EARLY AFTERGLOWS OF GAMMA-RAY BURSTS IN A STRATIFIED MEDIUM WITH A POWER-LAW DENSITY DISTRIBUTION

SHUANG-XI YI1,2, XUE-FENG WU1,3,4,5, AND ZI-GAO DAI1,2

1 School of Astronomy and Space Science, Nanjing University, Nanjing 210093, China; dzg@nju.edu.cn
2 Key Laboratory of Modern Astronomy and Astrophysics (Nanjing University), Ministry of Education, China
3 Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China
4 Chinese Center for Antarctic Astronomy, Chinese Academy of Sciences, Nanjing 210008, China
5 Joint Center for Particle Nuclear Physics and Cosmology of Purple Mountain Observatory-Nanjing University, Chinese Academy of Sciences, Nanjing 210008, China

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ABSTRACT

A long-duration gamma-ray burst (GRB) has been widely thought to arise from the collapse of a massive star, and it has been suggested that its ambient medium is a homogenous interstellar medium (ISM) or a stellar wind. There are two shocks when an ultra-relativistic fireball that has been ejected during the prompt gamma-ray emission phase sweeps up the circumburst medium: a reverse shock that propagates into the fireball, and a forward shock that propagates into the ambient medium. In this paper, we investigate the temporal evolution of the dynamics and emission of these two shocks in an environment with a general density distribution of $n \propto R^{-k}$ (where $R$ is the radius) by considering thick-shell and thin-shell cases. A GRB afterglow with one smooth onset peak at early times is understood to result from such external shocks. Thus, we can determine the medium density distribution by fitting the onset peak appearing in the light curve of an early optical afterglow. We apply our model to 19 GRBs and find that their $k$ values are in the range of 0.4–1.4, with a typical value of $k \sim 1$, implying that this environment is neither a homogenous ISM with $k = 0$ nor a typical stellar wind with $k = 2$. This shows that the progenitors of these GRBs might have undergone a new mass-loss evolution.

Key words: gamma-ray burst: general – radiation mechanisms: non-thermal

1. INTRODUCTION

Since their first discovery in 1997, gamma-ray burst (GRB) afterglows have been well understood (Wijers et al. 1997; Piran 1999; van Paradijs et al. 2000; Mészáros 2002; Zhang & Mészáros 2004), and are usually explained as being due to the interaction of an ultra-relativistic fireball with its surrounding medium. During such an interaction, there are two shocks when a relativistic fireball sweeps up the ambient medium: a forward shock (FS) that propagates into the circumburst medium, and a reverse shock (RS) that propagates into the fireball ejecta. The observed afterglow arises from the synchrotron emission of swept-up electrons accelerated by the FS and RS. GRBs can be classified into two types: short-duration hard-spectrum GRBs, which may originate from the mergers of two compact stars, and long-duration soft-spectrum GRBs, which may come from the core collapse of massive stars. The circumburst medium surrounding these two types of GRBs may be different due to their different origins. By assuming that GRB afterglows are produced by the fireball interacting with the circumburst medium, we can use GRB afterglows to probe their environments. In this paper, we assume a circumburst medium with a general density distribution of $n = AR^{-k}$. Such a circumburst medium is a homogeneous interstellar medium (ISM) when $k = 0$ and a typical stellar wind environment for $k = 2$. Much work has been done in terms of theoretical afterglow light curves for the case of an ISM environment ($k = 0$; Sarı et al. 1998; Kobayashi 2000; Panaitescu & Kumar 2004) and for the case of a typical stellar wind environment ($k = 2$; Dai & Lu 1998a; Mészáros et al. 1998; Panaitescu & Kumar 2000, 2004; Chevalier & Li 2000; Wu et al. 2003, 2004; Kobayashi & Zhang 2003; Zou et al. 2005).

Many early optical afterglows have been detected in the Swift era. The observations could provide important clues about the properties of the ambient medium of GRBs. Li et al. (2012) extensively searched for optical light curves from the literature and found that optical afterglows have different radiation components. These emission components may have distinct physical origins. In this paper, we consider smooth onset peaks in early optical afterglow light curves. The onset of an afterglow is assumed to be synchronous with the moment when the fireball is decelerated by the surrounding medium. Liang et al. (2010) found 20 optical light curves with such smooth onset features. We probe the type of GRB ambient medium with the rising and decaying slopes of the onset peak in the optical light curve. We study the emission of reverse–forward shocks both in the thick- and thin-shell cases for an environment with a general density distribution of $n = AR^{-k}$. We apply our model to 19 GRBs as a case study and find a typical value of $k \sim 1$ (see Figure 5). In Section 2, we discuss the hydrodynamic evolution of a fireball in both the thick-shell and thin-shell cases, and consider reverse–forward shocks in each case. Theoretical light curves of reverse–forward shock emission are derived in Section 3. We investigate 19 optical afterglow onset peaks in detail in Section 4. Discussion and conclusions are presented in Sections 5 and 6, respectively. A concordance cosmology with $H_0 = 71$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_M = 0.30$, and $\Omega_{\Lambda} = 0.70$ is adopted. $Q_\phi$ denotes $Q/10^9$ in cgs units throughout the paper.

2. HYDRODYNAMICS OF A RELATIVISTIC SHELL INTERACTING WITH ITS AMBIENT MEDIUM

For a relativistic shell decelerating in its circumburst medium, two shocks will develop: an RS that propagates into the shell, and an FS that propagates into the ambient medium. We assume that the shell and two shocks are spherical and the shocked fluid is uniform.
in the downstream. The shell is characterized by an initial kinetic energy $E$, initial Lorentz factor $\eta$, and a width $\Delta$ in the lab frame attached to the explosion center. Physical primed quantities are defined in the comoving frame. The comoving number density of the shell is then $n' = E/(4\pi m_p c^2 R'^2 \Delta n^2)$, where $R$ is the radius of the shell. The number density of the ambient stratified medium is assumed to have the following general distribution, $n_1 = A R^{-k} = n_0 (R/R_0)^{-k}$. We fix $n_0 = 1$ cm$^{-3}$ and let $R_0$ be variable. In this paper, we focus on the hydrodynamic evolution and emission of the reverse–forward shocks in arbitrary stratified ambient media with $0 < k < 3$. For $k \geq 3$, the energy-conservation shock solution cannot be applied, and the solution is limited between the shock front and the sonic point in the downstream of the shock (Sari 2006). As in the literature, we divide the two-shock system into four regions (Sari & Piran 1995): (1) the unshocked ambient medium ($n_1$, $c$, $p_1$, $\gamma_1$), (2) the shocked ambient medium ($n'_2$, $c'_2$, $p'_2$, $\gamma_2$), (3) the shocked shell ($n'_3$, $c'_3$, $p'_3$, $\gamma_3$), and (4) the unshocked shell ($n'_4$, $\gamma_4 = \eta$), where $n$ is the number density, $c$ is the internal energy density, $p$ is the pressure, and $\gamma$ is the bulk Lorentz factor. In the lab frame, the ambient medium is assumed to be static, i.e., $\gamma_1 = 1$ (the speed of the ambient medium can be neglected in our problem). The ambient medium and relativistic shell are assumed to be cold, i.e., the internal energy $e$ and pressure $p$ are negligible compared to the rest-mass energy density $\rho c^2$. The shocked ambient medium (region 2) and shocked shell (region 3) are assumed to have a relativistic equation of state, i.e., $p' = e'/3$. The jump conditions for the shocks are $e'_1 = (\gamma_2 - 1)n'_2 m_p c^2$, $n'_2 = 4(\gamma_2 + 3)n_1$ for the FS, and $e'_1 = (\gamma_3 - 1)n'_3 m_p c^2$, $n'_3 = 4(\gamma_3 + 3)n'_4$ for the RS. The Lorentz factor of the RS, $\gamma_3$, can be approximated as $\gamma_3 = (\gamma_3/\gamma_4 + \gamma_4/\gamma_3)/2$, as long as $\gamma_3 \gg 1$ and $\gamma_4 \gg 1$. The equilibrium of pressures and the equality of velocities along the contact discontinuity lead to $p'_2 = p'_3$ and $\gamma_2 = \gamma_3$, respectively. To solve the problem using the initial conditions, we adopt the ratio of the number density of the relativistic shell $n'_4$ to the number density of the ambient medium $n_1$ defined in Sari & Piran (1995), i.e.,

$$f = \frac{l^3}{(3 - k) R^3 \Delta n^2},$$

where the Sedov length $l$ is defined when the rest-mass energy of the swept ambient medium, $M_\text{sw} c^2$, equals the initial energy $E$ of the relativistic shell,

$$l = \frac{(3 - k) E}{4 \pi A m_p c^2}.$$  

On the other hand, the above jump conditions, equilibrium, and equality along the contact discontinuity lead to

$$f = \frac{(\gamma_2 - 1)(\gamma_2 + 3)}{(\gamma_3 - 1)(\gamma_3 + 3)} \simeq \frac{4 \gamma_2^2}{(\gamma_3 - 1)(\gamma_3 + 3)}.$$

For a relativistic reverse shock (RRS), i.e., $\gamma_3 \gg 1$ or $f \ll \eta^2$, we have $\gamma_3 \simeq \eta/2 \gamma_2 = \eta^{1/2} f^{-1/4}/\sqrt{2}$, $\gamma_2 = \gamma_3 \simeq \eta^{1/2} f^{1/4}/\sqrt{2}$. For a non-relativistic (Newtonian) reverse shock (NRS), i.e., $\gamma_3 \simeq 1$ or $\eta^2 \ll f$, we have $\gamma_3 - 1 \simeq 4 \eta^2 / 7 f$, $\gamma_2 = \gamma_3 \simeq \eta$.

