Research article

Option pricing of weather derivatives based on a stochastic daily rainfall model with Analogue Year component

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A R T I C L E   I N F O

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A B S T R A C T

In this study, we analyzed option pricing of rainfall derivatives based on stochastic daily rainfall model. We used Markov Chain Analogue Year model (MCAY) in order to describe occurrence process of daily rainfall. We have included the Analogue Year (AY) component in the Markov Chain (MC), which is a new component incorporated in this study and pricing rainfall derivatives. The inclusion of AY in the MC, provides excellent description of the occurrence process of daily rainfall. The amount of daily rainfall on wet days is obtained using Mixed Exponential distribution because it has the advantage of a better representation of extreme events. Combining the occurrence and amount model, we obtained Markov Chain Analogue Year Mixed Exponential model (MCAYMEM). Daily rainfall data from 2005 to 2017 were taken from Ethiopia National Meteorology Agency (ENMA) in order to assess the model performance. Based on the results of the daily rainfall models, we calculated an option price for different months. The price calculated using MCAYMEM gave an excellent result compared to the price calculated using MCMEM. This accuracy is mainly because of AY component included in the MC in the modeling of occurrence process.

1. Introduction

Rainfall derivative is a financial instrument used to hedge risks associated with rainfall fluctuation. It clearly and firmly states how payment will be settled between the parties involved based on the existing weather conditions during the contract period [1]. A financial weather contract is defined as a weather contingent contract whose payoff will be in an amount of cash determined by future weather events. The settlement value of these weather events is determined from a weather index expressed as values of a weather variable measured at a given location [2]. A financial weather contract can take the form of a weather derivative (WD) or of a weather insurance (WI) contract. Traditional insurance contracts are paid only after actual damages are quantified by proper loss adjustments and not simply upon the occurrence of the specific state of nature. This is an important difference that can make one of the instruments more attractive than the other [3]. Besides this, the problems of moral hazard and adverse selection that exist in traditional insurance disappear since the value of the weather index does not depend on the individual actions of market participants [4].

Weather variables are difficult to control especially for small-holder farmers in most developing and under-developed countries and mostly do have great impact on the farming activities of these farmers [5]. Thus, an effective and reliable risk management tool, weather derivative (WD) is needed to hedge farmers and stakeholders from the risk of weather uncertainties. By linking the payoffs to a fairly measured weather indices (e.g., temperature, rainfall, humidity, Sunshine), weather derivative reduces or eliminates the downside of traditional insurance [5]. Weather derivatives, unlike traditional derivatives, have no underlying tradable instrument or stock. Therefore, they cannot be used to hedge price risk since the weather itself cannot be priced, rather, they are used to hedge against other risks affected by weather conditions, such as agricultural yield risk [6]. They are usually swaps, futures and options based on different underlying weather measures [7]. When a weather event is a source of economic risk for agriculture, a weather derivative can become a hedging tool for agricultural producers. In order to develop weather derivatives for agriculture, just as for any other WD, the weather variable must be measurable, historical records must be adequate and available, and all parties involved in the transaction must consider such measures as objective and reliable.

In general, the development of weather derivatives in agriculture does not seem to be limited by the availability of adequate weather statistics, as many developed and developing countries have extensive...
and reliable weather data records. What may prove difficult is access to the data, both in terms of bureaucratic procedures and cost of purchase. Thus, the easier the access and the less expensive weather data, the more the opportunities for developing weather derivatives. The main point for the application of weather derivatives at the agricultural production level lies in the actual presence of a clear and satisfying relationship between the production variable and the weather factor [3].

Agriculture takes the lion’s share in Ethiopian economy. The agricultural sector contributes 45% to the Gross Domestic Product (GDP), 85% foreign earnings and provides livelihood to 80% of the population [8]. Ethiopian agriculture is extremely dependent on rainfall with irrigation agriculture accounting for less than 1% of the country’s total cultivated land [9]. Hence, the amount and temporal distribution of rainfall during the growing season are critical to crop yields and can induce food shortages and famine [9]. Rainfall variability, unreliable occurrences, insufficient amount and delay in onset dates contribute to a decline in crop yields with a reasonable amount in almost all parts of the country [10]. Rainfall variability usually results in a reduction of 20% production and 25% raise in poverty rates in Ethiopia [11, 12]. This rainfall variability has a great impact on the income of every household whose life is dependent on agriculture. Therefore, modeling and pricing rainfall derivative is very important for the country in order to reduce these risks.

