A new perspective on data-enabled predictive control

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Abstract—Data-enabled predictive control (DeePC) is a recently proposed approach that combines system identification, estimation and control in a single optimization problem, for which only recorded input/output data of the examined system is required. In this work we present a simple method to identify a multi-step prediction model from the same data required for DeePC. We prove that model predictive control based on this model is equivalent to DeePC in the deterministic case and that its solution has the same structure in the stochastic case. We investigate the advantages and shortcomings of DeePC as opposed to the sequential system identification and control approach for linear models with and without measurement noise. We find that DeePC adds significant complexity to the optimization problem and can also lead to deterioration in control performance for the non-deterministic case, as we illustrate with a simulation example.

I. INTRODUCTION

Model predictive control (MPC) is a popular strategy to control multivariable systems with constraints [1]. Traditionally, the main shortcoming of MPC is its computational complexity as it requires the online solution of an optimization problem. Due to advances in computing power, algorithms and efficient implementation strategies, solution time is often not an obstacle for its deployment anymore.

However, MPC still requires a dynamic model of reasonable accuracy to obtain a satisfactory performance. Such model can be obtained from physical modelling, using data-based system identification or a combination of both approaches. Learning from data has recently regained attention although it has been studied for a long time in the field of system identification [2], [3], [4], [5]. Important categories in this vast field are, among others, linear vs. nonlinear [2] and input/output vs. state-space models [3]. A concrete example of interest is the autoregressive model with exogeneous inputs (ARX) [5], a popular linear input/output approach. An adaptation of ARX, especially useful for MPC, is the linear multi-step ahead prediction model [6], which aims to find individual model parameters for each step of the prediction horizon.

In recent years, a new trend in data-based control seeks to integrate the traditional sequential system identification and control in a unified approach. From the machine learning community, reinforcement-learning is a popular approach following this paradigm [7]. However, reinforcement-learning comes with its own challenges such as the demand for large quantities of data and produces highly variable outcomes [8]. Another approach to unify identification and control is the newly proposed data-enabled predictive control (DeePC) [9] algorithm. The approach describes a simple configuration of a predictive control problem which operates directly on matrices of collected data. Furthermore, the required data has to satisfy only moderate requirements and can often be taken directly from a running process.

The initial work on DeePC [9] has thus sparked considerable interest in the control community with several extensions and modifications. The authors in [10] investigate DeePC with respect to stability and robustness and propose an adaptation to guarantee these properties. Distributionally robust DeePC is presented in [11], where it is assumed that data is sampled from a distribution of possible systems. In [12], DeePC is combined with an extended Kalman Filter to improve the performance in the case of noisy measurements.

However, DeePC remains in its nature a linear MPC scheme. In fact, the authors in [9] state that an equivalent classical MPC formulation exists for their proposed DeePC scheme. This equivalence is based on theorizing about the existence of an unknown system parameterization. But if such a parameterization is known, what are the advantages and potential shortcomings of the DeePC approach, especially in the linear setting, for which DeePC is derived?

The main contribution of this work is to help answer this question. We first prove that in the linear deterministic case, DeePC is equivalent to a MPC scheme based on an embedded least-square estimation of a multi-step linear prediction model. This multi-step model can be determined from the exact same data that DeePC requires to operate. Secondly, we show that in the non-deterministic case, output-feedback MPC based on the multi-step model and DeePC yield identical structures for their solutions. Finally, we compare both controllers numerically on a linear model with additive Gaussian noise. We include an analysis of the effect that the amount of available data and the regularization parameterization. But if such a parameterization is known, what are the advantages and potential shortcomings of the DeePC approach, especially in the linear setting, for which DeePC is derived?

