Energy Harvesting from Two Coupled Beams with Piezoelectric Patches

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Abstract. This paper presents the problem of energy harvesting from two coupled beams using piezoelectric patches. The energy harvester is subject to both harmonic and random excitation. The voltages across the piezoelectric patches are measured in real time in order to calculate the power. The coupling between the beams is an important factor for harvesting energy for a wider frequency band because new modes appear with this effect. This would result in widening the frequency bandwidth in which the energy can be harvested. Both numerical and analytical models are developed for the two coupled beams with two different lengths. The natural frequencies and mode shapes are derived and the harvested power is calculated for the random excitation. The resistance of the two piezoelectric patches is varied and the optimum resistance is derived. Experiments are carried out on two coupled beams to obtain the performance of the harvester to demonstrate the increase of the frequency bandwidth in which the energy can be harvested. It is shown that the harvester can collect energy at a wider frequency bandwidth compared to a single beam.

1. Introduction

The conversion between mechanical energy and electrical energy (and vice versa) has attracted the attention of researchers and industries for many years [1]. In addition to the electromagnetic and electrostatic phenomena, piezoelectric systems have been used for energy harvesting during the past two decades. Indeed, for a cantilever beam system, maximum energy can be harvested at resonance.

Piezoelectric materials can present a dual function as it follows: i) as a sensor with the output voltage, which can harvest energy at low frequencies; ii) as an actuator, in which energy can be supplied to the system for control purposes. These two functions are very relevant for an optimal and localized control. Moheimani and Fleming [2] described the characteristics of piezoelectric materials when they are connected with electrical shunts. Yun et al. [3] considered a beam, which is excited under its natural frequencies for active damping and its dynamic equations are derived using Euler-Bernoulli as shown in [4]. Guillot et al. [5] considered a non-linear Euler-Bernoulli beam which includes several piezoelectric patches on it. Actually, the addition of a patch to harvest and/or to control the vibratory energy changes the natural frequencies of the system due to the change of the geometric and inertial properties [5]. Collet and Jean [6]
investigated the phenomenon of electromechanical coupling induced by the piezoelectric material. The natural frequencies in short-circuit and in open-circuit are compared and shown in [7, 8], however in this paper, the focus is to show the increase of the frequency bandwidth using beams with different lengths. Other methods use non-linear strategies to extend the frequency band such as the bistable energy [9]. The Euler-Bernoulli model provides an approximation of the natural frequencies of a beam with and without a piezoelectric patch. Usually, in the literature the results are also compared with finite element method (FEM) for validation of models as carried out in [10, 11]. Piezoelectric patches provide good levels of damping when they are connected to an electrical shunt circuit. Mechanical energy is converted into electrical energy. This conversion, with the Joule's effect, is relevant for energy harvesting. Park [12] used the Hamilton’s principle and the equations were derived from the charge generated in piezoelectric due to the vibration of the beam in order to obtain the optimal shunt damping.

This paper focuses on harvesting energy for a two coupled beams system. The random excitation is the most representative excitation of natural hazards. So, the model is excited with random excitation. The paper is structured as follows: a example of two beams through a coupling stiffness is considered in section 2. This section shows the phenomenon of coupled modes with a simple model analytically. In section 3, electrical shunts from Ducarne [11] are used for the model using piezoelectric patches. Section 4 demonstrates experimental results. Finally, results is concluded in section 5.

2. Modelling of two beams coupled together

A model of two connected cantilever beams, depicted in figure 1, is considered. This system is interesting from energy harvesting viewpoint. This model can be condensed on a simple system with several degrees-of-freedom (dof) as shown in figure 2. In the following, we will treat the system analytically and numerically.

