Simulation of the Influence of Centrifugal Force and Fluid-structure Interaction on the Critical Speed of Agitator and the Applicability of the Theoretical Formula

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Abstract. On the ANSYS Workbench, the two-way fluid-structure interaction (FSI) process of the mixing system is simulated by combining the structure modules and flow field modules. According to the natural frequency of the shaft in the dry and wet mode which is obtained by modal analysis method, the critical speed of the shaft is solved. The effect of centrifugal force and FSI on the critical speed of the shaft is studied. The results show that under normal working conditions, the influence of centrifugal force and fluid pressure based on two-way FSI on the critical speed of the shaft is insignificant, while the added fluid mass leads to a significant decrease in the critical speed of the mixing shaft. These laws provide reliable evidence for the applicability of the theoretical formula and the optimal design of the shaft.

1. Introduction
Agitators are widely used in chemical, petrochemical and other fields, especially in many chemical equipments as a reactor or mixing device. The stirring device includes the shaft and accessories that rotate with the shaft (as shown in Fig.1). Due to the influence of fluid excitation force, input speed fluctuation and agitator eccentricity, there are three kinds of vibration: axial vibration, torsional vibration and bending vibration. Among them, bending vibration is highly destructive and its natural frequency is low, which is close to the fluid-induced frequency and easy to cause resonance.

The modal analysis of shaft has been paid much attention by scholars at home and abroad. Xiaojun Li [1] conducted modal analysis of ship axle-propeller combination on Workbench. Xiaodong Hu [2] analyzed the critical speed results of various methods when the action of flow field being considered. Liangliang Xu [3] studied the dynamic characteristics and influence of FSI of agitators used in chemical industry in ADINA. Zhenglin Zhou [4] used ANSYS Workbench platform to optimize and improve the structure of stirred reactor. There are few literatures on the modal analysis of stirring shaft in the process of two-way FSI. In this paper, for the agitator of open turbine with four inclined blades in the HG standard, the flow field characteristics are analyzed, the influence of centrifugal force of blade, fluid pressure of stirring medium and added fluid mass on critical speed of shaft is studied respectively.

Figure 1. The structure of agitator.
Comparing simplified theoretical formula and finite element analysis, the applicable range of the formula is defined.

2. Finite element model of agitator

In order to compare the difference between the critical speed obtained by the simplified theoretical formula in the HG standard and the results of finite element simulation, this paper refers to the standard HG3796.1-12 and HGT20569-2013 [5-6] to establish the agitator model. An agitator of open turbine with four inclined blades is selected. The agitator tank is a flat cylinder type. Specific size parameters are shown in Tab.1. Material of the shaft is stainless steel. Medium in the stirred tank is 46 # machine oil.

| Fluid domain diameter D (mm) | Blade diameter D_J (mm) | Blade width B (mm) | Distance from bottom h_1 (mm) | Blade tilt θ | Blade thickness δ (mm) | Hub height h (mm) | Hub outer diameter d (mm) | Shaft outer diameter d_0 (mm) |
|-----------------------------|------------------------|--------------------|-------------------------------|-------------|------------------------|-------------------|--------------------------|-----------------------------|
| 400                         | 200                    | 40                 | 100                           | 45°         | 3                      | 40                | 45                       | 28                          |

Grids in solid regions are shown in Fig.2. Hexahedral mesh is adopted, and the blade regions are regularized and encrypted. Grids in fluid regions are shown in Fig.3. The method of meshing by blocks reduces the computation and better simulates the real flow field behavior. In Fluent, multiple reference frame model (MRF) is adopted, the fluid regions are divided into the dynamic region near the blades and the static region around the blades, the dynamic region is encrypted. Unstructured tetrahedral elements are used in all fluid regions.

Boundary setting: the boundary of the shaft are established in cylindrical coordinate system. Radial and axial fixed constraints and circumferential motion constraint (according to the specified velocity of rotation) are applied to the fixed end of the shaft. Cylindrical Support is set at the bearing. Wall boundaries are set around and at the bottom of the agitator, symmetry boundary is set at the fluid level. The sliding surface of the dynamic and static regions is set as the interface type to ensure the transmission of data between the dynamic and static regions in the calculation process.

Formula of Reynolds number

\[ \text{Re} = \frac{\rho n_d^2}{\mu} \]  

Where \( d \) is the diameter of the blade, \( \rho \) is medium density, \( \mu \) is medium viscosity, \( n_d \) is the speed of the shaft. At the working speed (300rpm), the mixing Reynolds number is calculated to be 45900, so the flow field in the agitator is judged to be in a turbulent state. In this paper, a SIMPLE solution methods and RNG \( k-\varepsilon \) model are applied in Fluent.

