Hybrid inflation in high-scale supersymmetry

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Abstract

In hybrid inflation, the inflaton generically has a tadpole due to gravitational effects in supergravity, which significantly changes the inflaton dynamics in high-scale supersymmetry. We point out that the tadpole can be cancelled if there is a supersymmetry breaking singlet with gravitational couplings, and in particular, the cancellation is automatic in no-scale supergravity. We consider the LARGE volume scenario as a concrete example and discuss the compatibility between the hybrid inflation and the moduli stabilization. We also point out that the dark radiation generated by the overall volume modulus decay naturally relaxes a tension between the observed spectral index and the prediction of the hybrid inflation.
1 Introduction

The recent observations of the cosmic microwave background radiation (CMB) \cite{1} strongly suggest the inflationary era \cite{2} in the early Universe; the CMB temperature fluctuations extending beyond the horizon scale at the last scattering surface can be interpreted as the evidence for the accelerated expansion in the past.

The study of the CMB temperature anisotropies as well as the large-scale structure provide us with a clue about the mechanism that laid down the primordial density fluctuations. Whereas some models are already excluded or strongly disfavored, it is not enough at present to pin down the mechanism. From a minimalist point of view, the quantum fluctuations of the inflaton are the most plausible candidate for the origin of density fluctuations.

Among many inflation models proposed so far, a hybrid inflation scenario \cite{3} is of particular interest, especially in the supersymmetric framework. The inflationary trajectory is along the \( F \)-flat direction, and the inflation ends when waterfall fields become tachyonic and develop a large vacuum expectation value (VEV). The hybrid inflation has been extensively studied from various points of view \cite{4,5,6,7,8}.

It was pointed out in Refs. \cite{6,7} that the inflaton has a tadpole in proportional to the gravitino mass, because of the gravitational interaction in supergravity. In particular, the inflaton dynamics is significantly affected by the tadpole for the gravitino mass \( m_{3/2} \sim O(10^2) \) TeV and the successful inflation is possible only in a very tight corner of the parameter space \cite{7}. On the other hand, the recent discovery of the standard model (SM)-like Higgs boson with mass about 125 - 126 GeV \cite{9,10} may imply high-scale supersymmetry (SUSY) \cite{11,12}. If so, there will be a tension between the hybrid inflation and high-scale SUSY.

In this letter we point out that the tadpole can be cancelled if there is a SUSY breaking singlet \( Z \) with gravitational couplings, which is usually not required in high-scale SUSY because the gaugino masses can be generated by the anomaly mediation \cite{13,14,15}. In the presence of such a singlet, the gravitinos are generically produced by the inflaton decay \cite{16,17,18,19,20,21,22}, and moreover, coherent oscillations of the lowest component of \( Z \) are produced after inflation \cite{23} and mainly decay into gravitinos. Thus, the Universe
likely becomes gravitino-rich, which has various interesting implications \cite{24}. We will also see that the cancellation of the tadpole is automatic in the no-scale supergravity \cite{25,26}. However, the moduli stabilization is one of the central issue in the no-scale supergravity. Lastly we consider a LARGE volume scenario (LVS) \cite{27}, as a concrete and realistic model where all the moduli are stabilized successfully, in order to see whether the hybrid inflation can be successfully implemented. We will also point out that the dark radiation generated by the overall volume modulus decay \cite{28,29,30} naturally relaxes a tension between the observed spectral index and the prediction of the hybrid inflation.

2 Tadpole problem in hybrid inflation

In this section we briefly review the tadpole problem in the hybrid inflation \cite{6,7}. To see the essential features, we focus on the minimal supergravity model described by

\[
\begin{align*}
K &= |\Phi|^2 + |\Psi|^2 + |\bar{\Psi}|^2 + |Z|^2, \\
W &= \kappa \Phi (M^2 - \Psi \bar{\Psi}) + W_0(Z),
\end{align*}
\]

for the inflaton $\Phi$, and the waterfall fields $\Psi + \bar{\Psi}$ vector-like under a U(1) gauge symmetry, with the $R$ charges assigned by $R(\Phi) = 2$ and $R(\Psi) = R(\bar{\Psi}) = 0$. Here $Z$ is a SUSY breaking field, which we assume to have an $F$-term VEV to cancel the vacuum energy density, without specifying the detailed mechanism of stabilization. We also assume that the inflaton is charged under an additional global U(1)$_{\phi}$ symmetry which is explicitly broken only by the $M^2$ term. This approximate symmetry explains the hierarchy between the inflation scale and the Planck scale ($M_P = 1$). We take $M$ and $\langle W_0 \rangle$ to be real and positive, which is always possible through appropriate U(1)$_{\phi}$ and U(1)$_R$ transformations. Now the gravitino mass reads

