The selection strategy of optimized arrays for 3-D electrical resistivity tomography

Lincheng Jiang¹, Gang Tian¹, Bangbing Wang¹ and Amr Abd El-Raouf¹ ²

¹ School of Earth Sciences, Zhejiang University, Hangzhou, China
² Geology Department, Faculty of Science, Zagazig University, Zagazig, Egypt
Email: jianglc@zju.edu.cn

Abstract. Within electrical resistivity tomography (ERT), the selection of arrays and electrode locations can effectively enhance the resolution of imaging. By properly selecting and optimizing the survey design, a better resolution will be obtained with fewer electrodes and configurations than traditional arrays. Previous work has demonstrated that the optimized survey design using the 'Compare R' method can provide better resolution than conventional arrays. This paper adds target-oriented selection and modified the original 'Compare R' method. The modified method first selects the target area in the comprehensive data sets, then optimizes the choice by the modified CR method, and finally combines the optimization results of multiple sets of target areas. For the target area, this method can select fewer electrodes and arrays than the original CR method, get better resolution than conventional arrays, and take less calculation time.

1. Introduction
In recent decades, geoelectrical methods have played a really important role in near-surface exploration [4]. Gradually evolve from two-dimensional to three-dimensional, from a single survey line to a regular grid, from single-point measurement to high-density measurement [1,2]. 3D electrical resistivity surveys and inversion models are needed to accurately resolve structures in target areas with very complex geology where 2D models might suffer from human factors and survey lines locations. However, 3D electrical resistivity survey design still relies on data sets recorded using one or more of the standard electrode arrays (e.g., the conventional dipole-dipole array). They use an arrangement where the number of electrodes along one (x) direction is much larger than in the perpendicular (y) direction. The number of electrodes in various commercial multielectrode systems is frequently very limited by these arrangements. To get reasonable areal coverage, a roll-along method called The Abem SAS system is used [3]. Loke [5] and Uhlemann [8] utilize four cables for the SAS system to get comprehensive data sets.

Within geoelectrical exploration, the selection of measurement configurations and electrode locations is known to manage the image resolution. The most essential requirement in selecting the experimental parameters for a geophysical survey design is to be clear in specifying the geological and operational objectives. The geometry of the electrode array should be selected to fulfill the survey objectives. Consequently, it is necessary to determine whether limited combinations of electrode configurations are able to supply resistivity images that are comparable in quality to those provided by the comprehensive data sets.

As for the way to settle on the configurations suitable for exploration need, The 'Compare R' method is a good way [7,10]. The ‘Compare R’ algorithm proved to be the best method in terms of...
determining arrays that have the very best resolution among the techniques that were tested. At first, it used for 2D calculation but now it has been developed to calculate 3D ERT issues [5].

In this paper, the theory of the 'Compare R' method and also the target-oriented modified are briefly described. We numerically simulate the synthetic test model and present the findings of quantitative comparison.

2. Methodology

Based on the original 'Compare R' method, we do some modifies for target-oriented. The model resolution matrix $R$ quantifies the degree to which each model cell of a resistivity image is resolved by the measured data. For the linearized Gauss-Newton least-squares inversion method of electrical resistivity tomography issues, the model resolution matrix $R$ is defined as:

$$ R = (G^T G + C)^{-1} G^T G $$

(1)

Where the Jacobian matrix element $G_{ij}$ is the logarithmic sensitivity of the $i$th measurement to a small change in the resistivity of the $j$th model cell, and the constraint matrix $C$. The leading diagonal elements of $R$ give an estimate of the resolution of the individual model cells. The model resolution takes values range between 0 to 1, where 0 is unresolved and 1 is perfectly resolved.

The survey design should be focused on specific target areas, which require a priori information about the subsurface properties of the data collected from a large electrode space. We select electrodes and configurations from comprehensive set firstly which meet the requirements of the target area. The principle of selection is that the center measurement point of array is within the target area. After selection, the new comprehensive data set is the target set. The number of data points is far less than before and therefore the calculation time is additionally much less. For each measurement which is chosen by the target, the change in the model resolution matrix $\Delta R$ is calculated using Sherman-Morrison Rank-1 update [10]:

$$ \Delta R_s = \frac{z}{1 + (g^T z)} \left(g^T - y^T\right) $$

(2)

Where

$$ z = (G_s^T (G_s + C)^{-1} g, y = (G_s^T G_s) z $$

(3)

