Modelling of the energy transfer during scrap melting in microwave furnace

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Abstract. Efficient recovery and processing of metal waste is a priority for today’s economy. Several technologies are dealing with the recovery of metals through furnace melting but the extraction efficiencies are rather low. The paper studies the influence of heat transfer, produced by the action of microwaves, on the melting of metal waste batch. Correlation equations of the heat flux transmitted through the crucibles inside the oven were calculated by means of MATLAB modeling. Specific dimensions for the furnace enclosure were used in the paper. The simulation results were then compared to the experimental findings, to verify the model. The experimental stage consisted in the melting of aluminum and copper waste in the microwave installation, at laboratory level, and at the optimized parameters of the system.

1. Introduction
The recycling strategy is an important factor in today’s demand for a sustainable growth of the economy. Several aspects have to be taken into account during the development of a new recycling technology: diversity in recovery, recovery level, material consumption and energy efficiency. The energy that is spent in obtaining certain materials from scrap could be a major factor in deciding that the technology is industrially viable. The working temperature has a decisive influence on the quantity and quality of the metal / alloy produced. The increase of the temperature has the effect of accentuating the physical-chemical processes in the system.

Due to the superior efficiency in the heating process, microwave (MW) technology is increasingly used in various industrial fields [1]. The entire process of heat generation, transmission and conversion, that takes place in a MW furnace, is mainly carried out in two stages: microwaves field interaction, and heat transfer through the refractory material and the scrap batch. The heat generated by the microwaves in the ceramic material is uniform and efficient [2]. The heat transfer in crucible and charge is much slower and represents the rate-controlling step in the whole heating process.

The simulation of heat transfer in MW melting is very important for the metal recovery level from scrap and for reduction of energy consumption. Heat transfer processes are modeled either through analytical or numerical methods [3]. Recently, the analytical methods lost their efficiency due to the
complicated mathematical equations that result especially in systems with complex geometries. The numerical models are used extensively in research and industry for determining the temperature profile at any specific time and space, for various systems with complicated geometries. However, these methods apply several approximations and are not as precise as the analytical methods.

Previous research in the field showed various ways to determine the temperature profile of the MW furnace heating process by analytical modelling of particulate materials (sintering) but very little was studied about metal microwave melting. A direct estimation of heating time was provided in the paper by Chandrasekaran et al. [4]. A basic equation provided by overall energy balance of the system was solved for quadratic solutions. The model verification showed several discrepancies with the experimental results, which were attributed to the heat losses in the SiC susceptor, melting crucible and charge. A complete heat transfer study was not performed by authors. The underlaying equations for microwave direct heating of the crucible in a scrap melting furnace was provided by present authors in [5]. A detailed heat transfer, as taking into account the actual heat transfer between the system components, was not performed in the paper.

Present study is discussing the analytical modelling of the crucible-charge system during the metal scrap processing in a microwave melting furnace. The equations following the physical evaluations where calculated with MATLAB modelling program. The study is specific to the configuration of the crucible-charge system designed by our team. The modelling results were correlated to the experimental trials. The experimental stage consisted in the melting of aluminium and copper waste in the microwave installation, at laboratory level, and at the optimized parameters of the system. During the experimental works, heating tests of the oven with or without loading were carried out. Also, two sets of experiments were performed, one for aluminium waste and the other for Cu waste.

2. Experimental and methods
Heat transfer follows the two principles of thermodynamics: the first principle that expresses the law of heat energy conservation in the transfer processes and the second principle according to which the heat transfer is always performed from a higher temperature to a lower temperature. Heat transmission is carried out in three ways: conduction, convection and radiation. In order to determine the heating time of the crucible and the final melting time of the batch, the process of heat transfer through conduction under transient mode is considered. Modelling was performed based on the theoretical background for transient heat transfer of conduction, convection and radiation, with the Fourier, Newton and Boltzmann equations. By applying the energy balance to specific areas, there was determined the temperature-time gradient in the system. MATLAB® modelling software was used for solving mathematical equations and for graphical representation.

