Massive $c\bar{c}g$-Production in Diffractive DIS

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Abstract

We calculate the cross section for $q\bar{q}g$-production in diffractive DIS with finite fermion masses and zero momentum transfer. The calculation is done in the leading log$(1/x_\perp)$ approximation and is valid for the region of high diffractive masses (small $\beta$). We apply our cross section formula to diffractive charm production at HERA: in a Monte-Carlo-Simulation of diffractive $D^{*\pm}$ meson production we include both massive $q\bar{q}$- and massive $q\bar{q}g$-production, and we compare with preliminary H1 results. After adjusting an infrared cutoff parameter of our model which affects the overall normalization of the cross section we find reasonable agreement with the data. In particular, the slope of the $\beta$-distribution can be reproduced.

1. In the process of diffractive deep inelastic scattering, $\gamma^* + p \rightarrow p + X$, one can separate perturbative and non-perturbative contributions by filtering out particular diffractive final states. Examples of diffractive states which are perturbatively calculable are longitudinal vector particles or final states which consist of hard jets (and no soft remnant). In the latter case the hard scale which allows the use of pQCD is provided by the large transverse momenta of the jets, and the Pomeron exchange is modelled by the unintegrated gluon density. Another particularly interesting example is diffractive charm production, since the charm quark mass justifies pQCD, even for not so large transverse momenta of the outgoing quarks and gluons. Calculations for the diffractive production of massless open $q\bar{q}$ states and of massless $q\bar{q}g$ states have been reported in [1, 2, 3] and in [4], resp., and a comparison of diffractive two-jet and three-jet events observed at HERA with these calculations has been presented in [5]. Final states with finite quark masses have been calculated, so far, only for $q\bar{q}$ production [6] which is expected to be the dominant final state in the region of small diffractive masses (large $\beta$). However, as there are recent results from measurements of H1 and ZEUS at HERA on diffractive production of (charmed) $D^{*\pm}$ mesons, which extend into the small-$\beta$-region, gluon radiation can certainly not be neglected, and a comparison of the perturbative two gluon model with HERA data has not been possible yet. In this letter we report on a calculation of massive $q\bar{q}g$-production in DIS diffraction, and we present a comparison of our cross section formula with preliminary H1 data.

2. We will follow the study of massless $q\bar{q}g$-production presented in [4]. In particular, we again work in the leading-log $M^2$ approximation, which limits the applicability of our results to the small $\beta$-region. Fig. 1 shows the notations of the process. As in [4] we restrict ourselves to zero momentum transfer, $t = r^2 = 0$. 

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As usually, \( Q^2 \) denotes the virtuality of the photon, \( \sqrt{W^2} \) the energy of the photon proton system, \( M \) the mass of the diffractive system, and \( x = Q^2/(Q^2 + M^2) \), \( y = 2pq/2pl \) are the Bjorken scaling variables \( x \) and \( y \) (\( l \) is the momentum of the incoming electron). The variable \( \beta \) is defined as \( \beta = Q^2/(Q^2 + M^2) \), and it is convenient to introduce the momentum fraction of the Pomeron \( x_p = (Q^2 + M^2)/(Q^2 + W^2) \).

The limit we are interested in is defined as:

\[
Q^2 \ll M^2 \ll W^2. \tag{1}
\]

We use Sudakov variables \( k_i = \alpha_i q' + \beta_i p + k_i \) (with \( q' = q + xp, k_i = -k_i \)), and we express the phase space in terms of \( y, Q^2, M^2, m^2, t, k_1^2, k_2^2 \) with \( m^2 = m_{qg}^2 + k_2^2 \) (\( m_{qg} \) denotes the invariant mass of the \( q\bar{q} \)-subsystem). We obtain the following result:

\[
\frac{d\sigma_{\gamma^p}}{dydQ^2dm^2d^2k_1d^2k_2dt|_{t=0}} = \frac{\alpha_{em}}{yQ^2\pi} \\
\cdot \left[ 1 + (1-y)^2 \frac{d\sigma_{\gamma^p}}{dM^2dm^2d^2k_1d^2k_2dt|_{t=0}} - 2(1-y)\frac{d\sigma_{\gamma^p}}{dM^2dm^2d^2k_1d^2k_2dt|_{t=0}} \\
+ (1-y)^2\frac{d\sigma_{\gamma^p}}{dM^2dm^2d^2k_1d^2k_2dt|_{t=0}} + (2-y)\sqrt{1-y}\frac{d\sigma_{\gamma^p}}{dM^2dm^2d^2k_1d^2k_2dt|_{t=0}} \right], \tag{2}
\]

