Land-Use Filtering for Nonstationary Spatial Prediction of Collective Efficacy in an Urban Environment

J. Brandon Carter, Christopher R. Browning, Bethany Boettner, Nicolo Pinchak, and Catherine A. Calder

Abstract: Collective efficacy – the capacity of communities to exert social control toward the realization of their shared goals – is a foundational concept in the urban sociology and neighborhood effects literature. Traditionally, empirical studies of collective efficacy use large sample surveys to estimate collective efficacy of different neighborhoods within an urban setting. Such studies have demonstrated an association between collective efficacy and local variation in community violence, educational achievement, and health. Unlike traditional collective efficacy measurement strategies, the Adolescent Health and Development in Context (AHDC) Study implemented a new approach, obtaining spatially-referenced, place-based ratings of collective efficacy from a representative sample of individuals residing in Columbus, OH. In this paper, we introduce a novel nonstationary spatial model for interpolation of the AHDC collective efficacy ratings across the study area which leverages administrative data on land use. Our constructive model specification strategy involves dimension expansion of a latent spatial process and the use of a filter defined by the land-use partition of the study region to connect the latent multivariate spatial process to the observed ordinal ratings of collective efficacy. Careful consideration is given to the issues of parameter identifiability, computational efficiency of an MCMC algorithm for model fitting, and fine-scale spatial prediction of collective efficacy.

Keywords and phrases: Bayesian statistics, Data augmentation, Dimension expansion, Nonstationarity, Sociology, Spatial statistics.

1. Introduction

For decades, social science research has investigated how neighborhood residents can mobilize to address local problems (Shaw and McKay, 1942; Jacobs, 1961). The theory of neighborhood collective efficacy has been highly influential in this respect (Sampson, Raudenbush and Earls, 1997). Uniting Coleman’s (1988) concept of social capital and Bandura’s (1982; 1986) research on personal and group-level self-efficacy, neighborhood collective efficacy is a construct capturing
the collective capacity of community members to exert social control toward the realization of their shared goals (e.g., low levels of crime).

Empirical tests of the neighborhood collective efficacy theory often rely on data collected through sample surveys administered to residents of different neighborhoods within an urban center. Collective efficacy survey instruments vary, but typically ask urban residents to report perceptions of local trust, monitoring, and intervention norms in their neighborhood. We refer to these three categories of questions – "trust," "observation," and "defense," respectively – as the components of collective efficacy. Reports on these components are then aggregated within geographic units consistent with the notion of neighborhoods (e.g., census tracts). For example, the Project on Human Development in Chicago Neighborhoods (PHDCN), conducted in the mid 1990s, used neighborhood ratings from nearly 8,000 Chicago residents clustered within 343 census-based neighborhoods to capture collective efficacy (Raudenbush and Sampson, 1999; see also Matsueda and Drakulich, 2016; Wickes et al., 2019). In the PHDCN and other studies, multilevel/hierarchical regression models were used to capture between-neighborhood variation in collective efficacy through random effects, which are assumed to be spatially independent or dependent over the study region depending on the study design. Studies of neighborhood-level collective efficacy have found inverse associations with a host of neighborhood problems such as rates of homicide (Sampson, Raudenbush and Earls, 1997), intimate partner violence (Browning, 2002), child maltreatment (Molnar et al., 2015), and chronic disease (Cohen et al., 2006), even after adjusting for neighborhood sociodemographic factors.

In this paper, we extend the notion of collective efficacy as a continuously-indexed spatial feature of an urban area, which is defined by the collective impressions of places individuals visit as part of their normal, everyday routine. Using point-referenced collective efficacy data collected as part of the Adolescent Health and Development in Context (AHDC) Study, described in detail in Section 2, we estimate the components of collective efficacy across the city of Columbus, OH. Our approach also uses taxation-based records on the land use of parcels – small geographic units often used as a proxy for a “place.” Consistent with recent work suggesting that land-use compositions affect the levels of collective efficacy within neighborhoods (Corcoran et al., 2018), we use the category of land use to model spatial variation in the mean of the collective efficacy component processes. Our novel contribution from a statistical perspective is that we capture land-use driven (i.e. driven by spatially-referenced covariate information), second-order nonstationarity through a dimension-expansion strategy, building upon existing approaches for covariate-driven nonstationary spatial modeling (Calder, 2008; Schmidt, Guttorp and O’Hagan, 2011; Reich et al., 2011; Neto, Schmidt and Guttorp, 2014; Ingebrigtsen, Lindgren and Steinsland, 2014; Risser and Calder, 2015; Risser et al., 2019). In a similar spirit to the dimension expansion approach to nonstationary spatial modeling of Bornn, Shaddick and Zidek (2012), we augment the dimension of the latent collective efficacy component processes to the number of land-use categories and model a multivariate, stationary spatial process as if spatially-referenced data
on land-use specific collective efficacy components were observable everywhere. We then relate the observations to the higher dimensional random process using a filter defined by the land-use partitioned study area. We show that our land-use filtering model provides a better fit to the AHDC collective efficacy data than a traditional spatial generalized linear mixed model with a single latent second-order stationary Gaussian process (Banerjee, Carlin and Gelfand, 2014).

Our proposed methodology advances the measurement of collective efficacy in two ways. First, it is compatible with the more cost effective data collection strategy employed in the AHDC Study, where study participants report on collective efficacy levels at their routine activity locations, of which most have between five and eight. In addition, our model readily allows point-level prediction of the components of collective efficacy across the entire study area to better understand within neighborhood variation in collective efficacy. Point-level variability in collective efficacy may help explain phenomena such as crime which has been shown to concentrate at particular locations within neighborhoods (Weisburd et al., 2016). Finally, we note that while developed for the study of collective efficacy, our methodology can be readily applied in other spatial prediction settings where the study area can be partitioned into an arbitrary number of land-use categories.

The outline of the paper is as follows. In Section 2, we introduce the AHDC Study and the collective efficacy data. In addition, we describe exploratory analyses of the data that motivate our novel land-use filtering methodology. Section 3 introduces our land-use filtering model and details land-use filtering as applied to our ordinal response variable. We provide a summary of inferences and predictions from our fitted model to the AHDC data in Section 4 and explore model performance when data are generated from a land-use filtering process through a simulation study in Section 5. Lastly, we discuss implications of our modeling and data collection choices in Section 6.

2. Data and exploratory analyses

In this section, we describe the AHDC data in more detail and visually summarize the spatial patterning of the ratings of the components of collective efficacy. We also introduce the land-use data, which drive our proposed spatial filtering strategy for smoothing the ratings across the study area. This section concludes with summaries of preliminary models that motivate the more complex modeling strategy introduced in Section 3.

2.1. AHDC data

The AHDC Study is a longitudinal data collection project designed to improve understanding of how social, psychological, and biological processes shape youth developmental outcomes. Participants in the study are members of a representative sample of households with youth aged 11 to 17 residing within the I-270 belt loop in Franklin County, OH, which contains the city of Columbus and some
of its interior suburbs. In this paper, we use data from AHDC Wave 1, which was collected between 2014-2016. For each sampled household, one randomly-sampled youth aged 11-17 was selected for participation in the study. Upon enrollment, the primary caregiver, typically, but not always, the mother of the enrolled youth, filled out an entrance questionnaire which included questions on family and household composition, alcohol and substance use, employment and income, health, and social support. In addition to answering these demographic and socioeconomic background questions, caregivers also listed locations they frequent as part of their everyday routine. Caregivers indicated the location type (e.g. home, kid’s school, friends' and relatives' houses, work, grocery stores, etc.) and when they typically spend time at the location (e.g. daytime, nighttime, weekdays, weekends). They also answered multiple questions on each reported location to assess the social climate. These location ratings are the key data used in this current analysis to understand spatial variation in collective efficacy. From the location assessments, we focus on three collective efficacy questions posed as statements to which the caregiver rated agreement on a five-point Likert scale: If someone was being threatened near [location], other people around would come to their defense. You can trust people on the streets in the area near [location]. There are usually people watching what’s happening in the area near [location]. We refer to the responses to these questions and their corresponding latent spatial processes, respectively, as ”defense,” ”trust,” and ”observation.” While location ratings outside of the study area were provided, we restrict our analysis to the 4526 locations within the I-270 belt loop. Out of the 1369 caregivers in the data set, the number of reports by a single caregiver ranged from 1 to 27, with an average of 6.8 reports. With a few exceptions, all reports included ratings on all three components of collective efficacy along with the time of day the location is visited (daytime, nighttime, or both) and the day of week (weekday, weekend, or both). Caregivers were asked to rate their home location for both daytime and nighttime separately.

