Models of Millimeter and Radio Emission from Interacting Supernovae

Nitika Yadlapalli Yurk1, Vikram Ravi2, and Anna Y. Q. Ho3,4

1 Cahill Center for Astronomy and Astrophysics, MC 249-17 California Institute of Technology, Pasadena CA 91125, USA; nyadlapa@caltech.edu
2 Miller Institute for Basic Research in Science, 468 Donner Lab, Berkeley, CA 94720, USA
3 Department of Astronomy, University of California Berkeley, Berkeley, CA, 94720, USA
4 Lawrence Berkeley National Laboratory, 1 Cyclotron Road, MS 50B-4206, Berkeley, CA, 94720, USA

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Abstract

This work utilizes established models of synchrotron-powered light curves for core-collapse supernovae in dense circumstellar environments, namely, type IIn and Ibn, to demonstrate the potential for detecting millimeter emission from these events. The progenitor types of these supernovae are still an open question, but using the synchrotron light curves as probes for the circumstellar environments could shed light on the mass-loss histories of the progenitors and discern between different theories. Observations in millimeter bands are particularly fruitful, as they probe regions at smaller radii and higher ambient densities, where centimeter emission tends to be self-absorbed. In our application of these light curves, we explore a diversity of progenitor types and mass-loss profiles to understand their effects on the light-curve shapes. Additionally, we fit model parameters to the 8 GHz light curve of type IIn supernova 2006jd and then create millimeter light curves using these parameters to show the possibility of detecting an early millimeter peak from such an event. We predict that next generation millimeter surveys will possess the capability to detect nearby and extreme events. However, there is a pressing need for millimeter follow-up of optically discovered interacting supernovae to more completely sample the true population.

Unified Astronomy Thesaurus concepts: Core-collapse supernovae (304); Millimeter astronomy (1061)

1. Introduction

1.1. Millimeter Transients

To date, the transient millimeter sky remains largely unexplored. In recent years though, millimeter transients have been discovered on timescales from minutes to months. The first blind survey for millimeter transients was conducted by Whitehorn et al. (2016) using the South Pole Telescope (SPT), which surveyed 100 deg^2 of sky and discovered only one candidate transient at <3σ significance. An improved transient search was conducted with the SPT-3G camera, covering 1500 deg^2 with deeper sensitivity (Guns et al. 2021). This search yielded 10 unique transient sources of which eight were identified as flaring stars and two were identified as extragalactic events of unknown origin. Other blind detections of millimeter transients include three candidate stellar flares found serendipitously by the Atacama Cosmology Telescope (ACT; Naess et al. 2020). Though these works imply a higher event rate for millimeter stellar flares than previously known, the rates of extragalactic millimeter transients still pose an open question.

Extragalactic transient millimeter emission arises from energetic synchrotron sources. Metzger et al. (2015) and more recently Eftekhari et al. (2021) predict that wide-field millimeter surveys would be most sensitive to reverse shock emission from gamma-ray bursts (GRBs), such as those detected by De Ugarte Postigo et al. (2012) and Laskar et al. (2013). They also predict millimeter detections of tidal disruption events (TDEs), such as on-axis jetted sources like Swift J1644+9.3+573451 (Zauderer et al. 2011) or off-axis jetted sources like IGR J12580+0134 (Yuan et al. 2016). Optimistically, they additionally hope for a few detections of fast blue optical transients, such as nearby, luminous, millimeter transient AT2018cow (Ho et al. 2019). However, detections of typical core-collapse supernovae (CCSNe) at millimeter wavelengths are scarce. Here we demonstrate the potential of early targeted observations of interacting supernovae (SNe) to dramatically increase the number of detections and probe the environments in which these explosions occur.

1.2. Millimeter Detectable SNe

CCSNe result from the deaths of massive stars. There exists a broad diversity of flavors of CCSNe, usually arising from different progenitor types (seeSmartt 2009, and references therein). The most commonly observed SNe are type II-P, characterized by a plateau in the light curve following the initial decline from peak brightness. The plateaued light curve is powered by expansion of the photosphere due to recombination in the hydrogen envelope of a red supergiant (RSG) progenitor. Type II-L SNe, characterized by a linearly declining light curve rather than a plateaued one, are similar to type II-P but arise from RSG progenitors that have stripped hydrogen envelopes. Type Ibc/Ib SNe are also the products of stripped envelopes, but arise from massive Wolf-Rayet (W-R) stars rather than RSGs. Interacting SNe, however, are unique because they have no distinctive progenitor type.

