Prediction of Long Term Stability by Extrapolation

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Abstract

This paper studies the possibility of using the survival function, to predict long term stability by extrapolation. The survival function is a function of the initial coordinates and is the number of turns a particle will survive for a given set of initial coordinates. To determine the difficulties in extrapolating the survival function, tracking studies were done to compute the survival function. The survival function was found to have two properties that may cause difficulties in extrapolating the survival function. One is the existence of rapid oscillations, and the second is the existence of plateaus. It was found that it appears possible to extrapolate the survival function to estimate long term stability by taking the two difficulties into account. A model is proposed which pictures the survival function to be a series of plateaus with rapid oscillations superimposed on the plateaus. The tracking studies give results for the widths of these plateaus and for the separation between adjacent plateaus which can be used to extrapolate and estimate the location of plateaus that indicate survival for longer times than can be found by tracking.
Chapter 1

Introduction

This paper studies the possibility of using the survival function, to predict long term stability by extrapolation. The survival function is a function of the initial coordinates and is the number of turns a particle will survive for a given set of initial coordinates. To determine the difficulties in extrapolating the survival function, tracking studies were done to compute the survival function. The survival function was found to have two properties that may cause difficulties in extrapolating the survival turns function. One is the existence of rapid oscillations, and the second is the existence of plateaus. It was found that it appears possible to extrapolate the survival function to estimate long term stability by taking the two difficulties into account.

A model is proposed which pictures the survival function to be a series of plateaus with rapid oscillations superimposed on the plateaus. The tracking studies give results for the widths of these plateaus and for the separation between adjacent plateaus which can be used to extrapolate and estimate the location of plateaus that indicate survival for longer times than can be found by tracking.
Chapter 2

The survival function, $j_{\text{trns}}$.

For a given set of initial coordinate, $x_0, p_{x0}, y_0, p_{y0}$, one can find by tracking the survival time in turns, which is the number of turns the particle will survive before becoming unstable and which will be denoted by $j_{\text{trns}}$. This determines the function $j_{\text{trns}}(x_0, p_{x0}, y_0, p_{y0})$ which will be called the survival function [8, 9, 10, 11]. If one limits the tracking to $1 \times 10^6$ turns or less, one can find $j_{\text{trns}}$ for those $x_0, p_{x0}, y_0, p_{y0}$ for which $j_{\text{trns}}$ is less than or equal to $1 \times 10^6$.

A tracking study was done of particle motion with no rf present, using an older version of the RHIC lattice. Random and systematic field error multipoles are present up to order 10. The particle momentum is $dp/p = 0$. As the first case studied, the initial coordinates $x_0, p_{x0}, y_0, p_{y0}$ are chosen along the direction in phase space given by $p_{x0} = 0, p_{y0} = 0$ and $\epsilon_{x0} = \epsilon_{y0}$, where $\epsilon_{x0}, \epsilon_{y0}$ are the linear emittances in the absence of the error multipoles. Along this direction in phase space, $j_{\text{trns}}$ may be considered to be a function of $x_0$. For a given initial coordinate, $x_0$, one can find by tracking the survival time in turns, which is the number of turns the particle will survive before becoming unstable and will be denoted by $j_{\text{trns}}$. This determines the survival function [8, 9, 10, 11] $j_{\text{trns}}(x_0)$. If one limits the tracking to $1 \times 10^6$ turns or less, one can find $j_{\text{trns}}$ for those $x_0$ for which $j_{\text{trns}}$ is less than or equal to $1 \times 10^6$. For $dp/p = 0$, it is assumed that on the closed orbit $x = 0$ and $x_0$ is also the initial betatron oscillation amplitude.

The tracking was first done using $x_0$ which are separated by $dx_0 = .1$ mm. The results for $j_{\text{trns}}$ versus $x_0$ are shown in Fig. [2.1]. The results in this figure may be looked at as the results of a search in $x_0$ starting at large $x_0$ and decreasing $x_0$ in steps of $dx_0 = .1$ mm. The figure shows an apparent
stability limit for $10^6$ turns of $u_{sl} = 15.2$ mm.

