Numerical simulation of flow over a passive disturbance and backward-facing step

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Abstract
This research aimed to simulate the flow past a rib and backward-facing step (BFS). The Reynolds and sub-grid-scale (SGS) stresses were modelled. These models consisted of $k-\varepsilon$, Smagorinksy-Lilly, SGS turbulence kinetic energy ($k_{SGS}$), dynamic SGS and wall-adapting local eddy-viscosity (WALE) which implemented in Open FOAM. The simulation was carried out to compare against the experiment in literatures which gave a high Reynolds number of 15,500. The simulation results showed that the $k-\varepsilon$ model provided the best prediction of the turbulent flow over a passive disturbance and BFS. The absence of rib was carried out to compare with the presence of the rib. It was found that the rib positions had affected to flow reattachment behind the BFS.

Keywords: Simulation, Flow, Passive disturbance, Backward-facing step

1. Introduction
Separated flows are investigated in a wide range of devices or structures contacting air flow [1]. Generally, they are caused by a change of local structure, such as a channel with sudden expansion. The reattached flow location behind a backward-facing step (BFS) is an important phenomenon that attracts many researchers to manage [2]. The possibility of separated flow control is active and passive methods. The periodic disturbances inserted into the main flow or near wall of channel by a speaker [3] or a jet actuator [4-6] were the active method. These methods were effective but difficult to implement. The passive method is preferred. There are the additional elements with smaller sizes than the main separated flows or various step angles. The plate was additional on the step to study the passive control method. It changed the mean profiles of downstream reattached flows [7]. The rib was located on the channel entrance. An effect was found to happen to the downstream reattachment flow, when the distance between the rib and the BFS was proper [8, 9]. The numerical method is usually used for the simulation of turbulent flow over BFS. The turbulence modeling method is commonly used to simulate the large eddy flow in a high range of Reynolds number. The turbulence models are carried out to solve the Reynolds stresses in the Reynolds averaged Navier-Stokes (RANS) equation which governs the eddy flow [10]. Otherwise, simulation method, the spatial filtering operations are used to develop the Navier-Stokes equations. The sub-grid-scale (SGS) stresses are terms which happen by the filtering operation.
Many models were developed to calculate the SGS stresses. There were included Smagorinsky-Lilly, SGS turbulence kinetic energy ($k_{SGS}$), dynamic SGS and wall-adapting local eddy-viscosity (WALE) models [11, 12]. The selection of the SGS model play an important role in the large eddy simulation (LES) method. The flow over BFS with the expansion ratio of 2.00 and Reynolds number of 9,000 was investigated by $k - \varepsilon$ model and LES method. The Smagorinsky-Lilly model was used to calculate SGS stresses and successful to simulate the instantaneous large-scale vortices in BFS flow [13]. The dynamic SGS was selected to model the BFS with the expansion ratio of 2.25 and Reynolds number of 6,400. It was compared with the direct numerical simulation (DNS) and linear $k - \varepsilon$ model. The LES results were in good agreement with the DNS results in spite of the simulation domain was 1.7 times shorter than in DNS [14]. To investigate effects of a passive disturbance to the reattachment flow, the Smagorinsky-Lilly and one-equation models were used for the SGS stresses. The one-equation model described the velocity field characteristics on the step edge and the development dynamics of the separation region behind the step better than the Smagorinsky-Lilly model [15].

In this research, the Reynolds and SGS stresses models for the flow over the passive disturbance and BFS had been compared. There were $k - \varepsilon$, Smagorinsky-Lilly, $k_{SGS}$, dynamic SGS and WALE models in Open FOAM, open-source software. Particularly, effects of a location of the passive disturbance to the reattachment point behind the BFS were investigated. The best model found in this study will be used to design the passive disturbance of the BFS geometry, of the devices or structure which are in contact with air in future work.

