Continuous-variable quantum key distribution with non-Gaussian quantum catalysis

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The non-Gaussian operation can be used not only to enhance and distill the entanglement between Gaussian entangled states, but also to improve quantum communications. In this paper, we propose an non-Gaussian continuous-variable quantum key distribution (CVQKD) by using quantum catalysis (QC), which is an intriguing non-Gaussian operation in essence that can be implemented with current technologies. We perform quantum catalysis on both ends of the Einstein-Podolsky-Rosen (EPR) pair prepared by a sender, Alice, and find that for the single-photon QC-CVQKD, the bilateral symmetrical quantum catalysis (BSQC) performs better than the single-side quantum catalysis (SSQC). Attributing to characteristics of integral within an ordered product (IWOP) of operators, we find that the quantum catalysis operation can improve the entanglement property of Gaussian entangled states by enhancing the success probability of non-Gaussian operation, leading to the improvement of the QC-CVQKD system. As a comparison, the QC-CVQKD system involving zero-photon and single-photon quantum catalysis outperforms the previous non-Gaussian CVQKD scheme via photon subtraction in terms of secret key rate, maximal transmission distance and tolerable excess noise.

I. INTRODUCTION

Quantum key distribution (QKD) [1–5], as one of the mature practical applications in quantum information processing, allows two distant legitimate parties (normally a sender, Alice and a receiver, Bob) to establish a set of secure keys even in the presence of the untrusted environment controlled by an eavesdropper (Eve), and its unconditional security can be guaranteed by the laws of quantum physics, e.g., the uncertainty principle [6] and the non-cloning theorem [7]. In general, QKD mainly includes two families, namely discrete-variable quantum key distribution (DVQKD) and continuous-variable quantum key distribution (CVQKD) [2, 8–15]. Due to the limitation of single-photon detectors employed in DVQKD systems, in CVQKD, the sender Alice encodes the secret information on the quadratures of the optical field with Gaussian modulation and the receiver Bob decodes the secret information with high-speed and high-efficiency homodyne or heterodyne detection so that CVQKD has become the center of attention in recent years [16–17]. In addition, since the security proofs of the Gaussian-modulated CVQKD protocols against collective attacks [16–18] and coherent attacks [19–20] have been proven experimentally [21–23], the Gaussian-modulated CVQKD protocols take on the potential application prospects of long-distance communication. Among them, the Grosshans-Grangier 2002 (GG02) protocol [17] performs outstandingly over short distance, but seems unfortunately to be facing the problem of long-secure distance compared with its DVQKD counterpart.

Till now, many remarkable theoretical and experimental efforts have been devoted to extending the maximal transmission distance with high rate in CVQKD systems [23–28]. By the use of multidimensional reconciliation protocols in the regime of low signal-to-noise ratio (SNR) [23], it was demonstrated experimentally that CVQKD over 80 km transmission distance can be realized. The reason is that the multidimensional reconciliation is, in a sense, to design a suitable reconciliation code with high efficiency even at low SNR, which can increase the secure distance [28]. Alternatively, the discrete modulation protocols such as the four-state protocol [13, 27–29, 30] and eight-state protocol [31] were shown to improve the secure distance as there does exist suitable error-correlation codes with high efficiency for discrete possible values at low SNR. Especially for eight state protocol, not only can the secret key rate be improved, but the transmission distance more than 100 km can be achieved [31–32].

From a practical point of view, the maximal transmission distance and the unconditional security of the secret key are usually disturbed by the environmental noise and dissipation. To solve these problems, the methods of source monitoring [33] and linear optics cloning machine [34] have been proposed subsequently.

Thanks to the development of experimental techniques, on the other hand, some quantum operations have been used to improve the performance of CVQKD in terms of secret key rate and tolerable excess noise. For example, a heralded noiseless amplifier [26, 27, 35] was proposed to improve the maximal transmission distance roughly by the equivalent of $20\log_{10} g$ dB losses resulting from the compensation of the losses [27]. Recently, due to the fact that the non-Gaussian operation can be used for improving the entanglement [36–38] and quan-
tum teleportation in CV system \cite{39} \cite{40}, the photon-subtraction operation, which is one of the non-Gaussian operations, has been proposed to improve the secret key rate, the maximal tolerable excess noise and the transmission distance of CVQKD protocol \cite{11} \cite{14} \cite{15} \cite{29}. In particular, the single-photon subtraction operation in the enhanced CVQKD protocol outperforms other numbers of photon subtraction. Unfortunately, the success probability for implementing this single-photon subtraction operation at the variance of two-mode squeezed vacuum (TMSV) state \( V = 2 \) is limited to below 0.25, which may lead to loss more information between Alice and Bob in the process of extracting the secret key. In order to overcome the limitation, in this paper, we propose an improved modulation scheme for CVQKD by using another non-Gaussian operation, the quantum catalysis (QC) \cite{41}, which can be implemented with current experimental technologies. Attractively, the quantum catalysis operation is a feasible way to enhance the nonclassicality \cite{42} and the entanglement property of Gaussian entangled states \cite{43}, thereby has become one of the research hotspots in quantum physics. Different from the previous studied photon-subtraction operations, although no photon is subtracted and added in quantum catalysis process, quantum catalysis can be applied to facilitate the conversion of the target ensemble, which could prevent the loss of information effectively. Numerical simulation shows that the entanglement and the success probability for implementing quantum catalysis can be improved efficiently. Specifically, the success probability for implementing zero-photon quantum catalysis can be dramatically enhanced when compared with the previous CVQKD with single-photon subtraction. In addition, we illustrate the performance of QC-CVQKD with different photon-catalyzed numbers, and find that zero-photon and single-photon catalysis presents the well performance when optimized over the transmittance \( T \) of Alice’s beam splitter (BS).