The distance $d R$ over which the RS front travels and the length $d x$ of propagation of the RS in the unshocked shell satisfy the following equation (see also Sari & Piran 1995):

$$d R = \frac{d x}{\beta_3} = \beta_3 \left(1 - \frac{\gamma_2 n'_4}{\gamma_3 n'_2}\right),$$

where the second term on the right hand of the above equation reflects the shock compression of the fluid contained in the $d x$. In terms of $f$, we get (Kobayashi 2000; Wu et al. 2003)

$$d R = \alpha \eta \sqrt{f} d x,$$

in which the coefficient

$$\alpha = \frac{1 + 2 \gamma_3 / \eta}{\sqrt{4 (\gamma_3 / \eta)^2 + 6 \gamma_3 / \eta + 4}},$$

where $\alpha \simeq 1/2$ for RRS ($\gamma_3 \ll \eta$) and $\alpha \simeq 3/14$ for NRS ($\gamma_3 \approx \eta$), as given in Sari & Piran (1995). The increase of the electron number in the shocked shell (region 3) corresponds to the decrease of the electron number in the unshocked fireball shell (region 4), which reads

$$d N_3 = -d N_4 = 4 \pi R^2 n'_4 d x = 4 \pi \alpha^{-1} R^2 f^{1/2} n_1 d R.$$

The total number of electrons in the initial shell is $N_0 = E/\eta m_p c^2$. So the RS crossing radius $R_\Delta$ is determined by

$$N_0 = \int_0^{R_\Delta} 4 \pi \alpha^{-1} R^2 f^{1/2} n_1 d R.$$

In the observer’s frame, we have $d R = 2 T^2 c T/(1 + z)$, where $T$ is the observer time and $\Gamma$ is the Lorentz factor of the shock front. For an ultra-relativistic shock, the bulk Lorentz factor of the fluid just behind the shock front is $\gamma = \Gamma / \sqrt{2}$ (Blandford & McKee 1976). In this paper, we adopt the homogeneous-thin-shell approximation and assume that the bulk Lorentz factor of the whole shell is $\gamma$. Here, we use $d R = 4 \gamma^{-2} c T/(1 + z)$.

In general, we can work out the hydrodynamic evolution of the reverse–forward shocks by the above equations and initial conditions. Before we proceed to obtain analytical solutions for the problem, we compare four characteristic radii, which have been introduced to study this problem (Sari & Piran 1995 for $k = 0$; Wu et al. 2003; Zou et al. 2005; Granot 2012 for $k = 2$) as follows.
(1) The RS crossing radius $R_\Delta$, which can be approximated by

$$R_\Delta \simeq \Delta \eta \sqrt{f} \simeq \left( \frac{\Delta f^{3-k}}{3-k} \right)^{1/2}.$$  

(9)

(2) The transition radius $R_N$, which is defined when the RS changes from Newtonian to relativistic ($f = \eta^2$),

$$R_N \simeq \left[ \frac{f^{3-k}}{(3-k) \Delta \eta^4} \right]^{1/2}.$$  

(10)

(3) The spreading radius $R_S$, which is

$$R_S \simeq \Delta_0 \eta^2.$$  

(11)

Taking into account the spreading effect, the width of the shell is $\Delta \simeq \Delta_0 + R/\eta^2$. For $R < R_S$, $\Delta \simeq \Delta_0$; for $R > R_S$, $\Delta \simeq R/\eta^2$.

(4) The deceleration radius $R_\eta$, which is defined when the width of the swept-up ambient medium $M_{sw}$ by the FS equals $M_0/\eta$,

$$R_\eta = \frac{l}{\eta^{3-k}}.$$  

(12)

where $M_0 = E/\eta c^2$ is the initial mass of the fireball shell.

Therefore, we define

$$\xi \equiv \left( \frac{l}{\Delta} \right)^{1/2} \eta^{-1/2}.$$  

(13)

so the four radii follow the relation

$$\frac{R_N}{\xi^{3-k}} \simeq \frac{R_\eta}{R^2} \simeq \frac{R_\Delta}{R^2} \simeq \xi^2 R_S.$$  

(14)

In the case of $\xi < 1$, the order of the four radii is $R_N < R_\eta < R_\Delta < R_S$ ($0 \leq k \leq 2$) or $R_\eta < R_\Delta \leq R_S \leq R_N$ ($2 < k < 3$). $R_S > R_\Delta$ means that the radial spreading of the shell is unimportant, and $\Delta \simeq \Delta_0$. This is the so-called “thick-shell” case, as the initial width of the shell is thick enough so that the spreading can be neglected. In this case, $R_N < R_\Delta$ means that the RS is relativistic for $0 \leq k \leq 2$. However, for $2 < k < 3$, $R_S < R_N$ does not mean that the RS is Newtonian. For $2 < k < 3$, $f$ is proportional to $R^{-k/2}$, which is initially much smaller than $\eta^2$. The evolution of an RS for $2 < k < 3$ is thus from initially relativistic to non-relativistic later. This is because the ambient medium density drops steeply with radius. So $R_\Delta < R_N$ in the case of $2 < k < 3$ indeed means that the RS is relativistic. In general, the RS is always relativistic for $\xi < 1$.

In the case of $\xi > 1$, the order of the four radii is $R_S < R_\Delta < R_\eta < R_N$ ($0 \leq k < 2$) or $R_N \leq R_S \leq R_\Delta < R_\eta$ ($2 \leq k < 3$). $R_S < R_\Delta$ means that the radial spreading is important, and $\Delta \simeq \eta^2$. This is the so-called “thin-shell” case, because the initial width of the shell is thin enough that the spreading is dominant. In this case, we rewrite the expressions for the crossing radius and transition radius, and obtain

$$R_N \simeq R_\Delta \simeq R_\eta \simeq l/\eta^{3-k} \simeq \xi^2 R_S, \quad \xi > 1.$$  

(15)

The above relation shows that the RS becomes mildly relativistic when it just crosses the shell. This can also be drawn from $f \sim \eta^2$ at the crossing radius. Since $f \propto R^{4-k}$ in this case, we can see that $f$ is a decreasing function of $R$ for $k < 3$, or $f$ is much larger than $\eta^2$ at a smaller radius. In the following, in order to work out the analytical solution, we treat the thin-shell case by assuming that the RS is non-relativistic.

2.1. The Thick-shell Case ($\xi < 1$)

The RS in the thick-shell case can be assumed to be relativistic. The density ratio in the thick-shell case is

$$f = \frac{f^{3-k}}{(3-k) R^{2-k} \Delta_0 \eta^2}.$$  

(16)

The crossing radius is

$$R_\Delta = \left[ \frac{(4-k) f^{3-k} \Delta_0}{16(3-k)} \right]^{1/(4-k)}.$$  

(17)

and the crossing time of the RS in the observer’s frame is

$$T_\Delta = (1+z) \int_0^{R_\Delta} \frac{dR}{4 \gamma^2 c} = (1+z) \frac{\Delta_0}{4c}.$$  

(18)

At the crossing time, the bulk Lorentz factor of the shocked fluid (both the shocked shell and shocked ambient medium) is

$$\gamma_{3,\Delta} = \gamma_{2,\Delta} \simeq \frac{1}{\sqrt{2}} \eta^{1/2} f^{1/4} \left[ 2^k (3-k)(4-k)^{2-k} \right]^{-1/2(4-k)} \left( \frac{1}{\Delta_0} \right)^{(3-k)/2(4-k)}.$$  

(19)
For the RS, the relative Lorentz factor between the shocked shell and un-shocked shell at \(R_\Delta\) is
\[
\gamma_{3,\Delta} \simeq \frac{1}{2} \frac{\eta}{\gamma_{3,\Delta}} = \left[2^{3k-8}(3-k)(4-k)^{2-k}\right]^{1/2(4-k)} \frac{1}{\eta} \left(\frac{L}{\Delta_0}\right)^{-(3-k)/2(4-k)}.
\]
(20)
The number density and pressure of the shocked shell at the crossing time are
\[
n'_{3,\Delta} \simeq \frac{8\gamma_{3,\Delta} n_{1,\Delta}}{\eta} = \left[2^{3k-8}(3-k)^{2k-3}(4-k)^{-6+4k}\right]^{1/2(4-k)} \frac{A}{\eta} \left(\frac{(1-2k)(3-k)\Delta_0}{3}\right)^{1/2(4-k)},
\]
(21)
and \(e'_{3,\Delta} \simeq \gamma_{3,\Delta} n'_{3,\Delta} m_p c^2\), respectively. The total number of electrons at \(R_\Delta\) is \(N_{3,\Delta} = N_0\).

For the FS, the number density and pressure of the shocked surrounding medium at \(R_\Delta\) are
\[
n'_{2,\Delta} \simeq 4\gamma_{2,\Delta} n_{1,\Delta} = \left[2^{16+3k}(3-k)^{2k-1}(4-k)^{-6+4k}\right]^{1/2(4-k)} A \left(\frac{(1-2k)(1-2k)\Delta_0}{3}\right)^{1/2(4-k)},
\]
(22)
and \(e'_{2,\Delta} \simeq \gamma_{2,\Delta} n'_{2,\Delta} m_p c^2\), respectively. We assume a pressure balance across the contact discontinuity, \(e'_{2} \simeq e'_{3}\), so we have \(e'_{2,\Delta} \simeq e'_{3,\Delta}\).

