Steady natural convection heat and mass transfer flow through a vertical porous channel with variable viscosity and thermal conductivity

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Abstract
The present article studies the effects of suction/injection and variable physical properties on steady natural convection flow through a vertical channel. The investigation was done analytically using Differential Transformation Method (DTM). The variability in viscosity and thermal conductivity are considered linear function of temperature. The governing equations are transformed into a set of coupled nonlinear ordinary differential equations. Results obtained were compared with exact solution when some of the flow conditions were relaxed and results from DTM show an excellent agreement with the exact solution which was obtained analytically. The influence of the flow parameters on fluid temperature, concentration, and velocity are presented graphically and discussed for variations of the governing parameters. From the course of investigation, it was found that an increase in suction \((S > 0)\) through the heated plate causes fluid temperature, velocity as well as concentration to decrease. However, an increase in injection causes heat transfer rate as well as skin friction at the heated plate to decrease.

KEYWORDS
DTM, free convection, thermal conductivity, variable viscosity, viscous dissipation, suction/injection.

1 | INTRODUCTION

Problems involving Suction/Injection interestingly have been studied appreciably over some decades ago due to their practical and industrial applications. Jha et al.\(^1\) investigated natural convection flow in a vertical microchannel with Suction/Injection by obtaining closed form solution of the governing equations. Results from their work shows that heat transfer rate decreases upon increasing suction/injection on the channel surfaces. Jha and Aina\(^2\) used exact method in their study of Suction/Injection on steady fully developed mixed convection flow in a vertical parallel plate microchannel. It was observed from their work that as suction on the hot wall with a simultaneous injection on the cold wall of the channel increases, both fluid velocity and temperature decrease. Suction/injection on free convection flow in a vertical channel with thermal radiation has been studied analytically by Jha et al.\(^3\) using perturbation method. Result from the above work...
shows that the value of Nusselt number is higher in the case of suction in comparison with injection at $y = 0$. Sadri and Babaelahi\textsuperscript{4} carried out an analytical study of boundary layer flow with injection or suction using Optimal Homotopy Asymptotic Method (OHAM). They found that fluid velocity decreases with increasing suction/injection parameter. Ram-Reddy and Pradeepa\textsuperscript{5} solved numerically suction/injection on a free convection under convective boundary condition using Spectral Quasi-Linearization Method. Their work revealed that skin friction and heat and mass transfer increases as a result of increase in both cases of suction and injection. Uwanta and Hamza\textsuperscript{6} investigated the effect of suction/injection on a vertical porous plate using perturbation series method. They found that fluid concentration is high in the vicinity of the boundary wall ($y = 0$), where suction takes place more than at the boundary wall ($y = 1$) where injection takes place. In addition, it was observed that fluid concentration decreases due to suction but increases due to injection.

In all the aforementioned literature above, variable physical properties such as viscosity and thermal conductivity have been assumed constant. However, it is very important to take into account the variation of viscosity and thermal conductivity with temperature in order to avoid errors in the design of fluid machineries and various fluid flows. Meanwhile, major industrial problems involving flow of fluids such as geothermal systems, crude oil extraction, and machinery lubrication requires the variation of fluid properties with temperature. Salem and Odda\textsuperscript{7} carried out investigations on the influence of thermal conductivity and variable viscosity on the flow of a micropolar fluid in the presence of suction or injection. It is worth mentioning that temperature of the fluid increases upon increasing thermal conductivity for both cases of air and water. Furthermore, they found out that fluid velocity increases with decrease in injection and decreases with increase in suction. Thakur and Hazarika\textsuperscript{8} investigated variable viscosity and thermal conductivity on flow of micropolar fluid numerically using shooting method. Their work reveals that velocity of the fluid increases with the increase in viscosity. Attia\textsuperscript{9} studied numerically using Crank-Nicolson implicit method to investigate the influence of suction and injection on flow between two parallel plates with variable properties. One of the results from their work shows that fluid temperature decrease with increasing suction for any value of viscosity. Pinarbasi et al\textsuperscript{10} solved iteratively variable thermal conductivity and viscosity for nonisothermal fluid flow using the Chebyshev pseudospectral method. From the above work, it was concluded that increasing viscosity increases fluid temperature while fluid velocity decreases. Using the fourth order Runge-Kutta shooting method Hazarika and Gopal\textsuperscript{11} investigated the effect of variable viscosity and thermal conductivity of a flow past a vertical plate. Palani and Kim\textsuperscript{12} have carried out an extensive study on variable viscosity and thermal conductivity numerically on a vertical plate using finite difference method. They concluded that increase in viscosity and thermal conductivity causes fluid velocity to increase. Choudhury and Hazarika\textsuperscript{13} have discussed variable viscosity and thermal conductivity on free convection flow that is varying across boundary layer and Prandtl number using finite difference method. It was observed from the above work that an increase in thermal conductivity of the fluid causes the rate of heat transfer to decrease. Mola et al\textsuperscript{14} have numerically studied temperature dependent viscosity and thermal conductivity on natural convection using implicit finite difference method. Result from their work shows that increasing thermal conductivity causes the rate of fluid flow and temperature to increase across the boundary layer. Elmaboudy and Mekheimer\textsuperscript{15} discussed the simultaneous effects of variable physical properties such as variable viscosity and thermal conductivity on peristaltic flow of fluid in a vertical asymmetric channel was solved using perturbation method. The work, while adopted the perturbation method to carry out the investigation concluded that viscosity and thermal conductivity have significant influence on fluid velocity, temperature and mass transfer. Rudraiah et al\textsuperscript{16} used perturbation method to solve variable viscosity and viscous dissipation on Oberberk Magnetoconvection. A result from their work indicates that fluid velocity decreases with increase in viscosity. Gbadeyan et al\textsuperscript{17} discussed have discussed the effects of variable viscosity as well as thermal conductivity on Casson nanofluid flow using Galerkin weighted residual method. In the course of investigation, they found that increasing both viscosity and thermal conductivity lead to an increase in fluid velocity as well a decrease in fluid temperature.

