A novel methodology for high-frequency lumped equivalent circuit of an isolated transformer winding construction based on frequency response analysis signal morphology interpretation

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Abstract

A novel auto-synthesis method is proposed to construct the mutually coupled equivalent circuit of an isolated air-core transformer winding based on the signal morphology interpretation of its measured frequency response analysis (FRA) only. Therefore, this approach aims to determine the model parameters from the frequency sub-bands (low, medium and high frequency), each of which reflects its dominant parameters, where the parameter identification procedure is done from their corresponding sub-band separately. Besides, the optimisation algorithms have difficulties in estimating the nearly unique lumped circuit parameter in a large search space. For this reason, based on analytical simplifications, two novel dynamic autonomous methods have been proposed to restrict the search space bounds of the inductances and the capacitances. Finally, real case studies were used on actual air-core windings to validate the ability of the proposed method to perform an automated synthesis. Consequently, a nearly unique equivalent circuit model has accomplished automatically with high accuracy.

1 | INTRODUCTION AND BACKGROUND

Economic institutions that produce and distribute electrical energy strive to avoid transformer failures, which cause an interruption in production and possibly the damage of expensive equipment. The monitoring and diagnosis of its main component may predict, detect and locate their faults to avoid what could cause severe damage in the transformer. In order to provide monitoring systems for transformers, it is necessary to know all the available information and methods about them, such as modelling, diagnosis, synthesis and measurement methods etc., which are focused on the evaluation of the transformers’ state. The frequency response analysis (FRA) can be considered as one of the best techniques in this field [1–4] because it reflects the technical characteristics of the transformer and all their minor structure variations. Among the two main methodologies that exist to model, the frequency response of the transformer winding is the lumped parameter circuit model, which has been widely adopted in the literatures for FRA studies. It reflects the transformer internal physical structure and able to investigate the impacts of various winding deformations on the FRA response, generally used by [2,3,5–8] etc.

Obtaining precise winding behaviour requires precise determination of its parameters, where it is a problematic task to synthesise the unique circuit model. However, the accuracy of the synthesis methods depends on measurable amounts (such as equivalent inductance, equivalent capacitance and effective shunt capacitance [4,5,9]), as well as the numerical methods are used for parameters optimisation (finite element methods [10], intelligent optimisation algorithms, e.g. genetic algorithm [11,12], particle swarm optimisation [13], bat algorithm [5],

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artificial bee colony algorithm [4] etc.). Usually these researches are based on measurable parameters, then, attempts are made to determine the other parameters, by specifying their search space and defining the stopping criteria for the optimisation algorithm. However, on the one hand, the measured amount may be influenced by causes affecting the accuracy of the measurement. On the other hand, the use of optimisation algorithms with large search space affects the precision of the optimisation. To this end, this article aims to suggest a new method of auto-synthesis that depends only on the FRA measurement. Based on the FRA signal morphology interpretation (Figure 1), and the use of mathematical simplifications to sufficiently reduce the parameters search space, the unique equivalent circuit model of the transformer winding was accurately constructed.

The proposed method is mainly based on the FR signal subdivision into three frequency sub-bands (low, medium and high frequency); each sub-band contains dominant parameters [1,14] that are reflected on the winding response behaviour. First, at the low-frequency response range (LFR), the inductance effect is dominant, where the resistance and the inductances can be determined. Second, at medium-frequency response range (MFR), the effect is double (inductive and capacitive) which causes resonance and anti-resonance frequencies, where the initial section number was obtained from the number of resonance frequencies detected in this sub-band. Finally, at high-frequency response range (HFR), the effect is purely capacitive; by analytical relationship, the capacitances are determined.

Moreover, to ensure an appropriate synthesis of the adopted model, analytical simplifications have been made to restrict the search space bounds. The constraints and inequalities defining the model are also considered, which increases the precision and the convergence speed of the optimisation algorithm. Consequently, the FRA signal morphology interpretation method (FRA-SMI) is used to identify all the model parameters from the only input FR data measurement. Besides, its ability has been successfully tested using two experimental cases.

The objectives of this work can be listed as follows:

- Provide a novel iterative approach to ensure the synthesis of a quasi-unique mutually coupled equivalent model of transformer winding using only FR data measurement.
- Determine the model parameters with high accuracy using analytical simplifications to reduce their search space.
- Perform the synthesis task non-invasively and non-destructively, that is, using a single measurable amount (the FR impedance), without resorting to the detailed geometrical dimensions of the winding. This makes it applicable to monitoring/diagnostic techniques.

2 | LUMPED EQUIVALENT CIRCUIT MODEL

In order to monitor/diagnose a transformer winding, there must be represented by a physically realisable circuit model. The circuit should be synthesised such that it characterises the high-frequency behaviour of the transformer winding. The complexity during circuit synthesis could be simplified in two ways: if the multi-winding transformer is equivalently represented by a single ladder network and if all its parameters are considered static, their individual frequency dependencies have been neglected.

