The Fourier power spectrum is one of the most widely used statistical tools to analyze the nature of magnetohydrodynamic (MHD) turbulence in the interstellar medium. Lazarian & Pogosyan predicted that the spectral slope should saturate to $-3$ for an optically thick medium and many observations exist in support of their prediction. However, there have not been any numerical studies to date for testing these results. We analyze the spatial power spectrum of MHD simulations with a wide range of sonic and Alfvénic Mach numbers, which include radiative transfer effects of the $^{13}$CO transition. We numerically confirm the predictions of Lazarian & Pogosyan that the spectral slope of line intensity maps of an optically thick medium saturates to $-3$. Furthermore, for very optically thin supersonic CO gas, where the density or CO abundance values are too low to excite emission in all but the densest shock compressed gas, we find that the spectral slope is shallower than expected from the column density. Finally, we find that mixed optically thin/thick CO gas, which has average optical depths on the order of unity, shows mixed behavior: for super-Alfvénic turbulence, the integrated intensity power spectral slopes generally follow the same trend with sonic Mach number as the true column density power spectrum slopes. However, for sub-Alfvénic turbulence the spectral slopes are steeper with values near $-3$ which are similar to the very optically thick regime.

**Key words:** ISM: structure – magnetohydrodynamics (MHD) – radiative transfer – turbulence

**Online-only material:** color figures

1. INTRODUCTION

The interstellar medium (ISM) is turbulent on scales ranging from kiloparsecs to sub-astronomical unit (see Armstrong et al. 1995; Elmegreen & Scalo 2004; Chepurnov & Lazarian 2010), with an embedded magnetic field that influences its dynamics. Magnetohydrodynamic (MHD) turbulence is accepted to be of key importance for fundamental astrophysical processes, e.g., heat transport, star formation, and acceleration of cosmic rays.

Despite the clear importance of MHD turbulence to astrophysics, it is difficult to study. In light of this, numerical simulations have tremendously influenced our understanding of physical conditions and statistical properties of MHD turbulence (see Vázquez-Semadeni et al. 2000; Mac Low & Klessen 2004; Ballesteros-Paredes et al. 2007; McKee & Ostriker 2007 and references therein). Present codes can produce simulations that resemble observations in terms of structures and scaling laws, but because of their limited numerical resolution, they cannot reach the observed Reynolds numbers of the ISM (see, e.g., Shu et al. 2004; Lazarian 2009).

In this respect, observational studies of turbulence can test to what extent the numerical simulations are able to reproduce the conditions in the ISM. The turbulence power spectrum, which is a statistical measure of turbulence that quantifies how much energy resides on a given scale, can be used to compare observations with both numerical simulations and theoretical predictions.

However, the importance of obtaining the turbulence spectrum from observations extends beyond testing the accuracy of numerical simulations. In general, the shape of the energy spectrum is determined by a complex process of nonlinear energy transfer and observational studies of the turbulence spectrum are critical to determine sinks and sources of astrophysical turbulence. At large scales $l$, i.e., at small wave numbers $k \sim 1/l$, one expects to observe features in the energy spectrum $E(k)dk$ that reflect energy injection.

In terms of the ISM, the injection scale and main energy injectors are still unknown, but it is clear that turbulence in the Galaxy is driven on large scales (kpc) by supernova, magnetic rotational instability, galactic fountains, high-velocity cloud impacts, or some combination of these. At small scales, one should see the scales corresponding to the dissipation of energy.

The hydrodynamic counterpart of MHD turbulence theory is the famous Kolmogorov (1941) theory of turbulence. The transfer of energy from large-scale eddies to smaller scales continues until the cascade reaches eddies that are small enough to dissipate energy over an eddy turnover time. In the absence of compressibility, the hydrodynamic cascade of energy is $E \sim v_l^2 / \tau_{\text{casc},l} = \text{const}$, where $v_l$ is the velocity at the scale $l$ and the cascading time for the eddies of size $l$ is $\tau_{\text{casc},l} \approx l/v_l$. From this the well-known relation $v_l \sim l^{1/3}$ follows. In terms of the direction-averaged energy spectrum, this gives the famous Kolmogorov scaling $E(k) \sim 4\pi k^2 P(k) \sim k^{-5/3}$, where $P(k)$ is the three-dimensional energy spectrum.

