SPIDER OPTIMIZATION. II. OPTICAL, MAGNETIC, AND FOREGROUND EFFECTS

D. T. O'Dea1, P. A. R. Ade2, M. Amiri3, S. J. Benton4, J. J. Bock5,6, J. R. Bond7, J. A. Bonetti6, S. Bryan8, B. Burger3, H. C. Chiang9, C. N. Clark1, C. R. Contaldi1, B. P. Crill5,6, G. Davis5, O. Dore5,6, M. Farhang7,10, J. P. Filippini5, L. M. Fissel10, A. A. Fraisse9, N. N. Gandilo10, S. Golwala5, J. E. Gudmundsson9, M. Hasselfield10, G. Hilton11, W. Holmes6, V. V. Hristov5, K. Irwin11, W. C. Jones9, C. L. Kug12, C. J. MacTavish13, P. V. Mason4, T. E. Montroy8, T. A. Morford5, C. B. Netterfield3,10, A. S. Rahlin9, C. Reintsema11, J. E. Ruhl8, M. C. Runyan5, M. A. Schenker5, J. A. Shariff10, J. D. Soler10, A. Trangsrud3, C. Tucker2, R. S. Tucker3, A. D. Turner8, and D. Wiebe3

1 Theoretical Physics, Blackett Laboratory, Imperial College, London, UK
2 School of Physics and Astronomy, Cardiff University, Cardiff, UK
3 Department of Physics and Astronomy, University of British Columbia, Vancouver, BC, Canada
4 Department of Physics, University of Toronto, ON, Canada
5 Department of Physics, California Institute of Technology, Pasadena, CA, USA
6 Jet Propulsion Laboratory, Pasadena, CA, USA
7 Canadian Institute for Theoretical Astrophysics, University of Toronto, Toronto, ON, Canada
8 Department of Physics, Case Western Reserve University, Cleveland, OH, USA
9 Department of Physics, Princeton University, Princeton, NJ, USA
10 Department of Astronomy and Astrophysics, University of Toronto, Toronto, ON, Canada
11 National Institute of Standards and Technology, Boulder, CO, USA
12 Department of Physics, Stanford University, Stanford, CA, USA
13 Kavli Institute for Cosmology, University of Cambridge, Cambridge, UK

Received 2011 February 17; accepted 2011 June 8; published 2011 August 11

ABSTRACT

Spider is a balloon-borne instrument designed to map the polarization of the cosmic microwave background (CMB) with degree-scale resolution over a large fraction of the sky. Spider’s main goal is to measure the amplitude of primordial gravitational waves through their imprint on the polarization of the CMB if the tensor-to-scalar ratio, r, is greater than 0.03. To achieve this goal, instrumental systematic errors must be controlled with unprecedented accuracy. Here, we build on previous work to use simulations of Spider observations to examine the impact of several systematic effects that have been characterized through testing and modeling of various instrument components. In particular, we investigate the impact of the non-ideal spectral response of the half-wave plates, coupling between focal-plane components and Earth’s magnetic field, and beam mismatches and asymmetries. We also present a model of diffuse polarized foreground emission based on a three-dimensional model of the Galactic magnetic field and dust, and study the interaction of this foreground emission with our observation strategy and instrumental effects. We find that the expected level of foreground and systematic contamination is sufficiently low for Spider to achieve its science goals.

Key words: cosmic background radiation – cosmology: observations – gravitational waves – methods: analytical – methods: data analysis – polarization

Online-only material: color figures

1. INTRODUCTION

Observations of the cosmic microwave background (CMB) have been central to the development of a standard cosmological model over the past few decades. A key component of this model is inflation, a period of accelerated expansion that occurred very early in the history of the universe, which enables the model to reproduce the flatness and isotropy observed today. Importantly, inflation also predicts a nearly scale-invariant spectrum of primordial density (scalar) perturbations, in excellent agreement with recent observations of the anisotropies in the intensity of the CMB (Larson et al. 2011).

Although the inflationary hypothesis was first put forward several decades ago (Guth 1981), the details of the underlying physics that drives the expansion are still uncertain. Furthermore, despite the advances made through recent observations of the CMB, the constraints on the general parameters used to describe inflation are weak. As a result, a plethora of plausible inflationary scenarios have been advanced, drawing on a wide range of proposed physical mechanisms.

In addition to the density perturbations that seed large-scale structure, inflation generates a stochastic background of gravitational waves (tensor perturbations) that leave a unique imprint on the polarization of the CMB by introducing a curl or “B-mode” component. A detection of the B-mode signature would provide strong evidence for inflation and a measure of its energy scale, representing a major breakthrough in cosmology. The combination of this measurement with our current knowledge of the scalar perturbations would begin to elucidate the dynamics of inflation and place strong constraints on the underlying physics. As the current upper limit on the energy scale of inflation is around 10^{16} GeV, close to the grand unified theory scale, measuring the B-mode signature of gravitational waves in the CMB provides a unique opportunity to probe physics at energies far beyond the reach of terrestrial high-energy experiments.

The amplitude of inflationary gravitational waves is parameterized by the tensor-to-scalar ratio, r, which is defined as the ratio of the power in tensor modes to that in scalar modes at some pivot comoving scale, here taken to be 0.002 Mpc^{-1}.
The combination of CMB temperature data with measurements of large-scale structure sets the current upper limit of $r < 0.22$ at 95% confidence (Komatsu et al. 2009). However, cosmic variance ultimately limits the $r$ constraints that can be obtained from total intensity measurements, and further improvements will require direct measurement of CMB polarization.

A detection of the dominant $E$-mode component of CMB polarization was first reported around eight years ago (Kovac et al. 2002), and since then many experiments have reported further measurements (Readhead et al. 2004; Sievers et al. 2007; Barkats et al. 2005; Bischoff et al. 2008; Leitch et al. 2005; Montroy et al. 2006; Piacentini et al. 2006; Wu et al. 2007; Page et al. 2007). Recent observations have begun to characterize detailed features in the $E$-mode spectrum (Brown et al. 2009; Chiang et al. 2010; QUIET Collaboration 2010). However, a measurement of the more interesting $B$-mode component will require a substantial improvement beyond present experimental sensitivities.

The weakness of the $B$-mode polarization signature presents a major experimental challenge. Aside from the question of raw sensitivity, there are several other important obstacles that must be overcome. Systematic errors must be controlled with unprecedented accuracy if the small $B$-mode signal is not to be degraded by confusion with the much larger $E$-mode and total intensity signals. Diffuse Galactic foregrounds will also contaminate the observations, with synchrotron emission and thermal dust emission expected to significantly complicate the interpretation of polarization measurements across microwave frequencies.

SPIDER (Crill et al. 2008; Filippini et al. 2010) is a balloon-borne microwave polarimeter designed to search for the imprint of inflationary gravitational waves upon the polarization of the CMB. From a vantage point above the bulk of the atmosphere, SPIDER will map the CMB’s polarization over a large portion of the sky with a resolution close to 1 deg. SPIDER’s main goal is to measure the amplitude of gravitational waves with sensitivity down to $r = 0.03$ (with 99% confidence), or to place an upper limit at this value if no detection is made. To obtain this sensitivity, SPIDER will employ over 2500 bolometers.

