Reliability-based optimization of water distribution networks

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ABSTRACT

The water distribution system serves as a basic necessity for society. Due to its large size and involvement of various components, it is one of the most expensive civil infrastructures and thus demands optimization. Much work has been done for reducing the distribution system cost. However, with only one objective, the obtained solutions may not be practical to implement. Thus, improving cost along with the efficiency of the network is the demand of the hour. The present work introduces a unique parameter-less methodology for generating Pareto fronts without involving the concept of non-dominance. The methodology incorporates the Jaya optimization model for a bi-objective problem, one being the reduction in network cost and the other is improving the reliability index of the network. The efficiency of the proposed work is analyzed for three different benchmark problems. The Jaya technique is found to be very efficient and fast when compared with the other evolutionary technique applied for the same networks. The parameter-less nature of the Jaya technique smoothenes the process to a very large extent as no synchronization of algorithm parameters is required.

Key words: evolutionary algorithm, multi-objective, optimization, reliability, water distribution network

HIGHLIGHTS

- Jaya algorithm is applied for the very first time to a multi-objective optimization problem in the distribution network.
- Priori approach is proposed to generate a set of non-dominated solutions.
- The suitability of the Normalized objective function defined in previous research is validated.
- Jaya converges much faster as compared to another evolutionary algorithm.
- Present work results in highly reliable solutions.
INTRODUCTION

As humankind is evolving, there is always a demand for a much better solution. The solution that may fit only one objective is not seen as a satisfactory option as multiple goals need to be fulfilled. Meeting multiple goals from a single solution in technical terms is known as multi-objective optimization. Priori and posteriori are two approaches that can be used to solve any multi-objective optimization problem. In the priori approach, the decision-maker must decide the importance of all objectives i.e. weightage or priority before the start of the analysis, and corresponding solutions are obtained then. A different set of solutions can be obtained by varying the weights. However, a non-dominated set of Pareto Front is directly obtained in the posteriori approach and a decision-maker is allowed to choose a solution from this set. Priori approach uses the scalarization method to create the objective function which further utilizes an evolutionary algorithm (EA) to reach the optimal points. A multi-objective evolutionary algorithm (MOEA) is used to find the optimal results for a posteriori approach. MOEA’s working principle is similar to an EA except these are based on the concept of non-dominance & crowding distance to find the optimal solution which is considered complex when compared with the scalarization method (Jahanpour et al. 2018).

A water distribution system (WDS) comprises various parts such as reservoirs, pumps, pipes, valves, etc., that are required to supply the water at the consumer end with adequate pressure, of sufficient quality and quantity. Since WDS plays an important role for any developed society, it is obvious to state that the optimal design of a water distribution network is one of the most researched fields. The main objective in the design of any distribution network is to reduce network costs. Hence optimization of WDN is often seen as a single objective problem with the main aim of reducing the network cost. However, the optimal solution (optimal solution corresponds to the minimum network cost while satisfying all constraints of the problem) is not practical if implemented since it comprises diameters that are designed to just satisfy the minimum pressure head at the demand node. A slight change in this pressure may occur due to the pipe breakout or sudden increase in demand node that may lead to the network failure. Since WDN are a costly infrastructure, it is not possible to redesign them. In addition to this, the expansion of the network is not possible at all places due to the constraint of the space. Thus, at the design stage only if the efficiency of WDN is considered, it may save a lot of effort and cost at the later stage. Hence, minimizing the network cost should not be the only objective to satisfy a longer-running network. Problems that are associated with satisfying more than one objective from a solution are known as multi-objective optimization problems. It was not until the authors
Gessler & Walski (1985) that suggested the first multi-objective optimization model for the distribution system. In most cases, authors chose reduction of network cost as the first objective. The second objective can be a reduction in emission of greenhouse gases from any WDN, reduction of pressure deficit, improving the reliability of the network, etc (Jetmarova et al. 2018). Gheisi et al. (2016) presented detailed literature on different types of reliability and various measures to calculate them. Liu et al. (2017) conducted a comparative analysis of four different reliability indices in addition to introducing two new reliability indices namely API (available power index), PHRI (pipe hydraulic resilience index). In the present work, scalarization is adopted to optimize the network using different weights for a two-objective optimization problem. The scalarization method combines all the objectives of the problem into one by assigning different weights to different objectives (Gunanatara et al. 2017) carried out the study of comparing various reliability indices for the optimal design of WDN. Prasad & Park (2004) introduce the network resilience index with the addition of a demand or pipe failure, losses occurring in the pipes may increase dramatically and may affect the users downstream of the failure junction. Nonetheless, if the network can absorb this head loss and continue its supply, the failure may not lead to any drastic effects and may be fixed during odd hours of the day. These networks that absorb such failure losses are said to be more resilient (Rowell & Barnes 1984) and thus have immense hydraulic power. Todini (2000) took advantage of this surplus power in any distribution network and introduced a technique to determine the reliability of a WDN and therefore becomes the first to introduce the surrogate measure of calculating the reliability of the WDN. Thereafter many surrogate measures are developed and have been implemented for the optimal design of WDN. Prasad & Park (2004) introduce the network resilience index with the addition of diameter uniformity in Todini’s index of reliability. Raad et al. (2010) carried out the study of comparing various reliability indices and found Todini’s RI to be more reliable. Present work incorporates the RI for the optimal design of the three benchmark networks.

