b-Parity: counting inclusive b-jets as an efficient probe of new flavor physics

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We consider the inclusive reaction $\ell^+\ell^- \to nb + X$ ($n =$ number of b-jets) in lepton colliders for which we propose a useful approximately conserved quantum number $b_P = (-1)^n$ that we call b-Parity ($b_P$). We make the observation that the Standard Model (SM) is essentially $b_P$-even since SM $b_P$-violating signals are necessarily CKM suppressed. In contrast new flavor physics can produce $b_P = -1$ signals whose only significant SM background is due to $b$-jet misidentification. Thus, we show that $b$-jet counting, that relies primarily on $b$-tagging, becomes a very simple and sensitive probe of new flavor physics (i.e., of $b_P$-violation).

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The Standard Model (SM), despite its enormous success, is believed to be the low-energy limit of a more fundamental theory whose nature will be probed by the next generation of colliders. New physics effects have been studied in a variety of processes using model independent approaches \cite{1}, as well as within specific models \cite{2}. All such investigations aim at providing a clear unambiguous signal for non SM physics effects. In this letter we propose one such signal obtained through simple $b$-jet counting. The approach is best suited to lepton colliders but may be extended to hadron colliders, e.g., it applies to the Fermilab Tevatron $p\bar{p}$ collider to the extent that the sea $b$-quark content of the protons can be ignored.

Consider the inclusive multiple $b$-jet production in $\ell^+\ell^-$ collisions

$$\ell^+\ell^- \to n \cdot b + X,$$

where $n$ denotes the number of $b$ and $\bar{b}$-jets in the final state (FS) and $X$ stands for non $b$-jets, leptons and/or missing energy; it is understood that this represents the state after top-quark decay.

For the reactions \cite{1} we introduce a useful approximate symmetry we call $b$-Parity ($b_P$), defined as

$$b_P = (-1)^n.$$

In the limit where the quarks mixing CKM matrix $V$ \cite{3} satisfies $V_{3j} = V_{\bar{j}3} = 0$ for $j \neq 3$, all SM processes are $b_P$-even since in this case the third generation quarks do not mix with the others, and this leads to the conservation of the corresponding flavor number. Given the fast top decay and since $\text{Br}(t \to bW) \simeq 1$, the experimentally-observed flavor number is in fact carried only by the $b$-quarks. Therefore, the measured quantum number reduces to the net number of detected $b$-quarks; we find it convenient to use instead the derived quantity $b_P$.

The only SM processes that violate this conserved number necessarily involve the charged current interactions and are, therefore, suppressed by the corresponding small off-diagonal CKM elements $|V_{cb}|^2$, $|V_{ub}|^2$, $|V_{ts}|^2$ or $|V_{td}|^2$. As a consequence the irreducible SM background to $b_P = -1$ processes (induced by new flavor physics beyond the SM) is strongly suppressed: the SM is essentially $b_P$-even.

In the following we will look for experimental signatures of $b_P$-odd physics within multi-jet events. We will assume that a sample with a definite number of jets has been selected (we will use 2 and 4 jet samples) and determine (within each sample) the experimental sensitivity needed to detect – or rule out – new flavor physics of this type up to a certain scale. In contrast with other observables the determination of $b_P$ within a sample with a fixed number of jets relies primarily on the $b$-tagging efficiency and purity of the sample used and not on the particular structure of a given FS, nor does it require the identification of any other particle but the $b$. Thus, the main obstacle in this use of $b_P$ is the reducible SM background, due to jet mis-identification. This results from having a $b$-tagging efficiency $\epsilon_b$ below 1, and/or having non-zero probabilities $t_c$ and $t_j$ of misidentifying a $c$ or light jets for a $b$-jet, respectively. This type of background would of course disappear as $\epsilon_b \to 1$ and $t_c,j \to 0$, but even for the small value $t_c = 0.1$ and high $b$-tagging efficiency $\epsilon_b = 0.7$, can produce a significant number of (miss-identified) events in the detector. Since for most experiments $t_j$ is very small \cite{4}, the only relevant experimental parameters for this probe of new physics are $\epsilon_b$ and $t_c$.

