Magnetic skyrmions are topologically protected vortex-like spin textures that can be formed in non-centrosymmetric magnetic compounds [1]. Due to their stability, their extremely small size, and the possibility to drive them by low current densities, they are promising candidates for spintronic devices such as racetrack memories. In crystals lacking spatial inversion symmetry, the interplay of Heisenberg exchange interaction, antisymmetric Dzyaloshinskii-Moriya interaction, and an external Zeeman field may lead to the formation of vortex-like magnetic SKs. They have been predicted [2-4] years before they were experimentally discovered in magnetic layers with a strong spin-orbit interaction [1, 5, 6].

SKs carry a nonzero, integer value topological charge $Q$, also called SK number [7]. This number is an invariant that counts how many times the field configuration wraps around a unit sphere. It cannot be changed by continuous transformations. Due to this property, SKs are insensitive to imperfect fabrication or disorder. On a lattice, the argument of topological stability has to be replaced by a finite energy barrier, but despite their small size which is typically about 10-100 nm [8], SKs are quite stable. Due to the underlying emergent electromagnetic field induced by the Berry phase, SKs experience a Magnus force [9, 10] that strongly suppresses pinning by deflecting SKs from pinning centers [11]. Thus SKs can be driven at current densities of the order $10^5$ A/m$^2$, about four orders of magnitude lower than required, e.g., for domain walls [12, 13]. This makes SKs very promising candidates for future spintronic applications, especially for racetrack memories consisting of thin nanowires.

For the use in technical applications, however, several hurdles have to be overcome. First, the creation of SKs needs to be possible. This has been demonstrated by various mechanisms, e.g., by sweeping external magnetic fields [14] or by applying circular currents [15]. In addition, the controlled creation and annihilation of single SKs has been realized [16] and similar processes have been theoretically explained [17, 18]. Direct creation or annihilation of SKs suffers from the requirement of large currents or fields. Current-driven SKs on a two-lane racetrack memory devices [19, 20] have been proposed where SKs are placed on different “lanes” of a broad racetrack. However, current-driven SKs experience the SK Hall effect, in which the SKs develop a motion perpendicular to the direction of the applied current, just like charged particles in the standard Hall effect [21, 22]. In experiments, the corresponding SK Hall angle $\Theta_{\text{SHE}}$ between the transverse and the longitudinal velocity component, the latter is always antiparallel to the current, has exceeded 30° [21]. It depends on the Gilbert damping, the nonadiabaticity parameter, and the spin torques, but is independent of the external current density, at least when it overcomes some small threshold [22]. These facts have been previously explained based on a general SK equation of motion for the topological charge density [17], or the Thiele equation for a specific SK configuration [12]. A possible dependence on the external current density [21] is probably due to intrinsic pinning or SK deformation but is not yet fully explained.

The presence of the SK Hall effect limits the use of SKs on racetracks because the transverse velocity component can lead to annihilation of the SK at the edges of the track. For this reason, the SK Hall effect is typically seen as a detrimental effect. Several approaches have been proposed to keep SKs on the track. Most of them aim at creating potential barrier at the track edges, deflecting the SKs [23]. However, this can lead to an inefficient and hard to control zigzag path and to irregular SK motion.

In this work, we show that the Rashba spin-orbit interaction can be used to steer the SK Hall angle due to the interplay of current-induced spin-transfer torques and Rashba spin-orbit torques. This can even be used to completely suppress the SK Hall angle. Moreover, with an externally applied gate voltage it is possible to modify the magnitude of the spin-orbit torques allowing us to steer the SK Hall angle all-electronically. With this mechanism it is possible to move SKs on a broad racetrack at high speed, to efficiently steer their trajectories, e.g., to change lanes, and to realize conceptually new writing and gating operations with a tunable gate voltage.

