Few-photon optical diode in a chiral waveguide

Jin-Lei Tan,1 Xun-Wei Xu,1 Jing Lu,1 and Lan Zhou1,*

1Synergetic Innovation Center for Quantum Effects and Applications, Key Laboratory for Matter Microstructure and Function of Hunan Province, Key Laboratory of Low-Dimensional Quantum Structures and Quantum Control of Ministry of Education, School of Physics and Electronics, Hunan Normal University, Changsha 410081, China

We study the coherent transport of one or two photons in a one-dimensional waveguide chirally coupled to a nonlinear resonator. Analytic solutions of the one-photon and two-photon scattering is derived. Although the resonator acts as a non-reciprocal phase shifter, light transmission is reciprocal at one-photon level. However, the forward and reverse transmitted probabilities for two photons incident from either the left side or the right side of the nonlinear resonator are nonreciprocal due to the energy redistribution of the two-photon bound state. Hence, the nonlinear resonator acts as an optical diode at two-photon level.

I. INTRODUCTION

A diode that allows unidirectional propagation of signal is an indispensable circuit element for information processing. In the design of the circuit for optical information processing, optical isolators serving as optical diodes [1] are often used to realize nonreciprocal propagation of light. Conventional optical isolators are designed based on magneto-optic effect and work in a strong magnetic field [2]. The design and fabrication of a magnetic-free optical isolator on chip have attracted a great interest in the past decades. A number of alternative schemes have been proposed theoretically and realized experimentally based on diverse mechanisms, such as nonlinear optics [3–11], chiral quantum optics [12–16], optomechanics [17–28], atomic gases in motion [29–34], and non-Hermitian optics [35–38]. It has to be stressed that the previous studies of optical nonreciprocity have mainly focused on transmission rates of the incident classical light. Very recently, nonreciprocal quantum effects have been explored theoretically [39], including nonreciprocal photon blockade [39–44] and nonreciprocal quantum entanglement [45, 46], which open up a way to create and manipulate one-way nonclassical light.

Here, we propose to explore the controllability of the nonreciprocal transport light in quantum regime. Quantum switches that control the transport of a confined single photon have been proposed in a one-dimensional (1D) waveguide [47–49]. Strong photon-photon quantum correlations can also be created by few-photon transport in a 1D waveguide coupled to nonlinear scatterers, such as a two-level atom [50–52], a multi-level atom [53–58], multiple two-level atoms [59–62], a resonator coupling to a two-level atom [65–67], a multi-level atom [68], or a mechanical resonator [69–71]. The transport properties of a few photons inside a 1D waveguide coupled to a nonlinear resonator have been explored theoretically, and the bunching or anti-bunching behavior as a signature of strong quantum correlations at the few-photon level has been reported [63, 64]. However, most of the schemes are proposed based on the configuration that a 1D waveguide coupling to a nonlinear scatterers symmetrically, i.e., the coupling strengths are the same for photons propagating in different directions, which cannot provide quantum non-reciprocity at the few-photon level in these systems.

In this paper, we propose a scheme to realize an optical diode at the few-photon level based on the chiral coupling between a 1D waveguide and a Kerr-type nonlinear resonator, i.e., the coupling strengths between the resonator and the waveguide are different for photons propagating in different directions. The Laplace transform is applied to solve the photon scattering problems, and an analytic solution is obtained. It is found that, for a single photon coming from either side of the resonator, the transmission amplitudes are the same in magnitude, but their phases are different, which indicates that the resonator can act as a non-reciprocal phase shifter. However, the forward and reverse transmitted probabilities for two photons incident from either the left side or the right side of the nonlinear resonator are nonreciprocal due to the energy redistribution of the two photons bound state. The chiral coupling configuration therefore open a new avenue toward optical diode in few-photon level for quantum information processing on-chip.

This paper is organized as follows. In Sec. II, we introduce the model and establish the notation. Then we derive the analytic solutions of the one-photon and two-photon scattering in Sec. III and IV, and the forward and reverse transmitted probabilities for photon incident from either the left side or the right side of the nonlinear resonator are discussed at few-photon levels. We make a conclusion in Sec. V.

II. A CHIRAL WAVEGUIDE COUPLED TO A NONLINEAR RESONATOR

The system we consider is a 1D waveguide of infinite length coupled to a nonlinear resonator of the single-mode field at the origin, which is schematically shown in Fig. 1. The one-dimensional continuum of the waveg-
uider are formed by the right-going and left-going modes, the right-going mode are represented by the canonical creation and annihilation operators \( \hat{r}_\omega \) and \( \hat{\omega} \), and the left-going mode are represented by \( \hat{l}_\omega \) and \( \hat{\omega} \). Both modes obey the equal-time bosonic commutation relation \( [\alpha_\omega, \alpha^\dagger_\omega] = \delta (\omega - \omega') \) with \( \alpha_\omega = \hat{r}_\omega, \hat{l}_\omega \). Photons traveling in the 1D waveguide can tunnel into and out of a resonator with eigenfrequency \( \omega_c \). The mode of the resonator is denoted by the annihilation operator \( \hat{a} \). The resonator is filled with a Kerr medium with nonlinear interaction strength \( U \).

