High-contrast ZZ interaction using multi-type superconducting qubits

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For building a scalable quantum processor with superconducting qubits, the ZZ interaction is of great concern because of its relevance to implementing two-qubit gates, and the close contact between gate infidelity and its residual. Two-qubit gates with fidelity beyond fault-tolerant thresholds have been demonstrated using the ZZ interaction. However, as the performance of quantum processor improves, the residual static-ZZ can also become a performance-limiting factor for quantum gate operations and quantum error correction. Here, we introduce a scalable superconducting architecture to address this challenge. We demonstrate that by coupling two superconducting qubits with opposite-sign anharmonicities together, high-contrast ZZ interaction can be realized in this architecture. Thus, we can control ZZ interaction with high on/off ratio for implementing two-qubit CZ gate, or suppress it during the two-qubit gate operations using XY interaction (e.g. iSWAP). Meanwhile, the ZZ crosstalk related to neighboring spectator qubits can also be heavily suppressed in fixed coupled multi-qubit systems. This architecture could provide a promising way towards scalable superconducting quantum processor with high gate fidelity and low qubit crosstalk.

I. INTRODUCTION

Engineering a physical system towards fault-tolerant quantum computing demands quantum gates with error rates below the fault-tolerant threshold, which has been demonstrated in small-sized quantum processor with superconducting qubits [1]. At present, even high-performance superconducting quantum processor with dozens of qubits becomes available [2], but realizing fault-tolerant quantum computing is still out of reach, mainly as a result of the heavy overhead needed for error-correction with state-of-the-art gate performance. Since further reducing gate error rates enables more efficient scaling and lower overhead, the improvement of the gate performance is a leading task for realizing fault-tolerant quantum computing with superconducting qubits.

In today’s superconducting quantum processor, apart from increasing qubit coherence times, speeding up the gates can also fundamentally improve the gate performance. However, there is a fundamental limitation imposing a trade-off between gate speed and infidelity related to parasitic interactions. Since the current state of two-qubit gates typically tends to have slower gate speed and worse fidelity than single-qubit gates [3], this trade-off issue is particularly relevant for two-qubit gates. For implementing fast two-qubit gates with strong two-qubit coupling, one of the major parasitic interactions is the ZZ coupling, which is mostly related to the coupling between higher energy levels of qubits [4, 5]. This ZZ-type interaction, which describes that the frequency shift of one qubit depends on the state of the other, has been shown to act as a double-edged sword for superconducting quantum computing: it can be used to implement fast-speed and high-fidelity controlled-Z (CZ) gate [1, 6–8], yet it can also degrade performance of two-qubit gates using XY interaction (e.g. iSWAP) [2, 7–14]. Moreover, in fixed coupled multi-qubit systems, such as the one shown in Fig.1(a), where the circles denote qubits, and each gray line indicates the coupler for adjacent qubits, for gate operations in the two qubits enclosed by red rectangle, there are six neighboring spectator qubits, and the ZZ coupling related to these spectator qubits cannot be fully turned off by tuning qubits out of resonance [1]. The most obvious issue with its residual is manifested as crosstalk, resulting in addressing error and phase error during gate operations and error correction [15–21]. Further, these errors caused by the ZZ crosstalk are correlated multi-qubit errors, which is particularly harmful for realizing fault-tolerant scheme [22]. Consequently, combining these errors related to parasitic ZZ interactions leads to challenges for improving gate fidelity. In particular, as the gate performance improves, even the residual parasitic interaction can become performance limiting factor in the long term. Therefore, to avoid these detrimental effects, it is highly desirable to have high-contrast control over this parasitic coupling.

In this work, we introduce a superconducting architecture consisting of two-type superconducting qubits to address the change that comes from parasitic ZZ coupling. In our proposed architecture, two qubits with opposite-sign anharmonicities are coupled together, and high-contrast ZZ interaction can be realized by engineering the system parameters. Thus, by utilizing the ZZ interaction with high on/off ratio in this architecture, the CZ gate can be implemented with a speed faster than that of the traditional setup using only one type of qubit. Meanwhile, the parasitic ZZ coupling can also be deliberately suppressed during the two-qubit gate operations using XY interaction such as iSWAP gate, while leaving the XY interaction completely intact. The proposed architecture can also be scaled up to multi-qubit case, and in fixed coupled system, the ZZ crosstalk related to spectator qubits could also be heavily suppressed.
FIG. 1. (a) Layout of a two-dimensional nearest-neighbor lattice, where circles at the vertices denote the qubits, and gray lines indicate the coupler between adjacent qubits. The lattice consists of two-type qubits arranged in an -A-B-A-B- pattern in each row and column. (b) The circles with A and B label qubits with opposite-sign anharmonicities, and each one can be treated as a three-level anharmonic oscillator. Typically, transmon qubits and capacitive-shunted flux qubit can be modeled as an anharmonic oscillator with negative and positive anharmonicity, respectively. Qubits can be coupled to each other (c) directly via a capacitor or (d) indirectly using a resonator.

