Effects of pairing correlation on the low-lying quasiparticle resonance in neutron drip-line nuclei

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We discuss the effects of pairing correlation on quasiparticle resonance. We analyze in detail how the width of the low-lying ($E_x \lesssim 1$ MeV) quasiparticle resonance is governed by the pairing correlation in the neutron drip-line nuclei. We consider the $^{46}\text{Si} + n$ system to discuss the low-lying $p$-wave quasiparticle resonance. Solving the Hartree–Fock–Bogoliubov equation in coordinate space with the scattering boundary condition, we calculate the phase shift, the elastic cross section, the resonance width, and the resonance energy. We find that the pairing correlation has the effect of reducing the width of the quasiparticle resonance that originates from a particle-like orbit in weakly bound nuclei.

Subject Index D11, D13, D27

1. Introduction

Weakly bound nuclei near the drip-line have properties that are not seen in strongly bound stable nuclei. The neutron halo is a typical example [1,2]. Apart from quantal penetration caused by the small separation energy, the neutron pairing correlation plays crucial roles here; e.g., to determine the binding of two-neutron halo nuclei [3–7]. Note, however, that the pairing correlation in weakly bound nuclei is different from that in stable nuclei since it causes configuration mixing involving both bound and unbound (continuum) single-particle orbits, and this continuum coupling brings about novel features [5,8–17]. For example, the pairing correlation persists in drip-line nuclei only with the continuum coupling to allow binding of a two-neutron halo [5,8]. The continuum coupling is also necessary for the di-neutron correlation, a characteristic spatial correlation in neutron-rich nuclei [14,17–19]. On the other hand, the continuum coupling has a seemingly opposite mechanism to suppress the development of the halo radius, called pairing anti-halo effects [11,13,20].

Another interesting example is the possible manifestation of a new type of resonance generated by the pairing correlation and the continuum coupling, called the quasiparticle resonance [21,22]. If one describes a single-particle scattering problem within the scheme of Bogoliubov’s quasiparticle theory, even a scattering state becomes a quasiparticle state that has both “particle” and “hole” components. In other words, an unbound nucleon couples to a Cooper pair and a bound hole orbit, then forms a resonance. This quasiparticle resonance is also expected to exhibit new features in weakly bound nuclei since the continuum coupling becomes stronger as the separation energy decreases.

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In the case of well bound stable nuclei, the depth of the Fermi surface is around 8 MeV. Therefore, quasiparticle resonances, which emerge above the separation energy, have excitation energies larger than 8 MeV, and hence they correspond to deep-hole orbits. The excitation energy $E_{\text{stable}}^x$ of the quasiparticle resonance is much larger than the pair gap $\Delta$: $E_{\text{stable}}^x \gg \Delta$. In this case, the effect of the pairing correlation is treated in a perturbative way [21,22]. The resonance width $\Gamma$, for instance, is evaluated on the basis of Fermi’s golden rule. The width $\Gamma$ is predicted to be proportional to the square of the pair gap $|\Delta_{\text{average}}|^2$, and $\Gamma$ is estimated to be small (i.e., on the order of 1–100 keV) [21], much smaller than the experimentally known typical width (several MeV) of the deep-hole resonances [23]. Experimental identification of the pairing effect on the deep-hole resonances is not very promising in this respect [9].

In the case of small separation energy, in particular, in neutron-rich nuclei, the properties of the quasiparticle resonance may be different from those in stable nuclei. A neutron-rich nucleus has a shallow Fermi energy, with an extreme depth smaller than 1 MeV to be realized in neutron drip-line nuclei. In this case, the excitation energy of a quasiparticle resonance might be comparable with or smaller than the pair gap: $E_{\text{unstable}}^x \lesssim \Delta$. The pairing correlation may cause strong configuration mixing between weakly bound orbits and low-lying continuum orbits, since both are located near the Fermi surface. The perturbative description may not be applicable, and we expect an undisclosed relation between the quasiparticle resonance and the pairing correlation.

The small neutron separation energy provides another merit in studying the quasiparticle resonance. In this case, the quasiparticle resonance also appears in the low-lying region, where the level density is low. Other mechanisms beyond the mean-field approximation, for instance, the fragmentation due to coupling to complex configurations [24], are expected to be suppressed. This might increase the possibility of observing the quasiparticle resonance directly.

There exist several theoretical works that study the quasiparticle resonance in nuclei near the neutron drip-line [12,13,16,25–34]. Many of them employ the self-consistent Hartree–Fock–Bogoliubov (HFB) scheme [16,25–29], or its variation, in which the Hartree–Fock potential is replaced with the Woods–Saxon potential [12,13]. The quasiparticle resonance in deformed nuclei is also discussed [30,31]. Approximate schemes using the Hartree–Fock + BCS theory are also adopted both in nonrelativistic and relativistic frameworks [32–34]. Despite these previous studies, the effects of the pairing correlation on the low-lying quasiparticle resonance in weakly bound nuclei have not yet been revealed. We shall discuss this subject in order to understand the behavior of the pairing correlation in drip-line nuclei and unbound nuclei.

In the present study, we particularly aim to reveal the effects of the pairing correlation on the width of the low-lying quasiparticle resonance in drip-line nuclei. We focus on neutron resonances, in particular, in $p$ waves with small excitation energies $E_x \lesssim$ a few MeV. The continuum coupling is expected to be influential for neutrons in low-angular-momentum partial waves, i.e., in $s$ and $p$ waves, because of the existence of no (or small) Coulomb and centrifugal barriers. Additionally, neutrons in these partial waves play an important role in the neutron halo. Also, scattering neutrons in low-angular-momentum waves are a major contributor in the low-energy neutron-capture phenomena [35], important for astrophysical applications. In the present work, we discuss the $p$-wave quasiparticle resonance as the first step in a series of studies. The case of the $s$ wave, which involves a virtual state, will be discussed separately in a future publication.

