The calculation of the parameters of disc-shaped hydraulic fracturing crack propagating in hard rock roofs of a coal seam near the in-seam working

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Abstract. The model of geomechanical state of the disc-shaped hydraulic fracturing crack that propagates in hard rocks near the in-seam working was developed. The peculiarity of the model is in non-homogeneous stress field which is conditioned by the presence of the working. This field plays an important role in calculating the trajectory of the crack propagation and under certain conditions it drastically changes the direction of the initial crack. The problem of the rock massif stress state in the vicinity of the in-seam working is narrowed to the second exterior boundary value problem of the elasticity theory for integrated singular equation. It can be reached by substituting limit stress coal seam marginal zones by their acting stresses. Analyzing the results it is defined that when the ratio of the working liquid pressure to the gravitational hard rock pressure equals four units the trajectory of the crack is linear and coincides with the direction of the initial crack. If the ratio of the working liquid pressure to the gravitational pressure changes from three to four units the trajectory of the crack looks as a gently sloping curve that insignificantly changes its direction towards the initial crack.

1. Introduction

The problem of effective utilizing of hydraulic fracturing method is in setting reasonable parameters of the initial crack for its optimal propagation in the coal roof providing its controlled caving [1-4] and safety mining [5, 6].

The trajectory of the crack propagation is influenced by many factors and the most influencing one is the stress field in the vicinity of the growing crack as it propagates not in a homogeneous initial stress field of a virgin ground but in the vicinity of the workings that drastically changes this field.

In papers devoted to applying method of hard rock hydraulic fracturing, basically, the researches on hydraulic fracturing crack that propagates from the surface of the borehole situated in homogeneous gravitational field of stresses are introduced [7]. There are not many theoretical works devoted to the development of hydraulic fracturing cracks in rock massifs in the vicinity of the mine workings. Thus, in paper [4] the calculation of crack propagation direction in the vicinity of in-seam working is introduced. It is considered that this dome fold is an arch-type construction and the stresses in it are defined by the methods of frame structure construction mechanics later used for calculation of the hydraulic fracturing cracks trajectory. Such approach do not take into account either physical and mechanical properties of the massive or coal seam properties and this is the reason why this it can’t be considered only as a rough approximation in calculating hydraulic fracturing cracks.
In this regard setting the connection between the parameters of the massif, a working and operating performance equipment during hydraulic fracturing of the roof rocks by disc-shaped crack is an important and timely scientific issue.

The models for anisotropic coal rock massif geomechanical state enclosing the system of workings constitutes the basis of the hydraulic fracturing crack propagation mathematical model. The model is based on boundary element analysis and Coulomb-Mohr and Mohr–Kuznetsov strength criteria [8–11]. In the framework of these models a number of problems on the state of anisotropic massif enclosing system of workings in three- and two-dimensional settings were solved [12–16].

2. Problem statement and its solution

The problem is formulated as follows (figure 1): there is a rectangular-sectioned working 1 with the dimensions of \( b_v \times h_v \), which is driven at the depth of \( H \) along the coal seam 2 at its whole thickness in a rock massif simulated by weightless plain surface. The support response \( f \) is attached to a seam roof and a lying wall. The coal seam strength characteristics are less than the characteristics of enclosing rock massif strength but larger than the ones on the contact of a seam with other part of the massif which is loaded by the gravitational pressure on the top and the bottom \( \gamma H \) (\( \gamma \) – overlying rock weight-average unit specific gravity), and on the sides– \( \lambda \gamma H \) (\( \lambda \) – lateral thrust coefficient). In selvedges, zones of inelastic deformation 3 with the width of \( L \) are formed. The borehole 5 is drilled from the mine roof and out of it a disc-shaped crack 5 (initial crack) with the radius of \( b \) is formed by a crack forming device. The crack is loaded by the pressure \( p \) and inclined to the horizon at the angle of \( \theta \). Its coordinates in a system \( y_0z \), coincide with central axis of the working, \( y_t, z_t \).

In the process of solving the problem it is supposed that:

1) mine working trace and borehole axis are parallel and their dimensions along x-axis prevail significantly over the dimensions in the plane \( y_0z \), due to all these it can be considered that the rocks in the vicinity of the mine working and the borehole are under the state of plane strain deformation;

2) the seam strength is significantly lower that the strength of its enclosing rocks;

3) normal compressive stresses are positive;

4) hydraulic fracturing crack does not change the stress fields in the vicinity of the mine working;

5) liquid filtration processes in a massif and its other leakages are not taken into account.

