Estimation of the friction coefficient between wheel and rail surface using traction motor behaviour

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Abstract. The friction coefficient between a railway wheel and rail surface is a crucial factor in maintaining high acceleration and braking performance of railway vehicles thus monitoring this friction coefficient is important. Restricted by the difficulty in directly measuring the friction coefficient, the creep force or creepage, indirect methods using state observers are used more frequently. This paper presents an approach using a Kalman filter to estimate the creep force and creepage between the wheel and rail and then to identify the friction coefficient using the estimated creep force-creepage relationship. A mathematic model including an AC motor, wheel and roller is built to simulate the driving system. The parameters are based on a test rig at Manchester Metropolitan University. The Kalman filter is designed to estimate the friction coefficient based on the measurements of the simulation model. Series of residuals are calculated through the comparison between the estimated creep force and theoretical values of different friction coefficient. Root mean square values of the residuals are used in the friction coefficient identification.

Nomenclature

| Symbol | Description |
|--------|-------------|
| A, B   | System matrixes for the induction motor |
| a, b   | The contact ellipse semi-axes |
| C_{11,22,23} | Kalman coefficients |
| E      | Young’s modulus |
| F_x, F_y | Longitudinal and lateral creep forces at the wheel-rail surface |
| F_N    | Normal force at the wheel-rail surface |
| F_{crpf} | Moderated longitudinal creep force at the wheel-rail surface |
| f      | Frequency of the stator voltage at the motor |
| H      | Measurement matrix for the induction motor at the motor |
| I_{a_1}, I_{b_is}, I_{a_r}, I_{b_r} | Stator and rotor current at α and β phase at the motor |
| i      | Transmission ratio |
| J_{motor}, J_{wheel}, J_{roller} | Inertia of the motor, wheel and roller |
| K      | Kalman filter gain |
| L_n, L_r, L_m | Stator, rotor and mutual inductances |
| n_p    | Number of poles |
| P^{-}, P | Predicted and corrected error covariance matrix |
| Q      | Covariance matrix of system noise |
| R      | Covariance matrix of measurement noise |
1. Introduction

It is the creep force generated between the wheel and rail that determines the acceleration and braking performance of a railway vehicle. As the maximum creep force depends on the friction coefficient and mass of the vehicle, good monitoring of the friction coefficient is very important in estimating the maximum creep force and maintaining a satisfactory acceleration and braking performance.

A combination of the Luenberger observer and integrator was developed to estimate creep force and identify the skidding (sliding) phenomenon between the wheel and roller and were validated through experiments on a scaled roller-rig [1]. In the research, the sliding phenomenon was identified based on the sudden and significant change of the estimated friction force. The skidding phenomenon was then more thoroughly studied with the implementation of the 2nd order Luenberger observer [2]. The interaction between the wheel-roller slip and the torsional oscillations of the driving system was studied by using spectrum analysis, showing that the creep force was influenced by low frequency harmonics. These two researches focused only on the skidding phenomena and did not analyse the creepage and friction coefficient. Neural network methods were also used to estimate the friction coefficient [3]. The limitations of this method lie in the inherent inaccuracy of the neural network method. An approach using the Kalman filter was proposed to estimate the wheel-rail friction coefficient [4], but the slip ratio and vehicle speed were required in the estimation of friction coefficient. This method is made less practical due to the difficulty in measuring the slip ratio and vehicle speed. Researches have also been carried out in identifying low-adhesion between wheel and rail [5, 6]. These researches proposed ways to identify the low-adhesion scenario by studying the lateral behaviour of the vehicle, with the implementation of state observers. However, it can only identify the significant change in the friction coefficient. In later research, a series of state observers, which were tuned at the saturation points under different friction coefficients, are implemented to estimate the lateral dynamic behaviours of the vehicle and identified the friction coefficient by searching the least residuals produced by the observers [7]. In this way, a more accurate estimation of the friction coefficient can be made but problems can arise when the residual results give similar combinations.

