Cosmological \(N\)-body simulations with a large-scale tidal field

Andreas S. Schmidt, Simon D. M. White, Fabian Schmidt and Jens Stücker

Max Planck Institute for Astrophysics, Karl-Schwarzschild-Str. 1, D-85741 Garching, Germany

Accepted 2018 May 27. Received 2018 April 26; in original form 2018 March 8

ABSTRACT

In this paper we carry out anisotropic ‘separate universe’ simulations by including a large-scale tidal field in the \(N\)-body code GADGET4 using an anisotropic expansion factor \(A_{ij}\). We use the code in a pure particle-mesh mode to simulate the evolution of 16 realizations of an initial density field with and without a large-scale tidal field, which are then used to measure the response function describing how the tidal field influences structure formation in the linear and non-linear regimes. Together with the previously measured response to a large scale overdensity, this completely describes the non-linear matter bispectrum in the squeezed limit. We find that, contrary to the density response, the tidal response never significantly exceeds the large-scale perturbation-theory prediction even on non-linear scales for the redshift range we discuss. We develop a simple halo model that takes into account the effect of the tidal field and compare it with our direct measurement from the anisotropic \(N\)-body simulations.

Key words: methods: numerical – large-scale structure of Universe.

1 INTRODUCTION

Modern large-scale galaxy surveys offer a precise measurement of the density distribution of galaxies and matter, using a variety of probes like baryon acoustic oscillations (BAO), redshift space distortions (RSD), and gravitational lensing. With this data they aim to understand the cause of the accelerated expansion, and the physics of the early Universe (e.g. Inflation), as well as to measure the curvature of the Universe and the nature of primordial fluctuations.

This information is normally inferred from \(n\)-point statistics that compress the information contained in the underlying field. The simplest of these statistics is the two-point correlation function \(\xi(x)\) with its Fourier counterpart, the power spectrum \(P(k) \propto \langle \delta(k)\delta(k') \rangle\). Given the initial conditions provided by the cosmic microwave background (CMB), this provides a possibility to constrain the time evolution of structure in the universe. At early times, when linear perturbation theory accurately describes the structure evolution of large scales, the power spectrum does fully specify the underlying field. However at late times, when structure formation becomes non-linear, at least in the standard CDM model, perturbation theory breaks down and cannot fully describe the structure seen in galaxy surveys.

To unleash the full potential of large-scale galaxy surveys, a better understanding of the non-linear evolution is necessary. In finite volume surveys there are effects from large-scale perturbations which are not directly observable. These fluctuations, even though they have small amplitudes, modify structure on smaller scales due to the non-linear mode coupling that needs to be included in the analysis. There are two leading effects that come into play. The first is due to a coherent large scale over or underdensity in which the survey volume is embedded. The effect of a change in overdensity has been well studied using ‘separate universe simulations,’\(\backslash\)markboth{\(N\)-body simulations with a modified set of cosmological parameters implementing the gravitational effect of the large-scale overdensity (e.g. Frenk et al. 1988; McDonald 2003; Sirko 2005; Martino & Sheth 2009)}.

The anisotropy of the power spectrum also contains information on superhorizon perturbations (Byrnes et al. 2016) or statistical anisotropies in the two-point correlation function originating from physics of inflation (Jeong & Kamionkowski 2012). To disentangle these primordial effects from the late-time effects of tidal fields...
requires an accurate understanding of the latter. Further, the tidal response is an important ingredient in the covariance of the non-linear matter power spectrum (Bertolami & Solon 2016; Bertolami et al. 2016; Barreira & Schmidt 2017ab; Mohammed, Seljak & Vlah 2017). Finally, Barreira, Krause & Schmidt (2017) recently derived the supersample covariance of weak lensing power spectra using the response function approach. They showed that the supersample covariance contains significant contributions from the tidal field response. Hence, these have to be included in any cosmic shear analysis. To address these issues, we have therefore modified the particle-mesh (PM) part of the code GADGET4, to allow us to measure the response to large-scale tidal fields up to $k \sim 2$ h Mpc$^{-1}$, where here and throughout cMpc stands for Mpc in comoving coordinates, and hence into the non-linear regime. In the linear regime, we find that the measured response follows the theoretical prediction, but and hence into the non-linear regime. In the linear regime, we find that the measured response follows the theoretical prediction, but

where $K$ is the curvature, and both $\delta L$ and $\Pi_{ij}$ are the long-wavelength density and tidal perturbation corresponding to $\Phi$, respectively. Now consider the case where the wavelength of this mode is much larger than the size of the simulation box. Then, we can approximate $K_{ij}$ as spatially (but not temporally) constant. If $\Pi_{ij} \propto \delta_{ij}$, equivalently $K_{ijL} = 0$, the long-wavelength density perturbation can be absorbed in modified cosmological parameters, as derived in Baldauf et al. (2011) and Dai, Pajer & Schmidt (2015) and applied to simulations in Sirko (2005), Gnedin et al. (2011), Li et al. (2014) and Wagner et al. (2015). That is, even in the presence of the long-wavelength perturbation $\delta_{ij}$, the background metric within the simulation retains its FRW form,

