Some Aspects of Strange Matter In Astrophysics

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CERTIFICATE FROM THE SUPERVISOR

This is to certify that the thesis entitled "Some Aspects of Strange Matter in Astrophysics" submitted by Sri Shibaji Banerjee, who got his name registered on 16.11.2000 for the award of Ph.D. (Science) degree of Jadavpur University, is absolutely based upon his work under the supervision of Professor Sibaji Raha and that neither this thesis nor any part of it has been submitted for any degree/diploma or any other academic award anywhere before.

(Signature of the Supervisor & date with official seal.)
To my parents
Acknowledgements

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CHAPTER 1

Introduction and Overview

1.1 Outlook

The issues related to the origin, evolution, structure and constitution of the universe have attracted the attention of specialists and non-specialists for a very long time; however, the present epoch in the development of such theories is marked by a capacity to nail down everything to an astonishing precision. This has become possible largely due to the availability of today's observational and computational resources and it appears that at this time, the community has reached a consensus as to the queries related to the origin and composition of the universe. The major agreements (not yet really unanimous) belong mainly to three important areas, summarized in the following statements.

1. The Universe had its origin in a hot (and dense) big bang.
2. In the large scale the Universe appears to be flat, isotropic, homogeneous and expanding, possibly at an accelerated pace.

3. Most of the material content of the universe is carried by dark matter (i.e. non-luminous matter which marks its presence only through gravitational interaction.)

A large number of theories have been put forward to explain and interpret the above facts. Most of them seem to exist independently of the others, even when one takes into account constraints put on the individual theories. It appears that further experimental data are required before one is able to discern between them in an effective manner. In contrast to the many proposals on the composition of the Universe, which often border on exoticism, the present work lies entirely within the framework of the Standard Model of particle interactions.

The unifying theme of the current work is the fact that the STANDARD MODEL allows the existence of QUASIBARYONIC objects (hypothetical quark matter forms to which one can assign a baryon number, and in which strange quarks are an essential ingredient). This work focuses on the aspect of finding the relevance of such quasibaryonic objects in the issues related to the structure and composition of the universe in the light of modern astronomical observations.

The remaining part of this chapter is essentially a compact summary of the current ideas regarding the structure and composition of the universe along with an overview of the ideas leading to and linked with the strange (quark) matter hypothesis – how it can be correlated with the facts known (and being continually revealed) about the dark matter dominated universe. Specifically, I will try to outline the ways in which strange quark matter may form and
manifest itself, current searches for such matter and other related issues which lend perspective to the present work.

1.2 Bricks of the World

Not so long ago, the picture of the universe consisted of a single galaxy (the Milky Way) and was believed to be few million of years old. Today, we believe that the origin of everything can be traced back to the hot Big-Bang event that took place about 14 billion \((13 – 15 \times 10^9)\) years ago. The structure of the universe \([1]\) depends on the scale at which it is being observed: at scale sizes lower than \(\sim 100 \text{ Mpc}\) \(^i\) there exists hierarchical structures consisting of great *walls* of galaxies (accommodating more than 90% of the observed galaxies), super-clusters accommodating clusters of groups of galaxies, inter-spaced by *voids* or spaces devoid of any bright galaxies (Fig. 1.1), although at higher scales, all these structures appears to be replaced by a uniform homogeneous mass distribution. The questions regarding the structure and evolution of these objects appears to be intimately related.

Much of what is currently accepted as standard cosmological model \([2]\) can be traced back to the discovery of the cosmic microwave background radiation (CMBR) in 1964. This, together with the observed Hubble expansion of the universe, had established the hot big bang model as a viable model of the universe. The ultimate acceptance of the standard cosmological model was mostly due to the success of the theory of nucleosynthesis in reproducing the observed pattern of abundance of the light elements along with the proof of the black body character of the CMBR.

---

\(^i\)Mpc = 3.26 \(\times 10^6\) Light Years
1.2. BRICKS OF THE WORLD

Figure 1.1: Pie-diagram showing the hierarchical Structure of the Universe as revealed by the Las Campanas Redshift Survey [3]
For cosmic times larger than the Planck time $t_{P} = M_{Pl}^{-1} = 10^{-44}$ sec) gravitation can be described adequately by classical general relativity. The (Null) experimental evidence ([4]–[7]) regarding the anisotropy of the CBR (cosmic background radiation) allows one to assume the cosmological principle i.e the universe is homogeneous and isotropic (in the large scale). The four dimensional spacetime in the universe is then simply described by the Robertson - Walker metric [2]:

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \phi d\phi^2) \right)$$

where, $r, \theta, \phi$ are the co-moving polar coordinates which are fixed for objects that have no other motion other than the general expansion of the universe, $a(t)$ is the scale factor normalized to $a_0 = a(t_0) = 0$, $t_0$ is the present time and $k$ is the scalar curvature. The instantaneous physical radial distance is given by:

$$R(t) = a(t) \int_0^r \frac{dr}{\sqrt{1 - kr^2}}$$

and the physical velocity of an object (with no peculiar velocity with respect to the co-moving frame) then turns out to be

$$\vec{V} = \frac{\dot{a}}{a} \vec{R} \equiv H(t) \vec{R}$$

where $H(t)$ is the Hubble parameter. For a flat universe ($k = 0$) the relationship between the physical vectors $\vec{R}$ and the co-moving vectors $\vec{r}$ is simple revealed as $\vec{R} = a \vec{r}$. The radius of curvature is

\[\text{The Planck Mass is the mass value obtained by appropriately combining the fundamental constants } G, c \text{ and } \hbar \text{ to } \sqrt{\frac{\hbar c}{G}}. \text{ In Naturalized units, where } \hbar = c = 1, M_{Pl}^{-1} = \sqrt{c}. \text{ The Planck time is the time it takes light to travel a length equal to the Compton wavelength of a Planck mass particle, i.e } t_{Pl} = \sqrt{c}.\]
given by

$$R_{\text{curv}}^2 = \frac{H_0^{-2}}{\Omega_0 - 1}$$

where $H_0$ is the present value of the Hubble parameter ($H_0 = 100h \text{ Km S}^{-1} \text{ Mpc}^{-1}$ with $0.4 \leq h \leq 0.8$) and $\Omega_0$ is the ratio of the energy density $\rho_{\text{tot}}$ contributed by all forms of matter and energy to present value of the critical density $\rho_{\text{crit}}$. In other words,

$$\Omega_0 = \sum_i \Omega_i = \sum_i \frac{\rho_i}{\rho_{\text{crit}}} = \frac{\rho_{\text{tot}}}{\rho_{\text{crit}}}$$

where $\rho_i$ is the energy density contributed by the $i^{th}$ entity. The critical density, which appears in the above equation can be expressed in terms of the the constants $H_0$ and $G$ in the following form –

$$\rho_{\text{crit}} \equiv \frac{3H_0^2}{8\pi G} \simeq 1.88h^2 \times 10^{-29} \text{ g cm}^{-3}$$

and can as well be identified with the closure density. It follows that one can set the scalar curvature $k$ to zero if it is found that $\Omega_0 = 1$.

The value of $\Omega_0$ is most reliably estimated from the anisotropy of CBR (see [8, 9] and recent DASI observations [12]) (Fig.1.2) suggest that its magnitude can be quoted as $\Omega_0 = 1 \pm 0.2$. It is thus most likely the case that we really live in a flat universe. The universe in its early stage of evolution is temperature dominated and the temperature - time relationship in this era can be found from the Friedman equations (with $k = 0$) (assuming adiabatic universe evolution, i.e constant entropy in a co-moving volume):

$$T^2 = \frac{M_{\text{Pl}}}{2(8\pi c/3)^{1/2} \ell}$$

$\Omega_0 = 1$ is a requirement for the theory of Inflation[13]-[16]
1.2. BRICKS OF THE WORLD

Figure 1.2: The power spectrum of CMB, as obtained by recent experiments. The light curve is the one preferred by data and corresponds to \( \Omega_0 = 1 \) \[10\].

Figure 1.3: Hubble diagram based upon distances to supernovae of type 1a (SNe1a). The slope of the line is the presently accepted value of the Hubble constant \( H_0 = 64 \text{Kms}^{-1}\text{Mpc} \). \[11\]
and classically the beginning of time \( t = 0 \) coincides with \( T = \infty \). However, due to quantum effects, one can only say that the classical universe emerges at a cosmic time \( t \sim t_{Pl} \) with a temperature \( T \sim M_{Pl} \). The important events in the history of the universe are summarized in table 1.1.

| Time     | Temperature | Event                                                                 |
|----------|-------------|----------------------------------------------------------------------|
| \(10^{-37}\) sec | \(10^{16}\) GeV | Gut group \( G \) breaks down to the standard model gauge group \( G \rightarrow G_S = SU(3)_c \times SU(2)_L \times U(1)_Y \)       |
| \(10^{-10}\) sec | \(100\) GeV   | The Electroweak phase transition \( G_S \rightarrow SU(3)_c \times U(1)_{em} \)         |
| \(10^{-6}\) sec | \(100\) MeV   | QCD phase transition in which quarks became bound into hadrons     |
| 180 sec   | \(1\) MeV    | Nucleosynthesis, protons and neutrons begin to form nuclei           |
| 3000 year |              | Equidensity Point, matter begins to dominate over radiation.         |
| 200,000 year | \(3000\) K | Decoupling of matter and radiation and subsequent evolution of radiation as independent component. |
| \(2 \times 10^8\) year |         | Structure formation starts                                           |

Table 1.1: Events in the history of the Universe

The scope of the present work necessarily excludes the events that occurred before the cosmic Quark-Hadron phase transition but addresses the cosmic history from the subsequent time (i.e after \(10^{-6}\) sec) to the current time (i.e \(13 \times 10^9\) years).

### 1.2.1 Dark Matter

The total contribution to \( \Omega_0 \) can be split up into the matter and energy part

\[ \Omega_0 = \Omega_M + \Omega_E \]
1.2. BRICKS OF THE WORLD

(see Fig.1.4 - 1.5) The contribution of matter has been estimated in various ways: a short summary is given in table 1.2, the generally accepted value being $\Omega_M \simeq 0.4$. The apparent contradiction between the results $\Omega_0 \simeq 1$ and this value is attributed to a form of smooth dark energy component; the evidence for which is taken from the accelerated expansion of the universe, as shown by the Hubble diagrams (Fig.1.3) for several type Ia Supernovae (SNe Ia). The energy density contributed by the CBR and the massless neutrinos are too small to figure significantly in this part (energy sector) of the energy budget. Turning one's attention to the matter sector one finds that the big-bang nucleosynthesis can provide the most precise determination of the baryon density. Comparison of the primeval abundances of D, $^3\text{He}$, $^4\text{He}$ and $^7\text{Li}$ with their big-bang predictions defines an interval $\Omega_B h^2 = 0.007 - 0.024$ or $\Omega_B \approx 0.05 - 0.1$ [17, 18, 19, 20]. For the most part then, the baryons are unable to contribute significantly to the $\Omega_M$ and thus, most of the matter in the universe is dark.

There are actually two kinds of dark matter problems. The baryons carry only up to 5% of the total budget - it is however difficult to fulfill even this meager amount by collecting the baryons form all the visible stars (0.3 - 0.6 %) and hot intercluster gas (0.5 %). The nature of the remaining 90 % dark baryons is unknown: this is the first dark matter problem. In this work we are however concerned with the second problem in which one needs to account for the remaining 35 % contribution to the matter density (cold dark matter). The baryons that took part in nucleosynthesis are excluded from this sector and so it appears that the only possibilities are relic (non-baryonic) elementary particles left over from the big bang. One needs to postulate the existence of long lived or stable particles with very weak interactions so that their annihi-
1.2. BRICKS OF THE WORLD

Figure 1.4: A basic summary of the matter/energy content of the Universe. For a detailed account see fig. 1.5, below.

Figure 1.5: A standard summary of the composition of the Universe.
lations cease before their numbers are too small. The three most
discussed particles are a neutrino (of mass 30 eV) [8], an axion
[21] of mass $10^{-5\pm 1}$ eV and a neutralino [22] of mass between 50 -
500 GeV. These and the more exotic possibilities are invoked pri-
marily to cater to the viewpoint that CDM (Cold Dark Matter) must
be nonbaryonic (where the term nonbaryonic is usually taken as
synonymous with non participants in nucleosynthesis).

In the present work we present an alternative scenario for the
origin of dark matter. The most robust evidence for dark matter
is related to the nature of the rotation curve of spiral galaxies, in
which the velocities of some galactic component are plotted against
the distance from the galactic center. In practice one can obtain
the velocities of neutral hydrogen clouds using 21 cm emission.
The typical form of the curve appears as in Fig. [1.6]. The mass
distribution of the galaxy can be inferred from the Newton’s law
of circular motion $GM/r^2 = v^2/r$. The linear rise of $v(r)$ with $r$ near
$r = 0$ show that the mass density is essentially constant there. After
this brief spell, it is seen that the velocities remain constant out as
far as can be measured. This implies that the density drops like
$r^{-2}$ at large radius and that the mass $M(r) \propto r$ at large radii. Once
$r$ becomes larger than the extent of mass contributed by luminous
stars (The luminosity radii) the velocities should drop like $\propto r^{-1/2}$,
but this behaviour is not seen. Thus the spiral galaxies seem to
contain matter in its dark halo beyond its visible limit. Study of
the Milky Way Galaxy show that it’s rotation curves are consistent
with a flat rotation curve with $v = 220$ km /sec all way out to 50
kpc, so that it is a typical spiral galaxy with a large dark halo [24].

Recent experimental findings on the gravitational lenses in the
halo of our galaxy have lend support to this fact. Using the cue that
the Standard Model of particle interaction allows the existence of
1.3. A STRANGE UNIVERSE?

At the present moment, there is not enough experimental evidence to suggest that strangeness is a key ingredient for the matter present in the universe. Indeed, the local visible universe seems to be made entirely out of nuclear matter. The protons and neutrons (which form the majority of the baryons) readily form either tiny clumps of matter in the form of atomic nuclei or very large and ultradense conglomerates of neutron stars. There is a large "nuclear desert" in the middle mass range of Fig. 1.7, where no form of nuclear matter has been detected. All the more, as outlined above in Sec. 1.2, the visible universe consisting of visible stars and hot intercluster gas cannot even account for the full baryonic mass budget, which itself lends a very nominal quantity to the total matter density of the universe. The Standard Model of particle interactions is consistent

| Type of Analysis/Experiment                                      | Specific | Value of $\Omega_M$ |
|-----------------------------------------------------------------|----------|---------------------|
| Gas to total mass ratio in rich clusters                        | X Ray    | $0.3\pm0.05h^{-\frac{2}{3}}$ |
|                                                                | S-Z      | $0.25\pm0.1h^{-1}$   |
| Evolution of abundance of rich clusters with redshift           | 0.45±0.1 |
| Outflow of material from voids                                  | >0.3     |
| Evidence from structure formation                               | 0.4      |
| Power Spectrum                                                  | 0.4      |
| Mass to Light Ratio of Clusters                                 | 0.2±0.04 |

Table 1.2: Contribution of matter to $\Omega_0$

1.3 A Strange Universe?

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with the existence of new forms of matter in which stable entities can form from the combination of the third quark flavor (strange quark) with the two flavors (up and down) of quark matter found in ordinary matter. There are however, important structural differences between the two forms of matter. The difference lies in the content of the hadronic bags which hold the quarks free within the bag like enclosures but do not allow them to escape (Fig 1.8).

Experience suggests that it is unlikely to find stable bags of more than three up and down quarks. The deuteron, for example, exists in a stable configuration, comprising two distinct quark bags representing the proton and the neutron. If a quark bag capable of holding all these six quarks had a lower energy than deuteron, then the deuteron’s quarks would have spontaneously regrouped themselves in this state and matter as we know it would not exist. In 1971, A.R Bodmer [25] investigated the possibility of what might happen if strange quarks were added to the quark bag of up and down varieties and concluded that such forms of matter might exist as long-lived exotic forms of matter within compact stars, where they would be compressed much more than ordinary nuclei. S.A Chin and A.K. Kerman [26] and independently L.D McLerran and J.D. Bjorken [27] put forward some general arguments why strange matter should be stable, regardless of the possibility of the stabilization under pressure. Their main argument was that there would be no empty states to receive the down quarks that would result from the weak decay of the strange quark: this is the same principle that explains why the neutron is stable inside nuclei but decays into a proton in about 11 minutes outside the nuclei. In nuclei, the long range electrostatic repulsion between the protons will ultimately break it up, if the size grows beyond the attractive range of the short range internuclear forces. In contrast the differ-
ent quark flavors in a hadronic bag shares the energy equally, as a result of which the up, down and strange quarks come in almost equal numbers resulting in a (near) cancellation of charge.

The multi-quark hadronic bags are thus not subject to the size restrictions which are imposed on a ordinary nucleus and can easily fill in the range of sizes between the nucleus and the neutron star. The possibility of a strange universe, therefore, cannot be ruled out. As outlined below, the strangeness can occur at various scales, from forming heavier than usual isotopes of common elements, to larger strange ‘nuggets,’ compact stars composed largely (or completely) of strange matter, to entire ‘dark galaxies’.

The primary motivation for this work is therefore the exploration of the possible astrophysical consequences of the occurrence of this other kind of matter and its (close) encounter with the regular nuclear matter objects, on this planet, as well as elsewhere in the Universe.

