The derivation of the coupling constant in the new Self Creation Cosmology

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Abstract

It has been shown that the new Self Creation Cosmology theory predicts a universe with a total density parameter of one third yet spatially flat, which would appear to accelerate in its expansion. Although requiring a moderate amount of ‘cold dark matter’ the theory does not have to invoke the hypotheses of inflation, ‘dark energy’, ‘quintessence’ or a cosmological constant (dynamical or otherwise) to explain observed cosmological features. The theory also offers an explanation for the observed anomalous Pioneer spacecraft acceleration, an observed spin-up of the Earth and a problematic variation of G observed from analysis of the evolution of planetary longitudes. It predicts identical results as General Relativity in standard experimental tests but three definitive experiments do exist to falsify the theory. In order to match the predictions of General Relativity, and observations in the standard tests, the new theory requires the Brans Dicke omega parameter that couples the scalar field to matter to be $-3/2$. Here it is shown how this value for the coupling parameter is determined by the theory’s basic assumptions and therefore it is an inherent property of the principles upon which the theory is based.

1 Introduction

1.1 Notations and conventions

This paper adopts the "Landau-Lifshitz Spacelike Convention". A comma denotes ordinary differentiation, a semi-colon denotes covariant differentiation, and $\Box \phi$ is the d’Alembertian invariant, $\phi^{\sigma}_{\sigma}$. $\nabla$ is the normal gradient. The number of dimensions of the manifold $M$ is four throughout, the summation convention is followed, Greek indices indicate four-space-time and Latin indices indicate three-space. The metric signature is $(-,+,+,+)$, the Ricci
tensor is given in terms of the Christoffel symbols $\Gamma^\rho_{\mu\nu}$ by

$$+R_{\mu\nu} = +\Gamma^\rho_{\mu\nu,\rho} - \Gamma^\rho_{\rho\nu,\mu} + \Gamma^\sigma_{\mu\nu} \Gamma^\rho_{\sigma\rho} - \Gamma^\sigma_{\rho\nu} \Gamma^\rho_{\sigma\mu}$$

$G_N$ is the Newtonian gravitational constant as measured in "Cavendish" type experiments and the speed of light is unity unless otherwise specified as $c$. A tilde signifies the Einstein frame.

\section*{1.2 An historical introduction to Self Creation Cosmology}

In a recent paper this author, [Barber,(2002a)], developed a new Self Creation Cosmology (SCC), which superceeded and subsumed two earlier versions [Barber, (1982)]. All versions of SCC were modifications of the Brans Dicke theory (BD), [Brans & Dicke, (1961)] in which the equivalence principle was relaxed to allow for the continuous creation of mass out of self contained matter, gravitational and scalar fields. The new theory is a 'semi-metric' theory in that photons follow trajectories that are geodesics of the theory but particles do not, \textit{in vacuo} they follow orbits that are identical to General Relativity geodesics. The new theory was shown to predict a universe with a total density parameter of one third yet spatially flat, although it did require a moderate amount of 'cold dark matter'. It did not have to invoke the hypotheses of inflation, 'dark energy', 'quintessence' or a cosmological constant (dynamical or otherwise) to explain observed cosmological features. The new theory also predicted identical results as General Relativity (GR) in standard experimental tests. In a subsequent paper, [Barber, (2002b)], it was shown how the theory also offered an explanation for 'cosmic acceleration', the anomalous Pioneer spacecraft acceleration, an observed spin-up of the Earth and an apparent variation of $G$ observed from analysis of the evolution of planetary longitudes. Finally, it was shown, [Barber, (2003)], that measurement of geodetic precession, about to be performed on the Gravity Probe B satellite is a definitive experiment, which could falsify the theory as SCC predicts a value $\frac{2}{3}$ of that expected by GR.

For SCC to agree with observation, and hence GR, in the standard tests a coupling parameter $\omega$ was empirically set with the value

$$\omega = -\frac{3}{2}.$$
This paper will show why the value of $\omega$ has not been arbitrarily chosen but it is a definitive value required for consistency by the basic principles of the theory.

1.3 A summary of the theory

Mach’s Principle (MP) is incorporated in SCC by assuming the inertial masses of fundamental particles are dependent upon their interaction with a scalar field $\phi$ coupled to the large scale distribution of matter in motion in a similar fashion as BD. This coupling is described by a field equation of the simplest general covariant form

$$\Box \phi = 4\pi \lambda T_{\sigma \sigma}^M ,$$  \hspace{1cm} (2)

$T_{\sigma \sigma}^M$ is the trace of the energy momentum tensor describing all non-gravitational and non-scalar field energy. The BD coupling parameter $\lambda$ was found to be unity. [Barber, (2002a)] and in the spherically symmetric One Body problem

$$\lim_{r \to \infty} \phi (r) = \frac{\psi}{G_N} ,$$  \hspace{1cm} (3)

where $G_N$ is the normal gravitational constant measured in Cavendish type experiments and $\psi$ is a constant. $\psi$ was found to be unity in the subsequent development of the theory as described below.

In BD the Equivalence Principle was retained so that particle rest masses were invariant and $G$ would be observed to vary with position. By contrast, in SCC in which the Equivalence Principle is violated, it is $G_N$ that is measured to be invariant and it is particle rest masses that vary.

In all the SCC theories the Equivalence Principle is relaxed, specifically in the new theory it is replaced by the Principle of Mutual Interaction (PMI) in which

$$T_{\mu \nu ; \mu}^M = f_\nu (\phi) \Box \phi = 4\pi f_\nu (\phi) T_{\sigma \sigma}^M .$$   \hspace{1cm} (4)

Therefore in vacuo,

$$T_{em ; \mu}^\nu = 4\pi f_\nu (\phi) T_{em}^\sigma = 4\pi f_\nu (\phi) (3p_{em} - \rho_{em}) = 0$$   \hspace{1cm} (5)

where $p_{em}$ and $\rho_{em}$ are the pressure and density of an electromagnetic radiation field with an energy momentum tensor $T_{em; \mu \nu}$ and where $p_{em} = \frac{1}{3} \rho_{em}$. Thus the scalar field is a source for the matter-energy field if and only if
the matter-energy field is a source for the scalar field. Although the equivalence principle is violated for particles, it is not for photons, which still travel through empty space on (null) geodesic paths.

Particles do not have invariant rest mass and its variation is determined by a second principle, the Local Conservation of Energy. This requires the energy used in lifting a particle against a gravitational field to be absorbed into its rest mass; so that rest masses include gravitational potential energy. A particle’s rest mass is described by

\[ m_p(x^\mu) = m_0 \exp[\Phi_N(x^\mu)] , \]

where \( \Phi_N(x^\mu) \) is the dimensionless Newtonian potential and \( m_p(r) \to m_0 \) as \( r \to \infty \).

In SCC gravitational orbits and cosmological evolution are described in the Jordan frame. A conformal equivalence exits between the Jordan frame and canonical GR, which is the theory’s Einstein frame. This results in the geodesic orbits of SCC being identical with GR in vacuo. The Jordan (energy) frame \([JF(E)]\) conserves mass-energy, and the Einstein frame (EF) conserves energy momentum. The two conformal frames are related by a coordinate transformation

\[ g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} . \]

The JF of SCC requires mass creation, \( (T^\mu_{\nu;\mu} \neq 0) \), therefore the scalar field is non-minimally connected to matter. The JF Lagrangian density is,

\[ L^{SCC}[g, \phi] = \frac{\sqrt{-g}}{16\pi} \left( \phi R - \frac{\omega}{\phi} \phi,^\sigma \phi,^\sigma \right) + L^{SCC}_{\text{matter}}[g, \phi] , \]

and its conformal dual, \([Dicke (1962)]\), by a general transformation \( \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \), is

\[ L^{SCC}[^\Omega g, \tilde{\phi}] = \frac{\sqrt{-g}}{16\pi} \left[ \tilde{\phi} \tilde{R} + 6 \tilde{\phi} \Box \ln \Omega \right] + \tilde{L}^{SCC}_{\text{matter}}[^\Omega g, \tilde{\phi}] \]

\[ = \frac{\sqrt{-g}}{16\pi} \left[ (2\omega + 3) \frac{\Omega_{\sigma,\sigma} \Omega^\sigma}{\Omega^2} \tilde{\phi}^2 \right] \]

\[ + 4\omega \frac{\Omega_{\sigma,\sigma} \tilde{\phi}^2}{\Omega} + \omega \frac{\tilde{\phi}^2 \tilde{\phi}}{\tilde{\phi}} . \]
A mass is conformally transformed according to
\[ m(x^\mu) = \Omega \tilde{m}_0 , \] (10)

Equation (10) requires
\[ \Omega = \exp[\Phi_N(x^\mu)] , \] (11)

where \( m(x^\mu) \) is the mass of a fundamental particle in the JF and \( \tilde{m}_0 \) its mass in the EF.