From all the mentioned literature above, several techniques such as exact method, OHAM, perturbation method, Runge-Kutta method, shooting method Crank-Nicolson implicit method, Chebyshev pseudospectral method, finite difference method, and Galerkin weighted residual method have been devised for solving linear and coupled equations. On the other hand, Differential Transformation Method (DTM) technique has been proven to be accurate, more efficient, and requires less computational effort in comparison to other methods mentioned above. The DTM is a strong mathematical tool for solving systems of linear and nonlinear differential equations and requires significantly less computational resources. Mirzaaeghaian and Ganji\textsuperscript{18} carried out an application of DTM on micropolar fluid flow and heat transfer through permeable walls. Results obtained from their work were in excellent agreement when compared with Runge-Kutta method. Furthermore, they found that Reynolds number has little effect on fluid temperature and concentration. Domairry et al\textsuperscript{19} solved nonlinear differential equation governing Jeffery-Hamel flow with DTM. A result obtained from their work indicates that increasing Hartmann number lead to backflow reduction. Sheikholeslami and
Azimi applied DTM on a Nanofluid flow with magnetic field effect. Results obtained from DTM matched with results carried out using fourth-order Runge-Kutta method. Umavathi and Shekar investigated the combined effect of variable viscosity and thermal conductivity on free convection flow of a viscous fluid in a vertical channel using the DTM. The efficiency of the DTM was compared with Runge-Kutta method and findings reveal that both methods agree to the order of $10^{-6}$. Hatami et al. carried out a comparison between DTM with HPM and numerical solution on Newtonian and Non-Newtonian fluid flow. They concluded that the DTM is very effective and can achieve more suitable results than HPM in some areas of equations in engineering and science problems. Oke investigated the convergence of DTM for ordinary Differential Equation.

The work of Jha and Ajibade investigated free convection heat and mass transfer flow in a vertical channel with Dufour effect wherein fluid viscosity and thermal conductivity were assumed constant. In view of all the assumptions made by Jha and Ajibade, this present study is amplified to captures a real life situation with temperature dependent viscosity and thermal conductivity, viscous dissipation, heat source/sink as well as Suction/Injection. The governing equations of the flow fields are solved by DTM. The effect of different physical parameters on velocity, temperature, concentration, skin friction and rate of heat transfer are presented graphically and discussed for some carefully selected values of the governing parameters.

2 | MATHEMATICAL FORMULATION

We consider a steady flow of an incompressible fluid between two vertical parallel plates positioned at $y' = 0$ and $y' = h$ with uniform temperature $T_1$ (hot wall) and $T_0$ (cold wall). Fluid is assumed to be injected at a constant velocity $V_0$ through one wall and sucked off with the same velocity $V_0$ through the other wall. The injected fluid is expected to diffuse and influence the flow of fluid in the channel which is in the $x'$-direction which is taken vertically upward along the vertical plates and the $y'$-axis is taken normal to the plates as shown in Figure 1. Since the plates are infinite in length, the velocity, temperature, and concentration fields are function of $y'$ only.

The following assumptions were made in the present problem:

1. The flow is steady
2. The fluid flow is incompressible

![Figure 1: Schematic diagram of fluid flow](image-url)
3. The flow is assumed to be fully developed in terms of concentration, thermally, and hydrodynamically. No properties vary in the x direction.
4. The variable viscosity and thermal conductivity are considered to vary linearly with temperature.
5. The pressure of the fluid is taken to be constant except fluid density variation due to temperature differences used only to express the body force term as buoyancy term.
6. Velocity slip, concentration jump and temperature jump are not considered on the channel walls.
7. The Prandtl number is a variation of temperature dependent viscosity and thermal conductivity.

Under the following assumptions, the viscous dissipation is taken into account and using Boussinesq’s approximation, the following are the governing Equations:

Continuity equation:
\[
\frac{dV'}{dy} = 0. \tag{1}
\]

Momentum equation:
\[
\frac{d}{dy'} \left( \mu \frac{du'}{dy'} \right) - \rho_0 V' \frac{du'}{dy'} - \frac{dp'}{dx'} - \rho g = 0. \tag{2}
\]

Temperature equation:
\[
\frac{1}{\rho_0 c_p} \frac{d}{dy'} \left( k \frac{dT'}{dy'} \right) - V' \frac{dT'}{dy'} + \frac{Q_0}{\rho_0 c_p} (T' - T_0) + \frac{\mu}{\rho_0 c_p} \left( \frac{du'}{dy'} \right)^2 = 0. \tag{3}
\]

Concentration equation:
\[
D_m \frac{d^2 C'}{dy'^2} - V' \frac{dC'}{dy'} - K'(C' - C_0) = 0. \tag{4}
\]

The boundary conditions to the governing equations are:
\[
\begin{align*}
    u' &= 0, T' = T'_1, C' = C'_1 \quad \text{at} \quad y' = 0. \tag{5} \\
    u' &= 0, T' = T'_0, C' = C'_0 \quad \text{at} \quad y' = h
\end{align*}
\]

from Equation (1), setting \( u = 0 \), we get
\[
\frac{dp'}{dx'} = \frac{dp_0}{dx'} = -\rho_0 g, \tag{6}
\]

eliminating \(-\frac{dp'}{dx'}\) between Equations (2) and (6), we get
\[
\frac{d}{dy'} \left( \mu \frac{du'}{dy'} \right) - \rho_0 V' \frac{du'}{dy'} - (\rho - \rho_0)g = 0. \tag{7}
\]

From the equation of state, the body force term expressed as a buoyancy form is given by
\[
g(\rho - \rho_0) = -g \beta \rho_0 (T' - T'_0) - g \bar{\beta} \rho_0 (C' - C'_0), \tag{8}
\]

where \( \beta \) and \( \bar{\beta} \) are thermal and concentration coefficients of volume expansion.

