BPS equations in $\mathcal{N} = 2$, $D = 5$ supergravity with hypermultiplets

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Abstract

With the general aim to classify BPS solutions in $\mathcal{N} = 2$, $D = 5$ supergravities interacting with an arbitrary number of vector, tensor and hypermultiplets, here we begin considering the most general electrostatic, spherical-symmetric BPS solutions in the presence of hypermultiplet couplings. We discuss the properties of the BPS equations and the restrictions imposed by their integrability conditions.

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1 Introduction

In recent years five-dimensional \( N = 2 \) gauged supergravity theories have received considerable attention primarily for their relevance to the AdS/CFT correspondence [1]. In particular much interest has been directed toward the study of domain-wall supergravity solutions [2], [3] as duals of renormalization group (RG) flows in the corresponding field theory [4]. Also a strong motivation in this direction derives from phenomenological requests in brane-world scenarios obtained via M-theory compactifications and/or domain-wall type models [5].

Finding supersymmetric solutions of \( N = 2, D = 5 \) supergravities is never an easy exercise [6]; it becomes a quite difficult task if one considers general couplings to matter and general gaugings. Partial results have been obtained so far, i.e. for cases where only special vector or hypermultiplet gaugings have been considered [7], [8]. Here we start a systematic program with the general aim to classify BPS solutions with vector, tensor and hypermultiplet couplings. The introduction of the hypermultiplets is crucial for widening the variety of solutions as compared to the case where only vector multiplets are present [9], [10]. In particular, aside for the special example analyzed in [8], the existence of BPS black-hole solutions has not been investigated systematically. Of course, black-hole solutions would be especially relevant since, via the AdS/CFT correspondence, they could describe the RG flows between field theories in different dimensions [11].

In this paper we restrict ourselves to the case of hypermultiplet couplings, with generic gauging, and a static \( SO(4) \) symmetric ansatz for the metric. In this setting we study the integrability conditions that follow from the BPS equations and find a set of equations for the functions in the ansatz. The quaternionic geometries give equations for the scalars which are a generalization of the ones found in [9]. Then we analyze all these equations and check directly that they satisfy the equations of motion.

Our paper is organized as follows: in the next section we introduce the model and the basic ingredients. For the construction of the most general \( N=2 \) gauged supergravities and for technical details we refer the reader to the existing literature [12] and to references therein. We describe the form of the solutions we are looking for: obviously this choice determines the physics contained in the solution. Then we focus on the derivation of the BPS equations and study their integrability conditions. In section 3 we find the set of independent first order differential equations that are equivalent to the BPS conditions. In section 4 we show that the family of solutions we have found satisfy the equations of motion. In section 5 we discuss the properties of the BPS solutions in the special case of the universal hypermultiplet [13] and find an explicit result for a simple choice of the gauging. We conclude with some final remarks. Our notations and conventions are summarized in an Appendix.

2 The model and its BPS equations

We consider \( N=2 \) gauged supergravities in five dimensions interacting with an arbitrary number of hypermultiplets. (In this paper we do not study vector and tensor multiplet
coupings.) The field content of the theory is the following

- the supergravity multiplet

\[ \{ e^a_\mu, \psi^i_\mu, A_\mu \} \] (2.1)

containing the graviton \( e^a_\mu \), two gravitini \( \psi^i_\mu \) and the graviphoton \( A_\mu \), which is the only (abelian) gauge field present in the theory;

- \( n_H \) hypermultiplets

\[ \{ \zeta^A, q^X \} \] (2.2)

containing the hyperini \( \zeta^A \) with \( A = 1, 2, \ldots, 2n_H \), and the scalars \( q^X \) with \( x = 1, 2, \ldots, 4n_H \) which define a quaternionic Kähler manifold\(^1\) with metric \( g_{XY} \).

The bosonic sector of the theory is described by the Lagrangian density presented in \([12]\)

\[ \mathcal{L}_{\text{BOS}} = -\frac{1}{2} e \left[ R + \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + g_{XY} D_\mu q^X D^\mu q^Y \right] + \frac{1}{6\sqrt{6}} \epsilon^{\mu\nu\rho\sigma\tau} F_{\mu\nu} F_{\rho\sigma} A_\tau - e\mathcal{V}(q) \] (2.3)

with

\[ D_\mu q^X = \partial_\mu q^X + gA_\mu K^X(q) \]

where \( K^X(q) \) is a Killing vector on the quaternionic manifold and \( \mathcal{V}(q) \) is the scalar potential as given in Appendix.

We look for electrostatic spherical solutions that preserve half of the \( \mathcal{N} = 2 \) supersymmetries. To this end we make the following ansatz for the supergravity fields: we choose a metric which is \( SO(4) \) symmetric with all the other fields that only depend on the holographic space-time coordinate \( r \). Moreover we fix the gauge for the graviphoton keeping only the \( A_t \) component different from zero.

Introducing spherical coordinates \((t, r, \theta, \phi, \psi)\) we write \([8]\)

\[ ds^2 = -e^{2v} dt^2 + e^{2w} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\psi^2) \] (2.4)

where the functions \( v \) and \( w \) depend on \( r \) only. The variations of the fermionic fields under supersymmetry transformations give rise to the following BPS equations: for the gravitini we have \([14]\)

\[ 0 = \delta_\epsilon \psi_{\mu i} = \partial_\mu \epsilon_i + \frac{1}{4} \omega_{ab}^{\mu} \gamma_{ab} \epsilon_i - \partial_\mu q^X P^i_{Xj} \epsilon_j - gA_\mu P^i_{j} \epsilon_j + \frac{i}{4\sqrt{6}} (\gamma_{\mu\rho} - 4g_{\mu\nu}\gamma_\rho) F^{\mu\rho} \epsilon_i - \frac{i}{\sqrt{6}} g P^i_{j} \gamma_\mu \epsilon_j \] (2.5)

\(^1\)We collect the basic geometrical notions in the Appendix.
We note that in ref. [8] the corresponding equation contains an additional term. This extra term arises due to a incorrect interpretation of the covariant derivative acting on the spinor $\epsilon$ as given in [12], should not be present.

