In this paper, we shall investigate fractional partial differential equations with fractional moving boundary condition to study the dissociation of natural gas hydrate under heat injection. The moving boundary separates the hydrate reservoir into the dissociated zone and the hydrate one. By using the self-similar transformation and Wright function, we obtain the explicit solutions for two zones. We present simulations with steam and hot water injection and show the dissociation temperature in graphical mode from injection temperature to reservoir temperature with respect to the time, distance, and fractional order. Our analysis of fractional model turns out to be a successful generalization of the classical one; i.e., it can well describe the dissociation of natural gas hydrate and is theoretically consistent with the existing integer hydrate dissociation model. When the factional order tends to 1, the "limit solution" becomes the classical one.

1. Introduction

Amounts of natural gas hydrates have been found in permafrost and ocean bottom sediments. Natural gas hydrates are an unconventional source of energy and belong to the clean energy. Because of the characteristics of large storage scale and high energy density, natural gas hydrate become one of the potential new energy sources. There are many researchers focusing their study on the exploitation and mathematical model of natural gas hydrate.

Thermal models are investigated for the dissociation of natural gas hydrate. Holder et al. [1] established a zero-dimensional hydrate model. Ignoring energy loss, heat, and mass transfer, the authors revealed the feasibility of hydrate dissociation by thermal method. Selim and Sloan [2] viewed the hydrate decomposition process as a moving boundary ablation process and established one-dimensional hydrate decomposition models for temperature change under heat injection method. The classical parabolic differential equations based on the mass, momentum, and energy balances were used to describe the dissociation process. More precisely, let \( T \) be temperature and \( x = \chi(t) \) be a moving boundary, and the moving boundary condition was written as

\[
k_{HII} \frac{\partial T_{II}}{\partial x} - k_{HI} \frac{\partial T_I}{\partial x} = \phi \rho H Q_{Hid} \frac{dx}{dt}, x = \chi(t), t > 0, \tag{1}
\]

where some physical quantities \( k_{HII}, k_{HI}, \phi, \rho H \), and \( Q_{Hid} \) are of given values. An analytical solution was derived, and results were used to evaluate gas production. Later, the authors in [3–8] generalized the analytical model to radial symmetric decomposition model, well structure, etc. For hydrate decomposition model solved numerically, we refer to references [9, 10] and their related references.

Fractional calculus has been proposed to scientists and engineers for decades. And very soon it turned out to be a very powerful tool both in theoretical analysis and wide applications, as one can see its influence in the study of solute transport, proteins diffusion, viscoelasticity, and other problems [11–13]. For example, in [11], fractional
differential equations were established for fractal mobile/immobile solute transport. The solutions were gained by performing an integral transform. Results captured the anomalous behavior of tracer plumes in heterogeneous aquifers, which showed that a fractional model can describe the power law breakthrough curve more accurately than the classical model. Roscani et al. [14] investigated two kinds of fractional differential equations with Stefan-like boundary condition. One model was the Caputo one, written as

$$
\begin{align*}
\frac{d^\alpha}{dt^\alpha}u_1(x, t) &= \lambda_1^2 \frac{\partial^2 u_1(x, t)}{\partial x^2}, & 0 < x < \chi(t), 0 < t < T, \\
\frac{d^\alpha}{dt^\alpha}u_2(x, t) &= \lambda_2^2 \frac{\partial^2 u_2(x, t)}{\partial x^2}, & x > \chi(t), 0 < t < T,
\end{align*}
$$

(2)

where $\alpha \in (0, 1)$, $\frac{d^\alpha}{dt^\alpha}u(x, t)$ is the $\alpha$-order Caputo fractional derivative of $u(x, t)$ with respect to $t$. The author presented explicit solutions of fractional two-phase Stefan-like problems in terms of Wright functions.

Heat equations have many similarities to diffusion ones; therefore, fractional models are naturally introduced to study heat transfer problem [15]. Apart from the aforementioned fields, we found that very few study of fractional order models have been carried out on hydrate dissociation.

