Gauge mediation \cite{1,2,3,4,5,6} is very attractive for the supersymmetry (SUSY) breaking mechanism, since it is free from the serious flavor-changing neutral current (FCNC) problem in the supersymmetric standard model. It has been recently pointed out that the minimal gauge mediation model is still consistent with all experimental data and cosmological constraints \cite{8}. Thus it is worthwhile to discuss possible dark matter candidates in the gauge mediation model.

A well-known candidate is the gravitino. If the gravitino mass $m_{3/2}$ is lighter than 1 keV, the gravitino cannot saturate the dark matter abundance \cite{9}. Even for 1 keV $\lesssim m_{3/2} \lesssim$ 100 keV, the gravitino dark matter would be too warm, and the small-scale fluctuations will be erased \cite{10,11}. In addition, the reheating temperature after inflation should be $T_{RH} \gtrsim 10^4$ GeV for the thermally produced gravitinos to be dark matter \cite{12}.

To test the gravitino dark matter we need to produce first SUSY particles in collider experiments and see their decays into the almost massless gravitino. However, even if the SUSY particles are produced it is very challenging to observe the decay if the gravitino is heavier than 1 keV.

In this paper we discuss the other possibility that the $Q$ ball is the dark matter in the gauge mediation, and seek for the discovery potential for the $Q$-ball dark matter at the IceCube. There are a lot of flat directions in SUSY. They consist of some combinations of squarks (and sleptons) in the minimal supersymmetric standard model (MSSM), so that they carry baryonic charge. The $Q$ ball is a non-topological soliton, the energy minimum configuration of the flat direction for a finite baryon number \cite{13}. Since the $Q$ ball with large enough charge (the baryon number) is stable against the decay into nucleons in the gauge mediation, it is reasonable to consider the $Q$ ball as the dark matter candidate \cite{14}.

We show that the $Q$ ball would be the dark matter in the parameter region different from that for the gravitino dark matter: the gravitino mass could range for $3/2 \lesssim m_{3/2} \lesssim$ 3 GeV, or even smaller, and the reheating temperature must be $T_{RH} \lesssim 10^4$ GeV. Moreover, it has completely different detection procedure from that for other dark matter candidate. Very large volume neutrino detectors such as the IceCube can detect directly the dark matter $Q$ balls in the (near) future.

The sketch of the paper is as follows. In the next section, we show the set-up of the gauge-mediated SUSY breaking. In sec. 3, we review the $Q$ ball in the gauge mediation, where there are two types of $Q$ ball. We estimate the abundance of the $Q$ ball and show the parameter region where the $Q$ balls could be the dark matter in Sec. 4. In the same section, we seek for the possibility of the $Q$-ball detection in the IceCube. Sec. 5 is devoted to our conclusion.

II. GAUGE-MEDIATED SUSY BREAKING

We adopt the so-called minimal direct gauge mediation described only by a few parameters, and free from both the SUSY FCNC problem and the SUSY CP problem \cite{8}. The SUSY is spontaneously broken by the vacuum expectation value of the SUSY breaking field $Z$ as

$$\langle Z \rangle = 0, \quad \langle F_Z \rangle = F. \quad (1)$$

The SUSY breaking sector is connected to the observable sector by the messenger fields $\Psi$ and $\bar{\Psi}$, a pair of some representations and anti-representations of the minimal GUT group $SU(5)$, through the Yukawa interactions as

$$W = kZ\bar{\Psi}\Psi + M_{mess}\bar{\Psi}\Psi, \quad (2)$$

We study the $Q$-ball dark matter in the gauge-mediated supersymmetry breaking, and seek for the detection possibility in the IceCube experiment. We find that the $Q$ balls would be the dark matter in the parameter region different from that for the gravitino dark matter. In particular, the $Q$ ball is a good dark matter candidate for low reheating temperature, which may be suitable for the Affleck-Dine baryogenesis and/or nonthermal leptogenesis. The dark matter $Q$ balls are detectable by the IceCube-like experiments in the future, which is the peculiar feature compared to the case of the gravitino dark matter.

