Two Contact Binaries with Mass Ratios Close to the Minimum Mass Ratio

Kai Li1, Qi-Qi Xia1, Chun-Hwey Kim2, Shao-Ming Hu1, Di-Fu Guo1, Min-Ji Jeong2, Xu Chen1, and Dong-Yang Gao1

1 Shandong Key Laboratory of Optical Astronomy and Solar-Terrestrial Environment, School of Space Science and Physics, Institute of Space Sciences, Shandong University, Weihai, Shandong, 264209, People’s Republic of China; kaili@sdu.edu.cn
2 Department of Astronomy and Space Science, Chungbuk National University, Cheongju 361–763, Republic of Korea

Received 2020 March 6; revised 2021 September 2; accepted 2021 September 2; published 2021 November 25

Abstract

The cutoff mass ratio is under debate for contact binaries. In this paper, we present the investigation of two contact binaries with mass ratios close to the low mass ratio limit. It is found that the mass ratios of VSX J082700.8 +462850 (hereafter J082700) and 1SWASP J132829.37+555246.1 (hereafter J132829) are both less than 0.1 ($q \sim 0.055$ for J082700 and $q \sim 0.089$ for J132829). J082700 is a shallow contact binary with a contact degree of $\sim 19\%$, and J132829 is a deep contact system with a fill-out factor of $\sim 70\%$. The $O-C$ diagram analysis indicated that the two systems manifested long-term period decreases. In addition, J082700 exhibits a cyclic modulation which is more likely resulting from the Applegate mechanism. In order to explore the properties of extremely low mass ratio contact binaries (ELMRCBs), we carried out a statistical analysis on contact binaries with mass ratios of $q \lesssim 0.1$ and discovered that the values of $J_{\text{spin}}/J_{\text{orb}}$ of three systems are greater than $1/3$. Two possible explanations can interpret this phenomenon. One explanation is that some physical processes, unknown to date, are not considered when Hut presented the dynamic stability criterion. The other explanation is that the dimensionless gyration radius ($k$) should be smaller than the value we used ($k^2 = 0.06$). We also found that the formation of ELMRCBs possibly has two channels. The study of evolutionary states of ELMRCBs reveals that their evolutionary states are similar with those of normal W UMa contact binaries.

Unified Astronomy Thesaurus concepts: Close binary stars (254); Eclipsing binary stars (444); Stellar evolution (1599); Contact binary stars (297); Mass ratio (1012)

Supporting material: data behind figure

1. Introduction

Contact binaries which usually contain two late-type Roche lobe filling components are very common in the field, including open and globular clusters. It is estimated that about one of every 500 F, G, and K main-sequence stars is a contact binary (Rucinski 2002). Although contact binaries have been analyzed for almost 80 yr, there are still many issues to be settled, such as the formation, the evolution, the ultimate fate (e.g., Bradstreet & Guinan 1994; Eggleton & Kiseleva-Eggleton 2002; Qian 2003; Yakut & Eggleton 2005; Stepien 2006), the $0.22$ day short period limit (e.g., Rucinski 1992; Stepien 2006; Jiang et al. 2012; Qian et al. 2015; Li et al. 2019b), the low mass ratio cutoff (e.g., Rasio 1995; Li & Zhang 2006; Arbutina 2007, 2009; Jiang et al. 2010), and the magnetic activities (e.g., Applegate 1992; Qian et al. 2007; Zhou et al. 2016a; Pi et al. 2017). In order to solve these problems, we have to observe and study a large number of such systems.

The minimum mass ratio was first determined to be $q_{\text{min}} \sim 0.09$ by Rasio (1995) when neglecting the rotation of the less massive secondary component, and this value was dropped down to $q_{\text{min}} \sim 0.076$ when considering the rotation of the secondary component and the dimensionless gyration radii $k_1^2 = k_2^2 = 0.06$ (Li & Zhang 2006). Arbutina (2007, 2009) derived the minimum value of mass ratio to be around $0.094 \sim 0.109$ by assuming a radiative primary and a fully convective secondary, and this value was decreased to be $0.070 \sim 0.074$ when taking into account the effect of rotation. Jiang et al. (2010) suggested that the dimensionless gyration radii are decreasing with increasing mass and age, resulting in a minimum mass ratio different from $0.05$ to $0.105$. Very recently, based on a statistical study of 46 deep, low mass ratio contact binaries, Yang & Qian (2015) determined a minimum mass ratio of about $0.044$. Ultimately, such contact binaries are proposed to coalesce into a fast single-rotation star. It can be triggered not only when the spin angular momentum and orbital angular momentum meet the Darwin instability (i.e., $J_{\text{rot}} > \frac{1}{2}J_{\text{orb}}$) but also when the contact degree exceeds $70\%$ or $86\%$ (Hut 1980; Rasio 1995; Eggleton & Kiseleva-Eggleton 2001; Li & Zhang 2006). Qian et al. (2005a) first put forward the concept about “deep ($f \geq 50\%$), low mass ratio ($q \leq 0.25$) overcontact binaries” and proposed that this type of star is likely to be the progenitor of a blue straggler/FK Com-type star. At present, a lot of such type of binaries have been analyzed (e.g., Yang et al. 2009; Qian et al. 2011; Liao et al. 2017). However, there is only one target, V1309 Sco, whose merging progress has been observed (Tylenda et al. 2011), and it is making a blue straggler (Ferreira et al. 2019). Zhu et al. (2016) derived that V1309 Sco is a very deep contact binary ($f = 89.5\%$) with an extremely low mass ratio ($q \sim 0.095$) before the merge. Therefore, in order to comprehend the low mass ratio cutoff and search for progenitors of the merger, we should observe and analyze more contact binaries with mass ratios less than $0.1$. In this paper, we present the observations and investigations of two such systems, J082700 and J132829.

