A New Mechanism for Single Spin Asymmetries in Strong Interactions

N.I. Kochelev

Bogoliubov Laboratory of Theoretical Physics,
Joint Institute for Nuclear Research,
Dubna, Moscow region, 141980 Russia

Abstract

It is shown that the contribution of the instantons to the fragmentation of quarks leads to the appearance of a imaginary part in diagrams of quark-quark scattering at large transfer momentum. The imaginary part comes from the analytical continuation of the instanton amplitudes from Euclidean to Minkowsky space-time and reflects quasiclassical origin of instanton solution of QCD equations of motion. This phenomenon and instanton--induced quark spin-flip give a new nonperturbative mechanism for the observed anomalous single-spin asymmetries in hadron-hadron and lepton-hadron interactions.

Pacs: 12.39-x, 13.60.Hb, 14.65-q, 14.70Dj

Keywords: spin, pion, quark, gluon, instanton, asymmetry.

1 Introduction

The explanation of the large observed single-spin asymmetries (SSA) at large energies and momentum transfers in many inclusive and exclusive processes is one of most longstanding and outstanding problems in QCD [1],[2]. From naive point of view one can expect that with increasing of the energy and the momentum transfer the role of the quark spin in strong interaction should become smaller. At the same time the experimental data on spin-dependent cross sections reveal the opposite tendency: the spin asymmetries do not disappear at large energy showing in fact an anomalous growth with increasing of the momentum transfer.

Apart from our understanding of the origin of the large spin effects in QCD , the investigation of the fundamental mechanism which is responsible for SSA is also very important in view of future spin measurements at Brookhaven (RHIC-Spin Collaboration), CERN (COMPASS) and DESY (HERA-\vec{N}).

Within leading order of perturbative QCD it is impossible to obtain the large SSA because their values should be proportional to the current quark masses and decrease with energy and momentum transfer [1]. Additional suppression factors are those related with the loop integration generating the imaginary part of the amplitude and an extra power of $\alpha_s$.

The several attempts to explain the observed SSA have been undertaken. In [2] it was mentioned that twist-3 contribution can be important to explain this puzzle. Recently in [3] the convolution formula for single-spin asymmetry which include twist-3 quark-gluon correlation function has been obtained and the single-spin asymmetries for the pion production have been estimated. The main problem of this approach is unknown spin-dependent twist-3 distribution functions which in general are the functions of two variables and present the nonperturbative
part of the convolution formula together with rather well known twist-2 distribution functions of the partons in nonpolarized nucleon. These twist-3 distribution functions should be either extracted from other experiments or calculated within some nonperturbative approach.

There are several phenomenological approaches which also take into account the nonperturbative effects on single-spin asymmetries \[1\]. Some of them are using assumptions about quark transverse momenta in the distribution function (Sivers effect \[7\]) or in the quark fragmentation function (Collins effect \[8\]). In \[3\] the attempt to combine these two mechanisms for SSA has been made.

However all of these approaches are based on the phenomenological ways to introduce nonperturbative effects into SSA problem. The most of the parameters of these models were obtained from the fit of the available SSA experimental data, therefore the predictable power of such models is rather low.

In this Letter a new mechanism for single-spin asymmetries in strong interaction is suggested. This mechanism is based on the existence in the QCD vacuum of the strong nonperturbative fluctuations of gluon fields, so-called instantons (see recent review \[10\]). The instanton model of QCD vacuum describes very well not only the main properties of the vacuum state e.g. the values of the different quark and gluon condensates but it is also rather successful in the description of the hadron spectroscopy (see recent review \[10\] and references therein). Recent results of the lattice QCD \[11\] confirm the importance of the instantons in QCD vacuum.

The importance of the instantons for spin physics is related to their specific role in chiral structure of QCD vacuum. Thus the instantons describe subbarrier transitions between various classical minima of the QCD potential which correspond to different values of the axial vector charge. The changing of the value of the axial vector charge due to instanton transition leads simultaneously to the quark chirality flip. In \[12\] was mentioned that the quark chirality flip induced by instantons may give the natural explanation of the anomalous spin effects in strong interaction. In particular the instanton solution of famous “spin crisis” \[13\] has been suggested \[14\].