The total number of electrons in the shocked medium at \(R_\Delta\) is
\[
N_{2,\Delta} = \frac{4\pi}{3-k} n_{1,\Delta} R_{\Delta}^3 = \frac{4\pi}{3-k} A \left[\frac{(4-k)^3 3^k \Delta_0}{16(3-k)}\right]^{3-k/(4-k)}.
\]
(23)

### 2.1.1. The Shocked Shell

Before the RS crosses the shell, the hydrodynamic evolution of the RS can be characterized by \((T \leq T_\Delta)\)
\[
\gamma_3 = \gamma_{3,\Delta} \left(\frac{T}{T_\Delta}\right)^{-(2-k)/2(4-k)}, \quad R = R_{\Delta} \left(\frac{T}{T_\Delta}\right)^{2/(4-k)}, \quad N_3 = N_{3,\Delta} \left(\frac{T}{T_\Delta}\right),
\]
(24)
and
\[
n'_{3} = n_{3,\Delta} \left(\frac{T}{T_\Delta}\right)^{-(6+4k)/2(4-k)}, \quad p'_{3} = p'_{3,\Delta} \left(\frac{T}{T_\Delta}\right)^{-(2k)/4(4-k)}.
\]
(25)

After the RS crosses the shell, the shocked shell temperature and pressure are very high. The hydrodynamics of the shocked shell will be dominated by its adiabatic expansion. On the other hand, since the shocked shell is located not too far from the FS, it can be roughly regarded as the tail of the FS and so it follows the Blandford–McKee solution (Kobayashi & Sari 2000). Therefore, we assume \(\gamma_3 \propto R^{2k-7/4}, e'_3 \propto R^{(4k-26)/3}, n'_3 \propto R^{(2k-13)/2}\), and \(T \propto R^{1/\gamma_3^2 c}\). So the hydrodynamic evolution of the RS after crossing the shell is characterized by \((T > T_\Delta)\)
\[
\gamma_3 = \gamma_{3,\Delta} \left(\frac{T}{T_\Delta}\right)^{(2k-7)/4(4-k)}, \quad R = R_{\Delta} \left(\frac{T}{T_\Delta}\right)^{1/2(4-k)}, \quad N_3 = N_{3,\Delta},
\]
(26)
and
\[
n'_{3} = n_{3,\Delta} \left(\frac{T}{T_\Delta}\right)^{(2k-13)/4(4-k)}, \quad e'_{3} = e'_{3,\Delta} \left(\frac{T}{T_\Delta}\right)^{(2k-13)/3(4-k)}.
\]
(27)

### 2.1.2. The Shocked Surrounding Medium

Before the RS crosses the shell, the hydrodynamic evolution of the FS can be characterized by \((T \leq T_\Delta)\)
\[
\gamma_2 = \gamma_{2,\Delta} \left(\frac{T}{T_\Delta}\right)^{-(2-k)/2(4-k)}, \quad R = R_{\Delta} \left(\frac{T}{T_\Delta}\right)^{2/(4-k)}, \quad N_2 = N_{2,\Delta} \left(\frac{T}{T_\Delta}\right)^{2(3-k)/(4-k)},
\]
(28)
and
\[
n'_{2} = n_{2,\Delta} \left(\frac{T}{T_\Delta}\right)^{-(2+3k)/2(4-k)}, \quad e'_{2} = e'_{2,\Delta} \left(\frac{T}{T_\Delta}\right)^{-(2k)/4(4-k)}.
\]
(29)

After the RS crosses the shell, the hydrodynamics of the FS follows the Blandford–McKee self-similar solution. Because most of the energy and mass are contained within \(~ R/\gamma_2^2\), hereafter we adopt the uniform thin shell approximation. The hydrodynamics of the FS for \(T > T_\Delta\) is thus characterized by \((T > T_\Delta)\)
\[
\gamma_2 = \gamma_{2,\Delta} \left(\frac{T}{T_\Delta}\right)^{-(3-k)/2(4-k)}, \quad R = R_{\Delta} \left(\frac{T}{T_\Delta}\right)^{1/(4-k)}, \quad N_2 = N_{2,\Delta} \left(\frac{T}{T_\Delta}\right)^{(3-k)/(4-k)},
\]
(30)
and
\[
n'_{2} = n_{2,\Delta} \left(\frac{T}{T_\Delta}\right)^{-(3+3k)/2(4-k)}, \quad e'_{2} = e'_{2,\Delta} \left(\frac{T}{T_\Delta}\right)^{-3/(4-k)}.
\]
(31)
2.2. The Thin-shell Case ($\xi > 1$)

The RS in the thin-shell case can be assumed to be non-relativistic. Because the spreading effect is important in this case, the width of the shell is $\Delta \simeq R/\eta^2$. The density ratio is thus

$$f = \frac{1}{3-k} \left( \frac{l}{R} \right)^{3-k}.$$  \hspace{1cm} (32)

The crossing radius is

$$R_\Delta = \left[ \frac{9(3-k)}{56} \right]^{1/(3-k)} \frac{l}{\eta^{2/(3-k)}},$$  \hspace{1cm} (33)

and the crossing time of the RS in the observer’s frame is

$$T_\Delta = \left[ \frac{9(3-k)}{14 \times 4^{4-k}} \right]^{1/(3-k)} \frac{(1+z)l}{\eta^{2(4-k)/(3-k)}c},$$  \hspace{1cm} (34)

assuming $\gamma_2 = \gamma_3 \simeq \eta$ throughout the entire duration of the RS crossing the shell.

For the RS, the relative Lorentz factor between the shocked shell and unshocked shell at $R_\Delta$ is

$$\gamma_{34,\Delta} \simeq 1 + \frac{4\eta^2}{7f_\Delta} = 1 + \frac{9(3-k)^2}{98}.$$  \hspace{1cm} (35)

The number density and pressure of the shocked shell at the crossing time are

$$n_{3,\Delta}' \simeq 7f_\Delta n_{1,\Delta} = \left[ \frac{2976-k}{36(3-k)^{6-k}} \right]^{1/(3-k)} \frac{Al^{-k} \eta^{6/(3-k)}}{k},$$  \hspace{1cm} (36)

and $e_{3,\Delta}' \simeq (\gamma_{34,\Delta} - 1)n_{3,\Delta}' m_pc^2$, respectively. The total number of electrons at $R_\Delta$ is $N_{3,\Delta} = N_0$.

For the FS, the number density and pressure of the shocked surrounding medium at $R_\Delta$ are

$$n_{2,\Delta}' \simeq 4\gamma_{2,\Delta} n_{1,\Delta} = \left[ \frac{2976-k}{9^2(3-k)^{6-k}} \right]^{1/(3-k)} \frac{Al^{-k} \eta^{6/(3-k)}}{k},$$  \hspace{1cm} (37)

and $e_{2,\Delta}' \simeq \gamma_{2,\Delta} n_{2,\Delta}' m_pc^2$, respectively. We assume a pressure balance across the contact discontinuity, $e_2' \simeq e_3'$, so we have $e_{2,\Delta}' \simeq e_{3,\Delta}'$. The total number of electrons in the shocked ambient medium at $R_\Delta$ is

$$N_{2,\Delta} = \frac{4\pi}{3-k} n_{1,\Delta} R_\Delta^3 = \frac{9\pi}{14} \frac{Al^{3-k}}{\eta^2}.$$  \hspace{1cm} (38)

2.2.1. The Shocked Shell

Before the RS crosses the shell, the hydrodynamic evolution of the RS can be characterized by ($T \leq T_\Delta$)

$$\gamma_3 \simeq \eta, \quad R = R_\Delta \frac{T}{T_\Delta}, \quad N_3 = N_{3,\Delta} \left( \frac{T}{T_\Delta} \right)^{(3-k)/2},$$  \hspace{1cm} (39)

and

$$n_3' = n_{3,\Delta} \left( \frac{T}{T_\Delta} \right)^{-3}, \quad e_3' = e_{3,\Delta}' \left( \frac{T}{T_\Delta} \right)^{-k}.$$  \hspace{1cm} (40)

After the RS crosses the shell, similar to the thick-shell case, the hydrodynamic evolution of the RS is characterized by ($T > T_\Delta$)

$$\gamma_3 = \eta \left( \frac{T}{T_\Delta} \right)^{(2k-7)/(4(4-k))}, \quad R = R_\Delta \left( \frac{T}{T_\Delta} \right)^{1/2(4-k)}, \quad N_3 = N_{3,\Delta},$$  \hspace{1cm} (41)

and

$$n_3' = n_{3,\Delta} \left( \frac{T}{T_\Delta} \right)^{(2k-13)/(4(4-k))}, \quad e_3' = e_{3,\Delta}' \left( \frac{T}{T_\Delta} \right)^{(2k-13)/(4(4-k))}.$$  \hspace{1cm} (42)
2.2.2. The Shocked Surrounding Medium

