A two patch model for the population dynamics of mosquito-borne diseases

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Abstract. In this work, a model on the interaction dynamics of mosquitoes causing vector-borne diseases is formulated, without consider the human population as dynamic variables and including mobility from eulerian approach, in which the populations are not assigned to a place in particular, but they move between patches at certain rates. The results suggest that in order to eradicate the disease, the mosquito population in each patch must be controlled, but in order to reduce the magnitude of an epidemic outbreak, the transformation rate from susceptible to infected, the migration and immigration rate must be controlled.

1. Introduction
Vector-borne diseases are illnesses caused by pathogens and parasites in human populations, these diseases are transmitted by living organisms called vectors. Many of these vectors are bloodsucking insects that ingest disease-producing micro-organisms during a blood meal from an infected host (human or animal) and later inject them into a new host during their next blood meal. Mosquitoes are the best-known disease vector [1]. Major vector-borne diseases of humans include malaria and dengue. Malaria is caused by Plasmodium falciparum, P. vivax, P. ovale and P. malariae and it is transmitted by the bite of mosquitoes of the Anopheles genus (more than 60 known species can transmit diseases). Currently, the vector-borne disease with the highest global growth is dengue. Like the dengue virus, Zika virus, chikungunya and yellow fever are transmitted by Aedes aegypti and Aedes albopictus mosquitoes [2]. These diseases account for around 17% of the estimated global burden of communicable diseases and disproportionately affect poorer populations [1].

Gómez and Ibargüen in [3] formulated a model for the population dynamics between susceptible and infected mosquitoes. Let \(x(t)\) and \(y(t)\) populations of susceptible and infected mosquitoes at time \(t\), respectively. They assumed that the growth of susceptible mosquito follows a logistical regulation of total mosquito population \(x + y\) with carrying capacity denoted by \(k\) and intrinsic rate of growth \(\gamma\). On the other hand, susceptible and infected mosquitoes die at per capita constat rate \(\mu\). Finally, the transformation rate of susceptible to infected mosquitoes, due to contact with an infected individual with a vector borne disease is given by the function \(\beta\), which represent non constant transformation rate at time \(t\). The above lead to the following system of nonlinear differential equations (Equation (1) and Equation (2)).

\[
\frac{dx}{dt} = \gamma(x + y)\left(1 - \frac{x+y}{k}\right) - (\beta(t) + \mu)x
\]
\[ \frac{dy}{dt} = \beta(t)x - \mu \] (2)

As in [2,4-8], the qualitative analysis of the model, Equation (1) and Equation (2), was made in terms of threshold conditions. In this case, the threshold is \( \Phi = \gamma/\mu \); which is interpreted as the number of susceptible mosquitoes arise from total mosquito population during its average life. For the constant transformation rate \( \beta(t) \) for all \( t \), the results of existence and stability of equilibrium solutions are summarized in the following propositions.

- **Proposition 1.** If \( \Phi \leq 1 \), then \( E_0 = (0,0) \) is the only equilibrium solution of Equation (1) and Equation (2). If \( \Phi > 1 \), in addition to \( E_0 \) there exists the coexistence equilibrium (Equation (3)).

\[ E_1 = \left( \frac{k\mu(\Phi-1)}{(\beta+\mu)\Phi}, \frac{k\beta(\Phi-1)}{(\beta+\mu)\Phi} \right) \] (3)

- **Proposition 2.** If \( \Phi \leq 1 \), then \( E_0 \) is globally asymptotically stable, while if \( \Phi > 1 \), then \( E_1 \) is globally asymptotically stable.

The qualitative analysis revealed the existence of a forward bifurcation, in which mosquito-free equilibrium, \( E_0 \), loses its stability to \( \Phi = 1 \) and a transcritical bifurcation takes place. These results suggest that in order to eradicate the disease, the mosquito population must be controlled, but in order to reduce the magnitude of an epidemic outbreak, the transformation rate from non-carriers to carriers must be controlled.

### 2. Two patch model

With the purpose to study the impact of vector population spread into the spatial distribution of vector borne diseases, we deduce an approximation of displacement of vectors between two patches. Let \( S_i(t) \) and \( I_i(t) \) denote the number of susceptible and infected mosquitoes in patch \( i \) at time \( t \), respectively.