I. INTRODUCTION

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We show that the $Q$ ball would be the dark matter in the parameter region different from that for the gravitino dark matter: the gravitino mass could range for keV $\lesssim m_{3/2} \lesssim$ GeV, or even smaller, and the reheating temperature must be $T_{RH} \lesssim 10^4$ GeV. Moreover, it has completely different detection procedure from that for other dark matter candidate. Very large volume neutrino detectors such as the IceCube can detect directly the dark matter $Q$ balls in the (near) future.

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where $k$ is a coupling constant, and $M_{\text{mess}}$ is the messenger mass. Then the MSSM gaugino and scalar masses are estimated as

$$M_{\text{gaugino}} \sim M_{\text{scalar}} \sim \frac{g^2}{16\pi^2} \frac{kF}{M_{\text{mess}}},$$

(3)

On the other hand, the gravitino mass is given by

$$m_{3/2} = \frac{F}{\sqrt{3}M_P},$$

(4)

where $M_P \simeq 2.4 \times 10^{18}$ GeV is the Planck mass. The Higgs boson mass at around 126 GeV leads to

$$\Lambda_{\text{mess}} \equiv \frac{kF}{M_{\text{mess}}} \gtrsim 5 \times 10^5 \text{ GeV}.$$  

(5)

Since $kF < M^2$, it results in

$$\sqrt{kF} > \Lambda_{\text{mess}} \gtrsim 5 \times 10^5 \text{ GeV}.$$  

(6)

III. Q BALLS IN THE GAUGE MEDIATION

A $Q$ ball is a nontopological soliton, the energy minimum configuration of the scalar field for finite charge $Q$ [13]. In MSSM, this scalar field corresponds to some flat direction, which consists of some combination of squarks (and sleptons), and the charge $Q$ would be the baryon number. In the gauge mediation, the $Q$ ball with large enough charge is stable against the decay into nucleons, which will be seen later, it can be the dark matter of the universe. The potential of the flat direction is expressed as [14, 16, 17]

$$V = V_{\text{gauge}} + V_{\text{grav}} = M_F^2 \left( \log \frac{|\Phi|^2}{M_{\text{mess}}^2} \right)^2 + m_{3/2}^2 \left( 1 + K \log \frac{|\Phi|^2}{M_*^2} \right) |\Phi|^2.$$  

(7)

Here the first term comes from gauge mediation effects, and $M_F$ is related to SUSY breaking $F$-term as [16]

$$M_F \simeq \frac{g^{1/2}}{4\pi} \sqrt{kF},$$

(8)

where $g$ generically denotes the gauge coupling of the standard model. Thus the gravitino mass can be expressed as

$$m_{3/2} = \frac{1}{\sqrt{3M_P}} \frac{(4\pi)^2 M_F^2}{gk}.$$  

(9)

The second term of Eq.(7) originates from the gravity mediation effects, and the one-loop correction is included. $K$ is typically negative and $|K| = 0.01 - 0.1$, and $M_*$ is the renormalization scale.

Since the messenger scale $\Lambda_{\text{mess}}$ has the lower limit, Eq.(10), it should be

$$M_F \gtrsim 4 \times 10^4 g^{1/2} \text{ GeV}.$$  

(10)

Likewise, the gravitino mass has a lower bound as

$$m_{3/2} \gtrsim 6.1 \times 10^{-8} k^{-1} \text{ GeV}.$$  

(11)

We shall call them the $\Lambda_{\text{mess}}$-limit in the following.

