Relic density in SO(10) SUSY GUT models

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Abstract. Non-universal boundary conditions in grand unified theories can lead to non-universal gaugino masses at the unification scale. In R-parity preserving theories the lightest supersymmetric particle is a natural candidate for the dark matter. In this talk the composition of the lightest neutralino is studied, when nonuniversal gaugino masses come from representations of SO(10). In these cases, the thermal relic density compatible with WMAP observations is found.

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INTRODUCTION

The phenomenology of supersymmetric models depends crucially on the compositions of neutralinos and charginos. In addition to the laboratory studies, relevant input is obtained from the dark matter searches, where the WMAP satellite has put precise limits on the relic density. Supersymmetric theories which preserve R-parity contain a natural candidate for the cold dark matter particle. If the lightest neutralino is the lightest supersymmetric particle (LSP), it can provide the appropriate relic density.

In many supergravity type models the lightest neutralino is bino-like, which often leads to too high thermal relic density, as compared to the limits provided by the WMAP experiment. When the gaugino masses are not universal at the grand unification scale, the resulting neutralino composition changes from the usual universal gaugino mass case [1].

In this talk, the thermal relic density of the neutralino LSP is studied, when gaugino masses are due to nonuniversal representations of SO(10) grand unified theory (GUT) [2, 3]. SO(10) has many attractive features when compared to SU(5), in which the field strength superfield transforms in the adjoint representation. If the lightest neutralino is the lightest supersymmetric particle (LSP), it can provide the appropriate relic density.

Gaugino masses originate from the non-renormalizable terms in the $N = 1$ supergravity Lagrangian involving the gauge kinetic function $f_{ab}(\Phi)$ [6]. The gauge part of the Lagrangian contains the gauge kinetic function coupling with two field strength superfields $W^a$. The function $f_{ab}(\Phi)$ is an analytic function of the chiral superfields $\Phi$ in the theory. It transforms as a symmetric product of two adjoint representations under the GUT gauge group, since the field strength superfield transforms in the adjoint representation. In order to generate a mass term for the gauginos, the gauge kinetic function must be non-minimal, i.e. it must not be a constant [7]. The chiral superfields $\Phi$ consist of a set of gauge singlet superfields $\Phi^a$ and gauge nonsinglet superfields $\Phi^a$, respectively, under the grand unified group. If the auxiliary part $F_0$ of a chiral superfield $\Phi$ in the $f_{ab}(\Phi)$ gets a vev, then gaugino masses arise from the coupling of $f_{ab}(\Phi)$ with the field strength superfield $W^a$. The Lagrangian for the coupling of gauge kinetic function with the gauge field strength is written as

$$\mathcal{L}_{gk} = \int d^2 \theta f_{ab}(\Phi) W^a W^b + \text{H.C.,} \quad (1)$$

where $a$ and $b$ are gauge group indices (for example, $a, b = 1, 2, ..., 45$ for SO(10)), and repeated indices are summed over. The scalar fields in $f_{ab}(\Phi)$ are suppressed by the inverse powers of $M_P$, so the gauge kinetic function $f_{ab}(\Phi)$ can be expanded,

$$f_{ab}(\Phi) = f_0(\Phi^s) \delta_{ab} + \sum_n f_n(\Phi^s) \Phi^{a\bar{b}} M_P^n + \cdots, \quad (2)$$

where $\Phi^s$ and $\Phi^{a\bar{b}}$ are the singlet and nonsinglet chiral superfields, respectively. Here $f_0(\Phi^s)$ and $f_n(\Phi^s)$ are functions of gauge singlet superfields $\Phi^s$, and $M_P$ is some large scale. When $F_0$ gets a vev $(F_0)$, the interaction (1) gives rise to gaugino masses:

$$\delta m^2 = \frac{f_{ab}(F_0) \Phi^{a\bar{b}}}{M_P} + \text{H.C.,} \quad (3)$$

where $\lambda^a$ are gaugino fields. The non-universal gaugino masses are generated by the nonsinglet chiral superfield...
shows the area of preferred displays the ratios of resulting contributions \([\Phi]\) [2].