Transient module and Fluent module are connected through System Coupling to realize two-way FSI on Workbench. Set the parameters of FSI, create the data transfers, and iterate in the System Coupling. The solid regions and the fluid regions are solved simultaneously, and the time step is the same.
3. Flow field analysis of agitator based on two-way FSI

Figure 4. Velocity contour at Z=-20mm.  
Figure 5. Velocity contour at Y=0.

The velocity contours at cross-section Z=-20mm and vertical section Y=0 are shown in Fig. 4 and Fig. 5. The maximum velocity of the flow field (3.138 m/s) is located at the end of the blade, which is consistent with the theoretical calculation results, \( R_0 = 3.14 \text{m/s} \). The flow field presents the characteristics of double circulation flow. The fluid starts from the blade and moves upward and downward respectively, meets the fluid level and the bottom and then goes back to the blade. As can be seen from Fig. 5, under the blade, the flow velocity is higher and the stirring effect is better. However, the "dead zone" with low flow appears directly above the blade and below the shaft, which is not conducive to the mixture of the fluid.

4. Modal analysis and critical speed calculation

Mode is the natural vibration characteristic of structure, each mode has a specific natural frequency, damping ratio and mode shape. Through modal analysis of structure, mechanical faults caused by vibration can be diagnosed, vibration characteristics of structure can be understood, and theoretical basis can be provided for the design and optimization of structure. The static modal analysis does not consider the impact of stress on the stiffness of structure, only consider the boundary conditions, while the pre-stressed modal analysis needs to consider the load.

4.1. Static modal analysis

Firstly, the dry mode (excluding fluid) and wet mode (including fluid) under static state are analyzed to obtain the natural frequency and vibration mode of the shaft, and the results of modal analysis are compared with that of the simplified theoretical formula.

According to the design of shaft in standard HG3796.1-12 [6], the critical speed of an equal-diameter rigid cantilever shaft with m disks is as follows

\[
n_k = 114.7d_L^2 \frac{E(1-N_d^4)}{L_L^2 (L_a + a) W_s} 
\]

The above formulas only consider the added mass of the medium and do not consider the pre-stress caused by centrifugal force and fluid pressure, as well as the FSI effect on the natural frequency. The first critical speed of the shaft is calculated, \( n_{cr} = 6158 \text{rpm} \). The first natural frequency is 102.6HZ.

Block Lanczos algorithm in ANSYS is adopted in this paper for modal analysis. Lanczos algorithm has the characteristics of small computation, high precision, high speed and is more suitable for solving large symmetric eigenvalues. Only the fixed constraints and bearing constraints are applied to the shaft. The modal analysis of the shaft under the static condition is carried out. The first 10 natural frequencies are shown in Tab.2.
Table 2. The first 10 natural frequencies of shaft in static stage.

| Mode | 1          | 2          | 3          | 4          | 5          | 6          | 7          | 8          | 9          | 10          |
|------|------------|------------|------------|------------|------------|------------|------------|------------|------------|-------------|
|      | Frequency (Hz) | 147.42     | 147.43     | 263.77     | 274.04     | 280.12     | 280.51     | 802.28     | 1072.9     | 1073.1     | 1122.7     |

It can be seen from the results that the natural frequencies of the two successive orders are approximate, because the whole shaft is symmetrical in structure and has similar vibration mode. The first and second modes are the oscillation around the Y-axis in XOZ plane and the oscillation around the X-axis in YOZ plane respectively. The first natural frequency is 147.42Hz, and the critical speed obtained through calculation is 8845rpm, which is 44.8% higher than the calculation result of the formula in standard. It is preliminarily confirmed that the critical speed formula is conservative and has a large safety margin under the static condition. The influence of added fluid mass is investigated below.

Table 3. The first 10 natural frequencies of shaft under the action of added fluid mass.

| Mode | 1          | 2          | 3          | 4          | 5          | 6          | 7          | 8          | 9          | 10          |
|------|------------|------------|------------|------------|------------|------------|------------|------------|------------|-------------|
|      | Frequency (Hz) | 134.36     | 134.43     | 196.45     | 205.01     | 220.1      | 276.23     | 765.43     | 1010.      | 1011.7     | 1045.3     |

In the modal analysis module, the boundary conditions of fluid are applied by inserting APDL command. The fluid elements are changed to Fluid220 and Fluid221, and the density of fluid and the speed of sound in fluid are defined. The first 10 natural frequencies of are shown in Tab.3. The natural frequencies of the shaft at all orders have a significant decrease. Compared with the dry mode, the first natural frequencies have decreased 8.9%. The damping of the fluid plays a role in reducing the natural frequency. Compared with theoretical calculation, both of them take the influence of added mass of fluid into account, but the result 134.36Hz obtained by finite element simulation is 30.9% higher than 102.6Hz. It shows that the result of the theoretical formula still has a high safety factor.

