Resummation, Power Corrections and Prediction in Perturbative QCD

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Abstract

I give a pedagogical introduction to resummation and power corrections, using the thrust variable in electron-positron annihilation as an example, followed by a discussion of issues of predictability in perturbative QCD.

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1 The Thrust and Time Evolution

The session on the predictive power of QCD emphasized new developments in power corrections and resummation. This section presents material that introduced the session, with a discussion of some of the basic ideas and methods that underlie recent progress [1, 2, 3] on these topics, using the familiar example of the thrust in $e^+e^-$ annihilation. The following sections briefly treat some broad issues of predictive power. These comments were followed by an open discussion at the session.

**Thrust.** The thrust in $e^+e^-$ annihilation may be defined as

$$T = \max_{\hat{n}} \sum_{\text{particles}} \frac{p_i \cos \theta_i}{Q},$$

(1)

where the sum is over all particles in the final state, and where $\cos \theta_i$ is the angle between momentum $\vec{p}_i$ and the axis defined by the vector, $\hat{n}$, which is chosen to maximize the sum in the $e^+e^-$ center-of-mass (c.m.) frame. Neglecting particle masses, and taking $Q$ to be the total c.m. energy, the maximum value of $T$ is unity. In this configuration, the final state consists of two opposite-moving jets, $J_1$ and $J_2$, each consisting of perfectly collinear particles. For definiteness, we take the direction of $J_1$ along $\hat{n}$, with $J_2$ opposite. The probability for such a final state is suppressed by radiation, and it is this suppression, associated with the long-time evolution of the system, which we study using resummation methods. Let us see how this works.

We first observe that for massless particles $|\vec{p}_i| = E_i$, so that $\sum |\vec{p}_i| = Q$. Then, defining $p_i^\pm = E_i \pm |\vec{p}_i|\cos \theta_i$, we find

$$1 - T = \sum_{i \in J_1} \frac{p_i^-}{Q} + \sum_{j \in J_2} \frac{p_j^+}{Q} \sim \sum_{i \in J_1} \frac{1}{\tau_i^+Q} + \sum_{j \in J_2} \frac{1}{\tau_j^-Q},$$

(2)

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where, invoking the uncertainty principle, we identify \( \tau_i^\pm \sim 1/p_i^\mp \) as the space-time variable conjugate to the momentum component \( p_i^\mp \). Elementary relativistic kinematics then shows that \( \tau_i^\pm \) is the typical time that it takes to emit a particle of momentum \( p_i \), as seen in the c.m. frame. The sums in Eq. (2) are dominated by terms corresponding to the “earliest” emission, for which \( \tau^\pm \) is smallest.

**Factorization.** From the above, we can conclude that for small values of \( 1−T \), the evolution of the quark pair into the final state is a relatively long-time process. In this case, it is natural to propose that the overall cross section is a product of functions, one for each jet [4],

\[
\frac{d\sigma}{dT_1 dT_2} = J_1(1−T_1) J_2(1−T_2),
\]

with \((1/2)(1−T_1,2) \equiv \sum |p_i^\pm/Q|\). To find \( d\sigma/dT \), we only need to integrate over \( T_1 \) and \( T_2 \) in (3), subject to \( T = (1/2)(T_1 + T_2) \).

Now consider the significance of the observation that the \( T_i \)-dependence of jet \( J_i \) is determined by its earliest emission. We use one of the basic features of QCD (indeed of quantum field theory) – that processes occurring at different time scales are quantum-mechanically “incoherent”, and may therefore be treated probabilistically [5, 6, 7]. In this language, we write

\[
J_1(T_1) = P\left(1−T_1 = 2k^-/Q\right) \tilde{J}_0(k^-/Q),
\]

where \( P\left(1−T\right) \) is the probability density for the emission of a gluon with \( k^- = (1−T_1)Q/2 \), while \( \tilde{J}_0(k^-/Q) \) is the probability that there has been no emission of gluons before time scale \( \tau^+ \sim 1/k^- \sim 1/(1−T_1)Q \). Evidently, \( \tilde{J}_0 \) is a function that decreases with time, according to exactly the probability density \( P \),

\[
\frac{d\tilde{J}_0(k^-/Q)}{d(1−T)} = \left(\frac{Q}{2}\right) \frac{d\tilde{J}_0(k^-/Q)}{dk^-} = -P(2k^-/Q)\tilde{J}_0(k^-/Q).
\]