The ISM is both turbulent and magnetized, and therefore Alfvénic perturbations are vital. Contrary to Kolmogorov turbulence, in the presence of a dynamically important magnetic field, eddies become anisotropic along the magnetic field lines while the cascade of energy proceeds perpendicular to the local magnetic field. This observation corresponds to theoretical expectations of the Goldreich & Sridhar (1995, henceforth GS95) theory of Alfvénic turbulence. For the perpendicular

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3. Furthermore, for very optically thick CO gas, which has average optical depths on the order of unity, shows mixed behavior: for super-Alfvénic turbulence, the integrated intensity power spectral slopes generally follow the same trend with sonic Mach number as the true column density power spectrum slopes. However, for sub-Alfvénic turbulence the spectral slopes are steeper with values near $-3$ which are similar to the very optically thick regime.

4. For the perpendicular
eddies, the original Kolmogorov treatment is applicable resulting in perpendicular motions scaling as \( v_l \sim l_1^{1/3} \), where \( l_1 \) denotes eddy scales measured perpendicular to the local direction of magnetic field (see Lazarian & Vishniac 1999; Cho & Vishniac 2000; Maron & Goldreich 2001; Cho et al. 2002; see Cho et al. 2003 for review). These mixing motions induce Alfvénic perturbations that determine the parallel size of the magnetized eddies, i.e., the critical balance condition. Critical balance in GS95 is the equality of the eddy turnover time \( l_1/v_l \) and the period of the corresponding Alfvén wave \( \sim l_1/V_A \), where \( l_1 \) is the parallel eddy scale and \( V_A \) is the Alfvén velocity. Making use of the earlier expression for \( v_l \) one obtains \( l_1 \sim l_{\perp}^{2/3} \), which reflects the tendency of eddies to become more and more elongated as the energy cascades to smaller scales. The relations for incompressible MHD turbulence carry over to the Alfvénic and slow modes of compressible MHD (Cho & Lazarian 2002, 2003; Cho & Lazarian 2000; Lazarian & Pogosyan 2004). These theoretical and numerical predictions are almost always done under the assumption of an optically thin medium.

In light of the theoretical progress on studies of the power spectrum scaling of turbulence, there have been many investigations over the last 10 years which study the density/velocity power spectrum in radio position–position–velocity (PPV) cubes of neutral hydrogen in the Milky Way Galaxy, the Magellanic clouds, and other galaxies in the context of turbulence (see Table 1 for a summary of these studies). This is complex due to the entanglement of density and velocity fluctuations in PPV space. Attempts to use PPV data cubes to study fluctuations of intensity can be traced back to the work by Crovisier & Dickey (1983), Green (1993), and Stanimirovic et al. (1999). These studies suffered from a lack of theory with which to relate the statistics of PPV fluctuations with the underlying statistics of velocity and density fluctuations. Thus, the interpretation of the measured spectra of intensity fluctuations in channel maps was highly uncertain. These investigations not only study the two-dimensional column density spectrum, but also analyze the spectrum with varying channel thickness in the radio cube. This is the central idea behind the velocity channel analysis (VCA) developed by Lazarian & Pogosyan (2000, henceforth LP00) and in subsequent papers (Lazarian & Pogosyan 2004, 2006) which is intended to obtain both velocity and density turbulence spectra from the observations. The validity of the technique was verified numerically for optically thin gas in Esquivel et al. (2003) and at much higher precision in Chepurnov & Lazarian (2009).

However, no systematic numerical study has been performed so far for the study of the intensity power spectrum in the presence of self-absorbing media, several previous studies have discussed the importance of radiative transfer effects on the statistics of turbulence (e.g., Padoan et al. 2003; Shetty et al. 2011).