A simple Markov chain model (MC) for the occurrence of daily rainfall has been found by different authors [13, 14, 15, 16, 17, 18, 19]. In this paper, we model daily rainfall using the Markov chain Analogue year mixed exponential model (MCAYMEm). Our model is different from the existing models due to the inclusion of the Analogue Year (AY) component in the Markov chain (MC) in modeling the occurrence of daily rainfall. Thus, this paper has two main objectives, the first objective is developing a daily rainfall model. The second objective is to use the daily rainfall model in order to obtain rainfall index, that is, cumulative rainfall (CR) and based on the CR, we calculate an option price for rainfall derivatives. The paper is structured as follows: in section 2 we have discussed materials and methods, model formulation and description of option pricing of rainfall derivatives, in section 3 the obtained results are discussed under result and discussion and finally in section 4 we have made some concluding remarks.

2. Materials and methods

In this study, we considered thirteen years of daily rainfall data from January 1, 2005-December 31, 2017 of Bahir Dar city. The daily rainfall data were collected from Ethiopia National Meteorology Agency (ENMA). It is classified into two: the first twelve years of data (January 1, 2005-December 31, 2016) is used for model formulation, that is, for model parameter estimation and the last year (January 1, 2017- December 31, 2017) of the recorded data is used for model validation. Here, rainfall modeling is separated into two stages. In the first step, a Markov chain with analogue year component (MCAY) is applied in order to model the frequency (occurrence) of daily rainfall. Then, a mixed exponential distribution is fitted to the data in order to model the magnitude (amount) of daily rainfall conditional on the likelihood of a rainy day. By combining the occurrence and intensity (amount) model, we develop a Markov Chain Analogue Year Mixed Exponential model (MCAYMEm). The behavior of a Markov chain is governed by a set of probabilities known as the transition probabilities that determine the transition from one state to another state. The transition probabilities specify the likelihood for the system being in each of its possible states during the next time period.

The following four points are the basic characteristics of a daily rainfall that should be considered during modeling [20]. First, daily rainfall occurrence probabilities has a seasonal pattern. Second, rainy and dry days have autoregressive property. Third, the magnitude of daily rainfall changes with the season and finally the variation in amount of daily rainfall varies with season. In this paper, we model the occurrence or nonoccurrence of daily rainfall $X_t$ and amount of daily rainfall $Y_t$ separately and combine them to obtain a model for daily rainfall $R_t$ on day $t$ which is given as the product of the occurrence process $X_t$ and the amount process $Y_t$ as given in equation (1). Both components of the daily rainfall model are described in detail in the next subsections.

$$R_t = X_t Y_t$$ (1)

Both the occurrence and amount of daily rainfall vary with season and it seems to repeat itself year after year (see Fig. 1). Hence, we take into account these basic characteristics in our model.

The occurrence process and the amount process are modeled separately and fully described in the following subsections.

2.1. Rainfall occurrence process (frequency modeling)

Daily rainfall occurrence process $X_t$ is modeled as a zero-one process [14, 17, 18, 19, 21]. That is:

$$X_t = \begin{cases} 0, & \text{if day } t \text{ is dry,} \\ 1, & \text{if day } t \text{ is wet,} \end{cases}$$

where $X_t$ is assumed to follow a first-order two-state Markov process implying that the probability of rainfall occurrence depends only on the situation from the previous day. The process described by the following transition probability $\pi^0$ and $\pi^1$ which capture the probability of rain based on whether it rained during the previous day or not.

$$\pi^0 = \Pr(X_t = 1 | X_{t-1} = 0)$$
$$\pi^1 = \Pr(X_t = 1 | X_{t-1} = 1)$$

In order to handle the seasonal variation, the transition probabilities are modeled to change daily within a year and are approximated by truncated Fourier series.

