Analysis of Transmission Efficiency of Herringbone Gear with Elastohydrodynamic Lubrication Theory

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Abstract. This paper takes the herringbone gear as the research object, and analyzes the transmission efficiency of the herringbone gear by using the theory of non Newtonian fluid line contact elastohydrodynamic lubrication. Firstly, the oil film pressure and thickness of the contact area are obtained, then the friction coefficient distribution and transmission efficiency in the herringbone gear mesh area are obtained. Finally, the influence of various parameters on the transmission efficiency is analyzed.

Keywords: Herringbone gear, efficiency, non-Newtonian fluid, elastohydrodynamic lubrication

1. Introduction
Herringbone gears not only have the inherent advantages of traditional gears such as high transmission efficiency and compact structure, but also do not generate axial force, so they are often used in applications with heavy loads and high reliability requirements.

Gear tooth failure is the main failure form of gear transmission. When the lubrication is not ideal, it is easy to cause gear tooth fracture, tooth surface wear, tooth surface pitting, tooth surface glue, etc. Therefore, it is possible to improve lubrication performance and reduce friction by improving lubrication conditions Wear and improve service life. [1]

2. In the meshing area of herringbone gears Mathematical model
As shown in Figure 1, according to the comparative analysis of the forces on the herringbone gear and the helical gear, it is found that the total tangential force of the herringbone gear is equal to the total tangential force of the helical gear, and the axial forces of the herringbone gear cancel each other out to zero. In Figure 2, the shaded area is the meshing area of the herringbone gear, and the contact lines of different meshing positions are distributed in the meshing area. [2] Therefore, based on this idea, approximating the herringbone gear to two helical gears can simplify the calculation of the herringbone gear parameters.
According to the meshing characteristics of the herringbone gear, the herringbone gear is equivalent to two helical gears with opposite rotation directions. A mathematical model is established for the tooth surface parameters of the herringbone gear, as shown in Figure 3. The meshing area of the herringbone gear is a rectangular area, in which the length is equal to the tooth width \( b \), and the width is equal to the actual meshing area \([s_1, s_2]\). The contact line in the meshing region is symmetrical about the y axis, and the slope of the contact line is \( \pm \beta b \). Based on the middle end face, the distance from the meshing point of the \( i \) th contact line to the node is \( S_i \).

\[
\begin{align*}
    y_i &= s_i + \tan \beta b \cdot x \quad (-b/2 \leq x < 0) \\
    y_i &= s_i - \tan \beta b \cdot x \quad (0 \leq x < b/2) \\
    s_1 &< y_i < s_2 \\
    s_1 &\leq s_i \leq s_2 + \varepsilon \beta P_{bt}
\end{align*}
\]

(1)

\[
\begin{align*}
    R_1 &= r_s \sin \delta t + s \\
    R_2 &= r_s \sin \delta t - s' \quad s \in [s_1, s_2]
\end{align*}
\]

(2)

In the formula (2),

\[
\begin{align*}
    s_1 &= r_s \sin \delta t - \sqrt{r_h^2 - r_{b2}^2} \\
    s_2 &= r_s \sin \delta t - \sqrt{r_h^2 - r_{b1}^2}
\end{align*}
\]
Where \( y_i \) is the distance from each point on the \( i \)th contact line to the node. The radius of curvature of each point on the \( i \)th curve can be calculated by using the curvature radius formula (1) above. \( s_1 \) and \( s_2 \) are the value range of meshing area of face gear. See formula (2) for details. Furthermore, the comprehensive radius of curvature, entrainment velocity and sliding velocity of each point on the \( i \)th contact line can be calculated. Finally, the mathematical model of herringbone gear meshing area is obtained. [3]

3. Numerical Analysis of Non-Newtonian Fluid Line Contact Elastohydrodynamic Lubrication

3.1. Basic equation of elastohydrodynamic lubrication model

The elastohydrodynamic lubrication model of non-Newtonian fluid includes constitutive equation of non-Newtonian fluid, Reynolds equation, film thickness equation, viscosity pressure equation, density pressure equation and load balance equation. [4]

- **Constitutive equation of non Newtonian fluid.** The constitutive equation of REE Eyring rheological model can be written as follows:

\[
\frac{\partial u}{\partial x} = \frac{\tau_0}{\eta_0} \sinh \left( \frac{\tau}{\tau_0} \right)
\]

Where \( \tau_0 \) is the characteristic shear stress of Ree-Eyring rheological model, which is regarded as a constant there.

