Enhancement of axion decay constants in type IIA theory

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In this work, we investigate the possibility of enhancement of effective axion decay constant in well controlled constructions in string theory. To this end, we study the dynamics of RR axions arising in the compactifications of type IIA string theory on Calabi-Yau orientifolds with background fluxes (with non-perturbative effects included to ensure stabilization of all moduli). In this setting, we implement a recently studied variant of the original Kim-Nilles-Peloso (KNP) mechanism in which the axion alignment is induced through background fluxes. It is known that under these conditions, effective decay constant does not get parametrically enhanced with respect to the fundamental decay constants by adjusting the fluxes. We attempt to enhance the effective decay constant without changing the flux values but by looking for appropriate directions in field space along which the potential is sufficiently flat. This approach leads us to find some directions in field space in which the effective decay constant can always be enhanced.

I. INTRODUCTION

Recently, there have been a number of speculations regarding the possible constraints which any self-consistent theory of Quantum Gravity imposes on the low energy effective theory describing the Universe at longer distances. This has lead to various ideas such as the Weak Gravity Conjecture [1] (and its various manifestations), Swampland conjecture [2, 3] (and refined swampland distance conjecture [4]), de-Sitter swampland conjecture [5] and its refinements (see e.g. [6]).

Some of the arguments on which these conjectures rest are based on our general expectations from any self-consistent theory of Quantum Gravity while others are based on diligent studies of well known string compactifications [7–10] at leading order in $\alpha'$ and $g_s$ with well-controlled sub-leading corrections (see e.g. [11], [12] and [13]). In many such studies, an important role has been played by compactifications of type IIA theory mostly because in such a setting, all geometric moduli get fixed at the classical level (in this context, see e.g. [14–17] for some early papers on moduli stabilization in type IIA theory and [18–23] for some attempts to obtain cosmologically interesting solutions).

Another related subject, which has received a lot of attention is the possible non-existence of super-Planckian axion decay constants in well-controlled regimes of string theory [24]. This subject is of paramount importance given the possible connection of this to large field cosmic inflation, the only version of inflationary scenarios which can be observationally tested in the foreseeable future. One must recall that at the level of field theory, one could imagine several mechanisms which could boost the axion decay constant to super-Planckian values (such as in [25]), the same has not been convincingly established in well-controlled regimes of string theory (see e.g. [26] for a recent work and references). Moreover, it is worth noting that the recent attempts to search large axion decay constants [27] and large field excursions [28] in well understood regimes of type IIA theory have lead to refinements to the swampland distance conjecture [4].

In this paper, we shall focus on this issue of enhancement of axion decay constant in the context of well studied type IIA flux vacua [27, 28] (see also [29] for a recent work). In ref [27], the author used the idea first presented in [30] to realise a version of the well known Kim-Nilles-Peloso mechanism [25] in string theory by using fluxes. This proves to be a very convenient way of obtaining an effective axion decay constant in well-controlled regimes of string theory. Interestingly, it was found that unlike an randomly chosen field theory, in a low energy effective field theory arising from string theory, somehow, there seems to be no parametric enhancement of the decay constant.

In type IIA theory, moduli stabilization at leading order in $\alpha'$ and $g_s$ ensures that only a single linear combination of RR axions is fixed [14, 15]. When we have only two axions, this defines a unique straight line in field space which is flat direction at perturbative level. If distance along this direction is thought of as a field, this field is perturbatively massless. Non-perturbative effects such as Euclidean brane instantons will then lift this flat direction and generate a potential for the perturbatively massless field. On other hand, for the case with three axions, what gets fixed at leading order is a plane in three dimensional RR axion field space. Non-perturbative effects then generate a potential for the fields specifying points in this plane. In this plane, there are multiple straight line directions one could go and each of these can be thought of as the field of interest. The question we ask is, can we get the potential experienced along any such direction to be sufficiently flat? As we shall see, this can be done but notice that we have no guarantee that the field will actually go along such a straight line trajectory (though its dynamics is determined by, among other things, its potential).

In §II, we remind the reader the basic equations relevant for understanding the dynamics of RR axions in IIA flux vacua. Then, in §III, we attempt to enhance the
effective decay constant, in particular, in §, we present a method of doing so and study various consequences of this method in the later subsections. Finally, in §IV, we conclude with a discussion of various related issues.

II. TYPE IIA FLUX VACUA

In this work, we shall follow the notations and conventions used in ref [27]. It is well known that compactifying type IIA string theory on a Calabi-Yau three (CY3) orientifold gives rise to \( N = 1 \) supergravity theory in 1+3 spacetime dimensions.

