A model of intra-annual flow distribution with scanty observational data

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Abstract. A model of intra-annual (mean annual and mean monthly) flow distribution and methods of its application to reconstructing the zonal runoff in West Siberia Plain are developed with scanty initial information. Analysis of the results shows that the model allows us to satisfactorily describe the observable changes in the total runoff and its groundwater component. The observed increase in the groundwater levels in the taiga zone of Western Siberia may occur even with decreasing annual precipitation. Increasing air temperature at the beginning and end of the winter period leads to an increase in the underground runoff. In addition, during the winter period the average air temperature increases and, as a consequence, the soil ice content decreases and the filtration properties of the soil increase.

1. Introduction

At present there are many systems for computer modelling of hydrological processes – MIKE-SHE, SMS, ECOMAG, SWAP, and others [1-4]. The purpose of such systems is to solve problems of assessing the current state of water objects and forecasting the elements of the hydrological regime within the limits of predictable meteorological conditions or from qualitative analysis based on climate change scenarios. However, in all these cases a large amount of meteorological, hydrological, geobotanical and other types of information is required. Also, the laboriousness of obtaining certain types of such information is compensated by the uncertainty of others [3]. For this reason, there is a need for coherence of the models of hydrological processes being used and initial information.

This aspect is especially important when solving insufficiently developed questions of paleohydrological reconstructions and modelling of the hydrological conditions of mineral deposits formation. Therefore, the goal of the study is to develop a model for intra-annual flow distribution (mean annual and mean monthly values) and methods of its application suitable for reconstructing the zonal runoff in Western Siberia with minimum initial information.

2. Mathematical model

The model was obtained at the following assumptions: 1) the change in the moisture content reserves of the river catchment area \( W_U \) is proportional to the change in the water runoff \( V_Y \) and the basin lag tiltmeter \( \tau \) for the period \( dt \) (1); 2) the change in the catchment area \( F \) is insignificant and can be neglected (2); 3) the velocity of the lag tiltmeter \( \beta \) is proportional to the velocity of the groundwater flow, which is proportional to the filtration coefficient in the saturated zone \( k_0 \), the soil moisture \( \omega \) minus its ice content \( \varepsilon \), and to the average gradient of the river \( J \) and the river length \( L \), or the amplitude of the heights \( (Z_{\text{max}} - Z_{\text{min}}) \) and the catchment area \( F \) (3); 4) the change in the velocity of lag tiltmeter is proportional to the coefficient of subsurface water flow (the ratio of the groundwater runoff...
and the total runoff ($Y$); 5) the humidification of the catchment area $H$ occurs due to liquid atmospheric precipitation $H_r$ falling out at positive air temperature $T_a$, and the water yield of the snow cover $H_{sm}$ (5); 6) the scheme for equation (2) has the form (6):

$$V_H = V_H - V_E - V_Y = \frac{dW_H}{dt} = \tau \cdot \frac{dY}{dt} + V_Y \cdot \frac{dt}{dt},$$  \hspace{1cm} (1)

$$\frac{dY}{dt} = \frac{d(F \cdot Y \cdot a)}{dt} \approx a \cdot F \cdot \frac{dr}{dt} = \frac{1}{\tau} \left( H - E - Y \cdot (1 + \frac{dt}{dt}) \right),$$  \hspace{1cm} (2)

$$\beta = \frac{1}{\tau} \approx \frac{k_0 (\omega - \varepsilon)^b}{F} \approx \frac{k_0 \cdot k_1 (\omega - \varepsilon)^b (Z_{max} - Z_{wla})}{F},$$  \hspace{1cm} (3)

$$Y_{t+1} = Y_t \cdot \frac{1 - 0.5 \Delta t \beta_t}{1 + 0.5 \Delta t \beta_{t+1}} + \frac{\Delta t}{2 + \Delta t \beta_{t+1}} \cdot \left( \beta_{t+1} \cdot H_{t+1} + \beta_t \cdot H_t + (\beta_{t+1} + \beta_t) \cdot k_3 \cdot Y_{gr} \right) + \mu_0,$$  \hspace{1cm} (4)

where $V_H$, $V_E$, $V_Y$, $V_U$ are the volume of humidification, evaporation, total (surface and groundwater) runoff and change in the moisture content for the period $dt$; $H$, $E$, $Y$ are the depth of humidification, evaporation, and total runoff for the same period; $\alpha$ is the coefficient of dimension; $\Delta t$ is the time step (here $\Delta t = 1$); $k_1$, $k_2$, $b$ are the empirical coefficients; $\mu$ is the closing error of the calculation scheme with mathematical expectation $\mu_0$; $Z_{max}$ is the maximum height of the surface watershed; $Z_{wla}$ is the average river level in the calculated range. The values $k_0$, $k_1$, $k_2$, $b$, $\mu_0$ are determined by optimization methods.

