Pressure Transient Analysis and Transient Inflow Performance Relationship of Multiple-Fractured Horizontal Wells in Naturally Fractured Reservoirs by a Trilinear Flow Model

Huizhu Xiang,* Guoqing Han, Gaoqiang Ma, Zhiyong Zhu, Liying Zhu, and Long Peng

ABSTRACT: Trilinear flow model is an effective method to reproduce the flow behavior for horizontal wells with multistage hydraulic fracture treatments in unconventional reservoirs. However, models developed so far for transient analysis have rarely considered the inflow performance of wells. This paper introduces a new composite dual-porosity trilinear flow model for the multiple-fractured horizontal well in the naturally fractured reservoirs. The analytical solution is derived under a constant rate condition for analyzing transient pressure behaviors and generating the transient inflow performance relationships (IPRs). The plots of pressure profiles with time could provide insightful information about various flow regimes that develop throughout the entire production cycle. Sensitivity analysis of pressure and pressure derivative response was also performed by varying different parameters (such as hydraulic fracture width and permeability, reservoir configurations, etc.) and by which the impacts of different parameters on the durations of regimes as well as the productivity index can be confirmed. The main outcomes obtained from this study are as follows: (1) the ability to characterize naturally fractured reservoirs using a new composite dual-porosity trilinear flow model; (2) the application of analytical solutions of transient analysis to generate transient IPR curves for different flow regimes; (3) understanding the effect of reservoir configurations, fractures, and matrix characteristics on pressure distribution, flow regime duration, and transient IPR. More specifically, the pressure drop increases and the productivity index decreases with the decrease of the hydraulic fracture conductivity and the increase of matrix permeability and the skin factor. Also, the larger hydraulic fracture spacing and drainage area result in the later onset of the pseudo-steady-state regime. (4) A comprehensive study on transient pressure behaviors and transient inflow performance can provide valuable information to characterize the multifractured complex systems as well as some insights into the production.

1. INTRODUCTION

Hydraulic fracturing technology has been extensively used in the exploitation of unconventional reservoirs. Due to the rise of flow conductivity near wellbore, the seepage process in various porous media becomes more complex than that in conventional reservoirs. Over the past few decades, to solve the pressure or rate problem, there have been many well-developed models that can characterize the flow in the fractured formation.

Since the porous media of the fracture network in the enhanced fracture region consist of two independent systems, namely, induced fractures and the surrounding matrix, such region is usually characterized by dual-porosity models. Warren and Root1 first introduced the concept of dual porosity, in which natural fractures provide the main flow path, while the matrix provides the main storage space. The transfer of fluids from the matrix to the fracture was considered as pseudo-steady-state (PSS) flow by assuming a PSS matrix-fracture shape factor. Given that the fracture network is superimposed in the primary porosity,2 the pressure drops at any point of the matrix are the same.3

For the hydraulically fractured horizontal wells, the fluid movement in the formation is generally described by multilinear flow. Brown et al.4 introduced the classical trilinear model, whose analytical solution provides the information of the pressure transient response of 1-D flow in three regions: hydraulic fractures, the stimulated reservoir region surrounding
the hydraulic fractures (SRV), and original low-permeability reservoir region (USRV). Since then, even though lots of multilinear models were presented with different reservoir configurations, most existing linear flow models regard the USRV as single-porosity media. Since natural fractures exist not only in induced fracture network but also in the unstimulated reservoir region, such naturally fractured reservoirs should be idealized and mathematically treated as composite double-porosity regions. In this paper, the coupled dual-porosity model in both SRV and USRV was developed for naturally fractured reservoirs using the PSS approach.

Moreover, there have been numerous works analyzing the performance of fractured wells through the transient pressure behavior and production decline analysis. Kou et al. proposed a transient analysis model of a volume were also analyzed by solving the semianalytical permeability modulus and the radius of stimulated reservoir desorption, IPR based on the traditional trilinear-behavior and production decline analysis.8

The pattern is divided into three parts (Figure 2): the outer rectangular drainage region at an individual hydraulic fracture. As shown in Figure 1, the symmetrical pattern is one-quarter of a matrix system.

2. MATHEMATICAL MODEL

2.1. Trilinear Flow Model Description. As shown in Figure 1, the symmetrical pattern is one-quarter of a rectangular drainage region at an individual hydraulic fracture. The pattern is divided into three parts (Figure 2): the outer region (USRV), the inner region (SRV), and hydraulic fractures. In both outer and inner regions, the naturally fractured reservoir is characterized by a dual-porosity model, thus forming two porosity-permeability systems for the matrix and natural fractures. The analytical solutions are derived by using the continuity conditions for flux and pressure, as well as the boundary conditions for each flow process, respectively.

