The Present Status on $\sigma$ and $\kappa$ Meson Properties

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The recent experimental data of both $\pi\pi/K\pi$ scattering and production processes, suggesting the existence of scalar $\sigma$ and $\kappa$ mesons, are reviewed. In many $\pi\pi/K\pi$ production processes the direct effects of their productions are observed, while they are, because of chiral symmetry, hidden in scattering processes, and now $\sigma(500\sim 600)$ and $\kappa(800\sim 900)$ are considered to be confirmed experimentally. The recent criticism on our method of analyses, which is based on the long believed prejudice of universal $\pi\pi/K\pi$ phase through scattering and production amplitudes, is explained not to be valid.

§1. Introduction

The iso-singlet scalar $\sigma$ meson was introduced theoretically in the linear $\sigma$ model 1), and its existence was first suggested in one-boson-exchange potential model 2) of nuclear forces. The importance of $\sigma$ was stressed 3), 4) in relation with dynamical chiral symmetry breaking of QCD. However, its existence had been neglected phenomenologically for many years, being based on the negative results 5) of $\pi\pi$ scattering phase shift analyses.

Recently the $\pi\pi$ phase shift 6) was reanalyzed by many groups 7) including ours 8) and the existence of light $\sigma(450\sim 600)$ was strongly suggested. The result of the previous analysis with no $\sigma$ existence was pointed out 9) to be not correct, since in this analysis 5) there is no consideration on the cancellation mechanism between $\sigma$ amplitude and non-resonant $\pi\pi$ amplitude, which is guaranteed by chiral symmetry.

On the other hand, it is remarkable that, in contrast with the spectra of $\pi\pi$ scattering, the clear peak structure has been observed in mass region of $m_{\pi\pi} \sim 500$ MeV in the various $\pi\pi$ production processes, such as $J/\psi \rightarrow \omega\pi\pi$, $pp \rightarrow 3\pi^0$, $D^+ \rightarrow \pi^+\pi^-\pi^-$ and $\tau^- \rightarrow \pi^-\pi^0\pi^0\nu$, and this structure is shown to be well reproduced by the Breit-Wigner amplitude of $\sigma$ meson.

Thus, presently firm evidences 17) of $\sigma$ seem to be accumulated, and the column of $\sigma$ in particle data group table is corrected as “$f_0(600)$ or $\sigma$” in the newest’02 edition in place of $f_0(400\sim 1200)$ or $\sigma$ in the ’96–’00 editions. There are now hot controversies on the existence of $I = 1/2$ scalar $\kappa$ meson, to be assigned as a member of $\sigma$ nonet. Reanalyses 19), 20) of $K\pi$ scattering phase shift 21) suggest existence of the $\kappa(900)$, while no $\kappa$ is insisted in ref. 22). The existence of $\kappa$ is again suggested strongly in $K\pi$ production process of $D^+ \rightarrow K^-\pi^+\pi^+$, similarly to the case of $\sigma$.

In the analyses of the $\pi\pi$ (or $K\pi$) production processes mentioned above, the amplitudes are parametrized by a coherent sum of the Breit-Wigner amplitudes including $\sigma$ (or $\kappa$) and the non-resonant $\pi\pi$ (or $K\pi$) production amplitude. This
parametrization method is called VMW method.

Recently there have been raised some criticisms\(^24\),\(^25\) on the VMW method, especially in relation to the consistency with \(\pi\pi\) (or \(K\pi\)) scattering phase shift. However, as will be clarified in §3, we emphasize that the production amplitude is, in principle, independent from the scattering amplitude and that the analyses of production processes should be done independently of scattering processes. Also it is shown that our method of analyses, VMW method, is consistent\(^26\) with all the constraints from unitarity and chiral symmetry.

In this paper we review both of the \(\pi\pi\)(\(K\pi\)) scattering and production processes relevant to \(\sigma\) meson (\(\kappa\) meson), and report the present status of \(\sigma\) and \(\kappa\) meson properties. The phase motion of the production amplitude is also examined, and the above criticisms on VMW method will be clarified not to be correct.

§2. Experimental Evidences for \(\sigma\) and \(\kappa\)

\((\pi\pi\text{ scattering})\) We first review our reanalysis\(^8\) of \(\pi\pi\) scattering phase shift \(\delta\) obtained by CERN-Munich.\(^6\) The \(\delta\) of \(I = 0\) \(S\) wave amplitude, \(\delta^{(0)}_S\), is fitted by Interfering Amplitude method, where the total \(\delta^{(0)}_S\) below \(m_{\pi\pi} \approx 1\)GeV is represented by the sum of the component phase shifts,

\[ \delta^{(0)}_S = \delta_\sigma + \delta_{BG} + \delta_{f_0}. \]  

(2.1)

The \(\delta_\sigma\) is from \(\sigma\) Breit-Wigner amplitude and \(\delta_{f_0}\) is from \(f_0(980)\) Breit-Wigner amplitude with narrow width. The \(\delta_{BG}\) is from non-resonant repulsive \(\pi\pi\) amplitude, and is taken phenomenologically of hard-core type, \(\delta_{BG} = -p_1 r_c\) \((p_1 = \sqrt{s/4 - m^2_{\pi\pi}}\) being the CM momentum of \(\pi\)).

The experimental \(\delta^{(0)}_S\) passes through 90\(^\circ\) at \(\sqrt{s}(=m_{\pi\pi}) \approx 900\)MeV. This is explained by the cancellation between attractive \(\delta_\sigma\) and repulsive \(\delta_{BG}\). The result of the fit is given in Fig. 1. The mass and width of \(\sigma\) is obtained as \(m_\sigma = 585 \pm 20\)MeV and \(\Gamma_\sigma = 385 \pm 70\)MeV.