1.3.1 Strange Quark Matter or quasibaryons

Our usual experience with ordinary matter, suggests that the various forms of nuclear matter (nucleons, hyperons and baryons) show great stability. Such pure baryonic matter in the universe appears to be composed entirely of up (u) and down (d) quarks and held in stable configuration by strong interactions (nuclear forces). The most stable chemical element with the least energy per baryon ($^{56}$Fe) is therefore the natural choice for the ground state of such forms of matter. In the absence of any other form of matter in which strong interactions play the key role, this ground state can also be thought to be synonymous with the ground state of QCD (Quantum Chromo Dynamics), since QCD is the framework for the theory of strong interactions. In a seminal paper in 1984, how-
ever, Edward Witten [28] put forward a conjecture that a system of $3A$ up, down, and strange quarks with the number of $u$, $d$ and $s$ quarks roughly same, can have a lower energy per baryon compared to normal nuclear matter objects with mass number $A$. This form of quasibaryonic matter is known in the literature as Strange Quark matter (SQM). The term quasibaryonic was coined by ourselves [29] to imply that the quarks in this case would not form individual baryons, but would have wave functions ranging over the entire size of the system. It would still be possible to associate mass numbers with these objects (Color must still be confined, so it is still possible to talk about baryon number when discussing such a system) in the sense that a SQM blob of mass number $A$ is actually a system of $3A$ quarks kept in a color neutral configuration. This is also a system in which strong interactions play the dominant role and would therefore represent a new ground state of matter.

The above statement about the existence of an hitherto unknown form of matter is known in the literature as the STRANGE MATTER hypothesis. This assumption lies at the very foundation of the current work. The hypothesis illustrates that SQM, rather than nuclear matter, can very well represent the true ground state of strongly interacting matter.

1.3.2 Properties of quasibaryons or SQM

In ordinary circumstances, normal nuclear matter does not decay to this true QCD ground state because this would necessary require very high order weak processes in order to pass down to the novel state, as strange quarks must be generated in abundance from the $u$ and $d$ quarks; the timescale for the process being larger than the age of the universe [28]. On the other hand, SQM will
1.3. A STRANGE UNIVERSE?

readily absorb neutrons, since their stability is enhanced as they acquire larger mass numbers. They can therefore be distinguished from other forms of matter in their exceptional stability and an insatiable appetite for neutrons. It has been found efficient to divide the spectrum of strange matter into 3 categories arising mainly from size considerations [30].

1. **Bulk Strange Matter**: Bulk strange matter is sufficiently large \((A \sim 10^{44} \text{ or higher})\) and surface effects are usually small so that they may be disregarded in the first approximation. This is a system of free Fermi gas of 3A u, d and s quarks held in a quark bag which separates the collection from the vacuum by a phase boundary. The system has been found to be stable at zero temperature and pressure. The presence of the strange matter is an essential ingredient, as a system of u and d quark matter is known to be energetically unstable relative to the nuclear matter. It is only the presence of an extra third Fermi well (Fig.1.9) that can actually reduce the energy of a three flavor system relative to a two flavor system. Ideally, the system is large enough so that one may not consider the surface effects at all. Bulk strange matter is also electrically neutral as the number of the quark flavors are identical and the charge cancellations are near perfect \((\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s \text{ vanishes when } n_u = n_d = n_s)\). The charge cancellations in the above sense are actually perfect in the case when masses of the three quark varieties are equal, even when they are not, electrons ensure local charge neutrality (since the strange quark is heavier than the up and down varieties, there will be a net positive charge in the absence of electrons). Weak interactions maintain equi-
librium in the system through flavor conversions like:

\[
\begin{align*}
    d & \leftrightarrow u + e^- + \bar{\nu}_e \\
    s & \leftrightarrow u + e^- + \bar{\nu}_e \\
    u + s & \leftrightarrow d + u
\end{align*}
\]

The neutrinos generated in such reactions leave the system and are not ascribed any chemical potential. The model of bulk strange matter is useful because it can describe the naturally occurring quasibaryonic systems like strange stars and provides a limit which must hold for any model of SQM when the dimensions of the system tends to infinity. The model contains three free parameters, the Bag Constant $B$ representing an external pressure that keeps the system bound, the mass of the strange quark $m_S$ and the strong coupling $\alpha_S$. The Bag constant $B$ is essentially identical to the $B$ used in the MIT Bag model and is a parametrization of the long range QCD confinement force, being the difference between the perturbative and the nonperturbative vacua. The overall conclusion of such strange matter models is that bulk strange matter is stable over a certain region in the three dimensional parameter space $(B, m_S, \alpha_S)$. Strange matter can actually exist in a stable configuration if the values allowed by the model are consistent with the real world. The real-world values of the parameters are obtained from bag model fits to light hadron spectra. The renormalization point of the bag models is unknown, so neither $\alpha_S$ nor $m_S$ can be meaningfully compared. While it is not possible to compare the 'windows of stability' for strange matter to known values of the parameters $\alpha_S, m_S$ and $B$, the windows are quite large. It is therefore extremely
likely that bulk strange quark matter is bound and stable.

2. **Medium sized strange nuggets:** These nuggets have $A \leq 10^7$ and the models used to describe such objects are more detailed and take finite size effects into accounts. Strange quark matter in the medium range is still large enough to be treated as a Fermi gas, but small enough that effects relating to its finite size must be considered. The radius of such a strangelet is of the order of few 100’s of fm, which is less than the Compton wavelength of an electron. As a consequence, unlike bulk strange matter, electrons will not be found within strangelets and the strangelet therefore acquires a net positive charge. The electrons will now be found ‘orbiting’ the strangelet as in an atom. As a result, coulomb forces within the strangelet may no longer be neglected.

3. **Very small strangelets:** These resemble the isotopes of super-heavy elements in their mass. The study of such small strangelets mainly arose from the desire to explore a form of strange matter that can arise out from the collisions of the heaviest nuclei at the highest attainable energies. The models of bulk strange matter with surface effects are not suitable for their description, as one must take into account the possibility of formation of shell structure in their energy levels. The model begins by filling the energy levels in a bag, one quark at a time, minimizing the energy each time with respect to flavor. The bag radius is adjusted to balance a constant external pressure $B$. As the quarks are non-interacting, it is possible to ignore the perturbative QCD corrections. Ignoring coulomb interactions is also acceptable, since $Z$ will typically be very small for these small $A$ systems. This approach yields a relation-
ship between the energy per baryon (number) vs the baryon number, and reveals a shell structure analogous to nuclear structure. At first the quark bag begins to get filled by massless non-strange quarks. Soon the Fermi energy of the system becomes large enough compared to the strange quark mass so that it becomes energetically favorable to add massive strange quarks. At some future point the system will again become favorable for the acceptance of the non-strange quarks and the differential variation of the strange and non-strange levels with the radius of the bag produces level crossings at some $A$ where the strange quarks transform into strange quarks. There are also some values of $A$ where the $\varepsilon$ drops rapidly: at these points the decrease in energy from emitting a baryon is insufficient to offset the energy needed to climb out of the dip. The increased stability is a signature of the shell closure at these values of $A$. It is, in fact, possible to identify a sequence of magic numbers (See Fig.1.10) at $A = 6, 18, 24, 42, 54, 60, 84, 102$ etc for low $s$ quark masses, something strongly reminiscent of nuclear physics. For larger $s$ quark masses, it becomes more favorable to use $u$ and $d$ quarks instead of strange quarks, and these magic numbers change. The energy per baryon approaches the bulk limit when $A \to \infty$.

The vulnerability of such a small strangelet depends on their mass (related to their mode of production) and type of interaction in which they participate: to pass from one stable configuration to another, the time scales required are of the order of the weak interaction time scale and, in general larger $A$ implies greater stability (provided these objects are not big enough to have become medium sized nuggets). Thus, for example, stability of strangelets is of much concern in Heavy Ion
1.4 SQM AND THE MISSING MATTER

experiments (being governed by strong interaction time scale), while it is almost guaranteed for collisions of strangelets in the terrestrial atmosphere (The time delay between successive collisions is large enough so that they have enough time to switch over to one of such stable configurations).

1.4 SQM and the missing matter

The naturally occurring strangelets \( A \sim 10^{44} \) could have been formed in the early evolutionary phase of the universe. In the very same paper \(^{28}\) where Witten discussed the possible existence of stable multi-quark bags stabilized by strangeness, he had raised the possibility that the missing mass of the universe can be accounted for by SQM. In the following we present a brief synopsis of the original work – it must be mentioned here that although the essence of the agreement remain unaltered, later works have examined various related aspects in greater length and depth and the numbers quoted here are purely of historical significance. We discuss some more recent works in Chap.\(^{6}\).

Witten’s description of the process begins in the very early universe (row 3, table : 1.1) when the universe undergoes a a first order phase transition in the early Universe from a high temperature state of quasi-free \(^{iv}\) light quarks to a state of hadronic matter (The cosmic QCD Phase Transition). The Latent heat released during the first order phase transition holds the temperature to the steady value of \( T_c \) allowing low temperature bubbles of hadronic matter (Fig.1.11) to form. As the Universe expands, these bubbles grow as they absorb energy from their surroundings. At some

\(^{iv}\) in the sense that they are free to roam inside the bag, without having to be confined in individual hadronic bags
point the lower energy 'bubbles' will percolate, and soon after, it is the high energy regions that form bubbles. As the Universe continues to expand, the high energy free quark bubbles continue to give off energy. This release of energy may take two forms. If it comes from evaporation, i.e. the release of hadrons into the low temperature region, then the bubbles will continue to shrink until they disappear. If instead they lose energy via neutrino emission, the baryon number inside the bubble will remain constant while energy is released. The bubbles will continue to shrink in size, increasing the baryon density. Eventually the excess baryons inside will produce a pressure to resist further contraction. These lumps can now accommodate between 80% - 99% of all the baryon excess of the universe, but are only about $10^{-6}$ cm - 5 cm in radius and their mass lies somewhere between $10^9 - 10^{18}$ gms. In order for these lumps to survive they could not be composed of normal matter (quark matter is unstable without strange quarks), but instead would be composed of strange quark matter. But since these nuggets are so small, they would scatter very little light and would be impossible to observe directly. They would be exceptionally bright candidates for dark matter, being non participants in nucleosynthesis. After the pioneering work by Witten, E. Farhi and R.L Jaffe have shown that the chunks of the SQM could be stable for a much larger range of sizes than predicted by Witten. R Riisagar and J. Madsen found that the primordial quark nuggets had to be made of more than $10^{23}$ quarks if their existence were to be consistent with both the calculated amount of missing dark matter and the observed abundance of light isotopes.

\[\text{vi}\] Recent theories suggest larger sizes but hardly alters any of the following conclusions.

\[\text{vi}\] Henceforth referred to as SQNs or Strange Quark Nuggets.
1.4. SQM AND THE MISSING MATTER

1.4.1 Strange Quark Nuggets from secondary sources

The collisions of primordial nuggets and the mergers resulting from such events might also lead to strangelets of larger size. It is also possible that deep inside the neutron star, extreme densities and pressure might have triggered the conversion of nuclear matter to strange quark matter - it works in the following way. As mentioned above, non strange quark matter is unstable at zero pressure. The pressure inside a neutron star can make it possible to make the energies lower by few 10’s of MeV per nucleon at which point stable quark matter begins to form. However as soon as a small core of stable quark matter is formed, it can become more stable by converting some of the light quarks to strange quarks and grow by absorbing the surrounding nucleons, facing no Coulomb barriers in their way. In such cases the quark cores should be able to convert the whole star into a (strange) quark star, providing a contemporary source of strange matter. Even a droplet of SQM falling into a neutron star can initiate this conversion and can convert it into a quark star. The resulting star would be much more compact since it will be bound by intrinsic quark forces. It is imperative, in this situation, to figure out what the resulting size of the strange star is going to be. Quantitative estimates to this query have been obtained using the solution to the TOV equations of hydrodynamics with an assumed equation of state. We, on the other hand, tried to develop an analytic procedure based on the quark mass density dependent model (QMDD). The QMDD model was first proposed by Fowler, Raha and Weiner [33] to provide a dynamical description of confinement. In this model the quality of confinement is mimicked through the requirement that the mass of the quark becomes infinitely large as the volume increases to infinity, holding the energy.
density constant and is summarized in the effect

\[ m_q = \frac{B}{3n_B} \]

\[ m_s = m_{s0} + \frac{B}{3n_B} \]

where \( m_q \) is the (density-dependent) mass of the u and d quarks, \( m_s \) is the (density-dependent) mass of the strange quark, \( m_{s0} \) is the (nonzero) free mass of the strange quark, \( B \) is the Bag constant or the vacuum energy density within the bag and \( n_B \) is the baryon number density. With this model, the standard technique of balancing the gravitational attraction to the quark degeneracy pressure remains applicable even in the case of quark stars composed of massless u and d quarks, as these quarks are able to get their masses from the density effects. We discuss the application of these ideas in chapter 6.

### 1.4.2 Experimental searches for strange matter.

The search for quasibaryons, the living fossils of the early universe, is an important activity in the field of QCD and astrophysics. It necessarily has strong theoretical and experimental perspectives and the search for them is currently an active experimental pursuit that spans over diverse zones like the Milky Way Halo, The Earth’s crust, Geophysical specimens (meteorites), High altitude stations, Weather balloons and accelerators for heavy ion collisions. It is unfortunately a nontrivial task to distinguish between SQM and normal hadronic dust, since SQM constitute a new form of matter and not a specific type of particle having a definite mass; the sole criterion for distinction in this case being the extremely small charge to mass ratio of the strangelets in comparison to nuclear particles.
1.4. SQM AND THE MISSING MATTER

To find evidence for strange quark matter there are mainly four places of choice.

1. The Earth’s crust and rock samples: The idea that the SQM would have left their trace in the crust material was first advocated by A.D Rujula and S.L Glashow [34]. According to them there can be three possible consequences, depending on the size of the strangelets:

   (a) $10^7 < 3A < 10^{14}$ : These particles (nuclearites) would be slowed down and stopped by earth and could reveal themselves as

   i. Unusual meteoritic events caused by nuclearites traveling with tremendous velocities compared to usual meteorites and penetrating large depths of the atmosphere without burning off, making their way to the ground.

   ii. Earthquakes with special signatures characteristic of a point source and almost total absence of surface waves (since most of the energy loss is supposed to take place in the mantle rather than the crust, since the nuclearite traverses the Earth in less than a minute).

   iii. peculiar tracks in ancient mica

   (b) $3A > 10^{23}$ : They would pass through the earth leaving no traces.

   (c) $3A < 10^7$ : They might remain embedded in meteoritic or crustal material.

2. Heavy Ion Collisions: We have already discussed the prospect of production of strange quark blobs in experiments [35].
volving massive ion beams. Such experiments however, to date have no success to report. It seems that the main difficulty associated with the production of strangelets is that the strangelets produced in Heavy Ion Collisions will have very little time (governed by the time scale of strong interactions) to settle down on a stable configuration through their interaction. Their sizes would necessarily be very small (depending on the share of the incident energy density it will receive). However If they are detected, they can be resolved and discerned from other particles through a mass spectrometer on the basis of their extremely small charge-to-mass ratio.

3. The Atmosphere: Strangelets, generated from collisions between strange 'neutron' stars can also be set off in motion to be (possibly) revealed as exotic cosmic ray events with extremely small charge to mass ratios ($Z/A << 1$). Numerical simulations of head-on collisions of neutrons stars suggest that as much as 13% of the total mass of the system might be ejected [36]. These potentially relativistic strangelets would eventually be impingent upon the Earth’s atmosphere. Even if such collisions are rare, binary systems may eject some mass during mass transfer; they can also result from the decay of binary systems. These strangelets would have to overcome the effect of the magnetic field imposed by the earth in order to come down to the level of mountain altitudes, where they can be detected using a ground based large array of passive solid state detectors. It is also possible to find the evidence for strangelets in balloon borne experiments using active / pas-

\textsuperscript{viii}In contrast, the strange nuggets which appear in the early history of the universe gets enough opportunity in the cosmological scale, to stabilize through weak interactions.
sive detectors, but since the incidence rate of these objects is very small, the probability of detection must be extremely small. Nevertheless the balloon experiments are the first of these kind of experiments which showed the signature of exotic particles which match the charge to mass signature of the strangelets.

4. The Outskirts of our galaxy: Gravitational microlensing technique has arrived as a powerful tool for exploring the structure of our galaxy. The idea of microlensing rests upon the fact that the light from a distant star is lensed and form a ring around a massive object that is near and lies along the line of sight of the observer (Fig. 1.12).

In the more likely case in which the massive object is slightly displaced from the line of sight, it will form a double image of the distant star, separated by a small angle. In the usual case this angular shift is too small to be resolved since the lens masses are in the stellar mass range and distances to the lenses are the order of galactic lengths. In this case, however, the image of the star will appear to brighten up, since light from both images will pour into it. These observations reveal the size and the mass (Experimentally found to be $\sim M_\odot$) of the lenses, which can then be used to judge between possible physical candidates for the lens objects. The experiments have the potential to reveal objects which are not available for luminous interception and can reveal the nature of dark matter present in our galaxy.
Figure 1.6: Schematic diagram for the rotation curve of typical spiral galaxies.
Figure 1.7: In between the heaviest elements and neutron stars, there is a large range of atomic weights which does not contain any known forms of matter; this stretch can easily be filled with SQM [23].
Figure 1.8: Various combinations of quarks in hadrons. The lightest two, up and down are needed to make up ordinary matter. The strange quark has so far been found only in unstable particles. In ordinary nuclear matter, the individual hadrons retain their identity. According to Witten’s [28] conjecture, stable multiquark bags, in which the individual hadronic boundaries have dissolved, may exist as a more stable form of matter.
Figure 1.9: The presence of an extra Fermi well reduces the energy of a three flavor system relative to a two flavour system.
Figure 1.10: The Energy per baryon Vs. the atomic number $A$ for strangelets in a shell model calculation with $B = 145^{±} \text{ MeV}$. The different curves represent the values of the strange quark mass (0 - 300 MeV). The peaks represent *magic numbers* or the values of $A$ for which the stability is enhanced with respect to neighbouring atomic numbers [30].
1.4. SQM AND THE MISSING MATTER

Figure 1.11: The evolution of hadrons through bubble nucleation in the early universe according to Witten [28]
Figure 1.12: Since Light rays are bent when they pass close to a massive object, light from a distant source may be focused by a closer object to producing a sudden brightening. If the smaller object’s path takes it precisely in front of the other one, the image formed by the "gravitational lens" is a circular ring, referred to as an "Einstein Ring."
1.5 The outline of this thesis

In this work the content is arranged in the following manner. In Chap. 2 we propose a basic model of propagation of small lumps of strange matter through the terrestrial atmosphere. The model relies strongly on the peculiar properties of SQM. On the basis of this model we discuss the origin of a class of exotic cosmic ray events (characterized by very low charge to mass ratios) and show that it is possible to relate these events to the passage of small strangelets through the terrestrial atmosphere. The elementary model introduced in this chapter is extended in several ways in Chap. 3 in order to deal with the problem in a more satisfactory way and leads to results which appear to be in close agreement to the few experimental data available in this field. In Chap. 5 we examine the (possible) existence of a maximum mass limit for quark stars from an analytical standpoint on the basis of the density dependent quark mass model. Finally, in Chap. 6 we try to unfold a connection between the existence of massive gravitational lenses in the halo of our galaxy to the local density of dark matter by asserting that the lenses are made of coalesced quasibaryonic matter.