The assumptions using for the derivation of the trilinear flow model are made as follows

- The matrix permeability in the outer and inner regions is identical.
- The hydraulic fractures are assumed to have the same half-length, width, and spacing in the formation.
- Single-phase 1-D flow is developed in the three regions.
- The pseudo-steady dual-porosity model (sugar cube model) is applied in the outer and inner regions.

![Figure 2. Dimensions of the physical model.](https://doi.org/10.1021/acsomega.1c02659)

2.2. Analytical Model Development. To simplify the derivation process, the governing diffusivity equations and related initial/boundary conditions are developed in the Laplace domain.

Starting with the 1-D linear flow in the outer region, the diffusivity equation and the associated boundary conditions can be represented in the following dimensionless form

For a naturally fractured system

$$\frac{\partial^2 \bar{p}_{D}}{\partial x_D^2} + \lambda_0 (\bar{p}_{D} - \bar{p}_{oD}) = \frac{1}{\eta_0} \bar{p}_{oD}$$

(1)

For a matrix system

$$\bar{p}_{mD} = \frac{1}{1 + \frac{1}{s_f a m_{inD}}} \bar{p}_{oD}$$

(2)

Substituting eq 2 to 1, the differential equation is solved to give

$$\frac{\partial^2 \bar{p}_D}{\partial x_D^2} - sf(s) \bar{p}_{oD} = 0$$

(3)

where

$$f(s) = \frac{\lambda_0}{s_f a m_{inD} + s} + \frac{1}{\eta_0}$$

Also, the outer and inner boundary conditions are

$$\left( \frac{\partial \bar{p}_{oD}}{\partial x_D} \right)_{x_D=x_{oD}} = 0$$

(4)
The general solution for dimensionless pressure in the outer region is given by

\[ \bar{p}_{\text{OD}} = \bar{p}_{\text{OD}}(x_D) = \frac{\cosh[\sqrt{s}(x_D - x_D)]}{\cosh[\sqrt{s}(x_D - 1)]} \]

(6)

Flow in the inner region adjacent to the outer region is assumed as 1-D linear flow with the direction perpendicular to the hydraulic fracture. Following steps similar to those of the outer region, the diffusivity equation and related boundary conditions are given by the following equation

\[ \frac{\partial^2 \bar{p}_{\text{ID}}}{\partial x_D^2} - \epsilon \frac{\partial \bar{p}_{\text{ID}}}{\partial x_D} = 0 \]

(7)

where

\[ \epsilon = \frac{k_i s}{s + x_D \alpha h_{\text{md}}} + \theta \]

\[ \theta = \frac{k_w}{k_i} + s \]

\[ \beta = \sqrt{s} \tanh[\sqrt{s}(x_D - 1)] \]

and

\[ \left( \frac{\partial \bar{p}_{\text{ID}}}{\partial x_D} \right)_{x_D = x_{wD}} = 0 \]

\[ (\bar{p}_{\text{ID}})_{x_D = w_D/2} = (\bar{p}_{\text{FD}})_{x_D = w_D/2} \]

(9)

Then, the dimensionless pressure solution for the inner region is obtained as

\[ \bar{p}_{\text{ID}} = \frac{\cosh[\sqrt{s}(x_D - x_{wD})]}{\cosh[\sqrt{s}(x_D - w_D/2)]} \]

(10)

This dimensionless pressure solution of 1-D linear flow represents the pressure distribution inside the hydraulic fracture. The pressure behavior can be described by

\[ \frac{\partial^2 \bar{p}_{\text{FD}}}{\partial x_{FD}^2} - \theta \bar{p}_{\text{FD}} = 0 \]

(11)

where

\[ \theta = \frac{k_w}{k_i} + s \]

\[ \frac{\partial \bar{p}_{\text{FD}}}{\partial x_{FD}} \bigg|_{x_{FD} = 1} = 0 \]

\[ \left( \frac{\partial \bar{p}_{\text{FD}}}{\partial x_{FD}} \right)_{x_{FD} = 0} = -\frac{\pi}{sC_{\text{FD}}B} \]

(13)

The dimensionless pressure solution at the hydraulic fracture face can be obtained by

\[ \bar{p}_{\text{FD}} = (\bar{p}_{\text{FD}})_{x_{FD} = 0} = \frac{\pi}{sC_{\text{FD}}B \sqrt{\theta_F} \tanh(\sqrt{\theta_F})} \]

(14)

Considering the effect of non-Darcy flow and the mechanical skin factor, a total skin factor (SF) is introduced, so eq 14 can be rewritten as

\[ \bar{p}_{\text{FD}} = (\bar{p}_{\text{FD}})_{x_{FD} = 0} = \frac{\pi}{sC_{\text{FD}}B \sqrt{\theta_F} \tanh(\sqrt{\theta_F})} + \text{SF} \]

(15)

The solutions above in the Laplace domain can then be inverted into the time domain numerically using the Stehfest\textsuperscript{16} algorithm.