$$\psi_i = a_0 + \sum_{k=1}^{m} \left[a_{ik} \cos \left(\frac{2\pi k t}{365}\right) + b_{ik} \sin \left(\frac{2\pi k t}{365}\right)\right] ; i = 1, 2, \ldots, 3$$

where $\psi^0 = \frac{\exp(\psi_0)}{1 + \exp(\psi_0)}$ and $\psi^1 = \frac{\exp(\psi_1)}{1 + \exp(\psi_1)}$ and $m$ is the maximum number of harmonics needed to specify the seasonal cycles. Here we use the logistic function that is $f(x) = \frac{\exp(x)}{1 + \exp(x)}$ for the purpose of transforming the transition probability to the range $(0, 1)$. Based on Akaike information criterion (AIC) and Bayesian information criterion (BIC) we choose $m = 5$ for both $\pi^0$ and $\pi^1$. The coefficients of the Fourier series given in equation (3) are estimated by least squares method. In this model, in addition to the transition probability, we used the concept of analogue year, that is what was happened on the same date of the previous

Fig. 1. Time series plot of Bahir Dar’s daily rainfall data from 2005 to 2017.
year. Which improves the accuracy of our model and the concept is not yet used by previous researchers. In addition, the occurrence process $X_t$ can be generated recursively by using a uniform random variable $u_{1,t} \sim u(0,1)$ and a starting value $X_0$: For $1 \leq t \leq 365$

$$X_t = \begin{cases} 
1, & \text{if } p_t^{11} \geq u_{1,t}, \\
0, & \text{if otherwise,}
\end{cases}$$  \hspace{1cm} (4)

And for $366 \leq t \leq n$

$$X_t = \begin{cases} 
1, & \text{if } X_{t-365}^h \geq r_{\text{min}} \text{ and } p_t^{11} \geq u_{1,t}, \\
0, & \text{otherwise,}
\end{cases}$$  \hspace{1cm} (5)

where $p_t^{11}$ is an abbreviation of $p_t^{11}$ and $p_t^{11}$, $X_t^h$ the historical (observed) rainfall on day $t$ and $r_{\text{min}}$ describes the minimal amount that is detected as rain (0.1 inch=0.254 mm).

2.2. Rainfall intensity (magnitude modeling)

Here, we need to estimate the magnitude of the rainfall conditional on the fact that it rains on that particular day. This is done by fitting a distribution to the data. In order to identify a proper distribution to fit the data first the histogram of the daily rainfall is examined. As it can be depicted from Fig. 2, the histogram of the daily rainfall in Bahir Dar from January 1, 2005-December 31, 2017 is presented. A closer inspection of this figure reveals that the distribution of the rainfall is asymmetric that is skewed to the right with skewness of 3.90.

Different distributions with a nonnegative domain have been proposed to fit rainfall data in literature. Among these distributions, a Gamma distribution is proposed by [1, 22, 23, 24, 25, 26, 27], while a mixed Exponential distribution is proposed by [14, 26, 28, 29]. Mixed exponential distribution has the advantage of a better representation to extreme events [30]. In this paper, we used mixed exponential distribution to model the amount of daily rainfall $Y_t$ because mixed exponential distribution has the advantage of capturing extreme events (rainfall amounts). Moreover, mixed exponential distribution provide substantially better overall fits to daily rainfall data than the gamma distribution [14, 30, 31, 32]. Fig. 3 illustrates the procedure of superimposing fitted distributions and a histogram, for the 2005-2017 July and August rainfall at Bahir Dar. Superimposed on these histograms is PDF for the mixed exponential distribution fit using maximum likelihood (solid curve). The mixed exponential distribution represents these data much more closely, and provides a quite plausible summary of the year-to-year variations in the data. To determine the existence of extreme rainfall, we use box plots. Fig. 4 shows that the box plots of July and August to illustrate the presence of extreme rainfall. The presence of outliers in the box plots are one evidence for the existence of extreme rainfall in the area under study.

