C-Planning: An Automatic Curriculum for Learning Goal-Reaching Tasks

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Abstract

Goal-conditioned reinforcement learning (RL) can solve tasks in a wide range of domains, including navigation and manipulation, but learning to reach distant goals remains a central challenge to the field. Learning to reach such goals is particularly hard without any offline data, expert demonstrations, and reward shaping. In this paper, we propose an algorithm to solve the distant goal-reaching task by using search at training time to automatically generate a curriculum of intermediate states. Our algorithm, Classifier-Planning (C-Planning), frames the learning of the goal-conditioned policies as expectation maximization: the E-step corresponds to planning an optimal sequence of waypoints using graph search, while the M-step aims to learn a goal-conditioned policy to reach those waypoints. Unlike prior methods that combine goal-conditioned RL with graph search, ours performs search only during training and not testing, significantly decreasing the compute costs of deploying the learned policy. Empirically, we demonstrate that our method is more sample efficient than prior methods. Moreover, it is able to solve very long horizons manipulation and navigation tasks, tasks that prior goal-conditioned methods and methods based on graph search fail to solve.

1 Introduction

Whereas typical RL methods maximize the accumulated reward, goal-conditioned RL methods learn to reach any goal. Arguably, many tasks are more easily defined as goal-reaching problems than as reward maximizing problems. While many goal-conditioned RL algorithms learn policies that can reach nearby goals, learning to reach distant goals remains a challenging problem. Some prior methods approach this problem by performing search or optimization over subgoals at test time. However, these test-time planning methods either rely on graph search [7,31], which scales poorly with dimensionality [14], or continuous optimization over subgoals [26], which is expensive and can result in model exploitation.

In this paper, we take a different tack and instead use search at training time to automatically generate a curriculum. When training the agent to reach one goal, our method first determines some intermediate waypoints enroute to that goal. Then, it commands the agent to reach those waypoints before navigating to the final destination. Collecting data in this manner improves the quality of data, allowing the agent to learn to reach distant goals. Importantly, our method does not perform planning at test time, decreasing the computational demands for deploying the learned agent.

Our curriculum does not require manual engineering or prior knowledge of the tasks. Rather, the curriculum emerges automatically when we use expectation maximization (EM) to maximize a lower bound on the probability of reaching the goal. The M-step corresponds to a prior method for goal-conditioned RL, while the E-step corresponds to graph search. Thus, EM presents a marriage between previous goal-conditioned RL algorithms and graph-search methods.

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1Code and videos of our results: https://ben-eysenbach.github.io/c-planning/
Figure 1: C-Planning is an algorithm for goal-conditioned RL that uses an automatic curriculum of waypoints to learn policies that can solve complex tasks. In this manipulation task, the goal requires moving the green puck to the green dot and the red puck to the red dot. Our method learns to solve this task, manipulating multiple objects in sequence, without requiring any reward functions, manual distance functions, or human demonstrations.

The main contribution of this work is a goal-conditioned RL algorithm, C-Planning, that excels at reaching distant goals. Our method uses an automatic curriculum of subgoals to accelerate training. We show that our curriculum emerges from applying variational inference to the goal-reaching problem. Unlike prior work, our method does not require graph search or optimization at test-time. Empirically, C-Planning not only matches but surpasses the performance of these prior search-based methods, suggesting that it is not just amortizing the cost of graph search. We empirically evaluate C-Planning on temporally-extended 2D navigation tasks and complex 18D robotic manipulation tasks. C-Planning improves the sample efficiency of prior goal-conditioned RL algorithms and manages to solve more difficult manipulations tasks such as rearranging multiple objects in sequence (see Fig. 1.). To the best of our knowledge, no prior method has learned tasks of such difficulty without requiring additional assumptions.

2 RELATED WORK

The problem of learning to achieve goals has a long history, both in the control community Lyapunov and the RL community. Many prior papers approach goal-conditioned RL as a reward-driven, multi-task learning problem, assuming access to a goal-conditioned reward function. One unique feature of goal-conditioned RL, compared to other multi-task RL settings, is that it can also be approached using reward-free methods, such as goal-conditioned behavior cloning and RL methods that employ hindsight relabeling. While goal-conditioned behavior cloning methods are simple to implement and have shown excellent results on a number of real-world settings, they are not guaranteed to recover the optimal policy without additional assumptions (e.g., determinism, online data collection). While both methods excel at certain control tasks, both often struggle to solve tasks with longer horizons.