J082700 was first classified as a W UMa type star by G. Srdoc in 2010 (The International Variable Star Index, V SX$^3$). The period and amplitude of the light variation were determined to be $0.2717$ days and $0.157$ mag in the $B$ bandpass. J132829 was first classified as a W UMa type star by Télescope à Action Rapide pour les Objets Transitoires (T AROT; Damerdji et al. 2007), and the period was derived to be $0.384718$ day.  

https://www.aavso.org/vsx/index.php?view=detail.top&oid=251475
Recently, the orbital period and amplitude of the light variation were updated to be 0.2771582 day and 0.14 mag in the $V$ bandpass for J082700 and 0.3847052 day and 0.13 mag in the $V$ bandpass for J132829 by the All-Sky Automated Survey for SuperNovae (ASAS-SN; Shappee et al. 2014; Jayasinghe et al. 2018). In this paper, the multiple color light curves of the two systems were presented, then their orbital period changes were studied, and a statistic on contact binaries with mass ratios $q < 0.1$ is shown.

2. Observations and Data Reduction

The multiple color light curves of J082700 and J132829 were both observed in 2019 by using 1.0 m class telescopes. J082700 was observed using $VR_{I_c}$ filters by the Weihai Observatory 1.0 m telescope of Shandong University (WHOT; Hu et al. 2014) on March 23, 25, and 31 (the seeing was around 1.7″). J132829 was observed using $BVR_{I_c}$ filters by an 85 cm telescope at the Xinglong Station of National Astronomical Observatories (NAOs85cm) in China on March 15 (the seeing was around 4.5″) and by WHOT on April 25 and May 2 (the seeing was around 2.0″). During the observations, the PIXIS 2048B CCD camera and the Andor DZ936 CCD camera were equipped on WHOT and NAOs85cm, respectively. The scale of each image is approximately 0.35 per pixel for WHOT, resulting in a field of view of about $12'\times 12'$, and the scale of each image is approximately 0.94 per pixel for NAOs85cm, resulting in a field of view of about $32'\times 32'$. During the observations, J082700 was exposed for 80, 50, and 30 second in the $V$, $R$, and $I_c$ bands, respectively. The exposure times of J132829 were 50 s, 30 s, 20 s, and 15 s in the $B$, $V$, $R$, and $I_c$ bands, respectively, when using NAOs85cm, while 70 s, 40 s, 25 s, and 20 s in the $B$, $V$, $R$, and $I_c$ bands, respectively, when using WHOT. All images of the two stars were reduced by using the IRAF package. During the reduction, the comparison and check stars are GSC 3415-2327 ($J = 12.669$ mag, $H = 12.209$ mag, and $K = 12.102$ mag) and GSC 3415-2313 ($J = 12.852$ mag, $H = 12.583$ mag, and $K = 12.520$ mag) for J082700, while those for J132829 are GSC 3853-0012 ($J = 10.335$ mag, $H = 9.884$ mag, and $K = 9.828$ mag) and GSC 3853-0279 ($J = 10.616$ mag, $H = 10.226$ mag, and $K = 10.156$ mag).

The aperture photometry method was applied to reduce the observational data of J082700. When reducing the NAOs85cm data of J132829, we used the point-spread function (PSF) photometry method because there is a very close companion star (2MASS J13282957+5552471) which is about 7″ from J132829, and during the observations the seeing was around 4.5″, which is more than half of the distance between the two stars. For the WHOT data, we found that the photometric results derived by the aperture photometry method are much better than those derived by the PSF photometry method (the seeing during the observations was around 2.0″, which is less than one third of the distance), and we used the aperture photometry method to reduce the WHOT data of J132829. After the reduction, we determined one set of complete light curves for J082700 and two sets of complete light curves for J132829. All the light curves are shown in Figure 1. The photometric accuracy for the light curve of J082700 is around 0.006 mag in $V$ band, 0.005 mag in $R$, and 0.008 mag in $I_c$; that for the NAOs85cm light curve of J132829 is around 0.009 mag in $B$, 0.007 mag in $V$ band, 0.008 mag in $R$, and 0.007 mag in $I_c$; and that for the WHOT light curve of J132829 is around 0.006 mag in $B$ band, 0.005 mag in $V$ band, 0.005 mag in $R$, and 0.007 mag in $I_c$. Due to the light curve of J082700, we found that the amplitude of light variation in $V$ band is definitely 0.14 mag. Due to the light curves of J132829, the $V$ band amplitude of light variability is about 0.29 mag. The 0.13 mag light variability determined by ASAS-SN (Shappee et al. 2014; Jayasinghe et al. 2018) should be caused by the large pixel scale of the camera which is about 8″; thus the ASAS-SN camera cannot separate the target and the companion star. From our observations, we determined one heliocentric time of light minimum of J082700 to be 2458574.10494 ± 0.00055 and three heliocentric times of light minimum of J132829 to be 2458558.04146 ± 0.00089, 2458558.23706 ± 0.00088, and 2458599.20602 ± 0.00036. The method of Kwee & van Woerden (1956; hereafter K-W) was used to determine those times of minima. As seen in Figure 1, the two binaries are totally eclipsing binaries.

