Mission Design for the TAIJI mission and Structure Formation in Early Universe

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Abstract  Gravitational wave detection in space promises to open a new window in astronomy to study the strong field dynamics of gravitational physics in astrophysics and cosmology. The present article is an extract of a report on a feasibility study of gravitational wave detection in space, commissioned by the National Space Science Center, Chinese Academy of Sciences almost a decade ago. The objective of the study was to explore various possible mission options to detect gravitational waves in space alternative to that of the (e)LISA mission concept and look into the requirements on the technological fronts. On the basis of relative merits and balance between science and technological feasibility, a set of representative mission options were studied and in the end a mission design was recommended as the starting point for research and development in the Chinese Academy of Sciences. The

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mission design was eventually adopted by the current TAIJI mission as the baseline parameters for the project. Subject to technological constraints, the baseline parameters of the TAIJI mission were designed in such a way to optimise the capability of a spaceborne gravitational wave detector to probe high redshift light seed, intermediate mass black holes and thereby shed important light on the structure formation in early Universe.

Keywords

Gravitational wave detection in space; Mission design; Intermediate mass black hole binaries at high redshift Universe.

1. Introduction

In the year 2011, the general relativity group of the Morningside Center of Mathematics was commissioned by the National Space Science Center, Chinese Academy of Sciences to conduct a feasibility study on the detection of gravitational wave in space, as a project in the second phase of the pioneering (Xiandao) program in space. At then, the objective of the study was to explore various possible mission options to detect gravitational waves in space alternative to that of the (e)LISA mission concept and look into the requirements on the technological fronts. Though some preliminary study was made in the first phase of the pioneering program [1], a more complete and systematic study was needed to assess the feasibility for China to undertake further research and development in this direction, in tandem with Europe and US at that time.

A report was submitted at the end of the two year study to the National Space Center in which certain mission design was recommended as a starting point of the development. This mission design was later adopted by the current TAIJI mission as the baseline parameters for the project. In the intervening years, certain parts of the report were made public both in Chinese and English [2, 3]. The core of the report is concerned with science objectives and it was translated and published in English [4]. However the English translation left a lot to be desired and obscured the real content of the report. It is the aim of this article to put together the mission design and the primary science drivers of the TAIJI mission design recommended in the report in a more concise way in English, in the hope that the content of the report will be accessible to a boarder spectrum of readership. At the same time it serves to put on record the early phase of development of gravitational wave detection in space in China.

The outline of the article may be described as follows. Section 2 will be concerned with a catalogue of the gravitational wave sources at the frequency window $10^{-4} - 0.1 H_\odot$. In section 3 we will retrace the line of the thoughts leading eventually
to the mission design of the TAIJI mission. In section 4, we will present the simulation of the cosmic growth history of supermassive black holes and give event rate estimate for the detection of intermediate mass black holes at high redshift Universe. Section 5 is concerned with some simple event rate estimate on the detection of intermediate mass ration inspirals in dense star clusters. Some concluding remarks will be made in section 6 to conclude this article.

2. A survey of gravitational wave sources.

In comparison with ground-based detection, the characteristic mass and energy scale of gravitational wave sources in the detection of gravitational wave in space are generally much larger. Prospective gravitational wave sources in the frequency window $10^{-4}$ Hz to $0.1$ Hz are displayed in Table 1.

2.1 Compact Binary Star Systems.

Through optical and X-ray observations, a number of compact binary star systems with inspiral period shorter than one hour in the Milky Way were identified, including some binary systems with mass-transfer taking place. These binaries are confirmed gravitational wave sources at the millihertz band. It is expected that, with the increase in precision in our observations, more binaries of this kind [5, 6, 7, 8] will be discovered. As the astrophysical parameters and positions of these systems may be measured precisely, the gravitational waves generated by these sources may be predicted in a very accurate way [10, 11]. These sources, termed verification binaries, may be used in a reverse way as a standard candle to calibrate a gravitational wave detector in space.