### 2.1 Rapid oscillations in the survival function

Fig. 2.1 shows rapid oscillations in $j_{trns}$ with $x_0$. Large changes in $j_{trns}$ occur when $x_0$ changes by .1 mm. The oscillations extend over about .3 mm. Tracking studies show that the oscillations become more rapid when the search interval, $dx_0$, is decreased. This is indicated in Fig. 2.2 where $dx_0$ is decreased to .05 mm and in Fig. 2.3 where $dx_0$ is decreased to .025 mm. Results for different search intervals, $dx_0$, are shown in Table 2.1. $dx_0$ is decreased from .1 mm to .0001 mm. The wavelength of the oscillations, $\Delta x_0$, as measured from the stability limit for 1e6 turns, $u_{sl}$, to the location of the first peak in $j_{trns}$ decreases from about .3 mm to .0002 mm. Also listed for each value of $dx_0$ are the apparent stability limit for 1e6 turns, $u_{sl}$, and $j_{trns}$.
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Table 2.1: Results for different search intervals, $dx_0$. $dx_0$ is decreased from .1mm to .0001 mm. \( \Delta x_0 \) is the wavelength of the oscillations, as measured from the stability limit for 1e6 turns, $u_{sl}$, to the location of the first peak in $j_{trns}$. Also listed for each value of $dx_0$ are the apparent stability limit for 1e6 turns, $u_{sl}$, and $j_{trns}$ at $u_{sl} + dx_0$.

| $dx_0$ (mm) | $\Delta x_0$ (mm) | $u_{sl}$ (mm) | $j_{trns}$ at $u_{sl} + dx_0$ |
|-------------|------------------|--------------|----------------------------|
| .1          | .3               | 15.2         | 100000                     |
| .01         | .06              | 15.27        | 150000                     |
| .001        | .003             | 15.270       | 82605                      |
| .0001       | .0002            | 15.2793      | 60693                      |

Table 2.1 shows that the wavelength of the oscillations, \( \Delta x_0 \) is roughly proportional to the size of the search interval, $dx_0$. The value of $j_{trns}$ at $u_{sl} + dx_0$ shows that near $u_{sl}$, $j_{trns}$ changes appreciably in the small change in $x_0$ given by $dx_0$. Tracking results show that this seems to hold even at extremely small $dx_0$. The computed results appear to indicate that $j_{trns}(x_0)$ is not a continuous function of $x_0$. For a continuous function of $x_0$, one can find a small enough interval in $x_0$ such that the difference between the values of the function for any two $x_0$ in that interval is very small. This does not appear to be true for $j_{trns}(x_0)$.

The existence of the rapid oscillations in the survival function, $j_{trns}(x_0)$, would seem to make it difficult to extrapolate to find those $x_0$ that survive for more than 1e6 turns. However, one could view the survival function shown in Fig. 2.1 as being made up of the rapid oscillations superimposed on a smoother, more slowly varying function which could be used for the extrapolation. This is discussed further in section 2.3.

2.2 Plateaus in the survival function

Looking at Fig. 2.1, one can make out plateaus in the survival function, $j_{trns}(x_0)$. The plateaus are regions where $j_{trns}$ oscillates rapidly around an almost constant value of $j_{trns}$. The plateaus can be seen somewhat more clearly if one reduces the search interval $dx_0$, as shown in Fig. 2.2 where $dx_0$ is decreased to $dx_0 = .05mm$. One can make out 4 plateaus located at about $j_{trns}$=1.5e5, 2e4, 3500, 400 turns. Possible plateaus with $j_{trns}$ less than 100...
Figure 2.2: $j_{\text{trns}}$ versus $x_0$. $dp/p = 0$, $p_{x0} = 0$, $p_{y0} = 0$, $\epsilon_{x0} = \epsilon_{y0}$ direction, $dx_0 = 0.05 \text{mm}$. In the figure, $j_{\text{trns}}$, $x_0$, $dx_0$, $\epsilon_{x0}$, $\epsilon_{y0}$ represent $j_{\text{trns}}$, $x_0$, $dx_0$, $\epsilon_{x0}$, $\epsilon_{y0}$.