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In this chapter we will first consider a few cases in which exotic events were registered in cosmic ray experiments (2.1). It is always difficult to accommodate any of these events within a conventional framework of propagation of cosmic nuclei through the terrestrial atmosphere: we discuss these problems in Sec. 2.2 and finally, in Sec. 2.3 examine a new model of strangelet propagation, proposed recently by us [1].

2.1 Existing reports of exotic fragments

There have been several reports of exotic nuclear fragments, with highly unusual charge to mass ratio, in cosmic ray experiments. In the following, several such events are listed, from the least to the most recent. In 1978, an event (Price’s Event, [2]) was identified with $Z \sim 46$ and $A \sim 1000$ and was looked upon as a possible
2.1. EXISTING REPORTS OF EXOTIC FRAGMENTS

candidate for a magnetic monopole at that time. Around this time, the Centauro cosmic ray events [3] detected in emulsion exposures taken at Mt. Chacaltaya by the Brazil-Japan collaboration, raised great interest. These events were observed at atmospheric depths \( \sim 500 \, \text{gm/cm}^2 \) and accompanied by hundreds of baryons and almost no \( \pi^0 \) or \( \gamma \).

In 1990 Saito et al. [4] analyzed the data of a 1981 balloon borne experiment which carried Cerenkov and Scintillation counters and claimed to have identified two events which were consistent with \( A \sim 370 \) and \( Z \sim 14 \) and could not be explained within conventional premises. In Fig. 2.1 these two events are shown in a \( Z \) vs \( A \) plot. In order to accentuate the highly unusual characteristics of the deviants, the two events are shown along with a few normal nuclei like Fe, Pb and U.

In 1984, De Rújula and Glashow [6] considered the possibility of detecting large lumps of SQM, called "nuclearites", of \( A < 10^{15} \) and \( Z \) "well beyond any published periodic table". Their main conclusions have been presented briefly in the introduction (Sec. 1.4.2). Among the indications of these events, they considered the possibility of observing visible light produced through ionization of the atmosphere as well as epilinear seismographic events, along with the possibility of finding visible signatures of their tracks on ancient mica. It is interesting to note that Anderson et al. [7] analysed over 1 million seismic data recorded during the period 1990-1993, in search for an passage of a nuclearite through the mantle of the Earth and have tentatively identified one of them as a possible candidate event.

In 1993 Ichimura et al. [9] reported an event called the 'exotic track' event with \( Z \sim 20 \) and \( A \sim 460 \). The report was based on an analysis of a 1989 balloon borne experiment using solid state
2.1. EXISTING REPORTS OF EXOTIC FRAGMENTS

Figure 2.1: Z vs A relationship for SQM as given in [5]. The Z-A relationship for normal nuclei is shown for comparison.
nuclear track detectors (CR39).

There have been several other reports of events with $A \sim 350 - 500$ and $Z \sim 10 - 20$ in cosmic ray experiments [2, 8, 9, 10, 11], the so-called exotic cosmic ray events. In the following table (Tab. 2.1) we present a synopsis of the events in this range of charge and mass. All these events carry the signature of small charge to mass ratios ($Z/A << 1$) characteristic of SQM (1.4.2).

Although these observations come from different groups, the existence of such objects cannot yet be taken as confirmed, due to various experimental uncertainties like switch between gondolas, ambiguities associated with the calibration of Cerenkov counter output, detector noises, dead time etc, in the different experiments. These events, thus, are, at best, candidate events, although, in spite of that, it is important to understand what they could be, if they are eventually confirmed.

| Event                          | Mass   | Charge |
|-------------------------------|--------|--------|
| Counter Experiments ([8])     | 350-450| 14     |
| Exotic Track ([9])            | 460    | 20     |
| Price’s Event ([2])           | 1000   | 46     |
| Balloon Experiments ([10]-[11])| 370    | 14     |

Table 2.1: Summary of some exotic cosmic ray events

2.2 Problems related to propagation

In all the events discussed in the above section (2.1) the primary difficulty seems to the extent of penetration of these seemingly heavy nuclei in the terrestrial atmosphere, since all the events
were observed near (or slightly higher than) mountain altitudes. The cross sections of normal nuclides in this mass range would be too large for escaping the fate of catastrophic collision with the air nuclei before being intercepted unhampered into a detector (Fig. 2.2 panel A). In the following we discuss the evolution of the ideas leading to the interpretation of these objects as strangelets.

In [12] Bjorken and McLerran ruled out the possibility of a high Z primary nuclei as the source of the Centauro event mainly on the above ground (unusually high penetration) and the fact that the mean transverse momentum of the secondaries were much higher than the value typical of nuclear fragmentation. In order for an object to reach comparable altitudes they assumed the object to be a glob of nuclear matter of an unusual type with density $\sim 30 - 100$ times that of ordinary nuclear matter and radius 3 - 5 times smaller than ordinary nuclei. The cross-section of the object would thus be much smaller (Fig. 2.2 panel B) compared to usual nuclides in this mass range and would face little difficulty in reaching the atmospheric depths at which they had been observed. This will be favored all the more if the binding energies of the components of this peculiar nuclei were larger compared to conventional nuclear matter.

Compressed to such high densities, the constituents of the glob would be in a quark matter phase (Sec 1.3). In their model of propagation, the glob, on its way down the atmosphere, collides with air nuclei and gets heated up. It subsequently tries to get rid of the additional energy so acquired, either by the radiation of mesons or evaporation of the baryons. The fate of the glob depends crucially on its region of metastability: it explodes if the region of metastability extends only to a baryon number $N_{\text{crit}}$. On the other hand, the glob can propagate all the way down until it has fully
evaporated if it happens to be stable right up to $N_{\text{crit}} \rightarrow 1$. In this case if the binding energy per nucleon decreases with decreasing $N$ (due to the aforesaid radiation / evaporation) then the nuclei can explode if a central collision with an air nucleus imparts sufficient energy to the glob which is getting more and more loosely bound with time. However, if the energy per nucleon increases on evaporation, the glob can land safely on the terrestrial soil. It is clear that the most essential ingredient in the above model is the region of metastability of the globs. The quark matter globs were speculated to be (meta)stabilized by the presence of a fractionally charged free quark which might help compress the baryons to the required high density. This led to an unacceptable flux of quarks at the sea level together with an unacceptable rate for horizontal air showers. A slight variant of this model is capable of reducing the flux of globs at the sea level compared with the previous model but remains incompatible with the rate of horizontal air showers.

The above discussion serves to highlight the difficulties encountered in the interpretation of the exotic cosmic ray events along conventional lines. Another issue is that, unlike the above event, most other exotic events of the type given in table 2.1 were non-explosive and could not be dealt with properly within a fireball scenario. The requirement of metastability was also a stringent one, and this has led to various speculations on the composition of these quark blobs; e.g Chin and Kerman [13] proposed the existence of metastable multiquark states of large strangeness within the framework of the MIT bag model [14]. The existence of such objects has been postulated by other authors [15] too, but the seminal work of Witten [16] in 1984 provided the theoretical basis for the study of SQM (referred to in the following as strangelets) within the framework of QCD. We have already discussed part of this frame-
work in the introduction (1.3.1) leading to a classification of the
SQM in three mass groups (1.3.2). In the light of the prior disccus-
sion it seems natural to associate some of the events in Table. 2.1
with strangelets since the rather unusual $e/m << 1$ ratios which
appear in the table seems to be well correlated with the theoretical
estimates for $e/m$ of strangelets [17].

In spite of this, no consensus has yet emerged primarily be-
cause of ambiguities related to the mechanism of propagation of
these objects through the terrestrial atmosphere. For example if
a strangelet with baryon number $A \sim 1000$ appears at the top of
the atmosphere, there would be a serious problem with its pen-
etrability through the atmosphere, as the exotic events are observed
at quite low altitudes. One can assume arbitrarily that their geo-
metric cross sections are quite small (as in above). This conclusion
seems to be rather artificial because there is no compelling theo-
retical reason to believe that the mass-radius relation for SQN’s to
be much different from normal nuclides – at least not as dramatic
as suggested in [10]–[11] since the density of SQM is believed to be
not much large than ordinary nuclear matter [16].

As a way out of the situation Wilk et al. [18, 19, 20] and others
(e.g [21]) proposed a mechanism by which the strangelets will be
able to cover great atmospheric depths without having to rely on an
atypically small cross section. They explored the fate of strangelets
of initial masses of the order of $10^3$ a.m.u. incident on the up-
per layers of the atmosphere. The main assumption was that the
mass and hence the cross sections of such strangelets decrease
rapidly due to their collisions with air molecules in their downward
journey. In particular, they assumed that a mass equal to that of

\footnote{These are medium sized strangelets, according to the classification given in 1.3.2}
the nucleus of an atmospheric atom (\(A \sim 14.5\)) is ripped off from the strangelet in every such encounter, as if the atmospheric atom drills a bore through the strangelet (See Fig. 2.2, panel C). This model is characterized by a critical mass \(m_{\text{crit}}\) such that when the mass of the strangelet evolving out of an initially large strangelet drops below the above critical limit, it simply evaporates into neutrons: this happens when the separation energy \(dE/dA\) becomes larger than the mass of a baryon. In other words, the condition

\[
\left. \frac{dE}{dA} \right|_{m_{\text{crit}}} > m_n
\]

would fix the lower limit of the altitude upto which a strangelet would be able to penetrate (Fig. 2.3).

Let us reiterate the basic conclusions that can be derived from the earlier works (Fig. 2.2). Firstly, strangelets observed at the mountain altitudes typically have masses around 300 to 450 and charge between 10 to 20. But the experimental results obtained till date are inconclusive and hence they do not impose a strict bound on the mass and charge of strangelets that can be observed in future experiments. Secondly, although the correlation between penetrability and geometric cross sections is usually valid for ordinary nuclei, the same cannot be easily extrapolated to the case of strangelets since these massive objects are very tightly bound and are not expected to break up as a result of nuclear collisions. Indeed, in a typical interaction between a strangelet and the nucleus of an atmospheric atom, it is more probable for the strangelet to absorb neutrons so that the colliding nucleus, and not the strangelet, is likely to break up most of the time. Hence the scheme proposed in [18], namely that the mass of a strangelet decreases in every encounter, seems to be unrealistic. In a realistic model of propagation
2.2. PROBLEMS RELATED TO PROPAGATION

Actual depth of atmosphere penetrated

| (A) | (B) | (C) |
|-----|-----|-----|
| Usual scenario | Large mass but small c.s | Proposed by Wilk et al. |

Figure 2.2: Picture to illustrate the difficulty in the propagation of strangelets. Panel C shows the solution proposed by Wilk et al.
2.3. A NEW MODEL FOR PROPAGATION

one also has to consider the effect of the geomagnetic field which can act on a charged strangelet. Specifically, for medium to small sized strangelets (see Sec. 1.3.2) the charge on the strangelet will make the strangelet travel in twisted paths, increasing the effective length of the path and making the globs disappear long before mountain altitudes are reached if the cross-sections decrease according to the above prescription.

In the next section an alternative scheme will be introduced which attempts to include the above factors in consideration, specially for small sized strangelets.

2.3 A new model for propagation

The alternative scheme is based on the following premises:

1. The collision of a lump of SQM with ordinary matter results in the absorption of the neutrons from the colliding nucleus, as a result of which the mass of the strangelet increases in every collision and it becomes more tightly bound (Fig. 2.4).

2. The initial masses of the strangelets are assumed to be small in order to obtain final baryon numbers which are nearly equal to the observed ones at mountain altitudes. The discussion in the preceding chapter indicates that it is quite possible to have stable lumps of SQM with low mass numbers. This would also facilitate a somewhat larger flux in the cosmic rays.

3. The speed, and hence the kinetic energy of these particles, must be such that they would arrive at a distance of 25 km

\[\text{[ii]}\] These would fall in the class of very small strangelets 1.3.2.
2.3. A NEW MODEL FOR PROPAGATION

above the sea level, surmounting the geomagnetic effects. We start with such an altitude since the atmospheric density above 25 km is low enough to be neglected. The charge of the strangelet is also fixed by this assumption, corresponding to a certain strangeness fraction.

The simple assumptions proposed above give a picture more or less in accord with the observation of the propagation of the exotic nuclei in the atmosphere, which can give useful indications of the type of things to be expected in an actual experiment. The description of the model is given next. We consider a situation in which a strangelet with a low baryon number enters the upper layers (∼25 Km from the sea level) of the atmosphere. To arrive at this point, a charged particle must possess a speed determined by the formula (see, e.g., [22]).

\[
\frac{pc}{Ze} \geq \frac{M}{r_o^2} \frac{\cos^4 \vartheta}{(\sqrt{1 + \cos^4 \vartheta} + 1)^2}
\]

(2.1)

where \(M\) is the magnetic dipole moment of the Earth, \(r_o\) the radius of the Earth and \(\vartheta\) is the (geomagnetic) latitude of the point of observation (∼30°, which might represent a location in north eastern India). \(p\) and \(Ze\) represent the momentum and charge, respectively, of the particle. The magnetic field of the Earth is taken to be equivalent to that due to a magnetic dipole of moment \(M = 8.1 \times 10^{22} J/T\), located near the centre of the earth, the dipole axis pointing North-South. We have fixed the mass, initial speed and charge to be 64 amu, \(6.6 \times 10^7\) m per sec and 2 (electron charge), respectively, at the initial altitude of 25 km.

In the course of its journey, the strangelet comes in contact with air molecules, mainly \(N_2\). During such collisions, the strangelet absorbs neutrons from some of these molecules, as a result of which
2.3. A NEW MODEL FOR PROPAGATION

Figure 2.4: An alternate scheme, proposed by us, for the propagation of very small strangelets.

![Diagram showing alternate scheme]

Figure 2.5: Variation of mass of the strangelet with altitude, according to our scheme. The arrow corresponds to an altitude of 3.6 km from the sea level.

\[ M_0 = 64 \text{ amu} \]
\[ v_0 = 6.6 \times 10^6 \text{ m/sec} \]
it becomes more massive. The effect of such encounters is summarized in the formula

\[
\frac{dm_s}{dh} = f \times \frac{m_N}{\lambda} 
\]

(2.2)

where \(m_s\) is the mass of the strangelet, \(m_N\) the total mass of the neutrons in the atmospheric atom, \(\lambda\) the mean free path of the strangelet in the atmosphere and \(h\) the path length traversed. (It should be emphasized here that the strangelets would preferentially absorb neutrons, as protons would be coulomb repelled. Nonetheless the strangelet can absorb some protons in the initial phase of the descent, when the relative velocity between the strangelet and the air molecule is large. Thus, in this phase, both the mass and the charge of the strangelet will increase, while in the later phase the charge absorption is expected to become strongly inhibited. We do not address this issue in the present chapter \(^{iii}\) for the sake of simplicity, although it can be readily seen that the rate of increase in mass would obviously be faster than that in charge.) In the above equation, \(\lambda\) depends both on \(h\), which determines the density of air molecules and the instantaneous mass of the strangelet, which relates to the interaction cross section. The mean free path decreases as lower altitudes are reached since the atmosphere becomes more dense and the collision frequency increases. Finally the factor \(f\) determines the fraction of neutrons that are actually absorbed out of the total number of neutrons in the colliding nucleus. The expression for this factor has been determined by geometric considerations \(^{23}\) and is given by

\[
f = \frac{3}{4}(1 - \nu)^{1/2} \left(\frac{1 - \mu}{\nu}\right)^2 - \frac{1}{3} \left[3(1 - \nu)^{1/2} - 1\right] \left(\frac{1 - \mu}{\nu}\right)^3
\]

(2.3)

\(^{iii}\)We do address this issue in the next chapter
Figure 2.6: Variation of altitude with time. The arrow corresponds to an altitude of 3.6 km from the sea level.
2.3. A NEW MODEL FOR PROPAGATION

In eqn(2.3), $\mu = \frac{b}{R_1 + R_2}$ and $\nu = \frac{R_1}{R_1 + R_2}$ where $b$ is the impact parameter. $R_1$ and $R_2$ are the radii of the strangelet and the nucleus of the atmospheric atom, respectively. $f$ is initially small but grows larger and reaches the limiting value 1 when the strangelet grows more massive.

The above considerations lead us to a set of differential equations of the form

$$\frac{d\vec{v}}{dt} = -\vec{g} + \frac{q}{m_S} (\vec{v} \times \vec{B}) - \frac{\vec{v}}{m_S} \frac{dm_S}{dt} \tag{2.4}$$

In eqn(2.4), $-\vec{g}$ represents the acceleration due to gravity, $\vec{B}$ is the terrestrial magnetic field, $q = Ze$ and $\vec{v}$ represents the velocity of the strangelet. These equations were solved by the 4th order Runge Kutta Method for the set of initial conditions described above.

