Two-neutron densities obtained from microscopic wave functions of \(^6\)He and \(^8\)He are investigated to reveal di-neutron correlations. In particular, the comparison of the two-neutron density with the product of one-neutron densities is useful for a quantitative discussion of di-neutron correlations. The calculations show that the \(S = 0\) spatial two-neutron correlation increases at the surface of \(^6\)He(0\(^+\)) and \(^8\)He(0\(^+\)). The enhancement is remarkable in the \(^6\)He(0\(^+\)) ground state but not as prominent in the \(^8\)He(0\(^+\)) ground state. Configuration mixing of many Slater determinants is essential to describe the di-neutron correlations. Two-neutron densities in \(^{12}\)C wave functions with \(\alpha\)-cluster structures are also studied.

I. INTRODUCTION

Two-neutron (nn) correlations in neutron-rich nuclei are presently getting a lot of attention because the nuclear force shows a rather strong attraction in the \(1S_0\) channel which, however, is not strong enough to form a two-neutron bound state, instead there is a di-neutron scattering resonance at low energy. Thus neutrons are an example of the general problem how a low energy many-body system can be understood microscopically when the scattering length is larger than the extension of the many-body system. Interesting questions are: what kind of correlations are induced by the interaction and can one identify generic properties of loosely bound neutrons.

The experimental area of operation are weakly bound neutron-rich nuclei. For example, in neutron halo nuclei such as \(^{11}\)Li \([1]\), a spatially correlated two-neutron pair outside a core was theoretically predicted in many works \([2–11]\) and was supported by experiments \([12–16]\).

nn correlations have been intensively investigated for asymmetric nuclear matter \([17–24]\), and are discussed also in light neutron-rich nuclei such as \(^4\)He \([25–30]\) and in medium-heavy neutron-rich nuclei \([31, 32]\). These studies suggest that the nn correlations play an enhanced role at the nuclear surface of finite nuclei and in model studies of infinite neutron matter at low density (weak interactions turned off).

As the neutron-neutron interaction forms at low energies a localized resonance in the \(l = 0\) channel, it is expected that spatial correlations between two neutrons that are loosely bound to a core are enhanced compared to well bound nuclear many-body systems where they are in close contact to protons. In two-neutron halo nuclei, the nn correlations between valence neutrons are for example investigated by means of semi-microscopic three-body models describing the center of mass (c.m.) motion of a core and two neutrons. The antisymmetrization between a valence neutron and the core is not taken into account microscopically but it is treated semi-microscopically by considering Pauli forbidden orbits for the valence neutron motion. In such semi-microscopic models, two of the neutrons are regarded as valence neutrons and distinguished from the others. As their positions are the only degrees of freedom, one is able to discuss the nn correlations by simply analyzing the relative motion between the two valence neutrons. In reality, however, neutrons are indistinguishable fermions and the nuclear many-body system has to be expressed by fully antisymmetrized wave functions \([33–35]\). Therefore it is difficult to discern, at least in the inner region, valence neutrons from core neutrons.

Generally speaking, in quantum many-body systems consisting of indistinguishable particles, observable quantities are represented by hermitian operators that are symmetric under particle exchange. The two-body density satisfies this demand and contains all two-body information about the system. Thus it represents the basis for studies of nn correlations in theoretical frameworks and experimental observations.

Here we should remind the reader that microscopic wave functions always contain correlations that one regards as trivial but are also reflected in the two-body density. Firstly, antisymmetrization or the Pauli principle has significant effects on the two-body correlations. For example, if a spin-zero nn pair is chosen, only the parity-even state is allowed but the spatially odd components are forbidden in the relative wave function. Secondly, energy eigenstates of a finite nuclear system are also eigenstates of parity and total-angular-momentum. Therefore intrinsic states
have to be projected onto parity and total-angular-momentum eigenstates. This implies that even an intrinsic state without specific correlations leads to a many-body state that contains long-ranged many-body correlations. Thirdly, in case of an even-even nucleus, where in a shell-model picture with independent particles a single-particle $j$-shell is not completely filled, angular momentum coupling to $J^\pi = 0^+$ of the ground state leads to two-body correlations. Already these three examples show that it is essential to distinguish carefully trivial correlations from non-trivial ones that are induced by the nuclear interaction beyond the mean-field level.

One possible definition of two-body correlations is to take the difference of the two-body density calculated with the correlated many-body state minus the antisymmetrized product of its one-body densities so that the trivial part from the Pauli principle is not regarded as a correlation. This definition is in the spirit of classical probability theory where correlations between two random variables are defined by the difference between the joined distribution and the product of the reduced distributions (Sec. III C). Another possibility is based on the independent particle mean-field picture. The one-body density of the correlated many-body state can be used to calculate from the Hamiltonian a product of the reduced distributions (Sec. III C). Yet another possibility is based on the independent particle mean-field picture. The one-body density of the correlated many-body state can be used to calculate from the Hamiltonian a product of the reduced distributions (Sec. III C). Another possibility is based on the independent particle mean-field picture. The one-body density of the correlated many-body state can be used to calculate from the Hamiltonian a product of the reduced distributions (Sec. III C). Another possibility is based on the independent particle mean-field picture. The one-body density of the correlated many-body state can be used to calculate from the Hamiltonian a product of the reduced distributions (Sec. III C).

The aim of this paper is to study $nn$ correlations by analyzing microscopic many-body wave functions obtained by means of antisymmetrized molecular dynamics (AMD) [35–38]. In previous work AMD calculations described the structures of $^6$He and $^8$He in Ref. [25] quite well. The AMD wave functions are linear combinations of many Slater determinants and thus incorporate various types of correlations. An important advantage of the AMD wave functions is that the c.m. motion is completely decoupled from the intrinsic one.

After defining in Sec. II the one- and two-body densities that will be used and explaining the AMD many-body wave functions in Sec. II, we calculate the two-neutron densities of these correlated wave functions and discuss the characteristics of two-neutron densities calculated with AMD wave functions in comparison with the results of uncorrelated wave functions. We also perform a similar analysis for $^{12}$C wave functions and discuss in Sec. IV how $\alpha$-cluster structures show up in the two-neutron density. Finally, in Sec. V we summarize and give an outlook.