Reliability in Distribution Network

There is not a well-defined definition for the term reliability of distribution network (Xu & Goulter 1999). However, WDS is designed considering that total energy supplied by the pumps or elevated reservoir must be able to deliver the water at sufficient pressure despite all the losses occurring in the pipe. However, due to sudden breakout in the network, which may occur due to a sudden increase in the demand capacity such as fire demand or pipe failure, losses occurring in the pipes may increase dramatically and may affect the users downstream of the failure junction. Nonetheless, if the network can absorb this head loss and continue its supply, the failure may not lead to any drastic effects and may be fixed during odd hours of the day. These networks that absorb such failure losses are said to be more resilient (Rowell & Barnes 1984) and thus have immense hydraulic power. Todini (2000) took advantage of this surplus power in any distribution network and introduced a technique to determine the reliability of a WDN and therefore becomes the first to introduce the surrogate measure of calculating the reliability of the WDN. Thereafter many surrogate measures are developed and have been implemented for the optimal design of WDN. Prasad & Park (2004) introduce the network resilience index with the addition of diameter uniformity in Todini’s index of reliability. Raad et al. (2010) carried out the study of comparing various reliability indices and found Todini’s RI to be more reliable.

Present work incorporates the RI for the optimal design of the three benchmark networks.

Resilience Index (RI) – Todini’s RI is still the most popular surrogate reliability measure among all the indices developed (Raad et al. 2010). Resilience index is defined as the ratio of surplus hydraulic power to the available hydraulic power. The resilience index can range anywhere between 0 and 1 for the feasible solution, 0 being the least resilient & 1 being the maximum resilient. However, for a non-feasible solution where $h_{act} < h_{min}$, RI can even take a negative value. Such a situation arises more frequently while designing the distribution network by any evolutionary algorithm as it begins with random numbers. However, after certain iterations, the algorithms tend to give a positive value of RI.

\[
RI = \frac{\sum_{j=1}^{N} q_i (h_{act,j} - h_{min,j})}{\left( \sum_{i=1}^{R} Q_i h_{res,i} + \sum_{b=1}^{B} \frac{P_b}{g} \right) - \sum_{j=1}^{N} q_i h_{min,j}}
\]  

(1)

where $q_i$ is the demand at node $i$, $h_{act}$ & $h_{min}$ are the available & minimum pressure head respectively at any $j$, $Q_i$ flow from...
the reservoir to the network, $h_{res}$ head available at the reservoir, $P_h$ is pump capacity if any present in the network, $\nu$ is the specific weight of the liquid which is generally water, $N$ is the number of demand nodes in the network.

**OPTIMIZATION MODEL FOR DISTRIBUTION NETWORK**

The main objective of the problem is to minimize the total cost (TC) of the network per unit of reliability (R). Thus, the objective function is formulated as given in Equation (2)

$$\text{Min } CR = \frac{\text{Total Cost}}{\text{Reliability}}$$  \hspace{1cm} (2)

TC of the network depends on the per-unit cost of the commercial diameters obtained in optimization for the pipes and their corresponding length. TC can be calculated using Equation (3).

$$TC = \sum_{i=1}^{np} C_i(D_i)L_i$$  \hspace{1cm} (3)

where $L_i =$ length of pipe $i$ (m); $C_i(D_i) =$ per meter cost of a pipe for a given diameter; $D_i =$ diameter of chosen pipe (m); $np =$ total number of pipes in the network. Since the optimal design of WDN is a constrained problem, the following constraints are to be satisfied for each solution in the population.

**Continuity at nodes**

For any node, continuity of flow must be satisfied and can be written as

$$\sum Q_{in} - \sum Q_{out} = q_k, \quad \forall \ k \in nn$$  \hspace{1cm} (4)

where $Q_{in}$ and $Q_{out}$ = inflow & outflow at any node $k$ (m$^3$/s), $q_k =$ demand at node $k$ (m$^3$/s); $nn =$ number of nodes.

**Energy conservation in loops**

Total head loss for a closed-loop is always equal to zero. However, for serial pipes between two reservoirs with fixed heads, the head loss is calculated as the difference between those fixed heads.

$$\sum_{i \in loop} h_{fi} = 0, \quad \forall \ l \in nl$$  \hspace{1cm} (5)

where $nl =$ number of loops in the network; $h_{fi} =$ head loss because of friction in the pipe and fittings $i$ (m). Present work incorporates the Hazen-Williams formula for determining the pressure loss in pipes that occurs due to friction.

**Minimum pressure at nodes**

For the flow to occur, pressure at any node must be greater than the minimum pressure head required at that node.

$$H_k \geq H_k^{min}, \quad \forall \ k \in nn$$  \hspace{1cm} (6)

where $H_k =$ simulated pressure head at node $k$, $H_k^{min} =$ prescribed minimum pressure head at node $k$.

**Pipe size availability**

Since the optimal design of distribution networks belongs to the category of discrete optimization, the diameters must always be selected from the set of commercially available diameters. A larger set leads to an increase in search space.

$$D_i = [D(1), D(2), \ldots , D(S)], \quad \forall \ i \in np$$  \hspace{1cm} (7)

where $S =$ number of commercial pipe diameters, $np =$ number of pipes in the network.
NORMALIZED OBJECTIVE FUNCTION

The Scalarization method converts a multi-objective problem into a single objective by assigning weights to each objective and combining them. Selecting higher weight to any objective prioritize it over the other objectives. Tchebycheff scalarization using interactive weights was proposed by Stew & Choo (1983). Thereafter Tchebycheff has been used extensively for scalarization. Many new scalarization approaches that are used to generate Pareto solutions for non-convex problems can be found in Xia et al. (2021). The author also developed a combined scalarization technique. Kasimbeyli et al. (2019) compared six different scalarization techniques namely – weighted sum, epsilon constraint, Chebyshev method, Pascoletti- Serafini, conic & Benson method. Present work uses weighted sum by assigning higher weights to the cost and then reduce it subsequently. The decrease in weight of cost is equal to the increase in weight of reliability, such that the summation of all weights is always equal to one.

