The temperature dependence of the conductance of a quantum point contact has been measured. The conductance as a function of the Fermi energy shows temperature-independent fixed points, located at roughly multiple integers of \( e^2/h \). Around the first fixed point at \( e^2/h \), the experimental data for different temperatures can be scaled onto a single curve. For pure thermal smearing of the conductance steps, a scaling parameter of one is expected. The measured scaling parameter, however, is significantly larger than 1. The deviations are interpreted as a signature of the potential landscape of the quantum point contact, and of the source-drain bias voltage. We relate our results phenomenologically to the metal-insulator transition in two dimensions.

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Conductance quantization in short, quasi one-dimensional wires ('quantum point contacts'-QPCs) has been widely investigated since its discovery. Meanwhile, QPCs have become a key device for transport experiments in low-dimensional systems. Recently, QPCs have experienced renewed interest in relation to the metal-insulator transition (MIT) in two dimensions. It has been suggested that near the fixed point of the MIT, which typically occurs at a conductance of \( G \approx e^2/h \), the carrier gas segregates into conductive puddles, which are connected via quantum point contacts. Within such a percolation picture, a statistical ensemble of QPCs determines the transport properties of the inhomogeneous carrier system, since the applied external voltage drops predominantly across the QPCs. Of particular importance is the fixed point in the temperature-dependent conductance of QPCs at \( e^2/h \), which enters in the more general discussion of the quantum percolation model of Ref. [7].

\[
G(\mu, T) = \frac{2e^2}{h} \int d\Theta(E-E_1)[-\frac{\partial f(E)}{\partial E}]dE = \frac{2e^2}{h} \left[ 1 + e^{(E_1-\mu)/k_B T} \right]^{-1}. \tag{1}
\]

Here, \( f \) is the Fermi function, and \( E_1 \) is the energy of the first one-dimensional subband. Furthermore, we have assumed that the voltage drop across the QPC is small compared to \( k_B T \). Since within this model, \( G(\mu, T) \) has the shape of a Fermi function, the temperature-dependent conductance traces obtained for fixed \( \mu \) can be scaled onto a single curve \( G^*(\delta/T^{3/\alpha}) \), with \( \delta = (\mu-E_1)/E_1 \) and the scaling parameter \( \alpha = 1 \). \( G^* \) consists of two branches, corresponding to the metallic and the insulating region. Note that here, in contrast to the heavily debated scaling properties of two-dimensional systems, scaling does not imply a possible quantum phase transition at \( T=0 \). Nevertheless, it is interesting in view of Ref. [7] to compare \( \alpha \) obtained here with the scaling parameters found in two-dimensional systems showing a MIT.

The temperature dependence of the conductance quantization in QPCs has been addressed in several experiments, in particular with respect to the so-called “0.7-feature”. To our knowledge, the fixed point at \( G = e^2/h \) and the temperature dependence...
around it has not been investigated yet. However, fixed points have been detected occasionally, while the observed behaviors around these fixed points vary greatly and definitely differ from the expectation expressed by eq. (1). Yacobi et al.\textsuperscript{12}, for example, observe a fixed point at $G = 1.5e^2/h$ only, around which the metallic and the insulating phases appear reversed. This anomaly probably originates in the observed non-universal conductance quantization. Thomas et al.\textsuperscript{13} observe fixed points at $G = 3e^2/h$ and $G = 2e^2/h$, but not at $G = e^2/h$, which is possibly destroyed by the 0.7-feature. In the present paper, we report the observation of a series of clear fixed points ($n=1$...5) in a QPC. We investigate $G(\mu, T)$ around the fixed point at $G \approx e^2/h$ in further detail. $G(\mu, T)$ can be scaled reasonably well onto a single curve. The scaling parameter $\alpha$, however, is significantly larger than 1 (the value assumed in Ref. 6), and increases monotonically as the source-drain bias voltage is increased. Possible implications for describing the MIT in 2 dimensions in terms of a quantum percolation model are discussed.

The metallic and insulating regions around $G = e^2/h$ are significantly larger than 1 (the value assumed in Ref. 6), and definitely differ from the expectation expressed by eq. (1). Yacobi et al.\textsuperscript{12}, for example, observe a fixed point at $G = 1.5e^2/h$ only, around which the metallic and the insulating phases appear reversed. This anomaly probably originates in the observed non-universal conductance quantization. Thomas et al.\textsuperscript{13} observe fixed points at $G = 3e^2/h$ and $G = 2e^2/h$, but not at $G = e^2/h$, which is possibly destroyed by the 0.7-feature. In the present paper, we report the observation of a series of clear fixed points ($n=1$...5) in a QPC. We investigate $G(\mu, T)$ around the fixed point at $G \approx e^2/h$ in further detail. $G(\mu, T)$ can be scaled reasonably well onto a single curve. The scaling parameter $\alpha$, however, is significantly larger than 1 (the value assumed in Ref. 6), and increases monotonically as the source-drain bias voltage is increased. Possible implications for describing the MIT in 2 dimensions in terms of a quantum percolation model are discussed.

