Chemical Property of Colliding Sources in $^{124,136}$Xe and $^{112,124}$Sn Induced Collisions in Isobaric Ratio Difference and Isoscaling Methods

Chun-Wang MA, Shan-Shan WANG, Yan-Li ZHANG, and Hui-Ling WEI
Institute of Particle and Nuclear Physics, Henan Normal University, Xinxiang, 453007 China

The isoscaling and isobaric ratio difference (IBD) methods are used to study the $\Delta \mu / T$ ($\Delta \mu$ being the difference between the chemical potentials of neutron and proton, and $T$ being the temperature) in the measured $1.4$ GeV $^{124}$Sn + $^{124}$Sn, $^{112}$Sn + $^{112}$Sn, $^{136}$Xe + Pb and $^{124}$Xe + Pb reactions. The isoscaling phenomena in the $^{124}$Sn/$^{112}$Sn and the $^{136}$Xe/$^{124}$Xe reactions pairs are investigated, and the isoscaling parameter $\alpha$ and $\beta$ are obtained. The $\Delta \mu / T$ determined by the isoscaling method (IS–$\Delta \mu / T$) and IBD method (IB–$\Delta \mu / T$) in the measured Sn and Xe reactions are compared. It is shown that in most of fragments, the IS– and IB– $\Delta \mu / T$ are consistent in the Xe reactions, while the IS– and IB– $\Delta \mu / T$ are only similar in the less neutron-rich fragments in the Sn reactions. The shell effects in IB–$\Delta \mu / T$ are also discussed.

PACS numbers: 25.70.Pq, 21.65.Cd, 25.70.Mn
Keywords: isobaric yield ratio, isoscaling, symmetry energy

I. INTRODUCTION

Research on nuclear symmetry energy has attracted much attention both theoretically and experimentally because of its importance in nuclear physics and astrophysics (for recent reviews, see Refs. [1–3]). Among the various methods, the yield of light particles and heavy fragments (or ratios of these particles and fragments), are frequently used to study the symmetry energy of nuclear matter from sub-saturation to supra-saturation densities produced in heavy-ion collisions (HICs) [4–17]. The results obtained from these methods constrain the nuclear symmetry energy in a relative large scope [18]. As the density dependence of nuclear symmetry energy is still an open problem, an international collaboration called the “Symmetry Energy Project” has been launched in order to determine the density dependence of the symmetry energy beyond the normal nuclear matter based on the newly built radioactive ion beam facilities [19]. At the same time, it is also hoped that new new observable for studying the nuclear symmetry energy will be found.

The chemical potentials of the neutron ($\mu_n$) and proton ($\mu_p$) depend on nuclear density and temperature, and reflect the symmetry energy of different nuclear matter [20, 21]. In HICs, the different $\mu_n$ and $\mu_p$ in different reaction systems influence the yield of fragments, and result in isospin phenomena in reactions [22, 23]. In contrast, $\Delta \mu \equiv (\mu_n - \mu_p)$ can be determined from the yield of fragments. For examples, in the isoscaling and isobaric yield ratio (IYR) methods, $\Delta \mu / T$ ($T$ being the temperature) is considered as a probe to study the symmetry energy of nuclear matter in HICs [1, 21, 22, 24]. As discussed in previous works, $\Delta \mu / T$ can reflect the symmetry energy of both the colliding sources and the fragments using different scaling parameters. [8, 22, 27, 31, 32, 40, 41]. For convenience, $\Delta \mu / T$ is also called as the “symmetry energy” in this work. The $\Delta \mu / T$ determined by the isoscaling (IS–$\Delta \mu / T$) and isobaric yield ratio difference (IBD) methods (IB–$\Delta \mu / T$) have been compared in previous works [21, 27, 28]. Similar distributions of the IB–$\Delta \mu / T$ and IS–$\Delta \mu / T$ are found, which also reveals that $\Delta \mu / T$ depends on the neutron density ($\rho_n$) and the proton density ($\rho_p$) distributions, as well as the asymmetry of projectile.

