Fuzzy Unconstrained Optimization Problems with Triangular Fuzzy Numbers

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Abstract. In this paper, we consider an unconstrained optimization problem with fuzzy valued functions. Using generalized Hukuhara differentiability of fuzzy functions in Newton’s method, we obtain an optimal solution of fuzzy unconstrained optimization problem without transforming to crisp equivalent form. Numerical examples are provided to show the efficiency of the proposed method.

1. Introduction
In natural world to optimize the given systems, we need to deals the linear and nonlinear programming problems. Generally, we solve the nonlinear programming problems involving only crisp numbers in objective and its constrained coefficients. Nonetheless, to handled such natural problems, we aspect part of complications, because of uncertainties and inexactness tangled in the resulting parameters. Because of these uncertainties and inexactness, we are inadequate to apply the classical methods to handle such problems. Fuzzy sets introduced by Zadeh in 1965, its play a vital role to reverse the natural world positions. In 1970, Bellman and Zadeh [3] prospective the basic perception of dynamic under fuzzy situations. Thereafter, the concept of fuzzy mathematical programming introduced by Tanaka [13] in a generic level. Throughout the most recent decades, several researchers have designed optimization problems with fuzzy-valued objective functions. From that point onwards, different ways to deal with fuzzy linear programming problems (FLLP) and fuzzy nonlinear programming problems (FNLP) have been prospective in the literature. Umamaheswari [13] discussed the algorithm for solving fuzzy unconstrained optimization problems. In their paper, they used search direction for solving the unconstrained optimization problems. After, Ghosh [6] proposed the algorithm for solving interval-valued objective functions by using Newton’s method. Again, Newton’s method is also prospective by Pirzada and Pathak [9], in their proposed algorithm, they solved fuzzy-valued functions by using H- differentiability and max-ordering relations are determined on the set of triangular fuzzy numbers. Thenceforth, Chalco-Cano [4], discussed few disadvantages of Newton method proposed by Pirzada and Pathak[9] and they determined generalized Hukuhara differentiability (gH-differentiability) of fuzzy-valued functions for correcting these disadvantages.

This paper is composed as pursues: Section 2, introduces triangular fuzzy numbers, their arithmetic operations and a ranking method for solving Fuzzy unconstrained optimization problems. Some basic results on gH- differentiability of fuzzy–valued functions are depicted in section 3. Section 4 provides an Algorithmic Approach for solving unconstrained fuzzy optimization (UFO) problem and
in section 5, the adequacy of the prospective method is shown by methods of examples. Finally, section 6 contains some closing comments.

2. Preliminaries

We audit the documentation of fuzzy numbers and the basic results which will be useful to our further idea.

Definition 2.1

Let \( \mathbb{R} \) be the set of real numbers and \( \tilde{A} : \mathbb{R} \rightarrow [0,1] \) be a fuzzy set. We say that \( \tilde{A} \) is a fuzzy number if and only if it satisfies the following properties:

(i). \( \tilde{A} \) is normal, i.e., there exists \( x_0 \in \mathbb{R} \) such that \( \tilde{A}(x_0) = 1 \);
(ii). \( \tilde{A} \) is convex, i.e., \( \tilde{A}(tx + (1-t)y) \geq \min\{\tilde{A}(x),\tilde{A}(y)\} \) whenever \( x, y \in \mathbb{R} \) and \( t \in [0,1] \);
(iii). \( \tilde{A}(x) \) is upper semi-continuous on \( \mathbb{R} \), i.e., \( \{x / \tilde{A}(x) \geq \alpha\} \) is a closed subset of \( \mathbb{R} \) for each \( \alpha \in [0,1] \);
(iv). \( \text{cl}\{x \in \mathbb{R} / \tilde{A}(x) > 0\} \) is a compact set,

Where \( \text{cl} \) denotes closure of a set. The set of all fuzzy numbers on \( \mathbb{R} \) is denoted by \( F(\mathbb{R}) \). For all \( \alpha \in [0,1] \), \( \alpha \)-level set \( \tilde{A}_\alpha \) of any \( \tilde{A} \in F(\mathbb{R}) \) is defined as \( \tilde{A}_\alpha = \{x \in \mathbb{R} / \tilde{A}(x) \geq \alpha\} \). The 0-level set \( \tilde{A}_0 \) is defined as the closure of the set \( \{x \in \mathbb{R} / \tilde{A}(x) > 0\} \). By definition of fuzzy numbers, we can prove that, for any \( \tilde{A} \in F(\mathbb{R}) \) and for \( \alpha \in [0,1] \), \( \tilde{A}_\alpha \) is compact convex set of \( \mathbb{R} \), and we write \( \tilde{A}_\alpha = [\tilde{A}_\alpha^L,\tilde{A}_\alpha^U] \). Where \( \tilde{A} \in F(\mathbb{R}) \) can be recovered from its \( \alpha \)-level sets.

