Critical temperature of a ferromagnet/superconductor structures in a parallel magnetic field

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Abstract. Two- and three-layered structures consisting of ferromagnetic metal (F) film and superconductor (S) film are considered in the magnetic field \( H \) applied parallel to the plane of the F/S interface. Magnetic-penetration depth is presumed much greater than thicknesses of layers. Assuming that both S and F layers are dirty the boundary problem for the Usadel function is solved taking account of so called ”Umklapp processes” for momenta of superconducting pairs on the F/S interface. It is shown that the field \( H \) can significantly alter a functional form of the dependence of critical temperature \( T_c \) versus thickness of the F layer \( d_f \). Keeping in mind the possible applications like the superconducting spin-valve the behavior of \( T_c(d_f) \) for the F/S/F trilayer in the magnetic field is investigated.

1. Introduction

The thin-film heterostructures consisting of alternating ferromagnetic metal (F) and superconducting (S) layers are well explored systems (see the recent reviews [1, 2] and references therein). The competition of superconducting and magnetic states in the layered F/S structures leads to the pronounced non-monotonous dependence of the critical temperature \( T_c \) on a thickness of the F layers \( d_f \). The proximity effect of a single S film embedded in F metal in a magnetic field, both parallel and perpendicular to the layers, have been first studied in works [3, 4] (the modern references may be found in [1, 2]). In paper [5] (see also [2, 6]) authors have derived common boundary-value problem for the F/S nanostructures, in which a superconductivity in whole sample is a superposition of usual superconductivity with BCS type of pairing, when pairs in the S layers have zero total momentum, and unconventional one, when the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) pairs arise in the F layers with nonzero 3D coherent momentum. The processes of mutual transformation between BCS and FFLO pairs at the F/S boundary were shown to be the Umklapp processes during which coherent pair momentum becomes confined in the F layer [5, 6].

It is well known that the proximity effect of the thin-film F/S heterostructures depends heavily on some parameters of interface (the transparency \( \sigma \)) and the both metals (coherence lengths, electron zone structures, free length paths, details of pairing mechanisms in the S and F (BCS or FFLO, correspondingly), etc.) [1, 2]. In this issue we solve boundary problem for the Usadel function in the presence of the external magnetic field \( H \) parallel to the F/S interface for the F/S/F trilayer structures. Contrary previous investigations [1, 2] we investigate the behavior
of $T_c(d_f)$ in the magnetic field taking into account the Umklapp processes for the momentum of superconducting pairs on the F/S interface.

2. Boundary value problem for the Usadel’s equations.

We consider the F$_1$/S/F$_2$ trilayer consisting from layers with thicknesses $d_{f1}$, $d_s$ and $d_{f2}$, respectively. The axis $x$ is chosen perpendicular to the plane of interfaces, so that the external magnetic field $H(0, -H, 0)$ is parallel to this plane. We suppose that S layer is thin enough that no vortices appear [3, 4]. Near the second-order phase transition the magnetic field is uniform in the sample, so that we may choose the gauge $A = (0, 0, H(x - x_c))$ with a some gauge constant $x_c$. Assuming, that metals are dirty, we use Usadel version of the proximity effect to find critical temperature $T_c$ [1, 2]. The equations for Usadel function in the F and S areas and the equation on the superconducting order parameter $\Delta_s$ have the form (here and below $\hbar = k_B = \mu_B = 1$) 

$$\left[ \omega - \frac{D_s}{2} \left( \nabla + \frac{2\pi i A}{\Phi_0} \right)^2 \right] F_s(r, \omega) = \Delta_s(r), \quad \left[ \omega + iI - \frac{D_f(I)}{2} \left( \nabla + \frac{2\pi i A}{\Phi_0} \right)^2 \right] F_{1,2}(r, \omega) = 0, \quad \Delta_s(r) = 2\pi \kappa T_c \text{Re} \sum_{\omega > 0} F_s(r, \omega). \quad (1)$$

Here $I$ is exchange field; $\omega$ is the Matsubara frequency; $\kappa$ is a dimensionless constant of interparticle interaction; the diffusion coefficients $D_s$ and $D_f(I)$ are determined in [2]; $\Phi_0$ is the magnetic flux quantum. For simplicity we assume $\Delta_f = 0$. We seek the solution of the equations (1), (2) as $F_s(r, \omega) = F_s(x)/(\omega + k^2_s D_s/2)$, $F_{1,2}(r, \omega) = F_{1,2}(x) \exp(iq_1,2 s)/\omega$, where $s = s(y, z)$, and the wave vectors $q_1$ and $q_2$ describe the transverse component of wave vector of FFLO pairs and connect with Umklapp processes on the F/S interface. Their magnitudes and orientation are determined by optimization, i.e., from the condition of the maximum of $T_c$ (the minimum of free energy). After this substitution the equations (1) reduce to

$$\frac{\partial^2}{\partial x^2} F_s(x) = -k^2_s F_s(x), \quad \frac{\partial^2}{\partial x^2} \left( \frac{2\pi H}{\Phi_0} \right)^2 (x - x_c)^2 F_s(x) = -k^2_s F_s(x), \quad (I \gg T_c). \quad (3)$$

The temperature $T_c$ is defined from the equation

$$\ln t = \Psi \left( \frac{1}{2} \right) - \text{Re} \Psi \left( \frac{1}{2} + \frac{D_s k^2_s}{4\pi T_c t} \right), \quad (5)$$

where $\Psi(x)$ is the digamma function, $t = T_c/T_{cs}$, $T_{cs}$ is the temperature of the superconducting transition for isolated S layer.

