Exact Conductance through Point Contacts in the $\nu = 1/3$ Fractional Quantum Hall Effect

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The conductance for tunneling through a point contact between two $\nu = 1/3$ quantum Hall edges is described by a universal scaling function, which has recently been measured experimentally. We compute this universal function exactly, by using the thermodynamic Bethe ansatz and a Boltzmann equation.

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The gapless excitations (edge states) \([1]\) at the boundary of a fractional Quantum Hall droplet or bar provide one of the cleanest experimental realizations \([2]\) of a particular non-Fermi-liquid phase, often referred to as a Luttinger liquid. The Luttinger liquid concept \([3]\) was originally developed to describe the excitations about the left and right Fermi points of an interacting 1D electron gas \([1DEG]\). However, since random impurities destroy the non-Fermi-liquid phase of the 1DEG, it has been difficult to see such behavior experimentally. On the other hand, the right- and left-moving edge excitations of a Quantum Hall bar are identical to those near the right and left Fermi-points of the 1DEG. In the Hall bar, however, these right and left movers are spatially separated, so that backscattering due to disorder does not affect the non-Fermi-liquid state, which is therefore stable and experimentally observable.

One universal signature of the Luttinger state in a Hall bar of filling fraction \(\nu\) is the conductance \(G = \nu e^2/h\) \([4]\). Another experimental probe of the Luttinger state is the tunneling conductance through a (local) point contact. The experimental setup is a four-terminal Hall bar geometry (see \([5,2]\) for details). Perpendicular to the \(x\)-direction of the Hall bar, a gate voltage \(V_g\) induces a tunable constriction in the \(y\)-direction, i.e. a point contact. By adjusting the gate voltage to a particular value \(V_g^*\), the Hall conductance \(G\) (the current in the \(x\)-direction divided by the voltage drop in the \(y\)-direction) is tuned to its maximal (resonance) value \(G = \nu e^2/h\). Off resonance, \(G\) is predicted to vanish as \(T \to 0\) in a universal way, described by a universal scaling function of \(V_g - V_g^*\) and \(T\) \([4]\). This function, which has recently been measured \([2]\), is thus a fingerprint of the Luttinger non-Fermi-liquid state. It is computed exactly in the following.

Our method for computing the conductance exactly is based on the fact that at \(\nu = 1/3\), the model with the point-contact interaction is integrable \([6,7]\), and can thus be studied using exact \(S\) matrices and the thermodynamic Bethe ansatz (TBA) \([8]\). Integrability does not usually yield transport properties at non-zero temperature, because the conventional Kubo formula requires exact Green functions, which are in general unknown. Instead, we observe here that integrability defines a basis of massless charge-carrying ‘quasiparticle’ excitations of the non-interacting edges. These quasiparticles are scattered one-by-one off the point contact with a momentum-dependent one-particle scattering matrix \(S\) of transmission and reflection amplitudes. Furthermore, the quasiparticles are characterized by an exactly-calculable distribution function. This special behavior is a consequence of integrability, and it allows us to derive an exact rate (Boltzmann) equation for the conductance in this interacting theory.
We start by describing the left- and right-moving edge channels at filling fraction $\nu$ by left and right moving bosons $\phi_L$ and $\phi_R$. These are defined on a space $-l < x < l$ (we choose periodic boundary conditions for convenience). In the absence of the point-contact interaction, the dynamics of the bosons are generated by a Tomonaga Hamiltonian

$$H_0 = \frac{v_F/\pi}{\nu} \int_{-l}^{l} dx \left[ j_L^2 + j_R^2 \right], \quad (1)$$

quadratic in the two individually-conserved $U(1)$ currents, $j_L = -\frac{1}{4\pi}(\partial_t + \partial_x)\phi_L$ and $j_R = \frac{1}{4\pi}(\partial_t - \partial_x)\phi_R$. These currents are the charge densities in the two edges: for example, $eQ_L = e \int_{-l}^{l} dx j_L$ is the total charge of the left-moving edge.

