Mathematical Model for Transient Fluid Flow in a Continuous Casting Mold

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(Received on February 28, 2001; accepted in final form on April 24, 2001)

A mathematical model that can simulate the transient fluid flow phenomena in a continuous casting mold has been developed. In this mathematical model, multi-phase flow phenomena which consist of the Ar gas injected into the submerged entry nozzle, molten steel and solidified shell can be described. To verify the availability of the mathematical model, water cold model and fused metal hot model experiments were conducted and comparisons of the measured fluid flow phenomena were made with computational results. In the water cold model, the time fluctuation of the fluid flow just under the meniscus and the deformation of the meniscus shape were measured and in the fused metal hot model, effect of the Ar gas bubble on the fluid flow in the mold were investigated. Computational results gave good agreement with both experimental measured data. Real plant simulation were conducted to investigate the effect of Ar gas bubble on the fluid flow phenomena in the continuous casting mold by using this mathematical model.

KEY WORDS: transient fluid flow; gas bubble; solidified shell; multi phase; heat transfer; mathematical model; continuous casting; mold.

1. Introduction

The fluid flow and heat transfer phenomena in a continuous casting mold are very complex, and it is clear that these phenomena affect considerably on the product quality. Recently, requirement level toward the product quality has been rising more and more, therefore the understandings of the fluid flow and heat transfer phenomena in a continuous casting mold have become to be important. Many works\(^\text{1-6)}\) about the fluid flow and heat transfer phenomena in a continuous casting mold have been done from a long time ago, and recently, since the ability of the computer performance and mathematical simulation technique have improved rapidly, many works\(^\text{7-12)}\) by using the mathematical model have been active and given the basic understandings about the fluid flow and heat transfer phenomena in a continuous casting mold. But, those mathematical models are based on RANS (Reynolds–Averaged Navier–Stokes) and do not treat the transient fluid flow phenomena.\(^\text{12)}\) In a water cold model, the oscillation of the fluid motion injected through the SEN (submerged entry nozzle) was observed,\(^\text{11)}\) and it is recognized that transient phenomena in a mold are very important. Furthermore, it is interesting how Ar gas bubbles affect the transient fluid flow phenomena. However, there are few works\(^\text{13,14)}\) about the transient fluid flow phenomena in a continuous casting mold.

The purpose of this study is to develop a transient mathematical model fully coupled analysis of the molten steel flow with gas bubbles and heat transfer phenomena proceeding with solidification, and to understand the transient fluid flow phenomena in a continuous casting mold.

2. Mathematical Model

2.1. Governing Equations

The following assumptions and simplifications are made in this model.

(1) The fluid is incompressible, and liquid and solid phase density and specific heat are same and constant.
(2) Gas bubble is incompressible, shape is spherical, and diameter is not changed.
(3) Solid phase acts as a rigid body. (velocity of solid phase is equal to the casting speed.)
(4) In energy balance, enthalpy of gaseous phase is negligible.
(5) Momentum equation of gaseous phase is subject to Basset–Boussinesq–Oseen–Tchen equation.\(^\text{15)}\) However, Basset term was neglected.
(6) In momentum balance for liquid phase,\(^\text{15)}\) is very small and negligible.
(7) The area of\(^\text{f}_g\) = 0.8 is judged rigid body.
(8) LES (Large Eddy Simulation) is adopted as the turbulent model.\(^\text{16)}\)
(9) Turbulent Prandtl number is equal to 1.