Figure 1 shows the locations to which the ratings of defense, trust, and observation correspond, plotted separately for residential and nonresidential locations (as derived from the caregiver’s report on the location type). For the defense responses, the residential ratings are consistent with expectations if a spatially-dependent process underlies the ordinal ratings. Spatial clustering of higher defense ratings in residential areas are apparent in the northwest quadrant and a pocket of residences relative to the lower ratings of defense elsewhere in the city. Nonresidential locations also appear to exhibit the same general pattern. Caregivers indicated the location type (e.g. home, kid’s school, friends' and relatives' houses, work, grocery stores, etc.) and when they typically spend time at the location (e.g. daytime, nighttime, weekdays, weekends). They also answered multiple questions on each reported location to assess the social climate. These location ratings are the key data used in this current analysis to understand spatial variation in collective efficacy. From the location assessments, we focus on three collective efficacy questions posed as statements to which the caregiver rated agreement on a five-point Likert scale: If someone was being threatened near [location], other people around would come to their defense. You can trust people on the streets in the area near [location]. There are usually people watching what’s happening in the area near [location]. We refer to the responses to these questions and their corresponding latent spatial processes, respectively, as ”defense,” ”trust,” and ”observation.” While location ratings outside of the study area were provided, we restrict our analysis to the 4526 locations within the I-270 belt loop. Out of the 1369 caregivers in the data set, the number of reports by a single caregiver ranged from 1 to 27, with an average of 6.8 reports. With a few exceptions, all reports included ratings on all three components of collective efficacy along with the time of day the location is visited (daytime, nighttime, or both) and the day of week (weekday, weekend, or both). Caregivers were asked to rate their home location for both daytime and nighttime separately.

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Fig 1. Survey responses for the defense (top row), trust (middle row), and observation (bottom row) components plotted for residential locations (left column) and nonresidential locations (right column). The gray lines show census tract boundaries and the orange line is the 270 belt loop. Rated locations have been jittered within census block group to preserve the anonymity of study participants. Due to the jittering, some locations are plotted outside the I-270 belt loop. The plots were created in R (R Core Team, 2022; Pebesma, 2018; Wickham et al., 2019).
spatial patterning, yet not as well visually pronounced. While it is not always appropriate to draw conclusions about the strength of spatial dependence in a spatially-structured process underlying spatially-resolved ordinal data (see Sections 3.1 and 3.3 for a discussion of this issue), the visual differences are indicative of the process underlying the ratings.

A similar pattern is evident in Figure 1 for the trust and observation components – the spatial patterning of ratings for residential and nonresidential locations are distinct. For trust (middle row), both residential and nonresidential ratings exhibit clear spatial patterning, but the pattern is not identical across both location types. In contrast, the observation ratings (bottom row) visually show less of a distinctive spatial pattern. It is difficult from visual inspection of these maps alone to draw conclusions about the underlying spatial process that give rise to the observed ratings across residential and nonresidential locations. Yet, for all three components, it appears that allowing the mean process to vary as a parametric function of observed, spatially-referenced covariates would not capture the differences in the spatial patterning of observed ratings across residential and nonresidential locations.

2.2. Land-use data

In order to account for the effects of land use on differences in the spatial dependence structure of the components of collective efficacy across the study region when predicting collective efficacy components at unrated locations, we require the land-use designation of all prediction locations, which is not available in the AHDC data. In particular, caregivers’ reported location types are straightforward to classify as residential and nonresidential, however, we need an objective and systematic method to categorize all other locations in the study area not reported on. To this end, we utilized the May 2014 parcel data for the study area, downloaded from the Franklin County Auditor’s publicly accessible FTP site (, 2021). Each parcel has a designated tax code which we classified into one of three categories: residential, nonresidential, and other. We introduce the “other” category to distinguish nonresidential locations with minimal social presence, such as warehouses, quarries, parking lots and industrial centers, from locations where a social presence is expected. Figure 2 shows the partition of the study area into residential, nonresidential, and other categories as informed by the taxcodes of the parcel data. Most locations in the AHDC data lie within the boundaries of a parcel and received the same classification as the parcel within which they lie. The coordinates of other AHDC-reported locations fall on the road network and do not lie within the boundaries of a parcel. For these locations we assigned the land-use category of the nearest parcel. After initial assignments were made, we performed a second sweep of the data to verify that the reported caregiver location type matched the assigned parcel land-use type. Incongruencies existed between reported location types and assigned land-use types (e.g. location reported by caregiver as “neighborhood,” but assigned to parcel with tax designation “warehouse” and land-use category “other”). For
such locations, we reassigned the land-use category if any of the next 5 nearest parcels had a tax code and land-use assignment that was coherent with the reported location type. We deemed locations without a nearby congruent parcel as a reporting error and did not include them in the analysis. Table 1 gives the total number of ratings and locations used in our analysis broken down by collective efficacy component and land-use category.

Table 1
Breakdown of the number of locations, \( m \), and ratings, \( n \), by collective efficacy component. The subscripts on \( m \) and \( n \) further breakdown location and ratings counts by land-use type, with 1 for residential, 2 for nonresidential and 3 for “other”.

|       | Defense | Trust | Observation |
|-------|---------|-------|-------------|
| \( m \) |         |       |             |
| \( m_1 \) | 1770   | 1555  | 1777        |
| \( m_2 \) | 2045   | 1447  | 1997        |
| \( m_3 \) | 52     | 38    | 52          |
| \( n \) | 7580   | 5865  | 7548        |
| \( n_1 \) | 2906   | 2653  | 2917        |
| \( n_2 \) | 4616   | 3170  | 4573        |
| \( n_3 \) | 58     | 42    | 58          |

2.3. Preliminary exploratory modeling

To better understand the difference in spatial variation in collective efficacy components by land-use categories, we fit separate univariate Bayesian spatial ordinal regression models to each component of collective efficacy by land-use category (excluding the “other” category which has only approximately 50 ratings for each component). That is, we fit six models corresponding to the three components of collective efficacy (defense, trust, and observation) by the two land-use categories (residential and nonresidential, as determined by the parcel data). Section 3 provides details on the specification of spatial ordinal regression models but here we focus on inferences on key parameters describing the nature of the spatial dependence as a justification of our proposed model. Because Bayesian spatial ordinal regression models are partially identifiable (discussed in Section 3.3), we clarify that inferences on the spatial parameters cannot be interpreted as characterizing the true latent continuously-indexed spatial process; rather, these inferences serve as evidence of distinct spatial patterns by land-use category. With this caveat in mind, let \( \theta = (\phi, \tau^2) \) be the vector of spatial dependence parameters characterizing the exponential covariance function of the latent spatial process, where \( \phi \) represents the spatial range and \( \tau^2 \) is the parameter that governs the proportion of variance due to spatial variation, as opposed to independent error (in the Bayesian ordinal regression model the variance of the independent error is fixed at one). For all models, we adjust for the time-of-day (daytime, nighttime, or both) and the day-of-week (weekday only, weekend only, or mixed) variables in the mean function as indicated
by the caregiver in the survey response. We obtained draws from the posterior
distribution using the MCMC scheme described in Section 3.5.1.

Table 2 shows the estimated mean of the marginal posterior distribution of
the spatial range parameter, $\phi$, and spatial proportion-of-variance parameter,
$\tau^2$, for all six fitted preliminary models. The parameter estimates for the models
fitted to the residential data differ from the estimates from the nonresidential
data across all three collective efficacy components. A higher value of $\phi$ indicates
a shorter range of spatial dependence and a high value of $\tau^2$ indicates a larger
proportion of the variance is attributed to the spatial dependence rather than independent random error. We plot the estimated posterior mean correlation function with 95% pointwise credible intervals in Figure 3. For each pair of posterior samples of $\phi$ and $\tau^2$, we calculated the correlation function for a grid of distances from 0 to 5 miles and plot the pointwise mean and lower/upper bounds of a 95% credible interval. For the defense component, the estimated correlation function for the residential data is characterized by shorter term spatial dependence but a greater proportion of variation attributable to spatial dependence. In contrast, the correlation function for the nonresidential data of the defense component exhibits longer spatial dependence and a much larger proportion of independent error. A similar pattern characterizes the difference between the correlation functions for the trust and observation components. These differences in the estimated correlation functions between the two types of locations suggest that a model which accounts for the difference in spatial dependence between locations of different land-use types will better capture the overall spatial dependence structure of the data.

