Suppression of Lepton Flavor Violation from Corrections above $M_{\text{GUT}}$

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Abstract. We show how quantum corrections above the GUT scale can result in supersymmetric models where lepton flavour violation is naturally suppressed, in agreement with all experimental bounds. Within this framework, a soft term structure as predicted by an Abelian flavour symmetry combined with $SU(5)$ RGEs for scales above $M_{\text{GUT}}$, results to an efficient suppression of the off-diagonal terms in the scalar soft matrices, particularly for $m_0 < 100$ GeV.

1. Introduction

The purpose of this presentation is to analyse the predictions for lepton flavor violation (LFV) in models where SU(5) unification is combined with flavour symmetries, taking into account the RGE evolution above the GUT scale from Ref. [1] and focusing on the WMAP preferred area presented in [2]. We show that, by postulating the mass matrices at a high scale $M_X$ and evolving them down to $M_{\text{GUT}}$, the pathological situation encountered in the conventional models [3, 4] may be remedied. For regions of the parameter space with a low $m_0$, the pattern we end up with exhibits a sizable suppression in the off-diagonal terms as compared to the textures at $M_X$, yielding acceptable LFV predictions. We show that this is true, even in the case of maximal mixing in the charged lepton sector, which is the most dangerous one as far as LFV is concerned.

2. LFV in SUSY-SU(5) with see-saw neutrinos

In supersymmetric theories, charged lepton flavour violation may be generated at the loop-level, even in models with universal soft terms at a high scale. This is also true for the MSSM, when extended with a see-saw mechanism to generate small neutrino masses [5]. In this case, LFV terms are generated radiatively, since it is not possible to simultaneously diagonalise neutrino, charged lepton and slepton mass matrices [6].

In this section we consider a SUSY-SU(5) theory with see-saw neutrinos. Our starting point is the following SUSY $SU(5)$ superpotential:

$$W_X = T^T \mathcal{Y}^u u^T H + T^T \mathcal{Y}^d d^T \bar{H} + F^T \mathcal{Y}^\nu \nu^T S H + S^T \bar{M}_R S,$$

where $\mathcal{Y}_\alpha$ ($\alpha = u, d, \nu$) are the Yukawa matrices for the up-type quarks, down-quarks/charged-leptons and Dirac neutrinos, respectively. $M_R$ is the heavy Majorana mass matrix. The symbol
\( \delta \) stands for diagonal, the original fields rotated as
\[
T = U_{10} T_1, \quad F = U_{\nu L} F_1, \quad S = U_{\nu R} S_1,
\]
with the rotating matrices defined as
\[
\mathcal{Y}_u = U_{10} \mathcal{Y}_u^\delta U_{10}^T, \quad \mathcal{Y}_d = U_{5L} \mathcal{Y}_d^\delta U_{5R}^T, \quad \mathcal{Y}_\nu = U_{\nu L} \mathcal{Y}_\nu^\delta U_{\nu R}^T,
\]
and
\[
\mathcal{F}_d = V_{CKM}^\delta \mathcal{Y}_d^\delta V_E^\dagger.
\]
Here, \( V_{CKM} = U_{10}^\dagger U_{5L} \) and \( V_E = U_{\nu L}^\dagger U_{5R} \) denote the mixings in the quark and lepton sectors, while \( M_R = U_{\nu R}^\dagger M_\nu U_{\nu R} \).

The off-diagonal contributions to slepton mass matrices, when the superfields are rotated so that charged leptons become diagonal, can be understood through three rotations at different energy scales:

- **\( M_X \).** The rotations in the superpotential fields lead to the following transformation of the soft terms:
  \[
m^2_{\tilde{m}_{10}} = U_{10}^\dagger m^2_{\tilde{m}_{10}} U_{10}, \quad \tilde{m}^2_5 = U_{\nu L}^\dagger \tilde{m}^2_5 U_{\nu L}
\]

