Study on $\Upsilon(nS) \to B_c M$ decays

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Abstract

With anticipation of abundant Upsilons data sample at high-luminosity heavy-flavor experiments in the future, we studied nonleptonic two-body weak decays of $\Upsilon(nS)$ below the open-bottom threshold with $n = 1, 2$ and 3. It is found that branching ratios for $\Upsilon(1S, 2S, 3S) \to B_c \rho$ decays are relatively large among Upsilons decay into $B_c M$ final states ($M = \pi, K$ and $K^*$) and can reach up to $10^{-10}$, which is promisingly detected by experiments at the running LHC and forthcoming SuperKEKB.

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I. INTRODUCTION

About forty years after the discovery of Upsilon (the bound states of $b\bar{b}$ with quantum number of $I^GJ^{PC} = 0^{-1-}$) at Fermilab in 1977, the properties of bottomonium system continue to be the subject of intensive theoretical and experimental study. Major contributions were made recently by experiments at the asymmetric electron-positron colliders KEK-B with Belle detector and PEP-II with BaBar detector, and the hadron colliders Tevatron and LHC.

Some of the salient features of Upsilon are as follows: (1) In the center-of-mass frame of Upsilon, the relative motion of the bottom quark is sufficiently slow. Nonrelativistic Schrödinger equation can be used to describe well the spectrum of bottomonium system and thus one can learn about the interquark binding forces. (2) The $\Upsilon(nS)$ particles below the open-bottom threshold, with the radial quantum number $n = 1, 2$ and $3$, decay primarily via the annihilation of the $b\bar{b}$ quark pairs into three gluons, which also provide an entry to many potential final states including glueballs, hybrid and multiquark states. Thus the properties of the invisible gluons and of the gluon-quark coupling can be gleaned through the study of hadronic Upsilon decay. (3) Compared with the light $u, d, s$ quarks, the relatively large mass of the $b$ quark implies a nonnegligible coupling to the Higgs bosons, making Upsilon to be one of the best hunting grounds for light Higgs particles. By now, our knowledge of the properties of Upsilon comes mostly from $e^+e^-$ collision.

As is well known, Upsilon decay mainly through the strong and electromagnetic interactions. The coupling constant $\alpha_s$ for hadronic Upsilon decay is smaller than that for charmonium decay due to the Quantum Chromodynamics (QCD) asymptotic freedom. In addition, the coupling between Upsilon and photon is proportional to the electric charges of the bottom quark. So, one of the outstanding properties of Upsilon below $B\bar{B}$ threshold is their narrow decay width of tens of keV (see Table I). Besides, as an essential complement to Upsilon decay modes, the Upsilon weak decay is allowable within the standard model and might be accessible at experiments, although the branching ratio is tiny, about $2/\tau_B\Gamma_\Upsilon \sim 10^{-8}$ (see Table I). In this paper, we will estimate the branching ratios for nonleptonic two-body $\Upsilon(nS) \rightarrow B_cM$ weak decays, where $M = \pi, \rho, K$ and $K^*$. The motivation is listed as follows.

From the experimental point of view, (1) there is plenty of Upsilon at the high-luminosity
TABLE I: Summary of the mass, decay width and data samples of Upsilon below $B\bar{B}$ threshold collected by Belle, BaBar and CLEO Collaborations.

| meson | properties | data samples ($10^6$) |
|-------|------------|-----------------------|
|       | mass (MeV) | width (keV) | Belle [5] | BaBar [6] | CLEO [7] |
| $\Upsilon(1S)$ | 9460.30±0.26 | 54.02±1.25 | 102 | ... | 22.78 |
| $\Upsilon(2S)$ | 10023.26±0.31 | 31.98±2.63 | 158 | 121.8 | 9.45 |
| $\Upsilon(3S)$ | 10355.2±0.5 | 20.32±1.85 | 11 | 98.6 | 8.89 |

