‘Effective’ Pomeron model at large $b$

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In this paper we consider the influence of non-perturbative corrections on the large $b$ (impact parameter) behavior of the BFKL amplitude. This is done in the framework of a model where such “soft” corrections are taken into account in the BFKL kernel. We show, that these corrections lead to a power-like decreasing behavior of amplitude, which differs from the BFKL case.

1. Introduction

In this paper we consider the influence of the non-perturbative corrections to the behavior of the perturbative amplitude at large $b$. We do not have a consistent theoretical approach for taking into account non-perturbative corrections in QCD, so we examine our problem in the framework of an ‘effective’ Pomeron model introduced in [1], [2]. In this model we take for the perturbative amplitude the solution of the BFKL equation in leading order of QCD with fixed coupling constant [3]. Here the ‘effective’ Pomeron appears as the solution of the Bethe-Salpeter equation, resulting from the “ladder”, where a ‘soft’ kernel is included, together with the BFKL kernel. Therefore, this Pomeron represents a particular example of including non-perturbative corrections in the QCD high energy scattering amplitude.

It is well known, that the BFKL amplitude has a power-like decreasing behavior at large $b$, see [4], [5], [6]. Such behavior contradicts the general postulates of analyticity and crossing symmetry. From these postulates we know, that the absence in the hadron spectrum of particles with zero mass, dictates exponential $e^{-2b^2m_\pi}$ decrease of the scattering amplitude, see [10]. At the same time, the ‘soft’ Pomeron, as a Regge trajectory, provides sharp exponential $e^{-s^2B}$ decreasing behavior of the amplitude. There are different approaches which were suggested so as to achieve such an exponential decrease, see [6], [7], [8], [9]. In papers [6], [7] it was suggested, that to obtain the necessary result it is sufficient to include the non-perturbative corrections only in the Born term of the BFKL amplitude, leaving the kernel unchanged. On the other hand, there is a point of view, which claims that such a decrease may be achieved by only including non-perturbative QCD corrections in the BFKL kernel, see [8]. The model, considered in this paper, gives a particular example of such an approach, but will not lead to the unitarization of the cross section. Indeed, adding short range correction we do not modify the large $b$ behavior of the BFKL kernel, that necessary for the unitarization of the amplitude. But exploring the large $b$ behavior of the amplitude, which is the admixture of the non-perturbative and the BFKL kernels, we investigate the differences between this amplitude and the BFKL one at large $b$, that clarify the importance of the non-perturbative corrections even in the usual high-energy processes of pQCD.

In the next section we consider the BFKL amplitude for $q \neq 0$. In the following sections this amplitude will be used to construct the ‘effective’ Pomeron, and only the diffusion approximation for this ‘hard’ amplitude will be considered. In section 3, we consider the ‘effective’ Pomeron in the $q$ representation. In section 4 we discuss the $b$ representation of the amplitude which was found in the section 3. In section 5 we compare the large $b$ behavior of the BFKL and ‘effective’ Pomeron. In the last section we summarize our results.
2. Dipole amplitude in the diffusion approach

We start this section considering the dipole-dipole BFKL amplitude, which for large $b$ has the form:

$$N^{BFKL}(y, r_{1,t}, r_{2,t}; b) = \int \frac{d\nu}{2\pi i} \phi_{in} e^{\omega(\nu) y} 2^{4-8i\nu} \left( \frac{r_{1,t}^2 r_{2,t}^2}{b^4} \right)^{\frac{1}{2} + i\nu},$$

where $b > r_{2,t} > r_{1,t}$, see [4], [5]. The factor $2^{4-8i\nu}$ is the scale factor which is determined by the conformal invariance of the solution of the BFKL equation, see also [11], [12]. The Eq. (1) is divergent at small values of $b$ due to the fact that we use the approximate solution at large $b$ for the BFKL Green function. The correct solution of the BFKL equation, of course, gives finite result for all range of $b$, but has not been used in the treatment of the problem due the mathematical difficulties of such consideration. To avoid this divergence arising in integration over $b$ in our calculations, we will be use the following expression instead of Eq. (1):

$$N(y, r_{1,t}, r_{2,t}; b) = \int \frac{d\nu}{2\pi} \phi_{in} e^{\omega(\nu) y} 2^{4-8i\nu} \left( \frac{r_{1,t}^2 r_{2,t}^2}{(b^2 + r_{1,t}^2/4)^2} \right)^{\frac{1}{2} + i\nu}. \quad (2)$$

We see that the BFKL amplitude Eq. (1) has approximately a $1/b^4$ behavior at large $b$, and in integration over $b$ such behavior leads to the divergence of the integral in the region of small $b$. Therefore, we introduce a cut over $b$ in Eq. (2), writing $(b^2 + r_{2,t}^2/4)$ instead $b^2$ in denominator of expression. This substitution has no affect on the results obtained at large $b$, see [6]. The form of $\phi_{in}(r_{1,t}; b)$ is defined by the large $b$ behavior of the Born term of the BFKL amplitude:

$$\phi_{in} = \pi \alpha_s^2 \frac{N_c^2 - 1}{N_c^2} \frac{1}{1/2 - i\nu}. \quad (3)$$

In Eq. (2) $\omega(\nu)$ is the eigenvalue of the full BFKL kernel. In the following, to simplify the calculations, we will use the diffusion approximation for $\omega(\nu)$, (where $\omega(\nu) = \omega_0 - D\nu^2$), which is valid at small values of $\nu$. Together Eq. (2) and Eq. (3) give:

$$N(y, r_{1,t}, r_{2,t}; b) = \pi \alpha_s^2 \frac{N_c^2 - 1}{N_c^2} \int d\nu \frac{e^{\omega(\nu) y} 2^{4-8i\nu}}{2\pi (1/2 - i\nu)} \left( \frac{r_{1,t}^2 r_{2,t}^2}{(b^2 + r_{2,t}^2/4)^2} \right)^{\frac{1}{2} + i\nu}. \quad (4)$$

The Born term of Eq. (4) with the $\phi_{in}$ of Eq. (3) is:

$$N(y = 0, r_{1,t}, r_{2,t}; b)_B = \pi \alpha_s^2 \frac{N_c^2 - 1}{N_c^2} \frac{r_{1,t}^2 r_{2,t}^2}{(b^2 + r_{2,t}^2/4)^2}, \quad (5)$$

see [6].