For the hyperini the equations $\delta_i \zeta^A = 0$ lead to

\begin{equation}
\left[ \frac{i}{2} e^{-w} f_X^i q^X \gamma_1 - i \frac{g}{2} f_X^i K X e^{-v} A_t \gamma_0 \right] \epsilon_i = \frac{\sqrt{6}}{4} g K X f_X^i \epsilon_i \tag{2.6}
\end{equation}

where we have set $q^X = \partial_r q^X$

Without loss of generality it is convenient to parametrize the graviphoton as follows

\begin{equation}
A_t = \sqrt{\frac{3}{2}} a(r) e^v \tag{2.7}
\end{equation}

This allows to write explicitly the BPS equations for the gravitini

\begin{equation}
\left\{ \partial_t \delta^k_i + \frac{1}{2} v^r e^{-w} \delta^k_i \gamma_0 \gamma_1 + \frac{1}{6} g e^v P^r (\sigma_r)_i^k \gamma_0 + \frac{i}{2} e^{-v-w} (v' a + a') \delta^k_i \gamma_1 \right. \\
\left. - i \sqrt{\frac{3}{2}} g a e^v P^r (\sigma_r)_i^k \right\} \epsilon_k = 0 \tag{2.8}
\end{equation}

\begin{equation}
\left[ \partial_r \delta^j_i - i q^X p^r_X (\sigma_r)_i^j + \frac{1}{2} (v' a + a') (i \gamma_0) \delta^j_i \right. + \frac{g}{\sqrt{6}} e^w P^r (\sigma_r)_i^j \left. \right] \epsilon_j = 0 \tag{2.9}
\end{equation}

\begin{equation}
\left[ \partial_r \gamma_1 \gamma_2 \delta^j_i + i r e^{-w} \gamma_0 \gamma_1 \gamma_2 \delta^j_i + \frac{g r}{\sqrt{6}} P^s (\sigma_r)_i^j \gamma_2 \right] \epsilon_j = 0 \tag{2.10}
\end{equation}

\begin{equation}
\left\{ \delta^j_i \left[ \partial_0 - \frac{1}{2} e^{-w} \sin \theta \gamma_1 \gamma_3 - \frac{1}{2} \cos \theta \gamma_2 \gamma_3 + \frac{g r}{\sqrt{6}} P^s (\sigma_r)_i^j \gamma_3 \right] \right. \\
\left. + \frac{g r \sin \theta}{\sqrt{6}} \right\} \epsilon_j = 0 \tag{2.11}
\end{equation}

\begin{equation}
\left\{ \delta^j_i \left[ \partial_0 - \frac{1}{2} e^{-w} \cos \theta \gamma_1 \gamma_4 + \frac{1}{2} \sin \theta \gamma_2 \gamma_4 + \frac{g r e^{-w}}{4} (v' a + a') \cos \theta \gamma_0 \gamma_1 \right] \\
\right. + \frac{g r \cos \theta}{\sqrt{6}} \right\} \epsilon_j = 0 \tag{2.12}
\end{equation}

At this point, using (A.6) and the $SU(2)$ projection as in (A.8), we can rewrite the algebraic relations in (2.6) as

\begin{equation}
\left( i e^{-w} q^X \gamma_1 - i g \sqrt{\frac{3}{2}} a K X \gamma_0 - \sqrt{\frac{3}{2}} g K X \right) \left( g Z X \delta^j_i + 2 i R^s Z X (\sigma_r)_i^j \right) \epsilon_i = 0 \tag{2.13}
\end{equation}

We will make use of the above expression in the following.

\[\text{Note that in } [3] \text{ a was chosen to be a constant.} \]
2.1 Integrability conditions

Now we want to discuss the integrability of the gravitini equations in order to insure the existence of a Killing spinor (i.e. of residual supersymmetry). The standard procedure is to impose the vanishing of the various commutators. In this way one obtains equations that combined with the hyperini ones determine the unknown functions in the ansatz and impose restrictions on the geometry (gauging). We find it useful to adopt the following notation: given the vector \( P^s \ s = 1, 2, 3 \) we introduce the phase \( \vec{Q} \) so that \( \vec{Q} \cdot \vec{Q} = 1 \) and use the decompositions of the vector into its norm and phase

\[
P = \sqrt{\frac{3}{2}} W \vec{Q}.
\]  

We find four independent\(^3\) integrability conditions that we list below:

from the commutators between the BPS equations (2.10), (2.11) and (2.12) we obtain

\[
\left\{ i \gamma_0 \left[ 1 - e^{-2w} - \frac{r^2 e^{-2w}}{4} (v'a + a')^2 + g^2 r^2 W^2 \right] \delta^j_i + \left[ r e^{-2w} (v'a + a') \right] \delta^j_i + \gamma_1 \left[ g e^{-w} r^2 (v'a + a') W Q^s (\sigma_s)_i^j \right] \delta^j_i \right\} \epsilon_j = 0
\]  

(2.15)

The commutators between \( \text{[2.8]} \) and the angular components give the conditions

\[
\left\{ \gamma_0 [v'e^{-w} - g^2 e^w r W^2] \delta^j_i - i g r (v'a + a') W Q^s (\sigma_s)_i^j \gamma_1 - i (v'a + a') e^{-w} \delta^j_i \right\} \epsilon_j = 0
\]  

