Theory of rare charm decays

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Recent disagreement between experimental measurements of CP violating asymmetry in $D^0 \to K^+ K^-$ and $D^0 \to \pi^+ \pi^-$ and theoretical Standard model expectation motivated many studies within the Standard model and beyond. Rare charm decays offer new probe of possible signals beyond Standard model. CP conserving and CP violating contributions within the Standard model and beyond are reviewed for inclusive $c \to u\gamma$ and $c \to ul^+l^-$ and exclusive $D \to V\gamma$, $D^0 \to P^+P^-$, $D^+ \to \pi^+l^+l^-$, $D \to l^+l^-$ decays.

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1 Introduction

For more than two decades up-quark sector was not considered as interesting testing ground for new physics searches. The non-leptonic D mesons decay dynamics is polluted by the presence of many non-charm resonances in the vicinity of D meson masses. Flavour changing neutral current (FCNC) processes are specially interesting in searches of new physics. In the case of charm rare decays GIM mechanism plays special role. The interplay of CKM parameters and masses of down-like quarks leads to strong suppression in all rare D decays. The long distance contributions overshadow the short distance effects. The main issue is how to separate information on short distance dynamics, either within SM or in its extensions. This is a longstanding problem in rare charm decays. Discrepancy between measured and expected CP violating asymmetry in charm decays [1, 2, 3] triggered many studies of additional checks of the observed anomaly. Many theoretical studies were performed in order to explain the observed discrepancy [4, 5, 6, 7, 8, 9, 10, 11, 12]. Some of these approaches have explained observed asymmetry by the Standard model effects [11], while in the rest of them, possible new physics effects were considered. In [4] and [13] it was pointed out that most likely effective operators explaining CP asymmetry in charm decays are color-magnetic dipole operators. Most important result of these studies [4, 5, 6, 7, 8, 9, 10, 12] is that apparently one needs additional source of CP violation and in particular in the charm sector. The relevant question is: Is there any possibility to observe CP violation in charm rare decays? In Sec. 2 contributions to $c \to u \gamma$ and $c \to ul^{+}l^{-}$ decay modes are reviewed. The exclusive radiative weak charm meson decays and possibility to search for CP violation are discussed in Sec. 3. Tests of CP violation in charm meson decays with the leptons in the final state are discussed in Sec. 4. Last section contains the summary.

2 Inclusive decay modes: $c \to u \gamma$ and $c \to ul^{+}l^{-}$

The effective low-energy Lagrangian describing $c \to u \gamma$ and $c \to ul^{+}l^{-}$ transitions within SM is given by:

$$\mathcal{L}_{\text{eff}}^{SD} = \frac{G_{F}}{\sqrt{2}} V_{cb}^{*} V_{ub} \sum_{i=7,9,10} C_{i} Q_{i},$$  \hspace{1cm} (1)

The operators are then:

$$Q_{7} = \frac{e}{8\pi^{2}} m_{c} F_{\mu\nu} \pi_{\sigma} \sigma_{\mu\nu}(1 + \gamma_{5})c,$$

$$Q_{9} = \frac{e^{2}}{16\pi^{2}} \bar{\pi}_{L} \gamma_{\mu} c_{L} \gamma^{\mu}l,$$

$$Q_{10} = \frac{e^{2}}{16\pi^{2}} \bar{\pi}_{L} \gamma_{\mu} c_{L} \gamma^{\mu} \gamma_{5}l.$$  \hspace{1cm} (2)
In (1) $C_i$ denote as usual Wilson coefficients (they are determined at the scale $\mu = m_c$, $F_{\mu\nu}$ is the electromagnetic field strength and $q_L = \frac{1}{2}(1-\gamma_5)q$. In the case of $c \to u\gamma$ decay only $C_7$ contributes, while in the case of $c \to ul^+l^-$ all three Wilson coefficients are present. At one-loop level contributions coming from penguin diagrams is strongly GIM suppressed giving a branching ratio $\sim 10^{-18}$ [14, 17, 18, 19, 20]. The QCD corrections enhance this rate to $BR(c \to u\gamma)_{SM} = 2.5 \times 10^{-8}$ [15, 16]. Within Standard model the short distance contribution coming from $Q_7, 9$ leads to the branching ratio $BR(D \to X_u e^+ e^-)_{SM} \simeq 3.7 \times 10^{-9}$.