In comparison with the fragments produced by intermediate projectile fragmentation in Ref. [28], which have relative small masses, we will analyze the fragments produced by the $^{124,136}$Xe and $^{112,124}$Sn projectile fragmentation, which have larger masses and neutron-excess. This paper is organized as follows. In Sec. II the isobaric and isoscaling methods to determine $\Delta \mu / T$ are introduced; in Sec. III $\Delta \mu / T$ obtained from the studies of fragments in the $^{124,136}$Xe and $^{112,124}$Sn reactions are reported and discussed; In Sec. IV a summary is presented.

II. MODEL DESCRIPTION

The IBD method has previously been described in Ref. [28]. To illustrate the results more clearly, we briefly introduce the isoscaling and IBD methods based on the grand-canonical ensembles theory. The yield of a fragment with mass $A$ and neutron-excess $I$ ($I = N - Z$) in the grand-canonical limit is given by [43, 44].

$$Y(A, I) = C_0 A^\tau \exp\{[-F(A, I, T) + \mu_n N + \mu_p Z]/T]\}$$

(1)

where $C_0$ is a constant; $\mu_n$ and $\mu_p$ are the chemical potential of neutron and proton, and $N$ and $Z$ are the neutron and proton numbers. $\tau$ is nonuniform in different reaction systems [45]. $F(A, I, T)$ is the free energy of the cluster (fragment), and $T$ is the temperature.

The isoscaling phenomena are shown in the isotopic and isotonic ratios between two reactions of similar measurements. From Eq. (1), the yield ratio of one fragment

* Email: machunwang@126.com
between two reactions, $R_{12}^{IS}(N, Z)$ (1 and 2 denoting the reactions, usually 2 denoting the neutron-rich reaction system), can be defined as,

$$R_{12}^{IS}(N, Z) = Y_2(N, Z)/Y_1(N, Z) = C' \exp(\alpha N + \beta Z), \tag{2}$$

Taking the logarithm of Eq. (2), one can obtain,

$$\ln R_{12}^{IS}(N, Z) = C(\alpha N + \beta Z), \tag{3}$$

where $C'$ and $C$ are overall normalization constants which originate from the property of reaction system. $\mu_n$ and $\mu_p$ are assumed to change very slowly, and can thus be seen as the same in one reaction; $\alpha = \Delta \mu_n/T$ with $\Delta \mu_n = \mu_{n2} - \mu_{n1}$, and $\beta = \Delta \mu_p/T$ with $\Delta \mu_p = \mu_{p2} - \mu_{p1}$, which reflect the properties of colliding source. In the isotopic ratio, $\beta$ cancels out and $\alpha$ can be fitted; and in the isotonic ratio, $\alpha$ cancels out and $\beta$ can be fitted. $\alpha$ can be related to the symmetry energy ($C_{\text{sym}}$) in nuclear mass of colliding source by different scaling parameters.

For the IBD method, first, the IYR between isobars differing by 2 units in $I$, $R_{12}^{IB}(I + 2, I, A)$, can be defined in single reaction as,

$$R_{12}^{IB}(I + 2, I, A) = Y(A, I + 2)/Y(A, I) = \exp\{F(I + 2, A, T) - F(I, A, T) + \mu_n - \mu_p\}/T, \tag{4}$$

Unlike in the isoscaling method, the $C_0A^\nu$ term in Eq. (1) cancels out and the system dependence is removed in single reaction. Assuming that the isobars involved in the ratio have the same temperature, only the retained $\mu_n$ and $\mu_p$ are related to the colliding sources. Taking the logarithm of Eq. (1), one obtains,

$$\ln R_{12}^{IB}(I + 2, I, A) = (\Delta F + \Delta \mu)/T, \tag{5}$$

where $\Delta F = F(I + 2, A, T) - F(I, A, T)$, and $\Delta \mu = \mu_n - \mu_p$ as previously defined. Now we can define the difference between IYRs, which is labeled as the IBD, in two reactions as the follows,

$$\Delta \ln R_{12}^{IB} = \ln[R_{21}^{IB}(I + 2, I, A)] - \ln[R_{12}^{IB}(I + 2, I, A)] = [(\mu_{n2} - \mu_{n1}) - (\mu_{p2} - \mu_{p1})]/T = \Delta \mu_n/T - \Delta \mu_p/T = \alpha - \beta, \tag{6}$$

Eq. (6) reveals the relationship between the $\Delta \mu/T$ determined by isoscaling and IBD methods, i.e., IB–$\Delta \mu/T$ and IS–$\Delta \mu/T$ are linearly related as $\alpha - \beta$, and $\alpha$ and $\beta$ should equal the IS–$\Delta \mu/T$.

