Absorption Cross Section for S-wave massive Scalar

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Abstract

We examine the absorption cross section of the massive scalar field for the higher-dimensional extended object. Adopting the usual quantum mechanical matching conditions between the asymptotic and near-horizon solutions in radial equation, we check whether or not the universal property of the absorption cross section, which is that the low-energy cross section is proportional to the surface area of horizon, is maintained when the mass effect is involved. It is found that the mass effect in general does not break the universal property of the cross section if particular conditions are required to the spacetime geometry. However, the mass-dependence of the cross section is very sensitive to the spacetime property in the near-horizon regime.

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It is well-known that the low-energy absorption cross section of the massless particle for the black hole is proportional to the surface area of the horizon [1–6]. This universal property is shown to be maintained for the higher-dimensional objects such as extremal strings and black p-brane [4]. Furthermore, adopting the usual quantum mechanical matching conditions the authors in Ref. [5] have argued that the genuine physical reason for the occurrence of the universal property is the independence of the matching point between the asymptotic and near-horizon solutions.

In this letter we will examine whether the universal property of the low-energy absorption cross section is maintained or not in the higher-dimensional extended system when the mass effect of the scalar field is involved. In order to proceed we will adopt the quantum mechanical matching conditions

\[ \phi_\omega^\infty(R) = \phi_\omega^0(R) \]
\[ \frac{d}{dR}\phi_\omega^\infty(R) = \frac{d}{dR}\phi_\omega^0(R) \]

which is introduced in Ref. [5]. In Eq.(1) \( \phi_\omega^\infty \) and \( \phi_\omega^0 \) are the asymptotic and near-horizon solutions respectively of the massive scalar field and thus, Eq.(1) is the matching between them at arbitrary location \( r = R \).

We will follow Ref. [5] to check the universality when the scalar field is massive. The mass effect in the scalar field requires an explicit \( r \)-dependence of the time-time component of the metric for the derivation of \( \phi_\omega^0 \). (See Eq.(12)) The mass-dependence of the absorption cross section is very sensitive to the \( r \)-dependence of the metric. In spite of it the mass of the scalar particle does not break the universal property of the absorption cross section if particular conditions are required.

The spacetime generated by the higher-dimensional object is assumed to be

\[ ds^2 = \gamma_{\mu\nu}dx^\mu dx^\nu + f(r)dr^2 + r^2h(r)d\Omega_{n+1} \]

through this letter, where \( \mu, \nu = 0, 1, \cdots p \). Let us consider the minimally coupled massive scalar in this background, which should satisfy
\[(\Delta A \Delta^A - m^2)\Phi = 0. \quad (3)\]

If we assume \(\Phi = e^{-i\omega t} e^{i\omega r} \phi(r)\) which is valid for the low-energy s-wave, and introduce a “tortose” coordinate \(r_*\) as following,

\[
d r_* = dr \sqrt{-\gamma^{tt} f(r)}, \quad (4)
\]

one can show straightforwardly that Eq.(3) reduces to the following Schrödinger-like equation

\[
\left[-\frac{d^2}{dr_*^2} + V(r)\right] \psi = \omega^2 \psi \quad (5)
\]

where \(\gamma(r) \equiv \text{det} \gamma_{\mu\nu}\) and

\[
\psi(r) = \sqrt{U} \phi(r), \quad U(r) = \sqrt{\gamma^{tt} \{r^2 h(r)\}^{n+1}} \quad (6)
\]

\[V(r) = V_0(r) - \frac{m^2}{\gamma^{tt}} \quad V_0(r) = \frac{1}{\sqrt{U}} \frac{d^2 \sqrt{U}}{(dr_*)^2}.\]