It is not appropriate to treat the effects of the pairing correlation as a perturbation in the calculation of the resonance width in weakly bound nuclei. We therefore describe the continuum quasiparticle states by solving numerically the Hartree–Fock–Bogoliubov equation (equivalent to the
Bogoliubov–de Gennes equation) in the coordinate space \([9,21,22,36]\) to obtain the wave function of a neutron quasiparticle in the continuum. We impose the scattering boundary condition \([12,13,21,25]\). In this way, we calculate the phase shift for the continuum quasiparticle state and the elastic cross section for a neutron scattered by the superfluid nucleus. Then the resonance width and the resonance energy are extracted from the obtained phase shift. As a concrete example, we describe \(^{46}\)Si and an impinging neutron; in other words, a quasiparticle resonance in \(^{47}\)Si. Hartree–Fock–Bogoliubov calculations predict that this nucleus is located at or close to the neutron drip-line \([37]\). Also, it has neutron \(2p\) orbits in \(^{46}\)Si, which are expected to be weakly bound or located just above the threshold energy.

This paper is constructed as follows: In Sect. 2, we explain the HFB theory in the coordinate space, the scattering boundary condition of the Bogoliubov quasiparticle, and some details of the adopted model. In Sect. 3, we show the results of numerical analysis performed for the \(^{46}\)Si + \(n\) system. We also discuss the effects of the pairing correlation on the resonance width using a systematic calculation with various pairing strengths and nuclear potential depths. Finally, we draw conclusions in Sect. 4.

### 2. Theoretical framework

#### 2.1. The Hartree–Fock–Bogoliubov equation in coordinate space with the scattering boundary condition

We introduce the wave function of the Bogoliubov quasiparticle state in the notation of Refs. \([36,38]\). It has two components:

\[
\phi_i(\vec{r}\sigma) = \begin{pmatrix} \varphi_{1,i}(\vec{r}\sigma) \\ \varphi_{2,i}(\vec{r}\sigma) \end{pmatrix}. \tag{1}
\]

Here, \(\vec{r}\) is the spatial coordinate and \(\sigma\) represents the spin variable. Assuming that the system has spherical symmetry, we write the Bogoliubov quasiparticle wave function as

\[
\varphi_{1,i}(\vec{r}\sigma) = \frac{u_{ij}(r)}{r} \left[ Y_{l}(\theta, \varphi) \chi_{1}^{1}(\sigma) \right]_{jm}, \quad \varphi_{2,i}(\vec{r}\sigma) = \frac{v_{ij}(r)}{r} \left[ Y_{l}(\theta, \varphi) \chi_{1}^{1}(\sigma) \right]_{jm}, \tag{2}
\]

where \(l, j, m\) are the angular-momentum quantum numbers of the quasiparticle state, with \(Y\) and \(\chi\) being the spherical harmonics and the spin wave function. We also assume that the HF potential and the pair Hamiltonian \(\Delta(\vec{r})\) are local and real; then the Hartree–Fock–Bogoliubov equation in the coordinate space is written as

\[
\begin{pmatrix}
-h^2/2m \frac{d^2}{dr^2} + U_{ij}(r) - \lambda \\
\Delta(r)
\end{pmatrix}
\begin{pmatrix}
u_{ij}(r) \\
\Delta(r)
\end{pmatrix} = E
\begin{pmatrix}
u_{ij}(r) \\
\Delta(r)
\end{pmatrix}, \tag{3}
\]

where \(\lambda(< 0)\) and \(E\) are the Fermi energy and the quasiparticle energy, respectively. Here, the upper component of the quasiparticle wave function \(u_{ij}(r)\) represents an amplitude of the quasiparticle with the particle character, called hereafter the “particle” component for short. The lower component \(v_{ij}(r)\) represents the “hole” component. \(U_{ij}(r)\) is the mean-field potential and \(m\) is the mass of the neutron. The quasiparticle spectrum consists of discrete states with \(E < |\lambda|\) and continuum states with \(E > |\lambda|\) \([36]\).

We intend to describe a system consisting of a superfluid nucleus and an impinging neutron, which in principle should be treated as a many-body unbound state. However, we adopt an approximation in which the neutron is treated as an unbound quasiparticle state, governed by Eq. (3), built on a pair-correlated even–even nucleus. In other words, we neglect the self-consistent effect of an unbound
neutron on the mean-field and the pair correlation. Under this assumption, we focus on continuum quasiparticle states with \( E > |\lambda| \), which correspond to unbound single-particle states with positive neutron kinetic energy. We impose the scattering boundary condition on the Bogoliubov quasiparticle at distances far outside the nucleus as

\[
\frac{1}{r} \left( \frac{u_{lj}(r)}{v_{lj}(r)} \right) = C \left( \cos \delta_{lj} \frac{\tilde{j}_l(k_1 r) - \sin \delta_{lj} \tilde{n}_l(k_1 r)}{D h^{(1)}_l(i k_2 r)} \right) \xrightarrow{r \to \infty} C \left( \frac{\sin(k_1 r - \frac{\pi}{2} + \delta_{lj})}{k_1 r} \right),
\]

where \( k_1 = \sqrt{2m(\lambda + E)}/\hbar \), \( \kappa_2 = \sqrt{-2m(\lambda - E)}/\hbar \) [12,21,22,25,36]. The normalization factor \( C = \sqrt{2m k_1}/\hbar^2 \pi \) to satisfy \( \sum_{\sigma} \int d\vec{r} \phi^\dagger(\vec{r} \sigma, E) \phi(\vec{r} \sigma, E') = \delta(E - E') \). Here \( \delta_{lj}, j_l(z), n_l(z), h^{(1)}_l(z) \) are the phase shift, the spherical Bessel function, the spherical Neumann function, and the first-kind spherical Hankel function, respectively. The quasiparticle resonance can be seen in the elastic scattering of a neutron, and the elastic cross section \( \sigma_{lj} \) associated with each partial wave is

\[
\sigma_{lj} = \frac{4\pi}{k_1^2} \left( j + \frac{1}{2} \right) \sin^2 \delta_{lj}.
\]

