Energy Efficiency in Relay-Assisted Downlink Cellular Systems, Part I: Theoretical Framework

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Abstract

The impact of cognition on the energy efficiency of a downlink cellular system in which multiple relays assist the transmission of the base station is considered. The problem is motivated by the practical importance of relay-assisted solutions in mobile networks, such as LTE-A, in which cooperation among relays holds the promise of greatly improving the energy efficiency of the system. We study the fundamental tradeoff between the power consumption at the base station and the level of cooperation and cognition at the relay nodes. By distributing the same message to multiple relays, the base station consumes more power but it enables cooperation among the relays, thus making the transmission between relays to destination a multiuser cognitive channel. Cooperation among the relays allows for a reduction of the power used to transmit from the relays to the end users due to interference management and the coherent combining gains. These gain are present even in the case of partial or unidirectional transmitter cooperation, which is the case in cognitive channels such as the cognitive interference channel and the interference channel with a cognitive relay. We therefore address the problem of determining the optimal level of cooperation at the relays which results in the smallest total power consumption when accounting for the power reduction due to cognition. We focus on designing achievable schemes in which relay nodes perform superposition coding and rate-splitting while receivers perform interference decoding. For each given network configuration, we minimize the power consumption over all the possible cognition levels and transmission strategy which combines these coding operations. We employ an information-theoretical analysis of the attainable power efficiency based on the chain graph representation of achievable schemes (CGRAS): this novel theoretical tool uses Markov graphs to represent coding operations and allows for the derivation of achievable rate regions for a general network and a general distribution of the messages. A practical design examples and numerical simulation are presented in a companion paper (part II).
I. Introduction

Recently, driven by the explosive growth of wireless data traffic and the ever increasing economical and environmental costs associated with the network operating expenditure, energy efficiency has become an important design consideration in wireless network. The design of low-power wireless networks architectures and protocols has been the focus of much recent research [1], [2]. Although reducing energy consumption is an important goal in modern wireless networks, it should not hamper performance. A key way of simultaneously satisfying the energy efficiency requirement while attaining larger data rates is by increasing the density of networks. An increase in network densities can be attained by a variety of solutions such as: small cells, micro layer wireless nodes, femtocells and relay nodes. Wireless relay nodes, in particular, represent a simple and effective way of increasing the data rates and the energy efficiency of future cellular systems [3]. Both the spectral and the energy efficiency of wireless networks can be further boosted by allowing cooperation among the base stations and other nodes in the network. Coordination schemes such as Coordinated MultiPoint (CoMP) are being actively investigated for implementation in coming releases of LTE-Advanced networks [4]. Although dense and highly coordinated networks represent the most promising option to obtain high transmission rates at low energy, the design and analysis of such networks are challenging tasks.

Fig. 1. Architecture of Relay Assisted Downlink Cellular System.
The architecture of relay-assisted downlink cellular system in LTE-A is presented in Fig. 1. The system is comprised of a base station which is interested in communicating to multiple receivers with the aid of the relay nodes. The set of transmissions between the base station and the relay nodes is termed relay link while the one between relay nodes and receivers is termed access link. We consider the case where no direct link between the base station and receivers exists: this case can be easily obtained by considering an additional relay which is connected to the base station with an infinity capacity channel. In the relay link, transmissions take place over frequency separated channels and are thus non interfering. In the access link, instead, transmissions take place over the same frequency band and therefore are self interfering. When relays cooperate, the access link is analogous to a multi-terminal cognitive channel in which transmitting node are able to partially coordinate their transmissions.

In the literature two kinds of transmission strategies for this network model are usually considered: either the message of each user is known at only one relay nodes (uncoordinated case) or the message of one user is known at all the relays (fully coordinated case). We are interested in the intermediate scenario of partial, or unidirectional, transmitter cooperation which is usually embodied in cognitive channels such as the cognitive interference channel [5], [6] and the interference channel with cognitive relay [7], [8]. We are interested, in particular, in determining the message allocation at the relay nodes, also called the cognition level, which corresponds to the lowest overall power consumption. Minimizing the energy per bit required to achieve a given rate is the dual problem of maximizing the transmission rates for a fixed power consumption. For this reason capacity-approaching transmission strategies are also power efficient.

An exact solution to this problem is available only for very small and very regular networks and an exact solution appears infeasible. For this reason we consider the problem of deriving good communication strategies which achieve capacity in these simple and regular networks. We do so by automating the derivation of achievable rate regions using the Chain Graph Representation of an Achievable Region (CGRAS) [9]. The CGRAS generalizes the derivation of achievable rate regions based on superposition coding, interference decoding, binning and rate-splitting to a network with any number of transmitters and receivers. These fundamental random coding techniques are utilized to prove capacity for the vast majority of information theoretical channel models studied so far in the literature. Although no guarantee exists that these strategies are capacity achieving in general, no other achievable strategy is known to approach capacity for a general channel. For the relay-assisted downlink cellular system we consider the case in which relay nodes are able to perform superposition coding and rate-splitting and receivers are
able to perform interference decoding.

In a companion paper, [10], we apply this general approach to a simple channel with two relays and three receivers and derive explicit characterizations for the power consumption. We also perform numerical optimization and draw important insights on the structure of the optimal solution for larger networks.

A. Literature Overview

Cognition in the model we consider refers to partial, unidirectional transmitter cooperation among the relay nodes. This acceptance of the broad term “cognition” idealizes the ability of the relay to learn the message for the other users using the broadcast nature of the channel. Although unrealistic in some scenarios, this interpretation allows for the precise characterization of the limiting performance of a system in which some users in the network are able to gather information regarding the surrounding nodes.

The first channel which embodies this interpretation of cognition is the Cognitive InterFerence Channel (CIFC) [5] which is obtained from a classical two-user InterFerence Channel (IFC) when letting the message of one user to be know at the other user as well. This extra knowledge available at one of the transmitters (the *cognitive transmitter*) models its ability to acquire the message of the other user (the *primary transmitter*) through previous transmissions over the network. In this scenario unidirectional transmitter cooperation is possible: the cognitive transmitter can help the primary transmitter by using part of its power to transmit the same codeword as the primary transmitter. This strategy achieves capacity in a class of CIFC in the “very strong interference” regime [11], in which there is no loss of optimality in having both decoders decode both messages.