(2.16)

The commutators between \( \text{[2.9]} \) and the angular components give

\[
\left\{ -\frac{i}{2} g (v'a + a') r W Q^s (\sigma_s)_i^j \gamma_0 + \left[ \frac{1}{2} - w' e^{-w} - \frac{g^2}{2} e^w r W^2 \right] \gamma_1 \delta^j_i + \frac{g}{2} r q^X D_X (W Q^s) (\sigma_s)_i^j + \frac{i}{4} \partial_r \left[ r e^{-w} (v'a + a') \right] \gamma_0 \gamma_1 \delta^j_i \right\} \epsilon_j = 0
\]  

(2.17)

Finally the commutator between \( \text{[2.8]} \) and \( \text{[2.9]} \) gives

\[
\left\{ i g q^X D_X \left( \sqrt{\frac{3}{2}} e^w a W Q^s \right) (\sigma_s)_i^j - \gamma_0 \frac{g}{2} e^w q^X D_X (W Q^s) (\sigma_s)_i^j + \gamma_1 \left[ \frac{v'}{2} e^{-w} (v'a + a') \delta^j_i - \frac{i}{2} \partial_r \left( e^{-w} (v'a + a') \right) \delta^j_i \right] + \gamma_0 \gamma_1 \delta^j_i \left[ \frac{1}{2} g^2 e^{v+w} W^2 + e^{-w} \frac{1}{2} (v'a + a')^2 - \frac{1}{2} \partial_r (v'e^{-w}) \right] \right\} \epsilon_j = 0
\]  

(2.18)

\(^3\)Symmetry arguments show that the angular equations lead to only one independent condition.
We begin with the study of the equation in (2.15). Unless all coefficients vanish it can be written as

\[
(i f^0 \gamma_0 \delta^k + f^r (\sigma_r)^k \gamma_1) \epsilon_k = \epsilon_i \tag{2.19}
\]

where

\[
f^r = -ge^{w_r}rW^r \tag{2.20}
\]

\[
f^0 = -\frac{1 - e^{-2w} - \frac{4}{r^2} (w' a + a')^2 + g^2 e^{-2w} W^2}{r e^{-2w} (w' a + a')} \tag{2.21}
\]

The Killing equation in (2.19), viewed as a projector equation, leads to the consistency requirement

\[
(f^0)^2 + \sum_r (f^r)^2 = 1 \tag{2.22}
\]

Now we compare the above results with the content from the hyper ini equation in (2.13). Starting from (2.13), multiplying by the projector \( K\tilde{Z} \delta^i_j - 2i R^s_{\tilde{Z}X} K^Y (\sigma_s)^j_i \) and symmetrizing in \( Z, \tilde{Z} \), we obtain

\[
\left[ (KZK\tilde{Z} + 4K^X K^Y R^r_{XZ} R^s_{ZY} \delta_{rs}) \left[ a(\gamma_0 \delta^j_i - i\delta^j_i) - \left( \sqrt{\frac{2}{3}} g q^X g(X Z \tilde{Z}) \delta^j_i \right. \right. \right. \\
+2 \sqrt{\frac{2}{3}} e^{-w} q^X R^r_{|Z|X} K\tilde{Z}) (\sigma_r)^j_i - 2 \sqrt{\frac{2}{3}} e^{-w} q^X g(X R^s_{\tilde{Z}Y} K^Y (\sigma_r)^j_i \\
+4 \sqrt{\frac{2}{3}} e^{-w} q^X R^r_{(Z)[Y]X} K^Y \left( \delta_{rs} \delta^j_i + i\epsilon_{rst} (\sigma_r)^j_i \right) \gamma_1 \right] \epsilon_j = 0 \tag{2.23}
\]

The above equation is compatible with (2.19) only if the conditions

\[
q^X (g_X(Z \tilde{Z}) - 4R^s_{(Z)X} R^s_{\tilde{Z}Y} K^Y \delta_{rs}) = 0 \tag{2.24}
\]

are satisfied. If this is the case then (2.23) becomes

\[
\left[ -ia(\gamma_0 \delta^j_i - 2 \sqrt{\frac{2}{3}} e^{-w} q^X \frac{U^r_{XZ\tilde{Z}}}{g(KZK\tilde{Z} + 4K^X K^Y R^r_{ZT} R^s_{ZY} \delta_{rs})} \gamma_1 (\sigma_r)^j_i \right] \epsilon_j = \epsilon_j \tag{2.25}
\]

with

\[
U^r_{XZ\tilde{Z}} \equiv R^r_{(Z)[X]Z} - g_X(Z R^s_{\tilde{Z}Y}) K^Y + 2R^r_{(Z)X} R^s_{\tilde{Z}Y} K^Y \epsilon_{tsr} \tag{2.26}
\]

The second term in (2.25) must be independent of \( Z, \tilde{Z} \), so that one obtains

\[
q^X \frac{U^r_{XZ\tilde{Z}}}{KZK\tilde{Z} + 4K^X K^Y R^r_{ZT} R^s_{ZY} \delta_{ts}} = - \frac{q^X D_X P^r}{|K|^2} = - \sqrt{\frac{3}{2}} \frac{ge^{w}}{2} f^r \tag{2.27}
\]
The above equations have important consequences for the geometry of the moduli space; we will discuss them in the next section.