According to Van Everdigen and Hurst\textsuperscript{17} the dimensionless production rate with a constant bottom hole pressure is given by

\[ \bar{q}_D = \frac{1}{s \bar{p}_D} \]

(16)

and the dimensionless cumulated production is given by

\[ \bar{N}_D = \frac{1}{s \bar{p}_D} \]

(17)

Using the above equations, the transient production behavior of fractured horizontal wells can be analyzed. Details of analytical model development are given in Appendix B.

3. TRANSIENT BEHAVIORS AND FLOW REGIMES

Based on the model presented in the previous sections, the transient pressure behaviors can be obtained for constant rate wells, while transient rate behaviors can be obtained for constant bottom-hole pressure wells, which helps to distinguish between distinct flow regimes that develop throughout the production process.

These plots can provide information to estimate the starting and ending times of all developed flow regimes, as shown in Figures 3 and 4. These figures describe the pressure and pressure derivative curves, and the production rate decline curve and cumulative production profile obtained by using the reservoir parameters are listed in Table 1. Three flow stages developed within a fractured horizontal well in a fractured reservoir can be easily identified from the plots, including the bilinear flow, linear flow, and PSS flow. It is worth mentioning...
that the linear flow in the hydraulic fracture may hardly be captured due to its extremely limited duration.

Stage 1: the linear flow in hydraulic fractures usually lasts for a very short time and sometimes cannot even be observed due to the presence of the wellbore storage effect.

Stage 2: the bilinear flow is the first easily identifiable flow regime in the transient pressure and rate curves, and it flows linearly from both the hydraulic fractures and the inner region to the wellbore. This phase corresponds to a slope of 0.25 on the pressure derivative curves.

Stage 3: after the bilinear flow, the linear flow represents the flow from the inner region and can be identified by a 0.5 slope on the pressure derivative curves.

Stage 4: the PSS flow is the last flow regime observed during the whole process, indicating that the pressure pulse reaches the boundary of the reservoir. It can be characterized by a slope of one for pressure derivative curves. Meanwhile, both the pressure curve and the pressure derivative curve overlap with each other. Depending on the reservoir configurations and properties, this flow period usually takes a long time to arrive, sometimes even decades.

4. TRANSIENT IPR CURVES

Based on the above studies, the time when each flow regime occurs can be verified by transient behavior analysis. Therefore, the precondition of obtaining the time-variant IPR of each flow period can be implemented.

A procedure presented in previous studies \(^1\) for generating transient IPR under different production conditions is used here. According to the constant-rate pressure results obtained by using eq 14 established by the model above, at any time of interest, the flow rates with corresponding bottom hole pressures are known from the curves as shown in Figure 5.

5. RESULTS AND DISCUSSION

Based on the above theory, the transient behaviors and transient IPR curves of different flow regimes of a horizontal well with multiple hydraulic fractures in naturally fractured reservoirs were plotted and analyzed. In this part, sensitivity analysis is conducted to specify how and when the key parameters affect transient behaviors and IPR curves. The well is assumed to be under constant-rate conditions in this part.

Table 1. Reservoir and Fluid Data

| reservoir and fluid data | values |
|--------------------------|--------|
| reservoir pressure (MPa) | 50     |
| reservoir thickness (m)  | 10     |
| hydraulic fracture half-length (m) | 180 |
| hydraulic fracture width (m) | 0.0018 |
| reservoir size in x-direction, XeD | 2 |
| distance between hydraulic fracture, YeD | 2 |
| porosity of the matrix | 0.1 |
| porosity of fracture in SRV | 0.3 |
| porosity of fracture in USRV | 0.25 |
| porosity of hydraulic fracture | 0.1 |
| compressibility of the matrix (MPa\(^{-1}\)) | 2.30 \times 10^{-4} |
| compressibility of fracture in USRV (MPa\(^{-1}\)) | 3.00 \times 10^{-4} |
| compressibility of fracture in SRV (MPa\(^{-1}\)) | 3.00 \times 10^{-4} |
| compressibility of hydraulic fracture (MPa\(^{-1}\)) | 2.00 \times 10^{-4} |
| permeability of the matrix (mD) | 0.1 |
| permeability of fracture in USRV (mD) | 1 |
| permeability of fracture in SRV (mD) | 10 |
| permeability of hydraulic fracture (mD) | 10,000 |
| oil viscosity (mPa s) | 2 |
| oil formation volume factor | 1.05 |
| shape factor | 20 |
5.1. Permeability of Hydraulic Fractures. It is known that fracture permeability has a significant effect on the conductivity of fractures. With the increasing permeability, the fluid flows more easily in the porous medium toward the wellbore. Figure 9 shows the effect of different permeability on transient pressure and derivative pressure curve. The proposed model mainly yields different responses in the early period and the same responses at the late times. Knowing that hydraulic fracture permeability is a critical property related to flow in hydraulic fractures, it is reasonable to exhibit divergence in the bilinear flow regime on the curves. On the contrary, the shapes and trends of the later pressure behavior plots show no change with different values of hydraulic fracture permeability. Thus, it does not influence the linear flow and PSS flow which developed in the inner and outer regions.

