Modeling heat and mass transfer processes during evaporation of a water film in a horizontal channel with a cocurrent flow of moist air

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Abstract. Modeling of heat and mass transfer processes in a horizontal channel during evaporative cooling of a moist air flow with regard to the finite thickness of the liquid film is considered. The mathematical model consists of a system of differential equations in the boundary layer approximation. The simulation results have been obtained in a wide range of initial parameters: temperature $T_0=10\div50^\circ\text{C}$, humidity $\varphi_0=0\div100\%$, Reynolds number $Re=100\div2000$. Calculations were carried out at atmospheric pressure. Quantitative analysis of influence of initial parameters of flows on values of parameters of wet air flow at the outlet of the channel with and without taking into account the final thickness of the water film was carried out.

Introduction

Review of literature sources showed that in modeling of evaporative air cooling in channels, thermal resistance of water film is neglected [1, 2]. Such an assumption may not always be fulfilled, so taking into account the finite thickness of the water film on heat and mass exchange processes is of great interest. In the present work, a study of conjugate heat and mass transfer at the joint flow of the water film and the flow of moist air in a horizontal channel (Figure 1) is conducted. The dimensions of the channel were taken as follows: $H=15\text{ mm}$ and $L=100H$.

The following assumptions were made when modeling heat and mass transfer processes:
- the flow modes of the moist air and the water film are laminar. Wave effects on the water surface were not considered;
- the vapor-gas mixture was an ideal gas;
- at the interface there is equality of temperatures, velocities, heat fluxes and tangential stresses;
- diffusion and thermal conductivity of wet air and water flows in the longitudinal were not considered;
- the influence of radiation heat transfer is neglected;
The system of equations
Modeling of hydrodynamic and heat and mass exchange processes was performed in two-dimensional formulation. The following differential equations can be written for the liquid film:

- the equation of continuity:
  \[ \frac{\partial (\rho_L u_L)}{\partial x} + \frac{\partial (\rho_L v_L)}{\partial y} = 0; \]  
  (1)

- motion:
  \[ \rho_L u_L \frac{\partial u_L}{\partial x} + \rho_L v_L \frac{\partial u_L}{\partial y} = \frac{\partial}{\partial y} \left( \mu_L \frac{\partial u_L}{\partial y} \right) - \frac{\partial P}{\partial x}; \]  
  (2)

- energy:
  \[ \rho_L c_p u_L \frac{\partial T_L}{\partial x} + \rho_L c_p v_L \frac{\partial T_L}{\partial y} = \frac{\partial}{\partial y} \left( \lambda_L \frac{\partial T_L}{\partial y} \right); \]  
  (3)

- diffusion:
  \[ K_L = 1. \]  
  (4)

The system of differential equations for the flow of moist air, taking into account the assumptions made, can be written in the following form:

- the equation of continuity:
  \[ \frac{\partial (\rho_A u_A)}{\partial x} + \frac{\partial (\rho_A v_A)}{\partial y} = 0; \]  
  (5)

- motion:
  \[ \rho_A u_A \frac{\partial u_A}{\partial x} + \rho_A v_A \frac{\partial u_A}{\partial y} = \frac{\partial}{\partial y} \left( \mu_A \frac{\partial u_A}{\partial y} \right) - \frac{\partial P}{\partial x}; \]  
  (6)

- energy:
  \[ \rho_A c_{p_A} u_A \frac{\partial T_A}{\partial x} + \rho_A c_{p_A} v_A \frac{\partial T_A}{\partial y} = \frac{\partial}{\partial y} \left( \lambda_A \frac{\partial T_A}{\partial y} \right) + \rho_A D \left( \epsilon_{p_A} - \epsilon_{p_A} \right) \frac{\partial T_A}{\partial y}; \]  
  (7)

- diffusion:
  \[ \rho_A u_A \frac{\partial K_A}{\partial x} + \rho_A v_A \frac{\partial K_A}{\partial y} = \frac{\partial}{\partial y} \left( \rho_A D \frac{\partial K_A}{\partial y} \right). \]  
  (8)
Boundary conditions:

- at the entrance to the channel \((x = 0)\):
  \[u_L = u_{0,L}, \ T_L = T_{0,L}, \ K_L = 1, \ u_A = u_{0,A}, \ T_A = T_{0,A}, \ K_A = K_{0,A};\]  \(\text{(9)}\)

- on the walls:
  \[y = 0, \ u_L = 0, \ v_L = 0, \ q_{w1} = -\lambda_L \left(\frac{\partial T_L}{\partial y}\right)_{y=0}, \ K_L = 1;\]  \(\text{(10)}\)

  \[y = H, \ u_A = 0, \ v_A = 0, \ q_{w2} = -\lambda_A \left(\frac{\partial T_A}{\partial y}\right)_{y=H}, \ K_A = 0;\]  \(\text{(11)}\)

- on the water film surface \((y = \delta)\):
  \[u_L = u_A\]  \(\text{(12)}\)
  and

  \[\mu_L \left(\frac{\partial u_L}{\partial y}\right)_{y=0} = \mu_A \left(\frac{\partial u_A}{\partial y}\right)_{y=0};\]  \(\text{(13)}\)

  \[v_A = -\frac{D}{1 - K_y} \left(\frac{\partial K_A}{\partial y}\right)_{y=0};\]  \(\text{(14)}\)

  \[-\lambda_L \left(\frac{\partial T_L}{\partial y}\right)_{y=0} = -\lambda_A \left(\frac{\partial T_A}{\partial y}\right)_{y=0} - \frac{\rho_A D}{1 - K_{saty}} \left(\frac{\partial K_A}{\partial y}\right)_{y=0}.\]  \(\text{(15)}\)

According to Dalton's law for an ideal gas:

\[K_{saty} = \frac{m_L / m_A}{P / P_{sat} + m_L / m_A - 1},\]  \(\text{(16)}\)

where \(m_L = 18\) and \(m_A = 29\) are the molecular masses of water and air, respectively and \(P_{sat}\) – saturation pressure.

