CP-odd Neutral Higgs Effects in \( t\bar{t} \) Production

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ABSTRACT

We study CP violation in the process \( e^+e^- \rightarrow t\bar{t}\nu\bar{\nu} \) at an \( e^+e^- \)-TeV collider. As the source of CP violation we assume a two-Higgs doublet model with an explicitly CP-noninvariant Higgs potential. Sizeable CP-odd observables originating from the subprocess reaction, \( W^+W^- \rightarrow t\bar{t} \), may arise as a result of finite width effects of the neutral Higgs particles. CPT constraints due to final (initial) state interactions are also taken into account. Numerical estimates of the CP asymmetry are given.

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1. Introduction

$CP$ violation in the neutral Higgs sector with two or more Higgs doublets has recently received much attention due to some attractive features of these models [1-3]. For instance, these models can give rise to a large electric dipole moment ($EDM$) for the neutron [4] as well as for the top quark [5,6]. Non-negligible contributions to $EDMs$ for leptons [4,7] and gauge bosons $W^\pm$, $Z^0$ [8] have also been found. Last but not least, the $CP$-violating Higgs sector has entered the domain of cosmology, namely as a natural explanation of baryogenesis [9]. Note here that the strength of $CP$ violation from the Cabbibo-Kobayashi-Maskawa matrix is not sufficient to explain the baryon asymmetry in the universe [10]. Therefore, models manifesting $CP$ violation in the scalar sector offer an interesting field of investigation for $CP$-odd effects outside the well known $K^0 - \bar{K}^0$ system.

Beside the possibility of searching for sizeable signals of $EDMs$ one can also probe these models in top production and decay [7,11]. Here, however, we suggest a different way of testing $CP$ violation which can be mediated by neutral Higgs bosons. By considering the production of left- and right-handed $t\bar{t}$ pairs in a high energy $e^+e^-$-collider (i.e. $e^+e^- \rightarrow \nu_\ell\bar{\nu}_\ell t_\lambda\bar{t}_\lambda$, $\lambda\lambda = LL, RR$), one can show that the simultaneous presence of $CP$-even ($\bar{\psi}\psi$) and $CP$-odd ($i\bar{\psi}\gamma_5\psi$) fermionic operators in the Yukawa sector induces a $CP$ asymmetry in the event-rate difference of $N(t_L\bar{t}_L) - N(t_R\bar{t}_R)$ [5]. To obtain a non-zero asymmetry, one should have two interfering amplitudes with different weak phases as well. This requirement is effectively satisfied here by using complex Breit–Wigner ($BW$) propagators for resonant particles [11], i.e. Higgs scalars [12]. Nevertheless, it is known that the $CP$-even phases of final and (in our case) also initial state interactions cannot contribute to a $CP$ asymmetry [13,14]. This is sometimes called the $CP - CPT$ con-
nection \[14\]. We take such \textit{CPT} constraints into account in order to establish a genuine signal of \textit{CP} violation.

Our paper is organized as follows. In Section 2 we outline the basic features of a \textit{CP}-violating two-Higgs doublet model. In Section 3 we calculate the subprocess \(W^+W^- \rightarrow t\bar{t}\) with intermediate Higgs bosons in the \(s\)-channel. Section 4 deals with \textit{CPT} constraints and the gauge symmetry imposed on our process. Finally, in Section 5 we present numerical estimates and discussion of our results.

2. The \textit{CP}-violating two-Higgs doublet model

The simplest realization of \textit{CP} violation in the neutral Higgs sector is a two-Higgs doublet model where the usually imposed discrete symmetry \(D: \Phi_1 \rightarrow -\Phi_1, \Phi_2 \rightarrow \Phi_2\) is softly broken \[15\]. As usual the Higgs part of the Lagrangian reads

\[
\mathcal{L}_H = \sum_{i=1}^{2}(D_\mu \Phi_i)^+ (D^\mu \Phi_i) - V_H(\Phi_1, \Phi_2),
\]

where \(D_\mu\) is the covariant derivative. The potential \(V_H\) can be split into two terms according to the sum,

\[
V_H(\Phi_1, \Phi_2) = V_D(\Phi_1, \Phi_2) + \Delta V(\Phi_1, \Phi_2).
\]

\(V_D\) respects the discrete symmetry \(D\) and can be conventionally written in the form \[15\]

\[
V_D(\Phi_1, \Phi_2) = \mu_1^2(\Phi_1^\dagger \Phi_1) + \mu_2^2(\Phi_2^\dagger \Phi_2) + \frac{1}{2} \lambda_1(\Phi_1^\dagger \Phi_1)^2 \\
+ \frac{1}{2} \lambda_2(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\
+ \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_5(\Phi_1^\dagger \Phi_2)^2 \\
+ \frac{1}{2} \lambda_5^* (\Phi_2^\dagger \Phi_1)^2,
\]
whereas $\Delta V$ softly violates the $D$ symmetry by introducing linear $\Phi_1^\dagger \Phi_2$ terms

$$\Delta V = \lambda_6 (\Phi_1^\dagger \Phi_2) + \lambda_6^* (\Phi_2^\dagger \Phi_1).$$  \hspace{1cm} (2.4)$$

One can check explicitly by performing $CP$ transformation on the $\Phi_i$'s that $V_H$ is $CP$ invariant only if

$$\text{Im} \lambda_5 \lambda_6^* = 0$$  \hspace{1cm} (2.5)$$
holds. After spontaneous symmetry breaking with two complex vacuum expectation values ($VEVs$) $v_1$ and $v_2$ of the fields $\Phi_1$ and $\Phi_2$, respectively, eq. (2.5) becomes

$$\text{Im} \lambda_5 (v_1^* v_2)^2 = 0,$$  \hspace{1cm} (2.6)$$
which in turn is equivalent to

$$\text{Im} \lambda_6 (v_1^* v_2) = 0.$$  \hspace{1cm} (2.7)$$

Note that without loss of generality we can choose $\lambda_5$ real and, say, $v_1$ real. This follows from the fact that we have the freedom of redefining the fields by an arbitrary phase transformation

$$\Phi_i \rightarrow e^{i\alpha_i} \Phi_i.$$  \hspace{1cm} (2.8)$$

It is worth mentioning that due to this phase symmetry there is no $CP$ violation in $V_H$ if either $\lambda_5$ and $\lambda_6$ is identical to zero. This is also evident from (2.6) and (2.7). Writing the fields $\Phi_1$ and $\Phi_2$ in the form

