Double diffusion on peristaltic flow of nanofluid under the influences of magnetic field, porous medium, and thermal radiation

Asha Shivappa Kotnurkar | Sunitha Giddaiah

Department of Mathematics, Karnatak University, Dharwad, India

Correspondence
Asha Shivappa Kotnurkar, Department of Mathematics, Karnatak University, Dharwad, Karnataka, India 580003. Email: as.kotnur2008@gmail.com

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Abstract
The present article investigates the study of double-diffusive convection on peristaltic flow under the assumption of long wavelength and low Reynolds number. The mathematical modeling for a two-dimensional flow, along with double diffusion in nanofluids, is considered. The motivation of the present research work is to analyze the effects of the magnetic field and thermal radiation on a peristaltic flow through a porous medium in an asymmetric channel. The heat flux of the linear approximation employs the thermal radiation of the flow problem. The effect of thermal radiation and double diffusion is an important aspect of research due to its application in public health potential. Infrared radiation techniques are indeed used to treat many skin-related diseases. It can also be used as a measure of thermotherapy in some bones to enhance the blood circulation. It is found that approximately 80% of blood flow increases with radiation. The governing equations are analytically solved by using Homotopy analysis method with the help of the symbolic software Mathematica. The results of the velocity, pressure rise, temperature, solutal (species) concentration, and nanoparticle volume fraction profiles are graphically shown.

KEYWORDS
Double diffusion, magnetic field, nanofluid, peristaltic flow, porous medium, thermal radiation

1 | INTRODUCTION

Peristaltic transport problems have been special attention due to its wide range of applications in numerous fields such as chemical industries, biomedical, engineering, nuclear reactors, and physiology. In particular, peristaltic mechanism is involved in many physiological systems such as urinary system, movement of ovum in the female fallopian tubes, and vasomotion of small blood vessels such as capillaries, arterioles, and venues. Peristalsis was first initiated by Latham in 1966. Furthermore, the research work was extended by Shapiro et al and Jaffrin et al. Many researchers have been studied on both theoretical and practical aspects of peristaltic transport. The thermal energy vigorously depends on the temperature variation at the locomotion. However, the energy transfer from one body to another body is depends on the...
absolute temperature variation. Also, the thermal radiation has many applications in biological treatments such as lung cancer, bone cancer, breast cancer, thyroid, blood disorders, radiotherapy, and liver cancer. Research work on peristaltic motion with the influence of thermal radiation is reported in References.8-13 The radiation has been linearized using Roseland approximation.14

A homogeneous mixture of nanoparticles consists of a base fluid is known as nanofluids. The nanoparticles employed in nanofluids are made up of carbon nanotubes, metals, oxides, and carbides. The base fluid includes oil, water, and ethylene glycol. Nanofluids are produced by dispersing nanoparticles in base fluid; given it is superior thermophysical properties, nanofluids are gaining increasing attention and are showing promising potential in various applications. Choi15 was first to proposed nanofluids. Nanofluids include the enhanced thermophysical properties compared with ordinary fluids. The different types of nanoparticles are oxide ceramics, carbide ceramics, metals, and carbon nanotubes.16 In nanofluids, nanoparticles randomly move through liquids and possibly collide with Brownian motion and thus proposed to be one of the possible origins for thermal conductivity enhancement. The connection of peristalsis with the nanofluids has important applications in biomedical science (such as drug delivery and cancer treatment to treat radiotherapy), mechanical engineering, and chemical engineering (transport of chemicals). Recently, the combined study of peristaltic flow of nanofluids17,18 has been described to examine the role of nanofluids in physiological flows.

The heat and mass transfer occur concurrently with the complicity of the fluid motion is known as double diffusion. Double diffusion has important applications in solid-state physics, chemical engineering, geophysics, oceanography, astrophysics, and biology,19 as well as many engineering applications such as natural gas storage tanks, solar ponds, metal solidification processes, and crystal manufacturing. Nowadays, researchers have been focused on double diffusive convection of peristaltic transport. The peristaltic flow of double diffusive natural convection in nanofluid studied by Noreen et al.20 The research work on double diffusion was continued in References.21-26 The porous medium can be defined as solid bodies containing pores structures. The porous medium is the ratio of pore volume to the total volume of given sample materials. The study of porous medium on peristaltic transport has been attracted the many investigators in the field of biomedicine such as gall bladder with stones, human lungs, vascular beds, kidney, small blood vessels, bile duct, and also heat generation in industries.10,27-32

Limited works are found during the review literature on magnetic field combined with the double diffusive convection on peristaltic flow. So, in the current study, we have considered the magnetic field on peristalsis with double diffusive convection flow. Magnetic field is gained considerable important applications in bioengineering and industry. The important applications of magnetic field are in aerodynamic heating process, fluid droplet sprays, cancer tumor, electrostatic precipitation, removal blockage in the arteries, bleeding reduction in case of surgery, and hyperthermia. The recent research work on MHD.33-36

In all above-mentioned investigations reveal that the role of magnetic field and thermal radiation effects on double diffusive convection of nanofluid has not yet been studied in literature. Keeping in this mind, the aim of the current study is focused on the influence of thermal radiation and magnetic field on peristaltic flow with double diffusive convection through porous medium. Double diffusive convection problems have so many practical applications in oceanography, geophysics, biology, and astrophysics. Heat transfer rate is controlling by applying the radiation. The basic governing equations are highly nonlinear, which are solved by using Homotopy analysis method (HAM)37,38 and analysis of the embedded parameters on velocity, pressure rise, energy, solutal (species) concentration, and nanoparticle volume fraction is shown in the form of graphs.

