FRAGMENTATION OF A MOLECULAR CLOUD CORE VERSUS FRAGMENTATION OF THE MASSIVE PROTOPLANETARY DISK IN THE MAIN ACCRETION PHASE

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ABSTRACT

The fragmentation of molecular cloud cores a factor of 1.1 denser than the critical Bonnor-Ebert sphere is examined through three-dimensional numerical simulations. A nested grid is employed to resolve fine structure down to 1 AU while following the entire structure of the molecular cloud core of radius 0.14 pc. A barotropic equation of state is assumed to take account of the change in temperature during collapse, allowing simulation of the formation of the first core. A total of 225 models are shown to survey the effects of initial rotation speed, rotation law, and amplitude of bar mode perturbation. The simulations show that the cloud fragments whenever the cloud rotates sufficiently slowly to allow collapse but fast enough to form a disk before first-core formation. The latter condition is equivalent to $\Omega_0$ $t_{\text{ff}}$ $\geq$ 0.05, where $\Omega_0$ and $t_{\text{ff}}$ denote the initial central angular velocity and the freefall time measured from the central density, respectively. Fragmentation is classified into six types: disk-bar, ring-bar, satellite, bar, ring, and dumbbell types according to the morphology of collapse and fragmentation. When the outward decrease in initial angular velocity is more steep, the cloud deforms from spherical at an early stage. The cloud deforms into a ring only when the bar mode ($m$ = 2) perturbation is very minor. The ring fragments into two or three fragments via ring-bar type fragmentation and into at least three fragments via ring type fragmentation. When the bar mode is significant, the cloud fragments into two fragments via either bar or dumbbell type fragmentation. These fragments eventually merge because of their low angular momenta, after which several new fragments form around the merged fragment via satellite type fragmentation. This satellite type fragmentation may be responsible for the observed wide range of binary separation.

Subject headings: binaries: general — hydrodynamics — ISM: clouds — methods: numerical — stars: formation

1. INTRODUCTION

It is widely accepted that binary and multiple stars form as a result of fragmentation in collapsing molecular cloud cores (e.g., Bodenheimer et al. 2000). In the last two decades, fragmentation of the molecular cloud core has been investigated through numerical simulations by many authors. Criteria for the fragmentation of isothermally collapsing clouds have been investigated by Miyama, Hayashi, & Narita (1984), Boss (1993), Boss & Myhill (1995), and Tsuribe & Inutsuka (1999). Their criteria have converged as far as the isothermal phase is concerned; a cloud with $\alpha$ $\leq$ 0.2–0.5 collapses into fragments depending little on $\beta$, even when the initial density and velocity distributions differ. Here, $\alpha$ and $\beta$ denote the ratio of thermal energy to gravitational energy and that of rotation energy to gravitational energy, respectively. Tsuribe & Inutsuka (1999) pointed out that the criteria for fragmentation corresponds to the formation of a flat disk with flatness greater than 4$\pi$. On the other hand, a cloud with $\alpha$ $\geq$ 0.2–0.5 collapses self-similarly and shows no sign of fragmentation. Hanawa & Matsumoto (1999) and Matsumoto & Hanawa (1999) investigated deformation of the self-similarly collapsing cloud in search of the possibility that deformation of the central cloud to a bar might trigger fragmentation. The growth of the bar mode is slow compared with the timescale of the collapse, i.e., $\Delta \propto \rho_{\text{max}}$, where $\Delta$ and $\rho_{\text{max}}$ denote the amplitude of the bar mode and the maximum density of the cloud, respectively. The bar may indeed fragment, but only at a later stage.

As the collapse proceeds, the cloud core becomes optically thick and the efficiency of radiative cooling decreases. The temperature starts increasing when the central density exceeds the critical density of $\sim$10$^{-13}$ g cm$^{-3}$. This increase in temperature results in the formation of a quasi-static core, i.e., the first core of Larson (1969). The first core grows by accreting gas, and this accretion phase persists long enough for the core to fragment. Thus, the dynamics of the cloud changes qualitatively at the critical density. Stability against fragmentation is also likely to change at this critical density. In fact, the first core has been shown by recent simulations to be very unstable by taking account of the change in temperature (Burkert, Bate, & Bodenheimer 1997; Nelson 1998; Sigalotti 1998; Klapp & Sigalotti 1998; Boss et al. 2000).

Their simulations, however, assume rather small $\alpha$. When $\alpha$ is small, the cloud is Jeans-unstable and fragments easily (Tsuribe & Inutsuka 1999). The fragmentation of the first core may be due to the small $\alpha$ assumed. It is still unknown whether the first core fragments when the initial cloud has a moderately large $\alpha$. Thus, we investigate the fragmentation of the cloud with focus on the cloud with large $\alpha$ = 0.765. The initial cloud is only 1.1 times more massive than the critical...
Bonnor-Ebert sphere (Ebert 1955; Bonnor 1956), which is an equilibrium state of the isothermal cloud. The Bonnor-Ebert sphere provides a good fit to the density distribution of a dark globule. According to recent near-infrared observations (Alves, Lada, & Lada 2001), the model with $\xi = 6.9 \pm 0.2$ gives the best fit for B68, where $\xi$ denotes the non-dimensional radius. Similarly, those with $\xi = 12.5 \pm 2.6$ and $7.0 \pm 0.3$ give a good fit for B335 (Harvey et al. 2001) and the Coalsack (Racca, Gómez, & Kenyon 2002), respectively. When $\xi > 6.45$, the Bonnor-Ebert sphere is unstable against collapse. Thus, our initial model can be applied to these globules.

The model can also be applied to cloud cores embedded with molecular clouds. The masses of such cores evaluated from $^{13}$CO luminosity are similar to the virial masses (e.g., Onishi et al. 1996). This implies that the cores are gravitationally bound and nearly in equilibrium, and accordingly the parameter $\alpha$ is only slightly less than unity.

Recent numerical studies have lacked any survey of model parameters. In this study, 225 models with different rotation speed, rotation law, and amplitude of bar mode perturbation are considered. The simulations show many features of each type and the territory of each type in the parameter space are discussed.

In this paper, the collapse and fragmentation of molecular cloud cores are investigated using a nested grid. The nested grid has high spatial resolution near the center of the computation domain and allows fragmentation to be followed without violating the Jeans condition (Truelove et al. 1997). In § 2, the models of cloud cores are introduced. In § 3, the methods of numerical simulations are presented. In § 4, the results are shown and the fragmentation is classified. In § 5, the origins of different types of fragmentation are discussed. The simulations are compared with earlier numerical works, and the implications for binary formation are related. The paper is concluded in § 6.

2. MODELS

As a model for molecular cloud cores, we consider Bonnor-Ebert spheres, which belong to a sequence of equilibrium state isothermal spherical clouds confined by external pressure (Ebert 1955; Bonnor 1956). Given the external pressure ($P_{\text{ex}}$) and the sound speed ($c_s$), the Bonnor-Ebert sphere is stable against collapse only when the central density is lower than the critical value, 14.0 $P_{\text{ex}}c_s^2$. The critical Bonnor-Ebert sphere is used as a template for the model clouds examined in this study. In the models, the initial density distribution is given by

$$\rho(r) = \rho_c \phi_{\text{BE}}(r/a) ,$$

$$a = c_s \left( \frac{f}{4\pi G \rho_c} \right)^{1/2} ,$$

where $r, f, \rho_c$, and $G$ denote the radius, density enhancement factor, initial central density, and gravitational constant, respectively. The function $\phi_{\text{BE}}$ denotes the density distribution of the critical Bonnor-Ebert sphere and can be approximated as

$$\phi_{\text{BE}}(\xi) = 1 - \frac{\xi^2}{6} + \frac{\xi^4}{45} + O(\xi^6) .$$

The critical Bonnor-Ebert sphere has radius $\xi = 6.45$. A density enhancement factor of $f = 1.1$ is assumed in typical models because observed molecular clouds are nearly in the virial equilibrium. This slight density enhancement collapses rotating clouds when the initial cloud rotates slowly.