We will show below that the instanton leads to a precise behavior of effective quark-instanton vertices as functions of the incoming quark virtualities, which will be responsible for the magnitude of the SSA. More specifically an imaginary part arises for time-like virtualities of the quark in the diagrams induced by instantons, which is not suppressed at high energy and whose contribution enhances significantly the SSA.

2 Single-spin asymmetries in \(\pi\) meson production and instantons

Let us estimate the instanton contribution to the SSA for hadron production in quark-quark scattering. For definiteness we study \(\pi^+\) meson production in the scattering of two u-quarks, one of them transversely polarized. Our goal is to explain the large SSA in the fragmentation region of the polarized quark at high energies. In this kinematical regime only the diagrams of Fig.1 can contribute significantly \[1\]. The method for calculating these diagrams is standard (see for example \[15\]). The single spin asymmetry can be written in the following form \[16\]

\[
A = \frac{2Im(\Phi_5^*(\Phi_1 + \Phi_3))}{|\Phi_1|^2 + |\Phi_3|^2 + 4|\Phi_5|^2},
\]

\[1\] We assume that instanton induced quark-quark interaction determines the strength of \(\pi qq\) vertex. This is one of the important consequences of instanton model (see \[10\] and references therein).
Figure 1: The contribution of the instanton to the amplitude for $\pi^+$ production in the fragmentation region of the polarized quark in the scattering of two $u$-quarks. The label $I$ denotes instanton.

where we have neglected the contribution which comes from double spin-flip amplitudes $\Phi_2$ and $\Phi_4$, which are suppressed by factor $(m_q/\sqrt{S})$ with respect to the leading contributions; $m_q$ is the quark mass and $S = (p_1 + p_2)^2$. The helicity amplitudes entering (1) are

$$\Phi_1 = M_{+,+;+,+}, \quad \Phi_3 = M_{+,-,+,-}, \quad \Phi_5 = M_{++,+-}.$$  

By using for the gluon polarization tensor in Fig.1 its high-energy limit [15]

$$D^{\mu\nu} = g_{\mu\nu} + \frac{2}{S}(p_1^{\mu}p_2^{\nu} + p_1^{\nu}p_2^{\mu}) \approx \frac{2}{S}(p_1^{\mu}p_2^{\nu} + p_1^{\nu}p_2^{\mu}),$$  

the matrix element of the reaction

$$u(p_1) \uparrow + u(p_2) \rightarrow d(p_1') + u(p_2') + \pi^+(l)$$  

is given by

$$\tilde{M} = \frac{2g_s^2 g_{\pi^+ q q}}{S q^2} \bar{u}(p_2') \gamma_\mu t^a u(p_2) \bar{u}(p_1') p_2^\mu O_\mu u(p_1),$$  

where $g_s$ is the strong coupling constant, $g_{\pi^+ q q}$ is the $\pi^+$–quark coupling constant due to the instanton, and

$$Q_\mu = \gamma_5 \left( \frac{\hat{t} \gamma_\mu}{d_1} F(k_1^2) - \frac{\gamma_\mu \hat{t}}{d_2} F(k_2^2) \right).$$  

In order to obtain (6) the equations of motion has been used. In (6) $F(k^2)$ is form factor related with the finite size of instanton, $d_{1,2}$ are quark propagators in Fig.1, namely,

