Axion Landscape and Natural Inflation

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Abstract

Multiple axions form a landscape in the presence of various shift symmetry breaking terms. Eternal inflation populates the axion landscape, continuously creating new universes by bubble nucleation. Slow-roll inflation naturally takes place after the tunneling event, because a very flat direction with a super-Planckian decay constant arises due to the alignment mechanism. We study the vacuum structure as well as possible inflationary dynamics in the axion landscape scenario, and find that the inflaton dynamics is given by either natural or multi-natural inflation. In the limit of large decay constant, it is approximated by the quadratic chaotic inflation, which however is disfavored if there is a pressure toward shorter duration of inflation. If the spectral index and the tensor-to-scalar ratio turn out to be different from the quadratic chaotic inflation, there might be observable traces of the bubble nucleation. Also, the existence of small modulations to the inflaton potential is a common feature in the axion landscape, which generates a sizable and almost constant running of the scalar spectral index over CMB scales.
I. INTRODUCTION

Our Universe probably experienced the inflationary expansion at an early stage of the evolution [1–5]. The temperature and polarization anisotropies of the cosmic microwave background (CMB) encode the detailed information of the slow-roll inflationary dynamics [6, 7]. In particular, if the primordial B-mode polarization of the CMB is detected, it will be an important milestone toward a proof of inflation.

Recently the BICEP2 collaboration announced the detection of the B-mode polarization which could be due to the primordial gravitational wave with tensor-to-scalar ratio \( r = 0.20^{+0.07}_{-0.05} \) [8]. The level of dust contamination, however, has turned out to be rather high according to the Planck polarization data at 353 GHz [9], and an ongoing joint analysis of the Planck and BICEP2 data sets will clarify how much of the BICEP2 signal is due to the polarized dust emission. Still, a small but non-negligible fraction of the detected B-mode polarization might be due to the tensor mode, for instance, \( r \sim 0.1 \) may be allowed or even preferred [10] after subtraction of the dust contribution. Furthermore, even if the large fraction of the BICEP2 signal arises from dust emission, \( r = 0.01 \sim 0.1 \) will still be allowed [11, 12], and \( r \) of this order would point definitively to large field inflation such as chaotic inflation [13], where the inflaton field excursion exceeds the Planck scale [14]. Thus, there is still a room for large-field inflation, which is consistent with the current observations and will be probed by future satellite, balloon-borne, and ground-based CMB polarization experiments.

The central issue in large field inflation models is how to control the inflaton potential over super-Planckian field ranges. One plausible possibility is to introduce an approximate shift symmetry on the inflaton. The simplest inflation model along this line is natural inflation with the potential [15]

\[
V(\phi) = \Lambda^4 \left( 1 - \cos \left( \frac{\phi}{f} \right) \right),
\]

where the cosine potential is induced by some non-perturbative effects. In order to be consistent with the Planck data, the decay constant \( f \) must satisfy \( f \gtrsim 5M_P \) [16], where \( M_P \simeq 2.4 \times 10^{18} \text{GeV} \) is the reduced Planck mass.\(^1\) The inflationary energy scale is about \( 2 \times 10^{16} \text{GeV} \) close to the GUT scale. Thus, it is important to build a concrete inflation

\(^1\) There is no observational lower bound on the decay constant in the multi-natural inflation [17, 18].
model in a UV theory such as string theory (see e.g. Refs. [19–27], and also Ref. [28] for a recent view.).

The string theory is a promising candidate of unified theory for describing the quantum gravity. There appear many axions through compactification of the extra dimensions, and some of them may remain relatively light and play an important role in cosmology. In particular, one of such string axions could be the inflaton with the above potential (1). The required large decay constant, however, is not straightforward to realize because the fundamental decay constant of the string axions is no larger than the Planck scale in the limit of a weak coupling or a large extra dimension (see e.g. Refs. [29–31], also Ref. [32] for a strong coupling case, and Ref. [33] for a case with a warped extra dimension).

