The analogical reasoning analysis of Pesantren students in geometry

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Abstract. Analogical reasoning is very important for students in Pesantren. This paper describes the analogical reasoning of Pesantren Students in geometry and identifies their difficulties and obstacles. This research is a descriptive qualitative research with 78 respondents. It specifically analyses analogical reasoning in understanding the concepts of geometry, analogical reasoning in theorems and properties, and the use of analogical reasoning in geometry problems. Student difficulties are based on test results and interviews: 1) writing Pythagorean equations based on right triangle images in various contexts; 2) writing cosine equations based on verbal definitions and images; 3) drawing a right triangle based on Pythagoras equation; 4) doing analogical reasoning between Pythagorean theorem and law of cosine; and 5) doing the analogical reasoning based on the theorem.

1. Introduction

Pondok Pesantren (Islamic Boarding School) is the oldest educational system in Indonesia that is not only identical with the meaning of Islam but typical of Indonesia, whose existence has been tested by history and lasted until now [1] [2]. Every day, Santri (Students who study at Pondok Pesantren) study various Islamic religious knowledge in Pondok Pesantren to prepare themselves to become ulama, the person who is in charge of guiding the community in the issue of Islam. They are taught the various sciences of Islam to be able to perform ijtihad, that is, an Islamic legal term referring to independent reasoning or the thorough exertion of a jurist's mental faculty in finding a solution to a legal question. The person who performs ijtihad is called a mujtahid, in addition to having capabilities related to Islamic religious knowledge, a mujtahid must have reasoning ability, analogical reasoning, deduction, induction, conclusion [3] [4] [5] [6] [7].

Based on this, Santri are required to have good analogical reasoning abilities. Analogical reasoning is central to the cognitive abilities used in our daily lives because logical reasoning develops the skills to discover related aspects of new situations, Skills to apply the things known in new situations and generalization skills [8][9]. Analogical reasoning is also a core process of scientific discovery and problem-solving, as well as in categorization and decision making [10]. Analogical reasoning is important for studying abstract concepts [11].

One of the subjects believed be able to train and improve analogical reasoning abilities is mathematics because analogical reasoning is particularly prominent in mathematics [12]. According to Marcus (in [9]), analogical reasoning is one of the most important aspects of mathematical thinking. Analogy allows students to apply similarities between mathematical relationships to help understand
new problems or concepts through the contribution of an integral component of mathematical ability [13].

Analogical reasoning is still very low used in education. Mathematics teachers should encourage students to identify and use analogical reasoning as much as possible in various contexts [9]. Especially in Pondok Pesantren who learn mathematics two hours a week less than in public schools that study math 4 to 7 hours a week, based on the results of the author's observation, mathematics has not encouraged the students to use the abilities of analogical reasoning. Mathematics is only studied to meet the requirements only and has not been integrated with Islamic religious lessons. The gap in Mathematics subjects and religious subjects in Pondok Pesantren, thus affecting students in Pondok Pesantren difficulty understanding mathematics. Santri activities in Pondok Pesantren are always concentrated on activities based on religious education which is their routine, as a result of these habits, santri less interested in general subjects such as mathematics [14] [15].

1.1. Analogical Reasoning

Analogical reasoning is concerned with relevant information, extracting relationships within and across items, and creating precise mappings across domains to generate conclusions and/or obtain general principles [16]. Analogical reasoning is the ability to understand and use the relational similarity between two situations or events [10].

A person's cognitive structure consists of two systems, namely 1) symbolic system; and 2) associative reasoning systems. The symbolic system or rule-based reasoning is where the abstraction of real-world problems through symbolic representations and rules. While associative is a system of reasoning based on commonality where problems through association or similarity with other known information [17]. Analogical reasoning is an associative reasoning-based reasoning system, as shown in Figure 1.

Figure 1. Analogic Cognitive Structure

Analogical reasoning is a method of enabling stored schemes based on identification of connections, equations, or similarities between, which are usually regarded as distinct items. Analogy serves as a type of scaffold, where new information is anchored in the existing scheme. Therefore, analogical reasoning uses analogue schemes, or knowledge from previous experiences, to facilitate learning in new situations [19] [120].

According to Gentner & Smith [10], in general the analogical reasoning process is 1) Retrieval, is a process of a person considering analogous Mapping, i.e. situations (same) before in long-term memory with some current topics in working memory; 2) a two-case process that is present in working memory (either through analogical taking or simply through facing two cases together), mapping involves processes that align representations and project conclusions from one analog to another; and 3)
Evaluation, i.e. the process after the analogical mapping has been done, the results of the analogy and its conclusions are assessed.