To compare in the same real-time domain, select a specified time point to generate a transient IPR in the early flow regime, and the result is shown in Figure 10. The larger permeability, the larger the productivity index (the reciprocal of the slope).

5.2. Width of Hydraulic Fractures. Another parameter that dominates the fracture conductivity is the fracture width. The increased width means the strong storage capacity of hydraulic fractures and will result in more time for the fluid to flow within the hydraulic fractures. Similarly, the impact of the hydraulic fracture width on the pressure profile mainly concentrates in the early period, as shown in Figure 11. Meanwhile, according to the corresponding IPR curves in Figure 12, in the case of wider fractures, the reservoir could offer more flow rates at the same flowing bottom hole pressure. Based on the above results, the conductivity proportional to

Figure 7. Production rate profile at different flowing pressures ($P_{wf 1} < P_{wf 2} < P_{wf 3}$).

Figure 8. Transient IPR curves under constant flowing bottom hole pressures ($t_1 < t_2 < t_3$).

Figure 9. Pressure behaviors for different hydraulic fracture permeability.

Figure 10. Transient IPR curves for different hydraulic fracture permeabilities.

Figure 11. Pressure behaviors for different hydraulic fracture widths.
permeability and dimensionless hydraulic fracture width is 
proved to be conversely proportional with the dimensionless 
pressure drop and directly proportional to the productivity 
index.

5.3. Drainage Distance (Xe). Because the hydraulic 
fractures are not fully extended to the formation, the length 
of the drainage region in the X-direction is one of the 
important parameters of reservoir configuration. Figure 13 
indicates the effect of different sizes of the unstimulated region 
on the wellbore pressure behavior. It can be seen that in 
reservoirs with a small drainage area, the dimensionless time of 
the PSS flow starts earlier, while in those with a large drainage 
area, the pressure pulse needs more time to reach the 
boundary, which contributes to the late onset of the PSS period. Moreover, the pressure drops required for reservoir fluid to move in the outer region are higher in the small 
drainage area. The starting moment of the PSS regime to 
generate transient IPR are selected, and the curves are shown 
in Figure 14. This figure demonstrates that the productivity 
index of the small outer region is higher than that of the large 
outer region when the PSS regime starts.

5.4. Spacing between Hydraulic Factures (Ye). 
Another parameter that describes the dimensions of the 
drainage area is the hydraulic fracture spacing in the Y 
direction. Figure 15 indicates the effect of the different sizes of 
the stimulated region on the wellbore pressure behavior. Similarly, there is no difference in the beginning period until 
the pressure pulse reaches the no-flow boundary in the Y 
direction. The long-distance between fractures results in a long 
time for the whole inner region to participate in the flow and 
also leads to a delay in the PSS flow. The transient IPR at the 
start time of the PSS regime is given in Figure 16, where a 
smaller spacing corresponds to a relatively higher productivity 
index.

5.5. Permeability of the Matrix. Figure 17 shows the 
effect of different matrix permeabilities on the wellbore pressure. It can be observed that all curves show the same 
trend and shape but the absolute values differ, indicating that 
the permeability of the matrix makes no impact on the duration distribution of each flow regime. The pressure 
behavior and IPR curves in Figure 18 illuminate that an 
increase in the matrix permeability raises the pressure drop and decreases the production rate.
5.6. Skin Factor. As depicted in Figure 19, as the skin factor reduces, the dimensionless pressure drops decreases.

However, the curves converge to a stabilized value during the PSS period, meaning that this effect cannot reach the late time pressure profile. Meanwhile, different cases share the same pressure derivative plot. Thus, one can conclude that the skin factor does not affect the arrival time of each flow regime. Figure 20 indicates that the increasing skin factor contributes to a sharp IPR curve with a small productivity index.

6. CONCLUSIONS
In this paper, an analytical trilinear model was developed to investigate the complex dual-porosity system of a naturally fractured reservoir. The solution of the model was derived to analyze the transient pressure behavior, flow regimes, and IPR of multiple fractured horizontal wells. The following conclusions can be drawn:

1. The complex dual-porosity system model takes account both inner and outer natural fractures, thus providing...
more accurate results when characterizing the naturally fractured reservoirs.

