A Study on Course Timetable Scheduling and Exam Timetable Scheduling using Graph Coloring Approach

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Abstract: One of the most common academic scheduling problems which can be perceived in any educational system is the timetable generation. The presence of vast number of students and offered courses makes it difficult to schedule exams and class timetable in a limited epoch of time. An appropriate schedule can be designed by utilizing different resources like subjects, teachers, students and classroom in a way to evade conflicts by fulfilling special types of constraints. In this work we are using graph vertex coloring and edge coloring approach for generating exam time table and class time table schedule.

Keywords: Graph Coloring, Course Timetable Scheduling, Hard Constraints, Soft Constraints, Course matrix.

I. INTRODUCTION

In 1840, A.F mobius came up with the idea of complete graph and bipartite graph. In 1852, Thomas Guthere found the famous four-color problem. The first results about graph coloring deal exclusively with planar graphs in the form of the coloring of maps. Even though the four-color problem was invented it was solved only after a century by Kenneth Appel and Wolfgang Haken. In 1890, Heawood proved the five-color theorem, saying that every planar map can be colored with no more than five colors. In 1912, George David Birkhoff to study coloring problems in algebraic graph theory introduced the chromatic polynomial. Graph coloring has many real-time applications including map coloring, scheduling problem, parallel computation, network design, Sudoku, register allocation, bipartite graph detection. Exam time table is required in every educational institution. In every semester or year, the universities and colleges are required to generate exam time table for conducting the internal and the final semester exams. The presence of a large number of students and large number of offered courses sometimes makes it difficult to schedule the exam without having any conflict. Moreover, the exam time table could not be reused because the requirements and some restrictions of the problem keep changing. If done manually, generating an exam time table is very time consuming and requires a considerable amount of workforce which sometimes may lead to inefficiency. So it demands an automatic generation of time table with a limited number of user input parameters. The exam time table should be generated in such a way that one subject is scheduled only once. So this can be considered as a good constraint or restriction which must be satisfied while designing an automatic time table generation system. There is no restriction regarding the faculty members. That means any faculty member could be assigned to conduct the examination of any subject. We should also consider those students who have back papers (repeater exams) and thus, need to reappear in some of the papers. The exam time table should avoid such conflicts, like no two or three exams for the identical student should be scheduled at the same period of time. The exam schedule should not leave a big gap between exams for the students. Therefore, there is a much need of an effective and accurate timetable to the performance of any educational institute. Graph coloring is a method of assigning colors to certain elements of a graph subjected to certain constraints. Thus, optimal solutions to such problem may be found by determining minimal colorings for the corresponding graph.

A. Basic Concept Of Graph And Graph Coloring

1) Definition 1: A graph \( G \) is a non-empty finite set \( V \) of vertices and a non-empty finite set \( E \) of edges. A graph is a pair of sets \((V, E)\), where \( V \) is the set of vertices and \( E \) is the set of edges, connecting the pair of vertices.

![Fig 1. An example of a Graph.](image-url)
V = \{a, b, c, d\}
E = \{ab, ac, bd, de\}.

2) **Definition 2:** The minimum number of colors in the vertex coloring of a graph is called the chromatic number of G. A graph that can be assigned a proper k-coloring is called k-colorable, and it is k-chromatic if its chromatic number is exactly k.

3) **Definition 3:** Vertex coloring is nothing but, no two adjacent vertices have the same color. Two nodes are said to be adjacent if they are connected by an edge. Edge coloring is nothing but, no two adjacent edges have same color. Two edges are said to be adjacent if both of them share a vertex in common.

4) **Definition 4:** The set of vertices V can be partitioned into two disjoint sets V1 and V2. every edge of the graph connects a vertex in V1 to one in V2.

**B. Constraints**

To design and solving a scheduling problems, the essential conditions that are to be considered are the constraints. Based on these a schedule can be accepted or get rejected. Depending on the degree of strictness, constraints are broadly classified into two types hard and soft constraints. The class timetable scheduling and exam timetable scheduling should be generated by keeping in mind the number of teachers, number of subjects, number of students and number of faculty members available to create the timetable scheduling.