To solve longer-horizon tasks, prior work has combined goal-conditioned RL with graph search, noting that goal-conditioned value functions can be interpreted as dynamical distances and RL methods that employ hindsight relabeling. These prior methods typically proceed in two phases: the first phase learns a goal-conditioned policy, and the second phase combines that policy with graph search. While these methods have demonstrated excellent results on a number of challenging control tasks, including real-world robotic navigation, the stage-wise approach has a few limitations. First, the post-hoc use of graph search means that graph search cannot improve the underlying goal-conditioned policy. It is well known that the performance of RL algorithms is highly dependent on the quality of the collected data. By integrating search into the learning of the underlying goal-conditioned policy, our method will improve the quality of the data used to train that policy. A second limitation of these prior methods is the cost of deployment: choosing actions using graph search requires at least $O(|V|)$ queries to a neural network. In contrast, our method performs search at training time rather than test time, so the cost of deployment is $O(1)$. While this design decision does increase the computational complexity of training, it significantly decreases the latency at deployment. Our method is similar to RIS, which also performs search during training instead of deployment. Our method differs from RIS in how search is used: whereas RIS modify the objective, our method uses search to modify the data. This difference not only unifies the objectives for RL and graph search, but also significantly improves the performance of the algorithm. A subtle distinction that avoids favoring the learned policy and unifies the objectives for RL and graph search. We demonstrate the importance of this difference in our experiments.

While computing all pairwise distances requires $O(|V|^2)$ time, only $|V|$ edges change at each time step. The cost of computing the remaining edges can be amortized across time.
Effectively solving goal-conditioned RL problems requires performing good exploration. In the goal-conditioned setting, the quality of exploration depends on how goals are sampled, a problem studied in many prior methods [6,10,11,27,28,30,38,39]. These methods craft objectives that try to optimize for learning progress, and the resulting algorithms achieve good results across a range of environments. Our method differs from these methods in that the method for optimizing the goal-conditioned policy and the method for sampling waypoints are jointly optimized using the same objective. We demonstrate the importance of this difference in our experiments.

Our method builds on the idea that reinforcement learning can be cast as inference problems [2,19,29,35,36,40]. The observation that variational inference for certain problems corresponds to graph search is closely related to Attias [3]. Our work extends this inferential perspective to hierarchical models.

3 Preliminaries

We introduce the goal-conditioned RL problem and a recent goal-conditioned RL algorithm, C-Learning, upon which our method builds.

Goal-Conditioned RL. We focus on controlled Markov processes defined by states \( s_t \in S \) and actions \( a_t \). The initial state is sampled \( s_0 \sim p_0(s_0) \) and subsequent states are sampled \( s_{t+1} \sim p(s_{t+1} \mid s_t, a_t) \). Our aim is to navigate to the goal states, \( s_g \in S \), which are sampled from \( s_g \sim p_g(s_g) \). We define a policy \( \pi(a_t \mid s_t, s_g) \) conditioned on both the current state and the goal. The objective is to maximize the probability (density) of reaching the goal in the future.

To derive our method, or any other goal-conditioned RL algorithm, we must make a modeling assumption about when we would like the agent to reach the goal. This modeling assumption is only used to derive the algorithm, not for evaluation. Formally, let \( \Delta \in \mathbb{N}^+ \) be a random integer indicating when we reach the goal. The user specifies a prior \( p(\Delta) \). Most prior work (implicitly) uses the geometric distribution, \( p(\Delta) = \text{Geom}(1 - \gamma) \). The geometric distribution is ubiquitous because it is easy to incorporate into temporal difference learning, not because it is a particularly reasonable prior belief on when the agent will solve the task or when the episode will terminate. Prior work has also considered non-geometric distributions [9]. In this paper, we use the negative binomial distribution, \( p(\Delta) = \text{NegBinom}(p = 1 - \gamma, n = 2) \). Given a prior \( p(\Delta) \), we define the distribution over future states as

\[
p^\pi(s_{t+\Delta} = s_t \mid s_t, a_t) = \mathbb{E}_{p(\Delta)} \left[ p^\pi(s_{t+\Delta} = s_t \mid s_t, a_t) \right],
\]

where \( p^\pi(s_{t+\Delta} = s_t \mid s_t, a_t) \) is the probability density of reaching state \( s_{t+\Delta} \) exactly \( \Delta \) steps in the future when sampling actions from \( \pi(a_t \mid s_t, s_g) \). For example, the \( \gamma \)-discounted state occupancy measure \( \text{EG}(1 - \gamma) \) can be written as \( \text{pGeom}(1 - \gamma) \). Our objective for goal-reaching is to maximize the probability of reaching the desired goal:

\[
\max_{\pi} \mathbb{E}_{p_0(s_0)} \left[ p^{\pi(s_{t+\Delta} = s_g \mid s_0)}(s_{t+\Delta} = s_g) \right].
\]

C-Learning. Our method builds upon a prior method for goal-conditioned RL, C-Learning [8]. C-Learning learns a classifier for predicting whether a state \( s_t \) comes from a future state density \( p^\pi(\cdot \mid s_g)(s_{t+} = s_t \mid s_t, a_t) \) or a marginal state density:

\[
p^\pi(s_{t+} = s_t) = \int p^\pi(\cdot \mid s_g)(s_{t+} = s_t \mid s_t, a_t) p(s_t, a_t) ds_t da_t.
\]

The Bayes-optimal classifier can be written in terms of these two distributions:

\[
C_\theta(s, s_g) = \frac{p^\pi(s_{t+} = s_g \mid s)}{p_{\text{Geom}}(s_{t+} = s_g \mid s) + p_{\text{Geom}}(s_{t+} = s_g)}. \tag{3}
\]

In C-Learning, this classifier acts as a value function for training the policy. Our method will use this classifier not only to update the policy, but also to sample waypoints.

