A NEW STATE OF MATTER AT HIGH TEMPERATURE AS "STICKY MOLASSES"

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The main objective of this work is to explore the evolution in the structure of the quark-antiquark bound states in going down in the chirally restored phase from the so-called “zero binding points” $T_{zb}$ to the QCD critical temperature $T_c$ at which the Nambu-Goldstone and Wigner-Weyl modes meet. In doing this, we adopt the idea recently introduced by Shuryak and Zahed for charmed $\bar{c}c$, light-quark $\bar{q}q$ mesons $\pi, \sigma, \rho, A_1$ and gluons that at $T_{zb}$, the quark-antiquark scattering length goes through $\infty$ at which conformal invariance is restored, thereby transforming the matter into a near perfect fluid behaving hydrodynamically, as found at RHIC. We name this new state of matter as “sticky molasses”. We show that the binding of these states is accomplished by the combination of (i) the color Coulomb interaction, (ii) the relativistic effects, and (iii) the interaction induced by the instanton-anti-instanton molecules. The spin-spin forces turned out to be small. While near $T_{zb}$ all mesons are large-size nonrelativistic objects bound by Coulomb attraction, near $T_c$ they get much more tightly bound, with many-body collective interactions becoming important and making the $\sigma$ and $\pi$ masses approach zero (in the chiral limit). The wave function at the origin grows strongly with binding, and the near-local four-Fermi interactions induced by the instanton molecules play an increasingly more important role as the temperature moves downward toward $T_c$.

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1. Introduction

The concept that hadronic states may survive in the high temperature phase
of QCD, the quark-gluon plasma\(^1\), has been known for some time. In par-
ticular, it was explored by Brown et al.\(^2,3\). The properties of (degenerate) \(\pi\)
and \(\sigma\) resonances above \(T_c\) in the context of the NJL model was discussed
earlier by Hatsuda and Kunihiro\(^4\), and in the instanton liquid model by
Schäfer and Shuryak \(^5\). Recently, lattice calculations \(^6,7\) have shown that,
contrary to the original suggestion by Matsui and Satz \(^8\), the lowest char-
monium states \(J/\psi, \eta_c\) remain bound well above \(T_c\). The estimates of the
zero binding temperature for charmonium \(T_{J/\psi}\) is now limited to the inter-
val \(2T_c > T_{J/\psi} > 1.6T_c\), where \(T_c \approx 270\text{ MeV}\) is that for quenched QCD.
Similar results for light quark mesons exist but are less quantitative at the
moment. However since the “quasiparticle” masses close to \(T_c\) are large,
they must be similar to those for charmonium states.

In the chiral limit all states above the chiral restoration go into chiral
multiplets. For quark quasiparticles this is also true, but although the
chirality is conserved during their propagation, they are not massless and
move slowly near \(T_c\) where their “chiral mass” \(m = E(p \to 0)\) is large (\(\sim 1\)
GeV).

RHIC experiments have found that hot/dense matter at temperatures
above the critical value \(T_c \approx 170\text{ MeV}\) is not a weakly interacting gas
of quasiparticles, as was widely expected. We envision it to be “sticky
molasses.” Indeed, RHIC data have demonstrated the existence of very
robust collective flow phenomena, well described by ideal hydrodynamics.
Most decisive in reaching this conclusion was the early measurement of the
elliptic flow which showed that equilibration in the new state of matter
above \(T_c\) set in in a time < 1 fm/c \(^9\). Furthermore, the first viscosity
estimates \(^10\) show surprisingly low values, suggesting that this matter is the
most perfect liquid known. Indeed, the ratio of shear viscosity coefficient
to the entropy is only \(\eta/s \sim 0.1\), two orders of magnitude less than for
water. Furthermore, it is comparable to predictions in the infinite coupling
limit \(^11\) (for \(N=4\) SUSY YM theory) \(\eta/s = 1/4\pi\), perhaps the lowest value
possible.

Shuryak and Zahed \(^12\) (hereafter referred to as SZ whenever unambigu-
ous) have recently connected these two issues together. They have sug-
gested that large rescattering cross sections apparently present in hot mat-
ter at RHIC are generated by resonances near the zero-binding lines. In-
deed, at the point of zero binding the scattering length \(a\) of the two con-
stituents goes to $\infty$ and this provides low viscosity. This phenomenon is analogous to the elliptic flow observed in the expansion of trapped $^6\text{Li}$ atoms rendered possible by tuning the scattering length to very large values via a Feshbach resonance $^{13}$.