Though weak in strength in the gravitational wave amplitude in comparison with those generated by binary black hole mergers, due to the abundance in numbers (up to millions) and much shorter distance from a spaceborne detector, currently planned spaceborne detector will detect gravitational wave signals generated by millions of compact binaries in and outside of the Milky Way. Data analysis will enable us to identify signals from around ten thousand individual compact binaries and determine the astrophysical parameters of these wave sources, such as the mass, luminosity distance, position, and the orbital inclination of the binary stars. The rest unresolved signals will become a foreground (confusion) noise around the $mHz$ band and hamper the detection of other gravitational wave sources. At the same time, the foreground signal will be modulated by the annual motion of a spaceborne detector with respect to the orientation of the Galactic disk. The strength, spectral shape and annual variation of the foreground may then also be used to constrain and probe galactic structure, stellar evolution history and formation mechanism which are otherwise difficult to probe by conventional electromagnetic astronomy.
Table 1 Expected gravitational wave sources within the frequency window $10^{-4} - 0.1 Hz$.

| Gravitational wave sources | Brief description of source and the significance of its detection in astronomy and fundamental physics |
|----------------------------|--------------------------------------------------------------------------------------------------|
| Compact binaries (binary white dwarf and neutron star systems). | Tens of millions of compact binaries in the Milky Way and beyond. Binaries with precisely measured parameters will be used to calibrate the instruments of a gravitational wave detector and stellar dynamics. The unresolved signals generate a foreground that hampers the detection of other gravitational wave sources. At the same time, the foreground provides important information on the galactic structure of the Milky Way, stellar evolution history and formation mechanism. |
| Intermediate mass black hole binaries at high redshifts. | Intermediate mass black holes descended from the gravitational collapse of heavy Pop III stars, believed to be the seed black holes of the supermassive black holes in galactic centers at the present epoch. The detection will give clues on the galaxy-black hole co-evolution and distinguish various models and mechanisms for the cosmic growth of supermassive black holes and galaxies. |
| Supermassive black holes merger | The coalescence of supermassive black holes that accompany galaxy mergers. The detection provides new way to understand the dynamics of galaxy mergers. The high signal to noise ratios also means that it is a good candidate for multi-messenger astronomy and give precision measurement on the redshift-luminosity relation for cosmic evolution. |
| Extreme mass ratio inspirals. | Compact celestial bodies in the galactic centers captured by the supermassive black holes or stars generated by the formation and fast evolution of massive stars in the accretion disk surrounding the supermassive black hole. The detection presents a new way to probe galactic dynamics. The precision measurement of black hole parameters enables us to study spacetime structure in strong gravity field regime, check on the no-hair theorem of black holes and distinguish alternative gravity theories. |
| Intermediate mass ratio inspiral in dense star clusters | Inspiral systems formed by compact celestial bodies and intermediate-mass black holes in the center of dense star cluster. Provide unequivocal evidence for the existence of intermediate-mass black holes and reveal the dynamics of star cluster and the formation mechanism for intermediate-mass black holes. |
| Gravitational waves in early Universe | Gravitational waves generated by the big bang, inflation and possibly other cosmological processes like for instance the first order electroweak phase transition. |
| Burst signals. | Hyperbolic encounters of black holes or celestial bodies and possibly from unknown sources. |
| Unmodelled sources. | New physics and new astronomy |
2.2 Binary Black Hole Mergers

These are the majority of gravitational wave sources for detection in space and holds the promise of unravelling important and significant information concerning astrophysical and cosmological physics. Black hole binaries may be classified in accordance with the mass ratios between the two black holes undergoing inspiral and coalescence processes.

2.2.1 Intermediate mass black holes binaries at high redshift Universe

Astronomical Observations indicate that supermassive black holes are present in the centers of both normal and active galaxies, and that the velocity dispersion of stars in the galactic bulge is closely correlated with the mass of the supermassive black hole in the galactic center, i.e., the $M - \sigma$ relation [12, 13, 14]. The $M - \sigma$ relation reveals that a co-evolution history possibly exists between the galaxy and its central black hole. Quasars observed by the SDSS at redshifts larger than 6 powered by the accretion of supermassive black holes with $\sim 10^9 M_\odot$ in mass require that the seeds of supermassive black holes should form in the earlier period of the Universe [15, 16, 17]. This gives important constraints to the formation of supermassive black holes and the occurrence of the high-contrast density fluctuations in early universe.