The results are shown in figs. 2.5 – 2.9. Figure 2.6 shows the variation of Altitude with time, the zero of time being at 25 km. The time required to reach a place which is about 3.6 km above the sea level (height of a typical north east Indian peak like ‘Sandakphu’, where an experiment to detect strangelets in cosmic rays using a large detector array is being set up [24]) is indicated in the figure. The next figure (Fig.2.7) shows the change of the mass of the strangelet with time and figure 2.5 shows the growth of the strangelet mass with altitude. It can be seen from the figures that the expected mass at the aforementioned altitude comes out to be about 340 amu or so. Figure 2.8 shows the variation of the mean free time with altitude. The mean free time at all positions are more than the time scale for weak interactions ($10^{-8}$ sec) so that the strangelet gets enough time to stabilize and adjust itself to new baryon number configurations. Finally, figure 2.9 shows the variation of $\beta = v/c$ with time, showing that the speed of the
2.3. A NEW MODEL FOR PROPAGATION

Figure 2.7: Variation of mass with time. The time indicated by the arrow is the time taken to reach an altitude 3.6 km from the sea level, starting from 25 km.

\[ M_0 = 64 \text{ amu} \]
\[ V_0 = 6.6 \times 10^6 \text{ m/sec} \]
2.3. A NEW MODEL FOR PROPAGATION

Figure 2.8: Change of mean free time with altitude.
2.3. A NEW MODEL FOR PROPAGATION

strangelets decrease as they grow more massive.

In this chapter we have advocated a dynamical model of the propagation of strangelets through the terrestrial atmosphere. It is based strongly on the characteristic property of SQM which is responsible for regarding objects made of similar stuff as the ground state of QCD. It is realistic enough to include the difference in the interaction process which is expected when SQM and not ordinary nuclei, collide with atmospheric nuclei. The effects of Earth’s gravitational and magnetic fields are included in the equations of motion so that it is possible to derive meaningful information directly from the resulting trajectory. The main conclusion of the model is that the exotic cosmic ray events with very small Z / A ratios at mountain altitudes could result from SQM droplets which need not be large initially. Thus the flux of the strangelets may be appreciable enough to make their detection by a large area detector at mountain altitudes a real possibility.
2.3. A NEW MODEL FOR PROPAGATION

Figure 2.9: Change of $\beta = v/c$ of the strangelet with time.
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Energy loss of fast strangelets

In the previous chapter we have proposed a dynamical model for the propagation of strangelets of low mass numbers through the terrestrial atmosphere, where the stability of SQM plays a very important role. In that model the mass of the strangelet increases when it undergoes a collision with atmospheric atoms during the course of the journey. Using straightforward geometrical considerations, it has been shown [1] that the strangelet can grow from $A = 64$ amu to $A \sim 340$ amu by the time it reaches an altitude $\sim 3.5$ km, the altitude of a typical mountain peak with adequate accessibility for setting up a large detector array. This remarkable possibility makes it imperative to explore the consequences of this novel mechanism with greater care.

The basic model proposed earlier is summarized in the equation of motion eqn. [2.4]. While the first two terms in this equation have obvious significance, the third term accounts for the deceleration of
the strangelet due to its peculiar interaction with the air molecules; strangelets can readily absorb matter and become more strongly bound, unlike the normal nuclear fragments which tend to break up [2]. In this chapter the basic formalism will be extended [3, 4] in several ways, elaborated in the following section.

3.1 A more refined model

In the earlier model, the strangelet acquired mass only through the absorption of neutrons from the air molecule, since it was assumed that the repulsive Coulomb barrier on the surface of the strangelet will keep the protons off the strangelet. However, if we want to study initially fast moving strangelets, it appears quite possible that they absorb a few protons in the initial phase of their journey, when the relative velocity between the strangelet and the air molecule is large. In the previous chapter, the charge of the strangelet played only an indirect role, since it entered the equation of motion eqn. [2.4] only through the coupling with the geomagnetic field. Thus, although it was instrumental in steering the course of the strangelet it didn’t affect the strangelet speed. Therefore, although the presence of the charge increased the length of the trajectory before mountain altitudes were reached and led to larger mass increments compared to that for a neutral strangelet, it did not directly affect the instantaneous mass accumulation rate of the strangelet (eqn. [2.2]). In this chapter the formalism takes care of the accretion of charge of fast moving strangelets by formulating the problem in a relativistic setting. As a consequence of proton absorption, the issue of loss of energy of the strangelet through ionization of the surrounding media cannot be ignored any more. The accumulated charge thus will be able to influence the
3.1. A MORE REFINED MODEL

speed of the strangelet directly and it will be seen that the ionization losses become quite significant at comparatively low altitudes (where the atmosphere is dense and the strangelet is sluggish) and provide a lower limit to the height at which the strangelets can be detected successfully. In the previous analysis we have traced the fate of strangelets whose initial speeds at the upper layer of the atmosphere were only slightly higher than the lower bound on such velocities imposed by the geomagnetic field – however a larger spectrum of initial velocities can only be examined within a consistent relativistic framework. Although our model of propagation of strangelets rely heavily on the stability of small lumps of SQM [5, 6, 7, 8, 9], the probability of fission like fragmentation of such lumps cannot be ruled out, specially for highly energetic collisions of the strangelet with the air molecules, which can now occur at relativistic speeds. In this work we disregard the possibility of such events by providing a upper limit to the initial speed of the strangelet, above which it may no longer be practicable to evade the possibility of fragmentation. We have estimated that for our case (initial A larger than 40 for which the stability appears to be more robust due to an underlying shell like structure [9, 10]), this upper limit on the velocity comes out to be slightly above 0.7c (see App. B).

In order to incorporate the above effects, we first modify the term responsible for the absorption of neutrons in eqn. 2.4 to include the rate for proton absorption. It should be noted that absorption of neutrons would lead only to mass increase while that of protons would increase both the mass and charge of the strangelets.

In order to relate the proton absorption cross-section to the neutron absorption cross-section, we adopt the following simple model. The shell structure of N_2 dictates that only the single proton belong-
3.1. A MORE REFINED MODEL

ing to the outermost shell can be considered to be sufficiently free for the consideration of absorption by the strangelet. We describe the classical motion of the proton of energy $E$ in the vicinity of the strangelet after the model of the motion of a free particle of unit charge in the repulsive Coulomb field of the strangelet. The total energy of the proton at time $t$ is given by

$$E = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\phi}^2) + U(r) = \text{const} \quad (3.1)$$

where $U(r)$ represents the Coulomb energy of the proton and other quantities have their usual significance. The effective energy for the equivalent one dimensional problem is

$$U_{\text{eff}}(r, L) = \frac{L^2}{2mr^2} + U(r)$$

where $L = m\dot{r}^2\dot{\phi}$ is the angular momentum of the proton. The angular momentum $L$ also equals $mv_0 b$ where $v_0$ is the relative speed with which the $N_2$ nuclei (and hence, its constituent protons) approach the strangelet and $b$ is the impact parameter. The minimum separation along the trajectory occurs when

$$\frac{(mv_0 b)^2}{2mr_{\text{min}}^2} + U(r_{\text{min}}) = E = \frac{1}{2} mv_0^2 \quad (3.2)$$

Assuming that charge transfer can take place when $r_{\text{min}} \leq R_s$ (the radius of the strangelet), the corresponding value of $b$ ($\equiv b_c$) for which this occurs can be solved by substituting $b = b_c$ and $r_{\text{min}} = R_s$ in eqn. [3.2]. This yields

$$b_c^2 = R_s^2 \left(1 - U(R_s)/(\frac{1}{2}mv_0^2)\right)$$

The above assumption is equivalent to saying that all protons for
3.1. A MORE REFINED MODEL

which \( b > b_c \) misses the strangelet, while those with \( b_c \leq b_c \) are captured. Thus we can write the capture cross section for protons \( \sigma_p \) as

\[
\sigma_p = \pi b_c^2 = \pi R_s^2 \left[ 1 - \frac{Z_s e^2}{4 \pi \varepsilon_0 R_s} / \frac{1}{2} \frac{mv_0^2}{2} \right]
\] (3.3)

In contrast, the absorption cross section for neutrons \( \sigma_n \) is just \( \pi (R_n + R_s)^2 \) and hence we can infer that the accretion to the strangelet due to its interaction with neutrons \( \frac{dm_{sn}}{dt} \) is related to that due to protons \( \frac{dm_{sp}}{dt} \) by

\[
\frac{dm_{sp}}{dt} = \frac{\sigma_p}{\sigma_n} \frac{dm_{sn}}{dt} \equiv f_{pn} \frac{dm_{sn}}{dt}
\] (3.4)

whence,

\[
f_{pn} = \frac{R_s^2}{(R_n + R_s)^2} \left[ 1 - \frac{Z_s e^2}{4 \pi \varepsilon_0 R_s} / \frac{1}{2} \frac{mv_0^2}{2} \right]
\] (3.5)

Thus, \( f_{pn} \) determines the relative probability for a proton to undergo the above process vis-a-vis a neutron, and is less than one, on account of the coulomb barrier present at the surface of the strangelet.

We carry on the analysis by extending the formalism slightly to accommodate relativistic speeds. The equation of motion eqn. [2.4] generalized to a relativistic form leads to eqn. [3.6] in a straightforward manner (see App. A)

\[
\gamma m_s \frac{d\vec{v}}{dt} = -m_s \vec{g} + q(\vec{v} \times \vec{B}) - \gamma \vec{v} \left( \frac{dm_{sn}}{dt} + \frac{dm_{sp}}{dt} \right) - m_s \vec{v} \frac{d\gamma}{dt} - \frac{f(v)}{\sqrt{3}} \vec{v}
\] (3.6)

In the above equation the mass transfer rates for the proton and the neutron are related by an equation (eqn. [3.7]) similar to eqn. [3.5] but where \( \frac{1}{2} mv_0^2 \) is replaced by its relativistic equivalent \( E \):

\[
f_{pn} = \frac{R_s^2}{(R_n + R_s)^2} \left[ 1 - \frac{1}{E} \frac{Z_s e^2}{4 \pi \varepsilon_0 R_s} \right]
\] (3.7)
Finally, the last term of equation (3.6) accounts for the ionization loss. The expression for \( f(v) \) is given by \([11]\)

\[
f(v) = -\frac{dE}{dx} = \frac{Z_s^2 e^4 n Z_{med} l_n \left( \frac{b_{max}}{b_{min}} \right)}{4\pi \epsilon_0^2 m_e v^2}
\]

(3.8)

Here, \( n \) represents the number density of the atmospheric atoms at a particular altitude, \( Z_{med} \) is the number of electrons per atom of \( N_2 \) which can be ionized, \( m_e \) is the mass of the electron and \( b_{max} \) and \( b_{min} \) are the maximum and minimum values of the impact parameter. At large velocities, expression (3.8) reduces to, with \( I \) denoting the average ionizing energy,

\[
f(v) = \frac{Z_s^2 e^4 n Z_{med} l_n \left( \frac{2m_s v^2}{I} \right)}{4\pi \epsilon_0^2 m_e v^2} - \beta^2
\]

(3.9)

However, when the velocity of the strangelet falls below a critical value \( v \leq 2Z_s v_0 (v_0 = 2.2 \times 10^6 m/s \) is the speed of the electron in the first Bohr orbit), electron capture becomes significant which can be accounted for by the replacement \( Z_s \rightarrow Z_s^{\frac{1}{2}} v \) \([11, 12]\).

Equation (3.6) was solved by the 4th order Runge-Kutta method with different sets of initial mass, charge and \( \beta \). It may be mentioned at this point that the first term in eqn (3.6) is not important in magnitude, as is to be expected. We have nonetheless included it for numerical stability. This serves to define the downward vertical direction in the vector algorithm, especially for very small initial velocities.

### 3.2 Results

Figure 3.1 shows the final masses (for initial masses 42, 54, 60 and 64 amu and a fixed initial charge 2) as a function of initial \( \beta \). It is
Figure 3.1: Variation of final masses with initial $\beta(\beta_0)$ for different initial masses of the incident strangelet.
seen, especially for smaller initial masses, that the final mass decreases at first with increasing initial $\beta$ and then begins to increase again after a critical $\beta$ is reached and this critical value of $\beta$ shifts to the left for larger initial mass. Although mathematically delicate (it can be seen from eqn. 3.6 that a higher value of speed leads to an increasing value of the mass increment, which in turn slows down the particle), a qualitative explanation for this feature might be given as follows. One can think of the total region, through which the strangelet travels, being divided into two not-too-distinct subregions. In subregion I, corresponding to higher altitudes, the number of atmospheric particles is small, while this number is considerably larger in subregion II, corresponding to lower altitudes. For a strangelet of small initial mass (smaller size), the strangelet has a greater chance to escape subregion I if $\beta$ is higher, so that it will pick up lesser mass from this region. On the other hand if $\beta$ is very high, the volume that the strangelet sees will be contracted (the twisted tube through which it travels will be constricted), as a result of which it will interact with a greater number of atmospheric particles whence it will pick up a larger number of nucleons. It is clear that for an initially bigger (more massive) strangelet, this critical $\beta$ will be lower, as it will be able to sweep through a larger number of atmospheric particles right from the start. In a nutshell, this effect can thus be ascribed to higher speeds leading to larger mass increments, whose effect would be more pronounced for lower initial masses.

Let us now consider a representative set of data with initial mass 64 amu and charge 2 for detailed discussion. The results for $\beta_0 = 0.6$ are shown in figures 3.2 and 3.3, where the variation of speed ($\beta$) and the energy of the strangelet with altitude are depicted. The sharp change seen at $\sim 13$ km corresponds to the
Figure 3.2: Variation of final $\beta$ with altitude (a) for constant charge and without ionisation loss and (b) including proton absorption as well as ionisation loss

$m_{s_0} = 64$ amu
$\beta_0 = 0.6$
onset of electron capture, which is handled phenomenologically through the effective $Z_s$. The insets of figures 3.2 and 3.3 show a zoomed-up view of the respective quantities near the endpoint of the journey. It is apparent from the figures that the ionization term reduces the overall energy and speed considerably from the non-dissipative situation [1]. However, the zoomed-up insets in figs 3.2 and 3.3 show that the strangelets may have enough energy to be detectable at an altitude of 3.6 km from the sea level. For example, for the values of the initial quantities $m_{s_0}$ and $\beta_o$ shown here, the strangelet is left with a kinetic energy $\sim 8.5$ MeV (corresponding to $\frac{dE}{dx} \sim 2.35\text{ MeV/mg/cm}^2$ in a Solid State Nuclear Track Detector (SSNTD) like CR-39), which, although small, is just above the threshold of detection $\left(\frac{dE}{dx}\right)_{\text{crit}} \sim 1 - 2\text{MeV/mg/cm}^2$ for $\beta < 10^{-2}$ in CR-39 for the present configuration. Below this height, the possibility of their detection with passive detectors like SSNTD reduces to almost zero.

Table 3.1 lists the final values of the mass, the charge, $\beta$, and the energy of the strangelet at the end of the journey for different initial velocities. A comparison between table 2.1 first mentioned in the beginning of Chap. 2 and Tab. 3.1 shows that the final masses and charges are very similar to the ones found in cosmic ray events.

The experimental verification of SQM in cosmic ray flux (and the mechanism of their propagation through the earth’s atmosphere) is thus possible with a suitable ground based detector set up at high altitudes of about 3 to 5 km. At such altitudes, the predicted energy range of the resulting penetrating particles with mass $M$ between 300 and 400 and $Z$ between 10 and 15 should lie between 5 to 50 MeV. (This estimate corresponds to an averaging over all angles of incidence at the top of the atmosphere, taken to be 25 km here, as
3.2. RESULTS

Figure 3.3: Variation of kinetic energy of the strangelet with altitude

- $m_{s_0} = 64$ amu
- $\beta_0 = 0.6$
3.2. RESULTS

| $\beta_0$ | $m_{n_0}$ | $m_l$ (amu) | $q_l$ | $\beta_l \times (10^{-3})$ | $e_l$ [MeV] |
|-----------|-----------|-------------|-------|-----------------|--------|
| 0.2       | 42        | 294.7       | 3     | 2.8             | 1.05   |
|           | 54        | 369.4       | 4     | 3.0             | 1.55   |
|           | 60        | 415.8       | 4     | 3.0             | 1.80   |
|           | 64        | 446.5       | 5     | 3.1             | 1.98   |
| 0.4       | 42        | 246.4       | 6     | 4.9             | 2.84   |
|           | 54        | 359.5       | 8     | 4.7             | 3.73   |
|           | 60        | 415.6       | 8     | 4.7             | 4.25   |
|           | 64        | 452.0       | 9     | 4.6             | 4.63   |
| 0.6       | 42        | 235.8       | 10    | 7.4             | 5.97   |
|           | 54        | 357.1       | 12    | 6.6             | 7.15   |
|           | 60        | 416.0       | 13    | 6.4             | 7.87   |
|           | 64        | 453.6       | 14    | 6.3             | 8.39   |
| 0.7       | 42        | 236.4       | 12    | 8.6             | 8.16   |
|           | 54        | 359.1       | 14    | 7.6             | 9.59   |
|           | 60        | 418.3       | 15    | 7.3             | 10.46  |
|           | 64        | 456.3       | 16    | 7.2             | 11.11  |

Table 3.1: The final values, denoted with suffix $l$, are tabulated along with initial $\beta$ ($\beta_0$)
3.2. RESULTS

mentioned above.) A suitable locality for such observations at an altitude of about 3.5 km above the sea level has been identified at Sandakphu, in the middle ranges of the eastern Himalayas, with adequate accessibility and climatic conditions. Continuous exposure for months or years at a stretch of a detector assembly with stacks of SSNTDs like CR-39, covering a total area of about 400 m$^2$, is planned there. (The number of events due to strangelets may be as few as 5 - 10 per 100 m$^2$ per year, according to our approximate estimates (see Chap. 4.) The major considerations in this respect are cost, structural simplicity, and long time stability of the detection sensitivity against temperature fluctuations of several tens of Celsius degrees between summer and winter months and the ruggedness of the passive detectors. Regarding all these aspects, commercially available CR-39 appears to be the most suitable choice, which has been shown in NASA SKYLAB experiments [13, 14] to be capable of detecting heavy ions with energies upto 43 MeV/u. The signatures produced in such detectors in terms of mass, charge and energy of detectable strangelets can be evaluated in the expected $dE/dx$ range by measurements of track dimensions. For this purpose, additional calibration experiments, exposing CR-39 samples to heavy ions with variable charge states at almost similar energy ranges, are necessary which can be made at several existing heavy ion accelerator facilities. With efficient etching and automated track measurements, backgrounds of low energy secondary radiation with lower charge or mass are not expected to pose any serious problems. Due to specific inherent technical problems like "fading" of thermo-luminescent substances over a long interval of time, this kind of material do not seem to be practical in our experimental conditions. CR-39 has an additional advantage over the other types of passive semi-conductor detec-
A. RELATIVISTIC EQUATION OF MOTION

tors using co-polymers like SR6, CN85 or Lexan; a large amount of characteristic experimental data are already available for CR-39 in the existing literature. As alternatives, Mica or Overhead Transparency Foils may also be considered and calibration experiments using these materials will be conducted at accelerator facilities to judge their suitability. Other active detectors and devices do not appear to be suitable for installation at proposed mountain heights for stand-alone operation over long periods and are therefore not being considered at present.

