Constraints on a variable dark energy model with recent observations

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Abstract

We place, by the maximum likelihood method, constraints on a variable dark energy model with the equation of state \( w = w_0/[1 + b \ln(1 + z)]^2 \) using some recent observational data, including the new Sne Ia data from the SNLS, the size of baryonic acoustic oscillation peak from SDSS and the CMB data from WMAP3. We find that the SNLS data favor models with \( w_0 \) around \(-1\), in contrast to the Gold data set which favors a more negative \( w_0 \). By combining these three databases, we obtain that \( \Omega_m = 0.27^{+0.030}_{-0.038} \), \( w_0 = -1.11^{+0.21}_{-0.30} \) and \( b = 0.31^{+0.71}_{-0.31} \) with \( \chi^2 = 110.4 \) at the 95\% confidence level. Our result suggests that a varying dark energy model and a crossing of the \( w = -1 \) line are favored, and the present value of the equation of state of dark energy is very likely less than \(-1\).

PACS numbers: 98.80.-k, 98.80.Es

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I. INTRODUCTION

Since the Hubble diagram of Type Ia Supernovae (Sne Ia) [1] first indicated that the universe is undergoing an accelerating expansion, many works have been done, trying to explain this phenomenon. A large number of models for the cosmic acceleration are proposed by assuming the existence of an energy component with negative pressure in the universe, named dark energy, which at late times dominates the total energy density of the universe and drives its acceleration of expansion. The simplest candidate of dark energy is the cosmological constant $\lambda$ [2], with the equation of state $w = p/\rho = -1$. However, the cosmological constant has to be extremely fine-tuned in order to induce the observed cosmic expansion, and this has led many authors to use scalar fields, such as quintessence [3], phantom [4], quintom [5] and Chaplygin gas [6], as alternative models for dark energy. On the other hand there are also some other models where the observed cosmic acceleration is not driven by dark energy, such as modified gravity [7], theories with compactified extra dimensions [8], DGP model [9] and Cardassian cosmology [10] etc.

In order to determine the recent expansion history of our universe, one can also use a different approach, that is, to assume an arbitrary parametrization for the equation of state $w(z)$ for dark energy, where $z$ is the redshift. The parametrization may not be motivated by any particular fundamental physical theory and is thus "model-independent". It however needs to be designed to give a good fit to the observational data. The simplest parametrization is $w = constant$. Some other proposals, including $w(z) = w_0 + w_1 z$ [11], $w(z) = w_0 + w_1 z/(1 + z)$ [12] etc, are also made. Recently Wetterich [13] proposed an interesting phenomenological parametrization for a variable dark energy, in which the effective equation of state is expressed as:

$$w(z) = \frac{w_0}{[1 + b \ln(1 + z)]^2},$$

(1)

where $w_0$ represents the present value of the equation of state and $b$ is a positive constant characterizing the change of $w(z)$ with redshift. Apparently with this $w(z)$ there are three model parameters ($\Omega_m, w_0, b$) to be determined by the observations, where $\Omega_m = \rho_m/\rho_c$ represents the present matter density parameter and $\rho_c = 3H_0^2/8\pi G$ is the present critical density of our universe. The advantage of this parametrization over other parameterizations of a time varying equation of state, $w(z)$ is that it covers the whole available redshift range while other parameterizations proposed before cover only a restricted range of redshift [11].
Later, using Cosmic Microwave Background, Large Scale Structure, and SNe Ia data, Doran, Karwan and Wetterich [15] discussed the $w_0$ and the dark energy fraction at very high redshift $\Omega_d^e$ in this model, and found at the 95% confidence level $w_0 < -0.8$ and $\Omega_d^e < 0.03$, where $\Omega_d^e$ and $w_0$ are related to $b$ by $b = -3w_0(\ln \frac{1-\Omega_d^e}{\Omega_d^e} + \ln \frac{1-w_0}{3w_0})^{-1}$ with $\Omega_d^e$ being the present energy density of dark energy. Movahed and Rahvar [16] have used the Gold SNe Ia data [18], the position of first acoustic peak of the Cosmic Microwave Background radiation (CMB) and the size of baryonic acoustic oscillations peak to constrain this model and obtained at the 2$\sigma$ confidence level $\Omega_m = 0.27^{+0.04}_{-0.03}$, $w_0 = -1.45^{+0.65}_{-2.1}$ and $b = 1.35^{+6.30}_{-1.35}$.