The governing equations take the following form:

\[ f_g + f_l + f_s = 1 \]  \hspace{1cm} (1)
\[ \frac{\partial f'}{\partial t} + \frac{\partial}{\partial x_j} (f'U_{ij}) = -\text{Rate} \]  \hspace{1cm} (2)

\[ \frac{\partial f'}{\partial t} + \frac{\partial}{\partial x_j} (f'U_{ij}) = \text{Rate} \]  \hspace{1cm} (3)

\[ \frac{\partial f}{\partial t} + \frac{\partial}{\partial x_j} (fU_{ij}) = 0 \]  \hspace{1cm} (4)

\[ \frac{\partial U_{ij}}{\partial t} + U_{ij} \frac{\partial U_{ij}}{\partial x_j} = - \frac{1}{\rho_l} \frac{\partial p}{\partial x_j} + \frac{1}{\rho_l} \left[ (v_i + v_j) \left( \frac{\partial U_{ij}}{\partial x_j} + \frac{\partial U_{ij}}{\partial x_i} \right) \right] + F_i / f_i \rho_i \]  \hspace{1cm} (5)

\[ \rho \left( \frac{DU_{ij}}{Dt} \right) = \rho_1 \frac{\partial U_{ij}}{\partial t} + \frac{1}{2} \rho_1 \left[ \frac{\partial U_{ij}}{\partial t} - \frac{\partial U_{ij}}{\partial x_j} \right] \]

\[ - \frac{3 \mu}{4 d^2} \text{Re}_g C_{Dg} (U_{ij} - U_{ij}) + (\rho_1 - \rho_g) g_i \]  \hspace{1cm} (6)

\[ \frac{\partial T}{\partial t} + (f_i U_{ij} + f_j U_{ij}) \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ (\alpha + \alpha_r) \frac{\partial T}{\partial x_j} \right] + \frac{1}{C_p} \text{Rate}(\Delta H) \] \hspace{1cm} (7)

where

\[ p = (P + U_{ij} U_{ij} / 3), \quad v_i = (C_i \Delta) \left[ \frac{\partial U_{ij}}{\partial x_j} + \frac{\partial U_{ij}}{\partial x_i} \right]^{1/2}, \]

\[ C_i = 0.1, \quad \Delta = \text{Vol}^{1/3}, \quad \text{Vol} : \text{cell volume} \]

\[ F_i = \frac{3 \mu}{4 d^2} \text{Re}_g C_{Dg} (U_{ij} - U_{ij}). \]

\[ \text{Re}_g = d_i |U_i - U_l| / \mu_1, \quad C_{Dg} = 0.4 + 24 / \text{Re}_g + 6 / (1 + \sqrt{\text{Re}_g}) \]

Finally, relation between solidification rate and temperature is assumed linearly as the following form.

\[ T = T_s + (1 - f_s) T_l \] \hspace{1cm} (8)

where \( T_l \): liquidus temperature, \( T_s \): solidus temperature.

### 2.2 Free Surface

Meniscus movement can be represented by the height function method.\(^{17}\) Computational domain was divided into two parts. Grids at the upper part which includes the free surface was moved and at lower part was not moved as shown in Fig. 1. At the free surface, computational grids were moved with the fluid velocity in the vertical direction and were not moved in the horizontal direction. Intermediate grids were rearranged dividing equally the space between the free surface and the rigid line. Consequently in the area of moving grids, ALE (Arbitrary Lagrangian–Eulerian Computing) method\(^{18}\) was adopted for convective terms and regular method was adopted in the rigid area.

### 2.3 Solution Procedure

The solution algorithm was adopted SOLA method\(^{19}\) in the present mathematical model and basic equations are discretized on the staggered grid. The boundary fitted coordinate system was applied.\(^{20}\) The third-upwind scheme was used for convective term,\(^{21}\) and the second-order central scheme was used for diffusive term. All of equations were solved in time marching method by using the Euler method as the time integration, and time averaged field variables were obtained by summation of the computational results in each time step. This solution procedure is shown in Fig. 2.

### 3 Experiments and Model Validation

Two kinds of experiments were done to evaluate the
above mentioned mathematical model validity. One is the water cold model in which the fluid flow fluctuation originated in the turbulence was investigated and the other is the fused metal hot model in which Ar gas bubbles effect on the fluid flow phenomena was examined.

