Research Article

Irregularity Measures for Benzene Ring Embedded in P-Type Surface

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A topological index is an important tool in predicting physicochemical properties of a chemical compound. Topological indices help us to assign a single number to a chemical compound. Drugs and other chemical compounds are frequently demonstrated as different polygonal shapes, trees, graphs, etc. In this paper, we will compute irregularity indices for the benzene ring embedded in a P-type surface (BRp) and the simple bounded dual of the benzene ring embedded in a P-type surface (SBRp).

1. Introduction

ÓKeefe et al. [1] have dispersed around a quarter century ago a letter executing two 3D classification of benzene. From which, one is called 6.82P (polybenzene) and has a place with the space gather Im3m, contrast to the P-type surface, and this is due to an insertion of the hexagon fix in the surface of negative ebb and flow P. The P-type surface is facilitated to the Cartesian organizes in the Euclidean space. For further detail about this recurring surface, the author is referred to [2, 3]. This structure needed to be joined as 3D carbon solids; be that as it may, according to our knowledge, no such sequence was assumed before. The goal was to provoke the devotion of scientists to the atomic acknowledgment of such amiable thoughts in carbon nanoscience, as the graphenes took up a moment Nobel prize after C60, and also the immediate union of fullerenes is presently a reality, see for detail [4, 5].

Graph theory provides an interesting appliance in mathematical chemistry where it is used to compute the various kinds of chemical compounds and predict their various properties. One of the most important tools in the chemical graph theory is the topological index, which is useful in predicting the chemical and physical properties of the underlying chemical compound, such as boiling point, strain energy, rigidity, heat of evaporation, and tension [6, 7]. A graph having no loop or multiple edge is known as a simple graph. A molecular graph is a simple graph in which atoms and bounds are represented by vertices and edges, respectively. The degree of the vertex is the number of edges attached with that vertex. These properties of various objects are of primary interest. Winner, in 1947, introduced the concept of the first topological index while finding the boiling point. In 1975, Gutman gave a remarkable identity [8] about Zagreb indices. Hence, these two indices are
among the oldest degree-based descriptors, and their properties are extensively investigated. The mathematical formulae of these indices are

\[ M_1(G) = \sum_{uv \in E(G)} (d_u + d_v), M_2(G) = \sum_{uv \in E(G)} (d_u \times d_v). \]  

(1)

A topological index is known as an irregularity index [9] if the value of the topological index of the graph is greater than or equal to zero, and the topological index of the graph is equal to zero if and only if the graph is regular. The irregularity indices are given in Table 1. Most of the irregularity indices are from the family of degree-based topological indices and are used in quantitative structure activity relationship modeling.

For more about topological indices, one can read [10–33].

### 2. Irregularity Indices for \( BR_p \)

This section is about irregularity indices of \( BR_p \). The molecular graph of \( BR_p \) is given in Figure 1. We can observe from Figure 1 that there are two types of vertices present in the molecular graph of \( BR_p \), i.e., 2 and 3. The cardinality of the edge set is \( 32pq - 2p - 2q \).

The edge partition of \( BR_p \) is given in Table 2.

**Theorem 1.** Let \( G \) be \( BR_p \). The irregularity indices are

1. \( \text{VAR}(G) = 8p^2q^2 + 2p^2q + 2pq^2 - p^2 - 2pq - q^2 \) \( / (6pq)^2 \)
2. \( \text{AL}(G) = 16pq \)
3. \( \text{IR1}(G) = 10(8p^2q^2 + 2p^2q + 2pq^2 - p^2 - 2pq - q^2) / (6pq)^2 \)
4. \( \text{IR2}(G) = \sqrt{(240pq - 38p - 38q/32pq - 2p - 2q) - 32pq - 2p - 2q/12pq^2} \)
5. \( \text{IRF}(G) = 16pq \)
6. \( \text{IRFW}(G) = 16pq/240pq - 38p - 38q \)
7. \( \text{IR}(G) = 8/3(5 - 2\sqrt{6})pq \)
8. \( \text{IRB}(G) = 16(5 - 2\sqrt{6})pq \)
9. \( \text{IRC}(G) = 48\sqrt{6}(p^2q^2 + 112p^2q^2 + 2p^2q + 2pq^2 - p^2 - 2pq - q^2/6pq(16pq - p - q) \)
10. \( \text{IRDIF}(G) = 13.3328pq \)
11. \( \text{IRL}(G) = 6.4864pq \)
12. \( \text{IRLU}(G) = 8pq \)
13. \( \text{IRLF}(G) = 6.5312pq \)
14. \( \text{IRD1}(G) = 11.0896pq \)
15. \( \text{IRLA}(G) = 6.4pq \)
16. \( \text{IRGA}(G) = 0.3264pq \)

