SPIN-DEPENDENT STRUCTURE FUNCTION $g_1$ AT SMALL $x$

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Abstract

Accounting for double-logarithms of $x$ and running QCD coupling leads to expressions for both the non-singlet and singlet components of $g_1$. These expressions manifest the Regge asymptotics when $x \to 0$ and differ considerably from the DGLAP expressions at small values of $x$. 
1 Introduction

As is well known, deep inelastic scattering (DIS) is one of the basic processes for probing the structure of hadrons. From the theoretical point of view, the inclusive cross section of DIS is a convolution of the leptonic and hadronic tensors, with the information about the structure of the hadrons participating into DIS coming from the hadronic tensor. The forward Compton amplitude, when a deeply off-shell photon with virtuality $q^2$ scatters off an on-shell hadron with momentum $p$. The spin-dependent part, $W^{\text{spin}}_{\mu\nu}$, of the hadronic tensor is parametrized in terms of two structure functions, $g_1$ and $g_2$, as

$$W^{\text{spin}}_{\mu\nu} = i\epsilon_{\mu\nu\lambda\rho} \frac{q_\lambda m}{pq} \left[ S_\rho g_1 + (S_q - \frac{(S_q)}{pq} p_\rho) g_2 \right] \approx i\epsilon_{\mu\nu\lambda\rho} \frac{q_\lambda m}{pq} \left[ S_\rho^l g_1 + S_\rho^t (g_1 + g_2) \right],$$

so that $g_1$ is related to the longitudinal hadron spin-flip scattering, whereas the sum $g_1 + g_2$ is related to the transverse spin-flips. In Eq. (1), $m$ stands for the hadron mass, $S_\rho^l$ and $S_\rho^t$ are the longitudinal and transverse (with respect to the plane formed by $p$ and $q$) components of the hadron spin $S_\rho$. Both $g_1$ and $g_2$ depend on $x = -q^2/2pq$, $0 < x \leq 1$ and $Q^2 = -q^2 > 0$. Obviously, small $x$ corresponds to $s = (p + q)^2 \approx 2pq \gg Q^2$. In this case, $S_\rho^l \approx p_\rho/m$ and therefore the part of $W^{\text{spin}}_{\mu\nu}$ related to $g_1$ does not depend on $m$. When $Q^2 \gg m^2$, one can assume the factorization and regard $W^{\text{spin}}_{\mu\nu}$ as a convolution of two objects (see Fig. 1). The first one is the probability $\Phi$ ($\Phi = \Phi_q$ in Fig. 1a and $\Phi = \Phi_g$ in Fig. 1b) to find a polarized parton (a quark or a gluon) within the hadron. The second one is the partonic tensor $\tilde{W}^{\text{spin}}_{\mu\nu}$ defined and parametrized similarly to $W^{\text{spin}}_{\mu\nu}$. Whereas the partonic tensor $\tilde{W}^{\text{spin}}_{\mu\nu}$, i.e. the partonic structure functions $g_1$ and $g_2$, can be studied within perturbative QCD, $\Phi_q,g$ are essentially non-perturbative objects. The lack of knowledge of $\Phi$ is usually compensated by introducing initial parton distributions which can be found from phenomenological considerations. In doing so, the non-singlet component $g_1^{NS}$ of $g_1$ is usually expressed as a convolution of a piece which we denote $g_q^{NS}$ from purely evolution, and the initial polarized quark density $\Delta q$:

$$g_1^{NS} = g_q^{NS} \otimes \Delta q \ .$$

Similarly, the singlet structure function $g_1^S$ is expressed in terms of the evolution pieces $g_q$ and $g_g$ and the densities of the polarized quarks and gluons, $\Delta q$ and $\Delta g$:

$$g_1^S = g_q \otimes \Delta q + g_g \otimes \Delta g \ .$$

The subscripts $q, g$ in Eqs. (2,3) refer to the kind of the initial partons, as is shown at Fig. 1. We remind that there is no rigorous procedure for calculating
\[ g_1 \approx g_q + g_g \]