$$\begin{align*}
d s^2 &= -d t^2 + a^2(t)(1 + K r^2/4) \cdot \delta_{ij} \, d x^i \, d x^j, \\
\end{align*}$$

where we will also write

$$A_{ij}(t) = a_{bg}(t)\alpha_{ij}(t),$$

where $\alpha_{ij}$ is a symmetric matrix encoding the scale factor perturbation and $a_{bg}$ is an isotropic ‘background’ scale factor which we will specify later. Equation (4) is formally the metric describing a Bianchi I space–time. As shown in Ip & Schmidt (2017) however, a Bianchi I space–time is not equivalent to an FRW space–time with a tidal perturbation. Indeed, in order to source the $\alpha_{ij}$ in equation (5), a significant anisotropic stress is necessary, which is not present in standard N-body simulations containing only non-relativistic matter.

However, since motions in large-scale structure are non-relativistic, one can still use equation (4) to simulate the effect of a long-wavelength tidal field. The spatially homogeneous metric equation (4) offers the advantage of being compatible with the periodic boundary conditions employed in N-body simulations. For this, we choose $\alpha_{ij}(t)$ to match the time–time-component of the metric in the comoving (Fermi) frame of the particles induced by a long-wavelength tidal field $\Pi_{ij}(t)$. This approach is related to the ‘fake separate universe’ approach considered by Hu et al. (2016) and Chiang et al. (2016) for isotropic isocurvature perturbations due to dark energy and/or neutrinos.

In order to derive this matching for a general time dependence of the long-wavelength tidal field, we consider the geodesic deviation. Particle trajectories can be written as

$$\begin{align*}
x &= q + s(q, t),
\end{align*}$$

where all coordinates are comoving with respect to $a_{bg}$. $q$ is the initial position and $s(q, 0) = 0$. For non-relativistic particles in a perturbed FRW space–time with scale factor $a_{bg}$, the displacement obeys

$$\dot{s} + 2H_{bg}\dot{s} = -\nabla_{q}\Phi(q + s),$$

where $H_{bg} = a_{bg}/a_{bg}$, and $\nabla_{q}$ indicates the gradient with respect to the comoving coordinate equation (6). Taking the derivative of this equation with respect to $q$ yields the evolution of the geodesic deviation $M_{ij} \equiv \delta_{ij} - \alpha_{ij}$,

$$M_{ij} + 2H_{bg} M_{ij} = -\left(\dot{\delta}_{ij} + M_{ij}^s\right)\alpha_{ki}\alpha_{ji}\Phi.$$

Now consider the motion of comoving test particles in an unperturbed anisotropic space–time equation (4). In terms of physical coordinates, their acceleration is

$$\ddot{r}_i = \frac{d^2}{d\tau^2}\left(a_{bg}a_{ij}\right)x^i,$$

where $x^i$ is the comoving coordinate with respect to the metric equation (4), which is constant for comoving observers. On the other hand, in terms of a fictitious FRW space–time described by $a_{bg}(t)$, we have

$$r_i = a_{bg}(q_i + s_i),$$

so that this trajectory corresponds to a Lagrangian displacement of

$$\ddot{r}_i = \frac{d^2}{d\tau^2}\left[ a_{bg}(q_i + s_i) \right]$$

$$\quad = \delta_{bg}(q_i + s_i) + 2a_{bg}\delta_{bg} + a_{bg}\delta_{bg}.$$

Equating the previous two equations, and using the relation $\alpha_{ij}x^i = q_i + s_i$, we obtain

$$\ddot{s}_i = 2H_{bg}\delta_{bg} \left[ 2H_{bg}\delta_{bg} + \delta_{bg} \right] \left[ \alpha^{-1} \right] \left[ q_i + s_i \right].$$

Taking the derivative $\partial s / \partial q^i$, and comparing with equation (8), immediately yields

$$\dot{\delta}_{ij} = \dot{\alpha}_{ij}$$

$$\dot{\alpha}_{ij} = -2H_{bg}\delta_{bg} \left[ 2H_{bg}\delta_{bg} + \delta_{bg} \right] \left[ \alpha^{-1} \right] \left[ s_i \right].$$
Thus, when restricting to non-relativistic matter, any given large-scale tidal perturbation $\Pi_{ij}(t)$ [equation (2)] can be treated as an effective anisotropic metric, with anisotropic scale factors determined by an ordinary differential equation (ODE). So far, the ‘background’ scale factor $\alpha_0(t)$ was merely a bookkeeping factor without physical relevance. We now identify it as the scale factor of the background cosmology with respect to which the tidal perturbation $\Pi_{ij}$ is defined. We then obtain

$$\frac{d}{dt} \left( a_0^2 \alpha_{ij} \right) = -\frac{3}{2} \Omega_{m0} H_0^2 a_0^{-1}(t) \alpha_{ij}(t),$$

(13)

where we have rephrased the matter density $\rho_m \propto a_0^{-3}$ by using the Friedmann equation for $\alpha_0$ and defining the density parameter $\Omega_{m0}$.