2.2. Details of numerical calculation

We solve the radial HFB equation (3) in the radial coordinate space under the scattering boundary condition (4) of the Bogoliubov quasiparticle. In the present study, we simplify the HF mean-field by replacing it with the Woods–Saxon potential in a standard form:

\[
U_{lj}(r) = \left[ V_0 + (\vec{l} \cdot \vec{s})V_{SO} \frac{r_0^2}{r} \frac{d}{dr} \right] f_{WS}(r) + \frac{h^2 l(l + 1)}{2mr^2}, \quad f_{WS}(r) = \left[ 1 + \exp \left( \frac{r - R}{a} \right) \right]^{-1}.
\]

Although the self-consistency of the mean-field is neglected, an advantage of this treatment is that we can easily change the parameters of the potentials, facilitating systematic numerical analysis. On the other hand, the effects of weak binding on the potential, for instance, large diffuseness and long tail, are not taken into account in the present calculation. We also assume that the pair potential \( \Delta(r) \) has the Woods–Saxon shape:

\[
\Delta(r) = \Delta_0 f_{WS}(r).
\]

following Ref. [12]. The magnitude of the pair potential \( \Delta_0 \) is controlled by the average pair strength \( \tilde{\Delta} \) [12]:

\[
\tilde{\Delta} = \frac{\int_0^\infty r^2 \Delta(r) f_{WS}(r) dr}{\int_0^\infty r^2 f_{WS}(r) dr} = 0.0 - 3.0 \text{ MeV}.
\]

We change the strength \( \tilde{\Delta} \) from 0.0 MeV to 3.0 MeV in this study, considering the empirical systematics of the pair gap \( \Delta \sim 12.0/\sqrt{\tilde{\Delta}} \text{ MeV} \) [39] (\( \Delta \sim 1.7 \text{ MeV} \) for \( ^{46}\text{Si} \)). The parameters of the Woods–Saxon potential are taken from Ref. [39]. The radial wave function is numerically solved up to \( r_{\text{max}} = 40 \text{ fm} \), where it is connected to the Hankel functions, Eq. (4).

We consider the \( ^{46}\text{Si} + n \) system for the following reasons. First, \( ^{46}\text{Si} \) is predicted to be the drip-line nucleus in Si isotopes and the deformation of this nucleus is small according to the HFB calculations (see, for instance, Refs. [37,40,41]). It may be reasonable to assume that \( ^{46}\text{Si} \) has a spherical shape in the present calculation. Second, the neutron \( 2p_{3/2} \) or \( 2p_{1/2} \) orbits are expected to be either weakly bound or slightly unbound, and hence they are expected to form low-lying quasiparticle resonances. Note that \( ^{46}\text{Si} \) has not yet been observed experimentally [42].
Table 1. Neutron single-particle orbits in the Woods–Saxon potential of $^{46}$Si, obtained with the standard Woods–Saxon parameter [39].

| Single-particle orbit | Single-particle energy $e_{sp}$ [MeV] |
|-----------------------|--------------------------------------|
| $2p_{1/2}$            | $-0.056$                             |
| $2p_{3/2}$            | $-1.068$                             |
| $1f_{7/2}$            | $-2.821$                             |

Fig. 1. (a) Elastic cross sections $\sigma_{lj}$ for various partial waves in the case of $\Delta = 0.0$ MeV. (b) The same as (a), but for the case of $\Delta = 1.0$ MeV.

The neutron single-particle energies around the Fermi energy for $^{46}$Si in the Woods–Saxon potential are shown in Table 1. Both $2p$ orbits are bound very weakly for the original parameter set. In particular, the energy of the $2p_{1/2}$ orbit is very small: $e_{sp} = -0.056$ MeV. For the Fermi energy $\lambda$, we use a fixed value $\lambda = -0.269$ MeV, which is obtained by the Woods–Saxon–Bogoliubov calculation [30].

3. Results and discussion

3.1. Cross section and phase shift of neutron elastic scattering

Figure 1 shows the calculated elastic cross section that is obtained (a) without the pairing correlation ($\Delta = 0.0$ MeV) and (b) with the pairing correlation ($\Delta = 1.0$ MeV). In the case of $\Delta = 0.0$ MeV, single-particle potential resonances are found in the $f_{5/2}$ and $g_{9/2}$ waves, corresponding to the $1f_{5/2}$ and $1g_{9/2}$ orbits trapped by the centrifugal barrier. Note that configurations with the last neutron occupying the $2p_{3/2}$ or $2p_{1/2}$ orbits are bound states, and are not seen in Fig. 1(a).

On the other hand, in the case of $\Delta = 1.0$ MeV, we see narrow low-lying peaks in the $p_{1/2}$, $p_{3/2}$, and $f_{7/2}$ waves, which do not exist in the case of $\Delta = 0.0$ MeV. These peaks are not potential resonances caused by the centrifugal barrier. These characteristic resonances are the quasiparticle resonances, which are caused by the pairing correlation. They are associated with the weakly bound single-particle orbits $2p_{1/2}$, $2p_{3/2}$, and $1f_{7/2}$. With $\Delta = 1.0$ MeV, the quasiparticle states corresponding to $2p_{3/2}$ or $2p_{1/2}$ orbits become unbound resonances, seen as the low-lying peaks in Fig. 1(b). It is noted that the $2p_{1/2}$ resonance energy is lower than that of $2p_{3/2}$, with the ordering opposite to the standard single-particle states.

In the following discussion, we focus on the low-lying $2p_{1/2}$ resonance. Figure 2 shows the elastic cross sections and the phase shifts of the $2p_{1/2}$ resonance that are obtained for various values of the...
pairing strength $\tilde{\Delta}$. It is seen in these figures that the resonance is influenced significantly by the pairing strength $\tilde{\Delta}$.

