ISFL: Trustworthy Federated Learning for Non-i.i.d. Data with Local Importance Sampling

Zheqi Zhu, Pingyi Fan, Senior Member, IEEE, Chenghui Peng, and Khaled B. Letaief, Fellow, IEEE

Abstract—As a promising integrated computation and communication learning paradigm, federated learning (FL) carries a periodic sharing from distributed clients. Due to the non-i.i.d. data distribution on clients, FL model suffers from the gradient diversity, poor performance, bad convergence, etc. In this work, we aim to tackle this key issue by adopting data-driven importance sampling (IS) for local training. We propose a trustworthy framework, named importance sampling federated learning (ISFL), which is especially compatible with neural network (NN) models. The framework is evaluated both theoretically and experimentally. Firstly, we derive the parameter deviation bound between ISFL and the centralized full-data training to identify the main factors of the non-i.i.d. dilemmas. We will then formulate the selection of optimal IS weights as an optimization problem and obtain theoretical solutions. We also employ water-filling methods to calculate the IS weights and develop the complete ISFL algorithms. The experimental results on CIFAR-10 fit our proposed theories well and prove that ISFL reaps higher performance, as well as better convergence on non-i.i.d. data. To the best of our knowledge, ISFL is the first non-i.i.d. FL solution from the local sampling aspect which exhibits theoretical NN compatibility. Furthermore, as a local sampling approach, ISFL can be easily migrated into emerging FL frameworks.

Index Terms—federated learning, importance sampling, non-i.i.d. data, water-filling optimization.

I. INTRODUCTION

A. Backgrounds

As a novel paradigm for distributed intelligent system, federated learning (FL) has received great attention in both research and application areas [1]. The core idea of FL is to share the models of distributed clients, which train models using their local data. Basic federated learning systems consist of two stages: 1) local training, where clients update their local models with local data; and 2) center aggregation, where the cloud center aggregates the models uploaded from the clients and sends the global models back. Different from the traditional distributed learning methods that split the models or transmit the data, model parameters or gradients are designed to be the elements for client interaction in FL schemes. FL is recognized as a communication-efficient scheme for its full use of the client-center (client-client) communication, clients’ computation capability, and the distributed data sources [2], [3].

The structures of FL naturally fit the multi-tier intelligent systems where computing is coupled with communication [4]. On the one hand, FL can serve the hierarchical computing-communication systems such as edge computing or fog computing, where a cloud center and distributed devices/clusters participate. A number of applications emerge especially in the mobile scenarios such as artificial intelligence of Things (AIoT) and vehicular internet [5], [6]. The potentials of FL have been investigated in distributed inference [2], system optimization [9] and cooperative control [9]. In particular, due to the efficient sharing mechanism of FL, its combinations with reinforcement learning (RL) or multi-agent reinforcement learning (MARL) have been considered, which is referred to as federated RL (FRL) [10]. The corresponding frameworks have also been designed for collaboration and scheduling of intelligent edge computing systems in [11], [12]. Distributed computing systems also serve the federated learning tasks. With the development of wireless communication, intelligent devices and the lightweight machine learning models, FL is envisioned as one of the core technologies in 6G systems [13], [14]. These mobile systems support the FL to be deployed in numerous emerging applications, such as VR/AR, urban computing, blockchains, etc. [15].

Recently, FL has been evolving from many aspects. For the basic architecture, apart from the classic horizontal federated learning (HFL), vertical federated learning (VFL) is proposed for the scenarios where the clients possess different features of same data samples [3]. Some learning schemes of traditional deep learning areas are also migrated into FL. The typical concepts are the federated transfer learning (FTL) [15], and the aforementioned FRL. Specific FL algorithms have been widely studied, such as the clustered FL [15], hierarchical FL [18], decentralized FL [19]. Furthermore, some theoretical studies on FL exist. A main direction is the convergence analysis. For instance, Stich [20] derived the convergence of the local-SGD for the first time, and Wang et al. [21] bounded the gradient norms under several FL settings. Considering the communication factors, the joint analysis and the guide for system design are further studied in [22], [23]. These theoretical analysis guarantees the interpretability, robustness, generalization, privacy protection of FL and makes such learning schemes trustworthy [24].

B. Motivations

Distributed learning systems suffer from the heterogeneity of both the clients and data. Particularly, as mentioned in [25], the heterogeneity of distributed data, also referred to as non-i.i.d. (independent and identically distributed) data settings,
is one of the most crucial issues in FL. In this work, we consider a commonly acknowledged perspective of HFL non-i.i.d. setting, the data category distribution skew, which is typically represented in real world datasets. For example, the label distributions of clients’ training data make a difference on the global performance in classification tasks. Such non-i.i.d. data impacts FL from two aspects. First of all, the data distributions highly differ for different clients, resulting in the diversity of local models. Second is the imbalanced local data, which leads to bad generalization and the overfitting of the local models. Therefore, FL on non-i.i.d. data faces major challenges such as slow convergence and unsatisfactory performance.

In conventional centralized deep learning, importance sampling (IS) is regarded as an effective method to mitigate the data imbalance. Both theoretical and experimental analysis has verified its improvement on accuracy, as well as training speed. Besides, a number of centralized deep learning theories and developments have been migrated into FL studies.

In this paper, in order to address these challenges, we consider a way to improve the non-i.i.d. FL performance with data-level sampling during local updating. By doing so, we would be able to require less excessive communication and computation resources compared to the original FL. Specifically, we focus on the cases where each distributed client resamples the training samples from its local data under a set of importance weights. We also derive the theoretical guarantees of optimal IS weights for distributed clients and design the federated updating policies.

C. Related Works

In the literature, some related work on non-i.i.d. FL has been broadly studied from different aspects. The popular solutions contain data sharing, augmentation, model distillation, client selection, clustering, and sampling based methods. Here we mainly list the studies tightly related to ours.

As for FL with sampling approach to tackle non-i.i.d. data, Tudor et al. proposed a method to identify the data relevance at each client, based on which the data are sampled for local training before the learning tasks start. To extend the dynamic sampling strategies, Li et al. measured each sample’s importance by its gradient norms during the training and designed the FL algorithms with client+sample selection. Such data-driven method requires calculating the instant gradient norm of each training sample, which means that the extra backward propagation shall be processed locally. Another systematic work on importance sampling FL was done in, where both data and client sampling are considered. The importance weights of the mini-batches were derived theoretically and the convergence of algorithms was also analyzed. However, the theoretical guarantees were carried based on the convex function assumption which is not compatible with deep learning models such as neural networks. Besides, the experiments were tested on a simple regression problem. Thus, the theoretical analysis of importance sampling based FL in deep learning tasks is still an open problem.

There are other important theoretical results on non-i.i.d. FL. Li et al. established the connections between convergence rate and the factors including federated period, gradient Lipschitz, gradient norms, etc. Zhao et al. derived the upper bounds of the parameter deviations between FL on non-i.i.d. data and the centralized training on full data. They also pointed out in the theorems that the gradient Lipschitz of different labels/categories shall be introduced to describe the non-i.i.d. impacts on the performance gaps. This previous work has inspired us to start this work.

