Abstract—We study a two-tier macrocell/femtocell system where the macrocell base station is equipped with multiple antennas and makes use of multiuser MIMO (spatial multiplexing), and the femtocells are “cognitive”. In particular, we assume that the femtocells are aware of the locations of scheduled macrocell users on every time-frequency slot, so that they can make decisions on their transmission opportunities accordingly. Femtocell base stations are also equipped with multiple antennas. We propose a scheme where the macrocell downlink (macro-DL) is aligned with the femtocells uplink (femto-UL) and, Vice versa, the macrocell uplink (macro-UL) is aligned with the femtocells downlink (femto-DL). Using a simple “interference temperature” power control in the macro-DL/femto-UL direction, and exploiting uplink/downlink duality and the Yates, Foschini and Miljanic distributed power control algorithm in the macro-UL/femto-DL direction, we can achieve an extremely attractive macro/femto throughput tradeoff region in both directions. We investigate the impact of multiuser MIMO spatial multiplexing in the macrocell under the proposed scheme, and find that large gains are achievable by letting the macrocell schedule groups of co-located users, such that the number of femtocells affected by the interference temperature power constraint is small.

I. INTRODUCTION

It is widely recognized that spatial reuse is the single most valuable resource to dramatically increase the throughput of wireless cellular networks. However, deploying a very dense cellular infrastructure, with base station (BS) density that grows linearly with the user density, is not viable for a number of obvious practical and economical reasons. On the other hand, user-deployed WLANs (e.g., IEEE 802.11) achieve such dense spatial reuse in the unlicensed band. This solution has the advantage of providing very high data rates for short-range, mostly in-home, communication, but does not handle mobility as efficiently as cellular systems. Therefore, licensed cellular systems are naturally evolving towards two-tier architectures, where a large number of user-deployed femtocells operate under a common macrocell “umbrella”, that fills in the gaps of the small cells tier and supports mobility. A large body of theoretical and standardization studies on this topic has been produced in recent years (for a small sample, see [1], [2], [3], [4], [5]).

Most existing works focus on continuous transmission of macrocell user terminals (macro-UTs) and on the calculation of the pdf of the signal-to-interference plus noise (SINR) at given receivers, where the tail of the SINR pdf is related to the probability of outage. This approach disregards the fact that the forthcoming generation of cellular systems (notable, 3GPP LTE and IEEE 802.16m) is based on TDMA/OFDM, where dynamic scheduling is used in the macrocell tier for the the downlink (macro-DL) and for the uplink (macro-UL). For a macro-BS equipped with $M$ transmit antennas and serving up to $M$ macro-UTs on any time-frequency slot, the set of served macro-UTs (and therefore their location in the cell) may change on a slot by slot basis. This gives a statistical multiplexing opportunity to the femtocell tier: in the macro-DL slots, only the femtocells in the vicinity of a served macro-UT create significant interference to the macrocell tier; in the macro-UL slots, only the femtocells in the vicinity of an active macro-UT suffer from significant interference from the macrocell tier.

Fig. 1. Frame structure for the proposed cognitive femtocell system.

In order to exploit the implicit statistical multiplexing due to the macrocell dynamic scheduling, in [6] we proposed a “cognitive” approach to femtocells, where we assume that the femto-BSs and the femto-UTs can decode the macro-BS allocation map for both UL and DL. Fig. 1 shows a possible arrangement of macro and femto frames allowing cognitive operations, in the case where both tiers use time division duplexing (TDD). Assuming that the positions of all terminals are known, the femtocells can regulate their transmit power in order to guarantee a given “interference temperature” to the macro-UTs. In Section II we review the details of a scheme

1Femtocells are deployed in fixed positions, that can be made available through a database. Macro-UTs are mobile, but their position changes sufficiently slow so that through GPS and radio localization their position can be provided at a slow rate as protocol side information by the macro-BS itself.
proposed in [6], based on linear beamforming and UL/DL duality, and we extend it to the case of multi-antenna macro-BS. Then, in Section III we discuss the coexistence of the multiuser MIMO (MU-MIMO) spatial multiplexing in the macrocell tier with the cognitive femtocells. In fact, it is intuitively clear that there is a tradeoff between the macrocell and the aggregate femtocell throughput: if the macrocell serves many macro-UT using spatial multiplexing, correspondingly many femtocells have to turn down their transmit power because of the interference temperature requirement, and therefore the femtocell throughput is decreased. In contrast, if the macro-BS serves only one macro-UT at each time-frequency slot, only a few femtocells are affected by the power control requirement but the macrocell tier does not exploit the full multiplexing gain and its throughput is decreased. We shall see that this problem is alleviated by scheduling approximately co-located groups of macro-UTs.

