Double beta decay to excited states in $^{150}Nd$

Jorge G. Hirsch
Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN,
A. P. 14-740 México 07000 D.F.
Octavio Castaños and Peter O. Hess
Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México,
A. P. 70-543 México 04510 D.F.
Osvaldo Civitarese
Departamento de Física, Universidad Nacional de La Plata,
c.c.67 1900, La Plata, Argentina

Abstract:

The pseudo SU(3) model is used to study the double beta decay of $^{150}Nd$ to the ground and excited states of $^{150}Sm$. Low lying collective excitations of $^{150}Sm$ and its BE(2) intensities are well reproduced. Expressions for the two neutrino double beta decay to excited states are developed and used to describe the decay of $^{150}Nd$. The existence of selection rules which strongly restrict the decay is discussed.
Introduction

The neutrinoless double beta decay ($\beta\beta_{0\nu}$), undetected up to now, provides the more stringent limits to the Majorana mass of the neutrino $< m_{\nu_e} > \leq 1.1eV$\textsuperscript{[1]}. Its detection would imply an indisputable evidence of physics beyond the standard model and would be useful in order to select Grand Unification Theories\textsuperscript{[2]}.

Theoretical nuclear matrix elements are needed to convert experimental half-life limits, which are available for many $\beta\beta$-unstable isotopes\textsuperscript{[3]}, into constrains for particle physics parameters such as the effective Majorana mass of the neutrino and the contribution of right-handed currents to the weak interactions. Thus, these matrix elements are essential to understand the underlying physics.

The two neutrino mode of the double beta decay ($\beta\beta_{2\nu}$) is allowed as a second order process in the standard model. It has been detected in nine nuclei\textsuperscript{[3]} and has served as a test of a variety of nuclear models. The calculation of the $\beta\beta_{2\nu}$ and $\beta\beta_{0\nu}$ matrix elements requires different theoretical methods. Therefore a successful prediction of the former cannot be considered a rigorous test of the latter, but gives some confidence. However it is the best available proof we can impose to a nuclear model used to predict the $\beta\beta_{0\nu}$ matrix elements.

Many experimental groups have reported measurements of $\beta\beta_{2\nu}$ processes\textsuperscript{[3]}. Nearly for all the cases the ground state (g.s.) to ground state ($0^+ \rightarrow 0^+$) decay was investigated. In direct-counting experiments the analysis of the sum-energy spectrum of the emitted electrons allowed the identification of the different $\beta\beta$-decay modes\textsuperscript{[1]}.

Recently the possibility of detecting $\beta\beta$ decay into excited states of the daughter nucleus by measuring the gamma radiation has exerted some attraction among experimentalists. This is due to the fact that phase space integrals scale as the energy available for the decay and consequently decrease for excited states. In the case of the decay to a first excited $2^+$ state the phase space factor contains terms which are antisymmetric in the energies of the two outgoing electrons and antineutrinos, resulting in a large reduction of the corresponding integral. It makes such transitions very difficult to observe\textsuperscript{[3], [6]}. Also they are inclusive experiments which cannot distinguish between the different $\beta\beta$ decay modes. On the other hand the detection of
a gamma ray gives a much clearer signal than a continuous electron spectrum as in the case of the \( g.s. \rightarrow g.s. \) decay.

The pioneer work of Bellotti \textit{et al.} [7] determined lower limits to the \( \beta\beta \) decays to excited states of six nuclei. Lower limits for the decay of \( ^{126}Te \) and \( ^{130}Te \) to the first \( 2^+ \) state of the corresponding Xenon isotopes have been reported[3]. The \( \beta\beta \) decay of \( ^{76}Ge \) to excited states of \( ^{76}Se \) was studied looking for the detection of one or two photons in coincidence with two electrons[3]. The half-lives for the \( \beta\beta_{2\nu} \) decay from the g.s. of \( ^{100}Mo, ^{96}Zr \) and \( ^{150}Nd \) to the first excited \( 0^+ \) state of the daughter nuclei have been estimated for the first time in [11] assuming that the nuclear matrix elements of the transitions to the g.s. and excited \( 0^+ \) state are the same. Experimental studies of the decay to excited states looking for the \( \gamma \) ray signature have been performed for \( ^{96}Zr[12], ^{116}Cd[4, 13, 14], \) and \( ^{100}Mo[13, 15, 16, 17] \). The detection of the \( \beta\beta_{2\nu} \) to the first excited \( 0^+ \) state was reported for the first time in [17]. The factibility of studying the \( \beta\beta_{2\nu} \) to excited states in \( ^{150}Nd \) has been discussed in recent years[4, 11, 12, 19]. In [12] preliminary results were reported.

Theoretical analysis of the \( \beta\beta_{2\nu} \) to excited states have been performed in the context of the QRPA formalism for \( ^{100}Mo[20, 21], ^{136}Xe[22, 23], \) and \( ^{76}Ge[24, 25], ^{116}Cd[4] \) and also for \( ^{82}Se, ^{110}Pd \) and \( ^{128,130}Te[26] \). When the first excited \( 0^+ \) state was studied it was assumed that it is a member of a two quadrupole-phonon triplet. The QRPA calculation for \( ^{100}Mo \) exhibits an overestimation of the amplitude of the \( \beta\beta_{2\nu} \) decay to this excited state when the decay to the ground state is reproduced[21, 27]. For the case of \( ^{116}Cd[4] \) the matrix element of this decay is five times greater than that associated with the decay to the ground state.