Before the RS crosses the shell, the hydrodynamic evolution of the FS can be characterized by \((T \lesssim T_\Delta)\)

\[
\gamma_2 \simeq \eta, \quad R = R_\Delta \left( \frac{T}{T_\Delta} \right)^{3-k}, \quad N_2 = N_{2,\Delta} \left( \frac{T}{T_\Delta} \right)^{3-k},
\]

and

\[
n'_2 = n_{2,\Delta} \left( \frac{T}{T_\Delta} \right)^{-k}, \quad \nu'_2 = \nu'_{2,\Delta} \left( \frac{T}{T_\Delta} \right)^{-k}.
\]

After the RS crosses the shell, the hydrodynamics of the FS is similar to the case of the thick shell, which follows the Blandford–McKee solution and can be described as \((T > T_\Delta)\)

\[
\gamma_2 = \eta \left( \frac{T}{T_\Delta} \right)^{-3(k/2)(4-k)} - \nu_2, \quad R = R_\Delta \left( \frac{T}{T_\Delta} \right)^{1/(4-k)}, \quad N_2 = N_{2,\Delta} \left( \frac{T}{T_\Delta} \right)^{(3-k)/(4-k)},
\]

and

\[
n'_2 = n_{2,\Delta} \left( \frac{T}{T_\Delta} \right)^{-3(k/2)(4-k)}, \quad \nu'_2 = \nu'_{2,\Delta} \left( \frac{T}{T_\Delta} \right)^{-3/(4-k)}.
\]

3. EMISSION FROM THE REVERSE–FORWARD SHOCKS

We assume that the afterglow of a GRB is due to the synchrotron radiation of relativistic electrons with a power-law energy distribution, \(N(\gamma'_e) d\gamma'_e = N'_e \gamma'^{-p} d\gamma'_e \left( \gamma'_e > \gamma'_m \right)\), where \(\gamma'_m\) is the minimum Lorentz factor of the shock-accelerated electrons and \(p\) is the power-law index of the energy distribution. Assuming the two fractions \(\epsilon_e\) and \(\epsilon_B\), then the energy densities contained in the electrons and magnetic field are \(U'_e = \epsilon_e \epsilon' \) and \(U'_B = B^2/8\pi = \epsilon_B \epsilon' \), respectively. The minimum Lorentz factor and cooling Lorentz factor of electrons evolve as \(\gamma'_m \propto \epsilon_e \rho' / n'\) and \(\gamma'_c \propto B^{-2} \gamma^{-1} (1 + Y)^{-1}\). The Compton parameter \(Y\) is the ratio of the radiation energy density to the magnetic energy density. Cooling of electrons will change the energy distribution of the electrons (Sari et al. 1998). If the cooling Lorentz factor is smaller than the minimum Lorentz factor, then the energy distribution is altered to \(N(\gamma'_e) \propto \gamma'^{-2}\) for \(\gamma'_e \leq \gamma'_m\), and \(N(\gamma'_e) \propto \gamma'^{-p+1}\) for \(\gamma'_m < \gamma'_e\). Otherwise, the energy distribution is \(N(\gamma'_e) \propto \gamma'^{-p}\) for \(\gamma'_m \leq \gamma'_e \leq \gamma'_c\), and \(N(\gamma'_e) \propto \gamma'^{-p+1}\) for \(\gamma'_c < \gamma'_e\). The characteristic frequency radiated by an electron with \(\gamma'_e\) in the observer’s frame is \(\nu \simeq 0.5 \nu' \gamma'^{2} \nu'_L / (1 + z)\), where \(\nu'_L = q_e B^2 / 2 \pi m_e c\) is the Larmor frequency, where \(q_e\) and \(m_e\) are the charge and rest mass of an electron. Thus, the scaling laws for the typical frequency, cooling frequency, and peak flux density of synchrotron radiation are \(\nu'_e \propto B^2 \gamma \gamma'_m^2 \propto \epsilon'_e \gamma^{-2}\), \(\nu_c \propto 1 / B^3 \gamma T^2 (1 + Y)^2 \propto \epsilon'^{-3/2} \gamma^{-1} T^{-2} (1 + Y)^{-2}\), and \(F_{\nu,\text{max}} = (1 + z) (s - 1) / 2 \left( N_e P_{\text{v, max}} / 4 \pi D_L^2 \right) \propto N_e \gamma' \epsilon'_e^3\), respectively. \(N_e\) is the total number of electrons responsible for synchrotron radiation, where \(D_L\) is the luminosity distance of the source. The peak spectral power is \(P_{\text{v, max}} \simeq m_e c^2 \sigma_T \gamma B' / 1.5 q_e\). We note that only a fraction \((s - 1) / 2\) of the total electrons contribute to the radiation at the peak frequency of \(\nu'_e\), where \(s = 2\) for the fast cooling case \((\nu'_e < \nu'_m)\) and \(s = p\) for the slow cooling case \((\nu'_e > \nu'_m)\).

3.1. Reverse Shock Emission

We now consider the synchrotron emission from the shocked shell. When the RS crosses the shell, it heats the shell and accelerates electrons to form a relativistic non-thermal distribution in the shocked region. Although we investigate the RS emission in this paper, we will not pay too much attention to this emission component. This is because the RS emission is rarely identified in GRBs—only a very small fraction of GRBs have shown the RS emission component in their early light curves. However, tens of GRBs have been identified with an afterglow onset feature at early times, which is attributed to the FS emission. The evolution of the typical frequency, cooling frequency, and peak flux density of the RS follows the dynamics and the properties of the downstream medium, i.e., \(\nu'_m \propto \gamma'_3 \gamma'_3, \nu_c \propto \gamma'_3, T^{-2}, \) and \(F'_{\nu,\text{max}} \propto N_3 \gamma'_3 \gamma'_3\).

3.1.1. The Thick-shell Case

The RS is relativistic in the thick-shell case. The time of the RS crossing the thick shell is comparable to the duration of GRB prompt emission, i.e., \(T_\Delta \sim T_{90}\). The typical frequency, cooling frequency, and peak flux density of the RS at the RS crossing time \(T_\Delta\) are (e.g., Sari & Piran 1999; Waxman & Draine 2000)

\[
\nu'_m,\Delta = q_e \eta^2 \epsilon'_e \left( \frac{p - 2}{p - 1} \right) \frac{m_p^3}{m_e^2} (1 + z)^{-1} \left( \frac{\epsilon'_e}{8 \pi m_p} \right)^{1/2} \left[ \frac{(4 - k)^2 E T_\Delta}{16 \pi A m_p c (1 + z)} \right]^{-k/2(4-k)},
\]

\[
\nu'_c,\Delta = \frac{9(4 - k)^2 \pi m_e q_e}{2(1 + Y')^2 \sigma_T^2 (1 + z) [8 \pi \epsilon'_e A m_p]^{3/2}} \left[ \frac{(4 - k)^2 E T_\Delta}{16 \pi A m_p c (1 + z)} \right]^{(3-k)/2(4-k)}. \]
and
\[
F_{v,\text{max},\Delta}^{\text{RS}} = \frac{(s - 1)(1 + z)^2 m_p \sigma_T (8\pi \epsilon_B A m_p)^{1/2}}{12(4 - k)} \frac{E}{16\pi A m_p c(1 + z)} \left[ \frac{(4 - k)^2 E T_{\Delta}}{16\pi A m_p c(1 + z)} \right]^{-(2k-2)/(4-k)}.
\]

The scaling laws before and after the RS crossing time \(T_{\Delta}\) are
\[
T < T_{\Delta}: \quad v_m^f \propto T^{-\frac{2}{3}}, \quad v_c^f \propto T^{\frac{1}{2}}, \quad F_{v,\text{max},\Delta}^{\text{RS}} \propto T^{\frac{2}{3}-k},
\]
and
\[
T > T_{\Delta}: \quad v_m^f \propto T^{\frac{1}{3}}, \quad v_c^f \propto T^{\frac{1}{2}}, \quad F_{v,\text{max},\Delta}^{\text{RS}} \propto T^{\frac{1}{3}-k}.
\]

Due to the adiabatic cooling, the evolution of \(v_c^f (\gamma_c^f)\) is assumed to be the same as \(v_m^f (\gamma_m^f)\) after the RS crosses the shell (Kobayashi 2000).

