EFFECTS OF PERTURBING FORCES ON THE ORBITAL STABILITY OF PLANETARY SYSTEMS

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ABSTRACT

We consider dynamical effects of additional perturbative forces due to the non–point-mass nature of stars and planets: effects such as quadrupolar distortion and tidal friction in the systems of exoplanets. It is shown that one should not neglect these forces while modeling the dynamics of planetary systems, especially taking into account the undefined real masses of the planets due to unknown orbital inclinations and the unsatisfactory application of Keplerian fits to the radial velocity data in multiple planetary systems.

Subject headings: celestial mechanics, stellar dynamics — planetary systems — stars: individual (ν Andromedae)

1. INTRODUCTION

About 50 extrasolar planets have been discovered so far, among them at least five confirmed planetary systems around the stars ν Andromedae, with three Jupiter-mass planets (e.g., Laughlin & Adams 1999; Rivera & Lissauer 2000); HD 83443, with two Saturn-type companions (Mayor et al. 2001); GJ 876, orbited by two resonant planets (Marcy et al. 2001a), HD 82943, with two possibly resonant planets; and HD 74156 (two planets). In addition, there are two substellar mass companions orbiting HD 168443 (Marcy et al. 2001b; Udry et al. 2001), a system of Earth-mass planets around pulsar PSR 1257+12 (e.g., Konacki, Maciejewski, & Wolszczan 1999), and (for attempts at

2. THE MODEL

In order to estimate the dynamical effects on interacting bodies due to their mutual quadrupolar distortion (QD) and TF, based here on the near-equilibrium approximation, on the planetary systems treated as N-body systems with extra forces due to QD and TF in addition to the Newtonian gravity, we applied the following formulation for the force $F_{ij}$ of one body on the other, developed by Kiseleva, Eggleton, & Mikkola (1998, hereafter KEM98):

$$F_{ij} = -\left[\frac{Gm_i m_j}{r_{ij}^3} + \frac{6G(m_i A_i + m_j A_j)}{r_{ij}^5}\right] r_{ij} + \frac{27 \sigma_i \sigma_j A_i A_j}{2 r_{ij}^10} r_{ij}^2,$$

where

$$A_i = \frac{R_i^3 Q_i}{1 - Q_i}, \quad \alpha_i = \frac{\alpha_i}{Q_i^2 m_i R_i^3} \sqrt{\frac{Gm_i}{R_i^3}},$$

$$r_{ij} \equiv r_i - r_j.$$
Fig. 1.—Eccentricity of the outer planet in the \( \nu \) And planetary system with masses of planets C and D exchanged. The system is unstable (top panel) without QD and TF and (second and fourth panels) with relatively weak QD and TF, although QD increases the lifetime of the system. Third and bottom panels: QD with coefficients \( Q_p = 0.08 \) and \( \alpha = 10^{-4} \) make the system stable over the considered time interval (10^6 yr), with quasi-periodic fluctuations of \( e_c \).

We applied this model to the \( \nu \) And planetary system. Recently Jiang & Ip (2001) confirmed once more that the innermost planet does not affect very much the dynamics of the middle and outer planets, so we ignored it in our simulations. For calculation we used the regularized “chain” method (Mikkola & Aarseth 1993) with perturbations. The actual numerical integration of the equations of motion is carried out by a Bulirsch-Stoer integrator with a time step accuracy of 10^{-14}.

In our simulations we always used initial orbital parameters for orbital periods and eccentricities from the Lick data (Butler et al. 1999; Rivera & Lissauer 2000): days, \( P_p = 242 \) and \( e_p = 0.23 \) for the middle planet C and \( P_p = 1269 \) days, \( e_p = 0.36 \) for the outermost planet D. We always started simulations with all three components positioned in the same plane at aphelia of their orbits and with the two orbits out of phase with each other by 90° (so we did not use the observed values of \( \omega \)). Orbital parameters for the \( \nu \) And planetary system are not very precisely defined and may vary rather significantly (particularly the eccentricities) during the lifetime of the system (see below). Note, however, that differences between values of orbital parameters given by different authors may not be explained by real changes during a short observational time of a few years.

Any real and significant changes require a timescale of hundreds of years.

3. RESULTS

In our first set of simulations we use the nominal values of the planetary masses from the Lick data: for \( \sin i = 1 \), \( m_c = 1.98 \, M_\odot \) and \( m_d = 4.11 \, M_\oplus \). The adopted mass of the star was \( M = 1.3 \, M_\odot \). The relative inclination between the two orbital planes was taken to be \( \phi = 1° \) (models with higher relative inclinations were also tested, but we will discuss those results in our next paper). For this set of parameters, the system appears to be hierarchically stable over at least 10^7 yr, despite both orbital eccentricities fluctuating almost quasi-periodically within significant ranges: \( e_p = 0.5 \) and \( e_d = 0.3 \). These results are in good qualitative agreement with other simulations (Laughlin & Adams 1999; Rivera & Lissauer 2000). Neither TF or QD or their combination changes notably (except very minor details) the orbital evolution of this system.

