Distance-based formation control for multi-lane autonomous vehicle platoons

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Abstract
This paper investigates the formation control of connected autonomous vehicle (CAV) platoons moving in multi lanes using distance-based formation control techniques based on rigid graphs and V2V communication. A hierarchical architecture is proposed to decompose the cooperative control into velocity planning and vehicle dynamic control. A new velocity planning method is first developed via a distributed distance-based formation controller so that each vehicle can keep platoon and change lane. Then, for the vehicle dynamics with nonlinearities and bounded disturbances, an adaptive controller is designed for regulating driving/braking torque to achieve the longitudinal velocity output of the velocity planner. The steering controller is designed to adjust the yaw angle of each vehicle to track and change lanes. Furthermore, stability analysis is conducted based on the Lyapunov theory. Finally, the applications of the proposed control designs to various of automated highway system (AHS) scenarios including lane-change, curve lane and platoon overtaking, are simulated and numerically analysed to validate the effectiveness of theoretical results.

1 | INTRODUCTION

1.1 | Motivation
Cooperative control of connected and autonomous vehicle (CAV) systems is an interdisciplinary research topic which has attracted considerable attention in the field of control theory and automotive and transportation engineering disciplines, because of the high potential of improving traffic efficiency, enhancing road safety and reducing fuel consumption [1–4]. A CAV is defined as a system of vehicles, each of which is capable of automated driving and connecting to other vehicles in the system, road users, road infrastructure and the cloud [5]. The idea of driving automation and connectivity in vehicles is the foundation of intelligent transportation systems (ITS) [6]. For driving automation, most of the state-of-the-art autonomous vehicles utilise decision systems with the integration of perception, localization, planning and dynamic control [7, 8]. Global high-tech companies and vehicle engineers are enthusiastic to develop autonomous cars through various competitions and massive experiments [9]. For the connectivity in CAV system, a vehicle can communicate with infrastructure (V2I), other vehicles (V2V), cloud (V2C), and pedestrians (V2P) [10]. The development of autonomous driving and connected vehicles together would make the ITS more effective, thus relieve traffic congestion, abate traffic accidents and decrease environment pollution [11].

ITS aims to provide solutions to transportation challenges via autonomous driving and communication technologies for various roads and driving scenarios. Many of the current ITS research studies focus on highway driving scenarios due to their simpler road structure as well as the clearer lane definition compared to urban driving and unstructured roads [5]. Cooperative vehicle-highway automation system (CVHAS) [12] provides communication with other vehicles or from the infrastructure and driving assistance or driving autonomy based on on-board LiDAR and radar sensors. As a subsystem of CVHAS, AHS demands the full automation of vehicles and would be able to produce a virtually collision-free environment. The traffic congestion will be reduced especially by vehicle platooning, where such technique decreases the distances between cars or trucks on the highway system.
Multi-agent systems are common in nature and life, such as ant colonies, fish herds and the drone cluster. Multi-agent systems are defined that the system consisting of multiple agents can exchange information through the network. Therefore, multiple CAVs can be considered as multi-agent systems. In recent decades, the research on cooperative control of multi-agent systems is booming, such as the formation control of multi-agent systems, the game theory of multi-agent systems. It is not arduous that the cooperative control method of multi-agent systems can be applied to CAVs. Inspired by the hierarchy concept in [13], we propose a two-layer hierarchical architecture that decomposed this cooperative control problem of CAV platoons systems into two parts, velocity planning and vehicle dynamic control. For the planning layer, the collaboration of CAVs can be seen as a formation control problem of multi-agent systems. In [14], the methods of formation control have been categorized into position-, displacement- and distance-based according to types of sensed and controlled variables. The merit of distance-based formation control is that the method is driven by individual agent’s relative positions of their neighbouring agents with respect to their own local coordinate systems, which means the orientations of local coordinate systems are unnecessary to align with each other. Thus, compared with position-, displacement-based formation control, distance-based formation control has the natural advantages for AHS since LiDAR, radar and camera sensors are equipped on most of the autonomous vehicles for both performance and safety purposes. V2V technology could be potentially an additional sensor input that augments data available. The plan-out velocity based on methods of multi-agent systems is the input of the lower layer, namely the objectives of steering controller and speed controller. Based on the planning velocity, we further design the steering controller for lateral kinematics and adaptive speed controller with parameter estimation for longitudinal dynamics in the vehicle platoons.

Many issues about vehicle platooning have been studied recently including advanced control methods [15], power-train dynamics modelling [16], formation geometry and string stability [17]. However, the aforementioned researches only consider that the vehicle platoon operates at one straight single-lane. As mentioned in [5], modern challenges in AHS are platoon merging and splitting especially on multi-lane driving scenarios. The platoon merging and splitting can be regarded as switching formation control problems of multi-agent systems [14].

In this paper, we focus on the distance-based formation control, where vehicles are assumed to be capable of knowing the distance between their neighbouring vehicles to achieve the desired formation.

1.2 Literature review

The concept of AHS was first presented in 1939 New York World’s Fair, in the General Motors Pavilion [18]. After that, General Motors Corporation, the Bureau of Public Roads (the predecessor of the Federal Highway Administration) and California PATH have developed projects and researches about AHS for several decades [19–21]. For now, the literature about vehicle platoon and AHS are emerging with the development of autonomous vehicle technology and the internet of vehicles. Several advanced methods to resolve the vehicle platoon control problem have been studied in the literature concerned with sliding mode braking control [22], $\text{HOO}$ controller [23], event-triggered platoon control [24].