3.1. Water Cold Model

3.1.1. Comparison between Measured and Computational Results

The cross direction distribution of the velocity, velocity fluctuation just under the meniscus and meniscus profile were researched by using the 1/1 scaled water cold model as shown in Fig. 3. Two kinds of configuration of the SEN were prepared, since the shape of the SEN affects considerably the fluid flow phenomena in the continuous casting mold. Experiments were done in the condition shown in Table 1, and ① velocity fluctuation at the 1/2 thickness and 1/4 width just under the meniscus, ② the horizontal velocity distribution of the cross direction at the 1/2 thickness of the mold, and ③ the distribution of the meniscus shape of the cross direction were measured. The simultaneous measurement of the velocity was done by using two velocimeter on the velocity fluctuation measurement at the 1/2 thickness and 1/4 width. Propeller velocimeter was used for the measurement of the velocity, and a ruler was used for the measurement of the meniscus profile, and the distance to the meniscus from the base line was measured several times, and the mean value was adopted as the time averaged value. The time average operation of the velocity was made by the measured data for 10 min.

The cross direction distribution of the time averaged horizontal velocity at the 1/2 thickness of the mold and the meniscus profile are shown in Figs. 4, 5. As casting speed increases, horizontal velocity increases, and it is obvious that the deformation of the meniscus shape is large as well. The horizontal velocity just under the meniscus with Type B SEN which does discharge downward more is slower than Type A and the deformation of the meniscus is smaller, too. As the width of the mold is smaller, the horizontal velocity just under the meniscus is slower with the same SEN and same casting speed. Comparisons between measured and computational results are shown in the same figure, and the coincidence of both results is fairly good.

The momentary and time averaged computational flow fields are shown in Fig. 6. Though the flow field oscillates on either side, time averaged flow field is symmetrical. Figure 7 shows the comparison between the measured data at the 1/2 thickness and 1/4 width of the mold just under the meniscus and computational results. Remarkably, the hori-

![Fig. 3. Schematic view of 1/1 scaled water cold model.](image)

![Table 1. Experimental condition of water cold model.](table)

| Casting speed Vc (m/min) | 2.2, 3.3, 4.9 |
|--------------------------|--------------|
| SEN type                 | A, B         |
| Flow domain size (mm)    | width: 1000-1500, thickness: 90, length: 3500 |

![Fig. 4. Comparisons between computational and measured horizontal velocity under the meniscus.](image)
Horizontal velocity just under the meniscus fluctuates hardly, and the momentary maximum velocity reaches about two times of the time averaged value. Computational amplitude of the horizontal velocity coincides with measured data very well though the short time period of the horizontal velocity fluctuation which is under a second is different in the measured and computational results. The reason for this disagreement depends on the response speed of the velocimeter or time integration accuracy of the computational procedure and/or the others, and it is future problem. However, this mathematical model for transient fluid flow is able to give substantially meaningful results in the industrial design of the continuous casting machine.

3.1.2. Simple Estimation Method of the Meniscus Shape

Though the availability of this mathematical model was ascertained, the computational load is very heavy for a transient calculation with the moving boundary. Therefore, simple model was made to reduce this load. Meniscus height $h$ is estimated on the relation of $p = \rho gh$ from the calculated pressure under the flat shape meniscus in the simple model. Comparisons between the precise and simple model results are shown in Fig. 8, and both are consistent, therefore simple model is fairly precise. Furthermore, results of the simple model coincided with the precise model about the time fluctuation of the horizontal velocity just under the meniscus as shown in Fig. 9. Consequently, even if simple model is used, the horizontal velocity under the meniscus and the
meniscus profile can be estimated. This simple model was used in the following investigations.

3.2. Fused Metal Hot Model

As shown in Fig. 10, an experiment with the low melting point alloy was conducted to clear the effect of Ar gas bubble injected into the SEN on the fluid flow phenomena in a mold. The reason why low melting point alloy is used is to choose the fluid which is as close as possible to the actual molten steel. Characteristics of this low melting point alloy and molten steel are shown in Table 2. The experimental apparatus is the 1/4 scaled model for No. 3 CCM (Continuous Casting Machine) in Kashima steel works, and experimental conditions are selected so that Froude number might coincide with the actual plant conditions corresponding to casting speed about 2 m/min and experimental conditions are shown in Table 3. Temperature of fused metal was controlled to be 110°C. Ar gas was injected into the SEN, and diameters of the bubbles which leave through a mold top surface and number of bubbles were filmed by the high-speed video camera, and analyzed from the picture analysis. The approval method of the bubble diameter measurement is mentioned later. Gas flow rate through the meniscus which was divided into three parts in the width direction was measured by the mass flow meter. The fluid velocity just under the meniscus at the 1/2 thickness and 1/4 width was measured with Karman vortex velocimeter.