The initial central density is set at $\rho_c = 1 \times 10^{-19}$ g cm$^{-3}$, which corresponds to a number density of $n_c = 2.6 \times 10^4$ cm$^{-3}$ for the assumed mean molecular weight of 2.3. An initial temperature of $T = 10$ K is assumed, and hence $c_s = 0.19 \text{ km s}^{-1}$. The radius and mass of the cloud are thus $R_c = 0.144 \text{ pc}$ and $M_c = 3.24 M_\odot$ for $f = 1.1$.

The initial velocity includes only the $\varphi$-component, and the angular velocity depends on $R$ and $\varphi$ in cylindrical coordinates $(R, \varphi, z)$. The angular velocity $\Omega$ is expressed as

$$\Omega(R, \varphi) = \left[ \Omega_0 + \Omega_2 \cos 2\varphi + \Omega_3 \left( \frac{R}{a} \right)^3 \cos 3\varphi \right] \times \left[ 1 + 2C \left( \frac{R}{a} \right)^{2} \right]^{-1/2} ,$$

where $\Omega_0$ denotes the amplitude of the global rotation and $\Omega_2$ and $\Omega_3$ denote the amplitudes of the velocity perturbations of $m = 2$ and 3. The parameter $C$ specifies the dependence of $\Omega$ on $R$. When $C = 0$, the angular velocity is independent of $R$ and rotation is "rigid." When $C$ is larger, the angular velocity decreases more rapidly with increasing $R$. As shown later, fragmentation of the cloud core depends strongly on the parameter $C$. A small amplitude for the perturbation $m = 3$ is set, such as $\Omega_3t_0 = 1 \times 10^{-3}$, in all the models, where $t_0$ denotes the initial freefall timescale at the center and is defined as $(3\pi/32G\rho_c)^{1/2}$. This $m = 3$ mode breaks the point symmetry with respect to $R = 0$, and accordingly the fragments are slightly asymmetric in this simulation. The model parameters $\Omega_0, \Omega_2, \Omega_3$, and $C$ are varied to investigate the effects of rotation speed, amplitude of the bar mode, and the rotation law on fragmentation of the cloud cores.

To compare our initial models with those of earlier simulations, the ratios $\alpha = E_{\text{th}}/|E_{\text{grav}}|$ and $\beta = E_{\text{rot}}/|E_{\text{grav}}|$ are computed, where $E_{\text{th}}, E_{\text{rot}},$ and $E_{\text{grav}}$ are thermal energy, rotation energy, and gravitational energy, respectively (e.g., Tohline 1981; Miyama et al. 1984). In our model, the initial cloud has $\alpha = 0.765(1.1/f)$. The parameter $\beta$ is independent of $f$, and when $\Omega_3 = 0$, $\beta$ is described by

$$\beta = \beta_C = \frac{2}{\Omega_3t_0^2} \left( \Omega_0^2 + \frac{1}{2} \Omega_2^2 \right) ,$$

where the coefficient $\beta_C$ is a function of $C$ as shown in Figure 1. When a cloud rotates rigidly ($C = 0$), it has $\beta_C = 0.892$. When $C \sim 1$, $\beta_C$ decreases approximately in proportion to $C^{-1/2}$. The decrease in $\beta_C$ is due to slow rotation in the outer part of the cloud.

When $\alpha$ and $\beta$ are small, the cloud is unstable against fragmentation. When the cloud has uniform density and rotates rigidly, the criterion for fragmentation is $\alpha \lesssim 0.5$ (e.g., Tsuribe & Inutsuka 1999). When the cloud is centrally peaked and the axis ratio is 1.5, the criterion is $\alpha \lesssim 0.45$ for low $\beta$ (Boss 1993). When the cloud is more oblate, i.e., the axis ratio is 2.0, the criterion is $\alpha \lesssim 0.33$ for low $\beta$. This suggests that a cloud of $\alpha \geq 0.5$ is stable against fragmentation. In this study, the possibility of fragmentation of a cloud with $\alpha > 0.7$ is examined.

The dynamical evolution of a cloud is followed by taking account of the self-gravity and gas pressure. The magnetic
field is neglected for simplicity. The gas temperature is assumed to be 10 K below that of the critical density \( \rho_{\text{cr}} = 2 \times 10^{-13} \text{ g cm}^{-3} \) \( (n_{\text{cr}} = 5.24 \times 10^{10} \text{ cm}^{-3}) \) and to increase adiabatically in proportion to \( \rho^{7/5} \) above it. In other words, a barotropic equation of state is assumed, as expressed by

\[
P(r) = \begin{cases} 
\frac{c_s^2}{\gamma} & \text{for } \rho < \rho_{\text{cr}}, \\
\frac{c_s^2}{\gamma} \rho_{\text{cr}} \left( \frac{\rho}{\rho_{\text{cr}}} \right)^{7/5} & \text{for } \rho \geq \rho_{\text{cr}}.
\end{cases}
\]

This change in temperature reproduces the formation of the adiabatic core, which corresponds to the first core of Larson (1969). The value of the critical density \( \rho_{\text{cr}} \) is taken from the numerical results of Masunaga, Miyama, & Inutsuka (1997), where \( \rho_{\text{cr}} \) is taken from the

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r_{\text{cr}} = \frac{5}{3} \rho_{\text{cr}} \rho_{\text{crit}} \frac{1}{10^{-10} \text{ cm}}.
\]

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Thus, the total mass is conserved in our computations. The numerical fluxes are obtained by the method of Roe (1981) with modification to solve for the isothermal and polytrope gas. A MUSCL approach and predictor-corrector method are adopted for time integration (e.g., Hirsch 1990). The Poisson equation is solved by a multigrid iteration on a nested grid (Matsumoto & Hanawa 2003). This code solves for self-gravity consistently over all grids with different levels such that “the gravitational field line” is continuous at interfaces between coarse and fine grids. This consistency ensures that the obtained gravitational potential is accurate at least up to the quadrupole moment of a binary. Thus, the gravitational torque induced by a binary is accurately taken into account in our simulation.

Mirror symmetry with respect to the \( z = 0 \) plane is employed to reduce computation cost. A fixed boundary condition is set for the surface \( r = R_c \), representing a constant external pressure that confines the cloud during evolution. Gas is considered only in \( r \leq R_c \), when solving the Poisson equation.

In this paper, each grid has 256 \( \times \) 256 \( \times \) 32 cubic cells in high-resolution models and 128 \( \times \) 128 \( \times \) 16 cubic cells in low-resolution models in \( (x, y, z) \). The model parameters of the high-resolution models are shown in Table 1. The other models shown in this paper are the low-resolution models. The nested grid consists of grids of five levels at the initial stage. A new finer grid is introduced to maintain the Jeans condition of \( \frac{\lambda_1}{4} > h \) with ample margin (Truelove et al. 1997), where \( \lambda_1 \) and \( h \) are the Jeans length and cell width, respectively. Whenever \( \frac{\lambda_1}{4} \) becomes smaller than the cell width in the finest grid, a new finer grid is added to the nested grid. This means that a finer grid was added with ample margin of factor 2. Typical models have 14 grid levels at the last stage. The Jeans condition was violated in these simulations only when a high-density fragment escaped from the region covered by the finest grid, and the computation was terminated in the stage that this occurred. In the model shown in § 4.1.1, evolution was successfully computed up to the stage in which the mass of an adiabatic core (total mass in the region of \( \rho \geq \rho_{\text{cr}} \)) reached 0.07 \( M_\odot \).

