Sharp disentanglement in holographic charged local quench

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Abstract: We propose a charged falling particle in an AdS space as a holographic model of local charged quench generalizing model of arXiv:1302.5703. The quench is followed by evolving currents and inhomogeneous distribution of chemical potential. We derive the analytical formula describing the evolution of the entanglement entropy. At some characteristic time after the quench, we find that the entanglement shows a sharp dip. This effect if universal and independent of the dimension of the system. At finite temperature generalization of this model, we find that multiple dips and ramps appear.
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1 Introduction

The AdS/CFT correspondence is an important and universal tool in the description of quantum phenomena. The entanglement entropy is an important part of the holographic description of quantum systems [1]-[3]. An important feature of the AdS/CFT correspondence is the ability to capture the universal features of strongly-coupled systems in the regimes where the other methods have limitations. The striking example of such phenomena are systems out-of-equilibrium or at finite chemical potential. The holographic duality admits the non-perturbative description of such phenomena. The examples include physics of heavy-ion collisions[4, 5] and condensed matter theory[6, 7].

There are two exactly solvable canonical models of non-equilibrium process in CFT that are testing grounds for many holographic concepts and techniques. These are global and local quench. The global quench is the process when the system exhibits global perturbation. As an examples let us mention the instantaneous change of coupling constant, the homogeneous energy injection and the boundary state quench in 2d CFT (see [8] for review). From the holographic viewpoint, the global quench is described by the collapsing shell [9]-[11]. This black hole formation model can provide holographic setup for quite diverse type of physical situations including the models including, non-trivial initial states [12], hyperscaling [13, 14] and chemical potential[15].

The local quench is the process when the system evolves after the point-like perturbation. In this paper, our main focus will be on this type of quenches. The canonical examples of solvable local quenches in 2d CFT which have the holographic description are joining/splitting quenches and operator quench[17]-[26]. In the operator quench, one perturbs the system by the insertion of the local operator at some point and at time moment \( t_0 \). The holographic description of this quench has been proposed in [18] and explored in different versions in [21]-[26]. The model proposed in [18] consists of the point particle at the quench point \( x = x_0 \) falling in the bulk and deforming the \( AdS_3 \). One can find the exact analytic answer for the metric describing the Einstein equations with this point-like source. Moreover one can extend this description to the operator quench at the finite temperature[23, 24] and even many-dimensional quenches (at zero temperature)[18].

This work is devoted to the extension of the model [18] where particle perturbing the bulk carries \( U(1) \) charge. This implies that this model is dual to the local quench by the charged operator. Point-like charged perturbation in the bulk creates the distribution of the Maxwell field on the boundary and its presence corresponds to the presence of the nontrivial distribution of the chemical potential. Note that in [27] the inhomogeneous chemical potential and charge oscillations in the holographic context have been investigated. We calculate the chemical potential and currents following the
quench for perturbed $AdS_3$ and higher dimensional extensions explicitly.

Important results of [18] is the calculation of the entanglement entropy for some certain subregions. In [18] good approximation to the entanglement entropy that can be applied to holographic non-equilibrium processes has been proposed. As it was shown in [18] this approximation works very well as for lower-dimensional as for higher-dimensional quenches. We extend this calculation to our setup and find the universal effect in the evolution of the entanglement entropy. In zero charge case for the interval $x \in (-\ell, \ell)$ the entanglement entropy shows peaks around $t \approx \ell$. Turning on a non-zero charge of the operator leads to the sharp dip in the entanglement at this time. We find that this effect is also present in the higher-dimensional analog of this quench.

The finite temperature charged operator local quench is described by the charged particle falling on the BTZ black hole horizon [23, 24]. We find that the turning on non-zero temperature changes the evolution picture. Sharp entanglement dip is preceded by a sharp peak. After the dip the small smooth peak is also present.

