Microwave spectroscopic observation of a Wigner crystal of 1/2-charged quasiparticles

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Abstract

We have studied the microwave spectra of a wide quantum well for Landau level fillings, \( \nu \), just below 1/2, under conditions where the \( \nu = 1/2 \) fractional quantum Hall effect (FQHE) is present. One resonance in the spectra exhibits intensity variations with \( \nu \) in striking agreement with that expected for a pinning mode of Wigner solid of the charge 1/2 quasiholes of this FQHE state. This resonance is also quite sensitive to asymmetrization of the growth-direction charge distribution in the quantum well by gate bias. Another resonance in the spectra is associated with a different bilayer Wigner solid that also exists at much lower \( \nu \) than the 1/2 FQHE, and that appears to coexist with the 1/2 quasihole solid.
Wigner solids (WSs)\textsuperscript{[1,2]} are composed of charged carriers whose repulsive interaction dominates their zero-point and thermal motion. Two-dimensional electron systems (2DESs) in high magnetic fields offer a clear limiting case, since without disorder it is accepted that the ground state of a 2DES at sufficiently high magnetic field is a Wigner crystal\textsuperscript{2}. While there is experimental evidence\textsuperscript{[3–16]} and theoretical prediction\textsuperscript{[2,17,18]} for WSs in high but experimentally achievable magnetic fields, in these finite fields quantum correlation between carriers and finite overlap of single-carrier wave functions must be considered. One theoretical approach\textsuperscript{[19–22]} maintains the Wigner lattice nature of the state, in agreement with the experimental evidence pointing to an electron solid, but regards the carriers as composite fermions, which contain the correlations responsible for the fractional quantum Hall effect (FQHE) at rational fractional Landau level fillings, $\nu$. The existence of a WS demonstrates that the carriers composing it are interacting.

A central feature of the FQHE is that quasiparticles and quasiholes are fractionally charged; WSs of these carriers\textsuperscript{22} are expected to form close to (but not at) rational fractional $\nu$, so the concentration of these carriers is dilute. So far there is experimental evidence\textsuperscript{23} of such a solid only for the 1/3 FQHE, though even-denominator FQHE states are of current interest since some of them offer the possibility of non-abelian quasiparticle statistics. Fractional quasiparticle or -hole WSs are expected to be sensitive to the statistics of their constituent carriers, which may be non-abelian. In particular for non-abelian statistics, a WS has a density of states that allows for adiabatic refrigeration\textsuperscript{24}, but is likely as well to have different effective moduli from those of solids of carriers with abelian statistics. WSs of fractionally charged carriers are also relevant in implementing schemes for study of quasiparticle statistics, for example by mediating energy transmission through the bulk\textsuperscript{25,26}.

WSs in a 2DES at high magnetic field can be classified into two different types\textsuperscript{21}. Here we take a type-I WS to be a ground state of the entire 2DES, such as is found at the low $\nu$ termination of the FQHE series\textsuperscript{[2,17,18]}. The second, type-II WSs, are formed of quasiholes or quasiparticles in the presence of a gapped state such as a filled Landau level at an integer quantum Hall effect (IQHE) plateau\textsuperscript{[16,27,28]}, or a FQHE liquid\textsuperscript{23}. Crucially, the density of carriers in a type-II WS, but not a type-I WS, is dependent on the magnetic field, because the quasiparticle or quasihole density increases as flux is subtracted or added to the parent, gapped state.
Here we present evidence for a solid of 1/2-charged carriers, associated with the 1/2 FQHE in a wide quantum well (WQW) in GaAs \[29–36\]. While the possibility of one-component states in these systems has been considered \[37\] there is considerable evidence \[30, 33\] that it is a manifestation of the two-component Halperin-Laughlin state \(\Psi_{331}\) \[38\], which has ordinary abelian statistics. The 1/2 FQHE has also been observed in double quantum wells \[39\], in which it is likewise understood as a \(\Psi_{331}\) state.