Before turning to the backscattering interaction, we review quickly the calculation of the Hall conductance $G$ for decoupled edges. We put a voltage drop of $V$ in the $y$-direction, which has the effect of placing the charge carriers injected into the left- and right-moving edges at different chemical potentials. This is done by adding a term $-\Delta QeV/2$ to the Hamiltonian, where $\Delta Q \equiv e(Q_L - Q_R)$ is the difference between right- and left-moving charges. This charge difference is conserved ($\partial_t \Delta Q = 0$) when the edges do not interact, and the resulting current in the $x$-direction is

$$I_0(V) = ev_F \langle \Delta Q \rangle \frac{V}{2l}. \quad (2)$$

First-order perturbation theory in $V$ shows that the linear-response conductance is related to the charge susceptibility

$$G_0 = \lim_{V \to 0} \frac{1}{V} I_0(V) = \frac{e^2v_F \langle \Delta Q \Delta Q \rangle_{V=0}}{2T \langle \Delta Q \rangle_{V=0}}. \quad (3)$$

This, in turn, is related to the chiral $U(1)$ anomaly $\nu$, defined by $\langle j_L(x_1)j_L(x_2) \rangle = \nu/4\pi^2(x_1 - x_2)^2$. Using these expressions we recover the result of [4]: $G_0 = \nu e^2/h$. We use the TBA to give an alternate derivation below.

We now include a point-contact interaction coupling right and left edges at (say) the origin $x = 0$. This is represented in terms of the bosons $\phi_L, \phi_R$ by a backscattering contribution to the Hamiltonian [1,4,5,11]:

$$H_B = \lambda \{ e^{i\phi_L(x=0)}e^{-i\phi_R(x=0)} + e^{-i\phi_L(x=0)}e^{i\phi_R(x=0)} \}. \quad (4)$$

This can be viewed as the tunneling of Laughlin quasi-particles. Additional allowed terms are irrelevant when $\nu = 1/3$ [4]. In the presence of this term (i.e. for $\lambda \neq 0$) there will be
an additional (backscattering) contribution $I_B(V)$ to the current, so that the total current is $I(V) = I_0(V) + I_B(V)$. We will now derive an exact expression for $I(V)$. We proceed in four steps. (i) We map the problem with Hamiltonian $H_0 + H_B$ into two decoupled theories, one of which is affected by the backscattering interaction, and another one which is not. (ii) We find the exact quasiparticle spectrum and exact $S$ matrix, using the fact that the interacting theory is integrable. (iii) We derive an exact (Boltzmann) equation for the backscattering current $I_B$. This is possible, despite the non-Gaussian nature of the interaction, since due to integrability there is no particle production upon scattering off the point contact (in an appropriate basis). (iv) We use the thermodynamic Bethe ansatz to find the exact distribution functions, allowing us to evaluate the formula for the conductance explicitly.

Step(i): We see from (4) that the interaction involves only the combination $\phi_L(x = 0) - \phi_R(x = 0)$. It is therefore convenient to define a new basis of even and odd bosons by

$$
\phi^e(x + t) \equiv \frac{1}{\sqrt{2}}[\phi_L(x, t) + \phi_R(-x, t)], \quad \phi^o(x + t) \equiv \frac{1}{\sqrt{2}}[\phi_L(x, t) - \phi_R(-x, t)]
$$

($\phi^e$ and $\phi^o$ are both left-moving, and defined on $-l < x < l$). This leads to replacing $j_L, j_R$ with $j^e, j^o$ in the Hamiltonian (1), where $j^{e/o}(x + t) = (1/\sqrt{2})[j_L(x, t) \pm j_R(-x, t)]$ are the charge densities of even and odd bosons. The even and odd charges are thus related to the charges of the original left- and right-moving edges by

$$
\Delta Q = Q_L - Q_R = \sqrt{2}Q^o, \quad Q_L + Q_R = \sqrt{2}Q^e.
$$

Two major simplifications arise from this change of basis. First, the interaction involves only the odd boson $\phi^o$, while the even boson $\phi^e$ remains free \footnote{There is in fact a residual coupling between odd and even parts through boundary conditions \cite{12} but it does not affect the properties we discuss here.} (This was the crucial step in solving the X-ray edge problem in a Luttinger liquid \cite{13}.) Additionally, $Q^e$ measures the total charge on both edges and is conserved even in the presence of the interaction. Thus the backscattering current $I_B$ is related to $\Delta Q$ and can be entirely expressed in terms of the odd boson theory. Therefore, the even boson will play essentially no role in the following.
Step (ii): The odd boson theory is integrable. Integrability means that there is an infinite number of conserved quantities which commute with the Hamiltonian, even in the presence of the point interaction. The reformulation in terms of the odd boson is vital, because it is easy to show that the problem with interaction is not solvable in the original basis of (massless) Luttinger bosons $\phi_{L,R}$.

