Spin-parity assignments of excited $\Omega_b^-$-baryons in the Quark-Diquark Model

A Ali¹, A Dobrynina²,³, E Oleinik² and A Parkhomenko²

¹ Deutsches Elektronen-Synchrotron DESY, D-22607 Hamburg, Germany
² Department of Theoretical Physics, P. G. Demidov Yaroslavl State University, Sovietskaya 14, 150003 Yaroslavl, Russia
³ II. Institute for Theoretical Physics, University of Hamburg, Luruper Chaussee 149, D-22761 Hamburg, Germany

E-mail: ahmed.ali@desy.de, a.dobrynina@uniyar.ac.ru, lizok.glazok@mail.ru, parkh@uniyar.ac.ru

Abstract. Recently the LHCb Collaboration have found four baryonic states called $\Omega_b(6316)$, $\Omega_b(6330)$, $\Omega_b(6340)$, and $\Omega_b(6350)$ in the $\Xi_b^0 K^-$ invariant mass spectrum. Possible spin-parity assignments of these resonances in the Quark-Diquark Model as $P$-wave baryons are discussed. Parameters of the effective Hamiltonian are determined from the observed spectrum of $\Omega_b^-$-baryons and compared with the ones extracted from the mass spectrum of $\Omega_c^*$-baryons. Alternative interpretation, in which the three lowest-mass states have the spin-parities $J^P = 3/2^-$, $3/2^-$, $5/2^-$, predicts an $\Omega_b^-$-baryon with the mass $M \approx 6304$ MeV which is not yet seen experimentally.

1. Introduction

In the recent years a remarkable experimental progress has been achieved in the field of heavy hadrons. Many excited charmed baryons (the triplet and sextet based on the flavor $SU(3)_F$-group are shown in figure 1), their bottom partners (corresponding triplet and sextet can be obtained by changing the charm quark with the bottom one), several hidden-charmed tetra- and pentaquarks, double-charmed $\Xi_{cc}^{++}$-baryon as well as fully charmed tetraquark were discovered at the LHC. At the LHC Run-3, we anticipate that yet more heavy hadrons will be discovered.

The baryons we are going to discuss in this talk are $\Lambda_b^0 = (css)$ and $\Omega_b^- = (bss)$. In the quark-diquark model of baryons, which has received experimental support from the decays of $\Lambda_b^0 = (bud)$, these baryons are bound states of the heavy $c$- or $b$-quark and doubly-strange color-antitriplet vector diquark, $[ss]$, which is assumed to be in an $S$-wave. The ground baryonic state of such a system is a spin doublet with spin-parities $J^P = 1/2^+$ and $J^P = 3/2^+$. The number of orbitally excited heavy baryons is larger and includes five states: two $J^P = 1/2^-$ and $J^P = 3/2^-$ states, each, and a $J^P = 5/2^-$ state [1,2]. In 2017, the LHCb Collaboration measured precisely the masses and decay widths of five narrow $\Omega_b^*$-baryons in the $\Xi_c^+ K^-$ invariant mass spectrum [5]. Subsequently, Belle Collaboration confirmed the existence of four baryonic resonances in the same invariant-mass distribution [4]. The spin-parity of these states remains to be determined experimentally. The assignment of these states as $P$-wave resonances seems to be the most natural in the quark-diquark model [1,2,5,7], though interpretations in other models...
lead to different $J^P$ assignments. Experimental data\cite{3,4} and the assumed $J^P$-assignments \cite{2} are shown in table I.

\begin{table}
\begin{tabular}{|c|c|}
\hline
\textbf{$\Xi_c^+$} & \textbf{$\Sigma_c^{++}$} \\
\hline
uscdsc dsc usc & uucudc ssc ddc \\
\hline
\end{tabular}
\end{table}

\textbf{Figure 1.} Triplet (a) and sextet (b) representations of charmed baryons according to the flavor $SU(3)_F$-group.

This year, the LHCb Collaboration announced the discovery of four narrow $\Omega_b^-$-baryons $\Omega_b(6316)$, $\Omega_b(6330)$, $\Omega_b(6340)$, and $\Omega_b(6350)$ in the $\Xi_b^0 K^-$ invariant mass spectrum\cite{8}. Assuming that the $\Omega_b^0$- and $\Omega_b^-$-baryon mass spectra follow a similar pattern, Karliner and Rosner\cite{9} have suggested that these resonances are orbitally excited states of $\Omega_b^-$-baryons and the fifth state, being either the lightest or the heaviest one, is yet to be discovered experimentally\cite{9}. If the missing state has $J^P = 5/2^-$, its optimal mass is estimated as $M = 6358$ MeV\cite{9}, which would make it the heaviest of the multiplet.

