Superpotentials in IIA compactifications with general fluxes

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Abstract

We derive the effective $N = 1$, $D = 4$ supergravity for the seven main moduli of type IIA orientifolds with D6 branes, compactified on $T^6/(Z_2 \times Z_2)$ in the presence of general fluxes. We illustrate and apply a general method that relates the $N = 1$ effective Kähler potential and superpotential to a consistent truncation of gauged $N = 4$ supergravity. We identify the correspondence between various admissible fluxes, $N = 4$ gaugings and $N = 1$ superpotential terms. We construct explicit examples with different features: in particular, new IIA no-scale models and a model which admits a supersymmetric $AdS_4$ vacuum with all seven main moduli stabilized.

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1 Introduction

Compactifications of superstrings and M-theory \(^1\) may lead to four-dimensional vacua with exact or spontaneously broken supersymmetries. The pattern of residual and broken supersymmetries strongly depends on the set of moduli fields predicted by the compactification geometry and on the detailed dynamics of these moduli. Even for the phenomenologically attractive compactifications with spontaneously broken \(N = 1\) only, information on the dynamics of moduli is provided by the much larger symmetry of the underlying \(D = 10\) string theories, with sixteen or thirty-two supercharges. Similarly, in the effective \(D = 4\) low-energy supergravity theory, this information on moduli dynamics is encoded in the underlying \(N \geq 4\) supersymmetry. Thus, the Kähler potential of the \(N = 1\) effective supergravity follows from the scalar sigma-model induced by \(N = 4\) auxiliary field and gauge-fixing equations. And the \(N = 1\) superpotential for the moduli and matter fields is directly related to the \(N = 4\) supergravity \([2, 3, 4]\) gauging \([5]\), which in turn corresponds to a specific flux structure of the underlying ten-dimensional string theory or eleven-dimensional M–theory.

The generation of a scalar potential for the moduli fields is a crucial ingredient in supersymmetry breaking and in the determination of a stable \(D = 4\) background geometry, if any. It is also essential to reduce the number of massless scalars and/or undetermined parameters in the low-energy effective theory. Besides the curvature of the internal space itself, there are several well-known sources for a scalar potential in the compactified ten-dimensional (or eleven-dimensional) theory.

A first source is the Scherk-Schwarz mechanism \([6]\), and its generalization to superstrings via freely acting orbifolds \([7]\). The relevant fluxes are the geometrical ones, associated with the internal spin connection \(\omega_3\). Some of the corresponding effective theories are no-scale supergravity models \([8]\), with broken supersymmetry in a flat \(D = 4\) background. However, the gravitino and the other masses generated in this way are proportional (modulo quantized charges) to the inverse length scale of the compactified space, \(m \propto R^{-1}\). Therefore, to have supersymmetry breaking and/or preserving TeV scale masses, we need a very large internal dimension, \(R \sim 10^{15} l_P\), where \(l_P\) is the (four-dimensional) Planck length.

A second source is non-zero “fluxes” of antisymmetric tensor fields, as first identified

\(^1\)For an introduction, see e.g. \([1]\).
long ago for the three-form $H_3$ of the heterotic theory [9]. There is an extensive recent literature [10] on orientifolds of the IIB theory in the presence of three-form fluxes. For instance, simultaneous and suitably aligned NS-NS (NS = Neveu-Schwarz) and R-R (R = Ramond) 3-form fluxes, $H_3$ and $F_3$, can lead to no-scale supergravities, but now $m \propto l_p^2 R^{-3}$: as a result, TeV scale supersymmetry breaking and/or preserving masses can be obtained for $R \sim 10^5 l_p$. The richer flux content of the IIA theory has been studied to a lesser extent [11, 12].

Both sources, geometric and antisymmetric tensor fluxes, can be combined, as originally examined in the heterotic theory by Kaloper and Myers [13].

In this paper, we use the method of supergravity gaugings to describe in general terms the generation of moduli superpotentials in a specific compactification scheme, defined as follows. We consider compactifications of superstring theories on the orbifold $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$, combined for type-II strings with a compatible orientifold projection to reduce supersymmetry to four supercharges. The moduli spectrum includes then seven chiral multiplets from the closed string sector, and the orbifold has a natural permutation symmetry in the three two-tori ($T^2$) defined by the action of $\mathbb{Z}_2 \times \mathbb{Z}_2$ on the six-torus $T^6$. We then construct the gaugings associated to general flux structures respecting this ‘plane-interchange’ permutation symmetry (this assumption could be eventually relaxed, leading to a wider spectrum of possibilities). We include the fluxes generated by all antisymmetric tensor fields (NS-NS and R-R), and also geometrical fluxes associated to components of the internal spin connection, as in Scherk-Schwarz compactifications. We analyze here in detail the case of IIA strings (with $D6$-branes), since it offers the broadest choice of fluxes and breaking patterns. We establish the dictionary relating fluxes, gauging structure constants and superpotential terms, and the consistency conditions applying on gauging and flux coefficients. This general formulation allows us to study examples with selected phenomenological properties. We find in particular that gaugings and fluxes exist in IIA compactifications, such that all seven moduli are stabilized in a vacuum with $N = 1$, $D = 4$ Anti-de Sitter ($AdS_4$) supersymmetry. Other superstring theories and more general compactification schemes will be considered in a longer, companion paper [14].

This paper is organized as follows. The general method for obtaining $N = 1$ superpotentials from $N = 4$ gaugings, already anticipated in [15, 16], is studied and applied to our specific compactification scheme in Section 2. The familiar example of the heterotic theory is then used to define the relation between fluxes and superpotentials, and the consistency
conditions for a gauging (Section 3). We then turn to the general study of fluxes in type IIA compactifications (Section 4) and to the study of some selected examples (Section 5). We conclude in Section 6.

2 \( N = 1 \) superpotentials from \( N = 4 \) gaugings

The Lagrangian density describing the coupling of vector multiplets to \( N = 4, D = 4 \) supergravity [5] depends on two sets of numbers. The structure constants \( f^R_{ST} \) define the gauge algebra, and the duality phases \( \delta_R \) specify the duality-covariant coupling of each gauge field to the supergravity dilaton \( S \). With \( n \) vector multiplets, the gauge group is a \((6 + n)\)-dimensional subgroup of the natural \( SO(6, n) \) symmetry, inherited from the superconformal origin of the abelian theory. The structure constants must leave the \( SO(6, n) \) metric \( \eta_{RS} \) invariant, a condition which implies antisymmetry of \( f^U_{STR} \equiv f_{STR} \eta_{UR} \). Notice that \( \eta_{RU} \) is not in general the Cartan metric of the gauge group \(^2\). With the \( SU(1,1)/U(1) \) Kähler potential

\[
K(S, \overline{S}) = -\ln (S + \overline{S}),
\]

the \( S \)-dependent \( N = 4 \) (superconformal) gauge kinetic terms read

\[
\mathcal{L}_{gauge} = -\frac{1}{4} \sum_{R,S} \eta_{RS} S_{\delta_R} F^{R-}_{\mu\nu} F^{\mu\nu S-} + \text{h.c.} + \ldots,
\]

where \( F^{R+}_{\mu\nu} = F^{R}_{\mu\nu} \pm i \tilde{F}^{R}_{\mu\nu} \), and

\[
S_{\delta_R} = \frac{\cos \delta_R S - i \sin \delta_R}{-i \sin \delta_R S + \cos \delta_R}.
\]

Further gauge kinetic terms, depending on the scalars in the vector multiplets, arise from the elimination of superconformal auxiliary fields [5]. They also depend on the duality phases through the same \( S_{\delta_R} \). The duality phases \( \delta_R \) must respect the structure of the gauge algebra and discretization of the \( SU(1,1) \) \( S \)-duality group implies that only two choices of phases are allowed,

\[
\delta_R = 0 \leftrightarrow S_{\delta_R} = S, \quad \text{and} \quad \delta_R = \pi/2 \leftrightarrow S_{\delta_R} = 1/S,
\]

commonly associated with perturbative and non-perturbative sectors, respectively.

