Chen, Joe P.; Kudler-Flam, Jonah
Laplacian growth and sandpiles on the Sierpiński gasket: limit shape universality and exact solutions. (English) Zbl 1454.05043
Ann. Inst. Henri Poincaré D, Comb. Phys. Interact. (AIHPD) 7, No. 4, 585-664 (2020).

Summary: We establish quantitative spherical shape theorems for rotor-router aggregation and abelian sandpile growth on the graphical Sierpiński gasket (SG) when particles are launched from the corner vertex.

In particular, the abelian sandpile growth problem is exactly solved via a recursive construction of self-similar sandpile tiles. We show that sandpile growth and patterns exhibit a $(2 \cdot 3^n)$-periodicity as a function of the initial mass. Moreover, the cluster explodes – increments by more than 1 in radius – at periodic intervals, a phenomenon not seen on $\mathbb{Z}^d$ or trees. We explicitly characterize all the radial jumps, and use the renewal theorem to prove the scaling limit of the cluster radius, which satisfies a power law modulated by log-periodic oscillations. In the course of our proofs we also establish structural identities of the sandpile groups of subgraphs of SG with two different boundary conditions, notably the corresponding identity elements conjectured by S. Fairchild et al. ["The abelian sand-pile model on fractal graphs", Preprint, arXiv:1602.03424].

Our main theorems, in conjunction with recent results of J. P. Chen et al. ["Internal DLA on Sierpiński gasket graphs", Preprint, arXiv:1702.04017], establish SG as a positive example of a state space which exhibits "limit shape universality," in the sense of L. Levine and Y. Peres [Bull. Am. Math. Soc., New Ser. 54, No. 3, 355-382 (2017; Zbl 1451.37013)], among the four Laplacian growth models: divisible sandpiles, abelian sandpiles, rotor-router aggregation, and internal diffusion-limited aggregation (IDLA). We conclude the paper with conjectures about radial fluctuations in IDLA on SG, possible extensions of limit shape universality to other state spaces, and related open problems.

MSC:
05C20 Directed graphs (digraphs), tournaments
05C81 Random walks on graphs
05E18 Group actions on combinatorial structures
28A80 Fractals
31A15 Potentials and capacity, harmonic measure, extremal length and related notions in two dimensions
37B15 Dynamical aspects of cellular automata
60K05 Renewal theory
82C24 Interface problems; diffusion-limited aggregation in time-dependent statistical mechanics

Keywords:
Laplacian growth; rotor-router aggregation; divisible sandpiles; abelian sandpiles; sandpile group; internal diffusion-limited aggregation; harmonic measure; analysis on fractals; Sierpiński arrowhead curve; self-similarity; limit shapes; exact renormalization

Software:
GitHub; AutomataSG

Full Text: DOI arXiv

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