**C. Hard Constraints**

At any one period, Teacher cannot be in two classes at the same time.
At any one period, Students cannot attend two classes at the same time.
No more than two subjects or papers of the student can scheduled at the same time.
Subjects should be scheduled accordingly to the availability of class rooms and faculty members.

**D. Soft Constraints**

Teacher taking only two consecutive subjects to be scheduled.
More than two consecutive theory classes cannot be assigned to same teacher for teaching same subject.
There should not be any exam on holidays.
There should be a leave between two exams of particular stream.

**E. Case Studies**

In India, undergraduate and postgraduate universities offer a variety of subjects’ combinations to its students. To conduct such courses teachers of respective subjects are needed to be scheduled according to their availabilities in minimum number of time slot without any conflict. In the following subsections, we have presented two such typical cases of scheduling problems and their conflict free solution timetables. Some students have some arrear papers of some previous semesters. so those papers also will be
include while offering the subjects. While designing the time table availability of the number of room availability also needed. In this section, we have considered a scenario of an individual department of mathematics, where different subjects are offered by a university in an odd semester of a particular year. The number of papers or subjects offered in different semester number of faculties, number of students and number of available rooms are taken into consideration for generating a conflict and constraint free solution to final semester exam timetable.

1) Example 1
   a) Teacher- Subject Problem: For a given ‘X’ number of teachers, ‘Y’ number of subjects and available ‘P’ number of periods, a timetable should be prepared. The number of classes for each subject needed by a particular teacher is given table1. this problem is mentioned earlier in some papers but only a partial solution was provided in both.
   2) Input Data: Number of teachers -3
      Number of subjects -5

   Table 1 teacher – subject matrix
   | Periods P | Y_1 | Y_2 | Y_3 | Y_4 | Y_5 |
   |-----------|-----|-----|-----|-----|-----|
   | X_1       |    -| 1   | 1   |    -| 1   |
   | X_2       | 1   |    -| 1   | 1   |    -|
   | X_3       | 1   |    -| 1   |    -|    -|

3) Solution: In this problem we using bipartite graph which acts as the conflict graph. the two disjoint independent sets are number of teachers and number of subjects.

![Bipartite Graph](Fig 5. Bipartite graph)

Another way of solving the above problem is by converting the edge coloring problem into a vertex coloring problem. The bipartite graph is converted in line graph. The eight edges present in the bipartite graph in Fig.1 acts as the vertices of L(G) in Fig.2.

![Line Graph](Fig 6.Line graph L(G))
II. RESULT

The graph coloring of the line graph $L(G)$ in Fig.2 plotted in the solution table 3

Table 2 graph coloring solution table of Fig.2

| RED | BLUE | GREEN |
|-----|------|-------|
| V1  | V2   | V3    |
| V4  | V6   | V5    |
| V8  |      | V7    |

Now each of the colors in table 3 represents the periods in table 4. The vertex in $L(G)$ that corresponds to a particular edge in the bipartite graph $G$ represents the teacher–subject combination scheduled under that period. We obtain the final complete schedule is shown in table 4.

Table 3 Final Teachers – Subject Allotment Table

| Period 1 | Period 2 | Period 3 |
|----------|----------|----------|
| $x_1$-$y_2$ | $x_1$-$y_3$ | $x_1$-$y_5$ |
| $x_2$-$y_1$ | $x_2$-$y_4$ | $x_2$-$y_3$ |
| $x_3$-$y_1$ |         | $x_3$-$y_1$ |

A. Proof of Satisfaction

1) No common subjects in any column indicate that at any particular period, a subject is taught by only one teacher.

2) No duplicate data in any cell indicates that at any particular period, a teacher can teach only one subject.