If there are two (or more) axions, the effective decay constant can be enhanced by the so-called Kim-Nilles-Peloso (KNP) alignment mechanism [34]; the effective decay constant can be super-Planckian, even if the original ones are sub-Planckian. The KNP mechanism has attracted much attention especially after the BICEP2 result and it has been studied from various aspects [18, 35–45]. The original KNP mechanism [34] relied upon two axions with sub-Planckian decay constants, and a relatively large hierarchy in the anomaly coefficients was required for successful inflation. It was shown in Ref. [37] that, if there are more than two axions, large enhancement is possible even with the anomaly coefficients of order unity. The probability for the enhancement was studied in a case with hierarchical coefficients of the cosine functions [37] as well as general cases including the case in which the number of cosine functions $N_{\text{source}}$ is different from the number of axions $N_{\text{axion}}$ [38].

Multiple axions form a landscape if there are various shift symmetry breaking terms with $N_{\text{source}} > N_{\text{axion}}$, as the present authors proposed in Ref. [38]. Eternal inflation will populate a large number of local minima, continuously creating new universes by bubble nucleation [46]. Slow-roll inflation naturally takes place after the tunneling event, because a very flat direction with a super-Planckian decay constant arises due to the KNP mechanism. Thus, eternal inflation as well as the slow-roll inflation that follows the bubble nucleation can be realized in a unified manner in the axion landscape, while there is no clear connection between these two in the string landscape paradigm [47, 48]. Furthermore, since the flat direction appears accidentally, there is a pressure toward shorter duration of the slow-roll inflation [38]. If the total e-folding number is just about 50 or 60, it may be possible to

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2 In the most of the literature, the relation $N_{\text{axion}} = N_{\text{source}}$ was assumed.
observe some traces of the bubble nucleation such as negative curvature [49–53] and/or suppression of density perturbations at large scales [54–57].

The purpose of this paper is to study further the vacuum structure as well as possible inflationary dynamics in the axion landscape, both of which have not been examined in detail so far. We will first show that there are indeed numerous local minima in the axion landscape and that the energy density at local minima approaches a Gaussian distribution as $N_{\text{source}}$ becomes larger for a fixed $N_{\text{axion}}$. Next we examine the inflaton potential along the lightest direction in the axion landscape. As we shall see below, the possible inflaton dynamics depends on the properties of the cosine functions and the strength of pressure toward shorter duration of inflation. As a result, the predicted values of the spectral index and the tensor-to-scalar ratio are correlated with the existence or absence of the measurable remnant of the bubble nucleation.

II. AXION LANDSCAPE

We consider multiple axions with the following potential:

$$V(\phi_\alpha) = \sum_{i=1}^{N_{\text{source}}} \Lambda_i^4 \left( 1 - \cos \left( \sum_{\alpha=1}^{N_{\text{axion}}} n_{i\alpha} \frac{\phi_\alpha}{f_\alpha} + \theta_i \right) \right) + C,$$

where $\Lambda_i$ represents the dynamical scale of each non-perturbative effect, $n_{i\alpha}$ is an integer-valued anomaly coefficient matrix, $f_\alpha$ is the decay constant, $\phi_\alpha$ is the axion which satisfies a discrete shift symmetry,

$$\phi_\alpha \rightarrow \phi_\alpha + 2\pi f_\alpha,$$

and $\theta_i \in [0, 2\pi)$ denotes a CP phase of each non-perturbative effect. The constant term $C$ is adjusted so that the cosmological constant almost vanishes in the present Universe.

Now we study the vacuum structure in the axion landscape. The vacuum structure crucially depends on the values of $N_{\text{source}}$ and $N_{\text{axion}}$. In the case of $N_{\text{source}} < N_{\text{axion}}$, there are $N_{\text{axion}} - N_{\text{source}}$ flat directions implying continuous vacuum degeneracy, as long as the combination of the axions appearing in each cosine function is different from one another; if two of the combination are proportional to each other, i.e., $n_{i\alpha}/n_{j\alpha}$ is independent of $\alpha$ for some $i$ and $j$, there will be another flat direction. If $N_{\text{source}} = N_{\text{axion}}$, there are discrete potential minima with degenerate energy. If $N_{\text{source}} > N_{\text{axion}}$, there are many local minima
with different energy. In the following we focus on the last case to examine the distribution of the local minima for various $N_{\text{source}}$ and $N_{\text{axion}}$.