Interacting SNe are identified by their bright, narrow spectral lines that arise from shock interaction with a very dense circumstellar medium (CSM; Chugai 1990). Type IIn SNe spectra are dominated by hydrogen lines whereas type Ib SNe show weak hydrogen but strong helium lines (see Smith 2017, and references therein for more detail). High velocity shocks, reaching up to a few percent of the speed of light, propagating through regions of density orders of magnitude greater than interstellar medium (n_H \sim 10^3 \text{ cm}^{-3}) are optimal conditions for producing bright radio and millimeter emission. Millimeter emission is especially important, as it peaks at early times when the shock radius is smaller and the CSM density is higher. Early-time radio emission has been detected from type Ib/Ib SNe, such as the
nearby SN 1993J (Pooley & Green 1993), or from type Ib/c events, such as SN1998bw (Kulkarni et al. 1998) or SN 2009bb (Soderberg et al. 2010). A handful of Ib/Ibc SNe has been detected in millimeter observations. The very nearby events SNe iPTF13bvn (Cao et al. 2013) and SN 2011dh (Horeshi et al. 2013), for example, were detected with the Combined Array for Research in Millimeter-wave Astronomy, and were essential to constraining the early evolution of the shock radius and providing estimates of the pre-supernova (SN) mass-loss rate. To date, only a handful of type IIn SNe has been detected at radio wavelengths. Figure 7 in Chandra (2018) summarizes detections of IIn SNe at 8 GHz—we reproduce a few of these light curves here for reference in Figure 1. Notably, nearly all of the detections are made over 100 days post-explosion. Bietenholz et al. (2021) show that the luminosity-rise time parameterization of radio emission from type IIn SNe is characterized by a significantly later time to peak than that of type Ib/c or other type II SNe. Thus, observations at higher frequencies would prove useful in detecting the peak of the light curve at an earlier time. Type IIn and Ibn SNe have yet to be detected in millimeter observations, but are likely to be millimeter bright owing to their especially high CSM densities.

1.3. Diversity of Progenitor Models for Interacting SNe

The nature of the progenitors for interacting SNe is still unknown, owing to the spectactorally high observed mass-loss rates of up to $0.1 M_\odot$ yr$^{-1}$ and little understanding of the mechanisms driving this. Luminous blue variables (LBV), among the most luminous known stars, are commonly theorized to be IIn progenitors. Gal-Yam et al. (2007) provide the most direct evidence for this theory with pre-explosion Hubble Space Telescope observations of IIn SN 2005gl that revealed a source coincident with the SN position with a high luminosity that could only be explained by an LBV. Detections of pre-SN outbursts also hint toward an LBV origin for IIn SN. Namely, SN 2009ip, originally thought to be a type II SN, was discovered to be a bright LBV-like outburst prior to a true IIn SN explosion in 2012 (Mauerhan et al. 2013). The evolutionary pathway of LBVs, however, is still debated, casting doubt on their candidacy as IIn progenitors. Humphreys & Davidson (1994) suggest that LBVs are O-type stars experiencing a period of high mass loss en route to becoming W-R stars; stars in this evolutionary stage would not be expected to explode as SNe. Smith & Tombleson (2015) offer a model of LBVs arising from binary evolution, which better fits a picture of LBVs as IIn progenitors. Blue supergiants (BSGs) have also been hypothesized to be IIn progenitors as models of BSG explosions are consistent with the types of precursor outbursts observed in events such as SN 2009ip and SN 2010mc (Smith et al. 2014). Additionally, spatially resolved observations of the gas around VY CMa indicate that red supergiants (RSGs) may also have CSM structures that could give rise to IIn SNe (Smith et al. 2009). Thus, the ability to probe the CSM structure around observed SNe has the power to shed light on the nature of their progenitors.

Observations of synchrotron-powered light curves around the time of their peaks enable measurements of the radii and electron densities of the CSM the SNe is interacting with at that time, using equations published in works such as Duran et al. (2013) and Ho et al. (2021). Constraining the shock velocity and the CSM density by these measurements allows constraints to be placed on the properties and mass-loss history of the progenitor, narrowing down viable progenitor models. As spherically expanding synchrotron sources will peak earlier in the millimeter range than at lower frequencies, early millimeter observations are especially important in measuring the CSM density close to the surface of the progenitor, giving us a glimpse into the behavior of the progenitor in its final days to months.

In this paper, we model light curves of radio and millimeter emission from interacting SNe. Section 2 summarizes the emission and absorption models we use. Section 3 presents applications of this model to generate synthetic light curves of type IIn SNe and Section 4 shows the potential for detecting these events with next generation cosmic microwave background (CMB) surveys. We conclude in Section 5.

2. Model Summary

Radio emission from SNe originates in synchrotron radiation from free electrons in the CSM that are accelerated to relativistic energies by the forward SN shock. The brightness and evolution of the emission are dependent on the density and structure of the CSM, as will be seen in the equations presented below as well as in the light curves presented in the following section. The aim of this section is not to present novel emission models, but to give an overview of the well-established models that we utilize. We reproduce many relevant equations from the works cited in this section to highlight specific model parameters that we explore in the rest of this work.

For an interacting CCSN, the progenitor star undergoes pre-SN mass loss in the decades leading up to the explosion (Smith 2017, for example). The resulting CSM density can be modeled as a power law described by $M$, the mass-loss rate, and $v_w$, the wind velocity, as shown in Equation (1). $R_s$ is the radius of the progenitor star and $s$ describes the steepness of the CSM density profile. A steady wind can be modeled with $s = 2$, but nonsteady mass loss can lead to deviations from this value. Equations (1) and (4) and much of the following formalism is

![Figure 1. Reproduction of 8.5 GHz light curves of four type IIn SNe. This is meant to show the diversity of light curves that have been observed, highlighting the challenge in constraining a single progenitor model for these events. The data for these were taken from Chandra et al. (2012), Dwarkadas et al. (2016), Chandra et al. (2015), and Margutti et al. (2014).](image)
introduced and explained in detail in Chevalier (1982):

$$\rho_{\text{CSM}} = \frac{M}{4\pi v_w R^2} \left(\frac{R}{\bar{R}}\right)^3. \quad (1)$$

Following the collapse of the core, a radiation dominated shock travels through the progenitor star until the shock reaches a radius that is optically thin to high energy photons. When the shock breaks out of the stellar surface, these high energy photons propagate through the CSM, giving rise to an ionization front. Works such as those of Nakar & Sari (2010) and Kochanek (2019) go into detail about models and observations of the shock breakout, but we consider a simplified case for this work.