2. Simulation methods

2.1 Reynolds averaged Navier-Stokes

The mass conservative and Navier-Stokes equation for incompressible fluid have been averaged by time can be written by following equations.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0$$

$$\frac{\partial \mathbf{U}}{\partial t} + div(\mathbf{U} \mathbf{U}) = \frac{1}{\rho} \text{grad}(p) + v div(\nabla \mathbf{U}) - div(\rho \mathbf{U} \mathbf{U})$$

2.1.1 The $k - \varepsilon$ model

The Reynolds stresses ($-\langle \mathbf{U} \mathbf{U} \rangle$) as shown in equation (2) has been estimated by turbulence model, $k - \varepsilon$ model. The $k - \varepsilon$ model composed of the turbulent kinetic energy ($k$) and rate of viscous dissipation ($\varepsilon$) transport equations [16]. These equations can be written as follows:

$$\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho k \mathbf{U}_i)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \frac{\mu_{ed}}{\sigma_k} \frac{\partial k}{\partial x_i} \right) + 2\mu S_{ij} \cdot S_{ij} - \rho \varepsilon$$

$$\frac{\partial (\rho \varepsilon)}{\partial t} + \frac{\partial (\rho \varepsilon \mathbf{U}_i)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \frac{\mu_{ed}}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x_i} \right) + C_{1\varepsilon} \frac{\varepsilon}{k} 2\mu S_{ij} \cdot S_{ij} - C_{2\varepsilon} \rho \varepsilon^2$$

$$\mu_{ed} = \rho \sigma \frac{k^2}{2} S_{ij} = 1 \left( \frac{\partial \mathbf{U}_i}{\partial x_j} + \frac{\partial \mathbf{U}_j}{\partial x_i} \right)$$

where $C_{\mu} = 0.09$, $\sigma_k = 1.00$, $\sigma_\varepsilon = 1.30$, $C_{1\varepsilon} = 1.44$, $C_{2\varepsilon} = 1.92$

2.2 Large eddy simulation method

The LES method uses a spatial filtering operation to separate the larger and smaller eddies. The filtering function performs on the transient flow equation and gives rise to SGS stresses. The unsteady Navier-Stokes equations for a fluid which are performed by the filtering operation are given by:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0$$

$$\frac{\partial (\rho \mathbf{U})}{\partial t} + div(\rho \mathbf{U} \mathbf{U}) = -\text{grad}(p) + \mu \text{div}(\nabla \mathbf{U}) - \left\{ \text{div}(\rho \mathbf{U} \mathbf{U}) + \text{div}(\rho \mathbf{U} \mathbf{U}) \right\}$$

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The last term of the LES momentum can be written as follows:

\[ \text{div}(\rho u_i \ddot{u} - \rho_i \ddot{U}) = \text{div}(\rho u_i \ddot{u}_j - \rho_i \ddot{u}_j) = \frac{\partial \tau_{ij}}{\partial x_j} \]  \hspace{1cm} (8)

\[ \tau_{ij} = \rho u_i \ddot{u}_j - \rho_i \ddot{u}_j = L_{ij} + C_{ij} + R_{ij} \]  \hspace{1cm} (9)

\[ L_{ij} = \rho u_i \ddot{u}_j - \rho_i \ddot{u}_j \]  \hspace{1cm} (10)

\[ C_{ij} = \rho u_i \ddot{u}'_j + \rho u'_i \ddot{u}_j \]  \hspace{1cm} (11)

\[ R_{ij} = \rho u'_i \ddot{u}'_j \]  \hspace{1cm} (12)

where \( \tau_{ij} \) is the SGS stresses which contain three groups of stress comprising of 1) Leonard stresses \( (L_{ij}) \), 2) cross-stresses \( (C_{ij}) \), and 3) LES Reynolds stresses \( (R_{ij}) \).

2.2.1 Smagorinsky-Lilly model

The SGS stresses relate to the rate of strain \( (\ddot{S}_{ij}) \) which can be written by following equations \([17]\).

\[ \tau_{ij} = -2 \mu_{SGS} \ddot{S}_{ij} + \frac{1}{3} \tau_{ii} \delta_{ij} = -\mu_{SGS} \left( \frac{\partial \ddot{u}_i}{\partial x_j} + \frac{\partial \ddot{u}_j}{\partial x_i} \right) + \frac{1}{3} \tau_{ii} \delta_{ij} \]  \hspace{1.5cm} (13)

\[ \mu_{SGS} = \rho(C_{SGS})^2 \ddot{S} = \rho(C_{SGS})^2 \sqrt{2 \ddot{S}_{ij} \ddot{S}_{ij} \ddot{S}_{ij} = \frac{1}{2} \left( \frac{\partial \ddot{u}_i}{\partial x_j} + \frac{\partial \ddot{u}_j}{\partial x_i} \right)} \]  \hspace{1.5cm} (14)

where \( \mu_{SGS} \) is SGS dynamic viscosity, \( |\ddot{S}| \) is the average strain rate, \( C_{SGS} \) is constant and \( \Delta \) is the filter cutoff width.

