Non-Abelian Discrete Flavor Symmetries

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Abstract

This is an incomplete survey of some non-Abelian discrete symmetries which have been used recently in attempts to understand the flavor structure of leptons and quarks. To support such symmetries, new scalar particles are required. In some models, they are very massive, in which case there may not be much of a trace of their existence at the TeV scale. In other models, they are themselves at the TeV scale, in which case there is a reasonable chance for them to be revealed at the LHC (Large Hadron Collider) at CERN.
Introduction

Leptons and quarks come in three families. Their flavor structure, i.e. the specific values of their mass and mixing matrices, has long been a puzzle and a subject of study. In recent years, with the steady accumulation of data on solar and atmospheric neutrino oscillations, the lepton mixing matrix has been determined to a large extent and it came as a surprise to many that it does not resemble at all the known quark mixing matrix. Is there a way to understand this? One possible approach is the use of non-Abelian discrete symmetries, such as $S_3$ and $A_4$ among others. In this report, I will survey this topic, offering a basic recipe for constructing models, with an extensive (but nevertheless incomplete) list of references.

Finite Groups

To obtain a non-Abelian discrete symmetry, a simple heuristic way is to choose two specific noncommuting matrices and form all possible products. As a first example, consider the two $2 \times 2$ matrices:

$$ A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} \omega & 0 \\ 0 & \omega^{-1} \end{pmatrix}, $$

where $\omega^n = 1$, i.e. $\omega = \exp(2\pi i/n)$. Since $A^2 = 1$ and $B^n = 1$, this group contains $Z_2$ and $Z_n$. For $n = 1, 2$, we obtain $Z_2$ and $Z_2 \times Z_2$ respectively, which are Abelian. For $n = 3$, the group generated has 6 elements and is in fact the smallest non-Abelian finite group $S_3$, the permutation group of 3 objects. This particular representation is not the one found in textbooks, but is related to it by a unitary transformation [1], and was first used in 1990 for a model of quark mass matrices [2, 3]. For $n = 4$, the group generated has 8 elements which are in fact $\pm 1, \pm \sigma_1, \pm \sigma_2, \pm i\sigma_3$, where $\sigma_{1,2,3}$ are the usual Pauli spin matrices. This forms the group $D_4$, i.e. the symmetry group of the square, which was first used in 2003 [4, 5]. If the 8 elements are $\pm 1, \pm i\sigma_{1,2,3}$ instead, then they form the group of quaternions $Q$, which
has also been used [6] for quark and lepton mass matrices. In general, the groups generated by Eq. (1) have $2n$ elements and may be denoted as $\Delta(2n)$.

Consider next the two $3 \times 3$ matrices:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega^{-3} \end{pmatrix}. \quad (2)$$

Since $A^3 = 1$ and $B^n = 1$, this group contains $Z_3$ and $Z_n$. For $n = 1$, we obtain $Z_3$. For $n = 2$, the group generated has 12 elements and is $A_4$, the even permutation group of 4 objects, which was first used in 2001 in a model of lepton mass matrices [7, 8]. It is also the symmetry group of the tetrahedron, one of five perfect geometric solids, identified by Plato with the element “fire” [9]. In general, the groups generated by Eq. (2) have $3n^2$ elements and may be denoted as $\Delta(3n^2)$ [10]. They are in fact subgroups of $SU(3)$. In particular, $\Delta(27)$ has also been used [11, 12]. Generalizing to $k \times k$ matrices, we then have the series $\Delta(kn^{k-1})$. However, since there are presumably only 3 families, $k > 3$ is probably not of much interest.