The expansion of the ionization front is bounded by the speed of light and is given by Equation (2) where \( t \) is the elapsed time, \( Q \) is the number of ionizing photons per second, and \( m_p \) is the proton mass. The maximum radius of the ionization front, given in Equation (3), is dictated by \( T \), the duration of the breakout pulse. Assuming the shock breaks out at the stellar surface, the duration of the breakout pulse can be approximated by the light-crossing time of the progenitor, \( T = R_s/c \). For extreme CSM densities, the shock breakout may occur in the wind, but we ignore this possibility for now. We additionally ignore any recombination in the CSM.

$$R_{\text{ion}} = \frac{Qm_pv_w t}{M} \leq c t, \quad (2)$$

$$R_{\text{ion, max}} = \frac{Qm_pv_w}{M} T. \quad (3)$$

After the shock breakout, a forward shock continues to expand through the CSM. As the fastest moving unshocked ejecta reaches the decelerating shock front, a second shock known as the reverse shock forms. In the reference frame of the forward shock, the reverse shock propagates backward and reheats the ejecta. It should be noted though that in the observer frame, the reverse shock is mostly moving radially outward. The distinction between these two reference frames is important.

The boundary between the reverse shock and the forward shock is a contact discontinuity. We follow the discussion in Chevalier & Fransson (1994) and Section 5.6 of Vink (2020) for their derivations for shock radius and velocity. The shock behavior can be modeled by approximating the forward and reverse shocks as a thin shell, meaning we assume that the radius of both the forward and reverse shock is approximately equal to the radius of the contact discontinuity. The radius and velocity of this shell are calculated by balancing the pressure difference across the contact discontinuity with the amount of deceleration. The reverse shock pressure near the contact discontinuity is dependent on the density profile of the SN ejecta near the surface, \( \rho_0 \), where \( n \) defines the power-law index of the density profile:

$$\rho_j = \rho_0 \left(\frac{t}{t_0}\right)^{-3} \left(\frac{v_{j,s}^2}{r}\right)^{n}. \quad (4)$$

The inner regions of the ejecta are assumed to have a flat density distribution. The initial density and velocity of this core are described by constants \( \rho_j^3 \) and \( v_{j,s}^2 \), which are derived using total ejecta mass, \( M_{ej} \), and the kinetic energy of the ejecta, \( E_{\text{kin}} \).

$$v_{j,o} = \sqrt{\frac{\frac{10}{3} (n - 5) E_{\text{kin}}}{\frac{3}{2} (n - 3) M_{ej}}}. \quad (5)$$

$$\rho_j^{3,3} = \frac{M_{ej}^{3/2} E_{\text{kin}}^{3/2}}{3 \pi (n - 3)^{3/2}} \frac{1}{10 n - 5 (n - 3)} \frac{R}{R_s}. \quad (6)$$

The radius and velocity of the shell are then derived to be Equations (7) and (8), respectively.

$$R_i(t) = \left[ \frac{4\pi (3 - s)(4 - s) \rho_j^3 v_{j,s}^2 R_{\text{ion}}^2}{(n - 4)(n - 3)M} \frac{t^{n-3}}{t^{n-3}} \right]^{1/(n-s)}$$

$$V_i(t) = \frac{dR_i(t)}{dt} \propto t^{(3-s)/(2-n)}. \quad (7)$$

As the shock expands through the ionized CSM, the free electrons are Fermi accelerated through the shock to relativistic speeds with a power-law distribution of energies as described by Equation (9). The value of \( p \) is dependent on the details of the shock acceleration process. Table 1 in Chevalier (1998) compiles a list of SNe with observed indices. Many of these SNe have power-law indices close to 3, though some exhibit smaller values ranging as low as 2. In this work, we will consider the case of \( p = 3 \), but the results presented here do not change significantly based on the value of \( p \).

$$N(E) \propto E^{-p}. \quad (9)$$

We assume that all electrons are shocked to relativistic speeds and that the electrons always contribute a constant fraction, \( \epsilon_e \), of the total energy density of the shocked gas. Applying the Rankine–Hugoniot jump conditions for the case of a fast shock (see Draine 2011, for example), the post-shock energy density is

$$u_{\text{tot}} = \frac{9}{8} \rho_{\text{CSM}} V_i^2. \quad (10)$$

The strength of the emission also depends on the magnetic field behind the shock. We assume that the post-shock magnetic field density is a constant fraction, \( \epsilon_B \), of the total energy density.

