Charm mixing in the model-independent analysis of correlated \(D^0\bar{D}^0\) decays

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We investigate the impact of charm mixing on the model-independent \(\gamma\) measurement using Dalitz plot analysis of the three-body \(D\) decay from \(B^+ \rightarrow DK^+\) process, and show that ignoring the mixing at all stages of the analysis is safe up to a sub-degree level of precision. We also find that in the coherent production of the \(D^0\bar{D}^0\) system in \(e^+e^-\) collisions, the effect of charm mixing is enhanced, and propose a model-independent method to measure charm mixing parameters in time-integrated Dalitz plot analysis at charm factories.

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I. INTRODUCTION

Dalitz plot analysis of three-body decays of neutral \(D\) mesons is a useful tool in various measurements where coherent admixtures of \(D^0\) and \(\bar{D}^0\) are observed. This technique was initially proposed for the measurement of the unitarity triangle angle \(\gamma\) in \(B^+ \rightarrow DK^+\) decays [1,2]. Later it was applied to the measurement of charm mixing [3,4] and to the resolution of the quadratic ambiguity in the measurement of the angle \(\beta\) using a time-dependent analysis of the decay \(B^0 \rightarrow D\pi^0\) [4,5]. Most of these measurements are based on the \(D \rightarrow K^0_S\pi^+\pi^-\) decay which offers the best precision among three-body \(D^0\) decays.

The technique is model-dependent — it depends on the complex amplitude of the \(D^0\) decay which is obtained from the \(D^+ \rightarrow D^0\pi^+\) sample using model assumptions. The result of the measurement contains therefore model uncertainties. In the case of \(\gamma\) measurement, this uncertainty \((\sim 10^\circ)\) is already comparable to the statistical accuracy [7,8].

However, a modification of the analysis is possible that allows to perform a completely model-independent measurement [1]. It requires the phase space of the three-body \(D\) decay to be divided into bins. Information about the complete phase in each bin can be extracted from the quantum-correlated \(D^0\) decays from \(\psi(3770) \rightarrow DD\) process. The measurement of the strong phase in bins of the \(D \rightarrow K^0_S\pi^+\pi^-\) phase space was recently performed by the CLEO collaboration [9]. This measurement should allow to reduce the error of \(\gamma\) related to the uncertainty in the \(D \rightarrow K^0_S\pi^+\pi^-\) amplitude to \(1 \sim 2^\circ\).

Recently, charm mixing was observed by the Belle and BaBar experiments [10,11]. With degree-level precision, the effect of charm mixing can become significant in the measurement of \(\gamma\). It was shown that mixing contributes only of second order in the \(x\) and \(y\) parameters to the ADS, GLW and model-dependent Dalitz plot analysis methods, and thus can be safely neglected [12]. But the effect of mixing on the binned analysis with the phase terms extracted from quantum-correlated \(D^0\) decays is of separate interest.

In this paper, we investigate the impact of the charm mixing on the model-independent \(\gamma\) measurement, and show that ignoring the mixing at all stages of the analysis is safe up to a sub-degree level of precision. We also find that in the coherent production of the \(D^0\bar{D}^0\) system in \(e^+e^-\) collisions, the effect of charm mixing is enhanced compared to the case of \(D^0\bar{D}^0\) production, and propose a model-independent method to measure charm mixing parameters in time-integrated Dalitz analysis at charm factories. The method is sensitive to both mixing parameters, \(x\) and \(y\), as well as to \(CP\) violation parameters \(r_{CP}\) and \(\alpha_{CP}\). The sensitivity of the proposed method can be improved by adding doubly Cabibbo-suppressed three-body modes such as \(D^0 \rightarrow K^+\pi^-\pi^0\). We estimate the sensitivity of the proposed method using Monte-Carlo (MC) simulation.

II. MODEL-INDEPENDENT BINNED ANALYSIS OF THREE-BODY \(D^0\) DECAYS

To introduce the notation we briefly recap the technique of model-independent binned Dalitz plot analysis of \(B^+ \rightarrow DK^+\), \(D \rightarrow K^0_S\pi^+\pi^-\) decays used to extract the angle \(\gamma\). As usually presented, this does not take charm mixing effects into account.

The amplitude of the \(B^+ \rightarrow DK^+\), \(D \rightarrow K^0_S\pi^+\pi^-\) decay can be written as

\[
A_B = \overline{A} + r_B e^{i(\delta_A + \gamma)} A,  \quad (1)
\]

where \(\overline{A} = \overline{A}(m^2_{K_S\pi\pi}, m^2_{K_S\pi\pi}) \equiv \overline{A}(m^2_+, m^2_+)\) is the amplitude of the \(\overline{D}^0 \rightarrow K^0_S\pi^+\pi^-\) decay, \(A = A(m^2_+, m^2_-)\) is the amplitude of the \(D^0 \rightarrow K^0_S\pi^+\pi^-\) decay \((A(m^2_+, m^2_+), A(m^2_-, m^2_-))\) in the case of \(CP\) conservation in \(D\) decay, \(r_B\) is the ratio of the absolute values of the interfering \(B^+ \rightarrow D^0K^+\) and \(B^+ \rightarrow D^0K^-\) amplitudes, and \(\delta_A\) is the strong phase difference between these amplitudes. The density of the \(D\) decay Dalitz plot from \(B^+ \rightarrow DK^+\) decay is given by the absolute value...
squared of the amplitude

$$P_B = |A_B|^2 = \bar{A} + r_B e^{i(\delta_B + \gamma)} A^2 = \mathcal{P} + r_B^2 P + 2\sqrt{\mathcal{P}P}(x_B C + y_B S),$$

where

$$x_B = r_B \cos(\delta_B + \gamma); \ y_B = r_B \sin(\delta_B + \gamma).$$

The functions $C = C(m_+^2, m_-^2)$ and $S = S(m_+^2, m_-^2)$ are the cosine and sine of the strong phase difference $\delta_D = \arg \bar{A} - \arg A$ between the $D^0 \to K_S^0 \pi^+\pi^-$ and $D^0 \to K_S^0 \pi^+\pi^-$ amplitudes:

