Fermionic DBI and Chaplygin gas unified models of dark energy and dark matter from f-essence

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Abstract. In the present work, we study the cosmological model with fermionic field and with the non-canonical kinetic term (f-essence). We present some important reductions of the model as well as some of its generalizations. We also find the exact solution of the model and examine the influence of such gravity-fermion interaction on the observed accelerated expansion of our universe. Further, we discuss the cosmological implications of the exact models. We show that our f-essence model corresponds to \( \Lambda \)CDM, but not a standard cold dark matter model containing no radiation. Also, our particular f-essence model has no Einstein’s static universe. Further we discuss several fermionic Dirac-Born-Infeld (DBI) models as some particular reductions of f-essence, respectively and their Chaplygin gas counterparts are found.

1. Introduction
The recent data from type Ia Supernovae (SNIa) and cosmic microwave background (CMB) radiation have provided strong evidence for a spatially flat and accelerated expanding universe [1]-[2].
In the context of Friedmann-Robertson-Walker (FRW) cosmology with Einstein gravity, this acceleration is attributed to the domination of a component with negative pressure, called dark energy. So far, the nature of dark energy remains a mystery. Theoretically, the simplest candidate for such a component is a small positive cosmological constant, but it suffers the difficulties associated with the fine tuning and the coincidence problems [3]. Therefore, many authors are attracted by the idea of dynamical dark energy models, such as quintessence, phantom, k-essence, quintom etc (see Refs. [4]-[6] for recent reviews).

Assuming the validity of the theory of gravity, one attempt to explain the present stage of accelerated expansion of our universe is the existence of an exotic, currently dominant, ingredient of the energy content of the universe, known as dark energy, with unusual physical properties. The other possibility is modifying the general theory of relativity at large scales [7, 8, 9]. In cosmology, the investigation for the constituents responsible for the accelerated periods in the evolution of the universe is of great interest. The mysterious dark energy has been proposed as a cause for the late time dynamics of the current accelerated phase of the universe.

Recently, theories described by an action with non-standard kinetic terms, k-essence, attracted considerable interest. Such theories were first studied in the context of k-inflation [10], and then the k-essence models were suggested as dynamical dark energy for solving the cosmic coincidence problem [11]-[14]. The k-essence models have also attractor solutions: it means that the differential equations of k-essence model give solution representing the present state of the universe regardless of the choice of initial conditions imposed on the value of the scalar field. Thus the coincidence problem is resolved in k-essence model. Such models of k-essence with non canonical forms of the Lagrangian in some cases, cannot define a definite dynamics for the scalar field. Recently, several attempts were made to explain the accelerated expansion by choosing fermionic fields as the gravitational sources of energy (see e.g. refs. [15]-[33]). In particular, it was shown that the fermionic field plays a very important role in: i) keeping the isotropic shape of an initially anisotropic spacetime; ii) cosmological solutions without singularity; iii) prediction of late-time acceleration. In the present work, we study the cosmological model with fermionic field — the so-called f-essence. We examine the influence of such gravity-fermionic interactions on the accelerated expansion of the Universe. The formulation of the gravity-fermionic theory has been discussed in detail elsewhere [34]-[37], so we will only present the result here.

2. Basics of f-essence
In this work we consider the f-essence fermionic field $\psi$ minimally coupled to the gravitational field $g_{\mu\nu}$ given by the action

\[ S = \int d^4x \sqrt{-g} [R + 2K(Y, \psi, \bar{\psi})]. \quad (1) \]

Here the Lagrangian $K$ depends on the fermionic field $\psi$, its conjugate $\bar{\psi}$ and the canonical kinetic term $Y$. In general, the canonical kinetic term $Y$ reads as

\[ Y = 0.5i[\bar{\psi}\Gamma^\mu D_\mu \psi - (D_\mu \bar{\psi})\Gamma^\mu \psi]. \quad (2) \]

