Emulating exceptional-point encirclements using imperfect (leaky) photonic components

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Abstract – Non-Hermitian systems have recently attracted significant attention in photonics. One of the hallmarks of these systems is the possibility of realizing asymmetric mode switching and omni-polarizer action through the dynamic encirclement of exceptional points (EP). Here, we offer a new perspective on the operating principle of these devices, and we theoretically show that asymmetric mode switching can be easily realized – with the same performance and limitations – using simple configurations that emulate the physics involved in encircling EP’s without the complexity of actual encirclement schemes. The proposed concept of “encirclement emulators” may allow a better assessment of practical applications of non-Hermitian photonics.

1. INTRODUCTION – RECIPROCITY AND TRANSFER-MATRIX ASYMMETRY

The asymmetric omni-polarizers and asymmetric mode-switches recently demonstrated in [1,2] are devices that produce a certain output state (A), in the forward direction, regardless of the input state (A or B), whereas they produce the opposite output state (B) in the backward direction, again independent of the input state. Reciprocity (symmetry of the scattering matrix) imposes severe constraints on this process. In
particular, an ideal asymmetric mode-switch, with transmission amplitude in a certain output state equal for both input states, must be nonreciprocal.

To understand this better, let us consider a 2-port linear reciprocal device (not necessarily passive), as in Fig. 1(a), designed to produce output A when excited from the left and output B when excited from the right. Let us also assume unity transmission coefficient from port B on the left to port A on the right, i.e., $T_{BA}^{→} = 1$; then, it is necessary that $T_{AA}^{→} = \alpha$, with $\alpha \ll 1$ if the device is to produce (mostly) output B in the opposite direction. Indeed, because of reciprocity, $T_{AB}^{←} = T_{BA}^{→} = 1$ and $T_{AA}^{←} = T_{AA}^{→} = \alpha$. In other words, if $\alpha = 1$, the device can operate as an ideal mode-switch or omni-polarizer in the forward direction, with identical transmission amplitude for both input states (A or B), but it would then act as a signal splitter in the backward direction. To put it more bluntly, an ideal asymmetric mode switch or polarizer must be described by a forward transfer matrix

$$
\begin{pmatrix}
T_{AA}^{→} & T_{AB}^{→} \\
T_{BA}^{→} & T_{BB}^{→}
\end{pmatrix}
= 
\begin{pmatrix}
1 & 1 \\
0 & 0
\end{pmatrix}
\tag{1}
$$

which would mean that the backward transfer matrix would have to be

$$
\begin{pmatrix}
T_{AA}^{←} & T_{AB}^{←} \\
T_{BA}^{←} & T_{BB}^{←}
\end{pmatrix}
= 
\begin{pmatrix}
1 & 0 \\
1 & 0
\end{pmatrix}
\tag{2}
$$

which is a mode filter followed by a 3dB mode splitter, i.e. something of a very dubious utility.

Thus, any linear reciprocal device designed to operate as an asymmetric mode-switch or omni-polarizer must have transmission amplitude that is strongly dependent on the input state. This is true regardless of the device implementation, and it follows directly from reciprocity. In light of these considerations, we argue that, if only reciprocal components are used, there are much simpler ways to obtain the same asymmetric mode-switching action than dynamic encirclement of EP’s.

2. EXCEPTIONAL-POINT ENCIRCLEMENTS AND THEIR EMULATION
Let us now try to understand the basic physical principle that causes asymmetric mode switching during an EP encirclement, and whether this process can be emulated. For two coupled modes $E_{1,2}$ with complex propagation constants $\beta_{1,2} + i\gamma_{1,2}$, the characteristic matrix (Hamiltonian) of the system is

$$
H = H + H_{\text{int}} = \begin{pmatrix}
\bar{\beta} + i\bar{\gamma} & 0 \\
0 & \bar{\beta} + i\bar{\gamma}
\end{pmatrix} + \begin{pmatrix}
\delta\beta + i\delta\gamma & \kappa \\
\kappa & -\delta\beta - i\delta\gamma
\end{pmatrix},
$$

where $\kappa$ is the coupling coefficient, $\bar{\beta} = (\beta_1 + \beta_2)/2$, $\bar{\gamma} = (\gamma_1 + \gamma_2)/2$, $\delta\beta = (\beta_1 - \beta_2)/2$, $\delta = (\gamma_1 - \gamma_2)/2$. The two complex eigensolutions, $\tilde{\beta}_\pm = \bar{\beta} + i\bar{\gamma} \pm \sqrt{(\delta\beta + i\delta\gamma)^2 + \kappa^2}$, reveal two EP’s (shown in Fig. 1(b)) for $\delta\beta = 0$ and $\kappa = \pm\delta\gamma$, at which two solutions have identical eigenvalues and eigenmodes.

