JACOB’S LADDERS, CLASS OF CROSSBREEDING
CONSERVING CERTAIN CELL OF META-FUNCTIONAL
EQUATIONS AND EXISTENCE OF CANCEROUS GROWTH OF
THAT CELL

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Abstract. In this paper a set of 48 crossbreedings on certain cell of meta-
functional equations preserving the cell is obtained. In opposite direction we
presente an example of very complicated meta-functional equation not ob-
tained in basic cell though it is generated by sequence of internal crossbreed-
ings. Such a formula is called as cancerous growth on the basic cell.

DEDICATED TO GREGOR MENDEL AND IT’S PEA-CROSSBREEDING

1. Introduction

1.1. In the paper [14], eq. (1.10) we have considered the following set
\[
\begin{align*}
\{&\{\zeta(s), \Gamma(s), \text{cn}(s, k), J_p(s)\} \\
&\{\Gamma(2s), \text{cn}(2s, k), J_p(2s), \zeta(2s)\} \\
&\{\text{cn}(3s, k), J_p(3s), \zeta(3s), \Gamma(3s)\} \\
&\{J_p(4s), \zeta(4s), \Gamma(4s), \text{cn}(4s, k)\} \\
&\{\zeta(5s), \Gamma(5s), \text{cn}(5s, k), J_p(5s)\} \\
&\{\Gamma(6s), \text{cn}(6s, k), J_p(6s), \zeta(6s)\} \\
&\{\text{cn}(7s, k), J_p(7s), \zeta(7s), \Gamma(7s)\} \\
&\{J_p(8s), \zeta(8s), \Gamma(8s), \text{cn}(8s, k)\} \\
&\{\zeta(9s), \Gamma(9s), \text{cn}(9s, k), J_p(9s)\} \\
&\vdots
\end{align*}
\]
\[s \in \mathbb{C} \setminus \{N, P\}, \ k^2 \in (0, 1)\]

of foursomes, where \{N, P\} stands for the set of all zeros and all poles of the
functions contained in (1.1) for every admissible and fixed \(k\). We have constructed,
for example, the following set of exact meta-functional equations on the subset of
the first four sets in (1.1) (for more information see [14], (3.8) – (3.13)):

\[
\begin{align*}
|\zeta(s_1)| |\Gamma(s_2)| &||J_p(2s_3^2)|| + |\zeta(s_4^2)| |\text{cn}(s_3^1, k)| = \\
&= |\Gamma(2s_4^2)| |\text{cn}(s_3^1, k)| |\text{cn}(2s_2^2, k)| + |J_p(s_1^1)| |J_p(2s_2^1)|,
\end{align*}
\]

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i.e. there are the sets (1.7) 
\[ k \]

such that for their elements (variables) (1.8) 
\[ \tilde{\text{Z}}(s)\] 

following mother formula (exact complete hybrid formula):

\[ |\zeta(s)| |\Gamma(s)| |\Gamma(4s)| + |\text{cn}(s)| |\text{cn}(4s)|, \]

(1.9) 
\[ \text{J}(s)\] 

\[ |\zeta(3s)| |\Gamma(2s)| |\text{cn}(2s), k| + |\Gamma(3s)| |\text{J}(2s)|, \]

(1.10) 
\[ \text{cn}(3s), k| + |\zeta(2s)| |\zeta(3s)|, \]

(1.11) 
\[ \text{J}(4s) ||\text{J}(4s)| |\text{cn}(s)| + |\zeta(s)| |\zeta(4s)|, \]

(1.12) 
\[ \text{J}(4s) ||\text{J}(4s)| |\text{cn}(s)| + |\zeta(s)| |\zeta(4s)|, \]

\[ \text{J}(4s) ||\text{J}(4s)| |\zeta(3s) + |\zeta(3s)| |\zeta(4s)|, \]

\[ \text{J}(4s) ||\text{J}(4s)| |\zeta(3s), k| + |\zeta(3s), k| |\zeta(4s)|, \]

\[ |\zeta(3s)| |\zeta(4s)| |\text{J}(4s)| + |\Gamma(3s)| |\Gamma(4s)|, \]

\[ \text{J}(4s) ||\text{J}(4s)| |\zeta(3s), k| + |\zeta(3s), k| |\zeta(4s)|, \]

\[ |\zeta(3s)| |\zeta(4s)| |\text{J}(4s)| + |\Gamma(3s)| |\Gamma(4s)|, \]

i.e. there are the sets 
\[ \Omega^m \subset \mathbb{C}, m, l = 1, 2, 3, 4 \]

such that for their elements (variables) 
\[ s^m \in \Omega^m \]

equations (1.2) – (1.7) hold true.