\[ g_1^{NS} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} (1/x)^\omega e^{\gamma_{NS}(\omega) \ln(Q^2/\mu^2)}, \]
\[ g_1^{S} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} (1/x)^\omega \left[ C_q(\omega)\Delta q(\omega) + C_g(\omega)\Delta g(\omega) \right] e^{\gamma_S(\omega) \ln(Q^2/\mu^2)}, \]

with \( C_{NS}(\omega), C_{q,g}(\omega) \) being the coefficient functions and \( \gamma_{NS}(\omega), \gamma_S(\omega) \) the non-singlet and singlet anomalous dimensions respectively. The anomalous dimensions control the \( Q^2 \)-evolution and the coefficient functions govern the \( s \)-evolution which, at fixed \( Q^2 \), is equivalent to the \( x \)-evolution.

**Figure 1:** Representation of the hadronic \( g_1 \) as the convolution of the partonic \( g_1 \) and the initial parton densities.

\( \Delta q \) and \( \Delta g \). They have to be found from phenomenological considerations. On the contrary, there are regular perturbative methods for calculating the evolution parts \( g_1^{NS} \) and \( g_{q,g} \) of \( g_1 \). When \( Q^2 \) is much greater than the starting point \( \mu^2 \) of the \( Q^2 \)-evolution and at the same time \( x \ll 1 \), it is convenient to rewrite Eqs. (2,3) in the form of the Mellin integral:
The best known instrument to calculate the DIS structure functions is the DGLAP approach. In the DGLAP framework, both the coefficient functions and the anomalous dimensions are perturbatively known and represented by their one-loop, (or leading order (LO)) and two-loop (next-to-leading order (NLO)) contributions. The relevant Refs concerning the NLO calculations can be found in the review of Eq. \( \Delta q \) and \( \Delta g \) can be taken, for example, from Ref. DGLAP provides a quite good description of the experimental data. The extrapolation of DGLAP into the small-\( x \) region predicts an asymptotic behavior \( \sim \exp(\sqrt{C \ln(1/x)} \ln \ln Q^2) \) for all DIS structure functions (with different factors \( C \)). However, from a theoretical point of view, such an extrapolation at the small-\( x \) is rather doubtful. In particular, it neglects in a systematical way contributions of the type \( \sim (\alpha_s \ln^2(1/x))^k \) which are small when \( x \sim 1 \) but become relevant when \( x \ll 1 \). The total resummation of these double-logarithmic (DL) contributions was made in Refs. and Ref. for the non-singlet (\( g_1^{NS} \)) and singlet \( g_1 \) respectively, and it leads to the Regge (power-like) asymptotics \( g_1(1/x)^{\Delta DL} ((1/x)^{\Delta NS}) \), with \( \Delta DL, \Delta NS \) being the intercepts calculated in the double-logarithmic approximation (DLA). The weak point of this resummation in Refs. is the assumption that \( \alpha_s \) is kept fixed (at some unknown scale). It leads therefore to the value of the intercepts \( \Delta DL, \Delta NS \) explicitly depending on this unknown coupling, while \( \alpha_s \) is well-known to be running. The results of Refs. had led many authors (see e.g. Ref.) to suggest that the DGLAP parametrization \( \alpha_s = \alpha_s(Q^2) \) has to be used. However, according to results of Ref. such a parametrization is indeed correct for \( x \sim 1 \) only and cannot be used for \( x \ll 1 \). The appropriate dependence of \( \alpha_s \) suggested in Ref. has been successfully used to calculate both \( g_1^{NS} \) and \( g_1 \) singlet at small \( x \) in Refs. In the present talk we review these results.

Instead of a direct study of \( g_1 \) like it is done in DGLAP, it is more convenient to consider the forward Compton amplitude \( A \) for the photon-parton scattering related to \( g_1 \), with \( \Delta q = 1, \Delta g = 0 \), as follows:

\[
g_1(x, Q^2) = \frac{1}{\pi} \Im \lambda_s A(s, Q^2) .
\]

(5)

As already stated above, we cannot use DGLAP for studying \( g_1 \) or \( A \) at small \( x \) because it does not not include the total resummation of the double- and single-logarithms of \( x \) for the anomalous dimensions and coefficient functions, and also the \( \alpha_s \)-parametrization used is valid for \( x \) not far from 1.

