Evidence for Spatially Correlated Gaia Parallax Errors in the Kepler Field

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Abstract

We present evidence for a spatially dependent systematic error in the first data release of Gaia parallaxes based on comparisons to asteroseismic parallaxes in the Kepler field and provide a parameterized model of the angular dependence of these systematics. We report an error of 0.059±0.004 mas on scales of 0.3°, which decreases for larger scales to become 0.011±0.006 mas at 8°. This is consistent with the ∼2% zero-point offset for the whole sample discussed by Huber et al. and is compatible with the effect predicted by the Gaia team. Our results are robust to dust prescriptions and choices in temperature scales used to calculate asteroseismic parallaxes. We also do not find evidence for significant differences in the signal when using red clump versus red giant stars. Our approach allows us to quantify and map the correlations in an astrophysically interesting field, resulting in a parameterized model of the spatial systematics that can be used to construct a covariance matrix for any work that relies on TGAS parallaxes.

Key words: asteroseismology – catalogs – parallaxes – stars: distances

1. Introduction

The Gaia mission is expected to provide positions, parallaxes, and proper motions for a billion objects, with precisions of ~20 μas for stars down to 15th magnitude (Gaia Collaboration et al. 2016b). Though the final data release is scheduled for 2022, positions, parallaxes, and proper motions for 2 million stars common to Tycho-2 (Høg et al. 2000) and Gaia have been released as part of Data Release 1 (DR1) (Gaia Collaboration et al. 2016a). By using positions from Tycho-2 as priors on the astrometric solution, Michalik et al. (2015) demonstrated that sub-milliarcsecond accuracy could be achieved, resulting in the Tycho-Gaia Astrometric Solution (TGAS). Though the statistical errors may even be smaller than the 0.3 mas reported in DR1 (see Gould et al. 2016), systematic errors are expected to exist at the level of up to 0.3 mas (Lindegren et al. 2016). Various instrumental and modeling effects that may account for the systematic errors are explored in Lindegren et al. (2016), including the known chromaticity of the CCDs, inadequate temporal resolution of the satellite attitude model, and so-called “micro-clanks” due to mechanical jitter.

In this work, we compute asteroseismic parallaxes for more than 1000 red giants in the 10° × 10° Kepler field of view for comparison to TGAS parallaxes. Thanks to the order-of-magnitude better precision of asteroseismic parallaxes for red giants and the high stellar density of the Kepler field, we are able to investigate the presence of correlated errors in TGAS parallaxes on degree and subdegree scales in an effort to quantify the expected systematic spatial errors in TGAS parallax.

To date, other comparisons of the asteroseismic parallax scale to the TGAS parallax scale have considered global offsets—intra., nonspatially dependent differences. De Ridder et al. (2016), for instance, found good agreement between the two scales for a sample of 22 dwarfs and subdwarfs, but significant differences among 938 red giants. Huber et al. (2017) suggest that a global offset is partially mitigated when using a hotter temperature scale such as the infrared flux method (IRFM) temperature scale, and that radii inferred from TGAS parallaxes are consistent with asteroseismic radii to within 5% between 0.8 and 8 R⊙.

Compared to other parallax scales, the TGAS parallax scale exhibits a fractional offset. Davies et al. (2017), for example, attributed the red giant asteroseismic parallax offset from De Ridder et al. (2016) to errors in the TGAS parallax scale by comparing TGAS parallaxes to red clump (RC) distances. Their suggested correction agrees for ε ≥ 1.6 with that of Stassun & Torres (2016b), who compared TGAS parallaxes to parallaxes from eclipsing binaries. Huber et al. (2017) found that these offsets are too large and can be partially attributed to a too cool temperature scale, based on a larger sample of stars spanning from the main sequence to the red giant branch (RGB). At larger distances, Sasar et al. (2017) found no evidence for a global offset when comparing to RR Lyrae parallaxes at a median parallax of 0.8 mas, and neither did Casertano et al. (2017) when looking at Cepheid parallaxes. And at the smallest distances, Jao et al. (2016) found evidence for a correction consistent with that of Stassun & Torres (2016b) (amounting to ≈0.2 mas in the sense that TGAS overestimates distances) when compared to trigonometric parallaxes for 612 dwarfs at distances of less than 100 pc.

To date, two studies independent of the Gaia team have mentioned possible spatial dependencies in TGAS parallax scales. Casertano et al. (2017) found mild evidence for spatially correlated TGAS parallaxes at the level of 19 ± 34 μas on scales less than 10° using Cepheids, while Jao et al. (2016) reported a north–south ecliptic hemisphere difference in trigonometric and TGAS parallaxes. Despite the thorough investigation of the quantitative and qualitative existence of

http://www.cosmos.esa.int/web/Gaia/release
such systematic errors in Lindegren et al. (2016), the precise characterization of spatially dependent systematics in terms of a functional form and/or a characteristic scale at which the 0.3 mas systematic error applies was not released for DR1. Our exercise, then, is to identify and to characterize any spatial correlation of parallax errors in Gaia DR1.

The structure of this paper is as follows: we describe the provenance of and basic calibrations of the observables used to compute asteroseismic parallax in Section 2. In Section 3, we detail how we compute asteroseismic parallaxes, treatment of statistical errors therein, and how we test for the presence of spatially correlated offsets between asteroseismic TGAS parallaxes. We summarize our main findings in Section 4, discuss potential caveats to those findings in Section 5, and conclude in Section 6.

2. Data

Quantifying any systematic errors in Gaia parallaxes requires an independent and unbiased set of parallaxes to compare to the TGAS values. The Gaia team validated their parallaxes against Hipparcos parallaxes, which revealed the presence of systematic offsets (Lindegren et al. 2016). We attempt to present a complementary treatment of potential errors in the TGAS parallax scale for two main reasons. First, the Hipparcos parallaxes themselves have spatially correlated errors (see references in Section 5), which limits their usefulness when used as a validation set. More critically, a detailed model of the spatial correlations of TGAS parallax errors has not yet been published, which would be crucial to proper treatment of errors in work using TGAS between now and 2018 April, when the next Gaia data release is scheduled.

Our validation set consists of parallaxes of red giants in the Kepler field of view that have spectroscopic metallicities and asteroseismic data, which permits us to infer effective temperature, radii, and, by extension, luminosities. Adding reddening and bolometric flux information then yields distances and hence parallaxes. The resulting asteroseismic parallaxes have statistical errors an order of magnitude smaller than those in TGAS and hence permit a strong test of spatially correlated offsets in TGAS versus asteroseismic parallax scales.

Our sample consists of over 1000 red giants spread across the ~100 sq. deg. Kepler field of view, which means that we can probe systematic parallax offsets on scales less than a degree. This is the scale where Lindegren et al. (2016) indicate that systematic errors in the TGAS parallaxes are expected to be the largest.

The basis for our sample are TGAS stars that are also listed as asteroseismic giants in the APOGEE-Keppler Asteroseismic Science Consortium catalog (APOKASC; Pinsonneault et al. 2014), which combines infrared spectroscopic data from Data Release 13 of the Apache Point Observatory Galactic Evolution Experiment (APOGEE; Zasowski et al. 2013; Majewski et al. 2015) with asteroseismic data from the Kepler mission (Borucki et al. 2010). We now discuss the provenance of spectroscopic metallicities, asteroseismic parameters, reddening, and photometry in turn.

