Spare Parts Demand Analysis Method Based on Field Replaceable Unit

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Abstract. It is based on the reliability of spare parts optimization modeling, and the influence of failure rate, fixability, importance and availability on spare parts reserves is added to the model, and the weight of each spare part is calculated by fuzzy analytic hierarchy process.

1. Introduction
In the reliability analysis of the field replaceable unit, we found that it is very important to impbe able to get the replacement in time when the replacement of the replaceable unit components in the field is very important. Therefore, based on the analysis of the reliability of the field replaceable units, this paper analyzes the spare parts reserve of the field replaceable units.

2. Calculation Model of Spare Parts Requirement
On the optimization of the spare parts system [1], some important literature of relevant domestic and foreign mostly use Barlow’s theory and methods [2,3]. It considers components and their spare parts as cold storage subsystems, and the entire spare system is considered as a system of tandem systems of each subsystem. The reliability of the reserve-series system is the protection rate of the spare parts system. The optimization of the spare parts system is attributed to the minimum cost of the system under the constraints of the given system reliability. The model is as follows:

\[
\begin{align*}
\min & \sum_{i=1}^{n} c_i x_i, \quad i = 1, 2, L, n \\
\prod_{i=1}^{n} R_i(x_i, t_i) & > R_0
\end{align*}
\]

Where \( n \) is the type of spare part system; \( x_i \) indicates the number of type \( i \) spare parts; \( c_i \) indicates the spare parts on unit price of type \( i \); \( R_i(x_i, t_i) \) is the reliability of the type \( i \) spare parts equipped with \( x_i \) spare parts in \((0, t_i)\) time. \( R_0 \) is defined as the protection rate for the entire spare parts system. The problem with this computational model is that there are many types of spare parts, and if we use the evaluation function of the series model \( \prod_{i=0}^{n} R_i(x_i, t) \geq R_0 \) as the evaluation criterion, even if the reliability of each component reaches 0.9, the reliability of the 30 parts is only 0.424, which is not in line with the actual situation. In general, the protection rate of the spare parts system [4] is defined by the system service level:
Spare parts protection rate $ = \left( 1 - \frac{\text{The number of missing parts in time}}{\text{The number of received parts within a certain period of time}} \right) \times 100\%$ (2)

From the formula (2) we see the protection rate of spare parts is equivalent to the average number of single component protection rate, which is very different from the evaluation function in the series reliability model. An evaluation function is constructed from the concept of spare rate of spare parts. It can be seen from the formula (2) that the total spare rate is considered to be the probability that the spare parts can be provided in time. $N_i(t), (i = 1, 2, \cdots, n)$ indicates the number of times the type $i$ component has failed during the period $(0, t)$. Thus, the protection rate of the type $i$ spare parts is:

$$R_i = P\{N_i(t) \leq x_i\} \quad (3)$$

Spare parts system total security rate $R_s$ is:

$$R_s = P\{N_1(t) + N_2(t) + \cdots + N_n(t) \leq (x_1 + x_2 + \cdots + x_n)\} \quad (4)$$

The formula (4) is a reliability model constructed according to the basic definition of the spare rate of spare parts, and continues to derive its intrinsic form from this basic definition.

The observed value of the statistic $\hat{R}_i = \frac{N_i}{N_i(t)}$ is taken as the estimate of $R_i$, in formula:

$$x_i^0 = \begin{cases} x_i, & N_i(t) \geq x_i \\ N_i(t), & N_i(t) < x_i \end{cases} \quad (5)$$

For the entire spare system, the observed value of $\hat{R}_s$ is taken as an estimate of the total guaranteed rate $R_s$.

$$\hat{R}_s = \frac{\sum_{i=1}^n x_i^0}{\sum_{i=1}^n N_i(t)} = \frac{\sum_{i=1}^n N_i(t)}{\sum_{i=1}^n N_i(t)} R_s \quad (6)$$

We introduce $\lambda_i$ as the probability of type $i$ component’s failure, and by the update process theory we know: $\lambda_i = E(N_i(t))$. For parts of the service subject to the distribution of the situation, it is precisely established; for the non-exponential distribution of component life, it is asymptotically established. Therefore, the entire spare system protection rate can be expressed as

$$R_s = \sum_{i=1}^n \frac{\hat{\lambda}_i}{\sum_{i=1}^n \hat{\lambda}_i} R_i \quad (7)$$

We make $\rho_i = \frac{\hat{\lambda}_i}{\sum_{i=1}^n \hat{\lambda}_i}, i = 1, 2, \cdots, n$, so
\[ R = \sum_{i=1}^{n} \rho_i R_i \] (8)

\( \rho \) represents the weight of the protection effect of the type \( i \) component on the total protection rate of the system in formula (8). Obviously, in the same protection rate of each type of component, high failure rate has a great impact on the overall protection rate of the spare parts system. Thus, the reliability model is established based on the derived formula:

### 2.1. Protection Model Based on A Certain Amount of Follow-up Spare Parts

The objective function of the model is:

\[ R = \max \sum_{i=1}^{N} \rho(i) R(i) \] (9)

In the formula: \( R \) is the system protection rate, \( \rho(i) \) is the weight of a single component, \( R(i) \) is the guarantee rate of the individual components.

