An Experimental Study on Optimization in Permutation Spaces by Quantum-Inspired Evolutionary Algorithm Using Quantum Bit Representation

Yoshifumi Moriyama¹, Ichiro Iimura², Tomotsugu Ohno³ and Shigeru Nakayama⁴

¹, ³National Institute of Technology, Ariake College, 150 Higashihagio-machi, Omuta, Fukuoka 836-8585, Japan
²Prefectural University of Kumamoto, 3-1-100 Tsukide, Higashi-ku, Kumamoto 862-8502, Japan
⁴Kagoshima University, 1-21-40 Korimoto, Kagoshima 890-0065, Japan
E-mail: ¹yosifumi@ariake-nct.ac.jp, ²iimura@pu-kumamoto.ac.jp, ³e47207@ga.ariake-nct.ac.jp, ⁴shignaka@gmail.com

Abstract The quantum-inspired evolutionary algorithm (QEA) and QEA with a pair-swap strategy (QEAPS), where each gene is represented by a quantum bit (qubit) in both algorithms, have shown superior search performance to the classical genetic algorithm in the 0-1 knapsack problem. Also, from experimental results for the integer knapsack problem, a novel integer-type gene-coding method that can obtain an integer value as an observation result by assigning multiple qubits in a gene locus has shown superior search performance to the conventional binary-type gene-coding method. However, the integer-type gene-coding method cannot deal with permutations simply. Therefore, we have proposed two interpretation methods that can deal with permutations in order to expand the gene-coding method based on the qubit representation. From the results of a computer experiment using the proposed interpretation methods in the traveling salesman problem, we have clarified that the two proposed interpretation methods can search for the optimal solution, even with the gene-coding method based on the qubit representation. Moreover, there are suitable rotation angles for discovering the optimal solution that depend on the algorithm.

Keywords: quantum bit (qubit), quantum-inspired evolutionary algorithm (QEA), QEA with pair-swap strategy (QEAPS), permutation spaces, traveling salesman problem (TSP)

1. Introduction

Recently, some evolutionary algorithms incorporating principles of quantum computation have been proposed [1]-[4]. They are not suitable for a quantum computer but are efficient evolutionary algorithms for digital computers. The quantum-inspired evolutionary algorithm (QEA) [2], [3] is one of the evolutionary algorithms incorporating principles of quantum computation. In QEA, each gene is represented by a quantum bit (qubit), and the quantum superposition state is imitated. QEA can automatically shift from a global search to a local search similar to simulated annealing (SA) [5]. Han et al. have shown that QEA has superior search performance to a genetic algorithm (GA) [6] in the 0-1 knapsack problem (0-1KP). Nakayama et al. proposed a simpler algorithm that is referred to as QEA with a pair-swap strategy (QEAPS) [7], [8]. QEAPS requires fewer parameters to be adjusted than QEA. Nakayama et al. showed that QEAPS can find similar or even better quality solutions than QEA in 0-1KP. However, in QEA and QEAPS, each gene is represented by a qubit and both algorithms can only use a binary value as an observation result for a qubit.

Therefore, Iimura et al. proposed a novel integer-type gene-coding method [9], [10] that can obtain an integer value as an observation result by assigning multiple qubits in a gene locus. Moreover, they implemented the gene-coding method in both QEA and QEAPS and showed that it can search for a similar or a superior solution in a shorter time than a conventional binary-type gene-coding method in the integer knapsack problem (IKP). However, the integer-type gene-coding method cannot deal with permutations simply.

In order to expand the gene-coding method based on the qubit representation, this paper proposes two interpretation methods that can deal with permutations. The proposed interpretation methods interpret
integer values, which are obtained from the integer-type gene-coding method, as permutations.

Moreover, we have implemented the proposed interpretation methods in both QEA and QEAPS, showed the experimental results for the traveling salesman problem (TSP), and clarified their characteristics.

2. Solution Search Using Qubit Representation for Gene of Individual

This section overviews the process of QEA and QEAPS, which use the qubit representation for a gene of an individual. We first describe the qubit representation used for the gene coding. Next, we describe a binary-type gene-coding method and an integer-type gene-coding method based on the qubit representation. Using the binary-type gene-coding method can solve binary combinatorial optimization problems such as 0-1KP, and using the integer-type gene-coding method can solve integer combinatorial optimization problems such as IKP. After that, we describe the process of QEA and QEAPS.