D. Contributions & Paper Organization

The main contributions of this paper can be summarized as follows:

- We put forward a generalized Importance Sampling Federated Learning framework, abbreviated as ISFL, which introduces the local importance sampling to mitigate the effects of the non-i.i.d. data distribution.
- We derive the model deviation bound between ISFL and centralized training, which relaxes the restriction of convexity of the loss functions. The theoretical results indicate that not only the imbalanced distribution, but also the second moment of the gradient coefficients influence the performance gap for the non-i.i.d. setting.
- We formulate the weighting selection of each client as independent optimization sub-problems. The theoretical solutions for the optimal IS weights, as well as an adaptive water-filling approach for calculating the numerical solutions are also presented.
- We develop the corresponding ISFL algorithm and carry several experiments based on the CIFAR-10 dataset. It will be shown that the outcomes fit the theorems well and verify the improvement of the proposed algorithms. The experiments also suggest that ISFL is able to approach the accuracy of centralized training. Some insights on the generalization ability, personalized preference, as well as data efficiency are also discussed.

The rest of the article is organized as follows. In Section I, we introduce some necessary preliminaries and the basic idea of ISFL. In Section II, we firstly derive some theoretical results on the framework analysis and the weighting calculation. The optimal IS weighting solutions, as well as the corresponding ISFL algorithms are then developed. Experiment results and further discussions are presented in Section III. Finally, in Section IV, we conclude this work and point out several potential research directions.

II. Preliminaries and ISFL Framework

In this section, we present important preliminaries, as well as introduce local importance sampling into FL. We will then sketch the idea and the framework of ISFL. Some key notations are listed in Table I.

1The codes will be available at https://github.com/Zhuzzq/ISFL later.
| Notations | Description |
|-----------|-------------|
| $K$       | The number of the distributed clients. |
| $C$       | The category number of training data. |
| $T_f$     | Federated updating period. |
| $\pi = \{\pi_1, \ldots, \pi_K\}$ | Client weights for federated updating. |
| $\eta$    | Learning rate of each client. |
| $\mathcal{D}_k$ | Local training set of client $k$. |
| $\xi_i = (x_i, y_i)$ | A training data sample of category $i$. |
| $p = \{p_i\}$ | Global data proportion of category $i$. |
| $p^k = \{p^k_i\}$ | Local proportion of category $i$ on client $k$. |
| $w^k = \{w^k_k\}$ | IS weights for category $i$ on client $k$. |
| $\theta_t$ | Centralized model trained on full data. |
| $\theta^k_t$ | Global model at $t$-th epoch. |
| $\theta^k_t$ | Local model of client $k$. |
| $\ell(\xi; \theta)$ | Loss function of $\xi$ for model $\theta$. |

**A. Federated Learning for Non-i.i.d. Data**

By sharing the trained model parameters, FL serves as a communication-efficient paradigm for distributed learning, especially for mobile devices. The classical FL proceeds by merging the distributed models periodically to obtain a global model:

\[
\bar{\theta}_t = \frac{\sum_{k=1}^K \pi_k \theta^k_t}{\sum_{k=1}^K \pi_k} \quad \text{if } t \equiv 0 \mod T_f \tag{1}
\]

where $\pi_k$ is the weighting for client $k$, and $\theta^k_t$ is the local model for client $k$. By using FedAvg, we have $\pi_k = \frac{\mid \mathcal{D}_k \mid}{\sum_{i=1}^C \mid \mathcal{D}_i \mid}$. All clients also download the updated global model $\bar{\theta}_t$ every $T_f$ epochs. Since FL conducts a parameter-level fusion, the local training significantly affects the global performance. Each client tends to update the model towards the local optimum which fits its own local training data. Thus, one of the most crucial challenges for FL is the non-i.i.d. distribution of the local data, which causes the local gradients and model parameters to diverge as shown in Fig. 1. Compared to centralized training on complete data, the deviation of the distributed clients hinders the global model from reaching the optimum solution using the global data, which also leads to poor performance, as well as bad convergence.

![Fig. 1. A sketch for the impact of non-i.i.d. data: FedAvg with $K = 2$ and $T_f = 2$. The arrows represent the model evolution.](image1)

**B. Importance Sampling**

Importance Sampling (IS) provides an intuitive solution to make up the gap between the observed distribution and the inherent distribution [37]. Consider a certain function $f(x)$ of a random variable $x$, and let $q(x)$ be the latent target distribution and $p(x)$ be the known empirical distribution. Through sampling the data according to IS weightings $w(x)$ satisfying $\int w(x)p(x)dx = 1$, the re-sampled observation can be expressed as

\[
\mathbb{E}_{x \sim p}[w(x)f(x)] := \int p(x)w(x)f(x)dx. \tag{2}
\]

IS attempts to find proper weights which make the re-sampled observation approximate to the objective expectation, i.e.,

\[
\mathbb{E}_{x \sim q}[f(x)] := \int q(x)f(x)dx. \tag{3}
\]

Particularly, if $w(x)$ are set ideally as $\frac{q(x)}{p(x)}$, then the observed expectation with IS is equivalent to the objective. In general, IS algorithms adopt an iterative process to estimate the optimal IS weights. Numerous varieties of IS algorithms have been developed and their applications in machine learning area have also been exploited. This has inspired us to combine IS with local training to relieve the dilemma caused by non-i.i.d. data.

**C. ISFL Framework**

![Fig. 2. The workflow of ISFL framework.](image2)

As mentioned, the non-i.i.d. settings and the imbalanced distribution of the local data lead to the model deviation. We shall assume that there exists certain data sampling strategies from which all clients collaborate to make the global model reach a better performance. Hence, we introduce a set of dynamic IS weights for local training on each client and formulate FL with importance sampling, abbreviated as ISFL. In traditional FL settings, the clients sample their local training data uniformly to carry backward propagation with the mini-batch SGD based optimizer. Thus, the loss function of a batch in a single iteration on client $k$ can be expressed as

\[
\ell_k(t) = \mathbb{E}_{\xi \sim p^k}\ell(\xi; \theta^k_{t-1}) = \sum_{i=1}^C p^k_i \ell(\xi_i; \theta^k_{t-1}). \tag{4}
\]

The equivalent global loss in FL is

\[
\bar{\ell}(t) := \sum_{k=1}^K \pi_k \ell_k(t) = \sum_{k=1}^K \pi_k \mathbb{E}_{\xi \sim p^k}\ell(\xi; \theta^k_{t-1}). \tag{5}
\]
Meanwhile, the loss of centralized training on the full data is
\[
\hat{\ell}(t) = \mathbb{E}_{\xi \sim p} \ell(\xi; \hat{\theta}_{t-1}) = \sum_{i=1}^{C} p_i \ell(\xi_i; \hat{\theta}_{t-1}).
\] (6)

Comparing the FL global loss and the centralized loss, we can find that the main reason why non-i.i.d. FL performs worse than i.i.d. FL or centralized training is the imbalance of the local distribution. Furthermore, the non-i.i.d dilemmas cannot be tackled by simply resampling with the probability \(p\) because the model parameters \(\theta^k\), the gradients \(\nabla_{\theta} \ell(\xi_i; \theta^k)\), etc., affect the loss as the training carries on. Therefore, the optimal local sampling strategies \(q^k\) for each client are both unknown and dynamic. To improve the performance of non-i.i.d. FL, we consider a local sampling-level approach and introduce IS to the local training. Each client trains the model with the re-sampled data of different categories under a set of IS weights \(\{w_i^k \geq 0\}\), which satisfy \(\sum_{i=1}^{C} p_i^k w_i^k = 1\) for \(k = 1, \ldots, K\). Then, the expectation of the local loss in (6) shall be modified as
\[
\ell_k'(t) = \mathbb{E}_{\xi \sim q^k} \ell(\xi; \theta^k) = \sum_{i=1}^{C} p_i^k w_i^k \ell(\xi_i; \theta^k). \] (7)

The basic workflow of ISFL is shown in Fig. The Clients proceed their local training with the data sampled under the IS weights. The theorems in the next section will demonstrate that the optimal IS weights are related to not only the two distributions, but also the model parameters. Since the local models change as the training goes on, the IS weights shall be updated iteratively. Thus, ISFL conducts IS weight updates synchronously with the model sharing at every federated round.