The ‘effective’ Pomeron ‘ladder’, see Fig. 1, was studied in Ref. [2] at $q = 0$, the generalization to $q \neq 0$, which we need here, is very simple. Indeed, the equation that sums ‘ladder’ diagrams is of Bethe-Salpeter type, and the momentum transferred $q$, is a parameter which is preserved along a ‘ladder’. Therefore, the equation for $q \neq 0$ can be written in the same way as for $q = 0$, by introducing the kernel with $q$ dependence. Doing so, the form of solution will be the same as for $q = 0$, see next section. Consequently, in the further calculations we need to know the BFKL amplitude written in the $q$ representation:

$$N(y, r_{1,t}, r_{2,t}; q) =$$

$$\pi \alpha_s^2 \frac{N_c^2 - 1}{N_c^2} \int d\nu \frac{e^{\omega(\nu) y} (r_{1,t}^2 r_{2,t}^2)^{\frac{1}{2} + i\nu} 2^{4-8i\nu}}{2\pi (1/2 - i\nu)} \int \frac{db}{2\pi} J_0(|q||b|) \frac{b}{(b^2 + r_{2,t}^2/4)^{1+2i\nu}} =$$
\[ \alpha^2 \frac{N_s^2 - 1}{N_s^2} \left( r_{1,t} r_{2,t} \right) \int d\nu \frac{e^{\nu y} \left( r_{1,t}^2 q^2 \right)^{i\nu}}{2\pi (1/2 - i\nu)} \frac{K_{2i\nu}(|q|/|r_{2,t}|/2)}{\Gamma(1 + 2i\nu)}, \]

where \( K_{2i\nu} \) is the McDonald function. In the limit of the Born term at \( y = 0 \), performing contour integration and returning to the \( b \) representation, we again obtain the expression given by Eq. (5). Finally we have:

\[ N(y, r_{1,t}, r_{2,t}; q) = \int \frac{d\omega}{2\pi i} e^{\omega y} N_\omega (r_{1,t}, r_{2,t}; q) = \int \frac{d\nu}{2\pi i} e^{\nu y} \left( r_{1,t}^2 q^2 \right)^{i\nu} \frac{K_{2i\nu}(|q|/|r_{2,t}|/2)}{\Gamma(1 + 2i\nu)} . \]

In the further calculations we will use \( N_\omega (r_{1,t}, r_{2,t}; q) \) given by Eq. (7).

3. ‘Effective’ Pomeron

![Fig. 1. The diagrams and the graphic form of the ‘admixture’ of ‘soft’ and ‘hard’ kernels.](image)

In this section we consider the approach for the ‘effective’ Pomeron, which was introduced in the Refs. [1], [2]. The main idea behind this Pomeron is shown in Fig. 1. The resulting solution for the ‘ladder’ of Fig. 1, can be found as a solution of a Bethe-Salpeter type equation, and this solution is

\[ N_{T_{\text{full}}} (r_{1,t}, r_{2,t}; q) = N_\omega (r_{1,t}, r_{2,t}; q) + \frac{N_{0} (r_{1,t}, r_{2,t}; q)}{1 - A \int d^2 k_1 \int d^2 k_2 \phi(k_1, q) k_1^2 N_0 (k_1, k_2; q) k_2^2 \phi(k_2, q)}, \]

where is \( N_{0} (r_{1,t}, r_{2,t}; q) \) is defined in Appendix C, and where \( N_0 (k_1, k_2; q) \) in Eq. (10) is the Fourier transform of
We consider here the case of small $q$.

In the following calculations, for the sake of simplicity, we take the kernel in this form, but this form of the Eq. (10). Performing the Fourier transform for the transverse momenta $r$

From Ref. [2] we know, that in this case, the 'effective' amplitude has the form:

$$N_0^\omega (r_{1,t}, r_{2,t}; q) = (r_{1,t} r_{2,t}) \int d\nu \frac{(q^2)^{i\nu} 2^{4-8i\nu}}{2 \pi (\omega - (\nu)) (1/2 - i\nu)} \frac{K_{21,\nu} \{ |q| r_{2,t} / 2 \}}{1}.$$

The second term of Eq. (8) is the 'effective' Pomeron, which is an admixture of 'soft' and 'hard' kernels written in the $q$ representation:

$$N_{S-H}^\omega (r_{1,t}, r_{2,t}; q) = \frac{\hat{N}_0^\omega (r_{1,t}, r_{2,t}; q)}{1 - A \int d^2 k_1 \int d^2 k_2 \phi (k_1, q) k_1^2 N_0^\omega (k_1, k_2; q) k_2^2 \phi (k_2, q)}.$$

Throughout the rest of the paper we will concentrate on this part of the full solution for the 'effective ladder'. Eq. (10) has a different form than the expression obtained in [2]. Firstly, as mentioned in the previous section, the function of Eq. (10) depends on $q$, whereas in [2] it was obtained as a solution for the case $q = 0$. Another difference between Eq. (10) and the solution in paper [2], is in the form of the 'hard' Green's function. The rank of our 'effective ladder' in Fig. 1 contains two propagators, one from the soft kernel and another from the hard, see Fig. 1. In [2] the fully truncated Green’s function was used. Therefore, in Ref. [2], the additional propagator $1/k^2$ was included in integrations over each internal momenta. In our case, the function of Eq. (2) has four external propagators, and therefore, we have to truncate two of them. We do so, including $k^2$ in the integration over this internal momenta.

We also use the soft kernel of the model $K_{\text{soft}} (k_1, k_2; q) = \Delta_S \phi (k_1, q) \phi (k_2, q)$, where

$$\phi (k, q) = e^{-\frac{k^2}{2q^2} - \frac{(q - \vec{k})^2}{q^2}}. \quad (11)$$

In the following calculations, for the sake of simplicity, we take the kernel in this form, but this form of the kernel is also dictated by the instanton approach, see [1], and could have a more general basis. We will find later the numerical factor $A$ in Eq. (10), which provides the correct soft pole position in denominator of Eq. (10). Performing the Fourier transform for the transverse momenta $k_1, k_2$ to the size of dipoles $r_{1,t}$, $r_{2,t}$ in the denominator of Eq. (10), we obtain:

$$N_{S-H}^\omega (y, r_{1,t}, r_{2,t}; q) =$$

$$= \frac{\hat{N}_0^\omega (y, r_{1,t}, r_{2,t}; q)}{1 - A \int d^2 k_1 k_1^2 \int d^2 k_2 k_2^2 \phi (k_1, q) \phi (k_2, q) \int \frac{d^2 r_1}{(2\pi)^2} e^{-i \vec{k}_1 \cdot \vec{r}_1} \int \frac{d^2 r_2}{(2\pi)^2} e^{-i \vec{k}_2 \cdot \vec{r}_2} N_0^\omega (r_{1,t}, r_{2,t}; q)}.$$

We find the constant $A$ of Eq. (10) considering the Born term for the BFKL amplitude in $q$ representation:

$$N_0^\omega (k_1, k_2; q) = \frac{\delta^2 (\vec{k}_2 - \vec{q} + \vec{k}_1)}{k_1^4 k_2^4}.$$

We obtain:

$$\frac{A}{\omega} \int d^2 k_1 \int d^2 k_2 \phi (k_1, q) \delta^2 (\vec{k}_2 - \vec{q} + \vec{k}_1) \phi (k_2, q) = \frac{A}{\omega} \pi q_0^2 e^{-q_0^2 / 2 q^2}.$$

From Ref. [2] we know, that in this case, the 'effective' amplitude has the form:

$$\frac{1}{\omega} \frac{K_S / q^2}{\omega - \Delta_S + \alpha' q^2}.$$

We consider here the case of small $q$, $\frac{q^2}{\Delta_S} < 1$. The comparison with Eq. (14) gives:
\[
\frac{A}{2} \pi q_s^2 e^{-q^2/2q_s^2} = \Delta_s - \alpha' q^2, \tag{16}
\]

and
\[
A = \frac{2\Delta_s}{\pi q_s^2}, \quad \alpha' = \frac{A\pi}{4} = \frac{\Delta_s}{2q_s^2}. \tag{17}
\]

These expressions show the relationships of the parameters of the ‘soft’ Pomeron trajectory with the ‘soft’ kernel given by Eq. (11).

