AN INVESTIGATION OF NEUTRINO-DRIVEN CONVECTION AND THE CORE COLLAPSE SUPERNOVA MECHANISM USING MULTIGROUP NEUTRINO TRANSPORT

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ABSTRACT

We investigate neutrino-driven convection in core collapse supernovae and its ramifications for the explosion mechanism. We begin with a postbounce model that is optimistic in two important respects: (1) we begin with a 15 $M_\odot$ precollapse model, which is representative of the class of stars with compact iron cores; (2) we implement Newtonian gravity. Our precollapse model is evolved through core collapse and bounce in one dimension using multigroup (neutrino energy–dependent) flux-limited diffusion (MGFLD) neutrino transport and Newtonian Lagrangian hydrodynamics, providing realistic initial conditions for the postbounce convection and evolution.

Our two-dimensional simulation begins at 12 ms after bounce and proceeds for 500 ms. We couple two-dimensional piecewise parabolic method (PPM) hydrodynamics to precalculated one-dimensional MGFLD neutrino transport. (The neutrino distributions used for matter heating and deleptonization in our two-dimensional run are obtained from an accompanying one-dimensional simulation. The accuracy of this approximation is assessed.) For the moment, we sacrifice dimensionality for realism in other aspects of our neutrino transport. MGFLD is an implementation of neutrino transport that simultaneously (1) is multigroup and (2) simulates with sufficient realism the transport of neutrinos in opaque, semitransparent, and transparent regions. Both are crucial to the accurate determination of postshock neutrino heating, which sensitively depends on the luminosities, spectra, and flux factors of the electron neutrinos and antineutrinos emerging from their respective neutrinospheres.

By 137 ms after bounce, we see neutrino-driven convection rapidly developing beneath the shock. By 212 ms after bounce, this convection becomes large scale, characterized by higher entropy, expanding upflows and lower entropy, denser, finger-like downflows. The upflows reach the shock and distort it from sphericity. The radial convection velocities at this time become supersonic just below the shock, reaching magnitudes in excess of $10^9$ cm s$^{-1}$. Eventually, however, the shock recedes to smaller radii, and at $\sim$ 500 ms after bounce there is no evidence in our simulation of an explosion or of a developing explosion.

Our angle-averaged density, entropy, electron fraction, and radial velocity profiles in our two-dimensional model agree well with their counterparts in our accompanying one-dimensional MGFLD run above and below the neutrino-driven convection region. In the convection region, the one-dimensional and angle-averaged profiles differ somewhat because (1) convection tends to flatten the density, entropy, and electron fraction profiles, and (2) the shock radius is boosted somewhat by convection. However, the differences are not significant, indicating that, while vigorous, neutrino-driven convection in our model does not have a significant impact on the overall shock dynamics.

The differences between our results and those of other groups are considered. These most likely result from differences in (1) numerical hydrodynamics methods; (2) initial postbounce models, and, most important; (3) neutrino transport approximations. We have compared our neutrino luminosities, rms energies, and inverse flux factors with those from the exploding models of other groups. Above all, we find that the neutrino rms energies computed by our multigroup (MGFLD) transport are significantly lower than the values obtained by Burrows and coworkers, who specified their neutrino spectra by tying the neutrino temperature to the matter temperature at the neutrinosphere and by choosing the neutrino degeneracy parameter arbitrarily, and by Herant and coworkers in their transport scheme, which (1) is gray and (2) patches together optically thick and thin regions. The most dramatic difference between our results and those of Janka and Müller is exhibited by the difference in the net cooling rate below the gain radii: Our rate is 2–3 times greater during the critical 50–100 ms after bounce.

We have computed the mass and internal energy in the gain region as a function of time. Up to $\sim$ 150 ms after bounce, we find that both increase as a result of the increasing gain region volume, as the gain and shock radii diverge. However, at all subsequent times, we find that the mass and internal energy in the gain region decrease with time in accordance with the density falloff in the preshock region and with the flow of matter into the gain region at the shock and out of the gain region at the gain radius. Therefore, we see no evidence in the simulations presented here that neutrino-driven convection leads to mass.

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and energy accumulation in the gain region.

We have compared our one- and two-dimensional densities, temperatures, and electron fractions in the region below the electron neutrino and antineutrino gain radii, above which the neutrino luminosities are essentially constant (i.e., the neutrino sources are entirely enclosed), in an effort to assess how spherically symmetric our neutrino sources remain during our two-dimensional evolution, and therefore, in an effort to assess our use of precalculated one-dimensional MGFLD neutrino distributions in calculating the matter heating and deleptonization. We find no difference below the neutrinosphere radii. Between the neutrinosphere and gain radii we find no differences with obvious ramifications for the supernova outcome. We note that the interplay between neutrino transport and convection below the neutrinospheres is a delicate matter and is discussed at greater length in another paper (Mezzacappa and coworkers). However, the results presented therein do support our use of precalculated one-dimensional MGFLD in the present context.

Failure in our “optimistic” 15 $M_\odot$ Newtonian model leads us to conclude that it is unlikely, at least in our approximation, that neutrino-driven convection will lead to explosions for more massive stars with fatter iron cores or in cases in which general relativity is included.

Subject headings: convection — elementary particles — stars: interiors — supernovae: general

1. INTRODUCTION

1.1. Convection and the Supernova Mechanism

Much of the current research on the core collapse supernova mechanism is focused on the role of convection. This is motivated in part by a number of observations of SN 1987A, which indicate that extensive mixing occurred throughout much of the ejected material, which, by inference, points to fluid instabilities arising during the explosion itself. The early (1987 July) detection of continuum X-rays and $\gamma$-rays (Dotani et al. 1987; Sunyaev et al. 1987) and of the 847 and 1238 keV $^{56}$Co $\gamma$-ray lines (1987 August) (Matz, Share, & Chupp 1988; Matz et al. 1988), confirmed shortly thereafter by others (Cook et al. 1988; Gehrels, Leventhal, & MacCallum 1988; Mahoney et al. 1988; Sandie et al. 1988; Wilson et al. 1988; Teegarden et al. 1989), suggests that mixing of the $^{56}$Co in the ejecta must have occurred. $^{56}$Co arises from $^{56}$Ni, which is created in the deep silicon layer. Without the breaking of spherical symmetry, $^{56}$Co and other radioactive elements would remain buried under the massive stellar envelope for about a year, until the latter became transparent by expansion (Gehrels, MacCallum, & Leventhal 1987; McCray, Shull, & Sutherland 1988; Pinto & Woosley 1988; Xu et al. 1988).

