Self Coupling of the Higgs boson in the processes $pp \to ZHHH + X$ and $pp \to WHHH + X$

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To gain some sense about the likelihood of measuring the Higgs boson quartic coupling, we calculate the contribution to the triple Higgs production cross section from the subprocesses $q\bar{q} \to ZHHH$ and $q\bar{q}' \to WHHH$. Our results illustrate that determining this coupling, or even providing experimental evidence that it exists, will be very difficult.

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I. INTRODUCTION

The Standard Model (SM) has been very successful in explaining almost all experimental data to date, culminating in the discovery of the long awaited Higgs boson at the CERN Large Hadron Collider (LHC) \[1, 2\]. The most important experimental goals of Run 2 at the Large Hadron Collider are the investigation of Higgs properties and the search for new physics beyond the Standard Model.

Thus far the results from the LHC indicate that the couplings of the Higgs boson to other particles are consistent with the Standard Model. However the ultimate test as to whether this particle is the SM Higgs boson will be the trilinear Higgs coupling that appears in Higgs pair production and the quartic Higgs coupling that shows up in triple Higgs production.

The self interaction of the Higgs field, $H$, is

$$V(H) = \lambda v^2 H^2 + \kappa_3 \lambda v H^3 + \frac{1}{4} \kappa_4 \lambda H^4$$  \hspace{1cm} (1)

where $\lambda v^2 = \frac{1}{2} m_H^2$ and $v$ is the vacuum expectation value given by the $Z$ mass, $M_Z$, the weak mixing angle $\theta_W$, and the fine structure constant $\alpha$ as $v = M_Z \cos \theta_W \sin \theta_W / \sqrt{\pi \alpha}$. $\kappa_3$ and $\kappa_4$ are one in the standard model; these are what we would like to measure.

To get a feeling for the relative strengths of the terms in Eq. (1) above we consider here the contribution of the subprocesses $q\bar{q} \rightarrow ZHHH$ to $pp \rightarrow ZHHH + X$ and $q\bar{q} \rightarrow W^{\pm}HHH$ to $pp \rightarrow W^{\pm}HHH + X$. Typical diagrams for this process are shown in Fig. 1.

II. CONTRIBUTIONS FROM TRILINEAR AND QUARTIC COUPLINGS

The matrix element from the Feynman diagrams above has terms of the form

$$\mathcal{M} \sim A\kappa_4 + B\kappa_3 + C + D\kappa^2_3,$$  \hspace{1cm} (2)

where $A$ comes from diagram (a), $B$ from diagrams (b) and (c), $C$ from diagrams (d) and (e), and $D$ from diagram (f). The total cross section is given by

$$\sigma = \kappa^2_4 \sigma_{44} + \kappa^2_3 (\sigma_{33} + \sigma_{330}) + \sigma_0 + \kappa_4 \kappa_3 \sigma_{43} + \kappa_4 \sigma_{40} + \kappa_3 \sigma_{30} + \kappa^4_3 \sigma_{3333} + \kappa^2_4 \kappa^2_3 \sigma_{433} + \kappa^3_3 \sigma_{333},$$  \hspace{1cm} (3)
These separate cross sections for the various terms in Eq. (3), in femtobarns, for several center of mass energies, are given in Table I for $Z$ and Table II for $W^\pm$. These were derived using CTEQ6L1 distribution functions \cite{3} with scale $\sqrt{\hat{s}}$. We do not include any contribution from $gg$ or $gq$ initial states. A $K$ factor of \cite{4}

$$K = 1 + \frac{\alpha_s}{2\pi^2} \frac{16}{9} \approx 1.3$$

was included where

$$\alpha_s^{-1} = \frac{1}{0.130} + \frac{21}{12\pi} \log(\frac{\hat{s}}{M_t^2}) + \frac{46}{12\pi} \log(\frac{M_t}{M_Z}).$$

For the process $pp \rightarrow ZHHH + X$, the contents of Table I are illustrated in Fig. 2. The figure for $pp \rightarrow W^+HHH + X$ is similar.