However, this short distance contribution is overshadowed by long distance contributions [19, 21] of the size:

$$BR(D \to X_u e^+ e^-)_{SM} \sim O(10^{-6}).$$

One of the most popular extension of the SM is MSSM. Following discussion in [24] the model with non-universal soft-breaking terms, knowing that gluino (according to LHC bounds) cannot be lighter than $1.3$ TeV, would give $Br(c \to u\gamma)_{ gluino } \sim 5 \times 10^{-8}$. Rather high mass of gluino would also give rise to SM $BR(c \to ul^+l^-)$ by about factor 2. As noticed in [24] the other SM extensions might give larger increase of both inclusive branching ratios. However, regardless of the increase of short distance contribution the long distance effects screen their effect in the exclusive charm meson decay modes.

3 Exclusive decay modes: $D \to V \gamma$ and $D \to P^+P^-\gamma$

The amplitude for the $D \to V \gamma$ decay can be written as

$$A[D(p) \to V(p' e^+ e^-\gamma(q, \epsilon))] = -iA_{CP}\epsilon_{\mu\nu\alpha\beta}q^\mu\epsilon^{\nu\epsilon\epsilon'\gamma} + A_{PV}[(\epsilon^{\epsilon',\beta}) (\epsilon^{\epsilon\nu} \cdot q) - (p \cdot q)(\epsilon^{\epsilon\nu} \epsilon^{\epsilon'})].$$

(5)

In the recent analysis [25] the authors have reinvestigated long distance dynamics. Using QCD sum rules result for the tensor form factors ($T^\rho \simeq T^\omega \simeq 0.7 \pm 0.2$ from they found

$$(A_{PC, PV}^{\rho, \omega})_{SM} \simeq \frac{0.6(2) \times 10^{-9}}{m_D} | \frac{C_7(m_c)}{0.4 \times 10^{-2}} |$$

(6)

where superscripts $\rho, \omega$ denote appropriate vector meson state $V$. For the determination of short distance contribution one has to know matrix element of the $Q_7$ operator. In the calculations of it the tensor form-factors are present. In ref. [26] QCD sum rules were used to determine its structure. The long distance contribution was estimated by knowing that the relation $BR(D^0 \to K^{*0}\gamma)/BR(D^0 \to K^{*0}\rho^0) = BR(D^0 \to \phi\gamma)/BR(D^0 \to \phi\rho^0)$ is a consequence of vector meson dominance [25]

$$|(A_{PC, PV}^{V})_{LD}| = \frac{32\pi}{2m_D^2}(1 - \frac{m^2_D}{m^2_D})^{-3}\Gamma(D \to V\gamma)]^{1/2},$$

(7)
what gives, for $V = \phi_s |(A_{PC,PV}^\phi)^{LD}| = \frac{5.9(4) \times 10^{-8}}{m_D}$. The main concern of this study was to investigate CP asymmetry in these decay modes. The CP asymmetry is defined as

$$a_f \equiv \frac{\Gamma(D^0 \to f) - \Gamma(\bar{D}^0 \to f)}{\Gamma(D^0 \to f) + \Gamma(\bar{D}^0 \to f)}.$$  

(8)

A lot of attention has been paid to the recent measurements of the CP violating asymmetry in charm decays. The LHCb collaboration updated recently their analysis leading to decreased value of the world average CP asymmetry $[3,1,2]$, 

$$\Delta a_{CP} = (-0.329 \pm 0.121)\%.$$  

(9)

Assuming that only NP generates such CP asymmetry, the authors of [4] noticed that most likely candidate, among effective operators which can explain deviation, is the chromomagnetic operator $Q_8$. This operator, under QCD renormalization group can mix with the electric dipole operator $Q_7$ [25]. It results in the fact that both Wilson coefficients $C_7$ and $C_8$ are of comparable size at the charm scale. In particular their imaginary parts are then

$$\text{Im}[C_{NP}^7(m_c)] \approx \text{Im}[C_{NP}^8(m_c)] \approx 0.02 \times 10^{-2}.$$  

(10)