In WQWs the \(\nu = 1/2\) FQHE state exists for electron density, \(n\), in a certain range \[29–33, 40\]. Increasing \(n\) by means of symmetric front and back gate bias, causes the system to become more bilayer-like by reduction of the first-excited subband energy and by increasing the interlayer distance and the effective intralayer Coulomb interaction, \(e^2/4\pi\varepsilon\ell_B\), where \(\ell_B = \sqrt{\hbar/eB}\) is the magnetic length. A 1/2 FQHE state occurs when the system is sufficiently bilayer-like, but only when the inter- and intralayer Coulomb interaction energies are comparable \[29, 34\], hence for a given WQW the 1/2 FQHE occurs only within a particular range of \(n\). For larger \(n\) outside this range, the 1/2 FQHE gives way to an insulator \[31\] on increasing \(B\). The insulator is interpreted as a WS pinned by disorder, and a recent theory \[40\] of WQWs identifies this WS as a pinned rectangular lattice of the first excited subband, which is a type-I bilayer electron solid \[41\].

Microwave spectroscopy is of value for investigating solid phases because the spectrum of a WS exhibits a characteristic resonance due to the pinning of the WS by residual disorder. The resonance is understood as a pinning mode, a collective oscillation of solid domains about their pinned positions. Pinning modes have been used to study type-I WSs for \(\nu \lesssim 1/5\) \[3, 5, 7, 10, 13, 42\] and also for type-II WSs within the filling-factor ranges of the IQHE \[27, 28\], and for the WS of 1/3-charged carriers within the \(\nu = 1/3\) FQHE \[23\]. Of central importance for this paper is the recent observation \[41\] of a pinning mode of the low \(\nu\) insulator of a WQW. This insulator, which is found from \(\nu\) just below 1/2 to the lowest \(\nu\) that have been studied, is understood as a pinned, type-I, bilayer electron solid.

In this paper we present microwave measurements of a WQW with \(\nu\) near 1/2, under conditions for which the \(\nu = 1/2\) FQHE can be observed. For \(\nu < 0.47\) a resonance like the one observed in Ref. \[41\] is present and associated with a type-I bilayer electron solid. For a narrow range of \(\nu > 0.48\) there is a second, higher-frequency resonance, which we interpret as due to a type-II solid of 1/2 charged quasihole excitations. We find strong evidence for the type-II nature of this WS from a quantitative analysis of the resonance intensity vs \(\nu\).
The type-II resonance is more sensitive than the type-I resonance to the charge distribution in the growth direction of the well, so that small asymmetry in the well induced by gate bias weakens the $\nu = 1/2$ quasihole WS resonance concomitant with a relative strengthening of the type-I bilayer electron solid resonance. The observation of two resonance peaks in the same spectrum demonstrates coexistence of the type-I bilayer electron solid [41] and the type-II quasihole WS of the 1/2 FQHE state.

The measurements were performed on a GaAs/AlGaAs WQW of width $w = 80$ nm with an as-cooled density of $n = 1.1$ in units of $10^{11}$ cm$^{-2}$, which we use for density throughout the paper. Figure 1(a) shows a schematic representation of the top view of the microwave set-up. Our technique [12–15, 27, 42, 43] uses a coplanar waveguide (CPW) on the surface of the sample. We calculate the diagonal conductivity as $\sigma_{xx}(f) = (s/lZ_0) \ln(t/t_0)$, where $s = 30 \mu$m is the distance between the center conductor and ground plane, $l = 28$ mm is the length of the CPW, $Z_0 = 50 \Omega$ is the characteristic impedance without the 2DES, and $t$ is the transmitted signal amplitude and $t_0$ is the normalizing amplitude taken at $\nu = 0.5$. As elsewhere [28, 41], back and front gates are used to control the carrier density and the symmetry of the growth-direction charge distribution in the WQW. Measurements were performed at a bath temperature of $T = 50$ mK in the limit of low microwave power.