The equations (3) should be solved with following boundary conditions [2]

$$\left. \frac{4D_s \sigma_s \partial F_s(x)}{\partial x} \right|_{x=0} = \left. \frac{4D_f(I) \sigma_f \partial F_1(x)}{\partial x} \right|_{x=0} = F_s(0) - F_1(0), \quad (6)$$

$$\left. \frac{4D_s \sigma_s \partial F_s(x)}{\partial x} \right|_{x=d_s} = \left. \frac{4D_f(I) \sigma_f \partial F_3(x)}{\partial x} \right|_{x=d_s} = F_2(d_s) - F_3(d_s),$$
variants of the T superconductivity of the layered system. Changing the magnitude of \( q \) which allow for conditions at outer boundaries (7). Constants obtained.

In the single-mode approximation \[2\] the solutions take the form

\[
(x - x_c)^2_{s,f} \simeq \langle (x - x_c)^2 \rangle_{s,f} = \frac{1}{d_{s,f}} \int (x - x_c)^2 dx,
\]

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(x - x_c)^2_{s,f} \simeq \langle (x - x_c)^2 \rangle_{s,f} = \frac{1}{d_{s,f}} \int (x - x_c) dx.
\]

In the single-mode approximation \[2\] the solutions take the form

\[
F_1 = C_1 \cos(Q_1(x + d_1)); \quad F_s = A \cos Q_s x + B \sin Q_s x; \quad F_2 = C_2 \cos(Q_2(x - d_s - d_2)),
\]

which allow for conditions at outer boundaries (7). Constants \( C_{1,2}, A, B \) are determined from the boundary conditions (6). Corresponding wave vectors \( Q_s \) and \( Q_{1,2} \) are determined by

\[
Q_{1,2}^2 = k_s^2 - q_{1,2}^2 - \left( \frac{2\pi H}{\Phi_0} \right)^2 \langle (x - x_c)^2 \rangle - \left( \frac{2\pi H}{\Phi_0} \right) \langle x - x_c \rangle |q_{1,2}| \cos \varphi_{1,2},
\]

\[
Q_s^2 = k_s^2 - \left( \frac{2\pi H}{\Phi_0} \right)^2 \langle (x - x_c)^2 \rangle_s.
\]

Here \( \varphi_{1,2} \) is an angle between the wave vector \( q_{1,2} \) and the z-axis. It is obvious, that value of angles \( \varphi_1 = \pi \) and \( \varphi_2 = 0 \) lead to the maximum \( T_c \). Actually at such orientation of vectors \( q_1 \) and \( q_2 \) the Umklapp processes lower as far as possible the density of supercurrent and, hence, decrease the free energy of system.

3. Results and discussion

For numerical optimization we use usual values of parameters \[2\], i.e. for the S layer (\( \xi_s, \xi_0 \) is the BCS coherence length and \( l_s \) is the mean free path), for the F layer (the parameter \( 2l_f \tau_f \), where \( \tau_f \) is the free-path time; \( l_f = v_f \tau_f \)), the S/F interface transparency \( \sigma_s \), and the parameter \( n_{sf} = N_{sf} / N_{f} v_f \).

In Figure 1 the \( T_c(d_1) \) dependencies are shown for the S/F bilayer (\( d_2 = 0 \)) for different values of the external field \( H \). These results are obtained by optimization with respect to the magnitude of \( q_1 \). Here \( h = H / H_c \) is reduced magnetic field value, \( H_c \) is the thermodynamic critical magnetic field of the S metal. We see that the external magnetic field \( H \) suppresses a superconductivity of the layered system. Changing \( h \) we can get several qualitatively different variants of the \( T_c(d_f) \) dependence. At \( h = 0 \) we have upper solid curve transiting onto a plateau through one local minimum-maximum. The \( T_c(d_f) \) has reentrant character at sufficiently large values of the magnetic field \( h \approx 0.5 \). At last at \( h \gtrsim 0.65 \) the monotonic fall-off (up to zero) is obtained.
Figure 1. {Color online} Reduced transition temperature $t = T_c/T_{cs}$ as a function of the reduced thickness of the F layer $d_1/\xi_{s0}$ for the F/S system in external magnetic field $h$.

![Reduced transition temperature](image)

Figure 2. {Color online} Phase diagram $T_c(d_f)$ for the symmetrical F/S/F trilayer ($d_{f1} = d_{f2} = d_f$) for parallel (dashed line) and antiparallel (solid line) exchange fields.

![Phase diagram](image)

In Figure 2 the $T_c(d_f)$ dependencies are shown for the F/S/F trilayer at the relatively weak exchange field at parallel and antiparallel orientations of magnetizations in the left and right $F_{1,2}$ layers. This case is interesting for the possible spin-valve application [1], in which the external magnetic field may control two possible states of the trilayer. Indeed we see in Figure 2 that critical temperature $T_c$ is higher at antiparallel orientation of magnetizations than in case of parallel orientation. Note that transition temperature $T_c$ of the F/S/F system has maximum at these parameters if $q_{1,2}$ are equal to zero, i.e. the 1D FFLO states are realized [2, 6].

In this issue we have investigated the influence of the magnetic field on orbital motion of the conduction electrons only. But in specific cases it should be taken into account the interaction of the magnetic field with spin degrees of freedom of conduction electrons. In that case transition temperature $T_c$ of the F/S/F system will depend on a magnetic field sign ($\pm H$) and it will allow to govern by a F/S/F system conduction over a wide limits.

References

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