This paper proposes a different method that focuses on the torsional dynamics of a simplified driving system. Only the stator voltage, current and speed of the motor are required to measure. The creep force and creepage between the wheel and roller can be estimated by using a designed Kalman filter. By comparing the estimated creep force-creepage relationship to those under different friction
Coefficient, the root mean square of the residuals can be calculated. The friction coefficient identification is then based on the root mean square value.

2. The simulation model

The simulation model, which is based on the twin disc test rig at Manchester Metropolitan University, includes an AC induction motor, a transmission system, a wheel and a roller. The layout of the model is shown in Figure 1.

![Figure 1. Layout of the simulation model.](image)

The induction motor is modelled in a stationary frame and controlled by the Volts/Hz method. A voltage fed inverter is implemented in the motor drive circuit. And the supplied stator voltage is given as:

\[
U_{\alpha s} = U_m \cos(2\pi f)
\]

\[
U_{\beta s} = U_m \cos(2\pi f - \frac{2}{3}\pi)
\]

\[
U_{cs} = U_m \cos(2\pi f + \frac{2}{3}\pi)
\]

The stator voltage then transferred from the ABC axis to the stationary \(\alpha - \beta\) frame which the motor is modelled in, according to equation (2).

\[
\begin{bmatrix}
U_{\alpha s} \\
U_{\beta s}
\end{bmatrix} = \sqrt{3} \begin{bmatrix}
1 & \cos(-\frac{2}{3}\pi) & \cos(\frac{2}{3}\pi) \\
0 & -\sin(-\frac{2}{3}\pi) & -\sin(\frac{2}{3}\pi)
\end{bmatrix} \begin{bmatrix}
U_{\alpha s} \\
U_{\beta s} \\
U_{cs}
\end{bmatrix}
\]

The dynamic equations of the induction motor are shown in equation (3) to (12) [8].

\[
U_{\alpha s} = R_s I_{\alpha s} + \psi_{\alpha s}
\]

\[
U_{\beta s} = R_s I_{\beta s} + \psi_{\beta s}
\]

\[
U_{ar} = R_s I_{ar} + \psi_{ar} + \psi_{ar} \omega_r
\]

\[
\psi_{ar} = L_s I_{ar} + L_m I_{ar}
\]

\[
\psi_{br} = L_s I_{br} + L_m I_{br}
\]

\[
T_e = n_p L_m (I_{br} I_{ar} - I_{ar} I_{br})
\]

The induction motor and the wheel are connected by a pair of gears and the wheel and the roller interact with each other through the creep force that is generated between them. Thus the speed of the wheel and the roller can be calculated according to equations (12) to (14),

\[
J_{eq} \dot{\omega}_{motor} = T_e - iT_{crpf}
\]
\[ \omega_{\text{wheel}} = \frac{\omega_{\text{motor}}}{1} \]
\[ J_{\text{roller}} \dot{\omega}_{\text{roller}} = T_{\text{crpf}} \]

where \( J_{\text{eqv}} = J_{\text{rotor}} + \frac{J_{\text{wheel}}}{i^2} \), \( T_{\text{crpf}} = R_{\text{roller}} F_{\text{crpf}} \)

The creep force is acquired using Kalker's linear assumption and modified by the Vermeulen-Johnson equations [9]:

\[
F_x = -f_{11}y_x
\]
\[
F_y = -f_{22}y_y - f_{23}\omega_z
\]
\[
F_{\text{crpf}} = \begin{cases}
\frac{\mu F_x F_y}{\sqrt{F_x^2 + F_y^2}} & \sqrt{F_x^2 + F_y^2} \leq 3\mu F_N \\
\frac{\mu F_x F_y}{F_x^2 + F_y^2} & \sqrt{F_x^2 + F_y^2} > 3\mu F_N
\end{cases}
\]

The linear creep coefficients are defined as:

\[
f_{11} = E(a, b)C_{11}
\]
\[
f_{22} = E(a, b)^{1.5}C_{22}
\]
\[
f_{23} = E(a, b)^2C_{23}
\]
where \( a \) and \( b \) are calculated by the Hertz method and \( C_{11}, C_{22} \) and \( C_{23} \) are calculated from approximate formulae given by Kalker [10].

