On the impact of lensing magnification on redshift-space galaxy clustering analysis
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ABSTRACT

We study the impact of lensing magnification on the observed redshift-space three-dimensional galaxy clustering. For this, we use the RayGal suite of $N$-body simulations, from which we extract samples of dark matter particles and haloes in the redshift regime of interest for future large redshift surveys. Several magnitude-limited samples are built that reproduce various levels of magnification bias ranging from $s = 0$ to $s = 1.2$, where $s$ is the logarithmic slope of the cumulative magnitude number counts, in three redshift intervals within $1 < z < 1.95$. We study the two-point correlation function multipole moments in the different cases, in the same fashion as we would do on real data, and investigate how well one can recover the growth rate of structure parameter. In the analysis, we use an hybrid model that combines non-linear redshift-space distortions and linear curved-sky lensing magnification. We find that the growth rate is underestimated when magnification bias is not accounted for in the modelling. This bias becomes non-negligible for $z > 1.3$ and can reach 10\% at $z \approx 1.8$, depending on the properties of the target sample. However, adding the lensing linear correction allows us to recover an unbiased estimation of the growth rate in most cases, even when the fiducial cosmology is not that of the data. Our results show the importance of knowing in advance $s$ instead of letting this parameter free with flat priors, since in this case the error bars significantly increase. For large values of $s$, we find that one has to be careful with the weak-lensing limit, as it may not be a good approximation at high redshift.

Key words. Cosmology: theory, (Cosmology:) dark energy, (Cosmology:) dark matter, (Cosmology:) large-scale structure of Universe, Gravitation, Gravitational lensing: weak, Methods: numerical

1. Introduction

The observation of distant galaxies provides a wealth of information regarding the nature and content of our Universe. Since galaxies trace the underlying matter density field, their spatial distribution can be used to probe the evolution of the large-scale structure. Moreover, the Doppler effect induced by the peculiar motion of galaxies, also known as redshift-space distortions (RSD, Kaiser 1987), leaves a distinct imprint on their three-dimensional clustering, which can in turn be used to gain information on the linear growth rate of structure. The latter is directly sensitive to the theory of gravity (Guzzo et al. 2008).

Galaxy clustering has been used for more than two decades to put constraints on the $f_8$ parameter, that is the growth rate multiplied by the power spectrum normalisation, and found no significant deviations from General Relativity (e.g. Peacock et al. 2001, Tegmark et al. 2006, Blake et al. 2012, de la Torre et al. 2013, Alam et al. 2017, 2021). Future surveys such as Euclid (Laureijs et al. 2011), Subaru Prime Focus Spectrograph (Takada et al. 2014), Square Kilometre Array (Square Kilometre Array Cosmology Science Working Group et al. 2020), or the recently started DESI (DESI Collaboration et al. 2016) will probe the distribution of galaxies with unprecedented accuracy and at higher redshifts ($z > 1$) than before. In particular, Euclid will provide a $1 - 3\%$ level of precision on $f_8$ between $z = 0.9$ and $z = 1.8$ (Laureijs et al. 2011, Majerotto et al. 2012).

Because future galaxy surveys will probe higher redshifts with high accuracy, the theoretical modelling of galaxy clustering in form of summary statistics such as the two-point correlation function will need to be improved to provide an unbiased estimation of cosmological parameters. More precisely, the current modelling of the observed two-point correlation function (or power spectrum) only accounts for peculiar velocities (RSD), while analytical works have shown in linear theory that the next dominant term at high redshift will be due to gravitational lensing (Yoo et al. 2009, Challinor & Lewis 2011, Bonvin & Durrer 2011), also known as magnification bias (Schneider et al. 1992).

Indeed, gravitational lensing modifies the apparent angular position of sources due to the deflection of light rays along their path, and also magnifies their fluxes, so that in magnitude-limited surveys we are more likely to find objects in magnified regions.

It is interesting to remark that in galaxy–galaxy lensing, the theoretical framework to incorporate the magnification bias corrections was first devised in Ziour & Hui (2008), but it is only very recently that this has gained more attention (Duncan et al. 2014, Ghosh et al. 2018, Thiele et al. 2020, Unruh et al. 2020, Joachimi et al. 2021, Lee et al. 2021). The difference in the theoretical modelling between Dark Energy Survey 1-year (Prat et al. 2018) and 3-year (DESI Collaboration et al. 2021) galaxy-galaxy lensing analyses, where only the latter accounts for the effect of lensing magnification, is particularly telling. Similarly, several analytical works have investigated the effect of magnification bias on the three-dimensional two-point correlation function (Matsubara 2000, Hui et al. 2007, 2008), but has never been
implemented in observational analysis yet. The main reason for this is that, until now, galaxy redshift surveys have probed low-enough redshifts, where gravitational lensing does not have an
important impact on the overall three-dimensional distribution of galaxies. However, in the near future it might not be the case anymore. Of particular relevance for the present paper, Telić-Cizmek et al. (2021) investigated the effect of lensing magnification in spectroscopic surveys and found that neglecting magnification bias should lead to an under-estimation of the growth rate of structure. This work was restricted to pure analytical models in linear theory and a Fisher analysis. From a numerical point of view, Yousry Elkhashab et al. (2021) assessed the detectability of magnification bias on the power spectrum monopole only for a Euclid-like survey.

The present paper aims at investigating the impact of magnification bias on the determination of the growth rate from redshift-space clustering, in a typical observational set up. We use a suite of N-body simulations that accounts for the fully non-linear structure formation and perform realistic galaxy clustering analyses, similarly as in observations, in different regime of magnification bias. We investigate a minimal model to account for the magnification effect on the multipole moments of the redshift-space correlation function, and study the accuracy with which the growth rate of structure parameter can be recovered.

The paper is organised as follows. In Section 2 we discuss the theoretical modelling of the magnified correlation function multipole moments in redshift-space. The numerical data and methodology to perform a likelihood analysis are presented in Section 3. Finally, the results are shown in Section 4 and we conclude in Section 5.

2. Theory

The observed spatial distribution of galaxies in the Universe depends on two main aspects: first, galaxies are biased tracers of the matter density field, which means that in ‘real space’, (that is, the Universe as it is really), galaxies are located at local matter density peaks formed through gravitational instabilities. Second, the observation of galaxies through their emitting light modifies our perception of their distribution. In particular, their measured redshift can be shifted with respect to their Friedmann-Lemaître-Robertson-Walker counterpart due to peculiar velocities (RSD) and can therefore be deflected by gravitational potentials along their trajectories, which impacts the observed angular position of sources. In the following, we present how to model the two dominant effects leading to the observed redshift-space galaxy two-point correlation function, that is RSD and gravitational lensing magnification.