II. FORMULATION

A. One-body and two-body density

Let us consider an antisymmetrized many-body wave function $\Phi$ which represents an $A$-nucleon system. The one-body density is defined as

$$\rho^{(1)}(R) = \langle \Phi | \sum_{i=1}^{A} \delta(\tilde{r}_i - R) | \Phi \rangle .$$

(1)

Here $\tilde{r}_i$ is the position operator for the $i$th particle and $\Phi$ is normalized to one. Using the operator $\tilde{\tau}_z$ for the $z$-component of the Pauli matrices in isospin space the density can be decomposed into neutron and proton density.

$$\rho^{(1)}_{\tilde{\tau}_z}(R) = \langle \Phi | \sum_{i=1}^{A} \left( \frac{1 + \tilde{\tau}_z}{2} \right) \delta(\tilde{r}_i - R) | \Phi \rangle .$$

(2)

In a similar way the two-body density is defined as

$$\rho^{(2)}(\vec{r}_1, \vec{r}_2) = \langle \Phi | \sum_{i \neq j}^{A} \delta(\tilde{r}_i - \vec{r}_1) \delta(\tilde{r}_j - \vec{r}_2) | \Phi \rangle ,$$

(3)

where $\rho^{(2)}(\vec{r}_1, \vec{r}_2)$ is the probability density that one nucleon is found at the position $\vec{r}_1$ and another one at $\vec{r}_2$. The two-body density can be rewritten in terms of relative $\vec{r} \equiv \vec{r}_2 - \vec{r}_1$ and c.m. position $\vec{R} \equiv (\vec{r}_1 + \vec{r}_2)/2$,

$$\rho^{(2)}(\vec{r}_1, \vec{r}_2) = \rho^{(2)}(\vec{R}, \vec{r}) = \langle \Phi | \sum_{i \neq j}^{A} \delta(\tilde{R}_{ij} - \vec{R}) \delta(\tilde{r}_{ij} - \vec{r}) | \Phi \rangle ,$$

(4)
where \( \mathbf{r}_{ij} \equiv \mathbf{r}_j - \mathbf{r}_i \) and \( \mathbf{R}_{ij} \equiv (\mathbf{r}_j + \mathbf{r}_i)/2 \). The two-body density for neutron and proton pairs is given by
\[
\rho^{(2)}_{nn}(\mathbf{R}, \mathbf{r}) = \left( \Phi \right| \sum_{i \neq j}^A \left( 1 + \frac{\hat{\tau}_{zi}}{2} \right) \left( 1 + \frac{\hat{\tau}_{zj}}{2} \right) \delta(\hat{\mathbf{R}}_{ij} - \mathbf{R}) \\delta(\mathbf{r}_{ij} - \mathbf{r}) \left( \Phi \right).
\]
(5)

The two-neutron density \( \rho^{(2)}_{nn} \) (and likewise the one for protons) can be separated into densities for spin zero and for spin one pairs
\[
\rho^{(2)}_{nn}(\mathbf{R}, \mathbf{r}) = \rho^{(2)}_{nn,S=0}(\mathbf{R}, \mathbf{r}) + \rho^{(2)}_{nn,S=1}(\mathbf{R}, \mathbf{r}),
\]
(6)

with
\[
\rho^{(2)}_{nn,S=\{0,1\}}(\mathbf{R}, \mathbf{r}) = \left( \Phi \right| \sum_{i \neq j}^A \left( 1 - \frac{\hat{\tau}_{zi}}{2} \right) \left( 1 - \frac{\hat{\tau}_{zj}}{2} \right) \delta(\hat{\mathbf{R}}_{ij} - \mathbf{R}) \\delta(\mathbf{r}_{ij} - \mathbf{r}) \left( \Phi \right).
\]
(7)

\( \hat{P}_{tij} \) is the spin-exchange operator, and \( (1 \mp \hat{P}_{\tau ij})/2 \) projects on \( S = 0 \) and \( S = 1 \), respectively.

In order to reduce the six-dimensional list of arguments we expand the two-body density in spherical harmonics as
\[
\rho^{(2)}(\mathbf{R}, \mathbf{r}) = \sum_{LM,lm} \rho^{(2)}_{LM,lm}(\mathbf{R}, \mathbf{r}) Y_{LM}(\hat{\mathbf{R}}) Y_{lm}(\hat{\mathbf{r}})
\]
(8)

and consider in the present paper only the \( l = m = 0 \) and \( L = M = 0 \) component which we obtain by integrating over the orientations \( \mathbf{R} \) and \( \mathbf{r} \) as
\[
\rho^{(2)}(R, r) \equiv \rho^{(2)}_{00,00}(R, r) = \int d\Omega_R Y_{00}^*(\hat{\mathbf{R}}) \int d\Omega_r Y_{00}^*(\hat{\mathbf{r}}) \rho^{(2)}(\mathbf{R}, \mathbf{r}).
\]
(9)

For convenience we omit in the following the subscripts for \( L = M = 0 \) and \( l = m = 0 \). The lowest components of total two-neutron density, \( \rho^{(2)}_{nn}(R, r) \), and its \( S = 0 \) and \( S = 1 \) parts, \( \rho^{(2)}_{nn,S=0}(R, r) \) and \( \rho^{(2)}_{nn,S=1}(R, r) \), are calculated in the same way.

The integrated two-body density and two-neutron density equals \( A(A-1) \) and \( N(N-1) \), respectively,
\[
\int d^3r \int d^3R \ \rho^{(2)}(\mathbf{R}, \mathbf{r}) = 4\pi \int r^2 dr \int R^2 dR \ \rho^{(2)}(R, r) = A(A-1),
\]
\[
\int d^3r \int d^3R \ \rho^{(2)}_{nn}(\mathbf{R}, \mathbf{r}) = 4\pi \int r^2 dr \int R^2 dR \ \rho^{(2)}_{nn}(R, r) = N(N-1),
\]
(10)
(11)

which is twice the number of pairs.

The two-neutron c.m. density for the center of mass of a neutron pair coupled to \( S = \{0,1\} \) at a distance \( R \) from the center of the nucleus is obtained by integrating \( \rho^{(2)}_{nn,S=\{0,1\}}(R, r) \) over the relative distance \( r \).
\[
\rho^{(2)}_{nn,S=\{0,1\}}(R) = 4\pi \int dr r^2 \rho^{(2)}_{nn,S=\{0,1\}}(R, r).
\]
(12)

Here we give a comment on the total center of mass motion. In fully microscopic wave functions, the total c.m. motion is not contained in the wave functions, and the above mentioned two-body densities are defined with the coordinate operators, \( \mathbf{r}_i \), that measure from the c.m. of the total system. In AMD wave functions, the total c.m. motion separates from the intrinsic one when a common width is used for all single-particle Gaussian wave packets. In the present work, we eliminate the total c.m. motion in the calculations of two-body densities.

