Superdirective Antenna Pairs for Energy-Efficient Terahertz Massive MIMO
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Abstract—Terahertz (THz) communication is widely deemed the next frontier of wireless networks owing to the abundant spectrum resources in the THz band. Whilst THz signals suffer from severe propagation losses, a massive antenna array can be deployed at the base station (BS) to mitigate those losses through beamforming. Nevertheless, a very large number of antennas increases the BS’s hardware complexity and power consumption, and hence it can lead to poor energy efficiency (EE). To surmount this fundamental problem, we propose a novel array design based on superdirectivity and nonuniform inter-element spacing. Specifically, we exploit the mutual coupling between closely spaced elements to form superdirective pairs. A unique property of them is that all require the same excitation amplitude, and thus can be driven by a single radio frequency chain akin to conventional phased arrays. Moreover, they facilitate multi-port impedance matching, which ensures maximum power transfer for any beamforming angle. After addressing the implementation issues of superdirectivity, we show that the number of BS antennas can be effectively reduced without sacrificing the achievable rate. Simulation results demonstrate that our design offers huge EE gains compared to uncoupled arrays with uniform spacing, and hence could be a radical solution for future THz systems.

Index Terms—Antenna arrays, channel estimation, energy efficiency, hybrid beamforming, impedance matching, mutual coupling, superdirectivity, THz communications.

I. INTRODUCTION

MASSIVE multiple-input multiple-output (MIMO) is now a mature technology, which has been adopted by 5G new radio to provide superior network capacity and coverage. In parallel, millimeter wave (mmWave) systems start gaining ground as an effective way for delivering multi-gigabit rates thanks to their very large bandwidths [1], [2]. Toward this direction, communication above 100 GHz, i.e., terahertz (THz) frequencies, is widely deemed the next frontier of wireless systems with a plethora of promising applications, ranging from ultra-broadband femtocells to terabit-per-second links for wireless backhaul [3]. Nevertheless, THz signals are subject to severe propagation and molecular absorption losses, which can drastically limit the communication range and coverage [4]. To deal with this problem, large antenna arrays can be deployed at the base station (BS) to increase the signal power by means of sharp beamforming. As a result, massive MIMO is expected to be an integral component of future THz systems [5]. On the other hand, THz radio frequency (RF) circuits, e.g., power amplifiers, phase shifters, etc., exhibit significantly higher power consumption than their sub-6 GHz counterparts [6]. Additionally, baseband processing with multiple RF chains during channel estimation and data transmission is power intensive [7]. In conclusion, achieving a large beamforming gain in an energy efficient manner constitutes a major engineering challenge which calls for novel solutions.

The beamforming capabilities of multi-antenna systems have been extensively investigated in the past. For example, a typical $N$-element phased array offers a gain that scales linearly with $N$ when maximum ratio transmission (MRT) is used [8]. However, higher gains are possible by leveraging the mutual coupling between adjacent antennas. Specifically, Uzkov theoretically proved in his seminal work [9] that a uniform linear array (ULA) of $N$ isotropic radiators and vanishingly small inter-element spacing has an endfire directivity of $N^2$, a phenomenon now known as superdirectivity.

The theme of superdirectivity has been widely studied from both information theoretic and pure electromagnetic standpoints, e.g., [10], [11], [12], [13], [14], [15], [16], and [17], and references therein. However, most of the related literature considers arrays of uniform spacing. More importantly, it overlooks various communication and practical aspects, such as the EE, implementation limitations in hybrid analog-digital architectures, and channel estimation performance. Consequently, there are still critical questions about how THz massive MIMO could fully benefit from superdirectivity. Regarding other approaches, there is a stream of recent papers on arrays-of-subarrays (AoSA) with metallic antennas [18], [19], [20] and graphene-based plasmonic nanoantennas [21], [22], as well as on intelligent reflecting surfaces [23], [24], which can improve the EE of the system. Yet, they neglect the superdirective effects of closely spaced BS antennas.

This paper aims to show that superdirectivity can be ingeniously used to reduce the hardware complexity and boost
the EE of THz massive MIMO. The contributions of our work are summarized as follows:

- We introduce a coupling-aware array model based on antenna theory. In particular, we derive the input impedance matrix of the BS array assuming lossy dipole antennas. Note that directional antennas, such as linear dipoles, are mutually coupled even for half-wavelength spacing. Therefore, proper characterization of their electromagnetic interaction is crucial to beamforming [25].

- Based on the introduced array model, we study the implementation issues of superdirectivity. In particular, we look into the impedance matching problem as well as the realization of superdirective beamsteering in a hybrid analog-digital array architecture. We address both problems by proposing a novel array design relying on coupled antenna pairs. Specifically, the BS array is divided into multiple two-element groups, which are adequately separated so that inter-group coupling can be neglected. The resulting nonuniform linear array (NULA) greatly simplifies the optimal multi-port matching, and more importantly, requires uniform amplitude excitation. Consequently, it can be driven by a single RF chain to produce a pencil-like beam, similar to conventional phased arrays. The presented structure is also extended to the nonuniform planar array (NUPA) case. It is worth stressing that our method can be readily applied to an AoSA, wherein each subarray is a NULA or a NUPA.

- We exploit the excessive power gain of the proposed design to decrease the number of BS antennas. This leads to a low-dimensional massive MIMO system, whose performance is assessed in terms of the achievable rate under perfect and imperfect channel state information (CSI). For this purpose, approximate closed-form expressions for the signal and interference powers are provided assuming MRT. Moreover, the channel estimation problem is addressed by leveraging the popular orthogonal matching pursuit (OMP) algorithm.

- Extensive simulation results are provided corroborating our analysis. In particular, it is demonstrated that the proposed array design boosts the EE of THz massive MIMO without sacrificing the data transmission and channel estimation performances. As such, it has the potential to realize low-complexity and energy-efficient MIMO arrays with sharp beamforming capabilities.

The rest of the paper is organized as follows: Section II delineates the BS array model. Section III presents the impedance matching problem. Section IV delves into the implementation issues of superdirectivity and details the proposed solution. Section V introduces the system model used for performance evaluation. Section VI analyzes the signal and interference powers under the proposed array. Section VII addresses the channel estimation problem. Section VIII explains how to reduce the number of BS antennas. Section IX is devoted to numerical simulations. Finally, Section X summarizes the main conclusions of this work.

Notation: Throughout the paper, $D_N(x) = \frac{\sin(N\pi x/2)}{N\sin(\pi x/2)}$ is the Dirichlet sinc function; $\mathbf{A}(\cdot, \cdot, \cdot)$ is a vector field; $\mathbf{A}$ is a matrix; $\mathbf{A}^\dagger$, $\mathbf{A}^H$, and $\mathbf{A}^T$ are the conjugate, pseudoinverse, conjugate transpose, and transpose of $\mathbf{A}$, respectively; $[\mathbf{A}]_{i,j}$ is the $(i,j)$th entry of $\mathbf{A}$; $\mathbf{A}(i)$ is the $i$th column of $\mathbf{A}$; $\text{blkdiag}([\mathbf{A}_1, \ldots, \mathbf{A}_n])$ is a block diagonal matrix; $\mathbf{a}$ is a vector; $\|\mathbf{a}\|_1$ and $\|\mathbf{a}\|_2$ are the $l_1$-norm and $l_2$-norm of $\mathbf{a}$, respectively; $\text{mag}(\mathbf{a}) = [\|\mathbf{a}_1\|, \ldots, \|\mathbf{a}_N\|]^T$ for $\mathbf{a} = [\mathbf{a}_1, \ldots, \mathbf{a}_N]^T$; $\mathbf{I}_{N \times M}$ is the $N \times M$ matrix with unit entries; $\mathbf{I}_N$ is the $N \times N$ identity matrix; $\mathbf{0}_{N \times M}$ is the $N \times M$ matrix with zero entries; $\otimes$ denotes the Kronecker product; $\mathbf{a} \cdot \mathbf{b}$ is the inner product between $\mathbf{a}$ and $\mathbf{b}$; $\mathbb{E}\{\cdot\}$ denotes expectation; $\mathcal{CN}(\mu, \mathbf{R})$ is a complex Gaussian vector with mean $\mu$ and covariance matrix $\mathbf{R}$. Finally, $\text{Re}\{\cdot\}$ and $\text{Im}\{\cdot\}$ are the real and imaginary parts of a complex variable, respectively.

II. ANTENNA ARRAY MODEL

In this section, we present the BS array model which takes into account antenna mutual coupling.

A. Radiated Power

Consider an $N$-element ULA along the $z$-axis at the BS, as depicted in Fig. 1.1. The inter-element spacing is $d$. Each element is a linear dipole parallel to the $x$-axis, and is of length $\ell$

1We consider a linear array at the BS since it constitutes the building block of planar arrays. The planar case is investigated in Section VI-C and thereafter.
and radius $\rho$. According to [26, Ch. 4], the current distribution on each dipole $n$ has approximately the sinusoidal form

$$I_n(x') \approx I_n(0) \frac{\sin(\kappa N - \kappa|x'|)}{\sin(\kappa N)}, \quad |x'| \leq \ell/2, \tag{1}$$

where $I_n(0) \in \mathbb{C}$ is the input current, $\kappa = 2\pi/\lambda$ is the wavenumber, and $\lambda$ is the wavelength. We next focus on an arbitrary user who is in the far field of the BS array. The user’s location is described by the tuple $(r \cos \phi \sin \theta, r \sin \phi \sin \theta, r \cos \theta)$, where $r, \theta \in [0, \pi]$, and $\phi \in [0, 2\pi]$ are the radial distance, polar angle, and azimuth angle, respectively. The electric field at the user is then specified as (see Appendix A)

$$E(r, \theta, \phi) = -\frac{j\eta e^{-jkr}}{2\pi r} \sum_{n=0}^{N-1} e^{jkr_n} I_n(0) F(\theta, \phi), \tag{2}$$

where

$$F(\theta, \phi) = \frac{\cos(\kappa N - \kappa \sin \theta)}{\sin(\kappa N)} - \cos(\kappa N - \kappa \phi \sin \theta), \tag{3}$$

is the vector field pattern of each dipole, $e_\theta$ and $e_\phi$ are the unit vectors along the polar and azimuth directions, respectively, $\eta$ is the characteristic impedance of free-space, $\hat{r} = (\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta)^T$ is the unit radial vector along the user direction, and $r_n = (0, 0, n\ell)$ is the position vector of the $n$th antenna. The radiation intensity [W/sr] is written in vector form as

$$U \triangleq \frac{\|E(r, \theta, \phi)\|^2}{2\eta} = \frac{\eta}{8\pi^2} \|F(\theta, \phi)\|^2 \|a^H(\theta)\|^2, \tag{4}$$

where $a(\theta) = [e^{-jkr_n0}, \ldots, e^{-jkr_n(N-1)}]^T \in \mathbb{C}^N$ and $i = [I_0(0), \ldots, I_{N-1}(0)]^T \in \mathbb{C}^N$ are the far-field array response vector and the vector of input currents, respectively. Using (4), the power radiated by the antenna array is

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi U \sin \theta d\theta d\phi$$

$$= \frac{\eta^2}{8\pi^2} \int_0^{2\pi} \int_0^\pi a(\theta) a^H(\theta) \|F(\theta, \phi)\|^2 \sin \theta d\theta d\phi$$

$$= \frac{1}{2 \eta} \text{Re}\{Z_{\text{ideal}}\} i, \tag{5}$$

where $Z_{\text{ideal}} \in \mathbb{C}^{N \times N}$ is the input impedance matrix of the array assuming lossless antennas. Moreover, $R_i \triangleq \text{Re}\{Z_{\text{ideal}}\}_{ii} = 2\pi^2 \int_0^{2\pi} \int_0^\pi \|F(\theta, \phi)\|^2 \sin \theta d\theta d\phi$ is the input resistance of each lossless dipole.

