Color-octet mechanism in

$J/\psi$ productions

Pyungwon Ko *

Dep. of Physics, Hong-Ik University
Seoul 121-791, Korea

(March 26, 2022)

Abstract

I review recent developments in theoretical understandings of $J/\psi$ productions in various processes such as $p\bar{p} \to J/\psi + X$, $B \to J/\psi + X$ and $J/\psi$ photoproductions including the color-octet mechanism.

*To be published in the Proceedings of the KPS-JKPS Winter School held at Seoul, Korea during Feb. 21-28, 1996.

†pko@phyb.snu.ac.kr
I. INTRODUCTION

In the conventional approach, the inclusive $J/\psi$ productions in various high energy processes have been studied in the framework of perturbative QCD (PQCD) and the color singlet model \[1\]. In this model, one assumes that the color-singlet $c\bar{c}$ state is produced in the $^3S_1$ state in the short distance scale (i.e. via some hard process which is calculable in PQCD) from the beginning, and this object eventually evolves into the physical $J/\psi$ state in the long distance scale. However, this approach, when applied to the $J/\psi$ or $\psi'$ production at the Tevatron, severely underestimates the productions rate \[2\]. In order to reconcile the data and PQCD predictions, a new mechanism for heavy quarkonium productions has been suggested \[3\], the color-octet gluon fragmentation into $J/\psi$. Also, the color-octet mechanism in heavy quarkonium productions at hadron colliders through the color-octet $(c\bar{c})_8$ pair in various partial wave states $^{2S+1}L_J$ has been considered beyond the color-octet gluon fragmentation approach \[4\], \[5\]. In Refs. \[4\] and \[5\], a large class of color-octet diagrams has been considered which can contribute to the $J/\psi$ production at hadron colliders. Here, the basic picture is the following: at the parton level, one can have color-octet $c\bar{c}$ states with various $^{2S+1}L_J$, $q\bar{q} \rightarrow (c\bar{c})_8(^3S_1^{(8)}), \quad (1.1)$ $gg \rightarrow (c\bar{c})_8(^1S_0^{(8)} \text{ or } ^3P_J^{(8)}), \quad (1.2)$ at the short distance scale, and the subsequent evolution of the $(c\bar{c})_8(^{2S+1}L_J)$ object into a physical $J/\psi$ by absorbing/emitting soft gluons at the long distance scale \[1\]. The short distance process can be calculated using PQCD in powers of $\alpha_s$, whereas the long distance part is treated as a new parameter $\langle 0\vert \mathcal{O}_8^{\psi}(^3S_1^{(8)}) \vert 0 \rangle$ which characterizes the probability that the color-octet $(c\bar{c})_8(^{2S+1}L_J)$ state evolves into a physical $J/\psi$ by emitting/absorbing soft gluons \[2\]. By fitting the $J/\psi$ production at the Tevatron using the usual color-singlet production and the cascades from $\chi_c(1P)$ and the color-octet contribution, the authors of Ref. \[3\] determined $\langle 0\vert \mathcal{O}_8^{\psi}(^3P_0) \vert 0 \rangle = (6.6 \pm 2.1) \times 10^{-3} \text{ GeV}^3, \quad (1.3) \frac{\langle 0\vert \mathcal{O}_8^{\psi}(^1S_0) \vert 0 \rangle}{3} = (2.2 \pm 0.5) \times 10^{-2} \text{ GeV}^3 \quad (1.4)$ for $M_c = 1.48 \text{ GeV}$. Although the numerical values of two matrix elements $\langle 0\vert \mathcal{O}_8^{\psi}(^3P_0) \vert 0 \rangle$ and $\langle 0\vert \mathcal{O}_8^{\psi}(^1S_0) \vert 0 \rangle$ are not separately known in Eq. (1.4), one can still extract some useful information from it. Since both of the color octet matrix elements in Eq. (1.4) are positive definite, one has

---

1 This kind of processes is clearly possible, but has never been addressed before in a quantitative way. The color evaporation model \[6\] is in principle close to this picture, but there are some parameters which are introduced phenomenologically, and thus have no definitions in QCD.

2 See Ref. \[8\] for more details on NRQCD, and definitions of $\mathcal{O}_8^{\psi}$. 

2
\[ 0 < \langle 0 | O_8^\psi (1S_0) | 0 \rangle < (6.6 \pm 1.5) \times 10^{-2} \text{ GeV}^3, \quad (1.5) \]

\[ 0 < \frac{\langle 0 | O_8^\psi (3P_0) | 0 \rangle}{M_c^2} < (2.2 \pm 0.5) \times 10^{-2} \text{ GeV}^3. \quad (1.6) \]

These inequalities can provide us with some predictions on various quantities related with inclusive \( J/\psi \) productions in other high energy processes, which enables us to test the idea of color-octet mechanism.

Since the color-octet mechanism in heavy quarkonium production is a new idea proposed in order to resolve the \( \psi' \) anomaly at the Tevatron, it is important to test this idea in other high energy processes with inclusive heavy quarkonium productions. Up to now, the following processes have been considered: \( J/\psi \) production at the Tevatron and fixed target experiments \cite{4,5,8}, spin alignment of the color-octet produced \( J/\psi \) \cite{9}, the polar angle distribution of the \( J/\psi \) in the \( e^+e^- \) annihilations into \( J/\psi + X \) \cite{10}, inclusive \( J/\psi \) production in \( B \) meson decays \cite{11}, the \( Z^0 \) decays at LEP \cite{12,13}, and \( J/\psi \) photoproductions at the fixed target experiments as well as at HERA \cite{14,15,16}. These processes also depend on the aforementioned three color-octet matrix elements in different combinations from (1.4). Thus, one can check if the color-octet mechanism provides reasonable agreements between PQCD and the experimental data on inclusive \( J/\psi \) production rates from these processes.