Model – From a theoretical point of view, SKs are two-dimensional quasiparticles that obey the Landau-
Lifshitz-Gilbert equation [24–27], a partial differential equation describing the precessional motion of magnetic moments in a ferromagnetic material. To describe current driven SKs, it is extended by adiabatic and nonadiabatic spin torques, $T_{\text{ad}}$ and $T_{\text{nonad}}$, which are induced by spin-polarized currents[28–30], and reads

$$\partial_t n = -n \times B_{\text{eff}} + \alpha n \times \partial_t n + T_{\text{ad}} + T_{\text{nonad}}$$

(1)

with the normalized magnetization vector field $n = n(x, y, t)$ and the Gilbert damping constant $\alpha > 0$. The effective field $B_{\text{eff}} = -\partial H/\partial n$ contains all interactions of the system Hamiltonian $H$. Here, the gyromagnetic ratio $\gamma$ is absorbed in $B_{\text{eff}}$, $T_{\text{ad}}$, and $T_{\text{nonad}}$ and we set $\hbar = 1$.

Current-induced spin torques – We have calculated the current-induced spin-torques up to second order in the Rashba spin-orbit coupling parameter $\alpha_R$[31, 32] based on a semi-classical Boltzmann approach [33]. To zeroth order in $\alpha_R$, the adiabatic spin-transfer torque

$$T_{\text{STT}}^{\text{ad}} = v_s \partial_x n$$

(2)

is recovered. The prefactor $v_s = Pa^3j_e/(2e)$ with spin polarization $P$, lattice constant $a$ and elementary charge $e$ has the dimension of a velocity and is called effective spin velocity. The effective spin velocity is proportional to the external current density $j_e$ and can therefore easily be tuned. In addition to the spin-transfer torque, we find the adiabatic first-order spin-orbit torque

$$T_{\text{SOT}}^{\text{ad}} = \frac{2m\alpha_R}{\hbar^2} v_s (n \times \hat{y})$$

(3)

with the effective electron mass $m$. Up to first order, all relevant torques reported in the literature [30, 34–38] are recovered.

For the sake of simplicity, we neglect second order spin-orbit torques in the following discussion, thus $T^{\text{ad}} \approx T_{\text{STT}}^{\text{ad}} + T_{\text{SOT}}^{\text{ad}}$. As demonstrated in the Supplemental Material [33], second order torques can enhance the effects discussed in this work and should be considered under certain circumstances.

Damping of the spin dynamics of the localized electrons is described by the Gilbert damping term. Due to effects like impurity scattering or spin-orbit coupling, the itinerant electrons experience damping as well. The corresponding nonadiabatic spin torques are obtained as $T_{\text{nonad}} = -\beta n \times T_{\text{ad}}$ [35] with the nonadiabaticity parameter $\beta$. Since spin-orbit coupling is one of the main damping sources, the nonadiabatic spin-orbit torques can play a major role for the SK dynamics. This property is ultimately responsible for the possibility to steer SKs.

Controlling the skyrmion Hall angle – The topological charge of a SK leads to a theoretically predicted SK Hall effect, in which the SKs acquire a velocity component perpendicular to the applied electronic current. The SK dynamics is governed by Eq. (1) and is illustrated for the SK Hamiltonian

$$H = -J \sum_r n_r \cdot [n_{r+a\hat{x}} + n_{r+a\hat{y}}] - B \sum_r n_r$$

$$-D \sum_r \left[(n_r \times n_{r+a\hat{x}}) \cdot \hat{x} + (n_r \times n_{r+a\hat{y}}) \cdot \hat{y}\right]$$