### 3. NUMERICAL METHODS

In the simulations, the hydrodynamical equation and Poisson equation are solved by a finite difference method with second-order accuracy in time and space. A nested grid is employed to solve the central region with higher spatial resolution. The hydrodynamic code for the nested grid was developed by extending the simulation code of Matsumoto & Hanawa (1999). The nested grid consists of concentric hierarchical rectangular grids (Yorke, Bodenheimer, & Laughlin 1993), and the cell width of each grid decreases successively by a factor of 2. In the following, the coarsest grid is labeled level \( l = 1 \). The \( l \)-th level grid has \( 2^{l-1} \) times higher spatial resolution than the coarsest grid. All the fluxes are conserved at the interface between the coarse and fine grids as in the standard adaptive mesh refinement (AMR; Chiang, van Leer, & Powell 1992). Thus, the total mass is conserved in our computations. The numerical fluxes are obtained by the method of Roe (1981) with modification to solve for the isothermal and polytrope gas. A MUSCL approach and predictor-corrector method are adopted for time integration (e.g., Hirsch 1990). The Poisson equation is solved by a multigrid iteration on a nested grid (Matsumoto & Hanawa 2003). This code solves for self-gravity consistently over all grids with different levels such that “the gravitational field line” is continuous at interfaces between coarse and fine grids. This consistency ensures that the obtained gravitational potential is accurate at least up to the quadrupole moment of a binary. Thus, the gravitational torque induced by a binary is accurately taken into account in our simulation.

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### 4. RESULTS

#### 4.1. Rigidly Rotating Cloud

In this subsection, a total of 27 models of a rigidly rotating cloud \( (C = 0) \) in the region \( 0.03 \leq \Omega_0 \text{ff} \leq 0.3 \) and \( 0 \leq \Omega_2 \text{ff} \leq 0.3 \) are presented to study the dependence on \( \Omega_0 \) and \( \Omega_2 \).

Figure 2 summarizes the last stages of the 27 models. Each panel denotes the density distribution in the \( z = 0 \) plane. The panels are arranged such that \( \Omega_0 \) increases from left to right and \( \Omega_2 \) increases from bottom to top. The evolution of the clouds is classified into five types in the parameter space of the initial rotation \( (\Omega_0 \text{ff}) \) and the initial amplitude of bar mode \( (\Omega_2 \text{ff}) \). When \( \Omega_0 \text{ff} \leq 0.03 \) (left column), the cloud collapses to form a single disk (disk type...
Fig. 2.—Density distributions in the $z = 0$ plane at the last stages for models for which $C = 0$. Color denotes the density distribution on a logarithmic scale. The right color scale is for oscillation models, and the left color scale is for models of disk, satellite, ring-bar, and disk-bar types. Black contour curves denote the critical density $n_{cr}$. Panels are arranged in the order $\Omega_0 t_{ff} = 0.03, 0.05, 0.1, 0.2, \text{ and } 0.3$ from left to right, and $\Omega_2 t_{ff} = 0.0, 0.01, 0.03, 0.05, 0.1, 0.2, \text{ and } 0.3$ from bottom to top.
collapse). When $\Omega_0 t_{\text{ff}} = 0.3$ (right column), the cloud never collapses but instead oscillates. When $0.05 \leq \Omega_0 t_{\text{ff}} \leq 0.2$ (middle three columns), the cloud collapses into several fragments. The last type is further subdivided into three types: disk-bar, ring-bar, and satellite types. In the following, each type is discussed by showing a typical model.

4.1.1. Disk Type Collapse

In disk type collapse, the cloud collapses almost spherically in the isothermal collapse phase and forms a rotating disk after the central density exceeds the critical density $\rho_{\text{cr}}$. The disk grows by accretion and exhibits no sign of fragmentation. The model in which $(\Omega_0 t_{\text{ff}}, \Omega_0 t_{\text{ff}}, C) = (0.03, 0.03, 0.00)$ is shown in Figure 3 as a typical model of disk type collapse.

Figure 3a shows the initial stage, showing only the finest three grids ($3 \leq l \leq 5$), i.e., only $1/64$ of the full computation volume. The initial cloud has a spherical density distribution, and the cloud collapses almost spherically during the isothermal collapse phase ($\rho < \rho_{\text{cr}}$) as a result of the very slow rotation. When $\rho \approx \rho_{\text{cr}}$, the central cloud becomes slightly flattened by the rotation and deforms nonsymmetrically because of perturbation of the bar mode (Fig. 3b). The deformation is evaluated by measuring the moment of inertia,

$$I_g = \int_{\rho>0.1\rho_{\text{max}}} (r_g - r_{g,i})(r_g - r_{g,j})\rho(r) \, dr,$$

(7)

and the total mass,

$$M = \int_{\rho>0.1\rho_{\text{max}}} \rho(r) \, dr,$$

(8)

for the gas of density $\rho \geq 0.1\rho_{\text{max}}$. The subscripts $i$ and $j$ are coordinate labels, i.e., $x = r_1$, $y = r_2$, and $z = r_3$. The barycenter is defined as

$$r_{g,i} = \frac{1}{M} \int_{\rho>0.1\rho_{\text{max}}} \rho r_i \, dr.$$  

(9)

The long axis ($a_l$), short axis ($a_s$), and length along the $z$-axis ($a_z$) are defined by

$$a_l^2 = \frac{1}{2M} \left[ I_{11} + I_{22} + \frac{2}{4} [(I_{11} - I_{12})^2 + 4I_{12}^{1/2}] \right],$$

$$a_s^2 = \frac{1}{2M} \left[ I_{11} + I_{22} - \frac{2}{4} [(I_{11} - I_{12})^2 + 4I_{12}^{1/2}] \right],$$

$$a_z^2 = \frac{1}{2M} I_{33}.$$  

(10)

Figure 4 shows eccentricity, $(a_l/a_s) - 1$, and flatness, $[(a_l/a_s)^{1/2}/a_z] - 1$, as functions of the maximum number density $n_{\text{max}}$. The eccentricity oscillates with significant amplitude in the range $10^4 \text{ cm}^{-3} \leq n_{\text{max}} \leq 10^6 \text{ cm}^{-3}$ and increases roughly in proportion to $n_{\text{max}}^{1/6}$ in the range $10^6 \text{ cm}^{-3} \leq n_{\text{max}} \leq 10^{10} \text{ cm}^{-3}$ because of bar mode instability (Handawa & Matsumoto 1999; Matsumoto & Hanawa 1999). At the end of the isothermal collapse phase, the long axis is 12% longer than the short axis (eccentricity is 0.122). The flatness, $(a_l/a_s) - 1$, increases because of spin-up in the central cloud. In the range $10^6 \text{ cm}^{-3} \leq n_{\text{max}} \leq 10^{10} \text{ cm}^{-3}$, the flatness increases rapidly in proportion to $n_{\text{max}}^{0.7}$. At the end of the isothermal collapse phase, the flatness is 0.412 and the long axis is 50% longer than thickness in the $z$-direction.

Figure 4 also shows the central angular velocity in unit freefall time $\Omega_{\text{ff}}$. The angular velocity and the freefall time are measured as

$$\Omega = J_{\text{spin},z}/[M(a_l^2 + a_z^2)]$$

and $t_{\text{ff}} = (3\pi/32G\rho_{\text{max}})^{1/2}$, where $J_{\text{spin},z}$ denotes the $z$-component of the total spin angular momentum $J_{\text{spin}}$, which is defined as

$$J_{\text{spin}} = \int_{\rho>0.1\rho_{\text{max}}} (r - r_g) \times (v - v_g) \rho \, dr,$$

(11)

where

$$v_g = \frac{1}{M} \int_{\rho>0.1\rho_{\text{max}}} \rho v \, dr.$$  

(12)

This angular velocity represents the average angular velocity in the region of $\rho \geq 0.1\rho_{\text{max}}$ and is denoted $\Omega_{0.1}$. Another average angular velocity, $\Omega_{0.5}$, defined in the region of $\rho \geq 0.5\rho_{\text{max}}$, is also introduced to measure the value near the center. At the initial stage, $\Omega_{0.1}$ and $\Omega_{0.5}$ have the same value because the initial cloud rotates rigidly. In the early isothermal collapse phase, the central part rotates faster ($\Omega_{0.5} > \Omega_{0.1}$). For $n_{\text{max}} \approx 10^8 \text{ cm}^{-3}$, the cloud spins up according to $\Omega_{0.5} \propto n_{\text{max}}$; in other words, $\Omega_{0.5} \propto n_{\text{max}}$. This spin-up rate coincides with that expected for spherical collapse (Hanawa & Nakayama 1999). The cloud collapses almost spherically in the isothermal collapse phase, and significant deformation occurs only near the end of the isothermal collapse phase. The maximum value of $\Omega_{0.5} t_{\text{ff}}$ is 0.0973.