The waste processing plant consisted of a specialized microwave oven for melting metal alloys or wastes. The furnace contains 5 magnetrons with the individual power of 1kW. The melting chamber contains a 1 litter SiC crucible which is heated directly by the microwave field (susceptor) and a graphite crucible in which the melting batch is inserted. The aluminium or copper scrap was pressed into a mould on a hydraulic press, up to 75-80% compaction grades. Then the compacted metal waste/pieces were inserted into the melting crucible, mixed with the protective and refining flux. The process of melting the metal scrap started with oven generators at maximum power (5 × 850 W each). Temperature recording was done by means of a thermocouple inserted into the graphite crucible chamber.

3. Results and discussion
The heat transfer in the heating system of the furnace involves several stages and depends first and foremost on its design elements. The heating system consists of two crucibles placed concentric in the central area of the oven. The design elements are shown in figure 1.
Figure 1. The thermal flow generated by microwave field in the system SiC crucible – charge.

From figure 1 it is observed that the heat flow generated by the microwave is uniformly distributed over the entire surface of the crucible, but the process of heat transfer is different due to the geometrical configuration of the melting chamber. Thus, between the SiC and graphite crucibles there is a noticeable space at the lateral areas, of the cylindrical shape, which implies the existence of a heat transfer by convection, while the lower part of the crucibles is in the form of a circular plate, and the graphite crucible is supported by the SiC crucible and its walls come into contact, and the heat transfer is carried out by conduction. Therefore, we have two distinct sectors of heat transfer: a. The upper part of the crucible and b. The lower part of the crucible (circular surface).

To determine the modality of heat transfer, an energy balance of the system is performed [6].

For the charge subsystem, the energy required to heat the charge \( E_{ch} \) is given by the caloric energy received by the charge by heat transfer \( E_{st} \) minus the energy lost with the outside by convection \( E_{pc} \) and radiation \( E_{pr} \):

\[
E_{ch} = E_{st} - E_{pc} - E_{pr}
\]  

(1)

Since the charge is heated from the side and bottom the energy losses can only be through the top of the crucible. To determine the energy transferred to the charge, energy conservation will be applied to the SiC crucible and graphite system. Thus the energy transferred to the crucible charge is the energy generated by the microwave in the active areas of the SiC \( E_{m} \) crucible, less the energy consumed for SiC crucible heating \( E_{isc} \), less the energy consumed when heating the space between crucibles \( E_{isp} \), less the energy consumed when heating the crucible of graphite \( E_{igr} \), less the energy consumed when heating the pores (space) between the charge pieces \( E_{por} \), less the total energy lost to the heat transfer through materials \( E_{tr} \):

\[
E_{tr} = E_{m} - E_{isc} - E_{isp} - E_{igr} - E_{por} - E_{tr}
\]

(2)

Next, we will treat each portion separately to derive intermediate relations to find a function of time-related temperature \( T \) as a function of \( t \).

The heat transfer of a transient nature is characterized by a permanent variation of the temperatures at the interface of the transfer zones in the system. In order to define the system practically, the values of
these temperatures must be determined at any given time. Due to the high complexity of the connection relationships obtained by applying the energy balance on each transfer area, resulting in differential equations with many unknowns, numerical boundary conditions, characteristic to the described physical system, will be applied.

**Condition 1:** The initial temperature at which the heat transfer takes place in the system is the temperature produced by the microwave in the interaction areas of the SiC crucible and is equal to the \( T_{SiC} \). Because the heating time calculated by means of energy balance between the microwave input energy and heat energy produced is extremely small (18 seconds), it can be considered that the \( T_{SiC} \) is above the room temperature. Then:

\[
\frac{dT_{SiC}}{dt} = \text{constant} \tag{3}
\]

The other areas of the system are initially at room temperature

\[
T_{gr lat} (0) = T_{air 1} (0) = T_{charge lat} (0) = T_{air 2} (0) = T_{gr bot} (0) = T_{charge bot} (0) = T_{air 3} (0) = 20^\circ C \tag{4}
\]

Where: \( T_{gr lat} \) is the temperature of the graphite crucible on the lateral side, \( T_{air 1} \) is the air temperature between the SiC and graphite crucibles on the lateral side, \( T_{charge lat} \) is the temperature of the charge on the lateral side, \( T_{air 2} \) is the air temperature between the graphite crucible and the charge on the lateral side, \( T_{gr bot} \) is the temperature of the graphite crucible at the bottom, \( T_{charge bot} \) is the temperature of the graphite at the bottom and \( T_{air 3} \) is the air temperature between the graphite crucible and the charge at the bottom.

