Macroscopic Quantum Damping in SQUID Rings

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The measurement process is introduced in the dynamics of Josephson devices exhibiting quantum behaviour in a macroscopic degree of freedom. The measurement is shown to give rise to a dynamical damping mechanism whose experimental observability could be relevant to understand decoherence in macroscopic quantum systems.

\section*{I. INTRODUCTION}

Since Schrödinger, both theoretical and experimental efforts have been devoted to understand whether quantum mechanical laws still work at the macroscopic level \cite{1}. In particular, many avenues are currently investigated to create coherent superpositions of macroscopically distinguishable states starting either from microscopic degrees of freedom, as in the case of photons \cite{2-4}, electrons \cite{5}, neutrons \cite{6}, atoms \cite{7,8} and molecules \cite{9}, or macroscopic ones, for instance by using superconducting circuits \cite{10}. The common goal of these attempts is to grasp how the coherence properties, the crucial role of which is well established in the microworld, are washed out in the transition to the macroscopic realm, through what is commonly termed decoherence \cite{11,12}. This last concept, invoked to explain why superpositions of macroscopic states are so fragile to make their actual observation far from being a trivial task, is present whenever the system under consideration is coupled to the outside world, turning its evolution into an irreversible one. Among the possible sources of decoherence, the one attributable to the act of the measurement deserves special attention since, being any physical inquiry on the system performed via the coupling to an external probe, it is in principle unavoidable. In previous papers the measured induced decoherence has been analyzed in the case of microscopic systems, namely atoms confined either in optogrativational cavities \cite{13} or in electromagnetic traps \cite{14}. Observable quantities turn out to be modified giving rise to peculiar transient phenomena. In this work we extend the analysis to the dynamics of macroscopic quantum systems such as the Josephson devices which, as emphasized by Leggett \cite{10}, are promising candidates to study the coherence-decoherence transition by virtue of both their low operating temperature and the small intrinsic dissipations. In Section II we introduce the framework of measurement quantum mechanics and apply it to the measurement of energy in a radiofrequency superconducting quantum interference device (rf-SQUID). Decoherence signatures arising from the measurement process in the dynamics of the average magnetic flux are discussed in Section III. Final remarks on the feasibility and the differences from the microscopic systems complete the paper.

\section*{II. MEASUREMENT QUANTUM MECHANICS OF A SQUID RING}

The system that will be considered hereafter is a superconducting closed loop interrupted by a weak link of the Josephson type, in short an rf-SQUID. Neglecting dissipation mechanisms, its quantum mechanical description can be modeled through the effective Hamiltonian \cite{15}

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The quartic potential approximation (3), possessing two local minima at \( \hat{b} \) and \( \hat{c} \) for a bistable potential (3) corresponds to a physical SQUID. The ext magnetic flux threading the ring and the flux quantum \( \Phi_0 = \hbar/2e \approx 2.07 \cdot 10^{-15} \) Wb. In the circuit description (4), the total magnetic flux \( \hat{\Phi} \) and the total displacement flux (with units of electric charge) threading across the weak link capacitor, \( \hat{P}_b = -i\hbar d/\hat{\Phi} \), play the role of generalized coordinate and momentum respectively. Notice that \( C \) acts as the effective mass of the whole ring, treated as a single macroscopic quantum object evolving in \( \Phi \)-space under the potential \( \hat{V}(\Phi) \). If the adimensional variable \( x = (\Phi - \Phi^{ext})/\Phi_0 \) is introduced and a fixed value \( \Phi^{ext} \) of the external flux is chosen fulfilling the relation \( \Phi^{ext}/\Phi_0 = (n+1/2) \), with \( n \) integer to ensure \( \Phi \)-symmetry of the effective potential around \( \Phi = \Phi^{ext} \), the latter can be rewritten as

\[
V(x) = \frac{\Phi_0^2}{2L} x^2 + \frac{I_c \Phi_0}{2\pi} \cos(2\pi x).
\]

According to (3), monostable or multistable dynamics arises depending on the relative amplitude of the parabolic and periodic terms. In particular, a bistable regime is obtained if \( 1 < \beta < 5\pi/2 \), with \( \beta = 2\pi L I_c/\Phi_0 \). Under such conditions, the double well potential (4) can be approximated by a polynomial expansion,

\[
V(x) \approx V_0 - \frac{\mu}{2} x^2 + \frac{\lambda}{4} x^4, \quad \mu, \lambda > 0,
\]

being the parameters \( \mu, \lambda, V_0 \) related to the physical ones via the relationships:

\[
\begin{align*}
\mu &= 2\pi I_c \Phi_0 - \Phi_0^2/L, \\
\lambda &= 4\pi^3 I_c \Phi_0/3, \\
V_0 &= I_c \Phi_0/2\pi = 3\lambda/8\pi^4.
\end{align*}
\]