Structural mapping is a theory that explains analogical reasoning. Structural mapping theory suggests that analog schemas can be viewed as similar to relational structures or how they relate. In other words, the analogy is the identification of certain aspects of an item (referred to as a known or basic domain), similar to certain aspects of other items (unknown or target domains), as shown in Figure 2. The base and target domains are not similar across all accounts, but through the mapping structures of basic relational structures and target domains found similar [18].

Figure 2. Analogic Reasoning Structure Mapping ([17] p10)

Structural mapping allows the preparation of new schemes based on conclusions and predictions. The conclusion undergoes a transformation so that both items are close enough to allow mapping and transfer from base to target [21]. Causality can then be inferred and a causal mental model or schema developed.

2. Research Methods

This research is descriptive qualitative research with 78 students of grade 4 and 5 TMI (Tarbiyatul Mu'alin Wal Mu'alimat al Islamiyah-Islamic Education Teachers) at Pondok Pesantren in Bandung, Indonesia. The purpose of this study is to analyze the capabilities related to the analogical reasoning ability of the students. Researchers chose 78 students of grade 4 and 5 TMI or if in public schools the same as class 10 and 11 SMA (High School) with age 15-17 years. Unlike their friends in high school who study math 4-7 hours a week, but they learn math only 2 hours a week.

2.1. Research Instruments

The researcher analyzes the analogical reasoning ability of pesantren students in geometry and observes the difficulties faced by pesantren students in answering test questions. Problem test is given as many as 31 questions. The form of problem adopted from Magdaş [9]. consists of analogical reasoning in understanding the concepts of geometry, analogical reasoning in theorems and properties, and the use of analogical reasoning in geometry problems. In addition to the instrument test questions, also conducted interviews to determine the difficulties and obstacles faced by santri in answering test questions given.

2.2. Procedure

The test is given to all respondents as participants who agree to take the test. Each respondent is given a work sheet that they must fill. The test is given in two different times, the time given in each session is 90 minutes. Furthermore, the interviewed represent the low, high, and medium.

2.3. Data Analysis
Researchers gave scores of 0 and 1 for each question item. This score is given based on true or false if true 1 and false 0. But two items question, the score given between 0-3. The number of scores per person compared with the maximum number of scores to know the percentage. Here in after determined the percentage, biggest percentage, percentage, and standard deviation in every aspect. Also, analyze per item to know the correct percentage of respondents in each item.

3. Research Results
The analysis is carried out in three aspects, namely the analogical reasoning in understanding the concepts of geometry, analogical reasoning in theorems and properties, and the use of analogical reasoning in geometry problems. Here's a description of the three aspects of the analogical reasoning abilities of pesantren students in geometry.

Table 1. The Results of Analogical Reasoning Test of Pesantren Students in Geometry

| N=78 | Analogical reasoning in understanding the concepts of geometry | Analogy reasoning in theorems and properties | The use of analogical reasoning in geometry problems |
|------|-------------------------------------------------------------|---------------------------------------------|--------------------------------------------------|
| Mean | 44.84                                                       | 34.03                                       | 69.77                                           |
| Largest Score | 100                                                         | 81.81                                      | 91.67                                           |
| Smallest Score | 15.38                                                      | 9.09                                       | 16.67                                           |
| Standard Deviation | 25.20                                                      | 18.23                                      | 19.52                                           |

Based on Table 1 above, we can see on average of analogical reasoning in theorems and properties is the lowest, i.e. 34.03%. While the average of analogical reasoning in geometry problems is the highest, i.e. 69.77%. The average Analogical reasoning in understanding the concepts of geometry is only higher 10.71%, i.e. 44.84%.

3.1. Analogical Reasoning in Understanding the Concepts of Geometry
The questions given to the respondents relate to the Pythagoras and Cosine Theorems. The researcher analyzes the analogy abilities consisting of analogical reasoning from verbal expression to the expression of writing, analogical reasoning from the expression of writing to the expression of another form of writing, analogical reasoning from visual expression to the expression of writing in various contexts, and analogical reasoning from the expression of writing to visual expression.