Note that the above cancellation is shown\(^9\) to come from chiral symmetry in the linear \(\sigma\) model (L\(\sigma\)M): The \(\pi\pi\) scattering \(A(s,t,u)\) amplitude in L\(\sigma\)M is given by

\[ A(s,t,u) = \frac{(-2g_{\sigma\pi\pi})^2}{m^2_\sigma - s} - 2\lambda = \frac{s - m^2_\sigma}{f^2_\pi} + \frac{1}{f^2_\pi} \frac{(s - m^2_\sigma)^2}{m^2_\sigma - s}, \]  

(2.2)

as a sum of the \(\sigma\) amplitude \(A_\sigma\), which is strongly attractive, and of the non-resonant \(\pi\pi\) amplitude \(A_{\pi\pi}\) due to the \(\lambda\phi^4\) interaction, which is strongly repulsive. They cancel
with each other following the relation of $\sigma_{\pi\pi} = f_{\pi} \lambda = (m_{\pi}^2 - m_{\pi}^2)/(2f_{\pi})$, and the small $O(p^2)$ Tomozawa-Weinberg (TW) amplitude and its correction are left. The $A_{\sigma}(A_{\pi\pi})$ corresponds to $\delta_{\sigma}(f_{\pi})$.

Actually, as shown in Fig. 2, our theoretical curves for $\delta_{NR}^0$ and $\delta_{NR}^2$, obtained by unitarizing the respective Born amplitudes, given by $A_{\pi\pi}$, $A(t, s, u)$ and $A(u, t, s)$, in $\sigma_{\pi\pi}$ are consistent with our $\delta_{BG}$ of hard core type in our phase shift analysis, and with experimental $\delta_{0, 27}^2$ respectively.

Thus, it is shown that the $\sigma$ Breit-Wigner amplitude with non-derivative ($O(p^0)$) $\pi\pi$-coupling requires at the same time the strong ($O(p^1)$) repulsive $\pi\pi$ interaction to obtain the small $O(p^2)$ TW amplitude, satisfying chiral symmetry. This is the origin of $\delta_{BG}$ in our phase shift analysis.

There was an argument that a broad resonance with mass 1 GeV, denoted as $f_0(1000)$ or $\epsilon(900)$, instead of light $\sigma$, exists. We already investigated this possibility. The fit with $r_c = 0$ corresponds to the conventional analyses without the repulsive $\delta_{BG}^{Non.Res.}$. Actually the pole position of this fit, $\sqrt{s} = 970 - i320$, is very close to $\sqrt{s} = 1046 - i250(910 - i350)$ MeV of $f_0(1000)(\epsilon(900))$. The resulting $\chi^2$ is $\chi^2/N_F = 163.4/31$ (See Fig. 1). When we take into account the cancellation mechanism of chiral symmetry by including $\delta_{BG}^{Non.Res.}$, as was done in the present analysis, $\chi^2/N_F = 23.6/30$ is obtained. The greatly improved $\chi^2$ strongly suggests the $\sigma(600)$, rather than $f_0(1000)$.

($K\pi$ scattering) The similar cancellation is also expected to occur in $K\pi$ scattering since $K$ has also a property of Nambu-Goldstone boson. The $I = 1/2$ $S$ wave phase shift $\delta_S^{1/2}$ is parametrized by introducing the $\kappa$ Breit-Wigner phase shift $\delta_{\kappa}$ and its compensating repulsive non-resonant $K\pi$ phase shift $\delta_{BG}^{Non.Res.}$, as $\delta_S^{1/2} = \delta_{\kappa} + \delta_{BG}^{Non_Res.} + \delta_{\kappa}^{K\pi(1430)}$. The fit to $\delta_S^{1/2}$ below 1.6 GeV by LASS gives the mass and width of $\kappa$ meson as $m_{\kappa} = 905^{+65}_{-30}$ MeV and $\Gamma_{\kappa} = 545^{+235}_{-110}$ MeV.

($J/\psi \rightarrow \omega\pi\pi$) Following VMW method, the production amplitude is represented by a sum of Breit-Wigner amplitudes of $\sigma$ and $f_2(1275)$ and the non-resonant background. The $m_{\pi\pi}$ mass spectra clearly shows a peak structure, which is explained by a Breit-Wigner amplitude of $\sigma$ with $(m_{\sigma}, \Gamma_{\sigma}) = (482 \pm 3, 325 \pm 10)$ MeV. This is quite in contrast with the situation in the $m_{\pi\pi}$ spectra of $\pi\pi$ scattering, where the no direct $\sigma$ peak is observed because of the cancellation mechanism of chiral symmetry. The similar result is obtained by BES with $(m_{\sigma}, \Gamma_{\sigma}) = (390^{+60}_{-33}, 282^{+77}_{-50})$ MeV.