**Condition 2:** At the end of the heating process, the scrap temperature reaches an initially set value (in this case at \( T_{charge} = 1200^\circ C \)) and the process exits the transient regime and enters the constant regime. Therefore, the final temperatures of the system are between the values determined using the characteristics of the steady state heat transfer.

By applying the initial condition 1, we have:

\[
q_{lat} = 6760 \cdot \frac{dT_{SiC lat}}{dt} = q_{gen} \tag{5}
\]

Using MATLAB program and the regression method, an approximation of the heat transfer solution with a polynomial equation in \( T_{SiC lat} \) is obtained.

\[
2.63 \cdot 10^{-8} \cdot T_{SiC lat}^4 - 1.46 \cdot 10^{-4} \cdot T_{SiC lat}^3 + 0.379 \cdot T_{SiC lat}^2 - 36.8 \cdot T_{SiC lat} - 1.01 \cdot 10^5 = 0 \tag{6}
\]

The solution of the equation generates the value of 377°C. Therefore, the initial temperature from which the heat transfer begins is 377°C.

By applying condition 2, the application of steady state heat transfer delivers the following equation:

\[
q_{lat} = \text{const} \cdot U_{lat} \cdot \left( \frac{T_{SiC lat} - T_{charge} \text{a}}{\lambda_{SiC lat}} + \frac{1}{\alpha_{air 1}} + \frac{1}{\alpha_{air 2}} + \frac{1}{\alpha_{r1}} + \frac{1}{\alpha_{r2}} \right) \tag{7}
\]

Where: \( q_{lat} \) is the thermal flow at steady state, in the lateral side; \( U_{lat} \) is the total resistance of the system in the lateral side; \( \text{const} \) is the convection coefficient of the air between the crucibles; \( \alpha_{air 1} \) is the convection coefficient of the air between the crucibles; \( \alpha_{air 2} \) is the radiation coefficient between the crucibles; \( \alpha_{r1} \) is the radiation coefficient between the crucibles; \( \alpha_{r2} \) is the radiation coefficient between the crucibles; \( \lambda_{SiC lat} \) is the thermal conductivity of SiC, 120W/mK; \( \lambda_{gr lat} \) is the graphite crucible, 0.85cm; \( \lambda_{charge lat} \) is the thermal conductivity of the graphite, 168W/mK; \( L_{SiC lat} \) is the charge thickness, 4.5cm; \( L_{gr lat} \) is the charge thickness, 4.5cm; \( \lambda_{charge lat} \) is the thermal conductivity of the aluminium scrap charge, 120W/mK; \( \alpha_{air 1} \) is the convection coefficient of the air between the crucibles; \( \alpha_{air 2} \) is the convection coefficient of the air between the graphite crucible and the charge; \( \alpha_{r1} \) is the radiation coefficient between the crucibles; \( \alpha_{r2} \) is the radiation coefficient between the crucibles; \( L_{SiC lat} \) is the SiC crucible thickness, 1.5cm; \( \lambda_{SiC lat} \) is the thermal conductivity of SiC, 120W/mK; \( L_{gr lat} \) is the graphite crucible, 0.85cm; \( \lambda_{gr lat} \) is the thermal conductivity of the graphite, 168W/mK; \( L_{charge lat} \) is the charge thickness, 4.5cm; \( \lambda_{charge lat} \) is the thermal conductivity of the aluminium scrap charge, 120W/mK; \( \alpha_{air 1} \) is the convection coefficient of the air between the crucibles; \( \alpha_{air 2} \) is the convection coefficient of the air between the graphite crucible and the charge; \( \alpha_{r1} \) is the radiation coefficient between the crucibles; \( \alpha_{r2} \) is the radiation coefficient between the crucibles; \( \alpha_{r1} \) is the radiation coefficient between the crucibles; \( \alpha_{r2} \) is the radiation coefficient between the crucibles.
The convection coefficients were determined using the equations of Churchill and Chu for laminar and turbulent flow [8], for a temperature regime higher than 1000°C. Therefore, $\alpha_{air1}$ and $\alpha_{air2}$ were calculated and have the approximated values of $\alpha_{air1}=10\text{W/(m}^2\text{K)}$ and $\alpha_{air2}=5\text{W/m}^2\text{K}$.

