Towards data-free gating of heterogeneous pre-trained neural networks

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Abstract
The combination and aggregation of knowledge from multiple neural networks can be commonly seen in the form of mixtures of experts. However, such combinations are usually done using networks trained on the same tasks, with little mention of the combination of heterogeneous pre-trained networks, especially in the data-free regime. The problem of combining pre-trained models in the absence of relevant datasets is likely to become increasingly important, as machine learning continues to dominate the AI landscape, and the number of useful but specialized models explodes. This paper proposes multiple data-free methods for the combination of heterogeneous neural networks, ranging from the utilization of simple output logit statistics, to training specialized gating networks. The gating networks decide whether specific inputs belong to specific networks based on the nature of the expert activations generated. The experiments revealed that the gating networks, including the universal gating approach, constituted the most accurate approach, and therefore represent a pragmatic step towards applications with heterogeneous mixtures of experts in a data-free regime. The code for this project is hosted on github at https://github.com/cwkang1998/network-merging.

Keywords Neural networks · Mixture of experts · Gating network · Neural activations

1 Introduction
The transfer and combination of learning happens often in human learners, allowing the application of existing skills, knowledge and understanding to solve newly encountered problems, commonly under some guidance [1]. This process has inspired machine learning methodologies such as transfer learning and multi-task learning, which utilize prior knowledge to improve the learning of novel tasks. However, both methodologies concern themselves mainly with the improvement of specific tasks [2–4] leveraging additional information acquired from sources that were assumed to have some kind of similarity to their domain, and attempt to exploit this similarity to avoid negative transfer [3].

Moreover, in the vast majority of cases, the sharing or combining of knowledge is done during the training process itself, and assumes the availability of sufficient data.

We are motivated by an anticipated future, where the technology landscape is filled with an extremely large number of pre-trained models, many of which might be weakly or negligibly linked to the data sources that were used to train them. This weak linkage could be due to several reasons, no less the fact that many models might have learnt in an unsupervised manner, by freely interacting with an unconstrained environment. So, in a situation where we have several powerful heterogeneous models, but we don’t have the data sources that were instrumental to their training, and we can’t easily infer how to associate new inputs to specific models (i.e. inputs are thoroughly unknown), how do we then create effective combinations of these models? In other words, we are interested in the problem of combining heterogeneous pre-trained models in a data-free regime, where the inputs are completely unknown. It is important to tackle this problem when the agent designing the mixture of experts is a human being, but it is even more crucial when the designer is an artificial intelligence (AI) agent. Assuming that all ethical considerations and precautions have been duly considered,
how should we design an AI agent that is capable of combining heterogeneous pre-trained models in a data-free regime? An AI agent with this capability is useful since it contributes towards artificial general intelligence (AGI), and constitutes a faster approach to building systems for a broader and more diverse set of applications.

Arguably, the two most effective and well-known approaches to combining multiple models, are ensembles [5–8] and mixtures of experts [9–16]. Ensemble learning which is the application of the concept of “wisdom of the crowd”, is a common method of knowledge consolidation or combination which is widely used [17, 18]. Through the consolidation of knowledge from multiple learning machines, ensemble learning is able to improve overall performance when compared to the individual members of the ensemble [18]. Multiple different types of aggregation methods can be used to consolidate the predictions of the ensemble [19], each bringing a different benefit to address a certain problem usually related to the bias-variance decomposition [17], with the most common methods being bagging, boosting and stacking [17, 18, 20]. However, ensemble learning is more commonly used to improve performance on a single task, essentially providing a more robust prediction using the power of the combined learning agents [18, 20], each one giving a different interpretation to the same task.

The mixtures of experts (MEs) approach is more in line with our work. A traditional mixture of experts consists of a set of experts, focusing on equivalent, similar or different tasks, and an additional gating network, which is responsible for deciding which expert(s) should influence the output the most [9]. Both the gating network and all of the experts receive as input the same input pattern. The gating network generally provides a vector of gates, where each gate (a scalar) is multiplied by the output of a corresponding expert, and subsequently all of the modulated outputs are summed, in order to produce the final output. Given the age and flexibility of the approach, there are naturally many variations. Since our aim is to combine pre-trained models, each of which might be responsible for a different task, the ME approach is particularly suitable. In this context, the biggest challenge lies in how to implement and train the gating network, considering that we constrain ourselves to the data-free regime.