Motivated by the literatures listed above, we will discuss a time fractional heat conduction model in this paper to describe the hydrate decomposition process for heat injection case. Because the moving boundary is usually linked to melting (change from solid to liquid) or freezing (change from liquid to solid). We also consider the fractional moving boundary. Results from our model are used to calculate temperature distributions from hydrate reservoirs by steam or hot water thermal stimulation.

There are also many literatures discussing the depressurization model of hydrate decomposition [16–20]. As an attempt to focus on the discussion of fractional methods, in this paper, we shall not divert our attention away from the methods to cumbersome calculation. That is, we shall not consider the depressurization and the mass transfer model.

2. Hydrate Decomposition Model

Consider the hydrates that are uniformly distributed in a semi-infinite porous medium $(0 < x < \infty)$. It has an initial reservoir temperature $T_0$. At time $t = 0$, hot water is injected into the wellbore and reservoir temperature near the wellbore is immediately raised to injection temperature $T_{in}$ $(T_{in} > T_0)$. Then, the balance is disrupted and the hydrates begin to dissociate. The hydrate decomposition can be considered a moving boundary value problem. It has a decomposition front, which divides the reservoir into two parts: the decomposition zone (zone I, $0 < x < \chi(t)$) and the hydrate zone (zone II, $\chi(t) < x < \infty$), which are shown in Figure 1.

Before establishing the mathematical model, we give some assumptions here.

Figure 1: Hydrate decomposition diagram.

(1) The hydrate reservoir is a one-dimensional infinite region. It is saturated
(2) Only water and ideal state natural gas are dissociated from the hydrate, without carrying ice
(3) The thermal conductivity and diffusivity are constant. The thermal convection is ignored
(4) There is no energy loss for water dissociated diffusion, heat transfer from the down-hole to the dissociation and from hydrate zones into the top or bottom caps
(5) The gravity effect, the viscous dissipation, and reversible work of compression are all neglected

With the restrictions listed above, the energy conservation can be written as

$$
\rho u_t + \text{div } q = 0.
$$

(3)

where $u$ is the total internal energy and $q$ is heat conduction flux per unit volume. The relationship between total internal energy and temperature is $du = C_dT dt$, where $C$ is the specific heat capacity. Noticing $q = -kV_T$ and we have the one-dimensional heat conduction equation as follows:

$$
\frac{\partial T(x, t)}{\partial t} = \alpha \frac{\partial^2 T(x, t)}{\partial x^2},
$$

(4)

where $\alpha = k/\rho C$. Furthermore, we consider a memory effect of power law decay on temperature variation with respect to time. Let $f(t) = t^\alpha I'(1 - \alpha), \alpha \in (0, 1)$ be the ‘memory function.’ Then, the convolution of temperature derivative with respect to time and power function turns out to the Caputo fractional derivative, that is,

$$
\frac{\partial T(x, t)}{\partial t} * f(t) = \frac{\partial}{\partial t} D^\alpha T(x, t),
$$

(5)

where $*$ denotes the convolution. Here, we claim that the $\alpha$-order Caputo fractional derivative of $T(x, t)$ on $t$ for $\alpha \in (0, 1)$ is defined by

$$
\frac{\partial}{\partial t} D^\alpha T(x, t) = \frac{1}{\Gamma(1 - \alpha)} \int_0^t (t - \tau) \frac{\partial T(x, \tau)}{\partial \tau} d\tau.
$$

(6)
For more details, we refer the author to see the reference [21].