### B. AMD wave functions

In the following, we analyze the two-body density calculated with AMD many-body wave functions for the neutron-rich nuclei \(^6\)He, \(^8\)He and \(^{12}\)C. Details of the \(^6\)He and \(^8\)He calculations are explained in Ref. [25], and those \(^{12}\)C are described in Ref. [30]. In the framework of AMD, many-body states are represented by Slater determinants of single-particle Gaussian wave packets,
\[
\Phi_{AMD}(\mathbf{Z}) = A\{\varphi_1, \varphi_2, ..., \varphi_A\},
\]
(13)
where the \(i\)th single-particle wave function of the \(A\)-nucleon system is written as a product of spatial \((\phi X)\), intrinsic spin \((\chi)\), and isospin \((\tau)\) wave functions,

\[
\varphi_i = \phi X_i \chi_i \tau_i ,
\]

\[
\phi X_i(r_j) \propto \exp \left\{ -\nu (r_j - \frac{X_i}{\sqrt{\nu}})^2 \right\},
\]

\[
\chi_i = (\frac{1}{2} + \xi_i) \chi_\uparrow + (\frac{1}{2} - \xi_i) \chi_\downarrow .
\]

\(\phi X_i\) and \(\chi_i\) are characterized by complex variational parameters, \(X_i \equiv \{X_{i1}, X_{i2}, X_{i3}\}\), and \(\xi_i\). The isospin function \(\tau_i\) is fixed to be up (proton) or down (neutron). The width parameter \(\nu\) has a common value for each nucleus. Accordingly, an AMD wave function \(\Phi_{AMD}(Z)\) is expressed by the set of variational parameters, \(Z \equiv \{X_1, X_2, \cdots, X_A, \xi_1, \xi_2, \cdots, \xi_A\}\).

The many-body Hilbert space for a given total-angular-momentum \(J\) and parity \(\pi\) is spanned by a set of linearly independent \(J^\pi\)-projected AMD states. Those can be obtained by minimizing the energy of parity projected AMD states under various constraints. Another possibility to construct adequate many-body basis states is to vary the energy with respect to all parameters contained in \(Z\) after \(J^\pi\) projection.

Solving the many-body eigenvalue problem

\[
\hat{H} \Phi_n^{J^\pi M} = E_n^{J^\pi} \Phi_n^{J^\pi M} ,
\]

we obtain the eigenstates of the Hamiltonian \(\hat{H}\) represented as a linear combination of the parity and total-angular-momentum projected AMD states

\[
\Phi_n^{J^\pi M} = \sum_{k=1}^{k_{max}} \sum_{J=-J}^J c_{k,J} \hat{P}_{MK}^{J^\pi} \Phi_{AMD}(Z^{(k)}) ,
\]

where \(\hat{P}_{MK}^{J^\pi}\) is the parity and total-angular-momentum projection operator. This method is referred to as multiconfiguration AMD. For example the number of independent AMD configurations for the He-isotopes considered in the next section is \(k_{max} = 72\).

C. Correlations

The term “correlation” is used to express relations between entities. As it is utilized quite generally we want to be more specific what we mean by correlations among identical nucleons. For that it is necessary to distinguish between trivial correlations and those that are characteristic of the system. For example the relative distance of two nucleons in a nucleus will be limited by the diameter of the nucleus. This is certainly a correlation, but it just expresses that the two nucleons belong to the same nucleus which is trivial. The probability density to find two identical nucleons at distance zero vanishes because they are fermions. This correlation we would also like to consider as trivial. On the other hand, if for example the relative distance distribution of a spin \(S = 0\) neutron pair is more localized than the overall size of the nucleus or if it differs in the surface area from the one in the interior then we regard this correlation as non-trivial.

In general, independent fermions are represented by a single Slater determinant. Thus an AMD state \(\Phi_{AMD}(Z)\) represents \(A\) independent fermions. The only correlation among them is due to the Pauli principle which we consider as trivial. Therefore we want to regard the two-body density calculated with a single Slater determinant in general as uncorrelated. However, if the minimum energy state \(\Phi_{AMD}(Z)\) breaks the rotational symmetry of the Hamiltonian and is deformed it contains already non-trivial correlations: the nucleons are not distributed equally, i.e., isotropically around the center of mass. This deformed single Slater determinant is the intrinsic state and must not be regarded as an approximation to one of the eigenstates of the Hamiltonian which have good total angular momentum and parity.

Angular momentum and parity projection restores the symmetries and yields states which can be attributed to eigenstates. As the projected state is a superposition of many Slater determinants, namely the intrinsic state oriented in all directions given by the three Euler angles, there is no contradiction in regarding a single Slater determinant as uncorrelated. In this sense the intrinsically deformed state may already describe many-body correlations although it is a single Slater determinant. In the following we give explicit examples.

A parity projected AMD state (superposition of two Slater determinants), which minimizes the energy, has usually lower energy than the minimum energy AMD state. The additional binding energy is due to correlations which are present in the projected state but not in the single Slater determinant. The total energy is even lower when the
minimization is performed after projection on parity and total angular momentum (VAP). Here more many-body correlations, which are induced by the Hamiltonian, can be accommodated in the projected state. Finally it is obvious that in a configuration mixing calculation, where the many-body eigenstates of the Hamiltonian are given by superpositions of many $J^\pi$-projected Slater determinants (Eq. 18), one may be able to represent various kinds of correlations induced by the specific nature of the Hamiltonian. In order to distinguish trivial from non-trivial or less specific from more specific correlations, we define many-body reference states and compare their two-body densities with those of more correlated states. For example a reference state could be the Hartree-Fock like single AMD Slater determinant.

III. RESULTS OF $^6$He AND $^8$He

In this section we analyze one- and two-body densities obtained from $^6$He and $^8$He wave functions that have been calculated within the framework of AMD and discuss in particular the $nn$ correlations.

A. Wave functions of $^6$He, $^8$He, reference states and intrinsic densities

AMD calculations with multiconfiguration mixing (MC) for $^6$He and $^8$He were performed in Ref. [25]. In the present paper we analyze the MC wave functions that were obtained in Ref. [25] with the interaction parameter set "m56". It consists of the MV1 case(3) central force [39] with parameters $m = 0.56$, $b = h = 0.15$ and the G3RS-type spin-orbit force [40] with the strengths $u_I = -u_{II} = 2000$ MeV. This interaction set gives a good reproduction of the ground state properties of $^6$He and $^8$He and the neutron-halo structure in $^6$He as well as a reasonable description of the subsystem energies such as the $n$-$n$ scattering as shown in Ref. [25]. Hereafter, we call this parameter set "m56-ls2000". Each wave function of $^6$He and $^8$He is expressed by a linear combination of $k_{\text{max}} = 72$ parity and total-angular-momentum projected AMD configurations, see Eq. (18). In previous multiconfiguration AMD calculations for $^8$He, a second $0^+$ state with a well developed $^4$He+$2n$+$2n$ cluster structure was suggested, though there is no experimental data for the existence of this excited $0^+$ state yet. We investigate the two-neutron densities for the ground states of $^6$He($0^+\uparrow$) and $^8$He($0^+\uparrow$), and also that for the excited $^8$He($0^+_2$) state.