The contribution of the velocity fluctuations as outlined by LP00 may depend on whether the images of the eddies under study fit within a velocity slice (a “thick slice”) or if their velocity extent is larger than the slice thickness (a “thin slice”). The spectra of fluctuations that correspond to thin and thick slices are different and varying the thickness (i.e., effective channel width) of slices provides the possibility to disentangle the statistics of underlying velocities and densities in the turbulent volume. More recently, a technique that analyzes the density and velocity spectrum of turbulence by taking the spectrum along the velocity axis, the velocity coordinate spectrum (VCS), has also been developed and is complementary to the VCA technique (see Lazarian & Pogosyan 2008; Lazarian 2009; Chepurnov et al. 2010). Table 1 summarizes some of the variety of objects to which VCA has been applied.

How do these observed spectral slopes relate to predictions given by theory and numerics? The VCA/VCS techniques were developed from purely analytical considerations, and hence the spectral slope obtained from them can be related back to turbulence parameters such as the compressibility (due to shocks), the injection scale, the temperature of the medium, and energy contained in the turbulence. Furthermore, it is encouraging that the observed spectral indexes correspond to what is expected from simulations (see Beresnyak et al. 2005; Kowal et al. 2007; Burkhart et al. 2010), which show that, as the sonic Mach number increases, the density spectrum becomes increasingly flatter while the spectrum of velocity gets steeper. For incompressible hydrodynamic or super-Alfvénic turbulence, high-resolution simulations show very good agreement with the \( k^{-5/3} \) slope (Beresnyak 2012). In nearly incompressible motions with a relatively strong magnetic field, the spectrum of density scales similarly to the pressure as \( k^{-7/3} \) (Biskamp 2003; Kowal et al. 2007; Burkhart et al. 2010). These theoretical results are summarized in Table 2.

These theoretical and numerical predictions are almost always done under the assumption of an optically thin medium.

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**Table 1**

| Tracer | Object      | Type       | \( P_{\text{thin}}^{\text{PPV}} \) | \( P_{\text{thick}}^{\text{PPV}} \) | \( \tau \) | Reference                        |
|--------|-------------|------------|-----------------------------------|----------------------------------|----------------|----------------------------------|
| 1      | H\(_1\)    | Anti-center | Galactic | \( K^{-2.7} \)                  | N/A                         | Thin               | Green (1993); Lazarian & Pogosyan (2006) |
| 2      | H\(_1\)    | Toward CygA| Galactic | \( K^{-2.6} \)                  | \( K^{-2.8} \)               | Thin               | Deshpande et al. (2000)          |
| 3      | H\(_1\)    | SMC        | Extragalactic | \( K^{-2.7} \)                      | \( K^{-3.4} \)               | Thin               | Stanimirović & Lazarian (2001); Burkhart et al. 2010 |
| 4      | H\(_1\)    | Center     | Galactic | \( K^{-2.6} \)                  | \( K^{-3.4} \)               | Thin               | Dickey et al. (2001); Lazarian & Pogosyan (2004); Muller et al. (2004) |
| 5      | H\(_1\)    | B. Mag.    | Galactic | \( K^{-3} \)                    | \( K^{-3} \)               | Thin               | Lazarian (2006)                |
| 6      | H\(_1\)    | Arm        | Galactic | \( K^{-3} \)                    | \( K^{-3} \)               | Thin               | Lazarian (2006); Begum et al. (2006) |
| 7      | H\(_1\)    | DDO 210    | Extragalactic | \( K^{-3} \)                      | \( K^{-3} \)               | Thick               | Sun et al. (2006)               |
| 8      | 12CO       | L1512      | Galactic | \( N/A \)                       | \( K^{-2.8} \)               | Thick               | Stutzki et al. (1998); Dickey et al. (2001) |
| 9      | 12CO       | L1512      | Galactic | \( N/A \)                       | \( K^{-2.8} \)               | Thick               | Stutzki et al. (1998); Begum et al. (2006) |
| 10     | 12CO       | Perseus    | Galactic | \( K^{-2.7} \)                  | \( K^{-3} \)               | Thick               | Padoan et al. (2006)            |
| 11     | 12CO       | Perseus    | Galactic | \( K^{-2.6} \)                  | \( K^{-2.8} \)               | Thin               | Swift (2006)                    |

