Application of fractal theory in fault diagnosis of nonlinear mechanical equipment system: a review

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Abstract. Fractal theory takes the irregular and complex things as the research object, and uses the fractal dimension to describe the object. The review classifies and summarizes the application development of fractal theory in the diagnosis of common nonlinear mechanical equipment system faults such as diesel engines, gear boxes, hydraulic systems, etc. It is showed that fractal theory can extract and analyse irregular features and simplify complex research. It has certain advantages among various methods and is gradually being widely used. It provides a new reference and idea for the future research direction after fully understanding its current status and summarizing its characteristics. At present, most of the research on fault diagnosis of mechanical equipment is based on the signals under steady running state, while ignoring the transitional signals with high information content. Fractal theory has unique advantages in dealing with this kind of transitional signals such as the start and shutdown of equipment. It is a new trend to solve practical problems that study the short-time signals at transition process with strong non-linearity based on fractal theory to diagnose the fault of the mechanical equipment.

1. Introduction

The pace of development of modern industrial production is accelerating in the context of social economic development and technological progress. Mechanical equipment system plays a pivotal role in industry. Its characteristic advantages such as high speed, automation and continuity accelerate industrial development and bring great economic benefits. The operation of the equipment has a significant impact on the progress, achievements, and benefits of related industrial production. A failure in a certain part will affect the entire industrial production process, and then will have a different degree of negative impact on economic benefits. In severe cases, a safety accident may even occur, causing a huge harm to the personal safety of the staff and resulting in irreparable damage. Therefore, the reliability and safety of mechanical equipment are highly valued, the related research of fault diagnosis is particularly important. At the same time, the fault diagnosis technology is also put forward higher requirements.

Mechanical equipment is constantly being researched and innovated due to its importance. Its structure is more complex, the technical content is higher, and the complexity of its vibration signal is also increasing. The information content is more abundant, and most of them have nonlinear characteristics, so the single fault diagnosis method is more inadaptable to the feature recognition of increasingly complex mechanical equipment. For the fault diagnosis of nonlinear mechanical equipment systems, the fractal theory method has great application significance. So it is very necessary to introduce and apply the fractal theory to the fault diagnosis problems of nonlinear
mechanical equipment systems. In this regard, scholars have conducted a lot of research on this issue, innovated on the basis of existing methods and technologies, and achieved certain results.

2. Fractal theory

2.1. Concepts and characteristics of fractal
The fractal theory was founded by the American mathematician Mandelbrot. In the 1960s, he discussed the length of the British coastline in the American "Science" magazine, and proposed the self-similar nature of the coastline curve and other geographic boundaries. Since then, fractal thought has sprouted. Then he formally introduced the concept of fractal in 1975. Taking the natural irregular phenomena as the description object, he created the fractal geometry theory, which laid an important foundation for the analysis and research of irregular sets and complex sets in the future. And the theory has been widely used in various subject fields quickly.

Fractal theory is a subject with a high degree of activity and a wide range of applications in recent years. It takes irregular and complex things as the research object, and uses the fractional dimension to describe the object. Fractal can be reflected not only in geometric models, but also in mathematical models such as "information", "time", and "energy". If the rule set F is fractal, it must be satisfied that its part and the whole have self-similar properties [1]. This form of self-similarity can be either a strict one like the Koch curve, or a non-strict, statistically self-similar such as rolling mountains and coastlines. Through the above self-similarity differences, fractal can also be divided into regular fractal and random fractal. Fractal also has scale-free nature which means that the enlargement and reduction of the research object in the local area will not change the complexity, irregularity, shape and other characteristics of the original shape. Generally, the fractal dimension is larger than its own topological dimension.

2.2. Fractal dimension
In the 1910s, the mathematician Hausdorff proposed a concept that broke the stereotyped understanding of the concept of dimension. In traditional public opinion, people think that the spatial dimensions of objects are integers, however in reality, the dimensions of objects in nature exist in the form of fractions, and we can also regard the integer dimension as a special case in fractional dimension.

