Solidification in Stratified Flow of Two Immiscible Fluids in a Double-Bend

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Abstract. This article presents an investigation of solidification in stratified flow of two immiscible fluids in a double-bend. A combined enthalpy-level-set approach for solidification is employed. The fluid-solid and fluid-fluid interfaces are captured respectively using the enthalpy and the level-set methods. The approach is verified against the scenario of a thick water layer sandwiched between a wall and an oil layer with water solidifies on the wall where semi-analytical solution exists. With the approach verified established, the effects of Reynolds number, Stefan number, viscosity ratio, thermal diffusivity ratios on flow and heat transfer are investigated.

1. Introduction
Flow of two immiscible fluids are encountered frequently in engineering, i.e. liquid metal-air flow in mould, water-air flow in pipes and oil-water flow in production well. In such a flow system, when the temperature falls below the freezing point of one fluid, solidification occurs leading to the formation of a solid phase. The two-phase flow system now becomes a three-phase system with two fluid and a solid phases. Since the solid phase is impermeable to flow, solidification changes the flow passage geometry which in turn changes the heat transfer process and finally the solidification process itself. From a modeling point of view, the flow of the two fluids and the solidification process are fully-coupled together, i.e. a challenging moving boundary problems with dynamically evolving fluid-fluid interface and fluid-solid front. This type of problem is addressed here.

In this work, solidification in stratified flow of two immiscible fluids in a double-bend is investigated. Double-bends are found in many natural and engineering flow conduits, i.e. arteries, aircraft intake, heat exchanger and piping. Single-phase [1]-[3] and two-phase [4]-[5] flows in a double-bend have been explored. However, to the best knowledge of the authors, two-phase flow in a double-bend with melting/solidification has received very little attention and therefore is explored here.

The remaining of the article is divided into five sections. The problem of interest will be description in Section 2. In Section 3, the governing equations for heat, mass and momentum transfer for the current problem are presented followed by a brief description of the numerical solution procedure in Section 4. Then, results for various parametric studies are presented and discussed in Section 5. Finally, a few concluding remarks are given in Section 6.

2. Problem Description
Figure 1 shows a stratified flow of two fluids, +ve fluid and -ve fluid, in a double-bend. Initially, there is no -ve solid and the flow is steady with both fluids at temperature \( T_{hot} \). At \( t = 0^+ \), the temperature
of the outer walls of the second bend is reduced to \( T_{\text{cold}} \) (lower than the solidification temperature of -ve fluid \( T_m \)). Heat is then removed from the -ve fluid, its temperature particular near the wall drops to \( T_m \) and solidification occurs. A -ve solid layer forms and grows on the outer walls of the second bend.

![Figure 1. Schematic of solidification in two-phase stratified flow in a double-bend.](image)

3. Mathematical Formulation

The fluid-fluid interface is represented by a level-set function \( \phi \) [6] as

\[
\phi = \begin{cases} 
-d, & \text{if } x \in \Omega_f \cup \Omega_s \\
0, & \text{if } x \in \Gamma_f \\
+d, & \text{if } x \in \Omega_s
\end{cases}
\]  

(1)

and driven by fluid velocity \( \vec{u} \) as

\[
\frac{\partial \phi}{\partial t} + \vec{u} \cdot \nabla \phi = 0
\]

(2)

\( d \) is the shortest distance from the fluid-fluid interface. To maintain \( \phi \) remains as distance function, redistancing [7] is performed after solving Eq. (2). If needed, local mass correction [8] is performed.

The fluid-solid interface is represented by the -ve fluid volume fraction in a control volume (CV) as

\[
\varphi = \frac{V_f}{V_C}
\]

(3)

The evolution of \( \varphi \) is captured using the latent heat content in the energy equation.

The conservation equations governing the transport of mass, momentum and energy in the entire domain \( \Omega_f \cup \Omega_f \cup \Omega_s \) are given by

\[
\nabla \cdot \vec{u} = 0
\]

(4)

\[
\frac{\partial (\rho \vec{u})}{\partial t} + \nabla \cdot (\rho \vec{u} \vec{u}) = -\nabla p + \nabla \left[ \mu (\nabla \vec{u} + (\nabla \vec{u})^T) \right] + \rho \vec{g} - \frac{(1-\varphi)^2}{\varphi^3} \vec{u} - \sigma (\nabla \cdot \vec{n}) \vec{n} \delta(\phi)
\]

(5)

\[
\frac{\partial (\rho c T)}{\partial t} + \nabla \cdot (\rho c \vec{u} T) = \nabla \cdot (k \nabla T) + S_{\text{th}}
\]

(6)

where \( \vec{u}, p \) and \( T \) are respectively velocity, pressure and temperature. The thermo-physical properties are density \( \rho \), viscosity \( \mu \), surface tension \( \sigma \), specific heat \( c \) and thermal conductivity \( k \).

