Photon antibunching in a cavity-QED system with two Rydberg–Rydberg interaction atoms

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Abstract. We propose how to achieve strong photon antibunching effect in a cavity-QED system coupled with two Rydberg–Rydberg interaction atoms. Via calculating the equal time second-order correlation function $g^{(2)}(0)$, we find that the unconventional photon blockade and the conventional photon blockade appear in the atom-driven scheme, and they are both significantly affected by the Rydberg–Rydberg interaction. We also find that under appropriate parameters, one obtains the extremely strong photon antibunching by combining the conventional photon blockade and the unconventional photon blockade, and the mean photon number in the cavity can be improved significantly. In the cavity-driven scheme, the existence of the Rydberg–Rydberg interaction severely destroys the photon antibunching under the unconventional photon blockade mechanism. These results will help to guide the implementation of the single photon emitter in the Rydberg atoms-cavity system.

1 Introduction

Photon antibunching is a quantum mechanic effect in which emitted photons tend to be detected one by one [1–5]. This important effect can be used to implement single-photon emitter, which has important applications in quantum information processing [6,7] and quantum computing [8,9]. There are two methods to achieve strong photon antibunching in the cavity-QED system. One is called the conventional photon blockade (PB): A strongly coupled system between an optical cavity and another nonlinear degree of freedom possesses an anharmonic ladder in the energy spectrum. Thus, it can produce the photon antibunching when the single-photon nonlinearity is larger than the mode linewidth [10]. In the past few decades, a sequence of experimental and theoretical works related to the PB have been investigated in various systems, such as cavity-atom systems [11,12], superconducting qubit systems [13,14], optomechanical systems [15–17], etc.

In 2010, Liew and Savona found another mechanism called the unconventional photon blockade (UPB) originated from quantum interference which leads the probability amplitude of the two photon state to zero [18]. The UPB usually requires to add additional degrees of freedom, such as auxiliary cavities [19–21] or two-level atoms [22,23] into the system in order to establish multiple transition pathways to realize the destructive interference. In contrast with the PB, this interference-based photon blockade mechanism can achieve strong photon antibunching only requiring weak nonlinearity [19,24–26]. Due to this significant feature, a large number of works related to the UPB have been reported extensively in various system, such as single mode cavity with second-order nonlinearity [27], single mode cavity with Kerr-type nonlinearity [18,28,29], two-level emitter-cavity system [23], single mode cavity including a degenerate optical parametric amplifier [30], coupled optomechanical system [31,32] and Gaussian squeezed states [33].

Rydberg atoms with large principal quantum numbers have become an important tool of quantum information processing [34,35]. Rydberg blockade appears in the Rydberg atoms ensemble because of the strong Rydberg–Rydberg interaction, where Rydberg excitation of one atom prevents the excitation of atoms within the blockade radius. This phenomenon can be used to induce strong optical nonlinearity [36] which generates nonclassical states of light in the Rydberg atoms ensemble and the Rydberg atoms-cavity system [37–39]. In addition, photon antibunching induced by quantum interference can also be obtained in two-level atoms-cavity system [40,41]. Thus, one expects to obtain strong photon antibunching in two-level Rydberg atoms-cavity system by combining the above two effects.

This paper is organized as follows. In Sect. 2, we illustrate the theoretical model and Hamiltonian of the sys-
atom frequency detuning and \( \varepsilon \) represents the pump laser amplitude.

Defining the collective operators \([43]\)

\[
D_\pm = \frac{1}{\sqrt{2}}(\sigma^1_{rg} \pm \sigma^2_{rg}).
\]

and the atom–cavity interaction part of the Hamiltonian in Eq. \((2.1)\) can be reformulated as

\[
\begin{align*}
\hat{H}_I &= \hat{H}_+ + \hat{H}_- \quad (2.3) \\
\hat{H}_\pm &= \frac{g}{\sqrt{2}} (\cos(\kappa x_1) \pm \cos(\kappa x_2))(a D_\pm + a^\dagger D_\pm). \\
\end{align*}
\]

where \( \hat{H}_+ \) represents the coupling of the symmetric state and the cavity mode leading to the transitions \( |gg, n + 2\rangle \leftrightarrow |+, n + 1\rangle \leftrightarrow |rr, n\rangle \). \( \hat{H}_- \) represents the coupling of the antisymmetric state and the cavity mode, which gives rise to the transitions \( |gg, n + 2\rangle \leftrightarrow |-, n + 1\rangle \leftrightarrow |rr, n\rangle \). If we choose the distance between two Rydberg atoms as \( d = x_2 - x_1 = n\lambda \) \( (n = 0, 1, 2, \ldots) \), \( \hat{H}_- \) vanishes and the cavity mode only couples via \( |+\rangle \).