As reference states with less correlations, we prepare AMD wave functions by minimizing the energy of a single Slater determinant $\Phi_{\text{AMD}}(Z)$ without any projection or constraint. After the variation this intrinsic state is projected on $J^\pi = 0^+$. This procedure is denoted by PAV (projection after variation). The same interaction m56-ls2000 is used.

In order to study the effects of the spin-orbit interaction on the two-neutron correlations we also create sample states with PAV calculations without the spin-orbit force, $m = 0.56$, $b = h = 0.15$ and $u_I = -u_{II} = 0$ MeV (m56-ls0).

![FIG. 1: One-body density distribution of the intrinsic wave functions of the reference state (PAV) and the sample state (PAV-ls0) for $^6$He. Distribution of the matter, proton and neutron density is shown left, middle and right, respectively.](image-url)
By switching off the spin-orbit force, spin-zero neutron pairs are favored. In the following we label the PAV wave functions of the reference states by "PAV" and those obtained with no spin-orbit force by "PAV-ls0" (see Table I).

Let us briefly explain features of the intrinsic structure of the reference and sample states. In Fig. I, one-body density distributions of the intrinsic reference state $\Phi_{AMD}(Z)$ before projection (denoted by PAV) and those of the sample state with no spin-orbit force (denoted by PAV-ls0) are illustrated for $^6$He. The intrinsic proton density is isotropic, as spin up and down protons fill the s-shell. The dumbbell shape of the intrinsic neutron density of the sample state (PAV-ls0) indicates two pairs of spin up and down neutrons (Fig. 1(b)), while in the reference state (PAV), due to the spin-orbit force, the spin-zero neutron pairs are less separated and the $p_{3/2}$ components increase (Fig. 1(a)).

These effects are even more pronounced in the case of $^8$He displayed in Fig. 2. In the intrinsic wave function of the reference state (PAV), one finds that the 6 neutrons fill the $s_{1/2}$- and the $p_{3/2}$-shells and thus form a spherical distribution, see Fig. 2(a). This intrinsic wave function is already a good approximation to a $J^\pi = 0^+$ state, and hence the PAV reference state for $^8$He is a single Slater determinant even after $J^\pi$-projection. Therefore, we can regard this reference state as an uncorrelated state.

On the other hand, the intrinsic density of $^8$He in the sample state with no spin-orbit force (PAV-ls0) shows a triangular structure in the neutron density which indicates the three localized pairs of spin up and down neutrons (Fig. 2(b)). After projection on $J^\pi = 0$ all one-body densities will be spherical but the sample state contains this special kind of two-neutron correlations which will show up in the two-body density.

## B. One-body density

The one-body densities of neutrons, $\rho_n^{(1)}(R)$, calculated from the MC wave functions for $^6$He(0$^+_1$) and $^8$He(0$^+_1$) are displayed in Fig. 3 together with those of the reference states (PAV) and of the sample states (PAV-ls0). The density of the MC wave function for the excited state, $^8$He(0$^+_2$), is also shown.

In case of $^6$He(0$^+_1$), the neutron density of the MC wave function shows in the outer region a low-density tail, the neutron-halo, while that of the reference state (PAV) has no noticeable tail. The sample state (PAV-ls0) has more neutron density at the nuclear surface but the low-density tail does not extend so much as the MC one. The reason is that in the MC state configurations are admixed in which neutrons are further away from the core than in the reference state.

In the $^8$He(0$^+_1$) MC state a similar behavior of the neutron tail is found. However, the difference from the reference state (PAV) is not as pronounced as in $^6$He. On the other hand, the MC wave function of $^8$He(0$^+_2$) has a far reaching low-density tail in the neutron density.

One should keep in mind that the one-body density of a stationary eigenstate of the Hamiltonian can only provide an indication of many-body correlations but cannot prove their existence. The same halo could in principle also be possible in a mean-field picture where the last neutrons occupy weakly bound single-particle states. Such a picture
TABLE I: Adopted parameter sets of the effective nuclear force and labels of the wave functions for $^6$He, $^8$He and $^{12}$C. "MC" indicates the superposition of multiconfiguration AMD wave functions. "PAV" and "PAV-ls0" correspond to total angular momentum and parity projection after variation calculations for the reference states and the sample states, respectively. The details are described in text.

| nucleus | MC                  | PAV label of the wave functions | PAV calculations label of the wave functions |
|---------|---------------------|---------------------------------|------------------------------------------|
| $^6$He  | m56-ls2000          | m56-ls2000                      | m56-ls0                                  |
| $^8$He  | m56-ls2000          | m56-ls2000                      | m56-ls0                                  |
| $^{12}$C| m62-ls3000          | m62-ls3000                      | m62-ls0                                  |

Equation (12) defines the $S = 0$ two-neutron c.m. density $\bar{\rho}_{nn,S=0}^{(2)}(R)$ to find a $S = 0$ neutron pair with its c.m. position at $R$. In Fig. 4 the $S = 0$ two-neutron c.m. density is displayed for $^6$He and $^8$He. In the case of an uncorrelated gas of neutrons one anticipates a more narrow distribution for the c.m. positions of pairs than that of the positions of individual neutrons because particles on opposite sides of the nucleus contribute to c.m. positions at the center. Particles on opposite sides of the nucleus contribute to c.m. positions at the center. This effect is nicely visible in Figs. 3 and 4. The one-body density of the uncorrelated $^6$He($0^+_1$) PAV state drops to the one percent level of the central density around 3.6 fm while the two-body density does so at about 2.3 fm. Also for $^8$He($0^+_1$) case the difference is about 1.3 fm. If on the other hand due the interaction between them the neutrons like to form $S = 0$ pairs that are preferentially close in distance the two-body c.m. distribution will not be as narrow as in the uncorrelated case. Therefore, $\bar{\rho}_{nn,S=0}^{(2)}(R)$ is a useful measure of the spin-zero $nn$ correlations. An enhancement at the nuclear surface, for example, indicates that the halo contains preferentially correlated $S = 0 nn$-pairs.

When compared with the reference states (PAV) and also the sample states (PAV-ls0), the two-neutron c.m. densities calculated with the MC wave functions for the $^6$He($0^+_1$) and $^8$He($0^+_1$) states are very large at the nuclear surface. In particular, $^6$He($0^+_1$) shows a significant two-neutron c.m. density in the $R > 3$ fm region. This suggests more enhanced $nn$ correlations at the surface of $^6$He than of $^8$He. Even in the $^8$He($0^+_1$) state the two-neutron c.m. density $\bar{\rho}_{nn,S=0}^{(2)}(R)$ of the MC state at $R \sim 4$ fm is by a factor 100 larger than that of the reference state. The excited $^8$He($0^+_2$) state shows in the outer region remarkably large probabilities for $S = 0$ neutron pairs.

