Letter

Time-dependent kinetic analysis of trapped electrons in a magnetically expanding plasma

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Abstract

A deep understanding of the kinetic properties of the electrons in a magnetic nozzle (MN), which is attracting attention as an acceleration stage for thrusters, is of great significance as it directly contributes to the development of the MN performance. In the sense that a conversion of the electron momentum to the ion kinetic energy determines the characteristics of the MN, fundamental research on the kinetic feature of a magnetically expanding plasma has focused on the spatial distribution of the electron properties and proposed directions to the desired application. Unlike the common perception of this importance, various research groups have proposed contradictory arguments based on their theoretical approaches regarding the ion beam acceleration from the viewpoint of heat flow of electrons. We point out that the main reason for the absence of a theoretical consensus for the nozzle efficiency improvements arises from the lack of the clear interpretation of the plasma properties by focusing only on the final state of the electrons. In this Letter, time-resolved measurement of the electron energy distributions has been performed to grasp a detailed series of expansion processes. It has been revealed that the effective potential well gradually formed by the self-generated electric field acts as a limiting factor in the motion of electrons; this effect attributes to the changes of the electron energy distribution represented as the accumulation of the trapped electrons. The accumulation over the entire region diminishes the degree of the cooling rate of a system and decreases the electric field in the downstream region initially generated by the adiabatic expansion. The present study emphasizes that the kinetic features of an MN are strongly affected by the non-stationary motion of the trapped electrons; thus, the temporal behavior of the trapped electrons must be considered for prediction and analysis of nozzle performances.

Keywords: Magnetically expanding plasma, magnetic nozzle, thermodynamics, trapped electron, electron energy probability function, inductively coupled plasma, thruster

An in-depth understanding of the behavior of the plasma expanding along the magnetic field plays a crucial role in a variety of fields ranging from providing an understanding of space plasma [1–4] to applications for the purpose of space electric propulsion [5–9]. The magnetic nozzle (MN) is an attractive plasma accelerator for the space propulsion due to the characteristic of the electrode-free structure. In particular,
fundamental research on a typical feature of a magnetically expanding plasma described as convert of the thermal energy into directed kinetic energy is emphasized in connection with the engineering solution to the performance of the MN [10–20]. In the sense that a conversion of the electron momentum to the ion kinetics determines the characteristics of an electron-driven MN, analytic modeling of various points of view regarding the electron cooling mechanism is devoted to improvement of the nozzle performance. However, the blueprints of each study group based on modeling and experiment have proposed contradictory arguments regarding the thermodynamic state of the electron for a breakthrough of the MN performance in aspects of ion acceleration and plasma detachment [5–20]. To be more specific, Takahashi et al [9] claimed that the thermodynamic state of electrons involved in the electric field generation is close to isothermal, asserting that it is the inherent characteristics possessed by the magnetically expanding plasmas. In contrast, Kim et al emphasized that the electron thermodynamics in a system is affected by the presence of isothermally confined electron group, implying that the polytropic index and the nozzle efficiency cannot be directly inferred [8].

We point out that the main reason for the absence of a theoretical consensus for the improvements of the MN efficiency arises from the lack of the clear interpretation of the plasma properties by focusing only on the final state of the electrons. In this Letter, time-dependent kinetic property of magnetically expanding plasmas is investigated in order to overcome the limitations of analytical approach. The observation of the time evolution of the electron energy distribution is believed to provide a clue as to the understanding of the electron kinetic property along the magnetic field that has yet to be theoretically agreed upon.

The S-Nozzle device consists of three main components (figure 1(a)); an electrically floated plasma source tube made of quartz wound with a silver-plated 2-turn cylindrical antenna; a grounded expansion chamber; a mesh grid (60 line per inch) installed between the source tube and the expansion chamber. The mesh grid is capable of applying pulsed voltage signal to determine the plasma properties just ahead the expansion region. RF powers (13.56 MHz) of 180 W connected to the L-type impedance matching circuit is applied to the antenna, and the reflected power under 1% is sustained during the experiment. A square wave changing from –90 to 0 V (200 &mu;s for –90 V and 400 &mu;s for 0 V) is applied to the mesh grid through a signal generator (figure 1(b)). The signal generator with an extremely fast rise time (4 ns) is used to prevent expansion of the ejected plasma from the mesh grid during the rise time of the pulsed signal. The two different regions (the source tube and expansion chamber) are separated by the nozzle field magnet to form converging-diverging magnetic field configuration (figures 1(a) and 3(a)).