There are many bases for the quasiparticles of the odd-boson theory, which are related by not-necessarily-local mappings. The most obvious basis for a massless scalar field (the odd boson in our case) consists of plane waves, but they are not eigenstates of $H_0 + H_B$. A more useful basis is obtained by adding an auxiliary bulk mass term to the Hamiltonian, which is judiciously chosen so that bulk and point contact are both integrable. This defines a basis of massive bulk excitations in the odd-boson theory (a massive sine-Gordon model) which scatter off the point contact without particle production. By letting the auxiliary bulk mass tend to zero, a basis of massless particles is obtained. In this basis the conserved quantities are $\sum_i p_i^n$, where $p_i$ are the momenta of the individual particles, and where $n$ runs over an infinite subset of the positive integers. These conservation laws have important consequences for scattering. The ‘quasiparticles’ of this basis must scatter off the point contact without particle production, i.e. one-by-one. Away from the point contact, the quasiparticles scatter off of each other with a completely elastic and factorizable scattering matrix $S_{bulk}$. When $1/\nu$ is an integer, the bulk scattering is diagonal, so that the only allowed processes are of the form $|j(p_1)\rangle \otimes |k(p_2)\rangle \rightarrow |k(p_2)\rangle \otimes |j(p_1)\rangle$. Such a process is described by the $S$ matrix element $S_{jk}^{bulk}(p_1/p_2)$.

The conservation laws in fact allow determination of the exact quasiparticle spectrum and the $S$ matrix. This result is already known for the sine-Gordon model and its massless limit. The Hamiltonian written in terms of the odd boson corresponds to the case $\beta^2 = 8\pi\nu$ of . (It is crucial to take into account the $\sqrt{2}$ in .) At any value of $\nu$, there are a kink (+) and an antikink (−). These carry (odd) charges $Q^o = 1/\sqrt{2}$ and $-1/\sqrt{2}$, respectively. Moreover, for $j - 1 < 1/\nu \leq j$, there are $j - 2$ additional states, the breathers, which have no charge. These particles span the Hilbert space of the left-moving odd boson; we label them by indices $j, k, \ldots$ running over the kink (+), antikink (−) and breathers (b).

\[^2\] In the original formulations, it rather appears as a boundary problem. It is well known how to make an impurity problem into a boundary problem, by “folding” the (odd) system in half. We shall not do so here and refer to the treatment of the unfolded problem as in .
Scattering of a single kink by the point contact is described by a one-particle $S$ matrix with elements $S_{++}(p/T_B) = S_{--}(p/T_B)$ for kink $\rightarrow$ kink, and antikink $\rightarrow$ antikink, as well as $S_{+-}(p/T_B) = S_{-+}(p/T_B)$ for kink $\rightarrow$ antikink, and vice versa. Here $T_B \propto \lambda^{1/(1-\nu)}$ is the crossover scale introduced by the interaction. These were derived exactly in [6]:

$$S_{++}(p/T_B) = \frac{(p/T_B)^{(1/\nu)-1}}{1 + i(p/T_B)^{(1/\nu)-1}} \exp[i\alpha_\nu(p/T_B)]$$

$$S_{+-}(p/T_B) = \frac{1}{1 + i(p/T_B)^{(1/\nu)-1}} \exp[i\alpha_\nu(p/T_B)].$$

Here $\exp[i\alpha_\nu]$ is the phase of the expression given in Eq.(3.5) of [7] (where $\lambda$ there is $(1/\nu) - 1$). These two $S$ matrix elements are interchanged as compared to [7], because we have defined the charges here so that $S_{++} = 1$ at $T_B = 0$. This boundary $S$ matrix is unitary: $|S_{++}|^2 + |S_{+-}|^2 = 1$.

