Investigation of Load Sharing and Dynamic Load Characteristics of a Split Torque Transmission System with Double-Helical Gear Modification

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1.Introduction

Representing a new and advanced configuration designed to replace the planetary transmission, the split torque transmission system (STTS) is increasingly employed in helicopter main reducer [1–5]. STTS is characterized by a driving pinion which simultaneously meshes with two gears, and each path only carries an equal half of the original load under ideal conditions. Due to the power closed-loop features of the STTS and the inevitable manufacturing and installation errors of the gears, a problem of uneven load distribution in each path of the STTS is inevitably caused. The unequal load sharing of the STTS increases the vibration and noise, while simultaneously affecting the STTS’ reliability and durability.

Many research works have conducted numerous investigations on natural and load sharing characteristics of planetary and star transmission systems. Kahraman [6, 7] established a dynamic model of a planetary transmission. The author predicted natural modes of the system and investigated the influence of manufacturing and installation error on dynamic load coefficient of the system. Sondkar et al. [8] investigated the influence of double-helical (DH) gear stagger angle on dynamic response of a double-helical planetary gear system and found that the left and right sides of the DH gear can bear different dynamic load due to the effect of the stagger angle. Liu et al. [9] analyzed the influence of the profile error before and after the gear tooth separation on the vibration and dynamic meshing force of the herringbone planetary gears. Li et al. [10] established a model to predict the reliability of a helicopter planetary gear train under partial load and concluded that when the twin-engine helicopters are running on only one engine, the likelihood of
teeth fatigue fracture is increased. Ren et al. [11] established a
generalized dynamic model for a herringbone planetary gear
train which considers the manufacturing eccentric errors of
components and tooth profiles errors. Mo et al. [12, 13]
investigated the influence mechanism of multicoUpling er-
rors, flexible support stiffness, and floating components on
load sharing characteristics of a herringbone planetary
transmission system. Mo et al. [14] proposed a new dynamic
model for double-helical star gearing system and obtained
the vibration model, natural characteristics, and dynamic
response of the system. Wang et al. [15] established a dy-
namic model of a star herringbone gear transmission system
and discovered that the system has four typical vibration
modes. Wang et al. [16] investigated the influence of
bearing’s position and permutations on the load sharing
performance of star gearing system. Bechhoefer et al. [17]
processed the vibration signatures of a split torque gearbox
through a number of gear analysis algorithms to quantify the
gear fault performance. Gui et al. [18] analyzed the backlash
influence on load sharing coefficient of the two-input cy-
1 after [309]
lindrical gear STTS. Dong et al. [19] established a dynamic
model of the power-split transmission and obtained the
frequency domain and time domain responses of the system.
Zhao et al. [20] proposed a universal mathematical design
method for a torque-split gear transmission and verified the
correctness of the method through different types of gears.
Hu et al. [21] established the gear-shaft-bearing dynamic
model of two-path STTS and analyzed the influence of
factors such as shaft angle and DH gear stagger angle on the
system’s dynamic characteristics and load sharing charac-
teristics. Jia et al. [22] analyzed the influence of large
and weight of backlash and center distance error on STTS load
sharing characteristics.

Meshing error [11–14], time-varying meshing stiffness
(TVMS) [23–26], and meshing-in impact [27–30] are im-
portant internal excitation of gear transmission system.
Periodic TVMS excitation is the most obvious feature of gear
system vibration that distinguished it from most mechanical
systems. The finite element method and numerical analysis
method are usually used to calculate TVMS. The former has
high calculation accuracy but low calculation efficiency,
while the latter has high calculation efficiency but low cal-
culation accuracy. In this paper, the loaded tooth contact
analysis (LTCA) method is used to calculate the TVMS of
gear pair under static condition [31–33]. Gear modification
has proven to be an effective method for reducing vibration
and noise of the gear system. The modification is mainly
carried out around the three elements of modification
length, modification amount, and modification curve
[34–37]. DH gear is generally used as the final-stage
transmission in STTS because of its smooth transmission,
compact structure, large carrying capacity, and self-bal-
cancing axial force. The final-stage transmission reduction
ratio of approximately 10:1 to 14:1 can be achieved. Fur-
thermore, transmission performance of the final-stage
transmission has a crucial impact on the performance of
the entire STTS. The existing research results show that the
dynamic characteristics of a pair of DH gear transmission
system can be significantly improved through the modifi-
cation design of DH gear [38–40]. However, there are few
studies on the influence of DH gear modification on load
sharing and dynamic load characteristics of STTS.

In this paper, TVMS and meshing-in impact of each gear
pair are calculated accurately via tooth contact analysis
(TCA) and LTCA. Dynamic model of a two-input two-path
STTS accounting for meshing error, TVMS, and meshing-in
impact is established. The dynamic model can be employed
to analyze the load sharing and dynamic load characteristics
of STTS before and after modification under any working
conditions. The influence of gear eccentricity and installa-
tion error on load sharing characteristics of the system is
studied. By optimizing the amplitude of static loaded
transmission error and meshing-in impact, the modification
design of DH gear is carried out, and the topological
modification tooth surface of the DH gear is constructed.
Finally, the modification influence on load sharing and
dynamic load characteristics of two-input two-path STTS is
investigated. This research has important theoretical guid-
ance and significance for the load sharing design of STTS
and application of DH gear modification technology in
helicopter main reducer.

2. Internal Excitation Analysis of Two-Input
Two-Path STTS

2.1. Physical Model of Two-Input Two-Path STTS

Figure 1 shows the two-input two-path STTS for helicopter
main reducer. The helicopter main reducer has two engines
which transmit power to the input gear, one on the left and
one on the right side. The input power on both sides is
collected in the output gear, and the power is transmitted to
the main rotor and tail stabilator through the rotor shaft. The
two-input two-path STTS adopts three-stage transmission.
The first stage adopts a spiral bevel gear pair to achieve the
reversal of the transmission system. The second stage adopts
a spur gear pair; i.e., one spur pinion simultaneously meshes
with two large spur gears to achieve the power split. The
third stage adopts a DH gear pair; i.e., two small DH pinions
simultaneously mesh with one large DH gear to achieve
power convergence. For the convenience of the following
description, the gears of the two-input two-path STTS are
numbered in sequence from 1 to 8. The gears on the left and
right sides of the first stage, second stage, and third stage are
represented by \( z_j \) (\( i = L, R, j = 1 \ldots 7 \)). A large center output
DH gear is designated as the gear number 8, i.e., \( z_8 \).

As shown in Figure 2, coordinates of two-input two-path
STTS are established. Generalized STTS coordinate system is
O-XYZ, while the axis direction of gear 8 is defined as the Z-
axis direction. Parameters \( \sigma_1 \) and \( \sigma_2 \) represent system in-
stallation angles, and \( \psi \) represents the system layout angle.
The angle between the center line of each gear pair and the
positive X-axis direction is indicated by \( \gamma \) and the corre-
sponding subscript, i.e., \( \gamma_{L34} \) and \( \gamma_{R34} \). The angle between the
meshing line of each gear pair and the positive direction
of the X-axis is denoted as \( \eta \) and the corresponding subscript,

\( \eta_{L34} \) and \( \eta_{R34} \). The angular displacement is taken
counterclockwise from the positive $X$-axis as “+.” The remaining angles are not marked in Figure 2, and the local coordinate systems of the first-stage gear pair are omitted.

