Abstract

We investigate three aspects of the supposed problem of time: The disagreement between the treatments of time in general relativity and quantum theory, the problem of recovering time from within an isolated Universe and the prevalence of a unidirectional time flow (i.e., the so-called arrow of time). Under our interpretation, general relativity and quantum theory have complementary time treatments given that they emerge from a theory of a more fundamental nature. To model an isolated Universe, we use the Wheeler-DeWitt equation and then apply the Page-Wootters method of recovering time. It is argued that, if the recovery of an experience of time is indeed viable in this framework, interactions and quantum entanglement are both essential features, even though the former is normally an afterthought or altogether dismissed. As for the one-way arrow of time, this is, from our perspective, a consequence of including the aforementioned interactions. But underlying our interpretation, and pretty much all others, is the necessity for causality. It is this fundamental tenet which
accounts for our experience of time but yet can only be postulated. Our conclusion is that the ‘problem of causality’ is what should be the focal point of future investigations.

1 Introduction

The supposed ‘problem of time’ is an ongoing discussion in both physics and philosophy. Many different questions are raised under this encompassing topic, leading to disagreement amongst the wide range of contributors. Part of the trouble is the frequent use of buzzwords in lieu of clear-cut definitions. In order to avoid this pitfall, we will attempt to define any term that is used without relying on vague phrases. For examples of previous discussions on this topic, see [1, 2, 3, 4].

While some of these previous discussion present compelling arguments, there is no consensus as of yet. Pinning down a definition of time alone is troublesome, as pointed out in [5], amongst others. We start then by identifying what we believe to be the simplest possible definition of time, as well as any associated ingredients that are required to recover our experience of time. With these definitions in hand, we will assess three of the aspects of the problem of time. These are, one, the difference in how time is treated in quantum mechanics and general relativity, two, the phenomenon of a one-way arrow of time and, three, the recovery of time in an isolated and therefore timeless Universe.
1.1 Time and our experience of it

Time  The task of defining time presents one with a problem since it cannot be accessed directly. As with space, our knowledge of time comes about from studying the behavior of physical objects rather than directly measuring time (or space) itself. Since the main interest here is in describing the experience of time, we will avoid the debate about the potential existence of time and space as independent entities but, rather, focus on describing time as it affects physical systems. (But for a summary of this debate, see, e.g., [6].) We do note that the physical existence of an extended object — even for one that is not moving — would seem to imply that space is required to provide a physical meaning to the spatial dimensions of the object. If this is true, one might argue, by extension, that time should likewise be required, a point which will be elaborated on later.

To ensure that any features which are necessary for the experience of time do not become hidden assumptions, we begin with a very basic observation: The experience of time is one of change. In particular, we regard the ‘configuration’ as the feature of a physical system that undergoes change. By configuration, we really mean the state — which is defined by all the properties that the system might have at any given moment — but have tweaked the terminology as a reminder that the Schrödinger picture is then assumed for the quantum case. Given a physical system, which can be constructed from one or more parts, ‘change’ indicates the process by which the system

\footnote{As definitions of existence can lead to many philosophical issues, we will rely here on a notion of ‘physical being’ that is similar to the Parmenidean view of ‘what is’ as described in, for example, [7]. More nuanced definitions can be found in, for example, [8].}
transitions from any one configuration to another. For example, two configurations of a cup could be ‘in midair’ and ‘scattered in pieces on the floor’. The continual change in a system’s configuration leads to the experience of time but, as of yet, there is no compelling reason to insist that the configuration of any system must change. The implication is that the most basic definition of time lacks such a feature. With that said, it is important to distinguish between two related but still different situations where a system does not change:

On one hand, a system may have no functional dependence on time and therefore remains in the same configuration regardless of the value of the time coordinate. Such a system is normally said to be static; while it might exist in multiple moments of time, its configuration is the same in all. A stationary system would also fit this bill given that the transitions between configurations have been suitably coarse grained. On the other hand, a system with access to only one moment of time would also be incapable of change since it would have no secondary moment to ‘move’ into. We will call such a system ‘frozen’ to capture the idea that it is stuck in the one moment it happens to occupy. It could be argued that a frozen system can be functionally dependent on time as its configuration may depend on being stuck in this particular moment instead of that one. A ‘frozen’ system could then be considered to be a priori capable of change, which it would experience

\footnote{The debate over whether time has independent existence often separates ‘time’ as a thing-in-itself from ‘coordinate time’ as a relational concept which relates physical systems. For a summary of the difference, see, for example, \[\square\]. The discussion presented here, however, is not concerned with this debate and so no such distinction of coordinate time from ‘independent’ time is made.}
if it had access to more than one moment of time and was compelled to ‘move’ to a second moment. But, lacking these attributes, a frozen system, just like a static system, is unable to evolve.

But, on still another hand, one might ask if a non-evolving system needs to rely on time at all in order to exist. In other words, could an existent system have no opportunity for change as a strict matter of principle? If it could, this would provide us with a third distinguishable situation. Nevertheless, a compelling argument for the necessity of time in describing physical existence goes as follows: Since the configuration is what is affected as a system moves through time, one can reasonably assume that the configuration is what should be influenced by the removal of time. (However, see [6] for a contrary opinion.) Then, without the notion of a moment to ensure that a single configuration has been selected out of the many possibilities, the system could be viewed as occupying any one of its configurations. But this scenario is logically no different than claiming that the system is occupying all possible configurations. We would then argue that such a situation deprives the system of its physical existence and conclude that time is indeed necessary, regardless of any functional dependence on a parameter of time.

It is worth pointing out that depriving a system of the compulsion to change tends to put time and space on more even ground. This is because a basic definition of space might be that it is a feature of the Universe which

---

3 This view of frozen systems shares similarities with the Block (or timeless) Universe perspective. The latter is discussed in Section 4 and Appendix B.

4 This suggestion bears some similarity to the perdurantist view, a defense of which can be found in [9]. There it is argued that any object necessarily has a temporal aspect which must be taken into account, albeit with a different line of reasoning.
allows movement to occur but does not include the compulsion to move. Regardless of the impetus, whether it be the most basic definition of time or its unification with space, we will similarly define time as a feature of the Universe that allows change to occur but does not compel it. Given one’s own understanding of the experience of time, this definition immediately begs the question: what does provide the compulsion?

Causality  We still need to account for the continual change from one configuration to the next, and in a manner which is consistent with the experience of physical systems. The described process can be attributed to the principle of causality which, in most discussions, is either formally postulated or taken for granted to allow for an interpretation of physical theories.\footnote{We recognize that the status of causality is a contentious issue in the philosophy of physics and do not claim to present a proof of the principle. Rather, we are acknowledging causality’s role in our current descriptions of the experience of time even as an \textit{ad hoc} principle.} Since the meaning of causality has many different variations, let us first clarify the definition to be used here. What we have in mind is similar to the viewpoint of [10], which talks in terms of ‘interventions’ acting on systems to induce change. More to the point, causality will be taken to represent a process by which an external influence, the cause, compels a system to transition from one configuration to a different configuration, the effect.\footnote{As pointed out in [10], this suggests that causality cannot have meaning for a closed system for which there can be no outside influence. This concern will be addressed in Section[4]} Because one of the configurations is a consequence of the transitioning of the other, there is a natural precedence for the configurations, but this is not an inherently
temporal ordering until causality is combined with a suitable definition of
time, like the one above. Once this step is taken, the ordering in time can
be set so that the cause always precedes the event. A series of configurations
which are strung together with the requisite ordering provides the desired
picture of a system moving through time. This closely resembles the account
of time as a process of ‘becoming’, which was historically first suggested by
Heraclitus in ancient Greece and, more modernly, by (e.g.) Whitehead [11]
and Prigogine [12].