MacTavish et al. (2008) investigated the potential impact of many common sources of systematic error on the ability of SPIDER to achieve its main scientific goal, setting tolerance limits to guide the design of the instrument and observation strategy. In this paper, we continue this work, moving our focus onto new systematic effects that have proven to be important but that have so far received less attention. Instead of setting tolerance limits, our aim is to ensure that the measured performance of various instrument components and prototypes is sufficient to meet our main goal.

We also develop a detailed model of the foreground emission SPIDER will see. This makes the sky model against which the systematic effects are assessed more complete, and also enables us to further test the robustness of our observation and analysis strategy. In this work, we do not attempt to address the important issue of separating the foreground and CMB contributions to our data.

This paper is organized as follows. Section 2 gives a brief overview of the instrument and introduces the systematic effects we investigate. The model of polarized foregrounds we use is described in Section 3. Sections 4 and 5 introduce the simulation methodology and the baseline observation strategy we use, and present the results of these studies. Finally, Section 6 gives a discussion of the results and our conclusions.

2. THE INSTRUMENT

A detailed description of the SPIDER instrument is presented in Runyan et al. (2010). Here we provide a brief overview of the instrument and the sources of error we consider.

The SPIDER payload consists of six separate monochromatic instruments housed in a single liquid-helium cryostat. All six points in the same direction on the sky. For the initial long-duration balloon flight, SPIDER will observe with three telescopes at 90 GHz and three at 150 GHz. Subsequent SPIDER flights might involve observations at higher frequencies if required by the foreground situation informed by the first flight. The optical design is based on that of the successful BICEP telescope (Takahashi et al. 2010). Each instrument includes a telecentric refracting telescope consisting of two lenses cooled to 4 K. At 150 GHz this design produces a beam on the sky with full width at half-maximum (FWHM) close to 40 arcmin. Each telescope focuses light onto a focal-plane array of antenna-coupled bolometers, which are cooled to 300 mK using $^3$He closed-cycle sorption fridges. Each dual-polarization spatial pixel in the focal plane consists of two phased arrays of slot dipole antennas, which are sensitive to polarization along orthogonal axes. The power incident on these antennas is deposited onto a bolometer and detected with a superconducting transition-edge sensor (TES). Superconducting quantum interference devices (SQUIDS) are used to amplify the current produced by the TESs, which is then read out using time-domain multiplexing (de Korte et al. 2003; Battistelli et al. 2008). The focal-plane architecture is described in more detail in Kuo et al. (2008).

Note that the design of the beam-defining optical components, the lenses and the focal-plane arrays, is very similar to that of the BICEP2 experiment (Ogburn et al. 2010), although the architecture of the focal-plane unit is different; see Section 5.3.

A rotatable cold half-wave plate is mounted at the aperture of each telescope, on the sky-side of the primary lens. The plates are constructed from 330 mm diameter slabs of birefringent single-crystal sapphire, with a quarter-wave quartz coating to minimize reflections. The orientation of the half-wave plate determines the orientation of the instrument’s polarization sensitivity axes with respect to the sky (see Equation (6)). By making observations with the half-wave plate oriented over a range of angles during the flight, high polarization cross-linking can be achieved, improving the fidelity of the recovered polarization maps. The half-wave plates also provide powerful defense against several important sources of systematic error. Any error that does not transform as a true polarization signal under rotation of the half-wave plate can be distinguished from CMB signal during data analysis. This suppression will occur naturally using standard map-making techniques (provided that the polarization cross-linking is sufficient), as the systematic contributions to the data will have functional forms nearly orthogonal to that of the true sky signal. If necessary, further suppression may be achievable by modifying the map-making algorithm to model the contribution to the data induced by the error, and marginalizing over it if required.

In this paper, we consider several potential sources of systematic error in the SPIDER instrument. First, the behavior of the half-wave plate is weakly dependent on the frequency of the incident radiation, which leads to non-ideals when averaged over the SPIDER bandwidths, which are around 25% of the central frequency. Second, the focal plane is highly sensitive to external magnetic fields, and the motion of the instrument through the magnetic field of the Earth has the potential to generate...
significant spurious signals. Using laboratory measurements of the magnetic pickup in the focal plane, we estimate the in-flight response to Earth’s field. Finally, we consider imperfections in the optical beams defined by the antenna arrays and lenses, focusing on asymmetries and mismatches between the orthogonally polarized detectors in each focal-plane pixel. Optical characterization of the Spider and BICEP2 instruments have shown that these beam systematics are present at a small level, and we use these measurements to guide our studies of the expected impact on Spider’s science goals.

3. SKY MODEL: FOREGROUNDS

Galactic foreground emission is expected to contribute significantly to the polarized microwave emission across the sky, and may well dominate the CMB gravitational-wave signal at all frequencies. Therefore, it is important to include the foreground emission in our sky model for this work. Furthermore, Spider will observe a large fraction of the sky, potentially including regions close to the Galactic plane where the foreground emission is very bright in comparison to the CMB. The presence of this bright emission in our data may affect the performance of the observation and analysis strategy we use.

Unfortunately, these foregrounds are poorly constrained by current data and poorly understood, particularly above ~90 GHz, where the CMB emission is strongest and where Spider will operate. At these frequencies, the foreground emission is expected to be dominated by thermal emission from interstellar dust. Direct measurements by Archeops (Boëtis et al. 2004; Ponthieu et al. 2005) and by Wilkinson Microwave Anisotropy Probe (WMAP; Kogut et al. 2007) have shown that this emission is polarized at the few percent level at frequencies higher than 90 GHz. This is in agreement with our theoretical understanding of the underlying physical mechanism (Draine & Fraisse 2009), which can be summarized as follows. Dust grains are generally non-spherical, and preferentially emit radiation polarized along their longest axis. Mechanisms exist that align these grains with this axis perpendicular to the Galactic magnetic field. This leads to a net linear polarization of the observed emission. Independent observations of the polarization of starlight (Heiles 1996; Fosalba et al. 2002) are consistent with this picture.

Synchrotron emission, generated by the gyration of cosmic ray electrons in the Galactic magnetic field, is intrinsically polarized and constitutes the main polarized foreground at lower frequencies (Page et al. 2007). Since the polarized emission from interstellar dust is expected to dominate the polarized Galactic synchrotron emission at frequencies above ~70 GHz, we do not include the latter in our sky model.

Although little data are available regarding the polarized emission from dust, the same is not true of its total intensity. In particular, the IRAS satellite observed this emission across the sky at 100 µm and 240 µm, close to the peak in the dust emission. By constraining physically motivated extrapolations of these observations using further data, Finkbeiner et al. (1999, hereafter FDS) provided models of the emission at microwave wavelengths. At 94 GHz, these models have been shown to agree well with the WMAP observations, up to a small normalization factor and some minor structural differences in the Galactic plane (Gold et al. 2009). Data are more limited in the higher frequency bands relevant to Spider, but measurements agree well with the FDS predictions (Culverhouse et al. 2010; Veneziani et al. 2010). We use FDS’s model 8 to trace the total intensity of the dust emission.

