Gauged Yukawa Matrix Models and 2-Dimensional Lattice Theories

H. Hamidian\textsuperscript{a}, S. Jaimungal\textsuperscript{b}, G.W.Semenoff \textsuperscript{b}, P. Suranyi\textsuperscript{a}, and L. C. R. Wijewardhana\textsuperscript{a}

\textsuperscript{a}Department of Physics, University of Cincinnati
Cincinnati, Ohio, 45221 U.S.A.
\textsuperscript{b}Department of Physics, University of British Columbia
Vancouver, British Columbia, Canada V6T 1Z1

Abstract

We argue that chiral symmetry breaking in three dimensional QCD can be identified with Néel order in 2-dimensional quantum antiferromagnets. When operators which drive the chiral transition are added to these theories, we postulate that the resulting quantum critical behavior is in the universality class of gauged Yukawa matrix models. As a consequence, the chiral transition is typically of first order, although for a limited class of parameters it can be second order with computable critical exponents.
One of the most intriguing features of quantum spin systems is their relationship to
gauge theories. This connection was originally used to study chiral symmetry breaking in
quantum chromodynamics (QCD), where the strong coupling limit resembles a spin system
\[1\]. More recently, the analogy has been exploited to prove that certain gauge theories break
chiral symmetry in the strong coupling limit \[2, 3, 4\]. It has also been used to formulate
mean field theories for magnetic systems \[5\]. For the most part, these works use the formal
similarity between a gauge theory and a spin system at the lattice distance scale. Recently
it has been suggested that the analogy is much broader in that it can account for the quasi-
particle spectrum and other infrared features of the two systems \[6\]. In this letter we shall
present evidence for the latter by discussing a common feature of the phase diagrams of
2-dimensional quantum antiferromagnets and 3-dimensional QCD. The dependence of the
chiral symmetry breaking pattern on the number of flavors and colors of quarks in QCD is
similar to that of the antiferromagnet where the rank of the spin algebra and the size of
its representation play the same role as the number of flavors and colors, respectively. We
shall also study the critical behavior associated with a chiral or Néel phase transition. Such
a transition must be driven by operators which are added to the QCD or antiferromagnet
Hamiltonian and which have the appropriate symmetries. We argue that these transitions
fall into a universality class which can be analyzed using the epsilon expansion. We show
that in many cases they are fluctuation induced first order transitions.

It has been observed that, in both 2+1-dimensional QED \[7\] and QCD \[8\], there exists
a critical number of flavors such that if \(N_F < N_F^{\text{crit}}\) the model breaks chiral symmetry
spontaneously and if \(N_F > N_F^{\text{crit}}\) the theory is in a chirally symmetric, deconfined phase.
For large \(N_F\) and for large number of colors \(N_C\) the equation of the critical line is approximately
\[N_F - \frac{128}{3\pi^2}N_C = 0.\]
A heuristic argument for this behavior is that when \(N_F >> N_C\)
internal gluon exchanges and the gluon self-coupling are suppressed by factors of \(N_C/N_F\).
Resummation of leading order diagrams, which are chains of bubbles, produces an effective
interaction which falls off like \(1/|r|\), rather than the tree level \(\ln |r|\). The weak coupling of
order \(N_C/N_F\) and mild infrared behavior of this resummed theory result in a chirally symmetric,
de-confined phase. When \(N_F\) is small, the effective coupling is large and can generate
a condensate, which is already seen in QED \[7\]. In fact, in QCD, when \(N_F <<< N_C\) all planar
diagrams contribute to processes, making the effective interaction string-like \[9\] and the
theory is in a confining and chiral symmetry breaking phase \[10\]. Numerical simulations
of 3-dimensional QED support this scenario with $N_F^{\text{crit}} \sim 4$.

Mass operators for basic 2-component fermions in 2+1 dimensions are pseudoscalars and break parity explicitly \[12\]. Massless 2+1 dimensional QCD with an odd number of flavors of 2-component fermions is afflicted with the parity anomaly \[13\] which generates a parity violating Chern-Simons term and also fermion mass term by radiative corrections. With an even number of flavors, there exists a parity and gauge invariant regularization and QCD is the 2+1-dimensional analog of a vector-like gauge theory in 3+1 dimensions. In particular a kind of chiral symmetry can be defined. It is known that, in this case, parity cannot be broken spontaneously \[14\] and therefore to study chiral symmetry breaking it is necessary to seek parity conserving mass operators. Following \[7, 8, 15\] we shall use $N_F$ species of 4-component fermions. The flavor symmetry of massless QCD in this case is actually $SU(2N_F)$. We will add operators to the action which reduce the symmetry to $SU(N_F)$, for example, the gauged Nambu-Jona Lasinio (NJL) model with four-fermion interaction,

