STARQUAKE-INDUCED GLITCHES IN PULSARS

Richard I. Epstein
Los Alamos National Laboratory
epstein@lanl.gov

Bennett Link
Montana State University
Los Alamos National Laboratory
blink@dante.physics.montana.edu

Abstract
The neutron star crust is rigid material floating on a neutron-proton liquid core. As the star’s spin rate slows, the changing stellar shape stresses the crust and causes fractures. These starquakes may trigger pulsar glitches as well as the jumps in spin-down rate that are observed to persist after some glitches. Earlier studies found that starquakes in spinning-down neutron stars push matter toward the magnetic poles, causing temporary misalignment of the star’s spin and angular momentum. After the star relaxes to a new equilibrium orientation, the magnetic poles are closer to the equator, and the magnetic braking torque is increased. The magnitude and sign of the predicted torque changes are in agreement with the observed persistent spin-down offsets. Here we examine the relaxation processes by which the new equilibrium orientation is reached. We find that the neutron superfluid in the inner crust slows as the star’s spin realigns with the angular momentum, causing the crust to spin more rapidly. For plausible parameters the time scale and the magnitude of the crust’s spin up agree with the giant glitches in the Vela and other pulsars.

INTRODUCTION
Stresses in the crust of a neutron star could produce starquakes that affect the star’s spin evolution and generate high-energy emission. As the star’s spin rate increases or decreases, changes in the equilibrium shape of the star and the differential rotation between the crust and the interior neutron superfluid generate stress [1]. In “magnetars”, decay of the superstrong field ($B \gtrsim 10^{14}$ G) could break the crust and drive episodes of intense gamma-ray emission [2]. Recent studies showed that starquakes can change the magnetic spin-down torque acting on the star [3, 4]. Starquakes in slowing neutron stars drive matter toward
the magnetic poles, distort the star’s shape, and excite precession. As the precession damps, the star relaxes to a new rotational state with the magnetic poles closer to the equator. The new magnetic orientation enhances the braking torque on the star and may provide an explanation for the observed increases in the spin-down torque following glitches in the Crab pulsar, PSR1830-08 and PSR0355+54.

Here we investigate the physical processes that allow the star to relax to its post-starquake equilibrium. The most important processes are the coupling between the liquid core and solid crust and the creep of neutron superfluid vortices in the inner crust of the star. We find that the changes produced by large starquakes can trigger catastrophic unpinning of neutron superfluid vortex lines in the star’s inner crust [5]. As vortices move, the inner crust superfluid rapidly settles to a state of lower angular momentum, while exerting a spin-up torque on the crust. Our estimates show that starquake-triggered events may explain giant pulsar glitches as well as the persistent spin-down offsets. The next section summarizes earlier work on starquakes and spin-down offsets, and the following one describes the post-starquake spin relaxation and glitches.

**STARQUAKES AND SPIN-DOWN OFFSETS**

The crust of a spinning neutron star is oblate with an equatorial bulge. The moment of inertia of the bulge is $I_{eb} \sim I_{\text{crust}} R^3 \Omega^2/(2GM) \sim 4 \times 10^{-5} I_{\text{crust}} \Omega^2$, where $R \sim 10^6$ cm is the stellar radius, $\Omega = 100\Omega_2$ rad s$^{-1}$ is the star’s spin frequency, and $I_{\text{crust}}$ is the characteristic moment of inertia of the crust. The crust contains about 1% of the star’s total moment of inertia $I_{\text{total}}$. As the star spins down, the equatorial circumference shrinks and the polar radius grows. Because the crust is solid, strain develops as the star’s shape changes. As sketched in Figure 1, the strain in the crust is relieved along starquake faults that form at an angle to the star’s equator [3, 4]. Matter slides along these faults to higher latitudes, and magnetic stresses favor those faults that direct matter along field lines toward the magnetic poles.

