Diffusive Josephson junctions made out of multiwalled carbon nanotubes

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Abstract. We have investigated electrical transport in diffusive multiwalled carbon nanotubes (MWNT) contacted using superconducting leads made of Ti/Al/Ti sandwich structure. We measure proximity-induced supercurrents up to $I_{cm} = 1.3$ nA and find tunability by the gate voltage due to variation of the Thouless energy via the diffusion constant that is controlled by scattering in the MWNT. The modeling of these results by long, diffusive SNS junctions, supplemented with phase diffusion effects is discussed: the agreement between theory and experiments is tested especially on the basis of the temperature dependence of the Josephson coupling energy. In order to prove conclusively that the diffusive model works for MWNT proximity junctions, we propose an improved measurement scheme that is based on the kinetic inductance of superconducting junctions.

1. Introduction

Good, proximity-induced Josephson junctions have been difficult to realize in multiwalled carbon nanotubes (MWNT). In the first experiments, enhanced conductance was observed by Buitelaar et al. [1] near zero bias, which was interpreted in terms of multiple Andreev reflections (MAR) in the presence of inelastic processes [2]. Similar results were obtained by Kasumov et al. [3]. Recently, proximity induced supercurrent has been observed by Haruyama and coworkers [4, 5], most notably in multi-shell-contacted tubes grown within nanoporous alumina templates [5].

We have observed proximity-induced supercurrents in individual, diffusive MWNTs using bulk(side)-contacted samples with Ti/Al contacts. We interpret the gate-control of the supercurrents as originating due to tuning of the Thouless energy $\epsilon_{Th} = \hbar D/l^2$ via the diffusion constant $D$ that is controlled by scattering in the MWNT. In Ref. [6], we employed an analysis based on long, diffusive SNS junctions supplemented with phase diffusion effects, modeled in terms of the resistively and capacitively shunted junction model (RCSJ). The agreement between theory and experiments was tested especially on the basis of the temperature dependence of the Josephson coupling energy. Here we re-examine our previous analysis and propose an improved measurement scheme that is based on the kinetic inductance of the superconducting nanotube junctions.

2. Experimental details

The data that we present and analyze in this paper have been obtained on a tube of 4 $\mu$m in length and 16.6 nm in diameter. The 550 nm wide contacts had a Ti/Al/Ti sandwich structure.
Figure 1. Current as a function of the bias voltage $V$ at an intermediate gate voltage value of $V_g \simeq 3.2 \, \text{V}$ where clear hysteresis is visible and the switching current $I_{sw} = 0.15 \, \text{nA}$.

with thicknesses 5/70/10 nm, respectively, and the energy gap of aluminum was suppressed down to $\Delta = 139 \, \mu\text{eV}$. The length of the tube section between the contacts was 400 nm. The electrically conducting body of the silicon substrate was employed as a back gate, separated from the sample by 150 nm of SiO$_2$. The environment was governed by Al microstrip lines, 0.5 mm Thermocoaxial cables, and RC-filters with 1 kΩ resistors. Further details on the experimental arrangement and techniques can be found in Ref. [6].

3. Results

The measured IV-curves at intermediate critical currents are illustrated in Fig. 1. There is hysteresis which, according to the RCSJ model, can be explained if phase fluctuations and their damping fulfill certain conditions: the scaled temperature $\Gamma = \frac{k_B T}{E_J}$ has to be small enough and the parameter $\beta_J = \left(\frac{\omega_p R C_{tot}}{C_{tot}}\right)^{-1}$ must be $<< 1$ where $\omega_p$ is the plasma frequency, $R$ is the shunt resistance, and $C_{tot}$ is the total capacitance involved in the plasma oscillation. In Ref. [6], we estimated $R = R_J = \frac{dV}{dI} \sim 2 \, \text{k}\Omega$ from the IV of the junction above the plateau region at the largest critical current of $I_{cm} = 1.3 \, \text{nA}$, and $C_{tot} = 400 \, \text{fF}$ from the geometry of the experimental configuration. With these parameters and, by taking into account that $R$ increases strongly when $I_{cm}$ is reduced, the hysteresis could be qualitatively explained.

