Entropy generation and natural convection in a wavy-wall cavity filled with a nanofluid and containing an inner solid cylinder

Mohammed Hussein S. Alnajem¹, Ammar I. Alsabery¹,² and Ishak Hashim²
¹Technical Engineering Department, College of Technical Engineering, Islamic University, Najaf, Iraq
²School of Mathematical Sciences, Faculty of Science & Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia

Corresponding author’s: ammar_e_2011@yahoo.com (Ammar I. Alsabery)

Abstract. This work investigates the problem of entropy generation and natural convection in a wavy-wall cavity filled with a nanofluid and containing solid inner cylinder using the Galerkin finite-element method. An isothermal heat source fixed at the left vertical wall of the cavity and the horizontal walls have taken adiabatic, while the right wavy wall cooled isothermally. An Al₂O₃-water nanofluid fills the space between the wavy-wall cavity and the solid cylinder. To study this problem, the influence of three variables have taken into account. These variables are Rayleigh number \((10^3 \leq Ra \leq 10^6)\), nanoparticle volume fraction \((0 \leq \phi \leq 0.04)\), and the number of undulations \((1 \leq N \leq 4)\). The numerical results can be in the forms of streamlines, isotherms and local entropy generation as well as the average Nusselt number. The obtained results indicate that the effect of the nanoparticles addition on the heat transfer rate is essential for low Rayleigh number and number of undulations.

1. Introduction
Convective heat transfer within a closed cavity is considered a significant phenomenon in a wide range of engineering applications, such as electronic cooling, cooling of containment buildings, room ventilation, heat exchangers, storage tanks, double pane windows, solar collectors, etc. However, the low thermal conductivity of the pure fluid, such as water, ethylene glycol, and oil show a limitation for the heat transfer enhancement. To avoid this limitation, the nanoparticles submerged into the pure fluid which tends to change the thermophysical property of this fluid, therefore, the heat transfer rate enhanced. The used of nanofluids can be seen in many engineering applications, for example, cooling of electronics, heat exchangers, engine cooling, nuclear reactor safety, hyperthermia, biomedicine, vehicle thermal management, and many others [1]. Natural convection and heat transfer problems of CuO-EG-water nanofluid are in a square cavity are considered by Abu-Nada and Chamkha [2]. They reported that the viscosity model effects were more dominant on the average Nusselt number compare to the thermal conductivity model effects. The steady mixed convection and heat transfer are in a cavity containing an active rotating solid cylinder considered by Costa and Raimundo [3]. Basak and Chamkha [4] numerically investigated the heatline analysis on convective heat transfer of nanofluid in a square cavity with various thermal boundary conditions. Roslan et al. [5] reported a numerical investigation of the convective heat transfer of nanofluids in a square cavity with a rotating solid cylinder. They concluded that the maximum heat transfer rate obtained by the high nanoparticles concentration. Sheremet et al. [6] studied the entropy generation and natural convection heat transfer of nanofluid within square cavity containing a hot solid block. Alsabery et al. [7] used the finite difference method to investigate the transient natural convection and heat transfer in a trapezoidal...
cavity filled with nanofluid in the presence of spatial side-wall temperature. Khanafer and Aithal [8] numerically studied the effect of a rotating circular cylinder on the mixed convective heat transfer in a lid-driven cavity. Recently, Al Sabery et al. [9] reported a numerical investigation on the effect of two-phase nanofluid model of natural convective heat transfer in a square cavity filled with a nanofluid and containing an inner solid body.

The convective heat transfer in a wavy cavity is applicable in many engineering systems, for instance: cavity wall insulating systems, solar collectors, condensers in refrigerators, grain storage containers, and industrial heat radiators, and many others applications [1]. The entropy generation and natural convection problem in a wavy-wall cavity filled with nanofluid was reported by Cho and Chen [10]. The numerical study of Abu-Nada and Chamkha [11] investigated the steady mixed convective flow and heat transfer of water-CuO nanofluid in a lid-driven wavy-wall cavity. Gibanov et al. [12] have numerically studied the problem of natural convection in a differentially heated wavy cavity filled with micropolar fluid. They concluded that an increment of the undulation number tended to reduce the heat transfer rate. Jassim et al. [13] conducted a numerical investigation of the effect of sinusoidal walls on the mixed convection and heat transfer in a square cavity containing a heated rotating cylinder. They observed that the sinusoidal walls have a strong enhancement of heat transfer in comparison with the vertical walls. Calcagni et al. [14] made an experimental investigation of the problem of convection heat transfer in a square cavity heated from the bottom wall.