The quartic potential approximation (3), possessing two local minima at \( \hat{x}_\pm = \pm\sqrt{\mu/\lambda} \) separated by a barrier of height \( \Delta U = \mu^2/4\lambda \), has been widely exploited to model the macroscopic quantum tunneling of the flux through the junction (4). One can easily check by inverting (3) that only the range of values \( \mu < 3\lambda/2\pi^2 \) is allowed in order the bistable potential (4) corresponds to a physical SQUID.

Let us now include the effect of a measurement process in the description of the SQUID ring. We will focus the attention on a situation where the measurement most clearly possesses a dynamical content, namely the continuous monitoring of a generic system observable \( \hat{A} \). Significant progresses in modeling such a situation have been made in recent years through different approaches, all of which basically recognize that a measured system is not isolated but in interaction with a measurement apparatus. A detailed account of these approaches, together with the unified picture in which all of them can be incorporated, has been given elsewhere (16). Few remarks are in order to apply this general scheme to our particular problem at hand. Due to the fact that SQUIDs are macroscopic condensed matter objects and therefore two indiscernible rings (let alone an ensemble) cannot be constructed, measurement procedures are constrained to deal with just one SQUID and physical information emerges from averaging individual histories with the system each time prepared in the same initial macroscopic state. In the language of quantum measurement theory, such a procedure translates in the so-called selective \( a \) priori measurement scheme, whereby the instantaneous result of the measured observable corresponds to a particular, though \( a \) priori unknown, selection of the possible outcomes. Accordingly, the deterministic Schrödinger evolution for the state vector is replaced by a stochastic one, where single realizations closely mimic the fluctuating output of each experimental run. Two equivalent approaches may be used for unravelling the master equation, either a Quantum State Diffusion picture where stochasticity acts continuously (17), or the so-called Quantum Jumps approach resulting when stochasticity is chosen to act occasionally (18). Nonselective measurement results are recovered by averaging the selective outcomes according to their probability distribution. On the other hand, a different description of the dynamics in the presence of the measurement, although not viable in practice when no ensemble is at disposal, has been proven to be equivalent to the previous one insofar as averaged (hence nonselective) predictions are required. Being in this case technically much more simpler, it will be adopted for the SQUID problem.

The starting point for a direct description of nonselective measurements is the master equation, derived in the framework of open quantum systems, which rules the evolution of the density operator \( \hat{\rho}(t) \) of the system under observation (19),

\[
\frac{d\hat{\rho}(t)}{dt} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}(t)] - \frac{\kappa}{2} [\hat{A}, [\hat{A}, \hat{\rho}(t)]] ,
\]

where \( \hat{C} \) is the weak link capacitance, \( L \) the ring inductance, \( I_c \) the critical current of the junction, \( \Phi^{ext} \) is the external magnetic flux threading the ring and the flux quantum \( \Phi_0 = \hbar/2e \approx 2.07 \cdot 10^{-15} \) Wb.
where $\hat{H}$, given in (3), describes the dynamics of the unmeasured closed system ($\kappa = 0$) and the measured observable $\hat{A}$ is a so-called Lindblad operator representing the influence of the environment on the system. The parameter $\kappa$ expresses the strength of the coupling between the measured system and the meter, that will be assumed to be time-independent throughout the derivation (10). Let us restrict the discussion to the case $\hat{A} = \hat{H}$, corresponding to a continuous measurement of energy, with $\kappa = \kappa_E$. Other situations in which the measurement process has been taken into account for a bistable potential have been previously discussed in [20] and [21] for continuous and impulsive monitoring respectively, but choosing as measured observable the (possibly generalized) coordinate itself, time-independent throughout the derivation [16]. Let us restrict the discussion to the case $\kappa = \kappa_E$.

