INFLUENCE OF MAGNETIC FIELD ON EFFECTIVE ELECTRON-ELECTRON INTERACTIONS IN A COPPER WIRE

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We have measured in a copper wire the energy exchange rate between quasiparticles as a function of the applied magnetic field. We find that the effective electron-electron interaction is strongly modified by the magnetic field, suggesting that magnetic impurities play a role on the interaction processes.

In metallic wires, electron-electron interactions lead to quasi-elastic scattering, which modifies the electron phase-coherence time, and to inelastic scattering, which redistributes energy between electrons. Several recent experiments have shown that these properties can be sample-dependent. We have found furthermore that the low-temperature saturation of the phase-coherence time $\tau_\varphi$ of quasiparticles, observed in several experiments, is correlated with an anomalous dependence on the exchanged energy $\varepsilon$ of $K(\varepsilon)$, a function proportional to the averaged squared interaction between two quasiparticles. The theoretical predictions, for pure Coulomb interactions, $\tau_\varphi \propto T^{-2/3}$ and $K(\varepsilon) \propto \varepsilon^{-3/2}$, were observed in silver wires, whereas a “saturating” $\tau_\varphi$ and $K(\varepsilon) \propto \varepsilon^{-2}$ were systematically found in copper wires. In gold wires in which the presence of a large concentration of magnetic impurities ($\sim 50$ ppm of iron) could be assessed, $K(\varepsilon)$ was also found to behave as $\varepsilon^{-2}$, with a particularly large prefactor. On the theoretical side, it was recently understood that scattering by magnetic impurities, which limits the phase coherence time on a large range of temperature, can also provide another channel for energy exchange between quasiparticles. In order to probe if magnetic impurities indeed play a role in copper wires, we have measured the magnetic field dependence of the energy exchange rate.

1 Measurement set-up

1.1 Principle of the experiment

In order to measure the energy exchange rates between quasiparticles, we have determined the local electron energy distribution function $f(x, E)$ in a stationary out-of-equilibrium situation, as described in Fig. 1. A mesoscopic metallic wire is placed between two large electrodes implementing Landauer reservoirs. In the absence of interactions, the population of quasiparticles at a given energy interpolates linearly between the distribution functions in the contacts, leading, if $k_B T \ll eU$, to a double-step-shaped distribution function. In the opposite “hot electron” regime, i.e. when the typical interaction time is much shorter than the diffusion time $\tau_D = L^2 / D$, equilibrium is reached locally at each position along the wire: the energy distribution function $f(x, E)$ is a Fermi function, with a temperature dependent on the position along the wire.

Our experiments focus on the intermediate regime, in which interactions lead to a significant redistribution of the energy between quasiparticles, but not to a complete thermalization.
Figure 1: Experimental layout: A metallic wire of length \( L \) is connected to large reservoir electrodes, biased at potentials 0 and \( U \). In the absence of interactions, the distribution function in the middle of the wire displays a step with height \( 1/2 \) for energies between \(-eU\) and 0 (solid curve). When interactions are strong enough to thermalize electrons, the distribution function is a Fermi function (dashed curve). In the experiment, the distribution function is obtained from the differential conductance \( \frac{dI}{dV}(V) \) of the tunnel junction formed by the wire and a reference resistive electrode placed underneath.

In previous experiments\(^4\),\(^5\), the distribution function was inferred from the differential conductance \( \frac{dI}{dV}(V) \) of a tunnel junction between the wire and a superconducting probe electrode. Here the probe electrode, also fabricated with a superconducting metal (Aluminum), was designed as a long (25 \( \mu m \)), narrow (150 nm) and thin (12 nm) wire in order to present an appreciable resistance (\( \sim 1 \) k\( \Omega \)) in the normal state. In zero-magnetic field, the probe electrode was superconducting and the distribution function could be obtained as in the earlier experiments. In a magnetic field larger than 0.15 T, the probe electrode was normal and resistive. In this regime, dynamical Coulomb blockade of tunneling through the junction resulting from this “environmental” resistance\(^13\) can be exploited as a quasiparticle energy probe. The relationship between the differential conductance of the junction \( \frac{dI}{dV}(V) \) and the distribution function in the wire \( f(E) \) is then:

\[
\frac{dI}{dV}(V) = \frac{1}{R_T} \int dE \ f(E) \int d\varepsilon \ P(\varepsilon) \frac{\partial}{\partial E}((f_{ref}(E + eV + \varepsilon) - f_{ref}(E + eV - \varepsilon)) \quad (1)
\]

where \( R_T \) is the tunnel resistance of the junction, \( f_{ref}(E) \) the distribution function in the resistive electrode, assumed to be a Fermi function at temperature \( T_{ref} \), and \( P(\varepsilon) \) the probability for an electron to tunnel through the barrier while giving the environment the energy \( \varepsilon \):

\[
P(\varepsilon) = \int dt \ \exp[i\varepsilon t/\hbar] \ \exp J(t)
\]

with

\[
J(t) = 2 \int_0^\infty \frac{d\omega}{\omega} \ \frac{\text{Re}[Z_{env}(\omega)]}{R_K} \ \frac{\exp[-i\omega t] - 1}{1 - \exp[-\hbar\omega/k_B T]}
\]

where \( R_K = \frac{\hbar}{e^2} \approx 25.8 \) k\( \Omega \), \( Z_{env} \) and \( T \) are the environmental impedance and temperature. In our set-up, the environment consists of the parallel combination of the capacitance \( C \) of the junction and the resistance \( R \) of the probe electrode.

In order to evaluate the sensitivity of this novel technique, we show in Fig. 2 the calculated differential conductance for characteristics of the environment close to the experimental ones and for two distribution functions in the wire: a Fermi function, as expected at \( U = 0 \), and a typical double-step function for \( U = 0.3 \) mV. The zero-bias conductance dip splits into a double-dip for a double-step distribution function. In the experiment, we invert the transformation shown in Fig. 2 and determine the unknown \( f(E) \) from the measured \( \frac{dI}{dV}(V) \).
Figure 2: Computer-generated distribution functions at $U = 0$ and at $U = 0.3$ mV, with $kT = 0.004$ meV (left panel), and corresponding calculated differential conductance of the probe junction (right panel) for characteristics of the environment close to the experimental ones.

1.2 Experimental techniques

The sample was fabricated in a single pump-down, using three-angle shadow-mask technique through a PMMA suspended mask patterned using e-beam lithography. The substrate was thermally oxidized silicon. We first deposited a 12 nm-thick aluminum film defining the probe finger. It was then oxidized, in order to obtain the tunnel barrier. The wire and the reservoirs were obtained by the subsequent evaporation from a 99.999% purity copper target, at a pressure of $10^{-6}$ mb, at a deposition rate of 1 nm/s. The thickness and width of the wire are 45 nm and 105 nm. The reservoirs are 400 nm-thick, with an area of about 1 mm². From the low temperature wire resistance $R = 29.5$ Ω, we deduced from Einstein’s relation, assuming a rectangular cross-section, the diffusion constant $D = 90$ cm²/s and the diffusion time $\tau_D = 2.8$ ns. The sample was mounted in a copper box thermally anchored to the mixing chamber of a dilution refrigerator. Measurements were performed at 25 mK. Electrical connections were made through filtered coaxial lines.

Measurements proceed as follows. In a first step, the voltage $U$ is set to zero. The measured differential conductance of the tunnel junction $\frac{dI}{dU}(V)$, shown in Fig. 3, was found to be independent of the magnetic field at $B > 0.15$ T, a value at which all traces of superconductivity in the probe electrode are washed out. From the fit of $\frac{dI}{dU}(V)$ with Eq. (1), assuming $f(E) = f_{\text{ref}}(E)$ and $T_{\text{ref}} = T$, we deduce the tunnel junction conductance $R_T^{-1} = 23$ μS, the resistance of the probe electrode $R = 1.1$ kΩ, the tunnel junction capacitance $C = 0.8$ fF, and the temperature $T = 68$ mK. This excess temperature compared to the temperature of the mixing chamber of the dilution refrigerator (25 mK), might be due to an oversimplification of the environmental impedance, or to external noise. This question still has to be settled.