Recently Asitier et al. [19] released the data of high redshift supernovae from the Supernova Legacy Survey (SNLS). In this survey the systematic uncertainties and systematic errors are reduced. It is worth noting that the SNLS data set is a better agreement with the WMAP data compared to the Gold SNe Ia set [17]. Thus, in this paper we will reexamine this variable dark energy by using the 115 new SNLS SNe Ia data, the size of baryonic acoustic oscillations peak detected in the large-scale correlation function of luminous red galaxies from Sloan Digital Sky Survey (SDSS) [20] and the CMB data obtained from the three-year WMAP result [21]. We obtain at a 95% confidence level $\Omega_m = 0.27^{+0.036}_{-0.038}$, $w_0 = -1.11^{+0.21}_{-0.30}$ and $b = 0.31^{+0.71}_{-0.31}$.

II. THE BASIC EQUATION

By using Eq. (1) and the equation of energy conservation, it is easy to obtain the evolution of density of dark energy [13, 16]

$$\rho_d = \rho_{d0}(1 + z)^{3(1 + \bar{w}(z))},$$

where $\rho_{d0}$ denotes the present density of dark energy and $\bar{w}(z) = \frac{w_0}{1 + 3(ln(1+z))}$. Since WMAP observations strongly indicate that the geometry of our universe is spatially flat [22], we will ignore the term containing curvature factor in Friedman equation. If the radiation components in universe are further ignored, we find for the Hubble parameter

$$H^2(z; \Omega_m, w_0) = H_0^2[\Omega_m(1 + z)^3 + (1 - \Omega_m)(1 + z)^{3(1 + \bar{w}(z))}],$$
where \( H_0 \) is the present Hubble constant. Meanwhile one can show that for a flat universe the Luminosity distance, \( d^L \), can be expressed as

\[
d^L(z, H_0, \Omega_m, \omega_0) = \frac{c}{H_0} (1 + z) \int_0^z \frac{dz'}{E(z', \Omega_m, \omega_0)}.
\] (4)

Here \( c \) is the velocity of light and \( E(z, \Omega_m, \omega_0) = H(z; \Omega_m, \omega_0) / H_0 \).

### III. CONSTRAINTS FROM SNE IA, SDSS AND CMB DATA

The the 115 new Sne Ia data includes 44 previously published nearby Sne Ia and 71 distant Sne Ia released recently by the Supernova Legacy Survey (SNLS) [19] which is a planned five year survey of SNe Ia with \( z < 1 \). Constraints from Sne Ia can be obtained by fitting the distance modulus \( \mu(z) \)

\[
\mu(z) = 5 \log_{10}[D^L(z)] + \mathcal{M}.
\] (5)

Here \( D^L = H_0 d^L \) and \( \mathcal{M} = M - 5 \log_{10}(H_0) \), \( M \) being the absolute magnitude of the object.

Recently Eisenstein et al. [20] successfully found the size of baryonic acoustic oscillation peak using a large spectroscopic sample of luminous red galaxy from the SDSS and obtained a parameter \( A \), which is independent of dark energy models and for a flat universe can be expressed as

\[
A = \sqrt{\Omega_m} \int_{z_1}^{z_e} \frac{dz}{E(z)} \left( \frac{1}{z_1} \int_0^{z_1} \frac{dz}{E(z)} \right)^{2/3},
\] (6)

where \( z_1 = 0.35 \) and \( A \) is measured to be \( A = 0.469 \pm 0.017 \). Using parameter \( A \) we can obtain the constraint on dark energy models from the SDSS.

For the CMB data, the shift parameter \( R \) can be used to constrain the dark energy models and it can be expressed as [23]

\[
R = \sqrt{\Omega_m} \int_0^{z_e} \frac{dz}{E(z)},
\] (7)

for a flat universe, where \( z_e = 1089 \). From the three-year WMAP result [22], the shift parameter is constrained to be \( R = 1.70 \pm 0.03 \).