If we define the EF by \( G = G_N \) a constant, i.e. \( \tilde{\phi}_{,\sigma} = 0 \), and as \textit{in vacuo}
\[ \Box \ln \Omega = \Box \Phi_N(x^\mu) = \nabla^2 \Phi_N(x^\mu) = 0 , \] (12)

setting \( \omega = -\frac{3}{2} \) reduces the EF conformal dual, the EF Lagrangian density to
\[ L^{SCC}[\tilde{g}, \tilde{\phi}] = \frac{\sqrt{-\tilde{g}}}{16\pi} \left[ \tilde{R} \right] + L^{SCC}_{\text{matter}}[\tilde{g}, \tilde{\phi}] , \] (13)

which is canonical GR.

The question is, "Why should \( \omega = -\frac{3}{2} \)?" It is the purpose of the present paper to answer this question.

2 Incorporating the Principle of Mutual Interaction in BD

2.1 Constructing the SCC Field Equations

The first basis of the new Self Creation Cosmology is to fully incorporate Mach’s Principle in GR by following BD but then introducing the PMI, Equation (4), to quantify the violation of the Equivalence Principle and to allow for mass creation.

A scalar field \( \phi \) is defined by Equations (2) and (3). An energy momentum tensor for the scalar field \( T_{\phi \mu \nu} \) is added to the gravitational field equation in order to account for its presence and effect on the curvature of space-time
\[ R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = \frac{8\pi}{\phi} \left( T_{M \mu \nu} + T_{\phi \mu \nu} \right) . \] (14)

In order to determine \( T_{\phi \mu \nu} \) and \( f_{\nu}(\phi) \) Equation (14) is written in the following mixed tensor form
\[ T^\mu_{M \nu} = \frac{\phi}{8\pi} \left( R^\mu_{\nu} - \frac{1}{2} \delta^\mu_{\nu} R \right) - T^\mu_{\phi \nu} \] (15)
and the method described by Weinberg, [Weinberg, (1972), pages 158-160, equations 7.3.4-7.3.12] is followed. The most general form of $T_{\phi
u}^\mu$ using two derivatives of one or two $\phi$ fields and $\phi$ itself is

$$T_{\phi
u}^\mu = A(\phi)\phi^\mu;\phi_{\nu\sigma} + B(\phi)\phi^\mu;\phi_{\nu} + C(\phi)\phi^\mu;\phi_{;\nu} + D(\phi)\phi^\mu;\phi_{;\nu}$$

(16)

and covariantly differentiating produces

$$T_{\phi
u}^\mu = \left[ A'(\phi) + B'(\phi) \right] \phi^\mu;\phi_{\nu} + \left[ A(\phi) + D'(\phi) \right] \phi^\mu;\phi_{;\nu}$$

$$+ [A(\phi) + 2B(\phi) + C'(\phi)] \phi^\mu;\phi_{;\nu} + D(\phi) \Delta(\phi)$$

(17)

where a prime (') means differentiation w.r.t $\phi$. Now make use of the Bianchi identities and the identity (observing the sign convention)

$$\phi^\nu R_{\nu}^\sigma = \Delta(\phi^\nu) - (\Delta \phi)_{;\nu}$$

(18)

in order to examine the violation of the equivalence principle. Covariantly differentiating Equation 15 yields

$$T_{\phi
u}^\mu = \phi^\mu;\phi_{\nu} + 2\pi \left[ R_{\nu}^\mu - \frac{1}{2} \phi_{;\nu} \right] - T_{\phi
u}^\mu \ .$$

(19)

Taking the trace of Equation 14 gives

$$R = -\frac{8\pi}{\phi} \left[ T_{\phi\sigma}^\sigma + T_{M\sigma}^\sigma \right]$$

(20)

with

$$T_{\phi\sigma}^\sigma = [A(\phi) + 4B(\phi)] \phi^\sigma;\phi_{;\sigma} + [C(\phi) + 4D(\phi)] \Delta(\phi)$$

(21)

and from Equation 21

$$T_{M\sigma}^\sigma = \frac{1}{4\pi\lambda} \Delta(\phi)$$

(22)

Substituting Equations 21 and 22 in Equation 20 yields

$$R = -\frac{8\pi}{\phi} \left\{ [A(\phi) + 4B(\phi)] \phi^\sigma;\phi_{;\sigma} + \left[ C(\phi) + 4D(\phi) + \frac{1}{4\pi\lambda} \right] \Delta(\phi) \right\}$$

(23)
While Equations 17, 18 and 23 substituted in 19 produce
\[ T_{\mu \nu, \mu} = -\frac{1}{8\pi} (\Box \phi)_{;\nu} + \frac{1}{8\pi} \Box (\phi_{;\nu}) + \frac{1}{2\phi} [A(\phi) + 4B(\phi)] \phi_{;\mu} \phi_{;\nu} \]
\[ + \frac{1}{2\phi} \left[ C(\phi) + 4D(\phi) + \frac{1}{4\pi \lambda} \right] \phi_{;\nu} \Box \phi \]
\[ - [A'(\phi) + B'(\phi)] \phi_{;\mu} \phi_{;\nu} - [A(\phi) + D'(\phi)] \phi_{;\nu} \Box \phi \]
\[ - [A(\phi) + 2B(\phi) + C'(\phi)] \phi_{;\nu} \phi_{;\mu} - D(\phi) \Box (\phi_{;\nu}) - C(\phi) \Box (\phi_{;\nu}) . \]

If the Principle of Mutual Interaction, Equation 4, is applied
\[ T_{\mu \nu, \mu} = f_{\nu} (\phi) \Box \phi \]

So the coefficients of: \( (\Box \phi)_{;\nu}, \Box (\phi_{;\nu}), \phi_{;\mu} \phi_{;\nu}, \) and \( \phi_{;\mu} \phi_{;\mu}, \) must vanish in Equation 24 but those of \( \phi_{;\nu} \Box \phi \) must satisfy Equation 4. This yields five equations to solve for the five functions; \( A(\phi), B(\phi), C(\phi), D(\phi) \) and \( f_{\nu}(\phi). \)

| Term          | Coefficients ( = 0) | Solution |
|---------------|---------------------|----------|
| \( (\Box \phi)_{;\nu} \) | \( -\frac{1}{8\pi} - D(\phi) = 0 \) | \( D(\phi) = -\frac{1}{8\pi} \) (i) |
| \( \Box (\phi_{;\nu}) \) | \( +\frac{1}{8\pi} - C(\phi) = 0 \) | \( C(\phi) = +\frac{1}{8\pi} \) (ii) |
| \( \phi_{;\mu} \phi_{;\nu} \) | \( A(\phi) + 2B(\phi) + C'(\phi) = 0 \) | \( A(\phi) = -2B(\phi) \) (iii) |
| \( \phi_{;\mu} \phi_{;\mu} \phi_{;\nu} \) | \( \frac{1}{2\phi} [A(\phi) + 4B(\phi)] \]
| | \( - [A'(\phi) + 4B'(\phi)] = 0 \) | |