This paper is organized as follows. In Sec.II, we suggest a quantum catalysis operator, and detail the process of QC-CVQKD. In Sec.III, the success probability and the entanglement property for implementing quantum catalysis are analyzed, and security analysis for QC-CVQKD system is subsequently discussed. Finally, conclusions are drawn in Sec.IV.

II. QUANTUM CATALYSIS-BASED CVQKD

To make the derivation self-contained, we suggest quantum catalysis operation by using the technique of integral within an ordered product (IWOP) of operators \cite{44}, and then detail the QC-CVQKD.

FIG. 1. (Color online) Schematic diagram of the non-Gaussian operations. (a) The quantum catalysis (QC). An \( n \)-photon Fock state \(| n \rangle \) in auxiliary mode C is split on the asymmetrical beam splitter (BS) with transmittance \( T_2 \). Subsequently, a photon detector (PD) at the auxiliary mode is performed by the conditional detection of \(| n \rangle \), which is the so-called \( n \)-photon quantum catalysis represented by an equivalent operator \( O_n \). (b) The \( n \)-photon-subtraction operation. A vacuum state \(| 0 \rangle \) in auxiliary mode C is injected into the asymmetrical beam splitter (BS) with transmittance \( T_2 \). Likewise, a photon detector (PD) at the auxiliary mode is performed by the conditional detection of \(| n \rangle \).

A. Quantum catalysis operation

As shown in Fig.1(a), an \( n \)-photon Fock state \(| n \rangle \) in auxiliary mode C is injected at one of the input ports of BS with transmittance \( T_2 \), and simultaneously detected at the corresponding output port of BS, which is the so-called \( n \)-photon catalysis. In fact, this quantum catalysis process is often regarded as an equivalent operator \( O_n \) given by

\[
\hat{O}_n \equiv_C \langle n| B \ (T_2) \ | n \rangle_C ,
\]

where \( B \ (T_2) \) is the BS operator with transmittance \( T_2 \). To obtain the specific expression of the equivalent operator \( \hat{O}_n \), we employ the normally order form of \( B \ (T_2) \) by the IWOP technique and the coherent state representation of Fock state \(| n \rangle \), which are respectively expressed as 

\[
B \ (T) =: \exp((\sqrt{T_2} - 1)(b^\dagger b + c^\dagger c) + (c^\dagger b - cb^\dagger) \sqrt{1 - T_2}) : \]

and \(| n \rangle = 1/\sqrt{n!} \frac{\partial^n}{\partial \beta^n} | \beta \rangle \) where the notations 

\[
| \beta \rangle = \exp(\beta c^\dagger) | 0 \rangle
\]

represent a normally ordering of operator and an un-normalized coherent state, respectively. As a result, Eq.1 can be described as

\[
\hat{O}_n =: L_n \left( 1 - \frac{T_2}{T} b^\dagger b \right) \left( \sqrt{T_2} \right)^{b^\dagger b + n} ,
\]

where \( L_n (\cdot) \) denotes the Laguerre polynomials (see Refs.\cite{42} \cite{43} for the detailed calculation). By using the generating function of Laguerre polynomials, i.e.,

\[
L_n (x) = \frac{\partial^n}{\partial \gamma^n} e^{-x/\gamma} \left\{ \frac{e^{-\gamma x}}{1 - \gamma} \right\}_{\gamma = 0} ,
\]

\[
= \frac{\partial^n}{\partial \gamma^n} e^{-x/\gamma} \left\{ \frac{e^{-\gamma x}}{1 - \gamma} \right\}_{\gamma = 0} ,
\]
and the operator relation $e^{ab^*b} = \exp \{ (e^a - 1) b^*b \}$, Eq.(3) can be further rewritten as
\[ \hat{O}_n = G T_2 (b^*b) \left( \sqrt{T_2} \right)^{b^*b+n}, \tag{4} \]
where
\[ G T_2 (b^*b) = \frac{\partial^n}{n! \partial \gamma^n} \left\{ \frac{1}{1-\gamma} \left( \frac{1}{1-\gamma/T_2} \right)^{b^*b} \right\} |_{\gamma=0}. \tag{5} \]

From Eq.(4), we find that the quantum catalysis operation belongs to a kind of non-Gaussian operation. Moreover, as shown in Fig.1(a), for an arbitrary input state $|\varphi\rangle_{in}$ in mode B, the output state $|\psi\rangle_{out}$ can be expressed as $|\psi\rangle_{out} = \hat{O}_n |\varphi\rangle_{in}$, with the success probability $p$ for implementing the n-photon catalysis operation $\hat{O}_n$, which is beneficial for calculating the analytical expressions of the Alice output state and the covariance matrix between Alice and Bob in the following. In addition, different from the n-photon-subtraction operation shown in Fig.1(b), no photon is subtracted or added in n-photon catalysis operation. Such operation facilitates the transformation between input and output states, thereby effectively preventing useful information from being lost. However, no matter how many photons are catalyzed or subtracted, there is no quantum-catalysis or photon-subtraction effect when $T_2 = 1$.

