Numerical Analysis of Inner and Exterior Crack Relative Position for Stress Intensity Factors

Fang Xie, Dejun Liu* and Zhongbao Qin
Xi’an Hi-tech Institute, Xi’an 710025, China.

*Corresponding author email: 28003370@qq.com

Abstract. Establishing a cylinder pressure vessel model is to set a semi-ellipse crack on the inwall and ektexine of the vessel respectively. By changing the included angle values, standing for the inner and exterior crack relative position transformation, to get the inner and exterior crack stress intensity factors changing rules based on the results from finite element software ANSYS Workbench. The result indicates that the stress intensity factor value of inner and exterior cracks is minimum when their circumferential angle is 0° and the stress intensity factor value of inner and exterior cracks is maximum when their circumferential angle is 10°. In total, stress intensity factors of inner crack is more sensitive to the crack relative position transformation and can be easier to be effected by inner and exterior crack interaction.

1. Introduction
Pressure vessel is the special pressure-bearing equipment, so there exist initial cracks or cracks generating from working conditions inevitably. Because cracks can decrease the whole structural strength to some extent, it is necessary to study the relation between crack mechanism and multi-crack interaction [1, 2, 3]. Many scholars have done a lot of work on crack research. Zongjie Cao [4] conducts stress intensity factors of multiple 3-D cracks with co-plane utilizing minimum potential energy theorem and variational theorem. Anlin Yao [5] solves the inwall crack stress intensity factor for high pressure gas pipeline by crack virtual closure technique and conducts the main crack stress intensity factor calculation formula. Aiqin Wang [6] conducts the crack tip stress intensity factor expression for submarine pipeline with axial cracks using weight function methods. Feng Wang [7] proposes a simple method on 3-D crack stress intensity factors under the finite element software ANSYS modeling environment. Though scholars have done lots of research on 3-D crack interaction, the study on stress intensity factors variation of inner and exterior cracks which exist on the pressure vessel is very few. This paper studies the stress intensity factors of inner and exterior cracks changing rule and mechanics with cracks’ relative position change by finite element methods. This research possesses theoretical and practical value for engineering application.

2. Model Establishment

2.1. The whole model
The target to be researched is a thick-wall cylinder pressure vessel. The total length of model is 1500mm, the inside radius is 500mm and the wall thickness is 60mm. The pressure vessel material
is 15MnMoVN whose elastic modulus is 2.15e5 MPa, poission ratio is 0.28 and yield strength is 490MPa. The whole structure is shown as figure 1:

![Figure 1. Pressure vessel model](image)

### 2.2. The crack model

To study the effect of inner and exterior crack relative position transformation for stress intensity factors, a crack should be preset on the either inwall or ekstexine initially. It is shown as figure 2:

![Figure 2. Crack settings diagram](image)

Cracks are set on the middle position of the whole vessel. Through changing the included angle $\theta$ of inner and exterior cracks from $0^\circ$ to $180^\circ$, the relative position of inner and exterior cracks would be realized. Because semi-ellipse cracks are widely applied to crack research in practice, the crack surface is set to be semi-ellipse on the model. The semi-major axis length is 15mm while the semi-minor axis length is 5mm.

### 2.3. Mesh generation

The prerequisite that cracks can be successfully inserted into the model is that the body with cracks should be given tetrahedral mesh type in the ANSYS Workbench. In the previous ANSYS 18.0 versions, only a single crack can be inserted into the body with defect, while multi-crack analysis cannot be realized in the ANSYS Workbench. This paper analyzes multi-crack analysis resort to Workbench. To reduce calculation time, the cylinder pressure vessel model is divided into the top, middle and bottom part respectively. Because the middle part contains cracks, the mesh type of it is tetrahedral mesh, while the top and bottom parts’ mesh type is hexahedral mesh. The mesh generation is shown as figure 3:

![Figure 3. Mesh generation](image)
Then, cracks would be inserted into inwall and ektexine respectively and its mesh generation process would finish automatically. And the crack mesh generation is shown as figure 4:

![Figure 4. Crack mesh generation](image)

From the final mesh generation, the mesh element number is 300613, and node number is 890788.

3. Finite Element Analysis

3.1. Existing inner or exterior crack alone

To make the inner and exterior cracks’ coexisting results possess comparability, this part would only analyzes the solution of the pressure vessel with only a single crack. According to the practical pressure bore by vessel and constraint conditions, the inner pressure is set 30MPa and the bottom of the vessel is fixed to be the constraint condition.

