Numerical Simulation of Suspended Sediment Transportation Based on Particle Tracking Model

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Abstract. Coastal engineering that carried out on the muddy seabed were always accompanied by diffusion of suspended sediment, and that would impact on the surrounding marine environment. A 2-D tidal flow mathematical model of the Yueqing Bay was established based on the Lagrange particle tracking model, the diffusion of suspended sediment in pile foundation construction process of a new wharf in the Yueqing Bay was simulated through a continuous moving points method, the calculation results were compared with the one calculated by the traditional convection diffusion method, it showed that the results calculated from the two different methods were similar, therefore it proved the suitability of the Lagrange particle tracing model in the suspended sediment diffusion problems.

1. Introduction
Sediment would be suspended during the construction process of coastal engineering such as waterway dredging, pier piling, riprap embankment, etc. The suspended sediment can be diffused under the action of tidal current. High concentrations of suspended sediment in the water is harmful to marine organisms and environment. From the perspective of environmental protection, the impact of suspended sediment on the marine environment should be studied.

At present, the numerical simulation of suspended sediment diffusion was based on the convection-diffusion equation. The construction process of coastal engineering was generalized by some approximate methods, and the sediment caused by the process were used as the additional source term in the convection-diffusion equation. A distributed point source diffusion model combined with two-dimensional cohesive sediment mathematical model was established to simulate the transportation process of dumped mud in open estuary water[1], and transportation process of dumped mud in sea area around Macao was numerically calculated by the method. The large eddy simulation (LES) method was applied to the closure of N-S equations in the study of pollutant movement in water[2], and the numerical results was compared with the experimental results. A 2-D tide and suspended sediment numerical model was established to simulate the transportation process of spoiled mud from dredging up in the harbor basin and waterway in Cangnan Power Plant[3], the numerical simulation results were in good agreement with the monitoring results.

The Lagrangian particle tracking model was adopted to simulate the transportation process of suspended sediment in the construction process of pier piling, the result was compared with the result of traditional convection-diffusion method.
2. Numerical model

2.1. 2-D hydrodynamic model
The model was based on the solution of two-dimensional incompressible Reynolds averaged Navier-Stokes equations. Integration of the horizontal momentum equations and the continuity equation over depth \(h = \eta + d\) the following two-dimensional shallow water equations were obtained[4]:

\[
\begin{align*}
\frac{\partial h}{\partial t} + \frac{\partial h\vec{u}}{\partial x} + \frac{\partial h\vec{v}}{\partial y} &= hS \\
\frac{\partial h\vec{u}}{\partial t} + \frac{\partial h\vec{u}^2}{\partial x} + \frac{\partial h\vec{u}\vec{v}}{\partial y} &= f\vec{v}h - gh\frac{\partial \eta}{\partial x} - h\frac{\partial p}{\partial x} - \frac{gh^2}{\rho_0} \frac{\partial \rho}{\partial x} + \frac{\tau_{sx}}{\rho_0} - \frac{\tau_{sx}}{\rho_0} \\
&+ \frac{\partial}{\partial x}(hT_{sx}) + \frac{\partial}{\partial y}(hT_{sy}) + hu_s
\end{align*}
\]

\[
\begin{align*}
\frac{\partial h\vec{v}}{\partial t} + \frac{\partial h\vec{u}\vec{v}}{\partial x} + \frac{\partial h\vec{v}^2}{\partial y} &= -f\vec{u}h - gh\frac{\partial \eta}{\partial y} - h\frac{\partial p}{\partial y} - \frac{gh^2}{\rho_0} \frac{\partial \rho}{\partial y} + \frac{\tau_{sy}}{\rho_0} - \frac{\tau_{sy}}{\rho_0} \\
&+ \frac{\partial}{\partial x}(hT_{sx}) + \frac{\partial}{\partial y}(hT_{sy}) + hv_s
\end{align*}
\]

Where: \(t\) is the time; \(x, y\) are the Cartesian co-ordinates; \(\eta\) is the surface elevation; \(d\) is the still water depth; \(h = \eta + d\) is the total water depth; \(\vec{u}, \vec{v}\) are the velocity components in the \(x, y\) direction; \(f = 2\Omega \sin \phi\) is the Coriolis parameter(\(\Omega\) is the angular rate of revolution and \(\phi\) the geographic latitude); \(g\) is the gravitational acceleration; \(\rho\) is the density of water; \(\rho_0\) is the reference density of water; the lateral stresses \(T_{ij}\) include viscous friction, turbulent friction and differential advection.

2.2. Particle tracking module
In the Particle Tracking Module a Lagrangian discrete-parcel method was used, the suspended sediment was represented by a large ensemble of small parcels. Every particle’s transportation was governed by a random walk model. The position \(\vec{x}(t)\) of each particle in a random walk model was described by the non-linear Langevin equation[5]:

\[
\frac{dx}{dt} = A(\vec{x}, t) + B(\vec{x}, t)\xi(t)
\]

Where \(A(\vec{x}, t)\) is a known vector representing the deterministic forces that act to change \(\vec{x}(t)\), \(B(\vec{x}, t)\) is a known tensor that characterizes the random forces, and \(\xi(t)\) is a vector composed of random numbers that represent the random nature of tidal mixing which, when averaged over sufficiently long time and space scales, becomes effectively random.