$$B = \sqrt{8\pi u_{\text{tot}} \epsilon_B}. \quad (11)$$

For synchrotron sources, the frequency at which the source has an optical depth of unity is known as the synchrotron self-absorption (SSA) frequency, \( \nu_{\text{ss}} \). Above this frequency, the source will be optically thin and below this frequency the source will be optically thick. For a source of a given age, we can also define the cooling frequency, \( \nu_c \). This is defined as the characteristic frequency of an electron that has radiated an amount of energy equivalent to its total energy. It is defined in Equation (12), where \( m_e \) is the electron mass, \( q_e \) is the elementary charge, and \( \sigma_T \) is the Thomson cross section,

$$\nu_c = \frac{18\pi m_e c q_e}{\sigma_T B^2}. \quad (12)$$

The shape of the source spectrum depends on the ordering of \( \nu_{\text{ss}} \) and \( \nu_c \). The SN shock will begin in the fast cooling regime, where \( \nu_c < \nu_{\text{ss}} \). Referencing the equations in Appendix C of Ho et al. (2021), we can write expressions for \( \nu_{\text{ss}} \) and the corresponding
peak luminosity, \(L_a\) for the fast cooling case:

\[
\nu_a = R_s^{1/4} B^{3/4} \nu^{-1/4} \xi^{-1/8} \eta^{7/8},
\]

\(L_a = 4\pi R_s^{1/8} B_1^{1/8} \nu^{-5/8} \xi^{-5/16} \eta^{35/16} \zeta.
\)

As the shock expands, \(\nu_a\) decreases with time while \(\nu_c\) increases. Eventually, the shock will enter the slow cooling regime where \(\nu_c > \nu_a\). In this regime, \(\nu_a\) and \(L_a\) evolve as below.

\[
\nu_a = R_s^{2/7} B^{9/7} \eta,
\]

\(L_a = 4\pi R_s^{1/7} B^{9/7} \eta^{2/7} \zeta.
\)

The constants \(\xi, \eta,\) and \(\zeta\) are defined as follows:

\[
\xi = \frac{\sigma T}{18\pi m_e c q_e},
\]

\[
\eta = \left(\frac{\sigma T}{12\pi^2 m_e^2 c}\right)^{2/7} \left(\frac{q_e}{2\pi m_e c}\right)^{1/7},
\]

\[
\zeta = \frac{1}{3} \left(2\pi m_e\right)^{1/2} \left(\frac{\xi}{q_e}\right).
\]

In both the slow and fast cooling regimes, the optically thick part of the spectrum goes as \(L_{\nu} \propto \nu^{-n/2}\) in the case of fast cooling. When the shock transitions to slow cooling, the optically thin part of the spectrum goes as \(L_{\nu} \propto \nu^{-(p-1)/2}\) for \(\nu < \nu_c\) and shifts to \(L_{\nu} \propto \nu^{-p/2}\) for \(\nu > \nu_c\). See Spectra 2 and 3 within Figure 1 of Granot & Sari (2002) for a visualization of this. The functional definition of these spectra is given in Equation (C17) of Ho et al. (2021).

The emission is additionally absorbed in the ionized CSM due to free–free absorption (FFA). A frequency dependent correction must be applied to the spectrum by multiplying by \(e^{-\tau_{FFA}}\), where \(\tau_{FFA}\) is given in Equation (20). In the calculation of the free–free opacity, the value of \(\kappa_{FFA}\) is derived in Panagia & Felli (1975).

\[
\tau_{FFA} = \int_{R_i}^{R_{in}} \kappa_{FFA} n_e n_i ds,
\]

\[
\kappa_{FFA} = 4.74 \times 10^{-27} \left(\frac{\nu}{1 \text{ GHz}}\right)^{-2.1} \left(\frac{T_e}{10^5 \text{K}}\right)^{-1.35}.
\]

**3. Application to Type IIn/Ibn SNe**

**3.1. Model Light Curves**

In this section, we present radio light curves of forward shock emission arising from a variety of interacting SNe conditions. Given the broad range of viable progenitor models for interacting SNe, we consider three progenitor types: RSGs, BSGs, and W-R stars. We expect the RSG and BSG progenitors to represent light curves for type IIn SNe and W-R progenitors to represent light curves for type Ibn SNe. The synchrotron emission model we detail in Section 2 is applicable to both scenarios. Each progenitor model varies in its breakout radius, wind velocity, and shock breakout pulse luminosity. We adopt the same radii and breakout luminosities for the progenitor models as those used in Nakar & Sari (2010). We chose a characteristic RSG wind velocity of 20 km s\(^{-1}\) based on data on nearby RSGs (Maueron & Josselin 2011). OB type stars have wind velocities on the order of their escape velocities, so for our BSG and W-R models we choose wind velocities of 250 and 1000 km s\(^{-1}\) respectively. We assume an explosion energy of \(E = 10^{51}\) erg and an ejecta mass of \(M_{ej} = 5 M_\odot\). The fraction of energy in electrons and magnetic fields is fixed at \(\epsilon_e = \epsilon_B = 1/3\).

Our choices in parameter exploration are in part informed by modeling work done by Moriya et al. (2014) to fit bolometric light curves of type Ibn SNe. Figure 9 in this work shows a range of estimated mass-loss rates from various type Ibn SNe—from this, we choose to explore three different mass-loss rates in this work \(\dot{M} = 10^{-2}, 10^{-3},\) and \(10^{-4}\) \(M_\odot\) yr\(^{-1}\). Moriya et al. (2014) also infer the CSM density slope, \(s\), for each of these light curves. We explore parameters between \(1.6 < s < 2.2\), but refer to Moriya & Tominaga (2012) for a more detailed discussion of the important effect of the density profile on the resultant optical light curves. We note too that we assume an infinite wind and a fixed CSM density slope. For piecewise varying CSM models, we refer readers to works such as Matsuoka et al. (2019), which present hydrodynamical simulations of radio light curves resulting from shock interaction with a confined CSM.