In Eq. (5), the forth term on the right-side mimics Carman-Kozeny flow in porous media [9] for CVs partially filled with -ve solid where \( A = 10^9 \) and \( B = 0.005 \) [10]. The fifth term on the right-hand side of Eq. (5) represents surface tension effect modeled using Continuum Surface Force Model [11] where
\[
\delta(\phi) = \begin{cases} 
\frac{1 + \cos(\pi \phi / \varepsilon)}{2\varepsilon}, & \text{if } |\phi| < \varepsilon \\
0, & \text{otherwise}
\end{cases}
\]  
\[ \text{otherwise if } |\phi| < \varepsilon \]

\[ \hat{n} = \frac{\nabla \phi}{|\nabla \phi|} \]  

In Eq. (6), \( S_{Th} \) models the phase change of fluid/solid using the enthalpy formulation [12], [13]-[15], and is defined as the rate of change of the specific latent heat content

\[ S_{Th} = \begin{cases} 
-\rho h \frac{\partial \phi}{\partial t}, & \text{if } \bar{x} \in \Omega_f \cup \Omega_s \\
0, & \text{else}
\end{cases} \]  

where \( h \) is the specific latent heat of solidification.

Thermo-physical property can be expressed as \( \chi(\phi, \varphi) \). In particular, \( \rho \) and \( c \) are calculated as

\[ \chi = H\chi_+ + (1-H)\chi_- \]  

and \( \mu \) and \( k \) are determined as

\[ \frac{1}{\chi} = H \frac{1}{\chi_+} + \frac{1-H}{\chi_-} \]  

where

\[ \chi_-(\varphi) = \varphi \chi_f + (1-\varphi) \chi_s \]  

\[ H(\phi) = \begin{cases} 
0, & \text{if } \phi < -\varepsilon \\
\frac{\phi + \varepsilon}{2\varepsilon} + \frac{1}{2\pi} \sin \left( \frac{\pi \phi}{\varepsilon} \right), & \text{if } |\phi| \leq \varepsilon \\
1, & \text{if } \phi > +\varepsilon
\end{cases} \]

4. Solution Procedure

The governing equations are discretized using a finite volume method [16]-[17] on a staggered mesh arrangement with fully implicit scheme for transient terms, 2nd order upwind scheme for convective terms and 2nd order central difference scheme for diffusive terms. The SIMPLER algorithm is used to couple the velocity and pressure fields. A narrow-band [18] TVD-RK2 [19] WENO5 [20] approach is employed to treat the level-set function. The overall solution procedure for the presented method can be summarized as follows:

(1) Specify the initial conditions (i.e. \( t = 0 \)) for \( \bar{u}, \rho, T, \phi \) and \( \varphi \).

(2) Advance the time step to \( t + \Delta t \).

(3) Solve Eq. (2) for \( \phi |^{t+\Delta t}_t \) and perform redistancing.

(4) Solve Eqs. (4) and (5) for \( \bar{u} |^{t+\Delta t}_t \) and \( \rho |^{t+\Delta t}_t \).

(5) Solve Eq. (6) for \( T |^{t+\Delta t}_t \) with \( \phi |^{t+\Delta t}_t \) determined using enthalpy approach.

(6) Repeat steps (3) to (5) until the solution converges.

(7) If required perform local mass correction.

(8) Repeat steps (2) to (7) for all time steps.

5. Results and Discussions

Verification is performed for the scenario of a water layer sandwiched between a wall and an oil layer with water solidifies on the wall. Semi-analytical solution exists for one-dimensional semi-infinite domain with thick water layer. The present solution agrees well with the semi-analytical solution. Due to space limitation, this will not be presented here.
Figure 2: Solution for the standard case.

