A Study of the Charged Scalar in the Zee Model

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Abstract

An extension of the Zee model involving a light right handed neutrino, $\nu_R$ is considered. We update constraints on couplings between the bilepton scalar, the active neutrinos, $\nu_R$ and the charged leptons. We find that the most stringent constraint currently comes from measurements limiting the width of the decay $\mu \rightarrow e \gamma$. These are used to predict the upper bound on violation of lepton universality in leptonic $W$ boson decays and rare $Z$ decays, such as $Z \rightarrow e\mu$.

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Evidence is now mounting that points towards neutrinos having small but finite masses. The neutrino mixing explanations which have been invoked to explain the deficit of solar neutrinos, the atmospheric neutrino anomaly and the LSND results, require neutrino mass squared differences in the range, $\delta m^2$ of $10^{-6}$ to a few eV$^2$. The exact values depend on the details of the models used to analyze the data. The standard explanations for each phenomenon involve mixing between two flavors of neutrino. For example the atmospheric neutrino anomaly is considered to be large angle mixing between the muon neutrino and something else, which is not the electron neutrino. These three standard interpretations are not easily reconciled without four neutrino states. In an alternative, three neutrino mixing, it is difficult to accommodate the zenith angle dependence of the SuperKamiokande atmospheric results.

It is generally accepted that neutrinos with masses in the above mentioned range necessitate the extension of the Standard model (SM) of electroweak interactions. Early simple models involving neutrino mass and/or a fourth neutrino state were constructed in [3] and [4]. The simplest way to generate neutrino mass is to add at least one SM singlet right-handed fermion, denoted by $\nu_R$, to the matter content. Indeed Grand unified models such as SO(10) naturally contain such a neutral fermion. In such a model, masses for the ordinary neutrinos can be elegantly explained by the seesaw mechanism. For masses as small as those predicted by solar, atmospheric and LSND neutrino mixing, the SM singlet, $\nu_R$ is required to have a mass in the range of $10^{10} - 10^{12}$ GeV. The seesaw mechanism does not easily accommodate a light fourth neutrino. Therefore, in accepting such a scheme one also implicitly accepts the view that one or more of the explanations of the evidence for neutrino mixing is misleading. Furthermore, there are several uncertainties which arise when utilizing the seesaw mechanism to produce light neutrinos. The means of obtaining this high energy SM singlet is highly model dependent. The energy scale of the heavy $\nu_R$
is arbitrary and reproducing any of the mass and mixing schemes usually requires model embellishments.

In view of these uncertainties it is important to investigate scenarios or models which relate the small neutrino mass with new physics at the weak scale or just above, i.e. \( < O(\text{TeV}) \). This alternative has the phenomenological advantage that such models may be easily constrained or perhaps tested. The simplest scenario was constructed by Zee many years ago. In the original formulation only the ordinary left-handed neutrinos are employed and their masses are generated by radiative corrections. Since the masses produced in this way are naturally small, it is unnecessary to invoke a large mass scale. A light \( \nu_R \) can also be incorporated as an extension to the model. However, this is done at the price of giving up the predictability of the neutrino masses and mixings. To date the bulk of the literature on the Zee model is devoted to the study of the neutrino mass matrix and neutrino mixings. The charged scalar meson, \( S^- \), which we shall refer to as the bilepton scalar is the crucial agent that carries the lepton flavor violation (LFV) necessary for neutrino mass generation and has been relegated to a secondary role in all studies thus far.