(4)

defined on a square lattice with lattice sites $r = (x, y)$ and a fixed lattice constant $a = 0.5$ nm. $J = 1$ meV is the exchange interaction, $B$ an external Zeeman field, and $D = 0.18$ meV the Dzyaloshinskii-Moriya interaction strength. These values have been reported for MnSi [12]. The model Hamiltonian describes a system where Bloch SKs get stabilized by the Dzyaloshinskii-Moriya interaction in the bulk. The ground state of the system depends on the magnitude of the Zeeman field. A field $B = B_{\text{z}} \hat{z} = -0.03$ eV $\hat{z}$ (corresponding to $\sim -0.5$ T) applied orthogonal to the lattice is suitable to stabilize SKs with a radius of approximately 5 nm. Typical experimental values of the Gilbert damping parameter $\alpha$ and nonadiabaticity parameter $\beta$ cover a wide range. For $\alpha$, values ranging from $\alpha < 0.01$ to $\alpha \approx 1$[39, 40] and for $\beta$, values ranging from $\beta \approx 0.02$ up to $\beta > 4$[41, 42] have been reported for various materials. For our numerical simulations, we adopt values within these ranges and use a $80 \times 80$ square lattice with periodic boundary conditions.

When neglecting spin-orbit torques, $\Theta_{\text{SHE}}$ vanishes for $\alpha = \beta$ [17]. As the Gilbert damping $\alpha$ and the nonadiabaticity parameter $\beta$ have different origins and in general different magnitudes, this has to be considered a very special case and, so far, no material with $\alpha = \beta$ exhibiting a stable SK phase is known. Therefore, $\Theta_{\text{SHE}}$ does in general not vanish in the absence of further spin torques or external effects.

As shown in the following, the SK Hall angle $\Theta_{\text{SHE}}$ can be steered by the use of the spin-orbit torque which is generated by the spin-orbit interaction in the itinerant electrons. This clearly depends on the magnitude of the spin-orbit torque which is proportional to the Rashba coupling constant $\alpha_R$. Since $\alpha_R$ can be tuned relatively easily by gate voltages, we obtain the possibility to eventually steer $\Theta_{\text{SHE}}$ all-electronically. For many materials with either bulk or interfacial inversion symmetry breaking, a wide range of values for the phenomenological Rashba parameter $\alpha_R$ has been reported. Furthermore, it has been demonstrated experimentally that by applying an external gate voltage, the structure inversion symmetry of a crystalline lattice can be lifted, leading to variations of $\alpha_R$ in the range of $10^{-12}$ to $10^{-11}$ eV. Notably, this also works for metal interfaces as recent reports show [45, 46]. Thus, with a suitable gate voltage and an appropriate composition of materials in a metallic multilayer, a desired Rashba parameter can be realized and the SK Hall angle can be controlled.
To illustrate this idea by explicit results, we show the effect of $\alpha_R$ on $\Theta_{\text{SHE}}$ for various parameter choices of the pair of $(\alpha, \beta)$ in Fig. 1. Increasing $\alpha_R$ increases the magnitude of the spin-orbit torque without affecting the spin-transfer torque. In Fig. 1 (a), we show three curves with different values of $\alpha$ while the nonadiabaticity parameter is fixed at $\beta = 0.4$. At $\alpha_R = 0$, only the spin-transfer torque is present and $\Theta_{\text{SHE}}$ vanishes for $\alpha = \beta = 0.4$. Increasing the Gilbert damping $\alpha$ decreases $\Theta_{\text{SHE}}$ when tuning $\alpha_R$ for a fixed pair of $(\alpha, \beta)$ values, $\Theta_{\text{SHE}}$ decreases as $\alpha_R$ increases. The absolute decrease of $\Theta_{\text{SHE}}$ when tuning $\alpha_R$ is almost the same for different values of $\alpha$. This is surprising as it means that Gilbert damping barely affects the spin-orbit torques.

In Fig. 1 (b) the Gilbert damping is set to $\alpha = 0.4$ and the results for three different values of $\beta$ are shown. We have found that for small $\beta$, $\Theta_{\text{SHE}}$ remains almost constant as $\alpha_R$ changes because the adiabatic component of the spin-orbit torques barely affects $\Theta_{\text{SHE}}$ and the nonadiabatic component of the spin-orbit torque is the relevant one. Therefore, the impact of the spin-orbit torques on $\Theta_{\text{SHE}}$ grows when the nonadiabaticity parameter $\beta$ gets large. For $\beta \geq 0.6$, $\Theta_{\text{SHE}}$ can be decreased by about $10^6$ when tuning $\alpha_R$ within the experimentally accessible range from zero to $\alpha_R = 10^{-11}$ eVm. Only the nonadiabatic components of the spin-orbit torques have a significant impact on $\Theta_{\text{SHE}}$. This is in sharp contrast to the effect of the spin-transfer torque on $\Theta_{\text{SHE}}$. Both the adiabatic and the nonadiabatic component of the spin-transfer torque affect $\Theta_{\text{SHE}}$.