turns are being ignored. It will be seen that the width of the plateaus do not depend on the search interval, $dx_0$. This is also true of the location of the plateaus in $j_{\text{trns}}$. This is shown in Fig. 2.3 where $dx_0 = 0.025 \text{ mm}$.

The existence of the plateaus in the survival function, $j_{\text{trns}}(x_0)$, would seem to make it difficult to extrapolate to find those $x_0$ that survive for more than $1e6$ turns. If one does the extrapolation using points which are close to the stability limit for $1e6$ turns, $u_{st}$, one may get incorrect results as these points may lie on one of the plateaus.

2.3 Extrapolation of the survival function

The data given above leads to a model of the survival function, which pictures it as sequence of plateaus. Within the plateaus, $j_{\text{trns}}$ oscillates about some constant value of $j_{\text{trns}}$ which will be called the level of the plateau. The
existence of the plateaus makes extrapolation of the survival function appear
difficult. The last plateau that was measured has a level of about 1.5e5 turns
and a width of about 1.2 mm. An interesting question is what is the level of
the next plateau, at lower $x_0$, after the last plateau that was measured. In
the following this plateau will be referred to as the 'next plateau'.

Some help with the extrapolation is also provided by plotting $1/\log(j_{trns})$
against $x_0$ as shown in Fig. 2.4, where the search interval is $dx_0=.1$mm and
points with $j_{trns}$ less than 400 turns are omitted. It will be seen below
that the separation between adjacent plateaus does not vary greatly when
measured as the change in $1/\log(j_{trns})$.

To help locate the 'next plateau', long runs were done starting with
$x_0=15.2$mm, and decreasing $x_0$ in steps of .1mm. In order to detect the
beginning of the 'next plateau', the runs have to be long enough not to be
confused by the rapid oscillations in $j_{trns}$ that occur within each plateau.
Runs of length $2e7$ turns were chosen, and these runs take about 10 days for
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The results are shown in Fig. 2.5. One sees from this figure that the 'next plateau' appears to start at $x_0=15.3$ mm and ends at about 14.0 mm, and has a level of about $j_{trns}=15e6$ turns and a width of 1.2mm. The previous plateau goes from 16.5mm to 15.2 mm and has a width of 1.3mm and a level of $j_{trns}=1.5e5$ turns. The width of the plateau is measured here from the end of one plateau in $x_0$ to the end of the adjacent plateau and includes the transition region where the points move from one plateau to the next. The width of the 'next plateau' is difficult to measure, as the adjacent plateau at still lower $x_0$ is estimated to have a level of 3e9 turns, and cannot be found by tracking. One can see that the width is larger than 1.2mm. The $x_0$ at 14.0 mm survived more than 89.9e6 turns. The data given above for the 'next plateau' is somewhat in error but it seems better to use it than to throw away this information.

The results are also shown as a $1/\log(j_{trns})$ versus $x_0$ plot in Fig. 2.6.
Figure 2.5: \( j_{\text{trns}} \) versus \( x_0 \) including points with \( j_{\text{trns}} \) up to \( 4 \times 10^7 \). \( dp/p = 0, p_x = 0, p_y = 0, \epsilon_x = \epsilon_y \) direction, \( dx_0 = 0.1 \text{mm} \). In the figure, \( j_{\text{trns}}, x_0, dx_0, \epsilon_{x0}, \epsilon_{y0} \) represent \( j_{\text{trns}}, x_0, dx_0, \epsilon_{x0}, \epsilon_{y0} \).

