Lattice Gauge Theory and (Quasi)-Conformal Technicolor

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QCD with 2 flavours of massless colour-sixtet quarks is studied as a theory which might exhibit a range of scales over which the running coupling constant evolves very slowly (walks). We simulate lattice QCD with 2 flavours of sextet staggered quarks to determine whether walks, or if it has an infrared fixed point, making it a conformal field theory. Our initial simulations are performed at finite temperatures $T = 1/N_t a$ ($N_t = 4$ and $N_t = 6$), which allows us to identify the scales of confinement and chiral-symmetry breaking from the deconfinement and chiral-symmetry restoring transitions. Unlike QCD with fundamental quarks, these two transitions appear to be well-separated. The change in coupling constants at these transitions between the two different temporal extents $N_t$, is consistent with these being finite temperature transitions for an asymptotically free theory, which favours walking behaviour. In the deconfined phase, the Wilson Line shows a 3-state signal. Between the confinement and chiral transitions, there is an additional transition where the states with Wilson Lines oriented in the directions of the complex cube roots of unity disorder into a state with a negative Wilson Line.

Keywords: Lattice gauge theory, Walking Technicolor.

1. Introduction

Technicolor theories are QCD-like gauge theories with massless fermions, whose pion-like excitations play the role of the Higgs field in giving masses to the $W$ and $Z$. We search for Yang-Mills gauge theories whose fermion content is such that the running coupling constant evolves very slowly – walks. Such theories can avoid the phenomenological problems which plague other (extended-)Technicolor theories.

While many studies have used fermions in the fundamental representation, with large numbers of flavours, we are concentrating on higher representations of the colour group (in particular the symmetric tensor), where conformality/walking can be achieved at much lower $N_f$. There have been some studies with $SU(2)$ colour...
with two adjoint (symmetric tensor) fermions. We are considering QCD (SU(3) colour) with colour-sextet (symmetric tensor) quarks.

The 2-loop $\beta$-function for QCD with $N_f$ massless flavours of colour-sextet quarks, suggests that for $1 \frac{137}{120} \leq N_f < 3$, either this theory will have an infrared-stable fixed point, or a chiral condensate will form and this fixed point will be avoided. In the first case the theory will be conformal; in the second case it will walk. For $N_f = 3$ conformal behaviour is expected. $N_f = 2$ could, a priori, exhibit either behaviour. Because the quadratic Casimir operator for sextet quarks is $2^2$ times that for fundamental quarks, it is easier for them to form a chiral condensate.

Lattice QCD gives us a direct method to determine which option the $N_f = 2$ theory chooses. We are studying the $N_f = 2$ theory using staggered fermions. We are currently performing simulations at finite temperature ($T$). Finite temperature enables us to study the scales of confinement and chiral symmetry breaking, and yields information on the running of the coupling constant.

Simulations using Wilson fermions by DeGrand, Shamir and Svetitsky suggest that this theory is conformal. Our simulations suggest that it walks. The scales of confinement and chiral symmetry breaking appear to be very different. This also contrasts with what was reported by DeGrand, Shamir and Svetitsky for simulations using Wilson quarks. Hence the phenomenology is expected to be different from that of QCD with fundamental quarks and $N_f$ in the walking window, where these two scales appear to be the same. Preliminary studies of this theory using domain-wall quarks have been reported in.

In the deconfined phase we observe states where the phase of the Wilson Line is $\pm 2\pi/3$, and at weaker couplings, $\pi$ in addition to the expected states with positive Wilson Lines. These have since been predicted and observed by Machtey and Svetitsky using Wilson quarks.

2. Simulations and Results

We use the standard Wilson (triplet) plaquette action for the gauge fields and an unimproved staggered-fermion action for the quarks. The only new feature is that the quark fields are six-vectors in colour space and the gauge fields on the links of the quark action are in the sextet representation of colour. The RHMC algorithm is used to tune the number of flavours, $N_f$, to 2.

We run on $8^3 \times 4$, $12^3 \times 4$ and $12^4 \times 6$ lattices at quark masses $m = 0.005$, $m = 0.01$ and $m = 0.02$ in lattice units to allow extrapolation to the chiral ($m = 0$) limit. $\beta = 6/g^2$ is varied over a range of values large enough to include the deconfinement and chiral transitions. Run lengths of 10,000–200,000 trajectories per ($m, \beta$) are used.

More details of the results presented here are given in ref.

Since the results from the 2 $N_f = 4$ lattices are consistent, we present only results from our $12^3 \times 4$ simulations. Figure 1 shows the colour-triplet Wilson Line (Polyakov Loop) and the chiral condensate $\langle \bar{\psi}\psi \rangle$ as functions of $\beta = 6/g^2$, for each of the 3
Fig. 1. Wilson line and $\langle \bar{\psi}\psi \rangle$ as functions of $\beta$ on a $12^3 \times 4$ lattice.

quark masses on a $12^3 \times 4$ lattice. The deconfinement transition is marked by an abrupt increase in the value of the Wilson Line. Chiral symmetry restoration occurs where the chiral condensate vanishes in the chiral limit.

