TOPOLOGICAL DEFECTS WITH NON-SYMMETRIC CORE

Minos Axenides\textsuperscript{1}

\textit{Institute of Nuclear Physics,}
\textit{N.C.R.P.S. Demokritos}
\textit{153 10, Athens, Greece}

Leandros Perivolaropoulos\textsuperscript{2}

\textit{Department of Physics}
\textit{University of Crete}
\textit{71003 Heraklion, Greece}

Abstract

We demonstrate that field theories involving explicit breaking of continuous symmetries, incorporate two generic classes of topological defects each of which is stable for a particular range of parameters. The first class includes defects of the usual type where the symmetry gets restored in the core and vacuum energy gets trapped there. We show however that these defect solutions become unstable for certain ranges of parameters and decay not to the vacuum but to another type of stable defect where the symmetry in \textit{not} restored in the core. In the wall case, initially spherical, bubble-like configurations are simulated numerically and shown to evolve generically towards a \textit{planar collapse}. In the string case, the decay of the \textit{symmetric core vortex} resembles the decay of a semilocal string to a skyrmion with the important difference that while the skyrmion is unstable and decays to the vacuum, the resulting \textit{non-symmetric vortex} is topologically stable.

\textsuperscript{1}E-mail address: axenides@gr3801.nrcps.ariadne-t.gr
\textsuperscript{2}E-mail address: leandros@physics.uch.gr
1 Introduction

Topological defects\cite{1, 2} are stable field configurations (solitons\cite{3}) that arise during phase transitions\cite{4} in field theories with spontaneously broken discrete or continuous symmetries. Depending on the topology of the vacuum manifold $M$ they are usually identified as domain walls\cite{2} (kink solutions\cite{3}) when $M = Z_2$, as strings\cite{5} and one-dimensional textures (ribbons\cite{1, 6}) when $M = S^1$, as monopoles (gauged\cite{8, 9, 10} or global\cite{11, 12}) and two dimensional textures ($O(3)$ solitons\cite{13, 3}) when $M = S^2$ and three dimensional textures ($\text{SU}(2)$ skyrmions\cite{15}) when $M = S^3$. Topological defects appearing in GUT phase transitions and quantum field fluctuations produced during an inflationary phase\cite{16, 17, 18} constitute two physically distinct mechanisms for the production of the primordial fluctuations that gave rise to galaxies and large scale structure in the universe. The macrophysical predictions of topological defects (particularly cosmic strings) for structure formation have been studied extensively\cite{1, 19, 20, 21, 22, 23, 24} and compared with the corresponding observations with encouraging results.

Defects corresponding to all of the above vacua are characterized by a conserved quantity called the 'topological charge'\cite{3} which guarantees their stability. It essentially counts the number of times the field configuration covers the vacuum manifold of the theory. This is achieved by the variation of the field at either spatial infinity or over the whole of space. Thus topological defects could be classified in two broad categories. In the first category the topological charge becomes non-trivial due to the behavior of the field configuration at spatial infinity. In this case continuity, forces the field to remain out of the vacuum manifold at a localized region in space where the spontaneously broken symmetry get restored. This region is the core of the defect and is associated with vacuum trapped potential energy. Because the symmetric phase of the theory is retained in the core of these defects we will call them hereafter symmetric defects. Domain walls, strings and monopoles belong to this class.

In the second category the vacuum manifold gets covered completely as the field varies over the whole of coordinate space. The field variable thus remains in the vacuum at all points of coordinate space. Its value at infinity is moreover identified with a single point of the vacuum manifold. Textures \cite{14} (skyrmions \cite{15}), $O(3)$ solitons \cite{13} (two dimensional textures \cite{14, 25}) and ribbons \cite{1} (one-dimensional textures \cite{25}) belong to this class which we
will call for definiteness ’texture-like’ defects. Thus both the behavior of the field in the core of the defect (symmetric or non-symmetric phase) as well as its behavior at spatial infinity (covering or not the vacuum manifold) suffice to classify it as belonging to either the symmetric class or the texture class.

Even though this classification can incorporate most of the well known defects it is not difficult to think of configurations that belong to neither class and yet possess quite interesting properties. These are configurations, appearing mainly in models with explicit symmetry breaking. There the field variable covers the whole vacuum manifold at infinity with the core remaining in the non-symmetric phase. Thus it is possible to have domain walls, strings and monopoles with cores in the non-symmetric phase of the theory. For definiteness we will call these ’non-symmetric’ defects.

