Bubble chain resummation and universality

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Abstract

We propose a parameterization for contributions of an infinite set of diagrams (bubble chains) into physical observables represented as integrals of running coupling constant over the finite region in momentum space. The perturbation theory part of contributions is rendered well-defined by the principal value prescription for treating the Landau pole while the rest is connected to the gluon condensate in case of observables allowing the OPE analysis. The hypothesis of universality, i.e. the use of our parameterization for non-OPE cases, is discussed.

1. Introduction.

Perturbation theory (PT) in the running coupling constant of strong interaction is fully understood and well developed technically being an excellent tool for exploring hard processes in QCD. However the numerical value of the effective parameter of the expansion is quite large \( \alpha_s \) that forces one to compute several first terms of perturbative expansion to get a reasonable precision. Experimental data are permanently getting better while new higher order terms of PT expansions appear rarely because of computational difficulties. This is especially discouraging because the standard model fits existing data well \( \alpha_s \) and any sensible deflection will be noticeable at a high level of accuracy only.

Aiming to precise comparison with experiment one should keep in mind that PT is asymptotic and cannot provide unlimited accuracy at fixed \( \alpha_s \). In practice there is no indication yet on any asymptotic character of known series for physical observables, the main limitation being the technical complexity of getting new terms of an expansion. Few examples are known with several terms of \( \alpha_s \) expansion. For \( e^+e^- \) annihilation \( \alpha_s \) and the \( \tau \) semileptonic width \( \alpha_s \) the second subsequent correction of order
$\alpha_s^3$ is comparable with experimental precision in the $\overline{\text{MS}}$ scheme of subtraction. Terms of higher orders will hardly be available ever: perturbation theory seems to be saturated \cite{5}. And this is good because the computation of higher order terms is not worth it – being asymptotic the PT is also known to be incomplete. Theoretically recognized source of nonperturbative effects is the existence of instantons. Phenomenologically PT looks unsatisfactory as well – resonances can not be detected at any finite order in the coupling constant.

Thus, stuck to QCD as a fundamental model it is reasonable to go beyond PT from inside by summing particular infinite sets of diagrams or specific contributions. For simple cases this has been done on several occasions. The necessity to sum infinite series appears, for instance, when an analytic continuation from Euclidean to Minkowskian region is performed \cite{3, 7}. Some attention has been also drawn to fermionic bubbles summation in attempt to go beyond PT \cite{8} (as a recent review, see \cite{9}).

2. Fermionic bubble chains.

The motivation for choosing this particular set of diagrams is threefold:

i) in $1/N_f$ QED the corrections to photon propagator form a dominant set of graphs in $N_f \to \infty$ limit \cite{10},

ii) numerically the $\beta_0$ expansion (a QCD counterpart of $1/N_f$ expansion) works well in $\overline{\text{MS}}$ subtraction scheme for all known cases \cite{11},

iii) they constitute a well defined set of graphs that can be summed.

There are different and quite serious objections to above motivation but the third point is decisive; one wants to find diagrams that can be summed in a constructive way.

Trying to keep this approach alive one could put forward some other foundations for fermionic bubbles chains at least as possible indicator of leading diagrams. Thus, skeleton expansions \cite{12} or Schwinger-Dyson equations \cite{13} could lead in simple cases to the very same set of leading graphs in specific approximations. Early efforts to solve the confinement problem using a modified gluon propagator \cite{14} lie in the same bed.

3. Integral at small momenta.

Whatever the underlying motivation is there are observables that can be represented on a general ground as an integral of running coupling including an infrared region. Examples have been discussed in the literature, we mention three of them $e^+e^-$ annihilation \cite{8}, pole mass of heavy quark \cite{15} and event shape \cite{16, 17, 18}. Being infrared safe these observables can be formally written down as integrals over the running coupling that encounters the Landau pole present in the PT expression for $\alpha_s(t)$ at
small $t$. Therefore their expansion in $\alpha_s(Q)$, where $Q$ is a large scale involved in the process, generates a factorial growth of PT coefficients. The emerging series is sign-definite and cannot be summed with Borel technique. Thus for a number of observables the representation of the form

$$\int_0^{Q^2} \alpha_s(t)\omega(t)dt$$

(1)

can be formally obtained. The corresponding PT expression reads

$$\alpha_s(Q^2) \int_0^{Q^2} \omega(t)dt$$

and the difference between these two formulae reflects the result of “bubble summation improvement”. The weight function $\omega(t)$ depends on kinematical invariants that are of order $Q^2$. The problem reduces to a proper definition of the integral (1) that diverges due to the Landau pole if the perturbative running coupling is used in the infrared region of integration.