In a second step, the voltage $U$ is set to a finite value. We fit the measured $\frac{dI}{dU}(V)$ with the calculated differential conductance using Eq. (1). The function $P(\varepsilon)$ is calculated with the environmental characteristics $(R, C, T)$ determined at $U = 0$ and the fit parameters are the values of $f(E)$ at forty regularly spaced energy values between $-0.1$ meV and 0.1 meV. Note that setting the temperature $T_{\text{ref}}$ of the Fermi function $f_{\text{ref}}(E)$ in the probe electrode also to 68 mK causing numerical instabilities, the data shown here were obtained with $T_{\text{ref}} = 0$. 
Figure 3: Left panel: Measured differential conductance at $B = 0.8$ T for $U = 0$ (open symbols) and $U = 0.3$ mV (black symbols). Right panel: Data at $U = 0$ replotted on a log-scale, together with the best fit with Eq. 1 (solid line) obtained with $R_T^{-1} = 23 \mu$S, $R = 1.1$ kΩ, $C = 0.8$ fF, and $T = 68$ mK.

2 Experimental results

2.1 Distribution functions

We show in Fig. 4 the distribution functions for two bias-voltages $U = 0.1$ mV and $U = 0.3$ mV and for magnetic fields varying from 0.16 T to 2 T. Also shown with dotted lines are the zero-magnetic field curves, obtained with the probe in the superconducting state.

In zero-magnetic field, we find that $f(E)$ only depends on $E/eU$. Such a scaling property was found in all previous experiments on copper wires\textsuperscript{5,6}. In a magnetic field, scaling does not hold any more. Besides, the evolution of $f(E)$ with the magnetic field is voltage-dependent: at $U = 0.1$ mV, the plateau of the measured distribution functions near $f(E) = 0.5$ first gets steeper with the magnetic field up to 0.4 T, then flattens and is finally horizontal at $B = 2$ T; at $U = 0.3$ mV, it gets steeper as the magnetic field increases up to $B = 1.6$ T, then flattens again.

The excess rounding of the steps in a magnetic field compared to the zero-magnetic field data might be due to the choice $T_{\text{ref}} = 0$ in the deconvolution procedure, to an oversimplification of
the environmental impedance, to external noise, or, more fundamentally, to an increased energy exchange rate at low energies.

In the next section, we quantify the dependence of the interaction on $B$ and $U$.

2.2 Energy exchange rates

The distribution function can be calculated by solving the stationary Boltzmann equation in the diffusive regime:

$$\frac{1}{\tau_D} \frac{\partial^2 f(x,E)}{\partial x^2} + \mathcal{I}_{\text{coll}}^\text{in}(x,E,\{f\}) - \mathcal{I}_{\text{coll}}^\text{out}(x,E,\{f\}) = 0 \quad (2)$$

where $\mathcal{I}_{\text{coll}}^\text{in}(x,E,\{f\})$ and $\mathcal{I}_{\text{coll}}^\text{out}(x,E,\{f\})$ are the rates at which quasiparticles are scattered in and out of a state at energy $E$ by inelastic processes. Assuming that the dominant inelastic process is a two-quasiparticle interaction which is local on the scale of variations of the distribution function,

$$\mathcal{I}_{\text{coll}}^\text{in}(x,E,\{f\}) = \int d\varepsilon dE' K(\varepsilon) f_{E+E'} f_{E+E'-\varepsilon}(1-f_{E+E'})f_{E+E'}(1-f_{E+E'-\varepsilon}) \quad (3)$$

where the shorthand $f_{E+E'}$ stands for $f(x,E)$. The out-collision term $\mathcal{I}_{\text{coll}}^\text{out}$ has a similar form.

