On $\alpha$rw continuous and $\alpha$rw-Irresolute Maps in Topological Spaces

BASAVARAJ M. ITTANAGI and MOHAN V*

Department of Mathematics, Siddaganga Institute of Technology, Tumakuru-03, Affiliated to VTU, Belagavi, Karnataka state (India)
* Department of Mathematics, Gopalan College of Engineering and Management, Bangalore-48, Affiliated to VTU, Belagavi, Karnataka state (India)
Corresponding author E-mail: vengatachalam.mohan@gmail.com
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Abstract

In this paper, a new class of continuous maps called $\alpha$rw-continuous maps in topological spaces are introduced and studied. Also some of their properties have been investigated. We also introduce $\alpha$rw-irresolute maps, strongly $\alpha$rw-continuous maps, perfectly $\alpha$rw-continuous maps and discuss some properties.

Key words: $\alpha$rw-closed sets, $\alpha$rw-open sets, $\alpha$rw-continuous maps, $\alpha$rw-irresolute maps, strongly $\alpha$rw-continuous maps and perfectly $\alpha$rw-continuous maps.

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1. Introduction

The concept of continuous functions plays a very important role in general topology. The regular continuous and completely continuous functions are introduced and studied by Arya S P. Later, R S Walli et al. introduced and investigated $\alpha$rw-continuous functions in topological space. Recently, Basavaraj M Ittanagi et al. introduced and studied the basic properties of $\alpha$rw-closed sets in topological space. The aim of this paper is to introduce $\alpha$rw-continuous and $\alpha$rw-irresolute maps in topological space. Also, we study some of their basic properties of $\alpha$rw-continuous functions.

2. Preliminaries:

Throughout this paper, $(X, \tau)$ and $(Y, \sigma)$ (or simply $X$ and $Y$) represent a topological spaces on which no
separation axioms are assumed unless otherwise mentioned. For a subset A of a space X, cl (A) and int (A) denote the closure of A and the interior of A respectively. X-A or A^c denotes the complement of A in X.

We recall the following definitions and results.

**Definition 2.1:** A subset A of a topological space (X, τ) is called,

1) Semi-open set if A \(\subseteq\) cl (int (A)) and semi-closed set if int (cl (A)) \(\subseteq\) A.
2) Pre-open set if A \(\subseteq\) int (cl (A)) and pre-closed set if cl (int (A)) \(\supseteq\) A.
3) \(\alpha\)-open set if A \(\subseteq\) int (cl (int A)) and \(\alpha\)-closed set if cl (int (cl (A))) \(\subseteq\) A.
4) Semi-preopen set (\(\beta\)-open if A \(\subseteq\) cl (int (cl (A)))) and a semi-pre closed set (= \(\beta\)-closed) if int (cl (int (A))) \(\subseteq\) A.
5) Regular open set if A = int (cl (A)) and a regular closed set if A = cl (int (A)).
6) Regular semi open set if there is a regular open set U such that U \(\subseteq\) A \(\subseteq\) cl (U).
7) Regular \(\alpha\)-open set (briefly \(\alpha\)-open) if there is a regular open set U s.t U \(\subseteq\) A \(\subseteq\) \(\alpha\)-cl (U).

**Definition 2.2:** A subset A of a topological space (X, τ) is called

1) Semi \(\alpha\) regular weakly closed (briefly s\(\alpha\)r\(\omega\)-closed) set if scl (A) \(\subseteq\) U whenever A \(\subseteq\) U and U is \(\alpha\)rw-open in X.
2) Generalized pre regular closed set (briefly gpr-closed) if pcl (A) \(\subseteq\) U whenever A \(\subseteq\) U and U is regular open in X.
3) \(w\)-\(\alpha\)-closed set if \(\alpha\)-cl (A) \(\subseteq\) U whenever A \(\subseteq\) U and U is \(w\)-open in X.
4) \(\alpha\)-regular \(\alpha\)-closed set (briefly \(\alpha\r\alpha\)-closed) set if \(\alpha\)-cl (A) \(\subseteq\) U whenever A \(\subseteq\) U and U is \(\alpha\)-open in X.
5) Generalized closed set (briefly g-closed) if cl (A) \(\subseteq\) U whenever A \(\subseteq\) U and U is open in X.
6) Generalized semi-closed set (briefly gs-closed) if scl (A) \(\subseteq\) U whenever A \(\subseteq\) U and U is \(w\)-open in X.
7) Generalized semi pre regular closed (briefly gspr-closed) set if spcl (A) \(\subseteq\) U whenever A \(\subseteq\) U and U is regular open in X.
8) Strongly generalized closed set (briefly g*-closed) if cl (A) \(\subseteq\) U whenever A \(\subseteq\) U and U is g-open in X.
9) \(\alpha\)-generalized closed set (briefly \(\alpha\)-g-closed) if \(\alpha\)-cl (A) \(\subseteq\) U whenever A \(\subseteq\) U and U is \(g\)-open in X.
10) \(\alpha\)-regular weakly \(\alpha\)-closed set (briefly rw\(\alpha\)-g-closed) if \(\alpha\)-cl (A) \(\subseteq\) U whenever A \(\subseteq\) U and U is regular \(w\)-open in X.
11) Weakly generalized closed set (briefly \(w\)-g-closed) if cl (int (A)) \(\subseteq\) U whenever A \(\subseteq\) U and U is \(w\)-open in X.
12) Regular weakly generalized closed set (briefly rwg-closed) if cl (int (A)) \(\subseteq\) U whenever A \(\subseteq\) U and U is regular open in X.
13) Semi weakly generalized closed set (briefly swg-closed) if cl (int (A)) \(\subseteq\) U whenever A \(\subseteq\) U and U is regular semi-open in X.
14) Regular generalized weak (briefly rg\(w\)-g-closed) set if cl (int (A)) \(\subseteq\) U whenever A \(\subseteq\) U and U is regular semi open in X.
20) weak generalized regular–α closed (briefly wgrα-closed) set\(^{15}\) if \(\text{cl}(\text{int}(A)) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular α-open in \(X\).
21) regular pre semi–closed (briefly rps-closed) set\(^{22}\) if \(\text{spcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular α-open in \(X\).
22) generalized pre regular weakly closed (briefly gprw-closed) set\(^{16}\) if \(\text{pcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular semi-open in \(X\).
23) α-generalized regular closed (briefly αgr-closed) set\(^{30}\) if \(\alpha\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular in \(X\).
24) R*-closed set\(^{13}\) if \(rcl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular semi-open in \(X\).

