REASONING FROM DATA IN THE MATHEMATICAL THEORY OF EVIDENCE

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Abstract

Mathematical Theory of Evidence (MTE) is known as a foundation for reasoning when knowledge is expressed at various levels of detail. Though much research effort has been committed to this theory since its foundation, many questions remain open. One of the most important open questions seems to be the relationship between frequencies and the Mathematical Theory of Evidence. The theory is blamed to leave frequencies outside (or aside of) its framework. The seriousness of this accusation is obvious: no experiment may be run to compare the performance of MTE-based models of real world processes against real world data.

In this paper we develop a frequentist model of the MTE bringing to fall the above argument against MTE. We describe, how to interpret data in terms of MTE belief functions, how to reason from data about conditional belief functions, how to generate a random sample out of a MTE model, how to derive MTE model from data and how to compare results of reasoning in MTE model and reasoning from data.

It is claimed in this paper that MTE is suitable to model some types of destructive processes

Keywords: Approximate Reasoning, Learning System, Modeling Destructive Processes

1. INTRODUCTION

The Dempster-Shafer Theory or the Mathematical Theory of Evidence (MTE) [1, 2] shows one of possible ways of application of mathematical probability for subjective evaluation and is intended to be a generalization of bayesian theory of subjective probability [3].

This theory offers a number of methodological advantages like: capability of representing ignorance in a simple and direct way, compatibility with the classical probability theory, compatibility of boolean logic and feasible computational com-
MTE may be applied for (1) representation of incomplete knowledge, (2) belief updating, (3) and for combination of evidence [5]. MTE covers the statistics of random sets and may be applied for representation of incomplete statistical knowledge. Random set statistics is quite popular in analysis of opinion polls whenever partial indecisiveness of respondents is allowed [6].

Practical applications of MTE include: integration of knowledge from heterogeneous sources for object identification [7], technical diagnosis under unreliable measuring devices [8], medical applications: [9, 10].

An important example concerning difference between implications of bayesian reasoning and reasoning within MTE can be found in [11].

In spite of indicated merits, MTE experienced sharp criticism from many sides. The basic line of criticism is connected with the relationship between the belief function (the basic concept of MTE) and frequencies. A number of attempts to interpret belief functions in terms of probabilities have failed so far to produce a fully compatible interpretation with MTE - see e.g. [12, 13, 14] etc. Shafer [3] and Smets [15], in defense of MTE, dismissed every attempt to interpret MTE frequentistically. Shafer stressed that even modern (that meant bayesian) statistics is not frequentistic at all (bayesian theory assigns subjective probabilities), hence frequencies be no matter at all. Smets stated that domains of MTE applications are those where "we are ignorant of the existence of probabilities", and warns that MTE is "not a model for poorly known probabilities" ([15], p.324). Smets states further "Far too often, authors concentrate on the static component (how beliefs are allocated?) and discover many relations between TBM (transferable belief model of Smets) and ULP (upper lower probability) models, inner and outer measures (Fagin and Halpern [16]), random sets (Nguyen [17]), probabilities of provability (Pearl [18]), probabilities of necessity (Ruspini [19]) etc. But these authors usually do not explain or justify the dynamic component (how are beliefs updated?), that is, how updating (conditioning) is to be handled (except in some cases by defining conditioning as a special case of combination). So I (that is Smets) feel that these partial comparisons are incomplete, especially as all these interpretations lead to different updating rules." ([15], pp. 324-325). Ironically, Smets gives later in the same paper an example of belief function ("hostile-Mother-Nature-Example") which may be clearly considered as lower probability interpretation of belief function, just, at further consideration, leading to very same pitfalls as approaches criticized himself.

Wasserman [20] strongly opposed claims of Shafer [3] about frequencies and bayesian theory. Wasserman pointed out that the major success story of bayesian theory is the exchangeability theory of de Finetti, which treats frequency based probabilities as a special case of bayesian belief. Hence frequencies, as Wasserman claims, are inside the bayesian theory, but outside the Mathematical Theory of Evidence.