1. \( N = 1 \) supergravity

The dynamics of the scalar sector of any such theory is determined by a Kahler potential as well as a superpotential (along with other quantities which won’t play any role in what follows). Recall that if the complex scalar fields in the theory are denoted as \( \phi_i \), then, the Kahler potential \( K(\phi_1, \phi_2) \) is a real function of these fields and has mass dimension +2. Similarly, the superpotential \( W(\phi_i) \) is a holomorphic function of the fields and it has a mass dimension +3. The Lagrangian for the scalar sector (for those scalars which are not gauged i.e. no D-term potential) is given by the expression

\[
\mathcal{L} = K^{ij} \partial \phi_i \partial \phi_j - V_F
\]

where, the F-term scalar potential is given by

\[
V_F = e^{\frac{K}{M_p}} \left[ K^{ij} D_i W D_j W - \frac{3|W|^2}{M_p^2} \right],
\]

note that here, the Kahler covariant derivative is given by \( D_i W = \partial_i W + \frac{\partial_i K}{M_p^2} \) and \( K_{ij} = \frac{\partial^2 K}{\partial \phi_i \partial \phi_j} \), while \( K^{ij} \) is the inverse of \( K_{ij} \).

2. \( N = 1 \) supergravity from IIA: fundamentals

For type IIA supergravity on a CY3, we have a set of \( h^{1,1} \) complex scalars \( T_i = b_i + i t_i \) (called the complexified Kahler moduli) and a set of \( h^{2,1} + 1 \) complex scalars. The other complex scalars are \( S = s + i \sigma \) (here, \( s \) is the dilaton and \( \sigma \) is one of the axions) and \( U_\lambda = u_\lambda + i \nu_\lambda \), where \( \lambda = 1, 2, \ldots, h^{2,1} \) (here, \( u_\lambda \) are the complex structure moduli while \( \nu_\lambda \) are axions).

The Kahler potential for the resulting theory is given by a sum of three contributions

\[
K = - \ln 8V - \ln (S + \bar{S}) - 2 \ln V',
\]

here, \( V \) depends on the Kahler moduli \( t_i \) alone (see [27] for its exact expression) while \( V' \) depends on complex structure moduli \( u_\lambda \) alone. In this work, we focus our attention on two simple and quiet similar cases: (a) CY3 which is mirror of the quintic, for which \( h^{2,1} = 1 \), \( V' = u_1^{3/2} \) (we will call this the two axion case), and, (b) CY3 which is mirror to \( P_{1,1,6,9} \) manifold for which \( h^{2,1} = 2 \) while \( V' \) is given by [31]

\[
V' = u_1^{3/2} - u_2^{3/2},
\]

which we will call the three-axion case. It is easy to see that one could obtain \( V' \) of two-axion case from the \( V' \) of three-axion case by setting \( u_2 = 0 \).

3. Fluxes, superpotential and perturbative moduli stabilization

When one turns the fluxes on, a superpotential is induced in the four dimensional effective theory ([14–17], also, see [27] for the details relevant in our context). Notice that when one turns on the fluxes, the compactification manifold no longer stays Calabi-Yau, but for small enough fluxes, the backreaction could be ignored. One could then find the scalar potential using Eq (2) and look for supersymmetric critical points, by using the condition \( D_i W = 0 \). Notice that since we are looking for supersymmetric critical points, they can not be de-Sitter. By following this procedure, one finds that all the geometric moduli (Kahler moduli and complex structure moduli) get fixed while only one linear combination of RR axions \( (\sigma, \nu_\lambda) \) gets fixed, thus, unless \( h^{2,1} = 0 \), this leaves some axions unfixed.

For our purpose, the flux values themselves can be thought of as “free parameters” which we can adjust to obtain different solutions. When we focus attention on the \( (S, U_1, U_2) \) sector of the scalar field space, the corresponding “free parameters” are the fluxes denoted as \( q_1, q_2, f_0 \) and \( h_0 \) as well as the volume \( V \) (see [27] for details). In the three-axion case, moduli stabilization at the classical level fixes the vevs of the fields \( s, u_1 \) and \( u_2 \) to the values determined by the fluxes as follows [27]

\[
s = \frac{2f_0}{3h_0},
\]

\[
u_1 = \left( \frac{(q_1)^2}{(q_1)^3 + (q_2)^3} \right) 3h_0 s,
\]

\[
u_2 = \left( \frac{(q_2)^2}{(q_1)^3 + (q_2)^3} \right) 3h_0 s.
\]

On the other hand, only the following linear combination of the axions is fixed by this procedure.

\[
h_0 \sigma + q_1 \nu_1 + q_2 \nu_2 = \text{constant}.
\]

Here, the two-axion case can be arrived at by setting \( \nu_2 \) to zero.
4. Non-perturbative effects

The superpotential generated by fluxes receives non-perturbative corrections from Euclidean D2-brane instantons, which is of the form

$$ W = W_{\text{perturbative}} + \sum I A_I e^{-a_I^0 s - a_I^1 \nu}, \quad (9) $$

Given this generic form of the superpotential and the Kahler potential given in Eq (3), one can easily find the scalar potential by using Eq (2). For the correct choices of $K_{I}$ and $K_{I0}$, the potential would be of the form shown in Eq (11) below. These choices are determined by the choice of the compactification manifold.