2.2. Meshing Error of Second- and Third-Stage Gears Analysis. Since the load sharing of two-input two-path STTS is independent of the first-stage gear pair error, only equivalent displacement formula of the eccentricity and installation error along the meshing line of second- and third-stage gears is studied in this paper.

Second-stage spur gear pair (denoted as 34) of left-input two-path STTS is considered as an example. The projection relationship between the eccentricity error $E_{i3}$, $E_{i4}$ and installation error $A_{i3}$, $A_{i4}$ of spur pinion $z_{i3}$ and the spur gear $z_{i4}$, and the meshing line $ML_{i34}$ is shown in Figure 3. Parameters $\lambda_{i3}, \lambda_{i4}$ and $\mu_{L3}, \mu_{L4}$ represent phase angles of the eccentricity and installation errors, respectively. The eccentricity and installation errors of the spur pinion $z_{i3}$ and the spur gear $z_{i4}$ are projected to the meshing line to obtain their respective equivalent meshing errors. Then, the equivalent meshing errors of two gears are combined to obtain the equivalent cumulative meshing error on the meshing line of gear pair 34.

Therefore, the equivalent cumulative meshing error $e_{i34}$ ($i = L, R$) on the meshing line of the second-stage spur gear pair 34 can be expressed as

$$e_{i34}(t) = E_{i3} \sin(-\omega_{i3} t - \lambda_{i3} + \gamma_{i34} + \alpha_{i3})$$

$$+ A_{i3} \sin(-\mu_{i3} + \gamma_{i34} + \alpha_{i3})$$

$$+ E_{i4} \sin(-\omega_{i4} t - \lambda_{i4} + \gamma_{i34} + \alpha_{i4})$$

$$+ A_{i4} \sin(-\mu_{i4} + \gamma_{i34} + \alpha_{i4}).$$

\(1\)
Similarly, the equivalent cumulative meshing error on the meshing line of the second-stage spur gear pair 35 and the third-stage DH gear pair 68 and 78 can be expressed as

\[ e_{35}(t) = E_{35} \sin(-\omega_{35} t - \lambda_{35} + \gamma_{35} + \alpha_{35}) + A_{35} \sin(-\mu_{35} + \gamma_{35} + \alpha_{35}) + E_{35} \sin(-\omega_{35} t - \lambda_{35} + \gamma_{35} + \alpha_{35}) + A_{35} \sin(-\mu_{35} + \gamma_{35} + \alpha_{35}), \]

\[ e_{68}(t) = -E_{68} \sin(-\omega_{68} t - \lambda_{68} + \gamma_{68} + \alpha_{68}) - A_{68} \sin(-\mu_{68} + \gamma_{68} + \alpha_{68}) - E_{8} \sin(-\omega_{8} t - \lambda_{8} + \gamma_{68} - \alpha_{8}) - A_{8} \sin(-\mu_{8} + \gamma_{68} - \alpha_{8}), \]

\[ e_{78}(t) = -E_{78} \sin(-\omega_{78} t - \lambda_{78} + \gamma_{78} + \alpha_{78}) - A_{78} \sin(-\mu_{78} + \gamma_{78} + \alpha_{78}) - E_{8} \sin(-\omega_{8} t - \lambda_{8} + \gamma_{78} - \alpha_{8}) - A_{8} \sin(-\mu_{8} + \gamma_{78} - \alpha_{8}). \]

where \( \omega_{35} \), \( \omega_{68} \), and \( \omega_{78} \) are the angular velocities of spur gear \( z_{35}, z_{68}, \) and \( z_{78} \), respectively. \( \omega_{68}, \omega_{78}, \) and \( \omega_{8} \) are the angular velocities of DH gear \( z_{68}, z_{78}, \) and \( z_{8} \). Parameters \( \alpha_{68} \) and \( \alpha_{78} \) are the transverse pressure angle of spur gear and DH gear, respectively.

2.3. Calculation of Time-Varying Meshing Stiffness of Gears. The approach for LTCA model is used to solve the comprehensive meshing stiffness of gear pairs. The LTCA technology organically combines the geometric analysis and mechanical analysis of the gear. It has the following advantages [31]: (1) the whole tooth profile of large and small gear including tooth root transition surface is obtained by cutter tools generation, and the tooth profile is not simplified. At the same time, the flange structure is considered, and the finite element mesh is automatically generated by programming. The shape of the element is relatively regular, so the gear model itself has high accuracy. (2) The finite element method only needs one calculation to solve the nodal flexibility matrix of working tooth surface, while the normal flexibility matrix of contact line discrete points at different meshing positions in the meshing period can be obtained by interpolation. Compared with commercial finite element software, it takes less time and has higher efficiency. (3) It can reflect the micron-level (tooth modification) geometric feature changes of the tooth surface. (4) It can reflect the influence of the micron-level geometric features of the tooth surface on the mechanical properties (strength and vibration).

The finite element models of a driving gear for spiral bevel gear pair, spur gear pair, and DH gear pair are shown in Figure 4.

Taking DH gear as an example, the LTCA model of a DH gear is shown in Figure 5, and the LTCA models of a spiral bevel gear and a spur gear are detailed in [32, 33]. The LTCA model is to simulate and analyze the gear pair under static conditions and then obtain the static transmission error (STE) and static loaded transmission error (SLTE) of the gear pair. A mesh period of DH gear is divided into different mesh positions. Under the action of the total normal load \( P \), the normal deformation \( Z \) of different mesh positions can be obtained. The normal deformation \( Z \) is actually the SLTE expressed by the linear displacement. Converting it into the form of an angle displacement, it can be expressed as

\[ L_{TE} = 3600 \times 180 \left( \frac{Z}{\pi r_{62} \cos \beta_{b}} \right). \]

Here, \( r_{62} \) and \( \beta_{b} \) are the base cylinder radius and base helix angle of the DH gear, respectively.

Then, the amplitude of SLTE can be expressed as

\[ A_{LTE} = \max(L_{TE}) - \min(L_{TE}). \]

Finally, the comprehensive meshing stiffness \( k \) can be expressed as

\[ k(t) = \frac{F_n}{Z}, \]

where \( F_n \) is normal force acting upon the tooth flank.

According to (3)–(5), the SLTE and comprehensive meshing stiffness are two different description methods of the same nature of excitation. Thus, the two can be transformed into each other. Therefore, the fluctuation amplitude of the comprehensive meshing stiffness can be minimized by optimizing the amplitude of SLTE value in the subsequent optimization design.

The LTCA model can be used to accurately calculate the comprehensive meshing stiffness of each gear pair, and the time-varying meshing stiffness of each gear pair can be obtained by expressing it in the form of Fourier series, which can be directly introduced into dynamic equations of two-input two-path STTS.