Remark 1. Let us remind that the equations in question are generated by the following mother formula (exact complete hybrid formula):

\[ \tilde{Z}^2(\alpha_{11}) \sin^2(\alpha_{01}) + \tilde{Z}^2(\alpha_{12}) \cos^2(\alpha_{01}) = \tilde{Z}^2(\beta_1), \]

(see [10], (3.2), \( k_1 = k_2 = 1 \) and \( \frac{\pi}{2} \rightarrow \frac{\pi}{2} \) in our case), by this way: first this formula generates set of four transmutations (see [14], (3.3) – (3.6)), and secondly the operation of crossbreeding applies on the last set and noticed equations follow.

1.2. Let the symbol 
\[ [(1.2) \times (1.3)]|\zeta(s)| \]

denote crossbreeding on the set \{ (1.2), (1.3) \} of the meta-functional equations (1.2) and (1.3) for the neutral factor \(|\zeta(s)|\). Next, let \( C_0 \) denote the cell of exact meta-functional equations (1.2) – (1.7) (elements of \( C_0 \)).

Remark 2. The cell \( C_m \) is in this context generated by the transformation 
\[ g \rightarrow 4m + g, \quad g = 1, 2, 3, 4, \quad m \in \mathbb{N}_0 \]

in \( C_0 \) as follows 
\[ |\zeta(s)| \rightarrow |\zeta((4m + 1)s^m)|, \quad |\Gamma(s)| \rightarrow |\Gamma((4m + 1)s^m)|, \ldots \]

\[ |\Gamma(4s)| \rightarrow |\Gamma((4m + 4)s^m)|. \]

In this paper we show that there is a class of 48 crossbreedings of type (1.9) conserving the cell \( C_0 \). For example, we have 
\[ (1.12) \quad [(1.2) \times (1.3)]|\zeta(s)| = (1.5) \in C_0. \]
1.3. Let us notice, on the contrary, there is a chain of such crossbreedings in $C_0$ that gives, for example, the following result:

$$\Gamma(s_2)^2\Gamma(3s_4)\Gamma(4s_4)\Gamma(2s_3)\Gamma(4s_3)J_p(3s_3)J_p(4s_4)J_p(3s_2)J_p(4s_3)J_p(3s_1)J_p(4s_2)J_p(s_4)J_p(s_3)J_p(s_2)J_p(s_1)J_p(s_0)$$

(1.13)

Let the symbol $(3, 2) \leftrightarrow (3, 2)$ denote the type of elements of $C_0 = \{(1.2) - (1.7)\}$.

**Remark 3.** Then the meta-functional equation (1.13) - a cancerous growth of the basic cell $C_0$ - is of type

$$\{7, 7, 7, 6, 6\} \leftrightarrow \{7, 7, 7, 6, 6\}.$$  

The comparison of the norms of these types

$$|\{3, 2\} \leftrightarrow \{3, 2\}| = 10; |\ldots\}| = 2 \times (3 + 2),$$

$$|\{7, 7, 7, 6, 6\} \leftrightarrow \{7, 7, 7, 6, 6\}| = 66$$

gives us good reason to name it as cancerous growth of $C_0$.

**Remark 4.** We notice explicitly that the meta-functional equation (1.13) contains four squares $|\Gamma(3s_1)|^2$.

**Remark 5.** This paper is again based on new notions and methods in the theory of the Riemann’s zeta-function we have introduced in our series of 53 papers concerning Jacob’s ladders. These can be found in arXiv [math.CA] starting with the paper [1].

