangle = A(t) .
\]
In “alternative quantization” the role of \(J(t)\) and \(A(t)\) are interchanged. In this mass range both \(\Delta_{\pm}\) are positive and both the solutions of the linear equation vanish at the boundary. Thus the nonlinear terms in the equation \([2.10]\) are subdominant - which is why the leading solution near the boundary is the same as those of the linear equation, as written above.

We need to find time independent solutions of the equation \([2.11]\). Because of gauge invariance, we need to specify a gauge to qualify what we mean by time independence. For the equilibrium solution we require the solution to be \textit{real} - this fixes the gauge. Note that the tip of the soliton is locally two-dimensional flat space. Therefore we need to require the solution to be regular at the tip \(r = 1\). This leads to the following boundary condition at \(r = 1\)
\[
    \Phi(r) = \Phi_h + \Phi_h'(r - 1) + \cdots,
\]
where regularity requires
\[
    \Phi_h' = \frac{1}{(d+1)} \Phi_h (\Phi_h^2 + m^2) + \frac{1}{(d+1)} \Phi_h \mu^2 .
\]
To examine the phase structure we need to find time independent solutions with a vanishing source.

Clearly \( \Phi = 0 \) is always a solution. We have solved the equations numerically and found that there is a critical value of the chemical potential \( \mu_c \) beyond which there is another solution with a non-trivial \( r \) dependence which is thermodynamically preferred. This means that for \( \mu > \mu_c \), the operator dual to the bulk scalar has a vacuum expectation value, i.e., the global \( U(1) \) symmetry of the boundary theory is spontaneously broken. Although this could happen both in the standard and alternative quantizations, we need to check the critical value is less that that of the phase transition between the AdS soliton and AdS black hole: \( \mu_c < \mu_{c2} \). Otherwise, the scalar condensate phase is not available on the AdS soliton.

Figure 1 shows the behavior of the expectation value \( \langle O \rangle \) for \( m^2 = -15/4 \) for standard and quantization. We are plotting the condensation with respect to \( \mu q \) and the phase transition happens at \( \mu_c q \sim 1.89 \), which means the critical chemical potential is very small of order \( O(1/q) \) in the probe limit. It follows from (2.8) that \( \mu_c \) is always much smaller than \( \mu_{c2} \sim 1.86 \) and there exists a scalar condensate phase on the AdS soliton. Similarly for any given \( m^2 \), \( \mu = \mu_c \) is a critical point by letting \( q \) be large enough.

This transition was first found in [24] for a minimally coupled complex scalar - in this case the backreaction to the gauge field cannot be ignored, and the result follows from an analysis of the coupled set of equations for the scalar and the gauge field. In this case the gauge field introduces non-linearity in the problem which is necessary for condensation of the scalar. What we found is that a self-coupling does the same same job.
2.3 The zero mode at the critical point

To get some insight into this transition it is useful to write the equation (2.10) as a Schrödinger problem. First define a new coordinate

$$\rho(r) = \int_r^\infty \frac{ds}{s^2 f_{sl}^{1/2}(s)}, \quad (2.16)$$

which is the “tortoise coordinate” for the AdS soliton. $\rho(r)$ is a monotonic function of $r$ with the behavior

$$\rho \sim \frac{1}{r}, \quad r \to \infty,$$

$$\rho \to \rho_\star + \frac{2\sqrt{r-1}}{\sqrt{d+1}}, \quad r \to 1.$$  \quad (2.17)

For example, for $d = 3$ (asymptotically $AdS_5$ spacetime) soliton $\rho_\star = 1.311$. Let us now redefine the field by

$$\Phi(r, t) = \frac{1}{|r(\rho)|^{\frac{d-2}{2}}} \left( \frac{d\rho}{dr} \right)^{1/2} \Psi(\rho, t). \quad (2.18)$$