Observational results of the cosmic background radiation indicate that energy fluctuations of the fractional order of $10^{-5}$ exist on the isotropic background after the recombination era at $z \sim 1000$. The gravitational (Jeans) instability induced collapse and aggregation in these high-density places of the early Universe, and the first generation dark matter halos formed at around $z = 20 \sim 30$, in which the gas finally experienced cooling, fragmentation, and agglomerating of $H_2$ molecules and the Pop III (first generation) stars as a result [18, 19]. A lack of metal abundance nor efficient cooling mechanism mean that the Pop III stars are very massive. The first light from these stars ended the dark age era of the Universe, while its energetic UV component ionised the fragile neutral $H_2$ molecules and heralded the dawn of reionization. With zero metal abundance, the stellar wind loss caused by radiation may be neglected, the lost mass in the final stage of collapse is very small. Numerical simulations indicate that the Pop III stars of $100 \sim 260M_\odot$ disappear directly through the supernova explosions due to electron-positron pair instability, while the stellar bodies with heavier mass collapsed to form the black holes with mass greater than $\sim 150M_\odot$ [20]. This falls in the range of intermediate mass black holes which is one possible origin of the seed black holes for supermassive black holes observed at the present epoch. Alternatively, gas clouds may directly collapse to form the seed black holes with mass heavier than $10^4M_\odot$. Through an effective angular momentum transfer, the collapse of stellar cluster nuclei may also form seed black holes of $10^3 \sim 10^4M_\odot$ [21, 22].

According to the hierarchical growth model of cold dark matter, the supermassive black holes located at center of a galaxy at the present epoch evolved from density fluctuations before re-ionization: the small-mass sub-galactic structures formed first,
then merged with each other continuously to form bigger and bigger structures. In this process, if black holes existed in the galactic centers, these black holes would experience the continuous accretion and merger processes of the dark matter halos and host galaxies, and formed the currently observed massive black holes at different redshifts.

A detailed understanding of the formation and growth of supermassive black holes is of clear cosmological as well as astrophysical significance. It represents in part of our endeavor to understand galaxy evolution and structure formation of the Universe. Electromagnetic astronomy enables us to probe the dynamics of a galaxy at redshifts around 6 or a little higher. It is expected that the prospective launch of the JWST telescope will extend our horizon to higher redshifts in the infrared red regime of electromagnetic waves. At high redshifts beyond the reach of electromagnetic astronomy, gravitational waves detection provides a unique way to probe the growth of black holes and galaxies through probing the merger of intermediate mass black holes binaries descended from the Pop III stars. Gravitational waves may be considered as an entirely new window to observe the galaxy-black hole co-evolution process and obtain important information regarding the history of structure evolution beginning from the end of the dark age era.

2.2.2 Supermassive black hole mergers.

In the merger of two galaxies, provided there are supermassive black holes residing in the centers of both galaxies, the two supermassive black holes will be dragged into the newly formed galactic center due to the dynamic friction with the surrounding stars, and gradually a distance is reached within which gravitational interaction becomes dominant and a bound state is formed. Through the mutual three-body interaction with the surrounding small celestial bodies or the friction with gas, the potential energy is reduced and the orbit is further shrunk. When the binary black hole system transfers their gravitational energy and angular momentum to the surrounding stars through the interactions with them, the surrounding stars acquire higher energy and angular momentum and become capable of escaping from the galactic center. When the stellar environment is deficient in gas and the galactic structure is spherical symmetric, there is then a lack of dynamical drag that drives further the merger of two black holes (i.e., the final parsec problem). Current study indicates that after merger the structures of galaxies themselves are not spherical symmetric and the the stellar orbits are aspherical. There are enough stars capable of getting into the vicinity of the binary black holes, so that the two black holes can keep their closeness by ejecting stars, and the distance between the black holes further reduces; finally, gravitational waves generated by the gravitational interaction of the binary black holes begins to play a dominant role and brings away the energy and angular momentum, ending up finally coalescence of the binary black holes.

In the case when the stellar environment is gas rich, after the formation of binary black holes, a common accretion disk for the binary black holes forms from the surrounding gas. The binary black holes further accrete matter, transfer the angular
momentum to surrounding stars and finally forms a single black hole through the emission of gravitational waves [25, 26, 27, 21, 33]. Many complicated astrophysical factors and processes are involved, such as gas accretion, dynamic interaction with the small stars, structure of the accretion disk, accretion efficiency, and gravitational recoil of the final merger, etc. Detection of gravitational waves provides an entirely new way to the understanding of these physical processes [32, 34, 35].