Recall that the input resistance equals the radiation resistance divided by $\sin^2(\kappa N/2)$ [26, Ch. 8]. This is because all quantities are expressed in terms of the input currents rather than the current maxima $\{I_n(0)/\sin(\kappa N/2)\}_{n=0}^{N-1}$.

### B. Input Power and Array Gain

Realistic dipole antennas exhibit a conduction/loss resistance which leads to heat dissipation. Because of the skin effect of conductive wires carrying an alternating current, the loss resistance per unit length is given by [26, Eq. (2-90b)]

$$R_{\text{loss}} = \frac{1}{2\rho} \sqrt{\mu f \pi \sigma}, \tag{6}$$

where $f$ is the carrier frequency, $\mu$ is the permeability of free-space, and $\sigma$ is the conductivity of the wire material. Under the sinusoidal current distribution in (1), the loss resistance relative to the input current $I_n(0)$ is then specified as

$$R_{\text{loss}} = R_{\text{loss}} \int_{-\ell/2}^{\ell/2} \left| \frac{I_n(x')}{I_n(0)} \right|^2 dx' = \frac{\kappa N - \kappa \sin \theta}{4\kappa \rho \sin^2(\kappa N/2)} \sqrt{\frac{\mu f \pi \sigma}{\pi}} \|i\|^2, \tag{7}$$

which yields the overall power loss [27]

$$P_{\text{loss}} = \frac{1}{2} \sum_{n=0}^{N-1} R_{\text{loss}}|I_n(0)|^2 = \frac{1}{2} R_{\text{loss}} \|i\|^2. \tag{8}$$

Consequently, the input power at the antenna ports is

$$P_{\text{in}} = P_{\text{loss}} + P_{\text{rad}} = \frac{1}{2} R_{\text{loss}} \|i\|^2 + \frac{1}{2} \frac{1}{2 \eta} \text{Re}\{Z_{\text{ideal}}\} i = \frac{1}{2} \frac{1}{2 \eta} \text{Re}\{Z\} i, \tag{9}$$

where $Z \triangleq R_{\text{loss}} I_N + Z_{\text{ideal}}$ is the impedance matrix of the lossy array. Finally, the array gain is defined as

$$G(\theta, \phi) \triangleq \frac{4\pi U}{P_{\text{in}}} = G_e(\theta, \phi) (R_{\text{loss}} + R_i) \frac{|a^H(\theta) i|^2}{\text{Re}\{Z\} i}, \tag{10}$$

where $G_e(\theta, \phi) = \frac{\|F(\theta, \phi)\|^2}{\eta} \text{Re}\{Z\}$ denotes the gain of each dipole.

### C. Optimal Currents Under Fixed Input Power

According to Friis transmission formula, the power received by the user is given by [26]

$$P_r = P_{\text{in}} \left(\frac{\lambda}{4\pi f}\right)^2 G(\theta, \phi), \tag{11}$$

where an isotropic antenna has been assumed at the user for simplicity. We now seek to find $i$ that maximizes the received power (or equivalently $G(\theta, \phi)$) subject to the input power constraint $P_{\text{in}} \leq P_t$. The objective (10) is a generalized Rayleigh quotient. Thus, the optimal current excitation is obtained as [27]

$$i_{\text{opt}} = \sqrt{\frac{2P_t}{a^H(\theta) \text{Re}\{Z\} - a(\theta)}} \text{Re}\{Z\}^{-1} a(\theta), \tag{12}$$

and the maximum array gain is

$$G_{\text{max}}(\theta, \phi) = G_e(\theta, \phi) (R_{\text{loss}} + R_i) a^H(\theta) \text{Re}\{Z\}^{-1} a(\theta). \tag{13}$$

### Remark 1 (Uncoupled ULA): In the absence of mutual coupling, $\text{Re}\{Z_{\text{ideal}}\} = R_i I_N$ and $P_{\text{rad}} = \frac{1}{2} R_i \|i\|^2$. Moreover,

$$i_{\text{opt}} = \sqrt{\frac{2P_t}{2\pi N (R_{\text{loss}} + R_i)}} a(\theta) \text{ and } G_{\text{max}}(\theta, \phi) = G_e(\theta, \phi) N, \text{ which is the typical } O(N) \text{ power gain.} \tag{14}$$
III. IMPEDANCE MATCHING IN THE PRESENCE OF MUTUAL COUPLING

Mutual coupling alters the input impedance of each dipole. To see this, let \( \mathbf{v} = [v_0, \ldots, v_{N-1}]^T \in \mathbb{C}^{N \times 1} \) denote the vector of voltages at the antenna ports. Then, we have that \( \mathbf{v} = \mathbf{Z} \mathbf{i} = \mathbf{Z}_a \mathbf{i} \), where \( \mathbf{Z}_a \in \mathbb{C}^{N \times N} \) is a diagonal matrix with entries \([26, \text{Ch. 8}]\)

\[
[\mathbf{Z}_a]_{n,n} \triangleq \left[ \mathbf{Z} \right]_{n,n} + \sum_{m=0, m \neq n}^{N-1} \left[ \mathbf{Z} \right]_{n,m} \frac{I_m(0)}{I_n(0)},
\]

(14)

where \( [\mathbf{Z}_a]_{n,n} \) is the active impedance of the \( n \)th antenna. The active impedance of each dipole hinges on the excitation currents, and hence single-port conjugate matching is optimal only for a specific beamforming angle \( \theta \) [28]. For this reason, we resort to multi-port matching and model the BS array as in Fig. 2. The impedance network is a passive and lossless 2N-port network described by the matrix \( \mathbf{Z}_M \in \mathbb{C}^{2N \times 2N} \), which is partitioned as

\[
\mathbf{Z}_M = \begin{bmatrix}
\mathbf{Z}_{M11} & \mathbf{Z}_{M12} \\
\mathbf{Z}_{M21} & \mathbf{Z}_{M22}
\end{bmatrix}.
\]

(15)

The lossless property implies that \( \mathbf{Z}_M \) has only imaginary entries. The vector of voltage sources is denoted by \( \mathbf{v}_s = [v_{s,0}, \ldots, v_{s,N-1}]^T \in \mathbb{C}^{N \times 1} \). Each voltage source has an internal impedance \( Z_s \in \mathbb{C} \), with \( \text{Re}\{Z_s\} = R_s \). Likewise, the voltages and currents at the input ports of the network are denoted by \( \mathbf{v}_M = [v_{M,0}, \ldots, v_{M,N-1}]^T \in \mathbb{C}^{N \times 1} \) and \( \mathbf{i}_M = [i_{M,0}, \ldots, i_{M,N-1}]^T \in \mathbb{C}^{N \times 1} \), respectively. Based on basic circuit analysis, the relationship between the voltages and currents at the input and output of the matching network is [29]

\[
\begin{bmatrix} \mathbf{v}_M \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{M11} & \mathbf{Z}_{M12} \\
\mathbf{Z}_{M21} & \mathbf{Z}_{M22}\end{bmatrix} \begin{bmatrix} \mathbf{i}_M \\ -1 \end{bmatrix}.
\]

(16)

Using (16) and the relationship \( \mathbf{v} = \mathbf{Z} \mathbf{i} \) yields

\[
\mathbf{v}_M = (\mathbf{Z}_{M11} - \mathbf{Z}_{M12}(\mathbf{Z} + \mathbf{Z}_{M22})^{-1}\mathbf{Z}_{M21}) \mathbf{i}_M,
\]

(17)

where \( \mathbf{Z}_T \in \mathbb{C}^{N \times N} \) is the overall transmit impedance matrix accounting for power matching and mutual coupling. The problem of optimal multi-port matching is to find \( \mathbf{Z}_M \) so that \( \mathbf{Z}_T = \mathbf{Z}_a \mathbf{I}_N \). In this case, there are no reflection losses and half of the generated power enters the antenna array, i.e., maximum power transfer [30]. This is accomplished for [29]

\[
\mathbf{Z}_M = \begin{bmatrix}
- j \text{Im}\{\mathbf{Z}_a\} \mathbf{I}_N \\
- j \sqrt{R_s} \text{Re}\{\mathbf{Z}\}^{1/2} \\
- j \text{Im}\{\mathbf{Z}\}
\end{bmatrix}.
\]

(18)

Under (18), we have that

\[
\mathbf{v}_s = \mathbf{Z}_a \mathbf{i}_M + \mathbf{v}_M = (\mathbf{Z}_a \mathbf{I}_N + \mathbf{Z}_T) \mathbf{i}_M = 2R_s \mathbf{i}_M,
\]

(19)

and the total power generated by the voltage sources is

\[
P_{\text{total}} = \frac{1}{2} \text{Re} \left\{ \mathbf{v}_s^H \mathbf{i}_M \right\} = R_s \| \mathbf{i}_M \|^2.
\]

(20)

From (16) and (18), it also holds that

\[
\mathbf{i}_M = \mathbf{Z}_a^{-1}(\mathbf{Z} + \mathbf{Z}_{M22})^{-1} \mathbf{i}
\]

\[
= \frac{j}{\sqrt{R_s}} \text{Re}\{\mathbf{Z}\}^{1/2} \mathbf{i},
\]

(21)

and hence \( P_{\text{total}} = \| \mathbf{i}^H \text{Re}\{\mathbf{Z}\} \| \| \mathbf{i} \| = 2P_{\text{in}} \), which confirms the optimality of (18).