Compared with the results of the one-body density, the difference of the $S = 0$ two-neutron c.m. densities between the MC wave functions and the PAV ones is in both cases striking. This means that the $S = 0$ two-neutron c.m. density is a much better indicator for $nn$ correlations than the one-neutron density although both depend only on one variable, namely the distance from the center of the nucleus.

would of course not explain the Borromean behavior. Therefore we explore in the following two-body densities and study in which way correlations may affect two-body densities.
FIG. 3: One-body neutron density $\rho_n^{(1)}(R)$ of the $^6\text{He}(0^+_1)$ and $^8\text{He}(0^+_1)$ multiconfiguration states (MC), reference states (PAV) and sample states (PAV-ls0), as well as that of the $^8\text{He}(0^+_2)$ multiconfiguration state (MC).

2. Two-neutron probability densities $\rho_{nn}^{(2)}(R, r)$ and $\rho_{nn,S=0,1}^{(2)}(R, r)$.

The total two-neutron densities $\rho_{nn}^{(2)}(R, r)$ and their $S = 0$ and $S = 1$ parts $\rho_{nn,S=0,1}^{(2)}(R, r)$ are illustrated in Figs. 5 and 6 for the $0^+_1$ ground state of $^6\text{He}$ and for the $0^+_1$ and $0^+_2$ states of $^8\text{He}$, respectively. The scale of the horizontal axis for $R$ is taken to be $2\sqrt{A/(A-2)}$ times larger than that of the vertical axis for $r$. In the limit of a simple uncorrelated state where two neutrons are moving in a $0s$ orbit around a core with mass $A-2$, the two-neutron density should be a function of $4R^2/(A-2)/A + r^2$ and its contour lines become concentric circles in the $(R, r)$-plane in this scaling. For example in Fig. 6 the contour lines of the total density belonging to the $^8\text{He}$ PAV reference state are shown (most left panel in the second row). In the region far from the origin they look like concentric circles in the scaled $(R, r)$-plane, indicating almost no correlation in the outer low-density region.

3. $S = 1$ two-neutron probability density $\rho_{nn,S=1}^{(2)}(R, r)$

A first correlation that originates from the Pauli principle, and thus is regarded as trivial, can be seen in Figs. 5 and 6 at $r \approx 0$. While the the spin-zero component $\rho_{nn,S=0}^{(2)}(R, r)$ has large amplitude in the small $r$ region, the $S = 1$ two-neutron density $\rho_{nn,S=1}^{(2)}(R, r)$ vanishes at $r = 0$ and concentrates in regions with large $r$. This is easily understood because spatially odd (even) components of the relative motion are automatically selected for $S = 1$ ($S = 0$) pairs in the antisymmetrized wave functions.

It is interesting to see in Fig. 5 that for all three states MC, PAV, and even for the unphysical PAV-ls0 state the $S = 1$ two-neutron densities are rather similar for the $^6\text{He}$ ground state. The same holds true for the ground state of $^8\text{He}$. This can be seen in Fig. 6 which shows the densities for $^8\text{He}$ including the excited $0^+_2$ MC state. Only the $S = 1$
FIG. 4: S = 0 two-neutron c.m. density $\hat{\rho}_{nn,S=0}^{(2)}(R)$ of the $^6$He(0$_1^+$) and $^8$He(0$_1^+$) multiconfiguration states (MC), reference states (PAV) and sample states (PAV-ls0), as well as that of the $^8$He(0$_2^+$) multiconfiguration state (MC).

density of the excited 0$_1^+$ MC state (bottom row) extends significantly over a wider range of distances r between the neutron pair.

4. S = 0 two-neutron probability density $\rho_{nn,S=0}^{(2)}(R,r)$

Let us investigate now the S = 0 two-neutron density $\rho_{nn,S=0}^{(2)}(R,r)$ of the MC wave functions and compare with the results of the reference states (PAV) and the sample states (PAV-ls0). The S = 0 two-neutron densities of $^6$He are plotted in the second column of Fig. 5. A characteristic feature of the MC wave function is that its S = 0 two-neutron density extends beyond $R > 3$ fm without broadening in the relative distance r of the nn-pair. The occurrence of relatively small r values at large c.m. distance R indicates the presence of an extended long tail of a S = 0 pair of two neutrons that are closer to each other than their c.m. distance from the core. Such nn correlations are not clearly visible in the reference state (PAV) at large R. Let us remind the reader that the MC wave function shows the enhancement of the S = 0 two-neutron probability density already in the integrated $\hat{\rho}_{nn,S=0}^{(2)}(R)$ at the nuclear surface $R > 3$ fm, see Fig. 4. In the two-dimensional plot it becomes clear that the two neutrons reside in pairs with extensions considerably less than only Pauli-correlated pairs.

On the other hand the two-neutron densities of the unphysical PAV-ls0 sample states (third rows of Fig. 5 and 6) indicate that in these states, due to the absence of the spin-orbit force, the neutrons are grouped in S = 0 pairs in the outer regions at large R. These nn correlations in the surface one can already anticipate from the intrinsic one-body densities displayed in Fig. 1(b) and 2(b). But there one cannot judge in which regions the nn-pairs have predominantly S = 0 or S = 1. Looking in particular at the intrinsic one-body density of $^8$He in Fig. 2(b) which shows three di-neutron clusters and has a minimum at $R = 0$ one could even be misled and believe that the two-body density also should have a minimum at $R = 0$. But the two-body density has actually a maximum at $R = 0$. The correct interpretation is that nn-pairs that are located vis-à-vis from the center contribute at small c.m. $R$ and large
relative \( r \), while those sitting next to each other contribute at large \( R \) and small \( r \).

Many-body states obtained by configuration mixing (MC) are able to represent correlations beyond those residing in angular momentum projected intrinsic single Slater determinants. Especially the \( 0^+_1 \) and \( 0^+_3 \) MC states of \( ^6\text{He} \) in Fig. 5 (first and fourth row) show that the \( S = 0 \) neutron pairs tend to be concentrated at small \( r \) and to a lesser extent at small \( R \), quite in contrast to the uncorrelated PAV state or the unphysical PAV-ls0 state.

From the above analysis of the two-neutron density, we can conclude that in the surface of the neutron-rich nuclei \( ^6\text{He} \) and \( ^8\text{He} \) the nuclear interaction induces \( nn \) correlations of \( S = 0 \) di-neutron character.