2.4. Calculation of Gear Meshing-In Impact. In the actual transmission process of gear pair, due to the manufacturing error, installation error, and elastic deformation of gear
teeth, the normal pitches of the driving pinion \( p_b \) and the driven gear \( p_g \) along the line of action are no longer equal, resulting in a relative base pitch difference. The essence of meshing-in impact is that the tooth pair that enters meshing-in advance produces a relative velocity difference along the meshing line at the meshing point. At this time, the position of the driven gear that is about to enter the mesh can be regarded as a slight angle of retraction based on the theoretical meshing position, which is defined as the reverse angle.

The key to calculate the meshing-in impact is to accurately obtain the position of the meshing point on the tooth surface. The two-input two-path STTS includes spiral bevel gear, spur gear, and double-helical gear. According to the geometric position relationship shown in Figure 6, for spur and DH gears with standard involute tooth surface, the reverse angle of the driven gear can be obtained by employing the reversal method. Then, the position of the meshing-in point of the standard tooth surface can be obtained [28]. In Figure 6, \( \Delta \phi_{kg} \) is the reverse angle of the driven gear, and \( \Delta \phi_{kp} \) is the reverse angle of the driving pinion. Point \( D \) is the actual meshing point, and the standard tooth surface contains meshing interference at this point. Point \( E \) is the normal engagement point, and \( E' \) is the reversal point of the normal engagement point. \( r_p \) and \( r_g \) are pitch circle radii, and \( r_{bg} \) and \( r_{bp} \) are base circle radii of pinion and gear, respectively. \( r_{ag} \) is the addendum circle radius of the gear.

The exact position of the initial meshing-in point of engagement, \( D \), can be accurately determined as follows:

\[
r_{o,D} = \sqrt{r_{ag}^2 + (r_p + r_g)^2 - 2r_{ag}(r_p + r_g)\cos(y_g + \phi_k + \Delta \phi_{kg})},
\]

Figure 4: Finite element models of driving gear: (a) spiral bevel gear; (b) spur gear; (c) DH gear.

Figure 5: The LTCA model for DH gear.

Figure 6: Schematic diagram of meshing-in impact of standard tooth surface.
where \( \varphi_k = Z_l r_{a_k}, \quad \gamma_a = \pi/2 - \alpha_i - \phi \), \( \alpha_i \) is the transverse pressure angle.

Due to the complex tooth surface, it is difficult to determine the position of the initial meshing point through geometric analysis methods for spiral bevel gear and modified DH gear. In this paper, the reversal angle of the above-driven gear is calculated based on TCA and LTCA technology. Then, the initial meshing point position is determined. As shown in Figure 7, point A is the intersection of STE and SLTE curves, which represents the theoretical meshing point. \( L_{TE2} \) is the SLTE value of the current meshing tooth pair (tooth pair 2) at the meshing point. \( T_{E1} \) is the STE value of the previous meshing tooth pair (tooth pair 1) to the corresponding meshing point. Parameter \( \Delta \varphi \) is the backward angle of the driven gear in theoretical engagement position, which can be expressed as

\[
\Delta \varphi = |L_{TE2} - T_{E1}|
\]  

(7)

The rotation angle of the driven gear at the actual meshing point can be expressed as

\[
\varphi_2^* = \varphi_2 + \Delta \varphi,
\]

(8)

where \( \varphi_2 \) is the theoretical rotation angle of the driven gear at the engagement position.

By employing a new angle, \( \varphi_2^* \), the TCA of the tooth surface is performed again. Then, the position vector and normal vector of actual meshing-in impact point on the surface is performed again. When the position vector and \( \text{calculation} \), which can be expressed as

\[
\text{calculation}, \quad \text{calculation}
\]

(9)

where \( v_{np} \) and \( v_{ng} \) are normal velocities of pinion and gear at the meshing point, respectively.

Then, the meshing-in impact of gear pair is calculated via impact mechanics method. The maximum meshing-in impact can be expressed as [29]

\[
F_s = \left( \frac{n + 1}{2}, \quad \frac{I_p I_g}{J_p^2 k_y + J_g^2 k_y} \right)^{1/n},
\]

(10)

where \( I_p \) and \( I_g \) are the deformations of inertia of pinion and gear, respectively. \( N \) is the deformation coefficient under the static state, and \( k_s \) is single meshing stiffness at the engagement point.

The meshing-in impact of each gear pair is expressed as the sum of Fourier series and can be directly introduced into dynamic equation of two-input two-path STTS.

### 3. Dynamic Model of Two-Input Two-Path STTS

Figure 8 shows the dynamic model of the two-input two-path STTS. In Figure 8, \( m_i \) and \( I_i \) are the mass and moment of inertia of each gear, respectively. \( m_b \) and \( I_b \) are the mass and moment of inertia of the central output gear \( Z_b \). \( T_{L1} \) and \( T_{R1} \) are the left- and right-input torques. \( T_{out} \) is the output torque on the central output gear \( Z_b \). \( k_{i12}, k_{i34}, k_{i35}, k_{i68}, \) and \( k_{i78} \) are the comprehensive meshing stiffness of each gear pair. \( c_{i12}, c_{i34}, c_{i35}, c_{i68}, \) and \( c_{i78} \) are the normal meshing damping of each gear pair. \( k_{i23}, k_{i46}, \) and \( k_{i57} \) are the torsional stiffness of dual-gear shaft. \( c_{i23}, c_{i46}, \) and \( c_{i57} \) are the torsional damping of dual-gear shaft. \( \varphi_0 \) represents torsional freedom of each gear about its Z-axis. \( \varphi_8 \) represents torsional freedom of central output gear \( Z_b \) about its Z-axis. \( k_{i13}, k_{i24}, \) and \( k_{i54} \) denote the support stiffness of each gear in the X-direction. \( c_{i13}, c_{i24}, \) and \( c_{i54} \) denote the support damping of each gear in the X-direction. \( k_{i13}, k_{i24}, \) and \( k_{i54} \) indicate the support stiffness of each gear in the Y-direction. \( c_{i13}, c_{i24}, \) and \( c_{i54} \) indicate the support damping of each gear in the Y-direction. The rest of support stiffness and support damping are not shown in the figure.

Dynamic model of two-input two-path STTS represents a vibration system with 49 degrees of freedom. Its generalized displacement matrix \( X \) can be expressed as

\[
X = \begin{pmatrix}
X_{h1}, Y_{h1}, Z_{h1}, \varphi_{h1}, X_{h2}, Y_{h2}, Z_{h2}, \varphi_{h2}, X_{h3}, Y_{h3}, Z_{h3}, \varphi_{h3}, X_{h4}, Y_{h4}, Z_{h4}, \varphi_{h4}, X_{h5}, Y_{h5}, Z_{h5}, \varphi_{h5}, X_{h6}, Y_{h6}, Z_{h6}, \varphi_{h6}, X_{h7}, Y_{h7}, Z_{h7}, \varphi_{h7}, X_{h8}, Y_{h8}, Z_{h8}, \varphi_{h8}
\end{pmatrix}^T
\]  

(11)
where \( x, y, \) and \( z \) are the microdisplacements for two-input two-path STTS in each direction and \( \phi \) is the microrotation angle for two-input two-path STTS in Z-direction.

