Dynamical barrier for flux penetration in a superconducting film in the flux flow state

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The penetration of transverse magnetic flux into a thin superconducting square film in the flux flow state is considered by numerical simulation. Due to the film self-field, the governing equations are nonlinear, and in combination with the finite viscosity of the moving vortices, this sets up a dynamical barrier for flux penetration into the sample. The corresponding magnetization loop is hysteretic, with the peak in magnetization shifted from the zero position. The magnetic field in increasing applied field is found to form a well-defined front of propagation. Numerical estimates shows that the dynamical barrier should be measurable on films with low volume pinning.

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I. INTRODUCTION

The penetration of magnetic flux into superconductors is delayed due to the presence of surface barriers, such as the Bean-Livingston barrier and various barriers of geometric origin (The review by Brand lists 7 different mechanisms) The barriers are particularly important in thin films where the equilibrium field for existence of magnetic flux is much reduced from the bulk lower critical field to .

Numerical simulations show that in samples without volume pinning, the magnetic field near the edges is strongly reduced. The magnetic field in increasing applied field is found to form a well-defined front of propagation. Numerical estimates shows that the dynamical barrier should be measurable on films with low volume pinning.

II. MODEL

Let us consider a thin superconducting film with thickness d, shaped as a square with sides 2a > d. Due to absence of pinning, = 0, and the resistivity is solely given by the conventional flux flow expression, where is the normal state resistivity, is the upper critical field, and is the transverse component of the magnetic field. The magnetic field has two contributions, the applied field and self-field of the sample, where is the local magnetization.

Because the barrier does not prevent vortices from leaving the sample, the magnetization loop is asymmetric, and the magnetization irreversible.

The attention so far has mainly been paid to the static nature of barriers. Yet, dynamic effects might also give rise to barriers for flux penetration. In order to investigate if this is the case we consider dynamics of a superconducting film in transverse magnetic field. We assume that the film is sufficiently wide, so that the magnetic field can be treated as a continuum, and the spatio-temporal evolution of the system can be obtained by solution of the Maxwell-equations. In order to separate the dynamical barrier from other kinds of surface barriers, we disregard surface pinning, and assume that = 0 and the critical current density, = 0. Then, the only mechanism that gives loss in the system is the finite viscosity of the moving vortices, which gives a flux flow resistivity = | / |, where is the upper critical field. The corresponding dynamical barrier towards flux penetration will thus be strongly dependent on the rate of change the applied field.
Let us rewrite the equations on dimensionless form, assuming that the applied field is ramped with constant rate $|\dot{H}_a|$. We define a time scale and sheet current scale as

$$ t_0 \equiv \sqrt{\frac{\mu_0 H_c dw}{\rho_n HA}}, \quad J_0 \equiv \sqrt{\frac{\mu_0 H_c dw |HA|}{\rho_n}}. \quad (5) $$

The dimensionless quantities are defined as $\tilde{t} = t/t_0$, $g/J_0w$, $\tilde{H} = H/J_0$, $k = wk$. Eq. (3) becomes

$$ \frac{\partial \tilde{g}}{\partial \tilde{t}} = \mathcal{F}^{-1} \left[ \frac{2}{k} \mathcal{F} \left[ \frac{\partial \tilde{H}_z}{\partial \tilde{t}} - 1 \right] \right], \quad (6) $$

where

$$ \frac{\partial \tilde{H}_z}{\partial \tilde{t}} = \nabla \cdot \left[ \tilde{H}_z \nabla \tilde{g} \right], \quad (7) $$

valid inside the sample. As long as $|\dot{H}_a|$ is constant, there are no free parameters in the problem. We will henceforth omit the tildes in the dimensionless quantities, when reporting the results.

A total area of size $1.4 \times 1.4$ is discretized on a $512 \times 512$ grid. The additional vacuum at the sides of the superconductor is used to implement the boundary conditions.