So the Caputo fractional temperature distribution equations are established for the dissociation zone and hydrate zone, respectively.

\[ \zeta \partial_t^\alpha T_I(x, t) = \alpha \frac{\partial^2 T_I(x, t)}{\partial x^2}, \quad 0 < x < \chi(t), \quad 0 < t < T, \quad (7) \]

\[ \zeta \partial_t^\alpha T_H(x, t) = \alpha \frac{\partial^2 T_H(x, t)}{\partial x^2}, \quad x > \chi(t), \quad 0 < t < T. \quad (8) \]

The boundary and initial conditions associated with the above equations are

\[ T_H(x, 0) = T_i, \]

\[ T_I(0, t) = T_0, \quad T_H(+\infty, t) = T_i, \quad (9) \]

\[ T_H(\chi(t), t) = T_H(\chi(t), t) = T_d, \quad (10) \]

as well as the fractional-order moving boundary condition:

\[ \phi \rho_H \chi \frac{\partial}{\partial x} W(x, t) = k_I \frac{\partial}{\partial x} T_I(x, t) \left|_{(\chi(t), t)} \right. - k_H \frac{\partial}{\partial x} T_H(x, t) \left|_{(\chi(t), t)} \right. . \quad (12) \]

The unknown functions of our model are the temperatures \( T_I(x, t), T_H(x, t) \) and moving boundary \( \chi(t) \).

### 3. A Similarity Solution

For every \( x \in \mathbb{R} \), the Wright function is defined as

\[ W(x, \rho, \beta) = \sum_{k=0}^{\infty} \frac{x^k}{k! \Gamma(\rho k + \beta)}, \quad (13) \]

where the parameters satisfy \( \rho > -1 \) and \( \beta \in \mathbb{R} \). Mainardi function is a special Wright function, written as

\[ M_\rho(x) = W(-x, -\rho, 1 - \rho) = \sum_{k=0}^{\infty} \frac{(-x)^k}{k! \Gamma(-\rho k + 1 - \rho)}, \quad 0 < \rho < 1. \quad (14) \]

By direct calculations, we can obtain the following results. The proof is omitted here.

**Proposition 1.** Let \( \beta \geq 0, 0 < \rho < 1 \). The Wright function has the following properties:

1. \( W(-x, -\rho, \beta) \) is positive and strictly decreasing on \( x \in [0, +\infty) \):

\[ \rho x W(-x, -\rho, \beta - \rho) = W(-x, -\rho, \beta - 1) + (1 - \beta) W(-x, -\rho, \beta), \quad x \geq 0, \quad (15) \]

2. \( \lim_{x \to +\infty} W(-x, -\rho, \beta) = 0 \)

\[ \lim_{x \to +\infty} (1 - W(-2x, -\alpha/2, 1)) = \text{erf} (x), \quad \text{where erf} (x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-u^2} du \]

\[ \frac{\partial}{\partial x} W(x, \rho, \beta) = W(x, \rho, \rho + \beta), x \in \mathbb{R}. \quad (17) \]

Next, we will solve our fractional model (7)–(12) and present explicit solutions in terms of the Wright functions. Adopting the self-similar transformation:

\[ x = 2\eta \sqrt{\alpha_H t^{u/2}}, \quad (18) \]

and we assume that form of the moving boundary is \( \chi(t) = 2\xi \sqrt{\alpha_H t^{u/2}} \), where \( \xi > 0 \) is undetermined. In fact, we can solve equation (12) to get \( \xi \) later. The dissociation front of hydrate can be determined by \( \xi \), which is related to the physical properties of hydrate reservoir.

Suppose that the self-similar solution to problem (7)–(12) can be written by

\[ \begin{align*}
T_I(\eta) &= A + B \left[ 1 - W\left(-2\lambda \eta, -\frac{\alpha}{2}, 1 \right) \right], \\
T_H(\eta) &= C + D \left[ 1 - W\left(-2\eta, -\frac{\alpha}{2}, 1 \right) \right],
\end{align*} \quad (19) \]

where \( A, B, C, \) and \( D \) are some unknown constants, which just makes the solution a little bit more elegant.