Near the zero-binding points, to be denoted by $T_{zb}$, introduced by SZ the binding is small and thus the description of the system can be simple and nonrelativistic. The binding comes about chiefly from the attractive Coulomb color electric field, as evidenced in lattice gauge calculation of Karsch and collaborators$^{6,14}$, and Asakawa and Hatsuda$^7$, as we shall detail. The instanton molecule interactions, which we describe below, are less important at these high temperatures ($T \sim 400$ MeV). All changes as one attempts (as we show below) to discuss the more deeply bound states just above $T_c$.

In another work $^{15}$, Shuryak and Zahed have also found sets of highly relativistic bound light states in the strongly coupled $N=4$ supersymmetric Yang-Mills theory at finite temperature (already mentioned above in respect to viscosity). They suggested that the very strong Coulomb attraction can be balanced by high angular momentum, producing light states with masses $m \sim T$. Furthermore, the density of such states remains constant at arbitrarily large coupling. They argued that in this theory a transition from weak to strong coupling basically implies a smooth transition from a gas of quasiparticles to a gas of “dimers”, without a phase transition. This is an important part of the overall emerging picture, relating strong coupling, viscosity and light bound states.

In this work we wish to construct the link between the chirally broken state of hadronic matter below $T_c$ and the chirally restored mesonic, glueball state above $T_c$. Our objective is to understand and to work out in detail what exactly happens with hadronic states at temperatures between $T_c$ and $T_{zb}$. One important new point is that these chirally restored hadrons are so small that the color charges are locked into the hadrons at such short distances ($< 0.5$ fm) that the Debye screening is unimportant. This is strictly true at $T \gtrsim T_c$, where there is very little free charge. In this temperature range the nonrelativistic treatment of SZ should be changed to a relativistic one.

The relativistic current-current interaction, ultimately related with the classical Ampere law, is about as important as the Coulomb one, effectively doubling the attraction (see section 2.1). We also found that the spin-spin forces discussed in 2.2 are truly negligible. In effect, with the help of the instanton molecule interaction, one can get the bound quark-antiquark
states down in energy, reaching the massless $\sigma$ and $\pi$ at $T_c$, so that a smooth transition can be made with the chiral breaking at $T < T_c$.

The non-perturbative interaction from the instanton molecules becomes very important. Let us remind the reader of the history of the issue. The nonperturbative gluon condensate, contributing to the dilatational charge or trace of the stress tensor $T_{\mu\mu} = \epsilon - 3p$, is not melted at $T_c$. In fact more than half of the vacuum gluon condensate value remains at $T$ right above $T_c$. The hard glue or epoxy which explicitly breaks scale invariance but is unconnected with hadronic masses. The rate at which the epoxy is melted can be measured by lattice gauge simulations, and this tells us the rate at which the instanton molecules are broken up with increasing temperature$^1$.

As argued by Ilgenfritz and Shuryak$^16$, this phenomenon can be explained by breaking of the instanton ensemble into instanton molecules with zero topological charge. Such molecules generate a new form of effective multi-fermion effective interaction similar to the original NJL model. Brown et al.$^17$ (denoted as BGLR below) obtained the interaction induced by the instanton molecules above $T_c$ by continuing the Nambu-Jona-Lasinio description upwards from below $T_c$.

Our present discussion of mesonic bound states should not be confused with quasi-hadronic states found in early lattice calculations$^18$ for quarks and antiquarks propagating in the space-like direction. Their spectrum, known as “screening masses” is generated mostly by “dynamical confinement” of the spatial Wilson loop which is a nonperturbative phenomenon seen via the lattice calculations. Similar effects will be given here by the instanton molecule interaction.