In the case of a single wheel and roller without any lateral displacement, the creepage is defined as follow:

\[
\gamma_x = 2 \frac{R_{\text{roller}} \omega_{\text{roller}} - R_{\text{wheel}} \omega_{\text{wheel}}}{R_{\text{wheel}} \omega_{\text{wheel}} + R_{\text{roller}} \omega_{\text{roller}}}
\]
\[
\gamma_y = 0, \omega_z = 0
\]

3. The estimation of the creep force and creepage

To estimate the creep force and the creepage between the wheel and roller, the electric torque of the induction motor should be first estimated by the Kalman filter. The Kalman filter estimates a system with a subset of its state variables (measurements), and the input of the estimated system, as shown in Figure 2.

**Figure 2.** Block diagram for the system estimation using Kalman filter.

1.1 State model of the induction motor

To estimate the behaviour of an induction motor, the system and measurement equations can be expressed as:

\[
x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1}
\]
\[
z_k = Hx_k + v_k
\]
where:
\[ x_k = \begin{bmatrix} I_{as} & I_{bs} & \psi_{ar} & \psi_{br} \end{bmatrix}^T \]  
(21)

\[ z_k = \begin{bmatrix} I_{as} & I_{bs} \end{bmatrix}^T \]  
(22)

\[ u_k = \begin{bmatrix} U_{as} & U_{bs} \end{bmatrix}^T \]  
(23)

\[
A = \begin{bmatrix}
1 - \left( \frac{R_s}{\sigma L_s} + \frac{1 - \sigma}{\sigma L_r} \right) t_s & 0 & \frac{L_m}{\sigma L_s L_r} t_s & \frac{L_m}{\sigma L_s L_r} \omega_r t_s \\
0 & 1 - \left( \frac{R_s}{\sigma L_s} + \frac{1 - \sigma}{\sigma L_r} \right) t_s & -\frac{L_m}{\sigma L_s L_r} t_s & -\omega_r t_s \\
\frac{L_m}{t_r} t_s & 0 & 1 - \frac{1}{t_r} & -\omega_r t_s \\
0 & \frac{L_m}{t_r} t_s & \omega_r t_s & 1 - \frac{1}{t_r}
\end{bmatrix} \]  
(24)

\[
B = \begin{bmatrix}
\frac{1}{\sigma L_s} & 0 \\
0 & \frac{1}{\sigma L_s} \\
0 & 0 \\
1 & 0 & 0 & 0
\end{bmatrix} \]  
(25)

\[
H = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix} \]  
(26)

in which \( \sigma = 1 - \frac{L_m^2}{L_s L_r} \) and \( t_r = \frac{L_r}{R_r} \).

1.2 Creep force and creepage estimation

With the estimated state variables of the induction motor, creep force and creepage can be calculated according to equations (27) to (30).

\[ \hat{T}_c = \frac{L_m}{L_r} n_p (\hat{I}_{bs} \hat{\psi}_{ar} - \hat{I}_{as} \hat{\psi}_{br}) \]  
(27)

\[ \hat{T}_t = i(\hat{T}_c - J_0 \hat{\omega}_r) \]  
(28)

\[ \hat{\omega}_{\text{roller}} = \frac{1}{J_{\text{roller}}} \hat{T}_r \]  
(29)

\[ \hat{\omega} = \frac{R_{\text{wheel}} \hat{\omega}_{\text{wheel}} - R_{\text{roller}} \hat{\omega}_{\text{roller}}}{R_{\text{wheel}} \hat{\omega}_{\text{wheel}} - i R_{\text{roller}} \hat{\omega}_{\text{roller}}} \]  
(30)

where the symbol \( \hat{\cdot} \) indicates that the parameter is estimated.

1.3 The algorithm of the Kalman filter

Equations used in the Kalman filter can be classified into two groups, in the form of the “predictor-corrector” algorithm: time update equations which estimate the system state for the next time step based on the current system state and error covariance (the predictor part); and the measurement update equations which improve the state estimation with the measurements (the corrector part) [11].

