Maximal neutron star mass and the resolution of hyperon puzzle in modified gravity

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The so-called “hyperon puzzle” in the theory of neutron stars is considered in the framework of modified f(R) gravity. We show that for simple hyperon equations of state, it is possible to obtain the maximal neutron star mass which satisfies the recent observational data for PSR J1614-2230. In higher-derivative models with power-law terms as \( f(R) = R + \alpha R^2 + \beta R^3 \), the soft hyperon equation of state under consideration is usually treated as non-realistic in the standard General Relativity. The numerical analysis of Mass-Radius relation for massive neutron stars with hyperon equation of state in modified gravity turns out to be consistent with observations. Thus, we show that the same modified gravity can solve at once three problems: consistent description of the maximal mass of neutron star, realistic Mass-Radius relation and account for hyperons in equation of state.

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I. INTRODUCTION

The recent discovery of the pulsar PSR J1614-2230 [1] has set rigid constraints on various matter equations of state (EOS) for neutron stars at high densities. There are other indications in favor of the existence of massive neutron stars: 1.8\( M_\odot \) for Vela X-1 [2] and 2\( M_\odot \) for 4U 1822-371 [3]. In particular, this new limits on maximal mass of neutron star excluded many EOS, including hyperons and/or quarks EOS. According to the experimental data [4] and realistic models for strong interactions, the appearance of exotic particles occurs at densities \( 5 \times 10^{14} \text{ g/cm}^3 \). However, the hyperonisation softens EOS and the maximal allowable mass is reduced considerably [5-9]. The neutron stars with \( M > 2 M_\odot \) cannot be obtained in the framework of Thomas-Fermi model for non-uniform matter [10] with hyperon inclusion [11,12] or a quark-hadron phase transition [13].

The solution of such a “hyperon puzzle” can be searched, in principle, by constructing the hyperon EOS giving the maximal mass of neutron star around 2\( M_\odot \). The required stiffness of the EOS can be achieved in relativistic mean field theory (RMF) with hyperon-vector coupling larger than it follows from SU(6) symmetry models [14,15]. A model with chiral quark-meson coupling with \( M_{\text{max}} = 1.95 M_\odot \) has been recently considered [16]. The quartic vector-meson terms in the Lagrangian also lead to the stiffening of EOS and large neutron star mass [17]. The radius measurements of neutron stars could give more information about EOS for dense matter. Unfortunately, one has no such measurements for any neutron stars with a precise mass determination. Nevertheless, there are some astrophysical observations that could lead to the extraction of neutron star radii [18].

As shown in [19], based on data for radii and masses of three neutron stars (in EXO 1745-248 [20], in 4U 1608-52 [21] and in 4U 1820-30 [22]), the EOS with only nucleonic degrees (such as AP4, MP1 and MS1) are too stiff at higher density. A softer EOS describes these data with better precision. In Fig. 1 these data and the theoretical \( M - R \) relation for some hyperon EOS are shown. The following feature is obvious: although soft hyperon EOS predict the maximal value of mass \(< 2 M_\odot \), these EOS are more compatible with data by [20,22].

In Ref. [24], it is shown that, in the case of hyperonic matter with three exchange meson fields, the maximal mass is achieved only for low values of the effective nucleon mass. The addition of strange meson \( \psi \) allows to increase the maximal value of effective mass. Therefore, for the explanation of the new maximal limit of neutron star mass, one needs to complicate the simple ‘\( \rho \omega \sigma \)’-model. In fact, this complication leads to the stiff hyperon EOS which are close to pure nucleonic EOS (such as MP1, MS1) and we have contradiction with the data in [20,22].

One can assume that such contradiction can be considered as a further indication in favor of the necessity to re-examine gravity theory at the early/late universe or in strong field regimes.