$$C = \cos \delta_D(m_+^2, m_-^2); \ S = \sin \delta_D(m_+^2, m_-^2).$$

The expected number of events in the bin “$i$” of the Dalitz plot of $D$ from $B^+ \to DK^+$ is

$$N_i = h_B \left[ K_i + r_B^2 K_{-i} + 2\sqrt{K_i K_{-i}}(x_B C_i + y_B S_i) \right],$$

where $K_i$ is the number of events in the corresponding bin of the Dalitz plot of the $D$ meson in a flavor eigenstate (obtained using $D^{\pm \pm} \to D\pi^{\pm}$ samples) and $h_B$ is a normalization constant. The bin index “$i$” ranges from $-N$ to $N$ (excluding 0); the exchange $m_+^2 \leftrightarrow m_-^2$ corresponds to the exchange $i \leftrightarrow -i$. The terms $C_i$ and $S_i$ include information about the cosine and sine of the phase difference averaged over the bin region:

$$C_i = \frac{\int_{\mathcal{D}_i} |A||\bar{A}| \cos \delta_D \, dD}{\sqrt{\int_{\mathcal{D}_i} |A|^2 \, dD \int_{\mathcal{D}_i} |\bar{A}|^2 \, dD}}.$$  

Here $\mathcal{D}$ represents the Dalitz plot phase space and $\mathcal{D}_i$ is the bin region over which the integration is performed. The terms $S_i$ are defined similarly with cosine substituted by sine.

The expected number of events in each Dalitz plot bin is trivially obtained from the probability density by integrating over the bin area, which leads to the substitutions $P \to K_i, \mathcal{P} \to K_{-i}, C, S \to C_i, S_i$. In what follows, we only quote the number of events to save space. Normalization constants (such as $h_B$ in (5)) are also omitted.

The symmetry under $\pi^+ \leftrightarrow \pi^-$ requires $C_i = C_{-i}$ and $S_i = -S_{-i}$. The values of $C_i$ and $S_i$ terms can be provided by charm-factory experiments operated at the threshold of $D\bar{D}$ pair production. The wave function of the two mesons is antisymmetric, thus the four-dimensional density of two correlated $D \to K_S^0 \pi^+\pi^-$ Dalitz plots is

$$|A_{corr}(m_+^2, m_-^2)|^2 = |A_1 \bar{A}_2 - A_2 \bar{A}_1|^2 = P_1 \mathcal{P}_2 + \mathcal{P}_1 P_2 - 2\sqrt{P_1 \mathcal{P}_2 \mathcal{P}_1 P_2}(C_i C_j + S_i S_j),$$

where the indices “1” and “2” correspond to the two decaying $D$ mesons. In the case of a binned analysis, the number of events in the region of the $(K_S^0 \pi^+ \pi^-)^2$ phase space described by the indices “1” and “2” is

$$M_{ij} = K_i K_{-j} + K_{-i} K_j - 2\sqrt{K_i K_{-i} K_j K_{-j}}(C_i C_j + S_i S_j).$$

Once the values of the terms $C_i$ and $S_i$ are known from charm-factory data, the system of equations (5) contains only three free parameters ($x_B, y_B,$ and $h_B$) for each $B$ charge, and can be solved using maximum likelihood method to extract the value of $\gamma$.

Note that technically the system (5) can be solved without external constraints on $C_i$ and $S_i$ for $N \geq 2$. However, due to the small value of $r_B$, there is very little sensitivity to the $C_i$ and $S_i$ parameters in $B^+ \to DK^+$ decays, which results in a reduction in the precision on $\gamma$ that can be obtained.

### III. CONTRIBUTION OF CHARM MIXING TO MODEL-INDEPENDENT $\gamma$ MEASUREMENT.

In the case of $CP$ conservation, the mass eigenstates of the neutral $D$ system are given by

$$D_{1,2} = \frac{1}{\sqrt{2}} (D^0 \pm \bar{D}^0).$$

Charm mixing is described by two parameters, $x_D$ and $y_D$, which are defined as

$$x_D = \frac{m_2 - m_1}{\Gamma}, \ \ y_D = \frac{\Gamma_2 - \Gamma_1}{2\Gamma},$$

where $m_{1,2}$ and $\Gamma_{1,2}$ are the mass and decay widths of the mass eigenstates. We use notations $x_D$ and $y_D$ instead of the more common $x$ and $y$ in order not to confuse them with the $CP$-violating parameters $x_B$ and $y_B$ introduced before. The current world average values are: $x_D = (0.59 \pm 0.20)\%$, $y_D = (0.80 \pm 0.13)\%$. $CP$ violation modifies the expression (9) to

$$D_{1,2} = \rho D^0 \pm \eta \bar{D}^0,$$

where $\rho$ and $\eta$ satisfy $|\rho|^2 + |\eta|^2 = 1$. $CP$-violating mixing is thus described by two additional parameters $r_{CP}$ and $\alpha_{CP}$:

$$r_{CP} e^{i\alpha_{CP}} = q/p.$$
Below we present all the quantities that enter the model-independent analysis including the contribution of the CP-conserving charm mixing (the corresponding quantities are denoted with the prime mark). The full formalism including CP violation in mixing is given in Appendix A.

The number of events in the each bin of the flavor-tagged $D^0 \to K^0_S \pi^+ \pi^-$ Dalitz plot after integration over decay time is

$$K'_i = K_i + \sqrt{K_i}K_{-i}(y_D C_i + x_D S_i) + O(x_D^2, y_D^2).$$  \hspace{1cm} (13)

Similarly, one can obtain the number of events for the $D \to K^0_S \pi^+ \pi^-$ decay from $B^+ \to DK^+$ using symmetric beams is difficult. Therefore one can rewrite the mixing correction as

$$A_{\text{corr}}(t, m^2_+, m^2_-, m^2_{1+}, m^2_{1-}) = A_1(0)A_2(t) - A_1(0)A_2(t).$$  \hspace{1cm} (15)

Before the particle “1” decays, the amplitude stays antisymmetric and the mixing does not affect it. We can therefore assume that the particle “1” decays at the time $t = 0$ and count time from the moment of its decay.