For patch \( i \), all newborns in vector population are assumed to be into the susceptible class (no vertical transmission). The growth of susceptible mosquito, \( S_i(t) \), follow a logistical regulation of total mosquito population \( S_i + I_i(t) \) with carrying capacity denoted by \( k_i \) and intrinsic rate of growth \( \gamma_i \). On the other hand, susceptible and infected mosquitoes die at per capita constant rate \( \mu_i \). The population of infected mosquito grows as the susceptible mosquito acquires the parasite of the carrier individuals (humans or animals) at a ratio \( \beta_i S_i \) where \( \beta_i \) is the constant transformation rate. In the model, migration between patches is given by rates: \( \psi_{12}^L \geq 0 \) for \( L = S, I \) is the immigration rate from patch 2 to patch 1 of susceptible and infectious mosquitoes, respectively. \( \psi_{21}^L \geq 0 \) for \( L = S, I \) is the immigration rate from patch 1 to patch 2 of susceptible and infectious mosquitoes, respectively. \( -\psi_{11}^L \geq 0 \) for \( L = S, I \) is the emigration rate of susceptible and infectious mosquitoes in patch 1, respectively. \( -\psi_{22}^L \geq 0 \) for \( L = S, I \) is the emigration rate of susceptible and infectious mosquitoes in patch 2, respectively.

The immigration and emigration rates defined above satisfy \( \psi_{12}^L = -\psi_{22}^L \) and \( \psi_{21}^L = -\psi_{11}^L \) where \( L = S, I \). Under above assumptions we obtain the following system of ordinary differential equations, Equation (4) to Equation (7).

\[ \frac{ds_1}{dt} = \gamma_1(s_1 + i_1)\left(1 - \frac{s_1 + I_1}{k_1}\right) - \beta_1 s_1 - \mu_1 s_1 + \psi_{12} s_1 + \psi_{12} s_2 \] (4)

\[ \frac{ds_2}{dt} = \gamma_2(s_2 + i_2)\left(1 - \frac{s_2 + I_2}{k_2}\right) - \beta_2 s_2 - \mu_2 s_2 + \psi_{21} s_1 + \psi_{22} s_2 \] (5)

\[ \frac{di_1}{dt} = \beta_1 s_1 - \mu_1 i_1 + \psi_{11} i_1 + \psi_{12} i_2 \] (6)
\[
\frac{dl_2}{dt} = \beta_2 S_2 - \mu_2 l_2 + \psi_{21} l_1 + \psi_{22} l_2
\]  
(7)

with non-negative initial conditions \(S_1(0) = S_1^0\), \(S_2(0) = S_2^0\), \(I_1(0) = I_1^0\) and \(I_2(0) = I_2^0\). The set of biological interest is given by Equation (8).

\[
\Omega = \{(S_1, S_2, I_1, I_2) \in \mathbb{R}_+^4; S_1 + I_1 + S_2 + I_2 \leq 2k, k = \max\{k_1, k_2\}\}
\]  
(8)

The following lemma ensures that system, Equation (4) to Equation (7), has biological sense, that is, all solutions starting in remain there for all \(t \geq 0\).

- Lemma. Set \(\Omega\) defined in Equation (8) is positively invariant for solutions of the system, Equation (4) to Equation (7).
- Proof. Let \(M_1 = S_1 + I_1\) and \(M_2 = S_2 + I_2\), see Equation (9).

\[
\frac{d}{dt}(M_1 + M_2) = \gamma_1 M_1 \left(1 - \frac{M_1}{k_1}\right) - \mu_1 M_1 + \gamma_2 M_2 \left(1 - \frac{M_2}{k_2}\right) - \mu_2 M_2.
\]  
(9)

We obtain the following inequality, Equation (10).