Substituting Equation (iii) into (iv)
\[ \frac{B'(\phi)}{B(\phi)} = -\frac{1}{\phi} \]

which has the solution
\[ B(\phi) = \frac{k}{\phi}, \]

where \( k \) is a constant, and therefore by Equation (iii)
\[ A(\phi) = -\frac{2k}{\phi}. \]
If \( \kappa \) is now written as
\[ k = -\frac{\omega}{16\pi} \]
the non-unique solution is obtained
\[ A(\phi) = \frac{\omega}{8\pi\phi}, \quad B(\phi) = -\frac{\omega}{16\pi\phi}, \]
\[ C(\phi) = \frac{1}{8\pi}, \quad D(\phi) = -\frac{1}{8\pi}. \quad (28) \]

This solution looks the same as the BD solution except that \( \omega \) is as yet undetermined. A solution for \( T_{\mu \nu;\mu} \) is obtained by substituting Equation 28 into Equation 24 and examining the coefficients of \( \phi_{\nu;\mu} \square \phi \). This fifth equation which will determine \( \omega \) is
\[ T_{\mu \nu;\mu} = \left( \frac{1}{16\pi\phi} - \frac{1}{4\pi\phi} + \frac{1}{8\pi\lambda\phi} - \frac{\omega}{8\pi\phi} \right) \phi_{\nu;\mu} \square \phi, \quad (29) \]
which can be written as
\[ T_{\mu \nu;\mu} = \frac{\kappa}{8\pi} \frac{\phi_{\nu;\mu}}{\phi} \square \phi, \quad (30) \]
so
\[ f_{\nu}(\phi) = \frac{\kappa}{8\pi} \frac{\phi_{\nu;\mu}}{\phi} \quad (31) \]
where
\[ \kappa = \frac{1}{\lambda} - \frac{3}{2} - \omega. \quad (32) \]
\( \kappa \) can be thought of as an undetermined "creation coefficient". Note however that if \( \kappa = 0 \), i.e. when Equation 30 reduces to
\[ T_{\mu \nu;\mu} = 0 \]
which is the normal GR and BD conservation equation, then
\[ \omega = \varpi = \frac{1}{\lambda} - \frac{3}{2} \quad (33) \]
where \( \varpi \) is the standard BD parameter, and the BD field equations have been recovered as to be expected.

The complete set of field equations are now:
1. The scalar field Equation 2
2. The gravitational field equation

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi}{\phi} T_{M\mu\nu} + \frac{\omega}{\phi^2} \left( \phi_{;\mu} \phi_{;\nu} - \frac{1}{2} g_{\mu\nu} \phi_{;\sigma} \phi^{;\sigma} \right) + \frac{1}{\phi} (\phi_{;\mu} ;\nu - g_{\mu\nu} \Box \phi) \]  

(34)

where \( \omega = \frac{1}{\lambda} - \frac{3}{2} - \kappa \) is a constant and \( \lambda \) and \( \kappa \) are coefficients yet to be determined.

3. The "creation" field Equation (30) developed from the PMI, which replaces the conservation equation in GR and BD.

The source of curvature \( S_{\mu\nu} \) is defined by

\[ R_{\mu\nu} = \frac{8\pi}{\phi} S_{\mu\nu} \]  

(35)

where \( S_{\mu\nu} \) is derived from Equation (34) to be

\[ S_{\mu\nu} = T_{M\mu\nu} - \frac{1}{2} \left( 1 - \frac{1}{2} \lambda \right) g_{\mu\nu} T_M^{\sigma} + \frac{1}{8\pi\phi} \overline{\omega} \phi_{;\mu} \phi_{;\nu} + \frac{1}{8\pi\phi} \phi_{;\mu} ;\nu . \]  

(36)

The gravitational field equation can be written

\[ R_{\mu\nu} = \frac{8\pi}{\phi} \left[ T_{M\mu\nu} - \frac{1}{2} \left( 1 - \frac{1}{2} \lambda \right) g_{\mu\nu} T_M^{\sigma} \right] + \frac{\overline{\omega}}{\phi^2} \phi_{;\mu} \phi_{;\nu} + \frac{1}{\phi} \phi_{;\mu} ;\nu \]  

(37)

so \( R_{\mu\nu} \) can be written in terms of the BD parameter \( \overline{\omega} \) as follows

\[ R_{\mu\nu} = \frac{8\pi}{\phi} \left[ T_{M\mu\nu} - \left( \frac{1 + \overline{\omega}}{3 + 2\overline{\omega}} \right) g_{\mu\nu} T_M^{\sigma} \right] + \frac{\overline{\omega}}{\phi^2} \phi_{;\mu} \phi_{;\nu} + \frac{1}{\phi} \phi_{;\mu} ;\nu - \frac{\kappa}{\phi^2} \phi_{;\mu} \phi_{;\nu} \]  

(38)

which is the same as the equivalent equation in the BD theory except with the addition of the last term which includes the "creation coefficient" \( \kappa \). This expression is used in order to compare our solution with the standard BD theory below.
2.2 The Post-Newtonian Approximation

In order to develop the gravitational theory consider the gravitational and scalar fields around a static, spherically symmetric, mass embedded in a cosmological space-time. In such an embedding the value of the scalar \( \phi \) defining inertial mass is assumed to asymptotically approach a "cosmological" value \( G_0^{-1} \) which holds "at great distance" from any large masses. \( \phi \) is determined in the inertial, Lorentz frame of reference of the Centre of Mass using electromagnetic methods and this is the origin of our coordinate system.

\[
\phi = G_0^{-1} (1 + \epsilon)
\]

where \( G_0 \) is a constant of dimension and order \( G_N \), and \( \epsilon \) a scalar field defined by

\[
\Box \epsilon = \epsilon^{\sigma ; \sigma} = \frac{8 \pi}{3 + 2 \omega} G_0 T^\sigma
\]

in which \( \epsilon \to 0 \) as \( r \to \infty \) and we note \( \omega \) is the BD parameter \( \omega = \frac{1}{\lambda} - \frac{3}{2} \). \( T_{\mu\nu} \) is the energy-momentum tensor of ordinary matter and energy excluding the energy of the \( \phi \) field.

The gravitational field Equation 38 now becomes

\[
R_{\mu\nu} = 8 \pi G_0 (1 + \epsilon)^{-1} \left[ T_{M \mu\nu} - \left( \frac{1+\omega}{3+2\omega} \right) g_{\mu\nu} T^\sigma_{M \sigma} \right] + \frac{\omega}{(1+\epsilon)^2} \epsilon_{\mu ; \nu} + \frac{1}{(1+\epsilon)^2} \epsilon_{\mu ; \nu} \epsilon^{\sigma ; \sigma} - \frac{\kappa}{(1+\epsilon)^2} \epsilon_{\mu ; \nu} \epsilon^{\sigma ; \sigma}
\]

which again is the same as the equivalent BD equation except with the addition of the last term which includes \( \kappa \).

In the Post-Newtonian Approximation (PNA) slowly moving particles bound by gravitational forces are considered. If \( \tau \) and \( \tau^2 \) are typical distances and velocities of the system then the components of the metric and the Ricci tensor are expressed in powers of the parameters \( \frac{GM}{\tau} \) and \( \tau^2 \) and the PNA requires an expansion of these parameters to one order beyond Newtonian mechanics.

Now \( g_{ij} \) is of the order \( \tau^N \), \( \tau^2 \simeq \frac{GM}{\tau} \) and \( R_{\mu\nu} \) is of the order \( \frac{\tau^N}{\tau^2} \). Therefore in the PNA

we need to know \( g_{ij} \) to order \( \frac{2}{g_{ij}} \), \( g_{i0} \) to order \( \frac{3}{g_{i0}} \), and \( g_{00} \) to orders \( \frac{2}{g_{00}} \) and \( \frac{4}{g_{00}} \).