\(^2\)The \( SO(6, n) \) metric has six eigenvalues \(-1\) and \( n \) eigenvalues \(+1\).
The $n$ vector multiplets contain scalars in the representation $6$ of the $R$–symmetry group $SU(4)$. They live $[3, 4]$ on the coset $SO(6,n)/[SO(6) \times SO(n)]$:

$$\phi_{ij}^R = -\phi_{ji}^R = \frac{1}{2} \epsilon_{ijkl} \phi_{kl}^R, \quad \phi^{ij R} = (\phi_{ij}^R)^*, \quad (i,j,\ldots = 1,\ldots, 4, \quad R = 1,\ldots, 6 + n).$$

The structure of the sigma-model is dictated by the field equation of an auxiliary scalar, which leads to the constraint

$$\eta_{RS} \phi_{ij}^R \phi^{kl S} = \frac{1}{12} \left( \delta^k_i \delta^l_j - \delta^l_i \delta^k_j \right) \eta_{RS} \phi_{mn}^R \phi^{mn S},$$

and by the Poincaré gauge-fixing condition

$$\eta_{RS} \phi_{ij}^R \phi^{ij S} \equiv \phi_{ij}^R \phi_{ij}^R = -6.$$

These two conditions eliminate twenty-one scalar fields, and the local $SU(4)$ symmetry can be used to eliminate another fifteen. The remaining $6n$ physical scalars live on the announced coset.

As usual, gauging supergravity also generates a scalar potential, and gravitino mass terms $-(1/2) M_{3/2}^{ij} \overline{\psi}_i \sigma^{\mu\nu} \psi_j + \text{h.c.}$, with $^3$

$$M_{3/2}^{ij} = -\frac{4}{3} \varphi^*_R \delta_{RST} \phi^{ik R} \phi^{lj S} \phi_{ij T},$$

and

$$\varphi^*_R = \sqrt{\frac{2}{S+S}} \left( \cos \delta_R - i S \sin \delta_R \right).$$

To reduce supersymmetry to $N = 1$, we use a $Z_2 \times Z_2$ truncation, as in string orbifolds with the same discrete point group. This truncation leads to a moduli sector with seven chiral multiplets $S, T_A, U_A, (A = 1, 2, 3)$, for all string compactifications and compatible orientifolds and $D$–brane systems. We can also include an arbitrary number of matter multiplets, generically denoted by $Z_A^I, (I = 1,\ldots, n_A)$. The $N = 4$ sigma-model reduces to the Kähler manifold

$$M_{Z_2 \times Z_2} = \frac{SU(1,1)}{U(1)} \times \prod_{A=1}^{3} \frac{SO(2,2 + n_A)}{SO(2) \times SO(2 + n_A)}.$$ \hspace{1cm} (10)

Since

$$\frac{SO(2,2)}{SO(2) \times SO(2)} = \frac{SU(1,1)}{U(1)} \times \frac{SU(1,1)}{U(1)},$$

For $N = 4, D = 4$ supergravity, we mostly follow the conventions of [17], unless otherwise stated, and set the $D = 4$ Planck mass equal to one.
in the absence of further $Z^I_A$ fields each complex modulus is associated to an $SU(1, 1)/U(1)$ structure. In the Lagrangian, the truncation is performed by first rewriting the fields in an $SU(3)$ basis,

$$\phi^{RA} \equiv \phi^{RA4}, \quad \phi^{R}_{A} = (\phi^{RA})^* = \frac{1}{2} \epsilon_{ABC} \phi^{RBC}. \quad (12)$$

The three $SU(3)$ non-singlet gravitino multiplets are then truncated, and the remaining $N = 1$ gravitino mass term reads:

$$m_{3/2} = -\frac{4}{3} \varphi_{(R)}^{*} \epsilon_{ABC} \phi^{RA} \phi^{SB} \phi^{TC}. \quad (13)$$

This simple formula still depends on the constrained $N = 4$ scalar fields $\phi^{RA}$. However, once written in terms of the unconstrained fields, the expression of the gravitino mass term will considerably change (see below). These constrained states are truncated to $N = 1$ multiplets according to the $Z_2 \times Z_2$ action on the $SU(3)$ and $SO(6, n)$ indices $A$ and $R$, as in the sigma model truncation (10). Since our goal is to work with a fixed set of well-defined moduli and matter fields ($T_A, U_A, Z^I_A$), and to study various classes of gaugings of these multiplets, the next step is to solve the truncated constraints (6) and (7). We then introduce three sets of $4 + n_A$ complex scalars that we denote by

$$\sigma^1_A, \sigma^2_A, \rho^1_A, \rho^2_A, \chi^I_A, \quad A = 1, 2, 3, \quad I = 1, \ldots, n_A. \quad (14)$$

The truncated, $SO(2, 2 + n_A)$–invariant constraints, which for $\eta_{RS} = \text{diag} (-1_6, 1_n)$ read

$$|\sigma^1_A|^2 + |\sigma^2_A|^2 - |\rho^1_A|^2 - |\rho^2_A|^2 - \sum_I |\chi^I_A|^2 = \frac{1}{2},$$

$$(\sigma^1_A)^2 + (\sigma^2_A)^2 - (\rho^1_A)^2 - (\rho^2_A)^2 - \sum_I (\chi^I_A)^2 = 0, \quad (15)$$

are then solved in this basis by:

$$\sigma^1_A = \frac{1}{2} \frac{T_A U_A - (Z^I_A)^2}{|Y(T_A, U_A, Z^I_A)|^{1/2}}, \quad \sigma^2_A = \frac{i}{2} \frac{T_A + U_A}{|Y(T_A, U_A, Z^I_A)|^{1/2}},$$

$$\rho^1_A = \frac{1}{2} \frac{T_A U_A + (Z^I_A)^2}{|Y(T_A, U_A, Z^I_A)|^{1/2}}, \quad \rho^2_A = \frac{i}{2} \frac{T_A - U_A}{|Y(T_A, U_A, Z^I_A)|^{1/2}},$$

$$\chi^I_A = \frac{i}{2} \frac{Z^I_A}{|Y(T_A, U_A, Z^I_A)|^{1/2}}. \quad (16)$$

These expressions depend on the real quantity

$$Y(T, U, Z^I) = (T + \overline{T})(U + \overline{U}) - \sum_I (Z^I + \overline{Z}^I)^2. \quad (17)$$

As expected, the constraints eliminate six complex scalar fields.
The above equations allow to rewrite the scalar potential and the gravitino mass term as functions of the \( N = 1 \) complex scalars, the structure constants and the duality phases. The Kähler potential and the superpotential can then be obtained by separating the holomorphic part in the \( N = 1 \) gravitino mass term, using the relation \( m_{3/2} = e^{K/2} W \). The resulting Kähler potential is
\[
K = -\ln(S + \overline{S}) - \sum_{A=1}^{3} \ln Y(T_A, U_A, Z_A^I),
\]
while the superpotential is simply
\[
W = \frac{4}{3} \sqrt{2} \left[ \cos \delta_R - i \sin \delta_R S \right] \left[ \prod_{A=1}^{3} Y(T_A, U_A, Z_A^I) \right]^{1/2} f_{RST} \epsilon_{ABC} \phi^{RA} \phi^{SB} \phi^{TC}.
\]
It is a holomorphic function of \((S, T_A, U_A, Z_A^I)\), once the \( N = 4 \) scalars from the vector multiplets have been truncated to \( N = 1 \) and replaced by the solutions (16).

In this paper, we discard all matter fields \( Z_A^I \) for simplicity. However, many of the features encountered in the restricted cases studied here remain true with all matter fields included. Removing the \( Z_A^I \) fields, the generic superpotential is then a polynomial in the moduli fields with maximal degree seven. In particular, each monomial is of order zero or one in each of the seven moduli \( S, T_A, U_A \). The superpotential can then have up to \( 2^7 = 128 \) real parameters, which are structure constants and duality phases of the underlying \( N = 4 \) algebra \(^4\). These numbers will be identified with various fluxes of compactified string theories.

