The renormalization group effect to the bi-maximal mixing

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Abstract

We discuss whether the bi-maximal mixing given at GUT is consistent with the solar mixing at the normal side at the low energy. We consider the radiative corrections due to the $\tau$–Yukawa, the neutrino–Yukawa and the slepton threshold corrections and discuss in what situation the maximal solar angle rotates towards the normal side. In this scheme, the $|V_{13}|$ and the Dirac CP phase $\delta$ are induced radiatively and we estimate these values.

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1. INTRODUCTION

In view of the data by Super-K \cite{1}, SNO \cite{2}, the solar mixing angle will not be maximal and its best fit value is

\[ \tan^2 \theta_\odot \simeq 0.34, \] (1)

and the solar mass squared difference is precisely determined by the KamLAND data \cite{3} as

\[ \Delta m^2_\odot \equiv m^2_2 - m^2_1 \simeq 6.9 \times 10^{-5} \text{eV}^2. \] (2)

On the other hand, the bi-maximal mixing scheme \cite{4} of the neutrino mixing is quite attractive theoretically, but it predicts \( \tan^2 \theta_\odot = 1 \), which contradicts with the experimental data. However, if we consider that the bi-maximal mixing is derived by GUT, we have know the solar mixing angle at low energy where the neutrino oscillation experiments are preformed. This can be made by analyzing the renormalization group effect to the neutrino mass matrix.

In our previous paper \cite{5}, we analyzed the effect of the renormalization group due to \( \tau \)-Yukawa coupling to the neutrino mass derived from the see-saw mechanism, and we found that the solar angle increases as the energy scale decreases. That is, if \( \tan^2 \theta_\odot = 1 \) at GUT scale, \( M_X \), then the solar angle moves towards the dark-side, \( \tan^2 \theta_\odot > 1 \) at low energy, \( m_Z \), as the energy scale moves from \( M_X \) to \( m_Z \). This may be a serious trouble for the bi-maximal mixing scenario at GUT energy.

In this paper, we discuss the question whether we can achieve the normal-side solar mixing angle, \( \tan^2 \theta_\odot < 1 \), at low energy by quantum effects, starting form the maximal mixing at GUT energy. It is known \cite{6} that the quantum effects to the mixing angles is small when the neutrino mass spectrum is hierarchical. Therefore, we are forced to consider the quasi-degenerate neutrino scenario, if we seek this possibility.
2. THE RENORMALIZATION EFFECT TO THE BI-MAXIMAL MIXING

We take the basis where the charged lepton mass matrix is diagonal. The neutrino mass matrix which gives to the bi-maximal mixing at $M_X$ is given by

$$m_{\nu}(M_X) = O_B D_\nu O_B^T,$$

with

$$O_B = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{i}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{i}{\sqrt{2}} \end{pmatrix},$$

and

$$D_\nu = \text{diag}(M_1, M_2 e^{i\alpha_0}, M_3 e^{i\beta_0}).$$

where $M_i$ are neutrino masses at GUT scale and taken to be real positive and almost degenerate. The phases $\alpha$ and $\beta$ are CP violating Majorana phases. This mass matrix contains five parameters.

We assume that the neutrino mass matrix at the low energy, $m_{\nu}(m_Z)$ is related to that of GUT scale, $m_{\nu}(M_G)$ by

$$m_{\nu}(m_Z) = m_{\nu}(M_G) + K m_{\nu}(M_G) + m_{\nu}(M_G) K.$$

As we shall discuss in detail later, we parametrize $K$ as

$$K = \begin{pmatrix} \epsilon_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \epsilon_\tau \end{pmatrix}.$$