Definition 2.2

A fuzzy number \( \tilde{A} \) on \( \mathbb{R} \) is said to be a triangular fuzzy number (TFN) or linear fuzzy number if its membership function \( \tilde{A} : \mathbb{R} \rightarrow [0,1] \) has the following characteristics:

\[
\tilde{A}(x) = \begin{cases} 
  \frac{x-a_1}{a_2-a_1}, & \text{for } a_1 \leq x \leq a_2 \\
  \frac{a_3-x}{a_3-a_2}, & \text{for } a_2 \leq x \leq a_3 \\
  0, & \text{elsewhere}
\end{cases}
\]

We represent the triangular fuzzy number as \( \tilde{A} = (a_1, a_2, a_3) \). The pictorial representation of a triangular fuzzy number is:

\[
\mu_\tilde{A}(x) = \begin{cases} 
  1, & \text{for } a_1 \leq x \leq a_3 \\
  0, & \text{elsewhere}
\end{cases}
\]

Figure 1. Triangular fuzzy number.
We use $F(\mathfrak{F})$ to denote the set of all TFNs. The $\alpha$ level set of $\tilde{A}$ is defined as $\tilde{A}_\alpha = [(a_2 - a_1)\alpha + a_1, a_3 - (a_3 - a_2)\alpha]$. 

2.3 Arithmetic Operations

Fuzzy numbers represented in terms of their location index and fuzziness index functions are denoted as $\tilde{A} = (a_0, a_*, a^*)$. Arithmetic operations these fuzzy numbers are based upon both location index and fuzziness index functions. The location index numbers follows usual arithmetic and the fuzziness index functions follow the lattice rule. That is for any $a, b \in L$ (Lattice), we define $a \vee b = \text{lub}\{a, b\}$ and $a \wedge b = \text{glb}\{a, b\}$.

For arbitrary fuzzy numbers $\tilde{A} = (a_0, a_*, a^*)$, $\tilde{B} = (b_0, b_*, b^*)$, and $* = \{+, -, \times, \div\}$, the arithmetic operations on $\tilde{A} \ast \tilde{B} = (a_0, a_*, a^*) \ast (b_0, b_*, b^*) = (a_0 \ast b_0, \max(a_*, b_*), \max(a^*, b^*))$.

In particular for any two triangular fuzzy numbers $\tilde{A} = (a_0, a_*, a^*)$, and $\tilde{B} = (b_0, b_*, b^*)$ such that we define

(i). Addition $: \tilde{A} + \tilde{B} = (a_0, a_*, a^*) + (b_0, b_*, b^*) = (a_0 + b_0, \max(a_*, b_*), \max(a^*, b^*))$

(ii). Subtraction $: \tilde{A} - \tilde{B} = (a_0, a_*, a^*) - (b_0, b_*, b^*) = (a_0 - b_0, \max(a_*, b_*), \max(a^*, b^*))$

(iii). Multiplication: $\tilde{A} \times \tilde{B} = (a_0, a_*, a^*) \times (b_0, b_*, b^*) = (a_0 \times b_0, \max(a_*, b_*), \max(a^*, b^*))$

(iv). Division $: \tilde{A} \div \tilde{B} = (a_0, a_*, a^*) \div (b_0, b_*, b^*) = (a_0 \div b_0, \max(a_*, b_*), \max(a^*, b^*))$.

2.4 Ranking Functions

One of the ways for solving mathematical programming problems in a fuzzy environment is to compare fuzzy numbers. The comparison between fuzzy numbers is done by using a ranking function. An appropriate approach for ordering the elements of $F(\mathfrak{F})$ is to define a ranking function $R : F(\mathfrak{F}) \to \mathfrak{R}$, which maps each fuzzy number into the real line, where a natural order exists.