In previously mentioned literature, the vehicle platoon controllers are only concerned with longitudinal velocity in one straight lane. However, lane changing and car following follow are two essential units of microscopic traffic models in multi-lane. Thus, our algorithms consider not only longitudinal dynamics but also the yaw angle of each vehicle. There is much literature in relation to lane-change of vehicles [13, 25, 26], but our work first uses distance-based formation control of multi-agent systems based on rigid graphs to plan the velocity of CAVs when the vehicles need to keep platoon and change lane. Oh et al. proposed a series of theory about distance-based formation control [14, 27]. Dimarogonas et al. developed the stability of distance-based formation control [28] and exploited it for multi-robots systems [29]. Distance-based formation control combined with other foremost control method can produce new controllers such as adaptive control [30], and proportional-integral control [31]. In [32], they proposed a robust distance-based formation control method to solve the first-order system with unknown disturbance.

In addition to the planning problem, this paper also disentangles the control problems of longitudinal vehicle dynamics with nonlinear and unknown disturbances which are investigated in numerous literature. Vehicle dynamics have multivariate mature models such as a bicycle model and a 7-DOF model. This paper regards the longitudinal dynamics of vehicles as first-order dynamics with nonlinearity, unknown disturbance and unknown control gains [33]. To deal with this type of problems, many control algorithms have been proposed, for instance, adaptive neural output feedback control [34], adaptive actor–critic design-based integral sliding-mode control [35]. In general, the controllers in the above-mentioned literature are too complex for practical applications. Therefore, we propose a novel adaptive controller with parameter estimation to cope with this problem.

1.3 Contributions

To address the problem of cooperative control of CAV platoons in multi-lane, a novel hierarchical system framework is put forward in the foundation of the distributed distance-based formation controller and the technology of V2V. The contributions are as follows:

1. For the planning layer, a distributed distance-based formation controller is first devoted to planning the vehicular velocity in multi-lane. Furthermore, the scenario of vehicle platoon merging and splitting on multi-lane driving is settled by achieving a phase-varying formation configuration. Compared with the works of [32, 36, 37], we propose the
tightness of every edge in the multi-agent systems to solve the relationship of vehicle platoon and free car. Except for the rigid graph for the distance-based formation control[38], we add the traffic rule to constrain the formation geometry. 2. For the control layer, compared with the works of [2–4] where they focus on the research of platoon control, in this paper, adaptive controllers based on parameter estimation are designed for the longitudinal velocity of vehicles system with nonlinearity, unknown and bounded disturbance. Steering controllers are proposed to regulate the yaw angle of vehicles for the keeping platoon in the scenario of curve lane and changing lane. 3. The advantage of our designed system is that it is a decentralized and fully automated system. Compared with the works of [1, 19, 39] where the vehicles in multi-lane have to communicate with the roadside system, the server is non-essential to manage the vehicles on the highway environment. Each vehicle can utilize other vehicles’ positions and the distance with other vehicles by various sensors to plan and control themselves.

1.4 Organization

This paper is organized as the following. The problem statement is presented in Section 2. Section III designs the velocity planning algorithm for CAV platoons in multi-lane and the controllers for regulating the speed and the yaw of vehicles. The simulation results are shown in Section IV, and the conclusions are provided in Section V.

Notations: \( \mathbb{R} \) and \( \mathbb{R}^+ \) denote the sets of real numbers and positive real numbers, respectively. The set of \( m \times n \) real matrices is denoted by \( \mathbb{R}^{m \times n} \). \( \mathbb{R}^n \) represents the set of real \( n \times 1 \) vectors. The transpose of a vector or a matrix \( \mathcal{A} \) is denoted by \( \mathcal{A}^T \). The norm of \( p \in \mathbb{R}^n \) is defined as \( \|p\| := (p^T p)^{1/2} \).

2 PROBLEM STATEMENT

Consider the vehicle platoons consisting of \( N \) CAVs in a multi-lane highway scenario as shown in Figure 1. The red arrow signs indicate the direction of each vehicle intention the yellow dotted lines infer the communication between every two vehicles. As shown in Figure 2, the communication system is consisted of LiDAR/radar/cameras for detecting the distance with other vehicles and DSRC for broadcasting ego position and receiving other vehicles’ positions. The distance estimation error introduced by LiDAR/radar/cameras is negligible compared with the actual distance. Therefore, we do not consider the estimation errors introduced by these sensors. Each vehicle in the platoons has two options including car following (keeping platoon) and lane changing (splitting or merging). The above problem is a type of classical cooperative control problem of multi-agent systems. In response to the problem, we model multi-agent systems for planning and vehicle dynamics systems for control separately without considering the communication delay.  