The approval method of the bubble diameter which is obtained from picture analysis is as the following. Ar gas is injected with the constant flow rate into the low melting point alloy, and consequently gas bubbles are generated in the experimental apparatus as shown in Fig. 11. The meniscus surface was filmed by the high-speed video camera, and the number of bubbles and diameter are measured from the picture. The comparison between the diameter decided from the picture analysis and the results calculated from the gas flow rate and the bubble number is shown in Fig. 12. If a bubble diameter is less than about 6 mm, both diameters above mentioned are almost same, and the bubble shape is spherical and if greater than 6 mm, the bubble shape is assumed to be not spherical.

![Fig. 7. Comparisons between measured and calculated horizontal velocity under the meniscus at the 1/2 thickness and 1/4 width of the mold. (SEN: Type A, Mold width: 1500 mm)](image_url)

![Fig. 8. Comparisons between simple and precise simulation results of meniscus profile.](image_url)
Measured relation between the gas bubble diameter and flow rate ratio to the total flow rate is shown in Fig. 13 by using the apparatus as shown in Fig. 10. The bubble diameter of 1 mm and under is almost, and there is a little rate of the diameter of 5 mm and more, and the above condition is almost satisfied. It is very difficult to describe the behavior of the bubbles in the SEN because the wettability of the SEN wall and the existence of the free surface in the SEN affect the coalescence/breakup of the bubbles. Here, observed bubble diameter distribution through the mold top surface was adopted as a boundary condition at the SEN entry. This boundary condition is suitable if the coalescence/breakup of the bubbles can be ignored out of the SEN. Therefore, the coalescence/breakup of the bubbles in

| Casting speed V (cm/min) | 1.19 |
|-------------------------|------|
| Ar gas flow rate (NL/min) | 0.63 |
| SEN type | 2 square holes [23 × 23] (mm) |
| | discharge angle: 30° downward |
| Flow domain size (mm) | 300 × 68 × 140 |
| width × thickness × length | |

Fig. 9. Comparisons between simple and precise simulation results of the time fluctuation of the fluid velocity at the 1/2 thickness and 1/4 width of the mold.

Fig. 10. Schematic view of 1/4 scaled hot model.

Fig. 11. Schematic view of experimental apparatus for measurement of gas bubble diameter.

Table 2. Characteristics of low melting point alloy and molten steel.

|                | low temperature melting point alloy | molten steel |
|----------------|------------------------------------|--------------|
| Melting point (°C) | 70                                 | 1540         |
| Density (kg/m³)       | 9600                               | 7000         |
| Viscosity (kg/m s⁻¹) | 0.0032                             | 0.006        |
| Surface tension (N/m) | 0.29                               | 1.8          |
the SEN was taken into consideration indirectly but the behavior of the bubbles in the SEN wasn’t described properly in this mathematical model. The values indicated with filled circles in Fig. 13 are the bubble diameters used by the fol-

Fig. 12. Approval of measurement method of bubble diameter.

Fig. 13. Ratio of flow rate of each bubble diameter to total flow rate.

a) time changes of horizontal velocity under the meniscus.

b) comparisons between measured and calculated horizontal velocity under the meniscus.

c) time averaged velocity field at the 1/2 thickness of the mold

\[ \text{Ar=0. (NI/min)} \quad \text{Ar=0.63 (NI/min)} \]

Fig. 14. Effect of Ar gas flow rate on the fluid flow in the mold.
Following calculations, and five kinds of bubble diameters were considered. Computational results of fluid flow analysis are shown in Fig. 14. Upper circulating flow in the mold is suppressed by the buoyancy force caused by injected Ar gas as compared with no Ar gas injection. Comparison between the measured and calculated results of the horizontal velocity just under the meniscus at the 1/2 thickness and 1/4 width in the mold is shown in Fig. 14 and both results are almost in good agreement.