There are several origins for $\epsilon_e$ and $\epsilon_\tau$, the effect due to the $\tau$-Yukawa coupling, the effect due to the neutrino-Yukawa couplings, the threshold correction due to heavy right-handed neutrino mass splitting and the slepton mass splitting, which we discuss in the next section.
In below, we assume that $\epsilon_e$ and $\epsilon_\tau$ are small quantities of order $10^{-3}$ and consider the quasi-degenerate mass case,

$$M_1 \sim M_2 \sim M_3,$$

$$|\epsilon_e| \sim |\epsilon_\tau| << 1,$$

$$|\Delta_{31}| >> |\Delta_{21}| \sim \epsilon M_i.$$ \hfill (8)

where

$$\Delta_{21} = M_2^2 - M_1^2, \quad \Delta_{31} = M_3^2 - M_1^2.$$ \hfill (9)

Now, we discuss the effect of $K$ to the neutrino mass matrix in the first order approximation of $\epsilon_e$ and $\epsilon_\tau$.

We transform $m_\nu(m_Z)$ by $O_B$ as

$$\bar{m}_\nu(m_Z) = O_B^T m_\nu(m_Z) O_B$$

$$= D_\nu + O_B^T K O_B D_\nu + D_\nu O_B^T K O_B,$$ \hfill (10)

where

$$O_B^T K O_B = \frac{\epsilon_\tau}{4} \begin{pmatrix} 1 & 1 & \sqrt{2} \\ 1 & 1 & \sqrt{2} \\ \sqrt{2} & \sqrt{2} & 2 \end{pmatrix} + \frac{\epsilon_e}{2} \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$ \hfill (11)

Since $\bar{m}_\nu$ is a complex symmetric matrix, we consider the hermitian quantity $\bar{m}^\dagger_\nu \bar{m}_\nu$ which is given by

$$\bar{m}^\dagger_\nu \bar{m}_\nu \simeq (1 + 2\epsilon_e + \epsilon_\tau) M_1^2 + Y_0 + Y_e + Y_\tau$$ \hfill (12)

where elements of $Y_i$ are given by

$$Y_0 = \text{diag}(0, \Delta_{21}, \Delta_{31}),$$

$$Y_e \simeq -2\epsilon_e \begin{pmatrix} 0 & 1 + e^{i\alpha_0} & 0 \\ 1 + e^{-i\alpha_0} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} M_1^2,$$

$$Y_\tau \simeq \epsilon_\tau \begin{pmatrix} 0 & 1 + e^{i\alpha_0} & \frac{1+R^2+2Re^{i\beta_0}}{2\sqrt{2}} \\ 1 + e^{-i\alpha_0} & 0 & \frac{1+R^2+2Re^{i(\beta_0-\alpha_0)}}{2\sqrt{2}} \\ \frac{1+R^2+2Re^{-i\beta_0}}{2\sqrt{2}} & \frac{1+R^2+2Re^{i(\beta_0-\alpha_0)}}{2\sqrt{2}} & 2R^2 - 1 \end{pmatrix} M_1^2.$$ \hfill (13)
where \( R = M_3/M_1 \) and we neglected terms of order \( \epsilon_i \Delta_{21} \). We observe that \( \Delta_{31} \) dominates over all other terms in the matrix \( Y_0 + Y_e + Y_\tau \), so that we can employ the see-saw calculation. By the unitary matrix

\[
V_3 \simeq \begin{pmatrix}
1 & 0 & \frac{(Y_\tau)_{13}}{(Y_\tau)_{33}} \\
0 & 1 & \frac{(Y_\tau)_{23}}{(Y_\tau)_{33}} \\
-\frac{(Y_\tau)_{13}}{(Y_\tau)_{33}} & -\frac{(Y_\tau)_{23}}{(Y_\tau)_{33}} & 1
\end{pmatrix}, \tag{14}
\]

where \( Y_0 + Y_e + Y_\tau \) is block diagonalized in the approximation to neglect terms \((Y_\tau)_{3i}(Y_\tau)_{j3}/(Y_\tau)_{33}, (i,j \neq 3)\) which are of order \( \sim \epsilon^2 M_i^2 M_j^2/\Delta_{31} \). We diagonalize the remaining \( 2 \times 2 \) matrix by

\[
V_2 = \begin{pmatrix}
c & -se^{i\alpha_0/2} & 0 \\
se^{-i\alpha_0/2} & c & 0 \\
0 & 0 & 1
\end{pmatrix}, \tag{15}
\]

with \( c = \cos \theta \) and \( s = \sin \theta \), such that

\[
V_2^\dagger V_3^\dagger \bar{m}_\nu V_3 V_2 = \text{diag}(m_1^2, m_2^2, m_3^2). \tag{16}
\]