2 Mathematical Analysis

We have considered the two-dimensional peristaltic flow of uniform thickness \(d_1 + d_2\) on asymmetric channel. The fluid flow in the channel walls is produced, when the sinusoidal waves of the small amplitudes \(a_1\) and \(b_1\) propagates the speed of the channel walls. \(c\) is the constant speed of the channel (Figure 1).

The physical model of the channel is defined as

\[
\eta' = h'_{1} = d_1 + a_1 \cos \left( (\zeta' - ct') \frac{2\pi}{\lambda} \right),
\]

\[
\eta' = h'_{2} = -d_2 - b_1 \cos \left( \phi + (\zeta' - ct') \frac{2\pi}{\lambda} \right),
\]
Amplitudes of the waves are $a_1$ and $b_1$, $\lambda$ is the wavelength of channel walls, $\phi$ is the phase difference, considering the cartesian coordinate system $(\xi', \eta')$, where $\xi'$ and $\eta'$ are perpendicular to each other. Here, $d_1$ and $d_2$ are satisfies the below condition.

$$a_1^2 + 2a_1b_1 \cos \phi + b_1^2 \leq (d_1^2 + d_2^2).$$

(3)

Vector form of the velocity field $V$ is given as,

$$V = (U'(\xi', \eta', t'), V'(\xi', \eta', t')).$$

(4)

where $U'(\xi', \eta', t')$ and $V'(\xi', \eta', t')$ are the velocity components.

The radiative heat flux $q_r$ can be written as

$$q_r = -16\sigma^* T_1^3 \frac{\partial T'}{\partial \eta'}.$$  

(5)

In the above equation, $k^*$ is the Rosseland mean absorption coefficient and $\sigma^*$ denotes the Stefan-Boltzmann constant. Considering the nanofluid flow temperature is very small, therefore the term $T_1^3$ is a linear function of temperature.

The basic governing equations describing the peristaltic flow patterns for the nanofluid are as follows:

$$\frac{\partial U'}{\partial \xi'} + \frac{\partial V'}{\partial \eta'} = 0,$$

(6)

$$\rho_f \left( \frac{\partial U'}{\partial t'} + U' \frac{\partial U'}{\partial \xi'} + V' \frac{\partial U'}{\partial \eta'} \right) = -\frac{\partial p'}{\partial \xi'} + \mu \left( \frac{\partial^2 U'}{\partial \xi'^2} + \frac{\partial^2 U'}{\partial \eta'^2} \right) + \rho_f g (\phi' - \phi_0') + \rho_f g (T' - T_0') - g(\rho_p - \rho_0)(F' - F_0') - \sigma B_0^2 U' - \frac{\mu}{K_0} U',$$

(7)

$$\rho_f \left( \frac{\partial V'}{\partial t'} + U' \frac{\partial V'}{\partial \xi'} + V' \frac{\partial V'}{\partial \eta'} \right) = -\frac{\partial p'}{\partial \eta'} + \mu \left( \frac{\partial^2 V'}{\partial \xi'^2} + \frac{\partial^2 V'}{\partial \eta'^2} \right),$$

(8)

$$(\rho c)_f \left( \frac{\partial T'}{\partial t'} + U' \frac{\partial T'}{\partial \xi'} + V' \frac{\partial T'}{\partial \eta'} \right) = k_T \left( \frac{\partial^2 T'}{\partial \xi'^2} + \frac{\partial^2 T'}{\partial \eta'^2} \right) + \frac{D_T \alpha C_p}{C_s} \left( \frac{\partial^2 \phi'}{\partial \xi'^2} + \frac{\partial^2 \phi'}{\partial \eta'^2} \right) - \frac{\partial q_r}{\partial \eta'}$$

$$+ (\rho c)_p D_B \left( \frac{\partial F'}{\partial \xi'} + \frac{\partial F'}{\partial \eta'} \right) + (\rho c)_p D_T \left( \frac{\partial T'}{\partial \xi'} \right)^2 + \left( \frac{\partial T'}{\partial \eta'} \right)^2 \right].$$

(9)

$$\frac{\partial \phi'}{\partial t'} + U' \frac{\partial \phi'}{\partial \xi'} + V' \frac{\partial \phi'}{\partial \eta'} = \frac{\partial^2 \phi'}{\partial \xi'^2} + \frac{\partial^2 \phi'}{\partial \eta'^2}.$$  

