Exclusive $B \to \pi \ell^+ \ell^-$ and $B \to \rho \ell^+ \ell^-$ Decays in the Universal Extra Dimension

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Abstract

We investigate the influence of the universal extra dimension on the branching ratio in the $B \to \pi(\rho) \ell^+ \ell^-$ decay. Taking $1/R \sim \{200 - 1000\}$ GeV with one universal extra spatial dimension, which is consistent with the experimental data for $\mathcal{B}(B \to X_s \gamma)$, $\mathcal{B}(B \to K^* \mu^+ \mu^-)$ and the electroweak precision tests, we obtain that for both $(\mu, \tau)$ channels the branching ratio strongly depends on the compactification radius $1/R$.

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1 Introduction

The standard model (SM) has been a successful theory in reproducing almost all experimental data about the interaction of gauge bosons and fermions. However, the SM is not regarded as a full theory, since it cannot address some issues, i.e., gauge and fermion mass hierarchy, matter-antimatter asymmetry, number of generations, and so on. For these reasons, the SM can be considered as an effective theory of some fundamental theory at low energy.

Extra dimension model [1] is one of the candidates trying to shed light on some of those issues. It can be categorized in terms of the mechanism of new physics where the SM fields are constrained to move in the usual three spatial dimensions ($D_3$ bran) or can propagate in the extra dimensions (the bulk). The last one can be categorized as non-universal extra dimension (NUED) and universal extra dimensions (UED). In the non-universal model, the gauge bosons propagate into the bulk, but the fermions are confined to $D_3$ bran. In contrast, the UED allows fields to propagate into the bulk. The UED can be considered as a generalization of the usual SM to a $D_{3+N}$ bran where $N$ in the number of the extra dimensions [3]. The model introduced by Appelquist, Cheng, and Dobrescu (ACD) [2] is the most simple example of the UED where just single universal extra dimension is considered. This model has only one free parameter in addition to the SM parameters and that is the compactification scale $R$. Mass of the Kaluza-Klein (KK) particles are inversely proportional to $R$, then, at large value of $1/R$ the SM results can be almost recovered, since the KK modes, being more and more massive, are decoupled from the low-energy SM.

Two types of study can be conducted to explore extra dimensions. In the direct search, the center of mass energy of colliding particles must be increased to produce Kaluza-Klein (KK) excitation states, where KK excitation states are supposed to produce in pair by KK number conservation. On the other hand, we can investigate UED effects, indirectly. The indirect search at tree-level, where KK excitations can contribute as a mediator, is suppressed by KK number conservation. On the contrary, the same states can contribute to the quantum loop level where the KK number conservation is broken. As a result, flavor changing neutral current (FCNC) transition induced by quantum loop level can be considered as a good tool for studying KK effects. The collider signatures and phenomenology of UED have been studied by Ref. [3] and [4, 5], respectively. These studies have provided a theoretical framework to examine some inclusive and exclusive decays with the ACD.

FCNC and CP-violating are indeed the most sensitive probes of New Physics (NP) contributions to penguin operators. Rare decays, induced by flavor changing neutral current (FCNC) $b \rightarrow s(d)$ transitions is at the forefront of our quest to understand flavor and the origins of CP violation asymmetry (CPV), offering one of the best probes for NP beyond the Standard Model, in particular to probe extra dimension. In this regard, the semileptonic and pureleptonic B decays have been studied with UED scenario [4]–[9]. They have obtained that the inclusive and exclusive semileptonic and pureleptonic decays are sensitive to the new parameter coming out of the one universal extra dimensions i.e., compactification scale $1/R$.

New physics effects manifest themselves in rare decays in different ways: NP can contribute through the new Wilson coefficients or the new operator structure in the effective Hamiltonian, which is absent in the SM. Also, it may modify the SM parameters such as masses and CKM matrix elements. A crucial problem in the new physics search within
flavour physics in the exclusive decays is the optimal separation of new physics effects from uncertainties. It is well known that inclusive decay modes are dominated by partonic contributions; non-perturbative corrections are in general rather smaller[10]. Also, ratios of exclusive decay modes such as asymmetries for $B \to K( K^*, \rho, \gamma) \ell^+\ell^-$ decay [11]–[15] are well studied for new-physics search. Here, large parts of the hadronic uncertainties partially cancel out. The universal extra dimension with only one universal extra dimension belongs to the classes of NP, where the Wilson coefficients is modified by KK contributions [4, 5] in the penguin and box diagrams. Also, CKM matrix elements and masses are affected by ACD model. Obviously these modifications will affect the physical observables. In this connection, we try to investigate the effects of one universal extra dimension on the branching ratio(Br) of the $B \to \pi(\rho) \ell^+\ell^-$ decay.