We work with a space-time metric of the form

\[ ds^2 = -dt^2 + a^2(dx^2 + dy^2 + dz^2), \quad (3) \]

that is the FRW metric. For this metric, the vierbein is chosen to be

\[ (e^\mu_0) = \text{diag}(1, 1/a, 1/a, 1/a), \quad (e^a_0) = \text{diag}(1, a, a, a). \quad (4) \]
The Dirac matrices of curved spacetime $\Gamma^\mu$ are
\begin{align}
\Gamma^0 &= \gamma^0, \quad \Gamma^1 = a^{-1}\gamma^1, \quad \Gamma^2 = a^{-1}\gamma^2, \quad \Gamma^3 = a^{-1}\gamma^3, \\
\Gamma_0 &= \gamma^0, \quad \Gamma_1 = a\gamma^1, \quad \Gamma_2 = a\gamma_2, \quad \Gamma_3 = a\gamma_3.
\end{align}
Hence we get
\[\Omega_0 = 0, \quad \Omega_1 = 0.5\dot{a}\gamma^1\gamma^0, \quad \Omega_2 = 0.5\dot{a}\gamma^2\gamma^0, \quad \Omega_3 = 0.5\dot{a}\gamma^3\gamma^0.\]
We now ready to write the equations of the f-essence. We have
\begin{align}
3H^2 - \rho &= 0, \quad (8) \\
2\dot{H} + 3H^2 + p &= 0, \quad (9) \\
K_Y\dot{\psi} + 0.5(3HK_Y + \dot{K}_Y)\psi - i\gamma^0K_\psi &= 0, \quad (10) \\
K_Y\ddot{\psi} + 0.5(3HK_Y + \dot{K}_Y)\dot{\psi} + iK_\psi\gamma^0 &= 0, \quad (11) \\
\dot{\rho}_f + 3H(\rho_f + p_f) &= 0, \quad (12)
\end{align}
where $Y = 0.5i(\bar{\psi}\gamma^0\dot{\psi} - \ddot{\psi}\gamma^0\psi)$ and
\[\rho_f = YK_Y - K, \quad p_f = K.\]
are the energy density and pressure of the fermionic field. If $K = Y - V$, then from the system (8)-(12) we get the corresponding equations of the Einstein-Dirac model.

3. Solution
In this subsection we want to construct a solution of the particular f-essence model. Let $K = K(Y, u)$, where $u = \psi$.
We now consider the submodel, where we assume that $A_2 = \alpha Y^n, \quad B_2 = \beta u^n$ that is the case $K = \alpha Y^n + \beta u^n$. Let $a = a_0t^{\lambda}$. Then we have the following solution
\[Y = \left\{\left[-\frac{6m\lambda^2 - 4(m + 1)\lambda}{am}\right] t^{-2}\right\}^{\frac{1}{\lambda}}, \quad u = \left\{\left[-\frac{4\lambda}{\beta m}\right] t^{-2}\right\}^{\frac{1}{\lambda}}, \quad \text{(14)}\]
where
\[\lambda = \frac{2n - 2m + 2mn}{3mn}, \quad a_0 = \sqrt[3]{\frac{c}{an} \left[-\frac{6m\lambda^2 - 4(m + 1)\lambda}{am}\right]^{\frac{1}{4\lambda}} \left[-\frac{4\lambda}{\beta m}\right]^{\frac{1}{4\lambda}}}. \quad \text{(15)}\]
Finally we present the following formulas
\[u = ca^{-3}K^{-1}_Y, \quad \psi_j = c_j a^{-1.5}K^{0.5}e^{\gamma^0} \int K^{1.5}_Y dt, \quad \text{(16)}\]
where $c = |c|^2 + |c|^2_j - |c|^2_j - |c|^2_j, \quad c_j = \text{consts}.$

3.1. The cosmological implications of the f-essence model
Our model for some values of the parameters $(m, n)$ may be accelerating or decelerating. But in favor of the observational data we must have accelerating expansion today. The necessary and sufficient condition to have acceleration expansion is the $\dot{a} > 0$. From $a = a_0 t^\lambda$ we observe that for an acceleration phase we must have $\lambda(\lambda - 1) > 0$. It means that we must have
\( \lambda < 0, \quad \text{or} \quad \lambda > 1. \) By assuming that \( a_0 > 0 \), we must check the following function of the two variables \((m,n)\) for positivity
\[
f(m,n) \equiv \lambda(\lambda - 1) = -\frac{2}{9} \frac{(n - m + mn) (-2n + 2m + mn)}{m^2n^2}
\] (17)

Hence we find just two values of \( \lambda \) give us the accelerated expansion.