Normal relative displacement along the meshing line of each gear pair can be expressed as

\[
\begin{align*}
    p_{i12}(t) &= a_{ix}(x_{i1} - x_{i2}) + a_{iy}(y_{i1} - y_{i2}) + a_{iz}(z_{i1} - z_{i2}) + P_{ij}(r_{ab1}\varphi_{i1} - r_{ab2}\varphi_{i2}), \\
    p_{i34}(t) &= (x_{i3} - x_{i4})\cos \zeta_{i34} + (y_{i3} - y_{i4})\sin \zeta_{i34} + r_{ab3}\varphi_{i3} - r_{ab4}\varphi_{i4} - e_{i34}(t), \\
    p_{i35}(t) &= (x_{i3} - x_{i5})\cos \zeta_{i35} + (y_{i3} - y_{i5})\sin \zeta_{i35} + r_{ab3}\varphi_{i3} - r_{ab5}\varphi_{i5} - e_{i35}(t), \\
    p_{i68}(t) &= ((x_{i6} - x_{i8})\cos \zeta_{i68} + (y_{i6} - y_{i8})\sin \zeta_{i68} + r_{ab6}\varphi_{i6} - r_{ab8}\varphi_{i8} - e_{i68}(t))\cos \beta_{i6}, \\
    p_{i78}(t) &= ((x_{i7} - x_{i8})\cos \zeta_{i78} + (y_{i7} - y_{i8})\sin \zeta_{i78} + r_{ab7}\varphi_{i7} - r_{ab8}\varphi_{i8} - e_{i78}(t))\cos \beta_{i7}, \\
\end{align*}
\]

where \( p_{i12}(t) \) is the equivalent meshing line displacement of first-stage spiral bevel gear pair, \( p_{i34}(t) \) and \( p_{i35}(t) \) are the equivalent meshing line displacement of second-stage spur gear pair, and \( p_{i68}(t) \) and \( p_{i78}(t) \) are the equivalent meshing line displacement of third-stage DH gear pair. Parameters \( a_{ix}, a_{iy}, \) and \( a_{iz} \) are spiral bevel gear calculation coefficients.

The meshing force between the teeth of each gear pair can be expressed as

\[
\begin{align*}
    F_{i12} &= k_{i12} \cdot \dot{p}_{i12} + c_{i12} \cdot \ddot{p}_{i12}, \\
    F_{i34} &= k_{i34} \cdot \dot{p}_{i34} + c_{i34} \cdot \ddot{p}_{i34}, \\
    F_{i35} &= k_{i35} \cdot \dot{p}_{i35} + c_{i35} \cdot \ddot{p}_{i35}, \\
    F_{i68} &= k_{i68} \cdot \dot{p}_{i68} + c_{i68} \cdot \ddot{p}_{i68}, \\
    F_{i78} &= k_{i78} \cdot \dot{p}_{i78} + c_{i78} \cdot \ddot{p}_{i78},
\end{align*}
\]
3.1. Dynamic Equations of First-Stage Spiral Bevel Gear.
Dynamic equations for the left- and right-input first-stage spiral bevel gear can be expressed as
\[
\begin{align*}
\mathbf{m}_1 \ddot{x}_1 + c_{ix1} \dot{x}_1 + k_{ix1} x_1 &= -F_{i12x}, \\
\mathbf{m}_1 \ddot{y}_1 + c_{iy1} \dot{y}_1 + k_{iy1} y_1 &= -F_{i12y}, \\
\mathbf{m}_1 \ddot{z}_1 + c_{iz1} \dot{z}_1 + k_{iz1} z_1 &= -F_{i12z}, \\
\mathbf{I}_1 \ddot{\psi}_1 &= T_{i1} - F_{i12r_{01}} - F_{i12r_{01}}.
\end{align*}
\]
where \( T_{i1} \) is the input torque of a spiral bevel gear, \( F_{i12} \) is the meshing-in impact of spiral bevel gear, and \( r_{01} \) and \( r_{02} \) are the equivalent pitch radii of spiral bevel gear, respectively. Parameters \( F_{i12x_i} = a_{ix}(F_{in12} + F_{i12}), \) \( F_{i12y_i} = a_{iy}(F_{in12} + F_{i12}), \) and \( F_{i12z_i} = a_{iz}(F_{in12} + F_{i12}) \) are the component forces in each coordinate axis direction of a spiral bevel gear.

3.2. Dynamic Equations of Second-Stage Spur Gear.
Dynamic equations for the left- and right-input second-stage spur gear can be expressed as
\[
\begin{align*}
\mathbf{m}_3 \ddot{x}_3 + c_{ix3} \dot{x}_3 + k_{ix3} x_3 &= -(F_{m34} + F_{i34}) \cos \zeta_{i34} - (F_{in35} + F_{i35}) \cos \zeta_{i34}, \\
\mathbf{m}_3 \ddot{y}_3 + c_{iy3} \dot{y}_3 + k_{iy3} y_3 &= -(F_{m34} + F_{i34}) \sin \zeta_{i34} - (F_{in35} + F_{i35}) \sin \zeta_{i34}, \\
\mathbf{I}_3 \ddot{\psi}_3 &= c_{i32}(\psi_{i2} - \phi_{i3}) + k_{i32}(\psi_{i2} - \phi_{i3}) = -(F_{in34} + F_{i34}) r_{i3} - (F_{in35} + F_{i35}) r_{i5}, \\
\mathbf{m}_4 \ddot{x}_4 + c_{ix4} \dot{x}_4 + k_{ix4} x_4 &= (F_{m34} + F_{i34}) \cos \zeta_{i34}, \\
\mathbf{m}_4 \ddot{y}_4 + c_{iy4} \dot{y}_4 + k_{iy4} y_4 &= (F_{m34} + F_{i34}) \sin \zeta_{i34}, \\
\mathbf{I}_4 \ddot{\psi}_4 &= c_{i46}(\psi_{i4} - \phi_{i6}) + k_{i46}(\psi_{i4} - \phi_{i6}) = (F_{in34} + F_{i34}) r_{i4}, \\
\mathbf{m}_5 \ddot{x}_5 + c_{ix5} \dot{x}_5 + k_{ix5} x_5 &= (F_{m35} + F_{i35}) \cos \zeta_{i35}, \\
\mathbf{m}_5 \ddot{y}_5 + c_{iy5} \dot{y}_5 + k_{iy5} y_5 &= (F_{m35} + F_{i35}) \sin \zeta_{i35}, \\
\mathbf{I}_5 \ddot{\psi}_5 &= c_{i57}(\psi_{i5} - \phi_{i7}) + k_{i57}(\psi_{i5} - \phi_{i7}) = (F_{m35} + F_{i35}) r_{i5}.
\end{align*}
\]
where \( r_{i3}, r_{i4}, \) and \( r_{i5} \) are the base circle radii of second-stage spur gear, respectively. \( F_{i34} \) and \( F_{i35} \) are the meshing-in impact of spur gear pair.

