Non Degeneration of Fibonacci Series, Pascal’s Elements and Hex Series

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Abstract
Generally Fibonacci series and Lucas series are the same, they converge to golden ratio. After I read Fibonacci series, I thought, is there or are there any series which converges to golden ratio. Because of that I explored the interrelations of Fibonacci series when I was intent on Fibonacci series in my difference parallelogram. In which, I found there is no degeneration on Fibonacci series. In my thought, Pascal triangle seemed like a lower triangular matrix, so I tried to find the inverse for that. In inverse form, there is no change against original form of Pascal elements matrix. One day I played with ring magnets, which forms hexagonal shapes. Number of rings which forms Hexagonal shape gives Hex series. In this paper, I give the general formula for generating various types of Fibonacci series and its non-degeneration, how Pascal elements maintain its identities and which shapes formed by hex numbers by difference and matrices.

Keywords
Fibonacci Series, Lucas Series, Golden Ratio, Various Type of Fibonacci Series Generated by Matrices, Matrix Operations on Pascal’s Elements and Hex Numbers

1. Introduction
The Fibonacci sequence is named after Leonardo of Pisa (c. 1170 - c. 1250), popularly known as Fibonacci. He wrote a number of books such as Liber Abaci (The Book of Calculating) in 1202, Practica Geometriea (Practical Geometry) in 1220, Flos in 1225, and Liber Quadratorium (The Book of Squares) in 1225. Fibonacci sequence is a series of numbers in which each number is the sum of the two preceding numbers. First few numbers in the series are 1, 1, 2, 3, 5, 8, 13, 21, 34, 55 … In India, Fibonacci sequence appeared in Sanskrit prosody (a system of
versification). In the Sanskrit oral tradition, there was much emphasis on how long (L) syllables that are 2 units of duration mix with the short (S) syllables that are 1 unit of duration. Counting the different patterns of L and S within a given fixed length results in the Fibonacci numbers—the number of patterns that are $m$ short syllables-long is the Fibonacci number $F_m + 1$. According to Susantha Goonatilake of Royal Asiatic Society Sri Lanka, the development of the Fibonacci sequence “is attributed in part to Pingala (200 BC), later being associated with Virahanka (c. AD 700), Gopala (c. AD 1135), and Hemchandra (c. AD 1150).”

To find $F_n$ for a general positive integer $n$, we hope that we can see a pattern in the sequence of numbers already found. A sharp eye can now detect that any number in the sequence is always the sum of the two numbers preceding it. That is,

$$F_{n+2} = F_{n+1} + F_n, \text{ for } n = 0, 1, 2, 3, \ldots.$$  

Fibonacci series is helix like identity. It converges to golden ratio, we can show its existence in spiral shells but its elements never construct volumetric object. Fibonacci series elements construct Area only. Pascal triangle elements (Binomial series elements) construct area, volume and volumetric objects but whatever be it remains its identity which means, if we constructed a matrix with Pascal triangle elements, which would be a square matrix, its $k^{th}$ power or its inverse might have the same identity of Pascal triangle elements, hex series having different numbers, but all numbers will be derived by triangular series numbers.

The Fibonacci Numbers are also applied in Pascal’s Triangle. Entry is sum of the two numbers either side of it, but in the row above. Diagonal sums in Pascal’s Triangle are the Fibonacci numbers. We are getting some ideas from ([1] Jeffrey R. Chasnov (2016-19) - Fibonacci Numbers and the Golden ratio - Lecture Notes for Course - The Hong Kong University of Science and Technology, Department of Mathematics, Clear Water Bay, Kowloon - Hong Kong). We know Fibonacci Numbers and the Golden ratio ([2] Tom Davis, Exploring Pascal’s Triangle- tomrdavis@earthlink.net http://www.geometer.org/mathcircles, January 1, 2010; Relation between Pascal’s triangle and Fibonacci’s numbers; [3] Balasubramani Prema Rangasamy - Some extensions on numbers - Advances in Pure Mathematics, 2019, 9, 944-958. Difference table and [4] https://en.wikipedia.org/w/index.php?title=Golden_ratio&oldid=8374695178)

We know more about Fibonacci’s elements, Pascal’s elements, Hex numbers and Golden ratio. The Golden Section represented by the Greek letter Phi ($\phi$) = 1.6180339887.