\[
\frac{d\sigma_{\gamma^p}}{dM^2dm^2d^2k_1d^2k_2dt|_{t=0}} = \frac{9}{128\pi \sqrt{S(M^2 - m^2)m^2}} e^{2\alpha_{em}^2\alpha_1^2(1 - \alpha_1)} \cdot \\
\cdot [(\alpha_1^2 + (1 - \alpha_1)^2) M_t M_t' + m_{qg}^2 M_l M_l'] \tag{3}
\]

\[
\frac{d\sigma_{\gamma^p}}{dM^2dm^2d^2k_1d^2k_2dt|_{t=0}} = \frac{9}{128\pi \sqrt{S(M^2 - m^2)m^2}} e^{2\alpha_{em}^2\alpha_1^2(1 - \alpha_1)^2} \cdot \\
\cdot [M_t M_t' - M_{3t} M_{3t}'] \tag{4}
\]

\[
\frac{d\sigma_{\gamma^p}}{dM^2dm^2d^2k_1d^2k_2dt|_{t=0}} = \frac{9}{128\pi \sqrt{S(M^2 - m^2)m^2}} e^{2\alpha_{em}^2\alpha_1^2(1 - \alpha_1)^3 Q^2 M_t M_t'} \tag{5}
\]

\[
\frac{d\sigma_{\gamma^p}}{dM^2dm^2d^2k_1d^2k_2dt|_{t=0}} = \frac{9}{128\pi \sqrt{S(M^2 - m^2)m^2}} e^{2\alpha_{em}^2\alpha_1^2(1 - \alpha_1)^2(1 - 2\alpha_1)} \cdot \\
\cdot \sqrt{Q^2 [M_t M_t' + M_l M_l']} \tag{6}
\]
with
\[ S = \left(1 + \frac{k_1^2}{m^2} - \frac{(k_1 + k_2)^2}{m^2}\right) - 4 \left(\frac{k_1^2 + m_q^2}{m^2}\right)^2 \] (7)
and
\[ T_{il} = \left(\frac{1 + k_1 + k_2}{D(l + k_1 + k_2)} + \frac{k_1 + k_2}{D(k_1 + k_2)} - \frac{k_1 - 1}{D(k_1 - 1)} - \frac{k_1}{D(k_1)}\right) \left(\frac{1 + k_2}{(l + k_2)^2} - \frac{k_2}{k_2^2}\right)_l + (l \to -l) \] (8)
\[ T_l = \left(\frac{1}{D(l + k_1 + k_2)} + \frac{1}{D(k_1 + k_2)} - \frac{1}{D(k_1 - 1)} - \frac{1}{D(k_1)}\right) \left(\frac{1 + k_2}{(l + k_2)^2} - \frac{k_2}{k_2^2}\right)_l + (l \to -l). \] (9)

Here
\[ M_{il} = \int \frac{d^2l}{\pi l^2} F(x_p, l^2) T_{il} \] (10)
and
\[ D(k) = \alpha_1(1 - \alpha_1)Q^2 + k_1^2 + m_q^2. \] (11)

The function \( F \) denotes the unintegrated (forward) gluon density which is connected with the usual gluon density \( g(x, Q^2) \) through:
\[ \int d^2l \frac{d^2l}{\pi l^2} F(x, l^2) = xg(x, Q^2). \]

The parameter \( \alpha_1 \) is determined by the on-shell conditions for the final state particles:
\[ \alpha_1 = \frac{1}{2} \left[ 1 + \frac{k_1^2}{m^2} - \frac{(k_1 + k_2)^2}{m^2} \right] \pm \sqrt{S}, \] (12)
and it varies between 0 and 1. The values of the momenta \( k_1, k_2 \) and of \( m^2 \) decide which sign in eq.(12) holds.

The quark mass \( m_q \) enters the calculations in two places. First, the phase space of the diffractive system (and so the parameter \( \alpha_1 \) and the function \( S \)) depend upon the quark mass via the on-shell conditions for the outgoing particles. Secondly, the propagators of the internal fermion lines are modified by a nonzero quark mass which leads to changes in the matrixelements. Apart from the function \( D(k) \), eq.(11), which enters all the four \( \gamma^* p \)-cross sections, an additional term containing the quark mass emerges in \( d\sigma_{D,T^+} \), eq.(3).