3.3. Dynamic Equations of Third-Stage DH Gear.
Dynamic equations for the left- and right-input third-stage DH gear can be expressed as
\[
\begin{align*}
\mathbf{m}_6 \ddot{x}_6 + c_{ix6} \dot{x}_6 + k_{ix6} x_6 &= -(F_{in68} + F_{i68}) \cos \zeta_{i68} \cos \beta_6, \\
\mathbf{m}_6 \ddot{y}_6 + c_{iy6} \dot{y}_6 + k_{iy6} y_6 &= -(F_{in68} + F_{i68}) \sin \zeta_{i68} \cos \beta_6, \\
\mathbf{I}_6 \ddot{\psi}_6 + c_{i64}(\psi_{i4} - \phi_{i6}) + k_{i64}(\psi_{i4} - \phi_{i6}) &= -(F_{in46} + F_{i46}) \cos \beta_{i6}, \\
\mathbf{m}_7 \ddot{x}_7 + c_{ix7} \dot{x}_7 + k_{ix7} x_7 &= -(F_{in78} + F_{i78}) \cos \zeta_{i78} \cos \beta_7, \\
\mathbf{m}_7 \ddot{y}_7 + c_{iy7} \dot{y}_7 + k_{iy7} y_7 &= -(F_{in78} + F_{i78}) \sin \zeta_{i78} \cos \beta_7, \\
\mathbf{I}_7 \ddot{\psi}_7 + c_{i75}(\psi_{i5} - \phi_{i7}) + k_{i75}(\psi_{i5} - \phi_{i7}) &= -(F_{in57} + F_{i57}) \cos \beta_{i7}, \\
\mathbf{m}_8 \ddot{x}_8 + c_{ix8} \dot{x}_8 + k_{ix8} x_8 &= (F_{L68} + F_{L68}) \cos \zeta_{L68} + (F_{L78} + F_{L78}) \cos \zeta_{L78}, \\
\mathbf{m}_8 \ddot{y}_8 + c_{iy8} \dot{y}_8 + k_{iy8} y_8 &= (F_{L68} + F_{L68}) \sin \zeta_{L68} + (F_{L78} + F_{L78}) \sin \zeta_{L78}, \\
\mathbf{I}_8 \ddot{\psi}_8 &= -T_{out} + [(F_{L68} + F_{L68}) \sin \zeta_{L68} + (F_{L78} + F_{L78}) \sin \zeta_{L78}] \cos \beta_{L68} + (F_{L68} + F_{L68}) \sin \zeta_{L68} + (F_{L78} + F_{L78}) \sin \zeta_{L78} \cos \beta_{L68}.
\end{align*}
\]
where \( r_{i68}, r_{i68}, \) and \( r_{i68} \) are the base circle radii of DH gear, respectively. \( T_{out} \) is the output torque, while \( F_{i68} \) and \( F_{i78} \) are the meshing-in impact of DH gear pair, respectively.

3.4. Calculation of the Load Sharing Coefficient. The essence of load sharing coefficient (LSC) is to characterize the difference of power split in the two-path STTS, that is, the
difference in the transmission load of each gear caused by various factors (manufacturing error, installation error, and vibration). A Runge-Kutta method is used to solve the dynamic equations of the system. It should be noted that in the process of using coordinate transformation to eliminate the rigid body displacement of the system, for the two-input two-path STTS, there is a rigid body displacement in the left engine closed-loop system and the right engine closed-loop system, which need to be eliminated together. Finally, a dynamic model with 48 degrees of freedom can be obtained.

The instantaneous LSC in each tooth frequency cycle of each path of the left- and right-input second stage and third stage is defined as follows:

\[
\text{LSC}_{34} = \frac{2(F_{id34i})_{\text{max}}}{(F_{id34i})_{\text{max}} + (F_{id35i})_{\text{max}}},
\]

\[
\text{LSC}_{35} = \frac{2(F_{id35i})_{\text{max}}}{(F_{id34i})_{\text{max}} + (F_{id35i})_{\text{max}}},
\]

\[
\text{LSC}_{68} = \frac{2(F_{id68i})_{\text{max}}}{(F_{id68i})_{\text{max}} + (F_{id78i})_{\text{max}}},
\]

\[
\text{LSC}_{78} = \frac{2(F_{id78i})_{\text{max}}}{(F_{id68i})_{\text{max}} + (F_{id78i})_{\text{max}}},
\]

where \( k_1 = 1, \ldots, m_1 \) and \( k_2 = 1, \ldots, m_2 \). Parameters \( m_1 \) and \( m_2 \) are meshing gear frequency cycles of second stage and third stage within the system period, respectively. \( (F_{id34i})_{\text{max}} \) and \( (F_{id35i})_{\text{max}} \) are maximum meshing forces in the \( k_1 \) tooth frequency cycle of the second stage, while \( (F_{id68i})_{\text{max}} \) and \( (F_{id78i})_{\text{max}} \) are the maximum meshing forces in the \( k_2 \) tooth frequency cycle of the third stage.

The LSC of the system period of the left- and right-input second stage and third stage is defined as follows:

\[
\text{LSC}_{\text{left}} = \max(\text{LSC}_{34}, \text{LSC}_{35}),
\]

\[
\text{LSC}_{\text{right}} = \max(\text{LSC}_{68}, \text{LSC}_{78}).
\]

Therefore, the LSC of the left- and right-input two-path STTS can be expressed as

\[
\text{LSC}_i = \max(\text{LSC}_{\text{left}}, \text{LSC}_{\text{right}}).
\]

### 4. DH Gear Modification Optimization Design

Topological modification includes tooth profile modification and longitudinal modification, both of which are composed of two quadratic parabolas and a straight line, as shown in Figure 9. Parameters \( y_1 \) and \( y_2 \) are the maximum modification amount and modification length of the tooth root, respectively. Parameters \( y_3 \) and \( y_4 \) are the maximum modification amount and modification length of the tooth top, respectively. \( y_5 \) is the maximum modification amount at both tooth ends. \( y_6 \) is the length of the nonmodification area in the tooth direction. \( H \) and \( L \) are the effective tooth height and tooth length, respectively. \( s_1, s_2, s_3, \) and \( s_4 \) are the location coordinates of the modification points. For topological modification of DH gear (driving gear), left and right helical gear modification methods are the same.

Topological modification curve has to be defined prior to TCA simulation. The general form of the modification curve is as follows:

\[
C_u\left(\frac{s - s_2}{s_1 - s_2}\right)^c, \quad s_1 \leq s \leq s_2,
\]

\[
0, \quad s_2 < s < s_3,
\]

\[
C_q\left(\frac{s - s_3}{s_4 - s_3}\right)^c, \quad s_3 \leq s \leq s_4,
\]

where \( \xi \) is the modification amount, \( s \) is the modification point coordinate, \( s \in [s_1, s_2] \), \( C_u \) and \( C_q \) are modification values, and \( c \) is the index of the modification curve.