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The PNA formulas for the Ricci tensor are

\[
R_{00} = -\frac{1}{2} \nabla^2 g_{00} , \quad (42)
\]

\[
R_{00} = -\frac{1}{2} \nabla^2 g_{00} + \frac{1}{2} \frac{d^2 (\frac{1}{2} g_{00})}{dt^2} + \frac{1}{2} g_{ij} \frac{d^2 (\frac{1}{2} g_{00})}{dx^i dx^j} \quad (43)
\]

\[-\frac{1}{2} \left( \nabla^2 g_{00} \right)^2 ,
\]

\[
R_{0i} = -\frac{1}{2} \nabla^2 g_{i0} , \quad (44)
\]

and

\[
R_{ij} = -\frac{1}{2} \nabla^2 g_{ij} , \quad (45)
\]

where the indices \(i, j\) run from 1 to 3, covering only the space dimensions. From Equation 11 \(\epsilon\) has the expansion

\[
\epsilon = \epsilon + \epsilon + ... \quad (46)
\]

where \(N\) \(\epsilon\) is of the order \(v^N\) and in particular

\[
\nabla^2 \epsilon = -\frac{8\pi G_0}{3 + 2\omega} T^{00} \quad (47)
\]

the field equations for the components of the metric, \(g_{00}, g_{00}, g_{i0}\) and \(g_{ij}\) are now given by substituting Equations 42, 46 and 47 in Equations 42 - 45 as

\[
\nabla^2 g_{00} = -8\pi G_0 \left( \frac{2\omega + 4}{2\omega + 3} \right) T^{00} , \quad (48)
\]
\[ \nabla^2 4 g_{00} = \frac{\partial^2}{\partial t^2} \left( \frac{g_{00}}{2} \right) + g_{ij} \frac{\partial^2}{\partial x^i \partial x^j} \left( \frac{g_{00}}{2} \right) - \left( \nabla^2 \frac{g_{00}}{2} \right)^2 \]  
\[ + 8\pi G_0 \left( \frac{2\omega + 4}{2\omega + 3} \right) \frac{\partial^{00}}{\partial t} - 8\pi G_0 \left( \frac{2\omega + 4}{2\omega + 3} \right) ^{2ii} \]  
\[ + 16\pi G_0 g_{00} \left( \frac{2\omega + 4}{2\omega + 3} \right) ^{00} \]  
\[ - 8\pi G_0 \left( \frac{2\omega + 4}{2\omega + 3} \right) \frac{\partial^{00}}{\partial t} - 2 \left( \omega - \kappa \right) \left( \frac{\partial^2 \epsilon}{\partial t} \right)^2 \]  
\[ - 2 \frac{\partial^2 \epsilon}{\partial t^2} + 2 \Gamma_{00} \frac{\partial \epsilon}{\partial x^i} , \]  
\[ \nabla^2 3 g_{00} = 16\pi G_0 \left( 1^{10} - \frac{2\partial^2 \epsilon}{\partial x^i \partial t} \right) , \]  
\[ \nabla^2 2 g_{ij} = -8\pi G_0 \delta_{ij} \left( \frac{2\omega + 4}{2\omega + 3} \right) ^{00} - 2\partial^2 \epsilon \frac{\partial \epsilon}{\partial x^i \partial t} . \]  
From Equation 48 it follows that if the usual relation between \( g_{00} \) and the purely gravitational Newtonian potential \( \Phi_m \) holds by defining \( \Phi_m \) as
\[ \nabla^2 \Phi_m = 4\pi G_m \frac{\partial^{00}}{T} , \]  
so normalized that
\[ \Phi_m (\infty) = 0 \]  
where \( G_m \) is the metric gravitational "constant" associated with the curvature of space-time measured in the limit \( r \to \infty \). Then \( G_m \) is related to \( G_0 \) by the relationship
\[ G_m = \left( \frac{2\omega + 4}{2\omega + 3} \right) G_0 \]  
It is important to note that in the BD theory, where \( G_m \equiv G_N \), the definitions of \( G_0, \phi, \) and \( \psi \) in Equations 3 and 39 give the result
\[ \psi = \left( \frac{2\omega + 4}{2\omega + 3} \right) \]
Thus $\psi$ is not necessarily unity and has to be determined in this calculation. The relationship between $\epsilon$ and $\Phi_m$ can be derived from the Equations 47 and 52 to be

$$\frac{2}{\epsilon} = -\frac{1}{\varpi + 2}\Phi_m$$

Note the solution for $\Phi_m$ around a spherically symmetric mass in vacuo is given by

$$\Phi_m = -\frac{G_m M}{r}$$

therefore

$$\frac{2}{\epsilon} = +\frac{G_m M}{(2 + \varpi)r}$$

The field equations for $g_{00}$, $g_{i0}$ and $g_{ij}$ are

$$\nabla^2 4g_{00} = -2\left(\frac{\varpi + 1}{\varpi + 2}\right)\frac{\partial^2 \Phi_m}{\partial t^2} - 2g_{ij}\frac{\partial^2 \Phi_m}{\partial x^i\partial x^j} - 2\left(\frac{2\varpi + 5}{\varpi + 2}\right)(\nabla \Phi_m)^2 - 8\pi G_m \left(4 + \frac{1}{\varpi + 2}\right)\Phi_m \mathring{T}^{00} - 8\pi G_m \Phi_m \mathring{T}^{00} - 2\left(\frac{\varpi - \kappa}{\varpi^2}\right)^2 \left(\frac{\partial \Phi_m}{\partial t}\right)^2,$$

$$\nabla^2 3g_{i0} = 16\pi G_m \left(\frac{\varpi + 3}{\varpi + 2}\right)\mathring{T}^{i0} + \left(\frac{2}{\varpi + 2}\right)\frac{\partial^2 \Phi_m}{\partial x^i\partial t},$$

and

$$\nabla^2 2g_{ij} = -8\pi G_m \left(\frac{\varpi + 1}{\varpi + 2}\right)\mathring{\delta}^i_j \mathring{T}^{00} + \left(\frac{2}{\varpi + 2}\right)\frac{\partial^2 \Phi_m}{\partial x^i\partial x^j}.$$
is no difference between this theory and BD. Hence, as with BD, the gravitational field outside a static, spherically symmetric mass depends on \( M \) alone but not any other property of the mass. Also in this case the solutions of the PNA are exactly the same as with BD. Therefore the Robertson parameters for this theory are also given by the same formulas as in BD.

\[
\alpha = 1 \, , \, \beta = 1 \, , \, \gamma = \frac{\varpi + 1}{\varpi + 2} . \tag{60}
\]

### 2.3 Solving for \( \phi \)

The effect of allowing \( T_{M \nu \mu} \neq 0 \) has now to be calculated and its effect included in the modelling of experiments of slowly moving particles. That is the violation of the equivalence principle will produce a force \( G_\nu \) that will perturb particles, but not photons, from their geodesic world lines. The force density is given by

\[
G_\nu = T_{M \nu \mu} . \tag{61}
\]