Figure 2 shows light curves for combinations of the three progenitors and three mass-loss rates at two frequencies, 10 and 100 GHz, and two ionized CSM temperatures, \(T_e = 10^4\) and \(10^5\) K. Ejecta and CSM density profile indices are fixed to \(n = 10\) and \(s = 2\) in this example. The dotted and dashed lines in this figure represent emission that one may observe, taking into account FFA. Models including only synchrotron self-absorption, shown by solid lines, are included for comparison. These light curves illustrate the limits on searching for lower frequency radio emission from interacting SNe. For example, for an SN exploding from an RSG progenitor with a mass-loss rate of \(10^{-4}\) \(M_\odot\) yr\(^{-1}\), akin to a star like VY CMa, the 10 GHz emission could not reasonably be detected until around 200 days for the coolest CSM temperature of \(10^4\) K. The 100 GHz emission from the same event, however, peaks sooner than 100 days, yielding information about the CSM structure much closer to the progenitor surface than the 10 GHz emission. Similarly, in the BSG case, 100 GHz observations are essential to observe any early-time emission from progenitors with a mass-loss rate of \(10^{-2}\) \(M_\odot\) yr\(^{-1}\). For W-R progenitors, the high wind velocities and small progenitor radii decrease the CSM densities as well as limit the extent of the ionization front, making FFA much less of an issue in observing type Ibn SNe, compared to IIn. For all progenitor types, however, the millimeter light curve peaks much earlier than the radio light curve. This is because as the source radius increases and the shock decelerates, the SSA frequency decreases. As we will show in the next section, applying a model fitting approach to gleaning information from the light curve provides the most discerning information when detections are made around the peak. Thus, even independent of the problem of absorption, studies of interacting SNe would greatly benefit from early-time millimeter observations.

We also explore the effect of different ejecta and CSM density profiles as well as the initial explosion energy on the 100 and 10 GHz light curves. To do this, we start with an RSG model and fix \(\dot{M} = 10^{-2}\) \(M_\odot\) yr\(^{-1}\), \(T_e = 10^4\) K, and \(M_{ej} = 5 M_\odot\). In the upper panel of Figure 3, we fix \(s = 2\) and \(E = 10^{51}\) erg and vary the ejecta density power-law index in
Figure 2. Light curves at 10 and 100 GHz of synchrotron emission originating in the forward shock of an interacting SN. Three absorption cases are shown here: SSA only, SSA + FFA assuming a 10^5 K CSM, and SSA + FFA assuming a 10^4 K CSM. The extent of the ionized region contributing to FFA is calculated given the shock breakout pulse properties. We use progenitor radii of 500 R_e (RSG), 70 R_e (BSG), and 5 R_e (W-R) and pulse luminosities as shown in Figure 5 in Nakar & Sari (2010). We use steady wind velocities of 20 km s^{-1} (RSG), 250 km s^{-1} (BSG), and 1000 km s^{-1} (W-R). The RSG and BSG scenarios represent type IIn SNe while the W-R scenario represents a type Ibn SNe. For the 100 GHz light curves, we show a zoomed-in panel of the first 250 days to better visualize the timescale of the millimeter peaks. For scale, \sim 10^{30} \text{ erg s}^{-1} \text{ Hz}^{-1} \sim 1 \text{ mJy at } \sim 100 \text{ Mpc}. 
integer steps from eight to 12. In the middle panel, we fix
$n = 10$ and $E = 10^{51}$ erg, and vary the CSM density power-law
index $s$ from 1.6 to 2.2 in steps of 0.2. In the bottom panel, we
fix $s = 2$ and $n = 10$ and vary the initial explosion energy
between $5 \times 10^{49}$ erg $< E < 10^{52}$ erg. The trends discussed
below hold for both the BSG and W-R models as well,

Figure 3 shows that the millimeter and radio light curves are
most sensitive to changes in explosion energy and CSM density
profile. The middle panel of Figure 3 shows that for a fixed
mass-loss rate, decreasing $s$ increases the peak luminosity of
the millimeter light curve. This is because while a steady wind
is modeled with $s = 2$, a value of $s < 2$ represents that in
the past, the progenitor had a mass-loss rate higher than $M = 10^{-3}$
$M_\odot$ yr$^{-1}$ but the mass loss slowed leading up to the time
of explosion. A value of $s > 2$, however, indicates that the mass-
loss rate increased with time up to $M = 10^{-3}$ $M_\odot$ yr$^{-1}$ leading
up to the time of explosion. Thus, the trends shown here reflect
the fact that the $s < 2$ CSM scenarios presented result in the SN
shock interacting with more total mass which powers a brighter
light curve. As $s$ decreases, the shape of the light curve post-
peak flattens as well, resulting from the sustained higher CSM
densities at larger radial distances.