Usually the “tortose” coordinate \(r_*\) goes to \(\pm \infty\) in the asymptotic and near-horizon regions of \(r\) and the potential \(V_0(r)\) makes a barrier which separates these two regions. If, for example, we consider the usual 4d Schwarzschild spacetime by choosing \(f(r) = -\gamma^{tt} = (1 - r_H/r)^{-1}\), the “tortose” coordinate \(r_*\) and the potential \(V_0(r)\) reduce to \(r_* = r + r_H \ln(r - r_H)\) and \(V_0(r) = (r_H/r^3)(1 - r_H/r)\). Fig. 1 shows \(r_*\)-dependence of \(V_0(r)\) in this simple example when \(r_H = 1\). From Fig. 1 we understand that \(V_0(r)\) makes a barrier between the asymptotic and near-horizon regions.

We will solve Eq.(5) in the asymptotic region \((r \sim \infty)\) and near-horizon region \((r \sim 0)\) separately. Matching them using Eq.(1), we will derive the absorption cross section. Firstly, let us consider Eq.(5) in the asymptotic region with assumption that the geometry is asymptotic flat for simplicity;

\[
\lim_{r \to \infty} \gamma_{\mu\nu} = \eta_{\mu\nu}, \quad (7)
\]

\[
\lim_{r \to \infty} f(r) = \lim_{r \to \infty} h(r) = 1.
\]

Using Eq.(7), Eq.(5) reduces to
\[ \frac{d^2 u_\infty}{dr^2} + \frac{1}{r} \frac{du_\infty}{dr} + \left( \omega^2 - m^2 - \frac{n^2}{4r^2} \right) u_\infty = 0 \]  
\quad (8)

in this region where \( u_\infty \equiv \psi_\infty / \sqrt{r} \). Here the subscript denotes the region we consider for the solution of Eq.(5). Eq.(8) is easily solved in terms of Bessel function and hence we can derive

\[ \phi_\omega \equiv \sqrt{\frac{r}{U}} u_\infty = \frac{1}{(\omega vr)^{\frac{n}{2}}} \left[ AJ_{\frac{n}{2}}(\omega vr) + BJ_{-\frac{n}{2}}(\omega vr) \right] \]  
\quad (9)

where \( v = \sqrt{1 - m^2/\omega^2} \).

Now we define the flux of the massive scalar as

\[ \mathcal{F} = \frac{1}{2i} \frac{U}{\sqrt{-\gamma^{tt} f(r)}} \left( \frac{d\phi_\omega}{dr} - \phi_\omega \frac{d\phi_\omega^*}{dr} \right) \]  
\quad (10)

Inserting (9) into Eq.(10) yields the incoming flux in the form;

\[ \mathcal{F}_{in}^\infty = \frac{-1}{2\pi(\omega vr)^n} \left[ |A|^2 + |B|^2 + A^* B e^{-i\frac{n}{2}\pi} + A B^* e^{i\frac{n}{2}\pi} \right]. \]  
\quad (11)

When deriving Eq.(11) we used only the incoming wave, \textit{i.e.} \( e^{-i\omega vr} \), in the asymptotic formula of Bessel function.

Now we will solve Eq.(5) in the near-horizon region. Following Ref. [4,5] we take a following assumption

\[ \lim_{r \to 0} U \sim S r^{a-b} \]  
\[ \lim_{r \to 0} \sqrt{-\gamma^{tt} f} \sim T r^{b+1} \]  
\[ \lim_{r \to 0} \gamma^{tt} \sim \frac{W}{r^{2c}} \]  
\quad (12)

where \( S, T \) and \( W \) are some constant parameters. Especially, the parameter \( S \) is proportional to the area of the absorption hypersurface\(^1\). Thus, the universality means that the absorption cross section for the low-energy massless particle is proportional to the parameter \( S \). For example, the low-energy cross section for the massless scalar particle is found to be \( \sigma_L = \)

\(^1\)In the black hole spacetime this is same with area of the horizon surface.
\( \Omega_{n+1} S [4,5] \) when \( a = b \), where \( \Omega_{n+1} \) is an surface area of \( S^{n+1} \), which exactly coincides with the area of the absorption hypersurface.