For $\tilde{\Delta} = 0.0$ MeV, no single-particle resonance is seen in the $p_{1/2}$ wave since the $2p_{1/2}$ orbit is bound with the single-particle energy $E_{2p_{1/2}} = -0.056$ MeV and the corresponding quasiparticle energy $E_{2p_{1/2}} = |e_{2p_{1/2}} - \lambda| = 0.213$ MeV is smaller than the threshold $|\lambda| = 0.269$ MeV. As $\tilde{\Delta}$ increases ($\tilde{\Delta} \sim 0.5$ MeV), the $2p_{1/2}$ quasiparticle state acquires a quasiparticle energy $E$ larger than $|\lambda|$, and then appears in the continuum region as a resonance. On further increasing $\tilde{\Delta} \gtrsim 1$ MeV, both the resonance width and the resonance energy are found to increase. The increase in the resonance energy may be anticipated qualitatively, as the conventional BCS expression for the quasiparticle energy $E = \sqrt{(e_{sp} - \lambda)^2 + \Delta^2}$ suggests. The increase of the width $\Gamma$ as a function of the pair potential ($\propto |\tilde{\Delta}|^2$) is suggested in the perturbative analysis [21,22]. However, we found that nontrivial pairing effects are involved here, as we discuss below.

### 3.2. Resonance width and resonance energy

We evaluate the resonance width and the resonance energy in order to investigate quantitatively the effects of the pairing correlation on these values. We extract the resonance width and the resonance energy from the calculated phase shift using a fitting method. We employ the following function to fit:

$$\delta(e) = \arctan\left(\frac{2(e - e_R)}{\Gamma}\right) + a(e - e_R) + b,$$

where $e$, $\Gamma$, and $e_R$ are the kinetic energy of the scattering neutron, the resonance width (defined as the full width at half maximum (FWHM)), and the resonance energy, respectively, and the constants $a$ and $b$ represent a smooth background. We perform the fitting in the following two steps. First, we introduce a tentative energy interval and perform a fitting. Next, using zeroth-order values $e_R^{(0)}$ and $\Gamma^{(0)}$, we perform the second fitting for the interval $\max(e_R^{(0)} - \Gamma^{(0)}, 0) \leq e \leq e_R^{(0)} + \Gamma^{(0)}$. Figure 3 shows the resonance width $\Gamma$ and the resonance energy $e_R$ for various values of $\tilde{\Delta}$ corresponding to Fig. 2(b). The vertical axis is the resonance width $\Gamma$ and the horizontal axis is the resonance energy $e_R$. Both the resonance width $\Gamma$ and the resonance energy $e_R$ increase as the strength of the pairing correlation $\tilde{\Delta}$ increases. Although the resonance width $\Gamma$ becomes larger than the resonance energy $e_R$ for $\tilde{\Delta} \geq 2.0$ MeV, we regard it as a meaningful resonance since the fitting is of as good quality as that in the cases of $\tilde{\Delta} < 2.0$ MeV.
Fig. 3. The $e_R$–$\Gamma$ relation of the $2p_{1/2}$ quasiparticle resonance for various values of $\Delta$. The vertical axis is the resonance width $\Gamma$ and the horizontal axis is the resonance energy $e_R$.

Fig. 4. (a) The single-particle energy of the neutron $2p_{1/2}$ orbit for various depths of the Woods–Saxon potential. The vertical axis is the neutron single-particle energy and the horizontal axis is the variation of the potential depth $\Delta V_0$. Positive single-particle energy represents the resonance energy, and the length of the attached vertical bar represents the resonance width (FWHM). The dotted line indicates the height of the centrifugal barrier. (b) The $e_R$–$\Gamma$ relation of the $2p_{1/2}$ single-particle potential resonance for various potential depths $\Delta V_0$.

To investigate systematically the influence of the position of the single-particle orbit on the resonance, we change not only the strength of the pairing correlation $\Delta$ but also the single-particle energy of the $2p_{1/2}$ orbit. We vary the depth of the Woods–Saxon potential $V_0$ to change the single-particle energy. The variation from the original value is denoted by $\Delta V_0$. Figure 4(a) shows the $2p_{1/2}$ single-particle energy as a function of $\Delta V_0$. The length of the vertical bars in the figure represents the resonance width (FWHM). It is seen that the $2p_{1/2}$ orbit enters into the continuum as the depth is increased by $\Delta V_0 \sim 0.5$ MeV. The resonance width (vertical bars) increases with further increase of potential depth. The height of the centrifugal barrier $E_{\text{barrier}}$ for the $p_{1/2}$ wave (the dotted curve in Fig. 4(a)) is $\sim 0.5$ MeV, being approximately independent of $\Delta V_0$. Figure 4(b) shows the $e_R$–$\Gamma$ relation of the single-particle potential resonance corresponding to Fig. 4(a). For $\Delta V_0 \gtrsim 4.0$ MeV, the resonance width is very broad, $\Gamma \gtrsim 2e_R$, as expected from $e_R \gtrsim E_{\text{barrier}}$.

The resonance width and the resonance energy evaluated for various $\Delta$ and $\Delta V_0$ are plotted in the $e_R$–$\Gamma$ plane in Fig. 5. For reference, the $e_R$–$\Gamma$ relation of the single-particle potential resonance (Fig. 4(b)) is also shown.

Figure 5(a) is a plot displaying the dependence of $\Gamma$ on $\Delta$ for fixed values of $\Delta V_0$. We see that both the resonance width and the resonance energy increase with increasing $\Delta$ for all the values of $\Delta V_0$. 
dent of the resonance energy $e_R$ quasiparticle resonance, and the latter a hole-like quasiparticle resonance. Let us first analyze the hole-like quasiparticle resonances; i.e., those in the case of $\Delta > 0$. We shall examine these points in the following subsections.

