Secure Wireless Transmission for Reconfigurable Intelligent Surface Aided Full Duplex Systems

Pengxin Guan, Yiru Wang and Yuping Zhao

Abstract—This letter considers the secure communication in a reconfigurable intelligent surface (RIS) aided full duplex (FD) system. An FD base station (BS) serves an uplink (UL) user and a downlink (DL) user simultaneously over the same time-frequency dimension assisted by a RIS in the presence of an eavesdropper. In addition, the artificial noise (AN) is also applied to interfere the eavesdropper’s channel. We aim to maximize the sum secrecy rate of UL and DL users by jointly optimizing the transmit beamforming, receive beamforming and AN covariance matrix at the BS, and passive beamforming at the RIS. To handle the non-convex problem, we decompose it into tractable subproblems and propose an efficient algorithm based on alternating optimization framework. Specifically, the receive beamforming is derived as a closed-form solution while other variables are obtained by using semidefinite relaxation (SDR) method and successive convex approximation (SCA) algorithm. Simulation results demonstrate the superior performance of our proposed scheme compared to other baseline schemes.

Index Terms—RIS, full-duplex, secure communication, alternating optimization.

I. INTRODUCTION

RECONFIGURABLE intelligent surface (RIS) has been recognized as a key technology for 6G since it is capable of improving spectrum efficiency (SE) for wireless communication systems [1]. Recently, RIS is also implemented to enhance the physical layer security (PLS) performance [2]–[5]. By intelligently configuring the phase shifts at the RIS, which is also called passive beamforming, the signal power can be enhanced at legitimate receivers and attenuated at eavesdropper. In [2] and [3], the authors maximized secrecy rate by jointly optimizing the active and passive beamforming in a RIS-aided system. To further improve the PLS performance, artificial noise (AN) is also applied to the RIS assisted communication systems [4], [5].

However, the literature mentioned above only considered the half-duplex (HD) communication systems, which cause the SE loss. The full-duplex (FD) technology enables signal transmission and reception over the same time-frequency dimension and thus can improve the SE compared with HD [6]. The RIS-aided FD systems have gained some attention, such as sum rate maximization [7] and transmit power minimization [8]. Nevertheless, the secure communication in RIS-aided FD systems has not been well studied. The authors in [9] studied the robust secure beamforming in a RIS-aided point-to-point FD system. The authors in [10] used the deep reinforcement learning to maximize the sum secrecy rate (SSR) in a RIS assisted FD system. However, the AN, which has been shown to enhance system security, was not considered in these works. In addition, the authors in [11] only optimized the uplink secrecy rate optimization in a RIS aided FD system, while both uplink and downlink security needs to be considered in FD systems generally [12], [13].

In this letter, we consider a RIS aided FD system, where a FD base station (BS) communicates with an uplink (UL) user and a downlink (DL) user simultaneously in the presence of an eavesdropper. We aim to maximize the SSR by jointly optimizing the transmit beamforming, receive beamforming and AN covariance matrix at the BS, and passive beamforming at the RIS. The main contributions are summarized as follows:

• We formulate a SSR maximization problem, subjected to the maximum transmit power of the BS and the constant modulus constraints of RIS elements.
• We decompose the non-convex problem into tractable subproblems and propose an efficient algorithm based on alternating optimization (AO) framework.
• Simulation results show the benefits of our proposed scheme compared with other benchmark schemes.

Notation: $|x|$ and $\arg(x)$ are the absolute value of $x$ and phase of a complex number $x$. $\|x\|$ is the Euclidean norm of $x$. $x_{m}$ is the $m$-th element of $x$. $\text{Tr}(X)$, $X^H$, $\text{rank}(X)$ and $[X]_{i,j}$ denote the trace, conjugate transpose, rank and $(i,j)$-entry of the matrix $X$, respectively. $\text{diag}(x)$ is a diagonal matrix with the entries of $x$ on its main diagonal. $X \succeq 0$ denotes that $X$ is positive semidefinite. $\nabla$ represents the gradient operation.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model Description

As shown in Fig. 1, we consider a RIS-aided FD secure communication system, which consists of an FD BS, an UL

![Fig. 1. Illustration of the RIS aided FD secure system.](image-url)
user, a DL user, a RIS and an eavesdropper (Eve). The FD BS is equipped with $N_T$ transmit antennas and $N_R$ receive antennas, while the two users and Eve are equipped with a single antenna. The RIS with $M$ passive elements is deployed to improve system security performance.