Thus, the Hamiltonian of the system can be rewritten as

\[
\begin{align*}
\hat{H}_r &= - \sum_{j=1,2} \Delta_j a^\dagger a + g \sum_{j=1,2} (\sigma^1_{rg} a + \sigma^1_{gr} a^\dagger) \\
& \quad + V \sigma^2_{rr} + \hat{H}_d. \\
\end{align*}
\]

In the following, we will discuss the photon antibunching of the system with atom-driven and cavity-driven scheme, respectively.

### 3 Results and discussion

#### 3.1 Atom-driven scheme

First, in order to get instructive understanding of the photon antibunching, one seeks to analytical description of the equal time second-order correlation function \( g^{(2)}(0) \), which measures the photon statistic distribution of the cavity mode. Within the weak driving condition, only lower photons excitation states are occupied, then one can express the wave function of the system with truncated state as follows

\[
|\psi(t)\rangle = C_{gg0}(t)|gg0\rangle + C_{gg1}(t)|gg1\rangle + C_{gg2}(t)|gg2\rangle + C_{r+0}(t)|r+0\rangle + C_{r+1}(t)|r+1\rangle + C_{r-r0}(t)|rr\rangle.
\]

The coefficient \( C_{aan}(t) \) represents the probability amplitude of the state \( |aan\rangle = |aa\rangle \otimes |n\rangle \), where \( |aa\rangle (aa = gg, rr, +) \) is the collective states of the two coupled Rydberg atoms and \( |n\rangle (n = 0, 1, 2) \) is the Fock basis for the
cavity mode. The value $|C_{aaaa}(t)|^2$ denotes the probability of the corresponding state. In order to obtain the value of the probability amplitude, we need to solve the time-dependent Schrödinger equation $i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle$, where $\hat{H}$ is the effective non-Hermitian Hamiltonian

$$\tilde{H} = \hat{H}_r - \frac{i\kappa a^\dagger a}{2} - \frac{i\gamma}{2}(\sigma_x^1 \sigma_x + \sigma_z^1 \sigma_z). \quad (3.2)$$

$\kappa$ and $\gamma$ are loss rates of the cavity and the atoms, respectively. $\sigma_j(\sigma_j^\dagger)$ denotes the lowering (raising) operator of the $j$th atom. Substituting Eqs. (3.1) and (3.2) into the Schrödinger equation, one can obtain a set of equations of the time-dependent coefficients ($\hbar = 1$)

$$
\begin{align*}
&i\hat{C}_{gg1} = \sqrt{2}gC_{gg1} + \sqrt{2}\varepsilon C_{gg1} - \Delta _C C_{gg1} - i\kappa C_{gg1}, \\
&i\hat{C}_{gg2} = 2gC_{gg2} - 2\Delta _e C_{gg2} + i\kappa C_{gg2}, \\
&i\hat{C}_{rr0} = -2\Delta _a + V - i\gamma \hat{C}_{rr0} + \sqrt{2}gC_{rr0} + \sqrt{2}\varepsilon C_{gg1}, \\
&i\hat{C}_{r0} = -\Delta _e C_{r0} + \sqrt{2}gC_{gg1} + \sqrt{2}\varepsilon (C_{gg0} + C_{rr0}) - \frac{i\gamma}{2} C_{r0}, \\
&i\hat{C}_{a} = -\Delta _a C_{a} + \frac{1}{2} (\Delta _a + \Delta _c) C_{a} + \sqrt{2}gC_{rr0} + 2gC_{gg2} \\
&+ \sqrt{2}\varepsilon C_{gg1} - \frac{i(\gamma + \kappa)}{2} C_{r1}.
\end{align*}
$$