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \phi_1^0 + i\chi_1^0) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(|v_2|e^{i\xi} + \phi_2^0 + i\chi_2^0) \end{pmatrix},$$  \hspace{1cm} (2.9)$$
and collecting quadratic terms in the components of $\Phi_i$ we get, after diagonalizing the mass matrices, the following mass eigenstates

$$\begin{pmatrix} H^+ \\ G^+ \end{pmatrix} = U_H \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}, \quad \begin{pmatrix} H_1^0 \\ H_2^0 \\ H_3^0 \\ G^0 \end{pmatrix} = D^T O_H \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \\ \chi_1^0 \\ \chi_2^0 \end{pmatrix},$$  \hspace{1cm} (2.10)$$
where $H_i^0$ ($i = 1, 2, 3$) and $H^\pm$ are the physical Higgs particles and, $G^\pm$ and $G^0$ are charged and neutral Goldstone bosons, respectively. Defining

$$\tan \beta = \frac{|v_2|}{v_1},$$

(2.11)

the unitary transformation matrix $U_H$ in the charged Higgs sector reads

$$U_H = \begin{pmatrix} -s_\beta e^{i\xi} & c_\beta e^{i\xi} \\ c_\beta & s_\beta \end{pmatrix},$$

(2.12)

where $\sin x \equiv s_x$ and $\cos x \equiv c_x$. In eq. (2.12) the phase $\xi$ is trivial and can be removed away by appropriately rephasing the charged mass eigenstates $H^+, G^+$. The orthogonal transformation matrix in the neutral Higgs sector has been expressed for convenience as a product of two orthogonal matrices $D$ and $O_H$. The latter has the form

$$O_H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_\xi & 0 & s_\xi \\ 0 & -c_\beta s_\xi & -s_\beta & c_\beta c_\xi \\ 0 & -s_\beta s_\xi & c_\beta & s_\beta c_\xi \end{pmatrix}.$$  

(2.13)

The matrix $D$ acts only on the physical Higgses $H_i^0$ and can therefore be written as

$$D^T = \begin{pmatrix} d & 0 \\ 0 & 1 \end{pmatrix},$$

(2.14)

where $d$ is a $3 \times 3$ orthogonal matrix. This matrix can be parametrized by three Euler angles which are determined by the Higgs couplings $\lambda_i$. At this stage it is not necessary to do the diagonalization in full detail and to spell out $d$ in terms of $\lambda_i$. Instead, we mention that if condition (2.6) or alternatively (2.7) holds, then we get the matrix form

$$d = \begin{pmatrix} d' & 0 \\ 0 & 1 \end{pmatrix},$$

(2.15)
where $d'$ is a $2 \times 2$ dimensional matrix. In this case, $H_3^0$ decouples from the other Higgses and is of purely pseudoscalar nature in a $CP$-conserving two-Higgs doublet model. In general, as a consequence of $CP$-violating terms in $V_H$, the physical neutral Higgses are mixtures of $CP$-even and $CP$-odd components. It is then obvious that we have

$$d_{13}, d_{23}, d_{31}, d_{32} \propto \text{Im}\lambda_5(v_1^*v_2)^2. \quad (2.16)$$

Among the various vertices that are present in the full Lagrangian, we will only need the gauge-boson interactions with Higgses, Goldstone bosons $G^\pm$ and ghosts $c^\pm, \bar{c}^\pm$, as well as the Yukawa interactions for the top quark. It is straightforward to obtain the vertices from the Lagrangian (2.1) (considering also gauge-fixing terms)

$$V^{\mu\nu}_{W WH_i} = ig_W M_W g^{\mu\nu} (d_{1i} c_\beta + d_{2i} s_\beta), \quad (2.17)$$

$$V^{\mu\nu}_{ZZH_i} = ig_W M_Z^2 M_W g^{\mu\nu} (d_{1i} c_\beta + d_{2i} s_\beta), \quad (2.18)$$

$$V^{\mu}_{ZH_iH_j} = g_W M_Z (k_{H_i} - k_{H_j})^{\mu} (s_\beta d_{2i} - c_\beta d_{2j}), \quad (2.19)$$

$$V^{\mu}_{W \pm G^\mp H_i} = \pm i \frac{1}{2} g_W (k_{G_i} - k_{H_i})^{\mu} (d_{1i} c_\beta + d_{2i} s_\beta), \quad (2.20)$$

$$V^{\mu}_{c^+ \bar{c}^- H_i} = -i \frac{1}{2} \xi_W g_W M_W (d_{1i} c_\beta + d_{2i} s_\beta), \quad (2.21)$$

with the convention that all momenta are incoming and the definition

$$v^2 = |v_1|^2 + |v_2|^2. \quad (2.22)$$

In eq. (2.21) $\xi_W$ denotes the gauge parameter. In order to avoid flavor neutral currents at tree level, one has to require the Yukawa Lagrangian $L_Y$ to be invariant under the discrete symmetry $D$. We choose the coupling scheme where $u$-type quarks couple to $\Phi_2^0$ field via

$$L_Y = - \frac{1}{v_2} (U_R M_U U_L) \Phi_2^0 + h.c. + \ldots, \quad (2.23)$$

where $M_U$ is the diagonal mass matrix. Especially for the top quark we can rewrite (2.23) as follows [5]:

$$L^t_Y = - g_W \frac{m_t}{4 M_W} H_i^0 \bar{t} [Y_i (1 - \gamma_5) + Y_i^* (1 + \gamma_5)] t. \quad (2.24)$$
The Yukawa couplings $Y_i$ in (2.24) are given by

$$Y_i = \frac{d_{2i}}{s_\beta} + id_{3i} \cot \beta.$$  \hfill (2.25)

It is instructive to express (2.24) in an equivalent form [5]

$$L_Y = -g_W \frac{m_t}{2M_W} H_i^0 [\alpha_{i2} \bar{t}t - i\alpha_{i3} \cot \beta \bar{t}\gamma_5 t],$$  \hfill (2.26)

$$\alpha_{i2} = \frac{d_{2i}}{s_\beta}, \quad \alpha_{i3} = d_{3i}.$$  \hfill (2.27)

From (2.26) it is evident that $L_Y$ leads to CP violation, since $H_i^0$ couples simultaneously to CP-even ($\bar{t}t$) and CP-odd ($i\bar{t}\gamma_5 t$) currents.

