Faster Biological Gradient Descent Learning

Ho Ling Li

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School of Psychology
University of Nottingham, Nottingham NG7 2RD
U.K.

Abstract

Back-propagation is a popular machine learning algorithm that uses gradient descent in training neural networks for supervised learning, but can be very slow. A number of algorithms have been developed to speed up convergence and improve robustness of the learning. However, they are complicated to implement biologically as they require information from previous updates. Inspired by synaptic competition in biology, we have come up with a simple and local gradient descent optimization algorithm that can reduce training time, with no demand on past details. Our algorithm, named dynamic learning rate (DLR), works similarly to the traditional gradient descent used in back-propagation, except that instead of having a uniform learning rate across all synapses, the learning rate depends on the current neuronal connection weights. Our algorithm is found to speed up learning, particularly for small networks.

Introduction

In the past decades, back-propagation has become the go-to machine learning algorithm in training neural networks. It utilizes stochastic gradient descent (SGD) to minimize a cost function $C$ by adjusting the connection weights $w_{ij}$ with $\Delta w_{ij} = -\eta \frac{\partial C}{\partial w_{ij}}$ at each time-step. However, learning with the SGD originally purposed in back-propagation can be very slow. In addition, networks have higher risk to overfit when training is slow \[1\]. Therefore, algorithms have been developed to speed up convergence and improve robustness of learning. These algorithms are mainly separated into two categories: SGD and adaptive learning. Examples of adaptive learning algorithms are Adagrad \[2\], RMSprop \[3\], and Adam \[4\]. They are usually faster than SGD algorithms, such as momentum \[5\] and Nesterov momentum \[6\]. On the other hand, networks trained with SGD are better at generalization than with adaptive learning \[7\]. In order to benefit from both the training speed and generalization capability, several algorithms have been designed to transit from adaptive learning to SGD during training \[8\][9]. Regardless of which categories these algorithms belong, they require combining past updates with the current weight update, which make them complicated to implement biologically.

Inspired by synaptic competition in biology, we have come up with a simple and local gradient descent optimization algorithm that encourages the potentiation of strong synapses and suppresses the growth of weak synapses. This algorithm, named dynamic learning rate (DLR), works similarly to the traditional gradient descent used in back-propagation, except that instead of having a uniform learning rate across all synapses, the learning rate depends on the current connection weights of individual synapses and the $L_2$ norm of the weights of each neuron. It is found to speed up learning, particularly for small networks, with no demand on past information, hence making it biologically plausible.
Results

The design of DLR is based on the ideas that synaptic transmissions are metabolically expensive, thus pushing neurons to lower the number of strong synapses to save energy [10, 11, 12, 13]. Unlike the traditional SGD that uses the same learning rate $\eta$ for all synapses, to quicken the rise of strong synapses and speed up the diversification of the connection strength between neurons, DLR encourages neurons to form strong connections to a handful of neurons of their neighbouring layers by assigning higher learning rate ($\eta_{ij}$) to synapses with bigger weights ($w_{ij}$):

$$\eta_{ij} = -\eta_0 \frac{|w_{ij}| + \alpha}{||w_j|| + \alpha} \tag{1}$$

$$\Delta w_{ij} = -\eta_{ij} \frac{\partial C}{\partial w_{ij}}, \tag{2}$$

where $i$ represents the indices of the post-synaptic neurons and $j$ represents the indices of the pre-synaptic neurons. The parameter $\alpha$ is set at the range of values such that at the beginning of training $\alpha > ||w_j|| \gg w_{ij}$ so that all synapses have similar learning rate. As learning progresses, the learning rate of all synapses decreases, but large synapses would retain a relatively large learning rate while the learning rate of small synapses would become small. Here, $||w_j||$ is summing over all the post-synaptic weights of a pre-synaptic neuron, leading to each pre-synaptic neuron having strong connections to a limited amount of post-synaptic neurons only. However, DLR also works by replacing this term with $||w_i||$, hence

$$\eta_{ij} = -\eta_0 \frac{|w_{ij}| + \alpha}{||w_i|| + \alpha} \tag{3}$$

which promotes every post-synaptic neuron to form strong connections to a subset of pre-synaptic neurons instead. Whether Eq. 1 or Eq. 3 would perform better depends on the network architecture. Since most networks tend to have decreasing numbers of neurons for deeper layers, Eq. 1 is more applicable in general. We note that the proposed modulation of learning can easily be imagined to occur in biology, as it only requires each neuron to know the connection strength with its own pre- or post-synaptic neurons.

Training speed compared to standard methods

To test the performance of DLR, we implement a multi-layer network trained with back-propagation with one hidden layer to classify hand-written digits from the MNIST dataset to a benchmark accuracy of 96%. We compare the performance of DLR with the traditional SGD in back-propagation, Nesterov momentum [6], and the commonly used adaptive learning algorithm Adam [4]. All the algorithms involve one or two parameters except Adam, which involves four parameters: $\alpha$, $\beta_1$, $\beta_2$ and $\epsilon$. From a brief check, training becomes faster after changing $\alpha$ and $\epsilon$ from the default values suggested by Adam’s authors Kingma and Ba [4]. Therefore, Adam is optimized with respect to $\alpha$ and $\epsilon$. The other three algorithms have all of their parameters optimized. Networks with fewer than 100 hidden units train the fastest with DLR though Adam is not significantly slower than DLR, Figure 1. As the network size increases, Adam performs the best.

