Production of the triply heavy $\Omega_{ccc}$ and $\Omega_{bbb}$ baryons at $e^+e^-$ colliders

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Nonrelativistic quantum chromodynamics (NRQCD) factorization formalism is an important approach to investigate the production of the heavy quarkonium. In this paper, we study the production of the $\Omega_{ccc}$ and $\Omega_{bbb}$ at the $e^+e^-$ collider, using the NRQCD factorization formalism. We do the full calculation of the total and differential cross sections of the processes $e^+e^- \to \gamma^*/Z^* \to \Omega_{QQQ} + Q + Q + Q$, in the leading order at the $e^+e^-$ colliders with different energies. In this paper, $\Omega_{QQQ}$ means the triply heavy baryon $\Omega_{ccc}$ or $\Omega_{bbb}$. And $Q$ denotes the $c$ or $b$ quark. The results show that it is hard to observe them at the $e^+e^-$ colliders directly.

I. INTRODUCTION

The doubly charmed baryon $\Xi_{cc}$ has been observed in both SELEX and LHC [1, 2]. Then, different decay channels of the $\Xi_{cc}$ were observed in LHC [3-5]. The lifetime of the $\Xi_{cc}$ was measured in LHC also [6]. While, none of the triply heavy baryons has been observed in experiment. As the baryonic analogues of the heavy quarkonia, triply heavy baryons are helpful to understand the strong interaction between quarks, the hadron structures and weak decays of the heavy baryons. The triply heavy baryons have been studied in theory. About forty years ago, the heavy baryon spectroscopy was studied using the QCD bag model in [7], firstly. Then, the mass of the triply heavy baryons have been determined in kinds of theoretical models, such as, the QCD sum rules [8-11], the lattice QCD [15-20], the hypercentral quark model [21-24], the nonrelativistic quark model [25-30] and the relativistic quark model [31-34], the Faddeev equation [35-37], the symmetry-preserving Schwinger-Dyson equations [38, 39], the variational method [40], the Regge trajectories [41-44], etc.

As that two pairs of heavy quarks need to be produced in the production of the doubly heavy baryon [45-51], three pairs of heavy quarks need to be produced in the production progress of the triply heavy baryon. This makes it difficult to produce and observe the triply heavy baryon in experiment. The production cross sections of the triply heavy baryons at the LHC have been evaluated via fragmentation of the heavy quark in Refs.[52-55]. In [56, 57], the total and differential cross sections of the direct production processes of these baryons at the LHC have been calculated. And the results show that it is quite promising to discover the triply heavy baryons in LHC. The production of the $\Omega_{ccc}$ in heavy-ion collisions have been investigated in Refs.[58-60]. Compared with the hadron collider, the backgrounds at the $e^+e^-$ collider are much less. So, it is necessary to study the production of the triply heavy baryons at the $e^+e^-$ collider. The production of the triply charmed baryon in $e^+e^-$ collisions has been estimated in Ref.[61]. To simply the calculation, the authors in Ref.[61] have taken an approximation ignoring the mass of the charm quark in the numerator of the quark propagator and in all traces. However, as pointed out in Ref.[62], the approximation will lead to quite large errors. In this work, we report the study on the production of the triply heavy baryons $\Omega_{ccc}$ and $\Omega_{bbb}$ at $e^+e^-$ colliders. We calculate the total and differential cross sections of the direct production of the $\Omega_{ccc}$ and $\Omega_{bbb}$ exactly.

Because the constituent quarks of the triply heavy baryon are all heavy flavored, the NRQCD [63] can be used to describe the triply heavy baryon. The production of the $\Omega_{QQQ}$ can be factorized into two parts, the short-distance coefficient which describes the process of the production of the three heavy quark pairs, and the long-distance matrix element which describes the hadronization of three heavy quarks to the triply heavy baryon. The number of the Feynman diagrams corresponding to the short distance coefficient is large. To achieve the calculation, we utilize the characteristic of the constituent quarks as the authors have done in Refs.[56, 57].

The present paper is organized as follows. In Sec.II, we will show the details on the calculation of the two parts, the short-distance coefficient and the long-distance matrix element mentioned above. In Sec.III, we will list the numerical results and conclusions. And in Sec.IV, we will give a brief summary.

II. PRODUCTION OF THE $\Omega_{ccc}$ AND $\Omega_{bbb}$ AT $e^+e^-$ COLLIDERS

For the triply heavy baryons, the relative motions among the heavy valence quarks are nonrelativistic. The typical velocity $v$ of the heavy quark in the rest frame of the $\Omega_{QQQ}$ is small. And there are three distinct energy scales in the triply heavy baryons, i.e., the mass of the heavy quark $m$, the three-momentum of the heavy quark $mv$, and the energy of the heavy quark $mv^2$. And the three energy scales have the relationship of $m \gg mv \gg mv^2$. The direct production of the triply heavy baryons can be described as follows.
heavy quark pairs are produced firstly at energy scale $m$ or higher followed by the formation of the triply heavy baryon at the energy scale $m e$. Namely, the production cross section of the triply heavy baryon can be factorized into the short-distance coefficient and the long-distance matrix element. The short-distance coefficient describes the process of the three heavy quark pairs production which can be expanded as a power series of $\alpha_s$ and $\alpha$ at the energy scale $m$ or higher. The long-distance matrix element describes the hadronization of the triply heavy baryon from the pointlike three heavy quarks.