APOGEE temperatures, $T_{\text{eff,APOGEE}}$, and metallicities, [Fe/H], are taken from the Thirteenth Data Release of the Sloan Digital Sky Survey (SDSS DR13; SDSS Collaboration et al. 2016) and are corrected according to the metallicity-dependent term recommended in the DR13 documentation.8

Global asteroseismic parameters $\nu_{\text{max}}$ and $\Delta \nu$—which may be mapped onto stellar radii—were adopted from the SYD pipeline (Huber et al. 2009) values in version 3.6.5 of the APOKASC catalog (M. H. Pinsonneault et al. 2017, in preparation).

Because there is evidence that asteroseismic radii have evolutionary-state-dependent systematics (e.g., Miglio et al. 2012), we divide the TGAS-APOKASC giant sample into RGB and RC subsamples, to assess any differences in TGAS-asteroseismic parallax offsets as a function of evolutionary state. Evolutionary-state information is compiled from the asteroseismic classifications of Stello et al. (2013) or Mosser et al. (2014) (Y. Elsworth et al. 2017, in preparation).

Extinction corrections (described in Section 3.1) are made using the three-dimensional dust map of Green et al. (2015), as implemented in mwdust (Bovy et al. 2016). Shown here is $A_V$ in the region of the Kepler field of view, in Galactic coordinates. Choosing to include or not the higher-extinction region $\ell \lesssim 73^\circ$ does not eliminate spatially correlated offsets between asteroseismic and TGAS parallaxes.

Figure 1. Three-dimensional dust map from Green et al. (2015), as implemented in mwdust (Bovy et al. 2016). Shown here is $A_V$ in the region of the Kepler field of view, in Galactic coordinates. Choosing to include or not the higher-extinction region $\ell \lesssim 73^\circ$ does not eliminate spatially correlated offsets between asteroseismic and TGAS parallaxes.

Extinction corrections (described in Section 3.1) are made using the three-dimensional dust map of Green et al. (2015), as implemented in mwdust (Bovy et al. 2016). The $A_V$ extinction for the Kepler field of view is shown in Figure 1.

We opt to calculate an effective temperature and bolometric flux using the IRFM, as implemented in González Hernández & Bonifacio (2009), which was used to set the APOGEE effective temperature scale. For this purpose, we use near-infrared photometry in the $J$, $H$, and $K_s$ bands from the Two Micron All Sky Survey (2MASS; Skrutskie et al. 2006). Visual photometry is also required, which we derive from SDSS $g$ and $r$ photometry. We choose to convert these magnitudes to Johnson $B$ and $V$ according to Lupton (2005) rather than use Tycho $B$ and $V$. In doing so, the resulting visual photometry has less scatter than Tycho $B$ and $V$. Furthermore, the $griz$ photometry from the Kepler Input Catalogue (KIC; Brown et al. 2011), as recalibrated to be on the SDSS scale by Pinsonneault et al. (2012), is consistent with the 2MASS infrared photometry temperature scale for cool stars (see Pinsonneault et al. 2012).

The requirement that our sample of stars have $grJHK_s$, $\nu_{\text{max}}$, $\Delta \nu$, $T_{\text{eff,APOGEE}}$, [Fe/H], RGB, or RC evolutionary-state classifications and Gaia DR1 parallaxes ($\varpi_{\text{TGAS}}$) yields a base sample of 1592 giants.

8 http://www.sdss.org/dr13/lirspec/parameters/
9 https://github.com/jobovy/mwdust
10 https://www.sdss3.org/dr10/algorithms/sdssUBVRITransform.php
2.1. Quality Cuts

We omit stars known to be members of NGC 6791 and NGC 6819, as giants residing in these clusters could bias measurements of spatially correlated quantities.

Comparisons by Gould et al. (2016) of TGAS parallaxes to RR Lyrae parallaxes indicated that DR1 TGAS parallax statistical errors are inflated by \( \sim 30\% \). Because the calculation of a spatially correlated TGAS-asteroseismic parallax offset will be more robust with a proper treatment of the statistical errors, we modify those for the TGAS parallaxes according to their prescription. Reducing statistical errors in this way does not introduce a spatially correlated, systematic offset of the sort we present in this work.

Finally, TGAS parallaxes are required to have a signal-to-noise ratio greater than 1.6 (see Section 3.3).

The above quality cuts yield a total of 1392 giants, which compose the final TGAS-APOKASC sample used in the following.

3. Methods

3.1. Asteroseismic Parallax

Estimating errors in TGAS parallaxes requires an independent distance measure. Apart from the moving group or parallax methods, distance estimates of stars will require an estimate of stellar luminosity and its bolometric flux. For our purposes, we use asteroseismology to determine stellar luminosity and the IRFM to determine a bolometric flux, which are combined to yield a parallax/distance. As the following overview will show, asteroseismology effectively provides a radius, which, in combination with an effective temperature of the star, will determine its luminosity via the Stefan–Boltzmann equation; combined with the bolometric flux of the star, one can determine its distance.

In this work, we estimate stellar radius by way of two complementary scaling relations involving two different asteroseismic observables: \( \nu_{\text{max}} \) (roughly the frequency at which the largest-amplitude acoustic modes occur) and \( \Delta \nu \), the separation between acoustic modes of the same spherical harmonic degree, \( \ell \), but differing by one radial order number, \( n \).

It is well established (see, e.g., Tassoul 1980; Christensen-Dalsgaard 1993) that \( \Delta \nu \) is related to the mean density of a star via a scaling relation, assuming homologous behavior between the Sun and a given star, of the form

\[
\frac{\Delta \nu}{\Delta \nu_0} \approx \left[ \frac{M/M_\odot}{(R/R_\odot)} \right].
\]

Similarly, the frequency of maximum acoustic power, \( \nu_{\text{max}} \), has been found to scale as the acoustic cutoff frequency (Brown et al. 1991; Kjeldsen & Bedding 1995; Chaplin et al. 2008), i.e., as

\[
\frac{\nu_{\text{max}}}{\nu_{\text{max},0}} \approx \left[ \frac{M/M_\odot}{(R/R_\odot)} \right]^2 \left( \frac{T_{\text{eff}}}{T_{\text{eff},0}} \right).
\]

We can combine these two relations to yield an estimate of the radius, \( R \), of the star:

\[
(R/R_\odot) \approx \left( \frac{\nu_{\text{max}}}{\nu_{\text{max},0}} \right) \left( \frac{\Delta \nu}{\Delta \nu_0} \right)^{-1} \left( \frac{T_{\text{eff}}}{T_{\text{eff},0}} \right)^{1/2}.
\]

With a temperature and the radius, we can compute a luminosity and thus a luminosity distance/parallax, provided that we know the bolometric flux, \( F_{\text{bol}} \):

\[
\varpi_{\text{astero}} = \frac{\sqrt{F_{\text{bol}}}}{R \sqrt{\sigma_{\text{bol}} T_{\text{eff}}}},
\]

where \( \sigma_{\text{bol}} \) is the Stefan–Boltzmann constant. We adopt solar values consistent with those of Huber et al. (2009): \( \nu_{\text{max},0} = 3090 \mu \text{Hz} \), \( \Delta \nu_0 = 135.1 \mu \text{Hz} \), \( T_{\text{eff},0} = 5777 \text{ K} \), and \( \log g_0 = 4.438 \) (Mamajek et al. 2015).

