Probing Axion-like Particles via CMB Polarization

Tomohiro Fujita,1 Yuto Minami,2 Kui Murai,1,3 and Hiromasa Nakatsuka1

1 Institute for Cosmic Ray Research, The University of Tokyo, Kashiwa, 277-8582, Japan
2 High Energy Accelerator Research Organization, 1-1 Oho, Tsukuba, Ibaraki, 305-0801, Japan
3 Kavli Institute for the Physics and Mathematics of the Universe (WPI), The University of Tokyo, Kashiwa 277-8583, Japan

Axion-like particles (ALPs) rotate linear polarization of photons through the ALP-photon coupling and convert the cosmic microwave background (CMB) E-mode to the B-mode. We derive the relation between the ALP dynamics and the rotation angle by assuming that the ALP Φ has a quadratic potential, V = m^2Φ^2/2. We compute the current and future sensitivity of the CMB measurements to the ALP-photon coupling g, which can reach g = 4 \times 10^{-21}\,\text{GeV}^{-1} for 10^{-32}\,\text{eV} \lesssim m \lesssim 10^{-28}\,\text{eV} and extensively exceed the other searches for any mass with m \lesssim 10^{-25}\,\text{eV}. We also find that the fluctuation of the ALP field at the observer, which has been neglected in previous studies, can induce the significant isotropic rotation of the CMB polarization. The measurements of isotropic and anisotropic rotation allow us to put bounds on relevant quantities such as the ALP mass and the ALP density parameter Ωφ. In particular, if LiteBIRD detects anisotropic rotation, we obtain the lower limit on the tensor-to-scalar ratio as r > 5 \times 10^{-9}.

I. INTRODUCTION

Axion has attracted the wide range of interests in particle physics and cosmology. Peccei and Quinn originally introduced the QCD axion to solve the strong CP problem [1]. In recent decades, it has been found that the string theory predicts a number of axion-like particles (ALPs), which have the broad range of mass and the couplings to gauge fields. Such ALPs are expected to be ubiquitous in our universe, which provides a paradigm called small-scale crisis in cosmology [3–7]. Moreover, an extremely light ALP can be responsible for dark energy [8–12].

The axions and ALPs are intensively searched for in various methods [13–15]. Especially, the coupling to photon enables one of the most promising detection schemes. When photons travel through the axion background, their polarization angle rotates, which is known as “cosmic birefringence” [13–19]. To measure the rotation angle, we need to observe some known polarized photons, e.g. astronomical targets [20, 21], laser interferometers [22–24], and cosmic microwave background (CMB) [25–27]. The CMB photons acquire uncorrelated E- and B-mode polarization when they are emitted at the last scattering surface (LSS). The cosmic birefringence mixes the E- and the B-mode, which results in the EB cross correlation [10, 28–30]. Since the EB cross correlation vanishes in parity-conserving models, its detection is a smoking gun of parity-violating phenomena such as the ALP-photon coupling.

In this letter, we formulate the polarization rotation of CMB photons induced by the ALP field. We find that three components of the ALP field independently contribute to the rotation of the CMB polarization. We also discuss that the oscillation of axion during the LSS suppresses the polarization rotation. Finally we show the current and future sensitivity of the CMB observations to the ALP-photon coupling depending on the ALP mass.

II. BIREFRINGENCE BY ALP

Let us consider a Lagrangian with an ALP field φ coupled to photon:

\[ \mathcal{L} = -\frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{4} g \phi F_{\mu \nu} \tilde{F}^{\mu \nu}, \]

where V(φ) is the potential of φ, g is the coupling constant, F_{\mu \nu} is the field strength of photon, and \( \tilde{F}^{\mu \nu} \equiv \epsilon^{\mu \nu \rho \sigma} F_{\rho \sigma}/(2\sqrt{-g}) \) is its dual.

When photons travel in a non-zero axion background, the ALP-photon coupling rotates linear polarization plane of photons by [31]

\[ \alpha = \frac{g}{2} \Delta \phi \equiv \frac{g}{2} (\phi_{\text{obs}} - \phi_{\text{emit}}), \]

where the subscripts ‘obs’ and ‘emit’ denote the observation point and the emission point of photons in interest. Thus α does not depend on any dynamics between the boundaries and it purely measures the difference of the axion field value between the two positions in the space-time.