The stress results of a single exterior or inner crack set on the pressure vessel from the finite element analysis are shown by figure 5:

(a) Stress distribution of only an inner crack  (b) stress distribution of only an exterior crack

![Figure 5. Crack stress distribution of a single crack mesh generation](image)

From the crack stress distribution of the pressure vessel with a single crack, the figure indicates that no matter the inner crack or the exterior crack, the maximum stress value of the whole vessel locates on the crack tip which in the deepest position from the vessel surface. And the stress concentration is the most obvious. The maximum stress value of the vessel with a single inner crack is 1232.6MPa, and the maximum stress value of the vessel with a single exterior crack is 1154.9MPa. Because those stress values are higher than the material yield strength value, the crack tip is on the plasticity state. And the crack would continue to grow along the crack depth direction at this time. Furthermore, the extent of inner crack stress concentration is more intensive than the exterior crack stress concentration from the numerical comparison result.

The stress intensity factor results of a single exterior or inner crack set on the pressure vessel from the finite element analysis are shown by figure 6:
From the crack stress intensity factor distribution of the pressure vessel with a single crack, it indicates that the stress intensity factor distribution is similar to stress distribution. On the deepest position of the crack tip, the stress intensity factor value is maximum. That means the extent of crack growth along the depth direction is the most intensive. The maximum stress intensity factor value of the vessel with a single inner crack is 1015.3MPa*√mm, and the maximum stress intensity factor value of the vessel with a single exterior crack is 918.92MPa*√mm. It indicates that the influence of a single inner crack for the pressure vessel is stronger than a single exterior crack. And the stress intensity factor values are increasing from the shallow position to the deep position along the crack tip no matter for a single inner crack existing in the inwall or exterior crack existing in the ektextine.

3.2. Coexisting inner and exterior cracks

To study the crack tip stress intensity factor distribution of inner and exterior cracks impact for the crack relative position change, the exterior crack position is fixed so that the crack relative position can be changed by transforming the included angle \( \theta \) of the inner crack with the exterior crack. When the included angle is 0°, the tips of both the inner and exterior crack would be face to face directly. Cracks are preset on the model, and their stress intensity factor distribution is as figure 7 shown:

From the stress intensity factors distribution of both the inner and exterior cracks, its distribution is similar to the result from which there is only a single inner crack or exterior crack set on the model. The stress intensity factors of both the inner and exterior crack increase from the shallow position to the deep position of crack tips. And both cracks tend to grow along the depth direction. Therefore, this relative position would cause little effect for the stress intensity factor distribution. From the numerical value result, the maximum stress intensity factor value of the inner crack is 1009.6MPa*√mm less than the value 1015.3MPa*√mm which is got from the situation when there is only a single inwall crack on the vessel without the exterior crack. The maximum stress intensity factor value of the inner crack is 912.36MPa*√mm less than the value 918.92MPa*√mm which is got from the situation when there is only a single ektextine crack on the vessel without the inner crack. So when the included angle is set 0°, the stress intensity factors of both the inner and exterior crack would decrease. That means the extent of crack growth tendency would be constrained, making the crack growth process slower. To further
research the interaction between crack relative position and stress intensity factors, another two particular relative positions are chosen. The included angles are set 90° and 180° respectively. And the stress intensity factors results for the included angle 90° and 180° of the inner and exterior crack are shown as figure 8 and 9:

(a) SIF distribution of the inner crack                     (b) SIF distribution of the exterior crack

**Figure 8.** Stress intensity factor distribution of cracks under the included angle 90°

(a) SIF distribution of the inner crack                     (b) SIF distribution of the exterior crack

**Figure 9.** Stress intensity factor distribution of cracks under the included angle 180°

From the stress intensity factor distribution results of the inner and exterior crack under the included angles are 90° and 180°, the maximum values of stress intensity factors appear on the deepest location of the crack tip and the minimum values of stress intensity factors appear on the location which is near to the outside surface of the vessel. The maximum values of stress intensity factors of the inner crack when the included angles are 90° and 180° are 1013.7 MPa and 1015.2 MPa. And the maximum values of stress intensity factors of the inner crack when the included angles are 90° and 180° are 918.59 MPa and 918.61 MPa. If those values compare with the values when the included angle is set 0°, it finds that the stress intensity factor magnitude for the inner crack is rising while the stress intensity factor magnitude for the exterior crack is similar. Because of the differences for the inner crack and exterior crack, it is necessary to explore the change rule for the stress intensity factor values to relative position transformation of cracks. To consider overall relative positions of the inner and exterior crack, the circumferential included angle would be set at intervals of 15° so that the range of the included angle would be from 0° to 180° and can study the change rule in detail.