2.1. Simplified model with discretised system

Let us consider $n$ connected beams. If we condense each beam to its targeted mode, then the system can be represented as multi degree-of-freedom, shown in figure 2. $m_i, k_i, c_i, i = 1, \ldots, n$ stand for the mass, stiffness and damping of the $i^{th}$ mode, respectively. The $k_{c_j}, j = 2, \ldots, n-1$ represents the corresponding stiffness of each connection. Governing system equations referring
to [13] can be written as:

\[
\begin{align*}
    m_i \ddot{x}_i(t) + c_i \dot{x}_i(t) + (k_i - k_{ci})x_i(t) &= -m_i \ddot{x}_b(t), \\
    m_i \ddot{x}_i(t) + c_i \dot{x}_i(t) + (k_i - k_{ci(i-1)})x_i(t) &= -m_i \ddot{x}_b(t), \\
    m_i \ddot{x}_i(t) + c_i \dot{x}_i(t) + (k_i - k_{ci})x_i(t) &= -m_i \ddot{x}_b(t),
\end{align*}
\]

(1)

Neglecting the damping effects, for the model illustrated in figure 1, (1) yields:

\[
\begin{align*}
    m_1 \ddot{x}_1(t) + (k_1 - k_{c1})x_1(t) &= -m_1 \ddot{x}_b(t) \\
    m_2 \ddot{x}_2(t) + (k_2 - k_{c1})x_2(t) &= -m_2 \ddot{x}_b(t)
\end{align*}
\]

(2)

where the indexes 1 and 2 represent the beam 1 and the beam 2 respectively.

![Figure 2: Principle of a multiple branches of an energy harvesting system subject to base excitation.](image)

2.2. Structural response

In frequency domain, and for a two-degree of freedom model (2) we have,

\[
\begin{align*}
    -m_1 \omega^2 X_1 + k_1 X_1 + k_{c1}(X_1 - X_2) &= k_1 X_b \\
    -m_2 \omega^2 X_2 + k_2 X_2 + k_{c1}(X_2 - X_1) &= k_2 X_b
\end{align*}
\]

(3)

Consequently, (3) can be written in the matrix form as,

\[
\begin{pmatrix}
    -m_1 \omega^2 + k_1 + k_{c1} & -k_{c1} \\
    -k_{c1} & -m_2 \omega^2 + k_2 + k_{c1}
\end{pmatrix}
\begin{pmatrix}
    X_1 \\
    X_2
\end{pmatrix}
= 
\begin{pmatrix}
    k_1 \\
    k_2
\end{pmatrix}
\]

(4)

In this paper the masses and stiffnesses are tuned to match the first natural frequency of the beam. The first natural frequency of the beam 1 is obtained to be 41.1 Hz from analytical formulation using the parameters

\[
E = 7.0 \times 10^{10} Pa, \quad \rho = 2700 kg/m^3, \quad b = 0.02 m, \quad h = 0.002 m
\]

and \(L_1 = 0.2 m\) (aluminium material). Figure 3 is the frequency response of |\(X_1\)| and |\(X_2\)| for different coupling stiffness when the two beams have the same lengths and different lengths. First, when there is coupling effect the two resonances are evident because of the new peak appearing when we compare figure 3(b) to figure 3(a). Then, when there is no coupling effect, the two resonances coincide, as can be seen in figure 3(a). However, for beams with different lengths, the two resonances will be different as shown in figure 3(c). So, adding the coupling effect for two beam with different lengths can be beneficial for harvesting point of view because of the higher gain comparing figure 3(d) to figure 3(b).
Figure 3: Frequency response for a ratio $L_2/L_1 = 1$: (a) $k_{c1} = 0$, (b) $k_{c1} = 0.05k_1$ and for a ratio $L_2/L_1 = 1.05$: (c) $k_{c1} = 0$, (d) $k_{c1} = 0.05k_1$.

3. Finite Element Simulation of the energy harvesting using piezoelectric patches

In this part, we will investigate different configuration of piezoelectric patches for two connected beams. Using piezoelectric is one of the ways to harvest energy. The electromechanical coupling effect is relevant for our investigation because the addition of an electric shunt works as a mass spring damper. In this part let us consider electrical circuit composed of resistances connected to the piezoelectric patches. The investigation aims to show an optimal value of tuned resistance in order to harvest energy by joules effect. Numerical results will be emphasized by experimental tests for optimal case.

