The solution to Slavnov–Taylor identities in a general four dimensional supersymmetric theory

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Abstract

A solution to Slavnov–Taylor identities in a general four dimensional N=1 supersymmetric Yang-Mills theory containing arbitrary matter superfields is proposed. The solution proposed appears just a simple generalization of an analogous solution in the pure supersymmetric Yang-Mills theory.

Keywords:
Slavnov–Taylor identity, supersymmetric theory
1 Introduction

Recently the solution to Slavnov–Taylor (ST) identities has been proposed for supersymmetric N=1 theories without matter [1]. In this paper under the effective action we mean what is called the Legendre transformation of \(\ln Z[J]\) with respect to \(J\), where \(Z[J]\) is defined in the traditional way as the path integral in the presence of the external source \(J\).

The notation used for four dimensional supersymmetry are the same as those have been used in Ref. [2].

In the previous paper [1] we have described the procedure of deriving the solution to the Slavnov–Taylor identities for the case of the four dimensional Yang–Mills pure gauge theory with softly broken supersymmetry. In this section we make a short review of the method proposed there to proceed further with matter superfields. We consider for simplicity the case of the unbroken (rigid) supersymmetric N=1 Yang–Mills theory. The only difference from the softly broken case is that the “coupling” is really a coupling constant but not the external background superfield.

The path integral describing the quantum theory is defined as

\[
Z[J, \eta, \bar{\eta}, \rho, \bar{\rho}, K, L, \bar{L}] = \int dV dc d\bar{c} db d\bar{b} \exp i[S + 2 \text{Tr} \left( \int d^8 z J V + i \int d^6 y \eta c + i \int d^6 \bar{y} \bar{\eta} \bar{c} + i \int d^6 y \rho b + i \int d^6 \bar{y} \bar{\rho} \bar{b} \right) + 2 \text{Tr} \left( i \int d^8 z K \delta_{c,c} V + \int d^6 y Lc^2 + \int d^6 \bar{y} \bar{L} \bar{c}^2 \right)].
\]

The third term in the brackets is BRST invariant since the external superfields \(K\) and \(L\) are BRST invariant by definition. All fields in the path integral are in the adjoint representation of the gauge group. For the sake of brevity we use the following definition for measures in the superspace

\[
d^8 z \equiv d^4 x \; d^2 \theta \; d^2 \bar{\theta}, \quad d^6 y \equiv d^4 y \; d^2 \theta, \quad d^6 \bar{y} \equiv d^4 \bar{y} \; d^2 \bar{\theta}.
\]

The total gauge part of the classical action is

\[
S = \int d^6 y \frac{1}{g^2} \frac{1}{2} \text{Tr} \; W_\alpha W^\alpha + \int d^6 \bar{y} \frac{1}{g^2} \frac{1}{2} \text{Tr} \; \bar{W}_\dot{\alpha} \bar{W}^{\dot{\alpha}} + \int d^8 z \frac{1}{16 \alpha} \text{Tr} \left( \bar{D}^2 V \right) \left( D^2 V \right) + \int d^6 y \frac{i}{2} \text{Tr} \; b \; \bar{D}^2 \delta_{c,c} V + \int d^6 \bar{y} \frac{i}{2} \text{Tr} \; \bar{b} \; D^2 \delta_{\bar{c},\bar{c}} V,
\]

where \(b\) and \(\bar{b}\) are the antighost chiral and antichiral superfields, and \(c\) and \(\bar{c}\) are the ghost chiral and antichiral superfields. Such a choice of the gauge fixing and the ghost terms means that we fix the gauge arbitrariness by imposing the condition

\[
D^2 V(x, \theta, \bar{\theta}) = \bar{f}(\bar{y}, \bar{\theta}), \quad \bar{D}^2 V(x, \theta, \bar{\theta}) = f(y, \theta),
\]

where \(\bar{f}\) and \(f\) are arbitrary antichiral and chiral functions, respectively.
Having shifted the antighost superfields $b$ and $\bar{b}$ by arbitrary chiral and antichiral superfields respectively, or, having made the change of variables in the path integral (1) which are the BRST transformations of the total gauge action (2), we obtain ghost equations (3)