The nonselective master equation (5) can be conveniently solved by introducing the representation of the measured observable eigenstates, $\hat{A}$ in the form $\hat{a}(\hat{a}^\dagger)^n |\kappa\rangle = |\kappa\rangle$, where odd parity of the flux operator has been exploited. This allows us to write down the exact density operator evolution for the system undergoing the measurement process in the form

$$\frac{d}{dt} \rho_{nm}(t) = -\frac{i}{\hbar} (E_n - E_m) \rho_{nm}(t) - \frac{\kappa E}{2} (E_n - E_m)^2 \rho_{nm}(t),$$

whose general solution is

$$\rho_{nm}(t) = \exp \left\{ -\frac{i}{\hbar} (E_n - E_m) t - \frac{\kappa E}{2} (E_n - E_m)^2 t \right\} \rho_{nm}(0).$$

This allows us to write down the exact density operator evolution for the system undergoing the measurement process in the form

$$\dot{\rho}(t) = \sum_{nm} \rho_{nm}(0) \exp \left\{ -\frac{i}{\hbar} (E_n - E_m) t - \frac{\kappa E}{2} (E_n - E_m)^2 t \right\} |n\rangle \langle m|. \nonumber$$

where the quantum decoherence effect accompanying the measurement is manifested as the vanishing of the off-diagonal density matrix elements at a rate proportional to the associated energy spacing $\omega_{nm}$ [12]. According to Eq. (8), the evolution of the system has been altered with respect to the closed one, $\kappa_E = 0$; it is important to understand how such modifications can manifest in observable properties of the system. Our approach, based on a direct observation of the monitored quantum system with energy meter variables disregarded, can be considered as complementary to the adiabatic monitoring scheme already proposed in [25], where the quantum variables are instead traced out to leave a modified reduced evolution of the apparatus.

### III. QUANTUM DAMPING OF THE AVERAGE MAGNETIC FLUX

As a SQUID observable showing nontrivial dynamical behavior, let us consider the total average magnetic flux threading across the ring as a function of time, $\langle \hat{\Phi}(t) \rangle = \text{Tr}(\hat{\Phi} \hat{\rho}(t)) = \sum_{nm} \rho_{nm}(t) \langle n | \hat{\Phi} | n \rangle$. Owing to Eq. (8), we get

$$\langle \Phi(t) \rangle = \sum_{nm} \rho_{nm}(0) \exp \left\{ -\frac{i}{\hbar} (E_n - E_m) t - \frac{\kappa E}{2} (E_n - E_m)^2 t \right\} \langle m | \hat{\Phi} | n \rangle. \nonumber$$

At variance with the purely oscillatory behavior occurring when no measurement is performed, an exponential damping with state-dependent time constants $\tau_{nm} = 2/[\kappa_E (E_n - E_m)^2]$ arises, for which no classical counterpart can be envisaged. After a transient whose duration depends, for a fixed $\kappa_E$, on the Bohr frequencies $\omega_{nm} = (E_n - E_m)/\hbar$ picked up by the initial configuration, the average flux dynamics becomes completely inhibited leading, irrespective of the initial state, to an average localization around the asymptotic value

$$\lim_{t \to +\infty} \langle \hat{\Phi}(t) \rangle = \sum_n \rho_{nn}(0) \langle n | \hat{\Phi} | n \rangle = 0, \quad \kappa_E > 0,$$

where odd parity of the flux operator has been exploited.
In order to visualize the phenomenological features arising in the presence of quantum damping, we specialize the general description of Eq. (8) by choosing some representative initial SQUID configurations of the system. A picture of the double well potential (3) and its lowest energy eigenfunctions is reported in Fig. 1. Firstly, let the SQUID be initially prepared in a combination of the two lowest states, \( |\Psi, t = 0 \rangle = 1/\sqrt{2} (|0 \rangle + |1 \rangle) = |L \rangle \), where \(|0 \rangle\) and \(|1 \rangle\) are the symmetric ground state and the antisymmetric first excited state respectively and the notation \(|L \rangle\) has been introduced to stress that the dominant concentration of the state is in the left well. We will also consider the antisymmetric right-localized superposition state \(|R \rangle\), defined by \(1/\sqrt{2} (|0 \rangle - |1 \rangle) = |R \rangle\). Due to their localized nature, the states \(|L \rangle\), \(|R \rangle\) can be differentiated on a macroscopic level by the net left or right origin of the magnetic flux (also implying an opposite circulation sense for the superconducting current) and represent therefore a simple realization of two macroscopically distinguishable states. By evaluating the initial density matrix \(\hat{\rho}(0) = |L \rangle \langle L|\), a damped oscillation is found from Eq. (9) for the average flux motion,