In this paper we study the Zee model as the simplest model of lepton number violation at the weak scale. In view of all of the data on neutrino oscillations, we have augmented it with a \( \nu_R \). We focus on the physics involved with virtual effects of the charged scalar. Since no charged scalar has been found up to LEPII energies, we assume its mass to be greater than 100 GeV. Due to the simplicity of this model, the number of free parameters is relatively few. Furthermore, these can be tightly constrained by current precision measurements. We begin by updating all the constraints stemming from muon decays and tau decays. We also discuss the impact of the next generation of Michel parameters measurements on the model. We find that the strongest constraint currently comes from \( \mu \to e\gamma \) decay. We then present new results for leptonic universality tests involving
physical $W$ boson decays. This is particularly important in view of the large sample of $W$ bosons which will be obtainable from the LHC and the next linear collider (NLC). We also calculate corrections to the left-handed charged lepton and $Z$ boson couplings as well as the right-handed charged lepton and $Z$ couplings. As we shall see below these two types of corrections take very different forms. We also give the predicted widths of the rare $Z$ decays such as $Z \to e\mu$. As far as we can determine these results have not been presented before.

Without further ado the following Lagrangian is added to the SM:

$$\mathcal{L}_S = \left[ f_{12}(\nu^c e_L - \nu^c e_L) + f_{13}(\nu^c \tau_L - \nu^c e_L) + f_{23}(\nu^c \mu_L - \nu^c \mu_L) \right] + \left[ g_1 \nu^c \nu^c R + g_2 \nu^c \mu^c R + g_3 \nu^c \tau^c R \right] S^+ + h.c. , \tag{1}$$

where $S^+$ is the Zee scalar boson with hypercharge $Y=2$. The $f_{ij}$ and $g_i$ are Yukawa couplings and make up six free coupling parameters of the model. If all the g’s are set to zero one recovers the original Zee model. The SU(2)×U(1) charges and the lepton number $L$ of the leptons and $S^-$ are presented in Table I. It is seen that the Lagrangian conserves total lepton number; however, individual e, mu or tau number is violated.

We have not included the scalar potential for the $S^-$ and its interaction with the SM Higgs doublet as they are not needed here. It suffices to note that the Zee boson can develop a mass either by spontaneous symmetry breaking via coupling to the SM Higgs doublet and/or explicitly through a bare mass term. In either case the physical mass is another free parameter which we denote by $M_S$. The above Lagrangian has been used to study neutrino oscillations in [7, 9]. (It was found that the $\nu^c$ significantly changes the phenomenology of neutrino oscillations from that of the three light neutrinos case and a fit to all data can be achieved). It will be seen below that since we have assumed a light mass for this particle, it can impact precision electroweak measurements in unusual ways, due to its chirality. Significant bounds on the interaction strength of the $\nu^c$ can already be obtained with presently available data.
Since LEP II has set $M_S$ to be higher than the W mass at low energies we can integrate out the S boson and perform a Fierz transformation to obtain the following effective four-fermion Lagrangian in terms of weak eigenstates:

$$ - L_{eff}^S = \frac{1}{2M_S^2} \left\{ f_{12} | f_{12}^* |^2 \bar{e} L e L \bar{\nu} \nu e L \gamma^\rho \nu e L \gamma^\rho \mu L + f_{12}^* f_{13}^* \bar{e} L e L \bar{\nu} \nu e L \gamma^\rho \mu L + f_{12} f_{23}^* \bar{e} L e L \bar{\nu} \nu e L \gamma^\rho \mu L \right\} \right. \\
- \left[ f_{12}^* f_{23} \bar{e} L e L \bar{\nu} \nu e L \gamma^\rho \mu L + f_{13}^* f_{23} \bar{e} L e L \bar{\nu} \nu e L \gamma^\rho \mu L \right] \\
- \left[ g_{V}^* g_3 \bar{\nu} \nu e L \gamma^\rho \mu R \gamma^\rho \mu R - g_{L}^* f_{23} \bar{e} L e L \bar{\nu} \nu e L \gamma^\rho \mu R \right] \\
+ \left[ g_{L}^* f_{12} \bar{e} L e L \bar{\nu} \nu e L \gamma^\rho \mu R - g_{L} f_{12}^* f_{23} \bar{e} L e L \bar{\nu} \nu e L \gamma^\rho \mu R \right] + h.c. . 
$$

