Four-dimensional hybrid chaos system and its application in creating a secure image transfer environment by cellular automata

R. Parvaz $^a$, Y. Khedmati$^a$, Y. Behroo$^a$

$^a$Department of Mathematics, University of Mohaghegh Ardabili, 56199-11367 Ardabil, Iran.

Abstract

One of the most important and practical researches which has been considered by researchers is creating secure environments for information exchanges. Due to their structures, chaos systems are efficient tools in the area of data transferring. In this research, using a mathematical structure such as composing and transferring, we improve classical chaotic systems by creating a four-dimensional system. Then we introduce a new encryption algorithm based on the chaos and cellular automata. The security of the proposed environment which is evaluated using different types of security tests shows the efficiency of the proposed algorithm.

Keywords: Cryptography; Image; Chaotic system; Cellular automata.

1 Introduction

With the rapid development of social networks, a huge amount of various information in the form of text, image, audio and video is exchanged through these networks in any small period of time. Because of their visual features, digital images have been received more attention among other formats, and so it is important to present a new method or improve existing methods in order to transfer images safely. In addition to steganography, cryptography is a method of secure transfer of information in which the goal is to scramble pixels of data properly so that it is not detectable [4]. So far, various cryptographic methods have been proposed based on cellular automata, chaotic mapping, and so on for example see [8, 16, 24]. Chaos systems are nonlinear phenomena with random-like behaviors. These maps are very important in information security transforms with due regard to their special features,
which we will discuss in detail in future sections. Given that researchers have shown that algorithms that use existing chaotic maps are likely to be attacked [23], defining appropriate maps is a priority for cryptographic methods. The dimension of a chaos maps are determined based on the number of input and output variables. In general, it can be said that high-dimensional chaotic maps, although having high computational cost, perform better than one-dimensional maps due to their complex dynamic structure.

Cellular automata is one of the image encryption tools that has been used in this article due to its fast, easy and high speed process. In [2], Ray et al use chaos and 3D automation to encode digital images. First, the replacement operation is performed by a chaotic system, and then the diffusion operation is performed using a three-dimensional cellular automata. This method has shown good resistance to most known cryptographic attacks. Wang's method is an image encryption system based on chaos theory and cellular automation. This method uses a logistic map and reversible cellular automata. Pixel values divided in four-bit units, then in the permutation step of logistic map and the distribution step, cellular automation is also used. This method of cryptography, which is one of the symmetric methods, has good security and shows good resistance against differential attacks [17].

2 4D Hybrid Chaos Systems

In the first step of this section, we introduce a novel type of four-dimensional chaotic system by improving its overall structure, and then in the second subsection, we evaluate the proposed chaos behavior using the various tests such as lyapunov exponent and cobweb diagram.

2.1 Structure of the Proposed Hybrid System

In this subsection, more details about the proposed new hybrid chaos systems based on Tent, Sin and Logistic maps are given. The general structure of the proposed chaos system has been given in Fig. 1. The combination parts in the proposed system based on Tent, Sin and Logistic maps are shown in Fig. 2. The mathematics formulae for each of the parts can be written as follows.
Figure 1: The structure of the proposed chaotic system.

-First combination box:

\[
x_{i+1} := \begin{cases} 
\alpha x f_1^x \circ F_1^x(r, x_i) + g_1^x(r, x_i, y_i, z_i, w_i) \\
+ h_1^x \left( \frac{\beta x - r}{2} \right) \text{ mod } 1, \text{ when } w_i < 0.5, \\
\alpha x f_2^x \circ F_2^x(r, x_i) + g_2^x(r, x_i, y_i, z_i, w_i) \\
+ h_2^x \left( \frac{\beta x - r}{2} \right) \text{ mod } 1, \text{ when } w_i \geq 0.5.
\end{cases}
\] (2.1)

-Second combination box:

\[
y_{i+1} := \begin{cases} 
\alpha y f_1^y \circ F_1^y(r, y_i) + g_1^y(r, x_{i+1}, x_i, y_i, z_i, w_i) \\
+ h_1^y \left( \frac{\beta y - r}{2} \right) \text{ mod } 1, \text{ when } \zeta_1 < 0.5, \\
\alpha y f_2^y \circ F_2^y(r, y_i) + g_2^y(r, x_{i+1}, x_i, y_i, z_i, w_i) \\
+ h_2^y \left( \frac{\beta y - r}{2} \right) \text{ mod } 1, \text{ when } \zeta_1 \geq 0.5.
\end{cases}
\] (2.2)
-Third combination box:

\[
\begin{align*}
    z_{i+1} &:= \begin{cases} 
        a_1^z f_1^z \circ F_1^z(r, z_i) + g_1^z(r, x_{i+1}, x_i, y_i, z_{i+1}, z_i, w_i) \\
        + h_1^z \left( \frac{\beta_r^z - r_2}{2} \right) \mod 1, & \text{when } \zeta_2 < 0.5, \\
        a_2^z f_2^z \circ F_2^z(r, z_i) + g_2^z(r, x_{i+1}, x_i, y_i, z_{i+1}, z_i, w_i) \\
        + h_2^z \left( \frac{\beta_r^z - r_2}{2 \left(1 - \zeta_3\right)} \right) \mod 1, & \text{when } \zeta_2 \geq 0.5. 
    \end{cases}
\end{align*}
\]  

(2.3)

