Formation of Reflecting Surfaces Based on Spline Methods

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Abstract. The article deals with problem of reflecting barriers surfaces generation by spline methods. The cases of reflection when a geometric model is applied are considered. The surfaces of reflecting barriers are formed in such a way that they contain given points and the rays reflected at these points and hit at the defined points of specified surface. The reflecting barrier surface is formed by cubic splines. It enables a comparatively simple implementation of proposed algorithms in the form of software applications. The algorithms developed in the article can be applied in architecture and construction design for reflecting surface generation in optics and acoustics providing the geometrical model of reflex processes is used correctly.

1. Introduction

In designing of internal surfaces of buildings and facilities, especially communal facilities, it is often necessary to design surfaces in terms of the reflection of the rays falling on them. They can be light or sound rays. For the solution of such tasks two types of models are most often used: wave and geometrical [1-3]. Wave models are based on the fact that radiation (light or sound) is a wave. Such a model well describes the properties of radiation, phenomena occurring at its propagation, but is very complex and labor-consuming for calculations especially when it is used as the barrier of irregular shape surfaces. A geometrical model is considerably simpler. In this model the radiation is represented by segments of straight lines without considering its wave nature. Of course, such a model keeps from solving the problems connected with radiation interference or diffraction, but under certain conditions [4,5] it allows to calculate quite exactly the key optical or acoustic parameters of rooms based on reflex and refraction processes of rays [6,7].

Since, when forming such surfaces, it is necessary to simulate certain geometrical parameters in the set points of the surface, it is convenient to use spline methods for their creation. It enables implementation of algorithms in the form of software applications, and considerably accelerates software development process as the majority of modern systems have integrated functions for work with splines [8-13].

The article describes the algorithm of formation of a surface segment with given rectangular array of points that uniformly reflect the emission of a point source onto the specified surface segment.

2. Problem setting

Let in \( I \), a point source of radiation, be given array of points \( A_{ij} \), where \( i = 1, 2, \ldots, n \); \( j = 1, 2, \ldots, m \) and segment of surface \( \Omega \). Using a spline method [14,15], let us construct surface segment \( \Sigma \), containing points of array \( A_{ij} \) and uniformly reflecting the emission of source \( I \) on segment \( \Omega \). Let rays reflected from segment \( \Sigma \) at points \( A_{ij} \) strike corresponding points \( B_{ij} \).
3. Normals determination at points of incidence
Assume that the conditions enabling the use of the geometrical model of reflection processes are met. As is well-known [3], the incident and the reflected from a surface rays lie in one plane with a normal to a surface at the incidence point and make equal angles with it. Proceeding from this condition, we will define at points $A_{ij}$ vectors of normals $\vec{N}_{ij}$ of surface $\Sigma$ (Figure 1).

$$\vec{N}_{ij} = \frac{\vec{B}_{ij} - \vec{A}_{ij}}{||\vec{B}_{ij} - \vec{A}_{ij}||} \left( \vec{I} - \vec{A}_{ij} \right) \left( \vec{I} - \vec{A}_{ij} \right),$$

where $\vec{I}$, $\vec{A}_{ij}$, $\vec{B}_{ij}$ are radii vectors of corresponding points $I$, $A_{ij}$, $B_{ij}$.

![Figure 1. Determination of normal vector.](image)

4. Surface imaging
Let us take a range of points of array $A_{ij}$ with fixed value of $j$. We will construct a spline passing through these points so that tangents at them are perpendicular to the normal vector (1) and have the smallest angles $\phi$ with vectors $\vec{a}_{ij}$, where

$$\vec{a}_{ij} = \begin{cases} \vec{A}_{i+1j} - \vec{A}_{ij}, & \text{if } i = 1; \\ \vec{A}_{ij} - \vec{A}_{i-1j}, & \text{if } i \neq 1. \end{cases}$$

