CHIRAL RESTORATION IN
HOT AND/OR DENSE MATTER

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ABSTRACT

Chiral restoration phase transition in hot and/or dense hadronic matter is discussed in terms of the BR scaling based on chiral symmetry and scale anomaly of QCD. The precise connection between the scalar field that figures in the trace anomaly and the sigma field that figures in the linear $\sigma$ model is established. It is suggested that in hot and/or dense medium, the nonlinear $\sigma$ model linearizes with the help of a dilaton to a linear $\sigma$ model with medium-renormalized constants. The relevance of Georgi’s vector symmetry and/or Weinberg’s “mended symmetry” in chiral restoration is pointed out. Some striking consequences for relativistic heavy-ion collisions and dense matter in compact stars following stellar collapse are discussed.

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1 Introduction

One of the most intriguing problems in physics is how chiral symmetry is restored in matter as it becomes hot and/or dense. There is presently a lot of controversy and there are many interesting suggestions.

Some time ago [1], we proposed that scaling, which is a property of the Yang-Mills equations at tree level, applied to the dynamically generated masses of hadrons made up out of the chiral (up and down) quarks. Our result for vector (V) and scalar (σ) mesons in the low-energy sector was

$$\frac{f^*_V}{f^*_\pi} \approx \frac{m^*_V}{m_V} \approx \frac{m^*_\sigma}{m_\sigma} \approx \cdots$$  (1)

The nucleon effective mass scaled somewhat differently [2] as

$$\frac{m^*_N}{m_N} \approx \sqrt{\frac{g^*_A f^*_\pi}{g_A f^*_\pi}}.$$  (2)

In these equations the asterisk stands for in-medium quantity.

Since our results were obtained by introducing the breaking of scale invariance with the low-energy chiral Lagrangian through a scalar field denoted \(\chi\), which we called the “glueball” field, our paper has been largely interpreted erroneously as tying the scaling (1) and (2) to that of the gluon condensate. We wish to emphasize that this is not so and clarify where the error in interpretation is made. In our argument in [1], eq.(13), we split the glueball field into

$$\chi = \chi^* + \chi'$$  (3)

where \(\chi^*\) is the smooth gluon mean field and \(\chi'\) is the fluctuating scalar glueball field. This separation effectively splits \(\chi\) into the field \(\chi^*\) which scales the quark condensate and hadron masses, and the “non-smooth field” \(\chi'\) which governs the scaling of the gluon condensate. In [1], the role of the latter was not clearly specified. The more precise way in which the separation, guided by some results of lattice calculations, is made was given by Adami and Brown [3] who brought out clearly that there are effectively two scales in the broken symmetry sector of QCD [#2]. These scales can be given in terms of the two “bag constants”:

1. The bag constant for chiral symmetry restoration

$$B_{\chi_{SB}} \approx (140 \text{MeV})^4.$$  (4)

[#2] Several other authors have arrived at different conclusions. See, e.g., [4, 5] and footnote #12. To distinguish the scaling we are advocating from those favored by others, we will call ours “BR scaling” in this paper.
This bag constant is essentially the condensation energy of the vacuum in the Nambu-Jona-Lasinio (NJL) model. Basically, in this model this is the amount that the condensate of negative energy constituent quarks is lowered by developing masses. This discussion is, however, model-dependent and, while the physics seems sensible, it cannot be satisfactory from a formal point of view. Our aim is to improve on it.

2. The bag constant for gluon condensation

\[ B_{\text{glue}} \approx (250\text{MeV})^4. \] (5)

There is an intricate relationship between quark and gluon condensates, as evidenced by their behavior as the temperature is increased through chiral restoration \( T \sim T_{\chi SR} \). Lattice gauge calculations show that about half of the gluon condensate decondenses as the temperature rises through \( T_{\chi SR} \). Of course, all of the quark condensate \( \langle \bar{q}q \rangle \), which can be considered as the order parameter for the chiral restoration transition, goes to zero at the critical temperature.

In the chiral limit (defined as the limit in which bare quark masses are set equal to zero), all of the mass of hadrons such as the nucleon or vector meson can be expressed in terms of the gluon condensate \( T_{\mu}^\mu \), since the energy-momentum tensor involves only this condensate. In the QCD sum rule results, however, the dynamically generated hadron masses depend chiefly on the quark condensate \( \langle \bar{q}q \rangle \), the gluon condensate entering as a correction to this, at about the 10% level.

The intricate interplay between quark and gluon condensates in the determination of the \( \rho \)-meson was made clear by Adami and Brown. The argument given there was rather formal, phrased in temperature-dependent QCD sum rules. We present here a much simpler physical argument.

The role of the \( \langle \bar{q}q \rangle \) quark condensate is to produce most of the \( \rho \)-meson mass. It can be considered as generating the mass by producing a repulsive scalar potential, equal in magnitude to the mass \( m_\rho \), from the scalar quark condensate in the vacuum, as shown in Fig.1.

Clearly the quark condensates couple repulsively to the \( \rho \) meson, the repulsive scalar potential representing the \( \rho \)-meson mass. However, from the sign of the coefficient of gluon condensate, \( C_{G2}(T^2, M^2) \), this (gluon) condensate couples attractively. The dimension-4 operator \( G^2 \) where \( G_{\mu\nu} \) is the gluon energy-momentum tensor is scaled with \( M_B^{-4} \), where \( M_B \) is the Borel mass. With increasing density and deceasing quark condensate, \( M_B \rightarrow M_B^\ast \),

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\#3 We denote the current quark by \( q \) and the constituent quark by \( Q \).
\#4 An analogy is that of driving a car. The car moves because the wheels turn (gluon condensate). However, the driver determines the velocity at which he is moving from the speedometer (quark condensate). The wheels and speedometer are technically connected (QCD).
Figure 1: The role of the quark condensate $\langle \bar{q}q \rangle$ in generating the $\rho$-meson mass can be visualized in terms of a scalar field coupling the $\rho$ meson to the negative quark condensate in the vacuum. This simple picture was developed in [12], in order to relate the NJL model to this picture of the vacuum condensate producing the $\rho$ meson mass by this mean-field mechanism.

so decreasing. Although the gluon condensate enters into determining $m_\rho$ only at the $\sim 10\%$ level at zero density, at finite density, it would seem to become relatively more important because of the decreasing $(M_B^*)^{-4}$ effect. The gluon condensate itself is known to change rather little with density. At the point where the repulsive contribution from the quark condensate has dropped to the value of the attractive contribution from the gluon condensate, the $\rho$-meson mass $m_\rho^*$ would go to zero, before $\langle \bar{q}q \rangle^*$ goes to zero. It seems unreasonable that $m_\rho^*$ should go to zero before chiral restoration. (There is an exception to this in connection with Georgi’s vector limit which will be discussed in detail below.)

Adami and Brown [11] resolved this difficulty by showing that when the perturbative (black-body) temperature effects are summed and incorporated into the coefficient $C_{G^2}(T^2,M^2)$, then the modified coefficient goes to zero as $T \to T_{\chi SR}$. The contribution of the gluon condensate to the $\rho$-meson mass is always small, $\sim 10\%$ of that from the quark condensate. Pictorially this can be represented by Fig.2. Thus the gluon condensate enters as a “Lamb shift” correction to the $\rho$-meson mass. That is, it is the modification by the virtual gluon field in the mass dynamically generated from $\langle \bar{q}q \rangle$.

One can thus understand how the dynamically generated $m_\rho^*$ goes to zero as $\langle \bar{q}q \rangle^*$ goes to zero. However the scaling (1) and (2) – “BR scaling” – that results from splitting of $\chi$ into a $\chi_s$ and $\chi'$, has not been clearly explained. A recent paper by Beane and van Kolck [13] suggests a remedy to this deficiency. We describe their development in the next section.
2 A Dilatation-Invariant Low-Energy Lagrangian

The starting point of Beane-van Kolck treatment is an old “theorem” of Weinberg [14] which states that the full content of quantum field theory (here QCD) is given entirely by general physical principles like unitarity, cluster decomposition, Lorentz invariance etc. together with the assumed internal symmetries. QCD is characterized by the global $SU(2)_L \times SU(2)_R$ symmetry, which however is not obvious from the spectrum. Weinberg’s theorem suggests that an “equivalent” field theory exists, which employs constituent quark (quasiparticle) fields that realize chiral symmetry nonlinearly and therefore includes explicitly Goldstone boson fields. Brown and Rho [4] went further, beginning from the Skyrme Lagrangian out of which the fermions (nucleons) arose as chiral solitons. This would imply a replacement, at a later stage, of the nucleons made up out of constituent quarks in the Beane-van Kolck approach by baryon fields.

The low-energy effective theory that we wish to study must manifest the same conformal symmetry possessed by QCD. Yang-Mills theory possesses scale invariance which is broken by the anomaly while the scale invariance of the fermionic part of QCD is broken by the quark masses. Now the variables of the low-energy Lagrangian are the Goldstone bosons (pions) and constituent quarks (or nucleons). Low-energy vector and axial vector interactions can be viewed as made up out of correlated pions or in a more compact way introduced as hidden gauge bosons [15]. The latter allows us to relate the property of the vector mesons in hot and/or dense matter to Georgi’s vector symmetry [17] as we shall discuss later.

The dilatation invariant chiral Lagrangian supplemented by the trace anomaly that Beane and van Kolck arrive at is

$$\mathcal{L} = \bar{\psi}(\slashed{D} + \slashed{V})\psi + g_A \bar{\psi} A^\gamma_5 \psi - m_\chi \bar{\psi}\psi$$
\[ \frac{1}{4} f_\pi^2 \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) \chi^2 + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi \]

\[ -\frac{1}{2} \text{tr}(G_{\mu\nu} G^{\mu\nu}) - V(\chi) + \cdots \]  

(6)

Here \( m \) is the constituent quark mass (which will be defined more precisely later) and

\[ V_\mu = \frac{1}{2} (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger), \]

(7)

\[ A_\mu = \frac{i}{2} (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) \]  

(8)

with

\[ \xi^2 = U = e^{i\pi/f_\pi} \quad \text{with} \quad \pi \equiv \tau \cdot \pi(x). \]  

(9)

The trace “Tr” is over the flavor group and “tr” over the color group. Transformation properties of \( \xi \) and of the constituent quark doublet \( \psi \) under chiral and conformal transformation are given in [13]. What concerns us chiefly here is that a sufficient power of the scalar field \( \chi \) must be put into each term so that the operator scaling properties, under dilatation, of each term in the Lagrangian are those of QCD, \( i.e \), that the Lagrangian is dilatation-invariant, except for the potential term which breaks this invariance. This potential subsumes radiative corrections of high chiral order and hence can be very complicated, the only condition being that it gives precisely the trace anomaly of QCD in terms of the scalar field \( \chi \). The precise form of this potential is neither known nor necessary for our discussion. The basic assumption here is that the symmetry broken by the anomaly could be represented by that of spontaneous breaking and that the Coleman-Weinberg mechanism “chooses” the vacuum value of the scalar field \( \textit{in hot and/or dense medium} \).

In the “derivation” of the low-energy effective Lagrangian from a microscopic Lagrangian (say, QCD or a fundamental modeling of QCD), scaling is always broken in obtaining the current algebra term \( \frac{1}{4} f_\pi^2 \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) \). Generally, a regularization is made to incorporate high-energy degrees of freedom in low-energy effective theories and a momentum cut-off \( \Lambda \) that delineates the high-energy sector from the low-energy one gets transmuted to the physical constant \( f_\pi \), the pion decay constant. The scale breaking through the potential \( V(\chi) \) renders this transmutation automatic and more or less unique.

In the next section we shall review how Beane and van Kolck relate the scalar field \( \chi \) to an effective scalar \( \sigma \) field by introducing a linear basis

\[ \sigma + i \tau \cdot \pi = U \chi. \]  

(10)

It is in the linear basis that low-energy current algebra is reconciled with high energy Regge asymptotics and consequently becomes more relevant for high-T and high-density processes we are interested in.
As shown in Fig.1 the scalar $\sigma$ field can be thought of as generating a constituent quark mass $m_Q$ through

$$m_Q = g_{\sigma QQ} \langle \sigma \rangle / m_\sigma^2 \tag{11}$$

where the subscript $Q$ stands for constituent quarks of light flavors. Compare this to the mean-field model depicted in Fig.1,

$$\langle \sigma \rangle = -2 g_{\sigma QQ} \langle \bar{q}q \rangle / m_\sigma^2, \tag{12}$$

the factor 2 coming from the two light-quark flavors. In terms of the constituent quark mass $m_Q$ one can then write down the vacuum energy (see [3], eq.(3.11))

$$E_{\text{vac}} = -B_{\chi SR} = -12 \left\{ \int_0^\Lambda \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m_Q^2} - \frac{1}{2} \int_0^\Lambda \frac{d^3k}{(2\pi)^3} \frac{m_Q^2}{\sqrt{k^2 + m_Q^2}} \right\} + \frac{3\Lambda^4}{2\pi^2} \tag{13}$$

where $\Lambda$ is a cut-off set to obtain the known empirical value of the quark condensate

$$\langle 0|\bar{u}u|0 \rangle = -(240 \text{ MeV})^3. \tag{14}$$

Whereas the $E_{\text{vac}}$ of (13) is equivalent, physically, to the condensation energy in the NJL model which involves the quark condensate $\langle 0|\bar{u}u|0 \rangle$ that depends on the cut-off $\Lambda$ quadratically, eq.(13), however, diverges only logarithmically with $\Lambda$ which means that the $B_{\chi SR}$ is reliably determined in this formulation.

Determining $\Lambda$ from the quark condensate (14), one finds

$$B_{\chi SR} \approx (140 \text{ MeV})^4 \approx 50 \text{ MeV}/fm^3, \tag{15}$$

just the value of the MIT bag constant. We shall see in the next section that this is just the value obtained from the Beane-van Kolck theory.

We see that in the low-energy sector, dilatation invariance is broken by the development of the quark condensate. The scale of this breaking is $(B_{\chi SR})^{1/4}$. In QCD, however, scale breaking occurs in the gluon sector at QCD loop level. The magnitude of this scale breaking is an order of magnitude larger than the $B_{\chi SR}$ of eq.(15); namely

$$B_{\text{glue}} \approx 500 \text{ MeV}/fm^3, \tag{16}$$

obtained from the QCD trace anomaly. Although $B_{\text{glue}}^{1/4} \approx 250 \text{ MeV}$ is not so much larger than $B_{\chi SR}^{1/4} \approx 140 \text{ MeV}$, we know that the magnitude of the gluon condensate determines the glueball mass

$$M_{\text{GB}} \approx 1.5 \cdots 2 \text{ GeV}. \tag{17}$$
The QCD sum rules tell us \[1\] that the gluon condensate is relatively unimportant for the masses of hadrons made up out of up and down quarks, entering only through the Lamb-shift type effect of Fig. 2. What this implies is that the scalar $\chi$ field in the Lagrangian \[6\] has mostly to do with the quark condensate

$$B_{\chi_{SR}} \propto \chi^4$$

rather than with the gluon condensate $B_{\text{glue}}$. Just how $B_{\text{glue}}$ reduces in light-quark systems to $B_{\chi_{SR}}$ is not well understood. In any event, the $\chi_*$ of Brown and Rho \[1\] should be identified with the mean field of this *smoothed* field. This will be discussed in more detail in the next section. Here we consider the implication of the restoration of the correct scaling properties of the operators in the low-energy chiral Lagrangian.

The mean-field component of $\chi$ which we denoted $\chi_*$ previously will be a function of temperature and density. As in \[1\], we define

$$f_\pi^* \equiv f_\pi \chi_*.$$  \hspace{1cm} (19)

We must stress once again that the $\chi$ field is here already the *smoothed* field of the quark sector that includes no gluon fluctuation. An equivalent way of making the separation is to divide the scalar glueball field\[5\] associated with the trace anomaly to a “smooth” component $\chi_s$ and a “non-smooth” component $\chi_{ns}$, $\chi = \chi_s + \chi_{ns}$, and “integrate” out the non-smooth component $\chi_{ns}$ from the effective Lagrangian. What appears in the Beane-van Kolck theory is then $\chi_s$ which is relabeled as simply $\chi$. It is the mean-field component of this field that scales the quark condensate in the BR scaling. Its fluctuating part cannot in general describe a single local degree of freedom since it represents multi-pion excitations including the continuum but in hot and dense medium can be *interpolated* by a local scalar effective field denoted $\sigma$. We will have more to say on this point later.

Similarly we define

$$m_Q^* \equiv m_Q \chi_*.$$  \hspace{1cm} (20)

from which we see that the constituent quark mass $m_Q^*$ scales with temperature and density in the same way as the order parameter $f_\pi^*$. In particular, $m_Q^*$ must go to zero with chiral restoration, $f_\pi^* \to 0$.