The ‘effective’ pole of Eq. (10) is the solution of the following equation:

\[
1 - \frac{2\Delta_s}{\pi q_s^2} \int d^2k_1 k_1^2 \int d^2k_2 k_2^2 \phi(k_1, q) \phi(k_2, q) \int \frac{d^2r_1}{(2\pi)^2} e^{-i\vec{r}_1} \int \frac{d^2r_2}{(2\pi)^2} e^{-i\vec{r}_2} N_0^\omega(r_1, r_2; q) = 0. \tag{18}
\]

This equation is solved in Appendix A, and the solution obtained is:

1. In the region where \(2\nu_0 < 1\) and \(2\nu_0 \ln(q^2_s) < 1\) with

\[
\nu_0 = \frac{4\Delta_s}{D} \ln\left(\frac{q^2_s}{q^2}\right),
\]

the solution of Eq. (18) is given by

\[
\omega = \omega_{S-H} = \omega_0 + \left(\frac{\Delta_s}{2\nu_0 - 1}\right)^2 \frac{\ln^2(q^2_s) + 2q^2_s H^2}{2q^2_s H^2}.
\]

2. In the region \(2\nu_0 < 1\) and \(2\nu_0 \ln(q^2_s) > 1\), where

\[
\nu_0 = \sqrt{\frac{2\Delta_s}{D}},
\]

the solution for ‘effective’ pole reads:

\[
\omega = \omega_{S-H} = \omega_0 + \frac{\Delta_s}{2\nu_0 - 1} - 2\Delta_s \left(\frac{q^2}{16q^2_s}\right)^{2\nu_0}.
\]

Below we denote both solutions by \(\omega_{S-H}\), unless mentioned otherwise.

Returning to the Eq. (10), and integrating this equation over \(\omega\) we obtain:

\[
N_{S-H}(y, r_1, t, r_2, t'; q) = \int \frac{d\omega}{2\pi i} e^{y\omega} N_{S-H}^\omega(r_1, r_2; t; q) =
\]

\[
\int \frac{d\omega}{2\pi i} \frac{f(\omega_{S-H}, \omega)}{\omega - \omega_{S-H}} e^{y\omega} \tilde{N}^\omega(r_1, r_2; t; q),
\]

with the \(\tilde{N}^\omega(r_1, r_2; t; q)\) defined by Eq. (C4) and \(f(\omega_{S-H}, \omega) = 2\sqrt{\omega - \omega_0}\sqrt{\omega_{S-H} - \omega_0}\) in the case of solution Eq. (20), or \(f(\omega_{S-H}, \omega) = \omega - \omega_0\) in the case of solution Eq. (22).

For high energies we have that \(\omega_{S-H} - \omega_0 \gg D\nu_{SP}^2\), where \(\nu_{SP} = \frac{\ln(q^2_s/q^2)}{2Dq}\) or \(\nu_{SP} = \frac{\ln(q^2)}{2Dq}\), are the values of saddle points in integration over \(\nu_1\) or \(\nu_2\) in Eq. (C4) by the method of steepest descent. Therefore, after the integration over \(\omega\), we have:

\[
N_{S-H}(y, r_1, t, r_2, t'; q) = f(\omega_{S-H}, \omega_{S-H}) e^{y\omega_{S-H}} \tilde{N}^{(\omega=\omega_{S-H})}(r_1, r_2, t; q). \tag{24}
\]
We can calculate \( \tilde{N}(\omega = \omega_{S-H})(r_1,t, r_2,t; q) \), see Appendix D. In approximation of small \( \nu \) and \( \frac{b}{q_s} < 1 \) we obtain

\[
N_{S-H}(y, r_1,t, r_2,t; q_s) = C \Delta S e^{\omega_{S-H} y} \frac{(r_1,t r_2,t)}{16^{4\nu_0 - 1} D \nu_0} \left( \frac{q}{q_s} \right)^{2\nu_0} \left( \frac{q_s}{r_1,t q_s} \right)^{2\nu_0} K_{2\nu_0} (|q| r_{2,t} / 2).
\]

(25)

where \( \nu_0 = \sqrt{\frac{\omega_{S-H} - \omega_0}{B}} \) and \( C = 2 \) in the case of solution Eq. (20), or \( C = 1 \) in the case of solution Eq. (22). We see, that instead of the saddle point value of \( \nu \) in Eq. (6), the small \( q \) behavior of Eq. (25) is governed by non-perturbative contributions in \( \nu \), which are given by Eq. (19) or by Eq. (21).

4. Large \( b \) behavior of ‘effective’ Pomeron

In this section we will consider the large \( b \) representation of the amplitude Eq. (25). We define

\[
N_{S-H}(y, r_1,t, r_2,t; b) = \int_0^{q_s} d^2 q e^{i \vec{q} \cdot \vec{b}} N_{S-H}(y, r_1,t, r_2,t; q).
\]

(26)

The calculation of \( N_{S-H}(y, r_1,t, r_2,t; b) \) is performed in Appendix B. The results, obtained there, are the following.

1. In the region of \( b \) where

\[
4 b q_s \left( \frac{1}{2 \Delta S y} \right)^{1/2} < 1,
\]

(27)

or

\[
b < b_{max} = \frac{1}{4 q_s} (2 \Delta S y)^{1/4},
\]

(28)

the amplitude is

\[
N_{S-H}(y, r_1,t, r_2,t; b) = \frac{2^{4\nu_0 - 2}}{16^{4\nu_0 - 1}} \sqrt{\frac{2 \Delta S}{D}} \left( \frac{8}{\Delta S y} \right)^{1/2\nu_0} (r_1,t r_2,t q_s^2) \left( \frac{r_1,t}{r_2,t} \right)^{2\nu_0} e^{\omega_0 y + y \Delta S / 2} \Gamma(1 + \frac{1}{2\nu_0}),
\]

(29)

with

\[
\nu_0 = \sqrt{\frac{2 \Delta S}{D}}.
\]

(30)

We see, that for such \( b \), the amplitude does not depend on \( b \).

