Measurement of six degree motion error of linear stage

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Abstract. In this paper, we propose measurement system of the six degree of motion errors which is based on distance measurement by the laser interferometer. The system has six parallel laser beams and six corner cube mirrors on the linear stage, which reflect the corresponding laser beams. The error of axial direction is measured with the ordinary distance measurement method by laser interferometer. The vertical errors to the axial direction and the roll errors are measured by tilted beams using the wedge prism. The pitch and yaw errors are measured by the difference between distance of two corner cube mirrors. The experimental layout of the corner cube mirrors and the other optical devices are shown. As a result, the resolution of 66.5 nm for error of the axial direction, the resolution of 383 nm for the vertical errors to the axial direction, the resolution of 1.23 arc-sec for the roll error and the resolution of 0.429 arc-sec for the pitch and yaw errors are obtained in this system.

1. Introduction
Six Degree-of-freedom (6DOF) motion errors of a linear stage of a machine tool, a coordinate measuring machine and so on should be measured simultaneously. Accurate position and orientation can be obtained and higher compensation can be expected. However, 6DOF motion errors cannot be measured simultaneously because the stages have no enough space to setup conventional measuring instruments such as a displacement sensor and an autocollimator. Therefore, it is expected to develop the measuring instrument that can simultaneously measure the 6DOF motion errors is expected. For these reasons, many researchers have developed the measuring method of the 6DOF motion errors [1-5]. And, we developed the measuring system of 6DOF motion errors that employs the principle of the Michelson interferometer for all direction and rotational angle because of the advantage of constructing high accurate measuring system with comparative ease [6]. In this paper, we propose the 6DOF measurement system using ball lens as the target mirror of the interferometer.

2. Measurement principle
2.1. Optical system
The schematic diagram of the system is shown in Figure 1. The system is based on Michelson interferometer. The moving plate (MP) which consisted of six target mirrors (TM) and two plates, was movable and was mounted on a stage as a measuring object in situ situation. Six ball lenses were used as the target mirrors. The refractive indices of those are nearly 2. To fulfil retro reflection of a laser beam, the ball lenses were coated with silver film in a hemispheric part as shown in Figure 2. The other optical devices were fixed on an optical table. The laser beam emitted from laser source was expanded by two plano-convex lenses. The expanded laser beam was separated into the two laser
beams by a beam splitter. One of the two laser beams passed toward TM and was partly tilted by three wedge prisms (WP). The other laser beam passed toward a reference mirror (RM). Each laser beam entered and was reflected by the target mirrors and the RM. Finally, the laser beam reflected from the target mirrors interferes with the laser beam reflected by the RM. When the MP moved, optical path difference between the two laser beams was varied. The displacements of the 6DOF motion errors of MP could be measured by counting the number of the interference fringes due to the two laser beams. The number of the interference fringes could be obtained by photo diodes (PD).

Figure 1. Configuration of optical system of 6DOF laser interferometer.

Figure 2. Moving plate (MP).

2.2. Displacement measurement in x direction
The schematic diagram of the principle of displacement measurement in x direction is shown in Figure 3. When MP moves to the x direction with a value of $\Delta x$, the relation of the displacement $\Delta x$ and the number of the interference fringes which are caused by the interference of the laser beams reflected from the TM4 and the RM is shown as following equation (1).

$$\Delta x = \frac{\lambda}{2} n_4$$ (1)
Here, \( \lambda \) denotes a wavelength of the laser beam, \( n_4 \) denotes the number of interference fringe obtained by the PD which is irradiated by the reflected laser beam from TM4.

![Figure 3. The displacement measurement in x-direction.](image)

2.3. Displacement measurement in y direction

The schematic diagram of the principle of y direction displacement measurement is shown in Figure 4. A part of the laser beam is tilted with the angle \( \theta \) to the x axis in the x-y coordinate system by the WP2. When MP moves in the y direction with the value of \( \Delta y \), the optical path length of the tilted laser beam changes only \( \lambda n_5/2 \) where \( n_5 \) is regarded as the number of interference fringes obtained by the PD which is irradiated by the laser beam reflected from TM5. The relation of the displacement \( \Delta y \) and the number of the interference fringes which is caused by the interference of laser beams reflected from the TM5 and the RM is shown as following equation (2).

\[
\Delta y = \frac{\lambda}{2 \sin \theta} n_5
\]

(2)

Here, \( \lambda \) denotes the wave length of the laser beam.

![Figure 4. The displacement measurement in y direction.](image)

2.4. Displacement measurement in z direction and roll angle measurement around x axis

The schematic diagram of the principle of z direction and roll displacement measurement is shown in Figure 5. TM1, TM6, WP1, and WP3 are used in the measurement. The displacement in z direction can be measured by the same method as that in y direction. The displacement in z direction is measured by the both laser beams tilted by the both WP1 and WP3 of which primary axes are rotated with 90 degree around x axis. Thus, the displacement \( \Delta z \) and the roll displacement \( \Delta \theta_x \) can be measured as the following equation (3) and equation (4).

\[
\Delta z = \frac{\lambda}{4 \sin \theta} (n_1 + n_6)
\]

(3)
\[ \Delta \theta_x = \tan^{-1} \left[ \frac{\lambda}{2L_{1-6}} \sin \theta \right] \left( n_1 - n_6 \right) \]  

(4)

Here, \( \lambda \) denotes the wave length of the laser beam, \( L_{1-6} \) denotes the distance between TM1 and TM6.