In order to obtain the scaling of the vector meson masses, it is convenient to look as in \[1\] at the quartic Skyrme term

$$\frac{1}{32g^2} \text{Tr} \left[ U^\dagger \partial_\mu U, U^\dagger \partial_\nu U \right]^2.$$  \hspace{1cm} (21)

\[5\] Instead of dividing the field $\chi$, one could choose to divide the trace of the energy-momentum tensor $\sim \text{Tr} G^2$ (where $G_{\mu\nu}$ is the gluon field tensor) into a “quarkish” component $H_q$ and a gluonic component $H_g$ as in \[16\]. This separation may lead to a different result on scaling, although we have not investigated this possibility.
One can think of this arising when the vector mesons are integrated out from the effective Lagrangian. The constant $g$ can be identified as the hidden-gauge coupling constant. Now this term is scale-invariant as it is, therefore no $\chi$ field need be multiplied to it. When the scale invariance is spontaneously broken, the coefficient of this term remains independent of the factor $\chi_*$, so we have

$$g^* = g. \quad (22)$$

As shown recently by Harada, Kugo and Yamawaki [18], the effective chiral theory, when hidden gauge symmetry of the vector mesons is implemented, has the exact low-energy theorem

$$m_{V}^2 = 2g^2 f_\pi^2 \quad (23)$$

which is essentially the KSRF relation. This theorem is proven in zero-temperature and zero-density regime but we see no reason why it should not hold in medium. This would imply

$$m_{V}^* = 2g^{*2} f^{*2}_\pi \quad (24)$$

from which, with (22), follows

$$\frac{m_{V}^*}{m_V} \approx \frac{f^{*}_\pi}{f_\pi}. \quad (25)$$

The question of how the nucleon mass scales with temperature and density is a lot more subtle, because at finite density, a vector mean-field can develop and build up. There have been several studies on this issue [22] but no satisfactory answers have been obtained. We postpone this question until after we discuss the effective $\sigma$ field that results once the Lagrangian (6) is transformed to a linear basis.

We do not discuss the pion mass, which results from explicit chiral symmetry breaking, a phenomenon associated with the electroweak scale which is much higher than the chiral scale we are dealing with. From this point of view, the most reasonable thing to do is to assume that the pion mass does not scale within the range of temperature and density involved in the chiral Lagrangian.

In [1], we deduced the scaling of $\langle \bar{q}q \rangle$ with density from the assumed operator transformation properties of the explicit chiral symmetry breaking term in the low-energy Lagrangian. The result was

$$\left[ \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} \right]^{1/3} \approx \frac{f^{*}_\pi}{f_\pi}. \quad (26)$$

For the same reason as for the pion mass, this scaling relation does not follow immediately (without some strong assumptions) from the low-energy effective Lagrangian and in fact
may not be quite correct as we shall argue later in connection with the modeling of lattice results.

It should be remarked that the Brown-Rho consideration was at mean-field level. The question as to what that corresponds to in the sense of Weinberg’s “theorem” has not yet been addressed. In a broad sense of effective theories, the BR scaling is to represent the tree order in chiral perturbation expansion (more on this later). Higher-order corrections will surely modify the scaling relation. For example, from the Goldberger-Treiman relation, we find that

$$\frac{g_{\pi NN}}{m_N} = \frac{g_A}{f_\pi}. \quad (27)$$

As we shall review in the next section, the $g_A$ in medium drops from 1.26 to $\sim 1$ as density increases from zero to nuclear matter density $\rho_0$. The cause of this change has to do with the role of short-ranged interactions between baryons that involve multi-Fermi interactions absent in the matter-free space. This essentially cancels the change in $f_\pi$. Thus $g_{\pi NN}/m_N$ is expected to remain roughly constant up to nuclear matter density, even though the Brown-Rho scaling would have $m_N^*$ drop, but $g_{\pi NN}$ remain unchanged. Once $g_A^*$ has gone to $\sim 1$, however, one would not expect it to change much further, and the BR scaling should take over at higher densities.

Before proceeding, we should address the question as to what the precise role of the effective (density-dependent and temperature-dependent) masses and coupling constants is in confronting physical observables. As in all field theory problems, those renormalized quantities are parameters of the theory that have no physical meaning independently of the observables one is discussing. For instance, the $m^*$’s are not by themselves physical observables. In some special processes, they could be associated with quasiparticle parameters, that is, they can be taken as a pole of the particle Green’s function but in general, the pole of the Green’s function may not be approximated by the $m^*$’s. Residual interactions will modify the residues and pole positions. In this respect, it is important to realize that the $m^*$’s defined in one theory need not be the same as the $m^*$’s defined in another theory. Thus it makes no sense for someone to take a particular hadronic model to calculate what he/she defines as $m^*$’s and compare with another person’s $m^*$’s calculated in a different hadronic model. They can only compare physical amplitudes computed with the model wherein the parameters $m^*$’s appear. This also means that a particular scaling relation present at mean-field level of one person’s theory need not be reproduced in another person’s mean-field theory. As we shall note below, it is a particular reparametrization of the fields that gives rise to a simply scaling theory; other reparametrizations need not give the same scaling of the parameters although the physics can be identical. In fact, unless one defines how to compute corrections, the density (and temperature) dependence of the effective parameters with a particular model has no meaning. This point seems to be largely
overlooked in the field.

This then raises the question: In what sense the BR scaling is to be understood?

To answer this question, we recall the principal assumption made in [1] – and further elaborated in [20] – that at each density and/or temperature, we have an effective Lagrangian that has the same chiral and conformal symmetry as in free-space with the parameters of the theory density/temperature dependent. This can be considered as an application of Nambu’s “theorem” that whenever spontaneous symmetry breaking is involved, be that in condensed matter, nuclear or elementary particles, the physics involved is a generic $\sigma$ model characterized by different length scales [21]. It is not obvious that this is always possible, so in our case that is a strong assumption. However we propose one way of justifying this approach and it is the notion that the nuclear matter is a chiral Fermi liquid as proposed by Lynn [19].

It is fairly well-established empirically that the interactions of mesons and baryons inside nuclear matter are governed mainly by chiral symmetry. But it is also well-established that nuclear matter – and nuclear ground states – cannot be described starting from chirally symmetric Lagrangians, at least at low orders. So the ground-state (mean-field) properties which do not follow naturally from chiral symmetry and the excitation (fluctuation) properties which do are not compatible. These can be reconciled, however, if the nucleus is a soliton, that is, a chiral Fermi liquid drop with nuclear matter being a chiral Fermi liquid. The chiral Fermi liquid is a soliton solution of the chiral effective theory at quantum level. The parameters so fixed in this soliton structure can be identified with the $m^*$’s etc. (In the BR scaling, it is the potential $V(\chi)$ which is supposed to fix the parameters, but we do not really know how to calculate it. Lynn’s chiral liquid approach suggests how to do this.) Fluctuations around the soliton would then have the requisite chiral and conformal symmetry of the original Lagrangian, so must retain the generic $\sigma$ model form. (We argue below that it is a linear $\sigma$ model.) In looking at the scaling of the $\sigma$ field in terms of phenomenological meson theories that we describe in the next section, we are implicitly assuming this structure.

While we find the approach described above quite plausible, we must admit that we have not yet found a realistic chiral liquid soliton which could be used as the background field to build a theory to “determine” the parameters in a self-consistent way. One can however make the following conjecture. To the extent that Walecka’s mean field theory describes nature fairly accurately, the chiral liquid soliton, when correctly formulated, should resemble Walecka’s mean field theory. The parameters appearing in the effective Lagrangian describing fluctuations around the chiral liquid (mean field) will have BR scaling incorporated naturally. Indeed we shall see later that this is what is observed phenomenologically when we consider fluctuations in the strangeness flavor, i.e., in kaon-nuclear processes. For-
mulating this in a rigorous way remains an open problem.

3 Transformation to a Linear Basis; Interpretation of the $\sigma$ Field

Beane and van Kolck transform their Lagrangian to a linear Lagrangian by making the field redefinition

$$\Sigma = U \chi$$

where

$$\Sigma \equiv \sigma + i \tau \cdot \pi.$$  \hfill (29)

(Note that since the Sugawara field $U = e^{i\pi/f_\pi}$ is conformally invariant, the $\pi$ must scale as $f_\pi$ and consequently we have a new $\pi^*$ for each temperature or density. We shall not make this distinction here.) The Beane-van Kolck Lagrangian obtained by the above field redefinition is rather involved, but it simplifies considerably in the so-called “dilaton” limit ($m_\sigma \to 0, g_A \to 1$) to

$$L = i\overline{Q} \slashed{D} Q - \frac{1}{2} \text{tr}(G_{\mu\nu}G^{\mu\nu}) + \frac{1}{2} \partial_\mu \chi \cdot \partial^\mu \chi$$

$$+ \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{m_\sigma}{f_\pi} \overline{Q}[\sigma - i \gamma_5 \chi \cdot \pi]Q$$

$$+ \frac{m_\sigma^2}{16 f_\pi^2} (\sigma^2 + \pi^2)^2 - \frac{m_\sigma^2}{8 f_\pi^2} (\sigma^2 + \pi^2)^2 \ln[(\sigma^2 + \pi^2)/f_\pi^2] + \cdots$$  \hfill (30)

Since in the vacuum the pion field has zero VEV and we neglect pion fluctuation, we can identify

$$B_{\chi SR} = \frac{m_\pi^2}{16 f_\pi^2} \sigma_0^4 = \frac{m_\sigma^2 f_\pi^2}{16},$$

$$\sigma_0 \equiv \langle 0 | \sigma | 0 \rangle$$

since $\sigma_0 = f_\pi$ in the vacuum. We find immediately that

$$B_{\chi SR}^{1/4} \approx \frac{1}{2} \sqrt{m_\sigma f_\pi}.$$  \hfill (32)

For $B_{\chi SR}^{1/4} \approx 140$ MeV, $m_\sigma \approx 918$ MeV, in the range of $\sigma$ masses in the bare vacuum consistent with the Weinberg-Tomozawa relation. This $B_{\chi SR}$ is similar to the condensation energy in the NJL model, eq.(15), but has the advantage that it follows in a straightforward way from the development. Furthermore, no explicit cut-off is needed in this way of arriving
Figure 3: Uncorrelated two-pion exchange involving nucleon and $\Delta$ intermediate states at the condensate energy, with the bag constant arising naturally at the tree level. So, although we believe the NJL model captures the essential physics of the low-energy scale bag constant, the Beane-van Kolck treatment is highly more preferable and appealing.

Let us now discuss the $\sigma$ field which has emerged in going from the non-linear realization of chiral symmetry in the transformation to the linear basis. We should mention to start with that it is not a mere field redefinition which would leave physical quantities unchanged. In going from the nonlinear structure to the linear structure, a nontrivial physics information has been injected. This point was explained by Adami and Brown [3] using a different argument. The crucial point is that in the transformation, one has chosen the component of the $\chi$ field such that the $\sigma$ is now composed mainly of even powers of the pion field $\pi$ in the expansion of the $U = e^{i\pi/f}$ field. The fluctuating part of this $\sigma$ therefore interpolates two-pion excitations with a mixing to $4\pi, 6\pi$ etc. The physical implication of this structure can be appreciated by recalling the origin of the scalar attraction in the nucleon-nucleon interaction.

Beginning from the $N\bar{N} \rightarrow 2\pi$ helicity amplitude, which can be obtained by analytic continuation from $\pi N$ scattering amplitude, the scalar attraction can be built up [23]. A dynamical reconstruction of these results produces the scalar exchange in the Bonn potential [24]. In addition to the six uncorrelated two-pion exchange diagrams shown in Fig.3, there are the contributions in which the two pions scatter on each other in intermediate states, shown in Fig.4.

The $\pi - \pi$ rescattering necessary to form the correlated two-pion exchange is, of course,
Figure 4: Correlated two-pion exchange involving nucleon and Δ intermediate states automatically included in the dispersion theory formalism, where the \( N\bar{N} \rightarrow 2\pi \) helicity amplitudes are obtained by analytic continuation of the \( \pi N \rightarrow \pi N \) scattering. However it is more convenient for our purpose to use a dynamical model \(^\[25\]\) since we shall later introduce effects of finite density and/or temperature. The correlated two-pion exchange provides about \( \frac{2}{3} \) of the total \( 2\pi \) exchange in the exchange of scalar degrees of freedom. The mass distribution is broad, but the correlated exchange can be handled rather accurately by replacement by a sharp mass particle \( \sigma' \). We shall later drop the prime on this object, because we claim that this is just the low-mass dilaton of Beane and van Kolck. Effective coupling constants and masses for this effective scalar particle are given in various versions of the Bonn potential.

The main rescattering in the Durso-Jackson-Verwest work \(^\[25\]\) comes from crossed-channel \( \rho \)-meson exchange. Thus, we can picture the \( \sigma' \) as resulting from the summation of the diagrams shown in Fig.5.

We are now ready to investigate the density dependence of the \( \sigma \) mass. In eq.\(^\[25\]\), we showed that from the KSRF relation \( m_V^*/m_V \approx f_\pi^*/f_\pi \); i.e., the vector meson mass drops as the order parameter decreases. This density-dependent \( \rho \)-meson mass has been included in the integral equation, shown in Fig.3, determining the \( \sigma \) \(^\[25\]\). From the \( N\bar{N} \rightarrow 2\pi \) helicity amplitudes supplied to us by the authors of this work, we can track the downward movement in mass of the (distributed) scalar strength as the density increases. Since the downward shift of the scalar strength originates from the density dependence of the \( \rho \)-meson mass, it
Figure 5: Description of the scalar meson employed in terms of correlated two-pion exchange. The two pions interact chiefly through $\rho$-meson exchange in the crossed channel.

is clear that for low densities where the term linear in density suffices, that

$$\frac{m_\sigma^*}{m_\sigma} \approx \frac{m_\chi^*}{m_\chi}.$$  \hspace{1cm} (33)

The phase of the $f_{J=0}^0(t)$ helicity amplitude goes through $\pi/2$ for $\sqrt{t} \approx 500$ MeV, showing that the fictitious scalar particle of mass $\approx 600$ MeV used in boson-exchange models to mimic the enhancement in the $f_{J=0}^0(t)$ helicity amplitudes for $NN$ scattering becomes a real resonance of mass $\approx 500$ MeV by $\rho \approx \rho_0$.

Thus, for density $\rho \approx \rho_0$ there is a resonance in the scalar channel at $m_\sigma \approx 500$ MeV. This light mass $\sigma$ suggests that chiral symmetry be realized in the linear $\sigma$-model as a manifestation of “mended symmetry” as emphasized by Beane and van Kolck. While in the zero-temperature and zero-density regime, Nature prefers the nonlinear realization of chiral symmetry for which we have now rather compelling evidence, as density/temperature is increased in hadronic matter, the dilaton degree of freedom, frozen in the matter-free vacuum, gets identified with the effective scalar channel that is interpolated by a single scalar $\sigma$; at $\rho \approx \rho_0$ the nonlinear realization of chiral symmetry cedes to a linear realization with a low-mass $\sigma$ “seen” in boson-exchange models of the nucleon-nucleon interaction. An important point in our proposition is that in medium, chiral symmetry is still manifest in Goldstone mode up to the chiral phase transition and the dynamics can still be described by a chiral Lagrangian. This is in line with our proposition that Lynn’s chiral Fermi liquid (which we tend to identify with Walecka’s mean field) is a background around which fluctuations be made.

