Farhi-Susskind GUT model revisited in the technicolor coupled scenario

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The Farhi-Susskind GUT model with heavy color is revisited in the context of technicolor coupled models, where QCD and the heavy color (or technicolor) theories are coupled through different Schwinger-Dyson equations. As shown recently, when the strong interaction theories are coupled the technicolor and QCD self-energies are modified and the model phenomenology changes. The mass splitting between the different fermionic generations is implemented with a necessary horizontal (or family) symmetry. Pseudo-Goldstone boson masses are much higher than the ones obtained in the former version of the model. Although this model has only two fermionic families, it is a quite interesting laboratory to verify how the different fermion mass scales appear in this new scenario, and it is also a nice example to show how phenomenologically viable technicolor models can be built.

I. INTRODUCTION

The hierarchy and triviality problems related to the existence of fundamental scalar bosons have been discussed for a long time. The first attempts to solve these problems were proposed forty years ago in the seminal papers by Weinberg [1] and Susskind [2]. In these works the fundamental scalar boson that would be responsible for the Standard Model (SM) gauge symmetry breaking was substituted by a composite scalar boson generated by a new strong interaction dubbed as Technicolor (TC). This proposal was incorporated in the model of Farhi and Susskind [3] together with the idea that Nature may also have a Grand Unified Theory (GUT). These type of models were reviewed in Refs. [4, 5]. The possible existence of composite scalar bosons and a GUT are beautiful and naturally expected ideas. It is worth remembering that much that it was learned up to now about symmetry breaking involves a composite scalar boson (as in the QCD chiral symmetry breaking and in the microscopic BCS theory of superconductivity), and the SM convergence of interactions at high energy seems to indicate the presence of a GUT. Unfortunately, it is also known how difficult is to build a phenomenologically viable model along these lines [6, 7].

The Farhi-Susskind GUT model (FS) is not realistic because contains only two families of ordinary fermions. That was not the unique reason why the model was abandoned. It was also claimed that the model contained undesired pseudo-Goldstone bosons [8], flavor changing neutral currents incompatible with the experimental data, as well as the model failed to explain the observed ordinary fermion masses. However, we will claim in this work that some of these facts are not true; in particular, that the pseudo-Goldstone boson masses are heavier than usually thought. Actually, the GUT structure of the FS model is much more complex and interesting than what has been observed up to now. In the context of what we call Technicolor coupled models [9, 11] the mass generation of the model was not explored at full extent.

The many interactions of the FS model connect the different strong interactions (QCD and TC) in such a way that modifies the chiral symmetry breaking parameters of the theory as discussed in Ref. [8]. The fact that both strong interactions are coupled through different interactions modify their respective self-energies and the calculation of fermion masses. For instance, the techniquarks and quarks present in the FS model will have masses completely different than what has been assumed in the literature [4]. Actually, a similar type of model including three ordinary fermionic generations has been proposed by us in Ref. [11]. However that model is more complex than the FS one and its consequences were not exhausted in such work. On the other hand the FS model is simpler and a marvelous laboratory to demonstrate how phenomenologically viable TC coupled models can be built, as long as we introduce an extra horizontal or family interaction in the model.

In the next section we will present the FS model in a slightly different way, containing the ordinary first and third fermionic generations and not the first two generations as in the original work. We will than discuss their coupled Schwinger-Dyson equations (SDE) and show what fermion masses should be expected and what are the pseudo-Goldstone masses. As we shall see the first generation fermion masses will turn out to be incompatible with the known values. This problem will be solved in Section III with the introduction of a horizontal symmetry in order to correct the masses of the lighter fermions. The advantage of the FS model resides in the simplicity to see how the fermionic mass splitting between different generations can be obtained. In the last section we draw our conclusions.
II. FS MODEL FOR THE FIRST AND THIRD FERMIONIC GENERATION

The FS model is based on a non-Abelian $SU(7)$ gauge theory. This theory contains the $SU(2)_{TC}$ technicolor group and the Georgi-Glashow $SU(5)_{GG}$ GUT. Under the $SU(2)_{TC} \times SU(5)_{GG}$ group the $SU(7)$ anomaly free set of antisymmetric representations transform as

- $[2] = (1,10) + (2,5) + (1,1)$,
- $[4] = (1,10) + (2,10) + (1,5)$,
- $[6] = (1,5) + (2,1)$.