2.1 Qubit representation of gene

The classical GA (CGA) usually uses a definite binary, integer, or real value, or a character in a gene representation. However, the qubit representation can be used as a gene in QEA and QEAPS. In general, a qubit is described by a two-dimensional column vector in a complex vector space where the inner product is defined. It uses the following standard bases as orthonormal base vectors.

\[
|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

(1)

The qubit can have a stochastic superposition state (vector sum) of the two vectors \(|0\rangle\) and \(|1\rangle\) with each having a complex probability amplitude. The superposition state \(\varphi\) of the qubit can be illustrated by the Bloch sphere.

Figure 1 shows the genes with the qubit representation in an individual and the observation results of qubits. Chromosome \(q_i\) of individual \(i\) is described as \(q_i = q_{i1} \otimes q_{i2} \otimes \cdots \otimes q_{im} \quad (j = 1, 2, \cdots, m)\) and is assigned multiple qubits in gene locus \(j\). Gene \(q_{ij}\) of chromosome \(q_i\) is described as a tensor product of the qubits, \(q_{ij} = q_{ij1} \otimes q_{ij2} \otimes \cdots \otimes q_{ijh} \quad (k = 1, 2, \cdots, h)\).

Here, \(m\) is the number of genes and \(h\) is the number of qubits in a gene locus. If each qubit \(q_{ijk}\) is quantum-mechanically observed, then each binary value \(p_{ijk}\) of 0 or 1 can be obtained according to the probability amplitudes, \(\alpha_{ijk}\) and \(\beta_{ijk}\), of each qubit in the chro-
mosome. Qubit $q_{ijk}$ is as follows:

$$q_{ijk} = \alpha_{ijk} |0\rangle + \beta_{ijk} |1\rangle = \begin{bmatrix} \alpha_{ijk} \\ \beta_{ijk} \end{bmatrix}$$

where $|\alpha|^2$ is the probability that state $|0\rangle$ is observed and $|\beta|^2$ is the probability that state $|1\rangle$ is observed, which are normalized so that $|\alpha|^2 + |\beta|^2 = 1$.

2.2 Binary-type gene-coding method based on qubit representation

As shown in Fig. 2, the binary-type gene-coding method [2], [3] assigns one qubit in each gene locus $j$, that is, $h$ is set to 1. Therefore, the binary information $p_{ij}$, which is obtained by observing each qubit, is a 1-bit binary value. Then, the binary information $p_i$ is a solution, and the fitness $f(p_i)$ of chromosome $q_i$ is calculated from the binary information $p_i$ in the same way as in CGA.

2.3 Integer-type gene-coding method based on qubit representation

The integer-type gene-coding method [9], [10] uses multiple qubits in a gene locus $j$, obtains binary information as a result of observing the qubits, and obtains integer information by decoding the binary information. The idea is schematically shown in Fig. 3.

The integer-type gene-coding method interprets the binary information $p_{ij} = [p_{ij1}, p_{ij2}, \cdots, p_{ijn}]^T$, that is, the observation result obtained by assigning multiple qubits $q_{ij}$ in gene locus $j$, as a pure binary code. Finally, the integer-type gene-coding method decodes the pure binary code $p_{ij}$ to an integer value $x_{ij}$, and obtains integer information $x_i$ by assigning multiple qubits $q_{ij}$ in gene locus $j$, as a pure binary code. For example, the pure binary code “110” can be decoded to the integer value “6”. The integer information $x_i$ is a solution, and the fitness of the chromosome $f(x_i)$ can be evaluated from the integer information $x_i$ in the same way as in CGA.

2.4 Procedure of QEA and QEAPS in IKP

This section describes the procedure of QEA and QEAPS using an individual represented by the integer-type gene-coding method. Figure 4 shows the update cycle of an individual represented by the integer-type gene-coding method. In addition, this update cycle is for the case of a binary-type gene-
coding method, assuming that \( x_i \) is \( p_i \), \( b_i \) is \( s_i \), and \( h \) is set to 1 in order to constrain \( x_{ij} \) to only take a value of 0 or 1.

Individual \( i \) keeps chromosome \( q_i \) represented by multiple qubits as well as best solution information as the personal best obtained up to the present generation. The best solution information involves not only the integer information \( s_i \) but also the binary information \( b_i \) of the personal best.