However, to maintain a level of fairness among the objectives, it is important to normalize the objective function. Suribabu (2017) presents a normalized function incorporating the cost & resilience of the network.

\[ Z_{\text{min}} = w_1 \frac{C - C_{\text{min}}}{C_{\text{max}} - C_{\text{min}}} + w_2 \frac{RM_{\text{max}} - RM_{\text{min}}}{RM_{\text{max}} - RM_{\text{min}}} \]

(8)

where

\[ w_1 + w_2 = 1 \]  

(9)

\( C \) represents the cost of the network obtained, \( C_{\text{min}} \) is optimal network cost determine by substituting the optimal diameters that are obtained by considering minimizing the network cost as the only objective. \( C_{\text{max}} \) is the maximum network cost, obtained by substituting the maximum available commercial diameter of the network to all the pipes and thus calculate its cost, \( RM_{\text{max}} \) is a resilience measure of the network that corresponds to \( C_{\text{max}} \), \( RM_{\text{min}} \) is resilience measure of the network corresponds to the \( C_{\text{min}} \). \( RM \) is the resilience measure of the network at cost \( C \). \( w_1 \) & \( w_2 \) are weights assigned to cost and resilience measure of the network, respectively. The summation of \( w_1 \) & \( w_2 \) is always equal to 1. Since EA starts with random numbers, the probability that the \( RM \) value of a solution is less than \( RM_{\text{min}} \) is very high. For such situations, \( RM - RM_{\text{min}} \) will become negative which reduces the value of \( Z_{\text{min}} \) and such a solution will be selected as the best solution. To avoid this during any time of the optimal design, if \( RM < RM_{\text{min}} \) is found, \( RM \) is substituted as \( RM_{\text{min}} + 0.000001 \) so that \( RM - RM_{\text{min}} \) will always be equal to 0.000001. A similar concept may help in dealing with solutions that have zero resilience value. \( C_{\text{min}}, C_{\text{max}}, R_{\text{min}}, R_{\text{max}} \) values for the Two Loop, Hanoi, and Go-Yang networks are given in Table 1.

JAYA ALGORITHM

Almost all the evolutionary techniques developed involve various constants that need to be tuned for the proper functioning of the algorithm. The algorithm cannot function without these constants and may lead to incorrect results if tuned improperly. The tuning is a tedious process and involves huge computational efforts. In addition to this, these constants are to be newly tuned for each optimization problem. This led to the development of techniques that requires minimum interaction of algorithm constants such as parameterless or self-adaptive technique. Taking the above into consideration Rao (2016) developed the Jaya technique that is not only parameterless, i.e. no parameters/constants are involved thus totally escape the process of synchronization, but is extremely easy to incorporate into any optimization model. Like any other meta-heuristic technique, Jaya is also a population-based optimization algorithm that utilizes the best & worst solutions from the population set and used it to generate the new solutions. For a minimization problem, the solution with the minimum objective value is selected

| Network    | \( C_{\text{min}} \) | \( C_{\text{max}} \) | \( R_{\text{min}} \) | \( R_{\text{max}} \) |
|------------|---------------------|---------------------|---------------------|---------------------|
| Two Loop   | 4,19,000            | 44,00,000           | 0.2104              | 0.9038              |
| Hanoi      | 60,81,087           | 1,09,70,586         | 0.1920              | 0.3537              |
| Go-Yang    | 1,77,010            | 3,29,725.64         | 0.4944              | 0.9941              |
as the best and the maximum value is selected as the worst. Once the best and worst solutions are generated, a new generation of the population is obtained using Equation (10)

$$A(i + 1, j, k) = A(i, j, k) + r_1(i, j, 1)(A(i, j, b) - |A(i, j, k)|) - r_2(i, j, 2)(A(i, j, w) - |A(i, j, k)|)$$

(10)

$r_1$ and $r_2$ in Equation (10) are random numbers that are generated between 0 and 1. The best & worst solutions are represented by index $b$ & $w$ respectively. Index $i$ represents iteration number, $j$ represents the position of the variable in any solution, $k$ is the index of the position of solution in a population set. As the algorithm advances in the iterations, the best and worst values are improvised and an optimal solution is obtained, thus the algorithm becomes victorious as termination criterion is reached and hence named Jaya (A Sanskrit word that means – Victory over the evil). The pseudo-code for the methodology is given below.

Initialize trials, iterations, population size, number of variables, $w_1$ & $w_2$

Trial $t = 1$

while $t$ is less than $t$-maximum

Generate initial population matrix at random

Iteration $i = 1$

while $i$ is less than $i$-maximum

Initialize EPANET and determine cost matrix along with Resilience index (using Equations (3) and (1))

Determine objective function using Equations (2) and (8)

Identify best, worst and their corresponding indices

Update the population using Equation (10)

Compare Updated solution with previous solution

if updated solution is better than previous solution

update the population and move to next iteration

else

retain the population and move to next iteration

$i$ + +

end while

$n$ + +

end while

METHODOLOGY

1. The problem is initialized by the number of variables, number of iterations, weighing factor $w_1$ & $w_2$, lower and upper limit of the decision variable.

2. Initial solutions are generated randomly using Equation (11). The solutions are generated randomly to avoid any biasedness.

$$x_{ij} = d^L_i + rand_{ij} \times (d^U_i - d^L_i)$$

(11)

$x_{ij}$ is generated random value of the decision variable, $d^L_i$ is the lower and $d^U_i$ are the upper limits of the decision variables. $rand_{ij}$ is a random number generated between 0 & 1 and distributed uniformly.