The mixed exponential distribution is a weighted combination of two simple exponential distributions and inherits their properties. The mixed exponential distribution is defined by the following probability density function (PDF):

$$f_{\text{mix}}(Y) = \frac{\alpha}{\beta_1} \exp \left(\frac{-Y}{\beta_1}\right) + \frac{1-\alpha}{\beta_2} \exp \left(\frac{-Y}{\beta_2}\right)$$  \hspace{1cm} (6)

with $0 < \beta_1 \leq \beta_2$ and $0 < \alpha < 1$ and the cumulative density function (CDF) is defined by

$$f_{\text{mix}}(Y) = \alpha \exp \left(\frac{-Y}{\beta_1}\right) + (1-\alpha) \exp \left(\frac{-Y}{\beta_2}\right)$$  \hspace{1cm} (7)

where $\beta_1$ and $\beta_2$ are the parameters of the two exponential distribution respectively which represents the mean of the distributions and $\alpha$.
is the mixing parameter. Since rainfall amount varies seasonally, in order to maintain its seasonality, we determine the parameters for each month so that it varies monthly within a year and remains constant across different years, that is for each month we obtain its own distribution parameters. The parameters are determined using maximum likelihood estimators. When the parameters \( \alpha, \beta_1 \) and \( \beta_2 \) are estimated, the amount process can be simulated with two independent uniform random variables \( u_{1,1}, u_{1,2} \sim U(0,1) \) independent from \( u_{1,1} \sim U(0,1) \) using standard inverse transform method [1, 18, 33]:

\[
Y_t = -\delta_t \ln(u_{t,1}),
\]  

where \( Y_t \) describes the amount of rainfall on day \( t \) and \( \delta_t \) is given by

\[
\delta_t = \begin{cases} 
\beta_{1,t}, & \text{if } u_{1,1} \leq a_{t,k}, \\
\beta_{2,t}, & \text{if } u_{1,1} > a_{t,k}.
\end{cases}
\]  

After the estimation of the occurrence and the amount processes, they can be combined to simulate future rainfall using equation (1). Using this daily simulation, rainfall index is obtained. Rainfall index \( I_t \) over the period \( (t_1, t_2) \) is defined as the sum of the daily rainfall \( R_t \) for a particular location with accumulation period \( (t_1, t_2) \) which is known as cumulative rainfall (CR) as it is given by equation (10).

\[
CR = \sum_{t=t_1}^{t_2} R_t
\]  

(10)

2.3. Parameter estimation

Here, we describe maximum likelihood estimator for mixed exponential distribution. Probability density function (PDF) of the mixed exponential distribution is:

\[
f(x; \alpha, \beta_1, \beta_2) = \frac{\alpha}{\beta_1} \exp \left( -\frac{x}{\beta_1} \right) + \frac{1 - \alpha}{\beta_2} \exp \left( -\frac{x}{\beta_2} \right)
\]  

(11)

and the log-likelihood function is given by:

\[
l(x; \alpha, \beta_1, \beta_2) = \sum_{i=1}^{N} \ln f(x_i; \alpha, \beta_1, \beta_2)
\]  

\[
= \sum_{i=1}^{N} \left( \ln \left( \frac{\alpha}{\beta_1} \right) - \frac{x_i}{\beta_1} \right) + \sum_{i=1}^{N} \ln \left( \frac{1 - \alpha}{\beta_2} \right) - \frac{x_i}{\beta_2} \right)
\]  

\[
= N \ln \left( \frac{\alpha}{\beta_1} \right) - \sum_{i=1}^{N} \frac{x_i}{\beta_1} + N \ln \left( \frac{1 - \alpha}{\beta_2} \right) - \sum_{i=1}^{N} \frac{x_i}{\beta_2}
\]  

(12)

Then, set the derivative of equation (12) equal to zero to find the maximum

\[
\frac{\partial l(x; \alpha, \beta_1, \beta_2)}{\partial \alpha} = 0
\]  

(13)

\[
\frac{\partial l(x; \alpha, \beta_1, \beta_2)}{\partial \beta_1} = 0
\]  

(14)

\[
\frac{\partial l(x; \alpha, \beta_1, \beta_2)}{\partial \beta_2} = 0
\]  

(15)

The above derivatives give us the following equations

\[
\frac{N}{\alpha} - \frac{N}{1 - \alpha} = 0
\]  

(16)

\[
-\frac{N}{\beta_1} + \sum_{i=1}^{N} \frac{x_i}{\beta_1} = 0
\]  

(17)

\[
-\frac{N}{\beta_2} + \sum_{i=1}^{N} \frac{x_i}{\beta_2} = 0
\]  

(18)

2.4. Option pricing of rainfall derivatives

In this paper, we follow the approach used in [21] to calculate an option price for rainfall derivative. A weather derivative contract can be formulated by specifying the following seven basic parameters [34]: contract type, contract period, an official weather station from which the meteorological data is obtained, definition of the weather index \( i(t) \) underlying the contract, pre-negotiated threshold or strike level \( S \) for \( I_t \), tick size \( k \) or constant payment \( P_c \) per unit index and premium.