In Fig. 1(b) we plot $\text{Re}(\sigma_{xx})$ vs $f$ for several $\nu$, for a symmetric charge distribution at $n = 1.60$. Simulation of the charge distribution within the well is shown in the inset. The symmetric growth-direction charge distribution was maintained by biasing the gates such that individually they would change the density by the same amount. These spectra are obtained at several $\nu$ and are vertically offset for clarity, with $\nu$ marked on the right axis. At $\nu = 0.470$ we observe a well-defined resonance with $f_{pk} \sim 0.4$ GHz. We ascribe this resonance to a type-I WS, consistent in $f_{pk}$ with earlier studies [41]. As $\nu$ in Fig. 1(b) increases, the $f_{pk}$ of this type-I WS resonance does not change, and though its amplitude decreases, it is resolved through the entire range $0.470 < \nu < 0.498$. Also as $\nu$ increases a shoulder, visible for $\nu = 0.480$, develops into a second, independent resonance at $f_{pk} \sim 1.2$ GHz, which has nonmonotonic amplitude change as $\nu = 0.5$ is approached. Similar development of two resonances is observed in Fig. 1(c) at $n = 1.49$. The type-I WS resonance at $n = 1.49$ is less clear than it is for $n = 1.60$ and its amplitude decreases as $\nu$ approaches $1/2$, so that it is no longer observable at $\nu = 0.498$. The high-$f_{pk}$, broader resonance is also discernible for $0.480 < \nu < 0.498$. We interpret the two distinct resonances, with their different evolution
FIG. 1: (color online) (a) Schematic representation of the microwave setup. The gates are not shown. (b) and (c) Microwave spectra, plotted as the real part of the conductivity \( \text{Re}(\sigma_{xx}) \) vs the frequency, \( f \), for \( n = 1.60 \) and \( n = 1.49 \). Traces are obtained at fixed \( \nu \). The \( \nu = 0.470 \) trace is shown as a dashed line with values divided by 3. The weak bump near \( \sim 1.6 \) GHz is an artifact: the strong line at \( \nu = 0.470 \) has significant \( \text{Im}(\sigma_{xx}) \) at 1.6 GHz, which enables a standing wave between the line and a reflection near the sample mounting. \( |\text{Im}(\sigma_{xx})| \) is much smaller for the higher \( \nu \) data. Traces from \( \nu = 0.480 \) to \( \nu = 0.498 \) are shown with a \( \nu \) step of 0.002 between them, and are successively offset by 0.5 \( \mu \)S. Insets: The growth-direction charge distribution when it is symmetric about the well center.
with $\nu$ as due to two distinct solids, which can coexist as $\nu = 1/2$ is approached.

The $\nu$-region of coexistence of two solid phases indicates a first-order phase transition in the absence of macroscopic density inhomogeneity. A similar transition with two resonances in a $\nu$-range of coexistence was observed between the IQHE type-II WS and bubble solid in higher Landau levels of a single-layer 2DES [44]. We will interpret the higher-$f_{pk}$ resonance as due to a type-II WS composed of $1/2$ charged quasiholes.

To test this interpretation of the higher-$f_{pk}$ resonance we compare the charge density obtained from the intensity of the resonance to that expected for the type-II WS of $1/2$ charged quasiholes. We reproduce four spectra from Fig. 1(b) in Figs. 2(a)-(d). These data are shown by the solid curves and can be analyzed as two individual resonance peaks by fitting the spectra to a sum of two single-peak functions. The dashed and dotted curves show the components of the fit for the type-I and type-II WS peaks, respectively, and are

![FIG. 2: (color online) (a)-(d) Re ($\sigma_{xx}$) vs $f$ at several $\nu$ (solid traces) for $n = 1.60$. Each plot also contains individual peak fits for the bilayer resonance (dashed line) and the $\nu = 1/2$ composite fermion Wigner solid (dotted line), which are vertically offset for clarity.](image-url)
offset downward from the raw data for clarity. For this \( n \) the low-\( f_{pk} \) (type-I WS) resonance is relatively sharp and is best approximated by a Lorentzian while the higher-\( f_{pk} \) (type-II WS) resonance is broader and can be reasonably fit to either a Lorentzian or a Gaussian. Within a small error the type-I WS resonance \( f_{pk} \) remains constant with changing \( \nu \). For the fitting we allow the type-II WS resonance \( f_{pk} \) to vary. We perform this fitting procedure to extract these two resonances over the entire \( \nu \) range studied for \( n = 1.60 \) and \( n = 1.49 \) and calculate the integrated intensities, \( S = \int \text{Re} [\sigma_{xx}(f)] df \), from the fits.

