Factors affecting students’ capabilities in analyzing by using flow proof in real analysis lectures

H Helma¹, D Murni¹

¹Mathematics Department, FMIPA, Universitas Negeri Padang, Indonesia

Abstract. Real analysis is required for mathematicians. The competency is that students are able to analyze and think critically in solving a problem, and write their solutions in a logical statement. Students must have the definitions and theorems to solve the problems and perform a process which is often called the preliminary analysis. After that, students construct the proof based on the preliminary analysis. Based on the characteristics of the problem in real analysis, the solution given to the problem is using flow proof in composing preliminary analysis. Before the learning materials are prepared, it is important to know the characteristics of learners in following the learning of mathematics that affect the learning outcomes. There are five factors that influence the learning outcomes, namely background, interest, attitude, motivation and learning styles. The purpose of research is to determine the factors that influence learning outcomes. This research is descriptive. The instruments are assessment sheets for students’ characteristics and test. Results are the factors that influence the learning outcomes of students directly are background and learning style.

1. Introduction

Real analysis is one of the courses needed for prospective undergraduate mathematics and mathematics education. Expected competency is that students are able to analyze and think critically in solving a problem, and write their solutions in a logical statement [1]. Students can interpret reasoning and communicate their understanding, can make connections between definitions / theorems and use them to solve mathematical problems. The material of real analysis contains about definitions and theorems that must be proven by analysis. The example of the problem is

Let S be a nonempty bounded set in R.
Let a < 0, and aS = { ax : x ∈ S }.
Prove that sup ( aS ) = a inf S.

To solve this problem, students must understand the definition of supremum/infimum and perform a process of thinking analysis which is often referred to as a preliminary analysis. After a preliminary analysis, students construct evidence based on preliminary analysis.

Students often have difficulties when conducting preliminary analysis. The difficulty is examining the proof of a problem, structuring the mindset logically in doing evidentiary reasoning, and seeing the relationship between definitions, theorems, and sequential hierarchies [2]. Student difficulties in carrying out preliminary analysis will affect the ability of students to construct a proof which is the answer to the problems given. So, if given a problem of proof, students solve based on memorization.
Students are constrained in giving reasons for each statement given. They understand the procedure of evidence, but do not understand the steps of proof. As a result, they experience difficulties when starting the settlement of a proof, namely linking the statement given with logical reasons for the statement.

Proof of a problem presented in the books of real analysis does not provide an explanation of the reasoning process. Students find it difficult when reading the books. These results in less meaningful learning being carried out [3]. So, the interest and student learning outcomes in studying real analysis becomes low. Based on the problems found in the real analysis lecture, the solution that can be used is the use of flow proof in conducting a preliminary analysis. This is able to overcome the problem on the grounds that using flow proof will help students to train critical thinking that is used in the deductive process and structuring the mindset logically [4].

Many factors influence the ability of students to analyze evidence. These factors are background, interests, learning attitudes, learning motivation, and student learning styles. But, do these five factors affect the ability of students to analyze evidence using significant flow proof? This needs to be known so that meaningful learning occurs. Factors that influence the ability of students to analyze evidence using flow proof can be examined using regression analysis. Regression models are used [5], namely

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_k x_k + \epsilon$$

where $y$ is response variable, $\beta_0$ is intercept, $x_i$ is regressor variable, and $\epsilon$ is error. Estimate for $\beta$ is $b = (x'x)^{-1}(x'\ y)$, where

$$y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{pmatrix}, \quad x = \begin{pmatrix} 1 & x_{11} & x_{12} & \cdots & x_{14} \\ 1 & x_{21} & x_{22} & \cdots & x_{24} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{pmatrix}, \quad \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{pmatrix}, \quad \epsilon = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_k \end{pmatrix}$$

To know if there is a linear relationship between the variables $y$ with any variable $x_1, x_2, \ldots, x_k$, the test is used

$$F_o = \frac{SSR/k}{SSE/(n-k-1)}$$

In this case,

$$SSR = b'x'y - \left( \frac{\sum_{i=1}^{n} y_i}{n} \right)^2 / n$$

$$SSE = y'y - b'x'y$$

with $H_0: \beta_1 = \beta_2 = \ldots = \beta_k = 0$.

