New Taub-NUT-Reissner-Nordström spaces in higher dimensions

Robert Mann\textsuperscript{1} and Cristian Stelea\textsuperscript{2}

\textsuperscript{1}Perimeter Institute for Theoretical Physics
31 Caroline St. N. Waterloo, Ontario N2L 2Y5, Canada

\textsuperscript{1,2}Department of Physics, University of Waterloo
200 University Avenue West, Waterloo, Ontario N2L 3G1, Canada

Abstract

We construct new charged solutions of the Einstein-Maxwell field equations with cosmological constant. These solutions describe the nut-charged generalisation of the higher dimensional Reissner-Nordström spacetimes. For a negative cosmological constant these solutions are the charged generalizations of the topological nut-charged black hole solutions in higher dimensions. Finally, we discuss the global structure of such solutions and possible applications.

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\textsuperscript{1}E-mail: mann@avatar.uwaterloo.ca
\textsuperscript{2}E-mail: cistelea@uwaterloo.ca
1 Introduction

Despite their unphysical and bizarre properties, nut-charged spacetimes are receiving increased attention in recent years. Intuitively a nut charge corresponds to a magnetic type of mass. Its presence induces a so-called Misner singularity in the metric, analogous to a ‘Dirac string’ in electromagnetism [1]. This singularity is only a coordinate singularity and can be removed by choosing appropriate coordinate patches. However, expunging this singularity comes at a price: in general we must make coordinate identifications in the spacetime that yield CTCs in certain regions.

The presence of closed time-like curves (CTCs) in their Lorentzian section is a less than desirable feature. However, one could argue that it is precisely this feature that makes them more interesting. Recently the nut-charged spacetimes have been used in string theory as testbeds for the AdS/CFT conjecture [2, 3]. Another very interesting result was obtained via a non-trivial embedding of the Taub-NUT geometry in heterotic string theory, with a full conformal field theory definition (CFT) [4]. It was found that nutty effects were still present even in the exact geometry, computed by including all the effects of the infinite tower of massive string states that propagate in it. This might be a sign that string theory can very well live even in the presence of nonzero NUT charge, and that the possibility of having CTCs in the background can still be an acceptable physical situation. Furthermore, it has been recently noted that the boundary metric Lorentzian sector of these spaces in AdS-backgrounds is in fact similar with the Gödel metric [5, 6]. In addition, in asymptotically dS settings, regions near past/future infinity do not have CTCs, and NUT-charged asymptotically dS spacetimes have been shown to yield counter-examples to some of the conjectures advanced in the still elusive dS/CFT paradigm [7]—such as the maximal mass conjecture and Bousso’s entropic N-bound conjecture [8, 9, 10]. Moreover, they have been used to uncover deep results regarding the gravitational entropy [2, 11, 12, 13, 14, 15] and, in particular, they exhibit breakdowns of the usual relation between entropy and area [16] (even in the absence of Misner string singularities).

The first solution in four dimensions describing such an object was presented in [17, 18]. There are known extensions of the Taub-NUT solutions to the case when a cosmological constant is present and also in the presence of rotation [19, 20, 21]. In these cosmological settings, the asymptotic structure is only locally de Sitter (for a positive cosmological constant) or anti-de Sitter (for a negative cosmological constant) and we speak about Taub-NUT-(a)dS solutions.

Generalisations to higher dimensions follow closely the four-dimensional case [22, 23]. In constructing these metrics the idea is to regard Taub-NUT spacetimes as radial extensions of $U(1)$ fibrations over a $2k$-dimensional base space endowed with an Einstein-Kähler metric $g_B$. Then the $(2k + 2)$-dimensional Taub-NUT spacetime has the metric:

$$F^{-1}(r)dr^2 + (r^2 + N^2)g_B - F(r)(dt + A)^2$$

(1)

where $t$ is the coordinate on the fibre $S^1$ and the one-form $A$ has curvature $\mathcal{J} = dA$, which is proportional to some covariantly constant 2-form. Here $N$ is the NUT charge and $F(r)$ is a function of $r$. Recently NUT-charged spacetimes with more than one nut parameter have been obtained as exact solutions to higher-dimensional Einstein field equations [26]. These
solutions have been later generalised to arbitrary dimensions \cite{30}.