The flat direction has large amplitude during and after inflation, and starts its oscillation when the Hubble parameter becomes as large as the curvature of the potential [18]. Once the oscillation begins, the flat direction feels spatial instabilities, which grow into $Q$ balls very fast [14, 17, 19, 21]. Actually, the field rotates in the potential due to the so-called $A$-terms, so the orbit of the field may be circular or oblate depending on the size of the $A$-terms. However, in any case, $Q$ balls (and anti-$Q$ balls in the case with the oblate orbit) form with similar sizes [20, 21], so that the $Q$-ball abundance has almost no dependence on the size of the $A$-terms.
There are two types of $Q$ ball: the gauge-meditation type and the new type \cite{21, 22}. The former forms when the field begins the oscillation when the potential is dominated by $V_{\text{gauge}}$, where the field amplitude at that time is smaller than

$$\phi_{\text{eq}} \simeq \sqrt{2} \frac{M_F^2}{m_{3/2}},$$  \hspace{1cm} (12)

where $\Phi = \frac{1}{\sqrt{2}} \phi e^{i\theta}$. The charge of this type of $Q$ ball is given by \cite{21}

$$Q_G = \beta_G \left( \frac{\phi_{\text{osc}}}{M_F} \right)^4,$$  \hspace{1cm} (13)

where $\phi_{\text{osc}}$ is the amplitude of the field at the beginning of its oscillation, and $\beta_G = 6 \times 10^{-4}$ for a circular orbit ($\varepsilon = 1$), while $\beta_G = 6 \times 10^{-5}$ for an oblate orbit ($\varepsilon \lesssim 0.1$), where $\varepsilon$ denotes the ellipticity of the field orbit. The features of the gauge-mediation type $Q$ ball are as follows:

$$M_Q \simeq \frac{4\sqrt{2}\pi}{3} \zeta M_F Q_G^{3/4} \left( \frac{M_F}{m_{3/2}} \right)^{3/4},$$  \hspace{1cm} (14)

$$R_Q \simeq \frac{1}{\sqrt{2}} \zeta^{-1/2} M_F^{-1} Q_G^{1/4},$$  \hspace{1cm} (15)

$$\omega_Q \simeq \sqrt{\pi} \zeta M_F Q_G^{-1/4},$$  \hspace{1cm} (16)

where $M_Q$ and $R_Q$ are the mass and the size of the $Q$ ball, respectively, and $\omega_Q$ is the rotation speed of the field inside the $Q$ ball, and $\zeta$ is the $O(1)$ parameter \cite{23, 24}.

On the other hand, the new type $Q$ ball is produced if $V_{\text{grav}}$ dominates the potential when $\phi_{\text{osc}} > \phi_{eq}$. The charge of the $Q$ ball is estimated as \cite{21, 22}

$$Q_N = \beta_N \left( \frac{\phi_{\text{osc}}}{m_{3/2}} \right)^2,$$  \hspace{1cm} (17)

where $\beta_N = 0.02$ \cite{25}. The properties of this type of $Q$ ball are

$$M_Q \simeq m_{3/2} Q_N,$$  \hspace{1cm} (18)

$$R_Q \simeq |K|^{-1/2} m_{3/2}^{-1},$$  \hspace{1cm} (19)

$$\omega_Q \simeq m_{3/2}.$$  \hspace{1cm} (20)

The charge $Q$ is actually the $\Phi$-number and related to the baryon number as $B = bQ$, where the flat direction carries the charge $b$. For example, $b = 1/3$ for the $udd$ direction.

$Q$ balls can become the dark matter of the universe if they are stable against the decay into nucleons.\footnote{Q balls with $Q$ being the lepton number, so-called $L$ balls, can decay into neutrinos. Large $L$ balls could have lifetime longer than the present age of the universe, but the abundance becomes many orders of magnitude larger than the critical density. Therefore, $L$ balls cannot be the dark matter of the universe.} $\omega_Q$ can be regarded as the effective mass of the field inside the $Q$ ball. Because of kinematics, the $Q$ ball cannot decay into nucleons for $\omega_Q < bm_N$ with $m_N$ being the nucleon mass. For the gauge-mediation type $Q$ ball, the condition is rewritten as

$$Q_G > Q_D \equiv 4\pi^4 \zeta^4 \left( \frac{M_F}{bm_N} \right)^4 \simeq 1.2 \times 10^{30} \left( \frac{\zeta}{2.5} \right)^4 \left( \frac{b}{1/3} \right)^{-4} \left( \frac{M_F}{10^6 \text{GeV}} \right)^4.$$  \hspace{1cm} (21)