The Hamiltonian of the resonator-waveguide system reads

\[
\hat{H} = \frac{U}{2} \hat{a}^\dagger \hat{a} \hat{a} + \int \omega \left( \hat{r}_\omega \hat{r}_\omega + \hat{l}_\omega \hat{l}_\omega \right) d\omega + \int d\omega \left( \sqrt{\frac{\gamma_1}{4\pi}} \hat{r}_\omega^\dagger \hat{r}_\omega + \sqrt{\frac{\gamma_2}{4\pi}} \hat{l}_\omega^\dagger \hat{l}_\omega + h.c. \right)
\]

in the rotating frame with respective to \( \hat{H}_0 = \omega_c \hat{N} \), where

\[
\hat{N} = \hat{a}^\dagger \hat{a} + \int_{-\infty}^{+\infty} \left( \hat{r}_\omega^\dagger \hat{r}_\omega + \hat{l}_\omega^\dagger \hat{l}_\omega \right) d\omega
\]

is called the total excitation operator and commutes with \( \hat{H} \). As long as \( \omega_c \) is far away from the cutoff frequency of the photon modes, we can extend the lower limit of integration to \(-\infty\). The first line in Eq.(1) corresponds to the nonlinear interaction with strength \( U \) and free photon, the second line describes the interaction between the waveguide and the resonator. The coupling rates to right- and left-going photons are labeled by \( \gamma_1 \) and \( \gamma_2 \) respectively. The couplings are chiral as long as \( \gamma_1 \neq \gamma_2 \).

We further introduce the following operators of the waveguide

\[
\hat{b}_\omega = \sqrt{\frac{\gamma_1}{\Gamma}} \hat{r}_\omega + \sqrt{\frac{\gamma_2}{\Gamma}} \hat{l}_\omega, \quad \hat{c}_\omega = \sqrt{\frac{\gamma_2}{\Gamma}} \hat{r}_\omega - \sqrt{\frac{\gamma_1}{\Gamma}} \hat{l}_\omega
\]

where the total decay \( \Gamma = \gamma_1 + \gamma_2 \). The couplings are symmetric when \( \gamma_j = \Gamma/2 \). Eq. (3) decomposes the field of the waveguide into two decoupled modes. Then, the Hamiltonian in Eq.(1) is the sum of two parts \( \hat{H} = \hat{H}_c + \hat{H}_b \) with

\[
\hat{H}_c = \int_{-\infty}^{+\infty} \omega \hat{b}_\omega^\dagger \hat{b}_\omega d\omega
\]

\[
\hat{H}_b = \frac{U}{2} \hat{a}^\dagger \hat{a} \hat{a} + \int_{-\infty}^{+\infty} \omega \hat{b}_\omega^\dagger \hat{b}_\omega d\omega + \sqrt{\frac{\Gamma}{\pi}} \int d\omega \left( \hat{a} \hat{b}_\omega^\dagger + h.c. \right) d\omega
\]

The b modes couple to the resonator and the c modes evolve freely. The total number of excitations is conserved in both the b and c spaces separately.

### III. SCATTERING OF SINGLE PHOTON

We study the single-photon scattering with photons incoming from either left or right of the resonator [48, 49, 74]. As the photons in the c space propagate freely, the scattering process mainly occurs in the b space.

States \( \hat{a}^\dagger |0\rangle \) and \( \{ \hat{b}_\omega^\dagger |0\rangle \} \) are the basis in the b space with one excitation, where the ground state \( |0\rangle \) is a vacuum state of both the waveguide and the resonator. \( \hat{a}^\dagger |0\rangle \) is a state for single excitation in the resonator, \( \hat{b}_\omega^\dagger |0\rangle \) is a state for single excitation in the \( \omega \)th b mode. Then, the time-dependent wave function of the system reads

\[
|\Psi (t)\rangle = A(t) \hat{a}^\dagger |0\rangle + \int_{-\infty}^{+\infty} d\omega A_\omega (t) \hat{b}_\omega^\dagger |0\rangle
\]

where the coefficients \( A(t) \) and \( A_\omega (t) \) are the amplitudes of the corresponding state. The equations of motion read

\[
i\partial_t A(t) = \sqrt{\frac{\Gamma}{\pi}} \int_{-\infty}^{+\infty} d\omega A_\omega (t)
\]

\[
i\partial_t A_\omega (t) = \omega A_\omega (t) + \sqrt{\frac{\Gamma}{\pi}} A(t)
\]

By perform Laplace transform, Eq.(6) is transformed to a system of algebraic equations

\[
s A(s) - A(0) = -i \sqrt{\frac{\Gamma}{\pi}} \int d\omega A_\omega (s)
\]

\[
(s + i\omega) A_\omega (s) = A_\omega (0) - i \sqrt{\frac{\Gamma}{\pi}} A(s)
\]