According to above-mentioned studies and to the authors’ best knowledge, so far, there have been no studies of entropy generation and natural convective heat transfer in a wavy-wall cavity filled with a nanofluid and containing a solid inner cylinder. Thus, it believed that this work is valuable. The aim of this study is to investigate the entropy generation and heat transfer of Al$_2$O$_3$-water nanofluid in a wavy-wall cavity containing a solid inner cylinder.

2. Mathematical Formulation

Consider the steady natural convection problem in a wavy-wall cavity is with length $L$ and having a solid cylinder within the center with length $r$, Figure 1. An isothermal heater tends to take a position within the middle of the left vertical wall of the cavity with length $d$ while the wavy right wall is maintained at a constant cold temperature $T_c$. The remainder of the left vertical wall together with the bottom and top horizontal walls kept adiabatically. The boundaries of the domain have been taken to be impermeable; the space between the wavy cavity and the inner body filled with water-Al$_2$O$_3$ nanofluid. The Boussinesq approximation is applicable. By considering these assumptions, the governing equations of the laminar natural convection by using conservation of mass, momentum and energy equations can be shown in the dimensionless form as follows [9]:

\[
\nabla \cdot \mathbf{V} = 0 \quad (1)
\]

\[
\mathbf{V} \cdot \nabla \mathbf{V} = -\nabla P + Pr \frac{\rho_f \mu_f \nabla \mathbf{V} \cdot \nabla} {\rho_{nf} \mu_f} \nabla^2 \mathbf{V} + \frac{(\rho \beta)_{nf} \nabla \mathbf{V} \cdot \nabla} {\rho_{nf} \beta_f} \text{Ra Pr} \theta, \quad (2)
\]

\[
\mathbf{V} \cdot \nabla \theta = \frac{(\rho C_p)_{nf}}{(\rho C_p)_{f}} \frac{k_{nf}}{k_f} \nabla^2 \theta, \quad (3)
\]

\[
\nabla \theta = 0, \quad (4)
\]

where $\mathbf{V}$ is the dimensionless velocity vector $(U,V)$. 

\[
\nabla \cdot \mathbf{V} = 0 \quad (1)
\]

\[
\mathbf{V} \cdot \nabla \mathbf{V} = -\nabla P + Pr \frac{\rho_f \mu_f \nabla \mathbf{V} \cdot \nabla} {\rho_{nf} \mu_f} \nabla^2 \mathbf{V} + \frac{(\rho \beta)_{nf} \nabla \mathbf{V} \cdot \nabla} {\rho_{nf} \beta_f} \text{Ra Pr} \theta, \quad (2)
\]

\[
\mathbf{V} \cdot \nabla \theta = \frac{(\rho C_p)_{nf}}{(\rho C_p)_{f}} \frac{k_{nf}}{k_f} \nabla^2 \theta, \quad (3)
\]

\[
\nabla \theta = 0, \quad (4)
\]
Figure 1. (a) Physical model of convection in a wavy cavity together with the coordinate system and (b) 3D schematic diagram of the physical model together with the solid inner cylinder.

The governing equations of Navier Stokes equations Eqs. (1-4) are transformed into dimensionless forms using the following dimensionless variables:

\[
X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad V = \frac{vL}{\nu_f}, \quad \theta = \frac{T_w - T_c}{T_h - T_c}, \quad \alpha = \frac{T_c - T_i}{T_h - T_c}, \quad R = \frac{r}{L},
\]

\[
D = \frac{d}{L}, \quad \text{Pr} = \frac{\nu_f}{\alpha_f}, \quad \text{Ra} = \frac{g \beta_f (T_h - T_c) L^3}{\nu_f \alpha_f}, \quad P = \frac{pL^2}{\rho \alpha_f}.
\]

The dimensionless boundary conditions corresponding to Eqs. (1-4) given by:

\[
U = V = 0, \text{ for all of the cavity walls.}
\]