II. SUPERCONDUCTING CIRCUITS WITH OPPOSITE-SIGN ANHARMONICITY

We consider a superconducting architecture (hereinafter AB-type) where two qubits with opposite-sign anharmonicities are coupled together. The architecture can be treated as a module which can be easily scaled up to multi-qubit lattice case, and in Fig. 1(a), we show a case of nearest-neighbor-coupled qubit lattice, where circles with A and B label two-type qubits with opposite-sign anharmonicity arranged in an -A-B-A-B- pattern, i.e. only qubits with opposite-sign are coupled together. As shown Fig. 1(b), both qubits can be modeled as a three-level (i.e., $|0\rangle$, $|1\rangle$, $|2\rangle$) anharmonic oscillator for which the Hamiltonian is given as

$$H_1 = \omega_l q_l^\dagger q_l + \frac{\alpha_l}{2} q_l^\dagger q_l (q_l^\dagger q_l - 1),$$

where the subscript $l = a, b, c$ labels different-type anharmonic oscillator with anharmonicity $\alpha_l$ and frequency $\omega_l$, and $q_l$ ($q_l^\dagger$) is the associated annihilation (creation) operator truncated to the lowest three-level. Commonly, by ignoring higher levels, the transmon qubits and the capacitive-shunted flux qubit can be treated as anharmonic oscillators with negative and positive anharmonicity, thus can be described by the Hamiltonian in Eq. (1) with $\alpha_a < 0$ and $\alpha_b > 0$, respectively [23–25]. In principle, they can be coupled via a capacitor or a resonator, as shown in Fig. 1(c) and 1(d). For the direct coupled case, the Hamiltonian of the coupled two-qubit system is $H_3 = H_a + H_b + H_{1}$ with $H_{1} = g_{1}(q_{a}^\dagger q_{b} + H.c.)$ describing coupling terms, and $g$ is the coupling strength. While for the case of indirect coupled via a resonator, the Hamiltonian is $H_3 = H_a + H_b + H_{c.f}$, where $H_{c.f}$ is the Hamiltonian of the resonator, which can be treated as an anharmonic oscillator with zero anharmonicity, thus it can be described by Eq. (1) with $\alpha_c = 0$, and $H_{c.f} = (g_a q_a^\dagger c + g_b q_b^\dagger c + H.c.)$ describes the qubit-resonator coupling terms with strength $g_{a,b}$. For clarity, in the following discussion, we typically specialize in the superconducting architecture that consists of two coupled qubits with opposite-sign anharmonicity for which the dynamics can be described by the Hamiltonian $H_{1,2}$, but since the architecture we studied can be easily scaled up to multi-qubit lattice case, as shown in Fig. 1(a), the result we obtain can be totally applied to the whole lattice.

Before describing our main ideal for engineering high-contrast ZZ interaction in our proposed architecture, let us first examine the origin of the parasitic ZZ interactions in a traditional setup (hereinafter AA-type), where two transmon qubits are directly coupled, and can be described by the Hamiltonian $H_1$ with $\alpha_{a,b} < 0$. Fig. 2(a) shows numerically calculated energy-level of coupled qubits with anharmonicities $\alpha_{a,b} = -\alpha$, and $\alpha/2\pi = 250$ MHz ($\alpha$ is a positive number throughout this work). One can find that there are four avoided crossings, one corresponds to the XY interaction in one-excitation manifold, i.e., interaction between $|01\rangle$ and $|10\rangle$, and the other three associate with interactions among the two-excitation manifold consisting of qubit state $|11\rangle$ and noncomputational states $(|02\rangle, |20\rangle)$. The smallest one is the higher-order coupling between states $|02\rangle$ and $|20\rangle$, and the last two correspond to the interaction between noncomputational states $(|02\rangle, |20\rangle$) and qubit state $|11\rangle$ which changes the energy of $|11\rangle$, thus causing ZZ interaction. For quantitative analysis, the strength of the ZZ coupling is $\zeta = (E_{11} - E_{01}) - (E_{10} - E_{00})$ (the state with a tilde denotes the logical eigenstates, which has the maximum overlap with the bare basis states), and the analytical result is