Under the weak driving condition, given that $\{C_{gg0}\} \gg \{C_{gg1}, C_{gg2}\} \gg \{C_{gg2}, C_{gg1}, C_{rr0}\}$ and set $C_{gg0} = 1$, the steady-state solution can be obtained as follows

$$
\begin{align*}
C_{gg1} &= \frac{8g\varepsilon}{N}, \\
C_{gg2} &= \frac{16\sqrt{2}\varepsilon^2 [k + 2\gamma _a - 4\Delta _a - 2i\Delta _e + 2iV]}{MN},
\end{align*}
$$

where $M = 8g^2 - 2i\kappa \Delta _a - 2i\gamma _a - 4\Delta _a \Delta _e + \kappa \gamma _e$. $N = 4g^2 k - 16ig^2 \Delta _a + 2i\kappa \Delta _a - 4\kappa^2 \gamma _a - 8ig^2 \Delta _e + 8i\kappa \Delta _e + 8\kappa \Delta _a \gamma _e - 8\kappa \Delta _e \gamma _a - \kappa^2 \gamma _e + 2i\Delta _a \gamma _e + 8ig^2 V - ik^2 V + 2k\Delta _a V - 4i\Delta _a \Delta _e V - 4i\Delta _e \gamma _e V - i\kappa \gamma _e V + 2i\Delta _a \gamma _e V$.

Under the condition of the large laser-atom (laser-cavity) frequency detuning limit: $\Delta_a, \Delta_e \gg \gamma, \kappa$, an analytical expression of the equal time second-order correlation function can be represented as

$$g^{(2)}(0) = \frac{<a^\dagger a^\dagger a a>}{<a^\dagger a>^2} \approx \frac{2|C_{gg1}|^2}{|C_{gg2}|^2} \approx \frac{16(2g^2 - \Delta _a \Delta _e)^2(2\Delta _a + \Delta _e - V)^2}{N^2}. \quad (3.5)$$

which quantifies the joint probability of detecting two photons at the same time. In the limit of $g^2(0) \to 0$, one can obtain the optimal conditions of the photon antibunching

$$
\begin{align*}
\Delta_a &= \frac{1}{2}(V - \Delta _c), \\
\Delta_e &= \frac{2g^2}{\Delta_c}.
\end{align*}
$$

Fig. 2 Transition pathways with atom-driven scheme

The first expression in Eq. (3.6) is an optimal condition for the UPB induced by quantum interference, which can be obtained when the probability amplitude $C_{gg2} = 0$. As shown in Fig. 2, Rydberg state $|rr0\rangle$ is shifted by the Rydberg coupling strength $V$, and we find that UPB optimal condition is different from the non-interacting case [40]. One can see that it's relevant to the laser-atom frequency detuning, laser-cavity frequency detuning, and the Rydberg coupling strength. As shown in Fig. 2, two transition pathways $|gg0\rangle \sqrt{2\varepsilon} [|+0\rangle \sqrt{2g} |gg1\rangle \sqrt{2\varepsilon} [|+1\rangle \sqrt{2g} |gg2\rangle$ and $|gg0\rangle \sqrt{2\varepsilon} [|+0\rangle \sqrt{2g} |rr0\rangle \sqrt{2g} [|+1\rangle \sqrt{2g} |gg2\rangle$ are indistinguishable. Thus, destructive interference arises between these two pathways and one can expect to get a strong UPB effect [42]. The second formula in Eq. (3.6) is the optimal condition of the PB. As shown in previous studies [40], one can obtain a PB effect when the cavity-atom coupling strength $g$ is large enough.

Next, we will numerically calculate the equal time second-order correlation function $g^{(2)}(0)$ by solving the master equation of the system

$$
\frac{1}{\hbar} \frac{\partial\hat{\rho}}{\partial t} = [\hat{H}, \hat{\rho}] - i \sum_{j=1}^{2} \frac{\gamma_j}{2} \hat{D}[\sigma_j] \hat{\rho} - i\kappa \hat{D}[a] \hat{\rho}. \quad (3.7)
$$

where $\hat{D}[\sigma_j] \hat{\rho} = 2g \rho q^j - q^j \rho q - \rho q^j q$ denotes the Liouvillian superoperator that represents the loss of the Rydberg atoms (cavity), the collective damping of the Rydberg atoms is not considered for simplicity. The steady-state solution of the equal time second-order correlation function $g^{(2)}(0)$ is expressed as