3. WDN belongs to the category of non-combinatorial discrete optimization, but the solutions that are generated using Equation (11) are continuous and are thus converted to discrete values using Equation (12).

$$x_{ij} = \begin{cases} x_i(k) & \text{if } x_{ij} \leq \frac{x_i(k) + x_i(k+1)}{2} \\ x_i(k+1) & \text{otherwise} \end{cases}$$

(12)

$x_i$ is a set of commercially available discrete diameters, that are arranged in increasing order.
4. EPANET 2 (Rossman 2000) is called and simulated pressure heads, demand are determined at each node for every solution in the population set.

5. If solutions are found to violate the minimum pressure requirement constraint, it is penalized. The penalty for such a solution is determined using Equation (13).

\[ \text{Penalty} = \lambda \max (0, (P_{\text{req}} - P_{\text{min}})) \]  

(13)

where \( P_{\text{min}} \) is the minimum pressure value among all the nodes for a solution i.e. simulated using EPANET 2, \( P_{\text{req}} \) is the required minimum pressure for any node, \( \lambda \) is a penalty constant.

6. Total Cost i.e. the fitness function is calculated as a sum of penalty value and the cost of the network.

\[ \text{Total Cost} = \text{Cost} + \text{Penalty} \]  

(14)

7. Reliability is calculated using the resilience index defined in Equation (1).

8. Cost by Reliability is further calculated, which becomes the new fitness value for the Jaya algorithm. For normalized objective function \( Z_{\text{min}} \) is calculated.

9. The solution giving the minimum and maximum cost by reliability or \( Z_{\text{min}} \) value (normalized function) is taken as the best & worst solutions respectively.

10. Once the best and the worst solutions are identified, a new population is generated using Equation (10).

11. Steps 4–8 are followed for the new set of solutions. The new population is compared with the old population and the better-performing (with minimum fitness value) solutions are selected and passed to the next generation.

12. The process for a pair of weights is continued until a stopping criterion is reached.

**COMPUTATIONAL RESULTS**

**Two loop network**

Two loop network (Alperovits & Shamir 1977) is a hypothetical network with a reservoir fixed at an elevation of 210 m. The network has 14 commercially available pipe diameters and 8 pipes each 1,000 m long with a Hazen William coefficient of 130, arranged in two loops, which leads to a total search space of \( 1.48 \times 10^9 \). The minimum pressure head of 30 m is required for the network at each node. The other hydraulic data of the network such as demand, node elevation is given in Figure 1(a).

![Two Loop Network & its Pareto Front](image)

**Figure 1** | Two Loop Network & its Pareto Front.
The optimal cost of the network is 419,000 (Cunha & Sousa 1999) with the optimal diameters as given in Table 2. Different solutions obtained by the Jaya technique by varying the weights are given in Table 3. As can be seen from Table 3 for $w_1 = 0.95 \& w_2 = 0.05$, the optimized cost increase by 32.21% over the optimal cost, while reliability increases to 91.17%, which is very appealing. It is also to be noted that for $w_1 = 0.80, 0.75, 0.70$ similar cost and reliability are achieved. Similar trend is seen for $w_1 = 0.45, 0.40, 0.35$. No improvement in diameters is obtained for such solutions by varying the weights.

**Table 2 | Comparison of diameters for single objective & multi-objective including reliability**

| Pipe No. | Optimal Diameters (in mm) | Diameters for Minimizing Total Cost by Reliability (in mm) |
|----------|---------------------------|----------------------------------------------------------|
|          | Two Loop Network | Hanoi Network | Go-Yang Network | Two Loop Network | Hanoi Network | Go-Yang Network |
| 1.       | 457.2           | 1016.0       | 200             | 508.0           | 1016         | 300             |
| 2.       | 254.0           | 1016.0       | 125             | 406.4           | 1016         | 250             |
| 3.       | 406.4           | 1016.0       | 125             | 406.4           | 1016         | 200             |
| 4.       | 101.6           | 1016.0       | 100             | 25.4            | 1016         | 200             |
| 5.       | 406.4           | 1016.0       | 80              | 355.6           | 1016         | 200             |
| 6.       | 254.0           | 1016.0       | 80              | 25.4            | 1016         | 150             |
| 7.       | 254.0           | 1016.0       | 80              | 406.4           | 762          | 150             |
| 8.       | 25.4            | 1016.0       | 80              | 304.8           | 762          | 200             |
| 9.       | 1016.0          | 80           | 609.6           | 80              | 609.6        | 80              |
| 10.      | 762.0           | 80           | 762             | 762             | 762          | 80              |
| 11.      | 609.6           | 80           | 609.6           | 80              | 609.6        | 80              |
| 12.      | 609.6           | 80           | 508             | 80              | 609.6        | 100             |
| 13.      | 508.0           | 80           | 609.6           | 100             | 609.6        | 100             |
| 14.      | 406.4           | 80           | 762             | 80              | 762          | 80              |
| 15.      | 304.8           | 80           | 762             | 100             | 762          | 100             |
| 16.      | 304.8           | 80           | 1016            | 80              | 1016         | 80              |
| 17.      | 304.8           | 80           | 1016            | 100             | 1016         | 100             |
| 18.      | 304.8           | 80           | 1016            | 80              | 1016         | 80              |
| 19.      | 304.8           | 80           | 1016            | 80              | 1016         | 80              |
| 20.      | 304.8           | 80           | 1016            | 80              | 1016         | 80              |
| 21.      | 304.8           | 80           | 1016            | 80              | 1016         | 80              |
| 22.      | 304.8           | 80           | 1016            | 80              | 1016         | 80              |
| 23.      | 304.8           | 80           | 1016            | 80              | 1016         | 80              |
| 24.      | 304.8           | 80           | 1016            | 80              | 1016         | 80              |
| 25.      | 304.8           | 80           | 1016            | 80              | 1016         | 80              |
| 26.      | 304.8           | 80           | 1016            | 80              | 1016         | 80              |
| 27.      | 304.8           | 80           | 1016            | 80              | 1016         | 80              |
| 28.      | 304.8           | 80           | 1016            | 80              | 1016         | 80              |
| 29.      | 304.8           | 80           | 1016            | 80              | 1016         | 80              |
| 30.      | 304.8           | 80           | 1016            | 80              | 1016         | 80              |
| 31.      | 304.8           | 80           | 1016            | 80              | 1016         | 80              |
| 32.      | 304.8           | 80           | 1016            | 80              | 1016         | 80              |
| 33.      | 304.8           | 80           | 1016            | 80              | 1016         | 80              |
| 34.      | 304.8           | 80           | 1016            | 80              | 1016         | 80              |