The imaginary part of $C_{SM}^7$ is two orders of magnitude smaller. The NP contribution is comparable in size with the real part of SM $|C_{SM}^{7-\epsilon f}(m_c)| = (0.5 \pm 0.1) \times 10^{-2}$. This means that if the phase of long distance contribution can be neglected, and relative strong phase is maximal, the CP asymmetry can reach

$$|a_{V\gamma}| \sim 5\%.$$  

(11)

The amplitude for the $D \to P^+P^-\gamma$ decay can be decomposed into [28]

$$\mathcal{A}(D(P) \to P_1(p_1)P_2(p_2)\gamma(q,\epsilon)) = \frac{G_F}{\sqrt{2}} V_{ci}^* V_{uj} \left\{ F_0 \left[ \frac{p_1 \cdot \epsilon}{p_1 \cdot q} - \frac{p_2 \cdot \epsilon}{p_2 \cdot q} \right] + F_1 \left[ (p_1 \cdot \epsilon)(p_2 \cdot q) - (p_2 \cdot \epsilon)(p_1 \cdot q) \right] + F_2 \epsilon_{\mu\nu\alpha\beta} \epsilon_\mu P_\nu p_{1\alpha} p_{2\beta} \right\}.$$  

(12)

The first part is inner bremsstrahlung amplitude, the $F_1$ part denotes the electric transition and $F_2$ is the magnetic transition amplitude. The differential decay width is then

$$\frac{d\Gamma}{ds} = \frac{m_D^3}{32\pi} \left( 1 - \frac{s}{m_D^2} \right) \sqrt{s} \Gamma_0 \left[ |F_1(s)|^2 + |F_2(s)|^2 \right].$$  

(13)

The electric and magnetic dipole transitions were determined assuming vector meson exchange, knowing decay width of $V \to PP$ and using Breit-Wigner formula for the resonances present in the amplitude [25]. Following ref. [25] one can find that CP asymmetries for the region bellow and above $\phi$ resonance:

$$|a_{K^+K^-\gamma}|^{max} \simeq 1\%, \quad 2m_K \leq \sqrt{s} \leq 1.05 \text{ GeV},$$

$$|a_{K^+K^-\gamma}|^{max} \simeq 3\%, \quad 1.05 \leq \sqrt{s} \leq 1.20 \text{ GeV}.$$  

(14)

In Table 1, the recent or existing estimate of the branching ratios for $D$ rare decays are presented including the reference.
| Decay mode | Branching ratio | Reference          |
|------------|-----------------|--------------------|
| $D \to \rho(\omega)\gamma$ | $6 \times 10^{-5}$ | 1210.6546; 1205.3164 |
| $D \to K^+K^-\gamma$ | $1.35 \times 10^{-5}$ | 1205.3164 |
| $D \to X_\mu l^+l^-$ | $\mathcal{O}(10^{-5})$ | 1101.6053 |
| $D^+ \to \pi^+l^+l^-$ | $2 \times 10^{-6}$ | 1208.0795; 0706.1133 |
| $D^+_s \to K^+l^+l^-$ | $6 \times 10^{-6}$ | 0706.1133 |
| $D \to \pi^+K^-l^+l^-$ | $\mathcal{O}(10^{-5})$ | 1209.4253 |
| $D \to \pi^+\pi^-l^+l^-$ | $\mathcal{O}(10^{-6})$ | 1209.4253 |
| $D \to J/\psi l^+l^-$ | $\mathcal{O}(10^{-7})$ | 1209.4253 |
| $D \to \pi^-K^+l^+l^-$ | $\mathcal{O}(10^{-8})$ | 1209.4253 |
| $D \to \gamma\gamma$ | $(1 - 3) \times 10^{-8}$ | 1008.3141 |
| $D \to \pi^+\pi^-\mu^+\mu^-$ | $(7 - 8) \times 10^{-13}$ | 1008.3141; 0903.3650 |

Table 1: Branching ratios for charm meson decays. The second column contains the SM theoretical predictions in which long-distance contribution is dominant. The last column contains the most recent references.