2.1. Redshift-space distortions

The standard way of modelling the redshift-space clustering beyond linear theory (Peebles 1980; Kaiser 1987; Hamilton 1992) is to consider only peculiar velocities as redshift perturbation. The real-space quasi-linear clustering can be predicted with different flavours of perturbation theory, either Eulerian (Bernardeau et al. 2002; Croce & Scoccimarro 2006; Taruya et al. 2012; 2013), or Lagrangian (Zel’dovich 1970; Matsubara 2008; b; Carlson et al. 2013). On top of that, the mapping from real to redshift space can be done with different approaches, particularly common approaches in observational studies include TNS model (Taruya et al. 2010; 2013) and the streaming model (Scoccimarro 2004; Reid & White 2011).

In this work, we consider the Convolution Lagrangian Perturbation Theory (CLPT; Carlson et al. 2013) to predict real-space quantities. Generally, if the non-linear clustering in real space is well modelled with this approach, Lagrangian theories are not accurate enough on small scales in redshift space (White 2014). This is why, we adopt for the mapping from real to redshift space the Gaussian streaming model (Reid & White 2011). The joint use of Convolution Lagrangian Perturbation theory and Gaussian Streaming (CLPT-GS) has been shown to provide a good match to data and simulations (Wang et al. 2014) and is routinely used to model observations (for instance, Reid et al. 2012; Samushia et al. 2014; Zarrouk et al. 2018; Bautista et al. 2021).

2.1.1. Convolution Lagrangian perturbation theory

In the Lagrangian framework, the position $x$ of an infinitesimal volume element is given by

$$x(q,t) = q + \Psi(q,t), \quad (1)$$

where $q$ is the Lagrangian coordinate (initial position) and $\Psi(q)$ the displacement field. The latter encodes the displacement of any mass element. For conciseness, in the following we will omit its time dependence. In Lagrangian theories, the displacement field is assumed to be small, whereas for Eulerian theories it is the density contrast that is assumed to be small. The displacement field can thus be expanded as a perturbative series, $\Psi = \sum_{n=1}^{\infty} \Psi^{(n)}$, where the first-order solution is the well-known Zel’dovich approximation (Zel’dovich 1970).

Due to mass conservation, the relation between the density field of a volume element depends on its initial location through

$$[1 + \delta_1(x)] d^3 x = [1 + \delta(q)] d^3 q, \quad (2)$$

where $\delta$ is the matter density contrast. We have therefore that

$$1 + \delta(x) = \int d^3 q \, \delta_D(x - q - \Psi), \quad (3)$$

where $\delta_D$ is the Dirac delta function. However, the matter density field is not observationally relevant for galaxy clustering, but we are rather interested in biased tracers of the underlying matter field. By assuming a local Lagrangian bias, we can write

$$1 + \delta_b(q) = F(\delta(q)), \quad (4)$$

where $\delta_b$ is the density contrast of galaxies and $F(\delta)$ is the biasing function smoothed on a given scale. We note that at first order, the linear Eulerian and Lagrangian biases are simply related by $b^{(1)}_L = 1 + b^{(1)}_E$. The counterpart of Eq. (3) for biased tracers is thus

$$1 + \delta_b(x) = \int d^3 q \, F(\delta(q)) \, \delta_D(x - q - \Psi). \quad (5)$$

From this expression, it is possible to derive the real-space galaxy two-point correlation function, mean pairwise velocity,
and velocity dispersion as
\[ 1 + \xi(r) = \left[ 1 + \delta_{g}(x) \right] \left[ 1 + \delta_{g}(x + r) \right] \]
\[ = \int d^{3}q \, M_{0}(r, q), \quad (6) \]
\[ v_{12}(r) = \left[ 1 + \delta_{g}(x) \right] \left[ 1 + \delta_{g}(x + r) \right] \left[ v(x + r) - v(x) \right] \]
\[ = [1 + \xi(r)]^{-1} \int d^{3}q \, M_{1}(r, q), \quad (7) \]
\[ \sigma_{12}^{2}(r) = \left[ 1 + \delta_{g}(x) \right] \left[ 1 + \delta_{g}(x + r) \right] \left[ v(x + r) - v(x) \right]^{2} \]
\[ = [1 + \xi(r)]^{-1} \int d^{3}q \, M_{2}(r, q), \quad (8) \]
where \( r = |r| \) is the scale, \( v_{i} \) is the velocity along the line of sight \( u_{z} \), with \( v_{i}(x_{2}) - v_{i}(x_{1}) = (\hat{x}_{2} - \hat{x}_{1}) \cdot u_{z} \), and \( M_{0}, M_{1} \) and \( M_{2} \) are integration kernels containing the parameters of the model (Wang et al. 2014). The main difference between CLPT and other Lagrangian perturbation theories such as in Matsubara (2008b), lies in the resummation employed in the kernels.

2.1.2. Gaussian streaming model

The CLPT predictions for the real-space correlation function and the two first moments of the pairwise velocity distribution, that is Eqs. (6) to (8), can be used in the Gaussian streaming model (Reid & White 2011) to predict the anisotropic two-point correlation function in redshift space. In this model, the redshift-space correlation function is recovered by convolving the real-space correlation function along the line of sight, with a scale-dependent pairwise velocity distribution taken to be Gaussian. This gives

\[ 1 + \xi'(r_{\perp}, r_{\parallel}) = \int \frac{dy}{\sqrt{2\pi \sigma^{2}(r, v)}} \left[ 1 + \xi(r) \right] \]
\[ \times \exp \left\{ \frac{-|r_{\parallel} - y - uv_{12}(r)|^{2}}{2\sigma^{2}(r, v)} \right\}, \quad (9) \]

where \( r_{\parallel} \) and \( r_{\perp} \) are the components of the separation vector parallel and perpendicular to the line of sight, \( r^{2} = r_{\perp}^{2} + r_{\parallel}^{2} \), \( v = y/r \) is the end-point line-of-sight cosine angle definition, and \( \sigma^{2}(r, v) = \sigma_{g}^{2}(r, v) + \sigma_{v}^{2} \), with \( \sigma_{v} \) an additional velocity dispersion term that accounts for small-scale random motions in virialised objects. From Eq. (9), it is fairly easy to compute the multipole moments of the correlation function by integrating over the Legendre polynomials. We note that one limitation of the Gaussian streaming model as presented here, is that it assumes the distant-observer approximation, meaning that it neglects wide-angle effects (Szalay et al. 1998; Szapudi 2004; Papai & Szapudi 2008; Raccanelli et al. 2010; Reimberg et al. 2016; Castorina & White 2018; Beutler et al. 2019; Taruya et al. 2020). In our case, we restrict our analysis to relatively high redshifts \((z > 1)\) so that we can safely work under this approximation. Our implementation of the CLPT-GS model is publicly available 1.

2.2. Magnification bias

Beyond the redshift-space distortions induced by peculiar velocities, the next dominant effect at \( z > 1 \) that modifies the apparent distribution of galaxies is magnification bias. It originates from the fact that some regions of the sky are magnified (demagnified) due to gravitational lensing, meaning that they contain less (more) sources than on average. If we consider a survey of galaxies selected in flux, gravitational lensing will modify galaxy apparent fluxes, so that some of them will enter or exit from the sample. This selection effect will impact the observed clustering of galaxies. It is worth noting that magnification bias is not the only effect that arises, as there are also additional effects that depend on peculiar velocities or gravitational potential (Challinor & Lewis 2011). However, at high redshift, magnification bias is the dominant one and is the focus of this work.