**Cost (units & $)** | 4,19,000 | 60,81,087 | 1,77,010 | 5,54,000 | 70,67,326 | 1,87,378.93

**Reliability Index** | 0.2104 | 0.1920 | 0.4944 | 0.6091 | 0.3142 | 0.9594
Thus, considering the designer’s preference on the importance of the two objectives any solution can be picked. The reliability as high as 0.8998 is obtained when $w_2$ becomes 0.95 which is very near to the $R_{\text{max}}$, however, the cost at this reliability is 25,52,000, which is less when compared with the $C_{\text{max}}$ i.e. 44,00,000. Thus, maximum reliability can be attained with much less cost. Optimal diameters obtained for a certain set of weights are given in Table 4. No comparison is available in the literature for this normalized function and hence not been compared here. For every pair of weights, 10 trial runs are performed with a population size of 50 and 200 iterations as the termination criteria. Convergence is obtained in just 30–50 iterations for every trial run, which highlights the searching capability of Jaya for a non-linear discrete optimization problem. The Pareto Front obtained for the network is shown in Figure 1(b).

Table 3 | $Z_{\text{min}}$ values for different pairs of weights for Two Loop network

| S.No. | $W_1$ | $W_2$ | Cost of Network (in units) | Resilience Index | $Z_{\text{min}}$ |
|-------|-------|-------|----------------------------|-----------------|-----------------|
| 1.    | 0.95  | 0.05  | 5,54,000                   | 0.6091          | 0.1192          |
| 2.    | 0.90  | 0.10  | 6,42,000                   | 0.6800          | 0.1981          |
| 3.    | 0.85  | 0.15  | 6,72,000                   | 0.6952          | 0.2686          |
| 4.    | 0.80  | 0.20  | 8,32,000                   | 0.7771          | 0.3227          |
| 5.    | 0.75  | 0.25  | 8,32,000                   | 0.7771          | 0.3837          |
| 6.    | 0.70  | 0.30  | 8,32,000                   | 0.7771          | 0.4397          |
| 7.    | 0.65  | 0.35  | 9,02,000                   | 0.7957          | 0.4935          |
| 8.    | 0.60  | 0.40  | 9,54,000                   | 0.8063          | 0.5461          |
| 9.    | 0.55  | 0.45  | 9,94,000                   | 0.8133          | 0.5969          |
| 10.   | 0.50  | 0.50  | 12,84,000                  | 0.8590          | 0.6432          |
| 11.   | 0.45  | 0.55  | 13,24,000                  | 0.8642          | 0.6857          |
| 12.   | 0.40  | 0.60  | 13,24,000                  | 0.8642          | 0.7274          |
| 13.   | 0.35  | 0.65  | 13,24,000                  | 0.8642          | 0.7690          |
| 14.   | 0.30  | 0.70  | 15,12,000                  | 0.8780          | 0.8094          |
| 15.   | 0.25  | 0.75  | 15,12,000                  | 0.8780          | 0.8476          |
| 16.   | 0.20  | 0.80  | 15,82,000                  | 0.8809          | 0.8858          |
| 17.   | 0.15  | 0.85  | 16,82,000                  | 0.8846          | 0.9218          |
| 18.   | 0.10  | 0.90  | 20,42,000                  | 0.8934          | 0.9546          |
| 19.   | 0.05  | 0.95  | 25,52,000                  | 0.8998          | 0.9823          |

Thus, considering the designer’s preference on the importance of the two objectives any solution can be picked. The reliability as high as 0.8998 is obtained when $w_2$ becomes 0.95 which is very near to the $R_{\text{max}}$, however, the cost at this reliability is 25,52,000, which is less when compared with the $C_{\text{max}}$ i.e. 44,00,000. Thus, maximum reliability can be attained with much less cost. Optimal diameters obtained for a certain set of weights are given in Table 4. No comparison is available in the literature for this normalized function and hence not been compared here. For every pair of weights, 10 trial runs are performed with a population size of 50 and 200 iterations as the termination criteria. Convergence is obtained in just 30–50 iterations for every trial run, which highlights the searching capability of Jaya for a non-linear discrete optimization problem. The Pareto Front obtained for the network is shown in Figure 1(b).

