Research Article

Estimation of Water Saturation in Shale Formation Using In Situ Multifrequency Dielectric Permittivity

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Estimating water saturation via conventional logging tool such as resistivity cannot provide an accurate solution in a formation with low conductivity water and tight porosity. As an alternative, we employed a dielectric permittivity, which makes it easy to differentiate the water from the other fluids in pore structures. A multichannel frequency dielectric logging tool is used for measuring in situ permittivity. To simulate a dielectric permittivity, we used two analytic models: Lichtenecker-Rother (LR) and Stroud-Milton-De (SMD) models. The key goal of this research is to propose a workflow to evaluate an equivalent Archie’s parameter which can generate the same dielectric logging tool responses with core measurement results using a given analytic models. According to the results of the LR model curve-fitting, the estimated Archie’s parameter shows inversely proportional relationship with clay volumes. The estimated Archie’s parameter from the SMD model is sensitive to the lower frequency channels of the multifrequency dielectric logging tool. Nevertheless, utilizing the response of the dielectric logging tool in the frequency range, where interfacial polarization effect does not exist, can provide an alternative to estimate water saturation in shale formations with relatively less conductive waters.

1. Introduction

Accurate calculation of water saturation in a target formation is crucial for estimating the potential volume of hydrocarbon. Also, estimating the volume of water is a key factor in building a concrete plan for hydrocarbon exploration. A standard approach of estimating water saturation is using Archie’s equation with deep resistivity logging tools. When we apply the Archie’s equation for calculating water saturation, the cementation factor and tortuosity factor are measured in core samples that require numerous representative core samples of the formation. In addition, this measurement is not rigorous when dealing with tight formations that have a small pore volume. Also, when the matrix mineral components and pore structures are complex, the conventional Archie’s law based method cannot provide reliable fluid estimation results [1] since it assumes pure sandstone. Especially, Archie’s equation cannot handle the core samples with tight pore structures such as shale. Hence, many scientists have studied variety types of analytic models to explain the gaps between well logs and core data to estimate accurate total organic contents (TOC) and water saturation [2, 3].

For a common reservoir, geoscientists can convert the resistivity and conductivity information acquired by well logging to fluid saturation based on the given saturation model. Another limitation of the conventional method for calculating water saturation is that the resistivity logging tool cannot differentiate water from hydrocarbon when the water is less conductive (or lower salinity). This issue becomes more problematic when we work on unconventional reservoir due to clay bound water (Cho et al. [4, 5], Kadkhodaie and Rezaee [6], Sotelo et al. [7], and Tathed et al. [8]). As an alternative, for last decade, the measurement of dielectric permittivity has been gaining attention in the oil and gas industry. Most current logging techniques are not sensitive to the permittivity of formations, except for logging-while-
drilling (LWD) resistivity tools in specific conditions. The understanding of permittivity enables inferring the physical reasons for its main characteristics as a complex, frequency-dependent quantity. The relative permittivity of a formation is a real number when there is no dissipation of the energy carried by the electric field in the medium. However, when the formation dissipates energy, which is true in most of cases, the relative permittivity of the formation becomes a complex number which decreases from the low-frequency to the high frequency through several transitions, so-called frequency dispersion (Donadille et al. [9], Myers [10]).

Numerous lab measurements have been done to define the permittivity of rock samples. For example, Zinszner and Pellerin [11] made electrical measurements in sandstone and limestone. Also, there are several validated approaches for performing measurements of clay-bearing rocks (Fam and Dusseault [12], Myers [10, 13], Revil et al. [14], Seleznev et al. [15]), especially Josh and Clennell [16] demonstrated the lab measurement of the permittivity on clay and shale using a parallel plate measurement tool. However, evaluating in situ permittivity is still challenging. Donadille et al. [9] have developed a multifrequency dielectric logging tool to measure the dielectric permittivity at four different frequency components in in situ conditions. The responses from the lower frequency channels are still under the influence of interfacial polarization effect; however, we can get more stabilized signal as the frequency becomes higher. Baker and Kenyon [17] introduced an application of the data measured by multifrequency dielectric permittivity scanners to invert the cementation and the tortuosity factor by assuming that both terms are identical. In addition, Baker and Kenyon [17] estimated the salinity of the shallow zones using dielectric logging tools.