It should be stressed that this definition of causality does not implicitly
include any notion of determinism. Although determinism and causality are
often conflated, and many authors disagree that they can be separated, here
the distinction is maintained. The difference can be seen by considering
the case where a cause may have several possible effects, as is evident in
stochastic theories and, of course, in quantum mechanics. (See, e.g., [13] for
further elaboration.)

In spite of our claims about a causal ordering, the direction of the time
flow remains ambiguous. To understand why, let us to return to the example
of a falling cup. There, it is natural to identify gravity as the external
influence or cause and the transition from ‘in midair’ to ‘scattered in pieces on
the floor’ as the effect. This picture makes the ordering of the configurations
clear, but it is only natural because an observer would rarely (actually, never)
see the reverse ordering of configurations. As is well known, the equations of
motion in most physical theories are time-reversible invariant, and Newton’s
equations of motion are no exception. And it is just as well known that, with
the possible exception of the collapse the wavefunction, physics should not
and does not require a conscious observer to operate. Meaning that one could just as easily say that some unknown agent caused the plate to reassemble and project upwards. Who is to say what is the correct interpretation? This same logic can be extended to a chain of configurations: the sequence of the events remains clear but the labels of ‘cause’ and ‘effect’ can be arbitrarily assigned to either end of the chain. It then appears that time reversibility undermines the utility of causality in determining relationships between configurations. (This same observation is relevant to ‘timeless’ models of the Universe; see Section 4.)

The discussion above does, however, overlook thermodynamics, which identifies certain processes as being irreversible: those for which the entropy has increased. In other words, the second law of thermodynamics, which states that the entropy of a closed system can never decrease, enforces irreversibility and, with it, a unique direction in time. In the previous example, it is quite clear that the smashing of the plates is just such an irreversible process.

Before the thermodynamic argument can be accepted, there are (at least) two counter-arguments to consider: thermodynamics is not a fundamental theory, which makes it capable of hiding the intrinsic reversibility of physics, and there is no obvious reason why this so-called thermodynamic arrow of time should align itself with the cosmological time, which is taken to be the time experienced by a (typically large) group of gravitationally bound systems as a whole. But it will later be counter-counter-argued that irreversible processes are absolutely necessary if time is to emerge in an otherwise timeless (or isolated) Universe and that these very same processes are what accounts
for the direction of cosmological time. And, because the time evolution is itself emergent and not fundamental, it is no longer subject to the same symmetry properties of the underlying theory.

1.2 The ‘problem of time’

To summarize the discussion so far: Time (by itself) allows systems to change, causality compels them to change and, under the assumption of an isolated Universe, irreversible processes pick out a unique direction for change to occur in. Let us now return to the aforementioned three problems of time and briefly preview our proposed resolutions.

Our first task will be to address the different treatments of time in quantum mechanics and general relativity. As will be explained, there is reason to believe that these two time treatments descend from a common theory. To be clear, this common theory — for which the putative fundamental theory is its antecedent — does not have to contain time as its normally understood; a template will suffice. As for our reasoning, this is based on three observations: quantum mechanics is a descendant of quantum field theory, both quantum field theory and general relativity have a spacetime metric, and the Minkowski metric of quantum field theory should be regarded as a limiting case of a generic class of metrics and not merely a background structure. The very last claim will be shown to be a consequence of quantum field theory having no global symmetries, and it means that there is fundamentally no difference between the metrics — and therefore time parameters — of the

---

⁷The importance of irreversible processes in the context of time evolution has been advocated by others; for example, Prigogine [12] (also see [13, 15] for different perspectives).
two theories in question. Here, we are not claiming to offer a conclusive description of a ‘quantized time’ as it might appear in a theory of quantum gravity. Rather, we are presenting evidence of similarities between the two treatments of time, supporting the notion that they do in fact arise from a more fundamental source.

Now what about the arrow of time? As already discussed, our resolution of the timeless Universe problem (see below) also provides a built-in resolution to the arrow-of-time quandary, as it ensures that the cosmological times and thermodynamic time are in alignment. The central point is that the emergence of time requires irreversible processes as a matter of principle, and these provide a natural direction for the time flow. There is, however, another brand of time to consider; namely, psychological time. This typically refers to time as it is experienced by conscious beings who have an innate ability to remember the past but never the future. Although the argument that directly connects psychological time to its thermodynamic counterpart is well known (e.g., [16, 17]), we will summarize it here; both for the sake of completeness and because similar reasoning is used later in the paper. Briefly, the storage of a new memory first requires the erasure of an old one. But the latter comes at the cost of an increase in entropy due to an associated loss of heat [18]. It is often pointed out, by way of analogy, that this process is functionally no different than adding a new bit of information to a computer. What will be important to us is that the same can be said about any physical system that is capable of storing information, even if it is inanimate. For further discussion, see Section 3.

The last of the three issues is also the most involved: The problem of
providing time to the Universe if it is truly isolated. The Universe would then have to be in a timeless state because, simply put, it already contains ‘everything’. The real point though is the conservation of energy, which must be in effect for any closed system. For a closed system without gravity, whether classical or quantum, one is then free to set the on-shell value of the Hamiltonian to zero by adding a constant. If gravity is included, one no longer has this freedom, but general relativity handles it automatically through its constraint equations. And, deprived of a Hamiltonian (as far as physical solutions of the field equations are concerned), time evolution is impossible. To be clear, a particle in a well, for example, is in a static state and not a timeless one, because time is still provided by its external environment. But there can be no such environment to rely on when the Universe exists in isolation.

The model of an isolated Universe is captured by the Wheeler–DeWitt equation [19], which elevates general relativity into the quantum regime (more accurately, into the realm of semiclassical physics). We are not claiming (nor disputing) that this equation provides an accurate depiction of reality, but it does exhibit the essential feature of timelessness. Page and Wootters famously proposed a method of recovering time in the Wheeler–DeWitt framework; the basic idea is to allow a subsystem of the Universe to serve as a clock for the remainder [20]. The delineation of the subsystems is, at first glance, arbitrary under the proviso that the systems are maximally entangled (in the quantum sense) and weakly interacting. In subsequent

---

8Our discussion does not preclude parallel universes nor multiverse theories but instead assumes that one member of such an ensemble cannot influence any other.
descriptions of this method, the interactions are typically deactivated, at least as a limiting case. Our own investigations led to the conclusion that the interactions cannot be arbitrarily small and still allow for an adequate description of time \[21, 22\]. Moreover, as one subsystem is effectively measuring the other, the interactions are necessarily irreversible. And so what we have is an emergent cosmological time with a built-in arrow of time that automatically aligns with the thermodynamic arrow and then, vicariously, with the psychological arrow. (Similar links between general relativity and thermodynamics are touched upon in Section 4.)

What is still missing, however, is a mechanism that explains how either of the two subsystems can transfer from one of their configurations into the next. In other words, what is still lacking is an explanation of causality. Since causality is, to the best of our knowledge, always introduced as a postulate, a complete ‘theory of everything’ may be what is needed to pinpoint its origin. In the meanwhile, our suggestion would be to change the focus of future investigations from the problem of time to the mystery of causality, as it is the latter that is ultimately responsible for the experience of time in the Universe.