$$\zeta = -J(\tan \frac{\theta_a}{2} - \tan \frac{\theta_b}{2}),$$

with $\tan \theta_{a,b} = 2J/|\Delta \pm \alpha_{a,b}|$, and $\Delta = \omega_a - \omega_b$ denotes the qubit detuning, and $J = \sqrt{2}g_0$ is the coupling strength between $(|02\rangle, |20\rangle)$ and $|11\rangle$. When $J << |\Delta - \alpha|$, Eq. (2) can be approximated by $\zeta = -J^2/|\Delta - \alpha|$, which changes the energy of $|11\rangle$, and each one independently associates with the coupling between qubit state $|11\rangle$ and one noncomputational state $(|02\rangle$ or $|20\rangle$). This means that the two terms can be controlled independently by engineering the anharmonicities of the two qubits.

From the expression of the ZZ coupling for the traditional setup, one can find that by replacing one of the two transmon qubits by a superconducting qubit for which the magnitude of anharmonicity comparable to that of the transmon qubit, but with positive sign, the ZZ interaction from the two terms destructively interferes, thus the coupling can be heavily suppressed. In Fig. 2(b), we show numerically calculated energy level for this case with $\alpha_a/2\pi = 250$ MHz, and keep all other parameters the same as in Fig. 2(a). Compared with the traditional setup, the avoided crossing associated with the interaction between $|01\rangle$ and $|10\rangle$ is completely intact, but the interaction among two-excitation manifold forms an avoided crossing with triplets. At the triple degeneracy point shown in the inset of Fig. 2(b), the eigenstates are $(|02\rangle + |20\rangle - \sqrt{2}|11\rangle)/2$, $(|02\rangle - |20\rangle)/\sqrt{2}$, $(|02\rangle + |20\rangle + \sqrt{2}|11\rangle)/2$.
with the corresponding energies of $E_{11} - \sqrt{2}J$, $E_{11}$, and $E_{11} + \sqrt{2}J$ (see Appendix A for more details).

III. HIGH CONTRAST ZZ COUPLING

Before analyzing the ZZ coupling, particularly in the region close to the triple degeneracy point, we note that with different state labeling schemes applied to the $|11\rangle$, the result is fundamentally different. By choosing a labeling scheme from the point of view of adiabatically varying the qubit detuning, i.e., labeling the fifth eigenstate as the logical state $|11\rangle$, one can find that the ZZ coupling is completely eliminated in the whole regime including the triple degeneracy point. Thus, unlike in the traditional setup, there is no frequency shift (or accumulated phase) causing by the interaction between $(|02\rangle, |20\rangle)$ and $|11\rangle$. In the following discussion, we choose a simpler labeling scheme for the numerical analysis of ZZ coupling, where eigenstates with the maximum overlap with bare basis are taken as the corresponding logical states. Away from the triple degeneracy point, the result is the same as the former labeling scheme, and the strength can be expressed by Eq. (2). However, in the region close to the triple degeneracy point, the ZZ coupling is nonzero, and has coupling strength of $\sqrt{2}J$ at the degeneracy point, thus CZ gate can still be realized with diabatic scheme [4, 7] in this architecture.

Fig. 3(a) shows the numerical result of the ZZ coupling strength as a function of qubit detuning in our architecture, and the result for traditional setup is also shown for easy comparison. Away from the triple degeneracy point, the coupling is completely removed, while for region close to the degeneracy point, the coupling is non-zero, and coupling strength at degeneracy point is larger than that of traditional setup ($\sqrt{2}J$ vs $J$). In Fig. 3(b), we show the ZZ coupling as a function of the anharmonicity difference $\Delta_{\alpha} = \alpha_b - \alpha$ for coupling strength $g/2\pi = 15$ MHz and qubit detuning $\Delta = -150$ MHz. For anharmonicity difference $|\Delta_{\alpha}/2\pi| < 25$ MHz, the ZZ coupling can be suppressed roughly below 70 KHz, while for traditional setup, the strength is larger than 5 MHz, as shown in Fig. 3(a).