Let \( \lambda = \sqrt{\alpha_H/\alpha_I} \) and it is related to the diffusivity of decomposition zone and hydrate zone. From equations (4) and (5), noticing that when \( x = 0 \) or \( x = \chi(t) \), \( \eta = 0 \) or \( \eta = \xi \), respectively, it has

\[ A + B \left[ 1 - W\left(0, -\frac{\alpha}{2}, 1 \right) \right] = T_0, \quad (20) \]

which arrives at \( A = T_0, B = -(T_0 - T_d)/(1 - W(-2\lambda \xi, -\alpha/2, 1)) \). Similar, from

\[ T_H(x, 0) = C + D \left[ 1 - W\left(-\infty, -\frac{\alpha}{2}, 1 \right) \right] = C + D = T_i, \quad (21) \]

\[ T_H(\chi(t), t) = C + D \left[ 1 - W\left(-2\xi, -\frac{\alpha}{2}, 1 \right) \right] = T_d. \quad (22) \]

We can obtain that \( C = T_i + ((T_d - T_i)/W(-2\xi, -\alpha/2, 1) \), \( D = -((T_d - T_i)/W(-2\xi, -\alpha/2, 1) \).
temperature distribution model becomes the classical one [2].

Two cases are simulated in this section to show the temperature and temperature are investigated for both cases under given fractional order. Initial heat injection temperature and the decomposition front are also discussed. The influence of fractional order is also shown in the last for the hot-water-injecting case.

Choose \( \alpha_f = 2.89 \times 10^6 \mu m^2/s \), \( \alpha_H = 0.697 \times 10^6 \mu m^2/s \), \( \rho_H = 913 \text{ kg/m}^3 \), \( \phi = 0.3 \), \( k_I = 5.57 \text{ W/(m}\cdot\text{K}) \), \( k_{II} = 2.73 \text{ W/(m}\cdot\text{K}) \), and \( Q_{Hd} = 446120 + 132.638 \times 0.5(T_i + T_f) \). The injection temperature, boundary temperature, and reservoir temperature are 563.5 K, 282.5 K, and 275 K for steam case, respectively. And for hot water case, they are 450 K, 285 K, and 280 K, respectively.

Let \( \alpha = 0.9 \). When hot steam and hot water are injected, by Newton iterative computation, the values of \( \xi \) are 1.97 and 1.675, respectively. Furthermore, when decomposition distance and time are given, the temperature can be calculated from (23) and are shown in Figure 2. Figure 2(a) shows the temperature with distance for steam case and the right for hot water case.

Easily, the dotted line, in both figures of Figure 2, divides the temperature distribution into the above and the below area. The above one corresponds to the temperature of the hydrate decomposition area, and the below is the hydrate one. The temperature decreases rapidly in the decomposition zone and very slowly in the hydrate zone when the axial distance is increasing. It does not change and keeps the initial temperature all the time in most hydrate area. This is because of the thermal diffusion coefficient is very small in the hydrate zone and the heat here cannot be conducted out. Comparing the left and right Figure 1, it can be obtained that the higher the initial heat injection temperature is, the slower the decomposition zone temperature decreases, and the longer the axial distance of the decomposition leading edge reaches, the larger the decomposition zone is. Obviously, increasing the initial heat injection temperature can make hydrates decompose more quickly.

**Figure 2**: The temperature of steam and hot water varies with time.

So the solutions of problems (7)–(12) are given by

\[
\begin{align*}
T_I(\eta) &= T_0 - \frac{T_0 - T_d}{1 - W(-2\lambda\xi, -\alpha/2, 1)} \left[ 1 - W(-2\lambda\eta, -\alpha/2, 1) \right], \\
T_{II}(\eta) &= T_i + \frac{T_d - T_i}{W(-2\xi, -\alpha/2, 1)} W\left(-2\eta, -\frac{\alpha}{2}, 1\right).
\end{align*}
\]