The declaration parameter \( q \) in terms of the Hubble parameter is defined by
\[
q = -(1 + \frac{\dot{H}}{H^2})
\] (18)

Since \( H = \frac{\lambda}{t} \), then it reads
\[
q = -(1 - \frac{1}{\lambda})
\] (19)

To have an expanding universe, \( q > -1. \) Thus we must have \( \lambda > 0. \) This condition in comparison to (17) indicates that the only possible value of the parameter \( \lambda \) to having both acceleration and expansion is \( \lambda > 1. \) In terms of the set of the parameters \((m,n)\) it results
\[
\frac{2n - 2m + 2mn}{3mn} > 1
\] (20)

indeed for (20), there are four different kinds of the solutions which we summarize them here:

**Case 1:** \( m, n > 0. \) In this case, we have the following acceptable solutions
\[
m > \frac{2n}{n + 2}.
\] (21)

This is a condition which under it, the model describes the acceleration expansion.

**Case 2:** \( m, n < 0. \) In this case, we have two possibility
\[
m > \frac{2n}{n + 2}, \quad n + 2 > 0.
\] (22)
\[
m < \frac{2n}{n + 2}, \quad n + 2 < 0.
\] (23)

With both of these conditions, we conclude that the model describes the acceleration expansion.

**Case 3:** \( m > 0, n < 0. \) As the case 2, in this case, we have two possibility
\[
m < \frac{2n}{n + 2}, \quad n + 2 > 0.
\] (24)
\[
m > \frac{2n}{n + 2}, \quad n + 2 < 0.
\] (25)

Both of these conditions indicate that the model describes the acceleration expansion.

**Case 4:** \( m < 0, n > 0. \) As the case 1, in this case, there is only one possibility
\[
m < \frac{2n}{n + 2}.
\] (26)

Thus our solution for some values of the parameters \( m, n \) describes the acceleration expansion.

Now we discuss the statefinder parameters \( \{r,s\} \). They have been introduced to characterize the spatially flat universe \( k = 0 \) models with cold dark matter (dust) and dark energy [48]. They were defined by the following expressions:
\begin{align*}
    r & = \frac{\ddot{a}}{aH^2} = \frac{t(\lambda - 1)}{\lambda^2}, \\
    s & = \frac{r - 1}{3(q - 1)} = -\frac{2}{3} t\lambda^3 - t - \frac{\lambda^2}{3(3\lambda - 2)}. \quad (28)
\end{align*}

We obtain the relation between \( r \) and \( s \) as
\[ s = -\frac{2}{3} \frac{(r - 1) \lambda}{3\lambda - 2}. \quad (29) \]

The Fig. 1 shows the plot of the (29) for some values of the \( \lambda > 1 \).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Plot of the (29) for some values of the \( \lambda > 1 \)}
\end{figure}

The cosmological constant possesses a fixed equation of state parameter \( (w = -1) \) and a fixed Newton’s gravitational constant. Here \( \{1, 0\} \) corresponds to ΛCDM. Moreover \( \{1, 1\} \) represents the standard cold dark matter model containing no radiation while Einstein’s static universe corresponds to \( \{-\infty, \infty\} \) \[49\]. It is interesting to note that from (29) we can conclude that our f-essence model corresponds to ΛCDM, since it possesses the point \( \{1, 0\} \) for any value of \( \lambda \neq \frac{2}{3} \).

The point \( \{1, 1\} \) which represents the standard cold dark matter model containing no radiation is absent in our f-essence model. Also, our f-essence model has no analogue of the Einstein static universe corresponding to the fixed point \( \{-\infty, \infty\} \).

