Optical mode conversion in coupled Fabry–Perot resonators

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Low-loss conversion among a complete and orthogonal set of optical modes is important for high-bandwidth quantum and classical communication. In this Letter, we explore tunable impedance mismatch between coupled Fabry–Perot resonators as a powerful tool for manipulation of the spatial and temporal properties of optical fields. In the single-mode regime, frequency-dependent impedance matching enables tunable finesse optical resonators. Introducing the spatial dependence of the impedance mismatch enables coherent spatial mode conversion of optical photons at near-unity efficiency. We experimentally demonstrate a NIR resonator whose finesse is tunable over a decade, and an optical mode converter with efficiency >75% for the first six Hermite–Gauss modes. We anticipate that this new perspective on coupled multimode resonators will have exciting applications in micro- and nano-photonics and computer-aided inverse design. © 2020 Optical Society of America

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High-fidelity mode conversion and sorting are crucial tasks for quantum communication [1,2], as well as high-bandwidth mode-division multiplexed classical communication [3]. At the transmitting end of a communication network, mode conversion enables the encoding of information into the transverse spatial degrees of freedom of an optical field or fiber, thereby substantially increasing the bit rate. At the receiving end, mode sorting enables decoding of the previously encoded spatial information. While both mode conversion and sorting are fundamentally linear in the electromagnetic field, they are technically challenging because the necessary linear transformations are not generically quadratic in the transverse spatial coordinates and as such cannot be directly implemented with standard optics such as mirrors, lenses, and beam splitters.

At moderate efficiency, “mode shaping” can be achieved with a single phase plate [4,5] or digital micromirror device [6] that redirects a fraction of an incident optical field into a diffracted target mode. Near-unity efficiency requires implementing a unitary transformation of all of the incident mode to the target mode. In the special case of Hermite–Gauss (HG) ↔ Laguerre–Gauss (LG) interconversion, this unitary transformation can be realized via a pair of astigmatic lenses [7], since HG and LG modes are related to one another by only the relative phase of horizontal and vertical mode excitations. More general approaches to high-fidelity mode-converting unitaries include numerically optimized nanostructured couplers between waveguides [8,9]; adiabatically varying coupling between macroscopic optical fibers [10]; conformal beam transformations implemented in two or more holographic phase gratings [11–13]; meshes of Mach–Zehnder interferometers [14,15]; and long-period fiber gratings [16].

Implementing an arbitrary mode converter is formally equivalent to changing one quantum mechanical wave-function into another using only spatially local potentials that cannot themselves redistribute probability in space, but can impose phase gradients that result in such redistribution under the influence of a kinetic energy term. While lenses and mirrors can impart spatially varying phase profiles onto an incident optical field, it is the subsequent diffraction that must redistribute intensity; reshaping the mode via attenuation would irreversibly reduce the conversion efficiency. To our knowledge, prior work fits predominantly into three paradigms: adiabatically varying the system Hamiltonian such that an input mode/initial eigenstate is smoothly converted into the desired output mode/final eigenstate (as in coupled fibers); bang-bang unitaries that, in discrete steps separated by free evolution/diffraction, convert between input and output modes (as in cascaded diffraction gratings and long-period fiber gratings); or something in between that implements a “shortcut to adiabaticity” [17].

In this work, we explore a fourth paradigm relying on impedance mismatches between optical cavities to achieve near-unity efficiency mode conversion [18] without nanophotonics or non-quadratic optics. Using only lenses and mirrors, we demonstrate conversion of an HG_{00} mode into an arbitrary target HG_{m0} mode by simply varying the length of a Fabry–Perot resonator over a few nanometers. The large propagation distances required for other approaches are realized in our work by repeated round trips through the structure of the coupled cavities. Related ideas [19,20] in a nanofabricated setup allow mode conversion in a ring resonator coupled to a multimode waveguide, with mode-dependent phase matching providing mode selectivity; that approach affords ~5% bidirectional conversion,
as fabrication imperfections limit the (fine-tuned during fabrication) impedance match, and fiber-to-chip coupling induces losses. Indeed, resonant impedance matching between cavities has enabled efficient coupling of a Y-junction to a waveguide [21] without any mode control. In our work, coupled macroscopic resonators enable rapidly tunable impedance matching to arbitrary modes and remove the need to couple to a chip.

First, we introduce the simpler problem of coupled, impedance-mismatched Fabry–Perot cavities in the single-mode limit. Here the result is a cavity of tunable finesse $F$. We experimentally demonstrate such finesse tunability over a decade. We then consider the full problem of coupled, misaligned multimode Fabry–Perot cavities, where the resulting behavior corresponds to an optical mode converter. Implementing these ideas, we demonstrate optical conversion efficiency $>75\%$ for the first six HG modes, limited by mirror loss. Finally, we explore applications of these tools.