Now \( P(2k^-/Q) \) is just the square of the amplitude for a quark of energy \( Q/2 \), moving in the \( \hat{n} \)-direction to emit a gluon with minus momentum \( k^- \). Keeping only the leading logarithm in \( 1−T \sim 2k^-/Q \), we find from a lowest-order calculation that

\[
P\left(1−T\right) = C_F \frac{\alpha_s}{\pi} \frac{\ln(1−T)^{-1}}{1−T},
\]

where \( C_F = 4/3 \) in QCD. Substituting (6) into (3) we find that \( J_0 \) is a rapidly decreasing function of \( 1−T \),

\[
\tilde{J}_0(1−T_1) = \exp \left[-C_F \frac{\alpha_s}{2\pi} \ln^2(1−T_1)\right].
\]

Then for \( J_1 \), and similarly for \( J_2 \), we have

\[
J_1(1−T_1) = C_F \frac{\alpha_s}{\pi} \frac{\ln(1−T_1)}{1−T_1} \exp \left[-C_F \frac{\alpha_s}{2\pi} \ln^2(1−T_1)\right].
\]
Convoluting the two jets together, we find that the cross section $d\sigma/dT$ takes on a very similar form \([4]\), in terms of $1 - T$.

**Enter the running coupling.** At this point we ask, “what about the running of $\alpha_s$?” This effect enters through the calculation of the probability density $P(1 - T)$, which we recall is simply a branching probability for the emission of a single gluon of minus momentum $(1 - T)Q/2$. As an integral in $k$-phase space this is,

$$P(1 - T) = \frac{2C_F}{1 - T} \frac{\alpha_s}{\pi} \int_{(1-T)^2Q^2}^{(1-T)Q^2} \frac{dk_T^{2'}}{k_T^{2'}}.$$

The running coupling organizes quantum corrections to this process that come from momentum scales larger than $k_T$, the momentum transfer in the emission process. The natural extension of (9) to include these effects is thus \([8]\),

$$P(1 - T) \rightarrow C_F \frac{\alpha_s(k_T^2)}{1 - T} \int_{(1-T)^2Q^2}^{(1-T)Q^2} \frac{dk_T^{2'}}{k_T^{2'}} \frac{\alpha_s(k_T^2)}{\pi},$$

where the running coupling is the familiar $\alpha_s(k_T^2) = 4\pi/\beta_2 \ln(k_T^2/\Lambda^2)$, with $\beta_2 = 11 - 2n_f/3$. When $1 - T = \Lambda/Q$, this expression develops a singularity, as the perturbative running coupling diverges. This is not surprising, since in this region, we are looking at the emission of very soft gluons. The divergence is itself unphysical, but signals the entry of long-distance physics into the problem. The full cross section behaves in much the same way.

Fuller treatments, which include nonleading logarithms, virtual corrections and which pay closer attention to momentum conservation, show the same pattern in the thrust and other observables involving jets \([4, 9]\). Resummed perturbative cross sections almost always encounter a “corner” of phase space where the coupling blows up \([1, 2, 3]\). At the same time, such cross sections also often have a surprising feature, illustrated by Eq. (11). Suppose we regularize the $k_T$ integral in (10) by adding a mass to $k_T$: $k_T^2 \rightarrow k_T^2 + m^2$, with $m > \Lambda$. Then the regulated coefficient of $1/(1 - T)$ in $P(1 - T, m)$ is finite all the way down to $T = 1$. For large, fixed $1 - T$, however, $P(1 - T, m)$ can be expanded as a power series in $m/Q$, starting with $P(1 - T, m = 0)$. For $(1 - T)Q \gg \Lambda$, the effect of infrared regulation is to require power corrections to the perturbative result. “Freezing” $\alpha_s(k_T)$ at some fixed $\alpha_0$ for small $k_T$ has much the same effect.