The triply heavy baryon must be in color singlet. In the leading order, the color wave function of the $\Omega_{QQQ}$ must be $\frac{1}{\sqrt{6}}\xi_1 \xi_2 \xi_3 Q_{1\xi_1} Q_{2\xi_2} Q_{3\xi_3}$ with $\xi_i \ (i = 1, 2, 3)$ being the color index of the valence quark $Q_i$. As a ground state, the orbital angular momentum wave function is symmetrical. The exchange antisymmetry of the identical fermions implies that the $\Omega_{QQQ}$ must be the spin-symmetrical state and the spin of it must be $\frac{3}{2}$. As given in Ref. [56], in the leading Fock state description, the $S$-wave state of the nonrelativistic triply heavy baryon $\Omega_{QQQ}$ can be written as,

$$|\Omega_{QQQ}, \frac{3}{2}, S_2) = \sqrt{\frac{2M}{(2\pi)^3}} \int \frac{d^3 \vec{q}_1}{(2\pi)^3} \frac{d^3 \vec{q}_2}{(2\pi)^3} \frac{1}{\sqrt{3!}} \sum_{\xi_1, \xi_2, \xi_3} \sum_{\eta_1, \eta_2, \eta_3} \frac{\sqrt{6}}{\xi_1 \xi_2 \xi_3} |\Omega_{1\xi_1, \eta_1}, \eta_2, \eta_3)\rangle \langle \Omega_{2\xi_2, \eta_2}, \eta_1, \eta_3),$$

$$\langle \Omega_{2\xi_2, \eta_2}, \eta_1, \eta_3), |\Omega_{3\xi_3, \eta_3}, \eta_1, \eta_2)\rangle,$$

where $\vec{q}_3 = -\vec{q}_1 - \vec{q}_2$, and

$$\langle Q_1, \xi_1, \eta_1, \eta_2 | Q_2, \xi_2, \eta_2, \eta_3 | Q_3, \xi_3, \eta_3, \eta_2, \eta_3)\rangle = \delta_{\eta_1, \eta_2} \delta_{\eta_2, \eta_3} (2\pi)^3 3! E\epsilon(\vec{q}_1, \vec{q}_2) = 1,$$ (1)

In the heavy quark limit, the dependence of the short distance on the momenta, $q_1$ and $q_2$, can be neglected in the leading order. Namely, the momenta of the produced three identical heavy quarks can be treated as the same. As a result, the long-distance matrix is proportional to the wave function of the baryon at the origin,

$$\Psi(0, 0) = \int \frac{d^3 q_1}{(2\pi)^3} \frac{d^3 q_2}{(2\pi)^3} \psi(q_1, q_2) = \frac{1}{\sqrt{6}} \xi_1 \xi_2 \xi_3 f^{abc}(T^a)_{\xi_1, \eta_1} (T^b)_{\xi_2, \eta_2} (T^c)_{\xi_3, \eta_3} = 0,$$ (2)

In this paper, we consider only the contribution from the leading order short-distance coefficient, and the leading order long-distance matrix element. By the standard perturbation theory, the amplitude of the process $e^+ e^- \rightarrow \Omega_{QQQ} + \bar{\Omega} + \bar{Q} + \bar{Q}$ can be written as,

$$A(e^+ e^- \rightarrow \Omega_{QQQ} + \bar{\Omega} + \bar{Q} + \bar{Q}) = \frac{\sqrt{2M}}{\sqrt{(2\pi)^3}} \sqrt{3!} \Psi(0, 0)$$

$$\times M(e^+ e^- \rightarrow (QQQ)_{13, S_2} + \bar{\Omega} + \bar{Q} + \bar{Q}),$$ (4)

in which, $M(e^+ e^- \rightarrow (QQQ)_{13, S_2} + \bar{\Omega} + \bar{Q} + \bar{Q})$ is the matrix element in the leading order of the process $e^+ e^- \rightarrow (QQQ)_{13, S_2} + \bar{\Omega} + \bar{Q} + \bar{Q}$. It is the product of the short-distance coefficient with the color- and spin-wave functions in the long-distance matrix element.

