PRIMORDIAL MAGNETIC FIELDS AND ELECTROWEAK BARYOGENESIS

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Abstract

In this contribution we will shortly review the main mechanism through which primordial magnetic fields may affect the electroweak baryogenesis. It is shown that although strong magnetic fields might enhance the strength of the electroweak phase transition, no benefit is found for baryogenesis once the effect of the field on the sphaleron rate is taken into account. The possible role of hypermagnetic helicity for the electroweak baryogenesis is shortly discussed.

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1 Introduction

An outstanding problem in astrophysics concerns the origin and nature of magnetic fields in the galaxies and in the clusters of galaxies [1, 2]. The uniformity of the magnetic fields strength in the several galaxies and the recent observation of magnetic fields with the same intensity in high redshift protogalactic clouds suggest that galactic and intergalactic magnetic fields may have a primordial origin. Hopefully, a confirmation to this intriguing hypothesis will come from the forthcoming balloon and satellite missions looking at the anisotropies of the Cosmic Microwave Background Radiation. In fact, among other very important cosmological parameters, the observations performed by these surveyors may be able to detect the imprint of primordial magnetic fields on the temperature and polarizations acoustic peaks [3].

Besides observations, a considerable amount of theoretical work, based on the particle physics standard model as well as on its extensions, has been done which support the hypothesis that the production of magnetic fields may actually be occurred during the big-bang [4, 5].

Quite independently from the astrophysical considerations, several authors paid some effort to investigate the possible effects that cosmic magnetic fields may have for some relevant physical processes which occurred in the early Universe like the big-bang nucleosynthesis [6] and the electroweak baryogenesis (EWB). The latter is the main subject of this contribution.

Since, before the electroweak phase transition (EWPT) to fix the unitary gauge is a meaningless operation, the electromagnetic field is undefined above the weak scale and we can only speaks in terms of hyperelectric and hypermagnetic fields. The importance of a possible primordial hypercharge magnetic fields for the electroweak baryogenesis scenario is three-fold. In fact, we will show that hypercharge magnetic fields can affect the dynamics of the EWPT, they change the rate of the baryon number violating anomalous processes and, finally, hypermagnetic fields may themselves carry baryon number if they possess a non trivial topology. These effects will be shortly reviewed respectively in the section 2,3, and 4 of this contribution.
2 The effect of a strong hypermagnetic fields on the EWPT

As it is well known, the properties of the EWPT are determined by the Higgs field effective potential. In the framework of the minimal standard model (MSM), by accounting for radiative corrections from all the known particles and for finite temperature effects, one obtains that

\[ V_{\text{eff}}(\phi, T) \simeq -\frac{1}{2}(\mu^2 - \alpha T^2)\phi^2 - T\delta\phi^3 + \frac{1}{4}(\lambda - \delta\lambda T)\phi^4. \] (1)

where \( \phi \) is the radial component of the Higgs field and \( T \) is the temperature (for the definitions of the coefficients see e.g. Ref.\([7]\)).

A strong hypermagnetic field can produce corrections to the effective potential as it affects the charge particles propagators. It was shown, however, that these kind of corrections are not the most relevant effect that strong hypermagnetic fields may produce on the EWPT. In fact, it was recently showed by Giovannini and Shaposhnikov \([8]\) and by Elmfors, Enqvist and Kainulainen \([7]\) that hypermagnetic fields can affect directly the Gibbs free energy (in practice the pressure) difference between the broken and the unbroken phase, hence the strength of the transition. The effect can be understood in analogy with the Meissner effect, i.e. the expulsion of the magnetic field from superconductors as consequence of photon getting an effective mass inside the specimen. In our case, it is the \( Z \)–component of the hypercharge \( U(1)_Y \) magnetic field which is expelled from the broken phase just because \( Z \)–bosons are massive in that phase. Such a process has a cost in terms of free energy. Since in the broken phase the hypercharge field decompose into