The remainder of the discourse is organized as follows: Section 2 presents our argument that time in quantum mechanics and general relativity should not be regarded as independent entities, and that their respective time treatments can be traced to a common theory. Section 3 recalls how time can emerge in an otherwise timeless Universe and then shows how the requisite inclusion of interactions forms a link between the cosmological and thermodynamic time arrows. Section 4 presents additional discussion regarding the
problem of time in the context of our findings and conclusions. A supporting description of the Page–Wootters method is provided in Appendix A. Several possible objections to our conclusions are discussed in Appendix B, including the clock ambiguity [23], the Block Universe, and the inclusion of other universes.

2 Time in quantum mechanics and general relativity

The aim of this section is to reconcile the differing views of time which arise in different theories of physics. Historically speaking, the first physical definition of time appeared in Newtonian mechanics, and so we begin there.

Time in the classics

Time, as described by Newton, was an “absolute” quantity which existed outside and independently of physical systems [24]. As such, it could not be directly measured but only described as a relative quantity between events, where an event represents a change in the configuration of system and is quantified through the classical equations of motion. After establishing a coordinate system, one can use these equations to assign each event with a location in both space $\vec{r}$ and a time $t$. If different coordinate systems are moving relative to one another at a constant speeds, each one can be viewed as the reference frame for a co-moving observer.\footnote{Our use of terms such as ‘observer’ and ‘perspective’ should not be taken to imply a conscious experimenter. We will be explicit whenever such an observer is required to} Moreover, any two of
these inertial frames can be related through a Galilean transformation of their spatial coordinates. Note though that the time axes of two such reference frames can never be moving relative to one another, as $t$ applies globally to all frames (up to an arbitrary choice for $t = 0$), making the difference between time and space quite apparent. And so time in Newtonian mechanics, even as a parameter describing relations between events, remains external to the events. We will refer this parametrization as ‘external time’.

The external time $t$ requires a further assumption if it is to describe dynamics. Since the external time exists outside of the physical arena, there is no inherent motivation for a series of events to be ordered by, say, assigning each with a particular value of $t$. This mathematical framework, at best, only provides an elementary notion of time as already described in the introductory section. The principle of causality is still required to motivate a recognizable order for any series of events. Suppose that each configuration of a system is represented as a frame from a film reel. Without causal relationships, the frames may be shuffled into any arbitrary order without contradicting the mathematical rules of Newtonian mechanics but also without producing a recognizable description of reality. Only when causality is enforced (or postulated) will the frames be restricted to a specific order. This sequence accounts for the ‘moving’ from one configuration of the system to another; what is better known as dynamics.

But, as was unknown to Newton, his mechanics emerges from special relativity in the limit $c \to \infty$ or, alternatively, from quantum mechanics in the limit $\hbar \to 0$, both of which are illustrated later in Figure 3. Of make a point.
course, these two theories are themselves limiting cases of theories that are even more diverse in their scope. Meaning that external time and all its trappings should be viewed with a healthy dose of skepticism. With this in mind, the discussion will advance further up the ‘ladder of fundamentally’, starting with an examination of time in special relativity.

**Time in the relativistic theories**

If the Newtonian treatment of time is an artifact of its treatment in special relativity, how does the more fundamental notion of time differ?

Many discussions on special relativity (e.g., Chapter 2 of [25]) begin with the postulates that the laws of physics and the speed of light are the same in all inertial reference frames. Using these requirements, one finds that a pair of such frames can now be related by what are known as Lorentz transformations.\(^{10}\) These transformations are fundamentally different from the Galilean transformations of Newtonian physics in that time and space can both undergo transformations and can, indeed, even become mixed. To assure that the laws of physics remain intact, physical quantities are required to transform covariantly under Lorentz transformations. On this basis, one can rather construct the theory of special relativity by enforcing Lorentz covariance where appropriate. Importantly, there is a smaller class of quantities that, just like the speed of light, must remain unchanged under Lorentz transformations; these being the Lorentz invariants (typically scalar quan-

\(^{10}\)Lorentz transformations are described by rotations in space and/or boosts (rotations in four-dimensional Euclidean space). The complete set of transformations are the Poincaré transformations, which also include translations.
tities, but see below for a notable exception). As the set of all Lorentz transformations forms a group, a Lorentz-invariant quantity is a textbook example of a global symmetry.

An important example of a Lorentz-invariant scalar quantity is proper time; this being the amount of elapsed time between two points in spacetime as would be measured by a clock moving along a strictly timelike path (see below for a definition). A path-independent way of measuring proper time can be obtained by introducing the Minkowski metric tensor. Although not a scalar, the Minkowski metric is yet another object that is invariant under Lorentz transformations. It is also the uniquely coordinate-independent form of a more general tensor. In general, given a spacetime manifold (flat or otherwise), the associated metric tensor provides a means of measuring the square of the ‘distance’ between any two points in the manifold; this being the same, up to sign, as measuring the square of the proper time interval.

In discussions about spacetime geometry, the notions of space and time can become blurred because of their aforementioned mixing transformations. The proper time is one way of maintaining a distinction. Another, more geometric way is provided by null cones; this being an object whose surface is defined by the path of the light rays emanating from its apex. As light is the fastest-moving object from any observer’s perspective, the null cone serves as a means for delineating between timelike paths — those staying within the null cone — and spacelike — those passing through its outer surface. The boundary of the cone itself contains strictly null (lightlike) paths. For a more graphic description, see Figure[1]. It should also be kept in mind that the notion and utility of both proper time and light cones persists for more
general spacetime geometries.

![Diagram of a null cone](image)

Figure 1: A schematic diagram of a null cone, whose outer surface describes the path of light rays in spacetime.

It is clear that, at least in flat spacetime, the null cones identify just one of the four spacetime dimensions as temporal. But time, as it is experienced in Nature, is not included in this setup as long as the notion of causality is absent. Same as for Newtonian physics, special relativity does not necessarily produce recognizable dynamics. As per the previously described film analogy, special relativity might allow for the frames to be fixed together but it fails, on its own, to ensure that the sequence of frames will be arranged in any particular order.

Special relativity is, of course, the weak-gravitational or $G_N \to 0$ limit of its more fundamental description, the theory of general relativity. But translating between theories is much more involved than applying a simple limit. For one thing, in the general theory, inertial reference frames are no longer favored over their non-inertial (or accelerating) counterparts. For another, as
the effects of gravity are ‘turned on’, the spacetime geometry becomes curved and, as a result, the metric tensor will no longer be expressible in a constant form nor will it be invariant for an arbitrary Lorentz transformation. In fact, because two reference frames can have a relative acceleration between them, Lorentz transformations are no longer sufficient. Rather, diffeomorphisms (or generic coordinate transformations) are now required to map from one reference frame to another. Meaning that physical quantities are now required to transform covariantly under diffeomorphisms and there is a privileged class of quantities that are diffeomorphism invariant. The spacetime metric itself is, itself, not such an invariant and will generally change according to its position in spacetime but always in just the right way to ensure that scalar quantities, like proper time, remain diffeomorphism invariant.

And so, rather than saying that special relativity is a weakly gravitating limit of general relativity, one might be more accurate in claiming that general relativity comes about by breaking a global symmetry — that of Lorentz-invariant scalars — into a local (i.e., coordinate-dependent or gauged) symmetry — that of diffeomorphism-invariant scalars. From this point of view, the Minkowski metric is not so much the limiting case for a flat spacetime geometry but more like the metric whose associated gauge field can be fixed so as to trivially vanish. As the gauge field holds all the coordinate dependence, the Minkowski metric maintains a ‘hidden’ dependence on the spacetime coordinates. This dependence has, however, as a matter of choice, simply been gauged away.