The transmitted signal from the BS is given by

$$x = w_D + v,$$  \hspace{1cm} (1)

where $w \in \mathbb{C}^{N_T \times 1}$ is the transmit beamforming at the BS and $s_P$ is the DL signal information with normalized power. In addition, $v \in \mathbb{C}^{N_T \times 1} \sim \mathcal{C}\mathcal{N}(0, I)$ is the AN vector and $V$ is the corresponding covariance matrix.

Denote $h_B \in \mathbb{C}^{M \times N_T}$, $h_U \in \mathbb{C}^{N_R \times M}$, $h_D \in \mathbb{C}^{1 \times M}$, $h_E \in \mathbb{C}^{1 \times N_T}$, $h_B^H \in \mathbb{C}^{N_T \times M}$, $h_U^H \in \mathbb{C}^{M \times N_R}$, $h_U^H \in \mathbb{C}^{M \times N_T}$, $h_U^H \in \mathbb{C}$, $h_U \in \mathbb{C}$, and $h_U \in \mathbb{C}$ as the channel from the BS to the RIS, from the BS to the UL user, from the RIS to the UL user, from the BS to the DL user, from the UL user to the Eve, from the UL user to the DL user, from the BS to the Eve, from the BS to the DL user, and from the UL user to the RIS, respectively. $h_B \in \mathbb{C}^{N_R \times M}$ represents the residual SI channel at the BS [7]. $\Theta = \text{diag} \{ \phi_1, \phi_2, \cdots, \phi_M \}$ is the phase-shift matrix of the RIS and $\phi_m = e^{j\theta_m}$, where $\theta_m \in [0, 2\pi)$ denotes the phase-shift of the $m$-th reflecting element. Similar to [2]–[3], we assume full channel state information of the system is available.

Then, the signal received at the BS can be expressed as

$$y_U = w_R^H (h_B h_U + h_U h_B) y_U + w_R^H h_B y_B + n_D,$$ \hspace{1cm} (2)

and

$$y_E = (h_D h_B + h_B h_D) y_U + (h_D h_U + h_U h_D) y_E + n_E,$$ \hspace{1cm} (3)

where $n_D \sim \mathcal{C}\mathcal{N}(0, \sigma_D^2)$ and $n_E \sim \mathcal{C}\mathcal{N}(0, \sigma_E^2)$ are the AWGN at the BS and Eve, respectively.

Further, the signal-to-interference-plus-noise ratio (SINR) at the DL and UL users can be derived as

$$\gamma_U = \frac{P_U \|w_R \|^2}{\|h_B w_B\|^2 + \|w_R h_B V h_B^H + \sigma_D^2\|w_R\|^2},$$ \hspace{1cm} (5)

and

$$\gamma_E = \frac{P_U \|h_B^H w_B\|^2}{\|h_B^H h_U + h_U^H h_B \|^2 + \sigma_E^2},$$ \hspace{1cm} (6)

where $\hat{\mathbf{h}}_B = h_B^H\Theta h_U + h_U^H h_B$ for simplification.

We assume the Eve decodes the DL and UL signal separately, as in [12], [13]. Then, the SINR of the DL and UL signals at the Eve can be expressed as

$$\gamma_D = \frac{P_U \|h_B h_U + h_U^H h_B\|^2}{\|h_B^H w_B\|^2 + \|h_B^H h_U + h_U^H h_B + \sigma_E\|^2},$$ \hspace{1cm} (7)

and

$$\gamma_E = \frac{P_U \|h_B^H h_U + h_U^H h_B\|^2}{\|h_B^H w_B\|^2 + \|h_B^H h_U + h_U^H h_B + \sigma_E\|^2}.$$ \hspace{1cm} (8)

where $\hat{\mathbf{h}}_B = h_B^H \Theta h_B + h_B^H h_B$.