Here, in the above example there are maximum 3 allocations in any column i.e. maximum three classes can run parallelly.

a) Example 2: We have entered the following information, with core subjects, elective subjects and arrear papers.

i) First semester subjects: Real analysis (RA), Ordinary differential equations (ODE), Graph theory (GT), Automata theory (AT), Applied statistics (AS).

ii) Third semester subjects: Big data analysis (BA), Topology (TO), Statistical inference (SI), Fluid dynamics (FD), RA, ODE, AS.

a. Note: RA, ODE and AS are the first semester subjects. However, we are adding these subjects in the third semester because we are assuming that some third semester students have got arrear paper in these three papers and thus need to reappear.

iii) Fifth semester subjects: Resource management techniques (RMT), Mathematical methods (MM), Numerical analysis (NM).

a. Note: ODE, FD are first and third semester subjects. However, we are assuming that some fifth semester students have arrear papers in these subjects and thus need to reappear.

iv) Enter the number of students enrolled in all subjects

v) Number of rooms available is 8 (e.g., A35, B20, C30, D35, E40, F40, G20, H60).

vi) Number of faculties are available is 6.

vii) Starting exam date and time slots 10 a.m. to 1 p.m. and 1.30 a.m. to 4.30 p.m.

B. Generating The Course Matrix

In this course matrix we are adding the list of subjects offered in all semester including the arrear papers of the previous semester. The course matrix is shown in table7 and the graph shown in Fig. 3

C. Coloring Assignment

In this problem we are assign color to the subjects based on the course matrix. Two courses are said to be collide other if they are adjacent. No two adjacent courses are same color. In table 5, we observe the first row we see that RA and ODE could not be assigned in same color since they are adjacent. However, RA and RMT could be assigned the same color since they are not adjacent. Now we are taking the one dimensional array n, n=12, it’s shown in table 4. every index of array represents the subjects. Now we assign the initial array is 0, {1=Red, 2=Violet, 3=Blue, 4=Green, 5=Pink, 6=Yellow, 7=Orange}. Since, we are start with Real Analysis (RA); we assign color 1 to it. It is shown in table 5.
Table 4. Color the assignment of Real Analysis (RA)

| Subjects | RA | ODE | GT | AT | AS | BA | TO | SI | FD | RMT | MM | NM |
|----------|----|-----|----|----|----|----|----|----|----|-----|----|----|
| Color    | 1  | 0   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0  | 0  |

Table 5. Course matrix

| Subject | RA | ODE | GT | AT | AS | BA | TO | SI | FD | RMT | MM | NM |
|---------|----|-----|----|----|----|----|----|----|----|-----|----|----|
| RA      | -  | 1   | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0   | 0  | 0  |
| ODE     | 1  | -   | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1   | 1  | 1  |
| GT      | 1  | 1   | -  | 1  | 1  | 0  | 0  | 0  | 0  | 0   | 0  | 0  |
| AT      | 1  | 1   | 1  | -  | 1  | 0  | 0  | 0  | 0  | 0   | 0  | 0  |
| AS      | 1  | 1   | 1  | 1  | -  | 1  | 1  | 1  | 1  | 0   | 0  | 0  |
| BA      | 1  | 1   | 0  | 0  | 1  | -  | 1  | 1  | 1  | 0   | 0  | 0  |
| TO      | 1  | 1   | 0  | 0  | 1  | 1  | -  | 1  | 1  | 0   | 0  | 0  |
| SI      | 1  | 1   | 0  | 0  | 1  | 1  | 1  | -  | 1  | 0   | 0  | 0  |
| FD      | 1  | 1   | 0  | 0  | 1  | 1  | 1  | 1  | -  | 1   | 1  | 1  |
| RMT     | 0  | 1   | 0  | 0  | 0  | 0  | 0  | 0  | 1   | -   | 1  | 1  |
| MM      | 0  | 1   | 0  | 0  | 0  | 0  | 0  | 0  | 1   | 1   | -  | 1  |
| NM      | 0  | 1   | 0  | 0  | 0  | 0  | 0  | 0  | 1   | 1   | 1  | -  |

Fig 7. Course matrix graph

Now, we check the second row of the course matrix. Subject RA is adjacent to ODE. Thus we need to assign a different color to ODE and we assign color 2. It is shown in table .6.

Table .6 Color assignment of ODE

| Subjects | RA | ODE | GT | AT | AS | BA | TO | SI | FD | RMT | MM | NM |
|----------|----|-----|----|----|----|----|----|----|----|-----|----|----|
| Color    | 1  | 2   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0  | 0  |

Now, we check the third row of the course matrix where GT is adjacent to both RA and ODE. So, we assign a new color to GT. It is shown in table .7.