Another cognitive network studied in the literature is the InterFerence Channel with a Cognitive Relay (IFC-CR) [7] which is obtained from a classical two user IFC by adding an additional node in the network, the *cognitive relay*, which has knowledge of both messages to be transmitted and aids the communication of both users. In this channel model, the cognitive relay uses its powers to aid the transmission of both relays. As for the CIFC, using the power available at the cognitive relay to transmit the codeword of each users achieves capacity for a class of IFC-CR in the “very strong interference” regime [12], where again having all the receivers decode all the messages is optimal.

In general the power necessary to implement unidirectional transmitter cooperation is not considered as it is usually assumed that the transmitters opportunistically decode the messages that can be overheard over the wireless medium. Although this approach is valid in principle, it is conceivable that some architectures would actually invest resources to make cognition possible. One such architecture is the relay-assisted
downlink network in which the base station can invest additional power in distributing the message of one user to multiple relays, so as to transmitter cooperation in the access link. This additional power consumption in the relay link results in significant power saving in the access link, thus resulting in an overall reduction of the total power consumption.

B. Contributions

We focus on the problem of designing optimal cognition level and transmission strategies for a relay-assisted downlink cellular networks by considering the cooperation strategies among the relay nodes and interference decoding at the receivers. From available results for the IFC [13] and the CIFC [14], we know that superposition coding at the transmitters and interference decoding at the receivers are capacity achieving strategies. We choose to apply the insights provided by these classical channels to larger and more practical networks.

The overall contributions in the paper can be summarized as follow:

New Achievable Schemes: By considering the CGRAS of [9], we derive a set of achievable schemes for the downlink of a relay-assisted cellular system which employs superposition coding, interference decoding and rate-splitting. The schemes can be obtained for a system with any number of relay nodes and any number of receivers and for any combination of the transmission strategies mentioned above. Each transmission strategy is compactly represented using an acyclic directed graph which is useful both in specifying the encoding and decoding procedure and in deriving the achievable rate region.

A Lower Bound to the Power Consumption: We propose a lower bound to the power consumption of the model under consideration by generalizing the “max-flow min-cut” outer bound to the capacity of a general communication channel. Although not tight in general, this outer bound is useful in determining the overall energy efficiency of the system and show the superiority of the schemes involving relay cooperation as compared to the non cooperative scenario.

An example of our approach to a simple network with two relays and three receivers and insightful numerical simulations can be found in a companion paper [15].

C. Paper Organization

Section II introduces the channel model under consideration: the two-hop, relay-assisted broadcast channel. In Section III we present the transmission strategies considered in our approach. In Section
we introduce the automatic rate region derivation which allows us to design complex transmission
strategies for this channel model. In Section IV, we derive the lower bound on the energy consumption
that is obtained from the outer bound to the capacity of the relay link and the access link. Finally, Section
VI concludes the paper.

D. Notation

In the remainder of the paper we adopt the following notation:

- variables related to the Base Station (BS) are indicated with the superscript BS, moreover $i$ is the
  index related to BS,
- variables related to the Relay Nodes (RN) are indicated with the superscript RN, moreover $j$ is the
  index related to RNs and $j$ and $l$ are used to indicate subsets of RNs,
- variables related to the Receivers (RX) are indicated with the superscript RX, moreover $z$ is the
  index related to RXs and $z$ and $m$ are used to indicate subsets of RXs,
- $\mathcal{C}(\Sigma) = 1/2 \log (|\Sigma \Sigma^H + I|)$ where $X$ is a vector of length $k$ of jointly Gaussian random variables
  and $|A|$ indicates the determinant of $A$,
- $A_{ij}$ element of the matrix $A$ in row $i$ and column $j$.

II. Channel Model

We begin by introducing the channel model we consider: the two-hop, relay-assisted broadcast channel,
also depicted in Fig. 2. This model is inspired by the 3GPP recommendation for relays in LTE-A networks
[16], but it is also a viable model in many communication scenarios which make use of relay nodes to
increase the throughput and the power efficiency.

We consider the scenario in which a Base-Station (BS) transmits to $N_{RX}$ Receivers (RXs) via $N_{RN}$
Relay Nodes (RNs) while having no direct link to the RXs. Each RX $z$ is interested in the message $W_z$
at rate $R_z$ which is known at the BS and is to be transmitted reliably and efficiently to RXs through the
RNs. The BS-RNs and the RNs-RXs communication channels are referred to as relay link and access link
respectively, as in 3GPP standardization documents [16]. Motivated by LTE-A architecture, we assume
that relay link has separate and fixed frequency bands between the BS and each RN and that the fixed
frequency band which is different from the band assigned to the relay link and is shared among all the
RXs. The separation between relay and access link models a wireless backhaul connection between BS
and each RN which allows the RN to be transparent with respect to the RXs and among each other. This
facilitates the rapid deployment of the RNs and is useful in many scenarios, for instance when filling a coverage hole or when using the RNs for coverage extension.

The relay link is an Additive White Gaussian Noise (AWGN) channel in which the input/output relationship is

\[ Y^{RN} = D \cdot X^{BS} + Z^{RN}, \]

(1)

where \( D \) is a \( N_{RN} \times N_{RN} \) complex diagonal matrix of the channel gains, \( Z^{RN} \) is a vector of \( N_{RN} \) i.i.d. complex Gaussian random variables with zero mean and unitary variance and \( X^{BS} \) are the channel inputs from the BS. The matrix \( D \) is diagonal because of the assumption that the relay links utilize separate frequency bands. The channel inputs \( X^{BS} \) are subject to the second moment constraint:

\[ \sum_{i=1}^{N_{RN}} \mathbb{E} [ |X^{BS}_i|^2 ] \leq P^{BS}. \]

(2)

The access link is similarly defined as

\[ Y^{RX} = H \cdot X^{RN} + Z^{RX}, \]

(3)

where \( H \) is complex valued matrix of dimension \( N_{RX} \times N_{RN} \) of the channel gains, \( Z^{RX} \) is a vector of \( N_{RX} \) i.i.d. complex Gaussian random variables with zero mean and unitary variance and \( X^{RN} \) are the channel inputs. Each channel input \( X^{RN} \) is subject to the power constraint

\[ \mathbb{E} [ |X^{RN}_j|^2 ] \leq P^{RN}_j, \quad \forall \ j. \]