According to [12] and [13], the security rate of both links are defined as $R_i^{\text{Sec}} = \log_2(1 + \gamma_i) - \log_2(1 + \gamma_i^E)$, $i = \{U, D\}$, where $\cdot^* = \max(\cdot, 0)$.

**B. Problem Formulation**

In this work, we aim to maximize the SSR by jointly optimizing the transmit beamforming, receive beamforming, AN covariance matrix and passive beamforming. Mathematically, the optimization problem is formulated as

$$\mathcal{P}_1 : \max_{w, w_B, \Theta, V} \sum_{i = \{U, D\}} R_i^{\text{Sec}}$$ \hspace{1cm} (9a)

s.t. $Tr(ww^H + V) \leq P_{\text{max}},$ \hspace{1cm} (9b)

$\|w\|^2 = 1,$ \hspace{1cm} (9c)

$V \succeq 0,$ \hspace{1cm} (9d)

$|\phi_m| = 1, m = 1, \cdots, M,$ \hspace{1cm} (9e)

where $P_{\text{max}}$ is the maximum transmit power at the BS and (9e) imposes a unit modulus constraint on the RIS.

Note that the operator $[\cdot]^*$ has no impact on the optimization and hence it will be omitted in subsequent analysis. Problem $\mathcal{P}_1$ is still challenging to solve due to the non-convex objective function and unit modulus constraint, and then we will propose an efficient AO algorithm in the subsequent section.

**III. SUM SECURITY RATE MAXIMIZATION ALGORITHM DESIGN**

**A. Optimizing the Receive Beamforming Vector $w_R$**

In this subsection, we aim to optimize the receive beamforming $w_R$ with fixed $w$, $v$ and $\Theta$. Note that $w_R$ is only related to the UL SINR at the BS, thus the receive beamforming optimization problem can be expressed as

$$\mathcal{P}_2 : \max_{w_R} \gamma_U$$ \hspace{1cm} (10a)

s.t. $\|w_R\|^2 = 1.$ \hspace{1cm} (10b)

The problem $\mathcal{P}_2$ is a generalized eigenvalue problem. Let $W = \text{diag}(w, w, w)$ and then the optimal $w_R$ can be given as [11]

$$w_R = \frac{[h_B h_B^H + \sigma_B^2 I_N]^{-1}(h_B \Theta h_U + h_U)}{\|h_B h_B^H + \sigma_B^2 I_N\|^{-1}(h_B \Theta h_U + h_U)}.$$ \hspace{1cm} (11)

**B. Optimizing the Transmit Beamforming Vector $w$ and AN Covariance Matrix $V$**

In this subsection, we focus on the optimization of $w$ and $V$, while fixing the other variables. To facilitate formulation, we define $h_1 = \text{diag}(w_B^H h_B) h_U$, $h_2 = h_B^H w_R$, $h_3 = \text{diag}(h_B^H h_B) h_B$, $h_4 = h_U^H h_U$, $h_5 = \text{diag}(h_B^H h_B) h_U$, $h_6 = \text{diag}(h_B^H h_B) h_U$, $h_7 = h_B^H h_B$, $h_8 = h_B^H h_B$, $h_9 = h_B^H h_B$, $h_9 = h_B^H h_B$, and $q = [\phi_1, \cdots, \phi_M, 1]^H$.

\[1\]

If the achievable secrecy rate on a certain communication link is negative, we can turn off the transmission of this link and apply our proposed algorithm to complete the system optimization again, as in [13].
and \( Q = qq^H \). Then, after some intuitive operations, we can formulate transmit beamforming vector and AN covariance matrix optimization problem as

\[
P_3 : \max_{W, V} F(W, V) - G(W, V) \quad \text{subject to } \begin{align*}
\text{rank}(W) &= 1, \\
\text{Tr}(W + V) &\leq P_{\text{max}}, \\
W &\geq 0, \\
V &\geq 0,
\end{align*}
\]