The signal-noise ratio for the detection of the merger of two supermassive black holes is in general very large. The gravitational wave signals generated in the final merging process is detectable even in the time domain. Through the accumulation in time sufficient signal to noise ratio (for example, one month or one year before merger), certain astrophysical parameters like for instance the luminosity distance, orbital plane and possibly other parameters of the binary supermassive black hole merging system may be precisely measured and these otherwise will be difficult to obtain by electromagnetic means. Combined with the information of redshifts from measurements in electromagnetic wave astronomy, the binary supermassive black hole merger wave sources will become a standard siren with large signal-noise ratio and redshift range (in analogy to the standard candle of electromagnetic wave astronomy). This will enable us to determine precisely the redshift-luminosity relation and other cosmic evolution information [31, 32].

### 2.2.3 Extreme mass ratio inspirals.

In the center of a galaxy, through dynamical capture of a compact celestial body (stellar mass black hole, neutron star or a white dwarf) by the supermassive black hole located at the galactic center or possibly certain companion stars generated by the formation and fast evolution of massive stars in the accretion disk surrounding the supermassive black hole, a binary system, termed extreme mass ratio inspiral (EMRI), is formed with mass ratios $\sim 10^6$ or bigger between the supermassive black hole and the compact celestial body. EMRIs are an important gravitational wave source for detection in space around the mHz regime [36, 37, 38]. Though the event rate is highly uncertain, the general expectation is that EMRIs should be present in a galactic center. A long duration of observation is needed to accumulate the EMRI signal to noise ratio to a level above the threshold for detection.

The detection of gravitational waves from EMRIs provides a new window to study and explore the galactic center region which otherwise would be difficult to resolve by means of electromagnetic observations. Through probing the dynamics of EMRIs near the galactic center, we will be able to measure the mass, spin and higher order multipole moments of a supermassive black hole and the detectable range extends far beyond the local Universe. The detection is also a way to confirm the existence of a supermassive black hole in the center of an inactive galaxy in the $(10^4 \sim 10^7 M_\odot)$ regime and enable us to do precision measurements on black hole parameters.

The power of gravitational radiation generated by an EMRI system is rather weak and the orbital evolution of the compact celestial body is very slow. The adiabatic
evolution of Kerr black hole geodesics may be used to describe the orbital dynamics generated by gravitational waves. Though the periodically averaged frequency variation is generally unremarkable, due to the existence of eccentricity and a number of precession effects, the gravitational waves emitted by an EMRI system has rich short-period frequency variations and the phases with complex structures [39]-[43]. As a gravitational wave source, an EMRI system is characterised by 17 parameters. It is a great challenge to handle the adiabatic and multiscale evolution of the spacetime structure and the corresponding geodesics. The data processing in a large and complex parametric space together with a weak and long time wave train will also be a problem we have to contend with. The mass, angular momentum, mass quadrupole moment of a supermassive black hole retrieved from the gravitational wave signals of EMRIs [44, 45] also presents a way to check on the no-hair theorem of black holes [39, 46] and to distinguish alternative gravity theories.

2.2.4 Intermediate mass ratio inspirals.

The observations of ultraluminous X-ray sources (ULXs) and stellar dynamics in star clusters suggest the possible existence of intermediate-mass black holes of \(10^2 \sim 10^4 M_\odot\) in a dense star clusters. Observations indicate that ultraluminous compact sources are very common in our local Universe, the well known ones are M82-X1, M15, NGC3628, and the Centaurus \(\omega\) etc. [41, 47, 48, 49]. Generally, the ULX sources locate at a certain distance from the centers of galaxies, which means that their mass is not very large (does not exceed \(10^5 M_\odot\)). It is found that some ULX sources are surrounded by gaseous loops illuminated by X-rays, this means that at least the X-rays emitted by a part of ULX sources are not highly concentrated in the line of sight connecting the source to the Earth. If the energy released by the ULX sources is isotropic and the luminosity does not exceed the Eddington limit, it may be deduced that the mass for these X-ray sources is in the range of \(10^2 \sim 10^4 M_\odot\).

A strong quasi-periodic light variability has been detected for some ULX sources. The accretion by an intermediate-mass black holes at the center of a star cluster is one possible explanation for these ULX sources.