The paper encompasses three sections: In section 2, we recall the effects of the UED on the inclusive $b \to d \ell^+\ell^-$ decay and the general expressions for the matrix element and branching ratios of $B \to \pi(\rho) \ell^+\ell^-$ decay is presented. In section 3, we investigate the sensitivity of Br and CP asymmetry to the compactification scale($R$) and conclusion.

2 Matrix element $b \to d \ell^+\ell^-$ in ACD model

The QCD corrected effective Lagrangian for the decays $b \to d \ell^+\ell^-$ can be achieved by integrating out the heavy quarks and the heavy electroweak bosons in the SM:

$$M(b \to s \ell^+\ell^-) = \frac{G_F \alpha}{\sqrt{2} \pi} V_{tb} V_{td}^* \left\{ C_9^{eff} \left[ \bar{d} \gamma_\mu L b \right] \left[ \bar{\ell} \gamma^\mu \ell \right] ight. 
+ C_{10} \left[ \bar{d} \gamma_\mu L b \right] \left[ \bar{\ell} \gamma^\mu \gamma^5 \ell \right] 
- 2 \hat{m}_b C_7^{eff} \left[ \bar{d} i \sigma_{\mu\nu} \gamma^\mu R b \right] \left[ \bar{\ell} \gamma^\nu \ell \right] \right\}$$

(1)

where $C_9$ are Wilson co-efficients calculated in naive dimensional regularization (NDR) scheme at the leading order (LO), next to leading order (NLO) and next-to-next leading order (NNLO) in the SM[16]–[22].

As we mentioned above, in ACD model the new physics comes through the modification of the Wilson coefficients and the operator structures remain the same as SM. Considering the KK modes effects in the penguin and box diagrams, the above coefficients have obtained at NLO [4, 5]. Clearly, they depend on the additional ACD parameter i.e., $R$. These coefficients can be expressed in terms of the functions $F(x_t, 1/R)$, $x_t = \frac{m_t^2}{M_W^2}$, which is the generalization of the corresponding SM function $F_0(x_t)$ according to:

$$F(x_t, 1/R) = F_0(x_t) + \sum_{n=1}^{\infty} F_n(x_t, x_n)$$

(2)

with $x_n = \frac{m_n^2}{M_W^2}$ and $m_n = \frac{\pi}{R}$ [2] corresponding functions are $C(x_t, 1/R)$, $D(x_t, 1/R)$, $E(x_t, 1/R)$, $D'(x_t, 1/R)$ and $E'(x_t, 1/R)$, respectively. The Wilson coefficients in terms of these functions computed in [16]–[22]. We recall the formulae for the Wilson coefficients where we use the Wilson coefficients at the renormalization scale $\mu = m_b = 4.7 \text{ GeV}$. 
The functions $D$ where the wilson coefficients have been calculated in the leading order approximation.

$$
\eta = \frac{\alpha_s(\mu_w)}{\alpha_s(\mu_b)}, \quad \text{and}
$$

$$
C_7^{UED}(\mu_w) = 1, \quad C_7^{UED}(\mu_w) = -\frac{1}{2} D'(x_t, \frac{1}{R}), \quad C_8^{UED}(\mu_w) = -\frac{1}{2} E'(x_t, \frac{1}{R});
$$

where the wilson coefficients have been calculated in the leading order approximation. Moreover:

$$
\begin{align*}
\alpha_1 &= \frac{14}{23} \quad \alpha_2 = \frac{16}{23} \quad \alpha_3 = \frac{6}{23} \quad \alpha_4 = -\frac{12}{23} \\
\alpha_5 &= 0.4086 \quad \alpha_6 = -0.4230 \quad \alpha_7 = -0.8994 \quad \alpha_8 = -0.1456 \\
\end{align*}
$$

$$
\begin{align*}
h_1 &= 2.996 \quad h_2 = -1.0880 \quad h_3 = -\frac{3}{7} \quad h_4 = -\frac{1}{14} \\
\end{align*}
$$

$$
\begin{align*}
h_5 &= -0.649 \quad h_6 = -0.0380 \quad h_7 = -0.0185 \quad h_8 = -0.0057.
\end{align*}
$$