Examples of nonsymmetric defects have been discussed previously in the literature. Everett and Vilenkin [26], in particular, pointed out the existence of domain walls and strings with non-symmetric cores. In their model however the walls (strings) were bounded by strings (monopoles) formed at a phase transition at a higher energy scale. Thus, these defects were unstable to shrinking and collapse due to their string tension. More recently, Dvali et.al. [27] and Bachas et.al. [4] considered a model which admits nonsymmetric walls. They moreover showed the existence of classically stable bound states of such walls in this model which they called ’2\pi walls’ or membranes. They speculated that these states could be cosmologically disastrous even if the $Z_2$ symmetry is anomalous. These bound state wall configurations were previously identified with one dimensional textures [25] or ribbons [6] and are stabilized either by space compatification or by introducing a small anomalous term that breaks explicitly the $Z_2$ symmetry [28, 6].

A particular case of non-symmetric gauge defect was recently considered by Benson and Bucher [29] (see also Refs. [30, 31]) who pointed out that the decay of an electroweak semilocal string leads to a gauged ’skyrmion’ with non-symmetric core and topological charge at infinity. This skyrmion however, rapidly expands and decays to the vacuum.

Further back and in the context of $SU(5)$ GUT monopoles [10] higher $SU(2)$ embeddings into the fundamental quintuplet space of $SU(5)$ were shown to correspond to ’higher strength’ monopole configurations [32]. These were shown to be unstable and decay into the ’single-strength’ fundamental monopoles while preserving the overall topological number.

Our goal in this paper is to present more examples of topological defects
that belong to what we defined as the ‘non-symmetric’ class. We will study in detail the properties of the simplest such configurations. More specifically we will first consider a model with a $U(1)$ symmetry explicitly broken to $Z_2$ which is in turn spontaneously broken. We will demonstrate that the model admits all three types of defects (symmetric domain walls, ribbons and non-symmetric walls) and identify semi-analytically the range of parameters in which each type of domain walls is topologically stable.

In an attempt to study the cosmological effects of $2\pi$ walls (ribbons) we perform numerical simulations of collapsing bubbles of symmetric and non-symmetric walls. We do not see evidence of the formation of bound states ($2\pi$ walls). Instead, in the non-symmetric parameter range, the bubbles collapse asymmetrically (pancake collapse) and decay into the vacuum after a wall collision accelerated by the tension of the bubble. In the symmetric parameter range (symmetric phase in the wall core) the collapse is spherical and the bubble decays to the vacuum due to tension as expected. We give qualitative reasoning for this bubble behavior.

In section 3 we extend our analysis to higher dimensional defects and in particular to the case of non-symmetric global strings. We identify the parameter ranges for stability of symmetric and non-symmetric strings in a theory with a global $SU(2)$ symmetry explicitly broken to $U(1)$. These results are then extended to the case of the semilocal string [33, 34, 35, 36] where we identify the parameter range for the symmetric and non-symmetric phase. This result is in agreement with previous analyses [37, 36, 38]. We also briefly discuss the possibility of gauging the configurations considered which could lead to gauged strings and monopoles with non-symmetric core and thus to possible generalizations of electroweak strings [39, 40, 41, 42] and monopoles.

Finally, in section 4 we conclude, summarize and discuss the outlook of this work.
2 Domain Walls: Symmetric vs NonSymmetric Core

Consider a model with a $U(1)$ symmetry explicitly broken to a $Z_2$. This breaking can be realized by the Lagrangian density \[ L = \frac{1}{2} \partial_{\mu} \Phi^* \partial^{\mu} \Phi + \frac{2}{2} |\Phi|^2 + \frac{m^2}{2} Re(\Phi^2) - \frac{h}{4} |\Phi|^4 \] (1)

where $\Phi = \Phi_1 + i \Phi_2$ is a complex scalar field. After a rescaling

$$\Phi \rightarrow \frac{m}{\sqrt{h}} \Phi$$
$$x \rightarrow \frac{1}{m} x$$
$$M \rightarrow \alpha m$$

the potential takes the form

$$V(\Phi) = -\frac{m^4}{2h} (\alpha^2 |\Phi|^2 + Re(\Phi^2) - \frac{1}{2} |\Phi|^4)$$

For $\alpha < 1$ this is a saddle point potential i.e. at $\Phi = 0$ there is a local minimum in the $\Phi_2$ direction but a local maximum in the $\Phi_1$ (Fig 1). For $\alpha > 1$ the local minimum in the $\Phi_2$ direction becomes a local maximum but the vacuum manifold remains disconnected, and the $Z_2$ symmetry remains. This type of potential may be called a 'Napoleon hat' potential in analogy to the Mexican hat potential that is obtained in the limit $\alpha \rightarrow \infty$ and corresponds to the restoration of the $S^1$ vacuum manifold.