In this paper, I give the general formula for generating various types of Fibonacci series and its non-degeneration, how Pascal elements maintain its identities and which shapes formed by hex numbers by difference and matrices.

2. Row Matrix Building for Fibonacci’s Elements

Difference method
A $m \times 2$ matrix is given by:

$$
\begin{bmatrix}
1 & 1 \\
1 & 2 \\
2 & 3 \\
3 & 5 \\
5 & 8 \\
8 & 13 \\
13 & 21 \\
\vdots & \vdots \\
\end{bmatrix}
$$

In which odd rows numbers are Fibonacci series numbers and even rows numbers are difference of two consecutive odd rows numbers.

A $m \times 3$ matrix is given by:

$$
\begin{bmatrix}
1 & 1 & 2 \\
2 & 4 & 6 \\
3 & 5 & 8 \\
10 & 16 & 26 \\
13 & 21 & 34 \\
42 & 68 & 110 \\
55 & 89 & 144 \\
\vdots & \vdots & \vdots \\
\end{bmatrix}
$$

In which odd rows numbers are Fibonacci series numbers and even rows numbers are difference of two consecutive odd rows numbers.

A $m \times 4$ matrix is given by:

$$
\begin{bmatrix}
1 & 1 & 2 & 3 \\
4 & 7 & 11 & 18 \\
5 & 8 & 13 & 21 \\
29 & 47 & 76 & 123 \\
34 & 55 & 89 & 144 \\
199 & 322 & 521 & 843 \\
233 & 377 & 610 & 987 \\
\vdots & \vdots & \vdots & \vdots \\
\end{bmatrix}
$$

In which odd rows numbers are Fibonacci series numbers and even rows numbers are difference of two consecutive odd rows numbers.

A $m \times 5$ matrix is given by:

$$
\begin{bmatrix}
1 & 1 & 2 & 3 & 5 \\
7 & 12 & 19 & 31 & 50 \\
8 & 13 & 21 & 34 & 55 \\
81 & 131 & 212 & 343 & 555 \\
89 & 144 & 233 & 377 & 610 \\
898 & 1453 & 2351 & 3804 & 6155 \\
987 & 1597 & 2584 & 4181 & 6765 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\end{bmatrix}
$$

In which odd rows numbers are Fibonacci series numbers and even rows numbers are difference of two consecutive odd rows numbers.
We do the same again and again we get

1st series:  1 1 2 3 5 8 13 21 34 55 89
2nd series:  1 2 3 5 8 13 21 34 55 89 144
3rd series:  2 4 6 10 16 26 42 68 110 178 288
4th series:  4 7 11 18 29 47 76 123 199 322 521
5th series:  7 12 19 31 50 81 131 212 343 555 898
6th series:  12 20 32 52 84 136 220 356 576 932 1508

We can generate \( m \)th series by

\[
^m F_{n+2} = ^m F_{n+1} + ^m F_n
\] (1)

where

\[
^m F_n = 1^{m \times n} F_n - 1^{m \times n} F_n
\] (2)

where \( ^m F_n \) an \( m \)th element of a \( m \)th Fibonacci series, \( 1^n F_n \) is \( n \)th element of a 1st Fibonacci series and \( 1^{m+n} F_n \) is \( m+n \)th element of a 1st Fibonacci series.

1st series elements are known as Fibonacci numbers.
4th series elements are known as Lucas numbers.

Axiom 1: All the above series are converges to Golden ratio.

3. Addition Method

\[
\begin{bmatrix}
1 & 1 \\
3 & 4 \\
2 & 3 \\
7 & 11 \\
5 & 8 \\
18 & 29 \\
13 & 21 \\
\vdots & \vdots
\end{bmatrix}
\] is a \( m \times 2 \) matrix.

In which odd rows numbers are Fibonacci series numbers and even rows numbers are addition of two consecutive odd rows numbers.

\[
\begin{bmatrix}
1 & 1 & 2 \\
4 & 6 & 10 \\
3 & 5 & 8 \\
16 & 26 & 42 \\
13 & 21 & 34 \\
68 & 110 & 178 \\
55 & 89 & 144 \\
\vdots & \vdots & \vdots
\end{bmatrix}
\] is a \( m \times 3 \) matrix.

