DIRECT SIMULATIONS OF PARTICLE ACCELERATION IN A FLUCTUATING ELECTROMAGNETIC FIELD ACROSS A SHOCK

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Received 2008 October 22; accepted 2008 November 27; published 2008 December 29

ABSTRACT

We simulate the acceleration processes of collisionless particles in a shock structure with magnetohydrodynamical (MHD) fluctuations. The electromagnetic field is represented as a sum of MHD shock solutions ($B_0$, $E_0$) and torsional Alfvén mode spectra ($\delta B$, $\delta E$). We represent fluctuation modes in logarithmic wavenumber space. Since the electromagnetic fields are represented analytically, our simulations can easily cover as large as eight orders of magnitude in resonant frequency, and do not suffer from spatial limitations of box size or grid spacing. We deterministically calculate the particle trajectories under the Lorentz force for a time interval of up to 10 years, with a time step of $\sim 0.5$ s. This is sufficient to resolve Larmor frequencies without a stochastic treatment. Simulations show that the efficiency of the first-order Fermi acceleration can be parameterized by the fluctuation amplitude $\eta \equiv \langle \delta B^2 \rangle B_0^{-1}$. Convergence of the numerical results is shown by increasing the number of wave modes in Fourier space while fixing $\eta$. The efficiency of the first-order Fermi acceleration has a maximum at $\eta \sim 10^1$. The acceleration rate depends on the angle between the shock normal and $B_0$, and is highest when the angle is zero. Our method will provide a convenient tool for comparing collisionless turbulence theories with, for example, observations of bipolar structure of supernova remnants and shell-like synchrotron-radiating structure.

Key words: acceleration of particles – methods: numerical – MHD – turbulence

1. INTRODUCTION

Cosmic rays have a spectrum of $dN/dE \sim 10^5 (E/\text{GeV})^{-2.6} \text{cm}^{-2} \text{sr}^{-1} \text{s}^{-1} \text{GeV}^{-1}$ up to the so-called knee energy of $10^{15}$ eV. Cosmic-ray propagation theories suggest $dN/dE \propto E^{-2}$ energy spectra at the cosmic-ray acceleration sites (e.g., Strong et al. 2007).

The current description of cosmic-ray acceleration up to knee energy ($10^{15}$ eV) is the well-known first-order Fermi acceleration (Axford et al. 1977; Bell 1978). In the first-order Fermi acceleration model, magnetic turbulence is an important agent for particle acceleration. Turbulence makes the particle momenta isotropic, thus some of the particles cross the shock front many times. The expectation value of the kinetic energy after $N_j$ shock crossings is $E(N_j) = E_0(1 + h)^{N_j}$. On the other hand, the probability for a particle to survive $N_j$ shock crossings can be roughly estimated as $P(N_j) \propto (1 - p)^{N_j}$. This gives us the power-law spectrum of $dP/dE = E^{-\alpha - \beta}$.

Ellison et al. (1996), Lucek & Bell (2000), and Bell & Lucek (2001) have performed simulations to describe the self-consistent generation of turbulence, with approximations such as gyro-center approximation, random walk approximation, or lowering the dimension. On the other hand, the recent development of the particle-in-cell simulation has made it possible to describe the particle acceleration in an electron–positron plasma self-consistently (e.g., Spitkovsky 2008).

In this Letter, we propose an alternative approach to the simulation of cosmic-ray acceleration. We have calculated the motion of particles deterministically, solving the particles’ cyclotron motion from Larmor radii of thermal particles ($\sim 10^6$ cm) to that of knee energy particles ($\sim 10^{17}$ cm). According to the theories, we assume a turbulence spectrum in log $k$ space. This allows us to cover a large dynamic range of space and energy, which enables us to make a direct comparison of the accelerated cosmic-ray spectra with the observations.

2. NUMERICAL SCHEME

2.1. Representation of Turbulence

Upstream and Downstream Regions. In our method, the electromagnetic field and the velocity field of a continuous region are given by

$$ B(t, r) = B_0 + \sum_j B_{1,j} \exp(i(k_j \cdot r - \omega_j t + \phi_j)), $$

$$ u(t, r) = u_0 + \sum_j u_{1,j} \exp(i(k_j \cdot r - \omega_j t + \phi_j)). $$

where the amplitude and the wavenumber of the $j$th mode are

$$ B_{1,j} = (n_1 + i n_2)B_1 \left( \frac{k_j}{k_{\text{max}}} \right)^{P_1} $$

$$ u_{1,j} = \frac{u_A}{B_0} B_{1,j} $$

$$ k_j = k_{\text{min}} \left( \frac{k_{\text{max}}}{k_{\text{min}}} \right)^{j/(N_m-1)} $$

and the initial phase of the $j$th mode is $\phi_j$.

