About some problems regarding IKP-DKP of a 9R planar parallel mechanism

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Abstract. The paper presents some problems which might occur at different stages of the 9R(3RRR) planar mechanisms study (workspace characterizing, forward and direct kinematics, singularities analysis). At each of this stage, it has been identified certain problem which can occur and that need more attention and effort to be solved. In the paper classical methods, and formulas, known from technical literature, have been used, but occurred situations was depicted detailed and illustrated by graphical representations.

1. Introduction
This paper continues previous authors researches regarding 9R (3RRR) planar parallel mechanisms analysis, [1-6]. During these works, we dealt with workspace characterization depending on different constructive parameters, inverse and direct kinematic problems for certain trajectories and in each case, the singularities analysis has been done. It was chosen this kind of mechanism because of its incontestable advantages (higher speeds, better accuracy and strength reported to serial ones). These make this type of mechanism to be widely used in manipulator in technical purposes and well treated in technical literature also [7-35]. The major problem encountered to this mechanism is represented by singularity points located inside its workspace. In this paper, all presented numerical results have been obtained using classical, well-known methods and procedures. The main goal of the paper is to emphasise some problems or difficulties occurring to each of enumerated behind stages, in study of this type of mechanisms.

2. Theoretical preliminaries
In order to perform behind mentioned stages (workspace study, forward and direct analysis and singularities emphasizing) we done a set of interconnected programs (figure 1), which allows data to be taken from each another. In Figure 2 a kinematic scheme of the 9R (3RRR) planar parallel mechanism was shown [10]. In Figure 1, certain notations were made, with following meanings:

\( O_{i} = 1 \rightarrow 3 \) - fixed pairs depicting the fixed platform,

\( O_{xy} \) - referential system related to fixed platform,

\( B_{i} = 1 \rightarrow 3 \) - pairs of the mobile platform,

\( A_{i} = 1 \rightarrow 3 \) - pairs between proximal links \( O.A_{i} \) and the distal links \( A.B_{i} = 1 \rightarrow 3 \).

\( M_{x}, y_{m} \) - referential system related to mobile platform, \( M \) - point where end-effector is placed,
\( B_iB_j = b; i, j = 1,3, i \neq j \) - sizes of \( B_1B_2B_3 \) equilateral triangle,
\( O_iA_j = l_i; i, j = 1,3, i \neq j \) - lengths of proximal links,
\( AB_j = l_j; i, j = 1,3, i \neq j \) - lengths of distal links,
\( q = [x, y, \varphi] \) - input parameters (coordinates of point \( M \) reported to fixed system and angle between absciss axes of fixed and mobile system).
\( \Theta = [\theta_1, \theta_2, \theta_3] \) - output parameters representing position angles of driven (proximal) links,
\( \vec{s}_i = \vec{B}_iM, \ i = 1,3 \) - vectors establishing position of point \( M \) on mobile platform,
\( \vec{o}_i = \vec{O}_iO, \ \vec{r}_j = \vec{O}_jB_j, \ \vec{v}_j = \vec{OM}, \ \vec{s}_j = \vec{MB}_j \) - vectors defined using Figure 2.

Based on Figure 1, the following relationship can be written:
\[ \vec{r}_i = \vec{v} + R \cdot \vec{s}_i - \vec{o}_i, \]  \hspace{1cm} (1)
where \( R = \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix} \) is matrix of revolution by centre \( M \) and radius \( s_i \).

Squaring equation (1), the modulus of vector \( \vec{r}_i = \vec{O}_jB_j \) can be expressed:
\[ ||\vec{r}_i|| = \left[ \left( x + \cos \varphi \cdot \sin \varphi \right) - x_{\alpha i} \right]^2 + \left[ y + \sin \varphi \cdot \cos \varphi \right] - y_{\alpha i} \]^2 \]  \hspace{1cm} (2)
where \( x_{\alpha i}^M, y_{\alpha i}^M \) are the points \( B_i \) coordinates in mobile system and \( x_{\alpha i}, y_{\alpha i} \) - the fixed points \( O_i \) coordinates reported to fixed system.