The other necessary fields for a full description of the dust emission, namely, its degree and direction of polarization, are highly dependent on the Galactic magnetic field. Since the observed polarization signal is the sum of the emission from many independent regions along the line of sight, it is sensitive to the three-dimensional structure of this magnetic field. As a result, modeling the degree and direction of polarization of the dust emission requires the evaluation of appropriate line-of-sight integrals given three-dimensional models of the Galactic magnetic field and of the other relevant Galactic constituents.

Away from the Galactic center, the Galactic magnetic field is usually considered to have two nearly independent components: a large-scale coherent field associated with the Galactic disk and a small-scale field arising from turbulence in the interstellar plasma sourced by astrophysical events such as supernovae and stellar winds. The most informative probes of these fields are Faraday rotation measures of pulsars and extra-Galactic radio sources (Haverkorn et al. 2006; Han et al. 2006). While there is general agreement that the large-scale field follows a spiral pattern, its detailed structure is still uncertain. This uncertainty is unimportant when considering areas of the sky at high Galactic latitudes: since the dust is concentrated in a thin disk about the Galactic plane, we only see emission originating within around 1 kpc or so of the Sun, a region in which the large-scale field is reasonably well characterized. However, as Spider’s large sky coverage may include part of the Galactic plane, we require a model of the large-scale field structure in that plane. One popular candidate is the bisymmetric spiral (Han & Qiao 1994; Sun et al. 2008), which can be written as

\[
B_\rho = -B_0 \cos \left( \phi + \psi \ln \frac{\rho}{\rho_0} \right) \sin \rho \cos \chi_0 \\
B_\phi = -B_0 \cos \left( \phi + \psi \ln \frac{\rho}{\rho_0} \right) \cos \rho \cos \chi_0 \\
B_z = B_0 \delta \cos \chi_0.
\]  

Here, \(\rho, \phi, \text{ and } z\) are Galactocentric cylindrical coordinates with \(\phi\) measured from the direction of the Sun, \(\rho\) is the pitch angle of the field, \(\psi = 1/\tan \rho\), \(\rho_0\) defines the radial scale of the spiral, and \(\chi_0\) parameterizes the amplitude of the \(z\) component. We use the parameter values suggested in Miville-Deschênes et al. (2008): \(p = -8.5\ \text{deg}, \rho_0 = 11\ \text{kpc and } \chi_0 = 8\ \text{deg}\), with the field amplitude set to \(B_0 = 3\ \mu\text{G}\) and the distance between the Sun and the Galactic center taken to be 8 kpc. Although this field model is unlikely to provide a full description of our Galaxy (Men et al. 2008; Sun et al. 2008), it is sufficient for our current purpose since we do not require an accurate template of the sky, only a reasonable approximation against which to test the performance of the experiment.

The turbulent component of the Galactic magnetic field is somewhat less well understood. When constraining the above large-scale field, Miville-Deschênes et al. (2008) simultaneously fit a small-scale field component with best-fit rms amplitude \(B_{rms} = 1.7\ \mu\text{G}\). Since several different studies agree that the rms amplitude is similar to the amplitude of the large-scale field in the solar vicinity (Fosalba et al. 2002; Han et al. 2006), we set \(B_{rms} = 2\ \mu\text{G}\). Minter & Spangler (1996) examined the rotation measures of extra-Galactic sources across a small patch of the sky and concluded that the data were consistent with Kolmogorov turbulence on scales smaller than 4 pc, assuming a statistically isotropic and homogeneous Gaussian field. On larger scales, they found a somewhat flatter energy spectrum with an outer scale of up to 96 pc. Kolmogorov-type spectra up
to kilo-parsec scales in the interstellar magnetic field and other interstellar plasma components have also been reported by other studies (Armstrong et al. 1995; Lazarian & Pogosyan 2000; Cho & Lazarian 2008). Since it is numerically intractable to generate a full-sky realization of this turbulent field in three dimensions at sufficiently high resolution, we resort to independent one-dimensional realizations along the line of sight to each pixel. This model ignores correlations across the sky, but properly incorporates the line-of-sight depolarization. We choose an injection scale of 100 pc, assume the dissipation scale to be small, and use the one-dimensional Kolmogorov energy spectral index of $-5/3$.

We model the large-scale spatial distribution of the dust density, $n_d$, using a simplification of the model constrained in Drimmel & Spergel (2001),

$$n_d = n_0 \exp \left( -\frac{\rho}{\rho_d} \right) \text{sech}^2 \left( \frac{z}{z_d} \right).$$

(2)

For consistency with the WMAP polarization analysis (Page et al. 2007), we take the scale height $z_d = 100$ pc and the scale radius $\rho_d = 3$ kpc. We do not attempt to model the small-scale variations in the dust density and temperature, which may also affect the polarization degree and direction. Small-scale variations in the total intensity are included via the FDS model.

The model also requires a description of the physics of grain alignment and of the intrinsic polarization of the emission from an individual grain. In general, these are complex functions of the magnetic field and of various properties of the grains. Recently, good progress has been made in describing the details of the alignment using the theory of radiative torques (Lazarian & Hoang 2007; Hoang & Lazarian 2008). However, it is still difficult to produce a well-constrained quantitative description to apply to our model (Lazarian & Hoang 2009). We instead describe the alignment in an integrated manner, without recourse to the details of a particular physical mechanism. We assume that the polarization direction is always perpendicular to the component of the magnetic field in the plane of the sky, and that the degree of polarization depends quadratically on the magnetic field strength. This is similar to the behavior assumed in Page et al. (2007).

We compute the Stokes parameter maps associated with our three-dimensional model using the appropriate line-of-sight integrals,

$$I_{3D}(\theta, \phi) = \epsilon(\nu) \int_{0}^{r_{\max}} n_d(r) dr$$

$$Q_{3D}(\theta, \phi) = \epsilon(\nu) \int_{0}^{r_{\max}} n_d(r) p_0 [B_{\phi}(r)^2 - B_\theta(r)^2] dr$$

$$U_{3D}(\theta, \phi) = \epsilon(\nu) \int_{0}^{r_{\max}} n_d(r) p_0 [2B_{\phi}(r)B_\theta(r)] dr,$$

(3)

where the normalization $p_0$ is chosen to reproduce the average polarization fraction of 3.6% reported by WMAP outside their P06 mask (Kogut et al. 2007). Here, $r$, $\theta$, and $\phi$ represent spherical polar coordinates centered on the telescope, and $\epsilon$ is the emissivity of the dust as a function of frequency, $\nu$. Also, both the small-scale turbulent (ss) and large-scale (ls) magnetic field values are included here according to $B_r = B_{r,ss} + B_{r,ls}$, $B_\theta = B_{\theta,ss} + B_{\theta,ls}$, and $B_\phi = B_{\phi,ss} + B_{\phi,ls}$. Note that we conform to the default convention applied in the HEALPIX14 package (Górski et al. 2005) regarding the sign of $U$.