The CMB observations can detect the direction dependent rotation angle α(\hat{n}) of photons, which travel from the last scattering surface to an observation point. Then the rotation angle depends on

\[ \phi_{\text{obs}} = \phi(t_0, \mathbf{0}), \quad \phi_{\text{LSS}} = \phi(t_{\text{LSS}}, d_{\text{LSS}} \hat{n}), \]

where t_0 is the present time, and t_{\text{LSS}} and d_{\text{LSS}}\hat{n} denote the time and the position of the LSS, respectively. Note that the LSS has a finite thickness (~ 10^5 lightyears)
which may not be negligible, if \( \phi \) significantly evolves around \( t = t_{\text{LSS}} \),

We decompose the axion field into the background and the perturbation part, \( \phi(t, x) = \bar{\phi}(t) + \delta \phi(t, x) \). This decomposition reduces \( \Delta \phi \) in Eq. (2) into

\[
\Delta \phi = \Delta \bar{\phi} + \delta \phi_{\text{obs}} - \delta \phi_{\text{LSS}},
\]

with \( \Delta \bar{\phi} \equiv \bar{\phi}_{\text{obs}} - \bar{\phi}_{\text{LSS}} \). The first term in Eq. (4) can be easily calculated by solving the background dynamics of \( \phi \). The last term is directly given by the primordial perturbation. The second term \( \delta \phi_{\text{obs}} \) was overlooked in the literature. \( \Delta \bar{\phi} \) and \( \delta \phi_{\text{obs}} \) produce the isotropic birefringence, while \( \delta \phi_{\text{LSS}} \) leads to the anisotropic birefringence. Even when the background contribution \( \Delta \bar{\phi} \) is negligible, \( \delta \phi_{\text{obs}} \) statistically breaks the parity symmetry in the CMB polarization and can serve as an important probe, as we will see below.

Here we elaborate the decomposition of \( \phi \) into \( \bar{\phi} \) and \( \delta \phi \) in Fourier space. The contributions to \( \phi(t, x) \) from the Fourier modes with wavelengths longer than \( d_{\text{LSS}} \) take the same value at the observer and the LSS position. On the other hand, if the wavelengths of modes shorter than \( d_{\text{LSS}} \), their contributions are uncorrelated, because their phases are different at these distant positions. This fact can also be understood based on the quantum origin of the modes. When the long (short) modes exited the horizon during inflation, the two spatial points belonged to the same (different) horizon patch(es) and the quantum fluctuations freeze at the same (different) value(s) due to the causality. Thus, we decompose the background and perturbation in the following way:

\[
\bar{\phi}(t) := \int_0^{k_s} \frac{dk}{2\pi} e^{i k x} \phi_k(t),
\]

\[
\delta \phi(t, x) := \int_{k_s}^{\infty} \frac{dk}{2\pi} e^{i k x} \phi_k(t),
\]

where \( k_s = d_{\text{LSS}}^{-1} \approx H_0/3 \) is the splitting scale, \( H_0 \equiv H(t_0) \) denotes the Hubble constant at \( t_0 \), and we set the present scale factor \( a(t_0) = 1 \). Here, \( \phi_{\text{obs}}(t) \) and \( \phi_{\text{LSS}}(t) \) describe the same background dynamics, while \( \delta \phi_{\text{obs}} \) and \( \delta \phi_{\text{LSS}} \) are uncorrelated. We can separately calculate \( \delta \phi_{\text{obs}} \) and \( \delta \phi_{\text{LSS}} \) as independent perturbations. We assume that \( \bar{\phi}(t) \) has the Gaussian distribution and then its statistical information is all encoded in the power spectrum \( P_\phi(\ell, k) \) defined by \( \langle \phi_k(t)\phi_p(t) \rangle = (2\pi)^3\delta(k-p)2\pi^2 P_\phi(t, k) \).

The observational signatures of \( \delta \phi_{\text{obs}} \) and \( \delta \phi_{\text{LSS}} \) are distinct. The spatial distribution of \( \delta \phi_{\text{LSS}} \) results in the direction dependent rotation angle \( \alpha(\hat{n}) \), i.e., the anisotropic birefringence. The sensitivity of the CMB observations to the anisotropic birefringence is often characterized by \( A_\alpha \equiv L(L+1)C_{\ell\alpha}^2/(2\pi) \), where \( C_{\ell\alpha}^2 \) is the angular power spectrum of the rotation angle \( \bar{\phi} \). We assume that the scale-invariant power spectrum of the ALP field, \( P_\phi(t_{\text{init}}) = (H_1/(2\pi))^2 \), is produced with the Hubble parameter \( H_1 \) during inflation. Then we find a simple relation for \( L \lesssim 100, \)

\[
A_\alpha = \frac{q^2}{4} P_\phi(t_{\text{LSS}}),
\]

where we ignore the ALP mass. Thus, the power spectrum of \( \delta \phi_{\text{LSS}} \) is measured through the multipoles of the CMB polarization anisotropy.