\[
\frac{d}{dt}(M_1 + M_2) \leq \gamma (M_1 + M_2) - \frac{\gamma}{k} (M_1^2 + M_2^2) \leq \gamma (M_1 + M_2) \left(1 - \frac{M_1 + M_2}{2k}\right)
\]  
(10)

where \(\gamma = \max\{\gamma_1, \gamma_2\}\) and \(k = \max\{k_1, k_2\}\). Let \(M = M_1 + M_2\), then Equation (10) is rewritten as Equation (11).

\[
\frac{dM}{dt} \leq \gamma M \left(1 - \frac{M}{2k}\right)
\]  
(11)

The solution \(M(t)\) of inequality, Equation (11) in the interior of \(\Omega\) satisfies, Equation (12).

\[
M(t) \leq \left[\left(\frac{1}{M(0)} - \frac{1}{2k}\right) \exp(-\gamma t) + \frac{1}{2k}\right]^{-1},
\]  
(12)

In consequence \(M(t) \leq 2k\) for \(t \geq 0\). On the other hand, it can be verified that the vector field defined by right side of Equation (4) to Equation (7) points to the interior of \(\Omega\). Therefore, the solutions starting in remain there for all \(t \geq 0\).

3. Equilibrium solutions

In this section, we will characterize the equilibrium solutions of the system, Equation (4) to Equation (7). Before vector-borne disease, the system is at the equilibrium \(S_1 = 0, I_1 = 0, S_2 = 0\), and \(I_2 = 0\). Suppose that vectors have contact with host individuals of parasite. The existence of coexistence equilibria will depend of the following threshold quantities \(\Phi_1 = \gamma_1/\mu_1\) and \(\Phi_2 = \gamma_2/\mu_2\). The parameters are interpreted as the number of mosquitoes arise from total mosquito population during its average life in patch 1 and patch 2, respectively. The matrix formulation of the model, Equation (4) to Equation (7) is Equation (13).

\[
\begin{align*}
\dot{S} &= \text{diag} \left[\text{diag}(\gamma) \left[1 - \text{diag} \left(\frac{1}{k}\right) (S + 1)\right]\right] (S + 1) - \text{diag}(\beta + \mu) S + \psi^2 S I \\
\dot{S} &= \text{diag}(\beta) S - \text{diag}(\mu) I + \psi I
\end{align*}
\]  
(13)
Where $S = (S_1, S_2)^T, I = (I_1, I_2)^T, \gamma = (\gamma_1, \gamma_2)^T, \beta = (\beta_1, \beta_2)^T, \mu = (\mu_1, \mu_2)^T, 1/k = (1/k_1, 1/k_2)^T, \psi_s = (\psi_{ij}^s)_{1 \leq i,j \leq 2}, \psi^l = (\psi_{ij}^l)_{1 \leq i,j \leq 2}$ and $\text{diag}(\gamma) = \text{diag}(\gamma_1, \gamma_2)$. Setting the Equation (13) equal to zero we obtain the following matrix, Equation (14).

$$O = \text{diag}(\gamma) \left[ 1 - \text{diag} \left( \frac{1}{k} \right) (S + I) \right] (S + I) - \text{diag}(\beta + \mu)S + \psi_s S$$

$$O = \text{diag}(\beta)S - \text{diag}(\mu)I + \psi^l I$$

Solving is done, Equation (15) to Equation (18).

$$S_1 = a_1(\Phi_1 - 1) - b_1(\Phi_2 - 1)$$

$$S_2 = a_2(\Phi_2 - 1) - b_2(\Phi_1 - 1)$$

$$I_1 = a_1(\Phi_1 - 1) - b_1(\Phi_2 - 1)$$

$$I_2 = a_2(\Phi_2 - 1) - b_2(\Phi_1 - 1)$$

Where, $a_1, a_2, b_1, b_2, \bar{a}_1, \bar{a}_2$ are defined according to Equation (19) to Equation (24).