-Fourth combination box:

\[
\begin{align*}
    w_{i+1} &:= \begin{cases} 
        a_1^w f_1^w \circ F_1^w(r, z_i) + g_1^w(r, x_{i+1}, x_i, y_i, z_{i+1}, z_i, w_i) \\
        + h_1^w \left( \frac{\beta_r^w - r_3}{2} \right) \mod 1, & \text{when } \zeta_3 < 0.5, \\
        a_2^w f_2^w \circ F_2^w(r, z_i) + g_2^w(r, x_{i+1}, x_i, y_i, z_{i+1}, z_i, w_i) \\
        + h_2^w \left( \frac{\beta_r^w - r_3}{2 \left(1 - \zeta_3\right)} \right) \mod 1, & \text{when } \zeta_3 \geq 0.5. 
    \end{cases}
\end{align*}
\]  

(2.4)

where \(a_1 := \{r, x_i, y_i, z_i, w_i\}, a_2 := \{r, x_i, x_{i+1}, y_i, z_i, w_i\}, a_3 := \{r, x_i, x_{i+1}, y_i, y_{i+1}, z_i, z_i, w_i\}, a_4 := \{r, x_i, x_{i+1}, y_i, y_{i+1}, z_i, z_{i+1}, w_{i+1}\}, \) and \(\zeta_\tau = \tau_1 \) or \(\tau_{i+1}, \) for \(\tau = x, y, z, F^x_\zeta, \) for \(\tau = x, y, z, w, \zeta = 1, 2, \) can be considered as Sin or Logistic maps. \(\alpha^z_\zeta, \beta^z_\zeta, \) for \(\tau = x, y, z, w, \zeta = 1, 2, \)
1, 2, are arbitrary number in $R$, and $g^x_n, h^y_n$ are considered as arbitrary sufficiently smooth functions. In the proposed system, the best feature of the different chaos considered.

In many papers. In [1, 2, 10, 10, 20, 20, 10, 20, 10], $g^x_n, h^y_n, \beta^x_n, \beta^y_n, \beta^z_n$ are considered as arbitrary sufficiently smooth functions. In the proposed system, the best feature of the different chaos maps as Tent, Sin and Logistic maps have been improved by using Composition and transfer operator. In the following, the basic properties of the hybrid system have been studied. In order to study hybrid system, the following cases have been considered.

**Case i:** $\{a^x_1, a^x_2, a^x_3, a^x_4, a^y_1, a^y_2, a^y_3, a^z_1, a^z_2, a^w_1, a^w_2\} = \{1, 16, 10, 20, 10, 20, 10\}, \{\beta^x_1, \beta^x_2, \beta^y_1, \beta^y_2, \beta^z_1, \beta^z_2, \beta^w_1, \beta^w_2\} = \{6, 2, 50, 30, 50, 30, 50, 30\}, \{\xi^x, \xi^y, \xi^z\} = \{x_i, y_i, z_i\}$, $f^x_n(p) = \cosh(p), f^y_n(p) = \exp(\pi p)$, $f^z_n(p) = \exp(\pi p)$, $g^x_n(a_1) = 15 \tan(h x_i + z_i) + \sin((w_i) + 12 \cos(r x_i), g^y_n(a_1) = -7 y_i + \exp(1 + 2 w_i) + z_i + 14 \log((x x_i), g^z_n(a_2) = 2 \tan(x x_i) + y_i + 2 z_i + w_i), g^w_n(a_2) = z_i + w_i + 14 \exp(20 r x_i), g^x_n(a_3) = 2 \tan(x x_i) + y_i) + w_i + z_i, g^y_n(a_3) = 14 \exp(20 r x_i + w_i) + z_i, h^x_n(p) = \sin(2 p), h^y_n(p) = \exp(2 p), h^z_n(p) = \exp(2 p), h^w_n(p) = \cosh(4 p)$.

**Case ii:** $\{a^x_1, a^x_2, a^y_1, a^y_2, a^x_3, a^z_1, a^z_2, a^w_1, a^w_2\} = \{7, 12, 14, 3, 15, 15, 10\}, \{\beta^x_1, \beta^x_2, \beta^y_1, \beta^y_2, \beta^z_1, \beta^z_2, \beta^w_1, \beta^w_2\} = \{69, 28, 68, 36, 33, 2, 5, 7\}, \{\xi^x, \xi^y, \xi^z\} = \{x_{i+1}, y_{i+1}, z_{i+1}\}$, $f^x_n(p) = \cos(\sin(\pi p)), f^y_n(p) = \exp(\pi p), f^z_n(p) = \exp(\pi p), g^x_n(a_1) = 15 \tan(r w_i + x_i + 2 z_i) + \sin(w_i) + 12 \cos(r x_i), g^y_n(a_1) = 7 \sin(r y_i + w_i) - 7 r y_i + x_i + 2 w_i + z_i - 1, g^z_n(a_2) = 14 \cos(20 r x_i + x_{i+1}), g^w_n(a_2) = \tan(x_{i+1} + y_{i+1}) + w_i - 1, g^w_n(a_3) = 14 \cos(20 r x_i + x_{i+1}) + \log(x_i + w_i), g^w_n(a_3) = 14 \sinh(x_i + r x_{i+1} + w_i) + z_i + \sin(z_{i+1} + y_{i+1}), h^x_n(p) = \cos(2 p), h^y_n(p) = \sin(2 p), h^z_n(p) = \exp(2 p), h^w_n(p) = \exp(2 p), h^w_n(p) = \cosh(2 p), h^w_n(p) = \exp(4 p), h^w_n(p) = h^w_n(p) = \cosh(p).

