Kondo–lattice-like effects of hydrogen in transition metals

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We discuss the possibility of a Kondo like effect associated with H in metals resulting from the strong dependence of the H 1s orbital radius on the occupation number. We demonstrate that such a strong breathing property of the orbital radius, which translates directly into a strong occupation dependent hopping, results in the formation of local singlet-like bound states involving one electron on H and one on the surrounding metal orbitals. We also show that already at a mean field level an occupation dependent hopping integral leads to a substantial potential energy correction on hydrogen, and that the failure of band structure methods to incorporate this correction is responsible for the incorrect prediction of a metallic ground state for the YH3 switchable mirror compounds.

I. INTRODUCTION

The recent discovery [1] of the so called switchable mirror compounds based on YH3–x has renewed the interest in the electronic structure of transition metal and rare earth hydrides. These compounds undergo a metal to insulator transition as x changes from one to zero with the accompanying change from high optical reflectivity to optical transparency for photon energies below about 2 eV. LDA band structure calculations fail to reproduce the semiconducting gap for x = 3 in the widely accepted HoD3 structure or in the simpler cubic LaH3 structure. Although a gap can be obtained for more complicated distorted structures [2], it is still much too small; and although such distortions are not excluded for YH3, they do not appear to occur for LaH3, which has similar properties.

The failure to produce large enough band gaps in semiconductors by LDA is a well-known shortcoming which here however seems to take quite dramatic forms: The valence and conduction bands in LDA overlap by about 1 eV, so a total relative shift of about 3 eV is required to match the experimental value. Such dramatic discrepancies are reminiscent of the strongly correlated systems like the transition metal and rare earth oxides and point perhaps to the importance of correlation effects. Since however the H 1s orbitals are rather extended as compared to the 3d’s of the transition metals, especially for the negative ion, they are expected to form rather broad bands, and the on-site Coulomb interactions are strongly screened. Therefore the origin of the correlation effects may be quite different. In this paper we address this problem and come to the new suggestion that the correlation effects are a consequence of the large change in the H 1s orbital radius upon orbital occupation. This “breathing” property of the hydrogen ion is shown to introduce a new term in the mean field treatment of the electronic structure of hydrides. This term results in an opening up of the band gap in a quite natural way, with the retention of large band widths and nearly one particle behaviour of the excited states. Using a model Hamiltonian to demonstrate this behaviour we also show that for a range of parameter values the system behaves like a Kondo lattice insulator similar to that suggested by Ng et al [3].

II. THE BREATHING HYDROGEN ATOM

As is well known and referred to in most general chemistry text books, the so called effective radius of hydrogen is extremely strongly dependent on the charge state. The crystal radius of neutral H is 0.26 Å, whereas that of the negative ion H− is 1.54 Å [4]. The values of the average 1s orbital radius \( \sqrt{<r^2>} \), as obtained from free ion Hartree–Fock calculations, are 0.8 Å for H and 1.72 Å for H−. This very large change is not unexpected since in H, with its low nuclear charge of one, the screening of the nuclear Coulomb potential by a second s electron is very important and has a dramatic effect on the orbital radius. This large effect causes the effective hopping integrals or hybridizations with surrounding ions to be strongly different for the fluctuations involving H to H+ as compared to those involving H to H−, as pictured in Fig. 1.

![FIG. 1. Schematic representation of two electron and single electron wave function with (left column) and without (right column) taking into account the expansion of the Hydrogen wave function.](image-url)
If these instantaneous changes in the hopping integrals are larger than or comparable to other energy scales, like the orbital energy splittings, they must be treated explicitly. They can not be treated in a mean field like way (as is done in band theory), taking an average orbital radius corresponding to the average occupation as determined from self–consistent calculations.