\[ A^Y_\mu = \cos\theta_w A_\mu - \sin\theta_w Z_\mu, \] (2)

we see that the Gibbs free energy in the broken and unbroken phases are

\[ G_b = V(\phi) - \frac{1}{2} \cos^2\theta_w (B^\text{ext}_Y)^2, \] (3)

\[ G_u = V(0) - \frac{1}{2} (B^\text{ext}_Y)^2. \] (4)
where $B_Y^{ext}$ is the external hypermagnetic field. In other words, compared to the case in which no magnetic field is present, the energy barrier between unbroken and broken phase, hence the strength of the transition, is enhanced by the quantity $\frac{1}{2} \sin^2 \theta_w (B_Y^{ext})^2$. According to the authors of refs.\[8, 7\] this effect can have important consequence for the electroweak baryogenesis scenario.

In any scenario of baryogenesis it is crucial to know at which epoch do the sphaleronic transitions, which violate the sum $(B + L)$ of the baryon and lepton numbers, fall out of thermal equilibrium. Generally this happens at temperatures below $\bar{T}$ such that \[9\]

$$E(\bar{T}) \geq A,$$

(5)

where $E(T)$ is the sphaleron energy at the temperature $T$ and $A \simeq 35 - 45$, depending on the poorly known prefactor of the sphaleron rate. In the case of baryogenesis at the electroweak scale one requires the sphalerons to drop out of thermal equilibrium soon after the electroweak phase transition. It follows that the requirement $\bar{T} = T_c$, where $T_c$ is the critical temperature, turns eq. (5) into a lower bound on the higgs vacuum expectation value (VEV),

$$\frac{v(T_c)}{T_c} \geq 1.$$ 

(6)

As a result of intense research in the recent years \[10\], it is by now agreed that the standard model (SM) does not have a phase transition strong enough as to fulfill eq. (6), whereas there is still some room left in the parameter space of the minimal supersymmetric standard model (MSSM).

The interesting observation made in Refs.\[8, 7\] is that a magnetic field for the hypercharge $U(1)_Y$ present for $T > T_c$ may help to fulfill Eq.(6). In fact, it follows from the Eqs.(3), that in presence of the magnetic field the critical temperature is defined by the expression

$$V(0, T_c) - V(\phi, T_c) = \frac{1}{2} \sin^2 \theta_w (B_Y^{ext}(T_c))^2.$$ 

(7)

This expression implies a smaller value of $T_c$ than that it would take in the absence of the magnetic field, hence a larger value of the ratio (6). On the
basis of the previous considerations and several numerical computations, the
authors of Refs. [8, 7] concluded that for some reasonable values of the mag-
netic field strength the EW baryogenesis can be revived even in the standard
model. In the next section we shall reconsider critically this conclusion.

3 The sphaleron in a magnetic field

The sphaleron [11] is a static and unstable solution of the field equations
of the electroweak model, corresponding to the top of the energy barrier be-
tween two topologically distinct vacua. In the limit of vanishing Weinberg
angle, \( \theta_w \rightarrow 0 \), the sphaleron is a spherically symmetric, hedgehog-like con-
figuration of \( SU(2) \) gauge and Higgs fields. No magnetic moment is present
in this case. As \( \theta_w \) is turned on the \( U_Y(1) \) field is excited and the spherical
symmetry is reduced to an axial symmetry [11]. A very good approxima-
tion to the exact solution is obtained using the Ansatz by Klinkhamer and
Laterveer [11], which requires four scalar functions of \( r \) only,

\[
g'_a \, dx^i = (1 - f_0(\xi)) \, F_3 ,
\]

\[
gW_a^\sigma \, dx^i = (1 - f(\xi))(F_1 \sigma^1 + F_2 \sigma^2) + (1 - f_3(\xi))F_3 \sigma^3 ,
\]

\[
\Phi = \frac{v}{\sqrt{2}} \left( \begin{array}{c} 0 \\ h(\xi) \end{array} \right) ,
\]