Consider now the inclusive $b$ and $\bar{b}$-jet production process in \cite{1,4}. Focusing only on multi-jet FS, let $\sigma_{nm\ell}$ be the cross-section for

$$e^+e^- \to n \cdot b + m \cdot c + \ell \cdot j,$$

where $j$ is a light-quark or gluon jet and $c$ is a $c$-quark

*To be specific, we will consider reactions in $e^+e^-$ colliders, but the method is clearly extendible to muon colliders.
jet. Since our method does not require the detection of the charge of the $b$, $n$ is the number of $b + \bar{b}$-quarks and similarly $m$ and $\ell$ are the number of the corresponding jets in (3) irrespective of the parent quarks charges.

We denote by $t_i$ the $c$-jet mis-tagging probability (i.e., that of mistaking a $c$-jet for a $b$-jet) and by $t_j$ the light-jet mis-tagging probability (i.e., that of mistaking a light-jet or gluon-jet for a $b$-jet). Using these, the probability (or cross-section) for detecting precisely $k$ $b$-jets in the reaction (3) is given by

$$\bar{\sigma}_k = \sum_{n,v,w} P^m_{n} P^o_{v} P^e_{w} \left[ \epsilon_b^u(1 - \epsilon_b)^{n-u} \right] \left[ \epsilon_c^u(1 - \epsilon_c)^{m-v} \right] \left[ t_j^{v}(1 - t_j)^{v+u} \right] \left[ t_j^{u}(1 - t_j)^{u+v} \right] \sigma_{n,m} \delta_{n+u+v+k}, \quad (4)$$

where $P^i_j = \Pi/j!/(i - j)!$.

To experimentally detect $b\bar{b}$-odd signals generated by new physics one should simply measure the number of events with an odd number of $b$-jets in the FS. In particular, for the reaction (3), we define $N_{k,J}$ to be the number of events with $k$ (taken odd) $b$-jets in a FS with a total of $J$ jets. The sensitivity of $N_{k,J}$ to $b\bar{b}$-violating new physics is determined by comparing the theoretical shift due to the underlying $b\bar{b} = -1$ interactions with the expected error ($\Delta$) in measuring the given quantity. Thus, requiring a signal of at least $N_{SD}$ standard deviations, we have

$$\left| N_{k,J} - N_{k,J}^{(SM)} \right| \geq N_{SD} \Delta. \quad (5)$$

We will include three contributions to $\Delta$ which we combine in quadrature: $\Delta^2 = \Delta^2_{stat} + \Delta^2_{sys} + \Delta^2_{theor}$, where $\Delta_{stat} = \sqrt{N_{k,J}}$ is the statistical error, $\Delta_{sys} = N_{k,J} \delta_s$ is a systematic error and $\Delta_{theor} = N_{k,J} \delta_t$ is the theoretical error in the numerical integration of the corresponding cross sections. The quantities $\delta_s, \delta_t$ denote the statistical and theoretical errors per event; $\delta_s$ is estimated using experimental values from related processes (e.g., $R_b$ measurements), $\delta_t$ is derived from the errors in the Monte Carlo integration used in calculating the various cross sections.

There are various types of specific models beyond the SM (e.g., multi-Higgs models, supersymmetry, etc.) that can alter the SM prediction for the cross-section of reaction (3). In this letter we will take a model-independent approach in which we investigate the limits that can be placed on the scale $\Lambda$ of a new short-distance theory that can generate flavor violation, and which we parameterize using an effective Lagrangian (3)

$$L_{eff} = \frac{1}{\Lambda^2} \sum_i f_i O_i + O(1/\Lambda^3), \quad (6)$$

where $O_i$ are mass-dimension 6 gauge-invariant effective operators (some of which may have new flavor dynamics) and $f_i$ are coefficients that can be estimated using naturalness arguments (4).