$$
g^{(2)}(0) = \frac{Tr[\hat{\rho} a^\dagger a a^\dagger a]}{(Tr[\hat{\rho} a a^\dagger a])^2}. \quad (3.8)
$$
In the experiment, we consider two $^{87}\text{Rb}$ atoms placed in a plane cavity to produce nonclassical light. Such a single photon emitter can be realized by implementing a two-dimensional optical lattice in a cavity [44], and the two-dimensional optical lattice consists of a red-detuned laser beam perpendicular to the cavity axis and a blue-detuned laser beam parallel to the cavity axis. Rydberg atoms are distributed at several lattice sites, so they have well-defined positions. Moreover, excess Rydberg atoms in the optical lattice can be removed with a resonant push-out beam. The experimentally achievable parameters of the distance between two $^{87}\text{Rb}$ atoms can be adjusted from 15\,\mu m to 4\,\mu m [45]. For the Rydberg states $|r\rangle=|62D^2_2\rangle$, the van der Waals interaction coefficient is $C_9=730\,\text{GHz}\cdot\mu m^6$, and the Rydberg coupling strength $V$ can be tuned continuously between $2\pi\times(0.01, 28.33)\,\text{MHz}$ [45]. In addition, for principal quantum number $n \sim 60$, the $^{87}\text{Rb}$ atoms $|nD^2_2\rangle$ state has a lifetime of 203.80\,\mu s (the spontaneous decay rate is $\gamma = 2\pi \times 0.4\,\text{KHz}$ [46,47]). The pump laser amplitude $\varepsilon$ can be adjusted from $2\pi \times 500\,\text{KHz}$ to $2\pi \times 5\,\text{MHz}$. Moreover, the cavity QED experiments with $^{87}\text{Rb}$ atoms provide us other parameters: $(g, \kappa) = 2\pi \times (7.8, 2.5)\,\text{MHz}$ [44]. Next, we numerically calculate the photon antibunching by using the above experimental parameters.

In order to show the dependence of the $g^{(2)}(0)$ on the Rydberg coupling strength $V$, we plot the equal time second-order correlation function $g^{(2)}(0)$ (in logarithmic units) as a function of $\Delta_a/\kappa$ and $\Delta_c/\kappa$ with the Rydberg coupling strength $V = 2\kappa$ and $10\kappa$. The atom-driven strength $\varepsilon$ is assigned to 0.4$\kappa$ for weak driving condition. From Fig. 3a and b, one can see that the UPB effect appears and the position of the UPB can be changed by the Rydberg coupling strength $V$. We also find that the PB effect becomes significant in the case of the large cavity–atom coupling strength $g$. Analytical solutions for UPB (orange dashed line) and PB (black dashed line) are shown in Fig. 3a and b, and we observe that these numerical results are consistent with the analytical solution for the optimal conditions in Eq. (3.6).

To further clarify the effect of the Rydberg coupling strength $V$ on the UPB, we plot the equal time second-order correlation function $g^{(2)}(0)$ and the mean photon number $\langle a^\dagger a \rangle$ as a function of $\Delta_a/\kappa$ and $V/\kappa$ with $\Delta_a = -\frac{1}{2}(\Delta_c - V)$ in Figs. 4a and b. In Fig. 4a, one can see that UPB effect is greatly enhanced by the laser-cavity frequency detuning $\Delta_c$. In Fig. 2, two transition pathways are symmetric and indistinguishable when $\Delta_c = \Delta_a$ and Rydberg coupling strength $V = 0$. Thus, constructive interference presents and two photon state will be excited. Whereas, two transition pathways in Fig. 2 are distinct when $\Delta_c \neq \Delta_a$, and two photon state are inhibited because of the destructive interference [40]. To characterize the effect of detuning on photon statistical distribution more clearly, we substitute the UPB optimal conditions into the probability amplitude of two photon state and obtain

$$C_{gg2} \approx \frac{\sqrt{2\varepsilon^2(\kappa + 2\gamma)}}{\Delta_c^2(\frac{1}{\Delta_c^2} - \frac{\gamma}{\gamma_V})^2 + \frac{1}{4\gamma}}\frac{1}{(\Delta_c + 1)^2}.$$  \hspace{1cm} (3.9)

We find that probability of two photon state $|C_{gg2}|^2 \propto \Delta_c^{-10}$. That is, with the increase of $|\Delta_c|$, the two-photon state probability decreases rapidly, and this result is consistent with the numerical results obtained above.