dedicated bottomonium factories. Over $10^8$ Upsilon data samples have been collected at Belle and BaBar experiments (see Table I). Upsilon are also observed by the on-duty ALICE [8], ATLAS [9], CMS [10], LHCb [11] experiments at LHC. It is hopefully expected that more than $10^{11}$ $b\bar{b}$ quark pairs would be available per $fb^{-1}$ data at LHCb [12] and huge Upsilon data samples could be accumulated with great precision at the forthcoming SuperKEKB [13]. A large amount of data samples will provide opportunities to search for Upsilon weak decays which in some cases might be detectable. Hence, theoretical studies on Upsilon weak decays are very necessary to offer a ready reference. (2) For nonleptonic two-body $\Upsilon(nS) \rightarrow B_c M$ weak decay, the final states with opposite charges have definite energy and momentum in the $\Upsilon(nS)$ rest frame. In addition, identification of a single charged $B_c$ meson would provide an unambiguous signature of Upsilon weak decay, which is free from double tagging of the $b$-flavored hadron pairs. The small branching ratios make the observation of Upsilon weak decays very difficult, and evidences of an abnormally production rate of a single $B_c$ meson in Upsilon decay might be a hint of new physics beyond the standard model.

From the theoretical point of view, nonleptonic Upsilon weak decay could allow one to overconstrain parameters obtained from $B$ meson decay, test various models and improve our understanding on the strong interactions and the mechanism responsible for heavy meson weak decay. Phenomenologically, the $\Upsilon(nS) \rightarrow B_c M$ weak decays are monopolized by tree contribution and favored by the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix element $V_{cb}$, so they should have relatively large branching ratios. The amplitudes for the $\Upsilon(nS) \rightarrow B_c M$ decay are commonly written as factorizable product of two factors: one describing the transition between Upsilon and $B_c$ meson, and the other depicting the
production of the $M$ state from the vacuum. The earlier works, including Refs. [14, 15] based on a heavy quark effective theory and Ref. [16] based on the Bauer-Stech-Wirbel (BSW) model [17], riveted mainly upon the $\Upsilon(1S) \rightarrow B_c$ transition form factors. No research works devoted to nonleptonic $\Upsilon(2S), \Upsilon(3S)$ weak decays. In recent years, several attractive QCD-inspired methods have been developed to treat with the hadronic matrix elements of heavy flavor weak decay. In this paper, we will estimate branching ratios for nonleptonic two-body $\Upsilon(nS) \rightarrow B_c M$ weak decay, by considering nonfactorizable contributions to hadronic matrix elements with the QCD factorization (QCDF) approach [18], and calculating the transition form factor between Upsilon and $B_c$ meson with nonrelativistic wave functions.

This paper is organized as follows. In section II, we will present the theoretical framework and the amplitudes for $\Upsilon(nS) \rightarrow B_c M$ decays. Section III is devoted to numerical results and discussion. The last section is our summary.

II. THEORETICAL FRAMEWORK

A. The effective Hamiltonian

Using the operator product expansion technique, the effective Hamiltonian responsible for $\Upsilon(nS) \rightarrow B_c M$ decays is [19]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=d,s} V_{cb} V_{uq}^* \left\{ C_1(\mu) Q_1(\mu) + C_2(\mu) Q_2(\mu) \right\} + \text{h.c.},$$

where the Fermi coupling constant $G_F = 1.166 \times 10^{-5} \text{GeV}^{-2}$ [1]; The CKM factors can be expanded as a power series in the Wolfenstein parameter $\lambda = 0.22537(61)$ [1],

$$V_{cb} V_{ud}^* = A^2 \lambda^2 - \frac{1}{2} A \lambda^4 - \frac{1}{8} A \lambda^6 + \mathcal{O}(\lambda^8),$$

$$V_{cb} V_{us}^* = A \lambda^3 + \mathcal{O}(\lambda^8).$$

The Wilson coefficients $C_{1,2}(\mu)$ summarize the physical contributions above scales of $\mu$, which are calculable with the perturbation theory and have properly been evaluated to the next-to-leading order (NLO). Their values at scale of $\mu \sim \mathcal{O}(m_b)$ can be evaluated with the renormalization group (RG) equation [19]

$$C_{1,2}(\mu) = U(\mu, m_W) C_{1,2}(m_W),$$

4
where the RG evolution matrix $U(\mu, m_W)$ transforms the Wilson coefficients from scale of $m_W$ to $\mu$. The expression of $U(\mu, m_W)$ can be found in Ref. [19]. With the naive dimensional regularization (NDR) scheme, the numerical values of Wilson coefficients $C_{1,2}$ are listed in Table II.

