TARGET FRAGMENTATION AND FRACTURE FUNCTIONS.

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We analyze recent data on the production of forward neutrons in deep inelastic scattering at HERA in the framework of a perturbative QCD description for semi-inclusive processes, which includes fracture functions.

In the most naive quark-parton model picture, the semi-inclusive cross section for the production of a hadron \( h \) from the deep inelastic scattering of charged leptons is usually taken to be given by:

\[
\frac{d^2 \sigma}{dxdydz_h} = \frac{(1 + (1/y)^2)}{2y^2} x c_i f_{\pi\pi}(x) D_{h=i}(z_h); \quad (1)
\]

where, \( f_{\pi\pi} \) is the parton distribution of flavour \( i \), \( D_{h=i} \) is the fragmentation function of a hadron \( h \) from a parton \( i \) and \( c_i = 4 \epsilon_{\pi\pi}^2 (P + l)^2 \). The kinematical variables used to characterize these processes are:

\[
x = \frac{Q^2}{P^2} \quad y = \frac{P^2}{P^2} \quad z_h = \frac{P}{Q^2} \quad \frac{h}{P} = \frac{E_h}{P_l(1-x)} \quad \frac{1}{2} \cos h; \quad (2)
\]

where \( q \) is the transferred momentum \( (Q^2 = Q^2) \), and \( P \) and \( Q \) are the incoming lepton and proton momentum respectively. \( E_h, P_h \) and \( z_h \) are the produced hadron and target nucleon energies, and the angle between the hadron and the target in the centre of mass of the virtual photon-proton system, respectively.

Although next to leading order corrections to this cross section are also well known, and have been shown to give a very good description of the so-called current fragmentation region \( (h > 0) \), the target fragmentation region, which corresponds to \( h = 0 \) \( (z_h = 0) \), cannot be described with this simplified picture. First of all, it is easy to see that, at the lowest order, hadrons can only be produced antiparallel to the target nucleon \( (h = 1) \), excluding the forward configurations. On the other hand, going to next to leading order, the corrections to the cross section develop divergences proportional to \( 1-z_h \), related to soft emission \( (E_h = 0) \), and also to collinear configurations where hadrons are produced in the direction of the remnant target \( (h = 0) \). Since at
lowest order hadrons cannot be produced in that direction, it is not possible to
tfactorize the divergence, as usual, into parton distributions and fragmen-
tation functions. Then, in order to describe hadrons produced in the target frag-
m entation region even at the lowest order, and also to be able to perform at
higher orders a consistent factorization of divergences originated in the current
fragmentation region, a new distribution has to be introduced, the so-called
fracture functions, $M_{ih=1}(x; (1 - x)z)$ \[1\]. These distributions represent the
probability of finding a parton of flavour $i$ and a hadron $h$ in the target $N$
(here $z = E_h / E_p(1 - x)$).

Therefore the complete leading order expression for the cross section be-
comes

$$\frac{d^2 \sigma}{dx dy dz} = \frac{(1 + (1 - y)^2)}{2y^2} \left[ \sum_{i=1}^{Q^2} \int \frac{d \phi}{u} f^{(p)}(x) D_{ih=1}^{(z)}(2) + \int \phi^{(p)}(x) M_{ih=1}(x; (1 - x)z) \right]$$

Higher order corrections to this kind of cross section can be found in ref. \[1\].

The scale dependence of fracture functions at $O(\alpha)$ is driven by two kinds
of processes, which contribute to the production of hadrons in the remnant
target direction: the emission of collinear partons from those found in the
(target the usual source of scale dependence of parton distributions, often
called homogeneous evolution), and those where partons radiated from the
one to be struck by the virtual probe, fragment into the measured hadron (the
so-called inhomogeneous term). These two contributions lead at leading order
to the following equation:

$$\frac{\theta}{\theta \log Q^2} M_{ih=1} \phi^{(p)}(x) f^{(p)}(x) D_{ih=1}^{(z)}(2)$$

where $P_{ij}^{(p)}(u)$ and $\phi^{(p)}(u)$ are the regularized and real
A. L. T. Paris splitting
functions, respectively.

Recently, the ZEUS Collaboration has measured DIS events identifying
high-energy neutrons in the final state, at very small angles with respect to
the proton direction ($\theta_{lab} = 0.75$ m rad), in the kinematical range given by
$3 \times 10^{-4} < x < 6 \times 10^{-3}$, $10 < Q^2 < 100$ GeV$^2$ and high $x_h$, \[2\]. The ZEUS Collaboration have reported that events with $x_h > 0.50$
represent a substantial fraction (of the order of 10%) of DIS events. In the frame
of...
a picture for semi-inclusive processes including fracture functions, as the one outlined above, the ZEUS findings can be represented (at LO) by

\[ \frac{R_1}{d_x} \int_{0.5 R} d_y \frac{d_y}{d_x} dx \frac{M_{\text{LO}}}{F_2^P(x;Q^2)} \]

In eq. (5) we have also defined the equivalent to \( F_2 \) for fracture functions:

\[ M_{\text{LO}}^{n=p}(x; x_L; Q^2) = \int \frac{d_y}{d_x} M_{\text{LO}}^{n=p}(x; x_L; Q^2) \]

In Fig. 1 we show the experimental outcome for this fracture function (as defined in eq. (5) and at \( Q^2 = 10 \text{ GeV}^2 \), taking advantage of the negligible \( Q^2 \) dependence of the data), and we compare it to \( F_2^P \) and \( F_2^P \). We also show the contribution to the same observable coming from current fragmentation processes, which is pure NLO and it is about 8 orders of magnitude smaller than the experimental data.

