Non-Trivial Ghosts and Second Class Constraints

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Abstract

In a model in which a vector gauge field $W^a_\mu$ is coupled to an antisymmetric tensor field $\phi^a_{\mu\nu}$ possessing a pseudoscalar mass, it has been shown that all physical degrees of freedom reside in the vector field. Upon quantizing this model using the Faddeev-Popov procedure, explicit calculation of the two-point functions $<\phi\phi>$ and $<W\phi>$ at one-loop order seems to have yielded the puzzling result that the effective action generated by radiative effects has more physical degrees of freedom than the original classical action. In this paper we point out that this is not in fact a real effect, but rather appears to be a consequence of having ignored a “ghost” field arising from the contribution to the measure in the path integral arising from the presence of non-trivial second-class constraints. These ghost fields couple to the fields $W^a_\mu$ and $\phi^a_{\mu\nu}$, which makes them distinct from other models involving ghosts arising from second-class constraints (such as massive Yang-Mills (YM) models) that have been considered, as in these other models such ghosts decouple. As an alternative to dealing with second class constraints, we consider introducing a “Stueckelberg field” to eliminate second-class constraints in favour of first-class constraints and examine if it is possible to then use the Faddeev-Popov quantization procedure. In the Proca model, introduction of the Stueckelberg vector is equivalent to the Batalin-Fradkin-Tyutin (BFT) approach to converting second-class constraints to being first class through the introduction of new variables. However, introduction of a Stueckelberg vector is not equivalent to the BFT approach for the vector-tensor model. In an appendix, the BFT procedure is applied to the pure tensor model and a novel gauge invariance is found. In addition, we also consider extending the Hamiltonian so that half of the second-class constraints become first-class and the other half become associated gauge conditions. We also find for this tensor-vector theory that when converting the phase space path integral to the configuration space path integral, a non-trivial contribution to the measure arises that is not manifestly covariant and which is not simply due to the presence of second class constraints.
1 Introduction

Some time ago, a model in which a non-Abelian vector gauge field coupled to an antisymmetric tensor field that has a pseudo-scalar mass term was introduced [1]. The original motivation for considering this model was to see if the mass parameters occurring in this model could induce a pole in the propagator for the vector field that would be away from the massless limit, thereby providing an alternative to the Higgs mechanism for giving a mass to vector fields. Although this hope was not realized, it became apparent that this model is interesting for an unexpected reason: the presence of a pseudo-scalar mass term for the antisymmetric tensor field serves to eliminate all physical degrees of freedom from the model except for the usual transverse degrees of freedom present in the vector gauge field. This elimination of any physical degrees of freedom associated with the tensor field is surprising, notably because of the highly non-trivial way it occurs in the original action, but also because normally introduction of a mass term into a gauge invariant model (such as Yang-Mills theory) serves to increase the number of physical degrees of freedom, not decrease their number. The model we consider has the classical Lagrangian [1]

\[ \mathcal{L}_{c1} = -\frac{1}{4} F_{a \mu \nu}^a F^{a \mu \nu} + \frac{1}{12} G_{a \mu \nu \lambda}^a G^{a \mu \nu \lambda} + \frac{m}{4} \epsilon^{\mu \nu \lambda \sigma} \phi_{a \mu}^a \phi_{a \nu}^a \]

with \( m \) and \( \mu \) being mass parameters and

\[ F_{a \mu \nu} = \partial_\mu W_{a \nu} - \partial_\nu W_{a \mu} + f^{abc} W_{a \mu} W_{b \nu}^c \]

\[ G_{a \mu \nu \lambda}^a = D_{a \mu}^b \phi_{a \nu \lambda}^b + D_{a \nu}^b \phi_{a \mu \lambda}^b + D_{a \lambda}^b \phi_{a \mu \nu}^b \]

and

\[ D_{a \mu}^b = \partial_\mu \delta_{a b} + f^{acb} W_{a \mu}^p \quad ([D_{a \mu}, D_{a \nu}]^{ab} = f^{acb} F_{a \mu \nu}^p) \]

\((\eta_{\mu \nu} = \text{diag}(+ - - -), \epsilon^{0123} = +1)\).

In ref. [1], a constraint analysis [5-13] of this model shows that with twenty initial degrees of freedom (dof) in phase space (\( \phi_{a \mu \nu} - 6 \text{dof}, W_{a \mu}^a - 4 \text{dof}, \) plus associated momenta) and five first-class constraints, five gauge conditions and six-second class constraints, there are four net physical degrees of freedom, provided \( \mu^2 \neq 0 \). In the Abelian limit, explicit elimination of fields by use of equations of motion that are free of time derivatives shows that these are the usual transverse polarizations of the vector \( W_{a \mu} \) and their associated momenta.

It was clearly of interest to see if the interactions between the tensor and vector fields appearing in eq. (1) would somehow include dynamical degrees of freedom when radiative effects were taken into account. The one-loop contributions to the two-point functions \( < W_{a \mu}^a W_{a \nu}^b > [2], < \phi_{a \mu \nu} \phi_{a \lambda \sigma}^b > \)
\[ \delta W_a^\mu = D_{ab}^\mu \theta^b \]  
\[ \delta \phi_{\mu \nu}^a = f^{abc} \phi_{\mu \nu}^b \theta^c \]  
results in the necessity of gauge fixing. The Feynman gauge fixing Lagrangian

\[ L_{gf} = -\frac{1}{2} (\partial \cdot W)^2 \]  
leads normally to the introduction of the usual Faddeev-Popov (FP) ghost fields \( c^a \), \( \bar{c}^a \) whose action

\[ L_{gh} = \bar{c}^a (\partial \cdot D^{ab}) c^b \]  
involves a coupling of the ghosts to the vector. The effective action \( L_{c1} + L_{gf} + L_{gh} \) (eqs. (1,7,8)) has the usual BRST invariance [6-].

Using the Feynman rules based on \( L_{c1} + L_{gf} + L_{gh} \), an explicitly calculation of the one-loop two point function \( < W_a^\mu W_b^\nu > \) results in a complete cancellation of all diagrams involving the field \( \phi_{\mu \nu}^a \) [2]; the result is identical to what arises from a pure YM theory. This is not unexpected as the analysis of the canonical structure of the classical theory indicates that all degrees of freedom in the model reside solely in the vector field.