The first-order dynamics of \( i \)th vehicle, \( i \in \mathcal{V} \), \( \mathcal{V} = \{1, 2, \ldots, N\} \) is formulated as follows:

\[
\dot{p}_i(t) = v_i(t), \quad i \in \mathcal{V},
\]

where \( p_i = [p_{ix}^t, p_{iy}^t]^T \in \mathbb{R}^2 \), \( v_i = [v_{ix}^t, v_{iy}^t]^T \in \mathbb{R}^2 \) are the position and velocity of \( i \)th vehicle, respectively. In the multi-lane driving scenario, every vehicle can change lane or keep lane with two dimensions of position and speed. Consider the vehicle kinematics [40],

\[
\begin{align*}
\dot{p}_{ix}(t) &= s_i(t) \cos \psi_i(t), \\
\dot{p}_{iy}(t) &= s_i(t) \sin \psi_i(t), \\
\dot{\psi}_i(t) &= \frac{s_i(t)}{I_i} \tan \delta_i(t),
\end{align*}
\]

where \( p_{ix}(t) \in \mathbb{R} \) and \( p_{iy}(t) \in \mathbb{R} \) are the positions of the \( i \)th vehicle in the global coordinate system, \( s_i(t) \in \mathbb{R} \) denotes the speed of vehicle, and \( \psi_i(t) \in (-\frac{\pi}{2}, \frac{\pi}{2}) \), \( I_i \in \mathbb{R}^+ \) are yaw angle and wheelbase of \( i \)th vehicle, respectively, as illustrated in Figure 3. The steering angle of the front wheels is denoted by \( \delta_i(t) \in \mathbb{R} \). The following detailed longitudinal dynamics is considered for vehicle \( i \) [33]:

\[
\frac{\eta_{Ti}}{r_{ws}} T_i(t) = m_i s_i(t) + C_{A_i} \dot{s}_i^2(t) + m_i g f, \quad i \in \mathcal{V},
\]

where \( 0 < C_{A_i} < 0.4 \), \( m_i \in \mathbb{R}^+ \), \( g \in \mathbb{R}^+ \) and \( 0 < f < 0.02 \) are the lumped aerodynamic drag coefficient, vehicle mass, the acceleration due to gravity and the coefficient of rolling resistance, respectively. \( 0< \eta_{Ti} < 100\% \) is the mechanical
efficiency of the drive-line and \( r_{w_i} \in \mathbb{R}^+ \) denotes the wheel radius. \( T_i(t) \in \mathbb{R} \) is the actual driving/braking torque. By simplifying (3), we have

\[
j_i(t) = b_i u_i(t) + \theta_j v_i^2(t) + \omega_i, \forall i \in \mathcal{V},
\]

where \( n_i(t) = T_i(t) \), \( b_i = \frac{m_i}{r_{w_i} a_i} \), \( \theta_j = \frac{C_{d_i}}{m_i} \), \( \omega_i = -gf \) are the control input to be generated, unknown control gain, unknown damping coefficient and the bounded disturbance, respectively.

Remark 1. The vehicles are heterogeneous which means the parameters of each vehicle can be different. From the physics of (3), we know that \( b_i \) is positive although the magnitude of \( b_i \) is known.

The desired velocity, longitudinal speed and yaw angle of the \( i \)th vehicle are set as

\[
v_i(des) = f_1(p_i(t), \sum_{j \in N_i} p_j(t), d_{p,des}),
\]

\[
\begin{bmatrix}
    s_{i,des} \\
    \psi_{i,des}
\end{bmatrix} = \begin{bmatrix}
    \|v_i\|_2 \\
    \arctan \left( \frac{v_{i,y} \psi_{i,x} - v_{i,x} \psi_{i,y}}{v_{i,x}^2 + v_{i,y}^2} \right)
\end{bmatrix},
\]

and the steering and torque control signals are generated by

\[
\begin{bmatrix}
    \delta_i(t) \\
    u_i(t)
\end{bmatrix} = f_2(s_{i,des}, \psi_{i,des}),
\]

\[
\begin{bmatrix}
    \delta_i(t) \\
    u_i(t)
\end{bmatrix} = f_3(s_{i,des}, \psi_{i,des}),
\]

where \( f_1, f_2 \) and \( f_3 \) represents controller functions to be designed in Section 3, and \( d_{p,des}, p_i(t) \) represent the inter-vehicle distances of the optimal vehicle formation geometry and the position of neighbours for Vehicle \( i \), \( N_i \) is the set of neighbours of \( i \)th vehicle. Based on the above equations, the flow chart of the proposed control design is shown in Figure 4. Equations (5) and (6) constitute the velocity trajectory generator for formation control. Equations (7) and (8) represent the steering controller and speed controller respectively. The control design task can be summarized as finding appropriate \( f_1, f_2 \) and \( f_3 \) for guaranteeing the stability of the system, and is addressed in detail in the next section.

3 \CONTROL DESIGN

In this section, the design of the two-layer hierarchical control system illustrated in Figure 4, consisting of distance-based formation control, speed control and steering control, is presented.

3.1 \Vehicle planning algorithm

The motion planning part of vehicles consists of communication topology, formation geometry design, distance-based formation control method and the constraints of traffic rules.

A graph representation is used to describe the communication topology of the multi-vehicle platoon. A nonempty vertex set \( \mathcal{V} \) and an edge set \( \mathcal{E} \subseteq \mathcal{V} \times \mathcal{V} \) constitute the representative graph \( \mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A}) \), where \( \mathcal{A} = \{a_{ij}\} \in \mathbb{R}^{N \times N} \) is the adjacency matrix. Each vehicle is represented by a vertex in \( \mathcal{V} \). \( a_{ij} = d_{ij} > 0 \) means that vehicle \( i \) and \( j \) can communicate and \( \mathcal{G} \) has an edge between the vertices \( i \) and \( j \). \( a_{ij} = 0 \) indicates that there is no edge between vertices \( i \) and \( j \), i.e. there is no communication between vehicles \( i \) and \( j \). If every pair of different vertices in \( \mathcal{V} \) has an undirected path, i.e. a sequence of neighbour edges, in between, then the undirected graph is connected. If any edge is directed, i.e. \( a_{ij} > 0 \) but \( a_{ji} = 0 \), the graph is directed. The V2V communication system is constructed by LiDAR/radar sensors and camera measuring the distance of every two vehicles and
wireless communication for broadcasting position information mutually. The wireless communication for detecting the position information can construct the undirected graph while the communication topology of distance information could be a digraph if LiDAR/radar is loaded on the head of the vehicles as shown in Figure 6(a). Meanwhile, frequent switching network topology poses a cumbersome challenge to the design of the distributed controller because of high vehicle mobility[41]. Therefore, considering the position of sensors and the actual running situation of the CAVs, we have the following assumption:

**Assumption 1.** In a sequence of switching times, we assume that the switching topologies of wireless communication has an undirected graph representation and this topology representation graph always has a directed spanning tree.