3. CP violation in the process $e^+e^- \rightarrow t\bar{t}\nu_e\bar{\nu}_e$

First we will discuss the manifestation of CP violation in the process $W^+W^- \rightarrow t\bar{t}$ which must be understood as a subprocess of the reaction $e^+e^- \rightarrow t\bar{t}\nu_e\bar{\nu}_e$. Numerical results for such processes has been obtained by using the Effective Vector Boson Approximation (EVBA) [16] in ref. [17].

Regarding the signal of CP violation in this process we consider the production of left- and right-handed $t\bar{t}$ pairs. Under the assumption of CP conservation we have

$$< t_L(\vec{k}_t), \bar{t}_L(-\vec{k}_t) | \hat{\rho} | t_L(\vec{k}_t), \bar{t}_L(-\vec{k}_t) > = < t_R(\vec{k}_t), \bar{t}_R(-\vec{k}_t) | \hat{\rho} | t_R(\vec{k}_t), \bar{t}_R(-\vec{k}_t) >$$  \hfill (3.1)

in the centre of mass of the $t\bar{t}$ system, where $\hat{\rho}$ is the density-state operator defined as

$$\hat{\rho} = \sum_{\lambda_+, \lambda_-} T|W_{\lambda_+}^+(\vec{k}_+)W_{\lambda_-}^-(-\vec{k}_+) > < W_{\lambda_+}^+(\vec{k}_+), W_{\lambda_-}^-(-\vec{k}_+)|T^\dagger.$$  \hfill (3.2)
In eq. (3.2) \( \lambda \pm \) denote the polarizations of the \( W^{\pm} \) bosons and \( \mathcal{T} \) is the transition operator. In order to be condition (3.1) valid it is sufficient to have \( \mathcal{T} = \mathcal{T}^{CP} \), then \( \hat{\rho} = \hat{\rho}^{CP} \). As a consequence, a non-zero value of the asymmetry

\[ \hat{a}_{CP} = \frac{\hat{\sigma}(W^+W^- \to t_L\bar{t}_L) - \hat{\sigma}(W^+W^- \to t_R\bar{t}_R)}{\hat{\sigma}_{tot}} \]  

(3.3)

would signal \( CP \) violation. We have in mind the production of left- and right-handed fermions in a high-energy linear \( e^+e^- \) collider of 1–2 TeV centre of mass energy [18], where the polarization of the top quarks produced lies mainly along or opposite to their momenta. The above proposal of searching for \( CP \) signals has been extensively discussed in [5,6]. In fact, it has been argued that the final state \( t_L\bar{t}_L \) can be distinguished experimentally from \( t_R\bar{t}_R \) by looking at the energy distribution of the charged leptons (i.e. \( e, \mu \)) that result from the subsequent decays of the top quarks.

It is worth observing that only scalars and pseudoscalar currents lead to the production of left- and right-handed \( tt \) pairs. Indeed the currents \( J^\lambda_\chi^{H_i} (\lambda\lambda' = LL, RR) \) due to (2.24) read

\[ J^{LL}_H = Y_i^* \bar{u}_L(1 + \gamma_5)v_L, \]  

(3.4)

\[ J^{RR}_H = Y_i \bar{u}_R(1 - \gamma_5)v_R. \]  

(3.5)

The above two currents (i.e. eqs (3.3) and (3.4)) imply a difference in the amplitudes \( \mathcal{A}(W^+W^- \to t_\lambda\bar{t}_\lambda') \) for \( \lambda\lambda' = LL, RR \). This is, however, not enough to survive this difference in the squared matrix element. As is well known, an additional dynamical phase (beside the complex couplings \( Y_i \)) is required. Usually, such phases arise from the calculation of absorptive parts of higher order diagrams [11]. Here, we do that in an effective way by introducing complex \( BW \) propagators for the Higgs bosons similar to [12]. We, of course, assume that at least one of the Higgs masses is bigger than \( 2m_t \). In
general, it is known [13,14] that one must consider corrections due to final (initial) state interactions when discussing genuine \( CP \)-odd effects. In other words, scatterings going to intermediate states that are equal to final (initial) states will not contribute to the \( CP \)-violating parameter \( \hat{a}_{CP} \). These additional constraints, the so-called \( CPT \) constraints, will be taken into account in the next section. Here we simply quote the result for the squared matrix element using the effective approach of \( BW \) propagators. Defining

\[
\tilde{Y}_i = Y_i \, g_{HWW} \, , \quad \alpha_W = \frac{g_W^2}{4\pi} ,
\]

we obtain, for example, for the dominant subprocess \( W^+_L W^-_L \rightarrow t_L \bar{t}_L, \, t_R \bar{t}_R \) via Higgs exchanges

\[
|A_{LL,RR}|^2 = 3\alpha_W^2 \frac{m_t^2}{M_W^2} \frac{s_h^2 (s_h - 2m_t^2)}{m_W^2} \left[ \sum_{i=1}^{n_H} \frac{|\tilde{Y}_i|^2}{(s_h - M_i^2)^2 + M_i^2 \Gamma_i^2} \right] + 2 \sum_{i<j}^{n_H} \text{Re}(\tilde{Y}_i^* \tilde{Y}_j) \frac{(s_h - M_i^2)(s_h - M_j^2)}{[(s_h - M_i^2)^2 + M_i^2 \Gamma_i^2] [(s_h - M_j^2)^2 + M_j^2 \Gamma_j^2]} + 2 \sum_{i<j}^{n_H} \text{Im}(\tilde{Y}_i^* \tilde{Y}_j) \frac{(s_h - M_i^2)M_j \Gamma_j - (s_h - M_j^2)M_i \Gamma_i}{[(s_h - M_i^2)^2 + M_i^2 \Gamma_i^2] [(s_h - M_j^2)^2 + M_j^2 \Gamma_j^2]} \right].
\]