3.1. Mode shape

If the beam follows the Euler-Bernouilli model we have the following equation, with $w$ the vertical displacement

$$\frac{EI}{\rho A} \frac{d^4 w}{dx^4}(x,t) + \frac{\partial^2 w}{\partial t^2}(x,t) = F$$

(5)
For cantilever beams the boundary conditions are:

\[ w(0, t) = 0, \quad \frac{\partial w}{\partial x}(0, t) = 0, \quad \frac{\partial^2 w}{\partial x^2}(L, t) = 0, \quad \frac{\partial^3 w}{\partial x^3}(L, t) = 0 \]  

(6)

A general solution for this equation is (separate variable method),

\[ w(x, t) = \sum_{i=1}^{\infty} q_i(t) X_i(t) \]  

(7)

where \( q \) is the temporal function and \( X \) the mode function. Index \( i \) corresponds to \( i^{th} \) mode.

With (6) the mode shape is,

\[ X_i(x) = A_1 \sin(\beta_i x) + A_2 \cos(\beta_i x) + A_1 \sinh(\beta_i x) + A_1 \cosh(\beta_i x) \]  

(8)

with element inertia equation of,

\[ \beta_i^4 = \frac{\omega_i^2}{c^2}, \quad c^2 = \frac{EI}{\rho A}, \quad \omega_i^2 = \frac{\partial^4 X_i}{\partial x^4}(x, t) \frac{q_i}{X_i} \]  

(9)

Figure 4 shows the first four mode using finite element method (FEM).

Due to the interest of different lengths for our investigation, figure 5 shows the first six modes and the first sixth natural frequencies given by COMSOL package with the dimension used in figure 3 and a ratio \( \frac{L_2}{L_1} = 1.05 \). For example for the fourth mode the first beam (the shorter beam) is in its second vertical mode and the second beam in its first transversal mode.

### 3.2. Base piezoelectric patches

In most cases, piezoelectric materials are ceramic (PZT), which assumes that the device described in figure 6 is poled vertically, and the material is transversely isotropic. This assumption will simplify the matrix using in (10), most terms are zero or are equal. The behaviour of piezoelectric material can be described by,

\[ \begin{cases} 
\sigma_{11} = e_{11} \epsilon_{11} + e_{31} E_3 \\
D_3 = e_{31} \epsilon_{11} + \xi_{33} E_4 
\end{cases} \]  

\[ \omega_i^2 = \frac{\partial^4 X_i}{\partial x^4}(x, t) \frac{q_i}{X_i} \]  

(10)

where, \( e_{ij} \) the ratio of the stress in the axis \( j \) to the electric field applied along the axis \( i \) and \( \xi \) the matrix of permittivity. For our investigation we assume that only the stress \( \sigma_{11} \) is not zero, it is the same for the electric displacement with \( D_3 \). \( V \) is the voltage across piezoelectric patch, \( Q \) the charge and \( F \) a force excitation. \( \sigma \) and \( \epsilon \) are Stress and strain tensors, \( E \) and \( D \) are respectively the vectors the electric field and the electric displacement. Figure 6 shows a
simplify model for FEM of a piezoelectric patch as a plate. In fact, if all the nodes value vector can be split into two part; temporal and spatial such that;

$$\mathbf{U}(t) = \sum_{i=1}^{n} q_i(t) \phi_i$$  \hspace{1cm} (11)$$

With the normalisation and the orthogonality of the mode we have:

$$\forall i,j \quad \phi_i^t M \phi_j = \delta_{ij} \quad \text{and} \quad \phi_i^t K_m \phi_j = \delta_{ij}$$  \hspace{1cm} (12)$$

Next subsections aim to apply finite element method for investigation of piezoelectric patch behavior. Figure 7 shows that four case are investigated on our paper: only one patch at the connection, two patch in parallel, two patch in series and a "decentralized case".

