Complete Wetting in Supersymmetric QCD or Why QCD Strings Can End on Domain Walls

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Abstract

As argued by Witten on the basis of M-theory, QCD strings can end on domain walls. We present an explanation of this effect in the framework of effective field theories for the Polyakov loop and the gluino condensate in $N = 1$ supersymmetric QCD. The domain walls separating confined phases with different values of the gluino condensate are completely wet by a layer of deconfined phase at the high-temperature phase transition. As a consequence, even at low temperatures, the Polyakov loop has a non-vanishing expectation value on the domain wall. Thus, close to the wall, the free energy of a static quark is finite and the string emanating from it can end on the wall.

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Recently, Witten has argued that QCD strings can end on domain walls [1]. This effect follows from a calculation in M-theory, where domain walls are represented by D-branes on which strings can end. Witten’s argument applies to a theory in the universality class of \( N = 1 \) supersymmetric QCD. In this theory, a \( \mathbb{Z}(3)_\chi \) chiral symmetry — an unbroken remnant of the anomalous \( U(1)_R \) symmetry — is spontaneously broken at low temperatures by a non-zero value of the gluino condensate \( \chi \). As a consequence, there are three distinct confined phases, characterized by three different values of \( \chi \), which are related by \( \mathbb{Z}(3)_\chi \) transformations. Regions of space filled with different confined phases are separated by domain walls. Passing from one region to another through a domain wall, the QCD vacuum angle \( \theta \) changes by \( 2\pi \). This behavior is characteristic of axionic domain walls. Witten’s string theory arguments imply that a QCD string emanating from a static quark can end on such a wall. The properties of domain walls in supersymmetric theories have been investigated in great detail by Shifman and collaborators [2, 3] and topological defects ending on other defects have been studied in [4].

A qualitative explanation of why strings can end on axionic domain walls has been given by Rey [1] in the framework of field theory. His explanation is based on ’t Hooft’s picture of oblique confinement [5], which at \( \theta = 0 \) is due to monopole condensation. At \( \theta \neq 0 \) the monopoles turn into color-electrically charged dyons [6] and confinement is due to condensation of dyons. Hence, a confined-confined domain wall separates regions of monopole and dyon condensates. When a dyon passes through the domain wall, it turns into a monopole and leaves its color charge at the wall. If the walls can carry a color charge, a QCD string can end there. A more quantitative description of this effect in an axion toy model has been given in [6]. While Rey’s picture applies to the axion toy model, we think that it does not provide a correct field theoretic explanation of Witten’s M-theory calculation. We find that, in \( N = 1 \) supersymmetric QCD, strings can end on the walls not because the walls are charged, but because they can transport flux to infinity. A similar scenario involving a non-Abelian Coulomb phase was discussed qualitatively in [6].

In this paper, we provide the first quantitative field theoretic explanation for why strings can end on walls for models in the universality class of \( N = 1 \) supersymmetric QCD, i.e. for the same class of models to which Witten’s M-theory calculation applies. Often, topological defects, which result from spontaneous symmetry breaking, have the phase of unbroken symmetry at their cores. For example, magnetic monopoles have symmetric vacuum at their centers. Similarly, at the core of axionic cosmic strings the \( U(1)_{PQ} \) Peccei-Quinn symmetry is restored. Witten has constructed a model in which \( U(1)_{PQ} \) symmetry restoration leads to breaking of the \( U(1)_{em} \) gauge group of electromagnetism [7]. Hence, axionic strings in this model become superconducting. We proceed similarly for domain walls by constructing an effective field theory, in which \( \mathbb{Z}(3)_\chi \) restoration implies breaking of the \( \mathbb{Z}(3)_c \) center symmetry of the \( SU(3) \) gauge group. Then the high-temperature deconfined phase appears at the center of the domain wall, i.e. the Polyakov loop — as the
order parameter for $\mathbb{Z}(3)_c$, breaking — has a non-zero expectation value there. This means that a static quark has a finite free energy close to the wall and its string can end there.