This paper is intended to shed some light onto the dispute on relationship between MTE and frequencies. Section 2 introduces basic definitions of MTE. Section 3 clarifies the problem under consideration. Section 4 describes, how to interpret data in terms of MTE belief functions. Section 5 describes a proposal of a model.
for structuring n of MTE belief functions. Section 6 shows how to generate a random sample out of a MTE model of reality. Section 7 describes how to derive MTE model from data. Section 8 is devoted to comparison of results of reasoning in MTE model and reasoning from data. Section 9 indicates open research problem not considered so far within the presented interpretational framework of Mathematical Theory of Evidence.

2. BASIC DEFINITIONS OF MTE

Let us first remind basic definitions of MTE:

**Definition 1** Let \( \Xi \) be a finite set of elements called elementary events. Any subset of \( \Xi \) be a composite event. \( \Xi \) be called also the frame of discernment.

A basic probability assignment function is any function \( m: 2^\Xi \rightarrow [0, 1] \) such that

\[
\sum_{A \in 2^\Xi} |m(A)| = 1, \quad m(\emptyset) = 0, \quad \forall A \in 2^\Xi \quad 0 \leq \sum_{B \subseteq A} m(B)
\]

\(|.|\) - absolute value.

A belief function be defined as \( \text{Bel}: 2^\Xi \rightarrow [0, 1] \) so that

\[
\text{Bel}(A) = \sum_{B \subseteq A} m(B)
\]

A plausibility function be \( \text{Pl}: 2^\Xi \rightarrow [0, 1] \) with

\[
\forall A \in 2^\Xi \quad \text{Pl}(A) = 1 - \text{Bel}(\Xi - A)
\]

A commonality function be \( \text{Q}: 2^\Xi - \{\emptyset\} \rightarrow [0, 1] \) with

\[
\forall A \in 2^\Xi - \{\emptyset\} \quad Q(A) = \sum_{B \subseteq A} m(B)
\]

Furthermore, a Rule of Combination of two Independent Belief Functions \( \text{Bel}_1, \text{Bel}_2 \) Over the Same Frame of Discernment (the so-called Dempster-Rule), denoted

\[
\text{Bel}_{E_1 \oplus E_2} = \text{Bel}_{E_1} \oplus \text{Bel}_{E_2}
\]

is defined as follows: :

\[
m_{E_1 \oplus E_2}(A) = c \cdot \sum_{B,C: A = B \cap C} m_{E_1}(B) \cdot m_{E_2}(C)
\]

\((c - \text{constant normalizing the sum of } |m| \text{ to } 1)\)

Furthermore, let the frame of discernment \( \Xi \) be structured in that it is identical to cross product of domains \( \Xi_1, \Xi_2, \ldots, \Xi_n \) of \( n \) discrete variables \( X_1, X_2, \ldots X_n \), which span the space \( \Xi \). Let \((x_1, x_2, \ldots x_n)\) be a vector in the space spanned by
the variables $X_1, X_2, \ldots, X_n$. Its projection onto the subspace spanned by variables $X_{j_1}, X_{j_2}, \ldots, X_{j_k}$ ($j_1, j_2, \ldots, j_k$ distinct indices from the set $1, 2, \ldots, n$) is then the vector $(x_{j_1}, x_{j_2}, \ldots, x_{j_k})$. A projection of a set $A$ of such vectors is the set $A_{j_1, j_2, \ldots, j_k}$ of projections of all individual vectors from $A$ onto $X_{j_1}, X_{j_2}, \ldots, X_{j_k}$. $A$ is also called an extension of $A_{j_1, j_2, \ldots, j_k}$. A projection of a set $A$ of such vectors is the set $A_{j_1, j_2, \ldots, j_k}$ of projections of all individual vectors from $A$ onto $X_{j_1}, X_{j_2}, \ldots, X_{j_k}$. $A$ is also called an extension of $A_{j_1, j_2, \ldots, j_k}$.