### III. Main Results and Algorithms

In this section, the theoretical investigations of ISFL are conducted. We will present several theorems to explain why introducing local IS to FL and how to select the optimal IS weights. Based on these theoretical guarantees, the formal ISFL algorithms will also be designed.

#### A. Deviation Gap vs. Centralized Training

To discuss the proposed ISFL framework theoretically, we first state two basic assumptions. In addition, to improve the compatibility with neural networks, we relax the assumption of the loss functions’ convexity or \(\mu\)-strong convexity, which are usually used in the existing convergence analysis framework of FL.

**Assumption 1** (Conditional Gradient Lipschitz). For given training samples \(\xi_i = \{x_i, y_i\}\) of category \(i\), the gradient of the model is Lipschitz continuous, i.e.,
\[
\left\| \nabla_{\theta} \ell(\xi_i; \theta) - \nabla_{\theta} \ell(\xi_i; \theta') \right\| \leq L_i \left\| \theta - \theta' \right\|,
\] (8)
where the smallest \(L_i\) satisfying (8) is termed as the gradient Lipschitz of category \(i\).

**Assumption 2** (Bounded Gradient Norms). The gradient norm of model \(\theta_t\) at epoch \(t\) is bounded by:
\[
\left\| \nabla_{\theta} \ell(\xi_i; \theta_t) \right\| \leq G(t).
\] (9)

The gradient Lipschitz and bounded gradient norm are commonly-exploited assumptions in the theoretical studies of FL. Through the practical experiments, we find that the so-called gradient Lipschitz constants \(\{L_i\}\) actually change with the training process and the model parameters. Thus, the formal notation shall be \(\{L_{i,k}(t)\}\). For simplicity, we use \(\{L_i\}\) in the theorems and the dynamic gradient Lipschitz will be taken into account in the algorithm designs. With the above assumptions, we are now ready to investigate the parameter deviation between the ISFL and the centralized training in the following theorem.

**Theorem 1.** (Deviation Bound) Given the local IS weights \(\{w_i^k\}\). While \(t \equiv 0 \mod T_f\), denote \(\Delta_t = \|\hat{\theta}_t - \bar{\theta}_t\|^2\) as the model deviation between the ISFL scheme and the centralized full-data training after a federated updating period. Then, the deviation is bounded by:
\[
\Delta_t \leq 2 \sum_{k=1}^{K} \pi_k \left(1 + 2\eta^2 \sum_{i=1}^{C} p_i^k w_i^k L_i^2 \right) \left[1 + \eta^2 \sum_{i=1}^{C} (p_i^k w_i^k - p_i)^2 \right] \cdot \left(A_k^{2T_f - 2} \cdot \Delta_{t-T_f} + C \cdot B_k^2(1 - 1) \right) + \sum_{k=1}^{K} 4\pi_i^k \eta^2 G^2 (t - 1),
\] (10)
where \(A_k = 1 + \eta \sum_{i=1}^{C} p_i^k w_i^k L_i\) and \(B_k(t) = \sum_{\tau=1}^{T_f} A_k^2 G(t - \tau)\)

To derive the deviation bound in Theorem 1, we firstly show a lemma which contributes to the proof.

**Lemma 1.** Consider the local training at \(t\) when \(mT_f < t \leq (m + 1)T_f\), then the deviation between each client and the full-data training satisfies the following inequality,
\[
\left\| \theta_t^k - \bar{\theta}_t \right\| \leq \left[1 + \eta^2 \sum_{i=1}^{C} (p_i^k w_i^k - p_i)^2 \right] \cdot \left(A_k^{2(T_f - mT_f)} \Delta_{mT_f} + CB_k^2(t) \right).
\] (11)

**Proof of Lemma 1** Note that when \(mT_f < t \leq (m + 1)T_f\), all clients will carry their local training without federated updating. Hence, one can obtain
\[
\left\| \theta_t^k - \bar{\theta}_t \right\| = \left\| \theta_{t-1}^k - \eta \nabla_{\theta} \mathbb{E} \ell(\xi_i; \theta_{t-1}^k) - \bar{\theta}_t \right\| \leq \left\| \theta_{t-1}^k - \eta \nabla_{\theta} \mathbb{E} \ell(\xi_i; \theta_{t-1}^k) \right\| + \eta \left\| \nabla_{\theta} \mathbb{E} \ell(\xi_i; \theta_{t-1}^k) \right\| \leq \left\| \theta_{t-1}^k - \eta \nabla_{\theta} \mathbb{E} \ell(\xi_i; \theta_{t-1}^k) \right\| + \eta \left\| \nabla_{\theta} \mathbb{E} \ell(\xi_i; \theta_{t-1}^k) \right\|
\] (12)
\[
= \left\| \theta_{t-1}^k - \bar{\theta}_t \right\| \leq \left\| \theta_{t-1}^k - \bar{\theta}_t \right\| + \eta \left\| \nabla_{\theta} \mathbb{E} \ell(\xi_i; \theta_{t-1}^k) \right\| + \eta \left\| \nabla_{\theta} \mathbb{E} \ell(\xi_i; \theta_{t-1}^k) \right\|
\] (13)
where inequality (1) can be obtained by Assumption 1 and the triangle inequality of 2-norms. For simplicity, we denote $A_k = 1 + \eta \sum_{i=1}^C p_i^k w_i^k L_i$. Then, by recursion we obtain

$$
\|\theta_t - \hat{\theta}_t\| \leq A_k^{t-mT_f} \|\theta_{mT_f} - \hat{\theta}_{mT_f}\| + \eta \sum_{i=1}^C |p_i^k w_i^k - p_i| \sum_{\tau=1}^{t-mT_f} A_k^{\tau} G(t - \tau)
$$

(16)

where $B_k(t)$ is used to represent $\sum_{\tau=1}^{t-mT_f} A_k^{\tau} G(t - \tau)$, and $\theta_{mT_f} = \hat{\theta}_{mT_f}$ holds after every federated updating according to the FL rules. Finally, by applying the Cauchy-Schwarz inequality to the square of (17), we obtain the inequality in the lemma.

With the help of Lemma 1, we now complete the proof of Theorem 1.