Let $\tilde{A} = (a_1, a_2, a_3) = (a_0, a_*, a^*)$ be a triangular fuzzy number, then a special form of the ranking function is:

$$R(\tilde{A}) = \frac{a_* + 2a_0 - a^*}{2}.$$

3. Generalized Hukurara Differentiable of Fuzzy Functions

Definition 3.1.

Let $A, B \in F(\mathfrak{F})$ be any two fuzzy numbers, suppose C is said to be Generalized Hukukara difference of A and B such that, $A \Theta_{GH} B = C$ if and only if (i) $A = B + C$ and (ii) $B = A + (-1)C$.

If $A \Theta_{GH} B$ are exists then the $\alpha$ - levels of A&B are represented by
\[ [A \Theta_{GH} B]^\alpha = [A]^\alpha \Theta_{GH}[B]^\alpha = \min\{A^l_a - B_a^l, A^r_a - B_a^r\}, \max\{A^l_a - B_a^r, A^r_a - B_a^l\}\], for all \( \alpha \in [0,1] \).

Here the GH-difference of two intervals is denoted by \([A]^\alpha \Theta_{GH}[B]^\alpha\).

### 3.2. Differentiable of Fuzzy Functions

Let \( X \) be an open subset of \( \mathfrak{R}^n \) and \( F(\mathfrak{R}) \) be set of all fuzzy numbers. A mapping \( \tilde{G} : X \rightarrow F(\mathfrak{R}) \) is said to be a fuzzy valued function and is defined on \( X \). Corresponding such function \( \tilde{G}_a : X \rightarrow X_{[1]} \) is the group of interval-valued functions, here \( X_{[1]} \) represents the set of bounded closed intervals in \( \mathfrak{R} \) and each \( \tilde{G}_a(x) = [\tilde{G}(x)]^a = [g^a_1(x), g^a_2(x)] \). Then the endpoint of such functions \( g^a_1(x), g^a_2(x) : X \rightarrow \mathfrak{R} \) are called upper and lower functions of \( \tilde{G}_a(x) \).

**Definition 3.3.**

Let \( \tilde{G} : X \rightarrow F(\mathfrak{R}) \) be a fuzzy valued function and \( x_0, x_0 + h \in X \) such that the GH-derivative of \( \tilde{G} \) at \( x_0 \) is defined as

\[
\tilde{G}'(x_0) = \lim_{h \to 0} \frac{\tilde{G}(x_0 + h) \Theta_{GH} \tilde{G}(x_0)}{h} \tag{3.1}
\]

If \( \tilde{G}'(x_0) \in F(\mathfrak{R}) \) is satisfying (3.1) and it is exists then \( \tilde{G} \) is said to be GH-differentiable at \( x_0 \). If \( \tilde{G} \) is GH-differentiable for any value of \( x \in X \) then \( \tilde{G} \) is GH-differentiable over \( X \).

**Definition 3.4.**

Let \( \tilde{G}_a : X \rightarrow X_{[1]} \) is an interval-valued function, where \( X \) is an open subset of \( \mathfrak{R} \) is GH-differentiable at \( x_0 \in X \) such that \( \tilde{G}_a(x_0) \) is exists w.r.t. to the limit in metric space.

**Theorem 3.5.**

If a fuzzy-valued function is \( \tilde{G} : X \rightarrow F(\mathfrak{R}) \) GH-differentiable the interval-valued function \( \tilde{G}_a : X \rightarrow X_{[1]} \) is also GH-differentiable for every value of \( \alpha \in [0,1] \),

(i.e) \( \tilde{G}_a(x) = [\tilde{G}(x)]^a = [g^a_1(x), g^a_2(x)] \).

**Definition 3.6.**

Let \( \tilde{G} : X \rightarrow F(\mathfrak{R}) \) be a fuzzy-valued function and consider the fixed element

\( x^0 = (x^0_1, x^0_2, \ldots, x^0_n) \in X \). Suppose the function \( h(x_i) = G(x^0_1, x^0_2, \ldots, x^0_i, \ldots, x^0_n) \) is

GH- differentiable at \( x^0 \) then \( G \) has exists \( i \)th partial GH-derivative at \( x^0 \) and it is denoted by

\[
\frac{\partial G}{\partial x_i}(x^0).
\]

**Definition 3.7**
A fuzzy-valued function $\tilde{G}$ is defined on $X \subseteq \mathbb{R}^n$ and a fixed element $x^0 = (x^0_1, x^0_2, ..., x^0_n) \in X$, if $\tilde{G}$ is said to be GH-differentiable at $x^0$ then all the partial GH-derivatives $\frac{\partial \tilde{G}}{\partial x_1}(x^0), \frac{\partial \tilde{G}}{\partial x_2}(x^0), ..., \frac{\partial \tilde{G}}{\partial x_n}(x^0)$ exists and is continuous at $x^0$ in some neighborhood.