Table 2. Aspects of Analogical Reasoning Ability in Understanding Geometric Concepts

| Indicator | No | Percentage |
|-----------|----|------------|
| Be able to write Pythagorean formula based on verbal definitions | 1a | 97 |
| | 1b | 37 |
| | 1c | 40 |
| Be able to write Pythagorean formula based on the right triangle images in various contexts | 2a1) | 54 |
| | 2a2) | 31 |
| | 2a3) | 44 |
| Be able to draw a right triangle based on Pythagorean formula | 2b1) | 77 |
| | 2b2) | 22 |
| | 2b3) | 31 |
| Be able to write cosine formula based on verbal definitions and the right triangle image | 4a | 23 |
| | 4b | 33 |

Based on table 2 above, 97% of respondents can write Pythagorean equations based on the verbal definition in no. 1a, but no. 1b and 1c are only 37% and 40%. According to observations of
researchers and interviews with some respondents about no. 1a, they do not get difficulties because on the question, the hypotenuse is there, so they just simply write in accordance with the verbal definition, i.e. writing the sum of the squares of the other sides. Whereas when answering no 1b and 1c, they made an analogy mistake. When it appears before the "=\" sign is not the square of the hypotenuse, but they still answer the sum of the other side squares (including the hypotenuse). They have made the mistake on doing analogical reasoning from a verbal definition. Because on the verbal definition is the square of the hypotenuse is equal to the sum of the squares of the other sides, so they think everything listed in the matter is the square of the hypotenuse, as shown in Figure 3.

Figure 3. Two Examples of Answers 1a, 1b, and 1c

Number 2a1), 2a2), and 2a3) respectively 54%, 31%, and 44%. Respondents find it difficult to write Pythagorean equations based on a right triangle image in various contexts. Based on interviews with respondents they have difficulty determining the hypotenuse. During the interview, the researcher explained that the hypotenuse is the side that is in front of the right angle, some interviewed respondents became able to determine the Pythagoras formula based on a right triangle image. Examples of mistakes made respondents can be seen in Figure 4.

Figure 4. Example of Answers 2a1), 2a2), and 2a3)

77% of respondents at no 2b1) were able to draw a right triangle based on the Pythagoras formula, while at 2b2) and 2b3) respectively 22% and 31%. At no 2 b2), the difficulties faced by the respondent are Pythagoras's formula written on the question is the square of one right-angled side equal to the square of the hypotenuse minus the square of the other right-angled side. Based on interview results, the same difficulties as in 1b and 1c. In no. 2b3) Pythagoras formula written on the question is the name of its sides not based on the names of the vertices as in no 1, whereas respondents can do the analogy as in no 2b1). Figure 5 shows the respondents' mistakes.

The ability to write cosine formulas based on the verbal definitions and image in number 4a and 4b, i.e. 23% and 33%, means that 77% and 67% of respondents have trouble writing cosine formulas based on verbal definitions and image. The verbal definition reveals that the cosine of an angle in a right triangle is a ratio of triangle side located at an angle and the hypotenuse or the cosine of an angle in a right triangle is as the ratio of the lengths of the side of the triangle adjacent to the angle and the hypotenuse, so they assume that the length of the side of the triangle that is adjacent to the an
angle does not have to be a numerator and the hypotenuse does not have to be a denominator. The examples of wrong answers can be seen in Figure 6.

![Figure 5](image1.png)  
**Figure 5.** Example of Answers 2b1), 2b2), and 2b3)

![Figure 6](image2.png)  
**Figure 6.** Sample Answers Number 4a and 4b

3.2. Analogical Reasoning in Theorems and Properties

| Indicator                                      | No  | Percentage |
|------------------------------------------------|-----|------------|
| Be able to write Pythagorean formula based on the triangle image | 3a  | 94         |
|                                                | 3b  | 91         |
| Be able to write cosine formula based on the triangle image | 5a  | 34         |
|                                                | 5b  | 29         |
| Be able to do analogical reasoning between Pythagorean theorem and cosine rules | 3c  | 29         |
|                                                | 5c  | 11         |
|                                                | 5d  | 11         |
|                                                | 6a  | 11         |
|                                                | 6b  | 5          |

Based on Table 3 above, for 3a and 3b respectively 94% and 91% of respondents were able to write Pythagoras formula based on triangle images. In No 3a and 3b, the image of the triangle is almost identical to no 1a although it consists of two right-angled triangles, they have no difficulty determining the hypotenuse, so as to write the Pythagoras formula. One example of respondent's answer can be seen in figure 7.