Now, we can use the freedom of rotating the simulation box with respect to the global coordinates, in such a way that $K_{ij}$ becomes

$$K_{ij} \rightarrow K_{ij} \cdot \mathbf{\delta} \rightarrow \mathbf{\delta}.$$

In Fig. 2, we show an example of the evolution of the three scale factors. Note that $\lambda_i$ are chosen to be quite large here for illustration. As expected, at early times the $\zeta$-dovich approximation $\alpha_i(t) = 1 - \lambda_i D(t)$ (dashed lines) works well, while for later times the deviation from the numerical solution of the ODE’s (solid lines) becomes significant. Fig. 2 shows that a negative $\lambda_i$ is stretching (increasing the expansion) while a positive $\lambda_i$ is squeezing the simulation box (reducing the expansion).

3 ANISOTROPIC N-BODY SIMULATION

After describing the model for the tidal field and how the anisotropic scale factor is evolved, we now discuss the modified equations of motion that are used to evolve the simulation, and how they are implemented in GADGET4 (Springel in preparation), an updated version of GADGET2 (Springel 2005).

For this we define an anisotropic comoving frame which is related to physical coordinates by $x_i, \psi_i \rightarrow x_i, \text{com} \cdot a_i$.

Note that our implementation is fully non-linear in the tidal perturbation, although we will focus on an application to small linear tidal fields in this paper.

3.1 Equation of motion

In the anisotropic case, the dynamics of the collisionless particles are described by the Hamiltonian

$$H = \sum_i \left( \frac{1}{2m_i} \sum_k \alpha_{i,k}^2 \right) + \frac{1}{2} \sum_{i,j} \frac{m_i \phi(x_i)}{a(t) a_i a_j a_k},$$

(19)

with the canonical momentum $p_{i,k} = \alpha_{i,k}^2 m_i \delta_{i,k}$, where the index $k \in \{x, y, z\}$ defines the axis. From the Hamiltonian equation (19) we obtain the equations for the change in momentum

$$\dot{p}_i = -\frac{m_i}{2 \omega_{bg} a_i a_j a_k} \frac{\delta \phi}{\delta a_{bg}},$$

(20)

and the potential can be calculated by solving the Poisson equation, which in the anisotropic case is given as

$$\sum_i \alpha_{i}^2 \dot{a}_i \phi = 4 \pi G \rho_{\text{m0}} \delta a_{bg},$$

(21)

where the derivative $\nabla^2 \phi$ is with respect to the new rescaled comoving coordinate, $\rho_{\text{m0}} = \bar{\rho}(a_i a_j a_k)$ is the mean density of the box, $\bar{\rho}$ is the mean density of the universe without the imposed tidal

results are based on equations (13)–(14), which do not assume small tidal fields.

Fig. 1 shows a visualization of the effect of a large-scale tidal field $K_{ij}$ on the structure in a small simulation box with 80 cMpc h$^{-1}$. We show the results both in comoving and in physical space. In the comoving frame (upper panel), we see that the halos are stretched and squeezed forming ellipsoids while in the Eulerian frame (lower panel), where the box is rescaled according to the anisotropic scale factors $\alpha_i$, the halos appear spherical. This figure shows the result of the $N$-body implementation, which we will describe below. For the reminder of the paper we will drop the subscript $L$ and denote $K_{ij,L} \rightarrow K_{ij}$ and $\delta_i \rightarrow \delta_i$.

We now identify it as the scale factor of the background cosmology with respect to which the tidal perturbation $\Pi_{ij}$ is defined. We then obtain

$$\frac{d}{dt} \left( a_0^2 \alpha_{ij} \right) = -\frac{3}{2} \Omega_{m0} H_0^2 a_0^{-1}(t) \alpha_{ij}(t),$$

(13)

where we have rephrased the matter density $\rho_m \propto a_0^{-3}$ by using the Friedmann equation for $\alpha_0$ and defining the density parameter $\Omega_{m0}$.

Now, we can use the freedom of rotating the simulation box with respect to the global coordinates, in such a way that $K_{ij}$ becomes

$$K_{ij} \rightarrow K_{ij} \cdot \mathbf{\delta} \rightarrow \mathbf{\delta}.$$
The large-scale tidal field today and $\delta$ describes the overdensity. In Appendix we give additional code-specific modifications that need to be done for the time step integrals used in the evolution operator in GADGET4.

3.2 Potential calculations

The standard gravity solver used in GADGET4 is the TREEPM algorithm, where the long range force is calculated using the PM method, and the short range is computed using the Tree. We implemented a full TREEPM algorithm to handle the anisotropic coordinates, but as our interest in this paper focuses on the large-scale structure and the weakly non-linear regime, we use the code in a pure PM mode. In a subsequent paper we will focus on small-scale structure and haloes using the full TREEPM implementation.