Even more direct evidence of significant mixing is the very high Fe velocities ($\gtrsim 3000$ km s$^{-1}$) inferred from infrared observations of $^{56}$Fe II lines (Erikson et al. 1988; Rank et al. 1988; Hass et al. 1990; Spyromilio, Meikle, & Allen 2000), the very large $^{56}$Co velocities inferred from the 847 keV line profile by the GRIS experiment (Tueller et al. 1990), and hydrogen velocities as low as 800 km s$^{-1}$ (Höflich 1988). These observables are evidence that some of the $^{56}$Ni (the progenitor of $^{56}$Co and $^{56}$Fe) was mixed out to the hydrogen envelope. Because this degree of mixing is not reproduced by simulations in which the mixing arises only from Rayleigh-Taylor instabilities in the expanding envelope (Arnett, Fryxell, & Müller 1989; Den, Yoshida, & Yamada 1990; Hachisu et al. 1990; Yamada, Nakamura, & Oohara 1990; Fryxell, Arnett, & Müller 1991; Müller, Fryxell, & Arnett 1991; Herant & Benz 1991, 1992), it must be presumed that these instabilities were preceded by a prior round of instabilities, most likely occurring in the explosion itself (Herant & Benz 1992).

In addition to the observational evidence of extensive mixing due to fluid instabilities, there are compelling theoretical reasons to consider convection. The first and obvious reason is that there are several unstable regions that develop in the postcollapse core, both below and above the neutrinosphere (Epstein 1979; Arnett 1986, 1987; Bethe, Brown, & Cooperstein 1987; Burrows 1987; Bethe 1990). The second is the failure to explode of supernova simulations that do not incorporate fluid instabilities (Bruenn 1993; Cooperstein 1993; Wilson & Mayle 1993) and the expectation that fluid instabilities, by enhancing the transport of lepton-rich matter to the neutrinosphere or by augmenting the neutrino energy deposition efficiency above the neutrinosphere, will be helpful in generating explosions. These expectations derive from the nature of the explosion “power-up” phase, as envisioned by the current core collapse supernova paradigm. According to this paradigm, which is referred to as the “shock reheating mechanism” or “delayed mechanism” (Wilson 1985; Bethe & Wilson 1985), the shock launched into the outer core at core bounce stalls between 100 and 200 km because of nuclear dissociation and neutrino radiation. The shock then becomes an accretion shock, and within tens of milliseconds, a quasi-steady state structure is established in which infalling matter encountering the shock is dissociated into free nucleons and then heated by the transfer and deposition of energy by neutrinos radiating from the hot contracting core. As this matter continues to flow inward, neutrino and compressional heating increase its temperature until the cooling rate, which goes as the sixth power of the temperature, exceeds the heating rate. The inflowing matter therefore cools and eventually accretes onto the core. The radius at which the heating and cooling rates are equal is referred to as the “gain radius.” If the neutrino heating is sufficiently rapid in the region between the shock and the gain radius, the increased thermal pressure behind the shock will allow it to overcome the accretion ram pressure and propagate out through the envelope, thus producing a supernova.

1.2. Fluid Instabilities: Preliminaries

We emphasize again that spherically symmetric supernova simulations that have not incorporated fluid instabilities fail to produce the necessary heating to revive the shock. To appreciate the potential role of fluid instabilities in reviving the shock, we begin with the fact that the energy transfer between the neutrinos and matter behind the shock is mediated primarily by the charged current reactions:

$$v_e + n \rightarrow p + e^-, \quad \bar{v}_e + p \rightarrow n + e^+.$$  (1)
It follows that the heating and cooling rates by $v_s$'s are given by

$$\frac{\text{Heating}}{\text{Nucleon}} \propto L_{v_s} \langle \epsilon_{v_s}^e \rangle \left( \frac{1}{N_{v_s}} \right), \tag{2}$$

$$\frac{\text{Cooling}}{\text{Nucleon}} \propto T_m^6, \tag{3}$$

with similar expressions for the $v_e$'s. Here, $L_{v_s}$ is the $v_s$ luminosity; $\langle \epsilon_{v_s}^e \rangle$ is the $v_s$ mean square energy, averaged with respect to $\epsilon_{v_s}^e$; $1/N_{v_s} \approx 1$ is the inverse flux factor, which equals $c \times U_{v_s}/F_{v_s}$, where $U_{v_s}$ and $F_{v_s}$ are the $v_s$ energy density and flux, respectively; and $T_m$ is the local matter temperature. We have neglected the electron degeneracy of the matter in equation (3).

1.3. Fluid Instabilities below the Neutrinosphere

Fluid instabilities occur in several distinct regions in a postcollapse stellar core. Near and below the neutrinosphere, the dissipation of the shock because of nuclear dissociation and $v_e$ radiation will imprint a negative-entropy gradient and therefore destabilize this region to entropy-driven convection (Arnett 1986, 1987; Burrows 1987). While the convection will not be sustained, it can lead to a rapid initial turnover of the region. The material near and below the neutrinosphere will also be destabilized by a negative lepton gradient (Epstein 1979). This results from the lepton "trough" produced near the neutrinosphere by the combination of rapid electron capture and $v_e$ escape. A negative lepton gradient connects this region with the more lepton-rich material at smaller radii. Because electron capture and $v_e$ radiation continue near the neutrinosphere as the proto-neutron star evolves, the tendency of lepton-driven fluid motions to flatten the negative lepton gradient will be resisted, and the instability driving these fluid motions may tend to persist. Deeper in the core, the destabilizing effect of the negative lepton gradient will be counteracted by the stabilizing effect of the positive entropy gradient left over from the shock as it propagates through this region while gathering strength from the rebounding inner core. With the diffusion of both energy and leptons by neutrinos, this region can be destabilized on a diffusion timescale, if the diffusion rates for energy and leptons are different.

1.4. Fluid Instabilities above the Neutrinosphere

As infalling material encounters the shock, it is dissociated into free neutrons and protons if the shock is within a radius of ~200 km. (At larger radii, there will be an admixture of alpha particles.) As this material continues to flow inward, it will be heated by the charged-current reactions (eq. [1]), until it reaches the gain radius. Furthermore, the neutrino heating is strongest just beyond the gain radius and decreases farther out as the neutrino flow becomes radially diluted. These two factors conspire to create a negative entropy gradient between the gain radius and the shock, which will be unstable to entropy-driven convection. Because this convection is driven by neutrino heating, it will persist as long as neutrinos heat, or until an explosion develops. This convection, which is the subject of this paper, will be referred to as "neutrino-driven convection" or "ND convection."

Bethe (1990) pointed out that ND convection would reduce or eliminate the density inversions found at the outer edge of the hot bubble that formed in the successful supernova simulations of Wilson & Mayle (1993). ND convection
was not incorporated in these calculations, and an enormous negative density gradient developed behind the shock as it propagated out. He noted also that ND convection would bring cooler material down to the vicinity of the gain radius, where it would be more efficiently heated because the cooling rate, as given by equation (3), would be reduced; the outward flow of hot material would bring energy to the shock and thus help to support it.