However, an analogous calculation of the one-loop, two-point functions \( < \phi_{\mu \nu}^a \phi_{\lambda \sigma}^b > \) and \( < \phi_{\mu \nu}^a W_{\lambda}^b > \) using dimensional regularization show that these diverge [3,4]. Only by using non-local counter terms proportional to \( m \) can these divergences be removed, implying that \( \phi_{\mu \nu}^a \) develops degrees of freedom radiatively. This is most peculiar, especially since the shift in the antisymmetric tensor field so that

\[ \phi_{\mu \nu}^a = \chi_{\mu \nu}^a - \frac{m}{\mu^2} F_{\mu \nu}^a \]  
leads to

\[ L_{c1} = -\frac{1}{4} F_{\mu \nu}^a F^{a \mu \nu} + \frac{1}{12} H_{\mu \lambda}^a H^{a \mu \lambda} + \frac{\mu^2}{8} \epsilon_{\mu \nu \lambda \sigma} \chi_{\mu \nu}^a \chi_{\lambda \sigma}^a \]

\[ + \frac{m^2}{8 \mu^2} \epsilon_{\mu \nu \lambda \sigma} F_{\mu \nu}^a F_{\lambda \sigma}^a. \]  
\( (H_{\mu \lambda}^a \equiv D_{\mu \lambda \nu \sigma}^b + \ldots ) \)

The last term in eq. (10) is topological and does not contribute to perturbative calculations; the remaining terms are identical in form to those in eq. (1) with \( m = 0 \). Since by refs. [3,4] \( < \phi \phi > \) and \( < \phi W > \) appear to have divergent parts proportional to \( m \), this would mean that \( < \chi \chi > \) and
$< \chi W >$ should both be free of divergences. However, this would all be apparently inconsistent with eq. (9), as this equation superficially implies that

$$< \phi \phi > = < \chi \chi > - \frac{m}{\mu^2} (< \chi F > + < F \chi >) + \frac{m^2}{\mu^4} < FF >$$

(11)
as well as

$$< \chi \chi > = < \phi \phi > - \frac{m}{\mu^2} (< \phi F > + < F \phi >) + \frac{m^2}{\mu^4} < FF > .$$

(12)

The results of refs. [3,4] are inconsistent with eqs. (11,12).

These inconsistencies have motivated us to see if the structure of the model itself somehow invalidates the quantization procedure used to compute radiative effects in refs. [2-4]. It turns out that the presence of second-class constraints in the model leads to a non-trivial (field dependent) contribution to the measure of the path integral that is not taken into account when using the Fadeev-Popov procedure [17] to “factor out the superfluous degrees of freedom associated with gauge invariance of eqs. (5,6). Although the contribution of second-class constraints to the measure of the path integral have been understood for some time [21, 22], the model of eq. (1) provides the first example we know of in which the second-class constraints make a non-trivial (viz. field-dependent) contribution to the measure, thereby necessitating thereby necessitating the introduction of a new type of Grassmann “ghost” field to the effective action. In the following section we explicitly show how this new ghost arises. Following that, in section 3 we convert the path integral from phase space to configuration space using the approach of ref. [24]. In contrast to what happens in Yang-Mills theory [20, 43], the resulting path integral in configuration space is not manifestly covariant.

## 2 Second-Class Constraints

In attempting to compute radiative effects in YM theory using the same approach that worked in quantum electrodynamics, Feynman encountered an inconsistency that could be overcome by introducing Fermionic scalar “ghost” fields [14]. These FP fields were introduced more formally by Mandelstam [15], DeWitt [16] and Faddeev-Popov [17]. (For a more general discussion of gauge fixing which leads to having two Fermionic and one Bosonic ghost, see refs. [18,19].) This approach has been adequate for dealing with YM gauge theories coupled to matter (as is appropriate for the standard model) but a more general discussion is needed to eliminate superfluous degrees of freedom in more complicated models.

Beginning with the canonical analysis of constrained systems, Faddeev examined how a system with only first-class constraints could be quantized using the path integral [20]. He argued that for YM theory this approach is equivalent to that of ref. [17] in which “gauge orbits” are factored out of the path integral over all field configurations. His analysis was extended to systems with second-class constraints by Fradkin [21] and Senjanovic [22]. In a system with a denumerable number of
degrees of freedom \( (q^i(t), p_i(t)) \) in phase space, the matrix element for the \( S \)-matrix is

\[
< \text{out}|S|\text{in} > = \int \exp i \int_{-\infty}^{\infty} dt [p_i \dot{q}^i - H(q^i, p_i)] D\mu(q^i(t), p_i(t))
\]

where the measure is

\[
D\mu(q^i(t), p_i(t)) = \prod_a \delta(\phi_a) \delta(\chi_a) \det \{ \phi_a, \chi_b \} \prod_b \delta(\theta_b) \det^{1/2} \{ \theta_a, \theta_b \} \ Dq^i(t) Dp_i(t)
\]

where \( q^i(t) \rightarrow (q^i_{\text{out}}, q^i_{\text{in}}) \) as \( t \rightarrow \pm \infty \), \( H \) is the canonical Hamiltonian, \( \phi_a, \chi_a \) and \( \theta_a \) are the first-class constraints, associated gauge conditions and second-class constraints respectively, and \( \{ , \} \) denotes the Poisson Bracket (PB). Eq. (13) is independent of the choice of gauge conditions \( \chi_a \) provided \( \det \{ \phi_a, \chi_b \} \neq 0 \). Formally the factor of \( \Theta = \det^{1/2} \{ \theta_a, \theta_b \} \) appears in such theories as YM theories with a Proca mass, scalar theories quantized using light-cone coordinates and models with magnetic monopoles [22]; it is completely absent in massless YM theories [23]. However, there do not appear to be any examples in the literature for field theoretic models in which \( \Theta \) explicitly involves either \( q^i(t) \) or \( p_i(t) \) and hence \( \Theta \) can generally be absorbed into a normalization factor.

We note that it is possible to absorb the functional integration over the momentum \( p_i(t) \) in eq. (13) entirely into the measure, leaving the argument of the exponential to be \( i \int_{-\infty}^{\infty} dt L(q^i, \dot{q}^i) \) where \( L \) is the Lagrangian of the system [24]. It is also possible to generalize the quantization procedure to deal with gauge theories possessing open algebras. (For reviews, see refs. [7-13].)

It has also been demonstrated that second-class constraints can be converted into first-class ones by the introduction of new variables accompanied by an extension of the Hamiltonian so that the modified theory possesses a gauge invariance not originally present [25-28]. We will illustrate how this “BFT” procedure works by considering a massive Proca vector field whose action is

\[
\mathcal{L}_p = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{m^2}{2} A_\mu A^\mu.
\]

If \( \pi^0 \) and \( \pi^i \) are the momenta associated with \( A_0 \) and \( A_i \) respectively, it is easy to see that there is a primary second-class constraint

\[
\theta_1 = \pi^0 = 0
\]

and a secondary second-class constraint

\[
\theta_2 = \partial_i \pi^i + m^2 A_0 = 0
\]

along with the canonical Hamiltonian

\[
\mathcal{H}_c = \frac{1}{2} \pi^i \pi^i + \frac{1}{4} F_{ij} F_{ij} - \frac{m^2}{2} (A_0^2 - A_i A_i) - A_0 \partial_i \pi^i.
\]
There are no first-class constraints, which is consistent with there being no gauge invariance in the Proca Lagrangian.