**Proof**

\[
\text{VAR}(G) = \sum_{uv \in E(G)} (d_u - 2m/n)^2 = M_1(G)/n - (2m/n)^2
\]

\[
= \frac{(176pq - 20p - 20q)}{24pq} - \frac{(2(32pq - 2p - 2q)^2)}{24pq} \]

\[
= \frac{8p^2q^2 + 2p^2q + 2pq^2 - p^2 - 2pq - q^2}{(6pq)^2}
\]

(2)

\[
\text{AL}(G) = \sum_{uv \in E(G)} |d_u - d_v|
\]

\[
= |2 - 2|(|4p + 4q | + |2 - 3| |16pq|
\]

\[
+ |3 - 3| (16pq - 6p - 6q) = 16pq.
\]

(3)

\[
\text{IR1}(G) = \sum_{uv \in E(G)} d_u^3 - 2m/n \sum_{uv \in E(G)} d_u^2 = F(G) - (2m/n)^2 M_1(G)
\]

\[
= (496pq - 76p - 76q) - \frac{2(32pq - 2p - 2q)}{24pq}
\]

(4)

\[
\text{IR2}(G) = \sqrt{\sum_{uv \in E(G)} d_u d_v} - \frac{2m}{n} = \sqrt{M_2(G)/2} - \frac{2m}{n}
\]

\[
= \frac{(240pq - 38p - 38q)}{32pq - 2p - 2q} - \frac{2(32pq - 2p - 2q)}{24pq}
\]

(5)

\[
\text{IR3}(G) = \frac{(240pq - 38p - 38q)}{32pq - 2p - 2q} - \frac{2(32pq - 2p - 2q)}{12pq}
\]
Table 2: $E(BR_p)$. 

| $(d_u,d_v)$ | Frequency |
|-----------|-----------|
| (2,2)     | 4$p + 4q$ |
| (2,3)     | 16$pq$    |
| (3,3)     | 16$pq - 6p - 6q$ |

IRF($G$) = $\sum_{uv \in E(G)} (d_u - d_v)^2$

\begin{align*}
= (2 - 2)^2 (4p + 4q) + (2 - 3)^2 (16pq) \\
+ (3 - 3)^2 (16pq - 6p - 6q)
= 16pq,
\end{align*}

IRFW($G$) = $\frac{IRF(G)}{M_2(G)}$

\begin{align*}
= \frac{16pq}{240pq - 38p - 38q'}
\end{align*}

IRA($G$) = $\sum_{uv \in E(G)} \left( d_u^{1/2} - d_v^{1/2} \right)^2$

\begin{align*}
= n - 2R(G) = (24pq) - \frac{16}{3} (\sqrt{6} + 2)pq
= \frac{8}{3} (5 - 2\sqrt{6})pq,
\end{align*}

IRB($G$) = $\sum_{uv \in E(G)} \left( d_u^{1/2} - d_v^{1/2} \right)^2 = M_1(G) - 2RR(G)$

\begin{align*}
= (176pq - 20p - 20q) - 32 (\sqrt{6} + 3)pq
= 16 (5 - 2\sqrt{6})pq,
\end{align*}

IRC($G$) = $\sum_{uv \in E(G)} \sqrt{d_ud_v} = \frac{2m}{n}$

\begin{align*}
= 16 (\sqrt{6} + 3)pq \\
+ \frac{32pq - 2p - 2q}{24pq}
\end{align*}

