Decay Widths of $B_1$ and $B_2^*$ up to the order $O(1/m_Q)$ in HQET

Yuan-Ben Dai and Shi-Lin Zhu

Institute of Theoretical Physics, Academia Sinica, P.O.Box 2735, Beijing 100080, China

Abstract

We first improve the previous results on the masses of the doublets $(0^+,1^+)$ and $(1^+,2^+)$ with QCD sum rules up to the order of $O(1/m_Q)$ in HQET. With the mass matrix of the two $1^+$ states, we obtain the mixing angle for the two states and construct the interpolating currents which couple to only one of the two eigen states up to the order $O(1/m_Q)$. Using these results we derive the light-cone sum rules for the amplitudes of the pionic decay of $B_1$ and $B_2^*$ up to the order $O(1/m_Q)$ in HQET. The $O(1/m_Q)$ terms in these amplitudes are found to be small. When applied to $D_1$ and $D_2^*$ the theoretical results are in good agreement with experimental data though the theoretical uncertainty is sizable for the $D$ system.

PACS number: 12.39.Hg, 13.25.Hw, 13.25.Ft, 12.38.Lg

1 Introduction

Important progress has been achieved in understanding the properties of heavy mesons composed of a heavy quark and a light quark with the development of the heavy quark effective theory (HQET) \[1\]. HQET provides a systematic expansion of the heavy hadron spectra and transition amplitude in terms of $1/m_Q$, where $m_Q$ is the heavy quark mass. Of course one has to employ some specific nonperturbative methods to arrive at the detailed predictions. Among the various nonperturbative methods, QCD sum rules is useful to extract the low-lying hadron properties \[2\]. The properties of the ground state heavy mesons have been studied with the QCD sum rules in HQET in \[3, 4\]. In \[4\] the mass of the lowest excited heavy meson doublets $(2^+,1^+)$ and $(1^+,0^+)$ were studied with QCD sum rules in the heavy quark effective theory (HQET) up to the order of $O(1/m_Q)$. The widths for pionic decays of the lowest two excited doublets $(2^+,1^+)$ and $(1^+,0^+)$ is calculated with conventional QCD sum rules in the leading order of HQET in \[3\]. Properties of excited heavy mesons were investigated also with QCD sum rules in full QCD in \[4\]. Decay of $(1^+,0^+)$ were investigated with light-cone sum rules \[5\] in \[4\]. The pionic couplings between the lowest three doublets $(2^+,1^+), (1^+,0^+)$ and $(1^-,0^-)$ of heavy mesons are investigated in HQET using the light-cone sum rules \[10\]. In \[11\] the pionic couplings of the heavy baryons are calculated with the same technique.
One difficult problem encountered in studying the decay widths of excited heavy mesons with QCD sum rules is the following. Except for the lowest states $0^-, 1^-$, the spectra contains a pair of states for any spin-parity $J^P$. An interpolating current with definite spin-parity in general couples with both two states. This parity is especially serious for the two $1^+$ states. All theoretical calculations indicate that they are close in mass values but quite different in magnitude of their decay widths. One of the two $1^+$ states is a narrow resonance decaying mainly by emitting a $D$ wave pion, while the other one is a very wide resonance decaying mainly by emitting a $S$ wave pion. An interpolating current used for the narrow $1^+$ state with a small coupling to the other $1^+$ state may cause sizable error in the result of calculation. Only in the $m_Q \to \infty$ limit is there a conserved quantum number $j_\ell$, the angular momentum of the light component, which can be used to differentiate the two states. Therefore, HQET has the important and unique advantage for this purpose. Corrections for finite $m_Q$ can be calculated order by order in HQET. The interpolating currents which couple to only one of the two $J^P$ states have been given in [4] and used in [5].

In this work we shall calculate the decay widths of the doublet $(1^+, 2^+)$ to the first order in $1/m_Q$ using the light-cone sum rule approach. As the necessary steps we present the results from sum rules in HQET for masses of these excited states and their coupling constants to the interpolating currents at the leading and next-to-leading order in section 2 and section 3 respectively. A large part of contents of these sections have been obtained in [4]. The new things include the addition of the leading $\mathcal{O}(\alpha_s)$ perturbation term for the chromo-magnetic splitting $\Sigma$ and the results for corrections to coupling constants at the order $\mathcal{O}(1/m_Q)$. We also improve the numerical analysis of the sum rules at the leading order. The continuum threshold $\omega_c$ and the working region for the Borel parameter $T$ determined from the stability criterum of the sum rules in the leading order are used in the sum rules for $\Sigma$ term and the kinetic energy term $K$ in the corrections to the masses. Solving the mass matrix of the two distinct $1^+$ states, we arrive at the mixing angle for these two states. Using this angle the interpolating currents which couple to only one of the $1^+$ states up to the order $1/m_Q$ are constructed in section 4. In section 5 and 6 we employ the constructed currents and the light-cone QCD sum rules (LCQSR) in HQET to calculate the pionic decay amplitudes of $B_1$ and $B_2^*$ up to the order $\mathcal{O}(1/m_Q)$ taking into account the mixing of two $1^+$ states. The results are summarized and discussed in section 9.

2 Two-point QCD sum rules

The proper interpolating current $J_{j,P,j_\ell}^{a_1\cdots a_j}$ for the state with the quantum number $j, P, j_\ell$ in HQET was given in [4]. These currents were proved to satisfy the following conditions

$$
\langle 0|J_{j,P,j_\ell}^{a_1\cdots a_j}(0)|j', P', j_\ell'\rangle = f_{Pj} \delta_{jj'} \delta_{PP'} \delta_{j_\ell j_\ell'} T^{a_1\cdots a_j},
$$

$$
i \langle 0|T \left( J_{j,P,j_\ell}^{a_1\cdots a_j}(x) J_{j',P',j_\ell'}^{\beta_1\cdots \beta_j}(0) \right)|0\rangle = \delta_{jj'} \delta_{PP'} \delta_{j_\ell j_\ell'} (-1)^j S g_t^{a_1\beta_1} \cdots g_t^{a_j\beta_j} \times \int dt \delta(x - vt) \Pi_{P,j_\ell}(x)
$$
in the $m_Q \to \infty$ limit, where $\eta^{\alpha_1 \cdots \alpha_j}$ is the polarization tensor for the spin $j$ state, $v$ is the velocity of the heavy quark, $g_{\mu \nu} = g^{\mu \nu} - v^\mu v^\nu$ is the transverse metric tensor, $\mathcal{S}$ denotes symmetrizing the indices and subtracting the trace terms separately in the sets $(\alpha_1 \cdots \alpha_j)$ and $(\beta_1 \cdots \beta_j)$, $f_{P, j\ell}$ and $\Pi_{P, j\ell}$ are a constant and a function of $x$ respectively which depend only on $P$ and $j\ell$. Because of equations (1) and (2), the sum rule in HQET for decay widths derived from a correlator containing such currents receives no contribution from the unwanted states with the same spin-parity as the states under consideration in the $m_Q \to \infty$. Starting from the calculations in the leading order, the decay amplitudes for finite $m_Q$ can be calculated unambiguously order by order in the $1/m_Q$ expansion in HQET.

We shall confine our study to the doublets $(0^+, 1^+)$ and $(1^+, 2^+)$ here. There are two possible choices for currents creating $0^+$ and $1^+$ of the doublet $(0^+, 1^+)$, either

$$J_{0,+2}^1 = \frac{1}{\sqrt{2}} \bar{h}_t q , \quad J_{1,+2}^{\alpha} = \frac{1}{\sqrt{2}} \bar{h}_t \gamma^5 \gamma_t^\alpha q ,$$

or

$$J_{0,+2}^1 = \frac{1}{\sqrt{2}} \bar{h}_v (-i) \mathcal{D}_t q , \quad J_{1,+2}^{\alpha} = \frac{1}{\sqrt{2}} \bar{h}_v \gamma^5 \gamma_t^\alpha (-i) \mathcal{D}_t q .$$

Similarly, there are two possible choices for the currents creating $1^+$ and $2^+$ of the doublet $(1^+, 2^+)$. One is

$$J_{1,+1}^{\alpha} = \sqrt{\frac{3}{4}} \bar{h}_v \gamma^5 (-i) \left( \mathcal{D}_t^\alpha - \frac{1}{3} \gamma_t^\alpha \mathcal{D}_t \right) q ,$$

$$J_{2,+2}^{\alpha_1 \alpha_2} = \sqrt{\frac{1}{2}} \bar{h}_v (-i) \left( \gamma_t^{\alpha_1} \mathcal{D}_t^{\alpha_2} + \gamma_t^{\alpha_2} \mathcal{D}_t^{\alpha_1} - \frac{2}{3} g_t^{\alpha_1 \alpha_2} \mathcal{D}_t \right) q .$$

Another choice is obtained by adding a factor $-i \mathcal{D}_t$ to (7) and (8). Note that, without the last term in the bracket in (8) the current would couple also to the $1^+$ state in the doublet $(0^+, 1^+)$ even in the limit of infinite $m_Q$.

For the doublet $(0^+, 1^+)$, when the currents $J'_{0,+1/2}, J'_{1,+1/2}$ in (3), (4) are used the sum rule (same for the two states) after the Borel transformation is found to be

$$f^2_{+, 1/2} e^{-2\lambda/T} = \frac{3}{2^6 \pi^2} \int_0^{\omega_c} \omega^4 e^{-\omega/T} d\omega - \frac{1}{24} m_0^2 \langle \bar{q} q \rangle .$$

The corresponding formula when the current $J_{0,+2}$ and $J_{1,+2}$ in (3) and (4) are used instead of $J'_{0,+2}$ and $J'_{1,+2}$ is the following

$$f^2_{+, 1/2} e^{-2\lambda/T} = \frac{3}{16 \pi^2} \int_0^{\omega_c} \omega^2 e^{-\omega/T} d\omega + \frac{1}{2} \langle \bar{q} q \rangle - \frac{1}{8 T^2} m_0^2 \langle \bar{q} q \rangle .$$

3
When the currents (7) and (8) are used the sum rule for the $(1^+, 2^+)$ doublet is found to be
\[
f_{2/2^+}^2 e^{-2\Lambda/T} = \frac{1}{2\pi^2} \int_{\omega_c}^{\infty} \omega^4 e^{-\omega/T} d\omega - \frac{1}{12} m_0^2 \langle \bar{q}q \rangle - \frac{1}{25} \langle \alpha_s G^2 \rangle T .
\] (11)

Here \( m_0^2 \langle \bar{q}q \rangle = \langle \bar{q}g\sigma_{\mu\nu} G^\mu\nu q \rangle \). These sum rules have been obtained in [4]. In the derivations, only terms of the lowest order in perturbation and operators of dimension less than six have been included.