### 3.2.1. The Thick-shell Case

In the thin-shell case, the RS is non-relativistic, so it is too weak to decelerate the shell effectively. The spreading of the shell is significant in this case, so the time of the RS crossing the shell is much longer than the duration of GRB prompt emission, i.e., \(T_{\Delta} \gg T_{90}\). The typical frequency, cooling frequency, and peak flux density of the RS at the RS crossing time \(T_{\Delta}\) are
\[
v_m^f = \left[ \frac{9^{12/5} (3 - k)^{24 - 10k}}{29 - 6k 7^{24 - 9k}} \right]^{1/2} \left( \frac{p - 2}{p - 1} \right) m_p^2 q_T \eta^{(6-k)/(3-k)} \left( \frac{\epsilon_B}{m_p} \right)^{1/2} \left( \frac{E}{4\pi A m_p c^2} \right),
\]
\[
v_c^f = \left[ \frac{9^{2/3} (3 - k)^{8 - 6k}}{7^{24} (3 - k)^{10 - 6k}} \right]^{1/2} \left( \frac{\pi m_e}{\sigma_T} \right) \eta^{(4-k)/(3-k)} \left( \frac{E}{4\pi A m_p c^2} \right)^{3(k-4)/(2k-3)}.
\]
and
\[
F_{v,\text{max},\Delta}^{\text{RS}} = \left[ \frac{2^{15/2} 9^{2}}{3^{6} (3 - k)^{2k}} \right]^{1/2} (s - 1)(1 + z)^2 A^{3/2} \left( \frac{\epsilon_B}{m_p} \right)^{1/2} \frac{m_p^3 \sigma_T}{q_T D_L^2} \left[ \frac{E}{4\pi A m_p c^2} \right]^{3(2k-3)/(2k-3)}.
\]

The scaling laws before and after the RS crossing time are
\[
T < T_{\Delta}: \quad v_m^f \propto T^{\frac{1}{3}}, \quad v_c^f \propto T^{\frac{1}{2}}, \quad F_{v,\text{max},\Delta}^{\text{RS}} \propto T^{\frac{2}{3}-k},
\]
and
\[
T > T_{\Delta}: \quad v_m^f \propto T^{\frac{1}{3}}, \quad v_c^f \propto T^{\frac{1}{2}}, \quad F_{v,\text{max},\Delta}^{\text{RS}} \propto T^{\frac{1}{3}-k}.
\]

We assume that the equation of state of the shocked shell is mildly relativistic, so it can be regarded as the tail of the FS, satisfying the Blandford–McKee self-similar solution (see Kobayashi 2000 for an alternative treatment). However, since the RS emission is usually not observed (suppressed by the FS) in the thin-shell case, this assumption is unimportant.

### 3.2. Forward Shock Emission

Most GRBs have been detected with FS emission at early times. A significant fraction of GRBs have shown the afterglow onset feature (Liang et al. 2010). In this paper, we focus on the FS emission and investigate the effect of environments. The evolution of the typical frequency, cooling frequency, and peak flux density of the FS follows the dynamics and the properties of the downstream medium, i.e., \(v_m \propto \epsilon_B^{3/5} \gamma_2^{(3/2)} \gamma_2^{-3} T_{92}^{-2}, \quad v_c \propto \epsilon_B^{3/2} \gamma_2^{-5/2} T_{92}^{-1}, \quad \) and \(F_{v,\text{max},\Delta}^{\text{FS}} \propto N_z, \gamma_2^{(1/2)} T_{92}^{-1} \).

#### 3.2.1. The Thick-shell Case

The two characteristic frequencies and peak flux density at the RS crossing time \(T_{\Delta}\) in the thick-shell case are
\[
v_m^f = \frac{1}{2^{7/2}(4 - k)} \pi \epsilon_B^{3/2} \left( \frac{\epsilon_B}{m_p} \right)^{1/2} \left( \frac{p - 2}{p - 1} \right) m_p^2 q_T \left[ (1 + z) E \right]^{1/2},
\]
\[
v_c^f = \frac{9 \pi (4 - k)^2}{4k/(4-k)} \frac{q_T m_e}{(1 + z)^2 (1 + Y_f)^2 \left( 8\pi \epsilon_B A m_p \right)^{3/2}} \left[ \frac{(4 - k)^2 E \Delta}{4\pi A m_p c(1 + z)} \right]^{3(k-4)/(2k-3)}.
\]
and
\[
F_{v,\text{max},\Delta}^{\text{FS}} = \frac{(4 - k)^2}{3\pi (3 - k)^2 (1 + z)} \frac{1}{(1 + z)} m_p \sigma_T E \frac{q_T m_p D_L^2}{\left( 8\pi \epsilon_B A m_p c \right)^{1/2}} \left[ \frac{(4 - k)^2 E \Delta}{4\pi A m_p c(1 + z)} \right]^{-(2k-2)/(4-k)}.
\]

The scaling laws before and after the RS crossing time are
\[
T < T_{\Delta}: \quad v_m^f \propto T^{-1}, \quad v_c^f \propto T^{\frac{3}{2}}, \quad F_{v,\text{max},\Delta}^{\text{FS}} \propto T^{\frac{3}{2}-k},
\]
and
\[
T > T_{\Delta}: \quad v_m^f \propto T^{-\frac{1}{2}}, \quad v_c^f \propto T^{\frac{3}{2}}, \quad F_{v,\text{max},\Delta}^{\text{FS}} \propto T^{\frac{3}{2}-k}.
\]

As we can see, for the FS emission in the thick-shell case, the evolution of \(v_m^f\) is independent of \(k\), and hence does not depend on the distribution of the ambient medium.
3.2.2. The Thin-shell Case

The two characteristic frequencies and peak flux density at the RS crossing time $T_\Delta$ in the thin-shell case are

$$v_{m,\Delta} = \left[ \frac{2^{3+2k} \pi^k}{3(3-k)2^k} \right]^{1/2(3-k)} \left( \frac{e_i c}{\mu_k} \right)^2 \left( \frac{p-2}{p-1} \right)^2 \frac{m_p^3 \sigma T}{m_e^2} \frac{q_e(4-3k)/(3-k)}{1+z} \left( \frac{\epsilon_f A}{\pi m_p} \right)^{1/2} \left[ \frac{E}{4\pi A m_p c^2} \right]^{-k/(3-k)},$$

$$v_{c,\Delta} = \left[ \frac{2^{2+2k} \pi^k}{3(3-k)2^k} \right]^{1/2(3-k)} \left( \frac{m_e}{\sigma T^2 (\pi \epsilon_f m_p A)^{3/2}} \right) \frac{q_e (4-3k)/(3-k)}{1+z} (1+Y/f)^2 \left( \frac{E}{4\pi A m_p c^2} \right)^{(3k-4)/2(3-k)},$$

and

$$F_{v,\max,\Delta}^{FS} = \left[ \frac{9^{3-2k} (3-k)6-4k}{25-4k76-3k} \right]^{1/2(3-k)} (s-1)(1+z)\eta^{k/(3-k)} A^{3/2} (\pi \epsilon_f m_p)^{1/2} \frac{\sigma T}{q_e D_L^2} \left[ \frac{E}{4\pi A m_p c^2} \right]^{6+3k}. \tag{64}$$

The scalings laws before and after the RS crossing time are

$$T < T_\Delta : \quad v_m^f \propto T^{-\frac{1}{3}}, \quad v_c^f \propto T^{\frac{2}{3} \frac{4}{k}}, \quad F_{v,\max}^{FS} \propto T^{\frac{6+3k}{6}}, \tag{65}$$

and

$$T > T_\Delta : \quad v_m^f \propto T^{-\frac{1}{2}}, \quad v_c^f \propto T^{\frac{3-4k}{6}}, \quad F_{v,\max}^{FS} \propto T^{\frac{6+3k}{6} \frac{3}{2}}. \tag{66}$$

For $T > T_\Delta$, the FS enters the Blandford–McKee phase either in the thin-shell case or in the thick-shell case. So the hydrodynamics and temporal evolution of the characteristic frequencies and peak flux density are the same in both cases after the RS crosses the shell. The theoretical flux density before the RS crosses the shell is

$$F_{v}^{FS}(T < T_\Delta) \propto \left\{ \begin{array}{ll}
T^{\frac{k-3k-4p}{12}} v^{-\frac{5}{2}}, & v > \max \{ v_c^f, v_m^f \} \\
T^{\frac{k-2k-3p}{12}} v^{-\frac{7}{2}}, & v_m^f < v < v_c^f \\
T^{\frac{k-3k}{4}} v^{-\frac{7}{2}}, & v_c^f < v < v_m^f.
\end{array} \right. \tag{67}$$

The theoretical flux density after the RS crosses the shell is

$$F_{v}^{FS}(T > T_\Delta) \propto \left\{ \begin{array}{ll}
T^{-\frac{k-4}{6}} v^{-\frac{7}{2}}, & v > \max \{ v_c^f, v_m^f \} \\
T^{-\frac{k-3k-4p}{12}} v^{-\frac{5}{2}}, & v_m^f < v < v_c^f \\
T^{-\frac{7}{2}} v^{-\frac{7}{2}}, & v_c^f < v < v_m^f.
\end{array} \right. \tag{68}$$

Figures 1 and 2 present theoretical light curves of the FS emission at early times in the thin-shell case for $k = 1$.

4. CASE STUDY

In this paper, we investigate the temporal evolution of the dynamics and emission of these two shocks in a stratified medium with a power-law density distribution when both thick-shell and thin-shell cases are considered. The crossing time $T_\Delta \sim \Delta_0/c$ is comparable to the GB duration time for the thick shell, while for the thin shell, the crossing time $T_\Delta$ is larger than the GB duration. The observed peak time of early afterglow onset is typically larger than the GB duration in a statistical sense (Liang et al. 2010; Li et al. 2012). Therefore, these early onset peaks can be explained as the FS emission in the thin-shell case, and the peak time of the onset peak is interpreted as the RS crossing time. Recently, Liang et al. (2013) estimated the values of $k$ for a sample of early optical afterglow onset peaks by assuming $v_m^f < v < v_c^f$. They took $k$ to be free for the rising phase, but assumed $k = 0$ for the decay phase. They found that $k$ is generally less than 2 and the typical value of $k$ is $\sim 1$. In this paper, we consider a general power-law distribution of the ambient medium density during the whole afterglow phase and calculate the hydrodynamic evolution of forward–reverse shocks in both the thick-shell and thin-shell cases. This is different from Liang et al. (2013), as mentioned above. We do not consider the case of $v < \{ v_c^f, v_m^f \}$, because it is unlikely in optical and X-ray spectra. Synchrotron self-absorption can be neglected in the X-ray emission, and may also be unimportant most of the time for optical afterglows. For simplicity, we do not consider this effect in this paper. We select 19 GRBs as a sample to determine their $k$ values. Most of our sample is the same as Liang et al. (2013). In the following, we take three well-observed optical afterglows as example cases to test the FS model discussed in this paper. The fitting results of the three GRBs are presented in Figure 3 and the results of the remaining GRBs are shown in Figure 4 and Table 1.

