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Abstract. The method of constructing a hydrodynamic model of a large river is described in the work using the example of a section of the Volga River south of the Volzhskaya HPP. The construction of reliable hydrodynamic models of real river systems requires actual digital elevation models (DEM). Natural changes in the characteristics of river bed with time require updating and subsequent verification of DEM. We consider 100-kilometer section of the Volga River south of the Volzhskaya HPP, which determines the hydrological regime of the entire Volga-Akhtuba floodplain (VAF) right up to the Caspian Sea. The algorithm for constructing the DEM of bottom topography, based on a comparative analysis of the results of numerical hydrodynamic modeling and data on the dynamics of water levels at gauging stations, is proposed. In order to reconcile the changes in the water level $H(t)$ in the numerical hydrodynamic model with the observed dependence, we vary the characteristics of the local sectors of the bottom.

1. Introduction
Morphology and structure of the topography, characteristics of rocks and soils, as well as anthropogenic factors affect the deformation of river beds. Transfer of sediment has a strong influence on the change in the outlines and altitude characteristics of river beds [1]. The overregulation of the Volga due to the large number of hydroelectric power plants and reservoirs significantly changes the natural morphological processes [2, 3]. The considered section of the Volga River is located below the cascade of the HPP, and is an important part of the unique landscape of the Volga-Akhtuba floodplain [4, 5]. The sediment transfer changes the relief of the river bed, which affects the hydrological situation in the Volga-Akhtuba floodplain. These forms a need to update the available digital elevation models (DEM) to build reliable models of water distribution in the floodplain. The joint use of data from depth measurements (river lotto), gauging stations, remote sensing of the Earth and the results of numerical hydrodynamic modeling is an effective approach for updating the DEM.

Our goal is to create the high-quality hydrodynamic model of the Volga within 100 km to the south of the Volzhskaya HPP, since this region determines the nature of the spring flooding of the whole territory of the Volga-Akhtuba floodplain (VAF). This model is an important element of the research of the hydrological regime of the VAF for constructing an optimal hydrograph and improving the hydrological network of the floodplain.

2. Mathematical and numerical model
We use the shallow water equations in the form [4, 6, 7]:
\[
\frac{\partial H}{\partial t} + \frac{\partial Hu}{\partial x} + \frac{\partial Hv}{\partial y} = \sigma(x, y, t),
\]
\[(1)\]
\[
\frac{\partial (Hu)}{\partial t} + \frac{\partial (u^2 H)}{\partial x} + \frac{\partial (uvH)}{\partial y} = -gH \frac{\partial (H + b)}{\partial x} + 2vH\Omega_x \sin \theta + f_{s_x}^{\text{fric}} + f_{\sigma_x},
\]
\[(2)\]
\[
\frac{\partial (Hv)}{\partial t} + \frac{\partial (uvH)}{\partial x} + \frac{\partial (v^2 H)}{\partial y} = -gH \frac{\partial (H + b)}{\partial y} - 2uH\Omega_x \sin \theta + f_{s_y}^{\text{fric}} + f_{\sigma_y},
\]
\[(3)\]
where \(H\) is the depth, \(u\) and \(v\) are \(x\)- and \(y\)- components of the velocity, \(\sigma\) is the source function, \(g\) is gravitational acceleration, \(\Omega_x\) is the Earth’s angular velocity, \(\theta\) is the latitude, \(f_{s_x}\) and \(f_{\sigma_y}\) are specific force components due to the impulse of water source \(\sigma\).

We use the Chezy’s model for the bottom friction force: \(f_{s_x}^{\text{fric}} = -\frac{u}{2}\sqrt{u^2 + v^2}HA\), \(f_{s_y}^{\text{fric}} = -\frac{v}{2}\sqrt{u^2 + v^2}H\Lambda\), where \(\Lambda = \frac{2gn_H^2}{H^{4/3}}\), \(n_H\) is the Manning roughness coefficient.

