Efficient characterization of correlated SPAM errors

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State preparation and measurement (SPAM) errors limit the performance of gate-based quantum computers, but are partly correctable after a calibration step that requires, for exact implementation on a register of $n$ qubits, $2^n$ additional characterization experiments. Here we introduce an approximate but efficient method for SPAM error characterization requiring $2n(n-1)$ measurements. The technique assumes that multi-qubit measurement errors are dominated by pair correlations, which are estimated with $n(n-1)/2$ two-qubit experiments. We demonstrate this technique on the IBM and Rigetti online quantum computers, allowing comparison of their SPAM errors in both magnitude and degree of correlation. We find that the pair-correlation model is reasonably accurate on linear arrays of qubits. However, qubits arranged in more closely spaced two-dimensional geometries exhibit significant higher-order (such as 3-qubit) SPAM error correlations.

I. SPAM ERROR CORRECTION

Errors in a quantum computation are typically classified into state-preparation errors, gate errors, and readout or measurement errors. A standard approach for correcting some of these errors is to measure the overlap between initial and final classical states, and use this information to correct for measurement errors.

Here $T(x|x')$ denotes the quantum expectation value after preparing the classical state $x'$. Each column of $T(x|x')$ is the raw probability distribution $\text{prob}(x)$ measured immediately after preparing $x'$.

Ideally $T(x|x') = \delta_{x,x'}$, the $2^n \times 2^n$ identity. The exact implementation of the error correction technique is to measure $T$ and classically apply $T^{-1}$ to subsequently measured probability distributions. This forces an empty circuit to behave ideally in the noisy device. We refer to this technique as state preparation and measurement (SPAM) error correction to emphasize that errors in preparing the $|x\rangle$ get folded onto the $T$ matrix. We also note that the technique has only been applied in experiments where full probability distributions are measured.

However, there are two problems with the exact implementation: (1) It requires $2^n$ probability measurements, which is not scalable. (2) The matrix $T(x|x')$ typically becomes singular for large $n$, preventing direct inversion. In this work we address the first limitation by deriving an efficient method to estimate $T(x|x')$. The technique is efficient in the sense that it only requires $2n(n-1)$ probability measurements to estimate the entire set of $4^n$ matrix elements $\{T(x|x')\}_{x,x'}$. Note that even evaluating these $4^n$ matrix elements from measured data is classically inefficient; however, this does not currently present a practical limitation. Beyond its use in error correction, the estimated $T$ matrix can also be used to rapidly characterize and quantify correlated SPAM errors, including measurement crosstalk, which we demonstrate on three different devices.

II. TRANSITION MATRIX ESTIMATION

The exact $T$ matrix technique is not scalable because it does not make use of the tensor product structure of the problem and the fact that the qubits are physical devices that mainly interact pairwise. To this end, rewrite (1) as

$T(x|x') = \langle \pi(x_1) \otimes \pi(x_2) \otimes \cdots \otimes \pi(x_n) \rangle$, \hspace{1cm} (2)

where $\pi(x_i) = |x_i\rangle \langle x_i|$ is a single-qubit projector and the $x_i \in \{0,1\}$. The efficient method to estimate $T(x|x')$ is based on the assumption that the largest readout errors occur within an individual qubit or between pairs of qubits, and that higher-order (e.g., 3-qubit) error correlations are smaller. The data presented here confirm this for extended registers of qubits, such as chains, but not for clustered geometries where crosstalk is evidently stronger.

The zeroth-order (uncorrelated) approximation is to measure $n$ independent single-qubit $2 \times 2$ transition matrices

$T_i(x|x_i') := \langle \pi(x_i) \rangle$,

and approximate $T(x|x')$ by

$T_{\text{prod}}(x|x') = T_1(x_1|x_1') \otimes \cdots \otimes T_n(x_n|x_n')$. \hspace{1cm} (4)

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Note that we define (3) to be measured with an initial condition $0 \cdots x'_1 \cdots 0$ different from $x' = x'_1 \cdots x'_m$, which appears in (1) and (2). This reduction is necessary to avoid preparing exponentially many initial states, and is valid as long as the $X$ gates are unitary and local (see Appendix A). Measuring $T_{\text{prod}}(x|x')$ requires $2n$ measurements because there are two initial conditions (|0⟩ and |1⟩) for each of $n$ qubits. However, as we will explain below, in actual applications these $2n$ measurements are bypassed and are not included in the circuit count.

