Entropy of entanglement in the continuous frequency space of the biphoton state from multiplexed cold atomic ensembles

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Abstract

We consider a scheme of multiplexed cold atomic ensembles that generate a frequency-entangled biphoton state with controllable entropy of entanglement. The biphoton state consists of a telecommunication photon (signal) immediately followed by an infrared one (idler) via four-wave mixing with two classical pump fields. Multiplexing the atomic ensembles with frequency and phase-shifted signal and idler emissions, we can manipulate and can control the spectral property of the biphoton state. Mapping out the entropy of entanglement in the scheme provides the optimal configuration for entanglement resources. This paves the way for efficient long-distance quantum communication and for potentially useful multimode structures in quantum information processing.

Keywords: quantum communication, entropy of entanglement, cascade spontaneous emissions

(Some figures may appear in colour only in the online journal)

1. Introduction

Quantum communication relies on the coherent distribution of entanglement over long distances. This can be done by implementing a quantum repeater protocol [1, 2]. It distributes the entanglement to the end parties (A and B) of distance L by inserting M initially entangled pairs (A, C_1, C_2, \ldots, C_M, and B), respectively, with a distance L/M. Conditioning on locally joint measurements (e.g., Bell measurements) of the adjacent parties (C_i and C_{i+1}), the protocol succeeds and projects out the required entangled state of the end parties with some finite fidelity F \leq 1. Since then, long-distance quantum communication has been proposed in the setup of atomic ensembles [3] where entanglement swapping or quantum teleportation [4] becomes feasible. In about the last decade, correlated atom–light entanglement in the Raman-type [5–9] and diamond-type schemes [10–13] paved the way toward the realization of low-loss long-distance quantum communication.

The entanglement is a basic element in long-distance quantum communication, which has been focused on discrete degrees of freedom, either in light polarizations [14–16] or in central frequencies [17, 18]. Recently, the entanglement of continuous variables provides a richer capacity in quantum key distributions [19] and quantum information applications [20]. A plethora of continuous degrees of freedom involve light spectrum [21–23], transverse momentum [24, 25], space [26], and orbital angular momenta of light [27–32]. This higher-dimensional quantum capacity also manifests in the aspects of quantum memories using atomic ensembles [33, 34] and in proposed atomic [35] or optical frequency comb techniques [36]. In addition, multiplexing multimode quantum memories in space [37, 38] or time [39] can enhance the distribution rate of a quantum repeater protocol while the spectral shaping in spontaneous parametric downconversion [40, 41] or diamond-type atomic ensemble [42] helps facilitate the frequency encoding/decoding, which promises a potentially efficient multimode quantum communication.

This motivates us to investigate the entropy of entanglement in the continuous frequency space of the biphoton state from the multiplexed (MP) cold atomic ensembles where its telecommunication (telecom) bandwidth has the advantage of low-loss fiber transmission, while its spectrum can be manipulated in the MP scheme. Based on the studies of
and the unit direction of dipole moments

and effective

is the infrared one. The

eq\frac{D_1}{2i} - i \Delta \omega_i.

where

The pump fields are normalized Gaussian pulses

\Omega_{a,b}(t) = \frac{1}{\sqrt{\pi \tau}} \Omega_{a,b} e^{-t^2/\tau^2} \text{ where } \Omega_{a,b} is the pulse area. The coupling constants of the signal and idler photons are \( g_{A,i} \), with polarizations \( \epsilon_{A,i} \) and the unit direction of dipole moments \( \hat{d}_{A,i} \).

The FWM condition for two pump fields (of wave vectors \( k_a, k_b \) and two photons \( (k_i) \)) is

\Delta \omega_i \equiv \omega_i - \omega_3 - \Delta_2 = \omega_i - \omega_3 + \delta \omega_i

The telecom transition frequency is \( \omega_3 \equiv \omega_2 - \omega_3 \), and \( \omega_3 \) is the infrared one. The spectral function shows a Gaussian envelope that maximizes at energy conservation of two photons \( \Delta \omega_i + \Delta \omega_i = 0 \), which is modulated by a Lorentzian of an idler photon. The FWM condition guarantees the generation of directionally correlated signal and idler photons and selectively drives the atomic system into a symmetrically collective excitation [44–46].