In order to study the energy density distribution of the local minima, we simplify the axion potential (2) by setting $\Lambda_i = \Lambda$ and $f_\alpha = f$ for all $i$ and $\alpha$ and $C = 0$, and generate an integer-valued $N_{\text{source}} \times N_{\text{axion}}$ random matrix $n_{i\alpha}$ satisfying a constraint $|n_{i\alpha}| \leq 3$. Note that the structure of the axion potential does not depend on $C$. We have also generated a random real number between 0 and $2\pi$ for the relative phase $\theta_i$. Those simplifications are assumed in the following numerical analysis unless stated otherwise.

The KNP mechanism can be implemented if there appears an extremely light direction in the axion landscape.\(^3\) If all the relative phases are zero (or if they can be absorbed by the shift of the axions), this is the case if the smallest eigenvalues of the matrix $M_{\alpha\beta} \equiv \sum_i n_{i\alpha} n_{i\beta}$ happens to be much smaller than the others [38]. Let us denote by $R(\ll 1)$ the ratio of the smallest eigenvalue to the next smallest one. Note here that the eigenvalues of $M$ are non-negative. In general, all the relative phases cannot be absorbed by the shift of the axions if $N_{\text{source}} > N_{\text{axion}}$. Still, as we shall see shortly, when $R \gg 1$, there appears a very light direction whose typical decay constant is enhanced by a factor of $1/\sqrt{R}$. This can be understood as follows. First let us consider the case of $N_{\text{source}} = N_{\text{axion}}$. Then, $M_{\alpha\beta}$ is proportional to the mass matrix at the points where all the cosine functions are minimized. Since the typical potential height is of order $\Lambda^4$, $R \ll 1$ implies that the lightest direction has an enhanced decay constant. Now let us add an extra cosine function of some combination of the axions. Then, the matrix $M_{\alpha\beta}$ is no longer proportional to the mass matrix in the actual axion landscape, because all the cosine functions can not be generically minimized simultaneously. In this case, $R \ll 1$ implies that the combination of the axions appearing in the extra cosine function should be more or less orthogonal to the lightest direction obtained in the case of $N_{\text{source}} = N_{\text{axion}}$. Thus, the KNP mechanism can be implemented by requiring $R \ll 1$ even for the case of $N_{\text{source}} > N_{\text{axion}}$ with a non-zero relative phase. This argument will be valid unless $N_{\text{source}} - N_{\text{axion}}$ is not significantly larger than $N_{\text{axion}}$. In the following we use this condition to generate an axion landscape where there is a flat direction, but our results do not depend on how we implement the KNP mechanism in the axion landscape.

The energy density distribution of the local minima is shown in Fig. 1. Here the number

\(^3\) Note that this is because the typical potential height is of order $\Lambda^4$. If the dynamical scales are hierarchical, the lightest direction must be much lighter than the typical mass scale for the lightest direction, in order to realize the KNP mechanism.
of axions is fixed to be $N_{\text{axion}} = 8$, and we have varied the number of cosine functions as $N_{\text{source}} = 9, 11$ and 13. We have repeatedly generated the random matrix for $n_{i\alpha}$ until $R$ becomes smaller than $10^{-2}$ so as to implement the KNP mechanism. In each case we have searched for local minima in the vicinity of a randomly chosen initial position in the field space, and we have repeated this process until distribution of the found local minima converges. As a result we have found 14025, 30194 and 22939 minima for $N_{\text{source}} = 9, 11$ and 13, respectively. We can see from the figure that the distribution approaches a Gaussian distribution as $N_{\text{source}}$ increases for a fixed $N_{\text{axion}}$. The number of the local minima tend to increase and the energy density distribution approaches a Gaussian distribution as $N_{\text{axion}}$ and the upper bound on $|n_{i\alpha}|$ increase. We have confirmed that, even if we do not impose the KNP mechanism, the energy density distribution of the local minima exhibits a similar behavior.