The effect of a new phase appearing at a domain wall separating two bulk phases is well-known in condensed matter physics as complete wetting. For example, the interface between the human eye and the surrounding air is completely wet by a film of tears. Consequently, the eye-air solid-gas interface splits into a solid-liquid and a liquid-gas interface. The alternative is incomplete wetting. Then droplets of liquid form at the solid-gas interface, but no complete wetting film appears. Complete wetting is known to appear at the deconfinement phase transition of the $N = 0$ non-supersymmetric $SU(3)$ Yang-Mills theory [8, 9]. In that case, the $\mathbb{Z}(3)_c$ center symmetry is spontaneously broken at high temperatures, giving rise to three distinct deconfined phases, which are distinguished by different values of the Polyakov loop. When a deconfined-deconfined domain wall is cooled down to the phase transition, the low-temperature confined phase appears as a complete wetting layer that splits the deconfined-deconfined domain wall into a pair of confined-deconfined interfaces. In $N = 1$ supersymmetric QCD, there are three distinct confined phases. In this paper, we show that complete wetting also arises when a confined-confined domain wall is heated up to the phase transition. In this case, one of the three deconfined phases forms a complete wetting layer such that the Polyakov loop is non-zero at the domain wall. It continues to be non-zero even below the phase transition temperature. As a consequence, the free energy of a static quark located at the wall is finite. When the quark is displaced from the wall, its free energy increases linearly with the distance. Thus, the quark is confined to the wall and the string emanating from it ends there.

Complete wetting is a universal phenomenon of interfaces at first order phase transitions. In the case of $N = 0$ non-supersymmetric $SU(3)$ Yang-Mills theory the universal aspects of the interface dynamics are captured by a 3-d effective action [3],

$$S[\Phi] = \int d^3 x \left[ \frac{1}{2} \nabla \Phi^* \cdot \nabla \Phi + V(\Phi) \right], \quad (1)$$

for the Polyakov loop $\Phi(\vec{x})$, which is a gauge invariant complex scalar field. Its expectation value $\langle \Phi \rangle \propto \exp(-F/T)$, where $T$ is the temperature, measures the free energy $F$ of a static quark. In the confined phase, $F$ diverges and $\langle \Phi \rangle$ vanishes, while in the deconfined phase, $F$ is finite and $\langle \Phi \rangle$ is non-zero. Under topologically non-trivial gauge transformations, which are periodic in Euclidean time up to a center element $z \in \mathbb{Z}(3)_c = \{ e^{2\pi \imath n/3}, n = 1, 2, 3 \}$, the Polyakov loop changes into $\Phi' = \Phi z$. Hence, the $\mathbb{Z}(3)_c$ symmetry is spontaneously broken in the deconfined phase. Under charge conjugation, the Polyakov loop is replaced by its complex conjugate. The effective potential $V(\Phi)$ is restricted by $\mathbb{Z}(3)_c$ and charge conjugation symmetry, i.e.

$$V(\Phi z) = V(\Phi), \quad V(\Phi^*) = V(\Phi). \quad (2)$$
The most general quartic potential consistent with these symmetries takes the form

\[ V(\Phi) = a|\Phi|^2 + b\Phi_1(\Phi_1^2 - 3\Phi_2^2) + c|\Phi|^4, \tag{3} \]

where \( \Phi = \Phi_1 + i\Phi_2 \). One can restrict oneself to quartic potentials because they are sufficient to explore the universal features of the interface dynamics. At the deconfinement phase transition temperature (corresponding to \( b^2 = 4ac \)), the above potential has four degenerate minima, \( \Phi^{(1)} = \Phi_0 \in \mathbb{R} \), \( \Phi^{(2)} = (-1/2 + i\sqrt{3}/2)\Phi_0 \) and \( \Phi^{(3)} = (-1/2 - i\sqrt{3}/2)\Phi_0 \) representing the three deconfined phases and \( \Phi^{(4)} = 0 \) representing the confined phase. In ref.[9], it was shown that a deconfined-deconfined phase transition temperature (corresponding to \( N/Z \) = 1 supersymmetric QCD the \( Z(3)_c \) chiral symmetry is spontaneously broken in the confined phase. The corresponding order parameter is the complex valued gluino condensate \( \chi = \chi_1 + i\chi_2 \). Under chiral transformations \( z \in Z(3)_c \), the gluino condensate transforms into \( \chi' = \chi z \) and under charge conjugation it also gets replaced by its complex conjugate. At high temperatures, one expects chiral symmetry to be restored and — as in the non-supersymmetric theory — the \( Z(3)_c \) center symmetry to be spontaneously broken due to deconfinement. Consequently, the effective action describing the interface dynamics now depends on both order parameters \( \Phi \) and \( \chi \), such that

\[ S[\Phi, \chi] = \int d^3 x \left[ \frac{1}{2} \nabla \Phi^* \cdot \nabla \Phi + \frac{1}{2} \nabla \chi^* \cdot \nabla \chi + V(\Phi, \chi) \right]. \tag{4} \]

The most general quartic potential consistent with \( Z(3)_c, Z(3)_\chi \) and charge conjugation now takes the form

\[ V(\Phi, \chi) = a|\Phi|^2 + b\Phi_1(\Phi_1^2 - 3\Phi_2^2) + c|\Phi|^4 + d|\chi|^2 + e\chi_1(\chi_1^2 - 3\chi_2^2) + f|\chi|^4 + g|\Phi|^2|\chi|^2. \tag{5} \]