Just as discussed in the FS model, we will assume the symmetry breaking direction $SU(7) \rightarrow [SU(2)_{L} \times U(1)_{Y}] \times SU(5)_{s}$, where the strong $SU(5)_{s}$ gauge theory acts as a extended technicolor theory (ETC) $[4]$, and as considered in Ref. $[3]$ we shall not discuss the $SU(7)$ or $SU(5)_{s}$ symmetry breaking. This symmetry breaking could be even promoted by fundamental scalar bosons at GUT or Planck scale. This fact will become particularly clear because in the TC coupled scenario the extended TC boson masses generated in the $SU(5)_{s}$ and $SU(7)$ symmetry breaking will be pushed to very high energies, where such bosons may appear naturally in a supersymmetric theory.

The FS model was proposed to explain the SM gauge symmetry breaking and the fermion masses of the first and second ordinary fermionic generations. As mentioned previously we will change the model composition just exchanging the second by the third fermionic generation. For instance, the decompositions of the $[2]$ representation under $SU(2)_{TC} \times SU(5)_{GG}$ will now be described by

$$ (1,10) = \begin{pmatrix} 0 & \tilde{t}_r & -\tilde{t}_y & u_r & d_r \\ -\tilde{t}_r & 0 & \tilde{t}_b & u_y & d_y \\ \tilde{t}_y & -\tilde{t}_b & 0 & u_b & d_b \\ -u_r & -u_y & -u_b & 0 & \bar{e} \\ -d_r & -d_y & -d_b & -\bar{e} & 0 \end{pmatrix} = (1,1) = \bar{\nu}_r. $$

$$ (2,5) = \begin{pmatrix} D_r \\ D_y \\ D_b \\ E \\ N \end{pmatrix} / p $$

The $p$ subscript indicates that each technifermion is a TC doublet.

We also change the second by the third fermionic generation in the remaining $[4]$ and $[6]$ representations, everywhere else we stick to the same notation of the original FS model. At the Fermi scale the $SU(2)_{TC}$, that appears in the breaking of the $SU(5)_{s}$ strong group, undergoes chiral symmetry breaking. In this breaking the following non-zero TC condensates are formed: $\langle \bar{U}U \rangle, \langle \bar{D}D \rangle, \langle \bar{N}N \rangle, \langle \bar{E}E \rangle$. As we are considering all interactions that may be coupled it is possible to verify that the four vacuum expectation values of these condensates will be different.

At this point it is interesting to recall the results of Ref. $[3,10]$ to see what happens in the chiral symmetry breaking of coupled strong interaction theories. In Ref. $[2]$ we calculated numerically the self-energy ($\Sigma(p^2)$) of two coupled strong interaction theories (QCD and a $SU(2)$ TC theory), and verified that TC self-energy behaves as

$$ \Sigma_i(p^2) \approx \mu_{TC} \left[ 1 + \delta_i \ln \left( \frac{p^2 + \mu_{TC}^2}{\mu_{TC}^2} \right) \right]^{-\delta_i}, $$

where $\mu_{TC}$ is the dynamical TC mass, which should be of the order of the Fermi scale. $\delta_1$ and $\delta_2$ are parameters that depend on the QCD, TC and ETC theory. In the case with more interactions (e.g. electroweak) these parameters will contain corrections proportional to the charges of these theories.

Note that the behavior of Eq. (11) is not a surprise. The fact that another interaction added to the TC one changes the self-energy is known since the work of Takeuchi $[12]$. The reason for the behavior described above is that TC give masses to ordinary fermions QCD also give masses to the technifermions $[10]$, as well as other interactions may contribute to these masses when all their Schwinger-Dyson equations are coupled. For instance, the $\delta_2$ would be proportional to the mass anomalous dimension of a technifermion, but its value is connected to the QCD dynamics (that generates the technifermion mass) as well as to the other interactions present in the GUT.