The isolate symmetrical and homogeneous transformer winding can be modelled by a very specific number of sections $N$ that form a single ladder network magnetically and electrically coupled. Each identical quadrupolar section (Figure 2) is composed of internal winding resistance ($r$) in series with self-inductance ($l_i$), shunt and series capacitances ($C_g$ and $C_i$), and mutual inductances between sections $i$ and $j$ ($M_{ij}$).

The inductive effect in low frequencies can be modelled by an equivalent inductance in series with an equivalent resistance, which follows linear increasing values of impedance, and this may not be exactly linear due to the non-linearity of the core in case of the winding with iron core. Actually, in practice, it is possible to limit the influence of the core and the effect of neighbouring windings by choosing proper terminal connection [6,15]. There are two objectives to choosing the end-to-end short-circuit connection: the first is to significantly reduce the model of a multi-winding transformer to a single ladder network (reduced model) [6], and the second if when windings in transformers need to be checked individually as mentioned in the worldwide standardisation activities for the FRA test and

FiguRe 1 Model construction steps

FiguRe 2 High-frequency lumped equivalent circuit model
Kulkarni et al. [16], the response is influenced by the leakage inductance rather than the magnetizing inductance.

It is indicated in Refs. [6,17] that if all the ‘neighbouring windings (primary and secondary) are shorted and grounded’, their influence on the winding under investigation can be minimised. In this case, the transformer can be considered as equivalent to an isolated winding, and it can be represented by a single ladder network.

The authors in Refs. [18,19] indicate that the terminal behaviour of an iron-core transformer would become similar to that of an air-core transformer if all neighbouring windings are shorted. Thus, influence of one winding on other can be ignored with regard to mutual inductance. Therefore, their experimental comparison indicates that when the low-voltage winding is short-circuited, the response in the high-voltage winding with an iron core is similar to that with an air core.

However, the tested winding in an iron core transformer can be considered as an isolated coil [2] and without iron core [5,6,9,20,21] because if all the adjacent windings to the tested winding are short-circuited and grounded. The transformer winding under investigation can be represented by a single ladder network [6].

At the medium frequencies, the impedance can be modelled by parallel and series resonance circuits; the shunt capacitances represented the dielectric isolated between the winding and the core (earth), the series capacitance represented the capacitive effect between discs, and they are suitably combined in order to reproduce the peaks and troughs of the FR signal. As mentioned above, the end-to-end short circuit connection neglects the influence of adjacent windings and inter-winding capacitances.

In the high frequencies, the impedance can be modelled by a ladder network of shunt and series capacitances; the model is represented by capacitances only, and it follows decreasing values of impedance.

3 | FRA SIGNAL MORPHOLOGY INTERPRETATION

FRA-SMI method is based on the FR signal morphology, which is related to the transformer physical components (conductor section, spire numbers, dielectric etc.); therefore, it contains all technical characteristics specification of the transformer winding, which can be represented by electrical parameters (resistance, inductance, and capacitance). As shown in Figure 2, they can be determined from only the FR input data in wide frequency ranges: low-, medium- and high-frequency bands. The exactness analysis and interpretation in these regions can effectively improve the accuracy of equivalent model parameters identification. So, determination of the dominant parameter is based on its corresponding frequency sub-band, while the influence of other parameters is generally negligible [14].

In the context of transformer windings diagnostic, several works attempted to decompose the FR graphs, in order to understand the winding behaviour in each frequency zone. The authors in Ref. [14] proposed a new method of fault interpretation for transformer windings using the characteristics extracted from the equivalent gradient zones of the frequency response. Other researches have shown that an FR curve can be divided into several main frequency regions [3,21]. In this study, the objective is to identify the parameters located in each frequency sub-band (Figure 3).

- In the LFR sub-band, the effect is purely inductive, and the proper and mutual inductances are the dominant parameters.
- In the MFR region, dual effects are inductive and capacitive, influenced by the series \((C_g)\), shunts \((C_r)\) capacitances and the inductances.
- The HFR sub-band reflects the global capacitance \((C_{eq})\) caused by the two dominant parameters \(C_r\) and \(C_g\) capacitances.

Ragavan et al. [1] showed that an equivalent circuit of \(N\) sections can be represented by a transfer function (6). The mathematical expressions in each sub-band are defined as follows.

### 3.1 LFR sub-band

In this range, the isolated winding can be modelled by equivalent resistance in series with equivalent inductance, the latter represents the self and mutual inductances \(l_i\) and \(M_{ij}\) as shown in Figure 3. So, the parameters present in this region are \(R_{dc}\), \(L_{eq}\) \(l_i\) and \(M_{ij}\).