Rigid graph theory is an efficient tool for distance-based formation control, which is used to effectively formulate formation geometry[14, 27, 38]. A rigid graph can be utilised to ensure that all the vehicles in a multi-vehicle formation keep constant inter-vehicle distances during a continuous displacement. From Assumption 1, the formation geometry of CAVs can be modelled by a rigid graph. Next, we provide some two definitions about undirected graph rigidity and representations [38].

**Definition 1.** [38] A representation of a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a function $p : \mathcal{V} \rightarrow \mathbb{R}^2$, $p_i \in \mathbb{R}^2$ denotes the position of the node $i$ in representation $p$. The distance between two representations $p_1$ and $p_2$ of the same graph is defined by

$$d(p_1, p_2) = \max_{i \in \mathcal{V}} \|p_i(1) - p_i(2)\|.$$  (9)

For any representation $p$ of $\mathcal{G}$, the graph-representation pair $(\mathcal{G}, p)$ is called a framework. Moreover, two representations $p_1$ and $p_2$ are congruent if the distance between the position of every pair of nodes (connected by an edge or not) is the same in both of them: $\|p_i(1) - p_j(1)\| = \|p_i(2) - p_j(2)\|$ for all $i, j \in \mathcal{V}$. Such representations can be obtained one from the other by a rotation, a translation and/or a reflection.

**Definition 2.** [38, 42] Consider a framework $(\mathcal{G}, p)$ for a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and a representation $p$ of it. The set of representations $\mathcal{G}$ around $p$ such that $d(g, p) \leq \varepsilon$ is called $\varepsilon$-neighbourhood of $p$, and is denoted by $U_\varepsilon(p, \mathcal{G})$. The framework $(\mathcal{G}, p)$ is called rigid if there exists $\varepsilon > 0$ such that any representation $p' \in U_\varepsilon(p, \mathcal{G})$ is congruent to $p$. The graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is generically rigid if almost all its representations are rigid.

In a rigid graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with $m$ edges, every edge $e_k \in \mathcal{E}$, $k = 1, 2, ..., m$ can be assigned a weight which is equal to the desired distance of the two nodes that are joined by the edge $e_k$ in a certain representation $p$ of $\mathcal{G}$[43]. The weight of each edge $e_k \in \mathcal{E}$ is constrained to be constant in a rigid graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. But in our situation, the formation geometry of vehicles in the highway environment will keep different rigid graphs in different scenarios or activities.

The aim of the formation controller design is to ensure that the vehicles achieve and maintain rigid formation, satisfying the desired inter-agent distance $d_{k, \text{des}}$ mentioned in Section 2. Considering the CAV system of interest composed of $N$ vehicles introduced in Section 2 with motion kinematics (1), we present the distance-based formation controller in the sequel.

**Assumption 2.** The CAV system of interest is composed of $N$ vehicles with motion kinematics (1), with communication/formation graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, with the vertex set $\mathcal{V} = \{1, ..., N\}$ representing the vehicles and the edge set $\mathcal{E}$ representing the communication links and inter-vehicle distance constraints. It is assumed that $\mathcal{G}$ is a rigid graph and hence there exists a class $C(\mathcal{G}, d_{k, \text{des}})$ of rigid representations with graph structure $\mathcal{G}$ and edge lengths corresponding to $d_{k, \text{des}}$. It is further assumed that at the initial time $t = 0$, $p(0) = \{p_1(0), ..., p_N(0)\}$ is a rigid representation of $\mathcal{G}$ and there exists a representation $p^* \in C(\mathcal{G}, d_{k, \text{des}})$ such that $p(0) \in U_{p^*}(\mathcal{G}, \varepsilon_0)$ for a certain $\varepsilon_0 > 0$.

For every edge $e_k$, $k = 1, 2, ..., m$ connecting the two nodes, $i, j \in \mathcal{V}$, the weight of $e_k$ is the desired value $d_k > 0$ of the distance $\|e_k(t)\| = \|p_i(t) - p_j(t)\|$ between the nodes $i$ and $j$. The distance error is defined as

$$\varepsilon_k(t) = \|p_i(t) - p_j(t)\| - d_k.$$  (10)

Consider the semi-positive definite potential function[44]

$$\Phi(\varepsilon) = \frac{1}{2} \sum_{k=1}^{m} \|e_k(t)\|^2 - d_k^2 = \frac{1}{2} \sum_{k=1}^{m} \rho_k \varepsilon_k^2(t),$$  (11)

where $\rho_k > 0$ represents the tightness weight of $e_k$, for $k = 1, ..., m$ and $\varepsilon \in [\varepsilon_1, \varepsilon_2, ..., \varepsilon_m]^T$.