Now, let us consider the short-distance coefficient. The contribution to the production of the three heavy quark pairs comes from the $e^+ e^- \rightarrow \Omega_{QQQ}$ annihilation process mainly,

$$e^-(k_1) + e^+(k_2) \rightarrow Z^* / \gamma^* \rightarrow Q(p_1, \xi_1) + \bar{Q}(p_2, \xi_2) + Q(p_3, \xi_3) + \bar{Q}(p_4, \xi_4) + \bar{Q}(p_5, \xi_5) + \bar{Q}(p_6, \xi_6),$$ (5)

where, $k_1$ and $k_2$ are the 4-momenta of the electron and positron; $p_i \ (i = 1, 6)$ are the 4-momenta of the produced $Q$ and $\bar{Q}$ quarks, with $p_1 = p_2 = p_3$; $\xi_j \ (j = 1, 2, 3)$ are the color indices of the $Q_j$-quarks and $\bar{Q}_j$-quarks, respectively. The produced three pairs of heavy quarks in process (5) have six permutations denoted as, $(Q_1, Q_2, Q_3, \bar{Q}_1, \bar{Q}_2, \bar{Q}_3)$ with $i, j, k = 1, 2, 3$ and $i \neq j \neq k$. In the calculation, we will disregard the contribution of the electroweak interaction between the heavy quarks for the process (5). As a result, there are seven inequivalent topology structures for each of the six permutations as the same as the one in [56]. So, there are 42 topology structures totally for the produced heavy quarks. Inserting the $e^+ e^- \gamma^*$ vertex and $e^+ e^- Z^*$ vertex into the 42 topology structures in all the allowed positions in the tree level, we get 576 Feynman diagrams for the process (5). And, we show nine typical ones in Fig.1.

As pointed out above, the color configuration of the produced three heavy quarks is $T^a = \frac{\lambda^a}{2}$ with $\lambda^a \ (a = 1, \ldots, 8)$ being the Gell-Mann matrices and using the color-flow method in [56], we can get the color factors of the 576 Feynman diagrams. The color factors of the last two diagrams in Fig.1 are both,

$$\sum_{a,b,c} \sum_{\xi_1, \xi_2, \xi_3} \epsilon_{\xi_1 \xi_2 \xi_3} f^{abc}(T^a)_{\xi_1, \eta_1} (T^b)_{\xi_2, \eta_2} (T^c)_{\xi_3, \eta_3} = 0.$$

In which, $f^{abc}= 1$ (a, b, c = 1, ..., 8) is the structure constant of the $SU(3)$ group, and the index $\xi_i \ (i = 1, 2, 3)$.
appears twice, in $\varepsilon^1 \varepsilon^2 \varepsilon^3$ directly and as one of the indices $\xi_1$, $\xi_m$, and $\xi_n$ indirectly. And we see the result is independent onto the indices $i, j, k, l, m$, and $n$. As a result, we get the conclusion that the total contribution to the amplitude of the process (5) from the 72 Feynman diagrams involving the three-gluon vertex vanishes, because the color factors of these Feynman diagrams are all zero. 

The number of the Feynman diagrams which we need to consider reduces to 504. The color factors of the first seven Feynman diagrams in Fig.1 are all the same,

$$\sum_{a,b} \sum_{\xi_1, \xi_2, \xi_3} \frac{1}{\sqrt{6}} \varepsilon^{x_1 \xi_2 \xi_3} (T^a)_{\xi_1 \eta} (T^b)_{\xi_2 \eta} (T^c)_{\xi_3 \eta} = (-1)^N \frac{4}{3} \frac{4}{9} \varepsilon^{x_1 x_2 x_3},$$

in which $N$ is the number of permutations of transforming the set $(\xi_1, \xi_m, \xi_n)$ to the set $(\xi_1, \xi_2, \xi_3)$. For each of the Feynman diagrams, there is a Feynman factor $(-1)^N$, where $N^*$ equals the total number of permutations of transforming the set $(i, m, n)$ to the set $(1, 2, 3)$ and transforming the set $(i, j, k)$ to the set $(1, 2, 3)$. Subsuming the Feynman factor into the corresponding color factor, we find that all the color factors of the remaining 504 Feynman diagrams are the same,

$$C_{\text{col}} = (-1)^N \left(-1\right)^N \frac{4}{9} \frac{4}{9} \varepsilon^{x_1 x_2 x_3} = \frac{4}{9} \frac{4}{9} \varepsilon^{x_1 x_2 x_3}. $$