The impedance at low frequencies is [1]

\[
\lim_{s \to 0} Z(s) = R_{dc} + sL_{eq}
\]

Impedance \(Z(f)_{LFR} = R_{dc} + jX(f)_{LFR}\) \(\text{(1)}\)

Reactance \(X(f)_{LFR} = 2\pi f L_{eq}\) \(\text{(2)}\)

Equivalent inductance \(L_{eq} = Nl_i + 2\sum_{i=1}^{N-1} (N - i)M_{i,i+1}\)

\(\equiv l_i \left[ N + 2\sum_{i=1}^{N-1} (N - i)l_i \right]\) \(\text{(3)}\)

where \(M_{1,i+1} \equiv \lambda l_i\) according to Ref. [22]. \(R_{dc}\) is the direct current resistance, \(L_{eq}\) is the equivalent inductance, \(l_i\) is the self-inductance, \(N\) is the number of sections, \(M_{i,i+1}\) is mutual inductances between the \(N\) sections, and \(\lambda\) is coefficient of mutual inductances between 0 and 1.

Note: \(\lambda\) coefficient should be substantially constant for different mutual-inductances in case of isolated and uniform transformer winding \((M_{i,i+1} \equiv M_{i+1,i})\).

Considering the structure of the transformer winding, the magnetic coupling between the turns must follow specific rules. In the previous studies [21,23,24], the constraints of the inequalities shown in Equations (4) and (5) were used to identify the model parameters:

\[ M_{1,1} > M_{1,2} > M_{1,3} > M_{1,4} \]  \(\text{(4)}\)

\[ \lambda_{\text{min}}l_i < M_{i,i+1} < \lambda_{\text{max}}l_i \]  \(\text{(5)}\)

with \(0 < \lambda_{\text{min}} \leq \lambda \leq \lambda_{\text{max}} < 1\) and \(M_{1,2} = \lambda l_i, M_{1,3} = \lambda^2 l_i, M_{1,4} = \lambda^3 l_i\) … etc.

### 3.2 MFR sub-band

The increase in frequency provoked other phenomena, and the interaction between the inductive and the capacitive effect causes resonance and anti-resonance frequencies, where the impedance is represented by the transfer function in Equation (6) [1] as shown in Figure 3, with poles and zeros represent the open and short circuit natural frequencies \(O_{cnt}, S_{cnt}\) respectively. This band includes all the parameters of the winding (capacitances and inductances):

\[
Z(s) = \frac{\beta_1 (s - T) \Pi_{i=1}^{N-1} (s - z)(s - z^*)}{\Pi_{i=1}^{N} (s - p)(s - p^*)}
\]  \(\text{(6)}\)

with \(\beta_1\) being the scaling factor; \(S\) real zero; \(z_i, z_i^*\) complex-conjugate zero pair and \(p_i, p_i^*\) complex-conjugate pole pair.

However, Mukherjee et al. [25] have developed a relationship between the resonant frequencies and the winding parameters (capacitances and inductances), which is given as follows:

\[
\sum_{i=1}^{N} 1 = \sum_{i=1}^{N} (l_i)C_{ni} + \sum_{i=1}^{N} (M_{0,i})C_{gi}\]

\(\text{(7)}\)

\[
\omega_{\text{abi}} = 2\pi O_{cnt}\]

\(\text{The resonance frequencies of an FRA test with neutral shorted are equal to the anti-resonance frequencies with neutral open [25].}\)

### 3.3 HFR sub-band

At higher frequencies, the capacitive effect can be represented by Equation (8) [6], the curve decreases with increasing frequency as shown in Figure 3. The global capacitance represents the equivalent of shunt and series capacitances of all sections, and its value must remain considerably constant. Consequently, the existing parameters in this region are \(C_{eq}, C_g,\) and \(C_r\).

\[
Z(s)_{HFR} = \frac{\sin(\theta)}{sC_{eq}}
\]  \(\text{(8)}\)

With

\[
C_{eq} = \frac{C_g}{2} + \frac{1}{C_s} + \frac{1}{C_r} + \ldots
\]

\(\text{(9)}\)
4. DESCRIPTION OF FRA-SMI TO CONSTRUCT THE EQUIVALENT MODEL

Interpretation of the single FR measurement input data begins by digital filtering for signal noises elimination. Then, to clarify the dominance of parameters in each frequency range, a dynamic frequency sub-band division has been used to divide the FR data into three regions: LFR, MFR and HFR. Starting by specifying the LFR sub-band, it is important to ensure the linearity of the LFR curve to make sure the effect is purely inductive, normally below 1000 Hz in all windings, which is the lowest frequency when the winding behaves as a simple inductance [23]. Afterwards, the third band HFR is commenced after the last \( O_{\text{eq}} \) when the phase angle is very converged to 90°. Between LFR and HFR sub-bands, the MFR sub-band is identified.

The extraction of the different parameters constituting the model in each sub-band is done separately as shown in Figure 4. Starting by the section number \( N \) of the circuit to be synthesised, it must be initially equal or greater than the number of the resonance frequencies detected, which can be determined by direct extraction from the MFR sub-band data. Subsequently, the equivalent inductance is extracted directly from the slope of the reactance \( X(f)_{\text{LFR}} \). Then, in order to identify the self and mutual inductances, the optimisation algorithm requires an objective function and definition the parameters search space. So, using the BA algorithm [26], \( I_{l} \) and \( X_{l} \) in Equation (2) are determined and by substituting their values in Equation (3), all mutual inductances are easily obtained (more details in Section 5).