As already mentioned, null cones remain a part of general relativity but are generally deformed away from their flat-space geometries. An illustration
of this is provided in Figure 2. Other notions like timelike paths and proper time are similarly intact but, albeit, more challenging to (respectively) identify and calculate. However, ordering the configurations of a system into a physically significant sequence once again requires the postulation of causality. This is contrary to the idea that time, as it is experienced in Nature, emerges from the framework of general relativity as is sometimes claimed.

It can be anticipated that general relativity itself emerges from the fundamental theory or, perhaps, from an intermediary thereof (e.g., string theory [26]). Before further speculating on this possibility and its relevance to the emergence of time, let us first turn to another, seemingly unrelated series of physical theories.

Figure 2: A schematic diagram of null cones ‘warped’ by the curvature of spacetime.
Time in the quantum realm

Although the transition is not well defined, classical Newtonian behavior can be expected to emerge from quantum mechanics in the limit $\hbar \to 0$. But the transition of time does appear to go smoothly, as quantum mechanics, like its Newtonian counterpart, utilizes a global time parameter for the purpose of ordering the configurations of any given system. In the quantum case, the necessity for such an external time line can be traced to the theory’s prohibition on quantum time operators, which applies even as a strict matter of principle. The argument against a time operator is well known but is still worth reviewing: Any time operator would necessarily be conjugate to an energy operator (i.e., a Hamiltonian), as follows from the relation $[E][\Delta t] = [\hbar]$. Now let us suppose that a time operator $\hat{t}$ does indeed exist. Then one could construct a unitary operator $\hat{U} = e^{\pm i \hat{t} dE}$ which acts to translate states along the energy spectrum, $\hat{U} |E\rangle \to |E + dE\rangle$. Such a translation could be applied indefinitely, projecting the system into a state of arbitrarily negative energy and, thus, removing any notion of a stable vacuum state.

Things finally get interesting when the more fundamental theory of quantum fields is brought to the fore. As the synthesis of quantum mechanics and special relativity, quantum field theory can be expected to limit to quantum mechanics as $c \to \infty$. Note, though, that this limit, just like the quantum-to-classical case, lacks a well-defined transition as it would entail one to (somehow) ‘deactivate’ the so-called second quantization of the fieldless theory. But the takeaway point should be that the description of time in quantum field theory must mimic the time treatment of special relativity if the field theory is to maintain its integrity. Meaning that quantum field the-
ory, just like special relativity and all the others, depends on the postulation of causality if it is to describe the experience of time.

**Relating time across theories**

As illustrated in Figure 3 and discussed above, there are two ways to reach classical Newtonian physics from the more fundamental theories. Importantly, these are one-way paths; Newtonian mechanics can inherit time from either pathway, but one cannot go up along one path and then backtrack down the other in order to connect general relativity to quantum field theory. But this pair could still meet at the opposite end in an even more fundamental theory, as depicted in the figure. Although this is our expectation, we will proceed to argue that the two paths depend on the same basic notion of time, irrespective of whether or not they meet at the ‘top’.

Quantum field theory and general relativity are commonly viewed as disconnected entities in ‘theory space’. But yet they both include special relativity as a limiting case, which opens up the possibility of a hidden connection between the two theories. However, given the usual state of affairs, this would be a naive connection at best as the two theories provide much different interpretations of the spacetime metric which special relativity ultimately inherits. In the case of general relativity, the metric tensor is a physical field that describes the geometric and causal structure of spacetime. And so, from this point of view, the Minkowski metric of special relativity can be interpreted as the flat-spacetime limit of a physical object.

Meanwhile, the Minkowski metric of quantum field theory is generally regarded as a mathematical construct and not a real physical object. This
description falls in line with that of the gauge fields in classical electrodynamics, which seems sensible for a classical theory but somewhat contradictory for a theory of quantum (gauge) fields. We will argue next that this interpretation of the Minkowski metric is indeed misguided and that, just like in the flat-spacetime limit of general relativity, this tensor is harboring a hidden dependence on the coordinates of spacetime, making it a physically real field. Note, though, that any such dependence must remain hidden and undetectable so as not to jeopardize the requisite Lorentz invariance of quantum field theory nor the closely related theorem of Weinberg and Witten \[27\].

Our first argument is undoubtedly the simplest one: Quantum field theories can be expected, on very general grounds, to not have any unbroken global symmetries \[28\], and we see no good reason for Lorentz invariance to be exempt from this ‘policy’ (given that the effects of it breaking remain
hidden as just discussed). The relevant point here is the inevitable breaking of classical symmetries after the theory has been suitably quantized and renormalized; the so-called quantum anomalies [29].

A second argument comes from string theory, which is also void of global symmetries [30]. Even if string theory does not accurately depict reality, it is the only known self-consistent theory which accounts for both general relativity and quantum field theory and so it can be used as an indicator of what a more fundamental theory might look like. 11

Our third argument goes as follows: Suppose that there was a more fundamental theory which contains both general relativity and quantum field theory. We would then expect all of its observable and emergent features — including proper time, null cones and, by extension, the metric — to be subject to the effects of quantum fluctuations. Such fluctuations can not be prohibited from depending on the coordinates of spacetime.

Let us, finally, recall a standard argument that is routinely used against global symmetries: One considers particles that are forever lost inside of a black hole, which implies the breaking of globally conserved quantities such as the baryon number [32]. Note that the process of black hole evaporation process cannot resolve this situation because the emitted radiation is dominated by massless particles [33] and any such particle is incapable of carrying a baryon number.

The two middle arguments imply that there is some intermediary theory

---

11String theory is certainly more fundamental but we are not asserting that string theory is the fundamental theory. In fact, we would argue against such a suggestion (see, e.g., [31] for a relevant discussion.)
or, possibly, the fundamental theory itself from which both general relativity and quantum field theory are emergent (cf, Figure 3). Although not strictly necessary (as the other two arguments would suffice), this would be the natural expectation if one accepts our assertion that the two theories do indeed host similar time treatments. But, to be clear, we are not suggesting that a recognizable notion of time has to exist in all theories up to and including the fundamental theory, but only that some ‘blueprint’ for time is provided at a more fundamental level. This feature will be referred to as ‘elemental time’ later on in the paper.

To summarize, our contention is that all currently accepted theories are really talking about the same basic notion of time but, by the same token, all require that causality be postulated.

3 The thermodynamic arrow and the timeless Universe

Whereas the thermodynamic arrow of time points in the direction of increasing entropy, the closely aligned psychological arrow is taken to represent time as perceived by a conscious observer recording experiences in her memory. These arrows are well known but what may not be is that the concept of a psychological arrow can be extended beyond conscious observers such as ourselves and even beyond computers. The same idea applies just as well to any physical system that has changed its configuration as the result of an irreversible interaction. The point is that any such change can be viewed as the act of ‘recording information’ in the sense that one configuration (or
‘memory’) of the system is erased and replaced by a new one. The con-
straint of irreversibility ensures that the previous configuration can only be
restored with the sacrifice of a new one and accounts for the alignment with
the thermodynamic arrow. The latter because such interactions produce
heat and then, by virtue of the Clausius inequality, increase entropy. What
we intend to show here is that this arrow emerges as a consequence of the
Page–Wootters methodology \[20\], but only if one also insists on a physically
meaningful notion of time.

The phenomenon of time depends on one’s interpretation of the Universe.
If the Universe is regarded as a totally isolated system, as presumed here,
then the phenomenon of time cannot be imported from the ‘outside’ — it
must rather emerge from within. This isolated model of the Universe is
captured by the Wheeler–DeWitt equation \[19\],

$$\hat{H} |\Psi\rangle = 0,$$

where $\hat{H}$ is the Hamiltonian constraint from general relativity but elevated
to the role of a quantum operator and $|\Psi\rangle$ is the putative wavefunction of
the Universe. As a closed and stable gravitating system, the total energy of
the Universe is zero and unchanging. As such, the Hamiltonian annihilates
all physical states; meaning that the relevant states cannot evolve in time,
$$e^{i\hat{H}t} |\Psi\rangle = 1 |\Psi\rangle.$$ The conserved energy and the static nature of the states
should be regarded as a manifestation of the Universe’s isolation and not as
a unique feature of the Wheeler–DeWitt description.