$$S = \int d^3x \left( \frac{1}{4} F_{\mu\nu}^2 + \bar{\psi} \gamma_\mu D_\mu \psi + \frac{\lambda}{2} \left( \bar{\psi} T^A \psi \right)^2 \right)$$  \hspace{1cm} (1)

where $T^A$ is a generator of $SU(N_F)$ in the fundamental representation. The 4-fermi operator, which is renormalizable in the $1/N_C$ expansion \[18\], can drive the chiral phase transition with condensate $\phi^A = < \bar{\psi} T^A \psi >$. The results of \[7, 8\] indicate that if $N_F < N_F^{\text{crit}}$ the order persists even when $\lambda = 0$. Gauged NJL models, when analyzed by solving the gap equation \[16\], exhibit second order behavior at a surface in the space ($N_F, N_C, \lambda$). Our analysis will indicate that for a large range of parameters fluctuations make this transition first order. Our results do not apply to the hypothetical case where $N_F$ or $N_C$ are varied to drive the transition \[17\].

The 2-dimensional generalized antiferromagnet has Hamiltonian

$$H_{\text{spin}} = \kappa \sum_{<x,y>} \sum_{A=1}^{N_F^2-1} J^A(x) J^A(y)$$  \hspace{1cm} (2)

with $<x,y>$ nearest neighbor sites $x$ and $y$ on a square lattice and the spin operators $J^A(x)$ are in an irreducible representation of the $SU(N_F)$ Lie algebra

$$[J^A(x), J^B(y)] = i f^{ABC} \delta(x,y) J^C(x)$$  \hspace{1cm} (3)

When the representation at each site is a rectangular Young Tableau with $m$ rows and $N_C$
columns, it is convenient to represent the spin operators by the fermion bilinears

\[ J^A(x) = \sum_{\alpha=1}^{N_C} \sum_{a,b=1}^{N_F} \psi^\dagger_{a\alpha}(x) T^A_{ab} \psi_{b\alpha}(x) \]  

(4)

The fermions have the anticommutator,

\[ \{ \psi_{a\alpha}(x), \psi^\dagger_{b\beta}(y) \} = \delta_{ab} \delta_{\alpha\beta} \delta(x,y) \]  

(5)

Constraints which project out the irreducible representation of the spin algebra are

\[ G_{\alpha\beta}(x) \equiv \sum_{a=1}^{N_F} \psi^\dagger_{a\alpha}(x) \psi_{a\beta}(x) - \delta_{\alpha\beta} m \sim 0 \quad \forall x \]  

(6)

\( G_{\alpha\beta}(x) \) obeys the \( U(N_C) \) Lie algebra, commutes with the Hamiltonian and acts as the generator of gauge transformations with gauge group \( U(N_C) \).

The critical behavior of the antiferromagnet was examined by Read and Sachdev [19] using semiclassical methods. The only free parameters are the integers \( N_C \) and \( N_F \). \( N_C >> N_F \) is the classical limit of large representations, where the classical Néel ground state is stable with the staggered spin order parameter

\[ \mu_{ab} = (-1)^{\sum_i x_i} < \sum_{a=1}^{N_c} \psi_{aa}(x) \psi_{ba}(x) > \]  

(7)

On the other hand, the limit \( N_F >> N_C \) is the quantum limit where fluctuations are important and the system is in a spin disordered state. For both \( N_C \) and \( N_F \) large, they find a line of second order phase transitions in the \( (N_C, N_F) \) plane at \( N_F = \text{const.} \cdot N_C \) where the constant is a number of order one.

The relationship between the antiferromagnet and QCD is a very close one. There is an argument in ref. [20] which maps the strong coupling limit of lattice QCD onto the antiferromagnet with Hamiltonian [21]. The lattice regularization of the QCD Hamiltonian uses staggered fermions [22],

\[ H = \sum_{<x,y>} \left( \psi^\dagger_{a\alpha}(x) U_{xy}^{a\beta} \psi_{b\beta}(y) + h.c. + \frac{e^2}{2} \sum_{A=1}^{N_C} \left( E_A(x,y) \right)^2 \right) + \frac{1}{2 e^2} \sum_\square \text{tr} \left( \prod_\square U + \prod_\square U^\dagger \right) \]  