### B. The CVQKD protocol with quantum catalysis

In what follows, we elaborate the schematic diagram of the QC-CVQKD protocol, as shown in Fig.2. The sender, Alice generates a TMSV state (which is also called as an Einstein-Podolsky-Rosen (EPR) pair) involving two modes A and B with a modulation variance V, which is usually expressed as the two-mode squeezed operator $S_2 (r) = \exp \{ r (a^*b - ab) \}$ on the two-mode vacuum state $|00\rangle_{AB}$, i.e.,
\[ |TMSV\rangle_{AB} = S_2 (r) |00\rangle_{AB} = \sqrt{1 - \lambda^2} \sum_{l=0}^{\infty} \lambda^l |l,l\rangle_{AB}, \tag{6} \]

where $\lambda = \tanh r = \sqrt{(V - 1)/(V + 1)}$ for $V = 2a^2 + 1$, and $|l,l\rangle_{AB} = |l\rangle_A \otimes |l\rangle_B$ denotes the two-mode Fock state of both modes A and B. After that, she performs $m$-photon and $n$-photon catalysis operations in modes A and B, respectively, giving birth to the state $|\psi\rangle_{A,B}$.

Note that, in mode A, inserting another quantum catalysis operation $\hat{O}_m$ at the end of the transmission to Alice is designed to figure out what effect quantum catalysis has on the information between Alice and Bob, when comparing with the single-side quantum catalysis $\hat{O}_n$ case.

According to the afore-mentioned method of quantum catalysis operation, likewise in Eq.(4), we obtain the m-photon quantum catalysis operation, i.e.,
\[ \hat{O}_m = G T_1 (a^*a) \left( \sqrt{T_1} \right)^{a^*a+m}, \tag{7} \]
with the notation $G T_1 (a^*a)$ given by
\[ G T_1 (a^*a) = \frac{\partial^m}{m! \partial \tau^m} \left\{ \frac{1}{1-\tau} \left( \frac{1}{1-\tau/T_1} \right)^{a^*a} \right\} |_{\tau=0}. \tag{8} \]

Then, the yielded state $|\psi\rangle_{A,B}$ turns out to be
\[ |\psi\rangle_{A,B} = \frac{\hat{O}_m \hat{O}_n}{\sqrt{T_d}} |TMSV\rangle = \sum_{l=0}^{\infty} \frac{W_0}{\sqrt{T_d}} \frac{\partial^n}{\partial \gamma^n} W_l^{\gamma} \left( 1-\tau/(1-\gamma) \right) |l,l\rangle_{AB}, \tag{9} \]
where $P_d$ denotes the success probability of implementing quantum catalysis, which is an important indicator that affects the mutual information in the process of distilling a common secret key between Alice and Bob, and can be calculated as
\[ P_d = W_0^2 |\mathbb{R}^m,n \Pi \left( 1-W/W_1 \right) |, \tag{10} \]

with $\mathbb{R}^m,n, \Pi, W_0, W$ and $W_1$ defined in Eq.(A2). Detailed calculations of the success probability $P_d$ can be shown in Appendix A. From Eq.(9), the state $|\psi\rangle_{A,B}$ becomes a non-Gaussian entangled state.

At Alice’s station, the quadratures of both $x_A$ and $p_A$ are measured via heterodyne detection on the incoming one half of the state $|\psi\rangle_{A,B}$, and the other half of $|\psi\rangle_{A,B}$ is sent to Bob through an insecure quantum channel that can be controlled by Eve with the transmission efficiency $T_c$ and the excess noise $\varepsilon$. After receiving the state, Bob randomly chooses to measure either $x_B$ or $p_B$ via homodyne detection and informs Alice about the measured observable. Finally, Alice and Bob can share a string of secret keys by data-postprocessing.

Before deriving the performance of the QC-CVQKD protocol, we demonstrate the entanglement of both the Gaussian entangled state $|TMSV\rangle_{AB}$ and the transformed state $|\psi\rangle_{A,B}$. As a computable measurement of entanglement and an upper bound on the distillable entanglement, the logarithmic negativity is usually used to quantify the degree of entanglement, which is given by
\[ E_N = \log_2 \| \rho^{PT} \|, \tag{11} \]
in which $\rho^{PT}$ is the partial transpose of density operator $\rho$ about arbitrary subsystem, and the symbol $\| \|$ is the trace norm. By using the Schmidt decomposition, if an arbitrary state $|\Psi\rangle$ can be decomposed as $|\Psi\rangle = \sum_{n=0}^{\infty} w_n |n\rangle_A |n'\rangle_B$ with the positive real number $w_n$ and the orthonormal states $|n\rangle_A$ and $|n'\rangle_B$, its
logarithmic negativity can be calculated as

$$E_N = 2 \log_2 \left| \sum_{n=0}^{\infty} w_n \right|.$$  \hfill (12)

According to Eqs. (6) and (9), the logarithmic negativity of both the TMSV state [14, 15] and the resulted state \(|\psi_{A,B1}\rangle\) can be, respectively, calculated as

$$E_N ([TMSV]_{AB}) = -\log_2 \left(1 + \alpha^2\right) - 2 \log_2 \left(\sqrt{1 + \alpha^2} - \alpha\right),$$

$$E_N ([\psi]_{A,B1}) = 2 \log_2 \left| \sum_{n=0}^{\infty} W_n \frac{\partial^{m+n}}{\partial \tau^m \partial \gamma^n} W^d \right| \frac{1}{(1 - \tau)(1 - \gamma)}. \hfill (13)$$

III. PERFORMANCE ANALYSIS

In this section, we demonstrate the success probability regarding quantum catalysis operation, and derive the performance of the QC-CVQKD system in terms of secret key rate and tolerable excess noise. A performance comparison between the QC-CVQKD and the photon-subtracted CVQKD is made to highlight the merits of the QC-based system. Note that for a simple and convenient discussion, we consider two special cases, i.e., the bilateral symmetrical quantum catalysis (BSQC, in which \(T_1 = T_2 = T\) and \(m = n\)) and the single-side quantum catalysis (SSQC, in which \(T_1 = 1, T_2 = T\) and \(n\)).