For the QCD parameters entering the theoretical expressions, we use the following standard values
\[
\langle \bar{q}q \rangle = -(0.24 \text{ GeV})^3 ,
\langle \alpha_s G^2 \rangle = 0.038 \text{ GeV}^4 ,
m_0^2 = 0.8 \text{ GeV}^2 .
\] (12)

Here we redo the numerical analysis of the sum rules with the following criteria. The high-order power corrections is less than 30% of the perturbation term without the cutoff \( \omega_c \) and the contribution of the pole term is larger than 40% of the continuum contribution given by the perturbation integral in the region \( \omega > \omega_c \). \( \omega_c \) is constrained to be smaller than 3.2 GeV which is roughly the expected position of the pole of the radial excited state with the same quantum numbers. Subjected to these criteria we arrive at the stability region of the sum rules \( \omega_c = 2.8 - 3.2 \) GeV, \( T = 0.43 - 0.73 \) GeV for (3), \( \omega_c = 2.8 - 3.2 \) GeV, \( T = 0.82 - 1.1 \) GeV for (10), and \( \omega_c = 2.8 - 3.2 \) GeV, \( T = 0.54 - 0.73 \) GeV for (11). The results for \( \Lambda \) for three cases are the following
\[
\bar{\Lambda}(1/2^+) = 1.15 \pm 0.10 \text{ GeV}, \quad (13)
\]
for the doublet \((0^+, 1^+)\) when the currents (3), (4) without the derivative is used.
\[
\bar{\Lambda}(1/2^+) = 0.95 \pm 0.10 \text{ GeV}, \quad (14)
\]
for the same doublet when the currents (3), (4) with the derivative is used.
\[
\bar{\Lambda}(3/2^+) = 0.82 \pm 0.10 \text{ GeV}, \quad (15)
\]
for the doublet \((1^+, 2^+)\). These results shall be used in the following sections.

In the following sections we also need the values of f’s.
\[
f_{+3/2} = 0.19 \pm 0.03 \text{ GeV}^{3/2} , \quad (16)
\]
\[
f_{+1/2} = 0.37 \pm 0.06 \text{ GeV}^{3/2} , \quad (17)
\]
\[
f_{+1/2} = -(0.40 \pm 0.06) \text{ GeV}^{3/2} . \quad (18)
\]

Note it is impossible to determine the absolute sign of f’s from QCD sum rule approach. From the equation of motion we can derive [4]:
\[
f_{+,1/2} = -\bar{\Lambda}_{+,1/2} \frac{1}{2} f_{+,1/2} . \quad (19)
\]

In this work we adopt the convention of the positive \( f_{+,1/2}^*, f_{+,3/2} \).
3 The sum rules at the $O(1/m_Q)$ order

Inserting the heavy meson eigen-state of the Hamiltonian up to the order $O(1/m_Q)$ into the correlator

$$i \int d^4x e^{ik\cdot x} \langle 0 | T \left( J_{j,P,ji}^{\alpha_1 \cdots \alpha_j} (x) J_{j,P,ji}^\dagger \beta_1 \cdots \beta_j (0) \right) | 0 \rangle,$$

the pole term on the hadron side becomes

$$\Pi(\omega)_{pole} = \frac{(f + \delta f)^2}{2(\Lambda + \delta m) - \omega} = \frac{f^2}{2\Lambda - \omega} + \frac{2\delta mf^2}{(2\Lambda - \omega)^2} + \frac{2f\delta f}{2\Lambda - \omega},$$

where $\delta m$ and $\delta f$ are of the order $O(1/m_Q)$.

To extract $\delta m$ in (21) we follow the approach of [6] and consider the three-point correlation functions

$$\delta O \Pi(\omega, \omega') = i^2 \int d^4x d^4y e^{ik\cdot x - ik'\cdot y} \langle 0 | T \left( J_{j,P,ji}^{\alpha_1 \cdots \alpha_j} (x) O(0) J_{j,P,ji}^\dagger \beta_1 \cdots \beta_j (y) \right) | 0 \rangle,$$

where $O = K$ or $S$ is the kinetic energy or the spin dependent term of the $O(1/m_Q)$ lagrangian of the HQET. The scalar function corresponding to (22) can be represented as the double dispersion integral

$$\delta O \Pi(\omega, \omega') = \frac{1}{\pi^2} \int \frac{\rho(\omega, \omega') ds ds'}{(s - \omega)(s' - \omega')}.$$

The pole parts for them are

$$\delta_K \Pi(\omega, \omega')_{pole} = \frac{f^2K}{(2\Lambda - \omega)(2\Lambda - \omega')} + \frac{f^2G_K(\omega)}{2\Lambda - \omega} + \frac{f^2G_K(\omega')}{2\Lambda - \omega'},$$

$$\delta_S \Pi(\omega, \omega')_{pole} = \frac{d_M f^2 \Sigma}{(2\Lambda - \omega)(2\Lambda - \omega')} + d_M f^2 \left[ \frac{G_S(\omega)}{2\Lambda - \omega} + \frac{G_S(\omega)}{2\Lambda - \omega'} \right],$$

where

$$K_{j,P,ji} = \langle j, P, ji | \bar{h_v} (iD_\perp)^2 h_v | j, P, ji \rangle,$$

$$2d_M \Sigma_{j,P,ji} = \langle j, P, ji | \bar{h_v} g_{\sigma\mu\nu} G^{\mu\nu} h_v | j, P, ji \rangle,$$

$$d_M = d_{j,ji}, \quad d_{j,-1/2,ji} = 2ji + 2, \quad d_{j,+1/2,ji} = -2ji.$$

Let $\omega = \omega'$ in (24) and (25), and compare it with (21) one obtains

$$\delta m = -\frac{1}{4m_Q} (K + d_M C_{mag} \Sigma).$$

The simple pole term in (24) and (25) comes from the region in which $s(s') = 2\bar{\Lambda}$ and $s(s')$ is at the pole for a radial excited state or in the continuum. These terms are suppressed by
making double Borel transformation for both \( \omega \) and \( \omega' \). One obtains thus the sum rules for \( K \) and \( \Sigma \) as

\[
\begin{align*}
 f^2 K e^{-2\lambda T} &= \int_0^{\omega_c} \int_{\omega_c}^{\omega_c} d\omega d\omega' e^{-(\omega+\omega')/2T} \rho_K(\omega,\omega') , \\
 f^2 \Sigma e^{-2\lambda T} &= \int_0^{\omega_c} \int_{\omega_c}^{\omega_c} d\omega d\omega' e^{-(\omega+\omega')/2T} \rho_\Sigma(\omega,\omega') ,
\end{align*}
\]

(30)

(31)

where the spectral densities are obtained from straightforward calculations in HQET.

Confining us to the leading order of perturbation and the operators with dimension \( D \leq 5 \) in OPE, we find for the \( j^p_1 = \frac{1}{2}^+ \) doublet,

\[
\begin{align*}
 f^2 K e^{-2\lambda T} &= -\frac{3}{2^7 \pi^2} \int_0^{\omega_c} \omega^6 e^{-\omega/T} d\omega - \frac{3}{2^5 \pi} \langle \alpha_s GG \rangle T^3 , \\
 f^2 \Sigma e^{-2\lambda T} &= \int_0^{\omega_c} \int_0^{\omega_c} \rho(s, s') e^{-\frac{s+s'}{2\pi}} dsds' + \frac{1}{48\pi} \langle \alpha_s GG \rangle T^3 ,
\end{align*}
\]

(32)

(33)

with \( \rho(s, s') = \frac{1}{36} \alpha_s(2T) \pi^6 C_{mag} \{ s^2 (3s - s') \theta(s - s') + (s \leftrightarrow s') \} \) when the currents (\( 3 \)) and (\( 4 \)) are used and

\[
\begin{align*}
 f^2 K e^{-2\lambda T} &= -\frac{3}{2^7 \pi^2} \int_0^{\omega_c} \omega^4 e^{-\omega/T} d\omega - \frac{1}{2^5 \pi} \langle \alpha_s GG \rangle T - \frac{3}{8} m_0^2 \langle \bar{q}q \rangle , \\
 f^2 \Sigma e^{-2\lambda T} &= \int_0^{\omega_c} \int_0^{\omega_c} \rho(s, s') e^{-\frac{s+s'}{2\pi}} dsds' + \frac{1}{24\pi} \langle \alpha_s GG \rangle T + \frac{1}{48} m_0^2 \langle \bar{q}q \rangle
\end{align*}
\]

(34)

(35)

with \( \rho(s, s') = \frac{1}{23} \alpha_s(2T) \pi^6 C_{mag} \{ s^2 (3s - s') \theta(s - s') + (s \leftrightarrow s') \} \) when the currents (\( 3 \)) and (\( 4 \)) with no extra derivative are used. For \( j^p_1 = \frac{3}{2}^+ \) doublet, a straightforward calculation yields

\[
\begin{align*}
 f^2 K e^{-2\lambda T} &= -\frac{1}{2^7 \pi^2} \int_0^{\omega_c} \omega^6 e^{-\omega/T} d\omega - \frac{7}{3 \times 2^5 \pi} \langle \alpha_s GG \rangle T^3 , \\
 f^2 \Sigma e^{-2\lambda T} &= \int_0^{\omega_c} \int_0^{\omega_c} \rho(s, s') e^{-\frac{s+s'}{2\pi}} dsds' + \frac{1}{72\pi} \langle \alpha_s GG \rangle T^3 ,
\end{align*}
\]

(36)

(37)

with \( \rho(s, s') = \frac{1}{288} \alpha_s(2T) \pi^6 C_{mag} \{ s^4 (s - 3/5 s') \theta(s - s') + (s \leftrightarrow s') \} \). Combining (32)-(37) with (\( 5 \))-(\( 11 \)) we can obtain sum rules for \( K \) and \( \Sigma \) in the three cases.

The spin-symmetry violating term \( S \) not only causes splitting of masses in the same doublet, but also causes mixing of states with the same \( j, P \) but different \( j_l \). This mixing is characterized by the matrix element

\[
\langle j, P, j + \frac{1}{2} | \bar{h}_v \frac{g}{2} \sigma_{\mu\nu} G^{\mu\nu} h_v | j, P, j - \frac{1}{2} \rangle = -2 m_{++} (J^p) .
\]

(38)

This quantity can be extracted from the correlator

\[
\int d^4xd^4y e^{ik-x-i\vec{k} \cdot \vec{y}} \langle 0 | T \left( J_{j,P,\frac{1}{2}}(x) \bar{h}_v \frac{g}{2} \sigma_{\mu\nu} G^{\mu\nu} h_v(0) J_{j,P,-\frac{1}{2}}(y) \right) | 0 \rangle ,
\]

(39)
the double pole term of which is
\[
- \frac{2m_{++}(J^P)}{(2\Lambda_{+j+\frac{1}{2}} - \omega)(2\Lambda_{+j-\frac{1}{2}} - \omega')}. \tag{40}
\]
With the methods similar to that used above one can obtain the sum rule of \(m_{++}(J^P)\).
For two \(1^+\) states we find
\[
m_{+\frac{1}{2}}(1^+) f_{+\frac{3}{2}} e^{-\tilde{\Lambda}_{+\frac{1}{2}}/T} e^{-\tilde{\Lambda}_{+\frac{3}{2}}/T} = \int_0^{\omega_c} \int_0^{\omega_c} \rho(s, s') e^{-\frac{6m_Q}{72\pi} dsds'} + \sqrt{\frac{6}{72\pi}} \langle \alpha_s GG \rangle T^3. \tag{41}
\]
with \(\rho(s, s') = \frac{\sqrt{\alpha_s(2T)}}{s_s'} C_{mag} \{ s s' \theta(s - s') + s^3 (6s'^2 - 8ss' + 3s^2) \theta(s' - s) \} \) for the currents (3) and (7).
The sum rules (32)-(37) and (41) were obtained in [4] except that all loop corrections of the order \(\alpha_s\) were neglected there. In this work we have included the perturbative terms of the order \(\alpha_s\) in the sum rules for \(\Sigma\) and \(m_{1/2,3/2}\) which are leading perturbative terms for these quantities.