The corresponding equation of motion for the field $\Phi$ is

$$\ddot{\Phi} - \nabla^2 \Phi - (\alpha^2 \Phi + \Phi^*) - |\Phi|^2 \Phi = 0$$

which accepts the static kink solution

$$\Phi_1 = \Phi_R \equiv \pm (\alpha^2 + 1)^{1/2} \tanh((\frac{\alpha^2 + 1}{2})^{1/2} x)$$
$$\Phi_2 = 0$$

This solution corresponds to a symmetric domain wall since the core of the soliton is symmetric ($\Phi(0) = 0$) and the topological charge is resulting from
Figure 1: (a) The domain wall potential has a local maximum at $\Phi = 0$ in the $\Phi_1$ direction. (b) For $\alpha > 1$ ($\alpha < 1$) this point is a local maximum (minimum) in the $\Phi_2$ direction.

The behavior of the field at infinity ($Q = \frac{1}{2}(\Phi(-\infty) - \Phi(+\infty))/(\alpha^2 + 1)^{1/2}$). The form of the potential however implies that the symmetric wall solution may not be stable for $\alpha > 1$ since in that case the potential energy favors a solution with $\Phi_2 \neq 0$. However, the answer is not obvious because for $\alpha > 1$, $\Phi_2 \neq 0$ would save the wall potential energy but would cost additional gradient energy as $\Phi_2$ varies from a constant value at $x = 0$ to 0 at infinity. A stability analysis is therefore needed and may be performed as follows.

Consider an ansatz of the form

$$\Phi = \Phi_R(x) + \delta \Phi_1(x)e^{i\omega_1 t} + i\delta \Phi_2(x)e^{i\omega_2 t}$$

(9)

Substituting (9) in (6) we obtain to first order in the perturbations

$$-\delta \Phi_1'' + 3 \tanh^2\left(\frac{x}{\sqrt{2}}\right)\delta \Phi_1 - \delta \Phi_1 = \frac{\omega_1^2}{\alpha^2 + 1}\delta \Phi_1$$

(10)

$$-\delta \Phi_2'' + \tanh^2\left(\frac{x}{\sqrt{2}}\right)\delta \Phi_2 - \alpha^2 - 1 \delta \Phi_2 = \frac{\omega_2^2}{\alpha^2 + 1}\delta \Phi_2$$

(11)

For stability we demand that $\omega_i^2 > 0$ i.e that there are no negative eigenvalues to the Schroedinger-like equations (10) and (11). The same conditions are obtained by perturbing the static energy functional

$$E = \frac{m^4}{2h} \int_0^\infty dx (\Phi'^2 + \frac{1}{2}|\Phi|^4 - (\alpha^2|\Phi|^2 + Re(\Phi^2)))$$

(12)

to second order around the solution (7), (8) and demanding that the perturbations contribute no negative part to the energy.
The potential of the Schroedinger-like equation (11) is positive definite for \( \alpha < 1 \) and it becomes negative and deeper for \( \alpha > 1 \) as \( \alpha \) increases. To find the critical value of \( \alpha \) such that for \( \alpha > \alpha_{\text{crit}} \) (11) has negative eigenvalues we solve (11) numerically using the shooting method in the Mathematica \([13]\) system. The boundary conditions at the origin are \( \delta \Phi_2(0) = 1 \) and \( \delta \Phi'_2(0) = 0 \), corresponding to the ground state solution. We solve (11) for various values of \( c \equiv \frac{\alpha^2}{\alpha^2+1} \) with \( \omega_2 = 0 \) looking for \( c_{\text{crit}} \) such that \( \lim_{x \to \infty} \delta \Phi_2(x) = 0 \) which corresponds to a ground state with 0 eigenvalue. Clearly for \( c > c_{\text{crit}} \) the potential is deeper and there are negative eigenvalues. Using this method we find \( c_{\text{crit}} = \frac{1}{2} \). This solution corresponds to \( \alpha_{\text{crit}} = \sqrt{3} \simeq 1.73 \). As expected \( \alpha_{\text{crit}} > 1 \) due to the effects of the gradient energy discussed above. Using the same method we also solved equation (10) and found that it has no negative eigenvalues as expected due to the topological stability of the component \( \Phi_1 \).

