Finite Element State-Space Model of Edge Initiating Localized Interfacial Degeneration of Damped Composite Laminated Plates

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Abstract:
In presence of commonly encountered edge delamination problem, a new finite element formulation to examine the effects of a localized interfacial degeneration on the dynamic performance of the cross-ply laminated composite plate adopting the state-space approach is carried out. The stiffness term of the laminate is constructed via the assemblage of the local contribution from each ply sub-element. This involves also the inclusion of an interface layer employing the virtually zero-thickness formulation to model the interfacial degeneration. In addition, the proportional damping and kinematically consistent mass matrix are expressed to form the complete governing formulation. To invoke interfacial degeneration, the degenerated areas are defined from the edge propagating inwards in a localized manner. As the degenerated area of the interface increases, a decreasing pattern is observed in the natural frequency, damped frequency and damping ratio of the plate whereas there exists a rise correspondingly in the loss factor.

Keywords: dynamic finite element; kinematically consistent mass; localized interface degeneration; proportional damping; state-space

Introduction
Composites have rapidly become one of the chiefly applied materials in advanced engineering structures, as components in civil engineering, aerospace, and automotive, just to name a few. The most common type of composite is that of laminated structures where two or more laminae are bonded by a thin layer of adhesive material.

Although laminated composite has a greatly extensive potential in aforementioned applications, they may exhibit several peculiar modes of failure such as matrix crazing, delamination, fiber failure and interfacial bond failure due to debonding. In practice, it is considerably difficult to have a perfect interfacial bond especially during manufacturing process or the actual service life of composite laminates. The delamination remains as the most common failure observed, an interlayer separation damage mode that possibly occurs in the interface of a laminated composite. Due to this phenomenon, a reduction in the stiffness of material is customarily demonstrated. Therefore, a model of laminated composite with capability of describing the interfacial imperfection should be adopted for a better comprehension of such condition.

The dynamic properties such as natural frequency, loss factor and modal damping depend greatly on the material density, elastic constants, damping properties, geometry and layers orientations. Therefore, damping has become one of the important parameters related to the study of dynamic behavior of laminated composite structures. In general, damping can be divided into two main types: viscous damping and structural damping. It usually occurs as a mixture of two mechanisms in a composite laminate: damping between the fiber and adhesive layer within the laminated plies and that between the plies or between the laminae. The damping properties of

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laminated plate can be affected during the manufacturing process of gluing all layers. In relation, the manufacturing faults may cause the presence of debonding areas in the glued region. The presence of bonding-free areas could reduce the natural frequency and hence disturb the damping properties of material. Due to high labor and cost requirements of experimental studies, the predictions of changes in structural dynamic properties can be conducted by using the finite element method. With the modeling of degeneration of localized interfacial in composite laminated plate with an inclusion of damping, the accuracy to predict the failure will be improved and more realistic. An accurate modeling expression for damping is essential in describing better the dynamic response of laminated composite structures.

Up until now, several interface elements for modeling the behavior of the interfacial layers of laminated plates have been proposed. Sun and Pan\(^1\) characterized the mechanical behavior of composite laminate interfaces based on the generalized composite laminate theory. Bui, Marechal and Nguyen-Dang\(^2\) presented a numerical analysis for laminated composite plate with imperfect interlaminar interfaces. In relation to damping modeling, Rikards et al.\(^3\) applied two methods of damping analysis, complex eigenvalues and energy methods, for evaluating the dynamic performance of a sandwich structure. Hu and Dokainish\(^4\) presented two damping models, the viscoelastic damping (VED) and the specific damping capacity (SDC) models, to assess the damping behavior of composites. By considering the damping and delamination, Oh et al.\(^5\) performed a dynamic analysis of laminated composites with multiple delamination according to higher-order cubic zigzag theory.

It is noteworthy to see that all aforementioned modeling efforts have focused on single site interfacial imperfection rather than various locations. A discreet imperfection area has been possible since the introduction of numerical method such as the well-established finite element approach. In utilizing this technique, Abo Sabah and Kueh\(^6\) studied the effects of localized interface delamination on the behavior of laminated composite plates subjected to low velocity impact loading for different fiber orientations by means of virtually zero-thickness interface definition. They found that when the local delamination area increases, displacement increases. Also, a rise in top and bottom fiber orientations deviation increases both central deflection and energy absorption.