III. ENHANCEMENT OF AXION DECAY CONSTANT

In this section, we shall attempt to enhance axion decay constant in the set up of IIA theory presented in the last section. This problem was studied in ref [27] which we closely follow. As we shall see, there are important new lessons to be learnt even in the simple case of a CY$_3$ which is mirror to $P_{1,1,6,9}$ manifold [31], for which $h^{2,1} = 2$ and there are three RR axions.

To begin with, however, we remind ourselves of how to deal with a slightly different situation, the two-axion case mentioned in the last section i.e. mirror of the quintic for which $h^{2,1} = 1$ and $V' = u_i^{3/2}$. This case has already been studied in [27], but as we shall see, there are important observations to be made in order to study the more interesting case of mirror to $P_{1,1,6,9}$ with $h^{2,1} = 2$ i.e. the three axion case. In [27], the author only briefly mentions the three axion case (in particular, in [27] only the two-axion limit of the three axion case is mentioned). In the upcoming subsection, we revisit the two axion case while in the sub section after that, we analyse the three axion case.

A. The two-axion (i.e. $h^{2,1} = 1$) case

In this case, the fluxes we could vary (to understand axion dynamics) are $q^1$, $f_0$, $h_0$ and we could think of the volume of the compactification manifold $V$ as another “free parameter.” The values of moduli $s$ and $u_1$ can be found in terms of these variables.

1. Basics

Here, $V'$ can be obtained from Eq (4) by setting $u_2 = 0$ and so, using the equations presented in §II 1, we can show that

$$ K_{SS} = \frac{1}{4s^2}, \quad K_{U_1U_1} = \frac{3}{4u_1^2}. \quad (10) $$

At low energies, we can think of $s$ and $u_1$ as fixed quantities, the Lagrangian determining the dynamics of the remaining low energy fields is given by (see e.g. [27])

$$ \mathcal{L} = - f_0^2 (\partial \sigma)^2 - f_{\nu_1}^2 (\partial \nu_1)^2 - \left[ V_0 + A e^{-s} (1 - \cos \sigma) \right] + B e^{-u_1} (1 - \cos \nu_1), \quad (11) $$

where, the potential for the axions is generated by non-perturbative effects. On comparing Eq (1), Eq (10) and Eq (11), we find that $f_\sigma$ is dependent on $s$ while $f_{\nu_1}$ is dependent on $u_1$ i.e.

$$ f_\sigma = \frac{1}{2s}, \quad f_{\nu_1} = \sqrt{\frac{3}{u_1}}. \quad (12) $$

Needless to say, the canonically normalised axions are $f_\sigma \sigma$ and $f_{\nu_1} \nu_1$. Ignoring the non-perturbative effects, at leading order in $\alpha'$ and $g_s$, the linear combination $h_0 \sigma + q^1 \nu_1$ is fixed, by redefining the fields, one could ensure that

$$ h_0 \sigma + q^1 \nu_1 = 0. \quad (13) $$

In the $(f_\sigma \sigma, f_{\nu_1} \nu_1)$ plane of the canonically normalised fields, this describes a straight line passing through the origin i.e.

$$ \left( \frac{h_0}{f_\sigma} \right) f_\sigma \sigma + \left( \frac{q^1}{f_{\nu_1}} \right) f_{\nu_1} \nu_1 = 0. \quad (14) $$

The slope of this line is given by $-(h_0 f_{\nu_1})/(f_\sigma q^1)$ and the two direction cosines of the line are

$$ \ell_\sigma = \frac{q^1 f_\sigma}{N}, \quad (15) $$

$$ \ell_{\nu_1} = - \left( \frac{h_0 f_{\nu_1}}{N} \right), \quad (16) $$

where, $N = \sqrt{f_\sigma^2 (q^1)^2 + f_{\nu_1}^2 (h_0)^2}$. At this stage, it is worth recalling that in $N$-dimensional Euclidean space with Cartesian coordinates $(x_1, x_2, \ldots, x_N)$, the distance $r$ along any straight line passing through the origin (and with direction cosines $(\ell_1, \ldots, \ell_N)$) is $r = \ell_1 x_1 + \cdots + \ell_N x_N$. If we now call $\psi$ to be the distance along the direction described by line Eq (14), one finds that

$$ \sigma = \left( \frac{q^1}{N} \right) \psi, \quad \nu_1 = \left( -\frac{h_0}{N} \right) \psi. \quad (17) $$

If we go along the straight line direction Eq (14), non-perturbative effects shall generate a potential which can be found by substituting for $\sigma$ and $\nu_1$ from Eq (17) into Eq (11), one thus obtains,

$$ \mathcal{L} = - \frac{1}{2} (\partial \psi)^2 - \left[ V_0 + A e^{-s} \left( 1 - \cos \frac{\psi}{f_\sigma} \right) \right] + B e^{-u_1} \left( 1 - \cos \frac{\psi}{f_{\nu_1}} \right), \quad (18) $$
where, $f^s_\psi = N/q^1$ and $f^{u_1}_\psi = N/h_0$. Since in the low energy theory we can think of $s$ and $u_1$ as fixed quantities, the potential experienced if one moves along the straight line direction Eq (14) is a function of only one field, the distance $\psi$ along this direction. Then, Eq (18) suggests that the potential of $\psi$ is a sum of two cosines with different amplitudes and periods.