Table .7 Color assignment of GT

| Subjects | RA | ODE | GT | AT | AS | BA | TO | SI | FD | RMT | MM | NM |
|----------|----|-----|----|----|----|----|----|----|----|-----|----|----|
| Color    | 1  | 2   | 3  | 0  | 0  | 0  | 0  | 0  | 0  | 0   | 0  | 0  |
Table 8: Color Assignment of AT

| Subjects | RA | ODE | GT | AT | AS | BA | TO | SI | FD | RMT | MM | NM |
|----------|----|-----|----|----|----|----|----|----|----|-----|----|----|
| Color    | 1  | 2   | 3  | 4  | 0  | 0  | 0  | 0  | 0  | 0   | 0  | 0  |

Now, we check the fifth row of the course matrix, where AS is adjacent to RA, ODE, GT and AT. So, we assign a new color to AT. It is shown in table 9.

Table 9: Color Assignment of AS

| Subjects | RA | ODE | GT | AT | AS | BA | TO | SI | FD | RMT | MM | NM |
|----------|----|-----|----|----|----|----|----|----|----|-----|----|----|
| Color    | 1  | 2   | 3  | 4  | 5  | 0  | 0  | 0  | 0  | 0   | 0  | 0  |

Now, we check the sixth row of the course matrix. Here, BA is adjacent to both RA and ODE; however, it is not adjacent to GT. We could assign BA the same color which we have previously assigned to GT. It is shown in table 10.

Table 10: Color Assignment of BA

| Subjects | RA | ODE | GT | AT | AS | BA | TO | SI | FD | RMT | MM | NM |
|----------|----|-----|----|----|----|----|----|----|----|-----|----|----|
| Color    | 1  | 2   | 3  | 4  | 5  | 3  | 0  | 0  | 0  | 0   | 0  | 0  |

Now, we check the seventh row of the course matrix, where TO is adjacent to both RA and ODE, however, it is not adjacent to GT. Here, we need to note that BA and TO are in the same semester and thus they are adjacent, and BA has already been assigned the color 3 (which is also the color of OB). So, we could not assign the TO the color 3. So, we check the next subject (in the next column) which is AT. TO and AT are not adjacent and also the color assigned to AT is not yet assigned to any color assigned to any other subjects of the third semester. Thus, we could assign TO the same color that we have previously assigned to AT. It is shown in table 11.

Table 11: Color Assignment of TO

| Subjects | RA | ODE | GT | AT | AS | BA | TO | SI | FD | RMT | MM | NM |
|----------|----|-----|----|----|----|----|----|----|----|-----|----|----|
| Color    | 1  | 2   | 3  | 4  | 5  | 3  | 4  | 0  | 0  | 0   | 0  | 0  |

Similarly, we check the eighth row of the course matrix, where SI is adjacent to RA and ODE, but it is not adjacent to GT and AT. However, color 3 and 4 are already assigned to BA and TO, which are in the same semester as that of SI. And SI is adjacent to AS as well. So, we need to assign a new color for SI. It is shown in table 12.

Table 12: Color Assignment of SI

| Subjects | RA | ODE | GT | AT | AS | BA | TO | SI | FD | RMT | MM | NM |
|----------|----|-----|----|----|----|----|----|----|----|-----|----|----|
| Color    | 1  | 2   | 3  | 4  | 5  | 3  | 4  | 6  | 0  | 0   | 0  | 0  |

Similarly, for algorithm in row ninth of the course matrix, we need to assign a new color. It is shown in table 13.

Table 13: Color Assignment of FD

| Subjects | RA | ODE | GT | AT | AS | BA | TO | SI | FD | RMT | MM | NM |
|----------|----|-----|----|----|----|----|----|----|----|-----|----|----|
| Color    | 1  | 2   | 3  | 4  | 5  | 3  | 4  | 6  | 7  | 0   | 0  | 0  |

Similarly, for RMT in row tenth of the course matrix, Here, RMT is not adjacent to RA. So, we could assign the color 1. It is shown in table 14.