Traditionally the gating network is trained together with the experts, so how do we deal with the situation where the experts have already been trained? In principle one could train a gating network in order to optimize the combination of the pre-trained experts. But, what if the experts are heterogeneous (for different tasks), can be added at any time to the mixture, and we no longer have the original datasets that were used to train the experts? In this case we need the gating network to select a single expert (rather than a combination of experts) for some unknown input. Clearly, training a gating network over this mixture, is no longer feasible. What we need is a gating solution which itself is already pre-trained, allowing any number of new experts to be added, whereby given a new input, the correct expert that corresponds to that input, can automatically be selected. A solution to this problem, constitutes the main contribution of our paper. In other words, this paper proposes an approach that works towards “universal gating” for mixtures of heterogeneous experts, when the experts have already been trained, and when their data is no longer available. For simplicity, and partly because we aim to strengthen the links between Computational Neuroscience and Artificial Intelligence, we restrict our experts to being neural networks. From this point forward, whenever the term “expert” is used with respect to our work, the reader can assume that the expert is a neural network.

In our definition, the idea of “universal gating” involves: (1) the possibility of adding any number of new pre-trained heterogeneous experts, (2) one pre-trained gating network for all experts, and (3) the possibility that all experts are trained in a manner that is completely agnostic of any ME architecture or approach. In this paper, we propose two different gating approaches, one which addresses only the first point, whereby each expert has its own gating network, and the other which addresses all three points. For the first approach, we compare two main variants distinguished by how they implement the gating decision, i.e.: (1) via simple logic applied to different statistics over the output logits of the experts, (2) via a gating neural network trained to make a Boolean decision based on expert network activations (including activations of hidden nodes). Since the second variant proved to be significantly more accurate, we used it exclusively in the second approach. The main contributions of this paper include: (1) a novel partially-universal gating approach for mixtures of pre-trained experts (one gating network per expert), (2) a novel universal gating approach for mixtures of pre-trained experts (one gating network for all experts), and (3) a systematic comparison of different gating variants on multi-task problems constructed from well known datasets.

The next section provides an overview of the work that is most closely related to our proposed approach. The section after that describes the technical details of our approach and our experimental design, and is followed by the results section. In the final sections we discuss our interpretation of the results, and then conclude the paper.

2 Related work

One line of research that is related to our motivation pertains to Hybrid Expert Systems (HES) [21]. In the broadest sense, this field encompasses approaches that combine different
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As far back as 1991 [31], a single expert was also selected (or a dataset from the same distribution), lets refer to it as $E_j$, can we find $E_j$ by looking at a subset of the activation patterns generated by all experts $E_i \in E$ when processing $p$? Consider all relevant activation patterns as:

$$ A^* = \{ A(E_i(p)) | E_i \in E \} $$ (1)
where \( A(E_i(p)) \) refers to the relevant activations generated by expert \( E_i \) when processing \( p \), and \( A^* \) refers to the combination of all such activations from all experts.

Given (1) the core problem underlying this work involves building an overall classifier \( C \) (here referred to as a smart coordinator), such that:

\[
C(A^*) = (j, o)
\]

where \( j \) refers to the index of the correct expert (corresponding to \( p \)), and \( o \) refers to the classification output of \( E_j \) after processing \( p \).

### 3.2 Datasets

The experiments utilize the MNIST [33] and CIFAR10 [34] datasets. Two multi-task problems were created using these datasets, namely, the disjoint MNIST problem and the MNIST+CIFAR10 problem. The disjoint MNIST problem focuses on the combination of a network (expert) trained on the first 5 classes of MNIST and a network trained on the last 5 classes of MNIST, which represents the problem of combining heterogeneous networks trained on closely-related tasks. On the other hand, the MNIST+CIFAR10 problem focuses on the combination of a network trained on MNIST and a network trained on CIFAR10, representing the combination of heterogeneous networks trained on distantly related tasks. These problems allow us to compare the effect of task similarity on each combination method, allowing us to better understand the mechanisms of the proposed combination methods.

### 3.3 Experts

All of the pre-trained expert networks adopted the LeNet-5 convolutional neural network (CNN) architecture [35]. In general, LeNet-5 performs relatively well on MNIST, but the quality of this performance does not extend to the CIFAR10 dataset, for which it exhibits average accuracy. However, as this paper focuses on the performance of the gating solution and its improvement, the architecture was deemed suitable for the experiments. In all cases the networks employed the cross entropy loss for training.

The standard LeNet-5 architecture adopted consisted of: (1) a convolutional layer with 6 features, kernel size of 5x5, stride 1, and activation function ReLU, followed by (2) a max-pooling layer with kernel size 2x2 and stride 2, followed by (3) a convolutional layer with 16 features, kernel size of 5x5, stride 1, and activation function ReLU, followed by (4) a max-pooling layer with kernel size 2x2 and stride 2, followed by (5) a convolutional layer with 120 features, kernel size of 5x5, stride 1, and activation function ReLU, followed by (6) a fully connected layer with 84 nodes and ReLU activations, and (7) a final linear fully-connected layer connecting to the output nodes.

