Geometric scaling in leading neutron events at HERA

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This analysis provides new fits of the GBW model and the impact parameter-dependent saturation model (bSat or IP-Sat) to the leading neutron structure function HERA data in one pion exchange approximation. Both parametrizations of the dipole cross section provide good descriptions of the considered data. It is shown here for the first time that the experimental leading neutron production HERA data exhibits geometric scaling, which in this context means that the total $\gamma^p\pi^+$ cross section is a function of only one dimensionless variable $\tau = Q^2/Q^2_s(x)$. The geometric scaling region extends over a broad range of $Q^2$ and can be attributed to the presence of a saturation boundary which manifests at $Q^2 \geq Q^2_s$. The scaling behaviour in leading neutron events is profoundly similar to what has been observed for the inclusive DIS events.

I. INTRODUCTION

Over the last few decades, deep inelastic scattering (DIS) experiments have been essential in understanding the structure of the proton. The H1 and ZEUS experiments at HERA measured the proton structure function at an unprecedented precision to date. Mostly, this data is recorded in bins of the Bjorken variable $x$ and the photon virtuality $Q^2$ and available in a wide kinematic region reaching very small values of $x \sim 10^{-7}$. Though $x$ and $Q^2$ are independent variables, the total cross section $\sigma_{\text{tot}}^{\gamma^p}$ extracted from this inclusive data shows a striking feature that it depends only one dimensionless variable $\tau = Q^2/R^2_s(x)$ at low $x$. This was first observed by Staško, Golec-Biernat and Kwieciński in [1] and is commonly known as “geometric scaling”. This scaling behavior has a natural explanation in the dipole models with saturation for photon virtualities smaller than the saturation scale ($Q^2_s$) but for $Q^2 > Q^2_s$ this is not associated with saturation physics, rather this regularity exists in solution of evolution equations as demonstrated by Iancu, Itakura and McLerran for BFKL equations in [2] and in DGLAP equations with initial condition provided along the critical line $Q^2 = Q^2_s(x)$. The geometric scaling region extends as discussed in detail in [3]. Further in [4], the authors argued that the geometric scaling can be explained with generic boundary conditions for the standard DGLAP evolution. In general, the geometric scaling is expected to hold for $\ln Q^2/Q^2_s(x) << \ln Q^2_s(x)/\Lambda^2_{QCD}$, usually referred to as the extended geometric scaling regime. More detailed investigations of scaling behavior in inclusive DIS events are provided in [5]. In addition, the diffractive DIS data also exhibits similar scaling behavior [6].

In some of the DIS events, baryons carrying large fraction of the proton’s longitudinal momentum ($x_L > 0.3$) are produced in the far forward direction, commonly known as the leading baryons. These kind of events have been observed at HERA with leading neutrons, protons and photons [11-14]. Recently, the dipole framework has been extended to study the leading neutron events employing the one pion exchange (OPE) approximation [15-20]. In the dipole picture, the virtual photon emitted from the incoming electron splits into quark-antiquark pair forming a color dipole which then interacts with the target via strong interaction. In the case of leading neutrons, the color dipole interacts with the pion cloud of the proton, and the forward neutron comes from the proton as it splits into a neutron and a positive pion.

In our recent study [19], we showed using the saturated and non-saturated impact parameter dependent dipole models that the leading neutron data is insensitive to non-linear effects. This could be understood as the Bjorken $x$ value probed in this semi-inclusive measurement is considerably larger than the Bjorken $x$ in proton DIS events, where the latter has exhibited no clear signal for saturation. The next crucial step in this direction is to check whether the leading neutron events exhibit geometric scaling as observed in inclusive DIS events. This has not been tested thus far and is an important step from a phenomenological point of view. This paper aims at investigating the leading neutron events to find whether or not this regularity is observed in the experimental data. Here two well known parametrisation of the dipole models with saturation are used; the original Golec-Biernat and Wüsthoff (GBW) model [21] and the bSat (IP-Sat) model [22-23] which has an explicit DGLAP evolution. To obtain the saturation scale, new fits of the leading neutron structure function data are performed with both the models employing OPE.