The local tree four-quark operators are defined as follows.

$$\begin{align*}
Q_1 &= [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta] [\bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\alpha], \\
Q_2 &= [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta] [\bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\alpha],
\end{align*}$$

where $\alpha$ and $\beta$ are color indices and the sum over repeated indices is understood.

To obtain the decay amplitudes, the remaining and also the most intricate part is how to calculate accurately hadronic matrix elements squeezing the local operators between initial Upsilon and final $B_c M$ states.

B. Hadronic matrix elements

Analogous to the usual applications of hard exclusive processes in perturbative QCD proposed by Lepage and Brodsky [20], the QCDF approach is based on the collinear factorization approximation and power counting rules in the heavy quark limit, where hadronic matrix elements are written as the convolution integrals of hard scattering subamplitudes and universal wave functions [18]. The QCDF approach has been widely applied to $B$ meson weak decays. As for the $\Upsilon(nS) \to B_c M$ decay, using the QCDF master formula, hadronic matrix elements can be written as:

$$\langle B_c M | Q_1 | \Upsilon \rangle = \sum_j F_{\Upsilon \to B_c}^j \int dx H_j(x) \Phi_M(x),$$

where both transition form factor $F_{\Upsilon \to B_c}^j$ and wave function $\Phi_M(x)$ are universal and non-perturbative input parameters. For a light pseudoscalar and vector meson, the leading twist distribution amplitude can be expressed in terms of Gegenbauer polynomials [21]:

$$\phi_M(x) = 6x\bar{x} \sum_{n=0}^{\infty} a_n^M C_n^3/2(\bar{x} - x),$$

where $\bar{x} = 1 - x$; $a_n^M$ is the Gegenbauer moment and $a_0^M \equiv 1$.

Hard scattering function, $H_j(x)$, is assumed to be calculable order by order from the first principle of perturbative QCD theory. At order of $\alpha_s^0$, $H_j(x) = 1$ and the integral for wave
function in Eq.(7) results in decay constant $f_M$, which is the simplest scenario. At order of $\alpha_s$ and higher orders, expression of $H_j(x)$ is no longer trivial, part of strong phases and renormalization scale dependence of amplitude can be recuperated from hadronic matrix elements. The decay amplitudes could be written as

$$A(\Upsilon\rightarrow B_c M) = \langle B_c M | H_{\text{eff}} | \Upsilon \rangle = \frac{G_F}{\sqrt{2}} V_{cb} V_{ub}^* a_1 \langle M | J^\mu | 0 \rangle \langle B_c | J_\mu | \Upsilon \rangle.$$

The coefficient $a_1$ in Eq.(9), containing nonfactorizable contributions to hadronic matrix elements, is written as:

$$a_1 = C_1^{\text{NLO}} + \frac{1}{N_c} C_2^{\text{NLO}} + \frac{\alpha_s}{4\pi} C_F^{\text{LO}} C_2^{\text{LO}} V.$$

The explicit expression of parameter $V$ is the same as that in Ref. [22]. It has been shown that coefficient $a_1$ is infrared-safe and renormalization scale independent at order of $\alpha_s$ [22]. The numerical values of coefficient $a_1$ at scales of $\mu \sim O(m_b)$ are listed in Table II. From the numbers in Table II it is seen that one could get part information of strong phase by taking nonfactorizable corrections into account, though the strong phase is small and suppressed by factor $\alpha_s/N_c$.

C. Decay constants and form factors

The matrix elements of current operators are defined as follows:

$$\langle P(p) | A_\mu | 0 \rangle = -if_P p_\mu,$$

$$\langle V(p, \epsilon) | V_\mu | 0 \rangle = f_V m_V \epsilon^*_V.$$
where $f_P$ and $f_V$ are the decay constants of pseudoscalar and vector mesons, respectively; $m_V$ and $\epsilon_V$ denote the mass and polarization of vector meson, respectively.