The formal contribution of this work is to help answer this question. We first prove that in the linear deterministic case, DeePC is equivalent to a MPC scheme based on an embedded least-square estimation of a multi-step linear prediction model. This multi-step model can be determined from the exact same data that DeePC requires to operate. Secondly, we show that in the non-deterministic case, output-feedback MPC based on the multi-step model and DeePC yield identical structures for their solutions. Finally, we compare both controllers numerically on a linear model with additive Gaussian noise. We include an analysis of the effect that the amount of available data and the regularization term of DeePC have on the closed-loop performance. The remainder of this paper is structured as follows. In Section \textsection II we present the problem setup and review the data-enabled predictive control algorithm. We then present in Section \textsection III a data-based identification method for a multi-step ahead prediction model. With this model we formulate a MPC scheme for which we show equivalence with the DeePC method in Theorem \textsection III. We then proceed to analyse the case of a non-deterministic linear system in Section \textsection IV. Finally, we present a numerical comparison of both controllers in Section \textsection V. We finish this work with concluding remarks in Section \textsection VI.

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II. DATA-ENABLED PREDICTIVE CONTROL

We investigate an unknown discrete-time LTI dynamic system represented in state-space form:

\[ x_{k+1} = Ax_k + Bu_k, \quad y_k = Cx_k + Du_k, \tag{1a} \]

with \( x \in \mathbb{R}^n \) (states), \( u \in \mathbb{R}^m \) (inputs), \( y \in \mathbb{R}^p \) (measurements) and system matrices \( A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, C \in \mathbb{R}^{p \times n} \) and \( D \in \mathbb{R}^{p \times m} \). We assume that the system is given in its minimal representation. From this unknown system, we collect \( T \) sequences of input/output data of length \( L = T_{ini} + N \) and arrange them in matrices, such that:

\[ U_L = [u_L^1, u_L^2, \ldots, u_L^L], \quad Y_L = [y_L^1, y_L^2, \ldots, y_L^L], \tag{2} \]

where \( u_L^k = [(u_1^k)^\top, (u_2^k)^\top, \ldots, (u_L^k)^\top]^\top \in \mathbb{R}^{m \times T} \) and \( y_L^k = [(y_1^k)^\top, (y_2^k)^\top, \ldots, (y_L^k)^\top]^\top \in \mathbb{R}^{p \times T} \). These data matrices may or may not coincide with a Hankel or Page matrix (see [11]). Furthermore, we divide the input/output data in (2) in two parts, such that:

\[ U_L = \begin{bmatrix} U_{T_{ini}} & U_N \end{bmatrix}, \quad Y_L = \begin{bmatrix} Y_{T_{ini}} & Y_N \end{bmatrix}. \tag{3} \]

To clarify this notation, we now have \( U_{T_{ini}} \in \mathbb{R}^{m \times T}, U_N \in \mathbb{R}^{m \times N} \) and \( Y_{T_{ini}} \in \mathbb{R}^{p \times T} \) and \( Y_N \in \mathbb{R}^{p \times N} \).

Definition 1 ([9]): With \( L, T \) such that \( T \geq Lm \), the signals comprising matrix \( U_L \) are persistently exciting of order \( L \) if matrix \( U_L \) has full rank.

Definition 2 ([9]): We denote with \( l(A,B,C,D) \) the lag of system (1). It is defined as the smallest integer \( l \) for which the observability matrix:

\[ O_L(A,C) := \begin{pmatrix} C, CA, \ldots, CA^{l-1} \end{pmatrix}, \tag{4} \]

has full rank.

Assumption 1: Let \( T \geq Lm + n \) and \( U_L \) be persistently exciting of order \( L \) according to Definition [9].

In the following we state a variant (see Remark 1) of the fundamental lemma of behavioral system theory [13].