In order to calculate this effect \( \phi \) has to be determined to the third order of accuracy, \( (\epsilon^3) \), and this is possible both in BD and SCC. In the PNA solution to the One-Body problem the solution for \( \phi \) obtained from Equations 39, 46 and 56 is

\[
\phi = G_0^{-1} \left[ 1 + \frac{G_m M}{(2 + \varpi)} r + \ldots \right] \tag{62}
\]

and when the metric takes the standard form of the Robertson expansion

\[
d\tau^2 = \left( 1 - \frac{2G_m M}{r} + \ldots \right) dt^2 - \left( 1 + \frac{2\gamma G_m M}{r} + \ldots \right) dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 , \tag{63}
\]

where \( \gamma = \frac{(\varpi + 1)}{2(\varpi + 2)} \). Then, as \( \Box^2 \phi = 0 \) in vacuo,

\[
\Box^2 \phi = \frac{d^2 \phi}{dx^\lambda x^\lambda} + \Gamma_\mu^\lambda \frac{d\phi}{dx^\lambda} = 0 , \tag{64}
\]

where the affine connection \( \Gamma_\mu^\lambda \) is given by

\[
\Gamma_\mu^\lambda = \frac{1}{2} g^{\lambda \rho} \left( \frac{\partial g_{\rho \mu}}{\partial x^\nu} + \frac{\partial g_{\rho \nu}}{\partial x^\mu} - \frac{\partial g_{\mu \nu}}{\partial x^\rho} \right) . \tag{65}
\]
As \( g_{\mu\nu} \) is diagonal and \( \phi = \phi (r) \), the only non-vanishing components of the affine connection are

\[
\Gamma^t_{tt} = \frac{1}{2} g^{00} \frac{dg^{00}}{dr} = + \frac{G_m M}{r^2} \left[ 1 + O \left( \frac{G_m M}{r} \right) \right] ,
\]

\[
\Gamma^r_{rr} = \frac{1}{2} g^{rr} \frac{dg^{rr}}{dr} = - \frac{(\varpi + 1) G_m M}{r^2} \left[ 1 + O \left( \frac{G_m M}{r} \right) \right] ,
\]

\[
\Gamma^\phi_{\phi r} = \frac{1}{2} g^{\phi\phi} \frac{dg^{\phi\phi}}{dr} = \frac{1}{r} , \quad \text{and} \quad \Gamma^\theta_{\theta r} = \frac{1}{2} g^{\theta\theta} \frac{dg^{\theta\theta}}{dr} = \frac{1}{r} ,
\]

and Equation 64 becomes

\[
\Box^2 \phi = \frac{d^2 \phi}{dr^2} + \left\{ 1 - \frac{(\varpi + 1)}{(\varpi + 2)} \right\} \frac{G_m M}{r^2} \frac{2}{r} + \ldots \frac{d\phi}{dr} = 0 .
\] (67)

Integrating twice w.r.t. \( r \), and expanding the exponential with \( \frac{G_m M}{r} \ll 1 \), produces a solution with two integration constants, \( k_1 \) and \( k_2 \);

\[
\phi = k_1 + k_2 r + \frac{k_2}{2(\varpi + 2)} \frac{G_m M}{r^2} + \ldots .
\] (68)

Comparing coefficients with Equation 62 evaluates \( k_1 \) and \( k_2 \) and

\[
\phi = G_0^{-1} \left\{ 1 + \frac{G_m M}{(2 + \varpi) r} + \frac{1}{2} \left[ \frac{G_m M}{(2 + \varpi) r} \right]^2 + \ldots \right\} ,
\] (69)

so, to the accuracy of the post-post Newtonian approximation,

\[
\phi = \phi_0 \exp \left[ \frac{G_m M}{(2 + \varpi) r} \right] .
\] (70)

Therefore

\[
\frac{1}{\phi} \frac{d\phi}{dr} = - \frac{G_m M}{(2 + \varpi) r^2} .
\] (71)

### 2.4 The Scalar Field Acceleration

The expression for \( T^{\mu}_{\nu;\mu} \) for a system of \( n \) particles of rest mass \( m_n \) is given by

\[
T^{\mu}_{\nu;\mu} = G_\nu = \sum_n \delta^3 \{ x - x_n (t) \} g_{\nu\alpha} \frac{d\tau}{dt} \frac{dx^\alpha}{dt} \left[ m_n \frac{dx^\alpha}{d\tau} \right] ,
\] (72)
where $\delta^3 \{x - x_n (t)\}$ is the Dirac delta function, $d\tau$ the proper time defined by
\[
d\tau^2 = -g_{\mu\nu} dx^\mu dx^\nu
\] (73)
and $G_\nu$ is the force density.

Over the elemental volume of an individual test particle with density of inertial rest mass $\rho (r, t)$ becomes
\[
T^\mu_{\nu,\mu} = g_{\alpha\nu} \frac{d\tau}{dt} \frac{d}{d\tau} \left[ \frac{dx^\alpha}{dt} \right],
\]
that is,
\[
T^\mu_{\nu,\mu} = g_{\alpha\nu} \left[ \frac{d\rho}{dt} \frac{dx^\alpha}{dt} + \rho \frac{d^2 x^\alpha}{dt^2} \right].
\] (74)
Substituting Equation 2 in 30 yields
\[
T^\mu_{\nu,\mu} = \frac{\kappa \lambda}{\phi} \phi T^\sigma_{\nu,\sigma}
\] (75)
and using
\[
T^\sigma_{\nu,\sigma} = 3p - \rho = -\rho
\] (76)
for a perfect fluid in the rest frame when the pressure is negligible, the PMI solution for the non-conservation of the energy-momentum tensor, becomes
\[
T^\mu_{\nu,\mu} = -\frac{\kappa \lambda}{2} \frac{\phi}{\phi} \rho.
\] (77)
The effect of the scalar field force is therefore
\[
g_{\alpha\nu} \left[ \frac{d\rho}{dt} \frac{dx^\alpha}{dt} + \rho \frac{d^2 x^\alpha}{dt^2} \right] = -\frac{\kappa \lambda}{2} \frac{\phi}{\phi} \rho
\] (78)
Now consider the effect of this force on a mass particle momentarily at rest in the frame of reference of the Centre of Mass, that is: $\frac{dx^\alpha}{dt} = 0$. Equation 78 becomes
\[
\frac{d^2 x^\alpha}{dt^2} = -g^{\alpha\nu} \frac{\kappa \lambda}{2} \frac{\phi}{\phi} \rho.
\] (79)
2.5 Equations of Motion

It is now possible to examine the equations of motion in this theory. At every space-time event in an arbitrary gravitational field we can specify a set of coordinates $\xi^i$ in which the local description of space-time is Minkowskian, with a Special Relativity metric $\eta_{\alpha\beta}$ and in which a photon has an equation of motion

$$\frac{d^2\xi^\alpha}{d\sigma^2} = 0, \quad (80)$$

$$0 = -\eta_{\alpha\beta} \frac{d\xi^\alpha}{d\sigma} \frac{d\xi^\beta}{d\sigma}, \quad (81)$$

where $\sigma \equiv \xi^0$ is a suitable parameter describing the null-geodesic. We now consider the equation of motion of a distant particle, momentarily stationary, in the coordinate system $x^\mu$ of the frame of reference of the Centre of Mass. Transforming coordinates into this system the particle would also experience the scalar field acceleration described in Equation 79 and as the affine connection is defined by

$$\Gamma^\alpha_{\mu\nu} = \frac{\partial x^\alpha}{\partial \xi^\beta} \frac{\partial^2 \xi^\beta}{\partial x^\mu \partial x^\nu},$$

then, if the pressure is negligible,

$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -g^{\alpha\nu} \kappa \lambda \phi_{;\nu}, \quad (82)$$

Therefore for a slow particle

$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha_{00} (\frac{dt}{d\tau})^2 = -g^{\alpha\nu} \kappa \lambda \phi_{;\nu}, \quad (83)$$

also

$$\frac{d^2t}{d\tau^2} = 0, \quad \text{so} \quad \frac{dt}{d\tau} = \sqrt{-g_{00}^{-1}} \text{ which is a constant at } r = r_1.$$ 