$$\frac{\delta \Gamma}{\delta b} - \frac{1}{4} D^2 \frac{\delta \Gamma}{\delta K} = 0, \quad \frac{\delta \Gamma}{\delta \bar{b}} - \frac{1}{4} \bar{D}^2 \frac{\delta \Gamma}{\delta \bar{K}} = 0$$

and ST identities

$$\text{Tr} \left[ \int d^8z \frac{\delta \Gamma}{\delta V} \frac{\delta \Gamma}{\delta K} - i \int d^6y \frac{\delta \Gamma}{\delta c} \frac{\delta \Gamma}{\delta L} + i \int d^6\bar{y} \frac{\delta \Gamma}{\delta \bar{c}} \frac{\delta \Gamma}{\delta \bar{L}} - \int d^6y \frac{\delta \Gamma}{\delta b} \left( \frac{1}{32} \bar{D}^2 D^2 V \right) - \int d^6\bar{y} \frac{\delta \Gamma}{\delta \bar{b}} \left( \frac{1}{32} D^2 \bar{D}^2 V \right) \right] = 0,$$

respectively. All Grassmannian derivatives in this paper are left derivatives by definition. From the effective action $\Gamma$ we can extract a 2-point ghost proper correlator $G^{(2)}(z - z')$, and a 2-point connected ghost correlator $(-1)^{G^{(2)}}(z_2 - z')$, which is related to the previous one in the following way

$$\int d^8z' \ G^{(2)}(z - z') \ (-1)^{G^{(2)}}(z_2 - z') = \delta^{(8)}(z_2 - z_1).$$

In Ref. [1] it has been shown that the unique solution to ST identities (4) has the following form

$$\Gamma = \int d^6y \ f(g^2) \ \text{Tr} \ W_\alpha \left( V \ast (-1)^{G^{(2)}} \right) W^\alpha \left( V \ast (-1)^{G^{(2)}} \right) + \text{H.c.}$$

$$+ \int d^6y \ f_2(g^2) \ \text{Tr} \ W_\alpha \left( V \ast (-1)^{G^{(2)}} \right) W^\alpha \left( V \ast (-1)^{G^{(2)}} \right) \ W_\beta \left( V \ast (-1)^{G^{(2)}} \right) W^\beta \left( V \ast (-1)^{G^{(2)}} \right) + \text{H.c.} + \ldots$$

$$+ \int d^8z \ \frac{1}{32} \frac{1}{\alpha} \text{Tr} V \left( D^2 \bar{D}^2 + \bar{D}^2 D^2 \right) V$$

$$+ \int d^8z d^8z' \ 2i \ \text{Tr} \left( b(z) + \bar{b}(z) \right) G^{(2)}(z - z') \delta_{\bar{c},c} \left( V \ast (-1)^{G^{(2)}} \right)(z')$$

$$+ \int d^8z d^8z' \ 2i \ \text{Tr} \left( K \ast G^{(2)} \right)(z) \delta_{\bar{c},c} \left( V \ast (-1)^{G^{(2)}} \right)(z)$$

$$+ \int d^6y \ 2 \text{Tr} \ Lc^2 + \int d^6\bar{y} \ 2 \text{Tr} \ \bar{L}c^2.$$

Here we have introduced for the brevity the following notation for the integral convolutions

$$V \ast (-1)^{G^{(2)}} = \int d^8z' \ V(z') \ (-1)^{G^{(2)}}(z - z').$$

One can check that this action satisfies to the identity (4). Some comments are necessary here. First, $f(g^2)$ and $f_2(g^2)$ are functions of the gauge coupling. Second, the multidots $\ldots$ denote terms of higher orders in $W_\alpha$ invariant with respect to chiral rotations

$$W_\alpha \rightarrow e^{-A} W_\alpha e^A.$$
2 Including the matter

In this section we include the matter in the ST identity (4). The Lagrangian with the matter takes the form

\[
S = \int d^{6}y \, \frac{1}{g^{2}} \frac{1}{2} \text{Tr} \, W_{\alpha} W^{\alpha} + \int d^{6}y \, \frac{1}{g^{2}} \frac{1}{2} \text{Tr} \, \bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}} + \int d^{6}z \, \bar{\Phi} \, e^{V} \, \Phi
\]

\[
+ \int d^{6}y \, \left[ Y^{ijk} \Phi_{i} \Phi_{j} \Phi_{k} + M^{ij} \Phi_{i} \Phi_{j} \right] + \int d^{6}y \, \left[ \bar{Y}_{ijk} \bar{\Phi}^{i} \bar{\Phi}^{j} \bar{\Phi}^{k} + \bar{M}_{ij} \bar{\Phi}^{i} \bar{\Phi}^{j} \right].
\]

