Characterization of a heat exchanger by virtual temperature sensors based on identified transfer functions

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Abstract. Under certain conditions (steady flows, no temperature dependence of thermophysical properties), the temperature rise at any point (solid or fluid) of a heat exchanger, whatever its type, is a convolution product between the thermal source and the corresponding transfer function. Here a counter-flow heat exchanger is considered, where the hot fluid inlet temperature varies with time while the cold fluid inlet temperature is kept equal to the initial uniform temperature in the whole system. The transfer functions are estimated using synthetic temperature responses. This shows that temperature sensors located on the external surface of the exchanger can be used to detect fouling or to give informations about the outlet bulk temperatures of the fluids (steady state or transient thermal regimes).

1. Introduction

Heat exchangers play an important role in most of the industrial processes where production occurs using either heating or cooling phases. Fouling detection is a major problem for this type of unit. Several methods (filtration, chemical treatment, …) exist for slowing down fouling but, as soon as this has set in, complete cleaning becomes compulsory. Periodical cleaning to remove fouling is not always the best solution. If the cleaning period is short, it induces maintenance costs increase as well as production losses. Conversely, if this period becomes large, fouling may imply an increase of the energy consumed for the process, because of the associated reduction of the global conductance of the exchanger, and an increase of cooling water consumption, with its impact on the environment.

In the literature, several works can be found to design and implement techniques that allow to switch from systematical maintenance to predictive maintenance. One such a method is based on the mass variation of the exchanger. However, this method is not very precise if the fouling layer is thin with a possible high thermal resistance. It also requires a stop of the process for each diagnosis, with the associated costs. A classical method is based on the decrease of the global conductance of the unit, that can be detected in steady state regime through measurement of the inlet and outlet temperatures of the two fluids. However, bulk temperature measurement in each fluid flow is not always easy to implement (intrusive effects of the sensors, …). Another popular method is based on the increase of the pressure loss in each flow: for a constant flowrate of both fluid flows, this is caused by an increase
of the average velocities due to cross section reductions. However, the performance of this technique depends on the type of exchanger [1]. Other, more sophisticated, techniques have been developed for fouling thickness measurement: silicium sensors for biofilms [2], microstrip structures with microwave heating [3]. All these methods require a very specific instrumentation and are not always appropriate for industrial equipments, even if they perform well in a laboratory environment. So, the challenge is to design a method and associated sensors that can provide detection and assessment of fouling in real time, in an industrial unit.

In this work, a model, of the « grey box » type is proposed. It relies on transfer functions that can be identified on an experimental basis. This relies on the mathematical property of any transient linear integro-differential system of equations: if the coefficients of this system (a partial differential equation, for example, with its associated boundary and interface conditions) do not vary with time and if the time varying source term is unique, the solution of the system at any given location in the domain is a convolution product (in time) between the source and a transfer function (that depends on this location).

This can be applied to the transient thermal behaviour of a heat exchanger, whatever its type: as long as the two fluid flowrates inside a heat exchanger do not vary with time, the temperature rise at any point of the system (solid or fluid) is a convolution product between the thermal source and the corresponding transfer function. This result is valid if i) the variations of the thermophysical properties of both fluids and of the solid inner and outer walls do not vary with time and ii) in the case where only sensible heat is transferred between fluids and iii) the initial temperature in the whole system is uniform. The transfer functions depend on the location of the point considered, of the two fluid flowrates and on the thermophysical properties and geometry of the solid structure, and of the fluids in the system. The variation of one of these functions can be considered as a ‘signature’ of a change of a structural parameter of the system during operation, with its associate detection and possible quantification. This can allow to construct a fouling sensor using specific temperature sensor locations.

Since direct temperature measurement of outlet bulk temperatures of a flowing fluid using an internal sensor is not always possible, a non intruding technique using both a temperature sensor set on an appropriate location of the external exchanger surface connected by a transfer function to this fluid outlet temperature would allow to construct a virtual fluid temperature sensor.

The analytical expression of a thermal transfer function, in the Laplace domain, can be found using the Thermal Quadrupoles method [5] for a simple geometry. For real complex geometries corresponding to industrial heat exchangers, derivation of an explicit expression becomes impossible. However, knowledge of both input (thermal source, a heat power or a temperature) and output (local or averaged temperature) makes the identification of a transfer function possible. In the same way, knowledge of temperatures at two different locations in the system, connected to the thermal source by two different transfer functions, allows the identification of a new transfer function, a thermal transmittance here, provided that temperature input and output are well chosen.