($T^m \rightarrow \Pi\pi\pi$) The $m_{\pi\pi}$ spectra of $\Upsilon(2S) \rightarrow \Upsilon(1S)\pi\pi$, $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi$, $\Upsilon(3S) \rightarrow \Upsilon(2S)\pi\pi$, $\psi(2S) \rightarrow J/\psi\pi\pi$, $J/\psi \rightarrow \phi\pi\pi$ and $\phi K\bar{K}$ are commonly fitted. The Present Status on $\sigma$ and $\kappa$ Meson Properties
by the amplitude by VMW method, which is a coherent sum of $\sigma$ Breit Wigner amplitude $F_\sigma$ and non-resonant $\pi\pi$ amplitude $F_{2\pi}$. The $(m_\sigma, \Gamma_\sigma) = (526 \pm 48, 301 \pm 145)$ is obtained with $\chi^2/N_F = 86.5/(150 - 37) = 0.77$. The double peak structure in $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$ is explained by the interference between the $F_{2\pi}$ with constant phase and the $F_\sigma$ with moving phase.

$$(p\bar{p} \to 3\pi^0)$$

The $\pi^0\pi^0$ mass distribution\(^{13}\) and the $\cos\theta$ distributions in $m_{\pi\pi}$ around $KK$ threshold and 1.5 GeV are fitted\(^{14}\) by the amplitude of a superposition of $\pi^0R$ amplitudes with $R = \sigma, f_0(980, 1300, 1500)$ and $f_2(1275, 1565)$. The total amplitude symmetrized for three identical $\pi^0$ shows a peak in $m_{\pi\pi} \sim 700\text{MeV}$, which is explained by the contribution from $\sigma$ Breit Wigner amplitude with $(m_\sigma, \Gamma_\sigma) = (540 -29, 385 + 64)$. The $m_{\pi\pi} \sim 500 \text{MeV}$ region, which is explained by $\sigma$ contribution with $(m_\sigma, \Gamma_\sigma) = (478 +24, 17, 324 +42, 30 \pm 21)\text{MeV}$. The $\sigma\pi^+\pi^-$ decay is a main mode in this channel, and its fraction is 46.3 $\pm$ 9.0 $\pm$ 2.1\%.

Similarly the Dalitz plot of $D^+ \to K^+\pi^+\pi^+$ is analyzed by VMW method and the existence of $\kappa(800)$ is strongly suggested with $(m_\kappa, \Gamma_\kappa) = (797 \pm 19 \pm 42, 410 \pm 43 \pm 85)\text{MeV}.\text{\(^{23}\)}} The $\kappa\pi^+$ decay is a main mode in this channel, and its fraction is 47.8 $\pm$ 12.1 $\pm$ 3.7\%, which is almost equal to the fraction of $\sigma\pi^+\pi^+$ in $D^+ \to \pi^+\pi^+\pi^+$. This fact suggests the $\sigma(500)$ and $\kappa(800)$ belong to the same nonet.\(^{30}\) The Dalitz plot of $D^+ \to K^-\pi^+\pi^+$ is analyzed by VMW method and the existence of $\kappa(800)$ is strongly suggested with $(m_\kappa, \Gamma_\kappa) = (797 \pm 19 \pm 42, 410 \pm 43 \pm 85)\text{MeV}.\text{\(^{23}\)}} The $\kappa\pi^+$ decay is a main mode in this channel, and its fraction is 47.8 $\pm$ 12.1 $\pm$ 3.7\%, which is almost equal to the fraction of $\sigma\pi^+\pi^+$ in $D^+ \to \pi^+\pi^+\pi^+$. This fact suggests the $\sigma(500)$ and $\kappa(800)$ belong to the same nonet.\(^{30}\) The $\pi^0\nu\pi^0\nu$ mass distributions show clear peak structures of $\sigma$, while $\pi^-\pi^0$ mass distributions show clear peak of $\rho$. Branching fractions of $\rho\pi^0\nu\pi^0\nu$ and $\pi^-\pi^0\nu\pi^0\nu$ are 60.19\% and 18.76 $\pm$ 4.29\%, respectively. The $(m_\sigma, \Gamma_\sigma) = (555, 540)\text{MeV}.

\section*{§3. Method of Analyses of $\pi\pi/K\pi$ Production Processes}

\textit{(Essence of VMW method)} The analyses of $\pi\pi/K\pi$ production processes quoted in the previous section are done following the VMW method. Here we explain our basic physical picture on strong interactions and the essential point of this method.

The strong interaction is a residual interaction of QCD among all color-neutral bound states of quarks($q$), anti-quarks($\bar{q}$) and gluons($g$). These states are denoted as $\phi_i$, and the strong interaction Hamiltonian $\mathcal{H}_{\text{strong}}$ is described by these $\phi_i$ fields. It should be noted that, from the quark physical picture,\(^{31}\) unstable particles as well as stable particles, if they are color-singlet bound states, should be equally treated as $\phi_i$-fields on the same footing.

$$\mathcal{H}_{\text{strong}} = \mathcal{H}_{\text{strong}}(\phi_i)$$

$$\{ \phi_i \} = \{ \text{color singlet bound states of } q, \bar{q} \text{ and } g \}. \quad (3\cdot1)$$
The time-evolution by $\mathcal{H}_{\text{strong}}(\phi_i)$ describes the generalized $S$-matrix. Here, it is to be noted that, if $\mathcal{H}_{\text{strong}}$ is hermitian, the unitarity of $S$ matrix is guaranteed.

The bases of generalized $S$-matrix are the configuration space of these multi-$\phi_i$ states.

$$ S \text{ matrix bases } \{ \text{multi-}$\phi_i$-states} = \left\{ |\omega\pi\pi⟩, |\omega\sigma⟩, |\omega f_2⟩, |b_1\pi⟩, \cdots, |J/ψ⟩, \\ |N\pi⟩, (|N\pi\pi⟩)_{\text{Non.Res.}}, |Δ⟩, |Δ\pi⟩, |Δ\sigma⟩, \cdots, \right\}, \quad (3.2) $$

where the states relevant for $J/ψ → \omega\pi\pi$ decay and $N\pi$ scattering are respectively shown in 1st and 2nd lines as examples. The states including unstable particles shown with underlines are equally treated with non-resonant states $|\omega\pi\pi⟩$ and $|N\pi⟩, |N\pi\pi⟩$.