In this paper we give an alternative interpretation of the newly observed $\Omega_b^-$-baryons by the LHCb Collaboration. We also assume that the $\Omega_b^-$-baryons are $P$-wave resonances, but argue that all the five states are already present in the mass spectrum and two of these states having accidentally close masses are not resolved experimentally. Under this assumption, we have derived the coefficients of the effective Hamiltonian relevant for the heavy-baryon spectrum in the quark-diquark model. To do so, we first exemplify the procedure for the orbitally excited $\Omega_b^0$-baryons and then apply it to the $\Omega_b^-$-baryons. We also briefly discuss the other possibility of the $\Omega_b^-$-baryon assignments, according to which $\Omega_b(6316)$, $\Omega_b(6330)$, and $\Omega_b(6340)$ have spin-parities $J^P = 3/2^-$, $J^P = 3/2^-$ and $J^P = 5/2^-$, respectively, while the heaviest $\Omega_b(6350)$-baryon belongs to the radially excited states with $J^P = 1/2^+$. The consequences of this hypothesis are presented.

2. Orbitally excited $\Omega_b^0$-baryons in the quark-diquark model

Theoretical formulas for the $\Omega_b^{0\ast}$-baryon masses follow from the effective Hamiltonian\cite{7}:

$$H_{\text{eff}} = m_c + m_{\{ss\}} + 2\kappa_{ss} (S_{s1} \cdot S_{s2}) + \frac{B_Q}{2} L^2 + V_{SD},$$

$$V_{SD} = 2a_1 (L \cdot S_{\{ss\}}) + 2a_2 (L \cdot S_c) + b \frac{S_{12}}{4} + 2c (S_{\{ss\}} \cdot S_c).$$

In equation (1), $m_c$ and $m_{\{ss\}}$ are the masses of the $c$-quark and the spin-1 $\{ss\}$-diquark, respectively, $\kappa_{ss}$ is the spin-spin coupling of the quarks in the diquark, $L$ is the orbital angular momentum of the $\Omega_b^0$-baryon, and $B_Q$ is the orbital coupling. The coefficients $a_1$ and $a_2$ are the strengths of the spin-orbit terms involving the spin of the diquark $S_{\{ss\}}$ and the charm-quark spin $S_c$, respectively, $c$ is the strength of the spin-spin interaction between the diquark and the charm quark, and $S_{12}/4$ represents the tensor interaction.

1 The coefficients of the spin-orbit, $a_1$ and $a_2$, and the spin-spin interactions, $c$, in the spin-dependent part of the Hamiltonian, $V_{SD}$, differ by a factor 2 from the ones defined in \cite{3,5}.
Table 1. Experimental data from the LHCb [3] and Belle [4] collaborations. Masses are given in MeV. All the measurements have an additional uncertainty of +0.3 −0.5 MeV from the mass of the ground-state Ξ^+_c-baryon. The $J^P$ assignment assumes that all the states are $P$-wave baryons [2].

| Baryon | $J^P$ | LHCb                  | Belle                  |
|--------|-------|-----------------------|------------------------|
| Ω_c(3000) | 1/2− | 3000.4 ± 0.2 ± 0.1 | 3000.7 ± 1.0 ± 0.2 |
| Ω_c(3050) | 1/2− | 3050.2 ± 0.1 ± 0.1 | 3050.2 ± 0.4 ± 0.2 |
| Ω_c(3066) | 3/2− | 3065.5 ± 0.1 ± 0.3 | 3064.9 ± 0.6 ± 0.2 |
| Ω_c(3090) | 3/2− | 3090.2 ± 0.3 ± 0.5 | 3089.3 ± 1.2 ± 0.2 |
| Ω_c(3119) | 5/2− | 3119.0 ± 0.3 ± 0.9 | . . .                  |

