ac Stark shift and multiphoton-like resonances in low-frequency driven optical lattices

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We suggest that Bose-Einstein condensates in optical lattices subjected to ac forcing with a smooth envelope may provide detailed experimental access to multiphoton-like transitions between ac-Stark-shifted Bloch bands. Such transitions correspond to resonances described theoretically by avoided quasienergy crossings. We show that the width of such anticrossings can be inferred from measurements involving asymmetric pulses. We also introduce a pulse tracking strategy for locating the particular driving amplitudes for which resonances occur. Our numerical calculations refer to a currently existing experimental set-up [Haller et al., PRL 104, 200403 (2010)].

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I. INTRODUCTION

The study of multiphoton excitation and ionization processes so far has concerned the interaction of matter with strong electromagnetic fields [1, 2]. Extending our previous works [3, 4], we demonstrate in the present proposal that ideas and concepts developed in this field of research can also be applied for understanding the response of ultracold macroscopic matter waves in optical lattices to ac forcing with a slowly varying envelope. Such systems offer additional handles of control which are not available in experiments with atoms or molecules exposed to pulses of laser radiation. Therefore, they can give novel insights into multiphoton dynamics in general, and in particular may also allow one to systematically investigate the effects of interparticle interactions on such transitions.

The experimental investigation of Bose-Einstein condensates (BECs) in time-periodically forced optical lattices has been pushed forward with remarkable vigor within the last years, addressing quite diverse topics such as parametric amplification of matter waves [7], dynamic localization [8, 9], photon-assisted tunneling [10], coherent control of the superfluid-to-Mott insulator transition [11], quantum ratchets [12], super Bloch oscillations [13], quantum simulation of frustrated magnetism [14], controlled correlated tunneling [15, 16], and even the realization of tunable gauge potentials [17].

Thus, BECs subjected to time-periodic forcing constitute an emerging branch of research [18]. Here we outline how systematic use of the forcing’s envelope will allow one to extract crucial information about the systems’ dynamics.

II. DETECTION OF MULTIPHOTON-LIKE RESONANCES

The present theoretical study of multiphoton-like condensate dynamics refers to conditions recently realized by Haller et al. [13]. These authors have loaded BECs of Cs atoms into a vertically oriented 1D optical lattice $V(x) = (V_0/2)\cos(2k_Lx)$. Here $k_L = 2\pi/\lambda L$, where $\lambda L = 1064.49$ nm is the wavelength of the lattice-generating laser light, so that the lattice period is $d = \pi/k_L$.

We choose a comparatively shallow lattice with depth $V_0 = 2.3E_r$, where $E_r = (\hbar k_L)^2/(2m)$ is the single-photon recoil energy [19], with $m$ denoting the mass of the atoms. The first excited band then is of the “above-barrier” type, so that atoms in this and all higher bands do not need to tunnel through the barriers when moving over the lattice. Using a magnetically induced Feshbach resonance, the s-wave scattering length of the Cs atoms is tuned to zero [20], so that one is dealing with a condensate of effectively noninteracting particles.

By means of a time-periodic modulation of the levitation gradient employed for compensating gravity, an external oscillating force is introduced which acts on the atoms with maximum amplitude $F_{\max}$ and frequency $\nu = \omega/(2\pi)$, such that their dynamics are governed by the Hamiltonian

$$H(t) = \frac{\hbar^2}{2m} + V(x) - s(t)F_{\max}x \cos(\omega t)$$

with a dimensionless shape function $s(t)$ determining the envelope of the pulses applied. All calculations reported here are performed for the relatively low frequency $\omega/(2\pi) = 300$ Hz, so that $\hbar \omega = 0.23E_r$.

As indicated in Fig. 1 this means that the gap $\Delta_1 = 1.14E_r$ between the lowest two Bloch bands $E_1(k)$ and $E_2(k)$ of the unperturbed optical lattice, which occurs at the Brillouin zone edge $k/k_L = \pm 1$, amounts to $5.05\hbar \omega$: Exciting particles from the initially occupied lowest band to the first excited one requires the absorption of more than five “photons”.