The constant \( \xi > 0 \) can be calculated by the transcendental equation, which can be obtained by equation (23). The transcendental equation is expressed by

\[
\begin{align*}
\frac{k_{HI}(T_0 - T_d)\Gamma(1 - (\alpha/2))}{\sqrt{\alpha_H}} F_1(2\lambda\xi) - \frac{k_{HI}(T_d - T_i)\Gamma(1 - (\alpha/2))}{\alpha_H} F_2(2\xi) &= 2\Gamma\left(1 + \frac{\alpha}{2}\right) \phi \rho_H Q_{Hd} \xi, \\
F_1(\xi) &= \frac{M_{\alpha I}(\xi)}{1 - W(-\xi, -\alpha/2, 1)} , \\
F_2(\xi) &= \frac{M_{\alpha II}(\xi)}{W(-\xi, -\alpha/2, 1)}.
\end{align*}
\]

When \( \alpha = 1 \) in equations (7)–(12), the Caputo fractional temperature distribution model becomes the classical one [2].

**4. Result and Discussion**

Two cases are simulated in this section to show the temperature distribution when hot steam and hot water are injected, respectively. The relationship of time, decomposition distance, and temperature are investigated for both cases under given fractional order. Initial heat injection temperature and the decomposition front are also discussed. The influence of fractional order is also shown in the last for the hot-water-injecting case.

Choose \( \alpha_f = 2.89 \times 10^6 \mu m^2/s \), \( \alpha_H = 0.697 \times 10^6 \mu m^2/s \), \( \rho_H = 913 \text{ kg/m}^3 \), \( \phi = 0.3 \), \( k_I = 5.57 \text{ W/(m}\cdot\text{K}) \), \( k_{II} = 2.73 \text{ W/(m}\cdot\text{K}) \), and \( Q_{Hd} = 446120 + 132.638 \times 0.5(T_i + T_f) \). The injection temperature, boundary temperature, and reservoir temperature are 563.5 K, 282.5 K, and 275 K for steam case, respectively. And for hot water case, they are 450 K, 285 K, and 280 K, respectively.

Let \( \alpha = 0.9 \). When hot steam and hot water are injected, by Newton iterative computation, the values of \( \xi \) are 1.97 and 1.675, respectively. Furthermore, when decomposition distance and time are given, the temperature can be calculated from (23) and are shown in Figure 2. Figure 2(a) shows the temperature with distance for steam case and the right for hot water case.
Figure 3 shows that the temperature distribution at $x = 0.2$, $x = 0.5$, and $x = 0.8$ meter distances. The small figure in the box is the local enlarged one at the beginning.

With the increase of time, the reservoir temperature in some fixed location slowly rises to the hydrate decomposition temperature and then rises rapidly to tend to the initial heat injection temperature. Meanwhile, the closer the location is to the wellbore, the faster the temperature rises and the shorter the time needed to approach to $T_0$. On the contrary, the further away from the wellbore, the longer the time required for the temperature rise to approach the initial heat injection temperature.

When the fractional order is increasing, the value of $\xi$ is increasing too. In fact, for the hot-water-injecting case, when $\alpha = 0.8, 0.9, \text{and } 1$, the values of $\xi$ are 1.905, 1.97, and 2.01, respectively. The temperature distribution for dissociation, calculated by equations (23) and (24), are presented in Figures 4 and 5.

Figure 4 shows the change of temperature when $t = 10^5$ s and $\alpha = 0.8, 0.9, \text{and } 1$, respectively. When the fractional order $\alpha$ increases, the temperature decreases more slowly, and the dissociation interface $\chi(t)$ is farther from the wellbore. Figure 5 shows the change of temperature when $x = 0.8$ m and $\alpha = 0.8, 0.9, \text{and } 1$, respectively. And we also know from Figure 5 that when $\alpha$ increases, the temperature will reach the initial heat injection temperature in a shorter time.