A vacuum pumping system consists of an oil-sealed rotary pump (350 l min−1) and a turbo molecular pump (1000 l min−1). Using a mass flow controller with a maximum flow-rate of 100 ml min−1, argon gas is injected through a gas feeding port at a radial center of the quartz tube. The base pressure is 5.0 × 10−7 Torr and the operating pressure is fixed at 4.0 × 10−4 Torr, ensuring nearly collisionless expansion; the mean free path for electron-neutral collision (125 cm) exceeds the length of the expansion region (66 cm). Change of the operating pressure is not observed when various voltages are applied to the mesh grid, confirming that the pressure change during the experiments is negligible. An axially movable RF-compensated single Langmuir probe with a floating loop reference ring and two-stage LC resonance filter (100 kOhm at 13.56 MHz and 35 kOhm at that of the second harmonic to reduce the RF distortion) [21] is placed at the axial center of the plasma expansion region, where the magnetic field line is purely axial (figure 1). Probe tip is constructed from tungsten wire (0.05 mm in radius and 15 mm in length); the tip has a smaller radius than the electron Larmor radius (~1.6 mm at 3 cm from the nozzle throat; 95 &mu;s). A box-car averaging mode [22], which gives time-resolved diagnostics of the plasma parameters is used. Trigger to probe system is synchronized with the voltage signal to mesh grid (figure 1(b)), then at each cycle of the trigger signal, only some of the probe bias voltage to be measured is applied to the probe. Thus, the box-car mode can
acquire data points in each period of the voltage signal to the mesh grid and the procedure is repeated over time to obtain a complete $I-V$ curve. In all experimental conditions, the radius of the probe sheath is always shorter than the electron Larmor radius; thus, the electron energy probability functions (EEPFs) is obtained by calculating the second derivative of the measured $I-V$ characteristic $I''(\varepsilon)$ with the assumption of isotropic plasmas [21]. The second derivative $I''(\varepsilon)$ is proportional to EEPF $f_\varepsilon(\varepsilon)$ which is related to the electron energy distribution function (EEDF) $g_\varepsilon(\varepsilon) = \varepsilon^{1/2}f_\varepsilon(\varepsilon)$. The plasma potential $V_p$ is determined by the zero crossing of $I''(\varepsilon)$. The electron density $n_e$ and the effective electron temperature $T_{\text{eff}}$ corresponding to the mean electron energy are determined from the integral of the EEDF in a given electron energy range. Experiments were conducted after an hour of the chamber conditioning and repeatability of the experiment data was verified.

To observe a series of the electron expansion process, temporal evolution of the EEPFs are measured at 2.0 cm intervals from 3 to 35 cm from the nozzle throat. The measured EEPFs do not show bump structure in the high energy tail, implying negligible electron beam components. When $\approx 90$ V is applied to the mesh grid (0.5 $\mu$s before the rise up of the pulsed signal), confinement of the source plasma is strong that only a small amount of plasma can pass the mesh grid (figure 2(a)). Accordingly, the plasma in the expansion region has relatively low $n_e$ and $T_{\text{eff}}$, producing a negligible $V_p$ structure in the expansion region (figures 3(b)–(d)).

The expansion of the source plasma into the low density ambient plasma is achieved by pulsing the mesh grid to ground so that the confinement of source plasma is significantly reduced. Interestingly, as time elapsed, electrons begin to accumulate on the EEPFs (figures 2(b)–(e)); the accumulation phenomenon increases with time over the entire expansion region.

The accumulation of the electrons seen in the EEPFs directly attributes to $n_e$ and $T_{\text{eff}}$ changes over time (3.0 to 95 $\mu$s) throughout the entire expansion region (figures 3(c) and (d)). Especially, at the initial stage (3.0 $\mu$s), the cooling of the electrons proceeds rapidly (figure 3(c)), and the absolute value of $T_{\text{eff}}$ is reduced over time (the reduction near the nozzle throat is prominent). Unlike noteworthy changes in the absolute value of $T_{\text{eff}}$, the spatial gradient of $T_{\text{eff}}$ is appeared to be almost constant at each moment in the region closes to the nozzle throat. The spatial gradient of $V_p$ tends to develop until 10 $\mu$s (figures 3(b) and 5(a)). However, $V_p$ gradient begins to be reduced after 10 $\mu$s along with the reduced $n_e$ profile in the downstream region, implying that the accumulated electrons in the downstream region at the later phase of the expansion contribute to reducing $V_p$ structure. Eventually, the appearance of the thermal energy conserved electrons in the downstream contributes to reducing nozzle efficiency in terms of ion beam acceleration.