In theory the gating approach proposed in this paper can be applied to experts with any kind of neural architecture, however in practice this depends on the nature of the data used for training a pattern attribution network (PAN). If for instance a PAN is trained on abstract features of an output vector (e.g. mean and standard deviation), then it can be applied to practically any kind of neural expert, as long as such expert has an output vector (regardless of dimensionality), which is safe to assume in most cases. However, in the opposite extreme, one might decide to train a PAN based on a specific hidden representation (e.g. all activations of the final fully connected layer of some convolutional architecture), in which case we are then restricted to neural experts that exhibit the same key architectural features.

### 3.4 Gating

With regards to gating, our general approach entails attributing a new unknown pattern to its corresponding network by observing the node activations of all experts. Our goal is to attribute pattern \( p \) to the correct expert \( e \), where the correct expert is defined as the network that was trained on data originating from the same distribution that generated \( p \). The main assumption underlying our work is that the activations of the correct network are significantly different from the activations of the incorrect networks, and that there are certain unique features regarding correct networks that can be exploited to make the attribution decision. We systematically compare two main approaches which we abbreviate to “naive concatenation” and “smart coordinator” for the sake of convenience. The former is based on statistics of expert output nodes, and comes in two variants (basic and augmented), whereas the latter consists of training another neural network to associate features of expert activations with a Boolean label which states whether a particular input pattern belongs to an expert or not. The smart coordinator (SC) has two variants, namely SC1 and SC2, where the former uses partially universal gating (i.e. one gating network per expert), whereas SC2 uses a single universal gating network for all experts.

All neural networks in the experiments (both experts and gating networks) were trained with a learning rate of 0.01, utilizing a batch size of 512. The networks were trained for 10 epochs utilizing the stochastic gradient descent algorithm with a momentum of 0.9. These hyperparameters were determined via preliminary experimentation and were partially guided by common parameter settings found in the literature. Initialization was performed using Pytorch’s Kaiming Uniform method, and biases were initialized using the LeCun init method with a standard deviation of 1/\( \sqrt{fan_{in}} \).
3.4.1 Naive concatenation

The simplest way that one can combine multiple heterogeneous networks would be through naive concatenation. Naive concatenation operates with the assumption that the output logits produced by the networks, which refers to the output of the final layer of a network before applying the softmax function, would reflect the “state of mind” or uncertainty of the network when given a particular input. In general, if a network is given an input that it cannot recognize it should generally produce logits with values that are closer to each other (i.e. low confidence; high entropy), and if it is given an input that it can recognize it should generally produce logits with one of the values being distinctly higher than the others (i.e. high confidence; low entropy).

To concatenate the heterogeneous expert networks, the involved networks would each be given the same input data to be processed in order to produce the logits. All the produced output logits would then be concatenated together. A statistical function would then be applied on the concatenated logits in order to draw a prediction. Different types of statistical functions were experimented with in order to explore better methods to draw the prediction from the concatenated output logits, namely:

- **Argmax.** The correct network is deemed to be the one with the largest logit (Fig. 1).
- **Ratio.** For each expert we take the ratio between each logit and the sum of all logits in that expert. The expert with the largest ratio is deemed the correct expert.

- **Overall ratio.** The same as the above, except that each logit is divided by the sum of all logits in the whole concatenation.
- **Third quartile difference.** For each expert, we define this measure as the difference between its max logit, and its third quartile logit. The expert with the largest difference is deemed the correct expert.
- **Standard deviation.** In contradiction to our general intuition pertaining to output entropy and classification confidence, and for comparison purposes, the correct network is deemed to be the one with the smallest standard deviation of logits.

3.4.2 Multiple pass with post data augmentation

One possible issue associated with naive concatenation is the possibility of misclassification due to different experts exhibiting different output distributions. In order to try to counteract this, we experimented with augmenting the input into multiple variants, and then aggregating the results of the multiple decisions obtained by the naive methods described above. The outputs were aggregated either by taking the mean or by a voting mechanism. In the former case (i.e. mean) we computed the mean of the concatenations (resulting from the multiple augmentations), and then applied our standard decision methods, whereas in the latter case (i.e. voting) we computed decisions on each one of the concatenations, and then voted on the decisions. The augmentation techniques experimented with consisted of sharpening, Gaussian noise, Poisson noise, horizontal...
flip, vertical flip and random cropping. Note that the data augmentation was not used during expert training, but rather was applied only during the gating decision.