The rest of the paper is organised as follows. In the next section, a brief outline of the leading neutron production in GBW and bSat models is given and the fitting procedure is discussed. In section 3, the fit results, the extracted saturation scale and the geometric scaling results are presented. In section 4, we summarise and discuss the main conclusions of our study.
II. LEADING NEUTRON PRODUCTION IN THE DIPOLE MODELS

The dipole framework is formulated in the target rest frame where the incoming electron emits a photon which splits into a quark antiquark pair forming a color dipole which subsequently interacts with the target strongly. For leading neutron production, the dipole probes the pion cloud of the proton where the virtual pion comes from the proton as it splits into a neutron and a pion as illustrated in Fig. 1. In the one-pion exchange approximation, at high energies, the differential cross section for $\gamma^*p \rightarrow Xn$ can be written as [16]:

$$\frac{d^2\sigma(W,Q^2,x_L,t)}{dx_L dt} = f_{\pi/p}(x_L,t) \sigma_{\gamma^*\pi^*}^{}(\hat{W}^2,Q^2)$$  \(1\)

where $t$ is the four-momentum transfer squared at the proton vertex, $x_L$ is the proton’s longitudinal momentum fraction taken by the neutron, $W$ is the centre-of-mass energy for the photon-proton system, $\hat{W}^2 = (1-x_L)W^2$, $f_{\pi/p}(x_L,t)$ is the flux of pions emitted by the proton and $\sigma_{\gamma^*\pi^*}^{}$ is the cross section of $\gamma^*\pi^*$ interactions. The $t$ variable is related to $p_T$, the transverse momentum of the neutron, and $x_L$ as:

$$t \simeq -\frac{p_T^2}{x_L} - (1-x_L) \left( \frac{m_n^2}{x_L} - m_p^2 \right)$$  \(2\)

where $m_n$ and $m_p$ are the masses of neutron and proton respectively. The leading neutron structure function is given by [11]:

$$F_2^{LN}(W,Q^2,x_L) = \Gamma(x_L,Q^2)F_2^p(W,Q^2,x_L)$$  \(3\)

Here $\Gamma(x_L,Q^2) = \int_{t_{\text{min}}}^{t_{\text{max}}} f_{\pi/p}(x_L,t) dt$ is the pion flux factor integrated over the $t$-region of the measurement and $F_2^p(W,Q^2,x_L) = \frac{Q^2}{4\pi^2\alpha_{\text{EM}}} \sigma_{\gamma^*\pi^*}^{}(\hat{W}^2,Q^2)$ is the pion structure function.

A. The pion flux

The flux factor $f_{\pi/p}(x_L,t)$ describes the splitting of a proton into a $\pi n$ system. Following [15][19], the flux factor given by:

$$f_{\pi/p}(x_L,t) = \frac{1}{4\pi} \frac{2g_{pp}^2}{4\pi \left(m_n^2 + |t|\right)^2} (1-x_L)^{1-2\alpha(t)} |F(x_L,t)|^2$$  \(4\)

where $m_\pi$ is the pion mass, $g_{pp}^2/(4\pi) = 14.4$ is the $\pi^0 pp$ coupling. $F(x_L,t)$ is the form factor which accounts for the finite size of the vertex. The form factor is given as:

$$F(x_L,t) = \exp \left[ -R^2 |t| + m_n^2 (1-x_L) \right], \alpha(t) = 0$$  \(5\)

where $R = 0.6$ GeV$^{-1}$ has been determined from HERA data [24].

