Reasoning-and-proving and world-related problems in the mathematics textbook of Kurikulum 2013 revised in 2017

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Abstract. Reasoning-and-proving is believed by mathematics educators worldwide as an essential part of teaching and learning mathematics. In addition, relating a mathematics problem to real-world situation can help student making sense of a mathematics concept. One of the main sources used in Indonesia is mathematics textbook of Kurikulum 2013 which is provided by the government for free. Therefore, this study aims to look at students’ opportunities to solve real-world related problems that involves reasoning-and-proving activities in the mathematics textbook focusing on the grade 7 textbook, chapter 1 about Number. The methodological approach and analytical framework used is a combination between framework of reasoning-and-proving activities and the use of real-world related problems. The results show that mathematical problems involving reasoning-and-proving activities provided by the textbook are still limited, counting for just around 20% while 80% of the problems cannot be identified as reasoning-and-proving. Slightly higher, real-world related problems in the textbook count for approximately 30% while 70% of them are not related to real-world situations. The results suggest that grade 7 students who use the textbook of Kurikulum 2013 have inadequate opportunities to learn real-world related problems and reasoning-and-proving activities from the book.

1. Introduction
In Indonesian curriculum of education, Kurikulum 2013, which was revised in 2017, one of suggested teaching approaches is problem solving. This approach obviously requires students to reason every measure that they derive from their thoughts and prior knowledge so as to provide proofs to support their arguments. It is also suggested for mathematics teachers to use project-based approach, in which the students are required to solve a problem related to real-world situations. As it is commonly believed by mathematics educators that reasoning-and-proving and real-world related problems play a pivotal role in mathematics classrooms, this study aims to identify the opportunity of the students to solve real-world and reasoning-and-proving related problems in the mathematics textbook provided in Kurikulum 2013.

Since the widespread of an analytic framework of reasoning-and-proving that can be utilised to analyse mathematics textbooks and to examine teachers’ professional development [1], there has been a growing amount of related research conducted worldwide. However, one that is related to Indonesian mathematics textbooks is still limited, which is why this study must be conducted. Moreover, interesting part of this study is that there is a combined analysis between real-world related problems and reasoning-and-proving activities in the textbook. Examples of each analysis are provided.
2. Literature review and the framework

Learning is often defined as a process that includes a frequent interplay between learners and their environments [2]. The equilibrium process combines conceptual knowledge to be understood with cognitive structures. As such, in the field of mathematics, there are several stages that have to be undertaken during the establishment of new knowledge. First of all, there should be an identification of facts that are arranged into patterns. Next, the patterns will lead to the formulation of conjectures, which are going to be tested in order to discover the possibility of counterexamples. Finally, there should be arguments explaining the reasons of the knowledge establishment [3, 4]. As per virtually mathematicians, these reasoning activities help them to construct a fundamental understanding in developing proofs, which is why reasoning is considered as the essential part of making sense in learning mathematics.

This study uses an analytic framework [5] that has been created and used in several previous studies. The original framework includes three primary components, which are mathematical, psychological and pedagogical. However, the main focus of this study is solely on the mathematical component (table 1).

| Reasoning-and-proving | Making mathematical generalisation | Providing support to mathematical claims |
|-----------------------|----------------------------------|-----------------------------------------|
| Identifying a pattern | Making a conjecture | Providing a proof | Providing a non-proof argument |
| • Plausible pattern | • Conjecture | • Generic example | • Empirical argument |
| • Definite pattern | • | • Demonstration | • Rationale |

There are two main parts of this reasoning-and-proving framework: making mathematical conjecture and providing support to mathematical claims. The former consists of identifying a pattern, which is categorised into plausible and definite patterns, and making a conjecture whereas the latter is made up of providing a proof in form of generic example or demonstration and providing a non-proof argument that can be an empirical argument or rationale.