Let us consider the remaining 504 Feynman diagrams. For each one of the six permutations given above, there are 84 Feynman diagrams, in which 42 ones correspond to the process $e^- + e^+ \rightarrow Z^* \rightarrow Q_1 + Q_2 + Q_3$ and the other 42 ones correspond to the process $e^- + e^+ \rightarrow \gamma^* \rightarrow Q_1 + Q_2 + Q_3 + Q_1 + Q_2 + Q_3$. In the leading order, the momenta of the three heavy quarks are considered to be equal. As a result, the contribution of every 84 Feynman diagrams corresponding to the six permutations are the same. The total amplitude for the process $e^- e^+ \rightarrow (QQQ)_1^{(2, S_x)} + Q + Q + Q$

$$\mathcal{M}(e^- e^+ \rightarrow (QQQ)_1^{(2, S_x)} + Q + Q + Q) = 504 \sum_{k=1}^{504} C_{\text{col}} \Gamma_k$$

$$= 3! \sum_{k=1}^{84} C_{\text{col}} \Gamma'_k,$$

where $\Gamma_k (k = 1, 2, ..., 504)$ are the matrices without color factors of the Feynman diagrams, and $\Gamma'_k (k = 1, 2, ..., 84)$ are the matrices without color factors of the 84 Feynman diagrams corresponding to the permutation $(Q_1 Q_2 Q_3 Q_4 Q_5 Q_6)$.

### III. NUMERICAL RESULTS AND DISCUSSIONS

Now, the cross section of the progress (5) can be written as

$$\sigma = \frac{1}{3!} \int \sum_{S_x, \xi_1} \left( \begin{array}{c} 2 \pi \delta(k_1 + k_2 - P - p_4 - p_5 - p_6) \\ \frac{1}{2} \end{array} \right)$$

$$d \Pi_x \frac{1}{4} \sum_{\xi_1, \xi_2, \xi_3} |\mathcal{A}(e^- e^+ \rightarrow QQQQ'Q'Q)|^2,$$

with

$$d \Pi_x = \frac{d^3 P}{(2\pi)^3 E} \frac{d^3 p_4}{(2\pi)^3 E_{p_4}} \frac{d^3 p_5}{(2\pi)^3 E_{p_5}} \frac{d^3 p_6}{(2\pi)^3 E_{p_6}},$$

where $\xi_i (i = 1, 2, 3)$, $s_1$ and $s_2$ are the spins of the anti-quarks $\bar{Q}_1$, $e^+$ and $e^-$, respectively.

To do the numerical calculation, the parameters are taken as follows: $m_Z = 91.18$ GeV, $\Gamma_Z = 2.49$ GeV, $\alpha(m_Z) = 1/127.95$, $\sin^2 \theta_w = 0.224$, $m_c = 1.5$ GeV, $|\Psi_{\Omega_{ccc}}(0,0)|^2 = 0.36 \cdot 10^{-3}$ GeV$^6$ as in [61], and $m_b = 4.9$ GeV, $|\Psi_{\Omega_{bbh}}(0,0)|^2 = 0.189$ GeV$^6$ as in [57].

For the electromagnetic coupling constant, we adopt

$$\alpha(q) = \frac{\alpha(m_Z)}{1 - \frac{2 a_0(m_Z)}{3} \log\left(\frac{q}{m_Z}\right)},$$

where $q$ denotes the energy scale of the electromagnetic coupling constant, and we take $q = \sqrt{s}$ with $\sqrt{s}$ being the colliding energy in the center-of-mass frame. And the strong coupling constant, we adopt

$$\alpha_s(\mu) = \frac{\alpha_s(m_Z)}{1 - \frac{b_0}{2} \alpha_s(m_Z) \log\left(\frac{\mu}{m_Z}\right)},$$

with $b_0 = 11 - \frac{2}{3} n_f$, where $\mu$ is the energy scale and $\alpha_s(m_Z) = 0.118$. For the production of the $\Omega_{ccc}$, $n_f$ is taken to be 4 when $\mu \leq 2 m_b$ and 5 when $\mu$ is larger than $2 m_b$. And for the production of the $\Omega_{bbh}$, $n_f$ is taken to be 5. For comparison, we take two different values for $\mu$, i.e., $\mu = \sqrt{s}/2$ and $\mu = m_T/2$, where $m_T$ is the transverse mass of the produced baryon, namely $m_T^2 = p_T^2 + M^2$. And in our numerical calculation, the integrals of the phase-space are achieved by VEGAS [64].

SuperKEKB, the asymmetric beam energy $e^- e^+$ collider, is an upgrade of the KEKB accelerator facility. The target integrated luminosity of 50 ab$^{-1}$ to be collected by the Belle II experiment. And a high luminosity $e^- e^+$ collider named the Circular Electron-Positron Collider (CEPC) has been proposed by the Chinese particle physics community. The CEPC is designed to operate at three different modes, as a Higgs factory at

\[^3\text{Here, } p_T \text{ means the transverse momentum of the produced triply heavy baryon.} \]
\[ \sqrt{s} = 240 \text{ GeV}, \text{as a Z factory at } \sqrt{s} = 91.2 \text{ GeV and performing } WW \text{ threshold scans around } \sqrt{s} = 160 \text{ GeV} \ [65]. \]