3. Light-curve Solution

In order to determine the photometric elements of the two binaries, we should determine the effective temperature at first. Thanks to the Gaia mission (Gaia Collaboration et al. 2016, 2018), the temperatures have been determined to be 5847 K for J082700 and 6329 K for J132829, respectively. According to Andrae et al. (2018), the typical accuracy for the effective temperatures derived by Gaia between 3000 and 10,000 K is estimated to be ±324 K. The gravity-darkening and bolometric albedo coefficients of the two components are both

---

Figure 1. The left panel displays the $VR_{I_c}$ light curves of J082700 (the open circles, the solid circles, and the open triangles respectively refer to the data observed on 2019 March 23, 25, and 31), the middle panel plots the $BVR_{I_c}$ light curves observed by NAOs85cm of J132829, while the right one plots the $BVR_{I_c}$ light curves observed by WHOT of J132829 (the open and solid circles respectively represent the data observed on 2019 April 25 and May 2).

(The data used to create this figure are available.)
set as $g_1 = g_2 = 0.32$ and $A_1 = A_2 = 0.5$ due to Lucy (1967) and Rucinski (1969). The atmospheric models of Castelli & Kurucz (2004) were applied, and the limb-darkening coefficients were derived by a logarithmic law.

The mass ratio is one of the most important parameters for contact binaries and can be well determined by a radial velocity curve. If there is no radial velocity curve, the mass ratio can be well determined by using photometric light curves only for totally eclipsing contact binaries because they have a very steep relationship between mass ratios and relative radii (Terrell & Wilson 2005). Both of our targets manifest totally eclipsing phenomenon, indicating we can obtain accurate mass ratios. The Markov Chain Monte Carlo (MCMC) parameter search method by using PHOEBE 2.3 (Prša & Zwitter 2005; Prša et al. 2016; Horvat et al. 2018; Conroy et al. 2020; Jones et al. 2020) under the contact binary mode was employed to determine the most probabilistic physical parameters of the two binaries. The python-based MCMC software EMCEE (Foreman-Mackey et al. 2013) was used to adjust the parameters $q$, $i$, $T_2$, $r_1$, and $l_2$ for J082700 and $q$, $i$, $T_2$, $r_3$, $l_2$, $\theta$, $r_s$, $T_s$, and $l_3$ for J132829. The values of these parameters determined by the W-D code (Wilson & Devinney 1971; Wilson 1979, 1990) were served as priors for the MCMC sampling. A total of 32 parameter space Walkers were applied for the two binaries. Because the light curves shown in Figure 1 are obvious deviations from an ideal light-curve shape in pretty much all passbands, which inevitably means that the employed model (in this case PHOEBE) will suffer from systematics, we applied Gaussian processes during the modeling procedure. When we assigned the Gaia temperature to the primary component, we implicitly assumed that the surface brightness of the secondary component is negligible. This is only approximate; the surface brightness of the secondary star cannot be negligible. In order to solve this problem, we assumed blackbody radiation and used the following equations to determine the

Figure 2. The probability distributions of $q$, $i$, $f$, and $l_2/l_1$ determined by the MCMC modeling for J082700.
individual temperatures (Zwitter et al. 2003; Christopoulou & Papageorgiou 2013):

\[
T_1 = \frac{((1 + k^2)T_{\text{eff}}^4) / (1 + k^2(T_2/T_1)^4))^{0.25}}{T_1(T_2/T_1)},
\]

where \( T_{\text{eff}} \) is the temperature determined by Gaia, \( k \) is the radius ratio, and \( T_2/T_1 \) is the temperature ratio. However, the surface brightness ratio does not scale linearly with the temperature ratio. Therefore, we carried out three iterations to determine a relatively more precise primary temperature. During each iteration, the MCMC parameter search was run for 2000 steps for all three sets of light curves, resulting in 64,000 iterations (the burn-in part has been deleted when calculating the physical parameters). Chains longer than 10 times that of the integrated autocorrelation time were chosen as the convergent criterion following Conroy et al. (2020). We found that the chains are longer than 20 times that of the integrated autocorrelation time for each iteration, indicating all the MCMC runs are convergent. Each iteration needs the primary temperature as an input parameter. First, the Gaia temperature was assigned to the primary, and a more precise primary temperature was calculated by Equation (1). Second, a new iteration was carried out using the more precise primary temperature. Third, a new primary temperature was obtained by Equation (1) and was applied for the third iteration. We found that the physical parameters change very little during the three iterations, especially the primary temperature; the differences are less than 15 K for the three sets of light curves (5855, 5862, and 5867 K for J082700; 6313, 6299, and 6287 K for the NAOs85cm light curves of J132829; and 6315, 6305, and 6302 K for the WHOT light curves of J132829 for the three iterations). After that, 5867 K was assigned to the primary temperature of J082700, 6287 K was assigned to the primary temperature of the NAOs85cm light curves of J132829, and 6302 K was assigned to the primary temperature of the NAOs85cm light curves of J132829.
the primary temperature of the WHOT light curves of J132829. New MCMC parameter searching was carried out with a step of 15,000, resulting in 480,000 iterations. This time, the chains are longer than 60 times that of the integrated autocorrelation time, indicating very good convergence. When we finished the MCMC parameter searching, the first 160,000 iterations were discarded. Figures 2–4 show the probability distributions of $q$, $i$, $f$, and $l_2/l_1$ for J082700, the NAOs85cm light curves of J132829, and the WHOT light curves of J132829, respectively. All the physical parameters were obtained by using the median-value and are listed in Table 1. The final individual temperatures of the two components were also calculated by Equation (1) and are listed in Table 1; the uncertainty of the Gaia temperature was taken into account when estimating their uncertainties. For J132829, we found that the results derived by the NAOs85cm light curves and those derived by the WHOT light curves are consistent with each other. The photometric elements determined by the WHOT light curves were used for the following analysis because of the higher precision. The synthetic light curves calculated by PHOEBE 2.3 are all plotted in Figure 5. The left panel of this figure displays the fitted light curves of J082700, the middle panel plots the fitted NAOs85cm light curves of J132829, and the right one plots the fitted WHOT light curves of J132829. The $O - C$ residuals are plotted in the bottom panels; nearly flat residuals reveal that the theoretical light curves fit the observed ones very well.