II. SIMO/MISO INTERFERENCE CHANNEL

We consider a single macro-BS with $M$ antennas, serving $K \leq M$ macro-UTs. In the same coverage area, a set of femtocells share the same frequency band. Both tiers operate in TDD. The channel gains are formed by two components: a pathloss factor constant in time (over a large number of time-frequency slots) and frequency-flat, and a time-frequency selective small-scale fading that changes independently on a slot by slot basis. For simplicity, we focus here on a single subcarrier. By symmetry of the fading distribution, our results extend directly to an OFDM system with independent scheduling on each subcarrier.

In the proposed scheme we have two types of slots: macro-DL/femto-UL and macro-UL/femto-DL (see Fig. 2). The femtocells operate in TDMA. Therefore, the number of femto-UTs actually present in each femtocell is irrelevant, since only one of them is active at any given slot and, for the sake of simplicity, it is sufficient to consider a single femto-UT per femtocell. Notice that the femtocells form a SIMO/MISO interference channel, coupled with the vector broadcast (DL) and multiaccess (UL) channel corresponding to the macrocell.

**Macro-DL/Femto-UL slot:** Macro- and femto-UTs are equipped with a single antenna. The received signal at the $k$-th macro-UT is given in general by

$$y_k = \sqrt{g(k,0)} h_{mc,k}^H \sum_{i=1}^{K} v_i x_{mc,i} + \sum_{f \in C} \sqrt{g(k,f)} h_{f,k} x_f + z_k$$

(1)

where $h_{mc,i} \in \mathbb{C}^{M \times 1}$ is the channel vector from the macro-BS antenna array to macro-UT $i$, $v_i$ is the corresponding macro-BS beamforming vector, $C$ denotes the set of all femtocells, $h_{f,k}$ is the scalar small-scale fading coefficient from femto-UT $f$ to the macro-UT $k$, and $z_k \sim \mathcal{CN}(0,1)$ is AWGN. The coefficients $g(a,b)$ indicate pathloss between points $a$ and $b$, as detailed in Section III. The macro-BS is located at 0 (the origin of the cell coordinate system). The macro-BS calculates the beamforming vectors as functions of the matrix

$$H_{mc} = [h_{mc,1}, \ldots, h_{mc,K}],$$

formed by the $K$ active macro-UTs, enumerated without loss of generality from 1 to $K$. This can be obtained either by TDD reciprocity (open-loop) or by explicit channel state feedback [7]. In particular, here we consider Linear Zero-Forcing Beamforming (LZFB), such that $v_i$ is given by the $i$-th column of the Moore-Penrose pseudo-inverse of $H_{mc}$ normalized to have unit norm. Hence, $h_{mc,k}^H v_i = 0$ for all $i \neq k$. The macro-BS is subject to a total transmit power equal to $P_0$, equally allocated over the $K$ DL data symbols $x_{mc,i}$.

The femto-UT in the $f$-th femtocell transmits with power $\mathbb{E}[|x_f|^2] = P_f$, regulated such that the interference caused at all active macro-UTs users is less than the target interference temperature $\kappa$. Hence, we have

$$P_f = \min \left\{ \frac{\kappa}{\max_{k \in \{1, \ldots, K\}} g(k,f)} P_1 \right\},$$

(3)

where $P_1$ is the peak femtocell power. The SINR for macro-UT $k$ is given by

$$\text{SINR}_{mc-DL}^k = \frac{g(k,0) h_{mc,k}^H v_k |x_k|^2 P_f}{1 + \sum_{f \in C} g(k,f) |h_{f,k}|^2 P_f},$$