Making use of Eq.(12) we can transform Eq.(5) into

\[
\frac{d^2 \chi}{dy^2} + \frac{1}{y} \frac{d \chi}{dy} + \left[ 1 + V_1(y) + V_2(y) \right] \chi = 0 \tag{13}
\]

in the \( r \sim 0 \) region, where

\[
y = \frac{\omega T}{br} \quad \chi(r) = r^\frac{\nu}{2} \psi_0 \tag{14}
\]

\[
V_1(y) = -\frac{m^2}{W \omega^2} \left( \frac{\omega^2 T^2}{b^2 y^2} \right)^\frac{\nu}{2} \quad V_2(y) = -\frac{a^2}{4b^2} \frac{1}{y^2}.
\]

It seems to be impossible to solve Eq.(13) analytically if both of \( V_1(y) \) and \( V_2(y) \) are present. Thus we should take an approximation for the analytical approach.

If \( 0 < b < c \), \( V_2(y) \) is much greater than \( V_1(y) \), which makes \( \chi \) to be proportional to \( H^{(2)}_{\nu/2}(y) \) where \( H^{(2)}_\nu \) is usual Hankel function. Thus we can take a solution as

\[
\phi_\omega^0 = \frac{1}{(\omega r)^\frac{\nu}{2}} H^{(2)}_{\frac{\nu}{2}} \left( \frac{\omega T}{br} \right) \tag{15}
\]

in this region.

If \( b = c \), \( V_1(y) \) and \( V_2(y) \) are almost same order, which results in

\[
\phi_\omega^0 = \frac{1}{(\omega r)^\frac{\nu}{2}} H^{(2)}_{\nu} \left( \frac{\omega T}{br} \right) \tag{16}
\]

where

\[
\nu = \sqrt{\frac{a^2}{4b^2} + \frac{m^2 T^2}{WB^2}}. \tag{17}
\]

If \( 0 < c < b \), \( V_1(y) \) is a dominant term in the potential. In this case the solution of Eq.(13) cannot be solved in general. If \( b = 2c \), however, we can solve Eq.(13), which results in

\[
\phi_\omega^0 = \frac{1}{(\omega r)^\frac{\nu}{2}} \left[ F_\ell \left( \eta, \frac{\omega T}{br} \right) + i G_\ell \left( \eta, \frac{\omega T}{br} \right) \right] \tag{18}
\]

where \( F_\ell(\eta, z) \) and \( G_\ell(\eta, z) \) are the Coulomb wave functions and
\[
\eta = \frac{m^2 T}{2W_\omega b}, \quad \ell = \frac{a - b}{2b},
\] (19)

In Eq.(18) the coefficients of the Coulomb wave functions are chosen from a condition that we have a pure incoming wave at \( r \sim 0 \) region. In the following we will compute the low-energy absorption cross section for \( b < c, b = c, \) and \( b = 2c \) separately.

At \( 0 < b < c, \) the solution in near-horizon region is Eq.(15). Then it is easy to show that the incident flux for \( \phi_\omega^0 \) is

\[
F = \frac{1}{2i \sqrt{-\gamma^a}} \int \left( \phi_\omega^{0*} \frac{d\phi_\omega^0}{dr} - \phi_\omega^0 \frac{d\phi_\omega^{0*}}{dr} \right) = \frac{2bS}{\pi \omega^a T}.
\] (20)

Thus the absorption cross section defined as

\[
\sigma \equiv \frac{(2\pi)^{n+1} \Omega_{n+1}^{1/2}}{\omega^n T \Omega_{n+1}} \frac{|F_{\omega}^{in}|^2}{|F_{\omega}^{in}|^2}
\] (21)

becomes

\[
\sigma = \frac{4(2\pi)^{n+1}bS^{n+1}}{\omega^{a+1}T \Omega_{n+1}^{1/2}} \frac{1}{|A|^2 + |B|^2 + A^*Be^{-i\pi/2} + AB^*e^{i\pi/2}}
\] (22)

where \( \Omega_{n+1} \) is surface area of \( S_{n+1}, \) i.e. \( \Omega_{n+1} = 2\pi^{1+n/2}/\Gamma(1+n/2). \)