The first two-dimensional supernova simulation incorporating ND convection was performed by Herant et al. (1992) using a smooth-particle hydrodynamics (SPH) code. They found that, if ND convection is able to develop, it plays a crucial role in generating an efficient explosion. The convective flows in their calculations became large-scale “long-wavelength” flows, providing an efficient (separate hot and cold matter) way of conveying low-entropy matter back to the shock. Furthermore, the latent heat of the alpha–free-nucleon transition enables the storage and transport of large amounts of energy without large temperature increases. They noted finally that ND convection would allow accretion to continue while the shock is moving outward in radius, thereby maintaining an energy input to the shock from the neutrinos radiated by the accreting matter. Colgate et al. (1993) examined the accretion onto the neutron star during the explosion. (Herant et al. 1992 could not resolve this aspect of the problem in their simulations, and they ignored the contribution to the neutrino luminosity by the accreting matter and suggested that accretion would be unstable, leading to episodic bursts of high-energy neutrinos that would be efficiently absorbed by the overlying material. This conjecture has not yet been confirmed.) In a set of more refined and self-consistent calculations, Herant et al. (1994) confirmed the conclusions of Herant et al. (1992) and emphasized the large-scale character of ND convection and its role in producing robust and self-regulated supernova explosions. They concluded that ND convection is a powerful “convective engine” that feeds energy into the shock until the required explosion energy has been reached. Thus, ND convection almost guarantees a successful explosion.

A simulation using entirely different numerical techniques by Miller, Wilson, & Mayle (1993) reached a very different conclusion. They used a semi-Eulerian finite-difference method (Bowers & Wilson 1991), and a combination of lightbulb and two-temperature diffusion schemes for neutrino transport, calibrated by their sophisticated one-dimensional code. As in the Herant et al. (1992) and the Herant et al. (1994) simulations, they found that ND convection tended to evolve to a large-scale, long-wavelength flow. However, the growth rates for their convection were much slower, e.g., an e-folding time of 13 ms versus a fully developed convection in 20–25 ms after bounce. Because their convection growth rates were slow, and because the unstable conditions for convection’s growth are not well established until ~0.1 s after bounce, Miller et al. (1993) found that ND convection was not particularly important in their simulations. Acknowledging that the limitations of their two-dimensional model favored explosions, they concluded that more realistic two-dimensional simulations would show that the effects of ND convection are unlikely to revive a stalled shock.

Two-dimensional supernova simulations using hydrodynamics codes based on the piecewise parabolic method (PPM) of Colella & Woodward (1994) and incorporating ND convection were performed by Burrows et al. (1995), and by Janka & Müller (1995, 1996). In contrast to the results of Herant et al. (1992, 1994) and Miller et al. (1993), the PPM calculations found ND convection to be more turbulent, although a large-scale flow was exhibited. The flows consisted of high-entropy rising bubbles and balloons, distorted into mushrooms by Kelvin-Helmholtz instabilities, and low-entropy descending narrow flux tubes. The mode numbers ranged from 2 to 10. Burrows et al. (1995) implemented radial transport along their angular rays in a gray diffusion approximation for optical depths \( \geq 1 \), with a neutrino-matter coupling for optical depths \( \leq 1 \) that assumes particular electron neutrino and antineutrino energy spectra. Unlike the Herant et al. (1994) simulations, they found that neither mass nor energy accumulates in the convection region. They were unable to verify the basic “convective engine” paradigm of Herant et al. (1994) and found, instead, that if an explosion ensues, it is because of the decline in the accretion ram rather than an increase in the shock energy (see also Burrows & Goshy 1993).

Burrows et al. (1995) found that ND convection allows higher entropies to develop in the convective region than in one-dimensional simulations because of the longer time a given fluid element spends in the gain region (one or two cycle times before passing inward through the gain radius). This, combined with the dynamic pressure of the buoyant plumes, causes the stalled shock radius to equilibrate at a larger value and therefore at a lower gravitational potential. The corresponding reduction in the accretion ram makes it easier for two-dimensional supernova models to explode, but does not guarantee an explosion. Reducing the magnitudes of the neutrino-matter coupling terms by assuming a different neutrino spectrum led to failures.

Janka & Müller (1995, 1996) performed a very useful parameter study of one- and two-dimensional supernova simulations by varying a lightbulb neutrino source. Like Burrows et al. and unlike Herant et al. (1994), they did not find an accumulation of energy in the ND convection region until the typical value of \( 10^{54} \) ergs was reached. However, unlike the Burrows et al. simulations, their simulations did not exhibit the vigorous boiling that immediately preceded an explosion. They also found that accretion onto the proto–neutron star ceased when an explosion got underway; therefore, the explosion was not continuously fed by accretion luminosity. They did find that ND convection leads to a higher net efficiency of neutrino energy deposition and is an efficient mechanism of transporting energy to the shock. Their overall conclusions was that ND convection is important for generating explosions only in a rather narrow window \( (\Delta L/L \sim 20\%) \) of neutrino luminosities. Below this window, neither one-dimensional nor two-dimensional simulations would explode; above this window, both would explode.

1.5. This Work

It is apparent from the above discussion that there is considerable disagreement regarding the role of ND convection in the supernova mechanism. All of the above simulations were pioneering in the sense that multidimensional hydrodynamics was used. However, computer limitations necessitated that rather severe approximations be made in the neutrino transport and the coupling of neutrinos to matter. We feel that much of the disparity in the above...
results can be traced to differences in the treatment of neutrinos. In fact, we will show in this paper that the supernova simulations incorporating ND convection that give rise to explosions assume a hard neutrino spectrum (thus favoring explosions) when compared with multigroup calculations.

The purpose of this paper is to eliminate some of the uncertainties associated with neutrino transport approximations by “coupling” (the meaning of this term will be made clear below) a sophisticated multigroup flux-limited diffusion (MGFLD) neutrino transport code with the numerically nondiffusive high-order PPM hydrodynamics code, EVH-1, to examine the role of ND convection in the supernova mechanism. The importance of using multigroup neutrino transport is that the neutrino energy spectrum is part of the solution and need not be assumed. The importance of using flux-limited diffusion is that the transport of neutrinos from optically thick to optically thin regions is a part of the solution and need not be assumed. The importance of using flux-limited diffusion is that the transport of neutrinos from optically thick to optically thin regions is computed seamlessly and with sufficient realism.

Our procedure, described in more detail in Mezzacappa et al. (1998), is to perform a one-dimensional (Lagrangian) simulation of core collapse and the postbounce evolution (1) to generate running (i.e., time-dependent) inner and outer boundary conditions for all of the relevant variables at our fixed inner and outer Eulerian boundaries, and (2) to generate tables of the zeroth angular moments of the neutrino distributions as a function of time, radius, and neutrino energy. The two-dimensional simulation is then carried out using the inner and outer boundary conditions generated by our one-dimensional simulation, and the neutrino moment tables are used to compute the local energy and lepton exchange between the matter and the neutrinos.