(10)
\[
\frac{\partial F'}{\partial t'} + U' \frac{\partial F'}{\partial \xi'} + V' \frac{\partial F'}{\partial \eta'} = D_B \left( \frac{\partial^2 F'}{\partial \xi'^2} + \frac{\partial^2 F'}{\partial \eta'^2} \right) + \frac{D_T}{T_m} \left( \frac{\partial^2 F'}{\partial \xi'^2} + \frac{\partial^2 F'}{\partial \eta'^2} \right),
\]

where \((\rho c)_p\) is the heat capacity of the fluid, \(D_{TC}\) is the Dufour diffusivity, \(k_T\) is the thermal conductivity of the fluid, \((\rho c)_p\) is the effective heat capacity of the nanoparticle material, \(g\) is the gravity, \(\varphi'\) is the solutal (species) concentration, \(K_0\) is permeability constant of the porous medium, and \(\alpha\) is electrically conductivity of the fluid. Furthermore, \(\rho_l\) is the effective density, \(D_T\) is thermophoresis diffusion coefficient, \(D_s\) is the solutal diffusivity, \(C_p\) is the specific heat at constant pressure, \(\alpha\) is the thermal diffusion ratio, \(C_s\) is the concentration susceptibility, \(D_{CT}\) is the Soret diffusivity, and \(T_m\) is the mean fluid temperature.

The corresponding boundary conditions

\[
\begin{align*}
\psi' &= \frac{q}{2}, \quad U' = \frac{\partial \psi'}{\partial \eta'} = -c, \quad T' = T_0', \quad \varphi' = \varphi_0', \quad F' = F_0' \quad \text{at} \quad \eta' = h_1' = d_1 + a_1 \cos \left( \frac{2\pi}{\lambda} (\xi' - ct') \right), \\
\psi' &= \frac{q}{2}, \quad U' = \frac{\partial \psi'}{\partial \eta'} = -c, \quad T' = T_1', \quad \varphi' = \varphi_1', \quad F' = F_1' \quad \text{at} \quad \eta' = h_2' = d_2 + b_1 \cos \left( \frac{2\pi}{\lambda} (\xi' - ct') + \phi \right).
\end{align*}
\]

The connection between the wave frame and laboratory frame are introduced through

\[
u' = U' - c, \quad \eta' = \eta', \quad \nu' = V', \quad \xi' = \xi' - ct'.
\]

Introducing the following nondimensional variables

\[
\begin{align*}
\psi &= \frac{\psi'}{cd_1}, \quad \xi = \frac{\xi'}{\lambda}, \quad \eta = \frac{\eta'}{d_1}, \quad t = \frac{ct'}{\lambda}, \quad v = \frac{\nu'}{c}, \quad \delta = \frac{d_1}{\lambda}, \quad u = \frac{\nu'}{c}, \quad \alpha = \frac{k_c}{(\rho c)_p}, \quad \text{Pr} = \frac{u}{v}, \quad p = \frac{\rho d_1^2}{\nu c^2}, \\
\theta &= \frac{T - T_0}{T_1 - T_0}, \quad \varphi = \frac{\varphi - \varphi_0}{\varphi_1 - \varphi_0}, \quad \gamma = \frac{F - F_0}{F_1 - F_0}, \quad h_1 = \frac{h_1'}{d_1}, h_2 = \frac{h_2'}{d_1}, \quad d = \frac{d_1}{d_1}, \quad \text{Rd} = 16 \frac{\gamma T_1^3}{\beta k_c k_f}, \\
a &= \frac{a_1}{d_1}, \quad b = \frac{b_1}{d_1}, \quad \text{Gr}_T = \frac{\rho_f d_1^2 (T_1 - T_0)}{\nu c^2}, \quad \text{Gr}_c = \frac{\rho_f d_1^2 (\varphi_1 - \varphi_0)}{\nu c^2}, \quad \text{Da} = \frac{k_c}{d_1}, \\
\text{Gr}_F &= \frac{(\rho_f - \rho_l)d_1^2 (F_1 - F_0)}{\mu c^2}, \quad \text{Re} = \frac{\rho_f c d_1}{\mu}, \quad \text{Nt} = \frac{D_{CT} (T_1 - T_0)}{(\rho c)_p u T_m}, \quad \text{N}_{TC} = \frac{D_{TC} (T_1 - T_0)}{(\rho c)_p u T_m}, \quad \text{f} = \frac{q}{cd_1},
\end{align*}
\]

where \(M\) is the magnetic field, \(\delta\) is the dimensionless wave number, \(\theta\) is the dimensionless temperature, \(\varphi\) is the nanoparticle volume fraction, and the stream function taken as \(v = -\delta \frac{\partial \psi}{\partial \xi}\) and \(u = \frac{\partial \psi}{\partial \eta}\).