At this point we like to emphasize that neutrons are indistinguishable fermions and one cannot differentiate neutrons in the \( \alpha \)-core from those in the valence orbits, therefore one cannot filter out the contribution of the valence pairs alone. Let us consider the following spin structure

\[
[s1 \times s2]^{S=0}_{M=0} \times [s3 \times s4]^{S=0}_{M=0} = \frac{1}{2} [s1 \times s3]^{S=0}_{M=0} \times [s2 \times s4]^{S=0}_{M=0} + \frac{1}{2} [s1 \times s3]^{S=1}_{M=1} \times [s2 \times s4]^{S=1}_{M=-1} - \frac{1}{2} [s1 \times s3]^{S=1}_{M=0} \times [s2 \times s4]^{S=1}_{M=0} + \frac{1}{2} [s1 \times s3]^{S=1}_{M=-1} \times [s2 \times s4]^{S=1}_{M=1},
\]

where \( s1, s2 \) denote spin-1/2 neutron states with spatial orbits in the core and \( s3, s4 \) spin-1/2 states with spatial orbits in the valence space. The core-core pair is coupled to \( S = 0 \) and so is the valence-valence pair. But as Eq. (19) shows this state can also be written as a superposition of a four-neutron state with two \( S = 0 \) core-valence pairs and three four-neutron states with two \( S = 1 \) core-valence pairs.

In the extreme case where a system consists only of several \( S = 0 \) neutron pairs that are separated from each other, we can remove the inter-pair contributions by taking the difference between \( S = 0 \) and one third of \( S = 1 \) pairs,

\[
r^2 R^2 (\rho_{nn,S=0}^{(2)}(R,r) - \frac{1}{3} \rho_{nn,S=1}^{(2)}(R,r)).
\]
FIG. 6: (Color online) Same as Fig. 5 but for $^8$He. The forth row shows the densities of the MC ($0^+_2$) excited state.

The factor $\frac{1}{3}$ takes into account that there are three times more $S = 1$ inter-pair contributions than $S = 0$ ones. The differences calculated according to Eq. (20) are shown in Figs. 5 and 6 in the most right columns. As expected the contribution located at small $R$, which is supposed to come mainly from inter-pair nucleons, is strongly reduced, while the $nn$ correlations of the valence pairs are seen in the case of the MC wave functions as an enhancement of the amplitude in the ($R > 3$ fm, $r \approx 2$ fm) region when compared to the PAV state. In the case of $^8$He (see Fig. 6) this effect is very pronounced for the excited $0^+_2$ MC state.

5. Two-neutron density $\rho^{(2)}_{nn,S=0}(R, r)$ at $r = 0$

As discussed above, the $S = 0$ $nn$ correlations in the MC wave functions are characterized by a two-neutron density that extends toward large $R$ at small $r$ values. For a more quantitative discussion of the $S = 0$ $nn$-pairs with strong spatial correlations, it is useful to look at the two-neutron density at $r = 0$, $\rho^{(2)}_{nn,S=0}(R, r = 0)$. The quantity $\rho^{(2)}_{nn,S=0}(R, r = 0)/4\pi$ indicates the probability to find two neutrons at the same position $R$, averaged over the orientation of $R$

$$\frac{1}{4\pi} \rho^{(2)}_{nn,S=0}(R, r = 0) = \frac{1}{4\pi} \int d\Omega_R \rho^{(2)}_{nn,S=0}(R, r = 0).$$

Since the $S = 1$ two-neutron density vanishes at $r = 0$, $\rho^{(2)}_{nn,S=0}(R, r = 0)$ equals to the total two-neutron density $\rho^{(2)}_{nn}(R, r = 0)$.
For a single Slater determinant the two-body density can be written as an antisymmetrized product of one-body density matrices. It is easy to show that for $r = 0$, where only the $S = 0$ component contributes to the $nn$ density, this results in

$$\rho^{(2),SD}_{nn,S=0}(R, r = 0) = \rho^{(2)}_{nn}(R, r = 0) = 2\rho^{(1)}_{n\uparrow}(R)\rho^{(1)}_{n\downarrow}(R),$$

(22)

where $\rho^{(1)}_{n\uparrow}(R)$ denotes the one-body spin-up neutron density, and analogue for spin-down. The relation holds only if the total c.m. motion is not eliminated. For a $0^+$ state $\rho^{(2)}_{nn,S=0}(R, r = 0)$ and $\rho^{(1)}_{n\uparrow}(R)$ depend only on the absolute value $R$ so that $\rho^{(2)}_{nn,S=0}(R, r = 0) = \rho^{(2)}_{nn,S=0}(R, r = 0)/4\pi$ and $\rho^{(1)}_{n\uparrow}(R) = \rho^{(1)}_{n\uparrow}(R)$. As a result, the relation $\rho^{(2)}_{nn,S=0}(R, r = 0)/4\pi = 2\rho^{(1)}_{n\uparrow}(R)\rho^{(1)}_{n\downarrow}(R)$ is satisfied for an uncorrelated $0^+$ state given by a single Slater determinant. Therefore, comparing $\rho^{(2)}_{nn,S=0}(R, r = 0)/4\pi$ with $2\rho^{(1)}_{n\uparrow}(R)\rho^{(1)}_{n\downarrow}(R)$ gives another possibility to quantify $nn$ correlations.

Enhancement of $\rho^{(2)}_{nn,S=0}(R, r = 0)$ indicates that the many-body wave function contains correlations of $S = nn$-pairs beyond the mean field level. Here we should note that, when the total c.m. motion is properly removed from the uncorrelated $0^+$ state, the relation Eq. (22) is no longer satisfied because of the recoil effect.

In Figs. 7(a) and 7(b) the two-body densities at $r = 0$, $\rho^{(2)}_{nn,S=0}(R, r = 0)/4\pi$, are compared with the product of one-body densities $2\rho^{(1)}_{n\uparrow}(R)\rho^{(1)}_{n\downarrow}(R)$ of various many-body states for $^6$He and $^8$He, respectively. The total c.m. motion is removed from the wave functions in the present calculations as explained before. For comparison, we also include the results for the PAV reference states with the total c.m. motion. As seen in Fig. 7(c) for the uncorrelated PAV reference state with the total c.m. motion for $^8$He, $\rho^{(2)}_{nn,S=0}(R, r = 0)/4\pi$ agrees with $2\rho^{(1)}_{n\uparrow}(R)\rho^{(1)}_{n\downarrow}(R)$ because the PAV wave function is equivalent to a closed $p_{3/2}$ neutron-shell configuration and can be written by a Slater determinant. After removing the total c.m. motion from the reference state, $\rho^{(2)}_{nn,S=0}(R, r = 0)/4\pi$ is smaller than $2\rho^{(1)}_{n\uparrow}(R)\rho^{(1)}_{n\downarrow}(R)$ in the surface region (Fig. 7(c)), because scaling down of $R$ due to the recoil effect is larger in two-body density than in one-body density in general.