With the BFT procedure, we can introduce fields $\eta_1$ and $\eta_2$ such that \[ \{\eta_i, \eta_j\} = m^2 \epsilon_{ij} \] (19) and then replace eqs. (16-18) by \[ \bar{\theta}_1 = \pi^0 + \eta_1 \] (20) \[ \bar{\theta}_2 = \partial_i \pi^i + m^2 A_0 + \eta_2 \] (21) \[ \bar{\mathcal{H}}_p = \mathcal{H}_p - (\partial_i A_i) \eta_1 - \frac{1}{m^2}(\partial_i \pi^i + m^2 A_0) \eta_2 - \frac{1}{2m^2}(\eta_2^2 + \eta_1 \nabla^2 \eta_1). \] (22)

We now find that $\bar{\theta}_1$ and $\bar{\theta}_2$ are first-class constraints that have a weakly vanishing PB with $\bar{\mathcal{H}}_p$. A gauge invariance that consequently occurs in this modified theory can be worked out using either the approach of ref. [31] or ref. [32]. No second class constraint consequently occurs in the path integral of eqs. (13,14) which are associated with this model. In the gauge $\eta_1 = \eta_2 = 0$ the Proca model is recovered.

The introduction of extra fields to preserve a gauge invariance was an idea that originated with Stueckelberg [33], though not in the context of the constraint formalism. If the action of eq. (15) were replace by \[ \mathcal{L}_s = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} (A_\mu + \frac{1}{m} \partial_\mu \sigma)^2 \] (23) where $\sigma$ is the “Stueckelberg scalar”, then one has the gauge invariance \[ \delta A_\mu = \partial_\mu w \] (24) \[ \delta \sigma = -mw. \] (25)

One can show that the introduction of the Stueckelberg scalar into the Proca Lagranian is equivalent to the introduction of the fields $\eta_1$ and $\eta_2$ through the BFT procedure by performing a constraint analysis of $\bar{\mathcal{L}}_s = \mathcal{L}_s - m \partial_0 (\sigma A_0)$. From $\bar{\mathcal{L}}_s$ we find only the primary and secondary first-class constraints \[ \pi^0 + m \sigma = 0 \] (26) \[ \partial_i \pi^i + m^2 A_0 + m \sigma = 0 \] (27) which suggests, upon comparing eqs. (20,21) with eqs. (25,26), that \[ \eta_1 = m \sigma \] (28) \[ \eta_2 = m \pi. \] (29)
(There are no second-class or tertiary constraints.)

The Hamiltonian that follows from \( \overline{T}_s \) is \( \mathcal{H}_p \) in eq. (22) provided one employs the constraint of eq. (21). (The fields \( A_0, A_i \) and \( \sigma \) have conjugate momenta \( \pi_0, \pi^i \) and \( \pi \) respectively.)

The Stueckelberg model of eq. (23) can be quantized using the FP procedure. using the gauge fixing Lagrangian

\[
\mathcal{L}_{gf} = -\frac{1}{2\alpha}(\partial_\mu A^\mu - \alpha m\sigma)^2
\]

is particularly convenient as in this gauge \( A_\mu \) and \( \sigma \) decouple and the renormalizability of a model in which \( A_\mu \) is coupled to a conserved current becomes apparent. In the gauge \( \sigma = 0 \), we recover the Proca model and one should in principle consider how the second-class constraints of eqs. (16,17) contribute to the measure of the classical action, though in practice this measure can be ignored as \( \{\theta_1, \theta_2\} = -m^2 \) which is just a constant.

The constraints of the model in eq. (1) are much more complicated than those of the Proca model; in particular the PB of the second class constraints is no longer field independent. Following the procedure used in ref. [1] to analyze the constraints in the Abelian limit of \( \mathcal{L}_{cl} \), we define

\[
U^a = W^a_0, \quad V_i^a = W^a_i, \quad A^a_k = \phi^a_{0k}, \quad B^a_k = \frac{1}{2}\epsilon_{k\ell m}\phi^a_{\ell m}.
\]

When \( \mathcal{L}_{cl} \) is written in terms of these fields so that

\[
\mathcal{L} = -\frac{1}{4}(F_{ij}^a)^2 + \frac{1}{2}(\dot{V}_i^a - D_i^a U^b)^2 + \frac{1}{2}(\dot{B}_i^a + f^{abc}U^b V^c_i)^2
\]

\[
-(\dot{B}_i^a + f^{amn}U^m B_i^a)\epsilon_{ijk}(D_j A_k)^a \alpha \frac{1}{2}(\epsilon_{ijk}(D_j A_k)^a)^2 - \frac{1}{2}(D_i B_j)^a(D_j B_i)^a
\]

\[
+\mu^2 A_i^a B_i^a + \frac{m}{2}\left[\epsilon_{ijk}A^a_i F^a_{jk} + 2B^a_k(\dot{V}_k^a - \partial_k U^a + f^{abc}U^b V^c_k)\right]
\]

we find that their respective canonical momenta are

\[
\pi_i^V = 0, \quad \pi_i^V = \partial_0 V_i^a - (D_i U)^a + mB_i^a, \quad \pi_i^{Aa} = 0, \quad \pi_i^{Ba} = (D_0 B_i)^a - \epsilon_{ijk}(D_j A_k)^a,
\]

from which follows the canonical Hamiltonian

\[
\mathcal{H}_c = \frac{1}{2}\pi_i^V \pi_i^V + \frac{1}{2}\pi_i^{Ba} \pi_i^{Ba} + \pi_i^V (D_i U)^a + \pi_i^{Ba} \epsilon_{ijk}(D_j A_k)^a
\]

\[
+ f^{abc}U^a \pi_i^{Bb} B_i^c + \frac{1}{4}F_{ij}^a F_{ij}^a + \frac{1}{2}(D_i B_j)^a(D_j B_i)^a
\]

\[
- \mu^2 A_i^a B_i^a - m\pi_i^V B_i^a - \frac{1}{2}\epsilon_{ijk}A^a_i F^a_{jk} + \frac{m^2}{2}B_i^a B_i^a.
\]

The primary constraints of eqs. (32a,c) lead respectively to the secondary constraints

\[
S^a_i = (D_i \pi_i^V)^a + f^{abc}B_i^b \pi_i^{Bc}
\]
\[ S_i^a = \epsilon_{ijk} D_j^{ab} \pi_k^b - \mu^2 B_i^a - \frac{m}{2} \epsilon_{ijk} F_{jk}^a. \]  

(34b)

These constraints have the PB algebra

\[ \{ S_i^a, S_j^b \} = f^{abc} S_c, \quad \{ S_i^a, S_j^b \} = f^{abc} S_c. \]  

(35a - c)

The PB of \( S^a \) with \( \int \mathcal{H}_c dx \) weakly vanishes. However, the PB of \( S_i^a \) with \( \int \mathcal{H}_c dx \) yields the tertiary constraint

\[ T_i^a = -\mu^2 \pi_i^B + \mu^2 f^{abc} B_i^c + \epsilon_{ijk} \left[ f^{abc} (\pi_j^b \pi_k^c) + (D_j U)^b \pi_k^B - m B_j^b \pi_k^B \right] - D_j^a \left( f^{bcd} U^c_{,k} \pi_k^B \right) \\
+ (D_j D_i D_k D_l)^a - m(D_j D_k U)^a. \]  

(36)

We see that since

\[ \{ T_i^a, S_j^b \} = f^{abc} T_c. \]  

