Discrete and Finite Element Models for the Analysis of Unreinforced and Partially Reinforced Masonry Arches

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Abstract. In this work the behavior of masonry arches, without reinforcement and with partial reinforcement, is investigated by means of three different numerical models. The first one is a Discrete Element model based on rigid blocks, and elastic-plastic interfaces; the second one is a standard heterogeneous Finite Element Model, which is adopted for a detailed micro-modelling of arch voussoirs, joints, and reinforcements. The third model is analytic-numerical, and it is adopted for validating the other numerical results. The aim of the work is the comparison and validation of the numerical Finite and Discrete Element models for the correct simulation of masonry arch behavior, together with the evaluation of the effectiveness of these models in simulating the behavior of the partially reinforced arch.

Introduction

Masonry arches are one of the most important and complex structural elements that can be built with natural or artificial blocks. The assessment of their behavior has received new interest from the community of architects and civil engineers, with particular attention to limit analysis approaches, thanks to the contributions by Heyman [1]. Considering the typical pattern of the stones or clay bricks that are put together for making the load-bearing part of arches and vaults, Discrete Element Models (DEMs) are frequently adopted for assessing arch structural behavior [2], since they allow to model each voussoir and the corresponding contacts. The most common DEM computer codes are able to account for block deformability by adopting a mesh refinement for each discrete element (FEM-DEM [3]); furthermore recently, an approach able to account for block deformability with only one parameter has been proposed [4]. Masonry arches are often studied by means of analytical and numerical limit analysis approaches, which adopt the hypothesis of rigid blocks or rigid elements for defining collapse mechanisms [5]. In recent works dealing with masonry arch bridges, combined Finite-Discrete Element Models (FEM-DEM) were adopted and compared with limit analysis results [6], whereas standard Finite Element Models (FEMs) were adopted by defining material properties by means of homogenization procedures [7]. For restoration purposes and in order to allow masonry arches to stand both increasing vertical loads and/or seismic actions, strengthening interventions are often performed. Fiber-Reinforced-Polymers (FRP) are frequently applied to arch extrados or intrados, in order to increase the strength of the arch [8-12].

This work deals with the assessment of the behavior of masonry arches, both unreinforced and partially reinforced, by means of two numerical models and an analytical model. An existing Discrete Element Model (DEM) based on rigid blocks and elastoplastic interfaces, originally introduced for modelling masonry panels having regular texture [13, 14], is here extended to the
case of masonry arches in plane state. A heterogeneous FEM with standard quadrilateral elements and zero-thickness interface elements is also considered for calibrating DEM elastic and inelastic parameters. Such approach dedicated to DEM calibration with respect to other existing and commercial DE and FE models, was performed in the past for in-plane loaded masonry panels [3, 16]. Furthermore, an analytic model of a circular arch without and with a partial reinforcement along its extrados is developed for a further validation of the proposed numerical models. Numerical tests take into account an existing case study [17], which was already adopted for validating and comparing other numerical approaches.

**Numerical and Analytical Models for Masonry Arches**

**Discrete Element Model.** The original discrete model introduced by Cecchi and Sab [13] and already extended to the field of material nonlinearity [14] and random or quasi-periodic texture [15] is further extended to the case of blocks and interfaces having a generic position and orientation, with the main purpose of modelling the plane section of a masonry arch or a masonry vault. Analysis is carried on in a linearized two-dimensional (2D) framework, with 2D plane stress hypothesis. Blocks are considered as rigid bodies, whereas the elastic and inelastic behavior of the system is lumped at interface or joint level. Dry or mortar joints are modeled as elastic-plastic interfaces, which follow a Mohr-Coulomb yield criterion. Furthermore, the reinforcements applied along arch intrados and/or extrados are modelled as elements connected with the existing blocks.