We have shown only the terms relevant for $\mu$ decay. The dominant term for the muon lifetime is then given by

$$ - L_{eff} = \frac{4G_F^{SM}}{\sqrt{2}} \left\{ g_{LL}^V \bar{e} L e L \bar{\nu} \nu e L \gamma^\rho \mu L + \ldots \right\} , 
$$

where

$$ g_{LL}^V = 1 + \Delta g_{LL} \\
= 1 + \frac{2 | f_{12} |^2 M_W^2}{g^2 M_S^2} , 
$$

and $g$ is the SU(2) gauge coupling.

We have adhered to the notations of the Particle Data Group [11]. For the purpose of this paper it is convenient to work in the weak eigenbasis. One can easily see from Eq(2) that without $\nu_R$ the Zee model will give rise the same chiral structure as the SM [ see Eq.(3)]. The first term will interfere coherently with the SM amplitude and is the most important contribution to the muon lifetime, $\tau_\mu$. The terms involving $\nu_R$ add incoherently can be neglected in the muon lifetime $\tau_\mu$. Explicitly, [12][13]

$$ \frac{1}{\tau_\mu} = \frac{G_F^{SM} m_\mu^5}{192 \pi^3} \left( 1 + \Delta g_{LL} \right) \left[ 1 + \frac{\alpha}{2 \pi} \left( \frac{25}{4} - \pi^2 \right) \right] , 
$$
where we have used the radiatively corrected expression and have neglected terms involving $m_e^2/M_W^2$ and $m_\mu^2/M_W^2$. The SM fermi constant is given by

$$G_F^{SM} = \frac{\pi \alpha}{\sqrt{2} M_W^2 (1 - M_W^2/M_Z^2) (1 - \Delta r)}.$$  \hspace{1cm} (6)$$

Since $\tau_\mu$ is one of the most accurately determined quantity in particle physics a careful analysis is needed in order to obtain a bound on the coupling $f_{ij}$ versus $M_S$. In Eq. (6) the fine structure constant $\alpha(0)$ is very accurately known to be $1/137.03598959$ and $\Delta r$ is the SM correction and is found to be $\Delta r = 0.0349 \pm 0.0019 \pm 0.0007$. However, in computing $G_f^{SM}$ the largest error comes from $M_W$ measurement with current value given as $M_W = 80.39 \pm 0.06$ GeV and a much smaller error comes from $M_Z = 91.1867 \pm 0.0020$ GeV. Although the error in W mass measurement is only 0.075% it gets amplified in Eq.(6) to 0.36% and thus constitutes the biggest uncertainty. In contrast, the error in $M_Z$ measurement gives an error of 0.015% in $G_f^{SW}$. From the above we can obtain the error in the determination of $G_F^{SM}$ to be

$$G_F^{SM} = (1.166 \pm 0.005) \times 10^{-5} \text{GeV}^{-2}.$$  \hspace{1cm} (7)$$

In the above equations the LFV physics and the SM are assumed to be factorizable. This is a good approximation since the $S^-$ scalar does not couple to the W, because it is an SU(2) singlet, and hence does not affect the correction to the W propagator. We note that it does alter the running of $\alpha$, but the effect is much smaller than the uncertainty in $M_W$ and can be neglected. To get a bound on the couplings $f_{12}$ we demand that the corrections be no bigger the SM error given above. We use Eq. (6) to obtain a bound on $|f_{12}|^2$ as a function of $M_S^2$ which is displayed in Fig.(1).