We have verified this result using two other methods in order to both test the result and also test the accuracy of the methods. The first method is based on solving the full non-linear static field equations obtained from (6) with boundary conditions

\[
\Phi_1(0) = 0, \quad \lim_{x \to \infty} \Phi_1(x) = (\alpha^2 + 1)^{1/2} \quad (13)
\]

\[
\Phi'_2(0) = 0, \quad \lim_{x \to \infty} \Phi_2(x) = 0 \quad (14)
\]

Using a relaxation method based on collocation at gaussian points \([17]\) to solve the system (6) of second order non-linear equations we find that for \( \alpha < \sqrt{3} \) the solution relaxes to the expected form of (7) for \( \Phi_1 \) while
Φ_2 = 0 (Fig. 2). For α > \sqrt{3} we find Φ_1 \neq 0 and Φ_2 \neq 0 (Fig. 3) obeying the boundary conditions (13), (14) and giving the explicit solution for the non-symmetric domain wall. In both cases we also plot the analytic solution (7) stable only for α < \sqrt{3} for comparison (bold dashed line). As expected the numerical and analytic solutions are identical for α < \sqrt{3} (Fig. 2).

The second method is based on a numerical minimization of the energy functional (12) using the steepest descent method of Refs. [7, 44, 45]. We write the energy functional (12) on a one dimensional lattice of N points and a sum over 2(N − 1) variables

\[ E = \sum_{i=1}^{N-1} \left( \frac{\Phi_1(i+1) - \Phi_1(i)}{dx} \right)^2 + \left( \frac{\Phi_2(i+1) - \Phi_2(i)}{dx} \right)^2 + \frac{1}{2}(\Phi_1(i)^2 + \Phi_2(i)^2)^2 - \alpha^2(\Phi_1(i)^2 + \Phi_2(i)^2) - (\Phi_1(i)^2 - \Phi_2(i)^2) \]

We then evaluate numerically the quantities

\[ d\Phi_j(i) \equiv \frac{dE}{d\Phi_j(i)} \quad (j = 1, 2), \quad (i = 1, ...N) \]

(15)

and shift \( \Phi_j(i) \) to

\[ \Phi_j(i) = \Phi_j(i) - \epsilon \, d\Phi_j(i) \]

(16)

where

\[ \epsilon \simeq 10^{-3} \]

(17)
thus finding a new field configuration with lower energy. We repeat this procedure, keeping the boundary conditions, until the energy variation is negligible implying that we have reached a local minimum in configuration space. Our initial configuration was the analytic solution (7) perturbed by a small amount $\Phi_2 = 0.01 e^{-x^2}$. For $\alpha < \sqrt{3}$ the imposed perturbation decreased and the field configuration relaxed to the analytic solution (7). For $\alpha > \sqrt{3}$ the perturbation increased towards the solution obtained by the relaxation method (Fig. 3). The verification of the result that $\alpha_{cr} = \sqrt{3}$ not only makes it strongly established but is also a test for the validity of the methods used.

Bound states of pairs of non-symmetric walls have been discussed previously in the literature under different names (one-dimensional textures [25], ribbons [1, 2], 2$\pi$ walls [27]). By Derrick’s theorem [46] such configurations are normally unstable towards expansion induced by the gradient energy term [25] but can be stabilized by introducing a small explicit breaking of the $Z_2$ symmetry [1, 27] or by imposing periodic boundary conditions [6]. The cosmological effects of these states were discussed by Dvali et. al. [27]. They pointed out that the stability of a 2$\pi$ wall system in the presence of an anomalous $Z_2$ symmetry can lead to a cosmic overabundance of walls and therefore restrictions can be imposed on the allowed range of parameters. As the cosmic horizon expands however, the 2$\pi$ wall systems will emerge as bubbles of non-symmetric walls. It is the evolution and stability of such bubbles that needs to be studied in order to determine if these objects will cause a cosmological problem.

In order to address this issue we constructed a two dimensional simulation of the field evolution of domain wall bubbles with both symmetric and non-symmetric core. In particular we solved the non-static field equation (6) using a leapfrog algorithm [47] with reflective boundary conditions. We used an $80 \times 80$ lattice and in all runs we retained $\frac{dt}{dx} \simeq \frac{1}{3}$ thus satisfying the Cauchy stability criterion for the timestep $dt$ and the lattice spacing $dx$. The initial conditions were those corresponding to a spherically symmetric bubble with initial field ansatz

$$\Phi(t_i) = (\alpha^2 + 1)^{1/2} \tanh\left(\frac{\alpha^2 + 1}{2} (\rho - \rho_0)\right) + i \ 0.1 \ e^{-||x| - \rho_0|| x} \frac{x}{|x|} \quad (18)$$

where $\rho = x^2 + y^2$ and $\rho_0$ is the initial radius of the bubble. Energy was conserved to within 2% in all runs. For $\alpha$ in the region of symmetric core
stability the imaginary initial fluctuation of the field $\Phi(t_i)$ decreased and the bubble collapsed due to tension in a spherically symmetric way as expected.