There are two main types of options used in the weather risk-management market: call options and put options. A call contract involves a buyer and a seller who first agree on a contract period and a weather index \( (I_t) \) that serves as the basis of the contract. At the start of the contract, the seller receives a premium from the buyer. In return, during the contract or at the end of the contract period, if \( I_t \) is greater than the pre-negotiated threshold or strike \( S \), an amount that the seller pays to the buyer is

\[
P_{call} = k \max(I_t - S, 0)
\]  

(19)

where \( k \) (tick) is a pre-agreed upon constant factor that determines the amount of payment per unit of weather index [21]. A fixed amount \( P_c \) is paid if \( I_t \) is greater than \( S \) or no payment is made otherwise. In a contract, the payoff is determined by the difference between \( I_t \) and \( S \). A put option is the same as a call option except the seller pays the buyer when \( I_t \) is less than \( S \) [21].

\[
P_{put} = k \max(S - I_t, 0)
\]  

(20)

A call and a put are essentially equivalent to an insurance policy; the buyer pays a premium and in return receives a commitment of compensation when a predefined condition is met [6]. The price of an option (or its premium) is calculated from the expected payoff as [7]:

\[
c = \exp(-r(t_2 - t))P
\]  

(21)

where \( c \) is the premium that the hedgers (buyers) need to pay for a contract, \( r \) is a risk-free periodic market interest rate, \( t \) is the date that the contract is issued (purchased) and \( t_2 \) is the date the contract is claimed or the maturation date. \( P \) is the payoffs based on the predicted rainfall. Table 1 presents the option specification as a summary.

3. Result and discussion

Fig. 5a and 5b below revealed the comparison of the frequency of daily rainfall using Markov Chain (MC) (without Analogue Year component) and Markov Chain Analogue Year model (MCAYM) respectively. As clearly observed from the figures, the latter give us better fit to the observed frequency (occurrence) of daily rainfall. The maximum likelihood estimates of the mixed exponential distribution parameters for each month is presented in Table 2. As observed from the table, the values of the parameters are different for each month, this is due to the seasonal variation of the intensity of daily rainfall across different months. Fig. 6 presents the daily rainfall obtained by simulating using the developed model; Markov Chain Analogue Year Mixed Exponential model (MCAYMEM) and the observed daily rainfall. We use 50,000 paths simulation in order that sampling errors are virtually eliminated.

Fig. 7 shows the histogram of observed and simulated daily rainfall using MCAYMEM (Fig. 7a) and MCAYMEM (Fig. 7b). A closer inspection
Table 2
Parameter estimation for the mixed exponential distribution from 2005 to 2016.

| Months   | 𝛼   | 𝛽₁  | 𝛽₂  |
|----------|-----|-----|-----|
| January  | 0.3212 | 0.5013 | 2.8565 |
| February | 0.9939 | 0.01000 | 0.4915  |
| March    | 0.5689 | 1.5407 | 5.4560  |
| April    | 0.5038 | 2.2489 | 11.1370 |
| May      | 0.4541 | 2.9322 | 11.7050 |
| June     | 0.4356 | 4.2220 | 13.6680 |
| July     | 0.0500 | 4.4671 | 11.7680 |
| August   | 0.0621 | 2.7168 | 16.0620 |
| September| 0.5662 | 6.1245 | 13.2160 |
| October  | 0.3426 | 2.3976 | 11.7680 |
| November | 0.9113 | 0.0100 | 3.8600  |
| December | 0.5718 | 1.9127 | 9.7445  |

Fig. 5. (a) Frequency of rainfall occurrence using MC. (b) Frequency of rainfall occurrence using MCAY (that is our new model).

Fig. 6. Observed and estimated (MCAYMEM) daily rainfall.

Fig. 7. (a) Histogram of observed and estimated daily rainfall using MCMEM, (b) observed and estimated daily rainfall using MCAYMEM (the new model) of Bahir Dar from 2005 to 2017.