Figure 1. (a) 2-port system under study: ports A and B may correspond to different guided modes or polarization states. (b) EP’s and encirclement trajectory. Evolution during encirclement of (c) propagation constants, (d) extinction coefficient (dashed curves: eigensolutions; solid curves: eigensolutions weighted
by the respective population coefficient), and (e) population coefficients for the system initially prepared in mode B (top) and mode A (bottom). Red arrows indicate the direction of encirclement.

Let us then consider what happens when we encircle one of the EP’s, counterclockwise from a starting point S to a final point F = S. The results are shown in Fig. 1(c)-(e) for an encirclement along the contour with radius $0.3 \delta \gamma$. In Fig. 1(c),(d) one can see the evolution of the relative propagation constant, $\beta_\pm = \text{Re}(\tilde{\beta}_\pm) - \tilde{\beta}$, and the relative gain, $\gamma_\pm = \text{Im}(\tilde{\beta}_\pm) - \tilde{\gamma}$, while Fig. 1(e) shows the evolution of the population coefficients $|C_{\pm A,B}|^2$, i.e., the projection of the instantaneous state vector along the instantaneous eigenvectors $E_z(\theta)$. If we initially prepare the system in one of the eigenstates, $E_-(0)$ (or $E_+(0)$), associated with mode A(B), and assuming full adiabaticity, the mode should be converted into mode B(A) as a result of a full parametric encirclement. However, as the two instantaneous eigenstates evolve, they experience very different relative gain/loss; in addition, due to the non-Hermitian nature of the problem the eigenstates are not strictly orthogonal, and a portion of energy may leak from one state to the other. In particular, a leakage from the lossy state (originating from A, see Fig. 1) to the amplifying state cannot be ignored, as this fraction of energy will then experience a relative gain, while the remaining fraction in the lossy state will suffer extinction. This induces a state jump around point X in Fig. 1, consistent with Ref. [1,2] (the location of point X depends on the degree of non-adiabaticity, i.e., the speed of dynamic encirclement). As a result, toward the end of a full dynamic encirclement, the optical signal originating from A will be much stronger in mode A than in mode B. Conversely, the light that leaked from the amplifying state, originating from B, into the lossy state would suffer extinction and can be neglected (no state jump occurs in this case). Hence, no matter where the signal originates from (i.e., its input state), it ends up mostly in mode A. Following the same reasoning, it is also clear that reversing the direction of
encirclement (or of propagation) will bring the signal mostly into mode B, regardless of the input state. This is the mechanism behind the asymmetric mode-switching effect in [1]. However, it is also important to realize that when dynamic encirclement takes place, along one trajectory (going from B to A) the entire signal propagates with relative gain, while along the other possible trajectory with the same output (going from A to A) at first the signal experiences large loss (attenuation and leakage) and then it gets recovered by experiencing relative amplification. This combination of high loss followed by high gain may look benign to a theorist, yet it appears noxious in practice since the signal to noise ratio (SNR) is greatly deteriorated.