In eq. (3.7) \( n_H \) is the number of neutral Higgses (the result is then valid for multi-Higgs models) and \( s_h = k_h^2 = (k_t + k_t)^2 \). We are now in the position to define the physical \( CP \) asymmetry parameter \( A_{CP} \) as follows:

\[
A_{CP} = \frac{\Delta \sigma_{CP}(e^+e^- \rightarrow \nu_e \bar{\nu}_e t\bar{t})}{\sigma_{tot}(e^+e^- \rightarrow \nu_e \bar{\nu}_e t\bar{t})} ,
\]

\[
\Delta \sigma_{CP} = \sigma(e^+e^- \rightarrow \nu_e \bar{\nu}_e t_L \bar{t}_L) - \sigma(e^+e^- \rightarrow \nu_e \bar{\nu}_e t_R \bar{t}_R) .
\]

The result (3.7) merely shows that \( CP \) violation originates from the subprocess \( W^+ W^- \rightarrow H^0_i \rightarrow t\bar{t} \). After considering the \( CPT \) constraints mentioned above, we can exactly calculate \( \Delta \sigma_{CP} \) with the help of the Feynman graphs shown in fig. 1. This can be done quite easily if the full matrix element squared has the factorization property

\[
| \langle t_\lambda \tilde{t}_\lambda \nu_e \bar{\nu}_e | T | e^+e^- \rangle |^2 = F(s_h, M_{H_i}) \langle \nu_e \bar{\nu}_e H^*_i | T | e^+e^- \rangle |^2 ,
\]

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where $H^*_i$ denotes the $i$th virtual Higgs field with mass $\sqrt{s_h}$. Then, the phase-space integral $R_4$ for this particular $2 \rightarrow 4$ process can be expressed as follows [19]:

$$R_4(s_{tot}) = \frac{\pi^2}{16s_{tot}} \int_{\gamma_3(s_h)} \frac{ds_2dt_1dt_2ds_1}{\Delta_4^{1/2}} \frac{\lambda^{1/2}(s_h,m^2_1,m^2_2)}{2s_h},$$

(3.11)

where $s_i$ and $t_i$ are Mandelstam variables defined by

$$s_1 = (k_\nu + k_h)^2,$$

$$s_2 = (k_\nu + k_h)^2,$$

$$t_1 = (k_\nu - k_e^-)^2,$$

$$t_2 = (k_\nu - k_e^+)^2.$$  (3.12)

The phase-space boundaries $s_i^\pm$ and $t_i^\pm$ are calculated to be

$$s_2^\pm = \frac{1}{2} [(s_{tot} + s_h) \pm (s_{tot} - s_h)],$$

$$t_1^\pm = \frac{1}{2} [s_{tot} - s_2] \mp (s_{tot} - s_2),$$

$$t_2^\pm = \frac{1}{2s_2} [(s_2 - t_1)(s_2 - s_h) \mp (s_2 - t_1)(s_2 - s_h)],$$

$$s_1^\pm = s_{tot} - \frac{1}{(s_2 - t_1)^2} \left( D \mp 2\sqrt{F} \right)$$

(3.13)

with

$$D = (s_2 - t_1)[s_{tot}(s_2 - s_h) - t_2(s_{tot} + s_2)] + 2s_{tot}s_2t_2,$$

$$F = s_{tot} s_2 \prod_{i=1,2} (t_i - t_i^+)(t_i - t_i^-),$$

$$\Delta_4 = \frac{1}{16}(s_2 - t_1)^2(s_1^+ - s_1)(s_1 - s_1^-),$$

$$\lambda(x,y,z) = (x - y - z)^2 - 4yz.$$  (3.14)
4. **CPT constraints**

In the following we will assume for simplicity that there is only one intermediate Higgs state which is heavy and we choose to be $H_h \equiv H^0_3$.

Consider the amplitude $T$ of the diagrams depicted in fig. 1. Since we are interested in CP-odd observables that are sensitive to the Higgs width $\Gamma_h$, we do not calculate the dispersive parts of the one-loop corrections. Thus, we can list the different contributing amplitudes to $\Delta \sigma_{CP}$ as follows:

$$T_A = V_W \frac{g_{H,WW} Y_j^*}{s_h - M^2_j} V_{t_L} \left( \delta_{H_i H_j} - i \text{Im}(\Pi_{H_i H_j}) \frac{1}{s_h - M^2_j} \right), \quad (4.1)$$

$$T_B = V_W \frac{g_{H_h,WW} Y_h^*}{s_h - M^2_h + i M_h \Gamma_h} V_{t_L} \left( 1 - \frac{i \text{Im}(\Pi_{H_h H_h}) - i M_h \Gamma_h}{s_h - M^2_h + i M_h \Gamma_h} \right), \quad (4.2)$$

$$T_C = V_W \frac{g_{H_h,WW} Y_h^*}{s_h - M^2_h + i M_h \Gamma_h} i \text{Im}(\Pi_{H_h H_h}) \frac{Y_i^*}{s_h - M^2_i} V_{t_L}, \quad (4.3)$$

$$T_D = V_W \frac{g_{H_h,WW} Y_h^*}{s_h - M^2_h} i \text{Im}(\Pi_{H_h H_h}) \frac{Y_h^*}{s_h - M^2_h + i M_h \Gamma_h} V_{t_L}, \quad (4.4)$$

In addition, non-resonant contributions from $Z^0$, $\gamma$- and $b$-quark exchange graphs should also be considered, as well as absorptive parts of vertex corrections and box graphs. In eqs (4.1)-(4.4) we sum over those repeated indices $i,j$, which run over the non-resonant Higgs states; $i,j = 1,2$. $V_W$ is the amplitude of $e^+e^- \rightarrow \nu_\ell \bar{\nu}_\ell H^{0*}$ and $V_{t_L}$ the vertex function for $H^{0*} \rightarrow t_L \bar{t}_L$. Then, in the Born approximation we easily find that

$$\sum V_W^{(0)} V_W^{(0)*} = 16 \pi^3 \alpha_W^3 \frac{s_{\text{tot}}}{M_W^2} \frac{(s_{\text{tot}} - s_1 - s_2 + s_h)}{(t_1 - M_W^2)^2(t_2 - M_W^2)^2}, \quad (4.5)$$

$$\sum V_{t_L}^{(0)} V_{t_L}^{(0)*} = \sum V_{t_R}^{(0)} V_{t_R}^{(0)*} = 3 \pi \alpha_W (s_h - 2m_t^2) \frac{m_t^2}{M_W^2}. \quad (4.6)$$