In which odd rows numbers are Fibonacci series numbers and even rows numbers are addition of two consecutive odd rows numbers.
\[
\begin{bmatrix}
1 & 1 & 2 & 3 \\
6 & 9 & 15 & 24 \\
5 & 8 & 13 & 21 \\
39 & 63 & 102 & 165 \\
34 & 55 & 89 & 144 \\
267 & 432 & 699 & 1131 \\
233 & 377 & 610 & 987 \\
\vdots & \vdots & \vdots & \vdots \\
\end{bmatrix}
\]

is a \(m \times 4\) matrix.

In which odd rows numbers are Fibonacci series numbers and even rows numbers are addition of two consecutive odd rows numbers.

\[
\begin{bmatrix}
1 & 1 & 2 & 3 & 5 \\
9 & 14 & 23 & 37 & 60 \\
8 & 13 & 21 & 34 & 55 \\
97 & 157 & 254 & 411 & 665 \\
89 & 144 & 233 & 377 & 610 \\
1076 & 1741 & 2817 & 4558 & 7375 \\
987 & 1597 & 2584 & 4181 & 6765 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\end{bmatrix}
\]

is a \(m \times 5\) matrix.

In which odd rows numbers are Fibonacci series numbers and even rows numbers are addition of two consecutive odd rows numbers.

We do the same again and again we get

| Series | Elements |
|--------|----------|
| 1st    | 1 1 2 3 5 8 13 21 34 55 89 |
| 2nd    | 3 4 7 11 18 29 47 76 123 199 322 |
| 3rd    | 4 6 10 16 26 42 68 110 178 288 466 |
| 4th    | 6 9 15 24 39 63 102 165 267 432 699 |
| 5th    | 9 14 23 37 60 97 157 254 411 665 1076 |
| 6th    | 14 22 36 58 94 152 246 398 644 1042 1680 |

We can generate \(k^{th}\) series by

\[
{}^kF_{k+2} = {}^kF_{k+1} + {}^kF_n
\]

and

\[
{}^kF_n = {}^kF_{k+n} + {}^1F_n
\]

where \(^kF_n\) an \(n^{th}\) element of a \(k^{th}\) Fibonacci series, \(^1F_n\) is \(n^{th}\) element of a 1st Fibonacci series and \(^1F_{k+n}\) is \(k + 1^{th}\) element of a 1st Fibonacci series.

1st series elements are known as Fibonacc numbers.

2nd series elements are known as Lucas numbers.

3rd series elements are known as doubled Fibonacci numbers.

4th series elements are known as tripled Fibonacci numbers.
4. Difference between All Series Diagonal Elements

\[
d_n = s+1 \sum_{r=1}^s F_{n-r} - F_{n+1}, \quad n \geq 1
\]

(6)

and

\[
d_n = F_{n+1} - s+1 \sum_{r=1}^s F_{n-r}, \quad s \geq 2
\]

(7)

where \( d_n \) is \( n \)th element of a \( s \)th different series, \( s+1 \sum_{r=1}^s F_{n-r} \) is \( n \)th element of a \( s+1 \)th Fibonacci series and \( F_{n+1} \) is \( n+1 \)th element of a \( s \)th Fibonacci series.

From above those diagonal differences remains the extinct of Fibonacci’s elements.

5. Difference Chart of Above Series

| Diff 1 | Diff 2 | Diff 3 | Diff 4 | … |
|--------|--------|--------|--------|---|
| 1st series: | 1 1 2 3 5 8 13 21 34 55 89 | | | |
| 2nd series: | 2 2 4 6 10 16 26 42 68 110 | | | |
| 3rd series: | 3 4 7 11 18 29 47 76 123 199 322 | | | |
| 4th series: | 4 6 10 16 26 42 68 110 178 268 466 | | | |
| 5th series: | 6 9 15 24 39 63 102 165 267 432 699 | | | |
| 6th series: | 9 14 23 37 60 97 157 254 411 665 1076 | | | |
| 7th series: | 14 22 36 58 94 152 246 398 644 1042 1686 | | | |

From the above we chart,

| Series 1 | series 2 | series 3 | series 4 | series 5 | … |
|----------|----------|----------|----------|----------|---|
| Diff 1 series: | 0 1 2 3 5 | 8 | | | |
| Diff 2 series: | 1 1 2 4 7 | 12 | | | |
| Diff 3 series: | 1 2 4 7 12 | 19 | | | |
| Diff 4 series: | 2 3 6 11 18 | 31 | | | |
| Diff 5 series: | 3 5 10 18 31 | 55 | | | |