(4)

The transmission between the BS and the RNs as well as the transmission between the RNs and the RXs takes place over \( N \) channel transmissions. Each message \( W_z \) is uniformly distributed in the interval \([1 \ldots 2^{NR_z}]\). Let \( W \) indicate the vector containing all the messages to be transmitted, i.e. \( W = [W_1 \ldots W_{NRX}] \) and \( R \) the vector containing the rate of each message, i.e. \( R = [R_1 \ldots R_{NRX}] \). Additionally let \( W^{RN}_j \) be the set of messages decoded at relay node \( j \) and define \( W^{RN} = [W^{RN}_1 \ldots W^{RN}_{NRN}] \). A transmission on the relay link is successful if there exists an encoding function at the BS and a decoding function at each RN such that each relay can successfully decode the message in \( W^{RN}_j \) with high probability. Similarly, a transmission on the access link is successful if there exists an encoding function at each RN and a decoding function at each RX such that each receiver \( z \) can decode the message \( W_z \).
reliably. More formally, let \( \hat{W}_{RN j}^z \) be the estimate of \( W_z \) at relay \( j \) and \( \hat{W}_z \) the estimate of \( W_z \) at receiver \( z \) over \( N \) channel transmissions, then a communication error occurs when there exist \( \hat{W}_{RN j}^z \neq W_z \) or \( \hat{W}_z \neq W_z \) for some noise realization over the relay link or the access link.

A rate vector \( R \) is said to be achievable if, for any \( \epsilon > 0 \), there is an \( N \) such that

\[
\max_z \max_{W_{RN j}^z} \mathbb{P}[\hat{W}_{RN j}^z \neq \hat{W}_z \neq W_z] \leq \epsilon.
\]

Capacity is the closure of the union of the sets of achievable rates.

In the following we consider the problem of minimizing \( E_{TOT} \), the total energy power required to achieve a given transmission rate \( R \) defined as:

\[
E_{TOT} = \frac{P_{TOT}}{\sum_{z}^{N_{RX}} R_z}, \quad (5a)
\]

\[
P_{TOT} = P^{BS} + \sum_{j=1}^{N_{RN}} P^{RN}_j. \quad (5b)
\]

The channel we consider is meant to model the 3GPP-defined scenario for LTE-A networks according to [16], in particular for heterogeneous deployment of macro cells and outdoor out-of-band type 2 relays. The model considers downlink transmissions for the case in which the BS fully relies on the RNs and does not serve any RX directly. We assume fixed channel coefficients, thus taking into account distance-dependent path loss while disregarding other dynamic effects such as shadowing, penetration loss and fast fading. We assume that the BS has full channel state information of both relay and access link. Finally the model is coherent with the full buffer traffic assumption, in which there exists a continuous downlink transmission toward each RX.

### III. OVERVIEW OF THE TRANSMISSION STRATEGIES

We investigate the advantages offered by transmitter cooperation by focusing on three random coding strategies: superposition coding, interference decoding and rate-splitting. We introduce each coding technique in further detail next.

#### A. Superposition coding

Superposition coding [17] is a classical information theoretical coding strategy which consist of “stacking” codebooks on one another and it is known to achieve capacity in a number of channels. The bottom codeword can be decoded by treating the top codeword as noise while decoding of the top codeword is
possible only when the bottom codeword has been correctly decoded. When decoding the top codeword, the interference created by the bottom codeword is removed from the received signal thus facilitating correct decoding. In the system we consider, superposition coding can be applied at the RNs that have knowledge of multiple messages. It can also be applied across RNs when they have knowledge of the same messages: relays can cooperate in transmitting the common messages and additional codewords can be superimposed to the common codewords.

B. Interference Decoding

Interference decoding consists in having a receiver decoding an interfering codeword with the aim of removing its effect on the channel output. Superposition coding also imposes the decoding of a non-intended codeword at the users corresponding to the top codeword, but requires that the RN node encoding the top codeword also encodes the bottom codeword. Imposing the correct decoding of a codeword at a non-intended receiver adds an extra rate constraints on the rate of the interfering codewords: this means
that interference decoding is usually advantageous when the power at which the interfering codeword is received is much stronger than power of the intended codeword. In this scenario, the non-intended user can decode an interfering codeword without impacting the rate of the interfering user. This is indeed the intuition behind the capacity result in strong interference for the interference channel \[13\]: in this regime capacity is achieved by having each user in the interference channel decode the interfering codeword alongside the intended one. This can be done without loss of generality as the cross gains are much larger than the direct ones and the interfering codewords are received with a power much larger than the power of the intended signal.

C. Rate-Splitting

Rate-splitting was originally introduced by Han and Kobayashi in deriving an achievable region for the interference channel \[18\] which was later shown to be within one bit/s/Hz from capacity of the Gaussian channel in \[19\]. In the classical achievable scheme of \[18\] the message of each user is divided into a private and a common part: the private part is decoded only at the intended receiver while the common part is decoded by both receivers. If each message were private, each receiver would suffer from a level of interference which would hamper the communication from the intended receiver. If each message were public, then both rates would be limited by the decoding capabilities at both decoders. In general, the largest achievable rate is obtained by splitting the message in a public and private part and choosing the rate of each of the two resulting sub-messages according to the channel conditions.

In the following we consider the case in which rate-splitting can be performed at the relay nodes and the rate of each sub-message can be optimized to yield the smallest energy consumption. After rate-splitting, superposition coding and interference decoding can be applied among sub-messages: Sub-messages can also be merged when the set of encoders and decoders coincide and it is possible to show that merging messages when possible does not reduce the achievable region.

Rate-splitting interplays with transmitter cooperation in different ways. By splitting a message in multiple sub-messages, it is possible to increase the feasible coding strategies at the RNs. Sub-messages can be superimposed over each other and specific sub-messages can be decoded at different subsets of receivers. This also means that relays can cooperate in sending a particular part of a message and do not cooperate when sending others. Finally merging sub-messages mixes intended and non-intended messages in the same codeword which provides a different mechanism for performing interference decoding.
IV. THE CHAIN GRAPH REPRESENTATION OF ACHIEVABLE SCHEMES

In this section we present the class of transmission schemes which we consider for both relay and access link. Transmission over the relay link occurs on independent frequency bands and are thus non interfering: in this case one can apply coding as in the point-to-point channel and right away determine the power necessary to attain a certain message allocation at the RNs.