Going back to $k = 2$, but using instead the following two matrices:

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} \omega & 0 \\ 0 & 1 \end{pmatrix}. \quad (3)$$

Now again $A^2 = 1$ and $B^n = 1$, but the group generated will have $2n^2$ elements. Call it $\Sigma(2n^2)$. For $n = 1$, it is just $Z_2$. For $n = 2$, it is $D_4$ again. For $k = 3$, consider

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} \omega & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (4)$$

then the groups generated have $3n^3$ elements and may be denoted as $\Sigma(3n^3)$. They are in fact subgroups of $U(3)$. For $n = 1$, it is just $Z_3$. For $n = 2$, it is $A_4 \times Z_2$. For $n = 3$, the group $\Sigma(81)$ has been used [13] to understand the Koide formula [14] as well as lepton mass matrices [15]. In general, we have the series $\Sigma(kn^k)$. 

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Model Recipe

(I) Choose a group, e.g. $S_3$ or $A_4$, and write down its possible representations. For example
$S_3$ has $1, 1', 2$; $A_4$ has $1, 1', 1'', 2$. Work out all product decompositions. For example
$2 \times 2 = 1 + 1' + 2$ in $S_3$, and $3 \times 3 = 1 + 1' + 1'' + 3 + 3$ in $A_4$.

(II) Assign $(\nu, l)_{1,2,3}$ and $l'_{1,2,3}$ to the representations of your choice. If you want to consider
only renormalizable interactions, you will need to add Higgs doublets (and perhaps also
triplets and singlets) and assign them. You may also want to consider adding neutrino
singlets.

(III) Because of your choice of particle content and their representations, the Yukawa struc-
ture of your model is restricted. As the Higgs bosons acquire vacuum expectation values
(which may be related because of some extra or residual symmetry), your lepton mass ma-
trices will have certain particular forms. If the number of parameters involved are less than
the number of observables, you then have one or more predictions. Of course, the forms
themselves have to be consistent with the known values of $m_e, m_\mu, m_\tau$, etc.

(IV) Because there will be more than one Higgs doublet in such models, flavor nonconserva-
tion will appear at some level. You will need to work out its phenomenological consequences,
making sure that your model is consistent with present experimental constraints. You can
then proceed to explore its observability at the TeV scale.

(V) If you insist on using only the one Higgs doublet of the Standard Model, then you must
consider effective nonrenormalizable interactions to support the discrete flavor symmetry. In
such models, there are no predictions beyond the forms of the mass matrices themselves.

(VI) Quarks can be considered in the same way. The two quark mass matrices $M_u$ and $M_d$
must be nearly aligned so that their mixing matrix involves only small angles. In contrast,$M_\nu$ and $M_l$ should have different structures so that large angles can be obtained.
$S_3$

Being the simplest, the non-Abelian discrete symmetry $S_3$ was used already [16] in the early days of strong interactions. There are many recent applications [17, 18, 19, 20, 21, 22, 23, 24, 25, 26], some of which are discussed in my talk at VI-Silafae [27]. Typically, such models often require extra symmetries beyond $S_3$ to reduce the number of parameters, or assumptions of how $S_3$ is spontaneously and softly broken. For illustration, consider the model of Kubo et al. [17] which has recently been updated by Felix et al. [28]. The symmetry used is actually $S_3 \times Z_2$, with the assignments

$$(\nu, l), \ l^c, \ N, \ (\phi^+, \phi^0) \sim 1 + 2.$$ (5)

and equal vacuum expectation values for the two Higgs doublets transforming as 2 under $S_3$. The $Z_2$ symmetry serves to eliminate 4 Yukawa couplings otherwise allowed by $S_3$, resulting in an inverted ordering of neutrino masses with

$$\theta_{23} \simeq \pi/4, \ \theta_{13} \simeq 0.0034, \ m_{ee} \simeq 0.05 \text{ eV},$$ (6)

where $m_{ee}$ is the effective Majorana neutrino mass measured in neutrinoless double beta decay. This model relates $\theta_{13}$ to the ratio $m_e/m_\mu$.

