Information from quantum blackhole physics

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The study of BTZ blackhole physics and the cosmological horizon of 3D de Sitter spaces are carried out in unified way using the connections to the Chern Simons theory on three manifolds with boundary. The relations to CFT on the boundary is exploited to construct exact partition functions and obtain logarithmic corrections to Bekenstein formula in the asymptotic regime. Comments are made on the dS/CFT correspondence frising from these studies.

1. Introduction

For the past 10 years there has been tremendous progress in the understanding of quantum blackhole physics. Progress is seen at the attempts to understand the microstates which account for the entropy of blackhole which naturally emerges when we study the the quantum nature of the blackholes\(^1\). The classical studies of Hawking and Bekenstein led to the well known formula\(^2\):

\[
T = \frac{\kappa}{2\pi} \quad ; \quad S = \frac{\text{Area}}{4\pi}
\]  

(1)

All attempts to construct quantum theory of gravity string theory or canonical formulation or semiclassical methods, brick wall method etc uses this crucial data as the testing ground for the program. Similar situation arises in the study of quantum gravity in de Sitter spaces also even though the questions in this case are more complicated and far from good understanding. String theoretic considerations led to interesting link between AdS gravity and Conformal field theory on the boundary\(^3\). Herealso blackhole physics plays a good testing ground. On the other hand in the case of de Sitter gravity, String theory is yet to come out with suitable background

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to make any progress and relate to CFT on the boundary.

Observational evidence suggests the positive cosmological constant which has created interest in the gravitational dynamics of de Sitter space. De Sitter space which is maximally symmetric n dimensional space-time with positive cosmological constant has a cosmological horizon and regions of space-time which are not accessible to the observer. And the thermodynamics of such a horizon is similar to that of blackhole horizon. The difficulties with String theory in accommodating de Sitter space and other quantisation procedures like loop gravity in getting semiclassical descriptions forces one to look for new arena where some non perturbative quantum gravity answers can be provided. 3D gravity falls in such a category because of the connections to the Chern Simons gauge theory. The bonus is the known connection to Wess-Zumino model on the boundary will provide the required correspondences.

Here we focus on 3D gravity with cosmological constant with both positive and negative signatures corresponding to Anti deSitter and de Sitter spaces. In the first case we consider the BTZ blackhole and cosmological horizon in the second case. We treat the horizons as boundaries.

In the second section we point out that the self adjoint extension required for the definitions of Hamiltonians lead to states localised near the horizon which serve as the appropriate blackhole states. Then we consider BTZ BH and deSitter gravity in a unified way pointing out similarities and differences and set up the details of calculations for the partition function using CS gauge theory. In the fourth section we do explicit computation and obtain the leading Bekenstein Hawking contribution for the entropy as well as subleading logarithmic corrections. In the last section we point out the results in connections with AdS/CFT or dS/CFT correspondences. We also remark about issues in connection with Quasi normal modes and CFT etc. And we conclude with discussions on our work in relation to other contributions.

2. Self adjoint extensions and horizon states

We study the time-independent modes of a massless scalar field in various black hole backgrounds.

We find that for the non-extremal black holes, in the near-horizon limit is described by the Hamiltonian

\[ \left( -\frac{d^2}{dx^2} - \frac{1}{4x^2} \right) \chi = 0, \tag{2} \]

where \( x = (r - r_+) \) is the near-horizon coordinate. \( r_+ \) is the horizon. For the extremal Reissner-Nordstrom solution, however, we get near the horizon

\[ -\frac{d^2 \chi}{dx^2} = 0. \tag{3} \]

Another situation where we see the same equation is the near horizon geometry of the one-dimensional black hole discovered by Witten. The same behaviour is
exhibited in the BTZ black hole in $(2+1)-D$ gravity. In all these cases barring the extremal black holes, (2) is the near-horizon equation for the zero-mode solution.

This Hamiltonian $H$ is a special case of a more general Hamiltonian studied extensively in the literature. It is defined on a domain $L^2 \mathbb{R}^+, dx]$ and is of the form

$$H_\alpha = -\frac{d}{dx^2} + \frac{\alpha}{x^2}. \quad (4)$$

Classically, the system described by this Hamiltonian is scale invariant ($\alpha$ is a dimensionless constant). However, the quantum analysis of this operator is much more subtle. Proper analysis points out the existence of states localised near the boundary (in this case Horizon) and scale invariance is broken. These are the states to be identified as blackhole states.