In this procedure, mathematical simplifications were used to limit the search space in order to reduce the execution time and to increase the optimisation precision, and it is discussed in detail below (Section 5.2.3).

Thereafter, the estimation of the capacitances \( C_{q} and C_{l} \) is accomplished by Equation (7) using the results already obtained \( (O_{\text{eqfs}}, N, I_{l}, and M_{ij}) \). The search space of the capacitances is a problematic task, which has not been addressed until now [4,22]. Its restriction must be obtained automatically from the FR data, which has been discussed in detail in Section 5.

The estimation approach is based on a performance criterion, which consists of minimising the difference between the measured LFR, MFR, and HFR curves and those of the simulated model, as shown in the flowchart of Figure 4.

5. ESTIMATION OF MODEL PARAMETERS

5.1. Extraction of the resonance frequencies \( O_{\text{eqfs}} and section number \( N \)

Inspection of the FR curve shows at certain frequencies peaks and troughs, which present the \( O_{\text{eqfs}} \) and \( S_{\text{eqfs}} \) successively, where it is possible to extract their values, and corresponding to the number of the resonance frequencies, the initial value of the section number \( (N) \) can be chosen.

5.2. Determination of \( R_{dc}, L_{\text{eq}}, and M_{ijj} \)

The equivalent circuit at low frequencies under 1000 Hz is simplified and limited to the elements \( R_{dc}, L_{l} \) and \( M_{ij} \) [1], represented by the transfer function in Equation (1). The measured impedance LFR data elements are complex numbers at each frequency, where the real part represents the DC resistance \( R_{dc} \) (10), and the imaginary part represents the reactance \( X(f)_{\text{LFR}} \) in Equation (2).

5.2.1. Extract \( R_{dc} \) resistance

In order to experimentally verify the frequency dependence of the equivalent resistance and inductance in the LFR sub-band, using an LCR-meter, the impedance of the isolated winding (Case 1) was measured in the range (0–1000 Hz) in steps of 1 Hz; Figure 5 illustrates the impedance curve decomposition.

It should be noted that the eddy current effect (skin and proximity effects) on the resistance is negligible compared to DC resistance \( (R_{dc}) \) value. As shown in Figure 2, the difference \( \Delta R = 0.28 \Omega \) between the final value (at 1 kHz, 8 \( \Omega \)) and the initial value (at 0 Hz, 7.72 \( \Omega \)) is negligible. Also, after 100 Hz, the reactance and the impedance are superposed, where the equivalent resistance value is negligible compared to the reactance. The resistance is substantially constant and equal to the resistance in direct current \( (R_{dc}) \) [23]. It can be extracted directly from the real part in Equation (2):

\[
R_{dc} = \text{Re}(Z(f)_{\text{LFR}}) = Nr
\]

Here, \( r \) is resistance per section.

As shown in Figure 5, the linearity of the reactance curve at low frequencies proves that the inductance value is considered constant, where its frequency dependence is negligible compared to the reactance value \( (X(f)_{\text{LFR}}) \). According to Ref. [27], an acceptable hypothesis is that the frequency dependence of the resistance and the induction does not significantly affect the frequency response.

5.2.2. Extract \( L_{\text{eq}}, \) estimate \( I_{s} \) and \( M_{ij} \)

The imaginary part in Equation (2) represents the reactance \( X(f)_{\text{LFR}} \), where the equivalent inductance includes the self and mutual inductances, and its value remains constant until the limit of the linear region LFR sub-band.

Otherwise, Equation (2) can be written as

\[
X(f)_{\text{LFR}} = \omega N (I_{l} + I_{m})
\]

where \( I_{m} \) is the global mutual inductance.

To find the optimal solution that corresponds to the LFR sub-band measurement data, the expression (2) has been simplified to obtain the equation of the reactance (13) that
contains two unknowns $l_s$ and $\lambda$, which is considered the objective function of the search algorithm in order to estimate $l_s$ and $\lambda$ then the mutual inductances $M_{ij}$. However, each parameter ($l_s$ and $\lambda$) must be limited between an upper and lower bound of the search space.

**Search space of self and mutual inductances**

As indicated above, the search space is a fundamental task in the lumped equivalent circuit parameters estimation. However, Mukherjee et al. [4] indicated that there is no direct procedure for determining the search bounds to fix the population size of the search algorithm, and so it can be set according to experience, trial and error, and field knowledge.

Recent studies have used methods to fix the search space of $l_s$ and $M_{ij}$ by direct measurement of the equivalent inductance $L_{eq}$, where the upper and lower bounds value of self-inductance are chosen successively $l_{s_{min}} = 0$ and $l_{s_{max}} = L_{eq}/N$, [1,4,5,13,22], so that the self and mutual inductances satisfy the constraints of the inequalities of Equation (3). Anyway, estimating the parameters with this wide search space requires extensive computational time, and no unique solution will be found. Moreover, if the
The important advantage of the proposed method by authors in Ref. [22] is to reduce the search space by calculating \( I_s \) and \( M_{ij} \) using two measurements from internal points of external winding disks, where the other mutual inductances can be determined by substitution in Equation (3).