Equation (4) becomes equivalent to the stochastic differential equation

\[
\frac{d\vec{x}}{dt} = A(\vec{x}(t), t)dt + B(\vec{x}(t), t)dW(t)
\]

Where \(dW(t)\) is the random Wiener process. The mean value of \(dW(t)\) is zero and the mean square is proportional to \(dt\). The simplest discretization of equation (5) is the explicit Euler scheme for stochastic differential equations.

\[
\Delta \vec{x} = \vec{x}_n - \vec{x}_{n-1} = A(\vec{x}_{n-1}, t_{n-1})\Delta t + B(\vec{x}_{n-1}, t_{n-1})\sqrt{\Delta t}Z_n
\]

Where \(Z_n\) is vector of one, two, or three independent random numbers depending on the dimensionality of the problem form a distribution with zero mean and unit variance.

In the limit when the number of particles is infinite and the time step is infinitely small, equation(6) is equivalent to the Fokker-Planck equation:

\[
\frac{\partial f}{\partial t} + \frac{\partial}{\partial x_i}(A_if) = \frac{\partial^2}{\partial x_i x_j} \left( \frac{1}{2} B_{ij} f \right)
\]
Where $f(\vec{x}, t | \vec{x}_0, t_0)$ is the conditional probability density function for $\vec{x}(t)$.

3. Model application

3.1. Source of suspended sediment

The numerical model was applied to a wharf engineering of Yuhuan power plant in the Yueqing bay. The sediment of seabed would be disturbed while the construction of the piles. The wharf was consists of a coal unloading terminal and an integrated terminal, and both of the terminals were made of high piled beam-slab structure. There were 194 piles in the trestle of the coal wharf, and 24 source points were set up, each point represent 8 piles and each point continuous construction one day, the calculating order was from east to west; the coal dock platform has 314 piles, and it was generalized as 19 source points (each point represents 16 piles), each point continuous construction one day, the calculating order was from north to south; the trestle of the integrated terminal has 48 piles, and it was generalized as 6 source points (each point represents 8 piles), each point continuous construction one day, the calculating order was from east to west; the integrated terminal platform has 238 piles, and it was generalized as 15 source strengths (each point represents 16 piles), each point continuous construction one day, the calculating order was from north to south. The generalized source points of the two trestles was 0.3kg/s, and the generalized source points of the dock platforms was 0.6kg/s. The discharge time of each point is 24 hours.

The discharge sequence of the project was the trestle of the coal wharf, the coal dock platform, the trestle of the integrated wharf and the integrated wharf platform. The sketch of the project and the situation of the suspended sediment source points are shown in figure 1.

![Figure 1 The sketch of the project and the generalized source points](image)

3.2. The hydrodynamic condition

The main research object of the mathematical model to be established was in the middle of Yueqing Bay, the area of the model was about 2360km$^2$. Unstructured triangular grids were used in the model, the mesh grid was refined in the engineering area to ensure the accuracy of analog calculation, the model calculation scope and the grid was shown in Figure.2. The model was validated by the data of the fall of 2011. The tidal range represents the strength of hydrodynamic, so the tidal data of Kanmen tide station which is about 11km from the wharf was collected to found out the cumulative frequency of 90% and 10% of the tidal range, which was 2.68m and 5.56m respectively. Two circulation tide type were designed according to the cumulative frequency of the tidal range. The tidal level process of the designed tide types was showed in Figure 3.
3.3. The calculation result

Yueqing Bay is a semi-enclosed bay. The tidal current in the bay is of typical alternating current, while the current out of the bay is rotational. The flow fields of the bay at the moment of flood and ebb peak were shown in figure 4.

Due to the effect of flood and ebb tidal current, the suspended sediment which was caused by the construction of the piles would be drifted along with the tidal current. The transport direction of the suspended sediments was almost consistent with the direction of the tidal current. It was shown in figure 5 that the contour of concentration of suspended sediment at four typical tidal moment from the result of one of the source points in the trestle of the coal wharf. At the moment of flood or ebb peak, flow velocity was large, so the suspended sediment diffusion was fast, and the concentration was lower. At the moment of flood and ebb stack, flow velocity was quite small and the concentration was higher.
The same hydrodynamic condition and source parameters was applied in the convection-diffusion method. Statistics of the envelope areas of different concentrations levels was shown in Table 1. The calculation results from two methods was basically similar. The envelope area of high concentration levels (larger than 50 mg/L) which was calculated by the particle tracking method was bigger than the result calculated by the convection-diffusion method, and the area of low concentration levels was smaller.

Table 1. The envelope areas of different concentrations levels of the two methods

| Method               | Tidal type   | ≥10mg/L | ≥20mg/L | ≥50mg/L | ≥100mg/L | ≥200mg/L |
|----------------------|--------------|---------|---------|---------|----------|----------|
|                      |              | L       | L       | L       | L        | L        |
| Particle tracking    | Spring tide  | 1.36    | 0.73    | 0.19    | 0.02     | 0.00     |
|                      | Neap tide    | 1.33    | 0.72    | 0.22    | 0.07     | 0.01     |
| convection-diffusion | Spring tide  | 1.63    | 0.87    | 0.16    | 0.02     | 0.00     |
|                      | Neap tide    | 1.60    | 0.86    | 0.18    | 0.06     | 0.01     |

4. Conclusion

A 2-D hydrodynamic model of Yueqing bay was established and the suspended sediment in the construction of piles foundation of Yuhuan power plant was calculated by the method of particle tracking and convection-diffusion. The result showed that the suspended sediment would be drifted along with the tidal current; flow velocity was quite big at the moment of flood and ebb peak and the concentration was lower; flow velocity was quite small at the moment of flood and ebb stack and the concentration was higher. The calculation results from two methods was basically similar. Therefore, particle tracking method can be applied in the numerical simulation of transportation of suspended sediment.
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