Uncertainties in limits on TeV-gravity from neutrino-induced showers

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Abstract

In models with TeV-scale gravity, ultrahigh energy cosmic rays can generate microscopic black holes in the collision with atmospheric and terrestrial nuclei. It has been proposed that stringent bounds on TeV-scale gravity can be obtained from the absence of neutrino cosmic ray showers mediated by black holes. However, uncertainties in the cross section of black hole formation and, most importantly, large uncertainties in the neutrino flux affects these bounds. As long as the cosmic neutrino flux remains unknown, the non-observation of neutrino induced showers implies less stringent limits than present collider limits.

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I. INTRODUCTION

It has been proposed that in models of TeV-scale gravity [1], ultrahigh energy cosmic neutrinos colliding with atmospheric nuclei could form black holes (BHs) in the atmosphere [2]. (For a review on nonperturbative gravitational events at trans-Planckian energies and references, see e.g. Ref. [3].) The products of BH decay would then be detected as extensive air showers (EASs) of ultrahigh energy cosmic rays (UHECRs).

One of the signatures of BH formation in the atmosphere [4, 5] would be the observation of deeply penetrating quasi-horizontal EASs. Semiclassical calculations of BH cross sections suggest an interaction length for neutrino-nucleon events in air of the order of $10^5$ g cm$^{-2}$, about two orders of magnitude lower than the standard model (SM). This is considered enough to generate deeply penetrating horizontal air showers with little background from the SM. The nonobservation of these deeply penetrating horizontal air showers can set an upper bound on the BH cross section [6, 7]. Since the semiclassical BH cross section is inversely proportional to the fundamental gravitational scale $M_D$, a lower bound on $M_D$ follows. For a number of extra dimensions $n \geq 5$, the estimated lower bound on the gravitational scale is $M_D > 1.0 - 1.4$ TeV [8].

Limits on the fundamental gravitational scale from air showers depend on two main assumptions: i) the existence of a cosmogenic neutrino flux; and ii) the accuracy of estimates for the cross section for BH formation. The cosmogenic neutrino flux is not known with good precision. Many models have been proposed to estimate the flux, which varies by more than an order of magnitude (see e.g. Refs. [9, 10, 11]). Flux constraints made from experiments are quite generous in their range [12]. In addition, microscopic BH formation at trans-Planckian energies [13] is not understood nor has it been observed in particle collisions. Recent work assumes that the cross section for BH formation at parton level is approximately the semiclassical black disk (BD) area with Schwarzschild radius $r_s(M)$, where $M$ is the center of mass (c.m.) energy of the collision. However, a rigorous calculation of the BH cross section is still an open question.

The detection of microscopic BH formation in particle collisions would confirm TeV-gravity models. However, nonobservation of these events does not necessarily rule out TeV-gravity at this stage. A more stringent constraint on $M_D$ cannot be set until the neutrino flux and the physical process of BH formation are better understood. Microscopic BHs might
not form in the atmosphere even if the gravitational scale is of order of the TeV. Therefore, constraints on \( M_D \) from nonobservation of atmospheric BH events are not as stringent as what was previously considered. The perturbative predictions, on which current collider bounds are based \([14, 15]\), seem to be a much stronger basis than those invoking strong gravitational effects.

In this article we revisit and discuss lower bounds on \( M_D \) from nonobservation of BH-induced EASs in more detail, and find how various uncertainties may reduce these bounds.

II. CROSS SECTION

The total cross section for a neutrino-nucleon BH event in \((n + 4)\)-dimensions is obtained by summing the neutrino-parton cross section of BH formation \( \sigma_{\nu i \rightarrow BH}(xs, n, M_D) \) over the parton distribution functions (PDFs) \( q_i(x, -Q^2) \) \([16]\):

\[
\sigma_{\nu p \rightarrow BH}(s, x_m, n, M_D) = \sum_i \int_{x_m}^1 dx \, q_i(x, -Q^2) \sigma_{\nu i \rightarrow BH}(xs, n, M_D),
\]

where \( -Q^2 \) is the four-momentum transfer squared, \( \sqrt{x} \) is the fraction of the nucleon momentum carried by the parton, \( \sqrt{s x_m} = M_{BH, \text{min}} \), the minimal BH mass where semiclassical description is valid, and the parton cross section \( \sigma_{\nu i \rightarrow BH}(xs, n, M_D) \) is given by

\[
\sigma_{\nu i \rightarrow BH}(xs, n, M_D) = F \frac{1}{M_D^2} \left[ \frac{2^n \pi^{n-1} \Gamma \left( \frac{n+3}{2} \right)}{(2 + n)} \right] \frac{2^n}{n+1} \left( \frac{\sqrt{xs}}{M_D} \right)^{\frac{2}{n+1}},
\]

where \( F \) is a form factor.