One can see three plateaus near the stability boundary at the levels of about \( j_{\text{trns}} = 1.5 \times 10^6, 1.5 \times 10^5, \text{ and } 2 \times 10^4 \) turns. The properties of the plateaus are summarized in Table 2.2. The separation between the plateaus in the \( 1/\log(j_{\text{trns}}) \) plot are given by \( 0.054 \) and \( 0.039 \), so that the separation between the plateaus in the \( 1/\log(j_{\text{trns}}) \) are not too different. The widths of the plateaus that were measured were \( 1.2+, 1.3 \) and \( 1.7 \) mm.

The plateau model will now be used to extrapolate and investigate long term stability in RHIC. In RHIC, \( j_{\text{trns}} = 1 \times 10^9 \) turns coresponds to a survival

| Plateau level, \( j_{\text{trns}} \) | 1.5e6  | 1.5e5  | 2e4   |
|-----------------------------|--------|--------|-------|
| Plateau level, \( 1/\log(j_{\text{trns}}) \) | 0.139  | 0.193  | 0.232 |
| Plateau separation in \( 1/\log(j_{\text{trns}}) \) | 0.054  | 0.039  | —     |
| Plateau width, \( \Delta x_0 \) (mm) | 1.2+   | 1.3    | 1.7   |

Table 2.2: Plateau parameters for the \( \epsilon_{x0} = \epsilon_{y0} \) direction.
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Figure 2.6: $1/\log(j_{\text{trns}})$ versus $x_0$ showing the next plateau and the last measured plateau. $dp/p = 0$, $p_{x0} = 0$, $p_{y0} = 0$, $\epsilon_{x0} = \epsilon_{y0}$ direction, $dx_0 = 0.1\text{mm}$. In the figure, $1/\log(j_{\text{trns}})$, $x_0$, $dx_0$, $\epsilon_{x0}$, $\epsilon_{y0}$ represent $1/\log(j_{\text{trns}})$, $x_0$, $dx_0$, $\epsilon_{x0}$, $\epsilon_{y0}$.

time of about 3.5 hours. The data given above will be used to extrapolate to find the plateau whose level is greater than or equal to $j_{\text{trns}} = 1e9$ turns. This plateau will be called the $1e9$ plateau. The existence of the plateaux indicates that there are limits on the accuracy one can hope to achieve by extrapolation. The two parameters one needs to extrapolate the survival function are the plateau width and the plateau level separation. One cannot be certain what these parameters will be at $j_{\text{trns}}$ of the order of $1e9$ turns. However, one can use the data found for these two parameters at $j_{\text{trns}}$ of the order of $1e6$ turns, to make the best estimate for these parameters at $j_{\text{trns}}$ of the order of $1e9$ turns.

The plateau width and the plateau level separation were studied for three different cases corresponding to three different directions in phase space. The results are summarized in chapter 5. The widths of the plateaus were found to be roughly constant when measured in terms of $X_0 = [\beta_{x0}(\epsilon_{x0} + \epsilon_{y0})]^5$ with
Figure 2.7: \( \frac{1}{\log(j_{\text{trns}})} \) versus \( x_0 \) showing the plateaus found by extrapolation, including the 1e9 plateau. \( dp/p = 0, p_{x0} = 0, p_{y0} = 0, \varepsilon_{x0} = \varepsilon_{y0} \) direction, \( dx_0 = 0.1 \text{mm} \). In the figure, \( \frac{1}{\log(j_{\text{trns}})}, x_0, dx_0, \varepsilon_{x0}, \varepsilon_{y0} \) represent \( \frac{1}{\log(j_{\text{trns}})} \), \( x_0 \), \( dx_0 \), \( \varepsilon_{x0}, \varepsilon_{y0} \).