Hence, from Equations (7) and (8), we obtain
\[
\frac{1}{\rho_0} \frac{d}{dy'} \left( \mu \frac{du'}{dy'} \right) - V' \frac{du'}{dy'} + g \beta (T' - T'_0) + g \bar{\beta} (C' - C'_0) = 0. \tag{9}
\]
Integrating Equation (3), we get

$$V' = -V_0,$$  

(10)

where \(V_0\) is the constant velocity of suction or injection. In addition, \(V_0 > 0\) is for suction at \(y = 0\), \(V_0 < 0\) is for injection at \(y = 1\) and \(V_0 = 0\) represent the case of nonpermeable plate.

In view of Equation (10), Equations (3), (4), and (9) reduce to

$$\frac{1}{\rho_0} d\left(\frac{d\rho'}{dy'}\right) + V_0 \frac{d\rho'}{dy'} + g\bar{\rho}(T' - T_0') + g\bar{\rho}(C' - C_0') = 0,$$  

(11)

$$\frac{1}{\rho_0 c_p} \frac{d}{dy'} \left(k \frac{dT'}{dy'}\right) + V_0 \frac{dT'}{dy'} + \frac{Q_0}{\rho_0 c_p}(T' - T_0') + \frac{\mu}{\rho_0 c_p} \left(\frac{d\rho'}{dy'}\right)^2 = 0,$$  

(12)

$$D_m \frac{d^2C'}{dy'^2} + V_0 \frac{dC'}{dy'} - K'_c(C' - C_0') = 0,$$  

(13)

where \(Q_0\) is the heat generation constant which may be either positive (heat source) or negative (heat sink), \(\rho_0\) is the density in the hydrostatic state, \(g\) is the acceleration due to gravity, \(c_p\) is the specific heat of the fluid at constant pressure, \(\rho\) is the constant fluid density, \(\mu\) is the fluid viscosity, \(k\) is the thermal conductivity of the fluid and \(K'_c\) is the chemical reaction constant.

The viscosity (Rudraiah et al.\(^{16}\)) and thermal conductivity (Umavathi and Shekar\(^{21}\)) are considered to vary linearly with temperature as given below respectively

$$\mu = \mu_0[1 - a(T' - T_0')] \quad \text{and} \quad k = k_0[1 + b(T' - T_0')]$$  

(14)

where \((a < 0)\) for gases and \((a > 0)\) for liquids. Also, \((b > 0)\) for fluids such as water and gases while \((b < 0)\) for fluids such as lubricating oils. From Equation (5), the viscosity and thermal conductivity parameter is in the form:

$$\mu = \mu_0(1 - \lambda \theta) \quad \text{and} \quad K = k_0(1 + \gamma \theta).$$  

(15)

For the purpose of this research, viscosity and conductivity parameters are considered in the range of \(-0.7 \leq \lambda \leq 0, \quad 0 \leq \gamma \leq 6\) for air, \(0 \leq \lambda \leq 0.6, \quad 0 \leq \gamma \leq 0.12\) for water and for lubricating oil \(0 \leq \lambda \leq 3, \quad -0.1 \leq \gamma \leq 0\) (Palani and Kim\(^{12}\)).

Introducing the following nondimensional quantities

$$u = \frac{u'}{u_0}, \quad y' = y h, \quad \theta = \frac{T' - T_0'}{T_1' - T_0'}, \quad C = \frac{C' - C_0'}{C'_1 - C_0'}, \quad \lambda = a(T_1' - T_0'),$$  

(16)

$$K_c = \frac{K_c h^2}{\nu_0}, \quad \text{Pr} = \frac{\mu c_p}{\nu_0}, \quad \text{Sc} = \gamma_0 h^2, \quad Q = \frac{Q_0 h^2}{\nu_0 k_0}, \quad N = \frac{\bar{\rho}(C'_1 - C_0')}{\bar{\rho}(T'_1 - T_0')},$$

$$u_0 = \frac{g\bar{\rho}h^2(T_1' - T_0')}{\nu_0}, \quad \text{Pr}_0 = \frac{\mu c_p}{\nu_0}, \quad \text{Pr} = \frac{\mu c_p}{\nu_0} = \frac{\text{Pr}_0(1 - \lambda \theta)}{(1 + \gamma \theta)},$$

and using Equations (14) to (16), Equations (4), (11), (12), and (13) reduce to

$$\left(1 - \lambda \theta\right) \frac{d^2u}{dy'^2} - \lambda \frac{d\theta}{dy'} \frac{du}{dy'} + S \frac{du}{dy} + \theta + NC = 0.$$  

(17)

$$\frac{d^2\theta}{dy'^2} + \gamma(1 - \gamma \theta) \left(\frac{d\theta}{dy}\right)^2 + \frac{\text{Pr}(1 + \lambda \theta)S}{\nu_0} \frac{d\theta}{dy} + Q(1 - \gamma \theta)\theta + \frac{\text{Pr}\cdot \text{Ec}}{\nu_0} \left(\frac{du}{dy}\right)^2 = 0.$$  

(18)
\[
\frac{1}{Sc} \frac{d^2C}{dy^2} + S \frac{dC}{dy} - K_c C = 0, \tag{19}
\]

while the boundary conditions are:

\[
\begin{align*}
    u &= 0, \theta = 1, C = 1, \quad \text{at} \quad y = 0, \\
    u &= 0, \theta = 0, C = 0, \quad \text{at} \quad y = 1,
\end{align*}
\tag{20}
\]

where \( Pr \) is the Prandtl number, \( Pr_0 \) is the Prandtl number in the hydrostatic state, \( Ec \) is the Eckert number, \( N \) is buoyancy parameter, \( Sc \) is the Schmidt number. \( K_c \) is the chemical reaction parameter, \( S \) is the Suction/Injection parameter and \( Q \) is the heat source/heat sink parameter. Furthermore, the Prandtl number is a function of viscosity and thermal conductivity. As the Prandtl number varies, both fluid viscosity and thermal conductivity varies across the boundary layer (Choudhury and hazarika\textsuperscript{13}).