To begin with, the initialization is carried out by setting \( \alpha_{ijk} \) and \( \beta_{ijk} \) to \( 1/\sqrt{2} \) in order to observe states \(|0\rangle \) and \(|1\rangle \) with equal probability in individual \( i \) (\( j = 1, 2, \ldots, m \), and \( k = 1, 2, \ldots, h \)). Next, the evolution of an individual with qubits and the update of the best solution information in the individual are repeated in accordance with the following procedure until a given termination condition is satisfied.

First, binary information \( p_i \) is obtained from observing chromosome \( q_i \) according to its probability amplitudes. Then, integer information \( x_i \) is obtained from decoding the binary information \( p_i \). The binary information \( p_i \) and integer information \( x_i \) may be repaired into a feasible solution if \( x_i \) is not a feasible solution [10].

Next, the fitness \( f(x_i) \) is calculated from the integer information \( x_i \), and the fitness of the individual is decided. If the currently obtained integer information \( x_i \) is superior to the best solution information \( s_i \) as the personal best obtained up to the present generation, then the best solution informations \( b_i \) and \( s_i \) are replaced by \( p_i \) and \( x_i \), respectively.

If the best solution information \( s_i \) is superior to the integer information \( x_i \), qubit \( q_i \) is updated by a unitary transformation based on the rotation angle list \( u_i \) so that it becomes closer to the binary best solution information \( b_i \). Then, the rotation angle list \( u_i \) is created from each value of \( p_{ijk}, b_{ijk} \), and the magnitude correlation of \( f(x_i) \) and \( f(s_i) \). This list is used to increase and decrease the observation probabilities of \(|0\rangle \) and \(|1\rangle \). How to decide the rotation angle \( \theta_{ijk} \) is shown in Table 1 [2], [3]. A unitary transformation can be used to change the ratio of the probability amplitudes \( \alpha_{ijk} \) and \( \beta_{ijk} \) of the superposition state as follows:

\[
\begin{bmatrix}
\alpha'_{ijk} \\
\beta'_{ijk}
\end{bmatrix} = \begin{bmatrix}
\cos(\theta_{ijk}) & -\sin(\theta_{ijk}) \\
\sin(\theta_{ijk}) & \cos(\theta_{ijk})
\end{bmatrix} \begin{bmatrix}
\alpha_{ijk} \\
\beta_{ijk}
\end{bmatrix}
\]  

After that, an individual’s best solution informations \( b_i \) and \( s_i \) are transferred to the other individuals by the local migration process and the global migration process in QEA [2], [3], or the pair-swap process in QEAPS [7], [8].

In the initial stage of the search, QEA and QEAPS act as a random search. The algorithms can automatically shift from a global search to a local search, with \( \alpha_{ijk} \) and \( \beta_{ijk} \) gradually converging from \( 1/\sqrt{2} \) to 0 or 0. The magnitude of \( \theta_{C} \) affects the quality of the solution and the convergence speed. If \( \theta_{C} \) is too large, the solutions may diverge or converge prematurely to a local optimal solution. If \( \theta_{C} \) is too small, the convergence speed may become slow and the search performance will degrade [2], [3].

### 3. Interpretation Method Based on Integer-Type Gene-Coding Method

We have proposed two interpretation methods that can interpret integer information as permutation information based on the integer-type gene-coding method. This section describes the two proposed interpretation methods. We describe the search process in TSP using QEA and QEAPS for the search algorithms and describe in detail the two proposed interpretation methods.

#### 3.1 Procedure of QEA and QEAPS in TSP

Figure 5 shows the update cycle of an individual in TSP. Individual \( i \) keeps chromosome \( q_i \), which is represented by the integer-type gene-coding method, and best solution information as the personal best obtained up to the present generation. The best solution information involves not only permutation information \( s_i \) but also binary information \( b_i \) of the personal best.

To begin with, the initialization is carried out by setting \( \alpha_{ijk} \) and \( \beta_{ijk} \) to \( 1/\sqrt{2} \) in individual \( i \) (\( j = 1, 2, \ldots, m \), and \( k = 1, 2, \ldots, h \)). Next, the evolution of an individual using qubits and the update of the best solution information in the individual are repeated in accordance with the following procedure until a given termination condition is satisfied.