These results are in line with other works of local charged quenches. This topic is still quite unexplored in contrast to zero charge local quenches. In [25] the evolution of the higher-spin local quench has been investigated in the context of a particular higher spin theory (with the inclusion of higher spin three). They observe the effect similar to our entanglement dips, however, it is not clear whether there is a clear relation between them. The propagation of small charge density quench described by localized Gaussian profile of chemical potential has been considered in [28]. In [28] the background where the local perturbation propagates is the static Reissner-Nordstrom black hole and the perturbation is considered in the probe limit.

The paper is organized in the following way. In section 2.1 we discuss general setup and preliminary information about the holographic description of local quenches. Also we remind and discuss the general effect of static chemical potential on the entanglement entropy. The section 3 is devoted to description of the entanglement entropy evolution in the local charged quench at $T = 0$. In section 4 we discuss the case of $T > 0$ local charged quench. Last section is devoted to concluding remarks.

2 Preliminaries

2.1 Holographic description of local quenches: zero charge

In this section we remind well known facts from the description of quenches with zero charge. The metric of the form

$$ds^2 = -(1 - M + R^2)dt^2 + \frac{dR^2}{1 - M + R^2} + R^2 d\phi^2$$

(2.1)
corresponds to the solution of three-dimensional gravity with cosmological constant known as the BTZ black hole if $M > 1$ and if $M < 1$ we have the deficit angle space. Both of these spaces on the dual side corresponds to the primary operator insertion. According to the holographic dictionary this operator corresponds to the static massive particle. In 2d CFT this state can be expressed in the form given by the insertion of the primary operator

$$|\psi\rangle = \mathcal{O}(0)|0\rangle.$$  \hspace{1cm} (2.2)

The metric (2.1) corresponds to the deformed global AdS so the theory is dual to the CFT defined on the circle. To consider the CFT on the line deformed by the primary operator one has to consider the Poincare patch deformed by the particle. In this case particle cannot be static so the metric and the dual system become dynamical. The self-consistent equations motion for the point particle interacting with the gravity are complicated and in general it is hard to solve them.

However, we do not need to solve them in the straightforward manner. In [18] the following trick has been proposed. We know the metric for the static particle deforming the AdS in global coordinate frame we have. Then we have to make the change of variables to the Poincare patch. If $M = 0$ this change of variables (see the explicit form below in the text, (3.5)) transforms the metric (2.1) to the Poincare patch

$$ds^2 = \frac{L^2}{z^2} (-dt^2 + dx^2 + dz^2).$$ \hspace{1cm} (2.3)

If $M > 0$ the resulting metric has long and complicated form, however it is still the metric with constant Ricci scalar except the particle worldline. The particle itself from $R = 0$ in the initial metric is mapped to the parabolic worldline $z = \sqrt{\alpha^2 + t^2}$. Also there is the generalization of this framework to the finite temperature case. It is described by the one-sided BTZ black hole perturbed by the particle falling on the black hole horizon [23].

In [18] also the generalization of the local quench to many-dimensional case, i.e. the $AdS_{d+1}$ deformation by falling particle has been proposed. The static many-dimensional version of (2.1) describing the geometry outside the massive object is given by

$$ds^2 = -f(R)dt^2 + \frac{dR^2}{f(R)} + R^2 d\Omega_{d-2}^2,$$ \hspace{1cm} (2.4)

$$f(R) = 1 + R^2 - \frac{M}{R^{d-2}},$$ \hspace{1cm} (2.5)

which is the metric of many-dimensional global $AdS_{d+1}$-Schwarzschild black hole. This metric is dual to $d$-dimensional CFT perturbed by insertion of the operator in one
point. It was shown that the evolution of the system after such perturbation lead to the radial waves emitted from the perturbation point and the qualitative picture is the same as in the lower dimensional theory.

In the case of three-dimensional gravity it is known that the BTZ black hole is locally AdS space. This means that one can use this to construct the generalization of the operator local quench to finite temperature 2d CFT [23]. Instead of mapping (2.1) to the Poincare frame one can map it to the BTZ frame. This lead to the picture where the particle is falling on the black hole horizon and perturbing the static BTZ metric.