2. Binding of the $\bar{q}q$ states

2.1. The Coulomb interaction and the relativistic effects

At $T > T_c$ the charge is screened rather than confined$^19$, and so the potential has a general Debye form

$$V = \frac{\alpha_s(r,T)}{r} \exp\left(-\frac{r}{R_D(T)}\right)$$

(1)

(Note that we use a (somewhat nonstandard) definition in which $\alpha_s$ absorbs the 4/3 color factor.) The general tenet of QCD tells us that the strength of the color Coulomb should run. We know that perturbatively it should run as

$$\alpha_s \sim \frac{1}{\log(Q/\Lambda_{\text{QCD}})}$$

(2)
with $\Lambda_{QCD} \sim 0.25$ GeV. The issue is what happens when the coupling is no longer small. In vacuum we know that the electric field is ultimately confined to a string, producing a linear potential.

In the so-called “plasma phase” this does not happen, and SZ assumed that the charge runs to larger values, which may explain the weak binding at rather high $T$ we discussed in the introduction. Lattice results produce potentials which, when fitted in the form $V(r) = -A \exp(-mr) + B$ with constant $A, B$ indeed indicate that $A(T)$ grows above $T_c$ by a large factor, before starting to decrease logarithmically at high $T$. The maximal value of the average coupling $\text{max}(A) \approx 1/2$. This is the value which will keep charmonium bound, as found by Asakawa and Hatsuda, up to $1.6T_c$.

Running of the coupling is not very important for this work in which we are mostly interested in deeply bound states related with short enough distances. Therefore we will simply keep it as a non-running constant, selecting some appropriate average value.

It is well known in the point charge Coulomb problem (QED) that when $Z\alpha$ is increased and the total energy reaches zero there is a singularity, preventing solutions for larger $Z\alpha$. In the problem of the “sparking of the vacuum” in relativistic heavy ion collisions, the solution of the problem was found by approximating the nuclei by a uniformly charged sphere; for a review of the history see Rafelski et al.\textsuperscript{20}. As a result of such regularization, the bound electron level continues past zero to $-m$, at which point $e^+e^-$ production becomes possible around the critical value of $Z_{cr} = 169$. In short, the problem of the point Coulomb charge could be taken care of by choosing a distributed electric field which began from zero at the origin.

In QCD the charge at the origin is switched off by asymptotic freedom, the coupling which runs to zero value at the origin. A cloud of virtual fields making the charge is thus “empty inside”. We will model a resulting potential for the color Coulomb interaction by simply setting the electric field equal to zero at $r = 0$, letting it decrease (increase in attraction) going outward. We can most simply do this by choosing a charge distribution which is constant out to $R$, the radius of the meson. If the original $2m_q$ mass were to be lowered to zero by the color Coulomb interaction and instanton molecule interaction, then the radius of the final molecule will be

$$R \simeq \frac{\hbar}{2m_q},$$

although the rms radius will be substantially greater with the instanton
molecule interactions playing the main role around $T_c$.

\[
V = -\alpha_s \frac{1}{2R} \left( 3 - \frac{r^2}{R^2} \right), \quad r < R
\]

\[
= -\alpha_s \frac{1}{r}, \quad r > R.
\]  

(4)

This $V$ has the correct general characteristics. As noted above, the electric field $\vec{E}$ must be zero at $r = 0$. It is also easy to see that $V$ must drop off as $r^2/R^2$ as the two spheres corresponding to the quark and antiquark wave functions are pulled apart. Precisely where the potential begins the $1/r$ behavior may well depend upon polarization effects of the charge, the $+$ and $-$ charges attracting each other, but it will be somewhere between $R$ and $2R$, since the undisturbed wave functions of quark and antiquark cease to overlap here.

The $q\bar{q}$ system is similar to positronium in the equality of masses of the two constituents. Since the main term value is $ma^2/4$, the 4, rather than 2 in hydrogen, coming from the reduced mass, one might think that the Coulomb, velocity-velocity and other interactions would have to be attractive and 8 times greater than this term value in order to bring the $2m_q$ in thermal masses to zero. However, this does not take into account the increase in reduced mass with $\alpha$. Breit and Brown 21 found an $\alpha^2/4$ increase in the reduced mass with $\alpha$, or 25% for $\alpha = 1$, to that order. It should be noted that in the Hund and Pilkuhn 22 prescription the reduced mass becomes $\mu = m^2_q/E$, which increases as $E$ drops.

We first proceed to solve the Coulomb problem, noting that this gives us the solution to compare with the quenched lattice gauge simulations, which do not include quark loops.