B. GBW model

Using the optical theorem, the total $\gamma^*\pi^*$ cross section is given by the imaginary part of the forward elastic $\gamma^*\pi^* \rightarrow \gamma^*\pi^*$ amplitude as following:

$$\sigma_{\gamma^*\pi^*}^{}(\beta,Q^2) = \int d^2 r \int_0^1 \frac{dz}{4\pi} |\Psi_L,r(x_L,Q^2)|^2 I_{\gamma^*\pi^*}^{}(r,\beta)$$  \(6\)

where $\beta = \frac{Q^2 + m_n^2}{(1-x_L)W^2 + Q^2}$ is the scaled Bjorken variable for the photon-pion system. The photon wavefunctions are well known quantities calculated in [23] and $\sigma_{\gamma^*\pi^*}^{}(r,\beta)$ is the dipole-virtual pion cross section. The GBW model was proposed in [21], the first successful attempt to explain the inclusive HERA data in a saturation mechanism, the dipole cross section in GBW model given by [21]:

$$\sigma_{\gamma^*\pi^*}^{}(r,\beta) = \sigma_0 (1-e^{|Q_s|^2(\beta/\beta_0)^{\lambda}})$$  \(7\)

where the saturation scale $Q_s$ is defined as:

$$Q_s^2(\beta) = Q_s^2 (\beta/\beta_0)^{-\lambda}$$  \(8\)
with $Q_s^2 = 1 \text{ GeV}^2$. The above cross section has an important property of "geometric scaling" \cite{1}, which means that it depends only on one dimensionless variable $r Q_s$ or $\tau$ defined as:

$$\tau = Q^2 R^2_s(\beta) = \frac{Q^2}{Q_s^2(\beta)} \quad (9)$$

Here, there are three free parameters $\sigma_0, \lambda, x_0$, now either these can be fitted to the leading neutron structure function data or we could use the fit results of the inclusive proton data and make use of the assumption that the dipole-proton cross section and dipole-pion cross section are universal up to normalization at small $x$ \cite{19, 29–31}. This means

$$\sigma^*(r, \beta) = R_q \sigma^p(r, \beta) \quad (10)$$

where $R_q$ is determined by fit to the leading neutron structure function data and the dipole-proton cross section is taken from usual DIS fit of the GBW model from \cite{28}. Both these strategies are used in this analysis and the fit results are provided in Table I. For fit 1 of the GBW model, the parameters ($\sigma_0$, $\lambda$, and $x_0$) are fitted to the leading neutron structure function data, while for fit 2, these parameters are the same as determined from the usual inclusive DIS data and the assumption of hadronic universality at small $x$ between pions and protons (Eq. (10)) is used and the only parameter is $R_q$, which is determined from the fit. Though this assumption works well as shown in \cite{19, 29–31}, it does not yield a physical saturation scale other than the dilute limit of this cross section. Hence, we will use the saturation scale extracted from fit 1 of the GBW model.

### C. bSat model

Using the optical theorem, the total $\gamma^* \pi^*$ cross section is given by:

$$\sigma_{L,T}^{\gamma^* \pi^*}(\beta, Q^2) = \int d^2b \int_0^1 d^2r \int_0^{\beta \mu_0^2} d\zeta \left| \Psi_{L,T}(r, z, Q^2) \right|^2 \times \frac{d^2 \sigma(\pi^*)}{d^2 \mathbf{b}}(\mathbf{b}, r, \beta) \quad (11)$$

The bSat model contains DGLAP equation for evolution of gluon density for large scales and also has an explicit $b$-dependence. The dipole-pion cross section in the bSat model is given by \cite{22, 23}:

$$\frac{d^2 \sigma(\pi^*)}{d^2 \mathbf{b}}(\mathbf{b}, r, \beta) = 2 \left[ 1 - \exp \left( - F(\beta, r^2) T_p(\mathbf{b}) \right) \right] \quad (12)$$

with

$$F(\beta, r^2) = \frac{\pi^2}{2N_C} r^2 \alpha_s(\mu_0^2) \beta g(\beta, \mu_0^2) \quad (13)$$