The ordinary fermion masses generated by Eq. (11), as shown in Ref. $[2]$, will be given by

$$ m_i = \lambda_E \mu_{TC} \left[ 1 + \kappa_1 \ln \left( \frac{M_E^2}{\mu_{TC}^2} \right) \right]^{-\kappa_2}, $$

where $\lambda_E$ involves ETC couplings and a Casimir operator eigenvalue. $M_E$ is an ETC boson mass, and the $\kappa_i$ are also functions of the $\delta_i$ in Eq. (11) as well as other possible corrections (electroweak or other interactions). The QCD self-energy has the same behavior of Eq. (11) only changing $\mu_{TC}$ by $\mu_{QCD}$ (the QCD dynamical mass) and respective $\delta_i$ coefficients. With these comments in mind we can in the sequence discuss the mass generation in the FS model.

For simplicity, in the following we are assuming that all technifermion condensates are equal and proportional to $\mu_{TC}^2$ and the QCD ones equal to $\mu_{QCD}^3$. Usually the values assumed for these quantities are $\mu_{TC} \approx 250$GeV and $\mu_{QCD} \approx 250$MeV. The FS notation for the $SU(5)_{s}$ gauge bosons are changed to $E$ and $E'$, with masses $m_{E}$ and $m_{E'}$, interacting with a coupling constant $g_5$. Therefore, in their original model the first and (now) the third generation quark masses were

$$ m_u \approx \frac{\mu_{TC}^2}{m_{E}} \ , \ m_t \approx \frac{2\mu_{TC}^3}{m_{E}}. $$

The $E$ boson masses were claimed to be of $O(100)$TeV, and do not lead to a reasonable $t$ quark mass. In this work we will show that, as in the coupled systems discussed in $[8,10]$, the quark mass relationships are quite...
different from the ones shown in Eq. (4). In the coupled TC version (neglecting the log terms in Eq. (2)) these masses now are

\[ m_u \approx \lambda_\mu \mu_{TC} \quad , \quad m_t \approx 2\lambda_\mu \mu_{TC}, \]  

where \( \lambda_\mu \) is a product of coupling constant and Casimir operator related to the ETC (or GUT) interaction appearing in the mass diagrams of the 2/3 charged quarks.

Here we have the opposite case compared to the original model. The \( t \) quark mass, if we consider the effect of the logarithmic terms present in Eq. (2), that lowers the simple estimate of Eq. (4), is compatible with the experimental value. On the other hand the \( u \) quark mass is far away from the expected value. This inconsistency will be solved in the next section.

The determination of the electron mass is also quite different in both type of models. In Ref. [2] it was indicated that \( m_e = 0 \). However, the diagram of Fig. (1) will connect the electron to the \( d \) quark condensate, through the exchange of a SU(7) boson. This mass, that in the decoupled case would be extremely small, in the case where the SDE are coupled will be changed to

\[ m_e \propto a_{GUT} \mu_{QCD}, \]

which, considering the neglected logarithmic term involving a large GUT gauge boson mass, can be a little bit smaller than our simple estimate. Therefore the electron mass may be of order of a few MeV which is the right magnitude. This fact is a demonstration that the QCD quark condensates also participate in the process of mass generation of the ordinary fermions, which is a consequence of the logarithmic behavior of the self-energies in the coupled TC models.

The third generation charged lepton (\( \tau \)) obtain mass coupling to the \( U \) technifermion condensate, as shown in Fig. (1). In this case we are already generating a large mass splitting between the first and the third leptonic generation, what is peculiar to this model in the coupled scheme.

The most interesting case is to observe what happens with the technifermion masses in the TC coupled scenario. In the coupled TC scheme the technifermions will become quite massive and affect the pseudo-Goldstone boson spectra. In the FS model, when the chiral symmetry of the TC Lagrangian is broken by the technifermion condensates, 63 Goldstone bosons are formed. Three of these bosons will be absorbed by the weak bosons, sixty are pseudo-Goldstone bosons after radiative corrections, but some were expected to be quite light [3]. Fifty-six of these pseudo-Goldstone bosons that are not absorbed by the weak bosons carry color and have larger radiative corrections, while others may have only electroweak corrections to their masses. In the coupled scenario the lightest technifermion will be the neutral one (N). Apart the TC quantum number the technifermion N has the same quantum numbers of the ordinary neutrino. Its mass appears due to the diagrams of Fig. (2).