When the central density exceeds $\rho_{\text{cr}}$, the infall accelerates near the center and an adiabatic core, the first core, forms. The first core of Larson (1969) is quasi-static and spherical, whereas this adiabatic core is rotating and disklike. The adiabatic core formation ends the isothermal collapse phase. Infall still continues in the region far from the center, and the adiabatic core accretes gas from the envelope. Thus, the period after adiabatic core formation is called the accretion phase. Figure 3e shows the adiabatic core at the stage for which $t - t_{\text{cr}} = 216 \text{ yr}$, where $t_{\text{cr}}$ denotes the time at which $\rho_{\text{max}}$ exceeds $\rho_{\text{cr}}$. The adiabatic core consists of a flattened central kernel with an envelope of adiabatic gas. The kernel has a radius of 2 AU and a thickness of 1.5 AU. The mass of the kernel is $M_{\text{13}} = 7.4 \times 10^{-3} M_\odot$, where $M_N$ denotes the mass measured for $n = 10^9 \text{ cm}^{-3}$.

Figure 3d shows the adiabatic core at the stage for which $t - t_{\text{cr}} = 775 \text{ yr}$, after it has begun to accrete gas from the isothermal infalling envelope. The adiabatic core at this stage consists of a flattened central kernel with an extended disk and spiral arms. One of the dense spiral arms evolves into a dense clump, as can be seen to the upper left of the central kernel in Figure 3d. The dense clump falls into the central kernel and merges into it. Figure 3e shows the adiabatic core after the merge of the dense clump ($t - t_{\text{cr}} = 838 \text{ yr}$). The radius of the adiabatic disk increases to 15 AU by this stage. During the period $1.0 \times 10^3 \text{ yr} \leq t - t_{\text{cr}} \leq 1.1 \times 10^3 \text{ yr}$, this formation and merging of a clump occurs again. During the formation of the clump, the disk shrinks to 10 AU. However, after merging the radius of the adiabatic disk increases to 20 AU. Figure 3f shows the last stage of the simulation ($t - t_{\text{cr}} = 1.5 \times 10^3 \text{ yr}$). The radius of the adiabatic disk has increased to 20 AU, although the radius of the central kernel remains. The masses of the adiabatic core and the kernel are $6.9 \times 10^{-2}$ and $4.2 \times 10^{-2} M_\odot$ at this stage.
Fig. 3.—Density and velocity distributions for a model of disk type collapse, \((\Omega_{200}, \Omega_{200}, C) = (0.03, 0.03, 0.0)\). Upper and lower panels show the cross sections in the \(z = 0\) and \(y = 0\) planes, respectively. Color scale denotes the density distribution on a logarithmic scale. Contour curves denote the critical density \(n_{\text{cr}}\). Arrows denote the velocity.
the formation and merging of the dense clumps in the adiabatic disk. The disk has a small radius during formation and then expands greatly after merging. Second, $a_t$ and $a_z$ exhibit an anticorrelation with small amplitude on a short timescale. This anticorrelation is due to intermittent excitation of spiral arms, which transfer angular momentum from the adiabatic disk to the outer infalling envelope. These recurrently excited spiral arms are also seen in Saigo, Hanawa, & Matsumoto (2003), in which growth of the first core was investigated.

Figure 5 also shows the mass of the adiabatic core as a function of time. The mass increases monotonically because of accretion. The average accretion rate is approximately $5 \times 10^{-5} M_\odot \, \text{yr}^{-1}$.

The formation of the dense clump and excitation of the spiral arms are examined here on the basis of the linear stability of Toomre (1964). A thin disk is unstable when the following two conditions are satisfied simultaneously. First, the Toomre $Q$-value, defined as

$$Q = \frac{c \kappa}{\pi G \Sigma},$$

must be smaller than unity in the region of interest, where $c$, $\kappa$, and $\Sigma$ are the sound speed, the epicyclic frequency, and the surface density, respectively. Second, the unstable region should be larger than the critical Jeans length,

$$\lambda_c = \frac{2c^2}{G \Sigma} \left[ 1 + \left( 1 - Q^2 \right)^{1/2} \right].$$

These criteria are applied to our simulation by evaluating

$$\Sigma(x, y) = \int_{\rho > \rho_c} \rho(x, y, z) \, dz,$$

$$c(x, y) = \frac{1}{\Sigma(x, y)} \int_{\rho > \rho_c} \frac{dP}{d\rho} (x, y, z)^{1/2} \rho(x, y, z) \, dz,$$

$$\kappa(x, y) = \left( 4\Omega^2 + R \frac{d\Omega^2}{dR} \right)^{1/2},$$

$$\Omega(x, y) = \frac{1}{\Sigma(x, y)} \int_{\rho > \rho_c} v_z(x, y, z) \frac{R}{R} \rho(x, y, z) \, dz.$$

Figure 6a shows the distribution of the critical Jeans length $\lambda_c(x, y)$ for the stage shown in Figure 3d. The surface density $\Sigma$ takes a positive value within the domain indicated by the closed thick curve, i.e., edge of the adiabatic disk. The Toomre $Q$-value is less than unity in the gray regions, and larger than unity in the white regions. It is less than unity only in the central kernel, dense clump, and spiral arms. The rest of the disk has $Q > 1$, and the disk is globally stable against the ring mode. The critical Jeans lengths are less than 1 AU in both the central kernel and the dense clump, which is roughly 1 AU in size. Thus, both are self-gravitationally bounded. Figure 6b shows the same results for the last stage. The $Q$-value is less than unity only in the kernel and the spiral arms. The spiral arms have width 2.5 AU in the central region of $r \leq 5$ AU, where the critical Jeans length is less than 2.5 AU. In the outer region of $r \geq 10$ AU, the critical Jeans length is 5–7.5 AU, and the spiral arms have width 7.5 AU. Thus, the inner and outer spiral arms are self-gravitating and can be supposed to form by gravitational instability.
denotes the outline of the adiabatic disk. The gray scale denotes the critical type collapse, and the disk-bar type fragmentation.

The cloud collapse is almost axisymmetric in the isothermal disk-bar type. The model in which fragments into two fragments. This fragmentation is called shape in the early accretion phase. Thereafter, the bar deforms to a bar shape at \( t - t_c = 439 \) yr, as shown in Figure 7c, because of self-gravitational instability. The seed of the bar mode is discretization error in this model.

Figure 9 shows the distribution of critical Jeans length for the stage shown in Figure 7b. The elliptical disk of 12 AU \( \times 11 \) AU has a critical Jeans length of 2.4 AU, except in the region of the central holes, \( r \geq 2 \) AU. The disk thus suffers from gravitational instability and deforms to a bar. The bar fragments into two fragments at \( t - t_c \approx 500 \) yr, with masses of \( M_{13} = 1.0 \times 10^{-2} \) and \( 1.1 \times 10^{-2} M_\odot \) and \( M_{14} = 4.8 \times 10^{-3} \) and \( 3.0 \times 10^{-3} M_\odot \).

The fragments rotate around each other and accrete gas from the envelope. Figure 7d shows the fragments at the stage for which \( t - t_c = 1622 \) yr. Similar to the adiabatic core shown in Figure 3f, each fragment has a central kernel and spiral arms embedded in an extended disk. At this stage, the two fragments have masses of \( M_{13} = 3.0 \times 10^{-2} \) and \( 2.4 \times 10^{-2} M_\odot \) and \( M_{14} = 1.8 \times 10^{-2} \) and \( 1.9 \times 10^{-2} M_\odot \).

Figure 10a shows the loci of fragments for the period between the stages of fragmentation and Figure 7d (468 yr \( \leq t - t_c \leq 1622 \) yr). In this period of \( 1.1 \times 10^3 \) yr, both fragments rotate approximately 3 revolutions, increasing in separation. The time variation of the separation is shown quantitatively in Figure 11. The separation of the fragments increases from 11.6 AU with significant oscillation due to the eccentricity of the orbits. The fragments attain maximum separation of 30.6 AU at \( t - t_c = 1622 \) yr (at the stage of Fig. 7d). As the separation increases, the specific orbital angular momentum also increases by a factor of 4.9.