N body simulations of compact star clusters also suggests that intermediate-mass black holes can be produced by the fast collision and merging of stellar mass black holes in the central regions of star clusters, or by the repeated aggregation of massive stars. Some simulations even suggest that multi intermediate mass black holes may form in young star clusters [50]-[54]. In the star clusters containing intermediate-mass black holes, the compact celestial bodies, such as the stellar mass black holes, neutron stars, etc., can be captured by the intermediate-mass black hole through the dynamic processes of two-body exchange or hierarchical three-body interaction to form a binary system with mass ratios of a few tens to one thousand [52]-[57], which is termed the intermediate mass ratio inspiral (IMRI). Further, when a star cluster with an intermediate-mass black hole evolves toward the galactic nucleus, it will gradually be torn up by the tidal force, the remnant intermediate-mass black
hole and the supermassive black hole in the galactic center will form an IMRI on a bigger mass scale [56].

At present, the theory that an intermediate-mass black hole in the center of a dense star cluster provides a plausible explanation of the ULX observations. There are however many other possible explanations for the observations. Conventional electromagnetic astronomy often is not able to provide good resolution of the center of a cluster. However, gravitational waves can be violently emitted in the final inspiral stage of IMRI systems. The gravitational wave detection of IMRIs in dense star clusters will give equivocal evidence for the existence of intermediate-mass black holes in dense stellar environment. At the same time, the detection provides an entirely new avenue to probe the dynamics of star clusters and addresses important astrophysical problems like the formation mechanism of intermediate-mass black holes, the initial mass-function and dynamical evolution history of star clusters etc.

The characteristic frequencies of gravitational waves emitted by IMRIs in a dense star clusters fall into the $0.01 \sim 1Hz$ regime. To optimise detection of these sources, baseline parameters of a spaceborne detector is required to tune to this frequency range as the most sensitive regime of a detector. At the same time, better sensitivity in laser metrology is also needed to compensate for the weak signal strength due to smaller intermediate mass scale [56]-[59].

2.2.5 Gravitational waves from early Universe

Gravitational waves generated during the big bang and subsequent inflation era provide the earliest clue in our quest for understanding of the origin and creation of our Universe. The primary science objectives of The BBO (Big Bang Observer) [61, 62] and the DECIGO (Deci-Hertz Gravitational wave Observatory) [63, 64, 65] project are to detect directly the primordial gravitational waves from the early universe. However these projects pose extremely demanding requirements on the technological fronts. With the foreseeable development in technologies in the next two decades, it is more sensible to be content with giving an upper bound on this gravitational wave background. There are many competing theories and models which seek to describe the dynamics soon after the big bang, among these are the Big Bang-inflation process, the first-order electroweak phase transition, the extra-dimensional dynamics predicted by the superstring theory, the cosmic string network, [31, 60] etc. The current mission design will be able to impose meaningful constraints on the parameter space of these models.

2.2.6 Burst signals.

Burst signals are difficult to forecast their waveforms and foresee their detection at this stage. These signals may be originated from the short-distance capture/grazing among celestial bodies, or from cosmological process like the breaking up of a cos-
mic string and possibly some new and unmodelled astrophysical or even quantum gravitational processes.

### 3. Mission design

The starting point of the study is to explore the feasibility of detection in the band gap from \(0.1\, \text{Hz}\) to \(10\, \text{Hz}\) between LISA and LIGO. To this end, it is natural to consider in the mission design to shorten the armlength of the interferometer and increase the precision of the laser metrology by suppressing shot noise and enlarge the diameter of the telescopes. This happens to overlap with the ALIA mission concept proposed by Peter Bender [66, 67] which is conceivably in many ways the simplest adaptation of the LISA mission to a frequency band one order of magnitude higher than that of LISA, with the promise of mapping out mass and spin distribution of light seed black holes at high redshifts.

However, when the key technologies of the ALIA mission was further looked at, the sub-picometer interferometry requirement in the laser metrology part poses a major obstacle on the technological side of the mission. With a view that China will have a reasonable chance to realise the mission in the next few decades and to minimise possible risks in future research and development of the key technologies, further relaxation of the baseline parameters to a more realistic level seems to be a natural step to take.