4 Rare $D^+ \to \pi^+\mu^+\mu^-$, $D \to hh\ell^+\ell^-$ decays and CP violation

In this section rare $D \to \pi\ell^+\ell^-$ decays are reviewed with the goal to determine possibility to study CP violation observables [29]. The new CP violating effects in rare decays $D \to P\ell^+\ell^-$ might arise due to the interference of resonant part of the long distance contribution and the new physics affected short distance contribution. The appropriate observables, the differential direct CP asymmetry and partial decay width CP asymmetry are introduced in a model independent way. Among all decay modes the simplest one for the experimental searches are $D^+ \to \pi^+\ell^+\ell^-$ and $D^+_s \to K^+\ell^+\ell^-$. The short distance dynamics for $c \to u\ell^+\ell^-$ decay on scale $\sim m_c$ is discussed in details by the effective Hamiltonian given in [21, 4]. In the decay width spectrum of $c \to u\ell^+\ell^-$ two light generations dominate short distance dynamics. Only when third generation is included there is a possibility to obtain non-vanishing imaginary part: $\text{Im}(\lambda_b/\lambda_d) = -\text{Im}(\lambda_s/\lambda_d)$. The CP violating parts of the amplitude are suppressed by a very small factor $\lambda_b/\lambda_d \sim 10^{-3}$ with respect to the CP conserving ones and therefore the CP violating effects should be very small. Due to the rather large direct CP violation, measured in singly Cabibbo suppressed decays $D^0 \to \pi\pi, KK$, one might expect similar increase in charm rare decays. If the CP violation arises due to new physics effects, as it is mentioned already, it is due to the chromomagnetic operator $Q_8$ contribution at some high scale above $m_t$ [4]. This as in the case of radiative weak decays comes from mixing of $\hat{O}_8$ into $\hat{O}_7$ under QCD renormalization.

Close to the $\phi$ resonant peak the long distance amplitude for $D^+ \to \pi^+\mu^+\mu^-$ decay is, to a good approximation, determined by non-factorizable contributions of four-quark
operators in $\mathcal{H}^s$. The width of $\phi$ resonance is very narrow ($\Gamma_\phi/m_\phi \approx 4 \times 10^{-3}$) and well separated from other vector resonances in the $q^2$ spectrum of $D \to P\ell^+\ell^-$. Relying on vector meson dominance hypothesis the $q^2$-dependence of the decay spectrum close to the resonant peak follows the Breit-Wigner shape \[21, 23, 24\]

$$A_{LD}^\phi \left[ D \to \pi\phi \to \pi\ell^+\ell^- \right] = \frac{i G_F}{\sqrt{2}} \frac{8\pi\alpha}{3} \frac{a_\phi e^{i\delta_\phi}}{q^2 - m_\phi^2 + i m_\phi \Gamma_\phi} \pi(k_-) \bar{\psi}(k_+) . \tag{15}$$

Finite width of the resonance generates a $q^2$-dependent strong phase that varies across the peak. The strong phase on peak, $\delta_\phi$, and the normalization, $a_\phi$, are introduced in such a way, that both are assumed to be independent of $q^2$. Parameter $a_\phi$ is real and can be fixed from measured branching fractions of $D \to \pi\phi$ and $\phi \to \ell^+\ell^-$ decays \[24\]. We use PDG values for $BR(D^+ \to \phi\pi^+) = (2.65 \pm 0.09) \times 10^{-3}$, $BR(\phi \to \mu^+\mu^-) = (0.287 \pm 0.019) \times 10^{-3}$, and take into account the small width of $\phi$ by narrow width approximation as in \[29\]. With $A_{LD}^\phi = \overline{A}_{LD}^\phi$ the differential direct CP violation becomes

$$a_{CP}(q^2) = \frac{|A|^2 - |\overline{A}|^2}{|A|^2 + |\overline{A}|^2} = \frac{-3}{2\pi^2} \frac{f_T(q^2)}{a_\phi} \frac{m_c}{m_D + m_\pi} \frac{m_\phi}{m_\phi} \Im \left[ \frac{\lambda_b}{\lambda_s} C_7 \right] \left[ \cos \delta_\phi - \frac{q^2 - m_\phi^2}{m_\phi \Gamma_\phi} \sin \delta_\phi \right] . \tag{16}$$

The imaginary part in the above expression can be approximated as $\Im[\lambda_b C_7]/\Re \lambda_s$. The $\Im[\lambda_b C_7]$ was set to be $0.8 \times 10^{-2}$ in order to illustrate largest possible CP effect. Relative importance of the $\cos \delta_\phi$ and $\sin \delta_\phi$ for representative choices of $\delta_\phi$ is shown on the plot in fig 1. of \[29\]. As presented in \[29\] the CP asymmetry can be, depending on unknown $\delta_\phi$, even or odd with respect to the resonant peak position. The asymmetry can reach $a_{CP} \sim 1\%$ (see discussion in \[29\]).

