Arbitrary mechanical system description by a symbolic line

O Dmitrochenko¹, A Mikkola², A Olshevskiy¹,³

¹ Bryansk State Technical University, 7, 50 years of October st., Bryansk, 241035, Russia
² Lappeenranta University of Technology, 34, Skinnarilankatu, Lappeenranta, 53850, Finland
³ Konkuk University, 1, Hwanyang-dong, Gwangjin-gu, 143-701, Seoul, Korea

E-mail: dmitroleg@rambler.ru

Abstract. A single-line symbolic notation is proposed for description of an arbitrary multibody system. The kinematics is represented by a sequence of elementary transformations, each of those being marked by a reserved alphabetic character. Force and constraint links between the bodies are also defined by reserved characters. The parameters of the system, such as identifiers of degrees of freedom, inertia parameters and others, are assigned default names if not specified. However, user-defined names, parameters and functions can be placed instead if needed. The proposed description in its shortest form is suitable for academic purpose to identify only the essential properties of a multibody system. In an extended form, by explicit mentioning names of variables and parameters and other data like initial conditions, this description can serve as input data for a multibody analysis software. Lots of examples from the academic area and technical applications are given to show the applicability of the description.

Notation

- $g$ applies a uniform gravity force to all bodies (downwards the vertical axis by default)
- $i, j, k$ define a constant or known-as-function-of-time translation along local axes $x$, $y$, or $z$
- $l, m, n$ define a constant or known-as-function-of-time rotation around local axes $x$, $y$, or $z$
- $o$ obtains the orientation from body 0; obtains the orientation from body index 3: $o3$
- $q$ is a quaternion; adds four Euler parameters $q_0, q_1, q_2, q_3$ as DOFs and a constraint
- $u, v, w$ introduce a translational degree of freedom along local axes $x$, $y$, or $z$, accordingly
- $x, y, z$ introduce a rotational degree of freedom around local axes $x$, $y$, or $z$, accordingly
- $\theta, \phi, \psi$ names for a coordinate and its velocity in a functional definition of a force
- $>$ starts description of the next body relative to the previous one in the kinematical chain
- \ starts kinematic description of the next body after body 0; use $\\$ when starting from body 5
- $<$ starts right-to-left description; sometimes it is convenient to make the description shorter
- $[]$ defines the radius vector of the center of gravity by default; arbitrary translation $\{a, b, c\}$
- $\{}$ defines the initial value for a d.o.f. $\{v_0\}$; for body $\{v_{i0}, v_{i1}, v_{i2}\}$
- $\cdot$ velocity in expressions; initial velocity for d.o.f. $\{v_{i0}, v_{i1}, v_{i2}\}$; for body $\{v_{i0}, v_{i1}, v_{i2}\}$
- $\%$ an applied force: for d.o.f. $\{-c_{11}@d_{11}\}$; for body $\{f_x, f_y, f_z, t_x, t_y, t_z\}$, with torque
- $\div$ a spinning/spherical joint: $\#body_1 \div \{a_3, b_3, c_3\} \div \#body_2$; with joint points $\{\ldots\}$
- $\div$ a rolling without slipping: $\#body_1 \div \{surface_1\}$, $\div \{surface_2\} \div \#body_2$
- $\div$ the mass of the current body
- $\%$ a massless link; actuator $:\{L(t)\}$; a link with given length and mass $:\{L, M\}$ or $:\$ by default
- $\div$ a bipolar force; spring-damper $:\{-c_{11}@d_{11}\}$; also, a penalty force in contact

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

Published under licence by IOP Publishing Ltd
1. Introduction
Denavit–Hartenberg (D–H) method applied in robotics has been developed in [1]. The method allows representing the kinematical structure of a multibody system in a way, which is appropriate for automated generation of kinematical and dynamical equations of the system. The method is based on description of kinematics of a successive body relative to the previous one in the same kinematical chain. In a classic form, the kinematics is represented in the form of a diagram. The diagram representation is not suitable for the use as input data. Better appearance is achieved if Denavit–Hartenberg parameters are arranged in a table. The table already can serve as input data for a multibody software. However, this method is hard to apply for systems with closed kinematical loops; moreover, the table usually contains lots of zero values and rotations by ±90°, i.e. this description is redundant in practice. There have been successful attempts to extend the D–H formulation for mechanisms containing closed loops, for example in [2]. However, this leads to increasing the number of parameters used per kinematic link, and, consequently, to growing the size of the tables describing the mechanism. According to the current research, a short and pictorial text description of the mechanisms can be generated; the description represents kinematics and dynamics of the mechanisms completely and is considerably shorter than any of earlier notations available in literature [3, 4].

