Electroweak Baryogenesis with dimension-6 Higgs interactions\(^1\)

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**Abstract**

We present the computation of the baryon asymmetry in the SM amplified by dimension-6 Higgs interactions using the WKB approximation. Analyzing the one-loop potential it turns out that the phase transition is strongly first order in a wide range of the parameters. It is ensured not to wash out the net baryon number gained previously even for Higgs masses up to at least 170 GeV. In addition dimension-6 operators induce new sources of CP violation. Novel source terms which enhance the generated baryon asymmetry emerge in the transport equations. This model predicts a baryon to entropy ratio close to the observed value for a large part of the parameter space. My talk was mainly based on our recent work [1].

**KEYWORDS:** Dimension-6 operators, baryogenesis, electroweak phase transition.

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1 Introduction

The baryon asymmetry of the universe has been measured by combining measurements of the cosmic microwave background and large scale structures. The baryon to entropy ratio

$$\eta_B \equiv \frac{n_B}{s} = (8.9 \pm 0.4) \times 10^{-11}$$  \hspace{1cm} (1)

describes the abundance of matter over anti-matter in the universe. The origin of this value we want to explain.

In 1967 Sakharov formulated three necessary ingredients for baryogenesis, these are: baryon number non-conservation, C and CP violation and deviation from thermal equilibrium. The first condition is obvious, there must be a mechanism which violates B, since we assume the universe to be baryon symmetric in the beginning. Secondly, if C and CP were conserved, particles and anti-particles were produced with an equal rate so that a net baryon number cannot be generated. Thirdly we need deviation from equilibrium because CPT invariance ensures vanishing baryon density in strict thermal equilibrium. For electroweak baryogenesis a considerable departure from equilibrium is only possible from a first order phase transition (PT), since the expansion of the universe is slow during the electroweak epoch. The transition is triggered by an energy barrier in the Higgs potential which separates two energetically degenerate phases at the critical temperature $T_c$. At this temperature the formation of bubbles starts, they expand and finally percolate. The vacuum expectation value of the Higgs field is zero in the symmetric phase and changes rapidly at the bubble wall to a non-zero value $v_c$ inside the bubbles. Baryon number violation takes place outside the bubbles while within the sphaleron induced (B+L)-violating but (B-L)-conserving reactions must be strongly suppressed. To prevent baryon number washout after the phase transition the so called "washout criterion"

$$\xi = \frac{v_c}{T_c} \geq 1.1$$  \hspace{1cm} (2)

has to be satisfied [2]. This is the condition for a first order transition to be strong. The baryon asymmetry cannot be explained within the standard model (SM) due to the facts that there is not enough CP violation and the transition is only a smooth cross over. To overcome these difficulties we need extensions to the SM. One possible scenario is to introduce a non-renormalizable $\phi^6$ operator [3, 4]. The corresponding potential is

$$V(\phi) = -\frac{\mu^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4 + \frac{1}{8M^2} \phi^6,$$  \hspace{1cm} (3)

where $\phi^2 \equiv 2\Phi^\dagger\Phi$ with the SM Higgs doublet $\Phi$. In such a model the barrier in the potential which triggers the first order transition is no longer only provided by the cubic one-loop thermal corrections of the weak gauge bosons. It can also be generated from a negative $\phi^4$ term because the Higgs potential is stabilized by the $\phi^6$ interaction. This yields to a strong first order PT for Higgs masses larger than the
experimental lower bound. Another advantage of non-renormalizable interactions is that we get easily an extra mechanism for CP breaking to fuel baryogenesis, in addition to the usual breaking via the CKM matrix.

## 2 The phase transition

The effective Higgs potential is the crucial factor for studying the dynamics of the electroweak phase transition (EWPT). In the high temperature expansion we get

\[
V_{\text{eff}}(\phi, T) = \frac{1}{2} \left( -\mu^2 + \left( \frac{1}{2} \lambda + \frac{3}{16} g_2^2 + \frac{1}{16} g_1^2 + \frac{1}{4} y_t^2 \right) T^2 \right) \phi^2 \\
- \frac{g_2^3}{16\pi} T \phi^3 + \frac{\lambda}{4} \phi^4 + \frac{3}{64\pi^2} y_t^4 \phi^4 \ln \left( \frac{Q^2}{c_F T^2} \right) \\
+ \frac{1}{8M^2} (\phi^6 + 2\phi^4 T^2 + \phi^2 T^4)
\]