4 C-Planning: Goal-Conditioned RL with Planning
We now present our method, C-Planning. The main idea behind our method is to decompose the objective of Eq. 2 into a sequence of easier goal-reaching problems. Intuitively, if we want a suboptimal agent to navigate from some initial state $s_0$ to a distant goal state $s_g$, having the agent try to reach that state again and again will likely prove futile. Instead, we can command a sequence of waypoints and then propose a simple practical method to over waypoints is difficult. In Sec. 4.1, we derive the optimal waypoint distribution, and then propose a simple practical method to sample from this distribution in Sec. 4.2.

At a high-level algorithm summary, our method consists of two steps: using waypoint sampling to collect experience and performing standard goal-conditioned RL using the collected experience. The main challenge is the first step, as modeling the distribution over waypoints is difficult. In Sec. 4.1, we derive the optimal waypoint distribution, and then propose a simple practical method to sample from this distribution in Sec. 4.2.

### 4.1 Planning and Variational Inference

To formally derive our method for waypoint sampling, we cast the problem of goal-reaching as a latent variable problem. We assume that the agent starts at state $s_0$ and ends at state $s_g$, but the intermediate states that the agent will visit are unknown latent variables. This problem resembles standard latent variable modeling problems (e.g., VAE), where the intermediate state serves the role of the inferred representation. We can thus derive an evidence lower bound on the likelihood of reaching the desired goal:

**Lemma 1.** The objective $\mathcal{L}$, defined below, is a lower bound on the goal-reaching objective (Eq. 2):

$$
\log p_{\text{NDE 가운데}}(s_{t+} = s_g \mid s_0) \geq E_{q(s_w \mid s_g, s_0)} \left[ \log p_{\text{GEOM}}^\pi(s_{t+} = s_g \mid s_w) + \log p_{\text{GEOM}}^\pi(s_w \mid s_0) 
- \log q(s_w \mid s_g, s_0) \right] \triangleq L(\pi, q(s_w \mid s_g, s_0)).
$$

See Appendix A.1 for the proof. The lower bound, $\mathcal{L}$, depends on two quantities: the goal-conditioned policy and an inferred distribution over waypoints, $q(s_w \mid s_0, s_g)$. This bound holds for any choice of $q(s_w \mid s_0, s_g)$, and we can optimize this lower bound with respect to this waypoint distribution. We will perform expectation maximization [4] to optimize the lower bound, alternating between computing the optimal waypoint distribution and optimizing the goal-conditioned policy using this waypoint distribution. We now describe these two steps in detail.

**E-Step.** The E-step estimates the waypoint distribution, $q(s_w \mid s_0, s_g)$. The three terms in the lower bound indicate that the sampled waypoint should be reachable from the initial state, the goal state should be reachable from the sampled waypoint, and that the waypoint distribution should have high entropy. These first two terms resemble shortest-path planning, where distances are measured using log probabilities. Importantly, reachability is defined in terms of the capabilities of the current policy. We can analytically solve for the optimal waypoint distribution:

**Lemma 2.** The optimal choice for the waypoint distribution satisfies:

$$
q^*(s_w \mid s_g, s_0) = \frac{P_{\text{GEOM}}^\pi(s_g \mid s_w)P_{\text{GEOM}}^\pi(s_w \mid s_0)}{\int P_{\text{GEOM}}^\pi(s_g \mid s_{w'})P_{\text{GEOM}}^\pi(s_{w'} \mid s_0)ds_{w'}}.
$$

See Appendix A.1 for a full derivation. We illustrate this waypoint distribution in Fig. 2. In general, accurately estimating the optimal distribution, $q^*$, is challenging. However, in the next section, we develop a simple algorithm that only requires learning a classifier instead of a generative model.

**M-step.** The M-step optimizes the lower bound with respect to the goal-conditioned policy. We can ignore the $- \log q(s_w \mid s_0, s_g)$ term, which does not depend on the policy. The remaining two terms look like goal-conditioned RL objectives, with a subtle but important difference in how the
traj ectories are sampled. When collecting data, a standard goal-conditioned RL algorithm initially samples a goal and then collects a trajectory where the policy attempts to reach that goal. Our method collects data in a different manner. The agent samples a goal, then samples an intermediate waypoint that should lead to that goal. The agent attempts to reach the waypoint before attempting to reach the goal. After the trajectory has been collected, the policy updates are the same as a standard goal-conditioned RL algorithm.

**Two mental models.** The algorithm sketch we have presented, which will be fleshed out in the subsequent section, can be interpreted in two ways. One interpretation is that the method performs a soft version of graph search during exploration, using a distance function of \(d(s_1, s_2) = \log p_{\text{Geom}}(s_{t+1} = s_2 \mid s_1)\). From this perspective, the method is similar to prior work that performs graph search during test-time \([7, 31]\). However, our method will perform search at training time. Extensive ablations in Sec. 5 show that our method outperforms alternative approaches that only perform search during test-time.