There also exists a nut-charged generalisation of the Reissner-Nordström solution in four dimensions \cite{21,31}. However, there are no known higher dimensional generalisations of the nut-charged solutions in Einstein-Maxwell theory. The main purpose of this paper is to provide the generalisation of these spaces to higher dimensions. These solutions will represent the electromagnetic generalisation of the nut-charged spacetimes studied in refs. \cite{27,22,12,37,15} as well as the nut-charged generalisation of the higher dimensional Reissner-Nordström solutions \cite{32}.

Einstein-Maxwell theory in \( D \)-dimensions is described by the following action:

\[
I = -\frac{1}{16\pi G} \int d^Dx \sqrt{-g} [R - 2\Lambda - F^2] \tag{2}
\]

The equations of motion derived from this action can be written in the following form\(^1\):

\[
R_{\mu\nu} = 2 \left( F_{\mu\rho}F_{\nu}^{\rho} - \frac{1}{2(D-2)} F^2 g_{\mu\nu} \right) + \lambda g_{\mu\nu}
\]

\[
\nabla_\mu F^{\mu\nu} = 0 \tag{3}
\]

where \( F = dA \) is the electromagnetic 2-form field strength corresponding to the gauge potential \( A \). Our conventions are as follows: we take \((-+,+,\ldots,+)\) for the (Lorentzian) signature of the metric; in even \( D \) dimensions our metrics will be solutions of the sourceless Einstein-Maxwell field equations with cosmological constant \( \Lambda = \pm \frac{(D-1)(D-2)}{2l^2} \).

\section{The general solution}

Let us recall first the form of the 4-dimensional Taub-NUT-Reissner-Nordström metric. The metric is given by:

\[
ds^2 = -F(r)(dt - 2NA)^2 + F^{-1}(r) dr^2 + (r^2 + N^2) g_M \tag{4}
\]

where \( M \) is a 2-dimensional Einstein-Kähler manifold, which can be taken to be the unit sphere \( S^2 \), torus \( T^2 \) or the hyperboloid \( H^2 \). In each case we have:

\[
A = \begin{cases} 
\cos \theta d\phi, & \text{for } \delta = 1 \text{ (sphere)} \\
\theta d\phi, & \text{for } \delta = 0 \text{ (torus)} \\
cosh \theta d\phi, & \text{for } \delta = -1 \text{ (hyperboloid)}
\end{cases}
\]

while the function \( F(r) \) and the gauge field potential \( A \) have the following expressions:

\[
F(r) = \frac{r^4 + (l^2 + 6N^2)r^2 - 2ml^2 r - 3N^4 + l^2(q^2 - N^2)}{l^2(r^2 + N^2)}
\]

\[
A = -\frac{qr}{r^2 + N^2} (dt - 2NA) \tag{5}
\]

\(^1\)We use here \( \lambda = \pm \frac{D-1}{l^2} \).
Here $m$, $q$ and $N$ are respectively the mass, charge and the NUT parameter. As one can see directly from the expression of the 1-form gauge potential, one noteworthy feature of this solution is that the electromagnetic field strength carries both electric and magnetic components. Moreover, if we try to compute the electric and magnetic charges the results will depend on the radius of the 2-sphere on which we integrate (see also [34]). However, if we take the limit in which the 2-sphere is pushed to infinity we find that the magnetic charge vanishes and the solution is purely electric with charge $q$.

We are now ready to present the general class of electrically-charged Taub-NUT metrics in even dimensions $D = 2d + 2$. These spaces are constructed as complex line bundles over an Einstein-Kähler space $M$, with dimension $2d$ and metric $g_M$. The metric ansatz that we use is the following:

$$ds^2_D = -F(r)(dt - 2NA)^2 + F^{-1}(r)dr^2 + (r^2 + N^2)g_M$$  \hspace{1cm} (6)$$

Here $J = dA$ is the Kähler form for the Einstein-Kähler space $M$ and we use the normalisation such that the Ricci tensor of the Einstein-Kähler space $M$ is $R_{ab} = \delta g_{ab}$.

