Tunneling across mesoscopic Hall bars

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Abstract. At mesoscopic scale, the variable range hopping can be described by a tunnel effect through a single barrier. We have placed a GaALAs/GaAs HEMT in the quantum Hall regime (high magnetic field and very low temperature) and measured the components of the conductance tensor $\sigma_{xx}$ and $\sigma_{xy}$. We study the regime of the Shubnikov de Haas peaks and compare it with the same regime in large sample, commonly understood as a VRH regime. We analyse the variation in temperature in the range [0.1K-1K] for $kT$ to scale the fundamental energies, and demonstrate that in the mesoscopic scale, the measured longitudinal conductivity corresponds exactly to the tunneling through a single barrier. We deduce the barrier characteristics by fitting our data with the conductivity of a square tunnel barrier, and we retrieve the characteristic height and length of the bare disorder potential.

1. Introduction

In large Hall bar placed in the quantum Hall regime at very low temperature and high magnetic field, the absence of backscattering [1] guarantees a perfect quantization of the Hall resistance which allows to define the Ohm’s etalon [2]. In contrast, in nanoHall bars, backscattering between opposite edges of the sample gives rise to sharp peaks on the Shubnikov de Haas maxima and on the quantized Hall plateaus, when cooled to a very low temperature [3, 4, 5]. We have processed gated Hall bars (see fig.1) to make a spectroscopy of Landau levels by biasing the top-gate for fixed values of the magnetic field. Indeed the gate voltage allows to tune the electronic density and to modify the Fermi level position with respect to the long range impurity potential. We measured the longitudinal conductivity (SdH peak) as a function of the gate voltage for a set of temperature values in the range [0.1K, 1K] (see fig.2).

It is remarkable that unlike the case of wide samples, the longitudinal conductivity does not vanish when the temperature tends to zero [6, 7] as shown in fig.2. At mesoscopic scale, the conductivity tends to a skeleton function, which is a noisy image of the Landau level density of states [9, 10]. In a precedent work, we have analysed the temperature dependance of the longitudinal conductivity for fixed values of the gate voltage. On the high-$\nu$ side of the transition, the temperature dependence of the sharp peaks allows to identify this process with a double barrier tunnelling [5, 11]. On the low-$\nu$ side of the transition, the temperature dependence of the conductivity allows to identify this process with the variable range hopping [9]. We have
Figure 1. The magnetotransport coefficients are measured as a function of the magnetic field. Fluctuations are both on $R_{xx}$ and $R_{xy}$.

Figure 2. The longitudinal conductivity $\sigma_{xx}$ is deduced from the measurements of $R_{xx}$. At a fixed value of $B$, we have scanned the density of states by tuning the gate voltage. Arrows indicate the values of $V_g$ for which we analysed the temperature dependence of $\sigma_{xx}$.

demonstrated in ref [9] that the conductivity is well described by the relation [12]:

$$\sigma_{xx}(\nu, T) = \sigma_S(\nu) + \sigma_H(T) \exp \left( -\sqrt{T_0/T} \right)$$  

At vanishing temperature the hopping conductivity vanishes as well. Thus, the observed residual conductivity is obviously due to the recovering of wave functions between edges.

The purpose of this work is to demonstrate that the basic tunnel effects, which result in this residual conductivity, evolve in temperature in such a way that it reproduces the experimental variations. We will then have a description of the variable range hopping in terms of pure tunnel barrier transmission. We first give the description of the transmission through a tunnel barrier and then present our experimental results. We obtain the characteristics of the tunnel barriers for several values of $V_g$, and we retrieve the expected height and width of the barriers.

2. The conductance of a square tunnel barrier

Our two dimensional electrons are in the long range disorder potential created by the Si donor-impurities located in the GaAlAs barrier. Depending on the position of the Fermi energy with respect to the disorder potential, electrons will tunnel through different barriers located at different places in the sample. Of course, the shapes of tunnel barriers are not known, and our work is based on the idea that a square barrier would allow a reasonable fit of our data (as depicted in fig.3), an hypothesis which is verified a-posteriori. In the quantum Hall regime, the longitudinal conductivity corresponds to the conductivity across the sample, i.e. to the backscattering. Applying the Landauer formula [13]:

$$\sigma_{xx}(\nu, T) = \frac{2e^2}{h} \times t$$  

where $t$ is the transmission between sample edges through tunnel barriers, and $e^2/h$ the quantum of conductance. Let us make the hypothesis of a single square barrier. In the limit $t << 1$, the
transmission of an electron whose energy is $E$ through a square barrier of height $V_0$ and width $a$, is given by the well known formula where $m$ is the electron effective mass [14]:

$$t = \frac{16E(V_0 - E)}{V_0^2} \times \exp\left(-2a\sqrt{\frac{2m(V_0 - E)}{\hbar^2}}\right)$$  \hspace{1cm} (3)