The transition form factors are defined as follows [15–17]:

$$
\langle B_c(p_2)|V_\mu - A_\mu|\Upsilon(p_1, \epsilon)\rangle = -\epsilon_{\mu\nu\alpha\beta} \epsilon_\nu^\Upsilon q^\alpha (p_1 + p_2)^\beta \frac{V^{\Upsilon \rightarrow B_c}(q^2)}{m_\Upsilon + m_{B_c}} - i \frac{2 m_\Upsilon \epsilon_\tau^\Upsilon q}{q^2} q_\mu A_0^{\Upsilon \rightarrow B_c}(q^2) - i \epsilon_\Upsilon \epsilon_\mu (m_\Upsilon + m_{B_c}) A_1^{\Upsilon \rightarrow B_c}(q^2) - i \frac{\epsilon_\Upsilon \epsilon_\tau^\Upsilon q}{m_\Upsilon + m_{B_c}} (p_1 + p_2)_\mu A_2^{\Upsilon \rightarrow B_c}(q^2) + i \frac{2 m_\Upsilon \epsilon_\tau^\Upsilon q}{q^2} q_\mu A_3^{\Upsilon \rightarrow B_c}(q^2),
$$

where $q = p_1 - p_2$; and $A_0(0) = A_3(0)$ is required compulsorily to cancel singularities at the pole $q^2 = 0$. There is a relation among these form factors

$$
2m_\Upsilon A_3(q^2) = (m_\Upsilon + m_{B_c})A_1(q^2) + (m_\Upsilon - m_{B_c})A_2(q^2).
$$

The form factors, $A_{0,1}(0)$ and $V(0)$ at the pole $q^2 = 0$ are defined as [17],

$$
A_0^{\Upsilon \rightarrow B_c}(0) = \int d\vec{k}_\perp \int_0^1 dx \left\{ \Phi_\Upsilon(\vec{k}_\perp, x, 1, 0) \sigma_z \Phi_{B_c}(\vec{k}_\perp, x, 0, 0) \right\},
$$

$$
A_1^{\Upsilon \rightarrow B_c}(0) = \frac{m_b + m_c}{m_\Upsilon + m_{B_c}} I^{\Upsilon \rightarrow B_c},
$$

$$
V^{\Upsilon \rightarrow B_c}(0) = \frac{m_b - m_c}{m_\Upsilon - m_{B_c}} I^{\Upsilon \rightarrow B_c},
$$

$$
I^{\Upsilon \rightarrow B_c} = \sqrt{2} \int d\vec{k}_\perp \int_0^1 dx \left\{ \Phi_\Upsilon(\vec{k}_\perp, x, 1, -1) i\sigma_y \Phi_{B_c}(\vec{k}_\perp, x, 0, 0) \right\},
$$

where $\sigma_{y,z}$ is a Pauli matrix acting on the spin indices of the decaying bottom quark; $x$ and $\vec{k}_\perp$ denote the fraction of the longitudinal momentum and the transverse momentum carried by the nonspectator quark, respectively.

For Upsilons, the bottom quark is nonrelativistic with an average velocity $v \ll 1$ based on arguments of nonrelativistic quantum chromodynamics (NRQCD) [23]. For the double-heavy $B_c$ meson, both bottom and charm quarks are nonrelativistic due to $m_{B_c} \approx m_b + m_c$. Here, we will take the solution of the Schödinger equation with an isotropic harmonic oscillator potential as wave functions of Upsilons and $B_c$ states, i.e.,

$$
\phi_{1S}(\vec{k}) \sim e^{-\vec{k}^2/2\alpha^2},
$$

$$
\phi_{2S}(\vec{k}) \sim e^{-\vec{k}^2/2\alpha^2}(2\vec{k}^2 - 3\alpha^2),
$$

where $\alpha$ is the oscillator length.
\[ \phi_{3S}(\vec{k}) \sim e^{-\vec{k}^2/2\alpha^2}(4\vec{k}^4 - 20\vec{k}^2\alpha^2 + 15\alpha^4), \]  

(21)

where the parameter \( \alpha \) determines the average transverse quark momentum, \( \langle \phi_{1S}|\vec{k}_\perp^2|\phi_{1S} \rangle = \alpha^2 \). With the NRQCD power counting rules \[23\], \( |\vec{k}_\perp| \sim mv \sim m\alpha_s \) for heavy quarkonium. Hence, parameter \( \alpha \) is approximately taken as \( m\alpha_s \) in our calculation. Using the substitution ansatz \[24\],

\[ \vec{k}^2 \to \frac{\vec{k}_{\perp}^2 + \bar{x} m_q^2 + x m_b^2}{4 x \bar{x}}, \]  