The scattering process indicates that initially there is no excitation in the resonator, i.e. \( A(0) = 0, A_\omega (0) \neq 0 \). For the convenience of later discussion, we assume the probability amplitude at the initial time

\[
A_\omega (0) = \frac{\sqrt{\epsilon/\pi}}{\omega - \delta + i\epsilon}
\]

is symmetric with respect to the center \( \delta \) with width \( \epsilon \). After a long time with \( t >> \Gamma^{-1}, \epsilon^{-1} \), the resonator is in
The transmittance (a) and reflectance (b) as a function of $\omega$. Blue solid, red dashed, and black dotted curves are shown for $(\gamma_1, \gamma_2) = (1, 0), (0.8, 0.2), (0.5, 0.5)$, respectively. All parameters are in units of $\Gamma$.

Let us return to the scattering process in terms of the right and left-going modes. We first consider the single-photon wavepacket is in the right-going modes, traveling towards the resonator at origin from the left. Its state is described by

$$|\Psi(0)\rangle = \int_{-\infty}^{+\infty} d\omega A_\omega(0) \hat{A}_\omega |0\rangle$$

Then, the photon is absorbed by the resonator. Meanwhile, the resonator emits the photon in both the right and left directions, and relaxes to the ground state. The single photon wavepacket will be redistributed into a reflected and a transmitted part, which is described by the scattering amplitudes

$$t_\omega = \frac{\omega - i\gamma}{\omega + i\Gamma}, \quad r_\omega = -2\sqrt{\gamma_1\gamma_2} \frac{\omega + i\Gamma}{\omega + i\Gamma}$$

where we have defined $\gamma = \gamma_1 - \gamma_2$. If the single-photon wavepacket is in the left-going modes, traveling towards the resonator at origin from the right. Its state is described by

$$|\Psi(0)\rangle = \int_{-\infty}^{+\infty} d\omega A_\omega(0) \hat{a}_\omega^\dagger |0\rangle.$$

The reflection amplitude is still described by $r_\omega$, which can be easily understood by the fact that the expression of $r_\omega$ in Eq. (10) is the same under the exchange of $\gamma_1$ and $\gamma_2$. However, the transmission amplitude becomes

$$t'_\omega = \frac{\omega + i\gamma}{\omega + i\Gamma}.$$ (11)

Except different phases, transmission amplitudes are the same in magnitude for an incident photon coming from either side of the resonator. For symmetric coupling $\gamma_1 = \gamma_2$, the two transmission amplitudes are equal, i.e., $t'_\omega = t_\omega$, and we cover the results in Refs. [72–76].

Since the chiral coupling between the waveguide and the resonator introduces an imbalance of the probabilities between the right- and left-moving emitted photon, we plot the scattering coefficients as a function of the incident frequency $\omega$ for a fixed $\Gamma$. A Lorentzian line shape can be observed around the resonance, however, a single photon at resonance frequency is completely reflected only for symmetric coupling. As $\gamma$ increases, the reflection becomes lower and lower. If the chiral coupling is fixed, the propagating single photon get nearly total transmitted as its center is far way from the resonator mode.

### IV. TWO-PHONON SCATTERING

We now consider the scattering process in the double-excitation subspace. For photons incident from either side of the resonator in the $c$ space, they get transmitted directly. For two photons incident from either side of the resonator in the $b$ space, the two-photon wave functions have a complicated form due to the nonlinear interaction of the resonator. The general two-excitation state at arbitrary time in the $b$ space read

$$|\psi(t)\rangle = A_c(t) \frac{\hat{a}_c^\dagger \hat{a}_c^\dagger}{\sqrt{2}} |0\rangle + \int d\omega B_\omega(t) \hat{b}_\omega^\dagger |0\rangle$$

$$+ \int d\omega d\omega' C_{\omega\omega'}(t) \hat{b}_{\omega'}^\dagger \hat{b}_\omega^\dagger |0\rangle \theta (\omega > \omega')$$

where the function $A_c(t)$ stands for the amplitude of the state that both excitation are localized in the resonator, the functions $B_\omega(t)$ are the amplitudes of states with one of the excitations in the waveguide and the other...
in the resonator, functions $C_{\omega \omega'}(t)$ are the amplitudes of states that both photons are in the waveguide, they are symmetry to the permutation of photon frequencies $C_{\omega \omega'}(t) = C_{\omega' \omega}(t)$ due to the bosonic nature. The initial state is expected towards an asymptotic configuration with no excitation in the resonator and two traveling photons in the waveguide

$$C_{\omega \omega'}(0) = \left( \frac{C}{\omega - \delta + i \epsilon \omega' - \delta_2 + i \epsilon_2} \right)^{-1/2} \left( \frac{C}{\omega - \delta + i \epsilon \omega' - \delta_2 + i \epsilon_2} \right).$$

where the normalization constant

$$C = \sqrt{\frac{\epsilon_1 \epsilon_2}{2\pi^2}} \left[ 1 + \frac{4\epsilon_1 \epsilon_2}{(\delta_1 - \delta_2)^2 + (\epsilon_1 + \epsilon_2)^2} \right].$$