Because ND convection occurs between the gain radius and the shock, the convecting material has small neutrino optical depths for all of the relevant neutrino energies. Thus, we expect the feedback between the hydrodynamics and the neutrino transport in this region to be minimal. If this were the entire story, coupling one-dimensional MGFLD to two-dimensional hydrodynamics would be an excellent approximation. However, the neutrino radiation field in the gain region is largely determined by the neutrino transport below it, particularly above and below the neutrinospheres. Therefore, to fully assess the accuracy of one-dimensional transport, we must consider the feedback on the radiation field that results from (1) asymmetric accretion through the gain radius from the ND convection region and (2) non-spherically symmetric structure below the neutrinospheres that may result, for example, from proto-neutron star convection (see Wilson & Mayle 1993; Herant et al. 1994; Burrows et al. 1995; Keil et al. 1996; Mezzacappa et al. 1998). We consider the first effect here. The second effect has been considered at length in Mezzacappa et al. (1998), and the key results relevant to the claims made in this paper are mentioned again here (in § 4).

In § 2 we briefly describe our initial models, codes, and methodology. Our results are then presented in § 3. Section 4 is devoted to an assessment of our one-dimensional MGFLD neutrino transport approximation. In § 5 we summarize our results, compare them with those of other groups, and state our conclusions.

2. INITIAL MODELS, CODES, AND METHODOLOGY

We begin with the 15 $M_{\odot}$ precollapse model S15s7b (Woosley & Weaver 1995; Weaver & Woosley 1997). The initial model was evolved through core collapse and bounce using MGFLD neutrino transport and Lagrangian hydrodynamics, providing realistic initial conditions for the postbounce convection and evolution. The one-dimensional data at 12 ms after bounce (211 ms after the initiation of core collapse) were mapped onto our two-dimensional Eulerian grid. The inner and outer boundaries of our grid were chosen to be at radii of 20 and 1000 km, respectively. We used 128 nonuniform radial spatial zones, and for $\theta$ we used 128 uniform angular zones spanning a range of 180° (together with reflecting boundary conditions).

Because the finite differencing in our PPM scheme is nearly noise free, and because we cannot rely on machine roundoff to seed convection in a time that is short compared with the hydrodynamics timescales in our runs, we seeded convection everywhere on the grid by applying random velocity perturbations between $\pm 5\%$ to the radial and angular velocities. Our seeding included the initial Ledoux unstable regions below and around the neutrinospheres immediately following core bounce.

Time-dependent inner and outer boundary data for the enclosed mass, density, temperature, electron fraction, pressure, specific internal energy, and velocity were supplied by our accompanying one-dimensional MGFLD run, which was continued for this purpose for 700 ms after bounce. The inner and outer boundaries were chosen to be at 20 km (deep within the core) and at 1000 km (well outside the shock), respectively, which are regions in which the flow is spherically symmetric.

It is also important to note here that for the matter heating and deleptonization in our two-dimensional run, we use tables of precalculated neutrino distributions, $\psi_v(r, t; E_v)$, obtained from our accompanying one-dimensional run that implements MGFLD and Lagrangian hydrodynamics. Both our one- and two-dimensional simulations are Newtonian. The accuracy of using precalculated one-dimensional neutrino distributions in our two-dimensional models is discussed in § 4.

Details of the codes used in our simulations, and more detail on our methodology, can be found in Mezzacappa et al. (1998).

3. RESULTS

Figure 1 shows the velocity, density, entropy, and electron fraction profiles at the start of our run, 12 ms after bounce.

In Figures 2a–2d we plot the results of one-dimensional simulations using our MGFLD code and our PPM hydrodynamics code. (The PPM code has been generalized for realistic equations of state and for neutrino heating and cooling and deleptonization.) We compare density, entropy, electron fraction, and velocity at two different times during the course of a 500 ms run. The figures illustrate that the agreement is excellent, with the primary differences resulting from our PPM code’s ability to resolve the shock better. This gives us confidence (1) that the neutrino heating and cooling and deleptonization are simulated well by our PPM code and (2) that differences between two-dimensional and one-dimensional simulations of the shock dynamics will result from convection’s presence in the former case, not from numerical sources.

Figure 3a (Plate 19) shows the two-dimensional entropy profiles at three select times during our 500 ms run. At 137 ms after bounce, ND convection is rapidly developing below the shock, having entered the nonlinear regime.
There is evidence of expanding higher entropy rising flows, and denser lower entropy infalling matter. Roughly seven rising plumes can be counted at this time. The convection has not yet reached the shock. By 212 ms after bounce, ND convection is fully developed. A clear contrast is evident between the higher entropy upflows and the lower entropy downflows. At this time, the rising plumes have reached the shock and distorted it, and the convection is semiturbulent, although only four (versus seven) plumes in a range of 180° can be counted now. Although more turbulent, our flow patterns are in qualitative agreement with those obtained by Herant et al. using SPH (Herant 1994); the latter obtain a more orderly low-mode convection. However, differences between the SPH and PPM simulations are significant and most likely have important dynamical consequences. The low-mode convection obtained by Herant et al. (1992, 1994) led them to develop their “thermodynamic engine” interpretation of the supernova mechanism, although this “engine” apparently did not work in the Miller et al. (1993) simulations, in which low-mode convection was also exhibited. Such an interpretation is even less possible in the PPM simulations, where high- and low-entropy flows are not as well separated. We also mention here that our flow patterns are in qualitative agreement with those obtained by other groups that implement PPM hydrodynamics (Burrows et al. 1995, Janka & Müller 1996). Finally, at 512 ms after bounce, Figure 3a illustrates that our shock has receded to smaller radii, and that the convection beneath it has become even more turbulent, mixing to an even greater extent high- and low-entropy matter. It is clear that, even in the presence of large-scale ND convection, we do not, with our MGFLD neutrino transport, obtain an explosion.

In Figure 3b, we plot the one-dimensional entropy and two-dimensional angle-averaged entropy, both computed by the PPM code, as a function of radius at the same three postbounce times. The angle-averaged entropy is defined by

\[ \langle S \rangle(i) = \frac{1}{A(i)} \sum_{j=1}^{m_s} A(i, j) S(i, j), \]  

(4)

where

\[ A(i, j) = 2\pi r^2(i) \sin \theta(j) d\theta \]  

(5)

and where \( A(i) = 4\pi r^2(i) \) and \( d\theta = \pi/128 \). At 137 ms after bounce, as expected, there is very little difference between the one- and two-dimensional profiles; convection has not yet fully developed and one would not expect large differences between the two simulations. At 212 ms after bounce, when convection has fully developed, it has clearly flattened the peak in the entropy profile between 90 and 150 km, relative to the one-dimensional case, and it is also clear that the shock location is a bit farther out; convection does seem to have an effect on the shock radius in our simulations, albeit marginal, pushing it out to slightly larger radii. Moreover, at this time, the maximum angle-averaged entropy is 13, which is not particularly high, nor even marginal for an explosion. For example, Burrows et al. (1995) find that the entropies in their rising bubbles reach 25–35 prior to their explosions. Under explosive conditions, we expect entropies between the cooling proto-neutron star and the shock to rise well past ~ 30, a condition signaling the formation of a low-density radiation-dominated “bubble” behind the shock, as the shock separates itself from the proto-neutron star. At the last time slice, 512 ms after bounce, the shock front is located at smaller radii, and the entropy jump across it is greater. The entropy behind the shock rises in time because the preshock matter density decreases, and because the gravitational potential well becomes deeper as more material accumulates onto the nascent neutron star and the radius of the shock decreases. The peak entropies for this last slice reach 17–18. In addition, the differences between our one- and two-dimensional simulations are less pronounced than at the postbounce time \( t_{pb} = 212 \) ms.