### 2.2 Static charged backgrounds and entanglement entropy

To generalize the setup described in the previous section first one has to introduce the static geometry generalizing (2.1) and (2.4). The generalization of the BTZ black hole with the static charge has the form

\[
\begin{align*}
    ds^2 &= -f(R)d\tau^2 + \frac{dR^2}{f(R)} + R^2 d\phi^2 \\
    f(R) &= 1 - \mathcal{M} + R^2 - \frac{1}{2} Q^2 \log R
\end{align*}
\]

(2.6)

(2.7)

with the gauge field given by

\[
A_t = \Phi + Q \log R dt.
\]

(2.8)

Typically \( \Phi \) is set to be \( \Phi = -Q \log r_h \) and \( 1 - \mathcal{M} = -r_h^2 \) and for this choise this metric is known as charged BTZ black hole with the horizon at \( R = r_h \) and gauge field vanishing at \( r_h \). We leave the metric in this form to see the effects of the charge more clearly. To start with the effects let us consider the entanglement entropy for the subregions in the theory dual to the (2.6).

The standard Hubeny-Ryu-Rangamani-Takayangi (HRRT) prescription states that the entanglement entropy of the subregion \( \phi \in (-\ell/2, \ell/2) \) is given by the minimal surface (in \( d = 2 \) it is the geodesic)

\[
S_{EE} = \frac{A}{4G}
\]

(2.9)

We parametrize the geodesic by \( R = R(\phi) \) with \( R(\pm \ell/2) = \infty \). The geodesic has turning point such that \( R(0) = R_* \) and \( R'(0) = 0 \). The parametric expression for the
geodesic length $A$ has the form

$$A(R_*) = 2 \int_{R_*}^{R_{reg}} \frac{R}{\sqrt{R^2 - R_*^2} \sqrt{f(R)}},$$

(2.10)

and the interval size $\ell$ is

$$\ell(R_*) = 2 \int_{R_*}^{\infty} \frac{R_*}{R \sqrt{(R^2 - R_*^2)} f(R)}.$$  

(2.11)

Using these expressions one can obtain the entanglement entropy for fixed $\ell, M$ and $Q$

$$S = S(\ell, M, Q).$$

(2.12)

We present the dependence of $\Delta S = S(\ell, M, Q) - S(\ell, M, 0)$ in Fig.1. We see, that the excitation of the entanglement entropy is non-monotonous for different sign of $Q^2$.

![Figure 1](image)

**Figure 1.** The difference $\Delta S = S(\ell, M, Q) - S(\ell, M, 0)$ for $M = 0.2$ and $Q^2 = \pm 0.4$.

It is worth to comment the last choice corresponding to the imaginary gauge fields and consequently the imaginary chemical potential values. Let us mention where the imaginary chemical potential takes place in physics. For example the imaginary chemical potential has been investigated in the context of charged Renyi entropies in [29]. The Gross-Neveu model and phase diagrams with imaginary chemical potential has been investigated in [30–32].

The calculation of the entanglement entropy gives the real-valued answer also for $Q^2 < 0$ for background by (2.6).
Turning to the many-dimensional case the geometry on interest is the AdS-Reissner-Nordstrom black hole defined by the function $f(R)$ of the form

$$f(R) = 1 + R^2 - \frac{M}{R^{d-2}} + \frac{Q^2}{R^{2(d-2)}}.$$  \hfill (2.13)

To understand the behaviour of the observables defined by geometric quantities in this metric to compare with $d = 2$ charged black hole case we also consider the behaviour of the geodesic length connecting two points in black hole defined by (2.13). This quantity does not give the entanglement and is relevant to the equal-time Green function. However it still gives some intuition to correlations in the dual of (2.13). We present the geodesic length difference between charged and chargeless case in Fig.2. We choose $d = 3$ such that $d\Omega = d\theta^2 + \sin^2 \theta d\phi^2$ and fix $\phi = const$. In contrast with $d = 2$ it exhibits the monotonous dependence.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2.png}
\caption{The difference between the geodesic length for $M = 0.2$, $Q^2 = \pm 0.4$ and $Q = 0$ where $f(R)$ corresponds to (2.13).}
\end{figure}
3 Quench by the charged operator at zero temperature