The comparison between the calculated and measured results of gas flow rate through the meniscus in the width direction is shown in Fig. 15. Both results almost coincide though measured result seems to be more uniform compared with the calculated result. It is expected that the actual bubble diameter is smaller than the measured one since the small bubble diameter of 1 mm and under is counted as 1 mm and calculated gas flow rate profile will be more uniform. If the distribution of the bubble diameter can be given
precisely, effect of Ar gas injection into the SEN on the molten steel flow phenomena in the continuous casting mold can be evaluated quantitatively by using this mathematical model.

4. Real Plant Simulation

Model calculations corresponding to the actual real plant were carried out by using this mathematical model. Real plant simulations corresponding to No. 3 CCM in Kashima steel works are conducted in the condition shown in Table 4 and effect of Ar gas injection into the SEN on the fluid flow and heat transfer phenomena in the mold was investigated. The measured distribution of the gas bubble diameter above mentioned is adopted since the distribution of the bubble diameter in the real plant is unknown. It was supposed that injected gas temperature becomes molten steel temperature immediately, and temperature correction of the gas flow rate was done according to the ideal gas law. Computational results are shown in Fig. 16. Upper circulating flow in the mold is suppressed by the buoyancy force caused by the gas bubbles but the velocity fluctuation becomes greater than without Ar gas injection in the real plant mold as well as the hot model experiment.

Temperature at the meniscus is higher with Ar gas injection than without injection as shown in Fig. 16, but temperature under 1 400 mm below the meniscus is almost same with/without Ar gas injection. It is recognized that solidified shell is thin in the mold corner on which molten steel flow through the SEN impinges.

Finally, volumetric fraction distribution of Ar gas bubbles is shown in Fig. 17, and as bubble diameter is smaller, bubble reaches far away.

It is concluded that Ar gas injection into the SEN affects considerably on the fluid flow and heat transfer phenomena in a real plant mold.

5. Conclusions

A mathematical model that can simulate the transient fluid flow and heat transfer phenomena has been developed, and cold model and hot model experiments were conducted to verify the ability of the mathematical model. Furthermore, simple model in which meniscus profile is estimated from the calculated pressure under the flat meniscus shape has been constructed in order to reduce the computational load. Real plant simulations are made by using the mathematical model. The following results are obtained.

(1) The time averaged fluid velocity distribution in the width direction just under the meniscus, the time fluctuation of the fluid flow velocity, and the meniscus shape can be estimated precisely by the present mathematical model but the short time period of the fluctuation can’t be estimated.

(2) Simple model without moving free surface can simulate the transient fluid flow phenomena as well as the precise model.

(3) The horizontal velocity just under the meniscus fluctuates hardly, and the momentary maximum velocity reaches two times of the time averaged value.

(4) If the distribution of the bubble diameter can be given precisely, effect of Ar gas injection into the SEN on the molten steel flow phenomena in the continuous casting mold can be evaluated quantitatively by using this mathematical model.

(5) Effect of Ar gas on the fluid flow in the continuous casting mold is remarkable, and upper circulating flow in the mold is suppressed by the buoyancy force caused by injected Ar gas.

(6) Temperature at the meniscus is higher with Ar gas injection than without injection, but temperature under 1 400 mm below the meniscus is almost same with/without Ar gas injection.

