The H∞ Tracking with Preview for Linear Continuous-Time Systems

Driven by Wiener and Poisson Processes

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Abstract

In this paper, we study H∞ tracking with preview problems for linear continuous-time systems driven by Wiener and Poisson Processes. We introduce Riccati differential equations affected by Poisson parameters to solve the problems. We consider more elaborate and more rigorous statements and arguments than related previous work based on [18]. Finally we investigate tracking performances for various preview lengths by numerical examples.

Key Words: H∞ tracking with preview; Wiener processes; Poisson processes

1 Introduction

It is well known that, for design of tracking control systems, preview information of reference signals is very useful for improving performance of closed-loop systems, and recently much work has been done for preview control systems. Considering the effect of modeling uncertainties or disturbance is also very important on preview control theory.

Gershon et al. have presented the H∞ tracking theory with preview for continuous- and discrete-time systems with stochastic uncertainties [4, 5] developing the game theoretic approach for H∞ preview theory by U.Shaked et al [2, 16]. Gershon et al. have also presented the filtering theory for continuous- and discrete-time systems with stochastic uncertainties [3, 6]. They have pointed out that deterministic norm-bounded uncertainties cannot necessarily cope with parameters nonuniformly distributed around given average values, and uncertainties modeled as stochastic processes are encountered in many areas of applications, for example, nuclear fission and heat transfer, population dynamics models, and immunology and so on. However, in these papers, they have not considered the effects of Poisson processes.

Nakura has presented the stochastic preview H∞ tracking control and state estimation theory for linear impulsive systems [13, 14]. In these papers, he has not considered the effects of Poisson processes. He has also presented the preview tracking theory for linear continuous- and discrete-time Markovian jump systems where general stochastic and abrupt mode transitions are considered [9, 10, 11, 12]. However they are not restricted to Poisson processes.

In these previous research results, the direct effects of Poisson noises or uncertainties to the system dynamics have not been considered. The systems affected by Poisson processes may be found in the area of physical systems, manufacturing systems, financial systems and so on [1, 8, 17, 19, 20, 21]. Tracking theory with preview for systems driven by Poisson processes has not been fully investigated even for single mode systems, i.e., systems without any mode transitions.

In this paper, we study the stochastic H∞ tracking problems with preview by state feedback for linear continuous-time systems driven by both Wiener and Poisson processes. The systems are described by the Ito stochastic differential equations with jump parts and stochastic uncertainties to follow the Wiener and Poisson processes. These systems are also called the jump diffusion systems [7, 15, 20].

Recently B. Song et al. [18] have presented the H∞ filtering theory for the systems driven by Poisson process. In this paper we consider the preview H∞ tracking control problems based more rigorous statements and arguments than related previous work based on [18]. Finally we consider numerical examples and verify the effectiveness of the preview tracking theory presented in this paper.

2 Problem formulation

Let (Ω, F, P) be a probability space with filtration Ft, t ≥ 0, where Ω is the sample space, F is a σ-algebra of a subset of Ω called events and P is the probability measure on F. By (Ft)t≥0, we denote an increasing family of σ-algebras Ft ⊂ F. Consider the following linear continuous-time system with stochastic uncertainties or noise.

\[ dx(t) = [A(t)x(t) + B_1(t)w(t) + B_2(t)u_c(t) + B_3(t)r_c(t)]dt + B_3(t)r_c(t)dt + F_c(t)x(t)dβ + G_c(t)u_c(t)dζ + H_c(t)x(t^-)dη_p \]

\[ x(0) = x_0 \quad (1) \]

\[ z_c(t) = C_1(t)x(t) + D_{12}(t)u_c(t) + D_{13}(t)r_c(t) \]
where \( x \in \mathbb{R}^n \) is the state, \( w \in \mathbb{R}^p \) is the exogenous disturbance, \( u_c \in \mathbb{R}^{m_c} \) is the control input, \( z_c \in \mathbb{R}^{k_c} \) is the controlled output, \( r_c(t) \in \mathbb{R}^{k_c} \) is a known or measurable reference signal, \( x_0 \) is an unknown initial state. We assume that all matrices are of compatible dimensions. Throughout this paper the dependence of the matrices on \( t \) will be omitted for the sake of notation simplification.