Figure 5 shows the change of temperature of steam case when $x = 0.8$ and $\alpha = 0.8, 0.9, \text{and } 1$, respectively. Obviously, with the increase of time, the temperature of hydrate decomposition increases slowly and then increases rapidly close to the initial heat injection temperature. The higher the time fractional order, the faster the temperature increases.
The constant $\xi$ affects the move speed of the dissociation front. Figure 6 shows that, with the increase of initial heat injection temperature, $\xi$ will increase and the decomposition rate will also accelerate. Meanwhile, the value of $\xi$ is also increasing when the fractional order $\alpha$ is increasing. When $\alpha$ tends to 1, the value of $\xi$ is the same as that of the classical hydrate decomposition.

When $\alpha = 0.9$ and 1, the $\xi$ value of the corresponding hot steam injection is 1.97 and 2.01, respectively. Hydrate saturation can be calculated from moving boundary, and both are shown in Figure 7 for both fractional and integral hydrate models. They decrease gradually with the increase of time, while the fractional one has a tailing phenomenon.

5. Conclusions

By using time fractional derivative, a new mathematical model is investigated for the temperature variation in decomposed and undecomposed (hydrate) zones of the semi-infinite hydrate reservoir. The study shows that the fractional-order model can well describe the change rule of temperature on hydrate decomposition. When the fractional
order and the time are given, the temperature drops sharply to the dissociation temperature and then gradually to the reservoir temperature as the distance from the decomposition front increases. And when the decomposition distance is fixed, the temperature slowly rises to the decomposition temperature, and then rapidly to the initial heat injection temperature with the passage of time. With the increase of the fractional order at different times, the temperature gradient becomes smaller, the temperature drops more slowly, and the decomposition distance can be reached further. With the increase of the fractional order at different distances, the temperature will reach the initial heat injection temperature in a shorter time.

The variation of temperature with time during the classical hydrate dissociation model was obtained in [2, 4]. The fractional-order model is consistent with the classical dissociation model in terms of temperature variation and is an extension of the classical hydrate model. Furthermore, there is a need to extend the present analysis to both heat and mass transfer, which will be directly applicable to evaluate gas production by thermal stimulation from hydrate reservoirs.

### Abbreviations

- \( T_1 \): I zone temperature
- \( T_{II} \): II zone temperature
- \( \phi \): Porosity
- \( \alpha_1 \): I zone thermal diffusivity
- \( \alpha_{II} \): II zone thermal diffusivity
- \( \xi \): Hydrate dissociation rate
- \( k_1 \): I zone thermal conductivity
- \( k_{II} \): II zone thermal conductivity
- \( T_d \): Boundary temperature
- \( \rho_{II} \): Hydrate density
- \( Q_{hid} \): Hydrate heat of dissociation
- \( \lambda \): \( \sqrt{\alpha_{II}/\alpha_1} \)

### Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

### Acknowledgments

The authors are grateful to the National Natural Science Foundation of China (12001502, 51991365), Fundamental Research Funds for the Central Universities (2652019015, 2652020014), Key Program of Marine Economy Development (Six Marine Industries) Special Foundation of Department of Natural Resources of Guangdong Province [2021]56, and Guangdong Major Project of Basic and Applied Basic Research (2020B0301030003).

### References

[1] G. D. Holder, P. F. Angert, V. T. John, and S. Yen, “A thermodynamic evaluation of thermal recovery of gas from hydrates in the earth (includes associated papers 11863 and 11924),” *Journal of Petroleum Technology*, vol. 34, no. 5, pp. 1127–1132, 1982.

[2] M. S. Selim and E. D. Sloan, “Hydrate dissociation in sediment,” *SPE Reservoir Engineering*, vol. 5, no. 2, pp. 245–251, 1990.

[3] M. C. Li, S. S. Fan, Y. L. Su et al., “The Stefan moving boundary models for the heat-dissociation hydrate with a density difference,” *Energy*, vol. 160, pp. 1124–1132, 2018.

[4] L. G. Tang, G. Li, Z. P. Feng, and S. S. Fan, “Mathematic modeling on thermal recovery of natural gas hydrate,” *Natural Gas Industry*, vol. 26, no. 10, pp. 105–107, 2006.