Our vacuum may or may not be around the peak of the energy distribution. This crucially depends on the prior probability distribution of the parameters in the axion landscape as well as the constant $C$ which represents the other contributions to the cosmological constant. This issue is also related to the initial condition of the slow-roll inflation, but we do not pursue it further here.

The eternal inflation takes place if the Universe is stuck in one of the local minima with a positive energy. The bubble nucleation rate crucially depends on the energy difference between the adjacent vacua. We have evaluated the difference of the energy density and the distance between each local minimum and its nearest adjacent one in the case of $N_{\text{axion}} = 8$ and $N_{\text{source}} = 13$ studied above. The results are shown in Fig. 2. One can see that, while there is a peak at the vanishing energy density difference, the typical energy difference is of order unity in the units of $\Lambda^4$. Similarly, the typical distance between the two adjacent minima is of order $f$. The non-trivial distribution of the distance reflects the structure of the axion landscape in this example. The averaged energy density difference and distance are about $1.3\Lambda^4$ and $1.4f$, respectively. This is important for the initial condition of the subsequent slow-roll inflation, and we shall return to this issue later.
FIG. 1. Distribution of axion potential $V(\phi_\alpha)$ at the found local minima in the units of $\Lambda^4$. We set $N_{\text{axion}} = 8$, and we take $N_{\text{source}} = 9, 11$ and 13 from left to right. Each distribution is for one realization of the axion landscape satisfying $R < 10^{-2}$.

FIG. 2. Distribution of the difference of energy density ($\Delta V$) and the distance $D$ between each local minimum and its nearest adjacent one, in the units of $\Lambda^4$ and $f$, respectively. Here we used the axion landscape with $N_{\text{axion}} = 8$ and $N_{\text{source}} = 13$ in Fig. 1. The mean energy difference and distance are about $1.3\Lambda^4$ and $1.4f$, respectively.
III. INFLATION IN AXION LANDSCAPE

A very light axion with an effective super-Planckian decay constant can be realized with a certain probability in the presence of multiple axions owing to the KNP mechanism [37, 38]. Now we examine the inflaton potential in the axion landscape, where there are various shift-symmetry breaking terms with $N_{\text{source}} > N_{\text{axion}}$.

First let us show how the axion potential looks like when the KNP mechanism is operative. For visualization purpose we set $N_{\text{axion}} = 3$ and $N_{\text{source}} = 4$ and repeatedly generated an integer-valued matrix $n_{\text{ia}}$ until $R$ becomes smaller than $10^{-2}$. In Fig. 3 the surfaces show particular values of the axion potential, $V(\hat{\phi}) = 2\Lambda^4$ (purple) and $V(\hat{\phi}) = 2.5\Lambda^4$ (orange), where the three axis ($\hat{\phi}_1, \hat{\phi}_2, \hat{\phi}_3$) are chosen so that they coincide with the mass eigenstates at one of the local minima, which is shifted to coincide with the origin ($\hat{\phi}_1 = \hat{\phi}_2 = \hat{\phi}_3 = 0$) in the figure. Each shaded (purple) ellipsoid corresponds to one local minimum, while the light shaded (orange) casing represents the flat direction in the field space. Note that the scale of each axis is different; the range of each axis shown in this figure is $|\hat{\phi}_1| < 20f$ and $|\hat{\phi}_{2,3}| < 1.5f$. Therefore the potential is indeed flat (approximately) along the lightest direction ($\hat{\phi}_1$), while the other directions ($\hat{\phi}_{2,3}$) are heavier.

The inflaton is identified with the lightest degrees of freedom, while the other heavier degrees of freedom can be integrated out during inflation. There are many local minima and a light direction is attached to each minimum. As $N_{\text{axion}}$ increases, it becomes easier to obtain such a flat direction [38] when the relevant dynamical scales are comparable to each other. Thus, the large-field slow-roll inflation after the bubble nucleation is an automatic outcome in the axion landscape.