The masses of two \(1^+\) states are obtained by diagonalizing the mass matrix
\[
\left(\begin{array}{ccc}
\tilde{\Lambda}_{+\frac{1}{2}} - \frac{1}{4m_Q} (K_{+\frac{1}{2}} - C_{mag} \Sigma_{+\frac{1}{2}}) & \frac{1}{4m_Q} C_{mag} m_{+\frac{1}{2}} \Sigma_{+\frac{1}{2}} (1^+) \\
\frac{1}{4m_Q} C_{mag} m_{+\frac{3}{2}} (1^+) & \tilde{\Lambda}_{+\frac{3}{2}} - \frac{1}{4m_Q} (K_{+\frac{3}{2}} + 5 C_{mag} \Sigma_{+\frac{3}{2}})
\end{array}\right). \tag{42}
\]
where \(C_{mag} = (\frac{\alpha_s(m_0)}{\alpha_s(2T)})^{\frac{\beta_0}{3}}\), \(\beta_0 = 11 - \frac{2}{3}n_f\). In our analysis we keep four active flavors with the QCD parameter \(\tilde{\Lambda}_{QCD} = 220\text{MeV}\).

We can minimize the dependence of the sum rules obtained in Section 3 on \(f, \tilde{\Lambda}\) and \(\omega_c\) through dividing the sum rules in (32)-(37) by the sum rules in (9)-(11) respectively. The seemingly stability at large \(T\) values is not useful, since in this region the sum rules is strongly contaminated by higher resonance states. This continuum model contamination problem is quite severe also in the case of sum rules analysis of ground state heavy meson to \(1/m_Q\) [6, 18]. It originates from the high power dependence of the spectral densities. Moreover the stability plateau for the three-point sum rules does not necessarily coincide with that of two-point sum rules. We follow [4] and use the same working windows as those of two-point sum rules in the evaluation of \(O(1/m_Q)\) corrections.

Taking the working regions of the sum rules as that for the two-pion functions for the three cases, we obtain the set of values for \(K\) as following:
\[
K = -2.30 \pm 0.40 \text{ GeV}^2, \tag{43}
\]
for the doublet \((0^+, 1^+)\) when the currents (3), (4) without the derivative are used,
\[
K = -1.70 \pm 0.40 \text{ GeV}^2, \tag{44}
\]
for the same doublet when the currents (3), (4) with the derivative are used, and
\[
K = -1.60 \pm 0.40 \text{ GeV}^2, \tag{45}
\]
for the doublet \((1^+, 2^+)\).

The results for \(\Sigma\) for the doublet \((0^+, 1^+)\) in the working regions of the sum rules are

\[
\Sigma = 0.22 \pm 0.04 \text{ GeV}^2 ,
\]

when the currents \((\bar{3}) (\bar{4})\) without the derivative are used and

\[
\Sigma = 0.18 \pm 0.03 \text{ GeV}^2 ,
\]

when the currents \((\bar{3}), (\bar{4})\) with the derivative are used. For the doublet \((1^+, 2^+)\), the value of \(\Sigma\) is found to be

\[
\Sigma = 0.03 \pm 0.01 \text{ GeV}^2 .
\]

Dividing the sum rule in (41) by the sum rules in (10) and (11), we also obtain the expression for \(m_{1^+, 2^+}(1^+)\). Using the working windows as \(\omega_c = 2.8 - 3.2\) GeV and \(T = 0.53 - 0.73\) GeV, we obtain

\[
m_{1^+, 2^+}(1^+) = 0.08 \pm 0.02 \text{ GeV}^2 .
\]

The errors quoted above reflect only the variations with the Borel parameter \(T\) and the continuum threshold \(\omega_c\) within the working windows. They do not include other intrinsic errors in QCD sum rules in HQET. The corrections to the masses due to the mixing are formally of the order \(\frac{1}{m_Q}\). By diagonalizing the mass matrix we found that they are numerically negligible.

Since the experimental data is not yet available for the excited \(B\) mesons, we present our results for the \(D\) system assuming the HQET is good enough for the excited \(D\) meson too. For the doublet \((0^+, 1^+)\), we have

\[
\frac{1}{4} (m_{D_0} + 3m_{D_1}) = m_c + \Lambda + \frac{1}{m_c} [(0.57 \pm 0.10) \text{ GeV}^2] ,
\]

\[
m_{D_1} - m_{D_0} = \frac{1}{m_c} [(0.22 \pm 0.04) \text{ GeV}^2] ,
\]

when the currents \((\bar{3}) \text{ and } (\bar{4})\) are used and

\[
\frac{1}{4} (m_{D_0} + 3m_{D_1}) = m_c + \Lambda + \frac{1}{m_c} [(0.42 \pm 0.10) \text{ GeV}^2] ,
\]

\[
m_{D_1} - m_{D_0} = \frac{1}{m_c} [(0.18 \pm 0.03) \text{ GeV}^2] ,
\]

when the currents \((\bar{5}) \text{ and } (\bar{6})\) are used. As for the doublet \((1^+, 2^+)\), the result is

\[
\frac{1}{8} (3m_{D_1} + 5m_{D_2}) = m_c + \Lambda + \frac{1}{m_c} [(0.40 \pm 0.10) \text{ GeV}^2] ,
\]

\[
m_{D_2} - m_{D_1} = \frac{1}{m_c} [(0.06 \pm 0.02) \text{ GeV}^2] .
\]

The renormalization coefficient \(C_{mag}\) of the chromomagnetic operator for charmed meson in the above formulas is reasonably neglected. The results for \(B\) system are obtained by
replacing $m_c$ by $m_b$ and multiplying the right hand sides of (51), (53) and (55) by 0.8 since $C_{\text{mag}} \approx 0.8$ for B system.

The results in (54) with the $\bar{\Lambda}$ value in (15) and $m_c = 1.4 \text{GeV}$ agree reasonably with experimental data, though the uncertainties, especially for $\bar{\Lambda}$, are somewhat large.

In the following sections we will need the values of $r_1, r_2 = \delta f_{1,+,\frac{1}{2}}/f_{1,+,\frac{1}{2}}$, $r_1, r_2 = \delta f_{1,+,\frac{1}{2}}/f_{1,+,\frac{1}{2}}$ and $\delta f_{1,+,\frac{1}{2}}$, where $\delta f$ is the correction to $f$ due to the $K$ and $S$ terms in the Lagrangian at the order of $O(1/m_Q)$ and $\delta f_{1,+,\frac{1}{2}}$ is defined as:

$$\langle 0 | J_{1,+,\frac{1}{2}} | 1, +, \frac{3}{2} \rangle = \delta f_{1,+,\frac{1}{2}} \eta_{1,+,\frac{3}{2}}$$

which is also of the order $1/m_Q$.

Omitting the indices $j, P, j$, $r$ can be written in the form:

$$r = \frac{G_K}{2m_Q} + \frac{d_M}{2m_Q} \frac{G_S}{m_Q},$$

where $G_K$ and $G_S$ are defined as:

$$\langle 0 | i \int d^4x K(x) J_{j,P,j} (0) | j,p,j \rangle = f_{j,P,j} G_K \eta_{j,P,j},$$

$$\langle 0 | i \int d^4x S(x) J_{j,P,j} (0) | j,p,j \rangle = f_{j,P,j} d_M G_S \eta_{j,P,j}.$$  (58)

The mixing coupling constant $\delta f_{1,+,\frac{1}{2}} = G_{1,+,\frac{1}{2}}/2m_Q$ where $G_{1,+,\frac{1}{2}}$ is defined as:

$$\langle 0 | i \int d^4x S(x) J_{1,+,\frac{1}{2}} (0) | 1, +, \frac{3}{2} \rangle = G_{1,+,\frac{3}{2}} \eta_{1,+,\frac{3}{2}}.$$  (60)

$r_{1,+,\frac{1}{2}}$ for the ground state $1^-$ has been calculated in [13, 20]. Notice that $\delta f_{1,+,\frac{1}{2}}$ calculated in [13, 20] contains an additional term proportional to $\bar{\Lambda}/m_Q$. This term comes from the $O(1/m_Q)$ correction in the expansion of the physical vector current $Q\gamma_\mu q$ responsible to the leptonic decay of $B^*$, where $Q$ is the field in full QCD. For our purpose instead the matrix elements of the interpolating currents $J_{j,P,j}$ defined in [1] and [2] at the leading order are needed. Therefore this term is not included in (57). From the results of [20], we know $r_{1,+,\frac{1}{2}} = -(0.82 \pm 0.30)/m_Q$ for $1^-$ and $r_{0,+,\frac{1}{2}} = -(0.74 \pm 0.30)/m_Q$ for $0^-$ heavy meson at the leading order of $\alpha_s$ [20].

We follow the method used in [20] to derive the $G_K$, $G_S$ and $G_{1,+,\frac{1}{2}}$ for the $B_1$ meson and $B_2^*$ doublet. Letting $k = k'$ and $\omega = \omega'$ in (22)-(25) and (39)-(40) we arrive at the following sum rules:

$$f^2_{1,+,\frac{1}{2}} e^{-\frac{2K}{T}} (2G_K + \frac{K}{T}) = -\frac{3}{64\pi^2} \int_0^{\omega_c} s^5 e^{-\frac{s}{T}} ds + \frac{7}{96} \frac{\alpha_s}{\pi} G^2 T^2,$$  (61)

$$f^2_{1,+,\frac{1}{2}} e^{-\frac{2K_{1,+,\frac{1}{2}}}{T}} (2G_S + \frac{\Sigma}{T}) = \frac{\alpha_s}{32\pi^3} \int_0^{\omega_c} s^5 \left[ \frac{25}{81} - \frac{4}{15} \ln \frac{s}{\mu} \right] e^{-\frac{s}{T}} ds + \frac{1}{72} \frac{\alpha_s}{\pi} G^2 T^2,$$  (62)
Here the first error comes from the variation with $\omega_1^{10}_{11} + 2$. Diagonalizing the mixing mass matrix (42) we arrive at the physical eigenstates for the $r_i$. We arrive at one, which results in the curves in Fig. 3. The sum rule (63) the gluon condensate contribution is much bigger than the perturbative mass sum rules. In this region there exists stability plateau for the sum rules (61)-(62). For the sum rule (63) the gluon condensate contribution is much bigger than the perturbative one, which results in the curves in Fig. 3.

Numerically we have: $G_K = -(1.0 \pm 0.20 \pm 0.25)\text{GeV}$, $G_\Sigma = (0.013 \pm 0.005 \pm 0.002)\text{GeV}$, $G_{1/2,1/2} = (0.023 \pm 0.003 \pm 0.010)\text{GeV}$.

In the numerical analysis we choose the same stability region of $(T, \omega_c)$ as those from the mass sum rules. In this region there exists stability plateau for the sum rules (61)-(62). For the sum rule (63) the gluon condensate contribution is much bigger than the perturbative one, which results in the curves in Fig. 3.

In the discussion of the mixing of the two $1^+$ states we always use the current (6) with derivatives. The mixing parameters $\alpha, \beta$ are determined from the orthonormal conditions of the physical $1^+$ states:

\[
\langle 0 | J_{B_1}^B | B_1 \rangle = 0 ,
\]

\[
f_{+1/2} f_{+1/2} e^{rac{2G_{1/2,1/2}}{f_{+1/2}} + m_{+1}} = \frac{\sqrt{6} \alpha_s}{32 \pi^3} \int_0^{\omega_c} s^3 \left( \frac{4}{81} - \frac{1}{9} \ln \frac{s}{\mu} \right) e^{-\frac{s}{\mu}} ds + \frac{\sqrt{6}}{72} \left( \frac{\alpha_s}{\pi} G^2 \right) T^2 ,
\]

(63)

where we take $\mu = 2T$ as in the evaluation of $K$ and $\Sigma$.