**Theorem 3.8.**

If $\tilde{G}$ is GH-differentiable at $x_0 \in X$ then the real valued function $[g^a_l(x) + g^a_r(x)]: X \rightarrow \mathbb{R}$ is differentiable at $x^0$, for each $\alpha \in [0,1]$.

$$(i.e.) \quad \frac{\partial \tilde{G}_{l\alpha}^a}{\partial x_1}(x^0) + \frac{\partial \tilde{G}_{r\alpha}^a}{\partial x_1}(x^0) = \frac{\partial (g^a_l + g^a_r)}{\partial x_1}(x^0).$$

**Theorem 3.9.**

If a fuzzy-valued function $\tilde{G}: X \rightarrow F(\mathbb{R})$ is m-times GH-differentiable at $x_0 \in X$ then the real valued function $[g^a_l(x) + g^a_r(x)]: X \rightarrow \mathbb{R}$ is also m-times differentiable at $x^0$, for each $\alpha \in [0,1]$.

### 4. Newton Method

#### 4.1. Unconstrained Fuzzy Optimizations

Now we consider an unconstrained multi-variable fuzzy optimization problem

$$\min_{x \in X} \tilde{G}(x) \quad (4.1)$$

Where $\tilde{G}: X \rightarrow F(\mathbb{R})$ is a fuzzy-valued function defined on $X \subseteq \mathbb{R}^n$.

**Theorem 4.2:**

If $\bar{x}^*$ is local minimizer of a real valued function $[g^a_l + g^a_r]$, for all $\alpha \in [0,1]$ then $x^*$ is said to be locally non-dominated solution of the unconstrained fuzzy optimization (UFO) problem (4.1).

#### 4.2. Computational procedure for solution of Unconstrained Fuzzy optimization Problem using Newton Method

In this segment we propose a Newton technique to discover an answer of (UFO). For this reason we consider that at every estimation point $\tilde{x}_{(k)}$ we keep ascertain $\tilde{G}(x_k), \nabla \tilde{G}(x_k)$ and $\nabla^2 \tilde{G}(x_k)$. By utilizing Taylor’s method we obtained the formula for Newton Method.

$$(i.e.) \quad \tilde{x}_{k+1} = \tilde{x}_k - [\nabla^2 \tilde{G}(\tilde{x}_k)]^{-1} \nabla \tilde{G}(\tilde{x}_k). \quad (4.2)$$

Now we discuss the algorithm for Newton Method.

**Computational procedure:**

Step 1: Choose the initial point $\tilde{x}_0$.

Step 2: Compute the value of $\nabla \tilde{G}(x)$ and $\nabla^2 \tilde{G}(x)$.

Step 3: Substitute $k=0$ in (4.1).
Step4: Test the Optimality $\|\tilde{x}_{k+1} - \tilde{x}_k\| \leq \epsilon$, suppose $\tilde{x}_{k+1}$ is optimal “then stop the procedure. Otherwise go to step 3 and substitute $k=1, 2, 3$… repeat the procedure until to reach the non-dominated solution $\tilde{x}^*$.

Theorem 4.4:
Suppose $G$ be a fuzzy-valued function on $\mathbb{R}^n$ is three times continuously GH differentiable at $\tilde{x}^*$ in $\mathbb{R}^n$. If (i) $\nabla G(x^*) = 0$ (ii) $\nabla^2 G(x^*)$ is invertible then for all $\tilde{x}_0$ is closer to $\tilde{x}^*$, such that the Newton method is converges to $\tilde{x}^*$ with the order two and it is well defined for all $k$.