At this point from the angular integrability condition and the hyperini supersymmetry variation we have

\[
\sqrt{\frac{3}{2}} g^2 e^{2w} r W Q^r = -2 q^{jX} D_X P^r \quad |K|^2
\]

\[f^0 = -a\]  

(2.28)  

(2.29)

The above result and the relation in (2.22) fix \(f^r\) to be

\[f^r = \pm \sqrt{1 - a^2} Q^r\]  

(2.30)

In addition from the vector relation\(^4\) (2.28) we have

\[q^{jX} D_X Q^r = 0\]  

(2.31)

\[ge^w |K|^2 \sqrt{1 - a^2} = \pm 2W'\]  

(2.32)

If we use (2.29) and (2.30) in (2.21) and (2.20) then we find

\[\sqrt{1 - a^2} = \mp ge^w r W\]  

(2.33)

\[a = 1 - e^{-2w} - \frac{r^2 e^{-2w}}{4} (v'a + a')^2 + g^2 r^2 W^2\]  

(2.34)

Finally inserting (2.33) into (2.34) we obtain

\[1 = a^2 e^{-2w} \left[1 + \frac{r}{2} \left(v' + \frac{a'}{a}\right)\right]^2\]  

(2.35)

We postpone the discussion of the other consistency conditions and analyze next the implications of what we have just found for the geometric structure of the moduli space.

### 2.2 Geometric restrictions

Now we want to consider the equations in (2.24) and (2.27) and show that they determine the space-time \(r\)-dependence of the scalars, i.e. of all the quantities that enter in the description of the quaternionic geometry like the prepotential \(P^r\) and the Killing vectors. In order to elucidate their meaning in a transparent manner it is convenient to proceed as follows. First a double contraction of (2.24) with the Killing vector leads to \(q^{jX} K_X = 0\). Then using this result and contracting (2.24) with \(K^Z\) one obtains

\[q^{jZ} = \pm 3ge^w \sqrt{1 - a^2 \partial^Z W}\]  

(2.36)
The above equation is quite important: it describes the path in the moduli space associated to the BPS solution. It shows explicitly that, if we exclude the case $a^2 = 1$, the condition to have a fixed point is $\partial_Z W = 0$ which corresponds to a local minimum of the potential as observed in [9].

From (2.36) using $q'^Z W_Z = W'$ we obtain

$$|q'|^2 = \pm 3ge^{w} \sqrt{1 - a^2} W'$$

(2.37)

The contraction of (2.21) with $q'^Z$ gives

$$|q''|^2 K_Z = 2\sqrt{6}\delta_{rs} q'^X R^r_{sZ} Q^s W'$$

(2.38)

Acting now with $K^Z$ one obtains

$$|K|^2 |q'|^2 = 6W'^2$$

(2.39)

(2.39) together with (2.37) gives again (cfr 2.32)

$$|q'|^2 = \frac{3}{2} |K|^2 g^2 e^{2w}(1 - a^2)$$

(2.40)

which is in agreement with the fact that the Killing vector $K^X$ has to be null at the fixed point. Using (2.40) and (2.36) in (2.38) one easily obtains

$$K^Z = 2\sqrt{6}\delta_{rs} Q^r R^s_{XZ} \partial_X W$$

(2.41)

Now we consider (2.27) rewritten as

$$|K|^2 q'^X U^r_{XZZ} = - [K_Z K^- + 4K^T K^Y R^T_{ZT} R^s_{ZY} \delta_{ts}] q'^X D_X P^r$$

(2.42)

Contracting with $K^Z$ and using $K^X D_X P^s = 0$ (as follows from the definition (A.12)) we obtain

$$K^2 q'^X R^s_{ZX} = - \sqrt{\frac{3}{2}} W'Q^s K_Z - 3WW'Q^l D_Z Q^s \epsilon_{tr}$$

(2.43)

Moreover contracting (2.42) with $q'^Z$ and using (2.39) we find

$$|q'|^2 \partial^Z W = q'^Z W'$$

(2.44)

(which can be obtained also from (2.40) and (2.41)) and

$$|q'|^2 W D_Z Q^r + 2R^r_{ZX} W' q'^X Q^s \epsilon_{ts} = 0$$

(2.45)

which after use of (2.43) gives (2.39).

Note that (A.12) gives

$$K^Z = -\frac{4}{3} R^r_{ZX} D_X P^r$$

(2.46)
so that (2.41) can be written as

\[ K_Z = \sqrt{6}WR_r X_Z D_X Q_r \]  

(2.47)

Finally the contraction of (2.43) with \( Q^r \epsilon_{rst} \) gives

\[ 3WW'D_Z Q_t = |K|^2 Q^r q^X R^s_{ZX} \epsilon_{rst} \]  

(2.48)

Note that from (2.36) and (2.37) we also have

\[ |\partial W|^2 = \frac{|K|^2}{6} \]  

(2.49)

Let us collect the main results:

\[ q^Z = \pm 3ge^w \sqrt{1 - a^2} \partial^Z W \]  

(2.50)

\[ |K|^2 |q'|^2 = 6W \]  

(2.51)

\[ |q'|^2 K_Z = 2\sqrt{6}\delta_{rs} q^X R^s_{XZ} Q^W \]  

(2.52)

\[ K^Z = 2\sqrt{6}\delta_{rs} Q^r R^{sXZ} \partial_X W \]  

(2.53)

\[ K^2 q^X R^s_{ZX} = -\sqrt{3} \frac{W'}{2} Q^s K_Z - 3WW'Q^t D_Z Q^r \epsilon_{tr}^s \]  

(2.54)

From the above relations it follows

\[ |q'|^2 = \frac{3}{2} |K|^2 g^2 e^{2w} (1 - a^2) \]  

(2.55)

\[ K_Z = \sqrt{6} WR^r X Z D_X Q_r \]  

(2.56)

\[ |\partial W|^2 = \frac{|K|^2}{6} \]  

(2.57)

\[ 3WW'D_Z Q_t = |K|^2 Q^r q^X R^s_{ZX} \epsilon_{rst} \]  

(2.58)

\[ K^X q^X = 0 \]  

(2.59)

Now it is straightforward to verify that these conditions solve (2.24), (2.27) and (2.13) identically. Finally we note that (2.59) gives

\[ q^X D_X Q^r = 0 \]  

(2.60)

It is interesting and not at all obvious that (2.36) and the other relations we have found in this section look like a simple generalization of those obtained for flat domain wall solutions (where the gauge fields are zero). This is suggestive of an underlying general structure, independent of the form of the space-time solution.