In Figure 4a (Plate 20), we show the two-dimensional, electron fraction profiles at the same three postbounce times. These profiles are complementary to the entropy profiles. In particular, at \( t_{pb} = 512 \) ms we see low-Y_e (low-entropy) matter rising in dramatic expanding plumes, while high-Y_e (high-entropy) matter infalls in dense finger-like flows. The Y_e contrast between rising and falling flows is greatest at \( t_{pb} = 137 \) ms, when convection is developing and matter at disparate Y_e values in the Y_e profile beneath the shock, extending down to the trough near the neutrinospheres, is affected. In time, however, this contrast is lessened, and finally at \( t_{pb} = 512 \) ms the matter in the ND convection region has been sufficiently mixed that very little contrast is exhibited.
Figure 4b plots the angle-averaged electron fraction in our two-dimensional simulation against the corresponding one-dimensional results, both obtained with our PPM code. This quantity is defined by

$$\langle Y_e(i) \rangle = \frac{1}{A(i)} \sum_{j=1}^{n_e} A(i, j) Y_e(i, j).$$  \hspace{1cm} (6)$$

No differences are seen near the $Y_e$ trough, beneath the ND convection region; all differences occur directly below the shock, as expected. For example, a flattening in $Y_e$ is evident between $t_{pb} = 137$ ms and $t_{pb} = 512$ ms, which is clearly seen in the region between 90 and 150 km at $t_{pb} = 212$ ms. In addition, in both one and two dimensions, the electron fraction gradient steepens as the shock recedes and high-$Y_e$ matter advects inward through it. Similar to what we found in comparing the one- and two-dimensional entropy profiles, very little difference between our one- and two-dimensional electron fraction profiles is seen at the end of our simulation at $t_{pb} = 512$ ms.

In Figure 5a (Plate 21), we show the two-dimensional radial velocity at $t = 137, 212,$ and $512$ ms after bounce. Comparing Figures 3a and 5a, high-velocity portions of the flow, accelerated by the buoyancy force, are associated with both inflow and outflow. This is most easily seen looking at the second panel in both figures. The associations between the high inward radial velocity of the finger-like flow at $\theta \approx 60^\circ$ and the high-velocity outflow associated with a portion of the rising plume at $\theta \approx 120^\circ$ can easily be made.
In Figure 5b, we plot the angle-averaged radial velocity from our two-dimensional run against the radial velocity from our one-dimensional PPM run, as a function of time over our 500 ms window. The angle-averaged radial velocity is defined by

$$\langle v \rangle = \frac{1}{A(\theta)} \sum_{j=1}^{n_{\theta}} A(i, j) |v_r(i, j)| .$$

(7)

The two profiles agree quite well before convection has fully developed at $t_{pb} = 137$ ms; they differ most at $t_{pb} = 212$ ms, when convection is fully developed; and they are in closer agreement at the end of our run at $t_{pb} = 512$ ms. When convection has had a chance to develop, the angle-averaged shock radius is farther out relative to the one-dimensional shock radius, but not significantly so. Moreover, in both simulations, the shock recedes and strengthens, as evidenced by the increased velocity jump across it at the end of our run. (In both our one- and two-dimensional simulations, there is a small periodic inward and outward movement of the shock, i.e., a recession and a strengthening, followed by an advance and a weakening, and so on. As the shock moves out after strengthening, less material is advected onto the proto-neutron star, and the accretion luminosity drops. This decreases the neutrino heating behind the shock thus undermining its pressure support, and the shock moves inward. As the shock moves inward, the accretion luminosity rises and the shock is once again strengthened and moves out. This behavior has also been noted by others, e.g., Mayle 1985.)

In Figure 6, we plot the angle-averaged radial and angular convection velocities in our two-dimensional run for the
three postbounce slices we have focused on in this discussion, along with the angle-averaged sound speed. The convection velocities are defined by

$$\langle v_r \rangle_r = \frac{1}{A(i)} \sum_{j=1}^{n_h} A(i, j) |v_r(i, j)| - \langle v_r \rangle$$

and

$$\langle v_\theta \rangle_\theta = \frac{1}{A(i)} \sum_{j=1}^{n_h} A(i, j) |v_\theta(i, j)| .$$

At the time when convection is fully developed, at $t_{pb} = 212$ ms, the radial convection velocities in our simulation become supersonic just below the shock. Notice, also, the anticorrelation between the radial and angular convection velocity profiles, which is consistent with the convective flow shown, for example, in Figure 3a. The radial convection velocity is at a minimum at the top of the convecting region where the matter turns over, at which point the angular convection velocity is at a maximum. At $t_{pb} = 512$ ms, the radial convection velocity is still supersonic just below the shock, despite the recession of the shock and the more turbulent convection.

For matter in nuclear statistical equilibrium, the thermodynamic state is completely determined once the density, temperature (or equivalently, entropy), and electron fraction are known. In Figures 3b and 4b, we have already graphed the angle-averaged entropy and electron fraction from our two-dimensional simulation, together with the corresponding profiles from our one-dimensional MGFLD run. For completeness, in Figure 7, we plot the angle-averaged density. In the two-dimensional case, at $t_{pb} = 212$ and 512 ms, the shock is farther out in radius and the density jump therefore occurs at larger radii. Additionally,
the shock is distorted in our two-dimensional simulation, and this gives the appearance, in the angle-averaged density, of shock smearing. Away from the shock, however, the one- and two-dimensional profiles are very nearly the same.

The neutrino heating rate (in MeV nucleon\(^{-1}\)) in the region between the neutrinospheres and the shock can be written as (see also eq. [2])

\[
\dot{\varepsilon} = X_n \frac{L_{\nu_e}}{\lambda_0} \left( \frac{1}{\mathcal{F}} \right) + X_p \frac{L_{\bar{\nu}_e}}{\lambda_0} \left( \frac{1}{\mathcal{F}} \right),
\]

where \(X_n, X_p\) are the neutron and proton fractions; \(\lambda_0, \bar{\lambda}_0\) are the coefficients of the \(E_{\nu,\bar{\nu}}\); neutrino-energy dependences in the electron neutrino and antineutrino mean free paths, respectively; \(L_{\nu_e}, L_{\bar{\nu}_e}\); and \(\langle 1/\mathcal{F}, \mathcal{F} \rangle\) are the electron neutrino and antineutrino luminosities, mean square energies, and mean inverse flux factors, respectively, as defined by

\[
L_{\nu_e} = 4\pi r^2 \frac{2\pi c}{(hc)^3} \int dE_{\nu_e} d\mu_{\nu_e} E_{\nu_e}^3 \mu_{\nu_e} f.
\]  

\[
\langle E_{\nu_e}^2 \rangle = \frac{\int dE_{\nu_e} d\mu_{\nu_e} E_{\nu_e}^2 \mu_{\nu_e} f}{\int dE_{\nu_e} d\mu_{\nu_e} E_{\nu_e}^2 \mu_{\nu_e} f},
\]  

\[
\langle 1/\mathcal{F} \rangle = \frac{\int dE_{\nu_e} d\mu_{\nu_e} E_{\nu_e}^2 \mu_{\nu_e} f}{\int dE_{\nu_e} d\mu_{\nu_e} E_{\nu_e}^2 \mu_{\nu_e} f} = \frac{cU_{\nu_e}}{F_{\nu_e}}.
\]