The first diagram of Fig. (2) gives the usual dynamical TC mass to \( N \). It is the diagram (a2) of Fig. (2) that modify the running of the \( N \) technifermion self-energy, which turn out to be logarithmic due to the coupling of TC and QCD. The third diagram of Fig. (2) involves the TC condensate and a weak correction, leading to a current mass of the following order

\[ m_N \approx g_a^2 \mu_{TC}. \]  

This mass is of O(100)GeV and the logarithmic contribution appearing in the self-energy may even enhance this value due to the small Z boson mass compared to the scale \( \mu_{TC} \). All corrections to other technifermion masses are larger than this one. Therefore, the lightest pseudo-Goldstone that may contain this neutral technifermion (\( N \gamma_5 \gamma^\dagger N \), where \( i \) indicate electroweak indexes) will obtain a mass that may be computed with the help of the Gell-Mann-Oakes-Renner relation

\[ m_N^2 \approx m_N \langle N \bar{N} \rangle \frac{1}{2F_{\Pi}}, \]  

where \( \langle N \bar{N} \rangle \approx (250)^3 \text{GeV}^3 \) is the TC condensate, and \( F_{\Pi} \approx 190 \text{GeV} \) the technipion decay constant. This will give to the lightest neutral pseudo-Goldstone mass the following value

\[ m_N \approx 150 \text{ GeV}. \]  

Such estimate of the lightest pseudo-Goldstone boson mass is well above the ones found in the original FS model. In particular, such neutral boson could decay into weak gauge bosons and would probably be buried by the direct weak background. The other technifermion masses (carrying color and electric charges) are much larger, and will lead to even larger pseudo-Goldstone boson masses.

There are many other phenomenological consequences of the coupled TC scenario. The scalar composite boson that mimics the Higgs boson is also compatible with the data due to the reasons already exposed in Refs. [9] [11].
and the experimental oblique parameters will be also within the expected value [11]. The $SU(5)_s$ and $SU(7)$ gauge bosons may be quite heavy, since they barely affect the fermionic mass spectra, and we may say that the main problem in the coupled TC scheme applied to this model is the absence of a complete ordinary fermion mass splitting.

**III. SOLVING THE ORDINARY FERMION MASS SPLITTING**

In the previous section we have seen that the coupled SDE lead to different expressions for the fermionic self-energies, which generate masses that depend weakly on the ETG gauge boson masses, and are not able to explain the mass splitting of the ordinary fermions. The solution of this problem has already been advanced in Refs. [9, 11]. It is possible to introduce continuous or discrete symmetries of the horizontal or family type in such a way that the first family receives mass coupling to the QCD condensate, on the other hand the third fermionic generation will couple preferentially to the TC condensate. The idea to produce mass splitting between the fermionic generations is not different from the models where two fundamental scalar bosons are introduced to provide different masses to the different fermionic generations [14–17]. The difference here is that our scalar bosons are composite, and related to QCD and TC.

We would like to point out that we have found another particular formulation of the FS model where naturally the first fermionic family does not couple to technifermions. The fermionic content of the model continue to be described by the anomaly free [2], [4] and [6] of $SU(7)$, but now the decomposition of the [2] is

\[
(1, 10) = \begin{pmatrix}
0 & t_r & -t_y & t_r & b_r \\
-t_r & 0 & t_b & t_y & b_y \\
t_y & t_b & 0 & t_t & b_t \\
-t_r & -t_y & t_t & 0 & \tilde{\tau} \\
-b_r & -b_y & -b_t & -\tau & 0
\end{pmatrix}
\]

while the decomposition of the [4] is

\[
(1, 10) = \begin{pmatrix}
0 & \bar{u}_r & -\bar{u}_y & u_r & d_r \\
-\bar{u}_r & 0 & \bar{u}_b & u_y & d_y \\
\bar{u}_y & -\bar{u}_b & 0 & u_t & d_t \\
-u_r & -u_y & -u_t & 0 & \bar{\tilde{\tau}} \\
-d_r & -d_y & -d_t & -\bar{\tilde{\tau}} & 0
\end{pmatrix},
\]

\[
(2, 5) = \begin{pmatrix}
D_r & D_y \\
\bar{D}_b & \bar{D}_t \\
\bar{E} & \bar{N}
\end{pmatrix}_p
\]