The structure constants \( f_{RST} \) gauge a subalgebra of \( SO(6, 6) \), with dimension equal or less than twelve and compatible with the \( Z_2 \times Z_2 \) truncation. They verify Jacobi identities. The gauging structure constants with lower indices \( f_{RST} = f_{RQ} \eta_{QT} \) are fully antisymmetric for consistency of the gauging. The truncation to \( N = 1 \) provides further information. The residual Poincaré gauge fixing conditions solved by \( T_A \) and \( U_A \) are invariant under \( SO(3) \) rotations of the plane index \( A \) and \( SO(2, 2) \) rotations inside each plane. This means that structure constants can be classified using the \( SO(2, 2) \times SO(3) \) subgroup of \( SO(6, 6) \) with embedding \( 12 = (4, 3) \). One can rewrite the gauge algebra in this embedding by defining generators \( T_{Aa}, (A = 1, 2, 3, a = 1, \ldots, 4) \), and commutation relations
\[
[T_{Aa}, T_{Bb}] = f_{AaBb}^{C} T_{Cc}.
\]

\(^4\)The \( N = 1 \) truncation of the scalar fields \( \phi^{RA} \) associates to each fixed value of \( A = 1, 2, 3 \) only four values of the index \( R \), the four directions in each of the three \( SO(2, 2) \). Hence \( f_{RST} \) includes \( 4^3 = 64 \) real numbers.
The antisymmetric gauging structure constants are then
\[ f_{Aa Bb Cc} = f_{Aa Bb}^{Cd} \eta_{cd}, \]  
(21)

where \( \eta_{cd} \) is the \( SO(2,2) \) metric. The \( Z_2 \times Z_2 \) orbifold projection \(^5\) leads naturally to define a ‘plane-interchange symmetry’ in the moduli sector. Our purpose here is to study a particular class of gaugings which respect this plane-interchange symmetry. Ref. [14] will analyse more general gauging structures. The structure constants for these particular gaugings read
\[ f_{Aa_1 Bb_2 Cc_3} = \Lambda_{a_1 b_2 c_3} \epsilon^{ABC}, \quad (a_1, b_2, c_3 = 1, \ldots, 4), \]
\[ f_{Aa_1 Bb_2 Cc_3} = \Lambda_{a_1 b_2 c_3} \epsilon^{ABC}, \quad \Lambda_{a_1 b_2 c_3} = \eta^{c_3 d_3} \Lambda_{a_1 b_2 d_3}. \]
(22)

Each index \( a_1, b_2, c_3 \) is an \( SO(2,2) \) index, and there are in principle \( 4^3 = 64 \) possible combinations, as for the number of possible superpotential terms constructed with \( U_A \) and \( T_A \) and the rule that each term is either linear or independent of each modulus \( (2^6 = 64) \). Each \( SO(2,2) \) index refers to a specific complex plane of the \( N = 1 \) truncation: \( a_1 \) to the first plane, \( b_2 \) to the second, \( c_3 \) to the third. Antisymmetry of the gauging structure constants \( f_{RST} \) implies full symmetry of \( \Lambda_{a_1 b_2 c_3} \): this reduces the number of independent structure constants to 20, which is also the number of combinations of superpotential terms left invariant by any permutation of the plane index. The Jacobi identities verified by the structure constants \( f_{Aa Bb Cc} \) translate into a simple cyclicity property:
\[ \eta^{df} \Lambda_{abf} \Lambda_{cfe} = \eta^{df} \Lambda_{bcf} \Lambda_{afe} = \eta^{df} \Lambda_{cad} \Lambda_{bfe}, \quad \forall a, b, c, e. \]
(23)

Eq. (23) and symmetry of \( \Lambda_{abc} \) are the conditions applying to an \( N = 4 \) gauging respecting the plane-interchange symmetry.

There are two commonly used bases for \( SO(6,6) \). Firstly, the natural basis in which the Cartan metric is diagonal, as in eq. (15). Secondly, the S/A basis defined by
\[ ds^2 = \sum_{i=1}^{6} (dx^{i-} - dx^{i+}) \equiv \sum_{i=1}^{6} dx^{i+} dx^{i-} \equiv \sum_{i=1}^{6} dx^{is} dx^{ia}. \]
(24)

The Cartan metric in the S/A basis is off-diagonal,
\[ \eta = \frac{1}{2} \begin{pmatrix} 0_6 & I_6 \\ I_6 & 0_6 \end{pmatrix}. \]
(25)

\(^5\)And the \( SO(3) \) invariance of the constraints.
The analysis of the consistency conditions (23) is much simpler in the S/A basis, especially in view of the solutions (16) of the Poincaré constraints. The procedure to analyse a gauging and obtain the corresponding $N = 1$ superpotential is as follows. To begin with, select a symmetric set of constants $\Lambda_{a_1 b_2 c_3}$ that solve the cyclicity equations (23) in the S/A basis. Then, compute the resulting $N = 1$ superpotential, in two steps. Firstly, use the following correspondence between the indices $(a_1, b_2, c_3)$ and the twelve directions in $SO(6,6)$:

\begin{align*}
a_1 &= (1, 2, 3, 4) \leftrightarrow (5_S, 6_S, 5_A, 6_A), \\
b_2 &= (1, 2, 3, 4) \leftrightarrow (7_S, 8_S, 7_A, 8_A), \\
c_3 &= (1, 2, 3, 4) \leftrightarrow (9_S, 10_S, 9_A, 10_A).
\end{align*}

(26)

Secondly, use the solution of the Poincaré constraints, in the form

\begin{align*}
(5_S, 7_S, 9_S) &\rightarrow \frac{1}{\sqrt{(T_A + \overline{T}_A)(U_A + \overline{U}_A)}}, & A = 1, 2, 3, \\
(6_S, 8_S, 10_S) &\rightarrow iT_A/\sqrt{(T_A + \overline{T}_A)(U_A + \overline{U}_A)}, & A = 1, 2, 3, \\
(5_A, 7_A, 9_A) &\rightarrow T_A U_A/\sqrt{(T_A + \overline{T}_A)(U_A + \overline{U}_A)}, & A = 1, 2, 3, \\
(6_A, 8_A, 10_A) &\rightarrow iU_A/\sqrt{(T_A + \overline{T}_A)(U_A + \overline{U}_A)}, & A = 1, 2, 3.
\end{align*}

(27)

For each compactified string theory, the allowed fluxes will determine the set of allowed $\Lambda_{a_1 b_2 c_3}$ and the cyclicity equations (23) will impose the consistency relations between various fluxes. This method can be used to generate all superpotentials from fluxes verifying plane-interchange symmetry. Without invoking this symmetry, the analysis of a gauging would be similar, but with a set of nonzero gauging structure constants submitted to more complicated Jacobi identities, instead of the simple relations (23).

With our seven moduli fields, Kähler potential (18) and superpotential (19), the $N = 1$ supergravity scalar potential simplifies to

\[ e^{-K} V = \sum_{i=1}^{7} |W - W_i(z_i + \overline{z}_i)|^2 - 3|W|^2, \]

(28)

where $z_i = S, T_A, U_A$ and $W_i = (\partial W)/(\partial z_i)$. Each quantity $|W - W_i(z_i + \overline{z}_i)|$ is simply the superpotential $W$ with the corresponding field $z_i$ replaced by $-\overline{z}_i$.

### 3 Heterotic fluxes

Before moving to the discussion of the IIA theory, we recall some known results for $N = 1$ compactifications of the heterotic theory on the $T^6/(Z_2 \times Z_2)$ orbifold. This will be useful
to establish some notation and to illustrate our general method in a familiar case.

We begin with the identification of the seven main moduli. Conventionally, we split the space-time indices as $M = [\mu = 0, 1, 2, 3; i = 5, 6, 7, 8, 9, 10]$, and we take one $Z_2$ acting on the coordinates $x^{5,6,7,8}$, the other $Z_2$ on the coordinates $x^{7,8,9,10}$. This naturally defines three complex planes $A = 1, 2, 3$: $i_1 = 5, 6$, $i_2 = 7, 8$, $i_3 = 9, 10$. We follow the conventions of [1] unless otherwise stated.

