Statefinder parameters for quintom dark energy model

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Abstract

We perform in this paper a statefinder diagnostic to a dark energy model with two scalar fields, called “quintom”, where one of the scalar fields has a canonical kinetic energy term and the other has a negative one. Several kinds of potentials are discussed. Our results show that the statefinder diagnostic can differentiate quintom model with other dark energy models.

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It is widely believed nowadays that the present universe is undergoing an accelerating expansion. The converging evidences come from the analysis of data from supernova [1, 2], CMB [3, 4, 5, 6, 7, 8] and WMAP [9, 10]. In order to explain the cosmic accelerating expansion, a modified theory of gravity or the existence of dark energy is required. Perhaps the simplest candidate of dark energy is the cosmological constant with equation of state $w = p/\rho = -1$. However, there exist two problems with it, i.e., why the cosmological constant is so tiny compared to the theoretical expectations and why the remnant cosmological constant is becoming visible precisely at the present time in the cosmic history and why it does not exactly vanish. The inspiration coming from inflation has suggested that dark energy models are likely to be described by the dynamics of a (multi) scalar field(s), such as quintessence [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27] and phantom [28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65]. The quintessence scalar field has a positive kinetic term in its Lagrangian, which violates the strong energy condition but not the dominant energy condition, and the evolution of its equation of state parameter $w$ is in the range of $-1 \leq w \leq 1$. The phantom scalar field on the other hand possesses a negative kinetic term in its Lagrangian, which leads to some strange properties, such as the violation of the dominant energy condition and the occurrence of Big rip [66, 67, 68, 69, 70, 71, 72, 73, 74, 75]. (Let us note here that the possible avoidance of the Big Rip by various kinds of physical effects has been discussed by many authors [76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95]). The evolution of the equation of state parameter $w$ for phantom is in the range of $w < -1$. Other models including the Chaplygin gas [84, 85], braneworld models [86, 87, 88, 89, 90], holographic models [91] et al are also proposed to account for the present accelerating cosmic expansion.

The recent analysis of various cosmological data seems to indicate that it is mildly favored that the evolution of the equation of state parameter of the dark energy changes from $w > -1$ to $w < -1$ at small redshift [92, 93, 94, 95]. Although this result may be considered as tentative only since the standard $\Lambda$ CDM model remains inside $\approx 2\sigma$ error bars for all data, it is worthwhile to explore theoretical models which can bring $w$ crossing -1. In this regard, most models mentioned above can not do the job. “Quintom” [96, 97, 98], which assumes that the dark energy is composed of quintessence and phantom, on the other hand, can implement $w$ crossing -1 and in some cases fits the cosmological data better than models with $w \geq -1$. In addition ”quintom” model can reconcile the coincidence problem...
and predicts some interesting features in the evolution and fate of our universe \cite{96,97}. Considering a flat homogeneous and isotropic FRW universe filled with matter, radiation and “quintom” dark energy which is composed of a phantom scalar field $\phi$ and a normal scalar field $\varphi$, the dynamic equations for the universe, phantom and normal scalar fields can be described by

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3} \left[ \dot{\phi}^2 - \dot{\varphi}^2 + V(\phi, \varphi) - \frac{\Omega_{\text{mat}0}}{2a^3} - \frac{\Omega_{\text{rad}0}}{2a^4} \right],$$  

(1)

$$\ddot{\phi} + 3\frac{\dot{a}}{a} \dot{\phi} - V(\phi, \varphi)_{,\phi} = 0,$$ 

(2)

$$\ddot{\varphi} + 3\frac{\dot{a}}{a} \dot{\varphi} + V(\phi, \varphi)_{,\varphi} = 0,$$ 

(3)

where $V(\phi, \varphi)$ is the interacting potential between phantom and quintessence, $0$ denotes the present time, a dot means a derivative with respect to time $t$ and $a$ is the scale factor. The energy density and pressure for the quintom can be expressed as

$$\rho_{\text{qui}} = -\dot{\phi}^2 + \dot{\varphi}^2 + V(\phi, \varphi),$$ 

(4)

$$p_{\text{qui}} = -\dot{\phi}^2 + \dot{\varphi}^2 - V(\phi, \varphi).$$ 

(5)

Thus we can obtain the equation of state for the quintom

$$w_{\text{qui}} = \frac{p_{\text{qui}}}{\rho_{\text{qui}}} = \frac{-\dot{\phi}^2 + \dot{\varphi}^2 - V(\phi, \varphi)}{-\dot{\phi}^2 + \dot{\varphi}^2 + V(\phi, \varphi)}.$$ 

(6)

This expression implies $w_{\text{qui}} \geq -1$ when $|\dot{\varphi}| \geq |\dot{\phi}|$ and $w_{\text{qui}} < -1$ when $|\dot{\varphi}| < |\dot{\phi}|$.

