Existence conditions for a low-pressure high-current discharge in a cylindrical magnetron

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Abstract. In this paper an analytical model of a high-current form of a low-pressure glow discharge in a cylindrical magnetron is presented. Conditions of discharge self-sustainment, allowing to obtain a quantitative estimation of the discharge voltage were found. Were defined the critical values of the magnetic field and residual gas pressure below which the existence of this type of discharge is impossible. It is shown that the emission of fast electrons from the discharge gap changes the conditions of the discharge.

At present time for modification of materials are widely used beams of charged particles generated by plasma of a low-pressure gas discharge [1–3]. To the most promising technological sources of charged particle beams should be attributed plasma sources with magnetron [3–6] and hollow [7] cylindrical cathodes. Due to the oscillations of electrons in a magnetic field or an electrostatic trap discharges in such sources of charged particles exist at low pressures, providing a high degree of ionization of plasma gas and the stability of plasma. Reduction of gas pressure in a glow discharge with cold magnetron and hollow cathodes allows significantly increasing the electric strength of accelerating gap.

A physical model of high-current glow discharge with cylindrical hollow cathode is presented in work [8]. For discharge in the cylindrical magnetron such physical model is missing. The aim of this work is to develop an analytical model of a high-current discharge in cylindrical magnetron, taking into account emission of charged particles from the discharge gap, and to determine minimum values of the magnetic field \( B \) and residual gas pressure \( p \) at which the existence of this form of discharge is possible.

High-current discharge is modelled in a discharge cell, which is a hollow cylinder of radius \( R \), acting as anode, on the axis of which is located cathode in a form of a rod with radius \( r_0 \). A uniform magnetic field \( B \) is directed along the axis of the cylinder. In this work is investigated a stationary high-current mode in which the voltage drop is concentrated in a thin near-cathode layer, and the interelectrode space is occupied by quasi-neutral plasma.

A stream of fast electrons generated in the cathode layer, is fed at a speed \( V_f(R) = V_f(r_0) = (2eU/m)^{1/2} \) into the plasma region, where it maintains the necessary degree of ionization. Slow electrons of the plasma not capable to ionize atoms, provide short circuiting of discharge current to the anode.

For the solution is considered a one-dimensional problem in the cylindrical coordinate system. In determining the conditions of the discharge existence, assume that the movement of fast electrons...
from the cathode to the anode occurs in a diffuse mode, while the influence on the movement of fast particles of a superimposed electric field in quasi-neutral plasma is neglected. Using the stationary equations of continuity and the equations for the flow of fast electrons for radial distribution of particle concentration \( n_f \) in the discharge gap [9, 10] we will get the following equation:

\[
\frac{1}{\rho} \frac{d}{dp} \left( \rho \frac{dn_f}{dp} \right) - n_f = -\frac{\gamma n_e V_s \tau_{ef}}{2(R-r_0)},
\]

(1)

where \( \rho = r/\lambda; \lambda_e = (D_e \tau_0)^{-1/2}; D_e = eUv_{f0}/\left[3m_e \left( \omega_{Be}^2 + v_{f0}^2 \right) \right] - diffusion coefficient; \gamma_{f0} - frequency of elastic collisions of electrons with atoms; \omega_{Be} - Larmor frequency of rotation; \gamma - effective coefficient of ion-electron emission; \tau_{ef} = eU/(v_iW) - characteristic time of fast electrons relaxation; \gamma_{i} - ionization frequency; \gamma_{i} - energy spent by electrons on ionization of the gas atoms; \nu_{f0} - frequency of collisions; \tau_i - temperature of ions. Solution of the equation (1) is found in the form

\[
n_f(\rho) = n_{f0} \left[ 1 - A \left( \rho \right) - B K_0(\rho) \right],
\]

(2)

where \( I_0 \) and \( K_0 \) - modified Bessel functions of zero order of the first and second kind respectively; \( A \) and \( B \) - constants.

Distribution (2) is decreasing from the cathode to the anode in contrast to the growing solution from the cathode in the case of the hollow cathode effect in the electrostatic trap of a hollow cylindrical cathode [10].

Taking into account the boundary conditions at the anode \( n_f(r = R) = 0 \) and at the cathode \( n_{f0} = -D_e/(dn_f/dr)_{r=0} \), we will receive the calculated expression of the concentration distribution of fast electrons in the discharge gap:

\[
\frac{n_f(\rho)}{n_{f0}} = 1 - \frac{P + 2(\rho_{r} - \rho_{s0})G}{P},
\]

(3)

where \( P = I_0(\rho_{r})K_1(\rho_{s0}) + I_1(\rho_{s0})K_0(\rho_{r}); G = I_0(\rho_{r})K_1(\rho_{s0}) - I_1(\rho_{s0})K_0(\rho_{r}); \rho_{r} = R/\lambda_e; \rho_{s0} = r_0/\lambda_e \).