2.2.2 SGS turbulence kinetic energy model

The average strain rate is replaced by the SGS turbulent kinetic energy, therefore the SGS dynamic viscosity is written by:

\[ \mu_{SGS} = \rho C'_{SGS} \Delta \sqrt{k_{SGS}} \]  \hspace{1.5cm} (15)

where \( C'_{SGS} \) is 0.094.

The distribution of the SGS turbulent kinetic energy can be solved with the transport equation as follows:

\[ \frac{\partial k_{SGS}}{\partial t} + \text{div}(\rho k_{SGS} \ddot{u}) = \text{div} \left( \frac{\mu_{SGS}}{\sigma_k} \nabla k_{SGS} \right) + 2 \mu_{SGS} \ddot{S}_{ij} \ddot{S}_{ij} - \rho C_e \left( \frac{k_{SGS}}{\Delta} \right)^{\frac{3}{2}} \]  \hspace{1.5cm} (16)

where \( C_e \) is constant and \( \sigma_k \) is the turbulent kinetic energy stress.

2.2.3 Dynamic SGS model

Two different SGS stresses by different cutoff widths can be evaluated from following equations.

\[ \tau_{ij}^L - \tau_{ij}^H = \rho L_{ij} = \rho \ddot{u}_i \ddot{u}_j - \rho \ddot{u}_i \ddot{u}_j \]  \hspace{1.5cm} (17)

\[ L_{ij} = \frac{1}{3} \ddot{u}_{kk} \delta_{ij} = -C_{SGS}^2 \left( 2 \ddot{S}_{ij} \ddot{S}_{ij} - 2 \ddot{S}_{ij} \ddot{S}_{ij} \right), \quad C_{SGS}^2 = \frac{(L_{ij} M_{ij})}{(M_{ij} M_{ij})}, \quad M_{ij} = -2 \ddot{S}_{ij} \ddot{S}_{ij} + 2 \ddot{S}_{ij} \ddot{S}_{ij} \]  \hspace{1.5cm} (18)

where \( C_{SGS}^2 \) is constant, \( \Delta \) is the cutoff width.

2.2.4 WALE

The dynamic viscosity has been written by:

\[ \mu_{SGS} = \rho(C_w \Delta)^2 \left( \frac{S_{ij} S_{ij}}{S_{ij} S_{ij}} \right)^{\frac{3}{2}}, \quad S_{ij} = \frac{1}{2} \left( \frac{\partial \ddot{u}_k}{\partial x_i} \frac{\partial \ddot{u}_j}{\partial x_k} + \frac{\partial \ddot{u}_k}{\partial x_j} \frac{\partial \ddot{u}_i}{\partial x_k} \right) + \frac{1}{3} \delta_{ij} \frac{\partial \ddot{u}_k}{\partial x_i} \frac{\partial \ddot{u}_l}{\partial x_k} \]  \hspace{1.5cm} (19)

where \( C_w \) is 0.325.
3. Backward-facing step with and without a disturbance model

The flow investigation had been performed by modeling the 2D velocity field of the narrow air duct with a BFS before a rib was placed upstream of the BFS, whose geometry was described in [9]. The air duct had a length of 1,000 mm and the BFS had a constant height of 9 mm located at a distance of 600 mm from the air duct entrance. The expansion ratio of the air duct, the ratio between air duct outlet and inlet height, was 1.43. The rib high and width were 3 and 3 mm, respectively. The locations for installation of the ribs were varied between 0 to 60 m upstream of the BFS. The schematic diagram of the channel is shown in figure 1.

![Figure 1. The schematic diagram of the air duct section](image)

To model the narrow air duct without a rib, 221,400 hexagonal cells were carried out by the cell independent method to discrete this domain. In the same manner, the cell number of the air duct with a rib was 221,319. The inlet flow was constant and uniform with air velocity of 25 m/s. Consequently, the Reynolds number of flows over the BFS was 15,500. The outlet boundary condition was the zero gradient of pressure. The wall of air duct was set up with the no-slip boundary condition. The $k$–$\varepsilon$, Smagorinsky-Lilly, $k_{SGS}$, dynamic SGS and WALE models which implemented in OpenFOAM were carried out to simulate the velocity field in the air duct domain. The Euler implicit was used for time discretization. The upwind method was used to protect the high-speed effect of air flow for the pressure-velocity coupling problem. The PISO (Pressure Implicit with Split Operator) iteration was used for the accuracy solution. All simulation cases were computed by the personal computer with the computing property on Intel Core-i5 3.2 GHz CPU and 8 GB DDR2 SDRAM memory.