The second interpretation focuses on the task of reaching the goal from different initial states. The choice of waypoint distribution, and the policy’s success at reaching those waypoints, determines the initial state distribution for this task. Intuitively, the best initial state distribution is one that sometimes starts the agent at the true initial state \(s_0\), sometimes starts the agent at the final state \(s_g\), and sometimes starts the agent in between. In fact, prior work has formally shown that the optimal initial state distribution is the marginal state distribution of the optimal policy \([17]\). Under somewhat strong assumptions\(^1\), this is precisely what our waypoint sampling achieves. We refer the reader for the discussion and experiments in Appendix A.

### 4.2 A Practical Implementation

We now describe how we implement the policy updates (M-step) and waypoint sampling (E-step). For the policy updates, we simply apply C-learning \([8]\). The main challenge is waypoint sampling, which requires sampling a potentially high-dimensional waypoint distribution. While prior work \([11]\) has approached such problems by fitting high-capacity generative models, our approach will avoid such generative models and instead use importance sampling.

Let \(b(\cdot)\) be the background distribution, which we take to be the replay buffer. Our algorithm corresponds to the following two-step procedure. First, we sample a batch of waypoints from the buffer. Then, we sample one waypoint from within that batch using the normalized importance weights, \(\frac{g(s_w \mid s_g, s_0)}{b(s_w)}\). We can estimate these importance weights using the classifier learned by C-learning (Eq. 3):

\[
\frac{g(s_w \mid s_g, s_0)}{b(s_w)} = \frac{C_\theta(s_w, s_g)}{1 - C_\theta(s_w, s_g)} \frac{C_\theta(s_0, s_w)}{1 - C_\theta(s_0, s_w)} Z(s_0, s_g),
\]

where \(Z(s_0, s_g)\) is a normalizing constant.

See Appendix A.3 for a full derivation. Since we will eventually normalize these importance weights (over \(s_w\)), we do not need to estimate \(Z(s_0, s_g)\). We call our complete algorithm C-Planning, and summarize it in Alg. 1. Note that our algorithm requires changing only a single line of pseudocode: during data collection, we command the agent to the intermediate waypoint \(s_w\) instead of the final goal \(s_g\). The sampling procedure in Alg. 2 is scalable and easy to implement.

The core idea of our method is to sample waypoints enroute to the goal. Of course, at some point we must command the agent to directly reach the goal. We introduce a hyperparameter \(n_g\) to indicate how many times we resample the intermediate waypoint before commanding the agent to reach the goal. This parameter is not the planning horizon, which is always set to 2.

\(^1\)The assumptions are that (1) the policy always reaches the commanded waypoint, and that (2) the goal-reaching probabilities \(p_{\text{Geom}}(\cdot \mid \cdot)\) reflect the probability that the optimal policy reach some state.
5 Experiments

Our experiments study whether C-Planning can compete with prior goal-conditioned RL methods both on benchmark tasks and on tasks designed to pose a significant planning and exploration challenge. Our first experiments use these tasks to compare C-Planning to prior methods for goal-conditioned RL. We then study the importance of a few design decisions through ablation experiments. The benefits of C-Planning, compared with prior methods, are especially pronounced on these challenging tasks. To provide more intuition for why C-Planning outperforms prior goal-conditioned RL methods, we plot the gradient norm for the actor and critic functions, finding that C-Planning enjoys larger norm gradients for the critic and smaller variance of the gradient for the actor. Ablation experiments study the number of waypoints used in training. We finally measure the inference time of choosing actions, finding that C-Planning is up to 4.7× faster than prior methods that perform search at test time.

Environments. We investigate environments of various difficulty, some of which we visualize in Fig. 3. The first set of environments is taken directly from the Metaworld benchmark suite [37], a common benchmark for goal-conditioned RL. These tasks are challenging because of the manipulation skills required to push and reorient objects. While prior goal-conditioned RL methods do solve some of these tasks (e.g., Push and Reach), they stand at the edge of the capabilities of current methods. The second set of environments, 2D navigation mazes, are designed to stress-test the planning capabilities of different methods. Successfully solving these tasks requires reasoning over long horizons of up to 50 steps. These are challenging because, unlike many benchmarks, they require non-greedy exploration: greedy strategies get stuck in local optima. Our final set of environments combine the challenges of these environments. We extend the tasks in Metaworld to involve sequential manipulation of multiple objects, similar to the tasks in prior work [34]. For example, the Obstacle-Drawer-Close task requires the robotic arm to manipulate multiple objects in sequence, first pushing an object and then opening a drawer. Solving these tasks are difficult because the agent is given no demonstrations, no reward shaping, and no manually specified distance metrics.