### 3.3. One patch

First, the beams with one patch is considered and described in figure 7(a). The finite element method allows us to determine the coordinates of the $n$ nodes of the mesh. In fact, we defined the displacement field as,

$$\mathbf{u} = \mathbf{N} \mathbf{U}$$  \hspace{1cm} (13)$$

$$\mathbf{S} = \mathbf{D} \mathbf{u} = \mathbf{D} \mathbf{N} \mathbf{U} = \mathbf{B} \mathbf{U}$$
where $\mathbf{N}$ the shape function vector, $\mathbf{U}$ the nodes values vector, $\mathbf{S}$ the strain matrix, $\mathbf{D}$ the derivation operator and $\mathbf{B} = \mathbf{D}\mathbf{N}$.

Hamilton’s principle give:

$$
\begin{pmatrix}
M & 0 \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
\ddot{U} \\
\ddot{V}
\end{pmatrix}
+ \begin{pmatrix}
K_m & K_{em}^T \\
-K_{em} & K_e
\end{pmatrix}
\begin{pmatrix}
\dot{U} \\
\dot{V}
\end{pmatrix}
= \begin{pmatrix}
\dot{F} \\
\dot{Q}
\end{pmatrix}
$$

(14)

where:

$$
\mathbf{M} = \int_{\Omega} \rho \mathbf{N}^t \mathbf{N} d\Omega \; (n \times n)
$$

$$
\mathbf{K}_m = \int_{\Omega} \mathbf{N}^t \mathbf{C} \mathbf{N} d\Omega \; (n \times n)
$$

$$
\mathbf{K}_{em} = \frac{1}{h} \int_{\Omega} \mathbf{B} \mathbf{e}^t d\Omega \; (n \times 1)
$$

$$
\mathbf{K}_e = \mathbf{C} = \frac{b \xi_{33}}{h}
$$

$$
\mathbf{F} = \int_{\Omega} \mathbf{F} \mathbf{N}^t d\Omega \; (n \times 1)
$$

(15)

In [11] the optimal resistance is given for one beam, so,

$$
R = \frac{1}{\mathcal{C} \omega_i}
$$

(16)

Where $R$ is the optimal resistance entering into resonance at $\omega$.

3.4. two patches in parallel ($V_1 = V_2$)

In [11], (15) and (17) have the same form but the dimensions change. In fact, here $\mathbf{V}$ and $\mathbf{Q}$ is $2 \times 1$ dimension and

$$
\mathbf{K}_e = \begin{pmatrix}
\mathcal{C} & 0 \\
0 & \mathcal{C}
\end{pmatrix}
$$

(17)

Consequently, in the $r^{th}$ truncated mode or in other term by multiplying (14) by $\phi_r^t$ at the left, the system can be simplified because most of the matrices are diagonal now.

$$
\begin{cases}
\frac{\partial^2 q_r}{\partial t^2} + \omega_r^2 + \chi_{r1} V_1 + \chi_{r2} V_2 = F_r \\
CV_1 - q_r \chi_{r1} - Q_1 = 0 \\
CV_1 - q_r \chi_{r2} - Q_2 = 0
\end{cases}
$$

(18)

where

$$
(\chi_{r1}, \chi_{r2}) = K_{em}^t \phi_r = \phi^t_r K_{em}
$$

(19)

Here the stiffness is less than in the precedent case because the capacitance is multiplied by two.

$$
2CV - q_r (\chi_{r1} + \chi_{r2}) = V
$$

(20)
The optimal resistance for damping in frequency domain with \( V = Rj\omega Q \) is approximately,

\[
R = \frac{1}{2C\omega} \tag{21}
\]

3.5. Two patches in series \((Q_1 = Q_2 = Q)\)
With the same mathematical formulation in [11]. We can observe that the frequency changes, the stiffness is higher than in one patch because the capacitance is divided by two. The optimal resistance for damping is near to:

\[
R = \frac{2}{C\omega} \tag{22}
\]

In figure 8 we compare this two cases by using FE simulation in COMSOL. The higher mass of the damping is, the higher the damping is. However, with two patches in series the stiffness increases as far as the second mode is concerned.