An important result of the earlier studies is that starquakes shift the stellar matter asymmetrically, creating excess moment of inertia $\delta I$ about an axis different from any of the pre-starquake principal axes. This distortion changes the orientation of the principal inertial axis by an angle $\Delta\alpha \sim \delta I/I_{eb} \sim 2.5 \times 10^4 \Omega_2^{-2} \delta I/I_{\text{crust}} \sim 2.5 \times 10^{-3} \Omega_2^{-2} \delta_{-7}$, where $\delta I/I_{\text{crust}} \equiv 10^{-7} \delta_{-7}$ [3]. The distortion parameter $\delta_{-7}$ characterizes the size of the starquake.

When the principal axis of inertia of the crust is not aligned with the star’s angular momentum, the star precesses and wobbles. Eventually
Starquake-Induced Glitches

the star relaxes to a new equilibrium in which the axis and angular momentum are again realigned, and the magnetic pole is shifted by the angle $\Delta \alpha$ closer to the rotational equator. In some mechanisms for pulsar spin down, such as the magnetic dipole braking model, this angular shift increases the torque on the star, producing a long-lasting increase in the spin-down rate. The persistent spin-down offsets observed in the Crab pulsar can be explained by this mechanism if $\Delta \alpha \sim 10^{-3}$ \cite{6} corresponding to $\delta_\perp \simeq 1.6 \ (\Omega_2 \simeq 2$ for the Crab pulsar).

Figure 1 Starquakes relieve the stress that builds as a neutron star's spin slows. Matter can slide to higher latitudes along faults $F$ or $F'$. Magnetic stresses favor faults such as $F$ that move matter toward the magnetic poles. The accumulated matter, shown as snow-capped peaks, shift the principal axis of inertia by an angle $\Delta \alpha$ relative to the star.

THE ORIGIN OF GLITCHES

Starquake-induced asymmetry in the stellar crust excites precession. If the neutron star crust behaved as an isolated rigid body, it would precess or wobble at a frequency $\Omega_w \sim (I_{eb}/I_{\text{crust}})\Omega$ and the angle between the angular velocity and the angular momentum would be $\sim (\Delta \alpha)^2$. The spin behavior of a realistic neutron star is more complicated than this for several reasons. First, the pinning of superfluid vortices in the inner crust acts to stabilize the spin of the crust. Second, the crust and core of the star are not strongly coupled on the precession time scale. Third, the tilting of the crust accelerates vortex creep.

Pinned superfluid. In the inner crust of the star the neutron superfluid vortex lines may pin to the nuclei in the solid crust. The rotation of the superfluid is determined by the location of the vortex lines,
and, as long as the vortex lines remained pinned, the superfluid velocity field cannot change. The gyroscopic action of the pinned superfluid works with the equatorial bulge to further stabilize the star [7]. The moment of inertia \( I_{\text{pinned}} \) of the pinned superfluid is comparable to that of the crust and much larger than the moment of inertia of the equatorial bulge; \( I_{\text{pinned}} \sim 0.01I_{\text{total}} \gg I_{\text{eb}} \). The pinned superfluid decreases the equilibrium tilt of the star by a factor \( \frac{I_{\text{eb}}}{I_{\text{pinned}}} \) to an angle \( \Delta \alpha_p \sim \left[ \frac{\Delta \alpha_p}{\Delta \alpha} \right]^2 \sim 10^{-10} \left( \frac{\delta}{\delta_0} \right)^2 \ll \Delta \alpha \). This is the tilt angle of the star with completely pinned superfluid after the precession has damped (the tilt immediately after the starquake is \( \sim [\Delta \alpha_p]^2 \sim 10^{-10} \left( \frac{\delta}{\delta_0} \right)^2 \)). The precession frequency is proportional to \( I_{\text{pinned}} \) and inversely proportional to moment of inertia coupled to the crust. If the spin of the core and the crust are tightly coupled, the star’s precession frequency is \( \Omega_w = \left( \frac{I_{\text{pinned}}}{I_{\text{total}}} \right) \Omega \approx \Omega_2 \) rad s\(^{-1}\).