The radiation coefficients can be calculated depending on the emissivity ($\varepsilon_{SiC}$) and Boltzmann constant ($\sigma$):

$$
\alpha_{radair} = \varepsilon \cdot \sigma \cdot (T_{SiC} - T_{air}) \cdot (T_{SiC}^2 - T_{air}^2)
$$

(9)

The approximated values, on high temperatures, for both sections are: $\alpha_{rad1}=20\text{W/(m}^2\text{K)}$ and $\alpha_{rad2}=23\text{W/(m}^2\text{K)}$.

Applying these parameters in equation (8), it was calculated the total resistance of the Al charge system, $U_{latAl}=20.8\text{W/(m}^2\text{K)}$. In the same way is calculated the resistance for the Cu charge system $U_{latCu}=21.1\text{W/(m}^2\text{K)}$.

The thermal flow can be calculated using equation (10).

$$
q_{lat} = U_{lat} \cdot A_{lat} \cdot (T_{SiC lat} - T_{charge \ lat})
$$

(10)

It was established that the transfer area is equal with the interior area of the graphite crucible. It can be observed that $A_{lat}=0.02\text{m}^2$, the temperature of the charge is $T_{charge \ lat}=1000^\circ\text{C}$ and the lateral flow is equal with the generated flow, $q_{gen \ lat}=12.07\text{W}$. Using these parameters, it was calculated $T_{SiC lat}=1021^\circ\text{C}$ for Al charge. The temperature difference between the SiC crucible and the charge is 20°C, so the heat losses are minimal. For Cu charge the temperature difference is similar.

The heating time of the scrap charge and of each of the crucibles is determined by the correlations between the heat flow values of the system. Knowing the connection relations between thermal flows and temperatures for each point of the heat transfer path from the SiC crucible to the charge, we can determine direct dependencies of temperature - time variation for the transient state.

In this context, the lateral and bottom areas were studied separately, in order to determine the necessary heating time for the entire charge of metallic materials. Specific differential equations have been determined for the entire crucible-charge system.

In order to simplify the final differential relations, some justified approximations were made as follows:

- because the heat transfer by convection is two orders of magnitude smaller than that by radiation, it has been removed from the calculation
- because the space between the crucibles and between the crucibles and the charge is very small, the air temperatures can be approximated with the temperatures of the crucibles and the charge respectively.
- the heat transfer in the bulk charge was not taken into consideration. Numerical modelling is needed for further consideration

The heat transfer problem can be divided in three stages: the heat transfer to the SiC crucible, the heat transfer in the graphite crucible and the heat transfer in the charge. The calculation of final equations for each section of the system is based on known formulas for heat transfer (conduction, convection and radiation) and are not shown in the paper due the limited space available.

1. Lateral heat transfer

The heat energy balance between the SiC crucible and air is given by:

$$
q_{st \ SiC \ lat} = q_{gen \ lat} - q_{rad \ lat 1}
$$

(11)

Where: $q_{stSiClat}$ is the stored heat change of the SiC crucible, $q_{gen \ lat}$ is the heat flow generated by microwaves in the SiC crucible, $q_{rad \ lat}$ is the heat flow lost by radiation.