The coupling constants furnish an interesting interplay of the “vacuum structure” discussed up to now and short-range nuclear correlations that figure in nuclear many-body systems. At zero density, the pion coupling constant at tree level is known to be increased
by loop corrections, say, from $g_A = 1$ to $g_A = 1.26$; i.e.,

$$g_{\pi NN} = (g_{\pi NN})_{\text{tree}} g_A.$$ (34)

Sometime ago, Rho [27] and Ohta and Wakamatsu [28] independently predicted a strong density dependence in $g_A$

$$\frac{g_A(\rho)}{g_A(0)} = \left[1 + \frac{8}{9} \left(\frac{f_{\pi N\Delta}}{m_{\pi}}\right)^2 \rho \frac{\omega_R}{\omega} \right]^{-1}$$

$$= \left[1 + b \rho / \rho_0 \right]^{-1}$$ (35)

where $\omega_R \approx 2.1 m_\pi$ is the $\Delta N$ mass difference and

$$b \approx 0.8 (g_0)_{N\Delta}.$$ (36)

Here $(g_0)_{N\Delta}$ is the Landau-Migdal local field correction in the channel $NN \leftrightarrow N\Delta$ which can be interpreted as a four-Fermi interaction in the effective Lagrangian that figures in many-baryon systems [27]. The Landau-Migdal parameter is usually taken to be $\sim 1/3$; with this value we expect:

$$g_A(\rho_0) \approx 1.$$ (37)

This rapid drop in $g_A$ means that in nuclear matter calculations, it is better to use the pion-nucleon coupling given at tree level,

$$\frac{(g_{\pi NN})_{\text{tree}}^2}{4\pi} = \frac{1}{4\pi} \left(\frac{g_{\pi NN}}{g_A}\right)^2 \approx 8.8.$$ (38)

Since the $\sigma$ now forms the fourth component of $SU(2) \times SU(2) \sim O(4)$, i.e., the linear realization of chiral symmetry, it should have the same coupling constant [28]. Indeed this is essentially the coupling of the $\sigma$ in the Bonn potential [31] where the versions A, B and C of the relativistic OBEP have $g_\sigma^2/4\pi = 8.8$, $8.9$ and $8.6$, respectively. We thus come to the conclusion that at $\rho \approx \rho_0$, there is a satisfactory linear realization of chiral symmetry, with $\pi$ and $\sigma$ mesons coupled with approximately the tree-level pion coupling to the nucleon. (Note that the in-medium pion-nucleon vertex in the $\gamma_5$ coupling – which is what enters

**Careful analyses of Gamow-Teller transitions in nuclei reveal that the effective $g_A$ in nuclear $\beta$ decay is (modulo shell effects) near unity already in light nuclei where density is not high enough for the Rho-Ohta-Wakamatsu (ROW) mechanism to be fully effective. This can be understood as follows. In light nuclei, it is the core polarization effects induced by tensor forces [24] that play the main role in quenching the $g_A$, with the ROW mechanism being less effective. As density increases in heavy nuclei, the tensor force gets suppressed by the BR scaling as shown in [24] and hence becomes less effective in the quenching while the ROW builds up. In some sense, the tensor correlation and the ROW mechanism play a complementary role and both seem related – albeit indirectly – to the chiral restoration phenomenon.**

15
in the linear $\sigma$ model – is effectively of the form, $(g_{\pi NN})_{tree}/m_N^* \approx g_{\pi NN}/m_N$, so as long as the pion mass does not scale, the pion-exchange interaction remains unmodified. This feature of the linearly realized chiral symmetry is consistent with the BR scaling.) This indicates that nuclear matter manifests a “mended symmetry” envisaged by Weinberg in the large $N_c$ limit of QCD.

The density dependences of the type (35) are normally not included in the nuclear matter calculations, so it must be somewhat coincidental that the coupling constant, essentially that of eq.(38), can be used at zero density in the Bonn potential to describe the nucleon-nucleon scattering. At zero density, the low-mass $\sigma$ is not developed, and the full panoply of uncorrelated and correlated two-pion exchange had to be worked out. We note that in going from $\rho = \rho_0$ to $\rho = 0$ there are two opposing tendencies. The first is that the resonance in the correlated two-pion exchange moves up from $\sim 500$ MeV at $\rho \approx \rho_0$ to $\sim 1$ GeV [i.e, $f_0(975)$] at $\rho = 0$. The second is that the coupling increases from $(g_{\sigma NN}^2/4\pi)_{tree} \approx 8.88$ to $(g_{\sigma NN}^2/4\pi) \approx 14$. Even so, Durso, Kim and Wambach find that the scalar exchange at $\rho = \rho_0$ is substantially more attractive than that at $\rho = 0$. Brown and Machleidt show that density-dependent loop corrections must be introduced in order to achieve saturation of nuclear matter. If these higher-order corrections are introduced into the calculation of $m_\sigma(\rho_0)$, it will not come out as low as 500 MeV. This work is in progress.

The Bonn potential is undergoing some modifications at low momentum scales resulting from imposition of chiral constraints on the $\pi\pi$ scattering, which force the off-shell amplitude to go repulsive. In an earlier work, without these constraints, Schuck et al found that attractive $\pi\pi$ interactions produced very strong effects in the scalar-isoscalar ($\sigma$-meson) channel, correlating an unreasonable amount of S-wave strength in the region close to the threshold at $2m_\pi$. Aouissat et al show that when the $\pi\pi$ scattering amplitudes are constrained by the chiral symmetry à la linear $\sigma$-model the strong near-threshold strength disappears.

In the linear $\sigma$-model, from the potential

$$V = \frac{\lambda}{4} \left((\sigma^2 + \pi^2)^2 - f_\pi^2\right)^2$$

(39)

the $\sigma$ mass is

$$m_\sigma^2 = 2\lambda f_\pi^2.$$  \hspace{1cm} (40)

We do not expect the constant $\lambda$ to change with density or temperature, so $m_\sigma^2$ must scale with $f_\pi^2$:

$$\frac{m_\sigma^*}{m_\sigma} \approx \frac{f_\pi^*}{f_\pi}.$$  \hspace{1cm} (41)

This is the same scaling as obtained in eq.(25) for the vector mesons.
Once the low-energy $\pi\pi$ scattering is set by the linear $\sigma$-model \[7\], the introduction of the $\rho$-meson between pions, as discussed above, must be accompanied by subtractions, so that the S-wave $\pi\pi$ scattering phase shifts are not destroyed. Built in this way, the $\rho$-meson plays a relatively minor role, unlike that in the older work, in the composition of the effective $\sigma$-meson.

We should mention that the bag constant $B_{\chi SR}$ of eq.(15) is small compared with $B_{\text{glue}}$, but large in comparison with energies in low-energy nuclear physics. In the nuclear many-body system, it represents an energy of $\sim 50$ MeV/fm$^3$. At nuclear matter density, the density of the nucleon is $1/6$ per fm$^3$, so that the nucleon rest mass energy is $\sim 150$ MeV/fm$^3$, and the net binding energy per nucleon of 16 MeV is only $\sim 3$ MeV/fm$^3$. Thus $B_{\chi SR}$ is actually very appreciable.

4 Chiral Restoration at Finite Temperature

Lattice gauge calculations are now sufficiently accurate to make statements about finite temperature behavior of various quantities, as we discuss. The situation is not so good with finite density lattice calculations, so we shall not pursue this matter here.

We first put the finite-temperature lattice results into physical units following Koch and Brown \[9\] who used the results of Kogut $et\ al$ \[8\] calculated with 6 time slices. Later, more extensive results with 8 time slices \[37\] are found to more or less confirm the 6-time-slice results.

In lattice gauge calculations, the quark condensate $\langle \bar{q}q \rangle$ is measured for each temperature. In an attempt to understand the lattice results in terms of effective fields, the entropy was computed in \[38\] as a function of temperature, assuming the increase in entropy to result from the heavier hadrons, $\rho$, $\omega$ and $a_1$, going massless. The number of degrees of freedom in hadrons was, however, limited to 24, the number of quarks, since the latter are the fundamental degrees of freedom. In a modeling of the lattice data along the same line, Koch and Brown \[9\] calculated the rate of increase of entropy in the quarks under the two scaling assumptions:

1. That the hadron masses scale to zero as

$$\frac{m_h^*}{m_h} = \left( \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} \right)^1/3;$$  \hspace{1cm} (42)

\[We are aware of the fact that the linear $\sigma$-model without matter coupling is not consistent with chiral perturbation theory. Here we are focusing on low-energy S-wave $\pi\pi$ scattering in the presence of matter fields such as nucleons and vector mesons, so that the linear realization of chiral symmetry does not in practice suffer from this fundamental defect.\]
2. That they scale as

\[ \frac{m_h^*}{m_h} = \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle}. \]  

(43)

The scaling (42) is what was obtained in [1] with what we now believe to be too naive an assumption\(^*\) while the scaling (43) is obtained in the NJL model for the scalar field \(\sigma\).

The NJL model cannot make predictions for vector mesons whose mass scale is of the same order as the cut-off, so the model is moot on the \(\rho\) and \(a_1\).

The scaling (43) which we shall call “Nambu” scaling gives an excellent fit to the lattice results, the lattice entropy increasing with temperature substantially faster than that calculated from the scaling (42). However, one should be cautious in interpreting the results as a support for the Nambu scaling. The argument that goes into this theory, namely that the first 40 (24 quark and 16 gluons) degrees of freedom go massless and nothing happens to the masses of the other particles, is extremely crude. It resembles in some sense the Debye theory of phonons, where there is a sharp cut-off in the phonon spectra at the point where the degrees of freedom in it equal the number of underlying degrees of freedom. It must be admitted, however, that crude though it may be, our Debye-like theory, with linear dependence of the hadron masses on the quark condensate fits the lattice results for the entropy surprisingly well. It should be stressed that this fit uses only the linear relation between the hadron masses and the condensate. The temperature comes in only in the Boltzmann factor when one calculates the entropy. The temperature may be somewhat incorrect because Koch and Brown obtained it from the asymptotic scaling relation, \(i.e\), from the lowest-order perturbative expression. The color coupling constant \(\tilde{g}\) is not small enough for this to be quantitatively correct. Thus the temperature scale in Fig.6 may not be accurate. Using a better temperature scale will not, however, change the fit of the “Nambu” curve to the lattice data appreciably.

A caveat in what we have been discussing here is that it might not be a good approximation to have all the hadron masses scale universally and hence it would be premature to rule out one option in favor of the other. What may be considered solid in our argument is that the lattice results provide an empirical support for the scaling of the masses together with the connection to the quark condensate as given by (43).

Another interesting information one can get from the lattice results is the disappearance of the dynamically generated masses of constituent quarks (or hadrons). Lattice calculations\(^*\) The reasoning based on chiral symmetry and trace anomaly that leads to the BR scaling (1) does not uniquely lead to this relation. As mentioned below eq. (26), there is an additional assumption that goes into it, namely, a relation between the quark scalar density and the chiral field \(U\) which is not precisely given. As such this relation is not to be identified on the same footing as eq. (1) although we loosely call it a BR scaling.
Figure 6: The Koch-Brown analytical fit \[3\] to \(\langle \bar{\psi} \psi \rangle\), normalized to unity at \(T = 0\), is shown by the dropping dashed line. The data from lattice gauge calculations \[8\] is shown without error bars. The effects of the bare quark masses in ref.\[8\] were removed by Koch and Brown, so that their (dashed) line drops to zero. From the lattice results, the entropy can be calculated and the result is plotted, again without error bars, as the rising dashed line. The lower solid line follows from the scaling \((42)\); the upper solid line follows the “Nambu” scaling of \((43)\).
measure screening masses of hadrons. DeTar and Kogut [39] found in four-flavor calculations that the screening masses of the $\rho$ and $a_1$ mesons come together, as expected, at $T = T_{\chi_{SR}}$. Furthermore, to the accuracy measured, the common screening mass came out to be

$$\omega_{\text{scr}} \approx 2\pi T;$$

(44)

i.e., $\pi T$ per quark. Later two-flavor calculations [37] confirmed these results and found further that the common screening mass for the nucleon and its chiral (parity-doublet) partner was

$$\omega_{\text{scr}} \approx 3\pi T$$

(45)

for $T \geq T_{\chi_{SR}}$. As shown by Adami and Brown [3], if a residual mass $m_0$ remained above $T_{\chi_{SR}}$, the screening mass for the vector mesons would be

$$\omega_{\text{scr}} \approx \sqrt{\pi^2 T^2 + m_0^2} \approx \pi T + \frac{1}{2} m_0^2 \pi T.$$  

(46)

In fact to the accuracy calculated in lattice gauge simulations, no $m_0$ is needed, the conclusion being that – to this accuracy – $m_0$ is zero. Of course a small $m_0$ could easily escape detection.

Direct calculation of the constituent quark mass (not the screening mass) for four flavors [40] yields a mass which drops to the perturbative value

$$m_{\text{eff}} = gT/\sqrt{6}$$

(47)

for $T$ going from $T_{\chi_{SR}}$ to $1.75T_{\chi_{SR}}$. Similar calculations of the $\rho$-meson mass [41] find $m_{\rho}^*$ to decrease rapidly as $T$ increases. There is of course a caveat to this. One has to be careful in applying four-flavor results to the two-flavor world, since the chiral restoration transition is first-order in the four-flavor case and the quark condensate does not seem to change much below $T_{\chi_{SR}}$, whereas the transition is a smooth second-order one in the two-flavor case.

Calculations of the Bethe-Salpeter wave functions of $\pi$- and $\rho$-mesons [42] show these mesons to be more compact, i.e., more closely correlated, at $1.5T_{\chi_{SR}}$ than at zero temperature. Koch et al [43] have shown that this can be simply understood in terms of a dimensional reduction of QCD at high temperature.

The results of ref. [43] could be summarized as follows. The quarks propagating space-like experience a space-like potential that rises linearly at large distances, as first calculated on the lattice by Manousakis and Polonyi [44]. In a “funny space” obtained by interchanging $z$ and $t$, the quark-antiquark wave function is calculated from the non-relativistic Schrödinger equation in which the effective quark mass is

$$m_{\text{eff}} = \pi T.$$  

(48)
As with the screening masses, were a dynamically generated mass still present above \( T_{\chi SR} \), the effective mass to be used for the quark would be \( \sqrt{\pi^2 T^2 + m_0^2} \) where as before \( m_0 \) is the dynamically generated mass. Similar calculations of the \( \pi \)- and \( \rho \)-meson wave function were subsequently made by Schramm and Chu \[45\] who extended them to a wider range of temperature. They found that the wave functions changed even less with temperature than in the calculation of Koch et al.\[13\].

There are two remarkable features in the theoretical results of Koch et al: (1) Dimensional reduction of QCD seems to work well right down to \( T_{\chi SR} \approx 140 \text{ MeV} \), a surprisingly low energy for something which is supposed to happen at “asymptotic” energy\[9\]. (2) the helicity-zero state of the \( \rho \) meson comes out degenerate with the pion while the helicity \( \pm \) states of the vector meson are degenerate with each other. This is easily understood with the quarks massless in the dimensional reduction. In the “funny space” in which \( z \) replaces \( T \), at least asymptotically in temperature, the configuration space with the new \( z \) becomes two-dimensional with only \( x \) and \( y \) directions. Spin must be either perpendicular to the \( (x,y) \) plane or lie in the plane. The \( \rho \) meson has gone massless and behaves like a (charged) photon with helicities \( \pm 1 \) perpendicular to the plane. The helicity-zero state which originated from the longitudinally (in plane) polarized component of the \( \rho \) before it went massless now behaves as a scalar and forms a multiplet with the charged pion. We propose that this situation corresponds to the Georgi vector limit \[17\] in which the longitudinal components of the massive \( \rho \) meson decouple and become degenerate with the pseudoscalar Goldstone bosons, the pions, thereby the \( \rho \) becoming massless. Georgi envisaged this limit as arising in some particular limit (such as large \( N_c \) limit) of QCD. It appears from the results described above that the Georgi vector limit is relevant in the vicinity of the chiral phase transition. Note that this is also in line with (though not equivalent to) Weinberg’s “mended symmetry” scenario exploited by Beane and van Kolck.\[10\] We shall have more to say on this matter in connection with quark number susceptibility in the next section.

\[\#9\]Some insight into why the effective dimensional reduction works so well has been given by detailed calculations of Suzhou Huang and Marcello Lissia \[46\]. As noted above, the lowest Matsubara frequency becomes the chiral mass in our dimensionally reduced space. Corrections to this dimensional reduction involve only the higher Matsubara frequencies, the next one being \( 2\pi T \) higher in energy. They are prefixed by the running coupling constant \( \alpha_s \). Because of the \( 2\pi T \), one might expect the scale \( \Lambda^2 \) to be used in obtaining this \( \alpha_s = \alpha_s(\Lambda^2) \) to be a factor of \( \ln(2\pi)^2 \) higher than that for the \( \alpha_s \) in the dimensionally reduced space. The detailed calculation of Huang show it to be a factor of \( \sim 80 \) larger (for \( N_c = N_f = 3 \)), so that \( \alpha_s \) is cut down by \( (\ln 80)^{-1} \approx \frac{1}{4} \).

\[\#10\]S. Beane in private communication pointed out that the Georgi vector limit and Weinberg’s mended symmetry are basically different, particularly near a phase transition.
5 Quark Number Susceptibility; Description of the Vector Interaction for $T \geq T_{\chi SR}$

The quark number susceptibility [47, 48] is defined as

$$\chi_{\pm} = (\partial/\partial \mu_u \pm \partial/\partial \mu_d)(\rho_u \pm \rho_d) \quad (49)$$

where the + and − signs define the singlet (isospin zero) and triplet (isospin one) susceptibilities, $\mu_u$ and $\mu_d$ are the chemical potentials of the up and down quarks and

$$\rho_i = \text{Tr} N_i \exp \left[-\beta(H - \sum_{i=u,d} \mu_i N_i)\right] / V \equiv \langle\langle N_i \rangle\rangle / V \quad (50)$$

with $N_i$ the quark number operator for flavor $i = u, d$. Generalizing McLerran’s expression [49] for the susceptibility and changing his baryon number density to quark density, we obtain

$$\chi_{\pm} = (VkT)^{-1} \int d^3x \langle\langle (N_u(x) \pm N_d(x))(N_u(0) \pm N_d(0))\rangle\rangle. \quad (51)$$

The $\chi_{\pm}$ are called the singlet and triplet susceptibilities. We shall see that the $\chi_+$ is in the $\omega$-meson channel and the $\chi_-$ in the $\rho$-meson channel. We can thus call $\chi_{\pm}$ the isosinglet and isovector susceptibilities. The susceptibilities measured on lattice are given in Fig.7. For SU(2) symmetry, we expect $\chi_+ = \chi_-$ and this is what one observes in the lattice results.