First, binary information \( p_i \) is obtained from observing chromosome \( q_i \) according to its probability amplitudes. Then, integer information \( x_i \) is obtained from decoding the binary information \( p_i \). Here, in the case of an integer array \( x_i \) involving some overlapping numbers or some numbers greater than the number of cities \( m \), such as "6" at \( x_{i1} \) and \( x_{i3} \) in Fig. 5, \( x_i \) cannot deal with permutations simply. Therefore, the integer information \( x_i \) is interpreted as permutation information \( t_i \) using the interpretation method described in detail in the next section.
Next, the fitness $f(t_i)$ is calculated from the permutation information $t_i$, and the fitness of the individual is decided. If the currently obtained permutation information $s_i$ is superior to the best solution information $s_i$ as the personal best obtained up to the present generation, then the best solution informations $p_i$ and $s_i$ are replaced by $t_i$ and $s_i$ respectively. Then, if the best solution information $s_i$ is superior to the integer information $x_i$, qubit $q_i$ is updated by the unitary transformation shown in Eq. (3), which is based on the rotation angle list $u_i$ shown in Table 2.

After that, an individual’s best solution informations $p_i$ and $s_i$ are transferred to the other individuals by the two migration processes in QEA or the pair-swap process in QEAPS, similarly to in IKP.

### 3.2 Interpretation methods

As described above, the integer-type gene-coding method in the previous study [9], [10] cannot deal with permutations simply. In this study, we propose two interpretation methods for the integer-type gene-coding method that can convert integer information $x_i$ to permutation information. We describe the details of the interpretation procedures below.

**Cardinal number method** $I_1$ Figure 6 shows the interpretation process from integer information $x_i$ to permutation information $t_i$. $x_i$ is the integer information, which is decoded as the observation result $p_i$ of qubit $q_i$, in the integer-type gene-coding method.

First, the integers $x_i$ are ranked in ascending order. If there are equal values in $x_i$, the ranks are decided randomly. That is, there is a case that $(r_{i1}, r_{i3})$ is $(4, 5)$ or $(5, 4)$ in Fig. 6(a).

Next, each rank information $r_{ij}$ is interpreted as city $c_{r_{ij}}$, which is visited $j$th, and we obtain
tour information (permutation information) $t_i$. For example, in Fig. 6(a), city $c_{r_{i1}}$ named $c_5$ (“E”) is visited 1st, city $c_{r_{i2}}$ named $c_2$ (“B”) is visited 2nd, city $c_{r_{i3}}$ named $c_4$ (“D”) is visited 3rd, city $c_{r_{i4}}$ named $c_1$ (“A”) is visited 4th, and a city $c_{r_{i5}}$ named $c_3$ (“C”) is visited 5th. Finally, we obtain the tour information $t_i = E \rightarrow B \rightarrow D \rightarrow A \rightarrow C$, where city $c_1$ is “A”, city $c_2$ is “B”, city $c_3$ is “C”, city $c_4$ is “D”, and city $c_5$ is “E”.

**Ordinal number method $I_2$** Figure 7 shows the interpretation process from integer information $x_i$ to permutation information $t_i$. $x_i$ is the decoded integer information in the integer-type gene-coding method.

| Decoded integer information $x_i$ | Rank $x_i$ in ascending order | Fitness $f(t_i)$ |
|-----------------------------------|--------------------------------|------------------|
| $x_{i1}$ $x_{i2}$ $x_{i3}$ $x_{i4}$ $x_{i5}$ | $r_{i1}$ $r_{i2}$ $r_{i3}$ $r_{i4}$ $r_{i5}$ | $f(t_i) = 0.72$ |
| (a) The case of $x_{i3} = 6$ | | |

| Decoded integer information $x_i$ | Rank $x_i$ in ascending order | Fitness $f(t_i)$ |
|-----------------------------------|--------------------------------|------------------|
| $x_{i1}$ $x_{i2}$ $x_{i3}$ $x_{i4}$ $x_{i5}$ | $r_{i1}$ $r_{i2}$ $r_{i3}$ $r_{i4}$ $r_{i5}$ | $f(t_i) = 0.77$ |
| (b) The case of $x_{i3} = 2$ | | |