2.2.1. Impact on the number counts

The surface brightness of sources, defined as the flux per unit area, is conserved through gravitational lensing. This implies that the apparent size and flux of a source are simultaneously modified as

\[ S'(\theta) = \mu(S)S(\theta), \quad (10) \]
\[ d\Omega'(\theta) = \mu d\Omega(\theta), \quad (11) \]

where \( S, \mu \) and \( d\Omega \) are respectively the flux, magnification and solid angle, in the direction \( \theta \) on the sky (in the following we will omit the angular dependence), and a prime denotes a magnified quantity. The conservation of the number of sources can be written as

\[ n'(m')dm'd\Omega' = n(m)dm \Omega, \quad (12) \]

where \( n(m) \) is the number density of sources per unit of solid angle and per magnitude interval \( dm \), \( m = -2.5 \log(S) + C \) is the magnitude, and \( C \) a constant. From this, it is straightforward to infer the magnified magnitude \( m' = m - 2.5 \log(\mu) \). The total number of observed sources up to a given magnitude limit \( m_{l} \) is

\[ \int_{-\infty}^{m_{l}} n'(m')dm' = \mu^{-1} \int_{-\infty}^{m_{l}+2.5 \log(\mu)} n(m + 2.5 \log(\mu))dm \]
\[ = \mu^{-1} \int_{m_{l}}^{m_{l}+2.5 \log(\mu)} n(m)dm. \quad (13) \]

Ultimately, this can be rewritten in terms of cumulative number densities as

\[ n'(m_{l}) = \mu^{-1} n(m_{l} + 2.5 \log(\mu)). \quad (15) \]

One generally considers a simple model for the luminosity function, a power law such that (Schneider et al. 1992; Broadhurst et al. 1995)

\[ n(m) \propto 10^{m_{s}}, \quad (16) \]

where \( s \) is defined as the logarithmic slope of the cumulative distribution and is a property of the target sample. We note that for this specific function, cumulative and differential distributions have the same shape. By inserting Eq. (16) in Eq. (15), we obtain that

\[ \Delta_{len} = \mu^{2.5s-1} - 1 \]
\[ = (5s - 2)\kappa, \quad (17) \]
\[ \Delta_{len} = \mu^{2.5s-1} - 1 \]
\[ = (5s - 2)\kappa, \quad (18) \]

1 https://github.com/mianbreton/CLPT_GS
where $\Delta_{\text{len}} = n'(\langle m_1 \rangle)/n(\langle m_1 \rangle) - 1$ is the perturbation on the number count due to magnification bias in a given direction on the sky and is the lensing convergence. In the second line we have performed a first-order Taylor expansion on the magnification as $\mu = 1 + 2k$ (hereafter referred to as the ‘weak-lensing limit’), with $|k| \ll 1$. Gravitational lensing induces two competing terms on the observed number counts as seen in Eq. (13): the $s$-dependent term implies that in magnified regions ($\mu > 1, k > 0$), the flux of sources increases so that we are more likely to find objects (and conversely, there are less chances to find objects in demagnified regions). The second term, which is usually referred to as ‘dilution bias’, describes the change in size of solid angles on the sky. Magnified regions occupy more space, so that for a constant density, we should find less objects in those regions than on average. These two effects cancel exactly for $s = 0.4$.

2.2.2. Two-point correlation function correction

To be consistent, one should in principle derive the lensing correction associated to magnification bias on the correlation function, using the same theoretical framework as for RSD. Nonetheless, these developments are beyond the scope of the present paper, and instead, we propose a simple correction based on linear theory, which can be easily used on top of any RSD model.

We start from the observed galaxy number counts, which accounts for density, RSD, and lensing perturbations (the full expressions accounting for all the terms at first order in metric perturbations can be found in [Yoo et al. 2009] Challinor & Lewis 2011, Bonvin & Durrer 2011]

$$\Delta = \Delta_{\text{den}} + \Delta_{\text{rsd}} + \Delta_{\text{len}},$$

where $\Delta_{\text{den}} = \delta b, \delta b$ is the Eulerian linear bias, $\Delta_{\text{rsd}} = -\partial r/\partial H$ is the RSD component, where $\partial r/\partial H$ is respectively the gradient of the velocity field along the line of sight and the conformal Hubble parameter. The lensing perturbation $\Delta_{\text{len}}$ is that of Eq. (18). We note that the decomposition in Eq. (19) is only true at first order since it neglects higher-order lensing correlations.

Since the correlation function can be written as $\xi(r) = \langle \Delta(x)\Delta(x + r) \rangle$, the linear correction that comes from the addition of lensing magnification in the number counts is

$$\xi_{\text{corr}}(r) = \xi_{\Delta_{\text{den}}}(r) + \xi_{\Delta_{\text{rsd}}}(r) + \xi_{\Delta_{\text{len}}}(r).$$

where $\xi_{\Delta_{\text{A,B}}}(r) \equiv \langle \Delta_{\text{A}}(x)\Delta_{\text{B}}(x + r) \rangle$. The expressions for the different terms in Eq. (20) are derived in Matsubara (2000), Hui et al. (2007, 2008), and in Tansella et al. (2018a,b) for the curved-sky case. Precisely, in the latter case we have

$$\xi_{\Delta_{\text{A,B}}}(\theta, z_1, z_2) = \int \frac{dk}{k} P(k) Q_k^{\Delta_{\text{A,B}}}(\theta, z_1, z_2),$$

where $(\theta, z_1, z_2)$ defines the separation vector in observed coordinate, $P(k)$ is the primordial matter power spectrum, and the kernels $Q_k^{\Delta_{\text{A,B}}}$ with $A = \{\text{den}, \text{rsd}, \text{len}, \text{len}-\text{len}\}$ read

$$Q_k^\text{den-len}(\theta, z_1, z_2) = b(z_1) S_{\text{B}}(z_1) \left(\frac{2 - 5s}{2\chi^2}\right)$$

$$\int_0^{\chi_1} \frac{dk}{A} k^2 (2 - 5s) S_{\text{RSD}}(\xi_1, k\ell, \theta),$$

$$Q_k^\text{rsd-len}(\theta, z_1, z_2) = \frac{k}{\chi(z_1)} \left(\frac{2 - 5s}{2\chi^2}\right)$$

$$\int_0^{\chi_1} \frac{dk}{A} k^2 (2 - 5s) S_{\text{RSD}}(\xi_1, k\ell, \theta),$$

$$Q_k^\text{len-len}(\theta, z_1, z_2) = \left(\frac{2 - 5s}{2\chi^2}\right)$$

$$\frac{4}{\chi(z_1)}$$

$$\int_0^{\chi_1} \frac{dk}{A} k^2 (2 - 5s) S_{\text{RSD}}(\xi_1, k\ell, \theta),$$

In these equations $\zeta$ are pure geometrical functions provided in Appendix B of Tansella et al. (2018a), $\chi$ is the comoving distance to redshift $z_1$, and $S_{\text{B}}, S_{\text{RSD}}, S_{\text{RSD}}$ are the scaled transfer functions associated to density, peculiar velocity, and gravitational potentials, respectively. $\xi_{\Delta_{\text{A,B}}}(\theta, z_1, z_2)$ in Eq. (21) can be written in terms of the separations parallel and perpendicular to the line of sight, $r_\parallel$ and $r_\perp$, using that $r_\parallel = (\chi_2 - \chi_1) r_\bot = \sqrt{\chi_1^2 + \chi_2^2 - 2\chi_1\chi_2 \cos \theta}$, and $r = \sqrt{\chi_1^2 + \chi_2^2 - 2\chi_1\chi_2 \cos \theta}$.