Applying the determined parameters in equation (11), results the final equation (12).

$$
\frac{d}{dt}T_{SiC \ lat} = 2.47 \cdot 10^{-13} \cdot q_{gr \ lat} \cdot T_{SiC \ lat}^4 - 2.47 \cdot 10^{-13} \cdot T_{SiC \ lat}^4 + 0.296
$$

(12)

The heat energy balance between air and the graphite crucible is given by:

$$
q_{stgrlat} = q_{radlat2} - q_{radlat3} - q_{condlat3}
$$

(13)
Where: $q_{\text{gr lat}}$ is the stored heat change of the graphite crucible; $q_{\text{rad lat}}$ is the heat flow entered in the graphite crucible by radiation; $q_{\text{rad lat}}$ is the heat flow that exits the graphite crucible, by radiation; $q_{\text{cond lat}}$ is the heat flow that exits the graphite crucible, by conduction.

Applying the determined parameters in equation 13, results the final equations for Al charge (14) and Cu charge (15).

$$\frac{dT_{\text{gr lat}}}{dt} = -8.11 \cdot 10^{-12} \cdot T_{\text{gr lat}}^4 - 0.411 \cdot T_{\text{gr lat}} + 1.77 \cdot 10^{-12} \cdot T_{\text{charge lat}}^4 + 0.411 \cdot T_{\text{charge lat}} + 6.34 \cdot 10^{-12} \cdot T_{\text{SiC lat}}$$  

(14)

$$\frac{dT_{\text{gr lat}}}{dt} = -8.11 \cdot 10^{-12} \cdot T_{\text{gr lat}}^4 - 0.677 \cdot T_{\text{gr lat}} + 1.77 \cdot 10^{-12} \cdot T_{\text{charge lat}}^4 + 0.677 \cdot T_{\text{charge lat}} + 6.34 \cdot 10^{-1} \cdot T_{\text{SiC lat}}$$  

(15)

The heat transfer in the charge is given by:

$$q_{\text{st charge}} = q_{\text{rad lat}} + q_{\text{cond lat}}$$  

(16)

where: $q_{\text{st charge}}$ is the stored heat change of the charge; $q_{\text{rad lat}}$ is the heat flow entered in the charge by radiation; $q_{\text{cond lat}}$ is the heat flow that enters the charge, by conduction.

Applying the determined parameters in equation (16), results the final equation (17) for Al charge and (18) for Cu charge.

$$\frac{dT_{\text{charge lat}}}{dt} = 6.25 \cdot 10^{-13} \cdot T_{\text{gr lat}}^4 + 0.43 \cdot T_{\text{gr lat}} - 6.25 \cdot 10^{-13} \cdot T_{\text{charge lat}}^4 - 0.43 \cdot T_{\text{charge lat}}$$  

(17)

$$\frac{dT_{\text{charge lat}}}{dt} = 4.35 \cdot 10^{-13} \cdot T_{\text{gr lat}}^4 + 0.3 \cdot T_{\text{gr lat}} - 4.35 \cdot 10^{-13} \cdot T_{\text{charge lat}}^4 - 0.3 \cdot T_{\text{charge lat}}$$  

(18)

The differential equations presented for the three stages of the heating process were introduced in MATLAB program and were solved using numerical methods as Gaussian quadrature and Runge–Kuta relations. The simulated results for aluminium and copper charges are presented in figures 2, 3 and 4.

![Figure 2](image_url)  

**Figure 2.** Temperature gradient between SiC crucible and charge: (a) aluminum scrap; (b) copper scrap.
From the presented data (figures 2 and 3), it can be observed that the variation of SiC crucible and the graphite crucible presents an inflection in the range of 800-1000K, which represents the moment when the system arrives at steady state heating. There are no large differences between the Al and Cu charges heating profile. However, the Cu charge shows a slower heating slope and presents larger differences between the temperatures of system components. The Al charge heating time to 1200K at lateral side is of approximate 0.9 hours (54 min.) and the heating time for Cu charge is 0.8 hours (48 min). The heating time of the Al (figure 4) scrap is slightly higher than that of the Cu scrap even if Cu has much larger heat transfer coefficient. Nevertheless, if bulk heat transfer was also modelled, because of higher thermal conductivity, Cu scrap will absorb much faster the receiving heat.