**Coupling between the crust and the core.** Changes in the crust’s motion are communicated to the core by MHD-like waves. If the protons in the neutron star core form a type II superconductor, as expected, the magnetic field is confined to thin tubes of flux \( \pi \hbar c/e \sim 2 \times 10^{-7} \text{ G cm}^{-2} \), with characteristic dimensions \( \Lambda \sim 50 \text{ fm} \) and field strengths \( B_\phi \sim 10^{15} \text{ G} \). Signals travel from the crust to the core on an Alfvén time \( t_{\text{couple}} \sim (4 \pi \rho)^{1/2} R/(BB_\phi)^{1/2} \sim 4B_{12}^{-1/2} \text{ s} \), for an average field of \( B = 10^{12} B_{12} \text{ G} \) and a density of \( \rho \approx 10^{15} \text{ g cm}^{-3} \) [8]. Because of the imperfect coupling between the crust and the core, the precession is damped on a time scale [9]

\[
t_{\text{damp}} \approx \frac{\Omega}{\Omega_w} t_{\text{couple}} \sim \frac{I_{\text{total}}}{I_{\text{pinned}}} t_{\text{couple}} \sim 400 B_{12}^{-1/2} \text{ s}. \tag{1}
\]

The energy of the regular motion of precession is converted into irregular fluid motions in the core. The irregular motions of the superfluid neutrons and superconducting protons are then converted into thermal energy by processes such as electron scattering from vortex lines [10] or flux tube-vortex line interactions [11]. Since the core and crust are not strongly coupled on the precession time scale, the effective moment of inertia of the precessing material is less, and the precession frequency may be higher than we have used. The damping time scale would be correspondingly reduced.

After the precession has damped, the star’s crust has tilted by \( \Delta \alpha_p \). As the pinned vortex lines move to align with the crust’s rotation vector, the tilt angle grows until it equals \( \Delta \alpha \). We now examine the processes by which the vortex lines move and the effects of this motion on the star’s spin behavior.
Motion of vortex lines. In a steady state, the angular velocity of the superfluid closely follows that of the solid crust. As the superfluid rotation slows, the vortex lines move radially with a steady velocity $v_{ss} \simeq -R\dot{\Omega}/(2\Omega) = R/(4t_{age})$, where the spin-down age of the pulsar is $t_{age} \equiv \Omega/(2\dot{\Omega})$ \[14\]. For the Crab pulsar $v_{ss} \simeq 5 \times 10^{-6} \text{ cm s}^{-1}$ and for the Vela pulsar $v_{ss} \simeq 7 \times 10^{-7} \text{ cm s}^{-1}$. If the vortex lines move at $v_{ss}$ following a starquake, they would align with the new direction of the crust’s principal axis in a time $t_{align} \sim \Delta\alpha R/v_{ss} \sim 10^{-2}\delta_{-7}\Omega_{2}^{-2}t_{age}$, years for the Crab pulsar and decades for the Vela pulsar. The vortex velocity can be greatly enhanced by the perturbations produced by the tilt of the spin axis.\[ After the star’s spin axis tilts relative to the pre-starquake orientation, some parts of the star are further from the new spin axis; this shift can be as large as $\Delta\alpha_p R$. For these regions, the linear velocity from the star’s spin increases by $\Delta\alpha_p R\Omega$. The superfluid velocity, on the other hand, changes much less. If the neutron vortex lines in the star vortex were completely pinned, the superfluid velocity in the star’s frame would remain unchanged. Even though the superfluid in the core is not pinned, the vortex lines cannot move through the crust, so the crust superfluid velocity remains fixed. The velocity lag $v_\delta$ between the superfluid and the solid crust thus changes by as much as $\Delta v_\delta \sim \Delta\alpha_p R\Omega \simeq 10^3\delta_{-7} \text{ cm s}^{-1}$ in parts of the star; see Figure 2. Velocity differences of this magnitude may have dramatic effects on the

4Heat produced during a starquake can also increase the vortex velocity \[12\].
vortex velocity, large enough to produce the crustal spin-ups associated with glitches.