We take the same gauge fixing condition as we did in the previous section and the total gauge part is coinciding with (2). The matter superfield \( \Phi \) is in an appropriate, in general, reducible representation of the gauge group with a set of irreducible representations.

The path integral describing the quantum theory is defined as

\[
Z[J, \eta, \bar{\eta}, \rho, \bar{\rho}, j, \bar{j}, K, L, \bar{L}, k, \bar{k}] = \int dV \, dc \, dB \, d\bar{B} \, d\Phi \, d\bar{\Phi} \, \exp i \left[ S + 2 \text{Tr} \left( \int d^{8}z \, JV + i \int \int d^{6}y \, \eta c + i \int \int d^{6}y \, \bar{\eta} \bar{c} + i \int \int d^{6}y \, \rho b + i \int \int d^{6}y \, \bar{\rho} \bar{b} \right) \right.
\]

\[
+ \left( \int \int d^{6}y \, \Phi \, j + \int \int d^{6}y \, \bar{\Phi} \, \bar{j} \right) + 2 \text{Tr} \left( i \int \int d^{8}z \, K \delta_{c,c} V + \int d^{6}y \, L c^{2} + \int d^{6}y \, \bar{L} \bar{c}^{2} \right)
\]

\[
+ \int d^{6}y \, k \, c \, \Phi + \int d^{6}y \, \bar{k} \, \bar{c} \, \bar{\Phi} \right],
\]

where

\[
S = \int d^{6}y \, \frac{1}{g^{2}} \frac{1}{2} \text{Tr} \, W_{\alpha} W^{\alpha} + \int d^{6}y \, \frac{1}{g^{2}} \frac{1}{2} \text{Tr} \, \bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}} + \int d^{6}z \, \frac{1}{16} \frac{1}{\alpha} \text{Tr} \left( D^{2} \Phi \right) \left( D^{2} \Phi \right)
\]

\[
+ \int d^{6}y \, \frac{i}{2} \text{Tr} \, b \, D^{2} \delta_{\dot{c},c} V + \int d^{6}y \, \frac{i}{2} \text{Tr} \, \bar{b} \, D^{2} \bar{\delta}_{\dot{c},c} V + \int d^{6}z \, \bar{\Phi} \, e^{V} \, \Phi
\]

\[
+ \int d^{6}y \, \left[ Y^{ijk} \Phi_{i} \Phi_{j} \Phi_{k} + M^{ij} \Phi_{i} \Phi_{j} \right] + \int d^{6}y \, \left[ \bar{Y}_{ijk} \bar{\Phi}^{i} \bar{\Phi}^{j} \bar{\Phi}^{k} + \bar{M}_{ij} \bar{\Phi}^{i} \bar{\Phi}^{j} \right].
\]

The action (8) is invariant under the BRST symmetry (8),

\[
e^{V} \rightarrow e^{i\varepsilon} \, e^{V} \, e^{i\varepsilon},
\]

\[
c \rightarrow c + i \varepsilon^{2}, \quad \bar{c} \rightarrow \bar{c} - i \varepsilon^{2},
\]

\[
\delta b = \frac{1}{32} \frac{1}{\alpha} \left( D^{2} \, D^{2} V \right) \varepsilon, \quad \delta \bar{b} = \frac{1}{32} \frac{1}{\alpha} \left( \bar{D}^{2} \, \bar{D}^{2} V \right) \varepsilon,
\]

\[
\delta \Phi = -i \varepsilon \, \Phi, \quad \delta \bar{\Phi} = -\bar{\Phi} \, i \varepsilon
\]
with an Hermitian Grassmannian parameter $\varepsilon, \varepsilon^\dagger = \varepsilon$. The external sources $k, \bar k$ of the BRST transformations of chiral multiplets are BRST invariant by definition, so the last two lines in the eq. (7) are BRST invariant.