Fractal dimension is an important parameter to describe fractal. The results are presented in numerical form, which can be used to identify the research object quantitatively and intuitively. The complexity of fractal dimension is relatively high, and mathematicians have been conducting a lot of research on it, from a single fractal dimension to multiple fractal dimensions [2]. And through the exploration, a variety of fractal dimension theories and their corresponding dimension calculation methods have been developed. Among them, the box dimension is widely used, and the correlation dimension is also widely applied. The common calculation method of the correlation dimension is the G-P algorithm. When using the correlation dimension, attention should be paid to the noise of the vibration signal, because the correlation dimension is more sensitive to the noise of the vibration signal. As the noise increases, the correlation dimension's ability to recognize the signal state will be weakened. Therefore, the vibration signal of the research object should be processed to eliminate the noise [3]. In addition, there are information dimension, generalized dimension, capacity dimension and other fractal dimension theories.

3. Application of fractal theory in fault diagnosis of nonlinear mechanical equipment system
Fractal theory is one of the important means to study nonlinear science. In the research of equipment fault diagnosis based on vibration signals, many vibration signals are likely to mutate in practice, also are unstable and irregular. When this type of signal has strong nonlinear characteristics, the fractal theory method is suitable for this kind of irregular and complex phenomena, which can effectively solve the calculation of signal dimension and construct fractal filtering [4]. For example, Ziwen Jia et
al. [5] used mathematical morphological filters to analyse and modify the vibration signals of mechanical equipment in the time domain. Then used the G-P algorithm to calculate the correlation dimension of the signal, took it as the state identification characteristic index of the nonlinear system. Finally verified the feasibility of the algorithm through the simulation experiment of the Lorentz system and the actual simulation experiment on the Bentley-RK4 rotor test rig.

On the basis of studying a large number of relevant literatures, this paper classifies and summarizes the application of fractal theory in the following common nonlinear mechanical equipment system fault diagnosis.

3.1. Application in diesel engine fault diagnosis
Diesel engines have good economic performance and are widely used in large and medium-sized trucks, ships, engineering machinery and other manufacturing. It also plays an important role in the fields of agricultural machinery and aeroengine. Compared with other thermal engines, diesel engines also have advantages in energy saving and emission reduction. In addition, the small and high-speed diesel engines are getting new development gradually, so it is necessary to apply the advantageous fractal theory to the fault diagnosis technology. Sheng Yang et al. extracted the vibration signal characteristics of diesel engine. After signal noise reduction, the correlation dimension in fractal theory was selected as the characteristic value, and the fault signal was characterized by combining with PNN neural network. Hailong Lan et al. [6] combined the local wave decomposition method and the correlation dimension to perform fault diagnosis on the diesel engine. The signal was decomposed into the form of each basic mode component by the local wave method, and then the fractal theory was used to calculate the correlation dimension, which is used as the characteristic quantity to identify the diesel engine fault. In An Dong et al. [7]’s research on diesel engine fault diagnosis, the vibration signal of the diesel engine cylinder head was taken as the research object, and then the correlation dimension of the signal was calculated. The improved G-P algorithm is adopted in the calculation process, and the results obtained can identify the fault of the diesel engine.

