TWO-LOOP AND N-LOOP EIKONAL VERTEX CORRECTIONS *

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I present calculations of two-loop vertex corrections with massive and massless partons in the eikonal approximation. I show that the n-loop result for the UV poles can be given in terms of the one-loop calculation.

1 Introduction

The eikonal approximation is valid for emission of soft gluons. The approximation simplifies the usual Feynman rules for the quark propagator and quark-gluon vertex as follows:

\[
\bar{u}(p)(-i\gamma^\mu) \frac{i(y' k' + m)}{(p + k)^2 - m^2 + i\epsilon} \to \bar{u}(p)\gamma^\mu \frac{y' + m}{2p \cdot k + i\epsilon} = \bar{u}(p) \frac{v^\mu}{v \cdot k + i\epsilon} \tag{1}
\]

with \(k \to 0\) the gluon momentum, \(p\) the quark momentum after emission of the gluon, \(v\) a dimensionless vector \(v \propto p\), and I have omitted overall factors of \(g_s T_F\) with \(g_s^2 = 4\pi\alpha_s\) and \(T_F\) the generators of SU(3) in the fundamental representation.

The eikonal approximation has numerous phenomenological applications in QCD, including threshold resummations for a variety of QCD processes [1, 2, 3, 4, 5, 6]. In these applications we are mainly interested in the ultraviolet (UV) pole structure (in dimensional regularization) of one-loop, two-loop, and higher-loop eikonal vertex corrections. In this talk, I discuss explicit calculations of one-loop and two-loop eikonal vertex corrections for diagrams with massive and massless partons and show that the n-loop UV poles are given simply in terms of the one-loop result [7].

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2 One-loop calculations

Let us denote by $\omega_{ij}^{(n)}$ the kinematics, color-independent, part of the $n$-loop correction to the eikonal vertex with lines $i$ and $j$. At one loop, the expression

$$\omega_{ij}^{(1)}(v_i, v_j) = g^2 \int \frac{d^Dk}{(2\pi)^D} \frac{(-i)}{k^2 + i\epsilon} N_{\mu\nu}(k) \frac{\Delta_i}{\delta_i v_i \cdot k + i\epsilon} \frac{\Delta_j}{\delta_j v_j \cdot k + i\epsilon}$$

with $\delta = +1(-1)$ when $k$ flows in the same (opposite) direction as $v$, and

$$N_{\mu\nu}(k) = g_{\mu\nu} - \frac{n^\mu n^\nu}{n \cdot k} + n^2 \frac{k^\mu k^\nu}{(n \cdot k)^2},$$

where $n$ is the axial gauge vector. $\Delta = +1(-1)$ for a quark (antiquark) eikonal line, while for a gluon eikonal line $\Delta = +i(-i)$ for a gluon located above (below) the eikonal line.

Now let $I_1^{(1)}$ denote the contribution to $\omega_{ij}^{(1)}$ from the $l$-th term in the gluon propagator $N_{\mu\nu}(k)$ (i.e. $I_1^{(1)}$ is the contribution from the $g_{\mu\nu}$ term, $I_2^{(1)}$ from the $n^\mu n^\nu$ terms, and $I_3^{(1)}$ from the $k^\mu k^\nu$ term). In dimensional regularization with $\epsilon = 4 - D$, the UV poles in $\omega_{ij}^{(1)}$ for the case of massive quarks, with mass $m$, are given by [1]

$$I_1^{(1)}\text{UV} = \frac{\alpha_s}{\pi} \frac{1}{\epsilon} L_\beta, \quad I_2^{(1)}\text{UV} = \frac{\alpha_s}{\pi} \frac{1}{\epsilon} (L_i + L_j), \quad I_3^{(1)}\text{UV} = -\frac{\alpha_s}{\pi} \frac{1}{\epsilon}.$$

Figure 1: One-loop eikonal vertex correction diagram
and thus
\[ \omega_{ij}^{(1)\text{ UV}} = S_{ij}^{(1)} \left[ I_{1}^{(1)\text{ UV}} + I_{2}^{(1)\text{ UV}} + I_{3}^{(1)\text{ UV}} \right] = S_{ij}^{(1)} \frac{\alpha_s}{\pi \varepsilon} [L \beta + L_i + L_j - 1] \]

with \( S_{ij}^{(1)} = \Delta_i \Delta_j \delta_i \delta_j \) an overall sign and
\[ L \beta = \frac{1 - 2m^2/s}{\beta} \left[ \ln \left( \frac{1 - \beta}{1 + \beta} \right) + \pi i \right] \]

with \( \beta = \sqrt{1 - 4m^2/s} \). The functions \( L_i \) and \( L_j \) depend on the axial gauge vector \( n \) and are cancelled when we include the heavy-quark self energies.

When \( v_i \) refers to a massive quark and \( v_j \) to a massless quark we have [1]
\[ I_{1}^{(1)\text{ UV}} = \frac{\alpha_s}{2\pi} \left\{ \frac{2}{\varepsilon^2} - \frac{1}{\varepsilon} \left[ \gamma_E + \ln \left( \frac{v_{ij}^2 s}{2m^2} \right) - \ln(4\pi) \right] \right\}, \]
\[ I_{2}^{(1)\text{ UV}} = \frac{\alpha_s}{2\pi} \left\{ -\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon} \left[ 2L_i + \gamma_E + \ln \nu_j - \ln(4\pi) \right] \right\}, \]
\[ I_{3}^{(1)\text{ UV}} = -\frac{\alpha_s}{\pi} \frac{1}{\varepsilon}, \]

where \( \nu_a = (v_a \cdot n)^2/|n|^2 \), \( v_{ij} = v_i \cdot v_j \), and \( \gamma_E \) is the Euler constant. Note that the double poles cancel in the sum over the \( I^{(1)} \)'s and we get
\[ \omega_{ij}^{(1)\text{ UV}}(v_i, v_j) = S_{ij}^{(1)} \frac{\alpha_s}{\pi \varepsilon} \left[ -\frac{1}{2} \ln \left( \frac{v_{ij}^2 s}{2m^2} \right) + L_i + \frac{1}{2} \ln \nu_j - 1 \right]. \]