Note that high orders of perturbation theory are normally analyzed using the Borel transform in the coupling constant. Then nonsummable pieces reveal themselves as poles in the complex plane of a Borel parameter. The growth of coefficients in fact can be much slower than factorial – it is enough for the series to be sign-definite to be nonsummable and to produce a singularity. Such singularities are called renormalons with specification from what region they come – IR or UV. So, factorial divergences are called renormalons though singularities can be connected with other types of large $n$ behavior as well. In practice the simple way to generate coefficients at any $n$ is to use the running coupling and the presence of Landau ghost is equivalent to existence of renormalons as factorial divergences of PT series. One could say that renormalization group sums logs while the proper definition of the behavior of the coupling constant at small momenta sums bubble chains.

Qualitatively such chains are expected to produce nonperturbative pieces because $\alpha_s(Q^2)$ and $\alpha_s(t)$ are not connected perturbatively for $t \to 0$ and new terms are plausible, i.e. a new parameterization, different from purely perturbative, can enter the game.

Dealing with factorial divergence in quantum field theory (rewritten as singularities in the Borel plane) at present state of art means to define how to treat the Landau ghost. Nevertheless in the literature the interpretation of the pole in the Borel plane is widely used [19]. For instance, the principal value prescription is very popular [20]. In fact, the principal value prescription in the Borel plane is equivalent to the principal value prescription in momentum space at least at the level of one loop. Namely, a Borel represented observable of the form

$$PV \int_0^{\infty} \frac{e^{-\frac{1}{4}t}dt}{1-t}$$
after the change of the variable $t \to a \log(Q^2/t)$ goes to
\[ PV \int_0^{Q^2} \alpha(t)dt = \lim_{\epsilon \to 0} \left( \int_0^{\Lambda^2-\epsilon} + \int_{\Lambda^2+\epsilon}^{Q^2} \right) \frac{dt}{\log(t/\Lambda^2)} \]
where we defined $\alpha(t) = \beta_0 \alpha_s(t) = 1/\log(t/\Lambda^2)$. The last prescription is purely perturbative however.

Namely, consider the integral with $\omega(t) = 1$, other functions do not change things qualitatively. Then
\[ PV \int_0^{Q^2} \alpha(t)dt = \int_{Q_0^2}^{Q^2} \alpha(t)dt = \alpha(Q^2) \int_{Q_0^2}^{Q^2} \sum_{n=0}^{\infty} (\alpha(Q^2) \log(Q^2/t))^n dt \]
and
\[ PV \int_0^{Q_0^2} \alpha(t)dt = 0, \quad Q_0^2 = \Lambda^2 * 1.45... \]
The position of the zero $Q_0^2$ ($Q_0^2 = \Lambda^2 * 1.45...$) of the function
\[ f_\omega(Q^2) = PV \int_0^{Q^2} \alpha(t)\omega(t)dt \]
depends on the weight function $\omega(t)$ but the zero itself does exist for any function of the same sign at small $t$. The series \(^{(2)}\) is a convergent PT series. Indeed, inside the integration region we have
\[ |\alpha(Q^2) \log(t/Q^2)| < |\alpha(Q^2) \log(1.45\Lambda^2/Q^2)| = 1 - \frac{\log 1.45}{\log Q^2} < 1. \]
and the integrand converges homogeneously that allows one to integrate it getting again the convergent series. It is clear what happened – the most interesting region has been thrown away completely by choosing the PV prescription.

Still this is a definition of the perturbation series. The coefficients of $\alpha_s(Q)$ are not just numbers but are functions of $Q^2$ and contain powers of $\Lambda/Q$ as well. In fact any cut will introduce another scale that renders the coefficients to become some functions of the ratio of those scales, in spirit though the expansion remains purely perturbative. In this sense the cut with principal value prescription is minimal because it does not introduce any new scales. The integral in eq. \(^{(2)}\) can be computed explicitly
\[ \int_{Q_0^2}^{Q^2} \left( \log(Q^2/t) \right)^n dt = Q^2 \Gamma(n+1, \log(Q^2/Q_0^2)) \]
where $\Gamma(n, z)$ is an incomplete $\Gamma$-function
\[ \Gamma(n+1, z) = \int_0^z e^{-t} t^n dt = n! - \int_z^{\infty} e^{-t} t^n dt \]
that reduces to elementary functions for our particular case though. The last term in eq. \(^{(2)}\) behaves asymptotically at large $z$ as $\exp(-z)$, or $\exp(-1/\alpha(Q^s))$, where $Q^s = Q/\sqrt{1.45}$. Thus, any coefficient function of the new perturbative expansion \(^{(2)}\) contains “nonperturbative terms” (see, also \(^{(21)}\)).
Numerically this recipe is valid meaning that two expressions