If we neglect the $E_8 \times E_8$ or $SO(32)$ gauge bosons (which would generate multiplets of $Z_2^I$ type in the $D = 4$ theory, and would allow for additional fluxes associated with the internal components of their two-form field strengths), the bosonic fields of the $D = 10$ heterotic theory are just the universal ones of the NS-NS sector: the string-frame metric $g_{MN}$, the dilaton $\Phi$ and the two-form potential $B_{MN}$. Their $Z_2 \times Z_2$ invariant components can be decomposed as:

$$e^{-2\Phi} = s \left( t_1 t_2 t_3 \right)^{-1}, \quad g_{\mu\nu} = s^{-1} \bar{g}_{\mu\nu}, \quad (29)$$

$$g_{iAJA} = \frac{t_A}{u_A} \left( \frac{u_A^2 + \nu_A^2}{\nu_A} \right), \quad (A = 1, 2, 3), \quad (30)$$

$$B_{\mu\nu} \leftrightarrow \sigma, \quad B_{56} = \tau_1, \quad B_{78} = \tau_2, \quad B_{910} = \tau_3, \quad (31)$$

where $\bar{g}_{\mu\nu}$ is the metric in the $D = 4$ Einstein frame, and the symbol $\leftrightarrow$ indicates the four-dimensional duality transformation relating a two-form potential with an axionic pseudoscalar. Neglecting the dependence of the fields on the internal coordinates, absorbing an integration constant in the $D = 4$ Planck mass, conventionally set to unity, and making the identifications

$$S = s + i \sigma, \quad T_A = t_A + i \tau_A, \quad U_A = u_A + i \nu_A, \quad (A = 1, 2, 3), \quad (32)$$

we obtain $D = 4$ kinetic terms described precisely by the Kähler potential of eq. (18), for the case $Z_2^I = 0$ we have chosen to study:

$$K = -\ln (S + \overline{S}) - \sum_{A=1}^{3} \ln (T_A + \overline{T_A}) - \sum_{A=1}^{3} \ln (U_A + \overline{U_A}). \quad (33)$$

In view of what follows, we stress that the kinetic terms of the seven main moduli are invariant under both $O(7)$ rotations and $SU(1, 1) \times [SO(2, 2)]^3$ duality transformations.

We now summarize the different allowed fluxes, and identify the associated $N = 1$ superpotentials with the method illustrated in the previous section.
3.1 $H_3$ heterotic fluxes

As first recognized in [9], possible fluxes in the heterotic theory are those of the modified NS-NS three-form $\tilde{H}_3 = dB_2 + \ldots$, where the dots stand for the gauge and Lorenz Chern-Simons terms. There are eight independent real fluxes, invariant under the $Z_2 \times Z_2$ orbifold projection:

$$\tilde{H}_{579}, \tilde{H}_{679}, \tilde{H}_{589}, \tilde{H}_{689}, \tilde{H}_{5710}, \tilde{H}_{6710}, \tilde{H}_{5810}, \tilde{H}_{6810}. \quad (34)$$

The corresponding potential for the seven main moduli can be explicitly computed by dimensional reduction. Its generic structure is $V_{H_3} = e^K \prod_{A=1}^{3} f_A(\nu_A, u_A^2 + \nu_A^2)$, where each $f_A$ is a polynomial of at most degree one in its arguments. This is sufficient to deduce, similarly to what happens in IIB theories [10], the corresponding $N = 1$ effective super-potential $W_{H_3}(U)$, which carries no dependence on the $S$ and $T$ moduli. It is immediate to check that $\tilde{H}_3$ fluxes correspond to $N = 4$ gaugings for any choice of the parameters in (34). Leaving aside a systematic discussion, we just observe that, under the assumption of plane-interchange symmetry, there are four independent parameters, associated with four different structures in $W_{H_3}(U)$:

$$\tilde{H}_{579} \equiv \Lambda_{111} \leftrightarrow 1, \quad \tilde{H}_{579} = \tilde{H}_{679} = \tilde{H}_{5710} \equiv \Lambda_{114} \leftrightarrow i(U_1 + U_2 + U_3),$$

$$\tilde{H}_{689} = \tilde{H}_{5810} = \tilde{H}_{6710} \equiv \Lambda_{144} \leftrightarrow -(U_1 U_2 + U_2 U_3 + U_1 U_3), \quad \tilde{H}_{6810} \equiv \Lambda_{444} \leftrightarrow -i U_1 U_2 U_3. \quad (35)$$

3.2 Geometrical heterotic fluxes

The possible fluxes also include some geometrical ones, associated with the internal components of the spin connection $\omega_3$, and corresponding to coordinate-dependent compactifications [6]. These fluxes are characterized by real constants with one upper curved index and two lower antisymmetric curved indices:

$$f^i_{jk} = -f^i_{kj}. \quad (36)$$

These constants must satisfy the Jacobi identities of a Lie group, $f^i_{jk} f^k_{lm} + f^i_{lk} f^k_{mj} + f^i_{mk} f^k_{jl} = 0$, and the additional consistency condition $f^i_{ik} = 0$. The corresponding $D = 4$ potential in the heterotic theory can be easily calculated from the formulae in [6].
In agreement with the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold projection, we must assume here that

$$f^i_{A\,i\,B} = 0 \quad \text{for} \quad A = B \quad \text{or} \quad A = C \quad \text{or} \quad B = C,$$

which satisfies automatically the consistency condition $f^i_{ik} = 0$. Geometrical fluxes are then described by 24 real parameters,

$$C_{i\,A\,i\,B} \equiv f^i_{A\,i\,B} , \quad [(ABC) = (123), (231), (312)],$$

subject only to the Jacobi identities. Leaving aside a general discussion, we assume here plane-interchange symmetry, to reduce the number of independent parameters. Inspection of the resulting scalar potential singles out six different possible structures in the effective superpotential $W_\omega(T, U)$, always linear in the $T$ moduli and independent of $S$:

$$C_{679} = C_{895} = C_{1057} \equiv \Lambda_{112} \leftrightarrow i \, (T_1 + T_2 + T_3),$$
$$C_{579} = C_{957} = C_{795} \equiv \Lambda_{113} \leftrightarrow (T_1 U_1 + T_2 U_2 + T_3 U_3),$$
$$C_{6810} = C_{8106} = C_{1068} \equiv \Lambda_{244} \leftrightarrow -i \, (T_1 U_2 U_3 + T_2 U_1 U_3 + T_3 U_1 U_2),$$
$$C_{5810} = C_{7106} = C_{968} \equiv \Lambda_{344} \leftrightarrow -(T_1 + T_2 + T_3) U_1 U_2 U_3,$$
$$C_{896} = C_{1067} = C_{689} = \leftrightarrow -(T_1 U_2 + T_1 U_3 + T_2 U_1 + T_2 U_3 + T_3 U_1 + T_3 U_2),$$
$$C_{1058} = C_{6710} = C_{8105} \equiv \Lambda_{124} \leftrightarrow (T_1 U_1 U_2 + T_2 U_2 U_3 + T_3 U_3 U_1 + T_1 U_3 + T_2 U_1 + T_3 U_2).$$

In this case the Jacobi identities (23) impose some non-trivial constraints:

$$\Lambda_{112} \Lambda_{344} = \Lambda_{124} \Lambda_{134},$$
$$\Lambda_{113} \Lambda_{344} + \Lambda_{244} \Lambda_{134} = \Lambda_{344} \Lambda_{124} + \Lambda_{134}^2,$$
$$\Lambda_{112} \Lambda_{244} + \Lambda_{113} \Lambda_{124} = \Lambda_{112} \Lambda_{134} + \Lambda_{124}^2.$$