From this model we require maps of the polarization direction, $\gamma$, and degree, $P$, which are given by

$$P(\theta, \phi) = \frac{(Q_{3D}^2 + U_{3D}^2)}{I_{3D}}$$

$$\gamma(\theta, \phi) = \frac{1}{2} \arctan \left( \frac{U_{3D}}{Q_{3D}} \right).$$

Finally, we scale by the FDS intensity maps to obtain our final dust model expressions:

$$I_{dust}(\theta, \phi) = I_{FDS}$$

$$Q_{dust}(\theta, \phi) = I_{FDS} P \cos 2\gamma$$

$$U_{dust}(\theta, \phi) = I_{FDS} P \sin 2\gamma.$$

(5)

Figure 1 shows the Stokes parameter fields for this model evaluated at 150 GHz. In order to compare the dust emission to the CMB, we plot the pseudo-power spectra of these maps

---

14 See http://healpix.jpl.nasa.gov.
(calculated using the HEALPIX `anafast` facility) in Figure 2, masking regions within ~10 deg of the Galactic plane. For comparison the spectra from a smaller, relatively clean patch of sky are also plotted (red, dashed). This patch constitutes a sky fraction $f_{\text{sky}} \sim 0.08$ and encompases the region in the Southern Hemisphere observed by past experiments such as Boomerang (Masi et al. 2006). The CMB spectra for a universe consistent with current data with $r = 0.03$ are shown for comparison, (green, dot-dashed). In the right-hand panel, the contribution from gravitational lensing is also shown (blue, dotted).

(A color version of this figure is available in the online journal.)

Figure 2. Power spectra (from left I, E, and B) of our model of thermal dust emission at 150 GHz with (black, solid) masking regions within ~10 deg of the Galactic plane. For comparison the spectra from a smaller, relatively clean patch of sky are also plotted (red, dashed). This patch constitutes a sky fraction $f_{\text{sky}} \sim 0.08$ and encompasses the region in the Southern Hemisphere observed by past experiments such as Boomerang (Masi et al. 2006).

4. SIMULATION METHOD

We use the simulation pipeline and nominal observation and analysis scheme introduced and investigated in MacTavish et al. (2008). This in turn is based largely on the analysis pipeline developed for the 2003 flight of Boomerang (Masi et al. 2006), described in Jones et al. (2007).

To limit the computational requirements, data are simulated for only 8 spatial focal-plane pixels (16 bolometers) in a single, evenly spaced column spanning one SPIDER focal plane. The two antennas in each spatial pixel are sensitive to polarization parallel and perpendicular to the column. This limited simulation is sufficient for our current purpose, where we perform signal-only simulations and investigate systematic effects which are unlikely to worsen as the number of bolometers considered increases. Therefore, the contamination generated by the various systematic errors reported here can be treated as reasonable upper limits on the likely residuals in the experiment itself.

We simulate a mid-November launch from Alice Springs, Australia (longitude 128.5 deg east, latitude 25.5 deg south), with the balloon subsequently drifting in longitude at $3.76 \times 10^{-4}$ deg s$^{-1}$, the gondola is spun at 36 deg s$^{-1}$ at a fixed elevation of 49 deg, and we observe only when the Sun is 10 deg or more below the horizon. The half-wave plate is stepped once per day by 22.5 deg, and so we consider a four day observation period which allows us to fully sample the polarization modulation the wave plate induces; see Equation (6). With this scanning strategy, SPIDER observes approximately 60% of the sky.

Based upon this scenario, the flight simulator generates the time-ordered right ascension and declination observed by each spatial pixel, as well as its orientation angle $\psi$ with respect to the sky basis. The sky model is generated using the HEALPIX `synfast` facility and the foreground model described in Section 3 using $n_{\text{side}} = 1024$, i.e., pixels of size ~3.4 arcmin. The sky is also smoothed with a Gaussian kernel with FWHM of 40 arcmin to account for the angular resolution of SPIDER. The model is generated at 150 GHz, SPIDER’s main science channel. As we are not concerned with the detailed impact of instrumental noise or foreground separation, this is sufficient for this work.

Note that we use the same CMB realization for each of the various simulations we perform. Note also that as in MacTavish et al. (2008) no $B$-mode power is present in the original full-sky simulation from which the residual maps are obtained. Thus, any $B$-mode power in the final residual maps will be due to the mixing of modes from either systematic or cut-sky effects. The desired measure quantifies the size of the residual in the $B$-modes with respect to the expected original signal.

The pointing information is then used to generate the time-ordered data from the sky model and the wave-plate angle, $\theta$. The bolometer outputs are given by

$$d_{\pm} = I_{\text{sky}} \pm \left[ Q_{\text{sky}} \cos 2(\psi + 2\theta) + U_{\text{sky}} \sin 2(\psi + 2\theta) \right].$$

(6)

where the signs refer to the different bolometers in a spatial pixel. Including systematic effects requires modifications to the above procedures, which we describe in Section 5 for the systematic errors we consider.

SPIDER maps then are constructed in terms of the observed Stokes parameters, $P_{\text{obs}}, Q_{\text{obs}},$ and $U_{\text{obs}}$ with an iterative map maker, an adaptation of the Jacobi method (described in detail in Jones et al. 2007 and implemented in MacTavish et al. 2008), solving for the signal according to

$$\tilde{m} = (A^T N^{-1} A)^{-1} A^T N^{-1} d.$$  

(7)

Here, $\tilde{m}$ is the general least squares map triplet $(P_{\text{obs}}, Q_{\text{obs}},$ and $U_{\text{obs}}), A$ is the pointing matrix which maps time domain samples to pixels on the sky, and $N^{-1}$ is the inverse noise filter. Although the simulations are noiseless, the maps are made using a noise covariance matrix based on a 1/f spectrum with knee frequency 100 mHz, a typical value for the SPIDER detectors (Runyan et al.
We also apply a one-pole high-pass filter at 10 mHz to the time-ordered data during map-making, as this is expected to be necessary for the real data to mitigate long-timescale systematic errors such as incomplete knowledge of the system transfer functions. Thus, we properly include the expected loss of modes due to the low frequency noise and filtering and the resulting degradation of the recovered maps. These maps are produced at $n_{\text{side}} = 256$, i.e., a pixel size $\sim 13.7$ arcmin.

By examining maps of the residuals, i.e., the difference between the output Spider maps and the input model sky, we can begin to evaluate the consequences of the various sources of error. In order to properly assess their impact on our ability to meet our main goal of measuring the gravitational-wave signal, however, it is necessary to proceed to the power spectrum domain, where we can make direct comparisons with the expected CMB spectra.

We estimate spectra from our maps via a spherical harmonic transform using anafast. The decomposition of polarization fields into $E$- and $B$-modes is not unique on the cut sky, however, and so these so-called “pseudo”-spectrum estimates contain mixing between the two. The result of this is that the recovered $B$-mode spectrum is biased by the much larger input $E$-mode spectrum (Bunn et al. 2003; Lewis et al. 2002).