As a concrete example that clearly illustrates the significance of $b\bar{b}$, we consider the effects of the $b\bar{b}$-odd effective four-Fermi operator

$$O = (\bar{\ell} \gamma^\mu \ell) \left( \bar{q}_i \gamma^\nu q_j \right), \quad (7)$$

where $\ell$ and $q$ are the SM left-handed lepton and quark $SU(2)_L$ doublets and $i, j = 1, 2, 3$ label the generation. This operator gives rise to contact $e^+e^-\ell\gamma$ and $e^+e^-\ell\gamma$ vertices (and their charged conjugates). It can be generated, for example, by an exchange of a heavy boson in the underlying theory (see (3)). Although our method applies to any $b\bar{b} = -1$ process, in what follows we will investigate the effects of (7) on the reaction (3) as an illustration. In particular, on $N_{1,2}$ (i.e., 1 $b$-jet signal in a 2-jet sample, $J = n + m + \ell = 2$) and on $N_{3,4}$ and $N_{5,4}$ (i.e., 1 and 3 $b$-jet signals in a 4-jet sample, $J = n + m + \ell = 4$).

Consider first the 2-jet sample case: in the limit $m_t = 0$ for all $q \neq t$, the only relevant cross-sections are $\sigma_d = \sigma(e^+e^- \rightarrow dd)$, $\sigma_u = \sigma(e^+e^- \rightarrow uu)$, where $d = d, s, b$ and $u = u, c, t$, that are generated by the SM, and $\sigma_{bs} = \sigma(e^+e^- \rightarrow bs) = \sigma(e^+e^- \rightarrow \bar{b}s)$ generated by the $e\bar{e}$bs contact term. These cross-sections are calculated by means of the CompHEP package (1), in which we implemented the Feynman rules for the $e^+e^-\ell\gamma$ and $e^+e^-\ell\gamma$ vertices generated by the operator (7). Using (4), we get the following cross-section for the 2-jet events, one of which is identified as a $b$-jet ($\bar{\sigma}_1$ with $J = n + m + \ell = 2$)

$$\bar{\sigma}_1 = P^2 \left[ \epsilon_b(1 - \epsilon_b) + 2t_j(1 - t_j) \right] \sigma_d + P^2 \left[ \epsilon_c(1 - \epsilon_c) + 2t_j(1 - t_j) \right] \sigma_u + 2P^2 \left[ \epsilon_b(1 - t_j) + t_j + (1 - \epsilon_b) \right] \sigma_{bs} \quad (8)$$

that is used to calculate $N_{1,2}$. In Table 1 we give the largest $\Lambda$ (the scale of the new $b\bar{b}$ physics) that can be probed or excluded at the level of 3 standard deviations ($N_{SD} = 3$, derived using (3), for the three representative $b\bar{b}$-tagging efficiencies of 25%, 40% and 60% and fixing the $c$-jet and light-jet purity factors to 10% and 2%, respectively). Results are given for three collider scenarios: $\sqrt{s} = 200$ GeV with $L = 2.5$ fb$^{-1}$, $\sqrt{s} = 500$ GeV with $L = 100$ fb$^{-1}$ and $\sqrt{s} = 1$ TeV with $L = 200$ fb$^{-1}$. Both the systematic error $\delta_s$ and the theoretical uncertainty $\delta_t$ are assumed to be 5%. Also, an angular cut on the c.m. scattering angle of $|\cos \theta| < 0.9$ is imposed on each of the 2-jet cross-sections in (3). As

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1 We assume there are no significant lepton-number violation effects at scale $\Lambda$ that would generate dimension 5 operators.

2 Note that the limits derived here and throughout the rest of the paper assume $|f| = 1$ [see (1)]. Alternatively, they can be interpreted as limits on $\Lambda/\sqrt{|f|}$. 

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expected, we see from Table I that the sensitivity to the new flavor physics induced by the four-Fermi interaction increases with the b-tagging efficiency.

