Knowledge Representation in Agent’s Logic with Uncertainty and Agent’s Interaction

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Abstract. This paper studies knowledge representation in multi-agent environment. We investigate technique for computation truth-values of statements based at new temporal, agent’s-knowledge logic \(\mathcal{TL}^{KnI,U}_{Dist}\). A logical language, mathematical symbolic models and a temporal logic \(\mathcal{TL}^{KnI,U}_{Dist}\) based at these models are suggested. We find an algorithm which computes theorems of \(\mathcal{TL}^{KnI,U}_{Dist}\) and satisfiability of statements, this implies that \(\mathcal{TL}^{KnI,U}_{Dist}\) is decidable (i.e. – the satisfiability problem for \(\mathcal{TL}^{KnI,U}_{Dist}\) is solvable). Application areas are pointed and discussed.

Keywords: computation of truth values, multi-agent logic, uncertainty, temporal logics, decision algorithms

1 Introduction

Knowledge may have difference origin and substance, but often it is assumed to be based at judgments of an amount of agents. Actually we will interpret knowledge as result/output of interaction between agents (it will be modeled by tools of a hybrid of multi-modal agents’ knowledge logic and temporal logic). Areas of applications for multi-agent systems (MAS) and knowledge based systems (KBS) are indeed utterly diverse, but anyway, they are primarily focused to IT (Information Technologies) in various forms (cf. e.g. Badaracco et al \[2\], König et al \[18\], Håkansson \[12\], Håkansson, Hartung, \[13\] , Håkansson, \[15\], Burgin \[5\], see also \[14,11,16,19\]). Often logical instruments are useful, cf. eg. S.Cranefield \[6\] considering a logic for expression social expectations via conditional rules (individuals may be treated as agents with desired level of autonomy).

Often some variations of modal and multi-modal logics are used for formalizing agent’s reasoning. Such logics were, in particular, suggested in Balbiani et al \[3\], Vakarelov \[32\], Fagin et al \[9,8\], Rybakov et al \[20,21\]. Representing probabilistic features of reasoning, often some elements of fuzzy logic are efficiently implemented (cf. e.g. Ribaric et al \[31\]). Working with implementation
of various techniques in IT (for example in data mining) decision procedures and data elicitation again uses elements of logical reasoning (cf. e.g. Muyeba et al [21]). In the paper Rybakov, Babenyshev [26] some multi-agent logic modeling reasoning about distances in framework of temporal logic was suggested, it proves decidability of this logic (and, consequently, satisfiability problem).

This our current paper will study knowledge representation in logical terms. We extend results of [26] to the case of a temporal multi-modal logic, which also describes interaction of agents (knowledge by interaction) and uncertainty. But we consider uncertainty not via local knowledge, - as it was earlier in [26], – but now via interaction of agents. We will use some extension of linear temporal logic LTL (cf. for LTL origin and applications Pnueli [22], Manna and Pnueli [20], Barringer, Fisher, Gabbay and Gough [4], Vardi [35]). The mathematical theory of temporal logics overall formed a highly technical branch in the area of non-classical logic (cf. van Benthem [33,34], Gabbay and Hodkinson [10], Hodkinson [17], de Jongh et al. [7]).

This our paper is devoted to computation of truth statements in multi-agent environment within a temporal framework. We construct a new temporal logic \( \mathcal{TL}^{KnI,U,Dist} \) defined in a semantic way via special Kripke models which describes frames where transition periods are filled with intermediate states. The satisfiability problem for this logic (or dually – problem of decidability for \( \mathcal{TL}^{KnI,U,Dist} \)) is our prime aim. We find an algorithm which computes theorems of logic \( \mathcal{TL}^{KnI,U,Dist} \) (which implies that the logic is decidable, and the satisfiability problem for it is also decidable). The general methodology of this paper is borrowed from [24] and [26,25].