**Numerical methods**

**A. Model Description**

The configuration of the plate studied in this paper is shown in Figure 1. We consider here a rectangular [90/0] cross-ply laminate plate with two composite laminae, which are of the same thicknesses with an interfacial layer in between. Each lamina is formed by unidirectional E-glass fibers and Epoxy (3501-6) matrix material with a volume fraction of 0.4. The laminated composite plate is considered to be thin and flat such that the shear deformation is neglected.

The lamina is modeled and discretized by using a 4-node rectangular plate finite element. Also, the laminated composite plate is considered as a transversely isotropic solid material. There are five degrees of freedom at each node, which are displacement in x-direction \((u)\), displacement in y-direction \((v)\), displacement in z-displacement \((w)\), rotation about y-direction \((\theta_x)\) and rotation about x-direction \((\theta_y)\).

Furthermore, the interfacial layer is considered as an orthotropic material with null normal stress in x- and y-directions \((\sigma_x = 0 \text{ and } \sigma_y = 0)\) and in-plane shear stress in x-y plane \((\tau_{xy} = 0)\). It is modeled using a quadrilateral zero-thickness solid element with 8 nodes. However, there are only
three degrees of freedom for each node, which are the displacement in $x$-direction ($u$), displacement in $y$-direction ($v$), and displacement in $z$-displacement ($w$). The stiffness matrix of the lamina and interfacial element is computed using a $2 \times 2$ Gauss quadrature rule.

Proportional damping model is applied to describe the damping in this study. Eigenvalue analysis is conducted by means of state-space approach since the model is under free vibration environment. It is worth noting that a full degeneration is considered in a localized manner. From the eigenvalue analysis, natural frequency, damped natural frequency, loss factor and damping ratio of laminated composite plate, with the application of degeneration of localized degeneration, are to be determined.

B. Numerical Model Construction

The governing equation for the dynamic finite element in terms of the eigenvalue problem is

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = 0$$

where $[M]$ is the global mass matrix, $[\ddot{q}]$ is the nodal acceleration, $[C]$ is the global damping matrix, $[\dot{q}]$ is the nodal velocity, $[K]$ is the global stiffness matrix, and $[q]$ is the nodal displacement.

C. Stiffness Matrix of Elemental Lamina

The local stiffness matrix of the lamina is developed by combining the element strain-displacement matrix with the ABD matrix of the lamina shown below

$$K = \int \left[ B_i^T (A)_{ABD} B_i + B_i^T (B)_{ABD} B_i + B_i^T (C)_{ABD} B_i + B_i^T (D)_{ABD} B_i \right] d\zeta d\eta$$

where $B_i$ is the in-plane element strain-displacement matrix, $B_o$ is the out-of-plane element strain-displacement matrix, $(A)_{ABD}$ is the extensional stiffness, $(B)_{ABD}$ is the coupling stiffness, and $(D)_{ABD}$ is the bending stiffness of the lamina.

D. Stiffness Matrix of Elemental Interface

The stiffness matrix of the zero-thickness interface element is computed by

$$K_{\text{int}} = \frac{1}{h} \int B_{\text{int}}^T D_{\text{int}} B_{\text{int}} [d\zeta d\eta]$$

where $h$ is the thickness of the interface, $B_{\text{int}}$ is the combined element strain-displacement matrix, $D_{\text{int}}$
is the elasticity matrix, and \( J \) is the Jacobian matrix.

\[
J = \begin{bmatrix}
\frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\
\frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta}
\end{bmatrix}
\]  

(4)

\[
D_{\text{int}} = \begin{bmatrix}
G_{xz}(1 - R) & 0 & 0 \\
0 & G_{yz}(1 - R) & 0 \\
0 & 0 & E_z(1 - R)
\end{bmatrix}
\]  

(5)

where \( G_{xz} \) and \( G_{yz} \) are the out-of-plane shear modulus, \( E_z \) is the Young’s modulus in the \( z \)-direction and \( R \) is the imperfection factor.

E. Mass Matrix

The kinematically consistent mass matrix \([M]\) is defined to describe the mass contribution by summing all element mass matrices \([m]\).

\[
[M] = \sum_{e=1}^{n} [m]
\]  

(6)

\[
[m] = \int_{A} \rho N^T N \, dA
\]  

(7)

\( n \) is the total number of elements. \( \rho \) and \( N \) are the density and the combined shape functions, respectively.