Figure 10b shows the situation after the maximum separation. After the stage of Figure 7d, the separation begins to decrease, to \( \sim 7 \) AU in about 300 yr, as shown in Figure 11. The specific orbital angular momentum also decreases by a factor of 0.28. In this period, the fragments rotate only half a rotation. After the rapid decrease in the separation, the fragments rotate approximately 1.5 revolutions with a nearly constant separation of \( \sim 7 \) AU until the last stage. Figures 7e and 7f show the last stage of the simulation at different magnifications. At this stage, the fragments are separated by 6.7 AU and surrounded by a circumbinary disk with tightly winding spiral arms that transfer the orbital angular momentum of the fragments to the circumbinary disk. The decrease in separation is due to the formation of the circumbinary disk. At the last stage, the two fragments have the same mass, \( M_{14} = 2.4 \times 10^{-2} M_\odot \).

In the models of disk-bar type fragmentation, \((\Omega_{0}, \Omega_{2}, C) = (0.05, 0.0, 0.0)\) and \((0.05, 0.01, 0.0)\), the cloud collapse is almost axisymmetric in the isothermal collapse phase. Figure 7b shows the cloud at the end of the isothermal collapse phase. The central cloud is slightly flattened because of the rotation.

Figure 8 shows the evolution of flattness \( \left( \frac{(a/a_c)^{1/2}}{a_c} - 1 \right) \), eccentricity, \( \left[ (a/a_c) - 1 \right] \), and angular velocity \( \Omega_{0,ff} \) for the central cloud in the isothermal collapse phase. The flattening increases rapidly in proportion to \( \rho_{max} \). At the end of the isothermal collapse phase, a disk forms with flatness of 0.439. The eccentricity remains small because the initial cloud has only a small \( m = 3 \) perturbation, with no bar mode perturbation. The angular velocity \( \Omega_{0,ff} \) is relatively large at the beginning and increases according to \( \Omega_{0.5} \) as \( t \), up to \( \Omega_{0.5,ff} = 0.452 \) in the isothermal collapse phase. The growth rate of \( \Omega_{0,5,ff} \) is similar to that shown in Figure 4 for the disk type collapse model.

Figure 7b shows the adiabatic core at the stage for which \( t - t_c = 301 \) yr. The adiabatic core at this time consists of a flat disk with an envelope. The adiabatic disk deforms to a bar shape at \( t - t_c = 439 \) yr, as shown in Figure 7c, because of self-gravitational instability. The seed of the bar mode is discretization error in this model. Figure 9 shows the distribution of critical Jeans length for the stage shown in Figure 7b. The elliptical disk of 12 AU \( \times 11 \) AU has a critical Jeans length of 2.4 AU, except in the region of the central holes, \( r \geq 2 \) AU. The disk thus suffers from gravitational instability and deforms to a bar. The bar fragments into two fragments at \( t - t_c \approx 500 \) yr, with masses of \( M_{13} = 1.0 \times 10^{-2} \) and \( 1.1 \times 10^{-2} M_\odot \) and \( M_{14} = 4.8 \times 10^{-3} \) and \( 3.0 \times 10^{-3} M_\odot \).

The fragments rotate around each other and accrete gas from the envelope. Figure 7d shows the fragments at the stage for which \( t - t_c = 1622 \) yr. Similar to the adiabatic core shown in Figure 3f, each fragment has a central kernel and spiral arms embedded in an extended disk. At this stage, the two fragments have masses of \( M_{13} = 3.0 \times 10^{-2} \) and \( 2.4 \times 10^{-2} M_\odot \) and \( M_{14} = 1.8 \times 10^{-2} \) and \( 1.9 \times 10^{-2} M_\odot \).

Figure 10a shows the loci of fragments for the period between the stages of fragmentation and Figure 7d (468 yr \( \leq t - t_c \leq 1622 \) yr). In this period of \( 1.1 \times 10^3 \) yr, both fragments rotate approximately 3 revolutions, increasing in separation. The time variation of the separation is shown quantitatively in Figure 11. The separation of the fragments increases from 11.6 AU with significant oscillation due to the eccentricity of the orbits. The fragments attain maximum separation of 30.6 AU at \( t - t_c = 1622 \) yr (at the stage of Fig. 7d). As the separation increases, the specific orbital angular momentum also increases by a factor of 4.9.

Figure 10b shows the situation after the maximum separation. After the stage of Figure 7d, the separation begins to decrease, to \( \sim 7 \) AU in about 300 yr, as shown in Figure 11. The specific orbital angular momentum also decreases by a factor of 0.28. In this period, the fragments rotate only half a rotation. After the rapid decrease in the separation, the fragments rotate approximately 1.5 revolutions with a nearly constant separation of \( \sim 7 \) AU until the last stage. Figures 7e and 7f show the last stage of the simulation at different magnifications. At this stage, the fragments are separated by 6.7 AU and surrounded by a circumbinary disk with tightly winding spiral arms that transfer the orbital angular momentum of the fragments to the circumbinary disk. The decrease in separation is due to the formation of the circumbinary disk. At the last stage, the two fragments have the same mass, \( M_{14} = 2.4 \times 10^{-2} M_\odot \).
Fig. 7.—Same as Fig. 3 but for a model of disk-bar type fragmentation, \((\Omega_{\text{bar}}, \Omega_{\text{disk}}, C) = (0.05, 0.0, 0.0)\). Fig. 7f is an enlargement of Fig. 7e.
4.1.3. Ring-Bar Type Fragmentation

In the models of ring-bar type fragmentation, \((\Omega_{\text{eff}}, \Omega_{2\text{fr}}, C) = (0.1, 0.0, 0.0), (0.1, 0.01, 0.0), (0.2, 0.0, 0.0),\)
cloud collapse is almost axisymmetric, and a flat disk forms in the isothermal collapse phase. The disk deforms to a ring shape temporarily and then to a bar shape in the early accretion phase. Thereafter, the bar fragments into two or three fragments. This fragmentation is called ring-bar type fragmentation. The model in which \((\Omega_{\text{eff}}, \Omega_{2\text{fr}}, C) = (0.1, 0.0, 0.0)\) is shown in Figure 12 as a typical example of ring-bar type fragmentation. This type of fragmentation is similar to disk bar except for the transient formation of a ring and the number of fragments.

The initial stage is the same as for the previous model shown in §4.1.2 except for the initial uniform rotation speed. The cloud collapse is almost axisymmetric in the isothermal collapse phase. Figure 12a shows the cloud at the end of the isothermal collapse phase. The central cloud deforms to a disk shape because of the rotation.

Figure 13 shows the evolution of flatness \((\left[(a_1/a_2)^{1/2} / a_z\right] - 1)\), eccentricity \((|a_1/a_2| - 1)\), and angular velocity \((f_{\text{fr}})\) for the central cloud in the isothermal collapse phase. These evolutions are similar to those for the disk-bar model. The flatness increases rapidly in proportion to \(\rho_{\text{max}}\). At the end of the isothermal collapse phase, a thin disk has formed, with flatness of 1.96. The eccentricity remains small because the initial cloud has only a small \(m = 3\) perturbation and no bar-mode perturbation. The angular velocity \(f_{\text{fr}}\) is relatively large from the beginning and increases according to \(\Omega_{\text{fr}} \propto n_{\text{max}}^2\), to a maximum of \(\Omega_{\text{fr}} = 0.381\) at \(n_{\text{max}} = 5.46 \times 10^5 \text{ cm}^{-3}\). When the central cloud is dislikle, \(f_{\text{fr}}\) becomes saturated.
Figure 12b shows the adiabatic core at $t-t_{cr} = 563$ yr. The central adiabatic core is ringlike and surrounded by a flat isothermal envelope. The ring structure forms because of self-gravitational instability. Figure 14 shows the distribution of critical Jeans length for the stage shown in Figure 12b. The distribution is similar to that for the disk-bar model except that the adiabatic elliptical disk is larger roughly by a factor of 2 while the critical Jeans length is almost the same. The disk thus suffers from ring instability more strongly than in the disk-bar model.