3.2. Application in gearbox fault diagnosis
Liwei Tang et al. [8] applied the box-counting dimension algorithm based on fractal theory to the gearbox, and studied the fractal characteristics under the conditions of gearbox bearing cracks, gear pitting, etc., and then identified the gearbox fault. Saifei Zhang et al. [9] selected the correlation dimension as the characteristic value of the fractal, used G-P algorithm to perform fault diagnosis of the engine bearing of the gearbox, and verified the effective effect of the method on fault diagnosis by comparing the fractal dimension of each working condition. Pengcheng Yan et al. [10] extracted the characteristic information of vibration signal of spur gear transmission system with EMD and SVD methods to reduce signal noise. Then used Kolmogorov entropy, multiple fractal characteristic parameters to quantitatively identify the fault of the gearbox more comprehensively. And it is verified that the method can achieve the goal of identifying the nuance of faults. Zhichuan Liu et al. [11] used Kalman filter to reduce the noise of the vibration signal in the time domain of the gearbox, and then calculated its dimension based on the fractal dimension method of mathematical morphology. This extraction method is feasible for identifying gear wear, broken teeth and other faults. Qingqing Chu et al. [12] collected vibration signals in the normal state of gears and in several fault states such as gear wear, broken teeth and circumferential joint errors. Next used the EMD method to layer the signal and perform noise reduction. Then combined fractal theory with neural network to form a new fault diagnosis method which identify the status of gear more efficiently.

3.3. Application in wind turbine fault diagnosis
Wind turbines use renewable and non-polluting wind energy resources in nature, which is of great significance for alleviating energy shortages and achieving sustainable development [13].

Yang Li et al. [14] studied the fault diagnosis of the transmission system of wind turbines, analysed the multiple fractal characteristics of the vibration signals of fixed-speed bearings and variable speed bearings, and verified the effectiveness of the method for fault diagnosis of wind turbines through
experiments. Jianqin Fan [15] conducted fault diagnosis research on gears and rolling bearings that are prone to failure in wind turbines, constructed a nonlinear simulation signal for equipment operation, also measured and analysed the signals of direct-drive wind turbine and doubly-fed wind turbine. In the literature, MATLAB was used as a tool to calculate the correlation dimension of each signal, Kalman filtering and wavelet scale domain filtering were combined with the correlation dimension, which further improves the fault feature extraction of the wind turbine to achieve more effective fault diagnosis. Peiming Shi et al. [16] collected the vibration signal of the gearbox bearing of the wind turbine, extracted the time-domain characteristic parameters, and used the box dimension as the fault characteristic parameter, combined with the genetic algorithm support vector machine diagnosis algorithm, improved the identification rate of the wind turbine bearing fault.

3.4. Application in hydraulic system fault diagnosis
The hydraulic system has the characteristics of complexity, the occurrence of faults has greater randomness and non-linearity, the fault points are also more concealed, and there is a stronger correlation between faults. Simple condition monitoring is more limited for hydraulic systems, but the comprehensive application of fractal theory and other methods can effectively improve such problems.

Qinghua Wang et al. [17] simulated three kinds of faults such as hydraulic pump sliding shoe wear, collected and pre-processed vibration signals in the fault state and normal state of the hydraulic pump. Then extracted fractal characteristic parameters and found that the correlation dimension in different states was obviously different. The applicability of correlation dimension in hydraulic pump fault diagnosis is verified. Xiliang Liu et al. [18] measured simulated fault signal of the CB-KP63 hydraulic gear pump, used the packet decomposition method to pre-process the noise, and then calculated the box dimension of the vibration signal. The research showed that the box dimension is obviously different under different fault states which means that the box dimension is suitable for hydraulic pump state identification, and the accuracy is high. Tongguang Yang et al. [19] studied the problem of cracks in aviation hydraulic pipe. They captured the vibration signals of cracked and non-cracked hydraulic pipes through experiments. And used generalized fractal dimension to analyse whether the hydraulic pipe has cracks. This method can also effectively identify the early failure of hydraulic pipe cracks.