### 3. 3D gravity, BTZ blackhole and de Sitter space

The gravity action $I_{\text{grav}}$ written in a first-order formalism (using triads $e$ and spin connection $\omega$) is the difference of two Chern-Simons actions.

$$I_{\text{grav}} = I_{\text{CS}}[A] - I_{\text{CS}}[\bar{A}], \quad (5)$$

where

$$A = \left( \omega^a + i \frac{l}{M} e^a \right) T_a, \quad \bar{A} = \left( \omega^a - i \frac{l}{M} e^a \right) T_a \quad (6)$$

are $\text{SL}(2, \mathbb{C})$ or $\text{SU}(2) \otimes \text{SU}(2)$ gauge fields (with $T_a = -i \sigma_a/2$). Here, the cosmological constant $\Lambda = \pm (1/l^2)$. The Chern-Simons action $I_{\text{CS}}[A]$ is

$$I_{\text{CS}} = \frac{k}{4\pi} \int_M \text{Tr} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) \quad (7)$$

and the Chern-Simons coupling constant is $k = l/4G$. The Chern-Simons coupling constant $k = -l/4G$; Lorentzian gravity is obtained from the Euclidean theory by a continuation $G \rightarrow -G$.

Now, for a manifold with boundary, the Chern-Simons field theory is described by a Wess-Zumino conformal field theory on the boundary. We are interested in computing the entropy of the Euclidean BTZ black hole/ de Sitter space if cosmological constant is positive. The Euclidean continuation of the BTZ black hole as well as the de Sitter space has the topology of a solid torus. The metric for the Euclidean BTZ black hole in the usual Schwarzschild-like coordinates is

$$ds^2 = N^2 d\tau^2 + N^{-2} dr^2 + r^2 (d\phi + N\phi d\tau)^2 \quad (8)$$

where $\tau$ here is the Euclidean time coordinate and

$$N = \left( -M + \frac{r^2}{l^2} - \frac{J^2}{4r^2} \right)^{1/2}, \quad N\phi = -\frac{J}{2r^2} \quad (9)$$
The black hole metric is just the metric for hyperbolic three-space $H_3$

\[ ds^2 = \frac{l^2}{z^2} (dx^2 + dy^2 + dz^2), \quad z > 0, \quad (10) \]

Global $(2+1)-d$ de Sitter spacetime is described by the metric

\[ ds^2 = -l^2 d\tau^2 + l^2 \cosh^2 \tau d\Omega^2 \quad (11) \]

Equal time sections of this metric are two-spheres, and there are no globally timelike Killing vectors. However, there does exist a timelike Killing vector in certain patches of this spacetime. Figure 1 shows the Penrose diagram of global de Sitter space with these patches - II and IV. These regions are causally disconnected and the timelike Killing vector flows in opposite directions in these two patches. Each of these patches is bounded by the cosmological horizon, and described by the metric

\[ ds^2 = -N^2 dt^2 + N^{-2} dr^2 + r^2 d\phi^2 \quad (12) \]

where

\[ N^2 = (1 - \frac{r^2}{l^2}), \quad (13) \]

and $0 \leq r \leq l$. $\phi$ is an angular coordinate with period $2\pi$. The cosmological horizon in these coordinates is at $r = l$. Constant $t$ surfaces are discs $D_2$, and the topology of the patch is $D_2 \otimes \mathbb{R}$.

However, making the time periodic does convert it from $D_2 \otimes \mathbb{R}$ to a solid torus $D_2 \otimes S^1$ for each static patch II and IV. The interpretation of the Euclidean static patch metric covering the three-sphere completely (with the $r$ coordinate range being covered twice - once for each patch II and IV) is geometrically the gluing of the two solid tori (corresponding to the two patches II and IV) in such a way that the resultant manifold is closed; and a three-sphere. This can be done easily by gluing the two solid tori with oppositely oriented boundaries after performing a modular transformation on the boundary of one of them.