\[
\langle \hat{\Phi}(t) \rangle = \exp \left( -\frac{\kappa_E}{2} (\hbar \omega)^2 t \right) \Re \{ (0|\hat{\Phi}|1) e^{-i\omega t} \},
\]

where parity conservation has been again taken into account and \(\omega_{10} \equiv \omega = (E_1 - E_0)/\hbar\) denotes the angular tunneling frequency, related to the height barrier and the zero-point energy \(\hbar \omega_0\) in each well via a WKB estimate (3):

\[
\omega = \omega_0 \sqrt{\frac{\Delta U}{\hbar \omega_0}} \exp \left( - \frac{\Delta U}{\hbar \omega_0} \right).
\]

Transient phenomena in the average flux dynamics are ruled in this case by a time scale of the order \(2/(\kappa_E (\hbar \omega)^2)\). Some interesting insights on the action of the measurement process can also be gained by depicting the evolution of the \(|L\rangle\)-state directly in the \(LR\)-representation, which is the natural one for tunneling analysis. This may be accomplished by rewriting the relevant projectors in (3) according to the inverse transformation, e.g. \(|0 \rangle|0 \rangle = |L \rangle \langle L| + |R \rangle \langle R| + |L \rangle \langle R|/2, |0 \rangle|1 \rangle = |L \rangle \langle L| - |R \rangle \langle R| - |L \rangle \langle L| + |R \rangle \langle L|/2 and so on. As a result, the density operator of the measured system in the macroscopic \(LR\)-representation can be written as follows:

\[
\hat{\rho}(t) = \begin{pmatrix}
1 + e^{-\kappa_E (\hbar \omega)^2 t/2} \cos \omega t & -ie^{-\kappa_E (\hbar \omega)^2 t/2} \sin \omega t \\
-ie^{-\kappa_E (\hbar \omega)^2 t/2} \sin \omega t & 1 - e^{-\kappa_E (\hbar \omega)^2 t/2} \cos \omega t
\end{pmatrix}.
\]

The off-diagonal density matrix elements make evident the quantum interference between the macroscopically distinguishable states \(|L\rangle\) and \(|R\rangle\), implying the existence of a Schrödinger cat state (4). According to (13), the decay of this macroscopic coherence represents the off-diagonal effect of the coupling to the meter. In addition, the measurement influence extends to the diagonal matrix elements, expressing the probability for left and right localization of the state respectively. Asymptotically, the tunneling mechanism becomes frozen, a behavior showing close similarities with the result quoted in (20), where a dynamical suppression of tunneling in a symmetric double well potential arises as a consequence of an external perturbation - a coherent laser field driving transitions to an electronically excited bistable state. Of course, the same evolution given in (11) has to be recovered using the density representation (3),

\[
\langle \hat{\Phi}(t) \rangle = \frac{\langle L | \hat{\Phi} | L \rangle}{2} \left( 1 + e^{-\kappa_E (\hbar \omega)^2 t/2} \cos \omega t \right) + \frac{\langle R | \hat{\Phi} | R \rangle}{2} \left( 1 - e^{-\kappa_E (\hbar \omega)^2 t/2} \cos \omega t \right) = (0|\hat{\Phi}|0) + (0|\hat{\Phi}|1)e^{-\kappa_E (\hbar \omega)^2 t/2} \cos \omega t.
\]

A more realistic preparation of initially localized states usually takes place by considering Gaussian flux wavepackets of the form

\[
\psi_0(\Phi) = \frac{1}{(\pi \sigma_\Phi^2)^{1/4}} \exp \left( -\frac{(\Phi - \Phi_m)^2}{2 \sigma_\Phi^2} \right) = \frac{1}{\sqrt{\Phi_0} (\pi \sigma_x^2)^{1/4}} \exp \left( -\frac{(x - x_m)^2}{2 \sigma_x^2} \right),
\]

with the parameters \(\Phi_m = x_m \Phi_0 + \Phi_{ext}\) and \(\sigma_\Phi = \Phi_0 \sigma_x\) properly chosen to attain both the desired flux localization and a suitable initial average energy \((\langle \psi_0 | \hat{H} | \psi_0 \rangle < \Delta U\) in the tunneling regime). Starting from

\[
\rho_{nm}(0) = \int d\Phi d\Phi' \varphi_n^*(\Phi) \psi_0(\Phi) \psi_0^*(\Phi') \varphi_m(\Phi'),
\]