The Schrödinger equation yields the evolution of the amplitudes:

$$\partial_t A_\omega = -iU A_\omega - i \sqrt{\frac{2T}{\pi}} \int d\omega B_\omega \omega$$

$$\partial_t B_\omega = -i\omega B_\omega - i \sqrt{\frac{2T}{\pi}} A_\omega - i \sqrt{\frac{\Gamma}{\pi}} \int d\omega' C_{\omega \omega'} \omega$$

$$\partial_t C_{\omega \omega'} = -i(\omega + \omega') C_{\omega \omega'} - i \sqrt{\frac{\Gamma}{\pi}} (B_\omega + B_{\omega'})$$

During the scattering process the incident photons have a chance to jump into the resonator, however, We are mainly interested in the longtime behavior with $t \gg \gamma_i^{-1}, \epsilon_i^{-1}$. The asymptotic solution of $C_{\omega \omega'}$ is obtained by first Laplace transform of Eqs.(15) and then via the inverse Laplace transform up to the forth order of $\sqrt{T}$ after some algebra calculations. It has an overall phase factor $\exp[-i(\omega + \omega')t]$ multiplying an time-independent part

$$C_{\omega \omega'}(t) = e^{-i(\omega + \omega')t} \left[ \tilde{t}_{\omega \omega'} C_{\omega \omega'}(0) + D_{\omega \omega'} \right]$$

where we have introduced

$$D_{\omega \omega'} = \frac{U}{\omega + \omega' - U + 2i\Gamma \omega + \omega' - \delta + i(\epsilon_1 + \epsilon_2)}$$

For two incoming right-going photons from the left of the resonator with the initial state

$$|\psi(0)\rangle = \int d\omega d\omega' C_{\omega \omega'}(0) \tilde{t}_{\omega \omega'} |0\rangle,$$

there are four possibilities after the two incoming photons with frequencies $\omega$ and $\omega'$ colliding with the resonator: Both are transmitted with amplitude $\alpha_{tr}^{\omega \omega'}$; Both are reflected with amplitude $\alpha_{tl}^{\omega \omega'}$; One photon is transmitted and the other is reflected with amplitude $\alpha_{tr}^{\omega \omega'}$ and $\alpha_{tl}^{\omega \omega'}$. The state of out-going photons reads

$$|\psi(t)\rangle = \int d\omega d\omega' \left( \alpha_{tr}^{\omega \omega'} \tilde{t}_{\omega \omega'} |0\rangle + \alpha_{tl}^{\omega \omega'} \tilde{t}_{\omega \omega'} |0\rangle \right) e^{-i(\omega + \omega')t}$$

after a long time, the amplitudes are related to the single photon scatter amplitudes $\{t_{\omega}, r_{\omega} \}$ by

$$\alpha_{tr}^{\omega \omega'} = t_{\omega} t_{\omega'} C_{\omega \omega'}(0) + \frac{\gamma_1^2}{\Gamma^2} D_{\omega \omega'},$$

$$\alpha_{tl}^{\omega \omega'} = r_{\omega} r_{\omega'} C_{\omega \omega'}(0) + \frac{\gamma_2^2}{\Gamma^2} D_{\omega \omega'},$$

$$\alpha_{tr}^{\omega \omega'} = t_{\omega} r_{\omega'} C_{\omega \omega'}(0) + \frac{\sqrt{\gamma_1 \gamma_2}}{\Gamma^2} D_{\omega \omega'},$$

$$\alpha_{tl}^{\omega \omega'} = r_{\omega} t_{\omega'} C_{\omega \omega'}(0) + \frac{\sqrt{\gamma_1 \gamma_2}}{\Gamma^2} D_{\omega \omega'}$$

The first term on the right side of each line of Eq.(19) is the contribution from two noninteracting photons. The second part presents the correlation of output photons coming from the nonlinearity in the resonator, which is due to the existence of two-photon bound state[63, 64].

For two incoming left-going photons from the right of the resonator with the initial state

$$|\psi(0)\rangle = \int d\omega d\omega' C_{\omega \omega'}(0) \tilde{\bar{t}}_{\omega \omega'} |0\rangle,$$

the asymptotic state of the field for a long time after two photons colliding with the resonator can be written as

$$|\psi(t)\rangle = \int d\omega d\omega' \left( \beta_{tr}^{\omega \omega'} \tilde{t}_{\omega \omega'} |0\rangle + \beta_{tl}^{\omega \omega'} \tilde{t}_{\omega \omega'} |0\rangle \right) e^{-i(\omega + \omega')t}$$