(4)

and the corresponding instantaneous rate for macro-UT $k$ on the current slot is given by $R_k = \log(1 + \text{SINR}_{mc-DL}^k)$. For simplicity, we assume that the macro-BS schedules at each slot $K$ out of $U \gg K$ macro-UTs, picked at random with equal probability. Hence, by averaging over the fading realization and the $\binom{U}{K}$ sets of macro-UTs, we obtain the sum-throughput of the macrocell tier in the DL. When $K = M$ and the fading is Rayleigh i.i.d., using the results in [8], this can be given in closed form for fixed $U$ macro-UT positions.
The linear MMSE receive vector for estimating the desired symbol $w$ to the MISO/SIMO interference channel in \[10\], we have that

$$y_f = \sum_{j \in \mathcal{C}} \sqrt{g(f,j)} h_{f,j} x_j^f + \sum_{j=1}^K \mathbf{v}_k^x \mathbf{w}_{mc,k}^x + z_f$$

(5)

The linear MMSE receive vector for estimating the desired symbol $x_j$ from $y_f$ is given by $u_f = \alpha_f \Sigma_f^{-1} h_{f,j}^T$ where $\alpha_f > 0$ is chosen such that $||u_f|| = 1$, and $\Sigma_f$ is the interference-plus-noise covariance matrix in \[5\], given by

$$\Sigma_f = \mathbf{I} + \sum_{j \in \mathcal{C}, j \neq f} g(f,j) h_{f,j} h_{f,j}^H P_j$$

$$\quad + g(f,0) \frac{P_0}{K} \sum_{k=1}^K \mathbf{v}_k^H \mathbf{w}_k^H \mathbf{H}_{f,0}^H h_{f,0}^H.$$  

(6)

The receiver forms the scalar observation $\hat{y}_f = u_f^H y_f$, and the corresponding SINR is given by

$$\text{SINR}_{\text{f-UL}} = P_f h_{f,f}^H \Sigma_f^{-1} h_{f,f}. $$

(7)

Similarly to what argued before, the instantaneous rate of femtocell $f$ is given by $R_f = \log(1 + \text{SINR}_{\text{f-UL}})$. By summing over all the femtocells and averaging over the fading and the $(\mathcal{K})$ active macro-UTs sets (notice that they have an influence through the femtocell transmit powers $P_f$), we obtain the sum-throughput of the femtocell tier in the UL.

**Macro-UL/Femto-DL:** On the macro-UL/femto-DL slot, each femtocell has multiple antennas whereas the macro-UTs have single antenna each. Insisting on linear beamforming strategies, each femto-BS sends the $L$-dimensional signal vector $\mathbf{x}_f = \mathbf{w}_f s_f$, where $\mathbf{w}_f$ denotes the transmit beamforming vector and $s_f$ is the corresponding (coded) data symbol for its own intended femto-UT.

The received signal at the macro-BS is given by

$$y = \mathbf{H}_{f,0} \mathbf{w}_f s_f + \sum_{k=1}^K \mathbf{H}_{f,k} \mathbf{x}_{mc,k} + z_f$$

(8)

The BS forms the scalar observation $\hat{y}_k = r_k^H y_k$ for detecting $x_k$, where $r_k$ is the receive beamforming vector for macro-UT $k$.

The received signal at the femto user in femtocell $f$ is given by

$$y_f = \sum_{j \in \mathcal{C}} \sqrt{g(j,f)} h_{f,j}^H w_j s_j + \sum_{k=1}^K \sqrt{g(k,f)} h_{k,f}^H x_{mc,k} + z_f.$$  

(9)

Calculating the instantaneous SINRs $\text{SINR}_{\text{f-UL}}^{\text{mc-UL}}$ and $\text{SINR}_{\text{f-DL}}^{\text{mc-DL}}$ from \[8\] and from \[8\], respectively, is straightforward. In particular, from UL/DL duality (see \[9\]), extended to the MISO/SIMO interference channel in \[10\], we have that by letting $\mathbf{w}_f = u_f$ and $r_k = v_k$, it is possible to achieve $\text{SINR}_{\text{f-UL}}^{\text{mc-UL}} = \text{SINR}_{\text{f-DL}}^{\text{mc-DL}}$ and $\text{SINR}_{\text{f-DL}}^{\text{f-UL}} = \text{SINR}_{\text{f-DL}}^{\text{f-DL}}$ while preserving the total sum power, i.e., with

$$\sum_{f \in \mathcal{C}} Q_f + \sum_{k=1}^K Q_{mc,k} = \sum_{f \in \mathcal{C}} P_f + P_0,$$

(10)

where $Q_{mc,k}$ denotes the transmit power of macro-UT $k$ and $Q_f$ denotes the transmit power of the femto-BS $f$. The power allocation across the macro-UTs and the femto-BSs depends on the realization of the path losses and small scale fading components (which determine the beamforming vectors).