So multiplying through by $(\frac{dt}{d\tau})^2$ produces

$$\frac{d^2x^\alpha}{dt^2} + \Gamma^\alpha_{00} = +g_{00} g^{\alpha\nu} \kappa \lambda \phi_{;\nu}. \quad (84)$$
Now for a stationary field
\[ \Gamma_\alpha^\mu = -\frac{1}{2} g^\mu_{\nu} \frac{\partial g_{\nu 00}}{\partial x^\nu} \]  
(85)

and for a weak field, \( g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} \), where \( |h_{\alpha\beta}| \ll 1 \) and \( \eta_{\alpha\beta} \). The resulting affine connection, linearized in the metric perturbation \( h_{\alpha\beta} \), becomes in the spherically symmetric case, (as in GR)

\[ \Gamma_0^r = -\frac{1}{2} \eta^{rr} \frac{dh_{00}}{dr} \]  
(86)

so the only non zero component of Equation \( 84 \) is

\[ \frac{d^2r}{dt^2} = \frac{1}{2} \eta^{rr} \frac{dh_{00}}{dr} + g_{00} g^{rr} \frac{\kappa \lambda}{2} \frac{d\phi}{dr} \]  
(87)

The general standard form of the metric in both the BD and SCC theories is

\[ dr^2 = \left[ 1 - \frac{2GmM}{r} + 2 (1 - \gamma) \left( \frac{GmM}{r} \right)^2 + ... \right] dt^2 \]  
(88)

\[- \left( 1 + \frac{2\gamma GmM}{r} + ... \right) dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \]

where in both theories \( \gamma = \frac{(2 - \lambda)}{(2 + \lambda)} \), therefore

\[ g_{00} = -\left[ 1 - \frac{2GmM}{r} + \frac{4\lambda}{(2 + \lambda)} \left( \frac{GmM}{r} \right)^2 + ... \right] , \]  
(89)

\[ h_{00} = \frac{2GmM}{r} - \frac{4\lambda}{(2 + \lambda)} \left( \frac{GmM}{r} \right)^2 + ... \]  
(90)

and

\[ g_{rr} = 1 + \frac{2(2 - \lambda) GmM}{(2 + \lambda) r} + ... \]  
(91)

Substituting Equations 71, 89, 90 and 91 in Equation 87 yields

\[ \frac{d^2r}{dt^2} = -\left[ 1 - \frac{\kappa \lambda^2}{(2 + \lambda)} \right] \frac{GmM}{r^2} \]  
(92)

\[ + \left\{ \frac{4\lambda (2 + \lambda - 2\kappa \lambda)}{(2 + \lambda)^2} \right\} \left( \frac{GmM}{r^3} \right)^2 + ... \]
and therefore to first order the total acceleration experienced by a particle is
\[
\frac{d^2r}{dt^2} = - \left[ 1 - \frac{\kappa \lambda^2}{2 + \lambda} \right] \frac{G_m M}{r^2} + \ldots .
\] (93)

However Newtonian gravitational theory defines \( G_N \) by
\[
\frac{d^2r}{dt^2} = - \frac{G_N M}{r^2} ,
\] (94)
therefore the effect of violating the equivalence principle in accordance with the PMI is that every mass experiences an extra acceleration similar to Newtonian gravitation and which therefore is confused with it. According to SCC the Newtonian gravitational constant \( G_N \), as measured in a Cavendish type experiment, is a compilation of the effect of the curvature of space time, with its corresponding \( G_m \), and the action of the scalar field. The total Newtonian acceleration experienced by a mass particle is therefore
\[
G_N = \left[ 1 - \frac{\kappa \lambda^2}{(2 + \lambda)} \right] G_m .
\] (95)

Note that \( G_N \) and \( G_m \) refer to the total ”gravitational” accelerations experienced in physical experiments by atomic particles and photons respectively.

3 Incorporating the Local Conservation of Energy in BD

3.1 The Relationship Between \( \phi \) And \( m \).

Consider the general Gauss Divergence theorem applied to the gradient of the Newtonian potential \( \Phi_N \)
\[
\iiint_V \nabla \Theta \circ dV = \iint_S \Theta \circ dS ,
\]
put \( \Theta = \nabla \Phi_N \) and define \( \Phi_N \) by \( \nabla^2 \Phi_N = 4\pi G_N \rho \) with \( \lim_{r \to \infty} \Phi_N (r) = 0 \),
\[
\iiint_V \nabla^2 \Phi_N \, dV = \iint_S \nabla \Phi_N \cdot dS .
\] (96)
In the spherically symmetric One Body case the volume integral on the left
hand side is simply \(4\pi GM\) where \(M\) is the remote determination of the
total mass of the central body radius \(R\). Consider several concentric external
spheres of radius \(r_1, r_2\) etc. \(\geq R\) centered on the mass \(M\). As the contribu-
tions from the vacuum are zero the volume integrals over each sphere are
equal.

\[
\int \int \int_{V_1} \nabla^2 \Phi_N. dV = \int \int \int_{V_2} \nabla^2 \Phi_N. dV = 4\pi GM.
\]  

Therefore observers on the surface of each sphere will have different deter-
minations of the central mass, which will vary \(M \propto m_i^{-1}\) in the JF(E), that
is when comparing \(M\) to their locally determined atomic masses \(m_i\) by ob-
serving the red shift of photons that are emitted from the central mass with
invariant energy. As \(GM\) is constant for all \(r \geq R\) they will conclude

\[G(r) \propto M^{-1}(r) \propto m_i(r).\]

But \(G(r) = \frac{\omega}{\phi}\) therefore consistency demands

\[\phi(r) \propto m_i(r)^{-1}.\]  

### 3.2 The Gravitational Red-Shift of Light

In order to examine the measurement problem in both the EF and the JF(E)
the gravitational red shift of light is now considered. This analysis depends
on the assumption that if no work is done on, or by, a projectile while in
free fall then its energy \(E, P^0\) is conserved when measured in a specific
frame of reference, that of the CoM of the system. In a gedanken, ‘thought’,
experiment, construct a laboratory at the co-moving centroid, the CoM, of
the system. Connect it to the outside world by a radial tube through which
identical test masses and photons may be projected in vacuo. Launch such
projectiles, with rest masses, \(m_0\), at the CoM at various velocities to reach
maximum altitudes \(r_i\) where \(r_i\) varies from \(R\), the radius of the central mass,
out to infinity. The mass of the projectile \(m_c(r)\), the ’coordinate’ mass, is
in general a function of altitude measured in the CoM frame of reference.

First consider such a photon emitted by one atom at altitude \(x_2\) and
absorbed by another at an altitude \(x_1\). The emission and absorption fre-
cuencies of the photon, \(\nu(x_2)\) and \(\nu(x_1)\), are determined by comparing the
arrival times of two adjacent wave fronts emitted from one point in a gravitational field at \((x_2)\) and received at another at \((x_1)\). The standard time dilation relationship is thereby derived

\[
\frac{\nu(x_2)}{\nu(x_1)} = \left[ -g_{00}(x_2) \right]^{1/2}.
\]  

(99)

Hence substituting \(x_2 = r\) and \(x_1 = \infty\) in Equation 99, where \(g_{00}(x_1) = -1\), and writing \(\nu(\infty)\) as \(\nu_0\), yields the standard (GR) gravitational red shift relationship

\[
\nu(r) = \nu_0 \left[ -g_{00}(r) \right]^{1/2}.
\]  

(100)

Where the observer is at infinite altitude receiving a photon emitted at altitude \(r\).

Now consider the various projectiles. With the standard definition of proper time \(\tau\) from the metric

\[
d\tau^2 = -g_{\mu\nu}dx^\mu dx^\nu.
\]  

(101)

The 4-momentum vector of the projectile is defined

\[
P^\mu = m_c \frac{dx^\mu}{d\tau}.
\]  

(102)

The time component of 4-momentum \(P^\mu\) is the total 'relative' energy \(E\) and the space components form the 'relative' 3-momentum \(\vec{p}\). Now from Equation 101

\[
\frac{d\tau^2}{dt^2} = -g_{00} - 2g_{i0}v^i - v^2,
\]  

(103)

where \(v^i = \frac{dx^i}{dt}\) and \(v^2 = g_{ij} \frac{dx^i}{dt} \frac{dx^j}{dt}\).  