$A_4$

To understand why quarks and leptons have very different mixing matrices, $A_4$ turns out to be very useful. It allows the two different quark mass matrices to be diagonalized by the same unitary transformations, implying thus no mixing as a first approximation, but because of the assumed Majorana nature of the neutrinos, a large mismatch may occur in the lepton sector, thus offering the possibility of obtaining the so-called tribimaximal mixing matrix
which is a good approximation to present data. One way of doing this is to consider the decomposition

$$U_{l\nu} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -i/\sqrt{2} \\ 1/\sqrt{2} & 0 & i/\sqrt{2} \end{pmatrix}, \quad (7)$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$. The matrix involving $\omega$ has equal moduli for all its entries and was conjectured already in 1978 [31, 32] to be a possible candidate for the $3 \times 3$ neutrino mixing matrix.

Since $U_{l\nu} = U_{l\nu}^\dagger U_{l\nu}$, where $U_l$, $U_{\nu}$ diagonalize $M_l M_l^\dagger$, $M_{\nu} M_{\nu}^\dagger$ respectively, Eq. (7) may be obtained if we have

$$U_{l}^\dagger = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \quad (8)$$

and

$$M_{\nu} = \begin{pmatrix} a + 2b & 0 & 0 \\ 0 & a - b & d \\ 0 & d & a - b \end{pmatrix} \quad (9)$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -i/\sqrt{2} \\ 1/\sqrt{2} & 0 & i/\sqrt{2} \end{pmatrix} \begin{pmatrix} a - b + d & 0 & 0 \\ 0 & a + 2b & 0 \\ 0 & 0 & -a + b + d \end{pmatrix} \begin{pmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & 0 & 0 \\ 0 & -i/\sqrt{2} & i/\sqrt{2} \end{pmatrix}. \quad (10)$$

It was discovered in Ref. [7] that Eq. (8) is naturally obtained with $A_4$ if

$$(\nu, l)_{1,2,3} \sim \mathbf{3}, \quad l_{i,2,3}^\nu \sim 1 + 1' + 1'', \quad (\phi^0, \phi^0)_{1,2,3} \sim \mathbf{3} \quad (11)$$

for $\langle \phi_1^0 \rangle = \langle \phi_2^0 \rangle = \langle \phi_3^0 \rangle$. This assignment also allows $m_e$, $m_\mu$, $m_\tau$ to take on arbitrary values because there are here exactly three independent Yukawa couplings invariant under $A_4$. If we use this also for quarks [8], then $U_{u}^\dagger$ and $U_{d}^\dagger$ are also given by Eq. (8), resulting in $V_{CKM} = 1$, i.e. no mixing. This should be considered as a good first approximation because the observed mixing angles are all small. In the general case without any symmetry, we would have expected $U_u$ and $U_d$ to be very different.
It was later discovered in Ref. [33] that Eq. (9) may also be obtained with $A_4$, using two further assumptions. Consider the most general $3 \times 3$ Majorana mass matrix in the form

$\mathcal{M}_\nu = \begin{pmatrix} a + b + c & f & e \\ f & a + b \omega + c \omega^2 & d \\ e & d & a + b \omega^2 + c \omega \end{pmatrix}$, (12)

where $a$ comes from $\mathbf{1}$, $b$ from $\mathbf{1}'$, $c$ from $\mathbf{1}''$, and $(d, e, f)$ from $\mathbf{3}$ of $A_4$. To get Eq. (9), we need $e = f = 0$, i.e. the effective scalar $A_4$ triplet responsible for neutrino masses should have its vacuum expectation value along the $(1,0,0)$ direction, whereas that responsible for charged-lepton masses should be $(1,1,1)$ as I remarked earlier. This misalignment is a technical challenge to all such models [34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45]. The other requirement is that $b = c$. Since they come from different representations of $A_4$, this is rather ad hoc. A very clever solution [34, 39] is to eliminate both, i.e. $b = c = 0$. This results in a normal ordering of neutrino masses with the prediction [36]

$|m_{\nu e}|^2 \simeq |m_{ee}|^2 + \Delta m^2_{\text{atm}}/9$. (13)

Other applications [46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62] of $A_4$ have also been considered. A natural (spinorial) extension of $A_4$ is the binary tetrahedral group [63, 64] which is under active current discussion [65, 66, 67, 68].