2. Bottom side heat transfer

The heat transfer in the SiC crucible can be achieved by conduction. The conservation of energy in the system generates equation (19).

\[
\frac{q_{st\ SiC\ bot}}{d^2T/2.30} = \frac{q_{gen\ bot}}{d^2T/2.17/2.15} - \frac{q_{cond\ bot\ 1}}{d^2T/2.13/2.25/2.24/2.14} + 9.52
\]

(19)

Where: \( q_{st\ SiC\ bot} \) is the thermal flow of changing the SiC crucible state in the bottom area; \( q_{gen\ bot} \) is the thermal flow generated by microwaves in the SiC crucible, through the bottom side; \( q_{cond\ bot\ 1} \) is the thermal flow lost through the bottom side of the SiC crucible, by conduction.

Applying the known parameters in equation 19, results the final equation (20).

\[
\frac{dT_{SiC\ bot}(t)}{dt} = 0.462 T_{gr\ bot}(t) - 0.462 T_{SiC\ bot}(t) + 9.52
\]

(20)

The heat transfer in the graphite crucible is defined by equation (21):
\[ q_{st\ bot} = q_{cond\ bot\ 1} - q_{cond\ bot\ 2} - q_{rad\ bot} \]  

(21)

Where: \( q_{st\ bot} \) is the state change heat flow of graphite crucible, in the bottom side; \( q_{cond\ bot\ 1} \) is the heat flow that enters in the graphite crucible, through the bottom side, by conduction; \( q_{cond\ bot\ 2} \) is the heat flow that exits the graphite crucible through the bottom side, by conduction; \( q_{rad\ bot} \) is the heat flow that exits the graphite crucible through the bottom side, by radiation.

Applying the known parameters in equation (21), results the final equation (22) for Al charge and (23) for Cu charge.

\[
\frac{d}{dt} T_{gr\ bot}(t) = -1.67 \cdot 10^{-12} \cdot T_{gr\ bot}^4 + 2.44 \cdot T_{gr\ bot} + 1.67 \cdot 10^{-1} \cdot T_{charge\ bot}^4 + 0.116 \cdot T_{charge\ bot} + 2.33 \cdot T_{SiC\ bot}
\]

(22)

\[
\frac{d}{dt} T_{gr\ bot}(t) = -1.67 \cdot 10^{-12} \cdot T_{gr\ bot}^4 - 2.56 \cdot T_{gr\ bot} + 1.67 \cdot 10^{-12} \cdot T_{charge\ bot}^4 + 0.233 \cdot T_{charge\ bot} + 2.33 \cdot T_{SiC\ bot}
\]

(23)

The heat transfer in the charge is defined by equation 24:

\[ q_{st\ charge} = q_{rad\ bot\ 3} + q_{cond\ bot\ 3} \]  

(24)

Where: \( q_{st\ charge} \) is the state change heat flow of the charge; \( q_{rad\ bot\ 3} \) is the heat flow entered in the charge, by radiation; \( q_{cond\ bot\ 3} \) is the heat flow entered in the charge by conduction.

Applying the known parameters in equation 24, results the final equations (25) for Al charge and (26) for Cu charge.

\[
\frac{d}{dt} T_{charge\ bot}(t) = 1.91 \cdot 10^{-15} \cdot T_{gr\ bot}^4 + 2.07 \cdot 10^{-4} \cdot T_{gr\ bot} - 1.91 \cdot 10^{-15} \cdot T_{charge\ bot}^4 - 2.07 \cdot 10^{-4} \cdot T_{charge\ bot}
\]

(25)

\[
\frac{d}{dt} T_{charge\ bot}(t) = 1.33 \cdot 10^{-15} \cdot T_{gr\ bot}^4 + 1.44 \cdot 10^{-4} \cdot T_{gr\ bot} - 1.33 \cdot 10^{-15} \cdot T_{charge\ bot}^4 - 1.44 \cdot 10^{-4} \cdot T_{charge\ bot}
\]

(26)

The differential equations presented for the three stages of the heating process were introduced in MATLAB program, using numerical methods as Gaussian quadrature and Runge – Kuta relations. The simulated results for aluminum and copper charges are presented in figures 5, 6 and 7.