After the integration over time and taking the absolute value squared of the amplitude (16), we obtain the number of events $M'_{ij}$ in the bin “ij” of the phase space for the pair of $D$ mesons (still assuming that the particle “1” denoted here with the index “i” decayed first):

$$M'_{ij} = K_i K_{-j} + K_j K_{-i} \left( C_{ij} + S_{ij} \right) - 2\sqrt{K_i K_{-j} K_j K_{-i}} \left( C_{ij} + S_{ij} \right) - K_j \sqrt{K_{-i} K_{-j} (y_D C_i - x_D S_i)} - K_{-j} \sqrt{K_i K_{-j} (y_D C_i - x_D S_i)} + K_i \sqrt{K_{-j} K_{-i} (y_D C_i + x_D S_i)} + K_{-i} \sqrt{K_j K_{-i} (y_D C_i + x_D S_i)} + O(x_D^2, y_D^2).$$  \hspace{1cm} (16)

However, measurement of the decay time in an experiment with symmetric beams is difficult. Therefore one has to average over the decay order, which leads to the cancellation of all terms linear in mixing parameters:

$$M'_{ij} = M_{ij} + O(x_D^2, y_D^2).$$  \hspace{1cm} (17)

The real analysis performed at CLEO uses the values $M'_{ij}$ (which, as we have seen, are unaffected by mixing at first order), and values of $K_i$ obtained from correlated $D\bar{D}$ decays where one of the $D$ mesons serves as a flavor tag. The $K_i$ values extracted this way are also unaffected by mixing. Therefore, the values of $C_i$ and $S_i$ extracted in this analysis contain no linear mixing contribution.

As far as processes observed at $B$ factories are concerned, both the $D \to K^0_S \pi^+ \pi^-$ decay from $B^+ \to DK^+$ and the flavor-tagged $D \to K^0_S \pi^+ \pi^-$ decay contain mixing contributions, and therefore the observable numbers of events in the Dalitz plot bins are $N'_i$ and $K'_i$, respectively. Clearly, if one uses the values $K_i$, obtained from a charm factory, in the fit to obtain $\gamma$ from $N'_i$ described by Eq. 14, the resulting value contains contribution in first order in $x_D, y_D$. If the values $K_i$ are used, Eq. 14 can be rewritten as

$$N'_{i} = K'_i + r_{B} K'_{-i} + 2\sqrt{K'_i K'_{-i}} (x_D C'_i + y_D S'_i) + O(x_D^2, y_D^2).$$  \hspace{1cm} (18)

Thus, if the terms $C_i, S_i$ are left as free parameters in the fit to $B$ decay data, the mixing correction is only of second order (the effective parameters $C'_i, S'_i$ are measured in this case), but if these terms are obtained from correlated $D$ decays, first order mixing corrections to $\gamma$ appear. However, these corrections are additionally suppressed by factor $r_B \sim 0.1$, and the residual contribution of charm mixing to $x_B$ and $y_B$ is at a percent level.

A quantitative estimate of the effect was performed using the procedure described in Appendix B. Three analysis strategies have been considered:

1. Using $K_i$ (unaffected by mixing) from the coherent $D\bar{D}$ production.
2. Using $K'_i$ (Eq. 13) measured in $D^{*+} \to D^0 \pi^+, D \to K^0_S \pi^+ \pi^-$ decays.
3. Using $K_i$ and applying a linear correction for the mixing contribution according to Eq. 14 (assuming that $x_D, y_D$ are known).

The effect of mixing on the fitted value of $\gamma$ depends on the value $\alpha_D = \arctan(y_D/x_D)$, the ratio $\sqrt{x_D^2 + y_D^2}/r_B$, and the $\delta_B$ and $\gamma$ values. In our study, we use $\sqrt{x_D^2 + y_D^2}/r_B = 0.1$ (all biases are proportional to this quantity) and scan over the other parameters. The results are shown in Table 3. Clearly, if the $K'_i$ values from $D^{*+} \to D^0 \pi^+, D \to K^0_S \pi^+ \pi^-$ decays are used in the $\gamma$ fit, the mixing contribution can be neglected.

For clarity, we would like to compare our result with the conclusions of previous papers that have addressed the impact of charm mixing on $\gamma$ measurements 12 16.
TABLE I: Estimates of the charm mixing effect on the $\gamma$ value measured using the model-independent time-integrated Dalitz plot analysis method, for three different analysis strategies. The maximum $\gamma$ bias for varying $\alpha_D = \arctan(y_D/x_D)$, $\delta_B$ and $\gamma$, as well as the values of these parameters at which the maximum is reached, are shown. The estimation assumes $\sqrt{\frac{x_D^2 + y_D^2}{r_B}} = 0.1$.

| Strategy | $\Delta\gamma_{\text{max}}$ | $\alpha_{\text{max}}$ | $\delta_{B,\text{max}}$ | $\gamma_{B,\text{max}}$ |
|----------|-----------------|-----------------|-----------------|-----------------|
| 1. Using $K_i$ | 2.9° | 184° | 85° | 87° |
| 2. Using $K'_i$ | $-0.2°$ | 97° | 2° | 90° |
| 3. Linear correction | 0.07° | 324° | 72° | 73° |

Ref. [10] considers the case when the $D$ amplitude does not contain the mixing contribution (or is corrected for it), but the $B$ decay data are uncorrected for mixing. This corresponds to the analysis strategy 1 in our MC study. The systematic bias to $\gamma$ in that case is linear in $x_D$ and $y_D$ and can be numerically large. The treatment in [12] corresponds to the case where the mixing is neglected in both the flavor-tagged $D$ and $B$ data; the systematic bias in $\gamma$ is second order in $x_D$ and $y_D$ in that case. In the context of the model-independent binned Dalitz plot analysis, the conclusions of [12] can only be applied if the phase terms $C_i$ and $S_i$ are left as free parameters in the fit to $B$ data. The analysis procedure considered here is an intermediate case: part of information about the $D \rightarrow K_0^\pm \pi^\mp$ amplitude (namely, $K_i$ terms) is extracted from the flavor-tagged data uncorrected for mixing, while the $C_i$ and $S_i$ terms from the quantum-correlated $D\bar{D}$ decays contain no mixing contribution. This results in a bias linear in $x_D, y_D$ (but numerically small due to additional $r_B$ suppression).