On the PM, the Poisson equation can be simply solved by Fourier transforming equation (21). That leads to

$$\sum \alpha_i^{-2} k_i^2 \hat{\phi} = 4\pi G \rho_0 \hat{\delta},$$  \hspace{1cm} (22)

$\hat{\phi} = 4\pi G \rho_0 \sum \alpha_i^{-1} k_i^2 \hat{\delta}$,  \hspace{1cm} (23)

$= : 4\pi G \rho_0 \hat{\delta} G_s(k)$  \hspace{1cm} (24)

where $G_s$ denotes the Green’s function. In practice the potential calculation on the PM has to be modified by replacing the isotropic Green’s function $\tilde{G}(k) = \frac{1}{(\sum k_i^2)}$ by the anisotropic one $G_s(k) = \frac{1}{(\sum \alpha_i^{-2} k_i^2)}$. 

Figure 1. Thin slices of the density field of a sample simulation with a box size of 80 cMpc h$^{-1}$ and a strong tidal field with $\lambda = (-0.5, 0, 0.5)$. The left upper panel shows a standard simulation without large-scale tidal field. The right upper panel shows the same initial conditions evolving using a strong tidal field in comoving space. Here we see that most haloes seem more elliptical than spherical and some structures are merged that are still separated in the standard case along certain axes. The lower panel shows the tidal field simulation in physical space, where the axes are rescaled according to the anisotropic scale factors. In physical space most haloes appear spherical but on larger scales there is a clear alignment of structure with the tidal field. The colour represents the overdensity as given by the colourbar on the right.

Figure 2. The evolution of the relative scale factors $\alpha_i$ for an anisotropic region with deformation tensor eigenvalues $\lambda = (-0.7, 0.5, 0.2)$. The solid line represents the solution from the ordinary differential equation (14), while the dashed lines represent the Zel’dovich approximation. The different axes are colour coded according to the legend.
4 RESPONSE

Following the definition of the implementation, this section describes the property which we use to quantify the effect of the tidal field on the large-scale structure, which we will refer to as the tidal response function. We follow the procedures of Barreira & Schmidt (2017a), who defined a response function for the power spectrum, in particular the first-order expansion set out in their Section 3.2. The 3D power spectrum under the influence of a large-scale overdensity $\delta$ and an external tidal field $K_{ij}$ can be written as

$$ P(k) = P(k) \left( 1 + R_t(k)\delta + R_k(k)\hat{k}_i\hat{k}_j K_{ij} \right) $$

(25)

where $\hat{k}$ is a normalized $k$ vector such that $\sum_i \hat{k}_i^2 = 1$ and $K_{ij}$ is the traceless tidal tensor. This expression is valid at linear order in $\delta$ and $K_{ij}$, and is independent of the wavelength of the large-scale perturbations, as long as it is much larger than $1/k$. Further, the response $R_k(k)$ is independent of the eigenvalues of the deformation tensor $K_{ij}$.

We write

$$ R_k(k) = G_k(k) - k \frac{P'(k)}{P(k)}, $$

(26)

where $G_k(k)$ is the growth-only tidal response and $P(k)$ is the mean power spectrum, i.e. in the absence of any tidal effects. The growth-only tidal response is obtained when the modification of the power spectrum is measured in comoving coordinates. Akitsu et al. (2017) and Barreira & Schmidt (2017a) derived at leading order in perturbation theory,

$$ G_k^{LO}(k) = \frac{8}{7} $$

(27)

which is valid on large scales as $k \to 0$. We will compare our results to this result as a consistency test of the implementation. Otherwise higher order terms need to be taken into account and we would not require anisotropic $N$-body simulations.

Our simulations use a tidal tensor defined through the eigenvalues at $z = 0$

$$ (\lambda_x, \lambda_y, \lambda_z) = \left( -\frac{1}{2}, -\frac{1}{2}, 1 \right) \lambda_z. $$

(28)

We thus obtain

$$ \hat{k} \hat{k}' K_{ij} = D(t) \left( \lambda_x \hat{k}_x^2 - \frac{\lambda_y}{2} \hat{k}_y^2 - \frac{\lambda_z}{2} \hat{k}_z^2 \right) $$

$$ = \frac{D(t)\lambda_z}{2} (3\hat{k}_x^2 - 1) $$

$$ = \lambda_z D(t)Y_2(\mu) $$

(29)

where $Y_2$ is the second-order Legendre polynomial and $\hat{k} \cdot \hat{z} = \mu = \hat{k}_z$ is the cosine of the angle between the $k$ vector and the $z$ axis.

5 RESPONSE PREDICTIONS

We will consider two predictions for the response $G_k(k)$ on non-linear scales. First, Barreira & Schmidt (2017a) proposed that the shape of $G_k$ would follow that of the growth-only density response $G_1$ measured in Wagner et al. (2015a), with a normalization chosen so that the correct low-$k$ asymptote is obtained:

$$ G_k(k) = \frac{12}{13} G_1(k). $$

(30)

This was merely a simple ansatz to obtain numerical results for $R_k$ (and six further second-order response functions).

