Features of Quasi-frozen Spin Concept of Storage Ring for Searching for Electric Dipole Moment of Deuteron

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Abstract. One of the possible arguments for violating CP invariance is the existence of non-vanishing electric dipole moments (EDM) of elementary particles. To search for EDM, it was proposed to create a special storage ring [1], which can implement the “frozen spin” mode. An advantage of the “quasi-frozen spin” [2] concept developed by us is the ability to adapt existing storage rings to search for the deuteron EDM, in particular the accelerator complex NICA in JINR (Dubna).

1. Introduction
This article is devoted to the description of the storage ring for the search for the electric dipole moment of the deuteron. For the design of such a ring, we need to address three major challenges:
- the lattice should meet the conditions of stability of motion and minimization of beam loss, and it should incorporate straight sections to accommodate an accelerating station, beam injection and extraction equipment, a polarimeter;
- the beam polarization lifetime must reach approximately ~1000 seconds, by utilizing an RF cavity and a certain number of sextupole families;
- systematic errors have to be minimized to eliminate the induced fake EDM signal.

2. Frozen and quasi-frozen spin concepts
In this paper, we will mainly analyze the quasi-frozen spin (QFS) type of structure described in [2]. The concept of a “frozen spin” (FS) lattice had been suggested by BNL [1] and is based on elements that incorporate electric and magnetic fields in order for the spin of the reference particle to be always orientated along the momentum. We have developed our own version of the FS lattice (see fig.1) for comparison with the QFS lattice.
In all existing storage rings an implementation of the FS concept would require a complete optical upgrade. However, taking into account that the deuteron’s anomalous magnetic moment \( G = -0.142 \) has a small value and the fact that the spin oscillates around the momentum direction within half value of the advanced spin phase \( \pi \gamma G / 2 \) in the magnetic arc, each time returning to the same orientation in the electric-field elements on the straight sections, it is obvious that the effective contribution to the expected EDM effect is reduced only by a few percent. This allows us to proceed to the QFS concept [2], where the spin is not frozen with respect to the momentum vector but continuously oscillates around the momentum with a small amplitude of a few degrees.
In the case of the QFS lattice, we have two options (see Fig. 2). In the first option, the electric and magnetic fields are fully spatially separated in the arcs and the straight section elements. However, this concept inherits the drawback of cylindrical electrodes, namely, the whole set of high-order nonlinearities. Therefore, in the second option of the QFS lattice, we introduced a small magnetic field of ~100 mT, compensating the Lorentz force of the electric field on the arcs. Both QFS lattices consist of two arcs and two straight sections with approximately similar circumferences to that of the FS lattice. In both cases, the lattice includes straight sections with zero dispersion in the middle of the magnetic arcs for installation of the polarimeter, beam extraction and injection systems, and the RF cavity.

Since the second option is the most appropriate for the storage ring, we will only discuss this option here. In the magnetic arc, the particles are rotated by angle $\Phi_{\text{arc}}^B = \pi$, with simultaneous MDM spin rotation in horizontal plane relative to the momentum by angle $\Phi_{\text{arc}}^{s\pi} = \gamma G \cdot \Phi_{\text{arc}}^B$. On the straight section, the straight elements with E and B fields provide MDM spin rotation in the horizontal plane in the opposite direction relative to the momentum in E field by angle $\Phi_{\text{ss}}^E = - \left( \gamma G + \frac{\gamma}{\gamma + 1} \right) \beta^2 \cdot \Phi_{\text{ss}}^E$, where $\Phi_{\text{ss}}^E$ is the momentum rotation in electrical field, and in B field by angle $\Phi_{\text{ss}}^B = (\gamma G + 1) \cdot \Phi_{\text{ss}}^B$, where $\Phi_{\text{ss}}^B$ is the momentum rotation in magnetic field. Since the Lorentz force is zero, the angles $\Phi_{\text{ss}}^E = \Phi_{\text{ss}}^B$ are equal each to other. Therefore, they could be defined through one of them, for instance, through the magnetic field as $\Phi_{\text{ss}}^B = \frac{eB_{\text{ss}}}{m\gamma \beta^2} \cdot L_{\text{ss}}$, where $B_{\text{ss}}, L_{\text{ss}}$ are the magnetic field and the length of the straight element, respectively. To realize the QFS concept, we have to fulfill the condition $\Phi_{\text{ss}}^B - \Phi_{\text{ss}}^E = \Phi_{\text{arc}}^B$, i.e.

$$
(\gamma G + 1) \cdot \Phi_{\text{ss}}^B = \gamma G \cdot \pi
$$

Carrying out simple transformations, we obtain the basic relation for the straight element parameters:
where $L_G$ is the total length of the straight elements in one straight section.