### 3 Differential Transformation Method

We define the transformation of the \( k \)th derivative of a function as:

\[
F(K) = \frac{1}{k!} \left[ \frac{d^k f(\eta)}{d\eta^k} \right]_{\eta=\eta_0},
\tag{21}
\]

where \( f(\eta) \) is the original function and \( F(k) \) is the transformed function. The differential inverse transform of \( F(k) \) is given by:

\[
f(\eta) = \sum_{k=0}^{\infty} F(K)(\eta - \eta_0)^k
\tag{22}
\]

The concept of differential transformation is derived from a Taylor series expansion and in real applications; the function \( f(\eta) \) is expressed by a finite series as follows:

\[
f(\eta) = \sum_{k=0}^{m} F(K)(\eta - \eta_0)^k,
\tag{23}
\]

where the value of \( m \) is decided by the convergence of the series coefficient. The operations of the DTM for different functions are listed in Table 1 (Umavathi and Shekar\textsuperscript{21}).

| Original function | Transformed function |
|-------------------|----------------------|
| 1. \( y(x) = g(x) \pm h(x) \) | \( Y(k) = G(k) \pm H(k) \) |
| 2. \( y(x) = a g(x) \) | \( Y(k) = a G(k) \) |
| 3. \( y(x) = \frac{d^{(r)}g(x)}{dx^r} \) | \( Y(k) = (k+1)G(k+1) \) |
| 4. \( y(x) = \frac{d^{(r)}g(x)}{dx^r} \) | \( Y(k) = (k+1)(k+2)G(k+2) \) |
| 5. \( y(x) = g(x)h(x) \) | \( Y(k) = \sum_{l=0}^{k} G(r)H(k-l) \) |
| 6. \( y(x) = x^m \) | \( Y(k) = \delta(k-m) = \begin{cases} 1, & \text{if } k = m \\ 0, & \text{if } k \neq m \end{cases} \) |

**Table 1** Operations of differential transformation method
4 | SOLUTION WITH DTM

Taking the differential transforms of Equations (17) to (19), we obtain the following:

\[
\bar{U}(k + 2) = \frac{1}{(k + 1)(k + 2)} \left[ \sum_{r=0}^{k} (k - r + 1)(r + 1)\bar{U}(k - r + 1) \right.
\]
\[
+ \lambda^2 \sum_{r=0}^{k} \sum_{s=0}^{r} (k - r + 1)(r - s + 1)\bar{U}(k - r + 1)\bar{\theta}(r - s + 1) - \bar{\theta}(k) - \lambda \sum_{r=0}^{k} \bar{\theta}(k - r)\bar{\theta}(r) - N\bar{C}(k)
\]
\[
- \lambda N \sum_{r=0}^{k} \bar{\theta}(k - r)\bar{C}(r) - S(k + 1)\bar{U}(k + 1) - SA \sum_{r=0}^{k} (k - r + 1)\bar{U}(k - r + 1)\bar{\theta}(r) \right],
\]

(24)

\[
\bar{\theta}(k + 2) = \frac{1}{(k + 1)(k + 2)} \left[ -\gamma \sum_{r=0}^{k} (k - r + 1)(r + 1)\bar{\theta}(k - r + 1) \right.
\]
\[
+ \gamma^2 \sum_{r=0}^{k} \sum_{s=0}^{r} (k - r + 1)(r - s + 1)\bar{\theta}(k - r + 1)\bar{\theta}(r - s + 1)\bar{\theta}(s) - Q\bar{\theta}(k) + Q\gamma \sum_{r=0}^{k} \bar{\theta}(k - r)\bar{\theta}(r)
\]
\[
- \text{Pr} . Ec \sum_{r=0}^{k} (k - r + 1)(r + 1)\bar{U}(k - r + 1)\bar{U}(r + 1) - \text{Pr} . S(k + 1)\bar{\theta}(k + 1)
\]
\[
- \text{Pr} . S \lambda \sum_{r=0}^{k} (k - r + 1)\bar{\theta}(k - r + 1)\bar{\theta}(r) \right],
\]

(25)

\[
\bar{C}(k + 2) = \frac{1}{(k + 1)(k + 2)} [Sc.Kc\bar{C}(k) - Sc.S(k + 1)\bar{C}(k + 1)].
\]

(26)

where \( \bar{U}(k), \bar{\theta}(k), \) and \( \bar{C}(k) \) are the differential transform of \( U(y), \theta(y), \) and \( C(y) \) respectively. The transformed boundary conditions are:

\[
\bar{U}(0) = 0, \quad \bar{U}(1) = a, \quad \bar{\theta}(0) = 1, \quad \bar{\theta}(1) = b, \quad \bar{C}(0) = 1, \quad \bar{C}(1) = c,
\]

(27)

where \( a, b, \) and \( c \) are constants which are computed from the boundary conditions in Equation (20). The above equations for temperature, velocity, and concentration are solved and the results obtained are presented graphically from Figures 2 to 15, and numerically from Tables 2 and 7 for different governing parameters.

**Figure 2** Temperature profile for binary mixture of carbon dioxide in air for different values of Suction/Injection parameter \( S \) \( [Q = -10, N = 1, Ec = 0.2, Kc = 0.7, \lambda = -0.1 \text{ and } \gamma = 0.5] \)
FIGURE 3 Concentration profile for binary mixture of carbon dioxide in air for different values of Suction/Injection parameter $S$ [$Q = -10$, $N = 1$, $Ec = 0.2$, $Kc = 0.7$, $\lambda = -0.1$ and $\gamma = 0.5$]

FIGURE 4 Velocity profile for binary mixture of carbon dioxide in air for different values of Suction/Injection parameter $S$ [$Q = -10$, $N = 1$, $Ec = 0.2$, $Kc = 0.7$, $\lambda = -0.1$, and $\gamma = 0.5$]