In conclusion, we have presented a model for the propagation of cosmic strangelets of none-too-large size through the terrestrial atmosphere and shown that when proper account of charge and mass transfer as well as ionization loss is taken, they may indeed reach mountain altitudes, so that a ground based large detector experiment would have a good chance of detecting them.

A  Relativistic equation of motion

Starting with a relativistic form of the equation of rocket motion, we derive the equation of motion applicable for our case of a relativistic snowball. The equation of motion eqn. 2.4 is generalized to a relativistic form by interpreting $\vec{p}$ as the relativistic three momentum.

We consider a system of variable (proper) mass $M$ at the instant $t$ which changes to one with mass $M - dM$ at the instant $t + dt$ (see Fig. 3.4). The mass of the ejecta depicted by $\delta M^*$ is assumed to be different from $\Delta M$ since it moves with a different velocity $\vec{u}$. The velocities at the two points of time are denoted by $\vec{v}(\gamma)$ and $\vec{v}(\gamma + d\gamma)$ respectively, where $\gamma$ are the respective Lorentz factors. For the system enclosed in the dotted boundary,
A. RELATIVISTIC EQUATION OF MOTION

Figure 3.4: Relativistic rocket motion: (A) The rocket at time \( t \), (B) The rocket and its eject at time \( t + \Delta t \)

\[
\vec{F}_{\text{ext}} \simeq \frac{\vec{p}_f - \vec{p}_i}{\Delta t} \\
= \left( (\gamma + \Delta \gamma)(M - \Delta M)(\vec{v} + d\vec{v}) + \vec{u}\gamma \Delta M^* - \gamma M\vec{v} \right)/\Delta t
\]

In our case the particle is simply absorbed, so one can set \( \vec{u} \to 0 \)

and after discarding any terms \( O(\Delta^2) \)

\[
= \gamma M \frac{\Delta \vec{v}}{\Delta t} - \gamma \vec{v} \frac{\Delta M}{\Delta t} + \frac{\Delta \gamma}{\Delta t} M\vec{v}
\]
B. Threshold speed of strangelet for breakup

Replacing \( \frac{\Delta M}{\Delta t} \rightarrow -\frac{dM}{dt} \) and disregarding terms \( O(\beta^2) \) we get,

\[
\gamma M \frac{d\vec{v}}{dt} = \vec{F}_{\text{ext}} - \gamma \vec{v} \frac{dM}{dt} - M \vec{v} \frac{d\gamma}{dt}
\]

From (\( * \)) we recover the equation of motion 3.6 appropriate for the situation. It may be noted that for rocket motion the \( \frac{dM}{dt} \) is negative, while it is positive in the case for relativistic snowballs.

**B. Threshold speed of strangelet for breakup**

![Diagram](image)

(A) Lab Frame  (B) CM Frame

Figure 3.5: Collision of strangelet and air nuclei as seen from the (A) Lab frame (B) Center of mass frame. The small filled circle represents the center of mass of the colliding particles.

In this appendix we outline the calculations leading to the estimate of the upper limit of strangelet velocity. Roughly speaking, the
B. THRESHOLD SPEED OF STRANGELET FOR BREAKUP

reduced mass $\mu$ of the two body system of the strangelet ($m_s = 64$ GeV) and an average air nucleus ($m_N = 14$ GeV) is $\frac{m_s m_N}{m_s + m_N} \sim 11.48$ GeV. A crude estimate can be obtained by examining the kinetic energy $\mu(\gamma - 1)$ available in this system. Assuming that this is enough to break up the strangelet completely\footnote{we have taken the representative value of the binding energy per baryon of a strangelet from [10]}, one obtains a $\gamma \sim 1.28$ corresponding to a $\beta \sim 0.72$. To obtain a more careful estimate of the available energy we examine the energy $E_{\text{av}}$ available in the center of mass frame of the air molecule (suffix $N$) and the strangelet, (suffix $s$) (see Fig. [3.5]) which may be effective in breaking the strangelet up,

$$E_{\text{CM}} = m_s (\gamma_s^C - 1) + m_N (\gamma_N^C - 1) \quad (3.10)$$

The center of mass quantities (superfix $C$) can be expressed in terms of laboratory quantities (superfix $L$) as follows In the CM frame the strangelet and the air molecule approach each other with equal magnitude of three momentum, i.e

$$m_s \beta_s^C \gamma_s^C = m_N \beta_N^C \gamma_N^C \quad \text{or},$$

$$m_s \sinh \theta_s^C = m_N \sinh \theta_N^C \quad (3.11)$$

On the other hand $\theta_N^C = -\theta_{\text{CM}}^L$ and $\theta_s^C = \theta_s^L - \theta_{\text{CM}}^L = \theta_s^L + \theta_N^C$. Since $\theta_N^C$ is a negative quantity, redefining $\theta_N^C \rightarrow |\theta_N^C|$, one gets,

$$\theta_s^C = \theta_s^L - \theta_N^C \quad (3.12)$$

From [3.11] and [3.12] it follows that,

$$m_s \sinh (\theta_s^L - \theta_N^C) = m_N \sinh \theta_N^C \quad \text{or},$$

$$\coth \theta_N^C = \frac{m_N}{m_s} \csch \theta_s^L + \coth \theta_s^L \quad (3.13)$$
Finally, from [3.13] and using $\theta_s^L = \tanh^{-1} \beta$, $\theta^C_N$ can be expressed in terms of $\beta$ alone and the quantities $\beta^C_{s|N} = \tanh \theta^C_{s|N}$ can be similarly evaluated leading to an evaluation of $E_{\text{CM}}^{\text{avl}}$ in terms of the incident speed.

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CHAPTER 4

Strangelet event rates and abundances

In the earlier chapters (Chap. 2, 3) we have discussed some of the problems associated with the penetration of small lumps of SQM through the terrestrial atmosphere. In order to handle some of the quite unusual properties of SQM with respect to ordinary cosmic ray particles in a proper fashion, we have also put forward a dynamical model for the propagation of these objects through the terrestrial atmosphere. Working within the framework of the above model, we have been able to obtain the expected charge and mass range $i)$ with which these objects can arrive on the surface of the Earth. Within the scope of these chapters, we have also discussed some of the possible ground based experimental set-ups which can be decisive about the nature of the exotic cosmic ray events of the above type. In this chapter we, therefore try to provide an esti-

$\text{\textsuperscript{\textcircled{1}}} \text{These values compare reasonably well with the few experimental values quoted in Tab. 2.1}$
mate of the expected flux of the very small strangelets which can be intercepted by similar ground-based experiments. As a further consequence, we also try to estimate the relative abundance of the accumulated strangelets in the Earth’s crust.

4.1 Flux of galactic strangelets

In Chap. 1 we have indicated various ways by which small lumps of SQM may be formed. These include both galactic as well as extragalactic sources. However, most of the strangelets are expected to arrive from the local, galactic source of dark matter, whose density \( \rho \sim 10^{-24} \text{ gm } / \text{ cm}^3 \). From this, one can obtain an upper limit of the flux in the following way \[1\]. Assuming that all the dark matter consists of (spherical) strangelets of a certain radius \( r_s \), the number density \( n \) of such strangelets will be about

\[
n = \frac{3}{4\pi r_s^3} \frac{\rho}{\rho_n}
\]

where \( \rho_n \) is the typical density of nuclear matter objects. These objects move about randomly with a speed \( v \) determined by the depth of the gravitational potential of the galaxy, which is about \( 10^7 \text{ cm/sec} \). This results in a current density \( j \sim nv \)\[ii\] of strangelets in all possible directions. From this information one can find the number of such events expected during a year which may be registered by a passive detector array of a given size set up at mountain altitudes. In table 4.1 we list the event rate corresponding to strangelets of various sizes according to this scheme.

\[\text{ii}]\text{The expression for isotropic current density is actually } 1/4 \ nv, \text{ but dropping the numerical prefactor should not introduce any significant errors, which are already quite large, in view of the large approximations in effect.}
### 4.1. Flux of Galactic Strangelets

Table 4.1: Estimated flux of strangelets. Column 1, radius of strangelets in cm, column 2, mass number, column 3, \( n \) - the number density of strangelets, column 4, Flux of strangelets per cm\(^2\) per second, column 5, Flux on 400 m\(^2\) per year.

| \( r \)       | \( A \)       | \( n \)       | Flux      | Flux\(_{400}\) |
|--------------|--------------|--------------|-----------|--------------|
| 0.01         | \( 5.8 \times 10^{32} \) | \( 1.03 \times 10^{-38} \) | \( 1.03 \times 10^{-26} \) | \( 1.29 \times 10^{-12} \) |
| 10           | \( 5.78 \times 10^{31} \) | \( 1.03 \times 10^{-42} \) | \( 1.03 \times 10^{-35} \) | \( 1.29 \times 10^{-21} \) |
| \( 4.8 \times 10^{-13} \) | 64           | \( 9.35 \times 10^{-3} \)   | \( 9.3 \times 10^{3} \)     | \( 1.17 \times 10^{18} \) |
| \( 5.56 \times 10^{5} \)  | \( 1.0 \times 10^{56} \) | \( 5.9 \times 10^{-57} \)   | \( 5.9 \times 10^{-50} \)   | \( 7.5 \times 10^{-36} \) |

The procedure illustrated in table 4.1 yields an extreme overestimate for very small strangelets, since, in that case, the whole local dark matter density in the galaxy gets assigned to very small strangelets (e.g. row 3, column 2) of the same size. On the contrary, it is a well known fact (first mentioned in Sec. 1.4.2, item 4) that the primary constituents of the dark halo of the Milky Way are objects in the mass range of \( \sim M_{\odot} \), as indicated by microlensing experiments. In a subsequent chapter 6 we propose that there are reasons to believe that entire dark halo is composed of quasibaryonic lenses of \( A \sim 10^{55-56} \). If this is true, then the very small strangelets most probably originate from the collisions of these dark lenses (Sec. 1.4.2, item 3). In this section we present two estimates of the strangelet flux based on the above hypothesis.

The first estimate is based on the values of flux already given in table 4.1 but using a modified version of the procedure given above. If one envisages the collisions between two dark compact objects as being similar to two colliding nuclei, then a rough estimate of how much of the ejecta results in small strangelets can be obtained by using the fact that in a multifragmentation process \([2, 3, 4]\) the mass yield approximately obeys a power law behavior \( \propto A^{-\tau} \) with \( \tau \sim 2/3 \). From row 4 of table 4.1 we read that the flux
4.1. FLUX OF GALACTIC STRANGELETS

for objects of mass $M_\odot$ is $\sim 7.5 \times 10^{-36}$ for a 400 m$^2$ of detector area. However if we assume that this flux is the outcome of a “reaction” product of the collision of two objects in the Solar mass range, then the number for small fragments of a given mass number $A$ is this number, enhanced by a factor $(\frac{A}{1.0 \times 10^{36}})^{-2/3}$. For the typical mass of the strangelet assumed in chapter 3, the factor comes out to be $\sim 5.9 \times 10^{35}$, so that we get a flux of about 4 - 10 strangelets per year on a 400 m$^2$ passive detector layout.

The second estimate essentially borrows on the idea given in 5, in which the authors estimated the background flux obtainable from stellar collisions. Here the principal assumption is based on the observation that since several pulsars are members of binary systems, the two components of a binary are ultimately going to collide. If such collisions spread as little as 0.1 $M_\odot$ of non relativistic strangelets with baryon number $A$, the number of strangelets released in a single collision will be

$$N = \frac{0.1 M_\odot}{Am_n}$$

where, $m_n$ is the nucleon mass. If such objects are distributed homogeneously over a halo of radius $R_h \sim 10$ kpc, the number of particles flowing out isotropically per unit time, per unit area normal to the flow direction will be given by $\frac{3N}{4\pi R_h^3}v$, where $v$ is the flow velocity. The flux, per unit solid angle is then,

$$F = \frac{3}{16\pi^2} \frac{0.1 M_\odot}{R_h^3 m_n} A^{-1}v$$

or about $10^{-6}A^{-1}v_{250}$ cm$^{-2}$s$^{-1}$sterad$^{-1}$, where $v_{250}$ is the speed measured in units of 250 km s$^{-1}$, the typical speed of SQM in the galactic halo. The number of binary mergers in the galaxy have
4.2. RELATIVE ABUNDANCE OF STRANGELETS ON THE EARTH’S CRUST

been estimated [6] by a detailed computer simulation and can be quoted as \( R \sim 10^{-6} \text{yr}^{-1} \). Thus if one binary coalescence occurs per \( 10^6 \) years, but the disintegration products scatter over a region of radius 10 kpc, then we need to calculate the probability that one such event takes place within 10 kpc of the Earth. Since there would have been \( \sim 10^3 \) such events since the formation of the Milky Way Galaxy, the required probability is

\[
\mathcal{P} = 10^3 \times \frac{4\pi R_h^3}{4\pi R_R \times R^2} \sim 10^{-3}
\]

as the Milky Way Galaxy has the shape of a very flattened spheroid of major radius \( R \sim 10 \text{ Mpc} \) and minor radius \( R_h \lesssim 10 \text{ kpc} \). Then, the anticipated flux of strangelets of \( A \sim 64 \) would be \( F_{A=64} \times \mathcal{P} \) or about \( 5 / (\text{m}^2 \text{ Yr sterad}) \). The upper limit of flux on a 400 sq.m detector area, according to this estimate is as \( \text{Flux}_{400} \sim 10^2 \) per year. The actual expectation should be much lower, in view of the fact that the composition of the ejecta will necessarily accommodate strangelets of varying sizes.

### 4.2 Relative abundance of strangelets on the Earth’s crust

In this section we try to estimate the expected relative abundance of strangelets on the Earth’s surface. In an earlier chapter (Chap 3) we have presented a scheme of propagation of such strangelets through the terrestrial atmosphere and indicated the expected charge and mass with which these objects can arrive on the earth’s surface. The essential difference between the propagation of such strange quark balls and normal cosmic rays is the fact that the
strangelets can absorb nucleons (mainly neutrons) and become more stable when they interact with the atmospheric nuclei. In the course of propagation they lose energy both due to these collisions and the ionization loss (the strangelets have a small positive charge to start with, and they pick up some more from the protons in the initial phases of their journey through collisions with atmospheric nuclei) and as a result come down with very small velocities and land on the crust of the earth almost like a parachute. Starting with a mass of $\sim 64$ they might end up with a mass $\sim 350$ and charge $14$ (See Chap. 3, Tab. 3.1). It appears that the only way that they can propagate down the Earth’s surface is by the help of water percolation. In this section we make some not too unreasonable assumptions in order to estimate the relative abundance of such elements with respect to Silicon, since the charges of the strangelets are similar to it ($Z = 14$)\textsuperscript{110} and also because Si is the most abundant material on the Earth’s crust.

With the flux values calculated in Sec. 4.1 the total number of particles per unit area, $N_s$, that have accumulated on the earth’s crust, since the time of formation of the atmosphere (which we take to be $4.4 \times 10^9$ years) can be calculated. Out of these particles a smaller number can actually percolate within in a rectangular box of area 1 sq. m and depth $\sim 10$m (the depth up to which the water can come down) per year. Here we assume that a fraction of the particles $y \sim 1 - 10^{-3}$ which have been deposited over a unit area have actually come down through water percolation in this box. The total mass contributed by the strangelets in this way to the box mass, $m_s$ is then $N_s A_s m_n y$, where $m_n$ is the average nucleon mass and $A_s \sim 350$ is the mass number of the strangelet (we use SI

\textsuperscript{110}And hence might be separable using very high precision isotope separation methods.
units in these estimates). The mass contributed by Silicon, \( m_{Si} \) is roughly \( V \rho_e \times r_{Si} \), where \( \rho_e \) is the mean density of Earth \([7]\), \( V \) is the volume of the box and \( r_{Si} \) is the relative abundance of Silicon (27.7 \%). The relative abundance calculated for strangelets \( (r = \frac{m_s}{m_s + m_{Si}}) \) calculated in this way, come out roughly near \( 10^{-19} \) – \( 10^{-15} \).

In order to compare the values of relative abundances obtained above, we refer to the events analyzed by Saito et.al \([8]\) (see also the discussion at the beginning of Sec. 2.1). These events have a relative abundance of \( 2.1 \times 10^{-5} \) with respect to the total number of normal cosmic ray particles observed at the same total energy (including rest energy). Since the total energy of the strangelets lie somewhere in the 100 to 1000 GeV (corresponding to \( \beta = 0.2 – 0.7 \)) range we accept the corresponding flux window between \( 10^{-3} \) to \( 10 \) particles per \( m^2 \) per steradian per sec per GeV in the primary cosmic ray spectrum (Fig. 4.1). Thereafter, following the same methods as given above, the relative abundance turns out to be \( \sim 4.7 \times 10^{-16} \) – \( 10^{-9} \). Thus in this case, the expected relative abundance is considerably higher compared to our earlier estimate.