We begin by analyzing two single-mode Fabry–Perot cavities coupled through a shared mirror, as shown in Fig. 1(a). We will find that this arrangement acts as a tunable finesse cavity—it traps light for a short duration (low finesse) or a long duration (high finesse). The two cavities have identical waists and share a mutual axis to avoid inter-mode coupling.

The total optical transmission of this arrangement can be calculated in the $S$-matrix formalism (see Supplement 1) in terms of the lengths of the two cavities, the wavenumber $k$ of incident light, and the power reflection and transmission coefficients of the mirrors $M_1$, $M_2$, and $M_3$. A more intuitive understanding arises by observing that any single-mode scattering element is fully described by its (frequency dependent) reflection and transmission coefficients. It is thus valid to combine mirrors $M_2$ and $M_3$ with the propagation distance $L$ between them, into a single composite “effective mirror” $M_{23}$ with reflection and transmission coefficients $r_{23}(\delta), t_{23}(\delta)$ [Fig. 1(a), inset].

In this picture, what remains is the simple two-mirror “primary” cavity defined by the separation between $M_1$ and $M_{23}$. The total transmission of the primary cavity is thus precisely that of a single two-mirror Fabry–Perot with a frequency-dependent reflection coefficient for one end-mirror. The finesse of the primary cavity can be computed [23] as $F \approx \frac{2\pi}{T_1 + T_2 + X_1 + X_3}$ so long as the properties of $M_{23}$ remain $\sim$ constant across said resonance. Here $T_1$ and $T_3$ are power transmission coefficients, and $X_1, X_3$ are power loss coefficients. As $T_2$ is tuned, the finesse $F$ varies. Since $T_2$ is simply the transmission of the Fabry–Perot consisting of $M_2$ and $M_3$, it can range from unity to nearly zero as the length $L$ tunes the cavity from resonance to anti-resonance, thereby varying $F$ from small to large values.

Harnessing these principles, we construct a tunable finesse cavity using mirrors with reflectances $R_1 = 0.9997$ and $R_2 = R_3 = 0.990$ for 780 nm light. The finesse is tuned by varying $M_{23}$’s length with a piezoelectric actuator, and is then measured either spectroscopically or by cavity ringdown [22] [Fig. 1(c)]. The computed transmission of $M_{23}$, $T_{23}$ is shown in the black curve in Fig. 1(a), varying from unity on resonance to $\frac{1}{2} T_2 T_3 \approx 2.5 \times 10^{-5}$ at maximum detuning. The measured finesse [Fig. 1(b)] is in close agreement with a parameter-free theory (solid curve).

The finesse saturates at $1.7(2) \times 10^4$, limited by the reflectance of $M_1$, and is compared to theory for a perfect $M_1$ ($R_1 = 1$) in the dotted curve in Fig. 1(b). From there, the next bound on finesse is set by the minimum transmission of the variable reflector, $F_{\text{max}} = \frac{2\pi}{T_1 + T_2} \approx 2.5 \times 10^5$ for $R_2 = R_3 = 0.99$. In practice, we anticipate an ultimate finesse limit set by scattering and absorption losses of the mirror coatings, akin to a conventional Fabry–Perot cavity (see Supplement 1). Losses from the $M_2$ substrate and $M_3$ coating are strongly suppressed in high-finesse configurations, where very little power resides within $M_{23}$. A single-pass substrate loss of 1% (0.1%) limits only $F \leq 1 \times 10^4$ (2 $\times 10^5$), which improves further with higher $R_2$, $R_3$. Furthermore, fused-silica glass can exhibit loss $<1$ ppm/cm [24], obviating this limitation.

When the detuning between the cavities is smaller than the linewidth of the secondary cavity, the above picture breaks down, because (a) the $M_{23}$ transmission $T_{23}$ becomes strongly...
In the absence of transverse mode coupling, an input $HG_m \equiv HG_{m0}$ mode produces an output $HG_n$ mode, and the single-mode analysis applies. Introducing a transverse offset between the coupled cavities breaks orthogonality between their higher-order modes and generates inter-mode couplings [Fig. 2(a)]. In this case, the $HG_0$ mode of the primary cavity appears displaced on $M_{23}$, and thus has non-zero overlap with all modes of $M_{23}$. As such, $M_{23}$ now exhibits frequency- and mode-dependent transmission $T_{23}^{ij}(\omega) = |\alpha_{ij}|^2 T_{23}(\omega)$, with input and output modes $i$ and $j$ having an overlap integral $\alpha_{ij}$. The transverse modes of $M_{23}$ each have their own transmission function $T_{23}(\omega)$, all with identical linewidths, but different resonant frequencies due to the round-trip Gouy phase of $M_{23}$ [23]. The simulated $HG_0 \rightarrow HG_3$ transmission peak is shown in Fig. 2(b).