Then $\Psi(\rho, t)$ satisfies the equation

$$\left[ -\partial_t^2 + 2i\mu \partial_t - \frac{r^{2-d}}{\sqrt{f_{sl}(r)}} |\Psi|^2 \right] \Psi = \mathcal{P} \Psi - \mu^2 \Psi + \frac{r^{2-d}}{\sqrt{f_{sl}(r)}} |\Psi|^2 \Psi. \quad (2.19)$$

The operator $\mathcal{P}$ is

$$\mathcal{P} = -\partial_\rho^2 + V_0(\rho),$$

$$V_0(\rho) = \frac{m^2 r^2 + 4d(d+2)r^{2d+2} - 4d(d+3)r^{d+1} - (d-1)^2}{16r^{d-1}(r^{d+1} - 1)}, \quad (2.20)$$

where $r$ has to be expressed as a function of $\rho$ using (2.16).

The potential $V_0(\rho)$ has the following behavior near the boundary and the tip

$$V_0(\rho) = \frac{m^2 + \frac{d(d+2)}{4}}{\rho^2} + O(\rho^2), \quad \rho \to 0,$$

$$V_0(\rho) = -\frac{1}{4(\rho_\star - \rho)^2} + O(1), \quad \rho \to \rho_\star. \quad (2.21)$$

The behavior at the boundary $\rho = 0$ is of course the same as in pure $AdS_{d+2}$. The behavior near the tip $\rho = \rho_\star$ is in fact the correct behavior expected from a flat two dimensional space. Near the tip of the soliton the space becomes $\mathbb{R}^2 \times \mathbb{R}^{d-1}$ with $y \equiv (\rho_\star - \rho)$ playing
the role of a radial variable and $\theta$ playing the role of the polar angle. Indeed with the redefined field
\[ \tilde{\Psi}(y) = \frac{\Psi(\rho)}{\sqrt{\rho_* - \rho}}, \tag{2.22} \]
the operator $\mathcal{P}$ becomes, near $y = 0$, the zero angular momentum Laplacian in two dimensions
\[ \mathcal{P} \rightarrow -\frac{1}{y} \partial_y (y \partial_y) = - (\nabla^2)_0 \text{ constant} \tag{2.23} \]
In fact the eigenvalues of the operator $\mathcal{P}$ which acts on $\tilde{\Psi}$
\[ \mathcal{P} = -(\nabla^2)_0 + V_1(y), \quad \left( V_1(y) \equiv V_0(y) + \frac{1}{4y^2} \right), \tag{2.24} \]
are all positive. For $d = 3$ the proof is the following. Let us rewrite the potential $V_1(y)$ as follows
\[ V_1(y) = (m^2 + \frac{15}{4})r^2 + V_2(y) \tag{2.25} \]
where
\[ V_2(y) = \frac{1}{4} \left[ \frac{1 + 3r^4}{r^2 - r^6} + \frac{1}{y(r)^2} \right] \tag{2.26} \]
The term $V_2(y)$ is explicitly positive for all $r$. This may be seen as follows. The condition for positivity of $V_2(y)$ is
\[ \sqrt{r^6 - r^2} - y(r) \geq 0 \tag{2.27} \]
The inequality is saturated for $y = 0$ ($r = 1$). Furthermore the first derivative of the left hand side becomes
\[ \frac{1}{\sqrt{r^4 - 1}} \left[ \frac{3r^8 + 6r^4 - 1}{(1 + 3r^4)^{3/2}} - 1 \right] \tag{2.28} \]
This can be explicitly checked to be positive for all $r > 1$ (e.g. by squaring the expression). Therefore $V_2(y) \geq 0$ for all $r > 1$. The first term in $V_1(y)$ in (2.25) is the asymptotic potential in $AdS_5$ - when $m^2 + \frac{15}{4} > -\frac{1}{4}$ (which is the BF bound), this potential does not have any bound state. Since $V_2(y)$ differs from this asymptotic potential by a positive function, the full potential $V_1(y)$ does not have any bound state.