Proof of Theorem 1. By the definition of the federated updating in (1), we obtain

$$
\Delta_t = \|\theta_t - \hat{\theta}_t\|^2 = \left\| \sum_{k=1}^K \pi_k \theta^k_t - \hat{\theta}_t \right\|^2
$$

(18)

$$
= \left\| \sum_{k=1}^K \pi_k \left[ \theta^k_{t-1} - \eta \nabla_{\theta} \mathbb{E}_{\xi \sim q^k} \ell(\xi; \theta^k_{t-1}) - \hat{\theta}_{t-1} \right] + \eta \nabla_{\theta} \mathbb{E}_{\xi \sim q^k} \ell(\xi; \hat{\theta}_{t-1}) \right\|^2
$$

(19)

$$
\leq 2 \sum_{k=1}^K 2\pi_k \left( \|\theta^k_{t-1} - \hat{\theta}_{t-1}\|^2 + \eta^2 \|\nabla_{\theta} \mathbb{E}_{\xi \sim q^k} \ell(\xi; \theta^k_{t-1}) - \nabla_{\theta} \mathbb{E}_{\xi \sim q^k} \ell(\xi; \hat{\theta}_{t-1})\|^2 \right)
$$

(20)

$$
= 2 \sum_{k=1}^K 2\pi_k \left( \|\theta^k_{t-1} - \hat{\theta}_{t-1}\|^2 + \eta^2 \|\nabla_{\theta} \mathbb{E}_{\xi \sim q^k} \ell(\xi; \theta^k_{t-1}) - \nabla_{\theta} \sum_{i=1}^C p_i^k w_i^k \ell(\xi; \theta^k_{t-1})\|^2 \right)
$$

(21)

$$
= 2 \sum_{k=1}^K 2\pi_k \left( \|\theta^k_{t-1} - \hat{\theta}_{t-1}\|^2 + \eta^2 \|\nabla_{\theta} \sum_{i=1}^C p_i^k w_i^k \ell(\xi; \theta^k_{t-1}) - \nabla_{\theta} \sum_{i=1}^C p_i \ell(\xi; \hat{\theta}_{t-1})\|^2 \right)
$$

(22)

By substituting (26) into (25), we complete the proof of Theorem 1.

Remark 1. In principle, centralized training with full data is considered to be the ideal scheme for training the best neural models if the architectures and learning configures are fixed. Theorem 1 gives an upper bound of the parameter deviation between ISFL and the centralized full-data training. Note that for the neural network models, the parameter proximity of two models are neither able to provably guarantee a similar performance nor stability. Even so, the theorem still reflects the convergence of ISFL and guide us to investigate what factors determine the parameter deviations.

Theorem 1 implies that by adopting local IS, the parameter gap with centralized full-data training comes from three factors: the statistics of the gradient Lipschitz coefficients...
\{L_i\}, MSE (mean-square-error) between \(p^k\) and \(p\), as well as the gradient norm of centralized training. It is intuitive that the divergence between the local distributions and the global distribution causes a performance gap as the samples of minor categories are less used for training. We explain the involved expectation and the second moment of the gradient Lipschitz as the result of the federated operations. That is, on the one hand, local models tend to diverge due to the non-i.i.d. data. On the other hand, the models are forced to be identical after every federated aggregation. These two procedures interact and generate the terms of \{L_i\} in the upper bound \(\Omega\).

**B. Optimal Importance Sampling Weightings**

The aforementioned theorem gives an upper bound for the deviation between the ISFL scheme and the centralized full-data training. Guided by this, it is possible to find the optimal local IS weightings to minimize the parameter deviation upper bound which relieves the impact of the non i.i.d. data. Since the second term of the upper bound is independent from IS weightings \(w\), we only need to minimize the first summation term, denoted by

\[
\Lambda_t = \sum_{k=1}^{K} 2\pi_k \left(1 + 2\eta^2 \sum_{i=1}^{C} p_i^k w_i^k L_i^2 \right) \left[1 + \eta^2 \sum_{i=1}^{C} (p_i^k w_i^k - p_i)^2 \right]
\]

The factors which can be designed are the client weightings \(\pi\) for federated updating and the IS weightings \(w\) for local training. By considering the principle of IS, we formulate the general optimization problem as follows:

\[
P_0 : \min_{\pi, w} \Lambda_t (\pi, w) \tag{28}
\]

Subject to:

\[
\sum_{k=1}^{K} \pi_k = 1, \quad \pi_k \geq 0, \quad k = 1, \cdots, K
\]

\[
\sum_{i=1}^{C} p_i^k w_i^k = 1, \quad k = 1, \cdots, K
\]

\[
w_i^k \geq \sigma, \quad \text{for } k, i
\]

where the first two constraints in (28) are the agent weightings for federated updating and the last two are the constraints for local importance sampling. We force the IS weightings larger than a small constant \(\sigma\) to ensure that all local data contribute to the federated model. To solve the optimization problem \(P_0\), an iterative approach shall be employed to optimize \(\pi\) and \(w\) alternately. In the following, we will mainly be interested in the optimal selection of the local training sampling. In other words, the weightings \(\{\pi_k\}\) of the clients are preset and fixed. Thus, the general optimization problem \(P_0\) can be decomposed into \(K\) sub-problems which depend on the factors of each client. Note that the last term of \(\Lambda_t\) is related to the latest gradient deviation and the gradient norms in the current local epoch, which are determined by local training. Next we recognize the multiplier

\[
\rho_k = \left(1 + 2\eta^2 \sum_{i=1}^{C} p_i^k w_i^k L_i^2 \right) \left[1 + \eta^2 \sum_{i=1}^{C} (p_i^k w_i^k - p_i)^2 \right] \tag{29}
\]

as the magnification factor of the parameter deviation. Furthermore, we add the following assumption about the upper bounds of the expectation of the models’ gradient Lipschitz coefficients to eliminate the impact of the exponent term of \(A_k\), which is irrelevant to federated operations.

**Assumption 3** (Stable Expectation of \(L_i\)). The term \(A_k = 1 + \eta \sum_{i=1}^{C} p_i^k w_i^k L_i\) in (27) changes slowly with the training progress compared to the second moment of \(\{L_i\}\).

Hence, the third term in (27) is invariant in a single updating period under different IS weightings, and it is possible for us to find the explicit solution of the sub-problems. For simplicity, we use \(q_i = p_i^k w_i^k\) to denote the IS probability of the \(k\)-th client. We can then formulate the following sub-problem \(P^k\):

\[
P^k : \min_q \left(1 + 2\eta^2 \sum_{i=1}^{C} q_i L_i^2 \right) \left[1 + \eta^2 \sum_{i=1}^{C} (q_i - p_i)^2 \right] \tag{30}
\]

Subject to:

\[
\sum_{i=1}^{C} q_i = 1, \quad q_i \geq \sigma \cdot p_i^k, \quad i = 1, \cdots, C \tag{30a}
\]

From (30), we can find that there is a trade-off between two terms, the distribution similarity, and the second moment of the gradient Lipschitz. That is, to minimize the objective, we should take both factors into account. By solving each \(P^k\), we obtain the optimal local IS weightings for each client during one federated updating period. We sketch the optimal sampling selection as Theorem 2.