Although there is no reported calculation of the \( \beta\beta_{2\nu} \) of \( ^{150}Nd \) to excited states of \( ^{150}Sm \), this nucleus was mentioned as a suitable candidate for this decay. In [11] this conclusion has been reached assuming that the nuclear matrix elements for the \( \beta\beta_{2\nu} \) to the ground state and the first excited \( 0^+ \) state are equal. In [14] it was speculated that if the matrix element for the \( \beta\beta_{2\nu} \) of \( ^{150}Nd \) would show a similar enhancement over that of the g.s. decay as found for \( ^{116}Cd \), the decay rate into this excited level could even exceed that of the g.s. decay.

In the present paper we perform an analysis of the \( \beta\beta_{2\nu} \) decay of \( ^{150}Nd \) to excited \( 0^+ \) and \( 2^+ \) states of \( ^{150}Sm \) using the pseudo SU(3) formalism. This theoretical model is well suited to describe the collective spectra. Under this scheme the first \( 2^+ \) and \( 4^+ \) states are members of
the g.s. rotational band, while the second excited $0^+$ and $2^+$ states are members of an excited rotational band. The third excited $0^+$ state is the head of another rotational band. We will show that within the pseudo SU(3) model the $\beta\beta_{2\nu}$ decay to the first excited $0^+$ state will be cancelled while the other decays will be strongly suppressed. The necessary formalism to study the $\beta\beta_{2\nu}$ decay to $2^+$ states is also developed.

In Section 2 the pseudo SU(3) formalism and the model Hamiltonian are briefly reviewed. In Section 3 the summation method is used to obtain the $\beta\beta_{2\nu}$ matrix elements for the decay to the $2^+$ states. Section 4 contains the explicit formulae needed to evaluate the $\beta\beta_{2\nu}$ matrix elements in the pseudo SU(3) scheme. The nuclear structure analysis of $^{150}\text{Sm}$ is given in Section 5. In Section 6 the $\beta\beta_{2\nu}$ nuclear matrix elements and half-lives are presented. Conclusions are drawn in the last Section.

The pseudo SU(3) formalism

In order to obtain a microscopical description of the low lying energy states of $^{150}\text{Nd}$ and $^{150}\text{Sm}$ we will use the pseudo SU(3) model which successfully describes collective excitations in rare earth nuclei and actinides\cite{27} as well as the g.s. $\rightarrow$ g.s. $\beta\beta_{2\nu}$ and $\beta\beta_{0\nu}$ decays of six heavy deformed nuclei\cite{28, 29, 30}.

In the pseudo SU(3) shell model coupling scheme\cite{31} normal parity orbitals $(\eta, l, j)$ are identified with orbitals of a harmonic oscillator of one quanta less $\tilde{\eta} = \eta - 1$. This set of orbitals with $\tilde{j} = j = \tilde{l} + \tilde{s}$, pseudo spin $\tilde{s} = 1/2$ and pseudo orbital angular momentum $\tilde{l}$ define the so-called pseudo space. Recently it was found an analytic expression for the transformation of the normal parity orbitals to the pseudo space \cite{32}. Applying this transformation to the spherical Nilsson Hamiltonian it can be shown explicitly that the strength of the pseudo spin orbit interaction is almost zero for heavy nuclei and the orbitals $j = \tilde{l} \pm 1/2$ are nearly degenerate. For configurations of identical particles occupying a single $j$ orbital of abnormal parity a convenient characterization of states is made by means of the seniority coupling scheme.

The many particle states of $n_\alpha$ nucleons in a given shell $\eta_\alpha$, $\alpha = \nu$ or $\pi$, can be defined by the totally antisymmetric irreducible representations $\{1^{n_\alpha}_\nu\}$ and $\{1^{n_\alpha}_\pi\}$ of unitary groups. The
dimensions of the normal (N) parity space is $\Omega^N_\alpha = (\tilde{\eta}_\alpha + 1)(\tilde{\eta}_\alpha + 2)$ and that of the unique (A) space is $\Omega^A_\alpha = 2\eta_\alpha + 4$, with the constraint $n_\alpha = n^A_\alpha + n^N_\alpha$. Proton and neutron states are coupled to angular momentum $J^N$ and $J^A$ in both the normal and unique parity sectors, respectively.

The wave function of the many-particle state with angular momentum $J$ and projection $M$ is expressed as a direct product of the normal and unique parity ones, as:

$$|JM> = \sum_{J^N,J^A} [|J^N> \otimes |J^A>]|^J_M$$

We are interested in the low energy states of $^{150}Nd$ and $^{150}Sm$ which have $J = 0, 2, 3, 4, 6$.

For even-even heavy nuclei it has been shown that if the residual neutron-proton interaction is of the quadrupole type, regardless of the interaction in the proton and neutron spaces, the most important normal parity configurations are those with highest spatial symmetry $\{\tilde{f}_\alpha\} = \{2^{n^N_\pi}/2\}$ [27]. This statement is valid for yrast states below the backbending region. This implies that $\tilde{S}_\pi = \tilde{S}_\nu = 0$, i.e. only pseudo spin zero configurations are considered.

Additionally in the abnormal parity space only seniority zero configurations are taken into account. This simplification implies that $J^A_\pi = J^A_\nu = 0$. This is a very strong assumption quite useful in order to simplify the calculations. Its effects upon the present calculation are discussed below.