The compliment of the above mentioned closed sets are their open sets respectively.

**Definition 2.3:** A map \(f: (X,\tau) \to (Y,\sigma)\) is said to be
1) regular-continuous (r-continuous)\(^{3}\) if \(f^{-1}(V)\) is r-closed in \(X\) for every closed subset \(V\) of \(Y\).
2) Completely–continuous\(^{3}\) if \(f^{-1}(V)\) is regular closed in \(X\) for every closed subset \(V\) of \(Y\).
3) Strongly–continuous\(^{26}\) if \(f^{-1}(V)\) is clopen (both open and closed) in \(X\) for every subset \(V\) of \(Y\).
4) \(\alpha\)-continuous\(^{14}\) if \(f^{-1}(V)\) is \(\alpha\)-closed in \(X\) for every closed subset \(V\) of \(Y\).
5) Strongly \(\alpha\)-continuous\(^{32}\) if \(f^{-1}(V)\) is \(\alpha\)-closed in \(X\) for every semi-closed subset \(V\) of \(Y\).
6) \(\alpha\text{gr}\)-continuous\(^{19}\) if \(f^{-1}(V)\) is \(\alpha\text{gr}\)-closed in \(X\) for every closed subset \(V\) of \(Y\).
7) \(\alpha\text{gr}\)-continuous\(^{30}\) if \(f^{-1}(V)\) is \(\alpha\text{gr}\)-closed in \(X\) for every closed subset \(V\) of \(Y\).
8) \(\alpha\text{gr}\)-continuous\(^{15}\) if \(f^{-1}(V)\) is \(\alpha\text{gr}\)-closed in \(X\) for every closed subset \(V\) of \(Y\).
9) \(\alpha\text{gr}\)-continuous\(^{16}\) if \(f^{-1}(V)\) is \(\alpha\text{gr}\)-closed in \(X\) for every closed subset \(V\) of \(Y\).
10) \(\alpha\text{gr}\)-continuous\(^{22}\) if \(f^{-1}(V)\) is \(\alpha\text{gr}\)-closed in \(X\) for every closed subset \(V\) of \(Y\).

**Definition 2.4:** A map \(f: (X,\tau) \to (Y,\sigma)\) is said to be
1) \(\alpha\)-irresolute\(^{14}\) if \(f^{-1}(V)\) is \(\alpha\)-closed in \(X\) for every \(\alpha\)-closed subset \(V\) of \(Y\).
2) irresolute if $f^{-1}(V)$ is semi-closed in $X$ for every semi-closed subset $V$ of $Y$.
3) contra-o–irresolute if $f^{-1}(V)$ is $\omega$-open in $X$ for every $\omega$-closed subset $V$ of $Y$.
4) contra irresolute if $f^{-1}(V)$ is semi-open in $X$ for every semi-closed subset $V$ of $Y$.
5) contra r–irresolute if $f^{-1}(V)$ is regular-open in $X$ for every regular-closed subset $V$ of $Y$.
6) contra continuous if $f^{-1}(V)$ is open in $X$ for every closed subset $V$ of $Y$.
7) $r\omega$*-open (resp $r\omega$*-closed) map if $f(U)$ is $r\omega$-open (resp $r\omega$-closed) in $Y$ for every $r\omega$-open (resp $r\omega$-closed) subset $U$ of $X$.

Lemma 2.5:
1) Every closed (resp regular-closed, $\alpha$-closed) set is $\alpha$rw-closed set in $X$.
2) Every $s\alpha$rw-closed set is gs-closed set
3) Every $s\alpha$rw-closed set is $sg$-closed (resp $gsp$-closed, rps-closed, gspr-closed) set in $X$.

Lemma 2.6:
If a subset $A$ of a topological space $X$ and
1) If $A$ is regular open and $s\alpha$rw-closed then $A$ is $\alpha$-closed set in $X$.
2) If $A$ is open and $ag$-closed then $A$ is $s\alpha$rw-closed set in $X$.
3) If $A$ is open and $gp$-closed then $A$ is $s\alpha$rw-closed set in $X$.
4) If $A$ is regular open and $gpr$-closed then $A$ is $s\alpha$rw-closed set in $X$.
5) If $A$ is open and $wg$-closed then $A$ is $s\alpha$rw-closed set in $X$.
6) If $A$ is regular open and $rgw$-closed then $A$ is $s\alpha$rw-closed set in $X$.
7) If $A$ is regular open and $agr$-closed then $A$ is $s\alpha$rw-closed set in $X$.
8) If $A$ is $\omega$-open and $\omega\alpha$-closed then $A$ is $s\alpha$rw-closed set in $X$.

Lemma 2.7:
If a subset $A$ of a topological space $X$, and
1) If $A$ is semi-open and $sg$-closed then it is $s\alpha$rw-closed.
2) If $A$ is semi-open and $w$-closed then it is $s\alpha$rw-closed.
3) $A$ is $s\alpha$rw-open iff $U \subseteq \alpha$int $(A)$, whenever $U$ is $rx$-closed and $U \subseteq A$.