Based on the relative merits between science and technological viability, a number of mission options from both scientific and technological perspective are carefully studied. Mainly due to foreseeable technological limitations in laser interferometry in the next two decades, in the end a compromise between scientific significance and technological feasibility is reached and the following baseline parameters were chosen for further studies given in Table 2. Apart from the detection of gravitational wave sources in the \(10^{-4} - 0.1\, \text{Hz}\) frequency window, the primary science driver is set to probe high redshifts intermediate mass black hole binaries, with a view to understand structure formation in early Universe and galaxy-black hole co-evolution. In doing so, the mission will become a part of an astronomy program, working closely with future infrared red astronomy and radio astronomy programs in China to explore high redshift Universe after the dark age era. The baseline parameters are subject to minor variations, in particular the position noise may be relaxed further to 10pm or more, yet the science of the mission is still worth pursuing.

For reference purpose, the baseline design parameters of ALIA, LISA/eLISA are also given in Table 2. The relevant sensitivity curves are displayed in Figure 1. Apart from the instrumental noises, confusion noise generated by both galactic and extra-galactic compact binaries are also taken into consideration. Relevant confusion levels are converted from estimations by Hils and Bender [10] and Farmer and Phinney [11].
Table 2

| arm length (m) | telescope diameter (m) | laser power (W) | 1-way position noise (pm·Hz⁻¹/₂) | acceleration noise (m·s⁻²·Hz⁻¹/₂) |
|---------------|------------------------|----------------|----------------------------------|----------------------------------|
| 3 × 10⁹       | 0.45~0.6               | 2              | 5~8                              | 3 × 10⁻¹⁵ (> 0.1 mHz)             |
| 5 × 10⁶(ALIA) | 1.0                    | 30             | 0.1                              | 3 × 10⁻¹⁶ (> 1 mHz)               |
| 5 × 10⁹(LISA) | 0.4                    | 2              | 18                               | 3 × 10⁻¹⁵ (> 0.1 mHz)             |
| 1 × 10⁹(eLISA)| 0.2                    | 2              | 11                               | 3 × 10⁻¹⁵ (> 0.1 mHz)             |

Fig. 1 Sensitivity curves of mission designs with different choice of baseline parameters, with ALIA, LISA and eLISA included for the purpose of comparison.

For black hole binaries with mass ratio 1:4, typical of what one would expect from hierarchical black hole growth at high redshift, the all angle averaged detection range are plotted in Figure 2. Apart from galactic confusion noise, upper level (dashed curve) and lower level (dotted dashed curve) of confusion noise generated by extragalactic compact binaries as those estimated by [11] are also taken into account.

In calculating the averaged SNR, we have used hybrid waveforms in the frequency domain with black hole spin not taken into account [68, 69]. For one year of observation before merger, the contributions in SNR due to large spin is indeed negligible according to our calculations. Spin is relevant only in the parameter estimation stage, which will not be discussed in the present work. As may be seen from Figure 2, for a given redshift, the proposed mission concept is capable of detecting lighter black hole binaries in comparison with eLISA/LISA and thereby provides better understanding of the hierarchical assembling process in early Universe.

Apart from intermediate mass black hole binaries at high redshift, the designed sensitivity at around 0.01 Hz measurement band means that the instrument is also capable of detecting IMRIs harboured at globular clusters or dense young star clusters at low redshift (z < 0.6). See [57] for a further discussion of the capture dynamics of an IMRI in dense star clusters. Displayed in Fig 3 are the detection ranges of IMRIs with different mass ratios one year prior to merger. The stellar black hole is fixed to
Fig. 2 All-angle averaged detection range of a single Michelson channel with threshold SNR of 7 for 1:4 mass ratio intermediate black hole binaries, one year observation prior to merger. For each mission option, both upper and lower confusion noise levels (represented by the dashed curve and dotted dashed curve respectively) due to extragalactic compact binaries are considered.