![Figure 1. Computational scheme for in-seam working and a disc-shaped crack.](image-url)
It is well-known that in quasi-static processes a crack propagation in solid bodies takes place under the condition if \[ k_I^2 + k_{II}^2 = \frac{E \cdot \gamma_t}{1 - \mu^2} = K_{Ic}^2, \] (1)

where \( k_I \) is a coefficient of intensive stresses conditioned by the influence of normal loads \( p_I \) at the sides of the crack, \( k_{II} \) is a coefficient of intensive stresses influenced by tangential load \( p_{II} \) either at the sides of the crack. \( E \) is an elasticity modulus of the first genus, \( \mu \) is a rock massif Poisson’s ratio. \( \gamma_t \) is a tensile energy density, which is necessary for the formation of surface unit, \( K_{Ic} \) is a coefficient for crack resistance of the material (reference data for some types of rocks are given in [19]).

If the crack is little comparing to the size of the plane surface and is situated inside it then the loads \( p_I, p_{II} \) equal to normal and tangential stresses on the crack surface in case it would have been closed (hidden); so, the stresses are defined analytically or numerically for a crackless body.

The stress intensity coefficients for a disc-shaped crack with the radius \( a \), loaded by internal pressure at the section of radius \( b \) \((b < a)\), situated near in gravitational stress field are expressed by the following equations [20]

\[
k_I = \frac{2}{\sqrt{\pi a}} \left( \int_0^b \frac{r \cdot p_I}{\sqrt{a^2 - r^2}} dr - \int_0^a \frac{r \cdot p_I(r)}{\sqrt{a^2 - r^2}} dr \right), \quad k_{II} = \frac{2}{\sqrt{\pi a}} \int_0^a \frac{r \cdot p_{II}(r)}{\sqrt{a^2 - r^2}} dr, \quad (2)
\]

where \( r \) is a radial vertical axis measured from the center of the crack.

If the crack is in homogeneous field of stresses then \( p_I, p_{II} \) do not depend on \( r \) and \( k_I, k_{II} \) after integrating equations (2) are defined as follows

\[
k_I = 2p_I \sqrt{\frac{a}{\pi}} \left( 1 - \sqrt{1 - \frac{b^2}{a^2}} \right) - 2p_I \sqrt{\frac{a}{\pi}}, \quad k_{II} = 2p_{II} \sqrt{\frac{a}{\pi}}. \quad (3)
\]

Substituting equations (3) into the condition of Eq. (1) defines critical pressure of the liquid which corresponds to sustained growth of the crack.

\[
p_{kp} = \frac{1}{1 - \sqrt{1 - \frac{b^2}{a^2}}} \left( p_I + \frac{\pi \gamma_{II}}{4a \left( 1 - \mu^2 \right)} - p_{II}^2 \right). \quad (4)
\]

It’s obvious that on the surface of the initial crack there is no \( p_I, p_{II} \) and the crack initiation starts with the pressure equals \( p_{kp,\text{min}} \). Out of equation (4) when \( a = b \) it follows that

\[
p_{kp,\text{min}} = \sqrt{\frac{\pi \gamma_{II}}{4b \left( 1 - \mu^2 \right)}}. \quad (5)
\]

In this earlier set problem the characteristics \( \gamma_t \) can be considered single for all directions and for all types of the crack deformation and its deviation from the initial direction is expected when \( k_{II} \neq 0 \).

According to this supposition the direction of the crack propagation forms a certain angle with the original direction of a crack. This angle is defined while solving trigonometric equation towards angle \( \theta \), which is measured from the crack direction at the moment of its initiation [18]

\[
k_I \left( \sin \frac{\theta}{2} + \sin \frac{3\theta}{2} \right) + k_{II} \left( \cos \frac{\theta}{2} + 3 \cos \frac{3\theta}{2} \right) = 0. \quad (6)
\]
After finding the angle by solving equation (6) normal and tangential stresses acting on the sides of the crack are found according to stress component transformation formulas when rotating of coordinate axis as follows:

\[
\sigma_{II} = \left(\frac{\sigma_z - \sigma_y}{2}\right) \sin 2\theta + \tau_{yz} \cos 2\theta,
\]

\[
\sigma_{I} = \left(\frac{\sigma_z + \sigma_y}{2}\right) \cos 2\theta + \tau_{yz} \sin 2\theta,
\]

Thus, in the vicinity of the mine working a crack, in common, propagates nonlinearly. With each growing of the length (growth cycle) it also changes its direction.