For a more comprehensive analysis of ZZ coupling in this architecture, we explore the full parameter range in Fig. 3(c) with varying qubit detuning $\Delta$ and anharmonicity difference $\Delta_{\alpha}$. We identify three regions in parameter space with prominent characteristic. The two lighter regions indicate that the ZZ coupling becomes rather strong when the qubit detuning approaches the qubit anharmonicities, i.e., $\Delta = \alpha_{a,b}$, and the intersection region corresponds to the triple degeneracy point. The darker region shows where the ZZ coupling is heavily suppressed, or even completely removed for $\Delta_{\alpha} = 0$.
In the right panel of Fig. 3, we also show the result for indirect-coupled case in our proposed architecture, where qubits are coupled via a resonator, and the system is described by the Hamiltonian $H_2$ with spectrum similar to the direct-coupled case (see Appendix B for more details). As shown in Fig. 3(d), compared with the direct-coupled case, the ZZ coupling is not fully eliminated for $\Delta_\alpha = 0$, but still heavily suppressed as compared with traditional setup. Fig. 3(e) shows ZZ coupling strength as a function of anharmonicity difference $\Delta_\alpha$ for $\Delta/2\pi = -150\,\text{MHz}$, and we have found that the zero ZZ coupling point is at about $\Delta_\alpha/2\pi = 500\,\text{MHz}$ rather than 0 as in direct-coupled case. This characteristic is more prominent in the full parameter space, as shown in Fig. 3(f). The physics behind this characteristic is that since the two qubits are coupled via a resonator, the effective coupling strength between qubits depends on the qubit detuning, thus also on the strength of the interaction among the higher energy levels of qubits. Moreover, the higher energy level of the resonator also contributes to the ZZ coupling. To easily identify the contribution from the higher energy level of the resonator, we assume that the resonator has a nonzero anharmonicity $\alpha_c$, and the fourth-order result of the ZZ interaction strength from perturbative analysis [5, 26, 27] is

$$\zeta = 2g_a^2g_b^2\left[\frac{1}{\Delta^2} + \frac{1}{\Delta^2 - \alpha/\Delta} + \mu_c\right],$$  

where $\Delta_{a,b} = \omega_{a,b} - \omega_c$ is the qubit-resonator detuning, and $\mu_c = 1/(\Delta_a + \Delta_b - \alpha_c)(1/(\Delta_a + 1/\Delta_b))^2$. From Eq. (3), one can find that the first two terms in the bracket correspond to the contributions related to the qubit anharmonicity, thus resulting from the interactions among higher energy level of qubits, while the third one $\mu_c$ only involves $\alpha_c$, thus resulting from the interaction between the higher energy level of the resonator and qubit state [11]. Consequently, the zero ZZ coupling point dependents not only on anharmonicity difference, but also on the qubit detuning, as shown in Fig. 3(f) for $\alpha_c = 0$.

IV. ZZ INTERACTION WITH HIGH ON/OFF RATIO FOR IMPLEMENTING CZ AND iSWAP GATE

With the high contrast ZZ interaction in our proposed architecture, the following discussion focuses on studying the implementation of CZ gate and iSWAP gate with diabatic scheme [4, 7] in this architecture, and shows that the high on/off ratio of the ZZ coupling can dramatically improve the performance of these gates. Here, we typically specialize to direct-coupled system with always-on interactions described by the Hamiltonian $H_1$, but the method is general to other coupled system. For illustration purpose and easy reference, we use the same parameters as in Fig. 2(b) for implementing the two gates, and the control pulse associated with frequency tunable qubit (i.e., labeled by $a$) is [28]

$$\omega_a(t) = \omega_f + \frac{\Delta t}{2} \left[ \text{Erf}\left(\frac{t - t_p}{\sqrt{2}\sigma}\right) - \text{Erf}\left(\frac{t + t_p - t_f}{\sqrt{2}\sigma}\right) \right]$$  

where $\Delta = \omega_1 - \omega_f$, ramp time $t_r = 4\sqrt{2}\sigma$, $\sigma = 1\,\text{ns}$, and hold time $t_{hold} = t_f - t_t$ that is defined as the time-interval between the midpoints of the ramps.