Corresponding quantities are similarly defined for the electron antineutrinos. In equations (11)–(13), \(f\) is the electron-neutrino distribution function, which is a function of the electron-neutrino direction cosine, \(\mu_{\nu_e}\), and energy, \(E_{\nu_e}\). In equation (13), \(U_{\nu_e}\) and \(F_{\nu_e}\) are the electron-neutrino energy density and flux. Success in generating explosions by neu-
trino heating must ultimately rest on these three key neutrino quantities. In Figures 8 and 9, we plot them at a radius of 1000 km, as a function of time during our simulation. Figure 8a shows the electron neutrino and antineutrino luminosities and rms energies over the course of our entire simulation, where Figure 9 provides more detail over the crucial first 100 ms. The inverse flux factor is not plotted here because it is constant and unity at this radius. The luminosities and rms energies at $r = 1000$ km are representative of the corresponding values in the gain region because they flatten out above the neutrinosphere radii. (For completeness, Fig. 8b shows the muon and tau neutrino and antineutrino luminosities and rms energies over the same period.)

In Figure 10a, we plot the maximum net heating and net cooling rates as a function of time during the course of our simulation. Figure 10c provides more detail over the first 150 ms. The drop and flattening in our net heating rate between the start of our run and 100 ms are consistent with those exhibited by Janka & Müller (1996) in their Figure 13 for their exploding two-dimensional model “T4e.” Between 50 and 100 ms, our net heating rate levels off at a value $\sim 87$ MeV baryon$^{-1}$ s$^{-1}$, and the Janka & Müller rate levels off at a comparable, though somewhat larger, value $\sim 93$ MeV baryon$^{-1}$ s$^{-1}$. However, the most dramatic difference between our results and theirs surfaces when the magnitudes of the net cooling rate are compared. Between 50 and 100 ms, our rate is $\sim -193$ MeV baryon$^{-1}$ s$^{-1}$, whereas the Janka & Müller rate is 2–3 times less: $\sim -75$ MeV baryon$^{-1}$ s$^{-1}$. Our dramatic increase in net cooling below the gain radius, resulting from our use of MGFLD, must account in large part for the lack of an explosion in our model, despite the somewhat comparable net heating rate in the gain region, although Janka & Müller also begin with a larger initial shock radius that results from starting their Newtonian simulation with one of our general relativistic postbounce models.

One final note: Equation (10) is appropriate for neutrino emission and absorption. In our simulation, the heating contributions from neutrino-electron scattering (NES) are negligible, amounting to 3%–5% corrections for our postshock entropies (≤17–18). At typical postshock densities between $10^9$ and $10^{10}$ g cm$^{-3}$, entropies $\sim 30$ (almost twice as large as our entropies) would be required before the number density of pairs would become comparable to the baryon number density, i.e., before our NES heating contributions would double.

Figure 11 shows the evolution of the gain and shock radii in our one- and two-dimensional simulations. The shock follows the typical trajectory seen in all of our models: It moves out in radius to about 200 km in a quasi-hydrostatic way because of the rapid decline in the accretion ram pressure, and then recedes with some oscillation (most noticeably in the one-dimensional simulation) to a radius between 100 and 125 km at $t_{pb} = 500$ ms. Initially, the gain and shock radii diverge, giving rise to an increasingly larger gain region, but that trend reverses at $t_{pb} \approx 150$ ms. Initially, the gain volume increases with time.

More important, in Figures 12a and 12b, we plot the mass and internal energy in the gain region as a function of time during the course of our simulation. The initial rise in both mass and internal energy results primarily from the early dramatic increase in the gain volume, as the shock and gain radii diverge. However, at later times, both the gain region mass and internal energy decrease monotonically with time. This results because (1) the gain region is decreasing in size, and (2) the density ahead of the shock is falling off. The general trends exhibited by Figures 12a and 12b are indicative of a flow that passes through, rather than accumulates in, the gain region. Moreover, the ratio of the internal energy and mass in the gain region is roughly constant.
Fig. 10a.—(a) Plots of the angle-averaged \( \frac{dE}{dt} \) at 137, 212, and 512 ms after bounce; \( \frac{dE}{dt} \) is the net heating or cooling from electron neutrino and antineutrino absorption and emission. (b) Plots of the maximum net neutrino heating and maximum net neutrino cooling rates as a function of time during the course of our simulation. (c) Plots of the maximum net neutrino heating and maximum net neutrino cooling rates as a function of time during the critical 50–150 ms after bounce.
4. ASSESSING OUR USE OF PRECALCULATED ONE-DIMENSIONAL NEUTRINO DISTRIBUTIONS

Our results depend in part on the assumption that our electron neutrino and antineutrino sources remain to a good approximation spherically symmetric during the course of our two-dimensional run. This requires that there be no significant convection in the region encompassing or below the neutrinospheres and no significant influence of ND convection below the gain radii:

1. Convection below the neutrinospheres.—In a previous paper (Mezzacappa et al. 1998), we presented compelling evidence that, in the presence of neutrino transport, the convective transport of heat and leptons below the neutrinospheres by proto–neutron star convection will be significantly reduced. Our numerical results were supported by timescale analyses and by a simple analytical model, although final conclusions regarding the extent of proto–neutron star convection await fully self-consistent multidimensional multigroup radiation hydrodynamics simulations. Nonetheless, these results are mentioned here in support of the conclusions reached in this paper. In the absence of significant proto–neutron star convection, the imposition of a one-dimensional spherically symmetric neutrino radiation field in the region between the neutrinospheres and the shock, used to compute the neutrino heating and cooling there, should be a good approximation.

2. The influence of ND convection below the gain radii.—

Because our current prescription does not implement a self-consistent two-dimensional radiation hydrodynamics solution, we cannot capture enhancements in the neutrino luminosities emanating from the neutrinosphere region that
result from (1) non-spherically symmetric accretion through the gain radius and/or (2) inwardly propagating nonlinear waves that compress and heat the neutrinosphere region in a non-spherically symmetric way. For example, the dense finger-like low-entropy inflows in the ND convection region may penetrate the gain radius and strike the proto-neutron star surface (Burrows et al. 1995; Janka & Müller 1996). It has been suggested that the associated luminosity enhancements may help trigger explosions (Burrows et al. 1995), but conclusions regarding their benefit have been mixed (Janka & Müller 1996).