The amplitudes are related to the single-photon transmission and reflection $\{t_{\omega}, r_{\omega} \}$ by

$$\beta_{tr}^{\omega \omega'} = t'_{\omega} t'_{\omega'} C_{\omega \omega'}(0) + \frac{\gamma_1^2}{\Gamma^2} D_{\omega \omega'},$$

$$\beta_{tl}^{\omega \omega'} = r'_{\omega} r'_{\omega'} C_{\omega \omega'}(0) + \frac{\gamma_2^2}{\Gamma^2} D_{\omega \omega'},$$

$$\beta_{tr}^{\omega \omega'} = t'_{\omega} r'_{\omega'} C_{\omega \omega'}(0) + \frac{\sqrt{\gamma_1 \gamma_2}}{\Gamma^2} D_{\omega \omega'},$$

$$\beta_{tl}^{\omega \omega'} = r'_{\omega} t'_{\omega'} C_{\omega \omega'}(0) + \frac{\sqrt{\gamma_1 \gamma_2}}{\Gamma^2} D_{\omega \omega'}$$

The first term of the amplitudes $\beta_{tr}^{\omega \omega'}$ (a transmitted pair), $\beta_{tl}^{\omega \omega'}$ (a reflected pair), $\beta_{tr}^{\omega \omega'}$, and $\beta_{tl}^{\omega \omega'}$ (a pair of one transmitted and one reflected), is also the contribution from two noninteracting photons, which is in the same magnitude to the corresponding term for the both transmitted, both reflected, one transmitted and one reflected amplitudes in Eq.(19). However, The second part of each amplitude is different in magnitude to the corresponding term in Eq.(19) as long as $\gamma_1 \neq \gamma_2$. In this
ω, ω which has its extremum at the frequency ω = 2Γ/π. Eqs. (19) and (21) indicate that the interference is produced by two waves: one wave describes the two noninteracting photons (denoted as NP part), the other wave describes the two interacting photons (denoted as IP part) which is proportional to

$$D_{ωω'} = \frac{U}{\omega + \omega' - U + 2iΓ} \frac{ε/π}{2} \frac{i2Γ}{i2Γ} \frac{ω + \omega'}{ω + iΓω' + iΓω + ω' - δ + i(ε + Γ)}.$$ (23)

Once the nonlinear interaction U = 0, the IP part disappears. The value of the IP part are strongly dependent on the total energy E = ω + ω′ of the two incident photons for a given U ≠ 0. Its maximum may occur at E = 0, δ, 2δ, U respectively. In Fig.3, we have plotted the probability of the initial state (red solid line and green dashed line in (a)) and the norm square of D_{ωω′} in (b)-(c), and blue dotted line in (a) as function of the frequency ω for given U, ε. It can be observed that the IP part achieves its maximum value at δ = 0 and (ω, ω′) = (0, 0), i.e., both the total energy and the center frequency of each photon is resonant with the resonator, however the incident wave also achieves its maximum value at (ω, ω′) = (0, 0). It indicates that the interference between the NP part and the IP part occurs in the region around (ω, ω′) = (0, 0). In Fig.4, we have plotted the probability of the NP part (a), the IP part (b) of the two transmitted photons, and the norm square of the amplitude α_{ωω′} (c) and amplitude β_{ωω′} (d) as functions of ω, ω′. The different phases of both transmission amplitude introduced by the chiral coupling produce the different interference pattern of two transmitted photons incident from the right-going mode and the left-going mode.

In Fig.3(b-c), we have set E = U = 2δ for the red solid line and E ≠ U = 2δ for green dashed line, E = U ≠ 2δ.
for blue dotted line, where all $\delta \neq 0$. It can be found that the maximum value of the IP part is achieved at $E = 2\delta = U$, i.e., the center frequency of each photon is far away from the resonator mode, the sum of the centers of the two photons is equal to the energy of the resonator containing two photons and the total energy of the incident photon satisfies the two-photon resonance. When the nonlinear strength $U$ is in the order of magnitude of $\Gamma$, e.g. panel (b), the IP part has its maximum value around the $(\omega, \omega') = (1, 1)$ where the incident state also achieves its maximum value, but the width of the IP part is larger than that of the incident state. When the nonlinear strength $U$ is much larger than $\Gamma$, e.g. panel (c), the bound state achieves its maximum value around the $(\omega, \omega') = (0, 10), (10, 0)$ however, the incident wave achieves its maximum value at $(\omega, \omega') = (5, 5)$. Hence, the IP part gives rise to a redistribution of the energy of the photons. It is known in the previous section that the propagating single photon with the center around $\delta > \Gamma$ and width $\epsilon \ll \Gamma$ gets nearly total transmitted, in other word, the NP part is the domination of the amplitude of the both photons transmitted, however, the IP part plays the important role in the amplitude of the both photons reflected [63, 73, 74].

