The Anti-Unruh Effect

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We find that a uniformly accelerated particle detector coupled to the vacuum can cool down as its acceleration increases, due to relativistic effects. We show that in (1+1)-dimensions, a detector coupled to the scalar field vacuum for finite timescales (but long enough to satisfy the KMS condition) has a KMS temperature that decreases with acceleration, in certain regimes. This contrasts with the heating that one would expect from the Unruh effect.

Introduction.- In 1976, it was proposed that the inequivalence of field quantization schemes associated with inertial and accelerated observers implied that observers uniformly accelerating in the Minkowski vacuum (as seen by inertial observers) would detect a thermal bath of particles [1]. Specifically, an accelerated particle detector coupled to the Minkowski vacuum would experience a thermal response [2], a phenomenon known as the Unruh effect. The temperature $T$ of this thermal bath was found to be proportional to the magnitude $a$ of the proper acceleration of the detector, with $T = a/2\pi$. The Unruh effect has been predicted and derived in contexts as disparate as axiomatic quantum field theory [3], via Bogoliubov transformations [2], and in studies of the response of non-inertial particle detectors both perturbatively [2] and non-perturbatively [4–7], and even for non-uniformly accelerated trajectories [8, 9]. More recently non-perturbative techniques developed in [11] have been used to prove that within optical cavities in (1+1)-dimensions an accelerated detector equilibrates to a thermal state whose temperature is proportional to acceleration. This holds independently of the cavity boundary conditions, provided the detector is allowed enough interaction time [10].

Since all investigations so far have found that a particle detector coupled to the vacuum will detect more particles when it is accelerated than when undergoing inertial motion, we typically regard the Unruh effect as a universally true phenomenon: simply put, ‘accelerated detectors get hotter’. The common denominator in nearly all previous investigations is that the response of non-inertial detectors is studied for long interaction times, or for a field quantized in free infinite open space. However on empirical grounds, finite time studies with different boundary conditions are arguably relevant. Any experimental setup based on quantum optics (e.g. an atom accelerating through an optical cavity) will necessarily require particular boundary conditions rather than infinite space.

But do accelerated detectors always become hotter?

In this paper we address this question using both perturbative and non-perturbative tools. Previous numerical work on accelerating Unruh-deWitt detectors in cavities interacting for long times found that, as expected, a detector gets hotter and its temperature is proportional to its acceleration, $T \propto a$ [10]. However, due to the finite length and time scales, the slope was not found to be $1/2\pi$. In this paper we find that when shorter interaction times are considered, the transition probability of an accelerated detector can actually decrease with acceleration. This is possible because even an inertial detector switched on for a finite time in the ground state, and coupled to the Minkowski vacuum, will not remain completely ‘cold’ but will click due to switching noise and vacuum fluctuations (see [11] and [4] for a perturbative and non-perturbative analysis respectively). More surprisingly we find that this is not due to non-equilibrium effects; rather, the response of such detectors can be regarded as thermal.

Transition probability of an accelerated detector.- To model the field-detector interaction it is commonplace to use the Unruh-DeWitt (UDW) model [12], which consists of a point-like two-level quantum system that couples to a scalar field along its trajectory. We will first regard spacetime as a flat static cylinder with spatial circumference $L > 0$ (we will later consider the limit $L \rightarrow \infty$). This cylinder topology is equivalent to imposing periodic boundary conditions relevant to laboratory systems including closed optical cavities, such as optical-fibre loops [13], and superconducting circuits coupled to periodic microwave guides [14, 15].

The coupling of the field to the detector is described by the UDW Hamiltonian [12]

$$H_I = \lambda \chi(\tau)\mu(\tau)\phi(x(\tau), t(\tau))$$

(1)

where $\tau$ is the detector’s proper time, $\mu(\tau) = \sigma_z(\tau) = e^{i\theta(\tau)}\sigma^+ + e^{-i\theta(\tau)}\sigma^-$ is the detector’s monopole moment (with $\sigma^\pm$ being SU(2) ladder operators), and $\chi(\tau)$ is the switching function. For most of the paper we will consider $\chi(\tau)$ to be Gaussian

$$\chi(\tau) = e^{-\tau^2/2\sigma^2}$$

(2)

so that $\sigma$ establishes the timescale of the interaction between the field and the detector. The time evolution op-
erator under this Hamiltonian is given by the following perturbative expansion:

\[ U = \mathbb{1} + U^{(1)} + \mathcal{O}(\lambda^2) = \mathbb{1} - i \int_{-\infty}^{\infty} dt \, H_I(t) + \mathcal{O}(\lambda^2) \]

\[ = -i \lambda \sum_m (I_{+,m} a_m^+ + I_{-,m} a_m^- + \text{H.c.}) + \mathcal{O}(\lambda^2), \]