**Definition 2** Let $m$ be a basic probability assignment function on the space of discernment spanned by variables $X_1, X_2, \ldots, X_n$. $m_{j_1, j_2, \ldots, j_k}$ is called the projection of $m$ onto subspace spanned by $X_{j_1}, X_{j_2}, \ldots, X_{j_k}$ iff

$$m_{j_1, j_2, \ldots, j_k}(B) = c \cdot \sum_{A; B=A_{j_1, j_2, \ldots, j_k}} m(A)$$

($c$ - normalizing factor)

**Definition 3** Let $m$ be a basic probability assignment function on the space of discernment spanned by variables $X_1, X_2, \ldots, X_n$. $m^{+}_{j_1, j_2, \ldots, j_k}$ is called the vacuous extension of $m$ onto superspace spanned by $X_1, X_2, \ldots, X_n$ iff

$$m^{+}_{j_1, j_2, \ldots, j_k}(B^{+}_{j_1, j_2, \ldots, j_k}) = m(B)$$

and $m^{+}_{j_1, j_2, \ldots, j_k}(A) = 0$ for any other $A$.

We say that a belief function is vacuous iff $m(\Xi) = 1$ and $m(A) = 0$ for any $A$ different from $\Xi$.

Projections and vacuous extensions of Bel, Pl and Q functions are defined with respect to operations on $m$ function. Notice that by convention if we want to combine by Dempster rule two belief functions not sharing the frame of discernment, we look for the closest common vacuous extension of their frames of discernment without explicitly notifying it.

**Definition 4** (See [21]) Let $B$ be a subset of $\Xi$, called evidence, $m_B$ be a basic probability assignment such that $m_B(B) = 1$ and $m_B(A) = 0$ for any $A$ different from $B$. Then the conditional belief function $Bel(\cdot | B)$ representing the belief function $Bel$ conditioned on evidence $B$ is defined as: $Bel(\cdot | B) = Bel \oplus Bel_B$.

3. PROBLEM STATEMENT

We notice easily, that if the function $m$ is positive only for sets with cardinality 1 then Bel is the plain finite discrete probability function. On relationship between probability functions and bayesian rule consult e.g. [13]. As $m$ sums up to 1 on the
set of sets of elementary events we may be tempted to consider $m$ as an ordinary probability function. But, as cited previously from the work of Smetts, this view contradicts the Dempster rule of combination of independent $Bels$.

In fact, probability functions give rise to some expectations, which cannot be satisfied by a belief function. Generally, we feel that a theory of real world is correct if we make initial observations, let run the intrinsic real world process and the theoretical process on this set of observations and then results of both processes coincide. Probability theory matches this expectation. If we model the reality with a joint probability distribution $P()$ and set a selection condition to membership in set $B$, start a real world process generating independent events following distribution $P()$ and out of these events we strictly select those fitting condition $B$, and then within those selected events we estimate a joint probability distribution $P'()$, then $P'()$ and $P()$ are related (in the limit) by the theoretical model of conditional distribution $P'(A) = P(A|B)$. The major trouble with belief functions is that no such frequentist interpretation exists for them. Models criticized by Smets [15] and many other fail to define such an interpretation of initial data, the process and the results as to fit Dempster rule of independent evidence combination.

Smets and Shafer propose a simple solution to this problem: do not look for such real world processes at all. MTE shall exist as an nicely shaped abstract object in the space of philosophical thinking devoid of any practical meaning.

We propose here another way out. We shall insist on looking for processes meeting MTE requirements and rethink the type of process governing probabilistic reasoning.

4. A FREQUENTISTIC INTERPRETATION OF BELIEF FUNCTIONS

A detailed presentation of our interpretation is presented elsewhere [22].

Essentially, we assume (like in papers of Nguyen [17]) that each object of the population has set-valued attribute(s). But unlike other approaches we assume that objects are attached labels. Labels are subsets of the frame of discernment $\Xi$ and indicate which values are permissible for consideration for a given object. Then the belief function $\text{Bel}(A)$ measures for the set $A$ the share of objects for which the intersection of the attribute value of the object and of the label of this object (and NOT solely the attribute value) is contained in $A$ ($m$, $P_l$, and $Q$ are easily derived from Bel).