F. Damping Matrix

The proportional damping, \([C]\), also known as Rayleigh damping, is adopted and defined as a linear combination of global mass matrix and global stiffness matrix.

\[
[C] = \alpha [M] + \beta [K]
\]  

(8)

The coefficients, \( \alpha \) and \( \beta \), are computed by considering the required levels of proportional damping at two different frequencies, which are the first and second modes of free vibration.

G. State-Space Form

The linear equation of motion (Equation (1)) is solved employing the state-space method. For free vibration cases, the governing equation becomes an eigenvalue problem shown below.

\[
([A] - \bar{\omega}[B])\ddot{q} = 0
\]  

(9)

where \( \bar{\omega} \) is the frequency of the natural vibration in complex solution \((\bar{\omega} = \bar{\omega}_r + i\bar{\omega}_i)\), \( \ddot{q} \) is the mode shape vector and

\[
[A] = \begin{bmatrix}
[0] & -[K] \\
-[K] & -[C]
\end{bmatrix}
\]  

\[ [B] = \begin{bmatrix}
-[K] & [0] \\
0 & [M]
\end{bmatrix} \]  

(10)
H. Damping Properties

In terms of the structural response, the dynamic behavior of laminated composite plate is considered. Therefore, two important parameters, damping ratio, $\zeta$, and loss factor, $\eta$, are formulated in the followings.

$$\zeta = \frac{\alpha + \beta \omega_n^2}{2 \omega_n}$$

(11)

$$\eta = \frac{\omega_n}{\omega_n \zeta}$$

(12)

where $\omega_n$ = natural frequency and $\omega_d$ = damped frequency $= \omega_n \sqrt{1 - \zeta}$

I. Degeneration Pattern

To model the degeneration of interface, an area of interface degeneration is implemented and initiated from the edge of the laminate. This degeneration region is extended diagonally at the interface of the laminated composite plate as shown in Figure 2.

J. Validation

The present model is at first made similar with that of Hu and Dokainish for perfectly bonded case for validation purpose. Table 1 compares their computed natural frequencies and loss factor with those from the present study. Material properties used in the validation are: $E_1$ = 42.62 GPa, $E_2$ = 12.50 GPa, $G_{12}$ = 5.71 GPa, $G_{21}$ = 2.855 GPa, $v_{12}$ = 0.30, and $\rho = 1971.0$ kg/m$^3$. It is obvious that a good agreement is achieved in the comparison.

![Figure 2](image)

**Figure 2** Degeneration is initially implemented at the edge and then extended diagonally at the interface of the plate (shaded area indicates those degenerated)

|                  | Natural Frequency (Hz) | Loss Factor |
|------------------|------------------------|-------------|
| Hu and Dokainish\cite{Hu} | 9.3744                 | 6.75        |
| Present model    | 10.1535                | 6.07        |

Table 1 Comparison of natural frequency and loss factor
Results and discussion
The dynamic performances of the composite plate with degeneration initiated from the edge and extended diagonally at the interface of the laminated composite plate in terms of natural frequency, damped frequency, loss factor and damping ratio are shown in Fig 3.

As the degenerated area ratio increases, the natural frequency decreases. Therefore, the whole laminate accordingly becomes weaker. Similarly, the damped frequency follows the same dropping trend and has a large reduction when the degenerated area ratio is more than 0.1. The loss factor is a good way to express the damping characteristics of a material. The higher the loss factor, the more damping a material has. From the results, the loss factor increases as the degenerated area increases. This implies that the occurrence of the degenerated interface in the laminated composite plate promotes higher damping. This indicates also that a greater imperfection improves the damping behavior of a laminated material, owing to higher energy absorption by means of friction. Since the damping ratio and natural frequencies are dependent to each other, the decrease of damping ratio can be seen in Fig 3(d).

Figure 3 Change in (a) normalized natural frequency, (b) normalized damped frequency, (c) normalized loss factor, and (d) normalized damping ratio of cross-ply composite laminated plate due to various interfacial degenerated areas.

Conclusions
Finite element formulation for a two-layer cross-ply laminated composite plate incorporating a zero-thickness interfacial element and a proportional damping was developed. A state-space approach was employed to investigate the dynamic behaviors of the plate. It is found from the study that the natural frequency, damped frequency and damping ratio of the plate decrease as the
degenerated area of the interface increases. Correspondingly, the loss factor increases due to higher damping provided by a greater intensity in friction.

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