These observations are quite general; resummation almost always implies the presence of power corrections in terms of the relevant hard scale. It is easy to check, that in the range of a few to ten GeV, the quantity $\Lambda/Q$ is between the “leading order” quantity $1/\ln(Q^2/\Lambda^2) \sim \alpha_s(Q^2)$ and the “next-to-leading order”, $1/\ln^2(Q^2/\Lambda^2) \sim \alpha_s^2(Q^2)$ in magnitude, while in the same range $\Lambda^2/Q^2$ lies between next-to-leading and next-to-next-to-leading order corrections.

We may consider these power corrections as simply a signal of the limitations of perturbative methods. Alternately, we may treat them as a reflection of the structure of the actual behavior the full theory \([1, 2, 3]\). This viewpoint is explored in a number of the talks that followed in the session on the predictive power of QCD \([10]\).
2 The Predictive Power of Perturbative QCD: A Sketch

2.1 Review of basic methods

Infrared Safety. The quantitative predictions of perturbative QCD all require the identification of “infrared safe” quantities, which are dominated by the short-distance, partonic behavior of the theory. The perturbative expansion for such a quantity, labelled generically \( \sigma_{\text{IRS}} \), depending on a single hard scale, \( Q \), and some set of dimensionless parameters \( C_i \) is

\[
\sigma_{\text{IRS}} \left( \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), C_i \right) = \sum_{n \geq 0} a_n \left( \frac{Q^2}{\mu^2}, C_i \right) \left[ \frac{\alpha_s(\mu^2)}{\pi} \right]^2,
\]

where the \( a_n \) are dimensionless coefficients. Assuming that \( \sigma_{\text{IRS}} \) is observable, it is independent of the renormalization scale \( \mu \). We may then pick \( \mu = Q \), and get an expansion in the “small” coupling \( \alpha_s(Q^2) \). Here, the asymptotic freedom of QCD, by which \( \alpha_s \) decreases as its momentum scale increases, plays a central role. Note that \( \sigma_{\text{IRS}} \) need not be a cross section; we shall discuss other examples below. The first few \( a_n \) are known for many quantities. What Eq. (11) leaves out, however, are “power” corrections, that introduce nonperturbative scales. We have seen above an example of how such corrections arise, and have seen how perturbation theory can, at least in some cases, provide guidance as to their nature. To understand power corrections in a broader context, it is useful to review how results of the form (11) are derived.

Perhaps the purest example of this procedure is \( \sigma_{\text{tot}}^{e^+e^-} \), the total cross section for electron-positron annihilation into hadrons. In this case, the IR safety of the perturbative sum is ensured by a very fundamental property of the theory, its unitarity. Schematically, the optical theorem (a direct consequence of the conservation of probability) implies that

\[
\sigma_{\text{tot}}^{e^+e^-}(Q) = \frac{1}{Q^4} \text{Im} \Pi(Q^2),
\]

where \( \Pi(Q^2) \) is the contribution of all hadronic virtual states to the forward-scattering amplitude of a single off-shell photon (or Z). This forward-scattering amplitude is of the general form \( \int d^4x \exp[-iq \cdot x] \langle 0 | J^\mu(0) J_\mu(x) | 0 \rangle \), with \( J \) an electroweak current (including the electron charge) and \( q^2 = Q^2 \). Such a vacuum matrix element may be treated by the operator product expansion (OPE), an observation that lies at the heart of the many successes of the method of QCD sum rules. The OPE predicts in this case that nonperturbative corrections (proportional to vacuum condensates) begin to contribute only at the level of \( Q^{-4} \) compared to the leading perturbative expansion, (11). In fact, this result can also be derived by a variant of the reasoning given in Sec. 1 above. Perturbatively, power corrections appear through the coupling of soft gluons, with momenta of order \( \Lambda_{\text{QCD}} \), to off-shell partons. Mueller [11], applying observations of ’t Hooft [12] showed long ago how the running coupling, and
consequently divergences in the perturbative sum, appear from such configurations, with exactly the power behavior implied by the OPE.