Also, in the case i, $\{F^x_1, F^x_2, F^x_3, F^w_1\}$ are considered as Logistic map, and $\{F^y_1, F^y_2, F^y_3, F^w_1\}$ are considered as Sin map. For case ii, $\{F^x_1, F^x_2, F^x_3, F^w_1\}$ and $\{F^y_1, F^y_2, F^y_3, F^w_1\}$ are considered as Logistic and Sin maps, respectively.

### 2.2 Chaotic Behavior Analysis

In this subsection some important tests for the proposed chaos system are discussed. One of the important value in the study of the behavior of the chaos system is Lyapunov exponent or Lyapunov characteristic exponent. An $n$-dimensional chaos systems in general have $n$ values for Lyapunov exponent. There are many methods for calculating this value [21, 11, 3]. The method based on QR algorithm has been used for obtaining Lyapunov exponent in Fig. 3 for the case i. More details about this method can be found in [3]. The positive or negative values of the resulting values are related to the structure of a chaos system. This relation had been studied in many papers. In [14], the relation has been given as follows “In an $n$-dimensional dynamical system we have $n$ Lyapunov exponents. Each $\lambda_k$ represents the divergence of $k$-volume. The sign of the Lyapunov exponents indicates the behavior of nearby trajectories. A negative exponent indicates that neighboring trajectories converge to the same trajectory A positive exponent indicates that neighboring trajectories diverge” [14]. Also the following theorem has been given in [7] for this value.
Theorem 2.1. If at least one of the average Lyapunov exponents is positive, then the system is chaotic; if the average Lyapunov exponent is negative, then the orbit is periodic and when the average Lyapunov exponent is zero, a bifurcation occurs.

The results in Fig. 3 show that all four values of Lyapunov exponent in the proposed systems are positive. By using above studies, we can say that the proposed system in the all neighboring trajectories diverge. In order to compare proposed system, the Lyapunov exponent has been compared with 4D Chaotic Laser System \[9\] in Fig. 3. It is observed that the proposed system has better chaos behavior than Chaotic Laser System. Another tool for study chaotic behavior is bifurcation analysis. In the Fig.s 4-5, the results for the bifurcation analysis of the case i and ii of the proposed system have been shown. The chaotic attractors can be studied by this figure. The attractor for \( r \in (0, 1.2] \) is given in the vertical line at the chosen \( r \). Also, the cobweb plot (or Verhulst diagram) for case i have been given in the Fig. 6. By using this results, it is observed that for the given values, the resulting sequences of the proposed system has chaotic behavior.

Distribution is another important factor in evaluating chaotic system. One of the reasons for the weakness of chaos systems against the statistical attack is nonuniform distribution. The histogram plots of the proposed system for the case i are given in the Fig. 7. Also, the distribution patterns of the case ii are shown in Fig. 8. By using these results, it can be seen that the generated sequence of the proposed maps have flat distributions.
Figure 4: Bifurcation diagram results for the case i.

Figure 5: Bifurcation diagram results for the case ii.

Figure 6: Cobweb plots for the case i.
Remark 2.2. In the continuation of this paper outputs of the proposed four-dimensional chaos system with initial values $\gamma = (x_0, y_0, z_0, w_0)$ and $r$ is shown by using the following notation

$$\gamma \Psi^r_{\tau} := \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ y_1 & y_2 & \cdots & y_n \\ z_1 & z_2 & \cdots & z_n \\ w_1 & w_2 & \cdots & w_n \end{bmatrix}.$$ 

Also $\tau \Psi^r_{\tau}$ denotes $\gamma \Psi^r_{\tau}$, which the decimal part of the numbers is cut from the $\tau$-th
In this section, we introduce two types of functions named ULTS1D and S3D to intermix pixels of images. The former is upper and lower triangular shift function which is suitable for grayscale images and the latter is suitable for RGB images. Note that applying these functions on images does not change the original image size and only the pixel locations are moved. In the continuation of this section, \( n_1, n_2, n_3, n_4 \) are integers and \( n, m \) are natural numbers which \( m \) is divisible by 3 and \( \frac{m}{3} \) is perfect square. \( A, B \) and \( C \) are matrices of size \( n \times n \) and \( P \) is a permutation of size \( m \). The inputs of ULTS1D are \( A, n_1 \) and the output matrix ULTS1D(\( A, n_1 \)) is expressed as follows.

As seen in Figure 9, the pixel shuffling by ULTS1D starts from \( A(1, 1) \) entry and continues along the first row. Entries are shifted in the number of \( n_1 \) units. After reaching \( A(1, n) \), it changes direction to the secondary diameter until it reaches \( A(n, 1) \). Upon reaching \( A(n, 1) \), entries keep on moving across the last row and last column, respectively. In \( A(2, n) \), the route again change to diagonal, and this time along the sub-secondary diagonal. In this shift, there are some mutations of different lengths over secondary diameters that the first and shortest jump is from \( A(n-1, 3) \) to \( A(n-1, 1) \). After first jump, we move straight from bottom to top towards \( A(2, 1) \) and after entering this entry, we move towards \( A(2, n-3) \). After a unit of diagonal movement, we enter the third row to traverse \( A(3, 2) \). Now we enter \( A(4, 2) \) and similarly continue the displacement of entries. Note that in this process, we do not enter a situation that we have already passed through, and as mentioned before, we make jumps wherever necessary.
Now, by applying composition of functions and the shift function defined above, we will introduce FCS1D and SCS1D functions. The outputs \( B = FCS1D(A, n_1, n_2, n_3, n_4) \) and \( C = SCS1D(A, n_1, n_2, n_3, n_4) \) are obtained as follow.

\[
\begin{align*}
B & = ULTS1D(A, n_1); \\
B & = ULTS1D(B', n_2); \\
B & = ULTS1D\{\text{flipr}(B), n_3\} \\
B & = ULTS1D\{\text{flipud}(B), n_4\}; \\
C & = ULTS1D\{\text{flipud}(A), n_1\}; \\
C & = ULTS1D\{\text{flipr}(C), n_2\}; \\
C & = ULTS1D(C', n_3); \\
C & = ULTS1D(C, n_4);
\end{align*}
\]

which \( B' \) is transpose matrix of \( B \). For better understanding, examples for one-dimensional shift functions are given in Fig 10.