We find

\[
m_i \simeq M_i,
\]

\[
\Delta m^2_{\text{atm}} = m_3^2 - m_1^2 \simeq \Delta_{31},
\]

\[
\Delta m^2_{\odot} = m_2^2 - m_1^2 \simeq \frac{\Delta_{21}}{\cos 2\theta}, \tag{17}
\]

and the angle \( \theta \) is expressed by replacing \( M_i \) with \( m_i \) as

\[
\sin 2\theta = -\frac{2(-2\epsilon_e + \epsilon_\tau)m_1^2 \cos \frac{\alpha_0}{2}}{\Delta m^2_{\odot}}. \tag{18}
\]

By applying the transformation to \( m_\nu(m_Z) \), we find

\[
(O_B V_3 V_2)^T m_\nu(m_Z) O_B V_3 V_2 \simeq \text{diag}(m_1, m_2e^{i\alpha_0}, m_3e^{i\beta_0}), \tag{19}
\]

with \( m_i > 0 \), so that Majorana phases are determined. Thus, the mixing matrix which diagonalizes \( m_\nu(m_Z) \) is \( V = O V_3 V_{12}P \) with \( P = \text{diag}(1, e^{-i\alpha_0/2}, e^{-i\beta_0/2}) \). The neutrino mixing matrix is parameterized by

\[
V = V_{\text{MNS}} P_M, \tag{20}
\]
where $V_{MNS}$ is the MNS matrix \([?]\) and $P_M$ is the Majorana phase matrix

$$P_M = \text{diag}(1, e^{i\alpha_M}, e^{i\beta_M}),$$

which represents the Majorana CP phases \([?]\). From the above information, we find

$$V_{MNS} = \begin{pmatrix} c_\odot & -s_\odot \text{sgn}(\epsilon_\tau)|V_{13}|e^{-i\delta} \\ \frac{s_\odot}{\sqrt{2}} & \frac{c_\odot}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{s_\odot}{\sqrt{2}} & \frac{c_\odot}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

where

$$s_\odot = \frac{1}{\sqrt{2}}|c + se^{-i\alpha_0}|, \quad c_\odot = \frac{1}{\sqrt{2}}|c - se^{-i\alpha_0}|,$$ (23)

and the induced parameters, $|V_{13}|$ and the Dirac phase $\delta$ are given by

$$|V_{13}| = \frac{|\epsilon_\tau|m_1m_3\sin \frac{\Delta m_{\text{atm}}^2}{2}}{\Delta m_{\text{atm}}^2},$$

$$\delta = \xi_1 + \xi_2 + \frac{\alpha_0}{2} - \frac{\pi}{2} - \beta_0,$$ (25)

where $\xi_1 = \text{arg}(c - se^{-i\alpha_0})$ and $\xi_2 = \text{arg}(c + se^{i\alpha_0})$. The Majorana CP violation phases are given by

$$\alpha_M = \xi_2 - \xi_1 - \frac{\alpha_0}{2}, \quad \beta_M = \xi_2 - \frac{\beta_0}{2}.$$ (26)

It may be worthwhile to mention that the effect by $K$ in Eq.(6) is to rotate the solar angle, to change the solar mass-squared difference, to induce $|V_{13}|$ and $\delta$. This effect is possible only for the quasi-degenerate neutrino mass scenario.

3. THE EFFECT TO THE MIXING MATRIX

We found that the renormalization group affects to $\theta_\odot$ and $\Delta m_{\odot}^2$, and induces $|V_{13}|$ and $\delta$. In the following, we examine these quantities in detail.