2. The plots of the dimensionless transient pressure and pressure derivative can be helpful to identify different flow regimes that developed sequentially during the whole production period. Also, by inverting the constant rate results of the transient analysis to real-time units, a series of flow rates can be used in each flow regime to obtain the transient IPR curves.

3. The sensitivity analysis of different parameters is conducted to examine the effects on the pressure behavior as well as the productivity index of the wells.

4. The hydraulic fracture conductivity, mainly determined by hydraulic fracture permeability and width, plays a more pronounced role in the early period. The multiple-fractured horizontal wells with larger conductivity result in smaller pressure depletion and a larger productivity index.

5. The effects of reservoir configurations on the transient behavior are significant. The drainage distance (Xe) is investigated to influence the flow duration in the outer region, and the PSS flow occurs earlier with a decrease in the drainage distance. The hydraulic fracture spacing (Ye) controls the inner region, and the larger spacing results in a longer time for the whole inner region to participate in the flow and finally leads to a delay in the PSS flow.

6. The matrix permeability and skin factor are confirmed to have no impact on the duration of each flow regime. As the matrix permeability and skin factor increase, the pressure drop increases and the productivity index decreases.

■ APPENDIX A

Dimensionless Parameters

\[ P_{D} = \frac{2\pi k_{i}h_{f}(P_{m} - P_{o})}{\mu B} \]  
(A1)

\[ x_{D} = \frac{x}{x_{f}} \]  
(A2)

\[ y_{D} = \frac{y}{x_{f}} \]  
(A3)

\[ X_{D} = \frac{x_{e}}{x_{f}} \]  
(A4)

\[ Y_{D} = \frac{y_{e}}{x_{f}} \]  
(A5)

\[ w_{D} = \frac{w_{f}}{x_{f}} \]  
(A6)

\[ \eta = \frac{k_{i}}{(\phi\mu c)_{i}} \]  
(A7)

\[ \eta_{o} = \frac{k_{o}}{(\phi\mu c)_{o}} \]  
(A8)

\[ \eta_{p} = \frac{k_{p}}{(\phi\mu c)_{p}} \]  
(A9)

\[ \eta_{m} = \frac{k_{m}}{(\phi\mu c)_{m}} \]  
(A10)

\[ \eta_{ID} = \frac{\eta_{o}}{\eta_{i}} \]  
(A11)

\[ \eta_{ID} = \frac{\eta_{E}}{\eta_{i}} \]  
(A12)

\[ \eta_{ID} = \frac{\eta_{m}}{\eta_{i}} \]  
(A13)

\[ \omega_{o} = \frac{(\phi\mu c)_{o}}{(\phi\mu c)_{i}} \]  
(A14)

\[ \omega_{i} = \frac{(\phi\mu c)_{i}}{(\phi\mu c)_{i}} \]  
(A15)

\[ \omega_{m} = \frac{(\phi\mu c)_{m}}{(\phi\mu c)_{i}} \]  
(A16)

\[ t_{D} = \frac{\eta_{i}}{x_{f}^{2}P_{o}} \]  
(A17)

\[ (\phi\mu c)_{o} = (\phi\mu c)_{m} + (\phi\mu c)_{o} + (\phi\mu c)_{i} \]  
(A18)

\[ 1 = \omega_{m} + \omega_{o} \]  
(A19)

\[ 1 = \omega_{m} + \omega_{i} \]  
(A20)

\[ \lambda_{i} = \frac{k_{m}}{k_{i}}x_{f}^{2} \]  
(A21)

\[ \lambda_{o} = \frac{k_{m}}{k_{o}}x_{f}^{2} \]  
(A22)

■ APPENDIX B

Derivation of Solution for Pressure Drop in the Composite Dual-Porosity System

In the following part, the pressure solutions in the Laplace domain with the constant rate for the trilinear flow in a rectangular composite reservoir are derived. The multiple hydraulic fractured horizontal well extends in the naturally fractured formation, where dual-porosity media originally exist. Outer-Region System. The flow in the outer-region system is in the direction perpendicular to the inner-region system (X-direction). It is assumed that there is no flow beyond the outer drainage area.

