Quantum Cosmology and the Constants of Nature

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Abstract
In models where the constants of Nature can take more than one set of values, the cosmological wave function $\psi$ describes an ensemble of universes with different values of the constants. The probability distribution for the constants can be determined with the aid of the ‘principle of mediocrity’ which asserts that we are a ‘typical’ civilization in this ensemble. I discuss the implications of this approach for inflationary scenarios, the origin of density fluctuations, and the cosmological constant.

\footnote{Talk given at the RESCEU International Symposium “The Cosmological Constant and the Evolution of the Universe”}
1. Variable Constants

The observed values of the constants of Nature are conspicuously non-random (‘fine-tuned’). If particle masses are bounded by the Planck mass $m_p$ and the coupling constants by 1, then a random selection would give all masses $\sim m_p$ and all couplings $\sim 1$. The vacuum energy would then be $\rho_v \sim m_p^4$. In contrast, some of the particle masses are more than 20 orders of magnitude below $m_p$, and $\rho_v \sim 10^{-120}m_p^4$.

There is a popular view that all constants will in the end be determined from a unique logically consistent theory of everything. However, the constants we observe depend not only on the fundamental Lagrangian, but also on the vacuum state, which is likely not to be unique. For example, in higher-dimensional theories, the constants of the four-dimensional world depend on the way in which the extra dimensions are compactified. The number of different compactifications in superstring theories is believed to be $\gtrsim 10^4$. Moreover, Coleman (1988a) suggested that all constants appearing in sub-Planckian physics may become totally undetermined due to Planck-scale wormholes connecting distant regions of spacetime.

It has been argued (Carter 1974; Carr & Rees 1979; Barrow & Tipler 1986) that the values of the constants are, to a large degree, determined by anthropic considerations: these values should be consistent with the existence of conscious observers who can wonder about them. This ‘anthropic principle’ has not been particularly popular among cosmologists, since it appears to require the existence of an ensemble of universes with different values of the constants of Nature. It should be pointed out, however, that one is lead to the concept of such an ensemble if one adopts the view that the universe should be described quantum-mechanically. The approach I am going to describe here combines anthropic considerations with quantum cosmology and inflationary scenario.

2. Creation of Universes from Nothing

The world view suggested by quantum cosmology is that small closed universes, with all possible values of the constants of Nature, spontaneously nucleate out of nothing. Here, ‘nothing’ refers to the absence of not only matter, but also of space and time (Vilenkin 1982, 1986; Hartle & Hawking 1983; Linde 1984). After nucleation, the universes enter a state of
inflationary (exponential) expansion. It is driven by the potential energy of a scalar field \( \varphi \), while the field slowly ‘rolls down’ its potential \( V(\varphi) \). This vacuum energy eventually thermalizes, and inflation is followed by the usual radiation-dominated expansion.

The probability distribution for the initial states of nucleating universes can be obtained from the cosmological wave function \( \psi(a, \varphi) \) which satisfies the Wheeler-DeWitt equation

\[
\left[ \frac{\partial^2}{\partial a^2} - \frac{1}{a^2} \frac{\partial^2}{\partial \varphi^2} - a^2 \left( 1 - a^2 V(\varphi) \right) \right] \psi(a, \varphi) = 0, \tag{1}
\]

Here, \( a \) is the radius of the universe. Eq.\((1)\) is supplemented by the ‘tunneling’ boundary condition which requires that at \( a \to \infty \) \( \psi \) includes only outgoing waves (in other words, that \( \psi \) does not include any components describing universes contracting from an infinitely large size). Then one finds that the probability for a universe to nucleate with a set of constants \( \{\alpha_j\} \) and with the initial value of the scalar field between \( \varphi \) and \( \varphi + d\varphi \) is

\[
dw_\alpha = \rho_\alpha(\varphi) d\varphi, \tag{2}
\]

where

\[
\rho_\alpha(\varphi) \approx C_\alpha \exp \left( -\frac{3m_p^4}{8V_\alpha(\varphi)} \right) \tag{3}
\]

and \( C_\alpha \approx const. \) Here, the subscript ‘\( \alpha \)’ is a collective symbol for the constants \( \{\alpha_j\} \). The initial radius of the nucleating universes is \( a_0 = [V_\alpha(\varphi)]^{-1/2} \), and the overall normalization of \( \text{(3)} \) is determined by

\[
\sum_\alpha \int \rho_\alpha(\varphi) d\varphi = 1. \tag{4}
\]

For each set of \( \{\alpha_j\} \), the distribution \( \text{(3)} \) is peaked at the maximum of \( V_\alpha(\varphi) \). This is just the right initial condition for inflation.