More interesting transmission strategies can be developed for the access link, where simultaneous transmissions are self-interfering. For this link we consider any achievable strategy which combines superposition coding, interference decoding and rate-splitting for any given message allocation at the RNs. In order to obtain achievable schemes for any number of receivers and transmitters we employ the CGRAS, an automatic derivation of the achievable regions first introduced in [9]. Achievable regions based on random coding are derived using a few coding techniques which are specialized to the model under consideration. The derivation of the conditions under which the probability of encoding and decoding error goes to zero uses standard argument such as the covering lemma and the packing lemma [20] and leads, in turn, to the achievable region. The intuition in [9] is to generalize these derivations to a large class of channels with any number of transmitter, receivers and any distribution of messages. The achievable schemes are represented using chain graph and the distribution of the codewords in the codebook is obtained through a graphical Markov models associated with the given chain graph. The graphical Markov model can additionally be linked to the encoding and decoding error analysis and it is used to derive the achievable rate region for each possible scheme. Although conceptually simple, this idea makes it possible to obtain results valid for a large number of channels and fairly complex achievable schemes. The achievable schemes derived in this fashion offer no guarantees of approaching capacity but are helpful in lower bounding the performance limit of practical multi-terminal networks.

To compactly represent the achievable schemes using the CGRAS, it is convenient to use few graph theoretical notions that we introduce next.

A. Some Graph Theoretic Notions

A graph $G(V, E)$ is defined by a finite set of vertices $V$ and a set of edges $E \subseteq V \times V$ i.e. a set of ordered pairs of distinct vertices. An edge $(\alpha, \beta) \in E$ whose opposite $(\beta, \alpha) \in E$ is called an undirected edge, whereas an edge $(\alpha, \beta) \in E$ whose opposite $(\beta, \alpha) \not\in E$ is a directed edge. Two vertices $\alpha$ and $\beta$ are adjacent in $G$ if $(\alpha, \beta) \in E$ or $(\beta, \alpha) \in E$. If $A \subseteq V$ is a subset of the vertex set, it induces a
subgraph $S_A = (A, E_A)$, where the edge set $E_A = E \cap (A \times A)$. The parents of $\alpha$ in $A$ are those vertices linked to $\alpha$ by a directed edges in $E_A$, i.e.

$$\text{pa}_{E_A}(\alpha) = \{ \beta \in A \subseteq V | (\beta, \alpha) \in E_A, (\alpha, \beta) \notin E_A \},$$

(6)

This definition readily extend to sets as:

$$\text{pa}_{E_A}(B) = \bigcup_{\alpha \in B} \text{pa}_{E_A}(\alpha),$$

(7)

for $B \subseteq A$. Similarly, the children of $\alpha$ in $A$ are those vertices linked to $\alpha$ by a directed edges in $E_A$, i.e.

$$\text{ch}_{E_A}(\beta) = \{ \beta \in A | (\beta, \alpha) \in E_A, (\alpha, \beta) \notin E_A \},$$

(8)

This definition readily extend to sets as:

$$\text{ch}_{E_A}(B) = \bigcup_{\alpha \in B} \text{ch}_{E_A}(\alpha),$$

(9)

for $B \subseteq A$.

A path $\pi$ of length $n$ from $\alpha_0$ to $\alpha_n$ is a sequence $\pi = \{\alpha_0, \alpha_1, \ldots, \alpha_n\} \subseteq V$ of distinct vertices such that $(\alpha_{i-1}, \alpha_i) \in E$ for all $i = 1 \ldots n$. If $(\alpha_{n-1}, \alpha_n)$ is directed for at least one of the nodes $i$, we call the path directed. If none of the edges are directed, the path is called undirected. A cycle is a path in which $\alpha_0 = \alpha_n$. If all the edges are directed and the graph contains no directed cycles, the graph is said to be an Directed Acyclic Graph (DAG).

We will now briefly introduce the CGRAS of [9] for the case where superposition coding and rate-splitting is applied and we specialize it to the channel model under consideration.

B. CGRAS Definition and Notation

The Chain Graph Representation of Achievable Schemes (CGRAS) is defined for a general one-hop multi-terminal network without feedback or cooperation among terminals. The CGRAS is defined by

- a rate-splitting matrix $\Gamma$ which determines the relationship between original messages and sub-messages and by

- a DAG $\mathcal{G}(V, E)$ which describes the superposition coding steps among the codewords of each sub-message.
In the general formulation of [9], the CGRAS also considers interference pre-coding and, as a result, $\mathcal{G}(V, E)$ has undirected edges and is, more generally, a chain graph. In this context we only consider superposition coding which produces a DAG.

1) The Rate-Splitting Matrix: Rate-splitting consists in dividing the message $W_z$ into multiple sub-messages, each decoded by a different subsets of RXs. Sub-messages are further merged when the set of encoding RNs and decoding RXs coincide. In the following we use the notation $W_{j\rightarrow z}$ to indicate the sub-message encoded by the set of RNs $j$ and decoded by the set of decoder $z$. $W_{j\rightarrow z}$, just as $W_z$, is a uniform random variable over the interval $[1 \ldots 2^{N_{R_j\rightarrow z}}]$ and the mapping from $W_z$ into each sub-messages $W_{j\rightarrow z}$ can be obtained with any one to one mapping. For a given distribution of messages at the RNs $W_{RN}$, rate-splitting effectively transforms the problem of achieving a rate vector $R$ into the problem of achieving the rate vector $R'$ where

$$R' = \Gamma \cdot R$$

for $R' = \{R_{j\rightarrow z}\}$, that is $R'$ is the vector containing all the elements $R_{j\rightarrow z}$ (in any order), and the element in position $z \times (j, z)$ in the matrix gamma, $\Gamma_{z \times (j, z)}$, represents the portion of the message $W_z$ which is embedded in the sub-message $W_{j\rightarrow z}$. The coefficient $\Gamma_{z \times (j, z)}^{(j, z)}$ can be non-zero only when $z \in z$, that is when the portion of the message $z$ embedded in $W_{j\rightarrow z}$ is decoded at decoder $z$. This must hold since each sub-message of $W_z$ must be decoded at receiver $z$. Similarly $\Gamma_{z \times (j, z)}^{(j, z)}$ can be non-zero only when $W_z \in W_{RNj}$ for all $j \in j$, that is decoder $j$ can transmit a portion of message $z$ only when message $W_z$ is decoded at RN $j$. When the coefficient $\Gamma_{z \times (j, z)}^{(j, z)}$ is non-zero for multiple $z$ and the same $(j, z)$, this corresponds to the situation in which multiple sub-messages are merged to a single one.