We implement the magnification bias correction using the CorrE library (Tansella et al. 2018b), which provides directly $\xi_{\text{den-len}}, \xi_{\text{rsd-len}}, \xi_{\text{len-len}}$ in bins of $(r_\bot, r_\parallel)$ using curved-sky linear theory and given an input linear power spectrum. It was noted in Jelic-Cizmek (2021) that, although there is in general no large differences between the curved-sky and flat-sky prescriptions at the scales of interest for us, that is $r \leq 150 \ h^{-1}$Mpc, the $\xi_{\text{len-len}}$ component is quite sensitive to the adopted prescription. We therefore adopted the full curved-sky implementation.

Our theoretical model for the redshift-space correlation function therefore consists in CLPT-GS prediction for non-linear RSD, and the curved-sky linear theory prediction for the additional lensing magnification correction. Formally, the anisotropic correlation function model is given by

$$\xi_{\text{model}}(r_\bot, r_\parallel) = \xi_{\text{CLPT-GS}}(r_\bot, r_\parallel) + \xi_{\text{corr}}(r_\bot, r_\parallel).$$

A final step involves evaluating $\xi_{\text{model}}$ at coordinates $(r, v)$ using that $r = \sqrt{r_\bot^2 + r_\parallel^2}$ and $v = v_\parallel/r$, and computing associated multipole moments as

$$\xi_{\ell}(r) = 2\ell + 1 \int r^{\ell} \xi_{\text{model}}(r, v) L_\ell(v) dv,$$

where $L_\ell$ is the Legendre polynomial of order $\ell$.

We show the correlation function multipole moments computed with our model in Fig. 1. We considered here the matter at $z = 1.8$ in the LCDM model and $s = 1.2$. Both flat- and curved-sky linear lensing prescriptions are presented. We first remark that magnification bias adds a positive contribution to the correlation function multipole. Second, the full-sky and flat-sky implementations of the lensing correction give very similar results that are indistinguishable, except on the hexadecapole at large comoving separations. Overall, although we

\[ https://github.com/JCGoran/coffe \]
are dark-matter-only N-body simulations containing 4096 dark matter (DM) particles of mass $1.8 \times 10^{10} M_\odot$ in a volume of 2.625$^3$ Gpc$^3$. Both ΛCDM and wCDM versions are available and associated fiducial cosmological parameters are given in Table 1. The two cosmologies have different $\Omega_m$, and $\sigma_8$, and therefore different values of $f\sigma_8(z)$, since $f = \Omega_m(z)^{0.55}$ in General Relativity (Wang & Steinhardt 1998; Linder & Cahn 2007). This can be seen in Fig. 2, where the fiducial values of $f\sigma_8$ as a function of redshift for the two cosmologies, as well as the expectations from Planck Collaboration et al. (2016) ΛCDM best-fitting model assuming General Relativity are shown. Interestingly, the values of $f\sigma_8(z)$ for the RayGal ΛCDM (wCDM) model are close to Planck ones at high (low) redshift. It is worth emphasising the importance of analysing simulations with different cosmologies, since we can analyse them blindly in a fiducial cosmology, as in observations, and see whether one can recover unbiased estimates of the growth rate of structure.

3.1. RayGal light-cones

Several light-cones have been extracted from the RayGal simulations. In the present work, we use light-cones with an aperture of 2500 deg$^2$ extending to $z = 2$, which encompasses the redshift range probed by DESI (DESI Collaboration et al. 2016) and Euclid (Laureijs et al. 2011) surveys. Those light-cones contain DM particles, as well as DM haloes identified with the parallel Friend-of-Friend algorithm pFoF (Roy et al. 2014), using a linking length of 0.2. We imposed a minimum of 100 particles per halo, which leads to haloes with mass above $1.8 \times 10^{12} M_\odot$.

The gravitational lensing information is computed in the light-cones by using the ray-tracing code MagraThea-Pathfinder (Reverdy 2014). The latter implements an iterative algorithm that finds the null geodesics connecting the observer to each
source (Breton et al. 2019), that is, either particles or haloes. This allows the computation of RSD and lensing effects at the same time, in a general and accurate way. It is important to emphasize that the fact that the treatment of gravitational lensing does not involve the Born approximation, which is often used. In our light-cones, we have roughly $1.2 \times 10^7$ haloes in both cosmologies and we ray-trace about $4 \times 10^8$ randomly selected particles. Having both haloes and particles enable us to perform a redshift-space clustering analysis on a biased population for the former (and hence, closer to observations), and for the latter, to carry out a precise study where the number of matter tracers is maximised.

In Fig. 3, we show the redshift distribution of the halo and particle samples in the ΛCDM light-cone, as well as the adopted tomographic redshift bins. The distributions in the wCDM light-cone are very similar. The redshift bins cover a similar redshift range as present and future galaxy cosmological surveys, a regime where gravitational lensing effects on galaxy clustering start to be significant (at about $z > 1$). Regarding the shape of the redshift distribution, we see for particles that it monotonically increases, as expected in the case of constant density. One may however remark that at about $z = 2$, $N(z)$ seems to decrease. This is an edge effect due to the fact that we built our light-cones up to $z \sim 2$. To avoid any issue, we use in our analysis a maximum redshift of $z_{\text{max}} = 1.95$. For haloes, $N(z)$ reaches a maximum at around $z = 1.2$ and later decreases. This can be explained by the combined effect of the halo formation and limited mass resolution in the simulation. We do not impose any further selection in redshift to avoid discarding too many objects from our samples, and thus maximise RSD and lensing magnification signals.

### 3.1.2. Magnification bias implementation

To reproduce different levels of magnification bias, one could impose a halo mass-galaxy luminosity relation and estimate apparent magnified fluxes or magnitudes, on which one could later make selections. While it is clearly the appropriate methodology when constructing most realistic mock catalogues, our goal is to investigate the effect of magnification bias on galaxy clustering in a general way, independently of the properties of any target sample. Hence, we found that the easiest and most efficient way to mimic the effect of magnification bias is to directly use the magnification of sources. A possibility involves selecting objects using a probability function proportional to $\mu^2 s^5$ (see also Sect. 2.2.1). The advantages of this approach is that it discards less sources and does not depend on the mass resolution of the simulation. However, it depends on some normalisation and discards more sources at high values of $s$. This is why instead, we choose to weight each source by $\mu^2 s^5$. This allows to account for the effect of lensing magnification, while keeping all the objects in our sample.