**Algorithm 1** Algorithm for our augmentation approach with naive concatenation.

**Data:**
- Input data: $x$
- Network 1: $N_1$
- Network 2: $N_2$
- Output vector from network: $O$
- Augmentation functions: $A \leftarrow [aug_1, aug_2, \ldots, aug_i]$
- Aggregation function: $agg()$

**Output:** Final prediction, $\lambda$

$O_0 \leftarrow []$

$O_{N_1} \leftarrow N_1(x)$

$O_{N_2} \leftarrow N_2(x)$

$O_s \leftarrow append(O_{s}, concat(O_{N_1}, O_{N_2}))$

**while** not end of list $A$ **do**

$x_{aug_i} = A[i](x)$

$O_{N_{1, aug_i}} \leftarrow N_1(x_{aug_i})$

$O_{N_{2, aug_i}} \leftarrow N_2(x_{aug_i})$

$O_s \leftarrow append(O_{s}, concat(O_{N_{1, aug_i}}, O_{N_{2, aug_i}}))$

**end**

$O_{combined} \leftarrow agg(O_s)$

$\lambda \leftarrow argmax(O_{combined})$

Hypothetically, the output vectors of the “correct network” should be more consistent, varying minimally across the multiple augmented input passes, while the output vectors of “incorrect networks” should differ more across the augmentations, producing different predictions.

### 3.4.3 Smart coordinator

The methods mentioned above are generally concerned with simple statistics of the outputs of the involved heterogeneous networks to draw a combined prediction, which relies greatly on the assumption that the output patterns of the networks behave in a consistent manner. To move beyond this assumption, we propose to train an additional neural network, on features pertaining to key activations of an expert, when observing input pattern $p$, in order to determine whether $p$ is correctly attributed to that expert or not. For the sake of simplicity we term this additional gating network a pattern attribution network (PAN).

All of our PANs follow a simple Multilayer Perceptron architecture. For our partially universal gating solution, PANs adopt two hidden layers with 80 and 60 nodes respectively, with ReLU activations, and include a final linear layer connecting to two outputs. For our fully universal gating solution, PANs adopt three hidden layers with 80, 50, and 20 nodes respectively, again with ReLU activations and including a final linear layer connecting to two outputs.

For the partially universal gating case (i.e. SC1), each expert has its own PAN, and the combination of all PANs constitutes the “smart coordinator”. For the sake of clarity, lets assume that we have one expert for classifying cats, and another expert for classifying dogs. Given a new input pattern of a siamese cat, the PAN corresponding to the cat expert should return true, whereas the PAN corresponding to the dog expert should return false. The general architectural structure of an SC1 smart coordinator is shown in Fig. 2.

![Fig. 2 General structure of an SC1 smart coordinator with two PANs. Legend: Di - dataset i; Ei - expert i; Ai.z - activations from expert i after processing dataset z; T/F - true/false; yi - classification result of expert i](image)
Each PAN is trained by utilizing the combined datasets of all involved expert networks. Given two heterogeneous networks, network 1 and network 2, the PAN for network 1 (PAN 1) would be trained on the activation features produced by network 1 when given the examples from the combined training dataset of network 1 (i.e. data 1) and network 2 (i.e. data 2). For training PAN 1, the activation features generated from the training dataset of network 1 would be labeled true while the activation features generated from the training dataset of network 2 would be labeled false. A similar training procedure applies to the PAN of network 2 (PAN 2), which would instead have its activation features generated from dataset 1 labeled as false and the activation features generated from dataset 2 labeled as true.

Note that the above account constitutes another non-universal limitation of SC1, since experts are not agnostic of the overall smart coordinator solution, since all expert datasets are involved in the training of each PAN. Note however, that this could partially be addressed if PAN 1 were to be trained with data 1 and other datasets, where the latter are unrelated to the mixture of experts under consideration, and similarly with PAN 2.

Multiple types of activation features were explored in order to find the most effective features that could be used with the PAN. The first and simplest activation features to be tested were the output logits of the network. However, logits alone may not contain enough information for the PAN to recognise a concrete pattern, and therefore we also attempted to explore the outputs from the final fully connected layer. Statistical features of activations (rather than raw activations) were also experimented with in order to gain some insight into the potential usefulness of architecturally agnostic features (contributing further towards universal gating). Examples of the activation statistics include the mean, max and standard deviation.