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5.1 Maximum mass of compact objects

In this chapter we address the issue of the existence of a possible maximum mass limit for quark stars from an analytical standpoint. In the first half of the chapter we compare the state of the problem for different classes of compact objects with respect to the above limit. In the other half we illustrate the analytic procedure leading to an evaluation of the maximum mass limit, and finally comment on the nature of the results obtained.

The Quark stars, if they exist, belong to a family of compact objects like the white dwarfs and neutron stars. These objects have been a topic of interest for several decades. Compact objects like the ones mentioned above are produced as the end product of the stellar evolution, i.e., when the nuclear fuel of the normal stars has been consumed. The factor which decides whether a star ends
up as a white dwarf, neutron star or a black hole is primarily the star’s mass. White dwarfs originate from light stars with masses \( M \lesssim 4M_\odot \). They no longer burn nuclear fuel, but cool off gradually, radiating away the last bit of their thermal energy. The decreasing total energy leads to a gradual contraction of the star\(^{\text{ii}}\) to the point at which the density increases so much that the breakdown of Maxwell-Boltzmann relations give way to a degenerate electron gas which can exert pressure even at zero temperature by the virtue of Pauli exclusion principle. The pressure of the Fermi gas of electrons acts against the contraction of the star and might be able to bring it to a halt. The existence of a maximum mass limit for these objects can be inferred qualitatively (after Landau) as follows.

The total energy at the equilibrium point consists of a positive Fermi energy part which \( \sim N^{1/3} \) and a negative contribution due to the gravitational energy (\( \sim N \)), both of which scales as \( 1/R \). The much weaker dependence of the Fermi energy on \( N \) relative to the gravitational energy term is in fact the key factor responsible for the existence of a maximum mass for this class of objects. This implies that although stable equilibrium is possible for small values of \( N \), larger values will make the gravitational term dominate, making the net energy negative which increases toward zero as \( R \to \infty \). Thus, beyond a critical value of the mass the star cannot escape the fate of a gravitational collapse.

Neutron stars originate from stars more massive than the progenitors for the white dwarfs. At very high densities the electrons react with the protons to form neutrons via inverse beta decay. The incorporation of such effects in white dwarf matter leads to an instability which settles down to a stable configuration only when almost all the protons and electrons squeeze together to form neu-

\(^{\text{ii}}\)The contraction is a must for any gas with a polytropic index \( \Gamma > 4/3 \)
trons and the system can be once more supported against the huge gravitationally generated inward pressure by the pressure due to degenerate neutron gas.

Applying essentially similar qualitative arguments one forsees a maximum mass limit for the neutron stars also and both the collapsed star types turn out to have a maximum mass ($\sim 1.5M_{\odot}$), beyond which they collapse to black holes [1, 2, 3].

In a pioneering work in 1926, S. Chandrasekhar identified the pressure which holds up white dwarfs with the electron degeneracy pressure. Actual white dwarf models, taking the effects of relativistic speed of electrons in the degenerate electron equation of state were constructed by him in 1930 [4, 5]. In the course of these studies, Chandrasekhar made the very important discovery that the maximum mass limit of white dwarf stars have to be $\sim 1.4M_{\odot}$, the exact value depending on the composition of the stellar matter. This maximum mass limit is called the Chandrasekhar Limit in honor of its discoverer. In 1932, L.D. Landau [2] presented an elementary explanation of the Chandrasekhar limit and applied his arguments in a similar manner several months later to neutron stars.

Although the Chandrasekhar limit refers strictly to white dwarfs, the limiting mass for neutron stars is also loosely called the Chandrasekhar limit, primarily because the limits in the two cases turn out to be the same [2]; the sizes are however vastly different (white dwarfs have $R \sim 10^{-2}R_{\odot}$ [1] whereas for neutron stars $R \sim 10^{-5}R_{\odot}$ [3, Tab.1.1]). The maximum mass for the white dwarfs depend essentially on the fundamental constants while the maximum mass for neutron stars is a sensitive function of the yet-unknown equation of state of nuclear matter and requires the solution of the TOV

\[\text{\footnotesize\textsuperscript{[1]}}\text{ where, } R_{\odot} \text{ is the Solar Radius, } \sim 6.96 \times 10^{10} cm\]
5.1. MAXIMUM MASS OF COMPACT OBJECTS

equation for relativistic hydrodynamics along with an assumed equation of state.

The neutron star, so formed, is essentially a huge nucleus and the nucleons (mainly neutrons) which make it up can undergo a hadron - quark phase transition at high density and/or temperature (see sec. 1.3). This is likely, since the central densities in the neutron stars are high enough to favour such a transition. In this phase transition, the individual hadronic boundaries dissolve and the quarks get trapped within a larger bag whose radius coincides approximately with the star radius and the neutron star becomes a quark star in the process [6, 7, 8]. We have already mentioned that the central hypothesis of the present thesis is based on the suggestion of Witten [9] that strange quark matter may be the true ground state of the strongly interacting matter. In this circumstance, quark stars, if they are formed, would preferably convert to strange stars, comprising $u, d$ and $s$ quarks, under weak interaction. Several authors (for example, [10, 11]) have used different models to understand the properties of strange stars. For a review, see [8]. For such quark stars, the maximum mass would indeed be almost the same as that for neutron stars.

In contrast to other compact objects, the strange stars need not be the direct product of stellar evolution. This is an unique property which distinguishes them from other compact objects, e.g it is conceivable that if a large amount of quark matter exists in the universe as a relic of the cosmological quark-hadron phase transition [12], it could clump under gravitational interaction (see Chap. 6) and even form invisible quark galaxies [13]. The ‘stars’ of such a galaxy would be strange stars which do not evolve from neutron stars and thus are not governed by the Chandrasekhar limit for neutron stars. It is, therefore, highly relevant to inquire if
5.2. THE ANALYTICAL FORM OF THE MAXIMUM MASS LIMIT

there exists, just like the case of ordinary compact stars, an upper limit on the strange stars beyond which they would be gravitationally unstable. In the past, the problem has been approached numerically. Starting with the seminal work of Witten [9], most authors have concentrated on solving the Tolman-Oppenheimer-Volkov (TOV) equation (see, for example, [3]) for the quark matter equation of state. While the results show that there does exist a limiting mass for quark stars (which is very close to that for neutron stars), there is no a priori argument to prove that such a limit should exist or that it should depend mostly on fundamental constants, as is the case for the ordinary compact stars [3]. In this chapter we show analytically, from first principles, that such a limit exists for compact quark stars and that it is mostly determined by universal constants.

5.2 The Analytical Form of the maximum mass limit

In the following treatment we essentially follow Landau [2] and apply a general and simple picture of energy balance to a system of massless quarks, confined in a large bag [6] characterized by a constant energy density $B$. The above is adopted as a working model of a strange star for the present purpose.

As in the case of white dwarfs and neutron stars, the equilibrium should occur at a minimum of the total energy per fermion $e$, where $e \equiv e_F + e_G$, in which $e_F$ is the Fermi energy and $e_G$ is the gravitational energy per fermion. There is a crucial difference in the way the (Newtonian) gravitational energy can be estimated for quark stars and the ordinary compact stars. The Newtonian
5.2. THE ANALYTICAL FORM OF THE MAXIMUM MASS LIMIT

gravitational energy is a macroscopic quantity, and for ordinary compact stars, this mass is due almost entirely to the baryons. For quark stars, however, one needs to identify the total mass as the total (thermodynamic as well as the confining) energy in the star. In order to estimate the gravitational energy per fermion one needs a prescription for incorporating both contributions into an effective quark mass.

A suitable framework capable of handling this issue was formulated quite some time ago [14] in which the authors (The model is briefly discussed in Sec. 1.4.1) proposed a dynamical model of confinement in a many-body system of quarks. This was, in turn, motivated by an earlier work by Pati and Salam [15] who pictured confinement as the quark having a small mass inside a hadron and a very large mass outside. The standard description of confinement is provided by the bag model [16], which implies that, for a many-body system, the energy density inside the bag, for small total quark number density, differs by a positive constant \( B \) from that of the true vacuum outside. The QMD model, inspired by the Archimedean principle advocated in [15] parametrized confinement through a density dependent quark mass which varied so as to agree with the bag model limit of constant energy density, i.e.

\[
m_q \sim \frac{B}{n_q} \text{ as } n_q \to 0
\]

where \( m_q \) is the effective quark mass and \( n_q \) is the total quark number density. Thus the mechanism of confinement is mimicked through the requirement that the mass of an isolated quark becomes infinitely large so that the vacuum is unable to support it. The picture given in eqn. 5.1 then tells us that for a system of quarks at zero temperature the energy density tends to a constant
5.2. THE ANALYTICAL FORM OF THE MAXIMUM MASS LIMIT

value while the mass tends to infinity as the volume tends to infinity or the density tends to zero.

The number density of fermions is related to the chemical potential as

\[ n = \frac{g}{6\pi^2} \mu^3 \]

which dictates that

\[ \mu = \left( \frac{9\pi}{2g} \right)^{\frac{1}{3}} \frac{N^\frac{4}{3}}{R} \]  (5.2)

In the above relations, \( n \) is the number density, \( N \) the total number of fermions in a star of radius \( R \), \( g \) the statistical degeneracy factor and \( \mu \) is the chemical potential.

The fermion energy density is given by

\[ \varepsilon_F = \frac{g}{8\pi^2} \mu^4 \]  (5.3)

and hence the Fermi energy per particle of the quarks becomes

\[ e_F = \frac{\varepsilon_F}{n} = 3 \left( \frac{9\pi}{4} \right)^{\frac{1}{3}} \frac{N^\frac{4}{3}}{R} \]  (5.4)

The mass \( M \) of the star can be written in terms of \( N \) and \( B \) (the bag constant), if the density \( \rho(r) \) in the star is assumed to be roughly constant throughout the volume of the star. Hence using eq.(5.4),

\[ M = \int_0^R 4\pi r^2 \rho(r) \, dr = \frac{4}{3} \pi R^3 B + e_F N = \frac{3}{4} \left( \frac{9\pi}{2g} \right)^{\frac{1}{3}} \frac{N^\frac{4}{3}}{R} + \frac{4}{3} \pi B R^3 \]  (5.5)

Extremising the mass \( M \) (eq. 5.5) with respect to \( R \) gives,

\[ \left( \frac{9\pi}{2g} \right)^{\frac{1}{3}} \frac{N^\frac{4}{3}}{R^4} = \frac{16}{3} \pi B \Rightarrow \varepsilon_F = 3B \]
5.2. THE ANALYTICAL FORM OF THE MAXIMUM MASS LIMIT

Substituting eq. (5.6) in the expression for $M$ (eq. 5.5),

$$M = 4BV = \frac{16}{3} \pi BR^3$$

We note that this is very similar to the condition obtained for hadronic bags [16]. As a next step we find the $R$ for which the total energy per fermion would be maximum.

The gravitational energy per fermion $e_G$ is

$$e_G = -\frac{GM_{\text{eff}}}{R}$$

where $m_{\text{eff}}$ is the effective quark mass inside the star. Assuming that the effective quark mass contributes to the total star mass $M$, one can write for a strange star with $N$ quarks,

$$M = Nm_{\text{eff}} \Rightarrow m_{\text{eff}} = \frac{AB}{n}$$

As mentioned above, the effect of confinement in a quark matter system was shown [14] to be incorporable in the effective quark mass, which, the quarks being fermions, coincides with the quark chemical potential. As a result, one gets, in the limit of vanishing quark density [14],

$$\mu = \frac{B}{n}$$

This, together with the eq. (5.8), gives

$$m_{\text{eff}} = 4\mu$$

where all the energy (thermodynamic and confining) is included in the effective gravitational mass of the quarks inside the strange star.

Using equations (5.7), (5.8) and (5.9) we get
5.2. THE ANALYTICAL FORM OF THE MAXIMUM MASS LIMIT

\[ B^{1/4} (MeV) \quad R_{\text{max}} (Km) \quad \frac{M_{\text{max}}}{M_{\odot}} \quad N_{\text{max}} \]

| 145   | 12.11 | 1.54  | 1.55 \times 10^{14} |
|-------|-------|-------|---------------------|
| 200   | 6.36  | 0.81  | 5.90 \times 10^{16} |
| 245   | 4.24  | 0.54  | 3.21 \times 10^{16} |

Table 5.1: Computed values of the maximum Mass, Radius and Baryon number for quark stars for several values of the Bag constant

\[ e_G = -\frac{64}{3} \left( \frac{9\pi}{2g} \right)^2 g \pi B R N \]  

(5.10)

Minimising the total energy \( e = e_F + e_G \) with respect to \( N \), we get the expression for maximum value of \( R \) as

\[ R_{\text{max}} = \frac{3}{16} \frac{1}{\sqrt{\pi GB}} \]  

(5.11)

It may be observed, that in this case the \( N \) dependence of both \( e_F \), eqn. 5.4 and \( e_G \), eqn. 5.10 are the same, while the \( R \) dependence is different for the two terms. Finally, the maximum mass of the strange star is computed by substituting the value of \( R_{\text{max}} \) (from equation 5.11) in equation (5.6).

\[ M_{\text{max}} = \frac{16}{3} \pi B R_{\text{max}}^3 \]  

(5.12)

The chemical potential \( \mu \) can be evaluated in terms of \( B \) using equations 5.3 and 5.6. Substituting this in eq. 5.8 gives the value of \( N_{\text{max}} \). The values of \( R_{\text{max}}, M_{\text{max}} \) and \( N_{\text{max}} \) are tabulated below (Table 5.1) for various values of the Bag constant \( B \).

Thus we have demonstrated that there exists a limiting mass (the so called Chandrasekhar limit) for compact quark stars, beyond which they would be gravitationally unstable. As with other
5.2. THE ANALYTICAL FORM OF THE MAXIMUM MASS LIMIT

classes of compact objects, the maximum mass depends mostly on universal constants (\( G \) as well as \( \hbar \) and \( c \), which do not occur explicitly due to our use of the naturalised units) and on the bag energy \( B \). The bag energy is treated as a parameter here, but it is often regarded as a universal constant in its own right, since it represents the difference between the non-perturbative and perturbative vacua of Quantum Chromodynamics. It can be seen from the above equations (5.11, 5.12) that the physical radius \( R_{\text{max}} \), corresponding to the maximum mass as well as the maximum mass itself, are independent of the number of quark flavors. Although \( N_{\text{max}} \) depends on the statistical degeneracy factor \( g \) (or equivalently, the number of flavors), the dependence is extremely weak, as can be readily checked from equation (5.2). In fact, we have verified that there is almost no difference in \( N_{\text{max}} \) between the cases with 2 or 3 flavor quark matter. This, in turn, implies that the assumption of massless quarks (even for \( s \) quarks) does not materially affect these results. While it is true that the methods applied in this chapter are pedestrian in nature, the limits agree well with those found with the help of the numerical solutions of the TOV equation (see, for example, [9]). Although, in this work, we have adopted the simplifying assumption of a constant density profile for the quark star in order to have a simple analytic solution, we too get the characteristic scaling behavior [9] (\( R_{\text{max}} \propto B^{-1/2} \), \( M_{\text{max}} \propto B^{-1/2} \)) obtained from detailed numerical solutions. This shows that the simple picture presented here adequately incorporates the essential physics of the structure of the quark stars.
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MACHOs as quasibaryonic dark matter

6.1 CDM Objects in the halo

One of the mysteries that persist in the standard cosmological model is the nature of dark matter. It has long been conjectured that we live in a nearly critical density universe, although there had been no evidence of the required accumulation of matter through observations based on the spectrum. However indirect evidences (see 1.2.1) do suggest that there is an abundance of matter in the universe which is non-luminous, since it either does not interact, or does so very weakly, with any other forms of matter, except through gravitational interaction. The present consensus (for a short discussion, see 1.2.1 for an extended review, see 1 2) based on recent experimental data is that the universe is flat and that a sizable amount of the dark matter is "cold", i.e. nonrelativistic, at the time of decoupling. For example, The WMAP survey data 3 par-
tions the total matter-energy content of the universe roughly as 73% smooth dark energy, 23% cold dark matter leaving the rest 4% to luminous matter which goes into the making of bright galaxies and stars.