which includes the thermal mass term, the resummed one-loop contributions due to the transverse gauge bosons and the top quark as well as the leading one-loop and two-loop corrections due to the \( \phi^6 \) interaction. \( y_t \) is the top Yukawa coupling, \( g_2 \) and \( g_1 \) are the \( SU(2)_L \) and \( U(1)_Y \) gauge couplings, \( c_F \approx 13.94 \) and we have chosen \( Q = m_{\text{top}} = 178 \text{ GeV} \). It is possible to express the two unknown parameters \( \mu \) and \( \lambda \) by the physical quantities \( m_H \) and \( v = 246 \text{ GeV} \), so that we treat the Higgs mass \( m_H \) and the cut-off scale \( M \) as the free parameters of our model with the constraint \( m_H > 114 \text{ GeV} \). In addition we have to require \( M < 1 \text{ TeV} \) to be relevant at weak scale temperatures and to be consistent with electroweak precision data we should certainly have \( M \) bigger than a few hundred GeV.

To compute the strength of the PT we need the ratio of the non-zero value of the vacuum expectation value \( v_c \) and the critical temperature \( T_c \). To determine these two quantities the two conditions

\[
\left. \frac{\partial V_{\text{eff}}(\phi, T_c)}{\partial \phi} \right|_{\phi=v_c} = 0 \quad \text{and} \quad V_{\text{eff}}(v_c, T_c) = 0
\]

have to be fulfilled. Typically \( T_c \) is around 100 GeV in case of the EWPT.

In fig. 1 the strength of the PT is shown as a function of the model parameters. It becomes weaker for increasing Higgs masses and for the smallest allowed Higgs mass we need \( M \lesssim 825 \text{ GeV} \) to satisfy the washout criterion. Below the lowest line (indicated by “wrong global minimum”) the symmetric vacuum is the global one for every temperature meaning the non-zero vacuum is metastable and there is no longer a PT. Below the “metastability” line the probability for thermal tunneling gets too small compared to the Hubble expansion rate, that means in this region the universe remains stuck in the symmetric vacuum. But there is a large part of the parameter space satisfying the necessary condition for electroweak baryogenesis to succeed. In contrast to the SM we find a strongly first order phase transition, even for Higgs masses up to 170 GeV.
3 Bubble characteristics

The wall thickness $L_w$ and the velocity $v_w$ are the bubble properties which will enter the following computation of the baryon asymmetry.

What do we mean by the wall thickness and how to estimate it? The solution of the field equation can be approximately described by a kink,

$$\phi(z) = \frac{v_c}{2} \left(1 - \tanh \frac{z}{L_w}\right).$$

We could show that this kink solution using the estimate $L_w = \sqrt{v_c^2/(8V_b)}$, where $V_b$ is the height of the potential barrier, fits the wall profile quite well. $L_w$ varies in a wide range between 2 and $14T_c$ depending on the combination of the parameters. It becomes smaller as we decrease $m_H$ at fixed $M$ and it is the same the other way round if we decrease $M$ at fixed $m_H$. In fig. 2 this behaviour is presented in dependence of the model parameters. In addition lines of constant $\xi$ are shown.

The computation of the wall velocity is very complex. On the one hand the expansion of the bubbles is accelerated by the internal pressure and on the other hand it is slowed down by plasma friction. In addition $v_w$ is also reduced by latent heat of the nucleating bubbles. The wall propagates with a constant velocity when the forces are balanced and a stationary situation is reached. In general, the movement of the wall is faster in the case of a stronger PT. But it is only possible to give rough estimates with large uncertainties so that we decided to treat $v_w$ as a free parameter in the following computation of the baryon asymmetry.
A new source of CP violation is generated by introducing a dimension-6 Higgs-fermion interaction with the Lagrangian

$$\mathcal{L}_m = \bar{\Psi}_L \left( y \Phi + \frac{x}{M^2} (\Phi^\dagger \Phi) \Phi \right) \Psi_R + h.c. \quad (7)$$

where $y$ stands for the usual Yukawa couplings and $x$ for the new couplings containing complex phases relative to $y$. In general, these $x$ are of unknown flavour structure but we assume that they have the same structure as the corresponding Yukawa couplings. Then we can concentrate on the top quark and choose for simplification $y_t$ real and $x_t$ purely imaginary. (This is a good approximation since the real part of $x_t$ is negligible in comparison to $y_t$ because of the $M^2$ in the denominator. So only its imaginary part is of relevance.) The Lagrangian then reduces to