3.1 2d CFT: charged BTZ black hole

The charged BTZ metric describing the charged particle that rests in the center of the global $AdS_3$ has the form

$$ds^2 = -(1 - M + R^2 - \frac{1}{2}Q^2 \log R)d\tau^2 + \frac{dR^2}{1 - M + R^2 - \frac{1}{2}Q^2 \log R} + R^2 d\phi^2$$ (3.1)

with the gauge field

$$A_t = -Q \log \frac{R}{R_h} dt.$$ (3.2)

Also it is useful to rewrite this metric in the coordinates $Z = 1/R$. After some rescalings the metric (3.1) has the form

$$ds^2 = \frac{1}{Z^2} \left(- f(Z) dt^2 + \frac{dZ^2}{f(Z)} + dx^2\right),$$ (3.3)

$$f(Z) = 1 - Z^2 + \frac{Q^2}{2} Z^2 \log Z.$$ (3.4)

The global coordinate patch and coordinates of Poincare patch are related by the following coordinate transformation

$$z = \frac{\alpha}{R \cos(\phi) + \sqrt{R^2 + 1} \cos(\tau)},$$ (3.5)

$$t = \frac{\alpha \sqrt{R^2 + 1} \sin(\tau)}{R \cos(\phi) + \sqrt{R^2 + 1} \cos(\tau)},$$ (3.6)

$$x = \frac{\alpha R \sin(\phi)}{R \cos(\phi) + \sqrt{R^2 + 1} \cos(\tau)},$$ (3.7)

with the inverse mapping given by

$$\tau = \arctan \left( \frac{2\alpha t}{\alpha^2 - t^2 + x^2 + z^2} \right)$$ (3.8)

$$R = \sqrt{\alpha^4 + 2\alpha^2 (t^2 + x^2 - z^2) + (-t^2 + x^2 + z^2)^2}$$ (3.9)

$$\phi = \arctan \left( \frac{2\alpha x}{\alpha^2 + t^2 - x^2 - z^2} \right).$$ (3.10)
This mapping sends the center of the global coordinates $R = 0$ to the trajectory $z = \sqrt{\alpha^2 + t^2}$ of the charged particle falling in the $AdS_3$ bulk and deforming it. The gauge field corresponding to this dynamical background can be obtained in the straightforward manner also applying the mapping (3.8) to the static gauge field (3.2) and the explicit form can be found in the Appendix.A.

The solution with the metric (3.1) and corresponding gauge fields have the logarithmic terms which are absent in $d > 2$ case. In [33] it was shown that for the near-boundary expansion of the gauge field of the form

$$A_i \sim a_i(x) \log r + b_i(x) + \ldots$$

the functions $a_i(x), b_i(x)$ are identified with the current and the chemical potential as

$$\langle j_i(x) \rangle = a_i(x),$$

$$\mu = b_0(x).$$

Note, that $A_z$ component of the field (A.1) vanishes near the boundary as it should be. Applying (3.12) to gauge field from Appendix.A we get the currents after the perturbation in the form

$$\langle j_t \rangle = \frac{2\alpha Q (\alpha^2 + t^2 + x^2)}{(\alpha^2 + t^2)^2 + 2x^2(\alpha^2 - t^2) + x^4},$$

$$\langle j_x \rangle = -\frac{4\alpha Qtx}{(\alpha^2 + t^2)^2 + 2x^2(\alpha^2 - t^2) + x^4},$$

and the dynamical chemical potential distribution

$$\mu(t, x) = \frac{2\alpha Q (\alpha^2 + t^2 + x^2) \log \left( \frac{2\alpha R h \sqrt{(\alpha^2 + t^2)^2 + 2x^2(\alpha^2 - t^2) + x^4}}{(\alpha^2 + t^2)^2 + 2x^2(\alpha^2 - t^2) + x^4} \right)}{(\alpha^2 + t^2)^2 + 2x^2(\alpha^2 - t^2) + x^4}. \quad (3.16)$$

Applying map to the metric (3.1) we obtain the complicated metric which is the solution of the Einstein equations with the charged matter dual to the charged local quench. We are not going to present this metric here (for $Q = 0$ one can find it in [19]).