Table 14: Color Assignment of RMT

| Subjects | RA | ODE | GT | AT | AS | BA | TO | SI | FD | RMT | MM | NM |
|----------|----|-----|----|----|----|----|----|----|----|-----|----|----|
| Color    | 1  | 2   | 3  | 4  | 5  | 3  | 4  | 6  | 7  | 1   | 0  | 0  |
Now, the eleventh row of the course matrix, MM is not adjacent to RA, but color 1 has already been assigned to RMT, RMT and MM is offered in the same semester. Thus, they are adjacent and we could not assign the same color to two adjacent subjects. So, we check the next subject which is ODE. ODE is adjacent to MM. Then we check the next subject which is GT. GT is not adjacent to MM, and the color 3 is not yet assigned to any of the subjects of the fifth semester. So, assign MM the same color which we have previously assigned to GT. It is shown in table .15.

| Subjects | RA | ODE | GT  | AT  | AS  | BA  | TO  | SI  | FD  | RMT | MM | NM |
|----------|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|----|
| Color    | 1  | 2   | 3   | 4   | 5   | 3   | 4   | 6   | 7   | 1   | 3  | 0  |

Finally, after observing the twelve row of the course matrix, NM is being assigned the color which was previously assigned to AT. It is shown in table .16.

| Subjects | RA | ODE | GT  | AT  | AS  | BA  | TO  | SI  | FD  | RMT | MM | NM |
|----------|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|----|
| Color    | 1  | 2   | 3   | 4   | 5   | 3   | 4   | 6   | 7   | 1   | 3  | 4  |

For this case study, the number of subjects may be less, but in practical scenario sorting the subjects based on color values will enhance the computational performance. So, by considering the same example, after sorting the list of subjects are arranged in the following form which given in table .17.

| Subjects | RA | ODE | GT  | AT  | AS  | BA  | TO  | SI  | FD  | RMT | MM | NM |
|----------|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|----|
| Color    | 1  | 2   | 3   | 4   | 5   | 3   | 4   | 6   | 7   | 1   | 3  | 4  |

D. Generating the Exam Time Table Schedule and Allocate Rooms

We generate the exam time table based on the color value assigned to the subjects.

E. Allocation of Rooms

We calculate the total number of students registered in every subject which are scheduled in the same time slot. Finally, we print the exam time table schedule along with the rooms allocated. Continuing with our previous example, let us consider we have the following amount of data. The total number of room is 3.

1) A105 (here 45 is the room capacity)
2) B106
3) C108

Total room capacity is 150. the number of students register in every subject is given in table 14. The number of faculties available = 5.

| Subjects | No of students |
|----------|----------------|
| RA       | 43             |
| ODE      | 45             |
| GT       | 40             |
| AT       | 40             |
| AS       | 43             |
| BA       | 40             |
| TO       | 40             |
| SI       | 40             |
| FD       | 43             |
| RMT      | 40             |
| MM       | 40             |
| NM       | 40             |
F. Output

1) Sat 01 Dec 2018: Morning 10.00am to 01.00pm
   Subject: RA Subject: RMT   Room: A105, B106
   Afternoon 01.30pm to 04.30pm
   Subject: ODE   Room: A105

2) Mon 03 Dec 2018: Morning 10.00am to 01.00pm
   Subject: GT Subject: BA Subject: MM   Room: A105, B106, C108
   Afternoon 01.30pm to 04.30pm
   Subject: AT Subject: TO Subject: NM   Room: A105, B106, and C108

3) Tues 04 Dec 2018: Morning 10.00am to 01.00pm
   Subject: AS   Room: A105
   Afternoon 01.30pm to 04.30pm
   Subject: SI   Room: A105

4) Wed 05 Dec 2018: Morning 10.00am to 01.00pm
   Subject: FD   Room: A105

The final colored graph generated for the above data is shown in figure 8.4.

Fig 8. Final colored Graph

III. CONCLUSION

In this paper we are studied a teacher – subject scheduling problem and exam timetable scheduling problem where two alternative graph coloring methods were applied and a complete solution provided. We are also generated the exam time table scheduling and time table schedule with the help of graph coloring approach by taking the list of subjects number of faculties available to conducting the exams, number of room available for conducting the exams. Also it has successfully satisfied sum of the important hard and soft constraints.

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