We calculate the production cross sections of the baryons \( \Omega_{ccc} \) and \( \Omega_{bbb} \) at CEPC with the energy \( \sqrt{s} = 91.2 \text{ GeV and } \sqrt{s} = 160 \text{ GeV}, \) and at Belle II with the center-of-mass energy \( \sqrt{s} = 10.58 \text{ GeV}. \) The results are shown in Table I. And, we calculate the production of the \( \Omega_{ccc} \) and \( \Omega_{bbb} \) at the \( e^+e^- \) colliders with different colliding energies, which are shown in Fig.2 and Fig.3. From the Table I, Fig.2 and Fig.3, we see that the cross sections for the production of the \( \Omega_{ccc} \) and \( \Omega_{bbb} \) at \( \sqrt{s} = 91.2 \text{ GeV are larger than the corresponding ones at other colliding energies. This is because the cross sections contributed} \]

\[ \text{by the Z-boson exchange processes are proportional to a factor of } 1/\left( (s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2 \right), \text{which has a peak at the Z pole and decreases rapidly when the } \sqrt{s} \text{ deviates from } m_Z, \text{as pointed in Ref.[66].} \]

In this paper, we also calculate the differential cross sections \( d\sigma/dp_T \) of the production of the \( \Omega_{ccc} \) and \( \Omega_{bbb} \) at \( \sqrt{s} = 91.2 \text{ GeV, which are shown in Figs.4 and 5.} \]

As we see that both the integrated cross sections and differential cross sections are proportional to \( |\Psi(0,0)|^2, \alpha^2(q)\text{and }\alpha_s^4(\mu). \) So the numerical results can be changed by one even two orders using different the wave function at the origin of the triply heavy baryons, different running coupling constants and different energy scale.

**FIG. 1.** Nine typical Feynman diagrams for the process (5). The indices \( i, j, k, l, m, n = 1, 2, 3 (\text{with } i \neq j \neq k \text{ and } l \neq m \neq n). \)
FIG. 2. The production cross section of the $\Omega_{ccc}$ produced by $e^+e^-$ annihilation at different energies.

choices. We can get the conclusion from the numerical results shown in Figs.2 and 3, that the production cross sections of the $\Omega_{ccc}$ and $\Omega_{bbb}$ take their maximum values at $\sqrt{s} = m_Z$. From the numerical results in Table I, we know that it is impossible to observe $\Omega_{ccc}$ at SuperKEKB, if there is no other models to produce them at the $e^+e^-$ colliders. Also, we see there are around $32-198$ events for the production of $\Omega_{ccc}$ and $7-20$ events for the production of $\Omega_{bbb}$, at CEPC with $16 \text{ab}^{-1}$ integrated luminosity running at 91.2 GeV.

In this work, we also calculate the production cross section of the $\Omega_{ccc}$ using the same parameters as in Ref. [61]. And the numerical result is $0.20(2) \times 10^{-2}$ fb, which is about an order of magnitude smaller than the one $0.0404(4) \text{fb}$ in Ref.[61]. This huge difference may can be explained by the approximation adopted in Ref.[61].

|       | $\sqrt{s}$ | $\mu$         | $\Omega_{ccc}$       | $\Omega_{bbb}$       |
|-------|------------|---------------|----------------------|----------------------|
| CEPC  | 91.2 GeV   | $\sqrt{s}/2$ | 0.00204(5)           | 0.437(6) $\times 10^{-3}$ |
|       |           | $m_{Z}/2$     | 0.0124(3)            | 1.25(2) $\times 10^{-3}$ |
|       |           | $160 \text{ GeV}$ | $\sqrt{s}/2$ | 0.214(9) $\times 10^{-5}$ | 0.55(2) $\times 10^{-6}$ |
|       |           | $m_{Z}/2$     | 0.108(9) $\times 10^{-4}$ | 0.183(4) $\times 10^{-5}$ |
| Belle II | 10.58 GeV  | $\sqrt{s}/2$ | 0.249(1) $\times 10^{-5}$ | -                      |
|       |           | $m_{Z}/2$     | 0.707(3) $\times 10^{-5}$ | -                      |

TABLE I. Production cross sections (in unit fb) of the $\Omega_{ccc}$ and $\Omega_{bbb}$ at the CEPC and Belle II.

FIG. 5. The $p_T$ distributions of of the $\Omega_{bbb}$ produced at CEPC with $\sqrt{s} = 91.2$ GeV.

IV. SUMMARY

We have studied the production of the $\Omega_{ccc}$ and $\Omega_{bbb}$ at the SuperKEKB and the CEPC. From the numerical results, we conclude that it is impossible to find the triply heavy baryons at SuperKEKB, and it is hard to find $\Omega_{ccc}$ and $\Omega_{bbb}$ at the CEPC because of the few events.

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[1] M. Mattson et al. (SELEX), First Observation of the Doubly Charmed Baryon Ξ_{cc}^+, Phys. Rev. Lett. 89, 112001 (2002), arXiv:hep-ex/0208014.

[2] R. Aaij et al. (LHCb), Observation of the doubly charmed baryon Ξ^{++}_{cc}, Phys. Rev. Lett. 119, 112001 (2017), arXiv:1707.01621 [hep-ex].