The reason why the Im $\Pi_{hh}(q^2) - M_h \Gamma_h$ appears in the numerator of the second term of eq. (4.2) is that we expand the full propagator in the gauge-invariant complex pole
\(M_h^2 - i M_h \Gamma_h\) as follows [20]

\[
\frac{1}{q^2 - M_h^2 + i \text{Im} \Pi_{hh}(q^2)} = \frac{1}{q^2 - M_h^2 + i M_h \Gamma_h} \left( 1 + \frac{i \text{Im} \Pi_{hh}(q^2) - i M_h \Gamma_h}{q^2 - M_h^2 + i M_h \Gamma_h} \right)^{-1} \]

\[
= \frac{1}{q^2 - M_h^2 + i M_h \Gamma_h} \left( 1 - \frac{i \text{Im} \Pi_{hh}(q^2) - i M_h \Gamma_h}{q^2 - M_h^2 + i M_h \Gamma_h} \right) + \mathcal{O}(g^4_W). \tag{4.7}
\]

This also reflects the on-shell renormalization condition

\[
i \text{Im} \Pi_{hh}^R(q^2) \big|_{q^2 = M_h^2} = i \text{Im} \Pi_{hh}(q^2) - i M_h \Gamma_h \big|_{q^2 = M_h^2} = 0, \tag{4.8}
\]

at one-loop level. We have checked that the matrix element given by eqs (4.1)–(4.4) respects the CPT symmetry in the resonant as well as off-resonant region. Note that the way in which \(i \text{Im} \Pi_{hh}^R(q^2)\) enters eq. (4.7) is crucial to show CPT invariance in the off-resonant region. However, it has been argued by Sirlin and Stuart in [20] that the naive ansatz of BW propagators can induce gauge dependence in the matrix element \(\mathcal{T}\). They further claim that the residue of the transition element \(\mathcal{T}\) with respect to the complex-pole position \(M_h^2 - i M_h \Gamma_h\) and the background terms are separately gauge invariant. In this respect, keeping terms up to \(\mathcal{O}(g^2_W)\) at the resonant point, we can rewrite eqs (4.1)-(4.4) in a more convenient form, i.e.

\[
\mathcal{T}_A = V^{(0)}_W \frac{g_{H_h WW} Y_i^*}{M_h^2 - M_i^2} V_t^{(0)}, \tag{4.9}
\]

\[
\mathcal{T}_B = V^{(0)}_W \frac{g_{H_h WW} Y_h^*}{s_h - M_h^2 + i M_h \Gamma_h} \left[ V_t^{(0)} V_t^{(0)} + i \text{Im}(V^{(1)}_{W_i}(M_h^2)) V_t^{(0)} + i V^{(0)}_W \text{Im}(V^{(1)}_i(M_h^2)) \right. \\
\left. - i V^{(0)}_W \text{Im}(\Pi_{hh}^R(M_h^2)) V_t^{(0)} \right], \tag{4.10}
\]

\[
\mathcal{T}_C = V^{(0)}_W \frac{g_{H_h WW}}{s_h - M_h^2 + i M_h \Gamma_h} i \text{Im}(\Pi_{H_h H_i}) \frac{Y_i^*}{M_h^2 - M_i^2} V_t^{(0)}, \tag{4.11}
\]

\[
\mathcal{T}_D = V^{(0)}_W \frac{g_{H_h WW}}{M_h^2 - M_i^2} i \text{Im}(\Pi_{H_i H_h}) \frac{Y_h^*}{s_h - M_h^2 + i M_h \Gamma_h} V_t^{(0)}, \tag{4.12}
\]
and the remaining $Z$, $\gamma$- and $b$-quark mediated tree graphs can formally be written as

$$
T_{\text{tree}} = \frac{V^{Z(0)}_W V^{Z(0)}_t}{M^2_h - M^2_Z} + \frac{V^{\gamma(0)}_W V^{\gamma(0)}_t}{M^2_h} + T_b(M^2_h),
$$

(4.13)

where $T_b$ indicates the b-quark exchange graph at tree level. Box graphs and other non-resonant vertex corrections do not contribute at $O(g^2_W)$ accuracy. Thus, the sum of the amplitudes given by eqs (4.9)-(4.13)

$$
T = T_A + T_B + T_C + T_D + T_{\text{tree}},
$$

(4.14)

represents a gauge-invariant expression. However, some terms in $T$ will not contribute to $\Delta \sigma_{CP}$. Since the initial states $W^+W^-$ of the subprocess are unpolarized, it suffices to consider the tree-level value for the vertex $H - W - W$, i.e. $V^{(0)}_W$. We also omit terms proportional to $O((s_h - M^2_h)/s_h)$ in $|T_{CP}|^2$, i.e. $T_D$ and $\text{Im}\Pi_{hh}^I(M^2_h)$ drop out. A further simplification occurs by assuming that $M_i = O(M)$ for $i = 1, 2$, i.e. we neglect the mass difference $M_{H_1} - M_{H_2}$ as compared to the centre of mass energy $\sqrt{s_{\text{tot}}}$. Finally, the non-resonant amplitude $T_{\text{tree}}$ has been left out for simplicity. In general, the $Z^0(\gamma)$ particle will effectively operate only as an extra off-shell particle with mass $M_Z \simeq M$ (or zero) in the context of a multi-Higgs doublet model.