Lemma 1: Given input/output data of a deterministic system in the form of (1) which satisfies Assumption [1] and for which, according to Definition [2], \( T_{ini} > l(A,B,C,D) \), any arbitrary future system trajectory \( y_N = [y_{T_{ini}+1}, \ldots, y_{T_{ini}+N}]^\top \) can be predicted as the solution to the equation

\[ \begin{bmatrix} U_{T_{ini}} & U_N \\ Y_{T_{ini}} & Y_N \end{bmatrix} g = \begin{bmatrix} u_{T_{ini}} \\ u_N \\ y_{T_{ini}} \\ y_N \end{bmatrix}, \tag{5} \]

given initial sequences \( y_{T_{ini}} = [y_1^T, \ldots, y_{T_{ini}}^T]^T \), \( u_{T_{ini}} = [u_1^T, \ldots, u_{T_{ini}}^T]^T \) as well as the sequence of future control inputs \( u_N = [u_{T_{ini}+1}^T, \ldots, u_{T_{ini}+N}^T]^T \).

Remark 1: Note that the original statement of the fundamental lemma in [13] requires the matrices in (2) to be Hankel matrices. In [11] it is shown that (5) also holds for Page matrices which can extended to show that arbitrary sequences are sufficient as long as the space spanned by the initial states of these sequences has full rank.

Based on the fundamental Lemma [1] the authors in [9] proposed the elegant data-enabled predictive control scheme (DeePC), where the optimal control policy is obtained as the solution of:

\[ \min_{g \in \mathbb{R}^{n \times N}} f(g, u_N, y_N) \]

s.t. \[ (5) \tag{6} \]

\[ u_k \in U \forall k \in \{1, \ldots, N\}, \]

\[ y_k \in Y \forall k \in \{1, \ldots, N\}, \]

with arbitrary objective function \( f(g, u_N, y_N) \). Within the scope of this work, we only investigate the classical regulation task:

\[ f(g, u_N, y_N) = \sum_{k=1}^{N} (\|y_k\|_Q^2 + \|u_k\|_R^2). \tag{7} \]

III. PREDICTIVE CONTROL BASED ON A MULTI-STEP PREDICTION MODEL

We start this section by stating a well known data-based approach for linear system identification: The auto-regressive model with external inputs (ARX) [5], where

\[ y_{T_{ini}+1} = \bar{a}_1 u_1 + \cdots + \bar{a}_{T_{ini}} u_{T_{ini}} + \bar{b}_1 y_1 + \cdots + \bar{b}_{T_{ini}} y_{T_{ini}}. \tag{8} \]

Similarly as in Lemma [1] the equation can only hold if \( T_{ini} > l(A,B,C,D) \), according to Definition [2] as shown in [14]. To identify the parameters

\[ \hat{P} = [\bar{a}_1, \ldots, \bar{a}_{T_{ini}}, \bar{b}_1, \ldots, \bar{b}_{T_{ini}}] \tag{9} \]
based on the collected data in (2), one would typically solve the least-squares problem:

\[ \min_{\hat{P}} \| \hat{P} \begin{bmatrix} U_{T_{ini}} \\ Y_{T_{ini}} \end{bmatrix} - Y_+ \|_2. \tag{10} \]
in which the newly introduced matrix \( Y_+ \) simply represents the first \( p \) rows of \( Y_T \) corresponding to the temporal elements of the output sequences at \( T_{ini} + 1 \). Ruminiscant of the ARX model in (8), we propose to use the collected data (2) to directly identify a linear multi-step ahead predictor:

\[ y_N = a_1 u_1 + \cdots + a_{T_{ini}+N} u_{T_{ini}+N} + b_1 y_1 + \cdots + b_{T_{ini}+N} y_{T_{ini}+N}. \tag{11} \]

where the parameters are again summarized as

\[ P = [a_1, \ldots, a_{T_{ini}+N}, b_1, \ldots, b_{T_{ini}+N}]. \tag{12} \]

Our proposed structure shows strong similarities to the introduced multi-step prediction model in [6], with the main difference that (11) assumes a relation between the entire sequence of outputs and the entire sequence of inputs. This means that the predicted output at time \( T_{ini} + 1 \) also takes into consideration inputs from time \( T_{ini} + 2 \) to \( N \). This will not lead to a better system identification but this structure is necessary to establish an equivalence with the DeePC approach.
Assumption 2: Let $T \geq Lm + T_{\text{ini}}p$ and $U_L$ be persistently exciting of order $L$ according to Definition 1.