Motivated by the known four-dimensional solution we shall make the following ansatz for the electromagnetic gauge potential:

$$A = -\sqrt{\frac{D-2}{2}} \frac{qr}{(r^2 + N^2)^{D-1}}(dt - 2NA)$$  \hspace{1cm} (7)$$

Then the general solution to Einstein’s field equations with cosmological constant $\lambda = \pm(D - 1)/l^2$ is given by:

$$F(r) = \frac{r}{(r^2 + N^2)^{D/2}} \left[ \int (\frac{D-1}{l^2}(s^2 + N^2)) \frac{(s^2 + N^2)^{D/2}}{s^2} ds - 2m \right]$$

$$+ q^2 \frac{(D-3)r^2 + N^2}{(r^2 + N^2)^{D-2}}$$  \hspace{1cm} (8)$$

As in the 4-dimensional case the electromagnetic field strength has both electric and magnetic components. If we try to compute the electric and magnetic charges we obtain again results that depend on the radial coordinate $r$. However, if we push the integration surfaces to infinity the magnetic charge will vanish leaving us only with an effective electric charge.

As an example of this general solution, let us assume that the $(D - 2)$-dimensional base space in our construction is a product of $d$ factors, $M = M_1 \times \cdots \times M_d$ where $M_i$ are two dimensional Einstein-Kähler spaces or more generally $CP^n$ factors. In particular, we can use the sphere $S^2$, the torus $T^2$ or the hyperboloid $H^2$ as factor spaces. It is then easy to see that if $q = 0$ we recover the previously known higher dimensional Taub-NUT solutions studied in refs. [27, 22, 12, 37, 15]. On the other hand, if $N = 0$ then we recover the topological Reissner-Nordström-AdS solutions given in [32, 33].
3 Regularity conditions

Scalar curvature singularities have the possibility of manifesting themselves only in the Euclidean sections. These are simply obtained by the analytic continuations $t \rightarrow i\tau$, $N \rightarrow in$ and $q \rightarrow iq$, and can be classified by the dimensionality of the fixed point sets of the Killing vector $\xi = \partial/\partial\tau$ that generates a $U(1)$ isometry group. In four dimensions, the Killing vector that corresponds to the coordinate that parameterizes the fibre $S^1$ can have a zero-dimensional fixed point set (we speak about a ‘NUT’ solution in this case) or a two-dimensional fixed point set (referred to as a ‘bolt’ solution). The classification in higher dimensions can be done in a similar manner. If this fixed point set dimension is $(D-2)$ the solution is called a Bolt solution; if the dimensionality is less than this then the solution is called a NUT solution. Notice that if $D > 4$ then NUTs with larger dimensionality can exist [26, 27]. In general, fixed point sets need not exist; indeed there are parameter ranges of NUT-charged asymptotically $dS$ spacetimes that have no Bolts [35].

The singularity analysis performed here is a direct application of the one given in [23]. In order to extend the local metrics presented above to global metrics on non-singular manifolds the idea is to turn all the singularities appearing in the metric into removable coordinate singularities. For generic values of the parameters the singularities are not removable, corresponding to conical singularities in the manifold. We are mainly interested in the case of a compact Einstein-Kähler manifold $M$. Generically the Kähler form $J$ on $M$ can be equal to $dA$ only locally and we need to use a number of overlapping coordinate patches to cover the whole manifold. Now, in order to render the 1-form $d\tau - 2nA$ well-defined we need to identify $\tau$ periodically. We will require the period of $\tau$ to be given by:

$$\beta = \frac{4\pi np}{k\delta}$$

where $k$ is a positive integer, while $p$ is a non-negative integer, defined as the integer such that the first Chern class, $c_1$, evaluated on $H_2(M)$ is $\mathbb{Z} \cdot p$, i.e. the integers divisible by $p$. Among all the Einstein-Kähler manifolds the integer $p$ is maximised in $CP^q$, for which $p = q + 1$ [23]. It is also necessary to eliminate the singularities in the metric that appear as $r$ is varied over $M$. The critical points are to the so-called endpoint values of $r$: these are the values for which the metric components become zero or infinite. For a complete manifold $r$ must range between two adjacent endpoints and we must eliminate the conical singularities (if any) which occur at these points. The finite endpoints occur at $r = \pm n$ or at the simple zeros of $F_E(r)$. Quite generally, when the electrical charge $q$ is zero, $r = \pm n$ give the location of curvature singularities unless $F_E = 0$ there as well. By contrast with the uncharged case, it turns out that if $q \neq 0$ then the curvature singularities at $r = \pm n$ cannot be eliminated for any choices of the parameters. This can be seen from the fact that $F_E(r)$ diverges badly when $r \rightarrow \pm n$ for any values of $m$ and the components of the curvature tensor will diverge there as well. Therefore, in order to obtain non-singular Euclidean sections we have to restrict the range of the radial coordinate such that the values $r = \pm n$ are avoided. We should then restrict our attention to simple roots $r_0$ of $F_E(r)$ different from $\pm n$. In general, to eliminate a conical singularity at a root $r_0$ of $F_E(r)$ we must restrict the periodicity of $\tau$ to be given
by:
\[ \beta = \frac{4\pi}{|F_E'(r_0)|} \]
and this will generally impose a restriction on the values of the parameters once we match it with (9). This condition will also fix the location of the bolt, which will be given by a solution of the equation:
\[ npq^2 [(D - 3)^2 r_0^4 - 2n^2 r_0^2 + n^4] + \left[ np \left( \delta - \lambda (r_0^2 - n^2) \right) - k\delta r_0 \right] (r_0^2 - n^2)^{D-1} = 0 \quad (10) \]

For compact manifolds the radial coordinate takes values between two finite endpoints and we have to impose this constraint at both endpoints. If the manifold is noncompact then the cosmological constant must be non-positive and the radial coordinate takes values between one finite endpoint \( r_0 \) and one infinite endpoint \( r_1 = \infty \). Since for our asymptotically locally (A)dS or flat solutions the infinite endpoints are not within a finite distance from any points \( r \neq r_1 \) there is no regularity condition to be imposed at \( r_1 \). In this case the only regularity conditions are that \( F_E(r) > 0 \) for \( r \geq r_0 \) and \( \beta = \frac{4\pi}{|F_E'(r_0)|} \) to be satisfied. The only way to avoid a curvature singularity at \( r = n \) is to restrict the values of the radial coordinate such that \( r \geq r_0 > n \), i.e. the only non-singular Taub-NUT-RN spaces are the TNRN-bolt solutions\(^2\).

4 The Taub-NUT-RN solution in six dimensions

As an illustration of the general analysis performed in the previous section, in this section we shall look more closely at a six-dimensional Taub-NUT-RN solution constructed over the four-dimensional base \( S^2 \times S^2 \). Performing the analytical continuations \( t \to i\tau \), \( N \to i n \) and \( q \to iq \), the metric in the Euclidian sector can be written in the form:
\[
ds^2 = F_E(r) \left( d\tau - 2n \cos \theta_1 d\phi_1 - 2n \cos \theta_2 d\phi_2 \right)^2 + \frac{dr^2}{F_E(r)} + (r^2 - n^2) \left( d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2 \right) + (r^2 - n^2) \left( d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2 \right) \quad (11) \]
where the function \( F_E(r) \), respectively the 1-form potential \( A \) are given by:
\[
F_E(r) = \frac{3r^6 + (l^2 - 15n^2)r^4 + 3n^2(15n^2 - 2l^2)r^2 - 6ml^2r + 3n^4(5n^2 - l^2)}{3(r^2 - n^2)^2 l^2} - \frac{q^2 (3r^2 - n^2)}{(r^2 - n^2)^4} \]
\[
A = -\frac{\sqrt{2}qr}{(r^2 - n^2)^2} \left( d\tau - 2n \cos \theta_1 d\phi_1 - 2n \cos \theta_2 d\phi_2 \right) \quad (12) \]
\(^2\)We are focussing here on non-compact manifolds. For compact manifolds we could restrict the range of the radial coordinate \( r \) to the interval between two adjacent roots \( r_1 \leq r \leq r_2 \) of \( F_E(r) \) such that the values \( \pm n \) are avoided.
Regularity of the 1-form $d\tau - 2n A$ forces the periodicity of the Euclidian time to be $\frac{8\pi n}{k}$, for some integer $k$. As mentioned in the previous section, the NUT solution is singular hence we restrict our attention directly to the Bolt solutions. These solutions correspond to a four-dimensional fixed-point set located at a simple root $r_b$ of $F_E(r)$ and we restrict the values of the radial coordinate such that $r \geq r_b > n$. The periodicity of $\tau$ is given by $\frac{8\pi n}{k}$ and we have to match it with the periodicity obtained by eliminating the conical singularities at $r_b$. This will fix the location of the bolt as given by a root of