Gauginos belong to the adjoint representation of the gauge group, which in the case of \(SO(10)\) is the \(45\) dimensional representation. Because Eq. (3) must be gauge invariant, \(\Phi\) and \(F_\Phi\) must belong to some of the following representations appearing in the symmetric product of the two \(45\) dimensional representations of \(SO(10)\) [8]:

\[
(45 \otimes 45)_{\text{Symm}} = 1 \oplus 54 \oplus 210 \oplus 770. \quad (4)
\]

The representations \(54, 210\) and \(770\) may lead to non-universal gaugino masses, while the \(1\) dimensional representation gives manifestly the universal gaugino masses. The relations between the gaugino masses are determined by the representation invariants, and are specific for each of the representations. Because the gauge kinetic function (2) can get contributions from several different \(\Phi\)’s, a linear combination of any of the representations is also possible. In that case the gaugino masses are not anymore uniquely determined, in contrast to the contribution purely from one representation. Because of this it is more instructional to study the representations separately. In other words, we assume that the dominant component of the gaugino masses comes only from one representation, and more specifically, since we want to study non-universal gaugino masses, from one of the non-singlet representation.

DARK MATTER IN SO(10) REPRESENTATIONS

Breaking Chains: \(SO(10) \rightarrow H \rightarrow SM\)

The GUT group \(SO(10)\) breaks down to the Standard Model (SM) gauge group \(SU(3)\times SU(2) \times U(1)\) via some intermediate gauge group \(H\). Therefore the gaugino mass relations depend also on the gauge group breaking chain, in addition to the representation invariants coming from the gauge kinetic function. Moreover, the intermediate breaking scale affects also the generated gaugino masses. However, if the gauge breaking from the GUT scale to the SM scale takes place all at the GUT scale, these loop-induced messenger contributions [9] can be neglected in comparison to the tree-level contributions. Table 1 shows various possible \(SO(10)\) breaking chains [10, 8] for the two chosen representations. Some of the subgroups lead to universal gaugino masses, or to massless gauginos. In this talk we suppose that the gauge breaking from \(SO(10)\) to \(SU(3)\times SU(2) \times U(1)\) happens at the GUT scale, and that the GUT breaking does not affect the gauge coupling unification. We also limit our study to the representations \(54\) and \(210\) and to three specific intermediate gauge groups, \(SU(4)\times SU(2)\times SU(2)\), \(SU(2)\times SO(7)\) and \(SU(5)\times U(1)\). Table 2 displays the ratios of resulting gaugino masses at tree level as they arise when \(F_\Phi\) belongs to various representations of \(SO(10)\). The relations at the electroweak scale resulting from 1-loop renormalization group running are also displayed.

**TABLE 1.** Various breaking chains of \(SO(10)\)

| \(F_\Phi\) | \(H\) | Subgroup description |
|---|---|---|
| 54 | \(SU(4)\times SU(2)\times SU(2)\) | Pati–Salam |
| | \(SU(2)\times SO(7)\) | Universal gauginos |
| | \(SO(9)\) | |
| 210 | \(SU(4)\times SU(2)\times SU(2)\) | Massless gluino |
| | \(SU(3)\times SU(2)\times SU(2)\times U(1)\) | Massless \(SU(2)\) gauginos |
| | \(SU(3)\times SU(2)\times U(1)\) | |
| | \(SU(5)\times U(1)\) | 'Flipped' \(SU(5)\) |

**TABLE 2.** Ratios of the gaugino masses at the GUT scale in the normalization \(M^GUT_3 \equiv M_3(GUT) = 1\), and at the electroweak scale in the normalization \(M^EW_3 \equiv M_3(EW) = 1\)

| \(F_\Phi\) | \(H\) | \(M^GUT_3\) | \(M^GUT_5\) | \(M^GUT_7\) | \(M^EW_3\) | \(M^EW_5\) | \(M^EW_7\) |
|---|---|---|---|---|---|---|---|
| 54 | \(SU(4)\times SU(2)\times SU(2)\) | 1 | -1 | -1.5 | 0.14 | 0.29 | 1 |
| | \(SU(2)\times SO(7)\) | -1 | -1.5 | 0.14 | -0.15 | -0.44 | 1 |
| | \(SU(3)\times SU(2)\times U(1)\times U(1)\) | 1 | -7/3 | 0.15 | -0.68 | 1 |
| | \(SU(5)\times U(1)\) | -96/25 | 1 | -0.56 | 0.29 | 1 |