Baselines. We will compare C-Planning to a number of baselines. C-Learning is a recent goal-conditioned RL method that does not perform planning. C-Learning performs the same gradient updates as C-Planning, and only differs in how experience is collected. Comparing to C-Learning...
Figure 4: Comparison of goal-conditioned RL methods: We compare C-Planning to prior goal-conditioned RL algorithms on various tasks: (top) 2D maze navigation, (middle) benchmark manipulation tasks from Meta-world, and (bottom) robot manipulation tasks that require long horizon planning. For each task, we record the Euclidean distance to the goal, taking the minimum distance within an episode. All but the easiest task (Reach), C-Planning outperforms all prior methods, including those that perform planning. Only C-Planning is able to solve the most challenging navigation and manipulation tasks.

5.1 Comparison with Prior Goal-Conditioned RL Methods

To compare C-Planning to prior goal-conditioned RL algorithms, we start with three tasks from the MetaWorld suite: Reach, Push, Push-Wall. While C-Planning learns slightly slower than prior methods on the easiest task (Reach), we see a noticeable improvement over prior methods on the more challenging Push and Push-Wall tasks, tasks that require reasoning over longer horizons to solve. This observation suggests that that the waypoint sampling performed by C-Planning might be especially beneficial for solving tasks that require planning over long horizons.

To test the planning capabilities of C-Planning, we design a series of 2D navigation mazes. While the underlying locomotion motions are simple, these tasks post challenges for planning and are designed so that a greedy planning algorithm would fail. On all three mazes, C-Planning learns faster than all baselines and achieves a lower asymptotic distance to the goal, as compared to the baselines. On the most challenging maze, Maze-11x11, only our method is able to make any learning progress.
Figure 5: Planning at Training Versus Testing: We ablate the number of intermediate waypoints used to reach the goal, and compare against a variant of our method that performs planning during both training and testing (“C-Planning + SoRB”). This variant does not improve performance, indicating that the feedforward policy learned by C-Planning has already “internalized” these planning capabilities.

Of particular note is the comparison with SoRB, which uses the same underlying goal-conditioned RL algorithm as C-Planning, but differs in that it performs search at testing, rather than training. While SoRB performs better than C-learning, which does not perform planning, SoRB consistently performs worse than C-Planning. This observation suggests that C-Planning is not just amortizing the cost of performing search. Rather, these results suggest that incorporating search into training can produce significantly larger gains than incorporating search into the deployed policy.

As the last set of experiments, study higher dimensional tasks that require long-range planning. We design three environments: Obstacle-Drawer-Close, Obstacle-Drawer-Open and Push-Two. These tasks have a fairly large dimension (15 to 18) and require sequential manipulation of multiple objects. C-Planning outperforms all baselines on all three tasks. While C-learning makes some learning progress, SoRB (which modifies C-learning with search at test time) does not improve the results. RIS, which also samples waypoints during training but uses those waypoints differently, does not make any learning progress on these tasks. Taken together, these results suggest that waypoint sampling (as performed by C-Planning) can significantly aid in the solving of complex manipulation tasks, but only if that waypoint sampling is performed during training. Moreover, the comparison with RIS suggests that those waypoints should be used to collect new data, rather than to augment the policy learning objective. To the best of our knowledge, C-Planning is the first method to learn manipulation behavior of this complexity without additional assumptions (such as dense rewards or demonstrations).

5.2 Why Does C-Planning Work?

We ran many additional experiments to understand why C-Planning works so well. To start, we run ablation experiments to answer two questions: (1) Although C-Planning only applies planning at training time, does additionally performing planning at test time further improve performance? (2) How many times should we resample the waypoint before directing the agent to the goal?

Fig 5 shows the results of ablation experiments. C-Planning performs the same at test-time with and without SoRB-based planning, suggesting that our training-time planning procedure already makes the policy sufficiently capable of reaching distant goals, such that it does not benefit from additional test-time planning. Fig. 5 also shows that C-Planning is relatively robust to \( n_w \), the number of waypoints used before directing the agent to the goal. For all choices of this hyperparameter, C-Planning still manages to solve all the tasks.

**Gradient analysis.** We provide an empirical observation why C-Planning is better than vanilla goal-conditioned RL methods. We hypothesize that, by introducing waypoints, C-Planning provides a better learning signal, increasing the gradient norm for training the critic and decreasing the variance of the gradient for the policy. While prior work has theoretically analyzed the importance of gradient norms for RL [1], we will provide some empirical evidence for the importance of controlling gradient norms. In Fig. 7, we plot the norm of the gradient (mean and variance) of both C-Planning and C-Learning using the Maze-11x11 environment, finding that our method increases the norm of the critic norm and decreases the variance of the actor gradient norm.

**Test-time latency.** One important advantage of C-Planning is that it enables direct execution at test time, does not require any online planning. At test time, we directly set the agent’s goal to the
final goal state without commanding any intermediate states. This makes C-Planning considerably different from previous planning methods, such as SoRB \[7\]. Not only does our method achieve a higher return, but it does so with significantly less computing at deployment. We measure the execution time at evaluation time for C-Planning and SoRB with a various number of waypoints in Fig. 7. C-Planning is 1.74× faster than planning over four waypoints and 4.71× faster than planning over 16 waypoints in Push-Wall environment.