Pinned vortex lines in the inner crust can creep outward through thermal activation of vortex segments from their pinning barrier; a line segment unpins from one configuration and migrates to new sites where it repins [13, 14]. The general form of the creep velocity is $v_{\text{creep}} \propto \exp\left[-A/kT\right]$, where $A$ is the minimum activation energy for a segment of the vortex line to unpin [15]. The activation energy depends sensitively on the lag velocity $v_\delta$. If the vortex lines are moving at their steady state rate $v_{ss}$ and the lag velocity suddenly jumps by $\Delta v_\delta$, the new creep rate is

$$v_{\text{creep}} = v_{ss} \exp \left[ -\frac{dA}{dv_\delta} \frac{\Delta v_\delta}{kT} \right]. \quad (2)$$

The activation energy of a vortex line is characterized by the maximum pinning force $F_{\text{pin}}$ between a nucleus and a vortex line, the range of the pinning force $r_{\text{pin}}$ and the distance between pinning sites $\ell_{\text{pin}}$. The maximum lag velocity between the superfluid and the crust that the pinning forces can support is $v_{\text{max}} = F_{\text{pin}}/\rho_s \kappa \ell_{\text{pin}}$, where $\kappa = \pi \hbar/(m_n) = 2.0 \times 10^{-3}$ cm$^2$ s$^{-1}$ and $\rho_s$ is the mass density of the neutron superfluid; $\rho_s \sim 10^{14}$ g cm$^{-3}$ is the characteristic density of much of the inner crust superfluid. The activation energy required for a vortex line to unpin depends on the stiffness of the vortex line; if the line is flexible, it can unpin from one nucleus at a time, whereas if it is stiff, the line must unpin from many nuclei simultaneously [15]. The parameter $\tau \approx 0.4 \rho_s \kappa^2 r_{\text{pin}}/(F_{\text{pin}} \ell_{\text{pin}})$ characterizes the vortex line’s stiffness [15]; for the conditions in much of the crusts of the Crab and Vela pulsars $\tau > 1$ and vortex lines move by unpinning from many sites simultaneously. In this limit the appropriate expression for the activation energy is $A \approx 6.8 F_{\text{pin}} r_{\text{pin}} \tau^{1/2} (1 - v_\delta/v_{\text{max}})^{5/4}$, and it’s derivative is

$$\frac{dA}{dv_\delta} \approx -4.8 \frac{F_{\text{pin}} r_{\text{pin}} \tau^{1/2}}{v_{\text{max}}} \approx -3.0 \left( \frac{\rho_s^2 r_{\text{pin}}^3 \kappa^3}{v_{\text{max}}} \right)^{1/2}. \quad (3)$$

In obtaining this equation we set $(1 - v_\delta/v_{\text{max}})^{1/4} \sim 0.6$, which is a characteristic value for a variety of stellar models [14].

We can use the observations of the Vela pulsar to estimate $v_{\text{max}}$. This pulsar exhibited a string of 12 nearly evenly spaced glitches separated with an average interval of $t_{\text{int}} = 2.3$ years [16]. The regularity of these glitches suggests that the inner crust superfluid remains pinned between glitches until the velocity lag between the solid crust and the superfluid approaches $v_{\text{max}}$. With this interpretation, a lower limit to the critical lag velocity is $v_{\text{max}} > |\Omega_{\text{Vela}}| t_{\text{int}} R = 7.0 \times 10^3$ cm s$^{-1}$. The value of $v_{\text{max}}$
exceeds this limit because the inner crust superfluid might not relax to zero lag velocity at each glitch. For our estimates we use \( v_{\text{max}} = 10^5v_5 \) cm s\(^{-1}\). For \( v_5 \simeq 1 \) the pinning force is \( F_{\text{pin}} = \rho_s k t_{\text{pin}} v_{\text{max}} \simeq 13 (t_{\text{pin}}/100 \text{fm}) \) keV fm\(^{-1}\) at a superfluid density of \( \rho_s = 10^{14} \) g cm\(^{-3}\). This pinning force is much smaller than that obtained by recent microscopic calculations of vortex-nuclear interactions [17] assuming a perfect crystal, but it is larger than the average pinning force estimated for an amorphous crust [18].