In order to place limits on model parameters \( (\Omega_m, \omega_0, b) \) with the observation data, we make use of the maximum likelihood method, that is, the best fit values for these parameters can be determined by minimizing

\[
\chi^2 = \sum_i \frac{[\mu_{obs}(z_i) - \mu(z_i)]^2}{\sigma_i^2} + \frac{(A - 0.469)^2}{0.017^2} + \frac{(R - 1.70)^2}{0.03^2}.
\] (8)
For the SNLS Sne Ia data set, at a 95.4% confidence level we obtain $\Omega_m = 0.27^{+0.05}_{-0.07}$, $w_0 = -1.03^{+0.46}_{-1.42}$ and $b = 0.0^{+4.3}_{-1.42}$. These are different from the result obtained in Ref. [16] using 157 Gold Sne Ia data, where $\Omega_m = 0.01^{+0.10}_{-0.01}$, $w_0 = -1.90^{+0.75}_{-3.29}$ and $b = 6.00^{+7.35}_{-6.00}$. Apparently the SNLS data set favors models with $w_0$ around $-1$ while the Gold set favors a more negative $w_0$. Meanwhile the best fit value of $b(=0)$ for SNLS data show that a non-varying dark energy model is favored. However, if combining the SNLS Sne Ia, SDSS and CMB, we find that at a 95% confidence level $\Omega_m = 0.27^{+0.036}_{-0.038}$, $w_0 = -1.11^{+0.21}_{-0.30}$ and $b = 0.31^{+0.71}_{-0.31}$ with $\chi^2 = 110.4$. The best fit values show that a variable dark energy model is favored since $b$ is nonzero and it is very likely the present value of the equation of state is less than $-1$. In Fig. (1) the Hubble Diagram for 115 SNLS Sne Ia data set is shown with the $(\Omega_m, w_0, b) = (0.27, -1.11, 0.31)$. The contour plots of $\Omega_m$ and $w_0$ by fixing $b$ at its best fit value 0.31 are shown in Fig. (2). The contour plots of $\Omega_m$ and $b$ by fixing $w_0$ at its best fit value $-1.11$ are shown in Fig. (3). In Figs. (4) we give the evolutionary curves $w(z)$ vs $z$ with $1\sigma$ error bar based on SNLS Sne Ia+SDSS+CMB data. This figure shows graphically that SNLS Sne Ia +SDSS+CMB data favor a varying dark energy model. Comparing with the results obtained in Ref. [16], we find that in our results at the 95% confidence level stronger constraints on $w_0$ and $b$ are obtained. Meanwhile it is easy to see that we also obtained a stronger constraint on $w_0$ than that obtained in Ref. [15].

IV. CONCLUSION

In this paper we have placed constraints on a parameterized dark energy model [13] using the new SNLS Sne Ia data sets, the size of the baryonic acoustic oscillation peak from SDSS and the shift parameter from the CMB observation. It is found that a non-varying phantom dark energy model is favored if the SNLS Sne Ia data set is used in contrast to a varying dark energy when the Gold Sne Ia data is utilized [16], and different from the case of the Gold set, the model with $w_0$ around $-1$ is favored by SNLS data at a 95.4% confidence level. Combing three databases (SNLS Sne Ia, SDSS and CMB), we obtain a constraint on the model parameters $(\Omega_m, w_0, b)$, which suggests that a varying dark energy model and a crossing of the $w = -1$ line are favored [24], and the present value of the equation of state of dark energy is very likely less than $-1$. 
Acknowledgments

We would like to thank Z.H. Zhu for his helpful discussions. This work was supported in part by the National Natural Science Foundation of China under Grants No. 10375023 and No. 10575035, and the Program for NCET under Grant No. 04-0784.

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FIG. 1: The Hubble diagram for 115 SNLS Sne Ia data with the best fit parameters $(\Omega_m, w_0, b) = (0.27, -1.11, 0.31)$ obtained from the combination of SNLS, SDSS and CMB databases.
FIG. 2: The 1σ, 2σ and 3σ confidence contours for Ω_m and w_0 with b at its best fit value 0.31 from the combination of SNLS, SDSS and CMB databases.
FIG. 3: The $1\sigma$, $2\sigma$ and $3\sigma$ confidence contours for $\Omega_m$ and $b$ with $w_0$ at its best fit value $-1.11$ from the combination of Gold, SNLS, SDSS and CMB databases.

FIG. 4: The behavior of $w(z)$. The solid line plots $w(z)$ by using the best fit parameters ($w_0 = -1.11, b = 0.31$) obtained from the SNLS+SDSS+CMB data and the dotted lines are for $1\sigma$ errors. The solid line shows clearly a varying equation of state parameter.