The nature of the 23% cold non-luminous matter continues to be a mystery, at least within the standard framework of particle interactions, mainly because the required accumulation of baryons is clearly unaccountable by existing and extremely reliable data on the nucleosynthesis event responsible for the generation of nuclear matter. Most of the proposed dark matter candidates, therefore, rely on an extrapolated particle interaction model often venturing far into exotic domains. In recent years, there has been experimental evidence [4, 5] for at least one form of dark matter - the Massive Astrophysical Compact Halo Objects (MACHO) - detected through gravitational microlensing effects proposed by Paczynski [6] some years ago. As of now, there is no clear picture as to what these objects are made of in spite of a lot of efforts spent in studying them. Based on about 13 - 17 Milky Way halo MACHOs detected in the direction of LMC - the Large Magellanic Cloud (we are not considering the events found toward the galactic bulge), the MACHOs are expected to be in the mass range (0.15-0.95) $M_\odot$, with the most probable mass being in the vicinity of 0.5 $M_\odot$ [7, 8], substantially higher than the fusion threshold of 0.08 $M_\odot$. The MACHO collaboration suggests that the lenses are in the galactic halo. Assuming that they are subject to the limit on the total baryon number imposed by the Big Bang Nucleosynthesis (BBN), there have been suggestions that they could be white dwarfs [9, 10]. It is difficult to reconcile this with the absence of sufficient active progenitors of appropriate masses in the galactic halo. Moreover, recent studies have shown that these objects are unlikely to be white dwarfs,
even if they were as faint as blue dwarfs, since this will violate some of the very well known results of BBN \[10\]. There have also been suggestions \[11, 12, 13\] that they could be primordial black holes (PBHs) \((\sim 1 \, M_\odot)\), arising from horizon scale fluctuations triggered by pre-existing density fluctuations during the cosmic quark-hadron phase transition. The problem with this suggestion is that the density contrast necessary for the formation of PBH is much larger than the pre-existing density contrast obtained from the common inflationary scenarios. The enhancement contributed by the QCD phase transition is not large enough for this purpose. As a result a fine tuning of the initial density contrast becomes essential which may still not be good enough to produce cosmologically relevant amount of PBH \[14\]. Alternately, Evans, Gyuk & Turner \[15\] suggested that some of the lenses are stars in the Milky Way disk which lie along the line of sight to the LMC. Gyuk & Gates \[16\] examined a thick disk model, which would lower the lens mass estimate. Aubourg et al. \[17\] suggested that the events could arise from self-lensing of the LMC. Zaritsky & Lin \[18\] have argued that the lenses are probably the evidence of a tidal tail arising from the interaction of LMC and the Milky Way or even a LMC-SMC (Small Magellanic Cloud) interaction. These explanations are primarily motivated by the difficulty of reconciling the existence of MACHOs with the known populations of low mass stars in the galactic disks.

### 6.2  Stability of quark nuggets

In this chapter we accept the standpoint that the lensing MACHOs are indeed in the Milky Way halo and propose a theory which relates them to the quark nuggets which could have been formed in a first order cosmic quark - hadron phase transition, at a tem-
perature of $\sim 100$ MeV during the microsecond era of the early universe. In our picture, the MACHOS evolved out of these primordial quark nuggets. A few statements on the aforesaid quark-hadron phase transition may not be irrelevant here. The order of any phase transition carries the most significant bit of information about the phenomenon; however the order of the deconfinement phase transition is an unsettled issue till now. It is generally believed that a true second order phase transition is inconceivable in cosmological scenarios since nature does not provide an exact chiral symmetry [21]. In a pure (i.e. only gluons) $SU(3)$ gauge theory, the phase transition is of first order, as suggested by Lattice gauge theory. However, there exist no unequivocal approaches in the case when dynamical quarks are also present on the lattice, and instead of studying the deconfinement transition, one investigates the chiral transition. Although these two phase transitions are commonly treated to be equivalent, there is no definite reason why they should be simultaneous or of the same order [19]. The order of the chiral phase transition depends critically on the strange quark mass. Although the chiral phase transition is probably of first order for large strange quark mass, it may be of second order for lighter strange quarks. The situation remains controversial since the strange quark mass is of the order of the QCD scale [20]. In addition, the finite size effects of the lattice may tend to mask the true order of the transition. We are however, more concerned about the deconfinement transition and if it is really of the first order, the masking effect associated with it would be negligibly small in the early universe. In such circumstances, Witten (1984) argued, in a seminal paper [21], that strange quark matter could be the true ground state of Quantum Chromodynamics (QCD) and that a substantial amount of baryon number could be trapped in
6.2. STABILITY OF QUARK NUGGETS

the quark phase which could evolve into strange quark nuggets (SQNs) through weak interactions. (For a brief review of the formation of SQNs, see Alam, Raha & Sinha [22].) At this point, the most important question is whether the nuggets, so formed can be stable on cosmological time scales. The first study on this issue was addressed by Alcock and Farhi in 1985 [23]. They argued that a SQN can evaporate neutrons from its surface at \( T \geq I_N \) where \( I_N \sim 20 - 80 \text{ MeV} \) is the binding energy per baryon in SQM at zero temperature from the neutron mass. According to their calculations QN’s must have a baryon number in excess of \( 10^{51-53} \) in order to survive on cosmological time scales. This number is larger by a few orders of magnitude compared to the total baryon number of the universe at the aforesaid temperature and hence they concluded that it was apparently impossible to have any QN surviving till the present time. This conclusion was reexamined by Madsen et al. in 1986 [24], who pointed out that the neutron evaporation was a surface process in which the surface of a QN got gradually depleted of u and d quarks, as more and more neutrons were eliminated. In this process the flavor chemical equilibrium between the u,d and s quarks on the surface was lost and further evaporation would be suppressed till some s quark converts back to a u and d quark or there was some transport of u and d quarks from the core to the surface through convective process. Since both these processes are slow enough, the critical size of the nuggets which can survive is effectively lowered to \( \sim 10^{16} \). In a later work [25] it was suggested that the nuggets can also annihilate by boiling off hadronic bubbles from the bulk of the QN’s, but this was shown to have a rate much smaller than that for surface evaporation [26]. All of the above studies used idealized thermodynamic and binding energy arguments to calculate the baryon evaporation
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rate, with strong assumptions like geometrical cross sections, surface transparency of the QN to baryons etc. Later, studies using QCD - motivated dynamical models (like chromoelectric flux tube model) of baryon evaporation from SQNs have established \[27\] \[28\] that primordial SQNs with baryon numbers above \( \sim 10^{40-42} \) would be cosmologically stable.

In a previous work \[22\] by Alam, Raha and Sinha, it was shown that without much fine tuning, these stable SQNs could provide even the entire closure density (\( \Omega \sim 1 \)) \[22\]. Thus, the entire dark matter (CDM) (\( \Omega_{\text{CDM}} \sim 0.3-0.35 \)) could easily be explained by stable SQNs.

We can estimate the size of the SQNs formed in the first order cosmic QCD transition in the manner prescribed by Kodama, Sasaki and Sato \[29\] in the context of the GUT phase transition. For the sake of brevity, let us recapitulate very briefly the salient points here; for details, please see \[22\] and \[30\]. Describing the cosmological scale factor \( R \) and the coordinate radius \( X \) in the Robertson-Walker metric through the relation

\[
    ds^2 = -dt^2 + R^2 dx^2 = -dt^2 + R^2 \{ dX^2 + X^2 (sin^2 \theta d\phi^2 + d\theta^2) \}, \tag{6.1}
\]

one can solve for the evolution of the scale factor \( R(t) \) in the mixed phase of the first order transition. In a bubble nucleation description of the QCD transition, hadronic matter starts to appear as individual bubbles in the quark-gluon phase. With progressing time, they expand, more and more bubbles appear, coalesce and finally, when a critical fraction of the total volume is occupied by the hadronic phase, a continuous network of hadronic bubbles form (percolation) in which the quark bubbles get trapped, eventually
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evolving to SQNs. The time at which the trapping of the false vacuum (quark phase) happens is the percolation time $t_p$, whereas the time when the phase transition starts is denoted by $t_i$. Then, the probability that a spherical region of co-coordinate radius $X$ lies entirely within the quark bubbles would obviously depend on the nucleation rate of the bubbles as well as the coordinate radius $X(t_p,t_i)$ of bubbles which nucleated at $t_i$ and grew till $t_p$. For a nucleation rate $I(t)$, this probability $P(X, t_p)$ is given by

$$P(X, t_p) = \exp \left[ -\frac{4\pi}{3} \int_{t_i}^{t_p} dtI(t)R^3(t)[X + X(t_p, t_i)]^3 \right]. \quad (6.2)$$

After some algebra [30], it can be shown that if all the cold dark matter (CDM) is believed to arise from SQNs, then their size distribution peaks, for reasonable nucleation rates, at baryon number $\sim 10^{42-44}$, evidently in the stable sector. It was also seen that there were almost no SQNs with baryon number exceeding $10^{46-47}$, comfortably lower than the horizon limit of $\sim 10^{50}$ baryons at that time. Since $\Omega_B$ is only about 0.04 from BBN, $\Omega_{\text{CDM}}$ in the form of SQNs would correspond to $\sim 10^{51}$ baryons so that there should be $10^{7-9}$ such nuggets within the horizon limit at the microsecond epoch, just after the QCD phase transition [31, 22]. We shall return to this issue later on.

It is therefore most relevant to investigate the fate of these SQNs. Since the number distribution of the SQNs is sharply peaked [30], we shall assume, for our present purpose, that all the SQNs have the same baryon number.

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[^1]: For the QCD bubbles, there is a sizable surface tension which would facilitate spherical bubbles.
6.3 Coalescence of the primordial QN’s

The SQNs formed during the cosmic QCD phase transition at $T \sim 100$ MeV have high masses ($\sim 10^{44}$ GeV) and sizes ($R_N \sim 1$ m) compared to the other particles (like the usual baryons or leptons) which inhabit this primeval universe. These other particles cannot form structures until the temperature of the ambient universe falls below a certain critical temperature characteristic of such particles; till then, they remain in thermodynamic equilibrium with the radiation and other species of particles. This characteristic temperature is called the freezeout temperature for the corresponding particle. Obviously the freezeout occurs earlier for massive particles for the same interaction strength. In the context of cosmological expansion of the universe this has important implications; the ‘frozen’ objects can form structures. These structures do not participate in the expansion in the sense that the distance between the subparts do not increase with the scale size and only their number increases due to the cosmological scale factor.

For the SQNs, however, the story is especially interesting. Even if they continue to be in kinetic equilibrium due to the radiation pressure (photons and neutrinos) acting on them, their velocity would be extremely non relativistic. Also their mutual separation would be considerably larger than their radii; for example, at $T \sim 100$ MeV, the mutual separation between the SQNs (of size $\sim 10^{44}$ baryons) is estimated to be around $\sim 300$ m. It is then obvious that the SQNs do not lend themselves to be treated in a hydrodynamical framework; they behave rather like discrete bodies in the background of the radiation fluid. Due to their large surface area they experience quite substantial radiation pressure, in addition to gravitational forces due to the other SQNs.

In such a situation, one might be tempted to assume that since
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the SQNs are distributed sparsely in space and interact only feebly with the other SQNs through gravitational interaction, they might as well remain forever in that state. This, in fact, is quite wrong, as we demonstrate below.

The fact that the nuggets remain almost static is hardly an issue which requires justification. The two kinds of motion that they can have are random thermal motion and the motion in the gravitational well provided by the other SQNs. This other kind of motion is typically estimated using the virial theorem, treating the SQNs as a system of particles moving under mutual gravitational interaction [32, 33]. The kinetic energy ($K$) and potential energy $V$ of the nuggets at temperature $T = 100$ MeV can be estimated as,

\[
K = \frac{3}{2} Nk_b T
\]

\[
V = \sum_{i,j} G \frac{M_i M_j}{R_{i,j}} = \frac{GM_i^2 N^2}{2R_{av}}
\]

(6.3)

where $k_b$ is the Boltzmann constant, $M_i, M_j$ are the masses of the $i$th and $j$th nugget, $R_{i,j}$ is the distance between them and $R_{av}$ is the average inter-nugget distance. Substituting the number of nuggets $N = 10^7$, the baryon number of each nugget to be $10^{44}$ and $R_{av} = 300 m$, one gets $K = 2.4 \times 10^{-4}$ and $V = 3.09 \times 10^{35}$ (in MKS units) so that the ratio of $K$ and $\frac{V}{2}$ becomes $\sim 10^{-39}$. Thus it is impossible for these objects to form stable systems, orbiting round each other. On the other hand the smallness of the kinetic energy shows that gravitational collapse might be a possible fate.

Such, of course, would not be the case for any other massive particles like baryons; their masses being much smaller than SQN, the kinetic energy would continue to be very large till very low temperatures. More seriously, the Virial theorem can be applied only to
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systems whose motion is sustained. For SQNs, a notable property is that they become more and more bound if they grow in size (see Chap. [2]). Thus SQNs would absorb baryons impinging on them and grow in size. Also, if two SQNs collide, they would naturally tend to merge. In all such cases, they would lose kinetic energy, making the Virial theorem inapplicable.

One can argue that the mutual interaction between uniformly dispersed particles would prevent these particles from forming a collapsed structure, but that argument holds only in a static and infinite universe, which we know our universe is not. Also a perfectly uniform distribution of discrete bodies is an unrealistic idealization and there must exist some net gravitational attraction on each SQN. The only agent that can prevent a collapse under this gravitational pull is the radiation pressure, and indeed its effect remains quite substantial until the drop in the temperature of the ambient universe weakens the radiation pressure below a certain critical value. In what follows, we try to obtain an estimate for the point of time at which this can happen.

It should be mentioned at this juncture that for the system of discrete SQNs suspended in the radiation fluid, a detailed numerical simulation would be essential before any definite conclusion about their temporal evolution can be arrived at. This is a quite involved problem, especially since the number of SQNs within the event horizon, as also their mutual separation, keeps increasing with time. Our purpose in the present work is to examine whether such an effort would indeed be justified.

Let us now consider the possibility of two nuggets coalescing together under gravity, overcoming the radiation pressure. The mean separation of these nuggets and hence their gravitational interaction are determined by the temperature of the universe. If the en-
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tire CDM comes from SQNs, the total baryon number contained in them within the horizon at the QCD transition temperature (\( \sim 100 \) MeV) would be \( \sim 10^{51} \) (see above). For SQNs of baryon number \( b_N \) each, the number of SQNs within the horizon at that time would be just \( (10^{51}/b_N) \). Now, in the radiation dominated era the temperature dependence of density \( n_N \sim T^3 \), horizon volume \( V_H \) varies with time as \( t^3 \), i.e. \( V_H \sim T^{-6} \) and hence the variation of the total number inside the horizon volume will be \( N_N \sim T^{-3} \). So at any later time, the number of SQNs within the horizon (\( N_N \)) and their density (\( n_N \)) as a function of temperature would be given by:

\[
N_N(T) \approx \frac{10^{51}}{b_N} \left( \frac{100\text{MeV}}{T} \right)^3
\tag{6.4}
\]

\[
n_N(T) = \frac{N_N}{V_H} = \frac{3N_N}{4\pi(2t)^3}
\tag{6.5}
\]

where the time \( t \) and the temperature \( T \) are related in the radiation dominated era by the relation:

\[
t = 0.3g_*^{-1/2}\frac{m_{pl}}{T^2}
\tag{6.6}
\]

with \( g_* \) being \( \sim 17.25 \) after the QCD transition \[22\].

From the above, it is obvious that the density of SQNs decreases as \( t^{-3/2} \) so that their mutual separation increases as \( t^{1/2} \). Therefore, the force of their mutual gravitational pull will decrease as \( t^{-1} \). On the other hand, the force due to the radiation pressure (photons and neutrinos) resisting motion under gravity would be proportional to the radiation energy density, which decreases as \( T^4 \) or \( t^{-2} \). It is thus reasonable to expect that at some time, not too distant, the gravitational pull would win over the radiation pressure, causing the SQNs to coalesce under their mutual gravitational pull. The expression for the gravitational force as a function of temperature
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T can written as:

\[ F_{\text{grav}} = \frac{G b_n^2 m_n^2}{\bar{r}_{nn}^2} \]  

(6.7)

where \( b_n \) is the baryon number of each SQN and \( m_n \) is the baryon mass. \( \bar{r}_{nn}(T) \) is the mean separation between two nuggets and is given by the cube root of the ratio \( \kappa \) of total volume available and the total number of nuggets. One can roughly estimate how \( \kappa \) varies with temperature in the following way. The time-temperature relation eqn. 6.6 can be written in the form

\[ t = \frac{0.8324}{T^2} \]

where \( t \) is in seconds and \( T \) is the temperature in MeV. Using this, the horizon radius \( R_H = 2ct \), expressed in conventional units (m) is given by

\[ R_H = \frac{1.665c}{T^2} \]

and the horizon volume \( V_H = \frac{19.328 c^3}{T^6} \). Finally, writing \( b_n \approx 10^x \) one can get the following approximate expression for \( n_N \)

\[ n_N(T) = 10^{51-x} \left( \frac{100}{T} \right)^3 \]

In the following eqn. 6.8, we have substituted the value \( x \to 44 \) (or, \( b_n = 10^{44} \)) in the ratio \( V_H/n_N \) to get an estimate for \( \kappa \).

\[ \kappa = \frac{1.114 \times 10^{-12} c^3}{T^3} \]  

(6.8)

The force due to the radiation pressure on the nuggets may be roughly estimated as follows. We consider two objects (of the size of a typical SQN) approaching each other due to gravitational interaction, overcoming the resistance due to the radiation pressure. The usual isotropic radiation pressure is \( \frac{1}{3} \rho c^2 \), where \( \rho \) is the to-
6.3. COALESCEENCE OF THE PRIMORDIAL QN’S

total energy density, including all relativistic species. The nuggets will have to overcome an additional pressure resisting their mutual motion, which is given by $\frac{1}{3} \rho c^2 (\gamma - 1)$; the additional pressure arises from a compression of the radiation fluid due to the motion of the SQN. The moving SQN would become an oblate spheroid (with its minor axis in the direction of motion due to Lorentz contraction), whose surface area is given by

$$S = 2\pi a^2 + \frac{2\pi ab \sin^{-1} \epsilon}{\epsilon}$$

where $a$ is the length of the major axes perpendicular to the direction of motion, $b$ is the length of the smaller axis & where the ellipticity $\epsilon$ is given by

$$\epsilon = \sqrt{1 - \frac{b^2}{a^2}}$$

In this case $a \rightarrow R_N$ & $b \rightarrow R_N/\gamma$, where $\gamma$ is the Lorentz factor corresponding to the moving SQM. With these substitutions $\epsilon = \sqrt{\frac{\gamma^2 - 1}{\gamma}}$ and

$$S = 2\pi R_N^2 \left(1 + \frac{\sin^{-1} \epsilon}{\gamma \epsilon}\right)$$

for small values of $\epsilon$ (small $\gamma$), $\sin^{-1} \epsilon \sim \epsilon$, so that the surface area becomes $2\pi R_N^2 \frac{\gamma^2 - 1}{\gamma}$. Thus the total radiation force resisting the motion of SQNs is

$$F_{\text{rad}} = \frac{1}{3} \rho_{\text{rad}} c v_{\text{fall}} (\pi R_N^2) \beta \gamma$$

(6.9)

where $\rho_{\text{rad}}$ is the total energy density at temperature $T$, $v_{\text{fall}}$ or $\beta c$ is the velocity of SQNs determined by mutual gravitational field and $\gamma$ is $1/\sqrt{1 - \beta^2}$. The quantities $F_{\text{rad}}$, $\beta$ and $\gamma$ all depend on the temperature of the epoch under consideration. (It is worth mentioning at this point that the $t$ dependence of $F_{\text{rad}}$ is actually $t^{-5/2}$, sharper than the $t^{-2}$ estimated above, because of the $v_{\text{fall}},$
6.3. COALESCE OF THE PRIMORDIAL QN’S

which goes as $t^{-1/4}$.) The ratio of these two forces is plotted against temperature in figure [6.1] for two SQNs with initial baryon number $10^{42}$ each. It is obvious from the figure that ratio $F_{\text{grav}}/F_{\text{rad}}$ is very small initially. As a result, the nuggets will remain separated due to the radiation pressure. For temperatures lower than a critical value $T_{\text{cl}}$, the gravitational force starts dominating, facilitating the coalescence of the SQNs under mutual gravity.