First, we assume that deconfinement and chiral symmetry restoration occur at the same temperature and that the phase transition is first order. Then, three chirally broken confined phases coexist with three distinct chirally symmetric deconfined phases. The three deconfined phases have \( \Phi^{(1)} = \Phi_0 \), \( \Phi^{(2)} = (-1/2 + i\sqrt{3}/2)\Phi_0 \) and \( \Phi^{(3)} = (-1/2 - i\sqrt{3}/2)\Phi_0 \) and \( \chi^{(1)} = \chi^{(2)} = \chi^{(3)} = 0 \), while the three confined phases are characterized by \( \Phi^{(4)} = \Phi^{(5)} = \Phi^{(6)} = 0 \) and \( \chi^{(4)} = \chi_0 \in \mathbb{R} \), \( \chi^{(5)} = (-1/2 + i\sqrt{3}/2)\chi_0 \) and \( \chi^{(6)} = (-1/2 - i\sqrt{3}/2)\chi_0 \). The phase transition temperature corresponds to a choice of parameters \( a, b, ..., g \) such that all six phases \( \Phi^{(n)} \), \( \chi^{(n)} \) represent degenerate absolute minima of \( V(\Phi, \chi) \).

We now look for solutions of the classical equations of motion, representing planar domain walls, i.e. \( \Phi(\vec{x}) = \Phi(z) \), \( \chi(\vec{x}) = \chi(z) \), where \( z \) is the coordinate perpendicular to the wall. The equations of motion then take the form

\[ \frac{d^2 \Phi_i}{dz^2} = \frac{\partial V}{\partial \Phi_i}, \quad \frac{d^2 \chi_i}{dz^2} = \frac{\partial V}{\partial \chi_i}. \tag{6} \]
Figure 1: Shape of a confined-confined domain wall. Deep in the confined phase (a) $\Phi_1(0) \neq 0$, i.e. the center of the wall has properties of the deconfined phase. Close to the phase transition (b) the wall splits into two confined-deconfined interfaces with a complete wetting layer of deconfined phase between them.
Figure 1 shows a numerical solution of these equations for a domain wall separating two confined phases of type (5) and (6), i.e. with boundary conditions $\Phi(\infty) = \Phi(5)$, $\chi(\infty) = \chi(5)$ and $\Phi(-\infty) = \Phi(6)$, $\chi(-\infty) = \chi(6)$. Figure 1a corresponds to a temperature deep in the confined phase. Still, at the domain wall the Polyakov loop is non-zero, i.e. the center of the domain wall shows characteristic features of the deconfined phase. Figure 1b corresponds to a temperature very close to the phase transition. Then the confined-confined domain wall splits into two confined-deconfined interfaces and the deconfined phase forms a complete wetting layer between them. The solutions of figure 1 have deconfined phase of type (1) at their centers. Due to the $Z(3)$ symmetry, there are related solutions with deconfined phase of types (2) and (3).

For the special values $d = a = 0$, $e = b$, $f = c$, $g = 2c$ one can find an analytic solution for a confined-deconfined interface. Combining two of these solutions to a confined-confined interface, one obtains

$$
\Phi_1(z) = -\frac{1}{2} \Phi_0 [\tanh \alpha(z - z_0) - \tanh \alpha(z + z_0)], \quad \Phi_2(z) = 0,$$

$$
\chi_1(z) = -\frac{1}{4} \chi_0 [2 + \tanh \alpha(z - z_0) - \tanh \alpha(z + z_0)],$$

$$
\chi_2(z) = \sqrt{3} \frac{1}{4} \chi_0 [\tanh \alpha(z - z_0) + \tanh \alpha(z + z_0)],
$$

where $\Phi_0 = \chi_0 = -3b/4c$ and $\alpha = -3b/4\sqrt{c}$ (with $b < 0$). When $d = a = 0$, the critical temperature corresponds to $e^4/f^3 = b^4/c^3$. Near criticality, where $\Delta = e^4/f^3 - b^4/c^3$ is small, the above solution is valid up to order $\Delta^{1/2}$, while now $e = b$ and $f = c$ are satisfied to order $\Delta$. The width of the deconfined complete wetting layer,

$$2z_0 = -\frac{1}{2\alpha} \log \Delta + C,$$

where $C$ is a constant, grows logarithmically as we approach the phase transition temperature. This is the expected critical behavior for interfaces with short-range interactions \[9\]. It would be interesting if complete wetting could be studied in the framework of M-theory. If so, it should correspond to the unbinding of a pair of D-branes, which are tightly bound in the low-temperature phase.