Having laid out our procedure, we shall proceed with approximations. First of all, we ignore spin effects in getting a Klein-Gordon equation. The chirally restored one-body equation which has now left-right mixing is given by

\[
(p_0 + \vec{\sigma} \cdot \vec{p})\psi = 0.
\]  

(5)

Expressing $\psi$ in two-component wave functions $\Phi$ and $\Psi$, one has

\[
p_0\Phi = -\vec{\sigma} \cdot \vec{p}\Psi
\]

\[
p_0\Psi = -\vec{\sigma} \cdot \vec{p}\Phi,
\]  

(6)

giving the chirally restored wave function on $\Psi$

\[
\left( p_0 - \vec{\sigma} \cdot \frac{1}{p_0} \vec{p} \cdot \vec{\sigma} \cdot \vec{p} \right) \Psi = 0.
\]  

(7)
Here

\[ p_0 = E_V = E + \alpha_s/r. \]  

(8)

Neglecting spin effects, \( \vec{\sigma} \cdot \vec{p} \) commutes with \( p_0 \), giving the Klein-Gordon equation \( p_0^2 - \vec{p}^2 = 0 \). We now introduce the effective (thermal) mass, so that the equations for quark and hole can be solved simultaneously following 22:

\[ ([\epsilon - V(r)]^2 - \mu^2 - \hat{p}^2) \psi(r) = 0 \]  

(9)

where \( \hat{p} \) is momentum operator, and the reduced energy and mass are \( \epsilon = (E^2 - m_1^2 - m_2^2)/2E, \mu = m_1m_2/E \) with \( m_1 = m_2 = m_q \).

Furthermore from eq.(6),

\[ \langle \vec{\alpha} \rangle = \langle \Psi^\dagger \vec{\sigma} \Phi \rangle + \langle \Phi^\dagger \vec{\sigma} \Psi \rangle = \frac{\vec{p}}{p_0} - \frac{i}{p_0} \langle [\vec{\sigma} \times \vec{p}] \rangle. \]  

(10)

If \( \vec{\sigma} \) is parallel to \( \vec{p} \), as in states of good helicity, the second term does not contribute. From the chirally restored Dirac equation (5), ignoring spin effects such as the spin-orbit interaction which is zero in S-states we are considering, we find \( p_0^2 = \hat{p}^2 \). Brown 23 showed that in a stationary state the EM interaction Hamiltonian between fermions is

\[ H_{\text{int}} = \frac{e^2}{r} (1 - \vec{\alpha}_1 \cdot \vec{\alpha}_2), \]  

(11)

where the \( \vec{\alpha}_{1,2} \) are the velocities. Applying (11) to the chirally restored domain of QCD, we expect

\[ H_{\text{int}} = \frac{2\alpha_s}{r} \quad \text{for } \vec{\alpha}_1 \cdot \vec{\alpha}_2 = -1 \]
\[ = 0 \quad \text{for } \vec{\alpha}_1 \cdot \vec{\alpha}_2 = +1 \]  

(12)

### 2.2. The spin-spin interaction

The nonrelativistic form of the spin-spin interaction, in the delta-function form, may give an impression that it is maximal at the smallest distances. However this is not true, as becomes clear if the relativistic motion is included in full, and in fact at \( r \to 0 \) it is suppressed. At large \( r \), when particle motion is slow, it is of course again suppressed, thus contributing mostly at some intermediate distances.

This fact is clear already from the derivation of the 1s-state hyperfine splitting Fermi-Breit due to hyperfine interaction in hydrogen from 1930 24
given by
\[ \delta H = \frac{2}{3} (\vec{\sigma} \cdot \vec{\mu}) \int d^3r \frac{\psi \psi^\dagger d}{r^2} \frac{e}{E + e^2/r + m}. \] (13)

Note the complete denominator, which non-relativistically is just substituted by m alone, but in fact contains the potential and is singular at \( r \to 0 \). The derivative of the \( e^2/r \) in the denominator insured that the electric field was zero at \( r = 0 \). Here \( \vec{\sigma} \) is the electron spin, \( \vec{\mu} \) the proton magnetic moment. In eq (13) the derivative can then be turned around to act on \( \psi \psi^\dagger \), and to order \( \alpha = 1 \) and with the \( e^2/r \) neglected in the denominator, one has
\[ \delta H \simeq -\frac{8\pi}{3} (\vec{\sigma} \cdot \vec{\mu}) \frac{e}{2m} \psi^2(0), \] (14)
with \( \psi \) taken to be the nonrelativistic \( 1s \) wave function to lowest order in \( \alpha \).