FIGURE 5 Temperature profile for binary mixture of carbon dioxide in air for different values of Eckert number [$Q = -10$, $N = 1$, $Kc = 0.7$, $S = -0.4$, $\lambda = -0.1$, and $\gamma = 0.5$]
Figure 6  Velocity profile for binary mixture of carbon dioxide in air for different values of Eckert number \(Q = -10, N = 1, Kc = 0.7, S = -0.4, \lambda = -0.1\) and \(\gamma = 0.5\)

Figure 7  Temperature profile for binary mixture of carbon dioxide in air for different values of Viscosity parameter \(Q = -10, N = 1, Kc = 0.7, S = -0.4, \lambda = -0.2\), and \(\gamma = 0.5\)

Figure 8  Velocity profile for binary mixture of carbon dioxide in air for different values of Viscosity parameter \(Q = -10, N = 1, Kc = 0.7, S = -0.4, \lambda = -0.2\) and \(\gamma = 0.5\)
**FIGURE 9** Temperature profile for binary mixture of carbon dioxide in air for different values of Thermal conductivity parameter \( Q = -10, N = 1, Kc = 0.7, S = -0.4, \lambda = -0.2 \) and \( \gamma = 0.5 \)

**FIGURE 10** Velocity profile for binary mixture of carbon dioxide in air for different values of Thermal conductivity parameter \( Q = -10, N = 1, Kc = 0.7, S = -0.4, \lambda = -0.1 \) and \( \gamma = 0.5 \)

**FIGURE 11** Temperature profile for binary mixture of carbon dioxide in air for different values of heat source/sink parameter \( \gamma = 0.5, N = 1, Kc = 0.7, S = -0.4, Ec = 0.2 \) and \( \lambda = -0.1 \)
**FIGURE 12** Velocity profile for binary mixture of carbon dioxide in air for different values of heat source/sink parameter
\[ \gamma = 0.5, N = 1, K_c = 0.7, S = -0.4, E_c = 0.2 \text{ and } \lambda = -0.1 \]

**FIGURE 13** Velocity profile for binary mixture of carbon dioxide in air for different values of Buoyancy parameter
\[ \gamma = 0.5, Q = -10, K_c = 0.7, S = -0.4, E_c = 0.2 \text{ and } \lambda = -0.1 \]

**FIGURE 14** Concentration profile for binary mixture of carbon dioxide in air for different values of chemical reaction parameter
\[ Q = -10, N = 1, E_c = 0.2, \lambda = -0.1, \text{ and } \gamma = 0.5 \]
Figure 15  Velocity profile for binary mixture of carbon dioxide in air for different values of chemical reaction parameter \([\gamma = 0.5, Q = -10, N = 1, S = -0.4, Ec = 0.2, \text{and } \lambda = -0.1]\)

**Table 2**  Comparison of skin friction between the present method (Differential Transformation Method, DTM) and that of Jha and Ajibade\(^24\) when \(\lambda = \gamma = Q = Kc = Ec = 0\)

| N  | \(\tau_0\) Jha and Ajibade\(^24\), Df = 0 | \(\tau_1\) Present problem |
|-----|---------------------------------|---------------------|
| 0   | 0.33333333                      | 0.33333333          |
| 0.25| 0.41666667                      | 0.41666667          |
| 0.5 | 0.50000000                      | 0.50000000          |
| 0.75| 0.58333333                      | 0.58333333          |
| 1.0 | 0.66666667                      | 0.66666667          |

5 | **SOLUTION WITH EXACT METHOD**

Setting the viscosity parameter \((\lambda)\), Eckert number \((Ec)\) and thermal conductivity parameter \((\gamma)\) to zero, Equations (17) to (19) reduce to:

\[
\frac{d^2u}{dy^2} + S \frac{du}{dy} + \theta + NC = 0, \tag{28}
\]

\[
\frac{d^2\theta}{dy^2} + Pr \cdot S \frac{d\theta}{dy} + Q\theta = 0. \tag{29}
\]

\[
\frac{d^2C}{dy^2} + Sc.S \frac{dC}{dy} - Sc.KcC = 0. \tag{30}
\]

Solving Equations (28) to (30) with the boundary Equation (20), we get:

\[
\theta(y) = A_2 e^{m_1y} + A_1 e^{m_2y}, \tag{31}
\]

\[
C(y) = A_4 e^{m_3y} + A_3 e^{m_4y}, \tag{32}
\]

\[
U(y) = A_6 + A_5 e^{-Sy} - B_1 e^{m_1y} - B_2 e^{m_2y} - B_3 e^{m_3y} - B_4 e^{m_4y}. \tag{33}
\]

\[
\tau_0 = \left( \frac{du}{dy} \right)_{y=0} = -A_5 S - B_1 m_1 - B_2 m_2 - B_3 m_3 - B_4 m_4. \tag{34}
\]
\[ \tau_1 = -\left( \frac{du}{dy} \right)_{y=1} = A_5 S e^{-S} + B_1 m_1 e^{m_1} + B_2 m_2 e^{m_2} + B_3 m_3 e^{m_1} + B_4 m_4 e^{m_4}, \]  
\[ Nu_0 = -\left( \frac{d\theta}{dy} \right)_{y=0} = -A_2 m_1 - A_1 m_2, \]  
\[ Nu_1 = -\left( \frac{d\theta}{dy} \right)_{y=0} = -A_2 m_1 e^{m_1} - A_1 m_2 e^{m_2}, \]

where:

\[ m_1 = -\frac{Pr S + \sqrt{Pr^2 S^2 - 4Q}}{2}, \quad m_2 = -\frac{Pr S - \sqrt{Pr^2 S^2 - 4Q}}{2}, \]
\[ m_3 = \frac{e^{m_1}}{e^{m_1} - e^{m_2}}, \quad A_2 = 1 - A_1, \quad A_3 = \frac{e^{m_2}}{e^{m_1} - e^{m_2}}, \quad A_4 = 1 - A_3 \]
\[ A_5 = B_1 (1 - e^{m_1}) + B_2 (1 - e^{m_2}) + B_3 (1 - e^{m_1}) + B_4 (1 - e^{m_2}) \]
\[ A_6 = -A_5 + B_1 + B_2 + B_3 + B_4 \]