For $\alpha$ in the region of non-symmetric core stability the evolution of the bubble was quite different. The initial imaginary perturbation increased but even though dynamics favored the increase of the perturbation, topology forced the $Im \Phi(t)$ to stay at zero on two points along the bubble: the intersections of the bubble wall with the y axis (Figs. 4, 5). Thus in the region of these points, surface energy (tension) of the bubble wall remained larger than the energy on other points of the bubble. The result was a non-spherical collapse with the x-direction of the bubble collapsing first (Fig. 5). This asymmetric collapse of the bubble may be understood as follows: The bubble wall in a two dimensional projection may be approximated by a rectangle with dimensions x-y. Let the x sides of the rectangle be in the symmetric phase and therefore have energy per unit length $E_0$. Then the y sides would be in the non-symmetric phase with energy per unit length $E_1$.

Clearly $E_0 > E_1$ since we are in the parameter range where the non-symmetric wall is stable. Thus the energy of the bubble walls is

$$E = 2(E_0 x + E_1 y)$$

(19)
which implies that it is energetically favorable for the bubbles to collapse in the x direction rather than the y direction. This argument clearly does not hold in the range of symmetric wall stability since in that case the energy would be \( E = 2E_0(x+y) \). Also it does not hold for an imaginary perturbation without the \( \frac{1}{|x|} \) factor since in that case the energy would be \( E = 2E_1(x+y) \). Generically, the two types of perturbations are equally probable and therefore, in the parameter range of non-symmetric wall stability we anticipate about half of the wall bubbles to collapse asymmetrically (pancake collapse) while the rest would collapse in the usual spherical way.

In all the cases we simulated we saw no evidence for formation of a bound state and in all cases the bubbles collapsed to the vacuum accelerated by their tension. This decay would be even faster for an anomalous \( Z_2 \) symmetry. Thus the cosmological problem discussed in Ref. [27] appears not to be realized in the cases we examined.

Figure 5: Evolved field configuration \((t = 14.25, \) 90 timesteps\) for a non-symmetric initially spherical bubble wall with \( \alpha = 3.5 \).
3 Generalizations to Other Defects

It is straightforward to generalize the analysis of the previous section to higher dimensional defects like strings or monopoles. Consider for example a model with an $SU(2)$ symmetry explicitly broken to $U(1)$. Such a theory is described by the Lagrangian density:

$$
\mathcal{L} = \frac{1}{2} \partial_\mu \Phi^\dagger \partial^\mu \Phi + \frac{M^2}{2} \Phi^\dagger \Phi + \frac{m^2}{2} \Phi^\dagger \tau_3 \Phi - \frac{h}{4} (\Phi^\dagger \Phi)^2
$$

where $\Phi = (\Phi_1, \Phi_2)$ is a complex scalar doublet and $\tau_3$ is the $2 \times 2$ Pauli matrix. After rescaling as in equations (2)-(4) we obtain the equations of motion for $\Phi_{1,2}$

$$
\partial_\mu \partial^\mu \Phi_{1,2} - (\alpha^2 \pm 1) \Phi_{1,2} + (\Phi^\dagger \Phi) \Phi_{1,2} = 0
$$

where the $+$ ($-$) corresponds to the field $\Phi_1$ ($\Phi_2$).

Consider now the ansatz

$$
\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} f(\rho) e^{i\theta} \\ g(\rho) \end{pmatrix}
$$

with boundary conditions

$$
\lim_{\rho \to 0} f(\rho) = 0, \quad \lim_{\rho \to 0} g'(\rho) = 0
$$

$$
\lim_{\rho \to \infty} f(\rho) = (\alpha^2 + 1)^{1/2}, \quad \lim_{\rho \to \infty} g(\rho) = 0
$$

Figure 6: Field configuration for a symmetric-core global string with $\alpha = 2.6$. 
This ansatz corresponds to a global vortex configuration with a core that can be either in the symmetric or in the non-symmetric phase of the theory. Whether the core will be symmetric or non-symmetric is determined by the dynamics of the field equations. As in the wall case we expect the existence of a critical value for the parameter $\alpha$ such that for $\alpha < \alpha_{cr}$ the vortex core is in the symmetric phase ($g(\rho) = 0$) while for $\alpha > \alpha_{cr}$ the symmetric core configuration is unstable towards decay to a new vortex configuration with non-symmetric core ($g(0) \neq 0$). To determine the value of $\alpha_{cr}$ we solved numerically the system (21) of non-linear complex field equations with the ansatz (22) for various values of the parameter $\alpha$. We used the same relaxation technique discussed in the previous section for the case of walls.