In this paper, we introduce the background theory of dielectric permittivity and demonstrate the theory by applying them to a set of core samples acquired in shale reservoir. One of the key tasks is that we calibrate a dielectric permittivity log for water saturation that is generally higher than the core measurement. We then apply analytic models to

Figure 1: Comparison of different polarization types: (1) ionic contributions (a → d), (2) dipolar polarization (b → e), and (3) interfacial polarization (c → f). The interfacial polarization (f) is the most common type in the rock consisting of matrix and fluids.

(a) [Electronic \((\vec{E} = 0)\)]

(b) [Orientational \((\vec{E} = 0)\)]

(c) [Interfacial \((\vec{E} = 0)\)]

(d) [Electronic \((\vec{E} \neq 0)\)]

(e) [Orientational \((\vec{E} \neq 0)\)]

(f) [Interfacial \((\vec{E} \neq 0)\)]
Table 1: Dielectric constant ranges of common rocks, minerals, and fluids.

| Rocks, minerals, and fluids   | Dielectric constants |
|------------------------------|----------------------|
| Anhydrite                    | 6.3                  |
| Dolomite                     | 6.8                  |
| Barite                       | 7.0 – 12.2           |
| Sulphur                      | 3.6 – 4.7            |
| Biotite                      | 4.7 – 9.3            |
| Gneiss                       | 8.5                  |
| Limestone                    | 7.5 – 9.2            |
| Fluorite                     | 6.2 – 6.8            |
| Calcite                      | 7.8 – 8.5            |
| Apatite                      | 7.4 – 11.7           |
| Plagioclase feldspar         | 5.4 – 7.1            |
| Gypsum                       | 5.0 – 11.5           |
| Quartz                       | 4.2 – 5.0            |
| Pyrite                       | 10.9                 |
| Sandstone (dry to moist)     | 4.7 – 12.0           |
| Clay (dry to moist)          | 7.0 – 43.0           |
| Petroleum                    | 2.07 – 2.14          |
| Water (20°C)                 | 80.36                |

estimate the dielectric permittivity under the given condition, such as mineral composition and different water salinity. One of the simplest approaches to defining the dielectric permittivity of the solids may be to add volumetrically to the fluids’ dielectric permittivity (Hipel [18]); however, this approach is not applicable to a porous media since the interacting solid and fluid surfaces results in additional mobile charge carriers that can be polarized. Therefore, we employed two different analytic models: complex refractive index model (CRIM) (Birchak et al. [19], Seleznov et al. [15]) based approach for a mono frequency analysis and Stroud-Milton-De (SMD) (Stroud et al. [20]) model for frequency dispersion analysis using multichannel frequency measurement data. Then, we used the estimated permittivity value to define an optimum value of Archie’s parameter by applying nonlinear least square method. In each model case, we validated the analytic model via tests on core samples and then estimated Archie’s parameter in a field dataset.

2. Theory and Method

2.1. Physics of Dielectric Permittivity. When an electric field is applied to a material, redistribution of bound charges occurs locally to new equilibrium positions. This phenomenon of charge redistribution is called polarization. The reason for this phenomenon can be described in three cases as presented in Figure 1. An electronic contribution arises from the distortion of the electron shell relative to the atomic nucleus. Ionic contributions are attributed to the relative displacement and deformation of charged ions with respect to each other. Dipolar polarization arises when molecules possessing a permanent electric moment are oriented along an applied field. In addition, there can be a space-charge (interface or Maxwell-Wagner) polarization due to local migration of charged particles. This type of polarization is a common phenomenon in heterogeneous materials such as porous rocks saturated with different types of fluids (Sen [21]). The measured permittivity of common rocks (Manning and Athavale [22], Parasnis [23]) is displayed in Table 1. In addition to the matrix (solid) permittivity, the bound water molecules in the pore structure also contribute to the dielectric response (Goncalves and Tremosa [24]).

There is a similarity among all polarization phenomena in that they are effective at a low-enough frequency but vanish past a certain frequency which is called relaxation frequency (Lima and Sharma [25]). This term can be determined by the inertial moment of the particles in frictional and electrostatic forces. If the electric field rotates too fast, the particles cannot follow it; thus, it is difficult to induce polarization and permittivity is reduced. Therefore, relative permittivity decreases with increasing frequency [26, 27] demonstrates the existence of this phenomena by comparing the varying rates of decay of interfacial polarization due to clay and that due to conductive minerals as a function of frequency.