At this point, an astute reader may note that since all the “modal switching” action takes place in the vicinity of point X, the relative gain/loss preceding and following it does not have to be distributed and can simply be formed by lumped optical amplifiers and attenuators. Theoretically speaking, in the absence of gain/loss, adiabaticity for slowly varying systems tends to get restored, but in any real system ideal adiabaticity is never fully achieved. It is precisely this fact (leakage due to imperfect adiabaticity) that makes the remarkable properties of dynamic encirclement possible, while the specific evolution of the eigenmodes before/after point X is insignificant, as long as different trajectories have different gain and loss. Based on these considerations, it is clear that essentially the same behavior can be observed and emulated by using a leaky mode coupler preceded and followed by lumped gain/loss elements. As illustrated in Fig. 2(a), the coupler can simply be an imperfect directional coupler with coupling coefficient $|\kappa|<1$, preceded and followed by optical amplifiers with gain $\gamma/2$ and optical attenuators with loss $-\gamma/2$ (using two different levels of loss would also work). The transfer matrices for the proposed “encirclement emulator” are

$$T^{-} = \begin{pmatrix} 1-\kappa^2 & \kappa^2 - \gamma \\ \kappa^2 + \gamma & 1-\kappa^2 \end{pmatrix}, \quad T^{+} = \begin{pmatrix} 1-\kappa^2 & \kappa^2 + \gamma \\ \kappa^2 - \gamma & 1-\kappa^2 \end{pmatrix} \quad (4)$$
for forward and backward propagation, respectively. Let us assume a 90:10 directional coupler, so that 
\( \kappa^2 \approx -0.05dB, \ 1 - \kappa^2 = -10dB, \) and \( \gamma = 20dB. \) We performed numerical experiments for a 
representative example, and the results are reported in Fig. 2(b): For forward propagation, 
\( T_{AA}^\rightarrow / T_{AB}^\rightarrow \approx 10dB \) and \( T_{BA}^\rightarrow / T_{BB}^\rightarrow \approx 30dB; \) therefore, no matter where the signal originated from (A or B), 
it ends up mostly at output A. It is easy to see that, in the reverse direction, regardless of the input state, the 
signal ends up mostly at output B, hence realizing an asymmetric mode-switching effect as in the dynamic 
EP encirclement scheme discussed above. Indeed, if one compares these results with the observations of 
dynamic encirclement reported in [1], and replicated in Fig. 2(b), the results are nearly identical: almost all 
forward (backward) propagating light ends up in mode A(B). Notice that the transmission amplitude is 
highly asymmetric both in our scheme, with \( T_{AA}^\rightarrow / T_{BA}^\rightarrow = \alpha \approx -30dB, \) and in Ref. [1], with 
\( T_{AA}^\rightarrow / T_{BA}^\rightarrow \approx -25dB. \) Consistent with our discussion above, this transmission asymmetry is **unavoidable** in 
any linear and reciprocal asymmetric mode-switch. The asymmetry can be mitigated by using nonlinear 
saturable amplifiers that simply add a certain power to the signal no matter what the input is. Then a more 
equitable \( T_{AA}^\rightarrow / T_{BA}^\rightarrow \approx -3dB \) can be attained; however, this does not alleviate the main impediment to 
practical applications, namely, the fact that the signal following the AA pathway initially suffers a 
significant loss that deteriorates the SNR.

This approach to emulate the effects of an EP encirclement can be easily applied to other tasks, such as 
to realize the asymmetric omni-polarizers first explored in Ref. [2]. A possible configuration is shown in 
Fig. 2(c), based on two polarization-dependent gain/loss sections, on the two sides of an **imperfect** half- 
wave plate, which assures that rather than complete swap of polarization components there is a small 
leakage.

3. CONCLUSION
In summary, based on theoretical considerations and numerical experiments, we have shown that, at least in optics, the effects of a dynamic EP encirclement on the transfer matrix are essentially indistinguishable from the effects of a small signal leakage followed by large gain/loss asymmetry. We have also shown that, due to the inherent asymmetry of the transfer matrix, the SNR in all the “real” and “emulated” dynamic encirclement schemes is expected to sharply deteriorate, which, in our view, raises questions about the practical applicability of EP encirclements, but does not entirely preclude some niche applications depending on the specific requirements.

Figure 2. (a) Proposed scheme realizing asymmetric mode switching, emulating a dynamic EP encirclement. (b) Transmission amplitudes (blue) for a configuration as in (a), numerically simulated at
near-IR wavelengths, compared with the transmission amplitudes for the device in [1] (orange; values taken at the central frequency). (c) Asymmetric omni-polarizer based on the same mechanism as in (a).

**Funding.** National Science Foundation (NSF) (1741694).

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