We now proceed with the computation of the relevant absorptive parts of self-energies $\Pi_{H_iH_j}(M^2_h)$ and the vertex function $V^{(1)}_t(M^2_h)$ by adopting the Feynman–’t Hooft gauge. The advantage of this gauge is that one can have a safe estimate of the vertex $V^{(1)}_t$ for quite heavy Higgs masses by taking only into account the unphysical Goldstone bosons (see also fig. 2). We find that such corrections are either suppressed by a factor $m^2_t/M^2_h$ (e.g. fig. 2.a) or show a soft logarithmic dependence, $\ln(M^2_h/M^2)$, when increasing the heavy Higgs mass $M_h$ (e.g. graphs shown in figs 2.b–2.g). In a recent work [21], similar $CP$-odd observables have been calculated in the decay process $H_h \to t\bar{t}$. There, the $CP$ parameter $A_{CP}$ shows a quadratic behavior, i.e. $M^2_h/M^2$, different from that which
we obtain by computing the Feynman graphs in fig. 2. The reason lies in the fact that crucial \( H^*_i \)-mediated self-energy graphs like \( H_h \rightarrow (W^+W^-, Z^0Z^0) \rightarrow H^*_i \rightarrow t\bar{t} \) have been omitted in [21], with \( (W^+W^-, Z^0Z^0) \) denoting on-shell intermediate states. These graphs, however, will guarantee gauge invariance for the above process. Since we want to study the role of enhanced Higgs interactions in \( A_{CP} \), i.e. keeping terms of \( \mathcal{O}(g_W^2 M_h^2 / M_W^2) \), such self-energies have been consistently included in our theoretical considerations by going to the Feynman-'t Hooft gauge. We think that the resonant part of the Higgs-mediated amplitude will give the dominant contribution to the \( CP \) asymmetry \( A_{CP} \) in our fixed gauge (i.e. \( \xi_{W,Z} = 1 \)), in spite of the omission of the vertex corrections. In fact, \( A_{CP} \) will behave asymptotically according to the form

\[
|A_{CP}| \sim \frac{\bar{\Gamma}_h}{M_h}, \quad \bar{\Gamma}_h = \Gamma_h - \Gamma(H_h \rightarrow WW) - \Gamma(H_h \rightarrow ZZ), \tag{4.15}
\]

which increases quadratically with the heavy Higgs mass (i.e. \( M_h^2 / M_W^2 \)). Therefore, it is also important to notice that in a multi-Higgs scenario the total decay width of the heavy Higgs \( \Gamma_h \) can generally approximate the reduced width \( \bar{\Gamma}_h \) entering eq. (4.15). This observation supports the above approach of considering an amplitude in the Feynman-'t Hooft gauge, which is dominated by Higgs interactions. We finally remark that such a specific choice of gauge corresponds to the equivalence principle between longitudinal vector bosons and Goldstone fields by taking the limit \( g_W \rightarrow 0 \) with \( v \) fixed, i.e. \( g_W / M_W = 2 / v \).

We can now make use of the optical theorem and write down the absorptive parts of the self-energies in the following way:

\[
\text{Im } \Pi_{i_1i_2}(i_1 \rightarrow ab \rightarrow i_2)|_{q^2=s_h} = \frac{g_{i_1ab} g_{i_2ab}}{g_{H_h,ab}} M_h \Gamma_{h \rightarrow ab}(s_h), \tag{4.16}
\]

where \( g \)'s are Higgs coupling constants. Then, \( \Delta \sigma_{CP} \) can be cast into the form:

\[
\Delta \sigma_{CP} \simeq F_{CP}(M_h^2) \int dPS \frac{(\sum V_W V_W^\dagger)(\sum V_t V_t^\dagger)}{(s_h - M_h^2)^2 + M_h^2 \Gamma_h^2} \frac{4 M_h \Gamma_h}{M_h^2 - M^2} \tag{4.17}
\]
with

\[
F_{CP}(M_h^2) = \sum_{i \neq h} \text{Im}(Y_i^* Y_h) \sum_{ab} \frac{g_{H_h}^{WW}}{g_{H_h}^{ab}} \text{Br}(H_h \to ab) \det \left( \begin{array}{cc} g_{H_h}^{WW} & g_{H_h}^{ab} \\ g_{H_h}^{ab} & g_{H_h}^{WW} \end{array} \right). \tag{4.18}
\]

The above expression of \( F_{CP} \) is convenient to check CPT invariance. Although the quantity \( \text{Im}(Y_i Y_h^*) \) is not alone a genuine CP indicator (CP violation also manifests itself in the constants \( g_{H_h}^{ab} \)) one can, however, check that \( F_{CP} \) vanishes in the limit of CP invariance, i.e. \( \text{Im} \lambda_5 (v_1^* v_2) \to 0 \). Using eq. (3.11) and inserting (4.5) and (4.6) into (4.17) we arrive at the final expression for \( \Delta \sigma_{CP} \), i.e.

\[
\Delta \sigma_{CP} \approx \frac{\alpha_W^4}{256 \pi^2} \frac{m_t^2 M_t \Gamma_t}{s_{tot}} \int_{4m_t^2}^{s_{tot}} ds_h \frac{\lambda^{1/2}(s_h, m_t^2, m_t^2)}{s_h} \int_{\Gamma_s(s_h)} \frac{ds_2 dt_1 dt_2 ds_1}{\Delta_s^{1/2}} \left[ \frac{M_h^2 - 2m_t^2}{[(s_h - M_h^2)^2 + M_h^2 \Gamma_h^2][M_h^2 - M^2]} \right] \frac{s_{tot} - s_1 - s_2 + M_h^2}{(t_1 - M_W^2)^2(t_2 - M_W^2)^2}. \tag{4.19}
\]

Some comments concerning the function \( F_{CP} \) are now in order. In general one has

\[
F_{CP} = \sum_i F_{CP}^i, \quad \text{with} \quad i \in \{ t \bar{t}, WW, ZZ, H_iZ, H_iH_j, \ldots \}, \tag{4.20}
\]

where \( i \) stands for all intermediate states (including also Goldstone bosons and ghost fields) that can come on shell. From eq. (4.9) one easily derives that

\[
F_{CP}^{WW} = F_{CP}^{ZZ} = 0, \tag{4.21}
\]

which is a consequence of CP invariance. In eq. (4.18) it is also important to notice that intermediate states with weak couplings proportional to the initial couplings will not give any contribution to \( F_{CP} \), i.e. \( F_{CP}^{G^+ G^-} = 0 \), \( F_{CP}^{ZZ} = 0 \) etc. As far as the final state interactions is concerned, we observe that in the self-energy function \( \text{Im} \Pi_{i_1 i_2} (i_1 \to t \bar{t} \to i_2) \) there is no interference between scalar \((t \bar{t})_S\) and pseudoscalar \((t \bar{t})_P\) parts of the Yukawa Lagrangian (2.26). Recall that exactly this interference led to CP violation in our process. Therefore we can write

