Valuation of coefficient of rolling friction by the inclined plane method

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Abstract. A major objective of tribological researches is characterisation of rolling friction, due to various cases encountered in classical engineering applications, like gear transmissions and cam mechanisms or more recent examples met in bioengineering and biomedical devices. A characteristic of these examples consists in reduced dimensions of the contact zones, theoretically zero, the relative motion occurring between the contact points being either sliding or rolling. A characteristic parameter for the rolling motion is the coefficient of rolling friction. The paper proposes a method for estimation of coefficient of rolling friction by studying the motion of a body of revolution on an inclined plane. Assuming the hypothesis that moment of rolling friction is proportional to the normal reaction force, the law of motion for the body on the inclined plane is found under the premise of pure rolling. It is reached the conclusion that there is an uniformly accelerated motion, and thus for a known plane slope, it is sufficient to find the time during which the body runs a certain distance, starting from motionless situation. To obtain accurate results assumes finding precisely the time of motion. The coefficient of rolling friction was estimated for several slopes of the inclined plane and it is concluded that with increased slope, the values of coefficient of rolling friction increase, fact that suggest that the proportionality between the rolling torque and normal load is valid only for domains of limited variations of normal load.

1. Introduction

The interaction between two elements of a system can be made either by conform contact, when the bodies get contact on extended surfaces, or by nonconform contact, when the boundary surfaces coming into contact are different and the contact is established on set of points of null area (Hertzian contacts). Unlike conform contact, when between the points of contact region always exists relative velocity and thus sliding is present, for nonconform contacts, besides the sliding possibility there also exists the possibility of pure rolling - which occurs when between the theoretical contact points there is no relative motion. Characterizing friction in the case of pure rolling assumes establishing the moment of rolling. In dynamics literature, rolling friction torque is characterized assuming the proportionality between the friction torque and normal reaction force, the proportionality factor being the coefficient of rolling friction. Recent researches based on theory of elasticity lead to more complex dependencies [1-3].

Bioengineering research require tribological characterization of actual materials, [4], [5] which exhibit elastic or viscoelastic properties - like biological tissues or for the ones designed to replace these,
leading to energy losses due to internal friction phenomena [6], [7]. For pure rolling case, there are not losses due to friction [8] for rigid bodies; but for real materials like biological tissues, rolling is accompanied by hysteresis losses [9].

2. Theoretical considerations

In the present paper is considered the motion of a bearing ball on an inclined plane, assuming the proportionality between friction torque and normal force, Figure 1. The axis is chosen as the slope of the inclined plane and the motion of the ball is considered as plane motion, the position parameters of the ball are the displacement of the centre of mass of the ball \( x \) and the rotation angle about an axis normal to the plane of motion, \( \varphi \). The external force acting upon the ball is the force of gravity \( G \). In the theoretical contact point \( C \) between the ball and the inclined plane the normal reaction force, \( N \) the friction force (parallel to the plane) \( T \) and the rolling friction torque \( M_r \), will occur.

![Figure 1. Motion of ball on an inclined plane](image)

The equations of motion are represented by the center of mass theorem and the moment of momentum theorem, [10], written with respect to the centre of mass:

\[
\begin{align*}
mx' &= mg \sin \alpha - T \\
0 &= N - mg \cos \alpha \\
J_\varphi \ddot\varphi &= Tr - M_r
\end{align*}
\]  

(1)

For a sphere made by homogenous material, the moment of inertia \( J_\varphi \) with respect to the centre of mass, from equation (1), is:

\[
J_\varphi = \frac{2}{5} mr^2
\]  

(2)

where \( m \) and \( r \) are the mass and the radius of the ball, respectively. The unknown of the system are \( x, \varphi, N, T \) and \( M_r \). To ensure compatibility, two additional equations are required. The first one is a constitutive relation that accepts the proportionality between the rolling friction torque and the magnitude of normal compression force, the proportionality factor being the coefficient of rolling friction \( s_r \). The friction torque should oppose to the rotation tendency of the ball:

\[
M_r = s_r N
\]  

(3)
The second equation depends on the type of friction thought to occur in point \( C \). If sliding exists between the points contacting in \( C \), the friction force is known and found using the Coulomb law that considers the coefficient of dynamic friction \( \mu_d \):

\[
T = \mu_d N
\]  \hspace{1cm} (4)

In the case when pure rolling is supposed to be present, then, the point \( C \) should be the instant centre of rotation (instantaneous velocity center) with permanent zero velocity. A consequence of this fact is the simple relation between the velocity of the centre of mass of the ball, \( \dot{x} \) and the angular velocity \( \omega = \dot{\phi} \), explicitly:

\[
\dot{x} = \dot{\phi} r
\]  \hspace{1cm} (5)

In this case, the size of friction force is an unknown of the problem, the fulfillment of the following being required:

\[
|T| < \mu_s N
\]  \hspace{1cm} (6)

where \( \mu_s \) is the coefficient of static friction for the limiting case. The equations (1), (3) and (5) allow for finding the acceleration of the centre of mass of the ball:

\[
a = \ddot{x} = g \cos \alpha \left( \tan \alpha - \frac{s_r}{r} \right) \left( 1 + \frac{J_r}{m r^2} \right).
\]  \hspace{1cm} (7)

3. Experimental device

The equation (7) suggests a straightforward method for establishing the coefficient of rolling friction, similar to the method for finding the coefficient of sliding friction. Practically, the ball is set at rest on the plane, initially in horizontal position, and afterwards the plane is slowly inclined until the instant the ball starts to move. Quite prior to this instant, the ball was immobile at the limit; thus, form equation (7) it results directly:

\[
s_r = r \tan \alpha
\]  \hspace{1cm} (8)