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Diff $t^{th}$ series:
\[ i^{t+2}D_n = i^{t+1}D_n + i^tD_n \]  
(9)

where
\[ i^tD_n = ^tF_{n+1} - ^tF_n \]  
(10)

and
\[ i^{t+1}D_n = i^{t+1}F_{n+1} - i^{t+1}F_n \]  
(11)

where \(^tD_n\) an \(n^{th}\) element of a \(t^{th}\) different Fibonacci series, \(^tF_n\) is \(n^{th}\) element of a \(1^{st}\) Fibonacci series and \(^{t+1}F_{n+1}\) is \(k + 1^{th}\) element of a \(1^{st}\) Fibonacci series.

**Axiom 3**: All the above different series are converges to **Golden ratio**.

### 6. Difference Parallelogram of Fibonacci Numbers

\[
\begin{array}{cccccccccccc}
0 & 1 & 1 & 2 & 3 & 5 & 8 & 13 & 21 & 34 & \ldots \\
1 & 0 & 1 & 1 & 2 & 3 & 5 & 8 & 13 & 21 & \ldots \\
-1 & 1 & 0 & 1 & 1 & 2 & 3 & 5 & 8 & 13 & \ldots \\
2 & -1 & 1 & 0 & 1 & 1 & 2 & 3 & 5 & 8 & \ldots \\
-3 & 2 & -1 & 1 & 0 & 1 & 1 & 2 & 3 & 5 & \ldots \\
5 & -3 & 2 & -1 & 1 & 0 & 1 & 1 & 2 & 3 & \ldots \\
-8 & 5 & -3 & 2 & -1 & 1 & 0 & 1 & 1 & 2 & \ldots \\
13 & -8 & 5 & -3 & 2 & -1 & 1 & 0 & 1 & 1 & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots 
\end{array}
\]

Above difference parallelogram shows Fibonacci series never vanished, which means it exist everlastingly.

### 7. Matrices in Pascal’s Elements

Let

\[
A = \begin{bmatrix}
1 & 1 & 1 & 1 & \cdots \\
1 & 2 & 3 & 4 & \cdots \\
1 & 3 & 5 & 7 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots \\
^mC_0 & ^mC_1 & ^mC_2 & ^mC_3 & \cdots & ^mC_m
\end{bmatrix}
\]

be an \(n \times n\) matrix having Pascal’s elements. Where \(m = n - 1\). We called it as Pascal’s matrix.

Now we define Pascal matrix by any variable.

#### 1) NW (North-west Pascal’s matrix)

Let

\[
A = \begin{bmatrix}
a & a & \cdots \\
a & 2a & a & \cdots \\
a & 3a & 3a & a & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots \\
^mC_0a & ^mC_1a & ^mC_2a & ^mC_3a & \cdots & ^mC_m a
\end{bmatrix}
\]
be an \( n \times n \) matrix having Pascal's elements. Where \( m = n - 1 \). \( k \) is an exponent and “\( a \)” is variant.

Now,

\[
A^k = \begin{bmatrix}
  k^0 a^k & k^0 a^k & \cdots & k^0 a^k \\
  k^1 a^k & 2k^1 a^k & \cdots & k^1 a^k \\
  k^2 a^k & 3k^2 a^k & \cdots & k^2 a^k \\
  \vdots & \vdots & \ddots & \vdots \\
  m C_k k^{m-1} a^k & m C_k k^{m-2} a^k & \cdots & m C_k k^{0} a^k
\end{bmatrix}
\]

\[
A^{-1} = \frac{1}{a} \begin{bmatrix}
  1 & -1 & 1 & \cdots & 1 \\
  -1 & 1 & -2 & \cdots & 1 \\
  1 & -3 & 3 & \cdots & 1 \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  -1 & 3 & -3 & \cdots & 1
\end{bmatrix}
\]

Jordon normal matrix of \( A \)

\[
J_A = \begin{bmatrix}
  a & 1 & 0 & 0 & 0 \\
  0 & a & 1 & 0 & 0 \\
  0 & 0 & a & 1 & 0 \\
  \vdots & \vdots & \vdots & \vdots & \ddots \\
  0 & 0 & 0 & \cdots & a
\end{bmatrix}
\]