- **\( M_{GUT} \).** Assuming that \( SU(5) \) is broken down to the MSSM gauge group, the superpotential becomes
  \[
  \mathcal{W}_{MSSM} = Q^T \mathcal{Y}_u^\delta U H_2 + Q^T (V_{CKM}^\delta \mathcal{Y}_d^\delta) D H_2 + L^T (V_E^\dagger \mathcal{Y}_d^\delta) E H_2 + L^T \mathcal{Y}_\nu^\delta S H_2 + S^T M_R S
  \]
  where we have absorbed the matrices \( V_E^\dagger \) and \( V_{CKM}^\delta \) in the definitions of the superfields \( E \) and \( D \) respectively. The scalar soft masses then become:
  \[
m^2_E = V_{10}^\dagger m^2_{\tilde{m}_{10}} V_{10}, \quad m^2_L = \tilde{m}^2_5,
  \]
  \[
m^2_Q = m^2_\tilde{m}_5 = \tilde{m}^2_5, \quad m^2_D = V_E^\dagger \tilde{m}^2_5 V_E.
  \]

- **\( M_N \).** \( M_N \) is the scale at which the heavy right handed neutrinos decouple. Below this scale, the particle content is the one of the MSSM complemented with the neutrino mass operator resulting from the see-saw mechanism. Consequently, the superpotential can be written in a basis where the charged-lepton mass matrix becomes diagonal and the left slepton mass matrix becomes
  \[
m^2_L = V_E^\dagger m^2_L V_E \simeq U_{5R}^\dagger \tilde{m}^2_5 U_{5R}
  \]

3. **Runs above \( M_{GUT} \): Umbrella Effect.**

The introduction of a non-trivial flavour structure for the slepton soft terms at \( M_{GUT} \), as predicted by the family symmetry that also generates Yukawa couplings, typically results to a large violation of the bounds on \( l_j \rightarrow l_i \gamma \) [3, 4, 7]. This picture may be remedied by taking into account RGE effects from a scale \( M_X > M_{GUT} \). In this case, the cosmological requirement of having a neutral particle as the LSP imposes low values on \( m_0 \), such that \( m_{\tilde{\tau}} > m_X \) [2, 8, 9] (diagonal terms in the soft mass matrices have a large RGE growth, while non-diagonal elements remain almost unaffected by the runs). Thus, even assuming non-diagonal soft terms with matrix elements of the same order of magnitude at \( M_X \), the corresponding matrix at \( M_{GUT} \) exhibits dominant diagonal elements.

To some extent, the RGE effect is similar to the action of closing an umbrella: the general non-universal soft terms at \( M_X \) resemble an open umbrella that approaches a diagonal matrix at the GUT scale. The sleptons tend then to be aligned by the flavour blind gauge interaction,
Figure 1. Prediction for the charged-lepton flavour violating branching ratios showing the difference of taking either $M_X$ or $M_{GUT}$ as the starting point of the runs.

Driven by $\alpha_{GUT}$ above the GUT scale. An analogous effect is well known to take place in the squark sector [10] where the strong radiative corrections to the squark masses, proportional to the gluino mass, are family independent and counteract the splitting effect on the top Yukawa coupling (the effect is commonly referred to as the “gluino focusing effect”).

In Fig. 1, we show the differences between the following: i) SU(5) RGE evolution of the soft terms from a high scale $M_X$ down to $M_{GUT}$ and then to the MSSM with see-saw neutrinos (solid lines), and ii) Soft SUSY breaking terms given at $M_{GUT}$ and then the MSSM with see-saw neutrinos (dash-lines). In case ii) we stop the lines at the value of $m_0$ below which $m_{\tilde{\tau}}$ becomes the LSP. In contrast, $m_0$ can even vanish at $M_X$ in case i). The textures and soft terms we use are similar to Ref. [4]; in our work, however, we decouple the right-handed neutrinos below $M_{GUT}$. As a result, the predicted BR’s do not vanish in the limit $m_0 = 0$.

4. SU(5) textures

Let us consider an specific example of SU(5) Yukawa textures that match the fermion data and may also predict the pattern of soft terms to be expected.