(22)

one can obtain

\[ \phi_{1S}(\vec{k}_{\perp}, x) = A \exp\left\{ \frac{\vec{k}_{\perp}^2 + \bar{x} m_q^2 + x m_b^2}{-8 \alpha^2 x \bar{x}} \right\}, \]  

(23)

\[ \phi_{2S}(\vec{k}_{\perp}, x) = B \phi_{1S}(\vec{k}_{\perp}, x) \left\{ \frac{\vec{k}_{\perp}^2 + m_b^2}{6 \alpha^2 x \bar{x}} - 1 \right\}, \]  

(24)

\[ \phi_{3S}(\vec{k}_{\perp}, x) = C \phi_{1S}(\vec{k}_{\perp}, x) \left\{ \frac{2}{5} \left( \frac{\vec{k}_{\perp}^2 + m_b^2}{4 \alpha^2 x \bar{x}} - \frac{5}{2} \right)^2 - 1 \right\}, \]  

(25)

where parameters \( A, B \) and \( C \) are normalization factors.

**TABLE III:** Numerical values of transition form factors at \( q^2 = 0 \), where uncertainties of this work are from the masses of bottom and charm quarks, and numbers in Ref. \[16\] are computed with the flavor dependent parameter \( \omega \) the BSW model.

| transition | reference | \( A_0(0) \)  | \( A_1(0) \)  | \( A_2(0) \)  | \( V(0) \)  |
|-----------|-----------|---------------|---------------|---------------|-------------|
| \( \Upsilon(1S) \to B_c \) | [16]           | 0.46          | 0.62          | 0.38          | 1.61        |
| this work |           | 0.67±0.02     | 0.70±0.02     | 0.51±0.06     | 1.66±0.02   |
| \( \Upsilon(2S) \to B_c \) | this work | 0.65±0.02     | 0.69±0.02     | 0.48±0.04     | 1.44±0.03   |
| \( \Upsilon(3S) \to B_c \) | this work | 0.57±0.01     | 0.64±0.01     | 0.29±0.03     | 1.25±0.05   |

The numerical values of transition form factors at \( q^2 = 0 \) are collected in Table [III]. It is found that (1) form factors for the \( \Upsilon(1S) \to B_c \) transition are generally larger than those in Ref. [16]. (2) The value of form factor at \( q^2 = 0 \) decreases gradually with the increase of the radial quantum number of Upsilon.
D. Decay amplitudes

With the above definition of hadronic matrix elements, the decay amplitudes for $\Upsilon(nS) \to B_c M$ decays can be written as

$$
A(\Upsilon \to B_c^+ \pi^-) = \sqrt{2} G_F V_{cb} V_{ud}^* a_1 f_\pi m_\Upsilon (\epsilon_\Upsilon \cdot p_\pi) A_{0}^{\Upsilon \to B_c},
$$

$$
A(\Upsilon \to B_c^+ K^-) = \sqrt{2} G_F V_{cb} V_{us}^* a_1 f_K m_\Upsilon (\epsilon_\Upsilon \cdot p_K) A_{0}^{\Upsilon \to B_c},
$$

$$
A(\Upsilon \to B_c^+ \rho^-) = -i \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_1 f_\rho m_\rho \left\{ (\epsilon_\Upsilon \cdot e_\rho^*) (m_\Upsilon + m_{B_c}) A_{1}^{\Upsilon \to B_c} + (\epsilon_\Upsilon \cdot p_\rho) (\epsilon_\rho^* \cdot p_\Upsilon) \frac{A_{2}^{\Upsilon \to B_c}}{m_\Upsilon + m_{B_c}} - i \epsilon_{\mu \nu \alpha \beta} e_\Upsilon^\mu e_\rho^\nu p_\Upsilon^\alpha p_\rho^\beta 2 V_{\Upsilon \to B_c} \right\},
$$