4. Computation of Partition function

The connection can be written as

\[ A = \left( \frac{-i\pi \tilde{u}}{\tau_2} d\bar{z} + \frac{i\pi u}{\tau_2} dz \right) T_3 \quad (14) \]

where $u$ and $\tilde{u}$ are canonically conjugate fields and obey the canonical commutation relation:

\[ [\tilde{u}, u] = \frac{2\tau_2}{\pi(k+2)} \quad (15) \]

They can be related to the black hole parameters by computing the holonomies of $A$ around the contractible and non-contractible cycles of the solid torus. Then the
Fig. 1. Penrose Diagram
trace of the holonomies around the contractible cycle $A$ and non-contractible cycle $B$ are:

$$\text{Tr}(H_A) = 2 \cosh(i\Theta), \quad \text{Tr}(H_B) = 2 \cosh\left(\frac{2\pi}{l}(r_+ + i|r_-|)\right)$$

(16)

and for de Sitter space

$$\text{Tr}(H_a) = 2 \cos(\Theta), \quad \text{Tr}(H_b) = 2 \cos\left(\frac{\beta}{\tau}\right)$$

(17)

Now, we write the Chern-Simons path integral on a solid torus with a boundary modular parameter $\tau$. For a fixed boundary value of the connection, i.e. a fixed value of $u$, this path integral is formally equivalent to a state $\psi_0(u, \tau)$ with no Wilson lines in the solid torus. The states corresponding to having closed Wilson lines (along the non-contractible cycle) carrying spin $j/2$ ($j \leq k$) representations in the solid torus are given by $^{12, 13}$:

$$\psi_j(u, \tau) = \exp\left\{\frac{\pi k}{4\tau^2} u^2\right\} \chi_j(u, \tau),$$

(18)

where $\chi_j$ are the Weyl-Kac characters for affine SU(2). The Weyl-Kac characters can be expressed in terms of the well-known Theta functions as

$$\chi_j(u, \tau) = \frac{\Theta^{(k+2)}_{j+1}(u, \tau, 0) - \Theta^{(k+2)}_{j-1}(u, \tau, 0)}{\Theta^0_1(u, \tau, 0) - \Theta^2_1(u, \tau, 0)}$$

(19)

where Theta functions are given by:

$$\Theta^k_{\mu}(u, \tau, z) = \exp(-2\pi ikz) \sum_{n \in \mathbb{Z}} \exp 2\pi ik \left[ (n + \frac{\mu}{2k})^2 \tau + (n + \frac{\mu}{2k})u \right]$$

(20)

For both BTZ black hole and the de Sitter partition function are constructed from the boundary state $\psi_0(u, \tau)$. The construction is motivated by the following observations:

(a) In the Chern-Simons functional integral over a solid torus, we shall integrate over all gauge connections with fixed holonomy $H_b$ around the non-contractible cycle. This corresponds to the partition function with fixed period $\beta$ of the Euclidean time, that is, fixed inverse temperature. This in turn means we are dealing with the canonical ensemble. The variable conjugate to this holonomy is the holonomy around the other (contractible) cycle, which is not fixed any more to the classical value given by $\Theta = 2\pi$ for de Sitter space. We must sum over contributions from all possible values of $\Theta$ in the partition function. This corresponds to starting with the value of $u$ for the classical solution, i.e. with $\Theta = 2\pi$ in (17), and then considering all other shifts of $u$ of the form

$$u \rightarrow u + \alpha \tau$$

(21)
where \( \alpha \) is an arbitrary number. This is implemented by a translation operator of the form
\[
T = \exp \left( \alpha \tau \frac{\partial}{\partial u} \right)
\] (22)
However, this operator is not gauge invariant. The only gauge-invariant way of implementing these translations is through Verlinde operators of the form
\[
W_j = \sum_{n \in \Lambda_j} \exp \left( \frac{-n\pi \bar{\tau} u}{\tau_2} + \frac{n\tau}{k + 2} \frac{\partial}{\partial u} \right)
\] (23)
where \( \Lambda_j = -j, -j + 2, \ldots, j - 2, j \). This means that all possible shifts in \( u \) are not allowed. The only possible shifts allowed by gauge invariance are:
\[
u \rightarrow \nu + \frac{n\tau}{k + 2}
\] (24)
where \( n \) is always an integer taking a maximum value of \( k \). Thus, the only allowed values of \( \Theta \) are \( 2\pi(1 + \frac{n}{k + 2}) \).
We know that acting on the state with no Wilson lines in the solid torus with the Verlinde operator \( W_j \) corresponds to inserting a Wilson line of spin \( j/2 \) around the non-contractible cycle. Thus, taking into account all states with different shifted values of \( u \) as in (24) means that we have to take into account all the states in the boundary corresponding to the insertion of such Wilson lines. These are the states \( \psi_j(u, \tau) \) given in (18).