To show the effect of the asymmetric coupling on the scattering wave of the two photons coming from the left or right of the nonlinear resonator, we have plotted the norm square of the amplitudes of $\alpha_{\omega\omega'}^r$ and $\beta_{\omega\omega'}^r$ as functions of $\gamma_2$ by fixed $U = 10, \epsilon = 0.1$ at the single photon resonance $\omega = \omega' = \delta = 0$ and the two-photon resonance $(\omega, \omega') = (0, 10)$ or $(0, 10), \delta = 5$ as well as the two reflected photons as functions of $\gamma_2$ in Fig.5. The asymmetric light transmission can be obtained as long as $\gamma_1 \neq \gamma_2$. At the single photon resonance in Fig.5(a), the probabilities for two transmitted photons first decreases as $\gamma_2$ increases from 0 to 0.5, then they increases as $\gamma_2$ increases, they only differ each other in magnitude, we note that this phenomenon also appears when the position of the maximum value of the IP part is coincidence with the center of the incident wave. At the two-photon resonance in Fig.5(b), the probabilities for two transmitted photons $\alpha_{\omega\omega'}^r$ incident from the left of the nonlinear resonator decreases as $\gamma_2$ increases, but the probabilities for two transmitted photons $\beta_{\omega\omega'}^r$ incident from the right side increases as $\gamma_2$ increases. The monotonicity in Fig.5(b) is produced merely by the IP part because the incident photons are localized around $(\omega, \omega') = (5, 5)$ under the condition $\epsilon \ll \Gamma < \delta$. Since the asymmetric coupling permits unidirectional propagation of light and forbids the transmission in the reverse direction, the nonlinear resonator accomplishes the diode activity for two identical photons at the two-photon resonance with each energy far away from the incident center.

FIG. 5. (Color online) The norm square of the amplitudes of $\alpha_{\omega\omega'}^r$ and $\beta_{\omega\omega'}^r$ as functions of $\gamma_2$ for (a) $\omega = \omega' = \delta = 0$, (b) $(\omega, \omega') = (0, 10)$ or $(0, 10), \delta = 5$. (c) The probability for the two reflected photons as functions of $\gamma_2$. The other parameters are set as follow: $U = 10, \epsilon = 0.1$. All parameters are in units of $\Gamma$.

V. CONCLUSION

In conclusion, we have studied the coherent transport of one or two photons in a 1D waveguide asymmetrically coupled to a nonlinear resonator. We first construct the exact single-photon scattering states after a long time, and find that the asymmetric coupling leads to a phase difference on the amplitude of transmittance, a non-reciprocal phase shifter can be formed by the nonlinear resonator. Later, an analytic solution of two-photon scattering is derived, it is found that 1) the two-photon scattering state incident from either the left side or the right side of the nonlinear resonator contains the contribution of the NP part and the IP part. The different phases of transmission amplitude caused by the chiral coupling produce the different interference pattern of two-photon transmission at single-photon resonance. 2) The forward and reverse transmitted probabilities change
with the asymmetric coupling. The forward and reverse transmitted probabilities have the same monotonicity as the asymmetric coupling changes and only differ each other in the magnitude as the position of the maximum value of the IP part is coincidence with the center of the incident wave. However, the forward and reverse transmitted probabilities have different monotonicity under the condition $|\epsilon| < |\Gamma| < \delta$ when the position of the maximum value of the IP part is far away from the center of the incident wave, so a nonreciprocal two-photon transmission through a nonlinear resonator is found due to the energy redistribution of the two photons by the bound state, which indicates that the nonlinear resonator acts as an optical diode.

**ACKNOWLEDGMENTS**

This work was supported by NSFC Grants No. 11975095, No. 12075082, No.11935006, No.12064010, and the science and technology innovation Program of Hunan Province (Grant No. 2020RC4047)