The mathematical model in the fracture system

\[ \frac{\partial^{2}P_{o}}{\partial x^{2}} + \frac{k_{m}}{k_{o}}(P_{m} - P_{o}) = \frac{(\phi\mu c)_{o}}{\phi_{o}} \frac{\partial P_{o}}{\partial t} \]  
(B1)

In the dimensionless form, the equation is converted to

\[ \frac{\partial^{2}P_{ID}}{\partial x_{D}^{2}} + \lambda_{o}(P_{ID} - P_{o}) = \frac{1}{\eta_{ID}} \frac{\partial P_{ID}}{\partial t_{ID}} \]  
(B2)

In the Laplace domain, the equation is

\[ \frac{\partial^{2}P_{ID}}{\partial x_{D}^{2}} + \lambda_{o}(P_{ID} - P_{o}) = \frac{s}{\eta_{ID}} P_{ID} \]  
(B3)
The mathematical model in the matrix system

$$\alpha (P_m - P_o) + \frac{(\phi \mu c)_m}{k_m} \frac{\partial P_m}{\partial t} = 0$$  \hspace{1cm} (B4)

In the dimensionless form, the equation is converted to

$$\frac{(P_{mD} - P_{oD})}{\lambda_i} + \frac{1}{x_i^2 \alpha \eta_{mD}} \frac{\partial P_{mD}}{\partial t_{D}} = 0$$  \hspace{1cm} (B5)

In the Laplace domain, the equation is

$$\overline{P}_{mD} = \frac{1}{1 + \frac{s}{x_i^2 \alpha \eta_{mD}}}$$  \hspace{1cm} (B6)

Substituting eq B6 into B3, we can obtain

$$\frac{\partial^2 \overline{P}_{oD}}{\partial x^2_D} = sf(s) \overline{P}_{oD} = 0$$  \hspace{1cm} (B7)

where

$$f(s) = \frac{1}{x_i^2 \alpha \eta_{mD}} + \frac{1}{\eta_{oD}}$$

The no-flow outer-boundary condition is given by

$$\left( \frac{\partial \overline{P}_{oD}}{\partial x_D} \right)_{x = x_D} = 0$$  \hspace{1cm} (B8)

The inner-boundary condition is given by

$$\overline{P}_{oD}|_{x_D = 1} = \overline{P}_{oD}|_{x_D = 1}$$  \hspace{1cm} (B9)

The pressure solution of the outer drainage area in the Laplace domain can be obtained

$$\overline{P}_{oD} = \overline{P}_{oD}|_{x_D = 1} \frac{\cosh[\sqrt{sf(s)}(x_{oD} - x_D)]}{\cosh[\sqrt{sf(s)}(x_{oD} - 1)]}$$  \hspace{1cm} (B10)

**Inner-Region System.** The flow in the inner-region system is in the direction perpendicular to the hydraulic fracture (Y-direction). It is assumed there is a no-flow boundary between two hydraulic fractures.

The mathematical model in the fracture system

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{k_m}{k_i}(P_m - P) = \frac{(\phi \mu c)_m}{k_m} \frac{\partial P}{\partial t}$$  \hspace{1cm} (B11)

Flow rate continuity between the outer and inner regions is given by

$$k_i \left( \frac{\partial \overline{P}_{oD}}{\partial x_D} \right)_{x_D = x_i} = k_i \left( \frac{\partial \overline{P}_{oD}}{\partial x_D} \right)_{x = x_i}$$  \hspace{1cm} (B12)

Substituting in eq B11

$$\frac{1}{x_i^2} \frac{k_i}{k_i} \left( \frac{\partial \overline{P}_{oD}}{\partial x_D} \right)_{x_D = x_i} + \frac{\partial^2 \overline{P}_{oD}}{\partial y^2_D} + \frac{k_m}{k_i}(P_m - P) = \frac{(\phi \mu c)_m}{k_m} \frac{\partial P}{\partial t}$$  \hspace{1cm} (B13)

In the dimensionless form, the equation is converted to

$$\frac{k_i}{k_i} \left( \frac{\partial \overline{P}_{oD}}{\partial x_D} \right)_{x_D = 1} + \frac{\partial^2 \overline{P}_{oD}}{\partial y^2_D} + \lambda_i (P_{mD} - P_{oD}) = \frac{\partial P_{oD}}{\partial t_D}$$  \hspace{1cm} (B14)

In the Laplace domain, the equation is

$$k_i \left( \frac{\partial \tilde{P}_{oD}}{\partial x_0} \right)_{x_0 = 1} + \frac{\partial^2 \tilde{P}_{oD}}{\partial y^2_0} + \lambda_i (\tilde{P}_{mD} - \tilde{P}_{oD}) = \frac{\partial P_{oD}}{\partial t_0}$$  \hspace{1cm} (B15)

From eq B10

$$\frac{\partial \tilde{P}_{oD}}{\partial y_0} = -\sqrt{sf(s)} \frac{\partial P_{oD}}{\partial x_0} \frac{\sinh[\sqrt{sf(s)}(x_{oD} - x_D)]}{\cosh[\sqrt{sf(s)}(x_{oD} - 1)]}$$  \hspace{1cm} (B16)