2. Symbolic description of single-body systems
Translational motion of a material point is the simplest example of a mechanical system. The point moving along axis \( x \) can be described by single character \( u \). A point moving along axis \( y \) corresponds to \( v \) in the same figure, and, finally, \( w \) if along axis \( z \). Note that this single-character notation shows the direction only and leaves other parameters unidentified. If one wants to give all data for this degree of freedom, the description can be extended as follows:

\[
u(x_0) = (x_0) \cdot (v_0) \cdot (m_p) \cdot (\text{sign} ) \cdot (c_i u + c_i u' + d_i u' - d_i \text{sign} u + a \sin ft)
\] (1)

Here, \( u \) introduces a degree of freedom along axis \( x \), brackets ( ) rename it to \( x_0 \), symbols \( ( ) \) define initial value \( x_0 \), prime \( ' \) defines initial velocity \( v_0 \), currency sign \( $ \) gives mass \( m_p \) of the body, double prime \( '' \) imposes the applied force along the degree of freedom. The force is elastic-dissipative with parameters \( c_i \) and \( d_i \); it also has dry-friction part \( -d_i \text{sign} u' \) and a time-dependent harmonic part with amplitude \( a \) and frequency \( f \).

Two-dimensional point mass moving in plane can be represented by short code \( uv \) in Figure 1. Subscript \( g \) indicates that gravity force is to be added by default, see below. Free point moving in 3D can be described by \( uvw_g \) (Figure 1, right).

**Figure 1.** A free point in two and three dimensions.

Figure 1, center, represents a simple 2D ballistic problem taking into account viscous air drag, which is linear in velocities. Its description in terms of the proposed code looks like:

\[
u''(cu') = 0 (v_1 \cos \alpha_0) v''((-cv')_0). v_1(0)=0 \quad \text{and} \quad v_1(0)=0 \Rightarrow u_i(0)=0 \quad \text{and} \quad v_i(0)=0 \quad \text{and} \quad \alpha_0 \text{in} \quad \text{for} \quad \text{for} \quad \text{for} \quad \text{for} \quad \text{for}\n\]

(2)
In equation (2), description $uv'(-cu')v'(-cv')g$ contains forces along degrees of freedom separately. Alternatively, the point force can be given at once by all its components as $uv'(-cu',-cv')g$, or, using index notation for degrees of freedom, as $uv'(-c\dot{\gamma}_1,-c\dot{\gamma}_2)g$. This can be further shortened to $uv'\gamma g$, where prime ' refers to the vector of velocity, or, by dropping coefficient $c$ by default, even to shortest notation $uv'g$ shown in Figure 1, center.

Rotational motion of a simple pendulum is a frequently used benchmark multibody system. In Figure 2, several typical variants are presented. After rotation around axis $z$ by angle $\zeta_1$, translation along axis $x_1$ by constant value $i_1$ results in what is usually referred to as an R-link in robotics; in Figure 2 this case is denoted by $z_i g$. Alternatively, the shift along positive direction of axis $y_1$ by constant distance $j_1$ corresponds to inverted pendulum $z_j g$. If the latter shift occurs along the negative direction of axis $y_1$, the conventional clock pendulum is meant, denoted by $z_-j g$ in the figure. Eventually, at least eight pendulum designations are possible including $z-i g$, $z_-i g$, $z_j g$, $z_-j g$, which are not shown in the picture.

Moments of inertia are not included in the equations of motion by default if the last elementary transformation is translational, as for the case shown in Figure 2. To ultimately define the moments of inertia, square brackets $[\ ]$ are used. The empty brackets introduce three principal moments of inertia and assign default names $I_{x_1}, I_{y_1}, I_{z_1}$ to them. Default names like $I_{z_1}$ can be overridden by any name, let us say $C$, as follows: $z[C]g \rightarrow \dot{\gamma}_{\zeta_1} = -m_i g \sin \zeta_1$

Figure 2. Various descriptions of a simple pendulum.

Figure 3. Rigid bodies in two and three dimensions.