2) NE (North-East Pascal’s matrix)

Let

\[
A = \begin{bmatrix}
  a & a & a \\
  a & 2a & a \\
  a & 3a & 3a & a \\
  \vdots & \vdots & \vdots & \vdots & \ddots \\
  a C_k a & \cdots & a C_k a & a C_k a & a C_k a
\end{bmatrix}
\]

be an \( y \times y \) matrix having Pascal’s elements. Where \( x = y - 1 \). \( k \) is an exponent and “\( a \)” is variant.

Now, inverse for North-East matrix

\[
A^{-1} = \frac{1}{a} \begin{bmatrix}
  -1 & 1 & -3 & 1 & \cdots & 1 \\
  1 & -2 & 1 & \cdots & 1 \\
  -1 & 1 & \vdots & \vdots & \ddots & \vdots \\
  \vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
  1 & \cdots & \cdots & \cdots & \cdots & a
\end{bmatrix}
\]

3) SE (South-East Pascal’s matrix)
Let 

\[
A = \begin{bmatrix}
^nC_m a & \cdots & ^nC_j a & ^nC_i a & ^nC_0 a \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
a & 3a & 3a & a \\
a & 2a & a \\
a & a \\
\end{bmatrix}
\]

be an \( n \times n \) matrix having Pascal’s elements. Where \( m = n - 1 \). \( k \) is an exponent and “\( a \)” is variant.

Now, 

\[
A^{-1} = \frac{1}{a} \begin{bmatrix}
(-1)^{n-1}[^nC_m] & (-1)^{n-1}[^nC_3] & (-1)^{n-1}[^nC_2] & (-1)^{n-1}[^nC_1] & (-1)^{n-1}[^nC_0] \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
1 & -3 & 3 & -1 \\
1 & -2 & 1 & 1 \\
1 & 1 \\
\end{bmatrix}
\]

4) SW (South-West Pascal’s matrix)

Let 

\[
A = \begin{bmatrix}
^xC_y a & ^xC_y a & ^xC_y a & \cdots & ^xC_y a \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
a & 3a & 3a & a \\
a & 2a & a \\
a & a \\
\end{bmatrix}
\]

be an \( y \times y \) matrix having Pascal’s elements. Where \( x = y - 1 \). \( k \) is an exponent and ‘\( a \)’ is variant.

Now, inverse for south-west matrix

\[
A^{-1} = \frac{1}{a} \begin{bmatrix}
(-1)^{x-1}[^xC_y] & \cdots & (-1)^{x-1}[^xC_3] & (-1)^{x-1}[^xC_2] & (-1)^{x-1}[^xC_1] & (-1)^{x-1}[^xC_0] \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 1 & -1 \\
1 & -2 & 1 & 1 \\
1 & -3 & 3 & -1 \\
\end{bmatrix}
\]

Hex numbers:

| 1 | 7 | 19 | 37 | 61 | 91 | 127 | 169 | 217 | 271 | 331 |
|---|---|---|----|----|----|-----|-----|-----|-----|-----|
| 397 | 469 | 547 | 631 | 721 | 817 | 919 | 1027 | 1141 | 1261 | ... |

Let \( h \) be any hex number. We know mod 6 of any \( h \) is equal to 1.

Mod 6 of \( h_1 \equiv 1 \); Mod 6 of \( h_2 \equiv 1 \); \( \cdots \); Mod 6 of \( h_k \equiv 1 \);
Theorem 1: Difference between any two elements of Hex numbers is fully divided by 6.

Theorem 2: \( \sum_{k=0}^{\infty} H_{6k} \equiv a \mod 6 \), where \( n \) is integer and \( 0 \leq a < 6 \).

Theorem 3: Remainder of arbitrary product of any number of Hex series is always 1 when the product is divided by 6.

Proof:

\[
R = \prod_{i} \frac{H_i}{6} = 1.
\]

We can say above as

\[ R(h_1 \times h_2 \times \cdots \times h_i) \div 6 = (1 \times 1 \times \cdots \times 1) \mod 6 = 1. \]

Theorem 4: \( \sum_{k} H_{k} \mod (6) = \sum_{k} (H_{k} \mod 6) \) \( k \in \mathbb{Z} \)

Theorem 5: \( \sum_{k=0}^{\infty} H_{k} = k^3 \), where \( H_k \) is Hex series elements.