So, design of a fouling detector can be met in two steps. In a first stage, calculation of the transfer function connecting temperatures at two different locations can be made using either the input/output of a detailed model of a heat exchanger, or its identification (calibration) using experimental input/output measurements, of the transfer function: this is typically an inverse input problem here. The second stage, always used when development of a virtual sensor is looked for, consists in reconstructing the temperature at the point of interest, using both a measurement at the other location and the previously identified transmittance. In this work, identification of transfer functions have been implemented using Comsol [6], which allows to get synthetic temperature ‘measurements’ (outputs of a detailed model) for numerical experiments.
2. The studied system

Two fluids (hot and cold) flow in opposite directions in two channels of common length \( l \), and of thicknesses \( e_{f1} \) and \( e_{f2} \) respectively. The two channels are bounded by three parallel flat plates of thicknesses \( e_1 \), \( e_2 \), and \( e_3 \) (see figure 1). The two flows are assumed to be laminar and dynamically developed over the entire length of the heat exchanger. So the velocity distribution in each cross section is parabolic (Poiseuille flow). The five layers (\( S_1, S_2, S_3, f_1 \) and \( f_2 \)) are characterized by their thermal conductivities \( \lambda_i \), their volumetric heats \( \rho c_i \), and their thermal diffusivities \( \alpha_i = \lambda_i / (\rho c_i) \).

![Figure 1: Heat exchanger in counter-flow mode](image)

The top and bottom faces are submitted to heat losses by natural convection and (linearized) radiation with a corresponding heat exchange coefficient \( h \) with ambient air and the outside environment at temperature \( T_\infty \). The lateral cross-sections of the three solid plates at the left and right edges of the figure are insulated. The inlet bulk temperature of the two fluids (hot and cold) are imposed uniform and equal to \( T_1 \) and \( T_4 \) respectively, while the second derivatives of temperature (with respect to \( x \)) are equal to zero at the outlet sections of both fluids. Before the initial time \( (t \leq 0) \), temperature inside the two flowing fluids and inside the solid walls is uniform and equal to \( T_m \). At initial time \( (t = 0^+) \), the inlet temperature \( T_1 \) of the hot fluid starts to increase with time till it reaches a given asymptotic steady state level \( T_1^{ss} \). The temperature difference, \( \theta_i(t) = T_i(t) - T_\infty \), constitutes the forcing term (a temperature source) of the mathematical model of the corresponding physical system.

3. Estimation of the transfer functions

Let us remind here that the partial differential equation (PDE) system modelling the physical phenomena (here heat transfer) in transient regime in the heat exchanger depicted in figure 1, is linear, with time-invariant coefficients and a uniform initial condition. So, the temperature response (here the temperature increase \( \theta = T - T_m \)) at any given observation point is a convolution product between the thermal source and a transfer function \( H \). This one can be considered as the identity card of the response, whatever the time shape of the unique source (it depends on the location of the observation and of the coefficients of the system of equations, here the thermophysical properties of the fluids and of the solid walls, of the velocity profiles of both flows and of the geometry of the heat exchanger).

If the two quantities (source and response) have the same physical unit, we will call this transfer function a transmittance noted \( W (H = W) \). If these units are different, with a temperature response to a heat power, this transfer function is generally called an impedance, noted \( Z \) here \( (H = Z) \), as for Alternative Current electrical circuits.
In the present heat exchanger model, see figure 1, we have assumed that the bulk temperature of the inlet cold fluid $T_0$ is fixed and equal to $T_w$. So, the only temperature source is the bulk temperature of the hot fluid at the inlet, $\theta_i$ (in the opposite case, one could add the responses of the two temperature sources to get their cumulated response at any point). The transmittances $W_i$ (for $i = P_1, P_2, P_3, P_4$ and 2 and 3) that connect source $\theta_i$ to $\theta_{P_1}, \theta_{P_2}, \theta_{P_3}, \theta_{P_4}$, and $\theta_3$ (see the locations of the corresponding points in figure 1 and in Table 1) can be estimated using the following equation, where operator $\ast$ designates the convolution product between two functions defined on the same time interval:

$$\theta_i(t) = W_i(t) \ast \theta_i(t) = \int_0^t W_i(t-t') \theta_i(t') \, dt' \quad \text{with} \quad \theta_i = T_i - T_w \quad (1a, b)$$

Convolution integral (1b) can be written under a column vector/matrix form, once the three functions $\theta_i(t), \theta_j(t)$ and $W_i(t)$ discretized with a time step $\Delta t$ to form three corresponding column vectors $\theta_i, \theta_j$ and $W_i$, $m$ being the number of sampling times considered:

- Model used for identification of a transmittance (estimation of a transfer function based on prior knowledge of source $\theta_i(t)$ and response $\theta_j(t)$):

$$\theta_i = \mathbf{M} (\theta_j) W_i \Rightarrow W_i = (\mathbf{M} (\theta_j))^{-1} \theta_i \quad (2a, b)$$

- Model used for construction of a virtual temperature sensor (estimation of temperature source $\theta_i(t)$ based on a prior knowledge of source $\theta_j(t)$ and transmittance $W_i(t)$, for example):

$$\theta_i = \mathbf{M} (W_i) \theta_j \Rightarrow \hat{\theta}_i = (\mathbf{M} (W_i))^{-1} \theta_j \quad (3a, b)$$

where $\mathbf{M}(\cdot)$ is a matrix function (a Toeplitz matrix, of dimensions $m \times m$) of any column vector of size $m \times 1$:

$$\mathbf{M}(z) \equiv \Delta t \begin{bmatrix} z_1 & 0 & \cdots & \cdots & 0 \\ z_2 & z_1 & \ddots & \cdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ z m & z_{m-1} & \cdots & z_2 & z_1 \end{bmatrix} \quad \text{with} \quad z = \begin{bmatrix} z_1 = z(t_1) \\ z_2 = z(t_2) \\ \vdots \\ z_m = z(t_m) \end{bmatrix} \quad \text{and} \quad t_k = k \Delta t \quad \text{for} \quad 1 \leq k \leq m \quad (4)$$

and where $\hat{\cdot}$ designates the estimated value of the corresponding quantity in the inverse problem considered, that is either a transmittance identification (2) or a virtual sensor construction (3).

The advective term present in the heat equation makes the characteristic time between source and response decrease with respect to pure conduction. Since heat exchangers are usually run with fluid velocities that can not be neglected (high Péclet numbers), the thermal characteristic time is usually neglected with respect of the total duration of its operation. This is why most of the works present in the corresponding literature are focused on the steady state thermal behaviour of heat exchangers.

However, study of transient heat transfer in these units remains a very interesting subject, especially within the context of renewable energies with their intermittent nature. We will see briefly below how this kind of transient modeling can also allow, as a by-product, to get an identification of the steady state operation of a heat exchanger.A convolution product in the time domain corresponds to a simple product in Laplace domain. So, equation (1a) can be re-written as:

...
\bar{z}(p) = W^i(p) \bar{z}(p) \quad \text{where} \quad z(t) = \int_0^\infty z(t) \exp(-pt) \, dt \quad (5a, b)

where the upper bar on a given time function \( z \) designates its Laplace transform, \( p \) being the Laplace parameter. Application of the final value theorem for the Laplace transformation shows that, whatever the time variation of the source, the steady state (the asymptotic behaviour) of the response can be calculated as:

\[ \theta_{i,ss}^{\text{ss}} = W_{i,ss}^{\text{ss}} \theta_{i,ss}^{\text{ss}} \quad \text{with} \quad W_{i,ss}^{\text{ss}} = \int_0^\infty W_i^i(t) \, dt \quad (6a, b) \]

So, the transfer function \( W_{i,ss}^{\text{ss}} \) in steady state regime is the time integral of the transient transfer function \( W_i^i(t) \). Its physical unit is the unit of the transient transfer function multiplied by a time.

4. Identification results, virtual sensors and fouling detection

A numerical experiment has been run using the finite element code Comsol. It simulates the transient behaviour of the system shown in figure 1. The heat source is the bulk temperature of the inlet hot fluid and is taken as an exponential increase with time \( \theta_i(t) = \theta_i^{(1)} (1 - \exp(-t/\tau)) \) with \( \theta_i^{(1)} = 70^\circ\text{C} \) and \( \tau = 50 \, \text{s} \), while the inlet temperature of the cold fluid is kept to the ambient level \( \theta_3(t) = 0 \). The following responses have been simulated: bulk outlet temperatures of the hot fluid \( \theta_1 \) and of the cold fluid \( \theta_3 \), as well as local temperatures \( \theta_{P_1} \) to \( \theta_{P_4} \) of the corresponding points located on the external faces of the heat exchanger. Both source \( \theta_1 \) and these seven responses have been plotted in figure 2, with a time \( t \) varying between zero and \( t_{\text{final}} \), with a time step \( \Delta t = 0.5 \, \text{s} \). The dimensions of the studied system and the simulation parameters are given in Table 1. A triangular mesh of more than 3800 elements for each of the five layers of the system has been used.