4. Orbital Period Investigation

Orbital period investigation is a very powerful tool to study the dynamical evolution of the binary star itself as well as the additional companion(s) orbiting the binary star (e.g., Zhou et al. 2016b; Er-gang et al. 2019; Li et al. 2019a; Liao et al. 2019; Pi et al. 2019). Therefore, we analyzed the eclipsing time variations of J082700 and J132829. At present, there are many optical surveys that have observed these two targets. Northern
Sky Variability Survey (NSVS; Wozniak et al. 2004), Catalina Sky Survey (CSS; Drake et al. 2014), and ASAS-SN have observed J082700, and NSVS, CSS, TAROT, Wide Angle Search for Planets (WASP; Butters et al. 2010), and ASAS-SN have observed J132829. However, the time resolutions of these surveys are very poor. In order to derive as many eclipse timings as possible, we construct the complete-phase light curve or half-of-the-phase light curve over longer time intervals. One season of observations (several hundreds of data points) can be used because we are trying to identify the period variation as many years long, not the short-periodic signal as shorter than several years. Figure 6 shows an example of our calculated eclipsing times for J082700 using the CSS data. We divided the data into three segments shown in the upper panel. For each segment, we shifted the data into one period using the equation \( \text{MJD} = \text{MJD}_0 + P \times E \), where MJD is the observational time of the data, \( \text{MJD}_0 \) is the selected reference time, \( P \) is the orbital period, and \( E \) is the cycle number. Then, we derived the lower panel of Figure 6. Based on the light curves of the three segments displayed in the lower panel, five eclipsing minima were determined by the K-W method. Following this procedure, we determined two eclipsing minima by using the NSVS data, five eclipsing minima by using the CSS data, and four eclipsing minima by using the ASAS-SN data for J082700 and two eclipsing minima by using the NSVS data, two eclipsing minima by using the TAROT data, and four eclipsing minima by using the ASAS-SN data for J132829. The NSVS and CSS times were first transferred from MJD to JD (MJD = JD − 2,400,000.5), and then to HJD. G. Srdoc first identified J082700 as a binary, and uploaded the figure of its light curve to VSX. Then, we calculated two eclipsing minima from the light curves.4 For J132829, we can derive one eclipsing minimum from the original WASP data, and no eclipsing minimum can be determined by using the CSS data. All the derived times of eclipsing minima are listed in Table 2.

Using the following two equations,

\[
\text{Min.I} = 2455544.64369 + 0.2771582E, \quad (2) \\
\text{Min.I} = 2454604.41693 + 0.384705E, \quad (3)
\]

we constructed the \( O − C \) diagram for J082700 and J132829, respectively. The corresponding curves are shown in the left and right panels of Figure 7. As seen in the left panel of Figure 7, the \( O − C \) curve of J082700 exhibits a downward parabola plus a quasi-cyclic variation, so we used a second-order polynomial plus the light travel time effect (LTTE) model proposed by Irwin (1952) to fit the curve:

\[
O − C = \Delta T_0 + \Delta P_0 \times E + \frac{\beta}{2} \times E^2 \\
+ K \cdot \frac{1}{\sqrt{1 - e^2 \cos^2 \omega}} \left[ (1 - e^2 \sin \nu + \omega) + e \sin \omega \right],
\]

where \( \Delta T_0 \) and \( \Delta P_0 \) are respectively the corrections of the initial epoch and period with respect to the values in Equation (2), \( \beta \) is the long-term changing rate of the orbital period, and the details of other parameters can be found in Irwin (1952). As seen in the right panel of Figure 7, the \( O − C \) curve of J132829 exhibits a downward parabola; the following

---

**Table 1**

| Star     | J082700   | J132829−NAOs85cm | J132829−WHOT |
|----------|-----------|------------------|--------------|
| \( T_0 (K) \) | 5870 ± 335 | 6275 ± 337 | 6300 ± 339 |
| \( q(M_2/M_1) \) | 0.0550 ± 0.0006 | 0.0856 ± 0.0011 | 0.0889 ± 0.0011 |
| \( T_0 \) | 5828 ± 406 | 6375 ± 369 | 6319 ± 366 |
| \( i (\text{deg}) \) | 68.7 ± 0.6 | 81.4 ± 0.5 | 81.5 ± 0.9 |
| \( \Omega_1 = \Omega_2 \) | 1.801 ± 0.002 | 1.873 ± 0.003 | 1.883 ± 0.004 |
| \( \nu_2 \) | 0.0763 ± 0.0047 | 0.1354 ± 0.0035 | 0.1310 ± 0.0035 |
| \( \nu_3 \) | 0.0766 ± 0.0040 | 0.1337 ± 0.0032 | 0.1304 ± 0.0032 |
| \( \nu_4 \) | 0.0769 ± 0.0034 | 0.1320 ± 0.0022 | 0.1300 ± 0.0020 |
| \( \nu_5 \) | 0.0834 ± 0.0034 | 0.2097 ± 0.0049 | 0.2094 ± 0.0056 |
| \( \nu_6 \) | 0.0010 ± 0.0011 | 0.0012 ± 0.0016 |
| \( \nu_7 \) | 0.0008 ± 0.0004 | 0.0073 ± 0.0034 |
| \( \nu_8 \) | 0.6278 ± 0.0011 | 0.6118 ± 0.0011 | 0.6081 ± 0.0017 |
| \( \nu_9 \) | 0.1766 ± 0.0011 | 0.2207 ± 0.0024 | 0.2222 ± 0.0029 |
| \( f \) | 19.6 ± 5% | 74.3 ± 5% | 70.2 ± 5% |