When the proximal and distal links are in extension or superposes, \( ||\vec{r}_i|| = |l_i \mp l_j| \), after performing some calculations, relationship (2) become,
\[ \left( x - \left( -x_{\alpha i}^M \cos \varphi + x_{\alpha i}^M \sin \varphi + x_{\alpha i} \right) \right)^2 + \left( y - \left( -y_{\alpha i}^M \sin \varphi - x_{\alpha i}^M \cos \varphi + y_{\alpha i} \right) \right)^2 = (l_i \mp l_j)^2. \]  \hspace{1cm} (3)
This new relation represents equation of a circle with radius \( |l_1 \mp l_2| \) and centre of coordinates, 
\[
  a_i = -x_{B_i}^M \cos \varphi + y_{B_i}^M \sin \varphi + x_{O_i}, \quad b_i = -y_{B_i}^M \sin \varphi - x_{B_i}^M \cos \varphi + x_{O_i}.
\]
Consequently, the mechanism workspace can be thought as an intersection of three circular annuli, with exterior radius of \( l_1 + l_2 \) and interior ones of \( |l_1 - l_2| \).

Based on figure 1, the following relation can be write, 
\[
  \overline{A_iB_i} = \overline{OM} + R \cdot \vec{s} - \overline{O_iA_i} - \overline{OO_i}.
\]
Accomplishing calculations in relation (5) it obtains, 
\[
  \theta_i = 2 \cdot \tan^{-1} \left( \frac{y - b_i + d_i \sqrt{\Delta_i}}{x + K_i - a_i} \right),
\]
where, \( d_i = \pm 1, \quad K_i = \frac{1}{2l_i} \left[ (x - a_i)^2 + (y - b_i)^2 + l_i^2 - l_i^2 \right], \) and \( \Delta_i = (x - a_i)^2 + (y - b_i)^2 - K_i. \)

The angles \( \Theta = [\theta_1, \theta_2, \theta_3] \) represent the output data in the forward kinematic problem (IKP). All calculations performed until now refers to the first modulus in Figure 1.

In order to solve direct kinematics, the proximal links angles \( \Theta = [\theta_1, \theta_2, \theta_3] \) are supposed to be known (input data), and output data \( q = [x, y, \varphi] \), referring to mobile platform position, must be determined. To accomplish this demand, it has been used a method, detailed in a previous paper [\text{\textsuperscript{[\ldots]}}]. Here we will use this method but we remember it in very succinct manner. For accomplish DKP problem it been used Figure 3.

![Figure 3. The four bar contour \( A_1A_2B_1B_2 \) used in DKP.](image)

The \( A_2A_1B_1B_2 \) linkage has variable basis \( (A_2, A_1) \) and actuated joint \( A_1 \), the input parameter is angle \( \varphi_2 \) and output parameters – angles \( \varphi_b, \varphi_3 \) (not figured), \( \varphi \). The idea of this method is to determinate all data referring to point \( M \) and the mobile platform as a point belonging to connecting rod in four bar \( A_2A_1B_1B_2 \) mechanism with position of pairs \( A_2, A_1 \) known from IKP even if variable. The study of this four bar mechanism is done using technical literature [\text{\textsuperscript{\textsuperscript{[7-10]}}}]

In order to perform singularities study it has been used the following function, expressing the constant lengths of distal links. 
\[
  F_i(x, y) = \left( x + x_{B_i}^M \cos \varphi - y_{B_i}^M \sin \varphi - x_{O_i} - l_i \cos \theta_i \right)^2 - \left( y + y_{B_i}^M \sin \varphi + y_{B_i}^M \cos \varphi - y_{O_i} - l_i \sin \theta_i \right)^2 - l_i^2.
\]
As it is easy to notice this is a vector function \( F_i, i = 1, 3 \) with three variable \( q = [x, y, \varphi] \), the angle \( \varphi \) of the mobile platform being constant. The implicit function (7) can be concisely written \( F(\Theta, q) = 0 \). Differentiating this expression with respect the time, a relationship between input and output velocities is achieve \( J_q \cdot \dot{q} + J_\Theta \cdot \dot{\Theta} = 0 \). Writing \( \Delta J_q = 0 \), the singularities of 2\(^{nd}\) type can be studied [10]. It is obviously, with singularities problem deals the third category of programs (in Figure 1).