Considering two quadrilateral blocks connected by an interface (Fig. 1a) representing two generic voussoirs of a masonry arch, the displacements of the model are given by the rigid body motions of each block, namely the in-plane translations of each block center and the rotation of each block \( \omega_3 \) with respect to its center: \( \mathbf{q} = \{u_1, u_2, \omega_3\}^T \). The joint between the blocks is considered as a one-dimensional interface, and the relative displacements between the connected blocks are assumed as deformation measure. Due to the generic orientation of the interface, highlighted by its inclination \( \theta \) with respect to the horizontal axis (Fig. 1), the relative displacements at interface level (local) - relative normal and shear displacements and relative rotation - \( \mathbf{d}_{\text{loc}} = \{d_n, d_s, \delta\}^T \) are defined in terms of block (global) displacements by means of a rotation matrix and by a compatibility matrix [14] collecting the relative distances of each block center with respect to interface center in the global coordinates system (Fig. 1b).

Model deformability and nonlinearity are lumped at interface level by means of elastic-plastic laws. Actions at interface level (local) - a shear action, a normal action and a moment - \( \mathbf{f}_{\text{loc}} = \{f_n, f_s, m\}^T \) (Fig. 1b) are considered at interface midpoint. In the elastic field, they depend linearly on interface relative displacements, by means of an interface stiffness matrix \( \mathbf{f}_{\text{loc}} = \mathbf{K}_{\text{loc}} \mathbf{d}_{\text{loc}} \) collecting shear, normal and bending stiffness: \( \mathbf{K}_{\text{loc}} = \text{diag}\{k_n, k_s, k_r\} \). The material nonlinearity is guaranteed by limiting interface actions with a Mohr-Coulomb yield criterion and a tensile strength criterion.

Reinforcements are modelled as one-dimensional elements similarly to mortar joints, but they are characterized by a large normal stiffness depending on reinforcement elastic modulus, and by a
negligible shear and bending stiffness; furthermore, a criterion considering zero compressive strength and a maximum tensile strength can be defined. The interfaces between blocks and reinforcements can follow a shear-bond strength criterion.

**Heterogeneous Finite Element Model.** In order to model a masonry arch with a better detail with respect to the DEM, a heterogeneous FEM is adopted. Such a model is able to account both to block and joint elastic deformability and material nonlinearity; however, for simplicity, such nonlinearity is considered only at joint level. Differently with respect to DEM, this model allows to obtain more accurate information on structural behavior in terms of stresses and strains; on the other hand, it requires a large computational effort with respect to the DEM, due to the larger number of degrees of freedom involved in the analysis, which depend on the mesh refinement adopted. The heterogeneous FEM here considered was specially introduced for modelling masonry [19] and, among the different FEIs proposed by the element library, standard four-node quadrilateral elements are adopted for representing masonry blocks, whereas zero-thickness four-node interface elements are adopted for representing the joints between the blocks. Plane strain hypotheses are adopted, and blocks deformability is accounted by means of their elastic modulus $E_b$ and Poisson’s ratio $\nu_b$, whereas zero-thickness interfaces are characterized by a normal stiffness $k_n$ and a tangential stiffness $k_s$, together with joint tensile strength, cohesion and friction ratio.

Arch reinforcements can be modelled with one or two-dimensional elements and zero-thickness interfaces can be used for connecting the elements for the bricks with those of the reinforcements. However, a simpler approach for modelling reinforcements can be the application of standard or elastic restraints on the degrees of freedom involved by the contact with the reinforcement.