Substituting in eq B15

$$\frac{\partial^2 \tilde{P}_{oD}}{\partial y^2_0} + \lambda_i (\tilde{P}_{mD} - \tilde{P}_{oD}) - \frac{k_i}{k_i} \left( \frac{\partial \tilde{P}_{oD}}{\partial x_0} \right)_{x_0 = 1} - \frac{\partial P_{oD}}{\partial t_0} = 0$$  \hspace{1cm} (B17)

where

$$\beta = \sqrt{sf(s)} \tanh[\sqrt{sf(s)}(x_{oD} - 1)]$$

The pressure of the inner region is not a function of the distance in the x-direction; eq B17 is simplified to

$$\frac{\partial^2 \tilde{P}_{oD}}{\partial y^2_0} + \lambda_i (\tilde{P}_{mD} - \tilde{P}_{oD}) - \theta \tilde{P}_{oD} = 0$$  \hspace{1cm} (B18)

where

$$\theta = \frac{\lambda_i s}{k_i} + \frac{s}{x_i^2 \alpha \eta_{oD}}$$

The mathematical model in the matrix system

$$\alpha (P_m - P_o) + \frac{(\phi \mu c)_m}{k_m} \frac{\partial P_m}{\partial t} = 0$$  \hspace{1cm} (B19)

In the dimensionless form, the equation is converted to

$$\frac{(P_{mD} - P_{oD})}{\lambda_i} + \frac{1}{x_i^2 \alpha \eta_{mD}} \frac{\partial P_{mD}}{\partial t_{D}} = 0$$  \hspace{1cm} (B20)

In the Laplace domain, the equation is

$$\overline{P}_{mD} = \frac{1}{1 + \frac{s}{x_i^2 \alpha \eta_{mD}}}$$  \hspace{1cm} (B21)

Substituting eq. B21 into B18, we can obtain

$$\frac{\partial^2 \overline{P}_{oD}}{\partial y^2} - \epsilon \overline{P}_{oD} = 0$$  \hspace{1cm} (B22)

where

$$\epsilon = \frac{\lambda_i \sqrt{s}}{s + x_i^2 \alpha \eta_{oD}}$$

The no-flow boundary condition is given by

$$\left( \frac{\partial \overline{P}_{oD}}{\partial y_D} \right)_{y_D = y_D} = 0$$  \hspace{1cm} (B23)

The inner-boundary condition is given by

$$\overline{P}_{oD}(x_D = 0) = \overline{P}_{oD}(x_D = x_D)$$  \hspace{1cm} (B24)

The pressure solution of the inner drainage area in the Laplace domain can be obtained
Hydraulic fracture system

The flow in the outer-region system is in the direction perpendicular to the inner-region system (X-direction). It is assumed that there is no flow beyond the outer drainage area.

The mathematical model

\[
\frac{\partial^2 P_k}{\partial x^2} + \frac{\partial^2 P_k}{\partial y^2} + \frac{(\mu c) \beta}{k_F} \frac{\partial P_k}{\partial t} = f(y)
\]

Integrating both sides of eq B26 as follows

\[
\int_0^{W/2} \frac{\partial^2 P_k}{\partial x^2} dy + \int_0^{W/2} \frac{\partial^2 P_k}{\partial y^2} dy = \frac{(\mu c) \beta}{k_F} \int_0^{W/2} \frac{\partial P_k}{\partial t} dy
\]

using the following assumptions

\[
\frac{\partial P_k}{\partial x} \neq f(y)
\]

\[
\frac{\partial P_k}{\partial t} \neq f(y)
\]

results in

\[
\frac{2}{W} \frac{\partial P_k}{\partial y} \bigg|_{y=W/2} + \frac{\partial^2 P_k}{\partial x^2} = \frac{(\mu c) \beta}{k_F} \frac{\partial P_k}{\partial t}
\]

Flux continuity between the inner region and hydraulic fracture is given by

\[
k_i \left( \frac{\partial P_k}{\partial y} \right)_{y=W/2} = k_h \left( \frac{\partial P_k}{\partial y} \right)_{y=W/2}
\]

Substituting in eq B30

\[
\frac{\partial^2 P_k}{\partial x^2} + \frac{2}{W} \frac{k_i}{k_p} \left( \frac{\partial P_k}{\partial y} \right)_{y=W/2} = \frac{(\mu c) \beta}{k_F} \frac{\partial P_k}{\partial t}
\]

In the dimensionless form, the equation is converted to

\[
\frac{\partial^2 P_{kD}}{\partial x_{D}^2} + \frac{2}{W} \frac{k_i}{k_p} \left( \frac{\partial P_{kD}}{\partial y_{D}} \right)_{y_{D}=W/2} = \frac{1}{\eta_{FD}} \frac{\partial P_{FD}}{\partial t_{FD}}
\]