The idea of a label reflects subjectiveness within the framework of MTE. It happens in the real life that community attaches vicious labels to a man remembering only his weaknesses (which he surely has) and totally ignoring his virtues (which he may possibly have). So the label of an MTE object may express a kind of irreversible prejudice, or attitude, or belief of a community with respect to this object. We can also imagine a process of medical diagnosis. An initial set of hypotheses is proposed. Then various medical tests are run which label the patient with different sets of hypotheses, and the intersection of those labels and the initial finding is considered to be the actual illness (though no single test may be possible to distinguish this
single illness from all the other). Another type of labeling may be that of actual loosing some properties an object had once upon a time. Someone qualified today as a contemporary example good father, may lose this property in - say - 40 years (He may die by then or be just a good grandfather). A match may lose its feature as being able to light a fire etc.

The importance of a label for the MTE object is operational. Let us assume we run a deterministic MTE "selection" process proc\(_B\) as follows: if the intersection of B, the object’s attribute value and its label is empty then we reject the object from the population (negative selection, identical with a probabilistic conditioning process). Otherwise we change the label of the object to intersection of B and its pre-process label (this is unlike probabilistic conditioning where we leave accepted objects unchanged). Let us denote by Bel’ the belief function derived from the population obtained after the process Bel\(_B\). It is easily seen that then Bel' = Bel \(\oplus\) Bel\(_B\) = Bel\(_\cdot\cdot\cdot\leftarrow\|B\).\)

We may also consider a non-deterministic family of such labeling processes proc\(_{B_1}\), proc\(_{B_2}\), ..., proc\(_{B_k}\) where independently for each object randomly one of the processes is chosen with probability \(Pr(B_1), Pr(B_2), ..., Pr(B_k)\) respectively. Let Bel denote the belief function from frequencies in the initial population, Bel’ the belief function after the run of combined processes, and Bel\(_{proc}\) shall denote such a belief function that for \(j=1,\ldots,k\) \(m_{proc}(B_k) = Pr(B_k)\). Then again is is easily demonstrated that the expected value of Bel’ is equal Bel’’ = Bel \(\oplus\) Bel\(_{proc}\).

Let us illustrate the idea with the following example.

Let us assume that Mr.AX, Mr.BY, Mr.CZ and Mr.DT are major contributors of (imaginary) Formal Theory of Formality (FTF). We shall evaluate their relative contribution to the theory - just by measuring the number of publications. We collected 100 papers some of which were written by a single author, some by several of them. The overall statistics is presented in the subsequent table:

| Author(s)  | Number of publications |
|------------|------------------------|
| AX         | 5                      |
| BY         | 15                     |
| CZ         | 8                      |
| DT         | 2                      |
| AX, BY     | 24                     |
| AX, CZ     | 11                     |
| AX, BY, DT | 9                      |
| AX, BY, CZ, DT | 6                  |
| TOTAL      | 100                    |

The neighboring table shows the values of MTE measures for three sets of authors consisting of: (1) only AX, (2) only BY and (3) both AX and BY.

Let us assume that we obtained a "hint" from a "friendly person" that Mr.DT is in fact a fictive author of FTF. This information ("evidence") may be captured by the belief function Bel\(_1\) such that \(m_1(\{AX, BY, CZ\}) = 1\). How to use this hint
We may take the papers we collected, one by one, and delete Mr. DZ from the list of authors of the paper, and if no author is left, we throw the paper away. After such an operation we obtain the new ”statistics” of contributions:

| Author(s) | Number of papers: |
|-----------|-------------------|
| AX        | 5                 |
| BY        | 15                |
| CZ        | 8                 |
| AX, BY    | 33                |
| AX, CZ    | 11                |
| AX, BY, CZ| 26                |
| TOTAL     | 98                |

Bel’ be the new belief function for the new population. It is easily seen that $Bel' = Bel \oplus Bel_1$.

Let another, independent friendly person give us another hint that also CZ is a fictive contributor to FTF. But let us trust this person only to 70%. This fact should be represented by the belief function $Bel_2$ such that $m_2(\{AX, BY, DT\}) = 0.7$ and $m_2(\{AX, BY, CZ, DT\}) = 0.3$.