IRDIF($G$) = $\sum_{uv \in E(G)} \left| d_u - d_v \right|$

\begin{align*}
= \frac{2}{2} - \frac{2}{2} (4p + 4q) + \frac{2}{3} - \frac{3}{2} (24pq)
+ \frac{3}{3} - \frac{3}{3} (16pq - 6p - 6q)
= 13.3328pq,
\end{align*}

IRL($G$) = $\sum_{uv \in E(G)} \left| \ln d_u - \ln d_v \right|

= |\ln 2 - \ln 2| (4p + 4q) + |\ln 2 - \ln 3| (24pq)
+ |\ln 3 - \ln 3| (16pq - 6p - 6q)
= 6.4864pq,

IRLU($G$) = $\sum_{uv \in E(G)} \left| \frac{d_u - d_v}{\min(d_u,d_v)} \right|

= \frac{2}{2} (4p + 4q) + \frac{3}{3} (24pq)
+ \frac{3}{3} (16pq - 6p - 6q)
= 8pq.
The cardinality of the edge set of different types of edges are given in Table 3.

Here, we will discuss the irregularity indices for \( G \).

Let \( (G, \mathcal{E}(G)) \) be the molecular graph of \( (G, \mathcal{E}(G)) \) be the edge partition of \( G \).

\[ \begin{align*}
\text{IRLF}(G) &= \sum_{uv \in \mathcal{E}(G)} \left| d_u - d_v \right| / \sqrt{d_u d_v} \\
&= \frac{|2 - 2|}{\sqrt{4}} \left( 4p + 4q \right) + \frac{|2 - 3|}{\sqrt{6}} (24pq) \\
&\quad + \frac{|3 - 3|}{\sqrt{9}} (16pq - 6p - 6q) \\
&= 6.5312pq,
\end{align*} \]

\[ \begin{align*}
\text{IRL}(G) &= \sum_{uv \in \mathcal{E}(G)} 2 \left| d_u - d_v \right| / (d_u + d_v) \\
&= 2 \frac{|2 - 2|}{4} (4p + 4q) + 2 \frac{|2 - 3|}{5} (24pq) \\
&\quad + 2 \frac{|3 - 3|}{6} (16pq - 6p - 6q) \\
&= 6.4pq,
\end{align*} \]

\[ \begin{align*}
\text{IRD}(G) &= \sum_{uv \in \mathcal{E}(G)} \ln \left[ 1 + \left| d_u - d_v \right| \right] \\
&= \ln \left[ 1 + |2 - 2| \right] (4p + 4q) + \ln \left[ 1 + |2 - 3| \right] (24pq) \\
&\quad + \ln \left[ 1 + |3 - 3| \right] (16pq - 6p - 6q) \\
&= 11.0896pq,
\end{align*} \]

\[ \begin{align*}
\text{IRG}(G) &= \sum_{uv \in \mathcal{E}(G)} \ln \left( d_u + d_v \right) / \sqrt{2d_u d_v} \\
&= \ln \left( \frac{2 + 2}{\sqrt{2} \times 2} \right) (4p + 4q) + \ln \left( \frac{2 + 3}{\sqrt{2} \times 3} \right) (24pq) \\
&\quad + \ln \left( \frac{3 + 3}{\sqrt{3} \times 3} \right) (16pq - 6p - 6q) \\
&= 0.3264pq.
\end{align*} \]

3. Irregularity Indices for \( SBR_p \)

Here, we will discuss the irregularity indices for \( SBR_p \). The molecular graph of \( SBR_p \) is given in Figures 2 and 3. The cardinalities of different types of edges are given in Table 3. The cardinality of the edge set of \( SBR_p \) is \( 32pq - 2p - 2q \) (Figure 4).