Definition 2.8: A topological space $(X, \tau)$ is called $\alpha$-space if every $\alpha$-closed subset of $X$ is closed in $X$.

3. Sarw continuous Maps in Topological Spaces:

Definition 3.1: A function $f: X \rightarrow Y$ is called semi $\alpha$ regular weakly continuous ($s\alpha$rw–continuous) if $f^{-1}(V)$ is $s\alpha$rw–closed set in $X$ for every closed set $V$ in $Y$.

Theorem 3.2: Every continuous function is $s\alpha$rw–continuous but not conversely.

Proof: Let $f: X \rightarrow Y$ be continuous. Let $F$ be any closed set in $Y$. Then the inverse image $f^{-1}(F)$ is closed set in $X$. Since every closed set is $s\alpha$rw–closed by Lemma 2.5, $f^{-1}(F)$ is $s\alpha$rw–closed in $X$. Therefore $f$ is $s\alpha$rw–continuous.

Example 3.3: Let $X=\{a,b,c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$ Let $f: X \rightarrow Y$ defined by $f(a)=b$, $f(b)=c$, $f(c)=c$, then $f$ is $s\alpha$rw–continuous but not continuous, as closed set $F=\{b\}$ in $Y$, then, $f^{-1}(F)=\{a\}$ in $X$ which is not closed set in $X$.

Theorem 3.4: Every $\alpha$–continuous function is $s\alpha$rw–continuous but not conversely.
Proof: Let $f: X \to Y$ be $\alpha$–continuous. Let $F$ be any closed set in $Y$. Then the inverse image $f^{-1}(F)$ is $\alpha$–closed set in $X$. Since every $\alpha$–closed set is sarw–closed by Lemma 2.5, $f^{-1}(F)$ is sarw–closed in $X$. Therefore $f$ is sarw–continuous.

**Example 3.5:** Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ Let $f: X \to Y$ defined by $f(a)=b$, $f(b)=c$, $f(c)=c$, then $f$ is sarw–continuous but not $\alpha$–continuous, as closed set $F= \{b\}$ in $Y$, then $f^{-1}(F)= \{a\}$ in $X$ which is not $\alpha$–closed set in $X$.

**Theorem 3.6:** Every sarw–continuous function is gs–continuous but not conversely.

Proof: Let $F$ be sarw–continuous. Let $F$ be any closed set in $Y$. Then the inverse image $f^{-1}(F)$ is sarw–closed set in $X$. Since every sarw–closed set is gsp–closed by Lemma 2.5, $f^{-1}(F)$ is gsp–closed in $X$. Therefore $f$ is gsp–continuous.

**Example 3.7:** Let $X= \{a, b, c, d\}$, $Y= \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Let $f: X \to Y$ defined by $f(a)=b$, $f(b)=c$, $f(c)=a$, then $f$ is gsp–continuous but not sarw–continuous, as closed set $F= \{c\}$ in $Y$, then, $f^{-1}(F)= \{b\}$ in $X$ which is not sarw–closed set in $X$.

**Theorem 3.8:** Every sarw–continuous function is gsp–continuous but not conversely.

Proof: The proof follows from the fact that every sarw–closed set is gsp–closed set.

**Example 3.9:** Let $X= \{a, b, c, d\}$, $Y= \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Let $f: X \to Y$ defined by $f(a)=b$, $f(b)=c$, $f(c)=a$, then $f$ is gsp–continuous but not sarw–continuous, as closed set $F= \{c\}$ in $Y$, then, $f^{-1}(F)= \{b\}$ in $X$ which is not sarw–closed set in $X$.

**Theorem 3.10:** Every sarw–continuous function is gspr–continuous but not conversely.

Proof: The proof follows from the fact that every sarw–closed set is gspr–closed set.

**Example 3.11:** Let $X= \{a, b, c, d\}$, $Y= \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Let $f: X \to Y$ defined by $f(a)=b$, $f(b)=c$, $f(c)=a$, then $f$ is gspr–continuous but not sarw–continuous, as closed set $F= \{c\}$ in $Y$, then, $f^{-1}(F)= \{b\}$ in $X$ which is not sarw–closed set in $X$.

**Theorem 3.12:** Every sarw–continuous function is gspr–continuous but not conversely.

Proof: The proof follows from the fact that every sarw–closed set is gspr–closed set.

**Example 3.13:** Let $X= \{a, b, c, d\}$, $Y= \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Let $f: X \to Y$ defined by $f(a)=b$, $f(b)=c$, $f(c)=a$, then $f$ is s–continuous, but not sarw–continuous, as closed set $F= \{c\}$ in $Y$, then, $f^{-1}(F)= \{b\}$ in $X$ which is not sarw–closed set in $X$.

**Theorem 3.14:** Every sarw–continuous function is rps–continuous but not conversely.

Proof: The proof follows from the fact that every sarw–closed set is rps–closed set.

**Example 3.15:** Let $X= \{a, b, c, d\}$, $Y= \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Let $f: X \to Y$ defined by $f(a)=b$, $f(b)=c$, $f(c)=a$, then $f$ is rps–continuous. but not sarw–continuous, as closed set $F= \{c\}$ in $Y$, then, $f^{-1}(F)= \{b\}$ in $X$ which is not sarw–closed set in $X$.

**Remark 3.16:** The following examples shows that sarw–continuous maps are independent of pre–continuous, $\beta$–continuous, gp–continuous, gpr–continuous, swg–continuous, rwg–continuous, wg–continuous, gprw–continuous, , rgw–continuous, pgpr–continuous.

