Analysis of chaotic bright spin-wave soliton trains in magnetic films

A B Ustinov$^1$, A V Kondrashov$^1$ and A A Nikitin$^2$

$^1$St. Petersburg Electrotechnical University, St. Petersburg, 197376 Russia
$^2$ITMO University, St. Petersburg, 197101 Russia

E-mail: ustinov_rus@yahoo.com

Abstract. We have experimentally studied a chaotic dynamics of bright spin-wave soliton trains. Thin yttrium iron garnet films having pinned surface spins were used for the experiments providing propagation of highly dispersive dipole-exchange spin waves. Chaotic soliton trains were excited in the frequency band around low-frequency part of dipole gap of spin wave spectrum. Analysis of fractal dimension, embedding dimension, and Lyapunov exponents was carried out based on measured time profiles of chaotic soliton sequences. We find that the fractal dimension and Lyapunov exponents are weak function of carrier frequency around dipole gap whereas embedding dimension is almost constant.

1. Introduction

Envelope solitons are formed in the nonlinear dispersive waveguiding media with pulsed excitation if the dispersion spreading of a nonlinear wave packet is compensated by the media nonlinearity. Solitons are also formed with a monochromatic wave excitation through development of the spontaneous modulation instability (SMI) [1,2].

Among other media, high-quality magnetic films, such as single-crystal yttrium iron garnet (YIG) films, were proven to be excellent objects for experiments on nonlinear wave phenomena with microwave spin waves. We underline, that it was the YIG films where a formation of the stationary soliton trains from a monochromatic spin wave (SW) through development of the spontaneous modulation instability (SMI) has been discovered and studied [3-5].

Theoretically, the soliton as a stationary solution of the nonlinear Schrodinger (NLS) equation happens to be stable. Experimentally, the soliton is also usually considered as a stable excitation which preserves its shape and speed during propagation. At the same time, using an optical mode-locked laser, Bolton and Acton [6] have demonstrated in 2000 year a chaotic behavior of solitons which represented pulsations in amplitude of optical pulses circulating in the optical cavity. After that considerable amount of theoretical work was devoted to study of possible chaotic behavior of solitons in the different physical systems [7-13]. The concept of “dissipative solitons” [14,15] resolves the contradiction.

At present, studies on chaotic wave excitations in waveguiding media and auto-oscillators are of great interest both for basic research and for their possible applications in advanced optical and microwave communication technology [16-18]. Recently, the chaotic behavior of solitons have been observed experimentally for spin waves propagating in magnetic films [19]. It is resulted in a new wave of interest for both theoretical and experimental studies of chaotic dynamics of solitons [20-33].
Purpose of this work is analysis of the chaotic dynamics of solitons formed with monochromatic excitation of dipole-exchange spin waves through simultaneous development of the SMI and the chain of spin-wave parametric interactions in an yttrium iron garnet (YIG) film.

2. Experimental set-up

A 3.1-µm-thick, 1.5-mm-wide, and 30-mm-long YIG film waveguide was used for the experiments. The film was grown on a (111) plane single-crystal gadolinium gallium garnet substrate by the method of liquid phase epitaxy. The YIG film had a ferromagnetic resonance line-width of 0.4 Oe at 3 GHz, and a saturation magnetization $4\pi M_s$ of 1750 G. The YIG film waveguide was magnetized by external constant bias field $H_e$ of 2525 Oe directed perpendicular to the film plane. Note, that the YIG film is assumed to be magnetically isotropic and an internal bias field value calculated as $H_i = H_e - 4\pi M_s$ was 775 Oe that was in good agreement with measurements. The perpendicular magnetization provided a forward volume spin wave (FVSW) propagation in the waveguide.

The film had pinned surface spins and the excitations generated in the experiments were dipole-exchange spin waves demonstrating strong decrease in their group velocity at frequencies close to spin-wave resonances. The spin wave spectrum has so-called “dipole gaps” at these frequencies (see the figure 1). Similar to the paper [19], it is the existence of the dipole gaps that is the key condition for formation of the chaotic soliton trains.