On the other hand, any observations detect only a single realization of \( \delta \phi_{\text{obs}} \) at the observer’s point which contributes to the isotropic rotation angle. Since the mean value of the perturbation part always vanishes \( \langle \delta \phi \rangle = 0 \), the typical amplitude of \( \delta \phi_{\text{obs}} \) is estimated by its variance,

\[
\langle \delta \phi_{\text{obs}}^2 \rangle = \int_{k_s}^{\infty} \frac{dk}{k^3} P_\phi(t_0, k).
\]

Combined with the background contribution, the isotropic rotation angle \( \bar{\alpha} \) is given by

\[
\bar{\alpha} = \frac{q}{2} (\Delta \bar{\phi} + \delta \phi_{\text{obs}}).
\]

In summary, the cosmic birefringence by ALP has the following three contributions: (i) the ALP background motion \( \Delta \bar{\phi} \equiv \bar{\phi}_{\text{obs}} - \bar{\phi}_{\text{LSS}} \), (ii) the anisotropic distribution of the ALP field at the LSS \( \delta \phi_{\text{LSS}} \), and (iii) the ALP field fluctuation at the observer \( \delta \phi_{\text{obs}} \). \( \Delta \bar{\phi} \) and \( \delta \phi_{\text{obs}} \) generate the isotropic birefringence \( \bar{\alpha} \), and \( \delta \phi_{\text{LSS}} \) generates the anisotropic birefringence measured by \( A_\alpha \). Even when the axion field does not change overtime, (ii) and (iii) contributions still remain because of the spatial fluctuations, whereas \( \Delta \bar{\phi} \) vanishes.

### III. ALP FIELD DYNAMICS

In this letter, henceforth, we consider birefringence caused by an ALP field with the quadratic mass term,

\[
V(\phi) = \frac{1}{2} m^2 \phi^2.
\]

In the spatially flat Friedmann-Lemaître-Robertson-Walker universe, the metric perturbation is given by

\[
d^2s^2 = a^2(\eta) \left[ -d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j \right],
\]

where we choose the synchronous gauge and \( \eta \) is the conformal time. Note that \( h_{ij} \) includes both scalar components and tensor components. Then the equations of motion (EoMs) for the background and the perturbation are obtained as

\[
\ddot{\bar{\phi}}'' + 2H \dot{\bar{\phi}}' + a^2 m^2 \bar{\phi} = 0,
\]

\[
\ddot{\delta \phi}'' + 2H \dot{\delta \phi}' - \nabla^2 \delta \phi + a^2 m^2 \delta \phi = -\frac{1}{2} \delta \phi',
\]

\[
\delta \phi'' = \frac{2H \delta \phi'}{a^2 m^2}.
\]
where $h$ is the trace of $h_{ij}$. Here, the source term, $-\frac{1}{2} h \dot{\phi}'$, describes that the ALP perturbation is induced by the adiabatic perturbation in proportion to the ALP background motion $\dot{\phi}'$ [32, 33]. Nevertheless, we conservatively neglect the source term in this letter to estimate the robust contribution from $\delta \phi$ to birefringence, which is inevitably generated by inflation irrespective of the background dynamics $\phi(t)$. Note that $\delta \phi_{\text{LSS}}$ is not affected by the source term in any case, because $\phi$ does not evolve for $t < t_{\text{LSS}}$ in the mass region of our interest.

For pedagogical purposes, we solve the above EoMs for $\phi$ and $\delta \phi_{\text{obs}}$ in the Einstein-de Sitter (EdS) universe and discuss their behaviors based on the analytic expressions. We will show the numerical results with the $\Lambda$CDM model in the next section.

Solving Eq. (11), we obtain the simple expressions for $\Delta \phi \equiv (\phi_{\text{obs}} - \phi_{\text{LSS}})$ in two mass regions,

$$|\Delta \phi| \simeq \left\{ \begin{array}{ll} \frac{2}{3} \sqrt{\frac{2}{3} \Omega_\phi} \frac{m}{M_{\text{Pl}}} M_{\text{Pl}} & (m \ll t_0^{-1}) \\ \frac{2}{3} \sqrt{\frac{2}{3} \Omega_\phi} M_{\text{Pl}} & (t_0^{-1} \ll m \ll t_{\text{LSS}}^{-1}) \end{array} \right.,$$

where we introduced the density parameter of the ALP field, $\Omega_\phi \equiv (\dot{\phi}^2 + m^2 \phi^2)/6 M_{\text{Pl}}^2 H_0^2$. In the upper case in Eq. (13), since the ALP mass is too small to start oscillating until now, it is always in the slow-roll regime; thus $\phi$ evolves only a little. In the lower case, on the other hand, the ALP began to oscillate much later than $t_{\text{LSS}}$ and much earlier than the present epoch, and thus the ALP field value has been damped by the Hubble friction, which leads to $|\phi_{\text{obs}}| \ll |\phi_{\text{LSS}}|$. As a result, $\Delta \phi$ is suppressed by the factor $m/H_0$ in the lighter ALP case compared to the intermediate mass case with $t_0^{-1} \ll m \ll t_{\text{LSS}}^{-1}$.