(b) In order to obtain the final partition function, we must also integrate over all values of the modular parameter \( \tau \), i.e. over all inequivalent tori with the same holonomy around the non-contractible cycle. The integrand, which is a function of \( u \) and \( \tau \), must be the square of the partition function of a gauged \( SU(2)_k \) Wess-Zumino model corresponding to the two \( SU(2) \) Chern-Simons theories. It must be modular invariant – modular invariance corresponds to invariance under large diffeomorphisms of the torus. The partition function is then of the form
\[
Z = \int d\mu(\tau, \bar{\tau}) \left| \sum_{j=0}^{k} a_j(\tau) \psi_j(u, \tau) \right|^2
\] (25)
where \( d\mu(\tau, \bar{\tau}) = \frac{d\tau d\bar{\tau}}{\tau_2} \) is the modular invariant measure, and the integration is over a fundamental domain in the \( \tau \) plane. Coefficients \( a_j(\tau) \) must be chosen such that the integrand is modular invariant. These coefficients are given by
\[
a_j(\tau) = (\psi_j(0, \tau))^* \]
so that the partition function is uniquely fixed to be
\[
Z = \int d\mu(\tau, \bar{\tau}) \left| \sum_{j=0}^{k} (\psi_j(0, \tau))^* \psi_j(u, \tau) \right|^2
\] (26)
This is an exact expression for the canonical partition function in both the cases of BTZ and de Sitter spaces but with appropriate identification of holonomies.
To make a comparison with the semiclassical entropy of black hole, we evaluate the expression (26) for large horizon radius \( r_+ \) by the saddle-point method. Substituting from (18), (19) and (20), the saddle point of the integrand occurs when \( \tau_2 \) is proportional to \( r_+ \) and therefore large. But for \( \tau_2 \) large, the character \( \chi_j \) is

\[
\chi_j(\tau, u) \sim \exp \left[ \frac{\pi i (\frac{(j+1)^2}{4} - \frac{j}{2})}{\tau} \right] \frac{\sin \pi(j + 1)u}{\sin \pi u} \quad (27)
\]

We now use in (26) the form of the character for large \( \tau_2 \) from (27). In the expression for \( u \) in (16), we replace \( \Theta \) by its classical value \( 2\pi \). The computation has been done with positive coupling constant \( k \) and at the end, we must perform an analytic continuation to the Lorentzian black hole, by taking \( G \to -G \). It can be checked that after the analytic continuation, it is the spin \( j = 0 \) in the sum over characters in (26) that dominates the partition function. We obtain the leading behaviour of the partition function (26) for large \( r_+ \) (and when \(|r_-| < r_+\)) by first performing the integration over \( \tau_1 \) in this regime. The \( \tau_2 \) integration is done by the method of steepest descent. The saddle-point is at \( \tau_2 = r_+ / l \). Expanding around the saddle-point, by writing \( \tau_2 = r_+ / l + x \) and then integrating over \( x \), we obtain

\[
Z_{bh} = \frac{l^2}{r_+^2} \exp \left( -\frac{2\pi kr_+}{l} \right) \int dx \exp \left[ -\frac{\pi kl}{2r_+} x^2 \right] \quad (28)
\]

The integration produces a factor proportional to \( \sqrt{r_+} \). The partition function for the Lorentzian black hole of large horizon area \( 2\pi r_+ \) after the analytic continuation \( G \to -G \) is then

\[
Z_{Lbh} = \frac{l^2}{r_+^2} \sqrt{\frac{8r_+ G}{\pi l^2}} \exp \left( \frac{2\pi r_+}{4G} \right) \quad (29)
\]

upto a multiplicative constant. The logarithm of this expression yields the black hole entropy for large horizon length \( r_+ \):

\[
S = \frac{2\pi r_+}{4G} - \frac{3}{2} \log \left( \frac{2\pi r_+}{4G} \right) + \ldots \quad (30)
\]

For the case of de Sitter space we compute the partition function by substituting in the expression (26) the values of \( u \) and \( \bar{u} \) with \( \Theta = 2\pi \). We work in the regime where \( k \) (and therefore \( l \)) is large. Also, we must perform an analytic continuation to get the Lorentzian result - this is done by taking \( G \to -G \), and \( \beta \to i\beta \). For the regime when \( k \) is large, the leading contribution to the sum in the integrand comes from \( j = 0 \) as in (7). The \( \tau_2 \) integral can in fact be done exactly. We have