![Figure 7](image3.png)  
**Figure 7.** Sample Responses Number 3a and 3b

34% and 29% of respondents were able to write cosine equations based on triangle images, ie on 5a and 5b. Approximately 70% of respondents face difficulties, this is related to the difficulties faced by
respondents in answering no 4a and 4b. The triangular image in no 5a and 5b are almost identical to triangular image 4a and 4b, although they are composed of two right-angled triangles.

![Figure 8. Sample Responses 5a and 5b](image)

The respondents' ability to do analogical reasoning between the Pythagorean theorem and the cosine rules is illustrated in 3c, 5c, 5d, 6a, and 6b, respectively 29%, 11%, 11%, 11%, and 5%. Based on the results of interviews and analysis of respondents' answers, they have difficulty connecting Pythagorean theorem and cosine rules because they have to change some form of Pythagorean theorem and cosine formulas so that raises the rules of cosine. All numbers are so related that when they get into trouble on number 3c then the next number must be difficult too.

### 3.3. The Use of Analogical Reasoning in Geometry Problems

The average analogical reasoning ability in geometry problem is 69.77%. In detail can be seen in the table below.

| Indicator                                      | No | Percentage |
|------------------------------------------------|----|------------|
| Be able to do analogical reasoning based on given frontal activity |    |            |
| 1b                                             | 81 |
| 1c                                             | 81 |
| 2b                                             | 40 |
| 2c                                             | 65 |
| 3b                                             | 80 |
| 3c                                             | 87 |
| 4b                                             | 91 |
| 4c                                             | 91 |
| Be able to do analogical reasoning based on the theorem |    |            |
| 5a                                             | 88 |
| 5b                                             | 50 |
| 6                                              | 41 |

The table above illustrates the ability of respondents to perform analogical reasoning based on the frontal activity given in numbers 1b, 1c, 3b, 3c, 4b, and 4c, i.e. 81%, 81%, 80%, 87%, 91%, and 91% respectively. The respondents did not get into trouble because the frontal activity had been given in the matter, they only did an analogy based on the frontal activity. As for 2b and 2c only 40% and 6%, based on interviews with respondents who have difficulty answering, because they are difficult to determine the sides of the same length and should be analogous to the given frontal activity. This can be seen in Figure 9.
The ability of respondents in making analogical reasoning based on the theorem, in no 5a there are 88% of respondents who are able to do so, because they only define perpendicular bisector by definition. Whereas at 5b, only 50%, wherein those who answered incorrectly say they have difficulty connecting the perpendicular bisector theorem with the answers to no. 4a, 4b, and 4c. Some respondents only write the definition of perpendicular bisector on the triangle only, as can be seen in Figure 10. Whereas in the 6 where respondents are required to connect all their answers from No. 1 to No. 5, only 41% of respondents are able to do so. Some respondents simply write the perpendicular bisector definition.

4. Discussion and Conclusion

The average percentage of analogical reasoning ability in understanding geometric concepts is only 44.84% because according to Magdaş [9] to understand mathematical concepts, it is necessary to do an analogy between verbal expressions, symbols of definition (written expression) and visual representation (material models, drawings, etc.). Respondents still have difficulty in reasoning, especially when given a different context with verbal expressions. Analogy is the identification of certain aspects of an item (referred to as a known or basic domain), similar to certain aspects of another item (unknown or target domain) [17]. Therefore, when misidentifying, for example, identifying a hypotenuse on a right triangle or misidentifying a cosine definition, it would be wrong in another aspect. When an error occurs in the retrieval process, it will occur the mapping process errors. The average percentage of analogical reasoning ability in the theorem and geometry properties is only 34.03%. Analogical reasoning is to make mappings-a series of systematic correspondences that serve to align the source and target elements [16], so that when the source element is not properly understood it will have difficulty in aligning the target element with the target. Respondents have not really been able to identify Pythagorean theorems and cosine definitions as source elements, so they will have difficulty aligning with the target of the cosine rules.

While the average percentage of analogical reasoning ability in geometry problem is the highest, i.e. 69.77%. In this section, respondents are given frontal activity, so that the source element is clear, the respondent is quite aligned with the target. Analogical reasoning involves the identification of a
common relational system between two situations and yielding further conclusions driven by this similarity [10], so that when identification is generated in frontal activity it makes it easier for respondents to make conclusions that are driven by similarity.

The follow-up of this research is in developing geometric teaching materials to improve analogical reasoning ability, we need to pay attention to verbal expressions, symbol definitions (written expression) and visual representation (material models, drawings, drawings, etc.). We need to also strengthen the identification of source and mapping elements in the form of correspondence and systematic relational systems to align source and target elements.

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