Table 3
Comparison of Markov chain mixed exponential model (MCMEM) and Markov chain analogue year mixed exponential model (MCAYMEM or the new model) with corresponding absolute error (AE) in estimating the descriptive statistics.

| Statistics   | Observed | MCMEM | AE   | MCAYMEM | AE   |
|--------------|----------|-------|------|---------|------|
| Mean         | 3.84     | 5.81  | 1.97 | 3.84    | 0.00 |
| Standard deviation | 9.28  | 9.84  | 0.56 | 9.17    | 0.11 |
| Skewness     | 3.90     | 3.25  | 0.65 | 3.90    | 0.00 |
| Kurtosis     | 23.27    | 18.04 | 5.23 | 23.97   | 0.74 |

Table 3 reveals the comparison of the statistics of observed, MCMEM and using MCAYMEM (the new model). The statistics are very well approximated in MCAYMEM than MCMEM.

On the basis of the developed daily rainfall model, we obtain cumulative monthly rainfall for each month in 2017. Table 4 shows the estimated monthly cumulative rainfall using MCAYMEM and observed monthly cumulative rainfall along with the corresponding absolute error and Fig. 8 shows observed and simulated monthly cumulative rainfall in the year 2017 at Bahir Dar. As one can observe from Fig. 8, the
observed and simulated cumulative monthly rainfall using MCAYMEM are very close. Thus, this model can be used to simulate daily rainfall.

Fig. 9 shows that Bahir Dar experience much rainfall in the months of July and August, coupled with no rain records in the months of January, February, November and December which are the dry season in the target area of the study particularly and in Ethiopia generally. Due to this reason, cultivation or planting does not take place in the mentioned seasons (dry seasons). Cultivation takes place during the wet seasons specially in the kiremt season (June, July and August) all over in Ethiopia. Therefore, calculating an option price in the dry seasons are not important.

To price the rainfall derivative, the strike $S$ is obtained by taking the mean of the historical data of each month from 2005 to 2016 [6, 29]. For simplicity, we take the tick size $K$ to be one currency per unit index and for the contract period, we use the growing season from planting to harvesting (May to September). We set an option price for five months from May to September due to crop growth and cultivation in these months for most of Ethiopian crops, especially maize, wheat and Teff; they are highly correlated (have strong positive correlation) with the rainfall in the mentioned months [35]. Based on the option specification given in Table 1, the option prices are given in Table 5 and 6. The trading date for each month is the last day of the previous month. For instance, for May the trading date is April 30. In this study, we use European call and put option.

As indicated in Table 5 and 6, the price calculated using MCAYMEM gives an excellent result compared to the price calculated using MCMEM. Therefore, the inclusion of the Analogue Year (AY) in Markov Chain (MC) in the modeling of daily rainfall occurrence process has a great contribution in the improvement of the accuracy in the determination of wet and dry days.

### 4. Conclusion

In this paper, we present a daily rainfall model and calculate an option price for the underlying weather variable, that is rainfall. The modeling procedure is separated into two parts. In the first part, MCAY was employed in order to model the frequency of daily rainfall occurrence process. Thus, the inclusion of AY component in the MC, provides an excellent potential in capturing the dynamics of the occurrence process for daily rainfall which is not yet shared by previous researchers. In the second part of the model, in modeling the magnitude, mixed Exponential distribution was selected as an appropriate distribution. Combining the occurrence and amount model we developed MCAYMEM. Based on the developed daily rainfall model, we obtained rainfall index, that is cumulative rainfall (CR) and an option price for rainfall derivative is calculated using CR. The price calculated using MCAYMEM gave an excellent result compared to the price calculated using MCMEM. Therefore, the nobility of this study is the inclusion of the AY component in the MC in the modeling of daily rainfall occurrence process.

### Declarations

**Author contribution statement**

T. Berhane, N. Shibabaw, G. Awgichew, T. Kebede: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

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**Competing interest statement**

The authors declare no conflict of interest.

**Additional information**

No additional information is available for this paper.

**Appendix A**

See Table 7.
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Table 7

| m  | AIC  | BIC  |
|----|------|------|
| 1  | 30690| 30979|
| 2  | 30678| 30710|
| 3  | 30606| 30677|
| 4  | 30628| 30673|
| 5  | 30602| 30685|
| 6  | 30616| 30674|

Fig. 9. Histogram of observed and estimated cumulative rainfall of Bahir Dar in 2017.

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