### 2.3 Further restrictions

The equations obtained in the previous subsection are quite general. Now we have to consider the other integrability conditions together with (2.19). We start from (2.16): it
is easy to show that either all the coefficients vanish or it must be equivalent to \( (2.19) \). The first case reduces to the case in which all the coefficients of \( (2.19) \) vanish. The second case is verified when the following conditions are true:

\[
\begin{align*}
\mathit{f}^0 &= -\frac{v' - g^2e^{2w}rW^2}{(v'a + a')} \quad (2.61) \\
\mathit{f}^r &= -ge^wrW^r . \quad (2.62)
\end{align*}
\]

Other consequences of \( (2.16) \) are the following:

From \( (2.61) \) and \( (2.21) \) we find

\[
1 + 2g^2r^2W^2 + \frac{r^2e^{-2w}}{4}
\left[ v'^2 - (v'a + a')^2 \right] = \left( 1 + \frac{r}{2}v' \right)^2 e^{-2w} \quad (2.63)
\]

while inserting \( (2.33) \) into \( (2.61) \) we obtain

\[
a = \frac{rv' - 1 + a^2}{r(v'a + a')} \quad (2.64)
\]

Now we consider the integrability condition \( (2.17) \): by means of \( (2.19) \) we obtain the equations

\[
\begin{align*}
gr(v'a + a')W + garW' + \frac{1}{2} \sqrt{1 - a^2} \partial_r [re^{-w}(v'a + a')] &= 0 , \quad (2.65) \\
\mp gr(v'a + a')\sqrt{1 - a^2}W - aw'e^{-w} - g^2ae^wrW^2 + \frac{1}{2} \partial_r [re^{-w}(v'a + a')] &= 0 . \quad (2.66)
\end{align*}
\]

Similarly from \( (2.18) \) we have

\[
\begin{align*}
\mp \sqrt{1 - a^2}Q^s \left[ \frac{1}{2} g^2e^{v+w}W^2 + \frac{1}{2} e^{v-w}(v'a + a')^2 - \frac{1}{2} \partial_r (v'e^{-v-w}) \right] \\
- g^2aq'X D_x \left( \sqrt{\frac{3}{2}} e^v aP^s \right) + \frac{g}{2} e^vW'Q^s &= 0 , \quad (2.67) \\
\mp g\sqrt{1 - a^2}e^vW' + ae^v \partial_r [e^{-w}(v'a + a')] + g^2e^{v+w}W^2 \\
+ e^{v-w}(v'a + a')^2 - \partial_r (v'e^{-v-w}) &= 0 . \quad (2.68)
\end{align*}
\]

In the next section we analyze the system of first order differential equations obtained above.

\footnote{These two equations give again the condition \( (2.60) \).}
3 Static BPS configurations

Let us begin with equation (2.64) from which we easily obtain

\[ e^v = \frac{r}{r_0 \sqrt{1 - a^2}} \] (3.1)

where \( r_0 \) is an integration constant. Using (2.33) this can be rewritten as

\[ 1 = \mp gr_0 W e^{v+w} \] (3.2)

We focus on the equations derived in the previous section to obtain a set of independent ones. We start with the equation (2.65). Using (3.1) we find

\[ \partial_r \left( gae^v W \mp r \frac{e^{-w}}{2r_0} (v'a + a') \right) = 0 \] (3.3)

It is straightforward to verify that (3.3) is satisfied by (3.2) and (2.35). Thus (2.65) is identically satisfied.

Now we turn to the analysis of the equation (2.66). Using

\[ rv'a + ra' = a - \frac{ra'}{a^2 - 1} \] (3.4)

it becomes

\[ \partial_r \left( ae^{-w} + \frac{1}{2} re^{-w} (v'a + a') \right) = 0 \] (3.5)

which is solved by (2.35). Thus we conclude that the equation (2.66) is identically satisfied. We notice that also the equation (2.63) is identically satisfied. Indeed (2.35) gives

\[ 1 - \frac{r^2}{4} e^{-2w} (v'a + a')^2 = a^2 e^{-2w} + ae^{-2w} r(v'a + a') \] (3.6)

Inserting this expression into (2.63) we obtain

\[ 2g^2 r^2 W^2 - e^{-2w} (1 - rv') + ae^{-2w} r(v'a + a') + a^2 e^{-2w} = 0 \] (3.7)

Using (2.33) in the first term and multiplying by \( e^{2w} \) we have

\[ 1 - a^2 - rv' + ar(v'a + a') = 0 \] (3.8)

which is equivalent to (3.1).

In a similar way we can study the equation (2.68). If we use (2.33) in the first term of (2.68) we have

\[ g^2 e^{v+w} r WW' + g^2 e^{v+w} W^2 + ae^v \partial_r (e^{-w} (v'a + a')) + e^{v-w} (v'a + a')^2 - \partial_r (v'e^{v-w}) = 0 \] (3.9)
This can be rewritten in the form
\[ g^2 e^{v+w} r W W' + g^2 e^{v+w} W^2 + \partial_r (ae^{v-w}(v'a + a') - v'e^{v-w}) = 0 \] (3.10)
If we now multiply by \( r \) and then add and subtract the quantity \( e^{v-w}(a(v'a + a') - v') \) and finally use (3.8) we find
\[ g^2 e^{v+w} \frac{1}{2} \partial_r (r^2 W^2) - e^{v-w}(a(v'a + a') - v') + \partial_r (e^{v-w}(a^2 - 1)) = 0 \] (3.11)
Now we use \( g^2 r^2 W^2 = (1 - a^2) e^{-2w} \) and obtain
\[ \frac{e^{v+w}}{2} \partial_r [(1 - a^2) e^{-2w}] - e^{v-w}(a(v'a + a') - v') + \partial_r (e^{v-w}(a^2 - 1)) = 0 \] (3.12)
This shows that (2.68) is identically satisfied.