[3] R. Aaij et al. (LHCb), First Observation of the Doubly Charmed Baryon Decay Ξ_{cc}^{++} → Ξ^+_c π^+, Phys. Rev. Lett. 121, 162002 (2018), arXiv:1807.01919 [hep-ex].

[4] R. Aaij et al. (LHCb), Search for the doubly charmed baryon Ξ_{cc}^{++} in the Ξ^+_c π^− π^+ final state, JHEP 12, 107 (2021), arXiv:2109.07292 [hep-ex].

[5] R. Aaij et al. (LHCb), Observation of the doubly charmed baryon decay Ξ_{cc}^{++} → Ξ^+_c π^+, JHEP 05, 038 (2022), arXiv:2202.05648 [hep-ex].

[6] R. Aaij et al. (LHCb), Measurement of the Lifetime of the Doubly Charmed Baryon Ξ_{cc}^{++}, Phys. Rev. Lett. 121, 052002 (2018), arXiv:1806.02744 [hep-ex].

[7] P. Hasenfratz, R. R. Horgan, J. Kuti, and J. M. Richard, Heavy Baryon Spectroscopy in the QCD Bag Model, Phys. Lett. B 94, 401 (1980).

[8] J.-R. Zhang and M.-Q. Huang, Deciphering triply heavy baryons in terms of QCD sum rules, Phys. Lett. B 674, 28 (2009), arXiv:0902.3297 [hep-ph].

[9] Z.-G. Wang, Analysis of the Triply Heavy Baryon States with QCD Sum Rules, Commun. Theor. Phys. 58, 723 (2012), arXiv:1112.2274 [hep-ph].

[10] Z.-G. Wang, Analysis of the triply-heavy baryon states with the QCD sum rules, AAPPs Bull. 31, 5 (2021), arXiv:2010.08939 [hep-ph].

[11] T. M. Aliev, K. Azizi, and M. Savci, Masses and Residues of the Triply Heavy Spin-1/2 Baryons, JHEP 04, 042 (2013), arXiv:1212.6065 [hep-ph].

[12] T. M. Aliev, K. Azizi, and M. Savci, Properties of triply heavy spin-3/2 baryons, J. Phys. G 41, 065003 (2014), arXiv:1404.2091 [hep-ph].

[13] N. Alomayyah and T. Barakat, The excited states of triply-heavy baryons in QCD sum rules, Eur. Phys. J. A 56, 76 (2020).

[14] K. Azizi, T. M. Aliev, and M. Savci, Properties of doubly and triply heavy baryons, J. Phys. Conf. Ser. 556, 012016 (2014).

[15] S. Meinel, Prediction of the Omega_bbb mass from lattice QCD, Phys. Rev. D 82, 114514 (2010), arXiv:1008.3154 [hep-lat].

[16] R. A. Briceno, H.-W. Lin, and D. R. Bolton, Charmed-Baryon Spectroscopy from Lattice QCD with N_f = 2+1+1 Flavors, Phys. Rev. D 86, 094504 (2012), arXiv:1207.5356 [hep-lat].

[17] Y. Namekawa et al. (PACS-CS), Charmed baryons at the physical point in 2+1 flavor lattice QCD, Phys. Rev. D 87, 094512 (2013), arXiv:1301.4743 [hep-lat].

[18] M. Padmanath, R. G. Edwards, N. Mathur, and M. Peardon, Spectroscopy of triply-charmed baryons from lattice QCD, Phys. Rev. D 90, 074504 (2014), arXiv:1307.7022 [hep-lat].

[19] Z. S. Brown, W. Detmold, S. Meinel, and K. Orginos, Charmed bottom baryon spectroscopy from lattice QCD, Phys. Rev. D 90, 094507 (2014), arXiv:1409.0497 [hep-lat].

[20] N. Mathur, M. Padmanath, and S. Mondal, Precise predictions of charmed-bottom hadrons from lattice QCD, Phys. Rev. Lett. 121, 202002 (2018), arXiv:1806.04151 [hep-lat].

[21] B. Patel, A. Majethiya, and P. C. Vinodkumar, Masses and Magnetic moments of Triply Heavy Flavour Baryons in Hypercentral Model, Pramana 72, 679 (2009), arXiv:0808.2880 [hep-ph].

[22] N. Tazimi and A. Ghampour, Mass spectrum of triply heavy baryon in the hypercentral quark model, Mod. Phys. Lett. A 36, 2150270 (2021).

[23] Z. Shah and A. K. Rai, Mass spectra of triply heavy charm-beauty baryons, EPJ Web Conf. 202, 06001 (2019).

[24] A. K. Rai and Z. Shah, Regge Trajectories of triply heavy baryons, J. Phys. Conf. Ser. 934, 012035 (2017).