Table 4 | Optimal diameters (in mm) obtained for different pairs of weights for the Two Loop network

| Pipe No. | $W_1 = 0.8, W_2 = 0.2$ | $W_1 = 0.6, W_2 = 0.4$ | $W_1 = 0.4, W_2 = 0.6$ | $W_1 = 0.2, W_2 = 0.8$ |
|----------|------------------------|------------------------|------------------------|------------------------|
| 1.       | 558.8                  | 558.8                  | 609.6                  | 609.6                  |
| 2.       | 406.4                  | 508                    | 508                    | 508                    |
| 3.       | 508                    | 508                    | 508                    | 558.8                  |
| 4.       | 355.6                  | 254                    | 254                    | 406.4                  |
| 5.       | 406.4                  | 406.4                  | 508                    | 508                    |
| 6.       | 25.4                   | 25.4                   | 254                    | 25.4                   |
| 7.       | 355.6                  | 457.2                  | 508                    | 508                    |
| 8.       | 355.6                  | 355.6                  | 355.6                  | 457.2                  |
| Cost (units) | 8,32,000          | 9,54,000              | 13,24,000             | 15,82,000             |
| RI       | 0.7771                 | 0.8063                 | 0.8642                 | 0.8809                 |
Hanoi network

Hanoi network in the present work is taken from Fujiwara & Khang (1990). The network consists of 34 pipes and a reservoir at an elevation of 100 m (Figure 2(a)). There are six commercial sizes available for the Hanoi network as shown in Table 5, which leads to a search space of $2.865 \times 10^{26}$ which is large and computationally expensive and hence is a challenging problem for any computational technique. The hydraulic data of the network is given in the supplementary material. The optimal cost of the network is obtained as 6.081 M$ (Zheng et al. 2012) when only minimizing the network cost is considered. The diameters corresponding to this cost are shown in Table 2. The optimal diameters obtained when Equation (2) is optimized using Jaya are also shown in Table 2. The cost and resilience index for different weights to obtain a Pareto Front is shown in Table 6. As the weightage of the resilience index is increased, higher-cost solutions are obtained, since larger size diameters are needed to meet the resilience of the network. There is an increase of 83.85% in reliability for resilience as 0.3530 and cost of $9798084.93 i.e 61.21% more than optimized cost using cost and resilience index weights as 0.05 and 0.95 respectively. The designer can choose among any of the solutions given in Table 6 based on the availability of cost and desired level of reliability to be achieved. The results obtained by Jaya are similar to as obtained by Suribabu (2017). The slight change in

![Hanoi Network & its Pareto Front.](Figure 2)

**Table 5 |** Commercially available pipe diameters along with the unit cost for Two Loop, Hanoi, and Go-Yang Network

| Two Loop Network | Hanoi Network | Go-Yang Network |
|------------------|---------------|-----------------|
| Diameter (mm) | Cost (units) | Diameter (mm) | Cost ($/m) | Diameter (mm) | Cost ($/m) |
| 25.4 | 2 | 304.8 | 45.73 | 80 | 37.890 |
| 50.8 | 5 | 406.4 | 70.40 | 100 | 38.933 |
| 76.2 | 8 | 508.0 | 98.38 | 125 | 40.563 |
| 101.6 | 11 | 609.6 | 129.33 | 150 | 42.554 |
| 152.4 | 16 | 762.0 | 180.80 | 200 | 47.624 |
| 203.2 | 23 | 1016.0 | 278.30 | 250 | 54.125 |
| 254.0 | 32 | | | 300 | 62.109 |
| 304.8 | 50 | | | 350 | 71.524 |
| 355.6 | 60 | | | | |
| 406.4 | 90 | | | | |
| 457.2 | 130 | | | | |
| 508.0 | 170 | | | | |
| 558.8 | 300 | | | | |
| 609.6 | 550 | | | | |
diameter values from Suribabu (2017) is marked as bold in Table 7. DE applied by Suribabu (2017) took as many as 2,000 iterations to reach these results, Jaya converges to the same results in less than 100 iterations for all the successful trial runs, which again highlights the computational efficiency of the Jaya. The comparison of diameters obtained by Suribabu (2017) to the Jaya is shown in Table 7. For every pair of weights, 10 trial runs are performed with a population size of 75 and 200 iterations as the termination criteria. The Pareto Front obtained for the network is shown in Figure 2(b).

**Go-Yang network**

Go-Yang network (Figure 3(a)) consists of 30 pipes and 22 nodes arrange in 9 loops with a ground reservoir fixed at an elevation of 71 m. A pump of 4.52 kW capacity is provided to supply the water to the network. 15 m minimum pressure head is required above the ground elevation at all the nodes in the network. The Hazen-William coefficient for all the pipes in the network is taken as 100. The hydraulic data of the network such as ground elevation, nodal demand, etc, are given in the supplementary material. The optimal cost of the network (when only minimizing the network cost is considered) is reported as $177,010 (Zheng et al. 2012) with the optimal diameters as given in Table 2. Equation (2) is optimized using Jaya for the Go-Yang network and with just 5.85% increase in cost from the optimal value, 94.05% increase in resilience index is obtained, with the diameters given in Table 2, which is captivating and thus highlights the importance of considering the reliability in optimization model and to not rely on just minimizing the network cost. Table 8 shows the value of the cost and resilience index obtained for different pairs of weights. The maximum cost of the network is $329725.64 with an RI of 0.9941, however for the weights $w_1 = 0.95$ and $w_2 = 0.05$ the cost as high as $211604.5$ is reported with the RI of 0.9922 which highlights that by giving just 5% weightage to the reliability, maximum reliability can be achieved with a much lesser cost. With nearly the same resilience index, $118,121.14$ is saved which is huge and is more than the optimal network cost. The diameters obtained for a certain set of weights are shown in Table 9. For each set of weights 10 trial runs are performed with a population size of 75 and 200 iterations as the termination criteria. For all the trials the results are obtained in less than 100 iterations. The Pareto Front for the obtained results is shown in Figure 3(b) for the Go-Yang network.