Although the singlet susceptibility has large errors, it is statistically consistent with the more accurate nonsinglet susceptibility. We shall discuss the latter here. It can be seen that the susceptibility is small at low temperatures, rising rapidly as $T$ moves through the critical temperature $T_{\chi SR}$ up to $\sim 70\%$ of the free-quark value, designated $\chi^{(0)}$ in Fig.7.

We can distinguish three separate regimes in temperature, which we discuss one by one. In the very low temperature regime – in which we are not particularly interested – the nonsinglet susceptibility is saturated by the $\rho$ meson [48]

$$\chi_{\text{NS}}\Big|_{T \ll T_{\chi SR}} \approx (kT)^{-1} \int d^3x G_{\rho\rho}(x) \quad (52)$$

where $G_{\rho\rho}$ is the $\rho$-meson propagator.

As the temperature $T$ moves upwards to the onset of the phase transition, the constituent quark model would be more appropriate [50]. In RPA approximation of this model as depicted in Fig.8, the susceptibility below the critical temperature is

$$\chi = \chi_0 / (1 + g_V \chi_0) \quad (53)$$

where $g_V$ is the coupling of the constituent quark to the vector meson and $\chi_0$ is the susceptibility for non-interacting quarks which at $T \approx T_{\chi SR}$ where the dynamical quark mass $m_Q$
Figure 7: Quark number susceptibilities as calculated by Gottlieb et al [47]. The singlet (isoscalar) susceptibility is given on the left-hand side and, the non-singlet (isovector) on the right-hand side. The free-quark susceptibilities $\chi_{S,NS}^{(0)}$, corrected for effects of the finite lattice size, are also shown. Horizontal arrows label the values of $\chi$ in the continuum (where $\chi = 0.125$ in lattice units for 4 time slices) and corrected for the finite size of the lattice (labeled $8^3 \times 4$). The stars on the figure show our values calculated from perturbative gluon exchange, eq.(67) and equations that follow. The star at $T_c$ has to be moved somewhat to the right, because the phase transition is a smooth one, of width $\sim 20$ MeV [8] and our calculation applies only when the transition is completed.
Figure 8: The quark number susceptibility below $T_{\chi_{SR}}$ is described in RPA approximation by summing quark-antiquark bubbles interacting by exchange of $\rho$ mesons.

has dropped to zero has the value

$$\chi_0 \approx N_f T^2$$

with $N_f$ the number of flavors. Now expressing the constant $g_V$ in terms of the vector gauge coupling $g$ that figures in the hidden gauge symmetry Lagrangian \[15\] #11

$$g_V \approx \frac{1}{4} \frac{g^2}{m_V^2}.$$ \hspace{1cm} (55)

Thus

$$\chi(T) \approx \frac{\chi_0(T)}{1 + \frac{1}{4} \frac{g^2}{m_V^2} \chi_0(T)}.$$ \hspace{1cm} (56)

Kunihiro \[50\] investigated in the NJL model what happens to the susceptibility as expressed in eq.(56) as temperature approached $T_{\chi_{SR}}$ from below and concluded that the vector field should decouple to explain the rapid enhancement of the susceptibility observed in the lattice results. This means that $g^2/m_V^2$ must steeply go to zero. The NJL model cannot explain this decoupling since the constant $g_V$ in that model is a relic of degrees of freedom integrated out in arriving at the NJL form of effective theory and hence there is no way to know how this constant runs as a function of temperature or density. On the other hand the hidden gauge symmetry theory tells us at least qualitatively how the constant might run. The hint comes from the recent result of Harada and Yamawaki \[51\].

Using the hidden gauge symmetric Lagrangian \[13\], Harada and Yamawaki showed with one-loop $\beta$ functions that the gauge coupling $g = 2(1 + \kappa)g_{\rho\pi\pi}$ (where $\kappa$ is a parameter #11We are not putting asterisks but the temperature dependence of the constants and masses is understood.)
which takes the value $\kappa = -1/2$ in free space) scales to zero as $g(\mu) \sim (\ln \mu)^{-1}$ with $\kappa \to 0$ when $\mu \to \infty$. As the gauge coupling goes to zero, the vector-meson mass which is given by

$$m_V^2 = (1 + \kappa)^{-1} f^2 \pi g^2$$

(57)
goesto zero. We should not take this perturbative argument too literally for a quantitative understanding since there could very well be some important non-perturbative effects in this regime that could modify the running of the coupling constant but our assumption is that it is very possible that the gauge coupling constant drops – as suggested by the lattice data – faster than logarithmically. When $g = 0$, then the vector mesons decouple, the longitudinal component of the vectors becoming scalar Goldstone bosons degenerate with the pseudoscalar Goldstone bosons and the vector meson becomes massless. This is the Georgi vector limit \cite{17}. \#12 In this limit, we will be left with

$$\chi \approx \chi_0.$$  \hspace{1cm} (58)

Before the vector decoupling leading to \eqref{58}, we can use the KSRF relation at $T$ near $T_{\chi SR}$ (which seems to be justified by the work of Harada, Kugo and Yamawaki \cite{18}) and $\chi_0 \approx 2T^2$ for $T \approx T_{\chi SR}$ to get the ratio just before the critical point

$$\chi(T_{\chi SR}^c)/\chi_0(T_{\chi SR}^c) \approx \frac{1}{1 + \frac{1}{2} \left( \frac{T_{\chi SR}}{f_{\pi}} \right)^2} \approx 0.47$$  \hspace{1cm} (59)

for $T_{\chi SR} \approx 140$ MeV and $\kappa = 0$. Here we are assuming that as suggested by lattice results, $f_{\pi}$ remains at its zero temperature value up to near $T_{\chi SR}$. (The constant $f_{\pi}$ is believed to fall very rapidly to zero within a small range of $\Delta T$ near the critical temperature.) The ratio \eqref{59} is in agreement with the lattice data at $T \lesssim T_{\chi SR}$.

\#12 The phase with $g \neq 0$ preceding the Georgi vector limit contains the scalar Goldstone bosons ($s$) that are the longitudinal components of the massive vector mesons. Since $\kappa = 0$ with $f_s = f_{\pi}$, the symmetry $SU(2) \times SU(2)$ is restored for the would-be scalar Goldstone bosons $s$ and the pions. An interesting observation to make here is that when $\kappa = 0$, the direct photon coupling to the charged pion which is proportional to $(1 - \frac{1}{2} (1 + \kappa)) e$ becomes $\frac{e}{2}$ whereas in the normal phase where $\kappa = -1/2$ the direct coupling vanishes. This means that as temperature and/or density is raised, the photon coupling deviates from the canonical vector dominance picture. It also means that the photon couples half-and-half directly with the pion and through the massive $\rho$ for $g \neq 0$, $\kappa = 0$ and with the massless quark-antiquark pair of the $\rho$ quantum number for $\kappa = g = 0$. It should however be pointed out that as discussed by Pisarski\cite{6}, it is possible to preserve the vector dominance at all temperatures. This option would then violate hidden local symmetry with the consequence that the $\rho$ mass will go up – instead of down as in BR scaling – as temperature increases. While we favor the hidden gauge symmetry prediction which is consistent with the notion that the vector meson mass is dynamically generated, nothing rules out the vector dominance option and it will be up to experiments to decide which is chosen by nature. This makes the planned dilepton measurements of the mass shift of the vectors at GSI, CEBAF and other laboratories particularly tantalizing.
Figure 9: Perturbative calculation of $\chi$. The quark and antiquark coupled to the quark density $\rho_q$ interact by exchanging gluons depicted by wavy lines.

Let us finally turn to the third regime, namely above $T_{\chi SR}$. It has been shown by Prakash and Zahed [52] that with increasing temperature, the susceptibility goes to its perturbative value which can be calculated with the perturbative gluon-exchange diagrams of Fig. 9. The argument is made with the dimensional reduction at asymptotic temperatures, but as mentioned above, it seems to apply even at a temperature slightly above $T_{\chi SR}$.

In the following, we schematize the Prakash-Zahed argument. Let us assume, in accordance with the Georgi vector limit, that the vector meson exchanges are decoupled in this regime. Above $T_c$, the gluon exchanges are just those of Koch et al [43] [13]:

$$V(r_t) = \frac{4\pi e^2}{4m^2}\sigma_{z,1}\sigma_{z,2}\delta(r_t)$$

with

$$e^2 \to \frac{4}{3}\bar{g}^2 T$$

for QCD (with $\bar{g}$ the color gauge coupling) and $\delta(r_t)$ is the $\delta$-function in the two-dimensional reduced space. Here $m = \pi T$ is the chiral mass of quark or antiquark as explained in [43]. Possible constant terms that can contribute to eq. (60) will be ignored as in [13].

In order to evaluate the expectation value of the $\delta(r_t)$, we note that the helicity-zero $\rho$-meson wave function in two dimensions is well approximated by

$$\psi_\rho \approx Ne^{-r_t/a}$$

with $a \approx \frac{2}{3}$ fm and the normalization

$$N^2 = \frac{2}{\pi a^2}.$$  

#13Note that $\sigma_{z,1}\sigma_{z,2} = -1$ for both the pion and the helicity-zero component of the $\rho$ which is degenerate with it. This degeneracy of the helicity-zero states should be checked directly by lattice gauge calculations.
For the helicity $\pm 1$ $\rho$-mesons, $\sigma_{z,1}\sigma_{z,2} = 1$, so we find that the expectation value of $V$ is

$$\langle V \rangle = \frac{8}{3}\frac{g^2T}{\pi^2T^2a^2}. \quad (64)$$

Going from the lowest order process $\chi_0$ to the one involving the first rung in the ladder, Fig.9, means introducing the correction factor

$$1 - \frac{\langle V \rangle}{2\pi T}, \quad (65)$$

the denominator being the unperturbed energy of the quark-antiquark pair. Summing the rungs, as shown in Fig.9, to all orders gives us

$$\frac{\chi}{\chi_0} = \left(1 + \frac{\langle V \rangle}{2\pi T}\right)^{-1}. \quad (66)$$

The lattice calculations [47] use $6/\bar{g}^2 = 5.32$ which would give $\alpha_s = 0.07$ at scale of $a^{-1}$ where $a$ is the lattice spacing. (The relevant scale may be more like $2\pi/a$.) Calculations use 4 time slices, so the renormalized $\bar{g}$ is that appropriate to $a^{-1/4}$. Very roughly we take this into account by multiplying the above $\alpha_s$ by $\ln 4^2$; therefore using $\alpha_s \approx 0.19$. With this $\alpha_s$ and the above wave function, we find

$$\frac{\chi(T_c)}{\chi_0(T_c)} \approx 0.68. \quad (67)$$

As can be seen by the $\star$ at $T_c$ on the right-hand graph of Fig.9, this is just about the ratio obtained.

In view of the crudeness in our determination of $\alpha_s$, this quantitative agreement with lattice results may not be taken seriously. However, it should be noted that the perturbative correction $\langle V \rangle/2\pi T$ goes approximately as $T^{-2}$, neglecting the (logarithmic) change in $\alpha_s$, and it is seen from Fig.9 that this $T$-dependence fits that of the calculated $\chi/\chi_0$ quite accurately for $T > T_{\chi SR}$.

Going towards the vector limit, chiral $SU(2) \times SU(2)$ symmetry is “mended,” so

$$f_\pi = f_s \quad (68)$$

where $f_\pi$ and $f_s$ are the constants defined by

$$\langle 0|A^i_{\mu}\pi^j(q)\rangle = i f_\pi q_{\mu}\delta^{ij}, \quad \langle 0|V^i_{\mu}\pi^j(q)\rangle = i f_s q_{\mu}\delta^{ij} \quad (69)$$

where $V^i_{\mu}$ ($A^i_{\mu}$) is the vector (axial-vector) current. The isovector scalars $S^i$ correspond to the longitudinal components of the $\rho$. (We are ignoring here the dilaton $\sigma$ discussed above.) Thus while for $g \neq 0$, $m_\pi \neq m_S$, the equality (68) still holds by the mended symmetry.
Now assuming that the matter above $T_{\chi SR}$ corresponds to the vector limit with $g = 0$, is the relation (68) expected to hold?

As discussed above, in the dimensionally reduced “funny space,” the $\pi$ and the helicity-zero $\rho$ are degenerate (with their dynamical mass equal to zero) and their wave functions become identical \[43\]. Therefore we do expect (68) to hold trivially. This is of course consistent with the picture of the $\pi$ and $\rho$ made up of two non-interacting massless quarks with the Matsubara frequency $\pi T$ per each quark. A short calculation of the matrix element (68) with the $\pi$ wave function in the “funny space” gives, for large $T$:

\[
\tilde{f}_\pi \sim c\sqrt{\bar{g}T}
\] (70)

where $c$ is a constant $<< 1$ and $\bar{g}$ is the color gauge coupling constant. Of course just as the screening mass has no direct physical meaning – though perhaps related to physical quantities through analytic continuation, one cannot give a physical meaning to (70) in the sense of the pion decay constant. This may account for the fact that $\tilde{f}_\pi$ grows with $T$ just as the screening mass does, while one expects the physical $f_\pi$ to go to zero as one would for the dynamically generated mass denoted above as $m_0$. We clearly need to know how to go from “funny space” quantities to physical space ones. The link is lacking at the moment. What we can say however is that the results of the lattice calculations are consistent with Georgi’s vector limit encoded in (68).

### 6 Effects in Heavy-Ion Collisions

Consequences of the vanishing of the hadronic coupling $g_V$ of the hidden local symmetry, leaving on the colored gluon exchange, are strong for the relativistic heavy-ion experiments. Firstly, it should be noted that freeze-out in the Brookhaven AGS experiments has been determined to be \[53\] [54] [55] \#15

\[
T_{f_\pi} \approx 120 \sim 140 \text{ MeV.}
\] (71)

By freeze-out we mean the effective decoupling, in the sense of energy exchange, of pions and nucleons. (Less strongly interacting particles, such as the kaons, freeze out at a

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\#14 We denote the constant by $\tilde{f}_\pi$ to distinguish it from the physical pion decay constant $f_\pi$.

\#15 The original determination of $T > \sim 150$ MeV from the ratio of isobars to nucleons by Brown, Stachel and Welke [54] was corrected about 10 MeV downward by taking effects such as the finite width of the isobar into account. It should be also mentioned that as recently shown in a dilated chiral quark model [55], the scaling with temperature will be more rapid in dense matter than in matter-free space. Consequently, the freeze-out temperature in the AGS experiments where the reaction takes place at a high density must be lower than the critical temperature determined by lattice calculations that pertain to zero-density matter.
higher temperature, say, $T > T_{\chi_{SR}}$.) Lattice gauge calculations give chiral restoration at a temperature 

$$T_{\chi_{SR}} \approx 140 \text{ MeV}. \quad (72)$$

This suggests that freeze-out for particles other than the pion and nucleon is at a temperature higher than $T_{\chi_{SR}}$ and that the pion and nucleon freeze out at about $T_{\chi_{SR}}$. This means that interactions in the interior of the fireball will be at temperatures greater than $T_{\chi_{SR}}$.

We have already seen that the hadronic vector interaction essentially decouples, leaving only perturbative gluon exchange. Hatsuda and Kunihiro \[57\] show that the pionic and scalar degrees of freedom move smoothly through the phase transition and this has been verified by the behavior of the relevant screening masses as $T$ passes through $T_{\chi_{SR}}$. This is not surprising since as shown by Wilczek \[58\], the linear $\sigma$ model is the Ginzburg-Landau effective Lagrangian for the chiral restoration phase transition. It is in this sense that Beane and van Kolck \[13\] recover the linear $\sigma$ model in the “mended symmetry” regime (and also that the BR scaling arguments make sense at tree level). The pion and $\sigma$ fields are just the fields of this class of effective theories.

As noted above, the behavior of the pion mass $m_\pi$ at zero chemical potential ($\mu = 0$) is complicated \[58\]. A common bare quark mass $m_u \approx m_d = m$ plays the role of an external magnetic field. Basically one falls back onto the Gell-Mann-Oakes-Renner relation

$$m_\pi^2 \approx -\frac{2m_q\langle \bar{q}q \rangle}{f_\pi^2} \quad (73)$$

extended to finite temperatures and densities. Since both $\langle \bar{q}q \rangle$ and $f_\pi^2$ go to zero (in the chiral limit) as $T \to T_{\chi_{SR}}$, the behavior of the pion mass is a subtle matter. Indeed for $T = 0$ and low densities, one can use the fact that empirically $m_\pi$ changes only very little (if any) with density, which we know from the very small (and repulsive) scalar potential from the nucleons in pionic atoms, so we may turn matters around and say that $f_\pi$ scales as

$$f_\pi^* \propto |\langle \bar{q}q \rangle^*|^{1/2} \quad (74)$$

which would keep the pion mass unscaled \[59\].