C. The DAG

The DAG $\mathcal{G}(V, E)$ is used to represent the superposition coding step among sub-messages. Given any distribution of messages at the RNs, superposition coding among sub-messages can be applied whenever the bottom codeword is encoded by a larger set of RNs and decoded by larger set of RXs than the top codeword. Only under these circumstances the RNs encoding/(RXs decoding) the top codeword also encodes/(decodes) the bottom codewords. Additionally, if a codeword for $W_{i\rightarrow z}$ is superimposed over the codeword for $W_{1\rightarrow m}$, then any codeword superimposed over the codeword for $W_{1\rightarrow m}$ must also be superimposed over $W_{i\rightarrow j}$. The next lemma formally states these conditions.
Lemma IV.1. Let $U_{j \rightarrow z}^N$ and $U_{i \rightarrow m}^N$ be the codewords of length $N$ used to transmit message $W_{j \rightarrow z}$ and $W_{i \rightarrow m}$ respectively. Superposition coding of $U_{j \rightarrow z}^N$ over $U_{i \rightarrow m}^N$ can be performed when the following holds:

- $l \subseteq j$: that is, the bottom codeword is encoded by a larger set of RNs than the top codeword.
- $m \subseteq z$: that is, the bottom message is decoded by a larger set of RXs than the top message.

Moreover, if codeword $U_{j \rightarrow z}^N$ is superimposed over $U_{i \rightarrow m}^N$ and codeword $U_{l \rightarrow m}^N$ over $U_{i \rightarrow q}^N$, then $U_{j \rightarrow z}^N$ must be superimposed over $U_{i \rightarrow q}^N$.

All the achievable schemes where superposition coding is applied according to Lem. [IV.1] are feasible and the CGRAS provides an automatic tool to obtain the rate region associated with any such scheme. In the CGRAS communication, achievable schemes employing superposition coding are represented using a Directed Acyclic Graph (DAG) $\mathcal{G}(V, E)$ in which each node corresponds to a codeword and each edge to a superposition coding step, from the base codeword toward the top codeword.

The conditions in Lem. [IV.1] together with the fact that a codeword cannot be superimposed to itself, define the relation “being superimposed to” as a transitive relations which implies a further structure in the DAG.

Definition 1. Chain Graph Representation of an Achievable Scheme (CGRAS) For a given message allocation at the RNs $W_{RN}$ and rate-splitting matrix $\Gamma$, we defined the Chain Graph Representation of an Achievable Scheme (CGRAS) as a graph $\mathcal{G}(V, E)$ in which

- every vertex $v = (j, z) \in V$ is associated to the RV $U_{j \rightarrow z}$ from which the codeword $U_{j \rightarrow z}^N$ is generated (detailed in the following). $U_{j \rightarrow z}^N$ carries the message $W_{j \rightarrow z}$ at rate $R_{j \rightarrow z}$ obtained through the rate-splitting matrix $\Gamma$ from portion of the original messages $W_z$.
- the edge $e = ((l, m), (j, z))$ represent an edge from the node $U_{i \rightarrow m}$ to the node $U_{j \rightarrow z}$ which indicates that codeword $U_{j \rightarrow z}^N$ is superimposed over $U_{i \rightarrow m}^N$. This is also indicated as $U_{i \rightarrow m} \rightarrow U_{j \rightarrow z}$ and $U_{i \rightarrow m}$ is said to be a “parent” of $U_{j \rightarrow z}$, while $U_{j \rightarrow z}$ is the “child” of $U_{i \rightarrow m}$.
- The set of all edges in the graph $E \subset V \times V$ must satisfy the conditions in Lemma [IV.1].

Since superposition coding is a transitive relation, all the edges in the graph must be directed and there can be no cycle. The $\mathcal{G}(V, E)$ is then an DAG. The set of parent nodes of the vertex $U_{j \rightarrow z}$ is indicated as $\text{pa}(U_{j \rightarrow z})$, while the set of children as $\text{ch}(U_{j \rightarrow z})$.

The transmission scheme associated with a specific CGRAS is specified by describing how the codewords $U_{j \rightarrow z}^N$ are generated from the RVs $U_{j \rightarrow z}$ and how codewords are encoded and decoded at the receivers.
D. Codebook Generation

Given a CGRAS as defined in Def. 1, the codebook associated with the graph $G(V, E)$ is obtained by applying the following recursive procedure:

- At each step consider the node $(i, j)$ if either it has no parent nodes or if the codebook for all the parents nodes has already been generated. For each (possibly empty) set of parent codewords $\{U^N_{j\rightarrow z}, U_{1\rightarrow m} \mapsto U_{j\rightarrow z}\}$ repeat the following:
  1) generate $2^{NR_{j\rightarrow z}}$ codewords with i.i.d. symbols drawn from the distribution:

$$P_{U^N_{j\rightarrow z}|\text{pa}(U_{j\rightarrow z})},$$

(11)

2) index each codeword as

$$U^N_{j\rightarrow z}(w_{j\rightarrow z}, \{w_{1\rightarrow m}, U_{1\rightarrow m} \mapsto U_{j\rightarrow z}\}).$$

(12)

- Repeat the above procedure until the codebook of each vertex in $V$ has been generated.