2. For impact parameters which are such that

\[
b > b_{max} = \frac{1}{4 q_s} (2 \Delta S y)^{1/4},
\]

(31)

the amplitude reads as

\[
N_{S-H}(y, r_1,t, r_2,t; b) = \frac{2\pi^2}{16^{4\nu_0 - 1}} \sqrt{\frac{8 \Delta S}{D}} \left( \frac{r_1,t r_2,t}{y^2 + r_{2,t}^2/4} \right)^{1+2\nu_0} e^{\omega_0 y + y \Delta S / 2} \Gamma(1 + 2\nu_0),
\]

(32)

with the same \( \nu_0 \) as in Eq. (30).
5. Large b behavior of BFKL amplitude

We return to Eq. (4) and consider the large b behavior of this amplitude. The integration over \( \nu \), may be performed by the method of steepest descent, and for large b the answer is:

\[
N_{\text{BFKL}}(y,r_{1,t},r_{2,t};b) \propto \frac{1}{\sqrt{b}} \frac{r_{1,t}r_{2,t}}{b^2 + r_{2,t}^2/4} e^{\omega_0 y}.
\] (33)

The full expression for the ‘effective ladder’ of Fig. 1, is given by the sum of the usual BFKL amplitude and ‘effective’ Pomeron, which are Eq. (4), Eq. (29) and Eq. (32) correspondingly, see Eq. (8). From Eq. (8) it is also clear, that when \( \Delta_S = 0 \) we stay only with the BFKL amplitude, which is the lower bound for the ‘effective’ Pomeron. So, from the value of b larger than some \( b_0 \), the large b behavior of the ‘ladder’ is governed by the BFKL amplitude. The ‘effective’ Pomeron has a larger \( \sqrt{y} \) intercept than the BFKL one, but the BFKL amplitude decreases more slowly at large b. We find the value of \( b_0 \) by comparison Eq. (32) and Eq. (33). It is approximately:

\[
\frac{1}{\sqrt{y}} \simeq \left( \frac{r_{1,t}r_{2,t}}{b_0^2} \right)^{2\nu_0} e^{\Delta_S \nu_0 y} e^{y/2\nu_0}.
\] (34)

which gives

\[
b_0 = y^{1/2
u_0} \sqrt{r_{1,t}r_{2,t}} e^{y/2\nu_0 + \nu_0}.
\] (35)

For small \( \nu_0 \) and large \( y \), \( b_0 \) is a large number, from which the large b behavior of the full solution for the ‘effective ladder’ is governed by the BFKL amplitude.

6. Conclusion

In this paper we studied the influence of non-perturbative corrections on perturbative amplitude at large values of impact parameter. We investigated the influence in the framework of the model of the ‘effective’ Pomeron, which is given by Eq. (10). This Pomeron is the ‘admixture’ of the pQCD BFKL amplitude and the soft, non-perturbative Pomeron, and gives an example of the model where non-perturbative corrections are included in the perturbative QCD kernel.

The main result of this paper is given by Eq. (29) and Eq. (32). The impact parameter dependence of the amplitude, at large values of impact parameter is the following. In the region of b where

\[
b < b_{\text{max}} = \frac{1}{4q_s} \left( 2 \Delta_S y \right)^{1/4\nu_0},
\]

the b dependence of amplitude is defined by Eq. (29). For a constant value of rapidity this amplitude is constant, and does not depend on b, but only on values of \( \Delta_S \) and \( y \). In the region of b, from \( b_{\text{max}} \) to the \( b_0 \) of Eq. (35),

\[
b_0 > b > b_{\text{max}},
\]

the amplitude is given by Eq. (32). There is a power-like decrease of the amplitude in this region of b. From the point \( b = b_0 \) and to the some value of b, which defines the region of applicability of the diffusion approach, and which is \( b_{\text{diff}} = \sqrt{r_{1,t}r_{2,t}} e^{D y/2} \), the larger impact parameter behavior is governed by usual BFKL amplitude Eq. (33). Inserting the value \( b_0 \) of Eq. (35) in Eq. (33) we find, that at this point the intercept is very small:

\[
N_{\text{BFKL}}(y,r_{1,t},r_{2,t};b) \sim \text{Const} \frac{y^{\omega_0 - \Delta_S /2 + \nu_0}}{y^{1/4\nu_0}}.
\]
For small values of $\nu_0$, the amplitude decreases with $y$. It is also possible, that $b_0 > b_{diff}$, and only the ‘effective’ Pomeron will define the large $b$ behavior. This picture of the $b$ dependence is presented in the Fig. 2.

![Amplitude diagram](image)

**FIG. 2.** Amplitude behavior in impact parameter space.

Returning to equation Eq. (32) we see, that in the wide region of large $b$, the ‘effective’ Pomeron preserves a power-like decreasing structure and has the intercept $\omega_{S-H}$ larger than the intercept $\omega_0$ of the BFKL amplitude, see Section 5 of the paper. Nevertheless, this decrease is quite different from the structure of the pure BFKL large $b$ asymptotic behavior. Indeed, in the diffusion approach, the BFKL amplitude leads to the $1/b^2$ behavior, see Eq. (33) while the $b$ dependence of Eq. (32) is very different from this. This difference is in the sharper decrease, than the BFKL amplitude has, but it is even more important, that the power of $b$ is governed by $\nu_0$ function which depends on the value of $\Delta_S$, i.e. by the soft, non-perturbative physics.

Only from very large $b$, defined by Eq. (35), the asymptotic behavior of amplitude is again governed by the BFKL amplitude. Indeed, it was also mentioned in Introduction, that due the short range of the considered corrections, the behavior of ‘effective ladder’ from very large $b$ will be again defined by the BFKL amplitude. But still, the main result of this consideration is that there is a region of large $b$ where the admixture of the ‘soft’ and ‘hard’ Pomerons has the very different from the usual BFKL amplitude behavior.

Another observation is that the results of Eq. (29) and Eq. (32) are obtained in the approximation of small $\nu_0$, where $2\nu_0 < 1$ in the framework of the diffusion approach for the BFKL kernel. It easy to see, that in the case when $2\nu_0 > 1$ we will obtain a very different large $b$ behavior from those of Eq. (29) or Eq. (32), where the final answer will also depend on the value of $\nu_0$. In this case we need to include in the calculations the full BFKL kernel, instead of the diffusion approximation, and this task is beyond the framework of the paper.

**Acknowledgments:**
I wish to thank E. Levin and E. Gotsman, without whose help and advice this paper would not be written.

This research was supported by Israel Science Foundation, founded by the Israeli Academy of Science and Humanities.
APPENDIX A

In this Appendix we consider the expression for the denominator of Eq. (10), and will solve Eq. (18). In the denominator of Eq. (10) we must perform the integration over $k_1$, $k_2$, $r_{1,t}$ and $r_{2,t}$ variables. The integration over $k_1$, $r_{1,t}$ is simple:

$$\int k_1^2 d^2 k_1 e^{-\frac{k_1^2}{\sigma_s^2}} \int r_{1,t}^2 \frac{d^2 r_{1,t}}{(2\pi)^2} e^{-i\vec{k}_1 \cdot \vec{r}_1} r_1^{1+2i\nu} = (q_s)^{1-2i\nu} 2^{1+4i\nu} \frac{\Gamma\left(\frac{3}{2} + i\nu\right) \Gamma\left(\frac{1}{2} - i\nu\right)}{\Gamma\left(-\frac{1}{2} - i\nu\right)} {}_1F_1\left(\frac{1}{2}, -i\nu, 1, \frac{q^2}{4q_s^2}\right).$$

(A1)