- **Reynolds equation.** The Reynolds equation of steady state line contact is as follows:

\[
\frac{d}{dx} \left( \frac{\rho h^3 \frac{dp}{dx}}{\eta_0} \right) = 12u \frac{d(\rho h)}{dx}
\]

The boundary conditions are: \( p(x_0)=0, p(x_e)=0, dp(x_e)/dx=0 \).

- **Film thickness equation.** In the case of on-line Contact Elastohydrodynamic Lubrication, the film thickness equation in and near the contact zone is:

\[
\begin{cases}
    h(x) = h_0 + \frac{x^2}{2R} + v(x) \\
    v(x) = -\frac{2}{\pi^2} \int_{x_0}^{x_e} p(s) \ln(s-x)^2 ds + c
\end{cases}
\]

Where \( v(x) \) is the elastic deformation of the contact surface caused by the pressure change of the lubricating oil film; \( R \) is the equivalent radius of curvature of the friction pair; and \( E \) is the comprehensive elastic modulus of the friction pair.

- **Viscosity pressure equation.** The law equation between viscosity and pressure of lubricating oil is as follows:

\[
\eta = \eta_0 \exp \left\{ (\ln \eta_0 + 9.67) \left[ -1 + (1 + \frac{p}{\rho_0^{0.68}}) \right] \right\}
\]

Where \( \eta_0 \) is the ambient viscosity of lubricating oil and \( p_0 \) is the maximum contact stress of line contact.

- **Density pressure equation.** The law equation of the change between density and pressure of lubricating oil is as follows:

\[
\rho = \rho_0 \left( 1 + \frac{0.6\rho}{1 + 1.7p} \right)
\]

- **Load balance equation.** The resultant force of oil film pressure in the x-axis direction must be balanced with the load per unit length:

\[
w_i = \int_{x_0}^{x_e} p(x) dx
\]
3.2. Numerical calculation and analysis of herringbone gear elastohydrodynamic lubrication model

A set of parameters in Table 1 are selected to calculate the non-Newtonian fluid line contact elastohydrodynamic lubrication of herringbone gear. Firstly, the transmission parameters of herringbone gear are calculated by using the above formula, and the equivalent radius of curvature \( R \), unit load \( W_l \), entrainment speed \( U_e \) and sliding speed \( U_s \) of herringbone gear are obtained. Then the above parameters are substituted into the basic equations, and Matlab is used to solve the equations. Finally, the oil film pressure and the oil film thickness of each contact point in the gear meshing area are obtained. Figure 4–8 shows the distribution of the parameters of herringbone gear in the meshing area. Figure 9 shows the distribution of oil film pressure and oil film thickness at a certain point in the meshing area. [5]

Table 1. example of herringbone gear structure parameters and working conditions parameters

| Parameter                                      | Numerical value | Unit |
|------------------------------------------------|-----------------|------|
| Number of teeth \( z_1/z_2 \)                 | 33/48           | /    |
| Modulus \( m \)                                | 4               | \( mm \) |
| Pressure angle \( \alpha \)                    | 20              | degrees |
| Helix angle \( \beta \)                        | 20              | degrees |
| Addendum height coefficient \( ha \)           | 1               | /    |
| Top clearance coefficient \( c^* \)            | 0.25            | /    |
| Tooth width \( b \)                            | \( 30 \times 2 \) | \( mm \) |
| Elastic modulus \( E \)                        | \( 2.2 \times 10^{11} \) | \( Pa \) |
| Ambient viscosity \( \eta_0 \)                 | 0.038           | \( Pa \cdot s \) |
| Environmental density \( \rho_0 \)             | 871             | \( kg/m^3 \) |
| Viscosity pressure coefficient \( \alpha \)    | \( 2.2 \times 10^{-6} \) | / |
| Characteristic shear stress \( \tau_0 \)       | \( 1.0 \times 10^7 \) | / |
| Input torque \( M \)                           | 200             | \( N \cdot m \) |
| Driving wheel speed \( N_1 \)                  | 200             | \( r/min \) |

Figure 4. The equivalent radius of curvature of the herringbone gear.  
Figure 5. Herringbone gear entrainment speed.
3.3. Friction coefficient and transmission efficiency calculation of herringbone gear
The friction coefficient of the contact point is calculated by using the calculated oil film pressure and oil film thickness:

**Figure 6.** Sliding speed of herringbone gear.

**Figure 7.** Variation rule of contact linelength of herringbone gear.

**Figure 8.** Unit load distribution of tooth surface.

**Figure 9.** Oil film pressure thickness distribution in contact area.
\[ \mu = \frac{F_1 + F_2}{2w_l} \]  \hfill (9)

Where \( w_l \) is the load per unit length. The friction between the two contact surfaces is as follows:

\[ F_{1,2} = \int_{0}^{X_e} \tau \left|_{Z=0,h} \right. \, dx \]  \hfill (10)