A. Success probability for quantum catalysis

The explicit form of the success probability for implementing quantum catalysis operations has been given in Eq. (8). In particular, for the zero-photon BSQC \((T_1 = T_2 = T\) and \(m = n = 0\)) and SSQC \((T_1 = 1, T_2 = T\) and \(n = 0\)), the success probabilities for implementing such zero-photon quantum catalysis operations can be given by \(1/(1-(T^2 - 1)\alpha^2)\) and \(1/(1-(T - 1)\alpha^2)\), respectively. Given a high transmittance \(T = 0.95\), the success probabilities \(P_d\) can be plotted as a function of \(\alpha\) with several different photon-catalysis numbers such as \(m, n \in \{0, 1, 2\}\). Fig. 3 shows that the overall trend of success probability \(P_d\) decreases as \(\alpha\) increases. It indicates that for the increased modulation variance \(V = 2\alpha^2 + 1\), the success probability \(P_d\) of implementing quantum catalysis decreases. Meanwhile, the success probabilities decrease with the increased number of photon catalysis for both SSQC and BSQC. The above-mentioned phenomenon explains that the implementation of multiphoton catalysis \((m = n > 1\) and \(n > 1\)) may be relatively difficult to achieve. Whereas, the success probability \(P_d\) of SSQC provides better performance than that of BSQC when one considers same photon-catalyzed numbers. For the zero-photon SSQC \((n = 0\)) and BSQC \((m = n = 0\)), the success probabilities \(P_d\) for the given large \(\alpha\) \((\alpha = 3)\) are approximate 0.68 and 0.53, respectively. It is worth
noting that for the two-photon BSQC \((m = n = 2)\), the success probability \(P_d\) for the given large \(\alpha\) \((\alpha = 3)\) is below 0.2, which may leak much information in the CVQKD system.

Now we consider the effect of entanglement variation on the QC-CVQKD system, which can be evaluated by the logarithmic negativity in Eq.(13). For arbitrary photon-catalyzed numbers \(m\) and \(n\), we can obtain the logarithmic negativity of the state \(|\psi\rangle_{A_1B_1}\). Given a high transmittance \(T = 0.95\), we plot the logarithmic negativity of both \(E_N(|\psi\rangle_{A_1B_1})\) and \(E_N(|TMSV\rangle_{AB})\) as a function of \(\alpha\) involving different photon-catalyzed numbers, as shown in Fig.4. For the zero-photon and single-photon quantum catalysis, the entanglement property can be improved for \(\alpha = 3\), which may have an important impact on the correlation strength of mutual information between Alice and Bob. However, for \(\alpha = 3\), the gap of the enhanced entanglement in BSQC decreases with the increase of \(m, n \in \{0, 1, 2\}\). The similar trend occurs to SSQC and there is no improvement of the entanglement for \(n = 2\). Although the entanglement for \(m = n = 2\) can be improved at large region of \(\alpha\), there does exist the limitation of its success probability. These results show that the zero-photon and single-photon quantum catalysis (i.e. \(m = n \in \{0, 1\}\) and \(n \in \{0, 1\}\)) perform well in terms of the success probability and the entanglement property, when comparing with the two-photon cases (i.e. \(m = n = 2\) and \(n = 2\)). On the other hand, for the optimized \(T\), we give the optimal logarithmic negativity \(E_N\) as a function of \(\alpha\) for \(m = n \in \{0, 1\}\) and \(n \in \{0, 1\}\), as shown in Fig.5. We find that the optimal entanglements of different zero-photon and single-photon quantum catalysis cases overlap together, and then the gap of the improved entanglement increases with the increasing \(\alpha\).

To highlight the contribution of quantum catalysis operation, compared with single-photon subtraction, we illustrate the success probability and the entanglement property in Fig.6. For \(T \rightarrow 1\), although the improvement of the entanglement for single-photon subtraction [magenta surface] performs better than that for quantum catalysis operation, the success probability for the former is worse than that for the latter. As a result, the quantum catalysis operation is superior to the single-photon subtraction in terms of the success probability. These results indicate that the quantum catalysis, as a novel non-Gaussian operation, can be used to improve the entanglement property of Gaussian entangled states, and has an advantage of the success probability over the photon subtraction operation. Consequently, in what follows, we focus on quantum catalysis to enhance the performance of the CVQKD system.

B. Security analysis

To evaluate the performance of QC-CVQKD system, according to the detailed calculations of asymptotic secret key rate [see in Appendix B], we demonstrate the numerical simulations of the secret key rate and tolerable excess noise.

Fig.7 shows that for a given transmittance \(T = 0.95\), the asymptotic secret key rate \(K_R\) as a function of transmission distance can be plotted with different photon-catalyzed numbers \(m = n \in \{0, 1, 2\}\) and \(n \in \{0, 1, 2\}\). The black solid line denotes the secret key rate of the original protocol, which is exceeded by the QC-CVQKD system with zero-photon and single-photon quantum catalysis within the long-distance range. To be specific, the
proposed system of using zero-photon BSQC [blue dash-dotted line] has the longer transmission distance when compared with the zero-photon SSQC case [red dotted line]. While for the single-photon QC-CVQKD system, the BSQC [green dash-dotted line] in term of the maximum transmission distance is better than the SSQC case [yellow dotted line]. The reason may be that for the single-photon BSQC, extra adding the model of quantum catalysis $O_m$ before Alice taking heterodyne detection can be regarded as the generation of trusted noise controlled by Alice, thereby resulting in the diminution of the Holevo bound $S^G(B:E)$. However, for the two-photon QC-CVQKD system, the BSQC [cyan dash-dotted line] and SSQC [orange dotted line] are worse than the original one, resulted from the fact that the more photons are catalyzed, the higher the non-Gaussianity is, thereby making the more noise to the covariance matrix $[12, 13]$. In addition, Within the shortening distance, the secret key of the QC-CVQKD system is worse than that of the original system because of the limitation of the success probability of quantum catalysis. As a result, for a given large transmittance $T = 0.95$, the QC-CVQKD system of using zero-photon and single-photon quantum catalysis can lengthen the maximal transmission distance apart from the two-photon QC-CVQKD system.