IV. TIME-DEPENDENT CHARM MIXING MEASUREMENT

The charm sector is the only place where contributions to $CP$ violation from down-type quarks in the mixing diagram can be explored. While values of charm mixing parameters are not easy to predict in the SM, $CP$ violation in mixing is expected to be very small. However, there is a range of SM extensions which allow sizable $CP$ violation effects [17, 18]. Because of that, precise measurements of mixing as well as $CP$ violation parameters are essential.

The most accurate measurements of the charm mixing parameters to date have been performed by $B$-factories using time-dependent methods [10, 11]. For example, by observing the wrong-sign decay $D^0 \rightarrow K^+\pi^-$ [11], BaBar determines $R_D$, the ratio of doubly Cabibbo-suppressed (DCS) to Cabibbo-favored (CF) decay rates, and the mixing parameters $x_D^2$ and $y_D$, where $x_D' = x_D \cos \delta_{K\pi} + y_D \sin \delta_{K\pi}$, $y_D' = -x_D \sin \delta_{K\pi} + y_D \cos \delta_{K\pi}$ and $\delta_{K\pi}$ is the strong phase between the DCS and CF amplitudes. Since only $y_D'$ contributes linearly to the time dependence of the decay rate, and the strong phase $\delta_{K\pi}$ is close to zero [19], this kind of measurement is practically insensitive to the parameter $x_D$.

The Belle collaboration has performed a measurement of mixing parameters using time-dependent Dalitz plot analysis of the $D^0 \rightarrow K_S^0 \pi^+\pi^-$ decay [3]. A similar analysis has been performed by the BaBar collaboration using in addition $D^0 \rightarrow K_S^0 K^+ K^-$ [4]. Here the Dalitz plot distribution depends linearly on both mixing parameters. However, model assumptions in the decay amplitude description are necessary in this analysis, which unavoidably results in significant model uncertainties.

The model-independent binned Dalitz plot analysis may be extended to time-dependent measurements of charm mixing parameters and $CP$ violation in charm mixing. The time-dependent number of events in the bin “$i^\tau$” of $D^0 \rightarrow K_S^0 \pi^+\pi^-$ decay Dalitz plot is

$$K_i'(t) = h_D e^{-\Gamma t} \left[ K_i + \sqrt{K_i K_{i-1}} (y_D C_i + x_D S_i) \Gamma t + O((x_D + y_D)^2 (\Gamma t)^2) \right].$$

(20)

Using parameters $C_i$ and $S_i$ determined from independent measurements at a charm factory, we can eliminate completely the model uncertainty in $x_D$ and $y_D$ extraction.

We estimate the statistical sensitivity of the time-dependent fit to the mixing and the $CP$ violation parameters. Note that the values of $K_i$ can be measured independently with high precision at charm factory experiments. However, in that case one has to deal with systematic uncertainties due to detector efficiency and other effects. Depending on the magnitude of these uncertainties, one can consider a fit with $K_i$ as free parameters, where the statistical uncertainty increases, but the systematic effects are minimized.

Table II shows the results for both strategies. The toy MC simulation uses $10^6$ flavor-tagged $D$ mesons from $D^*$ decays corresponding to the samples available at the $B$-factories; the fit with fixed $K_i$ uses in addition $2 \cdot 10^6$ flavor-tagged $D$ decays in $C = -1$ state that can be obtained at a future charm factory experiment. The simulation uses the amplitude of $D \rightarrow K_S^0 \pi^+\pi^-$ decay determined by Belle [7], and the binning of the Dalitz plot

| Parameter | Precision |
|-----------|-----------|
| $K_i$ fixed | 17 | 22 |
| $K_i$ floated | 13 | 16 |
| $r_{CP}$ ($10^{-2}$) | 9 | 9 |
| $\alpha_{CP}$ (°) | 5 | 5 |
with 8 bins defined by the uniform division of the strong phase difference $\delta_D$ \cite{13}. The values of the phase terms $C_i, S_i$ are calculated from the $D \to K^0 S \pi^+ \pi^-$ amplitude; the contribution of their statistical precision in the final result is negligible with the current experimental data. Our estimates do not account for uncertainties in the time measurement and background effects.

V. TIME-INTEGRATED CHARM MIXING MEASUREMENT

The cancellation of terms linear in $x_D$ and $y_D$ for the correlated decays occurs only in the case of the antisymmetric wave function of two $D$ mesons produced with the charge conjugation quantum number $C = -1$. It is possible, however, to produce a pair of $D$ mesons with both quantum numbers $C = \pm 1$ in the process $e^+ e^- \to \psi(4040) \to D^0 \bar{D}^{*0}$ \cite{20}. The $e^+ e^- \to c \bar{c}$ cross-section at the $\psi(4040)$ mass is dominated by this process and is comparable to that at $\psi(3770)$ \cite{21}. Depending on the $D^{*0}$ final state ($D^0 s^0$ or $D^0 s^1$), the $D\bar{D}$ pair is either $C = -1$ or $C = +1$, respectively. In the case of $C = +1$, the amplitude of the decay is

$$A_{\text{corr}} = A_1 \bar{A}_2 + A_1 A_2.$$  \hspace{1cm} (21)