With a radius from asteroseismology, we turn to the bolometric flux and effective temperature, which we infer from the IRFM using \( BVJHK_s \) photometry, according to González Hernández & Bonifacio (2009). Calculating an effective temperature using the IRFM allows us to self-consistently estimate the reddening (and hence extinction) to each star, which is necessary to achieve a correct distance measure. The basic approach is to simultaneously fit a star’s spectral energy distribution from the optical to the infrared, taking advantage of the insensitivity of infrared stellar emission on effective temperature. First, the observed infrared flux is compared to the infrared flux for a model atmosphere, yielding an angular diameter. Next, a bolometric flux is computed by combining other photometric information (e.g., optical) with infrared photometry, based on an assumed stellar atmosphere model. Finally, a temperature is determined by using the previously computed bolometric flux and angular diameter.

The bolometric flux and temperature results converge iteratively. Since the IRFM requires stellar atmosphere lookups as a function of \([\text{Fe}/\text{H}], \log g, \) and \( T_{\text{eff}} \), we implement the IRFM using guesses for these quantities from APOGEE. For the whole process, we assume a fixed metallicity from APOGEE, \([\text{Fe}/\text{H}]\). Our initial guess for \( T_{\text{eff}} \) is \( T_{\text{eff, APOGEE}} \); our initial guess for \( \log g \) is calculated from Equation (2) using \( T_{\text{eff, APOGEE}} \). An IRFM temperature and bolometric flux are then computed iteratively, as described in González Hernández & Bonifacio (2009). The resulting IRFM temperature, \( T_{\text{eff, IRFM}} \), is used to compute a new \( \log g \), and the bolometric flux is used via Equation (4) to compute an asteroseismic distance/parallax, \( \varpi_{\text{astero}} \). An extinction for each band is then computed using the three-dimensional dust map of Green et al. (2015) using \( mvwdrust \) (Bovy et al. 2016), with which we correct the \( JHKL_s \) photometry, yielding dust-extincted \( J_0, H_0, K_s, B_0 \) and \( V_0 \) are computed by transforming corrected \( g \) and \( r \) magnitudes. This corrected photometry is then used in subsequent iterations to compute the bolometric flux and temperature, and the process is repeated until convergence in the asteroseismic parallax. We compute uncertainties on the derived quantities \( B_0, V_0, J_0, H_0, K_s, g, \) and \( r \), \( J, H, K_s, \nu_{\text{max}}, \Delta \nu, T_{\text{eff, APOGEE}}, \) and \([\text{Fe}/\text{H}]\) based on their statistical errors, and imposing a minimum uncertainty of 0.08 mag for \( AV \) to account for variations in \( RV \) within the Kepler field and for line-of-sight variations below the resolution of the Green et al. (2015) dust map (\( \sim 0.05 \)). Resulting IRFM temperatures are shown in Figure 2. When compared to APOGEE spectroscopic temperatures, the IRFM temperatures are on average \( \sim 70 \text{ K} \) hotter. As we have found in Huber et al. (2017), the IRFM temperature scale results in a smaller global offset between asteroseismic and TGAS parallaxes than when using, e.g., spectroscopic temperatures from APOGEE.
For purposes of illustration, Hertzsprung–Russell diagrams are constructed in Figure 3 from asteroseismic and TGAS parallaxes in combination with de-extincted \( V \)-band magnitudes, \( V_0 \). We employ an exponentially decreasing space density prior with a scale length of 1.35 kpc (Bailer-Jones 2015; Astraatmadja & Bailer-Jones 2016) for the conversion of parallax to distance. The RC is particularly sharp using asteroseismic parallaxes compared to the spread of RC luminosities assuming TGAS parallaxes. Figure 4 demonstrates the relative precision of asteroseismic and TGAS parallaxes as a two-dimensional histogram of statistical parallax error versus parallax for the TGAS-APOKASC giant sample; the median uncertainty for asteroseismic parallaxes is 0.03 mas, and the median uncertainty for TGAS parallaxes is 0.3 mas. Clearly, the error budget in the comparisons between the scales is dominated by TGAS parallax uncertainties.

### 3.2. Extinction

Extinction values are computed as a result of the iterative procedure described in Section 3.1, which we have compared to extinctions from the KIC (Brown et al. 2011). Previous studies suggest that the KIC extinctions are overestimated (Rodrigues et al. 2014; Zasowski et al. 2015), and we also find that our extinction values are smaller than those in the KIC. Figure 5 shows that the offset between our derived extinctions and KIC extinctions is comparable to the offset when checking against Bayesian extinction estimates of APOKASC giants in Rodrigues et al. (2014), who found \( A_V = (0.721 \pm 0.015)A_{V,KIC} - (0.139 \pm 0.007) \). This relation is plotted as a black dashed line on top of our extinctions. Our extinctions based on Green et al. (2015) dust maps also compare well to the extinctions derived from grid-based modeling in Huber et al. (2017). mwdust offers several dust maps, and our result does not significantly change if using the Green et al. (2015) map or a combination of individual maps from Marshall et al. (2006), Green et al. (2015), and Drimmel et al. (2003), as synthesized by Bovy et al. (2016).

### 3.3. Final TGAS-APOKASC Sample

In Figure 6, we show a direct star-by-star comparison of the two parallax scales. It is evident that at smaller \( \sigma \), there is a systematic offset between the two scales. This offset is

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**Figure 2.** APOGEE and IRFM temperature scales show a systematic offset that is temperature dependent. We find that \( T_{\text{eff, IRFM}} \) results in parallaxes more consistent with those from TGAS, and we use this temperature scale instead of \( T_{\text{eff, APOGEE}} \) throughout the paper. Gray dashed lines show one-to-one relations. Median errors on both quantities are shown by the error bar in the top panel. The bottom panel shows a binned median of the temperature difference (gray curve), with gray error bars representing the standard deviation of the difference within each bin and red error bars representing the statistical error on the median within each bin. The IRFM temperature is hotter than APOGEE spectroscopic temperatures by \( 65 \pm 13 \) K, on average.

**Figure 3.** Absolute magnitude in these Hertzsprung–Russell diagrams for the TGAS-ASPOKASC sample computed via the IRFM (see text) and either an asteroseismic parallax (left) or a TGAS parallax (right). Median error bars are shown in black. RC stars are shown in red and RGB stars in blue.