Since the graph is a DAG, it is always possible to generate the codebook for each message, starting from the nodes with no parents up to the nodes with no child nodes (i.e. with no outgoing edges). With the above procedure we obtain that a distribution of the codeword which corresponds to the $N$th memoryless extension of the distribution

$$P_U = P_{\{U_{j\rightarrow z}\}} = \prod_{(j, z)} P_{U_{j\rightarrow z}|\text{pa}(U_{j\rightarrow z})}.$$  (13)

E. Encoding procedure

Assume that the vector $W = [W_1 \ldots W_{N_{RX}}] = [w_1 \ldots w_{N_{RX}}]$ is to be transmitted from the RNs to the RXs, then each RN performs rate-splitting according to the matrix $\Gamma$ and maps the original messages to each sub-message. Successively, for each $(j, z)$ the codeword $U^N_{j\rightarrow z}(w_{j\rightarrow z}, \{w_{1\rightarrow m}, U_{1\rightarrow m} \mapsto U_{j\rightarrow z}\})$ is chosen for transmission. The channel inputs at each RN are obtained as a deterministic function of the messages known at the RN.

F. Decoding procedure

Decoding is performed using a jointly typical decoder, that is each receiver $z$ looks for the vector $\hat{w} = \{w_{j\rightarrow z}, z \in z\}$ such that its channel output appears jointly typical with the set of decoded codewords
\{\hat{U}_{j\rightarrow z}^N, \ z \in \mathbf{z}\}.

G. Achievable Rate Region

The achievable rate region of the transmission scheme associated with the CGRAS \(\mathcal{G}(V, E)\) can be obtained from the following theorem in [9]:

**Theorem IV.2. Achievable Rate Region** Consider any CGRAS \(\mathcal{G}(V, E)\) obtained from the message allocation at the RNs \(W_{RN}\) and rate-splitting matrix \(\Gamma\). Moreover let \(V^z\) be the index of all the messages decoded at receiver \(z\), that is

\[V^z = \{(j, z) \in V, \ z \in \mathbf{z}\},\]  

(14)

and let \(\mathcal{G}(V^z, E^z)\) be the subgraph induced by \(V^z\) for \(E^z = E \cap V^z \times V^z\). For any CGRAS \(\mathcal{G}(V, E)\), decoding is successful with high probability as \(N \to \infty\) if, for any receiver \(z\) and for any subset \(F \subseteq V^z\) such that

\[v = (j, z) \in F \implies \text{ch}_z(v) \in F,\]  

(15)

where \(\text{ch}_z(v)\) indicates the children of \(v\) in the subgraph \(\mathcal{G}(V^z, E^z)\), or equivalently

\[(j, z) \in F \implies (l, m) \in F \forall (l, m), \ U_{l \rightarrow m} \mapsto U_{j \rightarrow z},\]  

(16)

the following holds:

\[\sum_{(j, z) \in F} R_{j \rightarrow z} \leq I(Y_z^{RX}; p_{U_{j \rightarrow z}}|p_{U_{j \rightarrow z}}),\]  

(17)

with \(F = V^z \setminus F\) and for some \(U\) and \(X\) distributed according to any distribution that factorizes as in (13), any distribution \(P_{X_{RN}|U}\) defined as

\[P_{X_{RN}|U} = \prod_{k=1}^{N_{RN}} P_{X_{k_{RN}}|\{U_{j \rightarrow z}, \ k \in j\}},\]  

(18)

Although very compact, Theorem IV.2 offers the following simple interpretation: The CGRAS describes what superposition coding steps are performed in each particular scheme. Each RN \(j\) transmits a function of the sub-messages it knows, which is described by equation (18). Each RX \(z\) decodes the codewords in
the set \( \{U_{j,z}, z \in z \} \). The codewords in this set are superimposed one on top of the other which allows for a joint distribution of the codewords described by \( P_U \) in (13). After superposition, the channel inputs are obtained as a function of the codeword known at each encoder, which justifies the expression in (18).

At each decoder \( z \), the codewords \( U_{j,z}^N \) such that \( z \in z \) are decoded. Given how superposition coding is performed, a top codeword cannot be correctly decoded unless all the bottom codewords are also correctly decoded. For this reason the rate bounds are obtained by bounding the probability that each decoded codeword is incorrectly decoded given that all the base codewords are correctly decoded. Each bound in (17) indeed relates to the probability that the codewords in \( F \) are incorrectly decoded given that the codewords in \( \overline{F} \) are correctly decoded. This probability vanishes when the mutual information between the channel output and such incorrectly decoded codewords given the correctly decoded ones is greater than the rate of the incorrectly decoded codewords.

As previously mentioned, we restrict our attention to jointly Gaussian distributed \( U \)s and \( X \) which are linear combination of the \( U \)s. Additionally, for the case where a given \( U \) is transmitted by multiple RN, we fix the scaling coefficient of \( U \) in each \( X \) as to provide the largest ratio combining at the intended receiver.

**Lemma IV.3.** When evaluated for distribution \( P_U \) of (13) and the distribution \( P_{X|U} \) defined as

\[
U \sim \mathcal{N}_c(0, 1) \quad (19a)
\]

\[
X^{RN} = AU, \quad (19b)
\]

for some matrix \( A \) such that

\[
A_{j,(j,z)} \neq 0 \implies j \in \mathbf{j} \quad (20a)
\]

\[
\sum_{(j,z)} A_{j,(j,z)} = P_{j}^{RN}, \quad (20b)
\]

the rate bound in (17) reads

\[
\sum_{(j,z) \in F} R_{j,z} \leq \frac{1}{2} \log \left( \frac{|(H_zA_{H})^T + I|}{|(H_zA_{Vz})^T + I|} \right) \quad (21a)
\]

\[
= \mathcal{C}(H_zA_{|F}), \quad (21b)
\]

where \( H_z \) is the \( z^{th} \) row of the matrix \( H \) and \( A_{|S} \) is equal to the matrix \( A \) but entries corresponding to
the elements in $A_{j,(j,z)}$ is set to zero for every $(j,z) \in S$ and every $j$.

**Proof:** (20) is obtained by evaluating the mutual information term in (17) for the distribution of $P_{X^{RN}}$ in (19b).

With the choice of distribution in (19a), we restrict our attention to $U$ which are zero mean, unitary variance complex Gaussian RVs while the channel input at the RNs are linear combination of the codewords known at each RN. For this reason the assignment in Lem. IV.3 is usually considered a reasonable assignment although there is no guarantee that this assignment is optimal.