We note that in our theoretical modelling of magnification bias we assumed the weak-lensing limit, meaning that the convergence and magnification are small. As such, we used Eq. (15) instead of the exact Eq. (17) for the lensed number count. Fig. 4 shows the impact of this approximation on the averaged convergence, as a function of redshift. We first notice that the mean convergence in the unlensed case (that is when $s = 0.4$) is very close to zero, demonstrating the validity of our method to implement the effect of lensing magnification. For $s = 0.2$, we see that the difference between the exact and weak-lensing solutions is very small. For larger values, we can clearly see that the discrepancy grows. The difference between the exact and weak-lensing solutions increases with $s$, so that for $s = 1.2$ it reaches $\sim 40\%$ at $z = 1.9$. One might wonder if this is something to worry about, and how this affects the correlation function, since analytical prescriptions only work in the weak-lensing limit. We do see a difference on the multipoles of the correlation function in Fig. 4 for $s = 1.2$, however it does not reach the 40% of Fig. 4. This suggests that one should be careful while using the first-order lensing correction in the weak-lensing limit, as its validity depends on the redshift and value of $s$.

Finally, we point out that to account for the effect of lensing magnification we used a weight equal to $\mu^2 s^5$ on top of the observed angular positions in the galaxy clustering analysis. This is because our ray-tracing code computes the distortion matrix along the null geodesics that connect the observer to each source. In this case, the weak-lensing statistics in our sample are not the same as if we were using the Born approximation. Indeed, our
averaging procedure performs a ‘source averaging’ (Kibble & Lieu 2005; Bonvin et al. [2015; Kaiser & Peacock 2016; Breton & Fleury 2020). On average, light rays propagate in under-dense regions due to their path on the real null geodesics, which leads to a negative mean convergence. Had we used the Born approximation instead, our full sample would have been ‘unlensed’ and would have led to a vanishing mean convergence. In this case, to implement the effect of magnification in the mock catalogue one would rather have used the comoving angular positions and applied a weight equal to $\mu^{2.5z-1}$ to each source.

3.2. Cosmological analysis

We now turn to the analysis of the different samples described in Sect. 3.1. The total number of elements in all the studied cases are summarised in Table 2. Although Ray-Gal simulations provide a full redshift decomposition at first order in metric perturbations, in the present paper we only focus on the impact of lensing magnification beyond RSD and therefore ignore the more subtle effects that could affect the observed redshift. This means that we only perturb the redshift with the Doppler effect induced by peculiar velocities. The unlensed case corresponds to using observed angles. For one would rather have used the comoving angular positions and weight each source by $1/\mu^2$, as discussed in Sect. 3.1.2. In any configuration, the number of elements in Table 2 is the same (modulo tiny differences between observed and comoving angles due to the footprint). For particles, we select a random sub-sample of the $4 \times 10^6$ initial particles in our light-cones, as the calculation of the anisotropic correlation is very computationally expensive.

3.2.1. Anisotropic two-point correlation function

The estimation of three-dimensional clustering necessitates the assumption of a fiducial cosmology to convert angular positions and redshifts to comoving separations. In the present study, we assume as fiducial cosmology the ΛCDM cosmology of Ray-Gal (Table 1), which we use to analyse all the samples in Table 2 including those extracted from the wCDM light-cone. The anisotropic correlation function is estimated with the Landy-Szalay estimator (Landy & Szalay 1993) as

$$\xi_L(r, v) = \frac{DD(r, v) - 2DR(r, v) + RR(r, v)}{RR(r, v)},$$

where $DD$, $DR$, and $RR$ stand for data-data, data-random, and random-random pairs (weighted and normalised by the total number of elements), respectively. We use CORRFUNC (Sinha & Garrison 2020) to count the anisotropic number of pairs in bins of comoving separation $r$ and cosine angle $v$. The random samples contain 50 (20) times more objects than haloes (particles). We assign redshifts in the random catalogues using the shuffling method, which consists in randomly picking redshifts from the data catalogue. Eventually, the multipole moments of the correlation function are obtained from

$$\xi_l(r) = (2l + 1) \sum_{v=0}^{\nu} \xi_l(r, v) L_l(v) \Delta v.$$  

We are only interested in the first non-vanishing even multipole moments of the correlation function: the monopole ($l = 0$), the quadrupole ($l = 2$), and the hexadecapole ($l = 4$). In the present work, we consider the scale range 27.5 to 127.5 $h^{-1}$Mpc with bins of 5 $h^{-1}$Mpc, and 200 bins in $v$.

3.2.2. Covariance matrices

The covariance matrices on single multipole correlation function measurements are estimated analytically assuming Gaussianity as described in Grieb et al. (2016). Particularly, the covariance matrix between correlation function multipoles $\ell_1$ and $\ell_2$, and between scales $r_1$ and $r_2$, is

$$C_{\ell_1, \ell_2}(r_1, r_2) = \int \frac{i^{\ell_1 + \ell_2}}{2\pi^2} \int_0^\infty k^2 \sigma^2_{\ell_1, \ell_2}(k) \tilde{\eta}_{\ell_1}(kr_1) \tilde{\eta}_{\ell_2}(kr_2) dk,$$

where $\tilde{\eta}_l$ are the bin-averaged spherical Bessel functions and $\sigma_{\ell_1, \ell_2}$ are the per-mode covariance multipole moments, both given in Grieb et al. (2016). The latter function is an integral over the anisotropic power spectrum, which is set here to the corresponding best-fitting RSD model to measurements.

3.2.3. Likelihood analysis

We perform a likelihood analysis of the measured monopole, quadrupole, and hexadecapole correlation functions in each sample. The likelihood $L$ is defined as

$$-2 \ln L(\theta) = \sum_{i,j} N_p \Delta_i \Delta_j C^{-1}_{ij} \Delta_i \Delta_j,$$

where $\theta$ is the vector of parameters, $\Delta$ is the data-model difference vector, $N_p$ is the total number of data points, and $C$ is the covariance matrix. The model that we use has 6 free parameters: $\theta = (f, b_1, b_2, \sigma_m^2, \alpha_1, \alpha_2)$, which correspond respectively to the growth rate, first and second-order Lagrangian bias parameters, squared velocity dispersion, and two dilation parameters that accounts for Alcock-Paczyński distortions. We note that we vary $f$ and not directly $\sigma_8$ because the model takes as input the linear power spectrum associated to the ΛCDM simulation at the redshift of interest. We therefore need to use a fiducial value of $\sigma_8$ to compute the theoretical prediction. The main reason why we cannot let $\sigma_8$ free is because the value of $\sigma_8$ is degenerate with the growth factor $D_\ell(z)$. Within linear theory this is not a problem as $\sigma_8$ can factor out. Within the framework of CLPT, we cannot because the non-linear part of the power spectrum is redshift-dependent. In any case, although we let $f$ free in the likelihood analysis, we eventually extract $\sigma_8$ and compare it to the fiducial value.