According to the modification curve, the grid node amplitude of SLTE and meshing-in impact are the logical modification of DH gear (driving gear), left and right helical gear modification methods are the same.

The modified tooth surface is constructed by superimposing the theoretical tooth surface and the modification amount surface. Its position vector and normal vector can be expressed as

\[
R_1(u_1, l_1) = r_1(u_1, l_1) + \xi(u, v)n_1(u_1, l_1),
\]

\[
N_1(u_1, l_1) = \left(\frac{\partial r_1}{\partial u_1} + \frac{\partial \xi}{\partial u_1}n_1 + \frac{\partial n_1}{\partial u_1}\right) \times \left(\frac{\partial r_1}{\partial l_1} + \frac{\partial \xi}{\partial l_1}n_1 + \frac{\partial n_1}{\partial l_1}\right).
\]

where \( u_1 \) and \( l_1 \) are the tooth surface parameters, and \( r_1(u_1, l_1) \) and \( n_1(u_1, l_1) \) are the position and normal vectors of the DH pinion theoretical tooth surface, respectively.

The amplitude of SLTE and meshing-in impact are the main factors affecting the vibration and noise of the DH gear. In this paper, the optimization objective is to minimize the synthesis of the two. Parameters \((y_1, y_2, y_3, y_4, y_5, \) and \( y_6)\)
are employed for optimization. Modification optimization design model is represented as follows:

\[
\begin{align*}
F_1(y_1, y_2, y_3, y_4, y_5, y_6) &= \min \left( \frac{f_1}{f_{10}} \right), \\
F_2(y_1, y_2, y_3, y_4, y_5, y_6) &= \min \left( \frac{f_2}{f_{20}} \right), \\
\text{S.t} & \quad q_{\min} \leq y_1, y_2, y_5 \leq q_{\max}, \\
& \quad h_{\min} \leq y_3, y_4 \leq h_{\max}, \\
& \quad l_{\min} \leq y_6 \leq l_{\max},
\end{align*}
\]

(23)

where \(f_{10}\) and \(f_{20}\) are the amplitude of SLTE and meshing-in impact, respectively, of a nonmodified DH gear. \(f_1\) and \(f_2\) are the amplitude of SLTE and meshing-in impact, respectively, of a modified DH gear. Parameters \(q_{\min}\) and \(q_{\max}\) represent the lower and the upper bound, respectively, for \(y_1, y_2,\) and \(y_5\). \(h_{\min}\) and \(h_{\max}\) are the lower and the upper bound, respectively, for \(y_3\) and \(y_4\). \(l_{\min}\) and \(l_{\max}\) represent the lower and the upper bound, respectively, for \(y_6\).

NSGA-II employs fast nondominated sorting algorithm and a crowded distance comparison operator while introducing an elite strategy, which has low computational complexity and high computational efficiency. Hence, it is used in this paper to solve the aforementioned optimization model. The flowchart of topological optimization of DH gear is shown in Figure 10.

5. Numerical Analysis

Some specific parameters of two-input two-path STTS are shown in Table 1.

TVS and meshing-in impact of first-stage spiral bevel gear are shown in Figure 11. The maximum meshing-in impact is equal to 4080.284 N.

TVMS and meshing-in impact of second-stage spur gear are shown in Figure 12. The maximum meshing-in impact is equal to 8046.940 N.

Optimized DH pinion topological modification tooth surface is shown in Figure 13. The optimized modification parameters are \(y_1 = 0.023\) mm, \(y_2 = 0.035\) mm, \(y_3 = 2.30\) mm, \(y_4 = 2.01\) mm, \(y_5 = 0.013\) mm, and \(y_6 = 8.19\) mm.

TVS and meshing-in impact of third-stage DH gear before and after modification are shown in Figure 14. The maximum meshing-in impact is equal to 3909.481 N before modification and 1084.720 N after modification. The meshing-in impact is reduced by 72.25% after modification.

In Figure 14 and the following figures, “non-mod” represents the results of a DH pinion tooth surface with non-modification, while “mod” represents the results of a DH pinion tooth surface with the topological modification.

5.1. Dynamic Analysis of Two-Input Two-Path STTS. In two-input two-path STTS, the meshing period of the first-stage gear pair is the shortest, while that of the third-stage gear pair is the longest. In order to consider the influence of meshing-in impact of all gear pair levels, the following...
simulation calculation uses 1/100 of the meshing period of the first-stage gear pair as the step length. The simulation result employs the calculation results of the last 1500 meshing periods of the third-stage gear pair.

When the DH pinion is not modified, the influence of the installation error and eccentricity error of the second-stage spur gear and third-stage DH gear on the left-input two-path STTS LSC is shown in Figure 15. The law of the right-input two-path STTS LSC is similar to the law of the left-input two-path STTS. When a certain error emerges, the remaining errors are nullified and remain unaltered.

According to Figure 15, the LSC of the left-input two-path STTS linearly increases with an increase of the installation error (eccentricity error). In other words, the greater the installation error (eccentricity error), the greater the system’s LSC, and the worse the load sharing
The influence of third-stage DH gears installation error (eccentricity error) on LSC is greater than that of the second-stage spur gears. Under the action of gear comprehensive installation error, LSC may be less than or greater than that of gear installation error alone. However, under the action of gear comprehensive eccentricity error, LSC must be greater than that under the action of the gear eccentricity error alone. To summarize, the influence of the eccentricity error on STTS load sharing performance is greater than that of the installation error. In gear design, the eccentricity error of the third-stage DH gears must be strictly controlled.

In the subsequent analysis, errors of second and third-stage gears are selected according to the gear accuracy grade of 5, as shown in Table 2 [41].

Dynamic meshing force of first-stage gear pair of the left-input two-path STTS with and without considering meshing-in impact is shown in Figure 16. Dynamic force of the
first-stage gear pair slightly increases when meshing-in impact is accounted for. Since this paper only analyzes the load sharing and dynamic load of the second and third-stage gear pairs of the system, the dynamic characteristics of the first-stage gear pair are not discussed too much.

The maximum meshing force in the tooth frequency cycle of a second and third-stage gear pair of the left-input two-path STTS with and without considering meshing-in impact is shown in Figure 17.

According to the ISO standard for gears, the dynamic load factor is defined as the ratio of dynamic load to the static load. The dynamic load factor of the second-stage spur gear pair of the STTS is increased from 1.41596 without considering meshing-in impact to 1.42443 with considering meshing-in impact, and the dynamic load performance is reduced by 2.00%. The dynamic load factor of the third-stage DH gear pair of the STTS is increased from 1.20645 without considering meshing-in impact to 1.20669 with considering meshing-in impact, and the dynamic load performance is reduced by 0.12%.