Note that $\Phi(\varepsilon) = 0$ if and only if $\|e_k\| = d_k$, $\forall k = 1, ..., m$.

The control input, $v_{i, \text{des}}$ in (5) for each vehicle $i$ is proposed to be set as the negative gradient of the potential function,

$$v_{i, \text{des}} = f_i(p_i(t), \sum_{j \in N_i} p_j(t), d_{k, \text{des}})$$

$$= -\frac{\partial \Phi(\varepsilon)}{\partial p_i(t)}$$

$$= -\sum_{j \in N_i} \rho_k \|e_k(t)\|^2 - d_k^2)(p_i(t) - p_j(t)).$$  (12)

**Theorem 1.** Consider the multi-agent CAV platoon system composed of $N$ vehicles with motion kinematics (1) under Assumptions 1 and 2. The control structure (1), (5), (12), with $v_i = v_{i, \text{des}}$, asymptotically achieves the target formation described by the rigid graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and target distance set $d_{k, \text{des}} = \{d_1, ..., d_m\}$, i.e. $p(t)$ converges to a certain representation in $C(\mathcal{G}, d_{k, \text{des}})$. 
Proof. Applying the chain rule to (11), we obtain

$$\Phi(t) = \frac{d\Phi(t)}{dt}$$ (13)

with

$$\Phi(t) = \sum_{j=1}^{N} \frac{\partial \Phi(t)}{\partial p_j(t)} \frac{\partial p_j(t)}{\partial t}.$$ (14)

Substituting (5) and (12) into (14), for \(v_i = r_i, \text{des}, \) we obtain

$$\Phi(t) = -\sum_{j=1}^{N} v_j(t) \hat{p}_j(t)$$ (15)

and

$$= -\sum_{j=1}^{N} \|v_j(t)\|^2 \leq 0.$$ (16)

Hence we guarantee that \(e\) is bounded and \(\Phi(t)\) is non-increasing. Following the arguments in the proofs of Lemma 3.1, Theorem 3.2 and Theorem 3.3 of [42], local asymptotic stability and convergence of \(p(t)\) to \(C(G)\) is established under Assumption 2. \(\square\)

Remark 2. In implementation of the proposed algorithm, the neighbouring vehicle positions \(p_j(t)\) are obtained by DSRC and the distance \(e_k(t)\) is observed utilising Lidar/radar sensors and cameras. It should be noted that \(d_k\) value are set in advance and can be reset to new values as the environment changes. According to Theorem 1, the vehicles can keep rigid formation geometry in highway environment.

Remark 3. \(d_{p,des}\) in Figure 4 represents the formation geometry which needs to be achieved. As different with the time-varying formation control studies in [45] [46], the change of formation geometry is in the form of phase variation, which means the behaviours of vehicles can be disassembled into several phases with different formations.

Whereas Theorem 1 ensures the relative velocity of vehicles for keeping the formation, it does not guarantee the absolute velocity of each vehicle. Namely, the vehicles maybe keep the formation statically. Thus, vehicles must obey traffic law for safety purposes. For demonstration purposes, we design the following traffic rules for our agents in the CAV flocking. The vehicles on the highway need to (1) keep in the middle of the lane; (2) keep moving forward; (3) stay in under the speed limit. For the first rule, the vehicle should detect the lane lines and obtain the centreline for calculating the lateral position deviation of \(\Delta p_{i,j}(t)\). For the second rule, we need to add the reference velocity \(v_{ref}\) in addition to the formation plan-out longitudinal velocity. For the third rule, the maximum and minimum speed should be defined as \(v_{max}\) and \(v_{min}\) in accordance with the traffic laws.

Considering that the vehicle’s actual control inputs are driving/braking torque and steering angle of the front wheels, the final output should be transformed to longitudinal speed and yaw angle according to (1) and (2) as follows.

$$s_{i,des}(t) = \|v_i(t)\|$$ (17)

$$\tan(\psi_{i,des}(t)) = \frac{v_{i,y}(t)}{v_{i,x}(t)}$$ (18)

where \(s_{i,des} \in \mathbb{R}\) and \(\psi_{i,des} \in \mathbb{R}\) are the objective speed and yaw angle of the vehicle respectively.

Concluding the above, we can get the velocity planning algorithm flowchart as the Figure 5 where \(E_i\) is the set of edges connecting with \(i\)th vehicle defined as \(E_i = \{e_k \in E_i | i \in V, j \in N_i\}\).

3.2 Speed controller design

Based on the reference speed and yaw angle, we are able to design an adaptive controller to achieve control objective for the system (4). The objective of this controller is for the speed of vehicle \(s_i\) to track the planning speed \(s_{i,des}\). The tracking error is defined as

$$\gamma_i(t) = s_i(t) - s_{i,des}(t).$$ (19)

Specifically, the sign of \(b_i\) is positive since \(\eta_{T,i}, r_{p,i} \) and \(m_i\) are positive. There is a technique that we can found \(b_i = \frac{1}{b_i}\) because \(b_i > 0\). Taking the time derivative of \(\gamma_i(t)\), we obtain

$$\dot{\gamma}_i(t) = b_i n_i(t) + \theta \dot{r}_i^2(t) + \omega_i - s_{i,des}(t)$$

$$\dot{\gamma}_i(t) = \frac{1}{b_i} n_i(t) + \theta \dot{r}_i^2(t) + \omega_i - s_{i,des}(t).$$ (20)
The following controller is developed:

\[
u_i(t) = \hat{b}_i(t)(-\lambda\gamma_i(t) - \hat{\theta}_i(t)\dot{\gamma}_i(t) - \hat{D}_i(t) + \dot{s}_i(t)) + \dot{s}_i, \tag{21}\]

where \(\lambda \in \mathbb{R}^+\) is designed as a control gain and \(\hat{b}_i(t), \hat{\theta}_i(t)\) and \(\hat{D}_i(t)\) denote the adaptive estimate for \(b_i, \theta_i\) and \(\omega_i\), respectively and are explicitly defined as follows.

