Analysis and control measures for Lassa fever model under socio-economic conditions

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Abstract. Lassa fever is one of the animal-borne diseases that is affecting people in some of the West Africa countries. A mathematical model for dynamics of Lassa fever is formulated. It is assumed that individuals in the population comprises of two socio-economic classes (lower and higher socio-economic class). The significant features of the model are determined and analyzed accordingly. The effects of the control measures such as treatment, external protection, and rodent control are determined and their epidemiological implications discussed.

1. Introduction

Lassa fever is an infection caused by Lassa virus. The virus is hosted by a rodent called the multimamate rat (Mastomys natalensis). One of the medium of spreading Lassa fever to humans is by direct contact with infected rats or exposure to contaminated food or household appliances. Person to person transmission can occur when an individuals come in close contact with an infected person. For instance, skin breaks in humans or dust particles through the mucous membranes are medium through which individuals can contact the disease [1, 2].

Lassa fever a life threatening disease worldwide, especially in some West African countries such as Nigeria, Ghana, Benin, Sierra Leone, Guinea, Liberia and Mali where the disease seems to be endemic [2, 3]. Data from the World Health Organization (WHO), reveal that between January and February, 2020, Nigeria has approximately 172 confirmed cases with 72 deaths. Lassa fever death rate is approximately 14.8% [4]. Statistics from the Center for Disease Control and prevention (CDC) and WHO reveal that approximately 100,000 to 300,000 cases of Lassa fever with death records of about 5,000 were reported annually in West Africa [3].

Mathematical model is one of the major tools used in studying the dynamics of infectious disease such as Lassa fever [7, 8, 9]. Several mathematical models in the literature have studied Lassa fever disease dynamics theoretically [8, 9, 10, 11, 12, 13, 14, 15]. The dynamics of Lassa fever and impact of available control measures have been explored theoretically [16, 17, 18, 19, 14, 20, 21, 22]. Lassa fever have no approved vaccine presently. Further more, the treatment of Lassa fever is not 100% effective. Some research towards finding a vaccine for Lassa fever are reported in [24, 25, 26, 23]. In this work, we take into account other control measures in fighting Lassa fever disease as there is neither vaccination nor effective treatment available for the disease. Some of these control measures include proper environmental cleanliness, improved health care system, healthy living practices, effective rodent control and maintaining good life
style in general [1, 2]. Evidently, the above studies have contributed directly or indirectly in reducing Lassa fever. However, as far as we know, none of those research considered socio-economic classes (SECs) in studying the dynamics of Lassa fever disease. This study is our contribution towards filling this gap in the literature.

2. Model development

Consider a community where the individuals with total population \((N)\) comprises of two socio-economic classes. This total population of individuals \((N)\) is divided into 2 subpopulations denoted by \(N_i\) \((i = 1, 2)\) such that each subpopulations represent a socio-economic class (SEC). Each of this SEC is further divided into susceptible \((S_i)\), infected \((I_i)\), treated \((T_i)\) and recovered \((R_i)\) individuals. The total population of rodents \((Z(t))\) is partitioned into two classes: the susceptible rodents \((X)\) and the infected rodents \((Y)\). Infections occur when \(S_i(t)\) comes in contact with \(I_i(t)\) or \(Y(t)\) at a rate \(\beta_i\) and \(\alpha_i\) respectively [13, 19]. \(I_i\) get treated at a rate \(\tau_i\) and those that receive treatment recover at a rate \(\gamma_i\). \(X\) become infected when they come in contact with \(Y\) at a rate \(\sigma\). We assume that natural birth/death occurs in each SEC \(i\) at a rate \(\mu_i\) whereas natural birth/death occurs in rodents at a rate \(\nu\). To take SEC into consideration, it is important to know that humans in the higher socio-economic class are exposed to better living environment than humans in the lower socio-economic class [5, 6]. So, we assume that \(S_i(t)\) moves to \(S_j(t)\) at a rate \(\delta_{ij}\) whereas \(I_i(t)\) move to \(I_j(t)\) at a rate \(l_{ij}\). From the above assumptions, we derive the model