We use the dilation parameters along the parallel and transverse direction to account for any change in cosmology with respect to the fiducial one. Formally, these two parameters enter in our formalism as a multiplicative factor on the scales in the anisotropic correlation function: $\xi(\alpha_1, \alpha_2, \theta)$. If the fiducial cosmology is that of the data, we expect $(\alpha_1, \alpha_2) = (1, 1)$. Otherwise, these are given by the ratios

$$\alpha_1 = D_M(z)/r_d,$$

$$\alpha_2 = D_M^\perp(z)/r_d^\perp,$$

where the superscript ‘fid’ refer to an estimation using the fiducial cosmology, $D_M(z) = (1 + z)D_A(z)$ with $D_A$ the angular diameter distance, $D_M^\perp(z) = c/H(z)$, and $r_d$ is the sound horizon at the drag epoch. We note that $\zeta$ refers here to the effective redshift of the sample. In our case, for the four redshift bins these are $z = 0.857, 1.151, 1.448, 1.769$ for haloes and $z = 0.86, 1.155, 1.453, 1.777$ for particles.
As argued in Sánchez (2020), using σ₈ might not be ideal for galaxy clustering analysis, since it relies on a scale given in h⁻¹Mpc units that is cosmology-dependent. However, as noted in Bautista et al. (2021), one can still use σ₈ as long as we take into account the changes due to dilution parameters. Instead of computing σ₈ at 8 h⁻¹Mpc from a linear power spectrum at z = 0, one should rather evaluate this quantity at 8α h⁻¹Mpc, where α = α⊥/α∥ is the isotropic dilation parameter. In the following, our estimations of fσ₈ implicitly account for this correction, while the fiducial values are that of Fig. 2.

We perform three types of analysis. First, a standard analysis that only accounts for RSD using CLPT-GS model. This is similar to what is routinely done in galaxy clustering observational studies. Second, an analysis that includes the magnification bias correction in the model and where s is fixed to its fiducial value. Although it is possible to estimate s from the data itself, the standard method which consists in fitting the local slope in the faint-end of the luminosity function might not be accurate when the selection function is complex (von Wietheissem-Kramista et al. 2021). This is why we also propose an alternative analysis that includes the magnification bias correction in the model but where the s parameter is allowed to vary. Finally, we use the flat priors displayed in Table 3 (where only the last analysis allows s to vary). For each sample, we produce Monte Carlo Markov chains using the EMCEE algorithm (Foreman-Mackey et al. 2013) with 100 walkers and 50,000 steps per walker. The burn-in phase depends on the auto-correlation time and chains are thinned by removing highly-correlated steps. Eventually, the chains are analysed with GetDist (Lewis 2019) to get the final parameter constraints.

### 4. Results

In this section, we present the results of the redshift-space clustering analysis with and without magnification bias for the particle and halo samples. We first discuss the recovered value fσ₈ and its bias with respect to the fiducial value. We then turn to an overview of the results for the other five (or six when s is free) parameters of the model.

### 4.1. Bias on fσ₈

#### 4.1.1. Particles

Let us first consider the case where sources are DM particles, as these should give the clearest trend on the impact of magnification bias. The full results for the ΛCDM case can be found in Fig. 5.

In the standard analysis (RSD only), we see on the top row for the unlensed sample that we recover the expected value of fσ₈ within 1σ statistical uncertainty. When lensing magnification is incorporated in the data, we clearly see that it has almost no impact in the first redshift bin at z = 1.0 − 1.3, but strongly shifts the estimation of fσ₈ at higher redshifts. More precisely, magnification bias leads to an underestimation of the growth rate if it is not modelled. The shift on fσ₈ does not strictly increase with s, which makes sense as the most dominant term in the lensing correction is ζlens-lens that scales as (5s − 2²) (see Sect. 2.2). This means that s = 0 should lead to a similar effect as s = 0.8, while s = 0.4 completely cancels the lensing effect. Then, the shift on the growth rate monotonically increases for s > 0.8. We see that at z = 1.3 − 1.6 and z = 1.6 − 1.95, the shift on fσ₈ for s = 1.2 reaches approximately 9% and 11% respectively, with respect to the unlensed case, while it is 5% and 6% for s = 1. In the highest redshift bin, we find that the growth rate is not recovered within 1σ statistical uncertainty for s ≥ 1.

When magnification bias is accounted for in the modelling and s is known, we obtain an unbiased estimation of the growth rate, except for s = 1.2 in the highest redshift bin. This might come from the slight difference in the quadrupole seen in Fig. 4, where the theoretical prediction seems to underestimate the signal. This shows that overall, our linear-theory based lensing correction is good enough to correct for the effect of magnification bias. However, for very high values of s at high redshift, the weak-lensing limit that was used to compute the correction might not hold anymore. Therefore, accounting for higher-order terms may be important in the future for the most extreme configurations.

Lastly, when s is free in the fit, error bars on the parameters are larger, which is expected since there is more freedom in the model. Letting s free might not be the best strategy in the full-shape RSD analysis. One should rather rely on estimated values of s. Nonetheless, the best-fit values obtained when letting s free are very close to the case where s is fixed, meaning that we are able to estimate this nuisance parameter without any strong bias.

Overall, we find that depending on the sample selection, galaxy clustering surveys probing redshifts above z ~ 1.3 will need to properly include lensing magnification as part of the theoretical model to recover unbiased estimates of the growth rate of structure, and in turn be able to test gravity while keeping theoretical systematic errors under control.

In Fig. 6 we perform the same analysis but for the wCDM model. We see that the results are extremely similar to that of Fig. 5, which is reassuring since here we compare the esti-

### Table 2. Number of objects in the different cases as a function of the redshift bin, type of selection, cosmology, and source type (particles or haloes).

| Redshift bin | ΛCDM | wCDM |
|--------------|------|------|
|   | Particles (×10⁶) | Haloes (×10⁶) | Particles (×10⁶) | Haloes (×10⁶) |
| 1.0 < z < 1.3 | 9.6 | 2.2 | 9.7 | 2.5 |
| 1.3 < z < 1.6 | 11.3 | 2.2 | 11.3 | 2.4 |
| 1.6 < z < 1.95 | 14.5 | 2.1 | 14.2 | 2.4 |

### Table 3. Flat priors used in the MCMC likelihood analyses.

| Parameter | Flat prior |
|-----------|------------|
| f         | [0, 2]     |
| b₁        | [-0.5, 3]  |
| b₂        | [-70, 70]  |
| σ₁²       | [-50, 100] |
| α₉        | [0.5, 1.5] |
| α₉        | [0.5, 1.5] |
| s         | [0, 2.5]   |
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Fig. 5. Relative difference on $f\sigma_8$ with respect to its fiducial value, as a function of redshift bins and sample configuration for $\Lambda$CDM particles. The data points in blue, green and red refer to different analysis where we only account for RSD, add magnification bias with $s$ fixed and free respectively (see also Sect. 3.2.3). The central values are that of the overall best-fit, while error bars are that of the 66% confidence interval on the marginalised distribution. The grey shades highlight the $\pm 5\%$ and $10\%$ ranges.