| \( l \) (cm) | \( e_1 = e_2 = e_3 \) (cm) | \( e_{f_1} = 2e_{f_0} \) (cm) | \( x_{p_1} = x_{p_4} \) (cm) | \( x_{p_3} \) (cm) | \( x_{p_4} \) (cm) | \( U_{f_1} = U_{f_2} \) (cm/s) | \( t_{\text{final}} \) (s) |
|---|---|---|---|---|---|---|---|
| 50 | 0.3 | 2 | 0 | 10 | 50 | 0.2 | 1000 |

\[ \lambda_x = \lambda_{x_3} = \lambda_{x_2} \quad (\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}) \quad \rho c_{x_3} = \rho c_{x_2} = \rho c_{x_1} \quad (\text{kJ} \cdot \text{m}^{-3} \cdot \text{K}^{-1}) \quad \lambda_f = \lambda_{f_2} = \lambda_{f_3} \quad (\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}) \quad \rho c_{f_2} = \rho c_{f_3} \quad (\text{kJ} \cdot \text{m}^{-3} \cdot \text{K}^{-1}) \quad h \quad (\text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}) \]

43 3666 0.63 4186 1

Figure 2: Temperature variations for an exponentially varying bulk temperature \( \theta_1(t) \) of the hot fluid
4.1 Identification of the transfer functions (calibration):

Estimation of transfer functions is a preliminary step for constructing a virtual sensor or the fouling detector. Transmittances between $\theta_1$ and its temperature responses $\theta_2, \theta_3$ and $\theta_4$ are shown in figure 3 and the specific transmittances that link the pseudo-source $\theta_1$ to $\theta_2$ and $\theta_3$ are shown in figure 4.

All these transmittances have been estimated using the estimation principle given by equation (2a). Transmittance between $\theta_1$ and $\theta_4$ is not interesting since the temperature signal at $P_4$ is very weak in this type of configuration, especially if measurement noise is present in the real physical experiment.

![Figure 3: Calibration of virtual bulk temperature sensor](image)

![Figure 4: Calibration of the fouling detector](image)

The corresponding steady state transmittances have been calculated either by integration of the transient transmittances, see equation (6a), or using the asymptotic temperature outputs of the Comsol code. Table 2 sums up the values of these transmittances $W_{ij}^{ss}$.

| Transmittances identified from exponential excitation | $W_{ij}^{ss} = \int_0^\infty W_j(t) \, dt = \theta_j^{ss} / \theta_i^{ss}$ |
|------------------------------------------------------|-------------------------------------------------|
| $W_{11}^{ss}$                                       | 0.9993                                         |
| $W_{12}^{ss}$                                       | 0.7867                                         |
| $W_{13}^{ss}$                                       | 0.4146                                         |
| $W_{P2}^{ss}$                                       | 0.9953                                         |
| $W_{P3}^{ss}$                                       | 0.9759                                         |

4.2 Virtual sensor of bulk outlet fluid temperatures:

Bulk temperatures $\theta_1, \theta_2$ and $\theta_3$ of each fluid at each outlet may be difficult to measure during operation. So, the idea is to estimate these quantities using the previously estimated transmittance vectors $\tilde{W}_i$ (with $i = P_2, 2$ or 3) during calibration, as well as temperature measurement $\theta_{P_1}$ at a single point of the external face. In order to test this principle, a second numerical experiment has been made, with a change of the inlet bulk temperature of the hot fluid that is now a step function instead of the previous exponential used for estimating the transmittances. These are plotted in figure 3.

In a first step, only temperature $\theta_{P_1}$ is supposed to be known for the second excitation as well as transfer functions $\tilde{W}_i$. So, temperature responses of this second experiment, $\tilde{\theta}_i = (M(\tilde{W}_i))^{-1} \theta_{P_1}$, derived from the virtual sensor equation principle (3b) are calculated. Their variations $\theta_1, \theta_2$ and $\theta_3$ are plotted in figure 5, together with the corresponding output of the same numerical experiment.

The fit between virtual sensor temperatures and exact ones (Comsol outputs) is very good. Only reconstruction of the hot fluid inlet temperature $\tilde{\theta}_1$ has required a specific treatment because of numerical noise. The condition number of matrix $M(\tilde{W}_i)$ is very high ($3.72 \times 10^{16}$), that is of the same order of magnitude as the precision of the computer used ($10^{-16}$). So a TSVD (Truncated Singular Value Decomposition) approach has been used.
Value Decomposition) regularization [7, 8] has been made to solve the corresponding ill-posed problem, with a truncation to 1999 singular values for a number \( m = 2000 \) of times of observation.