**Example 3.17:** Let $X= \{a, b, c\}$, $Y= \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Let $f: X \to Y$ defined by $f(a)=b$, $f(b)=c$, $f(c)=a$, then $f$ is pre–continuous, $\beta$–continuous, gp–continuous, gpr–continuous, swg–continuous, rwg–continuous, wg–continuous, gprw–continuous, , rgw–continuous, pgpr–continuous. But not sarw–continuous, as closed set $F= \{c\}$ in $Y$, then, $f^{-1}(F)= \{b\}$ in $X$ which is not sarw–closed set in $X$. 

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Remark 3.18: From the above discussion and know results we have the following implications.

Theorem 3.19: Let \( f: X \rightarrow Y \) be a map. Then the following statements are equivalent:

i) \( f \) is sarw–continuous.

ii) the inverse image of each open set in \( Y \) is sarw–open in \( X \)

Proof:

i) Assume that \( f: X \rightarrow Y \) is sarw–continuous. Let \( U \) be open in \( Y \). The \( U^c \) is closed in \( Y \). Since \( f \) is sarw–continuous, \( f^{-1}(U^c) \) is sarw–closed in \( X \). But \( f^{-1}(U^c) = X - f^{-1}(U) \). Thus \( f^{-1}(U) \) is sarw–open in \( X \).

ii) Assume that the inverse image of each open set in \( Y \) is sarw–open in \( X \). Let \( F \) be any closed set in \( Y \). By assumption \( f^{-1}(F^c) \) is sarw–open in \( X \). But \( f^{-1}(F^c) = X - f^{-1}(F) \). Thus \( X - f^{-1}(F) \) is sarw–open in \( X \) and so \( f^{-1}(F) \) is sarw–closed in \( X \). Therefore \( f \) is sarw–continuous.

Hence (i) and (ii) are equivalent.

Theorem 3.20: If \( f: (X, \tau) \rightarrow (Y, \sigma) \) is map. Then the following holds.

i) \( f \) is sarw–continuous and contra \( r \)–irresolute map then \( f \) is \( \alpha \)–continuous

ii) \( f \) is ag–continuous and contra continuous map then \( f \) is sarw–continuous.

iii) \( f \) is gp–continuous and contra continuous map then \( f \) is sarw–continuous

iv) \( f \) is gpr–continuous and contra \( r \)–irresolute map then \( f \) is sarw–continuous.

v) \( f \) is wg–continuous and contra continuous map then \( f \) is sarw–continuous

vi) \( f \) is rwg–continuous and contra \( r \)–irresolute map then \( f \) is sarw–continuous

vii) \( f \) is agr–continuous and contra \( r \)–irresolute map then \( f \) is sarw–continuous

viii) \( f \) is wa–continuous and contra \( w \)–irresolute map then \( f \) is sarw–continuous

Proof:

i) Let \( V \) be regular closed set of \( Y \) as every regular closed set is closed, \( V \) is closed set in \( Y \). Since \( f \) is sarw–continuous and contra \( r \)–irresolute map, \( f^{-1}(V) \) is sarw–closed and regular open in \( X \). Now by
Lemma 2.6, $f^{-1}(V)$ is $\alpha$-closed in $X$. Thus $f$ is $\alpha$-continuous.

ii) Let $V$ be closed set of $Y$. Since $f$ is $\alpha g$-continuous and contra continuous map, $f^{-1}(V)$ is $\alpha g$-closed and open in $X$. Now by Lemma 2.6, $f^{-1}(V)$ is $\alpha rw$-closed in $X$. Thus $f$ is $\alpha rw$-continuous. Similarly, we can prove (iii),(iv),(v),(vi),(vii).

**Theorem 3.21:** If $f: (X, \tau) \to (Y, \sigma)$ is map. Then the following holds.

i) $f$ is $sg$-continuous and contra irresolute map then $f$ is $\alpha rw$-continuous.

Proof: Let $V$ be closed set of $Y$. Since $f$ is $sg$-continuous and contra irresolute map, $f^{-1}(V)$ is $sg$-closed and semi-open in $X$. Now by Lemma 2.7, $f^{-1}(V)$ is $\alpha rw$-closed in $X$. Thus $f$ is $\alpha rw$-continuous.

ii) The proof is in the similar manner.

**Theorem 3.22:** Let $A$ be a subset of a topological space $X$. Then $x \in \operatorname{srwcl}(A)$ if and only if for any $\operatorname{srw}$-open set $U$ containing $x$, $A \cap U \neq \emptyset$.

Proof: Let $x \in \operatorname{srwcl}(A)$ and suppose that, there is a $\operatorname{srw}$-open set $U$ in $X$ such that $x \in U$ and $A \cap U \neq \emptyset$ implies that $A \subseteq U^c$ which is $\operatorname{srw}$-closed in $X$ implies $\operatorname{srwcl}(A) \subseteq \operatorname{srwcl}(U^c) = U^c$. Since $x \in U$ implies that $x \notin U^c$ implies that $x \notin \operatorname{srwcl}(A)$, this is a contradiction. Conversely,

For any $\operatorname{srw}$-open set $U$ containing $x$, $A \cap U \neq \emptyset$. To prove that $x \in \operatorname{srwcl}(A)$. Suppose that $x \notin \operatorname{srwcl}(A)$, then there is $\operatorname{srw}$-closed set $F$ in $X$ such that $x \notin F$ and $A \subseteq F$. Since $x \notin F$ implies that $x \in F^c$ which is $\operatorname{srw}$-open in $X$. Since $A \subseteq F$ implies that $A \cap F^c = \emptyset$, This is a contradiction. Thus $x \in \operatorname{srwcl}(A)$.

**Theorem 3.23:** Let $f: X \to Y$ be a function from a topological space $X$ into a topological space $Y$. If $f: X \to Y$ is $\alpha rw$-continuous, then $f(\operatorname{srwcl}(A)) \subseteq \operatorname{cl}(f(A))$ for every subset $A$ of $X$.