A first glimpse at the condensate dynamics under the action of a force $F(t)$ is provided by Bloch’s acceleration theorem: The expectation value $\langle \dot{k}(t) \rangle$ of an atomic wave packet in k-space evolves in time according to

$$\frac{d}{dt} \langle \dot{k}(t) \rangle = F(t)$$

provided interband transitions remain negligible [21]. Assuming a sinusoidal force $F(t) = F \cos(\omega t)\Theta(t)$ instantaneously switched on at time $t = 0$, this gives $\langle \dot{k}(t) \rangle/k_L = (\langle \dot{k}(0) \rangle/k_L + (K/\pi)\sin(\omega t))$, where $K = F d/(\hbar \omega)$ is a dimensionless measure of the driving amplitude. Thus,
when considering a wave packet initially at rest in the lowest band, so that $\langle k \rangle(0)/k_L = 0$, the packet’s center $\langle k \rangle(t)$ just reaches the Brillouin zone edge when $K = \pi$. Because the zone edge gives rise to Zener-type transitions [22], one then expects pronounced excitation of higher bands. Therefore, one has $K \approx \pi$ as a rough order-of-magnitude estimate of the amplitude required for inducing multiphoton-like transitions.

This expectation is confirmed by the numerical calculations summarized in Fig. [2]. Here we consider driving pulses $s(t)F_{\text{max}}\cos(\omega t)$ as already incorporated into the Hamiltonian (1), with a smooth envelope

$$s(t) = \sin^2(\pi t/T_p) \quad ; \quad 0 \leq t \leq T_p.$$  

The pulse length is fixed at 60 driving cycles, $T_p = 60 T$ with $T = 2\pi/\omega$. In order to model the dynamics of a noninteracting BEC responding to such pulses, we start with an initial state

$$\psi(x, 0) = \sqrt{\frac{d}{2\pi}} \int dk g_1(k, 0) \chi_{1,k}(x)$$

made up from Bloch waves $\chi_{1,k}(x)$ of the lowest band, employing a Gaussian momentum distribution

$$g_1(k, 0) = \left( \sqrt{\pi \Delta k} \right)^{-1/2} \exp\left( -\frac{|k - \langle k \rangle(0)|^2}{2(\Delta k)^2} \right)$$

centered around $\langle k \rangle(0)/k_L = 0$ with width $\Delta k/k_L = 0.1$, and then solve the single-particle Schrödinger equation by means of a Crank-Nicolson algorithm [23]. Varying the maximum scaled amplitude $K_{\text{max}} = F_{\text{max}}d/(\hbar \omega)$ from pulse to pulse, we plot the escape probability from the lattice at the end of each pulse, at $t = T_p$. Here we assume that only atoms which finally still populate the lowest band remain in the lattice, since atoms which have been excited to the higher “above-barrier” bands tend to escape from the shallow lattice quite fast. Evidently, interband transitions start to make themselves felt at $K_{\text{max}} \approx 2.5$, and reach substantial strength when $K_{\text{max}} \approx 3$, confirming the above rough estimate. Thus, proof-of-principle experiments performed along these lines should establish the feasibility of using BECs in driven optical lattices as novel probes for multiphoton-like transitions: Take driving pulses with smooth envelopes, and measure the interband transition probability at the end of each pulse. In a series of such measurements with a constant pulse shape one then should observe a pronounced onset of interband transitions when the maximum amplitude is successively increased.