The hyperfine structure is obtained by letting the first \( \vec{p} \) in eq. (7) act on the \( p_0^{-1} \) and the second \( \vec{p} \rightarrow \vec{p} + \sqrt{\alpha_s} \vec{A} \) with
\[ \vec{A} = \frac{\vec{\mu} \times \vec{r}}{r^3} \] (15)
with \( \vec{\mu} \) the magnetic moment of the antiquark. One finds that the hyperfine structure is \(^{24}\)
\[ H_{\text{hfs}} = \frac{1}{p_0^2} \sqrt{\alpha_s} \frac{\vec{\sigma}}{r^2} \cdot |\vec{E} \times \vec{A}| \] (16)
where \( \vec{E} \) is color electric field. Thus,
\[ H_{\text{hfs}} = \frac{\sqrt{\alpha_s}}{p_0^2} \frac{|\vec{E}|}{r^2} \left( \frac{\vec{\sigma} \cdot \vec{\mu}}{r^2} - \frac{\vec{\sigma} \cdot \vec{r} \vec{\mu} \cdot \vec{r}}{r^4} \right) = \frac{2}{3} \frac{\sqrt{\alpha_s}}{p_0^2} \frac{|\vec{E}|}{r^2} \frac{\vec{\sigma} \cdot \vec{\mu}}{r^2}. \] (17)
where \( |\vec{E}| = 2\alpha_s/r^2 \). As in the hydrogen atom, the magnetic moments of quarks and antiquarks are
\[ \mu_{q,\bar{q}} = \mp \frac{\sqrt{\alpha_s}}{p_0 + m_{q,\bar{q}}} \] (18)
except that the Dirac mass \( m_{q,\bar{q}} = 0 \) and \( p_0 \), in which the potential is increased by a factor of 2 to take into account the velocity-velocity interaction, is now
\[ p_0 = E + 2(\alpha_s/r) \] (19)
for QCD so that in terms of the quark and antiquark magnetic moment
operators,

\[ H_{\text{hfs}} = -\frac{2}{3} \frac{|\vec{E}|}{p_0 r^2} (\vec{\mu}_q \cdot \vec{\mu}_{\bar{q}}). \]  

(20)

Of course, our \( p_0 \) for the chirally restored regime has substantial \( r \) dependence, whereas the \( e/r \) in the hydrogen atom is generally neglected, and \( E + m \) is taken to be \( 2m \), so that \( \mu_e = -e/2m_e \). From Fig. 1 it will be seen that (square of) the wave function is large just where \( \alpha_s/r \) is large.

For rough estimates we use averages. We see that, as in Table 1, if \( E \) is to be brought down by \( \sim 0.5 m_q \) for the \( \sigma \) and \( \pi \) by the Coulomb interaction, then

\[ 2\langle \alpha_s/r \rangle \simeq \frac{1}{2} m_q \simeq \frac{1}{4} p_0 \]  

(21)

so that with \( \alpha_s \sim 0.5 \),

\[ \langle r^{-1} \rangle \simeq \frac{1}{2} m_q. \]  

(22)

We next see that this is consistent with the spin splitting forming a fine structure of the two groups, the lower lying \( \sigma \) and \( \pi \), and the slightly higher lying vectors and axial vectors. Using our above estimates, we obtain

\[ \langle H_{\text{hfs}} \rangle \simeq \frac{1}{24} \frac{1}{16} \sigma_q \cdot \sigma_{\bar{q}} m_q. \]  

(23)

so that for the \( \sigma \) and \( \pi \) where \( \vec{\sigma}_1 \cdot \vec{\sigma}_2 = -3 \) we have

\[ \langle H_{\text{hfs}} \rangle \sim -\frac{m_q}{128}, \]  

(24)

the approximate equality holding when \( \alpha_s = 0.5 \). Note that the hyperfine effect is negligible for the \( \alpha_s \sim 0.5 \). Although formally eq. (23) looks like the hyperfine structure in the chirally broken sector, it is really completely different in makeup.