By using Equation (13), Equations (6) to (12) can be written as

\[
\frac{\partial p}{\partial \xi} = \frac{\partial^2 u}{\partial \eta^2} + \text{Gr}_T \varphi + \text{Gr}_c \varphi - \left( M^2 + \frac{1}{\text{Da}} \right) u, \quad (14)
\]

\[
\frac{\partial p}{\partial \eta} = 0, \quad (15)
\]

\[
\frac{\partial^3 \psi}{\partial \eta^3} + \text{Gr}_T \frac{\partial \theta}{\partial \eta} + \text{Gr}_c \frac{\partial \varphi}{\partial \eta} - \text{Gr}_F \frac{\partial \gamma}{\partial \eta} - \left( M^2 + \frac{1}{\text{Da}} \right) \frac{\partial \psi}{\partial \eta} = 0, \quad (16)
\]

\[
\frac{\partial^2 \theta}{\partial \eta^2} + \text{Nb} \text{Pr} \frac{\partial \theta}{\partial \eta} + \text{N}_{TC} \text{Pr} \frac{\partial^2 \varphi}{\partial \eta^2} + \text{N}_{CT} \text{Pr} \left( \frac{\partial \theta}{\partial \eta} \right)^2 + \text{Rd} \frac{\partial^2 \theta}{\partial \eta^2} = 0, \quad (17)
\]

\[
\frac{\partial^2 \varphi}{\partial \eta^2} + \text{N}_{CT} \frac{\partial \varphi}{\partial \eta} = 0. \quad (18)
\]
\[ \frac{\partial^2 \gamma}{\partial \eta^2} + \frac{N_t}{N_b} \frac{\partial^2 \theta}{\partial \eta^2} = 0. \] (19)

Boundary can be reduces to
\[ \begin{align*}
\psi &= \frac{f^*}{2}, \quad \frac{\partial \psi}{\partial \eta} = -1, \quad \theta = 0, \quad \varphi = 0, \quad \gamma = 0 \quad \text{at} \quad \eta = h_1 = a \cos[2\pi(\zeta - t)], \\
\psi &= -\frac{f^*}{2}, \quad \frac{\partial \psi}{\partial \eta} = -1, \quad \theta = 1, \quad \varphi = 1, \quad \gamma = 0 \quad \text{at} \quad \eta = h_2 = -d - b \cos[2\pi(\zeta - t) + \phi]
\end{align*} \] (20)

Here, we have considering \( f^* \) is the mean flow over a period is
\[ \Theta = f^* + 1, \quad f^* = \int_{h_1}^{h_2} \frac{\partial \psi}{\partial \eta} \, d\eta, \]
where \( \Theta = \frac{Q}{cd_1} \) and \( f^* = \frac{Q}{cd_1} \).

The coupled partial differential Equations (14) to (19) with the boundary conditions (20) are solved by using HAM with symbolic software Mathematica and the analysis of the present method of solution is mentioned the below section.

### 3 | METHOD OF SOLUTION

The governing Equations (14) to (19) are evaluated by Homotopy Analysis Method (HAM). This method is an analytical technique, which can be used to compute the nonlinear problems that contains small and large physical parameters (solution is obtained in terms of convergent series solutions). The method provides great freedom to select the initial approximations and auxiliary linear operators. By using this, any complicated nonlinear problems can be transformed into linear subproblems.

Initial approximations are chosen as follows:
\[ \psi_0(\eta) = \frac{\eta^2(3f^*(h_2 - h_1) - 2f^* \eta + h^4 - h_2 \eta)}{(h_2 - h_1)^3}, \] (22)
\[ \theta_0(\eta) = 1 + \frac{N_{TC} Pr}{(1 + Pr Rd)} \left( \frac{\eta - h_2}{h_1 - h_2} \right), \] (23)
\[ \varphi_0(\eta) = \frac{h_1 - \eta}{h_1 - h_2}, \] (24)
\[ \gamma_0(\eta) = \frac{h_1 - \eta}{h_1 - h_2}. \] (25)

Furthermore, the auxiliary linear operator of the problem is taken as
\[ L_{\psi} = \frac{\partial^4}{\partial \eta^4} - \frac{\partial^2}{\partial \eta^2}, \quad L_{\theta} = \frac{\partial^2}{\partial \eta^2}, \quad L_{\varphi} = \frac{\partial^2}{\partial \eta^2}, \quad L_{\gamma} = \frac{\partial^2}{\partial \eta^2}, \] (26)
which satisfies the properties
\[ L_{\psi} \left[ C_1 + C_2 \eta + C_3 \frac{\eta^2}{2} + C_4 \frac{\eta^3}{6} \right] = 0, \quad L_{\theta} [C_5 + C_6 \eta] = 0, \quad L_{\varphi} [C_7 + C_8 \eta] = 0, \quad L_{\gamma} [C_9 + C_{10} \eta] = 0. \] (27)

According to the methodology, the zeroth-order deformations of the problems are
\[(1 - q)L_w[\psi(\eta, q) - \psi_0(\eta, q)] = qH_w h_\psi N_w[\psi(\eta, q), \theta(\eta, q), \varphi(\eta, q), \gamma(\eta, q)],\]  
(28)

\[(1 - q)L_\theta[\theta(\eta, q) - \theta_0(\eta, q)] = qH_\theta h_\psi N_\theta[\theta(\eta, q), \varphi(\eta, q), \gamma(\eta, q)],\]  
(29)

\[(1 - q)L_\psi[\varphi(\eta, q) - \varphi_0(\eta, q)] = qH_\psi h_\psi N_\psi[\varphi(\eta, q), \theta(\eta, q)],\]  
(30)

\[(1 - q)L_\gamma[\gamma(\eta, q) - \gamma_0(\eta, q)] = qH_\gamma h_r N_\gamma[\gamma(\eta, q), \theta(\eta, q)],\]  
(31)