[25] J. Vijande, A. Valcarce, and H. Garciaizo, Constituent-quark model description of triply heavy baryon nonperturbative lattice QCD data, Phys. Rev. D 91, 054015 (2015), arXiv:1507.03735 [hep-ph].

[26] Z. Shah and A. K. Rai, Masses and Regge trajectories of triply heavy Ω_{ccc} and Ω_{bub} baryons, Eur. Phys. J. A 53, 195 (2017).

[27] G. Yang, J. Ping, P. G. Ortega, and J. Segovia, Triply heavy baryons in the constituent quark model, Chin. Phys. C 44, 023102 (2020), arXiv:1904.10166 [hep-ph].

[28] F. J. Llanes-Estrada, O. I. Pavlova, and R. Williams, Triply heavy baryon mass estimated within pNRQCD, Acta Phys. Polon. Supp. 6, 821 (2013), arXiv:1304.3636 [nucl-th].

[29] F. J. Llanes-Estrada, O. I. Pavlova, and R. Williams, A First Estimate of Triply Heavy Baryon Masses from the pNRQCD Perturbative Static Potential, Eur. Phys. J. C 72, 2019 (2012), arXiv:1111.7087 [hep-ph].

[30] A. Kakadiya, Z. Shah, and A. K. Rai, Spectroscopy of Ω_{ccc} and Ω_{bub} baryons, Int. J. Mod. Phys. A 37, 2250225 (2022), arXiv:2303.03771 [hep-ph].

[31] R. N. Faustov and V. O. Gulin, Triply heavy baryon spectroscopy in the relativistic quark model, Phys. Rev. D 105, 014013 (2022), arXiv:2111.07702 [hep-ph].

[32] A. P. Martynenko, Ground-state triply and doubly heavy baryons in a relativistic three-quark model, Phys. Lett. B 663, 317 (2008), arXiv:0708.2033 [hep-ph].

[33] S. Migura, D. Merten, B. Metsch, and H.-R. Petry, Charmed baryons in a relativistic quark model, Eur. Phys. J. A 28, 41 (2006), arXiv:hep-ph/0602153.

[34] T. Bhavsar, M. Shah, and P. C. Vinodkumar, A relativistic approach for triply heavy avour baryon, DAE Symp. Nucl. Phys. 63, 840 (2018).

[35] M. Radin, S. Babaghodrat, and M. Monemzadeh, Estimation of heavy baryons masses Ω_{ccc} and Ω_{bub} by solving the Faddeev equation in a three-dimensional approach, Phys. Rev. D 90, 047701 (2014).

[36] S.-x. Qin, C. D. Roberts, and S. M. Schmidt, Spectrum of light- and heavy-baryons, Few Body Syst. 60, 26 (2019), arXiv:1902.00026 [nucl-th].
[37] W. Zheng and H. R. Pang, *Momentum-space Faddeev calculations for ground-state triply and doubly heavy baryons in the constituent quark model*, Mod. Phys. Lett. A 25, 2077 (2010).

[38] P.-L. Yin, C. Chen, G. a. Krein, C. D. Roberts, J. Segovia, and S.-S. Xu, *Masses of ground-state mesons and baryons, including those with heavy quarks*, Phys. Rev. D 100, 034008 (2019), arXiv:1903.00160 [nucl-th].

[39] L. X. Gutiérrez-Guerrero, A. Bashir, M. A. Bedolla, and E. Santopinto, *Masses of Light and Heavy Mesons and Baryons: A Unified Picture*, Phys. Rev. D 100, 114032 (2019), arXiv:1911.09213 [nucl-th].

[40] Y. Jia, *Variational study of weakly coupled triply heavy baryons*, JHEP 10, 073 (2006), arXiv:hep-ph/0607290.

[41] K.-W. Wei, B. Chen, and X.-H. Guo, *Masses of doubly and triply charmed baryons*, Phys. Rev. D 92, 076008 (2015), arXiv:1503.05184 [hep-ph].

[42] K.-W. Wei, B. Chen, N. Liu, Q.-Q. Wang, and X.-H. Guo, *Spectroscopy of singly, doubly, and triply bottom baryons*, Phys. Rev. D 95, 116005 (2017), arXiv:1609.02512 [hep-ph].

[43] J. Oudichhya, K. Gandhi, and A. K. Rai, *Ground and excited state masses of Ωcc0, Ωcc+ and Ωccc++ baryons*, Phys. Rev. D 103, 114030 (2021), arXiv:2105.10647 [hep-ph].

[44] J. Oudichhya, K. Gandhi, and A. K. Rai, *Mass-spectra of singly, doubly, and triply bottom baryons*, Phys. Rev. D 104, 114027 (2021), arXiv:2111.00236 [hep-ph].

[45] Y. Jin, S.-Y. Li, Z.-G. Si, Z.-J. Yang, and T. Yao, *Colour connections of four quark QQQ′Q′ system and doubly heavy baryon production in e+e− annihilation*, Phys. Lett. B 727, 468 (2013), arXiv:1309.5849 [hep-ph].