$$d_1 = (p_1' + l)^2 - m_q^2, \quad d_2 = (p_1 - l)^2 - m_q^2.$$  

In principle (5) should include an integral of the density of instantons $n(\rho)$ over the instanton size $\rho$. However for estimating the SSA here we use the simple version of the instanton liquid model, $n(\rho) = n_0 \delta(\rho - \rho_c)$, with fixed instanton size, $\rho_c = 1.6 GeV^{-1}$. This model gives a good description of the hadronic properties and is very suitable for obtaining estimates[10]. In (6) the density of the instantons is in the numerator and the denominator. Therefore in the ratio it cancels. The structure in color space of all the helicity amplitudes in (1) is the same. Therefore
we can omit all global factors of \( \frac{p}{S} \) in the ratio as well. Thus it is enough to consider the matrix element
\[
M = \frac{1}{S q^2} \bar{u}(p_2') \hat{p}_1 u(p_2) \bar{u}(p_1') p_2'' Q_\mu u(p_1).
\]  
(8)

In the high energy limit its very suitable to use Sudakov variables
\[
\begin{align*}
l &= x_F \tilde{p}_1 + \beta_p \tilde{p}_2 + l_\perp \\
p_1' &= (1 - x_F) \tilde{p}_1 + \beta_1 \tilde{p}_2 + p_{1\perp}' \\
q &= \alpha \tilde{p}_2 + \beta_q \tilde{p}_1 + q_\perp,
\end{align*}
\]  
(9)

where
\[
\tilde{p}_1 = p_1' - \frac{m_q^2}{S} p_1^\mu, \quad \tilde{p}_2 = p_2' - \frac{m_q^2}{S} p_2^\mu,
\]  
(10)

and \( \tilde{p}_1^2 = \tilde{p}_2^2 \to 0 \) at \( S \gg m_q^2 \). In this limit, in the bottom part of the diagrams in Fig.1, due to (8) we have conservation of quark helicity
\[
\{ \bar{u}(p_2') \hat{p}_1 u(p_2) \} \lambda \lambda' \approx \delta_{\lambda_2 \lambda'_2} S
\]  
(11)

and therefore for helicity amplitude we have
\[
M_{\lambda_1,\lambda_2;\lambda'_1,\lambda'_2} = -\delta_{\lambda_2 \lambda'_2} \frac{1}{q^2 \beta_q} \{ \bar{u}(p_1') q_\perp^\mu O_\mu u(p_1) \} \lambda_1,\lambda'_1
\]  
(12)

where the current conservation condition for the quark current in the top line in Fig.1, \( q.J^1 = 0 \), has been used and
\[
\beta_q = \frac{m_q^2 x_F + l_\perp^2 + x_F (q_\perp^2 - 2 l_\perp q_\perp)}{S x_F (1 - x_F)}.
\]  
(13)

By using the on-shell conditions for outcoming quarks, \( p_1' = p_2' = m_q^2 \), and neglecting the mass of pion, \( l_\perp^2 = 0 \), one can easily obtain the following expressions for the quark propagators, \( d_{1,2} \), and quark virtualities in the intermediate states, \( k_{1,2} \), of Fig.1,
\[
\begin{align*}
d_1 &= \frac{m_q^2 x_F + (x_F q_\perp - l_\perp)^2}{x_F (1 - x_F)} d_2 = -\frac{m_q^2 x_F + l_\perp^2}{x_F} \\
k_1^2 &= \frac{m_q^2 x_F + (x_F q_\perp - l_\perp)^2}{x_F (1 - x_F)} \quad k_2^2 = -\frac{l_\perp^2 - x_F (1 - x_F) m_q^2}{x_F}.
\end{align*}
\]  
(14)

By using of the identity
\[
\gamma_\nu \gamma_\mu = g_{\nu \mu} + i \sigma_{\nu \mu}
\]  
(16)

one can write
\[
\bar{u}(p_1') \gamma_5 \hat{q}_\perp = -(q_\perp l_\perp) \bar{u}(p_1') \gamma_5 u(p_1) + l_\nu q_\perp^\nu \bar{u}(p_1') i \gamma_5 \sigma_{\nu \mu} u(p_1).
\]  
(17)