The spatial dependence structure of the model has implications for the main goal of our analysis: to predict levels of each component of collective efficacy at unobserved locations. A spatial generalized linear mixed model with a single latent stationary spatial process would most likely lead to a spatial dependence parameter estimate somewhere between the distinct land-use specific spatial dependence parameters shown in Table 2. We show this is indeed the case in Section 4 for all three components of collective efficacy. By expanding the dimension of the response to include a latent spatial process for each land-use type, we can better separate the distinct spatial dependence structures of each land-use process. This will allow us to have a better understanding of the spatial process of each component of collective efficacy at a finer resolution within census tract or block group. Furthermore, we can smooth over land-use boundaries through the cross correlations between the latent processes corresponding to land-use types.
3. Land-use filtering model

In this section, we introduce a novel land-use filtering model, a Bayesian spatial generalized linear mixed model that incorporates dimension expansion and multivariate spatial modeling strategies. We begin with a motivating schematic to demonstrate how land-use filtering captures nonstationary behavior in a latent univariate spatial process. Throughout this section we omit notation references to the three different components of collective efficacy since we will fit the same model to each component separately (see Section 6 for more details on the rationale for fitting the components separately).

3.1. Motivation

When the study area can be partitioned into different land-use categories (or some other spatial partition), we can expand the dimension of the latent spatial process in a spatial generalized linear mixed model to allow distinct spatial dependence structures for each land-use category. To motivate the land-use filtering model developed formally in the next section, consider the following generative model illustrated in Figure 4. We generate from a mean-zero latent multivariate spatial process with a known cross-covariance function on a fine grid of locations distributed across a two-dimensional region (a unit square in Figure 4). The assumed marginal spatial correlation functions for this illustrative example are shown on the left panel. We then discard the components of each sample that do not correspond to the land-use category of a particular location, producing the top three plots in the center column of Figure 4. The bottom plot shows the filtered process created by combining the land-use specific component processes at each location. The resulting image is fairly locally smooth across land-use boundaries due to the cross-component dependence of the latent process, but the spatial dependence structure is not identical across space. Lastly, we assign an ordinal rating to a set of random locations within the study area based on the value of the latent process at the locations. That is, these ratings are based on the non-discarded component of the simulated random vector associated with that location (top three plots on the right column). The simulated ratings (bottom right) arising from the filtered latent process (bottom center) are analogous to the AHDC collective efficacy ratings.

Note that in Figure 4 when the ratings across all three land-use categories are plotted together (bottom right column), it is difficult to visualize different spatial dependence structures which are (more) distinguishable when the latent processes are plotted together. By expanding the univariate response to allow for distinct spatial correlation functions for each land-use category and dependence across land-use categories, we can improve model fit for the AHDC collective efficacy ratings.
Fig 4. A demonstration of a generative model for ordinal data under land-use filtering. See Section 3.1 for a discussion.
3.2. Land-use filtering ordinal regression

In specifying our land-use filtering model, we consider only a single component of collective efficacy. Each of the AHDC caregivers provides their rating on a five-point ordinal scale, where a rating of $K \equiv 5$ is the “best” and 1 is the “worst.” To facilitate specification of our model, we introduce notation for the ratings, with the primary index indicating the location of the rating. We let $Y_{ij}$ denote the $j$th rating of the $i$th location, where $s_i$ is a unique location in the study area, $S$, for $i = 1, \ldots, m$ and $j = 1, \ldots, n_i$. The constant $m$ is the number of unique locations, and $n_i$ is the total number of caregivers who rated the $i$th location. We denote the total number of ratings as $n = \sum_{i=1}^{m} n_i$, and the collection of all ratings as $Y = \{Y_{ij} : i = 1, \ldots, m; j = 1, \ldots n_i\}$.

We model the elements of $Y$ using a Bayesian ordinal probit regression model, specified using the well known data augmentation scheme of Albert and Chib (1993, 1997), which has been extended to the spatial setting by De Oliveira (1997, 2000); Higgs and Hoeting (2010); Schliep and Hoeting (2015); Berrett and Calder (2012, 2016). Under this scheme, we introduce continuous latent variables $Z^*_{ij}$ defined at each of the $m$ locations and for each $n_i$ ratings and unknown cut points, $\gamma_1, \ldots, \gamma_{K-1}$ such that

$$Y_{ij} = \begin{cases} K & \text{if } \gamma_{K-1} < Z^*_{ij} \\ k & \text{if } \gamma_{k-1} < Z^*_{ij} < \gamma_k, \text{ for } k = 2, \ldots, K-1 \\ 1 & \text{if } Z^*_{ij} < \gamma_1 \end{cases}$$

Unlike the setting discussed in Higgs and Hoeting (2010), we have multiple latent random variables defined at each location due to the fact that some locations are rated by multiple individuals. Each of these random variables is related to a single latent spatial process $\tilde{Z}(s)$ defined for all $s \in S$. We assume that the $Z^*_{ij}$s are conditionally independent given the latent spatial process $\tilde{Z}(s)$, and parameters $\beta$, $\theta$, and $\sigma^2$:

$$Z^*_{ij} | \tilde{Z}(s_i), \beta, \theta, \sigma^2 \sim N(x_{ij}' \beta + \tilde{Z}(s_i), \tilde{\sigma}^2 g(s_i)),$$

where $x_{ij}$ is a $p \times 1$ vector of covariates associated with the $j$th rating of location $i$ and $\beta$ is the corresponding $p \times 1$ vector of regression coefficients. The vector $\tilde{\sigma}^2$ contains the variance parameters for the independent error of each land-use process and $g(s) : S \to \{1, \ldots, Q\}$ is a function that returns the index of the land-use category of location $s$. We use the tilde notation to highlight the fact that this latent process is constrained so that the total error variance of the $Z^*_{ij}$s is equal to one. Constraining the latent spatial process in ordinal regression model can aid in the identifiability of the covariance parameters $\theta$ (Schliep and Hoeting, 2015; Yan et al., 2007). We discuss additional identification considerations for spatial ordinal regression models in Section 3.3. The vector $\theta$ denotes the covariance parameters associated with the spatial process $\tilde{Z}(s)$, on which $\tilde{\sigma}^2$ also depends, due to the constraint on the total error variance of $Z^*_{ij}$.

Before defining the constrained $\tilde{Z}(s)$ and a model for its unconstrained analogue $\tilde{Z}(s)$, we introduce a $Q$-dimensional spatial process $\eta(s)$, where $Q$ is the...
number of land-use categories in the spatial partition. We define $\eta(s)$ to be a multivariate Gaussian process with mean zero and parametric covariance function $\Sigma^{(\eta)}(\theta)$ defined by the linear model of coregionalization (LMC). The LMC remains the standard approach to create covariance functions in multivariate spatial modeling (Goulard and Voltz, 1992; Grzebyk and Wackernagel, 1994; Schmidt and Gelfand, 2003; Wackernagel, 2003; Gelfand et al., 2004). While alternatives to the LMC exist (e.g., Gneiting, Kleiber and Schlather (2010) and Apanasovich, Genton and Sun (2012) propose Matérn cross-covariance functions for multivariate random fields), De Oliveira (1997, 2000) has shown that smoothness parameters (such as the smoothness parameter in the Matérn covariance function) are near nonidentifiable in spatial ordinal regression models. Since our $\eta$ will be a component of a spatial ordinal regression model, we prefer the LMC specification.

The LMC is a constructive approach for modeling a multivariate spatial process as a linear combination of independent spatial processes. Let $w(s)$ be a $Q$-dimensional spatial process comprised of independent component processes $w_1, \ldots, w_Q(s)$. Each $w_q(s)$ is a Gaussian process with mean 0, variance 1, and correlation function $\rho_q(s, s'; \phi_q)$, where $\phi_q$ are unknown parameters. Again, since smoothness parameters are near nonidentifiable, we specify $\rho_q(s, s'; \phi_q)$ as an exponential correlation function,

$$\rho_q(s, s', \phi_q) = \exp(-\phi_q||s-s'||),$$

but note that alternate correlation functions could be used instead. In the LMC specification, we assume that $\eta(s) = Aw(s)$, where $A$ is a $Q \times Q$ lower triangular matrix with $Q \times (Q-1)/2$ unknown parameters. It follows directly that the cross covariance for $\eta(s)$ and $\eta(s')$ is

$$\Sigma^{(\eta)}_{s,s'}(\theta) = A \Gamma(s, s') A',$$

where $\theta$ consists of the elements of $\phi$ and unknown elements of $A$ and $\Gamma(s, s')$ is a matrix valued function with $\rho_q(s, s'; \phi_q)$ on the $q$th diagonal.