| Decoded integer information $x_i$ | Rank $x_i$ in ascending order | Fitness $f(t_i)$ |
|-----------------------------------|--------------------------------|------------------|
| $x_{i1}$ $x_{i2}$ $x_{i3}$ $x_{i4}$ $x_{i5}$ | $r_{i1}$ $r_{i2}$ $r_{i3}$ $r_{i4}$ $r_{i5}$ | $f(t_i) = 0.75$ |
| (a) The case of $x_{i3} = 6$ | | |

| Decoded integer information $x_i$ | Rank $x_i$ in ascending order | Fitness $f(t_i)$ |
|-----------------------------------|--------------------------------|------------------|
| $x_{i1}$ $x_{i2}$ $x_{i3}$ $x_{i4}$ $x_{i5}$ | $r_{i1}$ $r_{i2}$ $r_{i3}$ $r_{i4}$ $r_{i5}$ | $f(t_i) = 0.75$ |
| (b) The case of $x_{i3} = 2$ | | |

4. Experimental Results and Consideration

To clarify the search performance of the proposed interpretation methods, we first used the relatively small-scale TSP named “burma14” (number of cities: 232 Journal of Signal Processing, Vol. 19, No. 6, November 2015
The experimental results are depicted as Fig. 8 in the case of using the cardinal number method $I_1$ and Fig. 9 in the case of using the ordinal number method $I_2$. Figures 8 and 9 show the rate at which the optimal solution is discovered in all trials $R$ as a bar graph and the average of the minimum number of generations required to search for the optimal solution in all trials $G$ as a line graph.

In these results, the discovery rate of the optimal solution $R$ using the interpretation method $I_2$ is higher than using $I_1$ in all cases of QEA and QEAPS. Suppose “6” is replaced with “2” for $x_{13}$ during the search process, as shown in Fig. 6. In the case of using $I_1$, the locations of cities “B”, “D”, and “C” (Fig. 6(a)), which correspond to the fluctuated ranks ($r_{12}$, $r_{13}$, and $r_{15}$), are respectively reallocated to “C”, “B”, and “D” (Fig. 6(b)) independently of their order. On the other hand, in the case of using $I_2$, even if “6” is replaced with “2” for $x_{13}$ (Fig. 7), the order of city “C” corresponding to $x_{13}$ is shifted to 2nd, and the orders of cities “B” and “E”, which correspond to the fluctuated ranks ($r_{12}$, and $r_{15}$), are kept appropriately. Therefore, $I_2$ can bequeath a superior order of the cities (building block) to the next generation than $I_1$.

The average number of generations $G$ tends to decrease with increasing rotation angle. It is observed that there are suitable rotation angles for discovering the optimal solution that depend on the algorithm. The suitable rotation angles are $\pi/700$ and $\pi/600$ [rad] in the case of QEA and $\pi/300$, $\pi/200$, and $\pi/100$ [rad] in the case of QEAPS.

5. Conclusions

In order to expand the integer-type gene-coding method based on the qubit representation, this paper has proposed novel interpretation methods for dealing with permutations. From the experimental results, we have clarified that the two proposed interpretation methods can search for the optimal solution of TSP, even with the gene-coding method based on the qubit representation. In further research, we plan to verify the search performance for a larger-scale TSP, to examine the application of the proposed interpolation methods to other combinatorial optimization problems, and to consider more suitable gene-coding method.

Acknowledgment

This work was supported by JSPS KAKENHI Grant Numbers 25330265 and 26580097.
References

[1] A. Narayanan and M. Moore: Quantum-inspired genetic algorithms, Proc. IEEE Int. Conf. Evolutionary Computation, pp. 61-66, 1996.

[2] K.-H. Han and J.-H. Kim: Quantum-inspired evolutionary algorithm for a class of combinatorial optimization, IEEE Trans. Evolutionary Computation, Vol. 6, No. 6, pp. 580-593, 2002.

[3] K.-H. Han and J.-H. Kim: On setting the parameters of QEA for practical applications: Some guidelines based on empirical evidence, Genetic and Evolutionary Computation — GECCO 2003, pp. 427-428, 2003.

[4] S. Nakayama, I. Iimura, M. Matsuo and M. Maezono: Consideration on interference crossover method in genetic algorithm, IPSJ Journal (Japanese edition), Vol. 47, No. 8, pp. 2625-2635, 2006.

[5] S. Kirkpatrick, C. D. Gelatt and M. P. Vecchi: Optimization by simulated annealing, Science, Vol. 220, No. 4598, pp. 671-680, 1983.