**Figure 4.** Two-dimensional histogram of statistical errors on parallax vs. parallax for asteroseismic parallaxes derived in this work (left) and for TGAS parallaxes (right). The former are an order of magnitude more precise than the latter, which makes the TGAS-APOKASC sample in this work a powerful calibrator for investigating systematic errors in TGAS parallaxes.
expected from comparing a precise asteroseismic parallax sample to a much less precise sample of TGAS parallaxes: the large fractional errors on TGAS parallaxes will tend to scatter to low parallax. We can mitigate the offset by applying a signal-to-noise cut such that the median of the difference between the two parallaxes is zero, to within the error on the median. We show the original distribution and the distribution of the parallax difference after a signal-to-noise cut of $S/N > 1.6$ in Figure 7. A potential zero-point offset in asteroseismic and TGAS parallax scales is discussed in Section 5.5 (see also Huber et al. 2017). We use the high signal-to-noise sample for the rest of the analysis (TGAS-APOKASC sample), which numbers 1392.

4. Results

4.1. Spatially Correlated Offsets in Asteroseismic and TGAS Parallaxes

Figure 8 shows our main result, which is a measure of the spatial correlation of the difference in the TGAS and asteroseismic parallax scales for all 1392 giants in the TGAS-APOKASC sample. Below, we first discuss the choice of a binned Pearson correlation coefficient as a metric for the spatially correlated parallax difference and how it is calculated, and then we present model fits to the observed signal.

4.2. Quantification Using a Binned Pearson Correlation Coefficient

Our measure of the parallax offset spatial correlation is a binned Pearson correlation coefficient,

$$
\xi(\theta) = \frac{\sum_{i,j} (x_{\text{TGAS},i} - \bar{x}_{\text{astero},i})(x_{\text{TGAS},j} - \bar{x}_{\text{astero},j})}{\sqrt{(\sum_{i,j} (x_{\text{TGAS},i} - \bar{x}_{\text{astero},i})^2)(\sum_{i,j} (x_{\text{TGAS},j} - \bar{x}_{\text{astero},j})^2)}}
$$

where $i$ and $j$ denote stars that are separated by an angle, $\theta'$, such that $\theta - \Delta\theta/2 < \theta' < \theta + \Delta\theta/2$ for a given angular bin size $\Delta\theta$. Equation (5) describes a Pearson correlation coefficient computed in bins of angular separation. A value of $-1$ indicates that $x_{\text{TGAS}}$ and $x_{\text{astero}}$ are perfectly anticorrelated at that separation; a value of $+1$ indicates that they are perfectly correlated; a value of 0 indicates that they are not correlated. In the absence of spatially correlated offsets between the parallax scales, then, we would expect a null signal of zero at all angular separations. For offsets that increase at small separations, there would be a rise in $\xi(\theta)$ with decreasing $\theta$. A zero-point offset would result in a flat, positive $\xi(\theta)$ for all $\theta$.

We compute error bars in the binned Pearson correlation coefficient via bootstrapping (Loh 2008). Briefly, the sample of objects was divided into $N$ spatial regions, which were then sampled with replacement (meaning that the same region could be used multiple times in a single bootstrap sample) $N$ times to create a bootstrap sample; we divided the sample into spatial regions according to their Kepler magnitude. For $N$ such samples, the $100(1-\alpha)$% confidence intervals on each point in the binned Pearson correlation coefficient were computed according to the bootstrap confidence interval (Davison & Hinkley 1997):

$$
2\hat{K} - K_{(B+1)(1-\alpha/2)} > 2\hat{K} - K_{(B+1)\alpha/2},
$$

where $K_A$ is the $A$th-ranked statistic (the binned Pearson correlation coefficient, $K = \xi$, in this case) computed from a bootstrap sample and $\hat{K}$ is the statistic computed using all the data.

In order to better characterize the errors on the statistic, we create mock TGAS stellar catalogs, whose positions are the same as those in the data, but whose parallaxes are drawn from the asteroseismic parallaxes, and injected with spatial correlations according to Equation (7), assuming Gaussian statistics, with best-fitting values from Table 1 (see Section 4.3). These fake TGAS parallaxes are then used to compute the binned Pearson correlation coefficient according to Equation (5), and the resulting distribution of values at each angular bin is used to compute a 68% confidence interval—a “systematic” error—for the statistic. Note that our mock catalog generation assumes Gaussianity in the distribution of TGAS-asteroseismic parallax difference, which we think is reasonable given the evident Gaussianity of the distribution shown in Figure 7. This “systematic” error is shown as a green band in Figure 8.

We also provide alternate representations of the TGAS-asteroseismic parallax offset in Figures 17 and 18. Both alternate representations indicate a spatial dependence in the offset, in agreement with the binned Pearson correlation coefficient. See the Appendix for details on how these alternate measures are computed.
For visualization purposes, a spatial map of the TGAS-asteroseismic parallax offset is plotted in Figure 9(a). Each point represents a star in the TGAS-APOKASC sample and is colored by \((\varpi_{\text{TGAS}} - \varpi_{\text{astero}})/\sigma\), where \(\sigma\) is the quadrature sum of the statistical errors from \(\varpi_{\text{TGAS}}\) and \(\varpi_{\text{astero}}\). A smoothed version of the data is calculated by convolving these values by a Gaussian with 0\(^\circ\)2 standard deviation. For comparison, a map of parallax offset with no spatial correlation is simulated in Figure 9(b). Whereas there is visible structure shown in the observed TGAS-asteroseismic parallax differences of Figure 9(a), there are no such correlated hot or cold spots in Figure 9(b). For comparison, a map injected with spatial correlations according to Equation (7),

\[
\xi(\theta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{d^2 \varpi}{\varpi^2} \frac{\varpi}{\sigma^2} e^{-\varpi^2/2\sigma^2} \cos(\theta) e^{-\theta^2/2\sigma^2} d\varpi
d\varphi
\]

is plotted in Figure 9(b). Error bars include a bootstrap and “systematic” error; the black line indicates the best-fitting model of the form in Equation (7); the green band indicates the 68\% confidence interval of the recovered binned Pearson correlation coefficient, as computed from a mock catalog of TGAS parallaxes assuming asteroseismic parallaxes as the true value and spatial correlations according to the black line. Refer to Section 4.2 for details.
Table 1
Fits to Models of the Spatially Correlated TGAS-asteroseismic Parallax Offset