Since there are more and more models proposed to explain the cosmic acceleration, it is very desirable to find a way to discriminate between the various contenders in a model independent manner. In this regard, Sahni et al. \cite{99,100} recently proposed a cosmological diagnostic pair \{r, s\} called statefinder, which are defined as

$$r \equiv \frac{\ddot{a}}{aH^2}, \quad s \equiv \frac{r - 1}{3(q - 1/2)},$$ 

(7)

to differentiate between different forms of dark energy. Here $q$ is the deceleration parameter. Apparently statefinder parameters only depend on $a$, and is thus a geometrical diagnostic. It is easy to see that statefinder differentiates the expansion dynamics with higher derivatives.
of scale factor $a$ and is a natural next step beyond $H$ and $q$. Since different cosmological models involving dark energy exhibit qualitatively different evolution trajectories in the $s-r$ plane, this statefinder diagnostic can differentiate various kinds of dark energy models. For LCDM (or ΛCDM) cosmological model, which consists of a mixture of vacuum energy and cold dark mass, the statefinder parameters correspond to a fixed point $\{r = 1, s = 0\}$. By far some models, including the cosmological constant, quintessence, phantom, the Chaplygin gas, braneworld models, holographic models, interacting and coupling dark energy models \textsuperscript{100, 101, 102, 103, 104, 105, 106}, have been successfully differentiated. For example, although the quintessence model with inverse power law potential, the phantom model with power law potential and the Chaplygin gas models all tend to approach the LCDM fixed point, for quintessence and phantom models the trajectories lie in the regions $s > 0$, $r < 1$ while for Chaplygin gas models the trajectories lie in the regions $s < 0$, $r > 1$, and on the other hand the quintessence tracker models and the Chaplygin gas models have typical trajectories similar to arcs of a parabola (upward and downward) respectively while for phantom model the trajectories are different \textsuperscript{100, 101, 102, 103}. For the coupled quintessence models the trajectories of $r(s)$ form swirl before reaching the attractor \textsuperscript{105}.

In this paper we apply the statefinder diagnostic to the quintom dark energy model. To begin with, let us use another form of statefinder parameters which can be written as

\begin{align}
    r &= 1 + \frac{9}{2} \left(\frac{\rho + p}{\rho}\right) \frac{\dot{\rho}}{\rho}, \\
    s &= \frac{(\rho + p)}{\rho} \frac{\dot{\rho}}{\rho}, \\
    q &= \frac{1}{2} (1 + 3w_{qui} \Omega_{qui} + \Omega_{rad}) .
\end{align}

Here $\rho$ is the total energy density and $p$ is the total pressure in the universe. Since the total energy, quintom energy and radiation are conserved respectively, we have $\dot{\rho} = -3H(\rho + p)$, $\dot{\rho}_{qui} = -3H(1 + w_{qui})\rho_{qui}$ and $\dot{\rho}_{rad} = -4H\rho_{rad}$. Thus we can obtain

\begin{align}
    r &= 1 - \frac{3}{2} \left[\frac{\dot{w}_{qui}}{H} - 3w_{qui}(1 + w_{qui})\right] \Omega_{qui} + 2\Omega_{rad} , \\
    s &= \frac{-3[\dot{w}_{qui}/H - 3w_{qui}(1 + w_{qui})]\Omega_{qui} + 4\Omega_{rad}}{9w_{qui}\Omega_{qui} + 3\Omega_{rad}} , \\
    q &= \frac{1}{2} (1 + 3w_{qui} \Omega_{qui} + \Omega_{rad}) .
\end{align}

Here $\Omega_{qui} = \rho_{qui}/\rho$ and $\Omega_{rad} = \rho_{rad}/\rho$. 

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In the following we will discuss the statefinder for the quintom model with several kinds of potentials. Firstly we assume that there is no direct coupling between the phantom scalar field and the normal scalar field with such a potential \( V(\phi, \varphi) = V_{\phi 0}e^{-\alpha \phi} + V_{\varphi 0}e^{-\beta \varphi} \), where \( \alpha \) and \( \beta \) are constants. In this case the universe has the phantom dominated late time big rip attractor \([98]\). In Fig. 1 we show the time evolution of statefinder pair \( \{r, s\} \) in the time interval \( t / t_0 \in [0.5, 4] \) where \( t_0 \) is the present time. The model parameters are chosen as \( V_{\phi 0} = 0.3 \rho_0 \) and \( V_{\varphi 0} = 0.6 \rho_0 \), where \( \rho_0 \) is the present energy density of our universe. We see that in the past and future the \( r - s \) is almost linear, which means that the deceleration parameter changes from one constant to another nearly with the increasing of time, and the parameters will pass the fixed point of LCDM in the future. These trajectories of \( r(s) \) are different from other dark energy models discussed in Refs. \([100, 101, 102, 103, 104, 105, 106]\).