Link is a common element of multibody systems. The same simple pendulum can be obtained if, as
shown in Figure 2, right, free planar point mass \( uv \) is constrained to the origin with the help of a massless link of a constant length, denoted by colon : so that the description: \( uv \) (or \( uv_1 \)) holds for the system. In this case, link: contributes constraint \( u_1^2 + v_1^2 - L_1^2 = 0 \) with default length \( L_1 \), and Lagrange multiplier \( A_1 \) into equations of motion, shown in Figure 2, right. The link with given length \( a \) has designation \( (a) \).

Kinematics of a rigid body in 2D is naturally described by combination \( uvz \), showing its three degrees of freedom, Figure 3, left.

The simplest form \( uvz \) results in situation when the body centre of mass \( C \) is placed in the origin of the rotating reference frame. This leads to decoupled translational and rotational motions. However, a shifted center of mass is common in applications. This aspect is designated by curly brackets \{ \}. The empty brackets introduce default values \( \rho_{x1} \), \( \rho_{y1} \), \( \rho_{z1} \) for the coordinates of the center of mass in the body reference frame,

Figure 3, middle. A 3D body using Cardan angles for orientation can be described by line \( uvwxyz \), not shown in the figure. Another case is Lagrange’s one, shown in Figure 3, right. As its designation \( \{u,v,t\}_A \) suggests, a specific symmetry of moments of inertia around the last rotation axis is required, and the center of mass is shifted along this axis.

3. Symbolic description of single-body systems

3.1. Open kinematical chains

The simplest multiple-body system is that consisting of several particles; say three, as in Figure 4, left. Its designation \( uvwxyz \) or just \( (uv)^3 \), contains degrees of freedom \( uv \) for each particle separated by backslash \ meaning that description of a new particle starts from the origin.

This system turns to a general Kepler’s three-body problem if so-called bipolar forces are accounted in between the bodies representing Newton’s gravitational forces, Figure 4, right.

![Figure 4. A three-point planar system, and a Kepler’s three-body problem.](image)

A bipolar force acting between bodies 1 and 2 has the following general description \( \{(f_{12}(\@,\@'))\}<2 \), where combination \( \@' \) represents the bipolar force, and scalar function \( f_{12} \) defines the force applied to body 1 depending on distance \( \@ \) between the bodies and its velocity \( \@' \). The same force with a negative sign is applied to body 2, as shown in Figure 4, right.

Applying gravitational bipolar forces between each pair of the above-mentioned particles, one can obtain

\[
uv \backslash uv \backslash uv: \{(uv)^3\} = (m_1, m_2, m_3) < 2 \text{ and } f_{12} < 3 \text{ or } f_{12} < 1
\]

\[
uv: (m_1, m_2, G/\@^2) < 2 \text{ and } (m_2, m_3, G/\@^2) < 3 \text{ or } (m_3, m_1, G/\@^2) < 1
\]

where the bipolar forces descriptions are inserted between the kinematical descriptions of the bodies. The forces are algebraic of degree –2, which can be shortened to \( \:@' \) as in Figure 4, right.

The next simple case is a multiple-pendulum system of two pendulums 1 and 2 that are not
interconnected kinematically, depicted in Figure 5, left.

In such case, kinematics of pendulum 2 is described starting from the fixed reference frame, referred to by index 0. The combination of characters \ or just single backslash \ is used for that.

![Figure 5](image)

Figure 5. Separate pendulums, relative and mixed connections of two pendulums.

Alternatively, kinematics of pendulum 2 can be described relative to the final coordinate frame of pendulum 1, as shown in the middle of Figure 5. This case of a consecutive connection of bodies is reflected by symbol > and now the two pendulums are kinematically interconnected and dynamically influence each other. The third, mixed, situation is possible when next pendulum 2 is attached to previous pendulum 1, but the orientation is described relative to other body, say body 0, as in Figure 5, right. This case is managed by using command o0 (value 0 can be omitted).

3.2. Closed-loop mechanisms

Closed-loop systems are usually studied starting with a well-known benchmark, the slider-crank mechanism in Figure 6. The closed loop can be treated in many ways. Starting from the slider body and continuing counter-clockwise along the mechanism’s contour, by returning to the origin, reference frame 0, designation \( zi > zi \) \( > zi > zi > zi > 0 \) is generated, Figure 6, left. Alternatively, going clockwise, description \( zi > zi > zi > zi > zi > 0 \) is postulated, Figure 6, middle. Combination >0 corresponds to the loop closure conditions, that, in the planar case, consist of three constraint equations.