The effective action $\Gamma$ is related to $W = i \ln Z$ by the Legendre transformation
\[ V \equiv -\frac{\delta W}{\delta J}, \quad ic \equiv -\frac{\delta W}{\delta \eta}, \quad i\bar c \equiv -\frac{\delta W}{\delta \bar \eta}, \quad ib \equiv -\frac{\delta W}{\delta \rho}, \quad i\bar b \equiv -\frac{\delta W}{\delta \bar \rho}, \]
\[ \Phi \equiv -\frac{\delta W}{\delta \bar j}, \quad \bar \Phi \equiv -\frac{\delta W}{\delta j}. \]
\[ \Gamma = -W - 2 \text{Tr} \left( \int d^6z \, JV + \int d^6y \, i\eta c + \int d^6\bar y \, i\bar \eta \bar c + \int d^6y \, i\rho b + \int d^6\bar y \, i\bar \rho \bar b \right) \]
\[ - \int d^6y \, \Phi \, j - \int d^6\bar y \, \bar \Phi \, \bar j \equiv -W - 2 \text{Tr} \left( X\phi - (\Phi) \right) \]
\[ (X\phi) \equiv i^{G(k)} \left( X^k \phi^k \right), \]
\[ X \equiv (J, \eta, \bar \eta, \rho, \bar \rho), \quad \phi \equiv (V, c, \bar c, b, \bar b). \]
where $G(k) = 0$ if $\phi^k$ is the Bose superfield and $G(k) = 1$ if $\phi^k$ is the Fermi superfield. Iteratively all equations (11) can be reversed,
\[ X = X(\phi, \Phi, \bar \Phi, K, L, \bar L, k, \bar k), \]
\[ j = j(\phi, \Phi, \bar \Phi, K, L, \bar L, k, \bar k), \]
\[ \bar j = \bar j(\phi, \Phi, \bar \Phi, K, L, \bar L, k, \bar k), \]
and the effective action is defined in terms of new variables, $\Gamma = \Gamma(\phi, \Phi, \bar \Phi, K, L, \bar L, k, \bar k)$.

Hence, the following equalities take place
\[ \frac{\delta \Gamma}{\delta V} = -J, \quad \frac{\delta \Gamma}{\delta \Phi} = -j, \quad \frac{\delta \Gamma}{\delta \bar \Phi} = -\bar j, \]
\[ \frac{\delta \Gamma}{\delta K} = -\frac{\delta W}{\delta \bar K}, \quad \frac{\delta \Gamma}{\delta k} = -\frac{\delta W}{\delta \bar k}, \quad \frac{\delta \Gamma}{\delta \bar k} = -\frac{\delta W}{\delta k}, \]
\[ \frac{\delta \Gamma}{\delta c} = i\eta, \quad \frac{\delta \Gamma}{\delta \bar c} = i\bar \eta, \quad \frac{\delta \Gamma}{\delta b} = i\rho, \quad \frac{\delta \Gamma}{\delta \bar b} = i\bar \rho, \]
\[ \frac{\delta \Gamma}{\delta L} = -\frac{\delta W}{\delta \bar L}, \quad \frac{\delta \Gamma}{\delta \bar L} = -\frac{\delta W}{\delta L}. \]

If the change of fields (3) in the path integral (1) is made one obtains the Slavnov–Taylor identity as the result of invariance of the integral (7) under a change of variables,

\[ \text{Tr} \left[ \int d^8z \, \frac{\delta}{\delta K} \delta \frac{\delta}{\delta J} - \int d^6y \, i\eta \left( \frac{1}{i} \frac{\delta}{\delta L} \right) + \int d^6\bar y \, i\bar \eta \left( \frac{1}{i} \frac{\delta}{\delta \bar L} \right) + \int d^6y \, i\rho \left( \frac{1 \frac{\delta}{\delta \bar L}}{32 \alpha D^2 D^2} \frac{\delta}{\delta J} \right) \right] \]
\[ + \int d^6\bar y \, i\bar \rho \left( \frac{1}{32 \alpha D^2 \bar D^2} \frac{\delta}{\delta J} \right) \]
3 Solution to ST identities