In dielectric logging, the logging tool generates an external electric field. The relative permittivity of a formation is a real number when there is no dissipation of the energy carried by the electric field in the medium. However, when the formation dissipates energy, then the relative permittivity of the formation becomes a complex number as follows:

\[ \varepsilon = \varepsilon' + i\varepsilon'', \]

where \( i \) is the imaginary unit defined by \( i^2 = -1 \). The real part \( \varepsilon' \) is related to the energy trapped within the formation by the electric field, while the imaginary part \( \varepsilon'' \) represents the amount of energy dissipation within the formation into other forms of energy. The complex dielectric permittivity consists of immobile part (\( \varepsilon_m \)) and mobile part (\( \varepsilon_w \) and \( \varepsilon_{sd} \)). Generally, the immobile part represented by the rock permittivity which can be found from Table 1 [28, 29] demonstrated the importance of the complex permittivity formulation and the interpretation of the dissipative components.

Fuller and Ward [30] demonstrated mathematically that the existence of imaginary conduction (Chelidze et al. [31], Garrouch and Sharma [32], Revil et al. [14]) with corresponding physical models. Both the real and imaginary terms of the complex permittivity are frequency-dependent since most formations are dispersive. The complex relative permittivity \( \varepsilon(\omega) \) of the formation with respect to angular frequency accounts for two types of current sources as follows:

\[ \varepsilon(\omega) = \varepsilon'(\omega) + i\frac{\sigma(\omega)}{\omega\varepsilon_0}, \]
combining the real-values permittivity and the frequency-dependent conductivity \( \sigma(\omega) \) yields

\[
\sigma(\omega) = \sigma_{\text{DC}} + i\omega\varepsilon_0\varepsilon''
\]

where \( \varepsilon_0 \) means the permittivity of a vacuum. The DC conductivity \( \sigma_{\text{DC}} \) comes in phase with \( \varepsilon'' \) in the imaginary part of the complex permittivity. In addition, DC conduction currents also dissipate in the form of heat energy. The conductivity and permittivity are both frequency-dependent due to the dispersive nature of the medium.

2.2. Measurement of Dielectric Permittivity. We employed multifrequency dielectric permittivity scanner (Ligneul et al. [33]) to measure the dielectric constant in in situ conditions. The tool makes measurements at high frequencies, in the megahertz (MHz) to gigahertz (GHz) range. Normally, low-frequency measurements are dominated by the conductivity of the formation, but as the frequency increases, dielectric effects begin to appear and then dominate. Measurements at high frequency (over \( 10^8 \) Hz) enable us to evaluate formation permittivity and conductivity simultaneously, which are used to define water saturation and salinity in invaded zones. Therefore, a multifrequency dielectric logging tool (Ligneul et al. [33], Seleznev et al. [34]) is designed to measure the permittivity and conductivity at four different frequency components (\( f_{\text{max}} = 1 \) GHz). The responses from the lower frequency channels are still under the influence of interfacial polarization effect. However, we can obtain more stable permittivity logs as the frequency becomes higher than 1 GHz (Figure 1). Thus, we can observe more obvious relationship between the water volume in formation and the response of permittivity scanners with multifrequency channels as presented in Figure 2.

2.3. Analytic Models of Dielectric Permittivity. We employed two different analytic models; complex refractive index model (CRIM) based approach for a mono frequency analysis and Stroud-Milton-De (SMD) for frequency dispersion analysis using multichannel frequency measurement data, to simulate the bulk dielectric permittivity. The CRIM and SMD model are common in that they can incorporate the volume fraction of fluids and minerals. However, they have different applicable frequency range. The CRIM is applied for a formation in the frequency range where textural effects have disappeared, which is at the highest-frequency (1 GHz) of the dielectric logging tool. In contrast, the SMD model can simulate a dispersion relation under the frequency range which can cover the frequency of multichannel dielectric permittivity scanner. For both models, the water permittivity \( \varepsilon_w^* \) is calculated based on analytic solutions (Stogryn [35]). Also, inspired by Baker and Kenyon [17], we assume that the cementation factor \( (m) \) and tortuosity factor \( (n) \) are identical in Archie’s equation (Archie [36]). For conciseness, we will refer to those terms; tortuosity and cementation factor, as Archie’s parameter for the rest of the paper.