[1] D. Jalas, A. Petrov, M. Eich, W. Freude, S. Fan, Z. Yu, R. Baets, M. Popovic, A. Melloni, J. D. Joannopoulos, M. Vanwolleghem, C. R. Doerr, and H. Renner, Nat. Photon. 7, 579 (2013).
[2] L. Bi, J. Hu, P. Jiang, D. H. Kim, G. F. Dionne, L. C. Kimerling, and C. A. Ross, Nat. Photon. 5, 758 (2011).
[3] L. Fan, J. Wang, L. T. Varghese, H. Shen, B. Niu, Y. Xuan, A. M. Weiner, and M. Qi, Science 335, 447 (2012).
[4] Z. Wang, L. Shi, Y. Liu, X. Xu, and X. Zhang, Sci. Rep. 5, 8657 (2015).
[5] A. S. Zheng, G. Y. Zhang, H. Y. Chen, T. T. Mei, and J. B. Liu, Sci. Rep. 7, 14001 (2017).
[6] C. Gonzalez-Ballestero, E. Moreno, F. J. Garcia-Vidal, and A. Gonzalez-Tudela, Phys. Rev. A 94, 063817 (2016).
[7] Q. T. Cao, H. Wang, C. H. Dong, H. Jing, R. S. Liu, X. Chen, L. Ge, Q. Gong, and Y. F. Xiao, Phys. Rev. Lett. 118, 033901 (2017).
[8] L. N. Song, Z. H. Wang, and Y. Li, Opt. Commun. 415, 39 (2018).
[9] X. Zhang, Q. T. Cao, Z. Wang, Y. X. Liu, C. W. Qiu, L. Yang, Q. Gong, and Y. F. Xiao, Nat. Photon. 13, 21 (2019).
[10] S. R. K. Rodriguez, V. Goblot, N. Carlon Zambon, A. Amo, and J. Bloch, Phys. Rev. A 99, 013851 (2019).
[11] P. Yang, X. Xia, H. He, S. Li, X. Han, P. Zhang, G. Li, P. Zhang, J. Xu, Y., and T. Zhang, Phys. Rev. Lett. 123, 236404 (2019).
[12] P. Lodahl, S. Mahmoodian, S. Stobbe, A. Rauschenbeutel, P. Schneeweiss, J. Volz, H. Pichler, and P. Zoller, Nature 541, 473 (2017).
[13] D. Roy, Phys. Rev. B 81, 155117 (2010).
[14] M. Scheucher, A. Hilico, E. Will, J. Volz, and A. Rauschenbeutel, Science 354, 1577 (2016).
[15] W. B. Yan, W. Y. Ni, J. Zhang, F. Y. Zhang, and H. Fan, Phys. Rev. A 98, 043852 (2018).
[16] Z. Wang, L. Du, Y. Li, and Y. X. Liu, Phys. Rev. A 100, 053809 (2019).
[17] S. Manipatruni, J. T. Robinson, and M. Lipson, Phys. Rev. Lett. 102, 213903 (2009).
[18] M. Hafezi and P. Rabl, Opt. Express 20, 7672 (2012).
[19] X. W. Xu and Y. Li, Phys. Rev. A 91, 053854 (2015).
[20] A. Metelmann and A. A. Clerk, Phys. Rev. X 5, 021025 (2015).
[21] M. Schmidt, S. Kessler, V. Peano, O. Painter, and P. Marquardt, Optica 2, 635 (2015).
[22] L. Tian and Z. Li, Phys. Rev. A 96, 013808 (2017).
[23] Z. Shen, Y. L. Zhang, Y. Chen, C. L. Zou, Y. F. Xiao, X. B. Zou, F. W. Sun, G. C. Guo, and C. H. Dong, Nat. Photon. 10, 657 (2016).
[24] F. Ruesink, M. A. Miri, A. Ali, and E. Verhagen, Nat. Commun. 7, 13662 (2016).
[25] K. Fang, J. Luo, A. Metelmann, M. H. Matheny, F. Marquardt, A. A. Clerk, and O. Painter, Nat. Phys. 13, 465 (2017).
[26] Y. Jiang, S. Maayani, T. Carmon, F. Nori, and H. Jing, Phys. Rev. Applied 10, 064037 (2018).
[27] D. G. Lai, J. F. Huang, X. L. Yin, B. P. Hou, W. Li, D. Vitali, F. Nori, and J. Q. Liao, Phys. Rev. A 102, 011502(R) (2020).
[28] Y. Chen, Y. L. Zhang, Z. Shen, C. L. Zou, G. C. Guo, and C. H. Dong, Phys. Rev. Lett. 126, 123603 (2021).
[29] D. W. Wang, H. T. Zhou, M. J. Guo, J. X. Zhang, J. Ev-ers, and S.Y. Zhu, Phys. Rev. Lett. 110, 093901 (2013).
[30] S. A. R. Horsley, J. H. Wu, M. Artoni, and G. C. La Rocca, Phys. Rev. Lett. 110, 223602 (2013).
[31] K. Xia, F. Nori, and M. Xiao, Phys. Rev. Lett. 121, 203602 (2018).
[32] S. Zhang, Y. Hu, G. Lin, Y. Niu, K. Xia, J. Gong, and S. Gong, Nat. Photon. 12, 744 (2018).
[33] E. Z. Li, D. S. Ding, Y. C. Yu, M. X. Dong, L. Zeng, W. H. Zhang, Y. H. Ye, H. Z. Wu, Z. H. Zhu, W. Gao, G. C. Guo, and B. S. Shi, Phys. Rev. Research 2, 033517 (2020).