Figure 5 is another plot showing the dependence on $\Delta V_0$ for fixed values of $\bar{\Delta}$. A distinctive feature seen in Fig. 5 is that the quasiparticle resonance exists even at energies $e_R$ higher than the barrier height $E_{\text{barrier}} \sim 0.5$ MeV. It is also seen that the $e_R - \Gamma$ relation displays two different features. One is seen in the bottom-right region of Fig. 5(b), where the resonance width changes only slightly with the resonance energy. The other is that the resonance width increases sensitively as the resonance energy changes, seen in the upper-left region. This difference in the $e_R - \Gamma$ relation is related to whether the $2p_{1/2}$ orbit is located above or below the Fermi energy. In other words, the difference originates from whether the original $2p_{1/2}$ orbit is particle-like or hole-like. More precisely, the $2p_{1/2}$ orbit is particle-like (hole-like) for $\Delta V_0 > -0.854$ MeV ($\Delta V_0 \leq -0.854$ MeV). The boundary $\Delta V_0 = -0.854$ MeV is plotted in Fig. 5(b) with open circles. In the following discussion, we call the former a particle-like quasiparticle resonance, and the latter a hole-like quasiparticle resonance.

Concerning the hole-like quasiparticle resonance, the resonance width is approximately independent of the resonance energy $e_R$. Deviation from this simple behavior is seen for $e_R \lesssim 1.0$ MeV. As for the particle-like quasiparticle resonance, the behavior is much more complicated and nontrivial. We shall examine these points in the following subsections.

### 3.3. Pairing effect on the hole-like quasiparticle resonance

Let us first analyze the hole-like quasiparticle resonances; i.e., those in the case of $e_{sp} < \lambda$. As already seen in connection with Fig. 5(b), the dependence of the resonance width $\Gamma$ on the average pairing gap $\bar{\Delta}$ appears rather simple: $\Gamma$ increases monotonically with $\bar{\Delta}$ while $\Gamma$ depends only weakly on the resonance energy $e_R$ or the single-particle energy $e_{sp}$. We shall now analyze the pairing dependence of the resonance width $\Gamma$ by comparing it with the analytical expression [21,22] that is derived for the hole-like quasiparticle resonance on the basis of the perturbation with respect to the pairing gap or the pairing potential.

The perturbative evaluation assumes that a single-hole state with energy $e_{sp}$ and wave function $\psi_i(\vec{r}\sigma)$ couples to unbound single-particle states $\psi_e(\vec{r}\sigma)$ only weakly via the pair potential $\Delta(\vec{r})$. This leads to the expression

$$\Gamma_i = 2\pi \left| \sum_{\sigma} \int d\vec{r} \psi_i^\dagger(\vec{r}\sigma) \Delta(\vec{r}) \psi_e(\vec{r}\sigma) \right|^2 \propto |\Delta_{\text{average}}|^2, \quad (10)$$

Fig. 5. The $e_R - \Gamma$ relation of the $2p_{1/2}$ quasiparticle resonance with various values of $\bar{\Delta}$ and $\Delta V_0$. (a) The $e_R - \Gamma$ relation for fixed values of $\Delta V_1$ with varying $\bar{\Delta}$ from 0.0 to 3.0 MeV. (b) The $e_R - \Gamma$ relation for given values of $\bar{\Delta}$ with varying $\Delta V_0$ from $-6.0$ to 4.0 MeV. The curve with $\bar{\Delta} = 0.0$ MeV is the $e_R - \Gamma$ relation of the $2p_{1/2}$ single-particle resonance, shown in Fig. 4(b).
The resonance energy in the zeroth order is 
\[ \varepsilon_R^0 = |e_i - \lambda| = |e_i| - |\lambda| \]
for the hole state. We shall now compare the resonance width \( \Gamma \) obtained from the numerical fit to the phase shift and that from the perturbative evaluation, Eq. (10). The results are shown in Fig. 6, which plots the evaluated widths as functions of the average pairing gap \( \bar{\Delta} \). The perturbative calculation using Eq. (10) is performed in two different ways, and they are plotted with the upward and downward triangles in Fig. 6. The curve with upward triangles is the case where the wave functions \( \varphi_i \) and \( \varphi_e \) of the hole and continuum orbits are fixed, and only \( \Delta(r) \) is changed. For the energy of \( \varphi_e \), we use the zeroth-order resonance energy \( \varepsilon_R^0 = |e_{2p1/2} - 2|\lambda| \). This scheme is called “Fermi’s golden rule 1” hereafter. In the calculation for the curve with downward triangles, we fix the single-particle wave function of the bound orbit \( \varphi_i \), but we choose the energy \( e \) of \( \varphi_e \) that reproduces the resonance energy \( \varepsilon_R(\bar{\Delta}) \) obtained from the phase shift for each \( \bar{\Delta} \) (called “Fermi’s golden rule 2”).

Figure 6(a) shows the \( \bar{\Delta} \) dependence of resonance width \( \Gamma \) for the resonance arising from the \( 2p_{1/2} \) hole state at \( \varepsilon_{sp} = -4.127 \text{ MeV} \) (\( \Delta V_0 = -10.0 \text{ MeV} \)). Figures 6(b) and (c) are the same as (a), but for the \( 2p_{1/2} \) hole orbits at \( \varepsilon_{sp} = -1.347 \text{ MeV} \) (\( \Delta V_0 = -4.0 \text{ MeV} \)) and \( \varepsilon_{sp} = -0.618 \text{ MeV} \) (\( \Delta V_0 = -2.0 \text{ MeV} \)), respectively. Figure 6(a) is for the case where the single-particle energy of the hole orbit is smaller than the Fermi energy \( \lambda = -0.269 \text{ MeV} \) by about 4 MeV. This is a typical hole-like quasiparticle resonance, since the resonance width \( \Gamma \) evaluated with perturbative calculations reproduces the nonperturbative evaluation of the resonance width \( \Gamma \). Deviations from the perturbative expression are seen in Figs. 6(b) and (c). The difference between the perturbative and nonperturbative evaluations becomes large as the single-particle energy \( \varepsilon_{sp} \) approaches the Fermi energy \( \lambda \) and the pair potential grows, as seen in Figs. 6(b) and (c).