As is well known the pinning of the surface spins arise from surface anisotropy (see e.g. [34]). Physical reason consists in the following. Fields affecting on magnetic moments in the vicinity of the interface between magnetic and non-magnetic materials are differ from those inside the magnetic material. If we do not want to investigate in details magnetic oscillations in these fields, we should take into account their influence on magnetic oscillations inside the ferromagnetic film with an introduction of the boundary conditions (assuming the thickness of the surface interface layer negligibly small). Using this approach it is possible to estimate frequencies of spin-wave resonances (see. e.g. [35]) which coincide with high precision with frequencies of dipole gaps of the spin-wave spectra (calculated with theory published in [36]) shown in figure 1(a) and with frequencies of dips in the amplitude-frequency characteristic shown in figure 1(b). Technological reasons of pinning of surface spins for magnetic film sample used in our experiments are out of the present research.

The spin waves in the YIG film waveguide were excited and detected by conventional microstrip transducers separated by a distance of 4.5 mm. The transducers had a width of 40 µm and a length of 3 mm.

![Figure 1](image_url)

**Figure 1.** Dipole-exchange spin wave spectrum (a) and amplitude-frequency characteristic (b) of the YIG-film structure
3. Experimental results and discussion

In the experiments a highly monochromatic signal was applied from a microwave source to the input microstrip transducer so as to excite a continuous wave (CW) spin wave in the YIG-film waveguide. The spin waves after propagation the distance of 4.5 mm were detected by the output transducer and then monitored with a microwave spectrum analyzer and a real-time digital oscilloscope having a high sample rate and a long acquisition. It allowed us to record the waveforms during the long time for further mathematical processing. During the measurements the CW input power $P_{in}$ was systematically varied from 0.1 to 40 mW. The CW frequency was preset in the SW excitation range, for the given experimental condition from the FWSW cutoff frequency of 2174 MHz to 2300 MHz at which output signal power level became small and comparable with noise level (see the figure 1).

A formation of the chaotic solitons was found at frequencies around 2247 MHz for the input power level $P_{in}$ around 35 mW. Detailed analysis of the shape of the observed nonlinear excitations was carried out by the method described in our previous work [19] in which chaotic spin-wave soliton trains were observed for the first time. This analysis confirmed solitonic nature of the chaotic pulses.

Figure 2 shows representative data obtained for the case when the monochromatic SW with power of 15.3 dBm at frequency 2247.3 MHz was excited in the waveguide. It is clear that peak power, period, and width of the solitons were chaotically varied.

![Figure 2](image)

**Figure 2.** Chaotic bright soliton train (a) and its frequency spectrum (b) at carrier frequency $f = 2247.3$ MHz.

The observed nonlinear dynamics of the soliton trains contained elements of both regular motion and chaos simultaneously. The regular motion was due to the development of the SMI of the spin waves excited in the film, whereas the chaotic motion was arisen by the development of the chain of parametric interactions (CPI) among the generated SW. It is clearly confirmed by the power-frequency spectrum of the chaotic solitons (see the figure 2(b)), which demonstrate elements of both SMI in the form of the well pronounced equidistant separate harmonics and CPI in the form of the pronounced continuous noise-like pedestal. The carrier frequency around which formation of the chaotic soliton trains took place is shown in the figure 1(a) by circle.

Numerical estimation of the obtained chaotic signal parameters for different values of input signal frequency was done. First multidimensional phase portraits for all frequencies were reconstructed. Conventional method of time delay was used [37]. Dynamic of the spin-system in the phase space is represented by a phase trajectories. Phase trajectory point coordinates were obtained using long waveforms as follows

$$x(t) = [U(t), U(t + \tau), ..., U(t + (d - 1)\tau)]$$

where $\tau$ is time delay, $d$ is a phase space dimension and $U(t)$ is the amplitude of analyzed signal. A form of phase trajectories is defined by an attractor which attracts trajectories of phase space. Regular
dynamic phase portrait is defined by so-called regular attractor and chaotic dynamics is defined by a so-called strange attractor. The 2D phase portrait of the observed solitonic chaos is shown in figure 3. According to the paper [37] the phase portrait obtained with the above mentioned method allows to define correctly such attractor parameters as fractal dimensions, minimum embedding dimensions, Lyapunov exponents etc. The values of the chaotic signal parameters allows to define whether the signal chaotic or noisy.