![Figure 8: Comparison between the three configurations: one big patch, two patches in parallel and two patches in series.](image)

3.6. Decentralized case
For decentralized case, both patches have their own electrical circuits. This is interesting at it gives some similar results, as the series case (optimal resistance, frequency resonance . . . ). Using FE simulation, shown in figure 9 the damping is observed to be higher; it is very relevant for energy harvesting. A slight difference can be observed in the characteristic values because the vibrations are not symmetric (even if the beams have the same dimensions).

4. Experiments
The experimental device is shown in figure 10 where the piezoelectric patches are smart materials ordered from [https://www.smart-material.com](https://www.smart-material.com). Due to beneficial interest for harvest energy, the system is composed of two beams with different lengths. The dimensions are the same of

\[\text{P2 type M2807-P2, length: 28 mm, width: 7 mm and a capacitance of 415.11 nF}\]
Figure 9: Comparison between the series case and decentralized

section 2 and $\frac{L_2}{L_1} = 1.05$. In addition, section 3 results showed the advantages of the decentralized case. So, let us consider the decentralized case for experiments. The system is under random excitation with the frequency $f \in [0, 1000]$ Hz. Piezoelectric shunts are connected to an amplifier and the signals are sent to DSPACE platform using Simulink package. After Fourier

Figure 10: Experimental device with $L_2 = 1.05L_1$ (Beam and shaker connected to dSpace).

transformation, the frequency response of collected voltages is presented in figure 11. We can observe the peak of each modes under random excitation except for the two first modes as the patches are at the base and the displacements are very low. The voltage output is higher for the beam one as its displacement in resonance. The optimal resistance value for energy recovery is calculated for the maximum frequency in figure 11 around 243 Hz. If we connect a resistance to the patch beam, with the relation in (16), the maximum power should be for $R = \frac{1}{243 \times 2\pi} = 43.3\, \Omega$. Figure 12 graphically describes the evolution of average power harvested
Figure 11: Frequency response of voltage under random excitation with two patches, one for each beam (experiments, decentralized case).

experimentally as a function of resistance defined, with $T$ the period of the system and $N \in \mathbb{N}$,

$$P(R) = \frac{1}{NT} \int_0^T \frac{V^2(t)}{R} \, dt$$  \hspace{1cm} (23)

Experimentally the results shows the broadening of the harvesting range via using beams with two different lengths.

Figure 12: Average Power under random excitation for the beam 1.
5. Conclusion
The vibratory energy harvesting from a two connected beam patched with one or several piezoelectric ceramics is studied. Two beams are identical in all properties just their length are different. For harvesting energy, the power produced by the patch will be optimal at the beam resonance. However, the optimal impedance should be modified for two connected beams. For energy harvesting, we studied four possible configurations: one patch, two patches in series, two patches in parallel and two patches with their own electrical circuit. The frequency response of voltage finite element simulation in COMSOL showed that the most relevant device for damping is the decentralized case, i.e. the configuration which corresponds to two patches with their own electrical circuit. Indeed, for two connected beams with different lengths and under random excitations, we can have highest harvested energy from each beam via tuning the resistance. In this paper, experiments validate the theoretical model built in COMSOL. The maximum harvested energy from the theoretical model is obtained from around the frequency which corresponds to the sixth and fourth modes of shorter and longer beams, respectively. This is validated experimentally as well. For having more precise predictions, one should consider a nonlinear theoretical model of the as considered by Guillot et al [5].

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