Nomenclature

\( C_p \): Specific heat
\( d_g \): Gas bubble diameter
\( f \): Volumetric fraction
\( g \): Gravitational acceleration
\( P \): Pressure
\( R \): Volumetric solidification rate
\( T \): Temperature
\( U \): Velocity

Greek symbols
\( \Delta H \): Latent heat

| Table 4. Computational condition Kashima No. 3 CCM. |
|-----------------------------------------------|
| Casting speed \( V_c \) (m/min) | 1.5 |
| Computational area size (mm) [width \times thickness \times length] | 1625 \times 270 \times 8000 |
| SEN type | 2 square holes(90 \times 90) (mm) |
| Discharge angle:30° downword |
| Ar gas flow rate (Nm3/min) | 0 , 10 |
| Liquidus temperature (°C) | 1536 |
| Solidus temperature (°C) | 1520 |
\( \alpha \): Thermal difusivity
\( \mu \): Viscosity
\( \nu \): Kinematic viscosity
\( \rho \): Density

Subscripts
\( g \): Gas
\( l \): Liquid
\( s \): Solid

REFERENCES

1) H. Tozawa, A. Idogawa, I. Nakato and K. Sorimachi: CAMP-ISIJ, 9 (1996), 604.
2) E. Takeuchi, H. Fuji, N. Miyasaka, T. Ohashi, T. Hiraoka and M. Yamahiro: Tetsu-to-Hagané, 69 (1983), 1607.
3) H. Tanaka, H. Kuwatori and R. Nishihara: Tetsu-to-Hagané, 78 (1992), 761.
4) Y. Otani, J. Fukuda, N. Iwata, N. Ishiwatari and K. Funato: CAMP-ISIJ, 7 (1994), 1194.
5) H. Shibata, H. Yin, S. Yoshinaga, T. Emi and M. Suzuki: ISIJ Int., 38 (1998), 149.
6) J. Kabota, N. Kubo, M. Suzuki, T. Ishii, R. Nishimachi and N. Aramaki: Tetsu-to-Hagané, 86 (2000), 271.
7) H. Harada, E. Takeuchi, M. Zeze and T. Ishii: Tetsu-to-Hagané, 86 (2000), 278.
8) N. Bessho, R. Yoda, H. Tamasaki, T. Fujii and S. Takatori: ISIJ Int., 31 (1991), 40; B. G. Thomas, X. Huang, and R. C. Sussman: Metall. Trans., 25B (1994), 527.
9) T. Ishii, N. Kubo, M. Suzuki, M. Nakata, J. Kubota and R. Nishimura: CAMP-ISIJ, 9 (1996), 212.
10) K. Takatani, Y. Ujisawa and Y. Tanizawa: Sumi tomokinzoku, 50 (1998), 84.
11) J. Anagnostopulos and G. Bergeles: Metall. Mater. Trans. B, 30B (1999), 1095.
12) X. K. Lan, J. M. Khodadadi and F. Shen: Metall. Mater. Trans. B, 28B (1997), 321.
13) I. Sawada, Y. Kishida, K. Okazawa and Y. Tanaka: Tetsu-to-Hagané, 79 (1993), 160.
14) A. F. Lehman, G. R. Tallback, S. G. Kollberg and H. R. Hackl: Int. Symp. on Electromagnetic Processing of Materials, ISIJ, Tokyo, (1994), 372.
15) J. O. Hinze: Turbulence, 2nd ed., McGraw Hill, New York, (1975), 460.
16) J. W. Deadrroff: J. Fluid. Mech., 41 (1970), No. 2, 545.
17) K. Takatani: Advances of MHD Application in Materials Processing, ISIJ, Tokyo, (1993), 324.
18) C. W. Hirt, A. A. Amsden and J. L. Cook: J. Comp. Phys., 14 (1974), 227.
19) C. W. Hirt, B. D. Nichls and N. C. Romero: SOLA-A Numerical Solution Algorithm for Transient Fluid Flows, Los Alamos Sci. Lab., Los Alamos, (1975), LA-5852.
20) K. Takatani: Tetsu-to-Hagané, 74 (1988), 1546.
21) K. Takatani: Progress in Simulation Tech. for Steel Ind., ISIJ, Tokyo, (2000), 6.
22) H. Mizukami, M. Hanao, A. Hiraki, M. Kawamoto, T. Watanabe, A. Hayashi and M. Iguchi: Tetsu-to-Hagané, 86 (2000), 265.