Figure 2: Response of multifrequency dielectric permittivity scanner in a borehole: (a) channel 1, (b) channel 2, (c) channel 3, and (d) channel 4, vs. water volume fraction color coded with shale volume. The frequency increases from channel 1 to channel 4.
2.3.1. Complex Refractive Index Model (CRIM). The most popular model to define a bulk permittivity in porous rock is the complex refractive index model (CRIM) (Birchak et al. [19], Greaves et al. [37], Seleznev et al. [15]). There have been several trials to incorporate additional term into the CRIM to demonstrate the superficial polarization (Knight and Abad [38], Knight and Endres [39]). In this paper, however, we start from a standard CRIM to generalize the model. The CRIM does not allow variation of the texture because the texture affects the interfacial polarization (or Maxwell-Wagner effect). For example, in porous rocks filled with saline water, the relative permittivity becomes dispersive under the frequency range of the dielectric logging tool device. The reason for the occurrence of dispersion is that they are subjected to interfacial polarization, which has a relaxation frequency within the frequency of interest in the dielectric logging tool. All the frequencies of the dielectric logging tool are close to the interfacial polarization relaxation frequency, so they are in a transition zone where permittivity decreases and conductivity increases as frequency increases. Therefore, the CRIM is applied for a formation in the frequency range where textural effects have disappeared, which is at the highest frequency (1 GHz) of the dielectric logging tool. The equation of standard CRIM can be written as follows:

\[
\sqrt{\varepsilon^*} = (1 - \phi)\sqrt{\varepsilon_m} + \phi \left[ S_w \sqrt{\varepsilon_w^*} + (1 - S_w)\sqrt{\varepsilon_{oil}} \right],
\]

where \(S_w\) denotes water saturation. \(\varepsilon^*\) is the effective complex dielectric permittivity of the rock, and corresponding subscript index denotes the materials; \(m\) and \(w\) denote matrix and water, respectively. \(\varepsilon^*\) and \(\varepsilon\) mean the complex and real number of permittivity, respectively. \(\phi\) means total

![Figure 3: Core measurements of dielectric permittivity showing different frequency dispersion trend according to the water saturation changes: (a) 29%, (b) 97%, and (c) 100%. The green (\(\alpha = 0.3\)) and blue (\(\alpha = 0.4\)) curves are calculated permittivity by applying the constant Archie’s parameter), and black dashed line means the measured permittivity from a core sample.](image-url)
porosity. According to equation (4), CRIM shows that the refractive index of the bulk formation is the volumetric average of its constituents’ refractive indices. Total organic carbon (TOC) needs to be included in immobile part, $\varepsilon_m$, which contains the volume of TOC ($v/v$) and permittivity of bitumen ($2.07 \sim 2.14$).

We applied CRIM to core samples which are fully saturated with brine under 1000 psi. Mineral fraction of the cores samples is measured using an X-ray diffraction (XRD) device. However, we need to note that there might exist uncertainty in the water saturation since injecting water under high pressure, over long periods of time, does not always guarantee 100% saturation of water in core samples.

For instance, it is hard to make a core sample fully saturated when the rock contains large portions of secondary porosity.

2.3.2. Lichtenecker-Rother (LR) Model. According to the results, the CRIM with the exponent 0.5 could not reproduce the measured permittivity. In this regard, we combined the Archie’s equation and CRIM to replicate the response of dielectric logging tool. By plugging in the Archie’s equation, we proposed the generalized CRIM, which can be derived as follows:

$$\varepsilon^{*}_\alpha = 1 - \phi \left( \varepsilon_m^{*\alpha} + \phi \left[ S_w \varepsilon_w^{*\alpha} + (1 - S_w) \varepsilon_{oil}^{*\alpha} \right] \right),$$

where $\alpha$ is inverse of Archie’s parameter ($\alpha = 1/m$). We set the water salinity 20kppm for the calculation of water permittivity. This generalized CRIM model is termed as Lichtenecker-Rother (LR) model [40]. Nevertheless, even in the LR model, a core sample with extremely small pore volume ($<1\%$) could not reproduce the measured permittivity. Theoretically, the calculated dielectric permittivity can be separated in real part and imaginary part as displayed in equation (1). For clear illustration of the frequency dispersion in dielectric permittivity, we presented the core measurement example in Figure 3. The green ($\alpha = 0.3$) and blue ($\alpha = 0.4$) curves are calculated permittivity by applying the constant Archie’s parameter, and black dashed line means the measured permittivity from a core sample. The red curve exhibits the dielectric permittivity acquired by the curve fitting. By calculating the optimum Archie’s parameter which minimizes the error between the calculated and observed dielectric permittivity, we can apply the correction to inaccurate water saturation which is estimated
mainly by using resistivity logs. In Figure 3, the shape of dispersion curve varies according to the water saturation (29%, 97%, and 100%). The diminishing curve from low-frequency end and concave curve with peak frequency denote imaginary and real part of the permittivity, respectively.