Taking \( \rho_s = 10^{14} \) g cm\(^{-3}\) and interaction distance \( r_{\text{pin}} = 10 \) fm [19], the change in the activation energy in parts of the crust after a starquake is

\[
\frac{dA}{dv_5} \Delta v_5 \sim -0.053v_5^{-1/2} \left( \frac{\Delta v_5}{\text{cm s}^{-1}} \right) \text{ keV} \sim -52 \delta_{-7} \Omega_2 v_5^{-1/2} \text{ keV.} \tag{4}
\]

The time scale for the post-starquake relaxation of the vortex lines is \( t_{\text{relax}} \sim \delta_{-7} \tau_{\text{age}} v_{\text{creep}} / v_5 \); for \( kT \) in keV, this gives

\[
t_{\text{relax}} \sim \frac{\delta_{-7} \tau_{\text{age}}}{100 \Omega_2^2} \exp \left[ -\frac{52 \delta_{-7} \Omega_2}{v_5^{1/2} kT} \right]. \tag{5}
\]

The vortex relaxation time is very sensitive to the magnitude of the quake-induced shape change. For example, taking \( v_5 = 1 \), we find that for the Crab pulsar (\( kT \sim 20 \) keV, \( \tau_{\text{age}} \sim 10^3 \) yr, \( B_{12} \sim 4 \)) the relaxation time is less than the damping time if the distortion parameter is \( \delta_{-7} \gtrsim 8.8 \), but it is more than 1000\( t_{\text{damp}} \) if \( \delta_{-7} \lesssim 4.3 \). The corresponding values for the distortion parameter for the Vela pulsar (\( kT \sim 8 \) keV, \( \tau_{\text{age}} \sim 10^4 \) yr, \( B_{12} \sim 8.8 \)) are \( \delta_{-7} \gtrsim 1.9 \) and \( \delta_{-7} \lesssim 1.2 \). For each pulsar, there is a critical value for the distortion parameter \( \delta_{-7} \). Starquakes that produce distortions above this threshold trigger rapid vortex motions while smaller events generate only gradual changes.

**Spin jumps.** In the large events, the rapid outward motion of the vortex lines produces a slowing of the superfluid and a corresponding spin up of the crust. In the regions of the inner crust where \( \Delta v_5 > 0 \) the vortex lines rapidly creep a distance \( \Delta \alpha R \sim 2.5 \times 10^{-3} \Omega_2^2 \delta_{-7} R \). The superfluid in the affected regions of the inner crust will now spin more slowly by corresponding amount: \( \Delta \Omega_4/\Omega \sim 2.5 \times 10^{-3} \Omega_2^2 \delta_{-7} \). The angular momentum lost by superfluid is imparted to the crust and the core which is strongly coupled to it [10], giving

\[
\frac{\Delta \Omega_{\text{crust}}}{\Omega} \sim -\frac{I_{\text{crust}}}{2I_{\text{total}}} \Delta \Omega_4 \sim 1.3 \times 10^{-5} \frac{\delta_{-7}}{\Omega_2^2}.
\]
The rapid creep of vortex lines following a large starquake could explain the giant glitches with $\Delta \Omega_{\text{crust}}/\Omega \sim 2 \times 10^{-6}$ observed in the Vela and other pulsars.