Under the condition of the large laser-cavity frequency detuning limit, there is also a strong UPB effect at a large range of values of $V$. Contrary to the second-order correlation function $g^{(2)}(0)$, the mean photon number $\langle a^\dagger a \rangle$ in the cavity is suppressed by the laser-cavity fre-
the Rydberg coupling strength. On the contrary, in the condition of laser-cavity frequency detuning, and its change with effect is. In addition, the mean photon number in the function
\[ g / \kappa \]
is opposite to the equal time second-order correlation function
\[ \gamma \]
Other parameters take as
\[ \Delta_c / \kappa \]
and mean photon number
\[ \langle a \dagger a \rangle \]
respectively. In Fig. 5a, one can see that in the condition of single photons under strong photon antibunching conditions caused by quantum interference.

To give a clearer picture of the effects of \( V \) on UPB and mean photon number \( \langle a \dagger a \rangle \), we plot the equal time second-order correlation function \( g^{(2)}(0) \) and the mean photon number \( \langle a \dagger a \rangle \) as a function of \( V / \kappa \) with \( \Delta_a = - \frac{1}{2} (\Delta_c - V) \) in Fig. 5a and b. Here, the laser-cavity frequency detuning \( \Delta_c \) is fixed at \(-40 \kappa, -20 \kappa, 20 \kappa, 40 \kappa\), respectively. In Fig. 5a, one can see that in the condition of the negative laser-cavity frequency detuning, the smaller the Rydberg coupling strength \( V \), the stronger the UPB effect is. On the contrary, in the condition of the positive laser-cavity frequency detuning, the larger the Rydberg coupling strength \( V \), the stronger the UPB effect is. In addition, the mean photon number in the cavity is small enough under both negative and positive laser-cavity frequency detuning, and its change with \( V \) is opposite to the equal time second-order correlation function \( g^{(2)}(0) \).

Next, we will investigate the effect of the Rydberg coupling strength \( V \) and the cavity–atom coupling strength \( g \) on the PB under the optimal condition \( \Delta_a = \frac{2g^2}{\Delta_c} \). In Fig. 6a, we plot the equal time second-order correlation function \( g^{(2)}(0) \) as a function of \( \Delta_c / \kappa \) and \( V / \kappa \). It’s clear that strong PB effect can be obtained in the positive laser-cavity frequency detuning interval when the Rydberg coupling strength \( V \) is large enough. Figure 6b displays the equal time second-order correlation function \( g^{(2)}(0) \) as a function of \( \Delta_c / \kappa \) and \( V / \kappa \). It’s clear that strong PB effect can be obtained with the increase of coupling strength \( g \) at the large laser-cavity frequency detuning limit. The physical mechanism of the PB is energy-level anharmonicity of the system. We analytically calculate the eigenvalue of \( H \) in the single-photon space with the driving term omitted, and we have analytical solution of eigenvalues
\[ E^{(1)}_{\pm} = \frac{1}{2} (\omega_a + \omega_c \pm \sqrt{8g^2 + (\omega_a - \omega_c)^2}) \]

When laser frequency \( \omega_L = E^{(1)}_{\pm} \), one obtains PB optimal condition \( \Delta_a = \frac{g^2}{\Delta_c} \). In addition, if the Rydberg coupling strength \( V \) and the cavity–atom coupling strength \( g \) are large enough, large energy level mismatch between the laser drive frequency and the second excited state can be obtained. Thus, strong photon antibunching result from the energy-level anharmonicity occurs in the system.