From the integrals over $k_2$, $r_{2,t}$, we see, that the main contribution comes from $r_{2,t} \sim 1/q_s$. Assuming $q/q_s < 1$ we replace the $K_{2i\nu}$ function by the first three terms of it’s expansion over $r_{2,t}q$. We will see later, that three terms are enough in order to achieve the desired precision $\left(\frac{q}{q_s}\right)^{2\nu}$. We have:

$$K_{2i\nu}\left(\frac{q r_{2,t}}{2}\right) =$$

$$2^{1+4i\nu} (q r_{2,t})^{-2i\nu} \Gamma(2i\nu) + 2^{-1-4i\nu} (q r_{2,t})^{2i\nu} \Gamma(-2i\nu) + 2^{-5+4i\nu} (q r_{2,t})^{2-2i\nu} \frac{\Gamma(2i\nu)}{1 - 2i\nu}.$$  

(A2)

Inserting expression Eq. (A2) back into the integral, and integrating over $k_2$, $r_{2,t}$, we obtain:

$$\int k_2^2 d^2 k_2 e^{-\frac{k_2^2}{\sigma_s^2}} \int r_{2,t}^2 \frac{d^2 r_{2,t}}{(2\pi)^2} J_0(k_1 r_{1,t})$$

$$\left(2^{1+4i\nu} (q r_{2,t})^{-2i\nu} \Gamma(2i\nu) + 2^{-1-4i\nu} (q r_{2,t})^{2i\nu} \Gamma(-2i\nu) + 2^{-5+4i\nu} (q r_{2,t})^{2-2i\nu} \frac{\Gamma(2i\nu)}{1 - 2i\nu}\right) =$$

$$= 2^{2i\nu} \frac{\Gamma(2i\nu) \Gamma(3/2 - i\nu) \Gamma(1/2 + i\nu)}{\Gamma(-1/2 + i\nu)} q_s^{1+2i\nu} q^{-2i\nu} {}_1F_1\left(\frac{1}{2}, -i\nu, 1, \frac{q^2}{4q_s^2}\right) +$$

$$+ 2^{-2i\nu} \frac{\Gamma(-2i\nu) \Gamma(3/2 + i\nu) \Gamma(1/2 - i\nu)}{\Gamma(-1/2 - i\nu)} q_s^{1-2i\nu} q^{2i\nu} {}_1F_1\left(\frac{1}{2}, -i\nu, 1, \frac{q^2}{4q_s^2}\right) +$$

$$+ 2^{-6+2i\nu} \frac{\Gamma(2i\nu) \Gamma(5/2 - i\nu) \Gamma(-1/2 + i\nu)}{\Gamma(-3/2 + i\nu)} q_s^{-1+2i\nu} q^{2-2i\nu} {}_1F_1\left(\frac{1}{2}, +i\nu, 1, \frac{q^2}{4q_s^2}\right).$$

So, now we are left with the integral over $\nu$:

$$\frac{\Delta_s}{\pi^2 q_s^2} e^{-\frac{q^2}{4q_s^2}} \int d\nu q_s^{1-2i\nu} q^{2i\nu} q^{25-4i\nu} \frac{\Gamma\left(\frac{3}{2} + i\nu\right) \Gamma\left(\frac{1}{2} - i\nu\right)}{\Gamma\left(1 + 2i\nu\right) \Gamma\left(-\frac{1}{2} - i\nu\right)} {}_1F_1\left(\frac{1}{2}, -i\nu, 1, \frac{q^2}{4q_s^2}\right)$$

$$\left(2^{2i\nu} \frac{\Gamma(2i\nu) \Gamma(3/2 - i\nu) \Gamma(1/2 + i\nu)}{\Gamma(-1/2 + i\nu)} q_s^{1+2i\nu} q^{-2i\nu} {}_1F_1\left(\frac{1}{2}, +i\nu, 1, \frac{q^2}{4q_s^2}\right) +$$

$$+ 2^{-2i\nu} \frac{\Gamma(-2i\nu) \Gamma(3/2 + i\nu) \Gamma(1/2 - i\nu)}{\Gamma(-1/2 - i\nu)} q_s^{1-2i\nu} q^{2i\nu} {}_1F_1\left(\frac{1}{2}, -i\nu, 1, \frac{q^2}{4q_s^2}\right) +$$

$$+ 2^{-6+2i\nu} \frac{\Gamma(2i\nu) \Gamma(5/2 - i\nu) \Gamma(-1/2 + i\nu)}{\Gamma(-3/2 + i\nu)} q_s^{-1+2i\nu} q^{2-2i\nu} {}_1F_1\left(\frac{1}{2}, +i\nu, 1, \frac{q^2}{4q_s^2}\right) \right)$$

(A4)
\[ + 2^{-6+2i\nu} \frac{\Gamma(2i\nu) \Gamma(5/2 - i\nu) \Gamma(-1/2 + i\nu)}{(1 - 2i\nu) \Gamma(-3/2 + i\nu)} q_s^{-1+2i\nu} q^2 - 2i\nu} \frac{\pi}{2i\nu} \cosh(\pi\nu) \) 

The terms with the gamma functions can be simplified. For the first term in the bracket we have:

\[
\frac{\Gamma(2i\nu) \Gamma(3/2 + i\nu) \Gamma(3/2 - i\nu) \Gamma(1/2 + i\nu)}{\Gamma(1 + 2i\nu) \Gamma(-1/2 - i\nu) \Gamma(-1/2 + i\nu)} = \frac{\pi (1/4 + \nu^2)^2}{2i\nu \cosh(\pi\nu)}.
\] (A5)

According to Eq. (A5), the main contribution in the integral of Eq. (A4) comes from the region of small \(\nu\), therefore, with \(\omega(\nu)\) in diffusion approximation, the first term in the bracket gives the following function in the integral:

\[
\frac{\pi (1/4 + \nu^2)^2}{2i\nu \cosh(\pi\nu) (\omega - \omega(\nu)) (1/2 - i\nu)} \frac{1}{2} + i\nu \right) \frac{\pi}{16i\nu} \approx \frac{9\pi e^{4\nu^2} I_0^2(q^2/4q_s^2)}{(211i) i\nu} (\omega - \omega_0 + D\nu^2).
\] (A6)

Now the equation Eq. (18) has the form:

\[
1 - \frac{\Delta s}{\pi} e^{-\frac{2\nu^2}{4\nu}} \int \frac{d\nu 2^{4-8i\nu} I_0^2(q^2/4q_s^2)}{\omega - \omega_0 + D\nu^2} \left( \frac{1}{8} - \frac{1}{8} \left( \frac{q^2}{q_s^2} \right)^{2i\nu} + \frac{9}{211i} \left( \frac{q^2}{q_s^2} \right) \right) = 0.
\] (A8)

We close the contour of integration in Eq. (A8) in the lower semi-plane, and after integration over \(\nu\), we have the following equation for the 'effective' pole position:

\[
1 - \frac{2\Delta s}{2\nu_0} e^{-\frac{2\nu^2}{4\nu}} + \frac{2\Delta s}{2\nu_0} e^{-\frac{2\nu^2}{4\nu}} \left( \frac{q^2}{q_s^2} \right)^{2\nu_0} + \frac{9\Delta s}{27 + 8\nu_0} e^{-\frac{2\nu^2}{4\nu}} \left( \frac{q^2}{q_s^2} \right) = 0.
\] (A9)

where \(\nu_0 = \sqrt{\frac{\omega - \omega_0}{D}}\). The solution of this equation depends on the value of \(\nu_0\). There are two possible cases, which we consider separately.