Now let us consider the matching between \( \phi_\omega^\infty \) and \( \phi_\omega^0. \) In Ref. [4] author uses

\[
\lim_{r \to 0} \phi_\omega^\infty = \lim_{r \to \infty} \phi_\omega^0.
\] (23)

This condition requires implicitly the assumption that there exists an intermediate region where \( \phi_\omega^0 \) and \( \phi_\omega^\infty \) can be matched. However, it is not clear at least for us to take this assumption \textit{ab initio}.

Instead of this the authors in Ref. [5] took Eq.(1) as matching conditions. Thus we do not need to assume the existence of the intermediate region from the beginning. If we solve Eq.(1) with the asymptotic solution (9) and the near-horizon solution (15), the coefficients \( A \) and \( B \) become

\[
A = (-1)^{\frac{n+1}{2}} \frac{\pi (\omega R)^{\frac{n-a}{2}} v^\frac{n}{2}}{2} \left[ -\frac{n + a}{2} J_{-\frac{n}{2}}(\omega v R) H^{(2)}_\frac{n}{2} \left( \frac{\omega T}{b R^8} \right) \right]
\] (24)
where the prime denotes the differentiation with respect to the argument. Inserting Eq. (24) into Eq. (22) one can compute the absorption cross section $\sigma^{(1)}$ straightforwardly.

In order to show that the low-energy cross section is independent of the matching point, we plot the $\omega$-dependence of $\sigma^{(1)}$ in Fig. 2, which indicates that in $\omega \sim m$ region $\sigma^{(1)}$ is independent of $R$.\(^2\) Thus the low-energy universality seems to be maintained when $0 < b < c$ if $m$ is not too large. To show this more explicitly, we compute the coefficients $A$ and $B$ in the low energy limit using the asymptotic formulae of Bessel and Hankel functions which results in

$$A = \frac{i}{\pi} \frac{2^n \pi}{2} \Gamma \left( \frac{a}{2b} \right) \Gamma \left( 1 + \frac{n}{2} \right) \left( \frac{2b}{T \omega_b^{b+1}} \right) \frac{\omega_T}{bR^b} \left[ \frac{-n + a}{2} J_{\frac{a}{2}}(\omega v R) H^{(2)}_{\frac{a}{2}}(\omega v R) + \frac{\omega T}{b R^b} J_{\frac{a}{2}}(\omega v R) H^{(2)}_{\frac{a}{2}}(\omega v R) \right]$$

$$B = (-1)^{n-1} \frac{\pi (\omega R)^{n-\frac{1}{2} v^n}}{2} \left[ \frac{-n + a}{2} J_{\frac{a}{2}}(\omega v R) H^{(2)}_{\frac{a}{2}}(\omega v R) + \frac{\omega T}{b R^b} J_{\frac{a}{2}}(\omega v R) H^{(2)}_{\frac{a}{2}}(\omega v R) \right]$$

at the leading order. It is worthwhile noting that the $R$-dependence disappears in $A$ and $B$ as indicated before as a physical origin of the universality in the low-energy cross-section. Computing the low-energy cross section by making use of Eq. (25), one can obtain easily

$$\sigma^{(1)}_L = \frac{\pi}{\Gamma^2 \left( \frac{a}{2b} \right)} \Omega_{n+1} S \left( \frac{\omega_T}{2b} \right)^{\frac{b-1}{2}} v^n$$

where $L$ in subscript stands for low-energy limit. If $a = b$, $\sigma^{(1)}_L$ becomes simply

$$\sigma^{(1)}_L = \Omega_{n+1} S v^n$$

which indicates that the mass in scalar particle decreases the absorption cross section. A similar decreasing behavior of the absorption cross section with respect to $m$ was also found by Unruh [7].