4.1. GRB 060605

GRB 060605 is a relatively faint GRB that was detected by Swift/Burst Alert Telescope (BAT), with a redshift of $z = 3.773$ (Ferrero et al. 2009). From Sato et al. (2006), the burst in the 15–350 keV band had a duration of $T_{90} = 15 \pm 2$ s. According to the traditional $T_{90}$ classification method, GRB 060605 belongs to a long duration burst. Because long bursts are widely believed to originate from the collapse of massive stars, the circumburst medium of GRB 060605 might have been a stellar wind environment. The peak time of this optical onset is $t_p = 399.1 \pm 13.0$ s, while the rising and decaying indices are $\alpha_1 = 0.90 \pm 0.09$ and $\alpha_2 = 1.17 \pm 0.05$,
respectively (Rykoff et al. 2009). Ferrero et al. (2009) studied the broadband spectrum of the afterglow of GRB 060605 at days and obtained a spectral index \( \alpha \approx 3.0 \pm 0.5 \). The correction for Galactic extinction at the \( R_c \) band was considered. We consider both \( \nu > \max\{\nu_C, \nu_m\} \) and \( \nu_{C} > \nu > \nu_m \) to interpret the spectral index \( \beta_o = 1.04 \pm 0.05 \). Thus, we have two possible values for the power-law index of energy distribution.

1. \( \nu > \max\{\nu_C, \nu_m\} \). In this case, \( \nu = 2\beta_0 = 2.08 \pm 0.10 \). The value of \( \nu \) can also be derived from the decay index, i.e., \( \nu = (4\alpha_2 + 2)/3 = 2.23 \pm 0.17 \), which is consistent with that derived from the optical spectrum. The theoretical rising index is \( \alpha_1 = (8 - 2k - kp)/4 \) (see Equation (67)), so \( k = (8 - 4\alpha_1)/(2 + p) = 1.08 \pm 0.11 \). The values \( k \) and \( p \) are both reasonable. We thus apply the \( \nu > \max\{\nu_C, \nu_m\} \) case of the FS model to fit this optical peak, adopting \( k = 1.08 \pm 0.11 \) and \( p = 2.08 \pm 0.10 \).

2. \( \nu_{m} < \nu < \nu_C \). In this case, \( \nu = 2\beta_0 + 1 = 3.08 \pm 0.10 \). The theoretical decaying index \( \alpha_2 = (12p - 3kp + 5k - 12)/(16 - 4k) \) (see Equation (68)). Therefore, we obtain \( k = (12p - 16\alpha_2 - 12)/(3p - 4\alpha_2 - 5) \approx -14.2 \) with the observed decaying index \( \alpha_2 = 1.17 \pm 0.05 \). This value of \( k \) is not reasonable. Thus, the model with \( \nu_{m} < \nu < \nu_C \) cannot explain the optical onset peak of GRB 060605.

From the two cases discussed above, we find that only the \( \nu > \max\{\nu_C, \nu_m\} \) case could be applied to explain the optical afterglow onset of GRB 060605. We derive \( k = 1.08 \pm 0.11 \) and \( p = 2.08 \pm 0.10 \). Figure 3 shows our model fitting to the observed afterglow of GRB 060605. We can see that the medium density profile \( n \propto R^{-1.1} \) is required to fit the data of GRB 060605. This implies that the circumburst medium of GRB 060605 is neither a homogenous ISM with \( k = 0 \) nor a typical stellar wind environment with \( k = 2 \), as previously assumed.

There is a total of seven physical parameters in our model, i.e., \( E, A(R_0), \eta, k, p, \varepsilon_B, \) and \( \varepsilon_e \). However, there are not enough observational conditions in GRB 060605 to derive the exact values of these parameters. We can only constrain the range of these parameters with available conditions. The values of \( k \) and \( p \) are estimated above for GRB 060605, so we can fix \( k = 1.1 \) and \( p = 2.1 \) in the following calculation. Then, we obtain

\[

v_{m,\Delta}^f = 1.41 \times 10^{15} \varepsilon_e^{-\frac{1}{2}} F_{\delta B,\gamma} E_{53}^{-0.029} \eta_2^{4.58} R_{0,17}^{0.87} \text{Hz},
\]

\[

v_{C,\Delta}^f = 1.69 \times 10^{14} \varepsilon_B^{-\frac{3}{2}} (1 + Y_f^2)^{-2} E_{53}^{-0.18} \eta_2^{0.37} R_{0,17}^{1.45} \text{Hz},
\]

\[

F_{v,\max,\Delta}^{FS} = 5.56 \times 10^{-3} \varepsilon_B^{-\frac{1}{2}} E_{53}^{0.71} \eta_2^{0.87} R_{0,17}^{1.17} \text{Jy}.
\]
Since the \( \nu > \max \{ \nu_f^l, \nu_m^l \} \) case is applied to explain the optical onset peak of GRB 060605, we get two constraints, i.e., \( \nu > \nu_m^l \) and \( \nu > \nu_f^l \), where \( \nu = 4.29 \times 10^{14} \) Hz is the optical frequency. The constraints are shown as follows.

1. The initial isotropically kinetic energy \( E = (1 - \eta_f)/\eta_f \nu_f,iso \), where \( \eta_f \) is the radiation efficiency of GRBs. The initial energy is \( E \sim \eta f \sim 10^{52} \) erg, as \( E_{\nu,iso,52} = 2.8 \pm 0.5 \) (Li et al. 2012; also see Ferrero et al. 2009) and \( \eta_f \sim 0.2 \).

2. \( \nu > \nu_m^l \). From Equation (69), we obtain \( R_{0,17} < 0.26E_{53}^{0.33} \varepsilon_{e,-1}^{-2.30} \varepsilon_{B,-1}^{0.58} \varepsilon_{\gamma,-1}^{-5.27} \).

3. \( \nu > \nu_f^l \). From Equation (70), we obtain \( R_{0,17} > 0.53E_{53}^{-0.02} \eta_2^{0.25} \varepsilon_{53}^{-0.13} (1 + \gamma f^{-1})^{1.38} \).

4. The crossing time \( T_o = t_p \). From Equation (34), we obtain \( R_{0,17} = 0.34E_{53}^{0.91} \eta_2^{0.27} \).

5. The peak flux density of the optical onset \( F_{\nu,p} \approx 3 \times 10^{-3} \) Jy. From Equations (69)–(71), we obtain

\[
F_{\nu,p}^{FS} = 6.71 \times 10^{-3} \varepsilon_{e,-1}^{1.10} \varepsilon_{B,-1}^{-0.03} (1 + \gamma f^{-1})^{1.61} E_{53}^{0.46} \eta_2^{3.28} R_{0,17}^{0.02} \text{ Jy},
\]

for \( \nu > \max \{ \nu_f^l, \nu_m^l \} \). So we have \( R_{0,17} \approx 0.27 \varepsilon_{e,-1}^{1.77} \varepsilon_{B,-1}^{-0.04} (1 + \gamma f^{-1})^{1.61} E_{53}^{0.74} \eta_2^{3.28} R_{0,17}^{0.02} \).

The allowed parameter values should satisfy the above constraints (2)–(5). Combining constraints (2) and (5), we have \( E_{53} > 0.53 \varepsilon_{e,-1}^{1.10} \varepsilon_{B,-1}^{-0.03} (1 + \gamma f^{-1})^{1.61} \). Combining constraints (4) and (5), we have \( E_{53} = 0.87 \varepsilon_{e,-1}^{1.10} \varepsilon_{B,-1}^{-0.02} (1 + \gamma f^{-1})^{0.98} \). Combining constraints (2) and (4), we have \( E_{53} < 0.75 \varepsilon_{e,-1}^{2.30} \varepsilon_{B,-1}^{-0.58} \). Combining constraints (3) and (4), we have \( \eta_2 < 0.92E_{53}^{0.19} \varepsilon_{B,-1}^{0.19} (1 + \gamma f^{-1})^{0.25} \). From the above analysis, we can see that the initial Lorentz factor \( \eta \sim 100 \), which is insensitive to other parameters. In Figure 3, we fit the optical data of GRB 060605 by adopting the following parameter values, \( k = 1.08 \pm 0.11, p = 2.08 \pm 0.10, R_{0,17} = 1 \times 10^{17} \text{ cm}, E = 8 \times 10^{53} \) erg, \( \eta = 120, \varepsilon_B = 0.2 \), and \( \varepsilon_e = 0.02 \).

4.2. GRB 081203A

GRB 081203A was detected and located by Swift/BAT with a duration of \( T_o = 294 \pm 71 \) s (Ukwatta et al. 2008). Optical spectroscopic observation led to the measurement of the redshift \( z = 2.05 \pm 0.01 \) (Kuin et al. 2009). The peak time of this afterglow onset is \( T_p = 367.1 \pm 0.8 \) s; the rise and decay slopes are \( \alpha_1 = 2.20 \pm 0.01 \) and \( \alpha_2 = 1.49 \pm 0.01 \), respectively (Kuin et al. 2009). The ultraviolet spectrum of this GRB was observed with the index \( \beta_0 = 0.90 \pm 0.01 \) for the early time. We also consider the two following scenarios \( \nu > \max \{ \nu_f^l, \nu_m^l \} \) and \( \nu_f^l > \nu > \nu_m^l \) to determine the environment of this GRB.