2 Semantics for \( \mathcal{TL}^{KnI,U,Dist} \), Modeling Runs of Time

We, first, will introduce mathematical models for description agent’s interaction (and later on we will base a logic upon this semantics). We will use the following notation: for any set \( A \) and a binary relation \( R \subseteq A \times A \),

- \( R^< \) be defined as follows: \( aR^< b \iff aRb \land \neg (bRa) \);
- \( R^2 = R \circ R \), \( R^{n+1} = R \circ R^n \) — finite compositions of the relation \( R \);
- \( R^+ = \bigcup_{n=1}^{\infty} R^n \) — transitive closure of \( R \);
- \( R^* = \bigcup_{n=0}^{\infty} R^n \) — reflexive and transitive closure of \( R \).

A Kripke (multi-relation) frame \( \langle C, R_1, \ldots, R_m, R \rangle \), is a set \( C \) with binary relations \( R_1, \ldots, R_m, R \). In the sequel, a multi-agent cluster is a Kripke frame \( \langle C, R_1, \ldots, R_m, R \rangle \), where 1) \( R = C \times C \) is the universal relation on a set \( C \); 2) \( R_1, \ldots, R_m \) are equivalence relations on \( C \). From this point on, we will call multi-agent clusters simply clusters, since we will not consider any other type of them. The class of all clusters we denote by \( Cl \). Given a cluster \( C \in Cl \), we denote \( R_1.C, \ldots, R_m.C, R_C \) the respective relations. A chain is a frame \( \langle \bigcup_{i=1}^{n} C_i, R_1, \ldots, R_m, R \rangle \), where \( C_1, \ldots, C_n \in Cl \) is a finite sequence of clusters,
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Let each $R_j$ be the union of individual $R_{j,C_i}$’s, and $aRy \iff \exists i, j (i \leq j \& a \in C_i \& b \in C_j)$.

Let $C = C(0), C(1), C(2), \ldots$ be a countable sequence of clusters. The basic semantic objects upon which we define our logic are the Kripke models based on the following frames:

$$N_C := \left( \bigcup_{i \in N} C(i) \cup \bigcup_{i \in N} [C(i), C(i+1)] , R_1, \ldots, R_m, R, \text{Next} \right)$$

where

1. for each $i \in N$, $[C(i), C(i+1)]$ is a collection (may be infinite) of chains $\langle C_1, \ldots, C_n \rangle$;
2. each $R_j, j = 1, \ldots, m$ is the union of the respective $R_{j,C_i}$, i.e.,
   $$R_j = \bigcup_{i \in N} R_{j,C(i)} \cup \bigcup_{i \in N} \{(R_{j,C} \mid C \in [C(i), C(i+1)]\}$$
3. $R = Q^+$, where
   $$Q = \bigcup_{i \in N} R_{C(i)} \cup \bigcup_{i \in N} \{R_{Chain} \mid \text{Chain} \in [C(i), C(i+1)]\}$$
   $$\cup \{(a, b) \mid a \in C(i) \& b \in C_1 \in \langle C_1, \ldots, C_n \rangle \in [C(i), C(i+1)]\}$$
   $$\cup \{(a, b) \mid a \in C_n \in \langle C_1, \ldots, C_n \rangle \in [C(i), C(i+1)] \& b \in C(i+1)\}$$
4. The relation Next is defined by
   $$a \text{Next} b \iff (a \in C(i) \& b \in C(i+1)) \lor (a \in C \in \text{Chain} \in [C(i), C(i+1)] \& b \in C(i+1)).$$

This semantics is similar to the one in [26], but now we extended the language of the logic offered in [26] to handle interactions the agents via uncertainty.

3 Syntax and Language for $\mathcal{TL}^{\text{KnI,U}}_{\text{Dist}}$

The logical language for our logic $\mathcal{TL}^{\text{KnI,U}}_{\text{Dist}}$ contains usual temporal operations Next (next) and Until (until), also we use new unary logical operations $K_i$ for agent’s knowledge, a special operator Today, together with a countable set of operations for measuring temporal distances $\{\Diamond_k^+ \}_{k \in N}$.