4. Results and discussion

The numerical results of the BFS flow without a rib show the velocity field by a color contour (figure 2). The air velocity was high at the mid height of air duct after past the BFS. The reattachment flow was investigated behind the step. There was the large recirculation flow. When the SGS models were investigated, a smaller one happened at the top wall and deep step corner of the narrow air duct behind the BFS. It was distinctly different flow behavior from the $k$–$\varepsilon$ model. However, the experimental result could not represent this phenomenon. The comparison of velocity field of simulation is carried out by non-dimensional longitudinal velocity profiles as shown in figure 3. The R-squared value was carried out to decide the accuracy of each model. It was found that the $k$–$\varepsilon$ model gave the best accuracy in agreement with the experiment. Its R-square was 0.9150. However, the middle range of velocity profile was incorrect slightly from the experimental results. The reattachment point could indicate by negative direction of velocity or reverse flow on bottom wall behind the BFS. It was showed at $X/H = 2.5$.

Figure 4 represents the numerical results of air flow past the BFS with a rib at $S = 60$. The $k$–$\varepsilon$ represented the velocity field like the air duct without BFS. The length of reattachment flow behind the BFS was reduced. There was effect by a rib. The air flow was disturbed by the rib; therefore, the vortex happened a lot after past the BFS for the SGS models. The air flow phenomenon by these SGS models could not measure by experiment. Figure 5 shows the comparison of velocity field of these simulations by non-dimensional longitudinal velocity profiles. There was found that the $k$–$\varepsilon$ model was a good agreement with the experiment because of it performed particularly well in confined flows [11]. Its R-square was 0.8716. The WALE model gave the R-square of 0.3964 which more accurate than other SGS
models. The rib position had affected to the length between the step and reattachment point extremely, therefore the recirculation length reduced distinctly when compared to the air duct without a rib. The reattachment point happened at \( X/H = 1.3 \).

The rib was removed and installed at 3 mm upstream of the step. Figure 6 shows the comparison of velocity field of simulation by non-dimensional longitudinal velocity profiles. The R-square of the \( k-\varepsilon \) model was 0.9176. The SGS models gave the minus of R-square value. The reattachment length increased distinctly when compared to the flow past BFS with a rib at \( S = 60 \) and without a rib. The reattachment point happened at \( X/H = 2.7 \). These simulations rendered the rib was installed at the location which added the height of the BFS. There was found that the height of the BFS increased the recirculation length.
Figure 4. The velocity field in the air duct with a rib at \( S = 60 \text{ mm} \) by: (a) \( k - \epsilon \), (b) Smagorinsky-Lilly, (c) \( k_{SGS} \), (d) dynamic SGS, and (e) WALE model.

Figure 5. Profiles of streamwise velocity behind the BFS in the presence of the rib at the distance between the rib and the BFS of 60 mm.

5. Conclusion

The velocity field of air flow inside a narrow air duct, with and without a rib upstream of a step, was modelled and investigated. The simulation models included the \( k - \epsilon \), Smagorinsky-Lilly, \( k_{SGS} \), dynamic SGS and WALE models. These models were implemented in Open FOAM. The R-square was carried out to compare with the experiment result for the accuracy of each model. It was found that the best model for simulating these cases was the \( k - \epsilon \) model by the average R-square of 0.9014. The 2D models of SGS stresses, the LES, in Open FOAM did not act properly, for the simulation of flow inside the narrow air duct with BFS, and also with a step and rib. The position for installing the rib had affected...
to the reattachment point of the recirculation flow behind the BFS. The length between the rib and BFS increased would reduce the recirculation length. Moreover, the height of the BFS also affected to the recirculation length. Therefore, the BFS height increased, would increase the reattachment point behind the BFS. The 2D model which was the $k-\varepsilon$ was found in this study will be used to design the passive disturbance of the BFS geometry for devices or structures in the future work.

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**Figure 6.** Profiles of streamwise velocity behind the BFS in the presence of the rib at the distance between the rib and the BFS of 3 mm.