The product form (4) neglects multi-qubit measurement error correlations/crosstalk, and is not reliable. We find this to be true even for non-adjacent qubit devices. To quantify the correlations we define a matrix

$$T_{\text{corr}} = T - T_{\text{prod}}.$$  (5)

To go beyond the product form we will calculate the leading-order approximation to $T_{\text{corr}}$. Let

$$\delta \pi_i(x_i|x'_i) := \pi_i(x_i) - T_i(x_i|x'_i)$$  (6)

be the single-qubit projector fluctuation, with $T_i$ defined in (3). Expanding (2) in powers of these fluctuations leads to

$$T(x|x') = \left\langle \bigotimes_{i=1}^n \left[ T_i(x_i|x'_i) + \delta \pi_i(x_i|x'_i) \right] \bigotimes_{k \neq i,j} T_k(x_k|x'_k) \right\rangle_{x'} = T_{\text{prod}}(x|x') + T_{\text{pair}}(x|x') + \cdots$$  (7)

where

$$T_{\text{pair}}(x|x') = \sum_{ij} C_{ij}(x_i, x_j|x'_i, x'_j) \bigotimes_{k \neq i, j} T_k(x_k|x'_k)$$  (8)

is the pair contribution, with

$$C_{ij}(x_i, x_j|x'_i, x'_j) = \langle \delta \pi_i(x_i|x'_i) \delta \pi_j(x_j|x'_j) \rangle_{0 \cdots x'_i \cdots x'_j \cdots 0} - T_i(x_i|x'_i) T_j(x_j|x'_j)$$  (9)

a two-qubit correlator that we call the pair-correlation function. In (9) we again reduced the initial condition $x'$ as discussed in Appendix A. Some useful properties of the $T$ matrices are given in Appendix B.

The pair contribution (8) is obtained by first measuring $\langle \pi_i(x_i) \pi_j(x_j) \rangle_{0 \cdots x'_i \cdots x'_j \cdots 0} \ (x'_i, x'_j \in \{0,1\})$ on each distinct pair of qubits. The single-qubit $T$ matrices (3) are then obtained by partial trace,

$$T_i(x_i|x'_i) = \sum_{x_j=0,1} \langle \pi_i(x_i) \pi_j(x_j) \rangle_{0 \cdots x'_i \cdots 0},$$  (10)

which is why they are not included in the total circuit count.

![FIG. 1. Ladder geometry of IBM Rüeschlikon device.](image)

III. SPAM ESTIMATION ON THE IBM AND RIGETTI DEVICES

Data was taken on the IBM Rüeschlikon, IBM Tenerife, and the Rigetti 8Q-Agave online quantum computers. The main features of the data are summarized in Table I. The first case we will consider is a linear chain $Q_1 Q_2 Q_3 Q_4$ of nearest-neighbor qubits on the IBM device, as shown in Fig. 1. The raw data for this case is given in Appendix C. Comparing $T_{\text{corr}}$ and $T_{\text{pair}}$ in Appendix C, we see that many features in $T_{\text{corr}}$ are correctly captured by the pair-correlation approximation, but higher-order correlations are observed as well.

A measure $\| T - I \|_F$ of the overall size of the SPAM error in this case is given in column 2. It is slightly larger than the other 4-qubit registers studied on the Rüeschlikon chip. This is because qubit $Q_2$ has an especially large single-qubit measurement error (Appendix C). SPAM errors on Rigetti Agave are similar to Rüeschlikon, but errors on IBM Tenerife are somewhat larger.

| Qubits   | $\| T - I \|_F$ | $\| T_{\text{corr}} \|_F$ | $\| T_{\text{pair}} \|_F$ | $\| T_{\text{corr}} - T_{\text{pair}} \|_F$ |
|----------|----------------|------------------------|------------------------|------------------------|
| $Q_1 Q_2 Q_3 Q_4$ | 1.279 | 0.106 | 0.187 | 0.082 |
| $Q_5 Q_6 Q_7 Q_8$ | 1.057 | 0.096 | 0.089 | 0.028 |
| $Q_1 Q_2 Q_3 Q_7$ | 1.014 | 0.057 | 0.068 | 0.035 |
| $Q_7 Q_8 Q_9 Q_{10}$ | 0.984 | 0.031 | 0.034 | 0.033 |
| $Q_1 Q_2 Q_3 Q_5 Q_7$ | 1.983 | 0.245 | 0.276 | 0.062 |
| $Q_1 Q_2 Q_3 Q_4$ | 1.631 | 0.625 | 0.610 | 0.096 |
| 0123 | 1.150 | 0.124 | 0.382 | 0.319 |
| 01234 | 3.254 | 1.849 | 4.731 | 6.124 |