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The sample is a Ga[Al]As heterostructure, grown by molecular beam epitaxy, with the 2DEG 34 nm below the surface. The QPC is defined by local oxidation with an atomic force microscope\textsuperscript{14} (insets in Fig.1). The lithographic channel width is 80 nm, and its length is given by the width of the oxide lines, i.e., 140 nm. The QPC can be tuned by voltages applied to a homogeneous top gate (tg) and to the planar gate (pg). A top gate voltage changes $\mu$ throughout the sample. However, $G$ is still determined by the QPC, as long as the surrounding 2DEG is not depleted. The sample is mounted in a $^4$He cryostat, in which the temperature can be varied between 1.7 K and room temperature, or in the mixing chamber of a $^3$He/$^4$He - dilution refrigerator with a base temperature of 90 mK, respectively. The electron density of the ungated 2DEG is $5.5 \times 10^{15}$ m$^{-2}$, and the mobility of the 2DEG is 93 m$^2$/Vs at T = 4.2 K.

\textbf{AC} resistance measurements are performed in a four-terminal setup. As the top gate voltage $V_{tg}$ or the temperature is changed, the voltage drop $V_b$ across the QPC is kept constant by a feedback loop which adjusts the current accordingly. This constant voltage drop is important, since $V_b$ modifies $G(\mu, T)$, as will be discussed below. Figure 1 shows $G$ as a function of $V_{tg}$ for different temperatures. The conductance quantization has a characteristic temperature of $\approx 15$K, which corresponds to an energy separation between the one-dimensional subbands of $\hbar \omega_y = 3.52 k_B T \approx 4.5$ meV. Here, $\omega_y$ denotes, within a parabolic approximation, the confining strength transverse to the transport direction. Fixed points occur at roughly multiple integers of $e^2/h$, and separate metallic regions from insulating regions. The traces contain additional conductance fluctuations between the plateaus, which depend on the cooldown cycle, but are always present and perfectly reproducible within one cooldown. Probably, they are due to imperfect adiabatic coupling between the QPC and the reservoirs, or due to nearby scatterers. At temperatures below 1K, the shapes of the steps are independent of temperature (not shown), indicating that the smearing of the steps is not solely determined by temperature.

We therefore model the QPC potential by a parabolic saddle point potential\textsuperscript{15} i.e. $V(x, y) = V_0 - \frac{1}{2} m^* \omega_x x^2 + \frac{1}{2} m^* \omega_y y^2$; $V_0$ is the potential energy of the saddle point, $m^*$ the effective electron mass, and $\omega_y$ denotes the curvature of the saddle point potential in transport direction. Using this model is justified since in the following, we focus on the regime $G < 2e^2/h$.

The transmission $T(\epsilon)$ for the lowest one-dimensional subband can be written as\textsuperscript{16}

$$T(\epsilon) = [1 + e^{-\epsilon}]^{-1}, \epsilon = 2(E - \frac{1}{2} \hbar \omega_y - V_0)/\hbar \omega_x. \quad (2)$$

Here, $E$ denotes the energy of the incident electrons. Since the separation of the conductance plateaus corresponds to $\hbar \omega_y$ in energy, we can estimate the lever arm inside the QPC and in the regime of the first conductance step to $\frac{dE}{dV_{tg}} \approx 50$ meV/V, which we use to transform $V_{tg}$ into energy. In the limit of $V_b = 0$ and for negligible temperatures, the width of the step edge corresponds to $\hbar \omega_x$. Hence, we can estimate $\hbar \omega_x \approx 2$ meV from Fig. 1.

We proceed by studying the “metal-insulator transition” around $G = e^2/h$ in more detail (Fig. 2). As $V_b$ is increased, the fixed point remains well-defined and shifts to smaller conductances. In addition, the step edge broadens significantly. In the insets of Figs. 2a and 2b, we show the best scaling achieved with the scaling variable

![Graphical representation of the conductance $G(\mu, T)$ vs. gate voltage $V_{tg}$ at different temperatures.](image)
|\delta|/T^\alpha|, using \alpha as a parameter. Scaling of the experimental data works well, although worse than for many two-dimensional systems showing an MIT. The scaling parameter \alpha, however, is larger than 1 and increases as \(V_b\) is increased. We estimate the error of \alpha to \(\delta \alpha \approx \pm 0.3\) at small \(V_b\), which increases for higher bias voltages. Thus, the MIT is clearly not solely determined by thermal smearing, as eq. (1) suggests.

In order to understand the origin of the step edge slope and the experimental scaling parameter in more detail, we consider a simple model, which includes thermal smearing, the potential landscape in transport direction, described by \(\omega_x\), and the bias voltage \(V_b\). We therefore follow Ref. 8 and model the current \(I\) by

\[
I = \frac{e}{\pi \hbar} \int T(\epsilon) \cdot \left[ f(E - (E_F + eV_b), T) - f(E - E_F, T) \right] dE,
\]

where \(T(\epsilon)\) is given by eq. (2), and use \(\omega_x\) as a parameter in order to obtain agreement with the experiment (Figs. 2c and 2d). With eq. (3), we can describe the experimental results, provided we allow \(\omega_x\) to decrease as \(V_b\) is increased. Such an effect is quite reasonable, since the potential drop in transport direction is super-imposed to the saddle point potential, which may result in a reduction of the curvature \(\omega_x\) at the saddle point. Furthermore, we have assumed in the model that \(\omega_y\) is independent of \(V_b\), as observed experimentally, i.e. the separation of the steps in \(V_{sd}\) does not depend on \(V_b\).