Proof: Since $f(A) \subseteq \operatorname{cl}(f(A))$ implies that $A \subseteq f^{-1}(\operatorname{cl}(f(A)))$. Since $\operatorname{cl}(f(A))$ is a closed set in $Y$ and $f$ is $\alpha rw$-continuous, then by definition $f^{-1}(\operatorname{cl}(f(A)))$ is a $\alpha rw$-closed set in $X$ containing $A$. Hence $\operatorname{srwcl}(A) \subseteq f^{-1}(\operatorname{cl}(f(A)))$. Therefore $f(\operatorname{srwcl}(A)) \subseteq \operatorname{cl}(f(A))$.

**Theorem 3.24:** Let $f: X \to Y$ be a function from a topological space $X$ into a topological space $Y$. Then the following statements are equivalent:

i) For each point $x$ in $X$ and each open set $V$ in $Y$ with $f(x) \in V$, there is a $\operatorname{srw}$-open set $U$ in $X$ such that $x \in U$ and $f(U) \subseteq V$.

ii) For each subset $A$ of $X$, $f(\operatorname{srwcl}(A)) \subseteq \operatorname{cl}(f(A))$.

iii) For each subset $B$ of $Y$, $\operatorname{srwcl}(f^{-1}(B)) \subseteq f^{-1}(\operatorname{cl}(B))$.

iv) For each subset $B$ of $Y$, $f^{-1}(\operatorname{int}(B)) \subseteq \operatorname{srwint}(f^{-1}(B))$.

Proof:

(i) $\Rightarrow$ (ii) Suppose that (i) hold and let $y \in f(\operatorname{srwcl}(A))$ and let $V$ be any open set of $Y$.

Since $y \in f(\operatorname{srwcl}(A))$ implies that there exists $x \in \operatorname{srwcl}(A)$ such that $f(x) = y$. Since $f(x) \in V$, then by (i) there exists a $\operatorname{srw}$-open set $U$ in $X$ such that $x \in U$ and $f(U) \subseteq V$.

Since $x \in f(\operatorname{srwcl}(A))$, then by theorem 3.22 $U \cap A \neq \emptyset$. $\phi \neq f(U \cap A) \subseteq f(U) \cap f(A) \subseteq V \cap f(A)$, then $V \cap f(A) \neq \emptyset$. Therefore we have $y = f(x) \in \operatorname{cl}(f(A))$. Hence $f(\operatorname{srwcl}(A)) \subseteq \operatorname{cl}(f(A))$.

(ii) $\Rightarrow$ (i) Let if (ii) holds and let $x \in X$ and $V$ be any open set in $Y$ containing $f(x)$. Let $A = f^{-1}(V^c)$ this implies that $x \notin A$. Since $f(\operatorname{srwcl}(A)) \subseteq \operatorname{cl}(f(A)) \subseteq V^c$ this implies that $\operatorname{srwcl}(A) \subseteq f^{-1}(V^c) = A$. Since $x \notin A$ implies that $x \notin \operatorname{srwcl}(A)$ and by theorem 3.22 there exists a $\operatorname{srw}$-open
A space \((X, \mathcal{T})\) is called s\(\alpha\)rw-space if every s\(\alpha\)rw–open set is semi-closed.

**Definition 3.25:** Let \((X, \tau)\) be topological space and \(\tau_{s\alpha rw} = \{V \subseteq X : s\alpha rw-cl(V^c) = V^c\}\). \(\tau_{s\alpha rw}\) is topology on \(X\).

**Definition 3.26:**

1) A space \((X, \tau)\) is called Ts\(\alpha\)rw-space if every s\(\alpha\)rw–closed set is closed.
2) A space \((X, \tau)\) is called s\(\alpha\)rwTsc – space if every s\(\alpha\)rw–closed set is semi-closed set.

**Remark 3.27:** The Composition of two s\(\alpha\)rw–continuous maps need not be s\(\alpha\)rw–continuous map

**Example 3.28:** Let \(X=Y=Z=\{a, b, c\}\). Let \(\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}\) be a topology on \(X,\)
\(\sigma = \{Y, \emptyset, \{a\}, \{b\}, \{a,b\}\}\) be a topology on \(Y\) and \(\eta = \{Z, \emptyset, \{a\}, \{b, c\}\}\) be a topology on \(Z\). Define \(f: (X, \tau) \rightarrow (Y, \sigma)\) by \(f(a) = a, f(b) = c, f(c) = b\) and \(g: (Y, \sigma) \rightarrow (Z, \eta)\) be the identity map. Both \(f\) and \(g\) are s\(\alpha\)rw-continuous but their composition \(g \circ f: (X, \tau) \rightarrow (Z, \eta)\) is not a s\(\alpha\)rw -continuous map as the closed set \(F = \{a\}\) in \((Z, \eta)\), but \((g \circ f)^{-1}(F) = \{a\}\) is not s\(\alpha\)rw-closed set in \(X\).

**Theorem 3.29:** Let \(f: X \rightarrow Y\) is s\(\alpha\)rw–continuous function and \(g: Y \rightarrow Z\) is continuous function then \(g \circ f: X \rightarrow Z\) is s\(\alpha\)rw–continuous.

**Proof:** Let \(g\) be continuous function and \(V\) be any open set in \(Z\) then \(g^{-1}(V)\) is open in \(Y\). Since \(f\) is s\(\alpha\)rw–continuous, \(f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)\) is s\(\alpha\)rw–open in \(X\). Hence \(g \circ f\) is s\(\alpha\)rw–continuous.

**Theorem 3.30:** Let \(f: X \rightarrow Y\) is s\(\alpha\)rw–continuous function and \(g: Y \rightarrow Z\) is s\(\alpha\)rw–continuous function and \(Y\) is T s\(\alpha\)rw–space, then \(g \circ f: X \rightarrow Z\) is s\(\alpha\)rw–continuous.

