Combining Nearest Neighbor Search with Tabu Search for Large-Scale Vehicle Routing Problem

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Abstract

The vehicle routing problem is a classical problem in operations research, where the objective is to design least cost routes for a fleet of identical capacitated vehicles to service geographically scattered customers. In this paper, we present a new and effective hybrid metaheuristic algorithm for large-scale vehicle routing problem. The algorithm combines the strengths of the well-known Nearest Neighbor Search and Tabu Search into a two-stage procedure. More precisely, Nearest Neighbor Search is used to construct initial routes in the first stage and the Tabu Search is utilized to optimize the intra-route and the inter-route in the second stage. The presented algorithm is specifically designed for large-scale problems. The computational experiments were carried out on a standard benchmark and a real dataset with 6772 tobacco customers. The results demonstrate that the suggested method is highly competitive.

1. Introduction

The Vehicle Routing Problem (VRP) is a well known NP-Hard problem in operational research, which has drawn enormous interests from many researchers during the last decades because of its vital role in planning of distribution systems and logistics in many sectors such as garbage collection, mail delivery, snow ploughing and task sequencing. The VRP can be briefly described as follows: Given on more depots, a fleet of vehicles, homogeneous or not, and a set of customers with known or forecast demands, find a set of closed routes, originating and, generally, ending at one of the depots, to service all customers at minimum cost, while satisfying vehicle and depot capacity constraints. The VRP is divided into many types. The important problems are VRP with Time Windows, VRP with Pick-Up and Delivery and Capacitated VRP. Other characteristics and requirements may be also considered, such as service and travel time restrictions, multiple commodities with different transportation requirements, time-dependent and uncertain demands or travel times, etc., yielding a rich set of VRP variants [1-5].
Recently many exact methods have been used to solve the VRP such as exact algorithms based on linear programming techniques and guided local search. Besides, heuristic techniques have drawn wide interests in researchers’ effort to solve large scale VRPs. Among the recently applied heuristic techniques are genetic algorithm, evolution strategies and neural networks. In the real application of large-scale VRP, Decomposition strategy is the most successfully approach recently [6, 7]. Generally, the solution divides into two parts (i.e., initial solution phase and optimizing phase).

Based on decomposition strategy, this paper presents a new and effective hybrid metaheuristic algorithm for large-scale vehicle routing problem. The algorithm combines the strengths of the well-known Nearest Neighbor Search and Tabu Search into a two-stage procedure. More precisely, Nearest Neighbor Search is used to construct initial routes in the first stage and the Tabu Search is utilized to optimize the intra-route and the inter-route in the second stage. The presented algorithm is specifically designed for large-scale problems.

The rest of the paper is organized as follows: The related work is given in Section 2. The proposed method is presented in Section 3. The experimental results are presented in Section 4. Finally, we conclude the paper.

2. Related Work

The VRP is to find the minimum cost routes to be traveled by a fleet of vehicles. All of the vehicles must start and terminate at a central depot. Each customer must be visited once, by one vehicle only. The VRP is a well known integer programming problem which falls into the category of NP-hard problems. A large number of approximation algorithms have been proposed to solve the problem. These algorithms are separated into two main categories - classical heuristics and metaheuristics. Tabu search strategy is the most widely used and efficient algorithm. Several variants of the standard Tabu search have been presented. During last decade, many nature inspired metaheuristic algorithms have been proposed for the VRP, for instance, genetic algorithms and ant colony optimization. For more detailed descriptions of these algorithms, survey papers by Bodin et al. [8], Gendreau et al. [9], Laporte et al. [10], Tarantilis [11], Li et al. [12], Laporte [13] may be referred.

There are literally hundreds of papers discussing vehicle routing problems and their variations [1-5]. Almost all papers focus on problems of relatively small size. Unfortunately, many of the proposed techniques do not scale well and some recent papers specifically address large-scale problems. Bouthillier et al [14] use parallel computation for scalability. Their main contribution is an architecture allowing different search strategies to run in parallel and to communicate their progress. To date, the most successful approach for solving large-scale VRPTWs is an advanced evolutionary technique [15] built upon the success of earlier algorithms (e.g., [16]). The main innovation in [15] are the incorporation of sophisticated diversification schemes (e.g., using guided local search) into an evolutionary framework.

For large-scale VRP, the recent work in [17, 18] who use the Variable Neighborhood Search (VNS) [19] and Route-Nearest Neighborhood (RNN) [18] shows that even large instances may be solved in an efficient way. The 2-phase hybrid meta-heuristic [20] also proved to successfully solve problems from small sizes large ones more than 1000 customers. Another possible approach is to use decomposition strategies like the POPMUSIC framework [6] that try to overcome size restrictions, by intelligently splitting the problem into sub problems and solving them separately. Decomposition strategies were recently also successfully applied to large scale real world problems [7]. The proposed approach in this paper is also based on the decomposition strategy.