[34] C. Liang, B. Liu, A. N. Xu, X. Wen, C. Lu, K. Xia, M. K. Tey, Y. C. Liu, and L. You, Phys. Rev. Lett. 125, 123901 (2020).
[35] C. E. Ruter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev, and D. Kip, Nat. Phys. 6, 192 (2010).
[36] N. Bender, S. Factor, J. D. Bodyfelt, H. Ramezani, D. N. Christodoulides, F. M. Ellis, and T. Kottos, Phys. Rev. Lett. 110, 234101 (2013).
[37] B. Peng, S. K. Ozdemir, F. Lei, F. Monifi, M. Gianfreda, G. L. Long, S. H. Fan, F. Nori, C. M. Bender, and L. Yang, Nat. Phys. 10, 394 (2014).
[38] L. Chang, X. S. Jiang, S. Y. Hua, C. Yang, J. M. Wen, L. Jiang, G. Y. Li, G. Z. Wang, and M. Xiao, Nat. Photon. 8, 524 (2014).
[39] R. Huang, A. Miranowicz, J. Q. Liao, F. Nori, and H. Jing, Phys. Rev. Lett. 121, 153601 (2018).
B. Li, R. Huang, X. W. Xu, A. Miranowicz, and H. Jing, Photon. Res. 7, 630 (2019).
[41] K. Wang, Q. Wu, Y. F. Yu, and Z. M. Zhang, Phys. Rev. A 100, 053832 (2019).
[42] H. Z. Shen, Q. Wang, J. Wang, and X. X. Yi, Phys. Rev. A 101, 013826 (2020).
[43] X. W. Xu, Y. J. Zhao, H. Wang, H. Jing, and A. X. Chen, Photon. Res. 8, 143 (2020).
[44] X. W. Xu, Y. Li, B. Li, H. Jing, and A. X. Chen, Phys. Rev. Applied 13, 044070 (2020).
[45] Y. F. Jiao, S. D. Zhang, Y. L. Zhang, A. Miranowicz, L. M. Kuang, and H. Jing, Phys. Rev. Lett. 125, 143605 (2020).
[46] F. X. Sun, D. Mao, Y. T. Dai, Z. Ficek, Q. Y. He, Q. H. Gong, New J. Phys. 19, 123039 (2017).
[47] Zhou L, Yang S, Liu Y X, Phys. Rev. A. 80, 062109 (2009).
[48] D.E. Chang et al., A.S. Sørensen, E.A. Demler, and M. D. Lukin, Nat. Phys. 3, 807 (2007).
[49] L. Zhou, Z.R. Gong, Y.-x. Liu, C.P. Sun and F. Nori, Phys. Rev. Lett. 101, 100501 (2008).
[50] J. T. Shen and S. H. Fan, Phys. Rev. Lett. 98, 153003 (2007).
[51] J. T. Shen and S. H. Fan, Phys. Rev. A 76, 062709 (2007).
[52] S. Xu and S. H. Fan, Phys. Rev. A 94, 043826 (2016).
[53] D. Roy, Phys. Rev. Lett. 106, 053601 (2011).
[54] H. X. Zheng, D. J. Gauthier, and H. U. Baranger, Phys. Rev. Lett. 107, 223601 (2011).
[55] H. X. Zheng, D. J. Gauthier, and H. U. Baranger, Phys. Rev. A 85, 043832 (2012).
[56] T. Y. Li, J. F. Huang, and C. K. Law, Phys. Rev. A 91, 043834 (2015).
[57] Y. Pan and G. Zhang, J. Phys. A: Math. Theor. 50, 345301 (2017).
[58] H. Xiao, L. Wang, L. Yuan, and X. Chen, ACS Photonics 7, 2010 (2020).
[59] E. Rephaeli, S. E. Kocbas, and S. H. Fan, Phys. Rev. A 84, 063832 (2011).
[60] D. Roy, Phys. Rev. A 87, 063819 (2013).
[61] D. Roy, Sci. Rep. 3, 2337 (2013).
[62] J. F. Huang, J. Q. Liao, and C. P. Sun, Phys. Rev. A 87, 023822 (2013).
[63] J. Q. Liao and C. K. Law, Phys. Rev. A 82, 053836 (2010).
[64] X.-W. Xu and Yong Li, Phys. Rev. A 90, 033832 (2014)
[65] T. Shi, S. H. Fan, and C. P. Sun, Phys. Rev. A 84, 063803 (2011).
[66] T. Shi and S. H. Fan, Phys. Rev. A 87, 063818 (2013).
[67] K. B. Joanesarson, J. Iles-Smith, M. Heuck, and J. Mork, Phys. Rev. A 101, 063809 (2020).
[68] W. B. Yan, Q. B. Fan, and L. Zhou, Phys. Rev. A 85, 015803 (2012).
[69] W. Z. Jia and Z. D. Wang, Phys. Rev. A 88, 066321 (2013).
[70] L. Qiao, Phys. Rev. A 96, 013860 (2017).
[71] J. Liu, W. Zhang, X. Li, W. Yan, and L. Zhou, Int. J. Theor. Phys. 55, 4620 (2016).
[72] Y. Xu, Y. Li, R. K. Lee, and A. Yariv, Phys. Rev. E 62, 7389 (2000).
[73] J. T. Shen and S. Fan, Opt. Lett. 30, 2001 (2005).
[74] J. T. Shen and S. Fan, Phys. Rev. Lett. 95, 213601 (2005).
[75] J. T. Shen and S. Fan, Phys. Rev. A 79, 023837 (2009).
[76] J. T. Shen and S. Fan, Phys. Rev. A 79, 023838 (2009).