Finally, when both \( v_i \) and \( v_j \) refer to massless quarks we have [8, 1]
\[ I_{1}^{(1)\text{ UV}} = \frac{\alpha_s}{\pi} \left\{ \frac{2}{\varepsilon^2} - \frac{1}{\varepsilon} \left[ \gamma_E + \ln \left( \delta_i \delta_j \frac{v_{ij}}{2} \right) - \ln(4\pi) \right] \right\}, \]
\[ I_{2}^{(1)\text{ UV}} = \frac{\alpha_s}{\pi} \left\{ -\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon} \left[ \gamma_E + \frac{1}{2} \ln(\nu_i \nu_j) - \ln(4\pi) \right] \right\}, \]
\[ I_{3}^{(1)\text{ UV}} = -\frac{\alpha_s}{\pi} \frac{1}{\varepsilon}. \]

Again, we note that the double poles cancel in the sum over the \( I^{(1)} \)'s and we get
\[ \omega_{ij}^{(1)\text{ UV}}(v_i, v_j) = S_{ij}^{(1)} \frac{\alpha_s}{\pi \varepsilon} \left[ -\ln \left( \delta_i \delta_j \frac{v_{ij}}{2} \right) + \frac{1}{2} \ln(\nu_i \nu_j) - 1 \right]. \]
3 Two-loop and \(n\)-loop calculations

For the two-loop diagram in fig. 2 we have

\[
\omega_{ij}^{(2)}(v_i, v_j) = g^4 \int \frac{d^D k_1}{(2\pi)^D} \frac{(-i)}{k_1^2 + i\epsilon} N^{\mu\nu}(k_1) \frac{\Delta_{1i} v_1^\mu}{\delta_{1i} v_1 \cdot k_1 + i\epsilon} \frac{\Delta_{1j} v_1^\nu}{\delta_{1j} v_1 \cdot k_1 + i\epsilon} \\
\times \int \frac{d^D k_2}{(2\pi)^D} \frac{(-i)}{k_2^2 + i\epsilon} N^{\rho\sigma}(k_2) \frac{\Delta_{2i} v_2^\rho}{\delta_{2i} v_2 \cdot (k_1 + k_2) + i\epsilon} \frac{\Delta_{2j} v_2^\sigma}{\delta_{2j} v_2 \cdot (k_1 + k_2) + i\epsilon} .
\]

(10)

Figure 2: Two-loop eikonal vertex correction diagram

When both partons are massive, an explicit calculation of the two-loop diagram gives the following results for the leading UV \((1/\epsilon^2)\) poles at two loops:

\[
I_{11}^{(2), \text{UV}} = \frac{\alpha^2}{\pi^2 \epsilon^2} \frac{1}{L_\beta}, \quad I_{21}^{(2), \text{UV}} = \frac{\alpha^2}{\pi^2 \epsilon^2} \frac{1}{L_\beta} (L_i + L_j) = I_{21}^{(2), \text{UV}} \\
I_{22}^{(2), \text{UV}} = \frac{\alpha^2}{\pi^2 \epsilon^2} (L_i + L_j)^2, \quad I_{13}^{(2), \text{UV}} = -\frac{\alpha^2}{\pi^2 \epsilon^2} L_\beta = I_{31}^{(2), \text{UV}} \\
I_{23}^{(2), \text{UV}} = -\frac{\alpha^2}{\pi^2 \epsilon^2} \frac{1}{L_i + L_j} = I_{32}^{(2), \text{UV}}, \quad I_{33}^{(2), \text{UV}} = \frac{\alpha^2}{\pi^2 \epsilon^2} .
\]

(12)
Then
\[ \omega_{ij}^{(2)} \text{UV} (v_i, v_j) = S_{ij}^{(2)} \frac{\alpha_s}{\pi^2} \frac{1}{\varepsilon^2} (L_\beta + L_i + L_j - 1)^2 + O \left( \frac{1}{\varepsilon} \right), \quad (13) \]

where \( S_{ij}^{(2)} = \Delta_{i1} \Delta_{j1} \Delta_{2i} \Delta_{2j} \delta_{i1} \delta_{j1} \delta_{2i} \delta_{2j} \). The calculation of the \( 1/\varepsilon \) terms is given in [7].

We now note that the leading two-loop UV poles are simply the square of the one-loop result since \( I_{mn}^{(2), \text{UV}} = I_{m}^{(1), \text{UV}} I_{n}^{(1), \text{UV}} \). Similar results hold for the cases when one or both partons are massless. For example, the leading poles in \( I_{23}^{(2), \text{UV}} \) when one of the partons is massless and the other is massive are \( (\alpha_s / \pi^2) (1/\varepsilon^3) \).

We noted that the coefficient of the leading UV pole in \( \omega_{ij}^{(2)} \) is simply the square of the coefficient of the leading UV pole in \( \omega_{ij}^{(1)} \). We can show by induction that this generalizes to \( n \) loops [7]. Thus the leading UV pole in the \( n \)-loop corrections for massive partons, \( \omega_{ij}^{(n)} \), is \( (\alpha_s / \pi)^n (1/\varepsilon^n) (L_\beta + L_i + L_j - 1)^n \). Furthermore a similar structure holds for non-leading UV poles [7].

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