The amplitude $C$ in Eq. \cite{4} comes from the $ZZH$ and $ZZHH$ couplings (diagrams (d) and (e)). Superficially these diagrams grow faster with energy than the diagrams that involve Higgs propagators. However, the largest energy behavior cancels between the diagrams with only $ZHH$ couplings and those that involve a $ZZH$ and a $ZZHH$ coupling. Explicitly implementing this cancellation of the large energy behavior seems essential for the calculation of $\sigma_0$; depending on the phase space integral to find the cancellation does not work for large center of mass energies. A similar high energy behavior occurs in the $W^+$ cross section and requires the same analytic cancellation. A detailed description of how this cancellation occurs is given in the next section.

III. CANCELLATION OF THE LEADING HIGH ENERGY BEHAVIOR

If we label the momenta as

$$q(p_1) + \bar{q}(p_2) \rightarrow H(k_1) + H(k_2) + H(k_3) + Z(P)$$

(7)
then the matrix element for $C$ can be written

$$M \sim \bar{v}(p_2)\gamma_\mu(g_\nu - \gamma_5)u(p_1)X^{\mu\lambda}\epsilon_\lambda(P)$$

(8)

The spinor factor goes as $E^4$ at large energy $E$. $X^{\mu\lambda}$ has two or three $Z$ propagators depending on the diagram. The propagator which couples to the spinor factor goes as $E^{-2}$ because the momentum in the $p^\mu p^\nu/M_Z^2$ term is $p_1 + p_2$ which is zero when dotted into spinor factor. The other one or two propagators do not have this cancellation and thus go as $E^0$. The $Z$ polarization vector can be longitudinal and thus go as $E^1$. So these diagrams go as $E^0$ for large $E$. The diagrams for contributions other than $C$ go as $E^{-2}$ or faster because they have Higgs propagators.

To see this $E^0$ behavior cancel we need to write out $X^{\mu\lambda}$

$$X^{\mu\lambda} = \frac{1}{2} A_1^{\mu\rho}(B_{2\rho}^\lambda + B_{3\rho}^\lambda + g_\rho^\lambda) + \frac{1}{2} A_2^{\mu\rho}(B_{1\rho}^\lambda + B_{3\rho}^\lambda + g_\rho^\lambda) + \frac{1}{2} A_3^{\mu\rho}(B_{1\rho}^\lambda + B_{2\rho}^\lambda + g_\rho^\lambda)$$

$$+ \frac{1}{2}(A_1^{\mu\rho} + A_2^{\mu\rho} + g^{\mu\rho})B_{3\rho}^\lambda + \frac{1}{2}(A_2^{\mu\rho} + A_3^{\mu\rho} + g^{\mu\rho})B_{1\rho}^\lambda + \frac{1}{2}(A_1^{\mu\rho} + A_3^{\mu\rho} + g^{\mu\rho})B_{2\rho}^\lambda$$

(9)

where

$$A_i^{\mu\lambda} = C_i(M_Z^2 g^{\mu\lambda} + k^\mu_i Q_i^\lambda) \quad \text{no sum on } i$$

$$B_i^{\mu\lambda} = D_i(M_Z^2 g^{\mu\lambda} - R_i^\mu k_i^\lambda) \quad \text{no sum on } i$$

for $i = 1, 2, 3$ with

$$Q_i^\mu = p_i^\mu + p_2^\mu - k_i^\mu$$

$$R_i^\mu = P^\mu + k_i^\mu$$

(10)

(11)

and

$$C_i = \frac{1}{Q_i^2 - M_Z^2}$$

$$D_i = \frac{1}{R_i^2 - M_Z^2}$$

(12)

(13)

(14)

(15)