The functions $D'$ and $E'$ in Eq. (5) are given as:

$$
D'_0(x_t) = \frac{-8x_t^2 + 5x_t^2 - 7x_t}{12(1-x_t)^3} + \frac{x_t^2(2-3x_t)}{2(1-x_t)^4} \ln x_t
$$

$$
E'_0(x_t) = -\frac{x_t(x^2_t - 5x_t - 2)}{4(1-x_t)^3} + \frac{3x_t^2}{2(1-x_t)^4} \ln x_t
$$

$$
D'_n(x_t, x_n) = \frac{x_t(-37 + 44x_t + 17x_t^2 + 6x_n^2(10 - 9x_t + 3x_t^2) - 3x_n(21 - 54x_t + 17x_t^2))}{36(x_t - 1)^3}
$$

$$
+ \frac{x_n(2 - 7x_n + 3x_n^2)}{6} \ln \frac{x_n}{1 + x_n}
$$

$$
- \frac{(x_t - 3x_t)(x_t + 3x_t^2 + x^2_n(3 + x_t) - x_n(1 + (-10 + x_t)x_t))}{6(x_t - 1)^4} \ln \frac{x_n + x_t}{1 + x_n}
$$

$$
E'_n(x_t, x_n) = \frac{x_t(-17 - 8x_t + x_t^2 + 3x_n(21 - 6x_t + x_t^2) - 6x_n^2(10 - 9x_t + 3x_t^2))}{12(x_t - 1)^3}
$$

$$
+ \frac{1}{2} x_n(1 + x_n)(-1 + 3x_n) \ln \frac{x_n}{1 + x_n}
$$

$$
+ \frac{(1 + x_n)(x_t + 3x_n^2 + x^2_n(3 + x_t) - x_n(1 + (-10 + x_t)x_t))}{2(x_t - 1)^4} \ln \frac{x_n + x_t}{1 + x_n}
$$

$\bullet C_9$

With regard ACD model and in the NDR scheme we have

$$
C_9(\mu) = P_0^{NDR} + \frac{Y(x_t, \frac{1}{R})}{\sin^2 \theta_W} - 4Z(x_t, \frac{1}{R}) + P_E E(x_t, \frac{1}{R})
$$
where \( P_0^{NDR} = 2.60 \pm 0.25 \) and the last term is numerically negligible\( (P_E \sim 10^{-2}) \). Besides

\[
Y(x_t, \frac{1}{R}) = Y_0(x_t) + \sum_{n=1}^{\infty} C_n(x_t, x_n)
\]

\[
Z(x_t, \frac{1}{R}) = Z_0(x_t) + \sum_{n=1}^{\infty} C_n(x_t, x_n)
\]

(11)

with

\[
Y_0(x_t) = \frac{x_t[x_t - 4]}{8} - \frac{3x_t}{(x_t - 1)^2} \ln x_t
\]

\[
Z_0(x_t) = \frac{18x_t^4 - 163x_t^3 + 259x_t^2 - 108x_t}{144(x_t - 1)^3}
\]

\[
+ \frac{32x_t^4 - 38x_t^3 + 15x_t^2 - 18x_t}{72(x_t - 1)^4} - \frac{1}{9} \ln x_t
\]

(12)

\[
C_n(x_t, x_n) = \frac{x_t}{8(x_t - 1)^2} [x_t^2 - 8x_t + 7(3 + 3x_t + 7x_n - x_tx_n) \ln \frac{x_t + x_n}{1 + x_n}]
\]

(13)

On the other hand, the effective coefficient \( C_9^{\text{eff}} \) is scheme independent. It can be parametrised as follows:

\[
C_9^{\text{eff}} = \xi_1 + \lambda_{tu} \xi_2,
\]