A couple of comments are in order. First the mass eigenstates as well as the mass eigenvalues generically depend on the position in the field space. Therefore, as one moves away from the local minima along the lightest direction, the composition of the inflaton gradually changes, and as a consequence, the inflaton potential is slightly modified from the simple cosine function. Rather, it is given by a certain superposition of the multiple cosine functions, i.e., multi-natural inflation [17]. This effect becomes significant especially if $N_{\text{source}} > N_{\text{axion}}$, because each local minimum has a different energy, in general. However, if the enhancement of the effective decay constant due to the KNP mechanism is large enough, the inflaton potential can be (only) approximately given by the natural inflation during the
FIG. 3. The three dimensional contour plot of the axion potential when the KNP mechanism is operative. The three axis ($\hat{\phi}_1, \hat{\phi}_2, \hat{\phi}_3$) are chosen so that they coincide with the mass eigenstates at one of the local minima at the origin. Note that the scale of each axis is different, and the flat direction is approximately along the direction of $\hat{\phi}_1$.

last 50 or 60 e-foldings, and in the limit of large enhancement it is approximated by the quadratic chaotic inflation. On the other hand, an extremely large decay constant is rare in the axion landscape.\footnote{That said, it is hard to make a definite statement concerning the probability because of the measure problem.} Thus, it depends on the pressure toward shorter inflation in the landscape whether the effective inflaton potential is given by the quadratic chaotic inflation or natural inflation. If the pressure is strong enough, the total duration of the inflation is likely close to just about 50 to 60, and we expect that both deviation from the quadratic chaotic inflation and the traces of the bubble nucleation may be observed together. The combination of these two observations is one possible outcome of the axion landscape.

Secondly, the position of the inflaton after the tunneling event is expected to be away from the local minimum by a factor of the effective decay constant along the flat direction. This is because the lightest direction does not participate the tunneling event, and there is no special reason for the inflaton to just sit on the local minimum after the tunneling \cite{49}. One can also understand this from the structure of the axion landscape. Suppose that there
is a very flat direction owing to the KNP mechanism, and that the Universe experiences eternal inflation when it is stuck in one of the local minima. Then, after a certain point of time, a bubble forms and the Universe tunnels to some point along the valley with an energy lower by $\mathcal{O}(\Lambda^4)$. Therefore, from the energy conservation point of view, the axion fields at the center of the bubble can be anywhere along the valley. See Fig. 4 for the schematic picture of the eternal inflation and subsequent slow-roll inflation in the axion landscape. On the other hand, if the energy difference between the two local minima were much smaller than $\Lambda^4$, the tunneling point would be limited to the vicinity of the lower local minimum. Thus, the typical duration of the slow-roll inflation after the bubble formation is determined by the effective decay constant along the lightest direction. If the effective decay constant is of order $f = 5 \sim 10M_P$, the total duration of inflation could be just 50 or 60.

We have so far assigned a common value $\Lambda$ to all the dynamical scales. However, there is no special reason to expect that this is the case, and indeed, some of the dynamical scales, $\Lambda_L$, can be (much) smaller than the others, $\Lambda_L \ll \Lambda$. Then, such cosine functions would give rise to small modulations to the inflaton potential. To be concrete, let us suppose that all the decay constants are of similar order, say, $f_\alpha = f \approx 10^{17}$ GeV, and the effective decay constant for the inflaton, $f_{\text{eff}}$, is enhanced by a factor of about $10^2$ owing to the KNP mechanism, i.e., $f_{\text{eff}} = \mathcal{O}(10^2)f = \mathcal{O}(10)M_P$. The small modulations are expected to have a decay constant of order $f$, as they do not participate in the KNP mechanism.