In \( j \)-cycles the crack looks as a jogged line in a form of a complex of straight lines in the limits of which stresses \( p_I \) and \( p_{II} \) are taken as constant values, and the critical pressure for cycle \( j \) is determined by the following formula

\[
p_{kpj} = \frac{1}{\left(1 - \frac{b_j}{a_j}\right)^2 - \frac{b_j^2}{a_j^2}} \left( J_I + \frac{\pi E\gamma_t}{4a_j (1-\mu^2)} - J_{II}^2 \right).
\]

Values \( J_I, J_{II} \) embedded into this formula are found as follows

\[
J_I = \frac{1}{a_j} \sum_{i=1}^{j} \int_{b_i}^{a_i} \frac{r}{\sqrt{a_j^2 - r^2}} dr,
\]

\[
J_{II} = \frac{1}{a_j} \sum_{i=1}^{j} \int_{b_i}^{a_i} \frac{r}{\sqrt{a_j^2 - r^2}} dr.
\]

In equations (8), (9)

\[a = a_j, \quad b = b_j = a_j - \Delta l_j,\]

where \( \Delta l_j \) - is a crack length growth that corresponds to cycle \( j \).

As the crack propagates in non-homogeneous stress field then stresses \( p_I \) and \( p_{II} \) at its sides in its two developing directions are different (it follows from equation (7)). Corresponding to this the entering values of equation (9) are taken as average from the stresses in points situated at the opposite (in the plane \( yz \) ) branches of the crack.

For directional hydraulic fracturing pumping sets with hard operating performances: delivery pressure \( p \) and rate of liquid throughput (per unit of time) \( Q \) are used. In such sets \( Q \) – is const. and \( p \) – is a variable value.

In the process of hydraulic fracturing a liquid pressure when going through the crack changes according to Poiseuille's law due to the viscosity \( \eta \) and due to the crack’s parameters (the opening before the following cycle of growing \( w \) and its length \( 2a \)) [21]

\[
p_j = p_{j-1} - \frac{3 \eta Q}{w_j^3} \Delta l_j.
\]

In equation (10) a crack opening \( w \) is found according to Sneddon’s equation [22]

Forming and developing the crack is a two-phase process. During the first phase the crack develops under gradually heightened liquid pressure starting from zero. Having reached value \( p_{kp,min} \), which is found according to equation (5), it starts «proceeding», and its direction coincides with the direction of the initial crack as only a liquid pressure acts at its end. While further increasing of the pressure on \( \Delta p \) to a certain current value \( p \) the crack grows in size on \( \Delta l \). This growth can be found by solving transcendental equation...
\[
p_j = \frac{1}{1 - \left(1 - \frac{b_j^2}{a_j^2}\right)} \left( J_I + \sqrt{\frac{\pi E \gamma t}{4 a_j (1 - \mu^2)}} - J_{II}^2 \right) = 0, \tag{11}
\]

where the second addend is a value of the critical pressure found by the equation (8) corresponding to the growth of the crack. In this equation the value of \( b \) should be put equal to the initial radius of the crack and after that it is possible to find value \( a \). Its solving can be done by successive approximations, for example, increasing with each iteration step the amount \( a \) on small quantity till the condition (11) is fulfilled. The growth \( \Delta l \) is found as a difference between a final value of the iteration process \( a \) and value \( b \). Then using equation (10) a new value of \( p \) is calculated and the successive pressure rise \( \Delta p \) is added after that a new cycle of the first phase for a crack propagation starts.

During spontaneous (uncontrolled) crack growing cycle value \( a \) of the previous cycle is assigned to value \( b \). Then \( k_I, k_{II} \) are found and through equation (6) an angle \( \theta \) between the crack direction of the previous cycle and a new direction is found. Further on, applying the method of successive approximation equation (11) is solved and the dimension \( a \) is found and then parameters \( w, p, p_{II}, k_I, k_{II}, \theta \) are calculated. Calculation process continues till the moment the liquid pressure \( p \) in a system reaches its maximum value \( p_0 \), which is defined by technical characteristics of the pumping equipment.