In the formula, \( \tau \left|_{Z=0,h} \right. \) is the shear stress on both surfaces of the lubricating oil film. The expression of the shear stress in the oil film is:

\[ \tau = \tau_a + Z \frac{\sigma_p}{\sigma_x} \]  \hfill (11)

In the formula, \( \tau_a \) is the shear stress acting on the surface of gear \( a \).

\[ \tau_a = \tau_0 \ln \frac{(u_a - u_b)^2 + (\xi_1 - \xi_2)^2 - (u_a - u_b)}{\xi_1 + \xi_2} \]  \hfill (12)

In the formula (13),

\[
\begin{align*}
\xi_1 &= \int_{0}^{h} \frac{\tau_0}{\eta} \cosh\left(\frac{z \, \partial p}{\tau_0 \, \partial x}\right) \, dz \\
\xi_2 &= \int_{0}^{h} \frac{\tau_0}{\eta} \sinh\left(\frac{z \, \partial p}{\tau_0 \, \partial x}\right) \, dz
\end{align*}
\]

Bring the relevant parameters into the calculation formula of the friction coefficient, the friction coefficient of each point in the meshing area of the herringbone gear can be calculated, as shown in Figure 10 and Figure 11.

**Figure 10.** Distribution of friction coefficient of herringbone gear.

**Figure 11.** Coefficient distribution of end face friction.

It can be seen from Figure 10 that the friction coefficient of the herringbone gear is larger at the meshing and meshing positions, where the meshing position is the maximum, and the node position friction coefficient is the smallest. In addition, the friction coefficient distribution of each end face is not exactly the same. Figure 11 is the friction coefficient distribution curve at one end face position.

After obtaining the friction coefficient and load of each point in the contact area, the transmission efficiency of the herringbone gear can be calculated according to the basic formula (13) of efficiency calculation.

\[ \eta = 1 - \frac{W_f}{W} = 1 - \frac{fR_0}{F \theta} = 1 - \frac{\sum_{i=1}^{n} \mu_i W_i R_i}{\sum_{i=1}^{n} W_i R_i \mu_i} \]  \hfill (13)
Figure 12. Force condition of herringbone gear.

Figure 12 Schematic diagram of the force situation of the human-shaped gear at \( i \). According to the nature of involute gears, the direction of \( F_{ni} \) is always the direction of the meshing line, and its magnitude is equal to the unit load \( W_{li} \) at this position. The direction of the friction force \( f_i \) is perpendicular to \( F_{ni} \) and the magnitude is \( \mu_i W_{li} \).

4. Influence of various parameters of herringbone gear on transmission efficiency

The efficiency distribution of helix angle 5°–35° is shown in Figure 13. With the increase of helix angle, the overall efficiency decreases gradually. When the helix angle increases, the normal load on the tooth surface increases, which is the main factor to increase the transmission power loss. [6]

The efficiency distribution of gear module 2 ~ 5 is shown in Figure 14. As the modulus increases, the transmission efficiency decreases. This is because, when other conditions remain unchanged, the increase of modulus leads to the increase of the overall size of the gear. At the same speed, the linear speed of the gear increases, and the relative sliding speed of the entry and exit points increases, so that the frictional loss power increases.

Figure 13. Influence of helix angle on transmission efficiency.

Figure 14. Effect of modulus on transmission efficiency.

The efficiency distribution under different working tooth widths is shown in Figure 15. The transmission efficiency increases with the increase of tooth widths. The main reason is that with the increase of working tooth width, the unit load of gear decreases and the friction coefficient decreases, which leads to the decrease of friction. However, when the tooth width is too large, the length of the single tooth contact line can not reach the full tooth width, which not only wastes material and processing
time, but also easily leads to the increase of the axial size of the transmission system. Therefore, the selection of tooth width is not easy to be too large, $\varepsilon_a = \varepsilon_b$ tooth width is more appropriate.

![Figure 15. Influence of tooth width on transmission efficiency](image)

5. Conclusion
Based on the theory of non-Newtonian fluid linear contact elastohydrodynamic lubrication, this paper studies the friction coefficient and transmission efficiency of the herring gear. Firstly, the mathematical model of the meshing region of the herringbone gear is proposed. Secondly, based on the theory of non-Newtonian fluid linear contact elastohydrodynamic lubrication, the friction coefficient distribution and transmission efficiency of the herring gear in the meshing region are obtained. Finally, the influence of each parameter on transmission efficiency is analyzed. This paper presents an analytical method of herring gear lubrication based on the theory of elastohydrodynamic lubrication to guide the gear design process.

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