**A. Implementation issues**

In order to calculate the beamforming vectors $\mathbf{u}_f$, using the matrix inversion lemma we can write $\mathbf{u}_f = \beta_f \mathbf{K}_f^{-1} \mathbf{h}_{f,f}$, where $\beta_f > 0$ is another normalizing proportionality constant and $\mathbf{K}_f$ is the received signal covariance matrix given by

$$\mathbf{K}_f = \Sigma_f + g(f,f) h_{f,f} h_{f,f}^H P_f.$$  

(11)

Hence, the MMSE beamforming vectors can be conveniently calculated by using a sample covariance estimate of $\mathbf{K}_f$, from the whole received femto-UL slot, and an estimate of the desired signal channel $h_{f,f}$ obtained by using UL pilots symbols, as in standard coherent detection for MIMO channels (e.g., currently implemented in IEEE 802.11n).

The other practical implementation problem of the proposed scheme consists of calculating the transmit powers $Q_{mc,k}$ and $Q_f$ in the macro-UL/femto-DL slot, for fixed unit-norm beamforming vectors $w_j = u_j$ and $r_k = v_k$. We propose to use the well-known Yates-Foschini-Miljanic distributed power allocation algorithm (see \[11\], \[12\]), that is guaranteed to converge to the solution.

For all femtocells $f$, fix the target DL SINR $\gamma_{\text{f-DL}} = \text{SINR}_{\text{f-DL}}^{\text{f-UL}}$. For all active users $k$ fix the target UL SINR $\gamma_{\text{mc-UL}} = \text{SINR}_{\text{mc-UL}}^{\text{f-DL}}$. Let $\text{SINR}_{\text{f-UL}}^{\text{mc-UL}}(\{Q_f\}, \{Q_{mc,k}\}, \{u_f\})$ denote the femtocell $f$ DL SINR for fixed beamforming vectors and transmit powers, and let $\text{SINR}_{\text{f-DL}}^{\text{f-UL}}(\{Q_f\}, \{Q_{mc,k}\}, \{u_f\}, \{v_k\})$ denote the macro-UT $k$ UL SINR for fixed beamforming vectors and transmit powers. Then, the iterative distributed power control algorithm, in our case, is given by:

1) **Initialization:** let $n = 0$ and let $Q_{mc,k}^{(0)} = P_0 / K$, $Q_f^{(0)} = P_f$ for $k = 1, \ldots, K$ and all $f \in \mathcal{C}$.

2) **Iterations:** for $n = 1, 2, 3, \ldots$ do

$$Q_{mc,k}^{(n)} = \frac{Q_{mc,k}^{(n-1)}}{\text{SINR}_{\text{mc-UL}}^{\text{f-UL}}(\{Q_f^{(n-1)}\}, \{Q_{mc,k}^{(n-1)}\}, \{u_f\})}$$

$$Q_f^{(n)} = \frac{Q_f^{(n-1)}}{\text{SINR}_{\text{f-DL}}^{\text{f-UL}}(\{Q_f^{(n-1)}\}, \{Q_{mc,k}^{(n-1)}\}, \{v_k\})}.$$  

(12)

In order to implement this scheme, a sequence of adjacent slots should be allocated to the same group of $K$ macro-UTs, and at each slot the receivers measure their SINR and report their measurements to the transmitters such that the power values can be updated according to \[12\].
TABLE I
SIMULATION PARAMETERS

| Parameter                      | Notation | Value  |
|--------------------------------|----------|--------|
| Macro Cell Side Length         | $L$      | 1000 m |
| Path Loss Parameter            | $\delta$ | 50 m   |
| FC Radius                      | $r_0$    | 10 m   |
| Distance between two FC         | $l$      | 40 m   |
| Path Loss Exponent             | $\alpha$ | 3.5    |
| Wall Partition Loss            | $\psi$   | 5 dB   |
| Min SNR at cell edge           | SNR$_{\text{min}}$ | 10 dB |
| Number of antennas at macro-BS | $M$      | 8      |
| Number of antennas at femto-BS | $L$      | 5      |