\[
Z_{dS} = \int_{-1/2}^{1/2} d\tau_1 \ 4\pi \ e^{\beta k/2l} \frac{1}{f(\tau_1)} \ K_1(-k/2 \ f(\tau_1)) \quad (31)
\]
where \( f(\tau_1) = \sqrt{\frac{\beta^2}{4\pi^2} - 4\pi^2 \tau_1^2} \), and \( K_1 \) is the Bessel function of imaginary argument. Using the approximation for the Bessel function with large argument

\[
K_1(z) = \sqrt{\frac{\pi}{2z}} e^{-z}[1 + O\left(\frac{1}{z}\right) + ...]
\]  

with the replacement \( \beta = 2\pi l \) for de Sitter space, we get, in the large \( k \) regime:

\[
Z_{dS} = 4\sqrt{\pi} \frac{4G}{2\pi l} e^{2\pi l/4G} 
\]  

Since this is the partition function in the canonical ensemble, we would have expected an additional term \( e^{-i\beta E} \) where \( E \) is the energy of de Sitter space. The notion of energy in asymptotically de Sitter spaces needs to be defined carefully, due to the absence of a global timelike Killing vector. The energy \( E \) that emerges in our formalism is defined on the horizon, and not at asymptotic infinity, as has been done, for e.g in 14. Our result seems to indicate that that energy \( E \) is zero for de Sitter space. Such a result coincides with the definition of energy as given by Abbott and Deser 15. The entropy is therefore

\[
S = (2\pi l)/4G - \log \frac{2\pi l}{4G} + .......
\]  

The leading term is the semi-classical Bekenstein-Hawking entropy that is proportional to the horizon “area”. The second term is the leading correction that is logarithmic in area.

The numerical coefficient of the logarithmic term for BTZ blackhole was \(-3/2\) whereas for the de Sitter case, it is \(-1\). This is somewhat puzzling at first glance. The black hole entropy was computed in the regime \( r_+ >> l \), where \( r_+ \) is the black hole horizon radius and \( l \) is the AdS radius of curvature. Then, there was an integral over the modular parameter similar to (26). The saddle-point for \( \tau_2 \), the imaginary part of the modular parameter occured when \( \tau_2 = r_+/l \). Thus this was the regime when \( \tau_2 \) was large. An interesting observation was made in 7 that replacing \( r_+/l \) in the black hole partition function by \( l/r_+ \), where now \( r_+ << l \), the AdS gas partition function was obtained, with the coefficient of the correction being \(+3/2\). This corresponds to a situation where the modular parameter \( \tau_2 = r_+/l \) is small. What happens when \( r_+ \sim l \), i.e \( \tau_2 \sim 1 \)? In fact, this is very similar to the de Sitter case, since the de Sitter horizon radius is exactly \( l \) ! The computation follows similar lines and leads to similar results. It can in fact be verified directly from (26) that the saddle-point is at \( \tau_2 = 1 \). Here, we see that the coefficient of the logarithmic correction is \(-1\). Thus, the coefficient of the correction seems to depend on the regime one is looking at. When, as in the above case, there are two independent length parameters \( l \) and \( r_+ \), only for \( r_+ >> l \) do we get the coefficient \(-3/2\).

Summarising our result for BTZ black hole we find:

\[
\text{For } r_+ >> l \quad S = \frac{2\pi r_+}{4G} - \frac{3}{2} \log \left(\frac{2\pi r_+}{4G}\right) + \cdots
\]
where the last expression in (35) for \( r_+ << l \) is the entropy of the AdS gas.

The above results are reminiscent of a duality proposed in [16] between the Euclidean BTZ black hole and Lorentzian de Sitter spaces.

Entropy of de Sitter space can also be studied from an alternative point of view by using dS/CFT correspondence [17]. In this framework all the information about quantum gravity in the bulk is expected to be contained in the conformal field theory at past or future infinity. The CFT is described by considering all possible metric fluctuations keeping the asymptotic behaviour to be de Sitter space. It consists of two copies of Virasoro algebras, each with central charge \( c = 3l/2G \). As shown in [14], the eigenvalues of the Virasoro generators \( L_0 \) and \( \bar{L}_0 \) for de Sitter space are both equal to \( l/8G \). Using the Rademacher expansion for modular forms, one can generalize the Cardy formula for growth of states in a CFT beyond the leading term. It has been shown [18] the sub-leading correction to the entropy of a BTZ black hole can be determined from this generalisation. We use these results to find the sub-leading corrections to the de Sitter entropy from the dS/CFT correspondence. The entropy obtained from a CFT with a given the central charge \( c \) and eigenvalue \( L_0 \) = \( N \), is given by [18]

\[
S_1 = S_0 - 3/2 \log S_0 + \log c + ...... \tag{36}
\]

where \( S_0 = 2\pi \sqrt{c}/(6(N - c/24)) \). This is the contribution from the Virasoro generator \( L_0 \). There is a similar contribution \( S_2 \) associated with the Virasoro generator \( \bar{L}_0 \), given by replacing \( N \) in the above by \( \bar{N} \), the eigenvalue of \( \bar{L}_0 \).