The analytic solution for Eq. (12) in the matter-dominated universe also gives the approximate solution of $\delta \phi_{\text{obs}}$. Ignoring the mass and source term, one finds the solution in Fourier space as

$$\phi_k(\eta) = \frac{3 \delta \phi_{\text{in}}}{k^2 \eta^2} \left[ \frac{\sin(k \eta) - k \eta \cos(k \eta)}{k \eta} \right].$$

where we used the initial condition $\phi_k(\eta_0) = \hat{\phi}_{\text{in}}^k$ and $\phi_k(\eta_0) = 0$ with a initial time $\eta_0$ which satisfies $k \eta_0 \ll 1$ for relevant wavenumbers. The variance of $\delta \phi_{\text{obs}}$ is computed as,

$$\langle \delta \phi_{\text{obs}}^2 \rangle = 9 P_{\phi_{\text{in}}} \int_{k_0}^{\infty} \frac{dk}{k} \left[ \frac{\sin(k \eta_0) - k \eta_0 \cos(k \eta_0)}{k \eta_0} \right]^2 \equiv P_{\phi_{\text{in}}} \left[ \frac{7}{4} - \gamma_E - \ln(2 k_0 \eta_0) + O(k_0^2 \eta_0^2) \right],$$

where we used $\hat{\phi}_{\text{in}}^k = (2\pi)^3 \delta(k - p) \frac{2}{p^2} P_{\phi_{\text{in}}}(k)$ and assumed that $P_{\phi_{\text{in}}}$ is scale-invariant. The above result implies that the typical size of $|\delta \phi_{\text{obs}}|$ is $(P_{\phi_{\text{in}}})^{1/2}$, while its actual value in our universe is determined only in a stochastic manner due to cosmic variance.

Finally we consider the damping effect from the thickness of the LSS which is relevant for $m \gtrsim t_{\text{LSS}}^{-1}$. If the ALP has already started oscillating at the time of last scattering, $\phi_{\text{LSS}}$ does not take a single value and its time variation should be taken into account. The visibility function, $g(T)$, describes the probability density that a CMB photon, now observed, scattered at the cosmic temperature $T$. We approximate $g(T)$ by a Gaussian function,

$$g(T) \simeq \frac{1}{\sqrt{2\pi \sigma_T^2}} \exp \left[ -\frac{(T - T_L)^2}{2 \sigma_T^2} \right],$$

where $T_L = 2941$ K and $\sigma_T = 248$ K are the fitting parameters of the visibility function [34]. Thus, the effective background value at the last scattering which replaces $\phi_{\text{LSS}}$ is given by

$$\langle \tilde{\phi}_{\text{LSS}} \rangle = \int dT g(T) \phi(t(T)).$$

We numerically find that $\langle \tilde{\phi}_{\text{LSS}} \rangle$ exponentially decays as $m$ increases for $m \gtrsim t_{\text{LSS}}^{-1}$. Note that, for an even heavier ALP, $\langle \tilde{\phi}_{\text{LSS}} \rangle$ becomes smaller than $|\phi_{\text{obs}}|$, and then $\phi_{\text{obs}}$ dominates $\Delta \phi$ in contrast to the previous cases with $m \ll t_{\text{LSS}}^{-1}$. We also apply this damping effect to $\delta \phi_{\text{LSS}}$. Since the large scale modes with $L \lesssim 100$ mainly contribute to $A_\sigma$, the mass dependence of $\delta \phi_{\text{LSS}}$ is effectively the same as the background $\tilde{\phi}$. Therefore, we multiply $\sqrt{A_\sigma}$ by a damping factor $\langle \tilde{\phi}_{\text{LSS}} \rangle/\tilde{\phi}_{\text{LSS}}$, where $\tilde{\phi}_{\text{LSS}}$ is computed for $t_0^{-1} \ll m \ll t_{\text{LSS}}^{-1}$.