In contrast, the below-proposed work aims to solve the problem of the search space reduction present in the previous researches [23–25,28]. The reduction should be automatically from the FRA only, and it can, therefore, be applied to methods of synthesis, parameters identification parameters and diagnosis.

However, the inequalities in Equation (5) can be written in the following form:

\[
2 \sum_{i=1}^{N-1} \left( \frac{N-i}{N} \right) l_{\min} l_i < 2 \sum_{i=1}^{N-1} \left( \frac{N-i}{N} \right) M_{1,i+1} \\
< 2 \sum_{i=1}^{N-1} \left( \frac{N-i}{N} \right) l_{\max} l_i 
\]

(12)

It can be expressed as follows: \( \beta_{\text{Nmax}} l_i < l_m < \beta_{\text{Nmin}} l_i \), with \( \beta_N = \frac{1}{2} \sum_{i=1}^{N-1} (N-i) l_i \) and \( l_m = \beta_N l_m \), where, \( 0 < \beta_N < N-1 \).

By substituting \( l_m \) in Equation (11) becomes

\[
X(f)_{\text{LFR}} = \omega N l_i \left( \frac{2}{N} \sum_{i=1}^{N-1} (N-i) l_i^2 + 1 \right) \\
= \omega N l_i (\beta_N + 1) 
\]

(13)

The search bounds of the self-inductance \( l_i \) is

\[
l_{\text{max}} = \frac{X(f)_{\text{LFR}}}{\omega N (\beta_{\text{Nmin}} + 1)} = \frac{1}{(\beta_{\text{Nmin}} + 1)} \frac{L_{\text{eq}}}{N} \\
l_{\text{min}} = \frac{X(f)_{\text{LFR}}}{\omega N (\beta_{\text{Nmax}} + 1)} = \frac{1}{(\beta_{\text{Nmax}} + 1)} \frac{L_{\text{eq}}}{N}
\]

However, previous studies [5,23,24] have used the constraints of the inequalities in Equations (4) and (5) to determine the self and mutual inductances. Furthermore, the authors in Ref. [28] have indicated that the upper and lower bounds of the mutual inductance \( M_{1,2} \) between the discs 1 and 2 can be chosen \( 0.4 l_{\text{max}} < M_{1,2} < 0.8 l_{\text{max}} \).

\( M_{ij} \) between the discs \( i \) and \( j \) may be bounded as follows:

\[
0.4M_{ij-1} < M_{ij} < 0.8M_{ij+1}, \forall i = 1, ..., N-1 \forall j = 2, ..., N-1. 
\]

Consequently, using the proposed method, the search space restriction of the self-inductance can be reduced to 73.9% in the case of \( 0.4 < \lambda < 0.8 \) and \( N = 6 \) as shown in Figure 6.

\[
0.245 \frac{L_{\text{eq}}}{N} < l_i < 0.509 \frac{L_{\text{eq}}}{N} \text{ compared to } 0 < l_i < \frac{L_{\text{eq}}}{N} 
\]

(see Figure 6).

Estimating of \( \lambda \) and \( l_i \) from Equation (13), in order to minimise the difference between the measured \( Z(f)_{\text{LFR}} \) curve and that the simulated. Then all the mutual inductances are calculated by substituting the \( l_i \) and \( \lambda \) values in Equation (3).

### 5.3 Model capacitance estimation \((C_s \text{ and } C_g)\)

In previous efforts, the estimation of the mutually coupled model capacitances has been highlighted in several kinds of researches. The authors in Ref. [29] proposed a method to estimate the series capacitance from the rational function in ‘s’ domain, in which the shunt capacitance is obtained by direct measurement. Besides, Shah et al. [9] have proposed a method.
for estimating the circuit model capacitances using measurable amounts (equivalent and shunt effective capacitances...etc).
The circuit capacitances may also be determined using the optimisation algorithms described in Refs. [5,24].

The determination of capacitances in this article is based on estimation and not on measurement, using the relationship between the resonance frequencies and the winding parameters (capacitances and inductances) accomplished by Mukherjee et al. [25] as mentioned above in Equation (7). It will be simplified in order to define an appropriate objective function.

