No scalar condensations outside reflecting stars with coupling terms from Ginzburg-Landau models

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Abstract

We consider static scalar fields coupled to the gradient where the coupling also appears in next-to-leading order Ginzburg-Landau models. We study condensation behaviors of scalar fields outside regular compact reflecting stars in the asymptotically flat background. For non-negative coupling parameters, we prove that the reflecting star cannot support coupled static scalar fields.

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I. INTRODUCTION

The famous black hole no hair theorem has attracted lots of interest from mathematicians and physicists. It states that static scalar fields cannot exist outside horizons of black holes in the asymptotically flat background, for progress see references [1]-[19] and reviews [20, 21]. It seems natural since horizons can absorb scalar fields outside black holes. However, such no scalar field property also appears in horizonless spacetime.

In the horizonless asymptotically flat background, Hod recently found that neutral compact reflecting stars cannot support formation of scalar field hairs [22]. It was further found that such no hair property appears for asymptotically flat neutral compact reflecting stars and scalar fields nonminimally coupled to the curvature [23]. This no scalar hair theorem was also extended to neutral compact reflecting stars in the asymptotically dS spacetime [24]. It was further found that large charged compact reflecting stars cannot allow the existence of scalar fields [25]-[32]. On the other side, in the extended Ginzburg-Landau superconductor model, there is a new term with scalar fields coupled to the gradient [33-35]. So it is of some interest to generalize the discussion in [22] by considering scalar fields coupled to the gradient.

This work is organized as follows. We firstly introduce the horizonless compact reflecting star gravity system with exterior scalar fields coupled to the gradient. For a non-negative coupling parameter, we prove that horizonless compact reflecting star cannot support the existence of static scalar field hairs. At last, we present our main conclusions.

II. NO COUPLED SCALAR FIELDS IN THE HORIZONLESS REFLECTING STAR BACKGROUND

According to extended Ginzburg-Landau models, the scalar fields may be coupled to the gradient [33-35]. We are interested in the case of coupled scalar fields in the asymptotically flat spacetime, whose Lagrangian density is given by [36-38]

\[ \mathcal{L} = R - |\nabla_\alpha \psi|^2 - \xi \psi^2 |\nabla_\alpha \psi|^2 - V(\psi^2). \] (1)

Here we label R as the Ricci curvature and \( \psi = \psi(r) \) as the scalar field. And \( \xi \) is the parameter describing coupling strengthen between scalar fields and the gradient. In order to obtain the final conclusion, we assume \( \xi > 0 \) in this work. The scalar field potential \( V(\psi^2) \) satisfying \( V(0) = 0 \) and \( \dot{V} = \frac{dV(\psi^2)}{d(\psi^2)} > 0 \), which is apparently satisfied by free scalar fields with \( V(\psi^2) = \mu^2 \psi^2 \) and mass \( \mu \).
The exterior spacetime of spherically symmetric horizonless compact star is
\[ ds^2 = -ge^{-\chi}dt^2 + \frac{dr^2}{g} + r^2(d\theta^2 + \sin^2\theta d\phi^2). \]  
(2)

The metric functions \( \chi \) and \( g \) only depend on the radial coordinate \( r \). The angular coordinates are labeled as \( \theta \) and \( \phi \) respectively. And the radius of the compact star is labeled as \( r_s \).

With the Lagrangian density (1), we obtain the equation of motion
\[ (1 + \xi\psi^2)\psi'' + [(1 + \xi\psi^2)(\frac{2}{r} - \frac{\chi'}{2} + \frac{g'}{g}) + 2\xi\psi\psi']\psi' - (\xi\psi^2 + \frac{\dot{V}}{g})\psi = 0. \]  
(3)

At the star radius, we impose the reflecting condition for the scalar field as
\[ \psi(r_s) = 0. \]  
(4)

At spatial infinity, metric functions have following asymptotical behaviors
\[ \chi \to 0, \quad g \to 1 \quad for \quad r \to \infty. \]  
(5)

