Comment on ”The nature of slow dynamics in a minimal model of frustration limited domains” by P. L. Geissler and D. R. Reichman: In a recent paper Geissler and Reichman investigated a model with the Hamiltonian

$$H = \int dr \left[ \frac{1}{2} \phi (r) \left( \tau + k_0^{-2} \left( \nabla^2 + k_0^2 \right)^2 \right) + \frac{\lambda}{2} \phi^4 \right]$$

(1)

with scalar field $\phi(r)$ in three dimensions using Monte Carlo simulations. The parameters of the model are the typical wave number, $k_0$, for modulations of $\phi$, the ”segregation strength”, $\tau$, between positive and negative $\phi$, and the interaction strength, $\lambda$. Many years ago by Brazovskii argued that this model may undergo a fluctuation induced first order transition into a lamellar state.

We recently argued that systems described by Eq.(1) may also undergo a self-generated glass transition, i.e. can become non-ergodic without the presence of quenched disorder. This result was based on our earlier work which demonstrated such glassiness in a related model discussed in the context of frustration domain formation by Kivelson et al. The qualitative features of that theory were confirmed in mode-coupling calculations and Monte Carlo simulations of the corresponding lattice model by Grousson et al. The theory is based on a replica formalism and two key assumptions were made in Refs.4,5,6: i) Glassiness can be studied within the mean-field replica formalism with one step replica symmetry breaking which was shown to be marginally stable. ii) The self-consistent screening approximation (SCSA), including de Gennes narrowing, for the summation of the perturbation series in $\lambda$ can be used. Recently we have been able to avoid this second assumption and found a complete solution of the mean field replica equations by generalizing the dynamic mean field approach (DMFT) of correlated electron models to the case of glassy systems. Given that our theory still relies on the applicability of the mean-field replica formalism, computer simulations like those of Refs.4,5,7,8 remain crucial for a final judgment on the existence of self generated glassiness in uniformly frustrated systems of this type.

Geissler and Reichman performed Monte-Carlo(MC) calculations and compared the results with two analytical approaches, one, a Hartree theory and the other, a mode coupling theory performed self-consistently up to second order in $\lambda$. For $\lambda = 1$, $k_0 = 0.5$ and $\tau$ between $-0.14$ and $0$, good agreement between the MC and Hartree approaches was obtained, whereas the primitive mode-coupling theory yields glassy dynamics already for $\tau \leq -0.1$, which was not seen in the Monte Carlo simulations. Thus, the Monte Carlo simulations could not confirm the glassy dynamics found in mode coupling calculations, but rather gave only fast, ”liquid-like” relaxations. Geissler and Reichman concluded that Eq.(1) likely does not exhibit glassy dynamics.

In Fig.1 we compare our results for the dynamic transition between the liquid and glass state obtained within the replica formalism and solved by using three different techniques: a self consistent second order theory (similar to the mode coupling theory of Ref.1), the scsa used in Refs.4,5,6 and the complete solution of the replica mean field problem (DMFT) of Ref.1. In the second order calculation we determined the liquid state correlation function within Hartree theory, which seems to be closest to the procedure of Ref.1, who used Monte Carlo results for the liquid structure facture and find that it is similar to the one obtained within the Hartree approach. The diamonds are the results of Geissler and Reichman (Figs.5 and 6 of Ref.1) where MC simulations is different from what the essentially exact Monte Carlo simulations finds. The key aspect of our theory was that...
even a simple mean field theory leads to a proliferation of metastable states and to an entropy crisis. Thus, self generated randomness emerges according to the random first order transition scenario.\(^1\)

That the parameters studied in Ref.\(^1\) are outside the region relevant for glassiness is consistent with their findings that no indication for the first order Brazovskii transition was detected either. We argued in Ref.\(^4\) that the glass transition occurs due to the same type of fluctuations and for similar parameters to the fluctuation induced first order transition proposed by Brazovskii.\(^3\) The absence of the glass transition could therefore most convincingly be demonstrated by showing the impossibility of achieving cooling rates that exceed nucleation rates.

In summary, the recent Monte Carlo calculations by Geissler and Reichman\(^1\) are not conclusive in ruling out the self-generated glass transition proposed in Ref.\(^5\). Nevertheless, if performed for parameters relevant for glassy behavior Monte Carlo calculations like theirs would be very useful in judging whether Eq.\(^1\) can be considered as a minimal model for glassiness or whether additional ingredients are needed. In addition they offer a powerful tool to study glassy dynamics beyond the limitations of replica mean field theory.

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