where \( F(W, V) \) and \( G(W, V) \) are the simplified notations of \( F(W, V, Q) \) and \( G(W, V, Q) \) with given \( Q \), which are shown at the bottom of this page. Note that \( F(W, V) \) and \( G(W, V) \) are both concave with respect to \( W \) and \( V \), and thus \((14a)\) is a difference-of-concave (DC) function. The problem \( P_3 \) is difficult to solve due to the non-convex objective function and rank-one constraint. However, we can utilize the first-order Taylor expansions to approximate \( G(W, V) \) around local point \((W^{(n)}, V^{(n)})\) at the \( n\)-th iteration, which can be written as

\[
G(W, V) \leq G(W^{(n)}, V^{(n)}) + \text{Tr}[V^{(n)} W^{(n)} \nabla^2 G(W^{(n)}, V^{(n)})] \\
+ \text{Tr}[V^{(n)} W^{(n)} (V - V^{(n)})] = G(W, V)|_{W^{(n)}, V^{(n)}},
\]

where \( G(W, V)|_{W^{(n)}, V^{(n)}} \) is a global upper bound of \( G(W, V) \). By dropping the rank-one constraint, the approximated problem at the \( n\)-th iteration can be reformulated as

\[
P_3' : \max_{W, V} F(W, V) - G(W, V)|_{W^{(n)}, V^{(n)}} \quad \text{subject to } \begin{align*}
(14c)-(14e).
\end{align*}
\]

Problem \( P_3' \) is a convex optimization problem, and thus CVX can be used to obtain the optimal solution \( W^{(n+1)} \) and \( V^{(n+1)} \). By iteratively solving problem \( P_3' \) based on the successive convex approximation (SCA) method, the objective function in \((14a)\) will increase and converges [15].

\textbf{Proposition 1:} The optimal transmit beamforming matrix of the Problem \( P_3' \) satisfies rank\((W^*) \leq 1\).

\textbf{Proof:} Please refer to the Appendix.

According to the Proposition 1, when the BS transmits the DL information, i.e., \( W \neq 0 \), we get rank\((W^*) = 1 \) and then the eigenvalue decomposition (EVD) can be applied to recover the transmit beamforming vector \( w^* \).

\section*{C. Optimizing the Phase Shift Matrix \( \Theta \)}

Now, we investigate the optimization of phase shift matrix \( \Theta \) with the other variables given. The phase-shifts of RIS optimization problem can be rewritten as

\[
P_4 : \max_{Q} F(Q) - G(Q) \quad \text{subject to } \begin{align*}
\text{rank}(Q) &= 1, \\
q_{m,m} &= 1, m = 1, \ldots, M + 1,
\end{align*}
\]

where \( F(Q) \) and \( G(Q) \) are the simplified notations of \( F(W, V, Q) \) and \( G(W, V, Q) \) with given \( W \) and \( V \). Note that \( F(Q) \) and \( G(Q) \) are both concave with respect to \( Q \). Then, similar to the optimization of transmit beamforming and AN covariance matrix, we can apply the SCA algorithm to solve this DC problem. Specifically, by using the first-order Taylor approximation for the concave function \( G(Q) \), we have

\[
G(Q) \leq G(Q^{(i)}) + \text{Tr}[V^{(i)} H^H G(Q)(Q - Q^{(i)})] \\
= G(Q^{(i)}),
\]

where \( Q^{(i)} \) is the solution at the \( i\)-th iteration. By dropping the rank-one constraint based on SDR method [2], we get

\[
P_4' : \max_{Q} F(Q) - G(Q)|_{Q^{(i)}} \quad \text{subject to } \begin{align*}
(17b)-(17c).
\end{align*}
\]

The problem \( P_4' \) is a convex semidefinite program which can be solved directly by exploiting the convex optimization tool, e.g., CVX. Using the SCA algorithm to iteratively solve the problem \( P_4' \), the objective function in \((17a)\) will increase and converges [15]. Then, we can apply the Gaussian randomization method to get a high-quality solution \( q^* \) [2]. Further, the phase-shifts of the RIS can be given by

\[
\theta_m = -\arg(q_m^*/q_{M+1}^*), \quad m = 1, \ldots, M.
\]

The corresponding phase shift matrix can be expressed as \( \Theta^* = \text{diag}(e^{i\theta^*}) \). However, the SDR method normally obtain a suboptimal solution to the phase shift optimization subproblems, which causes that the objective function’s value of the problem \( P_1 \) may not be non-increasing. Hence, in order to ensure the convergence of our algorithm, we only update the phase shift \( \Theta^* \) when it can improve the SSR of the system.