Secondly, we derive the prediction for the non-linear tidal response $G_k(k)$ in the halo model (see Cooray & Sheth 2002 for a review), paralleling the derivation of the density response in Takada & Hu (2013), Chiang et al. (2014), and Wagner et al. (2015a). Adopting the notation of Takada & Hu (2013), the halo model power spectrum, $P_{1\text{hal}}(k)$, is given by

$$ P_{1\text{hal}}(k) = P_{2\text{hal}}(k) + P_{1\text{hal}}(k) $$

(31)

where $P_{\text{hal}}(k)$ denotes the $n$-halo term,

$$ P_{n}(k_1, \ldots, k_m) = \int \frac{d\ln M}{n!} n M^{-\frac{1}{2}} \frac{b_n(M)}{b_1(M)} $$

(32)

and $n M$ is the mass function (comoving number density per interval in log mass), $M$ is the halo mass, $b_n(M)$ is the $n$-th order local bias parameter, and $u(M|k)$ is the dimensionless Fourier transform of the halo density profile, for which we use the NFW profile (Navarro, Frenk & White 1997) and $P_{\text{lin}}$ is the linear power spectrum. We normalize $u$ so that $u(M|k \to 0) = 1$. The notation given in equation (32) assumes $b_n \equiv 1$. $u(M|k)$ depends on $M$ through the scale radius $r_s$, which in turn is generated through the mass–concentration relation. All functions of $M$ in equation (32), along with $P_{\text{lin}}$, are also functions of $z$ although we have not shown this for clarity. In the following, we adopt the Sheth–Tormen mass function (Sheth & Tormen 1999) with the corresponding peak-background split bias, and the mass–concentration relation of Bullock et al. (2001). The exact choice of the latter only has a small impact on the predictions which does not affect our conclusions.

Now consider the tidal response. First, the linear power spectrum changes according to

$$ P_{\text{lin}}(k) \to \left[ 1 + \frac{8}{7} \hat{k}_i \hat{k}_j K_{ij} \right] P_{\text{lin}}(k). $$

(33)

Unlike the case of the response to a long-wavelength density perturbation, the halo number density is unchanged by a tidal field at linear order, since it is a scalar (McDonald & Roy 2009; Mirbabayi, Schmidt & Zaldarriaga 2015; Desjacques, Jeong & Schmidt 2016). Thus, the only remaining effect to consider is a possible change in the halo profiles.

A good zeroth-order assumption is that the inner regions of haloes are unaffected by the large-scale tidal field, since they virialize and decouple from large-scale perturbations at early times. Thus, the halo profiles are unchanged in physical coordinates, which, in terms of our comoving coordinates, implies

$$ u(M|k)_{K_{ij}} = u\left( M \left[ 1 + K_{ij} \hat{k}_i \hat{k}_j \right] k \right), $$

(34)

where we have expanded to linear order in $K_{ij}$ and used equation (18). Equivalently, since the NFW profile $u(M|k)$ is a function

MNRA$^{\text{S}}$ 479, 162–170 (2018)
of $kr_z$, where $r_z = R_{200}(M)/c(M)$ is the scale radius and $c$ is the concentration, we can rephrase this rescaling in terms of the concentration

$$c(M)_{K_i} = [1 + C_K K_i k^i] c(M), \quad (35)$$

where we have introduced a constant $C_K$ to allow for a more general behaviour. An unchanged halo profile in physical coordinates corresponds to $C_K = 1$, since $c \propto 1/r_z$ is the inverse of a physical length. Clearly, we expect $C_K$ to be in the approximate range of $0 \lesssim C_K \lesssim 1$.

Putting everything together, we obtain

$$G_K^H(k)P_{\text{im}}(k) = \frac{8}{7} [I_1(k)]^2 P_{\text{im}}(k) + C_K \left[ 2 (I_1)_{\text{in}c} \right. \times (k)I_1(k)P_L(k) + (I_2)_{\text{in}c} (k, k), \quad (36)$$

where

$$\left(I_1\right)_{\text{in}c} (k) = \int d\ln M m(\ln M) \left(\frac{M}{\bar{M}}\right) b_1(M) \left[ \frac{\partial u(M|k)}{\partial \ln c} \right]$$

$$\left(I_2\right)_{\text{in}c} (k, k) = 2 \int d\ln M m(\ln M) \left(\frac{M}{\bar{M}}\right)^2 u(M|k) \times \left[ \frac{\partial u(M|k)}{\partial \ln c} \right] \quad (37)$$

are the derivatives of the relevant mass integrals with respect to the halo concentration. Note that both of these integrals scale as $k^2$ in the large-scale limit, so that the effect of the tidal field on halo profiles (in comoving units) is only relevant on small scales, as expected.

If the inner regions of haloes indeed do not respond to the tidal field in physical space (corresponding to $C_K = 1$), then we expect the Eulerian response to asymptote to zero at large $k$. Via equation (26), this implies

$$G_K(k) \xrightarrow{k \to \infty} \frac{d\ln P(k)}{d\ln k}, \quad (38)$$

which is roughly $-2$. We will indeed see a change of sign in the simulation measurements of $G_K(k)$ on small scales.