The direct applicability of the OPE to an IR safe cross section is the exception rather than the rule, however, because it is rare to be able to reduce a physical cross section to a simple product of local operators. IR safe quantities for which the OPE does not control power corrections include event shapes in $e^+e^-$ annihilation, of which the thrust, discussed above, is a prime example. The proof of the IR safety of event shapes again depends upon the unitarity of QCD, but in a modified form [13]. Because different regions of the final-state phase space are weighted differently in an event shape, we cannot apply the optical theorem to the entire process. Essentially, unitarity is applied separately to each “jet” of outgoing partons moving in the same direction. The resulting sum is indeed IR safe, but the low momentum scales enter now through the couplings of soft gluons to lines that are generally close to the mass shell, as in the example of thrust above. Unprotected by the OPE, event shapes inherit, as above, larger power corrections than the total annihilation cross section, suppressed by lower powers of $Q$.

Factorization and evolution. The predictive potential of perturbative QCD is greatly enhanced by the factorization [7] of short-distance (perturbative) and long-distance (nonperturbative) dynamics, and by the computable evolution of cross sections with momentum transfer. Examples include the factorized structure functions in the deeply inelastic scattering (DIS) of hadron $A$ by vector boson $V$, with spacelike virtuality $q^2 = -Q^2$. These functions take the form

$$F^V_A(x, Q^2) = \sum_{\text{partons}} \int_x^1 \frac{d\xi}{\xi} C^V_a \left( \frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) \phi_{a/A}(\xi, \mu^2) \equiv \sum_a C^V_a \otimes \phi_{a/A}. \quad (13)$$

The $\phi_{a/A}$ are nonperturbative parton distributions for parton $a$ in hadron $A$, while each $C^V_a$ is a series in $\alpha_s(\mu)$, whose lowest order approximation is found from the Born cross section for $V(Q) + a$ scattering. $C^V_a$ is infrared safe, and therefore calculable in perturbation theory. The scale $\mu$ is called the factorization scale. A key observation is that $C^V_a$ depends on $\mu$ only through $Q/\mu$, and that all of the $Q$-dependence in $F$ is in the $C$’s. The convolution in (13) must be independent of the arbitrary choice of $\mu$, which leads, by simple separation of variable arguments, to the DGLAP evolution equations [6],

$$\mu \frac{dF^V_A}{d\mu} = 0 \quad \Rightarrow \quad \mu \frac{dC^V_a}{d\mu} = \sum_b C^V_b \otimes P_{ba} \left( \alpha_s(\mu^2) \right) \quad \mu \frac{d\phi_{a/A}}{d\mu} = -\sum_c P_{ac} \left( \alpha_s(\mu^2) \right) \otimes \phi_{c/A}. \quad (14)$$

The splitting functions $P_{ba}(\eta, \alpha_s(\mu))$ play the role of “separation constants”, which can depend only on parameters held in common by the parton distributions $\phi$ and the coefficient functions $C$.

Like the infrared safety of the total annihilation cross section, the factorization of the inclusive DIS structure functions follows from the optical theorem, and may be
treated by means of the operator product expansion. This opens the door for particularly well-organized treatments of power corrections, some of which were discussed in this session [14].

Closely related, but not covered by the OPE directly, are the factorizability and evolution equations of fragmentation functions,

\[ \mu \frac{dD_{C/c}(z, \mu)}{d\mu} = \sum_{d} \int_{z}^{1} \frac{dy}{y} \frac{\alpha_s}{\pi} P_{cd}(y) \frac{D_{C/d}(\frac{z}{y}, \mu)}{D_{C/c}(z, \mu)}. \]  (15)

A generic perturbative QCD cross section is factorized schematically as

\[ \sigma_{AB \rightarrow C} = \sum_{abc} \phi_{a/A} \otimes \phi_{b/B} \otimes H_{abc} \otimes D_{C/c}, \]  (16)

where in the case of a jet final state, the analog of the fragmentation function \( D_{C/c} \) is itself perturbatively calculable, and can be absorbed into \( H_{abc} \). This factorization reflects the mutual independence of QCD dynamics at different length scales, and expresses the same physical principles that are exploited in methods based on “effective theories” for heavy quark physics.