For $\alpha < \alpha_{cr} \approx 2.7$ the solution relaxed to a lowest energy configuration with $g(\rho) = 0$ everywhere corresponding to a vortex with symmetric core (Fig. 6).

For $\alpha > \alpha_{cr} \approx 2.7$ the solution relaxed to a configuration with $g(0) \neq 0$ indicating a vortex with non-symmetric core (Fig. 7). Both configurations are dynamically and topologically stable and consist additional paradigms of the defect classification discussed in the introduction.

The existence of a defect with non-symmetric core corresponding to each defect with a symmetric core and winding at infinity can be established for various other cases. For example, our discussion for global vortices with non-symmetric core can be trivially generalized to the case of global monopoles. Another interesting extension is the case of non-symmetric gauged defects like Nielsen-Olesen vortices or gauge monopoles. Extension of our work in
these directions is currently in progress.

A common feature of the defects we have discussed so far is the fact that they emerge in theories where a larger global symmetry is partially explicitly broken. This is not always necessary. In the case of semilocal strings a U(1) gauge symmetry is embedded in a SU(2) global one. The Lagrangian density is given by

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} (D_\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi)$$

with $\Phi = (\Phi_1, \Phi_2)$ a complex doublet, $D_\mu = \partial_\mu - ieA_\mu$ a U(1) covariant derivative and $V(\Phi) = \frac{1}{4}(\Phi^\dagger \Phi - \eta^2)^2$ i.e. the SU(2) global symmetry is spontaneously broken with the U(1) part of it being gauged. The SU(2) symmetry is not explicitly broken, instead only the U(1) part of it is gauged. This is an alternative way to single out an $S^1$ subspace of the $S^3$ vacuum manifold. A similar analysis as the one discussed above (solution of a coupled system of non-linear ode’s by a relaxation method) and an ansatz of the form (22) with $A_\mu = \hat{e}\theta v(\rho)$ leads to a critical value for the parameter $\beta \equiv \frac{2\lambda}{e^2}$ (the only parameter of the model) such that for $\beta < \beta_{cr} = 1$ the vortex with a symmetric core is stable. This result is in agreement with previous studies. For $\beta > \beta_{cr}$ the symmetric core vortex is unstable to a configuration with a non-symmetric core called the ‘skyrmion’. However in this case, since there is no explicit breaking of SU(2), the ‘skyrmion’ is not topologically stable (the vacuum is $S^3$ not $S^1$) and energetics favor expansion and eventual decay to the vacuum.

4 Conclusion

We have studied the existence and stability properties of topological defects with non-symmetric core and non-trivial winding at infinity. These defects possess cores which are either symmetric or non-symmetric depending on the range of parameters of the theory. They are distinct from previously discussed unstable hybrid defects as they are stable and are not necessarily connected to any other type of defect formed in a previous phase transition.

These results have several interesting implications and potential extensions. For example, the observed non-spherical collapse of wall bubbles with
non-symmetric core may imply that the domain wall network simulations need to be re-examined for parameter ranges where a non-symmetric core in energetically favored.

Our results are also relevant for baryogenesis mechanisms based on topological defects\cite{49, 50, 51}. These mechanisms are based on unsuppressed B+L violating sphaleron\cite{52} transitions taking place in the symmetric core of the defects during scattering processes \cite{53, 54, 55, 56}. Thus, the demand for symmetric core in such scenarios may lead to constraints in the parameters of the corresponding models.

The existence of gauge strings and monopoles with non-symmetric core is also an interesting extension of our results. Do monopoles exist with an $SU(3) \times SU(2) \times U(1)$ symmetric core and are they more stable than the fully symmetric $SU(5)$ one? The microphysical and cosmological implications of defects with non-symmetric cores is an interesting area for further investigation.