IV. SENSITIVITY OF THE CMB OBSERVATION

In this section, we numerically calculate the ALP field dynamics based on the $\Lambda$CDM model by solving Eqs. (11) and (12) as well as the Friedmann equation,

$$H = H_0 \sqrt{\Omega_\Lambda + \Omega_M (a^{-3} + \alpha_m a^{-4})},$$

where $\Omega_M \simeq 0.31$ is the density parameter of matter and $\alpha_m \simeq 1/3400$ is the scale factor at the matter-radiation equality. The upper bound of $\Omega_\phi$, which appeared in Eq. (13), is fixed as

$$\Omega_\phi \leq \begin{cases} \Omega_\Lambda & (m \leq 9.26 \times 10^{-34} \text{eV}) \\ 0.006 h^{-2} & (10^{-32} \text{eV} \leq m \leq 10^{-25.5} \text{eV}) \end{cases},$$

where $\Omega_\Lambda \simeq 0.69$ is the density parameter of dark energy [35]. $\phi$ can be responsible for all of dark energy if its mass is sufficiently small, while its equation of state (EoS) parameter $w_\phi$ deviates from $-1$ as $m$ increases. We obtain the condition that $\Omega_\phi$ can be $\Omega_\Lambda$ as $m \leq 9.26 \times 10^{-34} \text{eV}$ by requiring $w_\phi = (\dot{\phi}^2 - m^2 \phi^2)/(\dot{\phi}^2 + m^2 \phi^2)$
to lie within the Planck 95% confidence level constraint $w_A(t_0) = -1.04 \pm 0.10$ \cite{39}. For a larger mass region $m \geq 10^{-32}$ eV, the ALP behaves as dark matter once it starts oscillating at $H \approx m$. The observations of CMB and large scale structures constrain the ALP with such a transition of $w_A$ as $\Omega_\alpha h^2 \leq 0.006$ \cite{40}. For the intermediate mass region $9.26 \times 10^{-34} \text{eV} < m < 10^{-32}$ eV, we linearly connect these two upper limits on $\Omega_\phi$ in the log $m - \log \Omega_\phi$ plane. This treatment is compatible with the constraint given in Ref. \cite{36}. Moreover, we set the initial power spectrum of the ALP perturbation by fixing the tensor-to-scalar ratio $r = 2H_0^2/(\pi M_{\text{Pl}}^2 P_\zeta)$, where $P_\zeta = 2 \times 10^{-9}$ is the observed scalar power spectrum.

Substituting the numerically obtained $\Delta \phi$, $\delta \phi_{\text{obs}}$ and $\delta \phi_{\text{LSS}}$ into Eqs. (7) and (9), we obtain the precise predictions of the CMB polarization rotation in our ALP model. For simplicity, we evaluate $\delta \phi_{\text{obs}}$ in Eq. (9) by its root mean square (RMS). With these predictions, we can translate the sensitivities of the CMB experiments to $\alpha$ and $A_\alpha$ into the sensitivities to the ALP-photon coupling constant $g$. In Table 1, we summarize the projected sensitivities at 68% confidence level to $\alpha$ and $A_\alpha$ of Planck \cite{57}, South Pole Telescope \cite{39}, LiteBIRD \cite{58}, Simons Observatory (SO) \cite{11} and CMB-S4-like mission \cite{40} are summarized. In this letter, we only use LiteBIRD for the projected sensitivity to $|\tilde{\alpha}|$, because sensitivity of polarization rotation is degenerate with calibration uncertainties on artificial rotation of polarization sensitive detectors \cite{12} and we cannot find sensitivities including calibration uncertainties for the other projects.

Fig. 1 shows the current sensitivities to $g$. We obtain the best sensitivity owing to $\Delta \phi$ by saturating Eq. (19) and draw it as the purple line. It is proportional to $m^{-1}$ for $m \lesssim 10^{-32}$ eV and is almost independent of the mass for a higher mass range as explained in Eq. (13). When $\Omega_\phi$ is smaller, the sensitivity to $g$ is reduced as $\Omega_\phi^{-1/2}$. The red line denotes the sensitivity originating from $\delta \phi_{\text{LSS}}$ and the bound on $A_\alpha$. For $m \gtrsim 10^{-28}$ eV, the purple and red lines exponentially blow up, since the ALP oscillation effectively reduces $\phi_{\text{LSS}}$ as discussed around Eq. (17). Note that the purple line increases in proportion to $m$ for $m \gtrsim 10^{-27}$ eV, because $\phi_{\text{obs}}$ dominates $\phi_{\text{LSS}}$ there. The blue line represents the sensitivity contributed by $\delta \phi_{\text{obs}}$ which is numerically evaluated by its RMS $\langle \delta \phi_{\text{obs}}^2 \rangle^{1/2}$. The shaded regions show the current constraints from the light-shining-through-walls experiment OSQAR \cite{43}, the helioscope experiment IAXO \cite{14}, \cite{45}, and the X-ray bound by Athena \cite{49}. In Fig. 1 and 2, one observes that the CMB observations of $\tilde{\alpha}$ and $A_\alpha$ can achieve considerably better sensitivities to $g$ than the existing and upcoming axion search experiments. If a light ALP exists, inflation automatically generates its fluctuations and we can probe $g$ through $\delta \phi_{\text{LSS}}$ and $\delta \phi_{\text{obs}}$. Additionally assuming the background ALP field has a significant energy density, we find that $\Delta \phi$ provides a remarkable sensitivity up to $3 \times 10^8$ times better than the current one in the near future.