Table 2. The χ²-analysis of the orbitally-excited Ω^∗_c-baryon masses based on the measurements by the LHCb [3] and Belle [4] collaborations. This analysis is performed based on the LHCb data alone, as well as for the combined data set from both the LHCb and Belle collaborations presented in table 1. All the values of the parameters are given in MeV. The combined fit yields $\chi^2_{\text{min}}/\text{ndf} = 0.87/4$.

| $M_0^{(\Omega_c)}$ | $a_1$ | $a_2$ | $b$ | $c$ |
|-------------------|-------|-------|-----|-----|
| LHCb              | 3079.89 ± 0.40 | 13.47 ± 0.14 | 12.86 ± 0.38 | 13.48 ± 0.54 | 2.00 ± 0.22 |
| Combined           | 3079.80 ± 0.39 | 13.45 ± 0.13 | 12.94 ± 0.36 | 13.30 ± 0.48 | 2.01 ± 0.20 |

We note that several internal parameters of the Hamiltonian are fixed: charm-quark spin $S_c = 1/2$, diquark spin $S_{ss} = 1$, and Ω^∗_c-baryon orbital angular momentum $L = 1$. So, in the $L − S$ coupling scheme, two possible values of the total spin $S = 1/2$ and $S = 3/2$, after their coupling to $L$, allow us to get five $P$-wave states: two with the total angular momentum $J = 1/2$, two with $J = 3/2$ and the last one with $J = 5/2$ (see table 1). In this scheme the spin-dependent part, $V_{SD}$, is represented by the block-diagonal matrix, each block of which corresponds to the states with a fixed value of $J$ [7]:

$$\Delta M_{1/2} = \frac{1}{3} \left( 2 (a_2 - 4a_1) \frac{2\sqrt{2}}{2\sqrt{2}} (a_2 - a_1) - 5 (2a_1 + a_2) + \frac{b}{\sqrt{2}} \left( \begin{array}{c} 0 \\ 0 \\ 1 \\ -\sqrt{2} \end{array} \right) + c \left( \begin{array}{c} -2 \\ 0 \\ 0 \\ 1 \end{array} \right) \right), \quad (2)$$

$$\Delta M_{3/2} = \frac{1}{3} \left( 4a_1 - a_2 \frac{2\sqrt{5}}{2\sqrt{5}} (a_2 - a_1) - 2 (2a_1 + a_2) + \frac{b}{10} \left( \begin{array}{c} 0 \\ 0 \\ -\sqrt{5} \\ 2 \end{array} \right) + c \left( \begin{array}{c} -2 \\ 0 \\ 0 \\ 1 \end{array} \right) \right),$$

$$\Delta M_{5/2} = 2a_1 + a_2 - \frac{b}{5} + c.$$

After diagonalizing these matrices and adding the common mass term:

$$M_0^{(\Omega_c)} \equiv m_c + m_{ss} + \frac{\kappa_{ss}}{2} + BQ,$$  

(3)
one gets the following set of mass equations \[7\]:

\[
\begin{align*}
m_1^{(1/2)} &= M_0^{(\Omega_c)} - \frac{1}{2} (6a_1 + a_2 + b + c) - \frac{1}{6} \sqrt{(2a_1 + 7a_2 + 3b - 9c)^2 + 2 (4a_1 - 4a_2 - 3b)^2}, \\
m_2^{(1/2)} &= M_0^{(\Omega_c)} - \frac{1}{2} (6a_1 + a_2 + b + c) + \frac{1}{6} \sqrt{(2a_1 + 7a_2 + 3b - 9c)^2 + 2 (4a_1 - 4a_2 - 3b)^2}, \\
m_1^{(3/2)} &= M_0^{(\Omega_c)} - \frac{1}{10} (5a_2 - 4b + 5c) - \frac{1}{30} \sqrt{(40a_1 + 5a_2 - 12b - 45c)^2 + 5 (20a_1 - 20a_2 + 3b)^2}, \\
m_2^{(3/2)} &= M_0^{(\Omega_c)} - \frac{1}{10} (5a_2 - 4b + 5c) + \frac{1}{30} \sqrt{(40a_1 + 5a_2 - 12b - 45c)^2 + 5 (20a_1 - 20a_2 + 3b)^2}, \\
m^{(5/2)} &= M_0^{(\Omega_c)} + 2a_1 + a_2 - \frac{b}{5} + c.
\end{align*}
\]

There are five unknown variables \(\{M_0^{(\Omega_c)}, a_1, a_2, b, c\}\) in these five equations and at least five experimental measurements are required to determine them.