\section*{D. Overall Algorithm and Analysis}

The overall algorithm is implemented by iteratively solving the three subproblems, which is summarized in Algorithm 1.

Based on AO framework, Algorithm 1 will generate a monotonically increasing (at least nondecreasing) SSR sequence [2], [11]. Hence, our proposed algorithm is guaranteed to converge because the system SSR have a upper bound due to the limited resources.

The complexity of optimizing the receive beamforming vector is \( O(N_2^4) \) due to the matrix inversion in \((11)\). The complexity of solving the transmit beamforming vector and AN covariance matrix optimization subproblem is \( O(2I_1N_2^4) \) due to solving the SDP \( \text{[10]} \), where \( I_1 \) is the number of iterations for SCA algorithm. Similarly, the complexity of optimizing the phase shift is \( O(I_2(M+1)^3) \), where \( I_2 \) is the corresponding

\[
F(W, V, Q) = \log_2[P_1\text{Tr}(QH_1) + \text{Tr}[H_2(W + V)] + \sigma_0^2] + \log_2[P_1\text{Tr}(QH_4) + \text{Tr}[h_4^H Qh_3(W + V)] + \sigma_0^2] + \log_2[P_1\text{Tr}(QH_8) + \text{Tr}[h_8^H Qh_5(W + V)] + \sigma_0^2] + \log_2[\text{Tr}[h_5^H Qh_6(W + V)] + \sigma_0^2].
\]

\[
G(W, V, Q) = \log_2[\text{Tr}[H_2(W + V)] + \sigma_0^2] + \log_2[P_1\text{Tr}(QH_4) + \text{Tr}[h_4^H Qh_3(W + V)] + \sigma_0^2] + 2\log_2[P_1\text{Tr}(QH_8) + \text{Tr}[h_8^H Qh_5(W + V)] + \sigma_0^2].
\]
Algorithm 1 Alternating Optimization Algorithm for $\mathcal{P}1$

1. Initialization: set initial $\theta^{(0)}$, $\mathbf{w}^{(0)}$, $\mathbf{v}^{(0)}$, $\mathbf{w}_R^{(0)}$ and $k = 0$.
2. repeat
3. Update $k = k + 1$;
4. Solve problem $\mathcal{P}4$ based on SCA algorithm and then update phase shift $\theta^{(k)}$;
5. Solve problem $\mathcal{P}3$ to obtain $\mathbf{w}^{(k)}$ and $\mathbf{v}^{(k)}$ based on SCA algorithm and then get $\mathbf{w}_R^{(k)}$ by EVD;
6. Update the receive beamforming vector $\mathbf{w}_R^{(k)}$ by (11);
7. until The fractional increase of the SSR is below a threshold $\epsilon > 0$ or the maximum number of iterations $K$ is reached;
8. Output: $\mathbf{w}^*$, $\mathbf{v}^*$, $\mathbf{w}_R^*$ and $\theta^*$.

number of iterations. Hence, the overall complexity of our proposed algorithm is $O(L(N_R^3 + 2I_1N_S^2.5 + I_2(M + 1)^3.5))$, where $L$ is the number of iterations for AO algorithm.

IV. SIMULATION RESULTS

In this section, numerical simulations are provided to demonstrate the performance advantages of the proposed algorithm. We assume that the locations of BS, RIS, UL user, DL user and Eve are (0m, 0m), (20m,10m), (60m, 0m), (40m, 5m) and (20m, -5m), respectively. We assume that there are $N_T = 4$ transmit antennas and $N_R = 4$ receive antennas at the BS. The large-scale fading is modelled by $PL(d) = PL_0(d/d_0)^{-\sigma}$, where $PL_0 = -30$dB is the path loss at the reference distance $d_0 = 1$m, $d$ is the distance, and $\sigma$ is the path-loss exponent.