\[
\hat{\theta}_i(t) = \hat{\theta}_i(t) + \theta_i,
\]

\[
\hat{D}_i(t) = D_i(t) + \omega_i,
\]

\[
\hat{b}_i(t) = \hat{b}_i(t) + \hat{b}_i,
\tag{22}\]

where \(\hat{\theta}_i, \hat{D}_i\) and \(\hat{b}_i\) are the parameter estimation errors for vehicle \(i\). Substituting (21) and (22) into (20), we obtain

\[
\dot{\gamma}_i(t) = \frac{\hat{b}_i(t)}{b_i}(\gamma_i(t) - \hat{\theta}_i(t)\dot{\gamma}_i(t) - \hat{D}_i(t) + \dot{s}_i) + \theta_i - \dot{s}_i,
\]

\[
= \frac{\hat{b}_i(t)}{b_i}(\gamma_i(t) - \hat{\theta}_i(t)\dot{\gamma}_i(t) - \hat{D}_i(t) + \dot{s}_i) + \theta_i - \dot{s}_i,
\]

\[
\dot{\theta}_i(t) = \frac{\gamma_i(t) - \hat{\theta}_i(t)\dot{\gamma}_i(t) - \hat{D}_i(t) + \dot{s}_i}{\hat{b}_i(t)} + \hat{b}_i(t),
\tag{23}\]

where

\[
\hat{\theta}_i(t) = \frac{\gamma_i(t) - \hat{\theta}_i(t)\dot{\gamma}_i(t) - \hat{D}_i(t) + \dot{s}_i}{\hat{b}_i(t)} + \hat{b}_i(t),
\tag{24}\]

The adaptive laws are set as follows:

\[
\begin{cases}
\dot{\gamma}_i(t) = \frac{\gamma_i(t) - \hat{\theta}_i(t)\dot{\gamma}_i(t) - \hat{D}_i(t) + \dot{s}_i}{\hat{b}_i(t)} + \hat{b}_i(t), \\
\dot{\theta}_i(t) = \frac{\gamma_i(t) - \hat{\theta}_i(t)\dot{\gamma}_i(t) - \hat{D}_i(t) + \dot{s}_i}{\hat{b}_i(t)} + \hat{b}_i(t), \\
\dot{D}_i(t) = \gamma_i(t),
\end{cases}
\tag{25}\]

\[
\begin{align*}
\dot{V}_i(t) &= \frac{1}{2}\dot{\gamma}_i^2(t) + \frac{1}{2}\frac{\gamma_i^2}{|\hat{b}_i|} + \frac{1}{2}\hat{\theta}_i^2(t) + \frac{1}{2}\hat{D}_i^2(t),
\end{align*}
\tag{26}\]

Take time derivative of \(V_1(t)\) as the following,

\[
\dot{V}_i(t) = \gamma_i(t)\dot{\gamma}_i(t) + \frac{\gamma_i(t)}{|\hat{b}_i|}\dot{\gamma}_i(t) + \hat{\theta}_i(t)\dot{\theta}_i(t) + \hat{D}_i(t)\dot{D}_i(t).
\tag{27}\]

Substituting (23) into (27), we obtain

\[
\dot{V}_i(t) = -\lambda\gamma_i^2(t) + \frac{\gamma_i(t)}{|\hat{b}_i|}\dot{\gamma}_i(t) + \beta_i(t)\dot{\gamma}_i(t)
\]

\[
+ \hat{\theta}_i(t)\dot{\theta}_i(t) - \gamma_i(t)\dot{\gamma}_i(t) + \hat{D}_i(t)\dot{D}_i(t) - \gamma_i(t)(\dot{\gamma}_i(t) + \dot{s}_i)
\]

\[
= -\lambda\gamma_i^2(t) \leq 0.
\tag{28}\]

Since \(V_i(t) = V_i(0) + \int_0^t \dot{V}_i(t)dt = V_i(0) - \int_0^t \lambda\gamma_i^2(t)dt \geq 0\), (28) implies that \(\lim_{t \to \infty} \gamma_i(t) = 0\), which completes the proof.

\[
\text{Remark 4. Compared with the works} [15, 16] \text{which do not consider the parameter uncertainties, we design the adaptive controllers to compensate the effects of nonlinearities and parametric uncertainties in the longitudinal dynamics of vehicles.}\n\]

\[
\text{Remark 5. Positive systems are dynamical systems whose state variables are non-negative in value at all-time [47, 48]. In the whole process, the speed control system is a positive system because the state is} s_i. \text{However, in a planning sample time, the speed control system is not a positive system because the state is} \gamma_i(t) = s_i - s_{i,des}.\n\]

\[
\text{3.3 Steering controller design}\n\]

The steering controller is designed for achieving the objective yaw angle in lane-change or curve lane scenarios as follows. From the lateral kinematics (2) and objective yaw angle (18), the control protocol is designed as follows:

\[
\delta_i(t) = k_p\zeta_i(t)
\tag{29}\]

with

\[
\zeta_i(t) = \tan \psi_i(t) - \tan \psi_{i,des}(t),
\tag{30}\]

where \(k_p \in \mathbb{R}\) is a constant control parameter.