We denote by \( L_2(\Omega, \mathbb{R}^k) \) the space of square-integrable \( \mathbb{R}^k \)-valued functions on the probability space \((\Omega, \mathcal{F}, \mathbb{P})\). We also denote by \( L_2([0,T]; \mathbb{R}^k) \) the space of nonanticipative stochastic process \( \{f(t)\}_{t \in [0,T]} \) with respect to
\[
(F_t)_{t \in [0,T]} \quad \text{where} \quad f(t) \in L_2(\Omega, \mathbb{R}^k).
\]
\( \beta(t) \) and \( \zeta(t) \) are zero-mean real scalar Wiener processes.

Let \( \eta_p(t) \in \mathbb{R} \) be a scalar Poisson process with parameter \( \lambda_p \), i.e., \( \eta_p(t) \) follows the Poisson law
\[
P\{ \Delta \eta_p(t) = \eta(t + \Delta t) - \eta(t) = k \} = e^{-\lambda_p \Delta t} \frac{\lambda_p \Delta t^k}{k!}.
\]
We assume the following conditions:

\textbf{A1 :} \( \mathbb{E}\{d\beta(t)\} = 0 \), \( \mathbb{E}\{d\eta(t)^2\} = dt \),
\( \mathbb{E}\{d\zeta(t)\} = 0 \), \( \mathbb{E}\{d\zeta(t)^2\} = dt \),
\( \mathbb{E}\{d\beta(t)d\zeta(t)\} = \sigma dt \), \( |\sigma| \leq 1 \).

The \( \mathcal{H}_\infty \) tracking problems we address in this paper for the system (1) are to design control laws \( u_c(t) \in L_2(0,T] \) over the finite horizon \([0,T]\) using the information available on the known parts of the reference signals \( r_c(t) \) and minimizing the sum of the energy of \( z_c(t) \), for the worst case of the initial condition \( x_0 \), the disturbances \( w(t) \in L_2([0,T]; \mathbb{R}^p) \). Considering the average of the performance index over the statistics of the unknown parts of \( r_c \), we define the following performance index
\[
J_T(x_0, u, w) := -\gamma^2 x_0 R^{-1} x_0 - \gamma^2 \|w\|^2 + \mathbb{E}\left\{ \int_0^T \mathbb{E}_{R_s}\|z_c(s)\|^2 ds \right\}.
\]
where \( R = R' > 0 \) is a given weighting matrix for the initial state, \( \mathbb{E}_{R_s} \) means the expectation over \( R_{s+h} \), \( h \) is the preview length of \( r_c(t) \), and \( R_s \) denotes the future information on \( r_c \) at time \( s \), i.e., \( R_s := \{r_c(l); s < l \leq T\} \).

We consider the following \( \mathcal{H}_\infty \) tracking problem for the system (1) and the performance index (2):

**The Stochastic \( \mathcal{H}_\infty \) Fixed-Preview Tracking Problem by State Feedback:**

It is assumed that at the current time \( t \), \( r_c(s) \) is known for \( s \leq \min(T, s + h) \) where \( h \) is the preview length. Then find \( \{u^*_c\}, \{w^*\} \) and \( x^*_0 \) satisfying the following (saddle point) condition:
\[
J_T(x_0, u^*, w) \leq J_T(x^*_0, u^*, w^*) \leq J_T(x^*_0, u_c, w^*)
\]
where the control strategies \( u^*_c(s), 0 \leq s \leq T \) is based on the information \( R_{s+h} \) with \( 0 \leq h \leq T \).

### 3 \( \mathcal{H}_\infty \) Tracking Controllers by State Feedback

In this section we present the theory of stochastic \( \mathcal{H}_\infty \) tracking by state feedback and concrete a saddle point strategy.

First we present the following lemma shown in [18].

**Lemma 3.1** [18] If \( x(t) \) is the solution of the differential equation
\[
dx(t) = [A(t)x(t) + B_1(t)w(t) + B_2(t)u_c(t)]dt + B_3(t)r_c(t)dt + F_c(t)x(t)d\beta(t) + G_c(t)u_c(t)d\zeta(t) + H_c(t)x(t)\eta_p(t)dt
\]
with
\[
u_c(t) = K_x(t)x(t) + K_r(t)r_c(t)
\]
where \( K_x(t) \) and \( K_r(t) \) are some gain matrices, and \( A: \mathbb{R}^n \rightarrow \mathbb{R} \) is a continuous Borel measurable function, then, for any \( T > 0 \), we have
\[
\mathbb{E}\left\{ \int_0^T \mathcal{A}(x(t^-))d\eta_p(t) \right\} = \mathbb{E}\left\{ \lambda \int_0^T \mathcal{A}(x(t))dt \right\}.
\]