The initial motivation for this approach has been pursued starting from the observed accelerated expansion of the early/late universe. This fact has been confirmed by observations data. First of all, type Ia supernovae point out an accelerated expansion which cannot be obtained by standard perfect fluid matter as the source for the cosmological Friedman-Robertson-Walker equations [23-27]. Second, one can mention the observations of microwave background
radiation (CMBR) anisotropy \cite{28}, of cosmic shear through gravitational weak lensing surveys \cite{29} and, finally, data coming from Lyman alpha forest absorption lines \cite{30}. To explain the universe acceleration within General Relativity (GR), one needs to postulate the existence of some cosmic fluid with negative pressure (dark energy). In the framework of ΛCDM model, dark energy is nothing else but the Einstein Cosmological Constant and its density is about 70% of the global energy budget of the universe. The remaining 30%, clustered in galaxies and clusters of galaxies, should be constituted only for about 4% by baryons and for the rest by cold dark matter (CDM) the nature of which is, up to now, unclear.

Despite of the simplicity and the good agreement with observational data, the ΛCDM model has some fundamental problems at theoretical level. For example, one needs to explain the difference of 120 orders of magnitude between its observed value at cosmological level and the one predicted by quantum field theory/gravity \cite{32}.

From another viewpoint, the accelerated expansion of the universe (without dark components) maybe naturally explained by modification of gravity at the very early and very late universe. Indeed, modified gravity may provide viable gravitational candidate for dark energy (see refs. \cite{33,34} as well as for unification of dark energy and early-time inflation \cite{34} (for recent review of modified gravity, see \cite{36,37}). For instance, it has been shown that such theories give models which are able to reproduce the Hubble diagram derived from SNeIa observations \cite{38,40} and the anisotropies of CMBR \cite{41,42}.

Addressing the problem of exotic relativistic stars in modified gravity, in comparison with GR, could represent a testbed for modified gravity. For example, some models of $f(R)$ gravity do not allow the existence of stable star configurations \cite{43,44,45,46,47,48,49} and thus are considered unrealistic. However the existence of stable star configurations can be achieved in certain cases due to the so-called Chameleon Mechanism \cite{51,52} or may depend on the chosen EOS.

In this paper, we present the models of neutron star for simple hyperon EOS with maximal mass $\sim 2 M_\odot$ in the framework of analytic $f(R)$ models. We show that it is possible to address simultaneously the maximal value of neutron star mass as well as fit the data by \cite{19} assuming a hyperon EOS for dense matter. The paper is organized as follows. In Section II, we investigate the field equations for $f(R)$ gravity and the modified Tolman–Oppenheimer–Volkoff (TOV) equations. Then neutron star models with hyperon EOS in power-law modified gravity are considered. Mass-Radius diagram is derived and compared with the one of GR. The possibility to get maximal mass for neutron stars and consistent Mass-Radius relation for hyperon EOS within modified gravity is established. Conclusions and outlook are given in Section III.
II. MODIFIED TOV EQUATIONS IN $f(R)$ GRAVITY

The general action for $f(R)$ gravity is given by

$$S = \frac{e^4}{16\pi G} \int d^4x \sqrt{-g} f(R) + S_{\text{matter}}.$$ (1)

Here $g$ is the determinant of the metric $g_{\mu\nu}$ and $S_{\text{matter}}$ is the action of the standard perfect fluid matter. The variation of (1) with respect to $g_{\mu\nu}$ gives the field equations. The function $f(R)$ can be written as

$$f(R) = R + \alpha h(R),$$ (2)

putting in evidence the extra contributions with respect to GR. The field equations are

$$(1 + \alpha h R)G_{\mu\nu} - \frac{1}{2} \alpha (h - h R R) g_{\mu\nu} - \alpha(\nabla_{\mu} \nabla_{\nu} - g_{\mu\nu} \Box) h = 8\pi G T_{\mu\nu}/c^4.$$ (3)

Here $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ is the Einstein tensor and $h = \frac{dh}{dR}$.