(37a)

\[ \{ S_i^a, T_j^x \} = f^{apm} f^{exp} \left( \delta_i^a \pi_k^B \pi_k^B - \pi_i^B \pi_i^B \right) + \delta_i^a \left( D_j D_k D_l D_j - D_j D_j D_k D_l \right) \delta_j^a \\
+ (D_i D_i D_j D_j + D_j D_j D_i D_i - D_j D_i D_i D_j - D_l D_j D_j D_j) \delta_i^a \\
+ \mu^2 \left[ \mu^2 \delta_i^a \delta_j^a + \epsilon_{ilm} \left( -D_m^a f^{xyz} U^z \right) \right] - f^{aexp} \pi_m^p - f^{aexp} (D_m U)^p + (D_m^a f^{awz} U^w) + m f^{awx} B_m^y \right] \]  

(37b)

both \( S_i^a \) and \( T_i^a \) are irreducible second class even in the Abelian limit if \( \mu^2 \neq 0 \). There are five first-class constraints \( (\pi^U, \pi_i^A, S^a) \) and six second-class constraints \( (S_i^a, T_i^a) \) which accounts for why sixteen of the twenty degrees of freedom in phase space \( (W_\mu^a, \phi_\mu^a) \) and their associated momenta) are non-physical. The remaining four degrees of freedom are the two transverse polarizations of \( W_\mu^a \) and their conjugate momenta. The gauge transformation generated by \( (\pi^U, S^a) \) is the usual non-Abelian gauge transformation of eqs. (5,6) which is a closed, irreducible gauge transformation that forms a Lie algebra [13]. Just as in the canonical analysis of the first order Einstein-Hilbert action in \( d > 2 \) dimensions, the second class constraints must be eliminated before the algebra of first class constraints is fixed. In this case, we must first eliminate \( S_i^a \) and \( T_i^a \); otherwise we might conclude that \( \pi^U \) is second class as \( \{ \pi^U, T_j^a \} \neq 0 \).

When quantizing the model of eq. (1) using the path integral, eqs. (13,14) show that if second-class constraints are present, the factor of \( \Theta = \det^{1/2} \{ \theta_a, \theta_b \} \) contributes to the measure. This can be exponentiated by use of a Grassmann “ghost” field \( d^a \) and hence absorbed into the effective action,

\[ \Theta = \int Dd^a \exp i \int d^4x \, d^a \{ \theta_a, \theta_b \} \, d^b \]  

(38)

\[ = \int Dd^a \exp i \int d^4x \, \mathcal{L}_{\text{ghost}}. \]
Unlike the massive vector Proca model of eq. (15), this factor of $\Theta$ for the model of eq. (1) is not constant, but rather is field-dependent as can be seen from eq. (37). In fact, eq. (35a) shows that for this model

$$\Theta = \det \{ S_i^a, T_i^b \}$$

which by eqs. (37,38) becomes

$$= \int D\ell^a Dd^a \exp i \int d^4 x L_{\text{ghost}}$$

where

$$L_{\text{ghost}} = d^a \{ S_i^a, T_i^\ell \} d_i^\ell$$

where $\{ S_i^a, T_i^\ell \}$ is given by eq. (37b). Unfortunately, the calculations of $< W W >$, $< W \phi >$ and $< \phi \phi >$ in refs. [2-4] are deficient as the contribution coming from eq. (40) has been ignored.

One might try to circumvent having to include the contribution to radiative corrections coming from the field dependent of $\Theta$ by introducing a Stueckelberg field to restore gauge invariance, as was done in the case of the Proca model discussed above. This would hopefully eliminate all second-class constraints in the theory, making it feasible to employ the FP quantization procedure. Unfortunately, not all second-class constraints are eliminated upon introduction of a Stueckelberg field and so this hope is not realized.

To show this, we begin by introducing a Stueckelberg vector field $\sigma^a_{\mu}$. The most straightforward way of doing this is to replace $\phi^a_{\mu\nu}$ by $\phi^a_{\mu\nu} + D^a_{\mu\nu} \sigma^b_{\nu} - D^a_{\nu\mu} \sigma^b_{\nu}$ in eq. (1). This leads to

$$L = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{12} [G^{a\mu\lambda} + f^{abc} (F^{ab}_{\mu\nu} \sigma^c_{\lambda} + F^{ab}_{\nu\lambda} \sigma^c_{\mu} + F^{ab}_{\lambda\mu} \sigma^c_{\nu})]$$

$$\left[ G^{a\mu\lambda} + f^{abc} (F^{b\mu\nu} \sigma^c_{\lambda} + F^{b\nu\lambda} \sigma^c_{\mu} + F^{b\lambda\mu} \sigma^c_{\nu}) \right]$$

$$+ \frac{m}{4} \epsilon^{\mu\nu\lambda\sigma} \phi^a_{\mu\nu} F_{\lambda\sigma} + \frac{\mu^2}{8} \epsilon^{\mu\nu\lambda\sigma} (\phi^a_{\mu\nu} \phi^a_{\lambda\sigma} + 4 \phi^a_{\mu\nu} D^a_{\lambda\sigma} \sigma^b_{\lambda})$$

$$- 2 f^{abc} F^a_{\mu\nu} \sigma^b_{\lambda} \sigma^c_{\sigma} \right).$$

By construction, this Lagrangian is invariant under not only the gauge transformation of eqs. (5,6), but also the gauge transformation

$$\delta \phi^a_{\mu\nu} = D^a_{\mu} u^b_{\nu} - D^a_{\nu} w^b_{\mu}$$

$$\delta \sigma^a_{\mu} = -w^a_{\mu}$$

$$\delta W^a_{\mu} = 0.$$  

Despite the presence of this new gauge invariance we unfortunately still have second-class constraints in the model. To see this, we perform a constraint analysis of the model of eq. (41). It is sufficient to establish these points to set $W^a_{\mu} = m = 0$ and to look at the Abelian limit.
In this case, we begin by defining
\[ A_i = \phi_{0i}, \quad B_i = \frac{1}{2} \epsilon_{ijk} \phi_{jk}, \quad S = \sigma_0, \quad R_i = \sigma_i \quad (43a - d) \]
so that from eq. (41)
\[ L = \frac{1}{2} \dot{B}_k \dot{B}_k - \dot{B}_i \epsilon_{ijk} \partial_j A_k + \frac{1}{2} (\partial_k A_m \partial_k A_m - \partial_k A_m \partial_m A_k) \]
\[ - \frac{1}{2} (\partial_k B_k)^2 + \mu^2 \left( A_k B_k + \epsilon_{ijk} A_i \partial_j R_k + B_i \dot{R}_i - B_i \partial_i S \right). \]
The canonical momenta associated with these fields are respectively
\[ \pi^A_i = 0, \quad \pi^B_i = \dot{B}_i - \epsilon_{ijk} \partial_j A_k, \quad \pi^S = 0, \quad \pi^R_i = \mu^2 B_i \quad (45a - d) \]
and the Hamiltonian is
\[ H = \frac{1}{2} \pi^B_k \pi^B_k + A_i \epsilon_{ijk} \partial_j (\pi^B_i - \mu^2 R_k) - \mu^2 B_k (A_k - \partial_k S) + \frac{1}{2} (\partial_i B_i)^2. \]
The primary constraints \((\pi^A_i, \pi^S, \pi^R_i - \mu^2 B_i)\) yield respectively the secondary constraints
\[ B_i + \epsilon_{ijk} \left( \partial_j R_k - \frac{1}{\mu^2} \partial_j \pi^B_k \right) = 0 \quad (47a - c) \]
\[ \mu^2 \partial_i B_i = 0 \]
\[ -\mu^2 \pi^B_i = 0. \]
In turn, we see that since
\[ \{ \partial_i B_i, H \} = \partial_i \pi^B_i \quad (48a - b) \]
\[ \{ \pi^B_i, H \} = \partial_i \partial_j B_j + \mu^2 A_i - \partial_i S \]
there are no tertiary constraints. In total, when \(\mu^2 \neq 0\), there are five first-class constraints \((\pi^A_k, \pi^S \text{ and } \pi^R_k)\) and eight second-class constraints \((\pi^B_i, B^L_i, (\pi^R_i - \mu^2 B_i)^T \text{ and } \epsilon_{ijk} \partial_j R^T_k + B^T_i)\) where \(L\) and \(T\) refer to the longitudinal and transverse component of a vector in three dimensions. When we also include the five gauge conditions associated with the first-class constraints, we see that there are 18 constraints on the 20 variables \((\phi_{\mu\nu}, \sigma_\mu \text{ and their associated momenta})\) in phase space leaving us just two with physical degrees of freedom upon having introduced the field \(\sigma_\mu\) in addition to \(\phi_{\mu\nu}\). These degrees of freedom decouple from the tensor field, much as the Stueckelberg field in the Proca model (eq. (22)) decouples from the vector field. To see this, in \(\mathcal{L}\) given by eq. (44), the shift \(A_i \rightarrow A_i - \dot{R}^L_i + \partial_i S\) eliminates \(-\dot{R}^L_i + \partial_i S\) and its associated momentum from the model.