Besides $\tau_\mu$, the electron spectrum from muon decay also gives information on possible new physics. This is usually quoted in terms of the Michel parameter measurements. It is
interesting to note that only the following three Michel parameters get a correction from the Zee model

\[
1 - \frac{\xi \delta}{\rho} \approx 2 \left[ |g_{RR}^V|^2 + |g_{LR}^S|^2 \right] \geq 0 ,
\]

(8)

\[
1 - \xi' \approx 2 \left[ |g_{RL}^S|^2 + |g_{RR}^V|^2 \right] \geq 0 ,
\]

(9)

and

\[
1 - \xi'' \approx 2 \left[ |g_{RL}^S|^2 + |g_{LR}^S|^2 \right] \geq 0 ,
\]

(10)

here

\[
|g_{RL}^S|^2 = \frac{|g_1 M_W^2|}{g^2 M_S^2} \left( |f_{12}|^2 + |f_{23}|^2 \right) \leq 0.180 ,
\]

(11)

\[
|g_{LR}^S|^2 = \frac{|g_2 M_W^2|}{g^2 M_S^2} \left( |f_{12}|^2 + |f_{13}|^2 \right) \leq 0.0156 ,
\]

(12)

\[
|g_{RR}^V| = \frac{M_W^2 |g_1 g_2|}{g^2 M_S^2} \leq 0.033 .
\]

(13)

All the others take their canonical SM values. The left hand sides of Eq.(11 - 13) are experimental bounds taken from [11]. One other noteworthy point is that at the four-fermi level a right-handed current coupling is induced because of the $S$ and $\nu_R$ interactions in lepton sector. However, this model does not have a corresponding coupling in the quark sector. This leads to a possible interesting scenario in which an apparent contradiction can occur in the positive outcome of right-handed current searches in the purely lepton sector such as in $\mu$ and $\tau$ decays and a negative outcome in similar searches using hadrons such as in $\beta$ decays of nuclei or pion decays. In Fig. 1 we also display the bounds on the couplings of the $S$ boson from Michel parameters. These bounds are complementary to the limit one obtains from the lifetime study.
Another tree level process that probes lepton number violation is $\tau$ leptonic decay. The relevant effective four-fermi Lagrangian is obtained from Eq.(2) by substituting the subscripts 2 to 3 and 1 to 3 appropriately. Here we use the branching ratio $R_{e\mu}$ defined by $\tau \rightarrow e\nu\nu/\tau \rightarrow \mu\nu\nu$ and obtain [compare Eq.(5)]

$$R_{e\mu} = 1 + 2 \frac{M_W^2 (|f_{13}|^2 - |f_{23}|^2)}{g^2 M_S^2}.$$  (14)

Since experimentally $R_{e\mu}$ is close to unity the error $\pm 0.006$ only gives the absolute value of the difference between $|f_{13}|^2$ and $|f_{23}|^2$. Notice that there is no dependence on the strength of the couplings involving $\nu_R$. Thus, $\tau$ leptonic decays can be used to probe a different region of parameter space from $\mu$ decays. Moreover, Eq.(14) shows that the $f_{13}$ and $f_{23}$ are of the same order of magnitude. We also find the Michel parameter measurements here place less stringent bounds than from $\mu$ decays.

Next we discuss the one loop effect involving the S-scalar. The most accurately measured LFV neutral current process is the decay $\mu \rightarrow e\gamma$. The branching ratio is calculated to be

$$Br(\mu \rightarrow e\gamma) = \frac{\alpha (|g_1 g_2|^2 + |f_{13} f_{23}|^2)}{24 \pi g^4} \left( \frac{M_W}{M_S} \right)^4,$$  (15)

This is sensitive to $|g_1 g_2|^2 + |f_{23} f_{13}|^2$ and provides the most stringent limit [17] on these parameters; see Fig. 2. Since this is a sum of two positive terms and implies that $|g_1 g_2|$ and $|f_{13} f_{23}|$ must be of order 0.01 for $M_S = 800$ GeV. We note in passing that similar $\tau$ decays will test different combinations of coupling constants.