Let us examine $\rho^{(2)}_{nn,S=0}(R, r = 0)$ for the correlated states given by the MC wave functions. The MC wave function for the $^6$He($0^+_1$) state shows remarkable enhancement of $\rho^{(2)}_{nn,S=0}(R, r = 0)/4\pi$ at the surface compared with $2\rho^{(1)}_{n\uparrow}(R)\rho^{(1)}_{n\downarrow}(R)$ (Figs 7(a)). This enhancement indicates clearly the higher correlations beyond mean field, which are incorporated by the MC calculation, i.e., the superposition of many $J^π$-projected Slater determinants. By contrast, in the reference state (PAV) and the sample state (PAV-ls0), the surface tail of the two-neutron density $\rho^{(2)}_{nn,S=0}(R, r = 0)/4\pi$ is smaller than that of the squared one-neutron density just because of the recoil effect. Also in $^8$He, the MC wave functions for the $^8$He($0^+_1$) and $^8$He($0^+_2$) states show enhancement of the two-neutron density $\rho^{(2)}_{nn,S=0}(R, r = 0)$ at the surface. Compared with the results for the $^6$He($0^+_1$) state, it is found that the enhancement

FIG. 7: $\rho^{(2)}_{nn,S=0}(R, r = 0)/4\pi$ and $2\rho^{(1)}_{n\uparrow}(R)\rho^{(1)}_{n\downarrow}(R)$ for $^6$He. (a), (b), and (c): Densities in the $^6$He($0^+_1$) multiconfiguration state (MC), reference state (PAV), and the sample state (PAV-ls0), respectively, without the total c.m. motion. (d): Densities in the reference state (PAV) with the total c.m. motion.
FIG. 8: $\rho_{nn,S=0}^{(2)}(R, r = 0) / 4\pi$ and $2\rho_{nn}^{(1)}(R)/\rho_{nn}^{(1)}(R)$ for $^8\text{He}$. (a), (b), (c), and (d): Densities in the $^8\text{He}(0^+_1)$ multiconfiguration state (MC), the $^8\text{He}(0^+_0)$ one, the reference state (PAV), and in the sample state (PAV−ls0), respectively, without the total c.m. motion. (e): Densities in the reference state (PAV) with the total c.m. motion.

is less prominent in the $^8\text{He}(0^+_1)$ than in the $^6\text{He}(0^+_1)$. This means that the di-neutron correlations are weaker in the $^8\text{He}$ ground state than in the $^6\text{He}$ ground state. On the other hand, the excited $^8\text{He}(0^+_2)$ shows the remarkably enhanced two-neutron density $\rho_{nn,S=0}^{(2)}(R, r = 0)$ in the $R \geq 4$ fm region because of the well developed $^4\text{He}+2n+2n$ structure.

IV. RESULTS OF $^{12}\text{C}$

Multi-configuration AMD calculations of $^{12}\text{C}$ were performed in Refs. 36, 41 where the wave functions were obtained by variation after total-angular-momentum projection (VAP). It was shown that the AMD calculations successfully describe the ground state properties as well as various features of excited states with 3$\alpha$ cluster structure. In these studies of $^{12}\text{C}$, the $0^+_1$ state was found to be admixture of the $p_{3/2}$ sub-shell closure and $SU(3)$-limit 3$\alpha$ cluster components while the $0^+_2$ state turned out to be a well-developed 3$\alpha$-cluster state having the trend to form a gas like system of weakly interacting $\alpha$ particles. Very similar results have been obtained in FMD studies 42, 43 which investigated in detail elastic and inelastic form factors.

Because a $S = 0$ two-neutron pair is contained in each $\alpha$ cluster, the analysis of the $S = 0$ two-neutron density is expected to be also helpful in identifying $\alpha$-cluster correlations in $^{12}\text{C}$. In the following, we analyze the two-neutron densities of the $0^+_1$ and $0^+_2$ states of $^{12}\text{C}$ and discuss the relation to the 3$\alpha$ cluster features.

A. Wave functions of $^{12}\text{C}$

We use the wave functions of the $^{12}\text{C}(0^+_1)$ and $^{12}\text{C}(0^+_2)$ states that have been calculated in Ref. 36. These MC wave functions are expressed as a linear combination of 23 parity and total-angular-momentum projected AMD configurations which were obtained in VAP calculations. The interaction parameter set ”m62-ls3000” was used, see Table 4.
As a reference state with no or little correlations we take the AMD state with minimum energy obtained by a PAV calculation using the same interaction "m62-ls3000". Similar to the $^8$He case, the resulting $^{12}$C reference state has intrinsically an almost spherical shape as it is equivalent to $\frac{3}{2}$-shell closure, see Fig. 9(a). Therefore the $J^\pi$-projection for the reference state (PAV) changes little and the reference state is approximately a single Slater determinant which can be regarded as an uncorrelated state.

We also prepare the PAV-ls0 sample state by a PAV calculation with no spin-orbit force by using interaction set m62-ls0. The intrinsic structure of the sample state (PAV-ls0) is also illustrated in Fig. 9(b). It shows a triangular configuration of $3\alpha$ clusters because $\alpha$ clusters are energetically favored in absence of the spin-orbit force.

FIG. 9: One-body density distribution of the intrinsic wave functions of the reference state (PAV) and the sample state (PAV-ls0) for $^{12}$C.

FIG. 10: (a) One-body neutron density $\rho_n^{(1)}(R)$ of the $^{12}$C($0^+_1$), and $^{12}$C($0^+_2$) MC wave functions as well as those of the reference states (PAV) and the sample state (PAV-ls0). (b) $S=0$ two-neutron c.m. density $\bar{\rho}_{nn,S=0}^{(2)}(R)$ for the same states.
B. One-neutron densities and two-neutron c.m. densities

In Fig. 10(a) the one-body neutron densities, $\rho_n^{(1)}(R)$, of the MC wave functions are shown for the ground state $^{12}\text{C}(0^+_1)$ and the Hoyle state $^{12}\text{C}(0^+_2)$. The one-body density of the ground state differs not so much from that of the uncorrelated reference state as in the cases of $^6\text{He}$ and $^8\text{He}$. On the other hand, the density of the $^{12}\text{C}(0^+_2)$ Hoyle state is much lower in the interior and has a far out reaching tail. By just looking at the one-body density one can not decide if the many-body state is a shell model like state with individual nucleons moving in a shallow mean field or if, as is the case, the nucleons condense into $\alpha$-clusters which move in the outer regions like a weakly interacting gas of $^4\text{He}$ nuclei.