\[
m_{3/2} = \langle e^{K/2}W \rangle \simeq \langle W_0 \rangle,
\]

at the vacuum lying near the $F$-flat direction $\Psi \bar{\Psi} = M^2$, and thus is determined by $W_0$, which breaks U(1)$_R$ down to a $Z_2$ subgroup. Here one should be careful that $\langle e^{K/2} \rangle$ can be hierarchically small in the LVS as we shall see later.
For a sufficiently large field value of $|\Phi| (\gg M)$, the waterfall fields get large supersymmetric masses $\kappa|\Phi|$ and are stabilized at the origin, $\Psi = \bar{\Psi} = 0$. Then integrating out the waterfall fields, one obtains

$$\Delta K_{\text{eff}} = -\frac{\kappa^2}{16\pi^2} \ln \left( \frac{|\Phi|^2}{\Lambda^2} \right) |\Phi|^2,$$

which gives an important contribution to the scalar potential because the $F$-term of $\Phi$ includes $\kappa M^2$ induced by the linear superpotential term. The inflaton scalar potential is written

$$V = \kappa^2 M^4 + \frac{\kappa^4}{16\pi^2} M^4 \ln \left( \frac{|\Phi|^2}{\Lambda^2} \right) - 2m_{3/2}\kappa M^2(\Phi + \Phi^*)$$

$$+ m_\Phi^2 |\Phi|^2 + \frac{1}{2} \kappa^2 M^4 |\Phi|^4 + \cdots,$$

where $m_\Phi^2 = \mathcal{O}(m_{3/2}^2)$, and the ellipsis denotes higher order terms of $\Phi$ suppressed by $M_P$. The hybrid inflation is implemented by the first two terms in the potential. The inflaton $\Phi$ rolls down the (approximately) flat potential until the waterfall fields become tachyonic at $|\Phi| \approx M$ and end the inflation. Indeed, in the absence of the tadpole term, the successful inflation is achievable for a wide range of parameters, $10^{-7} \lesssim \kappa \lesssim 10^{-1}$ and $3 \times 10^{14} \text{ GeV} \lesssim M \lesssim 10^{16} \text{ GeV}$, where we have imposed the WMAP normalization on the density perturbation [1] and the cosmic string constraint [31].

However the hybrid inflation can be spoiled by the tadpole term of the inflaton [7]. It is generated proportional to the gravitino mass, more precisely to the $F$-term of the supergravity multiplet, independently of the details of supersymmetry breaking. This is because the $M^2$ term breaks the conformal symmetry explicitly. Let us see the difficulties caused by the tadpole. First, it generates an unwanted minimum at a large field value of $\Phi$, and thus once the inflaton is trapped in the wrong vacuum, the inflation will never end. This problem can be avoided by fine-tuning the initial phase of the inflaton. However, even in this case, a large tadpole makes the inflaton potential so steep that the duration of the inflation becomes shorter, requiring the inflaton to initially sit at larger field values. Then, in order to satisfy the WMAP normalization, the inflation scale must be higher.

\footnote{Note that the density perturbation $\zeta$ scales as $|V^{3/2}/V'|$, and the tadpole increases $|V'|$. Here the prime denotes the differentiation with respect to the inflaton.}
which is constrained by the observational upper bound on the tension of the cosmic string formed after the inflation ends. These imply that the tadpole is dangerous for the hybrid inflation, especially in the high-scale SUSY scenario where the gravitino mass is around or above $10^2$ TeV.