IV. HARDWARE-EFFICIENT IMPLEMENTATION OF SUPERDIRECTIVE BEAMSTEERING

In this section, we investigate the problem of generating a superdirective beam with a single RF chain and a low-complexity impedance matching network.

A. Problem Statement

From (12), (19), and (21), the vector of voltage sources maximizing the array gain is given by

\[
v_s^\text{opt} = 2R_s \mathbf{i}_M^\text{opt} = j2\sqrt{R_s} \text{Re}\{\mathbf{Z}\}^{1/2} \mathbf{i}_M^\text{opt}
\]

\[
= j2 \sqrt{\frac{2R_s P_t}{\mathbf{a}^H(\theta) \text{Re}\{\mathbf{Z}\}^{-1} \mathbf{a}(\theta)}} \text{Re}\{\mathbf{Z}\}^{-1/2} \mathbf{a}(\theta).
\]

(22)

In most massive MIMO studies, beamforming is conveniently described by a complex vector \( \mathbf{w} \in \mathbb{C}^{N \times 1} \) whose squared norm, \( \| \mathbf{w} \|^2 \), defines the transmit power. This representation originates from information theory and can be mapped to the voltage sources driving the array through the relationship [31]

\[
\mathbf{w} = -\frac{1}{j2\sqrt{R_s}} (v_s^\text{opt})^* = \frac{2P_t}{\mathbf{a}^H(\theta) \text{Re}\{\mathbf{Z}\}^{-1} \mathbf{a}(\theta)} \text{Re}\{\mathbf{Z}\}^{-1/2} \mathbf{a}(\theta).
\]

(23)

We next consider a hybrid analog-digital array with one RF chain and \( N \) analog phase shifters, as shown in Fig. 3.
In fact, \( Z_0 \) corresponds to the input impedance matrix of each \( N \)-element group. For \( N = 2 \), we have

\[
\text{Re}\{Z_0\} \triangleq \begin{bmatrix} R_{\text{self}} & R_m \\ R_m & R_{\text{self}} \end{bmatrix},
\]

where \( R_{\text{self}} = R_{\text{loss}} + R_1 \) denotes the self impedance for notational convenience, and \( R_m \) is the real part of the mutual impedance between two adjacent dipoles.

When the inter-group spacing is sufficiently large, the coupling between groups can be neglected. Thus, \( \text{Re}\{Z_{n_1,n_2}\} \approx 0 \) for \( n_{g1} \neq n_{g2} \), which makes \( Z \) approximately a block diagonal matrix, namely \( \text{Re}\{Z\} \approx I_N \otimes \text{Re}\{Z_0\} = \text{Re}\{Z_{\text{approx}}\} \). We finally stress that in addition to a large \( d_g \), the mutual coupling between antenna groups can be further reduced through various decoupling techniques, such as electromagnetic band gap structures placed between them [34].

2) Coupled Antenna Pairs: A major drawback of beamsteering under mutual coupling is the requirement of amplitude control at each voltage source. However, in a NULA, all antenna groups share the same voltage amplitudes. This is because from (22)

\[
\text{Re}\{Z_0\} = \sqrt{I_x} \cos \phi - \sqrt{I_y} \sin \phi,
\]

where \( \cos \phi = \frac{Z_{0,1} + Z_{1,0}}{Z_0} \) and \( \sin \phi = \frac{Z_{0,1} - Z_{1,0}}{Z_0} \). Let \( \phi = 0 \), then \( Z_{0,1} = Z_{1,0} = 0 \), and the following proposition justifies the choice of \( N = 2 \).

**Proposition 1:** For \( N = 2 \), \( \text{mag} \left( \text{Re}\{Z_0\}^{-1/2}a(0) \right) \) reduces to (31), as shown at the bottom of the next page.

**Proof:** See Appendix B.

According to Proposition 1, antenna pairs result in uniform excitation amplitudes \( \{v_{s,n}\}^{N-1}_{n=0} \), thereby enabling the realization of superdirective beamsteering with a single RF chain and \( N \) analog phase shifters. Additionally, each antenna pair is separately matched using the four-port network (see Appendix B)

\[
Z_{M,0} = \begin{bmatrix}
-\sqrt{Z_{0}}& -\sqrt{Z_{0}}& -\sqrt{Z_{0}}& -\sqrt{Z_{0}}
\end{bmatrix}^{1/2}
\]

where \( \text{Re}\{Z_0\} \in \mathbb{R}^{N \times N} \) and \( \text{Re}\{Z_{n_1,n_2}\} \in \mathbb{R}^{N \times N} \) are defined by (26) and (27), as shown at the bottom of the page, respectively.
\( \mathbf{Z}_0 \in \mathbb{C}^{2 \times 2} \), which holds for radiators beyond linear dipoles. The main reason we chose dipoles is mathematical tractability; otherwise, we would heavily rely on full-wave simulations to assess the antenna array gain. The problem of finding the optimal antenna type to facilitate superdirectivity is beyond the scope of the current work, and is a promising avenue for future research.

V. THZ MASSIVE MIMO MODEL

In this section, we introduce the high-level system model used to evaluate the performance of the proposed NULA in terms of the achievable rate and EE.

A. Signal Model

Consider a THz massive MIMO system, where the BS serves \( K \ll N \) single-antenna users. Let \( \mathbf{h}_k \in \mathbb{C}^{N \times 1} \) and \( \beta_k \) denote the small-scale fading channel and path loss of user \( k \), respectively. The downlink channel of user \( k \) is then specified as \( \sqrt{\beta_k} \mathbf{h}_k \). To facilitate hardware implementation, a fully connected analog-digital array with \( N_{RF} \) RF chains is considered at the BS [35]. Thus, the precoder is decomposed as \( \mathbf{W} = \mathbf{W}_{RF}\mathbf{W}_{BB} = [\mathbf{w}_1, \ldots, \mathbf{w}_{N_{RF}}] \in \mathbb{C}^{N \times N_{RF}} \), where \( \mathbf{W}_{RF} \in \mathbb{C}^{N \times N_{RF}} \) is the RF beamformer realized by analog phase shifters, whereas \( \mathbf{W}_{BB} \in \mathbb{C}^{N_{RF} \times N_{RF}} \) is the baseband precoder. Without loss of generality, we assume \( N_{RF} = K \) hereafter. Let \( \mathbf{x} = [x_1, \ldots, x_K]^T \sim \mathcal{CN}(\mathbf{0}_{K \times 1}, \mathbf{I}_K) \) be the vector of users’ data symbols. The transmitted signal is then given by \( \mathbf{Wx} \in \mathbb{C}^{N \times 1} \), and should satisfy the power constraint

\[
E\{||\mathbf{Wx}||^2\} = \sum_{k=1}^{K} ||\mathbf{w}_k||^2 \leq 2P_t, \tag{33}
\]

where \( 2P_t \) is the total power under perfect impedance matching. Given the above, the received baseband signal at the \( k \)th user is written as

\[
y_k = \sqrt{\beta_k} \mathbf{h}_k^* \mathbf{w}_k x_k + \sqrt{\beta_k} \sum_{i=1, i \neq k}^{K} \mathbf{h}_k^* \mathbf{w}_i x_i + n_k, \tag{34}
\]

where \( n_k \sim \mathcal{CN}(0, \sigma^2) \) is the additive noise. Finally, the signal-to-interference-plus-noise ratio (SINR) of user \( k \) is

\[
\text{SINR}_k = \frac{\beta_k ||\mathbf{w}_k||^2}{\beta_k \sum_{i=1, i \neq k}^{K} ||\mathbf{w}_i||^2 + B\sigma^2}, \tag{35}
\]

where \( B\sigma^2 \) is the noise power over the transmit bandwidth \( B \).

B. Channel Model

Because of the severe path attenuation in the THz band, multi-path scattering is very limited. We therefore assume line-of-sight (LoS) links between the BS and users, akin to [36]. In the presence of mutual coupling at the BS array, the channel vector of user \( k \) is expressed as [29, Eq. (105)]

\[
\mathbf{h}_k \triangleq \text{Re} \left\{ \mathbf{Z} \right\}^{-1/2} \mathbf{a}(\theta_k), \tag{36}
\]

where \( \mathbf{Z} \triangleq \frac{1}{R_{\text{loss}} + R_i} \mathbf{Z} \) is the normalized input impedance matrix of the BS array. Finally, because the molecular absorption losses are no longer negligible at THz frequencies, the path loss coefficient is calculated as [37]

\[
\beta_k = G_c(\theta_k, \phi_k) \left( \frac{\lambda}{4\pi r_k} \right)^2 e^{-\kappa_{\text{abs}} r_k}, \tag{37}
\]

where \( G_c(\theta_k, \phi_k) \) is the gain of each BS antenna, \( \lambda \) is the carrier wavelength, \( r_k \) is the distance from the BS to user \( k \), and \( \kappa_{\text{abs}} \) is the molecular absorption coefficient determined by the composition of the propagation medium [38].

C. Power Consumption Model

For a fully-connected array structure, the overall power consumption is given by [39]

\[
P_c = P_{BB} + K P_{RF} + KN P_{PS} + NP_{PA} + P_t + P_{CE}, \tag{38}
\]

where \( P_{BB}, P_{RF}, P_{PS}, P_{PA}, \) and \( P_{CE} \) denote the powers consumed by a baseband unit, an RF chain, a phase shifter, a power amplifier, and the channel estimation process, respectively. Moreover, each RF chain comprises a digital-to-analog converter (DAC), a local oscillator, and a mixer, and hence \( P_{RF} = P_{DAC} + P_{LO} + P_M \). We finally stress that the power consumption of splitters and combiners is negligible, and hence is ignored [39].