where \( g \) and \( g' \) are the \( SU(2)_L \) and \( U(1)_Y \) gauge couplings, \( v \) is the higgs
VEV such that \( M_W = gv/2, M_h = \sqrt{2} \lambda v, \xi = gvr, \sigma^a (a = 1, 2, 3) \) are the
Pauli matrices, and the \( F_a \)'s are 1-forms defined in Ref. [11]. The boundary
conditions for the four scalar functions are

\[
f(\xi), f_3(\xi), h(\xi) \rightarrow 0 \quad f_0(\xi) \rightarrow 1 \quad \text{for} \quad \xi \rightarrow 0
\]

\[
f(\xi), f_3(\xi), h(\xi), f_0(\xi) \rightarrow 1 \quad \text{for} \quad \xi \rightarrow \infty .
\]

It is known [11] that for \( \theta_w \neq 0 \) the sphaleron has some interesting electro-
magnetic properties. In fact, differently from the pure \( SU(2) \) case, in the
physical case a nonvanishing hypercharge current \( J_i \) comes-in. At the first
order in \( \theta_w, J_i \) takes the form

\[
J_i^{(1)} = -\frac{1}{2} g' v^2 h^2(\xi)[1 - f(\xi)] v_3 x_j ,
\]

(10)
where \( h \) and \( f \) are the solutions in the \( \theta \to 0 \) limit, giving for the dipole moment
\[
\mu^{(1)} = \frac{2\pi g}{3 g' v} \int_0^\infty d\xi \xi^2 h^2(\xi)[1 - f(\xi)] .
\] (11)

The reader should note that the dipole moment is a true electromagnetic one because in the broken phase only the electromagnetic component of the hypercharge field survives at long distances.

Following Ref.\[12\] we will now consider what happens to the sphaleron when an external hypercharge field, \( A^Y_i \), is turned on. Not surprisingly, the energy functional is modified as
\[
E = E_0 - E_{\text{dip}},
\] (12)
with
\[
E_0 = \int d^3 x \left[ \frac{1}{4} F_{ij}^a F_{ij}^a + \frac{1}{4} f_{ij} f_{ij} + (D_i \Phi)^\dagger (D_i \Phi) + V(\Phi) \right]
\] (13)
and
\[
E_{\text{dip}} = \int d^3 x J_i A^Y_i = \frac{1}{2} \int d^3 x f_{ij} f_{ij}^{\text{c}}
\] (14)
with \( f_{ij} \equiv \partial_i A^Y_j - \partial_j A^Y_i \). We will consider here a constant external hypermagnetic field \( B^Y_c \) directed along the \( x_3 \) axis. In the \( \theta \to 0 \) limit the sphaleron has no hypercharge contribution and then \( E_{\text{dip}}^{(0)} = 0 \). At \( O(\theta_w) \), using (10) and (11) we get a simple dipole interaction
\[
E_{\text{dip}}^{(1)} = \mu^{(1)} B^Y_c .
\] (15)

In order to assess the range of validity of the approximation (15) one needs to go beyond the leading order in \( \theta_w \) and look for a nonlinear \( B^Y_c \)--dependence of \( E \). This requires to solve the full set of equations of motion for the gauge fields and the Higgs in the presence of the external magnetic field. Fortunately, a uniform \( B^Y_c \) does not spoil the axial symmetry of the problem. Furthermore, the equation of motion are left unchanged \((\partial_i f_{ij}^{\text{c}} = 0)\) with respect to the free field case. The only modification induced by \( B^Y_c \) resides in the boundary conditions since – as \( \xi \to \infty \) – we now have
\[
f(\xi) , h(\xi) \to 1 , \quad f_3(\xi) , f_0(\xi) \to 1 - B^Y_c \sin 2\theta_w \frac{\xi^2}{8gv^2}
\] (16)
wheras the boundary condition for $\xi \to 0$ are left unchanged.