On the other hand, the new-type $Q$ ball is generically stable since $m_{3/2} \lesssim \text{GeV}.$

**IV. $Q$ BALLS AS DARK MATTER**

In this section, we investigate how the dark matter $Q$ ball can be realized, and seek for the possibility for their direct detections. We also show the parameter region where the gravitino can be the dark matter, and see that it resides in the different part in the parameter space.
A. Dark matter $Q$ balls

There are several conditions for the $Q$ ball to be the dark matter of the universe. The $Q$ ball should be (a) stable to be the dark matter, and (b) has correct dark matter abundance. Since there are two types of $Q$ balls, the amplitude at the oscillation, $\phi_{\text{osc}}$, should be at the right place in the potential, which corresponds to the condition (c) $\phi_{\text{osc}} < \phi_{\text{eq}}$ for the gauge-mediation type, and $\phi_{\text{osc}} > \phi_{\text{eq}}$ for the new type $Q$ ball, so that the potential must be dominated respectively by $V_{\text{gauge}}$ or $V_{\text{grav}}$ at the onset of the field oscillation. In addition, the $\Lambda_{\text{mess}}$-limit [10] or [11] should be satisfied.

Let us first consider the gauge-mediation type $Q$ ball. The $Q$-ball abundance is estimated as

$$\frac{\rho_Q}{s} = \frac{3T_{\text{RH}}}{4} \frac{\rho_Q}{\rho_{\text{inf}}} \bigg|_{\text{osc}} = \frac{3T_{\text{RH}} M_Q n_\phi / Q}{4} \frac{3}{3H_{\text{osc}}^2 M_P^2} \rho_{\text{inf}} \frac{3}{2} \pi \zeta \beta_{\text{G}}^{-1/4} T_{\text{RH}} \rho_{\text{osc}} = 2 \sqrt{\frac{\pi \zeta \beta_{\text{G}}^{-1/4} T_{\text{RH}} \rho_{\text{osc}}^2}{M_P^2}},$$

where $n_\phi = m_{\text{eff}} \phi_{\text{osc}}^2$, $m_{\text{eff}} = 2 \sqrt{7} M_F^2 / \phi_{\text{osc}}$, $3H_{\text{osc}} = m_{\text{eff}}$, and Eq.[14] are used. Observationally, the amount of the dark matter is $\rho_{\text{DM}} / s \simeq 4.4 \times 10^{-10}$ GeV [26]. Therefore, the amplitude of the field at the beginning of the oscillation becomes

$$\phi_{\text{osc}} = 5.80 \times 10^{-12} \text{GeV} \left( \frac{\zeta}{2.5} \right)^{-1/2} \left( \frac{\beta_{\text{G}}}{6 \times 10^{-4}} \right)^{1/8} \left( \frac{T_{\text{RH}}}{\text{GeV}} \right)^{-1/2}.$$  

(23)

The stability condition (21) and Eq.(13) lead to

$$\phi_{\text{osc}} > 2.13 \times 10^{14} \text{GeV} \left( \frac{\zeta}{2.5} \right)^{-3} \left( \frac{\beta_{\text{G}}}{6 \times 10^{-4}} \right)^{3/4} \left( \frac{b}{1/3} \right)^{2} \left( \frac{M_F}{10^6 \text{GeV}} \right)^{-4}.$$  

(24)