1. The first solution we obtain assuming that \(2\nu_0 < 1\) and \(2\nu_0 \ln(\frac{q^2}{q_s^2}) < 1\). In this case the equation may be rewritten in the form:

\[
1 - \frac{2\Delta s}{2\nu_0} e^{-\frac{2\nu^2}{4\nu}} \left( 1 - e^{-2\nu_0 \ln(\frac{q^2}{q_s^2})} \right) = 0,
\] (A10)

which has the following approximate solution:

\[
\omega = \omega_{S-H} = \omega_0 + (\Delta s)^2 \frac{\ln(\frac{q^2}{q_s^2})}{216\nu_0 - 4D},
\] (A11)

with

\[
\nu_0 = \frac{4\Delta s}{D} \ln(\frac{q^2}{q_s^2}).
\] (A12)
2. The second solution is where \( 2 \nu_0 < 1 \) and \( 2 \nu_0 \ln(\frac{q^2}{q^2}) > 1 \). Here we consider the following terms in our main equation Eq. (A9):

\[
1 - \frac{2 \Delta \mathcal{S}}{2 \Delta \mathcal{S}_0} e^{\frac{-3q^2}{4\nu_0}} + \frac{2 \Delta \mathcal{S}}{2 \Delta \mathcal{S}_0} e^{\frac{-3q^2}{4\nu_0}} \left( \frac{q^2}{q_s^2} \right)^{2\nu_0} = 0.
\] (A13)

The solution, which we obtain from Eq. (A13), is:

\[
\omega = \omega_{S-H} = \omega_0 + \frac{\Delta \mathcal{S}}{2 \Delta \mathcal{S}_0 - 1} - 2 \Delta \mathcal{S} \left( \frac{q^2}{16 q_s^2} \right)^{2\nu_0},
\] (A14)

where

\[
\nu_0 = \sqrt{\frac{2 \Delta \mathcal{S}}{D}}.
\] (A15)

**APPENDIX B**

The integral of Eq. (26) is

\[
N_{S-H}(y, r_1, t, r_2, t; b) = 2\pi C r_1 t r_2, t \frac{\Delta \mathcal{S}}{16^{4}\nu_0 - 1 D} (r_1 t q_s)^{2\nu_0} \int_0^{q_s} q d q J_0(|q| |b|) e^{i q S - H y} \frac{K_2 |r_2, t|/2}{q_0} \left( \frac{q}{q_s} \right)^{2\nu_0}.
\] (B1)

The value of this integral is defined by saddle points of exponent, together with \( J_0 \) function, or by initial point of integration. Initially, we consider the saddle points of this integral, which depend on the form of solution for \( \Delta S_{S-H} \), and the \( q \) region of applicability of this solution.

Let us consider the solution for \( \omega_{S-H} \) given by Eq. (20). The contribution from this solution comes from the region of \( q \) defined by condition \( 2 \nu_0 \ln(\frac{q^2}{q^2}) < 1 \). Taking the asymptotic expression for \( J_0(q b) \) in Eq. (B1), we have the saddle point equation:

\[
\frac{d}{dq} \left( \Delta \mathcal{S}^2 \frac{y}{216\nu_0} \frac{\ln^2(\frac{q^2}{q^2})}{D} + i q b \right) = 0,
\] (B2)

or

\[
-4 \Delta \mathcal{S}^2 \frac{y}{q} \frac{\ln(\frac{q^2}{q^2})}{216\nu_0 - 4 D} + i b = 0.
\] (B3)

The solution of this equation can be obtained by iteration and it is

\[
q_{SP}^0 = -4 i \Delta \mathcal{S}^2 \frac{y}{216\nu_0 - 4 b D},
\] (B4)

\[
q_{SP}^1 = -4 i \Delta \mathcal{S}^2 \frac{y}{216\nu_0 - 4 b D} \ln \left( \frac{b^2 q_s^2 D^2 216\nu_0 - 4}{16 \Delta \mathcal{S}^2 y^2} \right).
\] (B5)

We see, that in this case for large \( y \), we have \( |q_{SP} b| \propto y >> 1 \), therefore, the asymptotic expansion of \( J_0 \) is, indeed, justified. This solution gives:
The condition of applicability of this solution, $2\nu_0 \ln(q_s^2) < 1$, requires $|q_{SP}| > q_o = q_s e^{-\left(\frac{q_s^2}{2\Delta S}\right)^{1/2}}$, therefore Eq. (B6) is applicable in the limited range of $b$:

$$b < b_{max} = 4 \frac{\Delta S}{q_s} \frac{y}{D} \frac{1 + 2\nu_0}{2\nu_0} e^{\left(\frac{q_s^2}{2\Delta S}\right)^{1/2}},$$

and reaches the maximum value at the point $b = b_{max}$:

$$N_{S-H}(y, r_{1,t}, r_{2,t}; b) \propto \frac{r_{1,t} r_{2,t}^{1+2\nu_0}}{b^{2+2\nu_0}} e^{\omega_0 y + y \frac{\Delta S}{2\nu_0}} \frac{\Delta S}{2\nu_0} e^{\left(\frac{q_s^2}{2\Delta S}\right)^{1/2}}.$$  

(B8)

For other $\omega_{S-H}$, given by Eq. (22), we have from Eq. (B1) the following saddle point equation:

$$\frac{d}{dq} \left(-2\Delta S y \left(\frac{q^2}{16q_s^2}\right)^{2\nu_0} + iq b\right) = 0,$$

or

$$-4\Delta S y \frac{\nu_0}{q} \left(\frac{q^2}{16q_s^2}\right)^{2\nu_0} + iq b = 0.$$

(B10)

The equation Eq. (B10) has the approximate solution:

$$q_{SP} = -i 4q_s \left(\frac{\Delta S \nu_0 y}{b q_s}\right)^{1+4\nu_0},$$

(B11)

where again we check that $|q_{SP}b| \propto y > 1$. This solution contributes in the region of $q$, where $2\nu_0 \ln(q_s^2) < 1$, this gives $|q_{SP}| < q_o = q_s e^{-\left(\frac{q_s^2}{2\Delta S}\right)^{1/2}}$. We obtain for Eq. (B1):

$$N_{S-H}(y, r_{1,t}, r_{2,t}; b) \propto \frac{1}{b^2 q_s^4} e^{\omega_0 y + y \frac{\Delta S}{2\nu_0}} \frac{\Delta S}{2\nu_0} e^{\left(\frac{q_s^2}{2\Delta S}\right)^{1/2}}.$$

(B12)

We now consider the contribution in our integral, from the region of small $q$, which are close to the initial point of integration. Initially, we suppose that $q$ is so small that $q b < 1$. Later, we will define the region of $b$ where this condition is satisfied. For such small $q$ we have, that $J_0(q b) \approx 1$, and can expand $K_{2\nu_0}$ function around $q = 0$. We obtain:

$$N_{S-H}(y, r_{1,t}, r_{2,t}; b) = 2^{-1+4\nu_0} \pi^2 r_{1,t} r_{2,t} \Delta S \frac{D}{16\nu_0 - 1} \nu_0 \left(r_{1,t} r_{2,t}\right)^{2\nu_0} e^{\omega_0 y + y \frac{\Delta S}{2\nu_0}} \int_0^\infty dq^2 e^{-2\Delta S y \left(\frac{q^2}{2\Delta S y}\right)^{2\nu_0}}.$$