In order to obtain $G_K, G_\Sigma$ and $G_{1/2,1/2}$ we subtract (61), (62) and (63) by the expressions of (36), (37) and (41) after they are divided by $T$. The dependence of $G_K, G_\Sigma$ and $G_{1/2,1/2}/f_{+1/2}$ on the parameter $T$ and $\omega_c$ is shown in Fig. 1-3 with $\omega_c = 3.2, 3.0, 2.8$ GeV. In the numerical analysis we choose the same stability region of $(T, \omega_c)$ as those from the mass sum rules. In this region there exists stability plateau for the sum rules (61)-(62). For the sum rule (63) the gluon condensate contribution is much bigger than the perturbative one, which results in the curves in Fig. 3.

Numerically we have: $G_K = -(1.0 \pm 0.20 \pm 0.25)\text{GeV}, G_\Sigma = (0.013 \pm 0.005 \pm 0.002)\text{GeV}$.

Here the first error comes from the variation with $\omega_c$ and the second error is due to the variation with $T$ in the working region. $G_{1/2,1/2}/f_{+1/2} = (0.023 \pm 0.003 \pm 0.010)\text{GeV}$. Finally we arrive at $r_{1,+1/2} = -(0.52 \pm 0.22)/m_Q$, $r_{2,+1/2} = -(0.57 \pm 0.25)/m_Q$, $\delta f_{1/2,1/2} = (4.3 \pm 2.0) \times 10^{-3}/m_Q\text{GeV}^{5/2}$.

### 4 Mixing of the Two $1^+$ States

Diagonalizing the mixing mass matrix (12) we arrive at the physical eigenstates for the two $1^+$ states up to the first order of $1/m_Q$.

\[
|B_1^i\rangle = \cos \theta |1^+, +, \frac{1}{2}\rangle - \sin \theta |1^+, +, \frac{3}{2}\rangle ,
\]

(64)

\[
|B_1\rangle = \sin \theta |1^+, +, \frac{1}{2}\rangle + \cos \theta |1^+, +, \frac{3}{2}\rangle ,
\]

(65)

where $\theta$ is the mixing angle, which is determined by the following equation:

\[
\tan 2\theta = -\frac{2C_{mag}m_{1/2}}{\delta f_{1/2,1/2} + C_{mag}(\Sigma_{+,1/2} + 5\Sigma_{+,1/2})} .
\]

(66)

Numerically we get $\theta = -(0.10 \pm 0.05)$ for the $D$ system with $m_c = 1.4\text{GeV}$ and $\theta = -(0.03 \pm 0.015)$ for the $B$ system with $m_b = 4.8\text{GeV}$.

The corresponding interpolating currents which annihilate one of these two states and do not couple to the other up to the first order of $1/m_Q$ can be written as

\[
J_{B_1}^B = J_{1^+, +, 1/2} + \alpha J_{1^+, +, 3/2} ,
\]

(67)

\[
J_{B_1} = J_{1^+, +, 1/2} + \beta J_{1^+, +, 3/2} .
\]

(68)

In the discussion of the mixing of the two $1^+$ states we always use the current (6) with derivatives.
\[ \langle 0 | J_{B_1}' | B_1' \rangle = 0. \] (70)

Solving the above equations we obtain
\[ \alpha = -\frac{f_{1,3} \tan \theta + \delta f_{1,3}}{f_{1,3}} \] (71)
\[ \beta = \frac{f_{1,3} \tan \theta - \delta f_{1,3}}{f_{1,3}} \] (72)

With the values of \( f \)'s in (11)-(18), \( \delta f_{1,3} \) and \( \tan \theta \) we get \( \alpha = 0.22 \pm 0.10, \beta = -(0.06 \pm 0.03) \) for \( D_1', D_1 \) and \( \alpha = 0.065 \pm 0.032, \beta = -(0.02 \pm 0.01) \) for \( B_1', B_1 \) respectively.

5 Pionic decay amplitudes of \( B_1 \) up to the order of \( O(1/m_Q) \)

In this section we shall use the light-cone QCD sum rules (LCQSR) [8] to calculate the partial widths of the pionic decay processes of \( B_1 \) and \( B_2^* \). The decay width of \((1^+, 2^+), (0^+, 1^+)\) doublets have been calculated at the leading order in HQET with conventional QCD sum rules [3]. Different from the conventional QCD sum rules, which is based on the short distance operator product expansion (OPE), LCQSR is based on the OPE on the light cone which is the expansion over the twists of the operators.

Denote the doublet \((1^+, 2^+)\) by \((B_1, B_2^*)\). Let us consider first the decay process \( B_1 \to B^* \pi \). From (63) the decay amplitude can be written as
\[ M(B_1 \to B^* \pi) = \cos \theta M(\frac{3}{2}^+) + \sin \theta M(\frac{1}{2}^+) , \] (73)
where \( M(\frac{3}{2}^+) \) and \( M(\frac{1}{2}^+) \) denote contributions from the states \( j_\ell = \frac{3}{2}^+ \) and \( j_\ell = \frac{1}{2}^+ \) respectively. Up to the order of \( O(1/m_Q) \) we can confine us to the leading order for \( M(\frac{1}{2}^+) \) and up to the next-to-leading order for \( M(\frac{3}{2}^+) \) in the \( 1/m_Q \) expansion.

From covariance and conservation of the angular momentum of the light component in the \( m_Q \to \infty \) limit, the leading order amplitudes have the general form:
\[ M_0(B_1 \to B^* \pi) = M_0(\frac{3}{2}^+) = I \epsilon^*_\mu \eta_\nu (q^*_\mu q^*_\nu - \frac{1}{3} g^{\mu\nu} q^2) g(B_1, B^*) , \] (74)
\[ M(\frac{1}{2}^+) = I \epsilon^* \cdot \eta g'(B_1', B^*) , \] (75)
corresponding to the D-wave and S-wave for the pion respectively. Here \( \eta_\mu \) and \( \epsilon_\mu \) are polarization vectors for the states \( 1^+ \) and \( 1^- \) respectively. \( q_{\mu} = q_\mu - v \cdot q v_\mu \). \( I = \sqrt{2}, 1 \) for charged and neutral pion respectively.

The \( O(1/m_Q) \) correction \( M_1(\frac{3}{2}^+) \) arises from the the terms \( K \) and \( S \) in the Lagrangian in HQET. The operator \( K \) contributes only to the D-wave amplitude, while \( S \) contributes
to both the D-wave and S-wave amplitudes. Therefore the total $O(1/mQ)$ correction of the decay amplitude $M(B_1 \to B^* \pi)$ has the general form:

$$M_1(B_1 \to B^* \pi) = M_{1d} + M_{1s} = i \epsilon\mu\eta\nu\left\{ (q_t^{\mu} q_t^{\nu} - \frac{1}{3} g^{\mu\nu} q_t^2) g_{1d} + g^{\mu\nu} g_{1s} \right\}. \quad (76)$$

In order to get a consistent expansion over $1/mQ$ we consider the correlator

$$\int d^4 x \, e^{-ik \cdot x} \langle \pi(q) | T \left( J_{1,-\frac{1}{2}}^\beta (0) J_{B_1}^{\alpha \tau}(x) \right) | 0 \rangle, \quad (77)$$

where $J_{B_1}$ is the interpolating current for $B_1$. It is implied that the $O(1/mQ)$ terms in the Lagrangian of HQET are included in the calculation of $(77)$.

The pole term of $(77)$ can be expressed up to the order $O(1/mQ)$ as:

$$\frac{(f_{-\frac{1}{2}} + \delta f_{-\frac{1}{2}})(f_{+\frac{1}{2}} + \delta f_{+\frac{1}{2}})(M_0 + M_{1d} + M_{1s})^{\alpha\beta}}{[2(\Lambda_{-\frac{1}{2}} + \delta m_{-\frac{1}{2}}) - \omega'][2(\Lambda_{+\frac{1}{2}} + \delta m_{+\frac{1}{2}}) - \omega]}$$

which can be expanded as:

$$\frac{f_{-\frac{1}{2}} f_{+\frac{1}{2}} M_0^{\alpha\beta}}{(2\Lambda_{-\frac{1}{2}} - \omega')(2\Lambda_{+\frac{1}{2}} - \omega)} + \frac{f_{-\frac{1}{2}} f_{+\frac{1}{2}} (r_{1,-\frac{1}{2}} + r_{1,+\frac{1}{2}}) M_0^{\alpha\beta}}{(2\Lambda_{-\frac{1}{2}} - \omega')(2\Lambda_{+\frac{1}{2}} - \omega)} + \frac{f_{-\frac{1}{2}} f_{+\frac{1}{2}} (M_{1d} + M_{1s})^{\alpha\beta}}{(2\Lambda_{-\frac{1}{2}} - \omega')(2\Lambda_{+\frac{1}{2}} - \omega)}$$

where $M_0^{\alpha\beta}$, $i = 0, 1$, is defined by $M_i = \epsilon_\alpha^* \eta_\beta M_i^{\alpha\beta}$, and $r_{1,-\frac{1}{2}}$, $r_{1,+\frac{1}{2}}$ is defined in Section [4].

On the other hand $(78)$ can be expanded as:

$$\int d^4 x \, e^{-ik \cdot x} \langle \pi(q) | T \left( J_{1,-\frac{1}{2}}^\beta (0) J_{1,+\frac{1}{2}}^{\alpha \tau}(x) \right) | 0 \rangle$$

$$+ \beta \int d^4 x \, e^{-ik \cdot x} \langle \pi(q) | T \left( J_{1,-\frac{1}{2}}^\beta (0) J_{1,+\frac{1}{2}}^{\alpha \tau}(x) \right) | 0 \rangle$$

$$+ i \int d^4 x d^4 y \, e^{-ik \cdot x} e^{ik \cdot y} \langle \pi(q) | T \left( J_{1,-\frac{1}{2}}^\beta (0) \frac{q_\sigma}{4m_Q} \tilde{h}_v(0) \sigma \cdot G h_v(0) J_{1,+\frac{1}{2}}^{\alpha \tau}(x) \right) | 0 \rangle$$

$$+ i \int d^4 x d^4 y \, e^{-ik \cdot x} e^{ik \cdot y} \langle \pi(q) | T \left( J_{1,-\frac{1}{2}}^\beta (0) \frac{1}{2m_Q} \tilde{h}_v(0) (iD_\lambda)^2 h_v(0) J_{1,+\frac{1}{2}}^{\alpha \tau}(x) \right) | 0 \rangle, \quad (80)$$

where the first term is the correlator at the leading order of $1/mQ$ expansion, the second term is due to the mixing of two $1^+$ currents, the third and fourth term arises from the $O(1/mQ)$ corrections to the transition $j_\epsilon = \frac{3}{2}$ to $j_\epsilon = \frac{1}{2}^-$ due to the $O(1/mQ)$ terms of the Lagrangian in the order of $O(1/mQ)$. These correlators will be calculated with OPE and the leading order Lagrangian of HQET.
6 The sum rules for the coupling constants $g$, $g'$ etc

We consider the correlators appearing in (80). They have the general form:

$$
\int d^4x\ e^{-ik\cdot x}\langle \pi(q)|T\left( J^\beta_{1,-\frac{1}{2}}(0) J^{\alpha}_{1,+\frac{1}{2}}(x) \right)|0\rangle = g_\alpha^\beta q_\beta \bar{q}_t \ G(\omega, \omega'),
$$

(81)

$$
\int d^4x\ e^{-ik\cdot x}\langle \pi(q)|T\left( J^\beta_{1,-\frac{1}{2}}(0) J^{\alpha}_{1,+\frac{1}{2}}(x) \right)|0\rangle = g_\alpha^\beta G'(\omega, \omega'),
$$

(82)

$$
i\int d^4xd^4y\ e^{-i(k-k')\cdot y}\langle \pi(q)|T\left( J^\beta_{1,-\frac{1}{2}}(y) J^{\alpha}_{1,+\frac{1}{2}}(x) \right)|0\rangle = m_Q \left( q_\alpha^\beta q_t \bar{q}_t \ G^S_{1d}(\omega, \omega') + g_\alpha^\beta G^S_{1s}(\omega, \omega') \right),
$$

(83)

$$
i\int d^4xd^4y\ e^{-i(k-k')\cdot y}\langle \pi(q)|T\left( J^\beta_{1,-\frac{1}{2}}(y) J^{\alpha}_{1,+\frac{1}{2}}(x) \right)|0\rangle = m_Q \left( q_\alpha^\beta q_t \bar{q}_t \ G^K_{1d}(\omega, \omega') \right),
$$

(84)

where $k' = k - q$, $\omega = 2v \cdot k$, $\omega' = 2v \cdot k'$ and $q^2 = m_\pi^2 \approx 0$. The suffices $K$ and $S$ denote the contributions from the operators from $K$ and $S$ respectively. The forms of the right hand side of (81)-(82) are determined by that $\alpha$ and $\beta$ are transverse indices, $x - y = \nu t$ on the heavy quark propagator and the conservation of angular momentum of the light component.