TABLE I. Summary of SPAM errors measured on different chips and registers. Qubits/devices are grouped together in rows: the top 5 rows are IBM Rüeschlikon qubits. Row 6 is from IBM Tenerife. The bottom 2 rows are Rigetti 8Q-Agave qubits. The first column gives the qubit register. $\| T - I \|_F$ measures the overall magnitude of the SPAM errors, with $\| \cdot \|_F$ the Frobenius norm. $\| T_{\text{corr}} \|_F$ and $\| T_{\text{pair}} \|_F$ are measures of the error correlations, and $\| T_{\text{corr}} - T_{\text{pair}} \|_F$ measures the higher-order (such as 3-qubit) correlations. The matrices $T_{\text{corr}}$ and $T_{\text{pair}}$ are defined in (5) and (8).

Compare the $Q_1 Q_2 Q_3 Q_4$ case with $Q_5 Q_6 Q_7 Q_8$, another linear register of nearest-neighbor qubits at the other end of the Rüeschlikon device (Fig. 1). All SPAM errors are smaller on this register. However in both cases $\| T_{\text{pair}} \|_F$ gives a reliable estimate of $\| T_{\text{corr}} \|_F$.

Next we compare $Q_1 Q_2 Q_3 Q_4$ with $Q_1 Q_2 Q_3 Q_7$, a linear register of next-nearest-neighbor qubits. As expected, the correlations are noticeably smaller in this case.
So far we have discussed linear chains of qubits. The case $Q_7Q_8Q_9Q_{10}$ is interesting because the four qubits form a square at the end of the R" ueschlikon device. In this case $∥T_{\text{corr}} - T_{\text{pair}} ∥_F$ is the same size as $∥T_{\text{corr}}∥_F$, indicating the presence of significant higher-order correlations and a breakdown of the pair approximation. We interpret this breakdown as indicating the presence of strong measurement crosstalk.

The Agave chip geometry has qubits on the perimeter of a square. Qubits 0123 form nearest-neighbor register on the square and are in close proximity, similarly to the IBM register $Q_7Q_8Q_9Q_{10}$. Here we again observe strong higher-order correlations and a breakdown of the pair approximation, consistent with the interpretation that clusters (as opposed to extended registers) of qubits have stronger measurement crosstalk.

IV. CONCLUSION

Motivated by a common SPAM error correction technique [1–5], we develop and apply an $O(n^2)$ method to characterize correlated SPAM errors on a register of $n$ qubits. The technique assumes that correlated SPAM errors are dominated by pair correlations, which are estimated with $n(n - 1)/2$ two-qubit experiments. Although we find that the pair-correlation model is accurate for extended registers of qubits, such as chains, it is not reliable for clustered geometries on current devices where crosstalk is evidently stronger.

There are a few natural extensions of the pair-correlation technique. The first is to employ the pair-correlation functions in a Gaussian error model. In this case the odd-order correlations will vanish and the even-order ones are determined by Isserlis’s (or Wick’s) theorem [6]. However, we did not find this to significantly improve the accuracy of the estimate for $T_{\text{corr}}$. A second extension, which we have not implemented, would be to include and directly measure the 3-qubit correlations, which would require $O(n^3)$ measurements.