3. Illustrative examples

In this paragraph it has been presented exemplification of certain problems occurred in each of the three stages of study, according to stages from Figure 1. Let begin with first problem, referring to IKP, i.e. running the programs which deserve forward kinematics. There, the greatest occurred problem results from using trigonometric inverse of tangent function \( \tan^{-1} \). As is well-known, this function is defined as \( \arctan(x) : \mathbb{R} \rightarrow \left( -\frac{\pi}{2}, \frac{\pi}{2} \right) \). But real revolute motion of mechanisms elements do not take place into this interval only. Let see some diagrams:

Linear trajectory

![Figure 4](image)

**Figure 4.** L=240; l1=100; l2=110; b=50; \( \phi_i=\pi/6 \); d1=1; d2=1; d3=1; a) workspace and end-effector linear trajectory, b) IKP – results, c) DKP – results.
Triangular trajectory

![Triangular Trajectory Image]

**Figure 5.** L=240; l1=100; l2=110; b=50; fi=π/6; d1=1; d2=1; d3=1; a) workspace and end-effector triangular trajectory, b) IKP – results, c) DKP – results.

Quadrilateral trajectory

![Quadrilateral Trajectory Image]
Figure 6. L = 240; l1 = 100; l2 = 110; b = 50; \( \phi = \pi/6; \) d1 = 1; d2 = 1; d3 = 1; a) workspace and point \( M \) quadrilateral trajectory, b) IKP – results, c) DKP – results with \( \Delta \phi_2 = 1° \), d) DKP – results with \( \Delta \phi_2 = 1.9° \).

Circular trajectory
**Figure 7.** L=240; l1=100; l2=110; b=50; fi=π/6; d1=1; d2=1; d3=1; a) workspace and end-effector circular trajectory, b) IKP – results, c) DKP – results with Δϕ₂ = 1°, d) deviation of angle ϕ.

**Elliptical trajectory**

**Figure 8.** L=240; l1=100; l2=110; b=50; fi=π/6; d1=1; d2=1; d3=1; a) workspace and point M quadrilateral trajectory, b) IKP – results, c) DKP – with Δϕ₂ = 1°, d) DKP – with Δϕ₂ = 0.1°.
Butterfly trajectory

Figure 9. L=2.3; l1=1.1; l2=1.2; b=0.5; fi=1*pi/4; d1=1; d2=1; d3=1; a) workspace and point $M$ quadrilateral trajectory, b) IKP – results, c) DKP – results with $\Delta \phi_2 = 1^\circ$, d) DKP – results with $\Delta \phi_2 = 0.01^\circ$.

Figure 10. Singularities emphasizing L=2.4; l1=1; l2=1.1; b=0.5; $\phi = \pi / 5$; d1=1; d2=1; d3=1; a) $\Delta J_q$ – as a surface b) as level curves.
Figure 11. Singularities emphasizing $L=2.4; l_1=1; l_2=1.1; b=0.5; \varphi=0; d_1=1; d_2=1; d_3=1$; a) $\Delta J_q$ - as a surface  b) as level curves.

Figure 12. Singularities emphasizing $L=2.4; l_1=1; l_2=1.1; b=1.5; \varphi=\pi/6; d_1=1; d_2=1; d_3=1$; a) $\Delta J_q$ - as a surface  b) as level curves.

Figure 13. Singularities emphasizing $L=2.4; l_1=1; l_2=1.1; b=0.5; \varphi=\pi/6; d_1=-1; d_2=-1; d_3=1$; a) $\Delta J_q$ - as a surface  b) as level curves.
The second group of difficulties which can occur refers to the direct kinematics. Despite the method is quite simply and easy programmable, the results obtained after programs running sometimes are wrong (Figures 6, 7, 8, 9).

To the third level of programs (Figure1), the main difficulties that can occur refers to surfaces manipulation and its planar representations and also to the correlations between these data and those achieved in the previous level (DKP modulus).

4. Conclusions
The paper represent a continuation of authors researches begun some time ago. Thus we used a programme package, written during our researches and which has been under continuous improvement. Presented results were achieved using a classical mathematical apparatus, well-known in literature but programs packages were entirely made by us. On the whole, the work has a practical and didactical character, suitable for student training and practice. The main difficulties which can occur at 9R (3RRR) parallel mechanisms study were intentionally emphasized in this paper. These difficulties can occur to each stage of the study (forward and direct kinematics and singularities approaching). The results can and will be improved during the researches advances, especially by programs fitting and mathematical models improvement.

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