an average value \( \Delta X_0 = 2.00 \text{ mm} \). The plateau level separation measured in \( \frac{1}{\log(j_{\text{trns}})} \) varied from .054 to .019 with an average value of .033. It is suggested that in the extrapolation, the average value of these two parameters be used. For the case being considered here, it is assumed in the extrapolation that the plateau widths in \( x_0 \) will be 1.4 mm corresponding to the average value of \( \Delta X_0 = 2.00 \text{ mm} \), and the plateau level separation is .033 in \( \frac{1}{\log(j_{\text{trns}})} \). Note that in this case \( X_0 = 1.414 x_0 \)

The results found using these assumptions are shown in Fig. 2.7, where two new plateaus are shown that were found by extrapolation. The plateau just below the next plateau in \( x_0 \) has the level of \( j_{\text{trns}} = 3.3e9 \) turns and is the 1e9 plateau in this example. The 1e9 plateau goes from \( x_0 = 13.9 \text{mm} \) to \( x_0 = 12.6 \text{mm} \). The aperture for 1e9 turns may be taken as about 13.9mm, which may be compared to the 15.2 mm found using runs of 1e6 turns. This indicates a loss of about 1.3mm or about 9% due to the required survival
time of 1e9 turns. One may note that the level of the 1e9 plateau being 3.3e9
turns, one may expect that some of the \( x_0 \) on this plateau will not survive
1e9 turns due to the oscillations in \( j_{\text{TRNS}} \) that will occur on this plateau. To
be safer one could assume the aperture for 1e9 turns to be 12.6 mm, which
is the beginning of the adjacent plateau with a level of 1.2e14 turns, giving
a loss of 17%.

The procedure used indicates that in this case the result is not sensi-
tive to changes in the two assumptions made regarding the width and level
separation of the plateaus. If the plateau model continues to hold for the
extrapolated plateaus, and the width and level separation remain roughly
1.3 mm and .034 respectively, then the result for the aperture for 1e9 turns
will be about the same.

The plateau model developed above avoids certain problems that arise in
tracking studies. In trying to find the aperture for the survival time of 1e9
turns, the more usual approach is to try find the \( x_0 \) such that all smaller \( x_0 \)
will survive for 1e9 turns. If while doing a tracking search starting from large
\( x_0 \), one finds a \( x_0 \) that survives for 1e9 turn, then one has to ask whether all
smaller \( x_0 \) will survive for 1e9 turns. This is a difficult question to answer.
It is even possible, that there are are always smaller \( x_0 \) that will not survive
1e9 turns, although these \( x_0 \) may become very scarce at smaller \( x_0 \). In the
plateau model, in trying to find the aperture for the survival time of 1e9
turns, the approach is to try to find the plateau whose level is greater than
1e9 turns. This is a better defined target, and the expectation is that this
plateau will indicate a region of \( x_0 \) where most \( x_0 \) will survive 1e9 turns.
Chapter 3

Survival function along the $\epsilon y_0 = 0$ direction

Figure 3.1: $j_{trns}$ versus $x_0$. $dp/p = 0$, $p_{x0} = 0$, $p_{y0} = 0$, $\epsilon y_0 = 0$ direction, $dx_0 = .1$ mm. In the figure, $j_{trns}$, $x_0$, $dx_0$, $epx0$, $epy0$ represent $j_{trns}$, $x_0$, $dx_0$, $\epsilon x_0$, $\epsilon y_0$. 
CHAPTER 3. SURVIVAL FUNCTION ALONG THE $\epsilon Y_0 = 0$ DIRECTION