A similar calculation gives the correction to the anomalous magnetic moment of the charged leptons, $a_\ell$ where $\ell = e, \mu, \tau$. Then

$$a_\ell = \frac{\sum_{J\neq \ell} |f_J|^2 + |g_\ell|^2}{96 \pi^2} \left( \frac{m_\ell}{M_S} \right)^2.$$  (16)

Both $a_e$ and $a_\mu$ are very well measured quantities. However, since scalar interactions flip chirality and because of dimensional consideration we obtain two powers of lepton
mass in Eq. (16). These powers of lepton mass, when combined with the loop suppression factors make these limits less constraining than some of the other processes considered here. Some of the other processes considered here, such as the $\mu$ decay parameters and the right and left handed lepton coupling to the $Z$ discussed below, set tight limits on $|f_{12}|^2$, $|f_{13}|^2$, and $|f_{23}|^2$. Therefore, we can use Eq (16) to put limits on $g_1$ and $g_2$. The limits from measurements of the electron and muon are shown in Fig. 3. If the error can be reduced by a factor of 20 on $a_\mu$ the limit obtained is the lower curve in Fig. 3. This is the target precision of the Brookhaven experiment E821 [18].

We proceed to discuss the effect of the Zee model on precision measurement at the $Z$-pole. The main effect is on the left-right asymmetry of the charged leptons and lepton flavor changing neutral current decays. To one loop order the correction to the $Z\ell\ell$ vertex, $\delta \Gamma_\mu$ is given by

$$\delta \Gamma_\mu = \delta g_L \gamma_{\mu L} + \delta g_R \gamma_{\mu R}, \quad (17)$$

where

$$\delta g_L = \frac{b}{96\pi^2} \left( \sum_{j \neq \ell} |f_{\ell j}|^2 \right) \left( \frac{1}{3} s_W^2 - \frac{1}{3} - \ln b + i\pi \right), \quad (18)$$

and

$$\delta g_R = -\frac{g_2^2 b s_W^2}{288\pi^2}. \quad (19)$$

In the above we have used $b = M_Z^2/M_S^2$, $s_W = \sin \theta_W$ and $c_W = \cos \theta_W$. As expected the left-handed coupling is corrected in the Zee model without $\nu_R$. However, if $\nu_R$ exists it will also modify the right-handed electron coupling to the $Z$ in spite of the fact that, although $\nu_R$ is light, there is no tree level $Z\nu_R^c \nu_R$ coupling. Using Eqs. (18) and (19) one can calculate the left-right asymmetry, $A_{\text{LR}}$. It is given by

$$A_{\text{LR}} = A_{\text{LR}}^0 \left\{ 1 + \frac{32 s_W^2(1-2s_W^2)[2s_W^2\delta g_L - (1-2s_W^2)\delta g_R]}{(1-4s_W^2)[1+(1-4s_W^2)^2]} \right\}, \quad (20)$$
where $A_{LR}^0$ is the SM left-right asymmetry. The most stringent bounds are obtained by combined LEP [16] and SLC measurements [19] given in terms of the axial and vector couplings. This is shown in Fig. 4.

With the above limits on the various couplings we can also predict the upper limits for the rare decays $Z \rightarrow l_il_j$ where $l_i \neq l_j$ and $l_i = e, \mu, \tau$. The width is calculated to be

$$\Gamma(Z \rightarrow l_il_j) = \frac{g^2 M_Z b^2}{24\pi^5 (288c_W)^2} \left\{ |f_{ik}f_{jk}|^2 \left[ (3 \ln b - c_W^2) + \pi^2 \right] + |g_i g_j|^2 s_W^4 \right\}, \quad (21)$$

where $i \neq j \neq k$. With the limits given by the above considerations the upper limits for the $e\mu$, $\mu\tau$, and $e\tau$ are all unmeasurably small in the foreseeable future. For example the upper limit for the $Z \rightarrow e\mu$ branching ratio is of order $10^{-10}$ for a 800 GeV bilepton scalar.