To simplify Equation (7), the total inductance \( M_{0,N} \) can be given by Equation (14) and the sum of the total inductances by Equation (15). The relationship of Equation (7) after simplification is depicted in Equation (16):

\[
M_{0,1} = l, \quad \beta_1 = 0 \\
M_{0,2} = 2l_s(1 + \beta_2), \quad \beta_2 = \lambda \\
M_{0,3} = 3l_s(1 + \beta_3), \quad \beta_3 = \frac{2}{3}(2\lambda + \lambda^2) \\
M_{0,N} = Nl_s(1 + \beta_N), \quad \beta_N = \frac{2}{N} \sum_{i=1}^{N-1} (N - i)\lambda^i \\
M_{0,N} = \sum_{i=1}^{N} N(1 + \beta_N)
\] (14)

In Equation (7), if the winding structure is uniform, the capacitances are distributed symmetrically on the winding discs, where all the series and all the shunt capacitances are identical \( C_{si} = C_s \) and \( C_{si} = C_s \), respectively, and the same for the self-inductances \( l_{si} = l_s \).

After simplification, Equation (7) renders,

\[
NC_s + \beta C_g = \delta \quad (16)
\]

Such as \( \delta = \frac{1}{l} \sum_{i=1}^{N} \frac{1}{\omega_i} \) and \( \beta = \sum_{i=1}^{N} N(1 + \beta_N) \).

However, to reach a unique solution corresponding to the desired FR data, the search algorithm requires a sufficiently acceptable restriction of parameters search space.

5.3.1 Search space of capacitances \( C_g \) and \( C_s \)

To complete the construction of the lumped reference circuit of the winding, the capacitances \( C_s \) and \( C_g \) must be limited between upper and lower bounds.

The equivalent circuit in Figure 2 is on the form of an RLC ladder network representation that allows the reproduction of the FR signal which is composed of resonance and anti-resonance frequencies. However, the winding inductance and the shunt capacitance in the LC network show a series resonance. Consequently, the first peak is strongly influenced by the total shunt capacitance \( C_{p,s} \) [20,23]. Therefore, based on this interpretation, to reduce the capacitances search space, it can be used the capacitance \( C_{g,\text{ref}} \) of the first resonance \( \left( f_{\text{ref}} \right) \) in Equation (17), because it corresponding to an approximate value of the shunt capacitance \( C_g \)

\[
C_{g,\text{ref}} = \frac{1}{N(2\pi f_{\text{ref}})^2 L_{eq}}
\]

and by substituting its value in Equation (16), the approximate value of the series capacity \( C_s \) is also determined. The search space for the capacitances can be determined as follows

\[
\frac{C_{g,\text{ref}}}{n} < C_g < nC_{g,\text{ref}}
\]

where \( n \) is the coefficient of capacitance search space \( (n < N) \). \( C_{g,\text{ref}} \) and \( C_{g,\text{ref}} \) are the approximative series and shunt capacitances values, respectively.

In the HFR sub-band, the procedure leads to bringing the measured \( Z(f)_{\text{HFR}} \) data closer to that of the simulation in the same frequency range. Finally, all parameters are used to synthesise the equivalent circuit model, where the FRA-SMI method minimises the difference between the impedance measurement and that of the equivalent circuit model, and its ability has been tested by two isolated windings with air core in the research laboratory.

6 FRA-SMI METHOD APPLICATION ON EXPERIMENTAL CASES

The FRA-SMI method has been successfully applied to two isolated windings. The synthesis of the mutually coupled circuit begins by the frequency response data measurement, illustrated in Figure 7. Ideally, the FRA-SMI method should identify all parameters in order to reproduce the similar behaviour of the windings.

6.1 Case 01

The procedure of parameters identification is performed according to the flowchart in Figure 4, where all the input parameters of the algorithm are extracted directly from the impedance response measured data as follows:

- Extract the resonance frequencies \( f_{\text{ref}} \) from the MFR range shown in Figure 7a (109, 201 and 301 kHz).
- Determine the section number \( N \) of the equivalent model circuit, which is initially defined as \( N = 3 \).
- Extract the equivalent DC resistance \( R_{dc} \) of the winding from the LFR data. It was found to be \( R_{dc} = 7.72 \Omega \).
\begin{itemize}
\item Extract the equivalent inductance value ($L_{eq}$) of the winding from the LFR range as already explained in Section 3, which was obtained $L_{eq} = 53.5$ mH.
\item To find the optimal solution of the self and mutual inductances, the fitness function and the search space must be specified for the BA optimisation algorithm. In LFR subband, Equation (13) was used as the objective function in order to minimise the error between the measured reactance $X(f)_{LFR}$ and the model’s output (Figure 8a). Besides, to fix the search space bounds of the self and mutual inductances, the upper and lower values of $\lambda$ are chosen ($\lambda_{\min}$ and $\lambda_{\max}$), and then, the bounds of the self-inductance are obtained as follows:
\begin{itemize}
\item The upper bound for $\lambda$ $\lambda_{\max} = 0.8$;
\end{itemize}
\end{itemize}

\textbf{FIGURE 7} The frequency response measurement curves of the actual healthy windings: (a) Case 01, (b) Case 02
FIGURE 8  Comparison between the $X(f)_{LFR}$ measured to that estimated by frequency response analysis-signal morphology interpretation method: (a) Case 01, (b) Case 02
TABLE 1  Estimated inductances (mH)

| Parameters | Case 01 | Case 02 |
|------------|---------|---------|
| $N$        | 4       | 5       |
| $L_{eq}$   | 53.5    | 1.5     |
| $i_t$      | 5.6     | 0.17    |
| $M_{l,2}$  | 3.4     | 0.059   |
| $M_{l,3}$  | 2.1     | 0.02    |
| $M_{l,4}$  | 1.2     | 0.0072  |
| $M_{l,5}$  |         | 0.0025  |

- The lower bound for $\lambda_{\min} = 0.4$;
- The upper bound for $l_t$, $l_t_{\max} = 7.6$ mH;
- The lower bound for $l_t$, $l_t_{\min} = 4.44$ mH.