To investigate whether these effects would have been important in our simulation, we compared our one- and two-dimensional density, temperature, and electron-fraction snapshots at \( t_{\text{pb}} = 212 \) ms, i.e., at a time when ND convection was most vigorous. The results are represented graphically in Figure 13 (Plate 22). Up to the neutrinosphere radii (~51 km), we found no differences. Between the neutrinospheres and the gain radii (~90 km), we found hot spots in our two-dimensional simulation where \( \Delta T/T \sim 3\% \) over ~\( 1/4 \) of the volume and ~6\% over ~\( 1/2 \) of the volume, and Y\(_e\)-enhanced spots where \( \Delta Y_{e}/Y_{e} \sim 9\% \) over ~\( 1/4 \) of the volume. However, at \( t \sim 100-200 \) ms after bounce, \( L_{\nu_{\alpha}} (50 \) km) \( \approx 2.4 \times 10^{52} \) ergs s\(^{-1}\) and \( L_{\nu_{\tau}} (90 \) km) \( \approx 3.4 \times 10^{52} \) ergs s\(^{-1}\); therefore, only ~33\% of the neutrino luminosities would have been affected by these temperature and electron-fraction enhancements. (The percentages for electron antineutrinos are comparable ~50\% at 100 ms and ~33\% at 200 ms.) We also considered the enhancements to the electron neutrino and antineutrino pair emissivity from the hot spots, because of its strong \( T^{4} \) dependence on the local matter temperature. In our model the pair emissivity contributes only ~10\% to the total electron neutrino and antineutrino emissivity. The hot spots would increase this contribution, but only to ~15\% of the total emissivity.

Considering the small local enhancements in \( T \) and \( Y_{e} \), the small percentage of the volume in which they occur, and the fraction of the neutrino luminosities that would be affected by them, we do not expect these enhancements to have significant ramifications for the supernova outcome.

5. SUMMARY, COMPARISONS, AND CONCLUSIONS

With two-dimensional (PPM) hydrodynamics "coupled" to one-dimensional MGFLD neutrino transport, we see vigorous—in some regions supersonic—ND convection develop behind the shock. Despite this, we do not obtain explosions for what should be an "optimistic" 15 \( M_{\odot} \) model. Beginning with realistic postbounce initial conditions, our simulation has been carried out for ~500 ms, a period that is long relative to the 50–100 ms explosion timescales obtained by other groups, for models that explode.

An important and very interesting feature of our two-dimensional model is that, even in the presence of ND convection, the angle-averaged density, entropy, electron fraction, and radial velocity do not differ very much from their counterparts in our accompanying one-dimensional MGFLD run. The differences arise primarily because convection in our two-dimensional simulation has moved the shock somewhat farther out in radius. This indicates that, while vigorous convection may be present in our models, it does not contribute in any significant way to the angle-averaged shock dynamics.

The differences in outcome from group to group, and even from model to model for a given group, most likely result in large part from several factors:

1. Differences between numerical hydrodynamics methods, in particular, SP and PPM hydrodynamics, most likely contribute. With PPM hydrodynamics, ND convection is more turbulent, and consequently, the separation between high- and low-entropy matter in the gain region, exhibited in the Herant et al. (1992, 1994) simulations, does not obtain. This may have an impact on the neutrino heating efficiency, as discussed by Herant et al. (1994) in proposing their "Carnot engine" interpretation of the supernova mechanism. (Although we again note that Miller et al. found low-mode convection, but not a "Carnot engine.")

2. As stressed in Bruenn & Mezzacappa (1994), it is important to begin any two-dimensional simulation with initial postbounce conditions that have been generated by realistic core collapse and bounce, which necessitates the use of neutrino transport that includes all relevant neutrino interactions, particularly NES. NES is known to have a significant effect on the deleptonization of the core during infall, and consequently on the location and strength of the shock after bounce. We begin our simulations with postbounce configurations that have been evolved through collapse and bounce with MGFLD, and NES included. In Figure 5a of Burrows et al. (1995), the early shock trajectory behaves like an initially stronger "prompt" shock. In contrast, in our simulations the shock moves out quasi-hydrostatically as a result of the accretion of matter through it. The greater initial shock strength in the Burrows et al. simulation most likely results from the difference between simulating core collapse with a gray neutrino diffusion scheme (Burrows & Lattimer 1986), as opposed to MGFLD (Bruenn 1985). For the simulations presented in Herant et al. (1994), the same gray transport scheme is used both for their one-dimensional core collapse evolution and their two-dimensional postbounce evolution. It would be enlightening to investigate the differences in postbounce conditions, in particular, shock location and strength, obtained using gray and multigroup transport during core collapse and bounce. It is also important to stress that we use a one-dimensional Newtonian postbounce profile to start our two-dimensional Newtonian simulations. For Janka & Müller (1996), the initial shock radius increased from ~120 to ~200 km, i.e., by nearly a factor of 2, because they started their two-dimensional Newtonian simulation with our one-dimensional general relativistic postbounce model. This initial boost in shock radius will have an effect on the subsequent shock trajectory.

3. Ultimately, success or failure in generating supernova explosions rests on the right combination of neutrino luminosities, rms energies, and inverse flux factors. Whereas it is possible to list a number of inputs that might differ from group to group, and within each group, from simulation to simulation, the one that stands out the most is the neutrino rms energy. Of course, the neutrino heating rate depends on the square of the neutrino rms energy; therefore, differences in this quantity are magnified when folded into the final heating rate. For example, we find that with the Burrows et al. (1995) specification of both \( T_{e} \) and \( \eta_{e} \), the relationship between their neutrinosphere temperature and neutrino rms energy for their exploding "star" model is \( \langle E_{\nu}^{2} \rangle^{1/2} = \)
When we fit our electron neutrino spectrum at our neutrinosphere radius, we obtain a characteristic dependence on temperature of $3.0 T_{\nu}$. This difference translates to a $40\% - 50\%$ increase in the neutrino heating rate for the Burrows et al. “star” model, with commensurate ramifications for generating explosions. Burrows et al. note that explosions are not obtained in other models, presumably when the value of $\eta_{\nu}$ is chosen differently. Similarly, our mean electron neutrino and antineutrino energies, relative to those by Herant et al. (1994) for their $25 M_\odot$ model in their Figure 10, are significantly lower. At 100 ms after bounce, our mean electron neutrino and antineutrino energies are 10 and 13 MeV, respectively, compared with the 13–14 and $\sim 20$ MeV values obtained by Herant et al. (1994). The differences between our results and the results obtained by the two groups mentioned above result from (a) specification (Burrows et al.) rather than computation of the neutrino spectra in optically thin regions and (b) patching together optically thick and thin regions rather than having a transport scheme that transits through both regions. The maximum net heating rates obtained by Janka & Müller (1996) between their gain radii and shock are comparable to, but somewhat higher than, ours: $\sim 93$ MeV baryon$^{-1}$ s$^{-1}$ versus $\sim 87$ MeV baryon$^{-1}$ s$^{-1}$; however, their maximum net cooling rates below their gain radii are 2–3 times lower than ours: $\sim -75$ MeV baryon$^{-1}$ s$^{-1}$ versus $\sim -193$ MeV baryon$^{-1}$ s$^{-1}$. The MGFLD treatment we use gives a much greater neutrino cooling rate in deeper regions, which, as is well known, undermines efforts to generate explosions (e.g., see Herant et al. 1992).

