An Automatic Cycle-Slip Processing Method and Its Precision Analysis

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ABSTRACT On the basis of analyzing and researching the current algorithms of cycle-slip detection and correction, a new method of cycle-slip detection and correction is put forward in this paper, that is, a reasonable cycle-slip detection condition and algorithm with corresponding program COMPRE (COMpass PRE-processing) to detect and correct cycle-slip automatically, compared with GIPSY and GAMIT software, for example, it is proved that this method is effective and credible to cycle-slip detection and correction in GPS data pre-processing.

KEYWORDS GPS; cycle-slip; detection; correction; ambiguity

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Introduction

In the applications of GPS positioning, orbit determination or navigation, carrier phase and pseudo-range are two main measurements. Among others, pseudo-range is free to ambiguity and cycle-slip, and simply computing, but with low measurement precision; carrier phase wavelength is far shorter than code ranging and of higher precision, the precision of carrier phase measurements can be about 2 mm, the precision of point positioning can be cm-level and of relative positioning can be mm-level. So for the mm-level precise positioning and scientific study one must use carrier phase measurements, but should be aware of the problem of cycle-slip and ambiguity resolution. Therefore it is the main aim to detect and correct cycle-slip and resolve ambiguity in GPS pre-processing.

Presently, many methods of cycle-slip detection and correction have been developed, such as differential, wide-lane combination and pseudo-range combination, ionospheric combination, multi-normal fitting, linear fitting, dynamic model, differential between satellites and differential between stations, Autcl in GAMIT and Turboedit in GIPSY, and so on. Jia Peizhang analyzed the effective of Turboedit in GIPSY and put forward the Kalman filtering algorithm and wavelet analysis method of single frequency GPS data. A new method of cycle-slip detection and correction is put forward in this paper.

1 Measurement equations and their linear combinations

1.1 Measurement equation

In equations of carrier phase and pseudo-range, we neglected some effects of systematic error and random error, usually, the basic measurement equations of carrier phase and pseudo-range in consideration of these effects are shown as follows.

\[ L_1 = \lambda_1 \phi_1 = \rho - \frac{f_1}{f_1 - f_2} d_{\text{ion}} + \lambda_1 b_1 + \sum M_i \]

\[ L_2 = \lambda_2 \phi_2 = \rho - \frac{f_1}{f_1 - f_2} d_{\text{ion}} + \lambda_2 b_2 + \sum M_i \]  

(1)
\[ P_1 = \rho + \frac{f_1}{f_1 - f_2} d_{\text{ion}} + \sum M_{P_1} \]
\[ P_2 = \rho + \frac{f_1}{f_1 - f_2} d_{\text{ion}} + \sum M_{P_2} \]

where
\[
\sum M_{L_1} = C_1 + C_r + d_{\text{rao}} + d_{L_1} + R + e = M + e
\]
\[
\sum M_{L_2} = C_2 + C_r + d_{\text{rao}} + d_{L_2} + R + e = M + e
\]
\[
\sum M_{P_1} = C_1 + C_r + d_{\text{rao}} + d_{\text{rel}} + R + e = M + e
\]
\[
\sum M_{P_2} = C_2 + C_r + d_{\text{rao}} + d_{\text{rel}} + R + e = M + e
\]
\[ M' = C_1 + C_r + d_{\text{rel}} + R \]
is the item free from frequency; \( c \) is light velocity; \( f_1, f_2, f_3, f_4 \) are phase, frequency, wavelength and ambiguity of \( L_1, L_2, L_3, L_4 \) respectively; \( P_1, P_2 \) are pseudo-ranges of \( L_1, L_2 \) respectively; \( d_{\text{rao}} \) is ionospheric effect; \( \rho \) is geometry distance from satellite to station; \( C_r \) is satellite clock offset; \( C_r \) is receiver clock offset; \( d_{\text{rao}} \) is tropospheric delay; \( d_M \) is the multi-path effect; \( R \) is relative effect; \( e, \varepsilon \) are noise respectively; \( \sum M_{L_1}, \sum M_{L_2} \) are the total carrier measurements error of \( L_1, L_2 \), respectively; \( \sum M_{P_1}, \sum M_{P_2} \) are the total code measurements error of \( P_1, P_2 \) respectively.