Then, in order to account for the double-logs of both \( x \) and \( Q^2 \), we need to construct a kind of two-dimensional evolution equations that would combine both the \( x \) and \( Q^2 \) evolutions.
Such equations should sum up the contributions of the Feynman graphs involved to all orders in $\alpha_s$. Some of those graphs have either ultraviolet or infrared (IR) divergences. The ultraviolet divergences are regulated by the usual renormalization procedure. In order to regulate the IR ones, we introduce an IR cut-off $\mu$ in the transverse momentum space for the momenta $k_i$ of all virtual quarks and gluons:

$$\mu < k_{i\perp}$$  \hspace{1cm} (6)

where $k_{i\perp}$ stands for the transverse (with respect to the plane formed by the external momenta $p$ and $q$) component of $k_i$. This technique of regulating the IR divergences was suggested by Lipatov and used first in Ref. 9 for quark-quark scattering. Using this cut-off $\mu$, $A$ acquires a dependence on $\mu$. Then, one can evolve $A$ with respect to $\mu$, constructing the appropriate Infrared Evolution Equations (IREE). As $A = A(s/\mu^2, Q^2/\mu^2)$,

$$-\mu^2 A/s/\partial \mu^2 = \partial A/\partial \rho + \partial A/\partial y$$  \hspace{1cm} (7)

where $\rho = \ln(s/\mu^2)$ and $y = \ln(Q^2/\mu^2)$. Eq. (7) represents the lhs of the IREE for $A$. In order to write the rhs of the IREE, it is convenient to use the Sommerfeld-Watson transform

$$A(s, Q^2) = \int_{-\infty}^{+\infty} d\omega (s/\mu^2)^{\omega} \xi(\omega) F(\omega, Q^2)$$  \hspace{1cm} (8)

where $\xi(\omega)$ is the negative signature factor, $\xi(\omega) = [1 - e^{-i\pi\omega}]/2 \approx i\pi\omega/2$. It must be noted that the transform inverse to Eq. (8) involves the imaginary parts of $A$:

$$F(\omega, Q^2) = \frac{2}{\pi\omega} \int_{0}^{\infty} d\rho e^{-\rho\omega} \Im A(s, Q^2).$$  \hspace{1cm} (9)

Notice that, contrary to the amplitude $A$, the structure function $g_1$ does not have any signature and therefore $\xi(\omega) = 1$ when the transform (8) is applied directly to $g_1$.

2 infrared evolution equations for $g_1$

When the factorization depicted in Fig. 1 is assumed, the calculation of $g_1$ (for simplicity we will omit the superscript “s” for $g_1$ singlet, though we use the notation $g_1^{NS}$ for the non-singlet $g_1$) is reduced to calculating the Feynman graphs contributing the partonic tensor $\tilde{W}^s_{\mu\nu}$ depicted as the upper blobs in Fig. 1. Both cases, (a), when the virtual photon scatters off the nearly on-shell polarized quark, and (b), when the quark is replaced by the polarized gluon,
should be taken into account. Therefore, in contrast to Eq. (5), we need to introduce two Compton amplitudes: \( A_q \) and \( A_g \) corresponding to the upper blob in Fig. 1a and Fig. 1b respectively. The subscripts “q” and “g” refer to the initial partons. Therefore,

\[
g_1(x, Q^2) = g_q(x, Q^2) + g_g(x, Q^2),
\]

where

\[
g_q = \frac{1}{\pi} \Im s A_q(s, Q^2), \quad g_g = \frac{1}{\pi} \Im s A_g(s, Q^2)
\]