Having examined the constraints on the bilepton scalar we proceed to look into future tests. Short of discovering the $S$ scalar itself, which is best done in a linear $e^+e^-$ collider, we propose that $e, \mu, \tau$ universality for the W boson will be a good place to look this kind of LFV physics. Explicitly, the branching ratios $W \rightarrow e\nu/W \rightarrow \mu\nu$ and $W \rightarrow e\nu/W \rightarrow \tau\nu$ will be very valuable. In the SM these branching ratios are unity at the tree level but at the level of first order radiative correction this universality is broken. However, this breaking is suppressed by the factor of $\alpha(m_\mu^2/M_W^2)$ or $\alpha(m_\tau^2/M_W^2)$ and hence very small. The important point here is that the radiative corrections are accurately predicted in the SM, and the measurements of these branching ratios are very clean theoretical probes of LFV physics. We illustrate this in the Zee model by calculating the correction to the W leptonic decay widths. We find

$$\Gamma(W \rightarrow l_i \nu_i) = \frac{g^2 M_W}{48\pi} \left\{ \frac{\alpha}{2\pi} \left( \frac{2\pi^2}{3} - \frac{77}{12} \right) \right\}$$

$$\left\{ 1 + \frac{M_W^2}{72\pi^2 M_S^2} \left( 1 - 3 \ln \frac{M_W^2}{M_S^2} \right) \left[ \sum_{j \neq i} |f_{ij}|^2 \right] \right\}, \quad (22)$$

where $i = e, \mu, or \tau$. The first line is the radiatively corrected W boson leptonic width [21] and the second term in the curly bracket is the Zee model correction. As expected
the left-handed nature of the coupling is preserved and $\nu_R$ does not play a role. We have also kept only the leading term in $M_W^2/M_S^2$ in the calculation and neglected the lepton masses. Eq. (22) immediately gives rise to the following results for the leptonic branching ratios:

$$\text{Br} \left( \frac{W \to \mu \nu}{W \to e \nu} \right) = 1 + k(|f_{23}|^2 - |f_{13}|^2),$$

$$\text{Br} \left( \frac{W \to \tau \nu}{W \to e \nu} \right) = 1 + k(|f_{23}|^2 - |f_{12}|^2),$$

$$\text{Br} \left( \frac{W \to \tau \nu}{W \to \mu \nu} \right) = 1 + k(|f_{13}|^2 - |f_{12}|^2),$$

where $k = \frac{M_W^2}{72\pi^2 M_S^2} \left( 1 + 3 \ln \frac{M_W^2}{M_S^2} \right)$. As seen from $\tau$ universality the combination $|f_{23}|^2 - |f_{12}|^2$ is bounded to be small and so we determine that the first branching ratio of Eq. (23) can only accommodate a correction of less than $2 \times 10^{-5}$ for $M_S = 800$ GeV. Furthermore the constraints we obtained on $|f_{13}|^2$ and $|f_{23}|^2$ from the Z pole allow the universality violation in the last two branching ratios of Eq. (23) to be as large as $1 \times 10^{-3}$. At the LHC and the NLC where large samples of W decays are expected, these decays will be the most important and cleanest tool for probing LFV in the charged current sector. We note that the uncertainty due to the t-quark that plagues $\Delta r$ in the interpretation of the muon lifetime measurement does not enter here at the one loop level. Once sufficient statistics are obtained these measurements will supersede many low energy tests of LFV.

The last stop in our discussion of futuristic experiments is the production of the $S$-scalar. In particular we focus on pair production in $e^+e^-$ colliders. This is very similar to charged Higgs production in the two doublets model [22]. Since the Yukawa couplings $f_{12}$ and $f_{13}$ are seen to be small the t-channel process involving neutrino exchanges can be neglected and one needs only to consider the s-channel virtual photon and Z exchange graphs. The coupling are all determined by the SM charges [see Table I] the production cross section can be calculated with $M_S$ as the only free parameter. Explicitly, the cross
section for $e^+e^- \rightarrow S^+S^-$ is

$$
\sigma = \frac{\pi \alpha^2 \beta^3}{3s} \left[ 1 + \frac{s(-1 + 4s^2_{W^*})}{2c_W^2(s - M_Z^2)} + \frac{s^2(-1 + 4s^2_{W^*} + 8s^4_{W^*})}{4c_W^4(s - M_Z^2)^2} \right],
$$