For the star configurations, one can assume a spherically symmetric metric with two independent functions of radial coordinate, that is:

$$ds^2 = -e^{2\lambda}c^2 dt^2 + e^{2\lambda}dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2).$$ (4)

Then the following change of variable can result convenient [54, 55]

$$e^{-2\lambda} = 1 - \frac{2GM}{c^2 r}.$$ (5)

For the exterior solution, we assume a Schwarzschild solution and therefore the value of variable $M$ on the star surface is nothing else but the gravitational mass. For a perfect fluid, the energy-momentum tensor is $T_{\mu\nu} = \text{diag}(e^{2\phi} \rho c^2, e^{2\phi} P, r^2 P, r^2 \sin^2 \theta P)$, where $\rho$ is the matter density and $P$ is the pressure. The components of the field equations become

$$-8\pi G \rho/c^2 = -r^{-2} + e^{-2\lambda}(1 - 2r\lambda')r^{-2} + \alpha h R [-r^{-2} + e^{-2\lambda}(1 - 2r\lambda')r^{-2}]
\quad \frac{1}{2} \alpha (h - h R R) + e^{-2\lambda} \alpha [h' R r^{-1} (2 - r\lambda') + h'' R],$$ (6)

$$8\pi G P/c^4 = -r^{-2} + e^{-2\lambda}(1 + 2r\phi')r^{-2} + \alpha h R [-r^{-2} + e^{-2\lambda}(1 + 2r\phi')r^{-2}]
\quad \frac{1}{2} \alpha (h - h R R) + e^{-2\lambda} \alpha [h' R r^{-1} (2 + r\phi')],$$ (7)

where $' \equiv d/dr$.

The combination of the conservation law equation with Eq. (7) allows to obtain the second TOV equation. Finally, modified TOV equations take the following convenient form [59] (see, also [60])

$$\left(1 + \alpha h R + \frac{1}{2} \alpha h' R r \right) \frac{dm}{dr} = 4\pi rv^2 - \frac{1}{4} \alpha v^2 \left[h - h R R - 2 \left(1 - \frac{2m}{r}\right) \left(\frac{h' R}{r} + h'' R\right)\right],$$ (8)

$$8\pi p = -2 \left(1 + \alpha h R\right) \frac{m}{r^3} - \left(1 - \frac{2m}{r}\right) \left[\frac{2}{r} (1 + \alpha h R) + \alpha r^2 h' R \right] (\rho + p)^{-1} \frac{dp}{dr}$$ (9)

$$-\frac{1}{2} \alpha \left[h - h R R - 4 \left(1 - \frac{2m}{r}\right) \frac{h' R}{r}\right],$$

Here we use the dimensionless variables $M = m M_\odot$, $r \rightarrow r g$, $\rho \rightarrow \rho M_\odot/r g^3$, $P \rightarrow p M_\odot c^2/r g^3$, $R \rightarrow R/r g$, $\alpha r^2 h(R) \rightarrow \alpha h(R)$, where $r g = GM_\odot/c^2 = 1.47473$ km.

For the Ricci curvature scalar one can get the following equation:

$$3\alpha r^2 g \left\{ \left[\frac{2}{r} - 3m}{r^2} - \frac{dm}{r^2 dr} - \left(1 - \frac{2m}{r}\right) \frac{dp}{(\rho + p) dr}\right\} \frac{d}{dr} + \left(1 - \frac{2m}{r}\right) \frac{d^2}{dr^2} \right\} h R + \alpha r^2 h R R - 2\alpha r^2 h R - 8\pi (\rho - 3p) = -8\pi (\rho - 3p).$$ (10)
Using the mean-field approximation, one obtains the following equations for meson fields:

\[
\mathcal{L} = \sum_b \bar{\psi}_b \left[ \gamma_\mu (i\not{D} - g_{\sigma b} \not{\Omega} - \frac{1}{2} g_{\rho b} \not{\gamma} \rho \not{\gamma} \sigma) - (m_b - g_{\sigma b} \sigma) \right] \psi_b + \sum_i \bar{\psi}_i (\gamma_\mu (i\not{D} - m_i) \psi_i + \frac{1}{2} (\partial_\mu \sigma)^2 - m_\sigma^2 \sigma^2) - V(\sigma) + \frac{1}{2} \omega_\mu \omega^{\mu \nu} - \frac{1}{4} \rho_{\mu \nu} \rho_{\mu \nu} + \frac{1}{2} m_\rho^2 \rho_\mu^2.
\]