It should be noted that \( M_{BH, \text{min}} \) is not necessarily equal to the minimum allowed mass of the BH, \( \bar{M} \), and \( M_{BH, \text{min}} \geq \bar{M} \geq M_D \). \( M_{BH, \text{min}} \) and \( \bar{M} \) depend on quantum gravity physics and cannot presently be determined. \( x_m \) is generally assumed to be a constant parameter of order one. For spherically symmetric BHs a justification for this choice is provided by the following semiclassical argument \([8]\): For \( M_{BH, \text{min}}/M_D \gtrsim 3 \) and \( n \geq 5 \), the Hawking entropy of the BH is larger than 10, and therefore strong gravitational effects can be neglected. The semiclassical results based on Eq. (1) are then extrapolated for \( M_{BH, \text{min}}/M_D \lesssim 3 \) with the assumption that the BH or its Planckian progenitor decays on the brane, whatever the quantum theory of gravity may be. As \( \bar{M} \) and \( M_D \) are not necessarily equal, caution is required in extrapolating down the semiclassical results to \( M_{BH, \text{min}} \sim \bar{M} \sim M_D \), where
the physics is unknown and quantum effects are important [17]. The entropy estimation is based on Hawking’s semiclassical theory and is not valid at energies a few times above the fundamental scale. For example, it has been shown that the existence of a minimum length could dramatically increase the value of $M$, and thus also increase $x_m$ [18]. Finally, the estimate simply refers to static spherically symmetric BHs, and may be drastically affected if the geometry of the BH or its Planckian progenitor is different.

The form factor depends in principle on the energy of the process, on the gravitational scale, on the geometry and number of extra dimensions, and on the geometry and physical properties of the gravitational object. Some of the physical parameters that can affect the form factor are angular momentum, charge, geometry of the trapped surface, quantum corrections to classical gravity, unknown effects of super-Planckian particle physics, structure and topology of the compactified dimensions. With the lack of further insight, most authors simply set $F = 1$. Yoshino and Nambu (YN) [19] numerically investigated the formation of the BH apparent horizon. In the YN approach $F$ is a numerical factor depending on the number of extra dimensions ranging from $\approx 0.65$ ($n = 0$) to $\approx 1.88$ ($n = 7$). YN gives a relation between the impact parameter and the mass of the BH which is formed in the collision. The result is that the mass of the BH decreases as the impact parameter increases up to a maximum value. This behavior affects the computation of the total cross section by requiring the lower bound of the integral in Eq. (1) to depend on the impact parameter. The total cross section in the YN approach is

$$
\sigma'_{\nu p \rightarrow BH}(s, x_m, n, M_D) = \sum_i \int_0^1 2z dz \int_{x_m'}^1 dx q_i(x, -Q^2) \sigma_{\nu i \rightarrow BH}(xs, n, M_D),
$$

where $z$ is the impact parameter normalized to its maximum value and $x'_m = x_m/y^2(z)$, $y(z)$ being the fraction of c.m. energy that is trapped into the BH. The YN approach lowers the BD cross section (Fig. 1).

The YN result still relies on a number of assumptions that may affect the final estimate. (For recent criticisms, see Ref. [20].) The incoming partons, for example, are modeled as classical neutral point-like particles. Partons carry color and EM charge, and it has been shown that the physics of collisions between charged particles is quite different from that of uncharged ones [21]. Moreover, it is not clear what constitutes the energy that is not trapped inside the horizon. Recent studies seem to suggest that gravitational emission can account only for a part of the missing BH energy [22]. This could signal that the physics of
FIG. 1: BH cross section ($M_{BH,min} = M_D = 1.0$ TeV) for BD approximation (left upper curves) and for YN formalism (right lower curves). For each group, $n = 3 \ldots 7$ from below. $Q = r_s^{-1}$ and and CTEQ6 PDFs is used.

trans-Planckian collisions is more complex than the simple semiclassical picture.

Another source of uncertainty in Eq. (1) and Eq. (3) comes from the ambiguity in the definition of the momentum transfer for a BH event \cite{23}. The latter is usually chosen to equal the BH mass or the inverse of the Schwarzschild radius. However, there are no definite arguments to prefer either one or to exclude alternative choices. The uncertainty due to the ambiguity in the definition of the momentum transfer is evaluated as $\sim 10 - 20\%$ \cite{6}.

Finally, a minor but additional source of uncertainty in the total cross section is due to the PDFs. Different sets of PDFs are defined in the literature. The PDFs are not known for momentum transfer higher than a given value (see, e.g. Ref. \cite{24}), which is lower than the momentum transfer expected in BH formation. Equation (1) and Eq. (3) are calculated by imposing a cut-off at this energy. They also suffer from uncertainties at any momentum transfer that can contribute to the reduction in the total cross section. The
minimum uncertainty on the BH total cross section due to the PDFs can be estimated for a given distribution. The CTEQ6 distribution gives an uncertainty of $\sim 3 - 4\%$, a value that does not include the uncertainty due to the cutoff on the momentum transfer nor the uncertainty introduced by the use of different sets of PDFs. The MRST distribution for $M_D = 1.0$ TeV and $M_{BH,\text{min}}/M_D = 3$ gives results about $\sim 10 - 15\%$ lower than the CTEQ distribution.