The above analyses in Figs. 5 and 6 are based on the separate consideration of the UPB and the PB effect. Thus, we can only get a small mean photon number \( \langle a \dagger a \rangle \) when the equal time second-order correlation function \( g^{(2)}(0) \) is extremely small. It’s an adverse result for single photons detection. Here, we consider whether we can obtain stronger photon antibunching and larger mean photon number in the cavity by combining the UPB effect and the PB effect. In Fig. 7a and b, we plot the equal time second-order correlation function \( g^{(2)}(0) \) and the mean photon number \( \langle a \dagger a \rangle \) as a function of \( \Delta_c / \kappa \) and \( V / \kappa \) with \( \Delta_a = - \frac{1}{2} (\Delta_c - V) \). As shown in Fig. 7a, the value of the equal time second-order correlation function \( g^{(2)}(0) \) at the minimum is about \( 10^{-5.5} \). As assumed above, a stronger photon antibunching based on the UPB and the PB mechanism appears when the parameters satisfy \( V > 4g \). The white dashed line \( (V = \Delta_c + \frac{g^2}{\Delta_c}) \) in Fig. 7a corresponds to the intersection of the UPB optimal condition and the PB optimal condition. It analytically marks the minimum value of the equal time second-order correlation function \( g^{(2)}(0) \) in the parameter space. Thus, in
Fig. 5 The Rydberg coupling strength \( V \) has an opposite effect on \( g^{(2)}(0) \) \((\langle a^\dagger a \rangle)\) when the laser-cavity frequency detuning signs are opposite. (a) Plot of \( g^{(2)}(0) \) as a function of \( V/\kappa \) with \( \Delta_a = -\frac{1}{2}(\Delta_c - V) \) and \( \Delta_c = -40\kappa, -20\kappa, 20\kappa, 40\kappa \). (b) Plot of \( \langle a^\dagger a \rangle \) as a function of \( V/\kappa \) with \( \Delta_a = -\frac{1}{2}(\Delta_c - V) \) and \( \Delta_c = -40\kappa, -20\kappa, 20\kappa, 40\kappa \). Other parameters take as \( \gamma = 2\pi \times 0.4KHz \), \( \kappa = 2\pi \times 2.5MHz \), \( \varepsilon = 0.4\kappa \) and \( g = 2\pi \times 7.8MHz \).

Fig. 6 The Rydberg coupling strength \( V \) and the cavity–atom coupling strength \( g \) enhance PB effect. (a) Plot of \( \log_{10}[g^{(2)}(0)] \) as a function of \( \Delta_c/\kappa \) and \( V/\kappa \) with \( \Delta_a = \frac{2g^2}{\Delta_c} \), the atom–cavity coupling strength is given by \( g = 2\pi \times 7.8MHz \). (b) Plot of \( \log_{10}[g^{(2)}(0)] \) as a function of \( \Delta_c/\kappa \) and \( g/\kappa \) with \( \Delta_a = \frac{2g^2}{\Delta_c} \), the Rydberg coupling strength is given by \( V = \kappa \). Other parameters take as \( \gamma = 2\pi \times 0.4KHz \), \( \kappa = 2\pi \times 2.5MHz \), \( \varepsilon = 0.4\kappa \).

Finally, in order to fully characterize the quantum signatures of the single photon emitter, we need to consider the delayed second-order correlation function in the steady state

\[
g^{(2)}(\tau) = \frac{Tr[\hat{\rho}a^\dagger(0)a^\dagger(\tau)a(\tau)a(0)]}{(Tr[\hat{\rho}a^\dagger(0)a(0)])^2}. \tag{3.11}
\]

In Fig. 8a and b, we plot the delayed second-order correlation function \( g^{(2)}(\tau) \) for the UPB and the PB optimal conditions. One can observe that at \( \tau = 0 \),
Extremely strong photon antibunching and large mean photon number can be obtained by combining the UPB effect and the PB effect. 

\[ \log_{10}[g^{(2)}(0)] \]

as a function of \( \Delta_c/\kappa \) and \( V/\kappa \) with \( \Delta_a = -\frac{1}{2}(\Delta_c - V) \). 

Other parameters take as \( \gamma = 2\pi \times 0.4 KHz \), \( \kappa = 2\pi \times 2.5 MHz \), \( \varepsilon = 0.4\kappa \) and \( g = 2\pi \times 7.8 MHz \). The white dashed line is depicted by the function \( V = \Delta_c + \frac{4g^2}{\Delta_c} \).