H. On the Practical Implementation of the Proposed Achievable Strategies

The results in Th. IV.2 and Lemma IV.3 considered random codebook generation, joint typicality decoding and infinite block-length, which are common information theoretical tools to derive achievable rate regions. In practice, however, structured codebook, limited complexity decoding and finite block-length are necessary. Even though random coding cannot be directly employed in practical system, it provides significant insights on the relevant features of actual coding strategies. In the following, we provide some references to practical implementations of the three components used in the proposed achievable scheme, namely the (i) rate-splitting, (ii) superposition coding, and (iii) joint decoding.

**Rate-Splitting:** The mapping of a message into multiple sub-messages can be performed by dividing the binary representation of the original message into different portions which are then assigned to each sub-message. This operation has linear complexity and does not require any additional information to be sent over the channel. Each sub-message is coded separately to produce a codeword of block-length $N$ which, in general, results in an increase in encoding complexity with respect to the non rate-splitting case. The channel input can be obtained as a mapping of each symbol in the rate-split codewords to some symbol in the transmit constellation of choice.

**Superposition Coding:** The fundamental idea behind superposition coding is to generate a top codeword conditionally dependent on the base codeword(s). In the random coding construction, a different codebook is generated for each possible bottom codeword. In a practical scenario a similar coding strategy can be attained by letting the top codeword be the sum of two codewords: a codeword embedding the top message and one embedding the bottom one. This corresponds to the scenario in which the top codebook is obtained as a binary sum of the base codeword plus a reference codebook. This approach is considered in [21], [22] and [23] where it is shown to perform close to optimal in a number of scenarios.
Another important aspect of superposition coding is interference decoding: a decoder with high level of noisy is required to decode only the bottom codeword, while a decoder with a better SNR, can decode both top and bottom codeword, thus removing the effect of the interference when decoding its intended message. This suggests that the reference codebook for top codeword should be designed to be both a strong channel code but also to be a well-behaved interference for the weaker decoder.

**Sequential Decoding:** Low decoding complexity is the key behind the success of classical point-to-point codes such as turbo code [24] or LDPC code [25]. When interference decoding is considered, a decoder is required to simultaneously decode multiple codewords which is, in general, computationally expensive. In order to reduce the decoding complexity, sequential decoding can be considered but this usually results poor overall error performance. Constructions which allow for an efficient interference decoding have been considered in the literature: in particular [23] exploits the fact that the sum of two convolutional codewords is still a convolutional codeword to reduce the joint decoding of two codewords to the decoding of a single codeword from a larger codebook. This shows, at least empirically, that joint decoding can be performed with an overall complexity which is close to that of point-to-point codes and, thus, that interference decoding is feasible.

V. **Lower Bounds to the Energy Consumption**

We next derive a lower bound on the energy consumption for the channel model in Sec. II which makes it possible to evaluate the energy efficiency of Sec. IV. This bound is obtained by combining the capacity expression of the access link with an outer bound to the capacity of the relay link and minimizing the minimum of the two expressions over the message allocation at the RNs. Since the relay link employs frequency separated channels, the capacity of this link is trivial. The outer bound on the capacity of the access link, instead, is derived from an extension of the max-flow min-cut outer bound [26]. The max-flow min-cut outer bound assumes that the receivers are able to decode the interfering signals: for this reason this outer bound is usually tight when the level of the interfering codeword is either so low that it can be ignored or so high that it can be decoded while treating the intended signal as noise. Although this outer bound is loose in the general case, it still provides an approximate measure of the energy efficiency of the system under consideration.
A. Relay Link Capacity

The transmission links between the BS and each RN are assumed to be non-interfering; the capacity of the relay link thus reduces to the one of a parallel point-to-point channels with a common power constraint. The capacity of the latter channel is a straightforward function of the specific message allocation to be attained at the RNs.

**Theorem V.1. Relay Link Capacity** Consider the relay link as defined in (1) for a fixed message allocation $W^R_N$, the capacity of this channel is

$$
\sum_{z,W_z \in W_j^R} R_z \leq I(Y_j^R, X_j^{BS}) = C(d_j P_j^{BS}), \quad \forall \ j \in [1 \ldots N_{RN}],
$$

union over all the possible $P_j^{BS}$ such that $\sum_{j=1}^{N_{RN}} P_j^{BS} = P^{BS}$.

**Proof:** In the following we again drop the superscripts from $X$ and $Y$ for ease of notation. The channels in the relay link are non-interfering, so that

$$
Y_j = d_j^{RN} X_j + Z_j^{RN}.
$$

Each channel is a point-to-point channel for the transmission of the messages in the set $W_j^{RN} = \{W_i \in W_j^{RN}\}$ between the BS and RN $j$.

**Outer Bound:** In the following we drop the superscripts from $X$ and $Y$ for ease of notation. Using Fano’s inequality we obtain the rate bound

$$
N \sum_{z,W_z \in W_j^R} R_i \leq I(Y_j^N; X_j^N) \leq NI(Y_j^N; X_j), \quad \forall \ j \in [1 \ldots N_{RN}].
$$

The expression in (24) is maximized by Gaussian inputs $X_j^{BS}$ because of the “Gaussian maximizes entropy” property of the mutual information [26]. Note that the joint distribution among the inputs is not relevant as the RNs do not cooperate among each other. The largest achievable rate region is obtained by considering all the possible power assignments to the channel inputs $X_i^{BS}$ which satisfy the power constraint in (2).

**Achievability:** Random coding as in the Gaussian point-to-point channel on each orthogonal channel achieves the outer bound for a fixed $P_j^{BS}$. The union over all the possible $P_j^{BS}$ satisfying (2) attains the outer bound. ■
B. Access Link Outer Bound

While the transmissions on the relay link are assumed to be orthogonal, the transmissions on the access link interfere with one another and the capacity of this link is, therefore, determined by both the noise and the interference caused by simultaneous transmissions. A simple yet effective outer bound for such a channel is the max-flow min-cut outer bound in [26, Th. 14.10.1] and in [20, Th. 18.4]. The original outer bound is developed for non cooperatives sources, so that it is not directly applicable to the access link model under consideration. We need to develop a simple extension to this bound for the case in which the same messages can be distributed to multiple transmitters. The resulting bound is similar to the outer bound for the general multiple access channel with correlated sources in [27], in which an auxiliary random variable is associated to each of the transmitted messages. As for the capacity of the relay link, this outer bound is a function of the message allocation at the RNs.