To model the effect of the “breathing” hydrogen we introduce an occupation dependent hopping integral (following Hirsch [5]) between H and its nearest neighbours, in addition to the usual on-site Coulomb interaction of the Hubbard [6] or Anderson impurity [7] models. We consider the Hamiltonian of the form:

\[ H = -\Delta \sum_{\sigma} n_{h\sigma} h_{\sigma} + U n_{h\sigma} n_{\bar{h}\sigma} + \sum_{\sigma} \left[ (V_{1}\tau_{\sigma} h_{\sigma} h_{\bar{\sigma}} + V_{2}\tau_{\sigma} h_{\sigma} h_{\bar{\sigma}}) + H.c. \right], \]

which describes the hybridization of a single hydrogen atom with a single ligand orbital. Here \( h_{\sigma}^{\dagger} (l_{\sigma}^{\dagger}) \) creates an electron on the hydrogen (ligand), there is a charge transfer energy \( \Delta \) between hydrogen and ligand (we assume \( \Delta > 0 \)) and a Coulomb repulsion \( U \) between electrons on hydrogen. \( V_{1} \) and \( V_{2} \) are the occupation dependent hydrogen–ligand hopping integrals. We discuss the first ionization energy \( E_{-} \) for photoemission on a single cell, consisting of a hydrogen atom and a ligand orbital, filled with two electrons. The minimum ionization energy \( E_{-} \), together with the minimal electron affinity \( E_{+} \), determine the excitation gap \( \Delta_{\text{gap}} = E_{-} - E_{+} \). An underestimation of the magnitude of \( E_{-} \) thus may lead to a too small gap energy, as seems to be the case in LDA calculation for \( \text{YH}_{3} \).

For simplicity we take \( V_{2} = 0 \), so that the ground state of a single electron just corresponds to the electron being trapped in the collapsed hydrogen orbital, and has energy \(-\Delta\). A straightforward calculation then gives the corresponding ionization energy

\[ E = \frac{U - \Delta}{2} - \sqrt{\left(\frac{U - \Delta}{2}\right)^{2} + 2V_{1}^{2}}. \]

Taking for simplicity \( U = \Delta \) we obtain \( E = -\sqrt{2}V_{1} \). The ionization energy thus is predominantly due to the loss of kinetic energy, because the single electron in the final state cannot escape from the collapsed hydrogen orbital, so that the large gain in kinetic energy, which was possible for two electrons, is no longer possible.

On the other hand, constructing a single-particle Hamiltonian with an averaged hopping integral \( V_{MF} \approx \langle h_{\sigma}^{\dagger} h_{\bar{\sigma}} \rangle V_{1} \) and an “effective” on-site energy \( \Delta_{MF} \) (as it is done in an LDA calculation), the excitation energy would be simply the energy of the occupied mean-field orbital, i.e.,

\[ E_{MF} = \frac{\Delta_{MF}}{2} - \sqrt{\left(\frac{\Delta_{MF}}{2}\right)^{2} + 2V_{MF}^{2}}. \]

If the occupation of hydrogen 1s is significantly smaller than one per spin state, this way of calculation will miss a large part of the kinetic energy contribution to the excitation energy, unless the “effective” on-site energy is corrected to take this effect into account.

In a mean-field treatment of this Hamiltonian, it will become apparent that the occupation dependent hopping gives rise to very peculiar physics. Breaking down the conditional hopping terms into quadratic terms we obtain: \( \langle l_{\sigma}^{\dagger} h_{\sigma} l_{\bar{\sigma}} h_{\bar{\sigma}} \rangle \rightarrow \langle l_{\sigma}^{\dagger} h_{\sigma} \rangle \langle h_{\bar{\sigma}} l_{\bar{\sigma}} \rangle + \langle l_{\sigma}^{\dagger} h_{\bar{\sigma}} \rangle \langle h_{\sigma} l_{\bar{\sigma}} \rangle \). The first of these terms corresponds to weighting the “large” hybridization integral by the occupation of the hydrogen orbital, which is what one might have expected; the second term, however, is a correction to the on-site energy of hydrogen by a part of the kinetic energy. All in all we obtain:

\[
H_{MF} = \sum_{\sigma} [ -\Delta_{MF} n_{h\sigma} + ( V_{MF} n_{h\sigma} + H.c. ) ], \]

\[
V_{MF} = V_{1} n_{h} + V_{2} (1 - n_{h}) \]

\[
-\Delta_{MF} = -\Delta + U n_{h} + \alpha \langle T \rangle \]

\[
\alpha = \frac{V_{1} - V_{2}}{V_{MF}} \]

where \( n_{h} = \langle h_{\sigma}^{\dagger} h_{\sigma} \rangle \) and \( \langle T \rangle \) is the energy of hybridization between hydrogen and ligand. We thus find the surprising result that in this approximation the expectation value of the kinetic energy \( \langle T \rangle \) enters as an additional “potential” on the hydrogen sites, and is in fact even enhanced by the factor \( \alpha \). In the limit \( V_{1} \gg V_{2} \) we find \( \alpha \rightarrow n^{-1}_{h} > 1 \), so that the correction to the on-site potential of hydrogen becomes \( (V_{1}/V_{MF}) \langle T \rangle \), i.e., the kinetic energy for mixing with the hydrogen site, but calculated with the hopping integral for the “expanded” atom. This is clearly a huge energy, but it has a very clear physical interpretation: for two electrons in the cell, the hydrogen atom will oscillate between \( H^{0} \) and \( H^{+} \), so as to take maximum advantage of the expansion of the wave function, and the hybridization energy will be large. Removing one electron, the remaining electron will essentially be trapped in the “collapsed” hydrogen orbital, and there is practically no more hybridization energy. In the mean-field wave function both electrons are in the lower molecular orbital, which (due to its strongly negative effective on-site energy resulting from Eq. 3) has predominant hydrogen character. The ionization energy, which by Koopmans theorem should be given by the mean-field eigenvalue, then contains almost the entire kinetic energy of the two-electron state, because this kinetic energy has been put into the on-site potential of the hydrogen atom. To make this more quantitative, we have performed exact diagonalization calculations for a 1D chain of a model with occupation dependent hybridization between hydrogen and ligand. A schematic representation of the model is given in Fig. 3.
FIG. 2. Schematic representation of the 1D model used in the exact diagonalization.

Computer memory and CPU time limitations prohibit to diagonalize chains of more than six unit cells of this model, at least for the most interesting densities near two electrons per unit cell. For the given parameters, the ground state of this system corresponds closely to a state with two electrons in a local singlet state in each unit cell, with one electron primarily on H and the other primarily on the neighbouring ligands. Such a state is reminiscent of a Kondo-lattice insulator ground state. Figure 3 shows the \(k\)-resolved electron addition and removal spectra for the six-site chain at half-filling. To get a feeling for the dispersions of the different bands, we have combined spectra calculated with both periodic and antiperiodic boundary conditions. While there is no rigorous proof for this, inspection of Fig. 3 shows that in this way one obtains remarkably smooth “band structures”.

Unlike spectra for, e.g., the Hubbard model at half-filling, the calculated spectra are surprisingly “coherent”, with almost all of the spectral weight being concentrated in just three well defined “bands”. In inverse photoemission there is a band of predominant metal character, with little dispersion. In photoemission, there is a strongly dispersive band of mixed hydrogen-metal character, and a dispersionless low intensity band of practically pure hydrogen character. More detailed analysis shows, that the dispersionless band corresponds to \(H^+\) final states (i.e., it is a kind of “lower Hubbard band”) whereas the dispersive band corresponds to \(H^0\)-like final states. Next, Fig. 3 shows the spectral function for different values of \(U\). For comparison, the bands obtained by a mean-field solution of the model are also shown. Thereby the calculation has been done both with and without the kinetic energy correction \(V_T = \alpha \langle T \rangle\) to the hydrogen on-site potential. Quite obviously the calculation with \(V_T\) reproduces the exact band structure very well, whereas the bands without \(V_T\) while giving roughly the correct dispersion substantially underestimate the gap size. As explained above, we believe that LDA misses the kinetic energy correction \(V_T\), so that the LDA band structure rather corresponds to the bands without \(V_T\). In a phenomenological way, this suggests a kind of “scissors operator” approach to obtain the “correct” band structure of YH\(_3\) from the LDA result.