It is interesting to notice that the detection of $A_\alpha$ puts a lower bound on $r$. Since the ALP-photon coupling constant has the present upper bound, $g < 1.4 \times 10^{-12}$ GeV$^{-1}$, once the observation fixes $A_\alpha$, we obtain

$$r > 5 \times 10^{-9} \left( \frac{A_\alpha}{4 \times 10^{-3} \text{deg}^2} \right),$$

where we used Eq. (23). If Athena will improve the upper bound on $g$ by 10, for instance, the lower bound on $r$ increases by 100. Therefore, the observation of $A_\alpha$ combined with the axion search experiment chases up $r$ from below, and it is complementary to the CMB B-mode observation which pursues $r$ from above.

We can explore the further implications of the possible detection of either $\alpha$ or $A_\alpha$. If $A_\alpha$ is detected, using Eq. (22) and (23), we expect that $\tilde{\alpha}$ contributed at least by $\delta \phi_{\text{obs}}$ would be detected at

$$|\tilde{\alpha}| \approx 0.05^\circ \left( \frac{A_\alpha}{4 \times 10^{-3} \text{deg}^2} \right)^{1/2},$$

expressions:

$$g_{\Delta \phi}(m \lesssim H_0) = 3.0 \times 10^{-18} \text{GeV}^{-1}$$

$$\times \left( \frac{|\tilde{\alpha}|}{0.6^\circ} \right) \left( \frac{\Omega_\phi}{0.006} \right)^{-1/2} \left( \frac{m}{H_0} \right)^{-1},$$

(20)

$$g_{\Delta \phi}(H_0 \lesssim m \lesssim H_{\text{LSS}}) = 2.6 \times 10^{-20} \text{GeV}^{-1}$$

$$\times \left( \frac{|\tilde{\alpha}|}{0.6^\circ} \right) \left( \frac{\Omega_\phi h^2}{0.006} \right)^{-1/2},$$

(21)

$$g_{\phi_{\text{obs}}}(m \lesssim H_0) = 4.0 \times 10^{-14} \text{GeV}^{-1}$$

$$\times \left( \frac{|\tilde{\alpha}|}{0.6^\circ} \right) \left( \frac{r}{10^{-3}} \right)^{-1/2},$$

(22)

$$g_{\phi_{\text{LSS}}}(m \lesssim H_{\text{LSS}}) = 4.4 \times 10^{-15} \text{GeV}^{-1}$$

$$\times \left( \frac{A_\alpha}{8.3 \times 10^{-3} \text{deg}^2} \right)^{1/2} \left( \frac{r}{10^{-3}} \right)^{-1/2},$$

(23)

where $H_{\text{LSS}} \equiv H(t_{\text{LSS}})$. Note that Eqs. (20)–(22) do not reflect the fact that both $\Delta \phi$ and $\delta \phi_{\text{obs}}$ contribute to $\alpha$, because these expressions denote the individual sensitivities.
where the stochastic nature of $\delta\phi_{\text{obs}}$ blurs a simple prediction. However, if $\bar{\alpha}$ is not observed well below the above expected value, we can constrain the ALP mass, $m \gg H_0$, because the ALP has to already oscillate so that $\delta\phi_{\text{obs}}$ sufficiently decays by today. At the same time, we obtain the upper bound on $\Omega_{\phi}$, by equating Eq. (21) to Eq. (23).

$$\Omega_{\phi}h^2 < 1.8 \times 10^{-13} \left( \frac{|\bar{\alpha}|}{0.05^\circ} \right)^2 \left( \frac{A_{\alpha}}{4 \times 10^{-3} \text{deg}^2} \right)^{-1} \left( \frac{r}{0.06} \right)^{-1},$$

(26)

where not only $\bar{\alpha}$ but also $r$ is bounded above, $r < 0.06$ \cite{50,51}.