The large $E$ behavior comes from the $P^\mu P^\nu/M_Z^2$ part of the sum over $Z$ polarizations, so replace the polarization vector $\epsilon_\lambda(P)$ by $P_\lambda$ and dot $P_\lambda$ into $X^{\mu\lambda}$. Then use

$$D_i^{-1} = 2P \cdot k_i + \frac{1}{2}m_H^2$$

(16)

to eliminate $P \cdot k_i$ factors in favor of mass factors or the cancellation of $D_i$ terms. The remaining large $E$ terms will occur in the combination $C_i(p_1 + p_2 - k_i)^2$, which can be replaced by $M_Z^2$ and terms that vanish when contracted with the lepton factor. In particular if we define

$$F_i^\rho = D_i[M_Z^2 P^\rho + \frac{1}{2}M_H^2(P + k_i)^\rho]$$

(17)

then

$$X^{\mu\rho} P_\rho = \frac{1}{2} A_i^{\mu\rho}(F_{j\rho} + F_{k\rho}) + \frac{1}{2}[A_i^{\mu\rho} + A_j^{\mu\rho} + g^{\mu\rho}]F_{k\rho}$$

$$\equiv X^\mu$$

(18)

where

$$i, j, k = (1, 2, 3) + (2, 3, 1) + (3, 1, 2)$$

(19)
and $F^\rho_\mu$ is smaller than $B^{\mu\lambda}P_\lambda$ by two factors of mass rather than momenta. (The terms with the additional factors of momenta are proportional to $P^\mu + k^\mu_1 + k^\mu_2 + k^\mu_3 = p^\mu_1 + p^\mu_2$ dotted into the spinor factor.)

If we call the square of the spinor factor in Eq. (5), summed over spins, $L_{\mu\nu}$, then the square of the matrix element for $\sigma_0$, including the transverse polarizations of the $Z$, is

$$\sum_{\text{pol}} |M|^2 \sim L_{\mu\nu}X^{\mu\lambda}X^{\nu\eta}(-g_{\lambda\eta}) + L_{\mu\nu}X^{\mu\nu}/M^2$$

where $X^\mu$, defined in Eq. (18) above, can be simplified to

$$X^\mu = A^{\mu\rho}_1(F^\rho_2 + F^\rho_3) + A^{\mu\rho}_2(F^\rho_1 + F^\rho_3) + A^{\mu\rho}_3(F^\rho_1 + F^\rho_2) + \frac{1}{2}g^{\mu\rho}(F^\rho_1 + F^\rho_2 + F^\rho_3).$$

By explicitly implementing this cancellation, the integration over phase space is well behaved for all beam energies.

### IV. RESULTS AND CONCLUSIONS

The Tables show that the coefficients of $\kappa_4$ in the cross section Eq. (3) are small which makes a value for $\kappa_4$ almost impossible to determine independent of the value of $\kappa_3$. For example Figs. 3 and 4 show the cross section for the $Z$ process with $\sqrt{s} = 13$ and $\sqrt{s} = 100$ as a function of $\kappa_3$ for two values of $\kappa_4$. At $\sqrt{s} = 13$ TeV with $\kappa_3 = 1$ the difference in the cross section between $\kappa_4 = 1$ and $\kappa_4 = 10$ is $4.2 \times 10^{-4}$ fb. For $\sqrt{s} = 100$ TeV the same difference is $1.2 \times 10^{-2}$ fb and if $\kappa_3 = 10$ the difference is still less than 0.16 fb. Figures 5 and 6 illustrate this further by fixing $\kappa_3$ near 1 and varying $\kappa_4$ to find that the cross sections change by only small fractions of a femtobarn. For the $W^+$ process the contributions that include the quartic Higgs coupling are again too small to measure $\kappa_4$ or even to determine if it is nonzero. This is illustrated in Figs. 7 and 8.

The total cross section for the $W^-$ process is smaller than that for $W^+$ by a factor of 2.6.6, 2.24, 2.19, 1.75, 1.57, 1.47 for $\sqrt{s} = 8, 13, 14, 33, 60, 100$ TeV. The ratios of the individual cross sections (eg., $\sigma_{44}$) vary from these numbers by less than 10%.