| Sample | Model       | $\rho_{\text{max}}$ | $T_{1/2}$ | $C$   | $D$   | $E$   | $\chi^2$/dof | $\sigma_{\phi}(\theta = 0)\,\text{mas}$ | $\sigma_{\phi}(\theta = 0.5)\,\text{mas}$ | $\sigma_{\phi}(\theta = 1)\,\text{mas}$ | $\sigma_{\phi}(\theta = 8)\,\text{mas}$ |
|--------|-------------|----------------------|-----------|-------|-------|-------|---------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| ALL    | Equation (6)$^*$ | 0.059$^{+0.045}_{-0.049}$ | 0.110$^{+0.040}_{-0.035}$ | 0.003$^{+0.005}_{-0.003}$ | $\cdots$ | $\cdots$ | 7.287 | 0.050$^{+0.010}_{-0.012}$ mas | 0.076$^{+0.025}_{-0.035}$ mas | 0.030$^{+0.011}_{-0.011}$ mas | 0.018$^{+0.011}_{-0.011}$ mas |
| ALL    | Equation (7)$^{++}$ | 0.006$^{+0.002}_{-0.002}$ | $-0.025^{+0.035}_{-0.035}$ | 0.009$^{+0.001}_{-0.002}$ | $-0.007^{+0.001}_{-0.001}$ | 0.0010$^{+0.0003}_{-0.0003}$ | 4.850 | $\cdots$ | $\cdots$ | 0.059$^{+0.004}_{-0.004}$ mas | 0.024$^{+0.004}_{-0.004}$ mas | $-0.011^{+0.004}_{-0.004}$ mas |
| RC     | Equation (6)$^*$ | 0.084$^{+0.019}_{-0.019}$ | 4.970$^{+2.250}_{-2.172}$ | $-0.0310^{+0.0172}_{-0.0172}$ | $\cdots$ | $\cdots$ | 0.976 | 0.031$^{+0.006}_{-0.005}$ mas | 0.071$^{+0.006}_{-0.006}$ mas | 0.068$^{+0.007}_{-0.007}$ mas | 0.062$^{+0.006}_{-0.006}$ mas |
| RC     | Equation (7)$^*$ | 0.045$^{+0.004}_{-0.004}$ | $-0.025^{+0.035}_{-0.035}$ | $-0.014^{+0.006}_{-0.006}$ | $-0.005^{+0.004}_{-0.004}$ | 0.0028$^{+0.0011}_{-0.0011}$ | 1.215 | $\cdots$ | $\cdots$ | 0.058$^{+0.004}_{-0.004}$ mas | 0.065$^{+0.005}_{-0.005}$ mas | $-0.025^{+0.005}_{-0.005}$ mas |
| RGB    | Equation (6)$^*$ | 0.104$^{+0.027}_{-0.027}$ | 7.590$^{+1.780}_{-1.740}$ | $-0.0367^{+0.0206}_{-0.0206}$ | $\cdots$ | $\cdots$ | 0.230 | 0.037$^{+0.008}_{-0.009}$ mas | 0.078$^{+0.009}_{-0.009}$ mas | 0.078$^{+0.008}_{-0.008}$ mas | 0.073$^{+0.009}_{-0.010}$ mas |
| RGB    | Equation (7)$^*$ | 0.077$^{+0.018}_{-0.018}$ | $-0.038^{+0.036}_{-0.036}$ | $-0.005^{+0.004}_{-0.004}$ | $-0.009^{+0.004}_{-0.004}$ | 0.0027$^{+0.0014}_{-0.0014}$ | 0.044 | $\cdots$ | $\cdots$ | 0.091$^{+0.013}_{-0.013}$ mas | 0.082$^{+0.010}_{-0.011}$ mas | 0.043$^{+0.011}_{-0.011}$ mas |

Note. Best-fitting parameters for Equations (6) and (7) for the spatially correlated TGAS-asteroseismic parallax offset, with 68% confidence interval errors. A positive (negative) value in the last five columns indicates that the systematic offset is a positive correlation (an anticorrelation). An asterisk indicates preference of the model over a null signal based on the AIC criterion (see text). A plus sign indicates that the model of Equation (7) is preferred over the model of Equation (6) according to the AIC criterion.
assuming Gaussian statistics, with best-fitting values from Table 1, is shown in Figure 9(c). The spatial correlation model of Figure 9(c) qualitatively reproduces the patchwork structure seen in the data (Figure 9(a)).

4.3. Fitting Models to the Observed Spatially Correlated Offset

We fit an analytic form to the observed spatial correlation in order to determine a characteristic scale at which the systematic error is important. We consider two models—one with exponential spatial scale dependence, as might be expected from the Gaia scanning strategy, in which characteristic spatial scales could be strongly imprinted in the data. The first model we fit to the binned Pearson correlation coefficient is a purely exponential model of the form

$$\xi(\theta) = \rho_{\text{max}} \exp[-\theta/\theta_{1/2} \ln 2] + C.$$ (6)

Note that in the above expression $\rho_{\text{max}}$ is an overall amplitude to the spatially varying component of $\xi$, and that $\theta_{1/2}$ represents the angular scale at which spatial correlations are half of what they are at the smallest scales.

We also fit a polynomial of the form

$$\xi(\theta) = A + B \log \theta + C (\log \theta)^2 + D (\log \theta)^3 + E (\log \theta)^4.$$ (7)

We computed best-fitting parameters and their associated uncertainties by fitting with the Python MCMC routine of

Figure 9. Distribution of TGAS and asteroseismic parallax differences, in the sense of TGAS-asteroseismic, normalized by the statistical error on the difference, $\sigma \equiv \sqrt{\sigma_{\text{TGAS}}^2 + \sigma_{\text{astero}}^2}$, as a function of position on the sky. The smoothed field is calculated by convolving the observed data with a Gaussian filter of standard deviation 0.2°. Panel (a) shows the signal in the TGAS-APOKASC sample; panel (b) shows a model with no spatial correlation; panel (c) shows the best-fitting model of panel (a), with random spatial phase (see Section 4.3).
emcee (Foreman-Mackey et al. 2013) with a covariance matrix calculated from the bootstrap sample, whose diagonal is added in quadrature with the “systematic” error, described above.

In Table 1, we provide the resulting best-fitting parameters for the spatially correlated parallax offset models. We also provide best-fitting parameters for the RGB and RC subsamples separately (see Section 5.2). According to the Akaike Information Criterion (Akaike 1973), AIC = 2k − ln L, where k is the number of degrees of freedom and ln L = −0.5χ^2 is the log-likelihood, Equation (6) is preferred in the fits to the binned Pearson correlation coefficient for RGBs and RCs, but Equation (7) is preferred for the combined TGAS-APOKASC sample. We take preference of model 1 over model 2 to be AIC_{model1} − AIC_{model2} < −2. We refer to the best-fitting model as Equation (7) and recommend the fits to this polynomial model for characterizing covariance matrices. For completeness, we have also compared preference for both models to a null model of zero at all angular scales, finding that the null model is never preferred.

Systematic offsets at a given angular scale, θ, σ_{sys}(θ), are also reported in Table 1. They are calculated according to

\[ σ_{sys}(θ) = \sqrt{\langle ξ(θ) \rangle = \frac{1}{σ^2}}, \tag{8} \]

where σ = \sqrt{σ_{\text{MCMC}}^2 + σ_{\text{bootstrap}}^2}. Confidence intervals on σ_{sys} are computed using a covariance matrix built from the MCMC chains from model fitting (see above), according to which a representative distribution of model parameters is drawn, and for which a resulting distribution of possible σ_{sys} is computed. Note that the sense of this systematic offset between the two parallax scales is not indicated, as it will vary as a function of absolute position on the sky. One can see, for instance, the regions where the sign change of the offset switches in Figure 9. Rather, signs on this systematic offset provided in Table 1 indicate correlation (positive) versus anticorrelation (negative).

Our best-fitting polynomial model using the entire TGAS-APOKASC sample yields parallax offsets at the smallest separations of 0.059±0.004 mas and 0.011±0.006 mas at spatial scales of θ ≈ 0°.3 and θ ≈ 8°, respectively.