Let us discuss the $S = 0$ two-neutron c.m. densities $\rho_{nn, S=0}^{(2)}(R)$ of $^{12}\text{C}$ states shown in Fig. 10(b). In the $^{12}\text{C}(0^+_1)$ MC ground state the enhancement of the two-neutron c.m. density at the surface is not so remarkable when compared with the reference state (PAV) and it is even less when compared with the sample state (PAV-ls0). The reason is that the ground state of $^{12}\text{C}$ is well bound with respect to the $3\alpha$ threshold, and therefore formation of $\alpha$ clusters is not expected at the surface. This is in contrast to the cases of $^6\text{He}$ and $^8\text{He}$ which are loosely bound systems close to the two-neutron threshold. On the other hand, the MC wave function for the second $0^+$ state, which has an energy very close to the $3\alpha$ breakup threshold, shows a well developed $3\alpha$-cluster structure. This in turn leads to an enhanced $S = 0$ two-neutron c.m. density in the large $R$ region.

C. Two-neutron probability densities $\rho_{nn}(R, r)$ and $\rho_{nn, S=0,1}^{(2)}(R, r)$

The calculated densities for the various states of $^{12}\text{C}$ are summarized in Fig. 11. The two-neutron densities for the $^{12}\text{C}$ PAV reference state are quite similar to those for the $^8\text{He}$ PAV reference state because both states have the neutron configuration of a $p_{3/2}$-shell closure. For example, in the total density for the $^{12}\text{C}$ PAV reference state, the contour lines in the scaled $(R, r)$-plane look like concentric circles in the region far from the origin, indicating no correlations in the outer low-density region. As already discussed in the study of He isotopes, we can see the effect of two-body correlations, particularly, in $S = 0$ two-neutron densities and also in the difference $r^2 R^2\rho_{nn, S=0}^{(2)}(R, r) - \rho_{nn, S=1}^{(2)}(R, r)/3$ shown in the columns (b) and (d) in Fig. 11. Comparing the $S = 0$ two-neutron densities of the three wave functions for the $^{12}\text{C}(0^+_1)$ MC state, the PAV reference state and the PAV-ls0 sample state, it is found that the spin-zero $nn$ correlations in the MC state are slightly stronger than those of the uncorrelated reference state (PAV), but weaker than those of the sample state (PAV-ls0). This is consistent with the previous $^{12}\text{C}$ study which suggested that the $^{12}\text{C}$ ground state is an admixture of a shell model component with a $p_{3/2}$ sub-shell closure and $3\alpha$-cluster components.

The second $0^+$ state of the MC result (last row of Fig. 11) shows an enhanced amplitude of the $S = 0$ two-neutron density for $R > 3$ fm and below $r \approx 3$ fm. This is also clearly seen in the difference Eq. (20) displayed in the last column of Fig. 11.

For a more quantitative discussion we compare in Fig. 12 the two-neutron density at $r = 0$, $\rho_{nn, S=0}^{(2)}(R, r = 0)/4\pi$, with the product of one-neutron densities, $2\rho_n^{(1)}(R)\rho_p^{(1)}(R)$. The features of the two-neutron density are qualitatively similar to those in $^8\text{He}$. Namely, the MC wave functions show that the two-neutron density $\rho_{nn, S=0}^{(2)}(R, r = 0)$ in the ground state of $^{12}\text{C}$ is slightly enhanced at the surface. However the excited state, $^{12}\text{C}(0^+_2)$, shows a remarkable enhancement at all values of $R$ even in the center of the nucleus. The reason lies in the well-developed $3\alpha$ cluster structure and the fact that each $\alpha$ cluster houses an $S = 0$ $nn$-pair. Thus the enhancement of $\rho_{nn, S=0}^{(2)}(R, r = 0)$ can also be observed in states with strong $\alpha$-cluster correlations.
V. SUMMARY AND OUTLOOK

Two-neutron correlations in $^6$He and $^8$He are investigated by analyzing the two-body density of microscopic many-body wave functions obtained by antisymmetrized molecular dynamics (AMD). In order to visualize non-trivial spatial correlations, that are induced by the neutron-neutron interaction, the two-neutron density is calculated as function of the distance, $r$, and mean c.m. position, $R$, of the $nn$ pair. Results from correlated AMD wave functions are compared to those of uncorrelated (or less correlated) wave functions. These reference states are taken to be single Slater determinants because they represent independent fermions, and correlations induced by the Pauli principle are regarded as trivial. We find characteristic non-trivial two-neutron correlations in the $S = 0$ channel as an enhancement of the two-neutron density $\rho^{(2)}_{nn,S=0}(R, r)$ toward large $R$ at small $r$ values. These two-neutron correlations are weaker in the ground state of $^8$He than in $^6$He and are particularly pronounced at the surface of the excited $^8$He($0^+_2$) state. It is also found that superpositions of many angular momentum and parity projected Slater determinants are essential to incorporate the di-neutron correlations.

To see how $nn$ correlations are reflecting $\alpha$ cluster structures the $0^+_1$ ground state and the first excited $0^+_2$ state (Hoyle state) of $^{12}$C are also investigated. As the Hoyle state consists to a large extent of three loosely bound $\alpha$-particles at large distances from the center it contains spatially correlated $nn$ pairs which are not present in the ground state. In this case the cause for finding neutron pairs spatially close is the property of the nucleon-nucleon interaction to bind two protons and two neutrons particularly well so that strong 4-body correlations in form of $\alpha$ clusters are
FIG. 12: $\rho^{(2)}_{n,n,S=0}(R, r = 0)/4\pi$ and $2\rho^{(1)}_{n+}(R)\rho^{(1)}_{n+}(R)$ for $^{12}$C. (a), (b), (c), and (d): Densities in the $^{12}$C($0^+_1$) multiconfiguration state (MC), the $^{12}$C($0^+_2$) one, the reference state (PAV), and the sample state (PAV-ls0), respectively, without the total c.m. motion. (e): Densities in the reference state (PAV) with the total c.m. motion.

developed. Also these correlations are nicely visualized.

Thus, the two-neutron density is found to be a good probe to identify two-neutron correlations. In particular, the comparison of the two-neutron density with the squared one-neutron density, both calculated from the same many-body state, is useful for a quantitative discussion of the two-neutron correlations.

Although we do not expect that the general nature of the $nn$ correlations will be altered one should investigate if realistic effective interactions which reproduce the experimental phase shifts, like the ones obtained in the Unitary Correlation Operator Method (UCOM) [44–47] and successfully used in FMD calculations [48], give the same results as the more phenomenological effective potentials used here.

The other question is if large-scale shell model Hilbert spaces can equally well represent surface di-neutron correlations as the AMD states.

It would also be interesting to see in how far di-neutron correlations exist in heavier neutron-rich nuclei which possess a neutron skin but not necessarily a halo. Is there also a transition from mean-field dominated to correlation dominated dynamics as seen in lighter nuclei?

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