$$a_1 = \frac{\mu_1 (\beta_2 k_1 \gamma_2 \psi_{12}^l + \beta_2 k_1 \gamma_2 \mu_1 \gamma_2 \psi_{12}^s + \kappa_1 \gamma_2 \mu_2 \psi_{12}^s + \kappa_1 \gamma_2 \mu_1 \mu_2)}{(\beta_2 + \psi_{12}^l + \mu_2 + \mu_1 + \mu_1 \mu_2 + \mu_2 \psi_{12}^s + \mu_1 \mu_2) \gamma_1 \gamma_2}$$

$$a_2 = \frac{\mu_2 (\beta_2 k_2 \gamma_1 \psi_{12}^l + \beta_2 k_2 \gamma_1 \mu_2 \gamma_1 \psi_{12}^s + \kappa_2 \gamma_1 \mu_2 \psi_{12}^s + \kappa_2 \gamma_1 \mu_1 \mu_2)}{(\beta_2 + \psi_{12}^l + \mu_2 + \mu_1 + \mu_1 \mu_2 + \mu_2 \psi_{12}^s + \mu_1 \mu_2) \gamma_1 \gamma_2}$$

$$b_1 = \frac{\mu_1 \beta_2 k_1 \gamma_1 \psi_{12}^s}{(\beta_1 + \beta_2 + \psi_{12}^l + \mu_1 + \mu_1 + \mu_1 \mu_2 + \mu_2 \psi_{12}^s + \mu_1 \mu_2) \gamma_1 \gamma_2}$$

$$b_2 = \frac{\mu_2 \beta_2 k_2 \gamma_2 \psi_{12}^s}{(\beta_1 + \beta_2 + \psi_{12}^l + \mu_1 + \mu_1 + \mu_1 \mu_2 + \mu_2 \psi_{12}^s + \mu_1 \mu_2) \gamma_1 \gamma_2}$$

$$\bar{a}_1 = \frac{\mu_1 (\beta_1 k_2 \gamma_2 \psi_{12}^l + \beta_1 k_2 \gamma_2 \mu_1 \gamma_2 \psi_{12}^s + \kappa_1 \gamma_2 \mu_2 \psi_{12}^s + \kappa_1 \gamma_2 \mu_1 \mu_2)}{(\beta_1 + \beta_2 + \psi_{12}^l + \mu_1 + \mu_1 + \mu_1 \mu_2 + \mu_2 \psi_{12}^s + \mu_1 \mu_2) \gamma_1 \gamma_2}$$

$$\bar{a}_2 = \frac{\mu_2 (\beta_2 k_1 \gamma_1 \psi_{12}^l + \beta_2 k_1 \gamma_1 \mu_2 \gamma_1 \psi_{12}^s + \kappa_2 \gamma_1 \mu_2 \psi_{12}^s + \kappa_2 \gamma_1 \mu_1 \mu_2)}{(\beta_1 + \beta_2 + \psi_{12}^l + \mu_1 + \mu_1 + \mu_1 \mu_2 + \mu_2 \psi_{12}^s + \mu_1 \mu_2) \gamma_1 \gamma_2}$$

The coexistence equilibrium is given by $E_1 = (S_1, I_1, S_2, I_2)$. In the following result, it is verified that $E_1 \in \Omega$ under certain conditions.

- **Theorem.** If $\Phi_2 > 1$ and $\Phi_1 > 1$, then $E_1$ belongs to $\Omega$.

- **Proof.** Observe that if we verify that $S_1 > 0$ and $S_2 > 0$, then $E_1 \in \Omega$. To this end, we determine the conditions for which the expressions are positive, that is Equation (25) and Equation (26).

$$a_1 x - b_1 y > 0$$

$$-a_2 x - b_2 y > 0$$
Where \(a_1, a_2, b_1\) and \(b_2\) are defined in (7), \(x = \Phi_1 - 1\) and \(y = \Phi_2 - 1\). Since \(\Phi_1 > 1\) and \(\Phi_2 > 1\), then \(x > 0\) and \(y > 0\). Now, multiplying the first inequality of Equation (25) and Equation (26) by \(b_2\), the second inequality by \(b_1\) and adding the resulting inequalities we obtain \((b_2 a_1 - b_1 a_{12})x > 0\), since \(x > 0\) then \(b_2 a_1 - b_1 a_{12} > 0\). From Equation (19) to Equation (24), the previous inequality is verified.

4. Stability of equilibrium solutions

In this section we analyse the stability of equilibria. We begin by analysing the stability of the vector-free equilibrium. To this end, we will use the following stability test based on Gershgorin circles [9,10].