Results are presented in terms of dimensionless numbers: dimensionless time $t^* = tu_o / L$, Reynolds number ($Re = \rho_u u_o L / \mu_L$), Froude number ($Fr = u_o / \sqrt{gL}$), Weber number ($We = \rho_u u_o^2 L / \sigma$), Prandtl number ($Pr = \mu_L c_p / k_L$), Stefan number ($Ste = c_p (T_{hot} - T_{cold}) / h$), density ratios ($\rho_f / \rho_\infty$).
\( \rho_f / \rho_s \), viscosity ratio (\( \mu_f / \mu_s \)), thermal diffusivity ratios (\( \alpha_f / \alpha_s \), \( \alpha_s / \alpha_s \) and dimensionless solidification temperature (\( T^*_{m} = (T_m - T_{cold}) / (T_{hot} - T_{cold}) \)). No shrinkage is considered during solidification.

A standard case with \( Re = 0.2 \), \( Fr = 0.3193 \), \( We = 100 \), \( Pr = 5 \), \( Ste = 0.0025 \), \( \rho_f / \rho_s = \rho_f / \rho_s = 1 \), \( \mu_f / \mu_s = 0.2 \), \( \alpha_f / \alpha_s = 2.5 \), \( \alpha_s / \alpha_s = 10 \) and \( T^*_{m} = 0.5 \) will be first considered. The solution is shown in Fig. 2 with the fluid-fluid interface, fluid-solid interface, velocity and temperature fields over time plotted together. For mesh independent test, two different meshes: \( 60 \times 60 CVs \) with \( \Delta t^* = 5.0 \times 10^{-4} \) and \( 120 \times 120 CVs \) with \( \Delta t^* = 2.5 \times 10^{-4} \), are used. The fluid-fluid and fluid-solid interfaces at \( t^* = 0, 2.5, 5, 7.5 \) and 10 are superimposed in the last two plots. A mesh of \( 60 \times 60 CVs \) with \( \Delta t^* = 5.0 \times 10^{-4} \) is sufficient to resolve the solution. Initially at \( t^* = 0 \), the flow is steady and both fluids are at \( T^* = 1.0 \). The +ve fluid is more viscous than the -ve fluid, as such the +ve fluid layer is generally thicker. As the outer wall of the second bend is lowered to \( T^* = 0 \) at \( t^* = 0^\tau \) (below the solidification temperature of the -ve fluid \( T^*_{m} \)), heat is removed from both fluids as they flow. Along the streamwise direction, the temperature of both fluids decreases generally as more and more heat is removed. Of course, the -ve fluid has a lower temperature compare to that of the +ve fluid as the -ve fluid is directly in contact with the outer wall maintained at a lower temperature. When the temperature of the -ve fluid adjacent to the horizontal outer wall of the second bend decreases to \( T^*_{m} \), a small amount of -ve solid layer with almost uniform thickness starts to form. This is followed by the formation of a similar uniform -ve solid layer on the vertical outer wall of the second bend, although it is initially thinner than that on the horizontal outer wall. Beyond the first bend, the -ve fluid temperature is reduced to almost \( T^*_{m} \) after \( t^* = 0.3 \). Over time, the -ve solid layer on both horizontal and vertical outer walls of the second bend grows and becomes thicker. The double-bend is increasingly blocked by the -ve solid. Both +ve and -ve fluid layers becomes thinner accompanied by an increase in fluid velocity. Within the -solid layer, the temperature decreases from \( T^*_{m} \) at the fluid-solid interface to \( T^* = 0 \) at the outer wall of the second bend. It should be noted that at the inner corner of the first bend, -ve solid also forms and over time grows increasing upstream affecting the upstream temperature of both fluids.

The effects of some of the dimensionless numbers: \( Re \), \( Ste \), \( \alpha_f / \alpha_s \), \( \alpha_s / \alpha_s \) and \( \alpha_s / \alpha_s \), are now investigated. With all other dimensionless numbers unchanged, \( Re \) is increased from 0.2 (standard case), to 1 and then 5, see Fig. 3. Upon increasing \( Re \), more heat is convected downstream rather removed at the outer walls of the second bend. Therefore, at a given streamwise location, the average temperature is higher. The -ve solid layer forms later and grows slower. For example, to attain a comparatively thick -ve solid layer of \( Re = 0.2 \) at \( t^* = 10 \), \( t^* = 50 \) is needed for \( Re = 1 \) and \( t^* = 250 \) for \( Re = 5 \).