\[ PV \int_0^{Q^2} \alpha(t)\omega(t)dt \quad \text{and} \quad \alpha(Q^2) \int_0^{Q^2} \omega(t)dt \]

do differ. As we have shown however this difference can be accounted for perturbatively. We find

\[ \frac{PV \int_0^{Q^2} \alpha(t)\omega(t)dt}{\alpha(Q^2) \int_0^{Q^2} \omega(t)dt} = 1.39, \ 1.27, \ 1.23 \]

for \( \omega(t) = 1 \) and \( Q^2/\Lambda^2 = 100, \ 500, \ 1000 \). The convergence of the series looks like (for \( Q^2/\Lambda^2 = 100 \))

\[ 1.39 = 1 + 0.201 + 0.075 + 0.038 + 0.015 + \ldots \]

or in terms of \( \alpha(100) \)

\[ 1 + 0.92\alpha(100) + 1.59\alpha(100)^2 + 3.67\alpha(100)^3 + 10\alpha(100)^4 + 30\alpha(100)^5 + \ldots \]

instead of factorial growth

\[ 1 + 1!\alpha(100) + 2!\alpha(100)^2 + 3!\alpha(100)^3 + 4!\alpha(100)^4 + 5!\alpha(100)^5 + \ldots \]

The convergence is very slow. To reach a reasonable accuracy one needs almost as many terms as one could keep for the initial asymptotic series to get the best approximation. In this sense it imitates the asymptotic series very closely.

The convergence can be essentially improved by choosing some other expansion parameter that reduces to the change of the scale \( \alpha(100) \to \alpha(100/\xi), \ \xi > 1 \). The BLM choice \[22\] corresponds to vanishing of the first order correction; other optimization criteria lead to their own choice of the scale. On the whole, however, all these choices are perturbative in the sense that they give convergent series in the coupling normalized at some high scale. In practice convergence still can be slow and one is forced to use the integral formula but in principle this is a possible minimal way to define the perturbative series. Thus, the PV prescription does not create nonperturbative terms though it allows one to sum up some PT corrections in a closed form. Do the real nonperturbative terms exist in the chain?

Look at the running coupling more carefully. The PV prescription suffers of being nonpositively defined at small momenta that can contradict some general properties of quantum field theory \[23\]. For instance, in the case of current correlators the spectral density must be positive.

4. Extrapolation to infrared region.
We give several motivations and models for behavior of the coupling constant at small momenta keeping positivity \cite{24}.

First we use the freedom of choosing the renormalization scheme and coupling definition eventually. Consider the $e^+e^-$ annihilation. Corresponding $D(Q^2)$ function is known up to the $\alpha_s^3$ order. Redefining $\alpha_s$ in a scheme without higher order corrections we find

$$D(Q^2) = 1 + \frac{\tilde{\alpha}_s(Q^2)}{\pi}. $$

Evolution of the new charge $\tilde{\alpha}_s(Q^2)$ is governed by a new $\beta$ function $\tilde{\beta}(\tilde{\alpha}_s)$ that has an IR fixed point and the effective charge is frozen at small momenta. The same conclusion has been recently obtained in \cite{25} after analyzing the $e^+e^-$ annihilation cross-section $R(s)$ within the principle-of-minimal-sensitivity approach \cite{26}. Even without this reference one can choose a special $\beta$ function providing a smooth infrared behavior for the coupling. For instance,

$$\beta(\alpha) = -\frac{\alpha^2}{1 + \kappa \alpha^2}, \quad \kappa > 0 \quad (4)$$

where $\beta^{as}(\alpha) = -\alpha^2 + \ldots$ \cite{24}. Note, that this extrapolation is based on PT consideration only and on the formal use of the summed form of PT series in a region where it is not supposed to be valid.

One can think of eq. (4) as of a Pade approximation for the $\beta$ function in a particular scheme.