This means that, although the full analysis is performed using a $\Lambda$CDM fiducial model, we are still able to recover almost unbiased constraints on the cosmology of the data (see also Fig. 2 for the difference in $f\sigma_8$ between the two cosmological models). It is to be noted the very large error bars for $s = 0.2$ and $s = 0.6$ in the highest redshift bin. The fact that these configurations give similar results is expected since the leading magnification bias term scales as $(5s^2 - 2)^2$. Furthermore, it seems that for these particular configurations there is a multi-modal behaviour in the joint posterior probability for the growth rate and second-order Lagrangian bias parameters, leading to larger er-
Fig. 6. Same as Fig. 5 but for $w_{\Lambda}CDM$.

In the last two redshift bins, the shift on $f\sigma_8$ reaches approximately 12% and 10% for $s = 1.2$, when magnification bias is not accounted for. As previously, knowing the true value of $s$ in the modelling gives an unbiased estimate of $f\sigma_8$, except for the last redshift bin for $s = 0.8$, which is surprisingly off. This shows that although the lensing correction computed with the $\Lambda$CDM model is not exact for the $w_{\Lambda}CDM$ simulation as seen in Appendix A, it still includes most of the effect. In any case, adding the magnification bias correction at fixed $s$ gives a much better agreement to the fiducial value of $f\sigma_8$.

4.1.2. Haloes

We now turn to the analysis of DM haloes. While DM particles are useful since they allow characterising the effect of magnification bias with great precision, analysing haloes is more observationally relevant since these are biased matter tracers as ob-
served galaxies. This is particularly important because the relative effect of magnification bias on the total number counts scales as \((5s - 2)/b\), where \(b\) is the linear Eulerian galaxy bias. This means that we expect lensing to have less impact on samples with large bias. In our case, the best-fit RSD-only models for the unmagnified halo samples in the \(\Lambda\)CDM cosmology gives roughly \(b = 2.1, 2.5\) and 3 in the different redshift bins, from low to high redshifts. We show the results in Fig. 7. Qualitatively, the results are similar to those of Fig. 5. We find that the values of the growth rate tend to be slightly overestimated, even when the sample is not magnified. This can be attributed to residual theoretical uncertainties of the RSD model, and the modest volume probed by our lightcones, which can lead to non-negligible sample variance effects. Nonetheless, we can study and discuss the relative differences in the estimated parameters for the various cases. As previously, we find that the central values of \(f\sigma_8\) are shifted towards lower values when lensing is not accounted for in the modelling. The main difference between DM particles and haloes is that for the latter we do not clearly see the effect of lensing, as the magnified and unmagnified cases agree within 1\(\sigma\) error. In the highest redshift bin, the best-fit value for \(s = 1.2\) is shifted by roughly 3\% with respect to the unmagnified case. 

While it is still large, it does not reach the 11\% seen for the particles.

The results displayed in Fig. 8 for \(w\)CDM haloes are very similar. We find in the highest redshift bin a discrepancy of 3.5\% and 5.5\% for \(s = 1.0, 1.2\) with respect to the unmagnified case. While larger than for \(\Lambda\)CDM haloes, it still does not reach the 10\% difference that we have for particles.

Overall, we find that the effect of magnification bias is lower for haloes than for particles, which is expected due to galaxy bias. Moreover, the relative difference on \(f\sigma_8\) is suppressed by a factor close to \(b\). This means that very biased target samples might not be significantly subject to lensing effects at the considered redshifts in the present work.

### 4.2. Other parameters

In this section we study the impact of magnification bias on the parameters of our model beyond the growth rate of structure. We consider ACDM particles in two cases: \(s = 1.0\), as it is a realistic case for future spectroscopic surveys, and \(s = 1.2\) where the effect of magnification bias is the most significant. We present the posterior distributions for \(s = 1.0\) in Fig. 9. We see that not accounting for lensing leads to a biased estimation of the growth rate, while the lensing correction (either when \(s\) is fixed or free) allows us to recover the fiducial value within 1\(\sigma\) error. These results are approximately the same as those of Fig. 5 except that we show \(f\) (the free parameter of our model) and not \(f\sigma_8\). Since our fiducial cosmology is already that of the data, there should not be any qualitative difference, in the sense that we only need to multiply \(f\) by the fiducial \(\sigma_8\) (as in principle, the dilation parameters should be very close to unity). For DM particles, the first-order Lagrangian bias \(b_1\) should be equal to zero and this is indeed what we obtain. However, not accounting for lensing magnification leads to an overestimation of the bias. This comes from the fact that lensing adds a positive contribution to the multipole moments (as seen in Fig. 1), which is counter-balanced in the likelihood analyses by higher values of the galaxy bias. Moreover, we clearly see in the \((f, b_1)\) joint posterior probability an anti-correlation, where higher values of the linear bias induces lower values of \(f\). This comes from the fact that higher values of the galaxy bias leads to a larger amplitude of the quadrupole.

In order to fit the data, the likelihood analysis therefore converges towards a lower value of \(f\).

For the growth rate and galaxy bias parameters, we see that letting \(s\) free increases the error bars with respect to the case when \(s\) is fixed. However, this is not the case for the other nuisance parameters, that is \(b_2, \sigma_8^2\) and dilation parameters, where the two analyses give similar marginal distributions. Regarding the estimation of \(s\), we see a remarkable agreement between the expected and best-fit values. This shows that, in principle, we should be able to accurately recover \(s\) through galaxy clustering analysis. Furthermore, we notice that the marginal distribution of \(s\) exhibits a multi-modal behaviour around \(s = 0\). This comes from the fact that the leading correction term is proportional to \((5s - 2)^2\), and there are two plausible possibilities when \(s \leq 0.8\). This is a further evidence that it might not be optimal to let \(s\) free with a flat uninformative prior.

Finally, we find that in none of the cases we exactly recover \((a_\perp, a_\parallel) = (1, 1)\) within 1\(\sigma\) error, although our results are very close. This might be related to the fact that we have one particular realisation of the density field in a limited volume, which is subject to sample variance. We note that in the RSD-only case, the estimated \(a_\perp\) and \(a_\parallel\) parameters depart from fiducial values more than in the cases where lensing is accounted for. This means that in this case, the likelihood analysis prefers other cosmological models. This might further impact the estimation of the dilation-corrected \(\sigma_8\) as described in Sect. 3.2.3. However, this additional bias seems low for the redshift bins that we considered. It is worth mentioning the case of the second-order Lagrangian bias \(b_2\) whose fiducial value is only recovered when lensing is accounted for, while the inferred values of the velocity dispersion parameter \(\sigma_8^2\) are very similar in all cases. Nonetheless, these two parameters are considered as nuisance parameters over which we marginalise over.

