Extra S11 and P13 in the Hypercentral Constituent Quark Model

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Abstract

We report on the recent results of the hypercentral Constituent Quark Model (hCQM). The model contains a spin independent three-quark interaction which is inspired by Lattice QCD calculations and reproduces the average energy values of the $SU(6)$ multiplets. The splittings within each multiplet are obtained with a $SU(6)$-breaking interaction, which can include also an isospin dependent term. All the 3- and 4-stars resonances are well reproduced. Moreover, as all the Constituent Quark models, the hCQM predicts “missing” resonances (e.g. extra $S_{11}$ and $P_{13}$ states) which can be of some help for the experimental identification of new resonances.

The model provides also a good description of the medium $Q^2$-behavior of the electromagnetic transition form factors. In particular the calculated helicity amplitude $A_1$ for the $S_{11}(1535)$ resonance agrees very well with the recent CLAS data. More recently, the elastic nucleon form factors have been calculated using a relativistic version of the hCQM and a relativistic quark current.

1 Introduction

In recent years much attention has been devoted to the description of baryons in terms of three quark degrees of freedom. Starting from the classical Isgur-Karl model [1], many different CQMs have been developed: the algebraic one [2], the hypercentral CQM [3] and the GBE model [4, 5]. In the following will be shown the main features of the hCQM and will be presented some of the results obtained in the calculation of various baryon properties.
2 The Model

The internal quark motion is well described by the Jacobi coordinates \( \rho \) and \( \lambda \), or, in an equivalent way, by the hyperspherical coordinates \( x \) and \( t \): \[ x = \sqrt{\rho^2 + \lambda^2} \quad t = \arctan\left(\frac{\rho}{\lambda}\right) \] (1)

where \( x \) is the hyperradius and \( t \) is the hyperangle. In the hCQM the \( SU(6) \)-invariant part of the potential is assumed to be dependent only on the hyperradius and of the form \( V(x) = -\frac{\tau}{x} + \alpha x \). Interaction of the kind linear plus Coulomb-like have been used since time for the meson sector (e.g. the Cornell potential), and has been supported by recent Lattice QCD calculations \[7\]. The choice of an hypercentral potential (i.e. a potential which depends only on the hyperradius) has two different motivations: \( x \) is a collective coordinate, therefore an hypercentral potential contains also three body effects, moreover this potential can be read as the hypercentral approximation of a 2 body potential.

The splitting within each multiplet is produced introducing a perturbative \( SU(6) \)-breaking term, which, as a first approximation, can be assumed to be the standard hyperfine term \( H_{\text{hyp}} \).

The three quark Hamiltonian in the hCQM is then:

\[ H = \frac{p_\lambda^2}{2m} + \frac{p_\rho^2}{2m} - \frac{\tau}{x} + \alpha x + H_{\text{hyp}} \] (2)

where \( m \) is the quark mass (taken equal to 1/3 of the nucleon mass). The strength of the Hyperfine interaction is determined in order to reproduce the \( \Delta - N \) mass difference while the remaining parameters (\( \alpha \) and \( \tau \)) are fitted to the spectrum, leading to the following values: \( \alpha = 1.61 \text{ fm}^{-2} \), \( \tau = 4.59 \). Keeping these parameters fixed, the resulting wave functions have been used to calculate various physical quantities of interest: the helicity amplitudes \[9\], the electromagnetic transition form factors \[10\], the elastic nucleon form factors \[11\], the ratio between electric and magnetic proton form factors \[12\], and some interesting quantities related to the parton distributions \[13\].

3 Generalized \( SU(6) \)-breaking term

The Hamiltonian of eq.(2) give rise to a nice description of the spectrum, nevertheless in order to improve the quality of the reproduction, one can generalize the Hamiltonian operator introducing an isospin dependence in it.

The complete interaction used is \[14\]:

\[ H_{\text{int}} = V(x) + H_S + H_I + H_{SI} \] (3)
where $V(x)$ is the $SU(6)$-invariant part, $H_S$ is a smeared standard hyperfine term, $H_I$ is an isospin dependent term and $H_{SI}$ is a spin-isospin dependent term. The spectrum obtained with the interaction of eq. (5) is shown, in fig. 1. All the 3- and 4-stars resonances, in particular the Roper, are well reproduced.