### III. RESULTS AND DISCUSSIONS

In this paper, the yields of fragments in the measured $1.4$ GeV $^{124}$Sn + $^{124}$Sn, $^{112}$Sn + $^{112}$Sn reactions [49], and $1.4$ GeV $^{124,136}$Xe + Pb [43] will be analyzed using the isoscaling and IBD methods described above. First, the isoscaling phenomena in these reactions will be illustrated. The isotopic and isotonic ratios in the Sn and Xe reactions will also be analyzed to extract the isoscaling parameters $\alpha$ and $\beta$ according to Eq. (3). In Fig. 4, the isotopic and isotonic ratios between the $^{124}$Sn + $^{124}$Sn and $^{112}$Sn + $^{112}$Sn reactions are plotted, in which $Z$ changes from 16 to 31 and $N$ changes from 19 to 39. The different isotopes are shown by alternating full and open symbols for clarity. Quite good linear scalings are shown in the isotopic and isotonic ratios, which are denoted as lines from the linear fitting. In Fig. 4(b), the ratios of the isotones from $N = 33$ to 37 show different trends to those of other $N$–isotones, at the same time, less isotones are measured than the other isotones.

In Fig. 2 the isotopic and isotonic ratios between the $1.4$ GeV $^{136}$Xe + Pb and $^{124}$Xe + Pb reactions are plotted, in which $Z$ changes from 10 to 52 and $N$ changes from 12 to 66. Much more fragments in the Xe reactions are measured than the Sn reactions. The ratios of the $Z = 20$ and 21 isotopes show a little different trend to those of other $Z$-isotopes, at the same time, less isotopes are measured than other $Z$–isotopes.

In Fig. 3 the obtained isoscaling parameters $\alpha$ and $\beta$ are plotted. The values of $\alpha$ form a plateau around 0.4 in a wide range of $Z$ when $Z < 40$ both in the Sn and Xe reactions. When $Z > 40$, $\alpha$ increases with $Z$ in the Xe reactions. In particular, there is a minimum value around $Z = 20$ in the Xe reactions. For isotones, $\beta$ shows similar trend except the values are negative. When $N < 40$, $\beta$ forms a plateau around -0.45, and then decreases with increasing $N$. The $\beta$ distribution in the Sn reactions is not as smooth as that in the Xe reactions. The $\alpha$–plateau in the $25A$ MeV $^{86}$Kr + $^{124,112}$Sn reactions is around 0.4 [32], which coincides the result in this work. A decreasing of $\alpha$ is observed, however, when $Z > 30$, which is not shown due to the absence of data in this work. $\alpha = 0.36$ and $\beta = -0.4$ are obtained in the measured central $^{112}$Sn + $^{112}$Sn and $^{124}$Sn + $^{124}$Sn collisions at an incident energy of 50A MeV [31]. The $\alpha$ and $\beta$ values are also coincident with the results of the Sn reactions in this work.

Generally, $-\beta \approx \alpha$ is assumed [21, 47]. For one fragment, specific $\alpha$ and $\beta$ can be obtained from its $Z$-isotopes and $N$-isotones, respectively, and the correlation between $\alpha$ and $\beta$ can be obtained. The correlation between $-\beta$ and $\alpha$ in Sn and Xe reactions is plotted in Fig. 3. The $-\beta$ and $\alpha$ correlation in Xe reactions is fitted using a linear function, which reads $y = 1.08x + 0.06$ and satisfies the approximation $-\beta \approx \alpha$. For the Sn reactions, $-\beta$ does not depend on $\alpha$ linearly, thus the correlation is not fitted. Second, the IS–$\Delta \mu/T$ results will be compared. From Eq. (6), IS–$\Delta \mu/T$ should equal to $-\beta$. The value of IB–$\Delta \mu/T$ is calculated from IYRs according to Eq. (4). The values of $\Delta \mu/T$ are plotted in Fig. 5. Trends of $\Delta \mu/T$ distributions similar to the Ca and Ni reactions are found [28]. Generally, $\Delta \mu/T$ also shows the distribution of a plateau plus an increasing part. In most of the fragments, IB–$\Delta \mu/T$ agrees well with IS–$\Delta \mu/T$ in the Xe reactions. In the Sn reactions, IB–$\Delta \mu/T$ only
FIG. 1. (Color online) Isoscaling phenomena between fragments in the 1A GeV $^{124}$Sn + $^{124}$Sn and $^{112}$Sn + $^{112}$Sn reactions [46]. From left to right, in (a) the plotted are isotopic ratio of isotopes with $Z$ increase from 16 to 31, and (b) isotonic ratio of isotones with $N$ from 19 to 39. The linear fits to the isoscalings are shown as lines.

