Three-Fluid Simulations of Relativistic Heavy-Ion Collisions

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Abstract — A relativistic 3-fluid 3D hydrodynamic model has been developed for describing heavy-ion collisions at incident energies between few and $\sim 200$ A-GeV. In addition to two baryon-rich fluids which simulate mutually decelerating counterflows of target and projectile nucleons, the new model incorporates evolution of a third, retarded baryon-free fluid created by this decelerated baryonic matter. Different equations of state, including those with the deconfinement phase transition, are treated. A reasonable agreement with experiment is illustrated by proton rapidity spectra, their dependence on collision centrality and beam energy.

1 Introduction

During past twenty years, hydrodynamics proved to be quite a reasonable tool for describing heavy-ion collisions at moderate energies (see for example references in [1]). From general point of view this application of hydrodynamics seemed to be more successful at higher energies when the number of produced particles gets larger. However, as it has been noted by D.I. Blokhintsev\textsuperscript{2} in the beginning of the multiple-production era, the uncertainty principle introduces certain restrictions to this extension. Though a general conclusion of Ref. [2], that the very beginning of the expansion stage should be considered quantum–mechanically, is valid till now, the modern quark–gluon picture of interactions shifts the results of Blokhintsev’s simplified estimate to far ultra-relativistic energies. On other hand, it became clear that non-equilibrium processes are very important at this stage and that only a part of the total
collision energy can be thermalized. In this respect the standard one-fluid description assuming instantaneous local equilibrium seems to be quite limited.

In the present paper we demonstrate first results of a relativistic 3-fluid 3-dimensional hydrodynamic code developed for describing highly relativistic nucleus–nucleus collisions, i.e. up to energies reached at the CERN SPS where a partial deconfinement of hadrons may occur. Below, basic features of the hydrodynamic model will be presented and a numerical solution of 3-fluid hydrodynamics will first be compared with observables.

2 3-Fluid Hydrodynamic Model

A specific feature of the dynamic 3-fluid description is a finite stopping power resulting in a counter-streaming regime of leading baryon-rich matter. This counter-streaming behavior is supported by experimental rapidity distributions in nucleus–nucleus collisions and simulated by introducing the multi-fluid concept. The basic idea of a 3-fluid approximation to heavy-ion collisions [3, 4] is that at each space-time point \( x = (t, \mathbf{x}) \) the distribution function of baryon-rich matter, \( f_b(x, p) \), can be represented by a sum of two distinct contributions

\[
f_b(x, p) = f_p(x, p) + f_t(x, p),
\]

initially associated with constituent nucleons of the projectile (p) and target (t) nuclei. In addition, newly produced particles, populating the mid-rapidity region, are associated with a fireball (f) fluid described by the distribution function \( f_f(x, p) \). Note that both the baryon-rich and fireball fluids may consist of any type of hadrons and/or partons (quarks and gluons), rather than only nucleons and pions. However, here and below we suppress the species label at the distribution functions for the sake of transparency of the equations.

Using the standard procedure for deriving hydrodynamic equations from the coupled set of relativistic Boltzmann equations with the above-introduced distribution functions \( f_\alpha (\alpha = p, t, f) \), we arrive at equations for the baryon charge conservation

\[
\partial_\mu J^\mu_\alpha (x) = 0,
\]

( for \( \alpha = p \) and \( t \) ) and the energy–momentum conservation of the fluids

\[
\partial_\mu T^{\mu\nu}_p (x) = -F^{\nu}_p (x) + F^{\nu}_{fp} (x),
\]

\[
\partial_\mu T^{\mu\nu}_t (x) = -F^{\nu}_t (x) + F^{\nu}_{ft} (x),
\]

\[
\partial_\mu T^{\mu\nu}_f (x) = F^{\nu}_p (x) + F^{\nu}_t (x) - F^{\nu}_{fp} (x) - F^{\nu}_{ft} (x).
\]
Here $J^\mu_\alpha = n_\alpha u^\mu_\alpha$ is the baryon current defined in terms of proper baryon density $n_\alpha$ and hydrodynamic 4-velocity $u^\mu_\alpha$ normalized as $u_\alpha u^\mu_\alpha = 1$. Eq. (2) implies that there is no baryon-charge exchange between p- and t-fluids, as well as that the baryon current of the fireball fluid is identically zero, $J^\mu_f = 0$. The energy–momentum tensors $T^\mu\nu_\alpha$ in Eqs. (3)–(4) are affected by friction forces $F^\nu_p$, $F^\nu_t$, $F^\nu_{fp}$, and $F^\nu_{ft}$. Friction forces between baryon-rich fluids, $F^\nu_p$ and $F^\nu_t$, partially transform the collective incident energy into thermal excitation of these fluids ($|F^\nu_p - F^\nu_t|$) and give rise to particle production into the fireball fluid ($F^\nu_p + F^\nu_t$), see Eq. (5). $F^\nu_{fp}$ and $F^\nu_{ft}$ are associated with friction of the fireball fluid with the p- and t-fluids, respectively. Note that Eqs. (3)–(5) satisfy the total energy–momentum conservation