The emergence of time is problematic given that the Universe is prohib-
ited from ‘fetching’ any of its features from its (hypothetical) exterior. It
is useful to compare this situation to that of isolated quantum systems in a
more conventional setting. In standard quantum mechanics, the difficulty is overcome simply because the isolated system exists within an environment that does experience time, and so a notion of time can still be imported from this exterior region. Not unlike a Russian doll, any isolated system can be fitted inside a larger one so as to import a notion of time, with the procedure repeated indefinitely until some largest possible system is reached. In the case of the isolated Universe, however, one is beginning the iterative procedure already at this upper maximum.

Let us now move on to the Page–Wootters solution to this conundrum. Those authors proposed that the Universe be divided into two subsystems [20]: the clock $C$ and the rest of the Universe $R$. Here, we will only briefly summarize the method but have also included a more quantitative description in Appendix A. One begins by subdividing the Universe in such a way that $C$ and $R$ are maximally entangled and approximately isolated. The conjugate operator $\hat{P}_C$ to the clock Hamiltonian $\hat{H}_C = \text{Tr}_R\hat{H}$ can then serve as an effective time operator for $R$. The reason that this works is because the approximate isolation ensures that, as far as physical states are concerned, the respective Hamiltonians are related by $\hat{H}_R \approx -\hat{H}_C$, and so the effective time parameter for $C$ (namely, the eigenvalue of $\hat{P}_C$) and that for $R$ are the same up to a sign convention. The condition of maximal entanglement is itself necessary to ensure that the states of $R$ are indeed correlated with the eigenvalues of $\hat{P}_C$ in a one-to-one way (assuming no degeneracies). Any interaction effects, which would be governed by a Hamiltonian of the form $\hat{H}_I = \hat{H} - \hat{H}_C - \hat{H}_R \approx 0$, are negligible in this setup and the so-called ideal-clock limit implies that $\hat{H}_I$ vanishes identically. Although this treatment
suffered criticism from Kuchar [34], the issue has since been resolved by independent investigations [35, 36].

The Page–Wootters process thus restores the notion of time in the (otherwise) timeless Universe but not necessarily in a useful way. Given that the clock and its complement are roughly equal in size, \( \dim C \sim \dim R \), the emergent time parameter can only provide a time ordering for \( R \) as a whole; it cannot do so for an arbitrary subsystem of \( R \).

Figure 4: A representation of the Hilbert space of the Universe as used in the Page-Wootters method with clock system \( C \) maximally entangled with the rest of the Universe \( R \). The left image shows an isolated clock system. The right image shows a clock which is allowed to interact with an arbitrary subsystem \( R' \) contained in \( R \).

This point underlying this last claim is illustrated in Figure 4, where one can see that \( R \) has to ‘disperse’ the mutual entanglement throughout a much larger region of Hilbert space than that occupied by a smaller subsystem,

\[ \text{maximally entangled} \]

\[ \text{Interactions} \]

\[ \text{R} \]

\[ \text{R} \]

The consequences of weakening this assumption are discussed in Appendix B.
$R' \ll R$. That is, $R'$ can have only partial ‘knowledge’ of the mutual entanglement since this smaller subsystem is not, by itself, strongly entangled with $C$. So that, as far as $R'$ is concerned, an essential ingredient of the Page–Wooter’s framework is absent. This appears to limit the utility of their method to the case of a two-system picture. However, a way of circumventing this obstacle is still possible.

In some previous investigations \cite{21, 22}, we have made the case that $C$ cannot be completely isolated from $R$ if it is to efficiently provide a description of time. This conclusion depended on some specific physical examples, but the analytic findings are supported by a more general argument: As long as $R'$ is continuously interacting with $C$, its state (or configuration) will remain correlated with the states of $C$, which are in turn still correlated with the eigenvalues of $\hat{P}_C$. Hence, these very same eigenvalues can also serve as the time parameter for $R'$.

It is worth augmenting this argument with a particular example (\textit{cf.} Figure 4). Let us suppose that the role of the clock is played by the cosmological expansion of the Universe as often put forth in earlier literature; see, for example. \cite{37}. \textsuperscript{13} Let us also identify $R'$ and $R''$ as a pair of ‘tiny’ subsystems of $R$; namely, the Milky Way and some other galaxy that is close enough for there to be a mutual attractive force. There are now two possibilities: Either the gravitational pull between $R'$ and $R''$ is not strong enough to overcome the expansion of the Universe and the galaxies move apart, or the expansion effect loses out and the two galaxies move toward one another. In both

\textsuperscript{13}We are assuming that there is some mechanism and thus subsystem of the Universal Hilbert space that is responsible for the expansion.
scenarios, the resultant interaction between $R'$ and $R''$ is influenced by the expansion, and so by $C$. In other words, each galaxy is continually provided with a record of the expansion or, effectively, with a record of the time. In this setup, the light emitted from $R''$ and captured by $R'$ (and vice versa) is what represents the ‘clock readings’.

The expansion of the Universe has recently been adopted as a clock in a paper by Stupar and Vedral [38]. In that study, and in contrast to ours, the interaction effects were regarded as negligible to ensure that the clock is ideal (i.e., a completely isolated system). Although the clock in these examples is only one of many choices, we would contend that it is the natural one because, in this case, the clock parameter is literally the cosmological time. Moreover, this brand of interactions must apply to all systems as a matter of principle because of the uniquely universal nature of gravity; everything gravitates!

It is important to emphasize that any interaction between the various subsystems is associated with the ‘recording of information’ and, thus, with an accompanying production of heat. Hence, each successive configuration of the clock, as measured by $R'$, corresponds to an increase in entropy and so a thermodynamic arrow of time. The mutual entanglement between $C$ and $R$ is reduced by these same interactions, and so an arrow pointing in the direction of decreasing entanglement can be identified as the cosmological arrow of time. In this way, we anticipate the lining up of all the relevant arrows of time: cosmological, thermodynamic and psychological. But the elephant in the room continues to be the need for causality to be postulated.

While it might be argued that the entanglement can be discarded if in-
teractions are allowed to measure the clock directly, we briefly point out
that removing the entanglement would result in a breakdown of the Page–
Wootters description of time. Let us suppose that $|\Psi\rangle$ is a separable, rather
than entangled state of $C$ and $R$, which would then lead to

$$|\Psi_p\rangle = \alpha_p (e^{-ip\hat{P}_C} |\phi_p\rangle_C) \otimes |\phi_p\rangle_R$$

where $p$ is the eigenvalue of $\hat{P}_C$, $\alpha_p$ are coefficients, and $|\phi_p\rangle_C, |\phi_p\rangle_R$ are the
states of $C$ and $R$ respectively. Importantly, there is no intrinsic time that
can be utilized by $C$ in the absence of entanglement. If we consider $R$ as
the clock from $C$’s perspective, the removal of entanglement translates into a
given state of $C$ being able to ‘pick’ any state of $R$ as its ‘partner’, resulting
in a nonsensical description of time. The interactions would only be able to
recover a meaningful sense of time without entanglement if their strength
could be turned up such that they significantly influence $C$. This would
negate the weak interaction condition and thus cause a breakdown of the
formalism’s ability to describe time. There is, however, reason to think that
just such a scenario may be realized eventually.