4.2. Pre-stressed modal analysis

In order to investigate the influence of centrifugal force and fluid pressure which are not considered by the theoretical formula, the pre-stressed modal analysis is carried out.

Table 4. The first 10 natural frequencies of shaft under centrifugal force (Hz).

| Mode | Speed (rpm) | 1          | 2          | 3          | 4          | 5          | 6          | 7          | 8          | 9          | 10          |
|------|-------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|-------------|
|      | 300         | 147.32     | 147.35     | 263.71     | 273.96     | 280.16     | 280.34     | 802.23     | 1072.7     | 1072.8     | 1122.1     |
|      | 1000        | 146.5      | 146.53     | 264.16     | 274.48     | 280.68     | 280.87     | 802.37     | 1072.7     | 1072.8     | 1122.1     |
|      | 3000        | 139.08     | 139.11     | 268.12     | 278.98     | 285.22     | 285.41     | 803.42     | 1072.7     | 1072.9     | 1123.7     |
|      | 6000        | 110.15     | 110.19     | 280.88     | 293.62     | 300.12     | 300.31     | 807.05     | 1072.6     | 1072.8     | 1128.6     |

Before the modal analysis, the static pre-stressed analysis of the shaft at different rotating speeds is carried out, and different centrifugal force is obtained by giving different rotating speeds to the shaft. The first 10 natural frequencies under centrifugal force are shown in Tab.4. At the working speed of 300rpm, the centrifugal force has little influence on the natural frequency. As the rotating speed increasing, the natural frequency shows a significant downward trend. When the rotating speed reaches 3000rpm, the first natural frequency is 139.08Hz, 5.6% lower than that in the normal working condition. When the speed reaches 6000rpm, the first-order natural frequency is 113.6Hz, which is 25.2% lower than that in the normal working condition.

Table 5. The first 10 natural frequencies of shaft under fluid pressure (Hz).

| Mode | Speed (rpm) | 1          | 2          | 3          | 4          | 5          | 6          | 7          | 8          | 9          | 10          |
|------|-------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|-------------|
|      | 300         | 147.35     | 147.36     | 263.88     | 274.16     | 280.37     | 280.49     | 802.28     | 1073.      | 1073.1     | 1122.6     |
|      | 1000        | 146.52     | 146.58     | 263.97     | 274.26     | 280.32     | 280.81     | 802.28     | 1071.9     | 1072.5     | 1119.8     |
|      | 3000        | 139.77     | 139.8      | 268.12     | 278.88     | 285.24     | 285.26     | 803.6      | 1072.4     | 1072.6     | 1121.5     |
|      | 6000        | 113.6      | 113.64     | 284.49     | 297.29     | 303.96     | 304.08     | 808.79     | 1076.1     | 1076.3     | 1133.4     |
Through the FSI interface, the fluid pressure at the blade in the fluid domain is applied to the blade in the corresponding solid domain. As shown in Tab.5, under the speed of 300rpm, the natural frequency change is also small after considering the fluid pressure. As the speed increases, the natural frequency decreases significantly. At 3000rpm and 6000rpm, they are 5.1% and 22.9% lower than that under normal conditions. At 6000rpm, the first frequency is 3.1% higher than the result under centrifugal force. The fluid pressure makes the structure appear stress stiffening and improves the stiffness of the structure in all directions, but this effect is not large enough compared with centrifugal force, so the overall natural frequency still shows a downward trend.

4.3. The critical speed under the comprehensive influence of various factors
Due to the limitation of the software, it is not possible to realize the finite element simulation on the Workbench that takes fluid pressure, centrifugal force, fluid additional mass and other factors into consideration at the same time. In this paper, based on the above results, assuming that the influences of various factors can be added linearly, it is estimated that under the working condition of the agitator, the first natural frequency of the shaft is 134.2Hz, which is 8.94% lower than that of the static dry mode. At 6000rpm, the first natural frequency is 103.59Hz, which is 0.97% higher than the calculation result of the theoretical formula. When the speed is close to the critical speed, the deviation from the theoretical formula is very small.

5. Conclusion
An agitator of open turbine with four inclined blades is established in SolidWorks and Workbench. Combined Fluent and Workbench, finite element simulation of the two-way FSI process of the agitator system is completed, the dynamic characteristics of the agitator system are analyzed. By comparing the critical speed of mixing shaft obtained by modal analysis with the numerical calculation results of simplified theoretical formula, it is found that: under normal working conditions or low rotating speed, the influence of centrifugal force and fluid pressure based on two-way FSI on the natural frequency of the rigid shaft is insignificant. In the case of high speed, the natural frequency of the rigid shaft decreases with the increase of speed. The influence of added fluid mass on the critical speed is the most obvious among all factors. When these all above factors are taken into account, the natural frequency of the shaft tends to decrease. Under the condition of approaching the critical speed, the deviation of the modal analysis is small compared with the theoretical formula.

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