(14)

where

\[
\lambda_{tu} = \frac{V_{ub}V^*_{ud}}{V^*_{tb}V_{td}},
\]

and

\[
\xi_1 = C_9 + 0.138\omega(\hat{s}) + g(\hat{m}_c, \hat{s})C_0(\hat{m}_b) - \frac{1}{2}g(\hat{m}_d, \hat{s})(C_3 + C_4)
\]

\[- \frac{1}{2}g(\hat{m}_b, \hat{s})(4C_3 + 4C_4 + 3C_5 + C_6) + \frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6),
\]

\[
\xi_2 = [g(\hat{m}_c, \hat{s}) - g(\hat{m}_u, \hat{s})](3C_1 + C_2),
\]

(15)

where \( \hat{m}_q = m_q/m_b, \hat{s} = q^2 \), \( C_0(\mu) = 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6 \), and

\[
\omega(\hat{s}) = -\frac{2}{9}\pi^2 - \frac{4}{3}Li_2(\hat{s}) - \frac{2}{3}\ln(\hat{s})\ln(1 - \hat{s}) - \frac{5 + 4\hat{s}}{3(1 + 2\hat{s})} \ln(1 - \hat{s})
\]

\[- \frac{2\hat{s}(1 + 2\hat{s})(1 - 2\hat{s})}{3(1 - \hat{s})^2(1 + 2\hat{s})} \ln(\hat{s}) + \frac{5 + 9\hat{s} - 6\hat{s}^2}{3(1 - \hat{s})(1 + 2\hat{s})},
\]

(16)

represents the \( O(\alpha_s) \) correction coming from one gluon exchange in the matrix element of the operator \( \mathcal{O}_9 \) [19], while the function \( g(\hat{m}_q, \hat{s}) \) represents one–loop corrections to the four–quark operators \( \mathcal{O}_1–\mathcal{O}_6 \) [21], whose form is

\[
g(\hat{m}_q, \hat{s}) = -\frac{8}{9} \ln(\hat{m}_q) + \frac{8}{27} + \frac{4}{9}y_q - \frac{2}{9}(2 + y_q)
\]

\[- \sqrt{1 - y_q} \left\{ \theta(1 - y_q) \left[ \ln \left( \frac{1 + \sqrt{1 - y_q}}{1 - \sqrt{1 - y_q}} \right) - i\pi \right] + \theta(y_q - 1) \arctan \left( \frac{1}{\sqrt{y_q - 1}} \right) \right\}
\]

(17)
where \( y_q = 4\hat{m}_c^2/\hat{s} \). It should also be emphasized that \( C^\text{eff}_9 \) is in particular sensitive to the \( \hat{m}_c \) in the NLO. To reduce this dependency NNLO calculation is necessary.

Although long-distance effects of \( c\bar{c} \) bound states could contribute to \( C^\text{eff}_9 \), for simplicity, they are not included in the present study.

- **\( C_{10} \)**

  \( C_{10} \) is \( \mu \) independent and is given as:

  \[
  C_{10} = -\frac{Y(x_t, \frac{1}{\pi})}{\sin^2 \theta_w}. \tag{18}
  \]

We aim to calculate the decay rate for \( B \to \pi(\rho)\ell^+\ell^- \) using the experimental allowed region for \( 1/R \) from the \( B(B \to X_s\gamma) \) and \( B(B \to K^*\mu^+\mu^-) \) decays. One has to sandwich the inclusive effective Hamiltonian between initial hadron state \( B(p_B) \) and final hadron state \( \pi(p_\pi)(\rho(p_\rho)) \) to obtain the matrix element for the exclusive decay \( B \to \pi(\rho) \ell^+\ell^- \).

Following from Eq. (1), in order to calculate the decay width and other physical observable of the exclusive \( B \to \pi \ell^+\ell^- \) decay, we need to parametrize the matrix elements in terms of formfactors.