With the PANs prepared and trained, the smart coordinator can then be assembled. When given an input, the involved heterogeneous networks would process it as usual, however the activation features of each network would be passed to its corresponding PAN to be evaluated. If only one of the PAN returns true, it would imply that the input “belongs” to that network, and as such only the output from that network would be taken. However, if both PANs return true, or if both PANs return false, the smart coordinator would default to using the naive concatenation method with the common argmax function for making a combined prediction.

![Algorithm 2 Algorithm for smart coordinator.](image)

**Algorithm 2 Algorithm for smart coordinator.**

**Data:**
- Input data: \( x \)
- Network 1: \( N_1 \)
- Network 2: \( N_2 \)
- PAN 1: \( P_1 \)
- PAN 2: \( P_2 \)

**Output:**
- Final prediction, \( \lambda \)

\[ O_{N_1}, F_{N_1} \leftarrow N_1(x); O_{N_2}, F_{N_2} \leftarrow N_2(x); \]
\[ O_{P_1} \leftarrow P_1(F_{N_1}); O_{P_2} \leftarrow P_2(F_{N_2}); \]

- if (\( O_{P_1} \) is True) and (\( O_{P_2} \) is False) then
  - \( \lambda \leftarrow \text{argmax}(O_{N_1}); \)
- else if (\( O_{P_1} \) is False) and (\( O_{P_2} \) is True) then
  - \( \lambda \leftarrow \text{argmax}(O_{N_2}); \)
- else
  - \( \lambda \leftarrow \text{argmax}((O_{N_1}, O_{N_2})); \)

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**Fig. 3** Simplified structure of an SC2 smart coordinator, which by definition involves a single PAN. Legend: Di - dataset i; Ei - expert i; Aiz - activations from expert i after processing dataset z; T/F - true/false; yi - classification result of expert i
Table 1  Average test accuracy of the expert networks over 10 runs

| Dataset             | Average accuracy |
|---------------------|------------------|
| First 5 Classes of MNIST | 0.9948           |
| Last 5 Classes of MNIST | 0.9879           |
| MNIST               | 0.9863           |
| CIFAR10             | 0.6305           |
| Fashion-MNIST       | 0.8708           |
| Kuzushiji-MNIST     | 0.9180           |

For the fully universal gating solution (i.e. SC2), we take a similar approach, which is depicted in a simplified manner in Fig. 3. The main distinction lies in how we prepare the pattern-attribution dataset, and the fact that we use only one gating network. Briefly, the attribution datasets that are created for individual PANs in SC1 can be combined into one large dataset, with the additional constraint that features of activations (for PAN inputs) need to be abstract enough in order to be applicable to a broad range of new network (expert) architectures. The single dataset is then used to train a single universal PAN (UPAN), which is then combined with experts, their activations, and additional logic for making the final classification decision. The combination of all of these elements constitutes a smart coordinator, in this case SC2. For further universality, the gating network should be trained on activations from experts/datasets completely distinct from those in the mixture of experts under consideration. The experiments were implemented in Python using the Pytorch framework. All experiments were either executed on Google Colab, or on the first author’s laptop.

4 Results

The results were obtained by running experiments across 10 different seeds, each one of which was used for the entire pipeline from training experts to the implementation of the gating solution. In order to better understand the base performance of each expert network after training, we computed test accuracies on their corresponding test sets (i.e. MNIST (0-4), MNIST (5-9), MNIST (0-9), CIFAR10, Fashion-MNIST, and Kuzushiji-MNIST) as depicted in Table 1.

Ideally, assuming perfect gating and balanced data, we would want to obtain results with the average accuracy of both experts, that is, the target accuracy can be defined as $A_t = (A_1 + A_2)/2$, where $A_1$ and $A_2$ refer to the accuracy of the first and second network respectively. The ideal accuracy for the MNIST (0-4) / MNIST (5-9) mixture is thus 0.9914, and for the MNIST and CIFAR10 mixture is 0.8084.

When testing the combined experts trained on MNIST and CIFAR10, due to the difference in the number of input channels expected by each expert, with the MNIST expert accepting a single input channel input, and the CIFAR10 expert accepting 3 input channels, the combined testing data from MNIST and CIFAR10 has to be transformed to the correct number of channels for each expert network. More specifically, MNIST images must be transformed from 1 to 3 channels when being fed into the CIFAR10 expert, and CIFAR10 images must be transformed from 3 channels to 1 (i.e. grayscaled) when fed into the MNIST expert. In general, unconstrained mixtures of heterogeneous experts are likely to use many different input representations, and therefore input patterns will frequently need to be transformed and pre-processed with minimal assumptions in order to be compatible with each expert.