In this talk, I review two applications of the idea of the color-octet \( J/\psi \) productions, which I was working on, among many recent applications mentioned above \cite{3}. First in Sec. II, I discuss the \( B \rightarrow J/\psi + X \) using the factorization formula derived in Ref. \cite{11}, and find that the relations (1.5) and (1.6) overestimate the branching ratio for \( B \rightarrow J/\psi + X \), especially for (1.6). Then, we discuss the \( J/\psi \) photoproductions in the in Sec. III. For this process, the singlet contribution from \( \gamma + g \rightarrow J/\psi + g \) (the \( \gamma g \) fusion) has long been known. And we consider the color-octet subprocesses

\[ \gamma + g \rightarrow (c\bar{c})(1S_0^{(8)} \text{ or } 3P_j^{(8)}) \]

which have not been included in previous studies. These color-octet \( 2 \rightarrow 1 \) subprocesses can also contribute to the \( 2 \rightarrow 2 \) subprocesses through

\[ \gamma + g \rightarrow (c\bar{c})(1S_0^{(8)} \text{ or } 3P_j^{(8)}) + g, \]

\[ \gamma + q \rightarrow (c\bar{c})(1S_0^{(8)} \text{ or } 3P_j^{(8)}) + q. \]

\[ \gamma + g \rightarrow (c\bar{c})(2S+1L_j^{(8)}) + g. \]

All of these color-octet \( 2 \rightarrow 2 \) subprocesses are calculated in Refs. \cite{14,15}. Numerical analyses relevant to the fixed target experiments and HERA have been performed. We show that the relations (1.5) and (1.6) yields too large a cross section for the \( J/\psi \) photoproduction in the forward direction. They also leads to too rapidly growing \( d\sigma/dz \) distribution for high \( z \) region compared to the experimental observations. All of these seem to indicate that the relations (1.3) and (1.4), especially the latter, are probably overestimated by an

---

\[ ^3 \text{The case of } J/\psi \text{ productions at the Tevatron is covered in detail by Cho and Leibovich in Refs. [4,5].} \]
order of magnitude. This is not surprising at all, since the analyses in Ref. [5] employed
the leading order calculations for the color-singlet parton subprocess for the $J/\psi$ hadropro-
duction. We summarize our review and speculate the origins of these overestimates of $J/\psi$
photoproductions and $B$ meson decays in Sec. [IV].

II. INCLUSIVE $J/\psi$ PRODUCTION IN $B$ DECAYS

The effective Hamiltonian for $b \to c\bar{c}q$ (with $q = d, s$) is written as [17]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cq}^* \left[ \frac{2C_+ - C_-}{3} \bar{c}\gamma_\mu (1 - \gamma_5)c \bar{q}\gamma^\mu (1 - \gamma_5)b 
+ (C_+ + C_-) \bar{c}\gamma_\mu (1 - \gamma_5)T^a c \bar{q}\gamma^\mu (1 - \gamma_5)T^a b \right],$$

(2.1)

where $C_\pm$’s are the Wilson coefficients at the scale $\mu \approx M_b$. We have neglected penguin
operators, since their Wilson coefficients are small and thus they are irrelevant to our case.
To leading order in $\alpha_s(M_b)$ and to all orders in $\alpha_s(M_b) \ln(M_W/M_b)$, the above Wilson
coefficients are

$$C_+(M_b) \approx 0.87, \quad C_-(M_b) \approx 1.34.$$  

(2.2)

According to the factorization theorem for the $S$–wave charmonium productions in $B$
decays, one has [17]

$$\Gamma(b \to J/\psi + X) = \frac{\langle 0|O_1^{J/\psi}(3S_1)|0 \rangle}{3M_c^2} \hat{\Gamma}_1(b \to (c\bar{c})_1(3S_1) + X),$$

(2.3)

in the nonrelativistic limit, where $\hat{\Gamma}_1$ are rates for hard subprocesses of $b$ quark decaying
into a $c\bar{c}$ pair with suitable angular momentum and vanishing relative momentum in the
color-singlet:

$$\hat{\Gamma}_1(b \to (c\bar{c})_1(3S_1) + s, d) = (2C_+ - C_-)^2 \left( 1 + \frac{8M_c^2}{M_b^2} \right) \hat{\Gamma}_0,$$

(4)

$$\hat{\Gamma}_1(b \to (c\bar{c})_1(1S_0) + s, d) = (2C_+ - C_-)^2 \hat{\Gamma}_0,$$

(5)

with

$$\hat{\Gamma}_0 \equiv |V_{cb}|^2 \left( \frac{G_F^2}{144\pi} \right) M_b^3 M_c \left( 1 - \frac{4M_c^2}{M_b^2} \right)^2.$$  

(6)

The operator $O_1^H(2S+1S_J)$ is defined in terms of heavy quark field operators in NRQCD
[4]. Its matrix element $\langle 0|O_1^H(2S+1S_J)|0 \rangle$ contains the nonperturbative effects in the heavy
quarkonium production processes, and is proportional to the probability that a $c\bar{c}$ in a

---

We follow the notations in Ref. [7], and will not give explicit forms for these dimension-six
operators in this paper.
color-singlet $S$–wave state fragments into a color-singlet $S$–wave $c\bar{c}$ bound state such as a physical $J/\psi$, or $\psi'$. It is also related to the matrix element $\langle H | O_1 (2S^1 J) | H \rangle$ and the nonrelativistic quarkonium wavefunction as follows:

$$\langle 0 | O_1^{J/\psi} (3S_1) | 0 \rangle \approx 3 \langle J/\psi | O_1 (3S_1) | J/\psi \rangle \approx \left( \frac{9}{2\pi} \right) | R_\psi (0) |^2 ,$$

in the nonrelativistic limit. Note that dependence on the radial quantum numbers $n$ enters through the nonperturbative parameters, $\langle 0 | O_1^H (3S_1) | 0 \rangle$.

Using the leptonic decay width of $J/\psi$ and $\psi'$, one can determine

$$\langle J/\psi | O_1 (3S_1) | J/\psi \rangle \approx 2.4 \times 10^{-1} \text{ GeV}^3 ,$$

$$\langle \psi' | O_1 (3S_1) | \psi' \rangle \approx 9.7 \times 10^{-2} \text{ GeV}^3 ,$$

in the nonrelativistic limit with $\alpha_s (M_c) = 0.27$. From these expressions with $M_b \approx 5.3$ GeV, one can estimate the branching ratios for $B$ decays into $J/\psi + X$ and $\psi' + X$:

$$B (B \to J/\psi + X) = 0.23\%, \quad (0.80 \pm 0.08)\% ,$$

$$B (B \to \psi' + X) = 0.08\%. \quad (0.34 \pm 0.04 \pm 0.03)\% .$$

The recent data from CLEO [18] are shown in the parentheses, where the cascades from $B \to \chi_{cJ} (1P) + X$ followed by $\chi_{cJ} \to J/\psi + \gamma$ have been subtracted in the data shown. In view of these results, we may conclude there are some important pieces missing in the calculations of decay rates for $B \to (c\bar{c})_1 (3S_1) + X$ using the color-singlet model in the nonrelativistic limit.