To look for a condensate in standard quantization, we need to find time independent solutions of the equation (2.19) which satisfy the boundary condition $J = 0$ at $\rho = 0$ and is regular at the tip $\rho = \rho_*$. With these boundary conditions the operator $\mathcal{P}$ has a discrete and positive spectrum. This means that for sufficiently large $\mu$ the operator
\[ \mathcal{D} \equiv \mathcal{P} - \mu^2, \tag{2.29} \]
will have a negative eigenvalue. This is what we found numerically.
At the critical value $\mu = \mu_{c1}$ the operator $D$ has a zero eigenvalue, i.e. a zero mode which satisfies the appropriate boundary conditions both at the tip and at the boundary. This zero mode will play a key role in the following.

Note that even though the operator $D$ has negative eigenvalues in the condensed phase, the trivial solution does not become unstable. This is clear from (2.19) and from the fact the spectrum of $\mathcal{P}$ is positive, which shows that the frequencies of the solutions to the linearized equation are all real.

Following the arguments of [16] it can be easily checked that the transition is standard mean field. This means that

$$
\langle O \rangle_{J=0} \sim \sqrt{|\mu_{c1} - \mu|},
$$
$$
\langle O \rangle_{\mu=\mu_{c1}} \sim |J|^{1/3}.
$$

(2.30)

We expect that this transition extends to non-zero temperature, though we have not checked this explicitly.

### 3 Quantum quench with a time dependent source

We will now probe the quantum critical point by quantum quench with a time dependent homogeneous source $J(t)$ for the dual operator $O$, with the chemical potential tuned to $\mu = \mu_{c1}$. The function $J(t)$ will be chosen to asymptote to constants at early and late times, e.g.

$$
J(t) = J_0 \tanh(vt).
$$

(3.31)

Note that we are using units with $r_0 = 1$. The system then crosses the equilibrium critical point at time $t = 0$. The idea is to start at some early time with initial conditions provided by the *instantaneous solution* and calculate the one point function $\langle O(t) \rangle$. In standard quantization this means that we impose a time dependent boundary condition as in (2.11) and calculate $A(t)$. In alternative quantization the source should equal $A(t)$. In this paper we discuss the problem in standard quantization: the treatment in alternative quantization is similar.

#### 3.1 Breakdown of adiabaticity

With a $J(t)$ of the form described above (e.g. (3.31)), one would expect that the initial time evolution is adiabatic for small $v$ so long as $J_0$ is not too small. As one approaches $t = 0$ adiabaticity inevitably breaks down and the system gets excited. In this subsection we determine the manner in which this happens.
An adiabatic solution of (2.19) is of the form

\[ \Psi(\rho, t) = \Psi^{(0)}(\rho, J(t)) + \epsilon \Psi^{(1)}(\rho, t) + \epsilon^2 \Psi^{(2)} + \cdots , \]  

(3.32)

where \( \epsilon \) is the adiabaticity parameter. The leading term is the instantaneous solution of (2.19), which is (using the definition (2.29))

\[ D\Psi^{(0)} + G(\rho) |\Psi^{(0)}|^2 \Psi^{(0)} = 0 , \]  

(3.33)

satisfying the required boundary condition. Here we have defined

\[ G(\rho) \equiv \frac{r^{2-d}}{\sqrt{J_{sl}(r)}} . \]  

(3.34)

From (2.30) we know that for a real \( J(t) \), this is real and has a form

\[ \Psi^{(0)} \sim \rho^\alpha J(t) \left[ 1 + O(\rho^2) \right] + \rho^{1-\alpha} [J(t)]^{1/3} \left[ 1 + O(\rho^2) \right] , \]  

(3.35)

where

\[ \alpha \equiv \Delta_+ - d/2 . \]  

(3.36)