$$
A(\Upsilon \to B_c^+ K^{*-}) = -i \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* a_1 f_K m_K \left\{ (\epsilon_\Upsilon \cdot e_K^*) (m_\Upsilon + m_{B_c}) A_{1}^{\Upsilon \to B_c} + (\epsilon_\Upsilon \cdot p_K) (\epsilon_K^{*} \cdot p_\Upsilon) \frac{A_{2}^{\Upsilon \to B_c}}{m_\Upsilon + m_{B_c}} - i \epsilon_{\mu \nu \alpha \beta} e_\Upsilon^\mu e_K^{*\nu} p_\Upsilon^\alpha p_K^{*\beta} 2 V_{\Upsilon \to B_c} \right\}.
$$

III. NUMERICAL RESULTS AND DISCUSSION

In the center-of-mass frame of Upsilon, branching ratio for nonleptonic $\Upsilon(nS) \to B_c M$ weak decays can be written as

$$
Br(\Upsilon \to B_c M) = \frac{1}{12\pi} \frac{p_{\text{cm}}}{m_\Upsilon^2 \Gamma_\Upsilon} |A(\Upsilon \to B_c M)|^2,
$$

where the momentum of final states is

$$
p_{\text{cm}} = \frac{\sqrt{|m_\Upsilon^2 - (m_{B_c} + m_M)^2||m_\Upsilon^2 - (m_{B_c} - m_M)^2|}}{2 m_\Upsilon}.
$$

The input parameters, including the CKM Wolfenstein parameters, masses of $b$ and $c$ quarks, hadronic parameters including decay constant and Gegenbauer moment of distribution amplitudes in Eq. (8), are collected in Table IV. If not specified explicitly, we will take their central values as the default inputs. Our numerical results on branching ratios for $\Upsilon(nS) \to B_c M$ decays are displayed in Table V where theoretical uncertainties come from the CKM parameters, the renormalization scale $\mu = (1 \pm 0.5)m_b$, hadronic parameters, respectively. For the sake of comparison, previous results of Refs. [15, 16] are re-evaluated with $a_1 = 1.057$. The following are some comments.

(1) For the same final states, branching ratio of nonleptonic Upsilon weak decay increase with the radial quantum number of Upsilon below $B\bar{B}$ threshold, because of two facts, one
TABLE IV: Numerical values of input parameters.

| Wolfenstein parameters |
|------------------------|
| $\lambda = 0.22537\pm0.00061$ [1] |
| $A = 0.814^{+0.023}_{-0.024}$ [1] |

| masses of charm and bottom quarks |
|-------------------------------|
| $m_c = 1.67\pm0.07$ GeV [1] |
| $m_b = 4.78\pm0.06$ GeV [1] |

| decay constants |
|----------------|
| $f_\pi = 130.41\pm0.20$ MeV [1] |
| $f_\rho = 216\pm3$ MeV [21] |
| $f_K = 156.2\pm0.7$ MeV [1] |
| $f_{K^*} = 220\pm5$ MeV [21] |

| Gegenbauer moments at scale $\mu = 1$ GeV |
|------------------------------------------|
| $a_1^0 = 0$ [21] |
| $a_1^{1K^*} = 0.03\pm0.02$ [21] |
| $a_1^\pi = 0$ [21] |
| $a_1^K = 0.06\pm0.03$ [21] |
| $a_2^0 = 0.15\pm0.07$ [21] |
| $a_2^{1K^*} = 0.11\pm0.09$ [21] |
| $a_2^\pi = 0.25\pm0.15$ [21] |
| $a_2^K = 0.25\pm0.15$ [21] |

TABLE V: Branching ratios for $\Upsilon(nS) \to B_c M$ decays, where uncertainties of this work are from the CKM factors, scale $\mu = (1+0.5)m_b$, hadronic parameters, respectively; numbers of Ref. [15, 16] are evaluated with $a_1 = 1.057$.