Since it is so, for the optimal choice of $T$, we obtain the maximal secret key rate of the proposed system with zero-photon and single-photon quantum catalysis. In Fig.8, we show the maximal secret key rate as a function of transmission distance for $m = n = 0, 1$ and $n \in \{0, 1\}$, when compared with the original protocol [black solid line]. In Fig.8(b), it is a case of the optimal $T$ that achieves the maximal secret key rate. We find that, for the long-distance range, the zero-photon and single-photon QC-CVQKD systems at the optimal transmittance range ($0.86 \leq T \leq 1$) perform better than the original system, in terms of both secret key rate and transmission distance. It indicates that the quantum catalysis can be used for improving the performance of CVQKD. For the single-photon QC-CVQKD system [green dash-dotted line and yellow dotted line] at the long transmission distance, the range of the optimal $T$ is approximate $0.978 \leq T \leq 1$ in which there does exist a high success probability for single-photon quantum catalysis [see Fig.6a]. However, for the short-distance range, even if for the optimal choice of $T$, the secret key rate of the QC-CVQKD system is similar to that of the original system, because for $T_1 = T_2 = 1$ of Alice’s BS1 and BS2, there is no quantum catalysis effect resulting from the CVQKD system.

Additionally, the other factor that has an effect on the QC-CVQKD system is the tolerable excess noise. In Fig.9, we illustrate the tolerable excess noise as a function of transmission distance for the optimal choice of $T$. Analogous to Fig.8, at the long-distance range, the QC-CVQKD system with zero-photon and single-photon quantum catalysis exceed the original system with respect to the maximal tolerable excess noise for remote users. More specifically, the zero-photon QC-CVQKD system [blue dash-dotted line and red dotted line] presents the best performance since the maximal tolerable excess noise approaches to about 0.0292 at the transmission distance of 300 km. Besides, at the
transmission distance of 300 km, for the single-photon BSQC (i.e., $m = n = 1$) [green dash-dotted line] and SSQC (i.e., $n = 1$) [yellow dotted line], the maximal tolerable excess noises can approach to about 0.0261 and 0.0185, respectively. There results indicate that when the quantum channel is less noise ($\varepsilon \sim 0.0185$), the zero-photon and single-photon quantum catalysis can be applied to lengthen the maximal transmission distance up to hundreds of kilometers. In addition, we find that from Fig.8(a) and Fig.9, at the long distance range, for the single-photon QC-CVQKD schemes, the BSQC case [green dash-dotted line] performs better than the SSQC [yellow dotted line]. It indicates that the single-photon QC-CVQKD system, extra adding the model of quantum catalysis $\hat{O}_m$ in mode $A$ may be useful for improving the performance of CVQKD protocol, when compared with the SSQC $\hat{O}_n$ in mode $B$.

Interestingly, in Ref.[11], it was pointed out that for the photon-subtraction-involved CVQKD system, the single-photon subtraction operation can usually improve the performance of the related system. Therefore, in order to make comparisons of the QC-CVQKD and the single-photon-subtraction (SS)-CVQKD, here we give the schematic diagram of the SS-CVQKD system in Fig.11. We consider the asymptotic secret key rate $K_{asy}$ of reverse reconciliation under collective attack with the assistant of the IWOP technique [The more detailed calculations can be seen in Appendix C]. To display the effect of quantum catalysis on the performance of CVQKD, we plot the secret key rate and the tolerable excess noise of the CVQKD system involving quantum catalysis and single-photon subtraction as a function of transmission distance for photon-catalyzed numbers $m = n \in \{0, 1\}$ and $n \in \{0, 1\}$, as shown in Fig.10(a) and Fig.10(b), respectively. It is found that the performance of the SS-CVQKD system [magenta solid line] in terms of the maximal secret key rate and the maximal tolerable excess noise is outperformed by the QC-CVQKD system at the long transmission distance range. The reason may be that the success probability for single-photon subtraction is lower than that for quantum catalysis at the optimal
FIG. 10. (Color online) (a) The maximal secret key rate for $\epsilon = 0.01$ and (b) the maximal tolerable excess noise of the QC-CVQKD system [dash-dotted lines and dotted lines] and the CVQKD with the single-photon subtraction [magenta solid line] as a function of the transmission distance for bilateral symmetrical quantum catalysis (BSQC) cases ($T_1 = T_2 = T$ and $m = n = 0, 1$) [dash-dotted line] and single side quantum catalysis (SSQC) cases ($T_1 = 1, T_2 = T$ and $n = 0, 1$) [dotted line] for the optimal choice of $T$. The variance of EPR state is $V = 20$ and the reconciliation efficiency is $\beta = 0.95$.