Here, $P_1$ is the spectral index that reflects the nature of the turbulence and $N_m$ is the total number of the modes ($j \in \{1, \ldots, N_m\}$), $B_{1,j}$ is the amplitude for each mode, $k_j$ is its wavenumber, and $n_1, n_2$ are two mutually perpendicular unit vectors that are perpendicular to $k_j$. We choose $k_j$ to be either parallel or antiparallel to $B_0$. Equation (6) means that $k_i$ are logarithmically discrete.

We use $\eta = \langle \Sigma_j B_{1,j}^2 \rangle B_0^{-1}$ as the measure of the strength of the fluctuation, independent of $N_m$. Because increasing $N_m$...
while keeping \( \eta \) constant (1) keeps the magnetic energy in fluctuation mode, and (2) keeps the expectation value of the fluctuation field \( \langle \Sigma, B_{1,j} \rangle \) the same, if \( \phi_j \) are independent. We will confirm these properties in Section 3.

The argument to derive \( P_1 = -1/3 \) in \( \log k \) space is summarized below: variables in \( \log k \) space are marked by a tilde. The power-law energy spectrum is \( E(k) \propto k^{-\frac{1}{3}} \) in the Kolmogorov turbulence case. This energy spectrum is in the linear bin. In the log energy bin the spectral power is \( \tilde{E}(k) \propto k^{-\frac{1}{3}} \), and since \( \tilde{E} = 1/(8\pi)\tilde{B}^2 \), \( \tilde{B}(k) \propto \tilde{E}(k)^{-\frac{1}{2}} \sim k^{-\frac{1}{2}} \). Thus, we get \( P_1 = -1/3 \) for our discretization of the turbulent magnetic field.

We assumed strong shock junction conditions with low plasma \( \beta \) limit at the shock front: \( u_{dn} = J^{-1}u_{up} \), \( B_{1dn} = B_{1up} \), and \( B_{1dn} = B_{1LP} \), where \( J \) is the shock compression ratio, \( B_{1L} \) and \( B_{1LP} \) are components of the \( \mathbf{B} \) normal and tangential to the shock, respectively.

2.2. Initial Condition and Equation of Motion

For each set of initial conditions we introduce the electromagnetic fields described in Section 2.1. We choose a set of initial turbulence phase \( \phi_j \), sign of \( k_j \) and \( \omega_j \) from a uniform distribution. Then we put \( 10^5 \) protons in a Boltzmann distribution of temperature \( T \) at the upstream side of the shock.

We use the values in Table 1, based on Bamba et al.’s (2003) observation of SN 1006.

We make each turbulence mode propagate at Alfvéén velocity of the uniform field \( v_A = B_0/\sqrt{4\pi\rho} \) as in Equations (1)–(3), and update the particle with Lorenz force, with the fourth-order Runge–Kutta method. We choose time discretization \( dt \) for each particle at every time step, so that \( dt < 0.1(1 + \eta)eB_0\rho^{-1}c^{-1} \) and \( dt < 0.03e|B_0 + \sum B_j|\rho^{-1}c^{-1} \) always hold. A typical time step is 0.5 s whereas the Larmor period of the thermal particle for \( B_0 \) is \( \sim 10^2 \) s:

\[
\frac{d\mathbf{r}}{dt} = \frac{\mathbf{p}}{\gamma m} \tag{8}
\]
\[
\frac{d\mathbf{p}}{dt} = e\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right). \tag{9}
\]

3. RESULTS

We have examined the results of our method as follows. First, we have traced the particles’ energy \( E \) as the function of shock crossing number \( N_f \) (Figure 1). The inclination of the curves match the inclination of the theoretical prediction, \( E(N_f) = [1 + (2/3)(v_{up} - v_{inh})/c]N_f \). Second, we have traced the spatial location where the particles gained their kinetic energy. We have found that 94% of the final kinetic energy has been earned within one final Larmor radius away from the shock. This is consistent with the first-order Fermi acceleration picture. Third, we have studied the validity of our Fourier representation in \( \log k \) space. We have kept the physical parameters and increased the number of modes per decade \( \Delta \log k \equiv (N_{mode}/\log_{10})(k_{max}/k_{min}) \); we see that the spectra converge to the theoretical power-law spectrum (Figure 2). This justifies our use of the \( \log k \) space discretization.

We have performed a large number of simulations while varying the background conditions: \( \lambda_{max} \) from \( 10^{13} \) cm to \( 10^{17} \) cm, \( \eta \), the ratio of magnetic energy in fluctuation mode to that in the background field from 0.3 to 300, \( \theta \), the angle between the background field and the shock normal from 0 to \( \pi/2 \). Figure 3 shows the time evolution of the energy spectrum for 10 years. The high-energy end of the spectrum gradually grows, and reaches \( 2.5 \times 10^{13} \) eV after 10 years.