5. Discussion

5.1. “Systematic” Error in Pearson Statistic

As discussed in Section 4, we estimate “systematic” errors in the binned Pearson correlation coefficient by creating mock TGAS parallax catalogs. We do this because we expect bootstrap errors to underestimate the true error in the Pearson statistic for at least three reasons: (1) bootstrap sampling cannot account for the finite spatial extent of the Kepler field of view, which will affect the Pearson correlation coefficient in the largest angular separation bins (those comparable to the length of the side of the Kepler field of view); (2) by drawing distributions of parallaxes with correlated error patterns of a random spatial phase (where spatial phase determines the locations on the sky of the hot and cold spots in Figure 9), one marginalizes over the phase of the spatial correlation in a way that cannot be done with the single DR1 TGAS parallax catalog; (3) bootstrap sampling of the Pearson correlation coefficient does not take into account statistical errors in the parallaxes like creating sets of mock TGAS parallax catalogs does.

By adding the “systematic” and bootstrap errors in quadrature, we have likely overestimated the errors on the binned Pearson correlation coefficient, and so the significance of our result is conservative.

5.2. Correlation as a Function of Evolutionary Type

As noted in Section 2, there is evidence that RGBs and RCs obey different asteroseismic scaling relations, which could lead to systematic differences in their parallaxes, and perhaps a difference in spatially correlated offsets from TGAS parallaxes. We present Figures 10 and 11 in order to investigate whether the observed spatial correlations in parallax offset vary with stellar type. We find the two samples to yield consistent signals at all scales, with the RGB sample exhibiting a mildly larger amplitude. Moreover, when averaged over the entire Kepler field, there do not seem to be significant differences in the TGAS and asteroseismic parallax scales as a function of evolutionary type (see Section 5.5 and Huber et al. (2017)). We note, furthermore, that this observation also suggests that intrinsic spatial correlations of extinction are not contributing significantly to the signal, because in that case we would expect RC
parallaxes to be more spatially correlated than those of RGBs, since they have larger distances on average than RGBs. However, the RC sample shows mildly smaller, not larger, correlation coefficients than the RGB sample. We discuss asteroseismic parallax scale systematics further in Section 5.4.

5.3. Bias in Observables

As we note in Section 5.2, spatial correlations should not arise from global biases in the asteroseismic parallaxes. For completeness, however, we perform several checks on the reliability of the quantities that are used to compute asteroseismic parallaxes.

First, we confirmed that our re-reddened $V$ agree with those from APASS (Henden & Munari 2014) to within statistical errors, as shown in Figure 12.

We also tested the effect of using the APOGEE spectroscopic temperature versus an IRFM temperature in computing asteroeseismic parallaxes. The temperature scale does not remove the observed spatial correlation (see Section 5.4). Nevertheless, there is a different zero-point offset when using the IRFM temperature scale, which originates from the IRFM being systematically hotter than the APOGEE spectroscopic temperature (see Figure 2).

5.4. Spatial Correlation of Observables

It is possible that the observed spatial dependence of the quantity $\varpi_{\text{TGAS}} - \varpi_{\text{astero}}$ could be the result of spatial correlations in the observables that enter into the calculation of $\varpi_{\text{astero}}$. We plot all the observable quantities that are used to compute $\varpi_{\text{astero}}$, which are shown in Figures 13 and 14. We have computed a binned Pearson correlation coefficient for these quantities via a modified version of Equation (5):

$$
\xi(\theta) = \frac{\sum_{i,j} (X_i - \langle X \rangle)(X_j - \langle X \rangle)}{\sqrt{(\langle X_i - \langle X \rangle^2 \rangle)(\langle X_j - \langle X \rangle^2 \rangle)}}. \quad (9)
$$

where $\langle X \rangle$ is the average value of an observable quantity, $X$, for the entire TGAS-APOKASC sample. The above quantities, as a function of angular separation, how correlated an observable quantity, $X$, is. Table 2 shows that all observable quantities except $A_V$ have negligible spatially correlated systematic errors, $\sigma_{\text{sys}}$, based on fits to the binned Pearson correlation coefficient, which are mapped to systematic error according to Equation (8).

Unsurprisingly, there are non-negligible spatial correlations in our derived extinctions. In addition to intrinsic spatial clustering of dust in projection, the dust map in Figure 1 shows a region of enhanced extinction in the Kepler field, which is spatially concentrated. Crucially, we do not find significantly different results when we perform our analysis only on the region in which the extinction is the highest ($\xi \lesssim 0.3$).

Even a systematic spatial correlation at small angular scales (less than $0.1$) at the level of $0.07$ mag in $A_V$ would at most translate as a $0.035$ mas offset in parallax scales. However, we infer a systematic offset between TGAS and asteroseismic parallaxes of $0.127^{+0.010}_{-0.001}$ mas near $0.1$ for the best-fitting model for the entire TGAS-APOKASC sample. In other words, the correlations in dust cannot account for the correlation we see in TGAS parallaxes.

We performed additional tests to confirm that our result is not due to spatial correlations of extinction propagating into our asteroseismic parallaxes. Figure 15 shows the binned Pearson correlation coefficient for de-extincted $K_s$, $K_{s,0}$. We note that there is statistically no spatial correlation in $K_{s,0}$, as there is in $A_V$, which is due to the negligible dust extinction in $K_s$ ($A_K \approx 0.1A_V$). We therefore tested a single-band bolometric correction with $K_s$—instead of using $BVHK_s$ per our fiducial IRFM method described in Section 3—and found that it does not remove the spatially correlated parallax offset. We also confirm that using the reddenings derived from stellar models adopted in Huber et al. (2017) instead of those from a dust map does not change our results.

5.5. TGAS-asteroseismic Parallax Zero-point

Stassun & Torres (2016b) find that, when compared to 99 eclipsing binaries from Stassun & Torres (2016a) (which have parallaxes with uncertainties of $\sim 200$ mas), Gaia DR1 parallaxes are smaller by $\sim 200$ mas. Casertano et al. (2017), however, found no systematic offset above $1$ mas when comparing with 202 Galactic Cepheids with photometric parallaxes. We find the absolute zero-point correction suggested by Stassun & Torres

11 Here and for other parallax offsets quoted in mas in the paper, we assume the best-fitting model of Equation (7) fitted to the entire sample of red giants ("ALL"; this is the model plotted in Figure 8—see also Table 1) for the Pearson correlation coefficient, $\xi(\theta)$. The correlation coefficient is translated into an absolute offset in mas according to Equation (8).
to overcorrect the TGAS parallaxes by about $0.20 \pm 0.05$ mas (see Figure 7). Though this discrepancy in zero-points between TGAS and our parallax sample and that of Stassun & Torres (2016b) is likely because the zero-point offset should be fractional and not absolute, we take the chance here to discuss potential zero-point systematics in asteroseismic parallax.