1. \( \nu > \max \{ \nu_f^l, \nu_m^l \} \). In this case, we get \( k = -0.24 \pm 0.01 \), and \( p = 3.00 \pm 0.01 \) for the rising–decaying indices (see Equations (67) and (68)). This value of \( k \) is not reasonable, so this model is not suitable for GRB 081203A.
The two characteristic frequencies and peak flux density at the RS crossing time could be calculated with the above derived $k = 0.40$ and $p = 2.91$,

$$v_{m,\Delta}^f = 3.56 \times 10^{16} \epsilon_{e,-1} 2^{1/2} E_{54}^{-0.077} \eta_2^{4.15} R_{0.17}^{0.23} \text{Hz},$$  \hspace{2cm} (73)$$

$$v_{e,\Delta}^f = 8.16 \times 10^{13} \epsilon_{B,-1}^{3/2} (1 + Y_f)^{-2} E_{54}^{-0.54} \eta_2^{1.08} R_{0.17}^{-0.38} \text{Hz},$$  \hspace{2cm} (74)$$

$$F_{\nu,\text{max}}^{\text{FS}} = 0.204 \epsilon_{B,-1}^{1/2} E_{54}^{0.21} \eta_2^{0.15} R_{0.17}^{0.23} \text{Jy}.$$  \hspace{2cm} (75)$$

Since the $v_m < v < v_f$ case is applied to explain the optical onset peak of GRB 081203A, we get two constraints, i.e., $v > v_m^f$ and $v_e^f > v$, where $v = 4.29 \times 10^{14}$ Hz is the optical frequency. The constraints are shown as follows.

1. $E = (1 - \eta_\gamma) / \eta_\gamma E_{\gamma,\text{iso}}$. The initial energy is $E \sim \text{a few} \times 10^{54}$ erg as $E_{\gamma,\text{iso}} = 1.7 \pm 0.4$ erg (Li et al. 2012).

2. $v > v_m^f$. From Equation (69), we obtain $R_{0.17} < 4.53 \times 10^{-7} E_{54}^{0.35} \epsilon_{e,-1}^{8.69} \eta_2^{-1.21} \eta_2^{-18.04}$.

3. $v < v_e^f$. From Equation (70), we obtain $R_{0.17} < 0.013 \epsilon_{B,-1}^{3.95} \eta_2^{2.84} E_{54}^{-1.42} (1 + Y_f)^{-5.26}$.

4. The crossing time $T_{\Delta} = t_p$. From Equation (34), we obtain $R_{0.17} = 8.02 E_{54}^{0.40} \eta_2^{-18}$.

5. The peak flux density of the optical onset $F_{\nu,p} \sim 2.6 \times 10^{-2}$ Jy. From Equations (73)–(75), we get

$$R_{0.17} \approx 8.86 \times 10^{-7} \epsilon_{e,-1}^{-4.22} E_{B,-1}^{-2.17} E_{54}^{-1.89} \eta_2^{-9.09}.$$  \hspace{2cm} (76)$$

The allowed parameter values should satisfy the above constraints (2)–(5). Combining constraints (2) and (5), we have $\eta_2 < 0.556 E_{54}^{0.25} \epsilon_{e,-1}^{-1}$. Combining constraints (3) and (5), we have $\eta_2 > 0.45 E_{54}^{-0.04} \epsilon_{e,-1}^{-1} (1 + Y_f)^{0.44}$. Combining constraints (4)
and (5), we have $\eta_2 \sim 5.93 \epsilon_{54}^{0.25} \epsilon_{e,-1}^{0.47} \epsilon_{B,-1}^{0.24}$. In Figure 3, we fit the optical data of GRB 081203A by adopting the following parameter values, $k = 0.40 \pm 0.01$, $p = 2.91 \pm 0.01$, $R_0 = 1 \times 10^{17}$ cm, $E = 2 \times 10^{54}$ erg, $\eta = 120$, $\epsilon_B = 0.01$, and $\epsilon_e = 0.01$.

4.3. XRF 071031

The early light curve of the optical/near-infrared afterglow of the X-Ray Flash (XRF) 071031 at $z = 2.05$ with a duration of $T_{90} = 180 \pm 10$ s (Stamatikos et al. 2007; Kühler et al. 2009a) shows a slow increase with flux $\propto T^{-0.634 \pm 0.002}$ before the peak time $T_p = 1018.6 \pm 1.6$ s. After the peak time, the light curve decays with $T^{-0.845 \pm 0.001}$. The optical afterglow spectral index is $\beta_o = 0.9 \pm 0.1$.

1. $\nu > \max\{v_f^{\text{FS}}, v_m\}$. In this case, $p = 2\beta_0 = 1.8 \pm 0.2$. The value of $p$ can also be derived from the decay index, i.e., $p = (4\alpha_2 + 2)/3 = 1.793 \pm 0.001$, which is consistent with that derived from the optical spectrum. The theoretical rising index is $\alpha_1 = (8 - 2k - kp)/(2 + p) = 1.440 \pm 0.001$. The values of $k$ and $p$ are both reasonable. We thus apply the $\nu > \max\{v_f^{\text{FS}}, v_m\}$ case of the FS model to fit this optical light curve, adopting $k = 1.440 \pm 0.001$ and $p = 1.793 \pm 0.001$.

2. $v_m < \nu < v_f^{\text{FS}}$. In this case, $p = 2\beta_0 + 1 = 2.8 \pm 0.2$. The theoretical decay index $\alpha_2 = (12p - 3kp + 5k - 12)/(16 - 4k)$ (see Equation (68)). Therefore, we obtain $k = (12p - 162 - 12)/(3p - 4\alpha_2 - 5) \sim 404$ with the observed decay index $\alpha_2 = 0.845 \pm 0.001$. This value of $k$ is unreasonable. Thus, the model with $v_m < \nu < v_f^{\text{FS}}$ cannot explain the optical light curve of GRB 071031.

For $k = 1.440$ and $p = 1.793$, the two characteristic frequencies and the peak flux density at the RS crossing time are

$$v_{f,\Delta}^{\text{FS}} = 2.2 \times 10^{16} \epsilon_{B,-1}^{1.2} E_{53}^{-0.46} \epsilon_{\eta,2}^{-1} R_{0,17}^{1.38} \text{ Hz},$$

$$v_{c,\Delta}^{\text{FS}} = 9.65 \times 10^{13} \epsilon_{B,-1}^{-3/2} (1 + Y f)^{-2} E_{53}^{0.1} \epsilon_{\eta,2}^{-0.21} R_{0,17}^{-2.31} \text{ Hz},$$

$$F_{\nu,\text{max},\Delta}^{\text{FS}} = 0.01 \epsilon_{B,-1}^{1/2} E_{53}^{0.54} \epsilon_{\eta,2}^{0.92} R_{0,17}^{1.38} \text{ Jy}.$$

* For a flat energy distribution of electrons with $p < 2$, most of the energy of electrons is deposited in electrons with minimal Lorentz factors; for details see Dai & Cheng (2001). Here, we assume that the energy distribution of shock injected electrons has a broken power-law form, as introduced in Li & Chevalier (2001), thus the calculation of $v_m$ is the same as in Sari et al. (1998).
Since the $\nu > \nu_{\text{max}}$ case is applied to explain the optical light curve of GRB 071031, we get two constraints, i.e., $\nu > \nu_{\text{m}}$ and $\nu > \nu_{\text{c}}$, where $\nu = 4.29 \times 10^{14}$ Hz is the optical frequency. The constraints are shown as follows.

1. $E = (1 - \eta \gamma)/\eta \gamma E_{\gamma, \text{iso}}$. The initial energy is $E \sim 4 f_{\text{e}} w = 3.9 \pm 0.6$ erg (Li et al. 2012).

2. $\nu > \nu_{\text{m}}$. From Equation (69), we obtain $R_{0,17} < 0.058 E_{53}^{-0.33} f_{\text{e}}^{-1.45} \eta_2^{-0.36} \nu_{\text{c}}^{-3.57}$.

3. $\nu > \nu_{\text{c}}$. From Equation (70), we obtain $R_{0,17} > 0.52 \nu_{\text{c}}^{-0.65} \eta_2^{-0.09} E_{53}^{0.04} (1 + \nu_{\text{f}})^{-0.87}$.

4. The crossing time $T_{\Delta} = t_p$. From Equation (34), we obtain $R_{0,17} = 0.092 E_{53}^{0.69} \eta_2^{-3.56}$.

5. The peak flux density of the optical onset $F_{\nu,p} \sim 1.66 \times 10^{-4}$ Jy. From Equations (73)–(75), we get $R_{0,17} \sim 1.74 \times 10^{-3} f_{\text{e}}^{-1.03} \eta_2^{-0.06} E_{53}^{-0.53} \nu_{\text{c}}^{-3.59} (1 + \nu_{\text{f}})^{1.29}$.