(8)

where the first sum is over links and the second is over plaquettes \( \square \) of the lattice. The gauge fields, which are unitary matrices \( U_{xy} \) and electric field operators occupy links and satisfy the algebra

\[ [E^A(x,y), E^B(z,w)] = i f^{ABC} E^C(x,y) \delta(x,y, z,w) \]  

(9)
\[ [E^A(xy), U_{wz}] = i \delta(xy, zw) U_{xy} T^A. \]  

The Hamiltonian is supplemented by the Gauss’ law constraint

\[
\sum_{y \in N(x)} E^A(xy) T^A_{\alpha\beta} + \sum_{a=1}^{N_F} \psi^\dagger_{a\alpha}(x) \psi_{a\beta}(x) - \delta_{\alpha\beta} N_F / 2 \sim 0
\]

which enforces gauge invariance. Here the first summation is over nearest neighbors of \( x \).

Staggered fermions have a relativistic continuum limit when their density is 1/2 of the maximum that is allowed by Fermi statistics and the kinetic Hamiltonian has phases which produce an effective \( U(1) \) magnetic flux \( \pi \) per plaquette \[20, 3\]. In order to obtain these phases in the continuum limit, we have chosen the sign of the third term in the Hamiltonian so that it is minimized by the configuration of gauge fields with the property \( < \prod U > = -1 \). The constraint of half-filling \( m = N_F / 2 \) is enforced by (11). The naive continuum limit yields 2+1-dimensional QCD with gauge group \( U(N_C) \) and \( N_F \) species of massless four component fermions. The full chiral symmetry only emerges in the continuum limit. On the lattice, staggered fermions have a discrete remnant of chiral symmetry (translation by one site) which forbids explicit fermion mass terms \[3, 4\]. A fermion mass term is a staggered density operator. For example, a latticization of \( \bar{\psi}(x) T^A \psi(x) \) is obtained from the staggered magnetization density in eqn. (7) as \( \sum_{ab} T^A_{ba} \mu_{ab} \). Thus, the antiferromagnetic order parameter and the order parameter for chiral symmetry breaking with a flavor-vector condensate are identical.

The argument of \[3\] can be summarized as follows: The strong coupling limit, \( e^2 \to \infty \) suppresses fermion propagation. In the leading approximation, the Hamiltonian is minimized by the states which contain as little electric field as possible and which are compatible with the gauge constraint (11). When \( N_F \) is even \[21\], it is possible to solve Gauss’ law with \( E^A = 0 \). The occupation number of each site is \( N_F / 2 \) and \( < (-1)^x \psi^\dagger_{a\alpha} \psi_{a\alpha} >= 0 \). This is a degenerate state - any gauge invariant state with \( N_F / 2 \) fermions has the same energy. Because they are required to be color singlets, this is the same set of states as occurs in the antiferromagnet when \( m = N_F / 2 \), i.e. in the representation of \( SU(N_F) \) whose Young tableau has \( N_C \) columns and \( N_F / 2 \) rows. Furthermore, to resolve the degeneracy, one must diagonalize the matrix of perturbations. These are non-zero only at second order and the diagonalization problem is equivalent to solving for the ground state of the antiferromagnet Hamiltonian (2) with \( \kappa = t^2 / e^2 \). Finally, since the order parameters are identical, the
Néel ordered states of the antiferromagnet correspond to chiral symmetry breaking states of QCD. Thus, the infinite coupling limit of QCD is identical to the antiferromagnet. A main difference between QCD with finite coupling and the antiferromagnet is that QCD contains electric and gauge fields which allow a fermion kinetic energy and still retain gauge invariance, whereas in the antiferromagnet, the fermions are not allowed to move. One could regard the corrections to the strong coupling limit of QCD as the addition of degrees of freedom and gauge invariant perturbations in the antiferromagnet which allow fermion propagation. In fact, ref. [6] suggests even a stronger correspondence, that the additional degrees of freedom are generated dynamically.