To implement hybrid inflation successfully, one may add the inflaton coupling to the SUSY breaking field,

$$\Delta K = aZ + bZ|\Phi|^2 + \text{h.c.},$$

with constants $a$ and $b$ for $\langle |Z| \rangle \ll 1$. Note that the $aZ$ term contributes to the inflaton coupling since the supergravity action depends on the Kähler potential through $-3e^{-K/3}$, and also modifies the $F$-term of supergravity multiplet. Thus the potential includes

$$V_{\text{tadpole}} = -\left(2 - 3(a - b)\right)m_{3/2}\kappa M^2(\Phi + \Phi^*)^2,$$

where we have used $F^Z = -\sqrt{3}m_{3/2}M_P$, and assumed the approximate $U(1)_B$ symmetry. The above tells us that a cancellation between tadpoles for the inflaton occurs if one takes $a$ and $b$ to be

$$a - b = \frac{2}{\sqrt{3}}$$

which is nothing but fine-tuning, but there may be anthropic selection of the parameters for the successful inflation. Another way to avoid the tadpole problem, which would be more plausible, is to consider hybrid inflation within no-scale supergravity. Then, since the $F$-term of supergravity multiplet vanishes, no tadpoles are induced even though the $M^2$ term breaks the conformal symmetry explicitly. We will examine this possibility in more detail in the next section.

Lastly we mention the cosmological aspects of such SUSY breaking singlet with Planck-suppressed couplings. First, if $Z$ is stabilized during inflation at a place deviated from the low-energy minimum, it will start to oscillate after inflation [23]. The coherent oscillations of $Z$ may or may not dominate the Universe, depending on the initial amplitude. If kinematically allowed, $Z$ generically decays into gravitinos at a sizable rate. Second, we expect that there are generically following interactions,

$$K \supset |\Psi|^2ZZ + |\bar{\Psi}|^2ZZ + \text{h.c.},$$

(8)
which cause the gravitino production from the decay of the waterfall fields [16] [17] [18]. Note that the energy of the Universe after inflation is dominated by the inflaton and the waterfall fields. In fact, the inflaton and the waterfall fields are mixed with each other due to the superpotential term $W_0$ [16]. Thus, the Universe likely becomes gravitino-rich in the presence of such SUSY breaking singlet [24]. The gravitino-rich Universe has various interesting implications; the Wino-like LSP can account for dark matter if its mass happens to be of $\mathcal{O}(100)$ GeV, even though the other superparticles are much heavier. The singlet also makes the gravitino-induced baryogenesis [34] possible, if the R-parity is largely violated.

It is interesting that the SUSY breaking singlet $Z$ with gravitational couplings is required to cancel the tadpole of the inflaton, which enables the hybrid inflation in high-scale SUSY, while such singlet is not necessary from the phenomenological point of view because of the anomaly mediation contribution to the gaugino mass.

3 No-scale supergravity and LARGE Volume Scenario

In this section we consider concrete examples where the tadpole of the inflaton is naturally canceled or suppressed. First we consider the no-scale supergravity, and then move on to the LVS which has an approximate no-scale structure while all the modulus fields are stabilized.

Let us consider a no-scale model with a Kähler potential of the form [25] [26]

$$K = -3 \ln \left( T + T^\dagger - \frac{1}{3} \sum_i |\phi_i|^2 \right),$$  \hspace{1cm} (9)

and the superpotential

$$W_{\text{inf}} = \kappa \Phi (M^2 - \Psi \bar{\Psi}) + \omega_0,$$ \hspace{1cm} (10)

for a SUSY breaking field $T$, where $\phi_i$ denotes the inflaton $\Phi$ and the waterfall fields $\Psi$ and $\bar{\Psi}$, and we have included the constant superpotential. As the following argument does not depend on the the waterfall fields that are heavy during inflation, we will neglect them. Note that the superpotential does not contain $T$, and thus the tree-level potential
for $T$ remains flat at the minimum, which is one of the notable features in a no-scale supergravity model.

For the above no-scale model, one finds
\[
V_{\text{tadpole}} = -2 \left( 1 - \frac{1}{3} K^{IJ} K_I K_J \right) m_{3/2} \kappa M^2 (\Phi + \Phi^*) = 0,
\]
(11)
where $m_{3/2} \simeq e^{K/2} \omega_0$. Thus the tadpole problem is absent in the no-scale supergravity. In fact, it is well known that the scalar potential in a no-scale supergravity model resembles that in the global SUSY. Note however that the modulus $T$ has a run-away potential during inflation, and therefore it must be stabilized by modifying the Kähler potential \[36\] or adding the non-perturbative effects \[37\] or the $D$-term \[38\]. It is then important to see whether the modulus stabilization revives the tadpole problem or not.