2. ON THE STRUCTURE OF THE MOTHER FORMULA

2.1. Let us remind that

$$\alpha_1^{1,1} = \varphi_0^0(d_1), \quad \alpha_0^{1,1} = \varphi_1^1(d_1), \quad d_1 = d_1(U, \pi L; f_1),$$

(2.1)

$$\alpha_2^{1,1} = \varphi_0^0(d_2), \quad \alpha_0^{1,1} = \varphi_1^1(d_2), \quad d_2 = d_2(U, \pi L; f_2),$$

$$\beta_1^1 = \varphi_0^0(e), \quad e = e(U, \pi L),$$

(see [11, (4.5) – (4.17)], and

$$f_1 = f_1(t) = \sin^2 t, \quad f_2 = f_2(t) = \cos^2 t,$$

(2.2)

$$t \in [\pi L, \pi L + U], \quad 0 < U < \frac{\pi}{2}, \quad L \in \mathbb{N}.$$
Consequently, we have the following complicated composite functions
\[
\begin{align*}
\tilde{Z}^2(\alpha_1, \beta_1) &= \tilde{Z}^2(\varphi_0^1[d_1(U, \pi L; f_1)]), \\
\sin^2(\alpha_1^2) &= \sin^2(\varphi_1^1[d_1(U, \pi L; f_1)]), \\
\end{align*}
\]
(2.3)
\[
\begin{align*}
\tilde{Z}^2(\alpha_0, \beta_1) &= \tilde{Z}^2(\varphi_0^2[d_2(U, \pi L; f_2)]), \\
\cos^2(\alpha_0^2) &= \cos^2(\varphi_1^2[d_2(U, \pi L; f_2)]), \\
\tilde{Z}^2(\beta_1) &= \tilde{Z}^2(\varphi_0^1[e(U, \pi L)]), \\
\end{align*}
\]
where
\[
\begin{align*}
\tilde{Z}^2 = \frac{d\varphi_1(t)}{dt} = \left| \frac{\zeta(\frac{1}{2} + it)}{\omega(t)} \right|^2, \\
\omega(t) = \left\{ 1 + O\left( \frac{\ln \ln t}{\ln t} \right) \right\} \ln t,
\end{align*}
\]
see [4], (6.1), (6.7), (7.7), (7.8) and (9.1).

Now we have the following detailed form
\[
\begin{align*}
\tilde{Z}^2(\varphi_0^1[d_1(U, \pi L; f_1)]) \sin^2(\varphi_1^1[d_1(U, \pi L; f_1)]) + \\
\tilde{Z}^2(\varphi_0^2[d_2(U, \pi L; f_2)]) \cos^2(\varphi_1^2[d_2(U, \pi L; f_2)]) = \\
\tilde{Z}^2(\varphi_0^1[e(U, \pi L)])
\end{align*}
\]
(2.5)
of the exact complete hybrid formula that is a mother formula in this context for us.

Remark 6. Namely, formula (2.5) generates miscellaneous sets of its transmutations. Next, the operation of crossbreeding (see [9] – [14]) applied on these sets generates new miscellaneous sets of exact meta-functional equations.

Remark 7. Let us notice explicitly that the mother formula (2.5) is based mainly on the new transcendental functions
\[
\varphi_1(t), \quad \frac{d\varphi_1(t)}{dt}, \text{ i.e. } \Omega^m_n = \Omega^m_n[\varphi_1, \frac{d\varphi_1}{dt}],
\]
(comp. subsection 1.1) that we have introduced in our papers.

3. Jacob’s Ladders

3.1. Let us remind that the Jacob’s ladder
\[
\varphi_1(t) = \frac{1}{2} \varphi(t)
\]
we have introduced in [11] (see also [4]), where the function \( \varphi(t) \) is arbitrary continuous solution of the nonlinear integral equation (also introduced in [11])
\[
\int_0^{\mu[t(T)]} Z^2(t) e^{-\frac{\pi}{4} t} dt = \int_0^T Z^2(t) dt
\]
where
\[
\begin{align*}
Z(t) &= e^{i\psi(t)} \zeta(\frac{1}{2} + it), \\
\psi(t) &= -\frac{t}{2} \ln \pi + \text{Im} \left\{ \ln \Gamma \left( \frac{1}{4} + \frac{it}{2} \right) \right\},
\end{align*}
\]
where each admissible function $\mu(y)$ generates the solution

$$y = \varphi(T; \mu) = \varphi(T); \mu(y) \geq 7y \ln y.$$  

We call the function $\varphi_1(T)$ the Jacob’s ladder as an analogue of the Jacob’s dream in Chumash, Bereishis, 28:12.