the nonselective evolution of the average flux results from Eq. (3). By referring to (13) for general considerations on the Gaussian case still applying here, a quantitative description is made available by resorting to numerical calculations. The required energy eigenfunctions and eigenvalues have been computed through a selective relaxation algorithm.
extensively described in [27]. A representative evolution arising from a Gaussian state maximally contributed by the first four energy eigenstates is depicted in the sequence of Fig. 2, where the effect of an increasing coupling to the meter proceeds from 2(a) to 2(f). A more resolved picture for the first three cases follows in Fig. 3. The presence of the second doublet of almost degenerate eigenstates has been obtained by centering the Gaussian state around a value of magnetic flux intermediate between the ones corresponding to the left maxima of the eigenfunctions (Fig. 1). The transient strongly depends on the initial state and exhibits, as already remarked, many decay timescales. When \( \kappa_E > 0 \), after a time \( \tau \) only the oscillations involving levels with energies \(|E_n - E_m| < \sqrt{2/(\kappa_E \tau)}\) will survive. More in general, critical values of the measurement coupling constant can be introduced for each oscillation pattern as:

\[
\kappa_{nm}^{\text{crit}} = \frac{1}{T_{nm}(E_n - E_m)^2} = \frac{1}{\hbar(E_n - E_m)},
\]

where \( T_{nm} = 2\pi/\omega_{nm} \) are the periods of the Bohr oscillations. Each critical value rules the \( nn \)-subdynamics of the SQUID magnetic flux, giving rise to deviations from the oscillatory closed system evolution for \( \kappa_E > \kappa_{nm}^{\text{crit}} \). Since the smaller energy splitting is due to the ground state (\(|E_1 - E_0| > |E_n - E_m| \forall n,m\)), the overall dynamics will be affected if \( \kappa_E > \kappa_{10}^{\text{crit}} \). By varying \( \kappa_E \) it is thus possible to observe qualitatively different initial transients, selecting for instance the oscillations at the frequencies \( \omega_{32} \) and \( \omega_{10} \) (Figs. 2(b) and 3(b)), or the latter only (Fig. 2(c–f)). Further increase in \( \kappa_E \) causes the oscillations to disappear at all, Fig. 2(c-f).

Few remarks are worthwhile. Similar considerations can be in principle repeated for the evolution of any ensemble average quantity, the explicit solution of the appropriate stochastic Schrödinger equation (or Quantum Jumps equation) being required to calculate individual SQUID behavior. Note, however, that the transient nature of the expected phenomena would not allow us to exploit time averages of the monitored quantities sampled over a single stochastic trajectory, the former being only viable if the observed system is at thermal equilibrium as discussed in [28]. Indeed, having thermal fluctuations been neglected in our case, the same argument given in [17] can be applied to show that, for a SQUID density matrix evolving according to Eq. (6), convergence of individual runs to energy eigenstates is expected. Conceptually, the damping mechanism operating in the nonselective ensemble evolution of average properties can then be also interpreted as due to the random dephasing between different histories associated to the selective evolutions of the same properties - both dephasing and damping being signatures of irreversible dynamics. In addition, in order to make the predicted behavior qualitatively similar to the one occurring in practice, the physical parameters have to be chosen in such a way that both bistable and quantum tunneling regimes are guaranteed by the potential energy [2].

A quantum damping due to the interaction between a macroscopic quantum system and an informational environment has been discussed in the case of a SQUID ring undergoing continuous nonselective measurements of energy. This damping has no classical analog and is strongly imprinted by the decoherence process originating in the act of measurement.

The predicted effect can be made observable for instance by measuring the dynamics of the average magnetic flux in the SQUID or any other observable which does not commute with the Hamiltonian [3]. Since the system under observation is a single macroscopic object the procedure must consist in repeating the cycle of measurements, each characterized by the following steps. Firstly, the system is prepared in a flux state, say a Gaussian, centered around one of the two minima for instance by properly adjusting the external magnetic flux. At the same time the energy meter is turned on. The total magnetic flux is measured after a given duration \( t \) by means of an instantaneous von Neumann measurement. The cycle is repeated again at constant \( t \) allowing to build the magnetic flux distribution and therefore the average magnetic flux. The whole procedure is repeated again by sweeping the time \( t \), allowing a comparison with the predicted behaviour [3]. Concerning continuous measurements of energy, to our knowledge no experimental technique is by now available. The closest achievement has been reported by the Sussex group regarding the spectroscopy of quantum mechanical SQUID rings [28]. The kind of measurement required here is however quite different, since a dynamical monitoring of energy is demanded. In other words, even if the knowledge of the energy