We will find vector of tangent $\vec{t}_{ij}^v$ as the directing vector of the intersection line $\vec{t}_{ij}^v$ of planes $\alpha_{ij}$ containing vectors $\vec{a}_{ij}$ and $\vec{N}_{ij}$, and of $\beta_{ij}$, passing through point $A_{ij}$ and perpendicular to $\vec{N}_{ij}$ (Figure 2),

$$\vec{t}_{ij}^v = \alpha_{ij} \cap \beta_{ij}$$

where $\vec{a}_{ij} \in \alpha_{ij}$, $\vec{N}_{ij} \in \alpha_{ij}$, $\vec{a}_{ij} \in \alpha_{ij}$, $\vec{A}_{ij} \in \beta_{ij}$, $\vec{N}_{ij}$ is perpendicular to $\beta_{ij}$.

![Figure 2. Determination of vector of tangent $\vec{t}_{ij}^v$.](image)

When constructing a spline it is possible to define tangents explicitly only at the initial and the finite points [8-10]. Let us introduce additional points near $A_{ij}$ which will define tangents (2) in order
that the spline has given tangents at all points. Let us denote these points by $C^1_{ijk}$ and $C^2_{ijk}$, $k=1, 2, \ldots, M$ (Figure 3).

**Figure 3.** Construction of additional points.

Thus, having constructed the splines, passing through points rows of array $D_{ij}$ with fixed $j$, we will obtain the lines passing through points of $A_y$ and having at them tangents perpendicular to normals $\vec{N}_y$ (Figure 4). Let us denote the constructed splines by $n_j$. Their equations are

$$\vec{r}(u) = \vec{r}_y(u, \nu_j)$$

(3)

Points of array $A_y$ are shown in Figure 4, points of $D_{ij}$ are not shown.

**Figure 4.** Construction of splines $n_j$.

Let us construct at each point of $D_{ij}$ vectors of normals $\vec{N}_{ij}$, perpendicular to vectors $\vec{t}_{ij}$, where

$$\vec{t}^{\nu}_{ij} = \frac{d\vec{n}_{ij}(u, \nu_j)}{d\nu}$$

(4)

$\nu_j$ is a parameter corresponding to point $D_{ij}$ on the spline (3). At $D_{ij}$ array points, corresponding to the points of specified array $A_y$, vectors $\vec{N}_{ij}$ coincide with normals vectors at these points determined by relations (1). Let us construct $\vec{N}_{ij}$ at the nearest points (corresponding to additional points $C^1_{ijk}$ and $C^2_{ijk}$) so that rays reflected at them have the minimum deviation from $B_y$. We find vector $\vec{N}_{ij}$ by the formula
\[ \tilde{N}^t_{ij} = \frac{\tilde{B}_{ij} - \tilde{D}_{ij}}{\left| \tilde{B}_{ij} - \tilde{D}_{ij} \right|} - \frac{\tilde{I} - \tilde{D}_{ij}}{\left| \tilde{I} - \tilde{D}_{ij} \right|} \]  

(5)

where \( I^t = \text{Round}\left( \frac{i' + 2k}{2k + 1} \right) \). "Round" is a function, rounding argument value up to the nearest whole number. If vector \( \tilde{N}^t_{ij} \) were a normal to point \( D_{ij} \), the ray reflected at this point would hit precisely \( B_j \). But it is possible only for the points corresponding to \( A_j \), as in all other points \( \tilde{N}^t_{ij} \) is not perpendicular to the appropriate vector \( \tilde{t}^s_{ij} \) (4). Therefore we will construct it so that an angle \( \psi \) between it and \( \tilde{N}^t_{ij} \) were minimal. \( \tilde{N}^t_{ij} \) is defined as a directing vector of straight line \( n^t \) which is the intersection line of planes \( \gamma_{ij} \) and \( \delta_{ij} \). Plane \( \gamma_{ij} \) contains vectors \( \tilde{t}^s_{ij} \) (4) and \( \tilde{N}^t_{ij} \) (5). Plane \( \delta_{ij} \) contains point \( D_{ij} \) and is perpendicular to vector \( \tilde{t}^s_{ij} \) (Figure 5).

\[ n_{ij} = \gamma_{ij} \cap \delta_{ij} \]

where \( \tilde{t}^s_{ij} \in \gamma_{ij}, \tilde{N}^t_{ij} \in \gamma_{ij}, D_{ij} \in \delta_{ij}, \tilde{t}^s_{ij} \) is perpendicular to \( \delta_{ij} \).