VI. SIGNAL AND INTERFERENCE POWERS

A. Proposed NULA

We consider MRT at the BS. Under equal power allocation, we have that

\[
\mathbf{w}_k = \sqrt{\frac{2P_t}{K}} \mathbf{h}_k^* \frac{\text{Re} \left\{ \mathbf{Z} \right\}^{-1/2} \mathbf{a}(\theta_k)}{K}, \tag{39}
\]

and the power of the desired signal, normalized by \( K/(2P_t) \), is

\[
\frac{K}{2P_t} ||\mathbf{h}_k^* \mathbf{w}_k||^2 = \mathbf{a}^H(\theta_k) \text{Re} \left( \mathbf{Z} \right)^{-1} \mathbf{a}(\theta_k). \tag{40}
\]

The normalized input impedance matrix is used for notational convenience. In this way, the array gain is recast as \( G(\theta, \phi) = G_c(\theta, \phi) ||\mathbf{a}(\theta)||^2 \), and the path loss coefficient or channel vector will not include the term \( R_{\text{loss}} + R_i \). For example, \( \text{Re} \left\{ \mathbf{Z} \right\} = \mathbf{I}_N \) and \( \mathbf{h}_k = \mathbf{a}(\theta_k) \) in the absence of coupling.
Due to the block-diagonal structure of \( \bar{Z} \), (40) simplifies to

\[
\frac{K}{2P_t} |h_k^T w_k|^2 \approx a^H(\theta_k) \text{Re} \{ \bar{Z}_{\text{approx}} \}^{-1} a(\theta_k)
\]

\[
= (a^H(\theta_k) \otimes a^H(\theta_k)) \left( I_{N_x} \otimes \text{Re} \{ \bar{Z}_0 \} \right)^{-1}
\]

\[
= N_g a^H(\theta_k) \text{Re} \{ \bar{Z}_0 \}^{-1} a_0(\theta_k),
\]

where \( \bar{Z}_{\text{approx}} = \frac{1}{\rho^2 + \frac{1}{\rho^2}} \bar{Z} \) and \( \bar{Z}_0 = \frac{1}{\rho^2 + \frac{1}{\rho^2}} \bar{Z}_0 \). As seen from (41), the power gain of the NULA is \( N_g \) times the gain of a superdirective antenna group. In the sequel, we determine \( a^H(\theta_k) \text{Re} \{ \bar{Z}_0 \}^{-1} a_0(\theta_k) \) in closed-form for \( N = 2 \). We first have that

\[
\text{Re} \{ \bar{Z}_0 \} = \left[ \begin{array}{c} \bar{R}_m \\ 1 \end{array} \right],
\]

where \( \bar{R}_m = \frac{1}{\rho^2 + \frac{1}{\rho^2}} R_m \). Then, the inverse matrix is determined as

\[
\text{Re} \{ \bar{Z}_0 \}^{-1} = \frac{1}{1 - R_m^2} \left[ \begin{array}{cc} 1 & -\bar{R}_m \\ -\bar{R}_m & 1 \end{array} \right],
\]

and

\[
a^H(\theta_k) \text{Re} \{ \bar{Z}_0 \}^{-1} a_0(\theta_k)
\]

\[
= \frac{1}{1 - R_m^2} \left[ \begin{array}{cc} 1 & -\bar{R}_m \\ -\bar{R}_m & 1 \end{array} \right] e^{-j\bar{R}_m\cos \theta_k}
\]

\[
= \frac{1}{1 - R_m^2} \left[ 1 - \bar{R}_m e^{-j\bar{R}_m\cos \theta_k} e^{-j\bar{R}_m\cos \theta_k} \right],
\]

which gives

\[
a^H(\theta_k) \text{Re} \{ \bar{Z}_0 \}^{-1} a_0(\theta_k)
\]

\[
= \frac{1}{1 - R_m^2} \left( 1 - \bar{R}_m e^{-j\bar{R}_m\cos \theta_k} e^{-j\bar{R}_m\cos \theta_k} + 1 \right)
\]

\[
= \frac{2}{1 - R_m^2} \left( 1 - \bar{R}_m \cos (\bar{R}_m \cos \theta_k) \right).
\]

According to (45), \( a^H(\theta_k) \text{Re} \{ \bar{Z}_0 \}^{-1} a_0(\theta_k) \approx 2 \) for large \( \bar{d} \), i.e., \( \bar{R}_m \approx 0 \), which corresponds to the conventional power gain of a two-element array.

In a similar manner, the interference power at user \( k \) from the beam toward user \( i \neq k \), normalized by \( K/(2P_t) \), is given by

\[
K \frac{1}{2P_t} |h_i^T w_k|^2
\]

\[
= \left| a^H(\theta_k) \text{Re} \{ \bar{Z} \}^{-1} a(\theta_i) \right|^2
\]

\[
= a^H(\theta_k) \text{Re} \{ \bar{Z} \}^{-1} a_0(\theta_i)
\]

\[
\approx a^H(\theta_k) \text{Re} \{ \bar{Z}_{\text{approx}} \}^{-1} a_0(\theta_i),
\]

where (a) is proven in Appendix C. From (44) and (45), we finally have that

\[
\left| a^H(\theta_k) \text{Re} \{ \bar{Z}_0 \}^{-1} a_0(\theta_i) \right|^2
\]

\[
= \left| a^H(\theta_k) \text{Re} \{ \bar{Z}_0 \}^{-1} a_0(\theta_i) \right|^2
\]

\[
= \frac{1 + e^{-j\bar{R}_m\cos \theta_k} e^{-j\bar{R}_m\cos \theta_k}}{2(1 - R_m^2) (1 - \bar{R}_m \cos (\bar{R}_m \cos \theta_k))}.
\]

B. Comparison With Uncoupled ULA

In the massive MIMO literature, it is customary to consider a uniform inter-element spacing [40]. Furthermore, mutual coupling is avoided by employing a sufficiently large \( \bar{d} \) so that \( \text{Re} \{ Z \} \approx I_N \). In this case, the normalized power of the desired signal becomes

\[
K \frac{1}{2P_t} |h_k^T w_k|^2 \approx N,
\]

where \( K \) represents the number of active users and \( P_t \) is the total transmitted power.
NUPA is formed by placing arrays, which are of practical importance. Specifically, a vector of the NUPA is given by \( \mathbf{a} \), with inter-element spacing \( d \).\(^\text{4}\) The assumption of uncoupled elements under half-wavelength spacing does create coupling between directional antennas, and hence the input impedance matrix cannot be ignored.\(^\text{4}\) Lastly, according to the approximate expressions, the proposed NULA with \( N = 2 \) yields a relative power gain equal to

\[
K = N_d a_0^H(\theta) \mathbb{R} \{ \mathbf{Z}_0 \}^{-1} a_0(\theta) = N_d \left[ a_0^H(\theta) \mathbb{R} \{ \mathbf{Z}_0 \}^{-1} a_0(\theta) - 2 \right] = O(N_d),
\]

which scales linearly with \( N_d \) for some \( \theta \in [0, \theta_{\text{max}}] \); for example, \( \theta_{\text{max}} \approx 50^\circ \) in Fig. 5(a). This trade off between directivity and angular coverage is fundamental.\(^\text{42}\) Thus, increasing the former inevitably decreases the latter, and vice versa. Whether it is beneficial to have a highly directive array depends on the propagation environment and deployment scenario.\(^\text{42}\), \(^\text{43}\).

\(^\text{4}\)The assumption of uncoupled elements under half-wavelength spacing holds only for isotropic radiators.\(^\text{41}\).

In this section, we extend the proposed design to planar arrays, which are of practical importance. Specifically, a NUPA is formed by placing \( N_x \) NULAs along the \( x \)-axis with inter-element spacing \( d_x \), as depicted in Fig. 6(a). The total number of antennas is \( N = N_x N_y N_z \). Also, the response vector of the NUPA is given by \( \mathbf{a}(\theta, \phi) = a_x(\theta, \phi) \otimes a_y(\theta) \otimes a_0(\theta) \), where \( a_x(\theta, \phi) \triangleq [1, \ldots, e^{-j\kappa(N_x-1)d_x \cos \phi \sin \theta}]^T \in \mathbb{C}^{N_x \times 1} \). The coupling between the NULAs is very small for \( d_x \geq \lambda/2 \) since adjacent dipoles along the \( x \)-axis are collinear.

\[\text{Fig. 6. (a) Example of a NUPA with } N_x = N_y = N_z = 2; \text{ and (b) theoretical beampattern (52) against full-wave simulation for } d_x = 0.7\lambda, d_y = 1.5\lambda, \text{ and } d = \lambda/5. \text{ The desired beamforming direction is } (\theta_1, \phi_1) = (20^\circ, 0^\circ).\]

Given that, the impedance matrix of the NUPA reduces to \( \mathbf{Z} \approx \mathbf{I}_{N_x N_y} \otimes \mathbf{Z}_0 \), while the signal and interference powers in (40) and (46) are recast, respectively, as

\[
K = N_x N_y a_0^H(\theta) \mathbb{R} \{ \mathbf{Z}_0 \}^{-1} a_0(\theta) = N_x N_y \left[ a_0^H(\theta) \mathbb{R} \{ \mathbf{Z}_0 \}^{-1} a_0(\theta) - 2 \right] = O(N_x N_y),
\]

From (51), we see that the signal power is \( N_x \) times that of a NULA with \( N_y \) dipole pairs, as expected. Regarding the interference power, NUPA offers an additional degree of freedom compared to a single NULA along the \( z \)-axis, which is given by the term \( N_x N_y |D_{NN} \approx N_x N_y \mathbb{E} \{ \mathbf{Z}_0 \}^{-1} \mathbb{R} \{ \mathbf{Z}_0 \}^{-1} a_0(\theta) \}^2 \).

The good accuracy of (52) is confirmed in Fig. 6(b) considering the polar plane \( \phi_k = \phi_1 = 0 \). Note that the full-wave simulation was performed using the Antenna Toolbox of MATLAB.

\[\text{VII. CHANNEL ESTIMATION}\]

So far, we have assumed perfect channel knowledge at the BS and analyzed the performance of MRT in the presence of mutual coupling. In this section, we focus on the channel estimation problem, which is crucial to beamforming.

\[\text{A. Channel Reciprocity}\]

In massive MIMO, it is typical to invoke channel reciprocity for time-division duplex (TDD) operation.\(^\text{44}\) This enables
the BS to estimate the downlink channel through uplink pilots sent by users. Let \( \mathbf{h}_{k,DL} = \mathbf{h}_k^T \in \mathbb{C}^{1 \times \mathcal{N}} \) denote the downlink channel, where \( \mathbf{h}_k \) is given by (36). The TDD assumption is then that \( \mathbf{h}_{k,UL} = \mathbf{h}_k^T \). Although the physical channels (i.e., those defined by electromagnetic theory) are reciprocal, this does not generally hold for their information-theoretic counterparts in the presence of antenna mutual coupling [45]. In particular, the BS needs to employ a linear transformation to compute \( \mathbf{h}_{k,DL}^T \) from the estimated \( \mathbf{h}_{k,UL} \). Nevertheless, the transmit and receive array gains coincide with each other under isotropic background noise and noise matching at the receiver [29, Eq. (97)]. Thus, we can assume that \( \mathbf{h}_{k,UL} = \mathbf{h}_{k,DL}^T \), and that the reception strategy maximizing the signal-to-noise ratio (SNR) of each user \( k \) is the maximum ratio combiner \( \mathbf{v}_k = \mathbf{h}_k^H / \|\mathbf{h}_k\| \).

