Possible measurement of Quintessence and density parameter using strong gravitational lensing events

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We propose a possible measurement of the time variability of the vacuum energy using strong gravitational lensing events. As an example we take an Einstein cross lens HST 14176+5226 and demonstrate that the measurement of the velocity dispersion with the accuracy of ±5 km/sec will have a chance to determine the time dependence of the vacuum energy as well as the density parameter with the accuracy of order 0.1 if one fixes the lens model.

It is now fashionable to assume a non-vanishing value for the cosmological constant to explain some of the observations, such as the magnitude-redshift relation of high-z Type Ia supernovae. Its origin is not fully understood, but it is associated with the energy of the vacuum, and thus called the vacuum energy. It is sometimes argued that it comes from the energy of the zero-point fluctuation of quantum fields. However if it is so, the value expected by a naive theoretical argument based on the dimensional analysis is about $10^{122}$ larger than the observationally suggested value. It is also argued that the cosmological constant is identified with the potential energy of a scalar field. Then the natural energy scale will be again much higher than the observationally suggested value and one needs to explain such a small energy. Furthermore one is then naturally expect that the vacuum energy changes as Universe evolves. Thus to reveal the detailed nature of the vacuum energy, i.e., its existence and time variability, is very important not only for cosmology but also for fundamental physics.

Several methods have been proposed so far to determine the vacuum energy, e.g., the multiple imaged quasar statistics and the magnitude-redshift relation of distant Type Ia supernovae, but very few is able to measure its variability. Here we propose a possible method using strong gravitational lensing. As a matter of fact one of the authors has pointed out that Einstein ring system with suitable redshift combinations of the lens and source can be used as a powerful tool to measure the vacuum energy. We here point out that the system and other similar systems are also powerful to measure the variability as well. In the below we restrict ourself to the totally flat cosmology which means that $\Omega_m + \Omega_\Lambda = 1$, where $\Omega_m$ and $\Omega_\Lambda$ are the density parameter of matter and normalized cosmological constant at present time, respectively. This is supported by recent measurements of cosmic background anisotropy. We also take an “Einstein cross” gravitational lens system HST14176+5226 as our example.

HST 14176+5226 has been discovered with HST as a candidate of gravitational lens system in 1995, and subsequent spectroscopic observations have provided
confirmation that the system is indeed a lens. The elliptical lensing galaxy has a redshift of \( z = 0.803 \) with apparent magnitude 21.7 in V band and 19.8 in I band. The lensed source at redshift \( z = 3.4 \) appears to be QSO with an apparent magnitude 26.2 in V band and 25.7 in I band. The lens model based on a singular isothermal distribution gives a very good fit to the observed images, with normalized \( \chi^2 \) of unity.

The lens model gives us the (tangential) critical line which can be written in terms of the velocity dispersion of the lensing galaxy and the distance combination \( D_{ls}/D_s \), where \( D_{ls} \) is the angular diameter distance from a lens to a source and \( D_s \) is one from the observer to source. On the other hand, the distance combination has a strong dependence on the vacuum energy \( \Omega_\Lambda \), but has a week dependence on the matter contribution \( \Omega_m \) as pointed out by Ref. 21. Thus if the velocity dispersion of the lensing galaxy as well as the redshifts of the galaxy and the source are observed, the vacuum energy can be in principle well constrained when the redshift combinations are appropriate. In fact the idea using the tangential critical line as a method to determine the vacuum energy has been applied by Ref. 22 by observing the decrease of the number density of background galaxies in the cluster Cl0024+1654 at the critical line, and it is argued that the lower bound on the cosmological constant is obtained assuming the spherical symmetry of the cluster. However, the mass distribution of the cluster is likely to deviate from spherical symmetry and the effect does depend on the luminosity function and the evolution of the background galaxies, which makes this method difficult to withdraw any definite conclusion. On the other hand, systems like HST14176+5226 have a almost perfect symmetry which allows us to have a very good model fitting.