In Figs. 2 and 3 we show the regions in the $e_b - t_c$ plane (enclosed in the dark areas) where the flavor physics parameterized by $\langle \bar{b} l \rangle$ can be probed or excluded at the 3 standard deviation level (or higher); as an illustration we chose $\Lambda = 4s/\sqrt{5}$ ($s$ denotes the collider CM energy) for the collider scenarios mentioned above. The calculation was done using $\sigma_{\text{NN}}$ for $N_{1,2}$ with $\delta_s = \delta_t = 0.05$, $|\cos \theta| < 0.9$, $t_j = 0.02$. Evidently, $\Lambda$ as large as four times the c.m. energy of any of the three colliders may be probed or excluded even for rather small b-tagging efficiencies: typically $e_b > 25\%$ will suffice as long as the purity factors (in particular $t_c$ being the more problematic one) are kept below the 10% level.

For the 4-jet sample there are numerous processes that can contribute to $N_{1,4}$ and $N_{3,4}$. At the parton level the 4-jet events may be categorized as follows: (1) events containing 2 quark-antiquark pairs or one quark-antiquark pair and two gluons, $(q\bar{q})(q'\bar{q'})$, $(qq')gg$, where both $q$ and $q'$ denote any light quark ($q' \neq t$) including the case $q = q'$. (2) events with two charged quark pairs: $(ud)(\bar{u}'\bar{d}')$, where $u$, $u'$ are either $u$ or $c$-quarks and $d$, $d'$ are any of the down-type quarks, excluding the states $u = u'$ and $d = d'$ since these are induced in type (1) above. (3) the 4 combinations $(bs)gg$, $(bb)(bs)$, $(dd)(bs)$ and $(s\bar{s})(s\bar{b})$ (and the corresponding charged conjugate states) generated by the presence of the four-Fermi operator. It is worth noting that the $eectc$ contact term also contributes through graphs containing a virtual top-quark exchange.

In order to get a reliable jet separation within the 4-jet sample, we use the so-called Durham criterion $\bar{P}$, that requires the quantities $y_{ij}^D = 2\min(E_i^2, E_j^2)(1-\cos \theta_{ij})/s$, where $E_i$ and $E_j$ are the energies of the particles $i$ and $j$ and $\theta_{ij}$ is their relative angle ($i \neq j = 1, \ldots, 4$). We evaluate all 4-jet cross-sections using the CompHEP package with the cuts $y_{ij}^D \geq y_{\text{cut}}$ on all possible parton pairs $ij$ - we present our numerical results for $y_{\text{cut}} = 0.01$. In addition we neglect all quark masses except $m_{\text{top}}$, and the strong coupling constant $\alpha_s$ was evaluated to the next-to-next-to leading order at a scale $Q$ equal to half the CM energy for 5 or 6 active quark flavors depending on whether $Q < m_{\text{top}}$ or not respectively. For 6 flavors we used $\Lambda_{QCD} = 118.5\text{MeV}$ (see [4] for details).

The results for the 4-jet case, using $N_{1,4}$, are shown in Figs. 2 and 3 for a 200, 500 and 1000 GeV colliders, where, as in Fig. 1, any value in the $e_b - t_c$ plane inclosed by the dark area will suffice for probing or ruling out (at $3\sigma$) the new four-Fermi operator in (7) with a scale $\Lambda$ as indicated in the figure. As for the 2-jet sample, we take $t_j = 0.02$ and a systematic error of 5%. Using the results of the CompHEP Monte-Carlo integration we estimate that our calculated 4-jet cross-sections are accurate up to the level of about 10%, accordingly we choose $\delta_t = 0.1$ in Fig. 3.