This follows from the equations (2.17), (2.18) and (2.11). The adiabatic expansion now proceeds by replacing \( \partial_t \rightarrow \epsilon \partial_t \) in (2.19) substituting (3.32) and equating terms order by order in \( \epsilon \). The \( n \)-th order contribution \( \Psi^{(n)} \) satisfies a linear, inhomogeneous ordinary differential equation with a source term which depends on the previous order solution \( \Psi^{(n-1)} \). To lowest order we have the following equations for the real and imaginary parts of \( \Psi^{(1)} \)

\[ [D + 3G(\rho)(\Psi^{(0)})^2] (\text{Re } \Psi^{(1)}) = 0 , \]

\[ [D + G(\rho)(\Psi^{(0)})^2] (\text{Im } \Psi^{(1)}) = 2\mu \partial_t \Psi^{(0)} . \]  

(3.37)

Note that in these equations the time dependence of \( J(t) \) should be ignored. The full function \( \Psi \) must satisfy the boundary condition \( \lim_{\rho \to 0} [\rho^{-\alpha} \Psi(\rho, t)] = J(t) \). This means that the adiabatic corrections must start with the subleading terms, \( \Psi^{(1)} \sim \rho^{1-\alpha} \) as \( \rho \rightarrow 0 \) and has to be regular as \( \rho \rightarrow \rho_\star \). These provide the boundary conditions for solving the equations (3.37). Consider first the equation for \( \text{Im } \Psi^{(1)} \). Since the time dependence of \( \Psi^{(0)} \) is entirely through \( J(t) \) the solution may be written as

\[ \text{Im } \Psi^{(1)}(\rho, t) = 2\mu \dot{J}(t) \int_0^\rho \rho' G(\rho, \rho') \frac{\partial \Psi^{(0)}}{\partial J(t)}(\rho', J(t)) , \]  

(3.38)

(3.38)
where $G(\rho, \rho')$ is the Green’s function for the operator $D + G(\rho)(\Psi(0))^2$,

$$G(\rho, \rho') = \frac{1}{W(\psi_1, \psi_2)} \psi_1(\rho') \psi_2(\rho) , \quad \rho < \rho' ,$$  \hspace{1cm} (3.39)

$$= \frac{1}{W(\psi_1, \psi_2)} \psi_2(\rho') \psi_1(\rho) , \quad \rho > \rho' ,$$  \hspace{1cm} (3.40)

where $\psi_1$ and $\psi_2$ are solutions of the homogeneous equation $[D + G(\rho)(\Psi(0))^2] \psi_{1,2} = 0$ which satisfy the appropriate boundary conditions at the tip $\rho = \rho_*$ and at the boundary $\rho = 0$ respectively. The Wronskian $W(\psi_1, \psi_2)$ for this operator is clearly constant and is conveniently evaluated near the tip. Near $\rho = \rho_*$ these solutions behave as

$$\psi_1 \sim C \sqrt{\rho_* - \rho} , \quad \psi_2 \sim A \sqrt{\rho_* - \rho} + B \sqrt{\rho_* - \rho} \log(\rho_* - \rho) ,$$  \hspace{1cm} (3.41)

where $A, B, C$ are constants which depends on $J(t)^5$.

Thus the Wronskian is

$$W(\psi_1, \psi_2) = -BC .$$  \hspace{1cm} (3.42)

As noted in the previous section, the operator $D$ has a zero mode, i.e. $[D + G(\rho)(\Psi(0))^2]$ has a zero mode when $\Psi(0) = 0$, i.e. exactly at the equilibrium critical point. Thus, at this point we must have $B = 0$. This is why the first adiabatic correction $\text{Im} \Psi(1)(\rho, t)$ diverges at this point. For small $J(t)$ we can use perturbation theory to estimate the value of $B$. For small $J$ the zeroth order solution $\Psi(0)$ behaves as $J^{1/3}$ (the first term in (3.35) is subdominant). This is explicit to all orders in the expansion of the solution around the boundary. However, this is also justified by the results of the next section where we show that in the critical region the dynamics is dominated by a zero mode. The coefficient of the zero mode can be seen to be proportional to $J^{1/3}$ using a regularity argument similar to that in [16] so that the additional term in the operator behaves as $G(\rho)(\Psi(0))^2 \sim [J(t)]^{2/3}$. This yields $B \sim J^{2/3}$ as well. Thus the Green’s function which appears in (3.38) behaves as $J^{-2/3}$ so that the correction behaves as