**Theorem 2.** Consider the solution to \(P_0\) under Assumption 3. With the fixed weightings \(\pi\) for federated updating, the optimal IS weightings for local training sampling are

\[
q^*_i = \max \left\{ \sigma p_i^k, p_i + \frac{1 - \frac{C L^2}{\sum_{i=1}^{C} L_i^2}}{\frac{C}{\sum_{i=1}^{C} \left(1 - \frac{C L^2}{\sum_{i=1}^{C} L_i^2}\right)} \cdot \Gamma(\lambda)} \right\} \tag{31}
\]

where

\[
\Gamma(\lambda) = \frac{1}{\eta^2} \cdot \sqrt{\frac{C L - 2\eta^2 \sum_{i=1}^{C} L_i^2}{2 \sum_{i=1}^{C} L_i^2}}. \tag{32}
\]

**Proof of Theorem 2**. As discussed above, the fixed \(\pi\) and the new assumption reduce the iterative problem \(P_0\) to \(K\) sub-problems related to each client’s local settings. Denote the two terms, respectively by

\[
\Phi(q) = 1 + 2\eta^2 \sum_{i=1}^{C} q_i L_i^2; \tag{33a}
\]

\[
\Psi(q) = 1 + \eta^2 \sum_{i=1}^{C} (q_i - p_i)^2. \tag{33b}
\]

Then the objective function can be represented by \(\Phi(q) \cdot \Psi(q)\). It is easy to check that the objective function is convex about
\{q_i\}. Therefore, to derive the solution of \(P^k\), we formulate the Lagrangian function:

\[
\mathcal{L}(q, \lambda, \mu) = \Phi(q) - \lambda \left( \sum_{i=1}^{C} q_i - 1 \right) - \sum_{i=1}^{C} \mu_i \left( q_i - \sigma p_i^k \right)
\]

where the scalar \(\lambda\) is the multiplier for the equality constraint and the vector multiplier \(\mu = [\mu_1, \cdots, \mu_C]\) is for the inequality constraint. Hence, the Karush-Kuhn-Tucker (KKT) conditions of \(P^k\) can be written as

\[
\begin{align*}
\frac{\partial \mathcal{L}}{\partial q_j} & = 2\eta^2 L_j^2 \Psi + 2\eta^2 (q_j - p_j) \Phi - \lambda - \mu_j = 0 \quad (35a) \\
\sum_{i=1}^{C} q_i &= 1 \quad (35b) \\
\mu_j &\geq 0 \quad (35c) \\
q_j &\geq \sigma p_j^k \quad (35d) \\
\mu_j (q_j^* - q_j) &= 0 \quad (35e)
\end{align*}
\]

where \(j = 1, \cdots, C\). Note that for (35a)-(35e), \(\mu_j = 0\) holds if \(q_j > \sigma p_j^k\). Otherwise, \(q_j = \sigma p_j^k\) and \(\mu_j > 0\). Thus, we are able to adopt a water-filling method to solve the KKT conditions. Specifically, we can firstly assume that \(\mu_j = 0\) for all \(j\) and then adjust each \(q_j^*\) to satisfy (35d). Since \(\Phi > 0\) always holds, each \(q_j\) can be represented as

\[
q_j = p_j + \frac{\lambda - 2\eta^2 L_j^2 \Psi}{2\eta^2 \Phi} \quad (36)
\]

By summing up all (35a), it follows that

\[
\Psi = \frac{C \lambda}{C \sum_{i=1}^{C} L_i^2} \quad (37)
\]

Then, from the definition of \(\Psi\), \(\Phi\) can be solved as

\[
\Phi = \lambda \frac{\sum_{m=1}^{C} \left( 1 - \frac{CL_m^2}{\sum_{i=1}^{i} L_i^2} \right)^2}{\Psi - 1} \quad (38)
\]

By substituting (37) and (38) into (36), we obtain

\[
q_j(\lambda) = p_j + \frac{\sum_{m=1}^{C} \left( 1 - \frac{CL_m^2}{\sum_{i=1}^{i} L_i^2} \right)^2 \cdot \Gamma(\lambda)}{\sum_{m=1}^{C} \left( 1 - \frac{CL_m^2}{\sum_{i=1}^{i} L_i^2} \right)^2} \quad (39)
\]

Remark 2. Note that the \(k\)-th client’s optimal re-weighted sampling probability of the \(j\)-th category \(q_j^*\) is the maximum between \(\sigma p_j^k\) and \(q_j(\lambda)\). Moreover, \(\Gamma(\lambda)\) is a monotonic function of \(\lambda\) and \(\Gamma(\lambda) \geq 0\) always holds. Thus, we only need to consider the optimal non-negative \(\Gamma^*\). Then, the numerical solutions in (31) can be calculated through a classical water-filling approach as shown in Fig. 3.

FIG. 3. The water-filling sketch of the optimal IS weightings.

Now for each client and given its empirical gradient Lipschitz coefficients \(\{L_j\}\), the water-filling solution of the optimal IS weights are affected by the following two factors: \(\Gamma\) and \(\{\alpha_j\}\), where

\[
\alpha_j = \frac{1 - \frac{CL_j^2}{\sum_{i=1}^{i} L_i^2}}{\sum_{m=1}^{C} \left( 1 - \frac{CL_m^2}{\sum_{i=1}^{i} L_i^2} \right)^2} \quad (40)
\]

describes the gap between the empirical gradient Lipschitz coefficient of each category and the average value. Since \(\{\alpha_j\}\) are determined by \(\{L_j\}\), we only need to find a proper \(\Gamma\) to guarantee that the lowest sampling probability \(\sigma p_j^k\) falls below \(q_j^*(\Gamma)\), which is the water-filling level height of Category \(j\). Here, we derive the following Theorem 3 to show how we calculate the optimal sampling probability.

Theorem 3. For client \(k\), if the lowest IS weight is properly set to satisfy \(\sigma \leq p_j/p_j^k\) for any category \(j = 1, \cdots, C\), by carrying a water-filling approach to calculate the IS
probabilities in (31), then the optimal factor $\Gamma^*$ shall be chosen as
\[
\Gamma^* = \min_j \left\{ \frac{p_j - \sigma p_j^k}{-\alpha_j} \left| \frac{p_j - \sigma p_j^k}{-\alpha_j} \geq 0 \right. \right\}, \quad (41)
\]
if the set is not empty.

Proof of Theorem 3. Practically, $\sigma$ is set as a small value to ensure that all categories contribute to the local training. Besides, as the category distribution of all data, each $p_j$ shall not be too small. Thus, one can assume that $p_j > \sigma p_j^k$ holds in most cases. Moreover, note that the factors $\{\alpha_j\}$ satisfy the zero-sum property. The water-filling approach will then lead to $p_j + \alpha_j \Gamma \geq \sigma p_j^k$ for each $j$. That is,
\[
\begin{cases}
\Gamma \geq \frac{p_j - \sigma p_j^k}{-\alpha_j} & \text{if } \alpha_j > 0, \\
\Gamma \leq \frac{p_j - \sigma p_j^k}{-\alpha_j} & \text{if } \alpha_j < 0.
\end{cases} \quad (42a)
\]

always holds because the right-hand side is smaller than 0 and $\Gamma \geq 0$. Therefore, the optimal $\Gamma^*$ shall satisfy (42b). By setting $\Gamma$ as the smallest non-negative $\frac{p_j - \sigma p_j^k}{-\alpha_j}$, the numerical solutions to $P^k$ can be obtained. ■

C. ISFL Algorithms

Based on above theoretical propositions, we formulate the detailed ISFL algorithms. In brief, ISFL generalizes the original FL training process at the federated epoch. In addition to collecting, aggregating and sending back the updated parameters to clients, the center also estimates the gradient Lipschitz coefficients, calculates and hands out the IS weightings for each client. From Theorem 3, we first derive the water-filling algorithm as Algorithm 1 to calculate the optimal IS weightings for each client.