If unaccounted for, this $E$–$B$ mixing dominates the systematic contribution to the recovered $B$-mode spectrum. The development and implementation of an optimal unbiased power spectrum estimator which properly accounts for this mixing is beyond the scope of this work. Instead, we eliminate this bias by estimating the pseudo-spectra of the residual maps. This removes the true sky from our estimates, allowing us to concentrate on the contribution from systematic errors. We also lose a small contribution to the recovered $B$-mode spectrum from map-level errors which correlate with the sky (e.g., an absolute calibration error), but fully retain the much more worrisome systematic effects which couple the recovered $B$-modes to $E$-modes and total intensity. Using the power spectrum of the residuals has the added advantage of eliminating the cosmic variance contribution from the sky signal, allowing the systematic-induced bias to be accurately determined without the need for many independent simulations.

The resulting power spectrum estimate, $\tilde{C}^{B,\text{res}}_l$, should be compared to the expected CMB $B$-mode pseudo-spectrum measured by Spider without foregrounds, systematic errors or $E$–$B$ mixing. We estimate this by performing a systematic- and dust-free simulation with no input $E$-modes and calculating the resulting pseudo-spectrum, $\tilde{C}^{B,\text{noE}}_l$. Finally, to aid interpretation we form $R_l$,

$$R_l = \frac{\tilde{C}^{B,\text{res}}_l}{\tilde{C}^{B,\text{noE}}_l} C^B_l,$$  

so that the final comparison can be made against the input CMB spectrum, $C^B_l$.

We emphasize that this procedure is not designed to give a complete picture of Spider's ability to estimate the $B$-mode power spectrum but to allow us to ensure that the systematic-induced bias is small compared to the expected signal of interest.

For the above power spectrum estimations we use a mask covering the region observed, weighted by the time spent observing each sky pixel to reduce the impact of poorly sampled pixels near the map edges. We also add a $\pm 10$ deg Galactic-plane mask, as this area is heavily obscured by foregrounds and will not be available for cosmological analysis. This mask is shown in Figure 3.

5. SIMULATION RESULTS

5.1. Dust Model

We begin by investigating whether the addition of dust to our sky model reduces the effectiveness of the nominal observation and analysis strategy described in Section 4. To do so we perform two simulations, one using our full sky model and one with a CMB-only sky. Figure 4 compares the residual spectra, $R_l$, from these simulations. As demonstrated in MacTavish et al. (2008), 10 map-making iterations are sufficient to reduce the residuals to a negligible level for the CMB-only case. Using our updated sky model, the residuals remain significant after 10 map-making iterations, with $R_l$ greater than the CMB $B$-mode spectrum for $r = 0.03$ on large scales ($l < 20$). We find that increasing the number of iterations to around 30 suppresses the residuals to a negligible level. Figure 5 shows the residual $Q$ map for this case (the residual $U$ map has similar amplitude), and demonstrates that the recovered maps are of high quality, with little loss of information arising from the time-stream filtering and $1/f$ noise. Note that the small Galactic regions with relatively high residuals are behind our Galactic mask. In accordance with these results, we use 30 map-making iterations for the remaining simulations reported in this paper, unless otherwise stated. For completeness, Figure 4 also shows $R_l$ after 60 iterations, at which point the map-maker has fully converged.

With thousands of detector time streams, the reduction of the computational load of a map-making stage of Spider data analysis will be of utmost importance. Mitigating [systematic errors] pre-flight is advantageous both in terms of signal robustness and the ease and speed of data processing. For this exercise we have chosen a baseline of 30 iterations which will reflect realistic computational restrictions in the final data set.

5.2. Non-ideal Half-wave Plate

With the sky model thus established, we turn to our investigation of the impact of instrumental systematics on Spider's expected sensitivity. We begin with the non-trivial spectral response of the Spider half-wave plate.

The Spider half-wave plates consist of birefringent sapphire with thickness chosen such that the optical path length difference along the crystal axes is exactly a half wavelength at the center

Figure 3. Map showing the sky-mask used when calculating power spectra in our simulation pipeline in equatorial coordinates.

(A color version of this figure is available in the online journal.)
of the appropriate frequency band. The anti-reflection quartz coating is also optimized to this frequency. However, as we move away from this central frequency, the plate behavior becomes non-ideal; the optical path difference is no longer exactly a half wavelength, and the amplitude of reflections, although still suppressed, increases. When averaged over the frequency band, these non-idealities remain. Furthermore, the band-averaged non-idealities depend on the emission spectrum, $S(\nu)$, of the incident radiation. This spectrum is a priori unknown, as we do not know the relative contributions from the CMB and the various foregrounds. This complicates efforts to predict the effects of the half-wave plate non-idealities, and to account for them during data analysis should it prove necessary.

We use the analytic formula derived in Bryan et al. (2010b) to calculate the band-averaged Müller matrix describing the Spider half-wave plate for both CMB and dust emission spectra. This formula is based on a physical optics model of the half-wave plate. We use the refractive index measurements reported in Bryan et al. (2010a) for sapphire at 5 K, and take the quartz anti-reflection coatings to have a refractive index of 1.95. In performing the band averaging, the dust emission spectrum is modeled following model number 8 in FDS, using the average temperatures for the two model dust components, $T_1 = 9.7$ K and $T_2 = 16.2$ K, and so

$$S_{\text{CMB}}(\nu) = \left. \frac{dB(T, \nu)}{dT} \right|_{T_{\text{CMB}}}$$

$$S_{\text{dust}}(\nu) = \left( \frac{\nu}{\nu_0} \right)^{1.67} B(T_1, \nu) + 0.933 \left( \frac{\nu}{\nu_0} \right)^{2.7} B(T_2, \nu),$$  

where $B(T, \nu)$ is the blackbody spectrum and $\nu_0$ is 150 GHz. We also incorporate the Spider detector response spectrum reported in Filippini et al. (2010). The band-averaged half-wave plate Müller matrices are fully described by four independent
Some of these parameters have straight forward interpretations. \( T \) represents an overall transmission loss, and \( p \) arises due to differences between the transmission spectra for radiation polarized along the two crystal axes. The calculated parameter values for our 150 GHz band are given in Table 1, although note that as the CMB is expected to have no circular polarization, the \( s \) parameter should not be relevant to SPIDER.

The Müller matrix describing the transmission of radiation through the entire instrument from the sky to a bolometer can be expressed in terms of \( M_{\text{HWP, band}} \) and the matrix describing the antenna polarization, \( M_{\text{ant}} \), as

\[
M_{\text{ins}} = M_{\text{ant}} R_{a} R_{\psi} M_{\text{HWP, band}} R_{\psi} R_{a},
\]

where \( \epsilon \) is the angle between the antenna’s polarization direction and \( \psi \), i.e., its orientation in the focal plane (here taken to be 0 and 90 deg for the two bolometers in a spatial pixel), and the matrix \( R_{a} \) describes a basis rotation through an angle \( \alpha \). We assume the polarization of the antenna is perfect and so

\[
M_{\text{ant}} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.
\]

The bolometers measure the total power incident upon them, and so the ideal bolometer outputs, Equation (6), are replaced by (Bryan et al. 2010b)

\[
4d_{\pm} = 2T I_{\text{sky}} \pm (T - c)[Q_{\text{sky}} \cos 2(\psi + 2\theta) + U_{\text{sky}} \sin 2(\psi + 2\theta)] \\
\pm 2p[I_{\text{sky}} \cos 2\theta - Q_{\text{sky}} \cos 2(\psi + \theta) - U_{\text{sky}} \sin 2(\psi + \theta)] \\
\pm (T + c)[Q_{\text{sky}} \cos 2\psi + U_{\text{sky}} \sin 2\psi].
\]

To add the half-wave plate non-idealities to our simulations, we make this replacement when generating the time-ordered data.