In order to shuffle pixels of matrices \( A, B \) and \( C \) by swapping their parts based on \( P \), we define three-dimensional shift function which is shown by S3D. To get the output matrices of \( S3D(A, B, C, P) \), first, we block the matrices \( A, B \) and \( C \) to obtain the block matrices \( \tilde{A}, \tilde{B}, \tilde{C} \in (\mathbb{R}^{p \times p})^{1 \times 3 \times p^2} \), which \( p = \frac{\sqrt{m}}{\sqrt{3}} \). Then, we convert these block matrices to block vectors \( \tilde{V} \tilde{A}, \tilde{V} \tilde{B}, \tilde{V} \tilde{C} \in (\mathbb{R}^{p \times p})^{1 \times \frac{3p^2}{p^2}} \). Next, we consider the block vector \( \tilde{V} = [\tilde{V} \tilde{A}, \tilde{V} \tilde{B}, \tilde{V} \tilde{C}] \in (\mathbb{R}^{p \times p})^{1 \times \frac{3p^2}{p^2}} \). Now is the time to change the location.
of blocks of \( \tilde{V} \) based on \( P \) to get block vector \( \tilde{W} \in (\mathbb{R}^{p \times p})^{1 \times \frac{3n}{p^2}} \) as follow.

\[
\text{for } j=1:\text{size}(\tilde{V},2) \text{ do} \\
\quad \tilde{W}[1, j] = \tilde{V}\{1, P(1, j)\}; \\
\text{end for}
\]

Output matrices of S3D\((A, B, C, P)\) can be obtained by dividing cell vector \( \tilde{W} \) into three equal parts and deform them into matrices. In Fig. 11, the steps of blocking and relocating the blocks under a specific permutation are given.

Also Fig 12 is the step of relocating blocks of block matrices for \( m = 48 \). All the above operations are invertible and inverses of FCS1D and SCS1D and S3D are shown by InvFCS1D and InvSCS1D and InvS3D, respectively.

4 Cellular Automata

The study of cellular automata(CA) dates back to Von.Neumann [10]. CA is a model to describe a dynamic system composed of discrete cells where each cell can assume either the value 0 or 1. These cells create a lattice which changes in discrete time according local rules. Generally, a CA can be defined as \( \{C, S, V, F\} \), where \( C \) is the cell space and \( S \) is the discrete state sets, like \( \{0, 1\} \). \( V \) determines cellular neighborhoods and \( F \) is transfer function[5, 19, 20]. CA are categorized in different dimensions [12] and the process of running one dimensional CA with local rule 30
and triple neighborhoods are shown in Fig. 13. At this fig, 1 and 0 are shown with black and white squares, respectively, and \([0101110]\) is considered as the first input of CA. Eight possible states for \([0101110]\) with triple neighborhoods are shown in the second row. Third row shows the next state of cells. This process up to three steps, is shown in the last row of Fig. 13. Cellular automaton, which works based on simple logic computations, with pseudorandom hash behavior \([13]\), is highly parallel and distributed systems that can simulate complicated behaviors \([19]\). One of the ways to create a strong cipher system is to use cellular automata with the large number of rules.

In our proposed algorithm, one-dimensional CA with triple neighborhoods is used like Fig. 13, which the rule number for each input is obtained based on proposed chaotic system.

**Remark 4.1.** *The outputs of the decryption by reversible cellular automata is shown by*

\[
\begin{align*}
&\ (x', y') := \Phi(x, y, \text{rule}, \text{rep}),
\end{align*}
\]

*where \(x\) and \(y\) are inputs in the \(t-1\) and \(t\) times, respectively. "rule" is the rule number after "rep" repetitions.*

In the proposed algorithm, the matrix \(M_{n \times n}\) is encrypted based on reversible cellular automata by following algorithm.

\[
R = \begin{bmatrix}
\gamma^2 \psi^n / 2(1,:), \gamma^2 \psi^n / 2(2,:), \gamma^2 \psi^n / 2(3,:), \gamma^2 \psi^n / 2(4,:)
\end{bmatrix};
\]

for \(i = 1 : n \) do
  if \(i = 1 \) then
Figure 13: (a) Cellular automata with rule number 30, (b) Cellular automata process.