Some explicitly nonperturbative extrapolations exist also. The simple pattern is provided by the minimal analytic continuation

$$\alpha^{eff}(Q^2) = \frac{1}{ln\frac{Q^2}{\Lambda^2}} - \frac{1}{\frac{Q^2}{\Lambda^2} - 1}$$

that is regular everywhere outside a cut in the complex $Q^2$ plane but looks nonperturbatively in terms of an asymptotic charge $\alpha(Q^2) = (ln\frac{Q^2}{\Lambda^2})^{-1}$

$$\alpha^{eff}(Q^2) = \alpha(Q^2) - \frac{1}{e^{\alpha(Q^2)} - 1} = \alpha(Q^2) - e^{-\frac{1}{\alpha(Q^2)}} + \ldots$$

and makes an extraction of “purely” nonperturbative terms untransparent. The corresponding $\beta$ function

$$\beta(\xi) = -\xi^2 + (e^{\frac{1}{\xi}} - 1)^{-2}$$

also has an explicit nonperturbative term unlike eq. (4).

Thus, there are some arguments in favor of smooth behavior of the coupling constant in the IR region keeping positivity.

5. Nonperturbative terms and parameterization.
Going back to our main problem of finding a difference between
\[ \int_0^{Q^2} \alpha_s(t) \omega(t) dt \quad \text{and} \quad \alpha_s(Q^2) \int_0^{Q^2} \omega(t) dt \]
with a newly defined positive \( \alpha_s(t) \) one sees that new contributions should be added to the PV prescription – nonperturbative terms have appeared. It would be convenient to have a clear distinguish between PT and power corrections that are hidden in the exact formula (1). The simple parameterization could be also useful for comparison between different channels. We only need our function defined under the integration sign so we can account for the nonperturbative terms by localized distributions living at \( t = \Lambda^2 \). We write

\[ \alpha_{\text{eff}}(t) = \alpha(t) \big|_{\text{PV}} + A\Lambda^2 \delta(t - \Lambda^2) + B\Lambda^4 \delta'(t - \Lambda^2) + \ldots \]  

(5)

Parameters \( A, B, \ldots \) can be found for any particular model of extrapolation of running coupling constant into the infrared region. An important observation is that for a wide class of extrapolations an additional constraint \( A = 0 \) can be satisfied. It is not necessary but looks plausible because in simple cases such a condition corresponds to the absence of gauge invariant operators with the mass dimension two. In other words we can extend our running coupling without the large distortion of PT. Several models belonging to the above class are:

1. No interaction at small momenta

\[ \alpha_{\text{eff}}(z) = \alpha(z) \Theta(z - a\Lambda^2). \]

System of equations for determining the cutoff \( a \) and parameters \( A \) and \( B \) is

\[ li(a) + A = 0, \quad li(a^2) - B = 0 \]

where \( li(a) \) is a special function

\[ li(a) = \int_0^a \frac{dt}{\ln(t)} \]

with the PV prescription for the pole at real positive \( a > 1 \). A solution (with a constraint \( A = 0 \)) is \( a = 1.45, \; B = li(2.1) = 1.19 \).

2. Freezing (\textit{e.g.} [8])

\[ \alpha_{\text{eff}}(s) = \alpha(a\Lambda^2) \Theta(a\Lambda^2 - s) + \alpha(s) \Theta(s - a\Lambda^2). \]

The system of equations is

\[ \frac{a}{ln(a)} = li(a), \quad \frac{a^2}{2ln(a)} = li(a^2) - B. \]
with the solution \( a = 3.85, \ B = 2.6 \).

3. Minimal subtraction

\[
\alpha_{\text{eff}}(s) = \left( \frac{1}{\ln(s/\Lambda^2)} - \frac{1}{s/\Lambda^2 - 1} \right) \Theta(a\Lambda^2 - s) + \alpha(s) \Theta(s - a\Lambda^2) .
\]

(6)

Again

\[
\text{li}(a) - \ln(a - 1) = \text{li}(a), \quad \text{li}(a^2) - a - \ln(a - 1) = \text{li}(a^2) - B
\]

and \( a = 2, \ B = 2 \).

4. Models generated by the \( \beta \) function of eq. (4)

The solution depends on the parameter \( \kappa \) entering the expression for \( \beta \) function (4). For \( \kappa \sim 2 \) the solution is close to one of models (2) and (3) but for different \( \kappa \) there may be no solution at all or one with a nonzero \( A \).

Thus, for reasonable extrapolations the solution subjected to the condition \( A = 0 \) does exist and is stable enough, i.e. \( B \) does not change much. As for the model (3) one could allow the formula (4) be valid for any \( s \) without any cutoff \( a \). Then such an extrapolation, though quite legal, would be nonminimal in our sense and distort the PT strongly. It would introduce \( 1/Q^2 \) terms in cases where they seem to be forbidden by operator product expansion. But in fact they were just correspond to a definition of the perturbative series.