\[1\] For analogous definitions of critical measurement couplings in atomic systems see for instance [14,16].
levels is a prerequisite, a monitoring of the populations in each energy eigenstate is also necessary. For states formed by the lowest two levels, this could be achieved by coupling the two-level dynamics to a third level in a way similar to the one exploited to monitor the electronic populations in the study of atomic quantum Zeno effect [30]. In the SQUID problem, the lack of available schemes does not allow to make quantitative comparison between the predicted time decay due to quantum damping and the decays times due to classical sources of decoherence associated to intrinsic dissipations [31]. In addition, if fundamental energy localization is proven to occur in nature, the detection of measurement induced energy localization will only be possible if the measurement coupling $\kappa_E$ is sufficiently large compared to the minimum step time associated to intrinsic decoherence [24,25]. In any case, we guess that its actual observability is obtainable for values of the parameters not far from those required to make intrinsically quantum phenomena, such as macroscopic quantum coherence [32] or temporal Bell inequalities [10,33] detectable. It is also worth to point out that the predicted damping can be also implemented in a more quantitative way by considering tunneling phenomena at the microscopic level, for instance in the spectroscopy of molecular systems as considered in [26]. On the other hand, in this case the possibility to test decoherence of a macroscopic degree of freedom is lost.

The relevance of quantum damping in post-modern quantum mechanics is twofold. Firstly, it is an inequivocable test of the validity of current models on the quantum-to-classical transition which identify decoherence induced by the opening of the quantum system to the external world as the key concept. The possibility to prepare states in such a way to produce controllable decay patterns can support evidence for or against quantitative predictions of decoherence. Moreover, the decoherence issue in SQUID rings is crucial to exploit superconducting circuits as quantum computers [34]. The model presented here can be considered a highly idealized one describing the ultimate source of decoherence due to the informational input-output operations performable on superconducting quantum gates.

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FIG. 1. Plot of the double well potential $V(x) - V_0$ of Eq. (3) for the parameter choice $\mu = 1.80487(1)$ eV, $\lambda = 14.73360(1)$ eV, resulting in local minima at $|\hat{x}_\pm| = 0.35$ and height barrier $\Delta U = 0.055274$ eV. The four lowest energy normalized eigenfunctions $\varphi_n(x)$ ($n = 0, \ldots, 3$) are also shown together with their energies $E_0 = -0.0440591$ eV, $E_1 = -0.0440585$ eV, $E_2 = -0.0231600$ eV, $E_3 = -0.0230991$ eV.

FIG. 2. Predicted time dependence, in units of the tunneling period $T_{01}$, of the average magnetic flux for different values of the measurement coupling relative to the critical values $\kappa_{10}^{\text{crit}}$ and $\kappa_{32}^{\text{crit}}$ and for an initial Gaussian wavepacket having $x_m = -0.27$ and $\sigma_x = 0.06$. The state is contributed up to $\approx 97\%$ by the two lowest energy doublets. (a) $\kappa_E = 0$; (b) $\kappa_E = 10^{-2}\kappa_{32}^{\text{crit}} = 10^{-4}\kappa_{10}^{\text{crit}}$; (c) $\kappa_E = 10^{-1}\kappa_{32}^{\text{crit}} = 10^{-3}\kappa_{10}^{\text{crit}}$; (d) $\kappa_E = 10\kappa_{32}^{\text{crit}} = 10^{-1}\kappa_{10}^{\text{crit}}$; (e) $\kappa_E = 10^2\kappa_{32}^{\text{crit}} = \kappa_{10}^{\text{crit}}$; (f) $\kappa_E = 10^3\kappa_{32}^{\text{crit}} = 10^2\kappa_{10}^{\text{crit}}$. The dark band in (a)–(c) around the tunneling motion with $\omega_{10}$ is due to the high frequency secondary oscillations picked up by the initial state.

FIG. 3. Same as Fig. 2 on a time scale expanded by an order of magnitude for the previous cases (a), (b) and (c). In (a) all the Bohr frequencies are present; in (b) only the slower oscillations at $\omega_{10}$ and $\omega_{32}$ survive, with $\omega_{32}/\omega_{10} \sim 10^{-2}$. The $\omega_{32}$ component disappears within a tunneling period for a larger value of $\kappa_E$, as in (c).