Pinning modes roughly obey a sum rule \[ 45 \] \( S/f_{pk} = e^2\pi\tilde{\nu}/2\hbar \), where \( \tilde{\nu} \) is the relevant filling for the solid under consideration: \( \nu \) for the type-I WS, and \( \nu^* = \nu - 1/2 \) for a type-II WS of the 1/2 FQHE. In Fig.3 we plot the experimentally obtained \( S/f_{pk} \), \( (S/f_{pk})^{\text{exp}} \). The figure also shows the calculated value from the sum rule \[ 45 \], \( (S/f_{pk})^{\text{sr}} \), as a dotted line. The \( (S/f_{pk})^{\text{exp}} \) for the type-I bilayer electron solid resonance, plotted on the right, exceeds this \( (S/f_{pk})^{\text{sr}} \) value for a type-II 1/2 FQHE quasihole WS. The measured \( S/f_{pk} \) of the type-II WS is in good agreement with the predictions of the sum rule, with no adjustable parameters, for \( 0.485 < \nu < 0.5 \). For \( \nu < 0.485 \) \( (S/f_{pk})^{\text{exp}} \) of the type-II WS resonance deviates from \( (S/f_{pk})^{\text{sr}} \) and begins to decrease with increasing quasihole density. The good agreement between \( (S/f_{pk})^{\text{exp}} \) and \( (S/f_{pk})^{\text{sr}} \) provides compelling evidence for the interpretation of the higher-\( f_{pk} \) resonance as due to a type-II Wigner solid made up of quasiholes.

The close agreement for the type-II WS of \( (S/f_{pk})^{\text{exp}} \) and \( (S/f_{pk})^{\text{sr}} \) calculated using the measured density implies that only a small percentage of the carriers would be available for the type-I WS. For the type-I WS resonance, we compare \( (S/f_{pk})^{\text{exp}} \) to \( (S/f_{pk})^{\text{sr}} \) to obtain the participation ratio \( \eta \equiv (S/f_{pk})^{\text{exp}}/(S/f_{pk})^{\text{sr}} \). At \( \nu = 0.485 \), the filling at which \( (S/f_{pk})^{\text{exp}} \) begins to fall short of \( (S/f_{pk})^{\text{sr}} \) for the type-II WS, we find \( \eta \sim 10\% \) for \( n = 1.49 \) and \( \eta \sim 17\% \) for \( n = 1.60 \) for the type-I WS. Since the participation ratio of the type-I WS resonance continues to decrease as \( \nu \) goes from 0.485 toward 1/2, the observed near-full participation of the carriers in the type-II WS resonance is consistent with the observed type-I WS resonance intensity. A \( \nu \) goes below 0.48 the type-I WS resonance amplitude increases as the electrons solidify into the type-I WS; by \( \nu = 0.4 \) \( \eta \sim 100\% \) for the type-I WS \[ 11 \].

Figure 3(c) shows the extracted \( f_{pk} \) vs \( \nu \) for the type-II WS of 1/2 charged quasiholes, from the fitting procedure for the \( n = 1.60 \) data (\( n = 1.49 \) showed nearly identical values). \( f_{pk} \) is nearly constant for \( 0.485 < \nu < 0.5 \) but then drops as the quasihole density becomes
FIG. 3: (color online) (a) and (b) Integrated spectrum divided by peak frequency, $S/f_{pk}$, vs $\nu$ for the resonance of type-II WS of 1/2 charged quasiholes (left axis), and the type-I WS (right axis). The scales differ by a factor of 10. The dotted line in each plot corresponds to the sum rule $(S/f_{pk})^{sr}$ [45] for full participation of 1/2 charged quasiholes, and refers to the left axes. For (a) $n = 1.60$ and (b) $n = 1.49$. (c) $f_{pk}$ for the type-II WS vs $\nu$ for $n = 1.60$; on the top axis $\nu^* = 1/2 - \nu$ is shown.