In addition, a CP asymmetry of a partial width in the range $m_1 < m_{\ell\ell} < m_1$ can be introduced:

$$A_{CP}(m_1, m_2) = \frac{\Gamma(m_1 < m_{\ell\ell} < m_2) - \Gamma(m_1 < m_{\ell\ell} < m_2)}{\Gamma(m_1 < m_{\ell\ell} < m_2) + \Gamma(m_1 < m_{\ell\ell} < m_2)} , \tag{17}$$

where $\Gamma$ and $\overline{\Gamma}$ denote partial decay widths of $D^+$ and $D^-$ decays, respectively, to $\pi^\pm\mu^+\mu^-$. $A_{CP}$ is related to the differential asymmetry $a_{CP}(\sqrt{q^2})$ as

$$A_{CP}(m_1, m_2) = \frac{\int_{m_1^2}^{m_2^2} dq^2 R(q^2) a_{CP}(\sqrt{q^2})}{\int_{q_{\min}^2}^{q_{\max}^2} dq^2 R(q^2)} , \tag{18}$$

where

$$R(q^2) = \frac{1}{(q^2 - m_\phi^2)^2 + m_\phi^2 \Gamma_\phi^2} \int_{s_{\min}(q^2)}^{s_{\max}(q^2)} ds \sum_{s_+, s_-} |\bar{u}(s_-)(k_-) \phi v(s_+)(k_+)|^2 , \tag{19}$$
involves the resonant shape and the integral of the lepton trace over the Dalitz variable as in [29]. The asymmetry on the same bin for the $\pi^+\mu^+\mu^-$ final state can be defined as

$$C^\phi_{\text{CP}} \equiv A_{\text{CP}}(m_\phi - 20 \text{ MeV}, m_\phi + 20 \text{ MeV}).$$

(20)

The asymmetry $C^\phi_{\text{CP}}$ is most sensitive to the $\cos \delta_\phi$. Sensitivity is therefore optimized for cases when $\delta_\phi \sim 0$ or $\delta_\phi \sim \pi$. Its sensitivity would decrease if we approached $\delta_\phi \sim \pm \pi/2$, since the $a_{\text{CP}}(m_\ell\ell)$ would be asymmetric in $(m_\ell\ell - m_\phi)$ in this case. For that region of $\delta_\phi$ it was found that the following observable has good sensitivity to direct CP violation

$$S^\phi_{\text{CP}} \equiv A_{\text{CP}}(m_\phi - 40 \text{ MeV}, m_\phi - 20 \text{ MeV}) - A_{\text{CP}}(m_\phi + 20 \text{ MeV}, m_\phi + 40 \text{ MeV}).$$

(21)

The bins where the partial width CP asymmetries $C^\phi_{\text{CP}}$ and $S^\phi_{\text{CP}}$ are defined are shown in fig. 2 in [29] together with $a_{\text{CP}}(m_\ell\ell)$. The largest asymmetry $A_{\text{CP}}$ approaches $5\%$ for $\delta_\phi = \pm \pi/2$. The detailed analysis of the semileptonic four body $D \to hh\ell^+\ell^-$ decays was done in the work of ref. [30]. The dominant long-distance contributions (bremstrahlung and hadronic effects) are calculated and total branching ratios and the $(m^2_2, m^2_h)$ Dalitz plots are presented. Branching ratios turn out to be substantially larger than previously expected. Using vector meson dominance, it was found for the Cabibbo-allowed, singly Cabibbo-suppressed, and doubly Cabibbo-suppressed modes

$$BR(D^0 \to K^-\pi^+\ell^+\ell^-) \sim 10^{-5}$$
$$BR(D^0 \to \pi^-\pi^+\ell^+\ell^-) \sim 10^{-6}$$
$$BR(D^0 \to K^-K^+\ell^+\ell^-) \sim 10^{-7}$$
$$BR(D^0 \to K^+\pi^-\ell^+\ell^-) \sim 10^{-8}$$