6 | SKIN FRICTION

The dimensionless skin friction at the surface of the hot plate \((y = 0)\) and the cold plate \((y = 1)\) are given by:

\[ \tau_0 = (1 - \lambda \theta) \left( \frac{du}{dy} \right)_{y=0} \quad \text{and} \quad \tau_1 = -(1 - \lambda \theta) \left( \frac{du}{dy} \right)_{y=1}. \]  

7 | HEAT TRANSFER

The dimensionless Nusselt number at the hot plate \((y = 0)\) and the cold plate \((y = 1)\) are given by:

\[ Nu_0 = -(1 + \gamma \theta) \left( \frac{d\theta}{dy} \right)_{y=0} \quad \text{and} \quad Nu_1 = -(1 + \gamma \theta) \left( \frac{d\theta}{dy} \right)_{y=1}. \]  

8 | VOLUMETRIC FLOW RATE

The volumetric flow rate of the fluid is given by:

\[ V_m = \frac{1}{0} u(y) dy \]  

Also, results for the volumetric flow rate, skin friction and heat transfer are presented in Tables 5 to 7.

9 | VALIDATION OF RESULT

In order to verify the accuracy of the present solution method (DTM), the results for the skin friction at both boundary walls are compared with those reported earlier by Jha and Ajibade\(^{24}\) when \( S = Ec = \lambda = \gamma = Q = Kc = 0 \). The results of this comparison which are shown in Table 2 reveal an excellent agreement between the results in the present work and that of Jha and Ajibade.\(^{24}\)
TABLE 3 Comparison of skin friction at $y = 1$ using the present method (Differential Transformation Method, DTM) and Exact solution when $\lambda = \gamma = Ec = 0$, $Kc = 0.298068357179840$, $Q = −0.5$, $Pr = 0.71$, $N = 1$, and $Sc = 0.94$

| $S$ | DTM      | EXACT             |
|-----|----------|--------------------|
| 0.1 | 0.298068357179854 | 0.298068357179840 |
| 0.2 | 0.284363560225701 | 0.284363560225701 |
| 0.3 | 0.271031158207633 | 0.271031158207633 |
| 0.4 | 0.258079505565637 | 0.258079505565637 |
| 0.5 | 0.245515612210161 | 0.245515612210170 |

TABLE 4 Comparison of Velocity obtained using the present method (Differential Transformation Method, DTM) with Numerical solution when $\lambda = −0.1$, $\gamma = 0.5$, $Ec = 0.2$, $S = −0.4$, $Pr = 0.71$, $N = 1$, $Q = −0.5$, and $Kc = 0.7$

| $y$ | DTM        | Numerical               |
|-----|------------|-------------------------|
| 0   | 0.000000000000000 | 0.000000000000000       |
| 0.25| 0.103287066949775 | 0.103288421249588       |
| 0.5 | 0.125205453112129 | 0.125206425338577       |
| 0.75| 0.083369232969932 | 0.083366309418432       |
| 1.0 | 0.000000000000000 | 0.000000000000000       |

10 | CONVERGENCE OF THE DTM

The convergence of the DTM was established in Oke\textsuperscript{23} where they showed that solution obtained through the DTM converges to exact solution when the problem is linear. Comparison of the DTM with exact and numerical methods was carried out on temperature, velocity, and concentration fields for different values between the boundary walls in this work. From Table 3, results from the DTM shows a strong convergence with results obtained from exact method when viscous dissipation, viscosity as well as thermal conductivity parameters are zero. Table 4 also revealed a strong convergence to five decimal places of the solution obtained by DTM with that of the numerical method for different governing parameters. The Dsolve/Numeric for BVP from Maple Application package was used to obtain the numerical solution.

11 | RESULTS AND DISCUSSIONS

The present article investigated the effect of Suction/Injection on free convection heat and mass transfer flow through a vertical channel with temperature dependent viscosity and thermal conductivity. The nonlinear coupled governing equations have been solved by DTM to obtain results for temperature, velocity, and concentration fields. In addition, the exact and numerical solution are also obtained and considered for comparison with the DTM. For the purpose of discussion, the temperature, velocity, and concentration fields are presented graphically for different values of the governing parameters. The values of Prandtl number and Schmidt number are chosen to be $Pr = 0.71$ (air) and $Sc = 0.94$ (carbon dioxide), respectively, while other parameters in the flow are chosen arbitrarily. Therefore, throughout this discussion, the fluid considered is a binary mixture of CO\textsubscript{2} in air.

Figures 2 to 4 show the effect of suction/injection on fluid temperature, concentration, and velocity. In view of the investigations carried out, it is interesting to note that $S > 0$ signifies suction at the heated wall with a corresponding injection at the cold wall. In addition, $S < 0$ signifies injection at the heated wall with a corresponding suction at the cold wall and $S = 0$ represents the flow formation inside the channel. It is observed from Figures 2 and 3 that both fluid temperature and concentration increase with increasing injection on the heated porous plate. This is physically true since injection through the heated plate amplifies heat transfer and mass flux from the heated porous plate into the fluid. On the other hand, it is noticed that increasing suction at the heated wall decreases fluid temperature and concentration within the channel.\textsuperscript{6} This is expected since the horizontal fluid motion is in the direction opposing the heat and mass flux from the heated plate and as such, heated fluid is being sucked away from the channel only to be replaced with cold fluid through the cold plate. From Figure 4, it is observed that fluid velocity decrease as suction ($S > 0$) increases through the heated plate and this causes the convection current to be weakened as a result of decrease in temperature and concentration. In addition, increasing injection ($S < 0$) causes the fluid velocity to grow as shown in Figure 4.\textsuperscript{2,6}
Figures 5 and 6 represent the effect of Eckert number \((Ec)\) on temperature and velocity profiles. It is observed that increasing Eckert number increases both fluid temperature and velocity within the channel.\(^{21}\) Physically this is true since viscous dissipation produces heat as a result of hindrance caused between fluid particles which increase the initial fluid temperature. The resultant effect of temperature increase corresponds to the strengthening of the buoyance force which causes the fluid velocity to grow. A critical look at Figure 5 reveals that the viscous dissipation effect is more pronounced towards the cold plate which clearly affirms the dominance of the applied boundary temperature over the sheared heating at the fluid sections adjacent to the heated plate.