In order to see if the hybrid inflation can be implemented together with the successful moduli stabilization of $T$, let us study a LVS model based on the type IIB orientifold compactifications in flux vacua \[27\] as a more realistic example. We consider the model with relevant three Kähler moduli on a Calabi-Yau (CY) space with a singularity \[39\]
\[
K = -2 \ln \left( \mathcal{V} + \frac{\xi}{2} \right) + \frac{(T_v + T_v^\dagger + V_U(1))^2}{\mathcal{V}} + \frac{k_{\inf}}{\mathcal{V}^{2/3}} \left( 1 - \frac{\delta}{\mathcal{V}} \right) K_{\inf},
\]
\[
W = W_{\inf} + Ae^{-aT_s}.
\]
(12)
Here the three complexified Kähler moduli $T_i = \tau_i + i \sigma_i$ ($i = b, s, v$) describe the overall 4-cycle volume, the local 4-cycle volume, the singularity on the CY space respectively for the real parts $\tau_i$, while the imaginary parts $\sigma_i$ are given by the integrands of the RR 4-form potentials on the corresponding cycles. The CY volume $\mathcal{V}$ and the $\alpha'$-correction $\xi$ \[40\] are respectively given by
\[
\mathcal{V} = (T_b + T_b)^{3/2} - (T_s + T_s^\dagger)^{3/2}, \quad \xi = -\frac{\chi(\text{CY}) \zeta(3)}{2(2\pi)^3 g_s^{3/2}},
\]
(13)
where we will assume that $g_s = \mathcal{O}(0.1)$ is the string coupling and $\chi(\text{CY}) = 2(h^{1,1}(\text{CY}) - h^{2,1}(\text{CY})) = -\mathcal{O}(100) < 0$ is the Euler number on the CY; one then finds that $\xi = \mathcal{O}(1)$. For the inflaton sector, $k_{\inf} = 1 + \sum_{n \geq 1} b_n (T_v + T_v^\dagger)^n$ is $T_v$-dependent part with the Taylor expansion. $K_{\inf}$ and $W_{\inf}$ are given by Eq.(11), involving the constant superpotential.
\[\omega_0 = \mathcal{O}(1)\] which comes from the 3-form fluxes. We again assume the presence of \(U(1)_\Phi\) to simplify the following discussion. We have included the expected \(\alpha'\)-correction parameterized by \(\delta\) in the inflaton Kähler potential, because the matter wave function on the local model is not known well. The non-perturbative term \(A e^{-a T_s}\) is considered as instanton effect or gaugino condensation which arises from the E3-brane or D7-branes wrapping on the local cycle supported by \(\tau_s\). Then one finds that \(a = 2\pi/N\) and \(N\) is a natural number while \(A\) is expected to be order unity.

In this setup, both the visible and inflaton sectors are supposed to be realized on the singularity supported by \(T_v\), and therefore we will have the anomalous \(U(1)\) vector multiplet \(V_{U(1)}\) due to the chiral fermions. Then \(T_v\) has a supersymmetric Stuckelberg coupling associated with \(V_{U(1)}\) for the cancellation of the anomaly via the Green-Schwarz mechanism that \(T_v\) shifts under the \(U(1)\) transformation. Hence \(T_v\) becomes massive, being absorbed into the gauge multiplet: \(m_{T_v} = m_{V_{U(1)}} \sim 1/V^{1/2} = M_{\text{string}}\). Then one also finds that \(\langle T_v \rangle = 0\) through the D-term potential \(D_{U(1)} \propto \partial T_v K \propto \tau_v = 0\).

After integrating out the massive gauge multiplet which consists of \(T_v\) and \(V_{U(1)}\), the simplified action is obtained:

\[
\begin{align*}
K &= -2 \ln \left( V + \frac{\xi}{2} \right) + \frac{\tilde{K}_{\text{inf}}}{V^{2/3}} \left( 1 - \frac{\delta}{V} \right), \\
W &= W_{\text{inf}} + A e^{-a T_s}. 
\end{align*}
\]

Here note that \(K_{\text{inf}}\) can be modified to \(\tilde{K}_{\text{inf}}\); while the inflaton \(\Phi\) is assumed to be neutral under the anomalous \(U(1)\) and hence the \(\Phi\) part is not modified, \(\Psi + \bar{\Psi}\) can have quartic terms in the Kähler potential if they are charged. For the systematic study of the tadpole

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2. Although the tadpole \(M^2\) of the inflaton in the superpotential can be also written by moduli we will treat it as the constant in this paper, because the tadpole problem will not become better by such moduli without a fine-tuning.