The dipole cross section saturates for large dipole sizes $r$ and for large gluon densities. The scale at which the strong coupling $\alpha_s$ and gluon density is evaluated is $\mu_0^2 = \mu_0^2 + \zeta \tau$ and the gluon density at the initial scale $\mu_0$ is parametrised as:

$$\beta g(\beta, \mu_0^2) = A_g \beta^{-\lambda_s}(1 - \beta)^6$$

The transverse profile of the pion is assumed to be Gaussian with $T_p(\mathbf{b}) = \frac{1}{2 \pi \tau \beta^2} \exp \left( - \frac{b^2}{2 \tau \beta^2} \right)$ where $\tau \beta$ is the width of the pion. There is no available data which constrains this parameter. The width of pion is chosen to be $\tau \beta = 2 \text{ GeV}^{-2}$, motivated from the Belle measurements \cite{32, 33} of hadron-pair production in a two-photon process $\gamma^* \gamma \rightarrow \pi^0 \pi^0$ where it was found

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
GBW & $\sigma_0$ [mb] & $\lambda$ & $x_0/10^{-4}$ & $R_q$ & $\chi^2/N_{\text{dof}}$ \\
\hline
Fit 1 & $17.171 \pm 2.777$ & 0.223 & $0.018$ & 0.036 & $63.26/48 = 1.32$ \\
Fit 2 & 27.43 & 0.248 & 0.40 & 0.438 & $64.52/50 = 1.29$ \\
\hline
bSat & $A_g$ & $\lambda_g$ & $C$ & $R_q$ & $\chi^2/N_{\text{dof}}$ \\
\hline
Fit 3 & $1.208 \pm 0.012$ & 0.0600 & $0.038$ & 1.453 & $58.75/48 = 1.22$ \\
Fit 4 & 2.195 & 0.0829 & 2.289 & 0.520 & $66.19/50 = 1.32$ \\
\hline
\end{tabular}
\caption{Fit results of the GBW model and the bSat model to the leading neutron structure function HERA data for $\beta \leq 0.01$, with $N_p = 51$ points for $x_{L_{\text{min}}} = 0.6$. Quark masses are fixed and as given in \cite{25}.}
\end{table}
to be $B_\pi = 1.33 - 1.96 \text{ GeV}^{-2}$ \cite{34}. Also, H1 measurements \cite{35} of the $t$ spectrum for exclusive $\rho$ photoproduction with leading neutrons in $ep$ scattering suggests $B_\pi = 2.3 \text{ GeV}^{-2}$. For more detailed discussion on probing the transverse width of pion experimentally see \cite{19}. Similar to the GBW model, we follow both methods; first performing an independent fit of the gluon density parameters $A_g$, $\lambda_g$ and $C$ to the leading neutron structure function data and secondly making use of the assumption:

$$d\sigma_{q\bar{q}}^{(\pi)}(b, r, \beta) = R_g d\sigma_{q\bar{q}}^{(p)}(b, r, \beta)$$ \hspace{0.5cm} (14)

where only $R_g$ is fitted to the leading neutron structure function data and the dipole-proton cross section is taken from the fit of the bSat model to the usual DIS data from \cite{25}. Again, the saturation scale is extracted from the fit where $A_g$, $\lambda_g$ and $C$ are fitted to the leading neutron structure function data. The saturation scale in this case is given by \cite{22}:

$$Q_s^2(\beta, b) = 2/r_S^2$$ \hspace{0.5cm} (15)

where $r_S$ is defined by solving $1/2 = F(\beta, r_S^2)T_p(b)$ in the dipole amplitude.