3.2. Next, let us remind that the classical Hardy-Littlewood integral (1918)

$$(3.1) \quad \int_0^T \left| \zeta \left( \frac{1}{2} + it \right) \right|^2 dt$$

has the following expression

$$(3.2) \quad \int_0^T \left| \zeta \left( \frac{1}{2} + it \right) \right|^2 dt = T \ln T + (2c - 1 - \ln 2\pi)T + R(T),$$

with, for example, the Ingham’s estimate of the error term

$$(3.3) \quad R(T) = O(\sqrt{T} \ln T), \quad T \to \infty.$$  

However, by the Good’s $\Omega$-theorem (1977) we have that

$$(3.4) \quad R(T) = \Omega(T^{1/4}), \quad T \to \infty.$$  

Remark 8. It follows from (3.4) that

$$(3.5) \quad \limsup_{T \to \infty} |R(T)| = +\infty,$$

that is every expression of the type (3.2) of the Hardy-Littlewood integral possesses an unbounded error term at $T \to \infty$.

3.3. Under the circumstances (3.2) and (3.5) we have proved that the Hardy-Littlewood integral (3.1) has an infinite set of other completely new almost exact representations expressed by the following

Formula1.

$$(3.6) \quad \int_0^T \left| \zeta \left( \frac{1}{2} + it \right) \right|^2 dt = \varphi_1(T) \ln \{\varphi_1(T)\} +$$

$$+ (c - \ln 2\pi)\varphi_1(T) + c_0 + O \left( \frac{\ln T}{T} \right), \quad T \to \infty$$

with the property (comp. (3.5))

$$(3.7) \quad \lim_{T \to \infty} R_1(T) = \lim_{T \to \infty} \left\{ O \left( \frac{\ln T}{T} \right) \right\} = 0,$$

where $c$ is the Euler’s constant and $c_0$ is the constant from the Titchmarsh-Kober-Atkinson formula.

Remark 9. Comparison between (3.5) and (3.7) sufficiently characterizes level of exactness of our representation (3.6).

Now, let us remind other formulae demonstrating power of Jacob’s ladder $\varphi_1(t)$. 
3.4. First, we have obtained the following formula (see [2], (1.1)).

**Formula2.**

\[
\int_{T}^{T+U} \left| \zeta \left( \frac{1}{2} + i \varphi_1(t) \right) \right|^4 \left| \zeta \left( \frac{1}{2} + it \right) \right|^2 dt \sim \frac{1}{2\pi^2} U \ln^5 T,
\]

where \( U = T^{7/8+2\epsilon}, T \to \infty. \)

**Remark 10.** Our formula \((3.8)\) is the first asymptotic formula of the sixth order expression in \( |\zeta| \) on the critical line \( \sigma = \frac{1}{2} \) in the theory of the Riemann’s zeta-function.

3.5. Now, let

\[
S(t) = \frac{1}{\pi} \arg \left\{ \zeta \left( \frac{1}{2} + it \right) \right\}, \quad S_1(T) = \int_{0}^{T} S(t) dt,
\]

where arg function is defined by the usual way. We have obtained the following formula (see [3], (5.4), (5.5)).

**Formula3.**

\[
\int_{T}^{T+U} \left\{ \arg \zeta \left( \frac{1}{2} + i \varphi_1(t) \right) \right\}^{2k} \left| \zeta \left( \frac{1}{2} + it \right) \right|^2 dt = \frac{(2k)!}{k!2^{2k}} U \ln T \ln \ln T)^k, \quad T \to \infty \text{ and } k \text{ is fixed positive number.}
\]

**Formula4.**

\[
\int_{T}^{T+U} \left\{ S_1[\varphi_1(t)] \right\}^{2k} \left| \zeta \left( \frac{1}{2} + it \right) \right|^2 dt = a_k U \ln T, \quad T \to \infty.
\]

**Remark 11.** Formulæ \((3.3)\), \((3.8)\) – \((3.10)\) are not accessible within the current methods in the theory of the Riemann’s zeta-function.