$$\partial_\mu (T^\mu\nu_p + T^\mu\nu_t + T^\mu\nu_f) = 0. \quad (6)$$

In terms of the proper energy density, $\varepsilon_\alpha$, and pressure, $P_\alpha$, the energy–momentum tensors of the baryon-rich fluids ($\alpha = p$ and t) take the conventional hydrodynamic form

$$T^\mu\nu_\alpha = (\varepsilon_\alpha + P_\alpha) u^\mu_\alpha u^\nu_\alpha - g^{\mu\nu} P_\alpha. \quad (7)$$

For the fireball, however, only the thermalized part of the energy–momentum tensor is described by this hydrodynamic form

$$T^{(eq)}\mu\nu_f = (\varepsilon_f + P_f) u^\mu_f u^\nu_f - g^{\mu\nu} P_f. \quad (8)$$

Its evolution is defined by the Euler equation with a retarded source term

$$\partial_\mu T^{(eq)}\mu\nu_f(x) = \int d^4x' \delta^4 \left(x - x' - U_F(x')\tau\right) \left[F^\nu_p(x') + F^\nu_t(x')\right] - F^\nu_{fp}(x) - F^\nu_{ft}(x), \quad (9)$$

where $\tau$ is the formation time, and

$$U^\nu_F(x') = \frac{F^\nu_p(x') + F^\nu_t(x')}{|F^\nu_p(x') + F^\nu_t(x')|} \quad (10)$$

is a free-streaming 4-velocity of the produced fireball matter. In fact, this is the velocity at the moment of production of the fireball matter. According to Eq. (4), the energy and momentum of this matter appear as a source in the Euler equation only later, at the time $U^\mu_F \tau$ after production, and in different space point $x' - U_F(x')\tau$, as compared to the production point $x'$. 
The residual part of $T^{\mu\nu}_f$ (the free-streaming one) is defined as

$$T^{(fs)}_{f\mu\nu} = T^{\mu\nu}_f - T^{(eq)}_{f\mu\nu}. \quad (11)$$

The equation for $T^{(eq)}_{f\mu\nu}$ can be easily obtained by taking the difference between Eqs. (5) and (9). If all the fireball matter turns out to be formed before freeze-out (which is the case, in fact), then this equation is not needed. Thus, the 3-fluid model introduced here contains both the original 2-fluid model with pion radiation \[3, 5, 6\] and the (2+1)-fluid model \[7, 8\] as limiting cases for $\tau \to \infty$ and $\tau = 0$, respectively.

The nucleon–nucleon cross sections at high energies are strongly forward–backward peaked. Since the involved 4-momentum transfer is small, the Boltzmann collision term can be essentially simplified and in this case the friction forces, $F^\nu_\alpha$ and $F^\nu_t$, are estimated as

$$F^\nu_\alpha = \rho_\alpha \rho_t \left[ (u^\nu_\alpha - u^\nu_\bar{\alpha}) D_P + (u^\nu_p + u^\nu_t) D_E \right], \quad (12)$$

$\alpha = p$ and $t$, $\bar{p} = t$ and $\bar{t} = p$. Here, $\rho_\alpha$ denotes the scalar densities of the $p$- and $t$-fluids,

$$D_{P/E} = m_N V_{rel}^{pt} \sigma_{P/E}(s_{pt}), \quad (13)$$

where $m_N$ is the nucleon mass, $s_{pt} = m_N^2 \left( u^\nu_p + u^\nu_t \right)^2$ is the mean invariant energy squared of two colliding nucleons from the $p$- and $t$-fluids, $V_{rel}^{pt} = [s_{pt}(s_{pt} - 4m_N^2)]^{1/2}/2m_N^2$ is their mean relative velocity, and $\sigma_{P/E}(s_{pt})$ are determined in terms of nucleon-nucleon cross sections integrated with certain weights (see \[3, 5, 9\] for details).