In this section we present a general solution to the identity (13). The problem is to find a functional $\Gamma$ of the variables $\phi, \Phi, \bar{\Phi}, K, L, \bar{L}, k, \bar{k}$ that satisfies to the ST identity (13). There are several steps in the procedure of searching the solution. First, the gauge part of the solution that corresponds to the part of $\Gamma$ with zero values of $\Phi, \bar{\Phi}, k, \bar{k}$ is already found. It is the expression (5). Second, we suppose according to the no-renormalization theorem for supersymmetric theories that the chiral potential does not obtain any correction in the effective action in comparison with the classical action [5]. Thus, the form of the superpotential is

$$
\int d^6 y \left[ Y^{ijk} \Phi_i \Phi_j \Phi_k + M^{ij} \Phi_i \Phi_j \right] + \int d^6 \bar{y} \left[ \bar{Y}^{ijk} \bar{\Phi}_i \bar{\Phi}_j \bar{\Phi}_k + \bar{M}^{ij} \bar{\Phi}_i \bar{\Phi}_j \right] \\
+ \int d^6 y \ 2 \text{Tr} \ L c^2 + \int d^6 \bar{y} \ 2 \text{Tr} \ \bar{L} \bar{c}^2 + \int d^6 y \ k \ c \ \Phi + \int d^6 \bar{y} \ \bar{c} \ \bar{k} \ .
$$

(14)

As to the gauge-matter interaction part, it is easy to guess the most general form of a term of this part of $\Gamma$ which is

$$
\bar{\Phi} \ \bar{\nabla}^\alpha \ldots \bar{\nabla}^\beta \ e^{\tilde{V}} \ \nabla_\alpha \ldots \nabla_\beta \ e^{-\tilde{V}} \ \bar{\nabla}_{\bar{\alpha}} \ldots \bar{\nabla}_{\bar{\beta}} \ e^{\bar{V}} \ \nabla^\alpha \ldots \nabla^\beta \ \Phi,
$$

(15)

where $\tilde{V}$ means

$$
\tilde{V}(z) = \int d^8 z' \ \tilde{V}(z') \ G^{(2)}(z - z') .
$$

The argument why it must be so is very short. Indeed, as it has been shown in Ref. [4] after the change of variables

$$
K(z) = \int d^8 z' \ \bar{K}(z') \ (-1)^1 G^{(2)}(z - z') ,
$$

$$
V(z) = \int d^8 z' \ \tilde{V}(z') \ G^{(2)}(z - z')
$$

in the effective action functional $\Gamma$ the identity (4) recovers the functional $\Gamma$ in a gauge invariant manner with respect to new variables $\tilde{V}$ and $\bar{K}$. It means that in terms of these new variables the functional $\Gamma$ (3) has a gauge invariant structure for the field $\tilde{V}$ and all the action (5) as whole is BRST invariant in terms of the new variables. Having included the matter, one can see that the ST identity (13) recovers the effective action in a gauge invariant manner in terms of $\tilde{V}$ and $\bar{K}$ too, with a gauge transformation of matter fields defined by the terms in the last line in eq. (14). Apparently, the structure (13) is gauge invariant if we take into account the definitions [4] of the covariant derivatives

$$
\nabla_\alpha = e^{-\tilde{V}} \ D_\alpha \ e^{\tilde{V}} , \quad \nabla_{\bar{\beta}} = e^{\bar{V}} \ D_{\bar{\beta}} \ e^{-\bar{V}} .
$$

Any convolution of the spinor indices in (13) is allowed. For example, one can obtain

$$
\bar{\Phi} \ \bar{\nabla}^2 \ e^{\bar{V}} \ \nabla^2 \ e^{-\tilde{V}} \ \bar{\nabla}^2 \ e^{\bar{V}} \ \nabla^2 \ \Phi ,
$$

(15)
or

\[ \phi \, \nabla^\alpha \, e^V \, \nabla_\alpha \, e^{-V} \, \nabla_{\dot{\alpha}} \, e^V \, \nabla^\alpha \, \phi. \]

Thus, all the structures like (13), their degrees and convolutions in Lorentz indices can be present in the effective action. The coefficients before such structures can not be fixed in the framework of the approach proposed here. At the same time, this approach allows to restrict the functional structure of the effective action.