### 3.3 Combined heterotic fluxes

The combination of $\tilde{H}_3$ and $\omega_3$ fluxes in the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold of the heterotic string is then described, under the assumption of plane-interchange symmetry, by the ten real parameters of eqs. (35) and (39). According to eq. (23), consistency with an underlying $N = 4$ gauging amounts to requiring the Jacobi identities (cyclicity conditions) of eq. (40) and the additional condition

$$\Lambda_{111} \Lambda_{344} + \Lambda_{112} \Lambda_{444} + \Lambda_{113} \Lambda_{144} + \Lambda_{114} \Lambda_{244} = 2 \Lambda_{144} \Lambda_{124} + 2 \Lambda_{114} \Lambda_{134}.$$
The corresponding effective $N = 1$ superpotential would read

\[
W = \Lambda_{111} + i \Lambda_{114} (U_1 + U_2 + U_3) - \Lambda_{144} (U_1 U_2 + U_2 U_3 + U_1 U_3) - i \Lambda_{444} U_1 U_2 U_3 \\
+ i \Lambda_{112} (T_1 + T_2 + T_3) + \Lambda_{113} (T_1 U_1 + T_2 U_2 + T_3 U_3) \\
- i \Lambda_{244} (T_1 U_2 U_3 + T_3 U_1 U_3 + T_3 U_1 U_2) - \Lambda_{344} (T_1 + T_2 + T_3) U_1 U_2 U_3 \\
- \Lambda_{124} (T_1 U_2 + T_1 U_3 + T_2 U_1 + T_2 U_3 + T_3 U_1 + T_3 U_2) \\
+ i \Lambda_{134} (T_1 U_1 U_2 + T_1 U_1 U_3 + T_2 U_2 U_1 + T_2 U_2 U_3 + T_3 U_3 U_1 + T_3 U_3 U_2).
\]

(42)

Similar superpotentials were considered, motivated by $N > 1$ gaugings but without assuming plane interchange symmetry and without establishing the precise connections with fluxes, in [18]. The connection between Scherk-Schwarz compactifications, geometrical fluxes, $N = 4$ gaugings and $N = 1$ superpotentials was also discussed in [19], without assuming plane-interchange symmetry and in a different field basis. More results at the $N = 4$ level were obtained in [20]. A general analysis of combined fluxes in toroidal compactifications of the heterotic string was given in [13]: consistent $Z_2 \times Z_2$ truncations of their results are in complete agreement with our results.

It is important to recall that, in the heterotic theory, $\tilde{H}_3$ and $\omega_3$ fluxes, corresponding to perturbative $N = 4$ gaugings with trivial duality phases, can never generate $N = 1$ superpotentials with both constant and linear terms in $S$. We can then obtain, for example, no-scale models as in [7, 19], but never reach the full stabilization of all seven main moduli, including $S$. From the point of view of $N = 4$ supergravity, of course, we could also consider non-perturbative gaugings with non-trivial duality phases, which would give rise to both kinds of allowed $S$-dependences in the effective $N = 1$ superpotential. We may think of these gaugings as associated to possible non-perturbative effects such as gaugino condensation.

4 Fluxes in IIA superstrings

In type IIA and IIB superstring theories compactified on $T^6/(Z_2 \times Z_2)$, to produce $N = 1$, $D = 4$ supersymmetry we must introduce consistently an additional $Z_2$ orientifold projection. We discuss here only the case of the IIA theory, with a specific orientifold projection compatible with D6 branes.

The bosonic fields of the IIA theory are the universal ones of the NS-NS sector, plus
those of the R-R sector: a one-form $A_M$ and a three-form $A_{MNR}$. Five-forms and seven-forms are related to the previous ones by ten-dimensional duality, do not carry independent degrees of freedom and do not need to be included at this stage. Nine-form potentials do not carry any propagating degree of freedom in $D = 10$, even if they can play a role, as we shall see, in the classification of allowed fluxes. We consider here a specific $Z_2$ orientifold projection, involving the inversion of three out of the six internal coordinates and associated with D6 branes: it is not restrictive to take the odd coordinates to be $x^{5,7,9}$.

The independent invariant spin-0 fields from the NS-NS sector are then:

$$g_{ii}, \Phi, B_{56}, B_{78}, B_{910},$$

for which we can temporarily make the heterotic decomposition of eqs. (29)–(31), setting $\nu_A \equiv 0$ and disregarding the off-diagonal components of the internal metric. The independent invariant spin-0 fields from the R-R sector are:

$$A_{6810} = \sigma', A_{679} = -\nu_1', A_{589} = -\nu_2', A_{5710} = -\nu_3'.$$

Looking at the $D = 4$ kinetic terms of the fields in (44), we find

$$\mathcal{L}_R \rightarrow -\frac{1}{4} \tilde{e}_4 \tilde{g}^{\mu\nu} \left[ \mathcal{O}_0 (\partial_\mu \sigma') (\partial_\nu \sigma') + \sum_{A=1}^{3} \mathcal{O}_A (\partial_\mu \nu_A') (\partial_\nu \nu_A') \right],$$

where

$$\mathcal{O}_0 = \frac{u_1 u_2 u_3}{s}, \quad \mathcal{O}_1 = \frac{u_1}{su_2 u_3}, \quad \mathcal{O}_2 = \frac{u_2}{su_1 u_3}, \quad \mathcal{O}_3 = \frac{u_3}{su_1 u_2}. $$

This immediately suggests [12] the identification of the real parts $(s', u_1', u_2', u_3')$, associated by $N = 1$ supersymmetry to the imaginary parts $(\sigma', \nu_1', \nu_2', \nu_3')$:

$$s' = \sqrt{\frac{s}{u_1 u_2 u_3}}, \quad u_1' = \sqrt{\frac{s u_2 u_3}{u_1}}, \quad u_2' = \sqrt{\frac{s u_1 u_3}{u_2}}, \quad u_3' = \sqrt{\frac{s u_1 u_2}{u_3}}. $$

These identifications can be cross-checked by looking at the $F_{\mu\nu} F^{\mu\nu}$ and $F_{\mu\nu} \tilde{F}^{\mu\nu}$ terms in the effective four-dimensional action for the Yang-Mills vectors, generated by the Dirac-Born-Infeld and Wess-Zumino actions for the D6 branes, aligned along the (6810), (679), (589), (5710) O6-planes.

The theory under consideration exhibits a rich structure of possible invariant fluxes. As in the heterotic case, we first discuss each of them separately, then we look at a generic combination. The Jacobi identities of $N = 4$ gaugings, eq. (23), will be automatically satisfied if there are only NS-NS three-form fluxes, or only fluxes of the R-R forms: as we shall see, non-trivial constraints will arise only in the presence of geometrical fluxes or of combined fluxes.
4.1 \( H_3 \) fluxes in IIA

Only four out of the eight independent fluxes allowed for the NS-NS 3-form in the heterotic case, eq. (34), are also invariant with respect to the orientifold projection. In terms of \( H_3 = dB_2 \), they are:

\[
H_{579}, \quad H_{689}, \quad H_{6710}, \quad H_{5810}.
\]  

(48)

Inspection of the four-dimensional potential obtained from dimensional reduction shows that, after moving to the IIA field basis of eq. (47), and assuming plane-interchange symmetry, there are two independent superpotential structures:

\[
H_{579} = \Lambda_{111}^\prime \leftrightarrow i S, \\
-H_{689} = -H_{6710} = -H_{5810} = \Lambda_{114} \leftrightarrow i (U_1 + U_2 + U_3).
\]  

(49)

Notice that, in contrast with the heterotic case, two independent \( SU(1,1) \) phases are involved.