The algorithm of Combined Smoothed Particle Hydrodynamics – Total Variation Diminishing (CSPH-TVD) is used for numerical integration of the equations system (1) – (3). Our approach combines the advantages of the Lagrangian SPH method and the Eulerian TVD method [8]. Such combined approach has proved its effectiveness in solving problems of flooding on complex topography. The CSPH-TVD scheme is well balanced, conservative, has the second-order of accuracy for smooth solutions and the first-order accuracy approximation near breaks and fracture profiles. We use parallel software for GPUs, which is described in detail in Refs. [6, 9]. All numerical experiments have been performed on the NVIDIA Tesla K80 [6].

3. Digital elevation model
Data for space sounding SRTM and Earth observation satellites "Resurs-P" with accuracy of 5 meters have been used to build digital terrain models [10, 11]. Modern topographic and pilot maps and results of new depth measurements for the Volga River have been used as additional sources of data.

Figure 1 shows the sequence of steps for building our DEM.
To combine all different high-altitude data into one height matrix, we use a special approximation method based on solution of the boundary-value problem for the Poisson equation:

\[
\frac{\partial^2 b}{\partial x^2} + \frac{\partial^2 b}{\partial y^2} = \rho(x, y), \tag{4}
\]

where \(S\) is the shoreline, \(G\) is the river region, \(\rho\) and \(\psi\) are given functions. The source function \(\rho\) is determined by the altitude data (topographic isolines, depth measurements, etc.). The numerical solution of Eq. (4) is determined on a grid with square cells \((x_n, y_m) = (nh, mh)\), \(h = \frac{1}{M}\), where \(M\) is an integer number.

We use the iterative approach for problem (4), solving the nonstationary analogue of the Poisson equation (heat equation):

\[
\frac{\partial b}{\partial t} = \frac{\partial^2 b}{\partial x^2} + \frac{\partial^2 b}{\partial y^2} - \rho(x, y). \tag{5}
\]

The solution of the last equation tends to the stationary distribution \(b(x, y)\) in case \(t \to \infty\) in accordance with Eq. (4). We consider the nonstationary implicit numerical scheme:

\[
\begin{align*}
\frac{b_{n+1,m}^p - b_{n,m}^p}{\tau} &= \frac{b_{n+1,m}^p - 2b_{n,m}^p + b_{n-1,m}^p}{h^2} + \frac{b_{n,m+1}^p - 2b_{n,m}^p + b_{n,m-1}^p}{h^2} - \rho(x_n, y_m), \\
\left| b_{n,m}^p \right| &= \psi(S_{n,m}), \\
b_{n,m}^0 &= \varphi(x_n, y_m).
\end{align*}
\tag{6}
\]

The stop condition for the iteration procedure is: \(\max(b_{n,m}^{p+1} - b_{n,m}^p) < \varepsilon\) (accuracy). The function \(\rho\) is determined through the experimental observational data \(\{B_{n,m}\} : \rho = \frac{B_{\text{exp}}(x_n, y_m)}{h^2}\).

\textbf{Figure 2.} The water distributions and velocity fields for two zones in our numerical model
The approximation of the relief elevation characteristics with the use of (4) allows obtaining the DEM with sufficient accuracy and takes into account all individual measurements of the depths. To solve the hydrodynamic problem (1) – (3), it is necessary to recalculate the height matrix into a grid with larger cells. We use digital elevation models (DEM) for main hydrodynamic computations with cell sizes of $\Delta x = \Delta y = 25 \text{ m}$.

Figure 2 shows the results of numerical simulation of water dynamics in typical spring flood conditions for the northern part of the Volga-Akhtuba floodplain.

The top insert shows the water velocities near the source of the Akhtuba River, which is the left distributary of the Volga River 6 km south of the Volzhskaya HPP. The insert on the right displays the water velocity field near the eastern part of Sarpinsky Island opposite the city of Volgograd.