Attractively, from Figs.7, 8(a) and 10(a), we also consider the PLOB bound that stands for the fundamental rate-loss scaling (secret key capacity) [45]. By comparison, it is found that for a given transmittance $T = 0.95$, the performance of QC-CVQKD system using the zero-photon BSQC (i.e., $m = n = 0$) is closer to the PLOB bound than that of original protocol when the transmission distance reaches larger than $57.6851$ km. While for the optimal choice of the transmittance $T$, we can easily see that, our proposed QC-CVQKD system involving the zero-photon and single-photon quantum catalysis is closer to the PLOB bound when comparing with the SS-CVQKD. However, both of them are unable to exceed the PLOB bound at any transmission distance. Therefore, in order to beat the PLOB bound that is ultimate limit of repeaterless point-to-point communication, we can design the one way continuous-variable measurement-device-independent QC-QKD system acting as an active repeater.

IV. CONCLUSION

We have suggested the effect of quantum catalysis on the performance of the CVQKD system by using the IWOP technique. From the equivalent operator of quantum catalysis, the quantum catalysis that is a non-Gaussian operation in essence can be used for improving the CVQKD system. Different from the traditional TMSV, the entanglement of the resulted state using quantum catalysis can be improved significantly after optimizing the transmittance $T$ of Alice beamsplitters, and the success probability for quantum catalysis in high transmittance $T$ performs better than the single-photon subtraction case, especially for the zero-photon quantum catalysis. Taking into account the Gaussian optimality, we derive the lower bound of the asymptotic secret key rate of the QC-CVQKD for reverse reconciliation against the collective attack. Numerical simulations show that when comparing with the SS-CVQKD system, the QC-CVQKD system has an advantage of lengthening the maximal transmission distance with the raised secret key rates. For all the QC-CVQKD systems, the zero-photon quantum catalysis has the best performance. While for the QC-CVQKD system using single-photon quantum catalysis, the BSQC performs better than the SSQC due to the fact that extra adding the model of quantum catalysis $\hat{O}_m$ is useful for improving the performance of the CVQKD system. We make a comparison of the CVQKD systems involving quantum catalysis.
and single-photon subtraction. It is found that the QC-CVQKD system using the zero-photon and single-photon quantum catalysis is superior to the single-photon subtraction case in terms of the maximal transmission distance.

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Appendix A: Derivation of the success probability

\( P_d \)

In order to derive the analytical expression of the success probability \( P_d \) shown in Eq.(10), we rewrite the state in Eq.(9) as the density operator \( \rho = |\psi \rangle \langle \psi |, \)

\[
\rho_{A_1B_1} = \frac{1}{P_d} \hat{O}_m \hat{O}_n \langle TMSV | TMSV \rangle \hat{O}_n \hat{O}_m
\]

\[
= \frac{W_2^2}{P_d} \mathcal{R}^{m,n} \Pi \exp[a^\dagger b^\dagger W |00 \rangle \langle 00 | \exp[abW_1]
\]

we can obtain the success probability \( P_d \) given by

\[
P_d = W_0^2 \mathcal{R}^{m,n} \Pi \langle 00 | \exp[abW_1] \exp[a^\dagger b^\dagger W |00 \rangle
\]

\[
= W_0^2 \mathcal{R}^{m,n} \Pi \int \frac{d^2 \zeta}{\pi^2} \int \frac{d^2 \beta}{\pi^2} \exp \left[ \beta^2 + \xi \beta W_1 + \zeta \beta^* W \right]
\]

\[
= W_0^2 \mathcal{R}^{m,n} \left\{ \frac{\Pi}{1 - W_1 W} \right\}.
\]

(Appendix B: Calculation of asymptotic secret key rate)

Here, we present the calculation of asymptotic secret key rates of the QC-CVQKD system where Alice performs heterodyne detection and Bob performs homodyne detection. As mentioned above, state \(|\psi\rangle_{A_1B_1}\) belongs to a new kind of non-Gaussian state, thus we cannot directly use the results of the conventional Gaussian CVQKD to calculate its secret key rate. Fortunately, thanks to the extremity of Gaussian quantum states that the rendering secret key rate of the non-Gaussian state \(|\psi\rangle_{A_1B_1}\) is no less than that of a Gaussian state \(|\psi\rangle^{G}_{A_1B_1}\) with the same covariance matrix \(\Gamma_{A_1B_1} = \Gamma_{A_1B_1}^{G}\), we obtain \(K(|\psi\rangle_{A_1B_1}, \Gamma_{A_1B_1}^{G}) \geq K(|\psi\rangle_{A_1B_1}, \Gamma_{A_1B_1})\) [16, 18]. For reverse reconciliation, therefore, the lower bound of the asymptotic secret key rate under optimal collective attack can be given by

\[
\tilde{K}_R = P_d \left\{ \beta I^G (A : B) - S^G (B : E) \right\},
\]

where \(\beta\) denotes the reconciliation efficiency, \(I^G (A : B)\) denotes Alice and Bob’s mutual information, and \(S^G (B : E)\) denotes the Holevo bound, which is defined as the maximum information on Bob final key available to Eve.

In order to derive the analytical expression of the asymptotic secret key rate \(K(|\psi\rangle^{G}_{A_1B_1}, \Gamma_{A_1B_1})\), we consider the covariance matrix \(\Gamma_{A_1B_1}\) of the resulted state \(|\psi\rangle_{A_1B_1}\) given by

\[
\Gamma_{A_1B_1} = \left( \begin{array}{cc}
X_{A_{1 \alpha}} & Z_{A_{1 \sigma}} \\
Z_{A_{1 \sigma}} & Y_{B_{1 \beta}}
\end{array} \right),
\]

where \(II = diag(1,1), \sigma_{\zeta} = diag(1,-1), \) and \(X_{A_1}, Y_B \) and \(Z_{AB}\) can be derived by using the IWOP technique as follows: It is first required to derive the average values such as \(\langle a^\dagger a \rangle, \langle b^\dagger b \rangle \) and \(\langle ab \rangle\). According to Eq.(A1) and Eq.(A3), thus, it is straightforward to get