In practice, since the local empirical gradient Lipschitz coefficients change with the local model parameters $\theta^k$, $\{L_i\}$ shall be rewritten as $\{L_{i,k}\}$. Moreover, the local IS weightings shall be updated as the training progresses. Also, the IS-weight updating shall be synchronized with the model parameter updating at each federated epoch. When clients upload their parameters to the center, the gradient Lipschitz coefficients will be estimated using a selected dataset. Meanwhile, the IS weightings for next local training epochs will be renewed. The algorithm of FL with local IS, ISFL, will be referred to as Algorithm 2.

Specifically, Lines 3 to 6 in Algorithm 2 is the local training of each client with importance sampling. Lines 8, 9 and 14 comprise the traditional federated parameter updating. The IS weight updating is carried from Lines 11 to 13. ISFL can be regarded as a modified version of the original FL algorithms with two additional steps at the local training, as well as federated updating. That is,

1) ISFL introduces the dynamic IS of training data while each client processes local training, which weights the data of different categories to combat the impact of non-i.i.d. data.

Algorithm 1 Water-filling Solutions of Optimal Local IS Weightings for Client $k$.

\textbf{Input:} lowest weight $\sigma$, global distribution $\{p_i\}$, local distribution $\{p^k_i\}$, local empirical gradient Lipschitz $\{L_{i,k}\}$, category count $C$.

1: $\Gamma_k \leftarrow \emptyset$;
2: \textbf{for} each category $j = 1, \ldots, C$ \textbf{do}
3: \hspace{1em} Calculate the gradient Lipschitz deviation $\alpha_j^k$ as (40);
4: \hspace{1.5em} \textbf{if} $\alpha_j^k < 0$ \textbf{then}
5: \hspace{2em} $\Gamma_k \leftarrow \Gamma_k \cup \frac{p_j - \sigma p_j^k}{-\alpha_j^k}$;
6: \hspace{1.5em} \textbf{else}
7: \hspace{2em} Continue;
8: \hspace{2em} \textbf{end if}
9: \hspace{1em} \textbf{end for}
10: Select the optimal factor: $\Gamma^*_k = \min \{\Gamma_k \cup \max \{\Gamma_k \cup 0\}\}$;
11: \textbf{for} each category $j = 1, \ldots, C$ \textbf{do}
12: \hspace{1em} Calculate the IS weight: $w^k_j \leftarrow \frac{p_j + \alpha_j^k \Gamma^*}{p_j^k}$;
13: \hspace{1em} \textbf{end for}
\textbf{Output:} IS weights $\{w^k_j\}$.

Algorithm 2 ISFL: Federated Learning with Local Importance Sampling.

\textbf{System Settings:} global round $T_G$, federated period $T_f$, client weights $\{\pi_k\}$, local dataset $D_k$, learning rates $\{\eta\}$, etc.

\textbf{Local Initialization:} model parameters $\{\theta^k_0\}$, IS weights $\{w^k_1\}$.

1: epoch $t \leftarrow 1$;
2: \textbf{while} $t \leq T_G \cdot T_f$ \textbf{do}
3: \hspace{1em} \textbf{for} client $k$ in $\{1, \ldots, K\}$ \textbf{do}
4: \hspace{2em} Sample batches $\{\xi^k\}$ according to IS weights $\{w^k_1\}$;
5: \hspace{2em} Locally train $\theta^k_t$ with $\{\xi^k\}$;
6: \hspace{1em} \textbf{end for}
7: \hspace{1.5em} $t \mod T_f = 0$ \textbf{then}
8: \hspace{2em} Upload local parameters $\{\theta^k_t\}$;
9: \hspace{2em} Update global parameters: $\bar{\theta}_t \leftarrow \sum_{k=1}^K \pi_k \theta^k_t$;
10: \hspace{2em} \textbf{for} client $k$ in $\{1, \ldots, K\}$ \textbf{do}
11: \hspace{3em} Estimate gradient Lipschitz $\{L_{i,k}\}$;
12: \hspace{3em} Carry Algorithm 1 to calculate $\{w^k_{t+1}\}$;
13: \hspace{3em} Download IS weightings $\{w^k_{t+1}\}$;
14: \hspace{3em} Download global parameters: $\theta^k_{t+1} \leftarrow \bar{\theta}_t$;
15: \hspace{2em} \textbf{end for}
16: \hspace{1em} $t \leftarrow t + 1$;
17: \textbf{end while}
\textbf{Output:} Model parameters: $\bar{\theta}_t$ and $\{\theta^k_t\}$.

2) During the federated updating rounds, besides the traditional parameter upload/download, ISFL adds extra operations which can be executed at the central server: estimating the gradient Lipschitz coefficients, calculating the renewed IS weights, and sending them back to clients simultaneously through the parameter downlinks.
As for the cost of ISFL, the extra communication costs can be neglected because at each federated rounds, clients only need to pack in the IS weights, the $C$ length vectors, which take up far less capacity compared to the transmission of the model parameters. However, considerable computation costs are inevitable. The main computation consumption occurs while estimating the gradient Lipschitz coefficients.

Note that Theorem 1 requires the estimation of the conditional gradient Lipschitz between the local models and the centralized-trained model. However, in practical FL systems, clients are not allowed to share their local data for privacy protection in the federated mode. In this work, we consider an alternative solution which assumes that the global model tends to get close to the centralized model. Specifically, we adopt a vanilla approach to approximate $\{L_{i,k}\}$ by checking the maximum empirical $\hat{L}_{i,k}$ between $\theta^k_i$ and $\theta_t$ on a small selected dataset $D_L$, i.e.,

$$\hat{L}_{i,k}(t) := \max_{\xi_i \in D_L} \left\| \frac{1}{t} \sum_{k=1}^{t} \nabla \theta \ell(\xi_i; \theta^k_i) - \nabla \theta \ell(\xi_i; \theta_t) \right\| \theta^k_i - \theta_t, \quad (43)$$

when $t \equiv 0 \mod T_f$. All these operations could be deployed at the central server and therefore, neither private information nor local data of the clients will need to be transmitted and shared. In other words, the gradient Lipschitz estimation at the federated round causes additional computation at the central server. Since this work focuses on investigating the ISFL framework, the details of the gradient Lipschitz estimation and the corresponding computation scheduling will not be considered in this paper.

IV. EVALUATION

In this section, to evaluate our proposals, we carry out experiments on a popular image classification task, CIFAR-10. The corresponding FL algorithms are implemented with PyTorch\footnote{https://pytorch.org/}. The details of the experiment configurations, the results and the insights will be presented below.

A. Experiment Setups

In the basic experiments, we consider an FL system with 8 clients. The detailed settings and learning configurations are listed in Table II. We set the maximum global epoch and federated period as 25 and 4, respectively. That is, during each global epoch, clients train their models locally for 4 epochs. The local data of each client are divided into batches with size of 128 to carry the mini-batch SGD training and the global epoch, clients train their models locally for 4 epochs. Besides, as mentioned, the real empirical gradient Lipschitz between the local model and centralized model is agnostic because there is no model trained on the global data in practical FL. Therefore, we estimate the gradient Lipschitz using a small dataset $D_L$ using (43) with 2000 samples randomly selected from the datasets. However, to evaluate the developed theories and the algorithms, we synchronously train a centralized model on the complete data to calculate the precise gradient Lipschitz. To distinguish ISFL with the precise gradient Lipschitz from that with the approximate one in (43), we shall refer to it as ISFL-PL in the following experiments.