4. f-DBI

Let us return to the the action (1)
\[ S = \int d^4x\sqrt{-g}[R + 2K(Y, \psi, \bar{\psi})]. \quad (30) \]

For the FRW metric the corresponding system of equations takes the form (8)-(12). Now we want to present some examples of fermionic DBI models (or f-DBI for short. First it is necessary to review some essentials of the DBI and tachyonic models. In the past, the use of the tachyon
as a scalar field with negative and non canonical kinetic energy has been explored in certain string theories. Tachyons are used both for cosmology and specially for better understanding of the D-brane decay process [50]. This led to study of the role of tachyons in cosmology as a rolling tachyon, which is a scalar field $\phi$ has an EoS parameter between $(-1, 0)$ [51]. Thus a tachyon may be a suitable candidate for inflation in the high energy regime [52]. It can also be regarded as a candidate for dark energy even in some extended versions of Einstein gravity [53].

4.1. Tachyonic models
Let us we present some examples of tachyonic models that follow from f-essence. Consider examples.
i) First let us consider the following tachyonic models

$$K = -U \sqrt{1 - \alpha Y},$$

(31)

where

$$U = U(\bar{T}, T), \quad Y = 0.5i(\bar{T}\gamma^0 \bar{T} - \bar{T}\gamma^0 T)$$

(32)

and $T$ is the fermionic tachyon field. Then we get the following EoS

$$p = -\rho \pm \sqrt{\rho^2 - U^2},$$

(33)

where

$$\rho = \frac{0.5U}{\sqrt{1 - \alpha Y}} + 0.5U\sqrt{1 - \alpha Y}.$$  

(34)

ii) Let the f-essence Lagrangian has the form

$$K_2 = -U \sqrt{1 - \alpha Y^2},$$

(35)

where

$$U = U(\bar{T}, T), \quad Y = 0.5i(\bar{T}\gamma^0 \bar{T} - \bar{T}\gamma^0 T)$$

(36)

and $T$ is the fermionic tachyon field. Then we get the following EoS

$$p = -\frac{U^2}{\rho},$$

(37)

where

$$\rho = \frac{U}{\sqrt{1 - \alpha Y^2}}.$$  

(38)

So this model corresponds to the Chaplygin gas [38].

4.2. Dark energy models
a) Now we consider the following DBI model

$$K_2 = \epsilon U(\sqrt{1 - \alpha Y} - 1) + V,$$

(39)

where

$$U = U(\bar{\psi}, \psi), \quad Y = 0.5i(\bar{\psi}\gamma^0 \bar{\psi} - \bar{\psi}\gamma^0 \psi), \quad V = V(\bar{\psi}, \psi)$$

(40)

and $\psi$ is the classical commuting fermionic field. Then we get the following EoS

$$p = -2\rho \pm \sqrt{4\rho^2 - 2\epsilon^2U^2} + V - \epsilon U,$$

(41)
where $\bar{\rho} = \rho - \epsilon U + V$ and the energy density is given by
\[ \rho = -0.5\epsilon U \sqrt{1 - \alpha Y^2 U} - \frac{0.5\epsilon U}{\sqrt{1 - \alpha Y^2 U}} + \epsilon U - V. \] (42)

b) Our second example reads as
\[ K = \epsilon U(\sqrt{1 - \alpha Y^2 U} - 1) + V, \] (43)

where
\[ U = U(\bar{\psi}, \psi), \quad Y = 0.5i(\bar{\psi} \gamma^0 \psi - \bar{\psi} \gamma^0 \psi), \quad V = V(\bar{\psi}, \psi) \] (44)

and $\psi$ is the classical commuting fermionic field. Then we get the following EoS
\[ p = -\frac{\epsilon^2 U^2}{\rho - \epsilon U + V} - \epsilon U + V, \] (45)

where the energy density is given by
\[ \rho = -\frac{\epsilon U}{\sqrt{1 - \alpha Y^2 U}} + \epsilon U - V. \] (46)

As $V = \epsilon U$ this model becomes
\[ p = -\frac{\epsilon^2 U^2}{\rho} \] (47)

that corresponds to the Chaplygin gas [38]. We noted here that the correspondence between DBI essence and Chaplygin gas models has been discussed previously [54].