Examining Equation (13), we see that the total intensity contribution is reduced by the transmission loss caused by the half-wave plate, as expected. Similarly, the polarization efficiency is reduced to \((T - c)/2\). Both of these effects are degenerate with the more general calibration requirements of the experiment. Of greater interest are the remaining contributions to the bolometer outputs, which have dependencies on the half-wave plate orientation unlike those of the ideal outputs, being either independent of \( \theta \) or sinusoidal in 2\( \theta \). As these new contributions transform differently under rotation of the half-wave plate to the ideal bolometer outputs, these errors should be suppressed at the map level by the half-wave plate modulation. However, our nominal four day scan is not sufficient to fully sample these terms to take full advantage of this suppression, and so for this section we increase our scan duration to eight days.

In the following simulation results, we calibrate the SPIDER maps produced by the pipeline by applying the appropriate factor \([p \pm (T - c)]\) before calculating the residual maps and spectra. This enables us to investigate the more complicated and less well understood systematics introduced by the half-wave plate. Although the calibration factor varies between the sky components, the difference is small (0.1%) in comparison to our absolute calibration requirements and to the likely level of foreground residuals after foreground separation.

As the half-wave plate parameters vary between the sky components, we perform two independent simulations for the CMB and dust. The residual spectra for these simulations are shown in Figure 6. For the dust-only simulation the residuals are not negligible, rising to over 40\% of the expected CMB B-mode spectrum for \( r = 0.03 \) at low \( l \). The CMB-only residual is lower, but rises to around 20\% of the expected CMB spectrum. The residual \( Q \) map for the dust-only simulation is shown in Figure 7 (the residual \( U \) map has similar amplitude). By running a further dust-only simulation on an unpolarized sky (i.e., with \( Q = U = 0 \) everywhere) and setting \( T \) and \( c \) to their ideal values, we see that the residuals are dominated by leakage of the dust total intensity into polarization via \( p \), as shown in Figure 6.

Similar simulations isolating the contribution from the \((T + c)/2\) terms in Equation (13) confirm that such an error does not lead to significant residuals for any reasonable parameter values. Note that after 30 iterations, the map-maker has fully converged for these simulations.

Note that taking the unrealistic step of masking the Galactic plane on the input sky significantly reduces the residuals resulting from the half-wave plate non-idealities. Although the plane is always masked in the SPIDER maps when we calculate spectra, the filtering of the time-ordered data partially localizes the large systematic signal generated as the instrument scans across the plane, moving the resulting residuals to high latitude regions of the map.

Our simulations have shown that the errors introduced by our non-ideal half-wave plate model have the potential to mildly compromise SPIDER’s science results if not accounted for during map-making. However, the relevant parameters describing the half-wave plate non-idealities are not a strong function of the emission spectrum of the sky: comparing the values for the CMB and average dust spectrum, the most troublesome parameter, \( p \), changes by only around 10\%, and \( T \) and \( c \) change by less than 1\%. Similarly, variation in these parameters across the sky (as, for example, the dust temperatures vary) are expected to be smaller still. Therefore, we expect to be able to accurately correct for these errors during map-making by upgrading the algorithm to include the extra contributions to the bolometer outputs in the pointing matrix, using constant calibrated values of \( T, p, \) and \( c \), as proposed in Bryan et al. (2010b). This should reduce the map-level residuals to a negligible level, i.e., to those which are at the current science goals. For future experiments targeting \( r < 0.01 \), more detailed simulations will need to be undertaken to investigate the viability of this strategy. Modifying the map-maker in this manner may also provide greater flexibility in SPIDER’s scanning strategy, as
the increase from a 4 day cycle in our wave plate angle to an 8 day cycle might not be required.

5.3. Magnetic Field Sensitivity

A second potential concern for Spider is stray magnetic field pickup in the superconducting electronics chain. The SQUID amplifiers used in the signal readout chain are essentially highly sensitive magnetometers; left unshielded, they will not only amplify signal from the bolometers but also from Earth’s magnetic field. Furthermore, the transition temperature, $T_c$, of the TES detectors is weakly dependent on the magnetic field environment. Motion of the gondola through the terrestrial magnetic field may thus induce spurious signals in the Spider time streams. In order to mitigate these effects, considerable effort has been made to shield the focal plane so that pickup from Earth’s field will be sub-dominant to the CMB. Runyan et al. (2010) describes recent changes to the Spider design, notably a move from a flat focal-plane architecture to a shielded box scheme, with the aim of significantly reducing the magnetic pickup.

These improvements have been successful, and the magnetic response of a Spider telescope is now low enough that it has proven difficult to characterize through laboratory measurements. Response amplitudes have been measured for many of the detectors for the three magnetic field axes, with the remaining amplitudes being too small to detect against the measurement noise. These measurements were taken with the TES bolometers on transition (as they would be in flight), so that any changes in either $T_c$ or SQUID flux would be detected. In order to assign amplitudes to the eight detector pairs in our simulation, we have selected randomly from among the detected response amplitudes, taking care to ensure that the variation in amplitudes across the focal plane and across the different magnetic field axes is typical of that seen in the measurements.

It is not only the overall amplitude of the pickup that is important; any difference in magnetic response within a detector pair will lead to spurious polarization signals. We have therefore preserved detector pairings when selecting the amplitudes for our model. However, as these differences are typically much smaller than the overall amplitude, they are not
The Astrophysical Journal, 738:63 (14pp), 2011 September 1

O’Dea et al.

Figure 8. Residual Q map for a simulation including the signal induced by Earth’s magnetic field in equatorial coordinates. The black lines show the Galactic cut used when calculating power spectra. The magnetic field for each Spider pointing was determined using the World Magnetic Model code and data provided by the National Geophysical Data Center. The resulting time-ordered magnetic field data were used to construct the observed Q map.

(A color version of this figure is available in the online journal.)

Figure 9. Residual spectrum, $R_l$, for a simulation of the signal induced by Earth’s magnetic field (green, long-dashed). The spectrum for our nominal simulation (red) and the CMB $B$-mode spectrum for $r = 0.03$ (black) are shown for comparison.

(A color version of this figure is available in the online journal.)

well constrained by our measurements. In reality the differences may be significantly smaller, and so the simulations presented here represent an upper limit on the spurious signal Spider is likely to see.

The average response amplitudes for the three magnetic field axes for a typical detector have been measured to have values of $\sim 0.02 \mu K_{\text{CMB}}/\mu T$. Due to the very low signal levels, characterizing the frequency dependence of the pickup is not feasible, so instead we use a spectrum typical of those measured for the previous focal-plane design: a one-pole filter with a roll-off frequency of 0.3 Hz.