Now we consider the following Riccati differential equation.
\[
\dot{X} + A'X + XA + C_1'C_1 + \frac{1}{\gamma^2} XB_1B_1'X
\]
\[
- \dot{S}X + F_c'XF_c
\]
\[
+ \lambda_p[I + H_c]X\{I + H_c - X\} = 0
\]
where
\[
\dot{R}(t) = V_1 + G_c'X(t)G_c, \quad V_1 = D_1'D_12,
\]
\[
\dot{S}(t) = B_2'X(t) + D_1'C_1 + \alpha G_c'X(t)G_c.
\]

We obtain the following saddle point strategy for our game problem.

**Theorem 3.1** Consider the system (1) and suppose \( A1 \). Suppose there exists a matrix \( X(t) \) satisifying the conditions \( X(0) < \gamma^2 R^{-1} \) and \( X(T) = O \) such that the Riccati differential equation (3) with the Poisson rate parameter \( \lambda_p \) holds over \([0,T]\). Then the Stochastic \( \mathcal{H}_\infty \) Fixed-Preview Tracking Problem by State Feedback is solvable and a saddle point strategy which gives a solution of the Stochastic \( \mathcal{H}_\infty \) Fixed-Preview Tracking Problem by State Feedback is given by
\[
x^*_0 = [\gamma^2 R^{-1} - X(0)]^{-1} \theta(0)
\]
\[
w^* = \gamma^{-2} B_1'X + C_0 \theta
\]
\[
u^*_c = -\dot{R}^{-1} \dot{S} x - C_{10} r_c - C_{00} \theta_c
\]
where 
\[ C_\theta = -\gamma^{-2}B_1', \quad C_{\theta u} = \tilde{R}^{-1}B_2', \quad C_u = \tilde{R}^{-1}D_{12}D_{13}. \]

\( \theta(t), t \in [0, T], \) satisfies

\[
\begin{aligned}
\dot{\theta}(t) &= -\dot{A}_{c,p}(t)\theta(t) + \dot{B}_c(t)r_c(t), \\
\theta(T) &= 0
\end{aligned}
\]

(4)

where

\[
\begin{aligned}
\dot{A}_{c,p} &= A + \frac{1}{\gamma}B_1B_1'X - B_2\tilde{R}^{-1}\tilde{S} + \lambda_pH_c, \\
\dot{B}_c &= -(XB_3 + C_1'D_{13}) + S'C_u
\end{aligned}
\]

and \( \theta_c(t) \) is the 'causal' part of \( \theta(\cdot) \) at time \( t \). This \( \theta_c \) is the expected value of \( \theta \) over \( \tilde{R}_s \) and given by

\[
\begin{aligned}
\dot{\theta}_c(s) &= -\dot{A}_{c,p}(s)\theta(s) + \dot{B}_c(s)r_c(s), \\
\theta_c(t_f) &= 0
\end{aligned}
\]

(5)

where \( t_f = t + h\tau \quad \text{if} \quad t + h\tau < T \\
\quad \quad t_f = T \quad \text{if} \quad t + h \geq T \)

Moreover, the value of the game is

\[
\begin{aligned}
J_T(x_0^*, \mu^*, w^*) &= \mathbb{E} \left\{ \int_0^T R_{\theta}(\|\tilde{R}^{1/2}C_{\theta u}\theta(\cdot)\|^2) \, ds \right\} \\
&\quad + J_c(r_c)
\end{aligned}
\]

(6)

where \( \theta_1(t) = \theta(t) - \theta_c(t), \quad t \in [0, T], \)

\[
j_c(r_c) = \gamma^2 \mathbb{E}_{R_0} \left\{ \|\theta_0(0^-)\|^2 \right\}
\]

\[
\delta J_c(r_c) = \mathbb{E} \left\{ \int_0^T R_{\theta}(\delta J_c(r_c)) \, ds \right\}
\]

\[
\begin{aligned}
\delta J_c(r_c) &= \delta J_c(r_c) + 2\theta'B_3r_c + \gamma^2\|C_\theta\|^2 - 2\theta'C_{\theta u}\tilde{R}C_u'r_c - \|\tilde{R}^{1/2}C_{\theta u}\theta\|^2 \, ds,
\end{aligned}
\]

and \( P_0 = [R^{-1} - \gamma^{-2}X(0)]^{-1}. \)