For deriving QCD sum rules we first calculate the correlator (81), which is equal to

$$
\frac{\sqrt{6}}{8} \int_0^\infty dt \int dx e^{-ik\cdot x} \delta(-x - \nu t)\text{Tr}\left\{\gamma^s_\beta (1 + \hat{v})\gamma_5 (D^t_\alpha - \frac{1}{3} \gamma^t_\alpha \hat{D}^t)\langle \pi(q)|u(0)\bar{d}(x)|0\rangle\right\},
$$

(85)

By the operator expansion on the light-cone the matrix element of the nonlocal operators between the vacuum and pion state in (83) defines the two particle wave function of the pion. Up to twist four the Dirac components of this wave function can be written as $\bar{\pi}$:

$$
\langle \pi(q)|\bar{d}(x)\gamma_\mu \gamma_5 u(0)|0\rangle = -i f_\pi q_\mu \int_0^1 du \ e^{iux}(\varphi(u) + x^2 g_1(u) + \mathcal{O}(x^4))
$$

$$
\quad + f_\pi(x_\mu - \frac{x^2q_\mu}{qx}) \int_0^1 du \ e^{iux}g_2(u),
$$

(86)

$$
\langle \pi(q)|\bar{d}(x)i\gamma_5 u(0)|0\rangle = \frac{f_\pi m_\pi^2}{m_u + m_d} \int_0^1 du \ e^{iux}\varphi_p(u),
$$

(87)

$$
\langle \pi(q)|\bar{d}(x)\sigma_{\mu\nu}\gamma_5 u(0)|0\rangle = i(q_\mu x_\nu - q_\nu x_\mu) \frac{f_\pi m_\pi^2}{6(m_u + m_d)} \int_0^1 du \ e^{iux}\varphi_\sigma(u).
$$

(88)

The wave function $\varphi_\pi$ is associated with the leading twist two operator, $g_1$ and $g_2$ correspond to twist four operators, and $\varphi_p$ and $\varphi_\sigma$ to twist three ones. Due to the choice of
the gauge $x^\mu A_\mu(x) = 0$, the path-ordered gauge factor $P \exp (i g_s \int_0^1 dx^\mu A_\mu(ux))$ has been omitted. The coefficient in front of the r.h.s. of eqs. (87), (88) can be written in terms of the light quark condensate $< \bar{u}u>$ using the PCAC relation: $\mu_\pi = \frac{m^2}{m_u + m_d} = -\frac{2}{f_\pi^2} < \bar{u}u >$.

From (85), (86) and (88) we arrive at

$$G(\omega, \omega') = -\frac{\sqrt{6}}{8} f_\pi \int_0^\infty dt \int_0^1 du e^{i (1-u)\frac{\omega}{2} + i u \frac{\omega'}{2}} u \{ \varphi_\pi(u) + t^2 g_1(u) - \frac{i t}{q \cdot v} g_2(u) + \frac{i t}{6} \mu_\pi \varphi_\sigma(u) \} + \cdots .$$

(89)

For large euclidean values of $\omega$ and $\omega'$ this integral is dominated by the region of small $t$, therefore it can be approximated by the first a few terms.

Similarly we have:

$$G'(\omega, \omega') = -\frac{i}{4} g_\pi^2 f_\pi \int_0^\infty dt \int_0^1 du e^{i(1-u)\frac{\omega}{2} + i u \frac{\omega'}{2}} u \{ \varphi_\pi(u) + t^2 g_1(u) - \frac{i t}{q \cdot v} g_2(u) + \frac{i t}{6} \mu_\pi \varphi_\sigma(u) \} + \cdots ,$$

(90)

with the current (3) for $j_t = \frac{1}{2}^+$ state. Or

$$G'(\omega, \omega') = \frac{i}{4} f_\pi \int_0^\infty dt \int_0^1 du e^{i(1-u)\frac{\omega}{2} + i u \frac{\omega'}{2}} \{ \mu_\pi \varphi_P(u) - (q \cdot v)[\varphi_\pi(u) + t^2 g_1(u)] \} + \cdots .$$

(91)

with the current (4).

After double Borel transformation with the variables $\omega$ and $\omega'$ the single-pole terms in (88) are eliminated. Using (84), (85), (83) and subtracting the continuum contribution which is modeled by the dispersion integral in region $\omega, \omega' \geq \omega_c$, we arrive at:

$$g f_{-\frac{1}{2}} f_{+\frac{1}{2}} = -\frac{\sqrt{6}}{4} F_\pi e^{\frac{\lambda}{T} + \frac{\lambda_0}{T} + \frac{1}{T}} \{ u_0 \varphi_\pi(u_0) T(1-e^{-\frac{\omega_c}{T}}) - \frac{4}{T} u_0 g_1(u_0) + \frac{4}{T} G_2(u_0) + \frac{1}{3} \mu_\pi u_0 \varphi_\sigma(u_0) \} ,$$

(92)

with $F_\pi = \frac{f_\pi}{\sqrt{2}} = 92$ MeV, $u_0 \equiv \frac{T_1}{T_1 + T_2}$, $T \equiv \frac{T_1 T_2}{T_1 + T_2}$, where $T_1$, $T_2$ are the Borel parameters and $G_2(u_0) \equiv \int_0^{u_0} u g_2(u) du$. In obtaining (92) we have used the Borel transformation formula: $B_\omega^t e^{\omega t} = \delta(\alpha - \frac{1}{T})$.

Similarly, for the coupling constant $g'$ we have:

$$g' f_{-\frac{1}{2}} f'_{+\frac{1}{2}} = \frac{1}{8} F_\pi e^{\frac{\lambda}{T} + \frac{\lambda_0}{T} + \frac{1}{T}} \left\{ \frac{d^2}{du} \left( u \varphi_\pi(u) T^3 f_2(u_0) - 4u g_1(u) T f_0(u_0) + \frac{2}{3} \mu_\pi u \varphi_\sigma(u) T^2 f_1(u_0) \right) \right|_{u=u_0}$$

$$+ 4 \frac{d}{du} (u g_2(u)) \right|_{u=u_0} T f_0(u_0) \right\} ,$$

(93)

with the current (3), where $f_n(x) = 1 - e^{-x} \sum_{k=0}^n \frac{x^k}{k!}$ is the factor used to subtract the continuum. The derivative in (93) arises from the factor $q_\pi^2$ in (90). We have used integration by parts to absorb the factors $q \cdot v$ and $(q \cdot v)^2$. In this way we arrive at the simple form after double Borel transformation.

With the current (4) we have

$$g' f_{-\frac{1}{2}} f_{+\frac{1}{2}} = \frac{1}{4} F_\pi e^{\frac{\lambda}{T} + \frac{\lambda_0}{T} + \frac{1}{T}} \{- \varphi_\pi'(u_0) T^2 f_1(u_0) + 2 \mu_\pi \varphi_\sigma(u_0) T f_0(u_0) - 4 g_1'(u_0) \} ,$$

(94)
where \( \varphi'_a(u_0), g'_1(u_0) \) are the first derivatives of \( \varphi_a(u), g_1(u) \) at \( u = u_0 \).

For the \( \mathcal{O}(1/m_Q) \) correction to the decay amplitude for \( (j_\ell = \frac{3}{2}) \rightarrow (j_\ell = \frac{1}{2}) + \pi \), we need to consider the correlators \((83)\) and \((84)\). In calculations of these correlators enter the pion matrix elements of quark-gluon operators, which are parameterized in terms of pion wave functions defined in \( [8] \):

\[
< \pi(q) | \bar{d}(x) \sigma_{\alpha\beta} \gamma_5 g_s G_{\mu\nu}(ux) u(0) | 0 > =
\]

\[
f_{\pi}(q) \left[ \frac{g_\mu - \frac{q_\mu}{q \cdot x}}{g_\nu - \frac{q_\nu}{q \cdot x}} \right] \int D\alpha_i \varphi_{3\pi}(\alpha_i) e^{iqx(\alpha_1 + \alpha_2)} ,
\]

(95)

\[
< \pi(q) | \bar{d}(x) \gamma_\mu g_s G_{\alpha\beta}(vx) u(0) | 0 > =
\]

\[
f_{\pi}(q) \left[ \frac{g_\mu}{q \cdot x} \right] \int D\alpha_i \varphi_{3\pi}(\alpha_i) e^{iqx(\alpha_1 + \alpha_2)}
\]

and

\[
< \pi(q) | \bar{d}(x) \gamma_\mu g_s \tilde{G}_{\alpha\beta}(vx) u(0) | 0 > =
\]

\[
f_{\pi}(q) \left[ \frac{g_\mu}{q \cdot x} \right] \int D\alpha_i \varphi_{3\pi}(\alpha_i) e^{iqx(\alpha_1 + \alpha_2)} .
\]

(96)

(97)

The operator \( \tilde{G}_{\alpha\beta} \) is the dual of \( G_{\alpha\beta} \): \( \tilde{G}_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\rho\sigma} G^{\rho\sigma} \); \( D\alpha_i \) is defined as \( D\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3) \). The function \( \varphi_{3\pi} \) is of twist three, while all the wave functions appearing in eqs.\((96), (97)\) are of twist four. The wave functions \( \varphi(x_\mu, \mu) (\mu) \) is the renormalization point) describe the distribution in longitudinal momenta inside the pion, the parameters \( x_\mu \left( \sum x_\mu = 1 \right) \) representing the fractions of the longitudinal momentum carried by the quark and the antiquark.