$$2nq^2(9r_b^4 - 2n^2r_b^2 + n^4) + \left[2n \left(1 + \frac{5}{l^2}(r_b^2 - n^2)\right) - kr_b\right](r_b^2 - n^2)^5 = 0$$

As this is a polynomial equation of rank 12, an analytical solution for $r_b$ seems out of the question.

Finally, the value of the mass parameter is given by:

$$m_b = \frac{3n^2(6l^2 - q^2l^2 - 5n^8)}{6(r_b^2 - n^2)^2l^2r_b}$$

Generically there is a curvature singularity at $r = n$, which is simply avoided if we restrict the range of the radial coordinate such that $r \geq r_b > n$. If $q = 0$ we recover the six-dimensional cosmological Taub-NUT solution over the base space $S^2 \times S^2$, which was discussed in [37, 26]. If $n = 0$ the solution reduces to the topological Reissner-Nordström solution whose horizon topology is $S^2 \times S^2$.

Similar results are obtained for a fibration over $CP^2$. The only difference appears in the periodicity of $\tau$, which has to be now $12\pi n/k$ and this will also modify the equation for $r_b$ (as it can be read from the general expression (10) with $p = 3$ and $\delta = 1$). Unlike the uncharged case, the NUT solution is singular as there will be a curvature singularity at $r = n$.

5 Conclusions

In this paper we presented new families of higher dimensional solutions of the sourceless Einstein-Maxwell field equations with or without cosmological constant. These solutions are constructed as radial extensions of circle fibrations over even dimensional spaces that can be in general products of Einstein-Kähler spaces.

We have given the Lorentzian form of the solutions. However in order to understand the singularity structure of these spaces, we have concentrated mainly on their Euclidian sections – recognising that the Lorentzian versions are singularity-free – apart from quasi-regular singularities [36]. These correspond to the end-points of incomplete and inextensible geodesics that spiral infinitely around a topologically closed spatial dimension. However, since the Riemann tensor and all its derivatives remain finite in all parallelly propagated orthonormal frames, we take the point of view that these represent some of mildest of types of singularities and we ignore them when discussing the singularity structure of the Taub-NUT solutions.
In general the Euclidean section is simply obtained using the analytic continuations $t \rightarrow i\tau$, $N \rightarrow in$ and $q \rightarrow iq$. Generically the Taub-NUT solutions present themselves in two classes: ‘nuts’ and ‘bolts’. While in the uncharged case there can exist NUTs with intermediate dimensionality \cite{26,27,30}, for our NUT solutions the fix-point set has always dimension 0 and it is singular. Indeed, we found that at the NUT location there always exists a curvature singularity that cannot be eliminated for any choice of the parameters. This is clearly in contrast with the uncharged case: in absence of the electrical charge, there are NUT solutions that are non-singular – for example, if we use $M = CP^q$ then for an appropriate choice of the mass parameter we find that there is no curvature singularity at $r = n$. The only regular charged solutions are then the Bolt metrics. The regularity conditions require us to fix the periodicity of the Euclidian time $\tau$. This periodicity is determined in two ways and by matching the two obtained values we get restrictions on the values of the parameters in our solutions. These restrictions will also fix the location of the bolt. However, as in $D$-dimensions the equation that we have to solve is a polynomial equation of rank $2D$, there is no chance to obtain the bolt location in a closed analytical form.

Finally, we note that for the present solutions the boundary is generically a circle-fibration over base spaces that, being obtained from products of general Einstein-Kähler manifolds, can have exotic topologies. It would be interesting to see if they could be used as test-grounds for the $AdS/CFT$ correspondence and more generally in context of gauge/gravity dualities. We leave a more detailed study of this subject for future work.

\textit{Note added} After the completion of this manuscript we learned about the paper \cite{37} that presents similar higher-dimensional solutions.

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