We have previously argued that, although the effects of the interactions
between subsystems may be negligible to begin with, they would apply con-
tinually and eventually reach a point when the ideal-clock limit no longer
makes sense, even as a limiting case [21, 22]. The current situation, however,
does not face such a crisis. As the expansion continues, $R'$ and $R''$ will ei-
ther separate to a large enough distance to render the interactions as truly
negligible or they will merge into a single system for which the expansion
has no influence. Of course, either scenario prevents any further access to
the states of the clock. Extrapolating this idea to longer and longer scales, one comes to the realization that, for the inevitable Universal state of maximum entropy, the requisite interactions would become utterly irrelevant, as no new information about the state of $C$ could ever be recorded. In other words, the loss of the clock parameter would occur just when the evolution of the Universe has finally stagnated.

4 Concluding discussions

The status of time: Our previously stated definition of time, as the potential for change, is distinct from the experience of time. This is because real physical systems obey a compulsion to change through a particular sequence of configurations and not just the potential to do so. Moreover, the specific order of the sequence plays a role in that it maintains the consistency of physical laws, which are what determines how to relate one configuration to the next. Static systems are defined as those existing in a single configuration and so are not functionally dependent on time. This class of systems can be distinguished from those which are functionally dependent on time but have no compulsion to change; what we have referred to as being frozen in time. While it is not outside the norm to see the latter category labeled as ‘timeless’ (see below), we have argued, much to the contrary, that the physical existence of systems — even if fixed in this way — still require a moment of time just like a motionless object requires a position in space. If this assertion survives under closer scrutiny, theories which claim to remove time completely may be at a disadvantage.
The timeless Universe: The timeless description has its roots in the eternal Universe that was proposed by Parmenides in ancient Greece [7] but has since become associated with more modern representations.\footnote{See, for example, the discussion on Block-Universe models in Appendix B.4.}

The timeless interpretation of the Wheeler–DeWitt model is similar to some previous versions, except that it also incorporates the stochastic nature of quantum mechanics. A conceptual framework for this picture has been laid out by Barbour \cite{2}.\footnote{There are other applicable frameworks such as that in \cite{4}. Here, we are trying to capture the general features which are shared by most of the timeless interpretations.} That author presented a view in which there is an infinite ensemble of distinct frames, each of which includes a collection of physical systems in a very specific set of configurations. One could view the ensemble of frames as randomly ‘scattered’ in an infinitely large heap. The frames might then be ‘lined up’ to form an ordered sequence of configurations for each physical system. If time is indeed required for physical existence, then each such frame could still represent a moment of time. If this is indeed the viewpoint, then it is fully compliant with our notion of a frozen (but not static) version of the Universe.

Whereas the notion of a moment might survive in this manner, the concept of change cannot. An argument prohibiting change as a real physical process was presented long ago by McTaggart \cite{39}. Although disagreeing that the argument applies in general, we find that it works nicely in the context of such timeless descriptions and adapt it accordingly. Let us assume a time-frozen Universe (as described above) and consider a physical system with an infinitely large number of different configurations, each of which is contained in some ‘slice’ of time. The argument points out that all configu-
rations of the system can claim equal existence in any given slice as there is, *a priori*, no rule against this. The way out of this nonsensical scenario would be to insist that each configuration is constrained to exist in a single frame. As a result, the identity of the physical system would have to be separated into ‘parts’, with each of these being an individual physical entity that is existing, perpetually, in a particular frame with its single configuration. The physical system, as the sum of these parts, could not experience change in any objective way, as no single part of it ever transitions into any other. The conclusion is that any perceived change in a timeless Universe would have to be illusionary. This would be similar to the way in which individual frames in a sequence can be used to form the illusion of a moving picture, just like in twentieth-century animation.

To maintain this illusion of timeless change, the ‘memories’ — or the records of past interactions — that are present in each frame may be used to reproduce the specific order of configurations which are observed in Nature. But, for any given frame in the sequence, there could only ever be circumstantial evidence for the existence of (causally) older frames. Indeed, each individual frame isolates the physical entities within it — along with their respective records of past interactions — from any and all other frames, as each member of the ensemble exists independently of the others. Meaning that such a record cannot, in fact, be attributed to interactions; it can only

---

16 Here, we mean memories (and records as well) in the broadest terms possible. For instance, a sea-side rock might remember being eroded by the tides.

17 Other timeless interpretations focus on the relations between frames to describe an order, but these would presumably also rely on a record in each frame to account for any knowledge of other frames.
be related to the frame that hosts it.

And, if this state of affairs was not already problematic enough, an explanation is still needed to account for the specific order of configurations which appear in Nature. An advocate for timelessness would require something like, perhaps, a many-paths approach or an action principle to explain why the illusion of change (as perceived through the ordering of the frames) is consistent with the laws of physics. What cannot, however, be relied on is the notion of causality, as this conceptually breaks down when confronted with the timeless interpretation.

As pointed out in Section 1.1, causality runs amiss when the labels of ‘cause’ and ‘effect’ can be arbitrarily assigned to either end of a process. This would be the case in any theory that maintains time-reversal invariance under all circumstances, which is certainly the case here, and forms part of Hume’s concern that, in any such process, cause and effect would each be equally responsible for the other [40]. But, more importantly, without the notion of objective change, the motivation for connecting one configuration of a system to any other becomes *ad hoc*, never mind connecting them in a causally ordered way. What is needed for this to work would be the inclusion of an intervening process such as an irreversible interaction. In other words, maybe change could be an illusion but causality could never be.

**Change as a real process:** There is a school of thought that real physical change can be viewed as a process of ‘becoming’, aligning with ideas from [41, 42]. Our results seem to be in support of this viewpoint. For us, irreversible interactions serve as agents of change, inducing each configuration of a system
to transition to — or become — the next one in the sequence. This picture enabled us to recover time as an emergent phenomena, in spite of the isolated state of the Universe.

The aforementioned argument of McTaggart has been used to prohibit a description of the Universe as a place of ‘becoming’ due to the inclusion of a present-time or ‘now’ moment. However, the argument can only apply to the Parmenides Universe and other timeless versions for which every moment of time is to be regarded as existing equally (i.e., existing simultaneously and in perpetuity).

This distinction between ‘timefullness’ and timelessness reinforces the conclusion above: Treating change as a real process requires one to dismiss the notion that all moments of time could exist simultaneously. This treatment does, however, necessarily invoke causality if it is to be consistent with the experience of time in the Universe. While the description of interactions within a closed system is often said to be incompatible with causality — as no external influence is available to initiate a causal chain of events — there are still arguments to the contrary; see, for instance.

On the other hand, adopting the stance that change is an illusion, one would still require a mechanism that enforces the correct order on a sequence of configurations. There is then no added cost for postulating causality.

**Final thoughts:** The recurring theme of this work is that the basic definition of time does not appear to be an issue but the experience of time remains unexplained. Put differently, the necessity for causality and that it

---

18 The principles of relativity are sometimes used for the same purpose. But there are arguments that counter such a claim. See, for example. [15][43][44].
must be put in by hand is what appears to be the real ‘problem of time’.

The notion that time in quantum mechanics and time in general relativity are unrelated might be misleading given the expectation of an underlying fundamental theory. As explained earlier, the quantum and relativistic time treatments could plausibly be interpreted as emerging from the same source. Under this interpretation, it would not matter that the two treatments appear different, as their respective theories have different domains of applicability — except at the very scales where a more fundamental theory needs to be considered.