2. Flux independence of the slope of fixed direction

The coefficients of $\sigma$ and $\nu_1$ in Eq (13) are clearly flux dependent, hence, by changing the fluxes, we could change the slope of the line in $\sigma - \nu_1$ plane. Now consider the line in $(f_\sigma, f_{\nu_1}, u_1)$ plane of the canonically normalised fields, the slope of the straight line in Eq (14) is $-(h_0 f_{\nu_1})/(f_\sigma q^1)$. Using Eq (12), eq (5) and eq (6) (with $q^2$ set to 0), we find that this slope is equal to $-1/\sqrt{3}$. I.e the fixed direction in the space of canonically normalised fields makes an angle $-\pi/6$ w.r.t. the positive $f_\sigma$ axis. Thus, by changing the fluxes, we can not change the orientation of the straight line in the plane of canonically normalised fields.

3. Obstruction to flat potential

Now, in this context one could think about the potential along the $\psi$ direction and its possible flatness. One of the things we mean when we say that the potential along the straight line direction Eq (14) is pretty flat is that it is a cosine with very large period. Suppose that one of decay constants among $f^s_\psi$ and $f^{u_1}_\psi$, say the latter, is very large and that $s$ is large as compared to $u_1$, then, the amplitude of the first cosine in Eq (18) is exponentially suppressed as compared to the second cosine while the period of this second cosine is also large, thus, we could get a direction in which the potential is quite flat. Note that,

$$f^s_\psi = \frac{N}{q^1} = \frac{\sqrt{f^2_\sigma(q^1)^2 + f^2_{f_\psi}(h_0)^2}}{q^1}, \quad s = \frac{2f_\sigma \nu}{5h_0}, \quad (19)$$

$$f^{u_1}_\psi = \frac{N}{h_0} = \frac{\sqrt{f^2_\sigma(q^1)^2 + f^2_{f_\psi}(h_0)^2}}{h_0}, \quad u_1 = \frac{3h_0 s}{q^1}. \quad (20)$$

Thus, it appears that if one keeps $h_0$, $f_\sigma$ and $\nu$ fixed and increases $q^1$, then for large enough $q^1$, $s$ stays put while $u_1$ decreases and $f^s_\psi$ stays constant while $f^{u_1}_\psi$ increases. Thus, a low energy observer might conclude that the first cosine in Eq (18) shall become suppressed over the second one while the period of the second one could be made large, thus, flattening the potential. Furthermore, using eq (12), eq (5) and eq (6) (with $q^2$ set to 0), we conclude that

$$f^{u_1}_\psi = \frac{q^1}{\sqrt{3h_0 s}}, \quad (21)$$

so that increasing $q^1$ with fixed $s$ (by holding $h_0$, $f_\sigma$ and $\nu$ fixed) will cause $f^{u_1}_\psi$ as much as we like without any consequences. This happens to be not true, since it turns out that

$$f^s_\psi = \frac{1}{\sqrt{3}s} = \frac{2f_\sigma}{\sqrt{3}}, \quad (22)$$

$$f^{u_1}_\psi = \frac{\sqrt{3}}{u_1} = 2f_{\nu_1}. \quad (23)$$

Since $f^s_\psi$ and $f^{u_1}_\psi$ are simply proportional to the fundamental axion decay constants $f_\sigma$ and $f_{\nu_1}$, and since these fundamental decay constants can not be super-Planckian, we conclude that $f^s_\psi$ and $f^{u_1}_\psi$ shall also remain sub-Planckian. even though we could increase $f^s_\psi$ and $f^{u_1}_\psi$, we can not make them so large that $u_1$ and $s$ become too small. Since $s$ and $u_1$ are geometric moduli which determine the sizes and shapes of compactification manifold, they can not be made too small without leaving the regime of validity of low energy effective field theory.

For our purpose, we note that the factors relating $f^s_\psi$ to $f_\sigma$ and $f^{u_1}_\psi$ to $f_{\nu_1}$ are $O(1)$ numbers. We shall see that in two-axion limit of three-axion case, there is additional freedom which can cause these factors to be very large numbers.

B. The three axion case

In the rest of this subsection, we shall analyse this possibility and recover the two-axion case. We will find that the three axion case offers new features and there is scope for enhancement of decay constant.