5 Acknowledgements

We are grateful to T. Tomaras and B. Rai for many discussions as well as for providing us the efficient Fortran routine for energy minimization of Ref. \cite{7}. We are also thankful to C. Bachas for insightful comments in the initial stages of this work. M.A. acknowledges the hospitality of the U. of Crete Physics Department. This work was supported by the EEC grants $CHRX-CT93-0340$ and $CHRX-CT94-0621$ as well as by the Greek General Secretariat of Research and Technology grant 95$ΕΔ1759$.

References

[1] A.Vilenkin, E.P.S.Shellard, in "Cosmic Strings and other Topological Defects" Cambridge U. Press, 1994.

[2] A.Vilenkin, Phys.Rep. 121, 263, 1985;
J.Preskill, Ann.Rev.Nucl.Part. Sci. 34, 461, 1984.

[3] R. Rajaraman, 'Solitons and Instantons’ North Holland Pub., 1987.
[4] D.A. Kirzhnits, *JETP Lett.* **15**, 745, 1972;  
D.A. Kirzhnits and A.D. Linde, *Phys. Lett.* **42B**, 471, 1972.  

[5] H.B. Nielsen, P. Olesen, *Nucl. Phys.* **B61**, 45, 1973.  

[6] C. Bachas, T.N. Tomaras, *Nucl. Phys.* **B428**, 209, 1994.  

[7] C. Bachas, T.N. Tomaras, *Phys. Rev.* **D51**, 5356, 1995.  

[8] G. tHooft, *Nucl. Phys.* **B79**, 276, 1974.  

[9] A. Polyakov, *JETP Lett.* **20**, 194, 1974.  

[10] C. P. Dokos, T. N. Tomaras, *Phys. Rev.* **D21**, 2940, 1980.  

[11] M. Barriola, A. Vilenkin, *Phys. Rev. Lett.* **63**, 341, 1989.  

[12] L. Perivolaropoulos, *Nucl. Phys.* **B375**, 665, 1992.  

[13] A. Belavin, A. Polyakov *JETP Lett.* **22**, 245, 1975.  

[14] N. Turok, *Phys. Rev. Lett.* **63**, 2625, 1989.  

[15] T. Skyrme, *Proc. R. Soc.* **A262**, 233, 1961.  

[16] A.H. Guth, *Phys. Rev.* **D23**, 347, 1981;  
A.D. Linde, *Phys. Lett.* **108B**, 389, 1982;  
A. Albrecht, P.J. Steinhardt, *Phys. Rev. Lett.* **48**, 1220, 1982.  

[17] E. Kolb, M.S. Turner in *”The Early Universe”*  
(Addison-Wesley Pub.Co.1990)  

[18] G. Lazarides, R.K. Schaefer, Q. Shafi, IC-96-115, Aug 1996, 11pp. Submitted to *Phys. Rev. Lett.*, e-Print Archive: [hep-ph/9608236](http://arxiv.org/abs/hep-ph/9608236);  
G. Lazarides, Q. Shafi, *Nucl. Phys.* **B392**, 61, 1993;  
G. Lazarides, C. Panagiotakopoulos, N.D. Vlachos, *Phys. Rev.* **D54**, 1369, 1996;  
L. Randall, M. Soljacic, A. H. Guth, *Nucl. Phys.* **B472** 377, 1996.
[19] N. Turok, R. Brandenberger, Phys. Rev. D33, 2175 1986;  
R. Brandenberger, N. Kaiser, D. Schramm, N. Turok, Phys.Rev. D36, 2242, 1987;  
L. Perivolaropoulos, R. Brandenberger and A. Stebbins, Phys.Rev. D41, 1764, 1990.

[20] L. Perivolaropoulos, Astrophys. J. 451, 429 1995;  
R. Moessner, L. Perivolaropoulos, R. Brandenberger Astrophys.J. 425 365, 1994.

[21] D. H. Lyth, LANC-TH-93-19, Nov 1993. 52pp. Presented at Summer School in High Energy Physics and Cosmology (Includes Workshop on Strings, Gravity, and Related Topics 29-30 Jul 1993), Trieste, Italy, 14 Jun - 30 Jul 1993. Published in Trieste HEP Cosmol.1993:69-136 (QCD161:W626:1993) e-Print Archive: astro-ph/9312022

[22] 'TOPOLOGICAL DEFECTS AND THE FORMATION OF STRUCTURE IN THE UNIVERSE' by Robert Brandenberger (Brown U.). BROWN-HET-1037, Apr 1996. 15pp. Talk given at Pacific Conference on Gravitation and Cosmology, Seoul, Korea, 1-6 Feb 1996.