4. **Crossbreeding conserving a cell of meta-functional equations**

4.1. Let the cell \( C_0 \) be the set \((1.2)\) – \((1.7)\) of meta-functional equations (elements).

We apply the crossbreeding of type \((A_1)\) on two different elements of \( C_0 \) in case each of those two elements contains once a choice neutral factor. In this case the crossbreeding \((A_1)\) is simply elimination of the neutral factor. For example in the case

\[
[(1.2) \times (1.3)]|\zeta(s_1)|
\]

we have in the complete form

\[
(1.2) \Rightarrow |\zeta(s_1)| = \frac{|\Gamma(2s_3^2)| |\text{cn}(s_3^1, k)| |\text{cn}(2s_2^3, k)|}{|\Gamma(s_3^1)||J_p(2s_2^3)||J_p(2s_3^3)|} + \frac{|J_p(s_1^4)||J_p(2s_1^3)||\Gamma(s_3^1)|}{|J_p(2s_2^3)||\Gamma(s_3^1)|},
\]

\[
(1.3) \Rightarrow |\zeta(s_1)| = \frac{|\text{cn}(s_3^1, k)| |\text{cn}(3s_3^3, k)| |J_p(3s_2^3)|}{|\zeta(3s_2^3)||\Gamma(s_3^1)|} + \frac{|J_p(s_1^4)||\zeta(3s_2^3)||\Gamma(s_3^1)|}{|\zeta(3s_2^3)||\Gamma(s_3^1)|} - \frac{|\Gamma(3s_2^3)| |\text{cn}(s_3^1, k)|}{|\zeta(3s_2^3)||\Gamma(s_3^1)|},
\]
Theorem 1. The complete result is as follows.

Remark 12. The fact is that all the corresponding moduli are positive. This property follows from our construction of exact $\zeta$-factorization formulae (see [7], [8]).

Next we use the abbreviation (comp. (1.9))

$$[(1.2) \times (1.3)] [\zeta(s_1^1), |\Gamma(s_1^2)|, |J_p(s_1^1)|] = (1.5) \Leftrightarrow \\
\{([(1.2) \times (1.3)] [\zeta(s_1^1)] = (1.5)) \land ([(1.2) \times (1.3)] [\Gamma(s_2^1)] = (1.5)) \land \\
(\{(1.2) \times (1.3)] [J_p(s_2^1)] = (1.5))\}.$$

The complete result is as follows.

Theorem 1. For the cell $C_0 = \{(1.2) - (1.7)\}$ there is the following set of 36 crossbreeding of the type $(A_1)$:

$$[(1.2) \times (1.3)] [\zeta(s_1^1), |\Gamma(s_1^2)|, |J_p(s_1^1)|] = (1.5), \\
[(1.2) \times (1.4)] [\zeta(s_1^1), |\Gamma(s_2^1)|, |J_p(s_1^1)|] = (1.6), \\
[(1.2) \times (1.5)] [\zeta(2s_2^1), |\Gamma(2s_2^1)|, |\text{cn}(2s_2^1, k)|] = (1.3), \\
[(1.2) \times (1.6)] [\zeta(2s_2^1), |\Gamma(2s_2^1)|, |\text{cn}(2s_2^1, k)|] = (1.4), \\
[(1.3) \times (1.4)] [\zeta(s_1^1), |\Gamma(s_2^1)|, |J_p(s_1^1)|] = (1.7), \\
[(1.3) \times (1.5)] [\zeta(3s_2^1), |\Gamma(3s_2^1)|, |\text{cn}(3s_2^1, k)|, |J_p(3s_2^1)|] = (1.2), \\
[(1.3) \times (1.7)] [\zeta(3s_2^1), |\Gamma(3s_2^1)|, |\text{cn}(3s_2^1)|, |J_p(3s_2^1)|] = (1.4), \\
[(1.4) \times (1.6)] [\zeta(4s_2^1), |\Gamma(4s_1^1)|, |J_p(4s_2^1)|] = (1.2), \\
[(1.4) \times (1.7)] [\zeta(4s_2^1), |\Gamma(4s_2^1)|, |J_p(4s_2^1)|] = (1.3), \\
[(1.5) \times (1.6)] [\zeta(2s_2^1), |\Gamma(2s_2^1)|, |\text{cn}(2s_2^1, k)|, |\zeta(2s_2^1)|] = (1.7), \\
[(1.5) \times (1.7)] [\zeta(3s_2^1), |\Gamma(3s_2^1)|, |\text{cn}(3s_2^1, k)|, |J_p(3s_2^1)|] = (1.6), \\
[(1.6) \times (1.7)] [\zeta(4s_2^1, k), |\Gamma(4s_2^1)|, |\zeta(4s_2^1)|] = (1.5),$$