4. Modeling water levels at gauging stations

We use an updated digital relief model to simulate spring flooding, which is determined by a hydrograph of water out-flow through the Volga Hydroelectric Station [4, 5]. Comparison of the results of numerical dynamics of shallow water with data at gauging stations allows assessing the quality of DEM. Figure 3 shows a good agreement of the curves in 2017.

![Figure 3](image3.png)

Figure 3. Temporary changes in the water level at the gauging station "Tailwater" from measurements (solid line) and the results of our simulations (dotted line)

![Figure 4](image4.png)

Figure 4. Average water level at the "Tailwater" gauging stations (solid line), "River Port" (blue dotted line) and "Svetlyj Jar" (red dashed line) during the low water period for the last years

For the comparative analysis, the average yearly water levels were calculated at three gauging stations in the low water period, when the volume of discharge water from the Volga hydroelectric station was from 4500 to 7500 m³/s (Figure 4). The gauging stations data indicate systematic changes for average water levels in low-water conditions within 1 m. These deviations are apparently due to the morphological changes in the river bed as a result of the movement of the sediments.

We modify the function $b(x,y)$ for minimization of the discrepancy between the data at the gauging stations, using the approach for varying the bottom surface [12].

The river bed of the Volga was divided into square sections. By varying the side length of such segment $l$ and the altitude characteristics by a random variable within the given range $\Delta z_{\text{max}}$, we fit the water levels in the numerical model with the data at the gauging stations (Figure 5). For large discrepancies in the water level at the gauging station, we have to more strongly perturb the section of the relief elevation characteristics with the use of (4) allows obtaining the DEM with sufficient accuracy and takes into account all individual measurements of the depths. To solve the hydrodynamic problem (1) – (3), it is necessary to recalculate the height matrix into a grid with larger cells. We use digital elevation models (DEM) for main hydrodynamic computations with cell sizes of $\Delta x = \Delta y = 25 \text{ m}$.

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the river bottom in the model. We can obtain strong water level changes in the model with a decrease in the size of \( l \) or with an increase of \( \Delta z_{\text{max}} \).

We achieved the reproduction of observational data with an accuracy of 0.1 m (Table 1) at typical values of the Volga depth about 5-12 m.

![Figure 5](image.png)

**Figure 5.** Average water levels at the gauging stations of the Lower Volga in the period from 2009 to 2013 (solid line) and simulation results (dashed line): a) Tailwater of the Volga HPP; b) River Port in Volgograd; c) Svetlyj Jar settlement

| Table 1. The deviations of the water level in numerical experiments and at the Volga River gauging stations |
| --- |
| **Tailwater of the Volga HPP** | 2009-2010 | 2010-2012 | 2012-2013 |
| Deviations at gauging station (m) | -0.35 | 0.66 | -0.21 |
| Simulation results (m) | -0.30 | 0.64 | -0.18 |
| **River Port in Volgograd** | 2009-2010 | 2010-2012 | 2012-2013 |
| Deviations at gauging station (m) | -0.35 | 0.20 | 0.21 |
| Simulation results (m) | -0.33 | 0.23 | 0.22 |
| **Svetlyj Jar settlement** | 2009-2010 | 2010-2012 | 2012-2013 |
| Deviations at gauging station (m) | -0.59 | 0.34 | 0.24 |
| Simulation results (m) | -0.61 | 0.27 | 0.25 |

5. Conclusion

To simulate the dynamics of flooding in real territory, it is necessary to use high-quality digital terrain models. Because of the continuous morphological changes in river channels and coastal zones, we have to clarify the DEM, accounting for a new actual data. Our analysis of hydrological data shows the presence of systematic changes in the relief of the Volga bottom. We constructed the numerical hydrodynamic model of the Volga River flow with the adjacent territories of the northern part of the Volga-Akhtuba floodplain. The updating of the DEM was carried out using the data of three gauging stations on the Volga. We have found a discrepancy between the water levels measured at the gauging stations and the numerical simulation results within 0.1 m.

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