After creating an initial solution, an attempt is made to improve it by using local search with one or more neighborhood structures and a best-accept strategy. Most of the neighborhoods used are well known and were previously introduced in the context of various construction and improvement heuristics. Examples of such neighborhoods are 2-opt, Or-opt, 2-opt*, relocate and exchange, λ-interchange, CROSS-exchange, GENI-exchange and ejection chains, etc.

Tabu Search (TS) is a local search metaheuristic widely used in VRP [21]. TS explores the solution space by moving at each iteration from a solution to the best solution in a subset of its neighborhood \( N(s) \). Contrary to classical descent methods, the current solution may deteriorate from one iteration to the next.
Thus, to avoid cycling, solutions possessing some attributes of recently explored solutions are temporarily declared tabu or forbidden. The duration that an attribute remains tabu is called its tabu-tenure and it can vary over different intervals of time. The tabu status can be overridden if certain conditions are met; this is called the aspiration criterion and it happens, for example, when a tabu solution is better than any previously seen solution. Finally, various techniques are often employed to diversify or to intensify the search process. Many authors have presented numerous tabu search implementations involving sophisticated diversification and intensification techniques, explicit strategies for minimizing the number of routes, complex post-optimization techniques, hybridizations with other search techniques such as simulated annealing and genetic algorithms, parallel implementations, and allowance of infeasible solutions during the search.

3. The Proposed Approach

In this section, we firstly describe the capacity VRP model, and then depict the two phase approach based on the model.

3.1. The Capacity VRP Model

First, we define the following notation:

- \( A \) the set of arcs
- \( n \) the total number of nodes
- \( N \) the node-arc incidence matrix
- \( V \) the set of nodes
- \( T \) the route.
- \( c_{ij} \) the cost of using arc \((i,j)\) \( ((i,j) \in A) \)
- \( c(i, T) \) the cost of node \(i\) to route \(T\)
- \( b \) the demand vector.

The decision variables are:

- \( x_{ij} \) the flow on arc \((i,j)\),
- \( y_{ij} \) takes the value 1 if the vehicle travels from city \(i\) to city \(j\).

For VRP, the general objective is to minimize the total travel cost with some constraints, i.e.,

\[
\min : \sum_{(i,j) \in A} c_{ij} y_{ij}
\]  
(1)

Subject to:

\[
\sum_{i \leq j \leq n} y_{ij} = 1 \quad i = 1, ..., n
\]  
(2)

\[
\sum_{1 \leq i \leq n} y_{ij} = 1 \quad j = 1, ..., n
\]  
(3)

\[
Nx = b
\]  
(4)

\[
x_{ij} \leq (n-1)y_{ij} \quad (i, j) \in A
\]  
(5)

\[
x_{ij} \geq 0 \quad (i, j) \in A
\]  
(6)

\[
y_{ij} \in \{0,1\} \quad (i, j) \in A
\]  
(7)

Based on the above definition, given a route \(T\) and a node \(i \in V \setminus T\), \(T\) is initial a node random selected in \(V\). we select the farthest node to the depot. i.e.,

\[
s^* = \arg \min_{i \in V}(d(i,0))
\]  
(8)

Let
\[ c(i, T) = \min_{j \in T} c_{ij} \]  

and

\[ i^* = \text{arg}\left(\min\{c(i, T) : i \in V \setminus T\}\right). \]

Thus \( i^* \) is the closest node to \( T \). Now construct a sub-route \( T \cup \{i^*\} \) by inserting \( i^* \) into \( T \). This process terminates with a route.

3.2. The Nearest Neighbor Search

Let \( V^* \) denote the set of customer sites that have not been visited yet by a vehicle. Start at the depot (node 0) and construct a path consisting of nodes 0, \( i_1, \ldots, i_j \), where

\[ i_j = \text{arg}\left(\min\{c_{i_j-k} : k \in V^*\}\right) \]

Where \( u \geq d_{i_1} + d_{i_2} + \ldots + d_{i_j} \), and any other node \( s \in V^* \) is such that \( u < d_{i_1} + d_{i_2} + \ldots + d_{i_j} + d_s \).

Repeat the above step until all the sites are visited.

The total number of paths constructed presents the total number of vehicles needed to deliver the products to customers. The nearest neighbor heuristic is widely applied in VRP and Traveling Salesman Problem (TSP).

3.3. The Intra- and Cross-Exchange by Tabu Search

The initial solution produced by the Nearest Neighbor Search is improved by the Tabu Search (TS). We use the Intra-Exchange and Cross-Exchange in TS.