III. EFFECT OF CROSS SECTION UNCERTAINTIES ON $M_D$ BOUNDS

The BD approximation may be different than the actual cross section due to these uncertainties. To give a concrete example of how the determination of the cross section affects the $M_D$ bounds, let us look at a case where the parton cross section is arbitrarily reduced by a constant factor of order one. The cause of this reduction could be any one or a combination of the uncertainties previously discussed.

Consider high energy primary neutrinos with energy $E_\nu = 10^6 - 10^8$ TeV, $M_{BH,\text{min}} = 3M_D$ and $n = 5$. For the sake of simplicity, we temporarily neglect the YN results and consider Eq. (1). This is sufficient for a rough estimation. The total cross section (1) for $M_D = 1$ TeV and $F = 1$ lies within the shaded band of the total cross section for $M_D = 0.72$ TeV and $F = 1 \pm 2/3$. This is shown graphically in Fig. 2. Therefore, what is interpreted as the $M_D = 1.0$ TeV bound in the naive BD approximation can actually be a less stringent bound if the parton cross section is two-thirds smaller: $M_D = 0.72$ TeV, the lower bound from collider experiments derived from virtual graviton exchange [14, 15]. The situation is made even more complicated by the presence of the unknown parameter $M_{BH,\text{min}}$. Increasing $M_{BH,\text{min}}$ corresponds to decreasing the BH cross section. The lower bound on the eleven-dimensional $M_D$ for $M_{BH,\text{min}}/M_D = 5$ is approximately half the bound for $M_{BH,\text{min}}/M_D = 1$.

IV. NUMBER OF EVENTS AND NEUTRINO FLUX

The uncertainties in the BH cross section listed above apply to BH formation by UHECR events as well as by particle colliders. In cosmic ray events, however, the unknown neutrino flux adds further uncertainties. In the YN approach, the number of neutrino-nucleon BH
FIG. 2: The cross section for $M_D = 1.0$ TeV (thick solid line) and for $M_D = 0.72$ TeV (dashed line). Shaded band denotes $F = 1 \pm 2/3$ for $M_D = 0.72$ TeV. The cross section of $M_D = 1$ TeV with $F = 1$ lies within this band. CTEQ6 is used.

The number of events detected by a cosmic ray detector in time $T$ is:

$$N = N_A T \sum_i \int dE_\nu \int_0^1 2zdz \int_{x_m}^1 dx q_i(x, -Q^2) \sigma_{vi \rightarrow BH}(x_s, n, M_D) \frac{d\Phi}{dE_\nu} A(yE_\nu),$$

(4)

where $A(yE_\nu)$ is the experiment acceptance for an air shower energy $yE_\nu$, $N_A$ is Avogadro’s number, and $d\Phi/dE_\nu$ is the source flux of neutrinos.

The cosmogenic neutrino flux is considered to be the most reliable source of neutrinos. In this model, neutrinos are produced from ultrahigh energy protons interacting with the ubiquitous cosmic microwave background. However, this is not fully guaranteed as the existence of the cosmogenic neutrino flux relies on the assumption that cosmic rays with energies above $10^8$ TeV are extragalactic protons. Neither the source nor composition of cosmic rays above $10^8$ TeV are known (see, e.g., Refs. [27, 28]). If these are heavy nuclei or photons, or Lorentz invariance is violated [29], there may be no cosmogenic neutrino flux at ultrahigh energies. Even if UHECRs are protons from extragalactic sources, cosmological evolution, spatial distribution, abundance, and injection spectrum of UHECR sources can
change the cosmogenic neutrino flux by an order of magnitude. In order to derive a conservative lower bound on $M_D$, the lower end of plausible cosmogenic neutrino fluxes should be used in Eq. (4).

Here we consider a model that gives a relatively low neutrino flux in agreement with observations. The flux is calculated following the procedure of Ref. [11]. The source spectrum is proportional to $E^{-2.6} \times \exp(-E/E_c)$, where $E_c = 10^{8.5}$ TeV is the cutoff energy, and normalized to the cosmic ray luminosity at $10^7$ TeV. There is no cosmological evolution and the redshift integration is from $z = 0.05 - 8.00$. The parameters used are consistent with observations: A spectral index of 2.6 is the best fit to the highest energy data [30], and the cutoff energy is limited by the highest energy cosmic ray event observed to date. Since the ultrahigh energy proton sources are unknown, their evolution cannot be determined. No evolution is a reasonable assumption for the lower end of the neutrino flux that should be considered. Our model flux is in good agreement with the lower bound obtained from cosmic ray data analysis [12]. Figure 3 compares this flux to two other fluxes, by Protheroe and Johnson (PJ) [10] and Engel, Seckel, and Stanev (ESS) [11]. Our flux is lower by at least an order of magnitude than the PJ flux on the most relevant range of neutrino energies.