Firstly, we consider the implementation of CZ gate, and the main idea is as follows. By tuning the frequency of qubit $a$ from the parking point $\omega_f = 6.1\,\text{GHz}$ to the interaction point $\omega_f = \omega_b + \alpha$, i.e., the triple degeneracy point shown in Fig. 2(b), the CZ gate can be realized after a full Rabi oscillation between $|11\rangle$ and $|02\rangle + |20\rangle)/\sqrt{2}$. As mentioned above, the rabi rate is $\sqrt{2}J = 2g$, while for traditional setup it is $v/2g$. This is helpful for increasing the gate speed, and thus reducing the coherence error. The control pulse follows the one given in Eq. (4), and is plotted in Fig. 4(a). By initializing the system in $|11\rangle$ and $|01\rangle$, Fig. 4(b) shows the leakage $\epsilon_{\text{leak}} = 1 - P_{11}$ (where $P_{ij}$ denotes the population in state $|ij\rangle$) and swap error $\epsilon_{\text{swap}} = 1 - P_{01}$ as a function of the hold time, respectively. In Fig. 4(b), we also show the result for the system with an additional anharmonicity difference $\Delta_\alpha/2\pi = 10\,\text{MHz}$. In both cases, the leakage and swap error

![FIG. 4. Numerical result for CZ gate and iSWAP gate implementation in the proposed architecture with the same parameter as in Fig. 2(b). (a), (c) Typical pulse with small overshoot for realizing CZ gate and iSWAP gate with diabatic scheme. (b), (d) Leakage and swap error versus hold time, where the dashed and solid lines for system with and without an additional anharmonicity difference $\Delta_\alpha/2\pi$ of 10 MHz, respectively. By choosing optimal overshoot and hold time, both the leakage and swap error can be suppressed below $10^{-4}$. (e) Phase error caused by the parasitic ZZ coupling during iSWAP gate operations versus coupling strength $g$ and anharmonicity difference $\Delta_\alpha$. (f) Phase error versus anharmonicity difference for $g/2\pi = 15\,\text{MHz}$, where the solid line (the horizontal cuts through (e)) is for the proposed architecture, and the dashed line is for the traditional one.](image-url)
can be suppressed below $10^{-4}$, and even lower error below $10^{-5}$ should be possible with the procedure of synchronization [7]. Moreover, we can find that arbitrary control phase gate with swap error below $10^{-3}$ could be achieved as shown in Fig. 4(b). As noted in Ref.[7], the additional small overshoot to the interaction frequency $\omega_1$ is critical to optimize the leak error $\epsilon_{\text{leak}}$, and the swap error is insensitive to it (see appendix C for more details).

The implementation of iSWAP gate can be realized by tuning the two qubits into resonance, and the pulse used is plotted in Fig. 4(c). Similar to the case of CZ gate, by initializing the system in $|11\rangle$ and $|01\rangle$, we investigate the leakage error $\epsilon_{\text{leak}} = 1 - P_{11}$ and swap error $\epsilon_{\text{swap}} = P_{01}$ as a function of hold time, as well as the effect of the anharmonicity difference $\Delta_{\phi}$ on these errors. In both cases, the leakage and swap error can be suppressed below $10^{-4}$, and in principle can be further reduced with the procedure of synchronization [7]. Moreover, XY gate with arbitrary swap angle [29] with leakage error below $10^{-3}$ can be achieved as shown in Fig. 4(d). Note here that apart from the leakage error and control error, the coherent phase error resulting form parasitic ZZ coupling now is a performance limiting factor for realizing fast iSWAP gate with traditional setup [2, 7, 8]. By assuming no leakage error and swap error, Fig. 4(e) shows a rough result of the phase error during the implementation of iSWAP gate with rectangle pulse (i.e. setting $\sigma = 0$ in Eq. (4)) as a function of coupling strength $g$ in our proposed architecture (see appendix D for more details). The accumulated phase is $\phi = \zeta t_{\text{gate}} = \pi \zeta t / 2g$, where $\zeta$ denotes the strength of the parasitic ZZ coupling at the interaction point, and the associated error is defined as $1 - F$, where $F$ is the gate fidelity given by [30] $F = [\text{Tr}(U^\dagger U) + |\text{Tr}(U_{\text{ideal}}^\dagger U)|^2]/20 = [4 + |3 + e^{-i\phi}|^2]/20$. Fig. 4(f) shows the result of the system with a typical coupling strength $g/2\pi$ of 15 MHz, where the dashed line denotes the result for the traditional setup. As expected from the result shown in Fig. 3(c), since the parasitic ZZ coupling is heavily suppressed in our architecture, the infidelity related to phase error is dramatically reduced as compared with the traditional setup.