3. Such an \(U(1)_\Phi\) should be a global one, otherwise one will obtain the higher order terms of \(\Phi\) from the D-term potential. Therefore the \(U(1)_\Phi\) might be just accidental or a symmetry in a local geometry on a CY space.

4. In this paper, we will not consider the quantum effects associated with anomalous \(U(1)\) in addition to moduli-redefinitions for simplicity. However, it will be sufficient for the study of the tadpole problem. Even if included, the situation is not improved because the no-scale structure is broken not only by the \(\alpha'\)-correction but also by the quantum correction.
of $\Phi$ during inflation, let us rewrite the above effective action, neglecting $\Psi + \bar{\Psi}$:

$$
K = -3 \ln \left( T_b + T_b^\dagger - \frac{c_1}{3} |\Phi|^2 \right) + \frac{f \left( T_s + T_s^\dagger - \frac{c_3}{3} |\Phi|^2 \right)}{(T_b + T_b^\dagger - \frac{c_1}{3} |\Phi|^2)^n},
$$

$$
W = \omega_0 + A e^{-a T_s} + \kappa M^2 \Phi,
$$

(15)

for a positive rational number $n$. Note that the presence of $c_2$ implies an $\alpha'$-correction in the Kähler metric of the inflaton. Under the choice of $(n, c_1, c_2, c_3) = (3/2, 1, 2/3, 0)$ and $f(x) = 2x^{3/2} - \xi$, the above LVS model is reproduced and one then finds $\delta = \xi/3$; this is the sequestered case in which the inflaton Kähler potential is approximately given by $e^{K_{\text{moduli}}/3}$.

Using the effective action Eq.(15), the moduli are stabilized and the tadpole of $\Phi$ is then estimated as

$$
V|_{\text{tadpole}} = \left[ G_{\phi} e^G (G_i G_i - 3) + e^G (G_i \nabla_{\phi} G_i + G_\phi) \right]_{\Phi = 0} (\Phi + \Phi^*)
= r e^{K/2} m_{3/2} \kappa M^2 (\Phi + \Phi^*),
$$

(16)

with

$$
m_{3/2} = e^{K/2} W \sim \frac{\omega_0}{V},
$$

$$
r \simeq -\frac{n(n-3)(2n-3)x}{ax} c_1 - \frac{n^2(n-3)x}{ax} c_2 + t(n-1)(n+3) x c_3 f',
$$

(17)

where we have neglected small terms suppressed by $1/t^n$ in $r$, and defined $t \equiv T_b + T_b^\dagger$, $x \equiv T_s + T_s^\dagger$, and $f' = \partial_x f$. See the appendix A for the derivation. For $n = 3/2$, the first term in the numerator in Eq.(17) vanishes at the leading order of CY volume expansion, while the second and third terms do not. The third term will vanish when the inflaton is located on the singularity; $c_3 = 0$. However, it is expected that we will always have non-zero $c_2$ due to the $\alpha'$-correction, especially in the inflaton Kähler potential. In the original LVS model, one then finds

$$
r \simeq \frac{c_2 x f'}{c_1 ax t^{3/2}} \sim \frac{1}{V \ln V},
$$

$$
V|_{\text{tadpole}} = r e^{K/2} m_{3/2} \kappa M^2 (\Phi + \Phi^*) \sim \frac{1}{V^2 \ln V} m_{3/2} \kappa M^2 (\Phi + \Phi^*),
$$

(19)
while the vacuum energy during inflation is given by the no-scale one, $V_{\text{inf}} = \kappa^2 M^4 / \mathcal{V}^{4/3}$ at the tree-level. Here we have used $xf' \sim x^{3/2} \sim \xi = \mathcal{O}(1)$ and $ax \sim \ln \mathcal{V}$ obtained from the stationary conditions given in the appendix.

However, this is not the end of the story. The LVS model has a negative cosmological constant $\langle V_{\text{AdS}} \rangle \sim -m_{3/2}^2 / (\mathcal{V} \ln \mathcal{V})$ at the SUSY breaking minimum. Therefore the uplifting is required for obtaining the de Sitter/Minkowski vacuum [43] as in the KKLT case [45, 46, 47]. As can be seen in Eq. (16), there is an additional contribution from the uplifting sector to the tadpole of $\mathcal{O}(\mathcal{V}^{-3})$, because the sector contributes to the energy density of $|\langle V_{\text{AdS}} \rangle|$ in the scalar potential. In particular, $G^i \nabla_\Phi G_i$ in Eq. (16) depends on the coupling between the inflaton and the uplifting sector, and therefore is quite model-dependent, and so we are left with the tadpole of order $\mathcal{V}^{-3}$ unless fine-tuning between $\alpha'$-correction and the uplifting sector is assumed. (Furthermore, if $M$ originates from a heavy modulus, another contribution to the tadpole could be present.)