(104)

Therefore in a spherically symmetric, non-rotating, metric with \(g_{i0} = 0\),

\[-g_{00}E^2 = m_c^2 + \vec{p}^2.\]  

(105)

This is the spherically symmetric curved space-time equivalent to the SR identity

\[E^2 = m_c^2 + \vec{p}^2.\]  

(106)

Now consider two of the projectiles as they momentarily reach their respective apocentres at maximum altitude \(r\), and \(r + \delta r\). As they are momentarily stationary in the CoM frame \(\vec{p} = 0\). The difference between the two
adjacent projectiles at their apocentres is that one has a total energy and rest mass of $E(r)$, and $m_c(r)$, and the other $E(r + \delta r)$, and $m_c(r + \delta r)$.

Expanding for small $\delta r$, and where a prime ($'$) means $\frac{d}{dr}$, in the limit $\delta r \to 0$ we obtain

$$\frac{1}{2} \left[ -g_{00}'(r) \right] + \frac{E'(r)}{E(r)} \frac{m'_c(r)}{m_c(r)} .$$

(107)

Here two identical projectiles separated by an infinitesimal increase in altitude are compared. The only difference between them is the infinitesimal energy $\delta E$ required to raise such a projectile from $r$ to $r + \delta r$. Although the Newtonian potential $\Phi_N(r)$ is defined by

$$\nabla^2 \Phi_N(r) = 4\pi G_N \frac{T_{00}}{\rho} = 4\pi G_N \rho$$

(108)

which is normalized,

$$\Phi_N(\infty) = 0 ,$$

it is actually measured in a Cavendish type laboratory experiment by the force vector acting on a body, which is given by

$$\mathbf{F} = -m_p \nabla \Phi_N(r) .$$

(109)

Then if the mass $m(r)$ is raised a height $\delta r$ against this force, the infinitesimal energy $\delta E$ required is

$$\delta E = -\mathbf{F} \circ \delta \mathbf{r} = m_p(r) \nabla \Phi_N(r) \circ \delta \mathbf{r} .$$

(110)

That is in the radial case

$$\delta E = m_p(r) \Phi_N'(r) \delta r ,$$

(111)

where $m_p(r)$ is that physical mass entering into the Newtonian gravitational equation. Define such physical mass, momentarily at rest, as

$$m_p(r) = E(r) ,$$

(112)

so that the total "relative" energy at an altitude $r$ is its rest mass at that altitude, measured in the CoM frame of reference. In the limit $\delta r \to 0$ Equation (111) becomes

$$\frac{E'(r)}{E(r)} = \Phi'_N(r) ,$$

(113)
which when substituted in Equation 107 yields

\[
\frac{1}{2} \left[ -g^{00}(r) \right] + \Phi'_{N}(r) = \frac{m'_c(r)}{m_c(r)}.
\] (114)

This integrates directly,

\[
\frac{1}{2} \ln \left[ -g^{00}(r) \right] + \Phi_N(r) = \ln \left[ m(r) \right] + k
\] (115)

where \(k\) is determined in the limit \(r \to \infty\), \(g^{00}(r) \to -1\), \(\Phi_N(r) \to 0\) and \(m(r) \to m_0\). The rest mass, \(m(r)\), of a projectile at altitude \(r\), evaluated in the co-moving CoM frame is therefore given by

\[
m_c(r) = m_0 \exp \left[ \Phi_N(r) \right] \left[ -g^{00}(r) \right]^\frac{1}{2}.
\] (116)

This is the value, \(m_c(r)\), given by an observer at infinite altitude, where Special Relativity and a ground state solution to the theory are recovered, with well defined particle rest mass \(m_0\), 'looking down' to a similar particle at an altitude \(r\). From this expression it is obvious that with our assumption of the conservation of energy, \(P^0\), in the CoM frame gravitational time dilation, the factor \([-g^{00}(r)]^\frac{1}{2}\), applies to massive particles as well as to photons. As physical experiments measuring the frequency of a photon compare its energy with the mass of the atom it interacts with, it is necessary to compare the masses (defined by Equation 116) of two atoms at altitude, \(r\) and \(\infty\), with the energy (given by Equation 100) of a "reference" photon transmitted between them. This yields the physical rest mass \(m_p(r)\) as a function of altitude

\[
\frac{m_p(r)}{\nu(r)} = \frac{m_0}{\nu_0} \exp \left[ \Phi_N(r) \right].
\] (117)

Equation 117 is a result relating observable quantities, but how is it to be interpreted? In other words how are mass and frequency to be measured in any particular frame? In the GR EF (and BD JF) the physical rest mass of the atom is defined to be constant, hence prescribing \((\tilde{\omega}^{\mu})\), with \(m_p(\tilde{r}) = m_0\). In this case Equation 117 becomes

\[
\nu(\tilde{r}) = \nu_0 \left( 1 - \tilde{\Phi}_N(\tilde{r}) + \ldots \right).
\] (118)

Hence photons transmitted out of a gravitational potential well are said to exhibit a red shift which is equal to the dimensionless Newtonian potential \(\tilde{\Phi}_N\), and equal in GR, "coincidentally", to the time dilation effect, the
factor\([-\tilde{g}_{00}(\tilde{r})]\). That is, compared to reference atoms they mysteriously appear to lose (potential) energy.

However in the SCC JF(E) rest mass is given by the expression Equation \[6\] consequently a comparison of Equation \[117\] with the equation for rest mass in this frame yields

\[\nu(r) = \nu_0. \quad (119)\]

Therefore in the SCC JF(E), in which energy is locally conserved, gravitational red shift is interpreted not as a loss of potential energy by the photon but rather as a gain of potential energy by the apparatus measuring it. It is important to note that in this frame the frequency, and hence wavelength and energy, of a free photon is invariant, even when transversing space-time with curvature.

On the other hand, as experiments using physical apparatus refer measurements of energy and mass to the mass of the atoms of which they are composed, such observations interpret rest masses to be constant by definition. In SCC such experiments are conducted in its EF in which it is Equation \[118\] that describes gravitational red shift.

Using either frame the gravitational red shift prediction in SCC is in agreement with GR and all observations to date.

Time is the fundamental measurement in both frames, measured by the frequency of a reference photon in the JF(E) and by the Bohr frequency of an atom in the EF. By definition the speed of light is invariant in both.

### 3.3 At the Centre of Mass

Consider the origin of our coordinate system in the static, spherically symmetric, case, which is the centre of mass of the system. In Relativity theory the centroid of an isolated system with energy-momentum tensor \(T^{\mu\nu}\) and total 4-momentum \(P^\alpha\), when observed by an observer \(O\) with a 4-velocity \(U^\alpha\) at his Lorentz time \(x^0 = t\) and in his own Lorentz frame, is defined by

\[X^i_\mu(t) = \left(\frac{1}{P^0}\right) \int_{x_0=t} x^j T^{00} d^3x \quad (120)\]

and the co-moving centroid associated with the rest frame of the system is defined to be its Centre of Mass (CoM). At the CoM the resultant of all gravitational forces vanishes hence so does \(\sum \Phi_N\). Furthermore \(\phi = \phi(\Phi_N)\),

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therefore with $\nabla \Phi_N = 0$ at the CoM,

$$\nabla \phi = 0 .$$

(121)

As $\phi (x_\nu)$ is static and not dependent on time we have for all four $\nu$

$$\nabla_\nu \phi = 0 ,$$

(122)

thus at the CoM, by Equation 30 the PMI yields

$$\nabla_\mu T^\mu_M = \frac{1}{8\pi\lambda} \nabla_\nu \phi \Box \phi = 0 .$$

(123)

Hence the unique location of the centre of mass of the system, where the
ergy-momentum tensor of matter is conserved with respect to covariant
differentiation, can be considered a ‘proper laboratory’. Here the theory ad-
mits a ground state solution, $g_{\mu\nu} \to \eta_{\mu\nu}$ and $\nabla_\mu \phi = 0$ , here the equivalence

principle holds, even for a massive particle, and here a free falling physical
clock, remaining at rest, records proper time. Distances can be measured
by timing the echo of light rays (radar) using that clock and the metric
properly defined. Also Special Relativity is recovered as here the metric is
Minkowskian and standards of mass, length, time and the physical constants
defined for atoms, together with potential energy, retain their classical mean-
ing. Such a standard defined atom emits a ‘reference’ photon, which in the
JF(E) is transmitted across space-time with invariant energy and frequency.

