The $s$-wave repulsion and deeply bound pionic atoms: fact and fancy∗

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Fits to a large data set of pionic atoms show that the ‘missing’ $s$-wave repulsion is accounted for when a density dependence suggested recently by Weise is included in the isovector term of the $s$-wave pion optical potential. The importance of using large data sets is demonstrated and the role of deeply bound pionic atom states is discussed.

1. INTRODUCTION

Conventional phenomenological analyses of strong interaction effects in pionic atoms yield unexpectedly sizable $s$-wave repulsion [1]. Weise suggested that partial restoration of chiral symmetry in dense matter leads to enhancement of the free pion-nucleon isovector $s$-wave amplitude $b_1$ which could explain the anomalous repulsion [2]. The origin and magnitude of this enhancement have been discussed recently [3,4]. Weise’s suggestion was tested by Friedman [5] using a large set of pionic-atom data and was found to indeed account for most of the anomaly. This analysis has been extended significantly by increasing the number of data points from 60 to 106 and including several series of isotopes. Various sets of nucleon densities were used in addition to testing corrections due to a relativistic impulse approximation. Here we briefly review the results of the extended analysis [6] and, furthermore, focus attention on the significance of fits to large data sets and on the role played by the ‘deeply bound’ pionic atom states, particularly for the purpose of extracting the in-medium isovector $\pi N$ $s$-wave amplitude $b_1(\rho)$.

2. RESULTS

2.1. Implementation of Weise’s density-dependence for $b_1$

Highlights of the results of Ref. [6] are displayed in Fig. 1 for three potentials: (i) a conventional potential (C) with $s$-wave component

$$2\mu V_{opt}^{(s)}(r) = -4\pi(1 + \frac{\mu}{M})\{\bar{b}_0[\rho_n(r) + \rho_p(r)] + b_1[\rho_n(r) - \rho_p(r)]\}$$

$$-4\pi(1 + \frac{\mu}{2M})4B_0\rho_n(r)\rho_p(r);\quad \bar{b}_0 = b_0 - \frac{3}{2\pi}(\tilde{b}_0^2 + 2\tilde{b}_1^2)k_F(r),$$

where $k_F(r)$ is the local Fermi momentum. The second order term is included in $\bar{b}_0$ because of the extremely small value of the isoscalar amplitude $b_0$ [7]; (ii) the potential C where the isovector amplitude $b_1$ is made density dependent according to Ref. [2] (W, dashed);

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Figure 1. Best-fit $b_0$ and $b_1$ for 106 data points [6]. X marks free-space values [7].

Figure 2. Total $\chi^2$ vs. $\text{Re} B_0$ for the C and WB2 comprehensive-data fits [6].

and (iii) the latter potential where, in addition, a relativistic impulse approximation density-dependence effect is included (WB2). Eight points are given for each potential, corresponding to different models for the nuclear densities, different values for the rms radii of the neutron distributions and to different ways of handling the $p$-wave part of the potential. All the fits are equally good, with $\chi^2/F$ between 1.90 and 1.95. The figure shows clustering of points with particularly well-defined values for the isovector amplitude $b_1$ (constant for C, and at $\rho = 0$ for W and WB2). It is clearly seen that the conventional model C disagrees with the experimental free pion-nucleon values [7] which are marked as a boxed X, particularly with regard to $b_1$, whereas the Weise prescription W removes most of the discrepancy, notably for $b_1$. A close agreement is observed for the WB2 model. An important consequence of fitting to a large data base, 106 points in the present case, is that $b_1$ is determined extremely well with as small errors as $\pm 0.004 m_\pi^{-1}$. The errors on $b_0$ are larger, typically $\pm 0.010 m_\pi^{-1}$, due to including a quadratic-density dispersive term (with parameter $\text{Re} B_0$) in the potential [6]. The errors on $b_0$ could be artificially reduced by excluding such a term, but only at the unbearable cost of increasing $\chi^2$ by 90 units for potential C, as is clearly seen in Fig. 2. This obviously means that such a dispersive term is mandated by the data, contrary to recent claims [8,9]. The combined effect of $b_0$, $b_1$ and $\text{Re} B_0$, irrespective of which best-fit potential is adopted, is to produce a net repulsion of about 30 MeV inside of heavy nuclei such as Pb [6]; this is essentially the content of the phrase ‘anomalous repulsion’ for pionic atoms. We note that the size of the best-fit $\text{Re} B_0$, which is unacceptably large for C, decreases gradually upon introducing the density dependence appropriate to W and to WB2, becoming comparable for WB2 to the size of $\text{Im} B_0$ ($0.054 \pm 0.002 m_\pi^{-4}$) as appropriate to a dispersive term.
Table 1
Values of $b_1$ from fits to several data sets using potential C.