The relevant $J/ψ → \omega\pi\pi$ decay process has the 3-body final state $\omega\pi\pi$. This process is described by a coherent sum of amplitudes for various 2-body decays, $J/ψ → \omega\sigma$, $J/ψ → \omega f_2(1275)$, $J/ψ → b_1(1235)\pi$, · · ·, and for a non-resonant 3-body($\omega\pi\pi$) decay. These respective decay amplitudes correspond to different non-diagonal elements of the generalized $S$-matrix, and have independent coupling strengths.

The $H_{\text{strong}}$ induces the various final state interaction, reducing to the strong phases of the corresponding amplitudes. (See Fig.3.)

The remaining problem is how to treat unstable particles, as there is no established field-theoretical method for this problem. In VMW method unstable particles are treated intuitively by the replacement of propagator,

$$ \frac{1}{m_σ^2 - s - iε} \xrightarrow{\text{Strong Int.}} \frac{1}{m_σ^2 - s - i m_σ Γ_{σ}(s)}, \quad (3.3) $$

where we take the case of $\sigma$ as an example.

![Fig. 3. The final state interactions in $J/ψ → \omega\pi\pi$. The amplitude is a superposition of different $S$ matrix elements, such as $J/ψ → \omega\sigma$, $J/ψ → \omega f_2$, $J/ψ → b_1\pi$, · · ·, $J/ψ → \omega(\pi\pi)_{\text{Non.Res.}}$. The ellipses represent the final state interactions, and the corresponding amplitudes have independent strong phases.](image)

The effective $\omega\pi\pi$ amplitude is given by a coherent sum of all those decay amplitudes,

$$ F_{\omega\pi\pi} = F_{\omega\sigma} + F_{\omega f_2} + F_{b_1\pi} + \cdots + F_{\omega(\pi\pi)_{\text{Non.Res.}}}, \quad (3.4) $$

$$ F_{\omega\sigma} = r_{σ}e^{iθ_{σ}} \frac{m_{σ}Γ_{σ}}{m_σ^2 - s - i m_σ Γ_{σ}(s)} \sim \text{out} ⟨\omega\sigma | J/ψ ⟩ \text{in} $$

$^*)$ The strengths and phases of respective amplitudes are considered to be determined by quark dynamics. However, we treat them independent in phenomenological analyses.
\[
\begin{align*}
F_{\omega f_2} &= r_{f_2} e^{i\theta_{f_2}} \frac{m_{f_2} \Gamma_{f_2} N_{\pi\pi}(s, \cos \theta)}{m_{f_2}^2 - s - im_{f_2} \Gamma_{f_2}(s)} \approx \text{out} \langle \omega | J/\psi \rangle \text{in} \\
F_{\omega b_1} &= r_{b_1} e^{i\theta_{b_1}} \frac{m_{b_1} \Gamma_{b_1}}{m_{b_1}^2 - s - im_{b_1} \Gamma_{b_1}(s)} \approx \text{out} \langle \omega | J/\psi \rangle \text{in} \\
&\quad \vdots \\
F^{\text{Non}, \text{Res.}}_{\omega(\pi\pi)} &= r_{2\pi N.R.} e^{i\theta_{2\pi N.R.}} \approx \text{out} \langle \omega(2\pi)_{N.R.} \rangle | J/\psi \rangle \text{in}, \quad (3.5)
\end{align*}
\]

where \( F_{\omega\sigma} \) corresponds to the generalized S matrix element \( \text{out} \langle \omega | J/\psi \rangle \text{in} \), \( | \rangle \text{in} \langle \text{out} | \) denoting \text{in(out)-state of scattering theory. The} \( r_\sigma \) represents the corresponding coupling strength and the \( \theta_\sigma \) comes from the \( \omega \sigma \) rescattering (the final state interaction). The extra Breit-Wigner factor comes from the prescription Eq. (3.3). Similarly, \( F_{\omega f_2}, F_{\omega b_1} \) and \( F^{\text{Non}, \text{Res.}}_{\omega(\pi\pi)} \) correspond to the generalized S matrix elements, \( \text{out} \langle \omega f_2 | J/\psi \rangle \text{in}, \text{out} \langle \omega b_1 | J/\psi \rangle \text{in} \) and \( \text{out} \langle \omega(2\pi)_{N.R.} | J/\psi \rangle \text{in} \), respectively, and they have mutually-independent couplings, \( r_{f_2}, r_{b_1} \) and \( r_{2\pi N.R.} \), and strong phases, \( \theta_{f_2}, \theta_{b_1} \) and \( \theta_{2\pi N.R.} \). \( N_{\pi\pi} \) is angular function of \( f_2 \rightarrow \pi\pi \) D wave decay.

(\text{Relation between scattering and production amplitudes, and chiral constraint})

In \( \pi\pi \) scattering the derivative-coupling property of Nambu-Goldstone \( \pi \)-meson requires the suppression of the amplitude \( T_{\pi\pi} \) near threshold,

\[
T_{\pi\pi} \sim -p_{\pi 1} \cdot p_{\pi 2} \rightarrow m_{\pi}^2 \sim 0 \quad \text{at} \quad s \rightarrow 4m_{\pi}^2. \quad (3.6)
\]

This chiral constraint requires, as was explained in \( \S \)2, the strong cancellation between the \( \sigma \) amplitude \( T_\sigma \) and the non-resonant \( \pi\pi \) amplitude \( T_{2\pi} \), which means the strong constraints, \( r_{2\pi} \sim -r_\sigma, \theta_\sigma \sim \theta_{2\pi} \), in the corresponding formulas to Eqs. (3.4) and (3.5). The amplitude has zero close to the threshold and no direct \( \sigma \) Breit-Wigner peak is observed in \( \pi\pi \) mass spectra.