$$\mathcal{L}_m = \bar{t}_L m_t t_R + \bar{t}_R m_t^* t_L \quad (8)$$

with the complex top mass

$$m_t = y_t \frac{\phi(z)}{\sqrt{2}} + i |x_t| \frac{\phi(z)^3}{2 \sqrt{2} M^2} = m(z) e^{i \theta(z)} \quad (9)$$

where we have defined a CP violating phase

$$\theta(z) = \arctan \left( \frac{|x_t| \phi(z)^2}{2 y_t M^2} \right). \quad (10)$$
Obviously this is C conserving but P violating. In the WKB approach this leads to different dispersion relations for particles and anti-particles, depending on their complex mass. These dispersion relations induce different force terms in the transport equations which describe the evolution of the plasma. So the CP violating interactions create an excess of left-handed quarks over the corresponding anti-quarks. Sphaleron transitions where only left-handed particles are involved annihilate this asymmetry in the symmetric phase. The expanding bubble sweeps over this region and in the broken phase these (B+L)-violating reactions are suppressed because of the strongly first order PT. As a result a net baryon asymmetry is generated.

5 The evolution of the plasma

We model the evolution of the particle distributions \( f_i(t, x, p) \) by classical Boltzmann equations

\[
(\partial_t + \mathbf{x}\partial_x + \mathbf{p}\partial_p) f_i(t, x, p) = C[f_i].
\]

(11)

The index \( i \) stands for the type of the particle and \( C \) is the collision term. The dispersion relations come into play through

\[
\dot{x} = \partial_p E_i(x, p), \quad \dot{p} = -\partial_x E_i(x, p).
\]

(12)

Using the fluid-type ansatz in the rest frame of the plasma [5]

\[
f_i(t, x, p) = \frac{1}{e^{\beta (E_i - v_i p_z - \mu_i)} \pm 1},
\]

(13)

we introduce velocity perturbations \( v_i \) and chemical potentials \( \mu_i \) for each fluid. The velocity perturbations describe the particle movement in response to the force caused by the different dispersions relations.

We search for stationary solutions of these Boltzmann equations by expanding in derivatives of the fermion mass. If we weigh the Boltzmann equations with 1 and \( p_z \) we derive after momentum averaging to first order in derivatives

\[
\kappa_i v_w \mu'_{i,1} - K_{1,i} v'_{i,1} - \sum_p \Gamma_{p}^{\text{inel}} \sum_j \mu_{j,1} = K_{3,i} v_w (m_i^2)^j
\]

(14)

\[
-K_{1,i} \mu'_{i,1} + K_{2,i} v_w v'_{i,1} - \frac{K_{1,i}^2}{\kappa_i D_i} v_{i,1} = 0.
\]

(15)

\( \mu_{i,1} \) and \( v_{i,1} \) denote the perturbations to first order in derivatives. At this stage we do not have to distinguish between particles and anti-particles. To simplify the notation the symbols \( K_i \) and \( \kappa_i \) betoken some mass dependent momentum averages. The statistical factor \( \kappa_i \) would be 1 for massless fermions. A prime represents the derivative with respect to the relative coordinate perpendicular to the wall \( (\bar{z} \equiv z - v_w t) \).

Here the collision terms have already been expressed by inelastic rates \( \Gamma_{p}^{\text{inel}} \) for a
process \( p \) and by diffusion constants \( D_i \). Realize on the right-hand side of the first equation that the change in the particle mass along the wall causes the force term for the particles. (The used notation and a detailed computation can be found in [1].)

To second order in derivatives there is a difference between particles and anti-particles. The perturbations contain CP odd and even components but we are only interested in the CP odd ones. After subtracting the equations of particles and anti-particles we derive

\[
\kappa_i v_w \mu'_i - K_1,2 v'_i - \sum_p \Gamma_p^{\text{inel}} \sum_j \mu_j = -K_6,1 \theta_i m^2 \mu'_i
\]

\[
-K_1,2 + K_2,2 v'_i - \frac{K_2,1}{K_i} v_i = K_4,1 v_w m^2 \theta_i'' + K_5,1 v_w (m^2_i)\theta_i'
\]

Every CP violating source term on the right-hand side is proportional to derivatives of \( \theta_i \). The source terms proportional to the first order perturbations can enhance the asymmetry in the baryon number. In the following section we analyze their relevance.