Figure 5. Temperature gradient between SiC crucible and charge: (a) aluminum scrap; (b) copper scrap.
Figure 6. 3D representation of the SiC – graphite – charge temperature gradient for: (a) aluminium scrap; (b) copper scrap.

Graphical representation of the results for bottom side heating (figures 5 to 7) shows that the correlation between the temperatures of the SiC crucible and the graphite crucible follows a straight line due to the conduction type of heat treatment at crucibles interface. The Al charge heating time to 1200K at bottom side is of approximate 0.7 hours (42 min.) and the heating time for Cu charge is 0.9 hours (54 min). From the gradients determined for the lateral and the lower part of the crucibles, it is observed that the air layer on the lateral part determines a slower heating of the graphite crucible and therefore of the charge. The heating time to 1200K is similar to the lateral side. Even if the heat transfer by conduction between the two crucibles has higher efficiency than at the lateral side of the crucible, the contact area between the charge and the crucible is much smaller than for the lateral side of the crucible. By comparison with the temperature gradient the heating time for the bottom side is smaller and does not present a significant small slope as in the lateral side. The difference between the heating times for Al and Cu charges is explained also by the higher Cu heat capacity, but bulk heat transfer was not taken into consideration here.

Figure 7. Charge temperature versus time in transient state, for: (a) aluminum scrap; (b) copper scrap.

Even if the heat transfer by conduction between the two crucibles has higher efficiency than at the lateral side of the crucible, the contact area between the charge and the crucible is much smaller than for the lateral side of the crucible. By comparison with the temperature gradient the heating time for the bottom side is smaller and does not present a significant small slope as in the lateral side. The difference
between the heating times for Al and Cu charges is explained also by the higher Cu heat capacity, but bulk heat transfer was not taken into consideration here.

The laboratory experiments consisted in melting of the aluminium and copper wastes, in the microwave installation, in normal conditions of usage, at the optimized parameters of the system.

In order to obtain a better verification of the model simulated with MATLAB software, waste charges with a low content of impurities were selected. Once the applicability of the software is verified, it can be customized for the study of other wastes generated during the industrial processes. In figure 8 is presented the microwave installation used for the melting of nonferrous wastes.

The microwave furnace has five magnetrons, with the individual power of 1kW. The melting enclosure has a SiC crucible which is heated directly by the microwave field and a graphite crucible into which the charge is introduced.

The processing of nonferrous wastes in microwave field has three stages: the pressing of the wastes, mixture of the charge (the pieces of the metal waste are introduced in the melting crucible with the melting flux powder) and the melting process.

The aluminium wastes (figure 9(a)) were introduced in the charge as compacted doses and contain a high amount of metal (almost 99% gr. Al). The copper wastes (figure 9(b)) were cables with diameter of 1…3mm, without isolation layers and pressed to take the shape of the melting crucible. The purity of the copper in the cables is at least 99.8%. In figures 10 and 11 are presented the experimental results from the melting of aluminium and copper scrap, respectively.

Figure 8. The main components of the microwave furnace used for the melting of nonferrous wastes.

The aluminium wastes (figure 9(a)) were introduced in the charge as compacted doses and contain a high amount of metal (almost 99% gr. Al). The copper wastes (figure 9(b)) were cables with diameter of 1…3mm, without isolation layers and pressed to take the shape of the melting crucible. The purity of the copper in the cables is at least 99.8%. In figures 10 and 11 are presented the experimental results from the melting of aluminium and copper scrap, respectively.

Figure 9. The charge of Al (a) and Cu (b) waste.
From the graphs presented (figures 10 and 11), it can be seen that the temperature variation over time in the microwave furnace is slightly different for aluminium waste than for copper waste. Therefore, it is observed that the temperature of the copper charge increases much faster than that of aluminium, which shows a negative inflection during the beginning of the heating process. This is mainly due to the large difference between the calorific value of aluminium and copper, 900 J/kg K, respectively 400 J/kg K.