On the other hand, for general \( \pi\pi \) production processes the parameters \( r_i \) and \( \theta_i \) are independent of those in \( \pi\pi \) scattering, since they are concerned with different \( S \)-matrix elements. Especially we can expect in the case of \text{“}\( \sigma \)-dominance\text{“}, \( r_\sigma \gg r_{2\pi} \), the \( \pi\pi \) spectra show steep increase from the \( \pi\pi \) threshold, and the \( \sigma \) Breit-Wigner peak is directly observed. This situation seems to be realized in \( J/\psi \rightarrow \omega \pi\pi \) and \( D^+ \rightarrow \pi^- \pi^+ \).

Here we should note that the chiral constraint on \( r_\sigma \) and \( r_{2\pi} \) does not work generally in the production processes with large energy release to the \( \pi\pi \) system. We explain this fact in case of \( Y \) decays.\textsuperscript{26} Here we take \( J/\psi \rightarrow \omega \pi\pi \) as an example. We consider a non-resonant \( \pi\pi \) amplitude of derivative-type \( F_{\text{der}} \),

\[
F_{\text{der}} \sim P_{\psi} \cdot p_{\pi 1} P_{\psi} \cdot p_{\pi 2} / M_{\psi}^2, \quad (3.7)
\]

where \( P_{\psi}(p_{\pi 1}) \) is the momentum of \( J/\psi \) (emitted pions).\textsuperscript{8} This type of amplitude satisfies the Adler self-consistency condition, \( F \rightarrow 0 \) when \( p_{\pi 1\mu} \rightarrow 0 \), and consistent with the general chiral constraint. However, this zero does not appear in low energy

\textsuperscript{8} The equation (3.7) is obtained by the chiral symmetric effective Lagrangian, \( L_d = \xi_d \partial_\mu \partial_\nu \psi \lambda_\omega (\partial_\mu \partial_\nu \pi + \partial_\nu \sigma \partial_\mu \sigma) \). The possible origin of this effective Lagrangian is discussed in ref.\textsuperscript{26}. There occurs no one \( \sigma \)-production amplitude, cancelling the \( 2\pi \) amplitude, in this Lagrangian.
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region of actual $s$-plane. At $\pi\pi$ threshold (where $s = 4m_{\pi}^2$), $p_{\pi 1\mu} = p_{\pi 2\mu}$ and $P_\psi \cdot p_{\pi i}/M_\psi = E_{\pi i} \sim (M_\psi - m_\omega)/2$ ($E_{\pi i}$ being the energy of emitted pion), and thus,

$$ F_{\text{der}} \rightarrow \left( \frac{(M_\psi - m_\omega)}{2} \right)^2 \gg m_{\pi}^2 \quad \text{at} \quad s \rightarrow 4m_{\pi}^2. \quad (3.8) $$

The amplitude (3.7) is not suppressed near $\pi\pi$ threshold, and correspondingly there is no strong constraint between $r_\sigma e^{i\theta_\sigma}$ and $r_{2\pi} e^{i\theta_{2\pi}}$, leading to the threshold suppression. This is quite in contrast with the situation in $\pi\pi$ scattering, Eq. (3.6).

(“Universality” of $T_{\pi\pi}$ : threshold behavior) Conventionally all the production amplitudes $F_{\pi\pi}$, including the $\pi\pi$ system in the final channel, are believed to take the form proportional to $T_{\pi\pi}$ as

$$ F_{\pi\pi} = \alpha(s) T_{\pi\pi}; \quad \alpha(s) : \text{slowly varying real function}, \quad (3.9) $$

where $\alpha(s)$ is supposed to be a slowly varying real function. This implies that $F$ and $T$ have the same phases and the same structures (the common positions of poles, if they exist). The equation (3.9) is actually applied to the analyses of various production processes\(^5\),\(^{25}\),\(^{32}\),\(^{33}\), and it was the reason of overlooking $\sigma$ for almost 20 years in the 1976 through 1994 editions of Particle Data Group tables.

The equation (3.9) is based on the belief that low energy $\pi\pi$ chiral dynamics is also applicable to $\pi\pi$ production processes with small $s$, leading to the threshold suppression of spectra, as in Eq. (3.6), in all production processes because of the Adler zero in $T_{\pi\pi}$. This is apparently inconsistent with experimental data.

So, in order to remove the Adler zero at $s = s_0$ and to fit the experimental spectra, one is forced to modify\(^5\),\(^{25}\) the form of $\alpha(s)$ by multiplying artificially the rapidly varying factor $1/(s - s_0)$ without any theoretical reason.

When the scattering amplitude $T$ is unitarized by $N/D$ method, Adler zero in $N$ leads to the factor $s - s_0$ in the imaginary part of $D$. Then, in ref. 25), following Eq. (3.9), in addition to the (above mentioned) artificial factor in $\alpha(s)$, it is insisted that this $D$ function with the factor $s - s_0$ in its imaginary part should be applied to all the $\pi\pi$ production amplitudes $F$. However, such a requirement on $F$ has no relation with the Adler self-consistency condition which predicts the zero in total $F$. Moreover, that is not generally valid since in its approach only the $\pi\pi$ dynamics, that is, the final state interaction between two stable $\pi$ mesons is considered, and the various final state interactions, as explained in Fig. 3, are not taken into account.