Thus, the propositional language $\mathcal{L}$ for $\mathcal{TL}^{\text{KnI,U}}_{\text{Dist}}$ includes the following logical operations (logical connectives are given with their arities as upper-right indices):

$$\mathcal{L} := \langle \lor^2, \land^2, \rightarrow^2, \neg^1, N^1, \{K_i^1\}_{i=1}^m, \text{Unti}^1, \text{Kn}^1, U^1, \{\Diamond_k^+\}_{k \in N}, \text{Today}^1, \top^0, \bot^0 \rangle.$$
The alphabet of our logic uses propositional letters to denote not-identified statements: it contains an enumerable set $\textsf{Var} := \{x_1, x_2, x_3, \ldots\}$ of *propositional variables*. Formation rules for formulas over the propositional language $\mathcal{L}$ are below:

$$\alpha ::= x_i \mid \alpha_1 \land \alpha_2 \mid \alpha_1 \lor \alpha_2 \mid \alpha_1 \rightarrow \alpha_2 \mid \neg \alpha \mid K_i \alpha \mid N \alpha \mid \alpha_1 \text{Until} \alpha_2 \mid \diamond_k^+ \alpha \mid \text{KnI} \alpha \mid \text{U} \alpha \mid \top \mid \bot.$$ 

To describe this definition in less formal terms, this one means (i) any propositional letter $x_i$ is a formula; (ii) If $\alpha_1$ and $\alpha_2$ are formulas than $\alpha_1 \land \alpha_2$, $\alpha_1 \lor \alpha_2$, $\alpha_1 \rightarrow \alpha_2$, $\neg \alpha$, $K_i \alpha$, $N \alpha$, $\alpha_1 \text{Until} \alpha_2$, $\diamond_k^+ \alpha$, $\text{KnI} \alpha$, $\text{U} \alpha$ are again formulas. $\top$ and $\bot$ (logical constants - true and false) are formulas also.

### 4 Uncertain Statements, how we model

Now we would like to discuss the known approaches to handle logical uncertainty and to motivate our own approach. Maybe a first approach to work with logical uncertainty was based at multi-valued logics (symbolic approach; studied since the 1920s as infinite-valued logics notably by Lukasiewicz and Tarski), and fuzzy logics (numerical approach; which, in more modern descent, may be referred to Lotfi A. Zadeh, mid 1960s). Though, in such approaches, uncertainty is rather directly specified (so to say - enforced). It is easy to confess that nobody can ever determine with an absolute certainty whether a proposition concerning a scientific doctrine or even statistic observations is true or false. Besides, whenever the truth of a statement is declared, it is always done by an individual, and it can never be considered to represent a general and objective belief (though social environment often inclines an individual to join to most popular viewpoint).

**Example.** Consider a network with an admin serving it and users for this network, – as agents. Admit that these users and admin have an amount of assertions $\phi$ about the state of this network (written in the language of suggested logic (coding eg. constancy, presence of specific errors, attempts to crack it, and so forth). How we cold determine that a statement $\phi$ is uncertain? Consider the steps of inspections the network as a computation (indeed, the inspection may be undertaken by robots - software scripts - verifying some particular statements). Thus, agent’s inspection is a computation, how then we may define uncertainty of a statement $\phi$? There are several ways to approach it. For instance:

- (i) A statement $\phi$ is uncertain if in a future (after an interval of time in a computation) it will be a state when $\phi$ is true and a state where $\phi$ is false;

- (ii) A statement $\phi$ is uncertain if in current time cluster (e.g. – in a tick of time while multi-thread computation, or in a web search in current time point, etc.) it is a state where $\phi$ is true, and it is a state where $\phi$ is false;
– (iii) To handle multi-agents’ environment; a statement $\phi$ is uncertain if some agent consider it to be true now, but another one sees it is now false;

– (iv) A statement $\phi$ is uncertain if in the current time cluster (cf. (ii) for possible meaning) the following holds. Agents, passing information to each other (possible meaning: multi-thread computation and passing intermediate results via communication channels, communication of web admins via web pages available by admin logins and passwords (i.e access rules), human conversation by multiple phone calls, twitter etc) may achieve some state where $\phi$ is true, and using similar, but another procedure, they can find a state where $\phi$ is false.