The double beta decay, when described in the pseudo SU(3) scheme, is strongly dependent on the occupation numbers for protons and neutrons in the normal and abnormal parity states $n^N_\pi, n^N_\nu, n^A_\pi, n^A_\nu$ [28, 29]. These numbers are determined filling the Nilsson levels from below, as discussed in [28, 29]. In particular the $\beta\beta_{2\nu}$ decay is allowed only if it fulfils the following relationships

$$n^A_{\pi,f} = n^A_{\pi,i} + 2 , \quad n^A_{\nu,f} = n^A_{\nu,i} , \quad n^N_{\pi,f} = n^N_{\pi,i} , \quad n^N_{\nu,f} = n^N_{\nu,i} - 2 .$$

For $^{150}Nd$ it was assumed a deformation $\beta \approx 0.28$[33]. We have obtained the occupation numbers

$$n^A_\pi = 4 , \quad n^N_\pi = 6 , \quad n^A_\nu = 2 , \quad n^N_\nu = 6 .$$

$$\quad$$

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For $^{150}\text{Sm}$ it is found that $\beta \approx 0.19$ but we were forced to select a higher deformation to satisfy relations (2). According to [34] this higher deformation is more appropriate for $^{152}\text{Sm}$ and is related with some departure from a rotational behavior in the ground state band of $^{150}\text{Sm}$. The selected occupation numbers for $^{150}\text{Sm}$ are

$$n^A_\pi = 6, \quad n^N_\pi = 6, \quad n^A_\nu = 2, \quad n^N_\nu = 4.$$

In order to analyze the spectra and transitions amplitudes of $^{150}\text{Sm}$ we have selected the standard version of the pseudo SU(3) Hamiltonian[27]. It is constructed by a spherical central potential, a quadrupole-quadrupole interaction and a residual force. The latter allows the fine tuning of low lying spectral features like $K$ band splitting and the effective moments of inertia. The Hamiltonian looks

$$H = \sum_\alpha H_\alpha - \frac{1}{2} \chi Q^a \cdot Q^a \zeta_1 K^2 + \zeta_2 L^2.$$  \(5\)

The spherical Nilsson Hamiltonian which describe the single-particle motion of neutrons ($\alpha = \nu$) or protons ($\alpha = \pi$) is:

$$H_\alpha = \sum_s \hbar \omega \left\{ \eta_{\alpha s} \frac{3}{2} - 2k_\alpha \vec{L}_{\alpha s} \cdot \vec{S}_{\alpha s} - k_\alpha \mu_\alpha L^2_{\alpha s} \right\} - V_\alpha = \sum_{s,\alpha} \epsilon_{s\alpha} a^\dagger_{s\alpha} a_{s\alpha} \quad (6)$$

where $\eta = \tilde{\eta} + 1$ denotes the harmonic oscillator number operator and $\hbar \omega$ determines the size of the shell. A constant term $V_\nu$ ($V_\pi$) is included which represents the depth of the neutron (proton) potential well. In (6) the second quantization representation of $H_\alpha$ is given, $\epsilon_{s\alpha}$ being the single-particle energies.

The quadrupole operator $Q^a = \sum_s \{q_{\pi s} + q_{\nu s}\}$ acts only within a shell and do not mix different shells. The residual interaction, $K^2$, is a linear combination of $L^2$, $X_3$ and $X_4$, defined as

$$L^2 = \sum_i L_i^2$$
$$X_3 = \sum_{i,j} L_i Q^a_{ij} L_j$$
$$X_4 = \sum_{i,j,k} L_i Q^a_{ij} Q^a_{jk} L_k \quad (7)$$

They are rotational invariant and scalar operators built by generators of the algebra of SU(3)[27, 29] and $L_i$ and $Q^a_{ij}$ are cartesian forms of the total angular momentum and the quadrupole
operators, respectively. The $K$ is interpreted to be the third component of the total angular momentum on an intrinsic body-fixed symmetry axis of the system, which is given by

$$K^2 = (\lambda_1 \lambda_2 L^2 + \lambda_3 X_3 + X_4)/(2\lambda_3^2 + \lambda_1 \lambda_2)$$

(8)

with the parameters $\lambda_i$ denoting the eigenvalues of the mass quadrupole operator, which are related to the $SU(3)$ labels $(\lambda, \mu)$ through the expressions

$$\lambda_1 = \frac{1}{3}(\mu - \lambda), \quad \lambda_2 = -\frac{1}{3}(\lambda + 2\mu + 3), \quad \lambda_3 = \frac{1}{3}(2\lambda + \mu + 3).$$

(9)

Although the quantum number $K$ used to define the orthonormalised basis is not the same as the Elliot $\kappa$ the states studied in the present work satisfy quite accurately the relationships:

$$K^2|K = 1> = 0, \quad K^2|K = 2 > \approx 4|K = 2 > .$$

(10)

It would be possible to add the the hamiltonian (5) terms which distinguish different irreps. For the sake of simplicity we keep this simplest version.

With the occupation numbers determined in Eq (3) and (4) and the hamiltonian (5) the wave function of the deformed ground state of $^{150}Nd$ can be written\[28, 29\]

$$|^{150}Nd, 0^+ > \equiv |0^+ > = \{ (h_{11/2})^4, (i_{13/2})^2 \}_{\pi}, \quad J^A = M^A = 0; \quad (i_{13/2})^2, \quad J^A = M^A = 0 > \_A$$

$$\{1^6\}_{\pi}(12, 0)_{\pi}; \quad \{1^6\}_\nu(2^3)_{\nu}(18, 0)_{\nu}; \quad 1(30, 0)K = 1J = M = 0 > N ,$$

(11)

and the deformed low energy states of $^{150}Sm$ are described by the wave functions

$$|^{150}Sm, J^+_\sigma > \equiv |J^+_\sigma > = \{ (h_{11/2})^4, \quad J^A = M^A = 0; \quad (i_{13/2})^2, \quad J^A = M^A = 0 > \_A$$

$$\{1^6\}_{\pi}(2^3)_{\pi}(12, 0)_{\pi}; \quad \{1^4\}_\nu(2^2)_{\nu}(12, 2)_{\nu}; \quad 1(\lambda, \mu)_{\sigma}K J M > N ,$$