\(^2\)In order to apply our low-energy formulation we should require that the mass of the scalar particle is not large.
The authors in Ref. [5] applied the matching condition (1) to the case of the fixed scalar [8], where the low-energy absorption cross section does not obey the universality. The authors in Ref. [8] computed the low-energy cross section $\sigma_s$ by matching $\phi_0^0$ and $\phi_\infty^\infty$ through the solution in the intermediate region as Unruh did in his seminal paper [7] and obtained $\sigma_s = 2\pi\omega^2$. If one uses, however, the matching condition (1), one gets $\sigma_s = 2\pi\omega^2 R^2/(R-1)^2$. The explicit $R$-dependence indicates the non-universality, but its $\omega$-dependence is correct. This result may give us a confidence to use (1) to examine the $\omega$-dependence of the absorption cross section in the high-energy limit.

If one takes, for example, $\omega \to \infty$ limit in the coefficients $A$ and $B$ of Eq.(24), it is easy to show that these coefficients are explicitly dependent on the matching point $R$. Then, it is easy to show that the high-energy absorption cross section becomes

$$\sigma_H^{(1)} = \frac{(2\pi/\omega)^{n+1}}{\Omega_{n+1} R^{n-a}} \frac{4S/T}{\left[ \sqrt{\frac{4R^2}{T} - \sqrt{\frac{T^{a+1}}{vR^{b+1}}} \right]^2}. \quad (28)$$

The appearance of $R$ in Eq.(28) indicates that the high energy cross section loses the universality property. However, the $\omega$-dependence of $\sigma_H^{(1)}$, i.e. $\sigma_H^{(1)} \propto \omega^{-(n+1)}$ exhibits a decreasing behavior. A similar decreasing behavior was shown in Ref. [9] by adopting a numerical methods and in Ref. [10,11] by analyzing a modified Mathieu equation.

Now let us consider the case of $b = c$. In this case the asymptotic and near-horizon solutions are (9) and (16) respectively. The difference of order of Hankel function in (16) from (15) for $0 < b < c$ does not change the incoming flux of the near-horizon region because the order of Hankel function is only involved as a phase factor in the asymptotic formula. Thus, the low-energy absorption cross section has a same expression with Eq.(22). The difference of the low-energy cross section, however, from that for $0 < b < c$ case arises due to the matching between the asymptotic and near-horizon solutions. Applying the matching condition (1) we obtain

$$A = (-1)^{n+1} \frac{\pi (\omega R)^{n+1}}{2} v^{\frac{n}{2}} \left[ \frac{n+a}{2} J_{-\frac{n}{2}} (\omega v R) H_\nu^{(2)} \left( \frac{\omega T}{bR^b} \right) + \omega v R J_{-\frac{n}{2}} (\omega v R) H_\nu^{(2)} \left( \frac{\omega T}{bR^b} \right) \right] \quad (29)$$
\[ B = (-1)^{\frac{n-1}{2}} \pi (\frac{\omega R}{2})^{\frac{n-a}{2}} v^\frac{a}{2} \left[ \frac{-n + a}{2} J_\frac{a}{2} (\omega v R) H^{(2)}_{\nu} \left( \frac{\omega T}{b R^b} \right) + \omega v R J_\frac{a}{2} (\omega v R) H^{(2)}_{\nu} \left( \frac{\omega T}{b R^b} \right) \right]. \]

If we take \( \omega \to m \sim 0 \) limit in Eq.(29), it is easy to show that the coefficients \( A \) and \( B \) are explicitly dependent on \( R \) unlike massless and \( 0 < b < c \) cases. This means the low-energy cross section for \( b = c \) does not maintain the universality. The only way to keep the \( R \)-independence we should require the additional conditions

\[ W = -\frac{m^2 T^2}{n(a - n)} \quad a \geq 2n. \]  

If \( T \) is a real parameter, these additional conditions seem to change the Lorentz signature in the near-horizon region and thus may generate a serious causal problem. We guess this problem may be originated from the matching between the solutions whose valid regions are too distant. If our guess is right, the problem may be cured by introducing the intermediate region between near-horizon and asymptotic regions as Unruh did in Ref. [7]. This issue seems to need a careful treatment and we hope to discuss it in the future. In this paper we will not go further this causal problem.