On the other hand, if $\bar{\alpha}$ is detected and the corresponding $A_{\alpha}$ given by Eq. (25) is not observed, we know the detected $\bar{\alpha}$ is contributed by $\Delta\phi$. Substituting the detected $|\bar{\alpha}|$ and the experimental upper bound on $g$ into $g_{\Delta\phi}$, and using Eq. (19), we obtain the allowed range of $m$ as

$$10^{-8} \left( \frac{|\bar{\alpha}|}{0.3^\circ} \right) < \frac{m}{H_0} \lesssim 10^8 \left( \frac{|\bar{\alpha}|}{0.3^\circ} \right)^{-1},$$

(27)

where we used $\Delta\phi \simeq \gamma_{\text{obs}} = \sqrt{8\pi} M_P H_0 / m$ for $m > 3 \times 10^{-27}$ eV to derive the upper bound. In passing, we can develop similar arguments in the case where both $\bar{\alpha}$ and $A_{\alpha}$ are detected.

**V. SUMMARY AND DISCUSSION**

In this letter, we have investigated the cosmic birefringence of CMB photons as a probe of the ALP-photon coupling $g$ under the assumption that the ALP has a quadratic potential with an extremely light mass $m \lesssim \ell_{\text{LSS}}^{-1}$. Since the birefringence angle is proportional to $g$ and the difference of the ALP field values between the LSS and the observer, one can relate the observations of

---

**TABLE I: Current bounds and projected sensitivities to the polarization rotation parameters.**

| $|\bar{\alpha}|$ (°) | Current | LiteBIRD | SO | CMB-S4-like |
|----------------|---------|----------|----|------------|
| $A_{\alpha}$ (deg$^2$) | $< 0.6$ [37] | $0.1 \times 10^{-3}$ [38] | - | - |
| $m/H_0$ | $< 8.3 \times 10^{-3}$ [39] | $4.0 \times 10^{-3}$ [40] | $5.5 \times 10^{-4}$ [41] | $3.3 \times 10^{-5}$ [42] |

---

**FIG. 1:** Current sensitivities to the axion-photon coupling $g$ from $\Delta\phi$ (purple), $\delta\phi_{\text{LSS}}$ (red), and $\delta\phi_{\text{obs}}$ (blue). $\delta\phi_{\text{LSS}}$ and $\delta\phi_{\text{obs}}$ are calculated with $r = 10^{-3}$. The purple dots on the $\Delta\phi$ sensitivity show the masses at which the current EoS parameter $\delta\phi$ satisfies $|\delta\phi| < 3 \times 10^{-3}$. The shaded regions have been excluded by OSQAR [43] (orange), CAST [44] (light blue), SN1987A [45] (light green), and Chandra [46] (pink).

**FIG. 2:** Future sensitivities to the axion-photon coupling $g$ of LiteBIRD, Simons observatory (SO) and CMB-S4 are superposed on Fig. 1 as dotted lines. The horizontal dotted lines show the projected sensitivities of ALPSII [47] (orange), IAXO [14, 48] (light blue), and Athena [49] (pink).
the birefringence angle and $g$ by calculating the dynamics of the ALP field. The background dynamics and the fluctuation at the observer induce isotropic birefringence, while the fluctuation at the LSS induces anisotropic birefringence. The isotropic rotation induced by $\Delta \phi$ largely depends on the energy fraction of the ALP and, with the maximum allowed energy fraction, the sensitivity to $g$ extensively exceeds the current limits by other experiments. The same signal may be used to search for a quintessence field with a tiny $w + 1$. Even if the energy fraction of the ALP is negligible, the contributions of $\delta \phi_{\text{obs}}$ and $\delta \phi_{\text{LSS}}$ persist as long as the light ALP exists, and they have better sensitivities to $g$ than other experiments depending on $r$. In particular, we have found that $\delta \phi_{\text{obs}}$ stochastically breaks the parity symmetry and contributes to the isotropic rotation of the CMB polarization. If the ALP is heavy enough to oscillate during the last scattering, however, the birefringence effect is drastically suppressed. If isotropic or anisotropic birefringence is observed by the CMB observations, we can limit relevant parameters such as $r$, $\Omega_\phi$, and $m$. These limits will be substantially improved with the upcoming X-ray observation by Athena and the future CMB missions such as LiteBIRD, Simons Observatory, and CMB-S4.

We also comment on possible extensions of our work. The quadratic mass potential Eq. (10) used in our analysis should be seen as a toy model. For $\Omega_\phi \simeq \Omega_\Lambda$, this potential requires the ALP field value $\phi$ to be much larger than the Planck scale, which may not be favored by the UV completion. However, it is straightforward to compare $\alpha$ and $A_\alpha$ for the other ALP potential forms, by following our procedure. Although we evaluate $\delta \phi_{\text{obs}}$ by its RMS, $\delta \phi_{\text{obs}}$ in our universe may deviates from the RMS value by chance. A dedicated statistical treatment is needed for more precise predictions. We also ignored the source term in Eq. (12), while it might amplify $\delta \phi_{\text{obs}}$. We leave these intriguing problems for future work.