We now relate the $Q$-dimensional process $\eta(s)$ to the univariate process $Z(s)$ using land-use filtering (i.e., extracting the component of $\eta(s)$ corresponding to the land-use category of $s$). Formally, we define

$$Z(s) = f(\eta(s), g(s)) = \eta_{g(s)}(s) + \delta_{g(s)},$$

where $\delta = (\delta_1, \ldots, \delta_Q)$ is a vector of unknown mean shift parameters defining the mean of the $Z(s)$ process. Writing $Z(s) = a'_{g(s)} w(s) + \delta_{g(s)}$, where $a_q$ is the $q$th row of $A$, it is clear that $Z(s)$ is a Gaussian process with mean function $\delta(s)$, where

$$\delta(s) = \delta_{g(s)},$$

and cross-covariance function

$$\Sigma^{(Z)}_{s,s'}(\theta) = a'_{g(s)} \Gamma(s, s') a_{g(s')}.$$
between any two locations \( s \) and \( s' \) with land-use types \( g(s) \) and \( g(s') \). Note that \( Z(s) \) is nonstationary with both a mean function \( \delta(s) \) and covariance function \( \Sigma_{s,s'}^{(Z)}(\theta) \) that vary across land-use categories.

To complete the specification of the model, we need to define the covariance function for the constrained \( \tilde{Z}(s) \), which allows the \( Z_i(s) \) to have total variance of one. First we note that the variance of the unconstrained, land-use-category-specific process \( \eta_{g}(s) + \delta_q \) is \( ||a_g||^2 \). It follows then that the constrained covariance is

\[
\Sigma_{s,s'}^{(\tilde{Z})}(\theta^*) = \frac{a_g'(s) \Gamma(s, s') a_g(s')}{\sqrt{||a_g(s)||^2 + \sigma_g^2(s) ||a_g(s')||^2}},
\]

where \( \theta^* = \{\text{vec}(A), \phi, \sigma^2\} \). Additionally, the independent errors are also constrained so that

\[
\sigma_{g}^2(s) = \frac{\sigma_g^2(s)}{||a_g(s)||^2 + \sigma_g^2(s)}.
\]

The resulting constrained process, \( \tilde{Z}(s) \), is a nonstationary Gaussian process

\[
\tilde{Z}(s)|\delta(s), \theta^* \sim \text{GP}(\delta(s), \Sigma_{s,s'}^{(\tilde{Z})}(\theta^*))
\]

with mean function, \( \delta(s) \), and covariance function, \( \Sigma_{s,s'}^{(\tilde{Z})}(\theta^*) \).

### 3.3. Identifiability of model parameters

It is well known that Bayesian probit regression models for ordinal and binary data (and subsequently the spatial extensions of these models) contain non-identifiable parameters. That is, there is not a one-to-one mapping between the parameters and the value of the likelihood function. We describe below the necessary constraints to obtain likelihood identifiability in the land-use filtering model.

First, a global intercept (or cell means intercepts) and the cut points \( \gamma \) cannot be jointly identified. To address this identification issue we fix \( \delta_1 = 0 \), the mean of the first land-use process, and estimate the remaining \( \delta_q \) for \( q > 1 \). Additionally, we exclude a global intercept from \( \beta \), the vector of regression coefficients. Alternatively, one could fix one of the cut points, for example \( \gamma_1 = 0 \), which would allow the estimation of a global intercept in \( \beta \) or the estimation of \( \delta_1 \), but not both. As a last consideration on the cut points, in a standard multivariate probit model, one could use a different set of cut points for each multivariate process, although this would preclude the use of land-use specific intercepts. We opt for a single set of cut points across the land-use processes so that interpretation of the latent variable \( \tilde{Z}(s) \) is consistent across land-use categories.

In Bayesian spatial probit regression models with an independent error term, the variance of the independent error and the regression coefficients \( \beta \) are identified up to a multiplicative constant. In the hierarchical model specified above,
one could drop the constraint on the spatial random effects $Z(s)$ and fix each $\sigma_q^2$ to obtain likelihood identifiability. We instead choose to constrain the total error variance of $Z^*$ to be one, as this can aid the identifiability of the covariance parameters (Schliep and Hoeting, 2015) and facilitates a block update of the covariance parameters in our MCMC algorithm as described in Section 3.5.1. Under this scaling regime of equation 1, we are limited to estimating the proportion of the variance due to spatial dependence, and again the $\sigma_q^2$’s must all be fixed. By fixing $\sigma_q^2 = 1$ for all $q$, we can express the proportion of variance due to spatial correlation for each land-use category $q$ as 

$$\frac{||a_q||^2}{||a_q||^2 + \gamma}.$$ 

As the caregiver dataset has replication at the location level, we are required to include independent error for the latent process of the ratings. For a setting without replication, one could drop the independent error assumption and let 

$$Z^*_i = x'_i \beta + \tilde{Z}(s_i).$$

Now we constrain the variance of $\tilde{Z}(s)$ to be one, as again an unconstrained covariance and $\beta$ are nonidentified. The constrained covariance for the model without independent error results in the correlation function

$$R_{s,s'}(\theta) = \frac{a'_{g(s)} \Gamma(s, s') a_{g(s')}}{||a_{g(s)}|| ||a_{g(s')}||}.$$

Lastly, we discuss another identifiability concern that is uniquely Bayesian, that of partial identifiability. Partially identifiable models are characterized by posterior distributions that do not converge to a point mass as the sample size increases to infinity, yet are still informed by the data and differ from the prior distribution (Gustafson, 2015). Partially identified models contain partially or weakly identified parameters that are typified by substantive flat regions in the posterior or by a posterior that critically depends on the prior (Li, Ding and Mealli, 2022). Identifiability, in this context, is on a continuum reflecting the strength of learning from the data. Our model is partially identifiable in that our observed data is a corrupted version of the collective efficacy process that we are trying to predict. In particular, the covariance parameters are weakly identifiable. As a consequence, we cannot expect to precisely learn about these parameters even when the data are simulated from the model and the sample size is large. Section 5 investigates the implication of the partial identifiability of our model in terms of out-of-sample predictive performance.

3.4. Implications of the LMC specification

We now explore the implications of the LMC specification on the cross-covariance function of the multivariate latent process. In the continuous response setting, Gneiting, Kleiber and Schlather (2010) note that when all elements of the matrix $A$ are treated as unknown parameters (i.e. no structural zeros), the smoothness of each latent process is determined by the roughest latent component and that
by fixing the upper right elements of the matrix $\mathbf{A}$ to be zero, distinct smoothness properties can be estimated. In a similar vein, we find that the structural zeros also help in identifying distinct spatial range parameters for each component process.

Imposing this structure of $\mathbf{A}$, however, does indirectly place restrictions on the cross-covariance function. Consider the setting where the dimension of the latent process, $Q$, is three and $\mathbf{A}$ is lower triangular such that $a_{qq'} = 0$ for all $q' > q$, where $a_{qq'}$ denotes the $(q, q')$ element of $\mathbf{A}$. Let $\tilde{\eta}(\mathbf{s})$ be the constrained multivariate process defined by the LMC with the constraint given in Equation 1. The cross-covariance matrix-valued function for $\tilde{\eta}(\mathbf{s})$ and $\tilde{\eta}(\mathbf{s}')$ can be written as

$$
\begin{bmatrix}
\frac{\sigma^2_{11}\rho_1}{||a_{11}||^2 + \sigma_1^2} & \frac{a_{11}a_{21}\rho_1}{\sqrt{||a_{11}||^2 + \sigma_1^2}\sqrt{||a_{12}||^2 + \sigma_2^2}} & \frac{a_{11}a_{31}\rho_1}{\sqrt{||a_{11}||^2 + \sigma_1^2}\sqrt{||a_{13}||^2 + \sigma_3^2}} \\
\frac{a_{12}a_{21}\rho_1}{\sqrt{||a_{11}||^2 + \sigma_1^2}\sqrt{||a_{12}||^2 + \sigma_2^2}} & \frac{\sqrt{||a_{11}||^2 + \sigma_1^2}\sqrt{||a_{12}||^2 + \sigma_2^2}}{\sigma_2^2\rho_1 + \|a_{22}\|^2} & \frac{a_{12}a_{31}\rho_1}{\sqrt{||a_{12}||^2 + \sigma_2^2}\sqrt{||a_{13}||^2 + \sigma_3^2}} \\
\frac{a_{13}a_{21}\rho_1}{\sqrt{||a_{11}||^2 + \sigma_1^2}\sqrt{||a_{13}||^2 + \sigma_3^2}} & \frac{a_{12}a_{31}\rho_1}{\sqrt{||a_{12}||^2 + \sigma_2^2}\sqrt{||a_{13}||^2 + \sigma_3^2}} & \frac{\sqrt{||a_{12}||^2 + \sigma_2^2}\sqrt{||a_{13}||^2 + \sigma_3^2}}{\sigma_3^2\rho_1 + \|a_{33}\|^2}
\end{bmatrix},
$$