If the value of $F_o$ is greater than the value of $F$ table, then $H_o$ is rejected [5]. To determine whether the $x_i$ variable contributes to the $y$ variable, the test is used

$$t_o = \frac{b_j}{s(b_j)}$$

with $H_0: \beta_j = 0$. If value $|t_o|$ is greater than the value of $t$ table, then $H_o$ is rejected [6]. The coefficient determination is $R^2 = SSR/SST$, where $SST = y'y - \left( \frac{\sum_{i=1}^{n} y_i}{n} \right)^2 / n$.

Use of combination learning outcomes, background, interests, attitudes, motivations, and learning styles, determined by linear regression models that can be used. The regression model obtained is represented. So, there is a conclusion that refers to the purpose of the study. Therefore, the purpose of this study was to determine the factors that influence the ability of students to analyze evidence using flow proof in the real analysis lecture.
2. Research methods
This research uses an experiment with one shot case study design [7]. The subjects consisted of students’ Mathematics Program at Universitas Negeri Padang. The numbers was 67 people. Flow proof used is adjusted to the characteristics of the analysis in real analysis. Flow proof is made logically ordered according to thought procedures. The sequence of analysis procedures is intended to solve the problem. During the lecture process an observation was made on the flow proof design made by students to solve the problems. To achieve the stated research objectives, a data collection tool in the form of questionnaire sheets was used.

3. Results and discussions
Based on the results of the questionnaire given, student responses were obtained describing their backgrounds, interests, learning attitudes, learning motivations, and learning styles. Based on each of these factors, students’ responses to each indicator can be seen. There are four kinds of responses given, namely very agree, agree, not agree, and disagree.

The indicators in the background factor are the math scores obtained by students when they were in high school (A1), the value of the introduction to basic mathematics (A2) course, and living in an environment that encourages learning (A3). The results of the questionnaire about the background of students can be seen in Figure 1. There are 37% of students do not have a good Introduction to Basic Mathematics grades.

![Figure 1. Students’ background.](image)

In the interest factor, the indicators are to like the Real Analysis course (B1), be interested in learning the real analysis course (B2), always pay close attention to the real analysis lecture (B3), receive the real analysis lecture well (B4), and try to get involved active in lectures on real analysis (B5). The results of the questionnaire about student interest can be seen in Figure 2. There are 22% of students do not try to be actively involved in real analysis lectures, and 15% of students do not study the course well.

![Figure 2. Students’ interest.](image)
Indicators for student learning attitude factors are feeling satisfied because they can learn real analysis well (C1), can work together with friends when studying real analysis (C2), trying to be consistent using symbols and the term real analysis (C3), discipline using logic rules (C4), and confident in solving real analysis problems (C5). The results of the questionnaire about the background of students can be seen in Figure 3. There are 58% of students do not confident in solving real analysis problems.

![Figure 3. Students' learning attitude.](image)

For students' learning motivation factors, the indicators are if students do not find the answer to the assignment, they always try to find it in various ways (D1), try to succeed in doing the real analysis task (D2), have a strong drive in learning the real analysis (D3), sure real analysis gives good results for logic (D4), and always creates interesting activities in learning real analysis (D5). The results of the questionnaire about the background of students can be seen in Figure 4. There are 17% of students if they don't find the answer to the assignment, they don't try to find it in various ways, 26% of students lacked a strong urge to study real analysis, and 59% of students don't try to create interesting activities in studying real analysis.

![Figure 4. Students' learning motivation.](image)

In the learning style factor, the indicator is to understand the real analysis by seeing, reading, or writing (E1), doing the exercises (E2), listening to lecturers' explanations and discussions (E3), and prefers discussion and explanation (E4), and is mastering real analysis after doing the problem solving exercise (E5). The results of the questionnaire about student interest can be seen in Figure 5. There are 31% of students do not like to discuss and explain real analysis material.