The instantaneous LSC of the second-stage spur gear pair and third-stage DH gear pair with and without the consideration of meshing-in impact is shown in Figure 18. According to the LSC definition, its value cannot be lower than 1. In other words, the average system load without error is the load that the path transmission must bear. The LSC of the second-stage spur gear pair decreases from 1.06387 without considering meshing-in impact to 1.06332 with considering meshing-in impact, thus increasing the load sharing performance by 0.86%. The LSC of the third-stage DH gear pair decreases from 1.09669 without considering meshing-in impact to 1.09603 with considering meshing-in impact, thereby increasing the load sharing performance by 0.68%. The LSC of the left-input two-path STTS depends on the LSC of the third-stage, which is equal to $LSC_L = 1.09603$ with consideration of meshing-in impact. When accounting for meshing-in impact, although the dynamic meshing force of the system increases, the increase of denominator in (17) is larger than that of the numerator, which ultimately leads to a slight decrease in the LSC of the STTS. It should be noted that if the increase of the numerator in (17) is greater than the increase of the denominator after the meshing-in impact is considered, the LSC of the system will increase. However, after considering the meshing-in impact, the dynamic meshing force of the system will inevitably increase, and the dynamic load characteristics will inevitably decrease, which is unfavorable to the transmission characteristics of the entire system.

5.2 Influence of Modification on Load Sharing and Dynamic Load Characteristics. In this paper, the design load of DH gear pair is equal to 17500 Nm, and the theoretical contact ratio of DH gear pair is 3.42. Figure 19 shows the waveform change of SLTE under multiple loads for DH gear pair before and after modification. SLTE is illustrated three times by shifting the angular pitch. The rightmost column of Figures 19(a) and 19(b) shows the amplitude of SLTE of DH gear pair.

Figure 19(a) shows the case of the DH gear before modification. For DH gear with a standard tooth surface, the TE is approximately zero. In the process from entering meshing to exiting meshing, the meshing of quadruple tooth pairs and the meshing of triple tooth pairs alternately change. The SLTE of triple tooth pairs region is greater than that of quadruple tooth pairs. Because there is no STE to compensate for the difference of SLTE in different meshing zones, as the load increases, the change of the SLTE waveform gradually increases. In other words, the amplitude of SLTE increases linearly with the increase of the load, as shown by the black curve in Figure 19(c).

Figure 19(b) shows the case of DH gear after topological modification. When load is increased from the initial value of 1500 Nm to the design load, the amplitude of SLTE increases first and then decreases. This is due to the reduction of the contact ratio of the gear pair caused by the modification. When the load is small, there is an alternation of triple tooth pairs meshing and double tooth pairs meshing in the process of the tooth from meshing-in to meshing-out. When the load exceeds the design load, the amplitude of SLTE continues to gradually increase. This is because with the increase of the load, the actual contact ratio of the gear

![Graph showing the dynamic meshing force of first-stage gear pair on the meshing line.](image)

**Figure 16:** The dynamic meshing force of first-stage gear pair on the meshing line.
pair gradually increases and finally reaches the theoretical contact ratio. In the process of tooth entering meshing to exiting meshing, the quadruple tooth pairs meshing and triple tooth pairs meshing alternate, which is the same as the meshing situation of the DH gear before modification. However, because the STE after modification compensates the difference of SLTE in different meshing areas, the amplitude of SLTE after modification is still much smaller than that before modification. Before and after modification, the amplitude of SLTE changes from 2.495" to 0.220", respectively; i.e., it decreased by 91.18%, as shown in Figure 19(c).

The comprehensive meshing stiffness for DH gear pair under multiple loads before and after modification is shown in Figure 17: The maximum dynamic meshing force of gear pair with and without considering meshing-in impact: (a) second-stage spur gear pair; (b) third-stage DH gear pair.

Figure 17: The maximum dynamic meshing force of gear pair with and without considering meshing-in impact: (a) second-stage spur gear pair; (b) third-stage DH gear pair.

Figure 18: Instantaneous LSC of second and third stage with and without considering meshing-in impact: (a) instantaneous LSC for second-stage spur gear pair; (b) instantaneous LSC for third-stage DH gear pair.
| Torque (Nm) | Static Transmission Error (arc sec) |
|------------|------------------------------------|
| 1.150      | 0.223                              |
| 2.250      | 0.365                              |
| 3.350      | 0.507                              |
| 4.450      | 0.649                              |
| 5.550      | 0.791                              |
| 6.650      | 0.933                              |
| 7.750      | 1.075                              |
| 8.850      | 1.217                              |
| 9.950      | 1.359                              |
| 10.100     | 1.501                              |
| 11.125     | 1.785                              |
| 12.145     | 2.069                              |
| 13.175     | 2.495                              |
| 14.195     | 2.782                              |
| 15.215     | 3.059                              |
| 16.235     | 3.350                              |
| 17.255     | 3.630                              |
| 18.275     | 3.910                              |
| 19.295     | 4.200                              |
| 20.315     | 4.480                              |

**Figure 19:** Continued.
in Figure 20. Similarly, comprehensive meshing stiffness is illustrated three times by shifting the angular pitch. The rightmost column in Figures 20(a) and 20(b) indicates the average value of the meshing stiffness of DH gear pair.

Corresponding to Figures 19(a), and 20(a) represents the comprehensive meshing stiffness of the DH gear before modification. Since the DH gear has a standard involute tooth surface, the two tooth surfaces of the pinion and gear are completely conjugated under load. Furthermore, each meshing moment is characterized by a linear contact. Thus, with an increase in load, the average meshing stiffness increases slightly.

Corresponding to Figures 19(b), and 20(b) represents the comprehensive meshing stiffness of the DH gear after modification. The modification destroys the conjugate characteristics of the standard tooth surface, while pinion and gear tooth surface changes from a line contact to the point contact. The length of the contact line and the area of the contact ellipse of the modified tooth surface gradually increase with an increase in load. When load is increased from 1500 Nm to the design load, the average meshing stiffness of the modified DH gear pair rapidly increases. When subjected to the design load, the average meshing stiffness decreases from $1.24269 \times 10^6$ N/m before modification to $1.11100 \times 10^6$ N/m after modification; i.e., it decreases by 10.60%. When the applied load exceeds the design load, the actual contact ratio of the gear pair is close to the theoretical contact ratio. The average meshing stiffness slowly increases, and it approaches the average meshing stiffness before modification, as shown in Figure 20(c).

The research results in Figures 19 and 20 show that, on the one hand, modification reduces the amplitude of SLTE and the fluctuation amplitude of comprehensive meshing stiffness of the DH gear pair. On the other hand, modification leads to the decrease of the average meshing stiffness. The reduction of the fluctuation amplitude of comprehensive meshing stiffness is beneficial to the STTS’s dynamic load characteristics, while the reduction of the average meshing stiffness will have an impact on the STTS’s load sharing characteristics, which will be analyzed below.

The maximum meshing force in the tooth frequency cycle of second and third-stage gear pair of left-input two-path STTS before and after modification is shown in Figure 21. Only the fractional part of the dynamic load coefficient greater than 1 (indicating vibration) can be reduced via gear modification and dynamic design to improve the dynamic performance of gear transmission and reduce vibration level. Dynamic load factor of the STTS second-stage is reduced from 1.42443 before modification to 1.40194 after modification. However, third-stage dynamic load factor is significantly altered. The dynamic load factor is reduced from 1.20669 before modification to 1.13454 after modification, thereby increasing the dynamic load performance by 34.91%. The above research shows that the modification has a good effect on improving the dynamic load characteristics of the STTS third stage.