where the sum over \( m \) takes discrete values due to the periodic boundary conditions \( (k = 2\pi m/L) \), \( L \) is the scale of the natural IR cutoff (we neglect the interaction of the detector with the zero mode \( |0\rangle \)), \( a_m \) and \( a_m^\dagger \) are field mode annihilation and creation operators, and

\[ I_{\pm,m} = \int_{-\infty}^{\infty} \frac{d\tau}{\sqrt{4\pi|m|}} e^{i\Omega \tau + \pm \lambda \left( |m| t(\tau) - m x(\tau) \right) - \tau^2/2\sigma^2}, \]

which can be easily worked out from equation (1), expanding the field in plane-wave modes and substituting the expression for the monopole moment. If we consider a detector in its ground state, coupled to the vacuum state of the field, the transition probability at leading order in the perturbative expansion, will be given by

\[ P = \sum_{m \neq 0} |\langle 1_m, e \mid U^{(1)} \mid 0, g \rangle|^2 = \lambda^2 \sum_{m \neq 0} |I_{+,m}|^2 \]

**Evidence of the ‘Anti-Unruh’ effect.** For a uniformly accelerated two-level detector in a periodic cavity, the probability of transition takes the form

\[ P = \lambda^2 \sum_{n,\epsilon} \left| \int_{-\infty}^{\infty} \frac{d\tau}{\sqrt{4\pi n}} e^{i\Omega \tau + 2\pi i (\pm n|x(\tau)| - 1) - \tau^2/2\sigma^2} \right|^2, \]

upon substituting (3) into (4) and using

\[ \langle m | t(\tau) - m x(\tau) \rangle = \frac{n\epsilon}{a} [e^{\epsilon a \tau} - 1], \]

where \( m = -\epsilon n \) where \( n \in \mathbb{Z}^+ \), \( \epsilon = \pm 1 \), and \( t(\tau) = a^{-1} \sinh(\alpha \tau) \) and \( x(\tau) = a^{-1} (\cosh(\alpha \tau) - 1) \). As per our comments in the introduction, when \( a \rightarrow 0 \), \( P \) does not vanish since we are considering a finite time interaction 4 11.

Since the switching function is symmetric about \( t = 0 \), the overall contribution of the right-moving modes is equal to the overall contribution of the left-moving modes, so (5) simplifies to

\[ P = 2\lambda^2 \sum_{n > 0} \left| \int_{-\infty}^{\infty} \frac{d\tau}{\sqrt{4\pi n}} e^{i\Omega \tau + 2\pi i (\pm n|x(\tau)| - 1) - \tau^2/2\sigma^2} \right|^2, \]

which can be recast as

\[ P = \frac{-\lambda^2}{2\pi} \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' e^{i\Omega (\tau - \tau')} - \tau^2 + \tau'^2 + \log[1 - e^{-2\pi i \xi [\tau^2 - \tau'^2]}], \]

upon summing the series in \( n \). The first interesting feature to note in this expression is that the probability is not monotonically increasing with acceleration for all values of the parameters, contrary to expected intuition from the Unruh effect.

For illustration, before employing the Gaussian switching function, let us first compute the transition rate for sudden switching (which in \((1+1)\) dimensions is finite). Unlike our later results, this rate can be evaluated without requiring high-performance computing. Consider a detector suddenly switched on at time \( t = 0 \) and switched off at time \( t = T \). From (3) (substituting Gaussian by sudden switching) the transition rate is

\[ \mathcal{P} = \frac{-\lambda^2}{2\pi} \text{Re} \left( \int_0^T ds e^{i\Omega s} \log \left[ 1 - e^{-2\pi i \xi \left( e^{\pi s} - e^{-(\pi s)} \right)} \right] \right) \]

Plotting this expression as a function of acceleration in Fig. 1 we see that the rate at which this detector clicks can decrease with growing (small) acceleration.

![Figure 1](image-url)
Variance of the logarithm of the KMS ratio as a function of the thermality, we would need to show a linear dependence on the ground-state and excited detector comes from the equilibrium with a thermal background. To demonstrate the context of particle detectors, the KMS condition can be thought of as the postulation that the imbalance be between the excitation and de-excitation probabilities of a ground-state and excited detector comes from the equilibrium with a thermal background. To demonstrate thermality, we would need to show a linear dependence of the logarithm of the KMS ratio as a function of the gap \( \Omega \), where we define the KMS ratio as \( \frac{P(\Omega)}{P(-\Omega)} \), which for KMS states satisfies

\[
\frac{P(\Omega)}{P(-\Omega)} = e^{-\Omega/T}.
\]

For given values of \((\sigma, L)\) we computed the KMS ratio for differing values of \(\Omega\) via the same perturbative numerical integration used previously to find the temperature. A linear slope in the plots of the KMS ratio vs \(\Omega\) corresponds to a system that obeys the KMS condition. Our results are shown in Fig. 3. We see that the KMS condition is obeyed by the detector for the ranges of parameters considered in the figure.