Let us now construct the IREE for the amplitudes \( A_q, A_g \) related to \( g_1 \). To this aim, let us consider a virtual parton with minimal \( k_{\perp} \). We call such a parton the softest one. If it is a gluon, its DL contribution can be factorized, i.e. its DL contribution comes from the graphs where its propagator is attached to the external lines. As the gluon propagator cannot be attached to photons, this case is absent in IREE for \( A_q, A_g \). The second option is when the softest partons are a \( t \)-channel quark-antiquark or gluon pair. It leads us to the IREE depicted in Fig. 2. Applying the operator \(-\mu^2 \partial / \partial \mu^2\) to it, combining the result with Eq. (7) and using (8), we arrive at the following system of equations:

\[
(\omega + \partial / \partial y) F_q(\omega, y) = \frac{1}{8\pi^2} \left[ F_{qq}(\omega) F_q(\omega, y) + F_{qg}(\omega) F_g(\omega, y) \right],
\]

\[
(\omega + \partial / \partial y) F_g(\omega, y) = \frac{1}{8\pi^2} \left[ F_{qg}(\omega) F_q(\omega, y) + F_{gg}(\omega) F_g(\omega, y) \right].
\]

The amplitudes \( F_q, F_g \) are related to \( A_q, A_g \) through the transform (8). The Mellin amplitudes \( F_{ik} \), with \( i, k = q, g \), describe the parton-parton forward scattering. They contain DL contributions to all orders in \( \alpha_s \). We can introduce the new anomalous dimensions \( H_{ik} \). The subscripts “q,g” correspond to the DGLAP-notation. Solving this system of eqs. and using Eq. (11) leads to

\[
g_q(x, Q^2) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} (1/x)^\omega \left[ C_+(\omega) e^{\Omega_+ y} + C_-(\omega) e^{\Omega_- y} \right],
\]

\[
g_g(x, Q^2) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} (1/x)^\omega \left[ C_+(\omega) \frac{X + \sqrt{R}}{2H_{gg}} e^{\Omega_+ y} + C_-(\omega) \frac{X - \sqrt{R}}{2H_{gg}} e^{\Omega_- y} \right].
\]

The unknown factors \( C_\pm(\omega) \) have to be specified and will be discussed later. All other factors in Eq. (13) can be expressed in terms of \( H_{ik} \):

\[
X = H_{gg} - H_{qq}, \quad R = (H_{gg} - H_{qq})^2 + 4H_{qg}H_{gg},
\]

\[
\Omega_\pm = \frac{1}{2} \left[ H_{qq} + H_{gg} \pm \sqrt{(H_{qq} - H_{gg})^2 + 4H_{qg}H_{gg}} \right].
\]
Figure 2: Infrared evolution equations for the amplitudes $A_q, A_g$.

The anomalous dimension matrix $H_{ik}$ was calculated in Ref. 8:

$$
H_{gg} = \frac{1}{2} \left( \omega + Y + \frac{b_{qq} - b_{gg}}{Y} \right), \quad H_{qq} = \frac{1}{2} \left( \omega + Y - \frac{b_{qq} - b_{gg}}{Y} \right), \quad (15)
$$

$$
H_{gq} = -\frac{b_{gq}}{Y}, \quad H_{qg} = -\frac{b_{qg}}{Y}.
$$

where

$$
Y = -\sqrt{\left(\omega^2 - 2(b_{qq} + b_{gg}) + \sqrt{[(\omega^2 - 2(b_{qq} + b_{gg}))^2 - 4(b_{qq} - b_{gg})^2 - 16b_{qg}b_{gq}]} \right)/2}, \quad (16)
$$