In order to have a more accurate value for $C_{tot}$, we may use the analysis of the retrapping current $I_r$, which in Fig. 1 amounts to 70% of the switching current $I_{sw} = 150 \, \text{pA}$. According to Ref. [7] (see also [8, 9]), the IV-curve near the retrapping current (applicability of the limit $\beta_J << 1$ is assumed here) is given by

$$I - I_r \frac{I_r}{I} = 4\left(1 + \frac{V_0}{V}\right) \exp\left(-\frac{V_0}{V}\right)$$

where $V$ denotes the retrapping voltage and $V_0 = \frac{\hbar}{2e} \omega_p$. Using the shape of the retrapping current branch, we solve for $V_0 = 14 \, \mu\text{V}$ which corresponds to a plasma frequency of $\omega_p/2\pi = 1.1 \, \text{GHz}$. By estimating the nominal critical current $I^0_c = 3.2 \, \text{nA}$ we obtain $C_{tot} = 210 \, \text{fF}$ from the plasma resonance equation $C_{tot} = \frac{2e}{\omega_p^2}; I^0_c$ was estimated using the normal state resistance $R_n = 20 \, \text{k}\Omega$ of the junction and the product $e I^0_c R_n \simeq 7e_{Th} [10]$ with $e_{Th} = 9 \, \mu\text{eV} [6]$. The estimate for $C_{tot}$ is by a factor of two smaller than in Ref. [6] and this corroborates that the phase dynamics in these
junctions should be well described by classical phase dynamics. For the dissipative resistance we obtain $R = 1.3 \, \text{k}\Omega$ using the measured value of $\beta = \frac{\pi I_c}{4 I_{cm}} \simeq 0.55$ and the definition for it given above. This resistance indicates stronger dissipation than one would expect from the slope of the subgap IV curve; the enhanced dissipation may be caused by high frequency losses in the conducting substrate material. As this dissipative resistance is voltage dependent, it is hard to perform the phase diffusion analysis very accurately for our nanotube junctions. Moreover, the origin of the hysteresis may be partly of thermal nature [11], which further complicates the analysis based on the phase dynamics.

A crucial test for the validity of the diffusive junction model in “semiballistic” MWNTs is the temperature dependence of the Josephson coupling energy. The Josephson energy for a long diffusive junction, without interaction effects [12], can be calculated from the equation [13, 10]

$$I_c^0 = \frac{32}{3 + 2\sqrt{2} eR_n} \left( \frac{l}{l_T} \right)^3 \exp \left( -\frac{l}{l_T} \right) \tag{2}$$

which is valid in the limit $\Delta/\epsilon_{Th} \to \infty$ and where $l_T = \sqrt{\hbar D/2\pi k_B T}$. Fig. 2 illustrates how well we can account for our measurement results using Eq. (2) and the phase diffusion model. The agreement is quite good except for the lowest temperatures. The value for the diffusion constant that we obtain from the fitting is nearly equal to the value deduced from the conductance measured on the sample.

![Figure 2](image1.png)  
**Figure 2.** Temperature dependence of $I_{cm}$ (●) measured at the optimum back gate voltage value $V_g = 3.489 \, \text{V}$. The solid curve is a fit obtained from Eq. (2) and the phase diffusion model.

![Figure 3](image2.png)  
**Figure 3.** Schematics of the microwave reflection measurement setup that is employed to determine the current-phase relationships.

### 4. Discussion

As seen from the analysis above, we are able to present evidence for the diffusive junction model in MWNTs. Our analysis indicates that the gap of the contact material is irrelevant for the supercurrents, unless the contact spacing can be reduced and the ballistic limit will be reached. In order to make conclusions on these issues more solid, we have started investigations of MWNT junctions based on the kinetic inductance of the Josephson junctions.

The scheme for our kinetic inductance studies is displayed in Fig. 3. It is a microwave reflection measurement setup in which the resonant frequency of an $LC$ oscillator is modulated.
by a parallel Josephson inductance of the MWNT junction

\[ L_J^{-1}(\phi, V_g) = \left(\frac{2\pi}{\Phi_0}\right)^2 \partial^2 E(\phi, V_g)/\partial \phi^2, \]

(3)

where \( \Phi_0 = h/(2e) \) is the flux quantum. With a total shunting capacitance \( C \), the circuit thus forms a harmonic oscillator for small amplitude oscillations with the plasma frequency \( f_p = 1/(2\pi)(L_{TOT}C)^{-1/2} \) where \( L_{TOT} = L_J||L \). The change in the resonant frequency can be traced as a function of phase bias over the junction; then the current-phase relationship will be obtained by integration. The coupling capacitor \( C_c \) is chosen in such a way that the resonant circuit is rather well matched to 50 Ohms, which results in a rather small reflection amplitude at the resonance. First measurements have been performed and critical currents of 5 nA have been measured [14].

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