4.2 Geometrical IIA fluxes

Again, the IIA orientifold projection leaves invariant only half of the geometrical fluxes (38) that were allowed, modulo Jacobi identities, in the heterotic case:

\[
C_{5710}, C_{7105}, C_{1057}; \quad C_{679}, C_{796}, C_{967}; \quad C_{589}, C_{895}, C_{958}; \quad C_{6810}, C_{8106}, C_{1068}.
\]  

(50)

Inspection of the four-dimensional potential obtained from dimensional reduction shows that in this case, after moving to the IIA field basis of eq. (47), and assuming plane-interchange symmetry, there are three independent superpotential structures:

\[
C_{679} = C_{895} = C_{1057} \equiv \Lambda_{112}^\prime \leftrightarrow -S \left( T_1 + T_2 + T_3 \right), \\
C_{6810} = C_{8106} = C_{1068} \equiv \Lambda_{113} \leftrightarrow \left( T_1 U_1 + T_2 U_2 + T_3 U_3 \right), \\
C_{589} = C_{796} = C_{7105} = \leftrightarrow \left( -T_1 U_2 + T_1 U_3 + T_2 U_1 \right) + T_2 U_3 + T_3 U_1 + T_3 U_2, \\
C_{958} = C_{5710} = C_{967} \equiv \Lambda_{124} \leftrightarrow -T_2 U_3 + T_3 U_1 + T_3 U_2.
\]  

(51)

Notice that also for geometrical fluxes two different \( SU(1,1) \) phases appear, in contrast with the heterotic case. From eq. (23) we can easily derive the Jacobi identities required for a consistent \( N = 4 \) gauging with geometrical fluxes only:

\[
\Lambda_{124} \left( \Lambda_{124} - \Lambda_{113} \right) = 0.
\]  

(52)
4.3 \( F_0 \) flux

The mass parameter of massive IIA supergravity [21] can be regarded as a ten-dimensional zero-form flux \( F_0 \), dual to the ten-form field strength associated with a nine-form potential, which does not carry any propagating degree of freedom. Inspecting the \( F_0 \) contribution to the potential via dimensional reduction, and moving to IIA variables, we can identify the associated structure in the effective superpotential \( W \):

\[
F_0 = \Lambda_{222} \leftrightarrow -i (T_1 T_2 T_3). \tag{53}
\]

4.4 \( F_2 \) fluxes

The independent \( F_2 \) fluxes invariant under the orbifold and orientifold projections are:

\[
F_{56}, \quad F_{78}, \quad F_{910}. \tag{54}
\]

Looking at their contributions to the potential via dimensional reduction, moving to the IIA field basis of eq. (47), and assuming plane-interchange symmetry, we find that the corresponding structure in the effective superpotential \( W \) is

\[
F_{56} = F_{78} = F_{910} = \Lambda_{122} \leftrightarrow -(T_1 T_2 + T_1 T_3 + T_2 T_3). \tag{55}
\]

4.5 \( F_4 \) fluxes

The four-form fluxes with internal indices, invariant under the orbifold and orientifold projections, are:

\[
F_{5678}, \quad F_{78910}, \quad F_{91056}. \tag{56}
\]

Looking at their contribution to the potential via dimensional reduction, moving to the IIA field basis of eq. (47), and assuming plane-interchange symmetry, we can identify the corresponding structure in the effective superpotential:

\[
F_{5678} = F_{78910} = F_{56910} = \Lambda_{112} \leftrightarrow i (T_1 + T_2 + T_3). \tag{57}
\]

4.6 \( F_6 \) flux

Among the components of the R-R four-form field strength, invariant under both the orbifold and the orientifold projections, there is also \( F_{\mu \nu \rho \sigma} \), which is not associated with
any $D = 4$ propagating degree of freedom, and can be related by ten-dimensional duality to a ten-dimensional six-form flux $F_6$. A similar flux was considered in [22] to address the cosmological constant problem. Looking at the corresponding potential terms generated by dimensional reduction, and moving to the IIA variables of eq. (47), we can identify the corresponding structure in the effective superpotential:

$$F_6 = \Lambda_{111} \leftrightarrow 1.$$  (58)

Notice that the above flux generates a constant superpotential, not a constant potential, in the four-dimensional effective theory.

### 4.7 Combined IIA fluxes

Switching on simultaneously all the independent fluxes identified so far corresponds to having, as non-vanishing coefficients:

- $(\Lambda'_{111}, \Lambda'_{112})$ with $SU(1, 1)$ phase factor $iS$;
- $(\Lambda_{111}, \Lambda_{112}, \Lambda_{122}, \Lambda_{113}, \Lambda_{114}, \Lambda_{124})$ with $SU(1, 1)$ phase 1.

Under our simplifying assumption of plane-interchange symmetry, the Jacobi identities constraining such combined fluxes read

$$\Lambda_{222} \Lambda_{114} + \Lambda_{113} \Lambda_{122} = 2 \Lambda_{122} \Lambda_{124}, \quad \Lambda_{113} \Lambda_{124} = \Lambda^2_{124}. \quad (59)$$

Any combination of fluxes satisfying the above Jacobi identities corresponds to a $N = 4$ gauging, and can be easily translated into an effective $N = 1$ superpotential:

$$W = \Lambda_{111} + i \Lambda'_{111} S + i \Lambda_{112} (T_1 + T_2 + T_3) - \Lambda'_{112} S (T_1 + T_2 + T_3)$$
$$+ i \Lambda_{114} (U_1 + U_2 + U_3) + \Lambda_{113} (T_1 U_1 + T_2 U_2 + T_3 U_3) - \Lambda_{122} (T_1 T_2 + T_1 T_3 + T_2 T_3)$$
$$- \Lambda_{124} (T_1 U_2 + T_1 U_3 + T_2 U_1 + T_2 U_3 + T_3 U_1 + T_3 U_2) - i \Lambda_{222} T_1 T_2 T_3. \quad (60)$$

The results of eqs. (59) and (60) provide a powerful and practical tool for analyzing, directly in the $N = 1, D = 4$ effective theory, the different vacuum structures associated with the different allowed combinations of fluxes, in the chosen orbifold and orientifold of the IIA theory. The study of a large number of examples of flux configurations [14], in heterotic and type II strings, actually shows that the effective supergravity approach based upon
\( N = 4 \) gaugings can accurately reproduce the conditions imposed by the full field equations of the ten-dimensional theories. This of course requires to include all necessary brane and orientifold plane contributions to these equations. For the combinations of fluxes leading to stable vacua of our \( N = 1, D = 4 \) effective theory, it would be interesting to explicitly examine the corresponding combinations of D6-branes and O6-planes required to satisfy the \( D = 10 \) equations and Bianchi identities, and the associated tadpole cancellation conditions. This analysis goes beyond the scope of the present paper.

5 Some selected IIA examples

We present now some selected examples of admissible IIA fluxes that correspond to \( N = 4 \) gaugings with non-trivial \( SU(1,1) \) phases and give rise to physically different situations.

5.1 Flat gaugings, no-scale models: stabilization of four moduli

Switching on a system of \((\omega_3, H_3, F_0, F_2)\) fluxes, with nonzero parameters \((B, D > 0)\)

\[
\begin{align*}
\Lambda'_{112} &= -A, & \Lambda_{122} &= -AB, \\
\Lambda'_{111} &= C, & \Lambda_{222} &= -CD,
\end{align*}
\]

the Jacobi identities (59) are automatically satisfied, and the following effective \( N = 1 \) superpotential is generated:

\[
W = A \left[ S(T_1 + T_2 + T_3) + B(T_1 T_2 + T_2 T_3 + T_1 T_3) \right] + iC \left[ S + D T_1 T_2 T_3 \right].
\]

(62)

It is immediate to see that this corresponds to a no-scale model. Since \( W \) does not depend on \((U_1, U_2, U_3)\), the scalar potential is a sum of positive semi-definite terms,

\[
e^{-K}V = |W - (S + \bar{S})W_S|^2 + \sum_{A=1}^{3} |W - (T_A + \overline{T_A})W_{T_A}|^2,
\]

(63)

and stabilization of the \( S \) and \( T_A \) moduli occurs at

\[
\langle S \rangle = B T, \quad \langle T_1 \rangle = \langle T_2 \rangle = \langle T_3 \rangle = T, \quad T = \sqrt{\frac{B}{D}},
\]

(64)

with \( \langle V \rangle = 0 \). The \( U_A \) moduli remain as complex flat directions and supersymmetry is broken in the \( U_A \) sector, since the stabilization conditions lead to \( \langle W \rangle \neq 0 \), with a gravitino mass

\[
\langle m^2_{3/2} \rangle \propto \frac{|9 A^2 B + C^2 D|}{u_1 u_2 u_3}.
\]

(65)
To identify the gauging associated with the above fluxes and superpotential, it is convenient to rescale the fields according to

$$S \rightarrow B^{3/2}D^{-1/2}S, \quad T_A \rightarrow B^{1/2}D^{-1/2}T_A.$$  \hfill (66)

The stabilization of the rescaled fields occurs at $\langle S \rangle = \langle T_A \rangle = 1$, with a superpotential as in (62) with $B = D = 1$. The resulting group is $E_3 \times E_3$ [14], where $E_3$ is the three-dimensional Euclidean group, i.e. the $SO(3)$–invariant contraction of $SO(4)$ or $SO(3,1)$.