In SCC the CoM preferred frame of reference is selected if and only if
energy is to be locally conserved, otherwise the equations are manifestly
covariant.

Consider again the spherical shells around a central spherical mass $M$
Integrating the surface integral on the right hand side of Equation 96 over
the sphere at constant $r$ gives simply $4\pi r^2 \nabla \Phi_N (r)$, (In the standard general
form of the metric the surface area of a sphere is $4\pi r^2$. This fact is used both
here and below). Now the Newtonian potential is defined by a measurement
of acceleration in a local experiment

$$\frac{d^2r}{dt^2} = -\nabla \Phi_N (r) .$$

An observer at the Centre of Mass of the system, in the ‘proper laboratory’
where $M$ is constant, would conclude from Equations 96 and 97 that

$$\frac{d^2r}{dt^2} = -\frac{\psi M}{\phi r^2} .$$

(124)
Reiterating, in both BD and SCC the inertial masses of fundamental particles \( m_i \) are dependent on their interaction with the scalar field \( \phi \). However the difference is that whereas in BD a coordinate system is used in which \( m_i \) is constant and it is \( G \) which is observed to vary, in the One Body Problem of SCC, in which \( m_i \equiv m_i(r) \) and \( \phi \equiv \phi(r) \), it is the rest mass \( m_i \), as measured by electromagnetic radiation from the proper laboratory, which varies, and \( G_N \), as measured by atomic apparatus in a 'local laboratory', which is invariant.

### 3.4 Evaluating the Parameters in the Field Equations

The parameters \( \lambda, \kappa, \) and \( \psi \), will now be calculated. There are two methods of calculating the combined gravitational and scalar field acceleration, one derived from the equations of motion: Equation 92 and the other derived from the definition of the Newtonian potential applied to Gauss Divergence theorem: Equation 124. Consistency between these two methods places constraints on the three parameters. Using Equations 33 and 33 the relationship between \( G_m \) and \( G_0 \) may be written

\[
G_m = \left( \frac{2 + \lambda}{2} \right) G_0 ,
\]

and using this to substitute for \( G_0 \) in Equation 92 the combined gravitational and scalar field acceleration of a free falling massive body is given by

\[
\frac{d^2 r}{dt^2} = - \left\{ \frac{1}{2} \left( 2 + \lambda - \kappa \lambda^2 \right) - \left( 2 + \lambda - 2 \kappa \lambda \right) \frac{\lambda G_0 M}{r} + \ldots \right\} \frac{G_0 M}{r^2} .
\]  

(126)

But we also have an expression for this combined acceleration from Equation 124 together with the solution for \( \phi \) in Equation 70 expanded for small \( \frac{G_m M}{r} \). Using Equation 125 this becomes

\[
\frac{d^2 r}{dt^2} = -\psi \left[ 1 - \frac{\lambda G_0 M}{r} + \ldots \right] \frac{G_0 M}{r^2} .
\]  

(127)

Comparing coefficients between equations 126 and 127 sets two conditions on \( \lambda, \kappa, \) and \( \psi \). Consistency between the coefficients of \( \frac{G_0 M}{r^2} \) requires

\[
2 + \lambda - \kappa \lambda^2 = 2 \psi
\]  

(128)
and consistency between the coefficients of \( \frac{(G_0 M)^2}{r^4} \) requires

\[
2 + \lambda - 2\kappa \lambda = \psi . \tag{129}
\]

Furthermore we have two solutions for \( \phi \); one from the field Equation \( \ref{eq:field} \) and the other from the local conservation of energy. The solution from Equation \( \ref{eq:local} \) is

\[
\phi = \phi_0 \exp \left[ \frac{2\lambda}{2 + \lambda - \kappa \lambda^2} \frac{G_N M}{r} \right] \tag{130}
\]

the solution from Equations \( \ref{eq:field} \) and \( \ref{eq:local} \) is

\[
\phi = \phi_0 \exp \left[ \frac{G_N M}{r} \right] , \tag{131}
\]

so consistency between Equations \( \ref{eq:field} \) and \( \ref{eq:local} \) sets a third condition on the three parameters

\[
2 - \lambda - \kappa \lambda^2 = 0 . \tag{132}
\]

There are three simultaneous equations \( \ref{eq:field} \), \( \ref{eq:local} \) and \( \ref{eq:local} \) for \( \psi \), \( \lambda \) and \( \kappa \). Their unique solution is

\[
\psi = 1 , \quad \lambda = 1 \quad \text{and} \quad \kappa = 1 . \tag{133}
\]

Furthermore Equations \( \ref{eq:field} \) and \( \ref{eq:local} \) give the result

\[
G_N = \frac{1}{2} (2 + \lambda - \kappa \lambda^2) G_0 = G_0 = \lim_{r \to \infty} \frac{1}{\phi(r)} . \tag{134}
\]

Thus \( G_N \) is the proper value of \( \phi^{-1} \) as measured by atomic apparatus at infinity, and will be that value determined by physical apparatus in "Cavendish" type experiments elsewhere.

4 Conclusions

In conclusion, by following through carefully the consequences of introducing the principles of mutual interaction and the local conservation of energy we have determined that the three parameters introduced into the equations: \( \lambda \), \( \kappa \) and \( \psi \) are all unity. The values of \( \lambda \) and \( \kappa \) yield the following standard formulae: from Equations \( \ref{eq:field} \) and \( \ref{eq:local} \)

\[

\varpi = \frac{1}{\lambda} - \frac{3}{2} = -\frac{1}{2} , \tag{135}
\]

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and finally
\[ \omega = \frac{1}{\lambda} - \frac{3}{2} - \kappa = -\frac{3}{2}, \tag{136} \]
hence the value \( \omega = -\frac{3}{2} \) required in order to make the EF of the theory canonical GR is that value determined by consistency between Mach’s Principle and the Local Conservation of Energy in SCC.

With this value of \( \omega = -\frac{3}{2} \) the field equations of the theory become:

The scalar field equation
\[ \Box \phi = 4\pi T_M, \tag{137} \]
the gravitational field equation
\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi}{\phi} T_{M\mu\nu} - \frac{3}{2\phi^2} \left( \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi \right) + \frac{1}{\phi} (\nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \Box \phi), \tag{138} \]
and the creation equation,
\[ \nabla_\mu T^\mu_{M\nu} = \frac{1}{8\pi} \frac{1}{\phi} \nabla_\nu \phi \Box \phi \tag{139} \]

These field equations give rise to the cosmological and experimental features of the theory. The two conformal frames of the theory are both physical [see (Barber, 2002a)] one, the JF(E), conserves mass-energy and the other, the EF, conserves energy-momentum. Photons follow the geodesics of the JF, which determine cosmological evolution and particles follow the geodesics of GR, which determine all solar system experiments to date (February 2003). Those three definitive experiments [see (Barber, 2003)] that observe directly the curvature of space-time, or the false vacuum required by curvature, are the only ones able to distinguish between the two theories and are able to falsify one, or both, of them. The geodetic experiment of ‘Gravity Probe B’ is awaited with anticipation.

4.1 Acknowledgement

I wish to acknowledge my debt to Steven Weinberg as his textbook [Weinberg, (1972)] has been my guide. I have used and adapted his methods in the calculations incorporating the PMI, although I have used a sign convention that has reversed the sign of \( R_{\mu\nu} \) and \( R \).
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