Consequently we can define a meaningful KMS temperature as the slope of the plot of the KMS ratio as a function of \(\Omega\) within this parameter range. This way we can study the dependence of the KMS temperature on the detector’s acceleration to identify the regions where the Unruh effect is present. Concretely, we examine the KMS temperature for different values of \(\Omega\) considering both \(\Omega = 0\) (dashed) and \(\Omega = 1\) (solid). For given values of \(\sigma, L\) we vary \(\Omega\) and \(L\Omega\) considering both \(\Omega = 0.1\) (dashed) and \(\Omega = 1\) (solid). For \(L\Omega\) considering both \(\Omega = 0\) (dashed) and \(\Omega = 1\) (solid).

From Fig. 3 (top), we see that the temperature change with acceleration increases in magnitude as acceleration increases. Finally, (bottom) we also see that as acceleration increases, the region where the temperature’s derivative is negative shrinks, indicating that we recover the Unruh effect for large accelerations.

The derivative of the KMS temperature with respect to the acceleration is shown as a density plot in Fig. 4, where the derivative is zero as a dashed line. We see that for increasing interaction time (increasing \(\sigma\)) as well as increasing detector gap \(\Omega\), the negatively sloped region disappears, in line with our expectations that for long times the slope should approach the usual value of \(1/2\pi\). This indicates that turning the detector on for an infinite amount of time yields the Unruh effect. We also see that the Unruh effect is recovered for large accelerations.

1+1D continuum case.- The effect reported in this letter is not exclusive of cavity setups with periodic boundary conditions. We can examine the effect in the continuum just by replacing the expression (7) by its continuum analogue. We obtain

\[
P = \int_{-\infty}^{\infty} \frac{dk}{4\pi|k|} \left| \int dt e^{i\Omega(t-\frac{a}{2}k)\left[(e^{-i\Omega a} - 1) - \frac{e^{2i\Omega a} - 1}{2\pi^2}\right]} \right|^2,
\]

where \(s_k = \text{sgn}(k)\). We can expand this expression as

\[
P = \int_{-\infty}^{\infty} \frac{dk}{4\pi|k|} \int dt \int dt' e^{i\Omega(t-t')} e^{-\frac{e^{2i\Omega a} - 1}{2\pi^2} \times} e^{-i\frac{a^2}{2} |k|\left[(e^{-i\Omega a} - 1) - \left(e^{-i\Omega a'} - 1\right)\right]},
\]

a quantity well known to be IR divergent. Introducing
We can therefore evaluate the expression for the probability of transition for different values of the parameters characterizing the detectors. The results are depicted in Fig. 5. We see that as detector acceleration increases, the detector can register either more detection events or fewer, depending on the regime of parameter space, demonstrating that this phenomenon is also present in the continuum.

Rather than any kind of boundary conditions, the key ingredient responsible for the cooling of an accelerated detector is the finite time coupling, both for the cavity and the continuum. Further investigation is required in the latter case to determine if this is a consequence of the existence of an IR cutoff or the reduced dimensionality of spacetime.

**Nonperturbative Thermality.** - Independent of our study above, we also employed a completely different approach, using a non-perturbative Gaussian formalism [1] [10], to analyze this phenomenon. In this scenario the detector is modelled as a harmonic oscillator and ends up in a squeezed thermal state upon completion of its interaction with the field; thermality holds provided the squeezing contribution to the energy of this state is much smaller than the thermal contribution [4, 10]. We found this criterion to hold for all values of \((\sigma, \Omega)\) in the relevant parameter regimes of Fig. 3, consistent with our KMS perturbative analysis: thermality is indeed maintained, even in the regime where the detector cools with increasing acceleration. With full disclosure, this nonperturbative calculation was computationally taxing and we were not able to include enough field modes to guarantee full non-perturbative convergence. We therefore can only take this non-perturbative result as an indication, rather than a non-perturbative proof, of thermality. We emphasize that our previous KMS perturbative analysis above is devoid of these limitations.

**Conclusions.** - We have demonstrated that for finite-time interactions, a particle detector at constant acceler-
ation can experience (for certain parameter regimes), a cooler heat bath as compared to the same detector with a lower acceleration. Furthermore, when this phenomenon is manifest the KMS condition is satisfied in the same manner as the usual Unruh effect. We therefore conclude that the Anti-Unruh effect (a cooling down with acceleration) is not due to a non-equilibrium response stemming from the finite duration of the interaction.

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