### 6 SIMULATION SETUP

This section is dedicated to the simulation setup and discusses the main characteristics of the runs. To calculate the response of the power spectrum, we consider three choices for the imposed tidal field, two of which differ in the sign of each eigenvalue, $\lambda_i$, and one for which all the $\lambda_i$ are zero. For each, we consider 16 realizations of the initial density and velocity fluctuations at our starting redshift $z_{\text{init}} = 127$. These initial fluctuations are as expected in a fiducial flat $\Lambda$CDM cosmology with the Planck 2015 (Planck Collaboration XIII 2015) cosmological parameters, namely $\Omega_m = 0.308$, $\Omega_\Lambda = 0.692$, $\Omega_b = 0.04694$, $\sigma_8 = 0.829$, and $h = 0.678$. The evolution from each set of initial conditions is then calculated for each of our three choices of tidal field. This allows us to get an estimate of the cosmic variance introduced by the finite size of our simulation volume. For convenience all three simulations per response function estimate use exactly the same initial conditions, which are computed using the standard Zel’dovich approach for an isotropic expanding universe without imposed tidal field. We thus neglect the influence of the large-scale tidal field at the starting redshift. This introduces small, per cent-level artefacts, which we address later in Section 8.

Further, we will introduce a fully consistent model for the initial displacement in a ‘separate universe’ under the influence of a tidal field in a subsequent paper presenting the modified TREEPM. This will eliminate the aforementioned artefacts. The simulations are all run in pure PM mode with the modified Poisson equation, equation (24) and a grid for the PM of 2048^3 cells. Our simulation box has a size of 500 cMpc h^{-1} and we use 512^3 particles. With this, we get PM cells with a size of 244 ckpc h^{-1} for the PM mesh, which sets our force resolution limit.

To separate effects coming from the tidal field from those due to large-scale density offsets, we limit ourself to simulations with $\sum_i \lambda_i = 0 = \delta_L$ (traceless), where the $\lambda_i$ are the eigenvalues of the linear deformation tensor and $\delta_L$ is the linear overdensity at $z = 0$ for the runs including a tidal field. With this choice there are no contributions from $R_l$, since this part of the response is sourced by the overdensity. Effects from a large-scale overdensity were already discussed in Wagner et al. (2015a) and Wagner et al. (2015b).

Through the implementation of the tidal tensor presented here, we can simulate both a long-wavelength overdensity and the traceless tidal field. Choosing all three eigenvalues $\lambda$ equal (isotropic) we have, at linear order, $\delta(z = 0) = \lambda_1 + \lambda_2 + \lambda_3$ and no tidal field. Using this setup we ran an isotropic simulation to check the implementation and found good agreement with the growth-only response function $G_l$ from separate universe simulations.

The angle averaged (1D) power spectra for the three sets of simulations are shown in Fig. 3. The approximately 1 per cent difference seen for simulations of either sign of $\lambda_z$ relative to the isotropic case is most likely due to higher order terms in $K_{ij}$ [e.g. $(K_{ij})^2$], which are expected at the few per cent level. It is sufficient to consider the case where two eigenvalues are equal and the third is twice as large with the opposite sign so that the imposed tidal field is characterized by a single parameter. For the

![Figure 3](https://academic.oup.com/mnras/article-figures/479/1/162/5026633/3.png)
numerical calculation of the response we symmetrically combine the three simulations run from each set of initial conditions (see below). We thus end up with three simulations per measurement of $G_K$ and a total of 48 simulations for the full set.

To see the convergence and the scale where resolution effects become important, we run two more sets with the same particle number and initial conditions but with different PM resolution. The first set has 512$^3$ cells, while the second has 1024$^3$ cells. The comparison can be seen in Figs 4 and 5, where we show the 1D power spectrum and response function, respectively, for each force resolution. The response function and power spectrum are computed using a Fourier grid of 1024$^3$ cells that is unchanged for all PM resolutions. We note that the power spectra and response functions of such pure PM simulations do not vary monotonically as the resolutions. This is also true for the 1D power spectrum Fig. 4, which shows that changing the force resolution (PM grid size) influences the power spectrum. The 512$^3$ and 2048$^3$ cell runs agree quite well on most scales while the 1024$^3$ cell simulation is above both of them for nearly all scales. We see that the response function agrees for all three force resolutions up to $k \sim 1.3 h \text{Mpc}^{-1}$ at roughly the 10 per cent level. However, larger departures are seen on smaller scales (higher $k$), most likely due to anisotropic force-softening effects.

Given the unclear state of convergence of the PM results on very small scales evidenced in Fig. 5, we will limit ourselves to wavenumbers of $k \leq 2 h \text{Mpc}^{-1}$ in our discussions and in our main results, Figs 6 and 7.
Figure 7. Tidal response at higher redshifts: $z = 1.89$ (black symbols), $z = 0.98$ (magenta symbols), $z = 0.46$ (green symbols), and $z = 0$ (blue crosses). The blue line shows the logarithmic derivative of the power spectrum derived from simulations without large-scale tidal field [see equation (26)]. The shaded regions gives the 25–75 percentiles from the data set and computed using the same cosmological parameters as the simulations. Specifically, we take the 1D power spectrum from CosmicEMU (Heitmann et al. 2016) and calculate its logarithmic derivative. The result is shown in Fig. 6 as the green line.