Lemma 2: With data obtained from a deterministic LTI system in the form of (1) and, if Assumption 2 holds, we can obtain $P^* \in \mathbb{R}^{Np \times Lm + T_{\text{ini}}p}$, which allows to compute an arbitrary future system trajectory $y_N$ as:

$$y_N = P^* \begin{bmatrix} u_{T_{\text{ini}}} \\ u_N \\ y_{T_{\text{ini}}} \end{bmatrix},$$

(13)

for a given vector of inputs $u_N$.

Proof: For the identification of $P^*$ we construct a least squares problem:

$$P^* = \arg \min_P \| P \begin{bmatrix} U_{T_{\text{ini}}} \\ U_N \\ Y_{T_{\text{ini}}} \end{bmatrix} - Y_N \|_2^2.$$

(14)

The solution of this least-squares problem can be expressed explicitly by utilizing the Moore-Penrose inverse (denoted with the superscript $^\dagger$) as

$$P^* = Y_N M^T,$$

(15)

if Assumption 2 holds, since $M \in \mathbb{R}^{Lm + T_{\text{ini}}p \times T}$. Using the definition of $y_N$ and $P$ in (11) and (12) we obtain expression (13). Note that in general $M$ does not have full rank, which means the solution to problem (14) is not unique. This holds even though our matrix $U_L$ is persistently exciting according to Definition 1 due to the potential rank deficiency of $Y_{T_{\text{ini}}}$. Lemma 2 allows us to state an optimal control problem similar to (6) as

$$\min_{y_N, y_\text{ini}} \sum_{k=1}^N (\|y_k\|_G^2 + \|u_k\|_R^2)$$

s.t. $u_k \in U, \forall k \in \{1, \ldots, N\}$,

$y_k \in Y, \forall k \in \{1, \ldots, N\}$.  

(16)

Note that in the considered case of an underlying deterministic LTI system in the form of (1), the obtained regressor $P^*$ is perfect, yielding zero residuals in (14).

A. Equivalence of DeePC and MPC based on the multi-step prediction model

In the following we present our main contribution, a theorem stating that our proposed optimal control problem (16) is an identical representation of the DeePC (6) method.

Theorem 1: Let Assumption 1 and 2 hold. Given collected input-output data of a deterministic LTI system in the form of (2), the optimal control policy obtained as the solution of the DeePC problem (6) is equivalent to the optimal control policy as obtained from the solution of (16), based on the multi-step prediction model in Lemma 2.

Proof: To establish the equivalence, we modify problem (6) and eliminate the variable $g$ from the formulation. Remember that $g$ has to satisfy (5) which we split into:

$$\begin{bmatrix} U_{T_{\text{ini}}} \\ U_N \\ Y_{T_{\text{ini}}} \end{bmatrix} g = \begin{bmatrix} u_{T_{\text{ini}}} \\ u_N \\ y_{T_{\text{ini}}} \end{bmatrix},$$

(17)

and

$$y_N = Y_N g.$$

(18)

To eliminate $g$ from the formulation we need to find an explicit solution of (17) which we can then substitute into (18). To solve for $g$ we construct the least-squares problem:

$$\min_g \| \begin{bmatrix} U_{T_{\text{ini}}} \\ U_N \\ Y_{T_{\text{ini}}} \end{bmatrix} g - \begin{bmatrix} u_{T_{\text{ini}}} \\ u_N \\ y_{T_{\text{ini}}} \end{bmatrix} \|_2^2.$$

(19)

A solution is obtained again with the pseudo-inverse, yielding:

$$g^* = M^T b.$$

(20)

It is important to note that the least-squares solution is exact in the deterministic case, meaning $g^*$ satisfies (17). However, since $M$ does not necessarily have full rank, $g^*$ is not unique. All possible solutions satisfying (17) can thus be written as:

$$g^* = M^T b + \hat{g},$$

(21)

with $\hat{g} \in \ker(M)$, where we denote with $\ker(M)$ the null-space of matrix $M$ such that $M \hat{g} = 0 \forall \hat{g} \in \ker(M)$.