Unlike the case of the Proca field, we see that introduction of a Stueckelberg field to restore a gauge invariance absent in the original Lagrangian does not eliminate all second-class constraints in the model (There may be other, less obvious ways of introducing a Stueckelberg field that eliminates
all second class constraints). From our discussion of the constraints arising from the action of eq. (44), it is readily apparent that the full theory of eq. (41) has second-class constraints whose PB involves the fields $\phi_{\mu\nu}$ and $\sigma_{\mu}$. Consequently the contribution of the factor $\Theta$ to the measure of the path integral of eq. (13) will again be non-trivial even though the gauge invariance of eq. (42) is now present.

One might also attempt to eliminate the problems associated with incorporating $\Theta$ into the measure of the path integral by either directly converting second-class constraints into first-class ones \[34,35\], by treating half of the second-class constraints as begin first-class and the other half as being associated gauge conditions \[36-37\] or by introduction of new variables to convert second-class constraints into first-class ones \[25-28\]. (This is the BFT approach discussed above.) It doesn’t appear to be feasible to employ any of these three approaches to eliminate the second-class constraints present in the model of eq. (1) if both the constraints and the Hamiltonian are to be in closed form. However, in an appendix we will pursue the third approach (BFT) in the limit in which the gauge field is eliminated from this model and there is but a single tensor field $\phi_{\mu\nu}$, and to examine the feasibility of converting half of the second class constraints to gauge conditions as in refs. \[36,37\].

### 3 Conversion to the Lagrangian in the Path Integral

The path integral of eq. (13) is not in manifestly covariant form. For many models though, once the integral over $p_i(t)$ is performed, the phase of the exponential is given by the action

$$ S = \int dt (L(q^i(t), \dot{q}^i(t))) $$

and manifest covariance is restored. This is immediately true in a scalar theory with quartic self interaction. It is also true in YM theory as there the first-class constraints that are present make a contribution to the measure of eq. (14) that is equivalent to that which arises in the manifestly covariant FP procedure in the Lorenz-Feynman gauge.

However, this equivalence is not necessarily true in all gauge theories. The problem of reconciling the path integral of eqs. (13,14) which arises out of canonical quantization with the path integral derived by factoring out the integral over gauge equivalent field configuration with the phase factor taken to be $\exp iS$ \[16,17\] has been considered in \[16,42\] in the context of the action being the second order Einstein-Hilbert action. We wish to investigate more closely if the path integral of eqs. (13,14) when applied to the model of eq. (1) (with $m = 0$) is equivalent to the FP path integral used for this model in refs. \[2-4\]. The approach of Garczynski \[24\] will be used in this discussion.

In order to effect an integration over $p_i(t)$ in eq. (13), we begin by noting that in a system with no constraints and $n$ degrees of freedom

$$ p_i = \frac{\partial L(q^i, \dot{q}^i)}{\partial \dot{q}^i} $$

(49)
and so, using the standard property of the Dirac delta function
\[ dx \delta(f(x)) = dx \sum_i \delta(x - a_i) / |f'(a_i)| \quad (f(a_i) = 0) \] (50)
we see that
\[ \prod_{k=1}^{n} dv^k \delta(v^k - \dot{q}^k(q^k, v)) = |A_n(q, v)| \prod_{k=1}^{n} dv^k \delta \left( \frac{\partial L(q, v)}{\partial v^k} - p_k \right). \] (51)

where

\[ A_n(q, \dot{q}) = \det \left( \frac{\partial^2 L(q, \dot{q})}{\partial \dot{q}^i \partial \dot{q}^j} \right) \] (52)

is the Hessian for the system. In this case, eq. (13) becomes
\[ \langle \text{out} | S | \text{in} \rangle = \int \exp i \int_{-\infty}^{\infty} \left[ p_i (\dot{q}^i - v^i) + L(q^i, v^i) \right] \delta(v^i - \dot{q}^i(q, p)) Dq^i Dp_i Dv^i. \] (53)

The integral over \( p_i \) can be done in eq. (53) upon using eq. (51), leaving us with
\[ = \int |A_n(q, v)| \exp i \int_{-\infty}^{\infty} \left[ \frac{\partial L(q, v)}{\partial v^i} (\dot{q}^i - v^i) + L(q^i, v^i) \right] Dq^i Dv^i. \] (54)

If now we let \( v^i \rightarrow v^i + \dot{q}^i \), eq. (54) leads to
\[ \langle \text{out} | S | \text{in} \rangle = \int \exp i \int_{-\infty}^{\infty} dt \left[ L(q^i, \dot{q}^i) \right] m(q^i, \dot{q}^i) Dq^i. \] (55)

where
\[ m(q^i, \dot{q}^i) = \int |A_n(q^i, \dot{q}^i + v)| \exp i \int_{-\infty}^{\infty} dt \left[ L(q, \dot{q} + v) - L(q, \dot{q}) - v^i \frac{\partial}{\partial v^i} L(q, \dot{q} + v) \right] Dv^i. \] (56)

This approach to the path integral of eq. (13) can be adapted quite easily to the case in which constraints are present. In the presence of constraints, the Hessian \( A_n(q, \dot{q}) \) vanishes and the matrix \( \partial^2 L(q, \dot{q}) / \partial \dot{q}^i \partial \dot{q}^j \) has rank \( r < n \). If eq. (49) can be solved for \( i = 1 \ldots r \) then we have
\[ \dot{q}^i = f^i(q^1 \ldots q^n, p_1 \ldots p_r, \dot{q}^{r+1} \ldots \dot{q}^n) \quad (i = 1 \ldots r). \] (57)