To include magnetic pickup in our simulations, the response amplitudes and modeled frequency dependence are used to convert the input Earth’s magnetic field (in Tesla) to the output stray pickup observed in the Spider time streams (in $\mu$K). Earth’s magnetic field is determined for each Spider pointing (latitude, longitude, altitude, and time) using the World Magnetic Model code and data provided by the National Geophysical Data Center,15 which estimates the strength and direction of Earth’s main magnetic field for a given point. We use the pointing time streams to account for the changing orientation of the focal plane with respect to the terrestrial field. In order to isolate the systematic error induced by the pickup, for these simulations the only input signal is Earth’s magnetic field; the CMB and dust sky signals are not included. These magnetic field time-ordered data are the inputs to the map-maker. In this case there is no differencing of the final map with an input map; the output stray field map is treated as the residual map.

Figure 8 shows the resulting map for $Q$ polarization signal induced by Earth’s magnetic field. The $U$ map has similar amplitude. Figure 9 illustrates this magnetic response in multipole space. The power spectrum residuals, $R_l$, are less than 3% of the CMB $B$-mode spectrum for $r = 0.03$ across all multipoles, and so we conclude that with the improved focal-plane shielding, the magnetic pickup will not affect Spider’s science goals.

5.4. Mismatched Beams

In this section we examine the impact of non-idealities in the beams produced by the Spider optical system. Ideally, the

15 http://www.ngdc.noaa.gov/geomag/WMM/
instrumental beams would be azimuthally symmetric and would be independent of the antenna polarization. We consider several departures from this ideal situation, motivated by reported characterizations of the Bicep2 instrument (which shares a common optical and antenna design with SPIDER) and early measurements of the SPIDER beams. In particular, we use the results presented in Aikin et al. (2010) to guide us. Of most concern are mismatches between the beams seen by the two bolometers in a spatial pixel, as these have the potential to strongly couple total intensity on the sky into the observed polarization. In this work, we consider three such effects that have proven to describe accurately the main departures from ideality in the measured beams: a differential pointing error of 1.5 arcmin, a differential ellipticity of 0.012, and a differential beamwidth of 0.5%. The method we employ to include these effects in our simulations is designed to be sufficiently flexible to be applied to a more detailed description of the beams, such as will be available when full beam measurements of the SPIDER instruments are carried out in the near future.

Our baseline simulation method assumes that the instrumental beams are ideal, i.e., that each bolometer couples to the sky via the same azimuthally symmetric Gaussian kernel. This azimuthal symmetry ensures that the computationally intensive convolution operation need only be performed once for each sky pixel. For a general kernel this is not the case: the convolution is a function of the instrument orientation, which is constantly changing throughout the scan. Performing a convolution for each sample in the time-ordered data is not possible in a reasonable time. Therefore, to investigate the impact of non-azimuthally symmetric beams, we model them as the sum of a small number of Gaussians, each offset relative to the nominal pointing direction and with differing widths and amplitudes,

$$B(x) = \sum_{n=1}^{N} a_n G(x; x_n, \sigma_n). \quad (14)$$

Here, $B(x)$ is the beam as a function of position on the sky when the instrument is in some fiducial orientation, $G(x; x_0, \sigma)$ is a unit-normalized Gaussian centered on $x_0$ with width $\sigma$, and the amplitudes, $a_n$, sum to unity. As well as tracking the location of the nominal pointing direction on the sky, the pointing generator has been modified to also track the location of the center of each sub-beam, so that the full beam can be reconstructed at all times in the appropriate location and orientation.

To generate the bolometer outputs for this model, the contribution from each (azimuthally symmetric) sub-beam, $d_n$, is first calculated using Equation (6) or Equation (13), along with the sky-model convolved with a Gaussian of width $\sigma_n$. The convolution operator is linear, the final bolometer output is then given by

$$d = \sum_{n=1}^{N} a_n d_n. \quad (15)$$

This method only requires a factor of $N$ more convolution operations (or less if some of the $\sigma_n$ are equal to each other), as well as a factor of $N$ more operations in the pointing and time-ordered data generation. Note that, in keeping with the general approach of this paper, no attempt is made to correct for these beam errors during the map-making and power spectrum analysis.

So far, we have described a method to include non-azimuthally symmetric beams in our simulations. We also wish to consider the impact of differences in the beams seen by the two bolometers in each spatial pixel. This can be achieved by using different parameter values ($N, \sigma_n$ and $x_n$) for the two beams, $B_1(x)$ and $B_2(x)$. We can also allow the beams to vary across the focal plane by using different beam models for different focal-plane locations.

We must be careful with the applicability of this model. It is important that we only consider two different antenna-polarization orientations across our focal plane. For other polarization orientations (e.g., at ±45 deg), the instrument beams are functions of $B_1(x)$ and $B_2(x)$ in a manner which does not generally conform to Equation (14). Similarly, this model does not include (and cannot in general be extended to include) cross-couplings between the two polarization states induced by the optical system.

We perform simulations for each of the three beam mismatch errors we consider. In each case, we model the beams to have errors typical of those reported in Aikin et al. (2010). This represents the worst case for the likely performance of the SPIDER instruments, as further research and development is expected to improve the fidelity of the beams produced. Note that, in this section only, we slightly reduce the ideal beam FWHM used in the simulations to match those measured for Bicep2, from 40 to 31 arcmin.

We begin by considering differential beam pointing, i.e., differences in the beam centers seen by the two bolometers in a spatial pixel. Such errors couple gradients in the total intensity of the radiation incident on the optical system into linear polarization in the focal plane. If such contamination is not removed, the related tolerance limits are strict (Hu et al. 2003; O’Dea et al. 2007). SPIDER was designed with such concerns in mind, however, the half-wave plate is on the sky side of all the optical components. The effect of this coupling on the bolometer outputs will thus be independent of the half-wave plate orientation, and so will be suppressed in the map domain to a degree dependent on the half-wave plate rotation strategy. Note that unlike the pointing errors considered in MacTavish et al. (2008) where the pointing properties were defined to be time-invariant on the sky, here they are defined to be time-invariant in the focal plane.

Modeling the differential pointing is straightforward: we simply use one “sub-beam” for each bolometer in a spatial pixel, with an appropriate offset between them. For this offset we use a typical value of 1.5 arcmin. The residual spectrum for a simulation using this model is shown in Figure 10, and the residual $Q$ map is shown in Figure 11 (the residual $U$ map has similar amplitude). On large scales, where the gravitational-wave signal resides, the residuals are small, rising to around 20% of the CMB $B$-mode spectrum for $r = 0.03$ at $l \sim 100$. On smaller scales the residuals are significant, becoming comparable to the CMB spectrum around $l \sim 300$. This does not compromise SPIDER’s goals, however, as the gravitational lensing signal and instrumental noise are expected to dominate on these scales. Note that the residuals do not depend strongly on the orientation of the pointing error in the focal plane. Here we present the case where the pointing errors are parallel to the column of detectors we simulate. Repeating the simulation with the pointing errors perpendicular to the column, the residuals are generally lower for $l < 150$, and similar on smaller scales.