Now turn to the computation of the entanglement entropy evolution during the local quench by the charged operator. The HRRT prescription states in this case, that the entanglement entropy of the subsystem is given by the extremal geodesic connecting two points on the boundary. In [18] it was shown how to compute qualitatively (and for
small expansion parameter \( M \) even quantitatively good approximation to the length of the geodesic connecting two equal-time points on the boundary. Assume we have the expansion of the metric in the form

\[
g_{\mu\nu} \approx g^{(0)}_{\mu\nu} + g^{(1)}_{\mu\nu} + O(\text{parameters}) + ...
\]  

(3.17)

where "parameters" in our case are \( M \) or \( Q \). In the unperturbed metric \( g^{0}_{\mu\nu} \) the HRT surface has the form

\[
X^{(0)}_{\mu} = X_{\mu}(\xi),
\]

(3.18)

and the induced metrics are defined as

\[
G^{(0)}_{\alpha\beta} = \frac{\partial X^\mu}{\partial \xi^\alpha} \frac{\partial X^\mu}{\partial \xi^\beta} g^{(0)}_{\mu\nu}, \quad G^{(1)}_{\alpha\beta} = \frac{\partial X^\mu}{\partial \xi^\alpha} \frac{\partial X^\mu}{\partial \xi^\beta} g^{(1)}_{\mu\nu}.
\]

(3.19)

Finally the leading order perturbation to the area contributing to the entanglement entropy has the form

\[
\Delta S = \frac{1}{2} \int d^{d-1}\xi \sqrt{G^{(0)}} \text{Tr} \left[ G^{(1)}(G^{(0)})^{-1} \right].
\]

(3.20)

Note that we refer to this quantity as to the entanglement entropy, however, to be more precise this is the difference of entanglement between quench with fixed \( Q \) and \( Q = 0 \) local quench. Also in what follows this quantity is assumed to normalised on the factors like \( 4G \).

For relatively small \( M \) and \( Q \) this method gives good approximation (see also [19, 20] for applications of this method in the computation of complexity). One of advantages of this method is that it is straightforward to use it in the higher-dimensional generalization of the local quench that we will consider further in the text.

Applying this method it is straightforward to calculate the entanglement entropy for the single interval for the metric (3.1) and using (3.19) and (3.20) we get single integral expression for the entanglement entropy

\[
\Delta S = -Q^2 \int_{-\ell}^{\ell} \frac{\alpha^2 x^2}{2 \ell (\alpha^2 + t^2 - \ell^2)^2 + 4 \alpha^2 x^2} \log \left( \frac{4 \alpha^2 (\ell^2 - x^2)}{(\alpha^2 + t^2 - \ell^2)^2 + 4 \alpha^2 x^2} \right) dx.
\]

(3.21)

Performing the integration numerically we obtain the picture of the entanglement evolution and the summary of the operator charge effect presented in Fig.3 is as follows

- The entanglement evolution picture after the local quench is well known, and it looks (for the interval centered around the quench point) as follows: first the entanglement is growing, attains its maximum and there is sharp peak around
the time $t \approx \ell/2$. After that it exhibits the rapid decrease. The peak time is when the quasiparticles created after quench reach the boundary of the interval.

- In Fig. 3 we present the difference $\Delta S$ between the entanglement entropy of the charged (with the fixed $Q$) and chargeless quenches ($Q = 0$) with other fixed characteristics. We see that for $Q^2 > 0$ there is the sharp dip at the same time where chargeless quench have peak.