**Proof of Theorem 3.1**

**Sufficiency:** Let \( X(t) \) be a solution to (3) over \([0, T]\) such that \( X(0) < \gamma^{-2}R^{-1}. \) By applying the Ito formula to \( x'(t)X(t)x(t) \), and take expectation for every \( T > 0 \), we have

\[
\mathbb{E} \left\{ \int_0^T \mathbb{E}_{R_x} \left[ x'(s)X(s)x(s) \right] \, ds \right\}
\]

\[
= \mathbb{E} \left\{ \int_0^T \mathbb{E}_{R_x} \left[ \frac{dX(s)}{ds} \right] \right. \\
&\quad + x'(s)X(s)dx(s) \right. \\
&\quad + \int_0^T \text{tr} \{X(s)[F_c(s)x(s)G_c(s)u_c(s)] \} \\
&\quad \times \bar{P}[F_c(s)x(s)G_c(s)u_c(s)]' ds \right. \\
&\quad + \int_0^T [(x(s^-) + H_c(s)x(s^-))'X(s) \\
&\quad \times (x(s^-) + H_c(s)x(s^-)) - x'(s^-)X(s)x(s^-)] \, d\mu \right\}
\]

where

\[
\bar{P} = \begin{bmatrix} 1 & \alpha \\ \alpha & 1 \end{bmatrix}
\]

is the covariance matrix of the Wiener process vector \((\beta'(t)\zeta'(t))'\). Using the property

\[
\text{tr} \{X(s)[F_c(s)x(s)G_c(s)u_c(s)] \} \bar{P} \\
\times [F_c(s)x(s)G_c(s)u_c(s)] = x'(s)F_c(s)x(s)x(s) + 2\alpha x'(s)F_c(s)x(s)G_c(s)u_c(s) \\
+ u'_c(s)G_c'(s)x(s)G_c(s)u_c(s)
\]

and Lemma 3.1, we obtain

\[
\mathbb{E} \left\{ \int_0^T \mathbb{E}_{R_x} \left[ x'(s)X(s)x(s) \right] \right\} \\
= \mathbb{E} \left\{ \int_0^T \mathbb{E}_{R_x} d[x'(s)X(s)x(s)] \right. \\
&\quad + \int_0^T \text{tr} \{X(s)[F_c(s)x(s)G_c(s)u_c(s)] \} \\
&\quad \times \bar{P}[F_c(s)x(s)G_c(s)u_c(s)]' ds \right. \\
&\quad + \int_0^T [(x(s^-) + H_c(s)x(s^-))'X(s) \\
&\quad \times (x(s^-) + H_c(s)x(s^-)) - x'(s^-)X(s)x(s^-)] \, d\mu \right\}
\]

By considering (1) and (3), in the case of \( r_c(\cdot) \equiv 0 \), it can be shown that the following equality holds.

\[
\mathbb{E} \left\{ \int_0^T \mathbb{E}_{R_x} d[x'(s)X(s)x(s)] \right\} \\
= \mathbb{E} \left\{ \int_0^T \mathbb{E}_{R_x} \left[ \gamma^2[\|w\|^2 - \|w - \gamma^{-2}B_1Xx(s)\|^2] \\
- \|C_1x + D_{12}u_c\|^2 \\
+ \|u_c + \tilde{R}^{-1}\tilde{S}x(s)\|^2 \right] ds \right\}
\]

Moreover, in the general case that \( \{r_c(\cdot)\} \) is arbitrary, we have the following equality.

\[
\mathbb{E} \left\{ \int_0^T \mathbb{E}_{R_x} d[x'(s)X(s)x(s)] \right\}
where \( w \) is zero, including the 'causal' part of \( d \), and we have used the property of the expectation operator. Adding (8) to (7),

\[
\mathbb{E}\left\{ \int_0^T E_{R^c} \{ d(\theta'(s)x(s)) \} \right\}
= \mathbb{E}\left\{ \int_0^T E_{R^c} \{ \gamma^2 \|w\|^2 - \|\dot{w} - C_\theta(\theta(s))\|^2 \}
\right.
- \|C_1x + D_{12}u_c + D_{13}r_c\|^2
+ \|\dot{u}_c(s) + C_\theta u_c + C_{\theta u}(\theta(s))\|_R^2
+ \delta J_c(r_c)\} \right)\) \(ds\}
\]

where we have used the continuous part

\[
\dot{\theta} + \dot{A}_\phi \theta - \dot{B}_x r_c = 0
\]

of the dynamics (4) to get rid of the terms that mix \( r_c \), \( \theta \) and \( x \).