(a) The solar mixing angle

From Eq.(23), we find

$$\tan^2 \theta_\odot = \frac{1 + \sin 2\theta \cos \frac{\alpha_0}{2}}{1 - \sin 2\theta \cos \frac{\alpha_0}{2}},$$ (27)
so that \( \tan^2 \theta_\odot \) moves towards the normal side if \( \sin 2 \theta \cos(\alpha_0/2) = -\cos 2 \theta_\odot < 0 \) and to the dark side if \( \sin 2 \theta \cos(\alpha_0/2) = -\cos 2 \theta_\odot > 0 \). From Eqs.(18), we find

\[
\sin 2\theta \cos\left(\frac{\alpha_0}{2}\right) = -2(-\epsilon_e + \epsilon_\tau) \cos^2\left(\frac{\alpha_0}{2}\right) \frac{m_1^2}{\Delta m_\odot^2}.
\]  

(28)

Thus, we conclude for \( \Delta m_\odot^2 > 0 \)

\[
\tan^2 \theta_\odot = \begin{cases} 
\text{normal side} & -2\epsilon_e + \epsilon_\tau > 0 \\
\text{dark side} & -2\epsilon_e + \epsilon_\tau < 0
\end{cases}
\]  

(29)

Now, we consider the allowed range of \( \alpha_0/2 \). From \( \sin 2\theta = -\cos 2\theta_\odot / \cos(\alpha_0/2) \),

\[
|\cos \frac{\alpha_0}{2}| < \cos 2\theta_\odot.
\]  

(30)

By eliminating \( \sin 2\theta \), we find

\[
(-2\epsilon_e + \epsilon_\tau) = \frac{\cos 2\theta_\odot}{2} \frac{1}{\cos^2(\alpha_0/2)} \frac{\Delta m_\odot^2}{m_1^2}.
\]  

(31)

Numerically, we obtain

\[
-2\epsilon_e + \epsilon_\tau = 7.2 \times 10^{-3} \left(\frac{0.49}{\cos(\alpha_0/2)}\right)^2 \left(\frac{0.1 \text{eV}}{m_1}\right)^2 \left(\frac{\Delta m_\odot^2}{6.9 \times 10^{-5} \text{eV}^2}\right),
\]  

(32)

where we used \( \cos 2\theta_\odot = 0.49 \). The value of \(-2\epsilon_e + \epsilon_\tau \) should be smaller than \( 7.2 \times 10^{-3} \) for \( m_1 = 0.1 \text{eV} \), because \( |\cos \frac{\alpha_0}{2}| \) must be larger than \( \cos 2\theta_\odot \). Therefore, typically we need \( \epsilon_i \) of order \( 5 \times 10^{-3} \) in order to move \( \tan^2 \theta_\odot \) from 1 to 0.43. \(-2\epsilon_e + \epsilon_\tau \) is proportional to \( 1/m_1^2 \), the larger \( \epsilon_i \) is required if \( m_i \simeq m \) is smaller than 0.2eV.

(b) The size of the induced \( |V_{13}| \)

From Eq.(24), we find

\[
|V_{13}| = 0.018 \left(\frac{|\epsilon_\tau|}{5 \times 10^{-3}}\right) \left(\frac{m_1 m_3}{(0.1)^2 \text{eV}^2}\right) \left(\frac{2.5 \times 10^{-3} \text{eV}^2}{\Delta m_{\text{atm}}^2}\right) \left(\frac{\sin \frac{\alpha_0}{2}}{0.87}\right).
\]  

(33)

The induced \( |V_{13}| \) is small if the neutrino masses are less than 0.1 eV and if \( \epsilon_\tau \) is of order \( 5 \times 10^{-3} \). The larger \( V_{13} \) is expected when both \( \epsilon_e \) and \( \epsilon_\tau \) are of order of \( 10^{-2} \) and the neutrino masses are about 0.2eV.
(c) The size of the induced $\delta$