5. Integrable f-essence models

One of interesting class of f-essence models is integrable models (see e.g. [39]-[42]). Let the Lagrangian of f-essence has the form
\[ K_2 = F(Y) - V(\bar{\psi}, \psi). \] (48)

As an example, here we consider the following particular f-essence model when $F(Y)$ obeys the equation
\[ F_{YY} = 2F^3 + YF + \alpha, \] (49)

where $\alpha$ is some constant. It is nothing but the P\textsubscript{11} equation so that it is integrable. Now the full equations of f-essence take the form
\[
\begin{align*}
3H^2 - \rho_f &= 0, \\
2H + 3H^2 + p_f &= 0, \\
F_Y \dot{\psi} + 0.5(3HF_Y + F_Y)\psi + i\gamma^0 V_{\bar{\psi}} &= 0, \\
F_Y \dot{\bar{\psi}} + 0.5(3HF_Y + F_Y)\bar{\psi} - i\psi\gamma^0 V &= 0, \\
F_{YY} - 2F^3 - YF - \alpha &= 0, \\
\dot{\rho}_f + 3H (\rho_f + p_f) &= 0.
\end{align*}
\] (50-55)
To construct solutions of this system we start from the equation (54). The equation (54) has the following particular solutions (see e.g. [39]-[47])

\[ F \equiv F(Y; 1) = \psi - (2\psi^2 + Y)^{-1}, \]  
\[ F \equiv F(Y; 1) = -\frac{1}{Y}, \]  
\[ F \equiv F(Y; 2) = \frac{1}{Y} - \frac{3Y^2}{Y^3 + 4}, \]  
\[ F \equiv F(Y; 3) = \frac{3Y^2}{Y^3 + 4} - \frac{6Y^2(Y^3 + 10)}{Y^6 + 20Y^3 - 80}, \]  
\[ F \equiv F(Y; 4) = -\frac{1}{Y} + \frac{6Y^2(Y^3 + 10)}{Y^6 + 20Y^3 - 80} - \frac{9Y^5(Y^3 + 40)}{Y^9 + 60Y^6 + 11200}, \]  
\[ F \equiv F(Y; 0.5\epsilon) = -\epsilon\psi \]

and so on. Here

\[ \psi = (\ln \phi)_Y, \quad \phi(Y) = C_1 Ai(-2^{-1/3}Y) + C_2 Bi(-2^{-1/3}Y), \]

where \( C_1 = \text{consts} \) and \( Ai(x), Bi(x) \) are Airy functions.

The Fig. 2 shows the plot of the unnormalized \( \log(\phi(-2^{-1/3}Y)) \) for \( C_1 = 1, C_2 = 0 \) (we denote it by \( \log(\phi_1(-2^{-1/3}Y)) \)) and for \( C_1 = 0, C_2 = 1 \) (we denote it by \( \log(\phi_2(-2^{-1/3}Y)) \)).

**6. Conclusion**

The accelerating expansion of the universe is one of the biggest challenges in modern physics. If we keep the Einstein theory as the geometrical part of the model and think of the matter fields as the sources of such an acceleration, we have many choices for it. One of them is scalar field, with canonical or non canonical form. The canonical scalar field can not describe the acceleration expansion correctly and it is usefull only in inflationary scenarios. A natural extension of the canonical scalar field, is a non canonical ones in which we are working with a general form of the kinetic energy. There are different kinds of such models as k-essence. In this paper we considered a new model, f-essence in which the classical Dirac fermions play the role of the source of the dark energy. We studied the particular Lagrangian of such models. In the case of no
interaction between the scalar and fermion fields (no Yukawa-like interaction) we obtained some exact solutions. For this kind of solution we show that the model has an accelerating expansion and the scale factor behaves power law. By analyzing the statefinder parameters \( r, s \), we showed that our f-essence model corresponds to \( \Lambda \)CDM, but not a standard cold dark matter model containing no radiation. Also, our exact f-essence model can not has any Einstein’s static universe. Further we consider several fermionic DBI models as some particular reductions of f-essence and their Chaplygin gas counterparts are found. We showed that we can reconstruct the DBI and tachyonic and Chaplygin gas models in such a f-essence model with some specific forms of the Lagrangian.