3.5. Application of fault diagnosis in other nonlinear systems
Xianglin Hou et al. [20] defined the fractal calculation dimension based on the signal fractal dimension, combined with the neural network of the multilayer perceptron to form a fault diagnosis method for nonlinear systems. Then verified the feasibility and accuracy of the method through the example of rolling bearings. Meijun Zhang et al. [21] filtered and denoised the original engine signal, and used the correlation dimension to diagnose the fault, which improved the accuracy of the diagnosis. Then by comparison, analysed the correlation dimension of the engine start signal before and after noise reduction in normal state and in different degree of air leakage state to verify the reliability of the method. Yu Yuan et al. [22] established a new diagnosis principle for fault diagnosis of reciprocating compressor, decomposed the vibration signal in the time-frequency domain of local wave, determined the generalized dimension for multifractal, extracted the characteristic quantity of fault diagnosis, and obtained the practical conclusion of the method through case analysis. Zhongyuan Zhang et al. [23] proposed a fault diagnosis method for rolling bearings combining wavelet transform and fractal dimension. After collecting the vibration signals of rolling bearings, the wavelet transform method was used to carry out layered reconstruction. The fractal dimension calculated in the time-frequency domain is more suitable to be used as the basis for judging the failure of rolling bearing and is more accurate.

In the fault diagnosis of various nonlinear mechanical equipment systems, fractal theory has been applied more and more frequently due to the particularity of its research objects. It can effectively process nonlinear and non-stationary signals. In practical application, combining the fractal theory
with other analysis methods, the fault features can be more clearly extracted to describe the fault state of the system, which makes fault diagnosis more accurate, faster and more effective.

4. Development status and prospect of fractal theory
The research of fractal theory started late and is a relatively new and developing discipline. This paper summarizes the successful cases of the current fractal theory in the fault diagnosis of nonlinear mechanical equipment systems, but there is still a lot of research space in the application of fault diagnosis. Fractal theory usually needs to be combined with other theories in practical applications, which is its limitation. Therefore, improving the adaptive ability of itself is a major direction worth studying in the future. At the same time, fractal theory has not been developed for a long time, so some related schemes still need to be further verified in theory and practice. It may be the case that some schemes are feasible in the theoretical stage, but new problems may appear in practice, which still need to be explored and perfected.

Based on the research of the characteristics, advantages and the application of fractal theory in this paper, it can be seen that fractal theory has unique advantages in the practical application of nonlinear systems, and can process short-term, non-stationary and non-periodic signals more effectively. It is very promising to apply this theory to the short board of fault diagnosis of mechanical equipment at this stage. At present, there is still a lack of research on fault diagnosis of mechanical equipment based on transition process signals. Most are to study the vibration signal of the equipment in a stable running state, while ignoring the research on the transition signal of the equipment starting and stopping. The information content of the signal during the transition process is high. It is an important basis for fully grasping the status of mechanical equipment. The transient process has strong signal non-linearity and non-stationarity, and the process time is short. It is a good idea to study the transition process signal based on fractal theory to realize the fault diagnosis and recognition of mechanical system. It has a broad research prospect.

In addition, it is not only effectively applied in the fault diagnosis of nonlinear systems in the mechanical field described in this paper, but also plays an important role in fields such as geography, biology, chemistry, physics and social science, with a very wide range of applications [24, 25].

5. Conclusion
Due to the uniqueness of its research object, fractal theory is very suitable for extracting and analysing irregular features in fault diagnosis of nonlinear mechanical equipment systems. In practical applications, combined with other analysis methods, it can accurately and effectively describe the fault state of the system, with high accuracy and wide application. Based on a large number of related literatures, this paper summarizes the source, principle and application of the theory, and classifies and summarizes the successful cases of fractal theory’s application in fault diagnosis of nonlinear mechanical equipment systems. Then briefly describes the advantages and limitations of the theory in combination with the development status. Through the exploration, it can be seen that most of the current researches on fault diagnosis of nonlinear mechanical equipment are based on the signals during the smooth operation of mechanical equipment, while the signal information content of the transition process such as the start and stop of the equipment is rich, which has great research value. However, the nonlinearity and non-stationarity of the signal during the transition process are high, and the time is short, thus the research is relatively difficult. The fractal theory is suitable for dealing with such irregular and unstable research objects. Therefore, combining with the advantages of fractal theory, this paper puts forward an idea which is based on fractal theory to study the transition process signals to identify mechanical equipment faults. In general, fractal theory has a good prospect and a large development space.

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