| S.No. | $W_1$ | $W_2$ | Cost of Network ($) | Resilience Index | $Z_{min}$ |
|-------|-------|-------|--------------------|------------------|----------|
| 1.    | 0.95  | 0.05  | 64,39,485.15       | 0.2630           | 0.1837   |
| 2.    | 0.90  | 0.10  | 66,56,523.05       | 0.2908           | 0.2698   |
| 3.    | 0.85  | 0.15  | 67,63,503.90       | 0.3000           | 0.3435   |
| 4.    | 0.80  | 0.20  | 67,77,493.90       | 0.3010           | 0.4110   |
| 5.    | 0.75  | 0.25  | 67,94,518.05       | 0.3019           | 0.4776   |
| 6.    | 0.70  | 0.30  | 71,15,456.99       | 0.3166           | 0.5379   |
| 7.    | 0.65  | 0.35  | 72,27,908.24       | 0.3210           | 0.5917   |
| 8.    | 0.60  | 0.40  | 74,17,895.00       | 0.3281           | 0.6399   |
| 9.    | 0.55  | 0.45  | 75,61,088.00       | 0.3329           | 0.6838   |
| 10.   | 0.50  | 0.50  | 76,04,991.15       | 0.3341           | 0.7255   |
| 11.   | 0.45  | 0.55  | 77,35,780.40       | 0.3372           | 0.7657   |
| 12.   | 0.40  | 0.60  | 77,98,595.60       | 0.3384           | 0.8042   |
| 13.   | 0.35  | 0.65  | 79,65,846.80       | 0.3413           | 0.8402   |
| 14.   | 0.30  | 0.70  | 81,12,091.70       | 0.3432           | 0.8741   |
| 15.   | 0.25  | 0.75  | 84,67,134.20       | 0.3470           | 0.9058   |
| 16.   | 0.20  | 0.80  | 86,82,326.70       | 0.3488           | 0.9329   |
| 17.   | 0.15  | 0.85  | 89,67,423.89       | 0.3507           | 0.9560   |
| 18.   | 0.10  | 0.90  | 91,12,978.43       | 0.3514           | 0.9765   |
| 19.   | 0.05  | 0.95  | 97,98,084.93       | 0.3530           | 0.9934   |
CONCLUSIONS

In this paper, a new methodology is proposed that eases the process of generating a set of non-dominated solutions for a multi-objective optimization problem. The computational methodology presented is known to converge to the results previously reported in the literature in very fewer iterations. A normalized function is applied for multi-objective optimization in three different benchmark problems using the Jaya algorithm, a newly developed parameter-less technique. The proposed
The methodology can generate a set of non-dominated solutions by deciding the priority of the objectives between them. The technique is found to be more convenient when compared with the posteriori approach. No knowledge of the non-dominance is required, and different sets of Pareto Fronts are obtained that may be used as a pivot solution for further analysis. However, for the method described in the present work, the designer just needs to decide the weights of every objective to obtain the desired results. It is worth mentioning that the posteriori approach can lead to the non-dominance set in a single step.

| S.No. | W₁  | W₂  | Cost of Network ($) | Resilience Index | Zₘᵢₙ | Zₘᵢₙ |
|-------|-----|-----|---------------------|------------------|------|------|
| 1.    | 0.95| 0.05| 1,81,046.2          | 0.8731           | 0.0911|      |
| 2.    | 0.90| 0.10| 1,81,851.9          | 0.8927           | 0.1539|      |
| 3.    | 0.85| 0.15| 1,84,618.5          | 0.9347           | 0.2125|      |
| 4.    | 0.80| 0.20| 1,85,131.4          | 0.9407           | 0.2663|      |
| 5.    | 0.75| 0.25| 1,86,339.7          | 0.9524           | 0.3184|      |
| 6.    | 0.70| 0.30| 1,86,699.6          | 0.9552           | 0.3696|      |
| 7.    | 0.65| 0.35| 1,87,378.9          | 0.9594           | 0.4201|      |
| 8.    | 0.60| 0.40| 1,88,932.4          | 0.9665           | 0.4701|      |
| 9.    | 0.55| 0.45| 1,90,179.2          | 0.9717           | 0.5183|      |
| 10.   | 0.50| 0.50| 1,90,946.3          | 0.9742           | 0.5662|      |
| 11.   | 0.45| 0.55| 1,93,016.0          | 0.9794           | 0.6136|      |
| 12.   | 0.40| 0.60| 1,94,532.6          | 0.9827           | 0.6597|      |
| 13.   | 0.35| 0.65| 1,94,692.3          | 0.9829           | 0.7052|      |
| 14.   | 0.30| 0.70| 1,96,250.4          | 0.9852           | 0.7502|      |
| 15.   | 0.25| 0.75| 1,96,370.3          | 0.9854           | 0.7947|      |
| 16.   | 0.20| 0.80| 1,96,466.3          | 0.9855           | 0.8393|      |
| 17.   | 0.15| 0.85| 2,01,970.3          | 0.9893           | 0.8824|      |
| 18.   | 0.10| 0.90| 2,06,007.5          | 0.9910           | 0.9243|      |
| 19.   | 0.05| 0.95| 2,11,604.5          | 0.9922           | 0.9646|      |

Figure 3 | Go-Yang Network & its Pareto Front.
run while to generate the same Pareto Front in priori approach, different runs are made by varying the weights of the objective that makes the priori approach more convenient and simpler. Present work indicates that by including the resilience index in the optimization model, better solutions are obtained that are less expensive but highly reliable. Jaya belongs to a parameter-less algorithm and hence involves no constants which further ease the analysis as no efforts are spent on tuning the algorithm. Similar results are obtained in 40–50 iterations using Jaya when compared with the 2,000 iterations taken by Differential Evolution for the Hanoi network, this further reaffirms the trust in Jaya as an evolutionary algorithm capable of solving multi-objective optimization.

**DATA AVAILABILITY STATEMENT**

All relevant data are included in the paper or its Supplementary Information.
REFERENCES

Alperovits, E. & Shamir, U. 1977 Design of optimal water distribution systems. Water Resources Research 13 (6), 885–900. doi: 10.1029/WR013i006p00885.