Notes. \( P_{\text{thin}}^{\text{PPV}} \) corresponds to thin slices of the PPV cubes, which are dominated by velocity fluctuations, while \( P_{\text{thick}}^{\text{PPV}} \) corresponds to thick slices of the PPV cubes, which are dominated by density fluctuations. \( \tau \) gives the optical depth nature of data.
However, Lazarian & Pogosyan (2004, henceforth LP04) predict that absorption can induce a universal spectrum $k^{-3}$. Inspection of Table 1 shows that this prediction has already garnered observational evidence, although it has not yet been tested numerically. In this paper, we use numerical simulations with radiative transfer effects simulating the $^{13}$CO $J = 2$–1 transition. We vary our radiative transfer parameter space, including CO abundance ($^{13}$CO/H$_2$) and average density ($n$), to sample optically thin, optically thick, and mixed optically thin/thick lines. In this way, we can test how well the recovered two-dimensional density spectra match the theoretical predictions of the above-mentioned investigations.

This paper studies the effects of self-absorption on the recovery of the underlying density spectrum from integrated intensity maps. For this purpose, we do not use the idealized analytical model of radiation transfer adopted in LP04 but a radiative transfer code that is described in Ossenkopf (2002) and Burkhart et al. (2013). We compare our results with the theoretical predictions in LP04 and investigate the conditions in which the integrated intensity maps with varying optical depth reflect the underlying density spectrum of turbulence.

This paper is organized as follows: in Section 2 we describe the simulations and outline the main points of the radiative transfer algorithm. In Section 3 we present the analysis of the two-dimensional power spectrum of our simulations including optical depth effects. Finally, in Section 4 we discuss our results followed by the conclusions in Section 5.

### 2. DATA AND METHOD

We generate three-dimensional numerical simulations of isothermal compressible (MHD) turbulence by using the the Cho & Lazarian (2003) MHD code and varying the input values for the sonic and Alfvénic Mach numbers. We describe the simulations here as they are presented in code units; however, the isothermal simulations can be scaled to physical units easily as they are scale-free (see the Appendix of Hill et al. 2008 for more information on scaling). However, once radiative transfer is introduced, the simulations are no longer scale-free.

Turbulence is driven with large-scale solenoidal forcing. The magnetic field consists of the uniform background field and a fluctuating field: $B = B_{\text{ext}} + b$. Initially $b = 0$. For more details on the numerical scheme, see Cho et al. (2002), Cho & Lazarian (2003), Kowal et al. (2007), and Burkhart et al. (2009).

We divided our models into two groups corresponding to sub-Alfvénic ($B_{\text{ext}} = 1.0$) and super-Alfvénic ($B_{\text{ext}} = 0.1$) turbulence. For each group we compute several models with different values of gas pressure, which is our control parameter that sets both the sound speed and the sonic Mach number (see Table 3, second column). We run compressible MHD turbulent models, with 512$^3$ resolution, for $t \sim 5$ crossing times, to guarantee full development of the energy cascade. The models are listed and described in Table 3.

### Table 2

| Magnetic Nature | Type | Line Intensity Spectrum | Reference |
|----------------|------|-------------------------|-----------|
| $M_A < 1$      | Incompressible | $\approx k^{-13/3}$ | Biskamp 2003; Kowal et al. 2007 |
| $M_A > 1$      | Incompressible | $\approx k^{-11/3}$ | GS95; Lithwick & Goldreich 2001; Cho & Lazarian (2002, 2003) |
|                | Compressible   | Shallower than $k^{-11/3}$ | Beresnyak et al. 2005; Kowal et al. 2007 |
|                | Optically thick | $\approx k^{-3}$ | Lazarian & Pogosyan (2004) |