From (9), we have

\[
-\gamma^2 x'(0)R^{-1}x(0)
+ \mathbb{E}\left\{ \int_0^T E_{R^c} \{ \|z_r\|^2 - \|\dot{z}_r - C_\theta(\theta(s))\|^2 \} \right\} \right)
\]

where

\[\dot{u}_c(t) = u_c(t) + \hat{R}^{-1}\dot{S}x(t) + \mathbf{C}_u r_c(t).\]

Since the left hand side reduces to

\[
\mathbb{E}\left\{ \int_0^T E_{R^c} \{ \|z_r(s)\|^2 \} \right\} \right)
- \gamma^2 \|w\|^2 + \|u_c\|^2 + \dot{x}_0(R^{-1})x_0
- 2\theta'(0)x(0) - x_0X(0)x_0 \right)\}
\]

considering \( X(T) = 0 \) and \( \theta(T) = 0 \), we obtain

\[
J_T(x_0, u_c, w)
\]

\[
\mathbb{E}\left\{ -\gamma^2 E_{R^c} \{ \|x_0 - \gamma^{-2}P_0x_0(0)\|^2 \} \right\}
+ \mathbb{E}\left\{ \int_0^T E_{R^c} \{ -\gamma^2 \|\dot{w} - C_\theta(\theta(s))\|^2 \}
\right.
+ \|\dot{u}_c(s) + C_{\theta u}(\theta(s))\|_R^2\} \right) \right)\} \right)\}
+ J_c(r_c)\}
\]

where \( P_0 = [R^{-1} - \gamma^{-2}X(0)]^{-1}. \) Note that \( J_c(r_c) \) is independent of \( u_c \) and \( x_c \). Since the average of \( \theta_1 \) over \( R_0 \) is zero, including the 'causal' part \( \theta_c(t) \) of \( \theta(t) \) at time \( t \), we adopt

\[
\dot{u}_c(t) = -C_{\theta u}(\theta(t))
\]

i.e.,

\[
u_c(t) = -\hat{R}^{-1}\dot{S}x(t) - \mathbf{C}_u r_c(t) - \mathbf{C}_{\theta u}(\theta(t))
\]
as the minimizing control strategy. Then

\[
J_T(x_0, u_c, w^*)
= \mathbb{E}\left\{ \int_0^T E_{R^c} \{ \|\dot{u}_c(s) + C_{\theta u}(\theta(s))\|_R^2 \} \right\} + J_c(r_c)
\]

\[
\geq \mathbb{E}\left\{ \int_0^T E_{R^c} \{ \|C_{\theta u}(\theta(s))\|_R^2 \} \right\} \right)\} \right)\}
= J_T(x_0, u_c^*, w^*).\]

Finally we obtain

\[
J_T(x_0, u_c^*, w)
= \mathbb{E}\left\{ -\gamma^2 E_{R^c} \{ \|x_0 - \gamma^{-2}P_0x_0(0)\|^2 \} \right\}
+ \mathbb{E}\left\{ \int_0^T E_{R^c} \{ -\gamma^2 \|\dot{w} - C_\theta(\theta(s))\|^2 + \|C_{\theta u}(\theta(s))\|_R^2 \} \right\}
+ J_c(r_c)
\]

\[
\leq J_T(x_0, u_c^*, w^*)\]

which concludes the proof of sufficiency. (QED.)
4 Numerical Examples

In this section, we study numerical examples to demonstrate the effectiveness of the design theory presented in this paper.

We consider the following two mode systems and assume that the system parameters are as follows: cf.[2],[16]

\[ dx(t) = [Ax(t) + B_{1}w(t) + B_{2}u_{c}(t) + B_{3}r_{c}(t)]dt \]
\[ +F_{c}x(t)d\beta + G_{c}u_{c}(t)dz + H_{c}x(t^-)d\eta_{p}, \]
\[ x(0) = x_{0} \]...