**Analytic Model.** The analytic model adopted here has been already introduced by authors in a contribution dedicated to the collapse mechanisms of masonry domes [20]. The model envisages a method for the limit analysis of the arch, based on the kinematic approach, characterized by an algorithm that defines the position of the hinges, along the arch intrados and extrados, generated by a kinematic mechanism that may be activated by the minimum collapse multiplier of the live load $F$. As well known, the activation of a collapse mechanism of the arch is characterized by the formation of four hinges. The position of two of them is considered fixed along arch extrados (Fig. 2a), in particular the last one $(X_4,Y_4)$ is supposed to be at the base of the arch on the opposite side with respect to the position of the live load, which is typical of masonry arches; whereas the second one $(X_2,Y_2)$ is supposed to be matching the application point of $F$. The position of the other two hinges $(X_1,Y_1)$ and $(X_3,Y_3)$ is assumed to vary along the intrados. Each hinge is defined by an angle $\alpha_i$, with $i = 1, \ldots, 4$, hence angle $\alpha_1$ defines a portion of arch $A1$, which is not involved in the mechanism, whereas the other angles allow to define portions $A2$, $A3$, and $A4$ involved in the mechanism (Fig. 2b). The dead loads are represented by the self-weight $P_i$ of each arch portion involved in the mechanism and, if present, by the self-weight of the fill over each arch portion. These loads are defined by point forces applied at each portion center (Fig. 2c).

![Fig. 2 - Generic collapse mechanism for a circular arch: (a) hinge position; (b) arch portions involved in the collapse mechanism; (c) graphical determination of the displacements.](image-url)
The supposed mechanism allows to determine both graphically and numerically the displacement of the application point of each force of the model, namely $v_i$ for the arch portions and $v_F$ for the live load (Fig. 2c). Then, the virtual work principle may be written, and the minimum multiplier of the live load $F$ can be determined by defining a set of values for each unknown angle, namely $0 < \alpha_1 < \alpha_2$ and $\alpha_2 < \alpha_3 < \alpha_4$:

$$\lambda_{\text{min}} = \min\left\{ \{F \cdot v_F(\alpha_i, \alpha_j)\} / \left[ \sum_{i=2}^{n} P_i \cdot v_i(\alpha_i, \alpha_j) \right] \right\}$$

(1)

**Numerical Tests**

A well-known case study originally introduced by Orduna [17] and already adopted for the validation of other numerical models [18] is considered. The masonry arch (Fig. 3) has a span $L = 5.0$ m, rise $R = 2.5$ m, thickness $a = 0.3$ m and width $s = 1.0$. It is made of 31 voussoirs and it is covered by a back-fill up to a height $H = 3.0$ m, which represents only a further dead load for the structure. Masonry volumetric weight $\rho$ is equal to 20 kN/m$^3$, whereas back-fill volumetric weight is equal to 15 kN/m$^3$. The other mechanical properties of the blocks and the joints between the blocks are listed in Table 1.

![Fig. 3 - Geometry of the unreinforced masonry arch [17] (a) and of the partially reinforced masonry arch (b).](image)

**Table 1 - Mechanical properties of the masonry arch.**

| Blocks | Elastic modulus $E_b$ [MPa] | Poisson’s ratio $v_b$ | Density $\rho$ [kN/m$^3$] | Joints | Normal stiffness $k_n$ [N/mm$^3$] | Shear stiffness $k_s$ [N/mm$^3$] | Friction ratio $\mu$ |
|--------|-----------------|------------------|-----------------|--------|-------------------------------|-------------------|----------|
|        | 10000           | 0.2              | 20              | Normal stiffness | 2400             | 1000              | 0.75     |

**Unreinforced masonry arch.** The masonry without any reinforcement is subject to its self-weight, to arch fill self-weight and to an increasing vertical force $F$ acting along arch extrados, at one quarter of the entire arch span (i.e. $d = 1.25$ m in Fig. 3) from arch left end.