III. APPLICATION TO YH\(_3\)

We now use the above ideas for the case of YH\(_3\) and attempt to obtain reasonable parameters and subsequent estimates of the band gap. The three 5\(d\) electrons of Y will in the above scenario all be bound by the three H atoms per Y; this would again result in an insulating ground state. First, we obtain good estimates for the average hopping integrals and on-site energies, using a tight binding fit to an LDA band structure calculated with the LMTO method [9]. The upper panel of Fig. 4 shows the band structure calculated for YH\(_3\) in the LaH\(_3\) structure, which is practically identical to previous published results [10,11]. In the lower panel we show the result of a calculation for Y metal with the lattice constants of YH\(_3\), in order to establish the contribution of the H 1s orbitals.
To begin with, the lowest two bands which have predominant 5sp character, are nearly identical in both solids. Obviously, these free-electron-like states are not significantly affected by the insertion of hydrogen into the interstitial sites. In yttrium metal, the next group of states are the Y 4d bands, which form a dense continuum with a width of ~ 6 eV. The Fermi energy cuts into the lower part of this complex of d bands. In YH3 the situation is very different: the lowest Y 4d band and the Y 4s bands do have an appreciable width, they barely overlap with the lowermost of the following d bands. In fact, the band structure of YH3 already shows a very clear “gap” between the top of the hydrogen like valence band and the Y 4d-like conduction band throughout the entire Brillouin zone – YH3 thus is already “almost” a semiconductor. The Fermi energy cuts into the top of the hydrogen-like valence band and the bottom of the d-like conduction band, so that LDA predicts YH3 to be a semimetal. The shift of the lowermost d to considerably higher binding energy upon insertion of hydrogen, which is predicted by LDA, is in qualitative agreement with the photoemission data of Fujimori and Schlapbach [3].

For metallic yttrium, these authors found a high intensity structure at binding energies ≤ 2 eV, which probably corresponds to the occupied part of the Y 4d bands. For YH3 a similar structure occurs at a binding energy of 6 eV, indicating the shift of the d band away from EF. It should be noted, however, that the experimental shift is larger by ≈ 2 eV than that predicted by LDA. The picture thus is quite reminiscent of the well-known band-gap problem in semiconductors, where LDA fails to give correct values for the semiconducting gaps.

To extract additional information, we performed a tight-binding fit to the LDA band structure. It turned out that by using hybridization integrals only between nearest neighbours a surprisingly good fit of the first few valence and conduction bands could be obtained, as shown in Fig. 3. The tight binding fit was obtained using the following Hamiltonian in a mean field way, as discussed below:

\[
H = \sum_{k,\nu,\sigma} \epsilon_\nu(k)d_{k,\nu,\sigma}^\dagger d_{k,\nu,\sigma} + \sum_j \tilde{\epsilon}_j h_{j,\sigma}^\dagger h_{j,\sigma} + \sum_{i,j,\sigma} \left[ (V^{(1)}_{i,j,\sigma} d_{i,\nu,\sigma}^\dagger h_{j,\sigma}^\dagger h_{j,\sigma}) + H.c. \right] + U \sum_j h_{j,\uparrow}^\dagger h_{j,\downarrow}^\dagger h_{j,\uparrow} h_{j,\downarrow},
\]

Here \( \epsilon_\nu(k) \) denotes the Y 4d bands, which we obtained from the tight-binding fit.