The exterior crack would be fixed and then change the inner crack position at intervals of 15°. The results from the finite element analysis are shown as figure 10 and 11:
Figure 10. The inner crack SIF change with relative position

Figure 11. The exterior crack SIF change with relative position

From the figure 10, the stress intensity factor change rule of the inner crack is fluctuant. If making the single inner crack solution without an exterior crack appearance be the baseline to observe the stress intensity factors variation, the change rule is easier to research. The stress intensity factor is minimum when the included angle is set 0°, which indicates that the cracks’ interaction prohibits the inner crack growth. The stress intensity factor is maximum and the value is higher than the solution under the situation of the pressure vessel without the exterior crack when the included angle is set 10°, which indicates that the cracks’ interaction promotes the inner crack growth. When the included angle is over 30°, values of stress intensity factors are fluctuant obviously but their magnitude is less than the magnitude got from the solution under the situation of the pressure vessel without the exterior crack. Thus, exterior crack effect makes the inner crack stress intensity factors decrease, indicating that the exterior crack existence would weaken the inner crack growth.

From the figure 11, if making the single exterior crack solution without an inner crack appearance be the baseline to observe the stress intensity factors variation, the stress intensity factor variation of the
exterior crack is much more stable than the inner crack stress intensity factor variation. The stress intensity factor is minimum when the included angle is set 0°, which indicates that the cracks’ interaction prohibits the exterior crack growth. The stress intensity factor values would jump to the peak when the included angle range is from 0° to 10°, which indicates that when the included angle between the inner and exterior crack is small the interaction is remarkable. The stress intensity factor value is minimum when the included angle is set 0°, while the stress intensity factor value is maximum when the included angle is set 10°. That tendency is similar to the inner crack stress intensity factor variation, meaning that the interaction effect for the inner or exterior crack is same. When the included angle is over 15°, the stress intensity factors of the exterior crack vary stably and their magnitude is almost equal to the magnitude from the solution under the situation of the pressure vessel without the inner crack. Thus, if the inner and exterior cracks exist together on the pressure vessel and change their relative positions, the interaction effect for the inner crack is more intensive than the effect for the exterior crack.

4. Conclusion
For the cylinder pressure vessel with the inner crack and the exterior crack, this paper studies the stress intensity factors change rule with the relative position change of the inner and exterior crack.

(1) The stress intensity factor magnitude of both the inner and exterior cracks is minimum when the included angle is 0°, meaning that their interaction would prohibits the crack growth. While the magnitude of stress intensity factors would be maximum when the included angle is about 10°, indicating that the interaction of cracks would promote the crack growth.

(2) When the inner and exterior cracks exist together, the stress intensity factor change of the inner crack is more sensitive than the change of the exterior crack. And as the included angle increases, the exterior crack stress intensity factors are almost same while the inner crack stress intensity factors are fluctuant particularly.

References
[1] Zhangfeng Li, Nan Jiang. Calculation and Analysis of Fracture Parameters of Pressure Vessels Surface Crack with Finite Element Method [J]. Industrial Safety and Environmental Protection, 2011 (09): 58-60
[2] Siyuan Li, Zuocheng Li. Stress Calculation of Defect Location in Pressure Vessels and Pipings [J]. Petro-Chemical Equipment, 1996 (01): 27-30.
[3] Tongfeng Wei. Stress Analysis of Cracking Pipelines Based on ANSYS [J]. Electric Welding Machine, 2015 (03): 93-97.
[4] Zongjie Cao, Bing Xu. Interference of Stress Intensity Factor of 3-D Cracks with Co-plane [J]. Journal of Materials Engineering, 2003 (z1): 254-255.
[5] Anlin Yao, Yiping Hu, et al. Finite Element Numerical Analysis of Multiple Interacting Cracks in High Pressure Gas Pipeline [J]. Pressure Vessel Technology, 2011 (03): 28-32.
[6] Aiqin Wang, Taiyan Qin. Stress Strength Factor Analysis of Submarine Pipelines with Axial Surface Crack [J]. Chinese Journal of Applied Mechanics, 2008 (03): 439-444.
[7] Feng Wang, Qiqing Huang, et al. Analysis of Emulational Limited Element Method for the Three-dimensional Stress Intensity Factors [J]. Aeronautical Computing Technique, 2006 (03): 125-127.