The parameter $\kappa_3$ can be determined from processes with two Higgs bosons in the final state. For example, the subprocess $gg \rightarrow HH$ obviously depends on the three Higgs coupling as does $gg \rightarrow ttHH$ [5–21]. Processes with three Higgs bosons in the final state are necessary to determine $\kappa_4$. We show that the processes considered here are not sufficient at any energy to even verify the existence of a four Higgs coupling. This is most obvious from Figure 2 where the coefficients of $\kappa_4$ (dashed lines) are very small compared to most of the other partial

![Figure 3](image1.png)

**FIG. 3.** The cross section for $pp \rightarrow ZHHH + X$ from $q\bar{q} \rightarrow ZHHH$ for $\sqrt{s} = 13$ TeV is shown as a function of $\kappa_3$ for $\kappa_4 = 1$ (solid line) and $\kappa_4 = 10$ (dashed line).

![Figure 4](image2.png)

**FIG. 4.** The cross section for $pp \rightarrow ZHHH + X$ from $q\bar{q} \rightarrow ZHHH$ for $\sqrt{s} = 100$ TeV is shown as a function of $\kappa_3$ for $\kappa_4 = 1$ (solid line) and $\kappa_4 = 10$ (dashed line).
cross sections. In general the problem of determining $\kappa_4$ will be very difficult. Similar conclusions have been reached by Binoth, Karg, Kauer, and Rückl [22] and others [23, 24] for the gluon fusion process $gg \rightarrow HHH$.

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\begin{array}{cccccccccccc}
\sqrt{s} & \sigma_{44} & \sigma_{3333} & \sigma_{433} & \sigma_{40} & \sigma_{330} & \sigma_{43} & \sigma_{4} & \sigma_{333} & \sigma_{30} & \sigma_{33} & \sigma_{TOT} \\
8 & 4.72 \times 10^{-7} & 1.20 \times 10^{-6} & 1.43 \times 10^{-6} & 2.38 \times 10^{-6} & 3.38 \times 10^{-6} & 6.03 \times 10^{-6} & 7.69 \times 10^{-6} & 9.01 \times 10^{-6} & 3.05 \times 10^{-5} & 3.60 \times 10^{-5} & 9.80 \times 10^{-5} \\
13 & 1.57 \times 10^{-6} & 3.61 \times 10^{-6} & 4.47 \times 10^{-6} & 6.94 \times 10^{-6} & 9.31 \times 10^{-6} & 1.80 \times 10^{-5} & 2.32 \times 10^{-5} & 2.55 \times 10^{-5} & 9.05 \times 10^{-5} & 1.09 \times 10^{-4} & 2.92 \times 10^{-4} \\
14 & 1.85 \times 10^{-6} & 4.21 \times 10^{-6} & 5.22 \times 10^{-6} & 8.01 \times 10^{-6} & 1.07 \times 10^{-5} & 2.08 \times 10^{-5} & 2.70 \times 10^{-5} & 2.94 \times 10^{-5} & 1.05 \times 10^{-4} & 1.27 \times 10^{-4} & 3.39 \times 10^{-4} \\
33 & 9.37 \times 10^{-6} & 1.90 \times 10^{-5} & 2.46 \times 10^{-5} & 3.47 \times 10^{-5} & 4.38 \times 10^{-5} & 9.24 \times 10^{-5} & 1.23 \times 10^{-4} & 1.24 \times 10^{-4} & 4.64 \times 10^{-4} & 5.84 \times 10^{-4} & 1.52 \times 10^{-3} \\
60 & 2.43 \times 10^{-5} & 4.67 \times 10^{-5} & 6.16 \times 10^{-5} & 8.41 \times 10^{-5} & 1.04 \times 10^{-4} & 2.26 \times 10^{-4} & 3.06 \times 10^{-4} & 2.97 \times 10^{-4} & 1.14 \times 10^{-3} & 1.45 \times 10^{-3} & 3.74 \times 10^{-3} \\
100 & 5.15 \times 10^{-5} & 9.55 \times 10^{-5} & 1.27 \times 10^{-4} & 1.70 \times 10^{-4} & 2.07 \times 10^{-4} & 4.61 \times 10^{-4} & 6.26 \times 10^{-4} & 5.96 \times 10^{-4} & 2.31 \times 10^{-3} & 2.97 \times 10^{-3} & 7.62 \times 10^{-3} \\
\end{array}
\]