Fig. 4a shows the load-displacement curves obtained with the three models adopted, together with the ultimate load obtained with the limit analysis solution performed in [17, 18], and equal to 18 kN. The ultimate load obtained with the DEM is equal to 18 kN, and that obtained with the FEM is equal to 17.8 kN. The ultimate load given by analytic model turns out to be equal to 17.77 kN, and it is obtained with angles $\alpha_1 = 24^\circ$, $\alpha_3 = 149^\circ$. All the models are in good agreement with themselves and with the reference solution. Fig. 5a shows the collapse mechanism obtained with the DEM, which is characterized by the development of four hinges. The mechanism obtained with DEM is in excellent agreement with that obtained in [17] and hinge positions turn out to be alternately along arch intrados and extrados. The same mechanism is also obtained with the analytical model. Fig. 5b shows the collapse mechanism obtained with the FEM, which turns out to be in good agreement with that obtained with DEM and analytically, even if the opening of the third hinge is less evident, due to the deformability of the blocks. It is worth noting that the hinge formation with the DEM (Fig. 5a) is obtained when the bending moment of the interface reaches the corresponding tensile strength limit, and even if the current representation does not show an exact
hinge formation along arch intrados or extrados, it shows the tensile cracking of the interface starting from the opposite side of arch cross-section. The overlap between the blocks adjacent to a hinge, at this stage, does not imply further contact actions or elastic forces between the elements in contact.

Reinforced masonry arch. The masonry arch is then partially strengthened by applying a reinforcement along the extrados, from the left support up to the 7th block, with a reinforcement horizontal projection length $r$ equal to 0.60 m (Fig. 3b). In the DEM, reinforcements are modelled by additional nodes along the arch extrados reinforced portion, connected by elements having a normal stiffness 10 times larger than mortar joint stiffness, and assuming a perfect bond between the arch and the reinforcement. In the FEM, at this stage, the reinforcement has been simply considered by restraining the corresponding nodes along arch extrados.

Fig. 4b shows the load-displacement curves for the reinforced arch obtained with the different numerical models. The effect of the reinforcements increases the ultimate load of the arch; the DEM reaches an ultimate load equal to 22 kN, whereas the ultimate load obtained with the FEM is equal to 20.75 kN. In this case in the analytic model, the reinforcement of the arch is considered by increasing the arch portion $A_1$ not involved in the collapse mechanism: $40^\circ < \alpha_1 < \alpha_2$. With a first hinge position $\alpha_1 = 40^\circ$ and a third hinge position quite close to that obtained with the unreinforced case, $\alpha_3 = 149^\circ$, the analytic limit load multiplier turns out to be equal to 23.19 kN. Fig. 5c shows the collapse mechanism obtained with DEM. As expected, the new mechanism is characterized by
four hinges alternated along arch intrados and extrados, with the first hinge on the left that appears at the end of the reinforcement and of the other hinges are placed similarly to the unreinforced case. Fig. 5d shows the collapse mechanism obtained with the FEM, which is quite close to that obtained with DEM, with an evident hinge opening at the reinforcement end and under the applied load.

Conclusion

In this work, the behavior of masonry arches, without and with a partial reinforcement, has been studied by means of three numerical models: an existing DEM, originally introduced for modelling regular masonry and extended to the case of masonry arches; a heterogeneous FEM for obtaining more accurate results and validate DEM results; and an analytic model based on the kinematic theorem of limit analysis, adopted as a further benchmark for the previous results. The three models turned out to be accurate and in excellent agreement with themselves and with existing solutions, even if the heterogeneous FEM is more deformable and the corresponding ultimate load is slightly smaller than that obtained with the other models with the partially reinforced arch case. The analyses performed with DEM allowed to obtain collapse mechanisms where hinge formation was evident, also in the load-displacement curves characterized by sudden slope variations until the final horizontal branch highlighting the occurred collapse. Mechanisms also turned out to be in quite good agreement with those obtained analytically and represented by the angles of the unknown hinges along arch intrados or extrados. However, further developments of the proposed DEM will require to account for the overlaps between the blocks during the formation of a plastic hinge, in order to obtain more accurate deformed shapes. Other developments of this work will take into account existing case studies of unreinforced and reinforced arches tested in laboratory [9, 10], in particular, the simplified approach here adopted for representing the behavior of arch strengthening will be better investigated, and more accurate properties for restraining elements and arch-reinforcement interfaces will be adopted.