Eqs. (2)–(4) and (9), supplemented by a certain equation of state (EoS) and expressions for friction forces $F^\nu$, form a full set of equations of the relativistic 3-fluid hydrodynamic model. To make this set closed, we still need to define the friction of the fireball fluid with the $p$- and $t$-fluids, $F^\nu_p$ and $F^\nu_t$, in terms of hydrodynamic quantities and some cross sections.

To estimate the scale of the friction force between the fireball and baryon-rich fluids, similar to that done above for baryon-rich fluids, we consider a simplified system, where all baryon-rich fluids consist only of nucleons, as the most abundant component of these fluids, and the fireball fluid contains only pions. At incident energies from 10 (AGS) to 200 A-GeV (SPS) the relative nucleon-pion energies prove to be in the resonance range dominated by the $\Delta$-resonance. The resonance-dominated interaction implies that the essential
process is absorption of a fireball pion by a p- or t-fluid nucleon with formation of an $R$-resonance (most probably $\Delta$). As a consequence, only the loss term contributes to the kinetic equation for the fireball fluid. After momentum integrating this collision term weighted with the 4-momentum $p^\nu$, we get
\[ F^{\nu}_{f_\alpha}(x) = D^{f_\alpha} T^{(eq)0\nu}_f \rho_\alpha, \tag{14} \]
where transport coefficients take the form
\[ D^{f_\alpha} = W^{N\pi\rightarrow R}(s^{f_\alpha})/(m_N m_\pi) = V^{f_\alpha}_{rel} \sigma^{N\pi\rightarrow R}(s^{f_\alpha}). \tag{15} \]
Here, $V^{f_\alpha}_{rel} = [(s^{f_\alpha} - m^2_N - m^2_\pi)^2 - 4m^2_N m^2_\pi]^{1/2}/(2m_N m_\pi)$ denotes the mean invariant relative velocity between the fireball and the $\alpha$-fluids, $s^{f_\alpha} = (m_\pi u_f + m_N u_\alpha)^2$, and $\sigma^{N\pi\rightarrow R}(s)$ is the parameterization of experimental pion–nucleon cross-sections. Thus, we have expressed the friction $F^{\nu}_{f_\alpha}$ in terms of the fireball-fluid energy-momentum density $T^{(eq)0\nu}_f$, the scalar density $\rho_\alpha$ of the $\alpha$ fluid, and a transport coefficient $D^{f_\alpha}$. Note that this friction is zero until the fireball pions are formed, since $T^{(eq)0\nu}_f = 0$ during the formation time $\tau$.

In fact, the above treatment is an estimate of the friction terms rather than their strict derivation. As it is seen from Eq. (13) for the excited matter of baryon-rich fluids, a great number of hadrons and/or deconfined quarks and gluons may contribute into this friction. Furthermore, these quantities may be modified by in-medium effects. In this respect, $D_{P/E}$ and $D_{f_\alpha}$ should be understood as quantities that give a scale of this interaction.

3 Simulations of Nucleus–Nucleus Collisions

The relativistic 3D code for the above described 3-fluid model was constructed by means of modifying the existing 2-fluid 3D code of Refs. [3, 5, 6]. In actual calculations we used the mixed-phase EoS developed in [10, 11]. This phenomenological EoS takes into account a possible deconfinement phase transition of nuclear matter. The underlying assumption of this EoS is that unbound quarks and gluons may coexist with hadrons in the nuclear environment. In accordance with lattice QCD data, the statistical mixed-phase model describes the first-order deconfinement phase transition for pure gluon matter and crossover for that with quarks [10, 11]. For details concerning the used EoS’s, please, refer to [12].

In Fig. 1 global dynamics of heavy-ion collisions is illustrated by the energy-density evolution of the baryon-rich fluids ($\varepsilon_b = \varepsilon_p + \varepsilon_t$) in the reaction plane
of the Pb+Pb collision at $E_{\text{lab}} = 158$ A·GeV. Different stages of interaction at relativistic energies are clearly seen in this example: Two Lorentz-contracted nuclei (note the different scales along the $x$- and $z$-axes in Fig.1) start to interpenetrate through each other, reach a maximal energy density by the time $\sim 1.1$ fm/c and then expand predominantly in longitudinal direction forming a "sausage-like" freeze-out system. At this and lower incident energies the baryon-rich dynamics is not really disturbed by the fireball fluid and hence the cases $\tau = 0$ and $1$ fm/c turned to be indistinguishable in terms of $\varepsilon_b$.