This kind of rescalings can be applied in general, to shift the values at which the fields are stabilized. For simplicity, and without losing the full generality of the combination of fluxes, we choose that moduli are stabilized at value one in most of the following examples.

As a side remark, we notice here that the same phenomenology of the above example can be obtained without respecting the plane-interchange symmetry. As an example, we can consider as before a system of fluxes $(\omega_3, H_3, F_0, F_2)$, but this time corresponding to a superpotential:

$$W = A (ST_1 + T_2 T_3) + i B (S + T_1 T_2 T_3), \quad \text{with} \quad \langle m^2_{3/2} \rangle \propto \frac{A^2 + B^2}{u_1 u_2 u_3}. \hfill (67)$$

The complex flat directions are $(U_1, U_2, U_3)$, as in the first example.

We finally notice that, in the type–IIA theory, purely geometrical fluxes $\omega_3$ are not sufficient to stabilize all moduli explicitly appearing in the corresponding superpotential $W$, because the latter is always quadratic in the fields. As an example to illustrate this point, based on the two-dimensional Euclidean group $E_2$ [the $SO(2)$–invariant contraction of $SO(3)$ or $SO(2,1)$] and breaking the plane-interchange symmetry, we consider the superpotential

$$W = A (T_1 U_2 + T_2 U_1), \quad \text{with} \quad \langle m^2_{3/2} \rangle \propto \frac{A^2}{s t_3 u_3}. \hfill (68)$$

This corresponds to a $Z_2$ freely acting orbifold (generalized Scherk-Schwarz mechanism in string theory), with complex flat directions $(S, T_3, U_3)$. However, there are additional flat directions, because the auxiliary fields associated with $(T_1, U_1, T_2, U_2)$ are all set to zero by requiring $\tau_1 = \tau_2 = \nu_1 = \nu_2 = 0$ and $t_1 u_2 = t_2 u_1$. In this case the spectrum has 4 massive and 3 massless “axions”, 1 massive and 6 massless “dilatons”.

### 5.2 Gaugings with $V > 0$, cosmological models

Examples can be easily found, in which less than four moduli are stabilized and the potential is always strictly positive-definite, leading to runaway solutions (in time).
Superpotentials with a single monomial are of course examples where no modulus gets stabilized. For instance, we can choose the fluxes $\Lambda_{111} = F_6$, $\Lambda_{222} = F_0$ or $\Lambda'_{111} = H_3$, corresponding to

$$W = F_6, \quad W = -iF_0 T_1 T_2 T_3 \quad \text{or} \quad W = iH_3 S.$$  \hspace{1cm} (69)

This leads to $V = 4 e^K |W|^2$, with $|W| > 0$ and a gravitino mass term of the form

$$m_{3/2}^2 = \frac{1}{2^7 s t_1 t_2 t_3 u_1 u_2 u_3} \times \{|F_6|^2, \ |F_0 T_1 T_2 T_3|^2 \text{ or } |H_3 S|^2\},$$  \hspace{1cm} (70)

respectively.

An example where three moduli are stabilized is obtained by switching on a system of R-R fluxes $(F_0, F_2, F_4, F_6)$, with parameters

$$\Lambda_{111} = -\Lambda_{122} = A, \quad \Lambda_{112} = -\Lambda_{222} = B.$$  \hspace{1cm} (71)

The Jacobi identities (59) are automatically satisfied, and the following effective $N = 1$ superpotential is generated:

$$W = A (1 + T_1 T_2 + T_2 T_3 + T_3 T_1) + iB (T_1 + T_2 + T_3 + T_1 T_2 T_3).$$  \hspace{1cm} (72)

This choice of fluxes and superpotential is actually a gauging of $SO(1, 3)$. It is immediate to see that, since the superpotential does not depend on four of the seven main moduli (the $T$-moduli are stabilized at one), supersymmetry is broken and a positive-definite runaway $D = 4$ scalar potential is generated,

$$\langle V \rangle = \langle m_{3/2}^2 \rangle, \quad \text{with} \quad \langle m_{3/2}^2 \rangle = \frac{A^2 + B^2}{8 s u_1 u_2 u_3},$$  \hspace{1cm} (73)

possibly leading to time-dependent vacua of cosmological interest.

5.3 Gaugings with $V < 0$, stabilization of all moduli

We now look at situations where more than four moduli are stabilized, leading to negative-definite potentials once the stabilized moduli are set to their appropriate values.

We begin with a gauging of $E_3$ with fluxes $\Lambda_{113} = -\omega_3$ (geometric) and $\Lambda_{111} = F_6$ (R-R six-form), with $\omega_3, F_6 > 0$. The R-R six-form corresponds to the $SO(3)$ directions in $E_3$ while $\omega_3$ corresponds to the translations. The superpotential reads

$$W = -\omega_3 (T_1 U_1 + T_2 U_2 + T_3 U_3) + F_6.$$  \hspace{1cm} (74)
The six equations for the non-trivial supergravity auxiliary fields are solved at \( \langle \tau_A \rangle = \langle \nu_A \rangle = 0 \) and \( \langle t_1 u_1 \rangle = \langle t_2 u_2 \rangle = \langle t_3 u_3 \rangle = F_6/\omega_3 \). At these values, \( W = -2F_6 \), and the \( s \)-dependent scalar potential and gravitino mass term read

\[
V = -2 e^K |W|^2 = -\frac{\omega_3^3}{16 F_6^3} s , \quad m_{3/2}^2 = -\frac{1}{2} V .
\] (75)

At the string level, this is the well-known NS five-brane solution plus linear dilaton, in the near-horizon limit. The original gauging is \( SU(2) \), combined with translations, which emerge as free actions at the level of the world-sheet conformal field theory. It is remarkable that this \( E_3 \) algebra remains visible at the supergravity level. It is also interesting that, if we allow extra fluxes, induced by the presence of fundamental-string sources, we can reach \( AdS_3 \) background solutions with stabilization of the dilaton. All moduli are therefore stabilized. This has been studied recently at the string level [23].

Using all fluxes admissible in IIA, \( Z_2 \times Z_2 \) strings, we can obtain the stabilization of all moduli in \( AdS_4 \) space–time geometry. Switching on all fluxes \( (\omega_3, H_3, F_0, F_2, F_4, F_6) \), with parameters

\[
-\frac{1}{9} \Lambda_{111} = -\frac{1}{2} \Lambda'_{112} = \frac{1}{6} \Lambda_{113} = \Lambda_{122} = A ,
\]

\[
\frac{1}{2} \Lambda'_{111} = -\frac{1}{3} \Lambda_{112} = \frac{1}{2} \Lambda_{114} = -\frac{1}{5} \Lambda_{222} = B ,
\]

the Jacobi identities (59) are satisfied for

\[
6 A^2 = 10 B^2 ,
\] (78)

and the following effective \( N = 1 \) superpotential is generated:

\[
W = A [2 S (T_1 + T_2 + T_3) - (T_1 T_2 + T_2 T_3 + T_3 T_1) + 6 (T_1 U_1 + T_2 U_2 + T_3 U_3) - 9]

+ i B [2 S + 5 T_1 T_2 T_3 + 2 (U_1 + U_2 + U_3) - 3 (T_1 + T_2 + T_3)] .
\] (79)

Notice that condition (78) relates the terms with even and odd powers of the fields in the superpotential, thus its sign ambiguity is irrelevant. The superpotential (79) leads to a supersymmetric vacuum at \( \langle S \rangle = \langle T_A \rangle = \langle U_A \rangle = 1 \) \( (A = 1, 2, 3) \). Since at this point \( \langle W \rangle = 4 (3 A + i B) \neq 0 \), implying \( \langle V \rangle = -3 m_{3/2}^2 < 0 \), this vacuum has a stable \( AdS_4 \) geometry with all seven main moduli frozen.