(2)

where $\rho_q = \rho(s, s', \phi_q)$. From this expression for the cross-correlation function in Equation 2, it is clear that the cross-covariance between the first latent process and the other two only depends on $\rho_1$. That is, the structural zeros in $\mathbf{A}$ restrict the cross-covariance function between the first and subsequent latent process to only depend on the spatial dependence structure of the first spatial process. It immediately follows that the lower-triangular form of $\mathbf{A}$ implies that the ordering of the land-use categories will affect the estimated cross-covariance function. Additionally, we note that ordering of the components can affect the (partial) identifiability of distinct spatial range parameters of the spatial processes. We discuss our strategy for ordering land-use categories in Section 4.

Additionally, the implied cross-covariance function from the LMC model also has implications for the trade off between independent error and spatial variance when all $\sigma^2_q$s are set to one. For each land-use process the proportion of independent error is $\frac{\rho_j}{\|a_{jj}\|^2}$. A land-use process with greater independent error will have proportionally less spatial error. Because the total error variance is constrained to one, the land-use processes $\eta_q(\mathbf{s}) + \delta_q$ will have smaller spatial variance for land-use types with a greater proportion of independent error.

### 3.5. Model fitting

The full Bayesian model is specified by placing priors on the covariance parameters $\theta^*$, and mean function parameters $\delta$ and $\beta$. A normal prior for $\delta$ and $\beta$ is conditionally conjugate in the data augmented probit model. The diagonal elements of $\mathbf{A}$, $\phi$, and $\sigma^2$ have support on the positive real line, while the off diagonal elements of $\mathbf{A}$ have support on the real line.

We wrote a custom MCMC algorithm in Julia to fit the model (Bezanson et al., 2017; Rackauckas and Nie, 2017; Lin et al., 2019; Besançon et al., 2021;
As MCMC is computationally time consuming and burdensome we also propose an approximation method that does not utilize the data augmentation framework, but rather treats the ordinal ratings as a Gaussian outcome. We fit the approximate model by direct maximization of the approximate posterior. We say “approximate” as it is not appropriate to assume ordinal data follow a normal distribution, but for the AHDC collective efficacy data spatial predictions from this approximate model are nearly identical to those produced by the land-use filtering model and can be obtained much faster (computation time is hours instead of days).

To facilitate the discussion of each method used to fit the model we introduce vector notation. Let $\mathbf{Y}$ be a $n \times 1$ vector of ratings and $\mathbf{Z}^*$ be the $n \times 1$ latent response vector with $Y_r$ and $Z^*_r$ denoting the $r$th rating and latent variable. The vector $\tilde{\mathbf{Z}} = (\tilde{Z}_1, \ldots, \tilde{Z}_m)'$ contains the spatial random effects for each location, where $\tilde{Z}_i = \tilde{Z}(s_i)$. The $r$th row of the design matrix, $\mathbf{X}$, contains the covariates $x_r$ of the $r$th rating. $\mathbf{H}$ is a $n \times m$ matrix of ones and zeros that associates the spatial random effects to the corresponding locations of each rating. The $n \times 1$ vector $\epsilon$ of independent errors have a unique constrained variance $\tilde{\sigma}^2_q$, for each land-use type $q$. We then write the model as

$$\mathbf{Z}^* = \mathbf{X}\beta + \mathbf{H}\tilde{\mathbf{Z}} + \tilde{\epsilon},$$

where vector of spatial random effects, $\tilde{\mathbf{Z}}$, has a multivariate normal distribution,

$$\tilde{\mathbf{Z}} | \delta, \theta^* \sim \text{MVN}(\mathbf{M}\delta, \tilde{\Sigma}(\theta^*)), $$

and $\mathbf{M}$ is a $m \times Q$ matrix of zeros and ones that associates the $i$th location with the mean shift $\delta_q$ of the corresponding land-use type. Lastly, the $\tilde{\epsilon}$ are independent,

$$\tilde{\epsilon} | \theta^* \sim \mathcal{N}(0, \tilde{\mathbf{D}}),$$

with $\tilde{\sigma}^2_q$ on the $r$th diagonal of $\tilde{\mathbf{D}}$ for rating $r$ with land-use type $q$.

### 3.5.1. MCMC

To facilitate mixing of the MCMC algorithm, we integrate out the spatial random effects. After doing so, we write the model as

$$\mathbf{Z}^* = \mathbf{X}\beta + \mathbf{M}^* \delta + \nu,$$

with the error term $\nu$ having a multivariate normal distribution,

$$\nu | \theta^* \sim \mathcal{N}(0, R(\theta^*)), $$

where $\mathbf{M}^* = \mathbf{H}\mathbf{M}$ and

$$R(\theta^*) = \mathbf{H}\tilde{\Sigma}(\theta^*)\mathbf{H}' + \tilde{\mathbf{D}}$$

is a correlation matrix. First, we update the cut points and latent vector $\mathbf{Z}^*$. The cut points are drawn from a uniform distribution $\gamma_k \sim \text{U} (\max(Z^*_r | Y_r = 1), \max(Z^*_r | Y_r = 0))$. Then, we update the latent vector $\mathbf{Z}^*$ by drawing from a multivariate normal distribution:

$$\mathbf{Z}^* | \delta, \theta^* \sim \text{MVN}(\mathbf{M}\delta, \tilde{\Sigma}(\theta^*)).$$

Finally, we update the error terms $\tilde{\epsilon}$ from a normal distribution:

$$\tilde{\epsilon} | \theta^* \sim \mathcal{N}(0, \tilde{\mathbf{D}}).$$
While alternative schemes have been proposed for updating the cut points, these strategies prove to be impractical computationally for our relatively large data set in the spatial setting (Albert and Chib, 1997; Cowles, 1996; Higgs and Hoeting, 2010). Each $Z^*_r$ in the latent vector $Z^*$ is drawn from a normal distribution obtained by conditioning on $\beta$, $\delta$ and all other observations $Z^*_{-r}$ and truncated to support $[\gamma_{Y_{r-1}}, \gamma_{Y_r}]$. As the precision matrix, $\Lambda = R(\theta^*)^{-1}$, is used to evaluate the likelihood of $Z^*$ in the posterior for a Metropolis-Hastings update of $\theta$, we also write the variance of $Z^*_r | Z^*_{-r}$ in terms of the precision to avoid additional inverse calculations: $\text{var}(Z^*_r | Z^*_{-r}) = 1/\Lambda_{rr}$. After obtaining a draw of $Z^*$, the regression coefficients $\beta$ and mean shift $\delta$ can be drawn jointly from their full conditional distributions with Gibbs updates. We update $\theta$ jointly with a Metropolis-Hastings step evaluating the posterior conditional on the current draw of $Z^*$. We use a multivariate normal proposal distribution for $\theta$ centered around the current draw of $\theta$. We tune the covariance matrix for the proposal by running the algorithm to obtain at least ten thousand draws with a diagonal covariance. From this initial run, we scale the covariance of the draws of $\theta$ by 0.25 and use the resulting matrix as the covariance of our proposal distribution for the final run of the MCMC.

3.5.2. Approximate maximum a posteriori estimation

While running the full MCMC algorithm allows us to assess uncertainty in the model parameters, the algorithm is very time consuming to run due to the matrix inversions required to compute the likelihood of the latent vector $Z^*$. As an alternative, we consider a strategy to estimate the model parameters that finds the maximum a posteriori estimates of parameters from a model that approximates the full Bayesian ordinal specification. The approximate model treats the outcomes as Gaussian, so that

$$Y^* = X\beta + M^*\delta + \nu$$

$$\nu | \theta^* \sim N(0, \Sigma(\theta^*)),$$

where $\Sigma(\theta^*) = H\Sigma(\theta)H' + D$. The main difference between the ordinal model and the approximate model is that the covariance of $\nu$ is unconstrained. To simplify the calculations required to find the a posteriori estimates under this approximate model, we integrate out $\beta$ and $\delta$ and only estimate the covariance parameters $\theta^*$. The marginal distribution of $Y$ is then given by

$$Y | \theta^* \sim N(0, X\Sigma(\theta^*)X' + M^*\Sigma^* M^* + H\Sigma(\theta)H^* + D),$$

where $\Sigma_{\delta}$ and $\Sigma_{\beta}$ are the covariance from the priors placed on $\delta$ and $\beta$. We optimize for $\theta^*$ by placing priors on $\theta^*$ and then finding the maximum a posteriori estimate of the approximate marginal posterior.