V. CONCLUSION AND OUTLOOK

We have studied the parasitic ZZ coupling in a superconducting architecture [31–33] where two superconducting qubits with opposite-sign anharmonics are coupled directly or indirectly, and found that high-contrast control over the parasitic ZZ coupling can be realized. We further show that CZ gate with faster gate speed and iSWAP gate with dramatically lower phase error can be realized with diabatic scheme in the proposed architecture. Moreover, XY gate with arbitrary swap angle with leakage error below $10^{-3}$ and negligible phase error can be achieved, as well as the arbitrary control phase gate [29] with swap error below $10^{-3}$. Since these errors are caused by the off-resonant Rabi oscillation related to the associated parasitic interaction (XY for CZ gate, and ZZ for iSWAP gate), even lower error should be possible by increasing the value of qubit anharmonicity. Implementing these continuous set of gates natively with high-fidelity in our proposed architecture could be useful for near-term application of quantum processor [8, 29].

As one may expect, the high-contrast control over the parasitic ZZ coupling in this architecture could also improve the performance of parametric activated entangling gates [12–14] and cross-resonance gate [9, 11] as compared with that of the traditional setup. Extending to multi-qubit system, for fixed coupled case, the quantum crosstalk resulting from the parasitic ZZ coupling could be heavily suppressed, thus multiple quantum gate operations can be implemented simultaneously with low qubit crosstalk, while for tunable coupled case [34–37], the two-qubit gates using XY interaction can be implemented natively with negligible phase error caused by parasitic ZZ coupling [2, 7, 8].

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Appendix A: Triple degeneracy point

As shown in the inset of Fig. 2(b) in the main text, for architecture consisting of two direct-coupled qubits with opposite-sign anharmonicity, the interaction among the two-excitation manifold consisting of qubit state $|11\rangle$ and noncomputational state $|20\rangle$, $|02\rangle$ forms a triple degeneracy point, when the qubit detuning equals the value of the anharmonicity of qubits. Here, we give a detailed description of interaction among this two-excitation manifold. By assuming the constant energy of noncomputational state $|02\rangle$ is zero, i.e., $E_{02} = 0$, the Hamiltonian of the system truncated to the two-excitation manifold is

$$H_{\text{tri}} = \begin{pmatrix}
2\delta & J & 0 \\
J & \delta & J \\
0 & J & 0
\end{pmatrix}$$

(A1)

where $J = \sqrt{2}g$ is the coupling strength between $|11\rangle$ and noncomputational states $|02\rangle$, $|20\rangle$, and $\delta = \Delta - \alpha$. By defining $\theta = \arctan(\sqrt{2}J/\sqrt{\delta})$, the eigenstates of this Hamiltonian are

$$|\psi_1\rangle = \frac{[(1 + \sin \theta)|02\rangle + (1 - \sin \theta)|20\rangle - \sqrt{2}\cos \theta|11\rangle]}{2}$$

$$|\psi_2\rangle = \frac{[\cos \theta(|02\rangle - |20\rangle) + \sqrt{2}\sin \theta|11\rangle]}{\sqrt{2}}$$

$$|\psi_3\rangle = \frac{[(1 - \sin \theta)|02\rangle + (1 + \sin \theta)|20\rangle + \sqrt{2}\cos \theta|11\rangle]}{2}$$

(A2)
with the corresponding energies of $\delta - \sqrt{2J^2 + \delta^2}$, $\delta$, and $\delta + \sqrt{2J^2 + \delta^2}$.

At the triple degeneracy point where the qubit detuning equals the value of the anharmonicity of qubits, i.e., $\delta = \Delta - \alpha = 0$, the three eigenstates are $|02\rangle + |20\rangle - \sqrt{2}|11\rangle)/2$, $|02\rangle - |20\rangle)/\sqrt{2}$, $|02\rangle + |20\rangle + \sqrt{2}|11\rangle)/2$, with the corresponding energies of $-\sqrt{2}J$, 0, and $\sqrt{2}J$.

Appendix B: Qubits coupled via a coupler

In principle, the two qubits can be coupled directly, the spectrum of which is shown in Fig. 2 in the main text, and they can also be indirect-coupled via a coupler. Typically, the coupler circuit can be a resonator with or without anharmonicity (kerr interaction), a tunable inductor [34], or an effective tunable capacitor [36, 37] such as tunable coupler combining a capacitor and a bus resonator. In the following, we give a detailed analysis of the system with coupler using linear resonator and an effective tunable capacitor.