| final states | $\Upsilon(1S)$ decay | $\Upsilon(2S)$ decay | $\Upsilon(3S)$ decay |
|-------------|----------------------|----------------------|----------------------|
| $B_r(\Upsilon \to B_c \rho)\times 10^{10}$ | $0.93\pm0.07\pm0.05\pm0.03$ | $3.48\pm0.24\pm0.15\pm0.01$ | $5.27\pm0.40\pm0.23\pm0.24$ |
| $B_r(\Upsilon \to B_c \pi)\times 10^{11}$ | $1.04\pm0.07\pm0.05\pm0.03$ | $3.39\pm0.23\pm0.07\pm0.01$ | $5.26\pm0.38\pm0.11\pm0.24$ |
| $B_r(\Upsilon \to B_c K^*)\times 10^{12}$ | $2.47\pm0.18\pm0.11\pm0.07$ | $8.27\pm0.59\pm0.37\pm0.03$ | $12.28\pm0.94\pm0.54\pm0.57$ |
| $B_r(\Upsilon \to B_c K)\times 10^{12}$ | $3.71\pm0.26\pm0.17\pm0.10$ | $12.40\pm0.88\pm0.56\pm0.04$ | $19.09\pm1.47\pm0.84\pm0.89$ |

is that mass of Upsilon increases with the radial quantum number, which results in the final phase space increases with the radial quantum number; the other is that decay width of Upsilon decreases with the increase of the radial quantum number of Upsilon. Hence, branching ratio for $\Upsilon(3S)$ decay is the largest one among nonleptonic $\Upsilon(1S, 2S, 3S)$ weak decays into the same final $B_c M$ states.

(2) There is a clear hierarchical relation for the same decaying Upsilon, $B_r(\Upsilon \to B_c \rho) >$
$\mathcal{B}(\Upsilon \rightarrow B_c \pi) > \mathcal{B}(\Upsilon \rightarrow B_c K^*) > \mathcal{B}(\Upsilon \rightarrow B_c K)$. These are two dynamical reasons. One is that the CKM factor $|V_{cb}V_{ud}^*|$ responsible for $\Upsilon \rightarrow B_c \pi$, $B_c \rho$ decays is larger than the CKM factor $|V_{cb}V_{us}^*|$ responsible for $\Upsilon \rightarrow B_c K^{(*)}$ decays. The other is that Upsilons decay into two pseudoscalar mesons is suppressed by the orbital angular momentum with respect to Upsilons decay into $B_c V$ states with the same flavor structures.

(3) The $\Upsilon(1S, 2S, 3S) \rightarrow B_c \rho$ decays have large branching ratio, $\sim 10^{-10}$, which should be sought for with high priority and firstly observed at the running LHC and forthcoming SuperKEKB.

(4) There are many uncertainties. The first uncertainty from the CKM factors could be lessened with the improvement on the precision of the Wolfenstein parameter $A$ in the future. The second uncertainty from the renormalization scale should, in principle, be reduced by inclusion of higher order $\alpha_s$ corrections to hadronic matrix elements. The third uncertainty from hadronic parameters might be reduced with the relative ratio of branching ratios. For example, ignoring the kinematic effects, the relation

$$\frac{\mathcal{B}(\Upsilon(mS) \rightarrow B_c \pi)}{\mathcal{B}(\Upsilon(3S) \rightarrow B_c \pi)} \approx \frac{\mathcal{B}(\Upsilon(mS) \rightarrow B_c K)}{\mathcal{B}(\Upsilon(3S) \rightarrow B_c K)} \approx \left( \frac{A_0^{\Upsilon(mS) \rightarrow B_c}}{A_0^{\Upsilon(3S) \rightarrow B_c}} \right)^2,$$

(32)

can be used to check various phenomenological models and improve our understanding on the interquark binding forces for heavy quarkonium.

IV. SUMMARY

With anticipation of abundant Upsilons data sample at high-luminosity dedicated heavy-flavor factories, we studied the nonleptonic two-body bottom-changing $\Upsilon(nS) \rightarrow B_c M$ weak decays. Considering QCD radiative corrections to hadronic matrix elements with the QCDF approach, and using nonrelativistic wave functions to evaluate the $\Upsilon(nS) \rightarrow B_c$ transition form factors, we estimated the branching ratios for $\Upsilon(nS) \rightarrow B_c M$ weak decays. It is found that branching ratios for $\Upsilon(1S, 2S, 3S) \rightarrow B_c \rho$ decays is large, $\sim 10^{-10}$, which might be detectable at the running LHC and forthcoming SuperKEKB.

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