The educated reader might feel uncomfortable with condition (78), which seems to imply non-integer flux numbers. This is a consequence of our choice for presenting the model, with \( S = T_A = U_A = 1 \) at the minimum. One can recover integer flux numbers
by rescaling appropriately the moduli. A possible choice (among many others) is the following:

\[
(S, T_A, U_A) \rightarrow b(S, T_A, U_A), \quad b = \frac{B}{A} = \sqrt[5]{3}
\]  

(80)

With that choice

\[
W = N \left[ 2S (T_1 + T_2 + T_3) - (T_1 T_2 + T_2 T_3 + T_3 T_1) \\
+ 6 (T_1 U_1 + T_2 U_2 + T_3 U_3) - 15 \\
+i N [2S + 3 T_1 T_2 T_3 + 2 (U_1 + U_2 + U_3) - 3 (T_1 + T_2 + T_3)]
\]

(81)

where \( N = (3/5) A \).

We should emphasize here that this is the only known example of a complete stabilization of the moduli, reached in IIA by switching on fundamental fluxes (NS or R). We should also stress that this cannot happen in the heterotic string, because of the absence of \( S \)-dependence in the general flux-induced superpotential. Such a dependence could however be introduced under the assumption of gaugino condensation. In type IIB with D3-branes, the orientifold projection that accompanies the \( Z_2 \times Z_2 \) orbifold projection eliminates the \( \omega_3 \) fluxes, thus the \( T \) moduli are not present in the superpotential and cannot be stabilized by fluxes. The case of D9-branes (open string) is similar to the heterotic case, whereas the D7-brane set-up is not captured by the \( Z_2 \times Z_2 \) orbifold projection used here. The heterotic approach à la Horava–Witten is under investigation [14], whereas F-theory on Calabi–Yau four-folds can introduce exponential dependences in the superpotential [24] and stabilize the \( T \) moduli.

6 Conclusions and outlook

In this paper we proposed a novel, bottom-up approach for studying the infrared physics of superstring compactifications that preserve an exact or spontaneously broken \( N = 1 \) supersymmetry. The approach is in principle applicable to all ten-dimensional superstring theories and M–theory, and is based on the powerful constraints of the gauged \( N = 4 \) supergravity underlying all these compactifications. Since in \( N = 4, D = 4 \) supergravity the manifold of the scalar fields is unique, once the number of vector fields is given, our approach allows to identify unambiguously the Kähler potential of the \( N = 1, D = 4 \) effective theory. The various systems of fluxes allowed in the different superstring theories
are then used to determine, without solving the ten-dimensional equations of motion and Bianchi identities, the structure constants and duality phases that specify the gauging of the $N = 4$ theory. This in turn can be used to identify the superpotential of the resulting $N = 1$ theory. The search for the possible vacuum structures corresponding to the different systems of fluxes can then be performed in a very powerful and elegant formalism, by looking at the potential and auxiliary fields of the effective $N = 1$, $D = 4$ theory.

To be specific, we applied our strategy to situations where the reduction from $N = 4$ to $N = 1$ is achieved by a $Z_2 \times Z_2$ orbifold projection. In the present work, we kept for clarity the six $N = 4$ vector-multiplet geometrical moduli. Thus, the moduli sector of the resulting $N = 1$ theory (after the projection) contained seven distinguished chiral multiplets: $S,T_A,U_A$, $(A = 1,2,3)$. In the heterotic theory, the $Z_2 \times Z_2$ projection is enough to reduce the initial ($N = 4$) supersymmetry to $N = 1$. For describing type-II theories, and in particular the type-IIA compactifications on which we focused for this paper, an extra $Z_2$ orientifold projection is needed: we chose the one acting as a parity on three of the six internal coordinates, associated with $N = 1$ compactifications of the IIA theory with D6-branes and O6-planes. Type IIB can be treated similarly, by introducing a $Z_2$ orientifold projection associated with either D3– or D9–branes.

A major geometrical difference exists, however, which makes the type IIA compactifications far more interesting. In type IIB, the Calabi–Yau smooth manifold resolution of the orbifold holds even in the presence of $H_3$ and $F_3$ fluxes [10]. This explains why most of the literature deals with this kind of type IIB constructions. However, even in this case, the $\omega_3$ geometrical fluxes are incompatible with the $Z_2 \times Z_2$ orbifold projection (and its smooth Calabi-Yau resolution). The situation for type IIA is even more exotic, since no well-defined mathematical framework has yet been unravelled for understanding the “deformed Calabi–Yau” geometry.

Nevertheless, this is by no means an obstruction to us, when the situation is considered from the conformal field theory perspective of string theory. Many examples exist that demonstrate this: asymmetric orbifolds, fermionic constructions, twisted Gepner constructions, supersymmetric compactifications on manifolds with torsion, etc. Furthermore, these exact models, as well as the whole procedure we have developed so far for dealing with fluxes, point towards the existence of a generalized mirror symmetry, despite the absence of a Calabi–Yau geometrical interpretation for type IIA. A manifestation of that
symmetry emerges, for instance, when a non-trivial $\omega_3$ flux is switched on. In the (freely-acting) orbifold limit, a mirror-like $U \leftrightarrow T$ duality relates the type IIA to the type IIB side.

The gauging approach we propose here relies on the rich but constrained $N = 4$ structure. It enables us to bypass the above geometrical difficulties and to organize the fluxes in a systematic way. Indeed, the gauging procedure goes along with a set of structure constants and duality phases, where the former must satisfy Jacobi identities and antisymmetry conditions. These all enter the superpotential, which in turn determines the scalar potential. In the above framework, it is possible to list exhaustively the various choices for the structure constants and duality phases. The choice of the $Z_2 \times Z_2$ projection plays an important role, since it naturally induces an interchange symmetry among the three planes, and simplifies considerably the implementation of the full antisymmetry of $f_{RST}$. Releasing the assumption of plane-interchange symmetry, changing the orbifold and/or orientifold projections preserving $N = 1$ supersymmetry, moving to type-IIB or type-I superstring theories, all goes beyond the scope of the present paper, and is postponed to [14].

For each of the possible choices of the structure constants (this is equivalent to choosing the subgroup of $SO(6, 6)$ that is gauged) and duality phases, one can readily analyze the issue of moduli stabilization. Furthermore, and this is the core of the present paper, one can trace back the origin of the structure constants and duality phases in terms of fluxes in the underlying fundamental theory in ten dimensions. Although our main motivation was the analysis of the yet not unravelled type IIA, we applied our technique to the heterotic case, where we clarified the case where geometrical and NS-NS three-form fluxes are combined. Our pattern allows to reproduce systematically the various examples available in the literature, such as the no-scale models.

As far as type IIA is concerned, more possibilities exist, thanks to the presence of R-R fluxes, besides the geometric and NS-NS ones: $F_0$, $F_2$, $F_4$ and even $F_6$. They can be introduced one by one, or in combination, provided the Jacobi identities are still satisfied. A specific combination exists, which generates a solution where all seven moduli are stabilized, in an $AdS_4$ geometry. This is typical of type IIA and cannot happen in heterotic, where the allowed geometrical and three-form fluxes cannot create an $S$-dependence in the superpotential. More examples can be displayed with partial moduli stabilization: domain-wall solutions, runaway solutions, no-scale models ...
There are various directions that would be worth exploring, besides those already mentioned above. Among them, the detailed correspondence of the ten-dimensional equations of motion, Bianchi identities and tadpole cancellation conditions, with the equations and consistency conditions in the effective gauged four-dimensional supergravity theory. Also, the inclusion in our formalism of the scalar and vector fields associated to brane excitations, and the exploration of the new systems of fluxes associated with their field strengths.

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