[6] D. E. Goldberg: Genetic Algorithms in Search, Optimization and Machine Learning, Addison-Wesley, Reading, MA, 1989.

[7] S. Nakayama, T. Imabeppu and S. Ono: Pair swap strategy in quantum-inspired evolutionary algorithm, Genetic and Evolutionary Computation — GECCO 2006, Late Breaking Paper, Seattle, Washington, USA, 2006.

[8] S. Nakayama, T. Imabeppu, S. Ono and I. Iimura: Consideration on pair swap strategy in quantum-inspired evolutionary algorithm, IEICE Trans. Information and Systems (Japanese edition), Vol. J89-D, No. 9, pp. 2134-2139, 2006.

[9] I. Iimura, Y. Moriyama and S. Nakayama: Integer-type gene-coding method based on quantum bit representation in quantum-inspired evolutionary algorithm: Application to integer knapsack problem, J. Signal Processing, Vol. 16, No. 6, pp. 495-502, 2012.

[10] I. Iimura, Y. Moriyama and S. Nakayama: Search performance analysis of quantum bit representation method for integer-type gene according to difference in decoding process of observed bit sequence, IPSJ Journal (Japanese edition), Vol. 55, No. 2, pp. 1110-1115, 2014.

Yoshifumi Moriyama received his B. Eng., M. Eng. and Dr. Eng. degrees from Sophia University in 1992 and 1994, respectively, and his Ph.D. degree in engineering from Kagoshima University in 2004. From 1994 to 1997, he was a Research Scientist at Hitachi Research Laboratory, Hitachi, Ltd. From 1997 to 2002, he was a Lecturer at Kumamoto Prefectural College of Technology. From 2002 to 2003, he was a Research Associate at the Prefectural University of Kumamoto, from 2003 to 2006, he was a Senior Lecturer, from 2006 to 2012, he was an Associate Professor, and since 2012, he has been a Professor. He received the Best Paper Award for a Young Researcher from IPSJ in 2001, a Certificate of Merit for the Best Presentation from JSME in 2003, and the Best Paper Award for a Young Researcher from IPSJ Kyushu Chapter in 2003. His research interests include evolutionary computation, swarm intelligence, human interface, and learning environment design. He is a member of RISP, IPSJ, IEICE and IEEJ.

Ichiro Iimura received his B. Eng. and M. Eng. degrees from Sophia University in 1992 and 1994, respectively, and his Ph.D. degree in engineering from Kagoshima University in 2004. From 1994 to 1997, he was a Research Scientist at Hitachi Research Laboratory, Hitachi, Ltd. From 1997 to 2002, he was a Lecturer at Kumamoto Prefectural College of Technology. From 2002 to 2003, he was a Research Associate at the Prefectural University of Kumamoto, from 2003 to 2006, he was a Senior Lecturer, from 2006 to 2012, he was an Associate Professor, and since 2012, he has been a Professor. He received the Best Paper Award for a Young Researcher from IPSJ in 2001, a Certificate of Merit for the Best Presentation from JSME in 2003, and the Best Paper Award for a Young Researcher from IPSJ Kyushu Chapter in 2003. His research interests include evolutionary computation, swarm intelligence, human interface, and learning environment design. He is a member of RISP, IPSJ, IEICE and IEEJ.

Tomotsugu Ohno is a student on the Advanced Engineering Course of National Institute of Technology, Ariake College in Fukuoka. His research interests include evolutionary computation.

Shigeru Nakayama received his B.Sc.E.E. degree from Kyoto Institute of Technology in 1972 and his M.Sc.E.E. and Dr. Eng. degrees from Kyoto University in 1974 and 1977, respectively. From 1977 to 1981, he was a Research Associate at Kyoto Institute of Technology. From 1977 to 1981, he was a Research Associate at Sophia University in Tokyo. From 1981 to 1987 he was a Research Associate at Kyoto Institute of Technology. From 1987 to 1997, he was an Associate Professor at Hyogo University of Teacher Education. From 1997 to 2014, he was a Professor at the Department of Information Science and Biomedical Engineering, Kagoshima University, and since 2014, he has been an Emeritus Professor at Kagoshima University. His research interests include quantum computers, quantum-inspired algorithms, evolutionary computation, swarm intelligence, distributed parallel processing and image processing.

(Received May 17, 2015; revised July 29, 2015)