We consider $\alpha_0/2$ in the region of $0 < \alpha_0 < 2\pi$. From the relation, $\sin 2\theta \cos \frac{\alpha_0}{2} = -\cos 2\theta_{\odot} < 0$, we define

$$\sin 2\theta = -x, \quad x = \frac{\cos 2\theta_{\odot}}{\cos \frac{\alpha_0}{2}}.$$ \hspace{1cm} (34)

Then, the allowed region of $x$ is

$$\cos 2\theta_{\odot} < |x| < 1, \quad \left|\frac{\alpha_0}{2}\right| < 2\theta_{\odot}.$$ \hspace{1cm} (35)

We find two cases,

$$\tan \xi_1 = -\sqrt{1 - \left(\frac{\cos 2\theta_{\odot}}{|x|}\right)^2} \left(\frac{\sqrt{1+|x|} + \sqrt{1-|x|}}{\sqrt{1+|x|} - \sqrt{1-|x|}} + \frac{\cos 2\theta_{\odot}}{|x|}\right),$$

$$\tan \xi_2 = -\sqrt{1 - \left(\frac{\cos 2\theta_{\odot}}{|x|}\right)^2} \left(\frac{\sqrt{1+|x|} + \sqrt{1-|x|}}{\sqrt{1+|x|} - \sqrt{1-|x|}} - \frac{\cos 2\theta_{\odot}}{|x|}\right).$$ \hspace{1cm} (36)

Here, we consider $\cos \alpha_0/2 > 0$ case. For $x \to 1$, i.e., $\alpha_0/2 \to 2\theta_{\odot}$, we find for both cases $\xi_1 + \xi_2 = -\pi/2$ and thus $\delta = 2\theta_{\odot} - \pi - \beta_0 \sim -(2/3)\pi - \beta_0$. For $x \to \cos 2\theta_{\odot}$, i.e., $\alpha_0 \to 0$, we find $\xi_1 = \xi_2 = 0$ so that $\delta = -\pi/2 - \beta_0$. Therefore, we expect that $\delta + \beta_0$ takes values roughly between $-\pi/2$ and $-(3/2)\pi$. This is confirmed by the numerical computation and the result is shown in Fig.1, where the allowed region of $x$ is taken to be $0.49 < x < 1$. The phase $\delta = \beta_0$ varies roughly from $-\pi/2$ to $-(2/3)\pi$ as we expected.

For $\cos \alpha_0/2 < 0$, the $\delta$ is obtained just adding $\pi$ to values for $\cos \alpha_0/2 > 0$. The actual value of $\delta$ depends on $\beta_0$ which has to be fixed, maybe by the leptogenesis.

4. THE QUANTUM CORRECTIONS

We consider the MSSM model associated with the right-handed neutrinos. That is, we assume the neutrino-Yukawa couplings and the heavy right-handed Majorana neutrino masses, and the neutrino mass matrix is generated by the see-saw mechanism.
The value of $\delta + \beta_0$ as a function of $x = \cos 2\theta_\odot / \cos(\alpha_0/2)$. The region of $x$ is taken to be $0.49 < x < 1$.

(a) The renormalization group effect

The renormalization group for the neutrino mass matrix in MSSM is given for $M_X > \mu > M_R$ by

$$
\frac{dm_\nu}{d \ln \mu} = \frac{1}{16\pi^2} \left\{ \left[ (Y_\nu^\dagger Y_\nu)^T + (Y_e^\dagger Y_e)^T \right] m_\nu + m_\nu \left[ (Y_\nu^\dagger Y_\nu) + (Y_e^\dagger Y_e) \right] \right\},
$$

where $Y_\nu$ and $Y_e$ are the Yukawa coupling matrices for neutrinos and charged leptons, respectively, aside from the terms proportional to the unit matrix. When $\mu < M_R$, we have to omit the neutrino-Yukawa contribution, because the heavy neutrinos decouple form the interaction.