In the above equations, \( q \in [0, 1] \) is the embedded parameter, \( h_\psi, h_\theta, h_\varphi, \) and \( h_r \) are the nonzero auxiliary linear parameters, \( L_w, L_\theta, L_\psi, \) and \( L_r \) are the auxiliary linear operators, and \( H_w, H_\theta, H_\varphi, \) and \( H_\gamma \) are the auxiliary functions. The nonlinear operators \( N_\psi, N_\theta, N_\varphi, \) and \( N_r \) are written as.

\[
N_\psi[\psi(\eta, q), \theta(\eta, q), \varphi(\eta, q), \gamma(\eta, q)] = \frac{\partial^4 \psi(\eta, q)}{\partial \eta^4} - \left( M^2 + \frac{1}{Da} \right) \frac{\partial^2 \psi(\eta, q)}{\partial \eta^2} \\
+ Gr_T \frac{\partial \theta(\eta, q)}{\partial \eta} + Gr_C \frac{\partial \varphi(\eta, q)}{\partial \eta} - Gr_F \frac{\partial \gamma(\eta, q)}{\partial \eta},
\]  
(32)

\[
N_\theta[\theta(\eta, q), \gamma(\eta, q), \varphi(\eta, q)] = \frac{\partial^2 \theta(\eta, q)}{\partial \eta^2} + Pr N b \frac{\partial \theta(\eta, q)}{\partial \eta} \frac{\partial \gamma(\eta, q)}{\partial \eta} + Pr N t \left( \frac{\partial \theta(\eta, q)}{\partial \eta} \right)^2 \\
+ Rd \frac{\partial^2 \theta(\eta, q)}{\partial \eta^2} + N r C Pr \frac{\partial^2 \varphi(\eta, q)}{\partial \eta^2},
\]  
(33)

\[
N_\varphi[\varphi(\eta, q), \theta(\eta, q)] = \frac{\partial^2 \varphi(\eta, q)}{\partial \eta^2} + N c T \frac{\partial^2 \theta(\eta, q)}{\partial \eta^2},
\]  
(34)

\[
N_r[\gamma(\eta, q), \theta(\eta, q)] = \frac{\partial^2 \gamma(\eta, q)}{\partial \eta^2} + N t \frac{\partial^2 \theta(\eta, q)}{\partial \eta^2}.
\]  
(35)

Differentiating zeroth-order deformation \( n \)-times and setting \( q = 0 \), we obtain \( n \)-th order deformation equations

\[
L_w[\psi_n(\eta) - \xi_n \psi_{n-1}(\eta)] = h_\psi R_{n,\psi}(\eta),
\]  
(36)

\[
L_\theta[\theta_n(\eta) - \xi_n \theta_{n-1}(\eta)] = h_\theta R_{n,\theta}(\eta),
\]  
(37)

\[
L_\psi[\varphi_n(\eta) - \xi_n \varphi_{n-1}(\eta)] = h_\psi R_{n,\varphi}(\eta),
\]  
(38)

\[
L_r[\gamma_n(\eta) - \xi_n \gamma_{n-1}(\eta)] = h_r R_{n,r}(\eta),
\]  
(39)

where

\[
R_{n,\psi}(\eta) = \left. \frac{1}{(n - 1)!} \frac{\partial^{n-1} N_\psi[\psi(\eta, q), \theta(\eta, q), \varphi(\eta, q), \gamma(\eta, q)]}{\partial q^{n-1}} \right|_{q=0},
\]  
(40)

\[
R_{n,\theta}(\eta) = \left. \frac{1}{(n - 1)!} \frac{\partial^{n-1} N_\theta[\theta(\eta, q), \gamma(\eta, q), \varphi(\eta, q)]}{\partial q^{n-1}} \right|_{q=0},
\]  
(41)

\[
R_{n,\varphi}(\eta) = \left. \frac{1}{(n - 1)!} \frac{\partial^{n-1} N_\varphi[\varphi(\eta, q), \theta(\eta, q)]}{\partial q^{n-1}} \right|_{q=0},
\]  
(42)
Using Taylor series expansion, the equation of \( \psi(\eta, q), \theta(\eta, q) \varphi(\eta, q), \) and \( \gamma(\eta, q) \) with embedding parameter \( q \) can be written as

\[
\psi(\eta, q) = \psi_0(\eta) + \sum_{n=1}^{\infty} \psi_n(\eta) q^n, \quad \psi_n(\eta) = \frac{1}{n!} \frac{\partial^n \psi(\eta, q)}{\partial q^n}, \quad (44)
\]

\[
\theta(\eta, q) = \theta_0(\eta) + \sum_{n=1}^{\infty} \theta_n(\eta) q^n, \quad \theta_n(\eta) = \frac{1}{n!} \frac{\partial^n \theta(\eta, q)}{\partial q^n}, \quad (45)
\]

\[
\varphi(\eta, q) = \varphi_0(\eta) + \sum_{n=1}^{\infty} \varphi_n(\eta) q^n, \quad \varphi_n(\eta) = \frac{1}{n!} \frac{\partial^n \varphi(\eta, q)}{\partial q^n}, \quad (46)
\]

\[
\gamma(\eta, q) = \gamma_0(\eta) + \sum_{n=1}^{\infty} \gamma_n(\eta) q^n, \quad \gamma_n(\eta) = \frac{1}{n!} \frac{\partial^n \gamma(\eta, q)}{\partial q^n}, \quad (47)
\]

\[
\xi_n = \begin{cases} 
0, & n \leq 1 \\
1, & n > 1.
\end{cases} \quad (48)
\]

The above solutions are easily found coupled equations together with boundary conditions. The methodology of the given method, the solutions of equations (14) to (19) are written as follows.