(10)

where

\[ A = \begin{bmatrix} 0 & 1 \\ -1 & -0.4 \end{bmatrix}, B_{1} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, B_{2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \]

\[ B_{3} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, F_{c} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, G_{c} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \]

\[ H_{c} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \]

\[ C_{1} = \begin{bmatrix} -0.5 & 0.1 \\ 0 & 0 \end{bmatrix}, D_{12} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, D_{13} = \begin{bmatrix} -1.0 \\ 0 \end{bmatrix} \]

and

\[ C_{2} = \begin{bmatrix} 1 \end{bmatrix} \]

where it is assumed that \( r_{c}(\cdot) \) is not always a priori known over the whole time interval \([0, T]\) but has any distribution at the unknown part.

Let \( \alpha = 0.5 \). and we set \( x_{0} = col(0, 0) \).

Then we introduce the following objective function considering the input energy.

\[ J_{T}(x_{0}, u, r_{c}) = E\left\{ \int_{0}^{T} E_{R_{c}}\{ \|C_{1}x(s) + D_{13}r_{c}(s)\|^{2} + 0.12\|u(s)\|^{2} \} ds \right\} \]

By the term \( B_{3}r_{c}(t) \), the tracking performance can be expected to be improved as \([2, 16]\) and so on. The paths of \( \eta_{p} \) are generated randomly, and the performances are compared under the same circumstance, that is, the same set of the paths so that the performances can be easily compared.

We consider the whole system (10) with the Poisson rate parameter \( \lambda_{p} = 1.0 \) over the time interval \( t \in [0, 15] \). For this system, we apply the results of the stochastic \( H_{\infty} \) tracking theory presented in this paper for \( r_{c}(t) = \sin(\pi t/15) \) with various lengths of preview, and show the simulation results. We verify the effectiveness of the preview compensation by state feedback and compare the tracking performances for them. The square values \( \|C_{1}x(t) + D_{13}r_{c}(t)\|^{2} \) of the tracking errors for \( r_{c}(t) = \sin(\pi t/15) \) are shown in Fig. 1. While, in the case affected only by Gaussian noises, increasing the preview lengths from \( h = 0 \) to \( h = 0.5, 0.75, 1.0, 1.5 \) improves the tracking performance as shown in [12], in this case affected by both Gaussian and Poisson processes, we obtain worse tracking performance for \( h = 0.25 \) than \( h = 0 \). Then we obtain better tracking performances from \( h = 0.5 \) to \( h = 0.75 \) but we obtain worse tracking performance for \( h = 1.0 \) abruptly, and obtain worse tracking performance for \( h = 1.5 \) than for \( h = 1.0 \). This shows that we need more appropriate preview lengths to improve the tracking performances for the cases driven by the Poisson processes.

5 Concluding Remarks

In this paper we have presented the stochastic \( H_{\infty} \) tracking control theory considering the preview information by state feedback for the linear continuous-time systems driven by Wiener and Poisson processes, which are a class of jump diffusion systems.

The author had presented the solution of the stochastic optimal (LQ) and \( H_{\infty} \) preview tracking control theory by state feedback for the linear impulsive systems affected by Wiener Processes [12, 13]. However the stochastic \( H_{\infty} \) preview tracking theory for the systems affected by Poisson processes had not been yet fully investigated. Hence we have focused on it in this paper. We have introduced extended type of Riccati differential equations with initial conditions in order to solve the fixed-preview tracking control problem for the systems affected by Wiener and Poisson processes. Using the solution of this type of Riccati differential equation, we can design the state feedback controller. Note that, on the preview compensator, the term with the Poisson rate parameter is added.

Throughout this paper, it is assumed that the system states are fully observable over the whole time inter-
The preview tracking control theory in the case with partially observations is a very important further research issue.

References

[1] D. Applebaum: Levy processes from probability to finance and quantum groups, AMS, Vol. 51, pp. 1336-1342, 2004.

[2] A. Cohen and U. Shaked: Linear Discrete-Time $H_\infty$-Optimal Tracking with Preview, IEEE Trans. Automat. Contr., Vol. 42, No. 2, pp. 270-276, 1997.

[3] E. Gershon, U. Shaked and I. Yaesh: $H_\infty$ control and filtering of discrete-time stochastic systems with multiplicative noise, Automatica, Vol. 37, pp. 409-417, 2001.