At the end we consider the equation (2.67). Using (2.14) we have
\[ \pm \sqrt{1 - a^2} \left[ \frac{1}{2} g^2 e^{v+w} W^2 + \frac{1}{2} e^{v-w}(v'a + a')^2 - \frac{1}{2} \partial_r (v'e^{v-w}) \right] + \frac{3}{2} ga \partial_r (ae^w) + \frac{g}{2} e^w W' = 0 \] (3.13)
By means of (2.68) this can be written as
\[ -3 ga \partial_r (ae^w) + ga^2 e^w W' \pm a \sqrt{1 - a^2} e^v \partial_r (e^{-w}(v'a + a')) = 0 \] (3.14)
From (2.33) we have \( \mp g W = \sqrt{1 - a^2 e^{-w}} \) so that
\[ \pm g W' = \frac{a}{\sqrt{1 - a^2}} e^{-w} \frac{1}{r} + \frac{1 - a^2}{r^2} e^{-w} - \frac{1 - a^2}{r} \partial_r e^{-w} \] (3.15)
and then
\[ gae^v W' \pm \sqrt{1 - a^2 e^v \partial_r (e^{-w}(v'a + a'))} = \pm \frac{v'a}{r} \sqrt{1 - a^2 e^{-w} \mp \frac{a}{r} \sqrt{1 - a^2 e^v \partial_r e^{-w}}} \pm \sqrt{1 - a^2 e^v \partial_r (e^{-w}(v'a + a'))} \]
\[ = \pm \frac{1 - a^2}{r} (re^v \partial_r (e^{-w}(v'a + a')) + e^v v'ae^{-w} - ae^v \partial_r e^{-w}) \]
\[ = \pm \frac{1 - a^2}{r} e^v (-a \partial_r e^{-w} - e^{-w} a' + \partial_r (re^{-w}(v'a + a')) \]
\[ = \pm \frac{1 - a^2}{r} e^{v} \partial_r (ae^{-w} - re^{-w}(v'a + a')) = \mp \frac{3}{r_0} \partial_r (ae^{-w}) \] (3.16)
where in the last step we have used (2.33) and (3.11).
Then using \( \frac{1}{r_0} = \mp g e^v W \) we see that (3.14) is identically satisfied.

In the next section we verify that BPS solutions we have obtained so far satisfy the equations of motion.
4 Equations of motion

The equations of motion of our system are given by

\[-e^{v-w} \partial_r (v' e^{v-w}) - \frac{3}{r} e^{2(v-w)} + e^{2(v-w)} (v' a + a')^2 + 4 g^2 e^{2v} W^2 \]
\[- \frac{1}{2} g^2 e^{2w} |K|^2 + \frac{3}{2} g^2 e^{2v} |K|^2 a^2 = 0 , \quad (4.1)\]

\[e^{w-v} \partial_r (v' e^{v-w}) - \frac{3}{r} w' + |q'|^2 - (v' a + a')^2 - \frac{1}{2} e^{2w} (8g^2 W^2 + g^2 |K|^2) = 0 , \quad (4.2)\]

\[r e^{-2w} (v' - w') - 2(1 - e^{-2w}) + \frac{1}{2} r^2 e^{-2w} (v' a + a')^2 \]
\[+ \frac{2}{3} r^2 \left( -6g^2 W^2 + \frac{3}{4} g^2 |K|^2 \right) = 0 , \quad (4.3)\]

\[ae^v K_X q'_X = 0 , \quad (4.4)\]

\[e^{-(v+w)} r^{-3} \partial_r \left( r^3 e^{-v-w} q'^X \right) - \frac{1}{2} \partial^Z g_{XY} e^{-2w} q'^X q'^Y \]
\[+ \frac{3}{4} g^2 a^2 \partial^Z |K|^2 - \partial^Z \left( -6g^2 W^2 + \frac{3}{4} g^2 |K|^2 \right) = 0 , \quad (4.5)\]

\[3e^{-2w} (v' a + a') + r \partial_r (e^{-w} (v' a + a')) = g e^{-w} |K|^2 , \quad (4.6)\]

\[g e^{-2w} K_X q'_X = 0 . \quad (4.7)\]

where (4.1), (4.2), (4.3) and (4.4) are the Einstein equations, (4.5) is the equation for the scalar fields and (4.6), (4.7) are the Maxwell equations.

First we observe that both (4.4) and (4.7) are an immediate consequence of (2.59). Then we consider the sum of (4.1) and (4.2). Multiplying by \(e^{2(v-w)}\) and using (2.55) we obtain

\[g^2 |K|^2 = 2 \frac{e^{-2w}}{r} (v' + w') . \quad (4.8)\]

This is solved by (3.2), (2.33) and (2.32).