An alternative approach to boundary conditions for \( \psi \), which is based on Euclidean quantum gravity, has been developed by Hawking and collaborators. The expression for the probability distribution obtained using this approach is the same as \( \text{(3)} \) but with a different sign in the exponential. It is peaked at the minimum of \( V(\varphi) \) and does not favor initial states that lead to inflation. Here, I adopt the tunneling boundary condition.
We can think of the probability distribution $\rho_\alpha(\varphi)$ as describing an ensemble of universes which can be called ‘metauniverse’. The probability that a universe arbitrarily picked in this ensemble will have a particular set of $\{\alpha_j\}$ is

$$w_\alpha = \int \rho_\alpha(\varphi) d\varphi \propto \exp \left( -\frac{3m_p^4}{8V_\alpha^{(\text{max})}} \right),$$

(5)

where $V_\alpha^{(\text{max})} = \max\{V_\alpha(\varphi)\}$.

Those who are not comfortable with the idea of other universes can regard this ensemble of universes simply as a mathematical construct for calculating probabilities, just as we do in ordinary quantum mechanics. In either case, the calculations are done as if all the universes are ‘real’, and the simplest view to adopt is that they indeed are.

3. Principle of Mediocrity

It is quite possible that a universe randomly picked in the ensemble will be unsuitable for life, and therefore the distribution (5) is not adequate for predicting the observed values of the constants. Moreover, the number of civilizations in some of the universes may be much greater than in the others, and this difference should also be taken into account when evaluating the probabilities. The probability distribution of constants for a civilization randomly picked in the metauniverse is

$$P_\alpha = C^{-1}w_\alpha N_\alpha,$$

(6)

where $N_\alpha$ is the average number of civilizations in a universe with a set of constants $\{\alpha_j\}$ and $C = \sum_\alpha w_\alpha N_\alpha$ is a normalization constant. $N$ is taken to be the total number of civilizations through the entire history of the universe, rather than their number at some moment of time. (Comparing different universes at a given moment of time is not a very meaningful procedure: some universes may recollapse while the others are still expanding. Besides, the results of such a comparison are sensitive to the choice of the time variable).

If we assume that our civilization is a ‘typical’ inhabitant of the metauniverse, then we ‘predict’ that the constants of Nature in our universe are somewhere near the maximum

\footnote{This and the following sections are partly based on my papers (1995a,b,c). Related ideas were discussed by Albrecht (1995) and by Garcia-Bellido & Linde (1995).}
of the distribution \( \mathcal{E} \). The assumption of being typical, which I called the ‘principle of mediocrity’, is a version of the ‘anthropic principle’. The motivation for a new name was to emphasise the difference with the common practice of the anthropic principle where one merely assigns vanishing probabilities to the values of constants not suitable for life.

The number \( \mathcal{N} \) can be expressed as

\[
\mathcal{N}_\alpha = V_\alpha \nu_{\text{civ}}(\alpha),
\]  

where \( V_\alpha \) is the volume of the universe at the end of inflation, and \( \nu_{\text{civ}}(\alpha) \) is the average number of civilizations originating per unit thermalized volume. (More exactly, \( V \) is the volume of the 3-dimensional hypersurface separating the inflating and thermalized regions of spacetime).

The concept of ‘naturalness’ that is commonly used to assess the plausibility of elementary particle models is based on the assumption that the probability distribution for the constants is nearly flat, \( \mathcal{P}_\alpha \approx \text{const} \). The principle of mediocrity gives a very different perspective on what is natural and what is not. It predicts that the constants \( \{\alpha_j\} \) are likely to be such that the product

\[
\mathcal{P}_\alpha \propto w_\alpha V_\alpha \nu_{\text{civ}}(\alpha)
\]  

is maximized. The factors in this product have a strong (exponential) dependence on \( \{\alpha_j\} \), and the distribution \( \mathcal{P}_\alpha \) can be strongly peaked in some region of \( \alpha \)-space.

It should be emphasized that predictions of the principle of mediocrity are not guaranteed to be correct. After all, our civilization may be special in some respects. The predictions can be expected to have only statistical accuracy. That is, with a large number of predictions, only few of them are likely to be wrong.

4. Predictions for Finite Inflation

I will first assume that inflation has a finite duration. The more complicated case of eternal inflation will be discussed in Sec.5.

The inflaton potential

The volume factor \( V \) is given by \( V = V_0 Z^3 \), where \( V_0 \sim (GV_\alpha^{(\text{max})})^{-3/2} \) is the initial volume at nucleation and \( Z \) is the expansion factor during inflation. The maximum of \( Z \) is
achieved by maximizing the highest value of the potential $V_\alpha^{(max)}$, where inflation starts, and minimizing the slope of $V(\varphi)$: the field $\varphi$ takes longer to roll down for a flatter potential. [Note that the nucleation probability (5) is also maximized for the highest allowed value of $V_\alpha^{max}$].