The AGS and CERN relativistic heavy ion collisions offer exciting new perspectives on this problem. They make it possible to construct an environment of high densities, several times nuclear matter density, and high temperature, $T > T_{\chi_{SR}}$. As remarked above, essentially all of times of the fireball existence, which we shall shortly show to be $\Delta t \sim (25 - 30) \text{ fm/c}$, the temperature is greater than that for chiral restoration.
6.1 The $\Sigma$-Term Attraction

Some years ago, Kaplan and Nelson [60] showed that explicit chiral symmetry breaking – that gives masses to the Goldstone bosons – could be “rotated away” by condensing the Goldstone bosons in dense nuclear medium. Although they discussed kaon condensation, which we shall consider later, it is easy to grasp their key idea by looking at the pion. In fact, the effect we wish to study is interesting for both the pion and the kaon. Consider the “chiral circle,” Fig.10. At finite density, the energy density from the explicit chiral symmetry breaking is given by

$$H_{\chi\Sigma B} = \Sigma_{\pi N} N \bar{N} \cos \theta + \frac{1}{2} f_{\pi}^2 m_{\pi}^2 \sin^2 \theta.$$  \hspace{1cm} (75)

The first term on the right-hand side represents the sum of effects of the explicit chiral symmetry breaking contribution to each nucleon mass $\Sigma_{\pi N}$ and the second term the sum of the pion mass, with the pion field being given by $\pi = f_{\pi} \sin \theta$. Note that the physical pion results from small fluctuation about $\theta = 0$ on the chiral circle. Expanding $H_{\chi\Sigma R}$ in $\theta$:

$$H_{\chi\Sigma R} = \text{const.} + \frac{1}{2} m_{\pi}^2 \left( 1 - \frac{\Sigma_{\pi N} \bar{N} N}{f_{\pi}^2 m_{\pi}^2} \right) f_{\pi}^2 \theta^2 + \cdots$$  \hspace{1cm} (76)

where the ellipsis stands for higher orders in $\theta$. We see that the pion has developed an effective mass\(^{#16}\)

$$m_{\pi}^{*2} = m_{\pi}^2 \left( 1 - \frac{\Sigma_{\pi N} \bar{N} N}{f_{\pi}^2 m_{\pi}^2} \right).$$  \hspace{1cm} (77)

\(^{#16}\)As mentioned above and to be explained below, this effective mass is not “effective” at zero temperature because of energy-dependent repulsive interactions that enter at the same order of chiral perturbation theory canceling the sigma-term attraction but becomes observable as $T \rightarrow T_{\chi\Sigma R}$. 

30
It is, however, known from pionic atoms that the scalar interaction felt by the pion is very slightly repulsive in nuclei. This repulsion is brought about by Pauli blocking in the virtual pair terms, of the type shown in Fig. 11. In pion-nucleon scattering, a repulsion develops because of Pauli blocking when the nucleon in the $NN$ bubble in the pion self-energy tries to go into the state already occupied by the original nucleon (see Fig. 11a). With use of the Wick theorem, this is described by Fig. 11b as a $Z$-diagram, with the backward-going line representing the antiparticle.

To fully take into account the effect of this type, one has to include also the anti-$\Delta$ for pion-nuclear scattering and the anti-decuplet for kaon-nuclear scattering. The virtual pair terms of Fig. 11 and their decuplet counterparts effectively cancel the attraction expressed in the dropping $m_{\pi}^*$, eq. (77).\#17

Up to nuclear matter densities, the $\Delta$-nucleon energy difference does not change appreciably. In the Appendix of ref. [66], this is explained in terms of the local field correction splitting the nucleon and $\Delta$ at finite density. This correction, however, depends upon the coupling of vector mesons, $\rho$ and $\omega$, and as the vector mesons decouple at temperature $T > T_{\chi SR}$, the local field correction should go to zero. Therefore taking the decuplet to be degenerate with the octet should be literally correct at high temperatures.

Our above discussion can be carried over to the case of kaons, where the effects are

\#17In the chiral Lagrangian approach of refs. [61, 62, 63, 64], this repulsive contribution is a part of the $\mathcal{O}(Q^2)$ terms proportional to the square of the meson frequency that appear as “counter terms.” They are of the same order in the chiral counting as the $\Sigma$ term of Kaplan and Nelson [60]. This contribution is important for both pion-nuclear and kaon-nuclear scattering but as shown in [63, 64], it plays an insignificant role for kaon condensation in particular. The point of the discussion given here and in the following is that as argued in [63, 64] this term can be saturated by what we call “pair terms” involving the octet and decuplet whereas in general or more specifically in chiral Lagrangian approaches such a term can arise from complicated (uncalculable) sources.
larger and the situation is a lot more interesting.

The $K^+$ meson is

$$|K^+⟩ = |u\bar{s}⟩$$

(78)

containing a nonstrange quark and a strange antiquark. The vector interaction between a $K^+$-meson and the nucleon is, therefore, repulsive and in dense matter

$$V_{K^+N} \approx \frac{1}{3} V_{NN} \approx 90 \text{ MeV} \frac{\rho}{\rho_0} \approx -V_{K^-N}$$

(79)

where $\rho_0$ is nuclear matter density and $V_{NN}$ is the vector mean-field potential felt by a nucleon. The Kaplan-Nelson scalar attraction, comparable to that giving the pion its effective mass, eq. (77), is

$$S_{K^+N} \approx -\frac{\Sigma_{KN} \langle \bar{N}N \rangle}{2m_K f^2} \approx -64 \text{ MeV} \frac{\rho_s}{\rho_0} = S_{K^-N}$$

(80)

where $\rho_s$ is the scalar density. See also Appendix. At zero density, virtual pair corrections, of the type discussed for pions, remove about 37% of this attraction [61].

We note that the chief dependence in the $S_{K^+N}$ of eq. (80) is expected to come, at least initially, from that of $f$. Up to nuclear matter density the decrease in $f^*$ is given by eq. (74) with

$$\langle \bar{q}q \rangle^* \approx 1 - \Sigma_{\pi N} \rho \frac{f^*}{f^2} \frac{m_\pi^2}{m_\pi^2}.$$ 

(81)

Now if we take $\Sigma_{\pi N} \approx 46 \text{ MeV}$, then

$$f^2 \approx 0.6 f^2$$

(82)

thereby increasing the scalar attraction at $\rho = \rho_0$ by a factor of $\sim 1.6$. Whereas the $S_{K^+N}$ of eq. (80) is decreased $\sim 37\%$ by virtual pair correction [61], this factor of $\sim 1.6$ increase makes up for them, so we believe that the Kaplan-Nelson term (80) to be our best estimate for nuclear matter density $\rho_0$. Extrapolation to higher densities is then made linearly from here.

It should be noted that the (repulsive) virtual pair correction factor goes as $F \approx (1 - .37 \omega_K^2/m_K^2)$. In the case of the $K^-$-meson, the $\omega_K^2$ decreases with density, the correction dropping out. Taking into account the decrease in $(f^*)^2$ of (82), we then find that $S_{K^-N}$ becomes roughly equal to $V_{K^-N}$.

#18We are using $\Sigma_{KN} \approx 2.83 m_\pi$ obtained at tree level in [53], but see Appendix.
6.2 Subthreshold Kaon Production

Recent experiments on subthreshold kaon production in nucleus-nucleus collision at ~ 1 GeV/nucleon in SIS at GSI [67] give a factor of ~ 3 more $K^+$-mesons than predicted by conventional theories [68] with the best nuclear matter EOS – one with a conventional compression modulus and momentum dependence, as we discuss in more detail below.

Implementing (80) as a correction to the kaon mass, with $\Sigma_{KN} = 2.5m_\pi$, slightly less than the $\Sigma_{KN} = 2.83m_\pi$ obtained in [64], Fang et al [69] manage to explain the enhanced kaon production. Similar results have been obtained by Maruyama et al [70]. In this second paper, it appears that the GSI subthreshold $K^+$ events are reproduced without decreasing the kaon mass with density. But the authors employ the same scalar mean field for the $\Lambda$-particle as for the nucleon. However the coupling to the $\Lambda$ should be only 2/3 of that to the nucleon, the scalar field not coupling to the strange quark. In terms of lowering the in-medium threshold$^{19}$ for kaon production, employing the same scalar field for the $\Lambda$ as for the nucleon is, at least roughly, equivalent to using 2/3 of it for the $\Lambda$ and 1/3 for the kaon. Now 1/3 of the scalar field, for nuclear matter density $\rho = \rho_0$, is $\gtrsim 100$ MeV in magnitude, larger than the Kaplan-Nelson attraction (80)$^{20}$. So the physics is roughly equivalent.

Fang et al [69] do not include the (repulsive) virtual pair correction, nor the scaling in $(f^*_{\pi}^2)$ of eq.(82). Introducing these for $\rho \sim \rho_0$, they roughly cancel. For $K^+$ mesons, $\omega_K$ does not change much with density. Initially[64], it increases slightly. Thus, the factor $F \approx (1 - 0.37\omega_K^2/m_K^2)$ always cuts the $S_{K^+N}$ down somewhat. On the other hand, the $(f^*_{\pi}^2)$ continues to decrease as $\rho$ exceeds $\rho_0$, so that $\omega_{K'}$ will come back down to $m_K$ at $\rho \sim (2 - 3)\rho_0$, the densities relevant for subthreshold $K^+$ production. Thus, the scalar attraction and vector repulsion roughly cancel at these densities. This seems to be what is required to reproduce the subthreshold $K^+$-mesons [68, 70].

We remind the reader that chiral Lagrangians do not admit a scalar exchange between the kaon and the nucleon: Scalar fields have no role in chiral expansion. However, in phenomenological fits to $\bar{K}$-nucleon scattering [72], which do not include the Kaplan-Nelson term, an attraction of about the same magnitude [61] as would come from this term is introduced by an effective scalar exchange with a coupling constant combination

$$g_{\sigma NN}g_{\sigma KK}/4\pi \approx 0.9.$$  \hspace{1cm} (83)

The $\bar{K}$-nucleon scattering is, however, not so sensitive to this attraction as is the subthreshold kaon production, where an attractive scalar interaction substantially increases

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$^{19}$Randrup and Ko [1] showed that the subthreshold kaon production was chiefly determined by the maximum kaon momentum $P_{max}$ which in turn is determined by the threshold, given the input energy.

$^{20}$With Brown-Rho scaling and our $\Sigma_{KN}$, the Kaplan-Nelson attraction will grow up to 80 MeV for $\rho = \rho_0$. 

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Figure 12: Production of $K^+$ and $\Lambda$ in $NN$ and $\Delta N$ collisions from pion exchange.

the production of kaons.

From the above discussion, we see that it should be possible to explain enough sub-threshold $K^+$ production by using attractive scalar mean fields for each $u$ or $d$ quark, also the $u$-quark in the $K^+$-meson. This is essentially what Müller-Groeling et al. did, although we see from eq. (83) that the $g_{\sigma NN}g_{\sigma KK}/4\pi$ they used is somewhat less than the $\frac{1}{3}(g_{\sigma NN}^2/4\pi)\approx 1.5-2$ usually employed in nucleon mean fields. Our discussion of Maruyama et al. [69] shows that at least in the determination of $P_{\text{max}}$ of the $K^+$, this is essentially what they did.

Note that the vector mean field should be coupled to each $u$ or $d$ quark, since it couples to the baryon number; thus each $u$ or $d$ quark experiences a vector mean field equal to $\frac{1}{3}V_{NN}(\rho)$, as noted earlier. To the extent that the Kaplan-Nelson term (80) is equivalent to the scalar mean field of $\frac{1}{3}S_{NN}(\rho)$ at quark level, we have the simple picture of the mean fields familiar from, say, Walecka theory, being applied at (constituent) quark level. The enhanced subthreshold $K^+$ production then follows from the scalar mean field essentially canceling the repulsive vector one. The vector mean fields do not affect the determination of $P_{\text{max}}$ of the $K^+$, since the baryon number they couple to is the same before and after interaction.

Miškovic et al. [77] make no comparison of their results with the theoretical results mentioned above, because of uncertainties in the nuclear matter EOS, and in the $\Delta N \rightarrow K\Lambda N$ and $\Delta\Delta \rightarrow K\Lambda N$ cross sections, which produce $\sim 85\%$ of the subthreshold kaons. Questions concerning the EOS have been sorted out in the past few years [73] and the conclusion is that a conventional compression modulus $K_0 = 210 \pm 30$ MeV [74], together with momentum dependence (as follows from a nucleon effective mass), should be used.

The elementary processes $\Delta N \rightarrow K\Lambda N$ and $\Delta\Delta \rightarrow K\Lambda N$ have not been measured
experimentally. However, the $NN \rightarrow K\Lambda N$ cross section is given mainly by the process of Fig. 12. The $\Delta$’s that are present act as a reservoir of energy and the pion in the process of Fig. 12b is more nearly on-shell than in Fig. 12a. The crucial ratio of $\Delta\pi N$ to $N\pi N$ coupling is known to be $\approx 2$ from analyses of many experiments. Thus scaling in this way and averaging over the various charge states of the $\Delta$, as done by Randrup and Ko [71], should be sufficiently accurate to show that there is a real discrepancy between the experimental results [67] and the theoretical ones [68, 69, 70].

The $\Delta N \rightarrow N K \Lambda$ and $\Delta \Delta \rightarrow N K \Lambda$ cross sections are being recalculated in Jülich [75].

The strong interaction calculations, such as those of Ko and collaborators mentioned above, have been criticized recently by Schaffner et al. [76] and Maruyama et al. [77]. Firstly, it is pointed out that in calculations such as those of Fang et al. [69], the scalar density $\rho_s = \langle \bar{N}N \rangle$ was “incorrectly” replaced by the vector density $\rho_V$. It is clear that the scalar density decreases relative to the vector density as $m_N^* \rightarrow 0$. On the other hand, the ratio $\langle \bar{N}N \rangle/f^2$ appears in (80), and if the vector mass is brought down in medium by the Kaplan-Nelson term, then so is $f$ since, as noted earlier, they are related by the KSFR relation $m^2_\rho = 2f^2g^2_V$ and $g_V$ does not change at tree level. Thus, the $(f^*)^{-2}$ would be expected to more than counterbalance the decrease in $\langle \bar{N}N \rangle$ as compared with $\rho_V$. This presumption is supported by ref. [70] which mistakenly applied the same scalar mean field to the $\Lambda$ as to the nucleon. As noted, for subthreshold $K^+$ production, this amounts to using $1/3$ of the scalar field as the nucleon for the $K^+$; furthermore, this is a somewhat stronger attraction than would be given by the Kaplan-Nelson term without scaling the $f^{-2}$.

Schaffner et al. [76] further suggest associating the square of the vector potential with an increase in the kaon effective mass. This quadratic vector interaction was discarded by Brown et al. [78] on the ground that it meant using, to quadratic order, a term which had been derived in the chiral expansion only to linear order. Furthermore these authors argued that effective masses should involve scalar fields and that the vector mean field should give only a shift in the chemical potential, as in Walecka mean field theory.

These problems have been simply resolved in the calculation of kaon condensation in dense stellar matter [61, 62, 63, 64] where heavy-fermion chiral perturbation theory (HFChPT) was used. In HFChPT, the expansion is made with the velocity-dependent positive-energy baryon field $B_v = e^{im_v\gamma^\mu v}B(1 + \gamma \cdot v)B$. In [64], all terms up to $O(Q^3)$ in the chiral counting are summed. The square of the vector potential does not enter at this order. As explained in a note added in [64], the difference $\delta = \rho_s - \rho_V$ appears at $O(Q^4)$ in the chiral counting and taking into account that difference (and also the term of the
form of the square of the vector mean field) in chiral perturbation theory as developed in [64] would require, for consistency, including $O(Q^4)$ counter terms and calculating leading two-loop diagrams. A partial account of such terms as is done in [66, 70] would violate chiral Ward identities. We claim that the result of [64] corroborates this point: The large effects claimed by Schaffner et al [68] and Maruyama et al [69] do not appear, in that the scalar field here plays a relatively minor role. Its magnitude is varied more than a factor of two between calculations in [61] and [64], but the results are not so different. It thus appears that the schematization of [68] and [69] to relativistic mean field theory is too crude an approximation. We also point out that the argument of Schaffner et al [68] for repulsion arising from going off-shell in the interaction is incorrect. Such repulsions do not exist. As demonstrated in Appendix F of [74], the physics does not depend on the way the kaon field interpolates.