The algorithm was employed to optimise the self-inductance $l_t$ and the coefficient $\lambda$ to achieve the minimum fitness (13), where the estimate outputs are found to be, $\lambda = 0.6$ and $l_t = 5.6$ mH. It must be noted that the section number $N$ is equal or greater than the number of resonance frequencies; in this case, the algorithm results are more accurate with $N = 4$, and on the other hand, it must be corresponding to the number of winding discs ($N = 4$), in order to represent well the self and mutual inductances of the model.

- Calculate the mutual inductances $M_{l, i+1}$ by substituting $l_t$ and $\lambda$ values in Equation (3).

In the LFR sub-band, the optimisation algorithm minimises the difference between the reactance $X(f)_{LFR}$ and that of the model, in order to bring the measured curve closer to that of the simulation in this frequency range as shown in Figure 8a.

In the HFR sub-band, Equation (16) was used as the objective function in order to minimise the error between the measured impedance $Z(f)_{HFR}$ and the model’s output Figure 9. Besides, from the first resonance frequency, it can be extracted directly the approximate value of the shunt capacitance using Equation (17), and the result obtained was $C_{g, Ocnf1} = 10.52$ pF; and by substituting its value in Equation (16), the approximate value of $C_s$ is determined as $C_{s, Ocnf1} = 114.77$ pF. Consequently, the capacitances’ search space can be determined by multiplying and dividing the approximate values of capacitances by the coefficient $n$, which is determined in these cases ($n = 1.5$) in order to determine the upper and the lower bounds, respectively. However, a good limitation of the search space will allow a fine search around the optimum solution value. The search bounds of the shunt capacitance $C_s$

- $C_{s,\text{max}} = 1.5C_{g, Ocnf1} = 172$ pF;
- $C_{s,\text{min}} = \frac{C_{g, Ocnf1}}{1.5} = 76$ pF.

Moreover, the optimisation algorithm minimises the difference between the impedance $Z(f)_{HFR}$ and that of the model in this frequency range (Figure 9a). The optimal values for the series and shunt capacitances are used to calculate the equivalent capacity ($C_{eq}$) by substituting their values in Equation (9) (see Table 2). The approximate value $C_{s, Ocnf1} = 114.77$ pF is very closed to the estimated $C_s = 115.94$ pF.

Finally, the performance criterion procedure consists of bringing the measured FR curve closer to that of the model’s output simulation. The FRA-SMI method is used to minimise the difference between the impedance $Z(f)_{FR}$ curve and that obtained from the identified parameters of the model in large frequency range. Hence, the graphical comparison in DB is depicted in Figure 10.

Furthermore, the comparison of the measured resonance frequencies values to those estimated is shown in Table 3, where it is clear that they are identified with high accuracy using the FRA-SMI approach.

The four-section synthesised circuit has been constructed with these parameters (DC resistance, inductances and capacitances are in $\Omega$, mH and pF, respectively), shown in Figure 11. Its frequency response plots are presented in Figure 10a, in which all of the well pronounced natural frequencies of the model winding are seen.

### 6.2  Case 02

Applying the same procedure used in the first practical case, the ability of the FRA-SMI method is successfully tested by the second practical case. First, the resonance frequencies are extracted from the MFR data in Fig. 7(b), where the initial section number value is defined as $N = 5$. Then by direct extraction, the DC resistance $R_{dc} = 1.25 \, \Omega$ and the equivalent inductance $L_{eq} = 1.5 \, \text{mH}$ are determined. Second after choosing the upper and lower values of $\lambda_{\min}$ and $\lambda_{\max}$ the bounds are fixed as follows:

- The upper bound for $\lambda$, $\lambda_{\max} = 0.6$;
- The lower bound for $\lambda$, $\lambda_{\min} = 0.3$;
- The upper bound for $l_t$, $l_t_{\max} = 0.186$ mH;
- The lower bound for $l_t$, $l_t_{\min} = 0.115$ mH.

They are found to be $\lambda = 0.349$ and $l_t = 0.17$ mH, where by substituting their values in Equation (3), the mutual inductances $M_{l, i+1}(\text{mH})$ are given in Table 1 (Case 02).