(a-1) The $\tau$-Yukawa coupling

In $Y_e$, the term which gives a mass to $\tau$ dominates and we take $(Y_e^\dagger Y_e) = 2 \text{diag}(0,0,(m_\tau/v_d)^2)$. In MSSM, $(m_\tau/v_d)^2 = (m_\tau/v)^2(1 + \tan^2 \beta)$ and the $\tau$-Yukawa can give a sizable contribution for large $\tan \beta$ case. We note that in the standard model, there is no such enhancement factor and thus this contribution is negligible. In MSSM, we find the $\tau$-Yukawa contribution as
\[ \epsilon_e = 0 \] and
\[ \epsilon_\tau = \frac{1}{8\pi^2} (1 + \tan^2 \beta) (m_\tau / v)^2 \ln \left( \frac{m_Z}{M_X} \right) \]
\[ = -5.2 \times 10^{-3} \left( \frac{1 + \tan^2 \beta}{1 + 15^2} \right), \quad (38) \]
where we used \( M_X = 10^{16} \text{GeV}, m_Z = 91.187 \text{GeV}, m_\tau = 1.777 \text{GeV} \) and \( v = 245.4 \text{GeV} \). Since \(-2\epsilon_e + \epsilon_\tau < 0\), the solar mixing angle turns towards the dark side, which is unwanted for the maximal solar mixing case. This contribution becomes small if we take \( \tan \beta \) around 5.

(a-2) The neutrino-Yukawa coupling
For the contribution due to the neutrino-Yukawa couplings, there is no \( \tan \beta \) enhancement, so that we have to take relatively large \( Y^\dagger_\nu Y_\nu \) of order 0.1. If we take the scenario that the slepton mixing occurs mainly from the renormalization group effect from \( Y_\nu \), the lepton flavor violation process such as \( \mu \rightarrow e + \gamma \) tells that the off diagonal terms of \( (Y^\dagger_\nu Y_\nu) \) must be much smaller than \( 10^{-1} \). Therefore, it is reasonable to assume that \( Y^\dagger_\nu Y_\nu \) is almost diagonal and thus we parameterize
\[ Y^\dagger_\nu Y_\nu = \text{diag}(y^2_1, y^2_2, y^2_3) \]
\[ = y^2_\tau + \text{diag}(y^2_1 - y^2_\tau, 0, y^2_3 - y^2_\tau). \quad (39) \]
Then, we find
\[ \epsilon_i = \frac{1}{16\pi^2} (y^2_i - y^2_\tau) \ln \left( \frac{M_R}{M_X} \right) \]
\[ = -4.4 \times 10^{-3} \left( \frac{y^2_i - y^2_\tau}{0.1} \right) \ln \left( \frac{M_R}{10^{12} \text{GeV}} \right), \quad (40) \]
where \( i = e, \tau \) and \( M_R \) is the mass of the heavy right-handed neutrinos. In order to get \( \epsilon_e < 0 \) and \( \epsilon_\tau > 0 \), we need the anti-hierarchical structure for \( Y^\dagger_\nu Y_\nu \), i.e., \( y^2_1 - y^2_\tau > 0 \) and \( y^2_3 - y^2_\tau < 0 \) and the large coupling strength of order 0.3 is required.

(b) The threshold corrections
There may be two kinds of threshold corrections. One is due to the mass splitting among heavy right-handed neutrinos, which was discussed by Antusch, Kersten, Lindner and Ratz \[10\] for the inverted Dirac neutrino mass case. The other is due to the slepton mass splitting \[11\]. Here, we only discuss the effect due to the slepton mass splitting.

(b-1) The slepton mass splitting

We assume that the universal soft breaking terms aside from the corresponding term to the neutrino-Yukawa interaction which is non-universal. Since we assume that $Y_{\nu}$ is diagonal, the off diagonal elements of the slepton mass matrix is not induced. The lepton flavor violation occurs from the slepton mass splitting. The effect to the neutrino mass matrix is given by \[11\]

$$\epsilon_i = \frac{g^2}{32\pi^2} \left( -\frac{1}{z_i} + \frac{z_i^2}{z_i^4} \ln(1 - z_i) - (i \rightarrow \mu) \right) \quad (41)$$

where $i = e, \tau$ and $z_i = 1 - (\tilde{M}_i/\tilde{m})^2$, $\tilde{m}$ is wino mass and $\tilde{M}_i$ is the charged slepton mass.