**Numerical realization**

The solution of the specified system of differential equations \((1)-(8)\) together with boundary conditions \((9)-(16)\) was carried out numerically. The simulation program was implemented in the Fortran programming language. Test computational experiments have shown that it is reasonable to divide the computational domain into \(400 \times 80 \times 120\) cells (in the longitudinal direction -400 cells, in the transverse directions: 80 for the water film, 120 for the humid air flow).

The results of the numerical simulation program were the main parameters of the flows:

- bulk temperatures: \(T_{w>A} = \int_0^H \rho_A u_A T_A dy \ / \int_0^H \rho_A u_A dy, \ T_{w>L} = \int_0^H \rho_L u_L T_L dy \ / \int_0^H \rho_L u_L dy;\)

- bulk vapor concentration: \(K_{w>A} = \int_0^H \rho_A u_A K_A dy \ / \int_0^H \rho_A u_A dy.\)

Verification of the results of the developed modeling program was carried out by comparing the obtained data and the works [3, 4]. Figure 2 shows the local profiles of velocities, temperatures and...
concentrations obtained at $R_e = \frac{u_{0,A}^2 H}{v} = 2000$, $B_0 = 0.04$ kg/(m·s), $T_{0,A} = T_{0,L} = 20 ^\circ$C, $\varphi_g = 50 \%$, $q_{w1} = 1000$ W/m$^2$, $q_{w2} = 0$, $H = 15$ mm and $L = 100H$ mm. When analyzing the data, dimensionless profiles of temperatures and concentrations were used in the form of:

$$\theta = \frac{T - T_{0,A}}{T_{w1} - T_{0,A}}$$

$$\chi = \frac{K - K_{0,A}}{K_{surf} - K_{0,A}}$$

where $T_{w1}$ is the wall temperature at $y = 0$.

**Figure 2.** Comparison of simulation results and data of W.M. Yan [3, 4]:

- $a$ – velocity profile; $b$ – temperature profile; $c$ – vapour concentration profile

When studying heat and mass transfer processes in channels, an important issue is to assess the degree of influence of the initial flow rate (Reynolds number) on changes in the flow parameters along the length. Figure 3 shows the effect of the Reynolds number of air flow on the change in temperature and concentration on the surface of the water film when compared with the results of [3, 4].

It follows from this figure that a decrease in the initial flow rate (or thickness) of the water film leads to an increase in the parameters at the interface.

A comparison of the curves for the zero ($B_0 = 0$) and the final thickness of the water film indicates that the difference between the determined parameters increases with growth of $B_0$.

A comparison of the dependences shown in Figure 2 and Figure 3 allows concluding that the results obtained in this work and the data of Yan [3, 4] are qualitatively similar. Some quantitative deviations can be explained by the fact that different empirical dependences of the thermodynamic properties of moist air and water were used in the modeling.
Figure 3. Temperature (a) and concentration (b) on the surface of the water film
\( (R_{e_1} = 2000, T_{o,A} = T_{o,E} = 20\,^\circ C, \varphi_0 = 50\%, q_{w_1} = 1000\, W/m^2, \varphi_{w_2} = 0). \)
Solid lines are obtained in the present work. Points are data of Yan W.M. [3, 4]

Figure 4 shows the graphical dependences of the influence of the initial parameters (such as temperature, relative humidity and Reynolds number) on the changes in the average mass characteristics of the gas and liquid phases of the flows. Numerical results show that increasing the Reynolds number intensifies the processes of evaporation heat transfer, resulting in a stronger reduction of the water film temperature than the flow of moist air.

Figure 4. Influence of the initial parameters \( (T_{o,A} = T_{o,E}, q_{w_1} = q_{w_2} = 0):\)
\( a - R_{e_1} = 500, \varphi_0 = 0; b - T_{o,A} = T_{o,E} = 30\,^\circ C, \varphi_0 = 0; c - R_{e_1} = 500, T_{o,A} = T_{o,E} = 30\,^\circ C\)

It also follows from Figure 4 that an increase in the initial flow rate (initial thickness) of the water film leads to a sharp change in the thermophysical parameters of the moist air flow, which is
characterized by an increase in the temperature at the channel outlet, which ultimately leads to a deterioration in evaporative cooling of air or water as the flow rate of the liquid film increases.

**Conclusion**

The model of evaporative cooling considered in this work allows to analyze the influence of the water film thickness on the flow parameters at the outlet of the channel, as well as to quantify the influence of air humidity on the intensity of heat and mass exchange processes. It was found that the initial thickness of the water film significantly affects the cooling effect of the humid air flow. It is shown quantitatively that the maximum cooling effect of the flange air flow is observed at rather low values of Reynolds number (300-700 depending on initial conditions).

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