On the heated part of the left wall:

\[
\theta = 1, \quad X = 0, \quad (1 - D) / 2 \leq Y \leq (1 + D) / 2,
\]

On the adiabatic parts of the left wall:

\[
\frac{\partial \theta}{\partial X} = 0, \quad X = 0, \quad 0 \leq Y \leq (1 - D) / 2 \text{ and } (1 + D) / 2 \leq Y \leq 1,
\]

On the right wavy wall:

\[
\theta = 0, \quad A(1 - \cos(2N\pi X)), \quad 0 \leq Y \leq 1,
\]

On the adiabatic bottom and top walls:

\[
\frac{\partial \theta}{\partial X} = \frac{\partial \theta}{\partial Y} = 0, \quad 0 \leq X \leq 1, \quad Y = 0, Y = 1,
\]

\[
\theta = \theta_0, \text{ at the outer solid cylinder surface,}
\]

\[
\frac{\partial \theta}{\partial n} = K_r \frac{\partial \theta}{\partial n},
\]

where \(K_r = k_s/k_{nf}\) is the thermal conductivity ratio over the surface of the solid cylinder. The thermo-physical properties of the nanofluid are: heat capacitance \((\rho C_p)_{nf}\), effective thermal
diffusivity \((\alpha_{nf})\), effective density \((\rho_{nf})\), thermal expansion coefficient of the nanofluids \((\beta_{nf})\), the ratio of the thermal conductivity \(\left(\frac{k_{nf}}{k_f}\right)\) and the ratio of the dynamic viscosity \(\left(\frac{\mu_{nf}}{\mu_f}\right)\) which can be defined sequentially as follows \([19, 15]\):

\[
\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_f},
\]

\[
\rho_{nf} = (1-\phi)\rho_f + \phi\rho_p,
\]

\[
\beta_{nf} = (1-\phi)\beta_f + \phi\beta_p,
\]

\[
\left(\frac{k_{nf}}{k_f}\right) = 1 + 4.4Re^{0.4}Pr^{0.66}\left(\frac{T}{T_p}\right)^{10}\left(\frac{k_p}{k_f}\right)^{0.03}\phi^{0.66},
\]

\[
\left(\frac{\mu_{nf}}{\mu_f}\right) = 1/\left(1 - 34.87\left(d_p/d_f\right)^{0.3}\phi^{1.03}\right).
\]

The local Nusselt number evaluated at the heated part of the left vertical wall is defined by:

\[
Nu_{nf} = k_{nf} \left(\frac{\partial \theta}{\partial X}\right)_{X=0}. \tag{18}
\]

Finally, the average Nusselt number evaluated at the heated part of the left vertical wall of the cavity is given by:

\[
\bar{Nu}_{nf} = \int_{X=0}^{1+D/2} Nu_{nf} \, dY. \tag{19}
\]

The expression of local entropy generation in the dimensionless form is as follows:

\[
S_{GEN} = \frac{k_{nf}}{k_f} \left[\left(\frac{\partial \theta}{\partial X}\right)^2 + \left(\frac{\partial \theta}{\partial Y}\right)^2\right] + \frac{\mu_{nf}}{\mu_f} N_p \left[2 \left(\frac{\partial U}{\partial X}\right)^2 + \left(\frac{\partial V}{\partial Y}\right)^2\right] + \left(\frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 V}{\partial X^2}\right)^2\right]. \tag{20}
\]

The terms of Eq. (20) can be separated according to the following form:

\[
S_{GEN} = S_\theta + S_{\psi}, \tag{21}
\]

where \(S_\theta\) and \(S_{\psi}\) are the entropy generation due to heat transfer irreversibility (HTI) and the fluid friction irreversibility (FFI), respectively.