The representation [6] is not changed. The Feynman rules are similar to the ones of the FS model just changing the fermionic constituents as described above.

Note that in the above change the strong interactions are still coupled, although technifermions, just by construction, now couple to the third generation fermionic family. In particular the neutral technilepton ($\bar{N}$) continue to couple to the $b$ quark, which has a small condensation value due to its large mass, but still ensures that the TC self-energy is a hard one and produces a $N$ mass of the order shown in Eq. (4). Therefore, it is possible to see that the first and third generation ordinary fermions obtain different masses due to QCD and TC, as a consequence of their coupling to the $SU(7)$ and $SU(5)_s$ gauge bosons respectively.

With the above choice of fermionic representations we succeeded to obtain the mass splitting between the first and third generations. This is one of the main interesting points of this scheme: the different fermionic mass scales are a consequence of the different strongly interacting theories. Another interesting point is that a different number of diagrams contribute to the different quark masses, what not only generate the mass splitting between families but also between fermions with different isospin values, as observed in Fig.(3) and in Ref. [11].

There is a problem that has not been addressed up to now: the mixing between different generations has not been generated. The inclusion of a horizontal or family symmetry is indeed necessary to generate realistic models! Note that we could have kept the original FS fermionic representations and introduced a horizontal symmetry at that stage, but with the cost of more complicated fermionic representations in order to render the model free of anomalies. We simplified the problem with the choice of the representations shown in Eqs. (5) and (6). Continuing with the formulation proposed in this section, we can see that it is not difficult to introduce a
horizontal symmetry and obtain the mixing between the different fermion masses.

We will add to the modified FS model a horizontal symmetry based on the \( SU(2)_H \) group. In this case we can assume that all fermions that are in the [2], [4] and [6] representations of \( SU(7) \) are also in the same fundamental \( (2) \) representation of \( SU(2)_H \), since this is the simplest way to guarantee the cancellation of anomalies (i.e., \( 2A[10+10] = 0 \) and \( 2A[5+5] = 0 \)). The fermionic mass matrix in the \( SU(2)_H \) (or \( (u \ t) \) charge 2/3 quarks) basis is going to be given by

\[
m_{2/3} = \begin{pmatrix} a & b \\ -b & c \end{pmatrix},
\]

where we can identify

\[
a \approx \mu_{QCD}(C_H \alpha_H + 2C_7 \alpha_7) \quad (11)
\]
\[
c \approx \mu_{TC}C_5 \alpha_5 \quad (12)
\]
\[
b \approx \mu_{TC}C_5 C_H \alpha_3 \alpha_H. \quad (13)
\]

To obtain such mass matrix we assumed that in the \( [4] \) representation of \( SU(7) \), where are the first generation fermions, we have terms \( \bar{u}_i \gamma^\mu u^j_\mu \epsilon^{ijk}, \bar{d}_i \gamma^\mu u^j_\mu \epsilon^{ijk} \), that appear in the \((1,10)\) decomposition of that representation. The constants \( C_H, C_7 \) and \( C_5 \) are eigenvalues of the Casimir operator of the horizontal, \( SU(7) \) and strong \( SU(5)_s \) theories respectively, and \( \alpha_H \) and \( \alpha_3, \alpha_7 \) their coupling constants, where \( i,j,k = r,b \) are colors indexes.