Schaffner et al [76] point out the inconsistency of using the chiral vector and scalar interactions for the $K^-$ mean field in the Walecka-type mean-field formalism for the nucleons (or nucleons and hyperons). The chiral Lagrangian gives $V_{K^+N} \approx \frac{1}{6} V_{NN}$ rather than the $\frac{1}{3} V_{NN}$ we discussed in eq. (73). In Sakurai’s work and in the schemes treating the vector mesons as gauge particles in an effective theory, which we discussed in Section 5, the $\omega$-meson should couple to the baryon number lodged in nonstrange quarks. The $\rho$-meson couples to the isospin of the nucleon and this, plus the coupling to the pion, is responsible for the Weinberg-Tomozawa term in pion-nucleon scattering. The universal vector coupling can be obtained from the KSRF relation (23), and from it, the $\rho NN$-coupling:

$$\frac{(g/2)^2}{4\pi} = \frac{g_{\rho NN}^2}{4\pi} = 0.70(5),$$

(84)

the $(g/2)$ factor entering here because the isospin of the nucleon is 1/2. Via $SU(3)$ symmetry, which is incorporated in the chiral Lagrangian, we obtain for the $\omega NN$ coupling

$$\frac{g_{\omega NN}^2}{4\pi} = 9 \frac{g_{\rho NN}^2}{4\pi} \approx 6.34.$$ 

(85)

This is substantially smaller than the $\omega NN$ coupling constant employed in Walecka-type mean-field calculations:

$$\frac{g_{\omega NN}^2}{4\pi} \approx 10 \sim 12.$$ 

(86)

(The chiral $\omega NN$ coupling of (85) would give a vector mean field of only 178 MeV at nuclear matter density.) For the chiral calculation, we are justified in using the density dependence (82) for $f_\pi$ derived from chiral considerations. Inserting (82) with the KSRF relation, we find

$$\frac{g_{\omega NN}^2(\rho = \rho_0)}{4\pi} \approx 10.6.$$ 

(87)
In other words, the connection between the chiral mean fields and those used in Walecka mean-field theory, which generally works at $\rho \approx \rho_0$, can be made if and only if BR scaling is taken into account. Neither theory employs form factors, so that comparison of coupling constants is straightforward.

Calculations to date of kaon condensation in the Walecka mean field formalism have, indeed, been inconsistent in that they use empirical mean fields in baryonic interactions, but scalar and vector mean fields for the kaon from chiral Lagrangians without scaling $f^*_\pi$.

Our argument leading to (82) is difficult to employ for densities exceeding $\rho_0$. Even if the pion mass remains essentially unchanged, so that we can employ (74), the linear extrapolation (81) cannot be extended to higher densities. On the other hand the relativistic VUU calculations which employ Walecka mean fields work well up to several times nuclear matter density.

We can turn matters around, and use the Walecka mean fields determined by the VUU calculations. But then the $V_{K+N}$ should be obtained as in (79), by the scaling indicated by quark counting, from $V_{NN}$.

What to do with the Kaplan-Nelson scalar term is more intricate, since its origin is in the chiral symmetry breaking part of the chiral Lagrangian. The factor $F \approx (1 - 0.37 \omega^2_K/m^2_K)$ which arises from virtual pair corrections at the same order in chiral counting as the Kaplan-Nelson term, must be included in an ad hoc fashion in the relativistic mean field formalism. Aside from this factor, scaling (80) by the factor $(0.6)^{-1}$ of (82) gives $S_{KN} \approx -107$ MeV and our determination of $S_{KN}$ from the lattice calculation of $\langle N|\bar{s}s|N \rangle$, as described in the Appendix, would give $S_{KN} \approx -120$ MeV. Assuming quark scaling, this would give $S_{NN} \approx -360$ MeV for the nucleon mean field, not far from that in Walecka theory.

Although the scalar attraction on the kaon does not arise from a Walecka-type mean field mechanism, since the kaon is a Goldstone boson, use of the Walecka theory would give a rather similar effective scalar mean field.

The factor $F \approx (1 - 0.37 \omega^2_K/m^2_K)$ is, however, important. For the $K^+$-meson for which the $\omega_K$ increases above $m_K$ for low densities at least, this factor cuts down the scalar attraction and makes the overall mean field repulsive, as is observed in $K^+$-nucleus scattering. In the case of $K^-$-mesons, this factor moves towards unity as $\omega_K$ decreases with increasing density. Thus, it is not very important in determining the critical density for kaon condensation and the equation of state in the kaon-condensed phase.

The importance of the above development is that it enables us to directly use information obtained from the Bevalac and SIS heavy-ion experiments, as described by the relativistic VUU transport in the calculation of kaon condensation. Thus we have empirical determinations up to $\rho \sim 3\rho_0$. As noted, from the subthreshold $K^+$ production experiments, we can already say that the vector and scalar mean fields, the latter corrected by the factor
\[(1 - \omega_{K^+}^2/m_{K^+}^2), \text{ are roughly equal at } \rho \sim (2\frac{1}{2} - 3)\rho_0.\]

Within the Walecka-type mean field description, we can now understand the large attraction, \(-200 \pm 20\) MeV at \(\rho = 0.97\rho_0\), for \(K^-\)-mesons, found by Friedman, Gal and Batty\[90\]. This is just the sum of scalar and vector mean fields for the \(K^-\). The number should not be surprising, since the same contribution of mean fields comes in the spin-orbit interaction for the nucleon in Walecka theory, and the above interaction for the \(K^-\) would imply that \(S_{NN} + V_{NN} \approx 600\) MeV which is big, though somewhat smaller than needed for agreement with experiments. (Of course we should not forget that our virtual pair correction cuts down the \(K^-\) interaction somewhat, \(\sim 10\%\) at \(\rho \sim \rho_0\).)

6.3 "Cool" Kaons

The 14.6 GeV \(^{28}\text{Si} + \text{Pb} \rightarrow K^+(K^-) + X\) preliminary data \[79\] show cool components, with effective temperature of 12 MeV for \(K^+\) and 10 MeV for \(K^-\), which cannot be reproduced in the conventional scenarios employed in event generators. The latter give kaons of effective temperature \(\sim 150\) MeV. It is clear that some cooling mechanism is necessary to produce the cool kaon component. It is also clear that the above vector repulsion must essentially be absent for the temperature relevant for these experiments, as we discuss.

In an interesting article, V. Koch \[80\] has shown that given the attraction of (80) – although his results are not sensitive to the precise amount of attraction as long as it is as large as (80) – a cool kaon component can be reproduced. See Fig.14. Aside from the attractive interaction, it is necessary that the fireball expand slowly. The slow expansion results because the pressure in the region for some distance above \(T_{\chi SR}\) is very low \[9\], the energy in the system going into decondensing gluons rather than giving pressure. This results in an expansion velocity of \(v/c \sim 0.1\). In the case of 15 GeV/N \(\text{Si}\) on Pb transitions, the fireball has been measured \[81\] through Hanbury-Brown-Twiss correlations of the pions to increase from a transverse size of \(R_T(Si) = 2.5\) fm to \(R_T = 6.7\) fm, nearly a factor of 3, before pions freeze out. With an expansion velocity of \(v/c \sim 0.1\), this means an expansion time of \(\sim 25 - 30\) fm/c. (The full expansion time cannot be measured from the pions which occur as a short flash at the end.) Thus the fireball expansion lasts a very long time.

As long as the expansion is slow, the adiabatic invariant
\[ I = \int p \cdot dr \] (88)
remains a useful concept for the kaons. If the radius of transverse expansion \(r\) increases by a factor of 3, then the momentum will decrease by the same factor, and the energy by a factor of \(\sim 9\). Thus, as long as the expansion is slow, the kaons will be greatly cooled, say, by the factor of \(\sim 9\).

Before proceeding further, we pause here to elaborate on the slow-expansion scenario for which the low pressure is essential. In Fig. 13 we give the pressure and energy densities,
Figure 13: Entropy, energy and pressure densities deduced by Koch and Brown \cite{9} from the lattice gauge calculations of Kogut \textit{et al} \cite{8}.
along with the entropy, deduced by Koch and Brown \[9\] from Kogut et al’s lattice simulations \[8\]. Strictly speaking, the energy and pressure cannot be split up into a part from the quarks and a part from the gluons, for such a division is not gauge-invariant. So we should consider only the sum.

What we see from Fig. 13 is that the pressure is only slightly above zero in the region where the entropy increases rapidly, i.e., in the region of the phase transition at $T_c \sim 140$ MeV. As noted in \[9\], this small pressure results because the energy chiefly goes into decondensing the quarks and gluons, thereby producing almost no pressure. This can be expressed in terms of an effective bag constant $B$ for the transition. In order to see how this goes, consider the simplified case of a transition from pions to a quark/gluon plasma \[38\]. For simplicity, we neglect the pion pressure and energy, which can easily be corrected for at the end. The energy-density of a quark/gluon plasma is

$$
\begin{align*}
\epsilon_{QG} & = \frac{37}{30} \pi^2 T^4 + B_{eff}, \\
P_{QG} & = \frac{37}{90} \pi^2 T^4 - B_{eff}.
\end{align*}
$$

(89)

Now the phase transition occurs as soon as the $P_{QG}$ can be brought positive. From lattice calculations, we know that $T_c \approx 140$ MeV. From $P_{QG} = 0$ at $T_c$ we find

$$
B_{eff}^{1/4} = 199 \text{MeV},
$$

(90)

midway between $B_{\chi SR}$ of eq. (4) and $B_{glue}$ of eq. (5). Koch and Brown show that this “half-way house” results because only $\sim \frac{1}{2}$ of the gluon condensate is “melted” across the transition region. Note that if we were to calculate $T_c$ from first principles, we would need to know this latter fact. In other words, the determination of $B_{eff}$, that fraction of $B$ which is melted, must first be found. By using the lattice results, we have turned the problem around, obtaining $B_{eff}$. Equation (89) makes it clear why the pressure is so small in the region of $T_c$.

The smooth transition that we are finding implies that instead of having the “melting” occurring at a fixed $T = T_c$, it occurs over a region, determined as $\Delta T \sim 20$ MeV by Koch and Brown \[3\], although this number may change as nonperturbative effects are included in the $\beta$ function.

Estimates of the energy densities reached in the Brookhaven AGS collisions are quite model-dependent, but generally indicate that the highest temperatures are not more than $\sim 170$ MeV, and probably are less than this. Consequently, the systems do not reach temperatures much above $T_c$.

We should remark that while the relationship between hadron masses and the quark condensate, investigated in \[1\] does not involve knowing the temperature, the width of the
phase transition ($\sim 20$ MeV) does depend upon the asymptotic scaling relation being effectively valid. One can approximately check the validity of this relation as follows: Asymptotic scaling can be expressed as

$$\frac{8\pi^2}{\bar{g}^2} = (11 - \frac{2}{3}N_F) \ln \frac{k}{\Lambda_{QCD}}$$

(91)

giving the relation between the color gauge coupling $\bar{g}$ and the momentum $k$. For a lattice of $6 \times 12^3$, Kogut et al. found the transition at $\beta_c = 6/g_c^2 \approx 5.34$, whereas for $8 \times 16^3$, Gottlieb et al. find $\beta_c = 5.54$. Assuming the scale to depend chiefly on the number of time slices, one can get rid of $\Lambda_{QCD}$ using the two simulations, in order to express

$$\delta\beta_c = \frac{6}{8\pi^2} (11 - \frac{2}{3}n_F) \ln(\frac{8}{6}) \approx .21.$$ 

(92)

This “theoretical” $\delta\beta_c$, which assumes asymptotic scaling, is to be compared with the $\delta\beta_c \approx 0.2$ found in the lattice simulations. The good agreement indicates that the asymptotic scaling relation should be adequate for determining temperature differences.

Now in expansion of the system following maximum temperature, the expansion will “stick” for a long time (estimated to be $25 – 30$ fm/c) just in the region of $T_c$, where the pressure is nearly zero. Entropy density is decreased not by expansion, but by the hadrons, which are formed in this region, going back on-shell and the number of heavy hadrons then decreasing because of small Boltzmann factor $e^{-m_H/T}$. This picture seems to be supported by the work of Shuryak and Xiong who find evidence in the excess photons and dileptons in the SPS collisions (200 GeV/nucleon collision at CERN) for a long “mixed” phase of $\tau \sim 30 – 40$ fm/c.

Now returning to the problem of the cool kaons, the important point to note is that the kaons move through chirally restored matter, in which the hadronic (as opposed to gluonic) vector interactions are small or zero during the $25 – 30$ fm/c period. Kaons which do collide strongly inelastically will be removed from the cool kaon component and join the thermal kaons.

Given the cool kaon results, the collaboration E814 – now E877 – carried out a study of this component in Au+Au collisions at 10.8 A GeV/c at the AGS. Both $K^\pm$ spectra exhibited a cool, nonthermal component but with apparent temperatures in the ranges of $\sim 50 – 100$ MeV (depending on the rapidity), substantially higher than $\sim 10 – 12$ MeV found for Si+Au. This higher effective temperature was predicted by Koch. The Au system is larger, and the kaons have more chance to hit something and be scattered out of the cool component. What was unexpected was that, with decreasing $P_t$, the spectra began to drop in the case of the $K^+$ spectrum but not for the $K^-$ one. (Following the presentation of the work at the Quark Matter ’95 meeting, C.Y. Wong suggested that the drop in $K^+$ spectrum with $P_t$ decreasing below 140 MeV was due to the Coulomb interaction which
Figure 14: Kaon spectrum from a full transport calculation for 14.6A GeV/c Si + Pb collisions [79].
The full line is the result including mean fields for baryons and kaons, the dotted line the result for kaon mean field and no mean field for the baryons and the dashed line the pure cascade result. The calculated results are in arbitrary units. The nucleon vector and scalar couplings of the mean field were taken to be $g_V = 5.5$ and $g_S = 9.27$, the vector coupling being about half of the Walecka mean-field model, so that the kaons – which felt 1/3 of the nucleon mean fields – experience an attractive potential of $U_0 \simeq 50$ MeV. The field energy $\frac{1}{2}m_S\phi_S^2$, where $\phi_S$ is the scalar field, plays the role of the bag constant (4). Its value, taken to be about half of the $B$ of (4), was adjusted so that the pressure was low in the region of the phase transition, in accordance with the lattice results of Fig.13.
would be substantial in the large Au system. Work by Koch, in a schematic model, showed this to be a plausible explanation. Further work with full transport by Koch is in progress.)

It can be seen that the discovery of the cool kaon component tells us that the main part of the system is chirally restored (in the form of the Georgi vector limit), for a long time of \( \sim 25 - 30 \) fm/c, until the kaons freeze out. They freeze out well ahead of the pions, which equilibrate with the nucleons down to a density, of \( \rho_{fo} \approx 0.38\rho_0 \), because of their weak interaction.

Since masses of particles go back on-shell in free space before the particles reach the detector, the bulk of experiments is insensitive to what the masses were inside the hot and dense medium. Only carefully designed experiments will tell us that, and the cool kaon component is the most important of these to date.

7 Discussion of Chiral Restoration with Temperature

We have seen in the last section that the behavior of the quark number susceptibility with temperature, as calculated in lattice gauge calculations, shows that the hadronic vector coupling disappears as \( T \) moves upwards through \( T_{\chi_{SR}} \), and that the perturbative color gluon exchange describes the susceptibility well above \( T_{\chi_{SR}} \), as argued by Prakash and Zahed [52]. Somewhat surprising is the fact that the perturbative description, which gives a \( 1/T^2 \) behavior in the difference \( \chi(T) - \chi(0) \) between the susceptibility and that for free quarks, sets in just above \( T_{\chi_{SR}} \); i.e., for this purpose, asymptotia is \( T \geq T_{\chi_{SR}} \). Note that the screening mass of the \( \rho \)-meson goes to \( 2\pi T \), its asymptotic value, as soon as \( T \) reaches \( T_{\chi_{SR}} \).

As noted preceding eq.(57), the hidden local gauge coupling \( g \), at one loop order, scales to zero as \( g(\mu) \sim (\ln \mu)^{-1} \), i.e., the Georgi vector limit. As shown by Harada and Yamawaki [51], there is an ultraviolet fixed point \( \kappa = 0 \). Of course, the one-loop calculations are most certainly unreliable for learning what happens near the phase transition, where the higher loop effects and nonperturbative effects presumably pile up. Even so, the qualitative picture suggested by this one-loop calculation appears to be correct.