- Proposition 1. Let \(\bar{x}\) an equilibrium point of the system of ordinary differential equation \(x' = f(x)\), Equation (27).

\[
Df(\bar{x}) = \begin{pmatrix}
I_{11} & \cdots & I_{1n} \\
\vdots & \ddots & \vdots \\
I_{n1} & \cdots & I_{nn}
\end{pmatrix}
\]

(27)

The Jacobian matrix evaluated in \(\bar{x}\) and Equation (28).

\[
R_j = \sum_{i=1}^{n} |I_{ij}|
\]

(28)

For \(j = 1, \ldots, n\) if \(I_{jj} < 0\) and \(R_j < |I_{jj}|\) for \(j = 1, \ldots, n\) then \(\bar{x}\) is locally asymptotically stable. See [4] for a proof of Proposition 1. The stability of \(E_0\) depends on \(\Phi_1\) and \(\Phi_2\) defined formerly and the following parameters, Equation (29).

\[
\begin{align*}
\Phi_1 &= \frac{\gamma_1}{\beta_1 + \mu_1 + \psi_{21}^S}, \\
\Phi_2 &= \frac{\gamma_2}{\beta_2 + \mu_2 + \psi_{12}^S}
\end{align*}
\]

(29)

The parameter \(\Phi_1\) defined in Equation (29) is interpreted as the number of mosquitoes arise from susceptible mosquito population that immigrate from patch 2 to patch 1, and \(\Phi_2\) defined in Equation (29) is interpreted as the number of mosquitoes arise from susceptible mosquito population that immigrate from patch 1 to patch 2. The stability of the vector-free equilibrium is established in the following proposition.

- Proposition 2. If \(\Phi_1 < 1\) and \(\Phi_1 < 1\) for \(i = 1,2\), then \(E_0\) is locally and asymptotically stable in \(\Omega\).

- Proof. Jacobian of system (3) evaluated at the equilibrium \(E_0\) is given by Equation (30).

\[
J(E_0) = \begin{pmatrix}
\gamma_1 - \beta_1 - \mu_1 - \psi_{21}^S & \psi_{12}^S & \gamma_1 & 0 \\
\psi_{21}^S & \gamma_2 - \beta_2 - \mu_2 - \psi_{12}^S & 0 & \gamma_2 \\
\beta_1 & 0 & -\mu_1 - \psi_{21}^S & \psi_{12}^S \\
0 & \beta_2 & \psi_{21}^S & -\mu_2 - \psi_{12}^S
\end{pmatrix}
\]

(30)

Now, we will verify the hypothesis of Proposition 1. From (30) we observe \(I_{jj} < 0\) for \(j = 1,2\) if and only if \(\gamma_1 < \beta_1 + \mu_1 + \psi_{21}^S\) and \(\gamma_2 < \beta_2 + \mu_2 + \psi_{12}^S\) or equivalently \(\Phi_1 < 1\) and \(\Phi_2 < 1\). On the other hand, \(R_j < |I_{jj}|\) for \(j = 1, \ldots, n\) if and only if \(\gamma_1 < \mu_1\) and \(\gamma_2 < \mu_2\) or equivalently \(\Phi_1 < 1\) and \(\Phi_2 < 1\). In consequence, from Proposition 1 is concluded the local stability of \(E_0\). The proposition resumes the stability result of \(E_1\).
• Proposition 3. If $1 < \Phi_i < 1$ for $i = 1, 2$ then $E_1$ is locally and asymptotically stable in $\Omega$.