![Figure 5. Students' learning attitude.](image)

These factors can affect the ability of students to analyze evidence using flow proof. In theory, these five factors affect learning outcomes. For this reason, the five factors are regressor variables and learning outcomes are response variables. The results of student learning can be seen in Figure 6. Based on the results of variable correlation, it can be seen that learning outcome (N) is influenced by background (A), attitude (C), motivation (D), and learning style (E). In this case interest (B) affects N indirectly. Factor interest is influenced by background, attitude, and motivation.
Based on the data that has been obtained, fit the full model of regression. The regression equation is 
\[ N = -162 + 14.5 A - 19.4 B + 6.45 C + 12.8 D + 7.89 E \]
with the determination coefficient \( R^2 = 38.5\% \). This means, the total variation of students' learning outcomes is determined by background, interests, learning attitudes, learning motivation, and student learning styles.

The relationship between interest and background, attitude, and motivation also obtained a regression equation,

\[ B = 3.02 + 0.208 A + 0.341 C + 0.328 D \]

The coefficient of determination is \( R^2 = 60.6\% \), which means that the total variation of students' interests is determined by background, attitude, and motivation. Because the assumption of homogeneity of variance is not fulfilled, that is, there is a pattern in the plot of residuals versus fits, then we transform \( N \). By using Box-Cox Transformations, we obtained transformation \( N^{1/2} \).

The regression model is

\[ N^{1/2} = -8.15 + 0.996 A - 1.15 B + 0.409 C + 0.722 D + 0.545 E \]

Variation of learning outcomes scores (\( N^{1/2} \)) 42.5% is caused by background, interests, learning attitudes, learning motivation, and student learning styles. By using stepwise method, the best model is obtained

\[ N^{1/2} = -8.13 + 0.850 A + 0.598 E \]

with a level of error 5%. In this case,

\[ A = 4.26 + 0.311 B \]
\[ E = 9.75 + 0.416 B \]
\[ B = 4.17 + 0.378 C + 0.338 D \]
4. Conclusion
The result of this research is affect the ability of students to conduct preliminary analysis using flow proof directly are background and learning style. Background and learning styles are affected by interests. Interests is affected by learning attitudes and learning motivation.

References
[1] Bartle R G and Sherbert D R 2011 Introduction to real analysis fourth edition (Singapore: John Wiley & Sons Inc)
[2] Mukhni and Helma 2008 Peningkatan kualitas perkuliahan Analisis Real I melalui implementasi Model Pembelajaran Aktif menggunakan ALPS berbasis logical mind mapping (Indonesia: FMIPA Universitas Negeri Padang)
[3] Helma and Subhan M 2011 Upaya meningkatkan kemampuan mahasiswa dalam menyelesaikan permasalahan pada perkuliahan Real Analysis I dengan penggunaan mind mapping dan dipresentasikan dalam Bahasa Inggris (Indonesia: FMIPA Universitas Negeri Padang)
[4] Helma H 2019 Penggunaan flow proof pada perkuliahan analisis real untuk meningkatkan kemampuan mahasiswa dalam menganalisis pembuktian Jurnal Eksakta Pendidikan (JEP), 3(1), 55-60. doi:10.24036/jep/vol3-iss1/326
[5] Montgomery D C, Peck E A and Vining G G 2006 Introduction to linear regression analysis. (New York: Jhon Wiley & Sons Inc)
[6] Seber G A F and Lee A J 2003 Linear regression analysis second edition (New York: Jhon Wiley & Sons Inc)
[7] Sukmadinata and Syaodih N 2006 Metode Penelitian Pendidikan (PT Remaja Rosdakarya, Bandung)
[8] Cirillo M and Herbst P G 2011 Moving Toward More Authentic Proof Practices in Geometry The Mathematics Educator 21 11-33
[9] Miyazaki M, Fujita T, Jones K, and Iwanaga Y 2017 Designing a Web-based Learning Support System for Flow-chart Proving in School Geometry Digit Exp Math Educ 3 pp. 233-256
[10] Miyazaki M, Fujita T, and Jones K 2015 Flow-chart Proofs with Open Problems as Scaffolds for Learning About Geometrical Proofs ZDM: Int. J. on Math. Edu. 47 1211-24