In the case of $J/\psi \rightarrow \omega \pi\pi$, the emitted pion energy is of order $M_\psi$, and the new dynamics in $J/\psi$ energy region, which is beyond the scope of chiral dynamics, must also be considered. This is overlooked\(^5\),\(^{25}\) in Eq. (3.9).

Here I should like to stress the physical meaning of an example, Eq. (3.7), that Adler zero condition, even in the isolated final $2\pi$ system, does not necessarily lead to the threshold suppression. This implies that the above mentioned ad hoc prescription\(^ {25}\) to get rid of the undesirable zero near threshold of $F$ becomes not necessary, if we take the new dynamics duly into account.
§4. Phases of Production Amplitudes

(Generalized $S$ matrix and phase of production amplitude) It is often discussed that in order to confirm the existence of a resonant particle, it is necessary to observe the corresponding phase motion $\Delta \delta \sim 180^\circ$ of the amplitude like the case of $\rho$ meson. In the $\pi\pi$ $P$ wave amplitude a clear phase motion $\Delta \delta \sim 180^\circ$ due to $\rho$ meson Breit-Wigner amplitude is observed. However, in the case of $\sigma$ meson, because of the chiral cancellation mechanism (explained in §2), and because of its large width, the $\sigma$ Breit-Wigner phase motion $\Delta \delta \sim 180^\circ$ cannot be observed directly in the $\pi\pi$ scattering amplitude. While, in $\pi\pi$ production processes the amplitudes are the sum of various $S$ matrix elements as explained in Eqs. (3.4) and (3.5), and correspondingly the pure $\sigma$ Breit-Wigner phase motion may be generally difficult to be observed. However, only in some exceptional cases, when the amplitude is dominated by $\sigma$, the $\sigma$ phase motion may be directly observed.

On the other hand, as was mentioned at the end of §3, conventionally it is widely believed that all the $\pi\pi$ production amplitude $\mathcal{F}$ have the same phase as that of $\pi\pi$ scattering amplitude $\mathcal{T}$.

However, this belief (or Eq. (3.9)) comes from the incorrect application of elastic unitarity condition, which is not applicable to production processes, where the freedom of various strong phases, $\theta_{\sigma}, \theta_{b_1}, \theta_{f_2}, \cdots$ (in Eqs. (3.4) and (3.5)) allowed in generalized unitarity condition, is overlooked in Eq. (3.9).

Because of the effect of the above strong phases, generally $\mathcal{F}$ have different phases from $\mathcal{T}$. $\mathcal{F}$ have the same phase as $\mathcal{T}$ only in very limited cases when the final $\pi\pi$ (or $K\pi$) systems are completely isolated in strong interaction level. For 3-body decays such as $J/\psi \rightarrow \omega \pi\pi(K^*K\pi)$ and $D^- \rightarrow \pi^-\pi^+\pi^+(K^-\pi^+\pi^+)$, the above condition is not satisfied. Actually, the large strong phases in $J/\psi$ and $D$ decays are suggested experimentally. In order to reproduce the experimental branching ratio of $J/\psi \rightarrow 1^-0^-$ decays (that is, $J/\psi \rightarrow \omega\pi^0, \rho\pi, K^*K, \cdots$) it is necessary to introduce a large relative strong phase $^{34}\delta_{\gamma} = \arg\frac{a_{\gamma}}{a} = 80.3^\circ$ between the effective coupling constants of three gluon decay $a$ and of one photon decay $a_{\gamma}$. A similar result is also obtained in $J/\psi \rightarrow 0^-0^-$ decays. A large relative phase between $I = 3/2$ and $I = 1/2$ amplitudes of $D \rightarrow K\pi$ decays is observed: $\delta_{3/2}(m_D) - \delta_{1/2}(m_D) = (96 \pm 13)^\circ$, $^{35}$ (while in $B \rightarrow D\pi, D\rho, D^*\pi$ decays rather small relative phases are obtained). By considering these results we expect not small strong phases $\theta_{\sigma}, \theta_{b_1}, \cdots$ coming from $\sigma\omega, b_1\pi, \cdots$ rescatterings in $J/\psi$ decays.$^{4}$

($\cos \theta$ distribution in $J/\psi \rightarrow \omega\pi\pi$) In relation to this argument Minkowski and Ochs raise a criticism concerning the existence of light-mass $\sigma$-pole $^{24}$ in $J/\psi \rightarrow \omega\pi\pi$.

The $\cos \theta$ distribution is obtained in $m_{\omega\pi} = 250 \sim 750$ MeV by DM2. $^{10}$ They apply partial wave expansion (PWA) including $S$ and $D$ waves to obtain the cross section

$^{4}$ It is often argued that the amplitude of $J/\psi \rightarrow \omega\pi\pi$ (or $D \rightarrow \pi^-\pi^+\pi^+\pi^+$) near $\pi\pi$ threshold must take the same phase as the $\pi\pi$ scattering phase, since in this energy region the $m_{\omega\pi}(\sim M_\psi)$ is large, and $\pi\pi$ decouples from $\omega$ in final channel. $^{41}$ And this phase constraint is argued to come from $\pi\pi$ elastic unitarity condition. However, according to the work $^{34}$ $M_\psi$ is not sufficiently large for making $\pi\pi$ decouple from $\omega$, and the $\pi\pi$ elastic unitarity constraint actually does not work in the amplitude Eq. (3.4).
as \( \frac{d\sigma}{d\Omega} \sim |S|^2 + 10(3\cos^2\theta - 1)Re(SD^*) + O(|D|^2) \), where the \( \cos^2\theta \) term is proportional to \( Re(SD^*) \). Then, if the \( D(S) \) is dominated by \( f_2(1270) \) (\( \sigma \)) contribution, the angular distribution would vary with a sign change of the \( \cos^2\theta \) term (from + to −). Actually, contradictorily to this anticipation, the data do not show any sign change below 750 MeV, and they conclude that there is no indication for a Breit-Wigner resonance at 500 MeV. \(^{24}\)