### 2.2 Successes and limitations

The evolution of structure functions is probably the most “precise” and “predictive” of perturbative QCD results, tying together a multitude of experiments over a wide range of momentum transfers [15]. In this case, the cross section is nearly the simplest form possible in (16), involving the parton distributions of only a single hadron. Other striking phenomenological successes include the somewhat more qualitative predictions based on improvements of fragmentation analysis using on the concept of “coherence” [16].

In the experimental presentations during the first half of this conference, we have also seen:

- Good, sometimes outstanding, but not universal success (at the level of tens of percent) for jet and direct photon data over varying momentum transfers and magnitudes of cross sections [17].
- Estimates of theoretical uncertainties that more often than not exceed the data errors.
- Uneven success with heavy flavor production at high energies [18].

From the point of view of Eq. (16), the reasons why the predictions of perturbative QCD are sometimes of limited accuracy are fairly clear:

- The hard scattering functions \( H_{abc} \) in Eq. (16) have generally been computed to NLO in \( \alpha_s(\mu) \). Their variation with \( \mu \) is of the next highest order, \( \mu dH_{abc}/d\mu \sim \alpha_s(\mu)^{\text{NLO}+1} \).
Each nonperturbative function in Eq. (16) requires a factorization scale $\mu_f$. Although simplicity suggests that all of the $\mu_f$ be chosen equal, a single choice may not be the “optimal” for all, which can affect the size of higher-order corrections in the hard scattering function.

For complex processes, there are many “implicit” dimensionless scales. Examples include the $R$-parameters used to define jets in $p\bar{p}$ collisions. Dependence on such parameters is at once potentially important and hard to quantify systematically [19].

The ratios of partonic energies to heavy quark masses are often large, giving additional dimensionless parameters.

Parton distribution and fragmentation functions must be extracted from experiment, and include uncertainties that depend on choices of which data to use, and on starting parameterizations [20].

Power corrections of all kinds can be important, as we have suggested above. For event shapes in electron-positron annihilation there is ample evidence of this, and in this session experimental evidence and theoretical progress along these lines was discussed. In hadronic scattering at collider energies, energy flow from the “underlying event” is another potentially significant power correction [19]. “Intrinsic” transverse momentum is yet another [21]. It is the nature of power corrections to depend sensitively on kinematics, and they can be crucial in some kinematic ranges, and negligible not far away in phase space.

All of these limitations, particularly those associated with power corrections, sound a note of caution for our interpretation of NLO predictions based upon perturbative QCD. Let me make what I consider an important comment, however, in this connection. If we are interested in using existing perturbative predictions for new-particle production, or for measuring $\alpha_s$, power corrections are something of an embarassment, and we should emphasize quantities for which they are minimized. If, on the other hand, we are interested in the dynamics of QCD at the perturbative-nonperturbative interface, then power corrections are a boon. As we have observed above, at scales of a few GeV, power corrections are typically smaller than the leading order, but competitive with next-to-leading order. They are therefore readily observable but not dominant, and teach us new things about quantum chromodynamics. As with most new information, we must develop the tools with which to interpret them.

Let me turn briefly to some preliminary suggestions on how this might be done.

3 The Operator Content of Nonperturbative Parameters

If we had not known of the local gluon condensate from the general considerations of the OPE, we might have stumbled upon it in the course of analyzing infrared
renormalons in the total electron-positron annihilation cross section. It is natural to expect that infrared renormalons associated with event shapes and related semi-inclusive cross sections might imply a phenomenological role for as-yet unappreciated nonperturbative quantities, which, like the gluon condensate, are expressible in terms of the expectation values of operators in QCD. The literature routinely refers to “generalizations” of the OPE. Whether such generalizations constitute real progress remains to be seen, of course, but let me give an example or two, which illustrate a few of the possibilities.