**Representation 210**

In the representation \(210\) we inspected the breaking chain through the intermediate gauge group \(SU(5)\times U(1)\) (called flipped \(SU(5)\)). In Figure 1 the area of preferred thermal relic density in the representation \(210\) is plotted for a set of (GUT scale) parameters. For the chosen parameters, rather large WMAP preferred regions are found for the large values of \(M_2\) and/or \(m_0\) parameters. The dark shaded areas represent larger relic density than the lighter areas. \(\text{wmap}\) denoted filling is the WMAP preferred region, \(\text{lep}\) shows an area, where the experimental mass limits are not met, \(\text{rge}\) shows an area where there is no radiative \(\text{EWSB}\), and \(\text{lsp}\) an area where neutralino is not the LSP. For the relic density, we use here the WMAP combined three year limits [11]

\[
\Omega_{\text{CDM}} h^2 = 0.11054 \pm 0.00076 = 2\sigma. \quad (5)
\]

The tiny area enclosed by the \(bg\) contour is disallowed by the \(bg\) constraint. We have used the two sigma world average of \(BR(b \rightarrow s\gamma) = (355 \pm 24_{-10}^{+9}) \times 10^{-6}\) for the branching fraction [12]. The dash-dotted line (h) encloses an area with \(m_h < 114\) GeV. The \(\text{lep}\) limits for the particle masses are the same as in [13]. The spectrum was calculated with \(\text{SOFTSUSY}\) [14] and relic densities and constraints with \(\text{micrOMEGAs}\) [15].
The WMAP preferred region is very wide as compared
to e.g. models in typical mSUGRA or SU(5) [4, 13]. For
a large part of the parameter space the relic density is
below the WMAP limit. The lightest neutralino is either
wino or higgsino, which explains the low overall relic
density. In the diagonal of the figure the lightest chargino
mass is very close to the lightest neutralino mass leading
to the remarkable co-annihilation through the processes
$\tilde{\chi}_1^0 \tilde{\chi}_1^\pm \rightarrow q u d \bar{l}$ and
$\tilde{\chi}_1^\pm \tilde{\chi}_1^- \rightarrow q u d + \tilde{\nu} \tilde{l} W^+ W^-$. The spectrum is relatively heavy for the WMAP preferred
region, but the value of $\mu$ stays around a TeV, which
doesn’t necessarily require large fine-tuning.

**Representation 54**

The two chains of the 54 dimensional representation
do not lead to large areas with acceptable relic density. Because $\tilde{\chi}_1^0$ is mostly bino in SU(2)×SO(7) chain, the
spectrum with preferred RD is quite light and conflicts
with collider constraints in some parts of the parameter
space (although the higgsino component keeps the over-
all thermal relic density lower than for example in the
universal SUGRA model). In SU(4)×SU(2)×SU(2) the
higgsino component in the lightest neutralino is larger,
and also co-annihilation with chargino is present at some
points of parameter space, so the allowed region extends
also to heavier spectrum. For more detailed analysis
the reader is referred to an upcoming publication [16].

**SUMMARY**

We studied the dark matter allowed regions in the SO(10)
GUT representations, of which all but the singlet may
lead to non-universal gaugino masses. The WMAP preferred
relic density regions were quite distinct for different
representations, thus leading to quite different particle
spectra for each representation. As an example, we
showed the behavior of the thermal relic density in the
representation 210, since there the spectrum is quite
heavy as compared to the universal SUGRA case. It was
also shown, that the WMAP preferred relic density area is
very large in this case. Moreover, it is important to rea-
ize that there is no automatical theoretical preference for
the gaugino masses to be unified.

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