The edge partition of \( SBR_p \) is given in Table 3.

| \((d_u, d_v)\) | Frequency |
|----------------|-----------|
| (1,6)          | 4         |
| (2,6)          | 8         |
| (2,7)          | 8p + 8q - 24 |
| (4,6)          | 4         |
| (4,7)          | 8p + 8q - 24 |
| (4,8)          | 16pq - 24p - 24q + 36 |
| (6,7)          | 8         |
| (7,7)          | 4p + 4q - 16 |
| (7,8)          | 4p + 4q - 12 |
| (8,8)          | 24pq - 14p - 14q + 8 |

\[ 4 \ IRG(2) = \sqrt{(2048pq - 908p - 908q - 248/24pq - 14p - 14q + 8)} - (2(24pq - 14p - 14q + 8)/8pq - 2p - 2q + 1) \]

\[ 5 \ IRF(G) = 256pq - 108p - 108q - 4 \]

\[ 6 \ IRFW(G) = 256pq - 108p - 108q - 4/2048pq - 908p - 908q - 248 \]
Figure 4: Continued.
Figure 4: Continued.
Figure 4: Comparision of results. (a) VAR. (b) AL. (c) IR1. (d) IR2. (e) IRF. (f) IRFW. (g) IRA. (h) IRB. (i) IRC. (j) IRDIF. (k) IRL. (l) IRLU. (m) IRLF. (n) IRLA. (o) IRD1. (p) IRGA.

\[
\begin{align*}
(6) \quad & IRF(G) = 256pq - 108p - 108q - 4 \\
(7) \quad & IRA(G) = (2 - 4\sqrt{2})pq + 1/14(5 + 84\sqrt{2} - 16\sqrt{7} - 20\sqrt{14})p + 1/14(5 + 84\sqrt{2} - 16\sqrt{7} - 20\sqrt{14})q + 1/14(75 - 189\sqrt{2}/3 + 56\sqrt{6} + 72\sqrt{7} + 30\sqrt{14} - 8\sqrt{42}) \\
(8) \quad & IRB(G) = (192 - 128)pq + (192\sqrt{2} - 32\sqrt{7} - 32\sqrt{14} - 68)p + (192\sqrt{2} - 32\sqrt{7} - 32\sqrt{14} - 68)q = (288\sqrt{2} + 32\sqrt{3} + 24\sqrt{6} - 96\sqrt{7} - 96\sqrt{14} + 4\sqrt{42}) \\
(9) \quad & IRC(G) = (1/2(24pq - 2p - 2q + 1)(8pq - 2p - 2q) + 1))\{4(-16 + 16\sqrt{2}pq - 8\sqrt{3}p - 8\sqrt{7}q^3 + 48\sqrt{2}q^2 - 8\sqrt{3}q + 28\sqrt{7}p + 28\sqrt{7}q - 96\sqrt{2}p - 96\sqrt{2}q + 3\sqrt{6} - 12\sqrt{14} + 48\sqrt{2}p^2 - 8\sqrt{7}p^3 + 4\sqrt{3} + 8\sqrt{7}p^2 - 8\sqrt{14}p^2 + 32\sqrt{7}p^2q + 128\sqrt{2}p^2q^2 - 8\sqrt{14}q^2 + 32\sqrt{7}pq^2 - 6\sqrt{6}q + 28\sqrt{14}q + 2\sqrt{42} - 6\sqrt{6}p - 224\sqrt{2}p^2q + 400\sqrt{2}pq - \\
(10) \quad & IRDIF(G) = 24pq + 0.2128p + 0.2128q - 4.1624 \\
(11) \quad & IRL(G) = 11.0896pq - 1.6020p - 1.6020q - 1.3356 \\
(12) \quad & IRLU(G) = 16pq + 2.56p + 2.56q - 4.40 \\
(13) \quad & IRLF(G) = 11.3136pq - 9.2104p - 9.2104q + 22.4452 \\
(14) \quad & IRLA(G) = 10.6656pq - 2.21308p - 2.21308q - 8.1396 \\
(15) \quad & IRD1(G) = 11.1008pq + 11.5444p + 11.5444q - 27.5856 \\
(16) \quad & IRGA(G) = 0.9408pq + 0.3824p + 0.3824q + 0.1532 \\
\end{align*}
\]
Proof