Unlike Eq. 17, now we allow two $D$ mesons to decay into different final states, therefore we denote the numbers of events in bins of flavor-specific decays as $k_i$ and $K_i$, and the corresponding phase terms as $c_i, s_i$ and $C_i, S_i$. These can be either two-body (in that case only one bin makes sense, and the index "i" only takes the value $\pm 1$) or multibody final states. For non-self-conjugate final states, the index takes positive values for $D^0$ decay and negative values for $\bar{D}^0$ decay. The terms related to the strong phase difference are defined similarly to $C_i$ and $S_i$ in Eq. 6. The number of events in phase space bins with the contribution of mixing taken into account is

$$M_i^{\pm} = k_i K_{-i} + k_{-i} K_i$$

$$+ 2\sqrt{k_i k_{-i}} K_{-j} (c_i C_j + s_i S_j)$$

$$+ 2K_{-i} \sqrt{k_i} (y D_{Ci} - x D s_i)$$

$$+ 2K_{-j} \sqrt{k_{-i}} (y D_{Ci} + x D s_i)$$

$$+ 2k_i \sqrt{K_j K_{-j}} (y D_{Cj} - x D S_j)$$

$$+ 2k_{-i} \sqrt{K_j K_{-j}} (y D_{Cj} + x D S_j)$$

$$+ O(x_D^2, y_D^2).$$  \hspace{1cm} (22)

for $C = +1$, and

$$M_i^{\pm} = k_i K_{-j} + k_{-j} K_i$$

$$- 2\sqrt{k_i k_{-i}} K_{-j} (c_i C_j + s_i S_j)$$

$$+ O(x_D^2, y_D^2)$$

for $C = -1$.

Important special cases for the two-body decays are:

1. Flavor-specific decay ($e.g.$ $D^0 \to K^- e^+ \nu_e$): $K_1 = 1$, $K_{-1} = 0$.

2. $D^0 \to K^- \pi^+$ decay: $K_1 = 1$, $K_{-1} = r_{K\pi}$, $C_1 = \cos \delta_D$, $S_1 = \sin \delta_D$.

The effect of mixing is linear in the case of $C = +1$, which allows to measure the parameters $x_D$ and $y_D$ in the combined fit of the decays of both charge parities. The simplest strategy involves flavor-tagged $D \to K^0 S \pi^+ \pi^-$ decays produced in both $C = +1$ and $C = -1$ states. Decay of the $C = -1$ state yields the numbers of events in $D \to K^0 S \pi^+ \pi^-$ phase space bins unaffected by mixing, while the $C = +1$ state contains linear mixing contribution:

$$M_i^{\pm} = k_i K_{-i} + 2\sqrt{k_i k_{-i}} K_{-j} (y D_{Ci} - x D S_i) + O(x_D^2, y_D^2).$$  \hspace{1cm} (24)

Since there are bins with $|C_i| \sim 1$ and bins with $|S_i| \sim 1$, the sensitivity to both $x_D$ and $y_D$ should be of the same order. Note that the mixing term is twice larger with respect to that in $D^{\pm} \to D^0 \pi^+ \pi^-$, $D \to K^0 S \pi^+ \pi^-$ decays (Eq. 13).

We have made a quantitative estimate of the sensitivity to mixing parameters using MC simulation. The results of this study for a data sample equivalent to the integrated luminosity of $10^3$ fb$^{-1}$ are shown in Table III. This corresponds to one standard year of data taking with a peak luminosity of $10^{35}$ cm$^{-2}$s$^{-1}$. Projects of $c\tau$-factories with such luminosity are being considered \cite{22}. The numbers of events are estimated using the double-tagged event efficiency obtained by CLEO in the reconstruction of $\psi(3770)$ decays \cite{9} (see Table VII in Appendix B).

Our study involves only the flavor-tagged and double-tagged $D \to K^0 S \pi^+ \pi^-$ decays in the coherent $C = -1$ state (to extract $K_i$ and $C_i, S_i$, respectively), as well as

\begin{table}[h]
\centering
\caption{Statistical sensitivity to the mixing and $CP$ violation parameters for the time-integrated Dalitz plot analysis using a $10^3$ fb$^{-1}$ $e^+e^- \to \psi(4040)$ data sample.}
\begin{tabular}{|c|c|c|c|}
\hline
Parameter & $x_D$ (10$^{-4}$) & $y_D$ (10$^{-4}$) & $\tau_{CP}$ (10$^{-2}$) \\
\hline
Coherent only & 12.5 & 8.7 & 5.4 \\
Incoherent only & 18.4 & 12.9 & 5.2 \\
Both & 11.8 & 8.5 & 3.8 \\
\hline
\end{tabular}
\end{table}

\footnote{The optimal energy for this kind of experiment is 4.01 GeV, for which the $D\bar{D}^*$ cross section is maximized, and $D^*\bar{D}^*$ production is below threshold.}
the flavor-tagged $D \to K_S^0 \pi^+ \pi^-$ decays in the $C = +1$ state and the incoherent flavor-tagged $D \to K_S^0 \pi^+ \pi^-$ decays for the final extraction of mixing parameters. The incoherent events can be obtained at a charm factory from $\psi(4040) \to D^{\pm} D^{*\mp}$ decays. Table 11 shows that adding the incoherent decays to the coherent sample does not improve the mixing parameters precision significantly. It is explained by the fact that the precision is determined by the flavor-tagged $D \to K_S^0 \pi^+ \pi^-$ decays in $C = -1$ state. However, the inclusion of incoherent decays improves the precision of $CP$ violation parameters.

A more complicated analysis can also include the double-tagged $D \to K_S^0 \pi^+ \pi^-$ decays in $C = +1$ state which also include the mixing parameters at the first order. Our estimation of the mixing parameters precision is based on the assumption that systematic errors can be essentially suppressed since both $C = +1$ and $C = -1$ decays have similar kinematics and may be detected simultaneously in the experiment.