III. RESULTS

In Table I the fit results with the GBW and bSat models are shown. Fit 1 corresponds to the fit where the GBW model parameters ($\sigma_0$, $\lambda$, and $x_0$) are fitted to the leading neutron structure function data, while for fit 2, these parameters are the same as determined from the usual inclusive DIS data and the assumption of hadronic universality at small $x$ between pion and proton (Eq.(10)) is used and the only parameter is $R_q$, which is determined from the fit. Similarly, fit 3 corresponds to the fit where the gluon density parameters in the bSat model $A_g$, $\lambda_g$ and $C$ are fitted to the leading neutron structure function data, while for fit 4, these are the same as determined from the usual inclusive DIS data and the assumption of hadronic universality at small $x$ between pions and protons (Eq.(14)) is used where the only parameter $R_q$ is determined from the fit. The fits are performed using the MINUIT2 package \cite{36} and the corresponding $\chi^2/N_{\text{dof}}$ are shown, where $N_{\text{dof}} = N_p - (# \text{ of parameters})$ and $N_p$ is number of data points in the fit. We see that the GBW model in its original form, in addition to inclusive DIS data, also provide a good description of the leading neutron data for both the scenarios. The bSat model fit having an explicit DGLAP evolution provides the best description of the leading neutron data in all the fits and prefers a slower evolution of the gluon density, as well as almost half the number of gluons as compared to the inclusive DIS case as illustrated in Fit 3. For bSat, fit 4 with hadronic universality assumption also provides a reasonable description of the data.

In Fig. 2 the sensitivity of the fit 1 quality to the choice of the minimal value of the proton’s longitudinal momentum fraction taken by the neutron, $x_{L_{\text{min}}}$, in the data is shown. The blue solid curve represents the variation of $\chi^2/N_{\text{dof}}$ with the cutoff $x_{L_{\text{min}}}$ keeping the other parameters ($\sigma_0$, $\lambda$, and $x_0$) fixed from fit 1, while the red dashed curve shows the behaviour of $\chi^2/N_{\text{dof}}$ where all the parameters are kept free in the fit. We see that the choice $x_{L_{\text{min}}} = 0.6$ is optimal and the quality of the fit deteriorates as the $x_{L_{\text{min}}}$ is reduced further. This is not surprising as the one pion exchange approximation (OPE) holds good only for large momentum fractions carried by the neutron.

In Fig. 3 the saturation scale of the pion, $Q_s^2$, is plotted as a function of Bjorken $x$ at $x_L = 0.6$ in the left plot. The saturation scale in the GBW model is extracted from fit 1 as defined in Eq. (10) and from fit 3 for bSat model as defined in Eq. (15). The energy dependence of the sat-
uration scale of the pion is different in both the models, though both the models predict the saturation scale to be \( Q_s^2 \sim 1 \text{ GeV}^2 \) at \( x = 10^{-6} \). For reference, the ratio of the saturation scale of pions to that of the protons is also shown in the right plot. For the GBW model, we see that the saturation scale of the virtual pion probed in the leading neutron events is about half of the saturation scale of the proton for all \( x \) values, and the energy dependence is almost identical for both. For the bSat model, this is not the case and the saturation scale is half of the value of the proton’s saturation scale only at small \( x \). The evolution is considerably different for the two models. The small values of saturation scale of pions indicates that it is less sensitive to saturation and one needs to go to higher energies to probe non-linear effects in leading neutron events as compared to the usual DIS events.

In Fig. 4, the normalized dipole cross section \( \frac{\sigma}{\sigma_0} \) in the GBW model with the fit 1 parameters from Table I is plotted, evaluated at different scaled Bjorken variable \( \beta = 10^{-2}, 10^{-4}, 10^{-6} \). In the left plot, the dipole cross section is plotted as a function of dipole size \( r \) where we observe that the cross section saturates for large dipole sizes for all values of \( \beta \) as expected. In the right plot, the presence of geometric scaling in the GBW model is illustrated as all the curves from left plot with different \( \beta \) values merge into a single curve when the dipole cross section is plotted against the dimensionless variable \( r Q_s \) which shows that indeed the dipole cross section is a function of a single dimensionless variable rather than \( x \) and \( Q^2 \) independently. The geometric scaling is exact in the GBW model.

In Fig. 5, the dipole cross section \( \frac{d\sigma_q}{d^2b} \) in the bSat model is plotted, with parameters from the fit 3, eval-
The models have been fitted for kinematic region $x_L \geq 0.6$, where one pion exchange approximation holds good. The curves for both the GBW and the bSat model are practically on top of each other in the whole kinematic region.