Figure 7 shows the probability distributions \( |v(r)|^2 \) and \( |u(r)|^2 \) of the three examples of the hole-like quasiparticle resonance. The panels (a), (b), and (c) correspond to Figs. 6(a), (b), and (c), respectively (for \( \bar{\Delta} = 2.0 \text{ MeV} \)). Note that \( |u(r)|^2 \) is the probability distribution of the particle component while \( |v(r)|^2 \) is that of the hole component, and \( |u(r)|^2 + |v(r)|^2 \) is the total probability of...
finding the quasiparticle at position $r$. As expected, the probability $|u(r)|^2$ of the particle component is much smaller than the probability $|v(r)|^2$ of the main hole component in case (a), where the perturbation works well. In contrast, in case (c), where the perturbation breaks down, $|u(r)|^2$ is comparable to the probability $|v(r)|^2$ of the main hole component, indicating strong mixing of the particle component. For a more quantitative argument, we evaluate the probability distributions $|v(r)|^2$ and $|u(r)|^2$ integrated within the nuclear surface: $\bar{u}^2 = \int_0^R |u(r)|^2 dr$ and $\bar{v}^2 = \int_0^R |v(r)|^2 dr$, and evaluate the ratio $\bar{u}^2/\bar{v}^2$. The ratios are 0.021 and 0.254 for cases (a) and (c), respectively. In case (b), corresponding to the boundary region for the breakdown of the perturbation, the ratio is 0.091.

We have examined the applicability of the perturbative evaluation, Eq. (10), systematically for all the combinations of $\Delta$ and $\Delta V_0$ shown in Fig. 5. We adopt a criterion that both of the two evaluations of Eq. (10) with different choices of $\varphi_e$ agree with the nonperturbative numerical evaluation of the resonance width within $10\%$ error. We find then that the applicability of Eq. (10) is represented in terms of the single-particle energy $e_{sp}$, the Fermi energy $\lambda$, and the pair gap $\Delta$ as

$$e_{sp} \lesssim \lambda - 0.5\Delta. \quad (12)$$

We also examined the validity of Eq. (10) in terms of the ratio $\bar{u}^2/\bar{v}^2$. It is found that the applicability of Eq. (10) is also represented by

$$\bar{u}^2/\bar{v}^2 \lesssim 0.1. \quad (13)$$

The above analysis indicates that the perturbative evaluation works not only for the quasiparticle resonances associated with the deeply bound hole orbit, which has been considered previously [21,22], but also for quasiparticle resonances arising from a shallowly bound hole orbit; for instance, that with $e_{sp} \sim \lambda - 0.5\Delta$. Even in the latter case, the mixing of the particle component into the main hole component is small; $\bar{u}^2 \lesssim 0.1\bar{v}^2$. This is probably the reason why the perturbation works in the rather broad situation. In contrast, it is natural that the perturbation, Eq. (13), breaks down in the case of $e_{sp} > \lambda$, where the dominant component of the quasiparticle state is not the hole component $v(r)$, but the particle component $u(r)$. A quite different, probably nonperturbative, mechanism for the pairing effect on the resonance width is expected in this case.

### 3.4. Pairing effect on the particle-like quasiparticle resonance

We now analyze the particle-like quasiparticle resonances; i.e., those in the case of $e_{sp} \geq \lambda$. As typical examples, we examine two cases with $e_{2p1/2} = -0.056$ MeV ($\Delta V_0 = 0.0$ MeV) and...
Fig. 8. The $e_R$–$\Gamma$ relation of the $2p_{1/2}$ quasiparticle resonance in the case of the particle-like single-particle energies $e_{sp} = -0.056$ MeV ($\Delta V_0 = 0.0$ MeV) and $e_{sp} = 0.251$ MeV ($\Delta V_0 = 2.0$ MeV) (dashed and dotted curves), obtained by varying the average pairing gap $\Delta = 0.0$–3.0 MeV. The $e_R$–$\Gamma$ relation of the $2p_{1/2}$ single-particle potential resonance is also shown (solid curve).

Table 2. Resonance widths $\Gamma$ of the $2p_{1/2}$ quasiparticle and single-particle resonances that have $e_R = 0.300$, 0.375, and 0.450 MeV for three different values of $\Delta$. The single-particle resonance energy (or bound single-particle energy) $e_{sp}$ is also listed.

| $e_R$ [MeV] | 0.300 | 0.375 | 0.450 |
|-------------|-------|-------|-------|
| $\Delta$ [MeV] | 0.0   | 0.728 | 1.477 |
| $\Gamma$ [MeV] | 0.387 | 0.361 | 0.244 |
| $e_{sp}$ [MeV] | 0.300 | 0.251 | -0.056 |

with $e_{2p_{1/2}} = 0.251$ MeV ($\Delta V_0 = 2.0$ MeV). Note that $e_{2p_{1/2}} > \lambda$ in both cases. The curves from Fig. 5(a) corresponding to these cases are shown in Fig. 8. The $e_R$–$\Gamma$ relation of the single-particle potential resonance is also shown for reference.

As seen in Fig. 8 (and also in Fig. 5(a)), increasing the pairing potential increases monotonically both the resonance width $\Gamma$ and the resonance energy $e_R$, displaying a trend similar to that of the hole-like quasiparticle resonance. However, Fig. 5(b) also indicates that increasing the resonance energy with a fixed value of the pair potential leads to an increase in the resonance width in the particle-like case. We therefore suppose that two mechanisms are involved here. One is a kinematical effect: Due to the increase of the resonance energy, the penetrability of the centrifugal barrier increases, and consequently it leads to the increase of $\Gamma$. The other is a direct pairing effect, originating from the mixing between the particle and hole components caused by the pair potential.