The obtained $g^*$ can now be used in (18):

$$y_N = Y_N g^*,$$

(22)

$$\iff y_N = Y_N (M^T b + \hat{g}).$$

(23)

We then need to show that $\ker(M) \subseteq \ker(Y_N)$, such that

$$Y_N \hat{g} = 0 \quad \forall \hat{g} \in \ker(M).$$

(24)

The relationship $\ker(M) \subseteq \ker(Y_N)$ holds, because $P^*$ as obtained from (14), though not unique, satisfies exactly:

$$y_N = P^* M.$$  

(25)

For any $x \in \ker(M)$ we have:

$$Y_N x = P^* M x = 0,$$

(26)

which means $x \in \ker(Y_N)$ and consequently $\ker(M) \subseteq \ker(Y_N)$. Finally, we obtain from (23):

$$y_N = Y_N (M^T b)$$

(27)

$$= P^* b.$$  

(28)

We have thus eliminated $g$ from the formulation of (6), were the constraint now reads:

$$y_N = P^* \begin{bmatrix} u_{T_{\text{ini}}} \\ u_N \\ y_{T_{\text{ini}}} \end{bmatrix}.$$

(29)

This yields our proposed formulation (16) and therefore proves that (6) and (16) have the same solution.  

As a consequence of the equivalence between problem (6) and (16), we argue that the DeePC optimization problem
implicitly estimates the regressor $P^*$ at each control iteration. Since the underlying data-matrices $Q_k$ are collected prior to the control application and are unchanged afterwards, there appears to be no reason for this repeated estimation in the deterministic case. The DeePC optimization problem requires $T + (m + p)N$ optimization variables with $T \geq (T_{ini} + N)m + n$ (see Assumption 1) and $(m + p)(T_{ini} + N)$ equality constraints. Our proposed formulation has only $(m + p)N$ optimization variables and $pN$ equality constraints. The increased computational cost of DeePC can be also seen in the numerical example presented in Section V.

On the other hand, the advantage of DeePC is said to be its simplicity when compared to related model-based schemes [9], especially since the system identification step is known to be the most time consuming part of the process [11]. We argue that after having prepared the collected input-output data in the form of (2) which is required for both DeePC and our equivalent method, inferring the parameters $P^*$ with (15) poses a negligible challenge and can be done offline.

IV. NON-DETERMINISTIC CASE

We introduce the following linear system subject to zero-mean Gaussian noise $w_k \sim \mathcal{N}(0, \sigma_w^2)$:

$$x_{k+1} = Ax_k + Bu_k,$$  \hspace{1cm} (30a)

$$y_k = Cx_k + Du_k + w_k,$$  \hspace{1cm} (30b)

as for the deterministic case, data is collected, now resulting in the matrices:

$$U_L = \begin{bmatrix} U_{T_{ini}} \\ U_N \end{bmatrix}, \quad \tilde{Y}_L = \begin{bmatrix} \tilde{Y}_{T_{ini}} \\ \tilde{Y}_N \end{bmatrix}.$$

With

$$\tilde{M} = \begin{bmatrix} U_{T_{ini}} \\ U_N \end{bmatrix}.$$

In this scenario, the DeePC formulation must be adapted because constraint (5) can generally not be satisfied anymore. These modifications were already proposed in the original work on DeePC [9]. Following the authors in [10], we use here a slight modification using $\ell_2$-norms instead of $\ell_1$-norms to penalize the slack variables in the cost function,

$$\min_{g, u_N, \sigma} \sum_{k=1}^N \left( \|y_k\|_g^2 + \|u_k\|_g^2 \right) + \lambda_g \|\sigma\|_2^2$$

s.t.