In view of this, we first estimate the color-octet contributions to $B \to J/\psi + X$, motivated by the suggestion that the color-octet mechanism might be the solution to the $\psi'$ puzzle at the Tevatron. Although it is of higher order in $v^2$ ($\sim O(v^4)$), it can be important in the case of the inclusive $B$ decays into $J/\psi + X$, since the Wilson coefficient of the color-singlet part is suppressed compared to that of the color-octet part by a factor of $\sim \alpha_s$. (In Eq. (2.1), $(2C_+ - C_-) \approx 0.4$, and $(C_+ + C_-) \approx 2.20$.) In Ref. [11], a new factorization formula is derived for $B \to J/\psi + X$:

$$\Gamma (B \to J/\psi + X) = \left( \frac{\langle 0 | O_1^{J/\psi} (3S_1) | 0 \rangle}{3M_c^2} - \frac{\langle 0 | P_1^{J/\psi} (3S_1) | 0 \rangle}{9M_c^2} \right) \left( 2C_+ - C_- \right) \left( 1 + \frac{8M_c^2}{M_b^2} \right) \hat{\Gamma}_0$$

$$+ \left( \frac{\langle 0 | O_8^{J/\psi} (3S_1) | 0 \rangle}{2M_c^2} \right) \left( C_+ + C_- \right) \left( 1 + \frac{8M_c^2}{M_b^2} \right) \hat{\Gamma}_0$$

The radiative corrections in $\alpha_s$ has not been included here for consistency. To be consistent with the velocity counting rules in the NRQCD in the Coulomb gauge for the heavy quarkonia [11], one has to include the relativistic corrections as well, since $v \sim \alpha_s (M \nu)$ in heavy quarkonium system. If one includes the $O(\alpha_s)$ radiative corrections to $J/\psi \to l^+ l^-$ without relativistic corrections, one gets a larger $\langle 0 | O_1^{J/\psi} (3S_1) | 0 \rangle$ compared to the lowest order result, Eq. (2.8): $\langle 0 | O_1^{J/\psi} (3S_1) | 0 \rangle \approx 4.14 \times 10^{-1} \text{ GeV}^3$. Relativistic corrections gives a further enhancement. See Ref. [11] for further details.
\[
\begin{align*}
&+ \frac{3\langle 0| O_s^{J/\psi(1S_0)}|0 \rangle}{2M_c^2} (C_+ + C_-)^2 \hat{\Gamma}_0 \\
&+ \frac{\langle 0| O_s^{J/\psi(3P_1)}|0 \rangle}{M_c^4} (C_+ + C_-)^2 \left( 1 + \frac{8M_c^2}{M_b^2} \right) \hat{\Gamma}_0,
\end{align*}
\]
with $\hat{\Gamma}_0$ defined in (2.6). Using the relations (1.3) and (1.4), we estimate the above branching ratio to be (for $\alpha_s(M_{\psi}^2) = 0.28$ in Ref. [11])
\[
(0.42\% \times 12.8) < B(B \to J/\psi + X) < (0.42\% \times 13.8)
\]
which is larger than the recent CLEO data [18] by an order of magnitude.

\[
B_{\exp}(B \to J/\psi + X) = (0.80 \pm 0.08)\%.
\]

Here, the factor 0.42\% in Eq. (2.13) comes from the color-singlet and the color-octet $^3S_1$ contributions. Other factor comes from the color-octet $^1S_0$ and $^3P_J$ states, which are very large if one assumes the relations (1.5) and (1.6). The situation is the same for $B \to \psi' + X$. This is problematic, unless this large color-octet contributions are canceled by the color-singlet contributions of higher order in $O(\alpha_s)$ which were not included in Ref. [11]. If there are no such fortuitous cancelations among various color-octet and the color-singlet contributions, this disaster could be attributed to the relation (1.4) being too large compared to the naive velocity scaling rule in NRQCD, as noticed in Ref. [3]. It seems to be crucial to include the higher order corrections of $O(\alpha_s^4)$ for the color-singlet $J/\psi$ productions at the Tevatron, which is still lacking in the literature.

III. $J/\psi$ PHOTOPRODUCTION

The inelastic $J/\psi$–photoproduction has long been studied in the framework of PQCD and the color-singlet model [19] [20]. The lowest order subprocess at the parton level for $\gamma + p \to J/\psi + X$ is the $\gamma$–gluon fusion at the short distance scale
\[
\gamma + g \to (c\bar{c})(^3S_1^{(1)}) + g,
\]
followed by the long distance process
\[
(c\bar{c})(^3S_1^{(1)}) \to J/\psi,
\]
at the order of $O(\alpha\alpha_s^2\psi^3)$ in the nonrelativistic limit. Thus, the production cross section is proportional to the gluon distribution inside the proton. This is why the

\[6\]

Even if we use the new determination (3.38) by Fleming et al. [15], we still get a large branching ratio:
\[
(0.42\% \times 3.45) < B(B \to J/\psi + X) < (0.42\% \times 5.45),
\]
although the discrepancy gets milder than the case (2.13).
$J/\psi-$photoproduction has been advocated as a clean probe for the gluon structure function of a proton in the color-singlet model. Without further details, we show the lowest order color-singlet contribution to $J/\psi$ photoproduction through $\gamma-$gluon fusion in the nonrelativistic limit:

$$
\sum |\mathcal{M}(\gamma g \rightarrow J/\psi g)|^2 = N_1 \frac{\hat{s}^2(\hat{s} - 4M_c^2)^2 + \hat{t}^2(\hat{t} - 4M_c^2)^2 + \hat{u}(\hat{u} - 4M_c^2)^2}{(\hat{s} - 4M_c^2)(\hat{t} - 4M_c^2)(\hat{u} - 4M_c^2)},
$$

(3.3)

where

$$
z = \frac{E_\psi}{E_\gamma}|_{\text{lab}} = \frac{p_N \cdot P}{p_N \cdot k},$$
$$\hat{s} = (k + q_1)^2 = xs,$$
$$\hat{t} = (P - k)^2 = (z - 1)\hat{s}.
$$

(3.4)

The overall normalization $N_1$ is defined as

$$N_1 = \frac{32}{9} \left(4\pi\alpha_s\right)^2 \left(4\pi\alpha\right)^2 c_e^2 M_c^3 G_1(J/\psi).$$

(3.5)

The parameter $G_1(J/\psi)$, which is defined as

$$G_1(J/\psi) = \frac{\langle J/\psi | \mathcal{O}_1(3S_1) | J/\psi \rangle}{M_c^2},$$

(3.6)

in the NRQCD, is proportional to the probability that a color-singlet $c\bar{c}$ pair in the $3S_1^{(1)}$ state to form a physical $J/\psi$ state. It is related with the leptonic decay via

$$\Gamma(J/\psi \rightarrow l^+l^-) = \frac{2}{3} \frac{\pi c_e^2 \alpha^2}{\alpha_s} G_1(J/\psi),$$

(3.7)

to the lowest order in $\alpha_s$. From the measured leptonic decay rate of $J/\psi$, one can extract

$$G_1(J/\psi) \approx 106 \text{ MeV},$$

(3.8)

Including the radiative corrections of $O(\alpha_s)$ with $\alpha_s(M_c) = 0.27$, it is increased to $\approx 184$ MeV. Relativistic corrections tend to increase $G_1(J/\psi)$ further to $\sim 195$ MeV \[11\].