In order to extract the latter mixing effect, we compare these three curves at the same resonance energy. As an example, we make a comparison at $e_R = 0.45$ MeV. We then find that the resonance width for $\bar{\Delta} = 1.634$ MeV is narrower than that for $\bar{\Delta} = 0.0$ MeV and the width for $\bar{\Delta} = 1.897$ MeV is the smallest among the three cases. The resonance widths for these three cases are listed in Table 2, together with other examples compared at $e_R = 0.300$ and 0.375 MeV. This shows that the pairing correlation has the effect of reducing the resonance width if the comparison is made at the same resonance energy.

To examine the mechanism of the reduced resonance width, we look into the wave functions of the three resonances with $e_R = 0.450$ MeV. Figure 9 shows the probability distribution of the resonant quasiparticle states with $e_R = 0.450$ MeV. In the case of $\bar{\Delta} = 0.0$ MeV, the hole component $v(r)$ vanishes and $u(r)$ coincides with the single-particle wave function of the $2p_{1/2}$ potential resonance. With finite values of $\bar{\Delta}$, and increasing $\bar{\Delta}$, the probability $|u(r)|^2 + |v(r)|^2$ within the surface of the
nucleus \( r \lesssim R \) become larger. This is consistent with our finding that the resonance width become narrower with larger pair potential. In particular, it is seen that the increase of the probability inside the nucleus originates mainly from the increase of the hole component \( v(r) \).

The increase of the hole component \( v(r) \) is a natural consequence of the pairing correlation. Here we recall the simple BCS formula for the \( u \) and \( v \) factors: the amplitudes of the particle and hole components are

\[
v_{BCS}^2 = \frac{1}{2} \left( 1 - \frac{e - \lambda}{E} \right), \quad u_{BCS}^2 = \frac{1}{2} \left( 1 + \frac{e - \lambda}{E} \right),
\]

respectively, with the quasiparticle energy \( E = \sqrt{(e - \lambda)^2 + \Delta^2} \). The hole probability \( v_{BCS}^2 \), which vanishes for \( \Delta = 0 \), increases with increasing \( \Delta \) since the pair potential causes mixing between the particle and hole components. We consider that a similar mixing mechanism takes place in the present case. In Table 3, we show the ratio \( \bar{v}^2/\bar{u}^2 \) of the particle and hole components obtained from the HFB calculation, and \( v_{BCS}^2/u_{BCS}^2 \) evaluated by using the BCS formula (14). Here, the quasiparticle energy \( E \) is related to the resonance energy \( e_R \) as \( E = |\lambda| + e_R \). It is seen that the increasing trend of \( \bar{v}^2/\bar{u}^2 \) is consistent with that of the BCS formula, except for a difference by a factor of \( \sim 0.5 \). The consistency is also seen in examples at the other resonance energies.

The above observation leads to the following interpretation. The amplitude \( v(r) \) of the hole component increases due to the mixing of the hole and particle components via the pair potential. Since the hole component \( v(r) \) is localized inside and around the nuclear surface, the increase of \( v(r) \) leads to the increase of the probability distribution \( |u(r)|^2 + |v(r)|^2 \) inside the nuclear radius \( r \lesssim R \). This brings about the decrease of the resonance width.

As a secondary mechanism, we find that the particle component \( u(r) \) inside and around the surface increases with \( \bar{\Delta} \). This also contributes to the increase of \( |u(r)|^2 + |v(r)|^2 \). We will leave analysis of this mechanism for a future paper, since this contribution is small compared with the contribution from the hole component.
4. Conclusion

The quasiparticle resonance is predicted in Bogoliubov’s quasiparticle theory as an unbound single-particle mode of excitation caused by the pair correlation in nuclei. Expecting a strong influence from the pair correlation, in the present paper we have studied the properties of the quasiparticle resonance emerging in nuclei near the neutron drip-line. We focused on the resonance in the $p$-wave neutron with low kinetic energy in the $^{46}\text{Si}+n$ system, and analyzed in detail how the pair correlation controls the width of the quasiparticle resonance.

By solving numerically the Hartree–Fock–Bogoliubov equation in the coordinate space to obtain the quasiparticle wave function satisfying the scattering boundary condition, we calculate the phase shift of the neutron elastic scattering and then extract the resonance energy and the resonance width. Analyses are performed systematically for various strengths of the average pairing gap, and for different situations concerning whether the quasiparticle state is particle-like or hole-like; i.e., whether the single-particle orbit being the origin of the resonance is located above or below the Fermi energy.

We have discovered that the pairing effect on the width of the particle-like quasiparticle resonance is very different from that of the hole-like quasiparticle resonance, for which a perturbative treatment [21,22] of the pair potential is known. A peculiar feature of the particle-like quasiparticle resonance is that the resonance width for a strong pairing is smaller than that of a weaker pairing if comparison is made at the same resonance energy: The pairing correlation has the effect of reducing the resonance width. This is opposite to the pairing effect on the hole-like quasiparticle resonance. In the hole-like case, the pair potential causes coupling of the hole state to the scattering neutron states, leading to a decay of the hole state. In the particle-like case, in contrast, the pair potential causes the scattering state, represented by the particle component $u(r)$ of the quasiparticle wave function, to mix with the hole component $v(r)$, which is, however, confined inside and around the nuclear surface. Therefore, on increasing the strength of the pair potential, the probability of the quasiparticle state inside the nucleus increases, and hence the width (decay probability) decreases.