\[
F_{CP}^{t \bar{t}} = F_{CP}^{(t \bar{t})_S} + F_{CP}^{(t \bar{t})_P}. \tag{4.22}
\]
with

\[ F_{CP}^{(\bar{t}t)_R} = - \frac{3\alpha_W}{8\Gamma_h} \frac{m_t^2}{M_W^2} M_h \left( 1 - \frac{4m_t^2}{M_W^2} \right)^{3/2} \frac{\cot^2 \beta}{\sin^2 \beta} \cdot d_{23}d_{13}d_{33}(\cos \beta d_{13} + \sin \beta d_{23}), \]

\[ F_{CP}^{(\bar{t}t)_R} = \frac{3\alpha_W}{8\Gamma_h} \frac{m_t^2}{M_W^2} M_h \left( 1 - \frac{4m_t^2}{M_W^2} \right)^{1/2} d_{33} \cot^3 \beta \cdot \left( \cos \beta d_{13} + \sin \beta d_{23} \right) \left( 1 - d_{13}^2 + \cot \beta d_{13}d_{23} \right). \]

(4.23) (4.24)

For completeness we also give the $F_{CP}$ term for the decay channel $H_h \rightarrow H_i Z^0$ (including $G^0$)

\[ F_{CP}^{H_i Z^0} = \frac{\alpha_W}{16\Gamma_h} \frac{M_h^2}{M_W^2} M_h \lambda^{1/2} \left( 1 - \frac{M_Z^2}{M_h^2} \right)^2 \frac{M_Z^2}{M_h^2} \left( 1 - \frac{M_Z^2}{M_h^2} \right)^2 + \frac{M_Z^2}{M_h^2} \left( \frac{M_Z^2}{M_h^2} - 2 \right) \right] d_{33} \cot \beta \left( -s_{\beta} d_{1j} + c_{\beta} d_{2j} \right)^2 (c_{\beta} d_{13} + s_{\beta} d_{23}) \cdot \left( 1 - d_{13}^2 + \cot \beta d_{13}d_{23} \right). \]

(4.25)

Note that other decay channels, like $H_h \rightarrow H_i H_j$, $H_h \rightarrow H^+ H^-$, are, in principle, also possible. To avoid excessive complication, decay modes involving trilinear Higgs couplings have been not considered in our analysis. These decay modes, which depend strongly on the Higgs potential of the model under discussion, are also gauge invariant by themselves. In any case, due to the large number of free parameters existing in such multi-Higgs scenarios it is unlikely to expect that accidental cancellations or some kind of fine-tuning effect will occur between these additional decay channels and those which we have already computed in $F_{CP}$ such that $A_{CP}$ vanishes.

We close this section by making some important remarks. As we have seen, initial state interactions play a crucial role here. In general, it is important to know the production mechanism of the unstable particles when studying CP violation in their decays. In our case we have to calculate the whole scattering process mediated by the $H_h$ particle.
and the other light Higgses, and not only the decay of the $H_h$ particle, since $WW$ and $ZZ$ intermediate states will not contribute to $A_{CP}$ when Higgses are produced by $WW$ or $ZZ$ fusion in $e^+e^-$ collisions. Finally, the $CP$-violating scattering processes discussed above are not $s$-channel suppressed at very high energies and are therefore worth investigation.

5. Numerical estimates and discussion

It is not easy to give an accurate numerical value for the $CP$ asymmetry defining in eqs (3.8) and (3.9), since too many parameters entering $F_{CP}$ in (4.18) are unknown. However, in table 1 we have presented some typical numerical results. For definiteness, we have varied $M_h$ between 400 – 1000 GeV for different top masses, i.e. $m_t = 100, 120, 140$ GeV. In addition, we fix the light Higgs masses to be $M = 100$ GeV. Thus, for example, for $\sqrt{s_{tot}} = 2$ TeV we find numerically that

$$\frac{\Delta \sigma_{CP}}{F_{CP}} \sim (0.1 - 0.8) \times 10^{-2} \text{ pb}$$

whithin the range of parameters given above. To estimate $A_{CP}$ it seems reasonable to compute $\sigma_{tot}$, using Standard-Model values for the Higgs couplings. Doing so, we can have a first estimate for $\sigma_{tot}$ [19] and the magnitude of $A_{CP}$

$$\sigma_{tot} \sim \mathcal{O}(10^{-3} - 10^{-2}) \text{ pb}, \quad A_{CP} \sim \mathcal{O}(0.1 - 0.8) F_{CP}.$$  \hspace{1cm} (5.2)

From eq. (5.2) we also see that the value of $A_{CP}$ depends crucially on $F_{CP}$. Nevertheless, from eq. (4.20), the natural range of values for $F_{CP}$ can be constrained by the inequality

$$F_{CP} \leq \frac{\bar{\Gamma}_h}{\Gamma_h}$$

(5.3)

In our $CP$-violating two-Higgs doublet scenario, this would correspond to $F_{CP} \leq 1/2$. We must also note that the $CP$-odd observable constructed in eqs (3.8) and (3.9) is only
sensitive to absorptive parts at the one-loop level of the reaction. In other words, typical $CP$- and $CPT$-odd observables like $<\bar{s}_i\bar{k}_t>$, $<\bar{s}_i\bar{k}_\bar{t}>$ have to combine with the $CPT$-odd absorptive graphs in order to give a $CPT$-invariant contribution to the squared matrix element. $CP$-odd observables that appear at tree level (e.g. triple products of the form $<\vec{k}_t\cdot\vec{k}_t\times\vec{k}_\bar{t}>$) and are even under $CPT$, can be discussed in a similar way as given in [22].