In addition to the $z = 0$ response function discussed so far, we can look at earlier epochs. We use four additional snapshots at $z \sim 0.5$, $z \sim 1$, and $z \sim 2$, shown in Fig. 7. The wavenumber where the result deviates from the perturbation-theory prediction shifts to higher $k$ values with increasing redshift as expected (e.g. $k_{\text{break}} \approx 0.7$ h cMpc$^{-1}$ at $z \sim 2$). The deviation from the perturbation-theory prediction on the largest scales, which grows with redshift, is likely an artefact of the initial conditions that are taken to be identical for all simulations irrespective of the large-scale tidal field. As a result, the initial response is forced to be $G_k = 0$, whereas it should, in fact, be $G_k = 8/7$. This introduces a mismatch in the linear growing mode of order $D(z_{\text{init}})/D(z)$, where $z_{\text{init}} = 127$ is the starting redshift of the simulations.

Further, at higher redshifts $z$ we see the appearance of a bump at scales of $k \sim 0.5 - 0.6$ h cMpc$^{-1}$, which is most likely a decaying mode sourced by the unchanged velocity field in the initial conditions. This is clearly a problem if one is interested in the early evolution of the response function, although the expected effect at $z = 0$ is at the percent level, as mentioned above. This issue will be addressed in a follow-up study.

9 SUMMARY

We have simulated the effect of a large-scale tidal field on structure formation in an $\Lambda$CDM background cosmology using a modified version of GADGET4 with a PM force calculation adapted to an anisotropic background metric equation (4). The implementation is fully non-linear in the amplitude of the anisotropy.

As a first application, we computed the first order growth-only tidal response function $G_k$ induced by this tidal field in the power spectrum, up to $k \approx 2$ h cMpc$^{-1}$, using symmetric runs for the tidal field ($\lambda_{i1} = -\lambda_{i2}$), and recovered the predictions from perturbation theory on large scales. Going to smaller scales, the extrapolation number of haloes (at linear order), while a long-wavelength density perturbation does.

The shift between the $N$-body simulation and the simple halo model suggests that the outer parts of haloes as well as their environment, which are in the mildly non-linear regime, are significantly affected by a large-scale tidal field. But as this regime is not determined by haloes alone, this needs further investigation which will be the subject of further study focusing on intermediate and small scales as well as halo alignments.
using the growth only response function $G_1$ for overdensities from Wagner et al. (2015a) does not fit our measurements. In contrast to the interpolated solution, we find a suppression of the response on small scales compared to the large-scale value, which can be described approximately by the simple halo model in Section 5. The agreement with the simple halo model prediction is far from perfect, however, signalling a tidal response of haloes that is different in the inner and outer regions. A detailed analysis of this exceeds the scope of this paper, and will be the subject of an upcoming paper. We also show the first-order Eulerian response $R_k$ in Fig. 6 which was computed through the sum of $G_k$ and the logarithmic derivative of the isotropic power spectrum from CosmicEMU using the same background cosmology parameters as our simulations. This can now be used, for instance, in calculations of the covariance of the non-linear matter and weak lensing shear power spectra.

ACKNOWLEDGEMENTS

The authors thank Volker Springel for help with GADGET4 and helpful discussion about the convergence. Alexandre Barreira for providing the data for $G_1$, and Titouan Lazeyras for data of the separate universe simulations and helpful discussions. AS is supported by DFG ‘GrInflaGal’ from the European Research Council.