The tracking results presented above were all done along the $\epsilon x_0 = \epsilon y_0$ direction. Results will now be given for the $y_0 = 0, p_{y0} = 0$ or $\epsilon y_0 = 0$ direction. Tracking studies of other cases will test the consistancy of the plateau model. The direction in the space of $x_0, p_{x0}, y_0, p_{y0}$ is further defined by $p_{x0} = 0$. The particle motion is 4 dimensional because of the presence of skew multipoles. $j_{trns}$ may then be considered to be a function of $x_0$. Fig. 3.1 shows $j_{trns}$ as a function of $x_0$ as found with tracking runs of 1e6 turns. The apparent stability limit using 1e6 turns and $dx_0=.1$mm is 22.0 mm. To make the plateaus more visible, one can reduce the search interval $dx_0$ to $dx_0=.05$ mm. These results are shown in Fig. 3.2. One sees that there are two plateaus in the region shown with $j_{trns}$ greater than or equal to 1e4, whose levels are located at $j_{trns} = .9e6$, and 1e4 turns. The oscillations on the plateau near the stability boundary, appear to be smaller than those seen in the $\epsilon x_0 = \epsilon y_0$ case.

Runs of about 2e7 turns were done to find the shape of the 'next plateau'
Figure 3.3: \( j_{\text{trns}} \) versus \( x_0 \) including points with \( j_{\text{trns}} \) up to \( 4 \times 10^7 \). \( dp/p = 0, p_{x0} = 0, p_{y0} = 0, \epsilon_{y0} = 0 \) direction, \( dx_0 = 0.1 \) mm with \( dx_0 = 0.05 \) mm at larger \( x_0 \). In the figure, \( j_{\text{trns}}, x_0, dx_0, \epsilon_{px0}, \epsilon_{py0} \) represent \( j_{\text{trns}}, x_0, dx_0, \epsilon_{x0}, \epsilon_{y0} \) in the region where \( x_0 \) is less than or equal to 21.0 mm. Enough tracking runs were done to determine the beginning, and the level of the 'next plateau'. The results are shown in Fig. 3.3. Including these longer runs, one sees that the 'next plateau' begins at \( x_0 = 21.0 \) mm and the level of the 'next plateau' is about \( j_{\text{trns}} = 8 \times 10^6 \) turns. Here, the beginning of the plateau is the edge at larger \( x_0 \) and the end is the edge at lower \( x_0 \). The end of the 'next plateau' is somewhat difficult to determine, as the adjacent plateau at lower \( x_0 \) has a high level of about \( 1 \times 10^9 \) turns. The end of the 'next plateau' was taken to be at \( x_0 = 19.0 \) mm. The adjacent plateau at higher \( x_0 \) is at the level of \( 1.5 \times 10^6 \) turns and with the width of 1.8 mm. These two plateaus are separated by \( 0.017 \) in \( 1/\log(j_{\text{trns}}) \) which is smaller than the \( 0.054 \) found in the \( \epsilon_{x0} = \epsilon_{y0} \) case. The results are also shown as \( 1/\log(j_{\text{trns}}) \) versus \( x_0 \) plot in Fig. 3.4.

The data found for these two plateaus will be used to extrapolate and find the \( 1 \times 10^9 \) plateau in this case. In the extrapolation, the results found in chapter 5 for the average plateau width and level separation will be used. In
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Figure 3.4: $1/\log(j_{trns})$ versus $x_0$. $dp/p = 0$, $p_{x0} = 0$, $p_{y0} = 0$, $\epsilon_{y0} = 0$ direction, $dx_0 = 0.1$ mm. In the figure, $\log(j_{trns})$, $x_0$, $dx_0$, $\epsilon_{x0}$, $\epsilon_{y0}$ represent $1/\log(j_{trns})$, $x_0$, $dx_0$, $\epsilon_{x0}$, $\epsilon_{y0}$.

In this case, this leads to the assumptions that each plateau is 2.0 mm wide and a plateau level separation of 0.033 in $1/\log(j_{trns})$ will be used here.