However, (according to our preliminary analyses, \(^{36}\)) in this energy region there is almost no contribution from \( f_2(1275) \), and actually the \( l \geq 2 \) partial waves mainly come from \( b_1(1235) \) contribution, \( J/\psi \rightarrow \pi b_1 \) and \( b_1 \rightarrow \omega \pi \). In \( m_{\pi\pi} \gtrsim 500\text{MeV} \) the direct \( b_1 \) peak is seen in \( \cos \theta \) distribution, and it includes the large higher wave components. This fact means the above PWA does not work well in this energy region. Furthermore, each of the partial waves is expected to show a large phase motion (from \( b_1 \) pole) in the relevant energy region \( m_{\pi\pi} \sim 500\text{MeV} \), while in the above criticism the almost constant phase of \( D \) wave is assumed. Thus, the basic assumption is not applicable, and their criticism is not correct.

Recently a method extracting the \( \sigma \) phase motion from Dalitz plot data of \( D \) decays is presented in ref. 37), where the interference between \( f_0(980) \) (or \( f_2(1275) \)) and the remaining \( S \)-wave component is used to observe the \( \sigma \) phase motion. Similarly, in the relevant \( J/\psi \) decays, by using the Dalitz plot data, it may be possible to observe the \( \sigma \) phase motion, where the interference between \( b_1(1235) \) Breit-Wigner amplitude and \( \pi \pi \) \( S \)-wave component is used. The direct \( b_1 \) peak appears in \( m_{\pi\pi} \gtrsim 500\text{MeV} \) region of Dalitz plot, and in this energy region the \( S \)-wave phase motion is expected to be determined with good accuracy.\(^{4}\)

\[(pp \text{ central collision, } pp \rightarrow pp\pi^0\pi^0) \text{ A partial wave analyses of the } \pi^0\pi^0 \text{ system produced centrally in } pp \text{ collisions at } 450 \text{ GeV}/c \text{ are done by WA102} \({}^{38}\) \text{ and GAMS}\(^{39}\). A large peak structure around 500 MeV, observed in both \( S \) and \( D \) waves, is explained in ref. 24) by the one pion exchange (OPE).

It should be noted that the relative phase \( \phi_{S_0^-} - \phi_{D_0^-} \) between \( S_0^- \) and \( D_0^- \) amplitudes**\) in Fig. 4 is apparently different from the corresponding scattering phase \( \delta_0^\pi - \delta_2^\pi \), (shown by solid line in the figure) where \( \delta_0^\pi \) and \( \delta_2^\pi \) are isosinglet \( S \) and \( D \) wave \( \pi\pi \) phase shifts, respectively. (As

\[\text{Fig. 4. The relative phase } \phi_{S_0^-} - \phi_{D_0^-} \text{ between } S_0^- \text{ and } D_0^- \text{ in } pp \rightarrow pp\pi^0\pi^0 \text{ in (a) WA102} \({}^{38}\) \text{ and (b) GAMS. } \phi_{S_0^-} - \phi_{D_0^-} \text{ is different from the scattering } \delta_0^\pi - \delta_2^\pi \text{ shown by solid line.}\]

\(^{*)}\) In case \( (m_\pi, \Gamma_\pi) = (500, 350)\text{MeV} \) the \( \sigma \) Breit-Wigner amplitude gives the phase difference \( \Delta \delta = 83^\circ(67^\circ) \) between \( m_{\pi\pi}=450\sim850\text{MeV}(500\sim850\text{MeV}) \) which is somewhat larger than the corresponding \( \pi\pi \) scattering phase difference \( \Delta \delta \simeq 63^\circ(55^\circ) \).

**\) The relative phase between \( S_0^- \) and \( D_1^- \) in the relevant energy region is different from that between \( S_0^- \) and \( D_0^- \). The \( D_1^\pm \) components explain the \( \phi \) distribution, which suggests the pomerons have vector components.
shown in Fig. 1, \( \delta_0^0 \) gradually increases from 0 to 90° in \( m_{\pi\pi} = 2m_\pi \) through \( \sim 900 \) MeV, and \( \delta_0^2 \) dominated by \( f_2(1275) \) is almost 0 in this energy region.) Thus, it is experimentally confirmed that the phase of \( \pi\pi \) production amplitude \( F \) is different from \( \pi\pi \) scattering amplitude \( T \) in this process.

As shown in Fig. 4, in both experiments, the \( \phi_{S_0} - \phi_{D_0} \) is not constant, and shows some structure around \( 0.5 \sim 0.6 \) GeV, which seems to suggest the \( \sigma \) contribution, interfering with the background which may come from OPE. In VMW method the \( F \) is given by

\[
F = r_{BG}(s)e^{i\theta_{BG}} + r_{\sigma}e^{i\theta_{\sigma}} \frac{m_\sigma f_{\pi}(s)}{m_\sigma^2 - s - im_\sigma f_{\pi}(s)},
\]

where the 1st(2nd) term represents the background(\( \sigma \) BW) amplitude. An example of the mass spectra, \( |F|^2/\rho(s) \) (\( \rho(s) \) being \( \pi\pi \) state density \( p_1(s)/(8\pi\sqrt{s}) \)), and the phase \( \arg F \) is shown in Fig. 5. Here we use \( r_{BG}(s) = a(\sqrt{s} - 2m_0^2)\exp(-c\sqrt{s} - ds) \times \rho(s) \) (which is used in ref. 38), 39, and the parameters are selected by inspection of experimental mass and phase distributions.