In the above mentioned four equations, there have common error and different error, noise error of carrier phase measurement is 2 mm and of code measurement is 30 cm.

1.2 Wide-lane combination

According to the basic equations, let
\[ L_2 = \lambda_2 (\phi_2 - \phi_1) = \frac{f_1 L_1 - f_2 L_2}{f_1 - f_2} = \rho + d_{\text{ion}} - \frac{f_1 f_2}{(f_1 - f_2)} + \lambda_2 b_2 + e \]

where \( \lambda_2 = \frac{c}{f_1 - f_2} \approx 86, 2 \text{ cm} \) is wide wavelength; \( b_2 = b_2 - b_2, \) we define \( b_2 \) is the ambiguity residual between two frequencies.

What is more, let
\[ P_2 = \frac{f_1 P_1 + f_2 P_2}{f_1 + f_2} \]
\[ \rho + d_{\text{ion}} - \frac{f_1 f_2}{(f_1 - f_2)} + e \]

Then, combining the Eq. (3) and Eq. (4), we can get
\[ b_2 = b_1 - b_2 = \frac{1}{\lambda_2} (L_4 - P_4) = \frac{1}{\lambda_2} \left( \bar{P}_1 + \bar{P}_2 \right) + \frac{e - \varepsilon}{\lambda_2} \]

where \( \bar{f}_1 - \bar{f}_2 = \frac{17}{137} \bar{f}_1 \)
\[ \bar{P}_1 = \frac{P_1}{\lambda_1}, \bar{P}_2 = \frac{P_2}{\lambda_2} \]

In Eq. (5), wide-lane combination deletes geometry distance \( \rho \), ionospheric effect and systematic error, and just leaves ambiguity item, it is close to a constant if no cycle-slip.

From Eq. (2), geometry distance can be shown:
\[ \rho = \frac{f_1}{f_1 - f_2} \cdot P_1 - \frac{f_2}{f_1 - f_2} \cdot P_2 + e \]

Differential between adjacent epochs of \( b_2 \):
\[ \Delta b_2 (t_i, t_{i-1}) = \Delta b_1 - \Delta b_2 = \frac{1}{\lambda_2} (\Delta L_4 - \Delta P_4) = \Delta \phi_1 - \Delta \phi_2 - k(\Delta P_1 + \Delta P_2) \]

Ionospheric effect:
\[ d_{\text{ion}} = P_2 - P_1 \]

1.3 Single epoch ambiguity

According to Eq. (1), we can get the formulas of single epoch ambiguity as follows:
\[ b_1 = \phi_1 + \frac{1}{\lambda_1} \left( \frac{f_1}{f_1 - f_2} d_{\text{ion}} - \rho \right) \]
\[ b_2 = \phi_2 + \frac{1}{\lambda_2} \left( \frac{f_1}{f_1 - f_2} d_{\text{ion}} - \rho \right) \]

It is well known that there is no systematic error in single epoch ambiguity according to the Eq. (9). We can calculate single epoch ambiguity on \( L_1, L_2 \) with Eq. (9), if there is no cycle-slip, the ambiguity of adjacent epoch is similar. So we can detect cycle-slip by adjacent epoch ambiguity.

2 Cycle-slip detection and correction

2.1 Detection condition

The regressive computation formula of \( b_2 \) is
\[ \langle b_2 \rangle_i = \langle b_2 \rangle_{i-1} + \frac{1}{i} (b_2 - \langle b_2 \rangle_{i-1}) \]

where \( \langle \rangle \) means the mean value computed by regression method.

Besides, the regressive computation formula of ambiguity is
where \( k = 1, 2 \) means ambiguity of \( L_1, L_2 \), respectively.