FIG. 2. (Color online) Isoscaling phenomena in the fragment ratios between the 1A GeV $^{136}$Xe + Pb and $^{124}$Xe + Pb reactions [47]. From left to right, in (a) the atomic numbers $Z$ increase from 10 to 52, and in (b) the neutron numbers $N$ increase from 12 to 66. The linear fits to the isoscalings are shown as lines.

agrees with IS–$\Delta \mu/T$ in the $I=1$ and 2 fragments, but a relative large difference between them are shown in the $I=3$ and 4 fragments. The plateaus form around $\Delta \mu/T = 0.8$, as shown by the guiding lines. The $\Delta \mu/T$ plateaus in the $^{48}$Ca/$^{40}$Ca, $^{64}$Ni/$^{58}$Ni, $^{58}$Ni/$^{40}$Ca, and $^{48}$Ca/$^{54}$Ni reactions are around 2.0, 1.4, 0.7, and 0.5 [28]. The height of the plateau is explained as being due to the difference between the $\rho_n$ distributions, as well as the differences between the $\rho_p$ distributions in the core of the projectiles. The $\Delta \mu/T$ in the Sn and Xe reactions are
close to that of the $^{58}$Ni/$^{40}$Ca reactions, which indicates that in the core of $^{124}$Sn/$^{112}$Sn and $^{136}$Xe/$^{124}$Xe, the difference between the $p_n$ and $p_p$ distributions is similar to those between the cores of $^{58}$Ni/$^{40}$Ca.

Finally, we discuss the possible shell effects in IB–$\Delta \mu /T$ shown in Fig. 5 which are not obvious in the Ca and Ni reactions [28]. In a previous study, the shell effect is also found in the symmetry-energy coefficient ($a_{sym}$) of nucleus [13], which results in sudden changes in $a_{sym}$. Generally, $\mu_n$, $\mu_p$ and the resultant $\Delta \mu$ should vary very little, and there should be no nuclear structure effects in the colliding process near the critical point temperature [20]. In actual reactions, however, the primary fragment are greatly influenced by the secondary decay, and the nuclear structure effects can be manifested after the decay process. A relative low temperature ($T \sim 2 MeV$) is found for fragments with large mass using an isobaric ratio method [20]. At this low temperature, the shell effects may be obvious in $\Delta \mu /T$. It is clearly observed that around the magic number $Z = 20$, the isotopic ratios in the Xe reactions show different trends to the other isotopes. The same also happens in the $N = 20$ and $\sim 30$ isotopic ratios, which makes $\alpha$ and $\beta$ change suddenly, thus result in large gaps in $\Delta \mu /T$ of fragments with a magic number. IB–$\Delta \mu /T$, as has been noted, is more sensitive to the variation of symmetry energy than IS–$\Delta \mu /T$ since the IBD method uses only two isobars [28]. Taking IB–$\Delta \mu /T$ in the Xe reactions as an example, most of the IB–$\Delta \mu /T$ can be divided into different bands according to the shells. Some fragments with a magic number (or near the magic number) are labeled as $(Z,N)$ in Fig. 5. Large gaps are shown between the IB–$\Delta \mu /T$ near the closed-shell fragment and its neighbor, for example, the $I = 1$ fragments (18, 19) and (19, 20) in (b); the $I = 2$ fragments (18, 20) and (19, 21) in (c); the $I = 3$ fragments (25, 28) and (26, 29) in (d). The shell effects of $N = 28$ are not obvious when $I \geq 3$, and the same happens in $Z = 28$ fragment. The violent behavior is also observed in IB–$\Delta \mu /T$ of the neutron-rich $I = 6$ and $I = 7$ fragments within the range of $Z = 26 \sim 30$.