**Note.** Corresponding references to theoretical and numerical are in the far right column.

### Table 3

| Run | $\approx M_{\text{ch}}$ | $\approx M_{\text{ch}}$ | $N$ | $r$(norm) | $r$(n8250) | $r$(n9) | $r(x_{\text{co}}-5)$ | $r(x_{\text{co}}-8)$ |
|-----|-------------------------|-------------------------|-----|----------|-----------|--------|---------------------|---------------------|
| 1   | 0.4                     | 0.7                     | 512 | 3.6      | 71        | 0.13   | 37                  | 0.18                |
| 2   | 2.0                     | 0.7                     | 512 | 3.6      | 71        | 0.13   | 37                  | 0.18                |
| 3   | 4.0                     | 0.7                     | 512 | 3.6      | 71        | 0.13   | 37                  | 0.18                |
| 4   | 7.0                     | 0.7                     | 512 | 3.6      | 71        | 0.13   | 37                  | 0.18                |
| 5   | 8.0                     | 0.7                     | 512 | 3.6      | 71        | 0.13   | 37                  | 0.18                |
| 6   | 0.4                     | 2.0                     | 512 | 3.6      | 71        | 0.13   | 37                  | 0.18                |
| 7   | 2.0                     | 2.0                     | 512 | 3.6      | 71        | 0.13   | 37                  | 0.18                |
| 8   | 4.0                     | 2.0                     | 512 | 3.6      | 71        | 0.13   | 37                  | 0.18                |
| 9   | 7.0                     | 2.0                     | 512 | 3.6      | 71        | 0.13   | 37                  | 0.18                |
| 10  | 8.0                     | 2.0                     | 512 | 3.6      | 71        | 0.13   | 37                  | 0.18                |

**Notes.** Column 1 gives the model number. Columns 2 and 3 show the average sonic and Alfvénic Mach numbers. Column 4 shows the numerical resolution ($N$) of the simulations. In Columns 5–9, we show the map-averaged line center optical depths for the different values of our radiative transfer parameter space. For our radiative transfer parameter space we vary both the number density scaling factor (in units of cm$^{-3}$, denoted with the symbol $n$) and the molecular abundance ($^{13}$CO/H$_2$, denoted with the symbol $x_{\text{co}}$) in order to change excitation and optical depth $r$. To represent the typical parameters of a molecular cloud we choose standard values for density, $n = 275$ cm$^{-3}$, and abundance, $x_{\text{co}} = 1.5 \times 10^{-6}$ (which we will refer to as the normalized or norm parameter setup shown in Column 5). We vary density and abundance by values that are factors of 30 larger and smaller as compared to the $r$(norm) values. Thus Column 6, denoted by $n$8250, has parameters: $n = 8250$ cm$^{-3}$, and abundance, $x_{\text{co}} = 1.5 \times 10^{-6}$. Column 7, denoted by $n9$, has parameters: $n = 9$ cm$^{-3}$, and abundance, $x_{\text{co}} = 1.5 \times 10^{-6}$. Column 8, denoted by $x_{\text{co}}$ – 5, has parameters: $n = 275$ cm$^{-3}$, and abundance, $x_{\text{co}} = 4.5 \times 10^{-5}$. Column 9, denoted by $x_{\text{co}}$ – 8, has parameters: $n = 275$ cm$^{-3}$, and abundance, $x_{\text{co}} = 5 \times 10^{-8}$.

After we generate the simulations, including the full three-dimensional density and velocity cubes, we apply the SimLine-3D radiative transfer algorithm (Ossenkopf 2002) for the $^{13}$CO 2–1 transition. The code computes the local excitation of molecules from collisions with the surrounding gas and from the radiative excitation at the frequencies of the molecular transitions through line and continuum radiation from the environment. Instead of an exact description of the mutual dependence of the radiative excitation at each point in a cloud on the excitation at every other point, the code uses two approximations to describe the radiative interaction. First it computes a local radiative interaction volume limited by the velocity gradients (LGV). Cells with line-of-sight velocities different by more than the thermal line width cannot contribute to the excitation of the considered molecule, i.e., the interaction length is limited by the ratio of the thermal line width to the velocity gradient. This is a particularly useful approximation for supersonic turbulence. For the interaction of more remote points that accidentally have the same line-of-sight velocity, the second approximation applies, using the average isotropic radiation field in the cube for the excitation at every individual point. This best covers all cases with isotropic structure, i.e.,
turbulence simulations, which are not dominated by only a few large-scale structures. The driving of turbulence is done on large scales \((k \approx 2)\) and most structure (i.e., total power), including our inertial range, is at scales smaller than this. The error contributing to the power spectrum will be largest for large structures with internal radiative pumping as the detailed shape of the radiative interaction volume is ignored and instead replaced by an ellipsoid determined from the orthogonal velocity gradients. For all our cases, the approximations should be very good so that we expect an accuracy of better than 10% for supersonic turbulence and 20% for subsonic turbulence. This level of accuracy is sufficient for comparison with observational data as drifts in the receiver system and the atmosphere and the resulting temporal variation of the calibration parameters typically also provide calibration errors of that magnitude.