The current density of ions at the cathode, given that \( n_{f0} \) is defined in equation (1) in the conditions \( eV_s = j_e \), could be found using formula (3):

\[
j_e = \frac{1}{r_0} \int_{r_0}^{R} n_r e v r dr = j_e \left[ \frac{\gamma n_e V_s \tau_{ef}}{2R} \left( \frac{R^2 - r_0^2}{2} - \frac{\lambda_e R}{P} M - \frac{2(R-r_0)}{P} \left( \frac{RZ - r_0 P}{R} \right) \right) \right].
\]

(4)

where \( M = I_1(\rho_{r})K_1(\rho_{s0}) - I_1(\rho_{s0})K_1(\rho_{r}); Z = I_0(\rho_{r})K_0(\rho_{s0}) + I_1(\rho_{s0})K_0(\rho_{r}) \) - combinations of Bessel functions. From equation (4) it follows that for self-sustaining of a discharge the following condition should be met:

\[
1 = u \left[ 1 - \frac{R + r_0}{2P \rho_{s0} (R-r_0)} M \right] \left[ 1 - \frac{R - Z}{P \rho_{s0}} \right] + 1,
\]

(5)

where \( u = U/U_0 = \gamma_V \tau_{ef} \), \( b = B/B_0 \) - dimensionless variables; \( U_0 = W/(e\gamma); B_0 = 1.5(m_e Wv_{f0}/v_{f0})^{1/2}/(e\gamma(\rho_{s0} / r_0)); \gamma_{f0} \approx 2V; \rho_k = (2.61b)R/(R-r_0); \rho_{s0} = \rho_{s0} / R \); in the case of magnetized electrons when \( \omega_{Be}^2 >> v_{f0}^2 \).

The dependence of the discharge sustaining voltage on the magnetic field magnitude, following from (5), is shown in figure 1. For calculations the values of the anode radius \( R = 0.07 \) m and the
radius of the cathode $r_0 = 0.06$ m are taken from the system of “inverted magnetron” type [13]. It is seen that the minimum value of the magnetic field for which the existence of a discharge is still possible in cylindrical magnetron is more than in planar magnetron.

The condition of the discharge self-sustaining in a cylindrical magnetron can also be found in another way. As follows from [11], the multiplication factor of the plasma electrons $\alpha = eU/W$, is related with the ratio of the current of fast electrons at the anode to the total current discharge $\Psi = I_{fa}/I$ as follows:

$$\alpha \approx \frac{1}{\gamma - \Psi}. \quad (6)$$

Current of electrons to the anode is circuited mainly by the plasma electrons $I = I_{fa} + I_{ea} \approx I_{ea}$, i.e. (6) is valid provided that $\Psi \ll 1$. Determining the current density of fast electrons from the boundary conditions at the cathode $(n\nu_{j0})_0 = -D_{\beta}(dn/dr)_{r_0}$ and current density of plasma electrons at the anode

$$j_{ea} = \frac{1}{R_e} \int_{r_0}^{R} ev_\perp n_j r dr, \quad (7)$$

using (6) we will find the condition for discharge self-sustaining in the magnetron:

$$u = 1 + \frac{1}{0.5N\rho_N(1 + 4r_0/R - 5r_0^2/R^2) - 1}, \quad (8)$$

where $N = P[M + 2(\rho_R - \rho_o)Z]^{-1}$.

In figure 2 the dependence 2, following from equation (8), more accurately reflects the conditions of existence of the discharge in comparison with the corresponding dependence 2 (figure 1) because the minimum possible discharge voltage $u = 1$. This condition is achieved when all the fast electrons completely deplete their energy and only after that reach the anode [14].

Presented mathematical model does not allow identifying the effect of residual gas pressure on the voltage of high-current discharge. This can be done, if in equation (6) instead of expression (7) for a current density of plasma electrons at the anode use the expression following expression [11]:

$$j_{ea} = \frac{en_0 V_{Te}}{4\sqrt{1 + \beta_e^2}}, \quad (9)$$

where $n_0$ – plasma density in the interelectrode gap; $V_{Te} = (kT_e/m_e)^{1/2}$ – thermal velocity of the electrons; $\beta_e = 1/(r_{ce} n_{e0})$ – Hall parameter; $r_{ce} = m_e V_{Te} / (eB)$ – cyclotron radius of the plasma electrons rotation; $n_g$ – gas atoms density; $\sigma_{e0}$ – transport cross section of electron–atom collisions. Then in the assumption of constant concentrations of plasma in the interelectrode gap $n_0 \approx n_g$ instead of (8) we will have:

$$u(1 - \Psi/\gamma) = 1, \quad (10)$$
where

\[
\Psi = 4\beta c (m_e / m_i)\gamma^2 M / (\rho_e - \rho_i) + Z / p.
\]

\[\int \frac{d}{dr} \left( \rho e n_e V_e \right) = \frac{n_f}{\tau_f} - \frac{n_f}{\tau_f} + \frac{\gamma n_e V_e}{4(R - r_0)},
\]

Figure 2. Dependence of the discharge sustaining voltage from the magnetic field induction: 1 – for a planar magnetron with \(d = R - r_0 = 0.02\) m [14]; 2 – from equation (9) for the cylindrical magnetron with \(r_0/R = 0.7\) (\(r_0 = 0.05\) m, \(R = 0.07\) m); 3 – from equation (9) for the cylindrical magnetron with \(r_0/R = 0.2\) (\(r_0 = 0.014\) m, \(R = 0.07\) m).