Comparing these low residuals to the strict tolerance limits found for differential pointing errors in the absence of polarization modulation with a half-wave plate (Hu et al. 2003; O’Dea
et al. 2007), we can conclude that our half-wave plate rotation strategy leads to strong suppression of these errors at the map-making stage.

Next we consider differential beam ellipticity, which also couples the total intensity of the radiation incident on the optical system into linear polarization in the focal plane, through a local quadrupolar pattern. Defining the ellipticity of a beam as $e = (\text{FWHM}_{\text{maj}} - \text{FWHM}_{\text{min}})/(\text{FWHM}_{\text{maj}} + \text{FWHM}_{\text{min}})$, where the subscripts denote the major and minor axes of the ellipse, we take a typical value of the difference in ellipticity between the beams for the two bolometers in a spatial pixel of $e_1 - e_2 = 0.012$. We model an elliptical beam using two sub-beams with identical widths, $\sigma = 31$ arcmin, and centers displaced from the nominal pointing direction by $\delta/2$ in opposite directions along the desired orientation of the major axis. We find that for $\delta = 2$ arcmin, $e = 0.006$. Figure 12 shows the difference between this model and a true elliptical Gaussian with matching ellipticity; note that they agree to 0.1% everywhere. To produce the desired differential ellipticities in our simulations, we use this model with the major axis parallel to the polarization direction for each antenna. The resulting residuals, shown in Figure 10, are negligible across the multipole range relevant to SPIDER.

The relative orientation of the major axes of the elliptical beams and the polarization of the antennas is also an important factor in determining the impact of a differential ellipticity error. In the absence of half-wave plate modulation, differential ellipticity contributes a signal indistinguishable from a true polarization signal, i.e., one which cannot be mitigated through instrument rotation. The nature of this signal depends on the relative orientation of the major axes and the antenna polarization: if the major axes of the beams are at 45 deg to the corresponding antenna-polarization orientations this signal produces a B-mode pattern in the recovered maps (Shimon et al. 2008). Therefore, we also perform a simulation with the major axes of the elliptical beams at ±45 deg to the antenna-polarization orientations. This simulation produces residuals comparable to those shown in Figure 10, confirming that our half-wave plate rotation strategy is sufficient.
The Astrophysical Journal, 738:63 (14pp), 2011 September 1

O’Dea et al.

6. CONCLUSIONS

We have considered several potential sources of systematic error in SPIDER and used simulations to assess their likely impact on the experiment’s main science goal, assuming that no attempt is made to correct the effects during data analysis. Unlike previous examinations of systematic errors, the goal of this work is not to set tolerance limits to guide the design of the instrument, but instead to use measurements of particular instrumental non-idealities to assess their importance. Many of the systematics considered here have so far received little attention in the literature. We have also described a model of the polarized Galactic foreground emission SPIDER will observe, based on a three-dimensional model of the Galactic magnetic field and dust, and we have demonstrated that its presence will not compromise the performance of our baseline observation strategy.

We have considered three main sources of systematic errors that have been characterized through measurements of SPIDER (or SPIDER-like) hardware: the non-ideal spectral response of the half-wave plate, spurious signals generated by the motion of the instrument through Earth’s magnetic field, and mismatches in the instrumental beams generated by orthogonally polarized antennas.

We concluded that non-idealities in the spectral response of the half-wave plates result in errors in the recovered $B$-mode power spectrum that may mildly bias our measurement of $r$ if the gravitational-wave amplitude is close to our target sensitivity of $r = 0.03$ or below. This systematic effect, left uncorrected, will not significantly degrade the scientific results of the experiment, nor lead to a spurious detection of gravitational waves. However, it should be straightforward to remove this bias by introducing three parameters describing the half-wave plate non-idealities (which can be accurately measured pre-flight) into the pointing matrix during map-making.

Our simulations of the stray magnetic field pickup in the TES detectors and SQUID amplifiers have demonstrated that the recent changes to the focal-plane design to improve the magnetic shielding (Runyan et al. 2010) are sufficient to suppress the pickup to a negligible level. Using laboratory measurements of the focal-plane magnetic field response, including differential pickup within detector pairs, the simulated $B$-mode power spectrum residual is over an order of magnitude smaller than the CMB for $r = 0.03$ across the multipoles of interest.

Finally, we have shown that the differential beam non-idealities measured in SPIDER and Bicep2, which have a similar optical system, are not significant; the modulation of these errors introduced by the half-wave plate on the sky side of all the optical components ensures that our science goals are not affected by their presence. The method introduced here will also be used to perform similar tests on more sophisticated descriptions of the optical response of the SPIDER instruments when they become available.

We have considered these sources of error in their application to SPIDER, but the simulation methodology we have developed here is also applicable to other experiments with similar instrumental components attempting to measure the imprint of gravitational waves on the polarization of the CMB.

D.T.O. acknowledges support from STFC under the standard grant scheme (PP/E002129). The SPIDER project is supported by NASA award NNX07AL64G. W.C.J. acknowledges the support of the Alfred P. Sloan Foundation. Some of the results in this paper have been derived using the HEALPix package (Górski et al. 2005) as well as the FFTW package (Frigo & Johnson 2005).

REFERENCES

Aikin, R. W., Ade, P. A., Benton, S., et al. 2010, Proc. SPIE, 7741, 77410V
Armstrong, J. W., Rickett, B. J., & Spangler, S. R. 1995, ApJ, 443, 209
Barkats, D., Bischoff, C., Farase, P., et al. 2005, ApJL, 619, L127
Battistelli, E. S., Amiri, M., Burger, B., et al. 2008, J. Low Temp. Phys., 151, 908
Benolt, A., Ade, P., Amblard, A., et al. 2004, A&A, 424, 571
Bischoff, C., Hyatt, L., McMahon, J. J., et al. 2008, ApJ, 684, 771
Brown, M. L., Ade, P., Bock, J., et al. 2009, ApJ, 705, 978
Bryan, S. A., Ade, P. A. R., Amiri, M., et al. 2010a, Proc. SPIE, 7741, 77412B
Bryan, S. A., Monroy, T. E., & Ruhl, J. E. 2010b, Ap. Opt., 49, 6313
Bunn, E. F., Zaldarriaga, M., Tegmark, M., & de Oliveira-Costa, A. 2003, Phys. Rev. D, 67, 023501
Chiang, H. C., Ade, P. A. R., Barkats, D., et al. 2010, ApJ, 711, 1123
Cho, J., & Lazarian, A. 2008, arXiv:0812.2023
Croll, B. P., Ade, P. A. R., Battistelli, E. S., et al. 2008, Proc. SPIE, 7010, 70102P
Culverhouse, T., Ade, P., Bock, J., et al. 2010, ApJ, 722, 1057
de Korte, P. A. J., Beyer, J., Deiker, S., et al. 2003, Rev. Sci. Instrum., 74, 3807

Figure 12. Map showing the difference between the elliptical-beam model we implement in our simulations, with ellipticity $e = 0.006$, and a true elliptical Gaussian. The beams are normalized such that the true elliptical Gaussian peaks at one. Note that the difference is small, peaking at around 0.1%. (A color version of this figure is available in the online journal.)