Concerning the hole-like quasiparticle resonances, we have examined the applicability of the perturbative evaluation [21,22] of the resonance width. It is found that the perturbation can be applied not only to the quasiparticle resonances associated with the deeply bound hole state, as known previously, but also to hole-like quasiparticle resonances whose corresponding hole energy is close to the Fermi energy $\lambda$. More precisely, the applicability condition is evaluated as $e_{sp} \lesssim \lambda - 0.5\bar{\Delta}$.

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References

[1] I. Tanihata, H. Hamagaki, O. Hashimoto, Y. Shida, N. Yoshikawa, and N. Takahashi, Phys. Rev. Lett. 55, 2676 (1985).
[2] I. Tanihata, H. Savajols, and R. Kanungo, Prog. Part. Nucl. Phys. 68, 215 (2013).
[3] P. G. Hansen and B. Jonson, Europhys. Lett. 4, 409 (1987).
[4] G. F. Bertsch and H. Esbensen, Ann. Phys. (NY) 209, 327 (1991).
[5] J. Meng and P. Ring, Phys. Rev. Lett. 77, 3963 (1996).
[6] F. Barranco, P. F. Bortignon, R. A. Broglia, G. Coló, and E. Vigezzi, Eur. Phys. J. A 11, 385 (2001).
[7] T. Myo, S. Aoyama, K. Katō, and K. Ikeda, Prog. Theor. Phys. 108, 133 (2002).
[8] J. Meng, H. Toki, S.-G. Zhou, S. Q. Zhang, W. H. Long, and L. S. Geng, Prog. Part. Nucl. Phys. 57, 470 (2006).
[9] J. Dobaczewski, W. Nazarewicz, T. R. Werner, J. F. Berger, C. R. Chinn, and J. Dechargé, Phys. Rev. C 53, 2809 (1996).
[10] J. Dobaczewski, N. Michel, W. Nazarewicz, M. Płoszajczak, and J. Rotureau, Prog. Part. Nucl. Phys. 59, 432 (2007).
[11] K. Bennaceur, J. Dobaczewski, and M. Płoszajczak, Phys. Lett. B 496, 154 (2000).
[12] I. Hamamoto and B. R. Mottelson, Phys. Rev. C 68, 034312 (2003).
[13] I. Hamamoto and B. R. Mottelson, Phys. Rev. C 69, 064302 (2004).
[14] M. Matsuo, K. Mizuyama, and Y. Serizawa, Phys. Rev. C 71, 064326 (2005).
[15] M. Matsuo and T. Nakatsukasa, J. Phys. G: Nucl. Part. Phys. 37, 064017 (2010).
[16] Y. Zhang, M. Matsuo, and J. Meng, Phys. Rev. C 83, 054301 (2011).
[17] Y. Zhang, M. Matsuo, and J. Meng, Phys. Rev. C 90, 034313 (2014).
[18] K. Hagino and H. Sagawa, Phys. Rev. C 72, 044321 (2005).
[19] N. Pillot, N. Sandulescu, and P. Schuck, Phys. Rev. C 76, 024310 (2007).
[20] Y. Chen, P. Ring, and J. Meng, Phys. Rev. C 89, 014312 (2014).
[21] S. T. Belyaev, A. V. Smirnov, S. V. Tolokonnikov, and S. A. Fayans, Sov. J. Nucl. Phys. 45, 783 (1987).
[22] A. Bulgac, [arXiv:nucl-th/9907088] [Search inSPIRE].
[23] P. Ring and P. Schuck, The Nuclear Many-Body Problem (Springer, Berlin, 1980).
[24] G. F. Bertsch, P. F. Bortignon, and R. A. Broglia, Rev. Mod. Phys. 55, 1 (1983).
[25] M. Grasso, N. Sandulescu, N. V. Giai, and R. J. Liotta, Phys. Rev. C 64, 064321 (2000).
[26] S. A. Fayans, S. V. Tolokonnikov, and D. Zawischa, Phys. Lett. B 491, 245 (2000).
[27] N. Michel, K. Matsuyanagi, and M. Stoitsov, Phys. Rev. C 78, 044319 (2008).
[28] J. C. Pei, A. T. Kruppa, and W. Nazarewicz, Phys. Rev. C 84, 024311 (2011).
[29] Y. Zhang, M. Matsuo, and J. Meng, Phys. Rev. C 86, 054318 (2012).
[30] H. Oba and M. Matsuo, Phys. Rev. C 80, 024301 (2009).
[31] Y. N. Zhang, J. C. Pei, and F. R. Xu, Phys. Rev. C 88, 054305 (2013).
[32] N. Sandulescu, N. V. Giai, and R. J. Liotta, Phys. Rev. C 61, 061301(R) (2000).
[33] R. Id Betan, N. Sandulescu, and T. Vertse, Nucl. Phys. A 771, 93 (2006).
[34] N. Sandulescu, L. S. Geng, H. Toki, and G. C. Hillhouse, Phys. Rev. C 68, 054323 (2003).
[35] S. Raman, R. F. Carlton, J. C. Wells, E. T. Jurney, and J. E. Lynn, Phys. Rev. C 32, 1 (1985).
[36] J. Dobaczewski, H. Flocard, and J. Treiner, Nucl. Phys. A 422, 103 (1984).
[37] M. V. Stoitsov, J. Dobaczewski, W. Nazarewicz, S. Pittel, and D. J. Dean, Phys. Rev. C 68, 054312 (2003).
[38] M. Matsuo, Nucl. Phys. A 696, 371 (2001).
[39] A. Bohr and B. R. Mottelson, Nuclear Structure (Benjamin, New York, 1975), Vol. 1.
[40] T. R. Werner, J. A. Sheikh, M. Misu, W. Nazarewicz, J. Rikovska, K. Heeger, A. S. Umar, and M. R. Strayer, Nucl. Phys. A 597, 327 (1996).
[41] J. Terasaki, H. Flocard, P.-H. Heene, and P. Bonche, Nucl. Phys. A 621, 706 (1997).
[42] M. Thoennessen, At. Data Nucl. Data Tables 98, 933 (2012).