The results are shown in Fig. 3.5, where two extrapolated plateaus are shown with the plateau levels $8.5e8$, and $4.5e12$. The aperture for $1e9$ turns was taken to be 17.1 mm, the beginning of the plateau with a level of $4.5e12$ turns. 17.1 mm is to be compared with the aperture of 21.9 mm found with runs of $1e6$ turns. A loss of 4.8 mm or 22%.
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Figure 3.5: $1/\log(j_{trns})$ versus $x_0$ showing the plateaus found by extrapolation, including the $1e9$ plateau. $dp/p = 0$, $p_{x0} = 0$, $p_{y0} = 0$, $\epsilon_{x0} = \epsilon_{y0}$ direction, $dx_0 = 0.1mm$. In the figure, $1/\log(j_{trns})$, $x_0$, $dx_0$, $epx0$, $epy0$ represent $1/\log(j_{trns})$, $x_0$, $dx_0$, $\epsilon_{x0}$, $\epsilon_{y0}$. 
Chapter 4

Survival function along $\epsilon_{x0} = 0$ direction

Figure 4.1: $j_{\text{trns}}$ versus $X_0$. $X_0 = (\beta_{x0}/\beta_{y0})^5 y_0$. $dp/p = 0$, $x_0=0$, $p_{x0} = 0$, $p_{y0} = 0$, $\epsilon_{x0} = 0$ direction, $dx_0=.1$ mm and runs go up to $2e7$ turns. In the figure, $j_{\text{trns}}$, $X_0$, $dX_0$, $epx0$, $epy0$ represent $j_{\text{trns}}$, $X_0$, $dX_0$, $\epsilon_{x0}$, $\epsilon_{y0}$. 

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Results will now be given for the $x_0 = 0, p_{x0} = 0$ or $\epsilon_{x0} = 0$ direction. In order to be able to compare results with those of the above two cases, $j_{trns}$ will plotted against $X_0 = (\beta_{x0}/\beta_{y0})^5 y_0$. Fig. 4.1 shows $j_{trns}$ as a function of $X_0$ as found with tracking runs of about $2e7$ turns. The apparent stability limit using $1e6$ turns and $dx_0 = .1$ mm is $u_{st} = 24.9$ mm. In Fig. 4.1 one can make out two plateaus with levels larger than $1e4$ turns. The levels of these two plateaus are at $1.7e6$ turns and $1.5e7$ turns. The end of the $1.5e7$ plateau was taken as $X_0 = 21.4$ mm. The results are also shown as a $1/\log(j_{trns})$ plot in Fig. 4.2.

The data found for these two plateaus will be used to extrapolate and find the $1e9$ plateau in this case. In the extrapolation, the results found in chapter 5 for the average plateau width and level separation will be used. In this case, this leads to the assumptions that each plateau is $2.0$ mm wide and a plateau level separation of $.033$ in $1/\log(j_{trns})$ will be used here.

The results are shown in Fig. 4.3, where two extrapolated plateaus are
Figure 4.3: $1/\log(j_{trns})$ versus $X_0$. $dp/p = 0$, $p_{x0} = 0$, $p_{y0} = 0$, $\epsilon_{x0} = 0$, $\epsilon_{y0} = 0$ direction, $dX_0 = 0.1\text{ mm}$, $X_0 = (bex0/bey0)^5 y_0$, showing extrapolated plateaus. In the figure, $1/\log(j_{trns})$, $X_0$, $dX_0$, $\epsilon_{x0}$, $\epsilon_{y0}$ represent $1/\log(j_{trns})$, $X_0$, $dX_0$, $\epsilon_{x0}$, $\epsilon_{y0}$.

shown with the plateau levels $3.3e9$, and $7.7e13$ turns. Most of the particles on the $3.3e9$ plateau will be assumed to survive $1e9$ turns, and the aperture for $1e9$ turns is then $21.4\text{ mm}$. $21.4\text{ mm}$ is to be compared with the aperture of $24.9\text{ mm}$ found with runs of $1e6$ turns. A loss of $3.5\text{ mm}$ or $17\%$. 
Chapter 5