Let us consume this new ”evidence” as follows: we take papers (after last operation) one by one, throw a properly biased coin (70% heads, 30% tails), toss it and on heads we delete CZ from the list of authors (and throw the paper away if no other author is left) and on tails we accept the paper as is. The expected value over ”statistics” of a process like this is visible below.

| Author(s) | Number of papers: |
|-----------|-------------------|
| AX        | 8.3               |
| BY        | 15                |
| CZ        | 5.6               |
| AX, BY    | 40.8              |
| AX, CZ    | 7.7               |
| AX, BY, CZ| 18.2              |
| TOTAL     | 95.6              |

Let $Bel''$ denote the belief function after the second process. Provably its expected value may be calculated as $Bel'' = Bel' \oplus Bel_2$.

5. MTE MODELS

It is usually (next to) impossible to represent directly a (joint) probability distribution in a larger number of variables. E.g. 15 four-valued variables would require 1 Giga floating point cells. With MTE belief functions it is even worse. A direct representation of a belief function in 15 variables with four-valued frame of discernment each would require 1 billion Giga floating point cells. Therefore in both
domains alternative representations are looked for. Within probability domain so-called bayesian networks are used [23], exploiting conditional independences, where the joint probability distribution is represented as a combination of conditional probabilities of variables $X_i$ on their direct causes $X_{\pi(i)}$:

\[
P(x_1, \ldots, x_n) = \prod_{i=1}^{n} P(x_i|x_{\pi(i)})
\]

Within the MTE literature another kind of "belief networks" is used, due i.e. to Shenoy and Shafer [24]. It is so-called factorization along a hypergraph:

\[
Bel = Bel_1 \oplus Bel_2 \oplus \ldots \oplus Bel_m
\]

where no requirements are put on the form of components $Bel_i$. This approach has been criticized recently by Cano et al. [25] as - contrary to Pearl’s bayesian networks - these MTE belief networks don’t reflect (conditional) independences among variables. Shafer’s conditional belief function cannot be candidate for a factor in belief function factorization. Therefore they proposed a definition of (so-called apriorical) conditionality within MTE saying that $Bel$ is a conditional belief function conditioned on variables $X_1, \ldots, X_k$ iff $Bel^{\downarrow_{X_1,\ldots,X_k}}$ is a vacuous belief function. They require the factorization of a belief function to be in terms of such conditional belief functions. However, this definition is counterproductive as for the special case of belief function being a bayesian distribution the whole bayesian network may collapse into a single node of Cano’s belief network.

We proposed therefore another definition of apriorical conditional belief function in [26] and demonstrated there that Shenoy/Shafer factorization cannot be simpler than that into a belief network of ours. Apriorical conditional belief function $Bel^{\downarrow_{X_1,\ldots,X_k}}$ reflecting conditioning of $Bel$ on $X_1, \ldots, X_k$ is any pseudo-belief function satisfying

\[
Bel = Bel^{\downarrow_{X_1,\ldots,X_k}} \oplus Bel^{\downarrow_{X_1,\ldots,X_k}}
\]

(A pseudo-belief functions allows m’s to be negative, but only so that respective Q’s remain non-negative). We then define a belief network factorization of $Bel$ with frame of discernment spanned by variables $X_1, \ldots, X_n$ as

\[
Bel = \bigoplus_{i=1}^{n} Bel^{\downarrow_{\{X_{i}\} \cup X_{\pi(i)}|X_{\pi(i)}}}
\]

6. GENERATING RANDOM SAMPLES FROM MTE MODELS

Generating a random sample from the bayesian network is a relatively simple task. We have to consider an ordering of variables compatible with the partial order imposed by the network. To generate a single object, we take variable by variable following this ordering, look at values $x_{\pi(i)}$ already assigned to direct causes $X_{\pi(i)}$ of $X_i$, and then we select randomly one of the values $x_{i1}, x_{i2}, \ldots, x_{in}$ taken by variable
$X_i$ with probabilities proportional to $P(x_{i1} | x_{\pi(i)})$, $P(x_{i2} | x_{\pi(i)})$, \ldots, $P(x_{in} | x_{\pi(i)})$. That is, in a single pass we can generate one sample object. (All random selections should be independent of one another).