The path-loss exponent of the channels related to the RIS is set to 2.5 and that of other channels are set to 4 [11]. Similar to [3], [7], for small-scale fading, Rayleigh fading is assumed for each link. Besides, each entry of residual self-interference matrix $\mathbf{H}_BB$ is identically and independently distributed as $CN(0, \sigma_{SI}^2)$, where $\sigma_{SI}^2$ depends on the capability of the SIC techniques [13].

The proposed FD system with RIS and AN (FD-RIS-AN) is compared to the following benchmark schemes:

- **HD system with RIS and AN (HD-RIS-AN):** In this scheme, the UL and DL transmissions are allocated with half of the time as compared to the FD scheme. In addition, both the AN and RIS are adopted to improve the system SSR performance.
- **FD system with RIS but without AN (FD-RIS-NoAN):** The AN is not applied in the system, while only RIS is used to enhance the FD system SSR performance.
- **FD system with AN but without RIS (FD-AN-NoRIS):** The RIS is not deployed in the system, while only the AN is used to improve the system SSR performance.

Fig. 2 investigates the convergence performance of the proposed algorithm with different number of RIS elements. From the figure, it can be seen that as the number of iterations continues to increase, the system SSR increases to convergence in all scenarios, which verifies the effectiveness of the proposed algorithm.

Fig. 3 illustrates the SSR performance versus the number of reflecting elements. We can find that the system SSR increases with $M$ in the RIS assisted schemes, which indicates that with more reflecting elements, the more degrees of freedom of system design can be exploited and thus achieves higher gains. In addition, the SSR of proposed FD-RIS-AN scheme is always better than that of FD-RIS-NoAN scheme, revealing the effectiveness of AN in improving security performance.

Fig. 4 shows the SSR of different schemes versus the maximum transmit power of the BS. With the increase of transmit power, the SSR of all schemes increase. It is worth noting that the proposed scheme achieves the best performance, which shows the superiority of both AN and RIS assisted FD system in terms of security communications.

The impact of residual SI variance of the BS on the SSR performance is presented in Fig. 5. The SSR performance of all FD schemes decreases with the increase of $\sigma_{SI}^2$, which indicates the powerful SIC methods is necessary for the application of the FD system.

V. CONCLUSION

This letter studied the SSR maximization for RIS-aided FD systems and the AN is also considered. We propose an efficient algorithm based on AO to optimize the transmit beamforming, receive beamforming, AN covariance matrix at the BS and phase-shifts at the RIS iteratively. Simulation results show the performance advantages of the proposed FD system assisted by both RIS and AN compared with other baseline schemes.
where constraint (21b), (23c), (23d), (23f) and (14c), respectively.

Fig. 4. SSR performance versus the maximum transmit power of BS.

Fig. 5. SSR performance versus the residual SI variance.

APPENDIX

PROOF OF PROPOSITION 1

To prove the Proposition 1, we first rewrite the equivalent problem of $\mathcal{P}_Y$ as follows

$$
\mathcal{P} : \min_{\mathbf{W}, \mathbf{V}, \eta} \mu
$$

s.t. $- \sum_{i=1}^{4} \log_{2} (\eta_i + \sigma_i^2) + G(\mathbf{W}, \mathbf{V}|\mathbf{W}^{(n)}, \mathbf{V}^{(n)}) \leq \mu, \quad (21a)$

$$
\eta_1 \leq P_U \text{Tr}(\mathbf{QH}_1) + \text{Tr}[\mathbf{H}_2(\mathbf{W} + \mathbf{V})],
$$

$$
\eta_2 \leq P_U \text{Tr}(\mathbf{QH}_2) + \text{Tr}[\mathbf{h}_3^H \mathbf{Qh}_3(\mathbf{W} + \mathbf{V})],
$$

$$
\eta_3 \leq P_U \text{Tr}(\mathbf{QH}_3) + \text{Tr}[\mathbf{h}_5^H \mathbf{Qh}_5],
$$

$$
\eta_4 \leq \text{Tr}[\mathbf{h}_4^H \mathbf{Qh}_4(\mathbf{W} + \mathbf{V})],
$$

$$
14(c) - 14(e),
$$

(21b)