Lastly, we present the posterior distributions for \(s = 1.2\) in Fig. 10. The results are similar to that of Fig. 9 except that here we do not recover the value of \(f\) within 1\(\sigma\) error, even when we account for magnification bias as seen in Fig. 5. As in Fig. 9, we remark that we are able to recover \(s\) accurately, even in this configuration where the weak-lensing limit might start to no longer hold. We also notice that there is no multi-modal behaviour, contrary to the case with \(s = 1.0\). This is because \(s\) is large enough so that there is no ambiguity between the different solutions at \(s < 0.8\). This shows that, generally, adding the lensing contribution in the modelling allows for a considerably better recovery of the fiducial parameters.

### 5. Conclusion

In this paper, we studied the impact of magnification on the three-dimensional, redshift-space galaxy clustering and how it biases the estimation of the growth rate of structure when not properly accounted for in the modelling. This effect, commonly referred to as magnification bias, comes from the fact that gravitational lensing modifies the apparent angular positions of sources and magnify their fluxes. Therefore, magnitude-limited surveys are sensitive to it as it generates additional apparent spatial correlations in the data. We performed an exhaustive galaxy clustering analysis of the multipole moments of the correlation function, where the theoretical model relied on the Convolution Lagrangian Perturbation Theory and Gaussian Streaming model for non-linear RSD, and a curved-sky, linear-theory-based lensing correction. We confronted this model to high-resolution \(N\)-body simulations with two different cosmologies, \(\Lambda\)CDM and \(w\)CDM, and where the effects of RSD and magnification bias
were implemented in simulated catalogues of DM particles and haloes.

The main results of our paper are the following. First, magnification bias only becomes relevant at $z > 1.3$. Second, not accounting for magnification bias in the modelling gives a biased estimation of the growth rate. We found that analysing a magnified sample with a theoretical model that only contains RSD leads to an overestimation of the galaxy bias, and more importantly, an underestimation the growth rate. Depending on the redshift and slope of the galaxy luminosity function, the best-fit value of $f\sigma_8$ can be shifted by more than 10%. It appears that using the linear-theory lensing correction allows the recovery of unbiased estimates of the growth rate in most cases, if $s$ is accurately known a priori. If $s$ is unknown, it might be tempting to keep it as a free parameter of the model. While this allows the recovery of an unbiased estimation of $s$, it significantly increases the uncertainty on $f\sigma_8$, which we might refrain from. This shows that one should not let $s$ free (or at least, not use

Fig. 7. Same as Fig. 5 but for haloes.
a flat uninformative prior on that parameter) and rather find a way to estimate it differently. Finally, we find that we were able to recover the same results for a $w_{\Lambda}$CDM simulation although it was analysed with a fiducial $\Lambda$CDM model. This is encouraging and shows that our modelling is reliable even if we do not know exactly the underlying cosmology of the data.

Gravitational lensing will play an important role in galaxy clustering analyses in future high-redshift surveys, and its effect will have to be implemented in order to accurately recover cosmological parameters. This effect has not been considered so far in observational studies, even when the redshift of the sample was high (Okumura et al. 2016; Zarrouk et al. 2018; Hou et al. 2021). However, it does not necessarily mean that these works were not correct, since statistical uncertainties are still significant and the effect of magnification bias depends on the properties of the galaxy sample under scrutiny. An interesting prospect for future works would be to consistently incorporate the effect of gravitational lensing within the theoretical framework of non-

![Fig. 8. Same as Fig. 7 but for $w_{\Lambda}$CDM.](image-url)
linear RSD. Nonetheless, we showed that in our case an hybrid model of non-linear RSD and linear lensing is sufficient for the accuracy of our simulated data.

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Fig. 9. Posterior distributions for the parameters of our model, for the \( \Lambda \)CDM particles sample with \( s = 1.0 \) at \( z = 1.6 - 1.95 \).
Fig. 10. Same as Fig. 9 but with $s = 1.2$. 

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Appendix A: Magnification bias modelling on the correlation function multipoles

In this appendix we provide more details about the effect of magnification bias on the multipole moments of the correlation function. We consider the DM particles in the highest studied redshift bin at about $z = 1.8$, which maximises the magnification bias signal. The magnification bias corrections on the monopole, quadrupole and hexadecapole of the correlation function are shown in Fig. A.1. First, we notice that in any case, that is for $s = 0$ or 1.2, $\Lambda$CDM or $\omega$CDM, the lensing contribution to the correlation function is always positive at this redshift. It is interesting since it confirms the results of Fig. 4 and explains why, if lensing magnification is not incorporated in the model, the likelihood analysis tries to compensate this lack by increasing the value of the bias parameter (mostly due to the monopole) and therefore lower the value of the growth rate (as seen in Sect. 4) due to the quadrupole.

Secondly, for the case with $s = 0$, that is when we use observed angles instead of comoving ones, the monopole seem to agree with the theoretical prediction up to $60\ h^{-1}\text{Mpc}$ only. This is surprising as we would expect on the contrary a good agreement at large scales. This difference might be due to the large variance inherent to these scales or an inaccuracy in the modelling as well as the non-linearity of lensing corrections (see also [Hui et al. 2007] for a discussion). We also note the remarkable agreement between data and prediction for the quadrupole (while the hexadecapole seems underestimated in the prediction).

Thirdly, focusing on the case with $s = 1.2$ which give the most significant trend, we see that the theoretical prediction overestimates the monopole, and underestimate the quadrupole and hexadecapole above $80\ h^{-1}\text{Mpc}$. While this discrepancy is not seem necessarily large, especially for this high value of $s$, it impacts the estimation of the growth rate (see Fig. 5). This suggests that one should be careful about the first-order solution given under the weak-lensing limit, as higher-order terms might need to be accounted for at higher redshifts for large values of $s$.

Lastly, we see that in any case the theoretical prediction in the fiducial $\Lambda$CDM model does not agree very well with the data from the $\omega$CDM simulation. Although the shape is similar between data points and analytical prediction, the amplitude is different (this difference is clear for the quadrupole, where data points are consistently at least a factor two above the prediction for both $s = 1.2$). For more precise studies at higher redshift it will be important to find a way to account for this cosmology-dependant correction. Nonetheless, even if the present modelling in the fiducial $\Lambda$CDM cosmology is not perfect to analyse the $\omega$CDM, it is still much better than not accounting at all for lensing magnification.

Fig. A.1. Absolute difference on the correlation function multipoles for DM particles at the highest redshift bin (that is, $z = 1.6 - 1.95$), when accounting for magnification bias, with respect to the case where we have RSD only. The red and green (purple and blue) points refer to the monopole and quadrupole in $\Lambda$CDM ($\omega$CDM), while the red, green and black lines show the $\Lambda$CDM theoretical prediction for the monopole, quadrupole and hexadecapole computed with CorrF. Only for the $\Lambda$CDM case we show the hexadecapole, as the results for the $\omega$CDM model are very similar and in any do not impact much the likelihood analysis.