Let us now estimate the mass of the clumped SQNs, assuming that all of them within the horizon at the critical temperature will coalesce together. This is in fact a conservative estimate, since the SQNs, although starting to move toward one another at $T_{\text{cl}}$, will take a finite time to actually coalesce, during which interval more SQNs will arrive within the horizon.

In table [6.1] we show the values of $T_{\text{cl}}$ for SQNs of different initial baryon numbers along with the final masses of the clumped SQNs under the conservative assumption mentioned above.

It is obvious that there can be no further clumping of these already clumped SQNs; the density of such objects would be too small within the horizon for further clumping. Thus these objects would survive till today and perhaps manifest themselves as MA-

| $b_N$ | $T_{\text{cl}}$ (MeV) | $N_N$       | $M/M_\odot$ |
|-------|-----------------------|-------------|--------------|
| $10^{42}$ | 1.6                  | $2.44 \times 10^{14}$ | 0.24         |
| $10^{44}$ | 4.45                 | $1.13 \times 10^{11}$ | 0.01         |
| $10^{46}$ | 20.6                 | $1.1 \times 10^7$     | 0.0001       |

Table 6.1: Critical temperatures ($T_{\text{cl}}$) of SQNs of different initial sizes $b_N$, the total number $N_N$ of SQNs that coalesce together and their total final mass in solar mass units.
Figure 6.1: Variation of the ratio $F_{\text{grav}}/F_{\text{rad}}$ with temperature. The dot represents the point where the ratio assumes the value 1.
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CHOs. It is to be reiterated that the masses of the clumped SQNs given in table 6.1 are the lower limits and the final masses of these MACHO candidates will be larger. (The case for \( b_N = 10^{46} \) is not of much interest, especially since such high values of \( b_N \) are unlikely for the reasonable nucleation rates \([34, 30]\); we therefore restrict ourselves to the other cases in table 6.1 in what follows.) A more detailed estimate of the masses will require a detailed simulation, but very preliminary estimates indicate that they could be 2-3 times bigger than the values quoted in table 6.1.

The total number of such clumped SQNs (\( N_{\text{macho}} \)) within the horizon today is evaluated in the following way. With the temperature \( \sim 300 \, K \) and time \( \sim 4 \times 10^{17} \) seconds, the total amount of visible baryons within the horizon volume can be evaluated using photon to baryon ratio \( \eta \sim 10^{-10} \). The amount of baryons in the CDM will be \( \frac{\Omega_{\text{CDM}}}{\Omega_B} \) times the total number of visible baryons. This comes out to be \( \sim 1.6 \times 10^{79} \), \( \Omega_{\text{CDM}} \) and \( \Omega_B \) being 0.3 and 0.01\(^{ii}\) respectively. The total number of baryons in a MACHO is \( b_N \times N_N \) i.e. \( 2.44 \times 10^{56} \) and \( 1.13 \times 10^{55} \) for initial nugget sizes \( 10^{42} \) and \( 10^{44} \) respectively. The quantities \( b_N \) and \( N_N \) are taken from the Table 6.1. So dividing the total number of baryons in CDM by that in a MACHO, the \( N_{\text{macho}} \) comes out to be in the range \( \sim 10^{23-24} \).

We can also mention here that if the MACHOs are indeed made up of quark matter, then they cannot grow to arbitrarily large sizes. Within the (phenomenological) Bag model picture \([35]\) of QCD confinement, where a constant vacuum energy density (called the Bag constant) in a cavity containing the quarks serves to keep them confined within the cavity, we have earlier investigated \([36]\) the upper limit on the mass of astrophysical compact quark matter

\(^{ii}\)The visible baryons occur in two forms. As visible stars they make up about 0.3 - 0.6 % and as hot intergalactic gas, they contribute about 0.5 % of the total density, i.e about 0.8 - 0.11 % in all.
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jects. It was found that for a canonical Bag constant $B$ of $(145 \text{ MeV})^4$, this limit comes out to be $1.4 \, M_\odot$ (see Chap. 5). The collapsed SQNs are safely below this limit. (It should be remarked here that although the value of $B$ in the original MIT bag model is taken to be $B^{1/4} = 145 \text{ MeV}$ from the low mass hadronic spectrum, there exist other variants of the Bag model \cite{37}, where higher values of $B$ are required. Even for $B^{1/4} = 245 \text{ MeV}$, this limit comes down to $0.54 \, M_\odot$ \cite{36}, which would still admit such SQN.

As a consistency check, we can perform a theoretical estimate of the abundance of such MACHOs in the galactic halo which is conventionally given by the optical depth. The optical depth is the probability that at any instant of time a given star is within an angle $\theta_E$ of a lens, the lens being the massive body (in our case MACHO) which causes the deflection of light. In other words, optical depth is the integral over the number density of lenses times the area enclosed by the Einstein ring of each lens. The expression for optical depth can be written as \cite{38}:

$$\tau = \frac{4\pi G}{c^2} D_s^2 \int \rho(x) x (1-x) dx \quad (6.10)$$

where $D_s$ is the distance between the observer and the source, $G$ is the gravitational constant and $x = D_d D_s^{-1}$, $D_d$ being the distance between the observer and the lens (Fig. 6.2). In particular $\rho$ is the mass-density of the MACHOs, which is of the form $\rho = \rho_0 \frac{1}{\pi}$ in the naive spherical halo model, which we have adopted in our calculations. In the present case $\rho_0$ is given by

$$\rho_0 = \frac{M_{\text{macho}} \times N_{\text{macho}}}{4\pi R} \quad (6.11)$$

where $R = \sqrt{D_e^2 + D_s^2 + 2D_e D_s \cos \phi}$, $\phi$ and $D_e$ being the inclination of the LMC and the distance of observer (earth) from the Galactic
center respectively. $M_{\text{macho}}$ and $N_{\text{macho}}$ are the mass of a MACHO and the total number of MACHOs in the Milky Way halo.

The total visible mass of the Milky Way ($\sim 1.6 \times 10^{11} M_\odot$) is equivalent to the mass of $\sim 2 \times 10^{68}$ baryons. This corresponds to a factor of $\sim 2 \times 10^{-9}$ of all the visible baryons within the present horizon. Scaling the number of clumped SQNs within the horizon by the same factor yields a total number of MACHOs, $N_{\text{macho}} \sim 10^{13-14}$ in the Milky Way halo for the range of baryon number of initial nuggets $b_N = 10^{42-44}$. The value of $D_e$ and $D_s$ are taken to be 10 and 50 kpc, respectively. The value of the inclination angle used here is 40 degrees. Using these values for a naive inverse square spherical model comprising such objects upto the LMC, we obtain an optical depth of $\sim 10^{-6} - 10^{-7}$. The uncertainty in this value is mainly governed by the value of $\eta$, $\Omega_{\text{CDM}}$, and $\Omega_B$, and to a lesser extent by the specific halo model. This value compares reasonably well with the observed value [7, 8] and may be taken as a measure of reliability in the proposed model.

As an interesting corollary, let us mention that the scenario presented here could have other important astrophysical significance. The origin of cosmic rays of ultra-high energy $\geq 10^{20}$ eV continues to be a puzzle. One of the proposed mechanisms [39] envisages a top-down scenario which does not require an acceleration mechanism and could indeed originate within our galactic halo. For our picture, such situations could easily arise from the merger of two or more such MACHOs, which would shed the extra matter so as to remain within the upper mass limit mentioned above.

We thus conclude that gravitational clumping of the primordial SQNs formed in a first order cosmic quark - hadron phase transition appears to be a plausible and natural explanation for the observed halo MACHOs. It is quite remarkable that we obtain quan-
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Figure 6.2: Geometry of a gravitational lens system. The light ray propagates from the source S, and while passing the lens, gets deflected and reaches the observer at O. I is the image of the source S. The distances between the observer and the source, the observer and the lens and the lens and the source are $D_s, D_d, D_{ds}$ respectively. OA is the optic axis. The figure has been adapted from [38].
titative agreement with the experimental values without having to introduce any adjustable parameters or any fine-tuning whatsoever.

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Conclusions

The present work is based on our quest for understanding the possible role played by strange quark matter in the dark matter content of the universe, which is believed to comprise more than 90% of the total material content of the universe. The viewpoint which has emerged out of this effort can be summarized as follows.

At the age of a few microseconds, the universe underwent a possibly first order phase transition. This phase transition is responsible for generating the hadrons, which, before the transition existed in the form of Quark-Gluon Plasma; the baryon number of the universe was therefore contained in the quarks prior to this phase transition. These hadrons, namely the baryons among them, would eventually form the lighter elements through nucleosynthesis at an Universal age of $\sim 180$ seconds (Tab. [1.1]). Part of these light elements would be resynthesized by stars to heavier varieties and some of it would spill all over the world due to supernovae explosions. Some of these matter would be luminous and the rest of it would exist in the form of dark gas – but the total contribution to the matter sector, due to all such forms of matter, collectively called nuclear matter, would not be able to account for the fact the fact that we live in a nearly closure density universe. The missing matter can however be very well accounted by Quasibaryonic matter if one takes into account the fact that the bulk of the baryon
number content of the universe gets concealed within SQN’s which also form out of the aforementioned phase transition. It may be mentioned at this point that although all of what has been said before remains valid irrespective of whether the phase transition proceeds through a second order (or even continuous) process, but the formation of SQN’s require the phase transition to be first order. The SQN’s (The Acronym SQN first appear in [1,4], thus formed are massive objects which tend to clump together under mutual gravitational attraction, but prevented by the radiation pressure during the radiation dominated phase of the universe. As the universe expands, it cools down and as shown in Chap. 6, these objects could coalesce together forming objects in the half solar mass range. This happens when the Universe has cooled down to $\sim 1 - 10$ MeV, the exact time depending on the initial size of the SQN’s. The objects, so formed, would be more or less uniformly distributed through the volume of the universe and have properties which are characteristic for dark matter candidates. We have calculated the expected number of such candidates at the current time residing in the Milky way halo and estimated their optical depth for gravitational microlensing experiments which look for dark lenses in the galactic halo. The value of the optical depth, so obtained, compare reasonably well with the observed values and may be taken to be a measure of the reliability of the dark matter model proposed by us. It should be emphasized that a definite conclusion can only be reached after a detailed simulation is carried out. The central value of the mass range of the dark halo lenses are typically in the $0.5 M_\odot$ range. The masses of the clumped SQN’s can however be more or less than this value, depending on the time when they clump. The results obtained in Chap. 5 however indicates that the dark SQM lenses cannot be much heavier than this,
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since the maximum mass range of compact quark matter objects is in the same range. This so called Chandrasekhar Limit does not depend strongly on the number of quark flavors that goes into the making of the stars and is also applicable to the lens SQN’s. Thus although gravitational clumping might produce over-sized SQN’s, these would have to disintegrate, in order to maintain stability to sizes $\sim 0.5 M_{\odot}$ or less. The halo SQN’s are therefore suitable candidates for the MACHO’s found in the gravitational microlensing experiments. In the course of evaluation of the maximum mass limit for a configuration of massless quarks through an analytic procedure, we have adopted the Landau picture of energy balance through the density dependent quark mass model of confinement which was appropriate for the situation.

According to our picture, the local source of dark matter arises mainly from SQM blobs in the half solar mass range. It is therefore quite possible that occasional collisions of such objects can release bursts of small, atomic sized strangelets in every possible directions, and a few of them may be intercepted by our planet as well. In Chap.4 we examine the probability of observing such particles in Earth based experiments by various means. It has been emphasized earlier in this work (Chap.2 and Chap.3) that the detection of strangelets is crucial both from the standpoint of astrophysics as well as strong interaction physics. For astrophysics it can provide confirmation for the nature of dark matter - it’s detection will have the implication that dark matter is quasibaryonic in form. This will also help the Nuclear desert to be filled up with intermediate baryon number objects between atomic species to nuclear stars. For QCD it will provide experimental justification for Witten’s conjecture that the true ground state of strong interaction physics is SQM rather than the ordinary nuclear matter.
In Chap.2 and Chap.3 we have therefore examined thoroughly the problem of strangelet propagation through the terrestrial atmosphere. In order for the model to be consistent with the hypothesis of stability of strange matter with respect to ordinary nuclear matter, the strangelets have been invested with the extraordinary property of absorbing a fraction of atmospheric particles which are incident on it. This property is in stark contrast with the passage of an ordinary (heavy) cosmic ray particle which usually breaks up under such impact. In this model the strangelet grows like a snowball, absorbing mass and also some charge from the atmospheric particles; however its energy decreases due largely to the ionization loss of the surrounding media and partly due to the impacts. In fact the energy decreases so much that they go beyond the range of detectability of passive Solid State Nuclear Track Detector’s below typical mountain altitudes. The study reveals two important aspects: for one, it reproduces the observed pattern of several exotic cosmic ray events (very small $e/m$ ratio and detection at atmospheric depths much higher than ordinary cosmic ray particles, but not lower than typical mountain altitudes, as well as the value of the charges and masses found at those altitudes.) fairly well, suggesting that these events, previously unclassified, can now be associated, quite justifiably, with the passage of strangelets through the atmosphere. Secondly, the estimated energy deposition of the particles in SSNTD’s like CR-39 show that these are just above the threshold of detection at mountain altitudes, indicating that a ground based large area detector array of SSNTD’s might be quite capable of picking up strangelet signals.

Throughout our work we argue that strange matter is an essential component of the dark matter forms present in the Universe.
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The whole work, save for the part that deals with the maximum mass limit of quark stars, depend rather crucially on two factors:

1. The strange matter hypothesis.
2. The existence of the first order Quark-Hadron deconfining phase transition in the early universe.

The general agreement is that both are well founded hypothesis; however, none of them, so far, has passed the test of experimental verification. It thus appears that, once again, cosmological experiments will be able to disentangle issues which accelerator based experiments may not be able to address.

In the course of this work, several threads have emerged, which can extend the ideas developed in several ways. In Chap.6, as well as earlier in this chapter, we have mentioned that a detailed simulation is needed before one can reach a conclusion about the formation of half solar mass objects by the coalescence of the SQN’s. We propose to undertake an extensive numerical study of this case in near future. The idea behind the model of neutron absorption by SQM, used in Chap.2 is also applicable in a cosmological setting, since the depletion of baryons near SQN’s can cause local baryon inhomogeneities and affect the nucleosynthesis in unknown ways. The network of SQN’s, in some ways, form the first instances of structure in the early Universe. Inspired by the applicability of the density dependent quark mass model to quark star systems, we also propose to study the issue of the stability of SQM in the context of this model. In the present work we have not examined the role of SQM in the dark energy content of the universe, but there do exist some theoretical hints to assume that they might play an important role in providing for the acceleration of the universal expansion rate. Some work in this direction is already in progress,
but it already raises some intriguing questions which seem to be tied to the foundations of quantum mechanics; in particular, the effect of quantum entanglement in relativistic many body systems need to be explored much further before a definite commitment can be made.

In a nutshell, then, we have argued that the standard model of particle interactions and the strange matter hypothesis together can account for the cosmological dark matter problem, without having to resort to exotic reformulations of the physics of particle interactions.
List of Publications

1. Strangelets in terrestrial atmosphere, with S.K.Ghosh, S.Raha & D.Syam, *Journal of Physics*, 1999, **G25** L15

2. The Chandrasekhar limit for quark stars, With S.K.Ghosh & S.Raha, *Journal of Physics*, 2000 **G26** L1

3. Can cosmic strangelets reach the Earth?, With S.K.Ghosh S.Raha & D.Syam, *Physical Review Letters*, 2000, **85** 1384

4. Strange quark matter in cosmic rays and exotic events, With S.K.Ghosh, A.Mazumdar, S.Raha & D.Syam, *Astrophysics & Space Science*, 2000, **274** 655

5. Massive compact halo objects from the relics of the cosmic quark-hadron transition, With A.Bhattacharyya, S.K. Ghosh, S.Raha, Bikash Sinha and H.Toki, *Monthly Notices of the Royal Astronomical Society*, 2003 **340** 284

6. Relics of the cosmic quark-hadron phase transition and massive compact halo objects, With A.Bhattacharyya, S.K. Ghosh, S.Raha, Bikash Sinha and H.Toki, *Nuclear Physics*, 2003, **A715** 827

7. Some aspects of strangeness in astrophysics and cosmology, With A.Bhattacharyya, S.K. Ghosh, S.Raha, Bikash Sinha and H.Toki, *Nuclear Physics*, 2003, **A721**, 1028

8. Quantum chromodynamics, phase transition in the early universe and quark nuggets, With A.Bhattacharyya, S.K. Ghosh, S.Raha, Bikash Sinha and H.Toki, 2003, *Pramana - Journal of Physics* **60**, 909