**B. Problem Formulation**

We assume a block-fading model, where the channel coherence time is much larger than the training period. The BS estimates the uplink channel \( \mathbf{h}_k \) of each user \( k \) in rounds. Subsequently, we focus on an arbitrary user and omit the subscript “\( k \)”. Specifically, the training period for each user consists of \( N_{\text{slot}} \) time slots. At each time slot \( t = 1, \ldots, N_{\text{slot}} \), the user transmits the pilot signal \( x_t = \sqrt{P_p} \), where \( P_p \) is the power per pilot signal. In turn, the BS combines the received pilot signal using a training hybrid combiner \( \mathbf{v}_t = \mathbf{V}_{\text{RF},t} \mathbf{V}_{\text{BB},t} \in \mathbb{C}^{\mathcal{N} \times \mathcal{N}_{\text{RF}}} \). Therefore, the post-processed signal at slot \( t \), \( y_t \in \mathbb{C}^{\mathcal{K} \times 1} \), is written as

\[
y_t = \sqrt{\beta P_p} \mathbf{V}_t^H \mathbf{h} + \mathbf{v}_t^H \mathbf{n}_t, \tag{53}
\]

where \( \mathbf{n}_t \sim \mathcal{CN}(\mathbf{0}_{\mathcal{N} \times 1}, \sigma^2 \mathbf{I}_\mathcal{N}) \) is the additive noise vector. Let \( N_{\text{Beam}} = N_{\text{slot}} \mathcal{N}_{\text{RF}} \) denote the total number of pilot beams. After \( N_{\text{slot}} \) training slots, the BS acquires the measurement vector \( \bar{\mathbf{y}} \triangleq [\mathbf{y}_1^T, \ldots, \mathbf{y}_{N_{\text{slot}}}^T]^T \in \mathbb{C}^{\mathcal{N} \times 1} \) for \( \mathbf{h} \) as

\[
\bar{\mathbf{y}} = \sqrt{\beta P_p} \begin{bmatrix} \mathbf{V}_1^H \\ \vdots \\ \mathbf{V}_{N_{\text{slot}}}^H \end{bmatrix} \mathbf{h} + \begin{bmatrix} \mathbf{V}_1^H \mathbf{n}_1 \\ \vdots \\ \mathbf{V}_{N_{\text{slot}}}^H \mathbf{n}_{N_{\text{slot}}} \end{bmatrix} = \sqrt{\beta P_p} \mathbf{V}^H \mathbf{h} + \bar{\mathbf{n}}, \tag{54}
\]

where \( \mathbf{V} \triangleq [\mathbf{V}_1, \ldots, \mathbf{V}_{N_{\text{slot}}}] \in \mathbb{C}^{\mathcal{N} \times \mathcal{N}_{\text{Beam}}} \) is the effective noise matrix. More particularly, \( \mathbf{R}_{\mathbf{n}} \triangleq \sigma^2 \text{blkdiag}(\mathbf{V}_1^H \mathbf{V}_1, \ldots, \mathbf{V}_{N_{\text{slot}}}^H \mathbf{V}_{\text{Beam},N_{\text{slot}}}^H) \) is the covariance matrix of the effective noise, which is colored in general.\(^3\) Regarding the pilot combiners, due to the hybrid array architecture, \( \mathbf{V} = \mathbf{V}_{\text{RF}} \mathbf{V}_{\text{BB}} \) with \( \mathbf{V}_{\text{RF}} = [\mathbf{V}_{\text{RF},1}, \ldots, \mathbf{V}_{\text{RF},\mathcal{N}_{\text{RF}}}] \in \mathbb{C}^{\mathcal{N} \times \mathcal{N}_{\text{Beam}}} \) and \( \mathbf{V}_{\text{BB}} = \text{blkdiag}(\mathbf{V}_{\text{BB},1}, \ldots, \mathbf{V}_{\text{BB},\mathcal{N}_{\text{Beam}}}) \in \mathbb{C}^{\mathcal{N}_{\text{Beam}} \times \mathcal{N}_{\text{Beam}}} \) comprising the pilot RF beams and baseband combiners of the \( N_{\text{slot}} \) time slots, respectively.

**C. Orthogonal Matching Pursuit**

1) Sparse Formulation: We next consider a dictionary \( \mathbf{H} \in \mathbb{C}^{\mathcal{N} \times \mathcal{G}} \) whose \( \mathcal{G} \) columns are the channel vectors

\[
\text{associated with a predefined set of angles-of-arrival (AoA). Then, the uplink channel can be approximated as}
\]

\[
\mathbf{h} \approx \mathbf{H} \beta, \tag{55}
\]

where \( \beta \) is a \( \mathcal{G} \times 1 \) vector with a single nonzero entry corresponding to the LoS path. Therefore, (54) is recast as

\[
\bar{\mathbf{y}} = \mathbf{H} \beta + \bar{\mathbf{n}}, \tag{56}
\]

where \( \epsilon \leq \mathbb{E}(\|\bar{\mathbf{n}}\|) \) is an appropriately chosen bound on the mean magnitude of the effective noise. The \( l_1 \)-norm optimization problem in (56) can be readily solved by the popular OMP algorithm, which takes the following form for single-path channels

\[
g^* = \arg \max_{g \in \mathcal{G}} \| \mathbf{H}(g) \bar{\mathbf{y}} \|_1, \tag{57}
\]

where \( \mathcal{G} \) denotes the set of predefined AoA. Finally, the estimate of \( \mathbf{h} \) is obtained as \( \tilde{\mathbf{h}} = \mathbf{H}(g^*) \). It is worth stressing that the actual AoA might differ from the one defined by the dictionary. Nonetheless, this mismatch error can become negligible by adopting a high-resolution dictionary, as demonstrated in [46].

2) Dictionary and Pilot Beams: In the spirit of [47] and [48], we discretize the polar angle \( \theta \in [0, \theta_{\text{max}}] \) and azimuth angle \( \phi \in [0, \phi_{\text{max}}] \) as

\[
\theta_{gz} = \frac{\theta_{\text{max}}}{G_z} g_z, \quad g_z = 0, \ldots, G_z - 1, \tag{58}
\]

\[
\phi_{gz} = \frac{\phi_{\text{max}}}{G_z} g_x, \quad g_x = 0, \ldots, G_x - 1, \tag{59}
\]

where \( G = G_z G_x \) is the overall dictionary size, which results in the coupling-aware dictionary

\[
\tilde{\mathbf{H}} = \left[ \Re\{\mathbf{Z}\}^{-\frac{1}{2}} \mathbf{a}(\tilde{\theta}_0, \tilde{\phi}_0), \ldots, \Re\{\mathbf{Z}\}^{-\frac{1}{2}} \mathbf{a}(\tilde{\theta}_{G_z-1}, \tilde{\phi}_{G_z-1}) \right]. \tag{60}
\]
The elements of the RF combiner $\mathbf{V}_{RF}$ are selected from the set $\{-1/\sqrt{N},1/\sqrt{N}\}$ with equal probability. The reason we adopt a randomly formed RF combiner is that it will exhibit low mutual-column coherence, and therefore is expected to attain a high recovery probability according to the compressed sensing (CS) theory [49]. The columns of $\mathbf{V}_{RF}$ have been normalized so that the total power consumed during the channel estimation stage is $P_{CE} = KN_{\text{beam}}P_m$, as $KN_{\text{beam}}$ pilot beams are used for the $K$ users. The specific RF pilot design results in a colored effective noise. For this reason, $d_{\text{RF}}$ elements. Let $\mathbf{D}^H\mathbf{D}$ be the Cholesky decomposition of $\mathbf{V}_{RF}^H\mathbf{V}_{RF}$, where $\mathbf{D} \in \mathbb{C}^{N_{\text{RF}} \times N_{\text{RF}}}$ is an upper triangular matrix. Then, the baseband combiner of the $t$th slot is set to $\mathbf{v}_{BB,t} = \mathbf{D}^{-1}_t$, and hence $\mathbf{V} = \mathbf{V}_{RF}\text{blkdiag}(\mathbf{D}^{-1}_1,\ldots,\mathbf{D}^{-1}_{N_{\text{RF}}})$. Under this pilot beam design, the covariance matrix of the effective noise becomes $\mathbf{R}_n = \sigma^2 \mathbf{I}_{N_{\text{beam}}}$. 

VIII. REDUCING THE NUMBER OF BS ANTENNAS

As demonstrated in Section VI-B, the proposed NULA achieves larger gain than ULA thanks to the superdirective pairs. This excessive power gain can improve EE, because the number of antennas is kept fixed in both array designs. A more radical approach is to adopt a NULA with fewer elements than a ULA to substantially reduce the power consumption, akin to the paradigm of array thinning. Hereafter, we focus on the general case of planar arrays, and determine the parameters $N_g$, $d_g$, and $\bar{d}$ of NULA in order to attain a similar signal and interference power as a UPA of $N$ elements.

A. How Many BS Antennas Do We Need?

For the sake of fair comparison, we consider that both NULA and UPA have $N_x$ elements along the $x$-axis with inter-element spacing $d_x$. We then seek to find how many antennas along the $z$-axis are needed such that the two arrays offer the same signal power toward the endfire direction $(\theta, \phi) = (0^\circ, 0^\circ)$. This occurs when

$$N_xN_ga_0^H(0)\text{Re}\left\{\mathbf{Z}_0\right\}^{-1}a_0(0) = N, \quad (61)$$

which gives, after some basic algebra,

$$2N_g = \frac{1 - R_m^2}{\left(1 - R_m \cos(\kappa \bar{d})\right) N_x}. \quad (62)$$

Note that (62) hinges on the inter-element spacing $\bar{d}$ within dipole pairs, which determines the level of mutual coupling. From Fig. 7, we observe that the minimum number of antennas is attained for $\bar{d} = \lambda/5$.\(^6\) In this case, the NULA requires approximately 68% of the UPA antennas along the $z$-axis, and hence up to 32% saving in RF hardware is possible.