In the predictor part, where the time is updated, the state of the system and the error covariance matrix are predicted with equation (31) and (32).

\[ \hat{x}_{k+1} = A \hat{x}_k + B u_k \]  
(31)

\[ P_{k+1} = A P_k A^T + Q \]  
(32)
Then in the corrector part, where the measurement is updated, the system state estimation is improved with the Kalman filter gain, and the corrected system state and error covariance are used in the prediction of the next time step, as shown in (33), (34) and (35).

\[
K_k = P_k^r H^T (H P_k^r H^T)^{-1} \\
\hat{x}_k = \hat{x}_k + K_k (z_k - H \hat{x}_k) \\
P_k = (I - K_k H) P_k^r
\]

(33) (34) (35)

4. Friction coefficient identification

Series of creep force values under different friction coefficient are calculated according to the estimated creepage. These creep force values are compared to the estimated creep force and a series of creep force residuals can be calculated. By searching the minimum value of these residuals, the friction coefficient can be identified.

5. Results and analysis

5.1 Simulation results for the traction system

The parameters of the AC motor are given as:

\[
R_s = 3.2\Omega, \quad R_r = 2.2\Omega, \quad L_s = 0.377H, \quad L_r = 0.377H, \quad L_r = 0.361H, \quad n_p = 2 \quad \text{and} \quad J_{motor} = 0.02
\]

The parameters of the wheel and roller are given as:

\[
R_{wheel} = 0.2m, \quad R_{roller} = 0.2m, \quad J_{wheel} = 0.89kg\cdot m^2, \quad J_{roller} = 0.89kg\cdot m^2, \quad i = 4, \quad u = 0.27 \quad \text{and} \quad \text{material is mild steel.}
\]

The motor is controlled by the following command:

\[
f = \begin{cases} 
4f_0 & t \leq 0.25s \\
0 & t > 0.25s
\end{cases}
\]

(36)

\[
V_m = 30 + 5f
\]

(37)

where \(f_0 = 50Hz\).

The wheel and roller start to get in contact at 1s, thus there is no creep force between the wheel and roller before then.

Results of the rotating speed of the motor, wheel and roller are shown in Figure 3. The speed of the motor decreases slightly at 1s, when the wheel and roller start to interact with each other. The speed then increases slightly at about 2.6s, when the wheel and roller begin to rotate at a same speed. The change in the motor speed shows the change of load applied on it due to the effect of the creep force.

![Figure 3](image-url) Speed history of the motor, wheel and roller.

5.2 Estimation result of the creep force and creepage

Based on the result from the simulation model, rotor flux can be estimated by the Kalman filter. A comparison of the estimated results and simulated results of the electric torque is shown in Figure 4. The electric torque changes dramatically during its starting, where the error is most significant. The electric torque also changes at 1s and around 2.6s, influenced by the creep force between the wheel and roller, identical to the speed changes in Figure 3.
From the estimated electric torque, after the calculation with equation (28), (29) and (30), the estimated creep force-creepage curve is plotted and compared to the curve based on the simulation result, as shown in Figure 5. It can be seen that the estimation error for the electric torque is very small; therefore the estimated creep force and creepage are very accurate. The creepage-creep force result is then compared to series of simulation result to calculate the residual between them. In the last step, the root mean square value of the creep force residuals is calculated and shown in Figure 6. As can be seen, the root mean square value reaches the lowest value when the friction coefficient is 0.27, thus from the estimated creep force and creepage, the friction coefficient between the wheel and roller is identified as 0.27, which is the same as that in the simulation model. Therefore, this method is shown to be able to estimate and identify the friction coefficient between the wheel and roller.

6. Conclusion
This paper proposes a different method for estimating and identifying the friction coefficient between a rail wheel and rail based on the behaviour of the torsional dynamics of the traction system of the vehicle. Stator voltage, current and rotor speed of the motor are measured and processed through a Kalman filter and the creep force and creepage are then estimated. Residuals between the estimated creepage-creep force curve and those of theoretical values with different friction coefficients are calculated. By calculating the root mean square of the residuals and searching for the least root mean square, the friction coefficient is identified. The results have shown that the error is small and the estimation of the friction coefficient is accurate.
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