(22)

The new physics detection in these decay modes was also discussed. It was found that two angular asymmetries, namely the T-odd diplane asymmetry and the forward-backward dilepton asymmetry offer direct tests of new physics due to tiny Standard model backgrounds. If supersymmetric and $Z'$-enhanced scenarios are assumed, and if the size of Wilson coefficients $C_9$ and $C_{10}$ is compatible with the observed CP asymmetry in nonleptonic charm decays and flavor constraints, it was found in [30] that new physics effects in $D^0 \to h_1h_2\ell^+\ell^-$ might reach the $\%$ level. In Table 2 predictions for size of CP violating asymmetries in rare charm decays are presented.

The two body rare decays $D^0 \to \gamma\gamma$ and $D^0 \to \ell^+\ell^-$ were reconsidered in [31]. The result for the short and long distance contributions are $BR_{\text{2-loops}}(D^0 \to \gamma\gamma) = (3.6 - 8.1) \times 10^{-12}$. Short distance contributions in $D^0 \to \ell^+\ell^-$ decay lead to a very suppressed branching ratio in the SM. Therefore, it is natural to consider it as an ideal testing ground for NP effects. Ref. [31] considered contributions coming from $\gamma\gamma$ intermediate states due to long distance dynamics in $D^0 \to \mu^+\mu^-$ arriving at the value $BR(D^0 \to \mu^+\mu^-) \sim$
Decay mode | size | Reference
--- | --- | ---
$D \to \rho(\omega)\gamma$ | $\leq 5\%$ | 1210.6546
$D \to K^+K^-\gamma$ | $\leq 1\%$ ($\leq 3\%$) | 1205.3164
$D \to X_u l^+l^-$ | $\leq 1\%$ | 1212.4849
$D^+ \to \pi^+\mu^+\mu^-$ | $\leq 5\%$ ($1\%$) | 1208.0795
$D^+ \to hh\mu^+\mu^-$ | $\leq 1\%$ | 1208.0795

Table 2: CP violating asymmetries for charm rare decays, size and the original reference. The four last decay modes have the CP asymmetry in the vicinity $\phi$ resonance.

$(2.7 - 8) \times 10^{-13}$. According to calculations of the same authors, some NP models can enhance the branching ratio by a factor of 2. Recently LHCb improved bound on the branching ratio $BR(D^0 \to \mu^+\mu^-) \leq 6.2 \times 10^{-9}$ [52] and it offers an ideal possibility to test NP models.

5 Summary

The SM contribution to rare charm decays are rather well known. For all decay modes amplitudes are fully dominated by long distance dynamics. The possible presence of CP violation induced by new physics in charm nonleptonic decays open new window for new physics searches. The study of rare charm decays were revived and number of studies of CP violation in rare charm decays were done. Very interesting signals of new physics might arise in $D \to \rho(\omega)\gamma$ and $D \to K^+K^-\gamma$, as well as in decays with the leptonic pair in the final state $D \to X_u l^+l^-$, $D^+ \to \pi^+\mu^+\mu^-$, $D^+ \to hh\mu^+\mu^-$. The three body decays are particularly interesting, since one can focus on the CP asymmetry around the $\phi$ resonant peak in spectrum of dilepton invariant mass. The interference term between the resonant and the short distance amplitude drives the direct CP asymmetry. If there is no enhancement of CP violation in $D^+ \to \pi^+\ell^+\ell^-$ then one cannot judge whether CP violation in $D \to \pi\pi$, $KK$ is entirely due to SM dynamics or not. However, by not observing any CP asymmetry in $D^+ \to \pi^+\ell^+\ell^-$ around the $\phi$ peak would suggest that SM explanation of the observed CP violation in $D \to \pi\pi$, $KK$ is most likely. The study of CP violation in all rare charm decays might differentiate between possible explanations of the observed CP asymmetry in charm decays and constrain new physics in charm sector.

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