In the cycles of the second phase the growth of the pressure \( \Delta p \) is absent and the crack propagation starts from the pressure value \( p_0 \). The growth of the crack stops in case the pressure in it is less than the stresses in a virgin ground or in case the liquid injection into the crack under limiting cycle numbers is ceased.

3. The results of problem solution and their analysis

As the initial information the following data are taken: \( \gamma=25 \text{ kN/m}^3, H=700 \text{ m}, \lambda=1, f=2.5 \text{ kN/m}^2, b_v=5 \text{ m}, h_v=3 \text{ m}, E=20000 \text{ MPa}, \mu=0.25, \) uniaxial compression strength of the formation \( \sigma_0=10 \text{ MPa}, K_{1C}=1.66 \text{ MPa-m}^{1/2} (\gamma=12.92 \cdot 10^{-5} \text{ MPa-m}), b=0.066 \text{ m}. \eta=13.04 \cdot 10^{-10} \text{ MPa-s}, \gamma_t=3 \text{ m}, z_t=10 \text{ m}, \theta_t=-25^\circ, Q=0.005 \text{ m}^3/s. \) The rest data varied during the experiment.

The solution of the elastic-plastic problem is found in the framework of the model of geo-mechanic state of the coal-rock massif an in-seam working in it [14]. In this problem the distribution of normal and shear stresses in limit stress zones are obtained by the method of granular media mechanics and later they are approximated by analytical equations in the form of polynomials [16].

Graphs 1–4 in figure 2 are presented by diagrams of stresses \( \sigma_z \), built along a seam roof (along APB line in figure 1) in a limit-stress zone (graph 1) and in an elastic (graph 2). Analyzing the graphs it is seen that the value of the bearing pressure makes \( 1.6 \gamma H \), and the area of a limit-stress zone \( L \) equals to 4.15 m.

The trajectory of hydraulic fracturing crack for the values of pressure \( p_0=50 \text{ MPa} \) is built in figure 3. Number 1 indicates the coal working, number 2 is a limit stress zone, number 3 is an initial crack, number 4 is a crack trajectory branches. The numbers of the cycles that correspond to the growth of the crack are identified by the circles in the figure. It is seen that the trajectory is presented in the form of smooth, flat line basically close to a straight one. In the process of its growth it changes insignificantly its direction, deviating from the initial crack direction. Half the length of the crack is 11.35 m and corresponds to 19 cycles, thus the pressure in it, in the process of its growth, decreases on 0.022 MPa.
Figure 2. The curve of the stresses $\sigma_z$ spreading along the seam roof.

Figure 3. The crack trajectory when $p_0=50$ MPa.

Comparing the relation of $p_0/\gamma H$ with crack trajectory orientation and form it can be seen that when $p_0/\gamma H=3$ the crack trajectory is close to a straight line in a from and it insignificantly deviates from the direction of the initial crack.

Graph 1 that demonstrates the dependence of the crack half-length $a$ on the number of the cycles of its growth $j$, corresponding to trajectory in figure 3 is built in the figure 4.
The analysis of the graph shows that on the first, it is a non-linear one and on the second, during the last cycles its values increase sharply. The analysis of the crack growth demonstrates that the change in half-length with increasing the number of cycles is quite accurately described by the exponent function

\[ a_j = b_l \cdot \left( \frac{a_N}{b_l} \right)^{j/N}, \]

where \( N \) – is a finite number of cycles, \( a_N \) – is a finite half-length of the crack.

In figure 4 this graph is identified by number 2. It is well-seen that graphs 1 and 2 coincide with each other.

![Graph of the dependence of the disc-shaped crack half-length on the number of cycles](image)

**Figure 4.** The graphs of the dependence of the disc-shaped crack half-length on the number of cycles \( j \) when \( p_0 = 50 \) MPa.

### 4. Conclusions

1. On the basis of the deformable solid mechanics fundamental methods the model of disc-shaped hydraulic fracturing crack that propagates in hard roof of a seam was worked out. Under given characteristics of the media, working characteristics of the pumping equipment and initial parameters of the initial crack the model supports the calculation of the crack and the selection of its rational parameters.

2. When the value of a liquid pressure in a borehole equals to a threefold value of the gravitational pressure in a virgin massif at the depth of the coal working then the crack trajectory forms a flat curve close to a straight line and its direction insignificantly deviates from the initial crack direction.

3. The length of the hydraulic fracturing crack during its growth changes according to the exponential function law and the dimensions of the initial crack, its final length and the number of its growth cycles are included into it.
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