\[
\begin{align*}
\dot{S}_1 &= \mu_1 N_1(t) - (1 - \epsilon_1)S_1(t)(\beta_1 I_1(t) + \alpha_1 Y(t)) - (\mu_1 + \delta_{21})S_1(t) + \delta_{21}S_2(t), \\
\dot{I}_1 &= (1 - \epsilon_1)S_1(t)(\beta_1 I_1(t) + \alpha_1 Y(t)) - (\mu_1 + \gamma_1 + \tau_1 + l_{12})I_1(t) + l_{21}I_2(t), \\
\dot{T}_1 &= \gamma_1 I_1(t) - (\mu_1 + \theta_1)T_1(t), \\
\dot{R}_1 &= \gamma_1 I_1(t) + \theta_1 T_1(t) - \mu_1 R_1(t), \\
\dot{S}_2 &= \mu_2 N_2(t) - (1 - \epsilon_2)S_2(t)(\beta_2 I_2(t) + \alpha_2 Y(t)) - (\mu_2 + \delta_{21})S_2(t) + \delta_{12}S_1(t), \\
\dot{I}_2 &= (1 - \epsilon_2)S_2(t)(\beta_2 I_2(t) + \alpha_2 Y(t)) - (\mu_2 + \gamma_2 + \tau_2 + l_{21})I_2(t) + l_{12}I_1(t), \\
\dot{T}_2 &= \gamma_2 I_2(t) - (\mu_2 + \theta_2)T_2(t), \\
\dot{R}_2 &= \gamma_2 I_2(t) + \theta_2 T_2(t) - \mu_2 R_2(t), \\
\dot{X} &= \nu Z(t) - (1 - \eta)\sigma X(t)Y(t) - \nu X(t), \\
\dot{Y} &= (1 - \eta)\sigma X(t)Y(t) - \nu Y(t),
\end{align*}
\]

where \(N_i(t) = S_i(t) + I_i(t) + T_i(t) + R_i(t), \quad i = 1, 2\). The parameters of the model with their meaning are presented in Table 1. The subscript 1 and 2 are used to represent the lower and higher socio-economic classes respectively.
Table 1. Meaning of parameters used in model 1

| Parameter | Meaning                                                                 |
|-----------|-------------------------------------------------------------------------|
| \( \beta_i \) | Contact rate of \( S_i(t) \) with \( I_i(t) \)                        |
| \( \alpha_i \) | Contact rate of \( S_i(t) \) with \( Y(t) \)                           |
| \( \delta_{ij} \) | Rate at which \( S_i(t) \) migrates to \( S_j(t) \)                   |
| \( \iota_{ij} \) | Rate at which \( I_i(t) \) migrate to \( I_j(t) \)                    |
| \( \gamma_i \) | Natural recovery rate of \( I_i(t) \)                                  |
| \( \theta_t \) | Recovery rate due to treatment \((T_t(t))\)                           |
| \( \tau_i \) | Treatment rate of \( I_i(t) \)                                        |
| \( \sigma \) | Contact rate of \( X(t) \) with \( Y(t) \)                           |
| \( \mu_i \) | Natural birth/death rate of individuals in the SEC \( i \)               |
| \( \epsilon_i \) | Rate of reduction in susceptibility due to the use of external protection by \( S_i(t) \) |
| \( \nu \) | Natural birth/death rate of rodents                                    |
| \( \eta \) | Rate of decrease in contact rates with infected rodents                |

Initial conditions for model 1 are given as follows:

\[
S_i(0) > 0, \ I_i(0) \geq 0, \ T_i(0) \geq 0, \ R_i(0) \geq 0, \ X(0) > 0, \ Y(0) \geq 0, \ i = 1, 2. \tag{2}
\]

3. Model analysis

The dynamical system analysis of model 1 is presented in this section. The study will widen our knowledge on the dynamics and impact of available control measures for Lassa fever disease. The disease-free equilibrium (DFE) of model 1 is

\[
(S_1^0, I_1^0, T_1^0, S_2^0, I_2^0, T_2^0, X^0, Y^0) = \left( \frac{N \delta_{21}}{\delta_{12} + \delta_{21}}, 0, 0, \frac{N \delta_{12}}{\delta_{12} + \delta_{21}}, 0, 0, Z, 0 \right). \tag{3}
\]

The population of susceptible in each SEC at DFE depends on the movement rates \( \delta_{12} \) and \( \delta_{21} \). This demonstrates the importance of migration rates at DFE.