Using the mean-field approximation, one obtains the following equations for meson fields:

\[
m_\sigma^2 \sigma = \sum_b g_{\sigma b} n_b, \quad m_\omega^2 \omega_0 = \sum_b g_{\omega b} n_b, \quad m_\rho^2 \rho_0 = \sum_b g_{\rho b} n_b.
\]

Here \(n_b, n_b\) are the scalar and vector baryon number densities, correspondingly. We consider the GM2 and GM3 parametrization (the nucleon-meson couplings and scalar field potential \(V(\sigma)\) are given in [2]). The hyperon-meson couplings are assumed to be fixed fractions of nucleon-meson couplings, i.e. \(g_{hi} = x_{iH} g_{iN}\), where \(x_{iH} = x_{\rho H} = 0.600, x_{\omega H} = 0.653\) (see [55]).

For chemical potential of baryons and leptons, one has

\[
\mu_b = E_b^f + g_{\omega b} \omega_0 + g_{\rho b} \rho_0 + \Sigma^R, \quad \mu_i = E_i^f.
\]

Here \(E_b^f\) is the Fermi energy, for baryon \(E_b^f\) is related to the Fermi momentum \(k^2\) as \(E_b^f = (k^2 + m_b^2)^{1/2}\), where \(m_b = m_b - g_{\sigma b} \sigma\) is the effective mass of the baryon. The rearrangement self-energy term is defined by

\[
\Sigma^R = -\frac{\partial \ln g_{\sigma N}}{\partial n} m_\sigma^2 \sigma^2 + \frac{\partial \ln g_{\omega N}}{\partial n} m_\omega^2 \omega_0^2 + \frac{\partial \ln g_{\rho N}}{\partial n} m_\rho^2 \rho_0^2.
\]

Here \(n = \sum_b n_b\). The following conditions should be imposed on the matter for obtaining the EOS:

(i) baryon number conservation:

\[
\sum_b n_b = n,
\]

(ii) charge neutrality:

\[
\sum_i q_i n_i = 0, \quad i = b, l,
\]

(iii) beta-equilibrium conditions:

\[
\mu_n = \mu_\Lambda = \mu_{\Xi^0} = \mu_{\Sigma^0}, \quad \mu_\rho = \mu_{\Sigma^+} = \mu_\Lambda - \mu_e, \quad \mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + \mu_e, \quad \mu_e = \mu_\mu.
\]

At given \(n\), Eqs. (12)-(16) can be numerically solved. The resulting EOS are sufficiently soft (for analytical parametrization see [23]) and in GR, one cannot obtain the stars with maximal mass \(\sim 2M_\odot\) (see Fig. 1, curves labelled as gm2nph and gm3nph). However, as we demonstrate below, the situation is qualitatively different in modified gravity.

For the solution of Eqs. (8)-(10), one can use a perturbative approach (see [57, 58, 60] for details). For a perturbative solution, the density, pressure, mass and curvature can be expanded as

\[
p = \rho^{(0)} + \alpha \rho^{(1)} + ..., \quad \rho = \rho^{(0)} + \alpha \rho^{(1)} + ..., \quad m = m^{(0)} + \alpha m^{(1)} + ..., \quad R = R^{(0)} + \alpha R^{(1)} + ..., \quad dh_R\]

where functions \(\rho^{(0)}, \rho^{(0)}, m^{(0)}, \) and \(R^{(0)}\) satisfy the standard TOV equations. Terms containing \(h_R\) are assumed to be of first order in the small parameter \(\alpha\), so all such terms should be evaluated at \(O(\alpha)\) order.