**Suppression of the skyrmion Hall effect**  — Aiming to steer the SK Hall angle around $\Theta_{\text{SHE}} = 0^\circ$, $\beta$ needs to be sufficiently large, otherwise the spin-orbit torque does not have a significant impact and the parameter range in which steering is possible becomes very small. Furthermore, $\alpha < \beta$ has to be chosen properly so that $\Theta_{\text{SHE}} = 0^\circ$ is within the parameter range where steering is possible. For an estimate of a suitable parameter range, we show in Fig. 2 $\Theta_{\text{SHE}}$ as a function of the Rashba spin-orbit coupling parameter $\alpha_R$ and the nonadiabaticity parameter $\beta$ for the two cases $\beta/\alpha = 2$ (a) and $\beta/\alpha = 3/2$ (b). The blue contours depict a vanishing SK Hall angle $\Theta_{\text{SHE}} = 0^\circ$.
between the lanes of a two-lane SK racetrack.

**Concept of a skyrmion racetrack gate** – The possibility of an all-electronic steering of the SK Hall angle opens the doorway to the conceptual design of a gate in a SK two-lane racetrack, as it is sketched in Fig. 3. A SK can either be located in the left or in the right lane of the racetrack, the former corresponding to a “1” and the latter to a “0”. The racetrack could work as follows. A spin polarized current is applied to a broad quantum wire. By choosing suitable materials and possibly applying a gate voltage, the magnitude of the Rashba parameter is adapted such that for a running SK, \( \Theta_{\text{SHE}} = 0 \) and the SK can move along one lane of the wire on straight trajectories (blue areas in Fig. 3). Inevitably, due to material imperfections, fluctuations in the gate voltage, and other effects, a SK will not always move exactly antiparallel to the current flow. It could eventually switch lanes in an uncontrolled way, or get destroyed when hitting an edge of the track. To prevent destruction or loss of information, one can create potential barriers at the track edges and in the middle of the track, separating the wire into two lanes. Due to an effect similar to the Magnus effect, such barriers can deflect the SKs [10] and keep them in their lanes. Only within the writing area (green) the potential barrier separating the two lanes has to vanish. In this writing area, \( \alpha_R \) gets altered by a different gate voltage leading to a tunable SK Hall angle. Thus, the SK can change between the two lanes. After leaving the writing area, the initial configuration is restored and SKs move along their lanes.

![FIG. 3. Sketch of an all-electronically controlled two-lane SK racetrack. Incoming SKs (red circles) get driven by an external spin polarized current \( I_e \). In the blue regions, SKs are located either in the left or in the right lane of the track, the former corresponding to a logical “1” and the latter to a “0”. The two lanes are separated by a potential barrier which deflects the SKs and keeps them on their lanes. Alternatively, the lanes can be designed such that the SK Hall angle is completely suppressed by a constant spin-orbit torque for a distinct \( \alpha_R^0 \) such that \( \Theta_{\text{SHE}}(\alpha_R^0) = 0 \). This can be achieved, e.g., by some additional layer contributing to interlayer symmetry breaking (not shown in the sketch for simplicity). In the writing area (green), \( \alpha_R \) is altered by an external gate voltage \( V_G \) leading to a tunable \( \Theta_{\text{SHE}} \). In this area, the two lanes are not separated anymore. Therefore, incoming SKs can be transferred between the two lanes. After leaving the writing area, the initial configuration is restored and SKs move along their lanes.](image)
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