4 Conclusions

To conclude we would like to combine the pure gauge part (5), the superpotential (14) and the gauge-matter interaction part (15) together in order to obtain the total answer for the solution to the identities (13). Thus, the final result is

\[
\Gamma = \int d^8 y \, f(g^2, Y, \bar{Y}) \, \text{Tr} \, W_\alpha \left( V \ast ( -1 ) G^{(2)} \right) W^\alpha \left( V \ast ( -1 ) G^{(2)} \right) + \text{H.c.} \\
+ \int d^8 y \, f_2(g^2, Y, \bar{Y}) \, \text{Tr} \left( W_\alpha \left( V \ast ( -1 ) G^{(2)} \right) W^\alpha \left( V \ast ( -1 ) G^{(2)} \right) \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad W_\beta \left( V \ast ( -1 ) G^{(2)} \right) W^\beta \left( V \ast ( -1 ) G^{(2)} \right) \right) + \text{H.c.} + \ldots \\
+ \int d^8 z \, \frac{1}{32} \, \text{Tr} \, V \left( D^2 \bar{D}^2 + \bar{D}^2 D^2 \right) V \\
+ \int d^8 z \, d^8 z' \, 2i \, \text{Tr} \left( b(z) + \bar{b}(z) \right) \, G^{(2)}(z - z') \, \delta_{\epsilon,c} \left( V \ast ( -1 ) G^{(2)} \right)(z') \\
+ \int d^8 z \, 2i \, \text{Tr} \left( K \ast G^{(2)} \right)(z) \, \delta_{\epsilon,c} \left( V \ast ( -1 ) G^{(2)} \right)(z) \\
+ \int d^6 y \, \left[ Y^{ijk} \Phi_i \Phi_j \Phi_k + M^{ijk} \Phi_i \Phi_j \Phi_k \right] + \int d^6 \bar{y} \, \left[ \bar{Y}_{ij,k} \Phi_i \Phi^j \Phi^k + \bar{M}_{ij,k} \Phi_i \Phi^j \Phi^k \right] \\
+ \int d^6 y \, 2 \text{Tr} \, Lc^2 + \int d^6 \bar{y} \, 2 \text{Tr} \, \bar{L}c^2 + \int d^6 y \, k \, c \, \Phi + \int d^6 \bar{y} \, \bar{\Phi} \, \bar{c} \, \bar{k} \\
+ \int d^8 z \, \sum F(g^2, Y, \bar{Y}) \, \Phi(z) \, \nabla^\alpha \ldots \nabla^\beta \, e^{V_{+}\ast( -1 ) G^{(2)} \ast \nabla_\alpha \ldots \nabla_\beta \, e^{-V_{+}\ast( -1 ) G^{(2)}} \ast \nabla_{\dot{\alpha}} \ldots \nabla_{\dot{\beta}} \, \Phi(z). \]

Some comments about the last line of this expression which is responsible for gauge-matter interaction are necessary. First, we have taken the symbolic sum over all possible terms (15) allowed by Slavnov–Taylor identities. Second, \( F(g^2, Y, \bar{Y}) \) are some numbers which depend on couplings of the theory and can not be fixed in the framework of our approach. Third, in the covariant derivatives we have made the transformation from the variable \( \bar{V} \) to the initial variable \( V \), that is

\[
\bar{V}(z) = \int d^8 z' \, V(z') \, (-1)^{G^{(2)}}(z - z'),
\]

\[
\nabla_\alpha = e^{-V_{+}\ast( -1 ) G^{(2)}} \, D_\alpha \, e^{V_{+}\ast( -1 ) G^{(2)}}.
\]
One can check by a direct substitution that the result obtained here is the solution to the ST identity \([13]\). In case of Abelian vector superfield the ghost 2-point function turns out to be trivial \(\delta\)-function in the superspace, \(\delta(z - z')\), so that the correspondence with Abelian case is obvious. Traditional background technique \([4]\) usually used for calculations of perturbative characteristics in a general supersymmetric Yang–Mills theory can be also reproduced in the framework of the approach developed here with small modifications of the procedure. Thus, the proposed method could be useful in studying the quantum behaviour of a general supersymmetric quantum field theory with arbitrary matter contents.

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