From these two equations, we have the upper limit on the reheating temperature as

$$T_{\text{RH}} < 7.42 \times 10^{-4} \text{GeV} \left( \frac{\zeta}{2.5} \right)^{-3} \left( \frac{\beta_{\text{G}}}{6 \times 10^{-4}} \right)^{3/4} \left( \frac{b}{1/3} \right)^{2} \left( \frac{M_F}{10^6 \text{GeV}} \right)^{-4}.$$  

(25)

for the $Q$ balls to be the dark matter. The condition (c) can be rephrased, using Eqs.(23) and (12) together with Eq.(9), as the lower bound on the reheating temperature:

$$T_{\text{RH}} > 2.44 \times 10^{-8} \text{GeV} g^{-2} k^{-2} \left( \frac{\zeta}{2.5} \right)^{-1} \left( \frac{\beta_{\text{G}}}{6 \times 10^{-4}} \right)^{1/4}.$$  

(26)

We show the allowed region for the $Q$-ball dark matter in Fig.1. The red line denotes the upper limit (25), the magenta line shows the $\Lambda_{\text{mess}}$-limit [10] with $g = 1$, and the blue lines represent the lower bound [26] for each value of $k$ shown in the figure. We conservatively assume that $T_{\text{RH}} > 1$ MeV, so that we do not show the lower $T_{\text{RH}}$ than 1 MeV in the figures.

Next we consider the new type of $Q$ ball. Again, the $Q$ ball should be (a) stable to be the dark matter, and (b) has correct DM abundance. In this case, since the potential must be dominated by $V_{\text{grav}}$ at the onset of the field oscillation, (c) $\phi_{\text{osc}} > \phi_{\text{eq}}$.

The $Q$-ball abundance is estimated as

$$\frac{\rho_Q}{s} = \frac{3T_{\text{RH}}}{4} \frac{\rho_Q}{\rho_{\text{inf}} \bigg|_{\text{osc}}} \simeq \frac{9}{4} T_{\text{RH}} \left( \frac{\phi_{\text{osc}}}{M_P} \right)^2,$$

where we use $\rho_Q = M_Q n_Q = m_3 / m_{\text{eff}} \phi_{\text{osc}}^2 = m_3 / \phi_{\text{osc}}^2$ and $\rho_{\text{inf}} = 3H_{\text{osc}}^2 M_P^2 = m_3^2 / M_P^2 / 3$ at the onset of the field oscillation. Using $\rho_{\text{DM}} / s = 4.4 \times 10^{-10}$ GeV, we need the field amplitude when the oscillation starts as

$$\phi_{\text{osc}} = 4.75 \times 10^{13} \text{GeV} \left( \frac{T_{\text{RH}}}{\text{GeV}} \right)^{-1/2}.$$  

(28)

Together with the condition (c) $\phi_{\text{osc}} > \phi_{\text{eq}}$, the reheating temperature has an upper limit as

$$T_{\text{RH}} < 1.13 \times 10^3 \text{GeV} \left( \frac{M_F}{10^6 \text{GeV}} \right)^{-4} \left( \frac{m_3 / \text{GeV}}{2} \right)^2.$$  

(29)
We then obtain the $k$-dependent upper limit on $T_{RH}$ by inserting Eq.(9) into Eq.(29) as

$$T_{RH} < 1.63 \times 10^{-6} \text{ GeV} g^{-2} k^{-2}.$$  

(30)

The stability condition (a) is given by

$$m_{3/2} / b n = 0.333 \text{ GeV} \left( \frac{b}{1/3} \right),$$  

(31)

where Eq.(20) is used.

The $Q$ ball can be the dark matter in the rectangle region surrounded by the stability condition (31) in a red line, the lower limit on $m_{3/2}$ in magenta lines, the condition (c) Eq.(30) in blue lines, and $T_{RH} \gtrsim 1 \text{ MeV}$, as shown in Fig.2. Varying the value of $k$, the intersection of Eqs.(11) and (30) spans on the line

$$T_{RH} < 4.42 \times 10^8 \text{ GeV} \left( \frac{m_{3/2}}{\text{GeV}} \right)^2,$$

(32)

shown in a black line in the figure. We thus hatched the triangle surrounded by Eqs.(32) and (31) and $T_{RH} \gtrsim 1 \text{ MeV}$, as the allowed region.