\[
\text{Theorem 3. Consider the lateral kinematics (2) of Vehicle i, under the control law} (5), (6), (12), (29), (30). The yaw angle} \psi_i \text{of Vehicle i is ensured to be bounded and achieve asymptotic convergence to reference signal} \psi_{i,des} \text{if} k_p < 0.\n\]

\[
\text{Proof. Let} \alpha_i(t) = \psi_i(t) - \psi_{i,des}(t) \text{and design Lyapunov function as the following:}
\]

\[
V_\alpha(t) = \frac{1}{2}\alpha_i^2(t).
\tag{31}\]
There exists $k_1(t) \in \mathbb{R}^+$ such that $\tan \delta_i(t) = k_1(t) \delta_i(t)$, $\forall \delta_i(t) \in (-\frac{1}{2} \pi, \frac{1}{2} \pi)$. Take time derivative of $V_2(t)$ as the following

$$
\dot{V}_2(t) = \alpha_i(t) \dot{\psi}_i(t)
= \alpha_i(t) \frac{s_i(t)}{L_i} \tan \delta_i(t) - \alpha_i(t) \dot{\psi}_{i,\text{des}}(t)
= \alpha_i(t) \frac{s_i(t)}{L_i} k_1(t) k_p \dot{\psi}_{i,\text{des}}(t) - \alpha_i(t) \dot{\psi}_{i,\text{des}}(t)
= k_p k_1(t) \left( \psi_i(t) - \psi_{i,\text{des}}(t) (\tan \psi_i(t) - \tan \psi_{i,\text{des}}) - \alpha_i(t) \dot{\psi}_{i,\text{des}}(t) \right),
$$

(32)

The range of $\psi_i(t)$ is $(-\frac{1}{2} \pi, \frac{1}{2} \pi)$. Therefore, if $\psi_i(t) = \psi_{i,\text{des}}, \alpha_i(t) \dot{\psi}_{i,\text{des}}(t) = 0$; if $\psi_i(t) \neq \psi_{i,\text{des}}, \alpha_i(t) \dot{\psi}_{i,\text{des}}(t) > 0$. If $k_p < 0$, since $s_i > 0$, $L_i > 0$, $k_1(t) \in \mathbb{R}^+$ and $\lim_{t \to \infty} \dot{\psi}_i(t) = 0$ by Theorem 1, we have $\lim_{t \to \infty} \dot{V}_2(t) \leq 0$ and the proof is completed.

Remark 6. Different from the researches of [2, 3, 15, 16 23] which focus on longitudinal control, we design steering controllers for lateral control which means the vehicles not only follow the predecessor but also change the lane.

Remark 7. The merits of updating error by tangent function is to reduce calculation and improve computing speed.

4 AHS APPLICATION SIMULATIONS

This section presents simulations of some common scenarios in AHS for verifying the effectiveness of the proposed algorithm about autonomous vehicle platoons in multi-lane. For this purpose, based on the scenario shown in Figure 1, we test the function of lane-change according to the cooperative control of five CAVs in two straight lanes. Further, a curve lane scenario is considered to investigate the performance of the steering controller.

In addition, we also design a platoon overtaking scenario.

4.1 Lane-change scenario

In this work, we consider a merging and splitting as a change of formation in the vehicle platoons. In Figures 6 and 7, blue
The designed top-level formation controller and the low-level dynamic controllers should be able to complete the transition formation due to Theorem 1–3.

To verify the proposed algorithm in the lane-change scenario, a numerical simulation environment is designed with the starting platoon formation as Figure 6. For this simulation, every vehicle has its own information including positions, velocity and lane information. The initial positions and speed are set as \( p(0) = [(1, 6), (10, 6), (18, 6), (1, 2), (15, 2)]^T \) and \( s(0) = [20, 21, 22, 22, 20]^T \) on the basis of practical situation. As shown in the Figure 7(a), the vehicle 2 will change lane from the Figure 6(a) to Figure 6(b). For each vehicle, the unknown control parameters \( b_i \) and \( \Theta_i \) are set as \([0.001, 0.0025, 0.002, 0.0014]^T\) and \([-0.1, -0.11, -0.09, -0.13]^T\) heterogeneously. The disturbance \( d_i \) are set as \([0.1, 0.12, 0.09, 0.1, 0.098]^T\).

The results of the lane-changing behaviour with our control law are demonstrated in Figures 7 and 8. As shown in Figure 7, the vehicle 2 starts to change lane at around the longitudinal position of 140 m and the lane-change is completed at about 180 m. From Figure 8, the speed of vehicle 2 is about 27 m/s when it starts to change lane. The process of lane-change lasts about two seconds and the lane-change trajectory of vehicle 2 is shown in Figure 7(b), which is similar to a real-world driving scenario. During the process of lane-change, the vehicle 1 and the vehicle 5 accelerate while the vehicle 3 and 4 decelerate to ensure the formation and the front and rear cars are kept at a safe distance. At the same time, we can see from Figure 8 that the maximum yaw angle of vehicle 2 is about fifteen degrees, while the other cars are moving in the straight which the heading angle is maintained at zero degrees. The simulation results demonstrate our designed velocity planning algorithm is effective from the Figures 7 and 8. For the formation, we intercept four time points in Figure 7(a). Before lane-change, the five vehicles can keep formation as same as the formation in Figure 6(a). After the vehicles pass the longitudinal position of 150 m in Figure 7(a), the formation is the same as Figure 6(b). The velocities of five vehicles achieve consensus eventually from Figure 8.