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Appendix A: Initial condition reduction

Here we discuss the approximation
\[ \langle \pi_i(x_i) \rangle_{x'_1 \ldots x'_i \ldots x'_n} \approx \langle \pi_i(x_i) \rangle_{0 \ldots x'_i \ldots 0}, \tag{A1} \]
where, on the right-hand-side, qubits other than $i$ are initialized to $|0\rangle$. We justify (A1) by showing that
\[ \langle \pi_i(x_i) \rangle_{x'_1 \ldots x'_i \ldots x'_n} = \langle \pi_i(x_i) \rangle_{0 \ldots x'_i \ldots 0} \tag{A2} \]
if the $X$ gates ($\pi$ pulses) are unitary and local. In this case we have
\[ \text{Tr}[\rho_{x'_1 \ldots x'_i \ldots x'_n} \pi_i(x_i)] = \text{Tr}[U \rho_0 \ldots x'_i \ldots 0 U^\dagger \pi_i(x_i)], \tag{A3} \]
where $\rho_{x'_i}$ is the state obtained immediately after preparing classical state $x'$ and $U$ is the unitary corresponding to state preparation on qubits other than $i$. Because the gates are assumed to be local, $U$ commutes with $\pi_i$. We then obtain
\[ \text{Tr}[\rho_{x'_1 \ldots x'_i \ldots x'_n} \pi_i(x_i)] = \text{Tr}[\rho_{0 \ldots x'_i \ldots 0} \pi_i(x_i)], \tag{A4} \]
as required.

Appendix B: Properties of the transition matrices

The matrix $T(x|x')$ has the property that each column sums to unity. This is easily seen from the definition (1):
\[ \sum_x T(x|x') = \left\langle \sum_x |x\rangle \langle x| \right\rangle_{x'} = 1. \tag{B1} \]
The single-qubit transition matrices (3) and $T_{\text{prod}}$ also have this property. It follows that $T_{\text{corr}}$ satisfies $\sum_x T_{\text{corr}}(x|x') = 0$ for every $x'$. It is also easily seen that $T_{\text{pair}}$ satisfies
\[ \forall x': \sum_x T_{\text{pair}}(x|x') = 0. \tag{B2} \]
The neglected higher order terms in the series (7) also have this property, which guarantees that probability is conserved as higher-order fluctuations are included in the model.
Appendix C: Raw transition matrix data for IBM qubits $Q_1 Q_2 Q_3 Q_4$

On a chain $Q_1 Q_2 Q_3 Q_4$ of adjacent IBM qubits we find [7]

$$T_1 = \begin{pmatrix} 98.3\% & 13.5\% \\ 1.7\% & 86.5\% \end{pmatrix}, \quad T_2 = \begin{pmatrix} 99.0\% & 18.4\% \\ 1.0\% & 81.6\% \end{pmatrix}, \quad T_3 = \begin{pmatrix} 96.6\% & 5.8\% \\ 3.4\% & 94.2\% \end{pmatrix}, \quad T_4 = \begin{pmatrix} 96.5\% & 15.3\% \\ 3.5\% & 84.7\% \end{pmatrix},$$

$$T_{\text{prod}} = \begin{pmatrix} 90.7\% & 14.4\% & 5.5\% & 0.9\% & 16.8\% & 2.7\% & 1.0\% & 0.2\% & 12.5\% & 2.0\% & 0.8\% & 0.1\% & 2.3\% & 0.4\% & 0.1\% & 0 \\ 0.3\% & 79.6\% & 0.2\% & 4.8\% & 0.6\% & 14.8\% & 0 & 0.9\% & 0.5\% & 10.9\% & 0 & 0.7\% & 0 & 2.0\% & 0 & 0.1\% \\ 3.2\% & 0.5\% & 88.5\% & 14.1\% & 0.6\% & 0 & 16.4\% & 2.6\% & 0.4\% & 0 & 12.2\% & 1.9\% & 0 & 0 & 2.3\% & 0.4\% \\ 0.1\% & 2.8\% & 3.2\% & 77.6\% & 0 & 0.5\% & 0.6\% & 14.4\% & 0 & 0.4\% & 0.4\% & 10.7\% & 0 & 0 & 0 & 2.0\% \\ 0.9\% & 0.1\% & 0 & 0 & 74.8\% & 11.9\% & 4.5\% & 0.7\% & 0.1\% & 0 & 0 & 0 & 0 & 10.3\% & 1.6\% & 0.6\% \\ 0 & 0.8\% & 0 & 0 & 2.7\% & 65.6\% & 0.2\% & 4.0\% & 0 & 0.1\% & 0 & 0 & 0.1\% & 0 & 0.4\% & 0.9\% & 0.5\% \\ 0 & 0 & 0.9\% & 0.1\% & 2.6\% & 0.4\% & 72.9\% & 11.6\% & 0 & 0 & 0.1\% & 0 & 0.4\% & 0 & 10.0\% & 1.6\% \\ 0 & 0 & 0 & 0.8\% & 0 & 2.3\% & 2.6\% & 64.0\% & 0 & 0 & 0 & 0.1\% & 0 & 0.3\% & 0.4\% & 8.8\% \\ 1.6\% & 0.2\% & 0 & 0 & 0.3\% & 0 & 0 & 0 & 79.8\% & 12.7\% & 4.8\% & 0.8\% & 14.8\% & 2.4\% & 0.9\% & 0.1\% \\ 0 & 1.4\% & 0 & 0 & 0 & 0.3\% & 0 & 0 & 2.9\% & 70.0\% & 0.2\% & 4.2\% & 0.5\% & 13.0\% & 0 & 0.8\% \\ 0 & 0 & 1.5\% & 0.2\% & 0 & 0 & 0.3% & 0 & 2.8% & 0.4% & 77.8% & 12.4% & 0.5% & 0 & 14.4% & 2.3% \\ 0 & 0 & 0 & 1.3% & 0 & 0 & 0 & 0.2% & 0.1% & 2.5% & 2.8% & 68.3% & 0 & 0.5% & 0.5% & 12.7% \\ 0 & 0 & 0 & 0 & 1.3% & 0.2% & 0 & 0 & 0.8% & 0.1% & 0 & 0 & 65.8% & 10.5% & 4.0% & 0.6% \\ 0 & 0 & 0 & 0 & 0 & 1.1% & 0 & 0 & 0.7% & 0 & 0 & 0 & 2.4% & 57.7% & 0.1% & 3.5% \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.8% & 0.1% & 2.3% & 0.4% & 64.1% & 10.2% \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.1% & 0 & 0 & 0 & 0.7% & 0 & 2.0% & 2.3% & 56.3% \\ 