The partonic cross section for $\gamma + a \rightarrow J/\psi + b$ is given by

$$\frac{d\hat{\sigma}}{dt} = \frac{1}{16\pi \hat{s}^2} \sum |\mathcal{M}(\gamma + a \rightarrow J/\psi + b)|^2.$$

(3.9)

The double differential cross section is

$$\frac{d^2\sigma}{dzdP_T^2}(\gamma + p(p_N) \rightarrow J/\psi(P, \epsilon) + X) = xg(x, Q^2) \frac{1}{z(1-z)} \frac{1}{16\pi \hat{s}^2} \sum |\mathcal{M}|^2(\hat{s}, \hat{t}),$$

(3.10)

where

$$x = \frac{\hat{s}}{s} = \frac{1}{zs} \left[ M_\psi^2 + \frac{P_T^2}{1-z} \right].$$

(3.11)
One has the following constraints for $x, z, t$ and $P_T^2$:

$$\frac{M_{\psi}^2}{s} < x < 1, \quad (3.12)$$

$$-(\hat{s} - M_{\psi}^2) \leq \hat{t}(= t) \leq 0, \quad (3.13)$$

$$M_{\psi}^2 \leq \frac{M_{\psi}^2}{z} + \frac{P_T^2}{z(1-z)} \leq s. \quad (3.14)$$

The $z$ and $P_T^2$ distributions can be obtained in the following manner:

$$\frac{d\sigma}{dz} = \int_0^{(1-z)(s-M_{\psi}^2)} \frac{d^2\sigma}{dz dP_T^2} dP_T^2, \quad (3.15)$$

$$\frac{d\sigma}{dP_T^2} = \int_{z_{\text{min}}}^{z_{\text{max}}} \frac{d^2\sigma}{dz dP_T^2} dz, \quad (3.16)$$

$$z_{\text{max}} = \frac{1}{2s} \left( s + M_{\psi}^2 + \sqrt{(s - M_{\psi}^2)^2 - 4sP_T^2} \right), \quad (3.17)$$

$$z_{\text{min}} = \frac{1}{2s} \left( s + M_{\psi}^2 - \sqrt{(s - M_{\psi}^2)^2 - 4sP_T^2} \right). \quad (3.18)$$

There are two kinds of corrections to the lowest order result in the color-singlet model (3.3): the relativistic corrections of $O(v^2)$ and the PQCD radiative corrections of $O(\alpha_s)$ relative to the lowest order result shown in (3.3). We briefly summarize both types of corrections here, since they have to be included in principle for consistency, when one includes the color-octet mechanism in many cases.

The relativistic corrections to the $\gamma$–gluon fusion were studied by Jung et al. [20]. They found that relativistic corrections are important for high $z > 0.9$ at EMC energy ($\sqrt{s_{\gamma p}} \simeq 14.7$ GeV). Since it mainly affects the high $z$ region only, we neglect the relativistic corrections, keeping in mind that it enhances the cross section at large $z > 0.9$.

The radiative corrections to the $J/\psi$ photoproduction is rather important in practice. This calculation has been done recently in Ref. [21], and the scale dependence of the lowest order result ($Q^2$ in the structure function in Eq. (3.10)) becomes considerably reduced. For EMC energy region, the $K$ factor is rather large, $K \sim 2$. For HERA, it depends on the cuts in $z$ and $P_T^2$. We include the radiative corrections in terms of a $K$ factor suitable to the energy range we consider. Another consequence of the radiative corrections to the color-singlet $J/\psi$ photoproduction is that the PQCD becomes out of control for $z > 0.9$ at EMC energy. For HERA, one gets reasonable results in PQCD when one imposes the following cuts in $z$ and $P_T^2$: $z < 0.8$ and $P_T^2 > 1$ GeV$^2$. Thus, it does not make much sense to talk about the $z$ or $p_T$ distributions for such $z$ region in PQCD. One has to introduce cuts in $z$ as well as in $p_T$. Following the Ref. [21], we adopt the following sets of cuts:

$$z < 0.9, \quad \text{for EMC}, \quad (3.19)$$

$$0.2 < z < 0.8, \quad \text{for HERA}. \quad (3.20)$$

At HERA energies, the lower cut in $z(z > 0.2)$ is employed in order to reduce backgrounds from the resolved photon process and the $b$ decays into $J/\psi$. For these cuts, the $K$ factor is approximately $K \simeq 1.8$ both at HERA and the fixed target experiments. We include these radiative corrections to the subprocess (3.1) by setting $K \simeq 1.8$. 

8
Let us consider color-octet contributions to the $2 \rightarrow 1$ subprocesses via
\[ \gamma(k) + g_a(g) \rightarrow (c\bar{c})^{2S+1}L_j^{(8)}(P), \] (3.21)
followed by $(c\bar{c})$s fragmenting into $J/\psi$ with emission of soft gluons. This subprocess occurs at $O(\alpha_s^7)$. Here, $a, b$ are color indices for the initial gluon and the final color-octet $c\bar{c}$ state, and we are interested in $S = L = J = 0$ and $S = L = 1, J = 0, 1, 2$. There are 2 diagrams representing the vertex. Here we consider the process where only the gluon is off-shell. Following the conventions adopted in the previous section, we first write the matrix $O$ related to this effective vertex.
\[ O(P, q) = \frac{e e_c g_s \delta^{a\bar{b}}}{\sqrt{2}} \left[ \gamma^\mu \frac{P^\mu + \not{J} - \not{k} + M_c}{(P^\mu + q - k)^2 - M_c^2} \gamma^\nu \frac{P^\nu + \not{J} - \not{g} + M_c}{(P^\nu + q - g)^2 - M_c^2} \right]. \] (3.22)