[4] E. Gershon, D. J. N. Limebeer, U. Shaked and I. Yaesh: Stochastic $H_\infty$ Tracking with Preview for State-Multiplicative Systems, IEEE Trans. Automat. Contr., Vol. 49, No. 11, pp. 2061-2068, 2004.

[5] E. Gershon, U. Shaked and I. Yaesh, $H_\infty$ tracking of linear continuous-time systems with stochastic uncertainties and preview, Int. J. Robust and Nonlinear Control, Vol. 14, No. 7, pp. 607-626, 2004.

[6] E. Gershon, U. Shaked and I. Yaesh: $H_\infty$ Control and Estimation of State-Multiplicative Linear Systems, Lecture Notes in Control and Information Sciences, Vol. 318, Springer, London, 2005.

[7] I. Kolmanovsky and T. Maizenberg: Optimal Containment Control for a Class of Stochastic Systems Perturbed by Poisson and Wiener Processes, IEEE Trans. Automat. Contr., Vol. 47, No. 12, pp. 2041-2046, 2002.

[8] R. C. Merton: Option Pricing When Underlying Stock Returns are Discontinuous, J. Fin. Econ., Vol. 3, pp.125-144, 1976.

[9] G. Nakura: On Noncausal $H_\infty$ Tracking Control for Linear Continuous-Time Markovian Jump Systems, Proceedings of the 41st ISCIE International Symposium on Stochastic Systems Theory and Its Applications (SSS09), Kobe, Japan, pp. 172-177, 2009.

[10] G. Nakura: On Noncausal $H_\infty$ Tracking Control for Linear Discrete-Time Markovian Jump Systems, Proceedings of the 19th IEEE International Conference on Control Applications (CCA 2010), Yokohama, Japan, FrA10.4, pp. 1981-1986 (CD-ROM), 2010.

[11] G. Nakura: Stochastic Optimal Tracking with Preview by State Feedback for Linear Discrete-Time Markovian Jump Systems, International Journal of Innovative Computing, Information and Control (IJICIC), Vol. 6, No. 1, pp. 15-27, 2010.

[12] G. Nakura: Stochastic Optimal Tracking with Preview by State Feedback for Linear Continuous-Time Markovian Jump Systems, SICE Journal of Control, Measurement and System Integration (SICE JCMSI), Vol. 3, No. 3, pp. 164-171, 2010.

[13] G. Nakura: On Nocausal $H_\infty$ Tracking Control for Linear Impulsive Systems with Stochastic Uncertainties, Trans. of The Institution of Systems, Control and Information Engineers, Vol. 26, No. 12, pp. 456-465, 2013.

[14] G. Nakura: $H_\infty$ State Estimation for Linear Impulsive Systems with Stochastic Uncertainties, Proceedings of the 46th ISCIE International Symposium on Stochastic Systems Theory and Its Applications (SSS14), Kyoto, Japan, pp. 37-46, IAC3-2 (CD-ROM), 2014.

[15] B. Oksendal and A. Sulem: Applied Stochastic Control of Jump Diffusions, Universitext, Springer, 2005.

[16] U. Shaked and C. E. de Souza: Continuous-Time Tracking Problems in an $H_\infty$ Setting: A Game Theory Approach, IEEE Trans. Automat. Contr., Vol. 40, No. 5, pp.841-852, 1995.

[17] R. Situ: Theory of stochastic differential equations with jumps and applications. Springer-Verlag, Berlin, 2005.

[18] B. Song, Z.-G. Wu, J. H. Park, G. Shi and Y. Zhang: $H_\infty$ filtering for stochastic systems driven by Poisson processes, Int. J. Contr., Vol. 88, No. 1, pp. 2-10, 2015.

[19] T. M. Steger: Stochastic growth under Wiener and Poisson uncertainty, Economics Letters, Vol. 86, 311-316, 2005.

[20] J. J. Westman and F. B. Hanson: The LQGP Problem: A Manufacturing Application, Proc. Amer. Control Conf., Vol. 1, pp. 566-570, 1997.

[21] J. J. Westman and F. B. Hanson: The NLQGP Problem: Application to a Multistage Manufacturing System, Proc. Amer. Control Conf., Philadelphia, PA, pp. 1104-1108, 1998.