IV. SUMMARY

In this paper, the isoscaling and the IBD methods are used to study the $\Delta \mu /T$ by analyzing the fragment yield in the measured 1 A GeV $^{124}$Sn + $^{124}$Sn, $^{112}$Sn + $^{112}$Sn, $^{136}$Xe + Pb and $^{124}$Xe + Pb reactions. First, the isoscaling phenomena in the $^{124}$Sn/$^{112}$Sn and $^{136}$Xe/$^{124}$Xe reactions are analyzed, and the isoscaling parameter $\alpha$ and $\beta$ are obtained. $\alpha$ ($\beta$) is found to be almost constant in Sn reactions. The $\alpha$ ($\beta$) also changes very little in Xe reactions when $Z < 40$ ($N < 40$), but $\alpha$ ($\beta$) increases (decreases) when $Z > 40$ ($N > 40$). The $\alpha \sim -\beta$ approximation is satisfied in Xe reactions but not satisfied in Sn reactions.

The results of IS–$\Delta \mu /T$ and IB–$\Delta \mu /T$ in Sn and Xe reactions are compared. In most of the fragments, the IS–$\Delta \mu /T$ is consistent in Xe reactions, while they are only similar in the $I = 1$ and 2 fragments in Sn reactions. The values of the $\Delta \mu /T$ plateaus indicate that the difference between $p_n$ and $p_p$ distributions in the core of $^{136}$Xe/$^{124}$Xe and $^{124}$Sn/$^{112}$Sn is similar as that of the $^{58}$Ni/$^{40}$Ca.

The possible shell effects in the IB–$\Delta \mu /T$ of fragments are also discussed. The IB–$\Delta \mu /T$ in the Xe reactions can be divided into different bands according to different shells. The $Z = 20$ and $N = 28$ shell effects are obvious in IB–$\Delta \mu /T$ when the fragments are not very neutron-rich.
FIG. 5. (Color online) The values of IS– and IB– $\Delta \mu / T$ determined from the measured fragments ratios in Sn and Xe reactions. The numbers in brackets label the protons and neutrons of the nucleus $(Z, N)$ (see the text for explanation). The green circles represent the fragments having a magic number. The lines are just for guiding the eyes.

ACKNOWLEDGEMENTS

This work is supported by the National Natural Science Foundation of China (Grant No. 1005017), Program for Science & Technology Innovation Talents in Universities of Henan Province (13HASTIT046), and the Young Teacher Project in Henan Normal University.