2.3.3. Stroud-Milton-De (SMD) Model. Stroud et al. [20] proposed a textural model which is generally referred to as the SMD model. In this model, the spectral density function of the composites plays a key role in simulating the dispersive response of dielectric constant with respect to frequencies. The spectral density function can be determined by the distribution and composition of the interfaces among different phase, incorporating the depolarization factor of the phases. We used a two-phase model (Mavko et al. [41]), which consists of water and a mix of rock and oil. We can start developing SMD model by considering the complex relative fraction of water and

![Graphs showing the SMD model-fitting for the core data that are fully saturated with water.](image)

Figure 6: SMD model-fitting for the core data that are fully saturated with water. Note that the implementation of curve-fitting only focuses on real part of the permittivity $\varepsilon'$. $\sigma_{SMD}$ is calculated DC conductivity from the estimated permittivity. Each panel (a, c, e) shows the expanded part of the high frequencies (>10^6 Hz) as highlighted with red rectangles in each panel (b, d, f), respectively.
matrix permittivity \( (1 - \varepsilon_w / \varepsilon_m) \). However, working with the inverse of this relative fraction makes tasks easier

\[
s = \left( 1 - \frac{\varepsilon_w}{\varepsilon_m} \right)^{-1}.
\]  

By applying the above relation and integral representation, the analytic function \( f(s) \) (Stroud et al. [20]) can be set as

\[
f(s) = \frac{A}{s} + \int_0^s \frac{g(s')}{s - s'} ds',
\]

where the constant \( A \) consists of the actual volume fraction of water and Archie’s parameter. Therefore, \( A \) can be rewritten as \((S_w \phi)^m\), where \( \phi \) denotes the volume fraction occupied by pore structures. Since Stroud et al. [20] only introduced the test results with 100\% brine saturated case, they assumed that the \( A \) term only depends on the porosity \((S_w = 1)\).
The spectral density function \( g(s') \) takes positive real values, and the density function can be expressed as

\[
g(s') = C s'^{-0.5} (1 - s')^{0.5},
\]

where the exponent 0.5 is motivated by the standard CRIM (Birchak et al. [19], Seleznev et al. [15]). Stroud et al. [20] proposed different forms of the density function as shown in the following equation

\[
g(s') = C s'^{-b} (1 - s')^{e},
\]

where \( C, b, \) and \( e \) are parameters which can be determined as follows:

\[
C = \frac{\Gamma(2 - b + e)}{\Gamma(1 - b)\Gamma(1 - e)} (\phi_w - A), \tag{10}
\]

where

\[
b = 1 - \frac{\phi_w (1 - \phi_w)}{2\phi_w - A (3 - \phi_w)}, \quad e = \frac{\phi_w (\phi_w - A)}{2\phi_w - A (3 - \phi_w)}. \tag{11}
\]

Note that both the exponents \( b \) and \( e \) are positive values over the entire range of water volume fraction, when
Archie’s parameter is two ($m = 2$). Figure 4 exhibits the behavior of the density function $g(s')$ according to the change of water volume fraction and Archie’s parameter. Observing Figure 3, we can find that the value of $g(s')$ becomes unstable at large $s'$ as the water volume fraction $\phi_w$ decreases; however, as far as $\phi_w$ is greater than 7%, we could utilize SMD model for the estimation of dielectric permittivity. When we varied Archie’s parameter to demonstrate the behavior of the density function, the results rarely change as Archie’s parameter becomes greater than 1.8 as shown in Figure 3. The low-frequency components ($\omega \ll \sigma_w/\varepsilon_0\varepsilon$) correspond to small absolute values of $s$ as shown in Figure 5, where the values of $s$ are plotted for core samples saturated with two types of brine: one conductive (higher salinity) and the other moderately conductive (lower salinity). Stroud et al. [20] demonstrated the example of sandstone (black and red solid line) in Figure 5. We additionally displayed the example of clay samples by varying the amount of clay bound water and displayed the results with dashed line in Figure 5. According to the result, as the clay contains more water, the radius of the circular curve becomes greater. Also, when the core samples become more conductive, all the responses are concentrated to the smaller range of $s$, which means that the core samples saturated with higher salinity brine show smaller variations of $s$ with respect to the frequencies.