To summarize, for the canonically normalized inflaton field $\hat{\Phi} = \Phi / \mathcal{V}^{1/3}$, we generically have

$$V_{\text{inflaton}} = \kappa^2 \hat{M}^4 + \frac{\zeta}{\mathcal{V}} m_{3/2}^2 \hat{M}^2 (\hat{\Phi} + \hat{\Phi}^*) + \cdots, \quad (20)$$

at the tree-level. Here $\hat{M} = M / \mathcal{V}^{1/3}$, and $\zeta$ varies from $1 / \ln \mathcal{V}$ to order unity, depending on the details of the uplifting sector. Note that the Yukawa coupling between the inflaton and waterfall fields does not depend on $\mathcal{V}$ in the canonical basis. As expected from the fact that the approximate no-scale structure is broken at $\mathcal{V}^{-1}$, the tadpole suppressed by $\mathcal{V}$ appears. Since $\mathcal{V}$ is large, the tadpole problem is greatly relaxed in LVS, compared to the general case with the same gravitino mass.

Let us now discuss if the hybrid inflation works successfully in this framework with the suppressed tadpole. In principle the tadpole can be vanishingly small for sufficiently large $\mathcal{V}$. However, $\mathcal{V}$ cannot be too large, because the modulus $\text{Re}(T_b)$ would be extremely light, causing several difficulties. Note that the modulus obtains mass

$$m_{T_b} \sim \frac{m_{3/2}}{\mathcal{V}^{1/2} \sqrt{\ln \mathcal{V}}}. \quad (21)$$

In fact, the most important one is that from the modulus destabilization. Through the uplifting of the AdS to a Minkowski vacuum, the potential barrier with height $|\langle V_{\text{AdS}} \rangle| \sim$
$m_T^2$ appears. In order not to cause the decompactification of the modulus, $H_{\text{inf}} \sim \kappa \tilde{M}^2 \lesssim m_T$ should be satisfied \[48, 49\], where $H_{\text{inf}}$ is the Hubble parameter during inflation. Unless the tadpole is extremely suppressed, the inflation scale is bounded below as $H_{\text{inf}} \gtrsim \mathcal{O}(10^7)$ GeV. Then $V$ must be smaller than $\sim 10^7$, and the coefficient of the tadpole is given by

$$\frac{\zeta}{\sqrt{V}} \frac{m_{3/2}}{m_T} \gtrsim \zeta \times 10\text{ TeV}.$$  \tag{22}$$

In the case that there is an uplifting sector sequestered from the inflaton/visible sector as in the KKLT case, one will find $\zeta \sim 1/\ln V \gtrsim \mathcal{O}(10^{-2})$ without a modification of Eq.\(^{(19)}\). Then only a slight tuning at ten percent level is necessary. On the other hand, for $\zeta \sim 1$ in non-sequestered models, the tadpole is still so large that the inflation scale is bounded below as $H_{\text{inf}} \gtrsim 10^{10}$ GeV. Therefore $\zeta$ must be suppressed by several orders of magnitude with respect to the naive estimation. \(^{6}\)

In the discussion above we have assumed $\omega_0 \sim \mathcal{O}(1)$. Note that considering $\omega_0 \ll 1$ does not improve the tadpole problem, in spite of the fact that this apparently suppresses the tadpole. This can be understood as follows. The suppression of the tadpole in \(^{(20)}\) is a remnant of the no-scale structure, which is recovered in the large volume limit, $V \to \infty$. On the other hand, smaller $\omega_0$ reduces the mass of the overall volume modulus, and therefore the $V$ must be smaller in order not to destabilize the modulus for a fixed inflation scale. Thus, the suppression of the tadpole becomes weaker: $\frac{\zeta m_{3/2}}{m_T} \sim \zeta / V^{1/2}$.

The overall volume modulus decays before big bang nucleosynthesis if $V \lesssim \mathcal{O}(10^8)$, and the produced LSPs can account for the dark matter if $V \sim 10^7$ and the LSP is Wino-like neutralino \[30\]. Note that the cosmological moduli problem can be solved or relaxed by assuming additional entropy production such as thermal inflation \(^7\) or the R-parity violation, and in this case, the upper bound on the volume will be relaxed.