[23] V. Zanchin, J.A.S. Lima, R. Brandenberger, Phys.Rev. D54, 7129, 1996.

[24] L. Perivolaropoulos, MIT-CTP-2375, Oct 1994. 66pp. Contributed to Summer School in High Energy Physics and Cosmology, Trieste, Italy, 13 Jun - 29 Jul 1994. Published in Trieste HEP Cosmology 1994:204-270 (QCD161:W626:1994) e-Print Archive: astro-ph/9410097

[25] L. Perivolaropoulos, Phys.Rev. D46 1858, 1992.

[26] A.Vilenkin, A.E.Everett, Phys.Rev.Lett. 48, 1867 1982.

[27] G. Dvali, Z. Tavartkiladze, J. Nanobashvili, Phys.Lett. B352 214, 1995.

[28] J.Preskill,S.P.Trivedi, F.Wilczek, M.B.Wise, Nucl.Phys. B363, 207 1991.

[29] K. Benson, M. Bucher, Nucl.Phys. B406, 355, 1993.

[30] J. Preskill, Phys. Rev. D46, 4218 1992.
[31] M. Hindmarsh *Nucl.Phys.* **B392**, 461, 1993.

[32] A.N. Schellekens, C. Zachos, *Phys.Rev.Lett.* **50**, 1242, 1983.

[33] T. Vachaspati, A. Achucarro, *Phys. Rev.* **D44**, 3067, 1991.

[34] M. Barriola, T. Vachaspati, M. Bucher, *Phys.Rev.* **D50**, 2819, 1994.

[35] L. Perivolaropoulous, *Phys.Rev.* **D50**, 962, 1994.

[36] A. Achucarro, K. Kuijken, L. Perivolaropoulous, T. Vachaspati, *Nucl.Phys.* **B388**, 435, 1992.

[37] M. Hindmarsh, *Phys.Rev.Lett.* **68**, 1263, 1992.

[38] R.A. Leese, *Phys. Rev.* **D46**, 4677, 1992.

[39] Y. Nambu, *Nucl.Phys.* **B130**, 505, 1977; T. Vachaspati, *Nucl.Phys.* **B397**, 648, 1993.

[40] T. Vachaspati *Phys.Rev.Lett.* **68**, 1977, 1992, ERRATUM-ibid.69:216, 1992.

[41] M. James, L. Perivolaropoulous, T. Vachaspati, *Phys. Rev.* **D46**, 5232, 1992.

[42] M. James, L. Perivolaropoulous, T. Vachaspati, *Nucl.Phys.* **B395**, 534, 1993.

[43] S. Wolfram, *Mathematica*, Add. Wesley, 1992.

[44] C. Bachas, P. Tinyakov, T.N. Tomaras, *Phys.Lett.* **B385** 237, 1996.

[45] T.N. Tomaras, CRETE-96-18, May 1996. Talk given at 9th International Seminar on High-energy Physics: *Quarks 96*, May 1996. e-Print Archive: [hep-ph/9612341](http://hep-ph/9612341).

[46] G. Derrick, *J. Math. Phys.* **5**, 1252, 1964.

[47] W. Press et. al., *Numerical Recipes*, Cambridge U. Press, 2nd ed., 1993.

[48] M. Axenides, L. Perivolaropoulous, *in progress.*
[49] T. Prokopec, R. Brandenberger, A. C. Davis, M. Trodden, *Phys.Lett.* B384, 175, 1996.

[50] M. Trodden, A. Davis, R. Brandenberger, *Phys.Lett.* B349, 131, 1995.

[51] R. Brandenberger, A. Davis, T. Prokopec, M. Trodden, *Phys.Rev.* D53, 4257, 1996.

[52] J. Ambjorn, K. Farakos, S. Hands, G. Koutsoumbas, G. Thorleifsson, *Nucl.Phys.* B425, 39, 1994.

[53] V.A.Rubakov, *JETP Lett.* 33, 644, (1988); C.Callan, *Phys.Rev.* D25, 2141, 1982.

[54] W.B. Perkins, L. Perivolaropoulos, A.C. Davis, R.H. Brandenberger, A. Matheson, *Nucl.Phys.B* 353, 237, 1991; *Phys.Lett.*, B248, 263, 1990.

[55] R.Brandenberger, A.C.Davies, A.M.Matheson, *Phys.Lett.* 218B, 308, 1988.

[56] R.Brandenberger and L.Perivolaropoulos, *Phys.Lett.* 208B, 396, 1988.