Some issues about neutrino processes in color superconducting quark matter

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Abstract. Several relevant issues in computing neutrino emissivity in Urca processes in color superconducting quark matter are addressed. These include: (1) The constraint on $u$ quark abundance is given from electric neutrality and the triangle relation among Fermi momenta for participants. (2) The phase space defined by Fermi momentum reduction of quarks is discussed in QCD and NJL model. (3) Fermi effective model of weak interaction is reviewed with special focus on its form in Nambu-Gorkov basis.

INTRODUCTION

Supernova explosions are among the most violent and spectacular phenomena in our universe [for reviews, see, e.g. [1, 2] and references therein, or see the talk by Sumiyoshi on this workshop [3]]. In later stage of the evolution of massive stars, thermal-nuclear fusions which power the stars stop at the formation of irons, the most stable nuclei. The collapse takes place when the pressure cannot sustain gravitational forces due to drop of temperature. Compact stars or sometime called neutron stars are one possible product of such a collapse. Just before collapse the fraction of protons reach the level of the most neutron-rich terrestrial matter, about 0.4, much larger than that in nuclear matter in neutron stars. Therefore the electron capture process $e^+ + p \rightarrow n + \nu_e$ can occur during the collapse because the Fermi surface of protons is high enough to open up phase space. During the explosion the total energy released in neutrino bursts can reach as much as 20% of the solar mass. After the explosion, the proton abundance falls to the lower level characteristic of a compact star. The temperature of the newborn compact star exceeds some tens of MeV [4]. The compact star cools down mainly by neutrino emissions in its earlier age and gamma-ray emissions when it gets very old [for reviews of neutron star cooling, see, e.g. [4, 5, 6]].

The baryon density in the core of a compact star is likely to reach several times the nuclear saturation density, $\rho_0 \sim 0.16 \text{ fm}^{-3}$ or $2.7 \times 10^{14} \text{ g/cm}^3$. At such a high density, nucleons in nuclear matter are crushed into their constituents, i.e. quarks and gluons. This deconfinement transition to quark matter was suggested by Collins and Perry already in 1975 [7] based on the asymptotic freedom in quantum chromodynamics. In the same paper they also mentioned the possibility that quark matter could be a superfluid or a superconductor resulting from the attractive inter-quark force in some channels. Barrois, Bailin and Love developed this novel idea and studied the unusual variant of superconductivity in quark matter, which we now call color superconductivity.
FIGURE 1. The main neutrino processes in quark matter in normal and color superconducting states.

(CSC) [8, 9]. They did their calculations in the framework of weak coupling approach and did not take into account the dynamic screening of magnetic gluons which are dominant agents in pairing quarks. Therefore the gap or equivalently the transition temperature they obtained are too small to be of relevance to any sizable observables. About fifteen years later the color superconductivity had been re-discovered by several groups who found the gap could be large enough to bring some real effects in compact stars [10, 11, 12, 13, 14, 15]. [For recent reviews on color superconductivity, see, for example, [16, 17, 18, 19, 20, 21, 22, 23, 24]. Also see Zhuang’s talk on this workshop based on Ref. [25, 26].]

Considering that neutrino emissions are main source of cooling for young compact stars and that the star core is very likely to be CSC quark matter, it is important to study neutrino emissions in a CSC and how they influence the thermal history of the stars. It is well known that the most efficient or fast cooling processes in quark matter of normal state are the direct Urca processes, less efficient are the modified Urca and bremsstrahlung processes or slow processes. They are shown in Fig. 1. The direct Urca in color superconducting quark matter was studied in Ref. [27, 28, 29, 31, 30]. In a CSC, besides these neutrino processes, there are other neutrino sources such as decays of Goldstone modes in the color-flavor-locked phase [32, 33] or in particular kaon condensed phase of color flavor locked phase [18]. In this paper, we will briefly address several issues about direct Urca processes.

CONSTRAINT ON U QUARK ABUNDANCE FOR DIRECT URCA PROCESSES

The name of Urca was coined by Gamow and Schoenberg when they studied the star cooling mechanism by neutrino emissions [35]. As Gamow commemorated, the name actually came from a Casino in Rio de Janeiro considering that Urca processes’
accounting for rapid energy loss in stars is just like the casino’s exhausting money from gamblers’ pockets [4]. Later they gave Urca a physical meaning, i.e. the abbreviation of unrecordable cooling agent.

In nuclear matter, the proton abundance is crucial to phase space for Urca processes to proceed. In normal state quark matter, the abundance of $u$ quarks plays an equal role. Suppose there are only light quarks and electrons in the system and they are ultra-relativistic at high baryon densities. Let us estimate the fraction of $u$ quarks from the electric charge neutrality and $\beta$ equilibrium condition,

$$\frac{2}{3}n_u = \frac{1}{3}n_d + n_e, \quad \text{(electric neutrality)}$$

$$\mu_d = \mu_u + \mu_e, \quad \text{(}\beta\text{ equilibrium)}$$

$$p_{Fd} < p_{Fu} + p_{Fe}. \quad \text{(triangle relation)} \quad (1)$$

Here $n_i$, $\mu_i$ and $p_{Fi}$ are the number density, chemical potential and Fermi momentum for particle $i$ respectively. The $\beta$ equilibrium condition means that it costs no energy to convert a $u$ quark and an electron to a $d$ quark and vice versa. In $\beta$ equilibrium the abundances of quarks and electrons are stable and do not change with time. The chemical potential is just the Fermi energy, so in $\beta$ equilibrium and at zero temperature there is no phase space for Urca processes because participating particles on their respective Fermi surfaces do not satisfy energy conservation. At non-zero temperatures, quarks and electrons can be excited above their Fermi surfaces of order $T$, energy conservation for Urca processes can be reached in a small range. The third condition of Eq. (1) is the triangle relation among Fermi momenta of light quarks and electrons. We know that at low temperature momenta of participants are close to their Fermi surfaces. So in order to satisfy momentum conservation $p_d = p_u + p_e$ (the neutrino momentum is negligible), three momenta form a triangle in a plane, so the length of each side should be be less than the sum of lengths of other twos.

Rewriting the electric neutrality condition as $2p_{Fu}^3 = p_{Fe}^3 + p_{Fd}^3$ where the number densities are given by $n_i = p_{Fi}^3 / (3\pi^2)$ for quarks and $n_e = p_{Fe}^3 / (3\pi^2)$ for electrons. Applying the triangle relation, we have

$$2p_{Fu}^3 > (p_{Fd} - p_{Fu})^3 + p_{Fd}^3.$$  

In terms of the ratio $p_{Fu}/p_{Fd}$, the above inequality becomes

$$3(p_{Fu}/p_{Fd})^3 - 3(p_{Fu}/p_{Fd})^2 + 3(p_{Fu}/p_{Fd}) - 2 > 0,$$

which leads to

$$p_{Fu}/p_{Fd} > 0.8. \quad (2)$$

From the $\beta$ equilibrium condition in (1) and the inequality (2), we obtain

$$p_{Fe}/p_{Fd} < 0.2. \quad (3)$$

The above inequality means the Fermi momentum of electrons must be less than 20% of that of $d$ quarks. Following (2), the fraction of $u$ quarks then satisfies the inequality

$$x = p_{Fu}^3 / (p_{Fu}^3 + p_{Fd}^3) > 0.8^3 / (0.8^3 + 1) \approx 1/3. \quad (4)$$
Hence the fraction of $u$ quarks must exceed $1/3$ for Urca processes to proceed. The factor $1/3$ comes naturally if the electron density is neglected.

**PHASE SPACE FOR DIRECT URCA PROCESSES**

In ultra-relativistic case, the $\beta$ equilibrium condition is not compatible with the triangle relation in (1) since the former requires that $p_{F,d} = p_{F,u} + p_{F,e}$. This means phase space for neutrino emissions is zero for direct Urca. If the quark-quark interaction is switched on, Fermi momenta are not equal to chemical potentials any more, instead they get negative corrections from Landau Fermi liquid property [36, 37],

$$p_{iF} = (1 - \kappa)\mu_i, \quad i = u, d,$$

where $\kappa = \frac{C_F\alpha_S}{2\pi}$, $\alpha_S$ is the strong coupling constant and $C_F = (N_c^2 - 1)/(2N_c)$ with the number of colors $N_c = 3$. The reduction in Fermi momentum means a non-zero effective mass on the Fermi surface. This opens up phase space characterized by the triangle inequality $p_{F,d} < p_{F,u} + p_{F,e}$. As illustrated in Fig. 2, two dashed circles denote the Fermi surfaces for free $d$ and $u$ quarks. They shrink to two smaller solid circles after the interaction is turned on. The amount of reduction in Fermi momentum for $d$ quarks is larger than that for $u$ quarks, then there is a triangle among Fermi momenta implying non-vanishing phase space. The corrections proportional to $\alpha_s$ is from the quark-quark forward scattering via one gluon exchange with zero quark mass.

In Ref. [30] we re-analyzed phase space for DU processes by deriving $p_F$ as a function of $\mu$ in QCD with non-zero quark masses and some other features not considered in Ref. [36, 37]. We also carry out the same task in the NJL model. When $\alpha_S$ and $\kappa$ are small, $\kappa$ as a function $\mu$ can be solved,

$$\kappa(\mu) = \left[\kappa(\mu_0) - \frac{C_F\alpha_S}{2\pi}\right] \frac{\mu_0^2}{\mu^2} + \frac{C_F\alpha_S}{2\pi},$$

(6)

One sees that if $\kappa(\mu_0) < \frac{C_F\alpha_S}{2\pi}$ at $\mu_0$, then $\kappa(\mu) < \frac{C_F\alpha_S}{2\pi}$ is always true for any values of $\mu$. This is the physical branch, the other one corresponds to $\kappa(\mu_0) > \frac{C_F\alpha_S}{2\pi}$. A similar result can also be found in NJL model,

$$\kappa(\mu) = \left[\kappa(\mu_0) - \frac{4G_S\mu^2}{3\pi^2}\right] \frac{\mu_0^2}{\mu^2} + \frac{4G_S\mu^2}{3\pi^2},$$

(7)

where $G_S$ is the coupling constant for the scalar and pseudoscalar channels in NJL model. As in the QCD case, $\kappa(\mu) < \frac{4G_S\mu^2}{3\pi^2}$ is the physical solution. Following Eq. (6) and (7), both physical solutions show that the Fermi momentum reduction coefficient $\kappa$ are monotonously increasing functions of the chemical potential. Such a trend implies that phase space for neutrino emissions is quenched at lower baryon densities. The property seems robust and independent of specific models in computing the Landau coefficients.
FERMI MODEL OF WEAK INTERACTION

In calculating the neutrino emissivity in Urca processes, it is convenient to use the Fermi effective model of weak interaction since the characteristic energy scale is about a few hundred MeV, much less than the W-boson mass. The interaction Hamiltonian of the model is

$$H_I = \frac{G}{\sqrt{2}} J^\mu J^\dagger_\mu,$$

where the weak current is

$$J^\mu(x) = \bar{\psi}_l \gamma^\mu (1 - \gamma^5) \psi_e + \bar{\psi}_d \gamma^\mu (1 - \gamma^5) \psi_d$$

$$J^\dagger_\mu(x) = \bar{\psi}_l \gamma_\mu (1 - \gamma^5) \tau_+ \psi_l + \bar{\psi}_q \gamma_\mu (1 - \gamma^5) \tau_+ \psi_q,$$

and

$$|M|^2 \rightarrow \frac{G^2}{2} [J^\mu J^\dagger_\mu][J^\nu J^\dagger_\nu]$$

$$\rightarrow \frac{G^2}{2} \left[ \bar{\psi}_l \gamma^\mu (1 - \gamma^5) \tau_+ \psi_l \bar{\psi}_l \gamma_\nu (1 - \gamma^5) \tau_- \psi_l \bar{\psi}_q \gamma_\mu (1 - \gamma^5) \tau_- \psi_q \bar{\psi}_q \gamma^\nu (1 - \gamma^5) \tau_+ \psi_q + \bar{\psi}_q \gamma_\mu (1 - \gamma^5) \tau_- \psi_q \bar{\psi}_q \gamma_\nu (1 - \gamma^5) \tau_+ \psi_l \right.$$}

$$+ \bar{\psi}_q \gamma^\mu (1 - \gamma^5) \tau_- \psi_q \bar{\psi}_q \gamma^\nu (1 - \gamma^5) \tau_+ \psi_l]$$

$$\bar{\psi}_q \gamma^\mu (1 - \gamma^5) \tau_+ \psi_q \bar{\psi}_q \gamma_\nu (1 - \gamma^5) \tau_- \psi_q \right],$$

where $\psi_l = (\psi_e, \psi_e)^T$ and $\psi_q = (\psi_u, \psi_d)^T$. Here the flavor matrices $\tau_\pm$ are defined by

$$\tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$
The flavor sector of neutrino emissivity in CSC is more complicated than that in the normal phase because of pairings in flavor space. Nambu-Gorkov (NG) basis is a convenient mathematical tool to describe pairings. We have to write down the quark-quark-W-boson vertices in NG basis. To this end, we enlarge the spinor space by introducing a couplet field consisting of a quark field and its charge conjugate partner,

$$\Psi = \begin{pmatrix} \psi \\ \psi_c \end{pmatrix}, \quad \overline{\Psi} = (\overline{\psi}, \overline{\psi_c}),$$

$$\psi_c = C \overline{\psi}^T, \quad \overline{\psi_c} = \psi^T C,$$

where \( C = i \gamma^2 \gamma^0 \) is the charge conjugate operator. Here we suppress the quark index \( q \) in quark fields. Now we rewrite the bilinear term \( \overline{\psi}_q \gamma^\mu (1 - \gamma_5) \tau_+ \psi_q \) in the weak current in Eq. (8),

$$\overline{\psi} \Gamma^\mu_+ \psi = \psi_c^T C \Gamma^\mu_+ C \overline{\psi}_c = - \left[ \overline{\psi}_c C^T \Gamma^\mu_+ C^T \psi_c \right]^T$$

$$= - \overline{\psi}_c C^T \Gamma^\mu_+ C^T \psi_c \equiv \overline{\psi}_c \Gamma^\mu_+ \psi_c,$$

where we defined \( \Gamma^\mu_+ = \gamma^\mu (1 - \gamma_5) \tau_+ \) and \( \Gamma^\mu_- \equiv -C \Gamma^\mu_+ T C = -\gamma^\mu (1 + \gamma_5) \tau_- \). Therefore the weak vertex in NG basis is,

$$\Gamma^\mu_{NG, \pm} \rightarrow \begin{pmatrix} \gamma^\mu (1 - \gamma_5) \tau_\pm & 0 \\ 0 & -\gamma^\mu (1 + \gamma_5) \tau_\mp \end{pmatrix}.$$

The polarization tensor [see, e.g. Eq. (12) of of Ref. [29]] involved in computing the neutrino emissivity is written as
Table 1. Flavor traces in neutrino emissivity in color superconducting phases. The order parameter is denoted by $\Delta$, $I$ are $SU(3)_c$ generators in color space with $(I)_jk = -i\epsilon_{ijk}$ and $J$ are $SU(3)_f$ generators in flavor space with $(J)_jk = -i\epsilon_{ijk}$.

|         | $\Delta$ | $\text{Tr}[(\tau_+\Delta\tau_+\Delta^\dagger)]$ | off-diagonal |
|---------|----------|-----------------------------------------------|--------------|
| CFL     | $J \cdot I$ | 2                                             | $\sqrt{\cdot}$ |
| Single Flavor | $\delta_{fu}\delta_{su}$ | 0                                             | $\times$ |

\[
\Pi^{\mu\nu}(Q) = T \sum_{k_0} \int \frac{d^3k}{(2\pi)^3} \text{Tr}_{NG,c,f,s}[\Gamma^{\mu}_{NG,\tau\cdot}s(K)\Gamma^{\nu}_{NG,\tau\cdot}S(K+Q)] \\
= \Pi^{\mu\nu}_{11} + \Pi^{\mu\nu}_{22} + \Pi^{\mu\nu}_{12} + \Pi^{\mu\nu}_{21} \tag{10}
\]

where the trace is over NG, color, flavor and Dirac indices. Note that the factor 1/2 arising from averaging in NG basis cancels a factor of 2 from another identical term with a trace $\text{Tr}[\Gamma^{\mu}_{NG,\tau\cdot}s(K)\Gamma^{\nu}_{NG,\tau\cdot}S(K+Q)]$, see Eq. (18) of Ref. [29] and the argument that follows. $\Pi^{\mu\nu}_{11}$ and $\Pi^{\mu\nu}_{22}$ are diagonal components of the polarization tensor which are there in the normal phase, while $\Pi^{\mu\nu}_{12}$ and $\Pi^{\mu\nu}_{21}$ are off-diagonal components proportional to condensate square and vanishing in the normal phase. As is shown in Tab. 1 for the flavor trace, one can easily verify that for single flavor or spin-one pairings [14, 38, 39, 40, 41, 42, 43, 44, 45] off-diagonal parts are absent, but it is not the case for spin-zero pairings such as the color-flavor-locked phase [see, e.g. Ref. [11]] or the 2-flavor CSC phase [see, e.g. Ref. [14, 46]]. The reason is the electric charge conservation [29].

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