We have also proposed a means by which the experience of time can emerge by way of the Page–Wootters framework for clocks in an isolated Universe. The key new element in our proposal is the role played by irreversible interactions; not that their inclusion is in any way novel but, rather, that their importance in the framework has been under appreciated. It is this class of interactions that leads to the recovery of a Universal arrow of time; meaning that any system’s sequence of configurations will naturally line up with the thermodynamic and psychological arrows, and can even align with the cosmological arrow if the expansion of the Universe adopts the role of the clock. (To be clear, it is the entanglement between the clock and the remainder which ensures that all systems see the same set of arrows.) Our proposal is also indicative of a natural connection between gravity, as per the cosmological expansion, and entropy, through the associated recording of information. Although highly speculative, we are tempted to suggest that this is yet another manifestation of the close-knit relation between gravity and entropy that has become ubiquitous in the high-energy literature. For
instance, Jacobson’s derivation of Einstein’s equations via the first law of thermodynamics [46], the link between entanglement entropy and null surfaces [47] that follows from the gauge–gravity duality [48] and the recent recovery of the second law in the same holographic framework [49].

If we can establish the above interpretation of time, then what still remains? Once again, all such queries lead to the conspicuously absent explanation for causality. Without this fundamental principle, one cannot describe any process of change in Nature. Then, insofar as all sensible treatments of time are reliant on the concept of causal order, we would like to suggest that the problem of time be restated as the ‘problem of causality’. By removing the clutter from the discussion, our hope is that the understanding of time can advance just like time inevitably does itself.

Acknowledgments

The research of AJMM received support from an NRF Incentive Funding Grant 85353 and NRF Competitive Programme Grant 93595. KLHB is supported by an NRF bursary through Competitive Programme Grant 93595 and a Henderson Scholarship from Rhodes University. This work is based on the research also supported in part by the National Research Foundation of South Africa (Grant Numbers: 111616).
Section 3 introduced a timeless model of the Universe, as expressed by a pure state $|\Psi\rangle$, along with a Hamiltonian $\hat{H}$. To recover time in this setup, one can — following Page and Wootters [20] — partition the Universe into a clock $C$ and the rest $R$, which are governed by Hamiltonians $\hat{H}_C = \text{Tr}_R\hat{H}$ and $\hat{H}_R = \text{Tr}_C\hat{H}$ respectively. The conjugate to the clock Hamiltonian $\hat{P}_C$, as defined by $[\hat{H}_C, \hat{P}_C] = i$ (with $\hbar = 1$), plays a key role. Suppose that $\hat{P}_C |p\rangle = p |p\rangle$ and that $p$ is a continuous variable (at least for all practical purposes). Then $p$ can serve as the ‘evolution parameter’ whose associated evolution operator is $U_C = e^{i\hat{H}_C p}$.

In order for the parameter $p$ to serve as ‘time’ for both $C$ and $R$, their respective states should be maximally entangled; that is,

$$|\Psi\rangle = \sum_p \alpha_p |p\rangle |\phi_p\rangle ,$$

where $|\phi_p\rangle$ represents the states of $R$ (in terms of a suitable basis) and $\alpha_p$ represents complex coefficients.

Given that the Hamiltonian for the entire Universe vanishes, we have $\hat{H} = \hat{H}_C \otimes 1 + 1 \otimes \hat{H}_R + \hat{H}_I$, where $\hat{H}_I$ governs any interaction effects between $C$ and $R$. If $\hat{H}_I$ can be considered negligible, then $\hat{H} \approx \hat{H}_C \otimes 1 + 1 \otimes \hat{H}_R = 0$, which leads to the relation

$$\hat{H}_C \approx -\hat{H}_R .$$

This quasi-equality makes it clear that states of $C$ are correlated with those of $R$ in such a way that $p$ describes the evolution of both.
B Objections to the interacting clock solution

B.1 The clock ambiguity

The clock ambiguity has been one of the main criticisms against the Page–Wootters method [23]. Of particular interest to us is not only the ambiguity itself but a recently proposed resolution that is based on using an isolated clock system [50]. Because of our emphasis on the interactions between the clock and the rest of the Universe, we would like to propose a resolution that does not depend on isolating the clock.

The clock ambiguity is just as the name suggests: There are a multitude of ways, practically an infinite number, to partition the Universe into a clock $C$ and the rest $R$. It is then safe to say that different partitions would generally lead to different time parameters, as each choice of clock system will have its own unique succession of states. This is simply not an acceptable state of affairs for an emergent notion of time.

The choice of fully isolated subsystems does appear to circumvent this difficulty because, as made clear in [50], any such partition is related to all others via a unitary transformation. However, as shown in the main text of the current treatment, an interacting clock is an essential ingredient for the Page–Wootters framework to make sense. But, on the other hand, treating the isolated clock as a limiting case of a more general situation, one might be tempted to argue that the two views are consistent provided that the interactions are small enough. The problem with this argument is that the effects of the interactions would accumulate and would have to be taken into
account eventually; see Section 3 and also 21, 22. However, the problem with our counterargument is that the clock ambiguity must once again be confronted.

Yet, in the framework of our discussion, any ambiguity in the choice of clock is really besides the point. This is because there is no ambiguity in the arrow of time that emerges by way of ‘recording’ the interactions, and this arrow naturally aligns with that of thermodynamics, which certainly has no ambiguity in its meaning of time. We would then propose that the key to resolving the clock ambiguity is to make sure that one’s choice of clock is not isolated.

B.2 Size discrepancies and the large clock resolution

Let us first note that one can expect, on generic grounds, that \( \dim C \sim \dim R \) simply because of the condition of maximal entanglement. The same reasoning would imply that the two subsystems must basically agree on most measurable properties (such as the magnitude of each one’s energy), as maximal entanglement literally means that complete knowledge of one system provides one with complete knowledge of the other. But let us suppose that the dimensionalities do indeed differ. We can immediately discount a scenario like \( \dim C \ll \dim R \) because any one configuration of the clock would then correspond to a multitude of different \( R \) states, thus inhibiting the evolution of \( R \). But what about the opposite situation? More importantly, could one use the notion of a large clock to eliminate the need for interactions?

To address the second query, it is useful to recall from Section 3 that isolated clocks can only provide a sense of time for a two-system Universe.
The basic point is that, given a clock $C$, the remainder of the Universe $R$ and any parametrically small subsystem $R'$ of $R$, then $R'$ cannot have sufficient ‘knowledge’ of the entanglement between the clock and its complement. On the other hand, if $R'$ is of the same order in size as $R$, then there are still only two (or perhaps three) subsystems that are able to experience time.

It might appear that this problem can be avoided by taking $C$ to be very large, $\dim C \gg \dim R$, so that any other subsystem in the Universe could be as entangled as much with the clock system as it could ever be. But this would not work because each subsystem would then ‘see’ a different sequence of clock configurations and, therefore, would have its own distinct notion of time. For isolated subsystems this would indeed be the case — but not if the interactions between the subsystems are restored to provide them with a universally agreed upon thermodynamic arrow of time. And so, although the case of $\dim C \gg \dim R$ cannot be ruled out, it has no obvious selling points and seems rather unnatural.

**B.3 The multiverse and parallel universes**

We have, by intention, been restricting considerations to the case for which the Universe is in isolation. If one is invested in models with a large number of parallel universes and/or the now popular multiverse framework, then these external ‘verses’ would have to be precluded from influencing the Universe in question. Alternatively, should one or more of these external verses be shown to influence the Universe of interest, then the boundary of the system could be extended to include the influencing systems. Only an infinite number of such influencers would then produce any significant obstruction to our
B.4 The Block Universe

The so-called Block Universe is not too far removed from the timeless models of the Universe which were considered, and then mostly dismissed, in Section 4. Nevertheless, there is enough of a distinction to warrant a separate comment.