The ring-shaped adiabatic core deforms to a rotating bar as shown in Figure 12c. The bar is twice as long as that in the disk-bar model, and the bar fragments into three fragments. Figure 12d shows the fragments at $t-t_{cr} = 915$ yr. The central fragment is the most massive ($M_{13} = 2.0 \times 10^{-2} M_{\odot}$). The other fragments have masses of $M_{13} = 4.8 \times 10^{-3}$ and $5.4 \times 10^{-3} M_{\odot}$.

Figure 15 shows the loci of the three fragments. The red and blue fragments rotate at a distance of roughly 15 AU, while the green fragment rotates around the red-blue close binary at distance of roughly 40 AU (see also Fig. 12e). The three fragments form a hierarchical triple system.

Figure 12f shows the last stage of the simulation. The three fragments have similar masses, $M_{13} = 1.9 \times 10^{-2} M_{\odot}$ (red), $M_{13} = 1.7 \times 10^{-2} M_{\odot}$ (blue), and $M_{13} = 1.8 \times 10^{-2} M_{\odot}$ (green). The separation between the red and blue fragments is 14 AU, and that between their barycenters and the green fragment is 41.6 AU at the last stage.

The calculation was terminated at this stage because the green fragment escaped from the region covered by the fine grid ($l = 13$) to that covered by the coarser grid ($l = 12$).

4.1.4. Satellite Type Fragmentation

In the models with $0.05 \leq \Omega_{\text{fit}} \leq 0.2$ and with large $\Omega_{2\text{fit}}$, the cloud collapses to form a dense adiabatic core surrounded by an adiabatic disk. The disk suffers from self-gravitational instability and fragments into dense fragments orbiting around the central adiabatic core. The orbiting fragment is called a satellite fragment, and this fragmenta-

Figure 11.—Separation between fragments as a function of time for disk-bar type model, $(\Omega_{\text{fit}}, \Omega_{2\text{fit}}, C) = (0.05, 0.0, 0.0)$.
Fig. 12.—Same as Fig. 3 but for a model of ring-bar type fragmentation, \((\Omega_0, \Omega_2, C) = (0.1, 0.0, 0.0)\)
parameter $C$ is investigated. The azimuthally averaged angular velocity is independent of $R$ (uniform) at $C = 0$ and decreases with increasing $R$ as $C$ gets larger.

Figure 19 shows the models for which $\Omega_{0,\text{ff}} = 0.2$ for the central 230 AU $\times$ 230 AU square at the stage for which $\rho_c \approx \rho_{C2}$. On the basis of the morphology, these models can be classified into three types, i.e., ring, bar, and dumbbell types. Ring-shaped structures are seen in models with large $C$, while bar-shaped structures occur in models with large $\Omega_{2,\text{ff}}$. When both $C$ and $\Omega_{2,\text{ff}}$ are large, the density has two peaks and forms the dumbbell type. Both the dumbbell and ring forms in the model in which $C \geq 0.16$. Fragmentation, particularly the number of fragments, depends critically on the morphology.

The cloud with $\Omega_{0,\text{ff}} \leq 0.03$ collapses to form an adiabatic disk and exhibits no sign of fragmentation (disk type collapse). On the other hand, a cloud with $\Omega_{0,\text{ff}} \geq 0.05$ fragments by any of the disk-bar, ring-bar, satellite, ring, bar, or dumbbell types as far as it collapses. The parameter $\Omega_{0,\text{ff}}$ solely specifies whether the cloud fragments, whereas the other parameters specify only the type of fragmentation.

### 4.2.1. Ring Type Fragmentation

Ring type fragmentation takes place in models with large $C$ and small $\Omega_{2,\text{ff}}$. A ring forms during the collapse and fragments into more than three fragments. Figure 20 show the model with $(\Omega_{0,\text{ff}}, \Omega_{2,\text{ff}}, C) = (0.2, 0.0, 1.0)$ as a typical model of ring type fragmentation. Figure 20a shows the central cloud at the end of the isothermal collapse phase. The evolution is similar for ring type and ring-bar type as far as the isothermal collapse phase is concerned, with a flat disk forming in the cloud center. Figure 20b shows the stage for which $t-t_c = 634$ yr. The central disk suffers from ring instability, and the ring is more prominent than for the ring-bar type. Figure 20c shows the stage for which $t-t_c = 879$ yr, just at the moment of fragmentation. The ring fragments directly into four fragments, whereas the ring deforms into a bar before fragmentation in ring-bar type fragmentation. Figure 20d shows the stage for which $t-t_c = 1.26 \times 10^3$ yr (the last stage), in which four fragments can be seen. The loci of the four fragments are shown in Figure 21. The two central fragments (blue and red) exhibit close encounter while the outer fragments (green and purple) rotate in wide orbits. At the last stage, the masses of the fragments are $M_{13} = 2.0 \times 10^{-2} M_\odot$ (red), $9.4 \times 10^{-3} M_\odot$ (blue), $7.0 \times 10^{-3} M_\odot$ (green), and $7.4 \times 10^{-3} M_\odot$ (purple). The calculation was terminated here because the Jeans condition was violated after the escape of the outer fragments from the fine grid at $l = 12$.

### 4.2.2. Bar Type Fragmentation

Bar type fragmentation takes place in models with large $\Omega_{2,\text{ff}}$. The cloud collapses to form a narrow bar. Although the bar fragments into two fragments in many models, the fragments in this model merge to form a central adiabatic
Fig. 16.—Same as Fig. 3 but for a model of satellite type fragmentation, \((\Omega_{\text{Sr}}, \Omega_{\text{Sr}}, C) = (0.1, 0.05, 0.0)\)
core. The merger is due to the small angular momentum of the fragments. After the merger, the adiabatic core excites spiral arms and eventually satellite type fragmentation occurs as shown in § 4.1.4. Figure 22 shows the model with \( \Omega_0t_{\text{ff}}; \Omega_2t_{\text{ff}}; C = (0.1, 0.05, 0.0) \) as a typical model in which bar type fragmentation followed by satellite type fragmentation.

Figure 22a shows the cloud at the end of the isothermal collapse phase, where it collapses to form a dense bar. Figure 23 shows the evolution of the eccentricity, flatness, and rotation of the central part of the cloud in the isothermal collapse phase. The central angular velocity in unit freefall time, \( \Omega_0t_{\text{ff}} \) and \( \Omega_2t_{\text{ff}} \), increases in proportion to \( n_{\text{max}} \) and reaches \( \Omega_0t_{\text{ff}} = 0.460 \) at \( n_{\text{max}} = 9.04 \times 10^7 \text{ cm}^{-3} \) and \( \Omega_2t_{\text{ff}} = 0.291 \) at \( n_{\text{max}} = 1.14 \times 10^8 \text{ cm}^{-3} \). Meanwhile, the cloud collapses almost spherically. During \( n_{\text{max}} > 10^8 \text{ cm}^{-3} \), \( \Omega_{\text{ff}} \) decreases, and the flatness increases in proportion to \( n_{\text{max}}^{1/2} \) and exceeds unity at \( n_{\text{max}} \approx 10^8 \text{ cm}^{-3} \). The eccentricity also increases, although with significant oscillation. At the end of the isothermal collapse phase, \( a_l, a_s, \) and \( a_z \) are 56.0, 18.4, and 3.84 AU, respectively. The long axis \( a_l \) is 3.05 times longer than the short axis \( a_s \).

Figure 22b shows the central cloud at \( t - t_{\text{cr}} = 210 \text{ yr} \). The bar-shaped, adiabatic core is surrounded by the isothermal disk. At this stage, \( a_l, a_s, \) and \( a_z \) are 30.4, 2.80, and 2.99 AU, respectively. The long axis \( a_l \) is 10.9 times longer than the short axis \( a_s \).