Alsajri, M., Ismail, M. A. & Abdul-Baqi, S. 2018 A review on the recent application of Jaya optimization algorithm. In: 2018 1st Annual International Conference on Information and Sciences (AiCIS). IEEE, pp. 129–132. doi: 10.1109/AiCIS.2018.00034.

Cunha, M. D. C. & Sousa, J. 1999 Water distribution network design optimization: simulated annealing approach. Journal of Water Resources Planning and Management 125 (4), 215–221. doi: 10.1061/(ASCE)7033-9496(1999)125:4(215).

Dede, T. 2018 Jaya algorithm to solve single objective size optimization problem for steel grillage structures. Steel and Composite Structures 26 (2), 163–170. doi: 10.12989/scs.2018.25.2.163.

Fujiwara, O. & Khang, D. B. 1990 A two-phase decomposition method for optimal design of looped water distribution networks. Water Resources Research 26 (4), 539–549. doi: 10.1029/WR026i004p00539.

Gessler, J. & Walski, T. M. 1985 Water distribution system optimization. In: Army Engineer Waterways Experiment Station Vicksburg ms Environmental lab.

Gheisi, A., Forsyth, M. & Naser, G. 2016 Water distribution systems reliability: a review of research literature. Journal of Water Resources Planning and Management 142 (11), 04016047. doi: 10.1061/(ASCE)WR.1943-5452.0000690.

Goulter, I. & Morgan, D. 1984 Obtaining the layout of water distribution-systems-discussion. Journal of Hydraulic Engineering-ASCE 110 (1), 61–62. doi: 10.1061/(ASCE)0733-9429(1984)110:1(67).

Gunantara, N. 2018 A review of multi-objective optimization: methods and its applications. Cogent Engineering 5 (1), 1502242. doi: 10.1080/23319161.2018.1502242.

Hao, X., Gao, Y., Yang, X. & Wang, J. 2021 Multi-objective collaborative optimization in cement calcination process: a time domain rolling optimization method based on Jaya algorithm. Journal of Process Control 105, 117–128. doi: 10.1016/j.jprocont.2021.07.012.

Jahanpour, M., Tolson, B. A. & Mai, J. 2018 PADSs algorithm assessment for biobjective water distribution system benchmark design problems. Journal of Water Resources Planning and Management 144 (3), 04017099. doi: 10.1061/(ASCE)WR.1943-5452.0000875.

Kasimbeyli, R., Ozturk, Z. K., Kasimbeyli, N., Yalcin, G. D. & Erdem, B. I. 2019 Comparison of some scalarization methods in multiobjective optimization. Bulletin of the Malaysian Mathematical Sciences Society 42 (5), 1875–1905. doi: 10.1007/s40840-017-0579-4.

Liu, H., Savić, D. A., Kapelan, Z., Creaco, E. & Yuan, Y. 2017 Reliability surrogate measures for water distribution system design: comparative analysis. Journal of Water Resources Planning and Management 143 (2), 04016072. doi: 10.1061/(ASCE)WR.1943-5452.0000728.

Mala-Jetmarova, H., Sultanova, N. & Savic, D. 2018 Lost in optimisation of water distribution systems? a literature review of system design. Water 10 (3), 307. doi: 10.3390/w1003307.

Prasad, T. D. & Park, N. S. 2004 Multiobjective genetic algorithms for design of water distribution networks. Journal of Water Resources Planning and Management 130 (1), 73–82. doi: 10.1061/(ASCE)0733-9496(2004)130:1(73).

Raad, D. N., Sinke, A. N. & Van Vuuren, J. H. 2010 Comparison of four reliability surrogate measures for water distribution systems design. Water Resources Research 46 (5). doi: 10.1029/2009WR007785.

Rao, R. 2016 Jaya: a simple and new optimization algorithm for solving constrained and unconstrained optimization problems. International Journal of Industrial Engineering Computations 7 (1), 19–34. doi: 10.5267/j.ijiec.2015.8.004.

Rosman, L. A. 2000 EPANET 2. Users Manual. US Environmental Protection Agency (EPA). USA.

Suriab, C. R. 2017 Resilience-based optimal design of water distribution network. Applied Water Science 7 (7), 4055–4066. doi: 10.1007/s13201-017-0560-2.

Todini, E. 2000 Looped water distribution networks design using a resilience index based heuristic approach. Urban Water 2 (2), 115–122. doi: 10.1016/S1466-0629(00)00049-2.

Torres, A., Torres, D., Enriquez, S., De León, E. P. & Díaz, E. 2012 Evolutionary Multi-Objective Algorithms. In: Real-World Applications of Genetic Algorithms.

Xia, Y. M., Yang, X. M. & Zhao, K. Q. 2021 A combined scalarization method for multi-objective optimization problems. Journal of Industrial & Management Optimization 17 (5), 2669. doi: 10.3934/jimo.2020088.

Xu, C. & Goulter, I. C. 1999 Reliability-based optimal design of water distribution networks. Journal of Water Resources Planning and Management 125 (6), 352–362. doi: 10.1061/(ASCE)0733-9496(1999)125:6(352).

Zheng, F., Simpson, A. R. & Zecchin, A. 2012 A new NLP-DE hybrid optimisation model for water distribution system optimisation. In: WDSA 2012: 14th Water Distribution Systems Analysis Conference, 24–27 September 2012 in Adelaide, South Australia 66-76, Barton. ACT: Engineers Australia.

Zheng, F., Zecchin, A. C. & Simpson, A. R. 2013 Self-adaptive differential evolution algorithm applied to water distribution system optimization. Journal of Computing in Civil Engineering 27 (2), 148–158. doi: 10.1061/(ASCE)CP.1943-5487.0000208.

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