### 2.1 Decay rate for \( B \to \pi \ell^+\ell^- \) decay

The exclusive \( B \to \pi \ell^+\ell^- \) decay, which is described in terms of the matrix elements of the quark operators given in Eq. (1) over meson states, can be parametrized in terms of form factors \((f^+ f^-) \) and \( f_v \).

\[
\langle \pi(p_\pi)|\bar{d}\gamma_\mu(1 - \gamma^5)b|B(p_B)\rangle = f^+(q^2)(p_\pi + p_B)_\mu + f^-(q^2)q_\mu, \tag{19}
\]

\[
\langle \pi(p_\pi)|\bar{d}\sigma_{\mu\nu}q'^\nu(1 + \gamma^5)b|B(p_B)\rangle = [q^2(p_\pi + p_B)_\mu - q_\mu(m_B^2 - m_\pi^2)]f_v(q^2), \tag{20}
\]

Now, we can obtain the matrix element as:

\[
M_{B \to \pi} = \frac{G_F\alpha}{2\sqrt{2\pi}}V_{tb}V_{td}^*\left\{ (2Ap_\pi^\mu + Bq^\mu)\bar{\ell}\gamma_\mu\ell + (2Gp_\pi^\mu + Dq^\mu)\bar{\ell}\gamma_\mu\gamma^5\ell \right\}, \tag{21}
\]

where

\[
A = C^\text{eff}_9f^+ - 2m_BC^\text{eff}_7f_v, \tag{22}
\]

\[
B = C^\text{eff}_9(f^+ + f^-) + 2\frac{m_B}{q^2}C^\text{eff}_7f_v(m_B^2 - m_\pi^2 - q^2),
\]

\[
G = C_{10}f^+,
\]

\[
D = C_{10}(f^+ + f^-),
\]

From this expression of the matrix element, for the unpolarized differential decay width we get the following result:

\[
\left( \frac{d\Gamma^\pi}{ds} \right)_0 = \frac{G_F^2\alpha^2}{16\pi^5}|V_{tb}V_{td}^*|^2m_B^3v\sqrt{\lambda_\pi}\Delta_\pi, \tag{23}
\]


\[ \Delta_{\pi} = \frac{1}{3} m^2_{B} \lambda_{\pi}(3 - v^2)(|A|^2 + |G|^2) + 16m^2_{\rho}r_{\pi}|G|^2 + 4m^2_{\rho}s|D|^2 \]  
(24)

\[ + \ 8m^2_{\rho}(1 - r_{\pi} - s)Re[GD^{*}], \]

with \( r_{\pi} = m^2_{\pi}/m^2_{B}, \lambda_{\pi} = r^2_{\pi} + (s - 1)^2 - 2r_{\pi}(s + 1), v = \sqrt{1 - \frac{4t^2}{s}} \) and \( t = m_{\ell}/m_{B} \). We use the results of the constituent quark model [23], where the form factors \( f_{T} \) and \( f_{+} \) can be parametrized as:

\[ f(q^2) = \frac{f(0)}{(1 - q^2/T_f^2)[1 - \sigma_1q^2/M^2 + \sigma_2q^4/M^4]} \cdot \]  
(25)

In this model, \( f_{-} \) is defined slightly different and it is as:

\[ f(q^2) = \frac{f(0)}{[1 - \sigma_1q^2/M^2 + \sigma_2q^4/M^4]} \cdot \]  
(26)

The parameters \( f(0), \sigma_{i} \)'s can be found in Table 1.

| \( f(0) \) | \( \sigma_{1} \) | \( \sigma_{2} \) |
|---|---|---|
| \( f_{+} \) | 0.29 | 0.48 |
| \( F_{0} \) | 0.29 | 0.76 | 0.28 |
| \( f_{\nu} \) | 0.28 | 0.48 |

Table 1: \( B \rightarrow \pi \) transition form factors in the constituent quark model.

### 2.2 Decay rate for the \( B \rightarrow \rho \ \ell^{+}\ell^{-} \) decay

Similar to the \( B \rightarrow \pi \ \ell^{+}\ell^{-} \) decay the following matrix elements defined in terms of form-factors must be computed for the \( B \rightarrow \rho \ \ell^{+}\ell^{-} \) decay:

\[ \langle \rho(p_{\rho}, \varepsilon)|\bar{d}\gamma_{\mu}(1 - \gamma^{5})b|B(p_{B}) \rangle = -\varepsilon^{\mu}_{\nu\lambda\sigma}\varepsilon'_{\nu}p^{\lambda}_{\rho}p^{\sigma}_{B} \frac{2V(q^2)}{m_{B} + m_{\rho}} - i\varepsilon_{\mu}(m_{B} + m_{\rho})A_{1}(q^2) \]