We expect unity transmission to occur when $T_i = T_{23}^{ij}(\omega)$, where in-coupling occurs through the $HG_j$ mirror at $M_1$, and out-coupling occurs through the $HG_j$ mode of $M_{23}$. The multimode S-Matrix calculation shown in Fig. 2(c) (and detailed in Supplement 1) supports this intuition, showing nearly 100% conversion efficiency. As the length of $M_{23}$, $L$, is tuned, its higher-order modes individually approach resonance with the drive laser and primary cavity, satisfying the impedance matching condition at two drive-laser frequencies and thus permitting conversion of any input mode $i$ into any output mode $j$ as long as $|\alpha_{ij}|^2 > T_i$. Indeed, for the theory in Fig. 2, the mode overlap between $HG_0$ and $HG_3$ is only $\sim 6\%$, but near-unity conversion still occurs.

To demonstrate these principles, we construct a mode converter using mirrors with reflectances $R_1 = R_2 = 0.965(5)$ and $R_3 = 0.972(1)$ at 780 nm, whose performance is shown in Fig. 3. In each panel, the laser frequency is scanned to satisfy the resonance condition. Between panels, the length $L$ of $M_{23}$ is varied with a piezoelectric actuator to bring the target mode to resonance with the $HG_0$ mode of the primary cavity. The transmission is monitored on a large-area photodiode to determine conversion efficiency, and on a CCD camera to ascertain mode shape. With the cavities transversely offset by $\sim 1$ waist, $HG_0$ through $HG_5$ were generated with total conversion/transmission in excess of $75\%$. To access higher-order modes, the offset was increased to $\sim 2.5$ waists, yielding conversion of modes up to $HG_{12}$; the $HG_{10}$ mode is shown with $75\%$ total transmission.

We have presented a new framework for understanding coupled multimode optical resonators, where one resonator acts as a frequency- and mode-dependent mirror for the other resonator. Harnessing this new perspective, we demonstrate both a variable finesse cavity and an arbitrary spatial mode converter. By introducing an intracavity electro-optic modulator [25], we anticipate rapid tunability of finesse and output mode, enabling control of photon dynamics within a cavity lifetime.

In our approach, the mode conversion bandwidth is set by the cavity linewidth, and so can be increased by scaling down to micro-resonators. Working with small radius of curvature (ROC) fiber Fabry–Perots [26] should enable bandwidths up to $\sim 10$ GHz, and extending these ideas to nanophotonic platforms would allow further bandwidth gains [8,27]. Meanwhile, superpolished mirrors with loss $< 3$ ppm [28] should enable efficiency up to 99\%, limited by leakage into undesired modes. The mode converter is inherently bidirectional and can thus convert $TEM_{m0} \rightarrow TEM_{n0}$.  

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**Fig. 2.** Principle of optical mode conversion. (a) Two coupled Fabry–Perot resonators can act as an optical mode converter when a small transverse offset is introduced between their axes to couple their otherwise orthogonal transverse modes. Mirrors $M_2$ and $M_3$ act as a single “effective mirror” $M_{23}$ with frequency- and mode-dependent transmission $T_{23}^{ij}(\omega)$, for input/output modes $HG_{i/j,0}$. Near-unity efficiency $i \leftrightarrow j$ mode conversion through the full system $M_1 + M_{23}$ is achieved when the input and output couplings to the composite cavity $M_1/M_{23}$ are equal, $T_i = T_{23}^{ij}(\omega)$ (the “impedance matching” condition), and no light leaks out through other modes. (b) Simulated transmission of the effective mirror $M_{23}$ (in the absence of $M_1$), with a translated $HG_{0,0}$ input generating an $HG_{1,0}$ output. The transmission $T_{23}^{i0}$ is limited by the (translated) $0/3$ mode overlap of $\sim 6\%$, and the dashed horizontal line denotes $T_i$. The frequency dependence of the transmission guarantees that there are two frequencies where $T_i = T_{23}^{i0}$ (dashed vertical lines), resulting in perfect mode conversion at these frequencies once mirror $M_1$ is introduced, as shown in (c).
The techniques introduced in this work can be employed to interconvert between field profiles of any physical system in which coupled resonators can be realized whose eigenmodes are the desired input and output field profiles. Coupling to a twisted optical resonator [29] whose eigenmodes are LG would enable high-efficiency generation of optical orbital angular momentum states for optical communication. Similarly, the use of astigmatic cavities would allow control over both mode indices of the HG output. Indeed, these concepts transcend even light: by coupling together phononic resonators with disparate mode structures, it should be possible to deterministically and efficiently reshape acoustic waves [30].

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See Supplement 1 for supporting content.

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