and

$$(1.2) \times (1.7) = \emptyset, \ (1.3) \times (1.6) = \emptyset, \ (1.4) \times (1.5) = \emptyset,$$

(i.e. in these cases the corresponding two sets of moduli do not contain common modulus), and apart of these we have also

$$[(1.2) \times (1.3)] [\zeta(s_1^1), |\Gamma(s_2^1)|] = (1.5), \\
[(1.2) \times (1.4)] [\zeta(s_1^1), |\Gamma(s_2^1)|] = (1.6), \\
[(1.3) \times (1.4)] [\zeta(s_1^1), |\Gamma(s_2^1)|] = (1.7).$$

Here the neutral factor is the product of two moduli.

4.2. We apply the crossbreeding of type $(A_2)$ on two different elements of $C_0$ in case each of those two elements contains twice a choice neutral factor.

In this case, for example

$$[(1.2) \times (1.3)] |\text{cn}(s_3^1, k)|,$$
we have in the complete form
\[
(1.2) \implies \frac{|\text{cn}(s_3^1, k)|}{|J_p(2s_3^2)|} \{ |\zeta(2s_4^2)| - |\Gamma(2s_4^2)||\text{cn}(2s_2^2, k)| \} = |J_p(s_4^1)| - |\zeta(s_4^1)||\Gamma(s_2^1)|,
\]
(4.5)
\[
(1.3) \implies \frac{|\text{cn}(s_3^1, k)|}{|\zeta(3s_3^2)|} \{ |\Gamma(3s_4^2)| - |\text{cn}(3s_1^3, k)||J_p(3s_2^3)| \} = |J_p(s_4^1)| - |\zeta(s_4^1)||\Gamma(s_2^1)|,
\]
thus we obtain (1.5) by equating left-hand sides in (4.5).

The complete result is as follows.

**Theorem 2.** For the cell \(C_0 = \{(1.2) - (1.7)\}\) there is the following set of 12 crossbreedings of type \((A_2)\):
\[
[(1.2) \times (1.3)]|\text{cn}(s_3^1, k)| = (1.5), \quad [(1.3) \times (1.7)]|\zeta(3s_3^2)| = (1.4),
\]
\[
[(1.2) \times (1.4)]|\text{cn}(s_3^1, k)| = (1.6), \quad [(1.4) \times (1.6)]|\Gamma(4s_3^2)| = (1.2),
\]
\[
[(1.2) \times (1.5)]|J_p(2s_3^2)| = (1.3), \quad [(1.4) \times (1.7)]|\Gamma(4s_3^3)| = (1.3),
\]
\[
[(1.2) \times (1.6)]|J_p(2s_3^3)| = (1.4), \quad [(1.5) \times (1.6)]|J_p(2s_2^3)| = (1.7),
\]
\[
[(1.3) \times (1.6)]|\text{cn}(s_3^1, k)| = (1.7), \quad [(1.5) \times (1.7)]|\zeta(3s_3^2)| = (1.6),
\]
\[
[(1.3) \times (1.5)]|\zeta(3s_3^2)| = (1.2), \quad [(1.6) \times (1.7)]|\Gamma(4s_3^3)| = (1.5).
\]

Now, from (4.5) and (4.6) we have

**Corollary.** For the cell \(C_0\) there is the set of 48 crossbreedings of types \((A_1)\) and \((A_2)\) respectively, and these are conserving that basic cell. In other words, we have the set of 48 transmutations of the mother formula (1.8), \(= (2.5)\).

**Remark 13.** If we make use transformations (1.10), (1.11) (see Remark 2) on the results (4.2) – (4.4), (4.6) in \(C_0\) then we obtain corresponding results in \(C_m, m \in \mathbb{N}\).

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