Decreasing $E$ also dramatically changes the light-curve shape
by flattening the peak and decreasing the peak luminosity.
Comparing the curves at various energies to those light curves
shown in Figure 1 already shows preliminarily that for type IIn,
relatively lower energy explosions can still be responsible
for bright radio emission. The ejecta density profile, however,
has a minimal effect on either the radio or millimeter light curves,
indicating that detections of these events will not discern between
different models of $n$ for the progenitor. Taking into account all of these parameters is

3.2. Case Study: SN 2006jd

To illustrate an application of the modeling described in the
previous section, we fit model light curves to 8.5 GHz
detections of SN 2006jd. SN 2006jd was discovered on 2006
October 12 (Blondin et al. 2006). It was originally classified as
a type IIb SN and noted for its spectral similarity to SN 1993J,
though it was later reclassified as a type IIn. Most type IIn SNe
have very little, if any, monitoring with radio telescopes. For
2006jd, however, Chandra et al. (2012) report on 4 yr of
follow-up observations of the event with the Very Large Array,
ranging from 400–2000 days post-explosion. We convert
the flux densities reported in Chandra et al. (2012) to luminosities
using a distance of 79 Mpc to the event and utilize emcee
(Foreman-Mackey et al. 2013) to explore combinations of
model parameters that may fit the data. For a given progenitor
model, we allow parameter exploration of mass-loss rate, CSM
temperature, explosion energy, ejecta mass, ejecta density
profile index, and CSM density profile index. We use broad top
hat priors for the parameters, but we note that when applying
this modeling to newly discovered events, priors should be
informed by interpretation of early detections in optical and
other wavelengths. The parameter posterior distributions for
2006jd assuming an RSG progenitor are shown in Figure 4.

We note a few interesting characteristics of the posteriors
and compare them with Chandra et al. (2012) as well as Moriya
et al. (2013). The latter work uses a shock evolution model
similar to ours and analytically models bolometric light curves
for type IIn SNe model using a power-law declining
luminosity. Our results are generally consistent with both of
these works. We find that a CSM density profile around $s = 1.7$
is preferred for 2006jd, which matches closely with the \( s = 1.77 \) preferred in Chandra et al. (2012) but deviates slightly from the \( s = 1.4 \) preferred in Moriya et al. (2013). However, the fact that all of the values are significantly below 2 indicates a shared conclusion that episodic mass loss may be responsible for the CSM enrichment in 2006jd. We also find that the light-curve models are not very sensitive to \( M_{ej} \) or \( n \), similar to Moriya et al. (2013) where changing these values alters the explosion energy and mass-loss rate by only around \( \sim 10\% \).

Our results for the most likely explosion energy and mass-loss rate, however, deviate from those found in Moriya et al. (2013). We find an explosion energy requirement of only \( \sim \text{few} \times 10^{49} \text{erg} \) and a small mass-loss rate of \( M \sim 10^{-5} M_{\odot} \text{yr}^{-1} \). In contrast, the fits to the bolometric light curves of 2006jd in Moriya et al. (2013) yielded an energy requirement of \( \sim 10^{52} \text{erg} \) and a higher mass-loss rate of \( M \sim 10^{-3} M_{\odot} \text{yr}^{-1} \). Moriya et al. (2014) estimate the mass-loss history of several type IIn SNe and indicate a slowing in the already especially low amount of mass lost from the 2006jd progenitor in the final days before explosion. Thus, fitting one fixed value of \( M \) will likely lead to an imprecise model over longer periods. Another source of discrepancy likely arises from the fact that these light-curve models estimate the amount of energy needed to power just the synchrotron emission. As synchrotron emission traces only the fast moving ejecta, it does not represent the full energy budget for the explosion. However, the true fraction of the total energy it represents is impossible to constrain. As a final note on the posteriors, the light-curve model appears to weakly prefer a \( T_e \sim 4 \times 10^4 \) K, but we interpret this with caution as we do not expect late time data to constrain this parameter given that especially for a low mass-loss rate the shock would be in a regime that is optically thin to free-free by then.

These comparisons to Chandra et al. (2012) and Moriya et al. (2013) are meant only to confirm the validity of our posteriors, not to claim new insight on 2006jd. The aim of this section is to demonstrate the potential for observing millimeter flux from interacting SN by inferring from previous radio detections. In Figure 5, we show model 8.5 GHz light curves generated using parameters from 200 random samples from the Markov chain Monte Carlo (MCMC) chains. For the sample parameters, we generate light curves at 100 GHz as well. These light curves demonstrate that for parameters that generate late-time emission in a fairly narrow range of possible luminosities at 8.5 GHz, they generate early peaks at a diversity of possible luminosities for 100 GHz. A 100 GHz peak of \( \sim 10^{27} \text{erg s}^{-1} \text{Hz}^{-1} \) at the distance of 2006jd corresponds to a flux density of \( \sim 140 \mu \text{Jy} \). A significant detection of this flux density can easily be made with facilities like the Atacama Large Millimeter/submillimeter Array (ALMA).

### 4. Rates of Detection in Blind Surveys

Serendipitous millimeter transient detections are expected to increase in the era of wide-field CMB surveys. We explore their potential to detect type IIn SNe in a blind search by comparing the areal densities of the SNe with the surveys’ areas and point-source sensitivities. The areal density represents the number of sources we expect to detect in the sky per square degree at any given time. Thus, it is dependent on the volumetric event rate as well as the characteristic event timescale.