3.6. Prediction of the latent collective efficacy process

While the land-use filtering model is built on the assumption that we only observe a single land-use process at each location, when making predictions
we can predict the latent level of our response for all land-use categories at any location. These predictions may be useful in visualizing how the spatial processes differ from one another, however, predictions of land-use processes at locations that do not match the true land-use category may not be meaningful depending on the application at hand. It would not be correct, for our data example, to predict the change in the latent level of defense if a location changed land-use categories from residential to nonresidential.

We now detail how to obtain samples from the posterior predictive distribution for the spatial process $\tilde{Z}(s)$ at any location. Let $\tilde{S}$ be the ordered set of new locations at which we want to obtain predictions of the spatial process $\tilde{Z}(s)$ for $s \in \tilde{S}$ for a component of collective efficacy. Let $\tilde{M}$ be the matrix which associates prediction locations to a land-use category. Additionally, let draws from the posterior distribution be indexed by $b$. To obtain a draw $\tilde{Z}^{(b)}$ from the posterior predictive distribution we need to evaluate the following covariance matrices plugging in the posterior samples of model parameters (denoted by superscript $[b]$): $\tilde{\Sigma}_{S,S}(\theta^{*^{[b]}})$ and $\tilde{\Sigma}_{\tilde{S},S}(\theta^{*^{[b]}})$ from Equation 1 and $R_{S,S}(\theta^{*^{[b]}})$ from Equation 3. Then using properties of the multivariate normal distribution, we draw $\tilde{Z}^{(b)}$ from $\tilde{Z}^{(b)} \sim N(\mu^{[b]}, \Sigma^{[b]})$, where

$$
\begin{align*}
\mu^{[b]} &= \tilde{M} \delta^{[b]} + \tilde{\Sigma}_{S,S}(\theta^{*^{[b]}}) R_{S,S}^{-1}(\theta^{*^{[b]}}) (Z^{*^{[b]}} - X \beta^{[b]} - M \delta^{[b]}) \\
\Sigma^{[b]} &= \tilde{\Sigma}_{\tilde{S},S}(\theta^{*^{[b]}}) - \tilde{\Sigma}_{S,S}(\theta^{*^{[b]}}) R_{S,S}^{-1}(\theta^{*^{[b]}}) \tilde{\Sigma}_{\tilde{S},S}(\theta^{*^{[b]}}). 
\end{align*}
$$

The $\tilde{Z}^{(b)}$s are samples from the posterior predictive distribution $\tilde{Z}|\mathcal{Y}$.

4. Analysis of the AHDC caregiver data

We fit our proposed land-use filtering ordinal regression model to the caregiver ratings, separately for each of the three components of collective efficacy. As noted in Section 3.4, the ordering of the land-use processes in the LMC specification is consequential for identifiability of distinct spatial range parameters and has implications on the cross-covariance between land-use types. From our preliminary analyses in Section 2.3, we found that the residential processes had shorter spatial range than the nonresidential latent processes. Therefore, we selected residential as the first component in the LMC specification of the multivariate process $\eta(s)$ and nonresidential as the second. We then set the “other” category third, as the “other” category has very few observations ($\approx 50$) for all three components of collective efficacy. Moreover, we set the third latent component process in the LMC specification for the covariance function to be spatially independent (i.e. setting $\phi_3$ to positive infinity) as we found that the range parameter of the third latent process, $\phi_3$, is weakly identifiable for each component and MCMC chains did not appear to converge when we attempted to estimate it. As we expect the components of collective efficacy to differ in terms of the nature of the spatial dependence structure between nonresidential and “other” land-use areas and the “other” category covers a nonnegligible portion of the city, we could not justify collapsing the nonresidential and “other” categories into a single land-use category.
In the design matrix, $X$, we include time-of-day indicators (daytime, nighttime, or mixed) and the day-of-week indicators (weekday only, weekend only, or mixed) as reported by the survey participants. We allow the fixed effects corresponding to time-of-day and day-of-the-week to differ by land-use category. The baseline category is residential locations rated during the daytime. We do not include a day-of-week effect for residential locations as nearly all reports were day-of-week mixed. For the “other” category, we collapse day-of-week categories weekend-only and mixed and time-of-day categories night-only and mixed due to data availability.

We placed $N(0, 1)$ priors on each of the $\beta$’s and $\delta$’s, independent $N(0, 100)$ priors on the off diagonal elements of $A$, Cauchy$^+(0, 1)$ on each of the range parameters $\phi_i$ and diagonal elements of $A$, and a flat prior on the cut points. Ordering of the cut points in the posterior is imposed by the likelihood of the latent variables. For each land-use filtering model, we ran each MCMC for 200,000 iterations discarding the first 100,000 as burn in.

### 4.1. Model comparison

We compare the land-use filtering model for each collective efficacy component to a model that assumes a single stationary latent process. The only difference in the models is the specification of the correlation matrix. As such, we use the same $N(0, 1)$ priors on each of the $\beta$’s and $\delta$’s and a flat prior on the cut points. While the land-use filtering model defines the correlation matrix, $R(\theta^*)$, as in Equation 3, the stationary model defines the correlation as $R(\bar{\theta}) = (1 - \kappa)R(\phi) + \kappa I$, where $\kappa = 1/(1 + \tau^2)$, $I$ is the identity matrix, $R(\phi)$ is defined using the exponential correlation function and $\bar{\theta} = \{\phi, \tau^2\}$. We place Cauchy$^+(0, 1)$ priors on $\phi$ and $\tau^2$. As we are not interested in using predictions from the stationary model and the covariance parameters (i.e. only $\phi$ and $\tau^2$) converge more quickly, we obtain 10,000 samples and discard the first 2,000 as burn in.

To compare the fitted models, we use the Widely Applicable Information Criterion (WAIC) (Watanabe, 2010). Information criteria use the log predictive density and include a penalty for over-fitting. In our models, the predictive density (or likelihood) for the data augmentation model is recovered by integrating out the latent variable $Z^*$, resulting in an integral over the multivariate normal distribution:

$$p(Y|\theta^*, \delta, \beta, \gamma) = \int \cdots \int p(Y, Z^*|\theta^*, \delta, \beta, \gamma) dZ^*.$$  

WAIC is defined on the log pointwise posterior predictive density, which requires the posterior predictive density to be factored by individual observations. In the land-use filtered model this factorization is obtained by conditioning on the
spatial random effects $\tilde{Z}$ in the model,

$$p(Y|\theta^*, \tilde{Z}, \beta, \gamma) = \int \cdots \int p(Y, Z^*|\theta^*, \tilde{Z}, \beta, \gamma)dZ^*$$

$$= \int_{C_1} p(Z^*_1|\theta^*, \tilde{Z}, \beta, \gamma)dZ^*_1 \cdots \int_{C_n} p(Z^*_n|\theta^*, \tilde{Z}, \beta, \gamma)dZ^*_n$$

$$= \prod_{r=1}^{n} \Phi(\gamma_{Y_r} - x'_r \beta - h'_r \tilde{Z}) - \Phi(\gamma_{Y_r-1} - x'_r \beta - h'_r \tilde{Z})],$$

where $\Phi()$ is the standard normal cumulative distribution function and $h_r$ is the column vector of the $r$th row of $H$. The distribution of $Y_r$ given $Z^*_r$ is simply the indicator function $I(\gamma_{Y_r-1} < Z^*_r < \gamma_{Y_r})$, which gives the above bounds of integration

$$C_r = [\gamma_{Y_r-1}, \gamma_{Y_r}],$$

where we define $\gamma_0 = -\infty$ and $\gamma_K = \infty$. As a post processing step of the MCMC output, we obtain samples from the posterior distribution of the spatial random effects by drawing samples from the full conditional distribution given in Equation 4 for all observed locations.