Finally, perturbative TOV equations are, for mass \(m = m^{(0)} + \alpha m^{(1)}\):

\[
\frac{dm}{dr} = 4\pi \rho r^2 - \alpha^2 4\pi \rho^{(0)} h_R + \frac{1}{4} (h - h_R R) + \frac{1}{2} \alpha \left[ (2r - 3m^{(0)} - 4\pi \rho^{(0)},r^2) \frac{d}{dr} + r(r - 2m^{(0)}) \frac{d^2}{dr^2} \right] h_R.
\]
FIG. 2: The Mass-Radius diagram for GM2 model extended to hyperon sector (gm2nph) in modified gravity model $f(R) = R + \beta R^3$ and in GR, for comparison. For $\beta \approx -45$ (in units of $r^4$), the maximal limit of mass for star is around $2M_\odot$. The corresponding central density is $3.48 \times 10^{15} \text{ g/cm}^3$.

and for pressure $p = p^{(0)} + \alpha p^{(1)}$:

$$
\left( \frac{r - 2m}{\rho + p} \right) \frac{dp}{dr} = 4\pi r^2 p + \frac{m}{r} - \alpha r^2 \left[ 4 \pi p^{(0)} h_R + \frac{1}{4} (h - h_R R) \right] - \alpha \left( r - 3m^{(0)} + 2 \pi p^{(0)} r^3 \right) \frac{dh_R}{dr}.
$$

In [59], the Mass-Radius relation for the neutron stars, in particular, for modified gravity with $f(R) = R + \alpha R^2 (1 + \gamma R^2)$ is considered. In that case, we found that, for high central densities a second “branch” of stability emerges with respect to the one existing in GR. This stabilization of star configurations occurs due to the presence of cubic term in the Ricci curvature scalar.

For modified gravity with only cubic term (that is $f(R) = R + \beta R^3$) the maximal value of neutron star mass for given EOS increases for $\beta < 0$. This effect allows to construct neutron star models with maximal mass $\sim 2M_\odot$ even for those hyperon EOS which do not satisfy the observational constraints coming from standard GR. In other words, these stable star configurations can exist at higher central densities than in GR.

In Figs. 2 and 3, the Mass-Radius diagram for simple hyperon models (gm2nph and gm3nph) with realistic parameters is represented. We define the values of the parameter $\beta$ for obtaining the star configurations with $M \sim 2M_\odot$.

Note that the parameter $\alpha$ in modified TOV equations can be defined in our case as $\alpha = \beta |R^{(0)3}|_{\text{max}}$, where “max” means maximal value of cubic term at $O(\alpha)$ order. The scalar curvature $R^{(0)}$ is simply

$$
R^{(0)} = 8\pi (\rho^{(0)} - 3p^{(0)}).
$$

One can determine the dimensionless parameter

$$
\delta = \beta R^{(0)2}.
$$

In Fig. 4, the dependence of this parameter from density (for star configuration with maximal mass $\sim 2M_\odot$) is represented for gm2nph and gm3nph model. One can see that the cubic term is small if compared to $R$ even for high central densities.

The density profile even for star configuration with maximal mass $2M_\odot$ is almost the same as the corresponding profile for star model in GR (see Fig. 5). The increase of the “effective” density (and maximal mass) occurs due to the terms containing $\alpha$ in r.h.s. of Eq. (13).

The increase of maximal neutron star mass occurs for realistic $f(R)$ model of gravity with quadratic and cubic terms, that is

$$
f(R) = R + \alpha R^2 + \beta R^3.
$$

(20)
FIG. 3: The Mass-Radius diagram for gm3nph model in modified gravity \( f(R) = R + \beta R^3 \) and in GR for comparison. For \( \beta \approx -40 \) (in units of \( r_g^4 \)) the maximal limit of mass for star is around \( 2M_\odot \). The corresponding central density is \( 3.34 \times 10^{15} \) g/cm\(^3\).