The energy density can be expressed as
\[ \rho = -T_t^t = g\psi'^2 + g\xi\psi^2\psi'^2 + V(\psi^2). \]  
(6)

Physically acceptable solution requires that gravitational mass \( M = \int_0^\infty 4\pi r^2 \rho dr \) is finite, from which we obtain the following relation
\[ r^2 \rho \to 0 \quad for \quad r \to \infty. \]  
(7)

At the infinity, relations (6), (7) and \( \xi > 0 \) yields the condition
\[ \psi(\infty) = 0. \]  
(8)

According to relations (4) and (8), the scalar field \( \psi(r) \) must possess at least one extremum point \( r_{peak} \), where the function reaches a positive local maximum value or a negative local minimum value \[ 22. \] In the case that \( \psi(r) \) reaches a positive local maximum value, the scalar field satisfies the following relation
\[ \{ \psi > 0, \quad \psi' = 0 \quad and \quad \psi'' \leq 0 \} \quad for \quad r = r_{peak}. \]  
(9)

From relation (9) and \( \xi > 0 \), we deduce the characterized inequality
\[ (1 + \xi\psi^2)\psi'' + [(1 + \xi\psi^2)(\frac{2}{r} - \frac{\chi'}{2} + \frac{g'}{g}) + 2\xi\psi\psi']\psi' - (\xi\psi^2 + \frac{\dot{V}}{g})\psi < 0 \quad for \quad r = r_{peak}. \]  
(10)
In another case that the scalar field reaches a negative local minimum value, the relation becomes

\[
\{ \psi < 0, \quad \psi' = 0 \quad \text{and} \quad \psi'' \geq 0 \} \quad \text{for} \quad r = r_{\text{peak}}. \tag{11}
\]

From the relation (11) and \( \xi > 0 \), we deduce the characteristic inequality

\[
(1 + \xi \psi^2) \psi'' + [(1 + \xi \psi^2)(2 \frac{r'}{r} \chi' + \frac{g'}{g}) + 2 \xi \psi \psi'] \psi' - (\xi \psi'^2 + \dot{V} \psi) > 0 \quad \text{for} \quad r = r_{\text{peak}}. \tag{12}
\]

At the extremum point, the characteristic inequalities (10) and (12) are in contradiction with the equation of motion (3). It means the scalar field equation has no nonzero solution. Then we obtain a conclusion that horizonless compact reflecting stars cannot support static scalar fields coupled to the gradient with non-negative coupling parameters.

III. CONCLUSIONS

We investigated the condensation behaviors of static scalar fields outside asymptotically flat spherically symmetric compact stars. At the star radius, we take the reflecting boundary condition. In this work, we considered couplings between scalar fields and the gradient, where this type of coupling also appears in the next-to-leading order Ginzburg-Landau models. In the case that coupling parameters are non-negative, characteristic inequalities (10) and (12) are in contradiction with the equation of motion (3), which means that the reflecting star cannot support static coupled scalar fields.

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[1] J. D. Bekenstein, Transcendence of the law of baryon-number conservation in black hole physics, Phys. Rev. Lett. 28, 452 (1972).
[2] J. E. Chase, Event horizons in Static Scalar-Vacuum Space-Times, Commun. Math. Phys. 19, 276 (1970).
[3] C. Teitelboim, Nonmeasurability of the baryon number of a black-hole, Lett. Nuovo Cimento 3, 326 (1972).
[4] R. Ruffini and J. A. Wheeler, Introducing the black hole, Phys. Today 24, 30 (1971).
[5] S. Hod,Stationary resonances of rapidly-rotating Kerr black holes, The Euro. Phys. Journal C 73, 2378 (2013).
[6] S.Hod, The superradiant instability regime of the spinning Kerr black hole, Phys. Lett. B 758, 181(2016).
[7] Carlos Herdeiro, Vanush Paturyan, Eugen Radu, D.H. Tchrakian, Reissner-Nordström black holes with non-Abelian hair, Phys. Lett. B 772(2017)63-69.