Step (iii): We have shown that the bosonic field theory with Hamiltonian $H_0 + H_B$ can be studied in terms of a particular set of quasiparticles and their scattering. We now compute an exact equation for the conductance using this basis.

Without the backscattering, the left and right charges (or equivalently, the even and odd charges) are conserved individually. The backscattering allows processes where a charge carrier of the left-moving edge hops to the right-moving edge or vice versa. In the original basis, the current $I_B$ is the rate at which the charge of the left-moving edge is depleted. By symmetry, $\partial_t Q_L = -\partial_t Q_R$ in each such hopping event, so

$$I_B = \partial_t \left(\frac{e}{2} \Delta Q\right) = \partial_t \left(\frac{e}{\sqrt{2}} Q^o\right),$$

and we see that in the even/odd basis, the tunneling corresponds to the violation of odd charge conservation at the contact. In the $S$ matrix language this happens when $S_{+-} \neq 0$, so that a particle of positive odd charge (the kink) can scatter into one of negative charge (the antikink) at the contact. Neutral quasiparticles cannot transport charge and thus do not directly contribute to $\partial_t \Delta Q$.

To calculate the conductance, we start with a gas of quasiparticles with a chemical potential difference for kinks and antikinks corresponding to the voltage $V$. A positive voltage means that there are more kinks. When there are more kinks than antikinks, the backscattering will turn more kinks to antikinks than it turns antikinks to kinks. When a kink of momentum $p$ is scattered into an antikink (the conservation laws require
that it have the same momentum \( p \) this changes \( \Delta Q \) by +2. Since kink and antikink quasiparticles scatter off the point contact one-by-one, we may describe the rate at which this charge transport occurs in terms of two quantities: the probabilities of finding a kink or antikink of momentum \( p \) at the contact, and the transition probability \( |S_{+-}(p/T_B)|^2 \). We therefore study the density of states \( n_V(p) \) and the distribution functions \( f_{\pm}(p, V) \) in the thermodynamic limit \( (l \to \infty) \) and in the presence of an applied voltage \( V \). The number of allowed kink or antikink states per unit length with momentum between \( p \) and \( p + dp \) is given by \( n_V(p) d(p) \), while the number of states actually occupied by kinks or antikinks in this momentum range is \( n_V(p)f_+(p, V)d(p) \) and \( n_V(p)f_-(p, V)d(p) \), respectively. Because at most one of these quasiparticles is allowed per level [7,15] (a consequence of their being the massless limit of the sine-Gordon kinks), we have \( 0 \leq f_{\pm} \leq 1 \). Thus

\[
\frac{\langle \Delta Q \rangle_V}{2l} = \int_0^\infty d(p) n_V(p) [f_+(p, V) - f_-(p, V)].
\] (9)

These thermodynamic functions \( n_V(p) \) and \( f_{\pm}(p, V) \) are different from the free-fermion functions when the odd-boson kink theory is an interacting Luttinger liquid \( (\nu \neq 1/2) \), but they will be derived exactly from the TBA below.

Having all these definitions in place, it is now easy to compute the backscattering current from a rate (Boltzmann) equation. The number of kinks of momentum \( p \) which scatter into antikinks per unit time is given by \( |S_{+-}|^2 v_F n_V f_+ [1 - f_-] \); the factor \( [1 - f_-]f_+ \) accounts for the probabilities of the initial state being filled and the final state being open. The rate at which antikinks scatter to kinks is likewise proportional to \( [1 - f_+]f_- \), so the charge changes at a rate proportional to \( [1 - f_-]f_+ - [1 - f_+]f_- = f_+ - f_- \). Using (8) and (9) we have

\[
I_B(V) = -e \int_0^\infty d(p) n_V(p) v_F |S_{+-}(p/T_B)|^2 [f_+(p, V) - f_-(p, V)].
\] (10)