**Others**

Other recent applications of non-Abelian discrete flavor symmetries include those of $D_4$, $Q_4$, $D_5$, $D_6$, $Q_6$, $D_7$, $S_4$, $D_8$, $S_4$, $\Delta(27)$, $\Delta(75)$, $\Sigma(81)$, $B_3 \times Z_2^3$, and $B_3 \times Z_2$. This work was supported in part by the U. S. Department of Energy under Grant No. DE-FG03-94ER40837.
References

[1] E. Ma, arXiv:hep-ph/0409075.

[2] E. Ma, Phys. Rev. D 43 (1991) 2761.

[3] N. G. Deshpande, M. Gupta and P. B. Pal, Phys. Rev. D 45 (1992) 953.

[4] W. Grimus and L. Lavoura, Phys. Lett. B 572 (2003) 189 arXiv:hep-ph/0305046.

[5] W. Grimus, A. S. Joshipura, S. Kaneko, L. Lavoura and M. Tanimoto, JHEP 0407 (2004) 078 arXiv:hep-ph/0407112.

[6] M. Frigerio, S. Kaneko, E. Ma and M. Tanimoto, Phys. Rev. D 71 (2005) 011901 arXiv:hep-ph/0409187.

[7] E. Ma and G. Rajasekaran, Phys. Rev. D 64 (2001) 113012 arXiv:hep-ph/0106291.

[8] K. S. Babu, E. Ma and J. W. F. Valle, Phys. Lett. B 552 (2003) 207 arXiv:hep-ph/0206292.

[9] E. Ma, Mod. Phys. Lett. A 17 (2002) 2361 arXiv:hep-ph/0211393.

[10] C. Luhn, S. Nasri and P. Ramond, arXiv:hep-th/0701188.

[11] E. Ma, Mod. Phys. Lett. A 21 (2006) 1917 arXiv:hep-ph/0607056.

[12] I. de Medeiros Varzielas, S. F. King and G. G. Ross, Phys. Lett. B 648 (2007) 201 arXiv:hep-ph/0607045.

[13] E. Ma, arXiv:hep-ph/0612022.

[14] Y. Koide, Lett. Nuovo Cim. 34 (1982) 201.

[15] E. Ma, arXiv:hep-ph/0701016.

[16] Y. Yamaguchi, Phys. Lett. 9 (1964) 281.

[17] J. Kubo, A. Mondragon, M. Mondragon and E. Rodriguez-Jauregui, Prog. Theor. Phys. 109 (2003) 795 [Erratum-ibid. 114 (2005) 287] arXiv:hep-ph/0302196.
[18] J. Kubo, H. Okada and F. Sakamaki, Phys. Rev. D 70 (2004) 036007 [arXiv:hep-ph/0402089].

[19] S. L. Chen, M. Frigerio and E. Ma, Phys. Rev. D 70 (2004) 073008 [Erratum-ibid. D 70 (2004) 079905] [arXiv:hep-ph/0404084].

[20] W. Grimus and L. Lavoura, JHEP 0508 (2005) 013 [arXiv:hep-ph/0504153].

[21] L. Lavoura and E. Ma, Mod. Phys. Lett. A 20 (2005) 1217 [arXiv:hep-ph/0502181].

[22] T. Teshima, Phys. Rev. D 73 (2006) 045019 [arXiv:hep-ph/0509094].

[23] Y. Koide, Phys. Rev. D 73 (2006) 057901 [arXiv:hep-ph/0509214].

[24] R. N. Mohapatra, S. Nasri and H. B. Yu, Phys. Lett. B 639 (2006) 318 [arXiv:hep-ph/0605020].

[25] S. Morisi and M. Picariello, Int. J. Theor. Phys. 45 (2006) 1267 [arXiv:hep-ph/0505113].