---

4 We communicated with G. Srdoc to ask for the observational data by email. However, he told us that the data were lost due to backup disk failure. So, we derived the original data points from the VSX figures by using GetData Graph Digitizer version 2.25.0.32 (http://getdata-graph-digitizer.com/). The times were first transferred from UTC to JD, and then to HJD.
second-order polynomial was applied to fit the curve:

\[ O - C = \Delta T_0 + \Delta P \times E + \frac{\beta}{2} E^2. \]

The derived parameters are all listed in Table 3. We found that both of the binaries show long-term orbital period decreases, and removing the long-term decrease, J082700 exhibits a cyclic variation with a period of 12.5 yr and an amplitude of 0.0169 day. Because of the short time span and the scarcity of the eclipsing times of the two binaries, these results require more observations to be confirmed.

5. Discussion and Conclusions

Using the MCMC parameter search method based on PHOEBE 2.3, we analyzed one set of complete multiple color light curves of J087200 and two sets of complete multiple color light curves of J132829. The results show that these two binaries are extremely low mass ratio contact binaries (ELMRCBs), \( q \sim 0.055 \) for J087200 and \( q \sim 0.089 \) for J132829. J087200 is a shallow contact binary \( (f \sim 19\% < 25\%) \), while J132829 is a deep contact binary \( (f \sim 70\% > 50\%) \). The asymmetric light curves of J132829 can be interpreted by a dark spot on the more massive component. As seen in Figure 1, both of the light curves of J082700 and J132829 clearly show total eclipsing secondary minima. As proposed by Pribulla et al. (2003) and Terrell & Wilson (2005), the physical parameters of the two systems determined only by photometric light-curve synthetics should be reliable. We should state that none of the current models of contact binaries (W-D, PHOEBE, and so forth) correctly account for energy transfers in the envelope, yet at such low mass ratios, that has to be extremely significant. In addition, the fill-out factor at such low mass ratios is poorly defined because surface potentials through the inner Lagrange point \( (L_1) \) and the outer Lagrange point \( (L_2) \) are numerically very close. By calculating eclipsing times from all available photometric surveys and observations, we investigated the \( O - C \) variations of the two binaries. The orbital period of J082700 displays a long-term decrease at a rate of \( -9.52 \times 10^{-7} \) day yr\(^{-1} \) plus a cyclic oscillation with a period of 12.5 yr and an amplitude of 0.0169 day, while that of J132829 shows a long-term decrease at a rate of \( -4.46 \times 10^{-7} \) day yr\(^{-1} \).

5.1. The Period Changes

Both of the systems exhibit long-term period decreases. Such changes can be generally caused by a mass transfer from a more massive component to a less massive one or by the angular momentum loss (AML) or the combination. To discuss the reason for the long-term period decreases, we have to derive the absolute parameters of the two binaries. However, we cannot directly
determine their absolute parameters because of the lack of the radial velocity curve. Nevertheless, we try to estimate their global parameters based on the final results of the light-curve solutions. By assuming the more massive primary components of the two binaries are main-sequence stars, the masses can be estimated to be $M_1 = 1.06 M_\odot$ for J082700 and $M_1 = 1.23 M_\odot$ for J132829 based on the online table\(^5\) (Pecaut & Mamajek 2013). According to the adopted photometric elements and Kepler’s third law, the global parameters of the two binaries can be estimated as follows: $a = 1.83 R_\odot$, $M_1 = 1.06 M_\odot$, $M_2 = 0.06 M_\odot$, $R_1 = 1.15 R_\odot$, $R_2 = 0.32 R_\odot$, $L_1 = 1.40 L_\odot$, and $L_2 = 0.11 L_\odot$ for J082700 and $a = 2.45 R_\odot$, $M_1 = 1.23 M_\odot$, $M_2 = 0.11 M_\odot$, $R_1 = 1.49 R_\odot$, $R_2 = 0.55 R_\odot$, $L_1 = 3.15 L_\odot$, and $L_2 = 0.43 L_\odot$ for J132829. If the period decreases are caused by a mass transfer, the following equation can be used to estimate the mass transfer rate:

$$\frac{\dot{P}}{P} = -3 M_1 \left( \frac{1}{M_1} - \frac{1}{M_2} \right). \quad (6)$$

The mass transfer rate was calculated to be $dM_1/dt = -7.06(\pm 2.98) \times 10^{-8} M_\odot$ yr\(^{-1}\) for J082700 and $dM_1/dt = -4.64(\pm 0.85) \times 10^{-9} M_\odot$ yr\(^{-1}\) for J132829. The negative sign reveals that the more massive primary is losing mass. Assuming constant angular momentum and the total mass, the thermal timescale can be calculated to be $\tau_{th} = 2.20 \times 10^7$ yr for J082700 and $\tau_{th} = 1.01 \times 10^7$ yr for J132829 using the equation $\tau_{th} = GM_c^2 \frac{R_{Li}}{L_i}$. The thermal timescale mass transfer rate is determined to be $M_1/\tau_{th} = 4.82 \times 10^{-8} M_\odot$ yr\(^{-1}\) for J082700, while that for J132829 is derived to be $M_1/\tau_{th} = 1.22 \times 10^{-7} M_\odot$ yr\(^{-1}\). For J082700, the thermal mass transfer rate is similar with that calculated by Equation (6), meaning that the long-term orbital period decrease is possibly caused by mass transfer. However, we cannot rule out the possibility of AML via magnetic stellar wind. For J132829, the thermal mass transfer rate is very different from that calculated by Equation (6), meaning that the long-term orbital period decrease is possibly due to AML.