The cosmological literature abounds with remarks on the ‘unnaturally’ flat potentials required by inflationary scenarios. With the principle of mediocrity the situation is reversed: flat is natural. Instead of asking why $V(\varphi)$ is so flat, one should now ask why it is not flatter.

Low-energy physics

The ‘human factor’ $\nu_{\text{civ}}(\alpha)$ may impose stringent constraints on the constants \{\$\alpha_j\}$. We do not know what other forms of intelligent life are possible, but the principle of mediocrity favors the hypothesis that our form is the most common in the metauniverse. The conditions required for life of our type to exist [the low-energy physics based on the symmetry group $SU(3) \times SU(2) \times U(1)$, the existence of stars and planets, supernova explosions] may then fix, by order of magnitude, the values of the fine structure constant, and of electron, nucleon, and W-boson masses, as discussed by Carter (1974), Carr & Rees (1979) and Barrow & Tipler (1986).

Origin of structure

Superflat potentials required by the principle of mediocrity typically give rise to density fluctuations which are many orders of magnitude below the strength needed for structure formation. This means that the observed structures must have been seeded by some other mechanism. An alternative mechanism is based on topological defects: strings, global monopoles, and textures, which could be formed at a symmetry breaking phase transition. (For a review of topological defects and their cosmological implications see, e.g., Vilenkin & Shellard 1994). The required symmetry breaking scale for the defects is $\eta \sim 10^{16} \text{ GeV}$. With ‘natural’ (in the traditional sense) values of the couplings, the transition temperature $T_c \sim \eta$ is much higher than the thermalization temperature, and no defects are formed after thermalization. It is possible for the phase transition to occur during inflation, but the resulting defects are inflated away, unless the transition is sufficiently close to the end of inflation. To
arrange this requires some fine-tuning of the constants. However, since the dependence of 
the volume factor $V$ on the duration of inflation is exponential, we expect that the gain in 
the volume will more than compensate for the decrease in ‘$\alpha$-space’ due to the fine-tuning.\footnote{We note also that in some supersymmetric models the critical temperature of superheavy string formation can ‘naturally’ be as low as $m_W$ (Lazarides et. al. 1986).} Moreover, it has been recently shown (Kofman et. al. 1995) that large fluctuations of the 
inflaton field $\varphi$ prior to thermalization can result in the formation of superheavy defects, 
even in models with a low thermalization temperature.

Another possibility is to use more complicated models of inflation, such as ‘hybrid’ infla-
tion (Linde 1994), which involve several scalar fields and can give reasonably large density 
fluctuations even when the potentials are very flat in some directions in the field space.

The symmetry breaking scale $\eta \sim 10^{16} \text{ GeV}$ for the defects is suggested by observations, 
but we have not explained why this particular scale has been selected. The value of $\eta$ 
determines the amplitude of density fluctuations, which in turn determines the time when 
galaxies form and the matter density in the galaxies. Since these parameters certainly affect 
the chances for civilizations to develop, it is quite possible that $\eta$ is significantly constrained 
by the anthropic factor $\nu_{\text{civ}}(\alpha)$. It would therefore be interesting to study how structure 
formation would proceed in a universe with a very different amplitude of density fluctuations 
(and/or a very different baryon density). Some steps in this direction have been made by 
Rees (1980, 1995).

The cosmological constant

An anthropic bound on the cosmological constant has been first discussed by Weinberg 
(1987). In a spatially flat universe with a positive vacuum energy density $\rho_v$, gravitational 
clustering effectively stops at a redshift

$$1 + z_v \sim (\rho_v/\rho_{m0})^{1/3},$$

when $\rho_v$ becomes comparable to the matter density $\rho_m$. At later times, the vacuum energy 
dominates, and the universe enters a deSitter stage of exponential expansion. A bound on 
$\rho_v$ can be obtained by requiring that it does not dominate before at least a single galaxy had
a chance to form. There is evidence for the existence of quasars and protogalaxies as early
as $z \sim 4$, and Weinberg argued that the anthropic principle, interpreted in this way, cannot
rule out vacuum energy domination at $z \gtrsim 4$. This corresponds to the bound
\[ \frac{\rho_v}{\rho_{m0}} \lesssim 100, \] (10)
which falls short of the observational upper bound (see, e.g., Turner 1995),
\[ \left( \frac{\rho_v}{\rho_{m0}} \right)_{\text{obs}} \lesssim 5, \] (11)
by a factor $\sim 10$.