**Acknowledgments**

We would like to thank Ricardo Z. Ferreira, Masahiro Ibe, Masahiro Kawasaki, Eliechi Komatsu, Günter Sigl and Pranjal Trivedi for fruitful discussions and productive comments. K.M. was supported by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan and the Program of Excellence in Photon Science. N.H. was supported by Advanced Leading Graduate Course for Photon Science. This work was supported in part by the Japan Society for the Promotion of Science (JSPS) KAKENHI, Grant Number JP18K13537, JP19J21974, JP20K1449, and JP20J20248.
[26] M. A. Fedderke, P. W. Graham, and S. Rajendran, Phys. Rev. D 100, 015040 (2019), arXiv:1903.02666 [astro-ph.CO].
[27] G. Sigl and P. Trivedi, (2018), arXiv:1811.07873 [astro-ph.CO].
[28] M. Pospelov, A. Ritz, C. Skordis, A. Ritz, and C. Skordis, Phys. Rev. Lett. 103, 051302 (2009), arXiv:0808.0673 [astro-ph].
[29] W. Zhao and M. Li, Phys. Rev. D 89, 103518 (2014), arXiv:1403.3997 [astro-ph.CO].
[30] S. Lee, G.-C. Liu, and K.-W. Ng, The Universe 4, 29 (2016), arXiv:1912.12903 [astro-ph.CO].
[31] D. Harari and P. Sikivie, Physics Letters B 289, 67 (1992).
[32] R. R. Caldwell, V. Gluscevic, and M. Kamionkowski, Phys. Rev. D 84, 043504 (2011), arXiv:1104.1634 [astro-ph.CO].
[33] R. Dave, R. Caldwell, and P. J. Steinhardt, Phys. Rev. D 66, 023516 (2002), arXiv:astro-ph/0206372.
[34] S. Weinberg, Cosmology (2008).
[35] N. Aghanim et al. (Planck), Astron. Astrophys. (2018), arXiv:1807.06209 [astro-ph.CO].
[36] R. Hlozek, D. Grin, D. J. E. Marsh, and P. G. Ferreira, Phys. Rev. D 91, 103512 (2015), arXiv:1410.2896 [astro-ph.CO].
[37] N. Aghanim et al. (Planck), Astron. Astrophys. 596, A110 (2016), arXiv:1605.08633 [astro-ph.CO].
[38] Y. Minami and E. Komatsu, (2020), arXiv:2006.15982 [astro-ph.CO].
[39] F. Bianchini et al. (SPT), (2020), arXiv:2006.08061 [astro-ph.CO].
[40] L. Pogosian, M. Shimon, M. Mewes, and B. Keating, Phys. Rev. D 100, 023507 (2019) arXiv:1904.07855 [astro-ph.CO].
[41] S. A. Bryan et al., Proc. SPIE Int. Soc. Opt. Eng. 10708, 1070840 (2018), arXiv:1810.04633 [astro-ph.IM].
[42] E. Komatsu et al. (WMAP), Astrophys. J. Suppl. 192, 18 (2011), arXiv:1001.4538 [astro-ph.CO].
[43] R. Ballou et al. (OSQAR), Phys. Rev. D 92, 092002 (2015), arXiv:1506.08082 [hep-ex].
[44] V. Anastassopoulos et al. (CAST), Nature Phys. 13, 584 (2017), arXiv:1705.02290 [hep-ex].
[45] A. Payez, C. Evoli, T. Fischer, M. Giannotti, A. Mirizzi, and A. Ringwald, JCAP 02, 006 (2015), arXiv:1410.3747 [astro-ph.HE].
[46] M. Berg, J. P. Conlon, F. Day, N. Jennings, S. Krippendorf, A. J. Powell, and M. Rummel, Astrophys. J. 847, 101 (2017), arXiv:1605.01043 [astro-ph.HE].
[47] R. Bih et al., JINST 8, T09001 (2013), arXiv:1302.5647 [physics.ins-det].
[48] E. Armengaud et al., JINST 9, T05002 (2014), arXiv:1401.3233 [physics.ins-det].
[49] J. P. Conlon, F. Day, N. Jennings, S. Krippendorf, and F. Muia, Mon. Not. Roy. Astron. Soc. 473, 4932 (2018), arXiv:1707.00176 [astro-ph.HE].
[50] Y. Akrani et al. (Planck), (2018), arXiv:1807.06211 [astro-ph.CO].
[51] P. Ade et al. (BICEP2, Keck Array), Phys. Rev. Lett. 121, 221301 (2018) arXiv:1810.05216 [astro-ph.CO].