(12)

where $J^+_\sigma$ denotes a state with angular momentum $J$, positive parity and associated with the $SU(3)$ irrep $(\lambda, \mu)_{\sigma}$. In this approach we are assuming that the first $0^+, 2^+, 4^+$ states of $^{150}Sm$ are the low energy sector of a rotational band described by the normal $(\lambda, \mu)_{g.s.} = (24, 2)$ strong coupled pseudo $SU(3)$ irrep, the second $0^+, 2^+$ states belong to a second rotational band with $(\lambda, \mu)_{1} = (20, 4)$, and the third $0^+$ state is the head of another rotational band described by
the pseudo SU(3) irrep \( (\lambda, \mu)_2 = (22, 0) \). We will discuss also a gamma band associated with \( (\lambda, \mu)_{g.s.} \) with \( K=2 \).

The \( \beta\beta \) decay to excited \( 2^+ \) states

The inverse half life of the two neutrino mode of the \( \beta\beta \)-decay can be cast in the form

\[
\left[ \frac{1}{2} \tau_{2\nu}^{1/2} (0^+ \rightarrow J^+_{\sigma}) \right]^{-1} = G_{2\nu}(J^+_{\sigma}) \mid M_{2\nu}(J^+_{\sigma}) \mid^2 .
\]

where \( G_{2\nu}(J^+_{\sigma}) \) are kinematical factors. They depend on \( E_{J,\sigma} = \frac{1}{2} (Q_{\beta\beta} - E(J,\sigma)) + m_e c^2 \) which is the half of total energy released. The nuclear matrix element is

\[
M_{2\nu}(J^+_{\sigma}) \approx M_{2\nu}^{GT}(J^+_{\sigma}) = \frac{1}{\sqrt{J+1}} \sum \langle J^+_{\sigma} \mid \Gamma \mid \frac{1}{2}\rangle \langle \frac{1}{2}\mid \Gamma \mid 0^+_i \rangle \sum \langle 1^+_N \mid \Gamma \mid 1^+_N \rangle \langle 1^+_N \mid \Gamma \mid 0^+_i \rangle .
\]

with the Gamow-Teller operator \( \Gamma \) expressed as

\[
\Gamma_m = \sum \sigma_{msl_s} \equiv \sum \pi_{\nu} \sigma(\pi, \nu)[a_{\eta_{l_s}}^i a_{\eta_{l_s}}^j \otimes \bar{a}_{\eta_{l_s}}^{i} f_{1/2}^{i} f_{1/2}^{j} m].
\]

The energy denominator is \( \mu_N = E_{J,\sigma} + E_N - E_i \) and it contains the intermediate \( E_N \) and initial \( E_i \) energies. The kets \( |1^+_N \rangle \) denote intermediate states.

The mathematical expressions needed to evaluate the nuclear matrix elements of the \( g.s. \rightarrow g.s. \beta\beta \) decay in the pseudo SU(3) model were developed recently. The same formulae describe the decay to the first excited \( 0^+ \) state by replacing the values of the strong coupled irrep \( (\lambda, \mu)_{g.s.} \) of Eq. (12) with those corresponding to excited bands.

We will concentrate first on the derivation of the matrix element \( M_{2\nu}(2^+_\sigma) \) to the \( 2^+ \) excited states. The formulae for this decay resembles that of the decay to the \( 0^+ \) states but the energy denominator is up to the third power. Being in general this energy of the order of 10 MeV this power implies a factor 100 of suppression for this matrix element. The previous equation is rearranged as

\[
M_{2\nu}^{GT}(2^+_\sigma) = \sqrt{5} \sum_{\mu \mu'} \langle 1^+_1 m_1 | \Gamma_{\mu'} | 1^+_N m_1 \rangle \langle 0^+_1 m_1 | \Gamma_{\mu} | 0^+_i \rangle .
\]
Using
\[ \mu_N^{-3} = \frac{1}{2 \partial E_{J\sigma}^3} \mu_N^{-1} \]  \hspace{1cm} (17)
and the summation method described in [29, 33] it is possible to rewrite the second sum as
\[ \sum_{N,m_1} \frac{1}{2 \partial E_{J\sigma}^3} \{ \mu_N^{-1} \langle 2^+ m | \Gamma_{\mu'} | 1^+_N m_1 \rangle \langle 1^+_N m_1 | \Gamma_{\mu} | 0^+_i \rangle \} = \frac{1}{2 \partial E_{J\sigma}^3} \langle 2^+_\sigma | m, \mu' \rangle \sum_{\lambda=1}^{\infty} \frac{(-1)^\lambda}{E_{J\sigma}^{\lambda}} \Gamma_{\mu} [H, ..., [H, \Gamma_{\mu}]] \ldots \}^{(\lambda-times)} | 0^+_i \rangle \]  \hspace{1cm} (18)