For our tests we choose a total cube size of 5 pc and a gas temperature of 10 K. The cube is observed at a distance of 450 pc with a beam FWHM of 18″ and a velocity resolution of 0.05 km s\(^{-1}\). We vary both the density scaling factor \(\xi = 3, 6, 8, \text{and } 10\) from Table 3 for varying density (Columns 5–7 of Table 3) is shown in Figure 1. The left column shows the sub-Alfvénic case and the right column the super-Alfvénic case. We find almost identical “by eye” power spectral trends for cases of varying abundances (i.e., models in Columns 8 and 9 of Table 3) and hence only make the plot for the varying density parameter space (i.e., holding abundance constant). Additionally, we do not find significant differences between the power spectrum of different lines of sight relative to the mean magnetic field and only show here a line of sight taken perpendicular to the mean field.

The left and right top panels represent the most optically thick cases (dotted black and dash-dotted green lines) are saturated around \(-3\) regardless of the Son-c or Alfven Mach number. Saturated cases where the density or abundances are very low (dot-dashed line) show shallower slopes than the column density slopes. One might expect that the use of high-resolution simulations with radiative transfer might show slopes converging even closer to \(-3\).

Interestingly, the mixed optically thin/thick case or the norm case (yellow dashed lines) which has optical depth on the order of unity shows a trend with the slope that is dependent on the Alfven Mach number. For super-Alfvénic turbulence, the norm case follows the column density slopes and the slopes are much shallower than compared with the sub-Alfvénic counterpart (a difference in the slope of 0.6 for the \(M_s \approx 8\) simulations). However, for sub-Alfvénic turbulence, the norm case behaves as though it were generally optically thick, and the slopes remain somewhat more around \(-3\), although the \(M_s \approx 2.0, M_A \approx 0.7\) case shows a larger variation in its slope of \(-3.5 \pm 3\). Inspection of Table 3 reveals that the optical depths for the norm case are not substantially different between models with high and low magnetic fields (all hover around \(\tau \approx 3–4\)).

Finally, we also plot the slopes for the very optically thin cases where the density or abundances are very low (dot-dashed blue lines and thick dashed purple lines) in Figure 2. For the incompressible (subsonic) cases, the values of the slopes converge well with the true column density values, especially for super-Alfvénic turbulence (slopes of around \(-3.6\)). However, as the sonic Mach number increases, the slopes for these models become shallower than the column density slopes. The super-Alfvénic slopes are slightly shallower than the sub-Alfvénic slopes. We conclude that, for supersonic turbulent CO-integrated intensity maps with very low density/abundance values, the power of the total intensity map is increasing on smaller scales as compared with the true column density. This is due to the fact that, for these cases, excitation of the \(^{12}\)CO line is limited due to the density/abundance being too low to excite regions that are not in the highest density areas, i.e.,

The energy spectrum versus wave number for models 1, 3, 5, 6, 8, and 10 from Table 3 for varying density (Columns 5–7 of Table 3) is shown in Figure 1. The left column shows the sub-Alfvénic case and the right column the super-Alfvénic case. We find almost identical “by eye” power spectral trends for cases of varying abundances (i.e., models in Columns 8 and 9 of Table 3) and hence only make the plot for the varying density parameter space (i.e., holding abundance constant). Additionally, we do not find significant differences between the power spectrum of different lines of sight relative to the mean magnetic field and only show here a line of sight taken perpendicular to the mean field.