One possible bias in asteroseismic parallaxes are systematics in the scaling relations of Equations (1)–(3). The most evident assumption in using asteroseismic scaling relations to determine stellar radii is the assumption of homology in stellar structure relative to the Sun. Indeed, a growing body of literature indicates that the $\Delta\nu$ scaling relation can deviate by a couple percent when applied to derive RGB mean densities, compared to asteroseismic stellar models (e.g., White et al. 2011; Miglio et al. 2012; Guggenberger et al. 2016; Sharma et al. 2016). Among dwarfs, comparisons of asteroseismic radii

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**Figure 13.** Same as Figure 8, but for the observables on which asteroseismic parallax depends. Note the difference in scale for the $A_V$ panel.
to radii from *Hipparcos* parallax and bolometric flux (Silva Aguirre et al. 2012) and to radii from interferometry (Huber et al. 2012) show agreement to within 5%. RGB radii comparisons show mixed results (see Huber et al. 2012; Frandsen et al. 2013; Gaulme et al. 2014; Brogaard et al. 2016; Gaulme et al. 2016).

We investigate the validity of scaling relations in Huber et al. (2017) by comparing to *Gaia* parallaxes by averaging over the entire *Kepler* field of view. The spatially correlated offsets in asteroseismic and TGAS parallaxes at the level indicated in this work do not affect the result that asteroseismic radii are consistent with *Gaia* radii at the 5% level. We also find evidence for Δν corrections proposed by Sharma et al. (2016) improving global agreement between the two radius scales.

In light of evidence for red giant Δν scaling relation corrections, we conservatively apply the $T_{\text{eff}}$- and [Fe/H]-dependent corrections that Sharma et al. (2016) propose to our APOKASC Δν values. Such corrections do not significantly affect log $g$ calculations (Hekker et al. 2013), and the significance of our result is not affected by the choice of whether or not to apply Δν corrections.

Even if the asteroseismic scale is biased with respect to the TGAS parallax scale (which would manifest as a zero-point offset), the spatial correlation that we retrieve is still valid. Any required corrections to the asteroseismic scaling relations (Equations (1)–(3)) will only be spatially dependent insofar as the observable quantities that enter into them (i.e., $g$, $r$, $J$, $H$, $K_s$, $\nu_{\text{max}}$, Δν, [Fe/H], $T_{\text{eff}}$) are significantly spatially correlated. We demonstrate in Section 5.4 that no significant spatial correlations exist in these quantities, except in $A_V$. Importantly, we have confirmed that the result is insensitive to extinction corrections, which are, in fact, spatially correlated on the angular scales investigated in this paper.

### 5.6. Gaia Systematics

If the observed difference in TGAS and asteroseismic parallax scales is not due to spatial correlations of asteroseismic parallaxes themselves, then we interpret them as spatially correlated errors in TGAS parallaxes. There are a few reasons to come to this conclusion. As discussed in Lindegren et al. (2016), for instance, the attitude model of the astrometric solution does not have a high enough temporal resolution to remove small timescale attitude changes. As a result, the *Gaia* team expects that spatial correlations on scales of a few degrees and less are a result of unmodeled, correlated attitude changes on timescales of minutes, which translate into spatial correlations of degrees and less.

Though we do not have access to the astrometric solution model to independently demonstrate that observed TGAS-asteroseismic parallax offsets are a result of systematic errors in
### Table 2
Spatial Correlation in Observables

| Observable | Model | $\rho_{\text{max}}$ | $\theta_{1/2}$ | C | $\chi^2$/dof | $\sigma_{\text{sys}}(\theta = \theta_{1/2})$ | $\sigma_{\text{sys}}(\theta = 0^\circ)$ | $\sigma_{\text{sys}}(\theta = 0^\circ:3)$ | $\sigma_{\text{sys}}(\theta = 1^\circ:0)$ | $\sigma_{\text{sys}}(\theta = 8^\circ:0)$ |
|------------|-------|---------------------|----------------|---|-------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $v_{\text{max}}$ | Equation (6)* | $-0.004^{+0.010}_{-0.010}$ | $0.085^{+2.628}_{-2.628}$ | $0.0011^{+0.0011}_{-0.0011}$ | 4.160 | $-0.025^{+0.081}_{-0.049}$ $\mu$Hz | $-0.046^{+0.128}_{-0.057}$ $\mu$Hz | $-0.050^{+0.131}_{-0.053}$ $\mu$Hz | $-0.039^{+0.115}_{-0.050}$ $\mu$Hz | $-0.022^{+0.072}_{-0.042}$ $\mu$Hz |
| $\Delta \nu$ | Equation (6)* | $-0.0036^{+0.0027}_{-0.0025}$ | $5.872^{+1.297}_{-1.297}$ | $0.0009^{+0.0011}_{-0.0011}$ | 4.6685 | $-0.0008^{+0.0002}_{-0.0002}$ $\mu$Hz | $-0.0015^{+0.0001}_{-0.0001}$ $\mu$Hz | $-0.0014^{+0.0001}_{-0.0001}$ $\mu$Hz | $-0.0013^{+0.0001}_{-0.0001}$ $\mu$Hz | $-0.0007^{+0.0003}_{-0.0001}$ $\mu$Hz |
| $T_{\text{eff, BFM}}$ | Equation (6)* | $0.007^{+0.021}_{-0.001}$ | $0.84^{+3.170}_{-3.170}$ | $-0.0013^{+0.0013}_{-0.0013}$ | 1.445 | $1.65^{+5.83}_{-3.86}$ $K$ | $2.35^{+8.62}_{-5.86}$ $K$ | $2.34^{+9.65}_{-5.86}$ $K$ | $1.78^{+4.82}_{-3.45}$ $K$ | $-0.81^{+15.74}_{-1.74}$ $K$ |
| $A_V$ | Equation (6)* | $0.92^{+0.057}_{-0.057}$ | $3.80^{+0.516}_{-0.516}$ | $-0.38^{+0.056}_{-0.056}$ | 0.920 | $0.022^{+0.001}_{-0.001}$ mag | $0.05^{+0.001}_{-0.001}$ mag | $0.05^{+0.001}_{-0.001}$ mag | $0.05^{+0.001}_{-0.001}$ mag | $0.05^{+0.001}_{-0.001}$ mag |
| $\left[\text{Fe}/\text{H}\right]$ | Equation (6)* | $0.025^{+0.005}_{-0.005}$ | $8.14^{+1.581}_{-1.581}$ | $-0.015^{+0.002}_{-0.002}$ | 1.529 | $-0.001^{+0.0002}_{-0.0002}$ | $0.002^{+0.0002}_{-0.0002}$ | $0.002^{+0.0002}_{-0.0002}$ | $0.002^{+0.0002}_{-0.0002}$ | $0.001^{+0.0003}_{-0.0003}$ |
| $J$ | Equation (6)* | $0.002^{+0.001}_{-0.001}$ | $2.90^{+3.412}_{-3.412}$ | $-0.0040^{+0.0012}_{-0.0012}$ | 0.029 | $-0.0003^{+0.0002}_{-0.0002}$ mag | $0.0007^{+0.0001}_{-0.0001}$ mag | $0.0006^{+0.0002}_{-0.0002}$ mag | $0.0005^{+0.0002}_{-0.0002}$ mag | $0.0004^{+0.0002}_{-0.0002}$ mag |
| $K$ | Equation (6)* | $0.008^{+0.001}_{-0.001}$ | $0.63^{+0.311}_{-0.311}$ | $-0.0009^{+0.0014}_{-0.0014}$ | 2.299 | $0.0015^{+0.0014}_{-0.0014}$ mag | $0.0017^{+0.0014}_{-0.0014}$ mag | $0.0015^{+0.0014}_{-0.0014}$ mag | $0.0014^{+0.0014}_{-0.0014}$ mag | $0.0013^{+0.0014}_{-0.0014}$ mag |
| $g$ | Equation (6)* | $0.008^{+0.001}_{-0.001}$ | $2.48^{+0.351}_{-0.351}$ | $-0.0013^{+0.0020}_{-0.0020}$ | 15.329 | $0.0007^{+0.0002}_{-0.0002}$ mag | $0.0015^{+0.0002}_{-0.0002}$ mag | $0.0014^{+0.0002}_{-0.0002}$ mag | $0.0011^{+0.0002}_{-0.0002}$ mag | $0.0002^{+0.0003}_{-0.0003}$ mag |
| $r$ | Equation (6)* | $0.009^{+0.001}_{-0.001}$ | $4.40^{+0.327}_{-0.327}$ | $0.0029^{+0.0019}_{-0.0019}$ | 4.857 | $-0.0003^{+0.0002}_{-0.0002}$ mag | $-0.0007^{+0.0002}_{-0.0002}$ mag | $-0.0007^{+0.0002}_{-0.0002}$ mag | $-0.0006^{+0.0002}_{-0.0002}$ mag | $0.0002^{+0.0001}_{-0.0001}$ mag |

