Developing Guided Worksheet for Cognitive Apprenticeship Approach in teaching Formal Definition of The Limit of A Function

R Oktaviyanthi\textsuperscript{1,2}, J A Dahlan\textsuperscript{3}

\textsuperscript{1}School of Postgraduate Studies, Doctorate Program of Mathematics Education, Universitas Pendidikan Indonesia, Jl. Dr. Setiabudhi No. 229, Bandung, Jawa Barat 40164, Indonesia
\textsuperscript{2}Math Education Department, Universitas Serang Raya, Jl. Raya Serang – Cilegon Km. 5, Taman Drangong, Serang, Banten 42314, Indonesia
\textsuperscript{3}Department of Mathematics Education, Universitas Pendidikan Indonesia, Jl. Dr. Setiabudhi No. 229, Bandung, Jawa Barat 40164, Indonesia

\textsuperscript{1}rinaokta@student.upi.edu, \textsuperscript{2}rinaokta@unsera.ac.id, \textsuperscript{3}jarnawi@upi.edu

Abstract. This study aims to develop student worksheets that correspond to the Cognitive Apprenticeship learning approach. The main subject in this student worksheet is Functions and Limits with the branch of the main subject is Continuity and Limits of Functions. There are two indicators of the achievement of this learning that are intended to be developed in the student worksheet (1) the student can explain the concept of limit by using the formal definition of limit and (2) the student can evaluate the value of limit of a function using epsilon and delta. The type of research used is development research that refers to the development of Plomp products. The research flow starts from literature review, observation, interviews, work sheet design, expert validity test, and limited trial on first-year students in academic year 2016-2017 in Universitas Serang Raya, STKIP Pelita Pratama Al-Azhar Serang, and Universitas Mathla’ul Anwar Pandeglang. Based on the product development result obtained the student worksheets that correspond to the Cognitive Apprenticeship learning approach are valid and reliable.

1. Introduction

Research on the students’ understanding in the concept of limit of a function has been studied by researchers for decades. Some research focuses on how the understanding acquired by students and common misconceptions are made through the teaching of limit of a function and its reflection [1]. Other studies take attention to the perception of students on the practice of teaching that students have been done and its impact on the development of students’ understanding on the concept of limit of a function [2]. Another case emphasizes the design of learning a limit of a function that can be used as an alternative to teach it [3][4]. In Indonesia, the research area on the development of mathematics teaching materials about a certain material has grown almost more than ten years and still become one of the research trends. The development of instructional materials is aimed to support the implementation of learning that is adapted to the standard competencies to be achieved, the learning approach used and the indicators of learning achievement designed by the teacher.
The standard competencies to be accomplished in Calculus course with the subject matter of Function and Limit in Mathematics Education Program of Universitas Serang Raya is that students can recognize the basic concept of limit, arrange the calculation using limit and apply both in real-world problem solving by optimizing adaptive reasoning and decision making. The basic competencies to be realized that students are able to understand the limit of a function by formal definition and apply it in evaluating the limit values.

This study is part of a research on the development of mathematics teaching materials using the Cognitive Apprenticeship learning approach which is discussed separately in other papers. At a glance at the Cognitive Apprenticeship approach, Cognitive Apprenticeship instruction is more effective in teaching technical math than traditional instruction because it teaches learners to become problem solvers, emphasizes application-based issues, and encourages collaborative problem solving [5][6]. The Cognitive Apprenticeship contains interconnected aspects of the learning environment: modeling and explaining, multiple perspectives, exploration, self-study, collaboration and social negotiation, and authentic learning activities [7]. The development and implementation of Cognitive Apprenticeship by many researchers shows that Cognitive Apprenticeship has contributed to the problem-solving process.

The instructional material presented in this discussion is a student worksheet that corresponds to the Cognitive Apprenticeship learning approach that has the stages of modeling, coaching, scaffolding, articulation, reflection and exploring activities. The achievement indicator that became the focus of this worksheet design is that students are expected to (1) apply the concept of limit of a function by formal definition and (2) determine and evaluate the limit of a function using ε-δ. Based on the indicators of achievement, the objectives of this worksheet are (1) to guide the thoughts flow of the students in understanding the structure of the formal definition of limit of a function, (2) to guide the student's work steps to be directed and avoid misconceptions, and (3) to give a complete representation of the procedure of evaluating the limit using a formal definition of limit. Therefore the research question in this study is how to develop a guided worksheet in evaluating a limit of a function using a formal definition of the limit corresponding to the Cognitive Apprenticeship learning approach.

