Information entropy as an anthropomorphic concept

Panteleimon Rodis
Diploma in Computer Science, Hellenic Open University
rodispantelis@gmail.com
pantelisrodis.blogspot.com

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Abstract According to E.T. Jaynes and E.P. Wigner, entropy is an anthropomorphic concept in the sense that in a physical system correspond many thermodynamic systems. The physical system can be examined from many points of view each time examining different variables and calculating entropy differently. In this paper we discuss how this concept may be applied in information entropy; how Shannon’s definition of entropy can fit in Jayne’s and Wigner’s statement. This is achieved by generalizing Shannon’s notion of information entropy and this is the main contribution of the paper. Then we discuss how entropy under these considerations may be used for the comparison of password complexity and as a measure of diversity useful in the analysis of the behavior of genetic algorithms.

Keywords Information entropy, anthropomorphic, genetic algorithms, diversity

1. Introduction and definitions

Entropy as an anthropomorphic concept was first proposed in Jaynes seminal paper (Jaynes, 1965) and states that we may study entropy of many thermodynamic systems in a physical system. Let us extend this approach in information theory. Information entropy as defined by Claude Shannon (Shannon, 1948) it quantifies the amount of information that may be transmitted by a channel or more generally the amount of information that there is in a message $M$. Message $M$ consists of a set of $n$ symbols chosen from alphabet $\Sigma$. Let $p(i)$ be the probability of appearance of any symbol $i$ of $\Sigma$ in $M$ then for all $n$ symbols stands that entropy $H = -\sum_{i=1}^{n} p(i) \log p(i)$.

Probability $p(i)$ equals to the frequency of appearance of $i$ in $M$.

Let’s generalize this idea. Let $S$ be a set of objects that we may describe their properties using $v$ variables for each one. Let each variable take values from a range $[a, z]$ of discrete values. Now we have a lot of information about $S$ and we can use entropy in order to measure its complexity. Exactly as Jaynes describes the examination of physical systems we may choose any subset $v \subseteq v'$ in order to examine the information content of $S$. The choice of $v$ depends on how we care to examine $S$ and this provides the anthropomorphic characteristic to this analysis. Using information entropy we can then examine the complexity of $v$ properties of the objects in $S$.

Definition. Let $S$ be a set of $k$ objects, for each $o \in S$ let $V_o$ be a vector of $v$ variables that describe $o$. For variable $X$ in the $i$-th place of every $V_k$ we define probability $p(x) = p[X = x]$ and entropy $H(X) = -\sum_{i=1}^{k} p(x) \log p(x)$ which quantifies the information content of $X$ in $S$.

For the information entropy of $S$ with respect to all variables of every $V_o$ we denote $H_v = H(1, 2, \ldots, v)$. 
In the case where the \( v \) variables are independent it stands that \( H_v = - \sum_{i=1}^{k} \sum_{x=1}^{r} p(x) \log p(x) \).

In the case of any dependent variables \( X, Y \) in \( V \) and for \( x \in X, y \in Y \) the above equation is modified accordingly.

For joint probability \( p(x, y) \) stands \( H_v = - \sum_{i=1}^{k} \sum_{x=1}^{r} \sum_{y=1}^{s} p(x, y) \log p(x, y) \).

For conditional entropy \( H_s(X \mid Y) \) which stands for the probability \( p(y, x) \) when \( X \) depends on \( Y \) entropy is \( H_v(X \mid Y) = - \sum_{i=1}^{k} \sum_{x=1}^{r} p(x, y) \log p(x, y) \).

These considerations of entropy are different from the classical definitions of Shannon. Shannon defines conditional probability \( p(x, y) \) as the probability of appearance of symbol \( x \) which depends on the appearance of symbol \( y \). The above probabilities consider \( x \) and \( y \) as properties of discrete objects that the appearance of one property depends on the appearance of the other.

### 2. Applications

#### 2.1 Password rating for handheld devices

A strong password \( M \) must be complex this implies that one characteristic of \( M \) is higher entropy than easier passwords. In case that we focus in handheld devices the frequency of appearance of each character in \( M \) is not the only property we may consider. The user of each device has to change the keyboard appearance of the device in order to write upper case characters, lower case, symbols or numbers. This makes the input of \( M \) more complex than in desktop computers and this property may be considered when rating the complexity of \( M \).

The rating of password strength still regards characters of a simple string \( M \) but let us examine them as discrete objects of more than one property. Variable \( X \) is defined on the set of characters available to the user and \( p(x) \) is the probability of appearance of each character \( x \) in \( M \). Variable \( Y \) describes the property of each symbol of being upper case, lower case, symbol or number. So, \( p(y) \) is the probability of \( y \) being upper case character, lower case, symbol or number. The two variables are independent. As a result \( H_s(X, Y) = H(X) + H(Y) \). Computing entropy using the last equation provides a more accurate analysis of the information in \( M \).