In a mean-field treatment (3) would turn into

\[
H = \sum_{k,\sigma} \epsilon_\nu(k)d_{k,\nu,\sigma}^\dagger d_{k,\nu,\sigma} + \sum_j \Delta_j h_{j,\sigma}^\dagger h_{j,\sigma} \\
+ \sum_{i,j,\sigma} \left[ (V^{(1)}_{i,j,\sigma} d_{i,\nu,\sigma}^\dagger h_{j,\sigma}) + H.c. \right] + V^{(2)}_{i,j,\sigma} n_j + \sum_{i,j,\sigma} h_{j,\sigma}(1-n_j) + H.c.,
\]

with \( n_j = \langle h_{j,\uparrow}^\dagger h_{j,\downarrow} \rangle = \langle h_{j,\downarrow}^\dagger h_{j,\uparrow} \rangle \) and \( \Delta_j = \tilde{\epsilon}_j + Un_j \). We now introduce the parameter \( \lambda \), which we assume independent of \( j \), as \( \lambda = V^{(1)}_{i,j,\sigma}/V^{(2)}_{i,j,\sigma} \), i.e., the ratio of hopping integrals for the collapsed and expanded hydrogen atom. We then estimate the change as

\[
V^{TB}_{i,j,\sigma} \approx (n_j + \lambda(1-n_j))V^{(1)}_{i,j,\sigma},
\]

where \( V^{TB}_{i,j,\sigma} \) is the hybridization integral extracted from the tight-binding fit. Since \( n_j \) can be obtained from the tight-binding calculation as well, we can thus, for given \( \lambda \), obtain an estimate of \( V^{(1)}_{i,j,\sigma} \). Next, we estimate the “bare” on-site energies \( \epsilon_j \) of the hydrogen atoms from those of the tight-binding fit, \( \Delta_j^{TB} \), as follows:

FIG. 4. Band structures YH3 (top) and metallic yttrium with the lattice constants of YH3 (bottom). The Fermi energy is marked by the dashed line.
\[ \epsilon_j = \Delta_j^{TB} - n_j U. \] (6)

This introduces another unknown parameter: the onsite Coulomb repulsion \( U \) between two electrons on the hydrogen site. While an ab-initio calculation of \( U \) and \( \lambda \) would be highly desirable, this is outside the range of techniques available to us. We therefore will treat these quantities as implicit parameters, and consider the variation of possible results when \( U \) and \( \lambda \) are varied within “reasonable bounds.”

Using the parameters estimated in this way, we now proceed to an impurity–like calculation to determine the stabilization energy of Kondo-like local singlets, formed on a single hydrogen atom in the lattice of Y\(_4\) atoms. In the first step, we calculate the ground state energy \( E_{0}^{(2)} \) of a two-electron bound state from the ansatz

\[ |\Psi^{(2)}\rangle = \left[ \alpha h_{\uparrow}^{\dagger} h_{\downarrow}^{\dagger} + \frac{1}{\sqrt{2}} \sum_{\nu,k}^{\alpha} \beta_{\nu} \left( d_{k,\nu,\uparrow}^{\dagger} h_{\uparrow}^{\dagger} + h_{\downarrow}^{\dagger} d_{k,\nu,\downarrow} \right) \right] |\text{vac}\rangle. \]

Here summation over \( k \) and \( \nu \) refers to the Y\(_4\)\( d \) bands. The dispersion of these bands and the hybridization matrix elements between the band states and the hydrogen atom are calculated using the parameters from the tight binding fit, whereby the hydrogen-yttrium hybridizations depend on \( \lambda \) and the hydrogen on-site energy on \( U \). Then, we want to know the stability of this state against decay into a state with a single electron remaining in the hydrogen atom, and the second electron being in a free yttrium \( d \)-like state. The energy \( E_{0}^{(1)} \) of the single electron in the hydrogen is calculated from the ansatz

\[ |\Psi^{(1)}\rangle = \left[ \alpha' h_{\uparrow}^{\dagger} + \sum_{\nu,k}^{\beta} \beta_{\nu} \beta_{\nu}^{\dagger} d_{k,\nu,\uparrow} \right] |\text{vac}\rangle, \]

and for the energy of the \( d \)-like electron we simply choose the lower bound of the \( d \)-band complex, \( E_{\text{edge}} \). We then form the difference \( \Delta E = E_{0}^{(2)} - (E_{0}^{(1)} + E_{\text{edge}}) \) (see Fig. 4).