4.2 Curve-lane scenario

To verify the proposed algorithm in the curve-lane scenario, we also design a numerical simulation environment. From Figure 10, we can see blue curve lines denote two lanes which are eight meters wide. And the vehicles need to achieve the formation as shown in Figure 6(a). For this simulation, every vehicle has its own information including positions, velocity and lane information. The initial positions, speed and dynamics parameters of each vehicle are set the same as the case of lane-change scenario.

From Figure 10, the vehicles keep the formation as shown in Figure 6(a) although the lanes are curved. From Figure 11, the longitudinal velocities of five vehicles achieve consensus eventually while the yaw of each vehicle has phase difference because of the curve lanes. As shown in Figure 12, the error of velocity and yaw are almost zero which means the speed controller and steering controller are still effective in curve lane for the vehicle systems with nonlinear parameter and unknown disturbance.

4.3 Platoon overtaking scenario

In this part, we design a platoon overtaking scenario which a platoon consisted of two vehicles overtakes a vehicle in a two-lane highway. The initial positions and speed of black vehicles...
The speed and yaw angle of five vehicles in curve lanes.

The error of speed and yaw angle about five vehicles in curve lanes.

The initial position and speed of the red vehicle are (6,80) and 20, respectively. As shown in the Figure 13, we choose eight formation at $t = 1, 3, 5, 7, 9, 11, 13, 15$ s to present the platoon overtaking process. To finish this process, we design six formations for different phases. In the first phase, the two vehicles need to keep platoon but the leader in the platoon can detect the vehicle 3. For this phase, the tightness between the vehicle 1 and 2 is 100 but the tightness between vehicle 2 and 3 is 1. The formation matrix is designed as
\[
\begin{bmatrix}
0 & 20 & 70 \\
20 & 0 & 50 \\
70 & 50 & 0
\end{bmatrix}
\]

and the communication matrix is designed as
\[
\begin{bmatrix}
0 & 1 & 0 \\
1 & 0 & 1 \\
0 & 1 & 0
\end{bmatrix}
\]

The actual yaw of 3 vehicles in Figure 13 presents the vehicles in platoon change lane at $t = 1$ s and $t = 13$ s. In the first lane-change, the communication matrix changes to
\[
\begin{bmatrix}
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0
\end{bmatrix}
\]

and the formation matrix changes to
\[
\begin{bmatrix}
0 & 20 & 40 \\
20 & 0 & 40 \\
40 & 20 & 0
\end{bmatrix}
\]

Then, the formation matrix changes to
\[
\begin{bmatrix}
0 & 20 & 15 \\
20 & 0 & 4 \\
15 & 4 & 0
\end{bmatrix}
\]
at $t = 3$ s,
\[
\begin{bmatrix}
0 & 30 & 15 \\
30 & 0 & 15 \\
15 & 15 & 0
\end{bmatrix}
\]
at $t = 5$ s,
\[
\begin{bmatrix}
0 & 30 & 2 \\
30 & 0 & 30 \\
2 & 30 & 0
\end{bmatrix}
\]
at $t = 8$ s,
at \( t = 11 \) s in turn. After the second lane-change, the communication matrix changes to

\[
\begin{bmatrix}
0 & 1 & 1 \\
1 & 0 & 0 \\
1 & 0 & 0
\end{bmatrix}
\]

and the formation matrix changes to

\[
\begin{bmatrix}
0 & 20 & 50 \\
20 & 0 & 70 \\
50 & 70 & 0
\end{bmatrix}
\]

From the Figure 13, we can see that the vehicle 1 attempt to overtake the vehicle 3 after the vehicle 2 passes the vehicle 1. Before the second lane-change, the two vehicles in the platoon have been in the front of the purple car. Then, they come back to the previous lane. The actual speed of three vehicles in the Figure 14 presents the variety of the speed. With the change of the formation matrix, the speed of three vehicles fluctuates obviously for several times. This simulation results verify that the proposed systems have the ability to realise the function of platoon overtaking.

5.1 CONCLUSION

This paper presented a novel system framework to solve the problem of intelligent highway traffic systems using a two-layer structure including the planning layer and control layer. A velocity planning method based on a formation controller using rigidity graph and traffic rules was proposed for connected and autonomous vehicles in multi-lane. The proposed adaptive speed controller and steering controller could guarantee the velocity and yaw of each vehicle to achieve the planning velocity. The proposed velocity planning method combined formation control and traffic rules to achieve cooperative control of CAVs in the highway so that improved traffic efficiency and enhanced safety. Furthermore, a novel adaptive controller was proposed based on parameter estimation for the longitudinal dynamics with nonlinear and unknown disturbance. Meanwhile, this paper also resolved the problem of CAV platoons in the curve lanes because of the steering controller. The results of simulations verify the effectiveness of the velocity planning algorithm and the controllers.

The proposed system about CAVs in this paper is a huge project. There are still some unsolved problems. In the future, we will expand research on the design of optimal formation geometry, improving control performance and refining vehicle dynamics. In this paper, the speed of vehicles on the highway can achieve consensus without considering the difference between the fast lane and slow lane. Therefore, we will discuss the interaction of asymmetric vehicle platoons which means the different platoon has a different speed.

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