In principle we have to solve this set of differential equations (14), (15), (16) and (17) for every particle type \( i \) to compute the asymmetry in the left-handed quark density. The chemical potential \( \mu_{B_L} \) of the left-handed quarks is then given by the sum over the flavours. (For details see again [1].)

The weak sphalerons convert \( \mu_{B_L} \) into a baryon asymmetry by [6]

\[
\eta_B = \frac{n_B}{s} = \frac{405 \bar{\Gamma}_{ws}}{4\pi^2 v_w g_s T^4} \int_0^\infty d\bar{z} \mu_{B_L}(\bar{z}) e^{-\nu \bar{z}}.
\]

Here \( \bar{\Gamma}_{ws} = 1.0 \cdot 10^{-6} T^4 \) [8] is the weak sphaleron rate, \( g_s = 106.75 \) the effective number of degrees of freedom in the plasma and \( \nu = 45 \bar{\Gamma}_{ws}/(4v_w T^3) \).

6 Numerical results

In this section we present the results of our computation. The most important particles to be taken into account are the left- and right-handed top quarks and the Higgs bosons. We can neglect the other quarks and leptons due to their small masses. We found that also the Higgs bosons can be ignored because their influence on the generated baryon asymmetry is negligible. So we have to solve the equations (14), (15), (16) and (17) simultaneously only for the left-handed top and bottom quark as well as for the right-handed top quark to compute the asymmetry in the left-handed quark density. At this stage we assume that the gauge interactions are in equilibrium. Moreover we neglect the weak sphalerons, i.e. baryon and lepton...
As already mentioned there are different source terms appearing in the transport equations (16) and (17). In fig. 3 we compare their contributions to the generated baryon asymmetry for one typical set of parameters. The lower line represents the contribution to $\eta_B$ which arises from the source terms proportional to the first order perturbations $\mu'_{i,1}$ and $v'_{i,1}$. Note that these source terms are non-negligible, especially for small wall velocities. The middle line shows the contribution due to the other sources and the upper line is the total baryon asymmetry, that means the sum of the two parts. Another general result is that $\eta_B$ depends only slightly on $v_w$. We also found that $\eta_B$ decreases with increasing $L_w$. As expected $\eta_B$ increases rapidly for a stronger PT (larger values of $\xi$) since then the top mass becomes larger and the effect of the CP violating phase $\theta$ stronger. These are general remarks about the influence of the different parameters to the generated baryon asymmetry. If we look in detail at the model under consideration, we have to compute the strength of the PT and the bubble wall width for every value of the cut-off scale $M$. In our computation we take $|x_t| = 1$ for the CP violating phase (10). In fig. 4 we present
Figure 4: $\eta_B$ as a function of $M$ (in units of GeV) for two different Higgs masses respectively for $v_w = 0.01$ (solid) and $v_w = 0.3$ (dashed).

$\eta_B$ as a function of $M$ for two different Higgs masses $m_H = 115$ and 150 GeV. We have chosen one large $v_w = 0.3$ and one small wall velocity $v_w = 0.01$ to show again the small dependence on this parameter. The measured value (1) is signified by the horizontal lines in the figure. For every Higgs mass the baryon asymmetry gets smaller for increasing $M$. But we can easily generate the observed value for natural values of the parameters!

7 Conclusions

We have investigated the EWPT and baryogenesis in the SM extended by dimension-6 Higgs interactions. Non-renormalizable operators parameterize new physics beyond some cut-off scale $M$. This scale and the Higgs mass are the two free parameters of this model. Using the one-loop thermal potential we find a large part of the parameter space where the requirements of electroweak baryogenesis are satisfied. Even for Higgs masses up to 170 GeV the PT is strong enough to avoid baryon number washout.

A dimension-6 Higgs-fermion interaction arranges a new source of CP violation. We obtain a complex phase in the quark mass which varies along the bubble wall. This causes different forces on the quarks and corresponding anti-quarks as they pass through the phase boundary. As a result, an excess of left-handed quarks is created which afterwards is converted into a baryon asymmetry by the sphaleron transitions. We describe the evolution of the plasma in the WKB approach and expand in deriva-
tives of the wall profile. We found novel source terms in the transport equations. These source terms proportional to the first order perturbations enhance the generated baryon asymmetry especially for small wall velocities. The model under consideration provides the missing ingredients for electroweak baryogenesis, i.e. a strong first order phase transition and additional CP violation. The measured baryon to entropy ratio can be explained for natural values of the parameters.

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