Fracture functions, as parton distributions in general, are essentially of a non-perturbative nature and have to be extracted from experiment. However, their close relation with fragmentation and structure functions allows in certain extreme cases a model estimate for them, which can then be compared with actual measured data and evolved with the corresponding evolution equations.

As an example, recently, a very sensible model estimate for the production of forward hadrons in DIS was made, in which the idea of non-perturbative Fock components of the nucleon has been proposed. In this approach the semi-inclusive DIS cross sections, and through them the corresponding fragmentation functions at a certain input scale \( Q^2_0 \), can be interpreted as the product of a flux of neutrons in the proton (integrated over \( p_T^2 \)) times the structure function of the pion exchanged between them, i.e.

\[ M_{\text{LO}}^{n=p}(x; x_L; Q^2_0) = \int \frac{d_y}{d_x} M_{\text{LO}}^{n=p}(x; x_L; Q^2_0) \]

Using a non-perturbative computation of the flux, which is in very good agreement with experimental data on high energy neutron and \( \gamma^* \) production in hadron-hadron collisions and a parametrization for the pion structure function \( F_2^P \), in Fig. 1 we make a comparison between the model estimate and the data for \( Q^2_0 = 10 \text{ GeV}^2 \), finding a remarkable agreement between them.

The success of the model estimate encourages us to go further and use the functional dependence of fracture functions, induced by the model and corroborated by the data, to analyse also the \( Q^2 \) dependence.

We first analyse the more familiar process of neutron production. Since the probability of current parton fragmentation into a neutron (given by frag-
Figure 1: Fracture function of neutrons in protons as measured by ZEUS compared to the model prediction and current fragmentation contributions.

The evolution is mainly driven by the usual homogeneous term of the evolution equations leading to an almost constant ratio between the number of neutron tagged events and that of all DIS events, as observed by ZEUS.

However, the scale dependence induced in the cross section for the production of pions, at least in the kinematical region of very small $x$ and small $x_L$, can be considerably affected by the inhomogeneity, given that soft pions are produced more copiously from quarks than from neutrons. In order to analyse these features of the evolution, we estimate the proton to pion fracture function at some input scale $Q_0^2$ using the same ideas from early applied to neutron production, and noticing that the $\nu x$ can be straightforwardly obtained from the one used in the last section by means of the crossing relation $\nu_p (x_L) = \nu_n (1/x_L)$. Then, the proton to pion fracture function can be approximated by

$$M_{2,^p} \gamma^p x; x_L; Q_0^2 = \nu_p (x_L) F_{2,^n} \frac{x}{x_L}; Q_0^2$$

In Fig. 2a we show the model estimates for proton to pion fracture functions (taking $Q_0^2 = 4 \text{ GeV}^2$), integrated over two different bins of $x_L$, compared with the contribution coming from the current fragmentation processes. We
assume the same restrictions as in the data from the ZEUS Collaboration for neutron production.

Figure 2: a) Prediction for the fracture function of $^+$ in protons for two different bins of $x_L$ and the current fragmentation contribution. b) Evolution of the fracture function of $^+$ in protons for $x_L = 0.50$ and $Q_0^2 = 1 \text{ GeV}^2$, c) $x_L = 0.00$, and d) $Q_0^2 = 4 \text{ GeV}^2$

Of course, the model is not expected to work over the whole kinematical region and, in fact, any deviation from the scale dependence implied in eq. (8) (note that the $\Delta x$ is assumed to be $Q^2$ independent) would show the breakdown of the approximated factorization hypothesis. However, the ansatz in eq. (8) can be taken as an effective relation, valid at some initial value of $Q_0^2$, for which the estimated $\Delta x$ is adequate, and therefore provides a sensible input distribution. As usual, the correct scale dependence is that given by the evolution equations for fracture functions, and that is the aim of our next step.

In order to study the effect of the inhomogeneity in the evolution we take different values of $x_L$, and keep them fixed while we analyse the $x$ and $Q^2$ dependence of fracture functions induced by both the homogenous and the complete evolution equations. In g. 2b we show the result of an evolution from $Q_0^2 = 1 \text{ GeV}^2$ at $x_L = 0.50$. Both solutions, the homogeneous (dotted line) and the complete (solid line) are superimposed, the difference being less
than 0.1%. These behaviours are perfectly compatible with the results obtained by the ZEUS Collaboration in the case of neutron production (where the inhomogeneity contributes about 10 times less) in the same kinematical region, where no difference has been found in the evolution between $F_2$ and $M_2$.

However, for smaller $x_L$, the situation is completely different. As the fragmentation function increases with lower values of the argument, the inhomogeneous contribution becomes more relevant and its effect in the evolution is sizeable. In fact, g. 2c shows the evolution result for $x_L = 0.10$ and $Q^2_0 = 1$ GeV$^2$, where the full evolution results outside the homogeneous one by a factor of 4 at small $x$. These corrections are smaller if the ansatz of eq. (8) is assumed to be valid at values of $Q^2_0 = 4$ GeV$^2$ (g. 2d) but still remain considerable.

Concluding, in this paper we have analysed recent experimental data on the production of forward neutrons in DIS in terms of fragmentation functions, noting that the main features of the data can be fairly reproduced by this perturbative QCD approach, once a non-perturbative model estimate for the input fragmentation functions is given. Studying the evolution properties of these fragmentation functions in the specific case of forward pions in the final state, we have found that the effects of the inhomogeneous term in the evolution equations are large and measurable, particularly in the kinematical region of very small $x$ and all $x_L$. These effects are negligible for large values of $x_L$, justifying the use of the usual homogeneous Albrecht-Parisi equations for, as an example, the $t$-integrated differential structure function $F_{2p}^{(3)}(x_s, \tau Q^2)$, which is just the fragmentation function of protons in protons $F_{2}^{\text{pp}}(x_s, \tau Q^2)$.

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