In such an atomic system with an atomic density of \( \sim 10^{11} \text{ cm}^{-3} \), the idler photon is observed to be superradiant [10, 47, 48] due to induced dipole–dipole interactions [49] first proposed as Dicke’s radiation [50]. Superradiant decay constant is denoted as \( \Gamma_{A,i} \) [51], and \( \delta \omega_i \) is the associated cooperative Lamb shift [52–54].

The four atomic levels can be chosen as \( |0\rangle, |1\rangle, |2\rangle, |3\rangle \) = \( |5S1/2, F = 3\rangle, |5P1/2, F = 4\rangle, |4D3/2, F = 5\rangle, |5P3/2, F = 4\rangle \) where \( |2\rangle \) also could be \( 6S1/2, 7S1/2, \) or \( 4D3/2 \) that the telecom bandwidth resides in \( 1.3–1.5 \mu \text{m} \) [10].

The spectral property of this biphoton state can be analyzed by Schmidt decomposition [22] where the state vector can be expressed in terms of Schmidt eigenvalues \( \lambda_n \) and effective photon operators \( \hat{b}_n, \hat{c}_n \) that the biphoton state becomes

\[ \Psi = \int d\omega_i d\omega_j \hat{a}^\dagger \omega_i (\omega_i) \hat{a}^\dagger \omega_j (\omega_j) |0\rangle d\omega_i d\omega_j, \]

with signal and idler mode functions \( \psi_n, \phi_n \) that define the effective photon operators,

\[ \hat{b}_n^\dagger \equiv \int \psi_n (\omega_i) \hat{a}^\dagger \omega_i (\omega_i) d\omega_i, \]

2. Biphoton state in a continuous frequency space

We consider the biphoton state generated by a cold rubidium atomic ensemble with a diamond-type atomic level in figure 1. Two classical driving fields operate on the infrared and telecom transitions, and then within the FWM condition, correlated signal and idler photons in telecom and infrared bandwiths, respectively, are spontaneously emitted. The Hamiltonian and the coupled equations in the Schrödinger picture have been derived and have been solved [13] where we formulate the light–atom interactions with the dipole approximation [43]. The adiabatic approximation is assumed to be valid if the Rabi frequencies of the pump fields \( \Omega_a, \Omega_b \) are weak enough with large detunings \( \Delta_1 \equiv \omega_i - \omega_1 \) and \( \Delta_2 \equiv \omega_i + \omega_b + \omega_2 \) where \( \omega_{1,2} \) (and \( \omega_3 \)) are atomic level energies. The main result is the biphoton probability amplitude [13],

\[ D_0 (\Delta \omega_i, \Delta \omega_i) \]

where the spectral function of the biphoton state is

\[ f (\omega_i, \omega_j) = e^{-\left(\Delta \omega_i + \Delta \omega_i\right)^2 / 2} \]
\[
\hat{c}_n^+ = \int \phi_n(\omega_i) \hat{a}_{\omega_i}^+ \, d\omega_i.
\]

Furthermore, the entropy of entanglement can be written as
\[
S = - \sum_{n=1}^{\infty} \lambda_n \log_2 \lambda_n.
\]

This biphoton state in the continuous frequency space can be implemented in the DLCZ protocol [3], which is advantageous for its telecom bandwidth for low-loss fiber transmission. Combined with its infrared bandwidth suitable for the quantum storage locally, the entanglement swapping and quantum teleportation have been investigated using such cascade emissions [13]. In the next section, we consider a scheme of MP atomic ensembles that would allow us to manipulate and to control the entropy of entanglement. In addition, we may control the mode functions of the biphoton state.