6 Conclusion

In this paper, we introduced C-Planning, a method for incorporating search into the training of goal-conditioned agents. The method enables the automatic generation of a curriculum over intermediate waypoints. Unlike prior methods, our approach avoids the computational complexity of performing search at test time. C-Planning is simple to implement on top of existing goal-conditioned RL algorithms and achieves state-of-the-art results on a range of navigation and manipulation tasks.

Our work suggests a few directions for future work. First, C-Planning samples waypoint that correspond to the optimal state distribution, which is provably optimal in some settings \[17\]. How might similar ideas be applied to reward-driven tasks? Second, while this paper studied sought to learn agents for reaching distant goals, we assumed that those distant goals were provided as part of the task definition. Goal-conditioned algorithms that can propose their own distant goals, while already studied in some prior work \[11\]|28], remains an important direction for future work.
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A Proofs and Additional Analysis

A.1 Deriving the Lower Bound (Proof of Lemma 1)

We first formulate the sampling procedure on starting states $s_0$, waypoints $s_w$, goals $s_g$ and the corresponding time horizon variable $t_1$ and $t_2$. Then we derive a lower bound on the target log density of Eq. 1. We then show that optimizing the lower bound through an EM procedure is equivalent to breaking the goal-reaching task into a sequence of easier sub-problems. Finally, we wrapped up this section with a practical algorithm.

Data Generation Process. The generative model for which inference corresponds to our planning is to break the goal-reaching task into a sequence of easier sub-problems. We then show that optimizing the lower bound through an EM procedure is equivalent to this. We learn $q(s | s_0, s_g)$ by optimizing an evidence lower bound on our main objective (Eq. 2).

We learn $q(s_w | s_0, s_g)$ by optimizing a lower bound on the following question in the inference procedure: what intermediate states $s_g$? Formally, we will estimate a distribution $q(s_w | s_0, s_g)$. We first formulate the sampling procedure on starting states $s_0$, waypoints $s_w$, goals $s_g$ and the corresponding time horizon variable $t_1$ and $t_2$. Then we derive a lower bound on the target log density of Eq. 1. We then show that optimizing the lower bound through an EM procedure is equivalent to breaking the goal-reaching task into a sequence of easier sub-problems. Finally, we wrapped up this section with a practical algorithm.

Inference process. Under the formulation of the data generation process above, we then aim to answer the following question in the inference procedure: what intermediate states $s_g$? Formally, we will estimate a distribution $q(s_w | s_0, s_g)$. We first formulate the sampling procedure on starting states $s_0$, waypoints $s_w$, goals $s_g$ and the corresponding time horizon variable $t_1$ and $t_2$. Then we derive a lower bound on the target log density of Eq. 1. We then show that optimizing the lower bound through an EM procedure is equivalent to breaking the goal-reaching task into a sequence of easier sub-problems. Finally, we wrapped up this section with a practical algorithm.

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Before optimizing the lower bound, we introduce a subtle modification to the lower bound:

$$L_2(\pi, q(s_w \mid s_g, s_0)) \triangleq \int q(s_w \mid s_g, s_0) \left( \log p_{Geom}(s_{t+} = s_g \mid s_w) + \log p_{Geom}(s_w \mid s_0) - \log q(s_w \mid s_g, s_0) \right) ds_w.$$  

The difference, highlighted in orange, is that the probability of reaching the waypoint is computed for a goal-conditioned policy that is commanded to reach that waypoint, rather than the final goal. We show that this new objective is also an evidence lower bound on the same goal-reaching objective.
This section proves Lemma 3.

Proof. Recall that our goal is to solve the following maximization problem:

\[
\max_{q(s_w | s_g, s_0)} \mathbb{E}_{q(s_w | s_g, s_0)} \left[ \log p_{\text{geom}}(s_{t+} = s_g | s_w) + \log p_{\text{geom}}(s_w | s_0) - \log q(s_w | s_g, s_0) \right].
\]

(13)

Note that the waypoint distribution must integrate to one. The Lagrangian can be written as

\[
\mathbb{E}_{q(s_w | s_g, s_0)} \left[ \log p_{\text{geom}}(s_{t+} = s_g | s_w) + \log p_{\text{geom}}(s_w | s_0) - \log q(s_w | s_g, s_0) \right] + \lambda \left( \int q(s_w | s_0, s_g) ds_w - 1 \right),
\]

(14)

(15)

where \(\lambda\) is a Lagrange multiplier. We then take the derivative with respect to \(q(s_w | s_g, s_0)\):

\[
\frac{d}{dq}(q(s_w | s_0, s_g)) = - \frac{-q(s_w | s_0, s_g)}{q(s_w | s_0, s_g)} + \log p_{\text{geom}}(s_{t+} = s_g | s_w) + \log p_{\text{geom}}(s_w | s_0) - \log q(s_w | s_g, s_0) + \lambda
\]

(16)