$$\text{Im} \Psi(1) \sim \frac{\dot{J}(t)}{J^{2/3}} \frac{\partial \Psi(0)}{\partial J(t)} \sim \frac{\dot{J}(t)}{J^{4/3}} .$$  \hspace{1cm} (3.43)

The same argument shows that $\text{Re} \Psi(1) = 0$, so that $|\Psi(1)| \sim \frac{j}{j^{4/3}}$ as well. Therefore adiabaticity breaks when

$$|\Psi(1)| \sim |\Psi(0)| \Rightarrow \dot{J}(t) \sim J^{5/3} .$$  \hspace{1cm} (3.44)

\[5\text{Note that in the equations (3.37) the time is simply a parameter.}\]
For sources which vanish linearly at $t = 0$, i.e. \( J(t) \sim vt \) (e.g. of the form (3.31)) this means that if the source is turned on at some early time, adiabaticity breaks at a time

\[
t_{\text{adia}} \sim v^{-2/5}.
\]

while at this time the value of the order parameter \( \langle O \rangle \) is

\[
\langle O(t_{\text{adia}}) \rangle \sim [J(t_{\text{adia}})]^{1/3} = [vt_{\text{adia}}]^{1/3} \sim v^{1/5}.
\]

With the usual adiabatic-diabatic assumption, these exponents lead to Kibble-Zurek scaling for a dynamical critical exponent \( z = 2 \), even though the underlying dynamics is relativistic and non-dissipative. From the above analysis it is clear that this happened because the leading adiabatic correction is provided by the chemical potential term, which multiplies a first order time derivative of the bulk field.

### 3.2 Critical dynamics of the order parameter

The breakdown of adiabaticity means that an expansion in time derivatives fail. In this subsection we show, following closely the treatment of [15], that we now have a different small \( v \) expansion in fractional powers of \( v \) during the period when the sources passes through zero. This will lead to a scaling form of the order parameter in the critical region. In the following we will demonstrate this for the case where \( J(t) \sim vt \) near \( t \approx 0 \). However the treatment can be easily generalized to a \( J(t) \sim (vt)^n \) for any integer \( n \).

To establish this, it is convenient to separate out the source term in the field \( \Psi(\rho,t) \),

\[
\Psi(\rho,t) = \rho^\alpha J(t) + \Psi_s(\rho,t), \quad \alpha = \Delta - d/2,
\]

where we have used the relation (2.18) and the fact that near the boundary \( \rho \sim 1/r \). The equation of motion (2.19) then becomes

\[
- \partial_t^2 \Psi_s + 2i\mu \partial_t \Psi_s = (\mathcal{D} \rho^\alpha) J(t) + \mathcal{D} \Psi_s + G(\rho) \left[ \rho^{3\alpha} [J(t)]^3 + \rho^{2\alpha} [J(t)]^2 (2\Psi_s + \Psi_s^*) \right]
\]

\[
+ G(\rho) \left[ \rho^\alpha J(t) (2|\Psi_s|^2 + \Psi_s^2) + |\Psi_s|^2 \Psi_s \right]
\]

\[
+ \rho^\alpha [\partial_t^2 J - 2i\mu \partial_t J].
\]

This separation is useful because we know that in the presence of a constant source \( J(t) = \bar{J} \), the static solution has the asymptotic form

\[
\Psi_s \sim \rho^{1-\alpha} \left[ |\bar{J}|^{1/3} + O(\rho^2) \right] + \bar{J} \rho^\alpha [1 + O(\rho^2)],
\]

which follows from (2.30).
The scaling relations (3.45) and (3.46) suggest that we perform the following rescaling of the time and the field

\[ t = v^{-2/5} \eta, \quad \Psi_s = v^{1/5} \chi. \]  