The matrix elements of operators in (17) for the different helicity states at high energy are known:
\[
\begin{align*}
[\bar{u} \gamma_5 u]_{\lambda_1 \lambda'_1} &\approx \delta_{\lambda_1 \lambda'_1} 2m_q + \delta_{\lambda_1, -\lambda'_1} q_\perp - l_\perp, \\
[\bar{u} i \gamma_5 l_\nu q_\perp^\nu u]_{\lambda_1 \lambda'_1} &\approx -\delta_{\lambda_1, -\lambda'_1} \frac{l_\perp^2 |q_\perp|}{2x_F}.
\end{align*}
\]  
(18)
Therefore the final result for helicity amplitudes in (1) is

$$\Phi_{\lambda_1, \lambda_2; \lambda'_1, \lambda'_2} = \frac{\delta \lambda_2 \lambda'_2}{q^2 \beta_q} \left\{ \delta \lambda_1, -\lambda'_1 \right\} \left( q_{\perp} \cdot l_{\perp} \right) q_{\perp} - l_{\perp} \left( \frac{F(k^2_1)}{d_1} - \frac{F(k^2_2)}{d_2} \right) + \frac{p^2_{\perp} |q_{\perp}|}{2x_F} \left( \frac{F(k^2_1)}{d_1} + \frac{F(k^2_2)}{d_2} \right) + \delta \lambda_1, \lambda'_1 2m_q (q_{\perp} \cdot l_{\perp}) \left( \frac{F(k^2_1)}{d_1} - \frac{F(k^2_2)}{d_2} \right) \right\}. \quad (19)$$

The main feature of the instanton–induced form factor $F(k^2)$ in (19) is its nontrivial dependence on the virtualities of the incoming quarks into the instanton vertex. In the general case of the on-shell pion and off-shell quarks with virtualities $k^2_1$ and $k^2_2$ the effective quark-pion vertex has the following form

$$g_{\pi qq}(k^2_1, k^2_2) = g_{\pi qq} F(k^2_1) F(k^2_2), \quad (20)$$

where $F(k^2)$ is related to the Fourier transformed zero-mode of the quark in the instanton field in a singular gauge (see [10])

$$F(k^2) = -x \frac{d}{dx} \left\{ I_0(x) K_0(x) - I_1(x) K_1(x) \right\}, \quad (21)$$

where $x = \rho \sqrt{k^2}/2$.

The instanton is a classical solution of the QCD equations of motion in Euclidean space-time which is characterized by its size $\rho$ in this space-time. Therefore to obtain the result for the cross section the analytical continuation of instanton amplitudes to the physical Minkowsky space-time should be done. This continuation should be performed in the careful way because the instanton-induced amplitudes have the cut at the quark virtuality $k^2 = 0$ (see below). It is this cut which is responsible for the appearance of the imaginary part which is needed for SSA.

To eliminate the imaginary part it is more suitable to use a good approximation for this form factor which gives the correct behavior for quark zero-mode at $k^2 \to \infty$

$$F(k^2) \approx \frac{1}{1 + \rho^3 (\sqrt{k^2})^3/6}, \quad (22)$$

For a space–like value of the quark virtuality in the intermediate state in Fig.1a we have $k^2_2 < 0$ and therefore in Minkowsky space–time the form factor becomes

$$F(k^2_2) \approx \frac{1}{1 + \rho^3 (\sqrt{|k^2_2|})^3/6}. \quad (23)$$

At the same time for time–like virtuality in Fig.1b $k^2_1 > 0$ in Minkowsky space–time one obtains an imaginary part in form factor.

$$F(k^2_1) \approx \frac{1}{1 + i \rho^3 (|k_1|)^3/6}. \quad (24)$$

It is very well known that to get significant single spin asymmetry one must have both quark-spin flip and a large imaginary part in the amplitude [1]. Eqs. (19) and (24) show that in the

---

2 The origin of the effective quark-pion vertex within instanton model is the famous t’Hooft’s four-quark interaction related to the quark zero-modes in instanton field [17].