Figures 7 and 8 show the effect of variable viscosity \((\lambda)\) on temperature and velocity profiles. It is clear from this figures that both fluid temperature and velocity decreases within the channel with increase in viscosity \((\lambda < 0)\).\(^{7,9,13}\) This is physically true since growing viscosity creates impedance to free flow of fluid particles so that fluid velocity is decreased. This consequently lowers the viscous dissipation heating across the fluid sections so that fluid temperature decreases with growing viscosity.

The effect of thermal conductivity \((\gamma)\) is shown in Figures 9 and 10. Clearly from this figures, both fluid temperature and velocity increase with increase in thermal conductivity within the channel.\(^{12}\) This is expected since thermal flux into the fluid increases as a result of increase in thermal conductivity. This causes convection currents to be strengthened so that fluid velocity increases with increase in the thermal conductivity. It would be observed that the thermal boundary layer is significantly influenced toward the cold plate. This is premised around the physical fact that growing conductivity enhances thermal diffusivity which eventually expands the thermal boundary layer farther away from the heated plate.

The effect of heat source/ sink parameter \((Q)\) is illustrated in Figures 11 and 12. Both fluid temperature and velocity increases upon increasing heat source \((Q > 0)\) within the channel.\(^{5}\) This is physically true since increasing the heat source parameter amplifies the applied temperature causing the fluid temperature to increase and also, it strengthens the convection current within the channel leading to an increase in fluid velocity. Furthermore, increasing heat sink \((Q < 0)\) decreases both fluid temperature and velocity within the channel.

The influence of Buoyancy ratio parameter \((N)\) and chemical reaction parameter \((Kc)\) are illustrated in Figures 13 to 15. It is noticed from the figures that an increase in Buoyancy parameter increases fluid velocity\(^{24}\) while increasing the chemical reaction parameter decreases both fluid velocity and temperature. The physical meaning of the observed trend is that growing \(N\) increases buoyancy due to mass transfer and hence, an increase in fluid velocity as seen in Figure 13. In addition, increasing the chemical reaction causes a decrease in fluid concentration and this weakens convection due to mass transfer which consequently decrease fluid velocity as shown in Figure 15.

Table 5 presents the response of skin friction, rate of heat transfer, and volumetric flow rate to variations in Suction/Injection \((S)\) through the channel plates. Increasing Suction through the heated plate causes a decrease in volumetric flow rate. This is due to temperature as well as velocity decrease that characterize suction through heated plate.

The table further reveals that both heat transfer and skin friction decrease at the cold wall while they both increase at the heated plate. The observed trend is due to the physical fact that when suction takes place through the heated plate, heat transfer into the system is retarded by fluid flow in the opposite direction out of the system causing a decrease in the rate of heat transfer. It is also noticed that increasing Injection \((S < 0)\) through the heated plate increases volumetric flow rate, skin friction as well as heat transfer rate at the cold wall while rate of heat transfer and skin friction decreases at the heated wall.

| \(S\) | \(V_m\) | \(\tau_0\) | \(\tau_1\) | \(Nu_0\) | \(Nu_1\) |
|---|---|---|---|---|---|
| 0.4 | 0.07171805 | 0.66646809 | 0.26764415 | 1.59242941 | 1.00967589 |
| 0.3 | 0.07322275 | 0.66629883 | 0.28073799 | 1.54645404 | 1.04673109 |
| 0.2 | 0.07470556 | 0.66578973 | 0.29409814 | 1.50142853 | 1.08451661 |
| 0.1 | 0.07616534 | 0.66494648 | 0.30774032 | 1.45734775 | 1.12306679 |
| 0.0 | 0.07769062 | 0.66377439 | 0.32167404 | 1.41420817 | 1.16239981 |
| −0.1 | 0.07900973 | 0.66227873 | 0.33590317 | 1.37200723 | 1.20252051 |
| −0.2 | 0.08039084 | 0.66046487 | 0.35042660 | 1.33074279 | 0.35042660 |
| −0.3 | 0.08174200 | 0.65833857 | 0.36528899 | 1.29041275 | 1.28509328 |
| −0.4 | 0.08306122 | 0.65590609 | 0.38033094 | 1.25101467 | 1.32751053 |
4. The volumetric flow rate can be controlled effectively using the variations in viscosity as well as thermal conductivity.

1. Increasing fluid viscosity decreases both fluid temperature and velocity within the channel.
2. Increasing fluid conductivity increases the volumetric flow rate as well as skin friction and heat transfer.
3. Skin friction, Nusselt number, and volumetric flow rate all increase with increase in thermal conductivity of the working fluid.
4. The volumetric flow rate can be controlled effectively using the variations in viscosity as well as thermal conductivity of the working fluid.