To investigate the dispersion relationship between the dielectric permittivity and frequency, we can examine the asymptotic behavior of the complex dielectric permittivity. The asymptotic behavior of $\varepsilon'$ and $\sigma$ can be expressed as

$$\varepsilon' = \varepsilon^{1-b}\left(\frac{\sigma_w}{\omega\varepsilon_0}\right)^b \frac{C\pi}{2 \sin (b\pi/2)},$$

$$\sigma = A\sigma_w + \varepsilon^{1-b}\sigma_w(\varepsilon_0\omega)^{1-b}.$$
We use the above asymptotic behavior to simulate the frequency dependent dispersive dielectric permittivity and conductivity in the core samples. In this research, we applied the SMD model to the core data to validate the behavior of the SMD model in shale formation. In addition, we calculated the optimum value of Archie’s parameter through the curve-fitting using nonlinear least square inversion. We calculated the Archie’s parameter by fitting the permittivity values and applied the estimated Archie’s parameter to obtain conductivity. In other words, conductivity is not considered to optimize the Archie’s parameter, but only used to validate the optimum value of Archie’s parameter that is obtained from the model fitting of dielectric permittivity by comparing the calculated conductivity with the measured conductivity.

For utilizing the SMD model in observation of the dispersion relation as a function of frequency, note that the frequency range is a crucial factor to consider. Stroud et al. [20] introduced sandstone examples at high-frequency range (10⁷ Hz ~ 1 GHz). In such a high-frequency range, we cannot define the key parameters of energy dissipation and frequency dispersion. Accordingly, we extended the frequency range to cover the low-frequency part (10³ Hz ~ 10⁷ Hz) by applying the asymptotic approach of the SMD model as presented in equation (12) since there is little change on the slope of dispersion curve.

The result of the SMD model fitting with optimum Archie’s parameters is presented in Figure 6. Each figure shows the case of 100% water saturation. According to the measured data, as the amount of water saturation increases, the low-end permittivity ε₀ and the relaxation time τ also increase. Nonetheless, the SMD model could not simulate the ideal curve of frequency dispersion for the dielectric permittivity to define a knee point of the energy dissipation. Therefore, we put more weight on the high frequency band (>10⁹ Hz), the SMD model experiences a bias toward higher frequencies due to a higher density of data available at the high-frequency band. Therefore, the inversion algorithm used for curve-fitting puts more weight on higher frequency, which might result in inaccurate permittivity estimation on the low-frequency end.

3. Field Data Examples and Discussion

3.1. Lichtenecker-Rother (LR) Model. Since the LR model (generalized CRIM) yields more reliable results than the standard CRIM, we directly employed the LR method without demonstrating the CRIM. For the application of the LR model to the field data, we assume that the matrix consists of 5 lithologies: clay, calcite, quartz, dolomite, and pyrite. For the calculation of dielectric constant of water ε∗ w using [35]’s approach, temperature gradient is fixed to 0.03°C/m with 0°C at surface. Water salinity is set to 50 kppm. One of key factors that might have an influence on water
saturation is permittivity of clay. As the permittivity of clay varies depends on the amount of clay bound water, we set different conditions for the clay permittivity. The major difference between those assumptions is to use different dielectric constant: from dry clay \((\varepsilon_{\text{clay}}=7)\) to wet clay \((\varepsilon_{\text{clay}}=43)\), as shown in Figure 7. As the formation does not contain a large portion of clay bound water in the study area, the assumption with dry clay produces better fit with the reference water saturation, while wet clay assumptions yield somewhat smaller amount of water saturation than reference values. This phenomenon is attributed to the fact that clay bound water is considered as immobile part and included in \(\varepsilon_m\) term, which contributes to the decrease in the amount of water saturation under the given permittivity.