The kernel function $K(\varepsilon)$ is proportional to the averaged squared interaction between two quasiparticles exchanging an energy $\varepsilon$. We have neglected the possible dependence of $K(\varepsilon)$ on the energies of the initial and final states and on the position along the wire. The theory of Coulomb interactions in disordered wires predicts $K(\varepsilon) \propto \varepsilon^{-3/2}$. However, in copper wires, at zero-magnetic field, the scaling property implies, by a simple change of variables in Eq. 2, that $U^2 K(\varepsilon)$ is a function of $\varepsilon/eU$ only. If furthermore $K(\varepsilon)$ does not depend on $U$, one obtains $K(\varepsilon) = \gamma/\varepsilon^2$, with $\gamma$ a typical interaction rate, independent of $U$.

We have found that the curves $f(E)$ measured in a magnetic field could also be reasonably fitted assuming $K(\varepsilon) = \gamma(U,B)/\varepsilon^2$ where $\gamma(U,B)$ is now a fit parameter for each curve. It is only in zero-magnetic field that $\gamma(U,0)$ is independent of $U$. We show in Fig. 5 the evolution of this effective intensity $\gamma(U,B)$ with the bias voltage and the magnetic field, in a grayscale plot. One observes that the typical interaction rate has a non-monotonous behaviour with $B$. It increases roughly from $B = 0$ to $B \approx \frac{eU}{4\mu_B}$ and then decreases again. The zero-magnetic-field value $\gamma(U,0) = 0.52 \text{ ns}^{-1}$ is recovered near $B \approx \frac{eU}{2\mu_B}$ (see straight line on Fig. 5).

![Figure 5: Evolution of the interaction intensity $\gamma$ with the magnetic field $B$ and the bias voltage $U$ obtained from a fit assuming $K(\varepsilon) = \gamma/\varepsilon^2$. The straight line is $eU = 2\mu_B B$.](image)
3 Magnetic-impurity-mediated energy exchange between electrons

Kaminski and Glazman have proposed a second order process associated with magnetic impurities in which energy is exchanged between quasiparticles. In the top of Fig. 6, we describe this second-order process in the situation where, due to Zeeman effect, the spin-up and the spin-down states of the magnetic impurities are split by the energy \( g \mu_B B \). For simplicity, the impurities are assumed to be spin \( 1/2 \); \( g \) is the gyromagnetic factor of the impurities. In Fig. 6, the initial polarisation of the impurity is assumed to correspond to its lowest-energy state. The energy of the intermediate state is then \( -\varepsilon + g \mu_B B \), so that the rate of this process of second order in perturbation is proportional to \( (\varepsilon - g \mu_B B)^{-2} \). At \( B = 0 \), one obtains \( K(\varepsilon) \propto \varepsilon^{-2} \).

On the other hand, in a magnetic field, inelastic scattering also occurs in the first order process: the energy \( \pm g \mu_B B \) is exchanged directly between an electron and a magnetic impurity (see Fig. 6, bottom). The magnetic impurities then behaves as two-level systems, the occupation of the two states being determined self-consistently, at each position along the wire, by the coupling to the quasiparticles. This process could explain why the interaction rate \( \gamma \) first increases with magnetic field.

**Electron-electron interaction mediated by a magnetic impurity (second order process)**

**Direct interaction between an electron and a magnetic impurity (first order process)**

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If the splitting \( g \mu_B B \) becomes larger than \( eU \), starting from an occupied electron-state at energy \( E \), the state of energy \( E - g \mu_B B \) is occupied. Then, due to Pauli principle, the rate for first order processes vanishes, and the magnetic impurities remain in their ground state. The rate of the second-order processes also vanishes at large magnetic fields because of the \( g \mu_B B \) term in its denominator and of the Pauli constraint \( \varepsilon \lesssim eU \). This explains the decrease of the interaction rate at \( B \approx \frac{eU}{g \mu_B} \).

A quantitative description of these processes, taking into account the renormalisation of the coupling between quasiparticles and magnetic impurities due to Kondo effect, is under progress.
In conclusion, we have found that the anomalous energy exchange rate between quasiparticles in copper wire strongly depends on magnetic field. The magnetic field and bias-voltage dependence of the intensity of the interaction are in qualitative agreement with a magnetic-impurity-mediated interaction. The nature of the magnetic impurities is not known, but measurements of the phase-coherence time give evidence that the copper oxide might play a role\textsuperscript{13}.

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