Figure 22c shows the fragmentation of the narrow adiabatic bar at \( t - t_{\text{cr}} = 510 \text{ yr} \). The bar is wound because of the differential rotation and develops two density peaks. The separation between these peaks is 12.0 AU. Figure 22d shows the cloud at \( t - t_{\text{cr}} = 1.01 \times 10^3 \text{ yr} \). These density peaks merge to form a central core surrounded by an adiabatic disk with spiral arms. The spiral arms are the remnants of the wound bar. The disk is supported by centrifugal
force, and its radius is approximately 20 AU at this stage. The disk radius increases because of the accretion of gas from the infalling envelope, and by $t - t_{\text{cr}} \approx 1.5 \times 10^3$ yr the disk has a radius of $\sim 40$ AU and deforms into a ring. The ring is connected to the central kernel via the spiral arms, and the three satellite fragments form at the intersections of the ring and the spiral arms, as shown in Figure 22.

Figure 22/ shows the stage for which $t - t_{\text{cr}} = 1.25 \times 10^3$ yr (the last stage), in which five fragments can be seen. One of the two central fragments is formed by subsequent satellite type fragmentation. These fragments rotate in a close orbit of 17.1 AU. The masses of the fragments of the tight binary at the center are $M_{13} = 1.6 \times 10^{-2}$ and $1.7 \times 10^{-2} M_\odot$. The masses of the other fragments are $M_{13} = 9.4 \times 10^{-3}$, $5.8 \times 10^{-3}$, and $1.8 \times 10^{-3} M_\odot$ from inner to outer.

4.2.3. Dumbbell Type Fragmentation

Dumbbell type fragmentation takes place in models with large $\Omega_{2ff}$ and large $C$. The cloud collapses to form a dumbbell-shaped dense cloud having two density peaks at the end of the isothermal collapse phase. The dumbbell shape is a hybrid of the ring and bar forms. Although each of the density peaks evolves into a fragment, the fragments often merge as in bar type fragmentation. Satellite fragments form at a later stage whenever dumbbell type fragmentation occurs.

Figure 24 shows the model in which $(\Omega_{1ff}, \Omega_{2ff}, C) = (0.2, 0.2, 0.5)$, as a typical model of dumbbell type fragmentation followed by satellite type fragmentation. The evolution of dumbbell type fragmentation is similar to that of bar type fragmentation. Figure 24a shows the dense dumbbell-shaped cloud at the beginning of the accretion phase ($t - t_{\text{cr}} = 151$ yr). The two density peaks evolve into self-gravitationally bounded fragments, as shown in Figure 24b ($t - t_{\text{cr}} = 673$ yr), surrounded by the isothermal disk. The fragments then merge to form a central kernel. Figure 25 shows the loci of the fragments during the merger.

After the merger, the evolution of dumbbell type fragmentation is very similar to that of bar type. The central kernel is surrounded by a rotation-supported disk, which grows in radius by the accretion of gas from the infalling envelope. Figure 24c shows the disk at $t - t_{\text{cr}} = 1.03 \times 10^3$ yr. The disk radius increases to $\sim 40$ AU at this stage and
deforms into a ring. Figure 24d shows the last stage. Satellite fragments form at the intersections of the ring and the spiral arms as in model shown in Figure 22f.

4.3. Classification of Fragmentation Processes

The seven types of collapse and fragmentation described above—disk, disk-bar, ring-bar, satellite, ring, bar, and dumbbell types—are summarized as a means of classification. Figure 26 shows the branching of these fragmentation types schematically. In the isothermal collapse phase, cloud collapse is classified into four main types: disk, bar, dumbbell, and ring types. Disk type collapse is subdivided into disk, satellite, bar, and ring in the accretion phase.

For all types, fragments (self-gravitationally confined clumps) form only in the accretion phase. The isothermal collapse phase is so short that the cloud deforms into a disk, bar, dumbbell, or ring shape, but does not fragment.

Figure 27 summarizes domain of each type of collapse and fragmentation in three-dimensional phase space $(\Omega_{0\text{ff}}, \Omega_{2\text{ff}}, C)$. The models with $\Omega_{0\text{ff}} \leq 0.03$ exhibit disk type collapse (crosses) except for one model, while almost all models with $\Omega_{0\text{ff}} \geq 0.05$ undergo fragmentation. Some exceptional models exhibit oscillation (asterisks). The other parameters, $\Omega_{2\text{ff}}$ and $C$, specify the type of fragmentation.

The red symbols denote the models exhibiting satellite type fragmentation. The red cross denotes the model exhibiting disk type collapse followed by satellite type fragmentation, as shown in § 4.1.1. Similarly, the red triangle denotes the model of bar type fragmentation followed by satellite type fragmentation. Almost all the models proceed to satellite type fragmentation when the bar mode $\Omega_{2\text{ff}}$ of the initial cloud is significant.

Fragmentation could not be confirmed for the models indicated by filled symbols. Almost all these models have either a long bar, long dumbbell, or large ring in the beginning of the accretion phase. For the models indicated by filled triangles and inverted triangles, both the long bar and dumbbell are likely to fragment but could not be confirmed.
because of violation of the Jeans condition before fragmentation. From comparison with bar and dumbbell type fragmentation as shown in §4.2.2 and §4.2.3, these fragments appear to merge, and satellite type fragmentation should follow the merger. Similarly, for models indicated by filled circles the formation of a ring could be followed, but subsequent fragmentation could not.

5. DISCUSSION

5.1. Collapse, Fragmentation, Survival, and Merger

In this subsection, the fate of the collapsing clouds is discussed in detail. A cloud fragments whenever the cloud collapses and the initial rotation is faster than \( \Omega_{0\text{ff}} \simeq 0.05 \), independent of the other parameters \( \Omega_2 \) and \( C \). The cloud collapses to form a flat disk in the isothermal collapse phase, and fragmentation of the flat disk occurs by disk-bar, ring-bar, ring, bar, dumbbell, and satellite types. When \( \Omega_{0\text{ff}} \leq 0.05 \), a flat disk forms in the accretion phase. The flat disk formed in the isothermal collapse phase fragments, whereas that formed in the accretion phase does not.

The criterion for fragmentation is given by the initial angular velocity. The critical angular velocity, \( \Omega_{0\text{ff}} \simeq 0.05 \), is evaluated in terms of spin-up of the collapsing cloud. An isothermal cloud in runaway collapse spins up in proportion to \( \Omega_{r \text{ff}} \propto \rho_c^{1/6} \), where \( \Omega_c \) denotes the angular velocity at the center (Hanawa & Nakayama 1997). After the cloud changes its shape from a sphere to a disk, the angular velocity becomes saturated at \( \Omega_{r \text{ff}} \simeq 0.5 \) (Matsumoto, Hanawa, & Nakamura 1997; Matsumoto & Hanawa 1999). These quantities represent a good index of disk formation.

As shown in §4, the angular velocities \( \Omega_{0.5\text{ff}} \) in models of disk, disk-bar, ring-bar, and bar types have maximum values of 0.0973, 0.452, 0.381, 0.460, respectively, in the isothermal collapse phase. When \( \Omega_{0.5\text{ff}} \) is close to 0.5 (disk-bar, ring-bar, and bar types), the model forms a disk in the isothermal collapse phase and fragments in the later stages. The small difference between \( \Omega_c \) and \( \Omega_{0.5} \) is due to differential rotation in the core. The angular velocity is considerably smaller in the rest of the models (disk type).

Applying these quantities, the condition for formation of a disk in the isothermal collapse phase can be evaluated as:

\[
\Omega_{0\text{ff}} \gtrsim 0.5 \left( \frac{\rho_c}{\rho_{cr}} \right)^{1/6} = 0.045.
\]

This condition is consistent with our simulations.

The formation of a flat disk in the isothermal phase depends on the rotation of the central cloud but not on the rotation law specified by the parameter \( C \). In the isothermal collapse phase, the central velocity and density become more important to cloud collapse as the cloud shrinks (Matsumoto et al. 1997). During the isothermal (runaway) collapse phase, the mass of the central cloud decreases if defined as the mass contained in the isodensity sphere of \( \rho_{max}/2 \). The mass of the central cloud is only 0.01 \( M_\odot \) at the end of the isothermal phase. Since fragmentation takes place in the central 0.01 \( M_\odot \), the density and velocity thereof are important. The density and velocity in the envelope have little effect on the initial fragmentation but are involved in evolution of the fragments in the accretion phase.