![Figure 5: Exact temperatures (continuous lines) of the second numerical experiment and their virtual sensor estimations (interrupted line).](image)

### 4.3 Fouling detection

The transfer function approach can also yield an information about the appearance of fouling in both steady state and transient thermal regimes. In order to test the potential of a Non DestructiveTesting (NDT) technique, a 3\(^{rd}\) numerical experiment has been run on the same system as the two previous ones, except that two layers(common thicknesses \( e_c \)) of limestone deposit(\( \lambda_c = 1 \text{ W m}^{-1}\text{K}^{-1} \) and \( \rho_c c = 2 \times 10^6 \text{ J m}^{-3}\text{K}^{-1} \)), have been added on each inner wall of the hot fluid channel. Table 3 shows that the steady state transmittances, based on observation points \( P_1 \) (pseudo-source) and \( P_2 \) or \( P_3 \) (responses) on the external face of the hot fluid channel, are not very sensitive to the presence of the fouling layers: 2 different thicknesses \( e_c \) have been tested with corresponding ID thermal resistances (for a unit area) \( R_e = e_c/\lambda_c \). This low sensitivity can be explained by two competing effects: presence of a fouling layer makes the channel hydraulic diameter decrease, implying an increase of the internal convective heat transfer coefficient that can mask the introduction of the thermal resistance \( R_e \).

| Transmittances: \( W_{i,ss}^{i,ss} = \int_0^\infty W_{i}^{i,ss}(t) \ dt = \theta_{i}^{i,ss} / \theta_{i}^{i,ss} \) | \( W_{P_1,ss}^{P_1,ss} \) | \( W_{P_2,ss}^{P_2,ss} \) |
|---|---|---|
| without fouling layer: \( e_c = 0 \) | 0.9959 | 0.9766 |
| with fouling layer: \( e_c = 1 \text{ mm}, R_e = 1.0 \times 10^{-3} \text{ S.I.} \) | 0.9987 | 0.9967 |
| with fouling layer: \( e_c = 3 \text{ mm}, R_e = 3.0 \times 10^{-3} \text{ S.I.} \) | 0.9991 | 0.9973 |

In transient state, transmittance functions connecting \( P_1 \) (pseudo-source) and \( P_2 \) or \( P_3 \) (responses) are presented in figure 6 for different thicknesses of the fouling layers (\( e_c = 0 \) (no fouling), 1 and 3 mm). Let us note that without fouling, the peaks of the two transmittance curves, already shown in figure 4, occur at their corresponding advection times (\( x_{P_2}/U_{f_1} = 50 \text{ s for } W_{P_1,ss}^{P_2,ss} \) and \( x_{P_3}/U_{f_1} = 250 \text{ s for } W_{P_1,ss}^{P_3,ss} \)). This means that any of these two times can be used, for known locations of points \( P_2 \) or \( P_3 \), for estimating the internal mean velocity \( U_{f_1} \) of the hot fluid. Presence of the two fouling layers yields a
cross-section decrease by a factor \(2 e_r/e_{f1}\) (10\% for \(e_r = 1\) mm and 30\% for \(e_r = 3\) mm). This is precisely what is found for the decrease of the times of occurrence of the peaks of the two transmittance curves for these two values of \(e_r\). So, an increase of fouling thickness will imply first an increase of the maxima of the transmittances and a decrease of their times of occurrence. This is very interesting for quantifying the layers thickness through the induced velocity effect. As a conclusion, maximization of sensitivity, for NDT fouling detection, relies first on a pseudo-source sensor located at a point \(P_1\) of the external surface as close as possible to the inlet of the hot fluid, and on a second sensor as far as possible from the first sensor then.

**Figure 6:** Transient transmittances between observation points on the external surface: influence of the thickness of the fouling layer

5. **Conclusion**

We have shown that identification of a transfer function is possible for both transient and steady state regimes for a heat exchanger. This transmittance concept can be used to characterize the inner thermal behaviour of such a unit (virtual sensor methodology) or to detect and possibly quantify the presence of a fouling layer (NDT application). These results are encouraging, and deserve to be applied and validated for more complex heat exchanger geometries (shell and tubes, for example) than the simple 2D case that has been modelled here, with real (noisy) temperature measurements.

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