In our expression for \( \langle H_{\text{hfs}} \rangle \) we have the \( r \) dependence as \( (p_0 r)^{-4} r^{-1} \) and \( p_0 r = 4 \), basically because the Coulomb interaction lowers the \( \pi \) and \( \sigma \) only 1/4 of the way to zero mass. This explains most of the smallness of the spin-dependent interaction.

A recently renewed discussion of spin-spin and spin-orbit interactions in a relativistic bound states has been made by Shuryak and Zahed 25, who derived their form for both weak and strong coupling limits. Curiously enough, the spin-spin term changes sign between these two limits: perhaps this is another reason why at intermediate coupling considered in this work it happens to be so small.
2.3. The resulting $\bar{q}q$ binding

We first construct the bound states for $T \geq T_c$, at temperature close enough to $T_c$ so that we can take the running coupling constants at $T = T_c + \epsilon$. The fact that we are above $T_c$ is important, because the $\Lambda_{\chi SB} \sim 4\pi f_{\pi} \sim 1 \text{ GeV}$ which characterizes the broken symmetry state below $T_c$ no longer sets the scale. Until we discover the relevant variables above $T_c$ we are unable to find the scale that sets $\alpha_s = \frac{4\pi}{3\hbar c}$, the color Coulomb coupling constant.

Following SZ\textsuperscript{12}, we adopt quark-antiquark bound states to be the relevant variables and specifically, the instanton molecule gas\textsuperscript{26} as a convenient framework. In particular, Adami et al.\textsuperscript{27}, Koch and Brown\textsuperscript{28}, and BGLR\textsuperscript{17} have shown that $\gtrsim 50\%$ of the gluon condensate is not melted at $T = T_c$. The assumption motivated by Ilgenfritz & Shuryak\textsuperscript{16} is then that the glue that is left rearranges itself into gluon molecules around $T = T_c$, i.e., what BGLR call “epoxy”. We have quantitatively determined couplings for the mesons in the instanton molecule gas by extending the lower energy NJL in the chiral symmetry breaking region up through $T_c$\textsuperscript{17}. We set these couplings in order to fit Miller’s\textsuperscript{29} lattice gauge results for the melting of the soft glue.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{potential.pdf}
\caption{The color Coulomb potential $V$ and the corresponding wave function $\psi$ for relativistic Klein-Gordon case. The interaction eq. (4) with $R = \hbar/2m_q$ was used. The ground state energy with $\alpha_s = 1$ corresponds to $E_{\text{ground}} = 0.645 \, m_q$. The minimum of the potential at the origin is at $-3m_q$ here.}
\end{figure}
In Fig. 1 we show that if we choose $\alpha_s = 1$ (effectively $\alpha_s = 2$ by the doubling in Eq. (12)) as would be required to enter the strong coupling region considered by Shuryak and Zahed$^{30}$ we bring the meson mass down by $1.36 m_q$ from their unperturbed $2 m_q$. However, we switch to the region of $\alpha_s \sim 0.5$, which is required by charmonium (intermediate coupling). In Table 1 we summarize the Coulomb binding for a few choices of $\alpha_s$.

In the case of the instanton molecule interaction the coupling constant $G = 3.825 \text{ GeV}^{-2}$ is dimensionful, so that its contribution to the molecule energy scales as $G m_q^3$. (Since we take $\alpha_s = 0.5$ and will find that with inclusion of the velocity-velocity interaction the effective $\alpha_s$ will be 1, powers of $\alpha$ will not affect our answer. We will use $m_q = 1 \text{ GeV}$, essentially the lattice result for $3 T_c$ and $3 T_c^{14}$, which works well in our schematic model.) Of course, in QCD the Polyakov line goes to zero at $T_c$, indicating an infinite quark mass below $T_c$; i.e., confinement. Just at $T_c$ the logarithmically increasing confinement force will not play much of a role because the dynamic confinement holds the meson size to $\sim \hbar/m_q c$, or $\sim 0.2 \text{ fm}$ with our assumption of $m_q = 1 \text{ GeV}$. (Later we shall see that the rms radius is $\sim 0.3 \text{ fm}$.) Since we normalize the instanton molecule force, extrapolating it through $T_c$, and obtain the color Coulomb force from charmonium, our $m_q$ is pretty well determined. However, our $m_q = 1 \text{ GeV}$ is for the unquenched system and at a temperature where the instanton molecules play an important role.