The $Q_T$-distribution and “Wilson lines”. That resummation implies nonperturbative corrections is not a new observation. In fact, there is a quite sophisticated formalism, that predates the wave of interest in infrared renormalons, in at least one case of considerable phenomenological interest, the transverse momentum distributions for Drell-Yan pairs \cite{21, 22, 23}. In this cross section, the resummation is most transparent in Fourier transform (“impact parameter”) space, where it exponentiates into a form that reminds us of the thrust distribution above,

$$
\tilde{\sigma}(b) \sim \exp \left[ \int_0^Q \frac{d^2 k_T}{k_T^2} A(\alpha_s(k_T^2)) \ln \left( \frac{Q^2}{k_T^2} \right) \left( e^{-ik_T \cdot b} - 1 \right) + \ldots \right]
$$

$$
\sim \exp \left[ \int_{1/b_0^2}^Q \frac{d^2 k_T}{k_T^2} A(\alpha_s(k_T^2)) \ln \left( \frac{Q^2}{k_T^2} \right) + g_2(b_0)b^2 \ln (Qb) + \ldots \right]. \quad (17)
$$

The function $A = 2C_F(\alpha_s/\pi) + \ldots$ is a power series in the strong coupling with finite coefficients, so that all logarithms of the impact parameter, $b$, are generated by the explicit integrals in Eq. (17) and by expansions of the running coupling. The running coupling, however, diverges for $k_T \sim \Lambda_{QCD}$ in the first line. In the second expression, the $k_T$ integral has been regularized, in such a way that all leading (and in the full form) next-to-leading logarithms in $b$ are retained. This is done by introducing a modified impact parameter $b_* = b/\sqrt{1 + b^2/b_0^2}$, with $b_0$ a fixed distance scale \cite{23}. The “renormalon” singularity at small $k_T$ is now taken care of, but at the price of introducing a new mass scale, $1/b_0$, in the problem. This, however, is just what we expect for a full perturbative-nonperturbative cross section. In this case, the analog of the power expansion in $1/Q$ above is an expansion in powers of $b$. Of special interest is the “$g_2$” term shown in the second form in Eq. (17), in which a perturbative logarithmic dependence on $Q$ multiplies a nonperturbative power of $b$. In principle, it is possible to measure these, and related nonperturbative parameters. The derived values will depend on the choice of $b_0$. Considerable attention has been given to this problem recently in the context of electroweak vector boson production at the Tevatron \cite{21, 24, 25}.

Turning to the new parameter $g_2(b_0)$; what might it be telling us? One approach, which is close in spirit to the OPE analysis for annihilation, is to look for an operator vacuum expectation value which, when expanded in perturbation theory, gives an expression that is the same as the low-$k_T$ tail of the integral in Eq. (17) when the exponential in the first line is expanded to order $b^2$. Such an operator exists, and is, in fact, a reasonably natural generalization of the gluon condensate in the OPE. We can build up this object out of the elementary operators of QCD \cite{3}.
We begin by introducing a “Wilson line”, or ordered exponential of the gluon field, along a light-like path,

$$\Phi_v(x, x + sv) = P \exp \left[ -ig \int_0^s dt \, v \cdot A(x + tv) \right].$$  \hspace{1cm} (18)

Here $P$ stands for “path-ordering”, which simply means that we keep track of the color indices of each field $A^\mu$ in the expansion on (18). In $\Phi_v$, the gluon field is integrated along a straight path in space-time, defined by the “velocity” vector $v^\mu$, which begins at point $x$, and ends at point $sv^\mu + x^\mu$. We can think of such an operator as a model for the interaction of the gauge field $A^\mu$ with a fast-moving quark of velocity $v^\mu$, neglecting recoil. The analogous quantity for a fast-moving electron interacting with the photon field is a pure phase, and (18) is sometimes also referred to as the quark’s “nonabelian phase”.