A common feature of QCD and the antiferromagnet is that, aside from \( N_F \) and \( N_C \) they have no free parameters. We could imagine adding operators of the sort that, if their coupling constant is varied, it can induce the chiral transition. It is tempting to speculate that these transitions fall into a universality class which can take into account all such modifications, as long as they respect the symmetries of the theory. Here, we shall restrict our attention to those which lead to a Lorentz invariant continuum limit. We argue that the universality class is described by the 4\(-\epsilon\) dimensional Euclidean field theory,

\[
S = \int d^{4-\epsilon}x \left( \frac{1}{2} \text{tr} \nabla \phi \cdot \nabla \phi + \frac{8\pi^2 \mu^\epsilon}{4!} \left( \frac{g_1}{N_F^2} (\text{tr} \phi^2)^2 + \frac{g_2}{N_F} \text{tr} \phi^4 \right) + \frac{1}{4} \text{tr} F_{\mu \nu}^2 
+ \bar{\psi} \left( \gamma \cdot \nabla + i \mu^{\epsilon/2} e_1 \gamma \cdot A + i \mu^{1/2} e_2 \gamma \cdot \text{Tr} A + \frac{\pi \mu^{\epsilon/2} \sqrt{N_F N_C}}{\sqrt{N_F N_C}} \phi \right) \psi \right) \tag{12}
\]

The scalar \( \phi \) is an \( N_F \times N_F \) traceless Hermitian matrix. The 4-component spinor \( \psi \) is an \( N_F \times N_C \) complex matrix and \( A_\mu \) is a \( U(N_C) \) gauge field. In four dimensions this model has Euclidean Lorentz invariance, C,P and T, discrete chiral symmetry, \( (\psi \rightarrow \gamma^5 \psi, \phi \rightarrow -\phi) \) and global \( SU(N_F) \) flavor. (12) includes all operators which are marginal when \( D = 4 \).

The evidence that (12) describes the universality class comes from previous work where we examined a similar model where gauge couplings are absent [18]. We showed that the anomalous dimensions of operators computed in the model (12) with \( e_i = 0 \) were identical to leading order in \( 1/N_C \) and \( \epsilon \) to those of a \( 2 < D < 4 \) dimensional four-fermi theory. That a \( 4 - \epsilon \) dimensional Yukawa-Higgs theory has the same universal critical behavior as lower dimensional four-fermi theories with the same symmetries was originally suggested by Wilson [22]. For the case \( N_F = 1 \), where the chiral symmetry is discrete, higher order computations have been carried out [23, 24]. The results, as well as those of lattice simulations, support
the universality hypothesis \cite{23}. We conjecture that (12) represents the universality class of lower-dimensional four-fermi theories with $U(N_C)$ gauge invariance. We shall show that, as a consequence, for a large range of values of $(N_C, N_F)$, the chiral phase transition is a fluctuation induced first order transition. When it is second order, critical exponents are in principle computable in the epsilon expansion.

In order to analyze (12), we use the $\epsilon$ expansion and the renormalization group. The 1-loop beta functions to first order in $\epsilon$ are,

$$
\beta_1 = -\epsilon g_1 + \frac{N_F^2 + 7}{6N_F^2} g_1^2 + \frac{2N_F^2 - 3}{3N_F^2} g_1 g_2 + \frac{N_F^2 + 3}{2N_F^2} g_2^2 + \frac{1}{2N_F} y^2 g_1
$$

$$
\beta_2 = -\epsilon g_2 + \frac{2}{N_F} g_1 g_2 + \frac{N_F^2 - 9}{3N_F^2} g_2^2 - \frac{3}{8N_CN_F} y^4 + \frac{1}{2N_F} y^2 g_2
$$

$$
\beta_y = -\frac{\epsilon}{2y} - \frac{3}{16\pi^2} N_F^2 - \frac{1}{N_C} e_1^2 y - \frac{3}{8\pi^2} e_2^2 y + \frac{N_F^2 + 2N_FN_C - 3}{16N_F^2 N_C} y^3
$$

$$
\beta_{e_1} = -\frac{\epsilon}{2} e_1 - \frac{11N_F - 2N_F}{48\pi^2} e_1^2, \quad \beta_{e_2} = -\frac{\epsilon}{2} e_2 + \frac{N_C N_F}{12\pi^2} e_2^3 \tag{13}
$$

We can also compute the anomalous dimensions of the scalar and the fermion field. When $N_C$ and $N_F$ are large we obtain

$$
\Delta_S = 1 - \frac{\frac{2\gamma^2 - 11\gamma - 18}{(2\gamma - 11)(\gamma + 2)} \epsilon}{2} \tag{14}
$$

$$
\Delta_F = \frac{3}{2} - \frac{\frac{2\gamma^2 - 15\gamma - 50}{(\gamma + 2)(2\gamma - 11)} \epsilon}{4} \tag{15}
$$

respectively, where $\gamma \equiv N_F/N_C$. We shall see that the region where there can be second order behavior is $\gamma < 8.3$. This result is reliable for small epsilon.