**Figure 5.** Determination of vector of normal \( \tilde{N}^t_{ij} \).

Let us construct tangents vectors at points \( D_{ij} \) perpendicular to \( \tilde{N}^t_{ij} \) and making the smallest angle with \( \tilde{b}_{ij} \) where

\[ \tilde{b}_{ij} = \begin{cases} \tilde{D}_{ij+1} - \tilde{D}_{ij}, & \text{if } j = 1; \\ \tilde{D}_{ij} - \tilde{D}_{ij-1}, & \text{if } j \neq 1. \end{cases} \]

We will define \( \tilde{t}^n_{ij} \) in the similar way as we defined \( \tilde{t}^s_{ij} \) (2). We will find the intersection line of planes \( \alpha^t_{ij} \) and \( \beta^t_{ij} \).

\[ t^n_{ij} = \alpha^t_{ij} \cap \beta^t_{ij} \]  

(6)

where \( \tilde{b}_{ij} \in \alpha^t_{ij}, \tilde{N}^t_{ij} \in \alpha^t_{ij}, D_{ij} \in \beta^t_{ij}, \tilde{N}^t_{ij} \) is perpendicular to \( \beta^t_{ij} \). The directing vector of lines \( t^n_{ij} - \tilde{t}^s_{ij} \) is a required vector.

Let us construct the splines, passing through points of \( D_{ij} \) with fixed value of \( i' \) and having tangents at them (6). Let us introduce near \( D_{ij} \) additional points \( E^1_{jk} \) and \( E^2_{jk}, k = 1, 2, \ldots, M \) the radii vectors of which are determined by the relations
\[ \vec{E}_{ijk}^i = \vec{D}_{ij} - \frac{\vec{t}_{ij}^u}{\vec{u}_{ij}^u} d_k, \text{ where } j = 1, 2, \ldots, m - 1; \]
\[ \vec{E}_{ijk}^2 = \vec{D}_{ij} + \frac{\vec{t}_{ij}^u}{\vec{u}_{ij}^u} d_k, \text{ where } j = 2, 3, \ldots, m, \]
where
\[ d_1 = \frac{\vec{b}_{ij}}{N}, \quad d_2 = \frac{\vec{b}_{ij+1}}{N}, \]

Having introduced additional points, we have got array \( G_{ij} \) where \( j' = 1, 2, \ldots, 2M(m-1) + m \).

Points \( G_{2M(i-1)+1}(2M(j-1)+j) \) correspond to the points of array \( A_{ij} \). Through rows of points, with fixed \( i' \) we construct splines \( k_{ij} \) having given tangents (6) at these points (Figure 6).

Through points of \( G_{ij} \), with fixed \( i' \) we construct splines \( l_{ij} \) (Figure 7). We construct a surface according to the algorithm described by spline method in [14] using mesh \( k_{ij} \), \( l_{ij} \) as initial data.

**Figure 6.** The construction of the splines \( k_{ij} \). **Figure 7.** The construction of the splines \( l_{ij} \).

### 5. Example of surface generation

The described algorithm is implemented by technology Object ARX [16,17] for AutoCAD [18] in the language C++ [19,20]. The software developer was granted a copyright certificate.

Figure 8 shows the example of algorithm operation.

**Figure 8.** Example of algorithm operation.

### 6. Conclusion

The developed software on the basis of the algorithms, described in the article, can be used for different tasks solving in optics and acoustics, where a geometric model of reflection processes can be applicable. The software package can be used independently or be included in specialized CAD as a module.
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