$$
b_{ik} = a_{ik} + V_{ik}, \quad (17)
$$

$$
a_{qq} = \frac{A(\omega)C_F}{2\pi}, \quad a_{gg} = \frac{2A(\omega)N}{\pi}, \quad a_{gq} = -\frac{n_fA'(\omega)}{2\pi}, \quad a_{qg} = \frac{A'(\omega)C_F}{\pi}, \quad (18)
$$
and
\[ V_{ik} = \frac{m_{ik}}{\pi^2} D(\omega), \]  
(19)
with
\[ m_{qq} = \frac{C_F}{2N}, m_{gg} = -2N^2, m_{qg} = n_f \frac{N}{2}, m_{gq} = -NC_F. \]  
(20)
We have used here the notations \( C_F = 4/3 \), \( N = 3 \) and \( n_f = 4 \). The quantities \( A(\omega) \) and \( D(\omega) \) account for the running of \( \alpha_s \). They are given by the following expressions:
\[ A(\omega) = \frac{1}{b} \left[ \frac{\eta}{\eta^2 + \pi^2} - \int_0^\infty \frac{d\rho e^{-\omega \rho}}{(\rho + \eta)^2 + \pi^2} \right], \]  
(21)
\[ D(\omega) = \frac{1}{2b^2} \int_0^\infty d\rho e^{-\omega \rho} \ln \left( \frac{\rho + \eta}{\rho + \eta} \right) \left[ \frac{\rho}{(\rho + \eta)^2 + \pi^2} + \frac{1}{\rho + \eta} \right], \]  
(22)
with \( \eta = \ln(\mu^2/\Lambda^2_{QCD}) \) and \( b = (33 - 2n_f)/12\pi \). \( A' \) is defined as \( A \) with the \( \pi^2 \) term dropped out.

Finally we have to specify the coefficients functions \( C_\pm \) appearing in Eq. (13). When \( Q^2 = \mu^2 \),
\[ g_q = \tilde{\Delta} q(x_0), \quad g_g = \tilde{\Delta} g(x_0) \]  
(23)
where \( \tilde{\Delta} q(x_0) \) and \( \tilde{\Delta} g(x_0) \) are the input distributions of the polarized partons at \( x_0 = \mu^2/s \). They do not depend on \( Q^2 \). Eq. (23) allows us to express \( C_\pm(\omega) \) in terms of \( \Delta q(\omega) \) and \( \Delta g(\omega) \), which are related to \( \tilde{\Delta} q(x_0) \) and \( \tilde{\Delta} g(x_0) \) through the ordinary Mellin transform. Indeed,
\[ C_+ + C_- = \Delta q, \quad C_+ X + \frac{\sqrt{R}}{2H_{qg}} + C_- X - \frac{\sqrt{R}}{2H_{qg}} = \Delta g, \]  
(24)
with both \( \Delta q \) and \( \Delta g \) depending on \( \omega \). This leads to the following expressions for \( g_q \) and \( g_g \):
\[ g_q(x, Q^2) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} (1/x)^\omega \left[ \left( A(-) \Delta q + B \Delta g \right) e^{\Omega_+ y} + \left( A(+) \Delta q - B \Delta g \right) e^{\Omega_- y} \right], \]  
(25)
\[ g_g(x, Q^2) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} (1/x)^\omega \left[ \left( E \Delta q + A(+) \Delta g \right) e^{\Omega_+ y} + \left( -E \Delta q + A(-) \Delta g \right) e^{\Omega_- y} \right] \]  
(26)
with
\[ A(\pm) = \left( \frac{1}{2} \pm \frac{X}{2\sqrt{R}} \right), \quad B = \frac{H_{qg}}{\sqrt{R}}, \quad E = \frac{H_{gg}}{\sqrt{R}}. \]  
(27)
Eqs. (25, 26) express $g_1$ in terms of the parton distributions $\Delta q(\omega)$ and $\Delta g(\omega)$, which are related to the distributions $\Delta q(x_0)$ and $\Delta g(x_0)$ at very low $x$: $x_0 \approx \mu^2/s \ll 1$. Therefore, they hardly can be found from experimental data. It is much more useful to express $g_q, g_g$ in terms of the initial parton densities $\hat{\delta}q, \hat{\delta}g$ defined at $x \sim 1$. We can do it, using the evolution of $\Delta q(x_0), \Delta g(x_0)$ with respect to $s$. Indeed, the $s$-evolution of $\hat{\delta}q, \hat{\delta}g$ from $s \approx \mu^2$ to $s \gg \mu^2$ at fixed $Q^2$ ($Q^2 = \mu^2$) is equivalent to their $x$-evolution from $x \sim 1$ to $x \ll 1$.