The delayed second-order correlation function \( g^{(2)}(\tau) > g^{(2)}(0) \), and the photon antibunching appears. 

\[ g^{(2)}(\tau) > g^{(2)}(0) \]

\( \varepsilon = 0.2\kappa \), \( \varepsilon = 0.4\kappa \). Other parameters are set as \( \gamma = 2\pi \times 0.4 KHz \), \( \kappa = 2\pi \times 2.5 MHz \) for different pump laser amplitude.

3.2 Cavity-driven scheme

Here, we consider the cavity-driven scheme with \( \Delta_a = \Delta_c = 0 \), i.e., detunings of the two level Rydberg atoms transition frequency and the cavity resonant frequency from the driving laser vanish. Analytic expression for the time dependent coefficient can be obtained by solving the Schrödinger equations:
The UPB effect occurs in the cavity-driven scheme. In Fig. 9, two excitation pathways are shown in Fig. 10a. Where the equal time second-order correlation function \( g^{(2)}(0) \) as a function of the cavity–atom coupling strength \( g \) is plotted for different Rydberg coupling strength. Here, Eq. (3.14) is the optimal condition of the UPB induced by the destructive interference, which means that in order to get a strong photon antibunching effect, the strength of the Rydberg–Rydberg interaction must always be zero. That is, a finite Rydberg coupling strength \( V \) could weaken the photon antibunching.

In what follows, we will numerically study the UPB effect and compare with the analytic results, which are shown in Fig. 10a. Where the equal time second-order correlation function \( g^{(2)}(0) \) as a function of the cavity–atom coupling strength \( g/\kappa \) and the cavity–atom coupling strength \( \epsilon/\kappa \) are given by \( 0, 0 \) and \( 0, 0 \), respectively. Other parameters are given by \( \kappa, V \), and \( \epsilon, \gamma \), respectively. Other parameters are given by \( V = 0.1\kappa \) and \( \gamma = \kappa \).

The steady-state solutions of Eq. (3.12) can be obtained as follows:

\[
C_{gg2} = \frac{2\sqrt{2}\varepsilon(4\varepsilon^2 + \gamma(\gamma + \kappa))(\gamma + iV) - 4g^2(\gamma + 2iV)}{N},
\]

(3.13)

where \( N = (4\varepsilon^2 + \kappa^2)(4\varepsilon^2 + \gamma(\gamma + \kappa))(\gamma + iV) + 32g^4(\kappa + 2\gamma + 2iV) + 4g^2(\gamma(\kappa + 3\kappa + 4\gamma - 4\varepsilon^2(3\gamma + 4iV) + 2i\kappa(\kappa + 2\gamma)\sqrt{V}). \) Within the same methods mentioned above, one gets the equations

\[
4\varepsilon^2 - 4g^2 + \gamma^2 + \gamma^2 = 0,
\]

(3.14)

\[
V = 0.
\]

Here, Eq. (3.14) is the optimal condition of the UPB induced by the destructive interference, which means that in order to get a strong photon antibunching effect, the strength of the Rydberg–Rydberg interaction must always be zero. That is, a finite Rydberg coupling strength \( V \) could weaken the photon antibunching.

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ity, and hence of single photons number emitted from the cavity.

4 Conclusions

In summary, we have investigated the photon antibunching in a Rydberg atoms-cavity system in the atom-driven and cavity-driven scheme, respectively. By solving the Schrödinger equation and the Lindblad master equation, it can be found that in the case of the atom-driven, one can obtain a strong UPB effect when the laser-cavity frequency detuning is large enough. In the negative laser-cavity frequency detuning interval, the increase of Rydberg coupling strength weakens the UPB effect. On the contrary, the UPB effect can be enhanced with the increase of the Rydberg coupling strength in the positive laser-cavity frequency detuning condition. In addition, Rydberg–Rydberg interaction can also result in strong PB effect in the positive laser-cavity frequency detuning region. It’s worth noting that within the range of the appropriate parameters, while we get the extremely strong photon antibunching effect, the mean photon number in the cavity has also been improved significantly. In the cavity-driven case, the increasing interatomic Rydberg–Rydberg interaction strength could weaken the photon antibunching rapidly. Our study may provide guidance for the experimental construction of single-photon sources in the Rydberg atoms-cavity system.

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Author contributions

All authors contributed equally to the paper.

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