The left and right top panels represent the most optically thick cases (dotted black and dash-dotted green lines) are saturated around \(-3\) regardless of the Son-c or Alfven Mach number. Saturated cases where the density or abundances are very low (dot-dashed line) show shallower slopes than the column density slopes. One might expect that the use of high-resolution simulations with radiative transfer might show slopes converging even closer to \(-3\).

Interestingly, the mixed optically thin/thick case or the norm case (yellow dashed lines) which has optical depth on the order of unity shows a trend with the slope that is dependent on the Alfven Mach number. For super-Alfvénic turbulence, the norm case follows the column density slopes and the slopes are much shallower than compared with the sub-Alfvénic counterpart (a difference in the slope of 0.6 for the \(M_s \approx 8\) simulations). However, for sub-Alfvénic turbulence, the norm case behaves as though it were generally optically thick, and the slopes remain somewhat more around \(-3\), although the \(M_s \approx 2.0, M_A \approx 0.7\) case shows a larger variation in its slope of \(-3.5 \pm 3\). Inspection of Table 3 reveals that the optical depths for the norm case are not substantially different between models with high and low magnetic fields (all hover around \(\tau \approx 3–4\)).

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Figure 1. Energy spectrum vs. wave number on a log–log scale for models 1, 3, 5, 6, 8, and 10 from Table 3 for varying density (Columns 4–6 of Table 3). The top row represents the most optically thick cases (with $n > 2500$), the next two rows are for the norm case and $n \approx 9$ cases, respectively, and the very bottom row shows the column density power spectrum with no effects of radiative transfer. The sub-Alfvénic cases are organized on the left panels and the super-Alfvénic cases are organized on the right panels. Thick solid lines represent the theoretically expected three-dimensional power spectrum given in Table 2 (i.e., the Kolmogorov slope is $-11/3$). We find almost identical spectral trends for cases of varying abundances (Columns 7 and 8 of Table 3) and hence only make the plot for the density parameter space. We show insets with the power spectrum normalized both on an absolute scale to the amplitude at $\log k = 1.1$ and to the theoretical slopes expected from Table 2. The $k$ range plotted on the $x$-axis of the insets is identical to the range we take for the fitted slopes.

(A color version of this figure is available in the online journal.)
shock-compressed regions. Thus, a spectrum that is shallower than the column density power spectrum is obtained.

4. DISCUSSION

4.1. Optical Depth Studies on the Turbulence Power Spectrum

Our study supports the theoretical conclusion in LP04 that the spectrum of line intensity fluctuations in the case of optically thick lines tends to the universal spectral slope of $-3$, which does not reflect the actual underlying turbulence. In general, we also found that slopes shallower than the predictions of Table 2 can result from higher sonic Mach numbers, lower optical depths, and, to a lesser extent, super-Alfvénic turbulence.

We varied both density and CO abundance in our simulations to vary the optical depth in order to investigate its effects on the power spectrum. Our standard abundance represents the abundance in a fully molecular material. The transition region investigated by Clark et al. (2012) represents an intermediate regime toward our low-abundance case. For studies investigating the Fourier power spectrum of line intensity maps, we have shown that the main effect to be taken into account is the change of the optical depth, rather than the specific value of density or abundance.

However, one might speculate that the spatial variations of the CO abundance (Glover & Mac Low 2011) can be reflected in the power spectra. The abundance change will occur on the larger scales, i.e., $k \sim 1$, which is not in the inertial range that we consider here. Therefore, we can treat the abundance change in those simulations as a global change of the inertial power spectrum, similar to the global abundance scalings that we perform manually when going from $x_{\text{co}} - 8$ to norm and to $x_{\text{co}} - 5$. In that sense, our experiment can be directly used to study the effect of different abundance scalings in the process of molecular cloud formation. When interpreting our results, e.g., directly in terms of $^{13}$CO observations, we will therefore find a transition between the optically thin case and the norm case, i.e., we only expect a small change of the power spectrum slope in the process of the molecular cloud formation. In contrast, for the main $^{12}$CO isotope the molecular cloud formation process rather represents the transition from the norm case to the optically very thick cases, i.e., we predict a change of the power spectrum slope to $-3$ in the molecular clouds.