Table 2 summarizes the results of applying different naive concatenation approaches, whereas Table 3 extends these results based on different input augmentations. Table 4 summarizes the results of individual pattern attribution networks, whereas Table 5 reports the final smart coordinator results for the partially-universal gating approach (i.e. SC1). In other words, Table 4 depicts how successful PANs are in determining whether specific inputs belong to their experts or not, and Table 5 depicts the final classification performance after considering (by “smart coordination”) the PAN attributions and the expert classifications in an overall final decision. Table 6 reports preliminary attribution accuracy results for our universal pattern attribution network together with corresponding smart coordinator accuracies. The column denoted by “Outputs” refers to the condition where activation features correspond to output logits, whereas

Table 2  Average accuracy of naive concatenation with different statistical functions (argmax, standard deviation, ratio, overall ratio, and third quartile difference)

| Problem               | Statistical functions |
|-----------------------|-----------------------|
|                       | Argmax | Standard deviation | Ratio   | Overall ratio | Third quartile difference |
| disjoint MNIST (0-4, 5-9) | 0.9288 | 0.0941             | 0.3565  | 0.5085        | 0.9014                     |
| MNIST + CIFAR10       | 0.8039 | 0.0026             | 0.2030  | 0.4396        | 0.8061                     |
Table 3  Average accuracies of different augmentation approaches with mean and voting aggregations

| Problem            | Augmentation                                                                 | Aggregation method | Mean     | Voting   |
|--------------------|------------------------------------------------------------------------------|--------------------|----------|----------|
| MNIST (0-4/5-9)    | Single pass sharpen                                                          | Mean               | 0.8874   | 0.7388   |
|                    | 5 pass sharpen with differing alpha (0.1, 0.3, 0.5, 0.7, 1.0)               | Voting             | 0.4044   | 0.1156   |
|                    | Single pass gaussian noise                                                   |                    | 0.9183   | 0.9134   |
|                    | 5 pass gaussian noise                                                         |                    | 0.9119   | 0.9108   |
| MNIST/CIFAR10      | 6 pass gaussian noise with differing standard deviation (0.05, 0.1, 0.3, 0.5, 0.7, 1) |                    | 0.9261   | 0.9280   |
|                    | Single pass poisson noise                                                    |                    | 0.9193   | 0.9194   |
|                    | 5 pass poisson noise                                                          |                    | 0.9101   | 0.9085   |
| MNIST/CIFAR10      | 6 pass poisson noise with differing standard deviation (0.05, 0.1, 0.3, 0.5, 0.7, 1) |                    | 0.9156   | 0.9182   |
| MNIST/CIFAR10      | Horizontal and Vertical flip                                                 |                    | 0.7898   | 0.1045   |
| MNIST/CIFAR10      | Random cropping                                                              | Mean               | 0.8957   | 0.7289   |
|                    | Single pass sharpen                                                          | Voting             | 0.7383   | 0.5915   |
| MNIST/CIFAR10      | 5 pass sharpen with differing alpha (0.1, 0.3, 0.5, 0.7, 1.0)               |                    | 0.4320   | 0.0942   |
| MNIST/CIFAR10      | Single pass gaussian noise                                                   |                    | 0.7467   | 0.7209   |
| MNIST/CIFAR10      | 5 pass gaussian noise                                                         |                    | 0.6945   | 0.6805   |
| MNIST/CIFAR10      | 6 pass gaussian noise with differing standard deviation (0.05, 0.1, 0.3, 0.5, 0.7, 1) |                    | 0.7794   | 0.7951   |
| MNIST/CIFAR10      | Single pass poisson noise                                                    |                    | 0.7845   | 0.7616   |
| MNIST/CIFAR10      | 5 pass poisson noise                                                          |                    | 0.7663   | 0.7583   |
| MNIST/CIFAR10      | 6 pass poisson noise with differing standard deviation (0.05, 0.1, 0.3, 0.5, 0.7, 1) |                    | 0.7781   | 0.7777   |
| MNIST/CIFAR10      | Horizontal and Vertical flip                                                 |                    | 0.6765   | 0.2654   |

“Out. (F)” refers to simple features of the output logits (i.e. mean, max, and standard deviation).