We have computed the mass and internal energy in the gain region as a function of time to address the issue of whether or not ND convection leads to greater neutrino heating efficiency and the accumulation of mass and energy in the gain region (Bethe 1990; Herant et al. 1994). Other interpretations suggest that supernovae result as critical phenomena when the neutrino luminosities are sufficiently high, given the ram pressure of the preshock matter, to render the flow unstable to explosion (Burrows & Goshy 1993; Burrows et al. 1995). We find an increase in mass and internal energy in the gain radius up to about 150 ms after bounce, as a result of the increasing gain volume as the gain and shock radii diverge. After that, we find a monotonic decrease in both quantities, consistent with the density falloff in the preshock matter and with matter flowing through, rather than accumulating in, the gain region.

To assess our use of precalculated one-dimensional neutrino distributions for matter heating and deleptonization, we have considered the non–spherically symmetric luminosity enhancements that would occur from local temperature and electron fraction enhancements below the gain radii (which enclose the electron neutrino and antineutrino sources) in our two-dimensional run, which result either from non–spherically symmetric accretion through the gain radius or nonlinear inwardly propagating non–spherically symmetric waves. We see no enhancements below the neutrinosphere radii; between them and the gain radii, we see small enhancements that occur over a small fraction of the volume responsible for producing less than a third of the neutrino luminosities. Therefore, we do not expect these enhancements to have dynamical consequences.

We do not expect to obtain explosions for more massive stars. Moreover, our simulations are Newtonian. With general relativistic gravity conditions will be even more pessimistic. The neutrino luminosities will be redshifted, the increased infall velocities and the smaller width between the gain radii and the shock will allow less time for neutrino heating to reverse infall, and everything will occur in a deeper gravitational well, making explosion more difficult.

We are in the process of carrying out simulations with ray-by-ray MGFLD coupled to two-dimensional hydrodynamics in an effort to (1) “bracket” the approximation of using one-dimensional neutrino transport in a two-dimensional setting and (2) step toward a two-dimensional multigroup neutrino transport scheme. The imposition of spherical symmetry in our current model maximizes the lateral transport of neutrinos in regions that are optically thick, which would have a tendency to minimize convection in that region (Mezzacappa et al. 1998), whereas ray-by-ray transport, by definition, minimizes it. Our results regarding ND convection in the gain region hinge on our assumption that the neutrino radiation field there is realistically determined by one-dimensional MGFLD neutrino transport in the region near and below the neutrinospheres. Otherwise our use of one-dimensional precalculated distributions would not be a good approximation.) “Bracketing” our one-dimensional neutrino transport approximations will give us a better sense of how realistic these approximations are. Of course, final conclusions regarding our one-dimensional transport approximations await fully self-consistent two-dimensional multigroup radiation hydrodynamics simulations.

On a more optimistic note, recently we have obtained new results from comparisons of three-flavor Boltzmann neutrino transport and three-flavor MGFLD in post-bounce supernova environments (thermally frozen, hydrostatic). In particular, the Boltzmann net heating rate in the region directly above the gain radii is significantly larger (Messer et al. 1998). These results suggest that Boltzmann transport will yield greater neutrino heating and more vigorous ND convection; both would increase the chances of reviving the stalled shock.

Finally, it is well known that in two and three dimensions energy cascades in different directions, from short- to long-wavelength modes in two dimensions, and in the opposite direction in three dimensions (Porter, Pouquet, & Woodward 1992). Consequently, we expect three-dimensional simulations to “look” more like one-dimensional simulations than do our two-dimensional simulations. Therefore, it may be more, not less, difficult to obtain explosions in three dimensions, if success in generating explosions relies on convection. We are currently investigating the dependence of ND convection on the number of spatial dimensions (Knerr et al. 1998). To make matters worse, ultimately we will have the task of obtaining general relativistic three-dimensional explosions, with or without the aid of convection, and if one considers other complexities, such as (1) the uncertainties in the precollapse models (see, e.g., Bazan & Arnett 1994), high-density equation of state (e.g., see Keil & Janka 1995), and high-density neutrino opacities (see, e.g., Raffelt & Seckel 1995; Reddy, Prakash, & Latimer 1997), and (2) the noninclusion of rotation (e.g., see Shimizu et al. 1994) and other potentially important input physics in simulations that include realistic multidimensional hydrodynamics and neutrino transport, we are far from being able to say definitively how supernovae explode and whether or not a single component of the problem, like convection, is the key to unlocking it.
were carried out on the Cray C90 at the National Energy Research Supercomputer Center, the Cray Y/MP at the North Carolina Supercomputer Center, and the Cray Y/MP and Silicon Graphics Power Challenge at the Florida Supercomputer Center. We would like to thank Willy Benz, Adam Burrows, Chris Fryer, Wolfgang Hillebrandt, Thomas Janka, Ewald Müller, Michael Smith, Doug Swesty, and Friedel Thielemann for stimulating discussions, and especially the referee, Stirling Colgate, for many important comments, questions, and suggestions that improved the content of this paper significantly.
FIG. 3—(a) Two-dimensional entropy plots showing the evolution of neutrino-driven convection in our 15 $M_\odot$ model, at 137, 212, and 512 ms after bounce. (b) Plots of the angle-averaged entropy for the three entropy snapshots shown in (a). Also shown for comparison are the corresponding entropy profiles from our one-dimensional simulation.

Mezzacappa et al. (see 495, 915)
Fig. 4.—(a) Two-dimensional electron fraction plots showing the evolution of ND convection in our 15 $M_\odot$ model, at 137, 212, and 512 ms after bounce. (b) Plots of the angle-averaged electron fraction for the three electron fraction snapshots shown in (a). Also shown for comparison is the corresponding electron fraction from our one-dimensional PPM simulation.

Mezzacappa et al. (see 495, 916)
Fig. 5.—(a) Two-dimensional radial velocity plots showing the evolution of ND convection in our 15 $M_{\odot}$ model, at 137, 212, and 512 ms after bounce. (b) Plots of the angle-averaged radial velocity for the three radial velocity snapshots shown in Fig. 4a. Also shown for comparison is the corresponding radial velocity from our one-dimensional PPM simulation.

Mezzacappa et al. (see 495, 917)
FIG. 13.—Plots showing the fractional differences between our one- and two-dimensional PPM simulations. Shown are differences in temperature and electron fraction. The panels are a Cartesian mapping of our two-dimensional grid, with 128 angles along the \( y \)-axis and 128 radii along the \( x \)-axis. The contours mark fractional differences of 3\% (yellow), 6\% (red), and 9\% (black). The thick vertical lines mark the angle-averaged neutrinosphere and gain radii, in black and red, respectively.

Mezzacappa et al. (see 495, 924)