The influence of soft gluons on the amplitude for the annihilation of a quark-antiquark pair can be generated perturbatively by sewing together two Wilson lines, one representing the quark, the other the antiquark,

$$U_{v_1v_2}(0) = T \left[ \Phi_{v_2}^\dagger(0, -\infty) \Phi_{v_1}(0, -\infty) \right],$$  \hspace{1cm} (19)

with $T$ the time-ordering operation. We are interested in the transverse momentum of radiated gluons, which in classical terms is associated with the transverse force experienced by the quarks before their annihilation. Recalling the Lorentz force of electrodynamics, a gauge invariant measure of the nonabelian Lorentz force is

$$F_\alpha^\mu(x) = -ig \int_{-\infty}^0 ds \, \Phi_v(x, x + sv) \, v^\mu F_{\mu\alpha}^\nu(sv + x) \, \Phi_{-v}(x + sv, x),$$  \hspace{1cm} (20)

which we insert along the paths of the Wilson lines in $U$, to get

$$\langle 0 | \Phi_{v_2}^\dagger(0, -\infty) \left( \vec{F}_{v_1}(0) - \vec{F}_{v_2}^\dagger(0) \right) \Phi_{v_1}(0, -\infty) | 0 \rangle.$$  \hspace{1cm} (21)

This rather nontrivial generalization of the gluon condensate, $\langle 0 | F_{\mu\nu}(0) F_{\mu\nu}(0) | 0 \rangle$, generates the soft tail of the $k_T$ integral in Eq. (17), as desired. It is a matrix element that is sufficiently general to be found in many phenomenologically-interesting contexts. It is certainly a relevant question whether this “universality” can be put to good use.

Although (21) is nonlocal, it is contained along a one-dimensional path. This relative simplicity is associated with measuring a total momentum component of all the hadrons in the final state, in this case the total transverse momentum of hadrons recoiling against an electroweak boson. Other quantities, like the thrust, which measure details of the final state, are associated with even more complex operators [26, 27]. For example, power corrections to the thrust are sensitive to final-state interactions, in a way that the total transverse momentum is not. Power corrections to the thrust are associated with operators on the sphere at infinity, which is reached only after even the softest of interactions has ceased. Schematically, the first power correction to the moments of the thrust, $\tilde{\sigma}(N) \equiv \int dT T^N d\sigma/dT$, may be represented as [27]

$$\ln \tilde{\sigma}(N) \sim PT + \frac{N}{Q} \int_{-1}^1 d\cos \theta \, (1 - |\cos \theta|) \, E(\cos \theta).$$  \hspace{1cm} (22)
where “PT” denotes a perturbative contribution, and where $\mathcal{E}$ is a matrix element that measures energy flow in the presence of a pair of Wilson lines, which represent the incoming quarks. To be specific,

$$\mathcal{E}(\cos \theta) = \langle 0 | U_{v_1 v_2}^\dagger (0) \Theta(\cos \theta) U_{v_1 v_2} (0) | 0 \rangle,$$

where $U$ is given by Eq. (19), and where $\Theta$ is related to the energy-momentum tensor $\theta_{\mu\nu}(y)$ at infinity \[26, 27\],

$$\Theta(\cos \theta) = \lim_{|\vec{y}| \to \infty} \int_0^\infty dy_0 \ d\Omega_{ij}(\vec{y}) \epsilon_{ijk} \theta_{0k}(y') \delta(\cos \theta' - \cos \theta).$$

(24)

Such “maximally-nonlocal” correlation functions seem to be necessary to describe the nonperturbative information in power corrections to thrust. Again, more thought will be needed to test the utility of this observation.

## 4 Comments and Conclusions

Thanks to the program of computing a wide variety of hard-scattering cross sections at next-to-leading order \[28\], we can pose more questions of perturbative QCD than ever before. We must recognize, however, that NLO is the first serious approximation, and is not guaranteed to work, unaided, except in cases where there is only a single large scale. Beyond these situations, we may have to resort to higher orders, resummation and/or power corrections. Despite these limitations, pQCD at NLO has led to striking successes \[28\], even compared to a few years ago, many of which have been shown at this conference.

On the nonperturbative side, the success of the power correction analysis of event shapes in electron-positron annihilation and DIS surprised just about everyone. The reasons for these successes, and their real extent, are not yet fully understood. In this regard we should be encouraged, and at the same time more critical.

Much more work needs to be done on the relation of power corrections to perturbation theory in hadron-hadron scattering. In this connection, it will be important to sit back and think about what new measurements and theoretical developments will allow us to learn about quantum chromodynamics.

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