The water yield of the snow cover in month $t$ is determined by the difference in the moisture reserves $WS$ for the months $t$ and $(t-1)$ using the degree-day melting factor [5], data on the hydromorphological characteristics of the Western Siberia rivers [6], and the distribution of coniferous and deciduous forests in the territory of their catchments [7].

The evaporation from the catchment area surface $E_i$ in the $i$-th month was calculated by the Hargrave equation taking into account the recommendations [8]:

$$E_t = k_4 \cdot M_t \cdot R_0 \cdot (T_{at} + k_5) \cdot \sqrt{T_{max,t} - T_{min,t}},$$  \hspace{1cm} (7)

where $R_0$ is the extraterrestrial radiation, kJ/(cm$^2$ month); $T_{max,t}$, $T_{min,t}$ are the maximum and minimum monthly values of the air temperature, $^\circ$C; $k_4$ and $k_5$ are the empirical coefficients calculated by selection (the method of general decreasing gradient in MS Excel) under the condition of maximum approximation to the values of the mean annual monthly evaporation calculated by the Penman-Tortweitz method [8, 9] from the measurements of the elements of the heat balance at a meteorological station in Western Siberia. As an estimate of the degree of approximation $R^2$, the Nash and Sutcliffe criterion was used [10]. According to data for the watersheds of rivers of the taiga, forest-tundra, and forest-steppe zones of Western Siberia, $k_4=0.004$ and $k_5=30$.

According to [5], the calculation of soil moisture is carried out according to the empirical formulas (8-10) for the given average values of the measured moisture reserves in the one-meter layer of soil and ground for the third decade of May and August [7]:

$$\omega_t \approx \frac{\omega_{1b,t} + \omega_{1e,t}}{2000},$$  \hspace{1cm} (8)

$$\omega_{1e,t} = (\omega_{1b,t} + H_{r,t} + H_{sm,t}) \cdot \exp(-0.007 \cdot E_{0,t}),$$  \hspace{1cm} (9)

$$E_{0,t} = 0.18 \cdot (T_{at} + 25)^2 \cdot (1 - f_{at}),$$  \hspace{1cm} (10)

where $\omega$ is the monthly moisture content of the soil moisture, m$^3$/m$^3$; $\omega_{1b,t}$ and $\omega_{1e,t}$ are the moisture contents in the meter layer of soil at the beginning and end of the $i$-th month, mm; $E_{0,t}$ is the evapotranspiration, mm/month; $f_{at}$ is the average monthly relative atmospheric air humidity in fractions of unity. The calculation starts from the end of March.

The initial moisture content in the meter soil layer (at the end of March) is selected from the condition of minimum of the sum of the absolute deviation calculated and measured values. The
calculation of the initial moisture \( \varepsilon \) for months with a negative soil temperature is made by selection (the method of general downward gradient in MS Excel) according to [4, 11].

The average monthly soil temperature at a depth of 0.5 m from the surface is determined from the empirical dependence (11) obtained by the least squares method from the interpolation data of the mean monthly soil temperatures at depths of 0.4 and 0.8 m [12-13]:

\[
T_{s,t} = k_6 \cdot T_{a,t} + k_7,
\]

where \( k_6 = 0.589 \pm 0.057;\) \( k_7 = 5.586 \pm 0.216;\) the square of the correlation ratio between the calculated and measured values \( R^2 = 0.80.\) The groundwater component of the river flow \( Y_{gr} \) is determined according to [14]. The intra-annual groundwater flow distribution (based on observations and modeling data) is obtained as follows: 1) during the winter runoff low (from December to March) the groundwater runoff is assumed to be equal to the total one; 2) in the remaining months the groundwater runoff is determined by linear interpolation between the values of the groundwater runoff in March and December.

Thus, the initial information for the monthly water runoff model (1-11) is the average monthly air temperature, the monthly amount of precipitation, the average monthly relative humidity, and the initial moisture content at the end of the winter runoff low. The model can be used in two cases: 1) for studying the mechanism of water flow formation and the factors of its change; 2) for reconstructing the intra-annual flow distribution. In the first case the monthly values of the total runoff layer are also related to the initial data, and the initial data are taken for a conditionally homogeneous period according to climatic and hydrological directories. In the second case the calculation begins at the end of the winter runoff low, when a value of 11% of the annual groundwater flow \( Y_{gr} \) is taken as the monthly total runoff; the remaining initial data are taken from a typical intra-annual distribution for natural zones with mean annual precipitation and air temperature values determined by paleobotanical and other methods.