The Jacobian matrix $G_t$ is consisting of the sensitivities of the measurements of the target selected set. The vector $g$ contains the sensitivity values of the model cells for the new test configuration. Each measurement which from target selected set is ranked by the following equation:

$$ F_{cr} = \frac{1}{m} \sum_{j=1}^{n} \frac{\Delta R_{s,j}}{R_{s,j}} $$

(4)

Where $R_{s,j}$ is the resolution of the target selected set and $m$ is the number of model cells. The highest-ranking measurement is added to the ranked set and is that the first place. The next highest-ranked configuration added into ranked to be the second. The result of ranked configuration from target selected set is optimized set. We only need to calculate the sensitivity matrix of the target area once, and then calculate the contribution degree of every measurement separately from it. In this approach, we are able to reduce the time of iterative calculation of the overall the resolution matrix when measurements set changing.

The average relative model resolution $S$ shows the consequences, where $R_b$ is the resolution of the optimized set.
This evaluates for all cells within the target area. The optimization performance of this method shown by the average model resolution curve.

3. Synthetic test model

The methodology was tested on a simple synthetic 3D model consisting of three rectangular prisms. We consider a 10m×10m survey grid with an electrode spacing of 1m within the x-direction and line spacing of 1m within the y-direction. The electrodes are arranged along 11 parallel lines with 11 electrodes along each line, giving a total of 121 electrode positions. During this paper, the comprehensive data set for the 3D ERT survey grid, which consists of the possible inline alpha and beta arrays as well as their single offset and double and triple offset line versions [5]. For \( n \) electrodes, \( n(n-1)(n-2)(n-3)/8 \) unique four-point measurements can theoretically be acquired. Due to the stability of the inversion in the signal-to-noise, the comprehensive set is reduced by removing the Wenner-γ type measurements and all others with geometric factors greater than a specific limit [10]. We set the maximum geometric factor at 2261.9 (corresponding to the dipole-dipole array with \( a=1 \) and \( n=8 \)). The comprehensive sets have 30981 arrays. The conventional arrays data set, consisting of all the possible inline Wenner-Schlumberger and dipole-dipole arrays along the x-direction, it included 605 measurements. We use an open-source 'pyGIMLi' for modeling and inversion [6]. The model domain was discretized using an unstructured tetrahedral mesh, comprising 6830 elements. Prism A was defined as 4 m × 1 m × 1 m along the x, y and z directions and the depth range is 1m to 2m. Prism B was defined as 2 m × 2 m × 1 m and also the depth range is 1.5 m to 2.5 m. Prism C was defined as 1 m × 4 m × 1 m and the depth range is 0.5 m to 1.5 m. The resistivity of all three prisms is 100 Ωm and the background with 10 Ωm (Fig. 1).

![Figure 1](image)

**Table 1. Numbers of electrodes and measurements**

| Data set     | Number of electrodes | Number of measurements |
|--------------|----------------------|------------------------|
| Comprehensive| 121                  | 30981                  |
| Target A     | 43                   | 2840                   |
| Target B     | 54                   | 4777                   |
| Target C     | 72                   | 1416                   |
| Target all   | 105                  | 9033                   |
| Conventional | 121                  | 605                    |
| Optimized    | 98                   | 605                    |
According to the above method, we get the target set by target-oriented selection individually. For prism A, the number of measurements is 2840 and prism B, C are 4777, 1416, respectively. Table 1 shows the number of electrodes and measurements between comprehensive set and target set significantly decreased after selection. In order to get better compare with the comprehensive sets and conventional sets, we selected the same amount of data from the optimized set which obtained from the target selected set as the conventional sets.

Fig. 2 shows that the performance of target-oriented optimized for different targets. As the number of measurements increases, so does the resolution and therefore the number of electrodes used. After optimized ranking, the measurements at the forefront of the data set played a greater role in the resolution, and the value of resolution increased rapidly in the front part which indicating that fewer measurements can be selected to realize relatively good resolution results. The increase in the number of electrodes used soon approached the maximum value, indicating that the measurements composed of the electrodes selected by this method had a great effect on the target area, and therefore the selected electrodes were fully used.