Now one can normalize our parameterization in a particular case to express the nonperturbative parameter \( B \) through some known quantities and to predict new ones. The place to turn for normalization is \( e^+e^- \) annihilation where the operator expansion is known. We find

\[
\langle \frac{\alpha_s}{\pi} G^2 \rangle = \frac{12}{\pi \beta_0} B \Lambda^4.
\]

(7)

For \( \langle \alpha_s G^2 \rangle = (0.440 \ MeV)^4 \) the relation between the gluon condensate and \( \Lambda \) for this particular model is \( 0.330 \ MeV \sim 1.52 B^{1/4} \). For models (2) and (3) we find a constraint \( 0.330 \ MeV = (1.87 \pm 0.18) \Lambda_{\overline{\text{MS}}} \). Taken literary it gives \( \Lambda_{\overline{\text{MS}}} = 180 \pm 20 \ MeV \) that is in a reasonable agreement with the present data.

In general, our result means that the numerical values of the gluon condensate and of the parameter \( \Lambda_{\overline{\text{MS}}} \) are compatible with each other for smooth continuation into the infrared region (like models (2,3)). For the model (1) numerics will be slightly different and in case of the model (4) it depends on \( \kappa \) ranging from a bad solution through the standard values to some bad ones again.

The generalization to two loop approximation for the asymptotic charge is straightforward. The Landau pole still exists (though its location is changed a bit) and the above machinery can be applied.
6. Universality.

Here we assume the hypothesis of universality \[18, 28, 29\] to predict power corrections to a number of observables. We discuss the pole mass and the event shapes. In the framework of resummation of the perturbative corrections by a particular method of summation, the uncertainties, or the limit of accuracy, for physical observables are normally discussed. Our approach, connecting different channels, allows one to express one observable through another using the parameterization through the gluon condensate as an intermediate step and therefore gives the absolute magnitude of corrections.

First example is the pole mass of a heavy quark. Being well defined in the PT framework it allows the representation of the form \( (1) \) by inserting the bubble chain into the one loop mass operator. The difference between the pole mass and the running mass entering the renormalized QCD Lagrangian is given by \[15\]

\[
m_P - m_Q = \frac{8\pi}{3} \int_{|k|<\mu} \frac{d^3k}{(2\pi)^3} \frac{\alpha_s(k)}{k^2} = \frac{2}{3\pi} \int_0^\mu \frac{\alpha_s(t)dt}{\sqrt{t}}
\]

Using our parameterization \( (3) \) we get the result

\[
m_P - m_Q = \frac{B}{3\pi\beta_0} \Lambda = \frac{4B}{3b}
\]

at \( \beta_0 = b/4\pi \) which should be compared with the uncertainty found in \[15\]

\[
m_P - m_Q = \frac{8}{3b} \Lambda.
\]

The value of the parameter \( B \) is determined by the gluon condensate and is about two from \( (7) \).

Second example is the mean value of the thrust parameter \( T \) that has been computed up to the next-to-leading order in \( \alpha_s \) \[30\]

\[
\langle T \rangle = 1 - 0.355 \alpha_s(Q^2)(1 + 9.56 \frac{\alpha_s(Q^2)}{\pi}).
\]

Corresponding uncertainties due to bubble chain summation and estimates of generated power corrections have been considered in \[16, 17, 18\]. We use the representation of the form \( (1) \) given explicitly in \[17\]

\[
\delta \langle T \rangle = -\frac{16}{3\pi Q} \int_0^Q dk_\perp \alpha_s(k_\perp) = -\frac{8}{3\pi Q} \int_0^Q \alpha_s(t)dt\frac{1}{\sqrt{t}}
\]

Our parameterization leads to the result

\[
\delta \langle T \rangle = -\frac{16B}{3b} \frac{\Lambda}{Q}
\]

Note, that in this particular example formula \( (3) \) coincides with the previous case \( (8) \) up to numerical factors.
The dependence on the scheme and specification of \( \Lambda \) come from the analyses of the perturbative part of the expansion. If the particular scheme is chosen for PT expansion then the corresponding \( \Lambda \) appears in \([3]\). We are going to elaborate on this point elsewhere.

7. Conclusion.

We propose to parameterize the power corrections in a well-defined way. The universality hypothesis (physical justification of which we don’t discuss here) allows one to connect these corrections in different channels. The PT structure is reflected in the choice of the parameter \( \Lambda \): as soon as the PT scheme is fixed the corresponding parameter \( \Lambda \) appears in power corrections in our approximation.

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