If one takes \( \omega \to 0 \) limit in Eq.(29) with making use of Eq.(30), the coefficients \( A \) and \( B \) become

\[ A = 0 \quad B = \frac{(-1)^{\frac{n-1}{2}}}{2^\frac{a}{2} \pi^\frac{n}{2} - n} \left( \frac{2b}{\omega T} \right)^{\frac{a - 2n}{2b}} i \Gamma \left( \frac{n}{2} \right) \frac{\Gamma \left( \frac{a - 2n}{2b} \right)}{\Gamma \left( \frac{n}{2} \right)} v^n. \]  

Thus the coefficients \( A \) and \( B \) are independent of the matching point \( R \) as expected. Inserting Eq.(31) into Eq.(22) makes the low-energy cross section to be

\[ \sigma_L^{(2)} = \left( \frac{\omega T}{2b} \right)^{\frac{a - 2n}{2b}} 2^{2n+1} \pi^\frac{a}{2} n b S \frac{\Gamma^3 \left( \frac{n}{2} \right)}{\Gamma^2 \left( \frac{a - 2n}{2b} \right)} v^{-n}. \]  

It is interesting to note that \( \sigma_L^{(2)} \) is proportional to \( v^{-n} \) while \( \sigma_L^{(1)} \) in Eq.(26) is proportional to \( v^n \). This inverse power makes \( \sigma_L^{(2)} \) to exhibit an increasing behavior with respect to mass \( m \) unlike \( \sigma_L^{(1)} \).
If one takes a $\omega \to \infty$ limit in Eq. (29), one can compute the high-energy cross-section $\sigma_{H}^{(2)}$ for $b = c$ case. Although $\omega \to \infty$ limit of the coefficients $A$ and $B$ are different from those in $b < c$, this difference is only phase factor in leading order and therefore does not change the high-energy cross section, i.e. $\sigma_{H}^{(2)} = \sigma_{H}^{(1)}$. Thus the universality is not maintained in this case too, and the $\omega$-dependence is $\sigma_{H}^{(2)} \propto \omega^{-(n+1)}$.

Now let us discuss the absorption cross section when $b = 2c$. If one matches the asymptotic solution (9) and the near-horizon solution (18) at low energy, one can show straightforwardly the coefficients $A$ and $B$ become

$$A = i2^{\frac{n}{2}} \Gamma \left( 1 + \frac{n}{2} \right) D_{\ell}(\eta) \left( \frac{\omega^{b+1}T}{b} \right)^{-\frac{n+1}{2b}} B = 0$$

where

$$D_{\ell}(\eta) = \frac{\Gamma(2\ell + 1)}{2^{\ell + 1} \eta |\Gamma(\ell + 1 + i\eta)|}.$$  

Inserting (33) and $F_{0}^{m} = S/\omega^{a-b-1}$, which can be computed after tedious calculation with making use of the asymptotic formula of the Coulomb wave function [12], into Eq. (21), one can derive the following low-energy absorption cross section

$$\sigma_{L}^{(3)} = \left( \frac{\Gamma \left( \frac{a}{2b} \right)}{\sqrt{\pi} \omega^{a-b} \Gamma \left( 1 + \frac{a}{b} \right)} \right)^{2} \left( \frac{4}{\omega^{2T^{2}}} \right)^{\frac{n}{2}-1} e^{-\pi\eta} |\Gamma \left( \frac{a+b}{2b} + i\eta \right)|^{2} \sigma_{L}^{(1)}$$