In Fig. 5 we show the axion potential along the lightest axion in the landscape with $N_{\text{axion}} = 3$, $N_{\text{source}} = 5$, $\Lambda_i = \Lambda$ for $i = 1 - 4$ and $\Lambda_5 = 0.1\Lambda$. We required $R < 1/20$ to implement the KNP mechanism when the fifth cosine function is absent. One can see that there appear small modulations due to the mild hierarchy in the dynamical scales. Interestingly, such small modulations with mild hierarchy in the decay constants leads to a sizable running of the scalar spectral index, $|dn_s/d\ln k| = \mathcal{O}(0.01)$, which is almost constant over the CMB scales, and so, there is no contradiction with large-scale observations [58–62]. The presence of such small modulations, and therefore the running spectral index, is a common feature of inflation in the axion landscape. Such periodic modulations to the inflaton potential, if found, would definitively demonstrate that the inflaton is one of the axions with softly broken shift symmetry.
FIG. 4. The schematic view of eternal inflation and subsequent slow-roll inflation in the axion landscape. The flat direction $\hat{\phi}_1$ arises due to the KNP mechanism and $\hat{\phi}_2$ collectively represents the other heavy modes. First, the Universe is stuck in one of the local minima, and eternal inflation takes place. After a certain point of time, (1) tunneling takes place, (2) heavy axions fast roll and oscillate, and (3) slow-roll inflation starts along the flat direction.

IV. DISCUSSION AND CONCLUSIONS

After inflation, the inflaton needs to decay and reheat the standard model particles. This can be realized if one (or more) of the axions is coupled to the standard model sector via e.g.

$$\mathcal{L} = \frac{\phi_\alpha}{f_\alpha} F_{\mu\nu} \tilde{F}^{\mu\nu}. \quad (4)$$

Here, $F_{\mu\nu}$ is the gauge field strength in the Standard Model. The decay temperature is estimated to be of order $10^{10}$ GeV with the inflaton mass $m_{\phi_\alpha} \approx 10^{13}$ GeV and $f_\alpha \approx 10^{17}$ GeV. Thermal leptogenesis will be possible for such a high reheating temperature [63]. Note that the decay process should be considered in the mass eigenstate basis, and the couplings
FIG. 5. The potential along the lightest axion direction in the axion landscape with (solid) or without (dashed) small modulations, which is due to an extra cosine function with a suppressed amplitude.

may be suppressed by a mixing angle compared to a naive estimate (such a possibility is studied in the context of the axion-like particle dark matter [64]). Non-thermal leptogenesis may also be possible through the inflaton decay into the lightest right-handed neutrino $\nu^c$ with the interaction $\mathcal{L} = c(\phi_\alpha/f_\alpha) M \nu^c \nu^c$, where $M$ represents the right-handed neutrino mass and $c$ is a coupling constant.

The couplings of the inflaton like (4) are known to induce non-Gaussianity if the prefactors of $\hat{F} \tilde{F}$ change significantly during inflation [65, 66]. For the decay constant of order $f_\alpha \approx 10^{17}$ GeV, the current non-Gaussian constraint can be satisfied. If some of the decay constants might be smaller, or if the observational bound on the non-Gaussianity is improved, it may be possible to detect the non-Gaussianity generated through the interaction (4).

So far we have concentrated on the axions with the potential (2), assuming the other degrees of freedom such as their scalar partner “saxions” are much heavier. In fact, however, the saxions do not have to be hierarchically heavier, since the inflaton (the lightest axion) is much lighter than the typical axion mass when the KNP mechanism is operative. It is beyond the scope of this paper to study how light the saxions can be without modifying our results, but we expect that the basic features of the axion landscape scenario (eternal inflation, subsequent slow-roll inflation due to the KNP mechanism, the inflaton potential, etc.) will remain valid even if the saxions have a mass comparable to the axions.

Here let us comment on how the decay constant of string theorectic axions is determined.
The size of the decay constant depends not only on the wave function of an axion (tensor field) in the extra dimension, but also on how the axion couplings to gauge fields appear in four dimensions. The typical size of the decay constant is given by the string scale, the Kaluzua-Klein scale, or the winding scale. Further, the decay constant can be enhanced by one-loop factor, when the axion couples to the gauge field at the quantum level as in the case of Kähler moduli on a Calabi-Yau space in Heterotic string \[67\] or type IIA string.