We denote the first \( r \) variables with a prime \( (q'^i, p'_i; i = 1 \ldots r) \) and all others with a double prime \( (q''^i, p''_i; i = r + 1 \ldots n) \). Following ref. [24], we find that the general path integral of eq. (13) with constraints present leads to eq. (55) with \( m(q, \dot{q}) \) in eq. (56) now given by
\[ m(q^i, \dot{q}^i) = \int \left\{ |A_r(q, \dot{q}' + v', \dot{q}'')| \exp i \int_{-\infty}^{\infty} dt \left[ L(q, \dot{q}' + v', \dot{q}'') \right] \right. \]
\[-L(q, \dot{q}) - v^i \frac{\partial}{\partial v^i} L(q, \dot{q} + v', \dot{q}'') \quad (58)\]

\[S[q', \frac{\partial}{\partial v^i} L(q, \dot{q} + v', \dot{q}'')], v''] \right\} Dv'^i Dv''^i.\]

In eq. (58), \(S\) is given by

\[S[q_i, \frac{\partial}{\partial v^i} L(q, \dot{q}' + v', \dot{q}'')] = \det \frac{1}{2} \{\theta^a, \theta^b\} \det \{\phi^a, \chi^b\} \delta(\theta^a)\delta(\phi^a)\delta(\chi^a) \quad (59)\]

and

\[A_r(q, \dot{q}', \dot{q}'') = \det \left( \frac{\partial^2 L(q, \dot{q}', \dot{q}'')}{\partial \dot{q}^i \partial \dot{q}^j} \right). \quad (60)\]

We now can apply the path integral of eqs. (55) and (58) to the model of eq. (1). (We set \(m = 0\) for purposes of illustration.) The Lagrangian \(L\) is clearly manifestly covariant. We then identify \(q^i\) and \(q''^i\) with \((B^a_i, V^a_i)\) and \((A^a_i, U^a_i)\) respectively, and take \(v^i\) and \(v''^i\) to be \((\beta^a_i, \nu^a_i)\) and \((\alpha^a_i, \mu^a_i)\) respectively. This leads to the argument of the exponential in eq. (58) to being

\[\int dt \left[ L(q, \dot{q}' + v, \dot{q}'') - L(q, \dot{q}) - v^i \frac{\partial}{\partial v^i} L(q, \dot{q}' + v', \dot{q}'') \right] = -\frac{1}{2} \int d^4x \left[ (\nu^a_i)^2 + (\beta^a_i)^2 \right]. \quad (61)\]

The contribution of \(A_r\) to eq. (58) is just a constant and consequently can be factored out of the path integral. However, from eq. (59) we see that \(S\) is non-trivial. Upon choosing the gauge conditions

\[U^a = 0 \quad (62a)\]

\[A^a_i = 0 \quad (62b)\]

\[\partial_i V^a_i = 0 \quad (62c)\]

to be associated with the first-class constraints of eqs. (32a,c; 34a) respectively, we find that

\[\det \{\phi^a, \chi^b\} = \det \left[ \partial_i D^{ab}_i \right]. \quad (63)\]

We now can use eq. (37) to obtain the contributions of the second class constraints to \(S\).

In pure YM theory, one only has first-class constraints which lead to

\[S = \det(\partial_i D^{ab}_i) \delta(\pi^a U^a) \delta(U^a) \delta(\partial_i V^a_i) \delta(D^{ab}_i \pi^b_u) \quad (64)\]

for \(S\). As was shown in ref. [20] (see also ref. [43]), this can be replaced by the FP factor in the Lorenz-Feynman gauge \((\det(\partial^\mu D^{ab}_\mu) \delta(\partial^\mu W^a_\mu))\), which ensures that the path integral expression for \(<\text{out}|S|\text{in}>\) is manifestly covariant.
In contrast, the contribution from $S$ to the measure $m$ of eq. (58) for the tensor-vector model receives the contribution

$$S \left( q^i, \frac{\partial}{\partial \eta^m} L(q, \dot{q} + v', v'') \right)$$

$$= \left[ \delta(U^a) \delta(A^a_i) \delta(\partial_i V^a_\ell) \right] \left[ \delta(\mu^a) \delta(\alpha^a_i) \delta \left( D^a_{\ell i} (\dot{\beta} + \beta)_i^\ell - \mu^2 B^a_i - \frac{m}{2} \epsilon_{ijk} F^a_{\ell jk} \right) \right] \left[ \delta \left( - \mu^2 (\dot{\beta} + \beta)_i^\ell + \epsilon_{ijk} \left( (\dot{V} + mB + \nu)_j^a (\dot{\beta} + \beta)_k^c - mB^a_j (\dot{\beta} + \beta)_k^c \right) + (D_j D_j D_j D_k a) \right) \right] \left[ \det(\partial_i D^a_{ij}) \right]$$

$$\text{det}^{1/2} \left[ f^{apm} f^{xp} \left( \delta_{\ell i} (\dot{\beta} + \beta)_k^m (\dot{\beta} + \beta)_k^n - (\dot{\beta} + \beta)_k^m (\dot{\beta} + \beta)_{\ell}^n \right) + \delta_{\ell i} (D_j D_j D_k D_j - D_j D_j D_k D_k a)^{ax} + (D_j D_j D_j D_j + D_j D_j D_j D_j - D_j D_j D_j D_j - D_j D_j D_j D_j a)^{ax} + \mu^2 \left( \mu^2 \delta_{ax} \delta_{\ell i} - \epsilon_{ilm} f^{axp} (\dot{V} + mB + \nu)_m^p + mB^p_m \right) \right].$$

The terms in square brackets on the right side of eq. (65) come in turn from the gauge conditions (eq. (62)), the first class constraints (eqs. (32a,c; 34a)), the second class constraints (eqs. (34b, 36)), the PB of the first class constraints with the gauge conditions (eq. (63)), and the PB of the second class constraints (eq. (37b)) respectively. We have used the fact the

$$\frac{\partial L}{\partial \dot{B}^a_i} = \dot{B}^a_i + f^{abc} U^c \dot{B}^a_i - \epsilon_{ijk} (D_j A_k)^a$$

$$\frac{\partial L}{\partial \dot{V}^a_i} = \dot{V}^a_i - D_i^a U^b + mB^a_i.$$

When eq. (65) is combined with eq. (61), it is apparent that for this tensor-vector model, the contribution of $m$ to the path integral in configuration space is both non-trivial and is not manifestly covariant.
4 Discussion