3. MP cold atomic ensembles

Here, we consider a scheme of the MP atomic ensembles as shown in figure 1. We excite the atomic ensembles with common pump fields, and the spontaneously emitted cascade emissions are MP by the frequency and PSs that may control the central frequencies and relative phases of the photons. The frequency shifts can be done by AOMs, while the PSs can be implemented as in cross-phase modulation experiments by an electromagnetically induced transparency based Kerr medium [55], atomic ensembles [56, 57], or a hollow-core photonic band gap fiber [58] in the low light level. Similar to the DLCZ protocol where the excitation probability is made small so that multiphoton events are rare [3], we may also express the MP cascade emissions as product states of the biphoton state in equation (3). Along with a prefactor in equation (1) we denote as probability \( p \equiv D_{i,i} (\Delta \omega_1, \Delta \omega_2) / f(\omega_1, \omega_2) \) that depends only on excitation parameters, we have
\[
|\Psi\rangle_{\text{MP}} = (|0\rangle_1 + p |\Psi\rangle_1) \\
\otimes (|0\rangle_2 + p |\Psi\rangle_2) \otimes \cdots \otimes (|0\rangle_{N_{\text{MP}}} + p |\Psi\rangle_{N_{\text{MP}}}),
\]

where \( |0\rangle \) means the ground state and the subscripts denote the numbered atomic ensembles. We note that these atomic ensembles share the same excitation probability due to the common pump fields we apply, which, furthermore, remove the incoherent (random) relative phases that may deteriorate the state preparation in our scheme. After expanding the above product states and keeping the first two most significant terms, we derive
\[
|\Psi\rangle_{\text{MP}} = |0\rangle^{\otimes N_{\text{MP}}} + p_{\text{MP}}(\omega_i, \omega_i) + \mathcal{O}(p^2),
\]

where the events of more than two photons are on the order of \( p^2 \), which is extremely small in our assumption of weak excitations.

Therefore, the spectral function of the effective MP biphoton state can be written as
\[
f_{\text{MP}}(\omega_1, \omega_2) = \sum_{m=1}^{N_{\text{MP}}} e^{i \theta_m} \frac{e^{-\left((\Delta \omega_1 + \Delta \omega_2 + \delta \eta_m)^2 + \delta \eta_m \right)^2}}{2} - i (\Delta \omega_1 + \delta \eta_m),
\]

where \( N_{\text{MP}} \) is the number of the MP atomic ensembles, \( \delta \eta_m \) and \( \delta \eta_m \) are frequency shifts for idler and joint signal and idler photons, respectively. The phase shift is denoted as \( \theta_m \), which can be addressed independently for each atomic ensemble by PSs.

The spectral shaping for the above MP scheme has been investigated for symmetrical \((\delta \mu_0 = 0)\) and nonsymmetric \((\delta \mu_0 = \delta \mu_0)\) spectral functions [42]. In the symmetrical spectral function, the spectral weighting lies along the energy conserving axis \( \Delta \omega_1 = - \Delta \omega_2 \), which generates a larger entropy of entanglement \( S \). Here, we add another degree of freedom in the phase of the photons that provides richer information in \( S \) and more flexibility to control it.

4. Entropy of entanglement

The entropy of entanglement \( S \) is crucial in evaluating the capacity of the Hilbert space that the quantum system can access. In the setting of discrete quantum system, for a maximally entangled qudit state of dimensions \( D \) (or \( W \) state), we have
\[
|\Psi\rangle_{\text{MP}} = \frac{1}{\sqrt{D}} (|00\rangle + |11\rangle + \cdots + |10\rangle + \cdots + |01\rangle),
\]

where \( |0\rangle \) and \( |1\rangle \) represent a qubit space (e.g., polarizations for a single photon). The Hilbert space would involve \( 2^D \) states, while the entropy of entanglement \( S \) becomes \( \log_2 D \). Therefore, quantifying \( S \) allows us to analyze the capacity for quantum information processing and application. Other than a discrete quantum system, here, we focus on the capacity for long-distance quantum communication in our MP scheme in the continuous frequency space. There is no general analytical expression for \( S \) in the continuous space, which, however, can be quantified using Schmidt decomposition. In addition, we later will show in equation (14) that \( S \) in our MP scheme can be expressed approximately as a summation of the entropy for the qudit state of dimensions \( N_{\text{MP}} \) and the one in the continuous frequency space with \( N_{\text{MP}} = 1 \). Below, we consider two and three atomic ensembles, and we map out the entropy of entanglement in the setting of symmetrical spectral functions and the associated mode probability densities for the biphoton state.