5. Numerical Example

Example 5.1.
Let us consider the following Unconstrained Fuzzy optimization Problem:
$$\text{Min} \ G(x_1, x_2) = (1-1,3)x_1^3 \oplus (1,2,3)x_2^3 + (1-1,3)x_1x_2, \ x_1, x_2 \in \mathbb{R},$$
Let we take the initial approximation for given minimizer is $\tilde{x}_0 = (1, 1)$.

Step 1: Using our proposed arithmetic operation first we transform all the triangular fuzzy numbers in terms of their location index and fuzziness index functions then the given unconstrained optimization problem is written as:
$$\tilde{G}(x_1, x_2) = (1, 2 - 2\alpha, 2 - 2\alpha)x_1^3 \oplus (2, 1 - \alpha, 1 - \alpha)x_2^3 + (1, 2 - 2\alpha, 2 - 2\alpha)x_1x_2$$

Step 2: Now we calculate the sequence of solutions of $\tilde{x}_k, k = 1, 2, ...$ using our proposed algorithm we get,
$$\tilde{x}_{k+1} = \tilde{x}_k - [\nabla^2 G(\tilde{x}_k)]^{-1} \nabla G(\tilde{x}_k), k = 0, 1, 2, ... ,$$
Where,
$$\nabla G(\tilde{x}) = \begin{pmatrix}
(3, 6 - 6\alpha, 6 - 6\alpha)x_1^2 + (1, 2 - 2\alpha, 2 - 2\alpha)x_2

(6, 3 - 3\alpha, 3 - 3\alpha)x_2^2 + (1, 2 - 2\alpha, 2 - 2\alpha)x_1
\end{pmatrix}$$
and
$$\nabla^2 G(\tilde{x}) = \begin{pmatrix}
(6, 12 - 12\alpha, 12 - 12\alpha)x_1 & (1, 2 - 2\alpha, 2 - 2\alpha)

(1, 2 - 2\alpha, 2 - 2\alpha) & (12, 6 - 6\alpha, 6 - 6\alpha)x_2
\end{pmatrix}.$$

Step 3: Now apply the iterative procedure for Newton’s method we get the following iterations,

| $k$  | $\tilde{x}_1^k$       | $\tilde{x}_2^k$       | $\tilde{G}(x_1^k, x_2^k)$ |
|------|----------------------|----------------------|---------------------------|
| 0    | (1,0$\alpha,0\alpha$)| (1,0$\alpha,0\alpha$)| (-1,12-12$\alpha$,12-12$\alpha$)|
| 1    | (0.42254,12-12$\alpha$,12-12$\alpha$)| (0.46479,12-12$\alpha$,12-12$\alpha$)| (-0.17142,12-12$\alpha$,12-12$\alpha$)|
| 2    | (0.12871,12-12$\alpha$,12-12$\alpha$)| (0.20932,12-12$\alpha$,12-12$\alpha$)| (-0.019901,12-12$\alpha$,12-12$\alpha$)|
| 3    | (-0.14693,12-12$\alpha$,12-12$\alpha$)| (0.16315,12-12$\alpha$,12-12$\alpha$)| (0.03149,12-12$\alpha$,12-12$\alpha$)|
4 (0.01208, 12 − 12α, 12 − 12α) (0.07541, 12 − 12α, 12 − 12α) (−0.00048, 12 − 12α, 12 − 12α)
5 (0.03609, 12 − 12α, 12 − 12α) (−0.00218, 12 − 12α, 12 − 12α) (0.00003, 12 − 12α, 12 − 12α)
6 (0.00013, 12 − 12α, 12 − 12α) (0.00000, 12 − 12α, 12 − 12α)

\[ \therefore \text{The fuzzy optimal solution of the given UFO problem is} \]
\[ \tilde{G}(x^1_k, x^2_k) = (12α − 12, 0.000, 12 − 12α) \text{with } \tilde{x}^1_k = (12α − 12, 0.000, 12 − 12α) \text{ and} \]
\[ \tilde{x}^2_k = (12α − 12, 0.000, 12 − 12α). \]

**Example 5.2.**

Let us consider the following Unconstrained Fuzzy optimization Problem:

\[ \text{Min } \tilde{G}(x_1, x_2) = (1, 2, 4)x_1^3 + (1, 3, 5), \quad x_1, x_2 \in \mathbb{R}, \text{ let we take the initial approximation for given minimizer is } x_0 = (1, 1). \]