[26] S. Kaneko, H. Sawanaka, T. Shingai, M. Tanimoto and K. Yoshioka, Prog. Theor. Phys. 117 (2007) 161 [arXiv:hep-ph/0609220].

[27] E. Ma, [arXiv:hep-ph/0612013] 9

[28] O. Felix, A. Mondragon, M. Mondragon and E. Peinado, [arXiv:hep-ph/0610061].

[29] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B 530 (2002) 167 [arXiv:hep-ph/0202074].

[30] X. G. He and A. Zee, Phys. Lett. B 560 (2003) 87 [arXiv:hep-ph/0301092].

[31] N. Cabibbo, Phys. Lett. B 72 (1978) 333.

[32] L. Wolfenstein, Phys. Rev. D 18 (1978) 958.

[33] E. Ma, Phys. Rev. D 70 (2004) 031901 [arXiv:hep-ph/0404199].

[34] G. Altarelli and F. Feruglio, Nucl. Phys. B 720 (2005) 64 [arXiv:hep-ph/0504165].

[35] K. S. Babu and X. G. He, [arXiv:hep-ph/0507217].

[36] E. Ma, Phys. Rev. D 72 (2005) 037301 [arXiv:hep-ph/0505209].
[37] A. Zee, Phys. Lett. B 630 (2005) 58 [arXiv:hep-ph/0508278].
[38] E. Ma, Phys. Rev. D 73 (2006) 057304 [arXiv:hep-ph/0511133].
[39] G. Altarelli and F. Feruglio, Nucl. Phys. B 741 (2006) 215 [arXiv:hep-ph/0512103].
[40] X. G. He, Y. Y. Keum, R. R. Volkas, JHEP 0604 (2006) 039 [arXiv:hep-ph/0601001].
[41] B. Adhikary, B. Brahmachari, A. Ghosal, E. Ma and M. K. Parida, Phys. Lett. B 638 (2006) 345 [arXiv:hep-ph/0603059].
[42] B. Adhikary and A. Ghosal, Phys. Rev. D 75 (2007) 073020 [arXiv:hep-ph/0609193].
[43] F. Yin, Phys. Rev. D 75 (2007) 073010 [arXiv:0704.3827 [hep-ph]].
[44] G. Altarelli, F. Feruglio and Y. Lin, Nucl. Phys. B 775 (2007) 31 [arXiv:hep-ph/0610165].
[45] X. G. He, Nucl. Phys. Proc. Suppl. 168 (2007) 350 [arXiv:hep-ph/0612080].
[46] E. Ma, Mod. Phys. Lett. A 17 (2002) 289 [arXiv:hep-ph/0201225].
[47] E. Ma, Mod. Phys. Lett. A 17 (2002) 627 [arXiv:hep-ph/0203238].
[48] K. S. Babu, T. Kobayashi and J. Kubo, Phys. Rev. D 67 (2003) 075018 [arXiv:hep-ph/0212350].
[49] S. L. Chen, M. Frigerio and E. Ma, Nucl. Phys. B 724 (2005) 423 [arXiv:hep-ph/0504181].
[50] M. Hirsch, A. Villanova del Moral, J. W. F. Valle and E. Ma, Phys. Rev. D 72 (2005) 091301 [Erratum-ibid. D 72 (2005) 119904] [arXiv:hep-ph/0507148].
[51] E. Ma, Mod. Phys. Lett. A 20 (2005) 1953 [arXiv:hep-ph/0502024].
[52] E. Ma, Mod. Phys. Lett. A 20 (2005) 2601 [arXiv:hep-ph/0508099].
[53] E. Ma, Mod. Phys. Lett. A 20 (2005) 2767 [arXiv:hep-ph/0506036].
[54] E. Ma, H. Sawanaka and M. Tanimoto, Phys. Lett. B 641 (2006) 301 [arXiv:hep-ph/0606103].
[55] E. Ma, Mod. Phys. Lett. A 21 (2006) 2931 [arXiv:hep-ph/0607190].