The constraint function is:

\[ C = \sum_{i=1}^{N} C(i) X(i) \leq C(0) \] (10)

\[ R(01) \leq R(X(i),T) \leq R(02), i = 1, 2, \cdots, N \] (11)

\[ R(00) \leq R(X(i),T), i = 1, 2, \cdots, k, R(01) \leq R(00) \leq R(02) \] (12)

In the formula (10), \( C(i) \) is the unit price of the \( i \)-th spare part, which represents the sum of the product of the number and unit price of each component and makes the it less than or equal to the total cost of the system \( C(0) \); formula (11) indicates that the protection rate of each component in the order cycle is greater than or equal to the minimum protection rate, less than or equal to the maximum protection rate; formula (12) indicates that a number of particularly important spare parts protection rate is greater than the protection rate \( R(00) \), where \( R(00) \) is greater than or equal to the minimum protection rate, less than or equal to the maximum protection rate.

### 2.2. Protection Model Based on the Protection Rate of Follow-up Spare Parts

The objective function of the model is:

\[ C = \min \sum_{i=1}^{N} C(i) X(i) \] (13)

Constraint function:

\[ R(01) \leq R(X(i),T) \leq R(02) \] (14)

\[ \sum_{i=1}^{N} \rho(i) R(i) \geq R(0) \] (15)

\[ \sum_{i=1}^{J-1} \rho(i) R(i) + \sum_{i=J+1}^{N} \rho(i) R(i) + \rho(J) \times R(X(J) - 1, T) \leq R(0) \] (16)
3. Model Optimization of Spare Parts Requirement

In practice, spare parts are also affected by many factors, not only the impact of spare parts wear and tear, but also its importance in the equipment-critical impact. In addition, we need to consider the impact of economic and so on. The key to the component is the piece in the equipment system and the role of the impact on the size of the system performance. Parts of the wear and tear refer to the size of the degree on it, mainly related with the inherent reliability of parts, the use of environmental conditions and conditions. Some of these factors are not easy to quantify, and their relationship is a vague relationship with the spare parts, so it is suitable for fuzzy comprehensive evaluation method in the weight of the analysis.

In the calculation of this paper, we mainly consider the influence of the failure rate, maintainability, the importance and availability of components in the four aspects, and use the fuzzy analytic hierarchy process [5,6] to obtain the weight score of the component, and add up the weight score \( r_i \) of all the components to obtain a total weight fraction \( G \), 
\[
G = \sum_{i=1}^{n} r_i, \quad \text{so the weight of the component is} \quad \rho_i = \frac{r_i}{G}.
\]

3.1. Construction Judgment Matrix

Before the weight analysis, we must first establish a hierarchical structure to solve the problem, as shown in Figure 1.

![Figure 1. Weight Hierarchy Diagram](image)

Judgment matrix is the basic information of the analytic hierarchy process. It is an element value of the above layer as the criterion of judgment, and the two elements of the next layer compared to determine the element value of the matrix. The element \( a_{ij} \) in the judgment matrix \( A \) indicates that the relative importance of the element \( i \) to the element \( j \) from the viewpoint of the judgment criterion, which satisfies:
\[
a_{ij} = \frac{1}{a_{ji}}, a_{ii} = 1, (i, j = 1, 2, \cdots, n) (17)
\]

Judgment scale use AHP (Scale 1-9), with 1,3,5,7,9, to construct the fuzzy judgment matrix \( A^{(k)} \), respectively comparing element \( i \) to element \( j \) of the same, significant, important, more important, most important. For experts evaluation, they use fuzzy triangular number to express fuzzy judgment, with level 3 scale of fuzzy number. Experts have a certain degree of confidence \( \delta \) in the degree of importance of subjective judgments, when the confidence were "very sure", "more sure", \( \delta \) were the value of 0.5,1,1.5. \( A^{(k)} = (a_{ij})_{n \times n} \) expressed as:
where $x^-$ and $x^+$ are the normalized eigenvectors corresponding to the largest eigenvalues of the matrix $A^-$ and $A^+$. $\alpha$, $\beta$ for the value of:

$$\alpha = \left( \sum_{j=1}^{n} \frac{1}{\sum_{i=1}^{n} a_{ij}^+} \right)^{1/2}, \quad \beta = \left( \sum_{j=1}^{n} \frac{1}{\sum_{i=1}^{n} a_{ij}^-} \right)^{1/2}$$

(19)

We make a normalization of the calculated weight vector $w^{(k)}$, and get the normalized weight vector $w_0^{(k)}$. In order to ensure the reliability of decision-making, we need to carry on the consistency judgment to the fuzzy judgment matrix. The consistency test is carried out with the difference between $\lambda_{\text{max}}$ and $n$, that is,

$$C.I. = \frac{(\alpha \lambda_{\text{max}}^- + \beta \lambda_{\text{max}}^+)/2 - n}{n - 1}$$

(20)

If $C.I. \leq 0.1$, it is proved that the judgment matrix is consistent.

### 3.3. Calculate Decision Weights

In order to reduce the risk of decision-making, the assessment process has a number of experts to participate, while each expert's credibility is not the same, so we make each expert's own weight $y_k$, weight $w_0^{(k)}$, and we get decision-making weight

$$w = \frac{1}{\sum_{k=1}^{K} y_k} \sum_{k=1}^{K} y_k w_0^{(k)}$$

(21)

### 3.4. The Final Weight of the Component

We make the score of the failure rate, fixability, importance and availability of each component separately, and the scores of the four items are summed to obtain the weight fraction of the component, and then sum the weight scores of all components up, draw a total weight score $G = \sum_{i=1}^{n} r_i$. The final weight of the component is $\rho_i = \frac{r_i}{G}$.

### 4. Concluding Remarks
Through the fuzzy theory, the reliability of the field replaceable parts, the importance of the installation, the fixability and the availability of the weight in the outfield were weighted to obtain the weight of the components, we build up the optimization model based on the reliability of the spare parts requirements, and this method is in accordance with the practice of spare parts protection.

5. References
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