(3.50)

In the critical region we can now use \( J(t) = vt = v^{3/5} \eta \) and rewrite (3.48) as an expansion in powers of \( v^{2/5} \),

\[ D\chi = v^{2/5} \left[ 2i\mu \partial_\eta \chi - G(\rho) |\chi|^2 \chi - \eta (D^\alpha) \right] + O(v^{4/5}). \]  

(3.51)

As noted above, because of the boundary condition at \( \rho = 0 \) and the regularity condition at \( \rho = \rho_\star \) the spectrum of \( D \) is discrete. Let \( \varphi_n \) be the orthonormal set of eigenfunctions of the operator \( D \)

\[ D\varphi_n(\rho) = \lambda_n \varphi_n(\rho), \quad n = 0, 1, \ldots, \]  

(3.52)

with \( \lambda_0 = 0 \). \( \varphi_0(\rho) \) is the zero mode which we discussed earlier. Since \( \mu \) has been tuned to be equal to \( \mu_{c1} \), all the higher eigenvalues are positive.

We now expand

\[ \chi(\rho, \eta) = \sum_n \chi_n(\eta) \varphi_n(\rho), \]  

(3.53)

and rewrite the equation (3.51) in terms of the modes \( \chi_n(\eta) \)

\[ \lambda_n \chi_n = v^{2/5} \left[ 2i\mu (\partial_\eta \chi_n) - \sum_{n_1n_2n_3} C_{n_1n_2n_3}^{n} \chi_{n_3}^* \chi_{n_2} \chi_{n_1} + \mathcal{J}_n \eta \right] + O(v^{4/5}), \]  

(3.54)

where we have defined

\[ \mathcal{J}_n = \int d\rho \varphi_n^*(\rho) (D^\alpha), \]

\[ C_{n_1n_2n_3}^{n} = \int d\rho \varphi_{n_1}^*(\rho) \varphi_{n_2}^*(\rho) \varphi_{n_3}(\rho) G(\rho). \]  

(3.55)

It is clear from (3.54) that the zero mode part of the bulk field dominates the dynamics in the critical region. In fact for small \( v \) a solution is of the form

\[ \chi_n(\eta) = \delta_{n0} \xi_0(\eta) + v^{2/5} \xi_n + O(v^{4/5}). \]  

(3.56)

The zero mode satisfies a \( z = 2 \) Landau-Ginsburg equation

\[ -2i\mu \partial_\eta \xi_0 + C_{0000}^0 |\xi_0|^2 \xi_0 + \mathcal{J}_0 \eta = 0. \]  

(3.57)

Reverting back to the original variables we therefore have

\[ \Psi_s(\rho, t, v) = v^{1/5} \Psi_s(\rho, tv^{2/5}, 1), \]  

(3.58)
which implies a Kibble-Zurek scaling for the order parameter with $z = 2$

$$\langle O(t,v) \rangle = v^{1/5} \langle O(v^{2/5}t,1) \rangle .$$  \hspace{1cm} (3.59)

Note that the effective Landau-Ginsburg equation (3.57) is not dissipative because the first order time derivative is multiplied by a purely imaginary constant. In fact, in the absence of a source term the quantity $\frac{1}{2}(|\xi_0|^2)^2$ is independent of time.

Beyond the critical region, we cannot use the approximation $J(t) \sim vt$ and there is no useful simplification in terms of the zero mode. However, the boundary conditions at the tip are perfectly reflecting boundary conditions (as appropriate for the origin of polar coordinates in two dimensions) so that there is a conserved energy in the problem. This is in contrast to a black hole background where there is a net ingoing flux at the horizon causing the system to be dissipative. Indeed in the quench problem considered in [15] arguments similar to those used in this section also led to an effective Landau-Ginsburg dynamics with $z = 2$, but which is dissipative.