To derive a volumetric rate for type IIn SNe, we rely on the Zwicky Transient Facility’s Bright Transient Survey, which recently provided statistics for SN demographics, including corrections for various sources of inefficiency (Perley et al. 2020). They conclude that the rate of CCSNe is \( 1.01 \times 10^5 \text{Gpc}^{-3} \text{yr}^{-1} \) and that 10.2\% of those are classified as type IIn. Thus, we adopt a type IIn SN rate of \( 1.03 \times 10^5 \text{Gpc}^{-3} \text{yr}^{-1} \). We consider SNe with RSG and BSG progenitors and three different mass-loss rates, assuming fixed parameters of \( T_e = 10^4 \) K, \( E = 10^{51} \text{erg} \), \( M_{ej} = 5 M_{\odot} \), \( n = 10 \), and \( s = 2 \).

For each model, we set the characteristic timescale of the light curve by computing the amount of time between the...
luminosity reaching half the peak luminosity. We can compute the flux densities we expect to observe for a range of distances. The distances we consider are within a few hundred megaparsecs, chosen to roughly correspond to the flux density sensitivity limits of relevant millimeter surveys. Using the IIn SNe event rate given above, we can compute the number of SNe per year that exhibit a half-peak brightness above a given flux density. Multiplying by the light-curve timescale and dividing by the total number of degrees in the full sky then yields areal density. Figure 6 shows these lines of SN areal density as a function of half-peak flux density for the various models.

In Figure 6, we also show the limiting areal densities and flux densities of four CMB surveys of interest. Surveys are sensitive to sources with areal densities greater than one per field of view per square degree at a 5σ detection. The SPT observes a 1500 deg² area of sky down to σ = 2 mJy in their 95 GHz band (S. Gunn, 2021, personal communication) while the ACT observes 40% of the sky every week down to a σ = 25 mJy in their 90 GHz band (K. Huffenberger and S. Naess, 2021, personal communication). The SPT is within the threshold of being to observe SNe with progenitors undergoing among the most extreme mass-loss cases we consider in this work, while the ACT is unlikely to observe anything. This is consistent with the rate and types of extragalactic transients these surveys have already found. CMB-S4 is a next generation CMB survey with stations in both Chile and the South Pole. It is expected to conduct deep observations of 70% of the sky, covering frequencies between 30 and 280 GHz (Abazajian et al. 2019). The experiment also expects to deliver observations of transient sources using a difference imaging pipeline.

At 95 GHz, CMB-S4 will achieve σ = 3 mJy in a one week stack. Looking further to the future, CMB-HD is a new experiment proposed for the Astro2020 Decadal Survey. Projected to have more frequency bands and observe deeper, CMB-HD expects to observe 50% of the sky and achieve σ = 1 mJy for detecting transients. We expect that these two next generation CMB experiments to have the potential to detect bright and local type IIn SNe, as they lie within the requirements of sensitivity and areal density to observe a broader range of progenitor mass-loss rates.

We caution that the SN areal densities we present are likely optimistic. Several progenitor scenarios may contribute to the population of type IIn SNe that have been observed to date, and we do not believe that any of these lines of areal density are representative of the true sample of events. Additionally, we have assumed that ε_c = ε_b = 1/3. Lower energy fractions could yield significantly lower light-curve luminosities, decreasing the rates of detection we predict for these surveys. We also assume homogeneous CSM structure when in reality, CSM interaction is often complicated by inhomogeneous structure and asymmetry and may require more sophisticated modeling to properly describe it. Given this, blind surveys should be able to detect a sample of the most nearby and most extreme type IIn SNe. Follow-up millimeter observations of optically discovered SNe would supplement these detections, shedding light on the range of possible progenitors and mass-loss histories that produce these types of events.

5. Conclusion

The main goal of this work was to demonstrate the utility of early millimeter observations of SNe interacting with dense circumstellar environments. A variety of progenitor models have been proposed for these events, but discerning between different theories requires knowledge of the progenitor’s pre-explosion behavior. Probing the CSM into which an SN explodes can give us insight into the final days of stellar evolution for massive stars and better constrain the viability of different progenitor models.

Here, we apply a synchrotron emission model that takes into account slow versus fast cooling regimes and FFA to produce millimeter and radio light curves of interacting SNe. The light curves are generated by using the prescription of shock radius and velocity evolution outlined in Chevalier (1982) and assuming that a constant fraction of the total shock energy goes into electrons and magnetic fields. The model light curves of the first 1500 days post-explosion reveal that millimeter emission peaks significantly sooner than lower frequency radio emission, making it an important probe of the CSM density close to the surface of the progenitor. The light curves are most sensitive to changes in explosion energy and CSM density profile but are fairly insensitive to changes in the ejecta density profile.

For a practical application, we utilize emcee to fit parameters of synchrotron-powered light-curve models to an 8 GHz light curve of type IIn SNe 2006jd. We find that the light curve was most likely generated by a low energy explosion and a relatively low amount of mass loss enriching the CSM but with a profile indicative of nonsteady mass loss. The model fitting was not very sensitive to the CSM temperature or the total ejecta mass. Millimeter light curves generated by random samples of parameters from the analysis...
show the possibility of an early and luminous peak with potential for detection by sensitive instruments such as ALMA. In the upcoming era of wide-field CMB surveys, we believe there is potential for blind detections in the millimeter band of especially luminous and nearby interacting SNe. Combining these with millimeter follow-up observations of SNe discovered in optical surveys, a representative sample of interacting SNe can be built. Given millimeter light curves, fitting to the models described here presents a method to solve for first-order estimates of the viable progenitor and CSM parameters that describe the emission. Interpreting their millimeter light curves in conjunction with multiwavelength observations will be key in broadening our understanding of the evolution and mass-loss histories of massive stars that lead to these events.