In current paper we will consider the case (iv) as most complicated and cute. The other mentioned approaches also can be modeled in our technique, but we will consider (iv) not because it looks, so to say, most intricate. We think it reflects much better the essence of uncertainty (both from computational and philosophical viewpoint) in agents’ environment - via interaction of agents and a final conflict in opinions.

5 Rules for computation of truth values for statements

We turn now to description of rules for computation truth-values of formulas. For any collection of propositional letters $Prop \subseteq Var$ and any frame $N_C$, a valuation $V$ in $N_C$ is a mapping, which assigns truth values to elements of $Prop$ in $N_C$. Thus, for any $p \in Prop$, $V(p) \subseteq N_C$.

We will call any $\langle N_C, V \rangle$ a (Kripke) model. For any such model $M$, the truth values can be extended from propositions of $Prop$ to arbitrary formulas. For $a \in N_C$, we denote $(M, a) \models V \phi$ to say that the formula $\phi$ is true at $a$ in $M_C$ w.r.t. valuation $V$. Thus, $\forall p \in Prop$: $(M, a) \models V p \iff a \in V(p)$.

Rules for computation truth-values for boolean logical operations are defined as usual, e.g.: $(M, a) \models V \phi \land \psi \iff (M, a) \models V \phi \land (M, a) \models V \psi$; $((M, a) \models V \phi \land (M, a) \models V \psi \models ((M, a) \models V \phi$.

For other logical operations, suppose $a, b \in N_C$. Then

$(M, a) \models V K_i \phi \iff \forall b(aRb \Rightarrow (M, b) \models V \phi)$; $(M, a) \models V N \phi \iff \forall b(a\text{Next} b \Rightarrow (M, b) \models V \phi)$; $(M, a) \models V \phi \text{Until} \psi \iff \exists b(a\text{Next}^* b \& (M, b) \models V \phi \& \forall c(a\text{Next}^* c \text{Next}^+ b \Rightarrow (M, c) \models V \phi)$; $(M, a) \models V \phi \text{Next}^* \phi \iff \exists b(a(R^c)^k b \& (M, b) \models V \phi)$; $(M, a) \models V \phi \text{Today} \phi \iff \forall b \in C(a) ((M, b) \models V \phi)$; $(M, a) \models V \text{KnI} \phi \iff \exists a_{i_1}, a_{i_2}, \ldots, a_{i_k} \in C(a) \& R_{i_1} a_{i_1} R_{i_2} a_{i_2} \ldots R_{i_k} a_{i_k} \models V \phi$.

An important step in our approach is definition of logical uncertainty:

$(M, a) \models V U \phi \iff \text{KnI} \phi \land \text{KnI} \neg \phi$.

So, we assume that the logical truth of a statement $\phi$ is uncertain if agents may know via own interaction and passing knowledge one to other that $\phi$ may be true and also $\phi$ may be false. Usage of the logical operations has the following not-formal meaning:
Definition 1.  

Logic $\mathcal{TL}_{K\mathbf{n}I,U}^{Dist}$ is the set of all formulas which are valid in all frames $\mathcal{N}_C$.

We say a formula $\varphi$ is a theorem of $\mathcal{TL}_{K\mathbf{n}I,U}^{Dist}$ if $\varphi \in \mathcal{TL}_{K\mathbf{n}I,U}^{Dist}$; a formula $\varphi$ is satisfiable if there is frame and a valuation in this frame such that $\varphi$ is true at some state of this frame. Theorems of $\mathcal{TL}_{K\mathbf{n}I,U}^{Dist}$ are valid statements, – i.e. those which are always true.