In the appendix we analyze a Landau-Ginsburg toy model motivated by the results of this section.

4 Numerical results

In this section we summarize our numerical results. We have solved the bulk equation of motion numerically for $d = 3$. The results for different values of $m^2$ are similar. We present detailed results for $m^2 = -15/4$. In this case the critical value of the chemical potential is $\mu_c \approx 1.88$.

We discretize the partial differential equations (PDEs) (2.19) (written in the $y$-coordinate) in a radial Chebyshev grid to study the numerical problem. Once discretized in radial direction, the PDEs become a series of ordinary differential equations (ODEs) in the temporal variable. The resulting ODEs are solved with a standard ODE solver (e.g. CVODE). The time dependence is chosen to be of the form dependent source as in (3.31). In principle one may study with any kind of time dependent source.

We will consider the problem in two regimes. The first is the “slow” regime where we expect our analytic arguments to be accurate, the other is a “fast” regime where there is no adiabatic region whatsoever. In the slow regime we will try to zoom on the scaling region around the phase transition. In the fast regime we will find a large deviation from the adiabatic behavior and possible chaotic behavior.
4.1 Slow regime

Since our main interest is quench through the critical point, we concentrate mainly near the phase transition. We choose $\mu q = \mu_c q (\approx 1.88)$, so that the system is critical in the absence of any source. In the presence of a time dependent source of the form (3.31) we calculate the bulk field $\tilde{\Psi}(t)$ and extract from this the value of $\langle O(t) \rangle$ of the dual field theory. A typical plot of the real part of $\langle O(t) \rangle$ for slow quench through the phase transition is presented in Figure 2.

![Figure 2: The plot of Re $\langle O(t) \rangle$ with $v = 0.02$.](image)

Clearly the late time behavior is oscillatory, reflecting the fact that we are dealing with a closed and non-dissipative system.

We then zoom on the critical region near $t = 0$ for various value of $v$ to look for any scaling behavior. One way to look for this is to consider the behavior of $\langle O \rangle$ at $t = 0$. Equation (3.59) then predicts a scaling behavior $\langle O(0) \rangle \sim v^{1/5}$.

Figure 3 shows a plot of log(Re $\langle O(0) \rangle$) for different $v$. We fit the data points with a function $f(x) = A + Bx + C/x$, where $x$ is log($v$). Here we kept a sublinear ($O(1/x)$) term to understand how the fit function approaches a linear regime. From our analytic argument we expect $B = 1/5$. A fit of the numerical results yields $f(x) = 0.794 - 0.490/x + 0.206x$. Changing the number of fit points and range changes the values of fit parameters a bit, however we always get a value of $B$ which is close to 1/5 with only a few percentage deviation. The imaginary part (Im $\langle O(0) \rangle$) also satisfies the same scaling.

4.2 Fast regime

In the fast regime we see a large deviation from the adiabatic behavior, as shown in Figure 4.
Figure 3: The plot of $\log(\text{Re} \langle \mathcal{O}(0) \rangle)$ vs $\log(v)$. We also plotted the closest fit (see text).

Figure 4: Plot of $\text{Re} \langle \mathcal{O}(t) \rangle$ showing chaotic behavior with a large value of $v = 2.0$ and $\mu = 1$.

The motion in this regime becomes possibly chaotic. Here we have a system with a conserved energy. Once we put some energy in the system, the non-linearity possibly takes the system over the whole phase space (Arnold diffusion). It is expected that if we wait long enough the probe approximation actually breaks down \cite{26} and we have to consider the fully backreacted problem. We plan to attack this problem in the near future.