FIG. 4: The density dependence of dimensionless parameter \( \delta = \beta R_0^2 \) for gm2nph (\( \beta = -40 \)) and gm3nph (\( \beta = -45 \)) models in cubic gravity. The maximal value is less than 0.1 even for central regions of star.

The effect occurs for \( \alpha < 0 \) if cubic term is greater than quadratic at high densities. For a given value of \( \alpha \), one can define the parameter \( \beta \) where the maximal value of neutron star mass is \( \sim 2M_\odot \).

Furthermore, let us consider the case of gm3nph EOS. In Fig. 6, the realistic Mass-Radius diagram is represented for two values of \( \alpha = 5 \times 10^9 \) cm\(^2\), \( 1 \times 10^{10} \) cm\(^2\) (or \( \sim 0.22 \) and \( \sim 0.45 \) in units of \( r_g^2 \)). One can see that the two solar mass limit is reached for \( \beta \approx 35 \) and \( \beta \approx 30 \) for these values of \( \alpha \). In fact, the \( M - R \) relation, in this case, is close to the \( M - R \) relation for \( f(R) \) model without quadratic term. The analysis shows that there is a set of parameters \( (\alpha; \beta) \) at which we have the same \( M - R \) relation. Hence, analytical \( f(R) \) gravity models with quadratic and cubic terms may provide the resolution of neutron star maximal mass and hyperon puzzle problems, being consistent, at the same time, with the M-R diagram.
FIG. 5: The density profile for gm3nph model in modified gravity (for $\beta = -40$ and $\rho_c = 3.34 \times 10^{15}$ g/cm$^3$) in comparison with the one in GR. The difference is insignificant at high densities. The increase of “effective” ($\rho_{eff} = \frac{1}{4\pi} \frac{dm}{dr}$) density (and maximal mass) occurs due to the terms containing $\alpha$ in r.h.s. of Eq. (18). A similar picture takes place for gm2nph model.

FIG. 6: The Mass-Radius diagram for gm3nph EOS in modified gravity $f(R) = R + \alpha R^2 + \beta R^3$ with maximal mass $\sim 2M_\odot$ for two values of $\alpha$. These curves are close to M-R relation in the model with only the cubic term (see Fig. 3). Note that the quadratic term is smaller than the cubic one for given EOS if $\rho > \rho_0 \approx 2 \times 10^{14}$ g/cm$^3$.

### III. CONCLUSIONS AND PERSPECTIVES

In summary, we presented a possible solution of the “hyperon puzzle” in the neutron star theory. The softening of nucleon EOS, due to hyperonization, leads to the decrease of the upper limit mass of neutron star considerably below the two solar masses (in the simple model of hyperonic matter with realistic parameters) according to GR. However, in modified $f(R)$ gravity model with cubic and quadratic terms, it is possible to obtain neutron stars with $M \sim 2M_\odot$ for simple EOS from GM2 and GM3 model extended to hyperon sector. Of course, modified gravity under consideration is chosen to be in power-law analytic form as a simple example. However, the preliminary estimations indicate that similar effect may be expected for viable modified gravities where the analysis is more detailed and
realistic stellar models are considered. Note also that power-law $f(R)$ models are the standard approximation for more complicated non-linear $f(R)$ gravities. However, it is important to point out that the Mass-Radius relation significantly differs from GR only at high central densities. As consequence, the “effective” EOS is sufficiently soft to describe the radii and masses measurements for the three observed neutron stars EXO 1745-248, 4U 1608-52 and 4U 1820-30. In other words, the same modified gravity may solve simultaneously three problems of neutron stellar astrophysics: maximal mass of neutron star, realistic Mass-Radius relation and hyperon puzzle. As a next step, it would be very interesting to extend our results to non-perturbative treatment of TOV equations. However, up to now, it seems a very hard problem (see, for instance,[61]) which may need the development of qualitatively new numerical methods due to higher-derivative structure of $f(R)$ gravity and necessity to account for chameleon effects as well as quantum gravity effects at very high densities.

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