- For $Q^2 < 0$ case we have the opposite behaviour which is easily seen from (3.21). Instead of sharp dip one get the sharp peak and initially the entanglement decrease.

\[ \Delta S \]

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure3.png}
\caption{The difference between the entanglement entropy of $Q^2 > 0$ (blue curve) and $Q^2 < 0$ (red curve) and $Q = 0$ (with the other parameters being fixed). Here for all curves $\alpha = 0.3$, the subsystem $x \in (-3, 3)$ and $M = 0.3$}
\end{figure}

To understand the qualitative picture of the process it is useful to look at the bulk picture of perturbation caused by particle. We present the determinant of the metric on the constant time slice in Appendix A. In Fig. 7 We see that after some time particle position is surrounded by the negative "perturbation density" (with respect to the chargeless quench) and there are also positive contribution of the "metric density waves" closer to the boundary. So there is the competition in the contribution to the entanglement entropy between these perturbations.

This picture lead to quite sensitive behaviour of the entanglement for some cases. The entanglement for the interval $x \in (x_1, x_2)$ and $x_2 > x_1 > 0$ is presented in Fig. 4. One can see how small change of the left endpoint of the interval lead to the significant
change in the behaviour of the entanglement. The closer left endpoint is to the quench point the larger is dip in the entanglement.

![Graph](image)

**Figure 4.** The entanglement evolution for the interval $x \in (x_1, x_2)$ where $x_2$ is fixed. Curves of different color are in one to one correspondence with the coloring of the corresponding intervals depicted below. Here $Q = 0.2$, $M = 0.3$ and $x_2 = 3$. 
3.2 Local charged quench at higher dimensions: ”falling” Reissner-Nordstroem

Now turn to the many-dimensional generalization of the previous section. Zero charge analogue of many-dimensional local quench has been developed in [18]. In that setup one applies the construction from the previous subsection to the global AdS-Schwarzschild black hole. For the charged generalization of many-dimensional local quench one replaces this black hole with the global AdS\textsubscript{d+1}-Reissner-Nordstroem black hole. The global charged black hole solution in \(d+1\) dimensions is given in the form

\[
ds^2 = -f(R)dt^2 + \frac{dR^2}{f(R)} + R^2 d\Omega^2_{d-2},
\]

\[
f(R) = 1 + R^2 - \frac{M}{R^{d-2}} + \frac{Q^2}{R^{2(d-2)}},
\]

\[
A_t = \left(\sqrt{\frac{d-2}{2(d-2)}} \frac{Q}{R^{d-3}} + \Phi\right) dt,
\]

where \(\Phi\) is the constant. Let us focus here for definiteness on \(d = 4\) (i.e dual to the local perturbation on the plane). The transformation the Poincare patch in the general dimensions has the form

\[
R = \frac{\sqrt{\alpha^4 + 2\alpha^2 (\rho^2 + t^2 - z^2) + (\rho^2 - t^2 + z^2)^2}}{2\alpha z},
\]

\[
\tau = \arctan\left(\frac{2\alpha t}{\alpha^2 + \rho^2 - t^2 + z^2}\right),
\]

\[
\phi = \arctan\left(\frac{\alpha x}{\sqrt{\alpha^2 (x^2 + y^2)}}, \frac{\alpha y}{\sqrt{\alpha^2 (x^2 + y^2)}}\right)
\]

\[
\theta = \arctan\left(\frac{-\alpha^2 + \rho^2 - t^2 + z^2}{2\alpha \rho}\right).
\]

After the transformation to the Poincare coordinates and taking the near-boundary expansion of the gauge-field components (let us consider \(d = 3\) for definiteness) we obtain the radial current and the dynamical chemical potential perturbation

\[
\mu(t,x) = \frac{2\alpha \Phi (\alpha^2 + \rho^2 + t^2)}{(\alpha^2 + \rho^2)^2 + 2t^2(\alpha^2 - \rho^2) + t^4},
\]

\[
J_\rho(t, \rho) = -\frac{4\alpha \Phi \rho t}{(\alpha^2 + \rho^2)^2 + 2t^2(\alpha^2 - \rho^2) + t^4}.
\]
where we introduced the radial space coordinate $\rho = \sum_i^{d} \sqrt{x_i^2}$ (in our case $\rho = \sqrt{x^2 + y^2}$). As in the one-dimensional quench we have the current, however in this case it has the spherical symmetry. For arbitrary $d$ the answer is essentially the same up to some prefactor.