In the past I had used this notion of password rating for the development of a simple Android application (Rodis, 2014).

In this approach we may consider additional properties in the computation of the entropy of \( M \), extending the abilities of my application. We may examine the case that \( M \) has some meaning as a word in some human language like English; in this case its structure is not random. Following Shannon’s reasoning for any consecutive characters \( y, x \) in \( M \), we define conditional probability \( p(x, y) \) as the probability of \( x \) appearing after \( y \). The probability is determined in how often \( x \) follows \( y \) in English words. Thus, for each pair of consecutive characters in \( M \) we add to the above equation conditional entropy \( H_s(X \mid Y) \).

#### 2.2 Comparing diversity among distinct populations

Entropy can be used as a measure of comparison of diversity among different populations that share the same characteristics. The idea of using entropy for measuring diversity is not new and the benefits of this idea have been studied extensively; see (Heip & Engels, 1974)) and (Burke et al., 2004). The use of the definition of entropy presented in this paper for measuring diversity has the advantage of focusing in certain characteristics of the populations that we care about and we may base our analysis on them.
Let $b$ and $c$ be two distinct populations of not homogeneous members. The members of both populations are of the same kind; their characteristics can be described by a vector $V$ of $v$ variables that are valid for all members. It is clear that if the diversity of $b$ with respect to $V$ is higher than the diversity of $c$ then the entropy $H_V$ in $b$ is higher than in $c$.

According to Jayne there is no sense in saying that a system has high entropy. The measurement of entropy may be used as a mean of comparison among two or more systems. On the same way entropy as a measure of diversity may only be used effectively for the comparison of the diversities among different populations.

Next, we show how the application of entropy in the measurement of diversity is useful in the analysis of simple genetic algorithms.

The population $l$ created during the execution of genetic algorithm $G$ consists of genes or chromosomes that encode information about the possible solutions of the problem that $G$ solves. All the members of $l$ are built on the same pattern as they represent solutions of the same problem. Thus, each segment $s$ of each member $l_i$ of $l$ encodes the same characteristic. We then define vector $V$ of $i$ variables, where each variable $v_i$ corresponds to segment $s_i$ of the members of $l$. Each segment may consist of one or more bits that represent a specific part of information on each $l_i$.

The initial population $b$ of a simple genetic algorithm is created with randomly generated values. Experimentally and empirically it is widely acceptable that during the execution of a simple genetic algorithm its population tends to become homogeneous and the algorithm converges to a local or global optimum; indicatively see (Green et. al, 2013, p.94). Let us call $c$ the population that is generated when $G$ has converged.

From the evaluation of variable $X$ in the $i$-th place of every vector $V_b$ of population $b$ we construct string $d_i$ and for every vector $V_c$ of population $c$ we construct string $e_i$. Under the Kolmogorov perception of randomness (Kolmogorov & Uspenskii, 1988) string $d_i$ is more random than $e_i$. This also agrees with our definition of entropy as it is clear that $H(X_b) > H(X_c)$. Thus, the information entropy in population $b$ is higher than in population $c$.

As a result of all the above, for populations $b$ and $c$ produced during the execution of $G$ we may use entropy to compare randomness in their structures and the variation of their information content.

The above considerations about homogeneity and randomness clarify the following theorem and corollary.

**Theorem.** A simple genetic algorithm $G$ reduces the information entropy of its population of chromosomes while $G$ converges to a local or global optimum.

**Corollary.** The reduction of $H_V$ in its population is a property of an effective genetic algorithm.

A mathematically concrete proof of the theorem and corollary is not yet possible. The reason for this is that the functionality of genetic algorithms has not been completely clarified mathematically. The research in genetic algorithms is mostly based on experimental results. Nevertheless we may provide some reasoning on why the population of a genetic algorithm tends to become homogeneous, which is a basic issue for the theorem.

During the execution of $G$ the members of $l$ with higher fitness are promoted to participate in reproduction and crossover operations at the expense of the members with lower fitness. So it is reasonable to say that a chromosome with higher fitness will probably have more copies of itself in the next generation of $l$ resulting in a more homogenous population. Homogeneity implies that for any segment $s$ on the members of a homogenous population the information entropy will be lower than in a not homogenous population.

As a conclusion to the applications of the definition of entropy in this paper, we may say that it provides a mean to analyse information in larger extend than in the classical approach and this is a serious benefit of its applications.
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