**Proof:** Let \(g\) be s\(\alpha\)rw–continuous function and \(V\) is any open set in \(Z\) then \(g^{-1}(V)\) is s\(\alpha\)rw–open in \(Y\) and \(Y\) is T s\(\alpha\)rw–space, thus \(g^{-1}(V)\) is open in \(Y\). Since \(f\) is s\(\alpha\)rw–continuous, \(f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)\) is s\(\alpha\)rw–open in \(X\). Hence \(g \circ f\) is s\(\alpha\)rw–continuous.

**Theorem 3.31:** If a map \(f: X \rightarrow Y\) is completely-continuous, then it is s\(\alpha\)rw–continuous.

**Proof:** Suppose that a map \(f: (X, \tau) \rightarrow (Y, \sigma)\) is completely-continuous. Let \(F\) closed set in \(Y\). Then \(f^{-1}(F)\) is regular closed in \(X\) and hence \(f^{-1}(F)\) is s\(\alpha\)rw–closed in \(X\). Thus \(f\) is s\(\alpha\)rw–continuous.

**Definition 3.32:** A function \(f\) from a topological space \(X\) into a topological space \(Y\) is called perfectly semi regular weakly continuous (briefly perfectly s\(\alpha\)rw–Continous) if \(f^{-1}(V)\) is clopen (closed and open) set in \(X\) for every s\(\alpha\)rw–open set \(V\) in \(Y\).

**Theorem 3.33:** If a map \(f: X \rightarrow Y\) is continuous, then the following holds.

i) If \(f\) is perfectly s\(\alpha\)rw–continuous, then \(f\) is s\(\alpha\)rw–continuous.

ii) If \(f\) is perfectly s\(\alpha\)rw–continuous, then \(f\) is gs–continuous.

iii) If \(f\) is perfectly s\(\alpha\)rw–continuous, then \(f\) is gsp–continuous (resp gs–continuous, gspr–continuous, rps–continuous).
Proof:

i) Let \( F \) be open set in \( Y \), as every open is sarw–open in \( Y \) since \( F \) is perfectly sarw–continuous then \( f^{-1}(F) \) is both closed and open in \( X \), as every open is sarw–open, \( f^{-1}(F) \) is sarw–open in \( X \). Hence \( f \) is sarw–continuous.

ii) Let \( F \) be open set in \( Y \), as every open is sarw–open in \( Y \), since \( F \) is perfectly sarw–continuous, then \( f^{-1}(F) \) is both closed and open in \( X \), as every open is sarw–open that implies is gs–open, then \( f^{-1}(F) \) is gs–open in \( X \). Hence \( f \) is gs–continuous.

Similarly we can prove (iii).

Definition 3.34: A function \( f \) from a topological space \( X \) into a topological space \( Y \) is called semi \( \alpha \) regular weakly*- continuous (briefly sarw*-continuous) if \( f^{-1}(V) \) is sarw*-closed set in \( X \) for every semi-closed set \( V \) in \( Y \).

Theorem 3.35: If a map \( f: (X, \tau) \rightarrow (Y, \sigma) \) is

i) \( f \) is sarw–irresolute then it is sarw*-continuous.

ii) \( f \) is sarw*-continuous then it is sarw–continuous.

Proof:

i) Let \( f: X \rightarrow Y \) be sarw–irresolute. Let \( F \) be any semi-closed set in \( Y \). Then \( F \) is sarw–closed in \( Y \). Since \( f \) is sarw–irresolute, the inverse image \( f^{-1}(F) \) is sarw–closed set in \( X \). Therefore \( f \) is sarw–continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 4.4: Let \( X=Y=\{a, b, c\} \). Let \( \tau=\{X, \phi, \{a\}, \{b\}, \{a, b\}\} \) be a topology on \( X \), \( \sigma=\{Y, \phi, \{a\}, \{b, c\}\} \) be a topology on \( Y \). Then the identity map \( f: (X, \tau) \rightarrow (Y, \sigma) \) is sarw–continuous but not sarw–irresolute, as the inverse image of sarw–closed set \( \{a, b\} \) in \( Y \) is \( \{a, b\} \) which is not sarw–closed set in \( X \).

Theorem 4.5: If a map \( f: (X, \tau) \rightarrow (Y, \sigma) \) is sarw–irresolute, if and only if the inverse image \( f^{-1}(V) \) is sarw–
open set in X for every sarw–open set V in Y.

Proof: Assume that f: X→Y is sarw–irresolute. Let G be sarw–open in Y. The G^c is sarw–closed in Y. Since f is sarw–irresolute, f^(-1)(G^c) is sarw–closed in X. But f^(-1)(G^c) = X–f^(-1)(G). Thus f^(-1)(G) is sarw–open in X. Conversely, Assume that the inverse image of each open set in Y is sarw–open in X. Let F be any sarw–closed set in Y. By assumption f^(-1)(F) is sarw–open in X. But f^(-1)(F) = X–f^(-1)(F).

Thus X–f^(-1)(F) is sarw–open in X and so f^(-1)(F) is sarw–closed in X. Therefore f is sarw– irreolute.

Example 4.9: Let X=Y= {a, b, c}. Let τ = {X, φ, {a}, {b}, {a, b}, {a, c}} be a topology on X, σ = {Y, φ, {a}, {b, c}} be a topology on Y. Then the map f: (X, τ) → (Y, σ) defined by f(a)=b, f(b)=a, f(c)=c, is sarw–continuous but not strongly sarw–continuous, as the inverse image of sarw–closed set {b} in Y is {a} which is not α–closed set in X.
**Theorem 4.12:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) is strongly sarw–continuous if and only if \( f^{-1}(G) \) is open set in \( X \) for every sarw–open set \( G \) in \( Y \).