**Note.** Best-fitting parameters for Equation (6) for spatial correlations in observables, with 68% confidence interval errors. A positive (negative) value in the last five columns indicates that the systematic offset is a positive correlation (an anticorrelation). An asterisk indicates preference of the model over a null signal based on the AIC criterion (see the text).
the astrometric solution, we can make inferences based on the published DR1 data. In particular, we show in Figure 16 that the difference in TGAS and asteroseismic parallax scales correlates significantly with the fraction of “bad” across-scan direction observations to total across-scan direction observations. Taking this metric as a proxy for the uncertainty in the across-scan measurement, the correlation corroborates a note in Lindegren et al. (2016), indicating that the across-scan direction measurement error changes parallax solutions in a systematic way, for unknown reasons.

6. Conclusion

We have independently validated and have quantified spatially correlated errors in TGAS parallaxes, as predicted by the Gaia team. Our result complements warnings in the Gaia DR1 documentation that there exist systematic uncertainties of amplitude comparable to the statistical uncertainties. For convenience and comparison to future work, we have provided a characteristic scale and amplitude for the spatial correlations: an error of 0.059 ± 0.004 mas on scales of 0.3°, which decreases for larger scales to become 0.011 ± 0.004 mas at 8°. A covariance matrix for the correlated errors in parallax may be computed via Equation (8), using either of the models fit to the observed spatial correlation signal, \( \xi(\theta) \), which are provided in Table 1. For any pair of stars, \( i \) and \( j \), separated by angular distance, \( \theta \), their respective entry in a covariance matrix, \( \sigma_{ij}^2 \), would read \( \sigma_{ij}^2 = \xi(\theta) \sigma_i \sigma_j \), where \( \sigma_i \) are the statistical errors on each star’s TGAS parallax.

We have done several checks on our result, which is robust to the following:

1. the dust prescription that is used—without significant differences in the observed spatial correlation in parallax error when omitting the region of the Kepler field of view most affected by dust or when using a stellar model extinction instead of a dust map extinction;
2. the evolutionary status of the stars used to calculate asteroseismic parallaxes, with both first-ascent RGB and RC parallaxes indicating the same spatially correlated parallax offset with respect to TGAS parallaxes;

3. whether or not a \( BVJK \) bolometric correction is used to compute asteroseismic parallax or a \( K \)-band bolometric correction is used;
4. the temperature scale used—whether it be the spectroscopic \( T_{\text{eff,APOGEE}} \) scale or the IRFM scale, \( T_{\text{eff,IRFM}} \);
5. whether or not \( \Delta \nu \) corrections are applied to the asteroseismic scaling relations.

At this point, we cannot test the possibility of correlations on scales larger than 10°, due to the \( \sim 10^5 \times 10^5 \) spatial extent of the Kepler field of view. Future work could quantify spatial correlations on larger scales using, e.g., K2 asteroseismology, which would yield parallaxes for objects separated by the largest angular scales. We encourage the use of the spatial covariance functional form when computing quantities that depend on TGAS parallaxes, especially in light of the delay of Gaia DR2 to 2018 April.

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Appendix

Alternate Formulations of Spatial Correlation

Here we present representations of spatially correlated parallax offsets that are complementary to the Pearson correlation coefficient presented in the text. We prefer the Pearson correlation coefficient formulation because it is directly mappable to a covariance function, but we include the following representations for completeness.

A.1. Angular Correlation Function

The angular correlation function, often used in cosmological contexts, may be calculated using the Landy–Szalay estimator (Landy & Szalay 1993):

$$\xi(\theta) = \frac{\langle DD \rangle - 2 \langle DR \rangle + \langle RR \rangle}{\langle RR \rangle},$$

where $D$ in our case refers to an observed value of the quantity $(\varpi_{\text{TGAS}} - \varpi_{\text{astero}})/\sqrt{\sigma^2_{\text{TGAS}} + \sigma^2_{\text{astero}}}$ for a star, and $R$ refers to a sample drawn from the observed values of the normalized parallax difference, but with positions drawn randomly from within the Kepler field of view, making use of $\text{K2fov}^{12}$ (Mullally et al. 2016); $\langle \rangle$ represents the expected value of that quantity for pairs of points separated by angular distance $\theta$. We compute this statistic with $\text{TreeCorr}^{13}$ (Jarvis et al. 2004). Error bars for each angular bin are assigned based on Poisson statistics. This statistic is widely used in cosmology to compute correlation functions. Although complementary, the angular correlation coefficient will in general not be equivalent to the Pearson correlation coefficient. However, it does explicitly account for stochasticity in the spatial distribution of the TGAS-APOKASC sample. Results using this approach are qualitatively similar, as seen in comparing Figures 8 and 17.

A.2. Binned Absolute Difference

Most intuitive is the simple measure

$$\langle \delta \rangle(\theta) = \sqrt{\langle (\varpi_{\text{TGAS},i} - \varpi_{\text{astero},i})(\varpi_{\text{TGAS},j} - \varpi_{\text{astero},j}) \rangle/\theta},$$

which is a measure of the absolute difference in the parallax scales computed by binning pairs of stars, $i$ and $j$, separated by an angular distance, $\theta$. This scale will not necessarily be the

\[\text{https://github.com/mertonmyb/K2fov}\]

\[\text{https://github.com/mjvarek/TreeCorr}\]
same as the scale we present in the text, and in particular it is insensitive to the sign of the (anti)correlation. Figure 18 shows this measure for the TGAS-APOKASC sample.

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