The reference water saturation is calculated by combining the Luffel et al.’s (Luffel et al. [42]) and Archie’s (Archie [36]) equation. When applying Archie’s equation, we used the constant cementation and tortuosity factor \((m=n=1.8)\). The Luffel’s equation is employed only for the depth interval with high TOC (>2% in weight percent).

Given the water saturation estimated using dielectric permittivity log (Figure 7), the calculated water saturation using the constant cementation factor \((m=1.8)\) shows good agreement since it is computed using similar regime of the reference one. However, the corresponding response of calculated dielectric logging tool which is displayed in Figure 7 has a somewhat different range of amplitude compared to the measured permittivity response, though it shows similar shape of curve throughout the survey range. Hence, we are to estimate the amount of correction in water saturation by reproducing the measured dielectric permittivity.

![Figure 13: SMD model-fitting for the side core samples of the well.](image-url)
By applying equation (5), we can compute the dielectric logging tool response as shown in Figure 8. The nonlinear least square method is used to find an optimized Archie’s parameter, \( m \). The black dashed line in Figure 8 means the Archie’s parameter \( (m = 1/\alpha) \) which can minimize the error \( (\epsilon_{\text{measured}} - \epsilon_{\text{modeled}}) \) at each measured depth point. The optimized Archie’s parameter can generate the dielectric responses which show good agreement with the measured data as displayed in Figure 8 with red dashed lines. The corresponding water saturation from optimized \( m \) is presented in Figure 9. According to the calculated well logs, the optimized Archie’s parameter has higher value at the point where larger volume of organic carbon exits. This phenomenon is attributed to the low conductivity of organic carbon that is located among the pore volumes. One possible interpretation is that those organic carbon make current difficult to flow, so it results in the elevation of tortuosity level. Similarly, the optimized Archie’s parameter shows inversely proportional trend with clay volume. This is because the larger volume of clay may decrease the tortuosity level due to its relatively high conductivity. Given that TOC contributes considerably small portion to the bulk permittivity, it is intriguing that the optimized Archie’s parameter shows similar trend with TOC volume. We displayed a crossplot (Figure 10) to demonstrate the relation between the estimated Archie’s parameter and clay volume.

The log view displayed in Figure 11 shows difference between estimated and measured water saturation with relevant well logs. All the water saturation curves in Figure 11 are displayed in volume fraction. First, the reference water saturation curve is corrected at high TOC region, and Luf-fel’s equation is used for making this correction. The 8th track shows water saturation curve calculated by using the LR model with optimized Archie’s parameter which has better agreement with the core measurement compared to the reference one in 9th track. The amount of over- or underestimation is highlighted with green and orange colors in 7th track, respectively.

In general, dielectric constants that are measured in the lab tend to have smaller values than those that are measured using a dielectric logging tool. One of key goal of this research is to develop a method for calibrating the dielectric measurement of the core samples. Given that smaller values of dielectric permittivity in core samples are attributed to the loss of water volume, it is important to estimate the amount of water that is required for core samples to generate the same dielectric response with well logs. Below is a summary of the workflow we developed for calibrating core dielectric measurements:

(i) Defining clay permittivity by performing a modeling of water saturation (well log) using different type of clay assumption: dry clay or wet clay

(ii) Estimation of optimized Archie’s parameter (well log) using the LR model and non-linear least-square method

(iii) Calculate water saturation (well log) using CRIM with optimized Archie’s parameter

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Figure 14: (a) Optimized Archie’s parameter obtained by the SMD model fitting, and (b) the water saturation from the optimized Archie’s parameter.

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Figure 14: (a) Optimized Archie’s parameter obtained by the SMD model fitting, and (b) the water saturation from the optimized Archie’s parameter.
(iv) Applying the optimized Archie’s parameter to core data

(v) Estimation of water saturation that is required to replicate the dielectric logging tool responses under the given condition of core sample such as mineral composition and water saturation by using nonlinear least-square method. Note that we used log-based optimum Archie’s parameter to estimate the required amount of water saturation for the calibration of core measurement.

As shown in the core sample case, similarly, XRD measurement is performed to investigate mineral fractions for calculating the matrix permittivity. The optimized Archie’s parameter shown in 6th track of Figure 11 is obtained based on the trend of dielectric logs measured using multifrequency dielectric logging tool. As we described in well log examples, similarly, the optimized Archie’s parameter in core samples tends to diminish as depth becomes greater. The red dots presented in Figure 12 demonstrate that the optimized Archie’s parameter with dashed black line can replicate the complex permittivity which is obtained via lab experiments.