In this Letter we investigate the possibility to measure the variability of the vacuum energy using HST 14176+5226. Although we restrict ourselves to this system, the method can be applied to similar systems with symmetrical configurations and appropriate redshift combinations, such as Einstein ring system 0047-2808 and a quadruple lens system MG0414+0534. We shall describe the vacuum energy as a perfect fluid with the equation of state \( p = \omega \rho \) with \(-1 \leq \omega \leq 0\). The case \( \omega = -1 \) corresponds to the so called cosmological constant. Then the angular diameter distance in the totally flat universe with the vacuum energy is given by

\[
D(z_1, z_2) = \frac{c}{H_0} \frac{1}{1 + z_2} \int_{z_1}^{z_2} \frac{dx}{\sqrt{\Omega_m (1 + x)^3 + (1 - \Omega_m) (1 + x)^3 (1 + \omega)}}. \tag{0.1}
\]

The angular diameter distance comes in the lens equation through the lens potential which is modeled by an isothermal ellipsoid model.

\[
\Phi = 4\pi \left( \frac{\sigma_v}{c} \right)^2 \frac{D_{ls}}{D_s} \sqrt{(1 - \epsilon) \theta_1^2 + (1 + \epsilon) \theta_2^2}, \tag{0.2}
\]

where \( \sigma_v \) is the one-dimensional velocity dispersion. The ratio \( \epsilon \) between the minor-
and major-axis is related with the ellipticity \( \epsilon \) as

\[
e = \sqrt{1 - \epsilon^2}.
\]  

(0.3)

Model fitting gives \( e = 0.4 \) and \( \theta_E = 1".489 \), where

\[
\theta_E = 4\pi \left( \frac{\sigma_v}{c} \right)^2 \frac{D_{ls}}{D_s}.
\]  

(0.4)

Knowing the lens and source redshift \( z_l, z_s \) and \( \theta_E \), we plot the velocity dispersion \( \sigma_v \) in the \( \omega - \Omega_m \) plane on Figure 1. The region between the dotted vertical lines indicates the allowed range of the density parameter \( 0.2 \leq \Omega_m \leq 0.4 \) which is suggested from various observations concerning clusters.

![Fig. 1. Contours of constant velocity dispersion \( \sigma_v \) in the \( \omega - \Omega_m \) plane. Constant \( \sigma_v \) lines are drawn in steps of 5 km/sec for \( 265 \) km/sec \( \leq \sigma_v \leq 315 \) km/sec. Dotted vertical lines are \( \Omega_m = 0.2 \) and \( \Omega_m = 0.4 \).](image)

The figure shows that the velocity diversion is highly restricted in the case of \( \omega = -1 \) (the cosmological constant case), namely \( 291 \leq \sigma_v \leq 302 \) km/sec. Thus it shows that the system can be used as a good indicator for the existence of the cosmological constant when the observation of the velocity dispersion is performed within the accuracy of, say, 5 km/sec which is achievable by any of 8-10m telescopes with reasonable observational time. Put it other way, the accurate measurement of the velocity dispersion of the system, say ±5 km/sec gives us a determination of the density parameter \( \Omega_m \) with the accuracy of order of ±0.1.

The figure also shows the possibility of measurement of the parameter \( \omega \), namely the variability of the vacuum energy. If the measured value of the velocity dispersion is relatively large as \( 310 \pm 5 \) km/sec, then the \( \omega = -1 \) solution is inconsistent with the expected range of \( \Omega_m \). Thus it is extremely important to have an accurate measurement of the velocity dispersion and modeling of the lensing galaxy. Concerning
the accurate measurement, recent observation by Keck achieved an accurate measurement of the velocity dispersion for high redshift galaxies. On the other hand, the modeling of lensing elliptical galaxies is not easy task because of many theoretical ambiguities such as the choice of the dark halo potential and the existence of anisotropic velocity dispersions. However there have been steady progresses in this direction also. Although the isothermal distribution of dark matter halo is not chosen as the result of $\chi^2$ fitting by varying parameters in the dark matter potential in the original modeling of HST 14176+5226, it has been shown that isothermal distribution is consistent with both dynamical consideration and lensing data. Furthermore, dynamical observations and models of elliptical galaxies in the local Universe has been studied by Rix et al. and dynamical models of local early-type galaxies in SIS halos by Kochanek demonstrated that the observed stellar velocity dispersion ($\sigma_{ob}$) and the velocity dispersion associated with dark matter ($\sigma_v$) have nearly identical values using the observed distribution of image separation. If the dark matter halo has a finite core (within which the density is roughly constant), $\sigma_v$ is increased for a given observed $\sigma_{ob}$ because the central potential well is shallower. There have been some studies on this point and linear dependence on the core radius $r_c$ is obtained:

$$\sigma_v = \sigma_{ob} \left( A + B \frac{r_c}{r_E} \right)$$

(0.5)

where A and B are independent of core radius, and $r_E$ is the Einstein radius. Kochanek used $A = 1$ and $B = 2$. But this relation should be studied more carefully. Non-singular lenses tend to produce more highly magnified images and to produce characteristic images such as radial arcs so that detailed imaging by large telescopes is definitely to have more realistic model of the lensing system. It is hoped that further observational and theoretical progresses will resolve the above ambiguities in near future.

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