From the verification of the modelling results with the experimental data for the Al charge (figure 10), an obvious difference of the shape of the variation curves is observed. The experimental values are basically placed between the bottom and lateral modelling results. It seems that at the beginning of the heating process the heat transfer through lateral side is influencing mostly the global transfer while towards the end of the process the bottom side heat transfer has a greater importance. The experimental results show a slower heating time up to 400 K due to the aluminium high caloric capacity, when the charge is absorbing a large amount of heat. However, the experimental heating profile of the Cu charge (figure 11) is largely different than the Al charge profile. The temperature of the charge is increasing rapidly up to 800 K when a steady heating slope begins. The modelling results for the bottom and lateral side of the crucible are significantly different than the experimental results. This is explained by the relatively fast heating of the
Cu charge, where both heating methods contribute simultaneously. In this case the global heating of the charge can be approximated as a sum of both contributions. In order to determine the final heat transfer of particular waste charges in the microwave furnace is necessary a numerical modelling approach, which is dependable on the shape and dimensions of the bulk charge, to complete the present calculation results.

4. Conclusions
In this paper was performed the study of the influence of heat transfer on the melting of metalliferous waste in the microwave oven, by determining time-temperature differential relationships at lateral and bottom areas of the crucibles. The dependence relations of the heat flow transmitted through the crucibles inside the furnace were calculated with the help of the MATLAB modelling program. The specifications of the microwave oven and charge types were used in the calculations. Unlike the lateral area of the crucibles the bottom part is characterized by a heat transfer by conduction between the silicon carbide crucible and the graphite crucible which should lead to a faster heating of the system. Both sections of the system were divided into the silicon carbide crucible area, graphite crucible area and charge area. All areas were treated under conditions of transient heat transfer. Modelling using the MATLAB program generated three specific temperature-time dependence equations for each heating sections. Systems of differential equations were established for the integral physical interpretation of the heating system in the furnace. Solving the system of differential equations by specific numerical methods (Gaussian squaring and Runge-Kutta method) produced direct dependencies between each temperature in the system (SiC crucible, graphite crucible and charge) as well as between temperatures and process duration. The graphical representation of the time-temperature variations for each component of the system showed that the heat transfer in both areas give similar results, mainly due to the large difference between the transfer area of the two sections. The bottom heat transfer process is much more efficient due to the conduction surface between the SiC and the graphite crucible. Experimental work to verify the model showed a significant difference related to the complex geometry of the batch (pressed aluminium cans, pressed copper wires) and the large difference between the heat capacities of the Al and Cu charges. Both charges had different heating profiles. In order to determine a more precise configuration of the heating process it is recommended to finalize the modelling work by applying specific numerical methods that take into account the specific geometries of the waste loads.

5. References
[1] Appleton T. J., Colder R. I., Kingman S. W., et. al, 2005 Applied Energy 81(1) 85–113.
[2] Gupta M., Leong E. W. W., 2007 Microwaves and metals (Asia: John Wiley & Sons).
[3] Das S., Mukhopadhyay A. K., Datta S. and Basu D. 2009 Indian Academy of Sciences Bull. Mater. Sci. 32(1) 1–13.
[4] Chandrasekaran, S., Basak, T., and Ramanathan, S. 2011 Journal of Materials Processing Technology 211(3) 482–487.
[5] Şerban, B. A., Dumitrescu, D. V., Mitrică, D., Burada, M., Constantin, I., Olaru, M. T., and Drăgoescu, F. M. 2020 Bulletin of the Transilvania University of Brasov. Series I - Engineering Sciences 12(61)(2) 47–54.
[6] DeWitt D. P., Incropera F. P., Lavine A. S., Bergman T. L. 2007 Fundamentals of Heat and Mass Transfer, 6th Edition (New Jersey: John Wiley & Sons).
[7] Yunus A. C. 2002 Heat transfer. A practical approach, 2nd edition (New York: McGraw-Hill).
[8] Churchill, S. W., and H. H. S. Chu, 1975, Int. J. Heat Mass Transfer 18 1323-1329.

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