REFERENCES

Akitsu K., Takada M., 2017, Phys. Rev. D, 97, 063527
Akitsu K., Takada M., Li Y., 2017, Phys. Rev. D, 95, 083522
Baldauf T., Seljak U., Senatore L., Zaldarriaga M., 2011, J. Cosmol. Astropart. Phys., 10, 031
Barreira A., Schmidt F., 2017a, J. Cosmol. Astropart. Phys., 6, 053
Barreira A., Schmidt F., 2017b, J. Cosmol. Astropart. Phys., 11, 051
Barreira A., Krause E., Schmidt F., 2018, J. Cosmol. Astropart. Phys., 6, 015
Bertolini D., Solon M. P., 2016, J. Cosmol. Astropart. Phys., 10, 030
Bertolini D., Schutz K., Solon M. P., Walsh J. R., Zurek K. M., 2016, Phys. Rev. D, 93, 123505
Bullock J. S., Kolatt T. S., Sigad Y., Somerville R. S., Kravtsov A. V., Klypin A. A., Primack J. R., Dekel A., 2001, MNRAS, 321, 559
Byrnes C., Doménech G., Sasaki M., Takahashi T., 2016, J. Cosmol. Astropart. Phys., 1405, 048
Cooray A., Sheth R. K., 2009, MNRAS, 394, 2109
Chang C.-T., Dai L., Pajer E., Schmidt F., 2015, J. Cosmol. Astropart. Phys., 10, 059
Chang C.-T., Li Y., LoVerde M., 2016, Phys. Rev. D, 94, 123502
Cooray A., Sheth R. K., 2002, Phys. Rep., 372, 1
Dai L., Pajer E., Schmidt F., 2015, J. Cosmol. Astropart. Phys., 10, 059
Desjacques V., Jeong D., Schmidt F., 2016, Phys. Rep., 733, 1
Frenk C. S., White S. D. M., Davis M., Efstathiou G., 1988, ApJ, 327, 507
Gnedin N. Y., Kravtsov A. V., Rudd D. H., 2011, ApJS, 194, 46
Heitmann K. et al., 2016, ApJ, 820, 108
Hu W., Chiang C.-T., Li Y., LoVerde M., 2016, Phys. Rev. D, 94, 023002
Ip H. Y., Schmidt F., 2017, J. Cosmol. Astropart. Phys., 2, 025
Jeong D., Kamionkowski M., 2012, Phys. Rev. Lett., 108, 251301
Li Y., Hu W., Takada M., 2014, Phys. Rev. D, 89, 083519
Li Y., Schmittfull M., Seljak U., 2018, J. Cosmol. Astropart. Phys., 2, 022
Martino M. C., Sheth R. K., 2009, MNRAS, 394, 2109
McDonald P., Roy A., 2009, J. Cosmol. Astropart. Phys., 8, 20
Mirbabayi M., Schmidt F., Zaldarriaga M., 2015, J. Cosmol. Astropart. Phys., 7, 030
Mohammed I., Seljak U., Vlah Z., 2017, MNRAS, 466, 780
Navarro J. F., Frenk C. S., White S. D. M., 1997, ApJ, 490, 493
Planck Collaboration XIII, 2015, A&A, 594, 63
Sheh R. K., Tormen G., 1999, MNRAS, 308, 119
Sirko E., 2005, ApJ, 634, 728
Springel V., 2005, MNRAS, 364, 1105
Stüber J., Busch P., White S. D. M., 2017, MNRAS, 477, 3230
Takada M., Hu W., 2013, Phys. Rev. D, 87, 123504
Wagner C., Schmidt F., Chiang C.-T., Komatsu E., 2015a, J. Cosmol. Astropart. Phys., 8, 042
Wagner C., Schmidt F., Chiang C.-T., Komatsu E., 2015b, MNRAS, 448, 11

APPENDIX: MODIFIED EVOLUTION OPERATOR

This appendix is meant to give some code specific details for the implementation of anisotropic scale factors. In GADGET4, the particles are evolved using a standard kick-drift-kick (KDK) leapfrog algorithm where the drift ($D$), kick ($K$) and the final evolution ($E$) operators are given as (Springel 2005)

$$E(\Delta t) = K \left( \frac{\Delta t}{2} \right) D(\Delta t) K \left( \frac{\Delta t}{2} \right),$$

$$D(\Delta t) : \begin{cases} p_n \rightarrow p_n, \\ x_n \rightarrow x_n + \frac{p_n}{m_n} \int_{t}^{t+\Delta t} \frac{n}{a^2} \, dt, \end{cases}$$

$$K(\Delta t) : \begin{cases} x_n \rightarrow x_n, \\ p_n \rightarrow p_n + \int_{a}^{a+\Delta a} \frac{1}{a^2} \, da, \end{cases}$$

where we have the drift and kick integrals and the force $f_a = - \sum m_n \nabla_a \Phi(x_n)$. In the two integrals contained in the operators we have factors of $a$ which need to be translated to the anisotropic case. The transformed integrals to the actual time variable $a$ used in GADGET4 are

$$I_{kick} = I_1 = \int_{t}^{t+\Delta t} a^{-2} \, dt = \int_{a(t)}^{a(t+\Delta t)} \frac{1}{a} \frac{1}{H(a)} \, da,$$

$$I_{drift} = I_2 = \int_{a}^{a+\Delta a} \frac{1}{a^2} \frac{1}{H(a)} \, da,$$

which is the actual integral solved. Now we can change the scale factors to the anisotropic scale factors and end up with the corresponding integrals, keeping in mind that the factor $H(a) a$ is a switch from time to scale factor in the integration and is therefore unchanged. The new integrals in general depend now on the axis along which the integration is done, and we end up with six integrals (three for the kick and three for the drift) of the form

$$I_1 = \alpha_1^{-1} \int_{a_{b1}(t)}^{a_{b1}(t+\Delta t)} \frac{1}{H(a_{b1}) a_{b1}} \, da_{b1},$$

$$I_2 = \alpha_2^{-1} \int_{a_{b2}(t)}^{a_{b2}(t+\Delta t)} \frac{1}{H(a_{b2}) a_{b2}} \, da_{b2}.$$

We moved the $\alpha_i$’s out of the integral under the assumption that the change in the time step is small, which is reasonable for most sensible cases of the tidal field.3 By moving $\alpha$ from the integral, we can absorb it for the kick integral in the force calculation and end up with the standard integral which reduces computation overhead.

This paper has been typeset from a TEX/LATEX file prepared by the author.

3We also implemented a version, where $I_2$ is integrated fully without that assumption showing only a very minor code difference.