(17)

\[
= -1 + \log p_{\text{geom}}(s_{t+} = s_g | s_w) + \log p_{\text{geom}}(s_w | s_0) - \log q(s_w | s_g, s_0) + \lambda.
\]

(18)

(19)

We then set this derivative equal to zero and solve for \(q(s_w | s_g, s_0)\):

\[
q(s_w | s_g, s_0) = e^{\lambda-1} p_{\text{geom}}(s_{t+} = s_g | s_w) p_{\text{geom}}(s_w | s_0).
\]

Finally, we determine the value of \(\lambda\) such that \(q(s_w | s_0, s_g)\) integrates to one. We can then express the optimal waypoint distribution as follows:

\[
q^*(s_w | s_g, s_0) = \frac{p_{\text{geom}}(s_{t+} = s_g | s_w) p_{\text{geom}}(s_w | s_0)}{\int p_{\text{geom}}(s_{t+} = s_g | s_w') p_{\text{geom}}(s_w' | s_0) ds_w'}.
\]

\[
\square
\]

A.2 THE OPTIMAL WAYPOINT DISTRIBUTION (PROOF OF LEMMA 2)

This section proves Lemma 2

Proof. Define the normalizing constant as follows

\[
Z(s_0, s_g) = \frac{b(s_g)}{\int p_{\text{geom}}(s_{t+} = s_g | s_w') p_{\text{geom}}(s_w' | s_0) ds_w'}.
\]

A.3 ESTIMATING IMPORTANCE WEIGHTS (PROOF OF LEMMA 3)

This section proves Lemma 3
Figure 8: **Oracle experiment with the optimal generative model.** Goal-conditioned RL learns the task much faster if we can sample the waypoints from the expert policy’s marginal state distribution (C-Planning Optimal). Compared to planning with optimal state density distribution, planning using a learned classifier (C-Planning) shows comparable performance.

Substituting $Z(s_0, s_g)$ into the RHS of Eq. [4] and simplifying the result, we show that it equals the LHS of Eq. [4]

$$\begin{align*}
\frac{C_\theta(s_w, s_g)}{1 - C_\theta(s_w, s_g)} \frac{C_\theta(s_0, s_w)}{1 - C_\theta(s_0, s_w)} Z(s_0, s_g) &= \frac{C_\theta(s_w, s_g)}{1 - C_\theta(s_w, s_g)} \frac{C_\theta(s_0, s_w)}{1 - C_\theta(s_0, s_w)} \int P_{\text{geom}}(s_t = s_g | s_w) P_{\text{geom}}(s_t = s_w | s_0) ds_w \int P_{\text{geom}}(s_t = s_g | s_w) P_{\text{geom}}(s_t = s_w | s_0) ds_w \\
&= \frac{b(s_g)}{b(s_w)} \int P_{\text{geom}}(s_t = s_g | s_w) P_{\text{geom}}(s_t = s_w | s_0) ds_w \int P_{\text{geom}}(s_t = s_g | s_w) P_{\text{geom}}(s_t = s_w | s_0) ds_w \\
&= \frac{\pi(s_t = s_g | s_w) \pi(s_t = s_w | s_0)}{\int \pi(s_t = s_g | s_w) \pi(s_t = s_w | s_0) ds_w} \frac{1}{b(s_w)} \\
&= q(s_w | s_0, s_g) \frac{1}{b(s_w)}.
\end{align*}$$

\[\text{(20)}\]

\[\text{(21)}\]

\[\text{(22)}\]

\[\text{(23)}\]

\[\text{(24)}\]

### B  Additional Experiments

#### B.1  An Oracle Experiment

We run an experiment to confirm the well-known result [17] that the optimal initial state distribution for learning a task is the state distribution of that task’s optimal policy. Intuitively, if we could sample exactly from the state distribution of the optimal policy, then an RL agent could learn to reach goals more quickly. This is not surprising as it resembles behavioral cloning of the optimal policy. In this oracle experiment, it is assumed we are given the access to an optimal policy. We achieve this by following the sampling procedure from Algorithm [1] but replace the classifier $C$ with that learned from the optimal policy $C^*$. We then perform RL to reach waypoints and then the final goal. Results in Fig. [8] show that using the optimal policy as the initial state distribution results in faster learning than using the original state distribution. We also visualize the state density distribution of the optimal policy to provide more qualitative intuition on how the algorithm works.

In addition, we are also interested in measuring how well does planning through a learned model (C-Planning) performs if we don’t have access to the optimal generative model. Results in Fig. [8] show...
show that C-Planning achieves performance comparable to planning via optimal generative model in the Push and the Push-Wall environment.

B.2 Visualization of State Density Map of Optimal policy

We conduct this experiment on a 2D navigation task shown in Fig. 8 (left), where we have also visualized the original initial state distribution, the state distribution of an optimal policy, and the goal state. To conduct this experiment, we apply a state-of-the-art goal-conditioned RL algorithm (C-learning) in the two settings with different initial state distributions. For fair evaluation, we evaluate the policies learned in both settings using the original initial state distribution. The results shown in Fig. 8 (right) show that starting from the optimal initial state distribution results in 2.4x faster learning.