### 3.1 Convergence analysis of the HAM solution

The expression \( \psi(h_1), \psi(h_2), \theta(h_1), \theta(h_2), \varphi(h_1), \varphi(h_2), \gamma(h_1), \) and \( \gamma(h_2) \) contains the auxiliary parameters \( h_\psi, h_\theta, h_\varphi, \) and \( h_\gamma. \) The auxiliary linear parameters are adjusting and controlling the homotopic solutions. Plotting the \( h \) curves at 30th-order approximation to find the appropriate values of \( h_\psi, h_\theta, h_\varphi, \) and \( h_\gamma. \) (Figure 2A,B). In this problem, we have chosen the convergence solution as \( h_\psi = h_\theta = h_\varphi = h_\gamma = -0.7. \)

**FIGURE 2** A and B, \( h \) curves for the function of \( \psi(h_1), \psi(h_2), \theta(h_1), \theta(h_2), \varphi(h_1), \varphi(h_2), \gamma(h_1), \) and \( \gamma(h_2) \) at 30th-order approximations when \( b = 0.5, Da = 0.5, t = 0.2, a = 0.5, Pr = 7.0, \zeta = 0.2, d = 1.5, \phi = 0.3, Gr_\psi = 0.8, Gr_\theta = 0.8, Gr_\varphi = 0.8, N_\text{CT} = 0.5, Nt = 0.6, N_\text{TC} = 0.5, M = 0.5, Nb = 0.6.**
4 | DISCUSSION

The nonlinear partial differential equations are solved by utilizing the HAM. In the present article, we have used Mathematica as a computational software. The solutions of the HAM are obtained through code of the software Mathematica for velocity, pressure rise, temperature, nanoparticles volume fraction, and solutal (species) concentration profiles, and graphical results are plotted in Origin.

This section represents the analysis of different values of physical parameters for velocity, pressure rise, temperature, solutal concentration, and nanoparticle volume fraction.

4.1 | Velocity distribution

The effects of Hartmann number $M$, Darcy number $Da$, thermal Grashof number $Gr_T$, solutal Grashof number $Gr_C$, and nanoparticle Grashof number $Gr_F$ on velocity profile are representing through the Figures 3-7. Figure 3 shows a decreasing of the velocity profile when Hartmann number $M$ is enhanced. Physically, Lorentz force acts like retarding force for the fluid flow; hence, the magnetic force is slowing down the fluid flow. Such results found useful applications in medicine, that is, it regulates the blood flow and avoid the blood clotting. The influence of Darcy number $Da$ on velocity profile is presented in Figure 4. By enhancing Darcy number $Da$, then the velocity profile is increased. It is observed in Figure 4 that

**Figure 3** Velocity profile for different values of $M$ when $Gr_T = 0.8$, $Gr_C = 0.8$, $Gr_F = 0.8$, $Nt = 0.5$, $Nt = 0.5$, $Nb = 0.6$, $Pr = 7.0$, $\zeta = 1.0$, $Rd = 0.5$, and $\phi = 0.5$

**Figure 4** Velocity profile for different values of $Da$ when $Gr_T = 0.8$, $Gr_C = 0.8$, $Gr_F = 0.8$, $Nt = 0.5$, $Nt = 0.5$, $Nb = 0.6$, $M = 0.5$, $Pr = 7.0$, $\zeta = 1.0$, $Rd = 0.5$, and $\phi = 0.5
Figure 5  Velocity profile for different values of $Gr_T$
when $Gr_C = 0.8$, $Gr_F = 0.8$, $N_{CT} = 0.5$, $N_{TC} = 0.5$, $Nt = 0.6$, $Da = 0.5$, $Nb = 0.6$, $M = 0.5$, $Pr = 7.0$, $\zeta = 1.0$, $Rd = 0.5$, and $\phi = 0.5$

Figure 6  Velocity profile for different values of $Gr_C$
when $Gr_T = 0.8$, $Gr_F = 0.8$, $N_{CT} = 0.5$, $N_{TC} = 0.5$, $Nb = 0.6$, $Da = 0.5$, $M = 0.5$, $Nt = 0.6$, $Pr = 7.0$, $\zeta = 1.0$, $Rd = 0.5$, and $\phi = 0.5$