Finding a way of quantizing a model in a way that is manifestly consistent with covariance has been a long-standing problem, especially since the procedure in which a classical PB is converted into a quantum commutator and time evolution is governed by a Hamiltonian is always specific to one preferred reference frame. Stueckelberg [44] was the first to employ a manifestly covariant approach to quantizing electrodynamics; this was followed by the work of Feynman [45], Schwinger [46] and Tomonaga [47]. The path integral has long been seen as a way of quantizing any gauge model in a way consistent with manifest covariance. (It is not readily apparent if this is true for theories other than Yang-Mills theory which contain only first-class constraints.) The original work of Faddeev [20] for a restricted class of first-class constraints has been subsequently extended [42, 48-50, 7-13] to show how this can be done. However, incorporation second-class constraints (which lead to the factor of \( \det^{1/2} \{ \theta_a, \theta_b \} \) in eqs. (14) and (59)) into the path integral in a way that is manifestly covariant has not been done, though general discussions involving second-class constraints in the context of the path integral have appeared in the literature [51-54]. The model of eq. (1) provides for the first time an example where this particular problem becomes acute, as can be seen from eq. (64). We have examined the possibility of converting second-class constraints to being first-class through introduction of “Stueckelberg fields”, thereby making it feasible to employ techniques developed for models containing only first-class constraints, but unlike the Proca model, this is not possible for the non-Abelian model of eq. (1) as second-class constraints remain even after introduction of Stueckelberg fields. Consequently, this approach is distinct from the BFT approach in which all second-class constraints became first-class through introduction of new auxiliary fields. Furthermore, as shown in the appendix, it seems to be impossible in our non-Abelian model to devise a simple way of modifying the Hamiltonian so that our model can be viewed as merely being the gauge-fixed limit of a manifestly covariant model with only first-class constraints. This latter approach to handling second hand constraints has been employed in conjunction with BRST quantization [55]. The question of including secondary second class constraints into the path integral involving the exponential of the Lagrangian has also been examined in ref. [56].

The model of eq. (1) which we have been dealing with in this paper is clearly of limited physical interest. However, the problem of properly quantizing this model is highly non-trivial and provides insight that likely will give an understanding of how more significant theories should be properly quantized. It is apparent that one such theory is gravity, as is described by the Einstein-Hilbert action. Second-class constraints occur in the first-order (“Palatini”) formulation of the Einstein-Hilbert action [40,41]. From the results of this paper, we see that it is necessary to take into account the contribution of ghost loops arising from these second-class constraints when computing radiative affects using this first-order action. (The Faddeev-Popov procedure is inadequate to quantize this model.) The Palatini action is worth analyzing as it is just cubic in the interaction terms [57,58], while the second-order form of the Einstein-Hilbert action is non-polynomial [38,39].
We are examining this issue now. We also note the presence of second class constraints whose PB is non-trivial in the first order formulation of the three-dimensional Einstein-Cartan action [59].

The question of how to convert the path integral from an integral in phase space to an integral in configuration space is also important when quantizing the Palatini action for general relativity.

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Appendix

In this appendix, we will first discuss how the BFT approach of refs. [25-28] can be applied to a limiting case of the model of eq. (1) in which

\[ L = \frac{1}{12} (\partial_\mu \phi_\nu \lambda + \partial_\nu \phi_\mu \lambda + \partial_\lambda \phi_\mu \nu) \left( \partial^\mu \phi^{\nu \lambda} + \partial^\nu \phi^{\mu \lambda} + \partial^\lambda \phi^{\mu \nu} \right) \]

\[ + \frac{\mu^2}{8} \epsilon^{\mu \nu \lambda \phi} \phi_\mu \phi_\nu \phi_\lambda \sigma. \]  

(It is trivial to also include [1] the terms \(-\frac{1}{4} (\partial_\mu W_\nu - \partial_\nu W_\mu)^2 + \frac{m}{4} \epsilon_{\mu \nu \lambda \sigma} \phi^{\mu \nu} (\partial^\lambda W^\sigma - \partial^\sigma W^\lambda)\) into \( L \) in the following discussion.) Using the definitions

\[ A_i = \phi_{0i} \quad B_i = \frac{1}{2} \epsilon_{ijk} \phi_{jk} \]  

(A.2a, b)

it follows that

\[ L = \frac{1}{2} (\dot{B}_i - \epsilon_{ijk} \phi_{jk})^2 - \frac{1}{2} (\partial_i B_i)^2 + \mu^2 A_i B_i. \]  

(A.3)

The momenta corresponding to \( A_i \) and \( B_i \) are respectively

\[ \pi^A_i = 0, \quad \pi^B_i = \dot{B}_i - \epsilon_{ijk} \partial_j A_k. \]  

(A.4a, b)

The Hamiltonian is consequently

\[ H = \pi^A_i \dot{A}_i + \pi^B_i \dot{B}_i - L 
= \frac{1}{2} \pi^B_i a^B_i + \pi^B_i \epsilon_{ijk} \partial_j A_k + \frac{1}{2} (\partial_i B_i)^2 - \mu^2 A_i B_i \]  

(A.5)

and it follows that the primary constraint of eq. (A.4a) leads to the secondary constraint

\[ S_i = \epsilon_{ijk} \partial_j \pi^B_k - \mu^2 B_i \]  

(A.6)

and subsequently to the tertiary constraint

\[ T_i = \pi^B_i. \]  

(A.7)

It is apparent that \( \pi^A_i = 0 \) is a first-class constraint and \( S_i = T_i = 0 \) are second-class constraints as \( \{ S_i, T_j \} = -\mu^2 \delta_{ij} \).

In keeping with the BFT approach, auxiliary fields are introduced to convert the second-class constraints to first-class ones. Calling these fields \( Q_i \) and \( P_i \), with

\[ \{ Q_i, P_j \} = \delta_{ij} \]  

(A.8)

we now form

\[ \overline{S}_i = S_i + \mu^2 Q_i, \quad \overline{T}_i = T_i + P_i \]  

(A.9ab)
and

\[ \mathcal{H} = \mathcal{H} + (\partial_i \partial_j B_j + \mu^2 A_i)Q_i + 2\pi_i^B P_i \]

\[ -\frac{1}{2}Q_i \partial_i \partial_j Q_j + \frac{3}{2}P_i P_i. \]

These new quantities satisfy

\[ \{S_i, T_j\} = 0, \quad \{S_i, \mathcal{H}\} = \mu^2 T_i, \quad \{T_i, \mathcal{H}\} = 0 \]  

(A.11a - c)

so that in this new “bared” system \((\pi_i^A, S_i, T_i)\) are all first class constraints when considered in conjunction with the Hamiltonian \(\mathcal{H}\) with \((S_i, T_i, \mathcal{H})\) reducing to \((Q_i, P_i)\) as \((Q_i, P_i)\) go to zero.

This may be regarded as making a particular choice of gauge.