To clearly characterize the temporal evolution of these plasma parameters, we observe the electron thermodynamics by adopting the polytropic equation [8]. From a kinetics perspective, the effective electron pressure $p_{\text{eff}}$ and $n_e$ for each electron group at certain moment can be related to the polytropic equation using the measured EEPFs. Then, the polytropic equation with the exponent $\gamma$ is as follows:

\[
\log_{10} \int_{0}^{\infty} \varepsilon^{1/2}f(\varepsilon) \, d\varepsilon \propto \gamma \log_{10} \int_{0}^{\infty} \varepsilon^{1/2}f(\varepsilon) \, d\varepsilon.
\]

The log–log relationship of data shows that the adiabatic process dominates the electron thermodynamics near the nozzle throat at all moments (figure 4). That is, the thermodynamic states of the electrons along the divergent magnetic field near the nozzle throat is maintained over time. Up to 3.0 $\mu$s, a slope of the log–log plot is maintained at the entire expansion region (the expansion seen in the downstream region far from the nozzle throat also follows the polytropic law with the exponent closes to the adiabatic process). In contrast, a temporal variation of the slope is observed beyond the central region. The evaluated $\gamma$ becomes closer to unity as it approaches to the downstream, indicating that the electrons gradually accumulated in the downstream region behave to preserve the thermal energy with time.

In order to elaborate on the change of the EEPFs and plasma parameters, it is essential to consider the effective potential barrier which determines the electron motion in
For electrons, the effective potential structure is determined by the local maximum magnetic moment $m_{em}$, with a total electron energy $E_e$ as follows [8, 19]:

$$m_{em} = \frac{E_e}{e} \frac{V_p}{B} \frac{\partial Z}{\partial t}.$$ 

For simplicity, $m_{em}$ is normalized by the total potential drop along axial direction $V_{p, total}$ at each moment and the maximum magnetic field strength $B_{max}$. The local maximum magnetic moment can have minimum and maximum values at certain locations past the nozzle throat, and this generates the well of the effective potential barrier, in which the electrons can be trapped.

When the electrons with $E_e = 20$ eV are located at the nozzle throat, the effective potential well is not generated at all moment (3.0 – 95 μs) (figures 5(b)–(d)). In the case of relatively low energy electron ($E_e$ of 12 eV and 16 eV), the well of the effective potential structure is formed in the expansion region, while the electrons at 3 μs can still escape freely to the far-field region without trapping (figure 5(b)). Therefore, the behavior of electrons during the electric field formation in the expansion region can be described as follows; the electrons whose thermal energy is reduced through the adiabatic process are no longer involved in the cooling process and become disconnected to the source region. Then, they bounce back and forth as mirror effect, being accumulated in the expansion region with thermal energy conserved behavior.

As a result, the characteristics of the trapped electrons is superimposed on the initially generated axial profile of the plasma parameters. Eventually, the increased electron pressure due to the trapped electrons with low thermal energy in the downstream region begins to balance with the existing $V_p$ structure and eventually weaken the electric field in the region. Consequently, when total electrons are regarded as a single system in the MN, temporal changes in $\gamma$ can be introduced; the expansion along entire axial distance follows the polytropic law with $\gamma_e = 1.58$ at 3.0 μs and it reduces to $\gamma_e = 1.39$ at 95 μs. Thus, it is implied that as the thermal energy conserved electrons occupy most of the expansion region, the electron thermodynamics estimated for the entire expansion region can be seen as the isothermal process. It should be noted that the magnetically expanding electrons involved in the electric field generation behaves adiabatically.

The trapping of the cooled electrons via the effective potential structure is somewhat similar to the cut off effect during the afterglow of pulsed plasmas [26]. In general, the sequential diffusive cooling process of the electrons observed in the afterglow plasma is as follows: (1) the rapid loss of the energetic electrons results in the reduced plasma potential due to overall positive space charge, and (2) previously trapped electrons can overcome the reduced plasma potential and freely escape to the wall. In present study, the process we are interested in observing can be regarded as the reverse process to this; the effective potential structure generated by the adiabatic process of electrons gradually produces trapped electrons.
Figure 5. (a) Temporal variation of the total potential drop along axial direction $\Delta V_{p,\text{total}}$ and the axial profiles of $\mu_{e,m} B_{\text{Max}} / \varepsilon \Delta V_{p,\text{total}}$ at (b) 3.0 $\mu$s, (c) 10 $\mu$s, and (d) 95 $\mu$s, respectively.

In summary, the time-dependent kinetic property of the magnetically expanding plasma has been examined through the time-resolved measurement of the EEPFs. The developed effective potential well results in the trapping of the cooled electrons during the expansion process. As a result, the change of the EEPFs represented as the accumulation of the trapped electrons during expansion causes a spatial change in $\chi_e$ that becomes closer to unity as it approaches to the downstream. Until now, the non-adiabatic behavior has been believed to the presence of a heat flux from the plasma source to the downstream region, so that the combination of the polytropic equation and the momentum equation could determine an asymptotic value of the potential of the entire expansion region. However, unlike the previous studies regarding the subject of the heat transfer as a plasma source region, this study emphasizes that the non-adiabatic behavior of an electron system is the main result of the trapped electron group generated in the local region disconnected to the source. This effect reduces the nozzle efficiency; the stagnation of the trapped electrons in a limited region decreases the initially generated electric field in the downstream region.

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