$$\begin{bmatrix} U_{T_{ini}} \\ U_N \end{bmatrix} g = \begin{bmatrix} u_{T_{ini}} \\ u_N \end{bmatrix} + \begin{bmatrix} 0 \\ \sigma \end{bmatrix},$$

$$u_k \in U \ \forall k \in \{1, \ldots, N\},$$

$$y_k \in \mathcal{Y} \ \forall k \in \{1, \ldots, N\}.$$  \hspace{1cm} (31)

On the other hand, our method can be applied without further adaptations by first identifying $\tilde{P}^*$ similarly to (15) as:

$$\tilde{P}^* = \tilde{Y}_N \tilde{M}^T.$$  \hspace{1cm} (32)

and then solving:

$$\min_{\tilde{P}^*, \tilde{Y}_N, \tilde{M}} \sum_{k=1}^N \left( \|y_k\|_Q^2 + \|u_k\|_R^2 \right)$$

s.t. \hspace{1cm} (13)

$$u_k \in U, \ \forall k \in \{1, \ldots, N\},$$

$$y_k \in \mathcal{Y}, \ \forall k \in \{1, \ldots, N\}.$$  \hspace{1cm} (33)

The main difference in the non-deterministic setting is that we can expect $\tilde{M}$ to have full rank, such that the solution $\tilde{P}^*$ is now unique. Due to the underlying least-squares formulation (14), the obtained regressor has favorable statistical properties in this setting, such as being unbiased [15].

In the following we present a result regarding the obtained optimal trajectory of DeePC and the proposed method based on the multi-step prediction model. For this result we investigate slightly adapted formulations of both methods that facilitate the comparison. Most importantly, we drop the set constraints ($u_k \in U, y_k \in \mathcal{Y} \ \forall k \in \{1, \ldots, N\}$), which allows to find simple explicit solutions for both methods.

**Result 1:** In the unconstrained case: i) The optimal output trajectory obtained by the DeePC problem (31) and our method (33) is a linear function of the data ($y_{T_{ini}}, u_{T_{ini}}$) and ii) the additional regularization term $\lambda_{sg}$ in the DeePC formulation (31) is strictly necessary if

$$T > \min(Lm + T_{ini}p, Np).$$

as otherwise the solution requires the inverse of a singular matrix.

**Proof:** For both results, we further modify (31) by introducing an additional slack variable for the past inputs ($\sigma$) with respective penalization term in the cost function weighted by $\lambda_{sg}$. We argue that this is a reasonable addition as generally both inputs and measurements can be affected by noise. We then reformulate the DeePC problem by replacing

$$y_N = \tilde{Y}_{NG},$$

in the cost function, which yields:

$$f(g, y_N, \sigma) = \sum_{k=1}^N \left( \|y_k\|_g^2 + \|u_k\|_g^2 \right)$$

$$+ \lambda_g \|\sigma\|_2^2 + \lambda_{sg} \|\sigma\|_2^2 + \lambda_{sg} \|\sigma\|_2^2$$

$$= \|y_N\|_g^2 + \|\sigma\|_2^2.$$

We further consider (5) to obtain:

$$f(g, u_N, \sigma) = \|y_N\|_g^2 + \|u_N\|_g^2 + \lambda_g \|\sigma\|_2^2 + \lambda_{sg} \|\sigma\|_2^2 + \lambda_{sg} \|\sigma\|_2^2.$$  \hspace{1cm} (35)

$$= \|g\|_g^2 + \|\sigma\|_2^2.$$

Note that we newly introduced:

$$Q = \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix}, \quad R = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}.$$
We can thus rewrite the adapted DeePC problem as:

\[
\min_{\delta,\nu} \left\| g \right\|^2_G + \left\| \nu \right\|^2_V
\]

\[
s.t. \begin{bmatrix} U_{\text{ini}} \\ U_N \\ F_{\text{ini}} \end{bmatrix} g = \begin{bmatrix} u_{\text{ini}} \\ 0 \\ y_{\text{ini}} \end{bmatrix} \begin{bmatrix} \sigma_u \\ u_N \\ \sigma_y \end{bmatrix} =\]

where we combined the terms for \( u_N \) and \( \sigma_y, \sigma_u \) as \( v \) which is weighted with:

\[
V = \begin{bmatrix} \lambda_{\sigma_u} \\ \tilde{\lambda} \\ \lambda_{\sigma_y} \end{bmatrix}
\]