Fixed points occur at zeros of the beta functions, $\beta_i(g^*) = 0$. They are infrared (IR) stable or ultraviolet (UV) stable if all eigenvalues of the stability matrix, $\partial \beta_i / \partial g_j |_{g=g^*}$, are either positive or negative, respectively. The fixed points of the Higgs model ($e_1 = e_2 = y = 0$) were analyzed by Pisarski \cite{25} and of the Yukawa-Higgs model ($e_1 = e_2 = 0$) in ref. \cite{18}.

Second order phase transitions are possible when renormalization group trajectories flow to an IR stable fixed point. It is at the IR stable fixed points that the conformal field theory with all dimensional constants except the renormalization scale are set to zero, exists.

If there are no such fixed points, the only possible phase transition is a fluctuation induced first order one. Yamagishi \cite{26} formulated a criterion for this behavior. He showed that this
occurs when renormalization group trajectory crosses the surface in coupling-constant space given by

\[ P_i(g, y, e) = 0, i = 1, 2 \]  

where \( P_1(g, y, e) = (4 - \epsilon)(g_1 + g_2) + \beta_1 + \beta_2 \) and \( P_2 = (4 - \epsilon)(g_1 + N\frac{F}{2} g_2) + \beta_1 + N\frac{F}{2} \beta_2 \) depending on whether \( g_2 > 0 \) or \( g_2 < 0 \) respectively. When the renormalization group trajectory crosses these surfaces two further conditions must be met. To ensure that the extremum is a local minimum, rather than a maximum, it is necessary that

\[ D_i > 0 \quad \text{for} \quad i = 1, 2 \]

where

\[ D_1 = (4 - \epsilon)(\beta_1 + \beta_2) + \sum_i \beta_i \partial/\partial g_i \beta_1 + \beta_2 \]

\[ D_2 = (4 - \epsilon)(\beta_1 + N\frac{F}{2} \beta_2) + \sum_i \beta_i \partial/\partial g_i (\beta_1 + N\frac{F}{2} \beta_2) \]

depending on whether \( g_2 > 0 \) or \( g_2 < 0 \).

In order that this minimum has lower free energy than the trivial \( \phi = 0 \), it is necessary that the couplings at that scale obey

\[ g_1 + g_2 < 0 \quad \text{or} \quad g_1 + N_F g_2/2 < 0 \]  

(17)

For the beta function [13], there are two distinct cases. When \( N_F < 11N_C/2 \) the four dimensional non-Abelian gauge coupling is asymptotically free. The solution of \( \beta_{\epsilon_1} = 0 \) is at \( \epsilon_1 = 0 \) and this fixed point is UV, rather than IR stable. In this case, there should be a nonperturbative behavior associated with confinement which is inaccessible to our computation. When \( N_F > 11N_C/2 \) the four dimensional gauge coupling is not asymptotically free. \( \beta_{\epsilon_1} \) has two zeros, the UV attractive one located at zero and the IR attractive one at

\[ \epsilon_1^{*2} = \frac{24\pi^2}{2N_F - 11N_C} \epsilon \]  

(18)

Using this solution in \( \beta_y \), we see that the Yukawa coupling constant always has an UV attractive fixed point at zero coupling and an IR attractive fixed point at

\[ y_1^{*2} = \left( 1 - 9 \frac{N_C - \frac{11}{2N_F}}{11N_C - 2N_F} \right) \frac{8N_F^2 N_C}{N_F^2 + 2N_F N_C - 3} \epsilon \]  

(19)

Finally, using (19), the equations for fixed points of the matrix self-couplings are

\[ 0 = \left( \frac{y_1^{*2}}{2N_F} - \epsilon \right) g_1^* + \frac{N_F^2 + 7}{6N_F^2} y_1^{*2} + \frac{2N_F^2 - 3}{3N_F^2} y_1^* g_2^* + \frac{N_F^2 + 3}{2N_F^2} g_2^{*2} \]

\[ 0 = \left( \frac{y_2^{*2}}{2N_F} - \epsilon \right) g_2^* + \frac{2}{N_F} g_1^* g_2^* + \frac{N_F^2 - 9}{3N_F^2} g_2^{*2} - \frac{3}{8N_C N_F} y^{*4} \]  

(20)
Numerical investigations show that IR stable fixed points (in fact only one) exist when $(11/2) N_C < N_F < N_F^*(N_C)$. The upper critical $N_F^*(N_C)$ intersects the line $(11/2) N_C$ at $N_C = 1$ and when $N_C$ and $N_F$ are large, $N_F^*(N_C) \approx 10.7 N_C$.