### III. RESULT

Let us now consider the evolution of the sample as it completes a magnetization cycle. The external field driven with constant rate $|\dot{H}_a| = 1$ until the maximum field $H_a = 3$, starting from zero-field-cooled conditions. As applied field is changed, shielding currents are induced in the sample, giving it a nonzero magnetic moment $m$. The magnetic moment is calculated as $m = \int g(x, y) dxdy$. Figure [1] shows the magnetic moment as a function of applied field. The plot contains the virgin branch and a steady state loop. As expected for a superconducting film, the main direction of the response is diamagnetic. The absolute value of $|m|$ reaches a peak for $H_a = 0.54$ in the virgin branch and at $H_a = 0.35$ in the steady-state loop, while it decreases at higher magnetic fields. The shape of the loop is quite similar to superconductors with a field-dependent critical current $J_c$ except that the magnetization peak is shifted from $H_a = 0$.

In this respect the dynamical barrier is similar to other kinds of surface barriers.

Figure [2] shows $H_z$ and $J$ magnitude and stream lines at various applied fields. The state at $H_a = 0.5$ is close to the peak in magnetization in the virgin branch. The flux piles up close to the edges, and falls to zero on a well defined flux front, roughly penetrating one third of the distance to the sample center. The current stream lines are smooth, with highest density in the flux-penetrated region. The flux distribution has some similarity with the square in the critical state, but the most striking difference is the absence of dark $d$-lines at the diagonals. At $H_a = 1.1$, the flux front has reached the center of the sample. The edge of the sample is still white signifying piling up of flux there, but the flux distribution at this time is much more uniform than it was earlier, and the current density is correspondingly much lower. This is a feature caused by the short lifetime of currents of superconductors in the flux flow state. The rightmost panels show the remanent state after the field has been increased to max $H_a = 3$ and then back to $H_a = 0$. The distributions are star-shaped, with the inner part of the sample has low current and contains a lot of trapped positive flux. The flux is trapped due to a line with $H_z = 0$ inside the sample, where the strong shielding currents flow with zero resistivity. The shielding from the currents at this line prevents the trapped flux from leaving the sample.

Let us return to the dimensional quantities to determine how easy it is to measure the effect of the dynamical barrier. The most likely candidate for material are superconductors with low intrinsic flux pinning and low first critical field. One such material is MoGe thin-films. With the values $\mu_0 H_{c2} = 3T$, $\rho_n = 2 \cdot 10^{-6} \Omega m$, $d = 50 \text{ nm}$, $w = 2 \text{ mm}$, and driving rate $\mu_0 \dot{H}_a = 10 \text{ T/s}$, we get $J_0 = 35 \text{ A/m}$ and $t_0 = 4.3 \text{ ms}$. The characteristic current density will thus be $J_0/d = 6.9 \cdot 10^8 \text{ A/m}^2$ and the magnetic field values will be of order $\mu_0 J_0 = 0.043 \text{ mT}$. In this case the dynamical barrier will be larger than the geometric barrier obtained Ref. [10] which is of order $\mu_0 H_{ci} = \mu_0 H_{c2} \sqrt{d/w} = 0.01 \text{ mT}$, with $\mu_0 H_{c2} = 2 \text{ mT}$. Experimentally it will thus be easy to distinguish the geometric barrier from the dynamical barrier due to the ramp-rate dependency of the latter.
FIG. 2. (top) The magnetic flux distribution and (bottom) current density and stream lines, at $H_a = 0.5, 1.1,$ and 0 (Remanent state).

IV. SUMMARY

The penetration of magnetic flux into superconducting films can be delayed due to a dynamical barrier caused by the viscous motion of the vortices. In this work we have studied this effect on a thin film superconductor of square shape using numerical simulations. The point that makes the dynamics interesting is that in transverse geometry, the flux flow equations are non-linear due to the film self-field, contrary to parallel geometry where they are linear. In small applied magnetic field, the flux penetrates into the sample in an orderly manner with a well-defined flux front, similar to the critical state, but with absence of current discontinuity lines. When the applied field is changed there are fronts moving where the total magnetic field is zero, and shielding currents flow without resistivity. In particular in the remanent state, such a front will prevent magnetic flux from leaving the sample, so that the remanent state contains trapped flux. The magnetization loop is hysteric with the magnetization peak shifted from the zero position. Numerical estimates shows that the effect of the dynamical barrier should be possible to measure on thin films of materials with low volume pinning. The effect is easily distinguished from other kinds of barriers due to its dependence on the rate of change of the applied field.

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