Given these caveats, we may still try to compare our Coulomb result with the lowest peak of Asakawa et al.$^{31}$ which is at $\sim 2 \text{ GeV}$ for $T = 1.4 T_c \sim 0.38 \text{ GeV}$ and for Petreczky at $\lesssim 5 T \sim 2.030 \text{ GeV}$ for $T = 1.5 T_c \sim 0.406 \text{ GeV}$ where we used the Asakawa et al. $T_c$. We wish to note that: (i) These temperatures are in the region of temperatures estimated to be reached at RHIC, just following the color glass phase (which is estimated to last $\sim 1/3 \text{ fm}/c$). Indeed, Kolb et al. begin hydrodynamics at $T = 360 \text{ MeV}$. (ii) These are in the region of temperatures estimated by SZ$^{12}$ to be those for which bound mesons form. We find these mesons to be basically at zero binding, because the instanton molecule interactions although important at $T = T_c$ (unquenched) because of the smallness ($\sim 1/3 \text{ fm}$) of the Coulomb $\bar{q}q$ states, will be unimportant at $T \sim 400 \text{ MeV}$ where the molecules are much bigger. In the lattice calculations the scalar, pseudoscalar, vector and axial-vector mesons come at the same energy.

Whereas there seems to be consistency between our estimates and the giant resonances of both Asakawa et al.$^{31}$ and of Petreczky$^{32}$, we should note that with the $m_q \sim 1.6 \text{ GeV}$ by Petreczky et al.$^{14}$ the mesons would
still be bound by $\sim 1.2$ GeV at $T = 1.5T_c$ (quenched). We do not think that the instanton molecules should play an important role at such a high ($\sim 400$ MeV) temperature, so this seems to be a discrepancy. Such a high binding would seem to invalidate the SZ$^{12}$ need for the mesons to break up around this temperature. Earlier we have argued for a lower $m_Q \sim 1$ GeV.

We are unable to extend our consideration to higher temperatures, where the situation may move towards the perturbative one, but we believe that lattice calculations do support our scenario that the QGP contains large component of bound mesons from $T \sim 170$ MeV up to $T \sim 400$ MeV.

| $\alpha_s$ | $\Delta E_{\text{Coulomb}}$ [GeV] | $\sqrt{\langle r^2 \rangle}$ [fm] | $\Delta E_{\text{4-point}}$ [GeV] |
|-----------|----------------------------------|----------------------------------|-------------------------------|
| 0.50      | -0.483                           | 0.360                            | -0.994                        |
| 0.55      | -0.595                           | 0.313                            | -1.385                        |
| 0.60      | -0.707                           | 0.276                            | -1.834                        |
| 1.00      | -1.355                           | 0.143                            | -7.574                        |

For $\alpha_s = 0.5$, which is the value required to bind charmonium up through $T = 1.6T_c$, we find that the Coulomb interaction binds the molecule by $\sim 0.5$ GeV, the instanton molecule interaction by $\sim 1.5$ GeV. However, the finite size of the $\psi \Gamma \psi$ of the instanton zero mode could cut the latter down by an estimated $\sim 50\%$. As in the usual NJL, there will be higher order bubbles, which couple the Coulomb and instanton molecule effects. We draw the Coulomb molecule in Fig. 2, where the double lines denote the Furry representation (Coulomb eigenfunction for quark and antiquark in the molecule).

Figure 2. Coulomb molecule. The wavy line on the left represents the momentum transfer necessary to produce the molecule. The double line denotes the Furry representation, i.e., Coulomb eigenstate.

The four-point instanton molecule interaction is shown in Fig. 3. There will be higher-order effects as shown in Fig. 4, of the 4-point interaction
used in higher-order between Coulomb eigenstates which always end in a 4-point interaction. The energy of the propagators has been lowered from the 2 GeV of the two noninteracting quarks to 1.5 GeV by the Coulomb interaction. The series beginning with terms in Figs. 2−4 is

\[ \Delta E = -0.5 \text{GeV} - 1 \text{GeV} F - \frac{1 \text{(GeV)}^2 F^2}{1.5 \text{GeV}} - \frac{1 \text{(GeV)}^3 F^3}{(1.5 \text{GeV})^2} + \cdots \]

\[ = -0.5 \text{GeV} - \frac{1 \text{GeV} F}{1 - \frac{1.5 \text{GeV}}{1 \text{GeV}}} \]  

Now \( \Delta E = -1.25 \text{ GeV} \) is accomplished for \( F = 0.5 \).