B. Direct detection of the dark matter $Q$ balls

The $Q$ balls can be detected through the so-called KKST (Kusenko-Kuzmin-Shaposhnikov-Tinyakov) process [21, 27, 28]. It is similar to the Rubakov-Callan effect for the magnetic monopole [29]. When nucleons collide with a $Q$ ball, they enter the surface layer of the $Q$ ball, and dissociate into quarks, which are converted to squarks. During the course of the process, the $Q$ ball releases energy of $\sim 1 \text{ GeV}$ per collision by emitting soft pions. Therefore, the
charged particles are created along the path of the Q ball through the detector. Since the Q balls in our dark halo have the velocity \( v \sim 10^{-3} \), those experiments for the subrelativistic monopole search can be applied to the Q-ball detection.

The observable is the upper limit of the Q-ball flux at the certain cross section, since no Q ball has been detected so far. The flux and the cross section of the dark matter Q ball are given respectively by

\[
F < F_{\text{DM}} \simeq \frac{\rho_{\text{DM}} v}{4\pi M_Q^2}, \quad \sigma_Q \simeq \pi R_Q^2,
\]

where \( \rho_{\text{DM}} \sim 0.3 \text{ GeV/cm}^3 \) is the local dark-matter density, and they have relation to the charge \( Q \) though the Q-ball properties Eqs. (14) – (16) for the gauge-mediation type and Eqs. (18) – (20) for the new type. In Fig. 3, we show the theoretically expected regions for the gauge-mediation and new types of Q ball respectively by red and blue triangle areas. Also shown are the upper limits of the Q-ball flux by the BAKSAN [28] and the IceCube [30] hatched in dark green and green, respectively. We see that some region for the new type of Q ball is already excluded and there is good possibility for the direct detection of the Q ball in the near future.

Finally, we use these experimental bound to impose some constraints on the reheating temperature by using Eqs. (13) and (23) for the gauge-mediation type in Fig. 1, while using Eqs. (17) and (28) for the new type in Fig. 2. In this way, we plot the regions where the dark matter Q ball is excluded by the BAKSAN experiment (dark green) [28] and the IceCube (light green) [30]. For the gauge-mediation type Q ball in Fig. 1, the excluded regions are getting closer to the theoretically expected area. On the other hand, for the new type Q balls in Fig. 2, the upper part of the allowed region is now observationally excluded, and the Q-ball dark matter can be realized for \( T_{\text{RH}} \lesssim 10^4 \text{ GeV} \).

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2 We simply extrapolate the flux at the cross section \( \sim 10^{-22} \text{ cm}^{-2}\text{sec}^{-1}\text{sr}^{-1} \) to larger cross sections.
C. Comparing with gravitino dark matter

As mentioned above, the gravitino is also a natural candidate of the dark matter in the gauge mediation. We therefore estimate the parameters for which gravitinos account for the dark matter and show that the region is totally different from that where the $Q$ ball is the dominant component of the dark matter. The abundance of gravitinos thermally produced by scatterings is given by

$$Y_{3/2} = 7.67 \times 10^{-9} \left( \frac{T_{RH}}{10^8 \text{ GeV}} \right) \left( \frac{m_{\tilde{g}}}{\text{TeV}} \right)^2 \left( \frac{m_{3/2}}{\text{GeV}} \right)^{-2},$$