3.1. Basic reproduction number

The basic reproduction number can be understood as the expected number of secondary infections of Lassa fever produced when an infected individual is introduced into a given population that is susceptible to the disease. Using the next generation matrix approach [27], we obtain the basic reproduction number of model 1 to be

\[
R_0 = \max\{R^h_0, R^r_0\}, \tag{4}
\]

where, \( R^h_0 = \frac{(R_{11} + R_{33}) + \sqrt{(R_{11} + R_{33})^2 + 4(R_{13}R_{31} - R_{11}R_{33})}}{2}, \ R^r_0 = \frac{(1 - \gamma_1) \sigma Z}{\mu} \)

\[
R_{11} = \frac{(1 - \epsilon_1) \beta_1 S^0_{12}}{k_1 k_2 + k_1 l_{21} + k_2 l_{21} + \epsilon_1 b_1}, \ R_{31} = \frac{(1 - \epsilon_1) \beta_2 S^0_{12}}{k_1 k_2 + k_1 l_{21} + k_2 l_{21}}, \ R_{33} = \frac{(1 - \epsilon_2) \beta_3 S^0_{12}}{k_1 k_3 + k_1 l_{12} + k_3 l_{12}}, \ b_1 = k_1 + l_{12}, \ b_2 = k_2 + l_{21}, \ b_3 = k_3 + l_{12}.
\]

Note that the inequality \((R_{11} + R_{33})^2 + 4(R_{13}R_{31} - R_{11}R_{33}) \geq 0\) must hold for \( R^h_0 \) to exist. Biologically, \( R^h_0 \) is the basic reproduction number for humans while \( R^r_0 \) is the basic reproduction number related to rodents. From equation 4, we conclude that if \( R_0 = R^r_0 \), then Lassa fever outbreak is been driven by rats. On the other hand, if \( R_0 = R^h_0 \) then Lassa fever outbreak is been driven by humans. So for effective control measure, we must include control measures on both human and rats. Epidemiologically, once the basic reproduction number is less than one, the disease can be wiped out. But the disease seems difficult to eliminate whenever the basic reproduction number is above one. Therefore, to ensure total disease eradication, control measures that will decrease the basic reproduction number below one should be considered.
Table 2. Parameter values used for the numerical simulations

| Symbol of the parameters | Parameter values | Source |
|--------------------------|------------------|--------|
| $\mu_j$                  | 0.0000548        | [10]   |
| $\beta$                  | 0.00002          | [11]   |
| $\beta_1$                | 1.5$\beta$       | Estimated |
| $\beta_2$                | 0.5$\beta$       | Estimated |
| $\alpha$                 | 0.00001          | [11]   |
| $\alpha_1$               | 1.5$\alpha$      | Estimated |
| $\alpha$                 | 0.5$\alpha$      | Estimated |
| $\gamma$                 | 0.0476           | [10]   |
| $\gamma_1$               | 0.5$\rho$        | Estimated |
| $\gamma_2$               | 1.5$\rho$        | Estimated |
| $\tau_1$                 | 0.5              | Estimated |
| $\tau_2$                 | 1.5$\tau_1$      | Estimated |
| $\eta$                   | 0.0.667          | [15]   |
| $\epsilon_1$            | 0.25             | Estimated |
| $\epsilon_2$            | 0.45             | Estimated |
| $\theta$                | 0.1184           | [15]   |
| $\theta_1$              | 0.5$\theta$      | Estimated |
| $\theta_2$              | 1.5$\theta$      | Estimated |
| $\nu$                   | 0.2              | [10]   |
| $\sigma$                | 0.002            | [12]   |
| $\delta_{12}$            | 0.2              | [28]   |
| $\delta_{21}$            | 0.2              | [28]   |
| $l_{12}$                 | $\delta_{12}$    | [28]   |
| $l_{21}$                 | $\delta_{21}$    | [28]   |

3.2. Type reproduction number

The type reproduction number for each SEC is very crucial in finding the quantity of control measures required to reduce the transmission of Lassa fever in any of the SEC [29, 30]. The type reproduction number for SEC 1 using the method in [29, 30] is

$$T_1 = R_{11} + \frac{R_{13}R_{31}}{1 - R_{33}}. \quad (5)$$

Similarly, the type reproduction number for SEC 2 can be determined as

$$T_2 = R_{22} + \frac{R_{13}R_{31}}{1 - R_{11}}. \quad (6)$$

Epidemiologically, to eliminate the disease from SEC 1 we must keep $T_1$ below unity. Similarly, to eliminate the disease from SEC 2 we must keep $T_2$ below unity.

4. Numerical results

In this section we present numerical simulations to validate our analytical results. The values of the parameter used are given in Table 2.

Note: $\beta$, $\alpha$, $\gamma$ and $\theta$ in Table 2 represent person to person contact rate, person to rats contact rate, treatment rate and recovery rate of treated individuals respectively in a homogeneous population (i.e., population where SECs are not considered). We considered the above data for a homogeneous population due to limited access to data on SECs.
Figure 1. Plot showing the possible dynamics of Lassa fever with control measures and without control measures for SEC 1 and SEC 2: (a) $R_0^h = 0.1661$, $R_0^r = 0.6660$ (b) $R_0^h = 1.3292$, $R_0^r = 5.3280$ (see text for details).