With this matrix $O$ we can derive the effective vertices for the $\gamma g(c\bar{c})^{2S+1}L_j^{(8)}$ as
\[ \mathcal{M}'(1S_0^{(8)}) = 4i \frac{e e_c g_s}{g^2 - 4M_c^2} \delta^{a\bar{b}} \epsilon^{\mu\nu\kappa\lambda} \epsilon^a_{\mu} \epsilon^b_{\nu} P_\kappa k_\lambda, \] (3.23)
\[ \mathcal{M}'(3S_1^{(8)}) = 0, \] (3.24)
\[ \mathcal{M}'(3P_0^{(8)}) = \frac{2ee_c g_s \delta^{a\bar{b}}}{\sqrt{3}M_c} \left( g^2 - 12M_c^2 \right) \left( g_{\mu\nu} + 2 \frac{P_\mu P_\nu}{g^2 - 4M_c^2} \right) \epsilon^a_{\mu} \epsilon^b_{\nu}, \] (3.25)
\[ \mathcal{M}'(3P_1^{(8)}) = \frac{2ee_c g_s \delta^{a\bar{b}}}{M_c^2(g^2 - 4M_c^2)} \left( g^2 \epsilon^{\mu\nu\alpha\tau} + 2k_\kappa g^2(P_\mu \epsilon^\alpha\kappa\tau - P_\nu \epsilon^\mu\alpha\kappa\tau) + 4g_\nu M_c^2 \epsilon^\mu\alpha\kappa\tau \right) \epsilon^a_{\alpha}(J_2) \epsilon^b_{\nu} P_\tau, \] (3.26)
\[ \mathcal{M}'(3P_2^{(8)}) = \frac{16ee_c g_s \delta^{a\bar{b}}}{(g^2 - 4M_c^2)} M_c \left( g^{\mu\alpha} g^{\nu\beta} + 2k_\kappa g^{\mu\nu} P_\alpha g^{\nu\beta} - k_\nu g^{\mu\beta} \right) \epsilon^a_{\alpha\beta}(J_2) \epsilon^b_{\mu} \epsilon^g_{\nu}. \] (3.27)

Since $J/\psi$ can be produced via the $2 \rightarrow 1$ subprocesses with these effective vertices, we can obtain the $2 \rightarrow 1$ color octet contribution by using the following average squared amplitudes as
\[ \sum |\mathcal{M}'(1S_0^{(8)})|^2 = 2(e e_c g_s)^2, \] (3.28)
\[ \sum |\mathcal{M}'(3S_1^{(8)})|^2 = 0, \] (3.29)
\[ \sum |\mathcal{M}'(3P_0^{(8)})|^2 = \frac{6}{M_c^2}(e e_c g_s)^2, \] (3.30)
\[ \sum |\mathcal{M}'(3P_1^{(8)})|^2 = 0, \] (3.31)
\[ \sum |\mathcal{M}'(3P_2^{(8)})|^2 = \frac{8}{M_c^2}(e e_c g_s)^2. \] (3.32)

\[ \text{Our results agree with those obtained in Refs. [14] [15]. Note, however, that our convention of the invariant matrix is different from theirs.} \]
The \( J/\psi \) photoproduction cross section via \( 2 \rightarrow 1 \) process is given by

\[
\sigma \left( \gamma + p \rightarrow (c\bar{c})^{(8)} \rightarrow \psi \right) = \frac{7\pi (ee_c g_s)^2}{64 M_c^5} \left[ x f_{g/p}(x) \right]_{x=4M_c^2/s} \left( \frac{\langle 0|O^{(3)P_0^{(8)}}\psi|0\rangle}{M_c^2} + \frac{\langle 0|O^{(1)S_0^{(8)}}\psi|0\rangle}{7} \right). \tag{3.33}
\]

Since \( \hat{\sigma} \propto \delta(1-z) \), this \( 2 \rightarrow 1 \) color-octet subprocesses contribute to the elastic peak in the \( J/\psi \)–photoproduction. It is timely to recall that the color-singlet model with relativistic corrections still underestimates the cross section for \( z \geq 0.9 \) by an appreciable amount \[20\].

As \( z \rightarrow 1 \), the final state gluon in the \( \gamma \)–gluon fusion becomes softer and softer, although this does not cause any infrared divergence in the transition matrix element. Therefore, it would be more meaningful to factorize the effect of this final soft gluon into the color-octet matrix elements, \( \langle O^{(1)S_0} \rangle \) and \( \langle O^{(3)P_J} \rangle \). The color-octet \( 1S_0 \) and \( 3P_J \) states might reduce the gap between the color-singlet prediction and the experimental value of \( d\sigma/dz \) for \( 0.9 \leq z \leq 1 \).

The color-octet \( 2 \rightarrow 1 \) subprocess (3.21) considered before not only contributes to the elastic peak of the \( J/\psi \) photoproduction, but it also contributes to the resolved photon processes at \( O(\alpha^2 v^7) \), where the initial partons can be either gluon or light quarks \( (q = u, d, s) \). These processes are suppressed by \( v^4 \) but enhanced by \( 1/\alpha_s \), relative to the resolved photon process in the color-singlet model. Also, they can enhance the high-\( p_T \) \( J/\psi \)'s, which might be relevant to the \( J/\psi \) photoproduction at HERA. This can be a background to the determination of gluon distribution function of a proton, if the cross section is appreciable.

The resolved photon process in the color-singlet model is dominant over the \( \gamma \)–gluon fusion in the lower \( z \) region, \( z < 0.2 \), and it can be discarded by a suitable cut on \( z \). Since the color-octet contribution to the resolved photon process has not been studied in the literature, we address this issue here. When one considers

\[
\gamma + g \rightarrow (c\bar{c})_8^{(1)S_0 \text{ or } 3P_J}, \\
\gamma + q \rightarrow (c\bar{c})_8^{(1)S_0 \text{ or } 3P_J},
\]

one has to include

\[
\gamma + g \rightarrow (c\bar{c})_8^{(3)S_1}
\]

simultaneously, since both are the same order of \( O(\alpha^2 v^7) \). This diagram is the same as the color-singlet case except for the color factor of the \( (c\bar{c}) \) state.

It is straightforward, although lengthy, to calculate the amplitudes for the above three processes. Using REDUCE in order to the spinor algebra in a symbolic manner, we can get the averaged \( \mathcal{M} \) squared for various \( 2 \rightarrow 2 \) processes.