In the $\omega$-space, the system of IREE for the parton distributions looks quite similar to Eqs. (12). However, the eqs for $\Delta q, \Delta g$ are now algebraic because they do not depend on $Q^2$:

$$\Delta q(\omega) = \left( <e_q^2>/2 \right) \delta q(\omega) + \left( 1/\omega \right) \left[ H_{qq}(\omega) \Delta q(\omega) + H_{gg}(\omega) \Delta g(\omega) \right],$$

$$\Delta g(\omega) = \left( <e_q^2>/2 \right) \hat{\delta}g(\omega) + \left( 1/\omega \right) \left[ H_{gq}(\omega) \Delta q(\omega) + H_{gg}(\omega) \Delta g(\omega) \right]$$

(28)

where $<e_q^2>$ is the sum of the quark electric charges ($<e_q^2> = 10/9$ for $n_f = 4$) and $\hat{\delta}g \equiv -(A'(\omega)/\pi \omega^2)\delta g$ is the starting point of the evolution of the gluon density $\delta g$. It corresponds to Fig. 1b where the upper blob is substituted by the quark box. Solving Eqs. (28), we obtain:

$$\Delta q = \frac{\left( <e_q^2>/2 \right) \left[ \omega(\omega - H_{gg}) \delta q + \omega H_{gg} \hat{\delta}g \right]}{\omega^2 - \omega(H_{qq} + H_{gg}) + (H_{qq}H_{gg} - H_{gq}H_{gq})},$$

(29)

$$\Delta g = \frac{\left( <e_q^2>/2 \right) \left[ \omega H_{gq} \delta q + \omega(\omega - H_{qq}) \hat{\delta}g \right]}{\omega^2 - \omega(H_{qq} + H_{gg}) + (H_{qq}H_{gg} - H_{gq}H_{gq})}.$$  

(30)

Then Eqs. (25, 26, 29, 30) express $g_1$ in terms of the initial parton densities $\delta q, \delta g$.

When we put $H_{gg} = H_{gq} = H_{gg} = 0$ and do not sum over $e_q$, we arrive at the expression for the non-singlet structure function $g_1^{NS}$: Obviously, in this case $A^+(+) = B = E = \Omega_2 = 0, A^-(+) = 1, \Omega_+ = H_{qq}$. However, the nonsinglet anomalous dimension $H_{qq}$ should be calculated in the limit $b_{gg} = b_{gq} = b_{gg} = b_{gq} = 0$. We denote such $H_{qq} \equiv H^{NS}$. The explicit expression for it is:

$$H^{NS} = (1/2) \left[ \omega - \sqrt{\omega^2 - 4b_{gg}} \right].$$

(31)

Therefore, we arrive at

$$g_1^{NS} = \frac{e_q^2}{2} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \left( \frac{\omega \delta q}{\omega - H^{NS}} \right) \left( 1/x \right)^{\omega} \left( Q^2/\mu^2 \right)^{H^{NS}}.$$ 

(32)
3 Small-$x$ asymptotics for $g_1$

Before making use of Eqs. (25, 26 and 32) for calculating $g_1$ at small but finite values of $x$, let us discuss its asymptotics. When $x \to 0$ and $Q^2 \gg \mu^2$, one can neglect contributions with $\Omega_-$ in Eqs. (15). As is well known, $g_1 \sim (1/x)^{\omega_0}$ at $x \to 0$, with $\omega_0$ being the position of the leading singularity of the integrand of $g_1$. According to Eqs. (15), the leading singularity, $\omega^{NS}$ for $g^{NS}_1$ is the rightmost root of the equation

$$\omega^2 - 4b_{qq} = 0 \quad (33)$$

while the leading singularity, $\omega_0$ for $g_1$ is the rightmost root of

$$\omega^4 - 4(b_{qq} + b_{gg})\omega^2 + 16(b_{gg}b_{qq} - b_{qg}b_{gq}) = 0 \quad (34)$$