\[
\langle a^\dagger a \rangle = \text{Tr}(\rho^{N}_{A_1B_1} (aa^\dagger - 1) \rangle
\]

\[
= \frac{W_2^2}{P_d} \mathcal{R}^{m,n} \Pi \langle 00 | \exp[abW_1] \exp[a^\dagger b^\dagger W |00 \rangle
\]

\[
= \frac{W_2^2}{P_d} \mathcal{R}^{m,n} \left\{ \frac{\Pi}{1 - W_1 W} \right\} - 1.
\]
Furthermore, Eve accessible quantum information on
\[ \xi \]
where
\[ \langle b b \rangle = \text{Tr}[\rho_{A_1 B_1}^{N} (b b^\dagger - 1)] \]
\[ = \frac{W_0}{P_d} \text{Re}^m \Pi \int \frac{d^2 \alpha}{\pi^2} \int \frac{d^2 \beta}{\pi^2} \beta^* \times \exp[-|\alpha|^2 - |\beta|^2 + \alpha \beta W_1 + \alpha^* \beta^* W] - 1 \]
\[ = \frac{W_0}{P_d} \text{Re}^m \left\{ \frac{\Pi}{(1 - W_1 W)^2} \right\} - 1, \]
\[ = \langle a^\dagger a \rangle, \quad (B4) \]
\[ \langle a b \rangle = \text{Tr}[\rho_{A_1 B_1}^{N} ab] \]
\[ = \frac{W_0}{P_d} \text{Re}^m \Pi \int \frac{d^2 \alpha}{\pi^2} \int \frac{d^2 \beta}{\pi^2} \alpha \beta \times \exp[-|\alpha|^2 - |\beta|^2 + \alpha \beta W_1 + \alpha^* \beta^* W] \]
\[ = \frac{W_0}{P_d} \text{Re}^m \left\{ \frac{\Pi \text{W}}{(1 - W_1 W)^2} \right\}. \quad (B5) \]
Note that \( \langle a b \rangle = \langle a^\dagger b \rangle \). By combining Eqs.(B3)-(B5), therefore, we can directly obtain the elements of covariance matrix \( \Gamma_{A_1 B_1}^N \) as the following form
\[ X_A = \text{Tr} \left[ 1 + 2 a^\dagger a \right] \]
\[ = \frac{2W_0}{P_d} \text{Re}^m \left\{ \frac{\Pi}{(1 - W_1 W)^2} \right\} - 1, \]
\[ Y_B = \text{Tr} \left[ 1 + 2 b^\dagger b \right] = X_A, \]
\[ Z_{AB} = \text{Tr} [ab + a^\dagger b^\dagger] \]
\[ = \frac{2W_0}{P_d} \text{Re}^m \left\{ \frac{\Pi \text{W}}{(1 - W_1 W)^2} \right\}. \quad (B6) \]
After passing the untrusted quantum channel which is characterized by the transmission efficiency \( T_c \) and the excess noise \( \varepsilon \), the covariance matrix \( \Gamma_{A_1 B_2}^G \) reads
\[ \Gamma_{A_1 B_2}^G = \left( \begin{array}{c} X_A II \sqrt{T_c Z_{AB} \sigma_z} \\ \sqrt{T_c} \sigma_z \end{array} \right), \quad (B7) \]
where \( \xi = (1 - T_c) / T_c + \varepsilon \) denotes the channel-added noise referred to the input of Gaussian channel. The mutual information between Alice and Bob now can be expressed as
\[ I^G (A : B) = \frac{1}{2} \log_2 \frac{V_{A_1}}{V_{A_1|B_2}} \]
\[ = \frac{\log_2 \left\{ \frac{(X_A + 1) (X_A + \xi)}{(X_A + 1) (X_A + \xi) - Z_{AB}^2} \right\}}{2}. \quad (B8) \]
Furthermore, Eve accessible quantum information on Bob measurement can be calculated by assuming Eve can purify the whole system \( S^G (B : E) = S (E) - S (E|B) = \)
\[ S (AB) - S (A|B). \]
For the Gaussian modulation, the first term \( S (AB) \) is a function of the symplectic eigenvalues \( \lambda_{1,2} \) of \( \Gamma_{A_1 B_2}^G \), which is given by
\[ S (AB) = G \left[ (\lambda_1 - 1) / 2 \right] + G \left[ (\lambda_2 - 1) / 2 \right], \quad (B9) \]
where the Von Neumann entropy \( G [x] \) is
\[ G [x] = (x + 1) \log_2 (x + 1) - x \log_2 x \quad (B10) \]
and
\[ \lambda^2_{1,2} = \frac{1}{2} \left[ \Lambda \pm \sqrt{\Lambda^2 - 4D^2} \right], \quad (B11) \]
with the notation
\[ \Lambda = X_A^2 + T_c (X_A + \xi)^2 - 2T_c Z_{AB}^2, \]
\[ D = X_A T_c (X_A + \xi) - T_c Z_{AB}^2. \quad (B12) \]
Moreover, the second term \( S (A|B) = G \left[ (\lambda_3 - 1) / 2 \right] \) is a function of the symplectic eigenvalue \( \lambda_3 \) of the covariance matrix \( \Gamma_{A}^G \) of Alice mode after Bob performing homodyne detection, where the square of symplectic eigenvalue \( \lambda_3 \) is
\[ \lambda^2_3 = X_A \left[ X_A - \frac{Z_{AB}^2}{X_A + \xi} \right]. \quad (B13) \]
As a result, the asymptotic secret key rate can be written as
\[ \bar{K}_R = P_d \left\{ \beta I^G (A : B) - S (AB) + S (A|B) \right\}. \quad (B14) \]