On the other hand, real-world related problems are believed by several mathematics educators as way to trigger their students making sense of a mathematics concept because they can relate what they are learning to what the real situations are. As a result, more students will be attracted to learn mathematics since they think that mathematics concerns about the world problems as well [6]. In addition, students ought to work on some mathematics-based context tasks as if they were related to real-world situations albeit sometimes they have to neglect some factors that are related to the real situations of the tasks [7]. To illustrate this situation, a well-known context problem ignoring such factors describes that if there are 296 people who want to use a lift, which has a capacity of 14 people, during rush hours, how many times must the lift go up? The problem obviously ignores some facts such as people who probably use stairs or people’s weight that can affect the capacity of the lift.

Even though such real-world related problems cannot exactly pertinent to real-world situations, it is believed that they are still helpful to construct students’ understanding of a mathematics concept and to strengthen the students’ critical thinking. Besides, providing a dynamic context can bring the learners’ thinking to the imagined world [8]. This is the primary reason why in this study, I want to analyse the textbook based on the use of real-world related problems as well, so that I propose the following framework in figure 1.

Eventually, there are for possible problems are going to be found the textbook. First of all, the problems are real-world related and reasoning-and-proving activities. Secondly, the problems can be real-world related, but non-reasoning-and-proving activities. Thirdly, the problems are non-real-world related, but require reasoning-and-proving activities. The last is the problems that are not related to real world and have no reasoning-and-proving activities.
3. Methodology
This small-scale research focused on the grade 7 textbook of *Kurikulum 2013*, chapter 1 about Number [9]. By using the framework, mathematics questions in the textbook were examined to decide whether they are real-world related problems and reasoning-and-proving activities, real-world related problems but not reasoning-and-proving activities, not real-world related problems but reasoning-and-proving activities or not both real-world related problems and reasoning-and-proving activities (see figure 1). Examples of analysis will be provided in the next section.

4. Results

4.1. Examples of analysis
This part provides some examples of analysis, which reveal whether the questions involves reasoning-and-proving activities or not. Questions that solely require basic mathematical operations such as addition, subtraction, multiplication and division are not considered as a reasoning-and-proving activity. Since the language used in the analysed textbook is Bahasa Indonesia (Indonesian official language), the questions given here thus have been translated into English.

4.1.1. Real-world related problems and reasoning-and-proving activities. This category has very limited number of questions in the textbook. Here are some examples.

- Amalia’s height is 160 cm. Now her weight is 60 kg. In order to have an ideal height, she has to reduce her body mass index (BMI) to 20. How many kilograms should she lose? Explain your answer! (page 50)

- Edmund Halley (1656-1742) is the first person who saw a comet named after him “Halley Comet” in 1682. He precisely predicted that the comet will appear every 76 years afterwards.  
  a. According to Halley, what year did the comet appear in the past century? 
  b. When will the comet appear again? 
  c. Could Edmund Halley see the comet twice? Explain your answer! (page 77)
Obviously, these problems are related to real-life situations and require the students to reason and give proofs for their answers because in the end of the questions, there is a statement “explain your answer!”, which indicates that to solve the problems involves reasoning-and-proving activities [10].

4.1.2. Real-world related problems but non-reasoning-and-proving activities. These problems are related to real-life conditions, but they only need basic arithmetic operations to solve them. As such, they are not qualified to be categorised as a reasoning-and-proving activity.

- Mr. Abdul has borrowed IDR 700,000 of Mr. Boas’s money. Because Mr. Abdul’s son gets an accident, he has to borrow again IDR 200,000 from Mr. Boas.
  a. Describe this problem on the number line.
  b. Determine how much Mr. Abdul owes to Mr. Boas. (page 20)

- A tourist in Sunda Strait saw a dolphin jumping up to 4 metres above sea level. The fish then returns to the sea and dives up to 9 metres below sea level.
  a. Draw on the number line position of the dolphin from jumping to diving again.
  b. Determine the difference of the dolphin’s position while jumping and diving. (page 20)

4.1.3. Non-real-world related problems but involving reasoning-and-proving activities. Several reasoning-and-proving problems in the textbook are taken from TIMSS 1999 questions such the following example. This example can be categorised as making a conjecture. However, these questions are not real-world related problems because there are no real-life conditions mentioned.