[46] J. Jiang, X.-G. Wu, S.-M. Wang, J.-W. Zhang, and Z.-Y. Fang, *A Further Study on the Doubly Heavy Baryon Production around the Z0 Peak at A High Luminosity e+e− Collider*, Phys. Rev. D 87, 054027 (2013), arXiv:1302.0601 [hep-ph].

[47] J. Jiang, X.-G. Wu, Q.-L. Liao, X.-C. Zheng, and Z.-Y. Fang, *Doubly Heavy Baryon Production at A High Luminosity e+e− Collider*, Phys. Rev. D 86, 054021 (2012), arXiv:1208.3051 [hep-ph].

[48] J.-W. Zhang, X.-G. Wu, T. Zhong, Y. Yu, and Z.-Y. Fang, *Hadronic Production of the Doubly Heavy Baryon Ξbc at LHC*, Phys. Rev. D 83, 034026 (2011), arXiv:1101.1130 [hep-ph].

[49] J. P. Ma and Z. G. Si, *Factorization approach for inclusive production of doubly heavy baryon*, Phys. Lett. B 568, 135 (2003), arXiv:hep-ph/0305079.

[50] G. Chen, X.-G. Wu, Z. Sun, Y. Ma, and H.-B. Fu, *Photoproduction of doubly heavy baryon at the ILC*, JHEP 12, 018 (2014), arXiv:1408.4615 [hep-ph].

[51] Z.-J. Yang, P.-F. Zhang, and Y.-J. Zheng, *Doubly Heavy Baryon Production in e+e− Annihilation*, Chin. Phys. Lett. 31, 051301 (2014).

[52] V. A. Saleev, *Omega(ccc) production via fragmentation at LHC*, Mod. Phys. Lett. A 14, 2615 (1999), arXiv:hep-ph/9906515.

[53] M. A. Gomshi Nobary and R. Sepahvand, *An Investigation of triply heavy baryon production at hadron colliders*, Nucl. Phys. B 741, 34 (2006), arXiv:hep-ph/0508115.

[54] M. A. Gomshi Nobary and R. Sepahvand, *Triply heavy baryons*, eConf C0605151, 0010 (2006), arXiv:hep-ph/0609123.

[55] M. A. Gomshi Nobary and R. Sepahvand, *Fragmentation of triply heavy baryons*, Phys. Rev. D 71, 034024 (2005), arXiv:hep-ph/0406148.

[56] Y.-Q. Chen and S.-Z. Wu, *Production of Triply Heavy Baryons at LHC*, JHEP 08, 144 (2011), [Erratum: JHEP 09, 089 (2011)], arXiv:1106.0193 [hep-ph].

[57] S.-Z. Wu, Y.-W. Li, and R. Rashidin, *Heaviest bound baryons production at the Large Hadron Collider*, Phys. Rev. D 86, 114504 (2012).

[58] J. Zhao and P. Zhuang, *Multicharmed Baryon Production in High Energy Nuclear Collisions*, Few Body Syst. 58, 100 (2017).

[59] H. He, Y. Liu, and P. Zhuang, *Ωccc production in high energy nuclear collisions*, Phys. Lett. B 746, 59 (2015), arXiv:1409.1009 [hep-ph].

[60] F. Becattini, *Production of multiply heavy flavored baryons from quark gluon plasma in relativistic heavy ion collisions*, Phys. Rev. Lett. 95, 022301 (2005), arXiv:hep-ph/0503239.

[61] S. P. Baranov and V. L. Slad, *Production of triply charmed Omega(ccc) baryons in e+ e− annihilation*, Phys. Atom. Nucl. 67, 808 (2004), arXiv:hep-ph/0603090.

[62] C.-H. Chang and Y.-Q. Chen, *The Production of B(c) or anti-B(c) meson associated with two heavy quark jets in Z0 boson decay*, Phys. Rev. D 46, 3845 (1992), [Erratum: Phys.Rev.D 50, 6013(E) (1994)].

[63] G. T. Bodwin, E. Braaten, and G. P. Lepage, *Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium*, Phys. Rev. D 51, 1125 (1995), [Erratum: Phys.Rev.D 55, 5853(E) (1997)], arXiv:hep-ph/9407339.

[64] G. P. Lepage, *A New Algorithm for Adaptive Multidimensional Integration*, J. Comput. Phys. 27, 192 (1978).

[65] F. An et al., *Precision Higgs physics at the CEPC*, Chin. Phys. C 43, 043002 (2019), arXiv:1810.09037 [hep-ex].

[66] Z.-Y. Zhang, X.-C. Zheng, and X.-G. Wu, *Production of the Bc meson at the CEPC*, Eur. Phys. J. C 82, 246 (2022), arXiv:2111.13917 [hep-ph].