So far we have focused on the tadpole of the inflaton. For successful inflation, not only the tadpole but also the mass of the inflaton should be small enough. In general, there

\(^{5}\)This is the case for $\zeta m_{3/2}/V \gtrsim 1$ GeV \[4\].

\(^{6}\)This conclusion may be changed if the inflaton dynamics is significantly modified by considering higher order terms in the Kähler potential. We leave this issue for future work.

\(^{7}\)One can easily implement the thermal inflation \[50\] or second inflation \[51\] after the hybrid inflation in a unified manner.
is so called $\eta$-problem in the $F$-term inflation in supergravity, and some mild tuning of parameters is necessary to guarantee the light inflaton mass during inflation. In the LVS model, there might be an effective quartic coupling of the inflaton $|\Phi|^4$ with a positive coefficient in the Kähler potential, which leads to a negative mass of order the Hubble parameter during inflation (see also Ref. [8]). This can be cured by considering a coupling with the other moduli with an appropriate coefficient because these moduli have an $F$-term whose size is larger than that of the inflaton.

4 Discussion and Conclusions

So far we have assumed that the inflaton $\Phi$ is charged under the global $U(1)_\Phi$ symmetry, which is broken by the $M^2$-term. Here let us briefly discuss what if there is no such global $U(1)_\Phi$ symmetry. To be concrete, we consider the general case given in Sec. 2, and assume that $\Phi$ is also singlet under $U(1)_R$ symmetry. Then, the inflation scale is naturally related to the R-symmetry breaking. We expect naively $\kappa \sim m_{3/2}$ and $M \sim 1$, but such large $M$ would result in too large tension of cosmic strings. We may extend the gauge symmetry of the waterfall field to $SU(2) \times U(1)$ to avoid the cosmic string formation, which would allow larger values of $M$. In any case, the inflaton has a non-zero $F$-term of order $m_{3/2}M_P$, as the hybrid inflation and the Polonyi model are unified in some sense. Interestingly, the inflation scale is tied to the SUSY breaking scale, $H_{\inf} \sim m_{3/2}$, and the high-scale SUSY suggested by the SM-like Higgs boson may be simply due to the WMAP normalization of the density perturbations. Although significant fine-tuning would be required to realize the sufficiently small tadpole and the inflaton mass, our claim that there must be a SUSY breaking singlet holds even in the absence of the global $U(1)_\Phi$ symmetry.

The hybrid inflation is a simple and therefore attractive inflation model. In the minimal supergravity, the spectral index $n_s$ is predicted to be $0.98 - 1$, while it is possible to reduce $n_s$ slightly by adding a quartic coupling of the inflaton in the Kähler potential [52]. In fact, it is known that the spectral index close to unity is still allowed if there is additional relativistic degrees of freedom coined “dark radiation”, the existence of which is suggested by the current observations [1, 53, 54]. Interestingly, the real component of the overall volume modulus generically decays into its imaginary component, the axion [28].
In the presence of the Giudice-Masiero term, the modulus decay can indeed produce a right amount of dark radiation \cite{29, 30}. Thus, in light of the tadpole problem and the tension of the predicted spectral index with observation, the LVS is an interesting framework for hybrid inflation.

The inflation scale is constrained by the WMAP normalization of the density perturbation. If the density perturbation is generated by some other mechanism such as the curvaton or modulated reheating, the inflation scale can be lower, which relaxes the tension between the hybrid inflation and high-scale SUSY.

Alternatively, it is also possible to build a low-scale inflation model such as the two-field new inflation \cite{55, 56} or alchemical inflation \cite{51}. The inflation scale of the latter is necessarily lower than the gravitino mass, since it utilizes the SUSY flat direction as the inflaton, which is lifted only by the soft SUSY breaking mass, and therefore the moduli stabilization is not spoiled.

In this letter we have argued that the tadpole of the inflaton in the hybrid inflation, which causes a tension with high-scale SUSY, can be cancelled if there is a SUSY breaking singlet with appropriate couplings with the inflaton. It is interesting that the presence of such SUSY breaking singlet, which is not required in high-scale SUSY from phenomenological point of view, is favored by the inflation dynamics. We have also pointed out that the cancellation of the tadpole is automatic in a no-scale supergravity. As an realistic set-up with no-scale structure, we have considered the LVS and shown that the tadpole is indeed suppressed by the large volume, which enables us to implement the hybrid inflation in LVS without severe fine-tuning.