Possible applications of the semantics and suggested logic. This semantics may be applied for modeling various reasoning (decision making) concerning multi-agent environment. It could be multi-threads computations with intermediate channels for exchanging current results of computation. Some environment close to human reasoning involving many individuals, – as web and phone conferences, – good matches to accepted formalism as well. Another application areas could be web search for information via multiple web pages by many individuals with shared or distributed access rules. Any area, where communication and interaction of agents assumes passing intermediate information, can be efficiently modeled in our framework.

6 Satisfiability and Decidability, Computing Algorithms

Now we turn to computational problems for our suggested logical system. How to compute that a given statement is true, how to see that it is satisfiable, how to decide it? We will use techniques to handle inference rules from \cite{27,23,29,30}, since it works very well for our aims. To recall necessary definitions, an inference rule is a relation

$$r := \frac{\varphi_1(x_1, \ldots, x_n), \ldots, \varphi_l(x_1, \ldots, x_n)}{\psi(x_1, \ldots, x_n)},$$

where $\varphi_1(x_1, \ldots, x_n), \ldots, \varphi_l(x_1, \ldots, x_n)$ and $\psi(x_1, \ldots, x_n)$ are formulas constructed out of letters $x_1, \ldots, x_n$. The letters $x_1, \ldots, x_n$ are the variables of $r$, we use the notation $x_i \in Var(r)$.

Informal meaning of this rule is: $\varphi_1(x_1, \ldots, x_n), \ldots, \varphi_l(x_1, \ldots, x_n)$ are premises (assumptions) and $\psi(x_1, \ldots, x_n)$ is the conclusion of $r$: $r$ says that the conclusion follows from assumptions.
A rule \( r \) is said to be *valid* in a Kripke model \( \langle N_C, V \rangle \) (we will use notation \( N_C \vDash_V r \)) if
\[
\forall a \left( (N_C, a) \vDash_V \bigwedge_{1 \leq i \leq l} \varphi_i \right) \implies \forall a \left( (N_C, a) \vDash_V \psi \right).
\]

Otherwise we say \( r \) is *refuted* in \( N_C \) (or *refuted in \( N_C \) by \( V \)), and write \( N_C \nvDash_V r \).

A rule \( r \) is *valid* in a frame \( N_C \) (notationally, \( N_C \vDash r \)) if, for any valuation \( V \), \( N_C \vDash_V r \). Since our language \( \mathcal{L} \) includes conjunction we can consider only rules with one-formula premise.

Being given with an arbitrary formula \( \phi \), we can convert \( \phi \) into the rule \( x \rightarrow x/\phi \) and employ a technique of reduced normal forms for inference rules as follows. The following statement immediately follows from definitions.

**Lemma 1.** A formula \( \phi \) is a theorem of \( \mathcal{T}_{KnI,U}^{{K}n{I},U}^{Dist} \) iff the rule \( (x \rightarrow x/\phi) \) is valid in any frame \( N_C \).

So, instead of theorems we may consider valid inference rules.

A rule \( r_{nf} \) is said to be in *reduced normal form* if \( r_{nf} = \bigvee_{1 \leq j \leq s} \theta_j \), where each \( \theta_j \) has the form:
\[
\begin{align*}
\theta_j &= \bigwedge_{i=1}^{n} x_i^{t(j,i,0)} \land \bigwedge_{i=1}^{n} (\text{Next } x_i)^{t(j,i,1)} \land \bigwedge_{i=1}^{n} (K_i x_i)^{t(j,i,1)} \\
&\quad \land \bigwedge_{i=1}^{n} \phi_i^K x_i^{t(j,i,2)} \land \bigwedge_{i=1}^{n} (x_i \text{ Until } x_i)^{t(j,i,3)} \\
&\quad \land \text{KnIx}_i^{t(j,i,2)} \land \text{Ux}_i^{t(j,i,3)}
\end{align*}
\]
for some values \( t(j, i, z), t(j, i, k, z) \in \{0, 1\} \) and where, for every formula \( \alpha \) above, \( \alpha^0 := \neg \alpha \), \( \alpha^1 := \alpha \).