In the NICA accelerator complex under construction in Dubna, we would like to study the applicability of the concept of quasi-frozen spin, when standard magnetic dipoles that rotate the beam are placed on the arc. The spin on the arc deviates from the direction of motion by an angle on the order of ±10 degrees, which is compensated in straight sections by straight “E+B” elements with magnetic and electric fields. Using formula (2), we can estimate how much free space is required to place the EDM equipment in the straight sections. At the kinetic energy of deuteron 270 MeV when maximum figure of merit of polarimeter and the electric field in electrostatic deflector at the level of 12 MV/m the total length of all “E+B” deflectors is 28 m for each side of ring and a magnetic field in deflector is 80 mT only. With a total length of each straight section of 100 m “E + B” deflectors will occupy only a quarter of the straight section, which will allow us to place all the necessary equipment. The small magnetic field in deflector opens the prospects of simplifying the general construction of "E+B" deflector where a permanent magnet or an air core electric coil may be used.

Figure 3 shows the structure of the COSY and NICA rings. Equipment for EDM measurements can be placed in inside bypass straight sections. In the case of ring of the collider NICA, it is desirable to use arcs of ring at the same level, excluding passing through the IP point. Thus, using two rings, the lower and the upper, we will be able to collect statistics with double speed. In such an approach, when changes in the ring structure are not fundamental, the search for an EDM becomes a “non-binding” experiment for NICA.

![Figure 3: QFS in COSY ring and NICA](image)

3. Spin tune decoherence

Now, let us briefly mention the main causes of decoherence. Expanding in Taylor series the well-known expression for the spin tune in an electric field $v_S^B = 1/(\gamma^2 - 1) - G \cdot \gamma/\beta^2$ and in a magnetic field $v_S^B = \gamma G$ in the vicinity of an arbitrary point $\gamma_0$,

$$
\Delta v_S^B = \Delta \gamma \cdot G
$$

$$
\Delta v_S^E = \Delta \gamma \cdot [-G - (1 + G)/\gamma_0^2] + \Delta \gamma^2 \cdot (1 + G)/\gamma_0^3 + ...
$$

we see that the spin tune spread $\Delta v_S^E$ in an electric field has all orders of non-linearity. Obviously, the linear term $\Delta \gamma \cdot G$ in both fields gives the maximum contribution to the spin tune decoherence, and a simple estimate shows that the spin coherence time is limited to a few milliseconds.
Introduction of the RF cavity allows averaging and practically reducing the linear term contribution to zero. Despite the linear term in (3) being practically reduced to zero with RF, the term proportional to $\Delta \gamma^2$ restricts the spin coherence time to a few hundred seconds. The final step, to reduce the spin tune decoherence, is based on sextupoles, which change the orbit length dependent on momentum deviation and dispersion. Detailed numerical consideration of decoherence effects has been performed using code COSY Infinity.

4. Systematic errors

In the EDM ring experiment, systematic errors arise from misalignments of electric and magnetic elements and cause a “fake” EDM signal. The nature of origin being random errors, misalignments create conditions for systematic errors in EDM experiments. Installation errors (misalignments) are associated with limited capabilities of geodetic instruments. As is known, the bending magnet (or the electric deflector) can be rotated in three planes. We consider only the rotation around the longitudinal and the transverse axes, as the rotation around the vertical axis does not introduce a systematic error. Firstly, let us consider the case of the magnet rotated relative to the longitudinal axis. Due to such a rotation, the horizontal component of the magnetic field $B_x$ arises and causes the spin rotation $\Omega_x = \Omega_{Bx}$ in the same plane where we expect the EDM rotation. To illustrate, let us write solutions of the T-BMTE equations with the initial conditions as $S_x = 0, S_y = 0, S_z = I, \Omega_x = 0$ and $\Omega_x \neq 0$ in simplest form:

$$S_x(t) = \frac{\Omega_y \sin(\sqrt{\frac{\Delta \gamma^2}{\Omega_x^2 + \Omega_y^2}} \cdot t)}{\sqrt{\Omega_x^2 + \Omega_y^2}}; S_y(t) = -\frac{\Omega_x \sin(\sqrt{\frac{\Delta \gamma^2}{\Omega_x^2 + \Omega_y^2}} \cdot t)}{\sqrt{\Omega_x^2 + \Omega_y^2}}$$

We take the following designation of coordinates: $z$ longitudinal direction, $x$ horizontal and $y$ vertical direction. Taking into account the above, we can present the components $\Omega_x = \Omega_{coh} + \delta \Omega_{incoh}, \Omega_x = \Omega_{EDM} + \Omega_{Bx}$, where $\Omega_{EDM}$ is the frequency of spin rotation due to the presence of an EDM, $B_x$ is the horizontal component induced by the magnet rotation (misalignment), and $\Omega_{coh}, \delta \Omega_{incoh}$ are the coherent and incoherent components of the spin tune in the horizontal plane. The incoherent component is responsible for the spin decoherence due to which we observe the attenuation of the total signal on the polarimeter. We consider the sextupole correction to be successful when the total spin vector of all particles in the bunch doesn’t fall below half of its initial value in the course of 1000 seconds. The decoherence is allowed to reach the rms value $\langle \delta \Omega_{incoh} \rangle$ of 1 rad per spin coherence time $t_{CSP} > 1000$ sec. The coherent component reflects the oscillation of the total signal in QFS optics in respect to the momentum with a frequency of two times per turn, that is, $f_{coh} \approx 2 MHz$, and a small amplitude $a_{coh} \approx 0.2 rad$.