The two body terms of the Hamiltonian (15) commutes with the Gamow-Teller operator (13), thus the above multiple commutators are easy to evaluate. We obtain [29]
\[ [H, ..., [H, \Gamma_{\mu}] \ldots \}^{(\lambda-times)} = [H_\pi + H_\nu, ..., [H_\pi + H_\nu, \Gamma_{\mu}] \ldots \}^{(\lambda-times)} = \sum_{\pi, \nu} \sigma(\pi, \nu) [a^\dagger_{\pi} \otimes a^\dagger_{\nu}]^{1\mu} \{ \epsilon_{\pi} - \epsilon_{\nu} \}^\lambda \]  \hspace{1cm} (19)
where \( \pi \equiv (\eta_\pi, l_\pi, j_\pi) \) and \( \nu \equiv (\eta_\nu, l_\nu, j_\nu) \). Returning with this expression to the original formula, Eq. (13), resumming the infinite series and recoupling the Gamow-Teller operators it is found
\[ M^{GT}_{2\nu}(2^+_\sigma) = \sqrt{5} \frac{1}{2 \partial E_{J\sigma}^3} \left\{ \sum_{\pi, \nu, \pi', \nu'} \frac{\sigma(\pi, \nu)\sigma(\pi', \nu')}{{E_{J\sigma} + \epsilon_{\pi} - \epsilon_{\nu}}} \langle 2^+_\sigma | m, [a^\dagger_{\pi} \otimes a^\dagger_{\nu}]^{1\mu} [a^\dagger_{\pi'} \otimes a^\dagger_{\nu'}]^{1\mu} | 0^+_i \rangle \right\} = \frac{1}{2 \partial E_{J\sigma}^3} \sum_{\pi, \nu, \pi', \nu'} \frac{\sigma(\pi, \nu)\sigma(\pi', \nu')}{{E_{J\sigma} + \epsilon_{\pi} - \epsilon_{\nu}}} \langle 2^+_\sigma | m, [a^\dagger_{\pi} \otimes a^\dagger_{\nu}]^{1\mu} [a^\dagger_{\pi'} \otimes a^\dagger_{\nu'}]^{1\mu} | 0^+_i \rangle \]  \hspace{1cm} (20)

As it was shown in [29] the expression for the nuclear matrix element of the \( \beta \beta_{2\nu} \) decay to a \( 0^+ \) state is similar to (20), with a different power in the denominator. Equations (20) and (4.10) of [29] can be expressed in a compact form as:
\[ M^{GT}_{2\nu}(J^+_\sigma) = \sqrt{3} \sum_{\pi, \nu, \pi', \nu'} \frac{\sigma(\pi, \nu)\sigma(\pi', \nu')}{{E_{J\sigma} + \epsilon_{\pi} - \epsilon_{\nu}}} \langle J^+_\sigma | m, [a^\dagger_{\pi} \otimes a^\dagger_{\nu}]^{1\mu} [a^\dagger_{\pi'} \otimes a^\dagger_{\nu'}]^{1\mu} | J^+_\sigma \rangle \equiv \sum_{\pi, \nu, \pi', \nu'} \frac{1}{{E_{J\sigma} + \epsilon_{\pi} - \epsilon_{\nu}}} \langle J^+_\sigma | T^{Jm}(\pi \nu, \pi' \nu') | 0^+_i \rangle \]  \hspace{1cm} (21)
For practical purposes the tensor \( T^{Jm}(\pi \nu, \pi' \nu') \) was implicitly defined in the above equation. The \( \sqrt{3} \) for the \( J = 0 \) case comes from the relation between the scalar product of two vectors and their coupling to angular momentum zero.
The matrix elements $M_{2\nu}$

We want to evaluate the nuclear matrix element \((21)\) for the $\beta\beta_{2\nu}$ decay of the ground state of $^{150}Nd$, Eq. (11), to the ground and excited states of $^{150}Sm$ which are described by the wave functions of Eq. (12). Each Gamow-Teller operator \((15)\) annihilates a proton and creates a neutron in the same oscillator shell and with the same orbital angular momentum. In the case of the $\beta\beta_{2\nu}$ of $^{150}Nd$ it means that the operator annihilates two neutrons in the pseudo shell $\eta_{\nu} = 5$ and creates two protons in the abnormal orbit $h_{11/2}$. As a consequence the only orbitals which in the model space can be connected through the $\beta\beta_{2\nu}$ decay are those satisfying $\eta_{\pi} = \eta_{\nu} \equiv \eta$, that implies $l_{\pi} = l_{\nu} = \eta$, $j_{\pi} = j_{\nu} = \eta - \frac{1}{2}$ and $j_{\pi} = \eta + \frac{1}{2}$. These are the selection rules described by relations \((2)\) concerning the change in occupation numbers. Under this restrictions only one term in the sum over configurations $\pi\nu, \pi'\nu'$ survives and thus the nuclear matrix element $M_{2\nu}$ \((21)\) can be written as

\[
M_{2\nu}^{GT}(J_{\pi}^+) = \frac{1}{\mathcal{E}_{J_{\pi}^+}}\langle J_{\pi}^+ | T^{Jm} (\pi\nu, \pi\nu) | 0^+_i \rangle, \tag{22}
\]

where the energy denominator is determined demanding that the Isobaric Analog State in the intermediate odd-odd nucleus is an eigenstate of the Hamiltonian \((3)\). Its excitation energy is equal to the difference in Coulomb energies $\Delta C$. Their expressions are\[23\]

\[
\mathcal{E}_{J_{\sigma}} = E_{J_{\sigma}} + \epsilon(\eta_{\pi}, l_{\pi}, j_{\pi} = j_{\nu} + 1) - \epsilon(\eta_{\nu}, l_{\nu}, j_{\nu}) = E_{J_{\sigma}} - \hbar \omega k_{\pi} 2j_{\pi} + \Delta C.
\]

\[
\Delta C = \frac{0.70}{A^{1/3}} [2Z + 1 - 0.76((Z + 1)^{4/3} - Z^{4/3})] \text{MeV}. \tag{23}
\]

As it was discussed in \[29\] in the context of the g.s. $\rightarrow$ g.s. $\beta\beta_{2\nu}$ decay Eq. \((22)\) has no free parameters, being the denominator \((23)\) a well defined quantity. The reduction to only one term comes as a consequence of the restricted Hilbert proton and neutron spaces of the model. The initial and final ground states are strongly correlated with a very rich structure in terms of their shell model components.