From (10) we obtain the desired backscattering contribution to the conductance:

\[
G_B = \lim_{V \to 0} \frac{1}{V} I_B(V) = -2e \int_0^\infty d(p) n_0(p) v_F |S_{+-}(p/T_B)|^2 \partial_V f_+(p, V)|_{V=0}.
\] (11)

The density of states \( n_V(p) \) has been evaluated at \( V = 0 \) because \( f_+(p, V) - f_-(p, V) \) is already proportional to \( V \). The total conductance is thus \( G = \nu e^2/h + G_B \). (Note that (10) may also be used to compute the conductance \( G(V, T) \) at finite voltage \( V \), i.e. far out of equilibrium. We shall report on this shortly [16].)
To check our result, we consider $\nu = 1/2$, where the conductance was previously derived exactly [4]. Here the odd-boson kinks are simply free fermions [17, 13], so they have the Fermi distribution function $f_\pm(p, V) = 1/[1 + \exp(-p/T \pm eV/2T)]$. These fermions do scatter non-trivially off of the point contact, with $S$ matrix given by (3). The resulting expression for $G_B$ obtained from (11) is identical to the result in sect. VIII of [4].

Step (iv): We proceed by outlining the TBA computation of the densities of states $n_V(p)$ and the distribution functions $f_\pm(p, V)$ of the massless odd boson theory at $\nu = 1/3$. As explained above, these define the occupation numbers of quasiparticles, which are scattered without particle production by the point contact. The requirement of a periodic boundary condition results in an equation which relates the densities $n_j$ and $f_j$ of all these quasiparticles

$$n_j(p) = \frac{1}{v_F h} + \frac{1}{2\pi p} \sum_k \int_0^\infty dp' \Phi_{jk}(p/p') n_k(p') f_k(p'),$$  \hspace{1cm} (12)$$

where $\Phi_{jk}(p) = -i(d/d \ln p) \ln S_{jk}^{\text{bulk}}(p)$. For $\nu = 1/3$ there is only one breather $(b)$ and we have [15]

$$\Phi_{bb}(x) = 2\Phi_{++}(x) = 2\Phi_{+-}(x) = -\frac{4x}{x^2 + 1},$$

$$\Phi_{b+}(x) = \Phi_{+b}(1/x) = -\frac{4x^3 + 8x}{x^4 + 4},$$

where the others follow from the symmetry $+ \leftrightarrow -$.

One defines an auxiliary pseudoenergy variable $\epsilon_j$ to parametrize $f_j$ via $f_j \equiv 1/(1 + \exp(\epsilon_j - \mu_j/T))$, where the $\mu_j$ are the chemical potentials: $\mu_+ = -\mu_- = eV/2$; $\mu_b = 0$. By demanding that the free energy at temperature $T$ (expressible in terms of $f_j$ and $n_j$) be minimized, we find an equation for $\epsilon_j$ in terms of the (known) bulk $S$ matrix elements:

$$\epsilon_j(p/T, V/T) = \frac{p}{T} - \sum_k \int_0^\infty dp' \Phi_{jk}(p/p') \ln[1 + e^{\mu_k/T} e^{-\epsilon_k(p'/T,V/T)}].$$  \hspace{1cm} (13)$$

Solving this equation for $\epsilon_j$ gives the functions $f_j$. Even though the breather does not appear in (11), it interacts with the kink and antikink and affects the calculation of $f_\pm$.

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3 One can include the effects of the backscattering on $n$ and $f$, but this gives only contributions to $n$ and $f$ vanishing with the system size $2l$. For example, free energy $F(T/T_B)$ resulting from (3) was calculated in [7]. Since the conductance is computed in the limit $l \to \infty$ these contributions are not relevant here.
We can now evaluate the conductance explicitly. Using (13) and (12), it is easy to see that $\partial_V n_V(p)|_{V=0} = 0$, and that $\partial_V f_\pm(p,V)|_{V=0} = \pm \frac{e^2}{2}\tau f[1 - f]$, where $f(p) \equiv f_\pm(p,V = 0)$. Thus using (2) and (4) gives us the TBA relation for the conductance without a contact:

$$G_0 = \partial_V I_0(V)|_{V=0} = \frac{e^2}{T} \int_0^\infty d(v_F p) n_0(p)v_F[1 - f(p)]f(p).$$