In the Laplace domain, the equation is

\[
\frac{\partial^2 P_{kD}}{\partial x_{D}^2} + \frac{2}{W} \frac{k_i}{k_p} \left( \frac{\partial P_{kD}}{\partial y_{D}} \right)_{y_{D}=W/2} = \frac{s}{\eta_{FD}} P_{FD}
\]

From eq B25

\[
\left( \frac{\partial P_{kD}}{\partial t_{FD}} \right)_{y_{D}=W/2} = -P_{FD} \sqrt{\epsilon} \tanh(\sqrt{\epsilon}(y_{D} - w_D/2))
\]

Substituting in eq B34

\[
\frac{\partial^2 P_{FD}}{\partial x_{D}^2} + \frac{2}{W} \frac{k_i}{k_p} \left( \frac{\partial P_{FD}}{\partial y_{D}} \right)_{y_{D}=W/2} = \frac{s}{\eta_{FD}} P_{FD}
\]

The pressure of the hydraulic fracture is not a function of the distance in the y-direction; eq B36 is simplified to

\[
\frac{\partial^2 P_{FD}}{\partial x_{D}^2} - \theta_F P_{FD} = 0
\]

where

\[
\theta_F = \frac{2\beta_F k_i}{w_D k_F} + \frac{s}{\eta_{FD}}
\]

The no-flow outer boundary condition is given by

\[
\left( \frac{\partial P_{FD}}{\partial x_{D}} \right)_{x_{D}=0} = 0
\]

The inner-wellbore condition is given by

\[
\left( \frac{\partial P_{FD}}{\partial x_{D}} \right)_{x_{D}=w_D} = \frac{\pi}{sC_F D B}
\]

Finally, we obtain the bottom hole wellbore pressure in the Laplace domain:

\[
P_{w_D} = \left( P_{FD} \right)_{x_{D}=0} = \frac{\pi}{sC_F D B} \sqrt{\epsilon} \tanh(\sqrt{\epsilon} D)
\]

AUTHOR INFORMATION

Corresponding Author

Huizhu Xiang — State Key Laboratory of Petroleum Resources and Prospecting, China University of Petroleum-Beijing, Beijing 102249, China; orcid.org/0000-0001-6802-4939; Email: x51756732@163.com

Authors

Guoqing Han — State Key Laboratory of Petroleum Resources and Prospecting, China University of Petroleum-Beijing, Beijing 102249, China

Gaoqiang Ma — State Key Laboratory of Petroleum Resources and Prospecting, China University of Petroleum-Beijing, Beijing 102249, China

Zhiyong Zhu — State Key Laboratory of Petroleum Resources and Prospecting, China University of Petroleum-Beijing, Beijing 102249, China

Liyang Zhu — State Key Laboratory of Petroleum Resources and Prospecting, China University of Petroleum-Beijing, Beijing 102249, China

Long Peng — State Key Laboratory of Petroleum Resources and Prospecting, China University of Petroleum-Beijing, Beijing 102249, China; orcid.org/0000-0001-9654-953X

Notes

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**NOMENCLATURES**

- \( P \): pressure, MPa
- \( P_{Dy} \): pressure in Laplace domain, dimensionless
- \( Q \): flow rate, m\(^3\)/D
- \( \dot{Q}_{Dy} \): flow rate in the Laplace domain, dimensionless
- \( \alpha \): x-coordinate, m
- \( y \): y-coordinate, m
- \( x_0 \): fracture half-length, m
- \( w_f \): hydraulic-fracture width, m
- \( X_m \): reservoir boundary, m
- \( Y_m \): fracture half spacing, m
- \( \lambda \): matrix-fracture transfer parameter
- \( s \): Laplace operator
- \( \eta \): reservoir diffusivity ratio
- \( B \): oil formation volume factor, m\(^3\)/m\(^3\)
- \( c \): compressibility, MPa\(^{-1}\)
- \( h \): formation thickness, m
- \( k \): permeability, mD
- \( r_w \): wellbore radius, m
- \( t \): time, s
- \( \mu \): viscosity, cp
- \( \alpha \): matrix shape factor, m\(^{-2}\)
- \( \phi \): porosity, dimensionless
- \( \omega \): storage ratio, dimensionless

**SUBSCRIPTS**

- \( i \): inner region (SRV)
- \( o \): outer region (USRV)
- \( e \): external boundary
- \( f,F \): hydraulic fracture
- \( m \): matrix
- \( nf \): natural fracture
- \( w,f \): wellbore flow
- \( t \): total

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