large enough to destabilize the type-II solid. A nearly constant $f_{pk}$ vs $\nu$ was also observed in the coexistence range of the bubble and IQHE type-II WS [44]. One would expect [46–48] that $f_{pk}$ should decrease as $\nu^*$ (and the quasihole density) increases due to the increase of the shear modulus. However, the local density within domains of the type-II solid can remain constant while the total proportion of the area occupied by these domains increases
FIG. 4: (color online) Re($\sigma_{xx}$) vs $f$ at fixed $\nu$ for (a) $\nu = 0.490$, (b) $\nu = 0.494$, and (c) $\nu = 0.498$. Spectra are obtained at fixed total density, $n = 1.49$, and different charge configurations along the growth direction within the well, with $\Delta n$ denoting the charge difference between the top and bottom layers, specified as a percentage of the total charge density $n$.

as $1/2 - \nu$. From the spectroscopic data, the overall picture of the system near $\nu = 1/2$ is then of a composite, with 1) domains of local $\nu = 1/2$ FQHE, 2) domains of type-II WS of quasiholes of that state, and 3) domains of a type-I WS.

In WQWs the $\nu = 1/2$ FQHE state has been shown to be highly sensitive to the symmetry of the growth-direction charge distribution [30, 31, 33]. To further test the association of the higher-$f_{pk}$ resonance with the 1/2 FQHE we performed a series of measurements to investigate the role of forced charge asymmetry between the two layers. An asymmetric charge distribution was obtained by first biasing one gate to get half the desired charge asymmetry, $\Delta n/2$, and then biasing the other with opposite polarity to maintain the same total density.

In Fig. 4 we plot Re($\sigma_{xx}$) vs $f$ at $n = 1.49$ for three fixed $\nu$ as marked. In Fig. 4(a)
the symmetric case at $\nu = 0.490$ contains both resonances. Regardless of whether we asymmetrize with larger density at the well front, $\Delta n > 0$, or back, $\Delta n < 0$, we observe a loss of the higher-$f_{pk}$ type-II WS resonance, and a concurrent enhancement of the type-I WS resonance near $f \simeq 0.5$ GHz. This result is more dramatic at $\nu = 0.498$, as shown in Fig.4(c). The symmetric state only shows the higher-$f_{pk}$ type-II WS resonance and does not show the type-I WS resonance, but asymmetrization of the well destroys the type-II WS resonance and brings out the type-I WS resonance. The loss of the type-II WS with even slight asymmetry of the charge distribution is explained if the 1/2 FQHE and its excitations are much more sensitive to the symmetry than the type-I WS. The effect of asymmetry appears in Fig.4 by $|\Delta n| \approx 2.7\%$ and essentially complete by $|\Delta n| \approx 6.6\%$. These $\Delta n$ values are in reasonable agreement with Refs. [30, 31], in which the 1/2 FQHE is suppressed by $\Delta n$ of $3 - 5\%$. Larger asymmetries of the well (on the order of $|\Delta n| > 20\%$ [31]) destroy the type-I WS as well. This observed shift of intensity from the type-II to the type-I WS resonance is consistent with the constant local density within the type-II WS domains.

In dc transport measurements there was a small reentrant insulating range observed at $\nu$ slightly larger than $\nu = 1/2$ [31]. Our microwave measurements for $\nu > 1/2$ displayed no resonance; there was no observable pinning mode of the quasiparticles associated with $\nu = 1/2$ FQHE. A plausible explanation is that the quasiholes and quasiparticles have different interactions. Such a situation was predicted theoretically [21] near $\nu = 1/3$ for which the quasihole solid was calculated to have a higher melting temperature than the quasiparticle solid.

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