Since the Hilbert space has been divided in their normal and unique parity components we need to rearrange the creation an annihilation operators in the same way, \textit{i.e.}
\[
\left[ (a^\dagger_\pi \otimes \tilde{a}_\nu)^1 \otimes (a^\dagger_\pi \otimes \tilde{a}_\nu)^1 \right]^{JM} = \sum_{j_\pi j_\nu} \chi \left\{ \begin{array}{ccc} j_\pi & j_\nu & 1 \\ j_\pi & j_\nu & 1 \\ J_\pi & J_\nu & J \end{array} \right\} \left[ (a^\dagger_\pi \otimes a^\dagger_\pi)^{J_\pi} \otimes (\tilde{a}_\nu \otimes \tilde{a}_\nu)^{J_\nu} \right]^{JM} \quad (24)
\]

The \( \chi\{...\} \) is the unitary (Jahn-Hope) 9-j recoupling coefficient \[36\]. Introducing this expression in (22) together with the explicit form of the wave functions (11) and (12) we obtain

\[
M_{2\nu}(J^+_{\sigma}) = \sqrt{J + 3} \sigma(\pi, \nu)^2 E_{J\sigma}^{-1(J+1)} \sum_{j_\pi j_\nu} \chi \left\{ \begin{array}{ccc} j_\pi & j_\nu & 1 \\ j_\pi & j_\nu & 1 \\ J_\pi & J_\nu & J \end{array} \right\} 
\left[ < (h_{11/2})^6, J^A = M^A = 0 | (a^\dagger_\pi \otimes a^\dagger_\pi)^{J_\pi} | (h_{11/2})^4, J^A = M^A = 0 > \otimes \right.
\left. < (12, 0)_\pi; (12, 2)_\nu; 1(\lambda, \mu)_\sigma K = 1JM | (\tilde{a}_\nu \otimes \tilde{a}_\nu)^{J_\nu} \right. 
\left. | (12, 0)_\pi; (18, 0)_\nu; 1(30, 0)K = 1J = M = 0 > \right]^{JM} \quad (25)
\]

The matrix element (25) vanishes unless a pair of protons coupled to total angular momentum zero is created and two neutrons of the normal parity space coupled to pseudo orbital angular momentum \( \tilde{L} = J \) and pseudo spin equal to zero are annihilated. The above sum is thus restricted to \( J_\pi = 0, J_\nu = J \).

The operators in the normal space must be recoupled from the \( jj\)– to the \( LS\)–coupling scheme. The result is

\[
[\tilde{a}_\nu \otimes \tilde{a}_\nu]^{JM} = \sum_{L S} \chi \left\{ \begin{array}{ccc} l_\nu & 1 \\ l_\nu & 1 \\ \tilde{L} & \tilde{S} & J \end{array} \right\} \left[ (\tilde{a}(0\eta)\tilde{l}_{1/2} \otimes \tilde{a}(0\eta)\tilde{l}_{1/2})^{\tilde{L}\tilde{S}} \right]^{JM} \quad (26)
\]

The low energy levels are assumed to have pseudo spin \( \tilde{S} = 0 \), a fact that again simplifies the evaluation of the above sum by imposing \( \tilde{L} = J \).

Using that \( j_\pi = j_\nu + 1, l_\pi = l_\nu = \eta \) the reduced matrix elements \( \sigma(\pi \nu) \) read

\[
\sigma(\pi \nu)^2 = \frac{8\eta(\eta + 1)}{3(2\eta + 1)} \quad (27)
\]

In the seniority zero approximation the two particle transfer matrix element of the unique sector of Eq. (22) is evaluated by using the quasispin formalism and gives \[28, 29\]
\[ j_{\pi}^{n_{\pi}^A+2}, \; J_{\pi}^A = M_{n_{\pi}^A}^A = 0 \mid [a_{\pi}^\dagger \otimes a_{\pi}^\dagger]_0 \mid j_{\pi}^{n_{\pi}^A}, \; J_{\pi}^A = M_{n_{\pi}^A}^A = 0 > = \left[ \frac{(n_{\pi}^A + 2)(\eta + 1 - n_{\pi}^A/2)}{\eta + 1} \right]^{1/2} \] (28)

The evaluation of the matrix elements in the normal space of Eq. (25) is performed by using SU(3) Racah calculus to decouple the proton and neutron normal irreps, and expanding the annihilation operators of Eq. (26) in their SU(3) tensorial components. The final result is:

\[ M_{2\nu}^{GT} (J_\sigma^+) = a(J) b(n_{\pi}^A) \mathcal{E}_{J_\sigma}^{(J+1)} \sum_{(\lambda_0\mu_0)K_0} < (0\eta)1\ell, (0\eta)1\ell\mid (\lambda_0\mu_0)K_0 J > 1 \sum_{\rho} < (30,0)1\ell, (\lambda_0\mu_0)K_0 J\mid (\lambda\mu)\sigma 1J >_{\rho} \]

\[ \sum_{\rho'} < (12,2) \mid \mid \mid [\tilde{a}(0\eta), \frac{1}{2}\tilde{a}(0\eta), \frac{1}{2}](\lambda_0\mu_0) \mid \mid \mid (18,0) >_{\rho'} \] (29)