Un-bias estimate RMS of \( b_k \) is
\[
\sigma_k = \frac{1}{i-2} \sigma_{i-1} + \frac{1}{i} (b_k - \langle b_k \rangle_{i-1})^2
\]
(12)

Cycle-slip detection condition\(^4\) are
\[
| b_k - \langle b_k \rangle_{i-1} | \leq 4 \cdot \sigma_i
\]
(13)
\[
\Delta b_1 \leq 7, \Delta b_2 \leq 5
\]
(14)

It is shown that RMS is very small if there is no cycle-slip along the preceding \( i-1 \) epochs, RMS in \( i \)th epoch is larger when the cycle-slip occurred. What is more, it has proved that\(^4\) it is difficult to detect cycle-slip with detection Eq. (13) if the number of epoch is less than 16. Thus the detection condition of this paper is combination of Eq. (13) with Eq. (14).

### 2.2 Cycle-slip solution

Dividing the data series in some sub-arcs by above mentioned detection condition, and computing the mean value \( \langle b_m \rangle_m \) (\( m \) means \( L_1 \) or \( L_2 \)) of ambiguity of two frequencies and RMS \( \langle \sigma_j \rangle_k \) (\( k \) is the number of sub-arc) of \( b_k \) respectively, we can find the sub-arc with the minimum RMS \( \langle \sigma_j \rangle_k \) as base sub-arc from all sub-arcs that the number of epoch in this sub-arc is larger than 30. Now, we can compute the cycle-slip by ambiguity mean value of base sub-arc and those of others sub-arcs.

\[
\Delta b_{mJ} = \langle b_m \rangle_J - \langle b_m \rangle_J \quad (J \text{ is the base sub-arc})
\]
(15)

\[
\Delta b_m = \Delta b_{mJ} - \Delta b_{Jk}
\]
(16)

Eq. (15) stands for cycle-slip computation of \( L_1 \) or \( L_2 \), we can compute the cycle-slip in \( L_1 \) and \( L_2 \) by combining Eqs. (15) and (16), respectively.

During the cycle-slip computation, ionospheric combination is not used; the question of smoothing method and number of smooth order is avoided by polynomial in ionosphere combination.

### 2.3 Searching the reliable cycle-slip

From Eq. (9), we know that geometry distance \( \rho \) got from pseudo-range is an important parameter in the computation of ambiguity mean value, but the noise error of pseudo-range is about 30 cm, it make the uncertainty of cycle-slip computation reach to 3–4 cycles. So it is necessary to search the reliable cycle-slip. In this program, search bound is \([-4, +4]\], we take the minimum residual of difference of ambiguity mean value between two sub-arcs as the reliable cycle-slip. It is proved that it is meaningful for the search of reliable cycle-slip, besides by this it not only searches the reliable cycle-slip, but also avoids rounding error.

### 3 Precision analysis

#### 3.1 Compared with Turboedit in GIPSY about LC-PC residual

\[
L_C = \frac{f_1^2}{f_1^2 - f_2^2} L_1 - \frac{f_2^2}{f_1^2 - f_2^2} L_2
\]
\[
\rho - \frac{c}{f_1^2 - f_2^2} (f_1 b_1 - f_2 b_2) + \epsilon
\]
(17)
\[
P_C = \frac{f_1^2}{f_1^2 - f_2^2} P_1 - \frac{f_2^2}{f_1^2 - f_2^2} P_2 = \rho + \epsilon
\]
(18)
\[
V = LC - PC = \frac{c}{f_1^2 - f_2^2} (f_1 b_1 - f_2 b_2) + (\epsilon - \epsilon)
\]
(19)

From above mentioned formulas, we know that the residual of \( LC-PC \) may be fluctuant, and centered on 0 randomly without systematic error if detecting cycle-slip cleanly and solving free ambiguity correctly. So, \( V = LC - PC \) is one of the indexes to weigh the cycle-slip and ambiguity processing. Now, let us analyze \( LC-PC \) residual of some stations in domestic GPS net and IGS net.