### III. AVOIDED-QUASIENERGY-CROSSING SPECTROSCOPY WITH ASYMMETRIC PULSES

In a second step, this approach can be employed for getting more detailed insight into multiphoton dynamics. Namely, the single-band acceleration theorem [4] ignores an essential element: Not only does the wave packet’s center $\langle k \rangle(t)$ move within its band in response to the external forcing, but also the bands themselves are “dressed” by the drive, and therefore experience an ac Stark shift [24]. Hence, initially nonresonant bands may be shifted such that their separation in energy approaches an integer multiple of $\hbar \omega$ for certain Bloch wavenumbers $k$, possibly leading to strong resonant interband coupling, that is, to a multiphoton resonance. Theoretically, such resonances are found by computing

**FIG. 1:** Energy band structure of an unperturbed optical cosine lattice with depth $V_0 = 2.3 E_r$. Measured in terms of the “photon” energy $\hbar \omega = 0.23 E_r$, as corresponding to the driving frequency $\omega/(2\pi) = 300$ Hz, the gap between the lowest two bands is $\Delta_1 = 5.05 \hbar \omega$ at the Brillouin zone edge, and $\Delta_0 = 18.24 \hbar \omega$ at its center.

**FIG. 2:** Interband transition probabilities after pulses with the squared-sine envelope [3] and length $T_p = 60 \times 2\pi/\omega$, obtained from numerical solutions of the Schrödinger equation. All atoms which are excited to bands $n > 1$ after a pulse are assumed to escape from the lattice.
the quasienergy bands $\varepsilon_n(k)$ which emerge from the energy bands $E_n(k)$ in the presence of a drive with constant amplitude. These quasienergy bands reflect the ac-Stark-shifted energy bands, projected into an interval of width $\Delta \varepsilon = \hbar \omega$, so that a multiphoton resonance translates into an avoided quasienergy crossing \cite{4, 5}. For example, Fig. 3 shows the quasienergies $\varepsilon_n(0)$ with $n = 1, 2, 3$, which pertain to the pulses considered in Fig. 2 plotted versus the scaled amplitude $K$. The quasienergy originating from the ground-state energy $E_1(0)$ of the optical lattice undergoes two well-resolved avoided crossings when $K > 3$, signaling the presence of two individual multiphoton resonances. The observation that these “large” resonances begin to show up only for $K \approx \pi$ nicely relates the elaborate quasienergy approach to the previous elementary reasoning based on Eq. 2.

Inspecting that elementary reasoning once again, one expects multiphoton resonances to occur for smaller driving amplitudes when the initial packet is centered around a nonzero wavenumber, $\langle k \rangle(0)/k_L \neq 0$, since then smaller amplitudes are required for reaching the Brillouin zone edge. This expectation is confirmed by Fig. 4 which depicts quasienergies $\varepsilon_n(k)$ for $k/k_L = 0.8$, with the first resonance showing up already at $K \approx 0.9$. Experimentally, one can prepare an initial state with arbitrary $\langle k \rangle(0)$ by subjecting the condensate to a suitable “kick” \cite{23, 26}. Thus, one should also be able to detect the resonances predicted by Fig. 3. To this end, we propose a particular kind of “avoided-quasienergy-crossing spectroscopy” based on the use of asymmetric pulses $s(t)$. For illustration, we assume that the rising part of such pulses be given by the first half of the envelope \cite{3}, with fixed switch-on time $T_p^{(1)}/2 = 5T$, while their decreasing part is described by the second half of a squared-sine envelope, but with a different switch-off time $T_p^{(2)}/2$. Let the maximum scaled amplitude be $K_{\text{max}} = 1.2$. During the rising part of such a pulse, a wave packet initially centered around $\langle k \rangle(0)/k_L = 0.8$ then follows its quasienergy states adiabatically, until the instantaneous amplitude reaches the multiphoton resonance at $K \approx 0.9$ visible in Fig. 4. Then the packet undergoes a Landau-Zener transition to the anticrossing quasienergy state \cite{27}. Due to the rapid switch-on of the pulse, and to the narrow quasienergy separation $\delta \varepsilon$ at the avoided crossing, that transition is almost complete. Thereafter, the packet again adiabatically follows the pulse envelope, until the resonance is encountered a second time when the amplitude decreases. If then $T_p^{(2)} \gg T_p^{(1)}$, a major part of the wave function does not “jump over” the avoided crossing back to the initial state, but rather stays in the continuously connected quasienergy states. This implies that a major fraction of the condensate atoms is excited at the end of the pulse, escaping out of the lattice. When such an experiment is performed repeatedly with fixed rise time $T_p^{(1)}/2$ while varying the switch-off duration $T_p^{(2)}/2$, one should observe survival probabilities which drop exponentially with increasing $T_p^{(2)}$, allowing one to extract the quasienergy separation $\delta \varepsilon$ at the avoided crossing from the drop rate by means of the known Landau-Zener formula for quasienergy states \cite{27}.