For a rule \( r_{nf} \) in the reduced normal form, \( r_{nf} \) is said to be a *normal reduced form for a rule \( r \) iff, for any frame \( N_C \),
\[
N_C \vDash r \iff N_C \vDash r_{nf}.
\]

Based at the technique similar to one described in [28, Section 3.1], we can transform every inference rule in the language \( \mathcal{L} \) to a definably equivalent rule in the reduced normal form.

**Lemma 2.** Every rule \( r = \alpha/\beta \) can be transformed in exponential time to a definably equivalent rule \( r_{nf} \) in the reduced normal form.

We use this lemma for algorithms to solve satisfiability and decidability of \( \mathcal{T}_{KnI,U}^{{K}n{I},U}^{Dist} \). Notice that a formula \( \phi \) is satisfiable iff \( \phi \) is a theorem of \( \mathcal{T}_{KnI,U}^{{K}n{I},U}^{Dist} \).

So, decidability implicates solution for satisfiability problem. The decidability of \( \mathcal{T}_{KnI,U}^{{K}n{I},U}^{Dist} \) will follow (by Lemma 1) if we find an algorithm recognizing rules in the reduced normal form which are valid in all frames \( N_C \). For our approach to
complete our scheme, we need one more construction of special Kripke frames; in
a sense, they are looking similar to frames $N_C$, but have a bit another structure.
The structure of these frames is more complicated in comparison with used $N_C$
and we will omit their detail description due to size of paper limitation. We
will denote such frames by $N_C^\sharp$. To complete our approach we will apply proof
scheme from Rybakov & Babenyshev [26] specified more in [25].

**Lemma 3.** A rule $r_{nf}$ in reduced normal form is refuted in a frame $N_C$ if and
only if $r_{nf}$ can be refuted in a frame of the same sort but with clusters of size
square exponential from $r_{nf}$.

Using this lemma and structure of frames $N_C^\sharp$, following closely to proof from
[26], we derive

**Lemma 4.** A rule $r_{rf}$ in the reduced normal form is refuted in a frame $N_C$ iff
$r_{rf}$ can be refuted in some frame $N_C^\sharp$ by a valuation $V$ of special kind, where the
size of the frame $N_C^\sharp$ is effectively computable from $r_{rf}$.

Now, using Theorem 2, Lemma 1 and Lemma 4 we obtain

**Theorem 1.** The logic $TL^{K\cap I, U}_{Dist}$ is decidable. The algorithm for checking a for-
mula $\phi$ to be a theorem of logic $TL^{K\cap I, U}_{Dist}$ consists of verification for validity rules
in the reduced normal form at frames $N_C^\sharp$ of size $s$ effectively computable from
the size of the formula $\phi$.

As we noticed above, this theorem gives an algorithm which checks satisfi-
ability in $TL^{K\cap I, U}_{Dist}$. The algorithm is based at construction of the frame $N_C^\sharp$ in
Lemma 4.

7 Conclusions and Future Work

Our technique might be useful in applications to many areas. For example, it may
efficiently work in study and modeling reasoning (decision making) concerning
various multi-agent environments. A good example is multi-threads computa-
tions (where any thread is an agent) with intermediate channels for exchanging
by current results of computations. Reasoning, discussions at web and phone con-
ferences (actually any remote conversation (discussion)) may be in reasonable
depth formalized in suggested technique. Web (database) search for information
via multiple web pages (several/many databases) by sets of individuals or web
robots (as agents) with shared or distributed access rules very well suits again
for modeling in our suggested framework.

There are many prospective avenues to continue this research. First of all
the suggested technique has not too good computational efficiency as it uses
computation of truth values for rules in models – which is computationally very
costly. Thus, improvements of computational efficiency would be very desirable.
Next interesting problem is to transfer the suggested approach to non-linear
temporal logics: the cases when running of time (computational threads) is not linear.
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