\[
T = \text{diag}(M, i);
X = T(1 : n/2);
Y = T(n/2 + 1 : n);
\]

```plaintext
\textbf{else if } i \neq 1 \textbf{ then}
X = \text{diag}(M, i);
Y = \text{diag}(M, -i);
\textbf{end if}
\]

\[
[x, y] = \Phi(x, y, \text{rule} = R(i), 1);
\]

end for

5 Proposed encryption algorithm

The details of the encryption algorithm are given in this section. The proposed algorithm includes two parts. In the first part a sensitive algorithm for generating key space is introduced and in the second part, an algorithm is proposed for image encryption.

5.1 Key space generation algorithm

In the process of encryption and decryption methods, the important part that affects the output of the algorithm is the key space. In this section, an algorithm to generate an efficient key space is introduced. One of the most important features of the proposed key space is the sensitivity to slight changes in input images, which is
evaluated in the section related to simulation results. First, we design the proposed algorithm for image as follows.

Algorithm 1: Key space generation algorithm for $I \in \mathbb{R}^{n \times m}$.

Input: $I, r$;
$r_0 = 4 \text{rand}();$
$r_1 := [r_0 \times 10^3];$
$I_1 = \text{circshift} \{(\text{reshape}(I', 1, []), r_1);$
$\gamma_0 := I_1(1:4);$  
$q := 10^3 \Psi^{1}_r,$
\begin{itemize}
  \item [for] $i = 7:3:nm$ do 
  \begin{enumerate}
    \item $\gamma_1 := \big\{\text{mean}(q(2:4)), q(1), I_1(i-2), I_1(i-1), I_1(i)\};$
    \item $q = \gamma_1^i \Psi^{1}_r,$
  \end{enumerate}
\end{itemize}
end for
output: $r_0, q$;

This part of the algorithm is shown by the following notation according to the output and input values:

$$(r, r_0, q) = \text{key}(I, r).$$

For a color image $I_{n \times m \times 3}$, consider $I_1 \in \mathbb{R}^{n \times 3m}$ as follows.

$I_1(1:n, (i-1)m+1:im) := I(:, i), \quad i = 1, 2, 3.$

Then define

$I_2 = \text{circshift} \left(I_1, [r_0 \times 10^3]\right),$

where $r_0 := 4 \text{rand}()$. By using Algorithm 1, obtain

$$(r, r_i, q_i) = \text{key}\left(I_2(1:n, (i-1)m+1:im), r\right), \quad i = 1, 2, 3.$$  

In the last step of key generation for color image, consider key space as

$$(r_0, q) = \left(\text{mean}\left(|r_i|^3\right), \text{mean}\left(|q_i|^3\right)\right).$$

5.2 Encryption and decryption algorithm

The steps of the proposed encryption algorithm for a grayscale image $I \in \mathbb{R}^{n \times m}$ based on the new proposed chaos system are written as follows.
**Figure 14:** The structure of the proposed key generation algorithm.

**step 1.** By using key space generation algorithm obtain:

\[(r, r_0, q) = \text{key}(I, r),\]

where \(q = (x, y, z, w)\).

**step 2.** Define \(v_1 \in R^{1 \times n}, v_2 \in R^{1 \times 8m}\) and \(M \in R^{n \times m}\) as

\[
\begin{align*}
 v_1 & := \lfloor \chi^0_r \times 10^3 \rfloor, \\
 v_2 & := \lfloor \chi^m_{r_0} \times 10^3 \rfloor, \\
 M & := \text{mod}\left( \lfloor \text{reshape}\left( \chi^m_{\text{mean}(I,r_0)} \times 10^3, [n,m] \right) \rfloor, 255 \right),
\end{align*}
\]

where

\[
\chi^j_r = \left[ q \Psi^j_r (1,:), q \Psi^j_r (2,:), q \Psi^j_r (3,:), q \Psi^j_r (4,:) \right], \quad j = n, 8m, nm.
\]

**step 3.** Transfer \(I\) to binary image \(I_1 \in R^{n \times 8m}\), and shift as

\[
\begin{align*}
 &\text{for } i = 1:n \text{ do} \\
 &\quad I_1(i, 1:8m) = \text{circshift}\left( (I_1(i, 1:8m))^\prime, v_1(i) \right)^\prime; \\
 &\text{end for} \\
 &\text{for } i = 1:4 \text{ do} \\
 &\quad R[(i-1)n+1:2n, 1:2m] = I_1(1:n, 2(i-1)m+1:2im); \\
 &\text{end for} \\
 &\text{for } i = 1:2m \text{ do} \\
 &\quad SR(1:4n, i) = \text{circshift}\left( R(1:4n, i), v_2(i) \right); \\
 &\text{end for} \\
 &\text{for } i = 1:4 \text{ do} \\
 &\quad I_2(1:n, 2(i-1)m+1:2im) = SR((i-1)n+1:i n, 1:2m); \\
 &\text{end for}
\end{align*}
\]
step 4. Consider most and least significant bits in two separate matrices $I_2^m, I_2^l \in \mathbb{R}^{n \times 4m}$, respectively. Then define $I_5 \in \mathbb{R}^{n \times 4m}$ with $s = n/2 - \text{mod}(n/2, 3)$ by

$$I_5 = I_2^m (1 : 2 : 2s, :).$$

step 5. By using proposed algorithm based on cellular automata in Section 4, obtain $I_4$ for input values $I_3, q$ and $r$.

step 6. By inverting steps 3 and 4 on $I_4$ obtain $I_5 \in \mathbb{R}^{n \times m}$ and shift $I_5$ as

$$I_6 = \text{FCS1D} \left( I_5, v_1(10), -v_1(20), v_1(30), -v_1(40) \right).$$

step 7. Obtain final encryption image $EI$ by using bitwise XOR operation as

$$EI = I_6 \oplus M \oplus M_s,$$

where $M_s := \text{circhift} \left( M, [q(1), q(0)] \times 10^2 \right)$.

Now, we develop the above proposed algorithm for color image $I \in \mathbb{R}^{n \times m \times 3}$. In the first step, by using key space algorithm for color image obtain $(r, r_0, q)$. Next by S3D shift function obtain $I_1$ as

$$\{I_1(:, :, 1), I_1(:, :, 2), I_1(:, :, 3)\} = \text{S3D} \left( I(:, :, 1), I(:, :, 2), I(:, :, 3), p \right),$$

where $p = \text{sort}(q_{\Psi}^{nm})$. Finally, each layer of $I_1$ is encrypted using the encryption algorithm that described above.

Decryption processes are achieved simply by following reverse steps of the above algorithms.

Figure 15: The structure of the proposed encryption algorithm.

6 Simulation results

In this section, we review the results of the algorithm and to check the security of the proposed algorithm, we perform different types of security tests.
6.1 Key space analysis