Theorem V.2. Access Link Outer Bound  For a given message allocation at the RNs \( W^{\text{RN}} \), let \( Z \) be any subset of RXs, that is \( Z \subseteq [1 \ldots N_{\text{RX}}] \) then the region

\[
\sum_{z \in Z} R_z \leq I(\{Y_{z}^{\text{RX}}, \ z \in Z\}; \{U_z \in Z\}|\{U_z \notin Z\}), \tag{25}
\]

union over all the distributions of \( P_{UXY} \) for \( U = [U_1 \ldots U_{\text{RN}}] \) such that

\[
P_{UXY} = \prod_{z=1}^{N_{\text{RX}}} P_{Uz} \prod_{j=1}^{\text{RN}} P_{X_j|\{U_z, W_z \in W_j^{\text{RN}}\}} P_{Y|X}, \tag{26}
\]

is an outer bound to the capacity region.

In particular the distribution in \( P_{UX} \) can be chosen as

\[
U \sim \mathcal{N}(0, I) \tag{27a}
\]

\[
X = AU \tag{27b}
\]

\[
\forall A, \text{ s.t. } A_{jz} \neq 0 \Rightarrow W_z \in W_j^{\text{RN}}, \tag{27c}
\]

\[
\text{diag}(AA^T) = [P_1^{\text{RN}} \ldots P_{N_{\text{RN}}}^{\text{RN}}], \tag{27d}
\]

without loss of generality.
With the assignment in (27) we obtain that the outer bound can be expressed as

\[
\sum_{z \in Z} R_z \leq \frac{1}{2} \log \left| (H^Z A_{\mid Z})(H^Z A_{\mid Z})^T + I \right|,
\]

union over all the possible matrices \( A \), where \( H^Z \) corresponds to the matrix \( H \) restricted to the rows in \( Z \) and where the matrix \( A_{\mid Z} \) is equal to the matrix \( A \) but the entries corresponding to \( A_{\mid z} \) are set to zero for every \( z \in Z \).

**Proof:** In the following we drop the superscripts from \( X \) and \( Y \) for ease of notation. For each \( Z \) we can apply Fano’s inequality as follows

\[
N \sum_{z \in Z} R_z \leq I(\{Y^N_z, z \in Z\}; \{W_z \in Z\}) \leq I(\{Y^N_z, z \in Z\}; \{W_z \in \{Z \}, \{W_z \notin Z\})
\]

\[
= \sum_{i=1}^N (H(\{Y^N_z, i, z \in Z\}; \{W_z \in Z\}, Y^{i-1}) - H(\{Y^N_z, \{W_z, \ z \in [1 \ldots N_{RX}]\}))
\]

\[
\leq \sum_{i=1}^N (H(\{Y^N_z, i, z \in Z\}; \{W_z \in Z\}) - H(\{Y^N_z, \{W_z, \ z \in [1 \ldots N_{RX}]\}))
\]

\[
= NI(Y_z; \{U_z \in Z\}|\{U_z \notin Z\}, Q).
\]

For each \( Z \), the outer bound expression in (25) is maximized by Gaussian \( Us \) and Gaussian \( Xs \), also the maximum entropy is attained when the \( Xs \) are function of the \( Us \). For \( X \) to be both a deterministic functions of \( U \) and Gaussian, \( X \) must be obtained as a linear combination of the \( U \). Among all the possible linear combinations, only those satisfying the given power constraint should be considered. The time sharing RV \( Q \) can be dropped as it does not enlarge the outer bound region.

The idea behind Th. (V.2) is the following: each RV \( U_z \) relates to the message \( W_z \) which is decoded at all the RNs \( j \) for which \( z \in W^\text{RN}_j \). For each subset of receivers \( Z \), we upper bound the sum of the rates decoded by the set \( Z \) with the mutual information between all the channel outputs and the RVs \( U_z \) given that all interfering transmissions of \( U_z \) have been correctly decoded.

We can finally combine the results in Th. (V.1) and Th. (V.2) to determine a lower bound on the energy consumption.

**Lemma V.3. Energy Consumption Lower Bound** A lower bound on the energy consumption in transmitting the rate vector \( R \) is obtained by determining the smallest set of powers \( P^\text{BS} \) and \( [P^\text{RN}_1 \ldots P^\text{RN}_{N_{RN}}] \)
such that the rate vector $R$ is achievable for some message allocation $W^{RN}$.

**Proof:** The lemma follows from the fact that the energy minimization problem is the dual problem to the rate maximization problem. The capacity result in Th. [V.1] and the outer bound in Th. [V.2] are connected through the message allocation $W^{RN}$. Th. [V.2] bounds the powers $[P_{1}^{RN} \ldots P_{N}^{RN}]$ necessary to achieve a certain rate vector $R$ with the message allocation; while the BS power consumption $P^{BS}$ for this to be feasible is determined by Th. [V.1]

VI. CONCLUSION

We have investigated the relationship between cooperation and energy efficiency in relay-assisted downlink cellular system is studied through an information theoretical approach. In particular we consider the scenario in which the transmission between the end users and the base station is aided by multiple relays and no direct connection exists between the base station and the receivers. This scenario idealizes an LTE-style cellular network in which relay nodes are used to improve the energy efficiency of the network. Cognition, in this context, is attained by having the base station send the same message to multiple relays, which can be done at the cost of increasing the power consumption at the base station. Cognition allows cooperation among the relays which reduces the power consumption in the transmissions toward the end users. This, in turn, off-sets the increase in the power consumption at the base station. More messages are distributed to the relay nodes, more power is consumed at the base station and less power is used at the relays.

Our results show how to optimally design the messages allocation at the relays and the associated transmission strategies which result in the lowest overall power consumption. We focus on transmission schemes involving superposition-coding, interference decoding and rate-splitting and derive explicitly characterizations of the power consumption for this class of strategies. We do so by considering a novel theoretical tool which allows the automatic derivation achievable rate regions involving these coding techniques: the chain graph representation of achievable rate regions (CGRAS). Lower bounds to the energy consumption are also derived to evaluate the overall goodness of the proposed approach.

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