Assuming the \(J^P\) assignment as specified in table 1, we perform a \(\chi^2\)-analysis of the data. The best-fit values and 1\(\sigma\) uncertainties of free parameters, borrowed from \[7\], are presented in table 2. There are no free degrees of freedom for the fit based on the LHCb data alone (the first row in table 2) but combining them with the Belle data yields \(\chi^2_{\text{min}}/\text{ndf} = 0.87/4\) \[7\].

3. Possible assignments of excited \(\Omega_c^0\)-baryons in the quark-diquark model

Next, we apply the procedure used for the excited \(\Omega_c^0\)-baryons and presented in the previous section to the newly observed \(\Omega_c^0\)-baryons: \(\Omega_c^0(6316), \Omega_c^0(6330), \Omega_c^0(6340),\) and \(\Omega_c^0(6350)\). The first step in this direction within the quark-diquark model was done by Karliner and Rosner \[9\] who suggested to consider these resonances as orbitally excited well-separated states like in the case of the \(\Omega_c^0\)-baryons. The absence of the fifth state was explained by its rather large decay width which did not allow to extract the resonance parameters from the background.

In particular, if the observed states have spin-parities \(J^P = 1/2^-, 1/2^-, 3/2^-, 3/2^-\) and the missing one has \(J^P = 5/2^-\), its optimal mass is determined to be \(M = 6358\) MeV and the parameters entering the spin-dependent part \(V_{\text{SD}}\) of the effective Hamiltonian \[1\] are fitted as follows \[9\]:

\[
a_1 = 5.17\text{ MeV}, \quad a_2 = 3.18\text{ MeV}, \quad b = 7.02\text{ MeV}, \quad c = 1.54\text{ MeV}. \quad (5)
\]

The value of the parameter \(a_1\) is a factor 2.5 smaller than the one following from the \(\Omega_c^0\)-baryons (see table 2), which is not in line with the expectations of the quark-diquark model. This parameter is the strength of the spin-orbit interaction of \([ss]\)-diquark and it should remain approximately the same, being independent of the heavy quark flavor.

We give an alternative interpretation of the newly observed \(\Omega_c^0\)-baryons as \(P\)-wave resonances. We argue that the five anticipated states are already present in the mass spectrum but two of these states have accidentally close masses. More pointedly, the peak \(\Omega_c^0(6330)\) has a double humped structure, not yet resolved experimentally. From the assumption that both states are degenerate in mass, one can get the following spin-parity assignment of the LHCb resonances:

\[
\begin{align*}
M(\Omega_c^0(6316)) &= 6315.64 \pm 0.31 \pm 0.07 \pm 0.50; & J^P = 1/2^- \\
M(\Omega_c^0(6330)) &= 6330.30 \pm 0.28 \pm 0.07 \pm 0.50; & J^P = 1/2^- \\
M(\Omega_c^0(6330)) &= 6330.30 \pm 0.28 \pm 0.07 \pm 0.50; & J^P = 3/2^- \\
M(\Omega_c^0(6340)) &= 6339.71 \pm 0.26 \pm 0.05 \pm 0.50; & J^P = 3/2^- \\
M(\Omega_c^0(6350)) &= 6349.88 \pm 0.35 \pm 0.05 \pm 0.50; & J^P = 5/2^- 
\end{align*}
\]
From the mass equations \[4\], two physical solutions are allowed (all quantities are in MeV):

\[
\begin{align*}
I & : & M_0^{(\Omega_c)} & = 6337.3, & a_1 & = 2.4, & a_2 & = 5.1, & b & = 5.4, & c & = 3.8; \\
II & : & M_0^{(\Omega_b)} & = 6325.7, & a_1 & = 3.6, & a_2 & = 5.1, & b & = 1.3, & c & = 0.5.
\end{align*}
\]

This values are compatible with the ones in equation \[5\] and also demonstrate a suppression of \(a_1\) in \(\Omega_c^-\)-baryons relative to \(\Omega_b^-\)-baryons.