At the temperature $T$, the energy of the electron on one side of the tunnel barrier is $E = eV_H + kT$ as represented on fig.3 where $V_H$ is the Hall potential across the channel. Introducing this expression of $E$ into eq.(3), and making the approximation $eV_H \ll V_0$ and $kT \ll V_0$, leads to the following simplified equation (see Appendix):

$$\sigma_{xx}(\nu, T) = 32 \left(\frac{V_H + \beta T}{v_0}\right) \exp\left(-2a\gamma\sqrt{v_0 - \beta T}\right)$$  \hspace{1cm} (4)

where the Hall potential is known from the experimental conditions $V_H = R_H I$ where $R_H$ is the Hall resistance and $I$ the current through the sample; $\beta = k/e \approx 0.861 \times 10^{-4}$; $\gamma = \sqrt{2me/\hbar^2} \approx 10^{-9}$ an $v_0 = V_0/e$ the barrier height expressed in eV.

3. The discrete nature of variable range hopping

The transition $\nu = 2 \to \nu = 1$ is shown in fig.2 for various temperatures: we have varied the gate voltage to tune the electron density, and consequently to lower the Fermi level with respect to the long range impurity potential. So doing, we changed the paths followed by electrons through the tunnel barriers. We concentrated here on the low-$\nu$ side of the transition, where the conductivity obviously increases with the temperature.

For several values of the gate voltage, indicated by arrows on fig.2, we have reported on a graph (see fig.4), the conductivity versus the temperature. In Fig.4, all data were fitted successfully with the square barrier transmission (eq.(4)). We thus obtained both the residual conductivity at $T = 0$, and the temperature dependence.

Once we obtained the parameters of the tunnel barrier, we report them on a graph as a function of the gate voltage. In fig.5, we observed that all values concentrate around a mean value of $a = 250\mu m$ for the width, and $V_0 = 0.25eV$ for the barrier height. Those values are completely coherent with all parameters in our experiments. Indeed, taking into account the depletion length, the width of the channel is around $1\mu m$, and $250\mu m$ of tunnel barriers represents $1/4$ of the path length followed by electrons, which is reasonable. Also, $250\mu eV$ is $2.5$ times the Hall potential, a reasonable value for the barrier height.

We conclude from our results that the discrete nature of variable range hopping (VRH) has been revealed in these experiments. We have demonstrated that the VRH [8, 12, 15] corresponds in fact to a transport through tunnel barriers that we could characterize as a function of the filling.
For several values of the gate voltage, we reported the conductivity versus the temperature, and fitted the data with eq.(4).

Figure 4.

For a (stars) and $V_0$ (circles) obtained from the fit, as a function of the gate voltage.

Figure 5.

factor. These experiments can therefore be used to characterize precisely the bare disorder potential which is “seen” by two dimensional electrons, and which create the tunnel barriers in the VRH regime.

Appendix

Reporting the expression of the energy $E$ into eq.(3), using eq.(2) and assuming $eV_H << V_0$ and $kT << V_0$, leads to the equation:

$$\sigma_{xx} = \frac{32e^2}{hV_0} \left( e^2V_HV_0 + e(kT(V_0 - 2eV_H)) \right) e^{-2(a/h)\sqrt{2m(V_0 - eV_H - kT)}}$$  \hspace{1cm} (A.1)

We define $v_0 = V_0/e$, the value of the barrier height in $eV$.

$$\sigma_{xx} = \frac{32e^2}{h} \frac{1}{V_0} \left( e^2V_Hv_0 + e(kT(v_0 - 2V_H)) \right) e^{-2(a/h)\sqrt{2me(v_0 - V_H - (k/e)T)}}$$  \hspace{1cm} (A.2)

when $kT >> eV_H$ which is our case because we heat the crystal and not the electron gas, the above equation reduces to eq.(4).

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