Table 6 shows the skin friction, rate of heat transfer and the volumetric flow rate for different values of viscosity parameter ($\lambda$) when $Sc = 0.94$, $N = 1$, $Q = -0.5$, $Kc = 0.7$, $\gamma = 0.5$, $S = -0.4$, $Ec = 0.2$, and $Pr = 0.71$

| $\lambda$ | $V_m$ | $\tau_0$ | $\tau_1$ | $Nu_0$ | $Nu_1$ |
|-----------|-------|----------|----------|--------|--------|
| -0.1      | 0.08306122 | 0.65590609 | 0.38033094 | 1.25101467 | 1.32751053 |
| -0.3      | 0.07238881 | 0.64136892 | 0.35291011 | 1.27763395 | 1.31284297 |
| -0.5      | 0.06172147 | 0.59331576 | 0.32090552 | 1.30394709 | 1.29854730 |
| -0.7      | 0.05105867 | 0.51132754 | 0.28495353 | 1.33002839 | 1.28445574 |
| -0.9      | 0.04045021 | 0.39338671 | 0.24743840 | 1.35603847 | 1.26948786 |

Table 7 shows the skin friction, rate of heat transfer and the volumetric flow rate for different values of conductivity parameter ($\gamma$) when $Sc = 0.94$, $N = 1$, $Q = -0.5$, $Kc = 0.7$, $\gamma = 0.5$, $S = -0.4$, $Ec = 0.2$, and $Pr = 0.71$

| $\gamma$ | $V_m$ | $\tau_0$ | $\tau_1$ | $Nu_0$ | $Nu_1$ |
|-----------|-------|----------|----------|--------|--------|
| 0.1       | 0.08066834 | 0.64399218 | 0.36527459 | 1.07261430 | 1.11333451 |
| 0.3       | 0.08137028 | 0.64756481 | 0.36954525 | 1.11605192 | 1.16865117 |
| 0.5       | 0.08200449 | 0.65073912 | 0.37349699 | 1.15959678 | 1.22340556 |
| 0.7       | 0.08256883 | 0.65351880 | 0.37710221 | 1.20425466 | 1.27670156 |
| 0.9       | 0.08306122 | 0.65590601 | 0.38033094 | 1.25101467 | 1.32751053 |

**TABLE 6** Computation of variation of mass flow rate, skin friction, and Nusselt number for different values of viscosity parameter ($\lambda$) when $Sc = 0.94$, $N = 1$, $Q = -0.5$, $Kc = 0.7$, $\gamma = 0.5$, $S = -0.4$, $Ec = 0.2$, and $Pr = 0.71$

**TABLE 7** Computation of variation of mass flow rate, skin friction, and Nusselt number for different values of conductivity parameter ($\gamma$) when $Sc = 0.94$, $N = 1$, $Q = -0.5$, $Kc = 0.7$, $\gamma = 0.5$, $S = -0.4$, $Ec = 0.2$, and $Pr = 0.71$

12 | CONCLUSION

In this study, the influence of Suction/Injection on steady natural convection flow through a vertical channel was considered. Taken into account the effects of heat source/sink as well as variations in the thermo-physical fluid properties such as temperature dependent viscosity and thermal conductivity. The governing equations for temperature, velocity and concentration fields were solved analytically using the DTM. The results were verified with results from the exact and numerical methods and excellent agreement was found. The results of fluid flow within the channel reveal the following:

1. Increasing suction causes an increase in skin friction on the channel plates while injection causes a decrease.
2. Increasing fluid viscosity decreases both fluid temperature and velocity within the channel.
3. Skin friction, Nusselt number, and volumetric flow rate all increase with increase in thermal conductivity of the working fluid.
4. The volumetric flow rate can be controlled effectively using the variations in viscosity as well as thermal conductivity of the working fluid.

The investigations carried out in this article may be of great interest to industries, especially Oil and Gas, seeking to improve production capacities, reduce corrosion, separate oil/gas/water, as well as many other processes so as to improve the profitability of all exploration and recovery efforts using chemical Injection/Suction packages. In addition, the model for viscosity in Equation 14 is applicable in many processes where preheating of fuel are used as a mechanism to boost heat transfer effects.
PEER REVIEW INFORMATION

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NOMENCLATURE

- \( c_P \): specific heat at constant pressure
- \( D_m \): mass diffusivity
- \( D_f \): Dufour number
- \( Ec \): Eckert number
- \( g \): acceleration due to gravity, [m/s\(^2\)]
- \( h \): distance between the walls, [m]
- \( K_c \): chemical reaction parameter
- \( K_r \): chemical reaction constant
- \( k \): thermal conductivity, [W/m. K]
- \( k_0 \): thermal conductivity at \( T = T_0 \)
- \( N \): buoyancy ratio parameter
- \( Nu_0 \): nusselt number at \( y = 0 \)
- \( Nu_1 \): nusselt number at \( y = 1 \)
- \( Pr \): Prandtl number
- \( Pr_0 \): Prandtl number in hydrostatic state
- \( Q \): heat source/sink parameter
- \( Q_0 \): heat generation constant
- \( S \): Suction/Injection
- \( Sc \): Schmidt number
- \( T' \): temperature, [K]
- \( T'_1, T'_0 \): hot wall/ cold wall temperature
- \( u' \): dimensional velocity component
- \( u \): dimensionless velocity component
- \( V' \): velocity component
- \( V_0 \): constant velocity
- \( x' \): vertical coordinate, [m]
- \( y' \): horizontal coordinate, [m]

GREEK SYMBOLS

- \( \mu \): viscosity of the fluid, [kg. m\(^{-1}\). s\(^{-1}\)]
- \( \mu_0 \): viscosity at \( T = T_0 \), [kg. m\(^{-1}\). s\(^{-1}\)]
- \( \beta \): thermal expansion coefficient of fluid, [K\(^{-1}\)]
- \( \bar{\beta} \): concentration expansion coefficient
- \( \rho \): density, [kg. m\(^{-3}\)]
- \( \rho_0 \): density in the hydrostatic state
- \( \theta \): dimensionless temperature
- \( \lambda \): viscosity parameter
- \( \gamma \): thermal conductivity parameter
- \( \nu_0 \): kinematic viscosity of fluid, [m\(^2\). s\(^{-1}\)]
- \( \tau_0 \): skin friction at \( y = 0 \)
- \( \tau_1 \): skin friction at \( y = 1 \)

CONFLICT OF INTEREST

The authors declare no potential conflicts of interest with respect to the research, authorship, and publication of this article.

AUTHOR CONTRIBUTIONS

*Abiodun Ajibade*: Project administration; supervision. *Princely Ojeagbase*: Investigation.
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