We note that even in the case of optically thick lines, the power spectrum of channel maps may still reflect the velocity and density spectra for sufficiently large separations (see the corresponding criterion in LP04). However, the separation of density and velocity contributions may be more difficult using the VCA technique. For a steep density spectrum, which is expected for subsonic turbulence, we can recover the velocity spectral index. For supersonic turbulence that produces a shallow density spectrum, the recovery for the velocity spectral index from thin channels faces the problem that we do not know the actual density spectral index.

Thus, in order to know which regime we are in, we must know the sonic Mach number. The issues of the Mach numbers of turbulent clouds can be resolved with the use of other techniques (see Kowal et al. 2007; Burkhart et al. 2009; Burkhart & Lazarian 2012) or from independent velocity and temperature measurements (see Burkhart et al. 2010; Kainulainen & Tan 2013). In this case, if one is in the subsonic regime, one can get the velocity spectrum directly. In the case of supersonic turbulence, one may compare the fit of the velocity–density spectral to numerical simulations. Alternatively, the density spectrum can be obtained independently, e.g., from column density and dust extinction maps (see Lazarian 2009). This calls for multi-frequency and multi-instrument studies of turbulence. At the same time, comparing the spectra of density from dust and of total line emission can gauge the effects of optical absorption.$^6$

We note that for both VCA and VCS the thermal broadening is important. However, if we use sufficiently heavy species, the thermal broadening is reduced and therefore both subsonic and supersonic turbulence can be studied.

In addition, our study confirms that also for optically thin species, the spectral slope depends on the line excitation. When density and abundances are low, we undersample most of the gas, causing emission to be concentrated in the shock-dominated regions. This has the effect of shallowing out the spectral slope in the case of supersonic turbulence. Subsonic turbulence is hardly affected.

4.2. Differences in the Spectral Slope between Sub-Alfvénic and Super-Alfvénic Turbulence with Varying Optical Depth

While previous publications have predicted differences in the power spectral slopes between hydrodynamic/super-Alfvénic turbulence and highly magnetized turbulence in the incompressible limit with no radiative transfer effects (see Table 2) and in the case of fully optically thick lines (LP04), no predictions exist for determining the influence the magnetic field has on the slopes with optical depths around unity or less. In this work, we present the first measured differences between the slopes of these magnetic regimes with varying optical depth.

We find a significant difference in the regime of moderate optical depths (norm case), where the synthetic $^{13}$CO map shows a steeper spectrum for the supersonic, sub-Alfvénic
turbulence compared to the super-Alfvénic case. The spectrum shows more power on small scales similar to the transition to optically thick lines. This indicates a higher radiative excitation of the synthetic $^{13}$CO 2–1 transition in the sub-Alfvénic case, optically thick lines. This indicates a higher radiative excitation shows more power on small scales similar to the transition to turbulence compared to the super-Alfvénic case. The spectrum is increased as compared with their super-Alfvénic and hydrodynamic counterparts. Additionally, the stronger magnetic field decreases the compression of the media and mitigates the shock formation, which also increases the radiative interaction volume. Thus, we obtain a higher radiative pumping as a result of the increase in eddy volume, finally leading to a power spectrum that falls between the column density scaling and the optically thick case. In light of these effects, future work should further investigate the statistics of the velocity field (i.e., line widths) on the radiative transfer parameter space and the turbulence parameter space.

5. CONCLUSIONS

We analyze the observable two-dimensional power spectrum of integrated intensity $^{13}$CO maps created from three-dimensional MHD simulations with a range of sonic and Alfvénic Mach numbers and optical depths.

1. We confirm numerically the predictions of LP04 that the line emission spectral slope of a optically thick medium saturates to $-3$.

2. For very optically thin supersonic CO gas, where the density/abundance values are too low to excite emission in all but the densest shock-compressed gas, we find that the spectral slope is shallower than the expectations for column density.

3. We find that mixed optically thin/thick CO gas, which has optical depths on order of unity, shows mixed behavior: for super-Alfvénic turbulence, the spectral slopes follow the column density slope with $M^3$, while for sub-Alfvénic turbulence, the spectral slope is around $-3$, similarly to the very optically thick regime.

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