In our approach, all factors $b_{ik}$ depend on $\eta = \ln(\mu^2/\Lambda_{QCD})$, so the roots of Eqs. (33,34) also depend on $\eta$. This dependence is plotted in Fig. 3 for $\omega^{NS}$ and in Fig. 4 for $\omega_0$. Both the curve in Fig. 3 and the curve 1 in Fig. 4 have a maximum. We denote this maximum as the intercept. Therefore,

$$g^{NS}_1 \sim e_q^2\delta q(1/x)^{\Delta^{NS}}(Q^2/\mu^2)^{\Delta^{NS}/2},$$

$$g_1 \sim (< e_q^2 > /2)[Z_1\delta q + Z_2\delta g](1/x)^{\Delta}(Q^2/\mu^2)^{\Delta/2},$$

and we find for the intercepts

$$\Delta^{NS} \approx 0.4, \Delta \approx 0.86 \quad (36)$$
and $Z_1 = -1.2$, $Z_2 = -0.08$. This implies that $g_1^{NS}$ is positive when $x \to 0$ whereas $g_1^{S}$ can be either positive or negative, depending on the relation between $\delta q$ and $\delta g$. In particular, $g_1$ is positive when
\[ 15\delta q + \delta g < 0. \tag{37} \]
on otherwise it is negative. In other words, the sign of $g_1$ at small $x$ can be positive if the initial gluon density is negative and large.

4 $g_1^{NS}$ at finite values of $x$

Let us estimate the impact of the total resummation of DL and SL contributions on $g_1^{NS}$. According to Eq. (32), the value of $g_1(x, Q^2)^{NS}$ depends both on the perturbative terms and on the inputs $\Delta q$. The latter can be obtained by fitting the experimental data and it is known (see e.g. (3)) that widely different formulae for $\Delta q$ can be used. In order to avoid discussing the fitting procedure and as in this paper we present our results only the perturbative part of $g_1^{NS}$, we can assume
\[ \Delta q = \delta(1 - x). \tag{38} \]

Then let us calculate the ratio
\[ R = g_1^{NS} / \tilde{g}_1^{NS}, \tag{39} \]
where $g_1^{NS}$ is the LO DGLAP non-singlet $g_1$. The results of a numerical calculations for $R$ at $Q^2 = 20\text{GeV}^2$, $\mu = 1.5\text{GeV}$ are shown in Table 1.
Table 1: Ratio $R$ of structure functions $g_1^{NS}$ calculated in DLA and DGLAP LO.

| $x$   | $R$  |
|-------|------|
| 0.1   | 1.60 |
| 0.01  | 2.59 |
| 0.001 | 4.33 |
| 0.0001| 7.46 |

It shows that the impact of the total resummation of DL contributions is negligible for $x \geq 0.1$ but it grows fast with decreasing of $x$, achieving the values $R = 2.6$ at $x = 10^{-2}$ and $R = 7.5$ at $x = 10^{-4}$.

5 Conclusion

The total resummation of the most singular ($\sim \alpha_s^n/\omega^{2n+1}$) terms in the expressions for the anomalous dimensions and the coefficient functions leads to the expressions of Eqs. (10,25,26,32) for the singlet and the non-singlet structure functions $g_1$. It guarantees the Regge (power-like) behavior of $g_1$, $g_1^{NS}$ when $x \to 0$, with the intercepts given by Eq. (36). The intercepts $\Delta_{NS}, \Delta_S$ are obtained with the running QCD coupling effects taken into account. The value of the non-singlet intercept $\Delta_{NS} \approx 0.4$ is now confirmed by several independent analysis of experimental data and our result $\Delta_S \approx 0.86$ is in a good agreement with the estimate of Ref. [12]: $\Delta_S = 0.88 \pm 0.14$ obtained from analysis of the HERMES data. Eq. (32) states that $g_1^{NS}$ is positive both at $x \sim 1$ and at $x \ll 1$. The situation concerning the singlet $g_1$ is more involved: being positive at $x \sim 1$, the singlet $g_1$ can remain positive at $x \ll 1$ only if the initial parton densities obey Eq. (37), otherwise it becomes negative. The ratio of our results versus the DGLAP ones for non-singlet $g_1$ is given in Table 1. It shows explicitly that the impact of high-order DL contributions is small at $x \geq 0.1$ but it grows fast when $x$ is approaching $10^{-3} - 10^{-4}$.

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