Experimental data in this process seems to be consistent with the existence of \( \sigma \) meson.

\( D^+ \rightarrow K^-\pi^+\mu^+\nu \) and \( D^+ \rightarrow K^-\pi^+\pi^+ \)

The \( K^-\pi^+ \) spectra of \( D^+ \rightarrow K^-\pi^+\mu^+\nu \) by FOCUS \( 40 \) is dominated by \( P \) wave from \( K^{*0} \) interfering with a small \( S \) wave. Through the angular analysis this \( S \) wave component has almost constant phase \( \delta = \frac{\pi}{4} \) in mass region of \( \kappa \) meson, \( m_{K\pi} = 0.8 \sim 1.0 \) GeV. This \( \delta \) is suggested, by Minkowski and Ochs \( 24 \), to be the same as the \( K\pi \) scattering phase shift \( 21 \) by LASS in this mass region. Based on this result, they criticize the analysis of \( D^+ \rightarrow K^-\pi^+\pi^+ \) by E791 \( 23 \), where, as explained in \( \S 2 \), the \( \kappa \) Breit Wigner amplitude is applied and it has large phase motion in this mass region. They stated “Such a result (of E791) appears to contradict the above FOCUS result.”

However, this criticism is again premature because the effect of strong phases allowed in generalized unitarity condition is overlooked. In \( D^+ \rightarrow K^-\pi^+\mu^+\nu \) the final \( K^-\pi^+ \) is isolated in strong interaction level. Thus, as we explained in the first part of this section, the amplitude has the same phase as \( K\pi \) scattering amplitude due to Watson theorem. However, \( D^+ \rightarrow K^-\pi^+\pi^+ \) is a three body decay of heavy meson and \( K^-\pi^+ \) is not isolated in final channel, and the rescattering effects from various elements of generalized \( S \) matrix, such as, \( \text{out}(K^0\pi^+|J/\psi)_{in}, \text{out}(K^{*0}\pi^+|J/\psi)_{in}, \text{out}(K^{*0}\pi^+|J/\psi)_{in}, \cdots \), must be taken into account. In 2-body decays of \( D, J/\psi \) (\( B \)) mesons, the large (small) strong phase is suggested. \( 34 \) Thus, the phases of the above matrix elements are considered to be not small. Thus, total amplitude of \( D^+ \rightarrow K^-\pi^+\pi^+ \) has generally different phase from that of \( D^+ \rightarrow K^-\pi^+\mu^+\nu \).

Finally we should add a comment on \( D^+ \rightarrow K^-\pi^+\mu^+\nu \) in relation to \( K_{l4} \) decay.
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$K^+ \to \pi^+\pi^-e^+\nu$. The latter process is analyzed by Shabalin\textsuperscript{42} by using SU(3) linear $\sigma$ model, where the effect of direct $\sigma$ production, $K^+ \to \sigma e^+\nu$ (successively $\sigma \to \pi^+\pi^-$), explains the large width obtained experimentally (which is twice as large as the prediction by soft pion limit). At the same time this amplitude has the same phase as the $\pi\pi$ scattering phase shift. The $\sigma$ Breit Wigner phase motion is not observed due to Watson theorem, but its large decay width suggests the $\sigma$ production in this process. As can be seen in this example, the $\kappa$ phase motion is not observed in $D^+ \to K^-\pi^+\mu^+\nu$, but this fact does not mean no $\kappa$-existence. The analysis of $D^+ \to K^-\pi^+\pi^+$ by E791 does not contradict with FOCUS result, and strongly suggests the $\kappa(800)$ existence.

§5. Concluding Remarks

The masses and widths of $\sigma$ and $\kappa$ mesons quoted in the text are summarized in Table I. The peak structures of $\sigma$ and $\kappa$ meson productions are directly observed in these $\pi\pi$ and $K\pi$ production processes. It is remarkable that a clear peak structure from $\kappa$ production is observed in $K\pi$ mass spectra of $J/\psi \to K^{*0}K^-\pi^+$ by BES,\textsuperscript{43} which is added in the table.

The peak structures mentioned above are fitted well by the Breit-Wigner amplitudes of $\sigma$ and $\kappa$ following VMW method, independently from the $\pi\pi$ and $K\pi$ scattering. The criticisms\textsuperscript{24,25} on this method come from the confusion of the 3-body production processes with 2-body scattering process, and are not correct.

The recent belief that phases of all the $\pi\pi$ production amplitude $F$ is the same as that of $T$ also comes from the same kind of confusion, and thus, is not correct. The 3- (or multi-) body production processes including $\pi\pi$ or $K\pi$ system in the final channels are considered as superpositions of various two- (or more) bodies processes, which correspond to the different elements of general $S$-matrix. Thus, the total amplitude generally has different phase from that of the $\pi\pi$ and $K\pi$ scattering amplitudes.

The observed peak structures are considered as the strong evidences of $\sigma$ and $\kappa$ existence. Presently their existence seems to be established with the property $(m_\sigma, \Gamma_\sigma) = (\sim 500, \sim 300)$MeV and $(m_\kappa, \Gamma_\kappa) = (\sim 800, \sim 400)$MeV, respectively.

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