(B13)

Here, in the region of small $q$, we used the solution given by Eq. (22), since our function decreases strongly with $q$, the integration over $q$ in Eq. (B13) goes to infinity. We see from Eq. (B13), that the main contribution in the integral comes from the region $q$ where $q \sim 4q_s \left(\frac{1}{2\Delta S y}\right)^{1/4\nu_0}$. Therefore, the condition $q b < 1$ is satisfied for such $b$ which are

$$4b q_s \left(\frac{1}{2\Delta S y}\right)^{1/4\nu_0} < 1.$$

(B14)

In this region of $b$, after integration in Eq. (B13), we obtain:
\[ N_{S-H}(y,r_{1,t},r_{2,t};b) = \frac{2^{4\nu_0} \pi^2}{16^4 \nu_0 - 1} \sqrt{\frac{2\Delta S}{D}} \left( \frac{8}{\Delta S y} \right)^{1/2\nu_0} (r_{1,t} r_{2,t} q_s^2) \left( \frac{r_{1,t}}{r_{2,t}} \right)^{2\nu_0} e^{\omega_0 y + y \frac{\Delta S}{2\nu_0 - 1}} \Gamma(1 + \frac{1}{2\nu_0}), \]  

where

\[ \nu_0 = \sqrt{\frac{2\Delta S}{D}}. \]  

In the region of \( b \), where

\[ 4b q_s \left( \frac{1}{2\Delta S y} \right)^{1/4\nu_0} > 1. \]  

in the integration we can neglect the exponent and keep \( J_0(qb) \) function. In this case the integration over \( q \) is simple and gives:

\[ N_{S-H}(y,r_{1,t},r_{2,t};b) = \frac{2\pi^2}{16^4 \nu_0 - 1} \sqrt{\frac{8\Delta S}{D}} \left( \frac{r_{1,t} r_{2,t}}{b^2 + r_{2,t}^2/4} \right)^{1+2\nu_0} e^{\omega_0 y + y \frac{\Delta S}{2\nu_0 - 1}} \Gamma(1 + 2\nu_0). \]

Comparing Eq. (B15) and Eq. (B18) with Eq. (B6) and Eq. (B12), and taking Eq. (B14), Eq. (B17) into account, we see, that Eq. (B6) is suppressed at least by factor \( \frac{1}{b^2} \), in comparison to Eq. (B15), and Eq. (B12) is suppressed at least by factor \( e^{-y \frac{2\Delta S}{b^2 \nu_0} / y^{1/2\nu_0}} \), in comparison to Eq. (B18). Therefore, in the following we take the expression obtained by the contribution around the initial point of integration as the solution.

**APPENDIX C**

The function \( \tilde{\tilde{N}}(r_{1,t},r_{2,t};q) \) is the Fourier transform of the function \( \tilde{\tilde{N}}(k_1,k_2;q) \). Correspondingly to [2] this function is defined as:

\[ \tilde{\tilde{N}}(r_{1,t},r_{2,t};q) = \int d^2 k_1 \int d^2 k_2 e^{i\mathbf{k}_1 \cdot \mathbf{r}_{1,t} + i\mathbf{k}_2 \cdot \mathbf{r}_{2,t}} \tilde{\tilde{N}}(k_1,k_2;q) = \]  

\[ = \Delta_S q_s^2 \int d^2 k_1 \int d^2 k_2 e^{i\mathbf{k}_1 \cdot \mathbf{r}_{1,t} + i\mathbf{k}_2 \cdot \mathbf{r}_{2,t}} \int d^2 k' (k')^2 \phi(k',q) N_0^\omega(k',k_2;q) \int d^2 k'' (k'')^2 \phi(k'',q) N_0^\omega(k_1,k'';q), \]

where, as in second section, \( N_0^\omega(k_1,k'';q) \) is the Fourier transform of

\[ N_0^\omega(r_{1,t},r_{2,t};q) = (r_{1,t} r_{2,t}) \int d\nu \frac{(r_{1,t} q_s^2)^{i\nu}}{2\pi \left( \omega - \omega(\nu) \right) \left( 1/2 - i\nu \right)} \frac{K_{2i\nu}(|q||r_{2,t}|/2)}{\Gamma(1 + 2i\nu)}, \]  

and the function \( \phi \) is defined by Eq. (11). From Eq. (C1) we have

\[ \tilde{\tilde{N}}(r_{1,t},r_{2,t};q) = \Delta_S \int d^2 k_1 k_1^2 \phi(k_1,q) N_0^\omega(k_1,r_{2,t};q) \int d^2 k_2 k_2^2 \phi(k_2,q) N_0^\omega(r_{1,t},k_2;q) = \]

\[ = \Delta_S q_s^2 \int d^2 k_1 k_1^2 \phi(k_1,q) \int \frac{d^2 r'}{(2\pi)^2} e^{-i\mathbf{k}_1 \cdot \mathbf{r}'} N_0^\omega(r',r_{2,t};q) \int d^2 k_2 k_2^2 \phi(k_2,q) \int \frac{d^2 r''}{(2\pi)^2} e^{-i\mathbf{k}_2 \cdot \mathbf{r}''} N_0^\omega(r_{1,t},r'';q). \]

Performing the same integration as in Appendix A with the function Eq. (C2), we obtain with the precision of \( \left( \frac{q}{q_s} \right)^{2i\nu} : \)
\[ \hat{N}^\omega(r_{1,t}, r_{2,t}; q) = \Delta_S (r_{1,t}, r_{2,t}) e^{-q^2/4} \]

\[ \int \frac{d\nu_1}{2\pi} K_{2i\nu_1}(|q| |r_{2,t}|/2) \Gamma\left(\frac{3}{2} + i\nu_1\right) \Gamma\left(\frac{1}{2} - i\nu_1\right) 2^{4 - 8i\nu_1} \left( \frac{q}{q_s} \right)^{2i\nu_1} 2^{1 + 4i\nu_1} \frac{1}{\Gamma(\frac{1}{2} - i\nu_1)} \frac{1}{\Gamma(1 + 2i\nu_1)} \left( \frac{q}{q_s} \right)^{2i\nu_1} 2^{1 + 4i\nu_1} \frac{1}{\Gamma(\frac{1}{2} - i\nu_1)} \frac{1}{\Gamma(1 + 2i\nu_1)} \frac{1}{\Gamma(\frac{1}{2} + i\nu_1, 1, q^2/4q_s^2)} + \]

\[ \int \frac{d\nu_2}{2\pi} \Gamma(3/2 - i\nu_2) \Gamma(1/2 + i\nu_2) 2^{4 - 8i\nu_2} (r_{1,t} q_s)^{2i\nu_2} 2^{2i\nu_2} \frac{1}{\Gamma(\frac{1}{2} + i\nu_2)} \frac{1}{\Gamma(1 + 2i\nu_2)} \frac{1}{\Gamma(\frac{1}{2} - i\nu_2)} \frac{1}{\Gamma(1 + 2i\nu_2)} \frac{1}{\Gamma(\frac{1}{2} + i\nu_2, 1, q^2/4q_s^2)} + \]