The solution of the sphaleron equation of motions with the boundary conditions in the above were determined numerically by the authors of Ref. [12]. They showed that in the considered $B_Y^c$-range the corrections to the linear approximation

$$\Delta E \simeq \mu^{(1)} \cos \theta_W B_Y^c$$

are less than 5%. For larger values of $B_Y^c$ non-linear effects increase sharply. However, for such large magnetic fields the broken phase of the SM is believed to become unstable to the formation either of $W$-condensates [13] or of a mixed phase [14]. In such situations the sphaleron solution does not exist any more. Therefore, we will limit our considerations to safe values $B_Y^c \lesssim 0.4 \, T^2$.

From the previous considerations it follows that the conclusion that the sphaleron freeze-out condition (5) is satisfied and the baryon asymmetry preserved was premature. Indeed, in an external magnetic field the relation between the VEV and the sphaleron energy is altered and Eq. (6) does not imply (5) any more. We can understand it by considering the linear approximation to $E$,

$$E \simeq E(B_Y^c = 0) - \mu^{(1)} B_Y^c \cos \theta_W \approx \frac{4\pi v}{g} \left( z_0 - \frac{\sin 2\theta_w}{g} \frac{B_Y^c}{v^2} m^{(1)} \right)$$

where $m^{(1)}$ is the $O(\theta_W)$ dipole moment expressed in units of $e/\alpha_W M_W(T)$. From the Fig.1 we see that even if $v(T_c)/T_c \gtrsim 1$ the washout condition $E/T_c \gtrsim 35$ is far from being fulfilled. It follows that the presence of a strong homogeneous hypermagnetic field does not seem to help the EWB.

4 Baryons from hypermagnetic helicity

A more interesting scenario may arise if hypermagnetic fields are inhomogeneous and carry a nontrivial topology. The topological properties of the hypermagnetic field are quantified by the, so called, hypermagnetic helicity, which coincide with the Chern-Simon number

$$N_{CS} = \frac{\alpha}{\pi} \int_V \, d^3 x B_Y \cdot A_Y$$
where $A_Y$ is the hypercharge field. It is well known that the Chern-Simon number is related to the lepton and baryon number by the abelian anomaly. Recently it was noticed by Giovannini and Shaposhnikov [8] that the magnetic helicity may have some non-trivial dynamics during the big-bang giving rise, through the anomaly equation, to a variation of the fermion and baryon contents of the Universe. Magnetic configurations with $B_Y \cdot \nabla \times B_Y$ (“magnetic knots”) may have been produced in the early Universe, for example, by the conformal invariance breaking coupling of a pseudoscalar field with the electromagnetic field which may arise, for example, in some superstring inspired models [8, 15].

The interesting point that we would like to arise in the conclusions of this contribution is that a less exotic source of hypermagnetic helicity is provided by the Weinberg-Salam model itself. This source are electroweak strings. Electroweak strings are well known non-perturbative solutions of the Weinberg-Salam model (for a review see [16]). They generally carry a nonvanishing Chern-Simon number and, according to recent lattice simulations, they are copiously produced during the EWPT even if this transition is just a cross-over [17]. Although electroweak string have been sometimes invoked for alternative mechanism of EWB, it was not always noticed in the literature that CP symmetry is naturally broken for twisted electroweak strings without calling for extension of the Higgs structure of the model. This is just because the twist give rise to non-orthogonal hyperelectric and hypermagnetic fields. We suggest that primordial magnetic fields, even if they are uniform, could provide a bias for the baryon number violation direction. Finally, electroweak string decay might provide the third Sacharov ingredient for EWB, namely an out-of-equilibrium condition.

In conclusions, we think that a more careful study of the possible role of electroweak strings for the EWB is worthwhile.

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Figure 1: The VEV at the critical temperature, $v(T_c)$, and the sphaleron energy vs. the external magnetic field for $M_h = M_W$. 

\[ v(T_c)/T_c \]

\[ E/35T_c \]