The optimisation algorithm minimises the difference between the reactance $X(f)_{LFR}$ and that of the model in this frequency range (Figure 8b). The approximate value of the shunt capacitance is calculated by the first resonance frequency that is found to be $C_{g, Ocnf1} = 11.9$ pF; and by substituting its value in Equation (16),
FIGURE 9  Comparison between the \( Z_{f_{\text{HFR}}} \) measured to that estimated by frequency response analysis-signal morphology interpretation method: (a) Case 01, (b) Case 02
The approximate value of $C_s$ is $C_s^{\text{Ocnf}} = 168$ pF. So, by choosing the coefficient ($n = 1.1$), the bounds are fixed as follows.

- $C_{gmax} = 1.1 C_s^{\text{Ocnf}} = 13.1$ pF;
- $C_{gmin} = \frac{C_s^{\text{Ocnf}}}{1.1} = 10.8$ pF;
- $C_{smax} = 1.1 C_s^{\text{Ocnf}} = 184.8$ pF;
- $C_{smin} = \frac{C_s^{\text{Ocnf}}}{1.1} = 152.7$ pF.

**Table 2**  Estimated capacitances (pF)

| Parameters | Case 01   | Case 02   |
|------------|-----------|-----------|
| $C_s$      | 115.94    | 176.06    |
| $C_g$      | 7.13      | 12.47     |
| $C_{eq}$   | 38.2      | 54.5      |

**Figure 10** Comparison between the measured $Z(f)_{FR}$ to that estimated by frequency response analysis-signal morphology interpretation method:
(a) Case 01, (b) Case 02
The optimisation algorithm minimises the difference between the reactance \( Z(f_{HFR}) \) and that of the model Fig. 9(b). The optimal values of series, shunt and equivalent capacitances in (pF) are shown in Table 2. It can be seen that the approximate values of \( C_g \) and \( C_0 \) are very closed to the estimated values.

Finally, the FRA-SMI method minimised the difference between the impedance \( Z(f_{HFR}) \) curve and that obtained from the identified parameters of the equivalent model. The graphical comparison is depicted in Figure 10b. Furthermore, the comparison of the measured open-circuit natural frequency values to those estimated are shown in Table 3, where it is clear that the resonance frequencies \( O_{cnfs} \) are identified with high accuracy using FRA-SMI.

The five-section synthesised circuit has been constructed with these parameters (dc resistance, inductances and capacitances are in \( \Omega \), mH and pF, respectively), shown in Figure 12. Its frequency response plots are presented in Figure 10b.

**CONCLUSION**

The proposed FRA-SMI methodology was successfully applied to synthesise the mutually coupled equivalent ladder network of transformer winding based on FR signal morphology interpretation in the frequency regime. However, improvements were made to construct a quasi-unique equivalent circuit of the winding as follows:

- A novel auto-restriction of the search space was proposed to determine the inductance matrix. Mathematical simplifications are used to reduce sufficiently the search space bounds for the optimisation algorithm to obtain a precise solution with an efficient time in the low-frequency range.
- A novel auto-construction procedure of the capacitance matrix has been proposed. Mathematical simplifications are

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**TABLE 3** Comparison between the measured natural frequencies to those estimated using the frequency response analysis-signal morphology interpretation method

| Resonant frequencies | Case 01 | Case 02 | Deviation | Case 01 | Case 02 | Deviation |
|----------------------|---------|---------|-----------|---------|---------|-----------|
| \( O_{cnf1} \)      | 109     | 109     | 0.0%      | 5.36    | 5.36    | 0.0%      |
| \( O_{cnf2} \)      | 201     | 201     | 0.0%      | 7.83    | 7.82    | 0.12%     |
| \( O_{cnf3} \)      | 300     | 301     | 0.33%     | 9.80    | 9.81    | 0.13%     |
| \( O_{cnf4} \)      | -       | -       | -         | 11.51   | 11.53   | 0.17%     |
| \( O_{cnf5} \)      | -       | -       | -         | 12.70   | 12.71   | 0.08%     |

**FIGURE 11** Reference-circuit of model transformer with its healthy disc winding under test Case 1

**FIGURE 12** Reference-circuit of model transformer with its healthy disc winding under test Case 2
used to restrict the search space bounds of capacitances in a very limit domain.

In addition, the advantages of this approach can be listed as follows:

- The FRA measurement is the only input required.
- No needed for prior knowledge of winding construction.
- Significantly reduction in the search space of parameters.
- Improvement in time efficiency.
- Synthesis a nearly unique ladder network.
- Easy to implement.

Consequently, using a single measurement data of the frequency response magnitude and thanks to the novel procedures of the search space restriction, the construction of a nearly unique mutually coupled equivalent ladder network of a transformer winding was successfully carried out using the proposed method. The obtained results demonstrate that FRA-SMI can be considered as a promising autonomous tool for determining the transformer winding parameters in order to diagnose its internal faults.

So, the biggest advantage in this approach is to perform an autonomous synthesis task non-invasively and non-destructively, that is, by using a single measurable amount (FRA), from the transformer winding terminals. Currently, offline and online FRA measurement are possible, and this makes the proposed approach practically applicable in the monitoring/diagnostic techniques.

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