1. Resonator

For two qubits coupled via a resonator, the Hamiltonian of the system is given as (same as the one given in the main text, see $H_2$)

$H = \left[ \sum_{l=a,b,c} \omega_l q_l^\dagger q_l + \frac{\alpha_l}{2} q_l^\dagger q_l(q_{l-1}^\dagger q_l - 1) \right] + \sum_{l=a,b} g_l(q_l^\dagger q_l + q_l q_{l-1}^\dagger) + g_c(q_c^\dagger q_c + q_c q_{c-1}^\dagger),$  

where the subscript $l = a,b,c$ labels different-type anharmonic oscillator with anharmonicity $\alpha_l$ and frequency $\omega_l$, $g_l$ denotes the coupling strength between oscillators, and $q_l (q_{l-1}^\dagger)$ is the associated annihilation (creation) operator truncated to the lowest three-level.

Here, we consider that subscript $l = a,b$ labels the two qubits, and $c$ labels a linear resonator for $\alpha_c = 0$. Fig. 5 shows the numerical calculation of the energy level of the coupled system with traditional setup (AA-type) and in our proposed architecture (AB-type). The following parameters are used here: qubit frequency $\omega_q/2\pi = 4.914$ GHz, resonator frequency $\omega_r/2\pi = 6.31$ GHz, magnitude of qubit anharmonicity $\alpha_q/2\pi = 330$ MHz, and qubit-resonator coupling strength $g_{a,b}/2\pi = 138/(135)$ MHz.

Fig. 5 shows the numerically calculated energy level of coupled system, which is similar to the result for direct-coupled case. However, we note that for indirect-coupled case, the avoided crossing with triplets shown in the inset of Fig. 5(b) results from the interaction among two-excitation manifold consisting of six states, i.e., $|020\rangle$, $|200\rangle$, $|110\rangle$, $|101\rangle$, $|002\rangle$, and $|011\rangle$ (For $|ijk\rangle$, where the first two label two-qubit states, and the third one denotes state of the resonator), rather than three states in direct-coupled case. For qubit with weak anharmonicity coupled to a resonator in dispersive regime, i.e., $|\Delta_{a,b}| \gg g_{a,b}$, these states can be grouped into two distinct subsets, one with $|020\rangle$, $|200\rangle$, and $|110\rangle$, and the other with $|101\rangle$, $|002\rangle$, and $|011\rangle$ at an energy scale of $\omega + \omega_r \approx 2\omega_r$ mainly depends on the resonator frequency. In the dispersive regime, the two subsets are detuned on the order of $2\Delta_{a,b}$, which is assumed to be larger than the coupling strength between the two subsets. Thus, the avoided crossing with triplets shown in the inset of Fig. 5(b) can be approximately described by the result given in Appendix A, i.e., the avoided crossing with triplets results from the coupling among the two-excitation space of qubits ($|020\rangle$, $|200\rangle$, and $|110\rangle$).

2. Tunable Coupler

For two qubits coupled via an effective tunable capacitor combing a capacitor and a resonator, the Hamiltonian of the system is given as [36]
of coupled qubits with opposite-sign anharmonicity, i.e., $\alpha_a = -\alpha$, $\alpha_b = \alpha$. The inset shows the avoided crossing mainly resulting from the interaction among $|020\rangle$, $|200\rangle$ and $|110\rangle$.

FIG. 6. Numerical calculation of the energy levels of coupled qubits via a tunable coupler, as a function of the qubit detuning $\Delta = \omega_a - \omega_b$. (a) Energy levels of coupled qubits with same-sign anharmonicity $\alpha_{a,b} = -\alpha$ ($\alpha/2\pi = 250$ MHz). (b) Energy levels of coupled qubits with opposite-sign anharmonicity, i.e., $\alpha_a = -\alpha$, $\alpha_b = \alpha$. The inset shows the avoided crossing mainly resulting from the interaction among $|020\rangle$, $|200\rangle$ and $|110\rangle$.

FIG. 7. Numerical result for ZZ coupling strength $|\zeta|$ in the proposed architecture with a tunable coupler (AB-type). (a) $|\zeta|$ versus qubit detuning $\Delta$ for anharmonicity difference $\Delta_\alpha = 0$, where the dashed line is for traditional setup (AA-type). (b) $|\zeta|$ versus $\Delta_\alpha$ for $\Delta/2\pi = -150$ MHz. (c) The ZZ coupling strength $|\zeta|$ versus qubit detuning $\Delta$ and anharmonicity difference $\Delta_\alpha$ for $\alpha_c/2\pi = -100$ MHz. Horizontal (vertical) cuts through (c) correspond to the result shown in (a) and (b), respectively. (d) The ZZ coupling strength $|\zeta|$ versus coupler anharmonicity $\alpha_c$ and anharmonicity difference $\Delta_\alpha$ for $\Delta = 0$.