4.1. MP two atomic ensembles

For the MP scheme with two atomic ensembles in the symmetrical setting of frequency shifts where we set...
\[ \delta p_2 = - \delta p_1 \text{ with } \delta q_{1,2} = 0, \text{ the spectral function becomes} \]
\[ f_{\text{MP}}(\omega_i, \omega_f) = \frac{e^{-i(\Delta \omega_i + \Delta \omega_f)^2/\hbar}}{\Gamma^N/2 - i(\Delta \omega_f + \delta p_1)} + \frac{e^{i\theta_1}e^{-i(\Delta \omega_i + \Delta \omega_f)^2/\hbar}}{\Gamma^N/2 - i(\Delta \omega_f - \delta p_1)}. \]

where the phase \( \theta_1 \) is set to zero without loss of generality for the overall phase is irrelevant for the biphoton state. The entropy of entanglement \( S \) in such a scheme is shown in figure 2. For larger frequency shifts, we have a plateau of entropy distribution where its maximum appears at \( \Delta \omega_f = 0 \), while the minimum is at \( \theta_2 = \pi \). For smaller frequency shifts where MP spectral functions start to overlap, S has a maximum near \( \theta_2 = 2\pi \), while the minimum is still at phase \( \pi \), which indicates an antisymmetrical distribution to the axis of \( \Delta \omega_i = \Delta \omega_f \). We note that, as \( \delta p_1 \) approaches zero, which indicates no frequency shifts for the atomic ensembles, \( S \) at phase \( \pi \) has no physical meaning since the spectral function becomes null. At \( \delta p_1 = 0 \), the spectral function has no difference from the one with just a single atomic ensemble.

Now, we investigate, in detail, the eigenvalues and mode probability densities at some specific points in figure 2(a). In figure 3, we study the spectral functions with small frequency shifts. The relative large and minimal entropy \( S \) at \( \theta_2 = 0 \) and \( \pi \), respectively, in (a) and (b) can be understood in the descending Schmidt eigenvalues where the largest eigenvalue closer to 1 in figure 3(b) means a less entangled biphoton state. In the extreme case when \( \lambda_0 = 1 \), the biphoton state becomes an unentangled source such that the signal and idler photons are separate in their respective mode functions.
in contrast to (b) that the phase $\pi$ tends to distribute the spectral weighting on $\Delta \omega_i := 0$. We demonstrate three mode probability densities for this biphoton state where signal and idler photons show Gaussian and Lorentzian tails, respectively, as expected. The idler mode with the largest eigenvalue has double peaks, which would show interference patterns in time domains. This feature discriminates the characteristics of the signal mode of the largest eigenvalue from the idler one.

In figure 4, similar to figure 3, we show the corresponding results for large frequency shifts. A pair of degenerate eigenvalues appear along with degenerate mode probability densities. For large $\delta p_i$, $S$ does not deviate much on $\theta_2$ as can be seen in figure 2(b) and in the first ten eigenvalues in figure 4, which cannot be distinguished. The spectral distributions in figure 4(a) and (b) differ most in between these two spectral functions on $\Delta \omega_i = - \Delta \omega_i$, which reflects on the slightly different third and fourth mode probability densities, although the largest two modes show no difference. The well-separated mode functions (in contrast to the overlapped ones in figure 3) in the frequency space provide the possibility to address and to manipulate the frequency coding/encoding [40]. Therefore, the biphoton state in the MP scheme can potentially implement the Hadamard codes [41].

In the next subsection, we further study three multiplexed atomic ensembles where we show how the proposed scheme offers complexity in spectral properties with frequency and PSs and the potentiality in multimode quantum information processing.

4.2. MP three atomic ensembles

For the MP scheme with three atomic ensembles in the symmetrical setting of frequency shifts where we set $\delta p_2 = -\delta p_1$, $\delta p_3 = 0$, again, with $\delta \theta_{1,2,3} = 0$, the spectral function becomes

$$f_{\text{MP}}(\omega_s, \omega_i) = \frac{e^{i\theta_1} e^{-(\Delta \omega + \Delta \omega_i)^2/8}}{\gamma^2/2 - i (\Delta \omega + \delta p_1)} + \frac{e^{-(\Delta \omega + \Delta \omega_i)^2/8}}{\gamma^2/2 - i \Delta \omega_i} + \frac{e^{i\theta_2} e^{-(\Delta \omega + \Delta \omega)^2/8}}{\gamma^2/2 - i (\Delta \omega_i - \delta p_1)},$$

where again we set zero $\theta_3$ for the irrelevant overall phase of the biphoton state.