\[ + \ i(p_{B} + p_{\rho})(\varepsilon^{*}q)\frac{A_{2}(q^2)}{m_{B} + m_{\rho}} \]

\[ + \ iq_{\mu}(\varepsilon^{*}q)\frac{2m_{\rho}^{2}}{q^2}[A_{3}(q^2) - A_{0}(q^2)], \]  
(27)

\[ \langle \rho(p_{\rho}, \varepsilon)|\bar{d}i\sigma_{\mu\nu}q^\nu(1 \pm \gamma^{5})b|B(p_{B}) \rangle = 4\varepsilon^{\mu\nu\lambda\sigma}p^{\nu}_{\rho}q^{\sigma}T_{1}(q^2) \pm 2i[\varepsilon'_{\mu}(m_{B}^2 - m_{\rho}^2) \]

\[ - \ (p_{B} + p_{\rho})_{\mu}(\varepsilon^{*}q)T_{2}(q^2) \]

\[ \pm \ 2i(\varepsilon^{*}q)\left(q_{\mu} - (p_{B} + p_{\rho})_{\mu}\frac{q^2}{m_{B}^2 - m_{\rho}^2}\right)T_{3}(q^2), \]  
(28)

\[ \langle \rho(p_{\rho}, \varepsilon)|\bar{d}(1 + \gamma^{5})b|B(p_{B}) \rangle = -\frac{2im_{\rho}}{m_{b}}(\varepsilon^{*}q)A_{0}(q^2), \]  
(29)
where \( p_\rho \) and \( \varepsilon \) denote the four momentum and polarization vector of the \( \rho \) meson, respectively.

From Eqs.(27,28,29), we get the following expression for the matrix element of the \( B \to \rho \ell^+\ell^- \) decay:

\[
M^{B-\rho} = \left[ i\epsilon_{\mu\nu\lambda\rho}^{\ast\ast} \rho^\lambda \beta A + \epsilon_\mu^\ast B \pm \epsilon_\lambda^\ast E \pm (\epsilon_\rho^\ast \eta) (p_B) F \right] (\ell^- \gamma^\mu \ell^-) + G(\epsilon_\rho^\ast \eta)(\ell^- \gamma^5 \ell^-)
\]

where

\[
\begin{align*}
A &= \frac{4(m_b + m_d)T_1(q^2)}{m_B^2 s} + \frac{V(q^2)}{m_B + m_\rho} C_7^{\text{eff}}, \\
B &= -\frac{2(m_b - m_d)(1 - r_\rho)}{s} T_2(q^2) C_7^{\text{eff}} - \frac{(m_B + m_\rho)A_1(q^2)}{2} C_9^{\text{eff}}, \\
C &= \frac{4(m_b - m_d)}{m_B s} \left( T_2(q^2) + \frac{s}{1 - r_\rho} T_3(q^2) \right) C_7^{\text{eff}} + \frac{A_2(q^2)}{m_B + m_\rho} C_9^{\text{eff}}, \\
D &= \frac{V(q^2)}{m_B + m_\rho} C_10, \\
E &= -\frac{(m_B + m_\rho)A_1(q^2)}{2} C_10, \\
F &= \frac{A_2(q^2)}{m_B + m_\rho} C_10, \\
G &= \left( -\frac{m_\ell}{m_B + m_\rho} A_2(q^2) + \frac{2m_\rho m_\ell}{m_B^2 s} (A_3(q^2) - A_0(q^2)) \right) C_10.
\end{align*}
\]