However, because of the unknown values of the inclination of the orbital planes of the planets \( i_c \) and \( i_d \), which can differ very much from each other, even the mass hierarchy of the \( \nu \) And planetary system is questionable. If one accepts the values of \( i_c = 173° \pm 3° \) and \( i_d = 28° \pm 16° \) proposed by Pourbaix (2001) with a big uncertainty, this will lead to a very different system with component C being no longer a planet (\( m_c \approx 18 \, M_\oplus \)) and \( m_d \approx 8.5 \, M_\oplus \) (so the mass hierarchy is now changed). In agreement with earlier stability analyses (Stepinski, Malhotra, & Black 2000; Rivera & Lissauer 2000), the new system is highly unstable and disintegrates, ejecting component D within the first 1000 yr. The addition of QD increases.
significantly the lifetime of the system, although it does not change the chaotic and unstable character of its dynamics.

In order to study more systematically the effects of QD and TF (in this work we study these effect separately) on the dynamical stability of planetary systems unstable in point-mass approximation, we exchanged the masses of planets C and D in the \( \nu \) And system, leaving their initial eccentricities and orbital periods unchanged. Without QD and TF this system is unstable, ejecting the less massive \( m_p \) component D within \( \sim 3 \times 10^5 \) yr (top panel of Fig. 1). QD and TF may significantly change the dynamical evolution of the system. The scale of these changes depends very much on the chosen values of coefficients \( Q \) and \( \alpha \) from equation (2). The value of \( Q \) is defined by the polytropic index \( n \) of the body (eq. [3]). For the presumably radiative star of 1.3 \( M_\odot \) we assume \( n \sim 3 \) with corresponding \( Q \approx 0.02 \). For planets we took \( Q \approx 0.2 \) (\( n \sim 1.5 \)). The dynamics of the system during the first \( 1.5 \times 10^5 \) yr is significantly less chaotic than without QD, and planet D is not ejected to infinity until \( t \sim 5 \times 10^5 \) yr. However, the situation changes very dramatically when we increase the value of \( Q \) to 0.08 (so assuming a more convective interior of the star). The third panel of Figure 1 shows that in this case the system displays hierarchically stable dynamical behavior over at least \( 10^6 \) yr, with quasi-periodic fluctuations of both orbital eccentricities similar to ones of the original \( \nu \) And system.

The dynamical effect of TF depends on its coefficients \( \alpha \). Kiseleva et al. (1998) found that for stars similar to the ones in the \( \lambda \) Tau triple system (Fekel & Tomkin 1982) the most likely \( \alpha \sim 10^{-5} \). However, for planets \( \alpha \) can be significantly larger. We tested our model with \( \alpha = 10^{-5} \) and \( \alpha = 10^{-4} \) for all three bodies. The results shown in the two lower panels of Figure 1 are totally different. Setting \( \alpha = 10^{-5} \) does not seem to improve the stability of the system. However, for \( \alpha = 10^{-4} \) the bottom panel presents once more a hierarchically stable system with quasi-periodic behavior of its orbital parameters such as \( e_i \) and \( i \). The dynamical evolution of models with strong QD (\( Q = 0.08 \)) and with strong TF (\( \alpha = 10^{-4} \)) over \( 10^6 \) yr look in this case remarkably similar, despite different dynamical properties of these perturbations: TF is a dissipative force with respect to the total orbital energy and QD is conservative. However, such a similarity does not appear in other cases (see below), and we suspect that over a longer time the evolutionary patterns with QD and with TF will diverge.

We also studied models of \( \nu \) And with \( \sin i = 0.33 \) for both external planets C and D, so the mass hierarchy of the nominal system is preserved and \( M_p = 6.53 M_j, M_p = 13.56 M_j \). The results are shown in Figure 2. TF and especially QD significantly increase the lifetime (up to correspondingly \( \sim 5 \times 10^4 \) and \( \sim 1 \times 10^6 \) yr) of this unstable system.

4. Conclusions and Prospects

Our strongest conclusion is that the effects of perturbative forces such as QD and TF should not be neglected when investigating numerically the dynamical properties of extrasolar planetary systems, especially when their hierarchical stability is questionable and even weak additional forces may change the qualitative character of their dynamics. In our examples QD always had a stabilizing effect, but we cannot claim that it always works this way and more studies are needed.

We did not consider here the contribution of general relativity and intrinsic rotation of the bodies, which under some circum-
stances may be important. In any case, because of problems with the reliable determination of orbital parameters that cannot be considered as Keplerian in $N$-body systems, it would be very useful to apply a good dynamical chaos indicator (e.g., Lyapunov-type exponent), which can distinguish a long-term dynamical instability from relatively short term integrations, given a good-sized sample of possible initial parameters. One such indicator was suggested recently by Cincotta & Simó (2000). Our first attempts to apply it to extrasolar planetary systems are very encouraging, and we are going to discuss the results in our next paper (K. Goździewski, E. Bois, A. Maciejewski, & L. Kiseleva-Eggleton 2001, in preparation).

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