In contrast to what was found by DeGrand, Shamir and Svetitsky, we find well separated deconfinement and chiral-symmetry restoration transitions. The deconfinement transition occurs at $\beta = \beta_d$ where $
abla \beta_d(m = 0.005) = 5.405(5)$, $\beta_d(m = 0.01) = 5.4115(5)$ and $\beta_d(m = 0.02) = 5.420(5)$. The chiral transition, estimated from the peaks in the chiral susceptibility curves, occurs at $\beta_{\chi} = 6.3(1)$.

Figure 1 only accounts for the state with a real positive Wilson Line in the deconfined regime. However, from the deconfinement transition up to $\beta \approx 5.9$ there exist long-lived states with the Wilson Line oriented in the directions of the other 2 cube roots of unity. However, these states are metastable, eventually decaying into the state with a positive Wilson Line. Above $\beta \approx 5.9$, these complex Wilson Line states disorder into a state with a negative Wilson Line.

Let us now consider our $12^3 \times 6$ simulations. Again, the deconfinement and
chiral-symmetry restoring transitions are well-separated. Above the deconfinement transition, we again find a clear 3-state signal. This time, however, all 3 states appear equally stable. The system tunnels between these 3 states for the duration of the run until we are so far above the transition that the relaxation time for tunneling exceeds the lengths of our runs. We therefore artificially bin our ‘data’ according to the phase of the Wilson Line, into bins \((-\pi, -\pi/3), (-\pi/3, \pi/3), (\pi/3, \pi)\). Figure 2 shows the Wilson Lines and chiral condensates \(\langle \bar{\psi} \psi \rangle\) for the central ‘positive’ Wilson Line bin. Figure 3 shows the Wilson Lines and chiral condensates for the first and last ‘complex’ and ‘negative’ Wilson Line bins. The deconfinement transitions occur at \(\beta_d(m = 0.005) = 5.545(5), \beta_d(m = 0.01) = 5.550(5)\) and \(\beta_d(m = 0.02) = 5.560(5)\). Chiral-symmetry restoration occurs at \(\beta_\chi = 6.6(1)\).

As for \(N_t = 4\), there is further transition between the deconfinement and chiral transitions where the states with complex Wilson Lines disorder to produce a state with a negative Wilson Line. This transition occurs at \(\beta \approx 6.4\) for \(m = 0.01\) and
Fig. 3. Magnitude of the Wilson Line and chiral condensate for the state with a complex or negative Wilson Line as functions of $\beta$ for each of the 3 masses on a $12^3 \times 6$ lattice.

$\beta \approx 6.5$ for $m = 0.02$. This transition can be seen in fig. 3.

3. Discussion and conclusions

We are studying the thermodynamics of Lattice QCD with 2 flavours of staggered colour-sextet quarks. We find well separated deconfinement and chiral-symmetry restoration transitions. This contrasts with the case of fundamental quarks, where these 2 transitions are coincident, but is similar to the case of adjoint quarks where again these 2 transitions are separate.\textsuperscript{37,38}

We denote the value of $\beta = 6/g^2$ at the deconfinement transition by $\beta_d$ and that at the chiral transition by $\beta_\chi$. In the chiral limit $\beta_d \approx 5.40$ and $\beta_\chi = 6.3(1)$ at $N_t = 4$. At $N_t = 6$ these values become $\beta_d \approx 5.54$ and $\beta_\chi = 6.6(1)$. The increase in the $\beta$s for both transitions from $N_t = 4$ to $N_t = 6$ is consistent with their being finite temperature transitions for an asymptotically free theory (rather than bulk transitions). If there is an IR fixed point, we have yet to observe it. Our results
suggest a Walking rather than a conformal behaviour.

Why is this phase diagram so different from that for Wilson quarks (DeGrand, Shamir and Svetitsky)? Is it because there is an infrared fixed point, and we are on the strong-coupling side of it? Are our quark masses too large to see the chiral limit? Is it because the flavour breaking of staggered quarks does not allow a true chiral limit at fixed lattice spacing?

For the deconfined phase there is a 3-state signal, the remnant of now-broken \( Z_3 \) symmetry. For \( N_t = 4 \) the states with complex Polyakov Loops appear metastable. For \( N_t = 6 \) all 3 states appear stable. Breaking of \( Z_3 \) symmetry is seen in the magnitudes of the Polyakov Loops for the real versus complex states. Between the deconfinement and chiral transitions, we find a third transition where the Wilson Lines in the directions of the 2 non-trivial roots of unity change to real negative Wilson Lines. This transition occurs for \( \beta \approx 5.9 \) (\( N_t = 4 \)) and \( \beta \approx 6.4-6.5 \) (\( N_t = 6 \)). The existence of these extra states with Polyakov Loops which are not real and positive has been predicted and observed by Machtey and Svetitsky.

Drawing conclusions from \( N_t = 4 \) and \( N_t = 6 \) is dangerous. We have recently started simulations with \( N_t = 8 \). We should also use smaller quark masses. At \( N_t = 6 \), we need a second spatial lattice size. To understand this theory more fully, we need to study its zero temperature behaviour, measuring its spectrum, string tension, potential, \( f_\pi \). Measurement of the running of the coupling constant for weak coupling is needed.

We have recently started simulations with \( N_f = 3 \), which is expected to be conformal, to determine if it is qualitatively different from \( N_f = 2 \).

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