The wave function normalizations immediately follow from the definitions \((86)-(87)\):

\[
j_0^1 du \varphi_a(u) = j_0^1 du \varphi_a(u) = 1, \quad j_0^1 du g_1(u) = \delta^2/12, \quad \int D\alpha_i \varphi_{3\pi}(\alpha_i) = \int D\alpha_i \varphi_{3\pi}(\alpha_i) = 0,
\]

\[
\int D\alpha_i \varphi_{1\pi}(\alpha_i) = - \int D\alpha_i \varphi_{2\pi}(\alpha_i) = \delta^2/3 ,
\]

with the parameter \( \delta \) defined by the matrix element:

\[
< \pi(q) | \tilde{g}_a \tilde{G}_{\alpha\beta} \gamma^\alpha u | 0 > = i\delta^2 f_{\pi} q_\mu .
\]

For sum rules for the \( 1/m_Q \) corrections we shall compare the double Borel transform of \((79)\) with those of \((83)\) and \((84)\) for the D-wave and S-wave part separately. It is convenient to confine \( k_\mu \) to be longitudinal, i.e., \( k_\mu^t = 0 \). For the D-wave amplitude we have:

\[
G_{1\pi}^D(\omega, \omega') = \frac{3}{\sqrt{2} m_Q} \int dt_1 dt_2 D\alpha_1 \alpha_2 \left[ -\frac{f_{\pi}}{\sqrt{2}} (q \cdot v) \varphi_{3\pi}(\alpha_1) + F_\pi \varphi_{3\pi}(\alpha_2) + \tilde{F}_\pi \varphi_{3\pi}(\alpha_2) \right] e^{i\epsilon v t} e^{-iqv(t_1 + t_2)}
\]

\[
\cdot e^{i\epsilon v t} e^{i(q x_1 + q x_2 + \mu_\pi \alpha_1)}
\]

\[
= \frac{3}{\sqrt{2} m_Q} G_{1\pi}(\omega, \omega') ,
\]

(98)

\[
G_{1\pi}(\omega, \omega') = \frac{3}{\sqrt{2} m_Q} G_{1\pi}(\omega, \omega') ,
\]

(99)

\[
G_{1\pi}^S(\omega, \omega') = \frac{3}{\sqrt{2} m_Q} G_{1\pi}(\omega, \omega') ,
\]

(100)
where \((k - uq)^2 = -\frac{m^2}{4}(\omega - \omega')^2\).

After double Borel transformation we have:

\[
\frac{\lambda}{4} \left\{ -\frac{\sqrt{6}}{3} \frac{F_{\pi}}{m_Q} \int \mathcal{D} \alpha_i \varphi_3^\perp (\alpha_i) + \frac{1}{2} \tilde{\varphi}_{\parallel} (\alpha_i) \right\} - \frac{\sqrt{3}}{6} \frac{F_{\pi}}{m_Q} \int \mathcal{D} \alpha_i \frac{1}{\alpha_3} \frac{d}{d\alpha_2} \left( \alpha_2 \varphi_{3\pi} (\alpha_i) \right) T f_0 (\frac{\omega}{T}) + \frac{\sqrt{6}}{16} \frac{F_{\pi}}{m_Q} K(u_0, T),
\]

where the function \(K(u_0, T)\) arises from the kinetic operator \(\mathcal{K}\), which is defined as:

\[
K(u_0, T) \equiv \frac{d^2}{du^2} \left( u^3 \varphi_\pi (u) T^2 f_1 (\frac{\omega}{T}) - 4u^3 g_1 (u) + \frac{1}{3} \mu_\pi u^3 \varphi_\sigma (u) T f_0 (\frac{\omega}{T}) \right) |_{u = u_0} + 4 \frac{d}{du} (u^3 g_2 (u)) |_{u = u_0}.
\]

In order to extract \(g_{1d}\) we have to use the operator \(\frac{d}{dt} \{T \times (101)\}\).

For the S-wave part we find:

\[
\frac{\lambda}{4} \left\{ -\frac{\sqrt{6}}{3} \frac{F_{\pi}}{m_Q} q^2 \int \mathcal{D} \alpha_i \varphi_3^\perp (\alpha_i) + \frac{1}{2} \tilde{\varphi}_{\parallel} (\alpha_i) \right\} - \frac{\sqrt{3}}{6} \frac{F_{\pi}}{m_Q} q^2 \int \mathcal{D} \alpha_i \frac{1}{\alpha_3} \frac{d}{d\alpha_2} \left( \alpha_2 \varphi_{3\pi} (\alpha_i) \right) T f_0 (\frac{\omega}{T}) + \frac{\beta}{8} F_{\pi} \left\{ \frac{d^2}{du^2} \left( u \varphi_\pi (u) T^3 f_2 (\frac{\omega}{T}) - 4u g_1 (u) T f_0 (\frac{\omega}{T}) + \frac{1}{3} \mu_\pi u \varphi_\sigma (u) T^2 f_1 (\frac{\omega}{T}) \right) |_{u = u_0} + 4 \frac{d}{du} (u g_2 (u)) |_{u = u_0} T f_0 (\frac{\omega}{T}) \right\}.
\]

where \(\beta\) is defined in section \(3\) and its value is found through (52).

### 7 Sum rules for the pionic decay amplitudes of \(B_2^*\) up to the order of \(\mathcal{O}(1/m_Q)\)

In the following we shall consider the \(\mathcal{O}(1/m_Q)\) correction to the pionic decay amplitude of \(B_2^*\) meson. There are two decay channels. The decay amplitudes are:

\[
M(B_2^* \to B \pi) = I \eta_{\mu \nu} q^\mu \bar{q}^\nu [g^a(B_2^*, B) + g^1_1(B_2^*, B)]^\dagger,
\]

\[
M(B_2^* \to B^* \pi) = I \epsilon^a_{\alpha \beta \rho} \epsilon^{\alpha \beta \rho} q^\mu \bar{q}^\mu [g^b(B_2^*, B^*) + g^1_1(B_2^*, B^*)]^\dagger,
\]

where \(\eta_{\mu \nu}\) is the polarization tensor of \(B_2^*\) meson, \(g^a, g^b\) are the coupling constants at the leading order of \(1/m_Q\) and \(g^1_1, g^1_1\) is the \(\mathcal{O}(1/m_Q)\) correction to \(g^a, g^b\). It can be shown \([4]\) by combining heavy quark symmetry and chiral symmetry that at the leading order of \(1/m_Q\) the coupling constants in (74), (104) and (105) satisfy

\[
g^a(B_2^*, B) = g^b(B_2^*, B^*) = \sqrt{2} \frac{3}{3} g(B_1, B^*)^\dagger.
\]

Angular momentum conservation requires that both operators \(\mathcal{K}\) and \(\mathcal{S}\) contribute to the D-wave amplitude only, which is confirmed through explicit calculation. Moreover we do not consider mixing effects for \(B_2^*\) meson, which is expected to be small.
Since the detailed calculation is very similar to that for $B_1$ meson, we present the final results only:

$$
[r_0, u_{-1, \frac{1}{2}} + r_2, u_{+1, \frac{3}{2}}]g^a + g_i^a - \frac{\delta m_1 + \delta m_2}{T} g_i^a) \tilde{f}_{-1, \frac{1}{2}}f_{+1, \frac{3}{2}} = e^{-\frac{1}{2} + a_{\frac{1}{2}, \frac{3}{2}}} \left\{ D \alpha \frac{c}{m_Q} \left[ \tilde{d}_{\alpha \beta} \alpha_1 (\alpha_2) - \frac{1}{2} \tilde{d}_i (\alpha_i) \right] - \frac{\sqrt{7} T \psi}{4} m_Q \int D \alpha \frac{d}{d\alpha} (\alpha_2 \tilde{d}_{\alpha \beta} (\alpha_1)) T f_0 (\tilde{\psi}) + \frac{1}{8} m_Q F \bar{K}(u_0, T) \right\}, \tag{107}
$$

$$
[r_1, u_{-1, \frac{1}{2}} + r_2, u_{+1, \frac{3}{2}}]g^b + g_i^b - \frac{\delta m_1 + \delta m_2}{T} g_i^b) \tilde{f}_{-1, \frac{1}{2}}f_{+1, \frac{3}{2}} = \frac{1}{2} e^{-\frac{1}{2} + a_{\frac{1}{2}, \frac{3}{2}}} \left\{ D \alpha \frac{c}{m_Q} \tilde{d}_{\alpha \beta} \alpha_1 (\alpha_2) + \frac{1}{4} K(u_0, T) \right\}, \tag{108}
$$

where the function $K(u_0, T)$ is defined in (102).

Note that the corrections to the leading order coupling constants from the kinetic operator $K$ satisfy the same relation as (106), which is an expected result since $K$ does not depend on the spin.

8 Numerical analysis of the sum rules

In the numerical analysis we shall take $T_1 = T_2 = 2T$ and $u_0 = 1/2$ as done in [8] and [9]. Although the initial and final heavy meson states are different in their masses, this seems to be a reasonable approximation in view of the large value of $T_1$ and $T_2$ used below.

Note that our sum rules depend on the pion wave functions and the integrals or derivatives of them at the point $u_0 = \frac{1}{2}$. Since $\delta^2$ is numerically small, the uncertainty due to the integral term $G_2(u_0)$ is insignificant. The values of the wave functions at $u_0$ have been determined quite well and their dependence on the renormalization scale is very weak [8]. The first derivatives at $u_0$ are zero. The second derivatives are very sensitive to the detailed shape of the wave functions. Usually only the lowest few moments of the wave functions can be determined by QCD sum rules. There are many wave functions satisfying these constraints from moments. With the form of wave functions in [8], the final sum rules are sensitive to the renormalization scale and the coefficients $a_i$ in $\varphi_\pi(u)$ due to its large value. This fact indicates that the not-well-known nonperturbative contribution from condensates in the QCD vacuum and the even higher twist wave functions might be important. In order to estimate the corrections from the above sources we present the detailed expressions of the pion wave functions and their second derivatives in order to show the dependence on the parameters $\omega_{i,j}$ in them.

There are many discussions about $\varphi_\pi(u)$ in literature [12, 13, 8, 14, 13, 16]. The model wave function for $\varphi_\pi(u)$ based on the QCD sum rule approach was given in [8] as:

$$
\varphi_\pi(u, \mu) = 6u\bar{u} \left(1 + a_2(\mu) \frac{3}{2} [5(u - \bar{u})^2 - 1] + a_4(\mu) \frac{15}{8} [21(u - \bar{u})^4 - 14(u - \bar{u})^2 + 1] \right). \tag{109}
$$

Yet the authors pointed out that the oscillations of (109) around $u_0 = \frac{1}{2}$ is unphysical, which is due to the truncation of the series and keeping only the first a few terms when $\varphi_\pi(u)$ is
expanded over Gegenbauer polynomials. In [13] it was stressed that: (1) the expansion over Gegenbauer polynomials converges very slowly as can be seen from the large value of \(a_2\) and \(a_4\); (2) any oscillating wave function is not physical, since no detached scale is seen to govern such oscillations. Recently Mikhailov and Radyushkin [16] reanalysed the QCD sum rules for \(\varphi_\pi(u)\) taking into account the nonlocality of the condensates. They suggested the following wave function:

\[
\varphi_\pi(u) = \frac{8}{\pi} \sqrt{u(1-u)},
\]

which is close to the asymptotic form and the suggested pion wave function in [15]:

\[
\varphi_\pi(u) = N \exp \left( -\frac{m^2}{8\beta^2 u(1-u)} \right) \left[ \mu^2 + \mu \bar{\mu} + (\bar{\mu}^2 - 2)u(1-u) \right],
\]

where \(N = 4.53\), \(\mu = \frac{m}{2\beta}\), \(\bar{\mu} = \frac{1}{2\beta} \left( \frac{1}{4}m_\pi + \frac{2}{3}m_\rho \right) = \frac{\bar{m}}{2\beta}\) with \(m = 330\text{MeV}\), \(\bar{m} = 620\text{MeV}\), and \(\beta = 320\text{MeV}\). The Brodsky-Huang-Lepage wave function is also close to the asymptotic form except for the different behavior at end points [14]:

\[
\varphi_\pi(u) = 9.05u(1-u) \exp \left( -\frac{M^2}{8\alpha^2 u(1-u)} \right),
\]

where \(M = 289\text{MeV}\) and \(\alpha = 385\text{MeV}\). The asymptotic form of \(\varphi_\pi(u)\) reads:

\[
\varphi_\pi^{\text{asym}}(u) = 6u(1-u).
\]