In Fig. [4] the experimental data of the total cross section $\sigma^{\gamma^*}$ extracted from the leading neutron structure function, $F_2^{LN}$, employing the one pion exchange approximation is shown for $x_L > 0.6$ in the phase space defined by the photon virtuality in $6 < Q^2 < 100$ GeV$^2$ and the Bjorken $x$ values in $1.5 \cdot 10^{-4} < x < 3 \cdot 10^{-2}$. The total cross section $\sigma^{\gamma^*\pi^*}$ is plotted against the dimensionless variable $\tau = Q^2/Q_s^2(\beta)$ with saturation scale from Eq. [3] in the GBW model in the left plot and from Eq. [15] in the bSat model in the right plot. We observe that the experimental data exhibit geometric scaling behavior for the events with $x < 0.01$, $\beta < 0.1$ for the saturation scale values obtained from both GBW and bSat models. For the available data, the total cross section $\sigma^{\gamma^*\pi^*}$ shows the $1/\tau$ dependence at large $\tau$ which is very similar to what has been discovered for the total cross section $\sigma^{\gamma^*p}$ in usual DIS events. On the right plot, a few experimental data points at small values of $\tau$ are off from the scaling behaviour. This is due to the different mag-

$F_2^{LN}(\beta, Q^2, x_L)$

| $Q^2$ (GeV$^2$) | $x_L$ |
|----------------|-------|
| $7.3$          | $0.46$|
| $11$           | $0.55$|
| $16$           | $0.64$|
| $24$           | $0.73$|
| $37$           | $0.82$|
| $55$           | $0.91$|

FIG. 6. Comparison of the HERA data for leading neutron structure function $F_2^{LN}(\beta, Q^2, x_L)$ with the results from the GBW model (dashed black line) and the bSat model (solid blue line) with the parameters of Fit 1 and 3 respectively.
IV. CONCLUSIONS AND DISCUSSION

In this work, it is shown that the dipole model phenomenology can successfully describe the leading neutron DIS cross section with the new fits of the dipole model to the leading neutron structure function data. Using OPE, both the GBW and the bSat (IP-Sat) models provide a good description of the leading neutron data. Among all the fits, the fit with bSat model having an explicit DGLAP evolution describes the data best with a $\chi^2/N_{dof} = 1.22$, as depicted in Table I. The absorptive corrections are not included in this analysis as they are only dominant for small photon virtualities ($Q^2 < 6 \text{ GeV}^2$) which is outside the kinematic regime considered in this analysis and would just affect the normalization of the pion flux. In the first part, we have shown the presence of geometric scaling in the GBW model with the new fit parameters of the model from the leading neutron DIS. The scaling behaviour is not exact in the bSat model for small dipole sizes where the evolution effects play a dominant role, while for the large dipole sizes the model shows scaling behaviour similar to the scaling in the GBW model. The new fit results are used to extract the saturation scale of the pion which in general is about half of the proton’s saturation scale and the energy dependence of the pion saturation scale is identical to that of the proton in the GBW model. The main result of this study is presented in left plot of Fig. 7, where we have shown, using OPE, that the experimental data on $\sigma^{\gamma^* \pi^*}$ exhibits geometric scaling over an extended region of $Q^2$ and shows $1/\tau$ behaviour for large $\tau$ values. The presence of the scaling for $Q^2 > Q^2_s$ shows that the geometric scaling is more general than in the saturation model and can be attributed to the presence of a saturation boundary in the data which has its root in the evolution equations as described in detail in [2, 3]. This is profoundly similar to what has been previously seen in the $\sigma^{\gamma^* p}$ in inclusive...
DIS events. The presence of geometric scaling and similar behaviour of $\sigma^c\sigma^c$ as a function of $\tau$ hints toward the universality of small $x$ structure between pions and protons, though we need more experimental data at smaller $\beta$ values to further validate this statement. Future colliders such as EIC [26] and FCC or HeC [39] have the potential to probe this region of phase space and it will be interesting to see the differences and the similarities between inclusive DIS events and leading neutron DIS.

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