A better performance is achieved in Figure 6, right, by cutting the long kinematical chain into two branches, simple pendulum \( zi \), and elliptic pendulum \( zi \), then, the loop can be closed by means of a pin joint denoted by a small dot in the figure. Indeed, dot. is the best suitable symbol to mnemonically denote this type of linkage. Thus, notation \( zi > zi > zi > u \) is suggested here. As one can see, former line \( zi > zi \) is reversed to be directed towards central dot combination >i,i< representing the pin joint itself and two additional shifts \( i_2 \) and \( i_4 \) from centers of gravity of each body.

![Figure 6](image)

Figure 6. A slider-crank loop mechanism: counter-clockwise, clockwise and mixed (cut by a pin joint).
3.3. Multiple closed loops

Multiple closed loops can also be described in a systematic way. A typical example of such system is a parallel manipulator shown in Figure 7, left. It contains a central body, index 1, which has three degrees of freedom according to description \( uvz \), while four kinematic loops are identical, and the description of one of those is given in brackets. Leading backslash \( \backslash \) starts kinematic description from fixed reference frame 0, constant shift \( ij \) continues with double pendulum \( zi > izi \); remaining part \( > i.ij < 1 \) defines the pin joint connecting the last pendulum to body 1, where \( i \) and \( ij \) are shifts of a joint point relative to the two bodies.

A three-dimensional parallel mechanism with multiple closed loops is also presented in the same Figure 7, middle, where the famous Gough–Stewart hexapod platform is presented [5]. The position and orientation of the platform (body 1) is given by degrees of freedom \( uvwxyz \), while its six legs (cylinders) are modelled by links \( ^* \) in the brackets; \( ij \) denote translational shifts w.r.t. to the reference frame of the platform (body \( \backslash 1 \)) and the ground, respectively.

A spatial four-bar mechanism is shown in Figure 7, right. It is usually referred to as a RTSR mechanism, consisted of abbreviations of its joints: a rotational, a T-joint (or a universal joint), a spherical, and a rotational joint, again. Shortest description \( zi > izi > i.ij < jxik \) is obtained, if spherical joint \( S \) is modeled by three constraint equations. Almost all elementary transformations are marked in the figure, while the last two, \( ik \), are just constant shifts of second rotational joint \( R \) w.r.t. the first one in the \( xz \) plane. Alternative notation \( zi > iyzi > izyxj > jxik > 0 \) is possible, when spherical joint \( S \) is modeled by additional three rotations \( zyx \), and the kinematic chain after second rotational joint \( R \) around axis \( x \) and constant shifts \( ik \) are connected to the origin >0.

A model of the PATU 655 log crane

Selection of technical examples is concluded here by a model of the PATU 655 log crane considered in reference [6], and represented in Figure 8.
4. Conclusion
In this paper, a way to describe a multibody system by a short text line, fully representing the kinematics and dynamics of the system, has been proposed. There are two main motivations for this work: for relatively simple mechanical systems this description should be as short as possible to serve as the benchmark designations; and for complex technical systems, it should serve as a language of the input text file for multibody analysis preprocessors.

In the proposed multibody system description, kinematical elementary transformations, various force elements and constraints are represented by reserved alphabetic characters. The parameters of the system, such as identifiers of degrees of freedom, inertia parameters and others, can be assigned default names if not specified. However, user-defined names, parameters and functions can be introduced instead. Examples from the academic area and some technical applications are presented to show the applicability of the proposed text description.

5. Acknowledgement
The research is supported by the Russian Foundation for Basic Research (14-01-00662, 16-51-51022).

References
[1] Hartenberg R S and Denavit J 1955 J. of Applied Mechanics 77(2) 215–221
[2] Jalón J Gd and Bayo E 2005 Kinematic and dynamic simulation of multibody systems – The real-time challenge
[3] Dmitrochenko O, Matikainen M and Mikkola A 2014 Proc. of the 3rd Joint Int. Conf. on Multibody System Dynamics, and the 7th Asian Conf. on Multibody Dynamics (BEXCO, Busan, Korea)
[4] Dmitrochenko O, Matikainen M and Mikkola A 2014 Proc. of the ASME 2014 10th Int. Conf. on Multibody Systems, Nonlinear Dynamics, and Control, MSNDC2014 (Buffalo, New York, USA)
[5] Stewart D 1965 UK Institution of Mechanical Engineers Proceedings 180(1) 15
[6] Mikkola A 1997 Studies on fatigue damage in a hydraulically driven boom system using virtual prototype simulations, Ph.D. thesis (Lappeenranta University of Technology, Finland) p 61