Naively one would expect second order behavior for $N_C$ and $N_F$ in this entire region. However, in some cases, renormalization group trajectories can satisfy the conditions for first order behavior before they reach the IR fixed point. In particular, if the IR fixed point is such that, $g_1^* + g_2^* < 0$, then the far IR behaviour of the effective potential is approximately $V[\phi] \approx (g_1^* + g_2^*) \phi^{2(4-\epsilon)/(2-\epsilon)}$ and the trivial configuration $\phi = 0$ is a local maximum of the potential. In addition, if this $(g_1^*, g_2^*)$ is the limit of a flow originating from the ultraviolet stability wedge, $\{ g_1 + g_2 > 0 \cap g_1 + N_F g_2 / 2 > 0 \}$, the potential is bounded from below, and since it has a local maximum at the origin, it must have a minimum when $\phi \neq 0$. Hence, in this case, there is always a first order phase transition, even though an IR stable fixed point exists.

In fact, it is easy to see that the flow in this case intersects the surface $P = 0$. Since $g_1^* + g_2^* < 0$, any trajectory which flows from the UV fixed point at the origin (in a direction within the stability wedge) to the IR fixed point must intersect the surface $P = 0$ in $(\mathbf{13})$. Consider a trajectory which begins near the UV fixed point at the origin, where the couplings are small and $\beta_i \approx -\epsilon g_i$. Since the trajectory must be in the region $g_1 + g_2 > 0$, $P \approx (4 - 2\epsilon)(g_1 + g_2)$ which is positive. At the IR fixed point, the beta functions in $P$ vanish and it takes the form $P = (4 - \epsilon)(g_1^* + g_2^*)$ which is negative. Since $P(g, y, \epsilon)$ changes sign as we follow the trajectory from the UV to the IR fixed point, it must go through zero at least once.

We have verified numerically for a $(N_C, N_F) = \{(2, 15), (2, 18), (100, 1000)\}$, with $\epsilon = 0.1$ that they all satisfy the criteria for a first order phase transition. Generally, IR fixed points with $g_1^* + g_2^* < 0$ occur in the range $N_F(N_C) < N_F < N_F^*(N_C)$. In this region, the theory has an IR stable fixed point, but undergoes a first order phase transition. For large $N_F$ and $N_C$, $N_F(N_C) = 8.3 N_C$.

The other possibility is that the IR stable fixed point has the property $g_1^* + g_2^* > 0$. In this case, we have found that there are two kinds of trajectories, those which flow from the UV to IR fixed points without encountering the surface $P = 0$ and lead to second order transitions, and those which visit the region where $g_1 + g_2 < 0$. The latter trajectories cross the surface twice and satisfy the conditions for first order behavior at one of the intersections. In fig.1 we have plotted several RG flows for the case $N_C = 2$, $N_F = 13$ and $\epsilon = .1$. (The features
of the flow are insensitive to $\epsilon$ in the domain $0.1 < \epsilon < 1$. Near $\epsilon = 1$, the relevant coupling constants are large and one would not expect the 1-loop approximation that we have used to be accurate there.) The stars indicate that the flow has crossed the surface $P = 0$ in the region defined by $D_i > 0$ and (17). Hence the transition associated with those trajectories is first order. The crosses denote points where the flow intersects the surface (16), but not when $g_1 + g_2 < 0$. They do not correspond to global minima of the effective potential and therefore do not indicate a first order transition.

The physical quantum spin $j$ antiferromagnet corresponds to $N_F = 2$ and $N_C = 2j$. With $N_C = 1$ ($j = 1/2$) the non-abelian field must be removed, and the resulting theory has no IR fixed point, indicating a first order transition. $N_F = 2, N_C \geq 2$ (i.e. $j \geq 1$) is in the asymptotically free regime and hence these antiferromagnets cannot be analyzed by our techniques. We speculate that confinement is associated with a nonperturbative IR fixed point of the gauge coupling. In that case, it is likely that these antiferromagnets would have a second order transition.

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Figure caption: Fig. 1: Projection of the renormalization group trajectories in five dimensional coupling constant space onto the $g_1 - g_2$ plane for $N_C = 2$, $N_F = 13$ and $\epsilon = .1$. Stars indicate that the trajectory encounters the surface $P = 0$ in the region defined by eqns (17) whereas the crosses denote points of intersection where those criteria are not satisfied.
RG Flow

Nc = 2 Nf = 13 eps = 0.1