7. References

[1] Perlmutter S. et al. 1999 Astrophys. J. 517, 565-586
[2] Riess A G 1998 Astron.J. 116, 1009
[3] Peebles P J E, Ratra, B 2003 Rev. Mod. Phys. 75, 559
[4] Copeland E J, Sami M, Tsujikawa S 2006 Int. J. Mod. Phys. D, 15, 1753
[5] Frieman J, Turner M, Huterer D 2008 Ann. Rev. Astron. Astrophys. 48, 385
[6] Caldwell R R, Kamionkowski M 2009 Ann. Rev. Nucl. Part. Sci. 59, 397
[7] Nojiri S, Odintsov S D 2007 Int. J. Geom. Meth. Mod. Phys. 4, 115;
    Montelongo García N, Harko T, Lobo F. S. N, Mimoso J P 2011 J. Phys. Conf. Ser. 314:012060;
    Francisco T, Lobo S N, Nojiri S, Odintsov S D 2011 Phys. Rev. D84:024020;
    Raza M, Jamil M, Momeni D, Myrzakulov R 2011 (Preprint arXiv:1107.5807);
    Myrzakulov R 2011 Eur. Phys. J. C71: 1752)
[8] Nojiri S, Odintsov S D 2011 Phys. Rept. 500, 59-144
[9] Tsujikawa S 2010 Lect. Notes Phys. 800, 99-145
[10] Armendariz-Picon C, Damour T, Mukhanov V F 1999 Phys. Lett. B458, 209-218
[11] Armendariz-Picon C, Mukhanov V F, Steinhardt P J 2001 Phys. Rev. D63, 103510
[12] Armendariz-Picon C, Mukhanov V F, Steinhardt P J 2000 Phys. Rev. Lett. 85, 4438-4441
[13] Chiba T, Okabe T, Yamaguchi M 2000 Phys. Rev. D62, 023511
[14] Tsyba P Y, Kulhanarow I I, Yerzhanov K K, Myrzakulov T 2011 International Journal of Theoretical Physics, 50, 1876-1886
[15] Ribas M O, Devecchi F P, Kremer G M 2005 Phys. Rev. D72, 123502
[16] Samojeden L L, Devecchi F P, Kremer G M 2010 Phys. Rev. D81, 023501
[17] Samojeden L L, Kremer G M, Devecchi F P 2009 Europhys. Lett. 87, 10001
[18] Ribas M O, Kremer G M 2010 Grav. Cosmol. 16, 173-177
[19] Cai Y F, Wang J 2008 Class. Quant. Grav., 25, 165014
[20] Wang J, Cui S-W, Zhang C-M 2010 Phys. Lett. B683, 101-107
[21] Ribas M O, Devecchi F P, Kremer G M 2008 Europhys. Lett. 81, 19001
[22] Rakhi R, Vijayagovindan G V, Indulekha K 2009 A cosmological model with fermionic field, [Preprint arXiv:0912.1222]
[23] Rakhi R, Vijayagovindan G V, Noble P A, Indulekha K 2010 International Journal of Modern Physics A, 25, 1267-1278
[24] Chimento L P, Devecchi F P, Forte M, Kremer G M 2008 Class. Quant. Grav. 25, 085007
[25] Anischenko S V, Cherkas S L, Kalashnikov V L 2004 Cosmological Production of Fermions in a Flat Friedmann Universe with Linearly Growing Scale Factor: Exactly Solvable Model [Preprint arXiv:0911.0769]
[26] Saha B 2004 Phys. Rev. D69, 124006
[27] Esmaeikhanova K, Myrzakulov Y, Nugmanova G, Myrzakulov R 2011 International Journal of Theoretical Physics, [DOI: 10.1007/s10773-011-0996-3]
[28] Razina O V, Myrzakulov Y M, Serikbayev N S, Myrzakul Sh R, Nugmanova G N, Myrzakulov R 2011 European Physical Journal Plus, 126, N11, 1698
[29] Razina O V, Myrzakulov Y M, Serikbayev N S, Nugmanova G N, Myrzakulov R 2011 Central European Journal of Physics, [DOI:10.2478/s11534-011-0102-8]
[30] Vakili B, Sepangi H R 2008 Annals Phys. 323, 548-565
[31] Dereli T, Ozdemir N, Sert O 2010 Einstein-Cartan-Dirac Theory in (1+2)-Dimensions, [Preprint arXiv:1002.0958]
[32] Balantekin A B, Dereli T 2007 Phys. Rev. D75, 024039
[33] Armendariz-Picon C, Greene P 2003 Gen Relativ Gravit 35, 1637-1658
[34] Weinberg S 2007 *Gravitation and Cosmology* (John Wiley & Sons, New York, 1972), ibid. *Cosmology* (Cambridge, New York).