1. Diagonalisation of Kahler metric

Given the Kahler potential, the metric in the scalar field space can be found from

$$K_{ij} = \left( \frac{\partial^2 K}{\partial U_i \partial U_j^\dagger} \right), \quad (24)$$

which in the $(U_1, U_2)$ subspace of the scalar field space turns out to be

$$K_{ij} = \frac{1}{8} \left( \frac{3u_1}{u_1^{3/2} - u_2^{3/2}} \right)^2 \begin{pmatrix} 6u_1 + \frac{3u_1^{3/2}}{u_1^{3/2} - u_2^{3/2}} & -9\sqrt{u_1}u_2 \\ -9\sqrt{u_1}u_2 & 6u_2 + \frac{3u_2^{3/2}}{u_1^{3/2} - u_2^{3/2}} \end{pmatrix}. \quad (25)$$

We now restrict our attention to the subspace of $(U_1, U_2)$ which is spanned by $(\nu_1, \nu_2)$. In this two dimensional subspace, notice that the metric still depends on the vev of the moduli $u_1$ and $u_2$.

Given the Kahler metric $K_{ij}(u_1, u_2)$, we could find its eigenvalues (which we call $f^2_{\nu_1}$ and $f^2_{\nu_2}$) and eigenvectors. If one performs a change of basis such that the eigenvectors are used as the basis vectors, then the metric in
the new basis is diagonal. One can then make an additional anisotropic scaling transformation to turn the metric into an identity matrix. Let the normalised eigenvectors of the metric be denoted by \( \tilde{v}_1 \) and \( \tilde{v}_2 \) and let \( P \) be the matrix of change of basis from \( (\nu_1, \nu_2) \) to \( (\tilde{\nu}_1, \tilde{\nu}_2) \), i.e.,

\[ \nu_i = P_{ij}\tilde{v}_j, \quad (26) \]

since the metric is real-symmetric, \( P \) must be an orthogonal transformation. One must note that all these quantities depend on the moduli \( (u_1, u_2) \) which themselves depend on the fluxes.

2. Search directions and enhancement

Perturbative moduli stabilisation ensures that a plane in the \( (\sigma, \nu_1, \nu_2) \) space stays unfixed.

\[ h_0\sigma + q^1\nu_1 + q^2\nu_2 = 0, \quad (27) \]

Using Eq (26) and after scaling, this implies that

\[
\left( \frac{h_0}{f_\sigma} \right) (f_\sigma \sigma) + \left( \frac{q^1 P_{11} + q^2 P_{21}}{f_{\tilde{\nu}_1}} \right) (f_{\tilde{\nu}_1} \tilde{\nu}_1) \\
+ \left( \frac{q^1 P_{12} + q^2 P_{22}}{f_{\tilde{\nu}_2}} \right) (f_{\tilde{\nu}_2} \tilde{\nu}_2) = 0, \quad (28) 
\]

where, we have simply rewritten the previous equation in terms of normalised eigenvectors of the Kahler metric. This normalisation off-course also canonically normalises the axions we work with i.e. the fields \( f_\sigma \sigma, f_{\tilde{\nu}_1} \tilde{\nu}_1 \) and \( f_{\tilde{\nu}_2} \tilde{\nu}_2 \) are the canonically normalised axions. Needless to say, in the above equation \( f_\sigma \) is a function of \( s \) while \( f_{\tilde{\nu}_1} \) and \( P_{ij} \) are functions of \( (u_1, u_2) \).

Now, Eq (28) describes a plane in the \((f_\sigma \sigma, f_{\tilde{\nu}_1} \tilde{\nu}_1, f_{\tilde{\nu}_2} \tilde{\nu}_2)\) space of canonically normalised fields and from its defining equation, one can easily read off the components of the unit vector normal to the plane. Consider the line common between the plane Eq (28) and the plane \( \tilde{\nu}_2 = 0 \). Obviously, the equation of this line is given by

\[
\left( \frac{h_0}{f_\sigma} \right) (f_\sigma \sigma) + \left( \frac{q^1 P_{11} + q^2 P_{21}}{f_{\tilde{\nu}_1}} \right) (f_{\tilde{\nu}_1} \tilde{\nu}_1) = 0. \quad (29) 
\]

This is a direction in \((f_\sigma \sigma, f_{\tilde{\nu}_1} \tilde{\nu}_1)\) plane and one could go along this direction and ask whether the potential generated by non-perturbative effects could be sufficiently flat. In order to explore the other search directions, one could begin with a unit vector along the line given by the above equation and make a rotation by an angle \( \theta \) about the axis which is normal to the plane. For any choice of this angle \( \theta \), there will be a new search direction. Let the direction cosines of this new search direction be \( (\ell_\sigma, \ell_{\tilde{\nu}_1}, \ell_{\tilde{\nu}_2}) \), notice that these direction cosines depend on \( \theta \) in addition to depending on the fluxes. Since the search direction lies in the plane described by Eq (28), its direction cosines must satisfy the equation of the plane (since the plane passes through the origin)

\[
\left( \frac{h_0}{f_\sigma} \right) \ell_\sigma + \left( \frac{q^1 P_{11} + q^2 P_{21}}{f_{\tilde{\nu}_1}} \right) \ell_{\tilde{\nu}_1} \\
+ \left( \frac{q^1 P_{12} + q^2 P_{22}}{f_{\tilde{\nu}_2}} \right) \ell_{\tilde{\nu}_2} = 0. \quad (30) 
\]