**Figure 2.** The dashed line is the relative model resolution $S$ and the solid line is the number of electrodes, (a) target A, (b) target B, (c) target C, (d) target B (the number of measurements is only the first 200)

Fig. 3 shows the y=5 m slices of the inversion model. For the value of resistivity, the comprehensive set has the best performance, followed by the 'target all' set which consisting three targets, because the number of measurements is directly proportional to the resolution. For the conventional and optimized using the same number of data, the size of two blocks are better well resolved by the optimized arrays data set. Also, there are fewer false anomaly regions in the inversion results, the anomalous regions are more prominent and the size is closer to the real model. Note that although the number of arrays is the same between conventional and optimized, while, the number of electrodes used for optimized is less.
Figure 3. Inversion models for (a) comprehensive survey, (b) conventional survey, (c) target all survey (consisting A, B, C) and (d) optimized data set. The actual positions of the blocks are marked by black rectangles.

In order to quantitatively compare the inversion results of various surveys, the structural similarity (SSIM) method is used here [9]. Due to the different sets, the data involved within the inversion calculation are different, the traditional method of evaluating the RMS value of the inversion is not suitable for comparing the effects. SSIM gives more reliable measures of image similarity than the RMS difference metric. It gives the value 0 when comparing the target image to a random image with the same mean and variance, and 1 if the comparison image is identical to the target. We performed SSIM calculations along the XYZ direction to get the average structural similarity values in three directions, and finally got the overall average SSIM. These values can show the structural similarity between the inversion results in all directions in 3D and also the real model (Table 2).

| Data set       | X       | Y       | Z       | average |
|----------------|---------|---------|---------|---------|
| Comprehensive  | 0.8983  | 0.8936  | 0.8876  | 0.8932  |
| Target all     | 0.8965  | 0.8914  | 0.8852  | 0.8910  |
| Conventional   | 0.8927  | 0.8882  | 0.8836  | 0.8882  |
| Optimized      | 0.8928  | 0.8883  | 0.8846  | 0.8886  |

In three directions, the SSIM value also conforms to the objective law. The large amount of measurements used, the better result is going to be. With the same number of measurements, the optimized data set with fewer electrodes is even slightly better than conventional arrays. This quantitative comparison also shows that the target-oriented approach is feasible.

4. Conclusions
We present a modified compare R method to optimized survey design for electrical resistivity tomography. In the above of this paper, it demonstrated that target-oriented add into compare R method and the modified calculation can provide better resolution than conventional arrays. For the target area, this method selects fewer electrodes and arrays than the original CR method, get better resolution than conventional arrays, and reduces the calculation time. Furthermore, quantitative comparison of ERT imaging is an unresolved issue. This paper has made some attempts to it and obtained some results.

References
[1] Chambers J E, Wilkinson P B and Wardrop D, et al. 2012 Bedrock detection beneath river terrace deposits using three-dimensional electrical resistivity tomography Geomorphology 177 17–25.
[2] Chambers J E, Kuras O and Meldrum P I, et al. 2006 Electrical resistivity tomography applied
to geologic, hydrogeologic, and engineering investigations at a former waste-disposal site Geophysics 71 B231–B239.

[3] Dahlin T, Bernstone C and Loke M H 2002 A 3D resistivity investigation of a contaminated site at Lernacken in Sweden Geophysics 60 1682–1690.

[4] Loke M H, Chambers J E and Rucker D F, et al. 2013 Recent developments in the direct-current geoelectrical imaging method Journal of applied geophysics 95 135–156.

[5] Loke M H, Wilkinson P B and Uhlemann S S, et al. 2014 Computation of optimized arrays for 3-D electrical imaging surveys Geophysical Journal International 199 1751-1764

[6] Rücker C, Günther T and Wagner F M 2017 pyGIMLi: An open-source library for modelling and inversion in geophysics Computers and Geosciences 109 106-123

[7] Stummer P, Maurer H and Green A G 2004 Experimental design: electrical resistivity data sets that provide optimum subsurface information Geophysics 69 120–139

[8] Uhlemann S, Wilkinson P B and Maurer H, et al. 2018 Optimized survey design for electrical resistivity tomography: combined optimization of measurement configuration and electrode placement Geophysical Journal International 214 108-121

[9] Wang Z and Sheikh H R 2004 Image quality assessment: from error visibility to structural similarity: IEEE Trans. Image Process 13 600–612

[10] Wilkinson PB, Meldrum PI and Chambers JE, et al. 2006 Improved strategies for the automatic selection of optimized sets of electrical resistivity tomography measurement configurations Geophysical Journal International 167 1119-1126