Of course, analogous calculations can be performed for the process $e^+e^-\rightarrow Z^*Z^*\rightarrow e^+e^-t\bar{t}$ (see also table 2). This together with the process discussed in this paper could increase the statistics by considering the inclusive reaction $e^+e^-\rightarrow t\bar{t}X$, with $X=\nu\bar{\nu},e^+e^-$. We note that at the high energy $e^+e^-$ linear collider one has to worry about $t\bar{t}$ production through beamstrahlung photons [17]. This would increase the $CP$-even background and therefore a machine design with low beamstrahlung luminosity will be preferable [17]. Furthermore, the most optimistic value for $A_{CP}$, estimated by the $s$-channel suppressed process $e^+e^-\rightarrow Z^*\rightarrow t\bar{t}$, turns out to be quite small, i.e. $A_{CP}\simeq 10^{-3}$ [23], as compared to $A_{CP}\simeq 40\%$ in our case. On the other hand, the top-pair production cross section through $e^+e^-$ annihilation is of the order of 40 fb at $\sqrt{s_{tot}}=2$ TeV and is hence competitive with numerical estimates of cross sections for the vector-boson fusion processes as presented in table 1 and 2.

In principle, Higgs-mediated $CP$-odd processes could also play a crucial role in $pp$ colliders. However, the overwhelming top-production rate via gluon fusion will probably make such $CP$-odd tests more difficult.

In conclusion we have discussed in detail how the $CP$ violation of the Higgs poten-
tial with two Higgs doublets manifests itself in the scattering process $e^+ e^- \rightarrow \nu_e \bar{\nu}_e t \bar{t}$. Our analysis will be also valid for multi-Higgs doublet models. We have calculated the $CP$-odd resonant amplitudes of this reaction, paying special attention to constraints resulting from $CPT$ invariance. Using an approach which is $CPT$ and gauge invariant, we have demonstrated that $CP$ asymmetries induced by Higgs-width effects in $t \bar{t}$ production can be quite sizeable, $10\% - 40\%$, and could be probed at a high energy $e^+ e^-$ collider.

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Figure and Table Captions

Fig. 1: Feynman graphs of the resonant amplitudes contributing to $\Delta\sigma_{CP}(e^+e^- \rightarrow \nu_e\bar{\nu}_e t\bar{t})$.

Fig. 2: Typical diagrams, which give the leading contribution in the Feynman–t Hooft gauge to the one-loop coupling $H - t - \bar{t}$ in the limit of $M_h^2 \gg 4m_t^2$, according to the equivalence principle (see also text).

Tab. 1: Numerical results of the cross section $\sigma_{tot}^H$ and the $CP$-asymmetry parameter $A_{CP}$ for the process $e^+e^- \rightarrow H_i^* \rightarrow \nu_e\bar{\nu}_e t\bar{t}$ at $\sqrt{s_{tot}} = 2$ TeV. We fix $M = 100$ GeV. For $\sigma_{tot}^H$, we have assumed the Standard-Model values of the couplings, i.e. $g_{HWW} = g_{Htt} = 1$ in our notation. The three columns indicated by I, II, III stand for values of $m_t = 100, 120, 140$ GeV, respectively.

Tab. 2: Numerical results of the cross section $\sigma_{tot}^H$ and the $CP$-asymmetry parameter $A_{CP}$ for the process $e^+e^- \rightarrow H_i^* \rightarrow e^+e^- t\bar{t}$ at $\sqrt{s_{tot}} = 2$ TeV, using the same values for the set of parameters given in table 1.
Table 1

| $M_h$ [GeV] | $\sigma_{tot}^H$ [pb] | $A_{CP}/F_{CP}$ |
|-------------|-----------------------|-----------------|
|             | I         | II       | III      | I    | II   | III    |
| 400         | 7.2 $10^{-2}$ | 8.9 $10^{-2}$ | 1.0 $10^{-2}$ | 0.09 | 0.09 | 0.08  |
| 500         | 3.7 $10^{-2}$ | 4.9 $10^{-2}$ | 6.1 $10^{-2}$ | 0.15 | 0.14 | 0.13  |
| 600         | 1.9 $10^{-2}$ | 2.7 $10^{-2}$ | 3.4 $10^{-2}$ | 0.22 | 0.21 | 0.20  |
| 700         | 1.1 $10^{-2}$ | 1.5 $10^{-2}$ | 1.9 $10^{-2}$ | 0.29 | 0.28 | 0.26  |
| 800         | 5.6 $10^{-3}$ | 7.8 $10^{-3}$ | 1.0 $10^{-2}$ | 0.43 | 0.41 | 0.39  |
| 900         | 3.0 $10^{-3}$ | 4.2 $10^{-3}$ | 5.5 $10^{-3}$ | 0.59 | 0.57 | 0.55  |
| 1000        | 1.6 $10^{-3}$ | 2.2 $10^{-3}$ | 2.9 $10^{-3}$ | 0.82 | 0.80 | 0.78  |
Table 2

| $M_h$ [GeV] | $\sigma^H_{tot}$ [pb] | $A_{CP}/F_{CP}$ |
|-------------|------------------------|-----------------|
|             | I     | II    | III   | I     | II    | III   |
| 400         | $4.5 \times 10^{-3}$  | $5.6 \times 10^{-3}$ | $6.3 \times 10^{-3}$ | 0.09  | 0.09  | 0.08  |
| 500         | $2.4 \times 10^{-3}$  | $3.1 \times 10^{-3}$ | $3.8 \times 10^{-3}$ | 0.14  | 0.13  | 0.12  |
| 600         | $1.2 \times 10^{-3}$  | $1.7 \times 10^{-3}$ | $2.1 \times 10^{-3}$ | 0.21  | 0.20  | 0.19  |
| 700         | $6.6 \times 10^{-4}$  | $9.2 \times 10^{-4}$ | $1.2 \times 10^{-3}$ | 0.30  | 0.29  | 0.27  |
| 800         | $3.6 \times 10^{-4}$  | $4.9 \times 10^{-4}$ | $6.4 \times 10^{-4}$ | 0.41  | 0.39  | 0.37  |
| 900         | $1.9 \times 10^{-4}$  | $2.7 \times 10^{-4}$ | $3.5 \times 10^{-4}$ | 0.59  | 0.57  | 0.55  |
| 1000        | $1.0 \times 10^{-4}$  | $1.4 \times 10^{-4}$ | $1.9 \times 10^{-4}$ | 0.80  | 0.78  | 0.75  |