It is not that easy with belief function models in general. To generate a single sample object we have to assign it first the frame of discernment set $\Xi$, calling this set $O_0$. Then we take one after the other component $Bel_i$ function (ordering may be anyhow), eventually make the vacuous extension $Bel^\uparrow_i$ of it onto the frame of discernment. Then we select randomly a set $A_j \subseteq \Xi$ according to probability distribution proportional to $|m_i^\uparrow(A_j)|$. If at the time the object had the value $O_{i-1}$, then we assign it a new value $O_i = O_{i-1} \cap A_j$. Should $O_i$ be an empty set, then we terminate the pass without generating an object (a failure). That is, within a single pass we may or may not generate a sample object.

With pseudobelief functions the situation is even worse. The object is within each step assigned a set and a mark $+$ or $-$. If the $m_i(A_j)$ of selected $A_j$ is positive, then the mark is left unchanged, if it is negative, then the mark is inverted. If within the sample there are two identical objects with respect to the finally assigned set, but with opposed marks, then they are canceled out. After completion of sample generation process all remaining objects with marks $'$ are canceled.

However, if the model is in terms of a belief network as described in [26] (see section 5), then the problem of sample generation reduces to probabilistic case (one sample object per single pass), however one has to keep track of $'+/-'$ marks as indicated above.

7. EXTRACTING MTE MODELS FROM DATA

One of major advantages of the belief network model proposed by us for MTE is its capability to connect statistical independence from data with d-separation within the underlying directed acyclic graph of the belief network (see [27] for more on this point). d-separation property [23] means possibility of derivation of conditional independence from graphical structure of dependencies. Several algorithms have been proposed for derivation of belief network structure for MTE (see [22, 26, 27]).

Within the framework of DS-JACEK system [28] still another algorithm has been implemented. It is a version of CI algorithm of Spirtes et al. (see [29]). CI algorithm is known of capability to derive causal models under causal insufficiency. It has been adopted for generation of bayesian networks [30], and later for MTE belief networks [31]. The essential adaptation step for MTE is substitution of bayesian conditional independence test with MTE apriorical conditional independence test. If we test conditional independence of variables $X$ and $Y$ on the set of variables $Z$, then we have to compare empirical distribution $Bel^{x,y,z}$ with $Bel^{x,z} \oplus Bel^{y,z} \oplus Bel^{z}$. The traditional $\chi^2$ statistics is computed (treating the latter distribution as expected one). If the hypothesis of equality is rejected on significance level $\alpha = 0.05$ then $X$ and $Y$ are considered dependent, otherwise independent.

8. REASONING IN SHENOY-SHAFER AXIOMATIC FRAMEWORK VERSUS REASONING FROM DATA
The crucial point of the whole effort of developing data model for MTE was to provide a reference point for MTE reasoning engines. An MTE reasoning engine calculates (Shafer’s) conditional marginal distributions from an MTE model and a set of observations. Several such engines have been developed (see [24] for further references, see also [25]). Shenoy/Shafer method of local computations for MTE, described e.g. in [24] works with hypergraph factorization models of belief functions. It has been implemented in the reasoning engine of DS-JACEK [28].

Conditioning for probabilistic databases is achieved in a simple manner. It is sufficient to impose a filter (filtering expression, or select expression) and the calculation of marginals can be done on the physically same dataset just evaluating truth value of filtering expression.

It is not that simple for MTE databases. Neither are MTE constraints deterministic (they are usually non-deterministic) nor is conditioning just the matter of selection. One has to prepare the physical copy of the database. Then take each record case-by-case and apply the same procedure as when generating a random sample with two differences: we do not start with assignment of \( \Xi \) to the object, but rather start with its currently assigned set of values. Then we do not use Bels describing the model but rather the ones describing constraints (observations). So the record may be either discarded from the database or retained but changed (Due to possibilities of changes we just require that a physical copy of the original database is to be prepared for the process). Then marginals over the modified copy are just results of MTE reasoning from data.