In order to show the sensitivity of the key generation algorithm to the input values, Table 1 shows the output results of the algorithm for different inputs. According to the results, it can be seen that small changes in the input of the algorithm have caused changes in the key space. To withstand the brute force attacks the key space must be large enough. According to [1], this size must be greater than $2^{100}$. If the precision computing is considered as $10^{-15}$, then the key space size is $10^{90} = 2^{90 \log_{10}10} \approx 2^{298}$, and this is large enough to resist the attack.

Table 1: Key space generated by the proposed algorithm for various inputs and $r = 0.7$.

| Image    | Keys                     | $r_0$  | $x$    | $y$    | $z$    | $w$    |
|----------|--------------------------|--------|--------|--------|--------|--------|
| lena     | Original constant (0.7)  | 0.6038 | 0.2828 | 0.7086 | 0.4590 |
| (256 x 256 x 3) | One bit changed constant (0.7) | 0.5495 | 0.3590 | 0.9294 | 0.3930 |
|          | Original random          | 0.6600 | 0.7191 | 0.3064 | 0.8206 |
|          | One bit changed random   | 0.4336 | 0.4298 | 0.2942 | 0.4056 |
|          | First time run random    | 0.2499 | 0.4357 | 0.5456 | 0.5322 |
|          | Second time run random   | 0.4351 | 0.2396 | 0.6711 | 0.5193 |
| lena     | Original constant (0.7)  | 0.7753 | 0.3087 | 0.5087 | 0.1720 |
| (256 x 256) | One bit changed constant (0.7) | 0.6800 | 0.6547 | 0.2556 | 0.3093 |
|          | Original random          | 0.8145 | 0.4966 | 0.3931 | 0.0714 |
|          | One bit changed random   | 0.0451 | 0.4445 | 0.0295 | 0.7943 |
| pirate   | Original constant (0.7)  | 0.3329 | 0.9476 | 0.2881 | 0.5595 |
| (256 x 256) | First pixel changed constant (0.7) | 0.8755 | 0.1108 | 0.6811 | 0.4796 |
|          | Last pixel changed constant (0.7) | 0.3318 | 0.6714 | 0.1125 | 0.6366 |
|          | Original random          | 0.8654 | 0.2863 | 0.0157 | 0.5612 |
|          | First pixel changed random | 0.7584 | 0.9722 | 0.1689 | 0.9224 |
|          | Last pixel changed random | 0.7952 | 0.4099 | 0.1525 | 0.0080 |

6.2 Statistical analysis

Statistical test consists of three basic tests, correlation values, information entropy and histogram analysis. The correlation values for an image are calculated using the following formula

$$C_{x,y} = \frac{E(x - \bar{x})(y - \bar{y})}{\sigma_x \sigma_y},$$

where $E, \bar{x}, \bar{y}, \sigma_x$, and $\sigma_y$ represent expectation, mean values and standard deviation, respectively. If this number is close to zero, the encrypted image will not have significant information from the original image. The results of this value for original and encrypted image are given in Tables 2 and 3. By comparing values of encrypted and original images, it is observed that the results for the encrypted images are close to the ideal value, i.e., zero. Also, the correlation distributions for the original and encrypted images are shown in Figures 16-18. By using these figures, it can be seen that encrypted images have more uniform distribution.
Table 2: Correlation coefficient and Information entropy values in the plain and cipher grayscale images.

|                | Correlation coefficient | Information entropy |
|----------------|-------------------------|---------------------|
|                | Horizontal | Vertical | Diagonal (lower left to top right) | Diagonal (lower right to top left) |        |
| plain images   |            |          |                                |                                 |        |
| 5.1.12         | 0.95649    | 0.97408  | 0.93627                        | 0.93893                          | 6.7057 |
| 5.2.08         | 0.93707    | 0.89264  | 0.85427                        | 0.85572                          | 7.201  |
| boat (512 × 512) | 0.93812   | 0.97131  | 0.92585                        | 0.92216                          | 7.1914 |
| lena (256 × 256) | 0.92578   | 0.95926  | 0.92577                        | 0.90374                          | 7.4429 |
| lena (512 × 512) | 0.20131   | 0.21056  | 0.19815                        | 0.19016                          | 7.4455 |
| encrypted images |            |          |                                |                                 |        |
| 5.1.12         | −0.00047766 | 0.00061399 | 0.001538                      | −0.0001677                       | 7.9973 |
| 5.2.08         | −0.0003788  | 0.00013229 | −0.00082042                   | 0.00027628                       | 7.9993 |
| boat (512 × 512)| 0.00093687  | 0.00057866 | 0.00038615                    | −0.00084275                      | 7.9993 |
| lena (256 × 256) | −0.0021188 | −0.0025797 | −0.0015225                    | 0.00026687                       | 7.9972 |
| lena (512 × 512) | 0.00039856 | 0.00015902 | −0.00023439                   | 0.00069882                       | 7.9993 |

Figure 16: Correlation of neighborhood pixels at different directions before (a-c) and after encryption (d-f) of lena (256 × 256).

The uncertainty of information for an image is evaluated by using Shannon information entropy. This value is calculated by

\[
H = -\sum_i p(i) \log(p(i)),
\]

where \( p(i) \) is the probability of occurrence of each pixel. The ideal value for this test is 8. If the results of the algorithm are close to this number, it will be difficult to
Table 3: Correlation coefficient values in the plain and cipher grayscale images.

| Information          | Horizontal (lower left to top right) | Vertical (lower right to top left) | Diagonal (lower left to top right) | Diagonal (lower right to top left) | Entropy |
|----------------------|--------------------------------------|-----------------------------------|------------------------------------|------------------------------------|---------|
| **plain images**     |                                      |                                   |                                    |                                    |         |