Now, let \( \psi \) be the distance along the search direction, then, since \((\ell_\sigma, \ell_{\tilde{\nu}_1}, \ell_{\tilde{\nu}_2})\) are direction cosines (recall, discussion just before eq (17)),

\[
\ell_\sigma \psi = f_\sigma \sigma, \quad (31) \\
\ell_{\tilde{\nu}_1} \psi = f_{\tilde{\nu}_1} \tilde{\nu}_1, \quad (32) \\
\ell_{\tilde{\nu}_2} \psi = f_{\tilde{\nu}_2} \tilde{\nu}_2. \quad (33)
\]

Now, re-expressing the above relations in terms of the original axions \( \nu_1, \nu_2 \) tells us that

\[
(P^{-1})_{11} \nu_1 + (P^{-1})_{12} \nu_2 = \left( \frac{\ell_\sigma}{f_\sigma} \right) \psi, \quad (34) \\
(P^{-1})_{21} \nu_1 + (P^{-1})_{22} \nu_2 = \left( \frac{\ell_{\tilde{\nu}_1}}{f_{\tilde{\nu}_1}} \right) \psi, \quad (35)
\]

the above two equations can be used to solve for \( \nu_1 \) and \( \nu_2 \) in terms of \( \psi \), thus one gets (using the orthogonality of \( P \))

\[
\sigma = \left( \frac{\ell_\sigma}{f_\sigma} \right) \psi, \quad (36) \\
\nu_1 = \left[ \frac{P_{22} \ell_{\tilde{\nu}_1} f_{\tilde{\nu}_2} - P_{21} \ell_{\tilde{\nu}_2} f_{\tilde{\nu}_1}}{\det P f_{\tilde{\nu}_1} f_{\tilde{\nu}_2}} \right] \psi, \quad (37) \\
\nu_2 = \left[ \frac{P_{11} \ell_{\tilde{\nu}_2} f_{\tilde{\nu}_1} - P_{12} \ell_{\tilde{\nu}_1} f_{\tilde{\nu}_2}}{\det P f_{\tilde{\nu}_1} f_{\tilde{\nu}_2}} \right] \psi. \quad (38) 
\]

One expects that, after the diagonalisation of Kahler metric, the low energy effective theory is given by

\[
\mathcal{L} = -\frac{1}{2} f_\sigma^2 (\partial \sigma)^2 - \frac{1}{2} f_{\tilde{\nu}_1}^2 (\partial \tilde{\nu}_1)^2 - \frac{1}{2} f_{\tilde{\nu}_2}^2 (\partial \tilde{\nu}_2)^2 \\
- \left[ V_0 + A e^{-s}(1 - \cos \sigma) + B' e^{-u_1}(1 - \cos \tilde{\nu}_1) \\
+ C' e^{-u_2}(1 - \cos \tilde{\nu}_2) \right], \quad (39)
\]

which, when expressed in terms of the field \( \psi \) is

\[
\mathcal{L} = -\frac{1}{2} (\partial \psi)^2 - \left[ \frac{V_0'}{f_{\tilde{\nu}_1}^2} + A e^{-s} \left( 1 - \cos \frac{\psi}{f_{\tilde{\nu}_1}} \right) \\
+ B' e^{-u_1} \left( 1 - \cos \frac{\psi}{f_{\tilde{\nu}_1}} \right) + C' e^{-u_2} \left( 1 - \cos \frac{\psi}{f_{\tilde{\nu}_2}} \right) \right], \quad (40)
\]

Comparing Eq (39) with Eq (40) and using Eqs (36), (37), (38), one can thus read-off the effective decay con-
stants,
\[
\begin{align*}
  f^s_\psi & = \left( \frac{f_\sigma}{\ell_\sigma} \right), \quad (41) \\
  f^{u_1}_\psi & = \left[ \frac{\det P f_{\bar{v}_1} f_{\bar{v}_2}}{P_{22} \ell_{\bar{v}_2} f_{\bar{v}_2} \ell_{\bar{v}_1} f_{\bar{v}_1}} \right], \quad (42) \\
  f^{u_2}_\psi & = \left[ \frac{\det P f_{\bar{v}_1} f_{\bar{v}_2}}{P_{11} \ell_{\bar{v}_2} f_{\bar{v}_2} \ell_{\bar{v}_1} f_{\bar{v}_1}} \right]. \quad (43)
\end{align*}
\]

This set of equations tell us that in \((f_\sigma, f_{\bar{v}_1}, f_{\bar{v}_2})\) space of canonically normalised fields, if we go along a direction with direction cosines \((\ell_\sigma, \ell_{\bar{v}_1}, \ell_{\bar{v}_2})\) and if the distance travelled is the field \(\psi\), the potential experiences is given by the term in square brackets in Eq (40), where, the three effective axion decay constants \(f^s_\psi, f^{u_1}_\psi\) and \(f^{u_2}_\psi\) are given by the above equation. It is worth noting that in the above Eq, the matrix elements of \(P\) depend on the fluxes while, as mentioned above, the direction cosines depend on fluxes as well as \(\theta\). An important questions worth answering is could there be choices of fluxes and \(\theta\) which enhance the effective decay constants?