The code used to generate the figures shown in this paper can be found at https://github.com/nitikayad96/mmbrightsupernovae.

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**ORCID iDs**

Nitika Yadlapalli Yurk @ https://orcid.org/0000-0003-3255-4617
Vikram Ravi @ https://orcid.org/0000-0002-7252-5485
Anna Y. Q. Ho @ https://orcid.org/0000-0002-9017-3567

**References**

Abazajian, K., Addison, G., Adshead, P., et al. 2019, arXiv:1907.04473
Bietenholz, M. F., Bartel, N., Argo, M., et al. 2021, ApJ, 908, 75
Blondin, S., Modjaz, M., Kirshner, R., et al. 2006, CBET, 679, 1
Cao, Y., Kasliwal, M. M., Arcavi, I., et al. 2013, ApJL, 775, L7
Chandra, P. 2018, SSRv, 214, 27
Chandra, P., Chevalier, R. A., Chugai, N., et al. 2012, ApJ, 755, 110
Chandra, P., Chevalier, R. A., Chugai, N., Fransson, C., & Soderberg, A. M. 2015, ApJ, 810, 32
Chevalier, R. A. 1982, ApJ, 259, 302
Chevalier, R. A. 1998, ApJ, 499, 810
Chevalier, R. A., & Fransson, C. 1994, ApJ, 420, 268
Chugai, N. N. 1990, SvAL, 16, 457
De Ugarte Postigo, A., Lundgren, A., Martin, S., et al. 2012, A&A, 538, 44
Draine, B. T. 2011, Physics of the Interstellar and Intergalactic Medium (Princeton, NJ: Princeton Univ. Press)
Durán, R. B., Nakar, E., & Piran, T. 2013, ApJ, 772, 78
Dwarkadas, V. V., Romero-Cañizales, C., Reddy, R., & Bauer, F. E. 2016, MNRAS, 462, 1101
Eftekharí, T., Berger, E., Metzger, B. D., et al. 2021, arXiv:2110.05494
Foreman-Mackey, D., Hogg, D. W., Lang, D., & Goodman, J. 2013, PASP, 125, 306
Gal-Yam, A., Leonard, D. C., Fox, D. B., et al. 2007, ApJ, 656, 372
Granot, J., & Sari, R. 2002, ApJ, 568, 820
Guns, S., Foster, A., Daley, C., et al. 2021, arxiv:2103.06166
Ho, A. Y. Q., Margalit, B., Bremer, M., et al. 2021, arXiv:2110.05490
Ho, A. Y. Q., Phinney, E. S., Ravi, V., et al. 2019, ApJ, 871, 73
Horesh, A., Stockdale, C., Fox, D. B., et al. 2013, MNRAS, 436, 1258
Humphreys, R. M., & Davidson, K. 1994, PASP, 106, 1025
Kochanek, C. S. 2019, MNRAS, 483, 3762
Kulkarni, S. R., Frail, D. A., Wieringa, M. H., et al. 1998, NAtur, 395, 663
Laskar, T., Berger, E., Zauderer, B. A., et al. 2013, ApJ, 776, 119
Margutti, R., Milisavljevic, D., Soderberg, A. M., et al. 2014, ApJ, 780, 21
Matsukawa, T., Maeda, K., Lee, S.-H., & Yasuda, H. 2019, ApJ, 885, 41
Mauershan, J. C., Smith, N., Filippenko, A. V., et al. 2013, MNRAS, 430, 1801
Mauron, N., & Josselin, E. 2011, A&A, 526, 156
Metzger, B. D., Williams, P. K., & Berger, E. 2015, ApJ, 806, 224
Moriya, T. J., Maeda, K., Taddia, F., et al. 2013, MNRAS, 435, 1520
Moriya, T. J., Maeda, K., Taddia, F., et al. 2014, MNRAS, 439, 2917
Moriya, T. J., & Tominaga, N. 2012, ApJ, 747, 118
Naess, S., Battaglia, N., Bond, J. R., et al. 2020, arXiv:2012.14347
Nakar, E., & Sari, R. 2010, ApJ, 725, 904
Panagia, N., & Felli, M. 1975, A&A, 39, 1
Perley, D. A., Fremdling, C., Sollerman, J., et al. 2020, ApJ, 904, 35
Pooley, G. G., & Green, D. A. 1993, MNRAS, 264, L17
Smartt, S. J. 2009, ARA&A, 47, 63
Smith, N. 2017, Handbook of Supernovae (Berlin: Springer), 403
Smith, N., Hinkle, K. H., & Ryde, N. 2009, A3, 137, 3558
Smith, N., Mauershan, J. C., & Prieto, J. L. 2014, MNRAS, 438, 1191
Smith, N., & Tombleson, R. 2015, MNRAS, 447, 598
Soderberg, A. M., Chakraborti, S., Pignata, G., et al. 2010, Natur, 463, 513
Vink, J. 2020, Physics and Evolution of Supernova Remnants (Berlin: Springer), 87
Whitehorn, N., Natoli, T., Ade, P. A. R., et al. 2016, ApJ, 830, 143
Yuan, Q., Wang, Q. D., Lei, W. H., Gao, H., & Zhang, B. 2016, MNRAS, 461, 3575
Zauderer, B. A., Berger, E., Soderberg, A. M., et al. 2011, Natur, 476, 425