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A Adiabatic and scaling analysis of a toy model

In this appendix, we consider a (0 + 1)-dimensional toy model which follows the equation

$$2i\mu \dot{\phi} + (m^2 - \mu^2)\phi + \phi|\phi|^2 = J(t).$$  \hspace{1cm} (A.60)

The function $J(t)$ asymptotes to constants at early and late times and passes through zero in a linear fashion at some intermediate time, e.g.

$$J(t) = J_0 \tanh(vt).$$  \hspace{1cm} (A.61)

A.1 Adiabaticity

We first derive conditions for breakdown of adiabaticity near the critical point $m^2 = \mu^2$ and $J(t) = 0$. We carry out adiabatic expansion as following:

$$\partial_t \rightarrow \epsilon \partial_t, \quad \phi \rightarrow \phi_0(J(t)) + \epsilon \phi_1(t) + \cdots,$$  \hspace{1cm} (A.62)

where $\phi_0(J(t))$ is the (real) adiabatic solution given by

$$\phi_0(J(t)) = [J(t)]^{1/3}.$$  \hspace{1cm} (A.63)

The solution to $O(\epsilon^2)$ is,

$$\phi = \phi_0[J(t)] + \epsilon i \frac{2\mu}{\phi_0^2} \dot{\phi}_0 + \epsilon^2 \frac{1}{3\phi_0^2} \left(8\mu^2 \left(\frac{\ddot{\phi}_0}{\phi_0^3} - \frac{\dddot{\phi}_0}{2\phi_0^2}\right) - \frac{4\mu^2}{\phi_0^3} \dot{\phi}_0^2\right) + O(\epsilon^3).$$  \hspace{1cm} (A.64)

where The breakdown of adiabaticity happens when,

$$\frac{2\mu}{\phi_0^2} \dot{\phi}_0 \sim \phi_0,$$  \hspace{1cm} (A.65)

$$\frac{1}{3\phi_0^2} \left(8\mu^2 \left(\frac{\ddot{\phi}_0}{\phi_0^3} - \frac{\dddot{\phi}_0}{2\phi_0^2}\right) - \frac{4\mu^2}{\phi_0^3} \dot{\phi}_0^2\right) \sim \phi_0.$$  \hspace{1cm} (A.66)

For $J(t) = \tanh(vt) \sim vt$ the above two equations translate into,

$$\mu \sim v^{5/3}v^{2/3}.$$  \hspace{1cm} (A.67)

Thus if $\mu$ is of $O(1)$ then the above equations give us,

$$t \sim v^{2/5}.$$  \hspace{1cm} (A.68)
A.2 Scaling behavior

Now sitting at the critical point we study the behavior of the scaling solution with $\mu = O(1)$. From the adiabatic analysis we expect scaled time, $t = v^{2/5} t$.

We write the field $\phi$ as $\chi + i\xi$ and the source as $J_R + iJ_{Im}$, where both $J_R$ and $J_{Im}$ go as $vt$. To find the scaling exponents we extract the $v$ dependencies as,

$$ t = v^\alpha \bar{t}, \quad \chi = v^\beta \bar{\chi}, \quad \xi = v^\gamma \bar{\xi}. \quad (A.69) $$

Consistency of the equations demand,

$$ \alpha = -\frac{2}{5}, \quad \beta = \frac{1}{5}, \quad \gamma = \frac{1}{5}. \quad (A.70) $$

This determines the scaling behavior of the field $\phi$ at $m^2 = \mu^2$ and with $\mu$ of $O(1)$ as,

$$ \phi(t, v) = v^{1/5} \phi(v^{2/5}t, 1). \quad (A.71) $$

This agrees with our expectation from adiabatic analysis, and also has been confirmed numerically in Section 4.

A.3 Late time behavior

At late times the source $J(t)$ can be treated to be a constant and the solution can be obtained by perturbing around the static solution, $\phi_{static} = J^{1/3}$. It is then straightforward to see that the solution $\phi(t)$ is oscillatory with a frequency $\omega = \frac{r^{2/3}}{\sqrt{2\mu}}$.

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