The diagrams that contribute to the \( b \) entry in the Eq.\ref{eq:10} are displayed in Fig.\ref{fig:4}. The final mass matrix involves the diagrams of Figs.\ref{fig:3} and \ref{fig:4}.

\begin{figure}[ht]
\centering
\includegraphics[width=0.8\textwidth]{fig4.png}
\caption{ Contributions to the \( b \) entry in Eq.\ref{eq:10} due to the presence of the horizontal symmetry.}
\end{figure}

As we discussed before we are neglecting the contribution of logarithmic terms to the masses determination, what may change our evaluation of these equations, but not at a critical point. Continuing with our simple estimate, we can assume that the symmetry breaking of the \( SU(2)_H \) horizontal group happens at the same scale of the \( SU(5)_s \) theory. Furthermore, assuming \( C_5 \alpha_5 = O(0.1) \), and since the horizontal group is a small one we also assume \( C_H/C_5 = \frac{3}{4} \times \frac{10}{24} \approx \frac{1}{3} \), we can estimate \( C_H \alpha_H \approx \frac{O(0.1)}{3} \), and with other naive round values of \( \mu_{QCD} = 0.2 GeV \) and \( \mu_{TC} = 17 TeV \) we finally obtain

\[
m_{2/3} = \begin{pmatrix} 0.047 & 3 \\ -3 & 100 \end{pmatrix} \quad (14)
\]

which in the diagonal basis result in the following masses

\[
m_t \approx 100 GeV \quad (15)
\]
\[
m_u \approx 0.1 GeV. \quad (16)
\]

The \( 1/3 \) electrically charged quarks obtain similar masses.

It is important to note that the horizontal gauge boson masses (if the horizontal symmetry is based on a gauge theory) can be pushed to very high energies, since the horizontal gauge boson masses appear only in logarithms when we compute the fermion masses. They will introduce new contributions to the many fermion masses, but being very massive do not generate undesired flavor changing interactions. On the other hand this scheme allow us to understand the different fermionic scales: The splitting between the first and third fermionic family is a consequence of the different strongly interacting theories, where the mass of the first fermionic generation is connected to QCD, while the mass of the third fermionic generation will be related to the TC scale. Of course, in a more complex model including three generations, like the one of Ref.\cite{11}, the mixing between the different families can be obtained. It is also important to note that the lepton masses turn out to be lighter than the quark masses, since the colored particles have more diagrams contributing to their masses, as well as due to the different quantum charges of these particles.

\section{Conclusions}

We have reviewed the FS model in the context of the technicolor coupled scenario. Although the model is not realistic it gives a nice and simple example of how the different fermionic masses are generated. Relevant points of the model are: (a) the origin of the different fermionic mass scales is associated to the different strong interactions present in the model, (b) the horizontal symmetry is necessary in this scheme to generate the splitting of masses as well as the mixing between different fermionic families (c) a different set of diagrams may contribute to the origin of the isospin mass splitting in the same fermionic generation, what may appear naturally in more realistic models \cite{11}.

The mass splitting between the different families was easily obtained in Section III due to a particular choice of the GUT model fermionic representations. However, it is also possible to generate such masses with a more complex choice of horizontal quantum numbers as discussed in Ref.\cite{11}.

We also verified that pseudo-Goldstone boson masses are larger than previously discussed. Supposing that realistic TC models along this line could be realized in Nature, it is quite possible that such particles could have escaped detection up to now.

We have not discussed the scalar composite masses, but this subject was already discussed at length in
Ref. [11] and [18], and it can be consistent with the experimental data due to the decrease in the scalar mass value obtained from the homogeneous Bethe-Salpeter equation (BSE), originated by the normalization condition of the non-homogeneous BSE.

The gauge symmetry breaking of the unified strong and horizontal group was assumed to happen at very high energies, and could be even promoted by fundamental scalar bosons at scales where they may naturally appear. As a consequence we do not expect FCNC problems in this class of models.

The estimates we have made were just of order of magnitude. We systematically neglected the logarithmic contributions appearing in the self-energies and masses. A more detailed study will require these terms, although they will not affect strongly the mass values.

We hope that this simple example in the scenario of TC coupled models is able to show the path for the construction of realistic TC models.

**ACKNOWLEDGMENTS**

This research was partially supported by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) under the grants 302663/2016-9 (A.D.) and 303588/2018-7 (A.A.N.).

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