1) Datasets and Models: Our experiments are carried based on a popular image classification dataset, CIFAR-10. Since the goal is to prove the effectiveness of the proposed FL algorithms, we are not aiming at the state-of-the-art performance. As for the neural network model, we adopt a lightweight ResNet\footnote{https://pytorch.org/} with 2 residual convolutional blocks and 8 trainable layers in total (ResNet8). The centralized-full-data trained model reaches 91.25% accuracy.

2) Non-i.i.d. Settings: We implement commonly used label-level non-i.i.d. settings similarly to [36] as basic non-i.i.d. setting: (a) Clients randomly select a small proportion $(1 - NR)$ of the full data; (b) All the data samples are sorted by their labels and then divided into partitions with 500 images; and (c) Each client is randomly assigned 2 partitions. Overall, each client possesses 1125 training samples with 125 randomly assigned samples. The whole data of all clients is referred to as global data, $D_G$. Since the global data i.e., the ensemble of the clients’ data might still be imbalanced, we test the model performance not only on the standard CIFAR-10 test dataset $D_S$, but also on the global data $D_G$. The test on $D_G$ reflects the convergence and training efficiency of FL algorithms. The test on $D_S$ implies the generalization ability of the model because the data distribution differs from the integrated data for the non-i.i.d. settings.

B. Numerical Results

We firstly assign the non-i.i.d. data for distributed clients following the above steps. The local data distribution are shown in Fig. 4(a). To evaluate the performance of several training settings and methods, we compare ISFL and ISFL-PL with the original FL algorithm, namely, FedAvg, and the centralized learning (CL) scheme. The test accuracy curves are plotted in Fig. 4(b) and Fig. 4(c). The comparison of different learning schemes on $D_S$ is presented in Fig. 4(b). One can observe that for the non-i.i.d. data, FedAvg performs poorly in terms of accuracy, convergence, and stability. In contrast, ISFL improves about 20% accuracy and the fluctuations of repeated executions are much smaller. Most notably, the purple dash curve represents the centralized model trained on the integrated data of the same non-i.i.d. settings. As ISFL nearly reaches the centralized model with about 6% lower accuracy on standard CIFAR-10 test data, we can conclude that ISFL narrows the
Fig. 4. Non-i.i.d. distribution and comparison of test accuracy on CIFAR-10 dataset.

Fig. 5. Importance sampling probability \( \{q_i^k\} \) of the clients.

The detailed numerical results are listed in Table III. We also test the ISFL based algorithms under different values of \( \sigma \), the lowest IS weights. The outcomes show that \( \sigma \) has little influence on the model performance. Such phenomenon is comprehensible since we set \( \sigma \) as a small constant in the constraints \( P_0 \) to ensure that data in all categories could contribute to the federated model. Moreover, according to the water-filling solution in Theorem 3, the optimal IS probability is only related to the minimal of \( \Gamma \). Besides, as shown in Fig. 4 and Table III, the results of ISFL using the gradient Lipschitz estimated and ISFL-PL using the precise gradient Lipschitz are almost the same. This also proves the effectiveness of the gradient Lipschitz approximation using (43) and the proposed Algorithm 2.

| Training Settings       | Approach     | Test Accuracy (%) |
|-------------------------|--------------|-------------------|
|                         |              | \( D_S \)         | \( D_C \)         |
| i.i. data with the same volume | FedAvg      | 76.44             | /                  |
| full non-i.i. data      | centralized | 70.53             | /                  |
| non-i.i. data           | FedAvg       | 43.56             | 46.09              |
|                         | ISFL-PL-\( \sigma = 0.1 \) | **64.02**          | 74.39              |
|                         | ISFL-PL-\( \sigma = 0.02 \) | 63.84              | 72.87              |
|                         | ISFL-\( \sigma = 0.1 \) | 62.86              | **74.86**          |
|                         | ISFL-\( \sigma = 0.02 \) | 63.31              | 74.81              |

We further conduct several client-level analysis of ISFL with \( \sigma = 0.1 \). Fig. 6 is the evolution of the local IS probability \( \{q_i^k\} \) on each client. By Algorithm 2, ISFL updates the local IS weightings synchronously with the federated epoch. Each client also adopts its latest IS weightings for the local training.
epochs. One can observe that the IS probabilities of different categories change within small ranges. As shown, ISFL does not simply carry the re-weighting to sample the data of each category uniformly. Some categories are always sampled with a large probability, while some are occasionally re-weighted to nearly 0 probability. Moreover, the experiments illustrate that categories with a large proportion will be down-sampled but still get a larger IS probability than uniform sampling. On the contrary, some categories with a small proportion will be highly up-sampled such as Categories 3 and 5 for client 1. The experimental results meet the conclusion in Theorems 2 and 3. Through [31], the categories with large global proportion or small gradient Lipschitz might be utilized more frequently.

To verify the proposed theorems, we also plot the parameter deviation magnification factor $\rho_k$ and the parameter deviations in Fig. 6. As displayed in Fig. 6(a), the magnification factor $\rho_k$, i.e., the objectives [30] in $P_k$, hold the values near 1, which means that the parameter deviation $\Delta_k$ keeps its scale at the federated epochs. The deviation curves between each local model and the global model in Fig. 6(b) confirm that the model deviation and the convergence of ISFL are well tackled.

![Fig. 6. The parameter deviation analysis under ISFL ($\sigma = 0.1$) schemes.](image)

Since ISFL is a sampling-based approach, we further evaluate its data efficiency. We increase the size of the partition to 1000 for the non-i.i.d. settings and therefore each client possesses 2125 training samples. Different from training the full data in every local epoch, we consider the partial sampling with a sampling ratio $SR$, i.e., each client selects only $[SR \cdot |D_k|]$ samples for training at every local epoch. The results are shown in Fig. 7. ISFL not only gets a higher accuracy, but also its performance is stable and robust. The accuracy reduction is little with the $SR$ decreases. Particularly, even though each client only samples 0.1 ratio training data in local epochs, ISFL still preserves 52.46%, 70.27% accuracy on $D_S$ and $D_G$, respectively. Thus, ISFL exhibits a better sampling efficiency and data robustness, which is important for the scenarios where the local computation resources/timeliness are restricted.

![Fig. 7. The sampling efficiency evaluation of ISFL ($\sigma = 0.1$, partition size: 1000).](image)

V. Conclusion

In this paper, we investigated the label-skewed non-i.i.d. data quagmire in federated learning and proposed a local importance sampling based framework, ISFL. We derived the theoretical guarantees of ISFL, as well as the solutions of the optimal IS weightings using a water-filling approach. In particular, we improved the theoretical compatibility with the neural network models by relaxing the convexity assumptions. The ISFL algorithm was also developed and evaluated on the CIFAR-10 dataset. The experimental results demonstrated the improvement of ISFL for the non-i.i.d. data from several aspects. ISFL promotes 20%+ accuracy and nearly reaches the performance of the centralized model using the integrated non-i.i.d. data. ISFL also shows its potentials on sampling efficiency and data robustness. This is significant for FL, especially for the mobile light-weight scenarios.