3.2. SMD Model. In addition to the study on the core samples using the LR model as shown in the previous section, we also applied the SMD model fitting to the field dataset. We displayed several SMD model fitting of core samples from the borehole in Figure 13. In each subfigure, the dashed line shows the modeled permittivity using SMD model \((m = 2)\). The solid line is the result of curve-fitting with optimized Archie’s parameter. We performed model-fitting only in the high-frequency band since the behavior of the SMD model does not match well with the lab-measured data.

The black-dashed line in Figure 14(a) shows the optimized Archie’s parameter to replicate the dielectric permittivity in the SMD model. We also applied this cementation factor (or tortuosity factor) to calculate the water saturation by using Archie’s equation as shown in Figure 14(b). We also displayed the water saturation measured in core samples (red dots). We can find that there are several depth intervals which do not have optimum Archie’s parameter and water saturation. This is attributed to the noisy permittivity especially in lower frequency channel of the multifrequency dielectric logging tool. As the SMD model measures the dispersive response of dielectric constant, it is sensitive to the permittivity at low-frequency channels in the scanner. There occurs interfacial polarization of fluids in a porous rock, and this phenomenon becomes more dominant as the frequency decreases. Thus, it is still challenging to make a stable measurement of permittivity at low frequency under the borehole condition.

To further the validation of the calculated Archie’s parameter, we created crossplots showing correlation between the estimated Archie’s parameter and water volume.

![Crossplot showing the correlation between the optimized Archie’s parameter and formation properties: (a) water volume, (b) TOC, and (c) clay volume.](image_url)
(Figure 15). We could find exponentially decreasing relation from the crossplot with Archie’s parameter and water volume. However, although we expected the negative correlation between the estimated Archie’s parameter and clay volume as we demonstrated in the LR model examples, there is no tangible relation in the other crossplots of Figure 15 due to the inherent errors from the low-frequency channel measurement.

4. Conclusion

We applied two different analytic models to demonstrate (1) which model is the most appropriate to calculate the dielectric permittivity in shale formations, and (2) which model provides more reliable Archie’s parameter combined with dielectric permittivity logs. First, the LR model can be applied to the frequency range without the effect of interfacial polarization (over 1 GHz). As we tested with the standard CRIM with exponent 0.5, the model could not replicate the actual permittivity measured in a lab. Therefore, we used a combined model (LR model and Archie) to calculate the dielectric permittivity and water saturation using 1 GHz frequency component; where the exponent of the LR model is identical to the cementation (or tortuosity) factor of the Archie’s equation. Throughout an application of the LR model, we compared the dielectric permittivity from the core plugs and the well logs. We then demonstrated that core plugs has smaller value of dielectric permittivity than the in situ permittivity. Given that the measurement on core plugs is made one year after the core being acquired, there might be loss of water volume in core samples. Fitting the response of the dielectric logging tool, we could estimate the water volume being required to replicate the same dielectric permittivity response. While fitting the CRIM, we estimated the optimum Archie’s parameter. The estimated Archie’s parameter shows inversely proportional relationship with clay volumes. We also employed the SMD model to estimate Archie’s parameter and corresponding water saturation. We found a limitation of the model in that the SMD model, which could not generate the actual behavior of the permittivity dispersion (a sigmoidal dispersion curve) with varied frequencies, especially at the low-frequency channel. The SMD model is designed based on the spectral density model which is sensitive to the water volume. As the water volume becomes smaller (less than 7%), the behavior of the spectral density function also becomes unstable and it bears a greater level of uncertainty as we lower the target frequencies. Therefore, the SMD model is not applicable to the rock saturated with considerably small volume of water. In addition, estimated Archie’s parameter from the SMD model is extremely sensitive in the lower frequency channels of the multifrequency dielectric logging tool, and it is not trivial task suppressing the low-frequency noises that are associated with the borehole conditions in the subsurface. Nevertheless, utilizing the response of the dielectric logging tool in the frequency range where interfacial polarization effect does not exist can provide an alternative to estimate water saturation in shale formations with relatively less conductive waters.

Data Availability

The raw and processed data presented in this paper and the materials required to reproduce these findings cannot be shared since authors do not have the legal right to release the data.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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