Figure 1 illustrates the possible dynamics of Lassa fever across the SEC 1 and SEC 2. The figure reveals that the lower SEC will always have more Lassa fever infected individuals irrespective of the basic reproduction number. For $R_0^h = 0.1661$, $R_0^r = 0.6660$, we discover a small difference on the dynamics of the two SECs. In this case as the basic reproduction number is below one, the disease dies out gradually in the two SECs, so the results is reasonable. However, when $R_0^h = 1.3292$, $R_0^r = 5.3280$, we discover a significant difference on the dynamics of the two SECs. Under this condition when the basic reproduction number is above one, the disease is endemic in the two SECs, so the result is also reasonable. We also discovered from the figure that the basic reproduction number for rodents is greater for the two cases (i.e, when $R_0 < 1$ and $R_0 > 1$). This shows that rodents is driving the outbreak for these two cases. Therefore, effective control measure targeting rodents is required for possible eradication of the disease.

Figure 2. Figure illustrating the effect of considering the control measures across the two socio-economic classes.
Figure 2 demonstrates the impact of considering the three control measures (use of external protection, treatment and rodent control) simultaneously in reducing the spread of Lassa fever across the two socio-economic classes. The figure illustrates that considering the three control measures has significant impact in reducing the spread of Lassa fever in both the lower and higher socio-economic class.

Figure 3. Figure illustrating the impact of treatment of infected individuals in fighting Lassa fever disease in the lower and higher socio-economic classes.

Figure 3 demonstrates the impact of considering treatment for reducing the spread of Lassa fever across the lower and higher socio-economic classes. The figure illustrates that considering treatment has significant effect in curtailing Lassa fever infections in both the lower and higher socio-economic class. Particularly, the figure shows that increasing treatment rate decreases the number of Lassa fever infected individuals in the two socio-economic classes.

Figure 4. Figure illustrating the impact of use of external protection in fighting Lassa fever disease in the lower and higher socio-economic classes.

Figure 4 demonstrates the impact of using external protection as a control measure for reducing the spread of Lassa fever across the lower and higher socio-economic classes. The figure also reveals that using external protection has significant impact in decreasing Lassa fever disease...
in both the lower and higher socio-economic class. In particular, the figure shows that increasing the use of external protection decreases Lassa fever infections in the two socio-economic classes.

![Figure 5](image.png)

**Figure 5.** Figure illustrating the impact of rodent control in reducing Lassa fever disease in the lower and higher socio-economic classes.

Figure 5 demonstrates the impact of considering rodent control as a control measure for reducing Lassa fever across the lower and higher socio-economic classes. The figure also reveals that the use of rodent control significantly decreases the transmission of Lassa fever in both in both the lower and higher socio-economic class. In particular, the figure shows that increasing rodents control decreases Lassa fever infection in the two socio-economic classes.

5. Discussion

This study is carried out to fill the gap in the knowledge of the dynamics of Lassa fever disease and subsequently examine the effect of possible control measures in decreasing Lassa fever infections in a multiple socio-economic communities. To achieve this, we formulated an appropriate mathematical model representing Lassa fever disease dynamics that incorporates three possible control measures (treatment, use of external protection and rodent control). The model also took into consideration that the population comprises of individuals in the two socio-economic classes (lower socio-economic class and higher socio-economic class). From the analysis of the model, we discovered that the lower socio-economic class and higher socio-economic class have different dynamics with Lassa fever infections dominating in the lower socio-economic class. Therefore, control measures should target the lower socio-economic class for chances of eradicating the disease. Secondly, we discovered that the Lassa fever disease outbreak is driven either by rodents or humans depending on which of them have a greater basic reproduction number. For our study here, the rodent seems to have a greater basic reproduction number. Thus, control measures targeting the rodents is required for possible eradication of Lassa fever.

Thirdly, we discovered that each of the control measures: treatment, use of external protection or rodent control has significant impact in decreasing Lassa fever infections in the two socio-economic classes. So, any of the control measures can be introduced to reduce the spread of Lassa fever. Finally, we discovered that introducing the three control measures simultaneously yield a better result in decreasing Lassa fever infections in the two socio-economic classes. Thus, for effective eradication of Lassa fever infections from the entire population comprising of the lower socio-economic class and higher socioeconomic class, we recommend the combined control measures (treatment, use of external protection and rodent control).
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