[1] Li B A, Chen L W, Ko C M, 2008 Phys. Rep. 464 113.
[2] Chen L W et al., 2007 Front. Phys. China 2 327.
[3] Natowitz J B, Röpke G, Typel S et al., 2010 Phys. Rev. Lett. 104 202501.
[4] Kumar S, Ma Y G, Zhang G Q et al., 2012 Rev. C 85 024620; 2011 *ibid.*, 84 044620; 2012 *ibid.*, 86 051601(R).
[5] Pu J, Chen J H, Kumar S et al., 2013 Phys. Rev. C 87 047603.
[6] Toro M D et al, 2008 Int. J. Mod. Phys. E 17 1799.
[7] Ma Y G, Wang K, Wei Y B et al., 2004 Phys. Rev. C 69 064610.
[8] Xu H S, Tsang M B, Liu T X 2000 et al., Phys. Rev. Lett. 85 716.
[9] Tsang M B et al., 2004 Phys. Rev. Lett. 92 062701; Famiano M A et al., 2006 Phys. Rev. Lett. 97 052701; Tsang M B et al., 2009 Phys. Rev. Lett. 102 122701.
[10] Colonna M, 2013 Phys. Rev. Lett. 110 042701.
[11] Gautam S, Sood A D, Puri R K, Aichelin J 2011 Phys. Rev. C 83 034606.
[12] Huang M, Chen Z, Kowalski S et al., 2010 Phys. Rev. C 81 044620.
[13] Ma C W, Yung J B, Yu M et al., 2012 Chin. Phys. Lett. 29 062101.
[14] Ma C W, Wang F, Ma Y G and Jin C, 2011 Phys. Rev. C 83 064620.
[15] Ma C W, Pu J, Wei H L et al., 2012 Eur. Phys. J. A 48 78.
[16] Ma C W, Pu J, Wang S S, and Wei H L 2012 Chin. Phys. Lett. 29 062101.
[17] Ma C W, Song H L, Pu J et al., 2013 Chin. Phys. C 37 024102.
[18] Chen L W, arXiv:1212.0284 [nucl-th].
[19] https://groups.nscl.msu.edu/hira/sep.htm
[20] Huang M, Bonasera A, Chen Z et al., 2011 Phys. Rev. C 81 044618.
[21] Marini P, Bonasera A, McIntosh A et al., 2012 Phys. Rev. C 85 034617.
[22] Fang D Q, Shen W Q, Feng J et al., 2000 Phys. Rev. C 61 044610.
[23] Lukyanov S, Mocko M, Andronenko L et al., 2009 Phys. Rev. C 80 014609.
[24] Ma C W, Wei H L, Wang J Y et al., 2009 Phys. Rev. C 79 034606.
[25] Ma C W, Wei H L, and Wang J Y, 2009 Chin. Phys. B 18 4781.
[26] Tsang M B, Friedman W A, Gelbke C K et al., 2001 Phys. Rev. Lett. 86 5023.
[27] Marini P, Bonasera A, Souliotis G A et al., 2013 Phys. Rev. C 87 024603.
[28] Ma C W, Wang S S, Zhang Y L, Wei H L, 2013 Phys. Rev. C 87 034618.
[29] Ma C W, Pu J, Ma Y G et al., 2012 Phys. Rev. C 86 054611; Ma C W, Zhao X L, Pu J et al., 2013 Phys. Rev. C 88 014609.
[30] Botvina A S, Lozhkin O V, and Trautmann W et al., 2002 Phys. Rev. C 65 044610.
[31] Das C B, Gupta S D, Lynch W G et al., 2005 Phys. Rep. 406 1.
[32] Souliotis G A, Shetty D V, Veselsky M et al., 2003 Phys. Rev. C 68 024605.
[33] Souliotis G A, Shetty D V, Keksis A et al., 2006 Phys. Rev. C 73 024606.
[34] P Zhou, W D Tian, Y G Ma et al., 2011 Phys. Rev. C 84 037605.
[35] Chen Z, Kowalski S, Huang M et al., 2010 Phys. Rev. C 81 064613.
[36] A. Ono, P. Danielewicz, W. A. Friedman et al., 2003 Phys. Rev. C 68 051601(R); 2004 ibid, 70 041604(R).
[37] Souza S R, Tsang M B, Carlson B V et al., 2009 Phys. Rev. C 80 044606.
[38] Huang M, Chen Z, Kowalski S et al., 2011 Nucl. Phys. A 847 233.
[39] Li B A, and Chen L W 2006 Phys. Rev. C 74 034610.
[40] Fang D Q, Ma Y G, Zhong C et al., 2007 J. Phys. G: Nucl. Part. Phys. 34 2173.
[41] Fu Y, Fang D Q, Ma Y Get al., 2009 Chin. Phys. Lett. 26 082503.
[42] Mallik S and Chaudhuri G, 2013 Phys. Rev. C 87 011602(R).
[43] Das C B, Gupta S D, Liu X D, and Tsang M B 2001 Phys. Rev. C 64 044608.
[44] Tsang M B, Lynch W G, Friedman W A et al., 2007 Phys. Rev. C 76 041302(R).
[45] Huang M, Wada R, Chen Z et al., 2010 Phys. Rev. C 82 054602(R).
[46] Föhr V, Bacquias A, Casarejos E et al., 2011 Phys. Rev. C 84 054605.
[47] Henzl Hr F, Schmidt H K, Ricciardi M V et al., 2008 Phys. Rev. C 78 044616.