where $\sigma_{L}^{(1)}$ is given in Eq. (26). Using formulae of the gamma function [12], the final form of $\sigma_{L}^{(3)}$ reduces to

$$\sigma_{L}^{(3)} = \left( \frac{b}{a\omega^{a-b}} \right)^{2} \left( \frac{1}{\omega^{2T^{2}}} \right)^{\frac{n}{2}-1} e^{-\pi\eta/2} \sigma_{L}^{(1)} \prod_{n=0}^{\infty} \left[ 1 + \left( \frac{\eta}{\omega^{2} + n + \frac{1}{2}} \right)^{2} \right]^{-1}.$$  

Roughly speaking, therefore, we can say that the low-energy absorption cross section at $b = 2c$ is a multiplication of some damping factor to the low-energy cross section at $b < c$. This can be seen more clearly if $a = b$, where $\sigma_{L}^{(3)}$ simply reduces to

$$\sigma_{L}^{(3)} = \sigma_{L}^{(1)} \frac{\pi\eta e^{-\pi\eta}}{\sinh \pi\eta}$$
where $\sigma_{L}^{(1)} = \Omega_{n+1} S_{V}^{n}$. Thus if $\pi \eta << 1$ or $\pi \eta >> 1$, the damping factor becomes roughly $1 - \pi \eta$ or $2(\pi \eta)e^{-2\pi \eta}$ respectively.

The universal property of the low-energy absorption cross section is examined when the mass effect is involved. Taking an assumption (12) in the near-horizon region we have shown that the universal property is maintained when $0 < b < c$. If, however, $b = c$, the universal property is in general broken unless Eq.(30) is imposed. At $b = 2c$ the universal property is maintained and the final expression of the absorption cross section is a multiplication of some damping factor to the cross section at $b < c$.

We guess the peculiar behavior at $b = c$ is originated from the consideration of S-wave approximation. To go beyond S-wave approximation we should involve an angle-dependent term which generates an additional effective potential in the radial equation (5). This effective potential generally enables us to make an intermediate region between asymptotic and near-horizon regions, where the effective potential is dominant compared to other factors in the potential. Matching $\phi_{0}$ and $\phi_{\infty}$ via the solution in the intermediate region may remove the peculiar behavior at $b = c$.

Another interesting work in this issue is to go beyond the low-energy approximation. For the case of the 4d Schwarzschild black hole the absorption cross section in the entire range of $\omega$ is computed in Ref. [13,14] using a series solutions in the near-horizon and the asymptotic regions [15]. The most striking result when we study the absorption and emission problems of black hole from the viewpoint of the scattering theories is that the partial scattering amplitude loses its unitary property, which is closely related to the information loss. Thus one can apply the computational method of Ref. [13,14] to the higher-dimensional theories to go beyond the low-energy approximation. Work in this direction was done recently in Ref. [16,17]. In this way it may be possible to check our guess for the high-energy limit of the absorption cross section.

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FIGURES

FIG. 1. The plot of potential $V_0$ in terms of the “tortose” coordinate. Usually the potential $V_0$ makes a barrier which separates the asymptotic and near-horizon regions.

FIG. 2. Plot of $\sigma^{(1)}_L$ vs $\omega$ with various matching points when $m = 0.01$, and $n = a = b = T = S = 1$. This figure indicates the low-energy absorption cross section is independent of the matching point, which is the origin of universality.
Fig. 1

near-horizon region

asymptotic region
Fig. 2

\[ R = 5 \]

\[ \sigma_L^{(1)} \]

\[ \omega \]

\[ 0 \]

\[ 0.0 \]

\[ 0.1 \]

\[ 0.2 \]

\[ 0.3 \]

\[ 0.4 \]

\[ 0.5 \]