The weakness of the method consist in the fact that for common values of coefficient of rolling friction \( s_r = (10^{-5} + 10^{-4})m \) and a radius of the bearing ball \( r = 0.02m \), based on equation (8), it results that the angle needed for the start of ball motion is:

\[
\alpha = 0.038^\circ + 0.382^\circ
\]  \hspace{1cm} (9)

Consequently, accurate instruments for angle measurements are essential. Furthermore, the precision of achieving the horizontal position of the plane is directly affecting the value of coefficient of rolling friction. The present paper proposes the valuation of coefficient of rolling friction based on equation (7) by determining the acceleration with which the centre of mass of the ball moves.

A problem arising is how to eliminate the spin motion that the ball may possibly perform; in this case, the motion of the ball is no longer a planar one. To this purpose, instead of a single ball, two identical balls, connected via an aluminum rod are used, as shown in Figure 2. Another difficulty to be overcome is finding accurately the time of motion of the ball between two limits. Thus, in the launching position, the ball closes an electrical circuit. At the base of the plane there are two regions where on the surface of inclined plane, two conductive films are placed. When the rolling body reaches the base of the inclined plane, another electrical circuit closes. In this manner the instants of starting – opening of superior circuit, and ending – closing the second circuit, are precisely found. With known distance of running, \( d \), under the hypothesis of constant friction torque, the motion of the body is uniformly accelerated with the acceleration \( a = \frac{2d}{t^2} \).
In Figure 3 is presented the experimental test-rig. Each electrical circuit also contains a light bulb, placed on the inclined plane which is made from a plate of glass 10 mm thick. At the midst of the inclined plane, on the inferior side, a wire was placed, at certain distance from launching line. Thus, the hypothesis of constant acceleration was verified, Figure 4.

Figure 2. Principle of method

An experimental methodology implies video-capturing the region where the two light bulbs are placed. By analyzing the movie frame by frame, the time can be found. For a series of values of plane slope, three launches were made and then finding the accelerations and coefficients of rolling friction.

Figure 3. Experimental set-up before launching instant

Figure 4. Identification of instants when the moving body passes above the wire position

Figure 5. Final instant determined by optical method

Figure 6. Identification of initial and final instants of the motion
An essential aspect concerns the precision of identifying the initial and final instants of the motion. The variation of current in the electric circuit was evidenced using an electronic oscilloscope, Figure 6. It can be noticed that the instant of contact opening is very well identifiable while the instant corresponding to passing the imposed distance is difficult to spot, either by oscilloscope or by optical system. In Figure 5 there are presented three frames from the movie, correlated to the vicinity of final instant. It can be observed that the light of the bulb faints during an interval of approximately ten frames, \((0.08 \text{ s})\). It was made an attempt to mount a block at the base of the inclined plane for obtaining an impact at the finish of the motion (which is an identical situation with the start, but in reverse sense) but the result was not acceptable since after the collision the contact opens and the circuit is closed only for a period equal to the duration of collision, \(10^{-4} \text{ s}\), duration imperceptible, \([11], [12]\), both optically and electronically. In order to reduce and limit the effect of time measurement errors upon the precision of the results, it is required that the duration of experiments should be as great as possible and thus the absolute error, constant in this case, would affect as little as possible the relative error. It is required the construction of a test rig with greater length and smaller slope.

4. Experimental results and discussions

The values of the durations necessary for covering the two distances are presented in Figure 7, the points corresponding to the same slope of the inclined plane being practically overlapped. The repeatability of the results is shown in Figure 8; the accelerations obtained for different angles are the same for the three launchings under similar conditions and in the plot, the points overlie.

![Figure 7. Coincidence of times corresponding to repeated tests](image)

![Figure 8. Validation of acceleration steadiness and repeatability of results](image)

![Figure 9. Dependence of experimental coefficient of rolling friction \(s_r\) upon slope in Cartesian coordinates](image)

![Figure 10. Dependence of experimental coefficient of rolling friction \(s_r\) upon slope in logarithmic coordinates](image)
In Figure 9 there are presented the values the experimental dependencies of coefficient of rolling friction on the slope angle of the inclined plane and in Figure 10, using logarithmic coordinates, the data are interpolated by a line.

Two major conclusions result from these plots:
- it is confirmed the hypothesis of constant coefficient of rolling friction: the obtained values of $s$, are increasing with the increasing slope of the plane;
- the experimental values of $s$, are within the range $[24+163\mu m]$, values greater than the ones given in technical literature, [13], [14].

5. Conclusions
The paper presents a method and device proposed for evaluation of coefficient of rolling friction. The principle of the method consist in finding the acceleration, assumed constant, of a ball rolling over an inclined plane. Several difficulties had to be surpassed:
- elimination of spin motion, that was made using two identical balls connected by a rod;
- precise identification of the instants of start and end of motion; to this purpose, the mobile balls were introduced into electrical circuits;
- validation of hypothesis of constant acceleration: it was verified by measuring the periods in which two different distances were passed.

Though the time measurement errors influence considerably the accuracy of the experiments, the values of coefficient of rolling friction found by the present method are in good agreement with the ones given in literature. The main aspect underlined is the dependency of coefficient of rolling friction upon the slope of the inclined plane, in contradiction to a series of reference works where the hypothesis of constant value of $s$, is stated.

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