\[ \text{VAR}(G) = \sum_{u \in V} \left( d_u - \frac{2m}{n} \right)^2 \]
\[ = \frac{M_1(G)}{n} \left( \frac{2m}{n} \right)^2 = \left( \frac{576pq - 236q - 236q - 88}{8pq - 2p - 2q + 1} \right) \]
\[ - \frac{2(24pq - 14p - 14q + 8)}{8pq - 2p - 2q + 1} \left( \frac{576pq - 236q - 236q - 88}{8pq - 2p - 2q + 1} \right) \]
\[ = \frac{4(576pq^2 - 88pq^2 - 88pq^2 - 78p^2 - 572pq - 78q^2 + 209p + 209q - 86)}{(8pq - 2p - 2q + 1)^2} , \] (18)

\[ \text{AL}(G) = \sum_{u \in V} |d_u - d_v| \]
\[ = |1 - 6|(4) + |2 - 6|(8) + |2 - 7|(8p + 8q - 24) + |4 - 6|(4) + |4 - 7|(8p + 8q - 24) + |4 - 8|(16pq - 24p - 24q + 36) \]
\[ + |6 - 7|(8) + |7 - 7|(4p + 4q - 16) \]
\[ + |7 - 8|(4p + 4q - 12) \]
\[ + |8 - 8|(24pq - 14p - 14q + 8) \]
\[ = 64pq - 28p - 28q + 8 , \] (19)

\[ \text{IR1}(G) = \sum_{u \in V} d_u^3 - \frac{2m}{n} \sum_{u \in V} d_u^2 = F(G) - \left( \frac{2m}{n} \right) M_1(G) \]
\[ = (4352pq - 1924p - 1924q - 500) \]
\[ - \frac{2(24pq - 14p - 14q + 8)}{8pq - 2p - 2q + 1} (576pq - 236q - 236q - 88) \]
\[ = \frac{4(7792pq^2 - 2660pq^2 - 2660pq^2 - 690pq^2 - 540pq^2 - 690q^2 + 97p + 97q + 227)}{(8pq - 2p - 2q + 1)^2} , \] (20)

\[ \text{IR2}(G) = \sqrt{\frac{\sum_{uv \in E(G)} d_{uv}d_v}{m} - \frac{2m}{n}} = \sqrt{\frac{M_2(G)}{m} - \frac{2m}{n}} \]
\[ = \sqrt{\frac{2048pq - 908p - 908q - 248}{8pq - 2p - 2q + 1}} \left( \frac{2(24pq - 14p - 14q + 8)}{8pq - 2p - 2q + 1} \right) \]
\[ = \sqrt{\frac{2048pq - 908p - 908q - 248}{24pq - 14p - 14q + 8}} \left( \frac{2(24pq - 14p - 14q + 8)}{8pq - 2p - 2q + 1} \right) \] (21)

\[ \text{IRF}(G) = \sum_{uv \in E(G)} (d_u - d_v)^2 \]
\[ = (1 - 6)^2(4) + (2 - 6)^2(8) + (2 - 7)^2(8p + 8q - 24) + (4 - 6)^2(4) \]
\[ + (4 - 7)^2(8p + 8q - 24) + (4 - 8)^2(16pq - 24p - 24q + 36) \]
\[ + (6 - 7)^2(8) + (7 - 7)^2(4p + 4q - 16) + (7 - 8)^2(4p + 4q - 12) \]
\[ + (8 - 8)^2(24pq - 14p - 14q + 8) \]
\[ = 256pq - 108p - 108q - 4 , \] (22)
The document contains mathematical expressions and equations, including:

\[ IRF(G) = \frac{IRG(G)}{M_2(G)} = \frac{256pq - 108p - 108q - 4}{2048pq - 908p - 908q - 248} \]

\[ IRA(G) = \sum_{u \in E(G)} \left( d_u^{1/2} - d_v^{1/2} \right)^2 = n - 2R(G) \]

\[ IRB(G) = \sum_{u \in E(G)} \left( d_u^{1/2} - d_v^{1/2} \right)^2 = M_1(G) - 2RR(G) \]

\[ IRC(G) = \frac{\sum_{u \in E(G)} \sqrt{d_u d_v}}{m} - \frac{2m}{n} = RR(G) - \frac{2m}{n} \]