(26)

where $s$ is the cm energy squared and $\beta = \sqrt{1 - 4M_S^2/s}$. If the mixing of the bilepton scalar with charged Higgs bosons can be neglected its dominant decay mode would be into $l_i\nu_j$. The signal will be $\tau$ and $\mu$ plus missing energy and unmistakable.

As seen from the results we have presented, a bilepton scalar induces many interesting and testable effects. We have employed the Zee model augmented with a light $\nu_R$ for quantitative studies since it is relatively simple and the number of free parameters are relatively few. We find that the strongest bound on the Yukawa couplings in the model comes from $\mu \rightarrow e\gamma$ decay, lepton universality in $\tau$ decays and the lifetime of the muon. In particular $f_{12}$ and $f_{13}$ are both constrained to be less than of order 0.1 and 1 respectively for $M_S = 800$ GeV. However, the couplings $g_i$ have much looser bounds. Since Yukawa couplings are in general not universal we expect leptonic universality to be violated in W decays (see Eq. (22)). This illustrates the importance of W decays in probing LFV physics. Another effect which will be harder to discern is the modification of the U(1) coupling constant running. The $S$-scalar contributes an amount of $\frac{g'^4}{16\pi^2}$ to the U(1) $\beta$ function where $g'$ is the U(1) gauge coupling. This will upset the unification of the SM gauge couplings at very high energies which is usually taken to be a hint for supersymmetry. We do not consider this to be a serious impediment for the following reason. Although the Zee model is interesting in its own right, it carries the same arbitrariness as the Higgs sector of the SM. Hence, we expect it be part of a larger structure that entails supersymmetry. Promoting the bilepton scalar to a superfield necessitates the introduction of a second scalar in order to cancel the anomaly caused by the fermionic partner of the $S$. In short one would have to enlarge the minimal supersymmetric standard model by two bilepton superfields and also one singlet neutrino superfield. The details of how this can be achieved
and the ensuing intricate phenomenology is beyond the scope of the present paper and we shall defer the study of this issue to a later work.

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Table 1: Lepton number, SU(2), and U(1) charges of the left-handed lepton doublet, the $\nu_R$, the charged lepton conjugate, and the S-scalar

|     | $\ell$ | $\nu_R$ | $\ell^c$ | $S^-$ |
|-----|--------|---------|----------|-------|
| SU(2) | 1/2    | 0       | 0        | 0     |
| Y    | -1     | 0       | -2       | -2    |
| L    | 1      | 1       | 2        | 2     |
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14
Figure 1: Bounds on coupling constants derived from measurements of Michel parameters for muon decay are plotted as a function of the mass of the scalar, $M_S$. The constraint on $|f_{12}|^2$ comes from the measurement of the muon lifetime.

Figure 2: As in Figure 1, for the constants $|f_{23}f_{13}|^2 + |g_{1}g_{2}|^2$. This constraint comes from the experimental limit on the width of the decay process $\mu \rightarrow e\gamma$.

Figure 3: Coupling constants plotted as in Figure 1. These constraints come from precision measurements of the anomalous magnetic moment of the electron (top line), and of the muon (middle line). The lower line shows the bound which could be obtained if the muon measurement were twenty times more precise.

Figure 4: Coupling constants plotted as in Figure 1. This plot represents three different constraints as determined by the choice of leptons, $e, \mu, \tau$ for the parameters, $l, j, k$ (with $l \neq j \neq k$). These constraints are derived from the measurements of leptonic axial and vector couplings.
$\log_{10} \left( |f_{23}|^2 + |g_{12}|^2 \right)$