3. Using our analytic formulae, we have performed a numerical Monte Carlo calculation and compared with preliminary H1 data on \( D^{*\pm} \) production in diffractive DIS \( (m_{D^{*}} \approx 2.01\text{GeV}) \). There are (unpublished) HERA measurements on this process both from H1 and from ZEUS. Compared to other charmed mesons the \( D^{*\pm} \) mesons are easy to reconstruct. This makes them attractive objects for testing diffractive charm production. Both experiments made use of the decay channel
\[ D^{*+} \to D^0_{\pi^\pm} \to (K^+ \pi^\pm)_{\pi^\pm} \to (K^+ \pi^\pm)_{\pi^\pm} \] (and c.c.).
Figure 2: $d\sigma/dx_P$ (sum of $c\bar{c}$ and $c\bar{c}g$ production) at different values of the cut on the gluon transverse momentum, $k=k_{cut}$, and of the lower integration limit, $l=l_{min}$ (both in GeV$^2$).

which has a branching ratio of 2.63%. The kinematical regions where ZEUS and H1 have taken their data are slightly different. We will focus on the comparison with the preliminary H1 data which have been collected throughout the years 1995-1997. The amount of data is still quite poor due to the very rare occurrence of the $D^*$ in the considered process, and higher statistics will come from new data.

We have implemented the ep cross section for diffractive massive $c\bar{c}$ production both from and from our expression eq. for the massive $q\bar{q}g$ production into the Monte-Carlo-Program RAPGAP 2.08. This program includes hadronization based on the Lund model. We have added the simulated cross sections of $c\bar{c}$ and the $c\bar{c}g$ production; the cross sections for $D^{*+}$ and $D^{*-}$ production have also been added. $\alpha_s$ was fixed at a value of 0.25. We have used the GRV NLO gluon density to obtain the unintegrated gluon density. The boundary conditions imposed on the kinematic variables are the same as at H1. The electron and proton momenta (in the HERA system) are 27.6 GeV and 820 GeV, respectively, and we demand:

$$0.05 \leq y \leq 0.7$$
$$2 \leq Q^2 \leq 100 \text{ GeV}^2$$
$$x_P < 0.04.$$  

The momentum transfer of the proton has to be limited, because in the experimental analysis also events with diffractive excitations of the proton have been included in the diffractive cross section:

$$|t| \leq 1 \text{ GeV}^2.$$  

Finally, there are two restriction (of more technical origin) on the transverse momenta of the $D^*$ mesons and on their pseudorapidity values $\eta = -\ln \tan(\theta/2)$ ($\theta$ denotes the angle between the momentum of the
particle and the direction of the incoming proton):

\[ \eta(D^{*\pm}) < 1.5 \quad \text{and} \quad p_T(D^{*\pm}) > 2 \text{ GeV}. \]

In addition to these restrictions which come from the experimental side there is one further cut on the \(c\bar{c}g\) final state due to non-perturbative elements in the calculation. Since the transverse momentum of the final state gluon defines the scale for the strong coupling constant we have to impose a lower cutoff \(k_{\text{cut}}^2\) on the gluon transverse momentum. If it is chosen to be too large, we loose on the cross section; at too small values the reliability of perturbation theory becomes weak. In our calculations we have chosen \(k_{\text{cut}}^2 = 2 \text{ GeV}^2\). Finally, the \(l^2\) integration of the gluon loop in eq.(10) is not defined in the infrared region. We therefore choose a nonzero lower integration limit \(l_{\text{min}}^2\) at \(l_{\text{min}}^2 = 0.5 \text{ GeV}^2\). In the numerical treatment of the \(c\bar{c}\) production cross section neither \(k_{\text{cut}}^2\) nor \(l_{\text{min}}^2\) are needed.