The sensitivity of the detector for horizontal air showers also affects the computation of the lower bounds on $M_D$. For example, the Pierre Auger Observatory [31] uses 1.2 m high water Cherenkov detectors whereas AGASA [32] uses 5 cm thick scintillators. From purely geometrical arguments, AGASA’s detection capability rapidly goes down at large zenith angle. On the other hand, AGASA has a lower detector trigger threshold which gives higher sensitivity at lower energies. Scaling the aperture by the experiment size is an approximation that should be further improved.

V. REALISTIC BOUNDS ON $M_D$

The effect of the cosmogenic neutrino flux on $M_D$ lower bound is best illustrated by comparing the conservative flux of Sect. IV and the PJ flux. We performed a systematic analysis of the number of events using the above cosmogenic neutrino fluxes for different experiments (AGASA [33], HiRes [34], RICE [35]) and different choices of parameters in

\footnote{We thank Todor Stanev for providing the data.}
FIG. 3: Cosmogenic neutrino fluxes for the $\nu_\mu$ family. PJ (dashed line), ESS (dotted line), and our model (solid line) fluxes from the top.

the total cross sections from Eq. (1) and (3). For simplicity we used the apertures given by Refs. [6, 7, 8]. Though RICE is an experiment looking into “ice showers” rather than EASs, the technique used in EAS experiments is applicable. The use of the PJ flux gives a lower bound of $M_D \geq 1.0 - 1.4$ TeV, which is comparable to and sometimes more stringent than collider bounds. The most optimistic case with our model flux is when $Q = M_{BH}$ and $M_{BH, min} = M_D$. The lower bounds on the fundamental scale range from $M_D = 0.3 (M_{BH, min} = M_D)$ to $M_D < 0.2$ TeV ($M_{BH, min} = 5M_D$). These limits are much lower than the collider limits $^2 [14, 15]$.

We also considered changing the cosmological evolution of the conservative flux to be

\begin{itemize}
  \item The computations were performed using the CTEQ6 PDFs and YN approximation. The use of MRST PDFs give results identical within an uncertainty of 0.05 TeV and the BD approximation increases $M_D$ by 0.05 – 0.15 TeV.
\end{itemize}
that of Ref. \[36\], i.e.

\[
(1 + z)^3 \quad (z < 1.9) \\
(1 + 1.9)^3 \quad (1.9 < z < 2.7) \\
(1 + 1.9)^3 \exp[(2.7 - z)/2.7] \quad (2.7 < z)
\]

This increases the flux by about an order of magnitude. It is lower than the ESS flux for \(E_\nu \gtrsim 10^6\) TeV, and is considerably less by a couple of factors than the PJ flux in all energy ranges. The lower bounds increase by about 0.4 TeV, but are still below the collider bounds.

\section{VI. CONCLUSIONS}

We have reviewed the method of constraining the fundamental Planck scale from nonobservation of EASs and discussed the uncertainties on the \(M_D\) lower bound coming from the BH cross section and neutrino flux. The major source of uncertainties in the cross section is due to the lack of a definite theory of trans-Planckian scattering. Trans-Planckian gravity is not known at present to formulate a reliable model for BH formation. The BD approximation and its variants seem to provide a reasonable model for the BH formation cross section at parton level. However, the naive BD cross section relies on a number of crude assumptions and suffers from uncertainties that cannot be estimated. Neglected quantum gravity effects are expected to be relevant.

A conservative estimation of \(M_D\) lower bounds is obtained by taking into account these caveats and considering the range of cosmogenic neutrino fluxes which are compatible with observations. The conclusion is that in the absence of independent determination of the cosmogenic neutrino flux and of reliable theoretical computation of the BH cross section, nonobservation of BH-induced deeply penetrating EASs gives lower bounds on the fundamental gravitational scale less stringent than present collider bounds.

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Note added in Proof

Recent work by Hossenfelder [37] calculates the BH cross section suppression from minimal length at LHC. It is found that the ratio between the total cross section with and without minimal length effects is approximately 0.19 for the expected LHC energy, and increasing at higher energies. This result strengthens our arguments of section II, namely that the presence of a minimum length may substantially reduce the BD cross section.

In addition, progress in neutrino flux estimates with cosmic ray primaries other than protons have shown how uncertain the neutrino flux can be (see hep-ph/0409316 and hep-ph/0407618).

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