Working in the Furry representation, we have a −0.5 GeV shift already in the representation from the Coulomb wave functions. This means that we must obtain \( \Delta E = -1.5 \text{ GeV} \) to compensate for the \( 2m_q = 2 \text{ GeV} \), in order to bring the \( \pi \) and \( \sigma \) masses to zero. The four-point interaction is a constant, at a given temperature, so this problem is just the extended schematic model of nuclear vibrations (See Sec. V of Brown 33, where simple analytical solutions are given).

Our eq. (25) corresponds to the Tamm-Dancoff solution, summing loops going only forward in time. If \( \Delta E \) decreases −0.75 GeV in this approximation, then when backward going graphs are added 1, \( \Delta E \) will decrease by twice this amount 33, or the −1.5 GeV necessary to bring the \( \pi \) and \( \sigma \) energy to zero. Of course, forward and backward going loops are summed in the Bethe-Salpeter equation to give the NJL in the broken symmetry sector, but the actual summation is more complicated there, because the intermediate state energies are not degenerate. In the next section we shall show that the backward going graphs appear in lattice gauge calculations.
In detail, with our estimated $F = (0.75)^2$ and the 4-point energies from Table 1, our $\pi$ and $\sigma$ excitations without inclusion of backward going graphs are brought down 58% of the way from $-0.5$ GeV to $-2$ GeV; i.e., slightly too far. We have not made the adjustment down to 50%, because the uncertainties in our estimate of $F$, etc., do not warrant greater accuracy.

3. Conclusions

Shuryak and Zahed have discussed the formation of the mesonic bound states at higher temperatures, well above $T_c$. They pointed out that in the formation of the bound state, or any one of the molecular excited states, the quark-quark scattering length becomes infinite, similarly for the more strongly bound gluon-gluon states. In this way the nearly instantaneous equilibration found by RHIC can be explained. As we explained in the last section, lattice calculations seem to support the scenario of nearly bound scalar, pseudoscalar, vector and axial-vector excitation at $\sim 2T_c$ ($\sim 1.5$ times the quenched $T_c$).

In this work we are able to construct a smooth transition from the chirally broken to the chirally restored sector in terms of continuity in the masses of the $\sigma$ and $\pi$ mesons, vanishing at $T \to T_c$. In doing so we had to include relativistic effects. One of them – the velocity-velocity term related to Amper law for the interacting currents – nearly doubles the effective coupling. The spin-spin term happen to be very small. The crucial part of strong binding in our picture of $\bar{q}q$ mesons (or molecules) is the quasi-local interaction due to instanton molecules (the “hard glue”). We found that the tight binding of these mesons near $T_c$ enhances the wave function at the origin, and gives us additional understanding of the nonperturbative hard glue (epoxy) which is preserved at $T > T_c$.

Thus, we believe that the material formed in RHIC was at a temperature where although the matter is formally in a quark-gluon plasma phase, most of it is made of chirally restored mesons. Certainly this is not the weakly coupled quark-gluon plasma expected at high $T$.

Finally, in this paper we have focused on quantum mechanical binding effects in the vicinity of the critical temperature $T_c$ coming down from above. Nice continuity in the spectra of the light-quark hadrons – e.g., the pions and the $\sigma$ – across the phase boundary should also hold for other excitations such as the vector mesons $\rho, \omega, A_1$ which lie slightly above $\pi$ and $\sigma$ because of quantum corrections. Since going below $T_c$ from above involves a symmetry change from Wigner-Weyl to Nambu-Goldstone, there
is a phase transition and to address this issue, it would be necessary to treat the four-fermi interactions more carefully than in the pseudo-potential approximation adopted here. It seems plausible from the renormalization group point of view that the four-fermi interactions generated by the instanton molecules – attractive in all channels – will not only trigger the quark pairs to condense, thereby spontaneously breaking chiral symmetry but also bring down the mass of the vector mesons, as the temperature approaches $T_c$ from above. We will show in a future publication how this phenomenon can take place in a schematic model.

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