The Cross-Exchange swaps customers between two routes. Cross-Exchange generation mechanism can be described as follows. Given a solution for the problem represented by a set of routes

\[ S = \{r_1, \ldots, r_p, \ldots, r_q, \ldots, r_k\} \]

a Cross-Exchange between a pair of routes \( (r_p, r_q) \) is a replacement of a subset of customers \( S_1 \subseteq r_p \) of size \( |S_1| \leq \lambda \) by another subset \( S_2 \subseteq r_q \) of size \( |S_2| \leq \lambda \) to get two new routes

\[ r_p^* = (r_p - S_1) \cup S_2 \]

\[ r_q^* = (r_q - S_2) \cup S_1 \]

and a new neighboring solution

\[ S^* = \{r_1, \ldots, r_p^*, \ldots, r_q^*, \ldots, r_k\} \]

The neighborhood \( N_\lambda(S) \) of a given solution \( S \) is the set of all neighbors \( S^* \) generated in this way for a given value of \( \lambda \).

In CROSS-exchanges, the basic idea is to first remove two edges \((i-1, i)\) and \((k, k+1)\) from a first route while two edges \((j-1, j)\) and \((l, l+1)\) are removed from a second route. Then the segments \(i-k\) and \(j-l\), which may contain an arbitrary number of customers, are swapped by introducing the new edges \((i-1, j)\), \((l, k+1)\), \((j-1, i)\) and \((k, l+1)\).

Similarly, Intra-route optimization is equally important to find good solutions. In order to optimize individual routes, the Cross neighborhood is enlarged by including exchanges that apply to a single route. These exchanges are similar to those defined on a pair of routes. Namely, two edges are removed from a given route, and the segment between the two edges is moved at another location within the same route.
This approach generalizes the Or-opt exchanges, by allowing the relocation of segments of any arbitrary length.

4. Experiments

An experiment has been conducted on a standard benchmark dataset (i.e. Homberger 400 customer rc2_410) and a real dataset (i.e. the cigarette tradesmen in Suizhou city, Hubei province, P. R. China).

4.1. Experiment for standard benchmark

The standard benchmark dataset rc2_410 is with 400 customers, which is a combination of a random and clustered customer. Figure 1, Figure 2, Figure 3 and Figure 4 are the route construction results by NNS, NNS + INTRA-EXCHANGE, NNS + CROSS-EXCHANGE, NNS + INTRA- & CROSS-EXCHANGE, respectively. Obviously, Figure 2 reduces the density of lines compared with Figure 1, which means the total distance of the constructed routes is decreased. Therefore, NNS + INTRA-EXCHANGE algorithm has better constructed routes than NNS. Similarly, NNS + CROSS-EXCHANGE is superior to NNS + INTRA-EXCHANGE as shown in Figure 3 and Figure 2, and NNS + INTRA- & CROSS-EXCHANGE is superior to NNS + CROSS-EXCHANGE as shown in Figure 4 and Figure 3. In sum, our approach, NNS + INTRA- & CROSS-EXCHANGE can remarkably improve the constructed routes, which can further verify the superiority of the proposed approach in Table 1.

Table 1 show the four algorithms comparison in detail. They have the same number of routes, but their total distances are different remarkably. NNS + INTRA- & CROSS-EXCHANGE decreases the total distance greatly compared with NNS.

| Algorithm                        | Vehicle number | Distance    |
|----------------------------------|----------------|-------------|
| NNS                              | 9              | 14371.13    |
| NNS+INTRA-EXCHANGE               | 9              | 11742.06    |
| NNS+CROSS-EXCHANGE               | 9              | 6870.28     |
| NNS+INTRA-&CROSS-EXCHANGE        | 9              | 5915.50     |
Figure 1. NNS

Figure 2. NNS + intra-exchange
4.2. Experiment for real dataset

The real dataset has 6772 customers that distribute in the centre and suburb of the Suizhou city. We partition the whole customers into 5 areas accordingly to the real requirement.

Figure 5 shows the portioned five areas in Suizhou city, which is a real dataset with large number of customers. Figure 6-10 are the constructed routes by NNS + INTRA- & CROSS-EXCHANGE for five areas, respectively. The new approach takes a good effect in the real application, which greatly reduces the total distance for routes. Therefore, it decreases the costs and improves the efficiency for the tobacco delivery.
Figure 5. The partitioned five areas in Suizhou city.

Figure 6. The routes of area 1 by NNS + INTRA- & CROSS-EXCHANGE

Figure 7. The routes of area 2 by NNS + INTRA- & CROSS-EXCHANGE
Figure 8. The routes of area 3 by NNS + INTRA- & CROSS-EXCHANGE

Figure 9. The routes of area 4 by NNS + INTRA- & CROSS-EXCHANGE

Figure 10. The routes of area 5 by NNS + INTRA- & CROSS-EXCHANGE
5. Conclusion

For the large-scale VRP, we adopt a decomposition strategy to divide the big problem into two phases. The first phase is to construct the initial routes by nearest neighbor search (NNS), and then improve the routes by INTRA- & CROSS-EXCHANGE in the second phase. The experimental results demonstrate that the new algorithm improve the performance remarkably compared with the traditional algorithm.

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