For the other pion wave functions we use the results given in [8]. The detailed expressions of the pion wave functions relevant in our calculation are:

\[
\varphi_{3\pi}(\alpha_i) = 360\alpha_1\alpha_2\alpha_3 \left[ 1 + \omega_{1,0} \frac{1}{2}(7\alpha_3 - 3) + \omega_{2,0}(2 - 4\alpha_1\alpha_2 - 8\alpha_3 + 8\alpha_3^2) + \omega_{1,1}(3\alpha_1\alpha_2 - 2\alpha_3 + 3\alpha_3^2) \right],
\]

\[
\varphi_P(u, \mu) = 1 + B_2(\mu) \frac{1}{2} [3(u - \bar{u})^2 - 1] + B_4(\mu) \frac{1}{8} [35(u - \bar{u})^4 - 30(u - \bar{u})^2 + 3],
\]

\[
\varphi_\sigma(u, \mu) = 6u\bar{u} \left( 1 + C_2(\mu) \frac{3}{2} [5(u - \bar{u})^2 - 1] + C_4(\mu) \frac{15}{8} [21(u - \bar{u})^4 - 14(u - \bar{u})^2 + 1] \right),
\]

\[
\varphi_\perp(\alpha_i) = 30\delta^2(\alpha_1 - \alpha_2)\alpha_3^2 \left[ \frac{1}{3} + 2\epsilon(1 - 2\alpha_3) \right],
\]

\[
\varphi_{\parallel}(\alpha_i) = -120\delta^2\alpha_1\alpha_2\alpha_3 \left[ \frac{1}{3} + \epsilon(1 - 2\alpha_3) \right],
\]

\[
g_1(u) = \frac{5}{2} \delta^2 \bar{u}^2 u^2 + \frac{1}{2} \epsilon \delta^2 [\bar{u}u(2 + 13u\bar{u}) + 10u^3 \ln u(2 - 3u + \frac{6}{5}u^2) + 10\bar{u}^3 \ln \bar{u}(2 - 3\bar{u} + \frac{6}{5}\bar{u}^2)],
\]

\[18\]
where \( \bar{u} = 1 - u \), \( B_2 = 30R \), \( B_4 = \frac{3}{2} R (4\omega_{2,0} - \omega_{1,1} - 2\omega_{1,0}) \), \( C_2 = R (5 - \frac{1}{2} \omega_{1,0}) \), \( C_4 = \frac{1}{10} R (4\omega_{2,0} - \omega_{1,1}) \) and \( R = \frac{f_{3\pi}}{m_{\pi}^2} \). The coefficients \( f_{3\pi} \), \( \omega_{i,k} \), \( \delta^2 \) and \( \epsilon \) have been determined from QCD sum rules.

We shall take the scale \( \mu = \frac{3}{4} \omega_c = 2.4 \text{GeV} \), at which the various parameters are: \( a_2 = 0.35 \), \( a_4 = 0.18 \), \( \omega_{1,0} = -2.18 \), \( \omega_{2,0} = 8.12 \), \( \omega_{1,1} = -2.59 \), \( f_{3\pi} = 0.0026 \text{GeV}^2 \), \( \mu_\pi = 2.02 \text{GeV} \), \( \delta^2 = 0.17 \text{GeV}^2 \), \( \epsilon = 0.36 \).

\[
g_2(u) = \frac{10}{3} \delta^2 \bar{u}u(u - \bar{u}) , \tag{120}
\]
\[
G_2(u) = -\frac{10}{3} \delta^2 \bar{u}u (\frac{2}{5} u^2 - \frac{3}{4} u + \frac{1}{3}) , \tag{121}
\]

where \( \bar{u} = 1 - u \), \( B_2 = 30R \), \( B_4 = \frac{3}{2} R (4\omega_{2,0} - \omega_{1,1} - 2\omega_{1,0}) \), \( C_2 = R (5 - \frac{1}{2} \omega_{1,0}) \), \( C_4 = \frac{1}{10} R (4\omega_{2,0} - \omega_{1,1}) \) and \( R = \frac{f_{3\pi}}{m_{\pi}^2} \). The coefficients \( f_{3\pi} \), \( \omega_{i,k} \), \( \delta^2 \) and \( \epsilon \) have been determined from QCD sum rules.

We shall take the scale \( \mu = \frac{3}{4} \omega_c = 2.4 \text{GeV} \), at which the various parameters are: \( a_2 = 0.35 \), \( a_4 = 0.18 \), \( \omega_{1,0} = -2.18 \), \( \omega_{2,0} = 8.12 \), \( \omega_{1,1} = -2.59 \), \( f_{3\pi} = 0.0026 \text{GeV}^2 \), \( \mu_\pi = 2.02 \text{GeV} \), \( \delta^2 = 0.17 \text{GeV}^2 \), \( \epsilon = 0.36 \).

\[
\varphi_\pi(u_0) = \frac{3}{2} (1 - \frac{3}{2} a_2 + \frac{15}{8} a_4) , \tag{122}
\]
\[
\varphi_P(u_0) = 1 - \frac{1}{2} B_2 + \frac{3}{8} B_4 , \tag{123}
\]
\[
\varphi_\sigma(u_0) = \frac{3}{2} (1 - \frac{3}{2} C_2 + \frac{15}{8} C_4) , \tag{124}
\]
\[
g_1(u_0) = (\frac{5}{32} - 0.037 \epsilon) \delta^2 , \tag{125}
\]
\[
G_2(u_0) = -\frac{7}{288} \delta^2 , \tag{126}
\]
\[
\varphi''_\pi(u_0) = -12 (1 - 9 a_2 + \frac{225}{8} a_4) , \tag{127}
\]
\[
\varphi''_\sigma(u_0) = -12 (1 - 9 C_2 + \frac{225}{8} C_4) , \tag{128}
\]
\[
g_1''(u_0) = -(2.5 + 7.4 \epsilon) \delta^2 , \tag{129}
\]
\[
g_2'(u_0) = \frac{5}{3} \delta^2 , \tag{130}
\]
\[
\int D\alpha_1 \frac{d}{d\alpha_3} \varphi_\perp(\alpha_i) = -\frac{\delta^2}{12} (1 + 2 \epsilon) , \tag{131}
\]
\[
\int D\alpha_1 \frac{d}{d\alpha_3} \varphi_\parallel(\alpha_i) = -\frac{4}{3} + \epsilon \delta^2 , \tag{132}
\]
\[
\int D\alpha_1 \frac{1}{d\alpha_3} \frac{d}{d\alpha_2} [\alpha_2 \varphi_3(\alpha_i)] = 6 - 2 \omega_{1,0} - \frac{4}{7} \omega_{2,0} + \frac{8}{7} \omega_{1,1} , \tag{133}
\]

where the parameters \( a_i, B_i, C_i, \omega_{i,j} \) and \( \epsilon \) indicate the deviation from the asymptotic form of pion wave functions.

At \( u_0 = \frac{1}{2} \) the values of the functions appearing in \( \text{(92)-(107)} \) are: \( \varphi_\pi(u_0) = 1.22, 1.273, 1.25, 1.71, 1.5 \) for the wave functions \( \text{(109)-(113)} \) respectively. \( \varphi_P(u_0) = 1.07, \varphi_\sigma(u_0) = 1.55, g_1(u_0) = 0.022 \text{GeV}^2, g_2(u_0) = 0 \) and \( G_2(u_0) = -4.1 \times 10^{-3} \text{GeV}^2 \). Their first derivatives satisfy: \( \varphi''_\pi(u_0) = \varphi''_P(u_0) = \varphi''_\sigma(u_0) = g_1''(u_0) = G_2'(u_0) = 0 \). And their second derivatives are: \( \varphi''_\pi(u_0) = -35, -5.09, -0.02, -17.50, -12 \) for \( \text{(109)-(113)} \) respectively, \( \varphi''_\sigma(u_0) = -17, g_1''(u_0) = -0.88 \text{GeV}^2, g_2''(u_0) = 0.28 \text{GeV} \).
Note the theoretical side of the sum rules (101), (107) and (108) depends on \( \varphi_\pi(u) \) only through the combination \([u^3\varphi_\pi(u)]''_{u_0} = 3\varphi_\pi(u_0) + \frac{9}{4}\varphi_\pi''(u_0)\) in \( K(u_0, T) \). Its value is 3.18, 3.75, 2.93 and 3 for (101)-(113) respectively, which is relatively close to each other. We shall present numerical results using the wave function (112).

The dependence on the Borel parameter \( T \) of \( f_{-\frac{1}{2}}f_{+,\frac{1}{2}}g \), \( f_{-\frac{1}{2}}f_{+,\frac{1}{2}}g' \), \( m_Q f_{-\frac{1}{2}}f_{+,\frac{1}{2}}g_{1s} \), \( m_Q f_{-\frac{1}{2}}f_{+,\frac{1}{2}} \left( (r_{1, -\frac{1}{2}} + r_{1, +\frac{1}{2}})g + g_{1d} \right) \), \( m_Q f_{-\frac{1}{2}}f_{+,\frac{1}{2}} \left( (r_{0, -\frac{1}{2}} + r_{2, +\frac{1}{2}})g^a + g_1^f \right) \), and \( m_Q f_{-\frac{1}{2}}f_{+,\frac{1}{2}} \left( (r_{1, -\frac{1}{2}} + r_{2, +\frac{1}{2}})g^b + g_1^b \right) \) are shown in Fig. 4-10 with \( \omega_c = 3.2, 3.0, 2.8 \) GeV and the pion wave function (112). Requiring that the higher twist contribution is less than 30% of the whole sum rule, we get the lower limit for the Borel parameter \( T, T \geq 1.0 \) GeV. Requiring that the continuum contribution is less than 40% of the whole sum rule, we get the upper limit, \( T \leq 2.5 \) GeV.

Our results read:

\[
f_{-\frac{1}{2}}f_{+,\frac{1}{2}}g = -(0.23 \pm 0.02 \pm 0.01) \text{ GeV}^3, \tag{134}
\]

\[
f_{-\frac{1}{2}}f_{+,\frac{1}{2}}g' = -(0.41 \pm 0.03 \pm 0.04) \text{ GeV}^5 \tag{135}
\]

with the current (3), or

\[
f_{-\frac{1}{2}}f_{+,\frac{1}{2}}g' = (0.41 \pm 0.03 \pm 0.01) \text{ GeV}^4 \tag{136}
\]

with the current (4),

\[
f_{-\frac{1}{2}}f_{+,\frac{1}{2}}g_{1s} = \frac{0.038 \pm 0.005 \pm 0.006}{m_Q} \text{ GeV}^5 \tag{137}
\]

\[
f_{-\frac{1}{2}}f_{+,\frac{1}{2}} \left[ (r_{1, -\frac{1}{2}} + r_{1, +\frac{1}{2}})g + g_{1d} \right] = \frac{0.21 \pm 0.04 \pm 0.02}{m_Q} \text{ GeV}^3, \tag{138}
\]

\[
f_{-\frac{1}{2}}f_{+,\frac{1}{2}} \left[ (r_{0, -\frac{1}{2}} + r_{2, +\frac{1}{2}})g^a + g_1^f \right] = \frac{0.17 \pm 0.01 \pm 0.02}{m_Q} \text{ GeV}^3, \tag{139}
\]

\[
f_{-\frac{1}{2}}f_{+,\frac{1}{2}} \left[ (r_{1, -\frac{1}{2}} + r_{2, +\frac{1}{2}})g^b + g_1^b \right] = \frac{0.17 \pm 0.01 \pm 0.02}{m_Q} \text{ GeV}^3. \tag{140}
\]

The central values correspond to \( T = 1.3 \) GeV and \( \omega_c = 3.0 \) GeV. The first and second error refers to the variation with \( T \) and \( \omega_c \) respectively. The inherent uncertainties due to the method of QCD sum rules and the pion wave functions are not included here.