Also, our model does not make any assumptions on how the voltage drops along the QPC.

The conductance traces obtained from eq. (3) no longer scale exactly onto a single curve (Figs. 2c, d). However, the scaling analysis not only allows us to express the behavior around the fixed point in terms of a single parameter, but also shows that the imperfect scaling of the experimental data can be expected from models differing from the oversimplified one described by eq. (1). The insets in Fig. 2 show the scaled data (scaling functions) of the main figures. As \(V_b\) increases, \(\omega_x\) has to be reduced in order to obtain agreement between the scaling parameters in the experiment and in the model. This behavior is more clearly shown in Fig. 3, where we plot the results of many simulations and compare them with the experimental data. It can be seen that for \(eV_b \geq \hbar \omega_x\), the \(\omega_x\) needed in the simulation in order to reproduce the experimental scaling parameter becomes smaller. The reason for this is possibly again the simplifications made in our model, or nonlinearities in the current-voltage characteristics, which are expected as soon as \(e \cdot V_b\) gets of the order of the subband spacing \(\hbar \omega_y\). For small source-drain bias voltages, however, the experimentally obtained scaling parameter can be interpreted as a measure of \(\omega_x\).

The model developed by Meir in Ref. 8 describes the re-

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**FIG. 2.** Conductance of the QPC as a function of the Fermi energy, as measured for \(V_{sd} = 20 \mu V\) (a) and for \(V_{sd} = 2.8 \, mV\) (b). The Fermi energy was calculated from \(V_{sd}\) using the lever arm (see text). The insets show the corresponding scaling plots and the scaling parameters obtained. In (c) and (d), the calculated conductance within the model described in the text for the corresponding \(V_{sd}\) are shown, together with the scaling functions of the calculated conductance traces. In addition, in the inset in (c), the scaled thermally smeared Fermi function is shown for comparison, which has \(\alpha = 1\).
ently observed metal-insulator transition (MIT) in two dimensions in terms of a network of quantum point contacts with statistically distributed conductances. This network can be thought of being composed of two characteristic conductances, a metallic one ($\sigma_m$), and an insulating one ($\sigma_i$), which determine the scaling behavior of the network around the percolation threshold. Thus, this model cannot be applied to the MIT at a single QPC, as investigated in our experiment. Nevertheless, our results are related to Meir’s model, since they demonstrate that the shape of the saddle potential influences $\sigma_m$ and $\sigma_i$. Within a simple approximation, however, a connection between the MIT on a QPC and that one in a disordered two-dimensional carrier gas can be made. In the case of a disordered system consisting of conductive puddles, connected by identical QPCs, scaling parameter obtained experimentally from scaling the MIT of the disordered system is directly related to $\omega_x$. It is the characterizing parameter for the connections between the puddles (note that within our simple model, the macroscopic voltage drop measured corresponds to the voltage drop across a single QPC, except for a geometry factor).

![Graph](image)

**FIG. 3.** Comparison between the experimentally obtained values for $\alpha$ (full circles) and those obtained for various $\omega_x$ (see legend) within our model. The model predicts an increasing scaling parameter as $V_{sd}$ increases, which is also seen in the experiment. For $V_b \leq 2$ mV, the experimental numbers for $\alpha$ agree well with the model, assuming $h\omega_x \approx 2$ meV. For $V_b > 2$ mV, deviations occur (see text).

The samples which perhaps come closest to such a scenario are those studied by Ribeiro et al. There, self-assembled quantum dots are embedded in the plane of the electron gas. The dots, which act as repulsive scatterers, have a diameter of only 30 nm, a separation of $\approx 80$ nm on average, and are very homogeneous in size. It is thus quite possible that between the self-assembled dots, QPCs are formed, in which only the lowest subband is occupied. Analyzing the MIT observed in these disordered samples gives $\alpha = 2.6$ at small bias voltages, which, within our model, corresponds to a realistic value of $h\omega_x \approx 3.5$ meV.

In summary, we have observed a sequence of fixed points in a single quantum point contact. The scaling behavior of the temperature dependent conductance around the fixed point in the lowest subband has been studied, and significant deviations from simple Fermi function scaling have been found. Besides temperature, the scaling parameter is also determined by the potential shape of the QPC in transport direction, which we have quantified by the curvature $\omega_x$, and by the source-drain voltage. Our experiment and analysis suggests that in order to further check the validity of Meir’s model, an artificial, random network of quantum point contacts should be studied, which has become feasible by using lithography with an atomic force microscope.

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