Another color-octet \( (c\bar{c})^{(3)S_1^{(8)}} \) contribution to the \( J/\psi \)–photoproduction comes from the Compton scattering type subprocesses:

\[
\gamma(k, \epsilon) + q(p_1) \rightarrow (c\bar{c})^{(3)S_1^{(8a)}}(P, \epsilon^*) + q(p_2), \tag{3.34}
\]

where \( P \) and \( \epsilon^* \) are the four momentum and the polarization vector of the \( 3S_1 \) color-octet state, and \( a \) is its color index. This subprocess, if important, can be a background to the
determination of the gluon distribution function in a proton, since it is initiated by light quarks. From the naive power counting, however, we infer this subprocess occurs at $O(\alpha_s^2)$ in the coupling constant expansion, and also suppressed by $v^4$ compared to the color-singlet contribution (3.1) due to its color-octet nature. Thus, this subprocess is expected to be negligible.

One can actually quantify this argument by explicitly evaluating the Feynman diagrams for (3.34). The effective vertex for $q\bar{q} \to (c\bar{c})(^3S_1^{(8a)})$ is given by

$$M'(q(p_1)\bar{q}(p_2) \to (c\bar{c})(^3S_1^{(8a)})) = \frac{4\pi\alpha_s}{2M_c} \bar{v}(p_2)\gamma^n T^a u(p_1) \epsilon^*_\mu(p_1 + p_2, S_z), \quad (3.35)$$

where $\epsilon^*_\mu$ is the polarization of the produced spin-1 color octet object. Using this effective vertex, one can calculate the amplitude for $\gamma q \to (c\bar{c})S(^3S_1^{(8a)})q$

$$M'(\gamma q \to (c\bar{c})(^3S_1^{(8a)})q) = \frac{g_s^2ee_q}{2M_c} \bar{u}(p_2) \left[ \gamma^* (P, S_z) T_a \frac{(k + p_1 + M_c)}{(k + p_1)^2 - M_c^2} \ell_\tau + \gamma^* (p_1 + P + M_c) \ell_\tau (p_1 - P)^2 - M_c^2 \gamma^* (P, S_z) T_a \right] u(p_1). \quad (3.36)$$

where $ee_q$ is the electric charge of the light quark inside proton($q = u, d, s$). The average amplitude squared for the color-octet $^3S_1$ state is given by

$$\sum |M'(\gamma q \to (c\bar{c})(^3S_1^{(8)})q)|^2 = -\frac{2}{3M_c^2} (g_s^2ee_q)^2 \left( \frac{s}{\hat{s}} + \frac{\hat{u}}{\hat{s}} + \frac{M_c^2}{\hat{s}\hat{u}} \right). \quad (3.37)$$

This completes our discussions on the color-octet $2 \to 2$ subprocess for $J/\psi$ photoproductions.

Now, we are ready to show the numerical results using the analytic expressions obtained in the previous section. Let us first summarize the input parameters and the structure functions we will use in the following. The results are quite sensitive to the numerical values of $\alpha_s$ and $m_c$ and the factorization scale $Q$. We shall use $\alpha_s(M_Z^2) = 0.3$, $m_c = 1.48$ GeV and $Q^2 = (2m_c)^2$. For the structure functions, we use the most recent ones, MRSA [22] and CTEQ3M [23], which incorporate the new data from HERA [24], on the lepton asymmetry in $W$-boson production [24] and on the difference in Drell-Yan cross sections from proton and neutron targets [26]. For the $2 \to 1$ case, we show results using both structure functions. For the $2 \to 2$ case, we show the results with the CTEQ3M structure functions only, since the MRSA structure functions yield almost the same results within $\sim 10\%$ or so.

Let us first consider the $J/\psi$ photoproduction via the color-octet $2 \to 1$ subprocess. Since the subprocess cross section (3.33) vanishes except at $z = 1$, one can infer that it contributes to the $J/\psi$ photoproductions in the forward direction ($z \sim 1, P_T^2 \simeq 0$). In Figs. 1 (a) and (b), we show the $J/\psi$ photoproduction cross section in the forward direction ($\sigma_{\text{forward}}$) as well as the data from the fixed target experiments and the preliminary data from H1 at HERA, respectively. In each case, the upper and the lower curves define the region allowed by the relation (1.4) for two color-octet matrix elements, $\langle 0|O_8^\alpha(^3S_1)|0 \rangle$ and $\langle 0|O_8^\alpha(^3P_0)|0 \rangle$. In case of fixed target experiments, $\sigma_{\text{forward}}$ is usually characterized by $z > 0.9$, with the remainder being associated with the inelastic $J/\psi$ photoproduction. According to this criterion, the experimental value of $\sigma_{\text{exp}}(\gamma + p \to J/\psi + X)$ contains contributions
from inelastic production of $J/\psi$’s. Thus, the data should lie above the predictions from the color-octet $2 \rightarrow 1$ subprocess, (3.21). Fig. 1 (a) shows that the situation is opposite to this expectation. Color-octet contributions are larger than the data, which indicates that the numerical values of the color-octet matrix elements are probably too large. At HERA, one has the elastic $J/\psi$ photoproduction data, which can be identified with the color-octet $2 \rightarrow 1$ subprocess. By saturating the relation (1.4) by either color-octet matrix element, we get the $J/\psi$ photoproduction cross section in the forward direction (Fig. 1 (b)). We observe again that the color-octet contribution with (1.4) overestimates the cross section by a large amount. This disagreement can arise from two sources: (i) the radiative corrections to $p\bar{p} \rightarrow J/\psi + X$, which were ignored in Ref. [3] is important, and/or (ii) the heavy quark spin symmetry for $\langle O^\psi_{5}(3P_{J})|0\rangle \approx (2J + 1)$ $\langle O^\psi_{5}(3P_{0})|0\rangle$ may not be a good approximation. Although the heavy quark spin symmetry relation is used quite often in heavy quarkonium physics, it may be violated by a considerable amount [11].