Substituting \( c = 3l/2G \) and \( N = \bar{N} = l/8G \) in the above, we see that

\[
S = S_1 + S_2 = 2\pi l/4G - \log 2\pi l/4G + .... \tag{37}
\]

with the same coefficient \( -1 \) for the logarithmic correction as that obtained from the gravity partition function (33) in (34).

Thus, the quantum gravity calculation of de Sitter entropy and the entropy computation from the asymptotic CFT agree even in the sub-leading correction to the Bekenstein-Hawking term.

References

1. A. Peet, *Class. Quant. Gravity* **15** 3291 (1998) hep-th/9712253, 0008241.
2. J R David, G Mandal and S Wadia, *Phys. Repts* **369** (2002) 549.
3. J. Maldacena, Blackholes and D Branes, hep-th/9705078
4. S R Das, and S D Mathur, *Nucl. Phys* **478** (1996) 561,hep-th/9606185.
5. J. Maldacena and A. Strominger, *Phys. Rev.* **D55** (1997) 861,hep-th/9609026.
2. J. D. Bekenstein, *Phys. Rev.* **D7**(1973) 2333.

S. W. Hawking, *Comm. Math. Phys.* **43**(1975) 199.

3. J. Maldacena, *Phys. Rev. Lett.* **80**(1998) 4859, hep-th/9803002. *Adv. Theor. Phys.* **2**(1998) 231, hep-th/9711200.

4. E. Witten, hep-th/0106109.

R. Bousso, *JHEP* **0011**(2000) 038, hep-th/0010252.

5. A. Ashtekar, *Lect on Nonperturbative canonical gravity*, World Scientific, 1991.

R. Gambini, J. Pullin, *Loops, gauge theories and quantum gravity*, Cambridge University press, Cambridge.

6. E. Witten, *Commun. Math. Phys.* **117**(1988) 353.

7. T R Govindarajan, R. K. Kaul, V. Suneeta, *Classical and Quantum Gravity* **18**(2001) 2877; **19**(2002) 4195.

8. E. Witten, *Phys. Rev.* **D44**, 314 (1991).

9. M. Banados, C. Teitelboim and J. Zanelli, *Phys. Rev. Lett.* **69**, 1849 (1992);

M. Banados, M. Henneaux, C. Teitelboim and J. Zanelli, *Phys. Rev.* **D48**, 1506 (1993).

10. T R Govindarajan, V. Suneeta and S. Vaidya *Nucl. Phys. B* **583** (2000) 291; hep-th/0002036

K. Meetz, *Nuovo Cimento* **34** (1964) 690

H. Narnhofer, *Acta Physica Austriaca* **40** (1974) 306.

11. V. Suneeta, *JHEP* **0209**(2002) 040.

12. J. M. F. Labastida and A. V. Ramallo, *Phys. Lett.* **B 227**, 92 (1989).

J. M. Isidro, J. M. F. Labastida and A. V. Ramallo, *Nucl. Phys. B* **398**, 187 (1993). N. Hayashi, *Prog. Theor. Phys. Suppl.* **114**, 125 (1993).

13. S. Elitzur, G. Moore, A. Schwimmer and N. Seiberg, *Nucl. Phys. B* **326**, 108 (1989).

14. V. Balasubramanian, J. de Boer and D. Minic, hep-th/0110108.

15. I. F. Abbott and S. Deser, *Nucl. Phys. B* **195**, 76 (1982);

See also Tetsuya Shiromizu, Daisuke Ida and Takashi Torii, hep-th/0109057.

16. A. Corichi and A. Gomberoff, *Class. Quant. Grav.* **16**, 3579 (1999).

17. A. Strominger, *JHEP* **0110**, 034 (2001).

18. D. Birmingham and S. Sen, *Phys. Rev. D63*, 047501 (2000);

K. Gupta and S. Sen, *Phys. Lett.* **B526**, 121 (2002).