\(^6\)For spacings smaller than $\lambda/5$, the ohmic losses become dominant and decrease the array gain.
This performance can be readily obtained for a point-to-point link where the user is placed at the endfire direction of the BS. Conversely, the impact of reducing the number of BS antennas on the performance of multiuser transmissions is hard to analytically study, under either perfect or imperfect CSI. For this reason, we resort to numerical simulations in Section IX.

\subsection*{B. Spatial Resolution and Inter-Group Spacing}

It is known that spatial resolution is determined by the array size [26]. Thus, the NUPA with less elements will preserve its spatial resolution if its length along the \(z\)-axis is equal to that of UPA. This happens for \(N_g d + (N_g - 1) \bar{d}_g = \frac{(N/N_x - 1) d - N_g \bar{d}}{N_g - 1} \), or equivalently

\begin{equation}
\bar{d}_g = \frac{(N/N_x - 1) d - N_g \bar{d}}{N_g - 1},
\end{equation}

which ensures that the NUPA and UPA have the same physical size. To demonstrate this design methodology, we consider a \(32 \times 32\) element UPA at the BS with \(d = d_x = 0.7 \lambda\). Then, the NUPA will consist of \(N_g = 11\) dipole pairs, yielding a \(32 \times 22\) element array with inter-group spacing \(d_g = 1.95 \lambda\). We now compare these two arrays in terms of the overall signal and interference powers given by

\begin{equation}
\begin{aligned}
&\bar{a}_g^H(\theta_k) \text{Re} \{Z_0\}^{-1} \bar{a}_g(\theta_i) \quad \text{and} \\
&\bar{a}_d^H(\theta_k) \text{Re} \{Z_0\}^{-1} \bar{a}_d(\theta_i),
\end{aligned}
\end{equation}

and

\begin{equation}
B_x = N_x \left| D_{N_x} \left( \kappa d_x + (\bar{N} - 1) \bar{d} \right) \left( e^{j \theta_k} - e^{j \theta_i} \right) \right|^2
\end{equation}

are the beampatterns of the NUPA along the \(z\) and \(x\) directions, respectively [50].

\textbf{Remark 2:} The proposed NUPA enables beam broadening without sacrificing the maximum array gain by adopting a small inter-group spacing \(d_g\), i.e., by decreasing the length of the overall array along the \(z\)-axis. This feature is showcased in Fig. 9, and can be very useful for THz links where wide beamwidths alleviate the detrimental effect of beam misalignment [51].

\section*{IX. Simulation Results}

We conduct extensive numerical simulations to assess the performance of the proposed array design. In all numerical experiments, we consider a fractional bandwidth \(B/f \leq 0.1\) and LoS links, which ensure a spatially narrowband propagation channel [46]. The other simulation parameters are summarized in Table II. Also, all values in the power consumption model are taken from [20].
for any pair $n$ denotes the optimal excitation vector at each antenna pair. This impedance matrix is Toeplitz, and thus the BS acquires the channel estimates $h_{k}^\dag \hat{w}_n$ through the OMP estimator of Section VII-C. In the UPA case, the dictionary is $H = \{a(\theta_n, \phi_n), \ldots, a(\theta_{G_n-1}, \phi_{G_n-1})\}$. The BS treats those estimates as the true channels in the beamforming stage, i.e., $w_k = H_k^\dag / ||H_k||$. The achievable rate of user $m = 0$ is then specified as $R_{m} = B \log_2 (1 + P_m h_m^\dag H_m)$, where $H_m$ is given by (35). Note that this rate is achieved under the assumption that user $m$ knows $|h_k^2 w_k|^2$ and $\sum_{n\neq m} |h_n^2 w_n|^2$ in the decoding stage. These are scalars and, hence, are easy to be estimated. For the performance evaluation, the primary metrics are the sum-rate, minimum rate, and total EE defined as $\sum_{m=0}^{M} R_m$, $\min_{m} R_m$, and $\sum_{m=0}^{M} R_m / P_m$, respectively.

Figure 11 shows the results for a two-user transmission. As observed, the NUPA boosts the mean EE by 29% without method for two side-by-side dipoles [26, Ch. 8]. Since $Z_{\alpha} = \text{blkdiag}(Z_{0, a}, \ldots, Z_{0, a})$ using the relationship $Z_{\alpha}^{\text{opt}} = Z_{\alpha}^{\text{opt}}$, where $Z_{0, a}$

\begin{align}
    Z_{\alpha} &= \left[ Z_{\text{self}} + Z_m I_{a_n+1(0)}^T \right] \\
    &= \left[ Z_{\text{self}} + Z_m R_{m} Re^{-j\kappa d} \right] \\
    &= \left[ Z_{\text{self}} + Z_m R_{m} Re^{-j\kappa d} \right] \\
    &= \left[ Z_{\text{self}} + Z_m R_{m} Re^{-j\kappa d} \right]
\end{align}

Therefore, all dipole pairs share a common active impedance matrix whose entries are $[Z_{0, a}]_{1,1} = 21.84 + j32.89 \Omega$, and $[Z_{0, a}]_{2,2} = 40.87 + j83.89 \Omega$, for half-wavelength dipoles made of copper and inter-element distance $d = \lambda/5$. This is a unique feature of the proposed NULA/NUPA. In contrast, a superdirective ULA/UPA would need a different matching procedure for each port because the current amplitude is not uniform along the ports (akin to the source voltages). Thus, the derived architecture simplifies also single-port matching.

Finally, compared to an uncoupled ULA/UPA with $Z_{\text{self}} = 75.94 + j41.76 \Omega$, the active impedances $[Z_{0, a}]_{1,1}$ and $[Z_{0, a}]_{2,2}$ are not very large, and thus could be easily matched.

B. Multiuser Transmissions With Imperfect CSI

We now consider that the BS simultaneously transmits to $K$ users using MRT. The users’ directions are not fixed, and thus the BS acquires the channel estimates $\{h_k\}_{k=1}^{K}$ through the OMP estimator of Section VII-C. In the UPA case, the dictionary is $H = \{a(\theta_n, \phi_n), \ldots, a(\theta_{G_n-1}, \phi_{G_n-1})\}$. The BS treats those estimates as the true channels in the beamforming stage, i.e., $w_k = H_k^\dag / ||H_k||$. The achievable rate of user $k$ is then specified as $R_k = B \log_2 (1 + \text{SINR}_k)$, where SINR$_k$ is given by (35). Note that this rate is achieved under the assumption that user $k$ knows $|h_k^2 w_k|^2$ and $\sum_{n\neq k} |h_n^2 w_n|^2$ in the decoding stage. These are scalars and, hence, are easy to be estimated. For the performance evaluation, the primary metrics are the sum-rate, minimum rate, and total EE defined as $\sum_{k=1}^{K} R_k$, $\min_{k} R_k$, and $\sum_{k=1}^{K} R_k / P_k$, respectively.

Figure 11 shows the results for a two-user transmission. As observed, the NUPA boosts the mean EE by 29% without...
compromising the sum or minimum data rate. Regarding the OMP estimator, we calculate the normalized squared error (NSE) defined as $\text{NSE} = \frac{1}{K} \sum_{k=1}^{K} \left\| \mathbf{h}_k - \hat{\mathbf{h}}_k \right\|^2 / \left\| \mathbf{h}_k \right\|^2$. The training overhead per user is $N_{\text{beam}} = 0.8N$ for the NUPA, whilst $N_{\text{beam}} \approx 820$ beams for the UPA. Importantly, Fig. 12 indicates that channel estimation accuracy is similar for both arrays, although the NUPA employs 256 pilots less. Consequently, it has the potential to reduce also the CSI acquisition overhead of massive MIMO BS.

C. Comparison With Patch Antennas

Microstrip antennas, also known as patch antennas, are widely adopted in real-world mmWave and THz MIMO systems due to their planar geometry, directional radiation characteristics, and easy fabrication into printed circuit boards [52]. As such, it is of practical interest to investigate how our superdirective design compares with a typical patch antenna array. Here, we would like to stress that the proposed superdirective pair can be readily realized with printed dipoles in order to have a planar radiating structure [53]. In the sequel, we consider a copper patch of length $L$ and width $W$ on top of a grounded dielectric substrate of thickness $h$, dielectric constant $\epsilon_r$ and loss tangent $\tan \delta$. To ensure high radiation efficiency, the substrate thickness is selected in the range $0.025\lambda \leq h \leq 0.05\lambda$, where $\lambda$ is the free-space wavelength. Since the fields generated by the antenna propagate in two different media, a homogeneous medium is assumed with effective dielectric constant [26]

$$\epsilon_{\text{reff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \sqrt{1 + 12h/W}. \quad (71)$$

1) Antenna Gain: For the dominant transverse magnetic TM$_{010}$ mode, the width and length of the copper patch are chosen as [26]

$$W = \frac{c}{2f_r} \sqrt{\frac{2}{\epsilon_r + 1}}, \quad (72)$$

and

$$L = \frac{4c}{2f_r \sqrt{\epsilon_{\text{reff}}}} - 2\Delta L, \quad (73)$$

where $f_r$ denotes the resonant frequency of the antenna, and

$$\Delta L = 0.412h \left( \frac{\epsilon_{\text{reff}} + 0.3}{\epsilon_{\text{reff}} - 0.258} \right) \left( \frac{W/h + 0.264}{W/h + 0.8} \right). \quad (74)$$

Using these equations, the dimensions of the radiator are calculated for given $f_r$, $h$, $\epsilon_r$, and $\tan \delta$. Next, the azimuth $E_\theta$ and polar $E_\phi$ components of the electric field of the antenna are specified by [54, Ch. 4]

$$E_\theta = \frac{2h \sin \left( \frac{\pi W}{\lambda} \sin \theta \sin \phi \right)}{\pi} \cos \left( \frac{\pi L_{\text{reff}}}{\lambda} \sin \theta \cos \phi \right) \cos \phi, \quad (75)$$

$$E_\phi = -\frac{2h \sin \left( \frac{\pi W}{\lambda} \sin \theta \sin \phi \right)}{\pi} \sin \theta \sin \phi \times \cos \left( \frac{\pi L_{\text{reff}}}{\lambda} \sin \theta \cos \phi \right) \cos \theta \sin \phi, \quad (76)$$

where $L_{\text{reff}} \triangleq L + \Delta L$ is the effective length. Note that (75)-(76) are valid for polar angles $0 \leq \theta \leq \pi/2$ due to the presence of the ground plane. The directivity of the antenna is defined as $D(\theta, \phi) \triangleq 4\pi U/P_{\text{rad}}$, where $U = \frac{1}{2\pi} (E_\theta^2 + E_\phi^2)$ is the radiation intensity and $P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi/2} U \sin \theta d\theta d\phi$ is the radiated power.