Matrices of Hex numbers

Let we see the relation between hex numbers in matrix

1) Let \( A = \begin{bmatrix} 1 & h+1 \\ 3h+1 & 6h+1 \end{bmatrix} \) be a 2 \( \times \) 2 matrix which elements are hex numbers (where \( h = 6 \)) then \( |A| = \begin{vmatrix} 1 & h+1 \\ 3h+1 & 6h+1 \end{vmatrix} = 6h+1-3h^2-4h+1 = -3h^2+2h \)

2) Let \( A = \begin{bmatrix} h+1 & 3h+1 \\ 6h+1 & 10h+1 \end{bmatrix} \) be a 2 \( \times \) 2 matrix which elements are hex numbers (where \( h = 6 \)) then \( |A| = \begin{vmatrix} h+1 & 3h+1 \\ 6h+1 & 10h+1 \end{vmatrix} = -8h^2+2h \)

By above way we get, \(-15h^2+2h; -24h^2+2h; \ldots; -n(n+2)h^2+2h \)

1) Let we construct a difference triangle about above determinants

| Initial: \(-3h^2+2h; -8h^2+2h; -15h^2+2h; -24h^2+2h; -35h^2+2h \) | \( \ldots \) |
|-----------------------------------------------|----------------|
| 1st diff: \( 5h^2; 7h^2; 9h^2; 11h^2 \) | \( \ldots \) |
| 2nd diff: \( 2h^2; 2h^2; 2h^2 \) | \( \ldots \) |
| 3rd diff: \( 0; 0 \) | \( \ldots \) |

a) Let \( A = \begin{bmatrix} 1 & h+1 & 3h+1 \\ 6h+1 & 10h+1 & 15h+1 \\ 21h+1 & 28h+1 & 36h+1 \end{bmatrix} \) be a 3 \( \times \) 3 matrix which elements are hex numbers then \( |A| = \begin{vmatrix} 1 & h+1 & 3h+1 \\ 6h+1 & 10h+1 & 15h+1 \\ 21h+1 & 28h+1 & 36h+1 \end{vmatrix} = -27h^3 \)

b) \( |A| = \begin{vmatrix} h+1 & 3h+1 & 6h+1 \\ 10h+1 & 15h+1 & 21h+1 \\ 28h+1 & 36h+1 & 45h+1 \end{vmatrix} = -27h^3 \)
2) Let we construct a difference triangle about above determinants

| Initial: | 27h³ | 27h³ | 27h³ | 27h³ | 27h³ | ... |
|----------|------|------|------|------|------|-----|
| 1st diff: | 0    | 0    | 0    | 0    | 0    | ... |

a) Let

\[
A = \begin{bmatrix}
1 & h+1 & 3h+1 & 6h+1 \\
10h+1 & 15h+1 & 21h+1 & 28h+1 \\
36h+1 & 45h+1 & 55h+1 & 66h+1 \\
78h+1 & 91h+1 & 105h+1 & 120h+1 \\
\end{bmatrix}
\]

be a 4 × 4 matrix which elements are hex numbers then \( |A| = 0 \)

b) \( |A| = 0 \)

3) Let we construct a difference triangle about above determinants

| Initial: | 0 | 0 | 0 | 0 | 0 | ... |

From the above we can state:

a) Determinants of 2 × 2 matrix with hex series elements vanished at 2\text{nd} difference.

b) Determinant of 3 × 3 matrix with hex series elements vanished at 0\text{th} difference.

c) Determinant of 4 × 4 matrix with hex series elements and above are 0.

Which means hex series elements are forming hexagonal only.

8. Conclusions

1) Fibonacci series never dies. We can generate so many series like Fibonacci series, they also converge to golden ratio. By this way we find so many golden ratio pairs.

2) Matrix with Pascal elements never vanished at any “n” dimensional matrix calculation. For all arithmetic and matrix operation of matrix with Pascal elements never give up its frame. Here frame means the structure of matrix.

3) Sum of \( k \text{th} \) elements of hex series gives \( k^3 \) and hex series elements form hexagonal only.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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