[35] Wald R M 1984 *General Relativity*, (The University of Chicago Press, Chicago).

[36] Ryder L H 1996 *Quantum Field Theory* (Cambridge University Press, Cambridge).

[37] Birrell N D, Davies P C W 1982 *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge).

[38] Kamenshchik A Y, Moschella U, Pasquier V 2001 Phys. Lett. B, 511, 265

[39] Esmakhanova K, Myrzakulov N, Nugmanova G, Myrzakulov Y, Chechin L, Myrzakulov R 2011 *Dark energy in some integrable and nonintegrable FRW cosmological models*, International Journal of Modern Physics D, 20, N12, 1-28 [DOI No: 10.1142/S0218271811020445].

[40] Nugmanova G N, Myrzakul Sh R, Razina O V, Esmakhanova K R, Serikbayev N S, Myrzakulov R 2011 *Some cosmological aspects of Horava-Lifshitz gravity: integrable and nonintegrable models* [Preprint arXiv:1104.5374]

[41] Myrzakulov R 2011 Eur. Phys. J. C, 71, 1752

[42] Myrzakul Sh, Esmakhanova K, Myrzakulov K, Nugmanova G, Myrzakulov R 2011 *FRW cosmological models with integrable and nonintegrable differential equations of state*. [Preprint arXiv:1105.2771]

[43] Kulnazarov I, Yerzhanov K, Razina O, Myrzakul Sh, Tsyba P, Myrzakulov R 2011 Eur. Phys. J. C, 71, 1698

[44] Dzhunushaliev V, Folomeev V, Myrzakulov R 2010 Phys. Lett. B, 693, 209-212

[45] Elizalde E, Myrzakulov R, Obukhov V V, Saez-Gomez D 2010 Classical and Quantum Gravity, 27, 095007

[46] Myrzakulov R, Saez-Gomez D, Tureanu A 2011 General Relativity and Gravitation, 43, 1671-1684

[47] Jamil M, Myrzakulov Y, Razina O, Myrzakulov R 2011 Astrophys. Space Sci., 336, N2, 315-325

[48] Sahni V et al 2003 JETP Lett. 77 201

[49] Jamil M , Debnath U 2011 International Journal of Theoretical Physics 50:1602-1613[Preprint arXiv:0909.3689]

[50] Sen A 2003 International Journal of Modern Physics A 18 4869;

Gorini V, et al 2004 Phys. Rev. D 69 123512

[51] Gibbons G W 2002 Phys Lett B 537 1

[52] Mazumdar A, Panda S, Perez-Lorenzana A 2001, Nucl. Phys. B 614 101

[53] Jamil M et al 2011 Phys Lett B 702 315-319

[54] Debnath U , Jamil M 2011 Astrophys. Space Sci. 335:545-552.