For the comparison, we also adapt our proposed method \( (33) \) by introducing similar slack variables for the past measurements and inputs which are also penalized in the objective function. Similar to the modifications for DeePC above, we also drop the set constraints \( u_k \in \mathbb{U}, y_k \in \mathbb{Y} \; \forall k \in \{1, \ldots, N\} \). We obtain the following formulation:

\[
\min_{y_N, \nu} \left\| y_N \right\|^2_Q + \left\| \nu \right\|^2_V
\]

\[
s.t. \quad y_N = \tilde{P} \begin{bmatrix} u_{\text{ini}} \\ 0 \\ y_{\text{ini}} \end{bmatrix} \begin{bmatrix} \sigma_u \\ u_N \\ \sigma_y \end{bmatrix} =\]

With these reformulations and adaptations, we see that both DeePC \( (37) \) and our proposed method \( (39) \) come in the form of a weighted regularized least-squares (WRLS) problem for which an explicit solution can be obtained as shown in \([16]\). The optimal solution of the adapted DeePC problem \( (37) \) yields:

\[
g^* = (\tilde{M}^T \tilde{V} M + G)^{-1} \tilde{M}^T V \begin{bmatrix} u_{\text{ini}} \\ 0 \\ y_{\text{ini}} \end{bmatrix}.
\]

which allows to compute the optimal output sequence as:

\[
y_N^* = Y_N g^* = Y_N (\tilde{M}^T \tilde{V} M + G)^{-1} \tilde{M}^T V \begin{bmatrix} u_{\text{ini}} \\ 0 \\ y_{\text{ini}} \end{bmatrix}.
\]

In comparison, the adapted version of our proposed problem \( (39) \) yields:

\[
y_N^* = \left( (\tilde{P}^*)^T V \tilde{P}^* + \tilde{Q} \right)^{-1} (\tilde{P}^*)^T V \begin{bmatrix} u_{\text{ini}} \\ 0 \\ y_{\text{ini}} \end{bmatrix}.
\]

From the comparison of \( (41) \) and \( (42) \) we draw the following conclusions: Both DeePC and our proposed method obtain the optimal output trajectory as a linear function of the data as:

\[
y_N = K \begin{bmatrix} u_{\text{ini}} \\ 0 \\ y_{\text{ini}} \end{bmatrix}.
\]

While the matrix \( K \) differs for both methods, it is apparent that it is static thus raising the question whether a repeated implicit system identification as with DeePC is necessary.
A. Deterministic system

We first consider the described system without measurement noise and compare the DeePC algorithm (6) with our proposed method (16). The results are obtained with data from $T = 150$ captured sequences. In the deterministic setting, we have shown in Theorem 1 that both methods yield identical solutions. In Figure 2 we can see that the identical behavior also appears in practice. We present here an exemplary closed-loop trajectory for both methods with random (but identical) excitation phase to obtain $y_{\text{ini}}$ and $u_{\text{ini}}$. To further quantify the similarity, we compare the cumulated cost:

$$c = \sum_{k=0}^{N_{\text{seq}}} (y_k^T Q y_k + u_k^T R u_k).$$  

(48)

For the results in Figure 2 the cumulative cost differs between both controller variants on the order of $10^{-11}$ and equals to 5.617, which will serve as a benchmark for the non-deterministic case.

B. Non-deterministic system

In this subsection we investigate the previously described LTI system in the form of (30), where we choose $\sigma_w = 10^{-2}$ in all experiments. For DeePC (31) we choose $\lambda_g = 10^4$. Our proposed method in the form of (33) requires no additional tuning parameters.