Finding the exact HRRT surface in many dimensional case is complicated numerical problem and to get the insight in the entanglement we use the same method as in the previous section. We choose the subsystem to be the region bounded by the circle of radius $R$. The HRRT surface in this case has the form

$$z(\rho) = \sqrt{\ell^2 - \rho^2}.$$  

(3.28)

All calculation step of the entropy calculation are essentially the same as in the previous section and using formulas (3.21) we get the integral expression for the entanglement in the form

$$\Delta S \sim 2 \int_{0}^{\ell} \frac{8\alpha^4 Q^2 \rho^3 \sqrt{(\ell^2 - \rho^2)}}{\ell (\alpha^2 + 2\rho^2 + t^2 - \ell^2) + (\ell^2 - t^2)^2} \, d\rho,$$

(3.29)

and after integration we get the correction to the entanglement entropy

$$\Delta S \sim \frac{Q^2}{8\alpha} \left( \frac{3\alpha^4 + 2\alpha^2 (3\ell^2 + \ell^2) + 3 (\ell^2 - t^2)^2}{\ell^{\sqrt{(\alpha^2 + \ell^2)^2 + 2t^2(\alpha^2 - \ell^2) + t^4}}} \text{csch}^{-1} \left( \frac{\alpha^2 + \ell^2 - t^2}{2\alpha\ell} \right) - 6\alpha \right).$$

(3.30)

In Fig.5 we present the evolution corresponding to the quench in 3d-CFT by local charged operator We see that the qualitatively system resembles the entanglement evolution from the previous section. However there is the difference in some details of the evolution. The dip and ramp both are much more narrow and sharply peaked. Also one may notice that the behaviour is more monotonous - without additional maxima and minima. One can conclude that this is the universal behaviour.

4 Quench by the charged operator at finite temperature in 2d CFT

The operator local quench in 2d CFT admits finite temperature extension in the straightforward manner both on the gravity and quantum field theory sides. The holographic finite temperature local quench construction is based on the mapping of the metric (3.1) to the BTZ frame. The explicit form of the map between Poincare and
Figure 5. The entanglement entropy (approximate) evolution, for $d = 4$ for difference between $Q = 0.4$ and $Q = 0$. entanglement and the other parameters being fixed. The evolution is given for the interval $(-2, 2)$, $M = 0.4$, $\alpha = 0.25$.

BTZ frames is

$$\phi = \arctan \left( \frac{\varepsilon \sqrt{M} \sinh \left( \sqrt{M} x \right)}{\sqrt{1 - M z^2} \cosh \left( \sqrt{M} t \right) - \sqrt{1 - \varepsilon^2 M} \cosh \left( \sqrt{M} x \right)} \right),$$

$$\tau = - \arctan \left( \frac{\varepsilon \sqrt{M} \sqrt{1 - M z^2} \sinh \left( \sqrt{M} t \right)}{\sqrt{1 - \varepsilon^2 M} \sqrt{1 - M z^2} \cosh \left( \sqrt{M} t \right) - \cosh \left( \sqrt{M} x \right)} \right),$$

$$R = \frac{L}{2 \varepsilon M z} \sqrt{A_1 + A_2},$$

$$A_1 = -8 \sqrt{1 - \varepsilon^2 M} \sqrt{1 - M z^2} \cosh \left( \sqrt{M} t \right) \cosh \left( \sqrt{M} x \right) - 4 \varepsilon^2 M + 3 - M z^2,$$

$$A_2 = (2 - 2 M z^2) \cosh^2 \left( \sqrt{M} t \right) + (1 - M z^2) \cosh \left( 2 \sqrt{M} t \right) + 2 \cosh \left( 2 \sqrt{M} x \right).$$