With the central values of \( f' \)'s in (106)-(108) and the values of \( r \)'s in section 3, we arrive at:

\[
g = -(4.83 \pm 0.4 \pm 0.2) \text{ GeV}^{-2}, \tag{141}
\]

\[
g' = -(4.43 \pm 0.33 \pm 0.44) \tag{142}
\]

with the current (3), or

\[
g' = -(4.1 \pm 0.3 \pm 0.3) \tag{143}
\]

with the current (4),

\[
g_{1s} = \frac{0.77 \pm 0.19 \pm 0.19}{m_Q}, \tag{144}
\]
The leading order coupling constant \( g \) is about 9\% for the pionic widths of mesons in the \( (1^+, 2^+) \) doublet and discussions.

From the above results, it is straightforward to derive the decay widths for \( B_1 \) and \( B_2^* \). Summing over charged and neutral pion channels the decay widths are:

\[
\Gamma(B_1 \to B^* \pi) = \frac{1}{12\pi}(g^2 + 2gg_{1d})|\vec{q}|^5 + \frac{3}{8\pi}g_{1s}^2|\vec{q}|^5 , \tag{148}
\]

\[
\Gamma(B_2^* \to B \pi) = \frac{1}{20\pi}(g_s^2 + 2g^a g_1^a)|\vec{q}|^5 , \tag{149}
\]

\[
\Gamma(B_2^* \to B^* \pi) = \frac{3}{4\pi}(g_s^2 + 2g^b g_1^b)|\vec{q}|^5 . \tag{150}
\]

The last term in (148) is formally of \( \mathcal{O}(1/m_Q^2) \). However, due to the large space phase factor for the S-wave decay it is numerically not small.

The masses of \( B_1, B_2^* \) are not known experimentally. The decay width is sensitive to the \( B_1, B_2^* \) masses. If we use the pion wave function (112) in [14], the decay widths are:

\[
\Gamma(B_1 \to B^* \pi) = 7.0 \times \left( \frac{|\vec{q}|}{395\text{MeV}} \right)^5 + 1.2 \times \left( \frac{\beta}{0.02} \right)^2 \left( \frac{|\vec{q}|}{395\text{MeV}} \right) \text{MeV} , \tag{151}
\]
\[ \Gamma(B_2^* \to B\pi) = 5.4 \times \left( \frac{|q|}{450\text{MeV}} \right)^5 \text{MeV}, \] (152)

\[ \Gamma(B_2^* \to B^*\pi) = 4.9 \times \left( \frac{|q|}{405\text{MeV}} \right)^5 \text{MeV}, \] (153)

where the decay momentum \(|q|\) is in unit of MeV respectively.

If we assume the heavy quark expansion is also good for the charm system, the decay widths of \(D_1\) and \(D_2^*\) are:

\[ \Gamma(D_1 \to D^*\pi) = 6.2 + 9.7 \times \left( \frac{\beta}{0.06} \right)^2 \text{MeV}, \] (154)

\[ \Gamma(D_2^* \to D\pi) = 13.7 \text{ MeV}, \] (155)

\[ \Gamma(D_2^* \to D^*\pi) = 6.1 \text{ MeV}, \] (156)

where we have used the decay momenta \(|q| = 355, 503\) and \(387\) MeV respectively for the above three processes listed in [24]. Experimentally, the total decay widths of \(D_1\) and \(D_2\) are \(18.9^{+4.6}_{-3.3}\)MeV and \(23^{+5}_{-5}\)MeV respectively.

In summary, in this work we have improved previous results about the kinetic energy \(K\) and chromomagnetic splitting \(\Sigma\) of the excited heavy mesons in HQET. We have carried out a systematic expansion up to the order \(\mathcal{O}(1/m_Q)\) for the pionic decay amplitudes of \(B_1\) and \(B_2^*\) in the framework of HQET and light-cone sum rules. The mixing of the two \(1^+\) states has been properly taken into account. The \(\mathcal{O}(1/m_Q)\) correction to the decay amplitudes of \(B_1\) and \(B_2^*\) derived from the light-cone sum rule is small. When applied to \(D_1\) and \(D_2^*\) the central theoretical values are in good agreement with the experimental data, though the theoretical uncertainty is sizable for the \(D\) system.

Acknowledgements: S.Z. was supported by the National Postdoctoral Science Foundation of China and Y.D. was supported by the National Natural Science Foundation of China.
References

[1] B. Grinstein, Nucl. Phys. B339, 253(1990); E. Eichten and B. Hill, Phys. Lett. B234, 511(1990); A. F. Falk, H. Georgi, B. Grinstein and M. B. Wise, Nucl. Phys. B343, 1(1990); F. Hussain, J. G. Körner, K. Schilcher, G. Thompson and Y. L. Wu, Phys. Lett. B249, 295(1990); J. G. Körner and G. Thompson, Phys. Lett. B264, 185(1991).

[2] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B 147, 385, 448, 519(1979).

[3] E. Bagan, V.M. Brown and H.G. Dosch, Phys.Lett. B278,457(1992); M. Neubert, Phys. Rev. D45, 2451(1992); Phys. Rep. 245, 259(1994); D.J. Broadhurst and A.G. Grozin, Phys. Lett. B274, 421(1992).

[4] Y. B. Dai, C. S. Huang, M. Q. Huang and C. Liu, Phys. Lett. B390, 350(1997); Y. B. Dai, C. S. Huang and M. Q. Huang, Phys. Rev. D 55, 5719(1997).

[5] Y.B.Dai et al., report BIHEP-TH-97-005, [hep-ph/9705223].

[6] P. Ball and V. M. Braun, Phys. Rev. D 49, 2472(1994).

[7] P. Colangelo, G. Nardulli, A. A. Ovchinnikov and N. Paver, Phys. Lett. B269, 204(1991); P. Colangelo, G. Nardulli and N. Paver, Phys. Lett. B293, 207(1992); P. Colangelo, F. De Fazio, G. Nardulli, N. Di Bartolomeo and R. Gatto, Phys. Rev. D 52, 6422(1995).

[8] I. I. Balitsky, V. M. Braun and A. V. Kolesnichenko, Nucl. Phys. B 312, 509 (1989); V. M. Braun and I. E. Filyanov, Z. Phys. C 44, 157(1989); V. M. Braun and I. E. Filyanov, Z. Phys. C 48, 239(1990); V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Rückl, Phys. Rev. D 51, 6177(1995).

[9] P. Colangelo and F. De Fazio, report BARI-TH/97-250, [hep-ph/9706271], ( to appear in Euro. Phys. J. C ).

[10] Yuan-Ben Dai and Shi-Lin Zhu, [hep-ph/9802227], ( to appear in Euro. Phys. J. C ).

[11] Shi-Lin Zhu and Yuan-Ben Dai, [hep-ph/9802226], Phys. Lett. B 429, 72 (1998).

[12] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. 112, 173(1984).

[13] Z. Dziembowsky and L. Mankiewicz, Phys. Rev. Lett. 58, 2175(1987).

[14] S. J. Brodsky, T. Huang, and G. P. Lepage, in Quarks and Nuclear Forces, edited by D. Fries and B. Zeitnitz, Springer Tracts in Modern Physics, Vol. 100 (Springer-Verlag, New York, 1982).

[15] I. Halperin, Z. Phys. C56, 615(1992).

[16] S. V. Mikhailov and A. V. Radyushkin, Phys. Rev. D 45, 1754(1992).
[17] A. Falk and M. Luke, Phys. Lett. B292, 119(1992); U. Kilian, J.G. Körner, and D. Pirjol, Phys. Lett. B288, 360(1992).

[18] E. Bagan, P. Ball, V. M. Braun and H. G. Dosch, Phys. Lett. B278, 457(1992); M. Neubert, Phys. Rev. D 45, 2451(1992); D. J. Broadhurst and A. G. Grozin, Phys. Lett. B274, 421(1992).

[19] M. Neubert, Phys. Rev. D 42, 1076(1992).

[20] P. Ball, Nucl. Phys. B 421, 593(1994).

[21] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. 112, 173(1984).

[22] V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 237, 525(1984).

[23] V. M. Braun and I. B. Filyanov, Z. Phys. C 48, 239(1990).

[24] Particle Data Group, R. M. Barnett et. al., Phys. Rev. D 54, 1(1996).
Figure Captions

Fig. 1. The sum rules for $G_K$ as a functions of the Borel parameter $T$ for different values of the continuum threshold $\omega_c$. From bottom to top the curves correspond to $\omega_c = 3.2, 3.0, 2.8$ GeV respectively. The unit of $T$, $G_K$ is GeV.

Fig. 2. The sum rules for $G_\Sigma$ with $\omega_c = 3.2, 3.0, 2.8$ GeV with the curves from top to bottom.

Fig. 3. The sum rules for $G_{\frac{1}{2}, \frac{3}{2}}/f_{+, \frac{1}{2}}$ with $\omega_c = 3.2, 3.0, 2.8$ GeV.

Fig. 4. The sum rules for $f_{+, \frac{1}{2}}^\frac{1}{2} f_{+, \frac{3}{2}}^\frac{1}{2} g$ with $\omega_c = 3.2, 3.0, 2.8$ GeV with the pion wave function in (112) in [14].

Fig. 5. The sum rules for $f_{-, \frac{1}{2}} f_{+, \frac{1}{2}} g'$ with the current (3) and $\omega_c = 3.2, 3.0, 2.8$ GeV.

Fig. 6. The sum rules for $f_{-, \frac{1}{2}} f_{+, \frac{1}{2}} g'$ with the current (2) $\omega_c = 3.2, 3.0, 2.8$ GeV.

Fig. 7. The sum rules for $m_Q f_{-, \frac{1}{2}} f_{+, \frac{3}{2}} g_{1s}$ with $\omega_c = 3.2, 3.0, 2.8$ GeV.

Fig. 8. The sum rules for $m_Q f_{-, \frac{1}{2}} f_{+, \frac{3}{2}} \left( (r_{1, -\frac{1}{2}} + r_{1, +\frac{3}{2}}) g + g_{1d} \right)$ with $\omega_c = 3.2, 3.0, 2.8$ GeV.

Fig. 9. The sum rules for $m_Q f_{-, \frac{1}{2}} f_{+, \frac{3}{2}} \left( (r_{0, -\frac{1}{2}} + r_{2, +\frac{1}{2}}) g^a + g_1^a \right)$ with $\omega_c = 3.2, 3.0, 2.8$ GeV.

Fig. 10. The sum rules for $m_Q f_{-, \frac{1}{2}} f_{+, \frac{3}{2}} \left( (r_{1, -\frac{1}{2}} + r_{2, +\frac{3}{2}}) g^b + g_1^b \right)$ with $\omega_c = 3.2, 3.0, 2.8$ GeV.
\(\omega_c = 2.8\) GeV
\(\omega_c = 3.0\) GeV
\(\omega_c = 3.2\) GeV