$$T_{\text{corr}} = \begin{pmatrix} 0 & -1.3\% & 0 & -0.2\% & -1.1\% & -0.4\% & 0 & 0 & 0 & 0 & -0.5\% & -0.2\% & 0 & 0 & -0.2\% & -0.1\% & 0 \\ 0 & -0.6\% & 0 & 0.6\% & 0 & -1.1\% & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.5\% & 0 \\ 0 & 1.3\% & -0.2\% & -6.5\% & 0 & 0.2\% & -1.4\% & -1.5\% & 0.5\% & 0.2\% & 0 & -1.1\% & 0 & 0 & -0.4\% & -0.2\% \\ 0 & 0.6\% & 0 & 6.2\% & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.3\% \\ 0 & 0 & 0 & 0 & 0 & -1.3\% & 0 & 0.5\% & 0 & 0 & 0 & 0 & 0 & 0 & -1.8\% & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.9% & -1.0% & -4.8% & 0 & 0 & 0 & 0 & 0 & 0.3% & 0 & -2.1% & -0.9% \\ 0 & 0 & 0 & 0 & 0 & 0.4% & -0.1% & 4.9% & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.1% & 0.4% \\ 0 & 0 & 0 & 0 & 0 & 1.1% & 0.1% & 0 & 0 & -3.3% & -1.8% & 0.4% & 0 & -0.7% & -0.3% & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.1% & 0.1% & 0 & 0 & -3.3% & -1.8% & 0.4% & 0 & -0.7% & -0.3% & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0% & 0 & 3.2% & 0.1% & 7.2% & 0 & 0 & 0 & 0.6% \\ 0 & 0 & 0 & 0 & 0 & 0.1% & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.9% & 0 & 0.5% & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.8% & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1% & 1.9% & 0 & 0.8% \\ 0 & 0 & 0 & 0 & 0 & 1.3% & 0 & 0 & 0 & 0 & 0 & 0 & 1.5% & 1.0% & 2.1% & -3.6% \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.1% & 0 & 0 & 0 & 0 & 0.5% & 0.2% & 6.2% \end{pmatrix}$
and

\[
T_{\text{pair}} =
\begin{pmatrix}
0 & -1.3\% & 0 & -0.2\% & -1.1\% & -0.5\% & -0.1\% & 0 & -0.5\% & -0.3\% & 0 & 0 & -0.2\% & 0 & 0 & 0 \\
0 & -0.6\% & 0 & 0.6\% & 0 & -1.1\% & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.1\% & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix},
\]

where elements with magnitude smaller than $10^{-3}$ have been written as zero.

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