| R 0.94928            | 0.95616                              | 0.91631                           | 0.91764                            | 6.2499                              |         |
| G 0.93077            | 0.95338                              | 0.8968                            | 0.90017                            | 5.9642                              |         |
| B 0.91784            | 0.94421                              | 0.88696                           | 0.88898                            | 5.9309                              |         |
| R 0.97865            | 0.9879                               | 0.96875                           | 0.96841                            | 7.2549                              |         |
| G 0.96598            | 0.98201                              | 0.95147                           | 0.95072                            | 7.2704                              |         |
| B 0.95231            | 0.97178                              | 0.93069                           | 0.93057                            | 6.7825                              |         |
| R 0.92307            | 0.86596                              | 0.85187                           | 0.85434                            | 7.7067                              |         |
| G 0.86548            | 0.76501                              | 0.72493                           | 0.7348                             | 7.4744                              |         |
| B 0.90734            | 0.88089                              | 0.84244                           | 0.83986                            | 7.7522                              |         |
| R 0.97527            | 0.98531                              | 0.97339                           | 0.96484                            | 5.0465                              |         |
| G 0.96662            | 0.98017                              | 0.96303                           | 0.95357                            | 5.4576                              |         |
| B 0.93339            | 0.95579                              | 0.92643                           | 0.91863                            | 4.8001                              |         |
| **encrypted images** |                                      |                                   |                                    |                                    |         |
| R 0.0011619          | 0.0016396                            | 0.00053731                        | 0.00082608                         | 7.9973                            |
| G 0.0007757          | –0.00021986                         | 0.00069429                        | 0.00088685                         | 7.9971                            |
| B 0.00029618         | 0.0005109                            | 0.00098397                        | –0.0015792                         | 7.9972                            |
| R 0.00036129         | –0.00047313                         | –0.0015582                        | 0.00026443                         | 7.9973                            |
| G 0.00029232         | 0.0014998                           | 0.0010052                         | 0.001758                           | 7.9972                            |
| B 0.0001419          | 0.00005978                          | 0.0019824                         | –0.001847                          | 7.9973                            |
| R –0.00057646        | 0.00020068                          | 0.0010341                         | –0.00034744                        | 7.9993                            |
| G 0.00053615         | –0.00029491                         | 0.00061376                        | –0.00040533                        | 7.9993                            |
| B –0.000080725       | –0.0017738                          | –0.0002713                        | 0.00000724                         | 7.9993                            |
| R –2.3351e–06        | 0.00022357                          | –0.00074232                       | –0.0005947                         | 7.9993                            |
| G 0.00002728         | –3.7743e–05                         | 0.0012758                         | –0.00036377                        | 7.9993                            |
| B 0.00031603         | 0.00067082                          | –0.00071107                       | 0.0012129                          | 7.9993                            |

obtain valid information from the image. The results of this test for different images with different sizes are given in Tables 2 and 3. The results in these tables show that the information entropy of encrypted image by proposed algorithm are close to the ideal value. The distribution characteristics of pixel values is evaluated by histogram. Histograms of plaintext and encrypted images are given in Figure 19. Also the intensity histogram is shown in Figure 20. Using these results, it can be seen that the histograms of the cipher image are uniformly distributed in comparison with the original image. Therefore, according to the results the proposed algorithm has a good ability to resist statistical attacks.

### 6.3 Differential attack test

One of the most important attacks evaluated on cryptographic algorithms is differential attacks. This type of attack is studied by two tests, UACR test (number of pixels change rate) and NPCI test (unified average changing intensity). These values are calculated by...
Figure 17: Correlation of neighborhood pixels at different directions for 4.1.04.