[5] M. Roostae and Y. Leonenko, “Analytical investigation of gas production from methane hydrates and the associated heat and mass transfer upon thermal stimulation employing a coaxial wellbore,” *Energy Conversion and Management*, vol. 209, p. 112616, 2020.

[6] V. A. Kamath and S. P. Godbole, “Evaluation of hot-brine stimulation technique for gas production from natural gas hydrates,” *Journal of Petroleum Technology*, vol. 39, no. 11, pp. 1379–1388, 1987.

[7] M. C. Li, S. S. Fan, Y. L. Su, J. Ezekiel, M. Lu, and L. Zhang, “Mathematical models of the heat-water dissociation of natural gas hydrates considering a moving Stefan boundary,” *Energy*, vol. 90, pp. 202–207, 2015.

[8] M. Roostae and Y. Leonenko, “Analytical modeling of methane hydrate dissociation under thermal stimulation,” *Journal of Petroleum Science and Engineering*, vol. 184, p. 106505, 2020.

[9] I. Durgut and M. Parlaktuna, “A numerical method for the gas production process in gas hydrate reservoirs,” in *2nd International Conference on Natural Gas Hydrates, Proceedings*, pp. 549–556, 1996.

[10] Z. Su, Y. C. Cao, D. F. Chen, and N. Y. Wu, “A numerical model for vertical single well production from free gas zone beneath the hydrate layer: case history of the Blake Ridge,” *Natural Gas Industry*, vol. 29, no. 8, pp. 123–128, 2009.

[11] R. Schmer, D. A. Benson, M. M. Meerschaert, and B. Baeumer, “Fractal mobile/immobile solute transport,” *Water Resources Research*, vol. 39, no. 10, p. 1296, 2003.

[12] D. S. Banks and C. Fradin, “Anomalous diffusion of proteins due to molecular crowding,” *Biophysical Journal*, vol. 89, no. 5, pp. 2960–2971, 2005.

[13] F. Mainardi, *Fractional Calculus and Waves in Linear Viscoelasticity*, Imperial Collage Press, 2010.

[14] S. D. Roscani, N. D. Caruso, and D. A. Tarzia, “Explicit solutions to fractional Stefan-like problems for Caputo and Riemann- Liouville derivatives,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 90, p. 105361, 2020.

[15] V. R. Voller, “Anomalous heat transfer: examples, fundamentals, and fractional calculus models,” *Advances in Heat Transfer*, vol. 50, pp. 333–380, 2018.

[16] Y. H. Bai and Q. P. Li, “Simulation of gas production from hydrate reservoir by the combination of warm water flooding and depressurization,” *Science in China(Series E)*, vol. 41, no. 2, pp. 262–268, 2011.

[17] L. I. Lobkovskii and M. M. Ramazanov, "Mathematical model of axisymmetric quasi-steady-state heat and mass transfer in a..."
gas hydrate reservoir,” *Fluid Dynamics*, vol. 52, no. 4, pp. 536–546, 2017.

[18] X. Kou, Y. Wang, X. S. Li, Y. Zhang, and Z. Y. Chen, “Influence of heat conduction and heat convection on hydrate dissociation by depressurization in a pilot-scale hydrate simulator,” *Applied Energy*, vol. 251, p. 113405, 2019.

[19] M. J. Yang, J. N. Zheng, Y. Gao, Z. Ma, X. Lv, and Y. Song, “Dissociation characteristics of methane hydrates in South China Sea sediments by depressurization,” *Applied Energy*, vol. 243, pp. 266–273, 2019.

[20] S. M. Gil, H. J. Shin, J. S. Lim, and J. Lee, “Numerical analysis of dissociation behavior at critical gas hydrate saturation using depressurization method,” *Journal of Geophysical Research: Solid Earth*, vol. 124, no. 2, pp. 1222–1235, 2019.

[21] F. W. Liu, P. H. Zhuang, and Q. X. Liu, *Numerical methods for fractional partial differential equations with applications*, Science Press (In Chinese), 2015.