(21c)

(21d)

(21e)

(21f)

(21g)

where $\sigma_1^2 = \sigma_5^2$, $\sigma_2^2 = \sigma_6^2$ and $\sigma_3^2 = \sigma_4^2 = \sigma_7^2$. Problem $\mathcal{P}$ is a convex problem and hence the Slater’s condition holds. Then, the Lagrangian function of $\mathcal{P}$ in terms of $\mathbf{W}$ can be given by

$$
\mathcal{L} = -\text{Tr}(\mathbf{XW}) + \lambda_1 \text{Tr}[\mathbf{V}_n^H G(\mathbf{W}, \mathbf{V})(\mathbf{W} - \mathbf{W}^{(n)})] + \lambda_2 \text{Tr}(\mathbf{H}_2^H) - \lambda_3 \text{Tr}(\mathbf{h}_3^H \mathbf{Qh}_3^H) \mathbf{W} - \lambda_4 \text{Tr}(\mathbf{h}_4^H \mathbf{Qh}_4^H) \mathbf{W} + \lambda_5 \text{Tr}(\mathbf{W}) + \Delta.
$$

(22)

where $\lambda_1 \sim \lambda_5$ are the corresponding Lagrange multiplier to constraint (21b), (23c), (23d), (23f) and (14c), respectively. Note that $\mathbf{X} \succeq \mathbf{0}$ is the Lagrange multiplier to positive semi-definite constraint (14d). $\Delta$ in (22) is the collection of terms, which is not relevant for the proof. According to [14], we can get the KKT conditions about $\mathbf{W}$ as follows

$$
\mathbf{X}^* = \lambda_5^* \mathbf{I} - \mathbf{Y},
$$

$$
\mathbf{X}^* \mathbf{W} = \mathbf{0},
$$

(23)

(24)

where $\mathbf{Y} = [\lambda_2^* \mathbf{H}_2 + \lambda_3^* \mathbf{h}_3^H \mathbf{Qh}_3 + \lambda_4^* \mathbf{h}_4^H \mathbf{Qh}_4 - \lambda_5^* \mathbf{W}^H G(\mathbf{W}, \mathbf{V})]$. By Sylvester’s rank inequality and (24), we get

$$
0 = \text{rank}(\mathbf{X}^* \mathbf{W}) \geq \text{rank}(\mathbf{X}^*) + \text{rank}(\mathbf{W}) - N, \quad (25)
$$

Then, we assume $\delta$ denotes the largest eigenvalue of $\mathbf{Y}$. Based on (23), $\mathbf{X}^*$ is not a positive semi-definite matrix if the $\lambda_5^* \leq \delta$, which is contrary to KKT condition. Hence, we obtain $\lambda_5^* \geq \delta$. According to [14], the case where multiple eigenvalues have the same value $\delta$ occurs with probability zero, due to the randomness of the channels. Then, we get $\text{rank}(\mathbf{X}^*) \geq N - 1$. According to (25), we have

$$
\text{rank}(\mathbf{W}) \leq N - \text{rank}(\mathbf{X}^*) = 1, \quad (26)
$$

which thus completes the proof.

REFERENCES

[1] Q. Wu and R. Zhang, “Towards smart and reconfigurable environment: Intelligent reflecting surface aided wireless network,” IEEE Commun. Mag., vol. 58, no. 1, pp. 106–112, Nov. 2020.

[2] M. Cui, G. Zhang, and R. Zhang, “Secure wireless communication via intelligent reflecting surface,” IEEE Wireless Commun. Lett., vol. 8, no. 5, pp. 1410–1414, May 2019.

[3] H. Shen, W. Xu, S. Gong, Z. He, and C. Zhao, “Secrecy rate maximization for intelligent reflecting surface assisted multi-antenna communications,” IEEE Commun. Lett., vol. 23, no. 9, pp. 1488–1492, Jun. 2019.