In the above formula \(< ..,..\mid \mid \mid .. > \) denotes the SU(3) Clebsch-Gordan coefficients\[37\], the symbol \([..]\) represents a \(9 - \lambda \mu\) recoupling coefficient\[38\], and \(< .. \mid \mid \mid .. \mid \mid \mid .. > \) is the triple reduced matrix elements\[39\]. The energy denominator was defined in (23), and

\[ a(0) = \frac{4\eta}{(2\eta + 1)\sqrt{2\eta - 1}}, \quad a(2) = \frac{2}{2\eta + 1} \left[ \frac{5\eta(\eta - 1)(2\eta - 3)}{3(2\eta + 1)} \right]^{1/2}, \quad b(n_{\pi}^A) = [(n_{\pi}^A + 2)(\eta + 1 - n_{\pi}^A/2)]^{1/2} \] (30)

The rotational spectrum of \(^{150}Sm\)

The four parameters of the pseudo SU(3) hamiltonian \([\text{Ref.} 27]\) were fitted to reproduce the first \(2^+\) states in \(^{150}Sm\) as it was done in Ref.\[27\]. Their values are

\[ \chi = 3.47eV, \quad \zeta_1 = 215keV, \quad \zeta_2 = 50.4keV \] (31)

The right hand side of Fig. 1 exhibits nine of the lowest energy states which have been observed in \(^{150}Sm\), grouped in rotational bands. Angular momentum and parity are given for each level. The left hand side of Fig. 1 shows the calculated spectrum together with the
associated irreps. The gamma band is identified with $K=2$. The general trend is well reproduced but the experimental g.s. band does not show the rotational structure which is exhibited by the calculated one. This departure from an exact rotational behaviour was mentioned in Section 2 and it can be associated with the relatively small deformation reported for $^{150}Sm$. In this mass region the deformation suddenly jumps for $^{152}Sm$ to a value very similar to the deformation of $^{150}Nd$. The gamma band is present in the g.s. irrep because both $\lambda$ and $\mu$ are different from zero and is well fitted.

The excited $0^+$ states are the head of other rotational bands. The predicted energy gap between them is 125 eV while the experimental one is 454 eV. These numbers suggest that we have not a clear identification of these excited states. This fact has relevance in the study of the $\beta\beta_2\nu$ decay to these states.

The BE(2) transition intensities were evaluated using the effective quadrupole operator $^{[27]}$

$$Q_0 = e_{\pi}^{eff}Q_{\pi} + e_{\nu}^{eff}Q_{\nu}, \quad e_{\pi}^{eff} = e + e_{pol}, \quad e_{\nu}^{eff} = e_{pol}$$

with $e_{pol} = 0.93e$. The seniority zero condition imposed to the nucleons in abnormal parity orbitals inhibits them to participate in collective excitations. This restriction forces a slightly large value for the polarisation charge. A similar effect was found in a BE(2) study of rare earth and actinide nuclei$^{[27]}$.

The BE(2) intensities of the transition from the first and second $2^+$ to the ground state, and from the first $4^+$ to the $2^+$ state are shown in Table 1 and compared with their experimental values, in Weiskoff units (W.u.). The agreement is good except for the case of the transition from the second excited $2^+$ state to the ground state which fails in a factor four.

**The $\beta\beta_2\nu$ decay of $^{150}Nd$**

In this section we study the two neutrino mode of the double beta decay $\beta\beta_2\nu$ of $^{150}Nd$ into the ground state, the first excited $2^+$ and the first and second excited $0^+$ states of $^{150}Sm$.

In Table 2 the matrix elements, energy denominators, phase space integrals and predicted half-lives for the $\beta\beta_2\nu$ decay of $^{150}Nd$ to the ground state, the first $2^+$ and the first and second
excited $0^+$ states of $^{150}Sm$ are presented. The matrix elements (26) are given in units of $(m_e c^2)^{-(J+1)}$. Phase space integrals for the decay to $0^+$ states were evaluated following the prescriptions given in [30] with $g_A/g_V = 1.0$ and the kinematical factor for the decay to the $2^+$ state was taken from [3] renormalized by the above mentioned value of the axial vector coupling constant. It must be mentioned that these phase space factors differ in about 10% with those used in [28, 29] where a different renormalization procedure was used.

As it was mentioned in [29] the predicted half life for the $\beta\beta_{2\nu}$ to the ground state of $^{150}Sm$ is in reasonable agreement with the experimental data, which vary between 9 and $17 \times 10^{18}$ years [3, 41, 42].

The $\beta\beta_{2\nu}$ decay to the first excited $0^+$ state is forbidden. In this model this is imposed by the fulfilment of an exact selection rule. It can be understood by realizing that the pair of annihilation operators $\tilde{a}_{(0i)\frac{1}{2}^+}$, when expanded in their SU(3) components, have the couplings $(0,4) \otimes (0,4) = (0,8), (2,4)$ containing a $L = 0$ state. But acting over the $^{150}Nd$ g.s. irrep $(30,0)$ they cannot couple to the irrep $(20,4)$ which we associated with the first excited $0^+$ state. In other words the transition between members of these particular irreps are forbidden.

The decay to the second excited $0^+$ state is allowed but strongly cancelled. The reduction of the matrix element by a factor ten, in comparison with that associated with the decay to the g.s., is partly related with the fact that though the coupling $(30,0) \otimes (0,8) = (22,0)$ is allowed the coupling with $(2,4)$ is forbidden. The predicted half-life is four orders of magnitude larger than that of the decay to the g.s.