Recently, Amundson et al. performed the $\chi^2$ fit to the available fixed target experiments and the HERA data independently, and found that [13]

$$\langle O^\psi_{5}(1S_{0})|0\rangle + \frac{7}{M_{c}^{2}} \langle O^\psi_{5}(3P_{0})|0\rangle = (0.020 \pm 0.001) \text{ GeV}^{3},$$

(3.38)

using the MRSA\(^{(1)}\), and CTEQ3M structure functions with $\alpha_{s}(2M_{c}) = 0.26$ and $M_{c} = 1.5$ GeV. This determination is not compatible with the relation (1.4), since the resulting $\langle O_{5}(3P_{0})\rangle$ is negative. This is another way to say that the determination of the color-octet matrix elements from the $J/\psi$ productions at the Tevatron may not be that reliable. In fact, this is not very surprising, since the radiative corrections to the lowest-order color-singlet contributions to the $J/\psi$ hadroproductions are not included yet.

Next, we consider the $J/\psi$ photoproduction through $2 \rightarrow 2$ parton-level subprocesses. As discussed before, the PQCD corrections to the lowest order $\gamma + g \rightarrow J/\psi + g$ is not under proper control for $z > 0.9$. Therefore, we impose a cut $z < 0.9$ at EMC energy, $\sqrt{s_{\gamma p}} = 14.7$ GeV, and at HERA with $\sqrt{s_{\gamma p}} = 100$ GeV, we impose cuts on $z$ and $P_{T}^{2}$ [21]:

$$0.2 < z < 0.8, \quad P_{T}^{2} > 1 \text{ GeV}^{2}.$$  

In both cases, we set $K \approx 1.8$.

In Figs. 2 (a) and (b), we show the $d\sigma/dz$ distributions of $J/\psi$ at EMC (NMC) and HERA along with the corresponding data. In both cases, the color-octet $^{3}S_{1}$ contribution (Compton scattering type) is negligible in most regions of $z$, and thus can be safely neglected. The thick dashed and the thin dashed curves correspond to the cases where the relation (1.4) is saturated by $\langle O^\psi_{5}(3P_{J})|0\rangle$ and $\langle O^\psi_{5}(1S_{0})|0\rangle$, respectively. The thick and the thin solid curves represent the sum of the color-singlet and the color-octet contributions, in case that the relation (1.4) is saturated by $\langle O^\psi_{5}(1S_{0})|0\rangle$ and $\langle O^\psi_{5}(3P_{0})|0\rangle$, respectively. In either case, we observe that the color-octet $^{1}S_{0}$ and $^{3}P_{T}$ contributions begin to dominate the color-singlet contributions for $z > 0.6$, and become too large for high $z$ region considering we have not added the enhancements at high $z$ due to the relativistic corrections. Thus, this behavior of rapid growing at high $z$ does not agree with the data points at EMC and HERA, if we adopt the determination (1.4) by Cho and Leibovich [3].

In Fig. 3, we show the inelastic $J/\psi$ photoproduction cross section as a function of $\sqrt{s_{\gamma p}}$ with the cut, $z < 0.8$ and $P_{T}^{2} > 1 \text{ GeV}^{2}$. Again, the color-octet $^{3}S_{1}$ contribution is too small,
and thus not shown in the figure. Here again, the color-octet $^1S_0$ and $^3P_J$ contributions via $2 \to 2$ subprocesses dominate the color-singlet contribution, if the relation (1.4) is imposed. Although the total seems to be in reasonable agreement with the preliminary H1 data, direct comparison may be meaningful only if the cascade $J/\psi$’s from $b$ decays have been subtracted out. There are also considerable amount of uncertainties coming from $M_c$ and $\alpha_s$. Therefore, it is sufficient to say that the color-octet $^1S_0$ and $^3P_J$ states dominate the singlet contribution to the $J/\psi$ photoproduction, if the relation (1.4) is imposed.

IV. CONCLUSION

In summary, we considered the color-octet contributions to (A) the inclusive $J/\psi$ productions in $B$ decays, and (B) the $J/\psi$ photoproductions ($\gamma + p \to J/\psi + X$) through (i) $\gamma g \to (c\bar{c})_s(^1S_0$ and $^3P_J$) and the subsequent evolution of $(c\bar{c})_s$ into a physical $J/\psi$ with $z \approx 1$ and $P_T^2 \approx 0$, (ii) the subprocesses $\gamma + g$(or $q$) $\to (c\bar{c})_s(^1S_0$ or $^3P_J$) + $g$(or $q$). These are compared with (i) the measured $J/\psi$ photoproduction cross section in the forward direction, and (ii) the $z$ distributions of $J/\psi$ at EMC and HERA, and the preliminary result on the inelastic $J/\psi$ photoproduction total cross section at HERA. One finds that the relation (1.4) color-octet lead to too large contributions of the color-octet $^1S_0$ and $^3P_J$ states to the above observables. Especially, the first two observables contradict the observation. This is also against the naive expectation that the color-octet contribution may not be prominent as in the case of the $J/\psi$ hadroproductions, since they are suppressed by $v^4$ (although enhanced by one power of $\alpha_s$) relative to the color-singlet contribution. It is also pointed out that the same is true of the process $B \to J/\psi + X$, in which the relation (1.4) predicts its branching ratio to be too large by an order of magnitude compared with the data.

Therefore, one may conclude that the color-octet matrix elements involving $c\bar{c}_s(^1S_0, ^3P_J)$ might be overestimated by an order of magnitude. Since the relation (1.4) has been extracted by fitting the $J/\psi$ production at the Tevatron to the lowest order color-singlet and the color-octet contributions, it may be changed when one considers the radiative corrections to the lowest order color-singlet contributions.

ACKNOWLEDGMENTS

The author is grateful to Prof. H.S. Song and Mr. Jungil Lee for enjoyable collaborations on the subjects discussed in this article. This work was supported in part by KOSEF through CTP at Seoul National University, and by the Basic Science Research Program, Ministry of Education, 1996, Project No. BSRI–96–2418.
REFERENCES

[1] R. Baier and R. Ruckl, Z. Phys. C 19, 251 (1983); F. Halzen, F. Herzog, E.W.N. Glover and A.D. Martin, Phys. Rev. D 30, 700 (1984).

[2] M. Mangano, CDF Collaboration, presented at the 27th International Conference on High Energy Physics, Glasgow, July (1994), and references therein.

[3] E. Braaten and S. Fleming, Phys. Rev. Lett. 74, 3327 (1995).

[4] P. Cho and A.K. Leibovich, CIT Preprint, CALT–68–1988 (1995).

[5] P. Cho and A.K. Leibovich, CIT Preprint, CALT–68–2026 (1995).

[6] H. Fritzsch and K.H. Streng, Phys. Lett. B 72, 385 (1978).

[7] G.T. Bodwin, E. Braaten and P. Lepage, Phys. Rev. D 51, 1125 (1995).