In our simulation all the particles start their motion in the given time. Since we do not include the back-reaction from the particles to the electromagnetic field in our simulations, the time-integral of the energy spectra at each time slice gives the steady-state energy spectra. This steady-state spectrum is also shown in Figure 3 as a bold curve. The nonthermal component has an \( E(k) \propto k^{-1.6} \) power-law spectrum that meets the observational requirement mentioned in Section 1. We can
also estimate the “injection rate” to be the proportion of particles that have more than 2 GeV of energy after one year. For $T = 0.24$ keV, 24 keV, 2.4 MeV, and 0.24 GeV, the injection rate was $< 0.001, 1.9 \times 10^{-5}, 8.4 \times 10^{-2},$ and 0.378, respectively. The other parameters are $\eta = 30, \theta = 0,$ and $\lambda_{\text{max}} = 10^{17}$ cm.

In Figure 4, we show for the whole parameter range the ratio of the particle numbers that were accelerated to have energy greater than 2 GeV. We see that the acceleration is most efficient at the polar region ($\theta \approx 0$) when $\eta > 1$. We can understand this dependence of the spectra with background fluid parameters as follows: particles are trapped in Larmor motion and tend to move in the direction of $B_0$. Thus particles more easily cross the shock front when $B_0$ is parallel to the shock normal. If the turbulence amplitude is much weaker, fewer particles get reflected by pitch angle scattering, and Fermi acceleration is suppressed. The injection is more efficient for smaller $\lambda_{\text{max}}$, because more energy is distributed to modes with the smallest wavelengths which are resonant with the thermal particles.

If a spherical shock emerges in a uniform mean magnetic field, there are two polar regions where the mean magnetic field is parallel to the shock normal, and the equatorial region has the mean magnetic field perpendicular to the shock normal. Thus, we expect the Fermi acceleration process to be active only in the polar region pair. This might explain the bipolar structure we see at SN 1006.

We have also checked the acceleration rate in the three-dimensional (isotropic), rather than the one-dimensional (anisotropic) distribution of $k$. We have found a less significant dependence of injection rate on $\theta$ with larger $\eta$. We can interpret this as follows: if the turbulence spectrum is isotropic and the maximum turbulence wavelength is large, the turbulence modes with largest wavelength and strongest amplitude play the role of local $B_0$. Thus, we observe almost isotropic Fermi acceleration. This might explain the many supernova remnants (SNRs) with no typical orientation.

4. DISCUSSION

Some might question the validity of the value of $\eta$ being much greater than unity. However, Uchiyama et al. (2007) observed extremely fast varying X-ray images at SNR RX J1713.7-3946. Their observation may indicate that the magnetic field is locally enhanced up to 1 mG in about one year in SNRs, which corresponds to the $\eta = 100$ case in our model. Our simulations suggest that such rapidly varying spots in SNRs might be the sites of galactic ($E < 10^{15}$ eV) cosmic-ray acceleration.

Although we have ignored many of the Fourier modes by adopting a $\log k$ space discretization of the turbulence spectrum, the validity of the approximation can be argued from many viewpoints. Most importantly, we have confirmed that our measure in $\log k$ space leads to convergence. Figure 2 shows energy spectra for $\eta$ kept constant, with increasing $\Delta \log_{10} k$. The turbulent cascade, from which the actual turbulence arises, is by nature a logarithmic process: a mode of a certain wavelength couples with the mode of half the wavelength by a nonlinear term of the Euler equation. First-order Fermi acceleration also is a logarithmic process: particles gain energy as an exponential function of shock crossing number $E(N_f) = E_0(1 + h)^{N_f}$. Combining these observations, waves accelerate cosmic-ray particles even if they are logarithmically discretized.

The authors thank S. Inoue and K. Murase for useful comments. They also thank Center for Computational Astrophysics (CICA) of National Astronomical Observatory of Japan and Yukawa Institute for Theoretical Physics (YITP) in Kyoto University for their computing facilities. The numerical calculations were carried out on Cray XT4 at CICA and Altix3700BX2 at YITP. S.I. is supported by Grants-in-Aid (15740118, 16077202, and 18540238) from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan. This work was supported by the Grant-in-Aid for the Global COE Program “The Next Generation of Physics, Spun from Universality and Emergence” from the MEXT of Japan.

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Figure 3. The particle energy spectra for $\lambda_{\text{max}} = 10^{17}$ cm and $\eta = 10, \theta = 0$ at 1, 3, and 10 years of time evolution. The time-integrated energy spectrum is shown as a bold curve. The particle numbers that have more than 2 GeV of energy after one year.

Figure 4. Parameter dependence of particle acceleration efficiency. $\lambda_{\text{max}}$ is the longest wavelength of the turbulence modes, $\eta = (\Sigma B_i B_i B_0^{-2})^2 \in [0.3, 1, 3, 30, 300]$ is the ratio of turbulent magnetic field to unperturbed magnetic field, and $\theta$ is the angle between the shock normal and $B_0$. The other parameters are $\eta = 30, \theta = 0$, and $\lambda_{\text{max}} = 10^{17}$ cm.