TABLE I. Individual contributions to Eq. (3) for \(pp \rightarrow ZHHH + X\). \(\sqrt{s}\) is the center of mass energy in TeV. All cross sections are in femtobarns. \(\sigma_{TOT}\) is the sum of the contributions (the total cross section if \(\kappa_3 = \kappa_4 = 1\)).

\[
\begin{array}{cccccccccccc}
\sqrt{s} & \sigma_{44} & \sigma_{3333} & \sigma_{433} & \sigma_{40} & \sigma_{330} & \sigma_{43} & \sigma_{4} & \sigma_{333} & \sigma_{30} & \sigma_{33} & \sigma_{TOT} \\
8 & 6.58 \times 10^{-7} & 1.63 \times 10^{-6} & 1.96 \times 10^{-6} & 2.27 \times 10^{-6} & 3.17 \times 10^{-6} & 7.26 \times 10^{-6} & 6.14 \times 10^{-6} & 1.07 \times 10^{-5} & 2.87 \times 10^{-5} & 4.16 \times 10^{-5} & 1.04 \times 10^{-4} \\
13 & 2.03 \times 10^{-6} & 4.53 \times 10^{-6} & 5.65 \times 10^{-6} & 6.00 \times 10^{-6} & 7.96 \times 10^{-6} & 1.99 \times 10^{-5} & 1.70 \times 10^{-5} & 2.78 \times 10^{-5} & 7.79 \times 10^{-5} & 1.17 \times 10^{-4} & 2.85 \times 10^{-4} \\
14 & 2.36 \times 10^{-6} & 5.19 \times 10^{-6} & 6.52 \times 10^{-6} & 6.58 \times 10^{-6} & 9.02 \times 10^{-6} & 2.28 \times 10^{-5} & 1.96 \times 10^{-5} & 3.16 \times 10^{-5} & 8.93 \times 10^{-5} & 1.34 \times 10^{-4} & 3.27 \times 10^{-4} \\
33 & 1.08 \times 10^{-5} & 2.12 \times 10^{-5} & 2.78 \times 10^{-5} & 2.66 \times 10^{-5} & 3.34 \times 10^{-5} & 9.15 \times 10^{-5} & 8.08 \times 10^{-5} & 1.21 \times 10^{-4} & 3.55 \times 10^{-5} & 5.54 \times 10^{-4} & 1.32 \times 10^{-3} \\
60 & 2.66 \times 10^{-5} & 5.01 \times 10^{-5} & 6.65 \times 10^{-5} & 6.13 \times 10^{-5} & 7.56 \times 10^{-5} & 2.13 \times 10^{-4} & 1.91 \times 10^{-4} & 2.77 \times 10^{-4} & 8.29 \times 10^{-4} & 1.31 \times 10^{-3} & 3.10 \times 10^{-3} \\
100 & 5.43 \times 10^{-5} & 9.95 \times 10^{-5} & 1.33 \times 10^{-4} & 1.20 \times 10^{-4} & 1.64 \times 10^{-4} & 4.21 \times 10^{-4} & 3.81 \times 10^{-4} & 5.40 \times 10^{-4} & 1.63 \times 10^{-3} & 2.62 \times 10^{-3} & 6.14 \times 10^{-3} \\
\end{array}
\]

TABLE II. Same as Table I except for \(pp \rightarrow W^+HHH + X\).