Time–evolution of $\varepsilon_b$ in Fig.1 is calculated for the mixed phase model. Topologically results for EoS of pure hadronic and that of two-phase models look very similar. Due to essential softening the equation of state near the deconfinement phase transition, in the last case the system evolves noticeably slower what may have observable consequences [10, 11].

The energy released in the fireball fluids is of an order of magnitude smaller than that stored in baryon–rich fluids and depends on the formation time. At realistic values of the formation times, $\tau \sim 1$ fm/c, the effect of the interaction is substantially reduced. It happens because the fireball fluid starts to interact only near the end of the interpenetration stage. As a result, by the end of the collision process it looses only 10% of its available energy at $E_{\text{lab}} = 158$ A·GeV and 30%, at $E_{\text{lab}} = 10.5$ A·GeV. Certainly, this effect should be observable in mesonic quantities, in particular, in such fine observables as directed and elliptic flows. The global baryonic quantities stay practically unchanged at finite $\tau$ [1].

To calculate observables, hydrodynamic calculations should be stopped at some freeze-out point. In our model it is assumed that a fluid element decouples from the hydrodynamic regime, when its energy density $\varepsilon$ and densities in the eight surrounding cells become smaller than a fixed value $\varepsilon_{fr}$. A value $\varepsilon_{fr} = 0.15$ GeV/$fm^3$ was used for this local freeze-out density which corresponds to the actual energy density of the freeze-out fluid element of $\sim 0.12$ GeV/$fm^3$. To proceed to observable free hadron gas, the shock-like freeze-out [14] is assumed, conserving energy and baryon charge.

Proton rapidity spectra calculated for $Au + Au$ collisions are presented in Fig.2 for $E_{\text{beam}} = 6, 8$ and $10.5$ A·GeV. One should note that hydrodynamics does not make difference between bounded into fragments and free nucleons. In the results presented the contribution from light fragments ($d$, $t$, $^3$He and $^4$He) has been subtracted using a simple coalescence model [15]. This procedure allows one to reproduce reasonably evolution of the spectra shape with changing the impact parameter $b$ for all the energies considered. Some discrep-
Figure 1: Time evolution of the energy density, $\varepsilon_b = \varepsilon_p + \varepsilon_t$, for the baryon-rich fluids in the reaction plane ($xz$ plane) for the Pb+Pb collision ($E_{\text{lab}} = 158$ A·GeV) at impact parameter $b = 2$ fm. Shades of gray represent different levels of $\varepsilon_b$, as indicated at the right side of each panel. Numbers at this palette show the $\varepsilon_b$ values (in GeV/fm$^3$) at which the shades change. Arrows indicate the hydrodynamic velocities of the fluids.
Figure 2: Proton rapidity spectra from $Au + Au$ collisions at three bombarding energies and different impact parameters $b$ (given in fm). Three panels correspond to different equation of state. Experimental points are from [13].
ancy observed for peripheral collisions near the target and projectile rapidity are mainly due to a not-subtracted contribution of heavier fragments. As seen, the rapidity spectra are only slightly sensitive to the EoS used. The same conclusions can be drawn from Fig.3 for higher CERN SPS energies. One should notice that a general agreement here for central collisions is much better than that reached in any other 2-fluid hydrodynamic model. This originates mainly from larger energy-momentum transfer in (12) rather than from the account for the third fireball fluid.

4 Conclusions

In this paper we have developed a 3-fluid model for simulating heavy-ion collisions in the range of incident energies between few and about 200 A·GeV. In addition to two baryon-rich fluids, which constitute the 2-fluid model, a delayed evolution of the produced baryon-free (fireball) fluid is incorporated. This delay is governed by a formation time, during which the fireball fluid
neither thermalizes nor interacts with the baryon-rich fluids. After the for-
formation, it thermalizes and comes into interaction with the baryon-rich fluids.
This interaction is estimated from elementary pion-nucleon cross-sections. Im-
plementation of different EoS, including those with the deconfinement phase
transition, may open great opportunities for analysis of collective effects in rela-
tivistic heavy-ion collisions. Unfortunately, in spite of reasonable reproduc-
on of observable proton rapidity spectra in the wide range of bombarding ener-
gies and centrality parameters we are unable to favor any of considered EoS.
Analysis of more delicate characteristics is needed. This work is in progress
now.

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