On the other hand, the principle of mediocrity suggests that we look not for the value of
$\rho_v$ that makes galaxy formation barely possible, but for the value maximizing the number
of habitable stellar systems. As a rough measure of the latter, we can use the fraction of
baryonic matter, $f(\rho_v)$, that ends up in giant galaxies which contain most of the luminous
stars in the Universe. Dwarf galaxies with masses $M \ll 10^{11}M_\odot$ are vulnerable to losing much
of their gas through winds driven by supernovae from the first generation of star formation
(Dekel & Silk 1986; Babul & Rees 1992; Nath & Chiba 1995). (Note that planetary systems
and life do not form before the heavy elements produced in the first-generation stars are
dispersed in supernova explosions). In bottom-up structure formation scenarios, where giant
galaxies are formed mainly by aggregation of smaller components, the chances for life to
evolve may be greatly diminished if the vacuum energy dominates before the formation of
giant galaxies.\footnote{Observationally, there has been no strong evolution of giant galaxies since $z \sim 1$ (see, e.g., Ellis 1995). At the same time, there has been a marked decrease in the numbers of dwarf galaxies since $z \sim 0.5 - 1$. At least part of this decrease could be due to the absorption of dwarfs by larger galaxies, and thus it is conceivable that $f(\rho_v)$ grew, say, by a factor of a few at $z \gtrsim 1$. The width of the distribution \footnote{12} would then be comparable to the observational bound \footnote{11}.}

In top-down scenarios, such as hot dark matter with cosmic string seeds, first
structures can form at high redshifts, but the bulk of galaxy formation occurs at $z \sim 1 - 2$,
and thus the most probable values of $\rho_v$ are in the range $z_v \sim 1$ and $\rho_v/\rho_{m0} \lesssim 10$.

In both cases, the function $f(\rho_v)$ decreases from $f(0) \sim 1$ to negligibly small values at
$\rho_v/\rho_{m0} \gg 100$. At the same time, one expects the nucleation probability and the volume
factor $\mathcal{V}(\rho_v)$ to vary on a much greater scale (say, $\sim V_{\alpha}^{\text{max}}$), and therefore to be essentially constant in the range where $f(\rho_v)$ is substantially different from zero. Hence, the probability distribution for $\rho_v$, with all other constants fixed, is

$$d\mathcal{P} \propto f(\rho_v)d\rho_v,$$

where I have assumed that $\rho_v$ has a continuous spectrum and that it can vary independently of other constants.

A somewhat similar interpretation of the anthropic principle has been independently used by Efstathiou (1995) who defined the probability density for $\rho_v$ by estimating the number of giant ($L^*$) galaxies per photon, $N_g$, in a cold dark matter model with different values of $\rho_v$. An important difference between his approach and mine is that Efstathiou calculated $N_g(\rho_v)$ at the moment of time corresponding to a fixed (present) value of the Hubble parameter $H_0$, while my suggestion (Vilenkin 1995a,b) is to use the number of civilizations throughout the history of the universe, which would correspond to calculating the asymptotic value of $N_g(\rho_v)$ as $t \to \infty$. As noted by Rees (private communication), by fixing $H_0$ one eliminates all values of $\rho_v$ smaller than $3H_0^2/8\pi G$, and thus the analysis of $N_g$ at a fixed $H_0$ is not suitable for explaining the smallness of $\rho_v$.

For negative values of $\rho_v$, the scale factor behaves as $a(t) \propto [\sin(\pi t/t_c)]^{2/3}$, where $t_c = (\pi/6|\rho_v|)^{1/2}$, so that the universe recollapses on a timescale $t_c$. The width of the probability distribution for $\rho_v < 0$ is determined by requiring that the universe does not recollapse while stars are still shining and new civilizations are being formed. This gives a width comparable to (11).

### 5. Predictions for Eternal Inflation

I have assumed so far that inflation has a finite duration, so that the thermalized volume $\mathcal{V}$ and the number of civilizations $\mathcal{N}$ are both finite. This, however, is not generally the case. The evolution of the inflaton field $\varphi$ is influenced by quantum fluctuations, and as a

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5 A nucleation probability with an extremely sharp peak at $\rho_v = 0$ was obtained by Baum (1984), Hawking (1984), and Coleman (1988b). However, all these papers are based on Euclidean quantum gravity which has serious problems. For a discussion of the problems, see Fischler et al. (1989).
result thermalization does not occur simultaneously in different parts of the universe. In many models it can be shown that at any time there are parts of the universe that are still inflating (Vilenkin 1983; Linde 1986). Conclusions of Section 4 are directly applicable only if inflation is finite for all the allowed values of the constants.