and for $M = 0$ and $Q = 0$ this map brings the metric to the static BTZ form

$$ds^2 = \frac{L^2}{x^2} (- f(z) dt^2 + \frac{dz^2}{f(z)} + dx^2),$$

$$f(z) = 1 - M z^2, \quad T = \frac{1}{2 \pi z_h}, \quad M = 1/z_h.$$
For $Q \neq 0$ and $M \neq 0$ we get the dynamical metric corresponding to the BTZ black hole perturbed by the charged point-like object falling on the black hole horizon. The metric corresponding to this geometry has complicated form, so will not write down it explicitly here. The extension of the previous sections results concerning the evolution of the entanglement entropy of certain subregion is straightforward. We use the approximation (3.20) where the geodesic corresponding to the unperturbed metric has the form

$$z_{BTZ}(x, \ell_1, \ell_2) = \frac{2e^{\sqrt{M}(\ell_1+\ell_2)/2}}{\sqrt{M}} \frac{\sinh \left( \sqrt{M} (x - \ell_1) \right) \sinh \left( \sqrt{M} (\ell_2 - x) \right)}{e^{\sqrt{M}\ell_1} + e^{\sqrt{M}\ell_2}}. \quad (4.4)$$

We present the effect of the finite temperature on the evolution of the entanglement entropy after the quench by charged operator in Fig.6. We see that the evolution again is more complicated in comparison with the zero temperature case. One can observe the presence of two asymmetric peaks surrounding sharp entanglement dip. The first one is sharp and the second one is smooth.

![Figure 6](image)

**Figure 6.** The entanglement entropy evolution for interval of length $\ell$ after local charged operator quench at finite temperature for different values of $\varepsilon$. Here $M = 1$, $\mu = 0.1$, $Q = 0.8$, the interval is centered around quench point $x = 0$ and $\ell = 4$.

## 5 Concluding remarks

In this paper we have investigated the dynamics of the local charged operator quench. The quench is characterized by the initial nontrivial distribution of the chemical poten-
tial. During the evolution the chemical potential evolve and non-zero currents appear. We find that the entanglement of the symmetric interval around the quench point at some time shows sharp and sudden transition to significantly more disentangled state - the entanglement dip. After that it returns to the slowly decreasing entanglement regime. This result is universal and holds for higher dimension local quench at zero temperature. For a finite temperature 2d CFT the evolution is first shows sharp peak after some time evolution and then sharp dip. It would be interesting to obtain the generalization of this setup on the case of Lifshitz-like space and the generalization corresponding to the charged particle in the charged background.

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A The charged fields after the mapping

The gauge field corresponding to this dynamical background can be obtained in the straightforward manner also applying the mapping (3.8) to the static gauge field (3.2)

\[ A = A_t dt + A_x dx + A_z dz, \quad Q_{\text{eff}} = \left( \tilde{Q}_{\text{eff}}(t, x) + \log z \right), \]

\[ Q_{\text{eff}}(t, x) = Q \log \left( 2\alpha R_h \sqrt{\alpha^4 + 2\alpha^2 (t^2 + x^2 - z^2) + (-t^2 + x^2 + z^2)^2} \right), \]

\[ A_t = \frac{2\alpha (\alpha^2 + t^2 + x^2 + z^2)}{\alpha^4 + 2\alpha^2 (t^2 + x^2 + z^2) + (-t^2 + x^2 + z^2)^2} \cdot Q_{\text{eff}}(t, x), \]  

\[ A_x = -\frac{4\alpha x}{\alpha^4 + 2\alpha^2 (t^2 + x^2 + z^2) + (-t^2 + x^2 + z^2)^2} \cdot Q_{\text{eff}}(t, x), \]  

\[ A_z = -\frac{4\alpha z}{\alpha^4 + 2\alpha^2 (t^2 + x^2 + z^2) + (-t^2 + x^2 + z^2)^2} \cdot Q_{\text{eff}}(t, x). \]  

B The evolution of the metric on the constant time slice

![Figure 7](image-url)  

**Figure 7.** Here we present the density of the det of the metric of the constant time slice. We take the difference between \( Q = 0.1 \) and \( Q = 0 \) for \( M = 0.1 \) and fixed time moment \( t = 1 \). We see that around the particle position there is a negative contribution (colored by blue), while in the UV region there is a positive contribution.
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