The document appears to be discussing mathematical problems in engineering, with specific equations for various functions and constants. The equations involve square roots, fractions, and other algebraic expressions.
\[
\text{IRDIF}(G) = \sum_{u \in \mathscr{E}(G)} \left| \frac{d_u}{d_v} - \frac{d_v}{d_u} \right|
\]
\[
\begin{align*}
= & \frac{1}{6} \left( \frac{2}{8} - \frac{6}{2} \right) (4) + \frac{2}{6} (8) + \frac{2}{7} (8p + 8q - 24) + \frac{4}{6} (4) \\
+ & \frac{4}{7} (8p + 8q - 24) + \frac{8}{8} (16pq - 24p - 24q + 36) \\
+ & \frac{6}{7} (8) + \frac{7}{8} (4p + 4q - 16) + \frac{8}{7} (4p + 4q - 12) \\
+ & \frac{8}{8} (24pq - 14p - 14q + 8)
\end{align*}
\]
\[
= 24pq + 0.2128p + 0.2128q - 4.1624,
\]

\[
\text{IRL}(G) = \sum_{u \in \mathscr{E}(G)} \left| \ln d_u - \ln d_v \right|
\]
\[
\begin{align*}
= & \ln 2 - \ln 2 \frac{4}{4} + \ln 2 - \ln 6 \frac{8}{8} + \ln 2 - \ln 7 (8p + 8q - 24) + \ln 4 - \ln 6 \frac{4}{4} \\
+ & \ln 4 - \ln 7 (8p + 8q - 24) + \ln 4 - \ln 8 (16pq - 24p - 24q + 36) \\
+ & \ln 6 - \ln 7 (8p + 8q - 24) + \ln 6 - \ln 7 (4p + 4q - 16) + \ln 7 - \ln 8 (4p + 4q - 12) \\
+ & \ln 8 - \ln 8 (24pq - 14p - 14q + 8) = 11.0896pq - 1.6020p - 1.6020q - 1.3356,
\end{align*}
\]

\[
\text{IRLU}(G) = \sum_{u \in \mathscr{E}(G)} \left| d_u - d_v \right| \min \left( d_u, d_v \right)
\]
\[
\begin{align*}
= & \frac{1}{6} \left( \frac{2}{8} - \frac{6}{2} \right) (4) + \frac{2}{6} (8) + \frac{2}{7} (8p + 8q - 24) + \frac{4}{6} (4) \\
+ & \frac{4}{7} (8p + 8q - 24) + \frac{8}{8} (16pq - 24p - 24q + 36) \\
+ & \frac{6}{7} (8) + \frac{7}{8} (4p + 4q - 16) + \frac{8}{7} (4p + 4q - 12) \\
+ & \frac{8}{8} (24pq - 14p - 14q + 8)
\end{align*}
\]
\[
= 16pq + 2.56p + 2.56q - 4.40,
\]

\[
\text{IRLF}(G) = \sum_{u \in \mathscr{E}(G)} \left| \frac{d_u}{d_v} - \frac{d_v}{d_u} \right| \sqrt{d_u d_v}
\]
\[
\begin{align*}
= & \frac{1}{6} \left( \frac{2}{8} - \frac{6}{2} \right) (4) + \frac{2}{6} (8) + \frac{2}{7} (8p + 8q - 24) + \frac{4}{6} (4) \\
+ & \frac{4}{7} (8p + 8q - 24) + \frac{8}{8} (16pq - 24p - 24q + 36) \\
+ & \frac{6}{7} (8) + \frac{7}{8} (4p + 4q - 16) + \frac{8}{7} (4p + 4q - 12) \\
+ & \frac{8}{8} (24pq - 14p - 14q + 8)
\end{align*}
\]
\[
= 11.3136pq - 9.2104p - 9.2104q + 22.4452,
\]
IRLA (G) = \sum_{uv \in E(G)} 2 \frac{d_u - d_v}{d_u + d_v}
= 2 \left[ \frac{1}{7} (4) + 2 \frac{2}{8} (8) + 2 \frac{2}{9} (8p + 8q - 24) + 2 \frac{4}{10} (4) \right.
+ 2 \frac{4 - 7}{11} (8p + 8q - 24) + 2 \frac{4 - 8}{12} (16pq - 24p - 24q + 36)
+ 2 \frac{6 - 7}{13} (8) + 2 \frac{7 - 7}{14} (4p + 4q - 16)
+ 2 \frac{7 - 8}{15} (4p + 4q - 12)
+ 2 \frac{8 - 8}{16} (24pq - 14p - 14q + 8)
\left. \right]
= 10.6656pq - 2.21308p - 2.21308q - 0.81396,