In figure 5, we map out the complete entropy of entanglement $S$ for the biphoton state in equation (13) from small to large frequency shifts. The maximum $S$ approaches the four corners of the contour plots in phases, which are $\theta_{1,2,3} = 0, 2\pi$, as $\delta p_1$ increases. It also suggests that $S$ has relatively small deviations for larger frequency shifts similar to the case of two MP atomic ensembles. The map of $S$ indicates the asymmetry in two dimensions of $\theta_{1,2}$ with finite $\delta p_1$, and its minimum is observed to fix at $\theta_{1,2} = 4\pi/3$ and $2\pi/3$, respectively. Furthermore, even though $S$ can be manipulated to increase by increasing $\delta p_1$, from (a) to (c), we can see that the maximum $S$ shows up at some optimal $\theta_{1,2}$ that offer more degrees of freedom to generate maximal $S$ for some specific frequency shift.

In figure 6, we closely investigate the extreme points of figure 5(b). Again, the minimal $S$ reflects on its descending eigenvalues where the largest eigenvalue is closer to 1. Similar to two MP atomic ensembles, the first idler mode in (a) has triple peaks due to three atomic ensembles being MP. The large $S$ in (a) also reflects on its spectral function that aligns mostly on the axis that conserves photon energies in contrast to the centrally distributed one in (b). We also find an interesting feature in the first idler mode of (b), which has two small humps around the central peak, and they grow up as $S$ increases in the map of figure 5. For the first signal modes, we
Figure 5. Entropy of entanglement $S$ for three atomic ensembles. Contour plots of $S$ of the symmetrical spectral function in dimensions of $\theta_{1,2}$ with frequency shifts of $\Delta \omega/\Delta = (a) 3$, (b) 6, (c) 15, and (d) 30. The maximum of $S$ approaches $\theta_{1,2} = 0$, $2\pi$ as $\Delta \omega$ increases, while its minimum fixes at $\theta_1 = 4\pi/3$ and $\theta_2 = 2\pi/3$. The crosses represent the extreme points in $S$.

Figure 6. Schmidt decomposition and mode probability densities for three MP atomic ensembles. Schmidt eigenvalues and absolute spectral functions are shown at $\Delta \omega/\Delta = 6$ with $(\theta_1, \theta_2) = (\pi/3, 5\pi/3)$ and $(4\pi/3, 2\pi/3)$ in (a) and (b), respectively. The associated first three, $n = 1–3$ (solid curve, dashed curve, and +) mode probability densities of signal $|\psi_s|^2$ and idler photons $|\phi_f|^2$ are plotted accordingly.

Figure 7. Schmidt decomposition and mode probability densities for three MP atomic ensembles with $\theta_{1,2} = 0$. In (a) and (b), we demonstrate Schmidt eigenvalues and absolute spectral distribution at $\Delta \omega/\Delta = 50$. The associated first two degenerate, $n = 1$ and 2 (solid curve), and the next two, $n = 3$ and 4 (dashed curve and +), mode probability densities of (c) signal $|\psi_s|^2$ and (d) idler photons $|\phi_f|^2$ are plotted accordingly.
observe that the effect of narrowing in their linewidths is more significant by manipulating $S$ compared to figure 3 in the setting of two multiplexed atomic ensembles. In the scheme of multiplexed three atomic ensembles, we may have better and more flexible control over the spectral property due to the extra degree of freedom in the phases.