(34)

for $m_{3/2} \ll m_{\tilde{g}}$, where $m_{\tilde{g}}$ is the gaugino mass. Since $\rho_{DM}/s = 4.4 \times 10^{-10} \text{ GeV}$ and $\rho_{3/2} \leq \rho_{DM}$, we have

$$T_{RH} \leq 5.74 \times 10^6 \text{ GeV} \left( \frac{m_{\tilde{g}}}{\text{TeV}} \right)^{-2} \left( \frac{m_{3/2}}{\text{GeV}} \right) (100 \text{ keV} \lesssim m_{3/2} \lesssim \text{GeV}).$$

(35)

This is shown in a brown line in Fig.2. If the gravitino mass is less than 100 keV, the gravitino is mainly produced by the decay of the SUSY particle [9]. Then the upper limit on the reheating temperature becomes of the order of mass of SUSY particles for $\text{keV} \lesssim m_{3/2} \lesssim 100 \text{ keV}$, although the gravitino dark matter may be too warm. Therefore, the new-type $Q$ ball can be the dark matter for lower reheating temperature than the gravitino dark matter case, and even for the lighter gravitino mass region such as $\text{keV} \lesssim m_{3/2} \lesssim 100 \text{ keV}$.

On the other hand, comparing to the gauge-mediation type $Q$ ball, we need to rephrase Eq.(35) in terms of $M_F$. Using Eq.(9), we obtain the constraint as

$$T_{RH} \leq 2.18 \times 10^2 \text{ GeV} g^{-1} k^{-1} \left( \frac{m_{\tilde{g}}}{\text{TeV}} \right)^{-2} \left( \frac{M_F}{10^6 \text{ GeV}} \right)^2 (1.62 \times 10^6 \text{ GeV} k^{-1} \lesssim M_F),$$

(36)

which is shown in brown lines for each $k$ in Fig.1. As before, the constraint on $T_{RH}$ stays almost constant of the order of the SUSY particle mass for $m_{3/2} \lesssim 100 \text{ keV}$, displayed by the brown dashed line in the figure. As can be seen, this
type of $Q$ ball can be the dark matter for $T_{\text{RH}} \lesssim 10^3$ GeV, whereas the gravitino can be the (cold) dark matter for the opposite case $T_{\text{RH}} \gtrsim 10^3$ GeV. Also the $Q$-ball dark matter can exist for $60$ eV $\lesssim m_{3/2} \lesssim$ keV, in particular, for $k \simeq 1$. See Eq. (19).

V. CONCLUSION

We have investigated for the $Q$-ball dark matter in the gauge-mediated SUSY breaking, and sought for its direct detection. We have found that the $Q$ ball can be the dark matter in the region of parameter space different from that for the gravitino dark matter. The $Q$ ball will be the dark matter for the lower reheating case such as $T_{\text{RH}} \lesssim \sim 10^3$ GeV, whereas the higher $T_{\text{RH}}$ is necessary for the gravitino dark matter. Moreover, the dark matter $Q$ balls could be detected directly in the large volume neutrino telescopes, such as the IceCube [30], Baikal [31], KM3NeT [32], and so on, in the future. In particular, the IceCube experiment may detect the $Q$ ball with their full volume and the slow-particle trigger in the near future [30]. This is a distinctive feature of the $Q$-ball dark matter, whereas the gravitino dark matter cannot be found in collider experiments.

Finally we make a comment on baryogenesis. The $Q$-ball dark matter scenario requires low reheating temperature ($T_{\text{RH}} \lesssim 10^4$ GeV). Affleck-Dine baryogenesis usually works in such low reheating temperature. In this case, another flat direction than that forms dark matter $Q$ balls will produce the baryon asymmetry of the universe [33]. Alternatively, nonthermal leptogenesis by right-handed neutrinos with nearly degenerate masses may operate at such low reheating temperature, although it should be higher than $\sim 10^2$ GeV. The Affleck-Dine leptogenesis [34] may also work for $T_{\text{RH}} \gtrsim 10^2$ GeV. In these cases, the IceCube may detect the $Q$-ball dark matter in very near future experiments (see Figs.1 and 2).

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