2. Methods
This study is a development research that refers to the development of Plomp products with the stages of activity are literature review, field observation, design of guided worksheets, expert validation and limited testing [8]. Stages of literature review, field observation and design of guided worksheets produce prototype worksheets which are examples presented in the table below.

| Table 1. Example prototype of guided worksheet |
| Visualization of guided worksheet | Description |
| --- | --- |
| [Diagram showing modeling process] | Modeling, the lecturer explains the way of the formal definition and shows how to solve the problem so that students can observe, understand and build the crucial conceptual modeling process. |
Coaching lecturers guide students through the activities of guiding directions and stages of assistance, evaluating activities and diagnosing students’ difficulties, providing challenges and providing feedback.

Scaffolding, teachers support students through increasing the complexity of the problem and reducing the level of assistance so that students can complete their tasks independently.

The next procedure is validation made by experts both in terms of language and appropriateness of the problem with the indicator. In this study involved five experts consisting of a doctorate in mathematics, two math educations and two senior lecturers of Calculus. To distinguish the consistency of the validation results made the expert then used statistical test of Q-Cochran.

The last stage is limited trials to 90 first year students who take Calculus courses from three private universities in Banten namely Universitas Serang Raya, STKIP Pelita Pratama Al-Azhar Serang, and Universitas Mathla'ul Anwar Pandeglang. This limited trial aims to obtain validity and reliability of guided worksheets according to the preferred standards.
3. Results
To test the result of the validity of the work sheet written by the five valuators, used statistical test of Q-Cochran with hypothesis formulation as follows:

$H_0$: All five valuators give uniform consideration
$H_1$: All five valuators give different uniform consideration

The testing criteria for hypothesis formulation is that if the Asymp.Sig value is greater than $\alpha = 0.05$ then $H_0$ is accepted, meaning all five valuators give a uniform consideration.

**Table 2. Test the uniformity of the face validity of the guided work sheet**

| N   | Cochran's Q | Asymp. Sig. |
|-----|-------------|-------------|
| 6   | 6.857$^a$   | 0.144       |

*a. 1 is treated as a success.*

The table above shows the result that Asymp value. Sig = 0.144 is greater than $\alpha = 0.05$. Thus $H_0$ is accepted and $H_1$ is rejected, so it can be concluded that all five valuators have given uniform consideration to the validity of the face for this guided worksheet. This means that the use of language in the worksheet has met the effectiveness and then viewed the test of the validity and content of the guided worksheet. With the same steps and procedures, the results of the fifth valuator consideration for the contents obtained results as follows.

**Table 3. Test the uniformity of the validity of the contents of the guided worksheet**

| N   | Cochran's Q | Asymp. Sig. |
|-----|-------------|-------------|
| 6   | 5.007$^a$   | 0.287       |

*a. 1 is treated as a success.*

The result of Q-Cochran statistic test in table 3 shows that the value of Asymp. Sig = 0.287 is greater than $\alpha = 0.05$. Thus $H_0$ is accepted and $H_1$ is rejected so it can be concluded that all five valuators have given uniform consideration to the content validity for the guided worksheet.

Furthermore, after the uniformity test of the valuator, followed by a limited trial to 90 students who follow the Calculus course to obtain the validity and reliability of the worksheet according to the desired standards. For the validity of each item in the guided worksheet, the test criterion used is if $r_{xy} \geq r_{table}$ then it can be said the matter is valid. As for reliability, the test criteria used is if $R_{11} \geq r_{table}$, then the matter is said reliable. From the test results obtained data on the validity and reliability of guided worksheets as follows.

**Table 4. Test results validity and reliability of guided worksheets**

| Nomor Soal | Validitas | Keterangan | Reliabilitas | Kategori |
|------------|-----------|------------|--------------|----------|
|            | $r_{xy}$  |            | $R_{11}$     |          |
| 1          | 0.535     | Valid      |              |          |
| 2          | 0.432     | Valid      |              |          |
| 3          | 0.705     | Valid      |              |          |
| 4          | 0.518     | Valid      | 0.807        | Sangat Tinggi |
| 5          | 0.279     | Valid      |              |          |
| 6          | 0.324     | Valid      |              |          |

Table 4 shows that the questions in the guided worksheet can be used because all given questions are valid and have very high reliability.
4. Conclusion
Based on the results of the discussion it can be concluded that the guided worksheet in evaluating limit of a function using the formal definition of limit corresponding to the Cognitive Apprenticeship learning approach are valid and reliable, means that it can be used on a wider sample.

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