3. Results and discussion
Testing of the model is carried out by using the date of the hydrological and climatic conditions in the watersheds of medium rivers - the major tributaries of the Ob River (Table 1) for two periods - from 1971 to 1994 and after 1995. This choice is determined by the results of analysis of long-term changes in the annual rivers runoff in the region and its groundwater component, indicating a disruption in the homogeneity of the series around the border of the 1960s-1970s and by the groundwater runoff in the mid-1990s. The values of \( Z_{sela} \) are taken according to [15], the values of \( F, L, J, Z_{sela} \) according to [6], the values of \( Z_{max} \) according to topographic maps; and the remaining values are determined by selection (the method of general reducing gradient) in Excel.

| River – point of observation, characteristic | Tym River - v. Napas | Chaya River - v. Podgornoye | Vasugan River – v. Sredniy Vasugan | Ket River – v. Maksimkin Yar |
|---------------------------------------------|----------------------|-----------------------------|----------------------------------|----------------------------|
| Meteorological station                      | Vanil-Kynak          | Podgornoye                  | Sredniy Vasugan                  | Ust-Ozernoe                |
| Drainage area \( F, \text{ km}^2 \)        | 24500                | 31700                       | 25000                           | 38400                      |
| River length \( L, \text{ km} \)           | 678                  | 812                         | 434                             | 1010                       |
| Weight-average gradient of river \( J, \text{ m/km} \) | 0.12                | 0.07                        | 0.21                            | 0.14                       |
| Median elevation of basin \( Z_{bela}, \text{ m} \) | 120                 | 110                         | 120                             | 160                        |
| Mean water level                            | 63.84                | 55.36                       | 62.90                           | 87.12                      |
River – point of observation, characteristic

| River               | Tym River – v. Napas | Chaya River – v. Podgornoye | Vasugan River – v. Srednyi Vasugan | Ket River – v. Maksimkin Yar |
|--------------------|----------------------|-----------------------------|-----------------------------------|-------------------------------|
| \( Z_{ult}, \) m   | 163                  | 138                         | 151                               | 360                           |
| Maximum elevation in the basin \( Z_{max}, \) m |                      |                             |                                   |                               |
| \( R^2 \)           | 0.96                 | 0.74                        | 0.63                              | 0.93                          |
| \( k_0 \cdot k_1 \) in equation (3) | 24500               | 31700                       | 25000                             | 38400                         |
| \( b \) in equation (3) | 678                 | 812                         | 434                               | 1010                          |
| \( \Delta t \) in equation (6) | 0.12                | 0.07                        | 0.21                              | 0.14                          |
| \( k_3 \) in equation (6) | 120                 | 110                         | 120                               | 160                           |
| \( \mu_0 \) in equation (6) | 63.84               | 55.36                       | 62.90                             | 87.12                         |

Analysis of the results of testing of the models (1-11) showed that, first, the use of the model allows one to satisfactorily describe the observed change of the total runoff and groundwater component with minimum information. Second, there is an increase in the air temperature on average for the calculated multi-year periods and for March-April. A statistically significant increase in the atmospheric precipitation is not observed everywhere, but even with constant atmospheric moisture and warming in early spring the total run-off in these months increases (Fig. 1) due to an increase in the water yield of the snow cover and conditionally liquid precipitation. In addition, there is an underground component of the runoff for the year as a whole, which is explained by an increase in the groundwater runoff in the winter period, a decrease in the ice content (increase in the filtration properties), and an increase in the amount of atmospheric precipitation for the warm period - one of the important factors of infiltration.

Y, mm

![Graph showing runoff over months for different years](image-url)
Third, during the last decades there has been an increase in the resources of bog and groundwater, as evidenced by negative values of $k_2$ and $\mu_0$, and this is confirmed by data on the vertical growth of the peat deposits at about 1 mm/year [16] and increase in the groundwater levels in Tomsk [15, 17].

4. Conclusions
The above model is suitable mainly for studying the water balance of medium-sized rivers, in which changes in the runoff are, in general, subject to zonal regularities. For small rivers, some correction is required taking into account the local geological conditions, and for large rivers an integral assessment of the runoff transformation due to flows coming from various natural zones is needed. The above-developed methodology for calculating the intra-annual flow distribution and the results of groundwater flow reconstruction will allow further research on constructing a theory of mineral deposits formation.

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