if $s_1 < t_1$, $s_2 < t_2$. Let a family of random variables $\{X_t : s, t \in N^2, s \leq t\}$ be defined on some probability space. We say that the process $\{X_t\}$ is subadditive if

(i) When $s \leq u$ and $s_2 < u_2$, then $X_{su} \leq X_{s(u_1, t_2)} + X_{t_1, t_2}u$.

(ii) The joint distributions of the processes

\[
\{X_{(t_1, t_2)}(t_1, t_2)\} \quad \text{and} \quad \{X_{(t_1, t_2)}(t_1, t_2 + 1)\}
\]

are the same as those of $\{X_{su}\}$.

(iii) $\sup_t E(|Z_t|) < \infty$, where $Z_t = X_{0t}/t_1t_2$.

If instead of (i) the stronger condition (i') if $s \leq t \leq u$, $X_u - X_{(t_1, t_2)} - X_{(t_1, t_2)}u + X_u \leq X_u$ holds, we say that $X_u$ is strongly subadditive. We study the convergence as $t_1, t_2 \to \infty$ of $Z_u$.

**Theorem 1.** $\gamma = \lim_{t \to \infty} E(Z_t)$ always exists.

**Theorem 2.** If $\{X_{su}\}$ is subadditive then $Z_t$ is convergent in $L^1$ to a limit $\xi$, with $E(\xi) = \gamma$.

**Theorem 3.** If $\{X_{su}\}$ is subadditive and $E\{|Z_t| \log^+|Z_t|\} < \infty$ for all $t \in N^2$, then $\lim \sup_{t \to \infty} Z_t = \xi$ is a.s. finite, $E(\xi) = \delta$, and $Z_t$ converges in $L^1$ to $\xi$.

**Theorem 4.** For a wide class of strongly subadditive processes, the condition $\sup_t E(|Z_t| \log^+|Z_t|) < \infty$ implies that $Z_t$ is a.s. convergent.

Examples show that the conditions in Theorems 2–4 cannot be weakened. The results can be generalized to $N'$ if $\log^+|Z_t|$ above is replaced by $(\log^+|Z_t|)^{-1}$.

III. SYMPOSIUM ON STATISTICAL TECHNIQUES

The session on statistical techniques was organized to exchange views on a broad range of subjects on the relationship of statistics and stochastic processes. Six panelists, Professors C. L. Chiang, B. Epstein, D. L. Iglehart, M. Neuts, J. Th. Runnenburg, and G. Weiss each made a short statement, with questions and comments from the floor. The discussion moderators were Professors P. W. Mikulski and P. J. Smith.

1. Professor C. L. Chiang stated that one of the important statistical problems was estimation of parameters. He considered some unsolved estimation problems in the analysis of stochastic models, in particular a problem of estimation of intensity functions related to Markov processes with continuous-time parameter.
2. Professor B. Epstein discussed educational aspects, particularly the undesirable separation of the teaching of stochastic processes and statistics. He suggested introducing and discussing simple stochastic processes in undergraduate statistics courses. He also discussed some practical aspects of both fields, indicating that in life testing and reliability theory there is a close linkage between some of the key statistical techniques and stochastic processes.

Dr. P. Schweitzer commented that often in practice one faces two extremes. Either the conclusion is obvious from the data and no statistical analysis is desired, or the data are inconclusive and the consultee is not willing to believe in the results of the analysis.

Professor Ghosal commented that actual data often support several competing hypotheses simultaneously and this should be stressed to students.

3. Professor D. L. Iglehart suggested two areas of concern of stochastic processes and statistics: estimation of unknown parameters and simulation. Traditional statistical training often ignores simulation. Connections across the fields of statistics, probability and computing should be stressed and encouraged in the teaching process.

Professor B. Epstein commented that at the Technion, Israel Institute of Technology, there is under discussion an interdepartmental undergraduate program in Mathematical Sciences. It would include a judicious mix of required and elective courses in mathematics, statistics, operations research and computing. Professor Iglehart replied that such a program now exists at Stanford.

Professor Lepson commented that students often have technical knowledge of computing languages and techniques without understanding numerical analysis, which is also necessary.

4. Professor M. Neuts gave first some examples of recent research which, in his opinion, treated some fundamental problems using novel methods and leading to fertile areas of research. Among the authors were P. A. W. Lewis, S. Zacks, L. Aroian, R. Vitali, I. Greenberg, D. G. Kendall, G. Rossa and J. Heinhold. A general problem for which statistical analysis is badly needed is multi-dimensionality. This includes both data reduction for computing and simpler but applicable models involving fewer parameters.

Professor P. Todorovic pointed out that basic statistical theory (e.g. the Neyman–Pearson Lemma) often can be generalized to stochastic processes. An example is U. Grenander’s pioneering work in the theory of statistical inference for second-order stochastic processes.

5. Professor J. Th. Runnenburg discussed the difficulties inherent in practical problems, and the lack of attention given them by the profession. He reviewed briefly two problems. One, from his consulting practice, was a solution to the problem of the height of dikes in Holland. The other problem was from queueing theory suggesting a study of dependent input on queueing.
Professor R. Disney mentioned a practical problem involving stochastic processes and statistics which he encountered studying two types of calls to a police station (criminal and ambulance) using a two stage semi-Markov arrival queue model. Unfortunately, the length of the report on this study made it impossible to publish. This seems typical of applied research studies.

6. Professor G. Weiss's theme was 'how do you teach common sense in statistics?'. He described situations in which important but overlooked aspects of the problem led to erroneous or absurd solutions. Examples included excessively complicated models for which there is little data, 'mathematical' solutions which are impractical in concrete situations, and analyses which ignore obvious contamination of the data.

Professor P. Mikulski briefly summarized the discussion—noting that better integration of statistics and stochastic processes, improved education and attention to practical problems were the main themes of the discussion.