Equations (22) and (23) imply the absence of direct $CP$ violation in $D$ decays. It is, however, possible to take its effect into account. This requires doubling the number of parameters $K_i$, $C_i$, $S_i$ (by treating decays of $D^0$ and $\bar{D}^0$ separately for self-conjugate final states like $K_S^0 \pi^+ \pi^-$, or by decoupling the numbers of events in $CP$-conjugate modes for final states like $K^- \pi^+ \pi^0$). The number of equations used to constrain the phase terms $C_i$, $S_i$ from the double-tagged $D \to K_S^0 \pi^+ \pi^-$ decays will stay the same, but for a reasonably large number of bins the system of equations will remain solvable. Consequently, the method can be used to distinguish between direct $CP$ violation and $CP$ violation in mixing.

$CP$ violation in mixing of neutral kaons can mimic $CP$ violation in charm if the final state $K_S^0 \pi^+ \pi^-$ or other states including $K_S^0$ mesons are used in the analysis. $CP$-violating terms in the amplitudes $A_{1,2}$ are of the order $c\lambda^2$, where $\epsilon \simeq 2.2 \times 10^{-3}$ [24] is the magnitude of $CP$ violation in kaon mixing, and $\lambda^2 = \sin^2 2\theta_C \simeq 0.23^2$ gives the relative difference between the $D^0 \to K_S^0 \pi^+ \pi^-$ and $D^0 \to K_L^0 \pi^+ \pi^-$ amplitudes. If not accounted for, $CP$ violation in the kaon sector will introduce fake $CP$ violation in charm of the order $r_{CP} \sim c\lambda^2 / (x_D^2 + y_D^2) \sim 1\%$. However, once the amplitude of the $D^0 \to K_L^0 \pi^+ \pi^-$ decay is measured (this can be done at a charm factory exploiting kinematic reconstruction of the missing $K_L^0$ at $D\bar{D}$ threshold), this effect can be corrected for in the charm mixing measurement.

VI. EXTENSION TO OTHER $D^0$ DECAY MODES

The precision of the charm mixing parameters measurements can be improved by adding other three and four-body hadronic final states such as $K^- \pi^+ \pi^0$ and $K^- \pi^+ \pi^- \pi^+$ in addition to $D \to K_S^0 \pi^+ \pi^-$. In these cases the phase terms $C_i$, $S_i$ can be defined in the same way as for $D \to K_S^0 \pi^+ \pi^-$. The advantage of using flavor-tagged Cabibbo-favored and doubly Cabibbo-suppressed $D^0 \to K^{\pm} \pi^\pm \pi^0$ three-body decays for the mixing measurement is that the relative size of the interference term can be made much larger than in the case of $D \to K_S^0 \pi^+ \pi^-$. For example, if we observe decays where one of the $D$ mesons is tagged by its semileptonic decay to be $D^0$, and the other one is reconstructed in the $K^- \pi^+ \pi^0$ state ("wrong-sign" tag), the first term in Eq. (24) is of the order $R_{K^{+}\pi^{0}} \sim 0.06 \times 0.06$, whereas the interference term is of the order $\sqrt{R_{K^{+}\pi^{0}}(x_D, y_D)} \sim 0.06 \times 0.01$. While the statistical sensitivity to the mixing parameters should be similar compared to the previous analysis, the systematic error can be reduced since the relative magnitude of the interference term that contains the mixing parameters is larger, and thus is less sensitive to background and efficiency uncertainties.

Since the statistical sensitivity depends on the structure of the decay amplitude, we perform a MC study for the $D^0 \to K^{+} \pi^\pm \pi^0$ final state. The description of $CP$ and $D^0 \to K^{+} \pi^\pm \pi^0$ decay amplitudes is based on quasi two-body models obtained by CLEO [24] and BaBar [25].

As in the case of the $K_S^0 \pi^+ \pi^-$ decay, we use a uniform phase binning [19]: Fig. 1 shows the binning obtained from the $CP$ and $D$ decays used in our study. We use a sample corresponding to an integrated luminosity of $10^3$ fb$^{-1}$ (as for the $D \to K_S^0 \pi^+ \pi^-$ decay). The results of our calculation of the statistical uncertainties of the mixing parameters as functions of the average strong phase difference between $CP$ and $D$ decay amplitudes $\delta_D^{K^{+} \pi^{0}}$ are shown in Fig. 2. Apparently, there is a symmetry: $\sigma_{x_D} (\delta_D^{K^{+} \pi^{0}}) = \sigma_{y_D} (\delta_D^{K^{+} \pi^{0}} \pm \pi/2)$. The strong phase difference obtained by CLEO equals $227^{+14}_{-17}$. Our esti-
A model-independent method to measure charm mixing parameters from time-integrated analysis at a charm factory is proposed. Using a MC simulation we find that the sensitivity to both mixing parameters $x_D$ and $y_D$ is of order $10^{-3}$ for one year of data taking with luminosity $10^{35}$ cm$^{-2}$s$^{-1}$ at the peak of $\psi(4040)$ resonance. The method does not rely on absolute branching fraction measurements, and therefore does not contain uncertainties related to measurements of absolute efficiency or values of the branching fractions of the decays involved. Due to strong phase variations over the phase space of the $D \rightarrow K_S^0 \pi^+ \pi^-$ decay, nearly equal sensitivity to both $x_D$ and $y_D$ is obtained. The proposed method can also be used to measure the $C\bar{P}$ violation parameters of charm mixing.