The values of the fractal dimension and minimum embedding dimension were calculated using the standard Grassberger-Procaccia method [38,39]. First, correlation dimensions for several phase space dimensions $d$ were determined. Correlation dimension determines topology of the attractor projection into the phase space with certain dimension $d$. Due to increase of the phase space the form of attractor projection is encloses to the form of original attractor that leads to an increase in correlation dimension up to some saturation level. The saturation value is the fractal dimension of the attractor $D_f$. The value of the phase space dimension for which saturation begins is a minimum embedding dimension. Fractal dimension defines topology of the whole attractor. The attractor corresponding to the monochromatic signal is a stable point and its fractal dimension is zero. The attractor of a periodic signal is a limit cycle. Its fractal dimension equals 1. A noise signal phase trajectory fills up the phase space densely, therefore corresponding fractal dimension is infinite [40]. Dynamical chaos is characterized by appearance of the strange attractor in the phase space. Fractal dimension of the strange attractor is finite and no integer. Minimum embedding dimension defines a degrees of freedom quantity of the investigated spin system. Its value is proportional to required equation quantity for the system modeling.

Using described methods, values of fractal dimensions and minimum embedding dimensions for some particular frequencies of input signal were calculated. The results of calculation are represented in the table 1. The values of $D_f$ and $d$ demonstrate that in addition to regular soliton motion the chaotic modulation of the soliton shape takes place. Calculation results show that in the range of input frequency from 2246.0 MHz to 2247.7 MHz an increase in the frequency leads to fractal dimension weak increase from 5.9 to 6.2. It means that the behavior of the investigated spin-system becomes more complicate with frequency increasing. The values of the minimum embedding dimension are almost constant. It means that a change in the input frequency does not increases or decreases the number of degrees of freedom of the spin system.

Another parameter which was calculated is maximum Lyapunov exponent $\lambda$. It characterizes the rate of exponential divergence of phase trajectories. Dissipative systems are characterized by negative Lyapunov exponent. Zero exponents correspond to periodical motion. For the case of deterministic chaos maximum Lyapunov exponent is positive.

Figure 3. Attractor of the chaotic solitons at carrier frequency $f$ = 2247.3 MHz.
Table 1. Dependence of the fractal dimension, minimum embedding dimension, Lyapunov exponent, and modulation frequency from input frequency.

| Input signal frequencies (MHz) | 2246.0 | 2246.2 | 2246.3 | 2246.4 | 2246.5 | 2246.6 | 2246.7 | 2247.3 | 2247.7 |
|-------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $D_f$                         | 5.9    | 5.9    | 6.0    | 5.9    | 5.8    | 5.8    | 6.1    | 6.2    |        |
| $d_{em}$                      | 21     | 22     | 22     | 22     | 22     | 21     | 22     | 22     |        |
| $\lambda$                     | 0.08   | 0.073  | 0.087  | 0.08   | 0.09   | 0.081  | 0.086  | 0.083  | 0.091  |
| $\Delta f_{mod}$ (MHz)        | 2.9    | 2.95   | 2.93   | 2.98   | 2.95   | 2.95   | 2.98   | 2.97   | 2.92   |

For calculation of the Lyapunov exponents an algorithm introduced by Rosenstein and by Kanz was used [41]. The algorithm allows to calculate the average divergence of trajectories that starts from close placed points. For calculation the TISEAN package [42] was used.

Using the TISEAN values of maximum Lyapunov exponents $\lambda$ for different frequencies of the input signal were determined. In the table 1 the dependence of the Lyapunov exponent on input frequency is shown. Positive values of $\lambda$ confirm that observed solitonic signals demonstrates chaotic dynamics. Similar to fractal dimension an increase in the input signal frequency leads to weak increase of $\lambda$.

Calculations of the parameters shows that the chaotic soliton motion of the spin system depends weakly on the frequency of the input signal.

4. Conclusion
In this work the chaotic dynamics of soliton trains was studied. The results confirm deterministic nature of the observed chaotic soliton sequences.

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