• Proof. Jacobian of system, Equation (4) to Equation (7), evaluated at the equilibrium $E_1$ is given by Equation (31).

$$\begin{align*}
J(E_1) &= \begin{pmatrix}
I_{11}(E_1) & \psi_{12}^S & \gamma_1 \left(1 - 2 \frac{S_1 + I_1}{k_1}\right) & 0 \\
\psi_{21}^S & I_{22}(E_1) & 0 & \gamma_2 \left(1 - 2 \frac{S_2 + I_2}{k_2}\right) \\
\beta_1 & 0 & -\mu_1 - \psi_{21}^S & \psi_{12}^S \\
0 & \beta_2 & \psi_{21}^S & -\mu_2 - \psi_{12}^S \\
\end{pmatrix} \\
&= (31)
\end{align*}$$

Where $J_{11}(E_1) = \gamma_1 \left(1 - 2 \frac{S_1 + I_1}{k_1}\right) - \beta_1 - \mu_1 - \psi_{21}^S$ and $J_{22}(E_1) = \gamma_2 \left(1 - 2 \frac{S_2 + I_2}{k_2}\right) - \beta_2 - \mu_2 - \psi_{12}^S$. From non-trivial solutions we obtain Equation (32) and Equation (33).

$$\begin{align*}
S_1 + I_1 &= (a_1 + \bar{a}_1)(\Phi_1 - 1) = \frac{k_1(\Phi_1 - 1)}{\Phi_1} \\
S_2 + I_2 &= (a_2 + \bar{a}_2)(\Phi_2 - 1) = \frac{k_2(\Phi_2 - 1)}{\Phi_2} \\
&= (32) \\
&= (33)
\end{align*}$$

Substituting Equation (31) in Equation (32) and Equation (33), we obtain Equation (34).

$$J(E_1) = \begin{pmatrix}
\gamma_1 - \beta_1 - \mu_1 - \psi_{21}^S & \psi_{12}^S & -\gamma_1 + 2 \mu_1 & 0 \\
\psi_{21}^S & \gamma_2 - \beta_2 - \mu_2 - \psi_{12}^S & 0 & -\gamma_2 + 2 \mu_2 \\
\beta_1 & 0 & -\mu_1 - \psi_{21}^S & \psi_{12}^S \\
0 & \beta_2 & \psi_{21}^S & -\mu_2 - \psi_{12}^S \\
\end{pmatrix}$$

$$= (34)$$

Now, from (13) we observe $|J_{jj}| < 0$ when $\Phi_j > 1$ for $j = 1, 2$. On the other hand, $R_j < |J_{jj}|$ for $j = 1, \ldots, n$ if and only if $1 < \Phi_1 < 3$ and $1 < \Phi_2 < 3$. In consequence, from Proposition 1 is concluded the local stability of $E_1$.

![Figure 1. Numerical simulations of the model.](image)
5. Numerical solutions
In this section, we present numerical simulations and graphs illustrating the growth of susceptible and infectious mosquito population, respectively. The value of parameters $\gamma_1, \gamma_2 = 0.0558$, $\mu_1 = \mu_2 = 0.0039$, $\psi_1 = 0.002$ and $\psi_2 = 0.001$ were taken from [5,6,11,12]. For above values $\Phi_1 = \Phi_2 = 14.31$. The difference in migration rates between the patches produces an increase in the number of infected mosquitoes and a reduction in susceptible mosquito population in patch 2; in contrast to this, in patch 1 the susceptible mosquito population remains constant after a slight variation in the first 170 days (see Figure 1).

6. Conclusion
In this paper we formulate a two patch model trying to determine the effect of mobility in vector-borne diseases as Dengue, Zika or Malaria. From qualitative analysis of the model, we obtain two steady states: the mosquito-free state $E_0$ and the coexistence equilibrium $E_1$. The existence and stability of equilibria $E_0$ and $E_1$ are given in terms of the parameters $\Phi_1$ and $\Phi_2$. They are interpreted as the number of mosquitoes arise from total mosquito population during its average life in patch 1 and patch 2 (in patch 2 and patch 1), respectively. When $\Phi_1 < 1$ for $i = 1,2$, the solutions approach the mosquito-free equilibrium which means that susceptible and infected mosquito population tend to be controlled. If $1 < 1 < \Phi_1 < 3$ for $i = 1,2$, the mosquito population could not be controlled. As shown in Figure 1 the mosquito population could not be controlled to $\Phi_1 > 1$ for $i = 1,2$. These results suggest that in order to eradicate the disease, the mosquito population must be controlled, but in order to reduce the magnitude of an epidemic outbreak, the transformation rate from non-carriers to carriers must be controlled.

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