IRD1 (G) = \sum_{uv \in E(G)} \ln \left[ 1 + \frac{|d_u - d_v|}{2} \right]
= \ln \left[ 1 + \frac{1}{6} \right] (4) + \ln \left[ 1 + \frac{2 - 6}{8} \right] (8) + \ln \left[ 1 + \frac{2 - 7}{9} \right] (8p + 8q - 24)
+ \ln \left[ 1 + \frac{4 - 6}{4} \right] (4) + \ln \left[ 1 + \frac{4 - 7}{7} \right] (8p + 8q - 24)
+ \ln \left[ 1 + \frac{4 - 8}{16} \right] (16pq - 24p - 24q + 36)
+ \ln \left[ 1 + \frac{6 - 7}{8} \right] (8) + \ln \left[ 1 + \frac{7 - 7}{14} \right] (4p + 4q - 16)
+ \ln \left[ 1 + \frac{7 - 8}{15} \right] (4p + 4q - 12)
+ \ln \left[ 1 + \frac{8 - 8}{16} \right] (24pq - 14p - 14q + 8)
= 11.1008pq + 11.5444p + 11.5444q - 27.5856,

IRGA (G) = \sum_{uv \in E(G)} \ln \left( \frac{d_u + d_v}{2 \sqrt{d_u d_v}} \right)
= \ln \left( \frac{1 + 6}{2 \sqrt{1 \times 6}} \right) (4) + \ln \left( \frac{2 + 6}{2 \sqrt{2 \times 6}} \right) (8)
+ \ln \left( \frac{2 + 7}{2 \sqrt{2 \times 7}} \right) (8p + 8q - 24) + \ln \left( \frac{4 + 6}{2 \sqrt{4 \times 6}} \right) (4) + \ln \left( \frac{4 + 7}{2 \sqrt{4 \times 7}} \right) (8p + 8q - 24)
+ \ln \left( \frac{4 + 8}{2 \sqrt{4 \times 8}} \right) (16pq - 24p - 24q + 36) + \ln \left( \frac{6 + 7}{2 \sqrt{6 \times 7}} \right) (8)
+ \ln \left( \frac{7 + 7}{2 \sqrt{7 \times 7}} \right) (4p + 4q - 16) + \ln \left( \frac{7 + 8}{2 \sqrt{7 \times 8}} \right) (4p + 4q - 12)
+ \ln \left( \frac{8 + 8}{2 \sqrt{8 \times 8}} \right) (24pq - 14p - 14q + 8)
= 0.9408pq + 0.3824p + 0.3824q + 0.1532.

4. Graphical Representation

In this section, we will give the comparison of sixteen irregularity indices of BR_p and BR_q. The colour blue is fixed for BR_p, and the green colour is fixed for SBR_q. From the plots, one can observe the behaviour of computed results with respect to involved parameters.

5. Conclusions

The irregularity of a graph can be defined by different so-called graph topological indices [34, 39]. It is known that the irregularity measures are not always compatible. We foresee that our results could play an important role in determining properties of the understudy material such as enthalpy,
Toxicity, resistance, and entropy. It is hoped that this article will aid the reader in understanding the rationale and utility of a simple quantitative tool which could be used in occlusion assessment.

Data Availability

All data are including in this paper.

Conflicts of Interest

The authors do not have conflicts of interest.

Authors’ Contributions

All authors contributed equally in this paper.

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