Dependences of the discharge sustaining voltage from the gas pressure (xenon) at various values of the magnetic field induction obtained from equation (10) are shown in figure 3. With a decrease in the magnitude of the magnetic field increases the minimum gas pressure necessary to maintain high-current discharge form.

Figure 3. Dependence of the discharge sustaining voltage from the gas pressure (xenon) for different values of the magnetic field: 1 – \(b = 1.2\); 2 – \(b = 1.0\); 3 – \(b = 0.9\). \(B_0 = 1.4 \times 10^{-2}\) T, \(\gamma = 0.1\), \(W = 30\) eV, \(\sigma_0 = 1.5 \times 10^{-19}\) m².

Indeed, the increased departure of the plasma electrons across the magnetic field must be compensated by increasing the intensity of gas ionization by fast electrons. Let us note that in the reflective discharge in cylindrical geometry with a hollow cathode [11] takes place a reverse dependence of the minimum gas pressure on the magnitude of the magnetic field.

The conditions of the discharge are also influenced by the emission of fast electrons from the discharge gap. For the emission of fast electrons from the discharge gap along the magnetic field through the ends of the coaxial cylinder the continuity equation has the form:

\[
\frac{1}{r} \frac{d}{dr} \left( \rho e n_e V_e \right) = \frac{n_f}{\tau_f} - \frac{n_f}{\tau_f} + \frac{\gamma n_e V_e}{4(R - r_0)},
\]

where the characteristic time of complete loss of fast electrons, according to [14]
\[ \tau_{ef} = \frac{\tau_{ef}}{\tau_{ef} + \tau_{ef}}, \]

where \( \tau_{ef} = 2L(2m/eU_c)^{1/2} \) – time of particles loss due to the emission from the discharge; \( L \) – length of the discharge gap in the direction of the magnetic field (length of cylinder); \( \varepsilon \) – transparency of the end electrodes in the form of meshes, which are under the anode potential, and through which the emission of particles from the plasma takes place.

In this case, for the discharge self-sustaining the condition (8) must be satisfied, in which the dimensionless parameter \( u = U/U_0 = \gamma \nu \tau_{ef} \) should be replaced by \( u/(1 + \beta u^{1/2})^{1/2} \), where \( \beta = \varepsilon(2W/m)^{1/2}/(8\gamma v_i L) \).

The dependence of the voltage of the discharge existence from the magnetic field induction taking into account the emission of fast electrons from the discharge gap is shown in figure 4. With the increase of \( \beta \) the curves are shifted to the right and lower boundary of the magnetic field increases.

![Figure 4. Dependence of the discharge sustaining voltage from the magnetic field induction for \( \rho_0/R = 0.7 \): 1 – \( \beta = 0 \); 2 – \( \beta = 0.2 \); 3 – \( \beta = 0.3 \). \( B_0 = 1.4 \times 10^{-2} \) T, \( \gamma = 0.1 \), \( W = 30 \) eV, \( \sigma_{in} = 1.5 \times 10^{-19} \) m², \( \varepsilon = 0.6 \), \( L = 0.08 \) m.](image)

Thus, in the present work, the conditions for maintaining a high-current discharge in the cylindrical magnetron are defined. In this type of magnetron the minimum value of the magnetic field for which the existence of the discharge is yet possible, is higher than in planar magnetron. With increasing distance between the electrodes increases the magnitude of the minimum magnetic field. For the studied geometry the value of the minimum residual gas pressure \( (p \approx 10^{-3} \) Torr (0.133 Pa) at \( B = 1.7 \times 10^{-2} \) T), necessary to maintain high-current form of the discharge is on the order less than the critical pressure for the reflective discharge in cylindrical geometry with a hollow cathode \( (p \approx 2.6 \times 10^{-2} \) Torr (0.346 Pa) at \( B = 2 \times 10^{-3} \) T) [11]. With increasing gas pressure decreases the value of the minimum magnetic field needed to maintain a high-current type of discharge in cylindrical magnetron. In the reflective discharge in cylindrical geometry with a hollow cathode this dependency is reverse.

The emission of fast electrons from the discharge gap changes the conditions of the discharge in cylindrical magnetron. With the increase of the parameter \( \beta \) with increasing the transparency of the end electrodes \( \varepsilon \) and decreasing the length of the cylinder \( L \) increases the minimum value of the magnetic field for which the existence of the discharge is possible.

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