Next, two cases of potential \( V(\phi, \varphi) = V_{\phi 0}e^{-\alpha \phi} + V_{\varphi 0}e^{-\beta \varphi} + V_0 e^{-\kappa(\phi + \varphi)} \) and \( V(\phi, \varphi) = V_{\phi 0}e^{-\alpha \phi^2} + V_{\varphi 0}e^{-\beta \varphi^2} \) will be studied, as before where \( \alpha, \beta \) and \( \kappa \) are constants. The quintom model with both above potentials have the late time attractors but the former potential leads to a big rip attractor and the latter to a de Sitter attractor \([98, 107]\). In Fig. 2 the curves of \( r(s) \) are given. Apparently the left figure is very similar to Fig. 1. This shows that for uncoupling and coupling exponential potentials the evolutions of our universe are very
similar in the time interval we consider here. The right figure is different from Fig. 1 but has a common characteristic with the phantom with power law potential, quintessence with inverse power law potential and Chaplygin gas model that it reaches the point of LCDM with the increasing of time. The reason is that they all lead to the same fate of the universe–de Sitter expansion, but the trajectories to LCDM are different, therefore they can be differentiated.

FIG. 2: In the left figure curves $r(s)$ evolve in the time interval $\frac{t - t_0}{t_0} \in [0.5, 4]$ where $t_0$ is the present time for the potential $V(\phi, \varphi) = V_{\phi 0} e^{-\alpha \phi} + V_{\varphi 0} e^{-\beta \varphi} + V_0 e^{-\kappa (\phi + \varphi)}$. The model parameter are chosen as $V_{\phi 0} = 0.3 \rho_0$, $V_{\varphi 0} = 0.6 \rho_0$, $V_0 = 0.3 \rho_0$, $\alpha = 1$, $\beta = 1$ and $\kappa = 1$, 2 (solid line and dot-dashed line respectively). In the right figure the potential is $V(\phi, \varphi) = V_{\phi 0} e^{-\alpha \phi^2} + V_{\varphi 0} e^{-\beta \varphi^2}$. The model parameter are chosen as $V_{\phi 0} = 0.3 \rho_0$, $V_{\varphi 0} = 0.6 \rho_0$, $\alpha = 1$ and $\beta = 1$, 3 (solid line and dot-dashed line respectively). Dots locate the current values of the statefinder parameters.

Finally, we turn to the case of linear coupling potential $V(\phi, \varphi) = \kappa (\phi + \varphi) + \lambda \phi \varphi$, where $\kappa$ and $\lambda$ are two constants. The scalar field with a linear potential was firstly studied in Ref. [108] and it has been argued that such a potential is favored by anthropic principle considerations [109, 110, 111] and can solve the coincidence problem [112]. In addition if the universe is dominated by quintessence(phantom) with this potential it ends with a big crunch(big rip) [113]. The time evolutions of statefinder pair $\{r, s\}$ in the time interval $\frac{t - t_0}{t_0} \in [0.5, 4]$ are shown in Fig. 3 We see that in the case of negative coupling the diagram is very similar to Fig. 1 and the left figure in Fig. 2 which shows that in these cases the evolutions of our universe are similar in the time interval we consider here, as shown in Fig. 4. But for $\lambda > 0$ the figure of $\{r, s\}$ is very different. In order to well explain this we give the time evolutions of $\{r, q\}$ in Fig. 5 From Fig. 4 and Fig. 5 we find if the coupling
constant $\lambda$ is positive the accelerating expansion of our universe is temporal and the universe will decelerate again.

FIG. 3: The $r - s$ diagrams of linear potential case. Curves $r(s)$ evolve in the time interval $\frac{t}{t_0} \in [0.5, 4]$. In the left figure $\kappa = 0.8$ and $\lambda = -0.2, -0.8$(solid line and dashed line respectively). In the right figure $\kappa = 0.8$ and $\lambda = 0.2, 0.8$(solid line and dashed line respectively). Dots locate the current values of the statefinder parameters.

FIG. 4: The evolution of scale factor $a$ with $t/t_0$. 1 and 2 represent potentials $V(\phi, \varphi) = V_{\phi 0}e^{-\alpha \phi} + V_{\varphi 0}e^{-\beta \varphi}$ and $V(\phi, \varphi) = V_{\phi 0}e^{-\alpha \phi} + V_{\varphi 0}e^{-\beta \varphi} + V_0e^{-\kappa (\phi + \varphi)}$ respectively. 3 and dot dashed line denote the linear potential with negative and positive coupling respectively.

In summary, we study the statefinder parameters of the quintom dark energy model. Several kinds of potentials are discussed. It is found that the statefinder diagnostic can differentiate the quintom model with other dark energy models, but seems not very helpful to differentiate quintom dark energy models with some different kinds of potentials which lead to a similar evolution of our universe in the time interval we consider.
FIG. 5: The $r − q$ diagram of linear potential case. Curves $r(q)$ evolve in the time interval $\frac{t}{t_0} \in [0.2, 4]$, $\kappa = 0.8$ and $\lambda = 0.2, 0.8$ (solid line and dashed line respectively). Dots locate the current values of the statefinder parameters.

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