Finally, we investigate the spectral property with zero phases and large frequency shifts in figure 7, which has a large entropy of entanglement $S$. Pairwise eigenvalues appear again along with a comparable one for every third eigenvalue. We also see the degeneracies in mode probability densities that have peaks on the sides to the axis $\Delta \omega_i = 0$. For the symmetrical spectral functions, we consider here, $S$ increases as we MP more atomic ensembles. In the limit of a large number of atomic ensembles being MP, we can estimate $S$ as a combination of the entropy in terms of the qudit state of dimensions $N_{\text{MP}}$ with the excess entropy due to the entanglement in continuous frequency space, that is, $S = S_d + S_{N_{\text{MP}}=1}[42]$. $S_{N_{\text{MP}}=1}$ is the entropy of entanglement for our single biphoton state in the frequency space, while $S_d = \log_2(N_{\text{MP}})$ from the conventional qudit state,

$$\hat{\psi}_d = \sum_{m=1}^{N_{\text{MP}}} \frac{1}{\sqrt{N_{\text{MP}}}} \hat{a}_{m}^{\dagger} \hat{a}_{m}^\dagger \ |0\rangle,$$

(14)

where $m$ denotes the associated biphoton modes. In principle, we may generate large entropy of entanglement in high dimensions of photon frequency space from our scheme of the MP atomic ensembles.

5. Discussions and conclusions

We propose a scheme that controls the frequency and PSs of the cascade emissions from the MP atomic ensembles in which we can manipulate the spectral property of the biphoton state. We study the entropy of entanglement $S$ in detail for two and three atomic ensembles with dependences on frequency and PSs that can be controlled by AOMs and cross-phase modulation experiments, respectively. We can generate large $S$ by increasing the frequency shift until it saturates and locate the optimal phases to create the maximal $S$ with finite frequency shifts. The extra degrees of freedom in phases other than just frequency shifts provide a fruitful and versatile quantum information control. In addition, the mode probability densities show double or triple peaks indicating an interference pattern in time domains, which can be measurable and distinguishable from the other modes. We would expect a complexity arising in more than four atomic ensembles in the perspective of optimizing parameters of frequency and PSs since it would be harder to completely map out $S$ in the multidimensional frequency and phase spaces. In principle, our scheme opens up a new avenue for entropy control and manipulation in continuous frequency and phase spaces, which can generate large $S$, either by MP more atomic ensembles or by increasing $S_{N_{\text{MP}}=1}$, the entropy of entanglement for a single atomic ensemble, which can be done with a larger superradiant decay constant $\Gamma_{\text{Y}}^\dagger$ or a shorter pulse width $\tau$ [13].

To MP more atomic ensembles on a large scale, we may utilize the optical lattices to generate two- or three-dimensional arrays of ensembles that can be individually addressed by light–matter interactions. In this way, even larger $S$ can be created in our scheme to realize high-dimensional entanglement [30–32]. This provides the possibility for unlimited communication capacity that is useful in quantum key distribution [19] and quantum information application in continuous variables [20].

For the perspective of experimental measurements of Schmidt eigenvalues or the entropy of entanglement, it requires a technique that operates the mode selection. The spectral filtering technique [59] uses transmission gratings with predetermined spectral transfer functions to deflect the input optical pulses such that the spectral information is mapped to the spatial one. A similar spectrometer utilizing spectral-to-spatial mapping has been proposed in the planar holographic devices [60] and experimentally has been demonstrated on a disordered photonic chip [61]. After the spectral calibration, the reconstructed spectra can genuinely retrieve narrow spectral lines or multiple spectral lines with varying amplitudes [61]. The alternative technique of the quantum pulse gate [62] uses the sum frequency generation in the setting of parametric downconversion to select out the spectral modes. The input state and the shaped pump fields are coupled to the nonlinear waveguide that the selected mode (output) is converted to the sum frequency of both and is separated from the other orthogonal modes. It acts effectively as the tomographic reconstruction of the mode characteristics [63]. The Schmidt eigenvalues can then be retrieved from the probability for the mode selection procedure [64] if the quantum efficiency of the conversion is made high enough [63].

Our proposed scheme not only takes advantage of the telecom bandwidth that is favorable in low-loss long-distance quantum communication, but also offers an alternative frequency encoding/decoding platform in the MP biphoton state. In particular, for the modes in well-separated frequency domains, we can individually encode on the spectral property via frequency bins [40] such that Hadamard codes [41], for example, can be implemented and can be decoded via coincidence measurements on our signal and idler photons. Using the cascade emissions from the MP atomic ensembles, we expect a potentially efficient entropy manipulation in the biphoton state with controllability and flexibility in conventional quantum optical experiments. Our scheme provides an alternatively promising setup in accessing communication capacity in continuous variables and paves the way toward multimode quantum information processing.

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