The cyclic oscillation in the $O - C$ diagram of J082700 can be generally resulted from the magnetic activity of one or two components (Applegate 1992) or LTTE due to a third body (e.g., Liao & Qian 2010; Lee et al. 2013; Zhou et al. 2017). In order to check which one is the more likely reason, we calculated the quadruple moment variations of the two components using the equations taken from Applegate (1992) and Lanza & Rodonó (2002):

$$\frac{\Delta P}{P} = \frac{2\pi A}{P_{mod}}, \quad (7)$$

$$\frac{\Delta Q}{P} = -9 \frac{\Delta Q_1}{Ma^2}, \quad (8)$$

$\Delta Q_1 = 1.77 \times 10^{50}$ g cm\(^2\) and $\Delta Q_2 = 9.72 \times 10^{48}$ g cm\(^2\) were determined respectively for the primary and secondary components. The value of quadruple moment variation of the primary component is close to the typical value of $10^{51}$–$10^{52}$ g cm\(^2\). Therefore, the Applegate mechanism is a possible reason for the cyclic oscillation. Although the Applegate mechanism can explain the cyclic oscillation, we cannot rule out the possibility of LTTE via a third body. So we calculated the mass function of the third body using the following equation:

$$f(m) = \frac{(m_3 \sin \iota)^3}{(m_1 + m_2 + m_3)^2} = \frac{4\pi}{GP_3^2} \times (a_{12} \sin \iota)^3, \quad (9)$$

and we determined $f(m) = 1.61(\pm 1.68) \times 10^{-1} M_\odot$. If the third body is coplanar with the central eclipsing pair ($\iota' = \iota = 66.7^\circ$), the mass and the separation would be derived to be $M_3 = 0.96 \pm 0.75 M_\odot$ and $a_3 = 4.27 \pm 3.65$ au. Considering the mass of the tertiary companion, the luminosity would be 0.8 $L_\odot$ for a main-sequence star. However, when we searched third light during the light-curve modeling, no third

---

\(^5\) http://www.pas.rochester.edu/~emamajek/EEM_dwarf_UBVIJK_colors_Teff.txt
light was detected. Maybe the tertiary companion is a very faint star or a compact object.

5.2. Statistics on ELMRCBs

In order to investigate the properties of ELMRCBs, we carried out a statistics analysis on contact binaries with mass ratios \( q < 0.1 \). The name, the period, the mass ratio, the inclination, the effective temperatures of the two components, the contact degree, the orbital period change rate, and the spin angular momentum to the orbital angular momentum ratio, \( J_{\text{spin}}/J_{\text{orb}} \), are shown in Table 4. Sixteen such systems were collected. In this table, \( J_{\text{spin}}/J_{\text{orb}} \) was calculated using the following equation taken from Yang & Qian (2015):

\[
\frac{J_{\text{spin}}}{J_{\text{orb}}} = \frac{1 + q}{q} [(k_1 r_1)^2 + (k_2 r_2)^2 q],
\]

where \( r_1,2 \) and \( k_1,2 \) are the relative radii and the gyration radii to the stellar radii. For W UMa type contact binaries, \( k_1^2 = k_2^2 = k^2 \), and taking \( k^2 = 0.06 \) from Li & Zhang (2006), the values of \( J_{\text{spin}}/J_{\text{orb}} \) were derived. As seen in this table, the mass ratios of two stars (V1187 Her and J082700) are below the theoretically predicted cutoff mass ratio of contact binaries. And we found that the value of \( J_{\text{spin}}/J_{\text{orb}} \) has a tendency to decrease with the reduction of the mass ratio.

A very confusing result was obtained from these statistics; the values of \( J_{\text{spin}}/J_{\text{orb}} \) of three targets are greater than 1/3 (V1187 Her, J082700, and V857 Her). According to Hut (1980), binary systems will be dynamically unstable when the spin angular momentum is more than one-third of the orbital angular momentum. These three targets should be dynamically unstable. However, researchers have observed the stable light variability of eclipse, and their orbital period changes show no abnormal behavior. These three stars are stable contact binaries. There are two possible reasons. The first reason is that the dynamic stability criterion proposed by Hut (1980) should be corrected because some physical processes that are not known to date may not have been taken into account. If considering the unknown physical processes, it will possibly lead to the change of the present dynamical stability limit (the value of \( J_{\text{spin}}/J_{\text{orb}} \) would be greater than 1/3). The other reason, most likely, is that the ratio of the gyration radii to the stellar radii (\( k \)) should be smaller than \( k^2 = 0.06 \), meaning that the mass ratio limit of contact binaries will be affected by this ratio. The smaller the \( k \), the smaller the minimum mass ratio of contact binaries. This is consistent with the results derived by Rasio (1995) and Jiang et al. (2010). If V1187 Her is just at the boundary of stability, \( k^2 = 0.03249 \) was determined. Then, the revised values of \( J_{\text{spin}}/J_{\text{orb}} \) are listed in the tenth column of Table 4 by using \( k^2 = 0.03249 \). The correlations between the mass ratio and the initial and revised \( J_{\text{spin}}/J_{\text{orb}} \) are respectively displayed in the left and right panels of Figure 8. The dashed line represents the boundary between stable and unstable. A least-square method yields the following equations:

\[
\frac{J_{\text{spin}}}{J_{\text{orb}}} = 0.20(\pm 0.02) + 2.57(\pm 0.63) \times e^{-41.72(\pm 6.04) \times q},
\]

for the left panel, and

\[
\frac{J_{\text{spin}}}{J_{\text{orb}}} = 0.11(\pm 0.01) + 1.39(\pm 0.34) \times e^{-41.72(\pm 6.04) \times q},
\]

for the right panel. Based on Equation (11), a predicted minimum mass ratio was derived to be 0.0716, which is corresponding to the result derived by Li & Zhang (2006). Due to Equation (12), a predicted cutoff mass ratio was determined to be 0.0439, which is consistent with the result determined by Yang & Qian (2015).

By the statistics, not all contact binaries with mass ratios \( q \leq 0.1 \) are deep contact binaries, some of them are shallow or medium contact systems, even one of them has a fill-out factor of 1% (NSVS 2569022). This may mean that the formation of ELMRCBs has two channels. One channel is for the shallow or medium contact systems, their initial short period detached binaries had extremely low mass ratios and formed ELMRCBs by AML, and these ELMRCBs are newly formed contact binaries.
Figure 8. $J_{\text{spin}}/J_{\text{orb}}$ vs. mass ratio. Left panel shows the results when $k^2 = 0.06$, while the right one shows the results when $k^2 = 0.03249$. The dashed line represents the dynamic stability limit. The solid circles denote the 16 ELMRCBs. The solid line refers to an exponential fit to the 16 points.

### Table 4

| Parameters | Period (days) | $q$ | $i$ (deg) | $T_1$ (K) | $T_2$ (K) | $f$ | $dP/dt$(day yr$^{-1}$) | $J_{\text{spin}}/J_{\text{orb}}$ | $J_{\text{spin}}/J_{\text{orb}}^a$ | References |
|------------|--------------|-----|-----------|-----------|-----------|-----|-----------------|-----------------|-----------------|-------------|
| V1187 Her$^b$ | 0.310764 | 0.044 | 66.7 | 6250 | 6680 | 84% | $-1.50 \times 10^{-7}$ | 0.616 | 0.333 | (1) |
| J082700.8$^b$ | 0.277158 | 0.055 | 68.7 | 5870 | 5728 | 19% | $-9.52 \times 10^{-7}$ | 0.456 | 0.247 | (2) |
| V857 Her$^b$ | 0.382230 | 0.065 | 58.4 | 6530 | 6010 | 89% | $+2.90 \times 10^{-7}$ | 0.395 | 0.214 | (3) |
| ASAS J083241+2332.4$^b$ | 0.311321 | 0.068 | 52.1 | 7610 | 7140 | 98% | $+8.80 \times 10^{-7}$ | 0.333 | 0.139 | (4) |
| M4 V53$^b$ | 0.287797 | 0.074 | 76.3 | 6100 | 6100 | 1% | ... | 0.301 | 0.163 | (5) |
| V1309 Sco$^b$ | 0.29449 | 0.078 | 74.4 | 7415 | 6610 | 69% | $-5.89 \times 10^{-8}$ | 0.320 | 0.173 | (6) |
| SX Crv$^c$ | 0.316599 | 0.079 | 61.2 | 6340 | 6160 | 77% | $-1.05 \times 10^{-6}$ | 0.303 | 0.164 | (7) |
| V870 Ara$^c$ | 0.399722 | 0.082 | 70.0 | 5860 | 6210 | 96% | ... | 0.256 | 0.138 | (8) |
| NSV 13890b | 0.373880 | 0.080 | 76.2 | 6510 | 6426 | 9% | ... | 0.315 | 0.171 | (9) |
| XX Sexc | 0.540108 | 0.100 | 74.9 | 6881 | 6378 | 42% | ... | 0.236 | 0.128 | (14) |
| AW CrB$^b$ | 0.360935 | 0.101 | 82.1 | 6700 | 6808 | 75% | $+3.58 \times 10^{-7}$ | 0.242 | 0.131 | (15) |

Notes. The physical parameters of V1309 Sco were determined by the light curve before the merge. The mass ratio of AW CrB was determined to be 0.099 when no spot was considered, so we added this target to our list.

$^a$ The revised value of $J_{\text{spin}}/J_{\text{orb}}$.

$^b$ The mass ratios of these systems were determined photometrically.

$^c$ The mass ratios of these systems were determined spectroscopically.

References. (1) Caton et al. (2019); (2) This paper; (3) Qian et al. (2005b); (4) Srim et al. (2016); (5) Kjurkchieva et al. (2018); (6) Li et al. (2017); (7) Zola et al. (2004); (8) Szalai et al. (2007); (9) Wadhwa (2006); (10) Zasche & UHlar (2010); (11) Zhu et al. (2016); (12) Gazanas et al. (2006); (13) Zola et al. (2017); (14) Deb & Singh (2011); (15) Broens (2013).
are located very far below the TAMS line, meaning they are more evolved stars. We found that these two binaries (KIC 11097678 and V1309 Sco) have the longest period among the ELMRCBs; this is the reason why their secondary components are located below the TAMS line; they should be subgiants or red giants.