![Figure 3: The cross section of diffractive \(D^*\) production in the 2-gluon-model as a function of \(x_P\). The data points are preliminary H1 data.
](image-url)

In order to test the sensitivity of our results to the choice of the two cutoff parameters we have computed \(d\sigma/dx_P\) for different values. Results are shown in fig.2: the main change is in the overall normalization. As expected, lowering any of the two cutoffs leads to an increase of the cross section. In our subsequent analysis we have chosen to adjust our \(x_P\) distribution (\(c\bar{c}\) and \(c\bar{c}g\)) of the \(D^*\) mesons to the data in the lower \(x_P\)-bin. Since there is a certain arbitrariness in this choice, in our comparison with preliminary H1 data we will concentrate on the shape of the \(D^*\) distributions rather than its normalization.
Turning now to the results of our numerical analysis we start with the $x_P$ distribution. The contributions due to $c\bar{c}g$ and $c\bar{c}$ and the sum of both are shown in fig.3. (as we have said before, the value of the sum of both cross sections ($c\bar{c}$ and $c\bar{c}g$) has been adjusted to the data in the lower $x_P$-bin). In the upper bin the computed cross section is by a factor of about 10 smaller than the data point. In this $x_P$-region it is expected that, apart from Pomeron exchange (which, in our model, is the 2-gluon exchange) secondary exchanges have to be included: in a perturbative description such an exchange corresponds to $q\bar{q}$-exchange. Since such a contribution has not yet been included, it is not surprising that the two-gluon model undershoots the data.

![Graph](image)

Figure 4: The cross section of diffractive $D^*$ production in the 2-gluon-model as a function of $\beta$. Data points are preliminary H1 data [7].

The $D^*$ cross section as a function of $\beta$ is shown in fig.4. The measured cross section is rising with decreasing $\beta$. This property is not reproduced by the $c\bar{c}$ channel alone. Only when both $c\bar{c}$ and $c\bar{c}g$ production are included the $\beta$ distribution of the two gluon model increases with decreasing $\beta$. This fact is nearly independent of the choice of the cutoffs $l_{min}^2$ and $k_{cut}^2$ which mainly affect the overall strength of the cross section. Therefore the $c\bar{c}g$ contribution really improves the description of the $\beta$ distribution. The calculated sum of the cross sections overshoots the data in the upper $\beta$-bin and still remains below data in the lower $\beta$-bin. A reason for the disagreement in the lower $\beta$-bin could be the radiation of more than one gluon which might be present in the data but is not included in our model. Since in the small $\beta$-region $x_P$ gets larger, the disagreement could also be due to our neglect of secondary exchanges.
Two further distributions are shown in fig. 5. In contrast to the $c\bar{c}$ contributions alone the full cross sections in both bins lie close to the data points (except for the low-$y$ point in fig. 5b, even within the error bars of the data points). The ratio of the data values in the lower and upper $Q^2$ bins is about 15. For $c\bar{c}$ alone the model gives the value 8.3; when the $c\bar{c}g$ contribution is included it goes up to 10.7. Also in the $y$ distribution (fig. 5b) the agreement of the sum of both cross sections with data is better than for $c\bar{c}$ alone.

![Figure 5: The cross section of diffractive $D^*$ production in the 2-gluon-model as a function of $Q^2$ and as a function of $y$. Data points are the H1 preliminary data [7].](image)

4. In this letter we have analysed, within the perturbative two-gluon model, DIS diffractive charm production (production of $D^{*\pm}$ mesons). Compared to an earlier attempt where only $c\bar{c}$ production had been included in the theoretical analysis the present analysis contains, as the new ingredient, also (massive) $c\bar{c}g$ production and leads to a considerable improvement in the agreement with experimental data.

Despite this encouraging success, several improvements in the theoretical part should be made. First, the cross section formula for $c\bar{c}g$ production has been calculated in the leading log-$M^2$ approximation; an improvement which extends the applicability down to small-$M^2$ values would be very desirable. For consistency reasons, one then will need a NLO-calculation of $c\bar{c}$ production. Next, our comparison with data indicates the need of $c\bar{c}gg$ final states: such an extension (at least in the leading log-$M^2$ approximation) should be fairly straightforward. Finally, the region of $x_F > 0.02$ seems to require secondary exchanges which, in the framework of perturbative QCD, should be modelled by $q\bar{q}$ exchange.
A successful test of the two-gluon model in DIS Diffraction, apart from providing a description of charm or jet production at HERA, is also of general theoretical interest: the cross section formula for diffractive $q\bar{q} + ng$ production contains the perturbative triple Pomeron vertex which is expected to play a vital role in the unitarization of the BFKL approximation. It has been calculated both analytically and numerically, and these calculations can be tested experimentally in DIS diffraction dissociation.

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