Let us also briefly discuss the other possibility of the \(\Omega_c^-\)-baryon assignments, according to which \(\Omega_b(6316), \Omega_b(6330),\) and \(\Omega_b(6340)\) have spin-parities \(J^P = 3/2^-, J^P = 3/2^-\) and \(J^P = 5/2^-\), respectively, while the heaviest \(\Omega_b(6350)\)-baryon belongs to the radially excited states with \(J^P = 1/2^+\). It is not possible to work out the values of the strengths entering the effective Hamiltonian, as for five parameters we now have three input masses only. So, we need to either fix two parameters, or else relate them, with three unknown variables left for performing the numerics. Under the assumption that \(a_2 = a_1\) and \(c = 0\), we get the following two solutions of the mass equations:

\[
\begin{align*}
I & : & M_0^{(\Omega_c)} & = 6315.9 \text{ MeV}, & a_1 & = 10.0 \text{ MeV}, & b & = 30.2 \text{ MeV}; \\
II & : & M_0^{(\Omega_c)} & = 6325.7 \text{ MeV}, & a_1 & = 4.6 \text{ MeV}, & b & = -1.1 \text{ MeV}.
\end{align*}
\]

This allows us to calculate the masses of their missing light \(J^P = 1/2^-\) partners:

\[
\begin{align*}
M_1^{(I)} & = 6229 \text{ MeV}, & M_2^{(I)} & = 6303 \text{ MeV}; \\
M_1^{(II)} & = 6304 \text{ MeV}, & M_2^{(II)} & = 6317 \text{ MeV}.
\end{align*}
\]

The lowest state with the mass \(M_1^{(I)} = 6229 \text{ MeV}\) lies below the kinematic threshold in \(\Omega_b^- \to \Xi_b^0K^-\) decay which is \(M_{\text{thr}} = 6285.6 \pm 0.5 \text{ MeV}\). Both solutions predict a state with the mass \(M \simeq 6304 \text{ MeV}\) which is not yet seen experimentally. Most probably this resonance is wide and hidden in the background. The higher mass state in Sol. II is close in mass to \(\Omega_b(6316)\), so the first experimental peak can have a double humped structure if this assignment is correct.

4. Summary

The assignment of spin-parities \(J^P = 1/2^-, 3/2^-\) for the \(\Omega_c^-\)-baryons observed by the LHCb and Belle Collaborations allows us to fix all the coefficients in the effective Hamiltonian relevant for the \(P\)-wave heavy-baryon mass spectrum. This approach was used for the analysis of four excited \(\Omega_c^-\)-baryons observed by the LHCb Collaboration recently. The assignment of spin-parities \(J^P = 1/2^-, 3/2^-\) for \(\Omega_b^-\)-baryons with the assumption that the second observed peak can have a double humped structure allows us also to fix all the coefficients in the effective Hamiltonian. Alternative interpretation that three lowest mass states have spin-parities \(J^P = 3/2^-, 3/2^-\), \(5/2^-\) predicts an \(\Omega_c^-\)-baryon with the mass \(M \simeq 6304 \text{ MeV}\) which is not seen experimentally, and its discovery will give strong arguments in favor of this assignment. Further experimental study of the excited \(\Omega_b^-\)-baryons will allow us to test the quark-diquark model for heavy baryons.

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References

[1] Karliner M and Rosner J L 2015 Phys. Rev. D 92 074026 (Preprint 1506.01702)
[2] Karliner M and Rosner J L 2017 Phys. Rev. D 95 114012 (Preprint 1703.07774)
[3] Aaij R et al. (LHCb Collaboration) 2017 Phys. Rev. Lett. 118 182001 (Preprint 1703.04639)
[4] Yelton J et al. (Belle Collaboration) 2018 Phys. Rev. D 97 051102 (Preprint 1711.07927)
[5] Ali A, Maiani L, Borisov A V, Ahmed I, Jamil Aslam M, Parkhomenko A Y, Polosa A D and Rehman A 2018 Eur. Phys. J. C 78 29 (Preprint 1708.04650)
[6] Ali A, Maiani L, Borisov A V, Ahmed I, Aslam M J, Parkhomenko A Y, Polosa A D and Rehman A 2018 Acta Phys. Pol. B 49 1315–24
[7] Ali A, Ahmed I, Aslam M J, Parkhomenko A Y and Rehman A 2019 J. High Energy Phys. 10 256 (Preprint 1907.06507)
[8] Aaij R et al. (LHCb collaboration) 2020 Phys. Rev. Lett. 124 082002 (Preprint 2001.00851)
[9] Karliner M and Rosner J L 2020 Phys. Rev. D 102 014027 (Preprint 2005.12424)