Robust for small networks

Next, we are interested in knowing if DLR is more robust for small networks compared to other algorithms, i.e. whether DRL allows small networks to reach the designated accuracy that would otherwise not be able to reach if they are trained with other learning algorithms. To test it, we reduce the number of neurons at the hidden layer until the networks are no longer able to converge. Figure 2 shows the minimal network size that the network can still satisfy the accuracy requirement. Since the change in the network architecture may require different parameter values for the algorithms, we scan the performance of each algorithm across a parameter space. Compared to traditional SGD, Adam, and Nesterov momentum, which on average demands $36.3 \pm 1.2$, $35.3 \pm 1.6$
Figure 1: **Comparisons of training speed between traditional SGD, Nesterov momentum, Adam and DLR.** The networks with back-propagation are trained with one of the four algorithms. The network size is varied by adjusting the number of neurons in the hidden layer. The left figure shows the training time of the four algorithms. To illustrate the change in training time by switching from traditional SGD to the other three algorithms, the ratio of their training time to that of traditional SGD is displayed on the right. Networks equipped with DLR reach the designated accuracy the fastest when the networks have fewer than 100 hidden units. The uncertainties are standard deviations across multiple runs.

and 33.7 ± 2.0 hidden neurons respectively, networks trained with DLR only require 29.7 ± 1.6 neurons to reach 96% accuracy. Our speculated explanation is because DRL allows large weights to have high learning rates, in the case of networks being stuck at a local minimum, undesirable big weights can depress more quickly hence releasing the networks from that local minimum.

Figure 2: **Minimal requirement on network size.** By gradually reducing the number of neurons in the hidden layer, network sizes are decreased until the networks fail to reach an accuracy level of 96%. Networks trained with DLR needs on average fewer neurons than the other three algorithms. The uncertainties are standard deviations across multiple runs.
Synapse-specific learning rate benefits learning

During training, when implemented with DLR, the learning rate of most synapses would drop. To check that the improvement of training speed is not primarily due to a gradual global decrease in learning rate but instead due to the learning rate being synapse-specific, we have compared the learning time of networks trained with DLR with networks trained with the average learning rate of DLR. This is achieved by measuring the average learning rate between the input and hidden layers, and between the hidden and output layers of the networks trained with DLR, then fitting the average learning rate with \( a \exp(bt^{1/3} + ct) + d \), where \( t \) represents the training time. The fits are implemented into new networks, which have to learn to classify the MNIST dataset with that predetermined learning rate. Networks learnt with DLR reaches 96% accuracy with \((0.77 \pm 0.10)\) epoch in contrast to \((1.14 \pm 0.22)\) epochs when networks learnt with the average learning rate, showing the criticality of the learning rate being synapse-specific. Figure 3 shows the median of test accuracy over multiple runs, demonstrating how networks with DLR progress faster during training.

![Figure 3](image.png)

**Figure 3: Median of test accuracy over training time.** The median of test accuracy over multiple runs is shown until it reaches the designated accuracy. Networks equipped with DLR finish training within 0.8 epoch in majority of the runs. Networks trained with the average learning rate of DLR take at least 1.1 epochs in most of the runs.

**Discussion**

DLR has shown the possibility of a local and biological gradient descent optimization algorithm that can speed up neural network training with back-propagation. It only requires online information, which may have the benefits of lower memory usage at large networks compared to algorithms that require storing information from past updates.

It is shown to have comparable training speed with the popular adaptive learning algorithm Adam for networks with small and medium sizes, i.e. when the parameters in the networks are not redundant. In addition, DLR is found to be more robust than traditional SGD, Nesterov momentum and Adam as it allows small networks to acquire an accuracy level that otherwise would not be able to achieve. It shows that DLR can find the solution more efficiently even when the number of weight parameters is restrained. Here, we have conducted the tests on MNIST dataset with networks that are small compared to deep networks that are used to categorize much more complicated images. Therefore, it is uncertain how DLR will perform in deep networks. We wonder if it is also applicable in those networks, which even though have significantly larger network sizes, may still suffer the
issue of insufficient weight parameters. On the other hand, DLR may allow the use of smaller deep networks by guaranteeing similar performance as the larger ones, and provide the benefits of less computation time and memory.

In recent years, many algorithms that can effectively speed up learning are adaptive learning algorithms, which are found to have not as good generalization capability as SGD \cite{7}. It would be interesting to test if DLR, as a SGD, performs well in generalization while still has comparable training speed as adaptive learning algorithms.

**Methods**

We use networks with one hidden layer, logistic units without bias, and one-hot encoding at the output. Weights are updated according to the mean squared error back-propagation rule without regularization.

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