From this expression of the matrix element, we get the following result for the differential decay width:

\[
\left( \frac{d\Gamma^\rho}{ds} \right)_0 = \frac{G_\rho^2 \alpha^2}{3 \times 2^{10} \pi^5} |V_{tb} V_{td}^\ast|^2 m_B^5 v |\bar{\rho}_\Delta| \Delta^\rho.
\]

\[
\Delta^\rho = (1 + \frac{2t^2}{s}) \lambda_\rho \left[ 4m_B^2 s |A|^2 + \frac{2}{m_B^2 r_\rho} (1 + 12 \frac{sr_\rho}{\lambda_\rho}) |B|^2 \right] \\
+ \frac{m_B^2}{2 r_\rho} \lambda_\rho |C|^2 + \frac{2}{r_\rho} (1 - r_\rho + s) \text{Re}(B^* C) \right] + 4m_B^2 \lambda_\rho (s - 4t^2) |D|^2 \\
+ \frac{4(2t^2 + s) - 4(2t^2 + s)(r_\rho + s) + 4t^2(r_\rho^2 - 26r_\rho + s^2) + 2s(r_\rho^2 + 10sr_\rho + s^2)}{m_B^2 sr_\rho} |E|^2 \\
+ \frac{m_B^2}{2sr_\rho} \lambda_\rho \left[ (2t^2 + s)(\lambda_\rho + 2s + 2r_\rho) - 2 \{2t^2(r_\rho + 5s) + s(r_\rho + s) \} \right] |F|^2 \\
+ \frac{3}{r_\rho} \lambda_\rho |G|^2 + \frac{2\lambda_\rho}{sr_\rho} \left[ -2t^2(r_\rho - 5s) + (2t^2 + s) - s(r_\rho + s) \right] \text{Re}(E^* F) \\
+ \frac{12t}{m_B r_\rho} \lambda_\rho \text{Re}(G^* E) + \frac{2m_B t}{r_\rho} \lambda_\rho (1 - r_\rho + s) \text{Re}(G^* F)
\] (33)
with \( r_\rho = m_\rho^2/m_B^2, \lambda_\rho = r_\rho^2 + (s - 1)^2 - 2r_\rho(s + 1), v = \sqrt{1 - \frac{4t^2}{s}} \) and \( t = m_\ell/m_B \). The definition of the form factors are (see [24]):

\[
V(q^2) = \frac{V(0)}{1 - q^2/5^2},
\]

\[
A_1(q^2) = A_1(0)(1 - 0.023q^2),
\]

\[
A_2(q^2) = A_2(0)(1 + 0.034q^2),
\]

\[
A_0(q^2) = \frac{A_3(0)}{1 - q^2/4.8^2},
\]

\[
A_3(q^2) = \frac{m_B + m_\rho}{2m_\rho}A_1(q^2) - \frac{m_B - m_\rho}{2m_\rho}A_2(q^2),
\]

\[
T_1(q^2) = \frac{T_1(0)}{1 - q^2/5.3^2},
\]

\[
T_2(q^2) = T_2(0)(1 - 0.02q^2),
\]

\[
T_3(q^2) = T_3(0)(1 + 0.005q^2).
\]

with \( V(0) = 0.47, A_1(0) = 0.37, A_2(0) = 0.4, T_1(0) = 0.19, T_2(0) = 0.19, T_3(0) = -0.7 \).

### 3 Numerical analysis

In this section, we study the dependence of the total branching ratio on the compactification parameters \((1/R)\). The main input parameters in the calculations are the form factors. We use the results of Refs. [23] and [24] for \( B \to \pi \) and \( B \to \rho \) transitions, respectively. Also, we use the SM parameters shown in table 1:

| Parameter       | Value                  |
|-----------------|------------------------|
| \( \alpha_s(m_Z) \) | 0.119                  |
| \( \alpha_{em} \) | 1/129                  |
| \( m_W \)       | 80.41 (GeV)            |
| \( m_Z \)       | 991.18 (GeV)           |
| \( \sin^2(\theta_W) \) | 0.223                |
| \( m_b \)       | 4.7 (GeV)              |
| \( m_\mu \)     | 0.106 (GeV)            |
| \( m_\tau \)    | 1.780 (GeV)            |

Table 2: The values of the input parameters used in the numerical calculations.