The allowed parameter values should satisfy the above constraints (2)–(5). Combining constraints (2) and (5), we have $E_{53} > 0.017 f_{\text{e}}^{0.49} \eta_2^{0.49} (1 + \nu_{\text{f}})^{1.5}$. Combining constraints (4) and (5), we have $E_{53} \sim 0.039 f_{\text{e}}^{-0.84} \eta_2^{-0.05} (1 + \nu_{\text{f}})^{1.06}$. Combining constraints (3) and (4), we have $\eta_2 < 0.61 E_{53}^{0.18} \nu_{\text{c}}^{0.19} (1 + \nu_{\text{f}})^{0.25}$. In Figure 3, we fit the optical data of GRB 071031 by adopting the following parameter values, $k = 1.440 \pm 0.001$, $p = 1.793 \pm 0.001$, $R_0 = 2 \times 10^{16}$ cm, $E = 5 \times 10^{53}$ erg, $\eta = 90$, $\nu_{\text{c}} = 0.2$, and $\nu_{\text{f}} = 0.02$.

5. DISCUSSION

We have investigated the hydrodynamic evolution of a fireball in both thick-shell and thin-shell cases, and considered reverse–forward shocks in each case. According to the standard fireball model, the RS is initially non-relativistic for the thin-shell case, which is consistent with most of the onset observations. If the GRB ejecta is highly magnetized ($\sigma \gg 1$), then the RS will be significantly suppressed, and hence the FS evolution will also be altered. Although observations suggest that in some GRBs the ejecta is likely magnetized, the degree of magnetization is usually $\sigma < 1$ at the radius when the ejecta begins to decelerate. For simplicity, we assume that the ejecta has no magnetization ($\sigma = 0$) in this paper. For early afterglows from GRB ejecta with non-negligible magnetization, please see, e.g., Zhang et al. (2003) and Zhang & Kobayashi (2005). Our paper aims to present analytical solutions for the reverse–forward shock hydrodynamics and emission. In our numerical fit to some GRB afterglow onset,
we neglect the curvature effect. The curvature effect, or more strictly speaking, the equal-arrival-time-surface effect, has a minor effect on the rise-decay slope of GRB afterglows.

A large number of multi-waveband afterglows have been detected since the launch of Swift. The observations show that the optical and X-ray afterglows of some bursts have different temporal properties. A question thus arises: do afterglows at different wavebands have the same origin? Here, we analyze GRB 060605 as an example to discuss this question. The smooth optical afterglow of this burst is assumed to have been produced by the FS when the fireball was decelerated by a circumburst medium in the plateau phase with two break times \( t_{\text{fb}} \) and \( t_{\text{sw}} \). We applied our model to 19 GRBs and found that their \( k \) values are in the range of \( 0.4-1.4 \), with a typical value of \( k \approx 1 \) (see Figure 5). This implies that the circumburst medium of those GRBs is neither the ISM (\( k = 0 \)) nor a typical stellar wind (\( k = 2 \)). This could show a new

### Table 1

| GRB        | \( z \) | \( \beta_0 \) | \( k \) | \( p \) | \( t_B \) (s) | \( t_E \) (s) | \( R_0 \) (cm) | \( E \) (erg) | \( \eta \) | Emission Regime | Refs. |
|------------|--------|-------------|-------|------|-----------|-----------|------------|--------|-------|-----------------|------|
| 030418     | ...    | ... | 1.09 ± 0.12 | 1.73 ± 0.11 | 0.2 | 0.2 | E16 | 2E52 | 75 | \( v < v < v_f \) | (1) |
| 050730     | 3.96 | 0.56 ± 0.05 | 0.92 ± 0.11 | 2.16 ± 0.23 | 0.1 | 0.15 | E17 | 4E53 | 105 | \( v < v < v_f \) | (2) |
| 060605     | 3.77 | 1.04 ± 0.05 | 1.08 ± 0.11 | 2.08 ± 0.10 | 0.2 | 0.02 | E17 | 6E53 | 120 | \( v > v_f \) | (3, 4) |
| 060614     | 0.12 | 0.94 ± 0.08 | 1.19 ± 0.05 | 3.38 ± 0.28 | 0.2 | 0.02 | E15 | 5E53 | 30 | \( v < v < v_f \) | (5) |
| 060904B    | 0.703 | 1.11 ± 0.1 | 0.95 ± 0.17 | 1.80 ± 0.11 | 0.01 | 0.01 | 5E6 | 3E53 | 70 | \( v > v_f \) | (6, 7) |
| 070318     | 0.836 | 0.78 | 1.38 ± 0.06 | 2.11 ± 0.06 | 0.01 | 0.01 | 9E16 | 6E53 | 80 | \( v < v < v_f \) | (8, 9) |
| 070411     | 2.954 | ... | 1.43 ± 0.01 | 2.30 ± 0.00 | 0.01 | 0.01 | E17 | 2E54 | 110 | \( v < v < v_f \) | (10) |
| 070419A    | 0.97 | 0.82 ± 0.16 | 1.04 ± 0.05 | 2.37 ± 0.03 | 0.1 | 0.01 | 4E15 | 2E52 | 60 | \( v < v < v_f \) | (11, 12) |
| 070420     | ... | ... | 0.94 ± 0.25 | 2.13 ± 0.17 | 0.01 | 0.01 | 5E16 | 6E52 | 85 | \( v < v < v_f \) | (6) |
| 071010A    | 0.98 | 0.76 ± 0.23 | 0.37 ± 0.25 | 1.92 ± 0.05 | 0.3 | 0.01 | 3E16 | 6E52 | 70 | \( v < v < v_f \) | (13, 12) |
| 071031     | 2.05 | 0.9 ± 0.1 | 1.40 ± 0.00 | 1.79 ± 0.00 | 0.2 | 0.02 | 2E16 | 5E53 | 90 | \( v > v_f \) | (14) |
| 080319A    | ... | ... | 0.77 ± 0.02 | 0.76 ± 0.22 | 1.80 ± 0.11 | 0.1 | 0.1 | E15 | 5E51 | 80 | \( v < v < v_f \) | (15) |
| 080330     | 1.51 | 0.99 | 1.32 ± 0.03 | 3.03 ± 0.16 | 0.02 | 0.02 | 4E16 | 4E53 | 80 | \( v < v < v_f \) | (16, 9) |
| 080710     | 0.845 | 1.00 ± 0.02 | 0.92 ± 0.00 | 2.00 ± 0.00 | 0.2 | 0.01 | 1E16 | 4E53 | 60 | \( v > v_f \) | (17, 12) |
| 080810     | 3.35 | 0.51 ± 0.22 | 0.90 ± 0.03 | 2.41 ± 0.01 | 0.05 | 0.04 | 5E17 | 4E54 | 170 | \( v < v < v_f \) | (18) |
| 081203A    | 2.05 | 0.9 ± 0.01 | 0.40 ± 0.01 | 2.91 ± 0.01 | 0.01 | 0.01 | E17 | 2E54 | 120 | \( v < v < v_f \) | (19) |
| 090313     | 3.375 | 1.2 | 0.71 ± 0.09 | 2.33 ± 0.04 | 0.1 | 0.01 | 8E16 | 5E53 | 90 | \( v > v_f \) | (20) |
| 100906A    | 1.727 | ... | 0.63 ± 0.17 | 2.21 ± 0.14 | 0.01 | 0.01 | E17 | 8E53 | 180 | \( v < v < v_f \) | (22) |
| 110213A    | 1.46 | ... | 0.83 ± 0.04 | 2.04 ± 0.03 | 0.1 | 0.01 | 4E16 | 6E53 | 110 | \( v < v < v_f \) | (23) |

**References.** (1) Rykoff et al. 2004; (2) Pandey et al. 2006; (3) Rykoff et al. 2009; (4) Ferrero et al. 2009; (5) Della Valle et al. 2006; (6) Klotz et al. 2008; (7) Kann et al. 2010; (8) Roming et al. 2009; (9) Fynbo et al. 2009; (10) Ferrero et al. 2008; (11) Melandri et al. 2009; (12) Liang et al. 2010; (13) Covino et al. 2008; (14) Krühler et al. 2009a; (15) Li et al. 2012; (16) Guidorzi et al. 2009; (17) Krühler et al. 2009b; (18) Page et al. 2009; (19) Kuin et al. 2009; (20) Melandri et al. 2010; (21) Gorbovskoy et al. 2012; (22) Gorbovskoy et al. 2012; (23) Liang et al. 2013.

In the paper, we have investigated the evolution of the dynamics and emission of the forward–reverse shocks in the circumburst environment with general density distribution \( n_1 = AR^{-k} \) by considering thick- and thin-shell cases. The optical afterglow with one smooth onset peak at early times is usually attributed to an external shock when the fireball is decelerated by a circumburst medium. Long-duration GRBs may originate from the collapse of massive stars and their ambient medium may be stellar winds. We can infer the GRB circumburst medium from the rise and decay features of the early onset peak (see Equations (67) and (68)). We applied our model to 19 GRBs and found that their \( k \) values are in the range of 0.4–1.4, with a typical value of \( k \approx 1 \) (see Figure 5). This implies that the circumburst medium of those GRBs is neither the ISM (\( k = 0 \)) nor a typical stellar wind (\( k = 2 \)). This could show a new
mass-loss evolution of the progenitor of this GRB, that is, the mass-loss rate $\dot{M}$ and/or the wind velocity $v_w$ are varied at late times of the evolution of a massive star.

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