Following the $U(1)_F$ charge assignment in [11], the Yukawa matrices have the form

$$
\mathcal{Y}_u \propto \begin{pmatrix}
\varepsilon^6 & \varepsilon^5 & \varepsilon^3 \\
\varepsilon^5 & \varepsilon^4 & \varepsilon^2 \\
\varepsilon^3 & \varepsilon^2 & 1
\end{pmatrix}, \quad \mathcal{Y}_e^T \propto \mathcal{Y}_d \propto \begin{pmatrix}
\varepsilon^4 & \varepsilon^3 & \varepsilon^2 \\
\varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\
\varepsilon & 1 & 1
\end{pmatrix}, \quad \mathcal{Y}_\nu \propto \begin{pmatrix}
\varepsilon^{[1+ n_1]} & \varepsilon^{[1+ n_2]} & \varepsilon^{[1+ n_3]} \\
\varepsilon^{[n_1]} & \varepsilon^{[n_2]} & \varepsilon^{[n_3]} \\
\varepsilon^{[n_1]} & \varepsilon^{[n_2]} & \varepsilon^{[n_3]}
\end{pmatrix}
$$

(10)

where $n_i$ stand for the heavy Majorana neutrino charges. We will assume that the entire lepton mixing is arising from the charged-lepton sector, which is potentially the most dangerous case as far as LFV is concerned.

In addition, flavour symmetries generally imply non-universal soft terms [12], since the structure of the soft terms is linked to the family charges. For the Yukawa textures in Eq.(10)
the soft mass matrices $m_{10}^2$ and $m_{5}^2$ become

$$m_{10}^2 \propto \begin{pmatrix} 1 & \varepsilon & \varepsilon^3 \\ \varepsilon & 1 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix} m_0^2, \quad m_{5}^2 \propto \begin{pmatrix} 1 & \varepsilon \\ \varepsilon & 1 \\ \varepsilon & 1 \end{pmatrix} m_0^2,$$

(11)

Figure 2. Prediction for the $\tau \to \mu \gamma$ (top), $\tau \to e \gamma$ (middle) and $\mu \to e \gamma$ branching ratios for the cosmologically preferred area of values of $M_{1/2}$, $A_0$, for three different values of $\tan \beta$.

Some LFV predictions of the above model are displayed in Fig. 3. We should stress that the depicted ranges for both $m_0$ and $M_{1/2}$ are the cosmologically preferred parameter space, as found in [2]. We can see that the case $\tan \beta = 35$ is ruled out, as there is no overlapping region for the three decays. This is because of the RR sector-induced cancellations mentioned above, which arises for much larger values of $m_0$ for $\tau \to e \gamma$ than for the other two processes. The case $\tan \beta$
Figure 3. Allowed parameter space when both LFV and cosmological constraints are taken into account for $\tan \beta = 40$ and 45.

$\tan \beta = 40$ does possess a common area for $20 < m_0 < 50$ GeV. However, such an area lies outside the cosmologically preferred region, as shown on the left panel of Fig. 3. Thus, we conclude that the case $\tan \beta = 40$ is only marginally allowed. Finally, for $\tan \beta = 45$ the whole range for $m_0 < 100$ GeV is allowed, when suitable values for $M_{1/2}$, $A_0$ are chosen. Moreover, as shown on the right panel of Fig. 3, there exists an overlapping region when considering the cosmologically favoured parameter space (values of any parameters involved in these plots ($m_0$, $M_{1/2}$, $A_0$) beyond the ranges shown lead to tachyonic soft masses. Thus, GUT runs efficiently suppress the off-diagonal entries, yielding charged-LFV rates that render the $SU(5)$ model compatible with current experimental bounds.

5. Conclusions
We have shown that $SU(5)$ runs above the GUT scale, naturally suppress the off-diagonal entries of the soft matrices, through what has been called the umbrella effect, leading to a nearly flavour-independent contribution for $m_0 < 100$ GeV. Such low values of $m_0$ are discarded if $SU(5)$ runs are not taken into account, since in this case the lightest stau becomes the LSP. Within this framework, $SU(5)$ runs lead to acceptable LFV rates for the cosmologically preferred parameter space found in [2], favoring values of $\tan \beta$ around 45. Significant deviations from this value, in either way, are harder to reconcile with cosmological data. Our results indicate that even in this case, the umbrella effect leads to suppression to LFV rates, leading to a very significant enhancement of the available parameter space, as compared to the conventional schemes, with runs below $M_{GUT}$.

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