- The closest value of $P$ in the following number line is...

  ![Number line diagram]

  a. 1.1
  b. 1.2
  c. 1.4
  d. 1.5 (page 45)

In spite of this, there are also some reasoning-and-proving examples that require students to provide empirical argument such as the following examples.

- If “$b$” represents a number, find the possible values of “$b$” so that $63b452 > 635452$. Explain your answer! (page 9)
- If “$c$” represents a number, find the possible values of “$c$” so that $c45279 > 63545$. Explain your answer! (page 9)

4.1.4. Non-real-world related problems and non-reasoning-and-proving activities

The following are some examples that only need basic operations taken from page 35 of the textbook.

- Find the result of the following multiplication:
  a. $400 \times (-60)$
  b. $(-40) \times 600$
  c. $(-400) \times (-600)$ (page 35)

- Find the result of:
  a. $5 \times (15 - 6)$
  b. $12 \times (-7) + (-16) \div (-2)$
  c. $-15 \div (-3) - 7 \times (-4)$ (page 35)

These questions are not categorised as reasoning-and-proving activities because students just need to counting in order to solve them. Although it might be classified into a proof by calculating, the reasoning activities involved are reasonably limited. In addition, the problems are not related to real-world situations at all.
4.2. Quantitative analysis
The results reveal that 59 of 298 questions analysed are categorised into reasoning-and-proving activities or approximately 20%, precisely 19.8% whereas 70.2% (239) of which are not considered as reasoning-and-proving activities. In terms of contextual problems, 30.54% (91 of 298) questions are related to real-world situations while the proportion of questions unrelated to real life count for 69.46% or 207 of 298. The combination of these categories as shown in figure 1 can be seen in the following table 2.

Table 2. Percentage of questions for each category

| Reasoning-and-proving activities | Real-world related problems | Non-real-world related problems |
|----------------------------------|-----------------------------|---------------------------------|
| 1.68% (5)                        | 18.12% (54)                 |
| Non-reasoning-and-proving activities | 28.86% (86)          | 51.34% (153)                    |

A closer look at the table reveals that more than a half of the problems analysed are not related to real-world situations and do not require reasoning-and-proving activities. Following this number, the proportion of real-world related problems but non-reasoning-and-proving activities counts for 28.86%, which is approximately 10% more than that of non-real-world related problems but reasoning-and-proving activities. Interestingly, questions that are classified as real-world related problems and reasoning-and-proving activities are only 1.68% or 5 of 298.

5. Discussions and recommendations
The findings imply that year 7 students who are studying Chapter 1 about Number using mathematics textbook of Kurikulum 2013 have a very limited opportunity to learn real-world related problems and to experience reasoning-and-proving activities. However, this study does not intentionally claim that the more real-world related questions and reasoning-and-proving activities are in a mathematics textbook, the better it is. Hence, I strongly believe that there is a proportional number of such questions in a mathematics textbook, which is suggested to be conducted in future research. As such, in order to give more opportunities to the students, the role of mathematics teachers is also crucial, meaning that mathematics teacher should have a good understanding about real-world related problems and reasoning-and-proving activities, which can be taught in teacher education programs, for example [11]. It is also important to underline that classroom implementation of the textbook plays a pivotal role in teaching and learning mathematics [12].

To conclude, giving opportunity to students to learn a real-world related problem can improve their ability to solve daily life problems that are connected to mathematics concepts and providing them with reasoning-and-proving activities can sharpen their problem-solving skill. These are what the students need in the 21st century education to assist them in facing the complexity of the world [13].

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