5 Discussion

The naive concatenation methods were effectively the main control conditions for our experiments. The argmax function in particular, was initially expected to perform poorly, especially on similar tasks such as the disjoint MNIST problem, due to it simply taking the argmax of the concatenated outputs of multiple networks. However, it actually performed the best amongst all the statistical functions, and outperformed all of the multi-pass augmentation approaches as well. When comparing argmax with the ideal target combination accuracy (estimated by averaging individual accuracies), the accuracy of the naive concatenation method with the argmax statistical function was lower, as expected, but the difference between them was not very large, with the disjoint MNIST problem having a 0.063 difference, and

Table 4  Average pattern attribution accuracy for PAN

| Positive dataset | Negative dataset | Testing dataset | Outputs | Final FC | Out. (F) |
|------------------|------------------|----------------|---------|---------|----------|
| MNIST (0-4)      | MNIST (5-9)      | MNIST          | 0.9254  | 0.9761  | 0.8655   |
| MNIST (5-9)      | MNIST (0-4)      | MNIST          | 0.8943  | 0.9755  | 0.8419   |
| MNIST            | CIFAR10          | MNIST/CIFAR10  | 0.9999  | 0.9999  | 0.9999   |
| CIFAR10          | MNIST            | MNIST/CIFAR10  | 0.9983  | 0.9998  | 0.9716   |
Table 5  Average smart coordination accuracy for SC1

| Problems                | Coordinator feature | Outputs           | Final FC           | Out. (F)  |
|-------------------------|---------------------|-------------------|--------------------|-----------|
| disjoint MNIST (0-4, 5-9)|                     | 0.9458            | 0.9733             | 0.9294    |
| MNIST + CIFAR10         |                     | 0.8083            | 0.8084             | 0.8071    |

Another interesting observation can be seen in naive concatenation with third quartile difference, which performed worse than argmax on the disjoint MNIST problem, but when applied to the MNIST + CIFAR10 problem, it actually performed better than argmax, although it still did not reach the ideal target combination accuracy, lacking by 0.002. Judging by these observations, the assumption that the logits of a network reflect the “state of mind” of a network does partially hold true, with distinctly large activations more likely to correspond to correct predictions and thus indicating the “true” networks. However, looking at the performance of other naive mechanisms, namely standard deviation, ratio, and overall ratio, we can tentatively conclude that their underlying hypothesis that the remaining logits (i.e. not max) also contain useful information for the gating decision is not entirely confirmed, and that more work needs to be done if this information is to be extracted and used in a simplistic manner.

The multiple pass approach with augmentation is an extension to the naive concatenation with argmax method, taking its inspiration from ensemble learning, but instead of passing the input to multiple trained networks with individual strength and then consolidating their strengths to make a single prediction on the same task, it provides the same network with multiple views or perspectives on the same input data, allowing multiple predictions or “opinions” to be formed. The augmentation is meant to provide different perspectives however it was found it does not actually improve the predictions when aggregated. In fact, all of the augmentations caused a drop in accuracy, across both aggregation methods. The approach with 6-pass gaussian noise with differing standard deviations performs with an accuracy closest to the argmax baseline, however this might simply be due to the passes having less gaussian noise with the lower standard deviation, thus causing the augmented image to look similar to the original input after augmentation, which tips the majority vote to the predicted classes of the original data, causing it to resemble the naive concatenation baseline. As such, it can be concluded that the use of post data augmentation here does not actually highlight certain features out for the network to provide additional “opinions” or information that can contribute to a better rounded prediction, but instead simply confuses the network causing it to perform less accurately and have a less confident prediction in the end. Note that the expert networks used for this experiment were not trained with data augmentation, and as such if the networks were trained using some sort of data augmentation, the result of this experiment could potentially turn out differently.

From observing the test accuracy obtained from the smart coordinator SC1, it is apparent that the individual PANs were able to successfully learn the activation patterns from the individual networks, in order to identify the attribution of a given input to its corresponding network. When trained on the crafted feature dataset, the performance of PAN using

Table 6  UPAN and SC2 performance

| PAN trained on            | PAN tested on                | Activation feature | Outputs             | Out. (F)     |
|---------------------------|-----------------------------|--------------------|---------------------|--------------|
| MNIST (0-4, 5-9)          | MNIST (0-4, 5-9)            |                    | 0.9566 / 0.9515     | 0.9386 / 0.9363 |
| MNIST + CIFAR10           | MNIST + CIFAR10             |                    | 0.9998 / 0.8874     | 0.9932 / 0.8844 |
| MNIST (0-4, 5-9)          | MNIST+CIFAR10               |                    | N/A                 | 0.8859 / 0.8762 |
| MNIST+CIFAR10             | MNIST(0-4,5-9)              |                    | N/A                 | 0.8238 / 0.8155 |
| MNIST(0-4, 5-9)           | Fashion+Kuzushiji           |                    | N/A                 | 0.8763 / 0.8177 |
| MNIST+CIFAR10             | Fashion+Kuzushiji           |                    | 0.8103 / 0.7515     | 0.8797 / 0.8204 |