It is straightforward to check that all the equations are indeed satisfied:
Equation (4.1) is solved using (1.8), (3.9), (3.2), (2.33), (2.35) and (3.1).
Equation (4.3) is solved by (1.8) and (2.63).
Equation (4.5) is solved using (2.50), (1.8), (2.57), (2.33) and (3.1).
Finally (4.6) is solved by (2.35) and (1.8).
5 The Universal Hypermultiplet case

Now we collect the set of first order differential equations obtained by the BPS conditions:

\[ 1 = a^2 e^{-2w} \left[ 1 + \frac{r}{2} \left( v' + \frac{a'}{a} \right) \right]^2 \] (5.1)

\[ e^v = \frac{r}{r_0 \sqrt{1 - a^2}} \] (5.2)

\[ \sqrt{1 - a^2} = \mp ge^w r W \] (5.3)

\[ q^Z = \pm 3ge^w \sqrt{1 - a^2} \partial^2 Z W \] (5.4)

It is easy to reduce the above system to

\[ \begin{cases} 
q^Z = -3 \frac{1 - a^2}{r} \partial^2 \ln W \\
1 - a^2 = \frac{a^2}{4} (3a + r \frac{a'}{1 - a^2})^2 r^2 W^2 
\end{cases} \] (5.5)

(5.2) and (5.3) simply define \( v \) and \( w \) respectively in terms of \( a \) and of the scalars, hence as functions of \( r \).

In this form our problem is analogous to the domain-wall case: the main difference is that now the two differential equations are coupled equations and therefore finding an explicit solution is more involved. To solve (5.5) we have to specify \( W \) as a function of the scalars \( q^X \) i.e we have to choose which isometry of the quaternionic manifold represents the action of the \( U(1) \) gauge symmetry. As we have already argued in the previous sections the most interesting configurations are obtained for isometries in the isotropy group of some point of the quaternionic manifold. By definition this choice corresponds to a fixed point solution i.e one with asymptotic constant scalars. For (supersymmetric) black hole solutions it implies the existence of an horizon. The scale invariance appearing in the near-horizon limit gives rise to the enhancement of the unbroken supersymmetry associated to a fixed point.

In fact the configurations that have the most relevant role in the AdS/CFT correspondence are the ones with two fixed points: this type of solutions should describe a RG flow between two different CFT’s defined on the boundary of the five dimensional space-time. For black hole solutions this means that the space-time is maximally symmetric in the \( r \to \infty \) limit (for example AdS). Since as shown in [15], in order to obtain such configurations one needs the introduction of vector multiplets, we postpone the study of black hole configurations to future work.

Here, as an example, we construct an explicit solution of the BPS equations in the case of a \( n_H = 1 \) hypermultiplet, i.e. the so called universal hypermultiplet. This simple example is however important because this hypermultiplet (which contains the Calabi-Yau volume) appears in any Calabi-Yau compactification. We adopt for this manifold the notations and the coordinate system of [9].

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For simplicity we make the following choices for the gauged isometry and the graviphoton:

\[ K \equiv \vec{k}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \]  
\[ a' \equiv 0 \]  
\[ (5.6) \]

The Killing vector \( \vec{k}_1 \) has a simple interpretation: it represents the translation of the axionic scalar (\( \sigma \) in our notations). This solution has been already considered in [8]\(^6\).

We remark that since \( \vec{k}_1 \) is not in the isotropy group of any point of the manifold, the presence of fixed points is excluded. In general it is easy to see that fixed point solutions are ruled out by the assumption \( a' = 0 \).

We observe that, being the superpotential for \( \vec{k}_1 \) \( W_{\vec{k}_1} = \mp \sqrt{\frac{2}{3}} \sqrt{1 - a^2} \), the only dynamical scalar is \( V \). The others are space-time constants and can be set consistently equal to zero. Imposing (5.6) and (5.7) the system (5.5) becomes

\[ \begin{cases} V' = 6 \frac{1-a^2}{r} V \\ \frac{1-a^2}{r} = \frac{3}{32} \left( \frac{\Lambda}{V} \right)^2 \end{cases} \]  
\[ (5.8) \]

Since the metric of the quaternionic manifold (A.18) in our parametrization is singular for \( V = 0 \) we restrict ourselves to the branch \( V > 0 \). From (5.8) it follows that the scalar \( V \) has the form

\[ V = \mathcal{C} \, r^\Lambda \]  
\[ (5.9) \]

with \( \mathcal{C} \) and \( \Lambda = 6(1 - a^2) \) fixed, by consistency with (5.8), to be

\[ \Lambda = 1 \]  
\[ (5.10) \]
\[ \mathcal{C} = \sqrt{\frac{3}{32}} \sqrt{\frac{a^2}{1 - a^2}} \]  
\[ (5.11) \]

that in particular gives \( a = \sqrt{\frac{5}{6}} \). At the end we find

\[ V = \sqrt{\frac{15}{32}} gr \]  
\[ (5.12) \]

and using (5.2) and (5.3)

\[ e^v = \sqrt{\frac{6}{r_0}} \]  
\[ (5.13) \]
\[ e^w = \sqrt{\frac{15}{8}} \]  
\[ (5.14) \]

\(^6\text{As discussed in section 2 in [8] a mistake affects the calculations; however the final solution has the right functional form.}\)
Rescaling the time coordinate $t$ by the constant $\frac{\sqrt{6}}{r_0}$ the space-time metric and the electrostatic potential become

$$ds^2 = -r^2 dt^2 + \frac{15}{8} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\psi^2)$$  \hspace{1cm} (5.15)

$$A_t = \sqrt{\frac{5}{4} r}$$  \hspace{1cm} (5.16)

We notice that as expected the gauging constant $g$ appears only in the quaternionic scalar while the other space-time quantities do not depend on it.

6 Discussion and Outlook

In this work we have studied electrostatic spherical BPS ($N = 1$) solutions in $N = 2$ gauged supergravity in five dimensions with hypermultiplet couplings. In particular we have discussed the possibility to find (extreme) black-hole solutions. The main results we have obtained can be summarized as follows: first of all we have derived the BPS equations in a general setting, going beyond special cases treated previously with restrictive ansatz [8].