**Appendix C: The secret key rate of the single-photon subtraction EB-CVQKD protocol under collective attack**

In order to make a comparison of the proposed long-distance CVQKD scheme via quantum catalysis, here we review the CVQKD protocol of applying single photon subtraction, and then assume that these two schemes have the same quantum channel controlled by Eve. As
can be seen from Fig.11, Alice generates a two-mode squeezed vacuum state $|TMSV\rangle_{AB}$ (EPR), and performs heterodyne detection of the one half of $|TMSV\rangle_{AB}$. The other half of $|TMSV\rangle_{AB}$ after operating single-photon subtraction is sent to Bob through the same quantum channel marked by transmission efficiency $T_c$ and excess noise $\varepsilon$. Afterwards, Bob performs homodyne detection of the received state and then informs Alice about which observable he measured, so that two correlated variables, which are shared by both Alice and Bob, can be used to exact a common secret key.

In deed, starting from the concept of quantum operators, the single-photon subtraction operation can be seen as an equivalent operator $\Theta$ which is given by

$$\Theta = C \langle 1 | B (T) | 0 \rangle_C = \frac{1 - T}{T} b \exp \left[ b^\dagger b \ln \sqrt{T} \right]$$ \hspace{1cm} (C1)

Thus, the photon-subtraction state $|\Psi\rangle_{AB}$, after operating single-photon subtraction is expressed as

$$|\Psi\rangle_{AB} = \frac{1}{\sqrt{P_1}} \Theta |TMSV\rangle_{AB} = \frac{\tilde{A} \tilde{B}}{\sqrt{P_1}} \exp \left[ \tilde{B} a^\dagger b \right] a^\dagger |00\rangle_{AB}$$ \hspace{1cm} (C2)

where

$$\tilde{A} = \sqrt{\frac{(1 - \lambda^2)(1 - T)}{T}}, \quad \tilde{B} = \lambda \sqrt{T},$$ \hspace{1cm} (C3)

and

$$P_1 = \frac{\tilde{A}^2 \tilde{B}^2}{(1 - \tilde{B}^2)^2}$$ \hspace{1cm} (C4)

is the success probability of implementing the single-photon subtraction operation. After the photon-subtraction state $|\Psi\rangle_{AB}$ goes through the quantum channel, similarly to Eq.(B7), we also can obtain the covariance matrix $\Gamma^1$ as the following form

$$\Gamma^1 = \left( \begin{array}{ccc} XX & \sqrt{T_c} Z \sigma_z & \sqrt{T_c} Z \sigma_z \\ \sqrt{T_c} Z \sigma_z & T_c (Y + \xi) & 0 \\ \sqrt{T_c} Z \sigma_z & 0 & T_c (Y + \xi) \end{array} \right)$$ \hspace{1cm} (C5)

where $\xi = (1 - T_c) / T_c + \varepsilon$, and

$$X = \frac{4 \tilde{A}^2 \tilde{B}^2}{P_1 (1 - \tilde{B}^2)} - 1,$$

$$Y = \frac{2 \tilde{A} \tilde{B}^2 (1 + \tilde{B}^2)}{P_1 (1 - \tilde{B}^2)^3} - 1,$$

$$Z = \frac{4 \tilde{A}^2 \tilde{B}^3}{P_1 (1 - \tilde{B}^2)^3}. \hspace{1cm} (C6)$$

Now let us consider the calculation of asymptotic secret key rate of the single-photon subtraction EB-CVQKD protocol in the context of Gaussian optimality theorem. Thus, the lower bound of asymptotic secret key rate $K_{asy}$ of reverse reconciliation under collective attack is

$$K_{asy} = P_1 \{ \beta I^{Hom} (A:B) - S^{Hom} (B:E) \}$$ \hspace{1cm} (C7)

where $P_1$ has been derived in Eq.(C4), $\beta$ is the efficiency for reverse reconciliation, and the superscript Hom represents Bob taking homodyne detection. Additionally, the mutual information between Alice and Bob is given by

$$I^{Hom} (A:B) = \log_2 \left\{ \frac{(X + 1) (Y + \xi)}{(X + 1) (Y + \xi) - Z^2} \right\}$$ \hspace{1cm} (C8)

Under the assumption that Eve is able to purify the whole system, $S^G (B:E) = S(E) - S(E|B) = S(AB) - S(A|B)$, we can directly obtain the symplectic eigenvalues $\lambda_{1,2}$ of covariance matrix $\Gamma^1$ as the following form

$$\tilde{\lambda}_{1,2}^2 = \frac{1}{2} \left[ \tilde{C} \pm \sqrt{\tilde{C}^2 - 4 \tilde{D}^2} \right], \hspace{1cm} (C9)$$

with

$$\tilde{C} = X^2 + T_c^2 (Y + \xi)^2 - 2 T_c Z^2$$

$$\tilde{D} = XT_c (Y + \xi) - T_c Z^2$$ \hspace{1cm} (C10)

and

$$\tilde{\lambda}_3^2 = X \left[ X - \frac{Z^2}{Y + \xi} \right]. \hspace{1cm} (C11)$$

Furthermore, $S(AB) = G \left( \tilde{\lambda}_1 - 1 \right) / 2 + G \left( \tilde{\lambda}_2 - 1 \right) / 2$ and $S(A|B) = G \left( \tilde{\lambda}_3 - 1 \right) / 2$ where the Von Neumann entropy $G[x]$ is defined in Eq.(B10).

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