For sufficiently thin substrate, the surface wave losses can be neglected, and hence the radiation efficiency is determined by

$$\eta_{\text{rad}} = \frac{P_{\text{rad}}}{P_{\text{rad}} + P_c + P_d}, \quad (77)$$

where $P_c$ and $P_d$ denote the power dissipated in the conducting patch and dielectric substrate, respectively. These are given by the known closed-form expressions [54, Ch. 4]

$$P_d = \frac{1}{4} \omega \epsilon_0 \epsilon_r LW \tan \delta, \quad (78)$$

$$P_c = \frac{\omega \epsilon_0 \epsilon_r LW}{4\sigma R_s}, \quad (79)$$

where $\epsilon_0$ is the permittivity of free space, $\omega$ is the angular frequency, and $R_s = \sqrt{\omega \mu / (2\sigma_c)}$ is the sheet resistance of the copper patch. Finally, the element gain is $G_e(\theta, \phi, \phi') = \eta_{\text{rad}} D(\theta, \phi')$.

Figure 13(a) shows the gain pattern of a single half-wavelength dipole, a superdirective dipole pair, and a rectangular patch antenna with copper and a Polydimethylsiloxane substrate ($\epsilon_r = 2.35, \tan \delta = 0.03$) of thickness $h = 0.03\lambda$ [55]. The estimated patch antenna directivity at broadside is $D(0, 0) = 7.1$ dBi with radiation efficiency $\eta_{\text{rad}} = 60\%$, yielding a maximum gain $G_e(0, 0) = 4.86$ dBi in perfect agreement with [55] and [56]. On the other hand, the superdirective dipole pair offers a 6.6 dBi gain, and exhibits sufficient directionality at the element level. Regarding the footprint of the radiators, the width of the patch antenna is determined by the width of the grounded substrate, as depicted in Fig. 13(b), and is given by $W_g = 6h + W \approx 0.56\lambda$ [54]. Therefore, it is slightly larger than that of the half-wavelength dipoles comprising the superdirective pair.

Remark 3: A superdirective dipole pair is a multi-port antenna with reconfigurable radiation pattern. This reconfigurability could be potentially exploited to further increase the capacity of a multi-user link [57].

---

**Fig. 12.** CDF of the NSE for $K = 2$ users and partial beam training with $N_{\text{beam}} = 0.8N$ pilots per user.
Fig. 13. (a) Gain patterns at the vertical plane $\phi = 0$. The spacing between dipoles is $d = \lambda/5$. (b) Sketch of patch antenna and superdirective pair geometries.

Fig. 14. Broadside gain difference versus the number of BS antennas for various substrate heights.

2) Array Gain: We now consider a linear array of $N$ elements. Both the superdirective pair and the patch antenna have nulls in the radiation pattern at $\theta = \pi/2$, and hence the elements can be placed in a linear configuration with sub-wavelength spacing and negligible mutual coupling. Therefore, the maximum broadside array gain is $G(0, 0) = G_e(0, 0)N$. Figure 14 shows the broadside gain difference between the patch and superdirective antenna arrays. As seen for $h = 0.03\lambda$, the performance improvement is more than 20 dB for $N > 100$ antennas. More importantly, this is attained with a compact configuration as the patch antenna array will be $N(W_g - \lambda/2) > 6\lambda$ wider for $N > 100$. Consequently, the proposed grouping architecture can enable the realization of low-profile MIMO transceivers.

X. CONCLUSION

In this paper, we introduced the novel concept of superdirective dipole pairs. Specifically, we first derived a comprehensive array model that captures the physics of mutual coupling and ohmic losses of nonideal antennas. Capitalizing on the derived model, we then studied the implementation aspects of superdirectivity, namely the impedance matching and hybrid beamforming problems. To surmount the challenges of superdirective ULAs/UPAs, we proposed to partition the BS array into multiple two-element groups of sub-wavelength spacing. The resulting NULA/NUPA facilitates multi-port impedance matching, which is optimal for any beamforming angle. More importantly, it enables the realization of superdirectivity with a single RF chain per beam. Afterwards, we pursued a performance analysis in terms of achievable rate and EE under perfect and imperfect CSI. To this end, approximate closed-form expressions for the signal and multi-user interference powers were provided under MRT. The channel estimation problem was also addressed by employing OMP along with a coupling-aware dictionary. Numerical results were finally provided demonstrating that the proposed method boosts the EE of THz massive MIMO without compromising the data transmission and channel estimation performances. As a result, arrays of superdirective dipole pairs can be a promising approach for realizing energy-efficient MIMO antennas with sharp beamforming capabilities.

APPENDIX A

Based on the radiation equations, the electric field is specified as [26, Ch. 3]

$$E(r, \theta, \phi) = -j\eta \frac{k e^{-jkr}}{4\pi r} (A_\theta e_\theta + A_\phi e_\phi),$$

(80)

where

$$A_\theta = \int_{-\ell/2}^{\ell/2} I(x') \cos \theta \cos \phi e^{j\kappa x' \cos \phi \sin \theta} dx'$$

$$= \frac{I(0) \cos \theta \cos \phi}{\sin(\kappa \ell/2)} \int_{-\ell/2}^{\ell/2} \sin(\kappa \ell/2 - \kappa|x'|) e^{j\kappa x' \cos \phi \sin \theta} dx,'$$

(81)

and

$$A_\phi = \int_{-\ell/2}^{\ell/2} -I(x') \sin \phi e^{j\kappa x' \cos \phi \sin \theta} dx'$$

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\[= -\frac{I(0) \sin \phi}{\sin(\kappa \ell/2)} \int_{-\ell/2}^{\ell/2} \sin(\kappa \ell/2 - \kappa |x'|) e^{j\kappa x'\cos \phi \sin \theta} \, dx. \]

(82)

Utilizing the identity [26]

\[
\int e^{\alpha x} \sin(\beta x + \gamma) \, dx = \frac{e^{\alpha x}}{\alpha^2 + \beta^2} [a \sin(\beta x + \gamma) - \beta \cos(\beta x + \gamma)],
\]

(83)

for \( a = j \kappa \cos \phi \sin \theta, \beta = \kappa, \) and \( \gamma = \kappa \ell/2, \) and after some algebraic manipulations, we get

\[
\int_{-\ell/2}^{\ell/2} \sin(\kappa \ell/2 - \kappa |x'|) e^{j\kappa x'\cos \phi \sin \theta} \, dx
\]

\[
= \frac{2 \cos(\kappa \ell/2 \cos \phi \sin \theta) - \cos(\kappa \ell/2)}{\kappa \sin^2 \phi + \cos^2 \phi \cos^2 \theta}.
\]

(84)

Combining those equations yields the field expression in (2).

**APPENDIX B**

The inverse matrix of \( \mathbf{Z}_0 \) is

\[
\text{Re}\{\mathbf{Z}_0\}^{-1} = \frac{1}{R_{\text{self}} - R_m} \begin{bmatrix} R_{\text{self}} & -R_m \\ -R_m & R_{\text{self}} \end{bmatrix}.
\]

(85)

We now need to calculate the square root of the \( 2 \times 2 \) matrix \( \text{Re}\{\mathbf{Z}_0\}^{-1} \). To do so, we utilize the lemma [58]

\[
\mathbf{A}^{1/2} = \frac{1}{I} \begin{bmatrix} a_{11} + s & a_{12} \\ a_{21} & a_{22} + s \end{bmatrix},
\]

(86)

for

\[
\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},
\]

(87)

where \( s = \sqrt{a_{11}a_{22} - a_{12}a_{21}} \) and \( t = \sqrt{a_{11} + a_{22} + 2s}. \)

In our case, we have that

\[
s = \frac{1}{\sqrt{R_{\text{self}} - R_m}}, \quad t = \sqrt{2R_{\text{self}} + 2\sqrt{R_{\text{self}}^2 - R_m^2}}.
\]

(88)

Applying (86) to \( \text{Re}\{\mathbf{Z}_0\}^{-1} \), and after basic algebra, yields

\[
\text{Re}\{\mathbf{Z}_0\}^{-1/2} = \frac{1}{\sqrt{2R_{\text{self}} + 2\sqrt{R_{\text{self}}^2 - R_m^2}}} \begin{bmatrix} R_{\text{self}} & -R_m \\ -R_m & R_{\text{self}} \end{bmatrix}
\]

\[
\times \begin{bmatrix} R_{\text{self}} & -R_m \\ -R_m & R_{\text{self}} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

\[
= \frac{1}{\sqrt{2R_{\text{self}} + 2\sqrt{R_{\text{self}}^2 - R_m^2}}}.
\]

(89)

and

\[
\text{Re}\{\mathbf{Z}_0\}^{-1/2} \mathbf{a}_0(\theta)
\]

\[
= \frac{1}{\sqrt{2R_{\text{self}} + 2\sqrt{R_{\text{self}}^2 - R_m^2}}}.
\]

Lastly, calculating the magnitudes of the entries of \( \text{Re}\{\mathbf{Z}_0\}^{-1/2} \mathbf{a}_0(\theta) \), and some algebraic manipulations, we obtain (31). Also, because \( \mathbf{Z} \approx \mathbf{I}_{N_g} \otimes \mathbf{Z}_0 \) in the proposed NULA, (18) becomes

\[
\mathbf{Z}_M = \begin{bmatrix} -j\text{Im}\{\mathbf{Z}_0\} \mathbf{I}_{N_g} \otimes \mathbf{I}_2 - j\sqrt{R_{\text{self}}} \mathbf{I}_{N_g} \otimes \text{Re}\{\mathbf{Z}_0\}^{1/2} \\ -j\sqrt{R_{\text{self}}} \mathbf{I}_{N_g} \otimes \text{Re}\{\mathbf{Z}_0\}^{1/2} - j\mathbf{I}_{N_g} \otimes \text{Im}\{\mathbf{Z}_0\} \end{bmatrix}.
\]