It may be that the increase in \( f_\pi^* = f_s^* \) as \( T \) goes through \( T_{\chi_{SR}} \) found in the last section is an artifact of the spacelike propagated wave function. With timelike propagation, which has not been up to now carried out on the lattice, \( f_\pi^* \) and \( f_s^* \) may go to zero as \( T \to T_{\chi_{SR}} \), as suggested by the BR scaling [1]. What one does learn from the spacelike propagated wave function [43] is that the quarks in the \( \rho \)-meson behave, to lowest order, as free quarks, each with thermal energy of \( \pi T \), with corrections of \( \sim \alpha_s \) which give the Bethe-Salpeter amplitude for the \( \rho \). It is, in fact, this basically free quark kinematics which leads to the equality \( f_\pi^* = f_s^* \) for the spacelike wave functions. Wave functions propagated in the time direction will, presumably, have this same basically free quark kinematics. Indeed,
in four-flavor lattice calculations of the $\rho$-meson \cite{11}, the temporal correlators above $T_{\chi SR}$ were found to be consistent with a pair of quarks that are more or less free. Modulo the caveat with four-flavor lattice calculations mentioned before, given free quark kinematics, one would expect $f_\pi^* = f_s^*$, even if $f_\pi^* \to 0$ with chiral restoration. In the possible case of $f_\pi^* \to 0$ as $T \to T_{\chi SR}$, the Georgi limit could be realized either just before or simultaneously with the $f_s^*$ going to zero. Just what happens in the phase transition may be complicated to unravel. Nonetheless, the lattice calculations of the susceptibility do show that, to the accuracy we can analyze them, $g \to 0$ as $T \to T_{\chi SR}$, indicative of the Georgi vector limit.

While other viable explanations may be found in the future, as far we know, the soft kaons found in the E-814 experiment can be explained only if the vector-meson coupling is essentially absent at the relevant temperature, and this is a strong support for the Georgi picture in which the hidden gauge coupling constant “melts” at the temperature $T_{\chi SR}$.

Since Sakurai \cite{82} we have known that vector dominance and universal vector coupling worked well in describing vector interactions. It is in endowing the vector mesons an induced gauge structure \cite{18} that a contact (albeit an indirect one) with QCD is made. As pointed out by Georgi \cite{17}, the hidden local symmetry is useful because it keeps track of powers of $m_\rho/\Lambda_\chi$, where $\Lambda_\chi$ is the scale of the effective chiral theory, in the situation where the vector-meson mass $m_\rho$ is in some sense small. Indeed the successful KSRF relation follows simply \cite{17} if the vector mesons as well as the pseudoscalars are both light, so that we can stop with a Lagrangian with the lowest (that is, two) power of the derivative dictated by chiral invariance. This suggests rather strongly that the notion that the vector-meson mass becomes small as $T \to T_{\chi SR}$ is natural within the framework of effective chiral field theory. Remarkably, this seems to be supported by lattice gauge calculations at $T \sim T_{\chi SR}$. Now the gauge symmetry in the hidden symmetry scheme is an “induced” gauge symmetry lodged in hadronic variables. In this sector, the fundamental color gauge symmetry is not visible. It is the induced flavor one that is seen. What we observe is then that as $T$ goes towards $T_{\chi SR}$, the induced gauge symmetry gives way to the fundamental gauge symmetry. What is surprising is that this changeover seems to take place suddenly, with increasing temperature, the effective hadronic gauge symmetry applying for $T < T_{\chi SR}$, and the fundamental color gauge symmetry being realized perturbatively for $T > T_{\chi SR}$.

We have suggested that the Georgi vector symmetry with $\kappa = 0$ and $g \neq 0$ is also relevant for nuclear physics. It is the existence of such a symmetry that allows one to linearize the non-linear chiral Lagrangian of current algebra to the linear sigma model which justifies in some sense the separation of the scalar $\chi$ field, the low-frequency part of which giving rise to the previously obtained medium-scaling of effective chiral Lagrangians of ref.\cite{1}. Going beyond the tree approximation with this scaled chiral Lagrangian in hot and/or dense matter is an open problem which has attracted little attention up to date.
While consistent generally with both observations and theoretical prejudices, our discussion cannot be considered solid until higher-order calculations can be systematically carried out and compared with experiments. A small progress made in this direction is discussed in ref. [20].

Finally we should mention that an investigation of effective hadronic interactions mediated by “instanton molecules” carried out by Schäfer et al. [34] provides support to our thesis that chiral phase transition involves Georgi’s vector limit. In this work, it is found that all coupling constants in a NJL-type effective Lagrangian can be specified for \( T \gtrsim T_{\chi SR} \) in terms of a single parameter \( G \), signaling a swelling symmetry. At temperature \( T \gtrsim T_{\chi SR} \), the instantons can be taken to be completely polarized. In this case, the interaction in the longitudinal vector-meson channel becomes equally strong as the attraction in the scalar-pseudoscalar channel. Transversely polarized vector mesons are found to have no interaction. In detail how this comes about is as follows: The instanton molecules survive the chiral symmetry restoring transition; they leave chiral symmetry unbroken. According to Koch and Brown [9], about half of the gluon condensate remains for temperature \( T \gtrsim T_{\chi SR} \). This is assumed to reside in the molecules [34]. Quarks coupling through the instanton molecules experience, for randomly oriented molecules, an interaction

\[
\delta H = \frac{2G}{N_c^2}\{(\bar{\psi}\gamma_5\vec{\tau}\psi(x))^2 + \frac{1}{4}(\bar{\psi}\gamma_\mu\vec{\tau}\psi(x))^2 + \cdots\}
\] (93)

where we show only the terms that act in the pionic and \( \rho \)-meson sector. The \( \frac{1}{4} \) in front of the vector interaction comes from averaging the molecules over random directions. In this case, it is seen that the interaction in the \( \rho \)-meson channel is only \( \frac{1}{4} \) as large as pionic channel. If the instanton is completely polarized in the time direction – and the authors of [34] give reasons why this lowers the energy of the system – the interaction in the \( \rho \)-meson channel changes to

\[
\frac{1}{4}(\bar{\psi}\gamma_\mu\vec{\tau}\psi(x))^2 \rightarrow (\bar{\psi}\gamma_0\vec{\tau}\psi(x))^2
\] (94)

since directions are no longer averaged over. The \( \frac{1}{4} \) factor no longer appears, because there is no averaging over directions. This sets interaction in the pionic channel and that for the time-like \( \rho \) meson equal, decoupling the transverse \( \rho \) mesons. The time-like vector mesons are longitudinal in the sense that

\[
q^\mu \rho_\mu = 0
\] (95)

where \( q_\mu \) is the four-momentum of the \( \rho \) meson. We thus have a “realistic” dynamical model, giving the effective interaction in hadronic variables, which reproduces the Georgi vector limit under the assumption of a dilute gas of fully polarized instanton molecules and,
of course, the $\rho$ meson going massless.\(^{22}\) (In this model, the $\omega$ and the $\rho$ can be put into flavor $U(2)$ as well.) Note that the above considerations apply to the region of $T$ just above $T_c$, where the colored gluon exchange is present.

8 Chiral Restoration in Dense Matter in Stellar Collapse: Transition to Kaon Condensation and then to Quark Matter

In Section 6 we discussed effects of the attractive kaon-nucleon interaction, namely, the Kaplan-Nelson term, on subthreshold production of $K^\pm$ mesons in heavy-ion collisions of Au on Au, with 1 GeV/nucleon energy. These involve chiefly finite density effects, because the temperature is low, $\sim 75$ MeV, so that $T$ is well below $T_c$. We do not expect much effect of dropping masses, etc., because of the temperature.

Our consideration of chiral restoration with high temperature could be checked by lattice gauge simulations; more correctly, many of our ideas about the restoration stemmed from lattice gauge results. Unfortunately, lattice calculations cannot be carried out for finite density – at least not to date – because in changing to imaginary time (Euclidean space-time) the baryon chemical potential becomes imaginary and the probability is no longer positive, so that Monte Carlo methods no longer work, at least not effectively in their present formulation.

Effects from finite density in strong interactions have been reviewed extensively by Adami and Brown \(^3\) and so we will not repeat this review here. A very interesting situation is however provided by the collapse of stars, in which there is sufficient time for strangeness conservation to be violated. We shall review recent developments on this here.

For concreteness, let us sketch the scenario for Supernova 1987A. According to the scenario we shall pursue here, the core of the 18 $M_\odot$ progenitor collapsed, initially forming a neutron star, and blew off the outer material, in a time of $\sim 4$ seconds following collapse \(^{86}\). During this time the neutron star was stabilized by the trapped neutrinos. We know from the Kamiokande neutrino detector that the neutrinos came off for $\sim 12$ seconds; by this time they had dropped in energy below the $\sim 8$ MeV threshold for detectability. According to Burrows and Lattimer \(^{87}\) only about half of the thermal energy is carried away by the neutrinos during this $\sim 12$ seconds. About half of the thermal energy remains and creates substantial pressure, which helps to stabilize the neutron star. The central density of the neutron star grows to several times nuclear matter density.

\(^{22}\)Schäfer et al \(^{83}\) find that the molecules are not completely polarized (the polarization being $\sim 70\%$), so this model may be taken only as a caricature theory which, in a limit not far away from the physical world, reproduces the results of the Georgi vector limit.
Once the electron neutrinos have left, following the explosion, the electrons in the core turn into $K^-$-mesons and neutrinos, as we now explain. The neutrinos leave the core, again taking several seconds to do so. This cannot happen while the first set of neutrinos are trapped, because this would require

$$\omega_K = \mu_K = \mu_e - \mu_{\nu},$$

(96)

where the $\mu$'s are the chemical potentials, and as we shall see, it would be difficult to bring the kaon energy $\omega_K$ down this far. But when the $\mu_{\nu} \to 0$, it then becomes possible to bring $\omega_K$ down to the electron chemical potential $\mu_e$.

Prakash, Ainsworth and Lattimer \[88\] give simple parametrizations of the neutron rich equation of state for the situation after the neutrinos have left. The electron chemical potential is then established by the thermal equilibrium condition

$$e^- + p \leftrightarrow \nu + n,$$

(97)

with the neutrinos more or less freely leaving the star, so

$$\mu_e = \mu_n - \mu_p.$$  

(98)

The chemical potentials of the neutron and proton must be calculated, and $\mu_e$ determined from these and the condition of charge neutrality. Although $\mu_e$ depends somewhat on the compression modulus, etc., a good standard value for $\mu_e$ is

$$\mu_e \approx 117\text{MeV} \quad \text{for} \quad \rho = \rho_0.$$  

(99)

The $\mu_e$ increases slightly faster than as $\rho^{1/3}$ with density, reaching $\gtrsim 200$ MeV at about $3\rho_0$ for equations of state with a nuclear matter compression modulus of $K_0 = 200$ MeV. These numbers schematize the results of Thorsson, Prakash and Lattimer \[89\] and of \[64\].

Now the energy of a $K^-$-meson has been found to drop in nuclear matter. According to Friedman \textit{et al} \[90\], the real part of the $K^-$-nucleus potential is $-200 \pm 20$ MeV in the center of the $^{56}\text{Ni}$ nucleus at $\rho = .97\rho_0$. The $K^-$ feels double the attraction from a proton as it does from a neutron through the vector interaction \[79\] and the same attraction for neutron and proton through the scalar interaction \[80\]. Thus, for neutron-rich matter, one can say that the $K^-$ energy is decreased $\sim 150$ MeV at nuclear matter density $\rho_0$. What can happen is schematically described in Fig.15.

At the point where the $\omega_K$ line crosses the $\mu_e$ line, it becomes energetically possible for the “electrons” to turn into $K^-$’s accompanied by neutrinos

$$e^- \to K^- + \nu_e$$

(100)

with the neutrinos leaving the star. There will therefore be a second burst of electron neutrinos which should be detectable. Details (energy, time of emission, etc.) are being
Figure 15: Behavior of the $K^-$ energy and of the electron chemical potential $\mu_e$ as function of energy.

worked out by A. Burrows, J.M. Lattimer, M. Prakash and collaborators in Stony Brook at the present time.

We note that the large binding energies of Friedman, Gal and Batty [90] fit in very well with the scenario of Brown-Rho scaling, with $f \rightarrow f^*$ in (79) and (80) and as shown in Figs. 11-13 of the detailed calculations in [64]. One seems to need as much attraction as the theory can provide.

At densities above the kaon condensation threshold, the $K^-$ mesons of zero momentum form a Bose condensate, as discussed in [89, 64]. Sufficient binding energy is gained so that the equation of state is substantially softened, and the maximum neutron star mass is $\sim 1.5M_\odot$ as discussed by Brown and Bethe [91]. We will return to this matter at the end of this section.

It has been recently shown by Brown and Weingartner [92] that in the case of Supernova 87A if a neutron star were present, then we would see it with a luminosity $L$ of about $10^4$ times $L_\odot$, whereas the bolometric luminosity actually is $L \approx 10^2 L_\odot$ and can be explained as arising from radioactivity in the expanding shells. Thus their conclusion is that as in [91] the core has gone into a black hole. Furthermore Bethe and Brown [93] show that from the fact that 0.05 $M_\odot$ of $Fe$ was produced, together with the pre-supernova evolution, it is possible to determine the mass of the compact object to be in the range $(1.44 - 1.56) M_\odot$, confirming the suggestion by Brown and Bethe [91].

Recently Keil and Janka [94] have shown, based on the Glendenning mean-field equations [95], that hyperons enter into the equation of state at densities $\rho \sim 2\rho_0$. They find a delayed explosion scenario similar to that of Brown and Bethe [91] for cores in the range

$$1.58M_\odot < M_{\text{grav}} < 1.72M_\odot.$$ (101)
Here we have used the binding energy of

\[ E = 0.084 M_\odot (M/M_\odot)^2 \]  \hspace{1cm} (102)

of Lattimer and Yahil \cite{96} in order to connect baryon number masses given by Keil and Janka with gravitational masses. Undoubtedly parameters could be adjusted to bring the lower limit down to the 1.5 \( M_\odot \) of Bethe and Brown \cite{93}, which is required for observation. More extensive calculations with inclusion of exchange (Fock) terms \cite{97} bring the maximum neutron star mass above 2 \( M_\odot \), the Fock terms significantly stiffening the EOS once hyperons are present.

Introduction of hyperons into the EOS in Hartree approximation causes the electron chemical potential to saturate \cite{95} at \( \mu_e \sim 200 \text{ MeV} \) at \( \rho \sim 2\rho_0 \). If hyperons enter at densities before kaon condensation, they may hinder the condensation by bringing \( \mu_e \) down. On the other hand, they sufficiently soften the EOS that the system moves rapidly to a higher density, which brings the \( K^- \) energy \( \omega_K \) down, and these effects are likely to compensate. As noted, linear extrapolation of the binding energy of a \( K^- \) in nuclei to higher densities, or inclusion of Brown-Rho scaling \cite{64}, gives \( \rho_c \) for kaon condensation only slightly above 2\( \rho_0 \). Once finite temperature is included, a substantial number of thermal \( K^- \)-mesons are present, so the distinction between introduction of hyperons and kaon condensation is likely to be smeared out. Recently Ellis, Knorren and Prakash \cite{98} have introduced hyperons into the dense matter, largely confirming the calculations of Glendenning \cite{95} and of Keil and Janka \cite{94}. Ellis \textit{et al} find that the introduction of \( \Sigma^- \)'s brings down the electron chemical potential \( \mu_e \) and delays the kaon condensation until higher densities. We believe this to be incorrect for the following reasons: (1) \( \Sigma^- \)'s do not experience the same attraction in nuclei as the \( \Lambda \)'s do \cite{99}, although this is assumed in the quoted calculation; a possible explanation is that QCD sum-rule calculations \cite{100} find substantially more repulsive interaction on the \( \Sigma^- \) than on the \( \Lambda \), in contrast to quark scaling which predicts these to be the same; (2) the Ellis \textit{et al} calculations use Walecka mean fields for the hyperons, but interactions from chiral Lagrangians for the \( K^- \) mean fields. As noted at the end of Subsection 6.2, this is inconsistent. Given the Walecka mean fields for hyperons, the \( K^- \) mean fields should be a factor \( \sim 1.6 \) larger than employed. This not only makes kaon condensation competitive with the introduction of hyperons, but also means that it will probably precede any such introduction. To put it more directly, we believe that kaon condensation will win out over introduction of hyperons. Even if the latter did indeed enter first at lower densities, it would soften the EOS sufficiently in order to lead quickly to kaon condensation.

Aside from substantially softening the EOS, so that a compact object of mass 1.5\( M_\odot \) can go into a black hole, the strangeness condensed EOS has an important consequence, shown in Fig.16 \cite{101}. While the neutrinos are trapped, the leptons (neutrinos and electrons) produce substantial pressure, so that the maximum mass given by the solid line in Fig.16 is
Figure 16: The solid line shows the short-time maximum neutron star masses as function of central density, with an assumed lepton fraction of $Y_l = 0.4$, consisting of electrons and trapped neutrinos. The dashed line shows the long-time maximum cold mass, after the neutrinos have left. Note that the highest $n_{\text{cent}}$ is $\sim 12\rho_0$.

stabilized. Once the neutrinos leave, the electron fraction $Y_l$ drops to a low value, and since most of the lepton pressure has been carried away by neutrinos, the maximum mass that can be stabilized is substantially less, $1.5M_\odot$ in the case of Fig. 16. Consequently, compact objects with masses in the range

$$1.5M_\odot < M \lesssim 1.7M_\odot$$

will be stable during the time of neutrino emission; i.e., long enough to explode, and will then go into a black hole. Brown and Bethe [91] estimate that this delayed drop, after explosion and return of matter to the galaxy, into a black hole will take place for most stars with main sequence masses

$$18M_\odot < M < 30M_\odot.$$  \hspace{1cm} (104)

Supernova 87A lies near the lower end; in fact, it essentially sets the lower end. In PSR 1913 + 16 the pulsar, the larger of the neutron stars in the binary, has mass $1.44M_\odot$. Evolutionary calculations [102, 103] find that a $1.44M_\odot$ neutron star results from a $5 - 6$
Figure 17: A very soft EOS with $K_0 = 130$ MeV so that the maximum neutron star mass for cold matter (dashed line) is $1.5M_{\odot}$ as in Fig. 16. The short time maximum mass, given by the solid line, is now lower, even though the neutrinos are trapped with $Y_i = 0.4$. 
helium star, corresponding to a main sequence mass of 16 – 18 \( M_\odot \) for the progenitor. Thus, the pulsar in 1913 + 16 is very close to the maximum mass for a neutron star.