### 4.2. Results

We begin with a comparison of model fit using WAIC. The WAIC of the land-use filtering model and stationary model for each component of collective efficacy is given in Table 3. To calculate WAIC we drew the spatial random effect from thinned (every 8th draw after burn in for the stationary models and every 100th draw after burn in for the land-use filtering models) MCMC chains to give us 1000 draws from each model. Thinning facilitated a quicker computation of WAIC. For all three components, the WAIC is lower for the land-use filtering model, indicating a better model fit. The dimension expansion technique applied in the land-use filtering model allows us to estimate different spatial processes for each land-use type, allowing for a better fit of the underlying spatial process for each component of collective efficacy across land-use types.

| Table 3 | WAIC scores for the land-use filtering and the stationary model |
|---------|---------------------------------------------------------------|
|         | Defense | Trust | Observation |
| Land-use Filtering | 17013.3 | 13203.9 | 17998.1 |
| Stationary Covariance | 17732.3 | 13968.6 | 18871.0 |

Now we discuss how parameter estimates from our land-use filtering and stationary models compare, noting that these comparisons should be viewed heuristically given the partial identifiability of the covariance parameters characterizing the strength of spatial dependence. Tables 4 and 5 give the posterior mean estimates of the parameters in the land-use filtering and stationary models.
respectively. For the stationary models, the single estimate of $\phi$ is in between estimates of $\phi_1$ and $\phi_2$ from the land-use filtering model. The stationary model smooths evenly over the entire study region and masks variation by land-use category.

### Table 4

| Defense | Trust | Observation |
|---------|-------|-------------|
| Posterior mean | 95% Credible interval | Posterior mean | 95% Credible interval | Posterior mean | 95% Credible interval |
| $\phi_1$ | 325.06 (259, 408) | 405.40 (333, 486) | 2142.49 (1215, 4797) |
| $\phi_2$ | 2.54 (0.43, 7.88) | 24.29 (11.5, 41.7) | 92.70 (58.6, 136.0) |
| $a_{11}$ | 1.39 (1.22, 1.56) | 1.76 (1.61, 1.92) | 1.60 (1.48, 1.73) |
| $a_{21}$ | 0.32 (0.22, 0.41) | 0.31 (0.18, 0.43) | 0.05 (-0.12, 0.20) |
| $a_{22}$ | 0.58 (0.42, 0.72) | 0.68 (0.56, 0.81) | 0.34 (0.28, 0.39) |
| $a_{31}$ | 0.64 (0.02, 2.39) | 0.79 (0.06, 1.89) | 0.53 (0.03, 1.44) |
| $a_{32}$ | -0.39 (-3.91, 3.56) | 0.71 (-0.14, 1.64) | 0.63 (-0.07, 1.45) |
| $a_{33}$ | 2.42 (0.15, 6.89) | 0.67 (0.03, 1.89) | 0.64 (0.03, 1.76) |
| $\delta_2$ | 0.08 (-0.62, 0.77) | -0.37 (-0.74, 0.02) | 0.22 (0.10, 0.33) |
| $\delta_3$ | 0.03 (-0.76, 0.80) | -0.34 (-0.81, 0.15) | 0.10 (-0.52, 0.72) |

The regression coefficients for the two models are comparable. The baseline category is residential locations during the daytime. For all three components, we estimate a lower level in the latent process (i.e. higher probability of a lower rating) for residential locations at night. We are able to estimate this effect as caregivers were asked to rate their home neighborhood on all three components of collective efficacy for both the daytime and the nighttime. For nonresidential and “other” locations, we do not explicitly have this counter factual informa-
tion in the data set. For the land-use mean shift parameters in the land-use filtering model, we estimate an increase in observation for nonresidential locations compared to residential locations. Additionally, there is some evidence of a negative effect on trust for nonresidential locations compared to residential locations. The stationary covariance model estimates more clear effects (credible intervals that do not contain zero) of the mean shift parameters which may arise due to over-smoothing across land-use categories.

| Table 5 | Estimated posterior means and corresponding posterior 95% credible intervals of the parameters in the stationary Bayesian ordinal regression model. The numeric subscripts for the \( \beta \)'s and \( \delta \)'s correspond to the land-use processes: 1 for residential, 2 for nonresidential, 3 for the "other" category. The subscripts "tod" and "dow" stand for time of day and day of week with "n" for night, "m" for mixed, and "e" for weekend. |
|---------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|
|         | Defense                                          | Trust                                           | Observation                                     |
|         | Posterior mean                                  | 95% Credible interval                          | Posterior mean                                  | 95% Credible interval                          | Posterior mean                                  | 95% Credible interval                          |
| \( \phi \) | 103.21 (69.8, 140.0)                            | 120.15 (86.5, 155.1)                           | 494.95 (348.4, 716.5)                           |
| \( \tau^2 \) | 0.33 (0.26, 0.42)                               | 0.79 (0.66, 0.92)                              | 0.31 (0.25, 0.38)                               |
| \( \delta_2 \) | 0.09 (0.01, 0.16)                               | -0.33 (-0.41, -0.24)                           | 0.16 (0.09, 0.23)                               |
| \( \delta_3 \) | 0.09 (-0.24, 0.41)                              | -0.22 (-0.58, 0.15)                           | 0.09 (-0.54, 0.71)                              |
| \( \beta_{1, tod=n} \) | -0.41 (-0.48, -0.34)                            | -0.16 (-0.22, -0.09)                           | -0.24 (-0.30, -0.18)                            |
| \( \beta_{1, tod=m} \) | 0.13 (-0.01, 0.26)                              | -0.09 (-0.23, 0.06)                           | 0.09 (-0.05, 0.23)                              |
| \( \beta_{2, tod=n} \) | -0.04 (-0.12, 0.05)                             | -0.08 (-0.17, 0.01)                           | -0.12 (-0.21, -0.03)                            |
| \( \beta_{2, tod=m} \) | 0.01 (-0.08, 0.09)                              | -0.03 (-0.12, 0.06)                           | 0.01 (-0.08, 0.10)                              |
| \( \beta_{2, dow=e} \) | -0.06 (-0.14, 0.03)                            | -0.02 (-0.12, 0.07)                           | -0.23 (-0.32, -0.14)                            |
| \( \beta_{2, dow=m} \) | -0.08 (-0.08, 0.07)                            | 0.00 (-0.08, 0.09)                            | -0.10 (-0.18, -0.02)                            |
| \( \beta_{3, tod=n/m} \) | -0.10 (-0.76, 0.55)                             | -0.13 (-0.83, 0.59)                           | 0.02 (-0.62, 0.66)                              |
| \( \beta_{3, dow=e/m} \) | 0.03 (-0.57, 0.64)                              | -0.17 (-0.80, 0.45)                           | -0.40 (-1.00, 0.20)                              |
| \( \gamma_1 \) | -1.97 (-2.02, -1.92)                            | -1.77 (-1.86, -1.67)                           | -2.00 (-2.04, -1.95)                            |
| \( \gamma_2 \) | -1.34 (-1.37, -1.30)                            | -1.13 (-1.20, -1.06)                           | -1.34 (-1.37, -1.30)                            |
| \( \gamma_3 \) | -0.55 (-0.57, -0.53)                            | -0.24 (-0.31, -0.16)                           | -0.30 (-0.33, -0.29)                            |
| \( \gamma_4 \) | 0.75 (0.72, 0.78)                               | 0.90 (0.84, 0.95)                              | 0.59 (0.57, 0.61)                               |

### 4.3. Predictions

Visualization of the predicted levels of each component of collective efficacy also supports the use of the land-use filtering model. Figure 5 contains the point-wise posterior mean predictions of the spatial random effect \( \tilde{Z} \) across a fine grid of points (around 30,000) within the I-270 belt loop. The land-use category of each prediction location was assigned by the land-use category of the nearest parcel. We obtained predictions using Equation 4 for each of the draws of the thinned MCMC chains used in the WAIC calculations. Plotted is the point-wise posterior mean of the spatial random effect (\( \mu^{(b)} \) in Equation 4) at each prediction location averaged across the thinned draws. We calculated the posterior mean of the spatial random effect rather than the mean of posterior
predictive distribution due to memory constraints for obtaining a multivariate normal sample at a very fine grid of locations. The fine grid of locations allows us to better visualize within neighborhood variation across land-use categories. The predicted stationary process on the right is more smooth compared to the predicted land-use process on the left. Land-use filtering allows us to estimate distinct processes for each land-use type while also smoothing over land-use boundaries. There are discontinuities in the prediction at the boundaries from one land-use type to another, however, these discontinuities would be greater if we simply fit three independent process (i.e. the matrix of weights $A$ is a diagonal matrix) for each land-use category and then filtered the processes. The predicted land-use filtered process for the defense and trust components shows the shorter range spatial dependence within residential areas and the longer range spatial dependence for non-residential locations. For the observation component, the spatial range of residential locations is effectively zero, thus the predictions revert to the mean for residential locations while allowing for greater spatial smoothing over nonresidential locations.