One can see that for “reasonable” values of \( U \) and not very extreme values of \( \lambda \) the two-electron bound state attains a stabilization energy of several electron-volts. Drawing an analogy with the situation in cuprate superconductors, where the Zhang-Rice singlet has a stabilization energy of approximately 1 eV, it seems quite reasonable to adopt the picture of local bound states. Then, for YH\(_3\) one may expect that these bound states form a split-off band, with the Fermi energy lying in the gap between these states and the bottom of the 4\( d \) band; the physics is similar to our exact result on the 1D cluster.

![FIG. 7. Ionization energies \( \Delta E \) calculated with the impurity model for the metal plane hydrogen (left panel) and the tetragonal hydrogen (right panel). Note that the other tight binding parameters depend on the values for \( U \) and \( \lambda \) through Equations (5) and (6).](image-url)

IV. DOPING DEPENDENCE OF THE ELECTRONIC STRUCTURE

To simulate the physics of what happens as one removes hydrogen from YH\(_3\), we resort to an exact calculation of a small cluster as done above, but we now remove one hydrogen site and one electron. In Fig. 5 we show the spectral function for electron removal and electron addition before and after removal of the hydrogen. In the top panel, one can clearly recognize the large gap between the hydrogen like valence band and the metal like conduction band. Upon hydrogen removal, as shown in the bottom panel, the Fermi energy jumps into the metal band which implies that hydrogen behaves like an H\(^-\) ion in that it binds two electrons. This is consistent with the above discussion and also suggested by Ng et al [3].

In our previous discussion we came to the conclusion that H actually binds two electrons: one localized on H, and the other on the nearest neighbour metal atoms.
With the removal of a H atom, which takes only one electron with it, another “lonely” electron is left behind, which must then be in the conduction band. Removal of H from the trihydride insulator should then transfer spectral weight for electron removal from the top of the valence band to the bottom of the conduction band, some 2 eV higher in energy. This kind of behaviour upon doping is very similar to that predicted for [13] and observed [14] in the high $T_c$ cuprates. Consistent with this are the observations by Peterman et al. [15], who found that in hydrogen depleted trihydrides the Fermi energy falls into a “band” with very weak spectral weight, which grows upon further depletion.

V. CONCLUSIONS

We have shown that hydrogen is an extreme example of an atom with a large orbital occupation dependence of the orbital radius, leading to large occupation dependent hopping integrals in hydrides. We argue that explicit inclusion of such terms in the Hamiltonian results in a “scissors operator” like separation of the valence and conduction bands and, consequently, the opening of a substantial gap. We argue that the insulating character of YH$_3$ can be understood in this way. Using reasonable parameters obtained from tight binding fits to the band structure, we find that the ground state of YH$_3$ corresponds closely to that of a Kondo insulator, with each H binding two electrons in a singlet state.

![Graph](image)

**FIG. 8.** Single particle k-integrated spectral function for a three unit cell cluster of the 1D model with open boundary conditions. The spectra are calculated at “half-filling” (upper part) and with one charge neutral hydrogen atom removed from the central cell (lower part). The parts of the spectra to the right (left) of the vertical dashed-dotted line correspond to electron addition (removal). Parameter values are $\Delta = 1$ eV, $U = 2$ eV, $t = 0.5$ eV, $V_1 = 2$ eV, $V_2 = 0.2$ eV.

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