The 4-jet case is less sensitive to a $b\bar{p}$-odd signal induced by the four-Fermi operator in (7). For example, we see that for a 500 GeV collider with $t_c \sim 0.1$ and $t_j = 0.02$, a b-tagging efficiency of about 70% will be needed in order to probe or exclude a value of $\Lambda \sim 1300$ GeV by measuring $N_{1,4}$, while a 40% b-tagging efficiency will suffice for probing $\Lambda \sim 2000$ GeV using $N_{1,2}$ in the 2-jet sample. Equivalently, for a given value of $t_{c,j}$, higher $e_b$ will be required in the 4-jet measurement compared to the 2-jet one in order to detect a $b\bar{p}$-odd signal generated by the operator in (7) for the same value of $\Lambda$. (this may be somewhat improved by reducing the theoretical uncertainties). We also find that $N_{3,4}$ is less sensitive than $N_{1,4}$ to $\sigma_{\text{eq}}^{(1)}$ in (7).

Though $N_{1,2}$ is more efficient for probing type of new flavor physics which generate the four-Fermi operator at low energies, this is not necessarily a general feature: certain types of new physics will not contribute to the 2-jet FS and must be probed using the 4-jet sample. This is the case, for example, for an effective vertex generating a right-handed $Wbc$ coupling, that may alter the flavor structure of SM, and give rise to sizable $b\bar{p} = -1$ effects. Note, however, that this refers only to the exclusive 2 and 4 jet samples, since such a $Wbc$ right-handed coupling will give rise to a $b\bar{p}$-odd signal in inclusive 2-jet reactions such as $e^+e^- \rightarrow b + j + X$, where $j$ is a light jet. The analysis of these events is, however, considerably more complex.

We note that our cross sections include terms of order $1/\Lambda^4$ that will be modified by dimension 8 effective operators, which are in general present in (7). Note, however, that such dimension 8 operators, if generated by the underlying high energy theory, are expected to give an additional uncertainty of order $(s/\Lambda^2)^2$, below 3% for the results presented. In addition we note that the above analysis assumes unbiased pure samples with a fixed jet number, the effects of contamination from events with different jet number have not been included.

Before we summarize we wish to note that the following issues need further investigation:

- Our $b$-jet counting method can be used to constrain
specific models containing $b_P = -1$ interactions. For example, supersymmetry with R-parity violation or with explicit flavor violation in the squark sector and/or multi-Higgs models without natural flavor conservation can give rise to $t \to c$, $t \to u$ (or $b \to s$, $b \to d$) transitions, which may lead to sizable $b_P$-odd signals in leptonic colliders.

- In leptonic colliders with c.m. energies $\gtrsim 1.5$ TeV, $t$-channel vector-boson fusion processes become important. At such energy scales, the SM $b_P = -1$ reducible background needs to be reevaluated. At the same time, the $V_1 V_2$-fusion processes ($V_{1,2} = \gamma, Z$ or $W$) give rise to a variety of new possible $b_P = -1$ signals from new flavor physics (see e.g., [8]).

To summarize, we have shown that $b$-jet counting that relies on $b$-tagging (with moderate efficiency in a relatively pure multi-jet sample) can be used to efficiently probe physics beyond the SM. Reactions with a final $b$-jets can be characterized through the use of the quantum number $b_P = (-1)^n$ that we called $b$-Parity. Due to small off-diagonal CKM matrix elements, $b_P$ is conserved within the SM to very good accuracy; it follows that the SM contributions to the above reactions are $b_P$-even. Despite the presence of a (reducible) background, due to reduced $b$-tagging efficiency and sample purity, we showed that our method is sensitive enough to provide very useful limits on new flavor physics in a variety of scenarios (of which two examples are provided) using realistic values for $\epsilon_b$.

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FIG. 1. The shaded area denotes the region of the $\epsilon_b - t_c$ plane that can probe $\Lambda$ at the 3 standard deviation level (or higher). Results were obtained for three collider scenarios using $N_{1,2}$ as an observable and assuming $\Lambda < 4\sqrt{s}$. We took $\delta_s = 0.05$, $\delta_t = 0.05$ and $t_j = 0.02$ (see also text).

FIG. 2. Same as in Fig. 1 but for the 4-jet case and using $N_{1,4}$. The values of $\Lambda$ being probed are indicated in the figure for any of the three collider scenarios. Here we take $\delta_t = 0.1$ while $\delta_s$ and $t_j$ are kept as in Fig. 1.