Figure 18: Correlation of neighborhood pixels at different directions for encrypted 4.1.04

\[
NPCR = \frac{\sum_{i,j} D_{i,j}}{n \times m} \times 100\%,
\]

\[
UACI = \frac{\sum_{i,j} |C_{i,j}^1 - C_{i,j}^2|}{n \times m} \times 100\%,
\]
where $C^1$ and $C^2$ are two encrypted images such that their plain images differ only by one pixel. Also if $C^1_{i,j} \neq C^2_{i,j}$ then $D_{i,j} = 1$ otherwise $D_{i,j} = 0$. The ideal values for these tests are 100% and 33.33%, respectively. The results are shown in Tables 4-6. The best study on these values is discussed in [22], which critical intervals for NPCR and UACI are considered. In Tables 4 and 5, the results of the proposed algorithm for different images are compared to the critical values. These results indicate the success of the proposed method in these tests. Also in Table 6, the proposed method are compared to other studies.

![Figure 19](image1.png)  
**Figure 19:** (a)-(d) the original image and its histogram, (e)-(f) the output image of the proposed algorithm and its histogram

![Figure 20](image2.png)  
**Figure 20:** (a)-(c) 3D histogram representation of original image and (d)-(f) encrypted image for 4.1.04.
Table 4: Numerical results of the UACI and NPCR for different images (256 × 256).

| Image  | UACI critical values [22] | NPCR critical values [22] |
|--------|---------------------------|---------------------------|
|        | u*_{0.05} = 33.2824       | N*_{0.05} = 99.5693       |
|        | u*_{0.01} = 33.2255       | N*_{0.01} = 99.5527       |
|        | u*_{0.001} = 33.1594      | N*_{0.001} = 99.5341      |
| 5.1.12 | Lena 33.4570 Pass Pass Pass Pass 99.6127 Pass Pass Pass |
| 4.1.02 | R 33.4931 Pass Pass Pass Pass 99.6213 Pass Pass Pass |
| 4.1.04 | G 33.4763 Pass Pass Pass Pass 99.6000 Pass Pass Pass |
|        | B 33.4421 Pass Pass Pass Pass 99.6147 Pass Pass Pass |
|        | R 33.4882 Pass Pass Pass Pass 99.6191 Pass Pass Pass |
|        | B 33.5000 Pass Pass Pass Pass 99.6001 Pass Pass Pass |

Table 5: Numerical results of the UACI and NPCR for different images (512 × 512).

| Image  | UACI critical values [22] | NPCR critical values [22] |
|--------|---------------------------|---------------------------|
|        | u*_{0.05} = 33.3730       | N*_{0.05} = 99.5893       |
|        | u*_{0.01} = 33.3445       | N*_{0.01} = 99.5810       |
|        | u*_{0.001} = 33.3115      | N*_{0.001} = 99.5717      |
| 5.2.08 | Boat 33.4571 Pass Pass Pass Pass 99.6115 Pass Pass Pass |
| 4.2.03 | R 33.4546 Pass Pass Pass Pass 99.6215 Pass Pass Pass |
| 4.2.04 | Lena 33.4750 Pass Pass Pass Pass 99.6137 Pass Pass Pass |
|        | Lena 33.4683 Pass Pass Pass Pass 99.6046 Pass Pass Pass |
|        | R 33.4546 Pass Pass Pass Pass 99.6215 Pass Pass Pass |
|        | Lena 33.4683 Pass Pass Pass Pass 99.6046 Pass Pass Pass |
|        | R 33.4546 Pass Pass Pass Pass 99.6110 Pass Pass Pass |
|        | Lena G 33.4541 Pass Pass Pass Pass 99.6080 Pass Pass Pass |
|        | Lena B 33.4543 Pass Pass Pass Pass 99.6101 Pass Pass Pass |

Table 6: Correlation coefficient and Information entropy values in the plain and cipher grayscale images.

| Image   | Propose Ref. [15] | Ref. [18] | Ref. [6] | Propose Ref. [15] | Ref. [18] | Ref. [6] |
|---------|-------------------|-----------|----------|-------------------|-----------|----------|
| 5.1.09  | 99.6117 99.6078 99.6016 99.6064 | 33.4490 33.4663 33.4700 33.4456 |
| 5.1.10  | 99.6164 99.6098 99.6191 99.6154 | 33.4753 33.4510 33.4826 33.4946 |
| 5.1.11  | 99.6122 99.6077 99.6042 99.6244 | 33.4654 33.4832 33.5648 33.5541 |
| 5.1.14  | 99.6118 99.6129 99.6199 99.6364 | 33.5111 33.4848 33.4725 33.4655 |
| 7.1.01  | 99.6161 99.6040 99.6053 99.5992 | 33.4682 33.4779 33.4820 33.5037 |
| 7.1.02  | 99.6093 99.6016 99.6080 99.6075 | 33.4730 33.4172 33.4357 33.4237 |
| 7.1.09  | 99.6152 99.6061 99.6112 99.6162 | 33.4591 33.4814 33.4596 33.4177 |
| 7.1.10  | 99.6110 99.6052 99.6106 99.6045 | 33.4817 33.4852 33.4538 33.4344 |

6.4 Noise and data loss attack

In the transmission of images over the network and through physical channels, part of the data is naturally or intentionally lost due to noise and cropped attacks. Therefore, efficient cryptographic schemes are capable of retrieving the original images.
even after these types of attacks. In order to test the anti-noise ability of the proposed method, the decrypted images after the noise attack with different intensities 0.1 for gray and 0.2 for color images are given in Fig. 21. The strength of the proposed method against cropped attacks with varying degrees of data loss is shown in Fig. 22.

Figure 21: Salt & pepper noise attack results with different noise density and extracted secret images: (a), (d) decrypted images with noise intensity 0.1 and 0.2, respectively.

Figure 22: Results of extracting the secret image after slight change in the key: (a) almost 0.17 data loss, (b) almost 0.07 data loss, (c) almost 0.35 data loss, (d) almost 0.23 data loss, (e)-(h) decrypted images.

7 Conclusions

Create a secure environment for transferring images a burgeoning subject branch. To study this topic, in the first step, by the development and improvement of the chaos system, a new chaos system is introduced. The structure of the new system is studied with different tests and the results show the efficiency of the new system. In the next step, the proposed system is used to create a secure image transfer environment in the form of an encryption algorithm. The results of studying the proposed
algorithm by various security tests show that the proposed algorithm is efficient and safe.

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