Fig. 3 The signal-noise ratio contours for detecting IMRIs at different redshifts by a single Michelson channel with $3 \times 10^6$ km armlength and one year observation before merger. (a) $5 \text{pm} \cdot \text{Hz}^{-1/2}$, $z = 2$, (b) $5 \text{pm} \cdot \text{Hz}^{-1/2}$, $z = 6$, (c) $5 \text{pm} \cdot \text{Hz}^{-1/2}$, $z = 10$, (d) $8 \text{pm} \cdot \text{Hz}^{-1/2}$, $z = 2$, (e) $8 \text{pm} \cdot \text{Hz}^{-1/2}$, $z = 6$, and (f) $8 \text{pm} \cdot \text{Hz}^{-1/2}$, $z = 10$. Confusion noise considered is at an intermediate level between the upper and lower limit.

be $10M_\odot$, while the mass of an intermediate mass black hole is subject to variation in order to generate different mass ratios in the figure.
Fig. 4 All-angle averaged detection range under a single Michelson threshold SNR of 7 for IMRIs with reduced masses of $10M_\odot$, one year observation prior to merger. For each mission option, both upper and lower confusion noise levels (represented by the dashed curve and dotted dashed curve respectively) due to extragalactic compact binaries are considered.

Fig. 5 The signal-noise ratio contours for detecting IMRIs with the reduced mass of $10M_\odot$ by a single Michelson channel with armlength of $3 \times 10^6 km$, one year of observation before merger. (a) $5 pm \cdot Hz^{-1/2}$, and (b) $8 pm \cdot Hz^{-1/2}$. The confusion noise considered is at an intermediate level between the upper and lower limits.

4. Simulations of cosmic growth and merger of black holes and event rate estimates

To understand the detection capability in high redshift Universe of the mission options given in Table 2, we carry out a Monte Carlo simulation of black hole merger histories based on the EPS formalism and semi-analytical dynamics.

Pop III remnant black holes of $150M_\odot$ are placed in $3.5\sigma$ biased halos at $z = 20$ with initial spins of the seeds generated randomly. By prescribing VHM-type dynamics [25, 26], we trace downwards the black hole merging history. The halo mass
ratio criteria for major merger is set to be greater than 0.1. Both the prolonged accretion and the chaotic accretion scenario are considered. Black hole spins coherently evolve through both mergers and accretions processes and their magnitudes influence strongly the mass-to-energy conversion efficiency. We assume efficient gaseous alignment of the black holes so that the hardening time is short and only moderate gravitational radiation recoils take place. Numerical simulations [71] suggest that the hardening and merger time scales remain short even in gas free environment. In calculations relevant to GW observations, we assume a threshold SNR of 7 for detection in the sense of single Michelson interferometer and one year observation prior to merger.

The results are schematically given in Figure 6 and Figure 7.

![Graph](image)

**Fig. 6** Coalescence rate predicted by the Monte Carlo simulations.

We assess our simulations by fitting the black hole mass functions and luminosity functions at six almost equally divided successive redshift intervals ranging from $z = 0.4$ to 2.1 (see Figure 8). In the prolonged accretion scenario, the results deviate from the observational constrains given by Soltan type argument when going up to redshift $z > 1.5$. It may therefore underestimate the black holes growth rate and perhaps also the coalescence rate. Observationally the existence of very high redshift ($z > 6$) AGNs implies that feedback mechanisms may be very different at early epoch so that fast growth of the seed black hole could be possible.

In terms of coalescence rate, our result displayed in Figure 4 is in overall agreement with the results given by Sesana et al [32, 34] and Arun et al [70], though the coalescence counts given by their simulations are about two or three times higher. It is likely due to various numerical discrepancies in the simulations. Overall, our black hole mass growth is slower, particularly in the prolonged accretion scenario. At $z = 15$, the total mass of the black hole binaries typically are still less than $600M_\odot$. 


in the prolonged model and this may lead to a smaller counts in detectable sources. Our results are expected to give a very conservative (pessimistic) estimate of black hole binaries merger event rate.

The astrophysics encapsulated in our simulation represents the state of art understanding of structural formation after the dark age. Due to our poor understanding of the evolution of the Universe at this epoch, it is likely that the simulation overlooks many details of the physical processes involved. The event rate count should be
looked upon in a cautious way. Instead of reading into the precise numbers, it serves as an indication what spaceborne gravitational wave detector is capable of and in our case, the advantage of setting the most sensitive regime of the measurement band from a few $mHz$ to 0.01$Hz$.

5. Event rate estimate for the detection of IMRIs in dense star clusters

Consider the following scenario: [53, 54, 55]:

1. In the accessible range of the universe for the mission design, the spatial number density of star clusters is a constant in respect to the volume calculated by the luminosity distance. It has the same value as that of the local universe. For globular and young clusters: $n_{GC} \approx 8h^3 \cdot Mpc^{-3}$, $n_{YC} \approx 3h^3 \cdot Mpc^{-3}$, $h = H_0 / (100 km \cdot s^{-1} \cdot Mpc^{-1})$ and we take $h = 0.73$. In the event rate estimate in what follows, we consider only the globular clusters and assume that the total number density of dense star clusters is given by $n_C \approx 8h^3 \cdot Mpc^{-3}$, which will give a conservative estimation of the event rate.