In conclusion, multicolor light curves of two contact binaries, J082700 and J132829, have been analyzed. They are members of ELMRCBs. By analyzing the eclipsing times of the two binaries, we found that both of them show a long-term decrease. For J082700, a cyclic modulation is superimposed on the long-term variation, which is most likely caused by the Applegate mechanism. By the statistics of ELMRCBs, we found that the values of the spin angular momentum to the orbital angular momentum ratio of three systems are more than 1/3. Two possible reasons may be considered: One reason is the dynamic stability limit predicted by Hut (1980) would be changed. The other is the dimensionless gyration radius should be less than \( k^2 = 0.06 \), meaning that the cutoff mass ratio of contact binaries depends on the dimensionless gyration radius. Two channels of the formation of ELMRCBs are presented. The evolutionary states of ELMRCBs are also discussed; we found that, although they are very unusual, their locations on the color–density diagram are consistent with normal W UMa contact binaries. For J082700, the value of \( J_{\text{pin}}/J_{\text{orb}} \) is more than or very close to \( 1/3 \); it will be quickly merging. For J132829, the fill-out factor is 70%, and will be increased with the decreasing orbital period; a dynamic instability will be encountered when the surfaces of the components touch the outer critical Roche lobe, and then J132829 will be merging to a single fast-rotation star. Therefore, the two contact binaries are potential progenitors of the merger. Future observations, especially the high-resolution spectroscopic observations, are needed to confirm their extremely low mass ratios and to determine their more precise absolute parameters.

Thanks are due to the referee for the help and useful comments that improved our manuscript a lot. This work is supported by the Joint Research Fund in Astronomy (No. U1931103) under cooperative agreement between the National Natural Science Foundation of China (NSFC) and the Chinese Academy of Sciences (CAS), by NSFC (No. 11703016), by the Natural Science Foundation of Shandong Province (Nos. ZR2014AQ019, JQ201702), by the Young Scholars Program of Shandong University, Weihai (Nos. 20820162003, 20820171006), by the Chinese Academy of Sciences Interdisciplinary Innovation Team, and by the Open Research Program of Key Laboratory for the Structure and Evolution of Celestial Objects (No. OP201704). Work by C.H.K. was supported by a grant from the National Research Foundation of Korea (2020R1A4A2002885). The calculations in this work were carried out at the Supercomputing Center of Shandong University, Weihai.

We acknowledge the support of the staff of the Xinglong 85 cm telescope and WHOT. This work was partially supported by the Open Project Program of the Key Laboratory of Optical Astronomy, the National Astronomical Observatories, and the Chinese Academy of Sciences.

This work has made use of data from the European Space Agency mission Gaia (https://www.cosmos.esa.int/gaia), processed by the Gaia Data Processing and Analysis Consortium (DPAC, https://www.cosmos.esa.int/web/gaia/dpac/consortium). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the Gaia Multilateral Agreement.

This paper makes use of data from the DR1 of the WASP data (Butters et al. 2010) as provided by the WASP consortium, and the computing and storage facilities at the CERIT Scientific Cloud, registration No. CZ.1.05/3.2.00/08.0144, which is operated by Masaryk University, Czech Republic.

**ORCID iDs**

Kai Li @ [https://orcid.org/0000-0003-3590-335X](https://orcid.org/0000-0003-3590-335X)

Chun-Hwey Kim @ [https://orcid.org/0000-0001-8591-4562](https://orcid.org/0000-0001-8591-4562)

Shao-Ming Hu @ [https://orcid.org/0000-0003-3217-7794](https://orcid.org/0000-0003-3217-7794)

Dong-Yang Gao @ [https://orcid.org/0000-0001-6643-2138](https://orcid.org/0000-0001-6643-2138)

**References**

Andrae, R., Fouesneau, M., Creevey, O., et al. 2018, A&A, 616, 8

Applegate, J. H. 1992, ApJ, 385, 621

Arbutina, B. 2007, MNRAS, 377, 1635

Arbutina, B. 2009, MNRAS, 394, 501

Bradstreet, D. H., & Guinan, E. F. 1994, in ASP Conf. Ser. 56, Interacting Binary Stars, ed. A. W. Shafter (San Francisco, CA: ASP), 228

Brosens, E. 2013, MNRAS, 430, 3070

Butters, O. W., West, R. G., Anderson, D. R., et al. 2010, A&A, 520, 10

Castelli, F., & Kurucz, R. L. 2004, A&A, 419, 725

Caton, D., Gentry, D. R., Samec, R. G., et al. 2019, PASP, 131, 052403

Christopoulou, P.-E., & Papageorgiou, A. 2013, AJ, 146, 157

Coroy, K. E., Kochoska, A., Hey, D., et al. 2020, ApJS, 250, 34

Dameridji, Y., Klotz, A., & Boër, M. 2007, AJ, 133, 1470

Deb, S., & Singh, H. P. 2011, MNRAS, 412, 1787

Drake, A. J., Graham, M. J., Djorgovski, S. G., et al. 2014, ApJS, 213, 9

Eggleton, P. P., & Kiseleva-Eggleton, L. 2001, ApJ, 562, 1012

Eggleton, P. P., & Kiseleva-Eggleton, L. 2002, ApJ, 575, 461

Eg-gang, Z., Sheng-bang, Q., Soonthornthum, B., et al. 2019, ApJL, 871, L10

Ferreira, T., Saito, R. K., Minniti, D., et al. 2019, MNRAS, 486, 1220

Foreman-Mackey, D., Hogg, D. W., Lang, D., & Goodman, J. 2013, PASP, 125, 306

Gaia Collaboration, Brown, A. G. A., Vallenari, A., et al. 2018, A&A, 616, A1

Gaia Collaboration, Prusti, T., de Bruijne, J. H. J., et al. 2016, A&A, 595, 1