3. A few useful remarks

When our search direction is perpendicular to \(\sigma\) axis, we are in the region of field space where \(\sigma = 0\) and the scalar potential does not depend on \(\sigma\). Perpendicularity to \(\sigma\) axis also implies that \(\ell_\sigma\) is zero. So, in Eq (31), on LHS, \(\ell_\sigma = 0\) and on RHS, \(\sigma = 0\) (as we are in the plane perpendicular to \(\sigma\) axis. In such a case, Eq (31) becomes indeterminate and we do not expect to find \(f^s_\psi\) from Eq (41). Similarly, it is possible that for a fixed choice of fluxes, we happen to be exploring a direction such that the denominator in Eq (42) or Eq (43) becomes zero. Leaving such special cases where the denominator vanishes exactly, one could still ask whether there can be an enhancement of the effective decay constants.

Following the discussion at the beginning of §III.A.3, an important point worth noting is that in Eq (40), even if one of the decay constants, say \(f^{u_1}_\psi\), is large enough, in order to have a flat potential, we must also ask whether \(s\) and \(u_2\) are large enough that the contribution of the potentials (which will relatively more oscillatory since their decay constants are smaller) in the complete potential would be unimportant. If this can not be ensured, then, even if one of the decay constants, say \(f^{u_1}_\psi\) is large, we won’t get a flat potential. Suppose we choose the flux values such that e.g. \(u_2\) and \(s\) are sufficiently large as compared to \(u_1\), then the potential will be mostly dominated by the axion \(v_1\). For such a fixed choice of fluxes, one could go along any direction in field space (starting from the origin). If the direction cosines of the search direction happen to be such that the denominator in Eq (42) becomes small, then, we could have an enhancement of \(f^{u_1}_\psi\) as well as get an actual flat potential. From Eq (39), it is easy to see that the mass of each axion would be given by
\[
m_i^2 \sim \frac{e^{-u_i}}{f_i^2}, \quad (44)
\]
and typically, \(f_i \sim 1/u_i\), thus, \(m_i^2 \sim u_i^2 e^{-u_i}\), thus, large vev shall make the axions light (because of the exponential factor). Thus, it is conceivable that the potential can be flattened by this procedure. In fig (1), we have shown an example of this phenomenon.

4. Recovering the two-axion case

Starting from the formalism of three-axion case, one should be able to recover the two-axion case in some limit. This limiting case was briefly mentioned in [27] but we will find new effects not studied there. When one chooses \(q^1 \gg q^2\), one finds that \(u_1 \gg u_2\) and hence \(m_{\nu_1} \ll m_{\nu_2}\). If the other fluxes are also adjusted to also ensure that \(m_{\sigma} \ll m_{\nu_2}\), then, the axion \(\nu_2\) becomes too heavy. We then expect that we should be able to integrate out this heavy axion and recover the two-axion case in a limit. As we shall see, though this is true, there exist interesting subtleties. To take \(q^1 \gg q^2\) limit, we define
\[
\epsilon = \sqrt{\frac{u_2}{u_1}} = \frac{q^2}{q^1}, \quad (45)
\]
the desired limit is then \(\epsilon \to 0\) limit. Then, in terms of \(\epsilon\), the Kahler metric in Eq (25) takes the form
\[
K_{ij} = \frac{3}{4u_1^2 (1 - \epsilon^3)} \left( 1 + \frac{\epsilon^2}{2} - \frac{3\epsilon}{2} \frac{\epsilon}{1 + \frac{1}{2\epsilon}} \right), \quad (46)
\]
reaching only the leading powers of $\epsilon$, we can find the eigenvalues and hence see that

$$f_{\text{light}}^2 = \frac{3}{4\epsilon_1^2},$$

$$f_{\text{heavy}}^2 = \frac{f_{\text{light}}^2}{2\epsilon} = \frac{\epsilon^3}{2} \frac{3}{4\epsilon_2^2},$$

where, the $f_{\text{heavy}}$ does not become too large. At leading order, the matrix of change of basis is

$$P \approx \begin{pmatrix} 1 - \mathcal{O}(\epsilon^4) & -\mathcal{O}(\epsilon^4) \\ 3\epsilon^2 + \mathcal{O}(\epsilon^3) & 1 + \mathcal{O}(\epsilon^4) \end{pmatrix},$$