**Step 1:** Using our proposed arithmetic operation first we transform all the triangular fuzzy numbers in terms of their location index and fuzziness index functions then the given unconstrained optimization problem is written as:

\[ \tilde{G}(x_1, x_2) = (2, 1 − α, 2)(1 − α, 2)(3, 2 − 2α) \]

**Step 2:** Now we calculate the sequence of solutions of \( x_k, k = 1, 2, \ldots \) using our proposed algorithm we get, \( \tilde{x}_{k+1} = \tilde{x}_k - [V^2\tilde{G}(\tilde{x}_k)]^{-1} \cdot V\tilde{G}(\tilde{x}_k) \), \( k = 0, 1, 2 \ldots n \),

where, \( V\tilde{G}(\tilde{x}) = \begin{pmatrix} (6, 3 − 3α, 6 − 6α)x_1^2 \\ (6, 3 − 3α, 6 − 6α)x_2^2 \end{pmatrix} \) and

\[ V^2\tilde{G}(\tilde{x}) = \begin{pmatrix} (12, 6 − 6α, 12 − 12α)x_1 & 0 \\ 0 & (12, 6 − 6α, 12 − 12α)x_2 \end{pmatrix}. \]

**Step 3:** Now apply the iterative procedure for Newton’s method we get the following iterations,

| K | \( \tilde{x}^1_k \) | \( \tilde{x}^2_k \) | \( \tilde{G}(x^1_k, x^2_k) \) |
|---|---|---|---|
| 0 | (1.0α, 0α) | (1.0α, 0α) | (7.6 − 6α, 12 − 12α) |
| 1 | (0.5, 6 − 6α, 12 − 12α) | (0.5, 6 − 6α, 12 − 12α) | (3.5, 6 − 6α, 12 − 12α) |
| 2 | (0.25, 6 − 6α, 12 − 12α) | (0.25, 6 − 6α, 12 − 12α) | (3.0625, 6 − 6α, 12 − 12α) |
| 3 | (0.125, 6 − 6α, 12 − 12α) | (0.125, 6 − 6α, 12 − 12α) | (3.0078, 12 − 12α, 12 − 12α) |
| 4 | (0.0625, 6 − 6α, 12 − 12α) | (0.0625, 6 − 6α, 12 − 12α) | (3.001, 6 − 6α, 12 − 12α) |
| 5 | (0.03125, 6 − 6α, 12 − 12α) | (0.03125, 6 − 6α, 12 − 12α) | (3.00001, 6 − 6α, 12 − 12α) |
6 (0.015625, 6−6α, 12−12α) (0.015625, 6−6α, 12−12α) (3.0000, 6−6α, 12−12α) \\
7 (0.007813, 6−6α, 12−12α) (0.007813, 6−6α, 12−12α) (3.0000, 6−6α, 12−12α) \\
8 (0.003906, 6−6α, 12−12α) (0.003906, 6−6α, 12−12α) (3.0000, 6−6α, 12−12α) \\
9 (0.001953, 6−6α, 12−12α) (0.001953, 6−6α, 12−12α) (3.0000, 6−6α, 12−12α) \\
10 (0.000977, 6−6α, 12−12α) (0.000977, 6−6α, 12−12α) (3.0000, 6−6α, 12−12α) \\
11 (0.00096, 6−6α, 12−12α) (0.00096, 6−6α, 12−12α) (3.0000, 6−6α, 12−12α) \\

\[ \tilde{G}(x^1_k, x^2_k) = (-3 + 6\alpha, 3.000, 15−12\alpha) \text{ with } \tilde{x}^1_k = (6\alpha - 5.99903, 0.00097, 12.00097−12\alpha) \]
\text{and } \tilde{x}^2_k = (6\alpha - 5.99903, 0.00097, 12.00097−12\alpha).

6. Conclusion
In this paper, we have investigated the optimal solution for UFO problems, where as all elements of the considered UFO problems are triangular fuzzy numbers. Few numerical examples are work out by applying the prospective method without transforming the given problem to it’s crisp equivalent problem. But Pirzada and Pathak [9] and Chalco-Cano et.al [4] have converted the given UFO problem into crisp equivalent problem and then obtained the optimal solution. It is to be noted that the decision maker have the flexibility of choosing \( \alpha \in [0,1] \) depending upon the situation and his wish by using the proposed method.

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