[56] E. Ma, Mod. Phys. Lett. A 22 (2007) 101 [arXiv:hep-ph/0610342].

[57] L. Lavoura and H. Kuhbock, Mod. Phys. Lett. A 22 (2007) 181 (2007) [arXiv:hep-ph/0610050].

[58] I. de Medeiros Varzielas, S. F. King and G. G. Ross, Phys. Lett. B 644 (2007) 153 [arXiv:hep-ph/0610342].

[59] S. F. King and M. Malinsky, Phys. Lett. B 645 (2007) 351 [arXiv:hep-ph/0610250].

[60] S. Morisi, M. Picariello and E. Torrente-Lujan, Phys. Rev. D 75 (2007) 075015 [arXiv:hep-ph/0702034].

[61] Y. Koide, arXiv:hep-ph/0701018.

[62] M. Hirsch, A. S. Joshipura, S. Kaneko and J. W. F. Valle, arXiv:hep-ph/0703046.

[63] A. Aranda, C. D. Carone and R. F. Lebed, Phys. Lett. B 474 (2000) 170 [arXiv:hep-ph/9910392].

[64] A. Aranda, C. D. Carone and R. F. Lebed, Phys. Rev. D 62 (2000) 016009 [arXiv:hep-ph/0002044].

[65] P. D. Carr and P. H. Frampton, arXiv:hep-ph/0701034.

[66] F. Feruglio, C. Hagedorn, Y. Lin and L. Merlo, Nucl. Phys. B 775 (2007) 120 [arXiv:hep-ph/0702194].

[67] M. C. Chen and K. T. Mahanthappa, arXiv:0705.0714 [hep-ph].

[68] P. H. Frampton and T. W. Kephart, arXiv:0706.1186 [hep-ph].

[69] T. Kobayashi, H. P. Nilles, F. Ploger, S. Raby and M. Ratz, Nucl. Phys. B 768 (2007) 135 [arXiv:hep-ph/0611020].

[70] E. Ma, Fizika B 14 (2005) 35 [arXiv:hep-ph/0409288].

[71] C. Hagedorn, M. Lindner and F. Plentinger, Phys. Rev. D 74 (2006) 025007 [arXiv:hep-ph/0604265].
[72] Y. Kajiyama, J. Kubo and H. Okada, Phys. Rev. D 75 (2007) 033001 [arXiv:hep-ph/0610072].
[73] P. H. Frampton and T. W. Kephart, Phys. Rev. D 51 (1995) 1 [arXiv:hep-ph/9409324].
[74] K. S. Babu and J. Kubo, Phys. Rev. D 71 (2005) 056006 [arXiv:hep-ph/0411226].
[75] J. Kubo, Phys. Lett. B 622 (2005) 303 [arXiv:hep-ph/0506043].
[76] S. L. Chen and E. Ma, Phys. Lett. B 620 (2005) 151 [arXiv:hep-ph/0505064].
[77] E. Ma, Phys. Lett. B 632 (2006) 352 [arXiv:hep-ph/0508231].
[78] C. Hagedorn, M. Lindner and R. N. Mohapatra, JHEP 0606 (2006) 042 [arXiv:hep-ph/0602244].
[79] Y. Cai and H. B. Yu, Phys. Rev. D 74 (2006) 115005 [arXiv:hep-ph/0608022].
[80] H. Zhang, [arXiv:hep-ph/0612214].
[81] Y. Koide, [arXiv:0705.2275] [hep-ph].
[82] D. B. Kaplan and M. Schmaltz, Phys. Rev. D 49 (1994) 3741 [arXiv:hep-ph/9311281].
[83] M. Schmaltz, Phys. Rev. D 52 (1995) 1643 [arXiv:hep-ph/9411383].
[84] W. Grimus and L. Lavoura, JHEP 0601 (2006) 018 [arXiv:hep-ph/0509239].
[85] W. Grimus and L. Lavoura, [arXiv:hep-ph/0611149].