III. NUMERICAL RESULTS

In line with [6], we consider a unit-square cell $[-1/2, 1/2] \times [-1/2, 1/2]$, with the macro-BS located at the origin 0, and $F^2$ femtocells are centered at points of coordinates $\left(\frac{2i-F+1}{2F}, \frac{2j-F+1}{2F}\right)$, for $i, j = 0, \ldots, F - 1$. Femtocells are disk-shaped with radius $r_{fc}$, shielded from the outdoor environment by walls. For two points $a, b$, the distance dependent path loss component is given by

$$g(a, b) = \frac{\psi^{|n(a,b)|}}{1 + (d(a,b)/\delta)^\alpha},$$

where $d(a, b)$ denotes the distance between $a$ and $b$ modulo the centered unit square (torus topology); $n(a, b)$ counts the number of walls between points $a$ and $b$ (i.e., $n(a, b) = 0$ if both $a$ and $b$ are outdoor or they are in the same femtocell, $n(a, b) = 1$ if either $a$ or $b$ is indoor (inside a femtocell), and $n(a, b) = 2$ if $a$ and $b$ are in different femtocells); $\psi$ is the wall absorption factor; $\delta$ is the “3 dB” pathloss distance; $\alpha$ is the outdoor pathloss exponent.

We fix the macro-BS power $P_0$ such that the received SNR (without interference) for a macro-UT at the cell edge is 10 dB. By varying the value of the interference power temperature $\kappa$ and letting $P_1 = 30$ dB, we obtain the Pareto boundary of the throughput tradeoff region achievable with the proposed scheme in the macro-DL/femto-UL slot, as described before. The tradeoff region for the dual channel, i.e., macro UL/femto DL is obtained by using the same beamforming vectors and the iterative power control algorithm. We distinguish between the cases of colocated and non-colocated macro-UTs. In the first case, we macro-BS schedules $K$ users roughly located in the same position of the cell, such that they are separated enough to have independent small-scale fading, but they have the same pathloss with respect to the macro-BS and all the femtocells. In the second case, the $K$ macro-UT positions are independently selected with uniform probability over the cell. Fig. 3 shows the comparison of the Pareto boundaries of the tradeoff regions for the colocated and non-colocated case (supremizing over $K$), showing a clear advantage for the colocated case, which is possible when the macro-UT density is large enough so that $K$ approximately colocated users can be found. In all the results, femtocells are assumed to be “open access”, therefore, macro-UTs are located only outdoor since a macro-UT inside a femtocell would be automatically “swallowed” by the femtocell, and served as a femtocell user. We already showed in [6] that the impact of closed-access femtocells is minimal, in contrast to what observed for conventional “legacy” systems, thanks to the proposed cognitive scheme and the interference temperature power control.

Figs. 3 and 4 show the throughput tradeoff region (femtocell sum throughput vs. macrocell sum throughput) achieved by the proposed scheme in the macro-DL/femto-UL slot (averaged over random user positions), for colocated and non-colocated macro-UTs, respectively. As the number of served macro-UTs increases, the macrocell throughput increases initially up to a certain maximum value (in our case, the highest macrocell throughput is obtained for $K = 6$ users) and then decreases. This can be expected from the typical behavior of linear LZF precoding. For the non-colocated case, the
because of the imposed peak power constraint, duality does not strictly hold. Nevertheless, as seen from these plots, the impact of the peak power constraint on the macro-UL/femto-DL slot is basically negligible for this realistic range of system parameters.

IV. CONCLUSIONS

Overall, the ergodic rate region achievable by the proposed scheme is very competitive with other schemes proposed or analyzed in the current literature, considering that it can be achieved with a very simple protocol and low-complexity signal processing. For example, operating the system at the achievable throughput tradeoff point with macrocell throughput of 15 bit/s/Hz and femtocell throughput of 1000 bit/s/Hz, we can achieve 600 Mb/s over 40 MHz of system bandwidth of average macrocell symmetric data rate (both UL and DL), and 64 Mb/s per femtocell over the same system bandwidth (in our system geometry we have 625 femtocells per macrocell). Given these rather outstanding numbers, we believe that the proposed scheme is an attractive option for “beyond 4G” future wireless networks. Of course, several issues need further investigation, as for example the system operations and performance with multiple macrocells, the protocol overhead for implementing cognitive femtocells and adapting the power by the iterative algorithm, and the effect of scheduling co-located macrocell users on the macrocell user channel correlation, which may limit the effective macrocell multiplexing gain.

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