It is a very important issue to possess two different mechanisms for MTE reasoning: one from model and one from data. We may have diverse testing conditions where we may utilize them in suitable way. On the one hand we may have a model of reality available and want to develop/test a new/old MTE reasoning engine. It is usually hard to verify results of MTE reasoning for models with more than 10 variables by hand. And usually the implementation of sample generator in combination with sample constraint imposer is much simpler than that of a fine (time and/or space saving) model-based MTE reasoning engine. So we can generate a random sample from the model, run reasoning both via sample constraint imposer and via the developed knowledge-based reasoning engine and then compare resulting (aposterioric) conditional belief functions from data and from the model.

The other setting may be that we have data available and want to develop the proper model of reality. We may generate model from data either by automatic learning program, or in interactive manner or by “enlightened guess”. Then we can compare under various constrains the results of reasoning from this model and the data and eventually reject or adjust the model, change/improve model generation algorithms etc..

Last not least, we may have been provided with a model once upon the time and it was not until later that we collected a sufficient amount of data to verify its thorough or partial validity.
9. DISCUSSION AND CONCLUDING REMARKS

The frequentistic interpretation of Mathematical Theory of Evidence described in this paper has been fully implemented: major part of it within a knowledge-base development-and-test system DS-JACEK and partially within the test-bed for this system. For sparsely connected networks of up to 15 dempsterian variables, databases with up to 5,000 cases were randomly generated and thereafter in most cases successfully reconstructed using methodology described above. Cases of failure turned out usually to provide alternative but functionally equivalent structure. Run-times on an PC i486, 66MHz in extreme cases didn’t exceed 10 minutes.

Generation of random dempsterian samples served also as a test-bed for the reasoning machine of DS-JACEK. Following of test examples with more than 10 variables by hand may prove not feasible. Hence for larger MTE models random samples of 10,000 cases and more were generated and then results of knowledge-based DS-JACEK reasoning machine were compared with marginals from a sample-based test-bed reasoning machine. Comparisons of this form proved to be quite useful for program development.

With this presentation main application problems of MTE seem to be overcome. Processes changing data in a coherent way (reducing the space of possibilities for each individual) like those of stepwise medical diagnosis, or destructive processes have been identified as a class of processes which can be modeled using Mathematical Theory of Evidence. Belief functions may be calculated from data, verified against data and may serve as source for random generation of data. Independence of dempsterian-shaferian variables acquired statistical meaning. A new category of MTE belief networks has been defined paralleling bayesian belief networks in their capability to capture qualitatively independence and conditional independence in form of a directed acyclic graphs. Algorithms to decompose a data grounded belief function into a belief network have been elaborated.

A number of questions are still left open:

- how to interpret (if at all) pseudo-belief functions (that is with negative masses in basic probability assignment),
- how to obtain unbiased estimates of apriorical conditional belief functions weighing belief network nodes,
- how to accomplish statistically optimal approximations of belief functions by belief networks from noisy data,
- what should be optimal independence and conditional independence tests for belief network models.

These questions along with related issues are subject of further research. It is due to the work presented in this paper that questions of statistical estimations within the MTE may be posed at all.

Previous work, done among other by Shafer [21] by Kyburg [12], Halpern and Fagin [13], Fagin and Halpern [16, 14], Pearl [33], Grzymala-Busse [34], Nguyen [17],...
as well as critical evaluations of these approaches e.g. by Smets [15], contributed to clarification of similarities and dissimilarities between frequentistic approaches to probability and to MTE. The major contribution of this paper is to pinpoint the essential difference between probabilistic reasoning and the reasoning within MTE. Probabilistic reasoning can be understood - in terms of objects of the population - as a selection of objects with some properties. Whereas MTE reasoning means selection of objects combined with destruction of some of their properties. So, objects subject to a "probabilistic process" are the same before and after completion of the process. Whereas objects subject to an MTE process are changed, deformed after completion of the process. I think this explains difficulties encountered by alternative frequentistic interpretations of MTE presented in the literature. This fundamental insight allowed for enhancement of MTE with many features reserved so far for probabilistic models only (like representation in terms of a belief network, generation of random samples from belief function models, identification of belief networks from data, comparison of model-based and data-based reasoning etc.).

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