\[
S_\theta = \frac{k_{nf}}{k_f} \left[\left(\frac{\partial \theta}{\partial X}\right)^2 + \left(\frac{\partial \theta}{\partial Y}\right)^2\right],
\]

\[
S_{\psi} = \frac{\mu_{nf}}{\mu_f} N_p \left[2 \left(\frac{\partial U}{\partial X}\right)^2 + \left(\frac{\partial V}{\partial Y}\right)^2\right] + \left(\frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 V}{\partial X^2}\right)^2\right], \tag{22}
\]

and the Bejan number is defined as:
The governing dimensionless equations (1-4) subject to the boundary conditions (6-11) solved with the Galerkin weighted residual finite element method. The computational domain discretized into triangular elements as shown in Figure 2. Triangular Lagrange finite elements of different orders used for each of the flow variables within the computational domain. Residuals for each conservation equation obtained by substituting the approximations into the governing equations. To simplify the nonlinear terms in the momentum equations, a Newton-Raphson iteration algorithm used. The convergence of the solution assumed when the relative error for each of the variables satisfies the following convergence criteria:

\[
\frac{\Gamma_{i+1} - \Gamma_i}{\Gamma_{i+1}} \leq \eta,
\]

where \(i\) represents the iteration number and \(\eta\) is the convergence criterion. In this study, the convergence criterion set at \(\eta = 10^{-6}\).

For the intention of validating the data, the figures of the current work are compared with the ones reported by Costa and Raimundo [3] for the case of mixed convection in a square cavity with an inner solid cylinder and heated from sides, as shown in Figure 3. These results provide confidence in the accuracy of the present numerical method.
3. Results and Discussion

In this section, numerical results for the streamlines, isotherms and local entropy generation with various values of Rayleigh number ($10^3 \leq Ra \leq 10^6$), nanoparticle volume fraction ($0 \leq \phi \leq 0.04$), number of oscillations ($1 \leq N \leq 4$) reported. The values of the amplitude, the thermal conductivity of the solid cylinder (brickwork), the dimensionless radius of rotating cylinder, dimensionless length of the surface of the cylinder and Prandtl number fixed at $A = 0.1$, $k_s = 0.76$, $R = 0.2$, $\Theta = 360$, and $Pr = 4.623$. The values of $\overline{Nu}_{nf}$ calculated for different values of $Ra$ and $\phi$. The thermophysical properties of the pure fluid and solid $\text{Al}_2\text{O}_3$ phases shown in Table 1.

Table 1. Thermo-physical properties of water with $\text{Al}_2\text{O}_3$ nanoparticles at $T=310$ K.

| Physical properties | Fluid phase (water) | $\text{Al}_2\text{O}_3$ |
|---------------------|---------------------|------------------------|
| $C_p$, (J/kg K)     | 4178                | 765                    |
| $\rho$, (kg/m$^3$)  | 993                 | 3970                   |
| $k$, (W m$^{-1}$ K$^{-1}$) | 0.628            | 40                     |
| $\beta \times 10^5$, (1/K) | 36.2               | 0.85                   |
| $\mu \times 10^6$, (kg/ms) | 695                | -                      |
| $d_p$, (nm)         | 0.385               | 33                     |

Figure 4 illustrates the effects of various values of Rayleigh number on the streamlines, isotherms and local entropy generation maps for $\phi = 0.02$ and $N = 3$. At low Rayleigh number ($Ra = 10^3$), the flow within the wavy-wall cavity is characterized by three streamlines cells, two cells in clockwise direction, which appear close to the left wall while one cell anticlockwise direction located on the wavy right wall. The isotherm patterns appear with very low density next to the cold wavy wall while
in the middle of the cavity and the solid cylinder, the isotherm patterns tend to take almost a vertical line. The entropy generation of the considered cavity is primarily due to heat transfer irreversibility and it is almost negligible near the upper and lower portions due to insignificant heat flow in that region and the slow velocity. The increasing in Rayleigh number tends to increase the intensity of the streamlines and the number of cells due to the buoyancy forces are stronger compared to the viscous forces. The intensity of the isotherm patterns increases and occupied more space within the wavy-wall cavity. The isotherm patterns within the solid cylinder occur with horizontal lines. The local entropy generation is clearly higher compared to that of the low Rayleigh number, and the upper of the cavity participates in the entropy generation. Where the higher \( Ra \) lead to stronger fluid circulation, which causes high-velocity gradients. Therefore, more entropy generation due to fluid friction, as shown in Figure 4d.

![Figure 4](image_url)

**Figure 4.** Variations of the (left) streamlines, (middle) isotherms, and (right) local entropy generation evolution by \( Ra \) for \( \phi = 0.02 \) and \( N = 3 \).