The allowed range in the ACD model for the Wolfenstein parameters at \( 1/R = 200\text{GeV} \) are: \( 0.076 \leq \bar{\rho} \leq 0.260 \) and \( 0.305 \leq \bar{\eta} \leq 0.411 \) [4]. Note that, there is a small discrepancy with respect to the SM values in the higher values of \( 1/R \). In the present analysis, they are set as \( \bar{\rho} = 0.25 \) and \( \bar{\eta} = 0.34 \). In the Wolfenstein parametrization of the CKM matrix, \( \lambda_{tu} \)
\[\lambda_{tu} = \bar{\rho}(1 - \bar{\rho}) - \bar{\eta}^2 - i\bar{\eta} + O(\lambda^2). \quad (35)\]

Furthermore, we use the relation
\[
\frac{|V_{ub}V_{ub}^*|^2}{|V_{cb}|^2} = \lambda^2[(1 - \bar{\rho})^2 + \bar{\eta}^2] + O(\lambda^4) \quad (36)
\]
where \(\lambda = \sin \theta_W\).

From explicit expressions of the physical observables, one can easily see that they depend on both \(\hat{s}\) and the compactification radius \((1/R)\). One may eliminate the dependence on one of the variables. We eliminate the variable \(\hat{s}\) by performing integration over \(\hat{s}\) in the allowed region. The total branching ratio is defined as
\[
B_r = \int_{4m^2_{\ell}\ell/m^2_{B}}^{(1-\sqrt{r_{\pi(\rho)}})^2} \frac{dB}{d\hat{s}} d\hat{s}. \quad (37)
\]

The branching ratio given by Eq. (37) depends on compactification radius \(R\). The conservation of KK parity \((-1)^j\), with \(j\) the KK number, implies the absence of tree level contribution of KK states at low energy regime. This allows us to establish a bound: \(1/R > 250\text{GeV}\) by the analysis of Tevtron run I data\cite{2}. The same bound can be obtained by the analysis of measured branching ratio of \(B \to X_s \gamma\) decay\cite{4, 5}. A sharper constrain on \(1/R\) can be established by studying the zero point position of forward–backward asymmetry of \(B \to K^* \ell^+\ell^-\) decay the point of which is almost free of hadronic uncertainties\((\sim 5\%)\). In what follows, we consider \(200 < 1/R < 1000\text{GeV}\) and analyse the dependency of branching ratios in terms of inverse of the compactification radius \(R\). Also, physical observables are sensitive to \(\hat{m}_c\) where we use two different values of \(\hat{m}_c = 0.22\) and \(\hat{m}_c = 0.29\) in our numerical calculations.

Figs. (1)–(4) depict the dependence of the total branching ratio in terms of the compactification parameter \(1/R\) in two different values of \(\hat{m}_c = 0.22\) and \(\hat{m}_c = 0.29\) for \(B \to \pi \ell^+\ell^-\) and \(B \to \rho \ell^+\ell^-\) decays, respectively. Looking at these figures, we can see:

- \(B_r\) strongly depends on the compactification radius \(R\) for both \(\mu\) and \(\tau\) channels. Furthermore, \(B_r\) is decreasing function of the \(1/R\) where at large value of \(1/R\) the result of SM and ACD is almost the same as it is expected. Moreover, while the branching ratio of \(B \to \pi \ell^+\ell^-\) decay is sensitive to the value of \(\hat{m}_c\) for \(\mu\) lepton, the \(B \to \rho \ell^+\ell^-\) decay depicts a weak dependency on the value of \(\hat{m}_c\) for both \(\mu\) and \(\tau\) leptons.

To sum up, we presented the analysis of the \(B \to \pi(\rho) \ell^- \ell^+\) decay in ACD model with single universal extra dimension. The only free parameter of the model is compactification radius \(1/R\). We studied the dependence of the branching ratio on the inverse of compactification radius \(1/R\). The value of the branching ratio was obtained larger than the corresponding SM value.

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Figure captions

**Fig. (1)** The dependence of the total branching ratio of \( B \to \pi\mu^+\mu^- \) on \( 1/R \) for \( \hat{m}_c = 0.22, 0.29 \).

**Fig. (2)** The same as in Fig. (1), but for the \( \tau \) lepton.

**Fig. (3)** The dependence of the total branching ratio of \( B \to \rho\mu^+\mu^- \) on \( 1/R \) for \( \hat{m}_c = 0.22, 0.29 \).

**Fig. (4)** The same as in Fig. (3), but for the \( \tau \) lepton.
Figure 1:

Figure 2:
Figure 3:

Figure 4: