Adaptive versus Static Multi-oracle Algorithms, and Quantum Security of a Split-key PRF

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Overview

- Adaptive versus Static Multi-oracle Algorithms
- Our Results
  - Adaptive-to-static Compiler
  - Quantum Security of a skPRF
- Summary
Adaptive versus Static Multi-oracle Algorithms
Oracle Algorithms
An algorithm $\mathcal{A}^O$ querying a (possibly randomized) function $\mathcal{O}$ for free.

Assumption: a fixed upper bound $q$ on $\#$queries to $\mathcal{O}$. 
Multi-oracle Algorithms

An algorithm $A^{O_1,\ldots,O_n}$ querying multiple functions $O_1,\ldots,O_n$ for free.

Assumption: fixed upper bounds $q_1,\ldots,q_n$ on #queries to $O_1,\ldots,O_n$. 
Static Multi-oracle Algorithms
A multi-oracle algorithm with predetermined querying order.

In contrast, an adaptive algorithm can decide which oracle to query at what point dependent on previous oracle responses.
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- easier for analysis
Static Multi-oracle Algorithms
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In contrast, an adaptive algorithm can decide which oracle to query at what point dependent on previous oracle responses.

Why static algorithms?
- easier for analysis
- (sometimes) better bounds
Example: Multi-oracle Algorithms

Attackers $A$ against cryptographic schemes in the random oracle model:

- Encryption/KEM: $O_1 = \text{random oracle}$ and $O_2 = \text{decrpt/decap oracle}$.
- Signature: $O_1 = \text{random oracle}$ and $O_2 = \text{signing oracle}$.
- Pseudorandom function: $O_1 = \text{random oracle}$ and $O_2 = \text{evaluation oracle}$. 
Our Results
Our result consists of two parts

In the first part, we give a black-box, straight-line, efficient compiler transforming any (classical or quantum) multi-oracle algorithm $A$ to a static one $B[A]$, with a mild blow-up on its query complexity.
Our Results

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In the first part, we give a black-box, straight-line, efficient compiler transforming any (classical or quantum) multi-oracle algorithm $\mathcal{A}$ to a static one $\mathcal{B}[\mathcal{A}]$, with a mild blow-up on its query complexity.

$\mathcal{A}$ makes $q_1, \ldots, q_n$ respective queries to $\mathcal{O}_1, \ldots, \mathcal{O}_n$
Our Results
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In the first part, we give a black-box, straight-line, efficient compiler transforming any (classical or quantum) multi-oracle algorithm \( \mathcal{A} \) to a static one \( \mathcal{B}[\mathcal{A}] \), with a mild blow-up on its query complexity.

\[ \mathcal{A} \text{ makes } q_1, \ldots, q_n \text{ respective queries to } \mathcal{O}_1, \ldots, \mathcal{O}_n \]
\[ \Rightarrow \mathcal{B}[\mathcal{A}](1^{q_1}, \ldots, 1^{q_n}) \text{ makes } nq_1, \ldots, nq_n \text{ respective queries only.} \]
Our Results

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In the first part, we give a black-box, straight-line, efficient compiler transforming any (classical or quantum) multi-oracle algorithm $A$ to a static one $B[A]$, with a mild blow-up on its query complexity.

- $A$ makes $q_1, \ldots, q_n$ respective queries to $O_1, \ldots, O_n$
  $\Rightarrow B[A](1^{q_1}, \ldots, 1^{q_n})$ makes $nq_1, \ldots, nq_n$ respective queries only.
- Applications: simplifying existing results [ABB+17, ABKM21] but also obtaining an enhanced bound [JST21].

In the second part, we show the QROM security of a particularly efficient skPRF by Giacon, Heuer and Poettering [GHP18].

- Consequently, an efficient KEM combiner is QROM-secure.
- Our analysis crucially relies on the abovementioned compiler.
Our Results

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In the first part, we give a black-box, straight-line, efficient compiler transforming any (classical or quantum) multi-oracle algorithm $\mathcal{A}$ to a static one $\mathcal{B}[\mathcal{A}]$, with a mild blow-up on its query complexity.

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- Applications: simplifying existing results [ABB++17, ABKM21] but also obtaining an enhanced bound [JST21].

In the second part, we show the QROM security of a particularly efficient skPRF by Giacon, Heuer and Poettering [GHP18].

- Consequently, an efficient KEM combiner is QROM-secure.
- Our analysis crucially relies on the abovementioned compiler.
Part 1: Adaptive-to-static Compiler
Our Results: The Adaptive-to-static Compiler

Our compiler works by running an interactive oracle algorithm $\mathcal{B}$ as an interface between $\mathcal{A}$ and oracles $\mathcal{O}_1, \ldots, \mathcal{O}_n$ and re-routing the adaptive queries to the pre-determined static ones.
A Naive Compiler

Consider $n = 2$. Suppose $\mathcal{A}$ makes $q_1, q_2$ queries to $\mathcal{O}_1, \mathcal{O}_2$ respectively.

- Let $B_{\text{naive}}[\mathcal{A}](1^{q_1}, 1^{q_2})$ query in order

$$ (\mathcal{O}_1 \mathcal{O}_2)^{q_1+q_2} := (\mathcal{O}_1 \mathcal{O}_2) \ldots (\mathcal{O}_1 \mathcal{O}_2) \Bigg|_{q_1+q_2 \text{ times}} $$

- Forward the query of $\mathcal{A}$ and do a dummy query for mis-match.
A Naive Compiler

Consider $n = 2$.
Suppose $A$ makes $q_1, q_2$ queries to $O_1, O_2$ respectively.

- Let $B^{naive}[A](1^{q_1}, 1^{q_2})$ query in order

\[(O_1 O_2)^{q_1+q_2} := (O_1 O_2) \ldots (O_1 O_2) \underbrace{. \ldots .}_{q_1+q_2 \text{ times}}\]

- Forward the query of $A$ and do a dummy query for mis-match.

$B^{naive}[A](1^{q_1}, 1^{q_2})$ makes $q_1 + q_2$ queries to both $O_1, O_2$:

What if $q_1 = q_2^2$? Then it makes $\approx q_1 > > q_2$ queries to both. We want $\approx q_1$ queries to $O_1$ and $\approx q_2$ queries to $O_2$ instead!!
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Suppose $\mathcal{A}$ makes $q_1, q_2$ queries to $O_1, O_2$ respectively.

- Let $B^{\text{naive}}[\mathcal{A}](1^{q_1}, 1^{q_2})$ query in order

$$
(O_1 O_2)^{q_1 + q_2} := (O_1 O_2) \ldots (O_1 O_2) \cdot
dfrac{q_1 + q_2 \text{ times}}{
}
$$

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A Naive Compiler

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Suppose $A$ makes $q_1, q_2$ queries to $O_1, O_2$ respectively.

Let $B^{\text{naive}}[A](1^{q_1}, 1^{q_2})$ query in order

$$ (O_1 O_2)^{q_1+q_2} := (O_1 O_2) \ldots (O_1 O_2) \ . \ \text{for } q_1+q_2 \text{ times} $$

Forward the query of $A$ and do a dummy query for mis-match.

$B^{\text{naive}}[A](1^{q_1}, 1^{q_2})$ makes $q_1 + q_2$ queries to both $O_1, O_2$:

What if $q_1 = q_2^2$? Then it makes $\approx q_1 \gg q_2$ queries to both. We want $\approx q_1$ queries to $O_1$ and $\approx q_2$ queries to $O_2$ instead!!!
Abstract formulation: the string $s = (12)^{q_1 + q_2} = 1212\ldots12$ is a supersequence of every $s' \in \text{Char}(q_1, q_2)$ where

$$\text{Char}(q_1, q_2) := \{s' \in \{1, 2\}^* : \text{every } \sigma \in \{1, 2\} \text{ occurs in } s' \text{ for } q_\sigma \text{ times}\}.$$
Abstract formulation: the string $s = (12)^{q_1 + q_2} = 1212 \ldots 12$ is a supersequence of every $s' \in \text{Char}(q_1, q_2)$ where

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Problem of naive construction: $s \in \text{Char}(q_1 + q_2, q_1 + q_2)$
A Combinatorial Approach

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Problem of naive construction: $s \in \text{Char}(q_1 + q_2, q_1 + q_2)$

Goal: find such $s$ in, say $\text{Char}(2q_1, 2q_2)$. 
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Example

- for $(q_1, q_2) = (1, 3)$, pick $s = 2221222$
  - $s \in \text{Char}(1, 6) \subseteq \text{Char}(2q_1, 2q_2)$
A Combinatorial Approach

Abstract formulation: the string $s = (12)^{q_1 + q_2} = 1212 \ldots 12$ is a supersequence of every $s' \in \text{Char}(q_1, q_2)$ where

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- for $(q_1, q_2) = (1, 3)$, pick $s = 2221222$
  - $s \in \text{Char}(1, 6) \subseteq \text{Char}(2q_1, 2q_2)$
  - $s' = 2221 \sqsubseteq s$
A Combinatorial Approach

Abstract formulation: the string $s = (12)^{q_1+q_2} = 1212 \ldots 12$ is a supersequence of every $s' \in Char(q_1, q_2)$ where

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Problem of naive construction: $s \in Char(q_1 + q_2, q_1 + q_2)$

Goal: find such $s$ in, say $Char(2q_1, 2q_2)$.

Example

- for $(q_1, q_2) = (1, 3)$, pick $s = 2221222$
  - $s \in Char(1, 6) \subseteq Char(2q_1, 2q_2)$
  - $s' = 2212 \sqsubseteq s$
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Abstract formulation: the string \( s = (12)^{q_1+q_2} = 1212 \ldots 12 \) is a supersequence of every \( s' \in \text{Char}(q_1, q_2) \) where

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Problem of naive construction: \( s \in \text{Char}(q_1 + q_2, q_1 + q_2) \)
Goal: find such \( s \) in, say \( \text{Char}(2q_1, 2q_2) \).

Example

\begin{itemize}
  \item for \((q_1, q_2) = (1, 3)\), pick \( s = 2221222 \)
  \item \( s \in \text{Char}(1, 6) \subseteq \text{Char}(2q_1, 2q_2) \)
  \item \( s' = 2122 \sqsubseteq s \)
\end{itemize}
A Combinatorial Approach

Abstract formulation: the string \( s = (12)^{q_1+q_2} = 1212 \ldots 12 \) is a supersequence of every \( s' \in \text{Char}(q_1, q_2) \) where

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Problem of naive construction: \( s \in \text{Char}(q_1 + q_2, q_1 + q_2) \)
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Example

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  - \( s' = 1222 \sqsubseteq s \)
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\[ \text{for } (q_1, q_2) = (1, 3), \text{ pick } s = 2221222 \]
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Example

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  \begin{itemize}
  \item \( s \in \text{Char}(1, 6) \subseteq \text{Char}(2q_1, 2q_2) \)
  \end{itemize}
\item for \((q_1, q_2) = (3, 10)\), pick \( s = 2221222122221222122212222 \)
  \begin{itemize}
  \item \( s \in \text{Char}(6, 19) \subseteq \text{Char}(2q_1, 2q_2) \)
  \end{itemize}
\end{itemize}
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Abstract formulation: the string \( s = (12)^{q_1+q_2} = 1212 \ldots 12 \) is a supersequence of every \( s' \in \text{Char}(q_1, q_2) \) where

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Problem of naive construction: \( s \in \text{Char}(q_1 + q_2, q_1 + q_2) \)

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- for \( (q_1, q_2) = (3, 10) \), pick \( s = 2221222122212221222122212221222122212221222122212221222 \)
  - \( s \in \text{Char}(6, 19) \subseteq \text{Char}(2q_1, 2q_2) \)
  - \( 1222212222122 \subseteq s \)
A Combinatorial Approach

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  - \( s \in \text{Char}(6, 19) \subseteq \text{Char}(2q_1, 2q_2) \)
  - \( 11122222222222 \sqsubseteq s \)
A Combinatorial Approach

Abstract formulation: the string $s = (12)^{q_1+q_2} = 1212 \ldots 12$ is a supersequence of every $s' \in Char(q_1, q_2)$ where

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Problem of naive construction: $s \in Char(q_1 + q_2, q_1 + q_2)$
Goal: find such $s$ in, say $Char(2q_1, 2q_2)$.

Example

- for $(q_1, q_2) = (1, 3)$, pick $s = 2221222$
  - $s \in Char(1, 6) \subseteq Char(2q_1, 2q_2)$
- for $(q_1, q_2) = (3, 10)$, pick $s = 2221222122221222122212222$
  - $s \in Char(6, 19) \subseteq Char(2q_1, 2q_2)$
Our Embedding Lemma

Let \((q_1, \ldots, q_n) \in \mathbb{N}^n\).

**Lemma**

There exists a string \(s \in \text{Char}(nq_1, \ldots, nq_n)\) such that every string \(s' \in \text{Char}(q_1, \ldots, q_n)\) is a subsequence of \(s\).
Our Embedding Lemma

Let \((q_1, \ldots, q_n) \in \mathbb{N}^n\).

Lemma

*There exists a string \(s \in \text{Char}(nq_1, \ldots, nq_n)\) such that every string \(s' \in \text{Char}(q_1, \ldots, q_n)\) is a subsequence of \(s\).*

Furthermore, such \(s\) is polynomial-time computable given \((1^{q_1}, \ldots, 1^{q_n})\) in unary representation.
Our Embedding Lemma

Proof.

Idea: distribute each symbol $\sigma \in [n]$ evenly within the interval $(0, n]$ and collect them from left to right.
Our Embedding Lemma

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Idea: distribute each symbol $\sigma \in [n]$ evenly within the interval $(0, n]$ and collect them from left to right.

$s \leftarrow \epsilon$

0 \quad \{1\} \quad \{1\} \quad \{1\} \quad \ldots \quad 1/q_1 \quad 2/q_1 \quad 3/q_1 \quad \ldots$

Figure: Constructing the string $s$ (here with $3/q_1 = 2/q_2$)
Our Embedding Lemma

Proof.

Idea: distribute each symbol $\sigma \in [n]$ evenly within the interval $(0, n]$ and collect them from left to right.

$s \leftarrow \epsilon$

$0 \quad \frac{1}{q_1} \quad \frac{1}{q_2} \quad \frac{2}{q_1} \quad \frac{3}{q_1} = \frac{2}{q_2} \quad \cdots$

Figure: Constructing the string $s$ (here with $3/q_1 = 2/q_2$)
Our Embedding Lemma

Proof.

Idea: distribute each symbol $\sigma \in [n]$ evenly within the interval $(0, n]$ and collect them from left to right.

$s \leftarrow s\|1 = 1$

Figure: Constructing the string $s$ (here with $3/q_1 = 2/q_2$)
Our Embedding Lemma

Proof.

Idea: distribute each symbol $\sigma \in [n]$ evenly within the interval $(0, n]$ and collect them from left to right. 

$$s \leftarrow s\|2 = 12$$

\[
\begin{align*}
0 & \quad \frac{1}{q_1} & \quad \frac{1}{q_2} & \quad \frac{2}{q_1} & \quad \frac{3}{q_1} = \frac{2}{q_2} & \quad \ldots \\
\end{align*}
\]

**Figure:** Constructing the string $s$ (here with $3/q_1 = 2/q_2$)
Our Embedding Lemma

Proof.
Idea: distribute each symbol $\sigma \in [n]$ evenly within the interval $(0, n]$ and collect them from left to right.

$$s \leftarrow s\|1 = 121$$

\[0 \quad \frac{1}{q_1} \quad \frac{1}{q_2} \quad \frac{2}{q_1} \quad \frac{3}{q_1} = \frac{2}{q_2} \quad \ldots\]

Figure: Constructing the string $s$ (here with $3/q_1 = 2/q_2$)
Our Embedding Lemma

Proof. Idea: distribute each symbol $\sigma \in [n]$ evenly within the interval $(0, n]$ and collect them from left to right.

$s \leftarrow s\|1 = 1211$

Figure: Constructing the string $s$ (here with $3/q_1 = 2/q_2$)
Our Embedding Lemma

Proof.

Idea: distribute each symbol \( \sigma \in [n] \) evenly within the interval 
\((0, n]\) and collect them from left to right.

\[
s \leftarrow s\|2 = 12112
\]

Figure: Constructing the string \( s \) (here with \( 3/q_1 = 2/q_2 \))
Our Embedding Lemma

Proof.

Idea: distribute each symbol $\sigma \in [n]$ evenly within the interval $(0, n]$ and collect them from left to right.

$s \leftarrow s \parallel \ldots = 12112\ldots$

Figure: Constructing the string $s$ (here with $3/q_1 = 2/q_2$)
Our Embedding Lemma

Proof.

Idea: distribute each symbol \( \sigma \in [n] \) evenly within the interval \((0, n]\) and collect them from left to right.

\[
s \leftarrow s\| \ldots = 12112 \ldots \quad \text{until we reach time } n
\]

Figure: Constructing the string \( s \) (here with \( 3/q_1 = 2/q_2 \))
Our Embedding Lemma

Proof.

Idea: distribute each symbol $\sigma \in [n]$ evenly within the interval $(0, n]$ and collect them from left to right.

$s = 12112 \ldots$ until reach time $n$

\begin{figure}
\centering
\begin{tikzpicture}
\draw[->] (0,0) -- (6,0);
\draw (0,0.1) -- (0,-0.1) node[below] {0};
\draw (1.5,0.1) -- (1.5,-0.1) node[below] {$1/q_1$};
\draw (3,0.1) -- (3,-0.1) node[below] {$1/q_2$};
\draw (4.5,0.1) -- (4.5,-0.1) node[below] {$2/q_1$};
\draw (6,0.1) -- (6,-0.1) node[below] {$3/q_1 = 2/q_2$};
\end{tikzpicture}
\caption{Constructing the string $s$ (here with $3/q_1 = 2/q_2$)}
\end{figure}
Part 2: Quantum-security of a skPRF
Main Applications: skPRF

A skPRF is a function $\mathcal{F}(k_1, \ldots, k_n, x)$ such that:

- for each $i$: $\mathcal{F}$ is pseudorandom as a function with key $k_i$. (technical constraint: attacker never query the same $x$ twice)
- Implication: skPRF $\Rightarrow$ KEM-combiner [GHP18]
Main Applications: skPRF

A skPRF is a function \( \mathcal{F}(k_1, \ldots, k_n, x) \) such that:

- for each \( i \): \( \mathcal{F} \) is pseudorandom as a function with key \( k_i \).
  (technical constraint: attacker never query the same \( x \) twice)

- Implication: \( \text{skPRF} \Rightarrow \text{KEM-combiner} \) [GHP18]

Efficient hash-based instantiation by [GHP18]:

\[
\mathcal{F}(k_1, \ldots, k_n, x) := H(g(k_1, \ldots, k_n), x) \text{ for “key-mixing” } g.
\]

Already proven \textbf{classically} secure, \textbf{quantum} security unknown.
Main Applications: skPRF

A skPRF is a function $F(k_1, \ldots, k_n, x)$ such that:

- for each $i$: $F$ is pseudorandom as a function with key $k_i$.
  (technical constraint: attacker never query the same $x$ twice)

- Implication: skPRF $\Rightarrow$ KEM-combiner [GHP18]

Efficient hash-based instantiation by [GHP18]:

$$F(k_1, \ldots, k_n, x) := H(g(k_1, \ldots, k_n), x)$$

for “key-mixing” $g$.

Already proven classically secure, quantum security unknown.

Theorem (Our result: quantum security of $F$)

In the QROM, any skPRF attacker with at most $q_F, q_H$ respective queries to $F, H$ has advantage at most $4q_H\sqrt{2q_F}\epsilon + 4q_F\sqrt{2q_H}\epsilon$. 
Quantum-security of a skPRF

Proof idea (for random function $R$ and auxiliary oracle $H'$):

- initial querying pattern: $H' \ldots H' \mathcal{F} H' \ldots H' \mathcal{F} H' \ldots$
- from left to right, replace every $H'$ to $H$ and every $\mathcal{F}$ to $R$
Quantum-security of a skPRF

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Let’s look at the losses for replacing $H'$ to $H$:

- The loss replacing $H'$ to $H$ in each block: $2q_{H,i} \sqrt{q_{F} \epsilon}$
Quantum-security of a skPRF

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- The loss replacing $H'$ to $H$ in each block: $2q_{H,i} \sqrt{q_F \epsilon}$
- summing up, loss: $\sum_i 2q_{H,i} \sqrt{q_F \epsilon}$
Quantum-security of a skPRF

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- summing up, loss: $\sum_i 2q_{H,i} \sqrt{q_F \epsilon}$
- without any compiling, $q_{H,i} \leq q_H$ gives

$$q_H(q_F + 1) \sqrt{q_F \epsilon}$$
Quantum-security of a skPRF

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- naive compiler: $\sum_i q_{H,i} \leq q_H + q_F$ and $q_F$ becoming $q_F + q_H$ gives
  $$2(q_H + q_F) \sqrt{(q_H + q_F) \epsilon}$$
Quantum-security of a skPRF

Proof idea (for random function $R$ and auxiliary oracle $H'$):

- initial querying pattern: $H' \ldots H' \mathcal{F}' H' \ldots H' \mathcal{F} H' \ldots$
- from left to right, replace every $H'$ to $H$ and every $\mathcal{F}$ to $R$

Let's look at the losses for replacing $H'$ to $H$:

- The loss replacing $H'$ to $H$ in each block: $2q_{H,i} \sqrt{q_F \epsilon}$
- summing up, loss: $\sum_i 2q_{H,i} \sqrt{q_F \epsilon}$
- without any compiling, $q_{H,i} \leq q_H$ gives $q_H(q_F + 1) \sqrt{q_F \epsilon}$
- naive compiler: $\sum_i q_{H,i} \leq q_H + q_F$ and $q_F$ becoming $q_F + q_H$ gives $2(q_H + q_F) \sqrt{(q_H + q_F) \epsilon}$
- our compiler: $\sum_i q_{H,i} \leq 2q_H$ and factor 2 blow-up on $q_F$, gives $4q_H \sqrt{2q_F \epsilon}$
Quantum-security of a skPRF

Proof idea (for random function $R$ and auxiliary oracle $H'$):

- initial querying pattern: $H' \ldots H' \mathcal{F} H' \ldots H' \mathcal{F} H' \ldots$
- from left to right, replace every $H'$ to $H$ and every $\mathcal{F}$ to $R$

Let’s look at the losses for replacing $H'$ to $H$:

- The loss replacing $H'$ to $H$ in each block: $2q_{H,i} \sqrt{q_F \epsilon}$
- summing up, loss: $\sum_i 2q_{H,i} \sqrt{q_F \epsilon}$
- without any compiling, $q_{H,i} \leq q_H$ gives $q_H(q_F + 1) \sqrt{q_F \epsilon}$
- naive compiler: $\sum_i q_{H,i} \leq q_H + q_F$ and $q_F$ becoming $q_F + q_H$ gives $2(q_H + q_F) \sqrt{(q_H + q_F) \epsilon}$
- our compiler: $\sum_i q_{H,i} \leq 2q_H$ and factor 2 blow-up on $q_F$, gives $4q_H \sqrt{2q_F \epsilon}$

Our proof crucially relies on the compiler. ✔
Summary
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Our result consists of two parts

In the first part, we give a compiler transforming a multi-oracle algorithm $\mathcal{A}$ with $(q_1, \ldots, q_n)$ queries to a static one with $(nq_1, \ldots, nq_n)$ queries.

- simplifying existing results [ABB$^{+}$17, ABKM21] but also obtaining an enhanced bound [JST21].
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In the second part, we give the QROM security of the hash-based skPRF constructed by Giacon, Heuer and Poettering [GHP18].

- Consequently, the KEM combiner using $\mathcal{F}$ is QROM-secure.
- Our analysis crucially relies on the abovementioned compiler.
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- Consequently, the KEM combiner using $\mathcal{F}$ is QROM-secure.
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Take away: if you have adaptive adversaries, use our compiler!
Thanks for your listening!

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