In the literature, fragmentation of clouds is typically discussed in terms of the parameters \( \alpha \) and \( \beta \) (e.g., Bodenheimer & Burkert 2001), which are evaluated by volume integration of energy for the entire cloud. As described in equation (5), \( \beta \) depends not only on the central rotation \( \Omega_0 \) but also on the rotation in the envelope. As shown in our models, the epoch of flat-disk formation depends solely on \( \Omega_0 \) and is affected little by rotation in the envelope. Thus, the central angular velocity \( \Omega_0 \), rather than \( \beta \), describes the formation of the flat disk.

The cloud fragments when it satisfies the criterion of equation (19). The number of fragments and their orbital angular momentum then determine whether the fragments merge or survive. When the cloud fragments into three or more fragments, many of the fragments survive, as shown in ring-bar, satellite, and ring types. On the other hand, when the cloud fragments into only two fragments, the fate of the fragments depends on their orbital angular momenta: fragments with high orbital angular momentum survive (e.g., disk-bar type), while fragments with low orbital angular momentum merge (e.g., bar and dumbbell types). In the latter case, satellite fragments form after the merger.

The orbital angular momentum of the fragment depends on the timing of deformation into the bar shape. When the cloud deforms into a bar in the isothermal collapse phase, the bar does not have sufficient spin angular momentum to be supported by the rotation. In the isothermal collapse phase, the cloud undergoes runaway collapse and is never supported by the rotation (Saigo & Hanawa 1998). Thus, the orbital motions of the fragments are also never supported by the rotation. Furthermore, the bar loses angular momentum via gravitational torque in the accretion phase. In the model shown in §4.2.3 (dumbbell type), each fragment has specific orbital angular momentum \( 6.5 \times 10^{19} \text{ cm}^2 \text{ s}^{-1} \) at the stage shown in Figure 24b. This specific orbital angular momentum is only 20% of that required for support by rotation at \( R = 20 \text{ AU} \). On the other hand, when the cloud deforms into a bar in the accretion phase, the bar is supported by the rotation (e.g., disk-bar type). The adiabatic disk accretes gas with high specific angular momentum.
Fig. 22.—Same as Fig. 3 but for a model of bar type fragmentation followed by satellite type fragmentation, \((\Omega_{\text{bar}}, \Omega_{\text{sat}}, C) = (0.2, 0.0, 0.15)\)
and is already supported by rotation prior to its fragmentation.

5.2. Comparison with Earlier Numerical Simulations

Bar and dumbbell type fragmentation were also seen in the numerical simulation of Boss et al. (2000). They followed the evolution of clouds having an initial Gaussian density profile by three types of approximations: isothermal equation of state, barotropic equations of state, and the Eddington approximation of radiative transfer. The barotropic model of Boss et al. (2000) exhibits bar type fragmentation. The density profiles shown in their Figures 5c and 5d are similar to those shown in Figures 22b and 22d here. Using AMR code, they followed the formation of a bar, fragmentation of the bar, and merger of the fragments. This evolution resembles that of the model shown in §4.2.2, although the initial condition is quite different. They terminated the calculation at the stage of adiabatic disk formation after the merger, and the stage of satellite type fragmentation was not shown. Dumbbell type fragmentation is also seen in their Figure 2a, which

![Fig. 23: Same as Fig. 4 but for a model of bar type fragmentation followed by satellite type fragmentation. 
\( (\Omega_0, \Omega_2, C) = (0.2, 0.2, 0.15) \). Dotted curves denote the relationships \( n_{\text{max}} \) and \( n_{\text{max}} \) for comparison.](image)

![Fig. 24: Same as Fig. 3 but for a model of dumbbell type fragmentation followed by satellite type fragmentation. 
\( (\Omega_0, \Omega_2, C) = (0.2, 0.2, 0.5) \).](image)
is quite similar to Figure 24a here. Dumbbell type fragmentation in their study was computed under the Eddington approximation.

The initial cloud of Boss et al. (2000) is defined by α = 0.26, whereas α = 0.765 in the present study. Their cloud was thus colder or, in other words, more massive. Despite this difference, the fragmentation is very similar. The same mechanism of deformation and fragmentation therefore appears to be valid over a wide range of α.

Satellite type fragmentation has also been seen in many other simulations (Bonnell 1994; Burkert et al. 1997; Bate, Bonnell, & Bromm 2002). Bonnell (1994) followed the fragmentation in the second collapse, in which the first core collapses to form the second core. In their simulations, satellite fragments form through interaction of the spiral arms. The satellite fragments form in the same manner both in their and our simulations, even though different situations are considered. Burkert et al. (1997) followed the collapse and fragmentation of molecular cloud cores similar to this paper. They also computed the formation of satellite fragments and followed their orbits by using a nested grid. It was not explicitly mentioned whether the nested grid simulation satisfies the Jeans condition. Burkert et al. (1997) and Bate et al. (2002) confirmed the result by independent simulations using a smoothed particle hydrodynamics code.

5.3. Application to Formation of Binary and Multiple Stars

Some observations have indicated the rotation of molecular cloud cores. Goodman et al. (1993) found that 29 of 43 molecular clouds had a significant velocity gradient, corresponding to rigid rotation of 2 × 10^{-3} < β < 1.4 with typical values of β ∼ 0.02. These quantities correspond to 0.047 < Ω_{2tg} < 1.25 with typical values of Ω_{2tg} ∼ 0.15 for models for which (Ω_{2tg}, C) = (0, 0). Unfortunately, the observations were not of sufficient accuracy to specify the rotation law. Our simulations show that collapsing clouds having an initial rotation of Ω_{2tg} ≥ 0.05 fragment, which is consistent with observed high binary frequency.

Molecular cloud cores have internal motion often interpreted as turbulence. The internal motion should reflect the superposition of various modes of velocity perturbations. When the bar mode of a cloud is a significant, the cloud undergoes satellite type fragmentation as shown in our simulations. Therefore, satellite type fragmentation may be dominant. The satellite fragments merge and scatter while accreting gas. Consequently, the satellites have various binary separations. This may explain the wide range of separation for young and main-sequence binaries (e.g., Mathieu 1994).

It has recently been suggested that brown dwarfs may be formed by ejection of the seeds of stars from a parent cloud core (Reipurth & Clarke 2001; Bate et al. 2002). Satellite type fragmentation might be a corresponding case. In many cases of satellite type fragmentation, three or more fragments are formed. In these multiple systems, it is possible that a close encounter will eject the fragment from the cloud center. The satellite fragment has speed ~1 km s^{-1} at the last stage of the model in which (Ω_{2tg}, Ω_{2tg}, C) = (0.2, 0.2, 0.15) (for a fragment shown in Figure 22f, right). The velocity would be reduced substantially by ejection by the gravity of the molecular cloud core. The gravitational potential is evaluated to be ψ ≃ 2c^2 Π r, and the ejection speed would be v_{esc} ∼ 1 km s^{-1}. The ejected satellite fragment would have a velocity on the order of the escape speed if it exits.

6. SUMMARY

The collapse and fragmentation of molecular cloud cores was investigated for the case in which the initial cloud is almost in equilibrium, focusing on the effects of rotation speed, rotation law, and bar mode perturbation. The main results are summarized as follows.

A cloud 1.1 times denser than the critical Bonnor-Ebert sphere fragments when rotation of the initial cloud is slow enough to allow collapse but still significant, i.e., Ω_{2tg} ≥ 0.05. The latter condition gives rise to the formation of a flat disk in the isothermal collapse phase. This condition is independent of both the initial amplitude of the bar mode and the initial rotation law.

Six types of fragmentation were identified: disk-bar, ring-bar, satellite, bar, ring, and dumbbell types. The type of fragmentation depends on the initial amplitude of the bar mode and the initial rotation law. The fragments formed via bar or dumbbell type fragmentation merge because of their low angular momenta, and new fragments form via satellite type
fragmentation. In other words, a cloud forms satellite fragments whenever the bar mode of the initial cloud has an appreciable amplitude. Merger and close encounter of the satellite fragments may result in the wide range of the binary separation.

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