Now we wish to explain why the appearance of the deconfined phase at the center of the domain wall allows a QCD string to end there. We recall that an expectation value $\langle \Phi \rangle \neq 0$ implies that the free energy of a static quark is finite. Indeed, the solution of figure 1 containing deconfined phase of type (1) has $\Phi_1(0) \neq 0$, such that a static quark located at the center of the wall has finite free energy. As one moves away from the wall, the Polyakov loop decreases as

$$\Phi_1(z) \propto \exp(-F(z)/T) \propto \exp(-\sqrt{2a + 2g\chi^2} \cdot z).$$

Consequently, as the static quark is displaced from the wall, its free energy $F(z)$ increases linearly with the distance $z$ from the center, i.e. the quark is confined to
the wall. The string emanating from the static quark ends on the wall and has a tension

$$\sigma = \lim_{z \to \infty} \frac{F(z)}{z} = \sqrt{2a + 2g\chi_0^2 T}. \quad (10)$$

Still, there are the other domain wall solutions (related to the one from above by $Z(3)_c$ transformations), which contain deconfined phase of types (2) and (3). One could argue that, after path integration over all domain wall configurations, one obtains $\langle \Phi \rangle = 0$. However, this is not true. In fact, the wetting layer at the center of a confined-confined domain wall is described by a two-dimensional field theory with a spontaneously broken $Z(3)_c$ symmetry. Consequently, deconfined phase of one definite type spontaneously appears at the domain wall. This argument does not apply to an interface of finite area, e.g. to a bubble wall enclosing a finite volume of confined phase of type (5) in a Universe filled with confined phase of type (4). Indeed, due to quantum tunneling the deconfined wetting layer at the surface of such a bubble would change from type (1) to types (2) and (3). As a consequence, $\langle \Phi \rangle = 0$, such that a QCD string cannot end at the bubble wall. This observation is consistent with the $Z(3)_c$ Gauss law applied to the compact surface of the bubble. Color flux entering the bubble through a string must exit it somewhere else and then go to infinity. Therefore, the static quark at the origin of the string has infinite free energy and is confined in the usual sense. The deconfined wetting layer of an infinite domain wall, on the other hand, can transport flux to infinity at a finite free energy cost. Thus, a static quark at a finite distance from the wall has a finite free energy and the string emanating from it can end on the wall. The above arguments imply that a QCD string can also end at a confined-deconfined interface at the high-temperature phase transition. This is possible even in non-supersymmetric Yang-Mills theory.

We now understand that QCD strings can end on a confined-confined domain wall provided it is completely wet by deconfined phase. So far we have assumed that, above the phase transition, confinement is lost and simultaneously chiral symmetry is restored. In that case, indeed there is complete wetting. Now let us consider two alternative scenarios for the phase transition. First, we assume that chiral symmetry is still restored at temperatures above the phase transition, but that the theory remains confining. In that case, complete wetting still occurs, but now the wetting layer consists of chirally symmetric confined phase, which has $\Phi = 0$. Under these conditions, a QCD string could not end on the wall. Next, we assume that, above the phase transition, the theory is deconfined, but that chiral symmetry remains broken, such that there are nine high-temperature phases. Assuming that the gluino condensate does not vary drastically across the transition, one can show that complete wetting then does not occur. Hence, again $\Phi = 0$ at the center of the domain wall and a QCD string could not end there. It should be noted that a confined-confined domain wall could still be incompletely wet, i.e. droplets of deconfined phase could appear at the wall. Even in that case, a QCD string cannot end at a droplet. In contrast to a complete wetting layer, a droplet has a finite
volume. Hence, due to quantum tunneling a deconfined droplet of type (1) will turn into one of types (2) and (3). As a result, the average Polyakov loop vanishes for these configurations, and a static quark has infinite free energy even close to the wall. Given that QCD strings can end on the walls, as explained by Witten, we conclude that the two alternative scenarios for the phase transition can be ruled out. Interestingly, Witten’s M-theory arguments in combination with our results for complete wetting suggest that $N=1$ supersymmetric QCD has a high-temperature phase transition in which simultaneously confinement is lost and chiral symmetry is restored.

In conclusion, we have investigated $\mathbf{Z}(3)_c \otimes \mathbf{Z}(3)_\chi$ symmetric effective theories for the Polyakov loop and the gluino condensate. Due to complete wetting, deconfined phase appears at a confined-confined domain wall. Thus, close to the wall a static quark has a finite free energy and its string can end there. This is possible only when the wall is infinitely extended, because only then it can transport the quark’s color flux to infinity. Hence, without reference to M-theory, we can understand why a QCD string can end on the wall.

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