In the above estimate we assume that the lag between the superfluid and the crust is large enough to supply the needed angular momentum to the crust. As the vortex lines creep through a distance $\sim \Delta \alpha R$ the local superfluid slows by $\sim \Delta \alpha R \Omega$. If the pre-starquake lag velocity $v_{\delta}$ were less than this value, the vortex creep would stop before the vortex lines move the full distance $\sim \Delta \alpha R$. A necessary condition for the above estimate to be valid is that $v_{\delta} \gtrsim \Delta \alpha R$. Since $v_{\max} > v_{\delta}$ we have $v_{\delta} \gtrsim 2.5 \delta^{-7} \Omega^{-1}$. The limiting factor in the size of a starquake-induced pulsar glitch may be the pre-glitch lag velocity. The reason for the large difference between the size of the Crab glitches and the giant glitches of the Vela pulsar may be that in the Crab pulsar, with its higher internal temperature, vortex creep between glitches limits the build up of a sufficiently large $v_{\delta}$.

**SUMMARY**

Starquakes tilt the principal axis of inertia of a neutron star. As the star relaxes to its new equilibrium orientation, neutron superfluid vortex lines migrate outward, spinning up the rest of the star. The spin ups from large quakes may explain the giant glitches observed in isolated pulsars. The change in the direction of the magnetic axis may increase the spin-down torque, as observed in the Crab and other pulsars. Detailed calculations of the post-starquake relaxation for both large and smaller events may yield distinctive timing signatures to compare with observations.

**Acknowledgments**

This work was performed under the auspices of the U. S. Department of Energy, and was supported by IGPP at LANL, NASA EPSCoR Grant 291748, NASA ATP Grants NAG 53688 and NAG 52863, and by USDOE Grant DOE/DE-FG02-87ER-40317.

**References**

[1] Ruderman, M., ApJ, 203, 213 (1976).

[2] Thompson, C. & Duncan, R. C., ApJ, 473, 322 (1996); Thompson, C. & Blaes, O., Phys. Rev. D, 57, 3219 (1998).
[3] Link, B., Franco, L. M., & Epstein, R. I., ApJ, 508, 838-843 (1998).
[4] Franco, L. M., Link, B., & Epstein, R. I., preprint, astro-ph/9911105 (1999).
[5] Anderson, P. W., & Itoh, N., Nature, 256, 25 (1975).
[6] Link, B., Epstein, R. I. & Baym, G., ApJ, 390, L21 (1992); Link, B. & Epstein, R. I., ApJ, 478, L91 (1997).
[7] Shaham, J., ApJ, 214, 251 (1977); Sedrakian, A., Wasserman, I., and Cordes, J. M., ApJ524, 341 (1999).
[8] Abney, M., Epstein, R. I. & Olinto, A. V., ApJ, 466, L91 (1996); Mendell, G., MNRAS, 296, 903 (1998).
[9] Bondi, H., & Gold, T., MNRAS, 115, 41 (1955).
[10] Alpar, M. A. and Sauls, J. A., ApJ, 327, 723 (1988).
[11] Ruderman, M., Zhu, T., and Chen. K., ApJ, 492, 267 (1998).
[12] Link, B. & Epstein, R. I., ApJ, 457, 844 (1996).
[13] Alpar, M. A., Anderson, P. W., Pines, D., & Shaham, J., ApJ, 276, 325 (1984).
[14] Link, B., Epstein, R. I. & Baym, G., ApJ, 403, 285 (1993)
[15] Link, B. & Epstein, R. I., ApJ, 373, 592 (1991)
[16] Link, B., Epstein, R. I., & Lattimer, J. M., Phys. Rev. Lett., 83, 3362 (1999).
[17] Epstein, R. I. & Baym, G., ApJ, 328, 680 (1988); Pizzochero, P.M., Viverit, L. & Broglia, R. A. Phys. Rev. Lett., 79, 3347 (1997).
[18] Jones, P. B., MNRAS, 306, 327 (1999).
[19] Negele, J. W. & Vautherin, D., Nuc.Phys, A207, 298 (1973); DeBlasio, F. V. & Elgarøy, Ø. Phys. Rev. Lett., 82, 1815, (1999).