Not only do we believe that the Supernova 87A ended up as a black hole, but this was also likely for CAS A, a supernova explosion that took place in the 17th century. From the abundances of O, Mg, Ne in knots in the supernova remnant, the masses of the progenitor can be deduced to be \( M \approx 20M_\odot \). No compact object is found in the center of the remnant, although searches, with great sensitivity, have been carried out.

We shall now explain why the delayed explosion does not result from the standard neutron star scenario. This situation is shown in Fig.17. In this case the short-time maximum mass, with neutrinos trapped, and with \( Y_t = 0.4 \), is lower than the cold mass, given by the dashed line, after the neutrinos leave. The point is that, in the standard scenario, as the neutrinos leave, the original nuclear matter is converted to neutron matter, and neutron matter is much “stiffer” (that is, the pressure is higher) than nuclear matter. Thus, even though the pressure is lowered by the neutrinos departing, it is increased somewhat more, by the protons changing to neutrons. Thus, if the compact object is initially stable, it will remain stable, and will not go into a black hole.

Previous to the scenario involving the kaon condensed EOS, it was suggested that stars heavier than \( \sim 25M_\odot \) may leave black holes [104], and also that such stars might first explode, exhibiting light curves of Type II supernovae, and then collapse into black holes [105]. The compact core was, for certain range of masses, to be stabilized by the thermal pressure during the period of Klein-Helmholtz contraction, long enough to carry out nucleosynthesis, then going into a black hole after cooling and deleptonization. In terms of Fig.17, this scenario meant that the thermal pressure had to be sufficient, so that when thermal effects were added to the (short-time) solid line, it came close to the dashed line. Then, the compact object could have a mass lying above the dashed line (and below the solid line) and be stable for some time, before it went below the dashed line as it cooled.

**Table 1**

Energy gain \( \Delta E \) in MeV, chemical potential \( \mu_K \) in MeV, proton fraction \( x \) and electron fraction \( x_e \) as function of the density \( u = \rho/\rho_0 \) for a kaon condensed EOS with \( u_c = 4.2 \).

| \( u \) | \( \Delta E (\text{MeV}) \) | \( \mu_K (\text{MeV}) \) | \( x \) | \( x_e \) |
|------|----------------|----------------|------|------|
| 4.2  | 0              | 256            | 0.20 | 0.11 |
| 5.2  | −10            | 199            | 0.34 | 0.03 |
| 6.2  | −35            | 142            | 0.43 | 0.01 |
| 7.2  | −71            | 94             | 0.48 | 0    |
| 8.2  | −112           | 55             | 0.50 | 0    |
The trouble with this scenario, of thermal pressure stabilizing the compact object, is that the thermal pressure can stabilize only a small additional mass. Detailed calculations by Bombaci et al.\(^\text{[106]}\) give only an additional $\sim 2\text{--}3\%$ increase in stabilized mass, less than the distance between the solid and dashed lines in Fig.\(^\text{[17]}\). Similar results are obtained by Keil and Janka\(^\text{[94]}\) with their delayed explosion scenario in the EOS including hyperons. Thus, it is clear that in most cases, thermal pressure will not stabilize the compact object for some time, with the object later going into a black hole; rather, it will remain stable if it is so initially.

We see that a lot of observational evidence is explained with our kaon condensation EOS. Note that in this scenario, stars go “strange”; i.e., acquire a lot of strangeness already in the hadron sector. They do not, at the densities of $2\text{--}4\ \rho_0$, go to strange quark matter. The transition from kaon condensation to strange quark matter can, however, be constructed\(^\text{[107]}\). This was made under quite conservative assumptions of a small $\Sigma_{KN} = 1.3 m_\pi$ and no Brown-Rho scaling. We give in Table 1 the results of the calculation by Vesteinn Thorsson in his 1992 Stony Brook thesis. With small differences, these are the same numbers as in Table 3 of ref.\(^\text{[61]}\).

Of chief concern to us is the softening of the EOS by kaon condensation. In Fig.\(^\text{[18]}\) we plot the baryon number chemical potential vs. pressure. This is a convenient plot for investigating the transition to quark matter. In the following we use the procedure of Bethe et al.\(^\text{[108]}\). The PAL curve is taken from Prakash et al.\(^\text{[88]}\). It has a compression modulus $K_0 = 180\ MeV$, and the potential energy part of the symmetry energy rises linearly with $u = \rho/\rho_0$. The PAL21 curve from \(^\text{[88]}\) has a maximum neutron star mass of $M_{max} = 1.72M_\odot$. The curve including the kaon condensation with characteristics shown in Table 1 is plotted in the lower dashed line in Fig.\(^\text{[18]}\). In addition, the results for quark matter\(^\text{[108]}\) are given, but with strange quark mass $m_s = 200\ MeV$ included. (In \(^\text{[108]}\), $m_s$ was set equal to zero.) There $\alpha_s$ was scaled as

$$\alpha_s(k_F) = 2.2 k_F^{(0)}/k_F$$  \hspace{1cm} (105)

where $k_F^{(0)}$ is the Fermi momentum at nuclear matter density, $k_F$ the Fermi momentum at the density considered. This gave a rapid decrease in coupling constant, which Bethe et al.\(^\text{[108]}\) considered appropriate for the nonperturbative sector.

It should be noted that the M.I.T. $\alpha_s$ of 2.2 at $\rho = \rho_0$, even decreasing as rapidly as (105), gave an EOS which lies well above the PAL21 curve, so there is no hope of joining PAL21 to it, especially if PAL21 is decreased even further by kaon condensation. (A “conventional” compression modulus of $K_0 = 210 \pm 30\ MeV$\(^\text{[74]}\) would give an only slightly higher curve than PAL21.)

Quark EOS’s for $\alpha_s = 1.1$ and 0.55 are also shown. The $P_Q + B$ is plotted for the quark/gluon phase, so that introduction of a bag constant (which does not affect the chem-
Figure 18: Curves of chemical potential vs. pressure for hadronic matter (dashed curves). The upper dashed curve representing PAL21 \cite{88} has $K_0 = 180$ MeV, a symmetry energy for which the potential energy rises linearly with density. The lower dashed curve results when kaon condensation is included. Solid lines for quark matter are plotted for $p_Q + B$, instead of the pressure, but $B$ will be taken to be zero. The upper solid line is for \(\alpha_s = 1.1\); the lower one, for \(\alpha_s = 0.55\). On each line, the lowest black circle or square marks the density $2\rho_0$, and each successive dot or square indicates a density higher by $\rho_0$. 
ical potential \( \mu \) can be made by shifting the \( P_Q + B \) curve to the left by the amount \( B \), so as to obtain \( P_Q \). As outlined in [3], we believe the bag constant, the \( B_{\chi_S B} \) of eq. (4), to go to zero at chiral restoration. Note that the quark/gluon curve for \( \alpha_s = 1.1 \) is easily made equal to PAL21 over a wide range of pressure about \( P_N \sim 400 \text{MeV}/\text{fm}^3 \) in this way. Similarly, the quark/gluon curve with \( \alpha_s = 0.55 \) can be made equal to the PAL21 with kaon condensate curve, in the region of pressures which correspond to densities of \( \sim 8-10 \rho_0 \) for hadronic matter. (The density is not continuous at the transition.) We see immediately that with the M.I.T. value for \( \alpha_s \) used by Bethe et al [108], there is no hope of making a transition to quark matter; the quark matter EOS has a much higher energy than the hadronic one, especially when kaon condensation is introduced into the latter.

Fahri and Jaffe [109] have taken the position that the \( \alpha_s \) for the dense matter properties may be substantially smaller than 2.2, and have investigated the range of \( \alpha_s \) used in the two curves of our figure. Given these small \( \alpha_s \), it is possible to discuss the transition to quark matter.

With our conservative choice of the kaon condensed EOS, without Brown-Rho scaling, and the \( \alpha_s \) of 0.55 \( \frac{k_F^{(0)}}{k_F} \), a smooth join to quark matter can be made at a density of \( \sim 8-10 \rho_0 \). Whereas \( \alpha_s \) may not drop as rapidly as \( \frac{k_F^{(0)}}{k_F} \), an \( \alpha_s \) of \( \sim 0.25 \) at \( \rho \sim 10 \rho_0 \) is not unreasonable, although we have no way at present of determining a quantitative value at such high densities. We can thus say that, with a rather conservative kaon condensed EOS (with \( \rho_c \approx 4 \rho_0 \)), there is the possibility of a smooth cross-over transition to quark matter at the upper end of the densities obtained in compact objects, say, \( \rho \sim 10 \rho_0 \). With a softer EOS, obtained with Brown-Rho scaling, with kaon condensation taking place at \( \rho \leq 3 \rho_0 \), the transition density to quark matter would be substantially higher.

It is daring – and perhaps foolhardy – to extrapolate to densities \( \sim 10 \rho_0 \), but we believe the replacement of electrons by \( K^- \)-mesons at high densities to be a new idea with a solid foundation, which qualitatively changes the conceptual situation in compact star matter. We may expect the transition to quark matter at high densities to take place similarly to the transition with temperature to quark/gluon plasma, as discussed in Section 4; namely, the transition will be smooth and gradual. At the lower densities it will be more convenient – and cleverer – to use hadronic variables, but at the higher densities the quark language will become more efficient. As argued, we may expect the light-quark vector-meson masses to go to zero, with increasing density, and chiral symmetry to be realized in the Georgi vector limit preceding chiral restoration.

Given the kaon condensation phase transition, our EOS is so soft that it is difficult to stabilize stars of known masses. Indeed with the PAL21 EOS including kaon condensation, \( M_{\text{max}} = 1.42 M_\odot \), although raising \( K_0 \) to the more conventional value of 200 MeV allows us to obtain \( M_{\text{max}} = 1.5 M_\odot \). A slightly higher \( K_0 \) will be needed if the kaon condensed EOS
with Brown-Rho scaling is used, since this EOS is substantially softened by this.

Observed neutron star masses are shown in Fig. 19. Precisely the lower limit of the measured mass of Vela X-1 lay below $1.5 \, M_\odot$. New observation by Van Kerkwijk et al. [11] found that the observed velocities in Vela X-1 deviate substantially from the smooth radius-velocity curve expected from pure Keplerian motion. The deviations seem to be correlated with each other within one night, but not from one night to another. The excursions suggest something like pulsational coupling to the radial motion, and make it difficult to obtain an accurate mass measurement. The lower limit for the mass of the compact object in Vela X-1 is now found to be $1.43 M_\odot$ at 95% confidence level, or $1.37 M_\odot$ at this confidence interval around the most probable value. Consequently, Vela X-1 is no longer a big problem for our $M_{\text{max}} = 1.5 M_\odot$.

It is striking that well measured neutron star masses lie below $1.5 M_\odot$. (See Fig. 19.) However, the central value of the compact object in 4U 1700-37 lies at $1.8 M_\odot$, although the error bars encompass $1.5 M_\odot$. Brown et al. [103] give arguments that this compact object could be a low-mass black hole. (The principal aim of this paper is to show that stars in binaries can have much larger masses than the main sequence mass of $18 M_\odot$ of 1987A, and still end up as neutron stars. In other words, in binaries, because of the specifics of mass transfer, stars can evolve in quite a different way than single stars evolve. Since determination of masses has generally been carried out in binaries, this explains why it was not earlier recognized that a star with main sequence mass as light as $\sim 18 M_\odot$ (the progenitor of 1967A) could go into a black hole, and it was – and still is – considerable surprise that it probably did.)

In the past it has generally been thought that the reason accurately measured neutron star masses lie at $1.44 M_\odot$ and below is evolutionary in nature. Large stars collapse when the iron core exceeds the Chandrasekhar limit, $\sim 1.25-1.5 M_\odot$, depending on the main sequence mass of the star. In the past literature, accretion has been assumed to proceed only up to the Eddington limit

$$\dot{M}_{\text{Edd}} = 1.5 \times 10^{-8} M_\odot/\text{yr}$$

and there are relatively few situations where the compact core would be expected to accrete more than $\sim 0.1 M_\odot$ at this rate. (Even in the very old millisecond pulsars, only $\sim 0.1-0.2 M_\odot$ is estimated to have been accreted.) However, Chevalier [112] and Brown [113] have shown that in the common envelope phase of binary pulsar evolution, accretion can proceed at hypercritical rates

$$\dot{M} \geq 10^4 \dot{M}_{\text{Edd}}.$$ 

Thus, if evolutionary history determines their mass, neutron stars of higher mass than $1.5 M_\odot$ should exist, but none have been so far observed. Therefore, there should be an
Figure 19: Measured masses of 17 neutron stars from Arzoumanian et al [110], with the lower limit on the mass of Vela X-1 from Van Kerkwijk et al [111]. Objects in high mass X-ray binaries are at the top, radio pulsars and their companions at the bottom.
intrinsic limit on the mass of a neutron star, such as the one we find. It is difficult to explain the existence of such an intrinsic limit without softening of the EOS through a phase transition and we are proposing that kaon condensation is the key mechanism for it.

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8.1 Appendix: The Kaplan-Nelson Attraction

The attractive scalar mean field potential \( \Sigma_{KN} \), used first by Kaplan and Nelson [60] for kaon condensation, depends linearly on the \( KN \) sigma term \( \Sigma_{KN} \), which in turn depends on the strangeness content of the nucleon \( \langle N|\bar{s}s|N\rangle \). This is usually parametrized by

\[
y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle}.
\]

(108)

In the most extensive lattice gauge calculation to date, Liu finds [114]

\[ y = 0.33 \pm 0.09. \]

(109)

Given this, we can estimate \( \Sigma_{KN} \) using the information on the \( \pi N \) sigma term \( \Sigma_{\pi N} \). Since

\[
\frac{\Sigma_{KN}}{\Sigma_{\pi N}} = \frac{(m_s + m_u)\langle N|(\bar{s}s + \bar{u}u)|N\rangle}{(m_u + m_d)(\langle N|\bar{u}u + \bar{d}d|N\rangle)},
\]

(110)

taking the value for \( m_s \) of

\[
2m_s/(m_u + m_d) \approx 29
\]

(111)

from Bijnens et al [113], \( y = 0.33 \) and \( \Sigma_{\pi N} = 45 \text{ MeV} \), we find

\[
\Sigma_{KN} \approx 450 \pm 30 \text{ MeV}
\]

(112)

as the best current estimate of \( \Sigma_{KN} \). This is slightly larger than the 2.83 \( m_\pi \) found by Lee et al [64] from fitting the \( KN \) scattering amplitudes. We believe that the wild fluctuations in \( \Sigma_{KN} \) used in the literature have now settled down.
Using Liu’s value of

\[ \langle N|\bar{u}u + \bar{d}d|N \rangle = 8.22 \pm 1.1, \]  

we find as central value

\[ \langle N|\bar{s}s|N \rangle = 1.36. \]  

With \( m_s = 174 \text{ MeV} \), this would mean that \( m_s \langle N|\bar{s}s|N \rangle \), the contribution to the nucleon mass from the explicit chiral symmetry breaking in the strange sector, is 237 MeV which is sizable.

In chiral Lagrangians, the explicit chiral symmetry breaking in the strange sector is parametrized by a coefficient denoted \( a_3 \) in [116]. Politzer and Wise employed rather different values, \( a_3 m_s = 310 \text{ MeV} \) corresponding to a large \( \langle N|\bar{s}s|N \rangle \) and \( a_3 m_s = 140 \text{ MeV} \) corresponding to a small \( \langle N|\bar{s}s|N \rangle \)\(^{23}\). Using our above central values for \( m_s \) and \( \langle N|\bar{s}s|N \rangle \), we find, using the relations given by Politzer and Wise,

\[ a_3 m_s \approx 245 \text{ MeV} \]  

with an estimated uncertainty of \( \sim 10\% \).

Note that the \( \Sigma_{KN} = 450 \text{ MeV} \) of eq.(112) would give a coefficient of 72 MeV, rather than 64 MeV, in eq.(80). This difference is not significant, given the listed uncertainties.

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\(^{23}\)Note that we are using a different sign convention.
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