The instantaneous LSC of the second-stage spur gear pair and third-stage DH gear pair before and after modification is shown in Figure 22. Only the fractional part of the LSC greater than 1 (indicating vibration) can be reduced to improve the load sharing performance of the gear transmission through modification. The LSC of the second-stage spur gear pair decreased from 1.06332 before modification to 1.06183 after modification, thereby increasing the load sharing performance by 2.35%. The LSC of the third-stage DH gear pair decreased from 1.09603 before modification to 1.09203 after modification.
Figure 20: Continued.
modification, therefore increasing the load sharing performance by 4.17%. The LSC of the left-input two-path STTS depends on the LSC of the third stage. After modification, it is equal to \( LSC_L = 1.09203 \). The results indicate that DH gear modification has a certain effect on improving the system’s load sharing performance.

**Figure 21:** The maximum dynamic meshing force of gear pair before and after modification: (a) second-stage spur gear pair; (b) third-stage DH gear pair.

**Figure 20:** Comprehensive meshing stiffness for DH gear pair before and after modification: (a) comprehensive meshing stiffness under multiple loads before modification; (b) comprehensive meshing stiffness under multiple loads after modification; (c) comparison of average meshing stiffness under multiple loads before and after modification.
6. Conclusions

In this paper, the reversal method is used to calculate the meshing-in impact of standard involute spur gears and double-helical gears, and the meshing-in impact of spiral bevel gears and modified double-helical gears is accurately calculated based on tooth contact analysis and loaded tooth contact analysis. A dynamic model of two-input two-path split torque transmission system considering meshing error excitation, time-varying meshing stiffness excitation, and meshing-in impact excitation is proposed. Influence of double-helical gear modification on load sharing and dynamic load characteristics of two-input two-path split torque transmission system is investigated. The main conclusions are listed as follows:

(1) After considering the meshing-in impact, the dynamic meshing force of the gear pair increases, which is unfavorable to the transmission of the split torque transmission system. However, due to an increase in the average value of the maximum meshing force in each meshing period of the two paths, the load sharing coefficient of the system slightly decreases.

(2) After double-helical gear modification, the amplitude of static loaded transmission error, the fluctuation amplitude of comprehensive meshing stiffness, and the meshing-in impact are all reduced. Dynamic load characteristics of the third-stage gear pair are significantly improved, but dynamic load characteristics of the second-stage gear pair are less affected by double-helical gear modification.

(3) After double-helical gear modification, the average meshing stiffness is reduced. Simultaneously, due to an improvement of the split torque transmission system dynamic load characteristics, the load sharing characteristics of the system are also improved to a certain extent.

Abbreviations

| Symbol | Description                          |
|--------|--------------------------------------|
| $z_{ij}$ | Gears                               |
| $i = L_i, R_i$ | Center output double-helical gear  |
| $j = 1 \ldots 7$ | System installation angle          |
| $\theta_{\sigma_1}, \theta_{\sigma_2}$ | System layout angle               |
| $\psi$ | The angle between the center line of each gear pair and positive $X$-axis direction |
| $\eta$ | The angle between the meshing line of each gear pair and the positive direction of the $X$-axis |
| $E_{L3}$, $E_{L4}$ | Eccentricity error of the spur pinion $z_{L3}$ and the spur gear $z_{L4}$ |
| $A_{L3}$, $A_{L4}$ | Installation error of the spur pinion $z_{L3}$ and the spur gear $z_{L4}$ |
| $\lambda_{L3}$, $\lambda_{L4}$ | Phase angle                         |
| $\mu_{L3}$, $\mu_{L4}$ | The meshing line                    |
| $ML_{L34}$ | Equivalent cumulative meshing error on the meshing line of spur gear pair 34 and 35 |
| $e_{34}$, $e_{35}$ | Equivalent cumulative meshing error on the meshing line of double-helical gear pair 68 and 78 |
| $\omega_{L3}$, $\omega_{L4}$ | Angular velocities of spur gear $z_{L3}$, $z_{L4}$, and $z_{L5}$ |
| $\omega_{68}$, $\omega_{78}$ | Angular velocities of double-helical gear $z_{68}$, $z_{78}$, and $z_{8}$ |
| $\alpha_{L3}$, $\alpha_{L4}$ | Transverse pressure angle of spur gear and double-helical gear |
| $P_i$ | Total normal load                   |

Figure 22: Instantaneous LSC of second and third stage before and after modification: (a) instantaneous LSC for second-stage spur gear pair; (b) instantaneous LSC for third-stage DH gear pair.
Normal deformation

Static loaded transmission error

Amplitude of static loaded transmission error

Comprehensive meshing stiffness

Reverse angles of the driven gear and driving pinion

Pitch circle radii of pinion and gear

Base circle radii of pinion and gear

Addendum circle radius of the gear

Backward angle of the driven gear

Normal velocities of pinion and gear

Relative normal velocity

Meshing-in impact

Moments of inertia for pinion and gear

Single meshing stiffness at the engagement point

Mass and moment of inertia of gear $z_i$

Mass and moment of inertia of the central output gear $Z_g$

Left- and right-input torques

Output torque on the central output gear $Z_g$

Comprehensive meshing stiffness of each gear pair

Normal meshing damping of each gear pair

Torsional stiffness of dual-gear shaft

Torsional damping of dual-gear shaft

Torsional freedom of each gear about their Z-axis

Torsional freedom of central output gear $Z_g$ about its Z-axis

Support stiffness of each gear in the X-direction

Support damping of each gear in the X-direction

Support stiffness of each gear in the Y-direction

Support damping of each gear in the Y-direction

Equivalent meshing line displacement of spiral bevel gear pair

Equivalent meshing line displacement of spur gear pair

Equivalent meshing line displacement of double-helicical gear pair

Spiral bevel gear calculation coefficients

Meshing force of each gear pair

Meshing-in impact of spiral bevel gear

Equivalent pitch radii of spiral bevel gear

Base circle radii of second-stage spur gear

Meshing-in impact of spur gear pair

Base circle radii of double-helicical gear

Meshing-in impact of double-helicical gear pair

Modification parameters

Localization coordinates of the modification points

Modification values

Tooth surface parameters

Position and normal vectors of the double-helicical pinion theoretical tooth surface

Lower and upper bound for $y_1, y_2$, and $y_3$

Lower and upper bound for $y_3$ and $y_4$

Lower and upper bound for $y_5$

NSGA-II: Nondominated sorting genetic algorithm-II

STTS: Split torque transmission system

DH gear: Double-helicical gear

TVMS: Time-varying meshing stiffness

TCA: Tooth contact analysis

LTCA: Loaded tooth contact analysis

STE: Static transmission error

SLTE: Static loaded transmission error

LSC: Load sharing coefficient

Non-mod: Modification

Mod: Nonmodification

Data Availability

All the data supporting this paper are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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