For future work, more tests on the theorems and the performance of ISFL can be considered. For instance, the finer-grained analysis of the parameter deviation and the gradient Lipschitz approximation shall be studied. The other types of non-i.i.d. settings such as feature based non-i.i.d. cases shall also be considered. The potential strengths for local personalization can also be further developed by setting the $\{p_k\}$ different for each client. In addition, since ISFL is a local sampling federated algorithm, it can be easily migrated into other FL frameworks and the new developed sampling-based FL as a key building block can be combined with some emerging FL schemes.

References

[1] B. McMahan, E. Moore, D. Ramage, S. Hampson, and B. A. y Arcas, “Communication-efficient learning of deep networks from decentralized data,” in Artificial intelligence and statistics. PMLR, 2017, pp. 1273–1282.

[2] J. Konečný, H. B. McMahan, F. X. Yu, P. Richtárik, A. T. Suresh, and D. Bacon, “Federated learning: Strategies for improving communication efficiency,” arXiv preprint arXiv:1610.05492, 2016.

[3] Q. Yang, Y. Liu, T. Chen, and Y. Tong, “Federated machine learning: Concept and applications,” ACM Transactions on Intelligent Systems and Technology (TIST), vol. 10, no. 2, pp. 1–19, 2019.

[4] W. Y. B. Lim, N. C. Luong, D. T. Hoang, Y. Jiao, Y.-C. Liang, Q. Yang, D. Niyato, and C. Miao, “Federated learning in mobile edge networks: A comprehensive survey,” IEEE Communications Surveys & Tutorials, vol. 22, no. 3, pp. 2031–2063, 2020.

[5] S. Niknam, H. S. Dhillon, and J. H. Reed, “Federated learning for wireless communications: Motivation, opportunities, and challenges,” IEEE Communications Magazine, vol. 58, no. 6, pp. 46–51, 2020.

[6] D. C. Nguyen, M. Ding, P. N. Pathirana, A. Seneviratne, J. Li, and H. V. Poor, “Federated learning for internet of things: A comprehensive survey,” IEEE Communications Surveys & Tutorials, 2021.
T. Li, A. K. Sahu, A. Talwalkar, and V. Smith, “Federated learning: Challenges, methods, and future directions,” IEEE Signal Processing Magazine, vol. 37, no. 3, pp. 50–60, 2020.

L. U. Khan, S. R. Pandey, N. H. Tran, W. Saad, Z. Han, M. N. Nguyen, and C. S. Hong, “Federated learning for edge networks: Resource optimization and incentive mechanism,” IEEE Communications Magazine, vol. 58, no. 10, pp. 88–93, 2020.

S. Wang, T. Tuor, T. Salomidou, K. K. Leung, C. Makaya, T. He, and K. Chan, “Adaptive federated learning in resource constrained edge computing systems,” IEEE Journal on Selected Areas in Communications, vol. 37, no. 6, pp. 1205–1221, 2019.

X. Wang, Y. Han, C. Wang, Q. Zhao, X. Chen, and M. Chen, “In-edge AI: Intelligentizing mobile edge computing, caching and communication by federated learning,” IEEE Network, vol. 33, no. 5, pp. 156–165, 2019.

S. Yu, X. Chen, Z. Zhou, X. Gong, and D. Wu, “When deep reinforcement learning meets federated learning: Intelligent multisensory resource management for multiaccess edge computing in 5G ultradense network,” IEEE Internet of Things Journal, vol. 8, no. 4, pp. 2238–2251, 2020.

Z. Zhu, S. Wan, P. Fan, and K. B. Letaief, “Federated multiagent actor–critic learning for age sensitive mobile-edge computing,” IEEE Internet of Things Journal, vol. 9, no. 2, pp. 1053–1067, 2021.

K. B. Letaief, W. Chen, Y. Shi, J. Zhang, and Y.-J. A. Zhang, “The roadmap to 6G: AI empowered wireless networks,” IEEE communications magazine, vol. 57, no. 8, pp. 84–90, 2019.

Z. Yang, M. Chen, K.-K. Wong, H. V. Poor, and S. Cui, “Federated learning for 6g: Applications, challenges, and opportunities,” Engineering, 2021.

Q. Yang, Y. Liu, Y. Cheng, Y. Kang, T. Chen, and H. Yu, “Federated learning,” Synthesis Lectures on Artificial Intelligence and Machine Learning, vol. 13, no. 3, pp. 1–207, 2019.

Y. Liu, Y. Kang, C. Xing, T. Chen, and Q. Yang, “A secure federated transfer learning framework,” IEEE Intelligent Systems, vol. 35, no. 4, pp. 70–82, 2020.

F. Battler, K.-R. Müller, and W. Samek, “Clustered federated learning: Model-agnostic distributed multitask optimization under privacy constraints,” IEEE transactions on neural networks and learning systems, vol. 32, no. 8, pp. 3710–3722, 2020.

M. S. H. Abad, E. Ozfatura, D. Gunduz, and O. Ercetin, “Hierarchical federated learning across heterogeneous cellular networks,” in ICASSP 2020-2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP). IEEE, 2020, pp. 8866–8870.

A. Lalitha, S. Shekhar, T. Javidi, and F. Koushanfar, “Fully decentralized federated learning,” in Third workshop on Bayesian Deep Learning (NIPS), 2018.

S. U. Stich, “Local sgd converges fast and communicates little,” in International Conference on Learning Representations, 2018.

J. Wang and G. Joshi, “Cooperative sgd: A unified framework for the design and analysis of local-update sgd algorithms,” Journal of Machine Learning Research, vol. 22, no. 213, pp. 1–50, 2021.

A. Fallah, A. Mokhtari, and A. Ozdaglar, “Personalized federated learning with theoretical guarantees: A model-agnostic meta-learning approach,” Advances in Neural Information Processing Systems, vol. 33, pp. 3557–3568, 2020.

S. Wan, J. Lu, P. Fan, Y. Shao, C. Peng, and K. B. Letaief, “Convergence analysis and system design for federated learning over wireless networks,” IEEE Journal on Selected Areas in Communications, vol. 39, no. 12, pp. 3622–3639, 2021.

L. Floridi, “Establishing the rules for building trustworthy ai,” Nature Machine Intelligence, vol. 1, no. 6, pp. 261–262, 2019.

P. Kairouz, H. B. McMahan, B. Avent, A. Bellet, M. Bennis, A. N. Bhagoji, K. Bonawitz, Z. Charles, G. Cormode, R. Cummings et al., “Advances and open problems in federated learning,” Foundations and Trends® in Machine Learning, vol. 14, no. 1–2, pp. 1–210, 2021.

Q. Li, Y. Diao, Q. Chen, and B. He, “Federated learning on non-iid data silos: An experimental study,” arXiv preprint arXiv:2102.02079, 2021.

K. Hsieh, A. Panishayee, O. Mutlu, and P. Gibbons, “The non-iid data quagmire of decentralized machine learning,” in International Conference on Machine Learning. PMLR, 2020, pp. 4387–4398.

M. F. Bugallo, V. Elvira, L. Martino, D. Luengo, J. Miguez, and P. M. Djuric, “Adaptive importance sampling: The past, the present, and the future,” IEEE Signal Processing Magazine, vol. 34, no. 4, pp. 60–79, 2017.

A. Katharopoulos and F. Fleuret, “Not all samples are created equal: Deep learning with importance sampling,” in International conference on machine learning. PMLR, 2018, pp. 2525–2534.