The $\beta\beta_{2\nu}$ decay to the $2^+$ state is inhibited by the $\mu_3^2$ dependence of the matrix element as it is discussed in Section 3. The matrix element of the $\beta\beta_{2\nu}$ decay to the first excited $2^+$ state

\[ M_{2\nu}(2^+_{g.s.}) \]

is three orders of magnitude lesser than the matrix element of the decay to the g.s. $M_{2\nu}(0^+_{g.s.})$.

The present results contradicts those previously published [4, 11] were it was speculated that the $\beta\beta_{2\nu}$ decay of $^{150}Nd$ to the first excited $0^+$ state of $^{150}Sm$ could have a similar intensity of that to the g.s. We find that in the present formalism this decay is forbidden. If we select different occupation numbers for both $^{150}Nd$ and $^{150}Sm$, taken the deformation of the latter nucleus instead of that of the former, we find very similar results for the decay to the g.s. and the first $2^+$ state, but the matrix elements of the decay to the first and second $0^+$ states
becomes interchanged with essentially the same values. Considering the difference in the phase space integrals we predict a half live of the order $10^{21}$ years for the decay to the first excited $0^+$ state and the decay to the second excited one becomes forbidden.

The above discussed reduction of the matrix element of the $\beta\beta_{2\nu}$ decay to the excited $0^+$ state as compared with the decay to the g.s. is not a general result of the pseudo SU(3) scheme. A recent analysis of the case of $^{100}\text{Mo}$[13] shows that both matrix elements are very similar and that they are in agreement with the experimental information. In conclusion, the appearance of selection rules which can produce the suppression of the matrix elements governing a $\beta\beta_{2\nu}$ transition is a consequence of the details of the irreps involved.

The pseudo SU(3) model uses a quite restrictive Hilbert space. The model could be improved by incorporating mixing between different irreps, via pairing by example[14]. Also other active shells can be taken into account in the symplectic extension[15]. In both cases the selection rules that impose such strong restrictions to the $\beta\beta_{2\nu}$ decays of some nuclei can be superseded.

However if the main part of the wave function is well represented by the pseudo SU(3) model those forbidden decays will have, in the better case, matrix elements that will be no greater than 20% of the allowed ones, resulting in at least one order of magnitude cancellation in the half-life. In any case these results should be taken into account in the design of future experiments.

**Conclusions**

In the present paper we have studied the $\beta\beta_{2\nu}$ decay mode of $^{150}\text{Nd}$ to the ground and excited states of $^{150}\text{Sm}$. The transitions have been analyzed in the context of the pseudo SU(3) model. The experimental spectrum of the g.s. rotational band of $^{150}\text{Sm}$ was reproduced as well as the measured half-life of the $\beta\beta_{2\nu}$ decay to the g.s. but the excited rotational bands were not so well reproduced. The $\beta\beta_{2\nu}$ decay to the first excited $0^+$ state was found forbidden in the model and the decay to the second excited $0^+$ state has a half-life four orders of magnitude greater that that to the g.s.. The decay to the $2^+$ state is strongly inhibited due to the energy dependence of the matrix elements $M_{2\nu}(2^+)$, two powers greater than that of the matrix element $M_{2\nu}(0^+)$.  

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It is expected that improving the model would remove the exact selection rules which forbid some decays. In any case, if the pseudo SU(3) wave functions are a good representation of the low-lying energy states of $^{150}Nd$ and $^{150}Sm$, they will remain inhibited.
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**Table captions**

**Table 1** The BE(2) intensities, in Weiskoff units (W.u.), of the transition from the first and second $2^+$ to the ground state, and from the first $4^+$ to the $2^+$ state of $^{150}Sm$ are shown. The calculated and experimental values are exhibited in the second and third column, respectively.

**Table 2** The dimensionless $\beta\beta_{2\nu}$ matrix elements, energy denominators, phase space integrals and predicted half-lives for the decay of $^{150}Nd$ to the ground state, the first $2^+$ and the first and second excited $0^+$ states of $^{150}Sm$.

**Figure captions**

**Figure 1** Spectrum of the low-lying states of $^{150}Sm$. The levels are grouped in rotational bands and they are labeled by angular momentum and parity. The right hand side contains the experimentally determined levels. In the left hand side the calculated spectrum is exhibited with the associated irreps at the bottom.
### Table 1

| transition | theory | experiment |
|------------|--------|------------|
| $2^+_1 \rightarrow 0^+_{g.s.}$ | 55.7 | 55.8 |
| $2^+_2 \rightarrow 0^+_{g.s.}$ | 0.48 | 2.0 |
| $4^+_1 \rightarrow 2^+_1$ | 78.5 | 112.0 |

### Table 2

| $0^+ \rightarrow 0^+(g.s.)$ | $M^{GT}_{2\nu}(J^+_{\sigma})$ | $E_{J\sigma}[MeV]$ | $G_{2\nu}(J^+_{\sigma})[yr^{-1}]$ | $\tau^{1/2}_{2\nu}(0^+ \rightarrow J^+_{\sigma})[yr]$ |
|-----------------------------|-----------------|-----------------|-----------------|-----------------|
| $0^+ \rightarrow 0^+(1)$    | 0.0549          | 12.20           | $4.94 \times 10^{-17}$ | $6.73 \times 10^{18}$ |
| $0^+ \rightarrow 0^+(2)$    | 0.00499         | 11.58           | $9.33 \times 10^{-19}$ | $4.31 \times 10^{22}$ |
| $0^+ \rightarrow 2^+$       | $5.38 \times 10^{-5}$ | 12.04           | $4.78 \times 10^{-17}$ | $7.21 \times 10^{24}$ |