Blow is some GPS data from Pudong station of Shanghai (Fig. 1(a) – 1(b)), Hangzhou station of Zhejiang (Fig. 1(c) – 1(d)) in domestic GPS net on Aug. 24, 2002, contrasting residual of \( LC-PC \) processed by Turboedit in GIPSY software and COMPRE (we call the algorithm and corresponding program of this paper COMPRE) as follows (GPS23 means the GPS PRN number).
Fig. 1 Residual comparison between LC-PC of Turboedit in GIPSY and COMPRE on domestic GPS net

Below there are some GPS data from stations Ankara and Bogota on Jan. 1, 2003, raw observation is downloaded from ftp://lox.ucsd.edu, contrasting residual of LC-PC processed by COMPRE and Turboedit in GIPSY software are as follows.

From these figures, it is shown that result of COMPRE is cleaner than that of GIPSY. In the figures of cycle-slip detection and correction with Turboedit in GIPSY software, the cycle-slips are not detected cleanly; there is systematic error in LC-PC. What is more, some observation data is deleted in GIPSY but saved in COMPRE; algorithm of COMPRE can improve the use efficiency of raw observation.

Fig. 2 Residual contrast of LC-PC of Turboedit in GIPSY and COMPRE on IGS net
3.2 Compared with GAMIT software about RMS

3.2.1 Example 1

We analyze the one-day data on Aug. 24, 2002 from domestic GPS net with two algorithms, one is GAMIT software, the other is processed by GAMIT as clear data after pre-processing with COMPRE, and stations at which the data are observed are: Shanghai, Hangzhou, Huangshan, Xuancheng, Ma’anshan, Dangshan, Changzhou, Zhoushan, Gaoyou, and Dongtai. Sample rate is 30 s. The RMS of all parameters are listed in Table 1.

| Table 1 | Comparison between RMS of GAMIT and COMPRE |
|---------|------------------------------------------|
|         | COMPRE | GAMIT | Difference |
| Before iteration | RMS 13.4 | 14.1 | 0.7 |
| After iteration  | RMS 11.1 | 11.2 | 0.1 |

From the above table, it is shown that RMS of all parameters with COMPRE is little better than that of GAMIT, so it is proved that precision of COMPRE is similar to that of GAMIT.

3.2.2 Example 2

In addition, we analyze the one-day data on Jan. 4, 2005 from domestic GPS net with the same project as example 1, and stations are: Shanghai (Pudong, Sheshan, Fengxian), Zhejiang (Hangzhou, Zhoushan), Anhui (Huangshan, Xuancheng, Ma’anshan), Jiangsu (Gaoyou). Sample rate is 30 s. Note that, data of this day is worse, The RMS of all parameters are listed in Table 2.

| Table 2 | Comparison between of GAMIT and COMPRE for example 2 |
|---------|-----------------------------------------------------|
|         | RMS/mm | Difference/mm |
| COMPRE  | 101.5   | 47.6          |
| GAMIT   | 149.1   |               |

It is shown that RMS of all parameters with COMPRE is little better than that of GAMIT if the original observations are good (Table 1), but RMS of all parameters with COMPRE is much better than that of GAMIT if the original observations are worse (Table 2).

4 Conclusions

It must be noted that cycle-slip detection and correction with Turboedit in GIPSY is not brought to success in this stage, it will be finished to detect and correct by residual between observed range and computed range in adjustment stage. Similar to GIPSY software, cycle-slip detection and correction with COMPRE is not brought to success in this stage, it will be finished in adjustment stage, but, to the same stage, efficiency of COMPRE is better than GIPSY, it is predicted that the cycle-slip detection and correction would be easy in the later stage. The parameter RMS is equal to that of GAMIT software. All these show that it is valid and effective to GPS data pre-processing with COMPRE.

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