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**Appendix A: Formalism for coherent $D^0$ decays with charm mixing**

We denote the amplitude of $D^0 (\bar{D}^0)$ decay to the final state $f$ as $A (\bar{A})$. In the case of decay to three spin-zero particles, an amplitude $A$ depends on two kinematic parameters (here we use two squared invariant masses $m_{12}^2$ and $m_{13}^2$ of the decay products). The density of the $D^0$ decay Dalitz plot in the absence of charm mixing is

$$P (m_{12}^2, m_{13}^2) = |A (m_{12}^2, m_{13}^2)|^2 .$$  \(A1\)

Charm mixing modifies this expression as

$$P' (m_{12}^2, m_{13}^2, t) = \left| \chi(t) A + i \frac{y}{p} \sigma(t) \bar{A} \right|^2 ,$$  \(A2\)

where $t$ is the time between the production and the decay. Here we omit the dependence of $A$ and $\bar{A}$ on the Dalitz plot variables. The time-dependent terms are given by

$$\chi(t) + i \sigma(t) = \exp \frac{\Gamma t}{2} (-1 + x_D i y_D) .$$  \(A3\)

After integration over time we get

$$P' (m_{12}^2, m_{13}^2) = a_0 P + a_1 r_{CP} P + a_2 + a_3 \sqrt{P} .$$  \(A4\)

We have investigated the impact of charm mixing on all stages of the binned model-independent measurement of the CKM phase $\gamma$ from $B^+ \rightarrow DK^+$, $D \rightarrow K_S^0 \pi^+ \pi^-$ decays. We show that ignoring the mixing at all stages of the analysis is safe at the current statistical accuracy: the bias on $\gamma$ with the currently measured mixing parameters is of order 0.2$^\circ$.

A model-independent approach to perform a time-dependent Dalitz plot analysis of the $D \rightarrow K_S^0 \pi^+ \pi^-$ decay to extract charm mixing parameters was considered. The approach is promising for high-statistics analyses at the LHCb experiment and at SuperB factories.

We find that in the coherent production of the $D^0 (\bar{D}^0)$ system in $e^+ e^-$ collisions, the effect of charm mixing can be enhanced if the $\bar{D}^0$ is reconstructed in the $D^0 \gamma$ decay, which requires the $D^0 \bar{D}^0$ system to be in a $C = +1$ state.

A model-independent approach to perform a time-integrated analysis at a charm factory is proposed. Using a MC simulation we find that the sensitivity to both mixing parameters $x_D$ and $y_D$ is of order $10^{-3}$ for one year of data taking with luminosity $10^{35}$ cm$^{-2}$s$^{-1}$ at the peak of $\psi(4040)$ resonance. The method does not rely on absolute branching fraction measurements, and therefore does not contain uncertainties related to measurements of absolute efficiency or values of the branching fractions of the decays involved. Due to strong phase variations over the phase space of the $D \rightarrow K_S^0 \pi^+ \pi^-$ decay, nearly equal sensitivity to both $x_D$ and $y_D$ is obtained. The proposed method can also be used to measure the $C\bar{P}$ violation parameters of charm mixing.

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**Appendix A: Formalism for coherent $D^0$ decays with charm mixing**

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$$\chi(t) + i \sigma(t) = \exp \frac{\Gamma t}{2} (-1 + x_D i y_D) .$$  \(A3\)

After integration over time we get

$$P' (m_{12}^2, m_{13}^2) = a_0 P + a_1 r_{CP} P + a_2 + a_3 \sqrt{P} .$$  \(A4\)
where

\[ r_{CP} e^{i\alpha_{CP}} = \frac{q}{p}, \]

\[ S^\pm = \sin(\Delta \delta_D \pm \alpha_{CP}), \]

\[ C^\pm = \cos(\Delta \delta_D \pm \alpha_{CP}), \]

(A5)

\[ \Delta \delta_D = (\delta_D (m_{12}^2, m_{13}^2) - \delta_D (m_{12}^2, m_{13}^2)) \]

is a difference of strong phases for the decays \( D^0 \rightarrow f \) and \( \bar{D}^0 \rightarrow f \), and

\[ a_0 = \frac{1}{2} \left( \frac{1}{1 - y_D^2} + \frac{1}{1 + x_D^2} \right) \]

\[ = 1 + \frac{1}{2} (-x_D^2 + y_D^2) + O ((x_D + y_D)^3), \]

(A6)

\[ a_1 = \frac{1}{2} \left( \frac{1}{1 - y_D^2} - \frac{1}{1 + x_D^2} \right) \]

\[ = \frac{1}{2} (x_D^2 + y_D^2) + O ((x_D + y_D)^3), \]

\[ a_2 = \frac{y_D}{1 - y_D^2} = y_D + O ((x_D + y_D)^3), \]

\[ a_3 = \frac{x_D}{1 + x_D^2} = x_D + O ((x_D + y_D)^3). \]

The expressions for \( \bar{D}^0 \) decays can be obtained after the substitutions \( p \leftrightarrow q \) and \( P \leftrightarrow \bar{P} \):

\[ \bar{P}' \left( m_{12}^2, m_{13}^2 \right) = a_0 \bar{P} + a_1 r_{CP}^{-2} \bar{P} + r_{CP}^{-1} \sqrt{P \bar{P}} (C_{-a_2} + S_{-a_3}) \]

(A7)

Now we consider the decay of the coherent \( D^0 \bar{D}^0 \) pair in a \( C = -1 \) or \( C = +1 \) state. Assuming the particle denoted with the index “1” decayed first, the Dalitz plot density accounting charm mixing effects is given by the expression

\[ P^C_{corr} \left( (m_{12}^2), (m_{13}^2), (m_{12}^2), (m_{13}^2) \right) = b_0^C \left[ P_1 \bar{P}_2 + \bar{P}_1 P_2 + 2C \sqrt{P_1 \bar{P}_1 P_2 \bar{P}_2} (C_1 C_2 + S_1 S_2) \right] \]

\[ + b_1^C \left[ r_{CP}^{-2} P_1 P_2 + r_{CP}^{-2} \bar{P}_1 \bar{P}_2 + 2C \sqrt{P_1 \bar{P}_1 P_2 \bar{P}_2} (C_1^+ C_2^+ - S_1^+ S_2^+) \right] \]

\[ + b_2^C \left[ \sqrt{P_1 \bar{P}_1} C_1^+ (r_{CP} \bar{P}_1 + r_{CP}^{-1} P_1) + C \sqrt{P_1 \bar{P}_1} C_1^+ \right. \]

\[ \left. \left( r_{CP} \bar{P}_2 + r_{CP}^{-1} P_2 \right) \right] \]

\[ + b_3^C \left[ \sqrt{P_2 \bar{P}_2} S_2^+ (r_{CP} \bar{P}_1 - r_{CP}^{-1} P_1) + C \sqrt{P_2 \bar{P}_2} S_2^+ \left( r_{CP} \bar{P}_2 - r_{CP}^{-1} P_2 \right) \right]. \]