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A Tadpole in the LVS

In this appendix, we will show the derivation of Eq. (17). For this purpose, we need to consider the moduli stabilization at first. In the LVS model of Eq. (15), the scalar potential of moduli sector is given by

\[ V_{\text{moduli}} = \left| \frac{W_X}{W} \right|^2 - \frac{(n-1)f'}{t^n} - n(n-1)\frac{f''}{t^{2n}} \times \left( 1 + \mathcal{O}\left( \frac{1}{t^n} \right) \right). \]  

(23)

Here we defined \( t \equiv T_b + T_b^\dagger \), \( x \equiv T_s + T_s^\dagger \), \( W_X \equiv \partial T_s W \) and \( f' = \partial_x f \) and so on. So long as \( \kappa M^2 \Phi \ll 1 \), one can neglect the presence of the inflaton for moduli stabilization. The stationary condition is given by

\[ \frac{W_X}{W} = \frac{1}{t^n} \left[ (n-1)f' - n\frac{xf''}{ax} + \mathcal{O}\left( \frac{1}{(ax)^2} \right) \right], \]  

(24)

\[ f = \frac{2n}{3 + n} \frac{xf'}{ax} + \mathcal{O}\left( \frac{1}{(ax)^2} \right). \]  

(25)

Then, in the LVS model with \( n = 3/2 \), one obtains \( t^n \sim \mathcal{V} \sim e^{\alpha t_s} \gg 1 \) and \( t_s^{3/2} \sim \xi \). The vacuum energy density without any uplifting and the canonically normalized moduli masses are given by

\[ \langle V_{\text{AdS}} \rangle \sim -\frac{1}{ax t^{3+n}}, \quad m_{T_b} \sim \left( \frac{1}{ax t^{3+n}} \right)^{1/2}, \quad m_{T_s} \sim ax \frac{t_s^{3/2}}{t^{3/2}}. \]  

(26)

Next, the tadpole of \( \Phi \) is estimated as

\[ V_{\text{tadpole}} = \left[ G_\Phi e^G(G_i G^i - 3) + e^G(G_i \nabla \Phi G_i + G_\Phi) \right] |_{\Phi = 0} \Phi \]  

\[ = r^* e^{K/2} m_{3/2}^* \kappa M^2 \Phi, \]  

(27)

(28)

where

\[ m_{3/2} = e^{K/2} W \simeq \frac{\omega_0}{V}, \]  

(29)

\[ r = -2 + \left( K^u K_t^2 + 2K^t x K_t K_x + K^{xx} K_x^2 \right) \]  

\[ - K^{t \Phi} \left( K^u K_t K_{t \Phi} + K^t x K_t K_{x \Phi} + K^{t \Phi} K_{t \Phi} + K^{xx} K_x K_{x \Phi} \right) \]  

\[ + \frac{W_X}{W} \left( K^t x K_t + K^{xx} K_x - K^{\Phi \Phi} (K^t x K_{t \Phi} + K^{xx} K_{x \Phi}) \right). \]  

(30)
After substituting the stationary conditions of Eq. (24) and (25) into the above expressions of \( r \), one can obtain Eq. (17). Alternatively, once one notes that the tadpole in the scalar potential is understood as the relevant SUSY breaking term of \( W = \kappa M^2 \Phi \), more simple and general expression will be obtained as

\[
\frac{r^* m_{3/2}^*}{2} = -2F^\varphi - F^i \partial_i \left[ \ln \left( \frac{\kappa M^2}{e^{-K/3} K_{\Phi\Phi}} \right) \right]
\]

(31)

when the tadpole term of \( \Phi \) is absent in the Kähler potential because of \( U(1)_\Phi \). Here \( \varphi \) is the conformal compensator superfield and the SUSY breaking \( F \)-terms are given by

\[
F^\varphi = m_{3/2}^* + \frac{1}{3} (\partial_i K) F^i, \quad F^i = -e^{G/2} G^{ij} G_{ij}.
\]

(32)

Through this expression of \( r \), one can expect how the tadpole will be affected by the uplifting sector and other moduli \( T' \) generating the linear superpotential of \( \Phi \) such that \( \kappa M^2 = e^{-T'} \).

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