Results are portrayed as x/y where x denotes the average pattern attribution accuracy for UPAN, and y denotes the corresponding smart coordinator (SC2) accuracy. “Fashion” denotes the Fashion-MNIST dataset, whereas “Kuzushiji” refers to the Kuzushiji-MNIST dataset.
logits and hidden features generally achieved accuracy greater than 0.9, with only the PAN for the last 5 MNIST class using logits as features having a lower accuracy at 0.8943. For PANs using logit activation statistics instead of raw features from the network, the accuracy on the crafted dataset for the disjoint MNIST problem when tested was significantly lower than its counterpart, having an accuracy of 0.8655 and 0.8415 for the first 5 and last 5 MNIST classes respectively. Similar to the naive concatenation baseline, SC1 seems to work better in coordinating distinct tasks compared to similar tasks, with accuracy close to 100% when trained on the crafted dataset for the MNIST+CIFAR10 problem. Determining the attribution of a given input seems to be harder when the experts involved are trained on similar data, as the corresponding PANs have to learn specific features corresponding to the smaller differences between the different data distributions.

All types of PANs in the SC1 smart coordinator resulted in an overall improvement in accuracy for all problems when compared to naive concatenation using argmax. The output logit based PAN showed a performance increase of 0.017 for the disjoint MNIST problem and a performance increase of 0.0044 for the MNIST+CIFAR10 problem. The final FC based PAN performed the best, with a performance increase of 0.0445 for the disjoint MNIST problem and a performance increase of 0.0045 for the MNIST+CIFAR10 problem. The PAN based on output logit statistics exhibited the lowest increase in performance, having only a performance increase of 0.0006 for the disjoint MNIST problem and a performance increase of 0.0032 for the MNIST+CIFAR10 problem. The good performance of the final FC based PAN is most probably due to the availability of more information as compared to the other types of PAN, and similarly, the relatively poor performance of the output logit statistics is most probably due to a lack of discriminatory information. One final observation taken from the experiment on the final FC based PAN is that it reached the ideal target combination accuracy for the MNIST+CIFAR10 problem.

Preliminary results on universal pattern attribution networks (UPANs) depicted in Table 6 are indicative of the usefulness of the approach. Rows 3-6 exemplify cases that include, or are entirely made of, never before seen datasets, and therefore serve as the ultimate validation of the approach. Although attribution accuracy is lower relative to the easier conditions depicted in rows 1-2, the UPAN is still capable of performing significantly above chance level. The cells depicted by N/A refer to cases where the structure of output vectors are inconsistent between training and test cases (e.g. 5 outputs vs. 10 outputs), and therefore serve as a reminder of the importance of architecturally agnostic features. Additional unreported experiments were also conducted on what we call fast PANs (FPANs) whereby we use a UPAN to train a separate network to associate input images directly with specific experts, based on UPAN decisions. Preliminary results were positive, even outperforming the original UPAN accuracy. This development is important, because apart from its implications for accuracy, it represents a significant improvement in efficiency, given that we no longer need inputs to be processed by each expert in order for a gating decision to be made.

To wrap-up the discussion, a final point on the relative computational cost of the two main approaches proposed will be given here. The computational costs of SC1 and SC2 are in fact similar, since both approaches require processing a new pattern p through |E| experts. The difference lies in that each set of activation features (i.e. A(E_i(p), refer to (1)) for SC1 goes through a different corresponding PAN, whereas for SC2 each set of activation features goes through the same universal PAN. Thus it can be seen that the key determinant of the relative efficiency of the two approaches hinges on the size/complexity of each individual SC1 PAN versus the complexity of the universal PAN.

### 6 Conclusion

This paper has explored multiple approaches for the combination of heterogeneous pre-trained neural networks in a data-free regime, culminating in a universal gating network. Through the reported experiments, it was shown that it is possible to use neural activations themselves, or abstract features of these, in order to infer whether a particular input pattern belongs to a network/expert or not.

Given the potential of the universal gating approach to the rapid development of complex applications based on mixtures of heterogeneous experts, we recommend further research into: (1) training and testing on an even broader range of datasets and neural architectures, (2) more diverse and discriminatory network-agnostic features, and (3) fast pattern attribution networks (FPANs) where UPANs are used to train an additional network to associate inputs directly with gating decisions thus avoiding the need for inputs to be processed by all experts before making an attribution decision. Regarding the first point, we recommend focusing tests on high-impact applications such as medical diagnosis, since the data-free combination of heterogeneous pre-trained experts is likely to be most beneficial when it involves such high-impact applications.

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