(A8)

where

\[ b_0^C = \frac{1}{2} \left[ \frac{1 + C y_D^2}{(1 - y_D^2)^2} + \frac{1 - C x_D^2}{(1 + x_D^2)^2} \right] \approx a_0 + \frac{C + 1}{2} \left( x_D^2 + y_D^2 \right), \]

\[ b_1^C = \frac{1}{2} \left[ \frac{1 + C y_D^2}{(1 - y_D^2)^2} - \frac{1 - C x_D^2}{(1 + x_D^2)^2} \right] \approx (C + 2) a_1, \]

\[ b_2^C = \frac{(1 + C) y_D}{(1 - y_D^2)^2} \approx (1 + C) a_2, \]

\[ b_3^C = \frac{(1 + C) x_D}{(1 + x_D^2)^2} \approx (1 + C) a_3, \]

(A9)

and \( C = \pm 1 \) for the symmetric and antisymmetric states. Note that in the case \( C = -1 \) the interference term vanishes \([27]\), while for \( C = +1 \) it is doubled compared to the incoherent case.

The expression for the \( D \rightarrow K_S^0 \pi^+ \pi^- \) decay density in the \( B^\pm \rightarrow DK^\pm \) process accounting for the charm mixing
contribution is
\[
P_{B^\pm}(m_\pm^2, m_\mp^2) = a_0 \left[ P + r_B^2 P + 2\sqrt{P\overline{P}}(x_B C + y_B S) \right]
+ a_1 \left[ r_{C_F}^2 P + r_{C_F}^2 r_B^2 P + 2\sqrt{P\overline{P}}(x_B C^\pm - y_B S^\pm) \right]
+ a_2 \left[ x_B^\pm (r_{C_F}^1 P + r_{C_F}^1 r_B^1 P) + \sqrt{P\overline{P}}(r_{C_F}^1 C^\pm + r_{C_F}^1 r_B^1 C^\mp) \right]
+ a_3 \left[ y_B^\pm (r_{C_F}^1 P - r_{C_F}^1 r_B^1 P) + \sqrt{P\overline{P}}(r_{C_F}^1 - r_{C_F}^1 r_B^1 S^\pm) \right],
\]
where \(x_B^\pm = r_B \cos (\delta_B \pm \gamma \pm \alpha_{CP}), y_B^\pm = r_B \sin (\delta_B \pm \gamma \pm \alpha_{CP})\).

| Type | Number of events (10^6) |
|------|-------------------------|
| K^0 \pi^+ \pi^- | K^- \pi^+ \pi^0 |
| Incoherent flavor tagged | 6 | 30 |
| Coherent \(C = -1\) flavor tagged | 2.1 | 10.5 |
| Coherent \(C = +1\) flavor tagged | 1.4 | 7 |
| Coherent \(C = -1\) | 0.6 | 13 |

### Appendix B: Numerical calculation procedure

The numerical results presented in Sections III and VI are obtained using a procedure described below. In the model-independent approach, the phase space of the decay is divided into bins and we deal with numbers of events in each bin. The maximum likelihood method is used to obtain the parameters of interest from these numbers. The likelihood function is defined as

\[-2\log L = -2 \sum_i \log P(X_i, \langle X_i \rangle), \tag{B1}\]

where \(P(X, \langle X \rangle)\) is the Poisson probability function to observe \(X\) events with an expected number of events \(\langle X \rangle\).

The observed numbers of events \(X_i\) are obtained using the exact formulas with mixing (see Appendix A). When the systematic effect of charm mixing on the \(\gamma\) measurement is studied, the values of \(X_i\) are fixed (which corresponds to infinite statistics), while in the study of the statistical sensitivity of the mixing measurement in Sections V and VI we sample the \(X_i\) values according to the Poisson distribution.

The expected numbers of events \(\langle X_i \rangle\) include the free parameters of interest (\(x_B\) and \(y_B\) in Section III) or mixing parameters in Sections V and VI and are expressed via the numbers of events in flavor-specific \(D^0\) state \(K_i\) and the phase terms \(C_i, S_i\). The values of \(K_i\), \(C_i\) and \(S_i\) are calculated from the \(D \rightarrow K_S^0 \pi^+ \pi^-\) and \(D^0 \rightarrow K^+ \pi^+ \pi^0\) decay amplitudes [7, 24, 25] as

\[
K_i = \int_{D_i} p_D dm_x^2 dm_y^2, \tag{B2}
\]

\[
C_i = \frac{\int_{D_i} \sqrt{p_D} dm_x^2 dm_y^2 \int_{D_i} \overline{p}_D dm_x^2 dm_y^2}{\sqrt{\int_{D_i} \sqrt{p_D} dm_x^2 dm_y^2 \int_{D_i} \overline{p}_D dm_x^2 dm_y^2}}, \tag{B3}
\]

\[
S_i = \frac{\int_{D_i} \sqrt{p_D} dm_x^2 dm_y^2 \int_{D_i} \overline{p}_D dm_x^2 dm_y^2}{\sqrt{\int_{D_i} \sqrt{p_D} dm_x^2 dm_y^2 \int_{D_i} \overline{p}_D dm_x^2 dm_y^2}}, \tag{B4}
\]

The free parameters are extracted using MINUIT to minimize the likelihood function (B1).

Table V shows the numbers of events that we use to observe \(X\) events with an expected number of events \(\langle X \rangle\). To estimate the numbers we use the detection efficiency determined by CLEO [9] and the statistics corresponding to an integrated luminosity of 10^3 fb^{-1} at the \(\psi(4040)\) resonance.

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