Complex structure moduli in type IIB models also can induce the inflation. It depends on the detailed moduli stabilization with flux compactification whether such axions drive natural inflation. In particular, stringy corrections are important, and direct couplings of axions to branes/fluxes can induce explicit shift-symmetry breaking terms which cause the so-called monodromy inflation instead \[19\text{–}27\]. Although we have assumed canonical kinetic terms for the axions, the axion kinetic terms may depend on the axions through stringy or non-perturbative corrections. Then, the inflaton potential will be modified when expressed in terms of the canonically normalized axion fields.

The tadpole condition is another important issue for implementing the KNP mechanism in the string theory, which requires a number of (brane) charges through the alignment of wrapping numbers of branes or gauge fluxes on branes in the extra dimensions, while the net charge of branes should be vanishing in the compact extra dimension. In this respect, it is interesting to consider a possibility that the inflaton consists of RR two-from axions in type IIB model \[40, 42, 43, 45\] and fluxed seven branes, which produce non-perturbative inflaton potential, are wrapping on a curved extra dimension with orientifolds. In this case, a tight consistency condition on three brane charges can be relaxed by the curvature corrections due to the seven branes \[68, 69\]. Note that fluxes on the seven branes do not contribute to the seven brane charges and, therefore, it may ease the constraint on five brane charges if the flux contribution to the anomaly is canceled between branes and their mirror images \[70\]. Euclidean brane instantons may also relax this constraint because they will not produce real brane charges. Similarly, it is interesting to realize natural inflation with IIA/Heterotic Kähler moduli or IIB complex structure moduli through world-sheet instanton or its dual (classical) effect, respectively; they have another advantage that the decay constant is enhanced by one-loop factor.

The QCD axion may be a part of the axion landscape, but the isocurvature perturbations will be generically too large. In fact, too large isocurvature perturbation is a serious problem
for any scenarios including light and cosmological stable axions or axion-like particles, as long as the inflation scale is high. There are several ways to avoid the isocurvature bound. One possibility is that the shift symmetry is broken by a large amount during inflation so that axions are sufficiently heavy and their fluctuations are suppressed \cite{71,72,73}.

Throughout this paper we have focused on the large-field inflation in the axion landscape. In fact, it is also possible to implement small-field inflation based on an axion with multiple cosine functions. This is because, for a certain choice of the dynamical scales, the curvature of the axion potential around the hilltop can be vanishingly small, which leads to an axion hilltop inflation \cite{17,18}. The predicted tensor-to-scalar ratio is smaller; $r \lesssim 10^{-3}$ is expected for the sub-Planckian decay constant.

One of the central roles of the axion landscape is to provide a sufficiently flat direction suitable for large-field inflation based on the KNP alignment mechanism. If the number of axions are sufficiently large, the vacuum structure becomes so complicated that it may help to solve the cosmological constant problem with the aid of the anthropic principle \cite{74}.

In this paper we have studied the vacuum structure as well as possible inflationary dynamics in the axion landscape where there are many axions with various shift-symmetry breaking terms. The required flatness of the inflationary path is realized by the KNP alignment mechanism. The axion landscape provides us with a unified understanding of the eternal inflation in one of the local minima and the subsequent slow-roll inflation after the tunneling event. If the deviation from the quadratic chaotic inflation is found, it implies that there is pressure toward shorter duration of the slow-roll inflation, and so, it may be possible to observe remnants of the bubble nucleation such as negative spatial curvature as well as the suppression of density perturbations at large scales. The presence of small modulations to the inflaton potential is common in the axion landscape, which leads to a sizable running of the scalar spectral index of order $|dn_s/d\ln k| = \mathcal{O}(0.01)$ which is almost constant over CMB scales.

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\footnote{There is an infinite number of degenerate ground states in the case of an irrational axion \cite{75,76}.}
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