**Proof:** Assume that \( f: X \rightarrow Y \) is strongly sarw–continuous. Let \( G \) be sarw–open in \( Y \). The \( G^c \) is sarw–closed in \( Y \). Since \( f \) is strongly sarw–continuous, \( f^{-1}(G^c) \) is closed in \( X \).

But \( f^{-1}(G^c) = X - f^{-1}(G) \). Thus \( f^{-1}(G) \) is open in \( X \). Conversely, Assume that the inverse image of each open set in \( Y \) is sarw–open in \( X \). Let \( G \) be any sarw–closed set in \( Y \). By assumption \( G^c \) is sarw–open in \( X \). But \( f^{-1}(F^c) = X - f^{-1}(F) \). Thus \( X - f^{-1}(F) \) is open in \( X \) and so \( f^{-1}(F) \) is closed in \( X \). Therefore \( f \) is strongly sarw–continuous.

**Theorem 4.13:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) is strongly sarw–continuous then it is strongly sarw–continuous

**Proof:** Assume that \( f: X \rightarrow Y \) is strongly sarw–open and also it is any subset of \( Y \) since \( f \) is strongly sarw–continuous, \( f^{-1}(G) \) is open (and also closed) in \( X \). Therefore \( f \) is strongly sarw–continuous.

**Theorem 4.14:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) is strongly sarw–continuous then it is sarw–continuous.

**Proof:** Let \( G \) be open in \( Y \), every open is sarw–open and \( G \) is sarw–open in \( Y \), since \( f \) is strongly sarw–continuous, \( f^{-1}(G) \) is open in \( X \). Therefore \( f \) is strongly sarw–continuous.

**Example 4.15:** Let \( X=Y= \{a, b, c\} \). Let \( \tau= \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\} \) be a topology on \( X \), \( \sigma= \{Y, \phi, \{a\}, \{b, c\}\} \) be a topology on \( Y \). Then the map \( f: (X, \tau) \rightarrow (Y, \sigma) \) defined by \( f(a)=b, f(b)=a, f(c)=c \), is sarw–continuous but not strongly sarw–continuous, as the inverse image of sarw–closed set \( \{b\} \) in \( Y \) is \( \{a\} \) which is not closed set in \( X \).

**Theorem 4.16:** In discrete space, a map \( f: (X, \tau) \rightarrow (Y, \sigma) \) is strongly sarw–continuous then it is strongly continuous.

**Proof:** Any subset of \( Y \), in discrete space, Every subset \( F \) in \( Y \) is both open and closed, then subset \( F \) is both sarw–open or sarw–closed, i) let \( F \) is sarw–open in \( Y \), since \( f \) is strongly sarw–continuous, \( f^{-1}(F) \) is open in \( X \). ii) let \( F \) is sarw–open in \( Y \), since \( f \) is strongly sarw–continuous, \( f^{-1}(F) \) is open in \( X \). Therefore \( f^{-1}(F) \) is closed and open in \( X \). Hence \( f \) is strongly continuous.

**Theorem 4.17:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) and \( g: (Y, \sigma) \rightarrow (Z, \eta) \) be any two functions. Then

i) \( g \circ f: (X, \tau) \rightarrow (Z, \eta) \) is strongly sarw–continuous if \( g \) is strongly sarw–continuous and \( f \) is strongly sarw–continuous.

ii) \( g \circ f: (X, \tau) \rightarrow (Z, \eta) \) is strongly sarw–continuous if \( g \) is strongly sarw–continuous and \( f \) is continuous.

iii) \( g \circ f: (X, \tau) \rightarrow (Z, \eta) \) is sarw–irresolute if \( g \) is strongly sarw–continuous and \( f \) is sarw–continuous.

iv) \( g \circ f: (X, \tau) \rightarrow (Z, \eta) \) is continuous if \( g \) is sarw–continuous and \( f \) is strongly sarw–continuous.

**Proof:**

i) Let \( U \) be a sarw–open set in \( (Z, \eta) \). Since \( g \) is strongly sarw–continuous, \( g^{-1}(U) \) is open set in \( (Y, \sigma) \). As every open set is sarw–open, \( g^{-1}(U) \) is sarw–open set in \( (Y, \sigma) \). Since \( f \) is strongly sarw–continuous \( f^{-1}(g^{-1}(U)) \) is an open set in \( (X, \tau) \).

Thus \( (g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U)) \) is an open set in \( (X, \tau) \) and hence \( g \circ f \) is strongly sarw–continuous.

ii) Let \( U \) be a sarw–open set in \( (Z, \eta) \). Since \( g \) is strongly sarw–continuous, \( g^{-1}(U) \) is open set in \( (Y, \sigma) \).

Since \( f \) is continuous \( f^{-1}(g^{-1}(U)) \) is an open set in \( (X, \tau) \).

Thus \( (g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U)) \) is an open set in \( (X, \tau) \) and hence \( g \circ f \) is strongly sarw–continuous.

iii) Let \( U \) be a sarw–open set in \( (Z, \eta) \). Since \( g \) is strongly sarw–continuous, \( g^{-1}(U) \) is open set in \( (Y, \sigma) \).

Since \( f \) is sarw–continuous \( f^{-1}(g^{-1}(U)) \) is an sarw–open set in \( (X, \tau) \).

Thus \( (g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U)) \) is an sarw–open set in \( (X, \tau) \) and hence \( g \circ f \) is sarw–irresolute.

iv) Let \( U \) be open set in \( (Z, \eta) \). Since \( g \) is sarw–continuous, \( g^{-1}(U) \) is sarw–open set in \( (Y, \sigma) \). Since \( f \) is strongly sarw–continuous \( f^{-1}(g^{-1}(U)) \) is an open set in \( (X, \tau) \).
Thus \((g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))\) is an open set in \((X, \tau)\) and hence \(g \circ f\) is continuous.