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References

[1] J. Heyman, The masonry arch, John Wiley and Sons, 1982.
[2] N. Bicanic, C. Stirling, C.J. Pearce, Discontinuous modelling of masonry bridges, Comp. Mech. 31(1–2) (2003), 60–68.
[3] D. Baraldi, E. Reccia, A. Cecchi, In plane loaded masonry walls: DEM and FEM/DEM models. A critical review, Meccanica 53(7) (2018), 1613-1628.
[4] F. Cannizzaro, B. Pantò, S. Caddemi, I Caliò, A Discrete Macro-Element Method (DMEM) for the nonlinear structural assessment of masonry arches, Eng. Struct. 168 (2018), 243-256.
[5] M. Gilbert, C. Melbourne, Rigid-block analysis of masonry structures, Struct. Eng. 72(21) (1994), 356-361.
[6] E. Reccia, A. Cecchi, G. Milani, A finite element-discrete element approach for the analysis of the Venice trans-lagoon railway bridge, Civil-Comp Proceedings, 110, (2016).
[7] E. Reccia, A. Cecchi, G. Milani, A. Tralli, Full 3D homogenization approach to investigate the behavior of masonry arch bridges: The Venice trans-lagoon railway bridge, Constr. Build. Mat. 66 (2014) 567-586.
[8] M.R. Valluzzi, M. Valdemarca, C. Modena, Behaviour of brick masonry vaults strengthened by FRP laminates, J. Compos. Construct. 5(3) (2001), 163-169.

[9] P. Foraboschi, Strengthening of masonry arches with fiber-reinforced polymer strips, J. Compos. Constr. 8(3) (2004), 191-202.

[10] D. Oliveira, I. Basilio, P.B. Lourenço, Experimental Behavior of FRP Strengthened Masonry Arches, J. Compos. Constr. 14(3) (2010) 312-322.

[11] B. Pantò, F. Cannizzaro, S. Caddemi, I. Caliò, C. Chácara, P.B. Lourenço, Nonlinear modelling of curved masonry structures after seismic retrofit through FRP reinforcing, Build., 7(3) (2017) 79, 1-17.

[12] P. Zampieri, N. Simoncelo, C.D. Tetougueni, C. Pellegrino, A review of methods for strengthening of masonry arches with composite materials, Eng. Struct., 171 (2018) 154-169.

[13] A. Cecchi, K. Sab, A comparison between a 3D discrete model and two homogenised plate models for periodic elastic brickwork, Int. J. Solids Struct. 41(9-10) (2004) 2259-2276.

[14] D. Baraldi, A. Cecchi, Discrete approaches for the nonlinear analysis of in plane loaded masonry walls: Molecular dynamic and static algorithm solutions, Eur. J. Mech. A/Solids, 57 (2016) 165-177.

[15] D. Baraldi, A. Cecchi, Discrete model for the collapse behaviour of unreinforced random masonry walls, Key Eng. Mat. 747 (2017) 3-10.

[16] D. Baraldi, C.B. de Carvalho Bello, A. Cecchi, E. Meroi, E. Reccia, Non-linear behaviour of masonry walls: FE, DE & FE/DE models, Compos. Mech. Comp. Appl. Int. J. (2019) in press.

[17] A. Orduna, Seismic Assessment of Ancient Masonry Structures by Rigid Blocks Limit Analysis, Ph.D. Thesis, University of Minho, 2003.

[18] G. Milani, P.B. Lourenço, 3D non-linear behavior of masonry arch bridges, Comp. Struct. 110-111 (2012) 133-150.

[19] TNO DIANA, DIANA. Displacement method ANAlyser, release 9.4, User’s Manual.

[20] M. Pavlovic, E. Reccia, A. Cecchi, A Procedure to Investigate the Collapse Behavior of Masonry Domes: Some Meaningful Cases, Int. J. Arch. Herit. 10(1) (2016) 67-83.