The magnets are supposed to be installed at the technically feasible accuracy of $10 \div 100 \mu m$, which corresponds to the rotation angle of the magnet around the axis of about $\alpha_{max} = 10^{-4} \pm 5 \text{rad}$. Using COSY Infinity, we calculated the MDM spin rotation due to $B_x$, which is $\Omega_{Bx} \approx 3 \div 30 \text{ rad/sec}$. At the same time, at the presumable EDM value of $10^{-29} \text{e-cm}$, the EDM rotation should be $\Omega_{EDM} = 10^{-5} \text{ rad/sec}$, that is, $\Omega_{EDM}/\Omega_{Bx} \approx 10^{-9}$, and the expression (4) can be simplified without the loss of measurement accuracy of a possible EDM signal at the level of $10^{-9}$:

$$S_y(t) = -\sin(\Omega_{Bx} + \Omega_{EDM})t \text{ and } |S_x(t)| < a_{coh}.$$

We can see that the spin decoherence in the horizontal plane is not growing and is stabilized at the level of $S_x \sim a_{coh}$. This is a significant positive feature. But, to be fair, we should understand that, since $\Omega_{Bx} = \frac{e}{m} \gamma (G + 1)B_x$, we will now get, due to $\gamma = \gamma_0 + \Delta \gamma$, the spin frequency decoherence $\Omega_{Bx} = \frac{e}{m} \gamma_0 + \Delta \Omega_{x,dy}$ in the vertical plane around horizontal axis, which we can minimize using the same methods (sextupoles, RF) as in the horizontal plane. In addition, we are deprived of the ability to measure the accumulated EDM signal by growth of the vertical spin-vector component suggested in [3].
since spin rotation due to magnet errors is much faster than due to possibly-existing EDM, i.e. \( \Omega_{BX} \gg \Omega_{\text{EDM}} \). That is, \( S_y \) reaches a maximum for a very short time, whereas the signal EDM does not have sufficient time to be accumulated.

Therefore, the only solution is to measure the total frequency \( \Omega_{BX} + \Omega_{\text{EDM}} \) but, in order to isolate the EDM signal from the total signal, we need an additional condition. Such a condition is to measure the total spin frequency in the experiment with the counter clock-wise (CCW) direction of the beam \( \Omega_{\text{CCW}} = -\Omega_{BX} + \Omega_{\text{EDM}} \) and to compare with the clock-wise (CW) measurements \( \Omega_{\text{CW}} = \Omega_{BX} + \Omega_{\text{EDM}} \). Simultaneously, we must understand that the frequency measurement accuracy of \( \Omega_{\text{CW}}, \Omega_{\text{CCW}} \) determines the precision of the EDM measurement. For the statistical error of a spin oscillation measurement, one can use \( \sigma_\Omega = \delta \varepsilon_\Delta \sqrt{24/N/T} \) [6,7], where \( N \) is the total number of recorded events, \( \delta \varepsilon_\Delta \approx 0.03 \) is the relative error of asymmetry measurement, and \( T \approx 1000\text{sec} \) is the measurement duration. If we assume that we have a beam of \( 10^{11} \) particles per fill and the polarimeter efficiency of one percent, then we have the frequency error of \( \sigma_\Omega \approx 1.5 \cdot 10^{-7}\text{rad/sec} \). Taking into account the average accelerator beam time of 6000 hours per year, we can reach \( \sigma_\Omega \approx 1.0 \cdot 10^{-9}\text{rad/sec} \) with one-year statistics. Taking into account that an EDM value of \( d_d = 10^{-29}e\cdot\text{cm} \) results in the spin precession frequency of \( \Omega_{\text{edm}} \approx 1 \cdot 10^{-8} \) such accuracy of frequency determination is quite satisfactory and can provide to reach \( d_d = 10^{-29} \div 10^{-30}e\cdot\text{cm} \).

However, we need to be sure that when the sign of the driven magnetic field \( B_y \) for the CW-CCW is changed, the magnetic field component \( B_x \) is restored with the required relative precision of no less than \( 10^{-10} \). Therefore, we suggest calibrating the field in the magnets using the relation between the beam energy and the spin precession frequency in the horizontal plane, that is, determined by the vertical component \( B_y \). Since the magnet orientation remains unchanged, and the magnets are fed from one power supply, the calibration of \( B_y \) will restore the component \( B_x \) with the same relative accuracy \( 10^{-9} \), which applies to the difference \( \Omega_{\text{CCW}}^{\text{CW}} - \Omega_{BX}^{\text{BX}} \) as well.

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