We want to discuss two main questions: The effect of the tuning parameter $\lambda_g$ and of the number of recorded sequences $T$ on the cost, according to (48), and computation time.

In Figure 3 we present predicted optimal trajectories obtained with (31) and (33) for the same initial input/output data $y_{\text{ini}}$ and $u_{\text{ini}}$ (noise disturbed). These trajectories (denoted as pred. in Figure 2) are compared with the true system response resulting from the sequence of optimal inputs. We vary the number of recorded sequences $T = \{100, 150\}$ and change the value of $\lambda_g = \{0, 1\}$. For a more concise representation we only showcase the output of the second disc angle. Notice that $T = 100$ is the minimum number of sequences for which our proposed method is expected to work properly, according to Assumption 2 and barely exceeds $T = 96$, the lower bound to satisfy Assumption 1. With $T = 100$ we are also not violating condition (34) from Result 1, meaning that we expect a reasonable solution from DeePC without regularization on $g$ ($\lambda_g = 0$). This can be seen in Figure 3a), which covers exactly this case. On the other hand, case d) shows an unsatisfactory performance of DeePC, which stems from the fact that with $\lambda_g = 0$ and $T > 100$ a singular matrix arises in the computation of $g^*$. We also want to point out the similarities between case a) and c) in Figure 3, where c) shows the obtained solution with our method and $T = 100$. In this comparison we find that the obtained solutions are almost exactly identical, albeit far from ideal, due to the low number of samples. In this particular setting with $T = 100$, we find that the regularized DeePC approach with $\lambda_g = 1$ in Figure 3b) performs comparably well. We argue that the regularization helps to implicitly determine smoother system dynamics in this ill-conditioned case, showing a potential advantage of DeePC over the proposed method. We also see in Figure 3 that with $T = 150$ both DeePC (with $\lambda_g = 1$) and our proposed method perform well.

A further comparison is presented in Table I, where we analyze closed-loop trajectories obtained with DeePC (31), with $\lambda_g = 1$, and our proposed method (33). All results in this table are averages over ten simulation experiments, with independently sampled data matrices and measurement noise.

We take two main conclusions from the presented data.
in Table I. First and foremost, we see that our proposed method outperforms DeePC both in terms of overall cost and computation time in all scenarios. While the advantage of our method with respect to the cost is minor, the difference in computation time is significant. Especially for the case $T = 150$, which seems to be a good compromise between performance and cost for DeePC and our method, we notice a significant increase in computation time. Note that we compare only the online CPU time. The computation of the pseudo inverse is excluded, as it is performed offline and only once. As a second conclusion, we notice that even with $T = 100$, DeePC is outperformed by our proposed method. This is counter intuitive in comparison to Figure 3 case b) and c), where open-loop trajectories seem better with DeePC. However, open-loop predictions are not the same as closed-loop control and we find that the proposed method shows less oscillatory behavior around the origin, thus leading to a slightly lower closed-loop cost.

VI. CONCLUSIONS

In this work we presented a new perspective on the data-enabled predictive control (DeePC) approach, by deriving an equivalent formulation based on sequential identification and control. Our proposed formulation utilizes the exact same data matrices and identifies a multi-step prediction model at negligible additional computational cost. Through this equivalent formulation, we reason that DeePC implicitly estimates the same multi-step ahead prediction model at each iteration, thus adding significant additional computational cost to its online application. Furthermore, we find that the implicitly estimated system model has a disadvantageous structure, relating future control inputs to present measurements.

For the non-deterministic case, the exact equivalence between our method and DeePC does not hold anymore. By adapting both DeePC and our method slightly, we present explicit solutions to both problems highlighting their identical structure. In particular, we find that both methods linearly map the past measurements and inputs (a representation of the current state) to the optimal output sequence. Most importantly, this map, albeit different for both controllers, is static thus again raising the question whether a repeated implicit system identification is necessary.

In simulation studies on an exemplary LTI system with additive Gaussian noise, we show that our proposed method outperforms DeePC with respect to the closed-loop control objective cost and computation time.

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