2.5. Pitch and yaw angle measurement

The schematic diagram of the principle of pitch displacement measurement is shown in Figure 6. The pitch displacement can be calculated by x direction displacement of TM3 and TM4 and the distance between TM3 and TM4. The x direction displacement of TM3 and TM4 can be measured by the same method of \( \Delta x \) mentioned in 2.2. The yaw displacement can be measured as similar as the pitch displacement using TM2 and TM4. Thus, the pitch and yaw displacement \( \Delta \theta_y \) and \( \Delta \theta_z \) are expressed as following equation (5) and equation (6).

\[ \Delta \theta_y = \tan^{-1} \left[ \frac{\lambda}{2L_{3-4}} \left( n_3 - n_4 \right) \right] \]  

(5)

\[ \Delta \theta_z = \tan^{-1} \left[ \frac{\lambda}{2L_{2-4}} \left( n_2 - n_4 \right) \right] \]  

(6)

Here, \( \lambda \) denotes the wave length of the laser beam, \( L_{3-4} \) denotes the distance between TM3 and TM4, \( L_{2-4} \) denotes the distance between TM2 and TM4, \( n_2, n_3 \) and \( n_4 \) denotes the number of interference fringes obtained by the PD which are irradiated by the laser beam reflected from TM2, TM3 and TM4.

3. Experiment

The specifications of the optical devices are shown in Table 1. To evaluate the system, displacement of the MP was measured by the system. The displacement of the MP was also measured by 5530 Laser Calibration System manufactured by Agilent Technologies to be compared with the measured value of...
the system. For the purpose of giving the MP the displacement, the MP was set on positioning stages. P-733.3DD: the 3 axes piezo nanometer positioning stage manufactured by Physik Instrumente was used for the x, y and z direction movement. KGW06100, KGW06075 and KRW06360: the goniometer and rotation stages manufactured by SURUGA SEIKI were used for the roll, pitch and yaw angle movements. The specifications of the used positioning stages are shown in Table 2. The number of the interference fringes were calculated by dividing measuring time by a period of a sinusoidal shaped measurement data that was fitted to the function of sine curve using the least squares method in the experiment of translational displacement measurement. In angular displacement measurement, the number of the interference fringes were counted in 0.5 times period of the sine curve because the least squares method couldn’t be applied to the deformed measurement data due to low repeatability of the rotational and goniometer stages compared with the resolution of the interferometer. Regarding of translational displacement measurement, 1-10 μm displacements were given to the stage by 1 μm step. Regarding of rotational displacement measurement, the displacements from 50 to 250 arcsec were given to the stages by 50 arcsec step. Once displacement was measured, the stages were returned to the original point. Each displacement was measured five times. The results of the experiments of translational and rotational displacement measurements were shown in Figure 7 and Figure 8. The horizontal axis indicates the measured length by the Laser Interferometer (5530 Laser Calibration System). The vertical axis indicates the measured value of the proposed interferometer. The averages of five times measurements are shown. Let standard deviation be σ, the error bars indicate 3σ., The results of the movement of 1 μm in the y direction measurement and those of 1 μm and 2 μm in the z direction measurement were not recorded in the Figure 7 because the interferometric fringes are so obscure at these locations that the number of fringes could not be counted. From Figure 7 and Figure 8, linear outputs of the interferometer were obtained. It is proved that the proposed system has the ability to measure the 6DOF motion errors. It is assumed that the errors of the y, z and roll measurements were caused by misalignment of the WP because those errors were larger than the error of x measurement whose system does not employ WP.

| Table 1. The specifications of the optical devices. |
|-----------------------------------------------|
| λ: Wave length of the laser beam               | 632.57 nm |
| θ: Tilted angle of the laser beam              | 10 degree |
| Refractive indices of the ball lens            | 2.003     |
| L₁₆: Distance between TM₁ and TM₆              | 48.06 mm  |
| L₃₄: Distance between TM₃ and TM₄              | 7.92 mm   |
| L₂₄: Distance between TM₂ and TM₄              | 8.19 mm   |

| Table 2. The specifications of positioning stages. |
|-----------------------------------------------|
| XYZ-Stage [P-733.3DD] | Gonio-Stage [KGW06100] | Gonio-Stage [KGW06075] | θ-Stage [KRW06360] |
| Direction             | x, y, z               | roll                  | pitch                 | yaw                     |
| Resolution            | 0.1 nm                | 0.002466 degree       | 0.0032 degree         | 0.004 degree            |
| Repeatability         | 2 nm, 1 nm            | less than 0.003 degree | less than 0.003 degree | less than 0.01 degree   |
Six DOF motion errors measurement system that employed the interferometric principle using ball lens as the target mirrors was proposed. The experiment results showed that the relationship between outputs of the proposed system and those of 5530 Laser Calibration System manufactured by Agilent Technologies were linear. Consequently, the proposed system can measure the 6DOF motion errors. As the maximum errors between the outputs of the proposed system and those of the laser calibration system, 0.09 $\mu$m for x, 0.71 $\mu$m for y and 0.52 $\mu$m for z were obtained in the range of 10 $\mu$m travel of each direction. 11.1 arcsec for roll, 3.9 arcsec for pitch and 2.5 arcsec were obtained within the range of 250 arcsec of each rotational angle. as the maximum standard deviation, 0.13 $\mu$m for x, 0.18 $\mu$m for y, 0.30 $\mu$m for z, 3.9 arcsec for roll, 3.6 arcsec for pitch and 2.2 arcsec for yaw were obtained. Due to

**Figure 7.** Results of the translation displacements.

**Figure 8.** Results of the rotational angle measurements.

4. Conclusion
the interferometric principle of the proposed system, higher resolution and accuracy are expected if the commercial laser interferrometer are employed as the light source of the proposed system.

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