The Lagrangian in the “bared” system can be found by considering

\[ L = \pi_i A_i \dot{A}_i + \pi_i B_i \dot{B}_i + P_i \dot{Q}_i - \mathcal{H} \]  

(A.12)

and using the equations of motion for \(\dot{A}_i, \dot{B}_i, \dot{Q}_i\) to eliminate the momenta \(\pi_i^A, \pi_i^B\) and \(P_i\)

\[ \dot{A}_i = 0 \]  

(A.13a)

\[ P_i = 2(\dot{B}_i - \epsilon_{ijk} \partial_j A_k) - \dot{Q}_i \]  

(A.13b)

\[ \pi_i^B = 2\dot{Q}_i - 3(\dot{B}_i - \epsilon_{ijk} \partial_j A_k) \]  

(A.13c)

to express \(L\) in terms of the “position” variables \((A_i, B_i, Q_i)\) and their associated velocities. We obtain

\[ L = -\frac{1}{2} \dot{Q}_i \dot{Q}_i - \frac{3}{2}(\dot{B}_i - \epsilon_{ijk} \partial_j A_k)^2 + 2(\dot{B}_i - \epsilon_{ijk} \partial_j A_k)\dot{Q}_i \]

\[ -\frac{1}{2}(\partial_i B_i)^2 + \mu^2 A_i B_i - (\partial_i \partial_j B_j + \mu^2 A_i)Q_i + \frac{1}{2}Q_i \partial_i \partial_j Q_j. \]

(A.14)

Unlike the case considered in ref. [29,30], the gauge choice in which \(Q_i = 0\) does not reduce \(L\) in eq. (A.14) to \(L\) in eq. (A.3). It does not appear to be possible to express \(L\) in a manifestly covariant form.

To find the gauge transformation generated by the first class constraints \(\gamma_i^{(N)} = (\pi_i^A, S_i, T_i)\) we use the HTZ method [32]. (One could also employ the technique of ref. [31].) We begin by introducing a gauge generator

\[ G = \lambda_i^{(1)} \pi_i^A + \lambda_i^{(2)} S_i + \lambda_i^{(3)} T_i \]

(A.15)

and then employing the equation [32]

\[ \frac{D\lambda_i^{(N)}}{Dt} \gamma_i^{(N)} + \left\{ \lambda_i^{(N)} \gamma_i^{(N)}, \mathcal{H} + U_i^{(1)} \gamma_i^{(1)} \right\} - \delta U_i^{(1)} \gamma_i^{(1)} = 0 \]

(A.16)

to derive the equations

\[ \dot{\lambda}_i^{(1)} = \delta U_i^{(1)} \]

(A.17a)
\[ \dot{\lambda}_i^{(2)} = \dot{\lambda}_i^{(1)} \]  
(A.17b) 
\[ \dot{\lambda}_i^{(3)} = -\mu^2 \lambda_i^{(2)}. \]  
(A.17c)

If \( \lambda_i^{(3)} \equiv \epsilon_i \), then eq. (A.17) leads to the generator

\[ G = -\frac{1}{\mu^2} \dot{\epsilon}_i \pi_i^A - \frac{1}{\mu^2} \dot{\epsilon}_i (\epsilon_{ijk} \partial_j \pi_k^B - \mu^2 B_i + \mu^2 Q_i) + \epsilon (\pi_i^B + P_i), \]

(A.18)

and so

\[ \delta A_i = \{A_i, G\} = -\frac{1}{\mu^2} \dot{\epsilon}_i \]  
(A.19a) 
\[ \delta B_i = -\frac{1}{\mu^2} \dot{\epsilon}_i \partial_j \dot{\epsilon}_k + \epsilon_i \]  
(A.19b) 
\[ \delta Q_i = \epsilon_i. \]  
(A.19c)

One can verify that \( \overline{\mathcal{L}} \) in eq. (A.14) is invariant under the transformation of eq. (A.19).

In the Abelian limit considered in eq. (A.1), it is possible to treat \( S_i = 0 \) as being a first class constraint and take \( T_i = 0 \) as being the associated gauge condition provided we use that Hamiltonian

\[ \mathcal{H}_M = \mathcal{H} - \frac{1}{2} T_i T_i \]  
(A.20)

since \( \{S_i, \mathcal{H}_M\} = 0 \) and \( \mathcal{H}_M |_{T_i=0} = \mathcal{H} \) [36,37].

We now consider applying the techniques of refs. [36,37] to the full Hamiltonian of eq. (33). Finding the associated Hamiltonian \( \mathcal{H}_M \) in closed form is not possible, but one can determine \( \mathcal{H}_M \) in a perturbative fashion. With \( \mathcal{H}_C, S_i^a \) and \( T_i^a \) being given by eqs. (33,34b,36) respectively, we take

\[ \mathcal{H}_M = \mathcal{H}_C + \frac{1}{2} F_{i1i2}^{a1a2} T_{i1}^{a1} T_{i2}^{a2} + \frac{1}{3} F_{i1i2i3}^{a1a2a3} T_{i1}^{a1} T_{i2}^{a2} T_{i3}^{a3} + \ldots \]  
(A.21)

(where \( F_{i1\ldots in}^{a1\ldots an} \) are functions of the canonical variables and are symmetric in each set of indices) then the requirement

\[ \{S_i^a, \mathcal{H}_M\} = 0 \]  
(A.22)

leads to

\[ 0 = T_i^a + \left( F_{i1i2}^{a1a2} \Delta_{i1}^{a1a2} T_{i2}^{a2} + \frac{1}{2} \{S_i^a, F_{i1i2}^{a1a2}\} T_{i1}^{a1} T_{i2}^{a2} \right) 
+ F_{i1i2i3}^{a1a2a3} \Delta_{i1}^{a1a2a3} T_{i2}^{a2} T_{i3}^{a3} + \ldots \]  
(A.23)

where \( \Delta_{ij} = \{S_i^a, T_j^b\} \). We are led to a set of nested equations whose solution is

\[ F_{i1i2}^{a1a2} = -\Delta_{i1i2}^{-a1a2} \]  
(A.24)

\[ F_{i1i2i3}^{a1a2a3} = \frac{1}{6} \left[ \Delta_{i3i1}^{-a3a1} \{S_i^a, F_{i1i2}^{a1a2}\} + \Delta_{i2i1}^{-a2a1} \{S_i^a, F_{i2i3}^{a2a3}\} + \Delta_{i1i2}^{-a1a2} \{S_i^a, F_{i1i2}^{a1a2}\} \right] \]
etc.

These solutions are quite complicated as can be seen from the explicit expression for $\Delta^{ab}_{ij}$ in eq. (37b).

In the gauge $T^a_i = 0$, $\mathcal{H}_M$ of eq. (A.27) reduces to $\mathcal{H}_C$ of eq. (33).

We finally note that we can accommodate second class constraints by solving for the Lagrange multipliers with which they are associated in the extended Hamiltonian $\mathcal{H}_E$. For example, $\mathcal{H}_c$ for the Proca model in eq. (18) is associated with

$$\mathcal{H}_E = \mathcal{H}_c + \mu_1 \pi^0 + \mu_2 (\partial_i \pi^i + m^2 A_0); \quad (A.25)$$

in order that $\{\theta_i, \int \mathcal{H}_E dx\} = 0$ ($i = 1, 2$) we have

$$\mu_1 = \partial_i A_i \quad \mu_2 = (\partial_i \pi^i + m^2 A_0)/(2m^2). \quad (A.26)$$

With these expressions for $\mu_1$ and $\mu_2$ we find that

$$\dot{A}_0 = \left\{A_0, \int \mathcal{H}_E dx\right\} = \partial_i A_i \quad (A.27a)$$

$$\dot{A}_i = \left\{A_i, \int \mathcal{H}_E dx\right\} = (\delta_{ij} - \partial_i \partial_j/m^2) \pi_j. \quad (A.27b)$$

It does not appear to be possible to derive $\mathcal{H}_E$ from a covariant Lagrangian.