C Experimental Details

In this section, we provide the essential hyperparameters for reproducing our experiments in this section. We also introduce the hyperparameters used in baselines and provide a detailed description of environmental design.

C.1 Implementation Details

We introduce the hyperparameters used in C-Planning. Note that in C-Learning, only a classifier on \((s_t, s_{t+1}, a_t)\) is needed. In C-Planning, in order to sample waypoints from the distribution, an additional classifier on \((s_t, s_{t+1})\) is needed. We also introduce two hyperparameters: Maximum Steps Reaching Goal \((N_g)\) in Alg. 1 forces the agent to change the intermediate goal if the original goal hasn’t been reached for some steps; Distance Threshold Reaching Goal \((\epsilon_d)\) in Alg. 1 for determining whether a goal is reached or not. The rest are the standard hyperparameters for SAC algorithm and we list here for reference.

Table 1: Hyperparameters used for C-Planning in all the environments in MetaWorld.

| Hyperparameter                      | Value       |
|-------------------------------------|-------------|
| Actor lr                            | 0.0003      |
| Action-State Critic lr              | 0.0003      |
| State Critic lr                     | 0.00003     |
| Actor Network Size                  | (256, 256, 256) |
| Critic Network Size                 | (256, 256, 256) |
| Maximum Steps Reaching Goal         | 20          |
| Distance Threshold Reaching Goal    | 0.05        |
| Actor Loss Weight                   | 1.0         |
| Critic Loss Weight                  | 0.5         |
| Discount                            | 0.99        |
| Target Update Tau                   | 0.005       |
| Target Update Period                | 1           |
| Number Waypoints                    | 5           |
| Goal Relabel Next                   | 0.3         |
| Goal Relabel Future                 | 0.2         |

Note that we use a slightly different hyperparameters for 2D maze environment and we list below, all the other hyperparameters remains the same. Note that we follow the goal relabeling changes by C-Learning [8].
Table 2: Hyperparameters used for C-Planning in all the environments in 2D navigation maze.

| Hyperparameter            | Value |
|---------------------------|-------|
| Distance Threshold Reaching Goal | 1.0   |
| Number Waypoints          | 8     |
| Goal Relabel Next         | 0.5   |
| Goal Relabel Future       | 0.0   |

C.2 Environments

We follow the environment design of [8] with only one noticeable difference: in the original environments of C-Learning, for the ease of training, the author set the initial position of objects to be relatively near the arm so the arm can easily push the object, getting a better learning signal. We intentionally set the initial state of object to be far away from the arm. This significantly increase the difficulty of learning. We’ll release these environment with the code.

C.3 Baselines

We also provide the hyperparameters associated with the baselines. The two baselines we care about: C-Learning and RIS. We summarize their hyperparameters in the table:

Table 3: Hyperparameters used for C-Learning in all the environments in MetaWorld.

| Hyperparameter                  | Value                      |
|---------------------------------|----------------------------|
| Actor lr                        | 0.0003                     |
| Action-State Critic lr          | 0.0003                     |
| Actor Network Size              | (256, 256, 256)            |
| Critic Network Size             | (256, 256, 256)            |
| Actor Loss Weight               | 1.0                        |
| Critic Loss Weight              | 0.5                        |
| Discount                        | 0.99                       |
| Target Update Tau               | 0.005                      |
| Target Update Period            | 1                          |
| Goal Relabel Next               | 0.3                        |
| Goal Relabel Future             | 0.2                        |

Table 4: Hyperparameters used for RIS in all the environments in MetaWorld.

| Hyperparameter       | Value |
|----------------------|-------|
| epsilon              | 0.0001|
| Replay Buffer Goals  | 0.5   |
| Distance Threshold   | 0.05  |
| Alpha                | 0.1   |
| Lambda               | 0.1   |
| H lr                 | 0.0001|
| Q lr                 | 0.001 |
| Pi lr                | 0.0001|
| Encoder lr           | 0.0001|

D Visualization of the Learned Policy

In order to more intuitively visualize the behavior of the learned policy and emphasize the importance of the difficulty of the learning task, we plot the visualization of our learned policy for several snap shots in an episode for environment of Push-Two and Obstacle-Drawer-Open. We see that our method successfully guide the agent to learn the behavior of push the green object first, the push the red object; And first open the drawer then push the object to the desired location. We would
Figure 10: Visualization of the trained policy for the **Push-Two** environment. Our method successfully trains the agent to manipulate the two objects in the sequential manner without any offline data, expert demonstration and reward shaping. Prior baselines all fail to demonstrate such behavior.

Figure 11: Visualization of the trained policy for the **Obstacle-Drawer-Open** environment. Our method successfully trains the agent to first close the drawer and then push the object to the desired location. We emphasize that such behavior is hard to obtain and most of the prior methods only manage to close the drawer.

I like to emphasize that prior baselines all fail to demonstrate such behavior without any offline data, expert demonstration and reward shaping. The successfully learning of such behavior enables the robot to do more complex tasks without any human intervention.