We will verify shortly that this indeed gives the correct answer $G_0 = \nu e^2/h$. Moreover, this and the unitarity of the $S$ matrix allows us to write the full $G$ as

$$G = \frac{e^2}{T} \int_0^\infty d(v_F p)n_0(p)v_F|S_{++}(p/T_B)|^2[1 - f(p)]f(p).$$

(14)

By definition of $\epsilon$, $f(1 - f) = -\partial_\epsilon f$. The relations (13) and (12) imply that $n_j = \frac{T}{v_F h}\partial_\epsilon \epsilon_j(p/T, V = 0)$. Inserting (7) into (14), we find our final result for the conductance at $\nu = 1/3$:

$$G = -\frac{e^2}{h} \int_0^\infty dp \frac{p^4}{p^4 + T_B^2} \frac{\partial \epsilon_+(p/T, 0)}{\partial p} \frac{\partial f(p)}{\partial \epsilon_+}$$

$$= \frac{e^2}{h} \int_0^\infty dx x^3 \frac{4(T_B/T)^4}{(x^4 + (T_B/T)^4)^2} \frac{1}{1 + e^{\epsilon_+(x,0)}},$$

(15)

where we integrated by parts and defined the variable $x = p/T$.

As $T_B/T \to \infty$, we have $G \propto (T/T_B)^4$. Thus it goes to zero with the correct exponent, as in [4]. As $T_B/T \to 0$, we can also evaluate the conductance explicitly, becoming $[f(0) - f(\infty)]e^2/h$. The relation (13) gives $f(0) = 1/3$ and $f(\infty) = 0$, and we indeed recover $G_0 = e^2/3h$. For $T_B/T$ small, we can expand $f(x)$ in powers of $x^{4/3}$ based on the periodicity of the system (13) [18,7]; this means that $G - G_0 \propto (T_B/T)^{4/3}$, again in agreement with [4]. In fact, one can compute all of the perturbative coefficients using the TBA along with Jack-polynomial technology [19]. To plot the complete function, one solves (13) for $\epsilon_+$ and inserts the result into (15). We have done so numerically to double-precision accuracy. We compare our results with the experimental data as well as with the Monte Carlo results of [4] in fig. 1. The agreement between the Monte Carlo simulation and our exact scaling curve is excellent. The exact value of the universal parameter $K$ (defined so that $G(X) = KX^{-6}$ for $X$ large and $G(X) = (1 - X^2)/3$ for $X$ small) is $K = 3.3546...$ (where $X \approx .74313(T_B/T)^{2/3}$). (The value $K \approx 2.6$ quoted in [4] seems to have been slightly underestimated there.) The comparison with experiments is not completely straightforward, since the conductance $G_{pc}$ at the resonance peak decreases with temperature and is well below its resonance value $e^2/3h$. This difficulty arises since two parameters are needed
to tune to resonance in the absence of parity symmetry $^4$ (relevant for the experiment) whereas only one parameter, $V_g$, has been varied experimentally. This could be remedied by varying the magnetic field on the Hall plateau, as well as $V_g$, in a future experiment. Nevertheless, the data collapse well onto a single curve, and the so-obtained experimental scaling curve is in good agreement with the theoretical exact curve, given the large scatter of the data in the tail of the resonance curve. In particular, the data clearly show the predicted $G \propto T^4/(V_g - V_g^*)^6$ behavior in the tail.

To conclude, we have computed experimentally-measured transport properties at non-zero temperature exactly by using integrability, without the explicit knowledge of Green’s functions. Furthermore, our methods for computing the conductance can be immediately extended to non-zero voltage, providing exact transport properties out of equilibrium $^{16}$. This shows that integrability is a powerful method of addressing the many strongly-interacting problems of modern condensed-matter physics, for which reliable answers are notoriously difficult to come by. Integrability is not so exceptional in low dimensions, and we hope to report on exact results for other experimentally-important quantities in the near future.

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\[ G \pi e^2 \]

\[ X = 0.74313 \left( \frac{T_B}{T} \right)^{2/3} \]

Conductance

- exact curve
- Monte Carlo
- experimental data