where $g$ denotes the coupling strength between the two qubits via a capacitor. The system parameters used in the following discussion are: qubit $b$ frequency $\omega_b/2\pi = 4.914$ GHz, resonator frequency $\omega_c/2\pi = 6.514$ GHz with anharmonicity $\alpha_c/2\pi = -100$ MHz, magnitude of qubit anharmonicity $\alpha/2\pi = 250$ MHz, directed coupling strength $g/2\pi = 5$ MHz and qubit-resonator coupling strength $g_{\alpha(0)}/2\pi = 185(176)$ MHz.

FIG. 8. (a),(b) Swap error ($\epsilon_{\text{swap}} = P_{01}$) and leakage ($\epsilon_{\text{leak}} = 1 - P_{11}$) during the iSWAP gate operation with diabatic scheme as a function of hold time and frequency overshot ($\Delta_\omega$) for system initiated in $|11\rangle$ and $|01\rangle$, respectively. (c),(d) Leakage error ($\epsilon_{\text{leak}} = 1 - P_{11}$) and swap error ($\epsilon_{\text{swap}} = 1 - P_{01}$) during the CZ gate operation with diabatic scheme as a function of hold time and frequency overshot ($\Delta_\omega$) for system initiated in $|11\rangle$ and $|01\rangle$, respectively. The horizontal cuts (dashed lines) depict the optimal value of overshot adopted in the main text for the implementation of iSWAP gate and CZ gate.

In Fig. 7, we also show the numerical result of the ZZ coupling strength as a function of qubit detuning and anharmonicity difference $|\zeta|$ in this case, and the result for traditional setup is also shown for easy comparison. As shown in Fig. 7(a), (b), and (c), the numerical result is similar to that of resonator case shown in the right panel of Fig. 3. As analyzed for resonator case in the main text, the zero ZZ coupling point depends not only on the anharmonicity difference, but also on the coupler anharmonicity $\alpha_c$, as shown in Fig. 7(d), where the ZZ coupling strength $|\zeta|$ as a function of the coupler anharmonicity $\alpha_c$ and anharmonicity difference is plotted for $\Delta = 0$. This characteristic provided by this coupler circuit enables us to exploit a larger parameter space for engineering the ZZ coupling.
Appendix C: Optimal overshot for gate operation with diabatic scheme

As noted similar in Ref. [7], and also shown in Fig. 8(a) and 8(c), the small frequency overshot $\Delta f$ is critical to optimizing the leakage error and swap error for implementation of iSWAP gate and CZ gate with diabatic scheme, respectively. However, the leakage error for iSWAP gate and the swap error for CZ gate is insensitive to this small frequency overshot as shown in Fig. 8(b) and 8(d). The dashed line shows the optimal value of overshot adopted in the main text for the implementation of iSWAP gate and CZ gate.

Appendix D: Coherent phase error during iSWAP gate operation

For a system of two coupled qubits, by assuming no leakage to noncomputational states and control error, the intrinsic fidelity (excluding infidelity caused by decoherence error) of the iSWAP gate is mainly limited by the coherent phase error resulting from the parasitic ZZ interaction [2, 7, 8].

Here, in our proposed architecture, we consider the implementation of iSWAP gate with a rectangle pulse shown in Fig. 9(a). As shown in Fig. 9(b) with an optimal frequency overshot, both the swap error and leakage error can be below $10^{-3}$. In principle, by using the procedure of synchronization, both errors below $10^{-5}$ could be achieved. Thus, by assuming no leakage to noncomputational states, the $U$ implemented can be described by (up to single qubit rotations) [2, 7, 8]

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -i \sin(\theta) & 0 \\ 0 & -i \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & e^{-i\phi} \end{pmatrix}$$ (D1)

with $\theta = \pi/2$, and $\phi$ is the conditional phase resulting from the parasitic ZZ coupling. An ideal iSWAP gate $U_{\text{ideal}}$ can be described by Eq. (D1) with $\theta = \pi/2$ and $\phi = 0$. Thus the fidelity of the implemented iSWAP gate can be defined by $F = [\text{Tr}(U^\dagger U) + |\text{Tr}(U_{\text{ideal}}^\dagger U)|^2]/20 = [4 + |3 + e^{-i\phi}|^2]/20$ [30].

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