Extrapolation parameters for the plateau model

| direction in phase space | $\epsilon_{y0}=\epsilon_{x0}$ | $\epsilon_{y0}=0$ | $\epsilon_{x0}=0$ |
|--------------------------|------------------|-----------------|-----------------|
| plateau level, $j_{trns}$ | 15e6, 1.5e5, 2e4 | 8e6, 1.5e6, 1e4 | 1.8e7, 1.7e6, 4e3 |
| plateau level, $1/\log(j_{trns})$ | .139, .193, .232 | .145, .162 | .138, .160, .277 |
| plateau level separation in $1/\log(j_{trns})$ | .054, .039 | .017 | .022 |
| average plateau separation in $1/\log(j_{trns})$ | .033 | | |
| plateau width, $\Delta X_0$ (mm) | 1.70, 1.83, 2.38 | 2.0, 1.8 | 2.2+, 2.0 |
| plateau width, $\Delta X_0/X_0$ | .075, .051, .106 | .095, .086 | .092, .083 |
| average plateau width, $\Delta X_0$ | 2.00 mm | | |

Table 5.1: Plateau parameters for three directions in phase space. $X_0 = (\beta_{x0}(\epsilon_{x0} + \epsilon_{y0}))^5$.

The extrapolation depends on two parameters, the width of the plateaus and the separation between consecutive plateau levels as measured in terms of $1/\log(j_{trns})$. The behaviour of these two parameters was studied by doing tracking runs for 3 different cases, corresponding to the three different directions in phase space, $\epsilon_{y0}=0$, $\epsilon_{x0}=\epsilon_{y0}$ and $\epsilon_{x0}=0$. The two parameters were measured for these 3 cases using runs of about 2e7 turns. The results are summarized in Table 5.1. Altogether, 7 plateaus were found and the two parameters for these 7 plateaus were measured. In Table 5.1 one sees that the plateau width, as measured as $\Delta X_0$, $X_0 = (\beta_{x0}(\epsilon_{x0} + \epsilon_{y0}))^5$, is relatively
constant with an average value of $\Delta X_0 = 2.00$ mm. The plateau level separation when measured in $1/\log(j_{trns})$ varies considerably with an average value of .033.

Based on the above results, it is proposed that in extrapolating the survival function, it is assumed that the extrapolated plateau widths are given by $\Delta X_0 = 2.00$ mm, and the plateau level separations in $1/\log(j_{trns})$ are .033.
Chapter 6
Conclusions

Tracking studies lead to a model of the survival function, which pictures it as a sequence of plateaus. Within the plateaus, the survival time in turns, $j_{trns}$, oscillates about some constant value of $j_{trns}$ which will be called the level of the plateau. Studying the survival function along different directions in phase space, using an older version of the RHIC lattice, one finds that the width of the plateaus, $\Delta X_0$, $X_0 = (\beta_{x0}(\epsilon_{x0} + \epsilon_{y0}))^{1/5}$, remains roughly constant at about 2.00 mm. The separation between the levels of adjacent plateaus has the same order of magnitude when measured in terms of the change in $1/\log(j_{trns})$, and has an average value of .033. Using these results for the width of the plateaus and the separation between plateau levels, one can extrapolate to estimate the location of the plateaus that correspond to longer survival times than can be found by tracking. For the case treated, it was found that a required survival time of $1e9$ turns reduced the aperture by about 15% as compared to the aperture found by tracking using $1e6$ turns.

The plateau model also leads to new criteria to be used in tracking studies to find the aperture for particles to survive a given number of turns. In the plateau model, one finds the first plateau whose level is higher than the given number of turns, in order to find the aperture for the given number of turns. This is to be compared with often used methods, where one does a search starting at large amplitudes until one finds an amplitude that survives the given number of turns. In the latter method one cannot be sure that a finer search would not find unstable runs at smaller amplitudes or how frequently these unstable runs will occur. In the plateau model, while there may be unstable runs at smaller amplitudes, there is the assumption that they will not occur frequently.
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