Again with all other dimensionless numbers unchanged, the effect of \( Ste \) is shown in Fig. 4. \( Ste \) is the ratio of sensible heat to latent heat for solidification. Upon increasing \( Ste \) from 0.00125 to 0.0025 (standard case) and then 0.005, i.e. physically smaller latent heat for solidification, of course the -ve solid layer grows faster over time as less heat removal is needed for solidifying the same amount of -ve fluid. Therefore, for example at \( t^* = 10 \) the -ve solid layer for the case with \( Ste = 0.005 \) is the thickest.
Figure 3: Effect of Re.

Figure 4: Effect of Ste.
The effect of $\mu_f / \mu_+$ is illustrated via $\mu_f / \mu_+ = 0.2$ (standard case), 1 and 10 with the later two plotted in Fig. 5. Upon increasing $\mu_f / \mu_+$, the -ve fluid becomes more viscous, as such the -ve fluid layer becomes thicker with decreased velocity. At the same time, the +ve fluid layer becomes thinner and flowing faster. The effect of $\mu_f / \mu_+$ on the shape and amount of the -ve solid formed is less obvious even with $\mu_f / \mu_+$ varied over two order of magnitude, i.e. $10^{-1}$ to $10^1$.

Figure 6 shows the effect of $\alpha_f / \alpha_+$. It is varied from $\alpha_f / \alpha_+ = 0.25$, to 2.5 (standard case) and then up to 25. Upon increasing $\alpha_f / \alpha_+$, the -ve fluid becomes more thermally diffusive, allowing more heat to be conducted towards the outer walls of the second bend while it is flowing. Heat removal at the outer walls of the second bend is more effective. As such, the -ve solid layer grows faster.

The effect of increasing $\alpha_s / \alpha_+$ is similar to that of increasing $\alpha_f / \alpha_+$, although now that heat conduction towards the outer walls of the second bend becomes more effective within the -ve solid layer. As a results, the -ve solid layer grows faster upon increasing $\alpha_s / \alpha_+$, clearly illustrated in Fig. 7 with $\alpha_s / \alpha_+$ increased from 0.25 to 25.
Figure 6: Effect of $\alpha_f / \alpha_\infty$.

(a) $\alpha_f / \alpha_\infty = 0.25$

(b) $\alpha_f / \alpha_\infty = 25$

Figure 7: Effect of $\alpha_s / \alpha_\infty$.

(a) $\alpha_s / \alpha_\infty = 1$

(b) $\alpha_s / \alpha_\infty = 5$
6. Concluding Remarks
This article presents an investigation of solidification in stratified flow of two immiscible fluids in a double-bend. A combined enthalpy-level-set approach is employed. Parametric studies are performed for some of the dimensionless parameters. Although applied to solidification in stratified flow of two immiscible fluids, the approach is generic and applicable to other two-phase flow problems with solidification/melting and of various geometries. Besides, extension of the approach to three-dimension is straightforward.

Nomenclature

\begin{tabular}{|l|l|}
\hline
\(c\) & specific heat \\
\(d\) & shortest distance from \\
\(Fr\) & Froude number \\
\(g\) & gravitational acceleration \\
\(h\) & specific latent heat of solidification \\
\(H(\phi)\) & Heaviside function \\
\(k\) & thermal conductivity. \\
\(L\) & double-bend width \\
\(p\) & pressure \\
\(Pr\) & Prandtl number \\
\(Re\) & Reynolds number \\
\(STh\) & heat source/sink due to solidification \\
\(Ste\) & Stefan number \\
\(t\) & time \\
\(T_{hot}\) & temperature of hot wall \\
\(T_{cold}\) & temperature of cold wall \\
\(T_m\) & solidification temperature \\
\(T\) & temperature \\
\(u_o\) & inlet velocity \\
\(\bar{u}\) & fluid velocity \\
\(V_f\) & volume of \(-ve\) fluid \\
\(V_{CV}\) & volume of control volume \\
\(We\) & Weber number \\
\hline
\(\alpha\) & thermal diffusivity \\
\(\Gamma_f\) & fluid-fluid interface \\
\(\delta(\phi)\) & Delta function \\
\(\mu\) & viscosity \\
\(\rho\) & density \\
\(\sigma\) & surface tension \\
\(\varphi\) & \(-ve\) fluid volume fraction \\
\(\phi\) & level-set function \\
\(\Omega_+\) & \(+ve\) fluid region \\
\(\Omega_-\) & \(-ve\) fluid region \\
\(\Omega_s\) & \(-ve\) solid region \\
\hline
\end{tabular}

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