There are actually many variants of Block Universe (see, e.g., [6], for a summary), but any of these describe the same basic picture: A deterministic reality in which all ‘moments of time’ can be viewed as spacelike slices, with each one stacked on another to form a never-changing four-dimensional ‘block’ of spacetime. There is no possibility of distinguishing between past, present and future, as all such slices are meant to be equal in status. Consequently, any experience of time or any description of a transitory ‘now’ moment should be regarded as an illusion. As the concept of time becomes trivialized, there is indeed a sense of timelessness for these models. This timelessness is, however, different from that of our frozen Universe because the former cannot incorporate causality — its inclusion would inevitably require one to treat change as a real physical process. This point is elaborated on in Section 4.

\[\text{19}\] The same idea is also described by McTaggart’s B-series of time [39].
References

[1] S. Hawking and R. Penrose, *The nature of space and time* (Princeton University Press, 2010).

[2] J. Barbour, “The nature of time,” [arXiv:0903.3489 [gr-qc]].

[3] H. Price, “Cosmology, time’s arrow, and that old double standard,” [arXiv:9310022 [gr-qc]].

[4] C. Rovelli, *The Order of Time* (Penguin Books, 2018).

[5] J. D. Norton, “Time really passes,” Humana. Mente: Journ. of Phil. St. 13 (2010).

[6] B. Dainton, *Time and space* (Routledge, 2016).

[7] K. R. Popper, *The world of Parmenides*, (Taylor & Francis, 2012).

[8] M. Nelson, “Existence,” in The Stanford Encyclopedia of Philosophy, Winter 2016 Ed., Edward N. Zalta, ed., https://plato.stanford.edu/archives/win2016/entries/existence/.

[9] T. Sider, *Four-dimensionalism: An ontology of persistence and time* (Oxford University Press, 2001).

[10] J. Pearl, *Causality* (Cambridge University Press, 2009).

[11] A. N. Whitehead and D. W. Sherburne, *Process and reality* (Macmillan, New York, 1957).
[12] I. Prigogine, *From being to becoming: Time and complexity in physical systems* (W.H. Freeman, San Francisco, 1980).

[13] G. M. D’Ariano, “Causality re-established,” Phil. Trans. R. Soc. A **376**, 20170313 (2018).

[14] D. Z. Albert, *Time and chance* (Harvard University Press, 2003).

[15] G. F. R. Ellis, “On the flow of time,” [arXiv:0812.0240 [gr-qc]].

[16] S. W. Hawking, “The Direction of Time,” New Scientist **115**, 46 (1987).

[17] J. B. Hartle, “The physics of ‘now’,” Amer. Jour. of Phys. **73**, 101 (2005).

[18] R. Landauer, “Irreversibility and heat generation in the computing process,” IBM journal of research and development **5**, 183 (1961).

[19] B. S. DeWitt, “Quantum theory of gravity. I. The canonical theory,” Phys. Rev. **160**, 1113 (1967).

[20] D. N. Page and W. K. Wootters, “Evolution without evolution: Dynamics described by stationary observables,” Phys. Rev. D **27**, 2885 (1983).

[21] K. L. H. Bryan and A. J. M. Medved, “Realistic clocks for a Universe without time,” Found. Phys. **48**, 48 (2018) [arXiv:1706.02531 [quant-ph]].

[22] K. L. H. Bryan and A. J. M. Medved, “Requiem for an ideal clock,” [arXiv:1803.02045 [quant-ph]].
[23] A. Albrecht and A. Iglesias, “The Clock ambiguity and the emergence of physical laws,” Phys. Rev. D 77, 063506 (2008) [arXiv:0708.2743 [hep-th]].

[24] S. Hawking, A Brief History of Time (Bantam, 1988).

[25] R. d’Inverno, Introducing Einstein’s Relativity (Oxford University Press, 1992).

[26] J. Polchinski, String theory. Vol. 1: An introduction to the bosonic string, Vol. 2: Superstring theory and beyond (Cambridge University Press, 1998).

[27] S. Weinberg and E. Witten, “Limits on massless particles,” Phys. Lett. B 96, 59 (1980).

[28] E. Witten, “Symmetry and Emergence,” Nature Phys. 14, 116 (2018).

[29] G. ’t Hooft, “Symmetry Breaking Through Bell-Jackiw Anomalies,” Phys. Rev. Lett. 37, 8 (1976).

[30] T. Banks and L. J. Dixon, “Constraints on string vacua with spacetime supersymmetry,” Nucl. Phys. B 307, 93 (1988).

[31] J. Polchinski, “M theory: Uncertainty and unification,” [arXiv:0209105[hep-th]].

[32] J. D. Bekenstein, “Nonexistence of baryon number for static blackholes,” Phys. Rev. D 5, 1239 (1972).
[33] D. N. Page, “Particle emission rates from a black hole. I: Massless particles from an uncharged, nonrotating hole,” Phys. Rev. D 13, 198 (1976).

[34] K. V. Kuchar, “Time and interpretations of quantum gravity,” in Proc. 4th Canadian Conference on General Relativity and Relativistic Astrophysics, G. Kunstatter, D. E. Vincent and J. G. Williams, eds. (1992).

[35] C. E. Dolby, “The Conditional probability interpretation of the Hamiltonian constraint,” [arxiv:0406034 [gr-qc]].

[36] V. Giovannetti, S. Lloyd and L. Maccone, “Quantum Time,” Phys. Rev. D 92 045033 (2015) [arXiv:1504.04215 [quant-ph]].

[37] A. Vilenkin, “Quantum cosmology and the initial state of the universe,” Phys. Rev. D 37, 888 (1988).

[38] S. Stupar and V. Vedral, “Was inflation necessary for the existence of time?,” [arXiv:1710.04260 [quant-ph]].

[39] J. E. McTaggart, “The unreality of time,” Mind 17, 457 (1908).

[40] D. Hume, “An enquiry concerning human understanding,” in Seven Masterpieces of Philosophy, 191 (Routledge, 2016).

[41] L. Smolin, “Temporal relationalism,” arXiv:1805.12468 [physics.hist-ph].

[42] S. M. Carroll, “What if Time Really Exists?,” [arXiv:0811.3772 [gr-qc]] (2008).
[43] C. Bouton, “Is the Future already Present? The Special Theory of Relativity and the Block Universe View,” in *Time of Nature and the Nature of Time*, 89 (Springer, 2017).

[44] J. Erasmus, “Can Cosmology Justify Belief in an Eternal Universe?,” in *The Kalam Cosmological Argument: A Reassessment*, 129 (Springer, 2018).

[45] M. Frisch, *Causal reasoning in physics* (Cambridge University Press, 2014).

[46] T. Jacobson, “Thermodynamics of spacetime: the Einstein equation of state,” Phys. Rev. Lett. **75**, 1260 (1995).

[47] S. Ryu and T. Takayanagi, “Holographic derivation of entanglement entropy from AdS/CFT,” Phys. Rev. Lett. **96**, 181602 (2006) [hep-th/0603001].

[48] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, “Large N field theories, string theory and gravity,” Phys. Rept. **323**, 183 (2000) [hep-th/9905111].

[49] N. Engelhardt and S. Fischetti, “Losing the IR: a Holographic Framework for Area Theorems,” [arXiv:1805.08891 [hep-th]].

[50] C. Marletto and V. Vedral, “Evolution without evolution and without ambiguities,” Phys. Rev. D **95** 043510 (2017) [quant-ph].