Then we have discussed the structure of the integrability conditions and in particular of the hyperini equations. We have obtained relations which appear to be a generalization of those found in [9] for flat domain walls. This result is somewhat surprising since our ansatz is quite different from the one in [9]. In addition we have considered a configuration with a non-vanishing gauge field whose presence complicates considerably the structure of the equations.

We have verified that our BPS solutions satisfy the equations of motion.

The above results leave much space for further studies. First of all it would be interesting to explore if and under which conditions the structure found for the hyperini equation is maintained when more general ansatz are considered and vector multiplets are introduced. In these general settings one would like to explore the existence of black hole solutions leading to nontrivial RG flows.

Finally it would be interesting to consider such explicit solutions and extend them to non extreme black holes along the lines of what has been done for the case of vector multiplets [10].

These open problems are currently under investigations.

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A Conventions

In this appendix we present some definitions and properties that we use in our work. With

\[ q^X \quad x = 1, \ldots, 4n_H \]  

we denote the scalars of the hypermultiplets which are the coordinates of a quaternionic manifold. We introduce the 4

\[ H \]  

beins as

\[ f^A_X(q^Y), \quad i = 1, 2 \in SU(2), A = 1, \ldots, 2n_H \in Sp(2n_H) \]  

The splitting of the flat indices in \( i \) and \( A \) reflects the factorization of the holonomy group in \( USp(2)(\simeq SU(2)) \otimes USp(2n_H) \) which is the main feature of those spaces. The indices as a consequence of the symplectic structure are highered and lowered with the antisymmetric matrices

\[ \epsilon_{ij}, \quad C_{AB}, \quad \epsilon_{ij} = \epsilon^{ij}, \quad \epsilon_{12} = 1 \]  

\[ C_{AB}C^CB = \delta_A^C, \quad C^{AB} = (C_{AB})^* \]  

following the NW-SE convention [9].

The important relation

\[ f_{Xic}f_{Yj}^C = \frac{1}{2} \epsilon_{ij}g_{XY} + R_{XYij} \]  

can be viewed as a definition for the quaternionic metric \( g_{XY} \) and for the \( SU(2) \) curvature \( R_{XYij} \).

We use the symbols \( p_{Xi}^j \) for the \( SU(2) \) spin connection whereas \( \omega^{ab}_\mu \) denotes the usual Lorentz spin connection. The covariant derivative which appears in the gravitini supersymmetry variation acts on the symplectic Majorana spinors \( \epsilon_i \) as

\[ D_\mu \epsilon_i = \partial_\mu \epsilon_i + \frac{1}{4} \omega^{ab}_\mu \gamma_{ab} \epsilon_i - \partial_\mu q^X p_{Xi}^j \epsilon_j - gA_\mu P_i^j \epsilon_j \]  

where the generalized spin connection receives the following contributions: the first term represents the Lorentz action while the others can be identified with the \( SU(2) \) action plus a term due to the R-symmetry \( SU(2) \) gauging. \( A_\mu \) is the graviphoton 1-form and \( P^r \) are the prepotentials while \( g \) is the electric gauge coupling.

It is useful to introduce the projection on the Pauli matrices for quantities in the adjoint representation of \( SU(2) \), for example

\[ R_{XYij} \simeq R^r_{XY}(i\sigma_r)_i^j \]  

where \( (\sigma_r)_i^j \) are the usual Pauli matrices

\[ \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \]  

\[ (A.9) \]
which satisfy

\[(σ_r)_i^j (σ_s)_j^k = δ_{rs}δ_i^k + iε_{rs}^t (σ_t)_i^k \tag{A.10}\]

\[[σ_r, σ_s] = 2iε_{rst}σ^t \tag{A.11}\]

The prepotentials are defined by the relation

\[R^r_{XY}K^Y = D_X P^r \tag{A.12}\]

\[D_X P^r := \partial_X P^r + 2ε^{rst}p^s_X P^t \tag{A.13}\]

where \(D_X\) is the \(SU(2)\) covariant derivative. They can be expressed in terms of the Killing vectors

\[P^r = \frac{1}{2n_H}D_X K_Y R^r_{XY} \tag{A.14}\]

The scalar potential is defined as

\[V = -4P^r P^s g^{2} + 2N_{Ai}N^{Ai} g^{2} \tag{A.15}\]

with

\[N^{Ai} = \frac{\sqrt{6}}{4} K^X f^{Ai}_X = \frac{2}{\sqrt{6}} f^{Ai}_X R^r_{YX} D_Y P^r \tag{A.16}\]

Defining the superpotential \(W\) by \(P^r = \sqrt{\frac{2}{3}} WQ^r\) the potential becomes

\[V = -6g^2W^2 + g\frac{9}{2}g^{XY} \partial_X W \partial_Y W \tag{A.17}\]

The universal hypermultiplet \((n_H = 1)\) corresponds to the quaternionic Kähler space \(SU(2,1)/SU(2)U(1)\). A significant parametrization, from a M-theory point of view, is \(q^X = \{V, σ, θ, τ\}\)

with the metric

\[ds^2 = \frac{dV^2}{2V^2} + \frac{1}{2V^2} (dσ + 2θ dτ - 2τ dθ)^2 + \frac{2}{V} (dτ^2 + dθ^2) \] \(\tag{A.18}\)

Using the general properties of quaternionic geometry it is possible from \((A.18)\) to derive explicitly all the quantities presented above, in particular the Killing vectors and the prepotentials of the eight isometries of manifold. For the axionic shift we have:

\[\bar{k}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \bar{P}_1 = \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{4V} \end{pmatrix} \tag{A.19}\]
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