where, we follow the convention that the first column of $P$ is the eigenvector corresponding to smaller eigenvalue. Let us suppose that when we try to retain the two-axion limit, the search direction we explore is the intersection of the plane of perturbatively unfixed axions and $\sigma - \hat{\nu}_1$ plane, this makes sense since this is equivalent to $\hat{\nu}_2 = 0$. Using the above form of the $P$ matrix in Eq (28), it is easy to see that, in the limit $\epsilon \to 0$, the line which is common to this plane (in the space of canonically normalised scalar fields) and the plane $\nu_2 = 0$ has along the vector $(1, -1/\sqrt{3}, 0)$. Since this is the direction along which $\psi$ is defined in the two-axion limit, this indicates that we have recovered the flux independence of the slope of this line (mentioned in §III A 2) in the two-axion limit. Moreover, if we keep the leading order terms in $\epsilon$ and follow the procedure described in §III B 2, we can see that

$$f_{\psi}^{\text{eff}} = 2f_{\nu_1} + \mathcal{O}(\epsilon^3).$$

This result was also mentioned in [27]. Now, having recovered the results in the two-axion case, let us apply the ideas presented in §III B 2.

To this end, we begin to explore other directions in the three axion field space. In generating fig (2), we have fixed the fluxes to the following values $q_1 = 60, q_2 = 12, h_0 = 10, f_0 = 10$ and the volume $V \approx 200$. This gives $\epsilon = 0.2$ and the initial direction of exploration (the intersection of the plane of perturbatively unfixed axions and $\sigma - \hat{\nu}_1$ plane) makes an angle of $-29.25$ deg w.r.t. $\sigma$ direction, which is pretty close to the angle obtained in §III A 2.

As we explore other directions in the plane by changing the angle $\theta$ (see discussion below eq (29)), we experience enhancement of the decay constants. Furthermore, the case $\theta = 0$ gives results approximately in agreement with the two-axion case. In fig (2) the minimum of the green curve (the variation of $f_\psi^v$ against $\theta$) lies very close to the dotted green horizontal line (which specifies $f_{\nu_1}$). But the the minimum of the red curve (the variation of $f_\psi^v$ against $\theta$) is far above the dashed orange horizontal line (which specifies $f_{\nu_1}$). This is a manifestation of eq (22) and Eq (23). In particular, in fig (2), at $\theta = 0$, the extreme left region, $f_\psi^{\text{eff}} \approx 2f_{\nu_1}$. Similarly, is is easy to see from fig (2) that, as mentioned in §III B 3, as we vary $\theta$, the effective decay constants blow up and this happens for $\theta = \pi/2$ for $f_\psi^v$.

5. Beyond the two-axion limit

While it was insightful to recover the two-axion model in an appropriate limit of the three-axion model, one must understand that one could vary fluxes such that $\epsilon$ is no longer small. For every choice of fluxes, we could vary $\theta$ to look for directions to enhance the effective decay constants. The results presented here establish that the decay constants can be enhanced this way. In particular, no matter what choice of fluxes on begins with, one could always vary $\theta$ and find directions in field space along which the potential is quite flat.

IV. DISCUSSION

In this work, we tried to find directions in RR axion field space such that the scalar potential along the direction is sufficiently flat. This is done by (a) making sure that the effective decay constant due to one of the axions is large and (b) the vev of the saxion corresponding to the rest of axions are so large that their contribution to the scalar potential is negligible. We found that this can always be done for any fixed choice of fluxes. Note that just because there is a straight-line direction in axion field space along which the potential is sufficiently flat, it does not mean that the motion in field space would be along that direction. Moreover, when the number of RR axions in the model is larger than three, we will have a lot more freedom to enhance the decay constant by the approach presented in this paper. Also, note that the type IIA flux vacua we have been dealing with are AdS, one must check what uplifting will do to the enhancement described here. Thus, one needs to understand uplifting mechanisms better before one can make any statements about large field inflation based on the work of this paper.

It must also be mentioned that we have not taken into account the backreaction caused by Kahler moduli as pointed out in [29] very recently. Among the most important issues being explored in the recent literature is the question whether one could obtain super-Planckian axion decay constants in controlled regimes of string theory. In this paper, however, we have deliberately not said anything about super-Planckian decay constants. We have presented a way to realise large effective decay constants which has the attractive feature that it works even if the fluxes are completely fixed. So, if the fluxes are fixed to acceptable values such that we are in a well-controlled regime (such as the ones we dealt with), it is hard to imagine how simply exploring the field space in different directions could induce any backreactions which will forbid large decay constants. On the other hand, one must think more carefully to understand the possible ways in which this “mechanism” of enhancement would fail to provide super-Planckian effective decay constants. There may be some ways by which the refined swampland distance conjecture [4] forbids any super-Planckian enhancement and hence super-Planckian field excursion.
by e.g. making a tower of states very light leading to the breakdown of effective theory. These issues are worth exploring in future.

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