3 It is interesting that only nonspin-flip amplitude in (19) is proportional to the quark mass. In pQCD approach [3] we have opposite situation e.g. spin-flip amplitude is proportional to the quark mass. The difference comes from the additional quark spin-flip at quark-pion vertex in Fig.1.
instanton induced diagrams have both of these components. The SSA is proportional to the interference of diagrams in Fig.1. One can interpret the contribution from first diagram (Fig.1a) as a Sivers effect \cite{7} in the quark distribution function and the contribution from the second diagram (Fig.1b) as a contribution to quark fragmentation function, the so-called Collins effect \cite{8}.

To obtain the final result for the asymmetry one should integrate in (1) the numerator and the denominator over $q_\perp$ and regularize in some way the gluon propagator at small $q^2$. The usual procedure (see for example \cite{15}) is to substitute in the gluon propagator, $q^2 \to -(q^2_\perp + \mu^2)$, where we have used for the infrared regulator $\mu \approx \Lambda_{QCD} \approx m_q \approx 0.35$ GeV. The value of $m_q = 0.35$ GeV is the constituent quark mass within instanton liquid model \cite{10}.

The result of the calculation of the SSA is presented in Fig.2 as a function of $x_F$ and in Fig.3 as a function of both $x_F$ and $p_\perp$. The value of the asymmetry is rather large $A \approx 30\%$ in the large $x_F$ and $p_\perp$ region. The magnitude and sign of the asymmetry of $\pi^+$ mesons is in qualitative agreement with the experimental data \cite{1}. At the same time the negative and smaller polarization of d-quark in comparison with u- quark polarization in proton should lead to the negative and small positive SSA for the $\pi^-$- and $\pi^0$- meson production, respectively. This feature also was observed by the E704 Collaboration. For more detailed comparison with total set of data one should include in the calculation the u-and d-quark distribution functions in the polarized and unpolarized nucleon and take the modern result for the density of instanton from lattice calculations \cite{11}. This will be the subject for a forthcoming paper.

We should stress that the instantons give the large SSA at large transfer momentum. The scale of the transfer momentum where one can expect the large SSA in instanton approach is determined by the average size of the instanton in QCD vacuum. This size is much smaller than the confinement size. Therefore the typical values of the transfer momentum in instanton–induced quark fragmentation to hadrons are substantially larger than the usual value $p_\perp \approx 0.2$ GeV related to the confinement scale. It is this $p_t$ dependence of SSA that poses one of the main problems in the most attempts to explain the phenomena. One can also easy understand the origin of the observed enhancement of the SSA at large $x_F$ region. Indeed the value of the SSA

Figure 2: The instanton contribution to the single spin asymmetry for pion production as a function of $x_F$.

Figure 3: The instanton contribution to the single spin asymmetry for pion production as a function of $x_F$ and $p_t = |l_\perp|$. Solid line is for $x_F = 0.9$, dashed line is for $x_F = 0.6$ and dotted line is for $x_F = 0.3$. 
is determined by product of the imaginary part of the diagram in Fig.1b and real part of the diagram in Fig.1a. The imaginary part of diagram in Fig.1b is proportional to the virtuality of quark in Fig.1b coming into the instanton vertex \((24)\). This virtuality is \(k_2^2 \approx p_\perp^2 / (x_F (1 - x_F))\). At the same time the real part of the diagram in Fig.1a is small at low \(x_F\) due to form factor \(F(k_2^2)\), where \(k_2^2 \approx -p_\perp^2 / x_F\). As result the instanton approach predicts the large SSA only in large \(x_F\) region. This prediction is confirmed by the data \([1]\).

It should be mentioned that instanton (antiinstanton) transition defines the particular time direction \([4]\) and therefore the possible connection of the instanton mechanism of the SSA with T-odd fragmentation functions \([18]\) should be clarified. Recently, a large azimuthal asymmetry in semi-inclusive polarized electroproduction of pions was observed at HERMES \([19]\). It can be shown that the instanton mechanism for single spin asymmetries suggested in our paper allows to explain these data \([20]\) as well.