Figure 7  Velocity profile for different values of $Gr_F$
when $Gr_T = 0.8$, $Da = 0.5$, $N_{CT} = 0.5$, $Gr_C = 0.8$, $N_{TC} = 0.5$, $Nb = 0.6$, $M = 0.5$, $Nt = 0.6$, $Pr = 7.0$, $\zeta = 1.0$, $Rd = 0.5$, and $\phi = 0.5$
the velocity increases in the Darcy number $Da$. Physically, Darcy number $Da$ will provide less resistance to the fluid motion. Figures 5 and 6 are plotted to show the effects of thermal Grashof number $Gr_T$, solutal Grashof number $Gr_C$ on the velocity. Opposite behavior can be observed in both $Gr_T$ and $Gr_C$. The magnitude of the velocity decreases with an increasing the thermal Grashof number $Gr_T$, and also the velocity profile increases, when higher values of solutal Grashof number $Gr_C$. Due to facts that the thermal Grashof number $Gr_T$ satisfies the relative influence of thermal buoyancy force and viscous hydrodynamic force. Thus, increasing of $Gr_T$ cause the viscosity reduces. The opposite behavior can be seen in the case of solutal Grashof number $Gr_C$. Figure 7 shows the effect of nanoparticle Grashof number $Gr_F$ on velocity profile. It is obvious that the velocity profile decreases with increasing the $Gr_F$. It is due to the fact that $Gr_F$ increases the viscosity of nanoparticles reduces, which leads to reduction in velocity for $M = 0.5$ and $Da = 0.5$.

4.2 | Pressure rise

Figures 8-12 are plotted to the different physical parameters are Hartmann number $M$, Darcy number $Da$, thermal Grashof number $Gr_T$. Solutal Grashof number $Gr_C$ and nanoparticle Grashof number $Gr_F$ on pressure rise $\Delta p$ vs flow rate $Q$. From Figures 8 and 9, we observed that for higher values of Hartmann number $M$, the pressure rise is enhanced, whereas decay in the pressure rise with higher values of Darcy number is observed in Figure 9. Figure 10 shows the pressure rise for different values of nanoparticle thermal Grashof number $Gr_T$. In this figure, it is noticed that the pressure rise increases with a
higher value of Grashof number \( Gr_T \). Figures 11 and 12 show the pressure rise for different values of nanoparticle thermal solutal Grashof number \( Gr_C \) and Grashof number \( Gr_T \). The same effect of solutal Grashof number \( Gr_C \) and nanoparticle Grashof number \( Gr_F \) can also be observed in Figures 11 and 12. That is, the pressure rise decreases with an increase in solutal Grashof number \( Gr_C \) and nanoparticle Grashof number \( Gr_F \). Physically, it is valid because concentration of nanoparticles in the fluid increases, which cause decreasing the pressure.

4.3 | Temperature distribution

Figures 13-17 depicted for different values of Brownian motion parameter \( Nb \), thermophoresis parameter \( Nt \), Dufour effect \( N_{TC} \), Soret number \( N_{CT} \) and the thermal radiation parameter \( Rd \). From Figures 13 and 14, it is observed that there is enhancement in temperature profile for different values of Brownian motion parameter \( Nb \) and thermophoresis parameter \( Nt \). The increase of Brownian motion parameter \( Nb \) cause the random motion of fluid particles that produce more heat, so there is temperature rises in the system, which can be seen in Figure 13, whereas, in Figure 14, the temperature profile increases as the fluid particles are moved away from the cold surface to hot surface by increasing the thermophoresis parameter \( Nt \). The same behavior can be observed in Dufour effect \( N_{TC} \) and Soret number \( N_{CT} \). In Figure 17, the opposite behavior can be seen, that is, the decay in the temperature profile with an increase of thermal radiation \( Rd \). It is due to
fact that the thermal radiation is inversely proportional with thermal conduction parameter \( \kappa_T \), therefore maximum heat is radiated away from the system, which leads to reduction in the heat conduction of the fluid.

### 4.4 Solutal (species) concentration distribution

Figures 18-21 are shown to examine the solutal (species) concentration profile by \( Nb, Nt, N_{CT} \), and \( N_{TC} \). Effects \( Nb \) and \( Nt \) are discussed using Figures 18 and 19. From these two figures, one can see that the solutal (species) concentration profile has the similar behavior on both \( Nb \) and \( Nt \), and solutal (species) concentration decreases with an enhancing of \( N_{CT} \) (Figure 20) and also the same behavior \( N_{TC} \) in Figure 21.

### 4.5 Nanoparticle volume fraction distribution

Figures 22-25 are depicted the nanoparticle concentration visualizations for the influences of \( Nb, Nt, N_{TC} \), and \( N_{CT} \) on peristaltic flow in the presence of nanofluids. Figure 22 shows that the nanoparticle volume fraction of fluid increases with increase in Brownian motion parameter because the temperature distribution is large in the case of nanofluids, which can lead to the distribution of the system. However, the opposite results are seen in the case of thermophoresis parameter.
**FIGURE 14** Temperature profile for different values of $N_t$ when $N_{CT} = 0.5$, $N_{TC} = 0.5$, $Nb = 0.6$, $Pr = 7.0$, and $Rd = 0.5$

**FIGURE 15** Temperature profile for different values of $N_{CT}$ when $N_{TC} = 0.5$, $N_t = 0.6$, $Nb = 0.6$, $Pr = 7.0$, and $Rd = 0.5$

**FIGURE 16** Temperature profile for different values of $N_{TC}$ when $N_{CT} = 0.5$, $N_t = 0.6$, $Nb = 0.6$, $Pr = 7.0$, and $Rd = 0.5
**Figure 17** Temperature profile for different values of $Rd$ when $N_{CT} = 0.5$, $N_{TC} = 0.5$, $Nt = 0.6$, $Nb = 0.6$, and $Pr = 7.0$.