The results of a series of solutions to the Schrödinger equation corresponding to this scenario are plotted in Fig. 5. From the slope of the numerical data we deduce a quasienergy gap $\delta \varepsilon/(\hbar \omega) = 0.0099$, in agreement with the value 0.01 read off from Fig. 4. These findings clearly underline that this quasienergy gap $\delta \varepsilon \ll \hbar \omega$ is the actually relevant energy scale for the multiphoton transitions under scrutiny here, not the band separation $\Delta_1 = 5.05 \hbar \omega$ indicated in Fig. 1.

There are further options offered by BECs in driven optical lattices which have no match in laser-based multiphoton studies. For example, one can switch off the driving force abruptly at any moment, and analyze the state of the wave packet at that particular instant.
with varying switch-off durations \( T \) are determined by the spectrum shown in Fig. 4; its quasienergy states adiabatically, not encountering the escape probabilities versus switch-off time for the previous calculations shown in Fig. 2, and plot \( \langle k \rangle(0)/k_L = 0.8 \), so that the dynamics are determined by the spectrum shown in Fig. 4 \( K_{\text{max}} = 1.2 \) for all pulses.

Specifically, we again utilize envelopes of the form \( \langle k \rangle (0)/k_L \approx 3 \) with \( T_p = 60T \) abruptly at \( t = T_{\text{out}} \), and recording the escape probability at this moment. The onset of interband transitions then reveals the presence of a multiphoton resonance at the instantaneous amplitude reached at \( t = T_{\text{out}} \).

To summarize, we have argued that BECs in shallow optical lattices exposed to ac forcing with frequencies of some 100 Hz and smooth envelopes can be employed for mimicking multiphoton processes. The enormous degree of controllability realizable with such set-ups enables one to obtain information not reachable with laser-irradiated crystalline solids \( \cite{3} \); in particular, we have suggested the use of asymmetric pulses for performing avoided-quasienery-crossing spectroscopy. Moreover, we have shown how pulse tracking by abruptly switching off the driving amplitude allows one to monitor the dynamics at each moment during an individual pulse, and thus to locate multiphoton resonances between ac-Stark-shifted Bloch bands. Obviously, our approach also lends itself to systematic explorations of the effects of pulse shaping. Even more interestingly, one can activate interparticle interactions by suitably tuning the s-wave scattering length of the Cs atoms \( \cite{20} \). Hence, the detailed experimental investigation of the influence of such interactions on multiphoton transitions has come into immediate reach.

**IV. SUMMARY AND OUTLOOK**

To summarize, we have argued that BECs in shallow optical lattices exposed to ac forcing with frequencies of some 100 Hz and smooth envelopes can be employed for mimicking multiphoton processes. The enormous degree of controllability realizable with such set-ups enables one to obtain information not reachable with laser-irradiated crystalline solids \( \cite{3} \); in particular, we have suggested the use of asymmetric pulses for performing avoided-quasienery-crossing spectroscopy. Moreover, we have shown how pulse tracking by abruptly switching off the driving amplitude allows one to monitor the dynamics at each moment during an individual pulse, and thus to locate multiphoton resonances between ac-Stark-shifted Bloch bands. Obviously, our approach also lends itself to systematic explorations of the effects of pulse shaping. Even more interestingly, one can activate interparticle interactions by suitably tuning the s-wave scattering length of the Cs atoms \( \cite{20} \). Hence, the detailed experimental investigation of the influence of such interactions on multiphoton transitions has come into immediate reach.

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