As the full Bayesian spatial ordinal regression model is time consuming to fit we also compared the predictions from the ordinal land-use filtering model to predictions from the approximate land-use filtering model described in Section 3.5.2. The approximate model treats the rating response directly as a Gaussian outcome, thus the scales of predictions are different. Additionally, treating the outcome directly as Gaussian allows one to drop the restrictions on the covariance function when defining the land-use filtering process. Predictions from both models are very highly correlated for each component of collective efficacy: predictions for the defense component had a correlation of 0.93, for trust a correlation of 0.90, and for observation a correlation of 0.92. In practice, when uncertainty estimates of the collective efficacy process are not needed, using the approximate model will give nearly the same results as the full Bayesian spatial ordinal model with a significant reduction in computation time.

5. Simulation Study

To further understand the uses and limitations of our model we explore model performance when data are generated from a land-use filtering process as described in Section 3. In this model generated data setting we seek to understand the limits of our partially identified model.

As we have a partially identified model, the constraints explained in Section 3.3 do not guarantee that we will be able to recover the values of model parameters used to generate the data. Indeed, all spatial probit regression models are partially identifiable: a direct result of binning the latent spatial process into ordinal or binary categories. While this loss of information weakens identifiability of the spatial parameters, we can still evaluate the predictive performance of our model by comparing predictions from the model to the true latent process of the generative model over a fine grid of points. This evaluation of model performance aligns with the main goal of our analysis to understand the fine grain variation of collective efficacy within census units.
For our simulation study we use the same parameter settings as in the demonstration of the generative model in section 3.1. Our test set to evaluate predictive performance is the same fine grid of ten thousand points over the unit square domain. For each iteration of the simulation study we randomly generate a training set of one thousand $x$ and $y$ coordinates drawn uniformly from the unit square. Each training set location is assigned the land use category of the nearest test set location. Next, we draw a realization of the latent variable $Z^*$ at the combined set of test and training locations with mean zero and covariance function defined by equation 3 with

$$A = \begin{bmatrix} 1.8 & 0.0 & 0.0 \\ 0.8 & 1.2 & 0.0 \\ 0.9 & 1.0 & 1.25 \end{bmatrix}, \quad \phi = [40.0 \quad 10.0 \quad 2.0]' \text{, and } \sigma^2 = [1.0 \quad 1.0 \quad 1.0]' \, .$$

From the latent process realization we apply the cutoffs $\gamma = [-2.0 \quad -0.5 \quad 0.1 \quad 1.1]'$ to obtain the ordinal response. Then we estimate the parameters of the approximate model as described in section 3.5.2 for both the land-use filtering and stationary specification (note that for both models $\delta$ and $\beta$ are set to be zero). Using the approximate maximum a posteriori estimates we generate predictions for the test set conditional on the observed ordinal responses. We use the approximate model for computational efficiency and because the approximate model predictions are highly correlated with the full Bayesian model predictions. Since the predictions conditional on the ordinal data will be on a different scale than the latent process, to evaluate model performance we compare the centered and scaled predictions given the ordinal response to the true latent process values at the test set of locations. Out of 100 replications of the simulation study (generating data from the land-use filtering model and fitting both the approximate land-use model and stationary model to the simulated data at the training locations) the land-use filtering model had a lower mean (over the test set of locations) absolute error in 76% of the replications and higher correlation coefficient between the predictions and true values at the test locations in 81% of the replications. By both the mean absolute error and correlation coefficient, the land-use filtering model consistently performs better than the stationary spatial model in predicting the components of collective efficacy at unobserved locations even though the model is only partially identifiable.

6. Discussion

As demonstrated in the analysis section, our proposed land-use filtering model for the AHDC collective efficacy data better fits the observed data than the default approach using a spatial generalized linear model with a single stationary latent process. This superiority holds for all three components of collective efficacy. Due to the lack of software for fitting alternative models with a single nonstationary latent process, as opposed to the default latent stationary model discussed in Section 4, our model comparison exercise is somewhat limited in scope. For example, Risser and Calder (2015)’s covariance-regression
model could be used to allow the parameters of the latent spatial process to vary smoothly with spatially-referenced land-use covariate information.

In our analysis, we fit our model separately to the ratings of each component of collective efficacy. While one could consider modeling the ratings of the three components jointly as a multivariate response, in our opinion, doing so would limit the applicability of our findings in addressing important sociological questions. Collective efficacy was originally conceived as a multidimensional construct comprised of separately operating social processes (Sampson, Morenoff and Earls, 1999). Modern studies of crime and other outcomes have looked at the effects of separate components of collective efficacy (Hipp, 2016; Wickes et al., 2019). Therefore, as an input to downstream analyses, separate predictions of the component processes will allow for a richer and more nuanced collection of questions about collective efficacy to be addressed.

As we had very few reports on locations that fell into the “other” category, we choose to fix \( \phi_3 = \infty \) the spatial dependence parameter of the third latent process in the LMC construction. This had the effect of adding additional independent error to the spatial process for the components of collective efficacy at the “other” locations. Alternatively, one could consider dropping the third latent process entirely by fixing \( a_{33} = 0 \), so that the spatial process of our three land-use categories is an expansion of a two dimensional latent spatial process.

We choose to keep \( \phi_3 \) in the model to represent our belief that there truly are three underlying, land-use-specific processes for the components of collective efficacy, and we fix \( \phi_3 \) due to data availability.

A limitation of our analysis is that we were not able to account for dependence in the collective efficacy component ratings made by the same individual. Conceptually, it may be possible to account for differences in how individuals use the component response scales since individuals report on several locations, which may be rated by multiple individuals. See Linero, Bradley and Desai (2018) for one approach to address individual rater effects. However, since many routine locations will be closer to the caregiver’s home location, we found that an individual random effect is essentially confounded with the spatial random effect. Since the latter is of primary interest and necessary for spatial prediction at unrated locations, we decided to forego an individual-level random effect in our analyses.

Finally, we note that our prediction of the collective efficacy components across the study area implicitly assume that the caregivers who rate locations other than their residence are a representative sample of individuals who frequent the locations as part of their daily routine. We view this assumption as foundational to our notion of collective efficacy as a continuously-indexed spatial process. Alternatively, one could define collective efficacy of a location from the perspective of an objective observer, who may or may not spend time at the location. This notion is not compatible with the AHDC Study design and, more importantly, is arguably not consistent with sociological theory which emphasizes perceptions of space among users of the space.

In future work, we will use the estimated continuously-indexed spatial collective efficacy component processes to examine the relationship between collective
efficacy and point-level crime data. We will also explore the hypothesis that within-neighborhood variation in collective efficacy relates to crime (Weisburd et al., 2016). Such an analysis is only possible with our fine-grained maps of collective efficacy across the study area.

In conclusion, while our proposed land-use filtering model is a novel method for learning about small-scale variability in collective efficacy across an urban environment, we acknowledge that the approach may be more generally applicable. Other spatial prediction problems in which the spatial dependence structure may depend on land use (e.g., pollution mapping) might benefit from the modeling approach. More broadly, any spatially-dependent, categorical variable that defines a partition of a well-defined study area could be used instead of land use in the approach.

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Supplementary Material

Example dataset and code

AHDC data will be deposited to Inter-university Consortium for Political and Social Research (ICPSR) in publicly available and restricted access forms (expected in Summer 2023). To ensure participant privacy and maintenance of data confidentiality, the ADHC caregiver location reports needed to reproduce the analyses in this paper will only be available in the restricted access version of the data. Qualified researchers will be able to submit an application to ICPSR for access to the data. In our supplementary material, we instead provide a synthetic data set generated from the posterior predictive distribution of the trust component at randomly selected locations out of the grid of points used for our predictive maps. Accompanying the example data set is code to reproduce the analyses done in the paper.
Fig 5. Point-wise posterior means of the spatial random effects for defense (top row), trust (middle row), and observation (bottom row). On the left are the predictions of the collective efficacy components derived from the land-use filtering model. Predictions from the stationary model are on the right. The scales are the same for plots on the same row.
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