All the CQMs predict states which don’t have (yet) experimental confirmations. In particular (see Table 1 and Fig. 1) the hCQM predicts 5 missing resonances with energies below 1900 MeV. Recent analysis (see for example [16, 17] and references quoted therein) show that there are some indications for the presence of a third $S_{11}$ and a third $P_{13}$ with masses comparable with the predictions of the hCQM.

Table 1: hCQM prediction for $S_{11}$, $D_{13}$, $P_{13}$ and $P_{33}$ resonances, compared with PDG data[15].

| State | PDG | hCQM | hCQM+Iso | State | PDG | hCQM | hCQM+Iso |
|-------|-----|------|---------|-------|-----|------|---------|
| $S_{11}$ | 1535 | 1507 | 1524 | $P_{13}$ | 1720 | 1797 | 1848 |
| 1650 | 1574 | 1688 | | 1900 | 1835 | 1816 |
| (2090) | 1887 | 1861 | | 1853 | 1894 |
| | 1937 | 2008 | | 1863 | 1939 |
| $D_{13}$ | 1520 | 1526 | 1524 | $P_{33}$ | 1232 | 1240 | 1232 |
| 1700 | 1606 | 1692 | | 1600 | 1727 | 1723 |
| (2080) | 1899 | 1860 | | 1920 | 1843 | 1921 |
| | 1969 | 2008 | | 1856 | 1955 |
| | | | | 2104 | 2049 |

4 The electromagnetic transition form factors

The helicity amplitudes for the e.m. excitation of baryon resonances, $A_{1/2}$, $A_{3/2}$ and $S_{1/2}$ are calculated as the transition matrix element of the transverse and longitudinal part of the e.m. interaction between the nucleon and the resonance states given by this model. A non relativistic current for point quarks has been used.

The longitudinal and transverse transition form factors have been systematically calculated for all the resonances (including the missing ones) predicted by the hCQM. The results for the $A_{1/2}^P$ and $A_{3/2}^P$ amplitudes for all the negative parity resonances are reported in Ref. [10]. In fig. 2 the result for the $A_{1/2}$ amplitude for the $S_{11}(1535)$ is shown; the prediction agrees quite well with the data except for some discrepancies at small $Q^2$. These discrepancies could be ascribed to the non-relativistic character of the model, or better to
the lack of explicit quark-antiquark configurations which are expected to be
important at low $Q^2$.

5 Relativity and elastic nucleon form factors

It is well understood that in order to obtain a better description of the baryon
properties one has to introduce relativity in CQMs. Starting from a CQM
one can introduce relativistic effects by: a) using a relativistic kinetic energy
operator; b) boosting the baryon wave functions from the initial and final
rest frames to a common frame; c) using a relativistic quark current. In the
hCQM the potential parameters have been refitted using a relativistic kinetic
energy operator, the resulting spectrum is not much different from the non
relativistic one. The boosts and a relativistic quark current expanded up to
the lowest order in quark momenta has been used both for the elastic form
factors[11] and for the helicity amplitudes [20]. While in the latter case the
effect of these relativistic corrections is small, for the elastic form factors the
relativistic effects are more important, in particular, as we have shown for
the first time in [12], they are responsible for the decreasing $Q^2$-behaviour
of the ratio between the electric and magnetic proton form factors.
More recently a relativistic quark current with no expansion in the quark mo-
menta and the boosts to the Breit frame have been applied to the calculation
of the elastic form factors. The resulting theoretical curves [21], calculated
without free parameters and with pointlike quarks, agree very nicely with
the experimental data except for some discrepancies at low $Q^2$. The decrease
of the ratio between electric and magnetic proton form factors is stronger
than in the previous cases and reaches almost the 50% level, not far from the
recent TJNAF data [22].

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Figure 1: The spectrum obtained with the hypercentral model using the interaction in eq. (3) (top figure) compared with the spectrum obtained by Capstick [1] (bottom figure). The boxes are the experimental data of PDG with their uncertainties; dark grey boxes are 3- and 4-stars resonances, light grey boxes are 1- and 2-stars resonances[15].
Figure 2: The helicity amplitude $A_{1/2}^p$ for the $S_{11}(1535)$ resonance calculated with the hCQM (dashed curve) and with the model of Ref. [18] (full curve). The data are taken from the compilation of Ref. [19].