We find that $\epsilon_i > 0$ if $\tilde{M}_i > \tilde{M}_\mu \sim \tilde{m}$ and $\epsilon_i < 0$ if $\tilde{M}_i < \tilde{M}_\mu \sim \tilde{m}$ Thus, $\epsilon_e < 0$ and $\epsilon_\tau > 0$ are realized when the slepton mass splitting is hierarchical, i.e., $\tilde{M}_e < \tilde{M}_\mu < \tilde{M}_\tau$. In Fig.2, we showed values of $-2\epsilon_e + \epsilon_\tau$ as the function of $\tilde{M}_e/\tilde{m}$ and $\tilde{M}_\tau/\tilde{m}$ with $\tilde{M}_\mu = \tilde{m}$. Value of order $3 \times 10^{-3}$ can be obtained for the ordinary mass ordered case, typically for $\tilde{M}_e/\tilde{m} \sim \tilde{m}/\tilde{M}_\tau \sim 1/2$.

5. THE SUMMARY

We considered the fate of the bi-maximal mixing scheme which is realized at GUT in the light of the recent experimental implication that the solar neutrino mixing angles is in the normal side. We considered three kinds of quantum effects in the MSSM: a) The effect due to the $\tau$-Yukawa coupling. We showed that this effect turns the solar angle towards the dark side as the energy scale decreases form the GUT scale to the weak scale. b) The effect due to the neutrino-Yukawa couplings $Y_\nu$. This contribution becomes
FIG. 2: The $\bar{M}_e/\bar{m}$ and $\bar{M}_\tau/\bar{m}$ dependence of the value of $-2\varepsilon_e + \varepsilon_\tau \cdot x = \cos 2\theta_\odot / \cos(\alpha_0/2)$. The region of $x$ is taken to be $0.49 < x < 1$. In this figure, we take $\bar{M}_\mu = \bar{m}$ for simplicity.

effective when elements of $Y_\nu$ are as large as 0.3, which gives the Dirac neutrino masses of order 50GeV. If we adopt the simple case where the soft breaking term is universal at the GUT scale. Then the only source for the the lepton flavor violation is from the neutrino-Yukawa couplings. In this story, the lepton flavor violation processes such as $\mu \rightarrow e + \gamma$ occurs from the renormalization group effect from $M_X$ to $M_R$ [13], which is proportional to $Y_\nu^\dagger Y_\nu$. We argued that $Y_\nu^\dagger Y_\nu$ must be almost diagonal in order to suppress the lepton flavor violation process. In addition, the inverted hierarchy for the diagonal elements of $Y_\nu^\dagger Y_\nu$ is required, in order to obtain the normal side solar angle at low energy.

c) We also considered the slepton threshold effect. We showed that the slepton threshold effect rotates the solar angle towards the normal side if the slepton mass spectrum is hierarchical.

The bi-maximal mixing scheme which is achieved at GUT scale is quite interesting in the following reasons: 1)This mixing scheme can be obtained in the GUT scheme. 2)The neutrino mass matrix contains only seven parameters, three mixing angles, two Majorana phases and three masses. By using the renormalization group, the parameters at the low energy are obtained. In particular, the $|V_{13}|$ and the Dirac phase $\delta$ are induced by the renormalization group. Furthermore, if we assume that $Y_\nu^\dagger Y_\nu$ is almost diagonal, we can
show that all CP violation phases are given by two Majorana phases, which are phases of neutrino masses. That is, we can show that the CP violation phases which appear in the leptogenesis are given by two Majorana phases, in the see-saw scheme. We gave this argument to demonstrate that the bi-maximal mixing provides the possibility that we understand CP violation phases. This work is now under preparation.

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