2. The probability of a small compact celestial body being captured by an intermediate-mass black hole at the center of a cluster is given by

$$\nu(M, \mu, z) \approx 10^{-10} \frac{M}{\mu} a^{-1},$$

i.e., it is directly proportional to the mass of the intermediate-mass black hole, and inversely proportional to the reduced mass. This assumption is motivated by the dynamical analysis on globular clusters [51]-[54].

In the event rate calculations, for wave sources with non-negligible redshift effect, a scaling factor $1 + z$ is required in principle, but the coalescence rate itself is just an order of magnitude estimate. Further, the accessible range of redshifts considered by the current mission options is quite small. This scaling factor will be neglected here.

3. In dense star clusters where the intermediate-mass black holes is located, the mass distribution function of the intermediate-mass black holes is assumed to be

$$f(M) = \frac{f_{tot}}{\ln \frac{M_{max}}{M_{min}}} \frac{1}{M},$$

$M_{min}$ and $M_{max}$ are respectively the lower and upper limits of the mass-distribution range of intermediate mass black holes, they are taken respectively as $10^2 M_\odot$ and $10^4 M_\odot$, and beyond this range $f(M) = 0$. $f_{tot}$ is the fraction of dense star clusters containing intermediate-mass black holes. Its value is highly uncertain and we shall take a conservative estimate that $f_{tot} = 0.1$ [53, 55].

Take the observational time of one year before merger, the reduced mass of an IMRI system is taken to be $10M_\odot$ and the threshold value of signal-noise ratio of
a single Michelson detection is taken as 7. The event rate may then be calculated using the following formula [53, 56]:

\[ R = \frac{4\pi}{3} \int_{M_{\text{min}}}^{M_{\text{max}}} |D_L(M, \mu)|^3 v(M, \mu, z) n_c f(M) dM. \]

and the result is given in Table 3.

| Mission option | Upper level of confusion | Lower level of confusion |
|----------------|--------------------------|--------------------------|
| ALIA          |                         |                          |
| \( z_c = 5 \) | \( \sim 8000 \)         | \( \sim 12000 \)         |
| \( z_c = 3 \) | \( \sim 6000 \)         | \( \sim 7000 \)         |
| 5pm(D=0.6m)   | \( \sim 90 \)           | \( \sim 130 \)           |
| 8pm(D=0.45m)  | \( \sim 26 \)           | \( \sim 32 \)           |
| LISA (5 \times 10^6 \text{ km in armlength}) | | \( \sim 3 \) |

The above event rate estimate is subject to many uncertainties and perhaps we should not attach too much importance to the precise numbers. Instead, the calculations serves as an indication of the detection potential of the mission concept as far as IMRIs at low redshifts are concerned. Further, as event rate goes up as the cubic of the improvement in sensitivity, it also brings out the advantage of shifting slightly the most sensitive region of the measurement band to a few hundredth Hz, as far as detection of IMRIs is concerned. It should also be remarked that collision of dense star clusters [57] constitutes a possible intermediate mass black hole binaries gravitational wave sources, while the inspiral of massive black holes (\( \sim 10^3 M_\odot \) to \( \sim 10^4 M_\odot \)) into the supermassive black hole at the center of a galaxy is also a promising IMRI source [56, 72]. However, the corresponding event rates would be difficult to estimate.

**Concluding remarks**

Gravitational wave detection in space promises to open a new window in the quest for understanding of our Universe, in particular as a new way to probe the formation of galactic structure at early Universe discussed here. With the recent revised definition of the LISA mission in which the armlength of the laser interferometer is shortened to 2.5 million kilometers, modulo some minor variations in the baseline parameters, there is basically no longer any difference between LISA and TAIJI in terms of the mission definition. A global effort to realise such a mission seems to be the next natural step forward, though it does not seem possible in view of the current political climate. Still it is an option we should keep in mind, in the hope that when the occasion is right this becomes a realistic step to be taken.
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