[4] S. Hong, C. Pan, H. Ren, K. Wang, and A. Nallanathan, “Artificial-noise-aided secure MIMO wireless communications via intelligent reflecting surface,” IEEE Trans. Commun., vol. 68, no. 12, pp. 7851–7866, Sep. 2020.

[5] H. Niu, X. Lei, Y. Xiao, M. Xiao, and S. Mumtaz, “On the efficient design of ris-assisted secure MISO transmission,” IEEE Wireless Commun. Lett., vol. 11, no. 8, pp. 1664–1668, Apr. 2022.

[6] A. Sahbarwal et al., “In-band full-duplex wireless: Challenges and opportunities,” IEEE J. Sel. Areas Commun., vol. 32, no. 9, pp. 1637–1652, Jun. 2014.

[7] H. Shen, T. Ding, W. Xu, and C. Zhao, “Beamformin design with fast convergence for IRS-aided full-duplex communication,” IEEE Commun. Lett., vol. 24, no. 12, pp. 2849–2853, Aug. 2020.

[8] Y. Ge and J. Fan, “Robust secure beamforming for intelligent reflecting surface assisted full-duplex communication,” IEEE Commun. Lett., vol. 21, no. 1, pp. 383–397, Jul. 2022.

[9] Y. Ge and J. Fan, “Robust secure beamforming for intelligent reflecting surface assisted full-duplex mimo systems,” IEEE Trans. Inf. Forensics Security, vol. 17, pp. 253–264, Dec. 2021.

[10] Z. Peng et al., “Deep reinforcement learning for RIS-aided multiuser full-duplex secure communications with hardware impairments,” IEEE Internet Things J., pp. 1–1, May 2022.

[11] Y. Ge and J. Fan, “Robust secure beamforming for intelligent reflecting surface assisted full-duplex mimo systems,” IEEE Trans. WIRELESS COMMUN., vol. 21, no. 1, pp. 383–397, Jul. 2022.

[12] Y. Ge and J. Fan, “Robust secure beamforming for intelligent reflecting surface assisted full-duplex mimo systems,” IEEE Trans. Inf. Forensics Security, vol. 17, pp. 253–264, Dec. 2021.

[13] Z. Peng et al., “Deep reinforcement learning for RIS-aided multiuser full-duplex secure communications with hardware impairments,” IEEE Internet Things J., pp. 1–1, May 2022.

[14] X. Tang, X. Lan, D. Zhai, R. Zhang, and Z. Han, “Securing wireless transmissions with RIS-receiver cooperation: Passive beamforming and active jamming,” IEEE Trans. Veh. Technol., vol. 70, no. 6, pp. 6260–6265, May 2021.

[15] Y. Wang, R. Sun, and X. Wang, “Transceiver design to maximize the weighted sum secrecy rate in full-duplex swip systems,” IEEE Signal Process. Lett., vol. 23, no. 6, pp. 883–887, Apr. 2016.

[16] F. Zhu, F. Gao, M. Yao, and H. Zou, “Joint information- and jamming-beamforming for physical layer security with full duplex base station,” IEEE Trans. Signal Process., vol. 62, no. 24, pp. 6391–6401, Oct. 2014.

[17] D. Xu, X. Yu, Y. Sun, D. W. K. Ng, and R. Schober, “Resource allocation for secure IRS-aided multiuser mimo systems,” in 2019 IEEE Globalcom Workshops (GC Wkshps), Dec. 2019, pp. 1–6.

[18] N. Sun, P. Babu, and D. P. Palomar, “Majorization-minimization algorithms in signal processing, communications, and machine learning,” IEEE Trans. Signal Process., vol. 65, no. 3, pp. 794–816, Aug. 2017.

[19] T. Terlaky, Interior Point Methods for Nonlinear Optimization. Berlin, Heidelberg: Springer Berlin Heidelberg, 2010, pp. 215–276.
This figure "fig1.png" is available in "png" format from:

http://arxiv.org/ps/2206.08797v2