(91)

We now partition the vectors of voltages and currents at the input and output of the impedance matching network as \( \mathbf{v}_M = [\mathbf{v}_{M,0}, \ldots, \mathbf{v}_{M,N_g-1}]^T, \mathbf{i}_M = [\mathbf{i}_{M,0}, \ldots, \mathbf{i}_{M,N_g-1}]^T, \) and \( \mathbf{v} = [\mathbf{v}_0, \ldots, \mathbf{v}_{N_g-1}]^T, \) and \( \mathbf{i} = [\mathbf{i}_0, \ldots, \mathbf{i}_{N_g-1}]^T, \) where \( \mathbf{v}_{M,n} = [\mathbf{v}_{M,n}, \mathbf{v}_{M,n+1}]^T \) is the vector of voltages at the \( n \)th input port pair; \( \mathbf{i}_{M,n}, \mathbf{v}_{n}, \) and \( \mathbf{i}_n \) are defined similarly. By using (91) and the previous decompositions, (16) is recast as

\[
\begin{bmatrix} \mathbf{v}_{M,0} \\ \mathbf{v}_0 \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{M,0} & \mathbf{Z}_{M,1} \mathbf{i}_{M,0} \\ \mathbf{i}_0 & \mathbf{Z}_{M,0} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{M,N_g-1} \\ \mathbf{v}_{N_g-1} \end{bmatrix}^T
\]

(92)

which shows that \( \mathbf{Z}_M \) comprises \( N_g \) four-port matching networks, which are independent of each other.

**APPENDIX C**

We have that

\[
\left| \mathbf{a}_H(\theta_k) \text{Re}\{\mathbf{Z}_{\text{approx}}\}^{-1} \mathbf{a}(\theta_i) \right|^2
\]

\[
= \left| \left[ \left( \mathbf{a}_H(\theta_k) \otimes \mathbf{a}_H(\theta_k) \right) \mathbf{I}_N \otimes \text{Re}\{\mathbf{Z}_0\}^{-1} \mathbf{a}_g(\theta_k) \otimes \mathbf{a}_0(\theta_i) \right] \mathbf{a}_g(\theta_k) \otimes \mathbf{a}_0(\theta_i) \right|^2
\]

\[
= \left| \left[ \mathbf{a}_H(\theta_k) \otimes \mathbf{a}_H(\theta_k) \right] \mathbf{I}_N \otimes \text{Re}\{\mathbf{Z}_0\}^{-1} \mathbf{a}_g(\theta_k) \otimes \mathbf{a}_0(\theta_i) \right|^2
\]

\[
= \left| \mathbf{a}_H(\theta_k) \text{Re}\{\mathbf{Z}_0\}^{-1} \mathbf{a}_g(\theta_k) \right|^2 \left| \mathbf{a}_H(\theta_k) \mathbf{a}_g(\theta_k) \mathbf{a}_0(\theta_i) \right|^2
\]

\[
= \mathbf{a}_H(\theta_k) \text{Re}\{\mathbf{Z}_0\}^{-1} \mathbf{a}_g(\theta_k) \mathbf{a}_0(\theta_i)
\]

\[
= \frac{N_g}{N_g} \sum_{n_g=0}^{N_g-1} e^{-j\kappa n_g(\tilde{d} + (N-1)\tilde{d})\cos \theta_k - \cos \theta_i}
\]

\[
= \frac{1}{N_g} \left[ 1 - e^{-j\kappa N_g(\tilde{d} + (N-1)\tilde{d})\cos \theta_k - \cos \theta_i} \right]^2
\]

\[
= N_g \left[ 1 - e^{-j\kappa (\tilde{d} + (N-1)\tilde{d})\cos \theta_k - \cos \theta_i} \right]^2
\]

(94)

which completes the proof.
REFERENCES

[1] W. Roh et al., “Millimeter-wave beamforming as an enabling technology for 5G cellular communications: Theoretical feasibility and prototype results,” IEEE Commun. Mag., vol. 52, no. 2, pp. 106–113, Feb. 2014.

[2] S. D. Assimonis, M. A. Abbas, and V. Fusco, “Millimeter-wave multi-mode circular antenna array for uni-cast multi-cast and OAM communication,” Sci. Rep., vol. 11, no. 1, p. 4928, Mar. 2021.

[3] T. S. Rappaport et al., “Wireless communications and applications above 100 GHz: Opportunities and challenges for 6G and beyond,” IEEE Access, vol. 7, pp. 78729–78757, 2019.

[4] M. Matthaiou, O. Yurduseven, H. Q. Ngo, D. Morales-Jimenez, S. L. Cotton, and V. Fusco, “The road to 6G: Ten physical layer challenges for communications engineers,” IEEE Commun. Mag., vol. 59, no. 1, pp. 64–69, Jan. 2021.

[5] J. Zhang, E. Björnson, M. Matthaiou, D. W. K. Ng, H. Yang, and D. J. Love, “Prospective multiple antenna technologies for beyond 5G,” IEEE J. Sel. Areas Commun., vol. 38, no. 8, pp. 1657–1660, Aug. 2020.

[6] H.-J. Song and N. Lee, “Terahertz communications: Challenges in the next decade,” IEEE Trans. Terahertz Sci. Technol., vol. 12, no. 3, pp. 105–117, Mar. 2022.

[7] E. Björnson, L. Sanguinetti, J. Hoydis, and M. Debahb, “Optimal design of energy-efficient multi-user MIMO systems: Is massive MIMO the answer?” IEEE Trans. Wireless Commun., vol. 14, no. 6, pp. 3059–3075, Jun. 2015.

[8] M. L. Morris, M. A. Jensen, and J. W. Wallace, “Superdirectivity and low-resolution PSs for mmWave MU-MISO systems,” IEEE Trans. Antennas Propag., vol. 66, no. 10, pp. 5979–5985, Oct. 2018.

[9] S. Shen and R. D. Murch, “Impedance matching for compact multiple antenna systems in random RF fields,” IEEE Trans. Antennas Propag., vol. 64, no. 2, pp. 920–925, Feb. 2016.

[10] T. L. Marzetta, “Limits of transmit and receive array gain in massive MIMO,” in Proc. IEEE WCNC, May 2020, pp. 1–8.

[11] S. D. Assimonis, T. V. Yooltis, and C. S. Antonopoulos, “Design and optimization of unipolar EBG structures for low profile antenna applications and mutual coupling reduction,” IEEE Trans. Antennas Propag., vol. 60, no. 10, pp. 4944–4949, Oct. 2012.

[12] X. Yang, M. Matthaiou, J. Yang, C.-K. Wen, F. Gao, and S. Jin, “Hardware-constrained millimeter-wave systems for 5G: Challenges, opportunities, and solutions,” IEEE Commun. Mag., vol. 57, no. 1, pp. 44–50, Jan. 2019.

[13] X. Gao, L. Dai, Y. Zhang, T. Xie, X. Dai, and Z. Wang, “Fast channel tracking for terahertz beamspace massive MIMO systems,” IEEE Trans. Veh. Technol., vol. 66, no. 7, pp. 5689–5696, Jul. 2017.

[14] C. Lin and G. Y. Li, “Indoor terahertz communications: How many antenna arrays are needed?” IEEE Trans. Wireless Commun., vol. 14, no. 6, pp. 3097–3107, Jun. 2015.

[15] C. Han, A. O. Bicen, and I. F. Akyildiz, “Multi-ray channel modeling and wideband characterization for wireless communications in the terahertz band,” IEEE Trans. Wireless Commun., vol. 14, no. 5, pp. 2402–2412, May 2015.

[16] L. N. Ribeiro, S. Schwarz, M. Rupp, and A. L. F. de Almeida, “Energy efficiency of mmWave massive MIMO precoding with low-resolution DACs,” IEEE J. Sel. Topics Signal Process., vol. 12, no. 2, pp. 298–312, May 2018.

[17] E. Björnson, J. Hoydis, and L. Sanguinetti, “Massive MIMO networks: Spectral, energy, and hardware efficiency,” Found. Trends Signal Process., vol. 11, nos. 3–4, pp. 154–655, 2017.

[18] M. T. Ivrlac and J. A. Nossek, “Physical modeling of communication systems in information theory,” in Proc. IEEE Int. Symp. Inf. Theory, Jun. 2009, pp. 2179–2183.

[19] M. A. Jensen and J. W. Wallace, “A review of antennas and propagation for MIMO wireless communications,” IEEE Trans. Antennas Propag., vol. 52, no. 11, pp. 2810–2824, Nov. 2004.

[20] C. Hermosilla, R. Feick, R. A. Valenzuela, and L. Ahumada, “Improving MIMO capacity with directive antennas for outdoor-indoor scenarios,” in Proc. VTC Spring - IEEE Veh. Technol. Conf., May 2008, pp. 41–44.

[21] C. Hermosilla, E. G. Larsson, and H. Yang, “Performance of Massive MIMO,” in Proc. IEEE Int. Symp. Inf. Theory, Jun. 2009, pp. 2179–2183.

[22] C. Hermosilla, R. Feick, R. A. Valenzuela, and L. Ahumada, “Improving MIMO capacity with directive antennas for outdoor-indoor scenarios,” in Proc. VTC Spring - IEEE Veh. Technol. Conf., May 2008, pp. 41–44.
K. R. Jha and S. K. Sharma, “Waveguide integrated microstrip patch antenna at THz frequency,” in *Proc. IEEE Antennas Propag. Soc. Int. Symp. (APSURSI)*, Jul. 2014, pp. 1851–1852.

S. Abu-Surra et al., “End-to-end 6G terahertz wireless platform with adaptive transmit and receive beamforming,” in *Proc. IEEE Int. Conf. Commun. Workshops (ICC Workshops)*, May 2022, pp. 897–903.

K. Ying et al., “Reconfigurable massive MIMO: Harnessing the power of the electromagnetic domain for enhanced information transfer,” *IEEE Wireless Commun.*., early access, Mar. 20, 2023, doi: 10.1109/MWC.014.2200418.

B. W. Levinger, “The square root of a 2 × 2 matrix,” *Math. Mag.*, vol. 53, no. 4, pp. 222–224, 1980.

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