In summary, the instanton–induced contribution to quark–quark scattering amplitude leads to a large quark single–spin asymmetry at high energy and large momentum transfer. The origin of SSA is in the large imaginary part of the instanton–induced amplitudes in the time–like region of quark virtuality. This is related to the quasiclassical origin of the instanton which stems from the fact that it is a soliton–like solution of the QCD classical equation of motion in Euclidean space–time. We should also emphasize that appearance of the imaginary part in the quark-quark scattering amplitudes which include the quark lines with time-like momenta is the common feature of the instanton–induced processes. The consideration of the different manifestations of this phenomenon in polarized and unpolarized lepton-hadron and hadron-hadron interactions will be the subject of forthcoming papers.

Acknowledgements

The author is thankful to M.Anselmino, A. De Roeck, A.E.Dorokhov, S.B.Gerasimov, E.A.Kuraev, A.V.Efremov, E.Leader, M.G.Ryskin and V.Vento for helpful discussions. He also very grateful to Prof.A.Di Giacomo for the warm hospitality at Pisa University where this paper was started. The work was supported in part by the Heisenberg–Landau and INFN-BLTP JINR exchange programs.

References

[1] D.L.Adams et al.,, Phys. Lett., B264 (1991) 7; Phys. Rev. Lett. 77 (1996) 2626.

[2] A.D. Krisch, Proc. 9th Int. Symp. on High Energy Spin Physics, Bonn, (1990) 20 ;
G. Bunce et al., Part. World 3 (1992) 1.

[3] G.L.Kane, J.Pumplin, and W.Repko, Phys. Rev. Lett. 41 (1978) 1689.

[4] A.V.Efremov, O.V. Teryaev, Yad.Fiz. 39 (1984) 1517.

[5] J.W.Qiu and G.Sterman, Nucl.Phys. B353 (1991)137;
Phys. Rev. D59 (1999) 014004.

[6] S.M.Troshin and N.E.Tyurin, Phys. Rev. D54 (1996) 838;
C.Boros, Z.-T.Liang, T.-C.Meng and R.Rittel, J. Phys. G24 (1998) 75.

The instanton describes the subbarier transition in time from \(-\infty\) to \(+\infty\) while the antiinstanton is the transition in opposite direction.
[7] D. Sivers, *Phys. Rev.* **D41** (1991) 261.

[8] J. C. Collins, *Nucl. Phys.* **B396** (1993) 161.

[9] M. Anselmino and F. Murgia, [hep-ph/9901442](http://arxiv.org/abs/hep-ph/9901442).

[10] T. Schäfer and E. V. Shuryak, *Rev. Mod. Phys.* **70** (1998) 323.

[11] D. A. Smith and M. J. Teper, *Phys. Rev.* **D 58** (1998) 014505.

[12] A. E. Dorokhov, N. I. Kochelev, Yu. A. Zubov, *Inter. Jour. of Mod. Phys.* **A8** (1993) 603, and references therein.

[13] M. Anselmino, A. Efremov, and E. Leader *Phys. Reports* **261** (1995) 1.

[14] N. I. Kochelev, *Phys. Rev.* **D57** (1998) 5539.

[15] A. Ahmedov, I. V. Akushevich, E. A. Kuraev, P. G. Ratcliffe, [hep-ph/9902418](http://arxiv.org/abs/hep-ph/9902418).

[16] See, e.g., C. Bourrely, J. Soffer, and E. Leader, *Phys. Rep.* **59** (1980) 95.

[17] G. ’t Hooft, *Phys. Rev.* **D14** (1976) 3432.

[18] D. Boer and P. J. Mulders, *Phys. Rev.* **D57** (1998) 5780; D. Boer, R. Jakob and P. J. Mulders, *Phys. Lett.* **424** (1998) 143.

[19] H. Avakian, [hep-ph/9908490](http://arxiv.org/abs/hep-ph/9908490).

[20] N. I. Kochelev, A. V. Vinnikov, T. Morii, to be published.