**Figure 18** Solutal (species) concentration profile for different values of $Nb$ when $N_{CT} = 0.5$, $N_{TC} = 0.5$, $Nt = 0.6$, $Pr = 7.0$, and $Rd = 0.5$.

**Figure 19** Solutal (species) concentration profile for different values of $Nt$ when $N_{CT} = 0.5$, $Nb = 0.6$, $N_{TC} = 0.5$, $Pr = 7.0$, and $Rd = 0.5$. 
**FIGURE 20** Solutal (species) concentration profile for different values of $N_{CT}$ when $N_{TC} = 0.5$, $Nt = 0.6$, $Nb = 0.6$, $Pr = 7.0$, and $Rd = 0.5$

**FIGURE 21** Solutal (species) concentration profile for different values of $N_{TC}$ when $N_{CT} = 0.5$, $Nt = 0.6$, $Nb = 0.6$, $Pr = 7.0$, and $Rd = 0.5$

**FIGURE 22** Nanoparticle volume fraction profile for different values of $Nb$ when $N_{CT} = 0.5$, $N_{TC} = 0.5$, $Nt = 0.6$, $Pr = 7.0$, and $Rd = 0.5
FIGURE 23 Nanoparticle volume fraction profile for different values of $N_t$ when $N_{CT} = 0.5$, $N_{TC} = 0.5$, $Nb = 0.6$, $Pr = 7.0$, and $Rd = 0.5$

FIGURE 24 Nanoparticle volume fraction profile for different values of $N_{CT}$ when $N_{TC} = 0.5$, $N_t = 0.6$, $Nb = 0.6$, $Pr = 7.0$, and $Rd = 0.5$

FIGURE 25 Nanoparticle volume fraction profile for different values of $N_{TC}$ when $N_{CT} = 0.5$, $N_t = 0.6$, $Nb = 0.6$, $Pr = 7.0$, and $Rd = 0.5$
Figure 23 represents the nanoparticle volume fraction, and the nanoparticle volume fraction reducing when $Nt$ is enhanced. Figure 24 described the influence of $N_{CT}$ on nanoparticle volume fraction; here, nanoparticle volume fraction increases when Soret parameter $N_{CT}$ is increased. Figure 25 shows that the influence of $N_{TC}$ on nanoparticle volume fraction of fluid enhanced with the enhancing of the Dufour effect $N_{TC}$.

5 | CONCLUSION

The current research work analyzes the double diffusion on peristaltic flow of nanofluid in presence of porous medium, magnetic field, and thermal radiation through asymmetric channel. The important points are listed below.

- The current research work has potential in biomedical, engineering, and industrial applications.
- The behavior of relaxation to retardation time on velocity and pressure rise are opposite.
- Opposite behaviors on temperature is noted for $Nb$ and $Rd$.
- Behavior of $Gr_F$ and $Gr_C$ on pressure rise is too similar, that is, increases for $N_{TC}$ and $N_{CT}$ slightly enhanced the temperature of the wall surface.
- Opposite behavior of $Gr_F$ and $Gr_C$ on velocity profile and pressure rise profile.
- The behavior of $Gr_F$ and $Gr_C$ on pressure rise are similar.
- The similar behavior of $Nb$, $Nt$, $N_{TC}$, and $N_{TC}$ on solutal (species) concentration profile. Here, solutal concentration profile enhanced with higher values of $Nb$, $Nt$, $N_{TC}$, and $N_{CT}$.
- Opposite behavior of $Nt$ and $Nb$ on nanoparticle volume fraction profile.

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CONFLICT OF INTEREST

The authors declare no potential conflict of interest.

NOMENCLATURE

- $t'$: time (seconds)
- $D_T$: thermophoresis diffusion coefficient ($m^2/s$)
- $p'$: pressure in fixed frame (Pa)
- $h_1'$: right wall
- $Re$: Reynolds number
- $u'$, $v'$: velocity components (m/s)
- $Pr$: Prandtl number
- $h_2'$: left wall
- $T'$: temperature (K)
- $\phi'$: nanoparticle volume fraction
- $M$: Hartmann number
- $D_B$: Brownian diffusion coefficient ($m^2/s$)
- $Gr_F$: nanoparticle Grashof number
- $T_m$: fluid mean temperature (K)
- $Gr_T$: thermal Grashof number
- $F'$: solutal species concentration
- $N_{CT}$: Soret parameter
- $Nb$: Brownian motion parameter
- $Gr_C$: solutal Grashof number
Darcy number

thermophoresis diffusion parameter

stress tensor (N/m²)

AUTHOR CONTRIBUTIONS

Asha Kotnurkar Supervision-Equal; Sunitha G Investigation-Equal, Validation-Equal, Writing-review & editing-Equal.

ORCID

Asha Shivappa Kotnurkar © https://orcid.org/0000-0001-6755-7629

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