[8] S. Fleming and I. Maksymyk, MADPH-95-922, UTTG-13-95, hep-ph/9512320 (1995).

[9] P. Cho and M.B. Wise, Phys. Lett. B346, 129 (1995).

[10] E. Braaten and Yu-Qi Chen, NUHEP-TH-95-9, hep-ph/9508373 (1995).

[11] P. Ko, J. Lee and H.S. Song, Phys. Rev. D 53, 1409 (1996).

[12] K. Cheung, W.-Y. Keung and T.C. Yuan, Fermiab-Pub-95/300-T (1995); P. Cho, CALT 68-2020 (1995).

[13] S.W. Baek, P. Ko, Jungil Lee and H.S. Song, SNUTP 96-061, hep-ph/9607236 (1996).

[14] M. Cacciari and M. Krämer, DESY 96-005, hep-ph/9601276 (1996).

[15] J. Amundson, S. Fleming and I. Maksymyk, UTTG-10-95, MADTH-95-914, hep-ph/9601298 (1996).

[16] P. Ko, J. Lee and H.S. Song, SNUTP 95-116, hep-ph/9602223 (1996), to appear in Phys. Rev. D.

[17] G.T. Bodwin, E. Braaten, T.C. Yuan and P. Lepage, Phys. Rev. D 46, R3703 (1992).

[18] R. Balest et al., CLNS 94/1315, CLEO 94-26 (1994).

[19] E.L. Berger and D. Jones, Phys. Rev. D 23, 1521 (1981).

[20] H. Jung, D. Krucker, C. Greub and D. Wyler, Z. Phys. C 60, 721 (1993).

[21] M. Krämer, DESY 95-155 (1995).

[22] A.D. Martin, R.G. Roberts and W.J. Stirling, Phys. Rev. D 50, 6734 (1994).

[23] H. Lai et al., Phys. Rev. D 51, 4763 (1995).

[24] ZEUS Collaboration, Phys. Lett. B 316, 412 (1993); H1 Collaboration, Nucl. Phys. B 407, 515 (1993).

[25] CDF Collaboration, F. Abe et al., Phys. Rev. Lett. 74, 850 (1995).

[26] NA51 Collaboration, A. Baldit et al., Phys. Lett. B 332, 244 (1994).

Figure Captions

Fig.1(a) The cross sections for $\gamma + p \rightarrow J/\psi + X$ in the forward direction at the fixed target experiments as a function of $E_\gamma$. The solid and the dashed curves were obtained using the CTEQ3M and the MRSA structure functions. Here, TOT$_s$ is the $^1S_0^{(8)}$ saturated curve and TOT$_p$ is the $^3P_J^{(8)}$ saturated one.

Fig.1(b) The cross sections for $\gamma + p \rightarrow J/\psi + X$ in the forward direction at HERA as a function of the square root of $s_{\gamma p}$. The solid and the dashed curves were obtained using the CTEQ3M and the MRSA structure functions. Here, TOT$_s$ is the $^1S_0^{(8)}$ saturated curve and TOT$_p$ is the $^3P_J^{(8)}$ saturated one.
Fig. 2(a) The differential cross sections \( d\sigma/dz \) for \( \gamma + p \rightarrow J/\psi + X \) at EMC as a function of \( z \equiv E_{J/\psi}/E_\gamma \). The singlet contributions are in the thick dotted curve, the color-octet \( ^1S_0 \) contributions in the thick dashed curve (with \( \langle O_8^{\psi}(^1S_0) \rangle = 6.6 \times 10^{-2} \text{ GeV}^3 \)), and the color-octet \( ^3P_J \) contributions in the thin dashed curve (with \( \langle O_8^{\psi}(^3P_J) \rangle/M_c^2 = 2.2 \times 10^{-2} \text{ GeV}^3 \)). The total is shown in the solid curve. The relation (1.4) allows the region between two solid curves. Here, TOT\(_s\) is the \( ^1S_0^{(8)} \) saturated curve and TOT\(_p\) is the \( ^3P_J^{(8)} \) saturated one.

Fig. 2(b) The differential cross sections \( d\sigma/dz \) for \( \gamma + p \rightarrow J/\psi + X \) at HERA as a function of \( z \equiv E_{J/\psi}/E_\gamma \). The singlet contributions are in the thick dotted curve, the color-octet \( ^1S_0 \) contributions in the thick dashed curve (with \( \langle O_8^{\psi}(^1S_0) \rangle = 6.6 \times 10^{-2} \text{ GeV}^3 \)), and the color-octet \( ^3P_J \) contributions in the thin dashed curve (with \( \langle O_8^{\psi}(^3P_J) \rangle/M_c^2 = 2.2 \times 10^{-2} \text{ GeV}^3 \)). The total is shown in the solid curve. The relation (1.4) allows the region between two solid curves. Here, TOT\(_s\) is the \( ^1S_0^{(8)} \) saturated curve and TOT\(_p\) is the \( ^3P_J^{(8)} \) saturated one.

Fig. 3 Total inelastic \( J/\psi \) photoproduction cross section for \( z < 0.8 \) as a function of the square root of \( s_{\gamma p} \). The singlet contributions in the thick dotted curve, the color-octet \( ^1S_0 \) contributions in the thick dashed curve (with \( \langle O_8^{\psi}(^1S_0) \rangle = 6.6 \times 10^{-2} \text{ GeV}^3 \)), and the color-octet \( ^3P_J \) contributions in the thin dashed curve (with \( \langle O_8^{\psi}(^3P_J) \rangle/M_c^2 = 2.2 \times 10^{-2} \text{ GeV}^3 \)). The total is shown in the solid curve. The relation (1.4) allows the region between two solid curves. Here, TOT\(_s\) is the \( ^1S_0^{(8)} \) saturated curve and TOT\(_p\) is the \( ^3P_J^{(8)} \) saturated one.
FIG. 1 (a).
FIG. 1 (b).
EMC ( $\sqrt{s_{\gamma p}} = 14.7$ GeV )

\[ \frac{d\sigma}{dz} (p_T > 0.1 \text{ GeV}) \]

FIG. 2 (a).
HERA ( $\sqrt{s_{\gamma p}} = 100$ GeV )

\[ \frac{d\sigma}{dz} \left( p_T > 1 \text{ GeV} \right) \]

\[ \text{nb} \]

FIG. 2 (b).
$\sigma(\sqrt{s_{\gamma p}})$

$p_T > 1$ GeV
$z < 0.8$

FIG. 3.