| data         | light $N = Z$ +$^{205}$Pb | light $N = Z$ + light $N > Z$ | global 1 $^{16}$O to $^{238}$U | global 2 $^{12}$C to $^{238}$U |
|--------------|----------------------------|--------------------------------|---------------------------------|---------------------------------|
| points       | 14                         | 16                             | 106                             | 114                             |
| $\chi^2/F$   | 3.0                        | 2.9                            | 1.9                             | 2.1                             |
| $b_1(m_\pi^{-1})$ | -0.113±0.025              | -0.096±0.014                   | -0.113±0.004                    | -0.112±0.004                    |

2.2. Global vs. partial data-set analyses

In order to assess the significance of using a large data base, we have studied the consequences of using severely reduced data sets. Following Yamazaki et al. [8,9], we chose the 1s states in the $N = Z$ nuclei $^{12}$C, $^{14}$N, $^{16}$O, $^{20}$Ne, $^{24}$Mg and $^{28}$Si to which we added the ‘deeply bound’ 1s state of $^{205}$Pb [10] or, alternatively, the ‘normal’ 1s states in the neighbouring $N > Z$ nuclei $^{18}$O and $^{22}$Ne, in order to determine the isovector amplitude $b_1$. With fits to such greatly reduced data sets one must assume a fixed $p$-wave potential, which we took from Ref. [6]. The results of best-fit values for $b_1$ using potential C within these partial data sets are shown in Table 1, together with similar results using potential C within global data sets: ‘global 1’ from Ref. [6] and ‘global 2’ which extends it by including also $^{12,13}$C and $^{14}$N. It is clear that, with the fairly small errors on $b_1$ in the global fits, the best-fit value of $b_1$ is enhanced over 20% with respect to the free-space value of $b_1 = -0.0885^{+0.0010}_{-0.0021} m_\pi^{-1}$ [7], whereas the larger errors associated with using restricted data sets can hardly qualify for making such a statement. We would like to emphasize that, with the realistic uncertainties which are typical of the restricted data sets, one cannot argue for a solid evidence of in-medium enhancement of $b_1$. It is interesting to note, for the restricted fits, that smaller errors are obtained using the two neighbouring $^{18}$O and $^{22}$Ne than using the deeply bound $^{205}$Pb. This suggests that the ‘deeply bound’ pionic 1s states do not carry new information over that already contained in the ‘normal’ pionic 1s states, as demonstrated below.

2.3. The role of deeply bound pionic atom states

Finally we remark on the role played by the deeply bound 1s and 2p states of pionic atoms in providing information on the pion-nucleus interaction. Global fits have shown that the 1s and 2p ‘deeply bound’ states in $^{205}$Pb follow the general trend observed for more than 100 ‘normal’ states and that the agreement between calculation and experiment for them was actually better than the average (Fig. 3 of Ref. [6]). Similar conclusions were obtained earlier for $^{207}$Pb [11].

Figure 3 shows that this conclusion follows naturally from the properties of the atomic wavefunctions and their relation to the nuclear density. The figure displays absolute values squared of the radial wavefunctions multiplied by the nuclear density for a normal 1s state (in $^{20}$Ne) and for a deeply bound 1s state (in $^{208}$Pb). It is seen that the Coulomb wavefunction would have indeed penetrated deeper into the heavy Pb nucleus, but due to the strong interaction it is repelled such that its overlap with the nucleus is sufficiently small to make the width of the state relatively narrow and thus making the state observable. In fact, the ‘deeply bound’ 1s wavefunction does not overlap with inner regions.
of the nucleus more so than a ‘normal’ 1s wavefunction does. It is thus concluded that deeply bound states do not play any special role in the determination of pionic atom potentials. This is not surprising since the same mechanism which causes the deeply bound states to be narrow, namely, the strong repulsion [12,13] of the wavefunction out of the nucleus, also masks the nuclear interior such that the penetration of the deeply bound pionic atom wavefunction is not dramatically enhanced compared to the ‘normal’ states.

REFERENCES

1. For a recent review see C.J. Batty, E. Friedman, A. Gal, Phys. Rep. 287 (1997) 385.
2. W. Weise, Nucl. Phys. A 690 (2001) 98c.
3. E.E. Kolomeitsev, N. Kaiser and W. Weise, nucl-th/0207090.
4. G. Chanfray, M. Ericson and M. Oertel, nucl-th/0211035.
5. E. Friedman, Phys. Lett. B 524 (2002) 87.
6. E. Friedman, Nucl. Phys. A 710 (2002) 117.
7. H.-Ch. Schröder et al., Eur. Phys. J. C 21 (2001) 473.
8. P. Kienle and T. Yamazaki, Phys. Lett. B 514 (2001) 1.
9. H. Geissel et al., Phys. Lett. B 549 (2002) 64; see also T. Yamazaki and S. Hirenzaki, nucl-th/0210040.
10. H. Geissel et al., Phys. Rev. Lett. 88 (2002) 122301.
11. E. Friedman and A. Gal, Phys. Lett. B 432 (1998) 235.
12. E. Friedman and G. Soff, J. Phys. G: Nucl. Phys. 11 (1985) L37.
13. H. Toki and T. Yamazaki, Phys. Lett. B 213 (1988) 129.