Figure 5 presents the effects of various values of oscillation \( (N) \) on the streamlines, isotherms and local entropy generation maps for \( Ra = 10^5 \) and \( \phi = 0.02 \). From this figure, it observed that the number of oscillation strongly influenced the geometric shape of the flow cell, as well as the
distribution of isotherms and local entropy generation. The flow within the wavy-wall cavity shows four streamlines cells at the upper and lower segments. The isotherm patterns observe with a high density next to walls of the wavy-wall. The distribution of the entropy generation shows a weak behavior within the cavity, entropy generation lines show a high density within the wavy-wall cavity affected by the lower thermal gradient. The intensity of the streamlines increases with the increase in the number of oscillation, and due to that, the strength of the flow circulation increases. The density of the isotherm patterns increases close to the right wavy wall with the increasing of the number of oscillation due to the increment of the gradient of the boundary layer. Increasing the number of oscillation up to the high values enhanced the entropy generation due to the strong fluid circulation. More lines move toward the vertical left wall and the right wavy wall.

Figure 5. Variations of the (left) streamlines, (middle) isotherms, and (right) local entropy generation evolution by $N$ for $Ra = 10^5$ and $\phi = 0.02$.

Figure 6(a) presents the effects of $\phi$ on the average Nusselt number with $Ra$ for $N = 3$. It observed that the heat transfer rate rises with increasing of Rayleigh number due to the increment of the buoyancy forces compared to the viscous forces. The heat transfer rate has enhanced with increasing nanoparticle volume fraction. However, at $Ra \geq 10^4$ there is an optimum volume fraction of
nanoparticles for a maximum heat transfer rate. This is because the addition of nanoparticles causes an improvement in both thermal conductivity and viscosity of the fluid. Figure 6(b) shows the effects of various values of $\phi$ on Bejan number with $Ra$. The average Bejan number decreases as Rayleigh number increases which clarify the dominance of the irreversibility due to the fluid friction with the high Rayleigh numbers.

![Graph](image)

**Figure 6.** Variations of (a) the average Nusselt number and (b) Bejan number with $Ra$ for different $\phi$ at $N = 3$.

Figure 7(a) explains the effects of $N$ on the average Nusselt number with $\phi$ for $Ra = 10^5$. The heat transfer rate has augmented with the growth of the nanoparticle volume fraction. The values of average Nusselt number denote that the heat transfer rate increases and then decreases with the increment of nanoparticle volume fraction. This is due to the various flow velocity speeds with different amount of nanoparticles. The average Nusselt number is maximum for $N = 2$ affected by the available space between the solid cylinder and the nanofluid segment. Figure 7(b) illustrates the effects of various values of $N$ on Bejan number with $\phi$ for $Ra = 10^5$. It observed an enhancement on average Bejan number with the growing of nanoparticle volume fraction. The enhancement on the average Bejan number has obtained with the high numbers of oscillations.

![Graph](image)

**Figure 7.** Variations of (a) the average Nusselt number and (b) Bejan number with $\phi$ for different $N$ at $Ra = 10^5$.

4. Conclusions

Natural convection and entropy generation in a wavy-wall cavity filled with a nanofluid and containing a solid inner cylinder has considered in this work. The space between the wavy-wall cavity and the solid cylinder has filled with an Al$_2$O$_3$-water nanofluid. The problem was solved using the
Galerkin weighted-residual finite-element method. The main conclusions of the study can be summarized as follows:

The inner solid cylinder tends to influence the flow behavior, temperature distribution and local entropy generation due to the resistance of the solid cylinder to the conductive heat transfer. The heat transfer rate is an increment function of the nanoparticles volume fraction. Average Bejan number reduces by the increment of the Rayleigh number when the fluid friction irreversibility (FFI) is dominant.