**Theorem 4.18:** Let \(f: (X, \tau) \rightarrow (Y, \sigma)\) and \(g: (Y, \sigma) \rightarrow (Z, \eta)\) be any two functions. Then

i) \(g \circ f: (X, \tau) \rightarrow (Z, \eta)\) is strongly sarw–continuous if \(g\) is perfectly sarw–continuous and \(f\) is continuous.

ii) \(g \circ f: (X, \tau) \rightarrow (Z, \eta)\) is perfectly sarw–continuous if \(g\) is strongly sarw–continuous and \(f\) is perfectly sarw–continuous.

**Proof:**

i) Let \(U\) be a sarw–open set in \((Z, \eta)\). Since \(g\) is perfectly sarw–continuous, \(g^{-1}(U)\) is clopen set in \((Y, \sigma)\). \(g^{-1}(U)\) is open set in \((Y, \sigma)\). Since \(f\) is continuous \(f^{-1}(g^{-1}(U))\) is an open set in \((X, \tau)\). Thus \((g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))\) is an open set in \((X, \tau)\) and hence \(g \circ f\) is strongly sarw–continuous.

ii) Let \(U\) be a sarw–open set in \((Z, \eta)\). Since \(g\) is strongly sarw–continuous, \(g^{-1}(U)\) is open set in \((Y, \sigma)\). \(g^{-1}(U)\) is open set in \((Y, \sigma)\). Since \(f\) is perfectly sarw–continuous, \(f^{-1}(g^{-1}(U))\) is a clopen set in \((X, \tau)\). Thus \((g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))\) is a clopen set in \((X, \tau)\) and hence \(g \circ f\) is perfectly sarw–continuous.

**Theorem 4.19:** Let \((X, \tau)\) be any topological space and \((Y, \sigma)\) be a Tsarw -space and \(f: (X, \tau) \rightarrow (Y, \sigma)\) be a map. Then the following are equivalent:

i) \(f\) is strongly sarw–continuous.

ii) \(f\) is continuous.

**Proof:**

(i) \(\Rightarrow\) (ii) Let \(U\) be any open set in \((Y, \sigma)\). Since every open set is sarw-open, \(U\) is sarw-open in \((Y, \sigma)\). Then \(f^{-1}(U)\) is open in \((X, \tau)\). Hence \(f\) is continuous.

(ii) \(\Rightarrow\) (i) Let \(U\) be any sarw-open set in \((Y, \sigma)\). Since \((Y, \sigma)\) is a Tsarw-space, \(U\) is open in \((Y, \sigma)\). Since \(f\) is continuous. Then \(f^{-1}(U)\) is open in \((X, \tau)\). Hence \(f\) is strongly sarw–continuous.

**Theorem 4.20:** Let \((X, \tau)\) be a discrete topological space and \((Y, \sigma)\) be any topological space. Let \(f: (X, \tau) \rightarrow (Y, \sigma)\) be a map. Then the following statements are equivalent:

i) \(f\) is strongly sarw–continuous.

ii) \(f\) is perfectly sarw–continuous.

**Proof:**

(i) \(\Rightarrow\) (ii) Let \(U\) be any sarw-open set in \((Y, \sigma)\). By hypothesis \(f^{-1}(U)\) is open in \((X, \tau)\). Since \((X, \tau)\) is a discrete space, \(f^{-1}(U)\) is also closed in \((X, \tau)\). \(f^{-1}(U)\) is both open and closed in \((X, \tau)\). Hence \(f\) is perfectly sarw–continuous.

(ii) \(\Rightarrow\) (i) Let \(U\) be any sarw-open set in \((Y, \sigma)\). Then \(f^{-1}(U)\) is both open and closed in \((X, \tau)\). Hence \(f\) is strongly sarw–continuous.

**Theorem 4.21:** Let \(f: (X, \tau) \rightarrow (Y, \sigma)\) be a map. Both \((X, \tau)\) and \((Y, \sigma)\) are Tsarw-space. Then the following are equivalent:

i) \(f\) is sarw-irresolute.

ii) \(f\) is strongly sarw-continuous

iii) \(f\) is continuous.

iv) \(f\) is sarw-continuous.

**Proof:** Straight forward.

**Theorem 4.22:** Let \(X\) and \(Y\) be sarwTsc-spaces, then for a function \(f: (X, \tau) \rightarrow (Y, \sigma)\), the following are equivalent:

i) \(f\) is sc-irresolute.

ii) \(f\) is sarw-irresolute.

**Proof:**

(i) \(\Rightarrow\) (ii): Let \(f: (X, \tau) \rightarrow (Y, \sigma)\) be a sc-irresolute. Let \(V\) be a sarw -closed set in \(Y\). As \(Y\) sarwTsc-space, \(V\) is a semi-closed set in \(Y\). Since \(f\) is sc-irresolute, \(f^{-1}(V)\) is semi-closed in \(X\). But every semi-closed set is sarw-closed in \(X\) and hence \(f^{-1}(V)\) is a sarw-closed in \(X\). Therefore, \(f\) is sarw-irresolute.

(ii) \(\Rightarrow\) (i): Let \(f: (X, \tau) \rightarrow (Y, \sigma)\) be a sarw -irresolute. Let \(V\) be a semi-closed set in \(Y\). But every semi-closed set is sarw-closed set and hence \(V\) is sarw-closed set in \(Y\) and \(f\) is sarw-irresolute.
implies $f^{-1}(V)$ is $s_{arw}$-closed in $X$. But $X$ is $s_{arw}$-Tsc-space and hence $f^{-1}(V)$ is semi-closed set in $X$. Thus, $f$ is sc-irresolute.

Future work:

The extension of the paper will be carried as the future work with $s_{arw}$-closed and open maps, homeomorphism in topological spaces.

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