In an eternally inflating universe, the thermalization volume \( V \) is infinite and has to be regulated. If one simply cuts it off by including only parts of the volume that thermalized prior to some moment of time \( t_c \), with the same value of \( t_c \) for all universes, then one finds that the results are extremely sensitive to the choice of the time coordinate \( t \). For example, cutoffs at a fixed proper time and at a fixed scale factor \( a \) give drastically different results (Linde et. al. 1994). An alternative procedure (Vilenkin 1995; Winitzki & Vilenkin 1995), which is free of this problem, is to introduce a cutoff at the time when all but a small fraction \( \epsilon \) of the initial (co-moving) volume of the universe has thermalized. The value of \( \epsilon \) is taken to be the same for all universes, but the corresponding cutoff times \( t_c \) are generally different. The limit \( \epsilon \to 0 \) is taken after calculating the probability distribution \( P_\alpha \). It can be shown that the resulting distribution is not sensitive to the choice of \( t \).

I will omit the calculation of the regularized volume \( V \) and even the rather lengthy expression for \( V \) obtained as a result of that calculation. The essence of the result can be expressed as

\[
V \propto \epsilon^{-d/(d-3)} Z^3. \tag{13}
\]

Here, \( Z \) is the expansion factor during the slow-roll phase of inflation, when quantum fluctuations are small, and \( d < 3 \) has the meaning of the fractal dimension of the inflating region.

In the limit \( \epsilon \to 0 \), non-vanishing probabilities are obtained only for \( \{\alpha_j\} \) corresponding to the largest value of \( d \),

\[
d(\alpha) = \max. \tag{14}
\]

The fractal dimension \( d \) increases as the potential \( V(\varphi) \) becomes flatter, and thus the condition (14) tends to select maximally flat potentials.

It is possible that the condition (14) selects a unique set of \( \{\alpha_j\} \). Then all constants of Nature can, at least in principle, be predicted with 100% certainty. On the other hand, if
the maximum of $d$ is strongly degenerate, then Eq.(14) selects a large subset of all $\{\alpha_j\}$. All values of $\alpha$ not in this subset have a vanishing probability, and the probability distribution within the subset is proportional to $w_\alpha Z_\alpha^3 \nu_{\text{civ}}(\alpha)$ [see Eq.(8)]. The probability maximum is then determined by the same considerations as in the case of finite inflation.

This situation will arise, for example, if the potential $V(\varphi)$ has a large number of minima, parametrized by some subset of the constants $\{\beta_j\} \subset \{\alpha_j\}$. Thermalization will then occur to different minima in different parts of the universe. The regularized volume in this case is still given by Eq.(13), but now $d$ has the same value everywhere and is independent of $\{\beta_j\}$ (Linde et. al. 1994). From Eq.(13), $V_\beta \propto Z_{\beta}^3$, and the corresponding probabilities are

$$P_\beta \propto Z_{\beta}^3 \nu_{\text{civ}}(\beta).$$

(15)

The probability distribution (15) has the same dependence on the slow-roll expansion factor $Z$ and on the anthropic factor $\nu_{\text{civ}}$ as we found in the case of finite inflation. The predictions for $\{\beta_j\}$ are, therefore, essentially the same (see Section 4).

Another possibility is that the ‘constants’ of low-energy physics are affected by some very weakly coupled fields. These could be the moduli fields of superstring theories, one example being the dilaton which determines the value of the Newton’s constant $G$. Such fields should be included in $\{\beta_j\}$ as continuous variables.6

6. Conclusions

When quantum mechanics is applied to the entire universe, we are inevitably lead to the concept of an ensemble of universes, with a probability distribution for different initial conditions and for different constants of Nature. The principle of mediocrity asserts that we are typical inhabitants of this ensemble. The values of the constants suggested by this principle are the ones that give a very flat inflaton potential, a non-negligible cosmological constant, and density fluctuations seeded either by topological defects or by quantum fluctuations in models like ‘hybrid’ inflation. We can expect to find these features in our own universe, provided that they are consistent with the spectrum of the allowed constants $\{\alpha_j\}$.

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6The probability distribution for a Brans-Dicke field (which is similar to the dilaton) was discussed, using a different approach, by García-Bellido et. al. (1994, 1995).
The spectrum of \( \{\alpha_j\} \) will hopefully be determined from the fundamental particle theory, and until then no reliable predictions can be made for the values of the constants. However, this preliminary analysis suggests that the predicted values may be very different from the choices considered ‘natural’ by particle physicists.

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