Nomenclature

\begin{align*}
A & \quad \text{amplitude} \\
C_p & \quad \text{specific heat capacity} \\
D & \quad \text{length of the heat source} \ (D = 0.5) \\
d_f & \quad \text{diameter of the base fluid molecule} \\
d_p & \quad \text{diameter of the nanoparticle} \\
k & \quad \text{thermal conductivity} \\
L & \quad \text{width and height of the cavity} \\
N & \quad \text{number of oscillations} \\
\rho & \quad \text{density} \\
\nu & \quad \text{kinematic viscosity} \\
\mu & \quad \text{dynamic viscosity} \\
\theta & \quad \text{dimensionless temperature} \\
p & \quad \text{solid nanoparticles} \\
f & \quad \text{base fluid} \\
c & \quad \text{cold} \\
nf & \quad \text{nanofluid} \\
s & \quad \text{solid cylinder}
\end{align*}

Greek symbols

\begin{align*}
\phi & \quad \text{Volume fraction}
\end{align*}

References

[1] Shenoy A, Sheremet M, and Pop I 2016 Convective Flow and Heat Transfer from Wavy Surfaces \textit{Viscous Fluids, Porous Media, and Nanofluids}.

[2] Abu-Nada E and Chamkha A J 2010 Effect of nanofluid variable properties on natural convection in enclosures filled with a cuo–eg–water nanofluid \textit{International Journal of Thermal Sciences} \textbf{49} 2339–2352.

[3] Costa V and Raimundo A 2010 Steady mixed convection in a differentially heated square enclosure with an active rotating circular cylinder \textit{International Journal of Heat and Mass Transfer} \textbf{53} 1208–1219.

[4] Basak T and Chamkha A J 2012 Heatline analysis on natural convection for nanofluids confined within square cavities with various thermal boundary conditions \textit{International Journal of Heat and Mass Transfer} \textbf{55} 5526–5543.

[5] Roslan R, Saleh H, and Hashim I 2012 Effect of rotating cylinder on heat transfer in a square enclosure filled with nanofluids \textit{International Journal of Heat and Mass Transfer} \textbf{55} 7247–7256.

[6] Sheremet M A, Oztop H F, Pop I and Abu-Hamdeh N 2015 Analysis of entropy generation in natural convection of nanofluid inside a square cavity having hot solid block: Tiwari and das model \textit{Entropy} \textbf{18} 9.

[7] Alsabery A I, Hashim I, Chamkha A J, Saleh H and Chanane B 2017 Effect of spatial side-wall temperature variation on transient natural convection of a nanofluid in a trapezoidal cavity \textit{International Journal of Numerical Methods for Heat & Fluid Flow} \textbf{27} 1365–1384.

[8] Khanafer K and Aithal S 2017 Mixed convection heat transfer in a liddriven cavity with a rotating circular cylinder \textit{International Communications in Heat and Mass Transfer} \textbf{86} 131–142.

[9] Alsabery A I, Sheremet M, Chamkha A J and Hashim I 2018 Conjugate natural convection of Al\textsubscript{2}O\textsubscript{3}–water nanofluid in a square cavity with a concentric solid insert using buongiornos two-phase model \textit{International Journal of Mechanical Sciences} \textbf{136} 200–219.
[10] Cho C and Chen C 2013 Natural convection heat transfer and entropy generation in wavy-wall enclosure containing water-based nanofluid *International Journal of Heat and Mass Transfer* **61** 749–758.

[11] Abu-Nada E and Chamkha A J 2014 Mixed convection flow of a nanofluid in a lid-driven cavity with a wavy wall *International Communications in Heat and Mass Transfer* **57** 36–47.

[12] Gibanov N S, Sheremet M A and Pop I 2016 Natural convection of micropolar fluid in a wavy differentially heated cavity *Journal of Molecular Liquids* **221** 518–525.

[13] Jassim H M, Ali F H, Mahdi Q A, and Hadi N J 2017 Effect of parallel and orthogonal sinusoidal walls on mixed convection inside square enclosure containing rotating cylinder in Mechanical and Aerospace Engineering (ICMAE) 8th International Conference on. IEEE, pp. 365–370.

[14] Calcagni B, Marsili F, and Paroncini M 2005 Natural convective heat transfer in square enclosures heated from below *Applied Thermal Engineering* **25** 2522–2531.

[15] Corcione M 2001 Empirical correlating equations for predicting the effective thermal conductivity and dynamic viscosity of nanofluids *Energy Conversion and Management* **52** 789–793.

**Acknowledgment**

The work was supported by the Universiti Kebangsaan Malaysia (UKM) research grant DIP-2017-010. We thank the respected reviewers for their constructive comments which clearly enhanced the quality of the manuscript.