\[ \int \frac{d\nu_2}{2\pi} \Gamma(3/2 + i\nu_2) \Gamma(1/2 - i\nu_2) 2^{4 - 8i\nu_2} (r_{1,t} q_s)^{2i\nu_2} 2^{2i\nu_2} \frac{1}{\Gamma(\frac{1}{2} + i\nu_2)} \frac{1}{\Gamma(1 + 2i\nu_2)} \frac{1}{\Gamma(\frac{1}{2} - i\nu_2)} \frac{1}{\Gamma(1 + 2i\nu_2)} \frac{1}{\Gamma(\frac{1}{2} + i\nu_2, 1, q^2/4q_s^2)} \]

where \( \omega(\nu) = \omega_0 - D\nu^2 \). We perform the integration over both variables \( \nu_1 \) and \( \nu_2 \) by closing the contours of integration in the lower half-plane, and taking into account the poles \( \nu_1 = -i\nu_0 = -i\sqrt{\frac{\omega_0 - \omega}{D}} \), \( \nu_2 = -i\nu_0 \) and \( \nu_2 = 0 \). In the limit of small \( \nu_0 \) the integration over \( \nu_1 \) gives:

\[ \frac{2^{5 - 8\nu_0} e^{q^2/8q_s^2}}{D\nu_0} K_{2\nu_0}(|q| |r_{2,t}|/2) I_0(q^2/4q_s^2) \frac{\Gamma(3/2)}{\Gamma(1/2)} \frac{\Gamma(1/2)}{\Gamma(-1/2)} \left( \frac{q}{q_s} \right)^{2\nu_0} \].

Integration over \( \nu_2 \) gives:

\[ \frac{2^{3 - 8\nu_0} e^{q^2/8q_s^2}}{D\nu_0^2} I_0(q^2/4q_s^2) \frac{\Gamma(3/2)}{\Gamma(1/2)} \frac{\Gamma(1/2)}{\Gamma(-1/2)} \left( r_{1,t} q_s \right)^{2\nu_0} \].

Therefore, we obtain:

\[ \hat{N}^\omega_S(r_{1,t}, r_{2,t}; q) = \frac{\pi \Delta_S}{16^4\nu_0 - 1} \frac{16^4\nu_0 - 1}{\omega_0 - \omega_0} e^{-3q^2/4q_s^2} I_0(q^2/4q_s^2) \left( \frac{q}{q_s} \right)^{2\nu_0} \left( r_{1,t} q_s \right)^{2\nu_0} K_{2\nu_0}(|q| |r_{2,t}|/2) \].

\textbf{APPENDIX D}

We have to calculate the following expression:

\[ \hat{N}^\omega_S(r_{1,t}, r_{2,t}; q) = \Delta_S (r_{1,t}, r_{2,t}) e^{-q^2/4} \]

\[ \int \frac{d\nu_1}{2\pi} K_{2i\nu_1}(|q| |r_{2,t}|/2) \Gamma\left(\frac{3}{2} + i\nu_1\right) \Gamma\left(\frac{1}{2} - i\nu_1\right) 2^{4 - 8i\nu_1} \left( \frac{q}{q_s} \right)^{2i\nu_1} 2^{1 + 4i\nu_1} \frac{1}{\Gamma(\frac{1}{2} - i\nu_1)} \frac{1}{\Gamma(1 + 2i\nu_1)} \left( \frac{q}{q_s} \right)^{2i\nu_1} 2^{1 + 4i\nu_1} \frac{1}{\Gamma(\frac{1}{2} - i\nu_1)} \frac{1}{\Gamma(1 + 2i\nu_1)} \frac{1}{\Gamma(\frac{1}{2} + i\nu_1, 1, q^2/4q_s^2)} + \]

\[ \int \frac{d\nu_2}{2\pi} \Gamma(3/2 - i\nu_2) \Gamma(1/2 + i\nu_2) 2^{4 - 8i\nu_2} (r_{1,t} q_s)^{2i\nu_2} 2^{2i\nu_2} \frac{1}{\Gamma(\frac{1}{2} + i\nu_2)} \frac{1}{\Gamma(1 + 2i\nu_2)} \frac{1}{\Gamma(\frac{1}{2} - i\nu_2)} \frac{1}{\Gamma(1 + 2i\nu_2)} \frac{1}{\Gamma(\frac{1}{2} + i\nu_2, 1, q^2/4q_s^2)} + \]

\[ \int \frac{d\nu_2}{2\pi} \Gamma(3/2 + i\nu_2) \Gamma(1/2 - i\nu_2) 2^{4 - 8i\nu_2} (r_{1,t} q_s)^{2i\nu_2} 2^{2i\nu_2} \frac{1}{\Gamma(\frac{1}{2} + i\nu_2)} \frac{1}{\Gamma(1 + 2i\nu_2)} \frac{1}{\Gamma(\frac{1}{2} - i\nu_2)} \frac{1}{\Gamma(1 + 2i\nu_2)} \frac{1}{\Gamma(\frac{1}{2} + i\nu_2, 1, q^2/4q_s^2)} \]

where \( \omega(\nu) = \omega_0 - D\nu^2 \). We perform the integration over both variables \( \nu_1 \) and \( \nu_2 \) by closing the contours of integration in the lower half-plane, and taking into account the poles \( \nu_1 = -i\nu_0 = -i\sqrt{\frac{\omega_0 - \omega}{D}} \), \( \nu_2 = -i\nu_0 \) and \( \nu_2 = 0 \). In the limit of small \( \nu_0 \) the integration over \( \nu_1 \) gives:

\[ \frac{2^{5 - 8\nu_0} e^{q^2/8q_s^2}}{D\nu_0} K_{2\nu_0}(|q| |r_{2,t}|/2) I_0(q^2/4q_s^2) \frac{\Gamma(3/2)}{\Gamma(1/2)} \frac{\Gamma(1/2)}{\Gamma(-1/2)} \left( \frac{q}{q_s} \right)^{2\nu_0} \].

Integration over \( \nu_2 \) gives:

\[ \frac{2^{3 - 8\nu_0} e^{q^2/8q_s^2}}{D\nu_0^2} I_0(q^2/4q_s^2) \frac{\Gamma(3/2)}{\Gamma(1/2)} \frac{\Gamma(1/2)}{\Gamma(-1/2)} \left( r_{1,t} q_s \right)^{2\nu_0} \].

Therefore, we obtain:

\[ \hat{N}^\omega_S(r_{1,t}, r_{2,t}; q) = \frac{\pi \Delta_S}{16^4\nu_0 - 1} \frac{16^4\nu_0 - 1}{\omega_0 - \omega_0} e^{-3q^2/4q_s^2} I_0(q^2/4q_s^2) \left( \frac{q}{q_s} \right)^{2\nu_0} \left( r_{1,t} q_s \right)^{2\nu_0} K_{2\nu_0}(|q| |r_{2,t}|/2) \].
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