Charged radiative pion capture on the nucleon in heavy baryon chiral perturbation theory

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The differential cross sections and s-wave and p-wave multipoles for $\pi^- p \to \gamma n$ and $\pi^+ n \to \gamma p$ have been calculated through $O(p^3)$ in heavy baryon chiral perturbation theory (HBChPT). Fits to existing data allow several of the low energy constants to be determined. Generally results of the calculation compare well with dispersion relation predictions.

1. INTRODUCTION

Radiative pion capture by the nucleon, via the associated pion photoproduction process, has been extensively investigated for neutral pions in the context of heavy baryon chiral perturbation theory (HBChPT). Much less is known however about the charged pion case, which has contributions from the leading order, $O(p)$. There the s-wave multipole has been calculated \cite{1} and is in reasonable agreement with experiment. A calculation of the p-wave multipoles can provide additional interesting information about the convergence of the HBChPT expansion above absolute threshold and also an opportunity to compare this approach with others, for example dispersion relation approaches. Such a calculation is also important as a way to obtain some more of the low energy constants. The values of such constants are necessary if we are to continue the program begun for ordinary muon capture in Ref. \cite{2} and extend it to radiative muon capture \cite{3}.

2. OUTLINE OF CALCULATION

We begin with the HBChPT Lagrangian $\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)}$ to $O(p^3)$. The lowest order Lagrangian is given by the standard form $\mathcal{L}_{\pi N}^{(1)} = \overline{N}_v (iv \cdot \nabla + g_A S \cdot u) N_v$. The higher order terms are taken in the specific representation of Eckert and Možiš [4] exactly as used in Ref. [2]. To $O(p^3)$ it is necessary to calculate all tree level diagrams together with the one loop diagrams constructed from the low order parts of the Lagrangian. We performed the calculation in an arbitrary gauge, so that we could check gauge invariance explicitly, and only at the end reduced to the specific gauge $v \cdot \epsilon = 0$. The calculation was
also performed in a general isospin basis, so that we could verify that the π⁰ amplitudes agreed with previous work.⁴

The reaction of interest here is the radiative pion capture process, whereas almost all previous work has dealt with the inverse, photoproduction, reaction. The cross sections are related trivially via the usual detailed balance equation. However the relations between the amplitudes for the two processes, and thus between the multipoles, are not so simple and involve complex conjugation and various sign changes. Thus we have actually evaluated the amplitude and multipole amplitudes for the photoproduction process, making connection with the radiative capture data at the cross section level. This means that we can compare results for amplitudes and multipoles directly with the conventional ones derived for photoproduction.

With this understanding the amplitude can be written, in terms of the T-matrix, in the Coulomb gauge with \( \epsilon_0 = 0 \) and the transversality condition \( \vec{\epsilon} \cdot \vec{k} = 0 \) as

\[
\mathcal{M}^{N\rightarrow\pi N} = \frac{m_N}{4\pi\sqrt{s}} T \cdot \epsilon = F_1(E_\pi, x) i\chi^\dagger \vec{\sigma} \cdot \vec{\epsilon} \chi + F_2(E_\pi, x) \chi^\dagger \vec{\sigma} \cdot \vec{q} \vec{\sigma} \cdot (\hat{k} \times \vec{\epsilon}) \chi + F_3(E_\pi, x) i\chi^\dagger \vec{\sigma} \cdot \hat{k} \vec{\epsilon} \cdot \vec{q} \chi + F_4(E_\pi, x) i\chi^\dagger \vec{\sigma} \cdot \hat{q} \vec{\epsilon} \cdot \hat{q} \chi ,
\]

where \( \sigma^i \) is a Pauli matrix in spin space between the two-component spinors of the incoming/outgoing nucleon (\( \chi/\chi^\dagger \)), \( \epsilon \) is the photon polarization vector and \( x = \cos \theta \) corresponds to the cosine of the angle between the photon and the pion momenta and \( E_\pi \) is the pion's center of mass energy. Furthermore, each structure amplitude \( F_i(E_\pi, x) \) (\( i=1,2,3,4 \)) can be decomposed into three isospin channels, corresponding, when the appropriate linear combinations are taken, to the physical processes. The s-wave electric, p-wave electric and two p-wave magnetic multipoles can then be projected from these using standard formulas.⁶

3. RESULTS

By squaring the amplitude we obtain expressions for the differential cross sections which can be fitted to recent data from TRIUMF ⨂ and SAL ⨃ to obtain values for the three unknown low energy constants, \( b_{10}, b'_{21}, b'_{22} \), which contribute. The \( O(p) \) result does not fit the data. Adding \( O(p^2) \) terms improves the fit, but it is necessary to include the full \( O(p^3) \) result to get a good fit. This is consistent with applications of HBChPT to other processes which also often find that the \( O(p) \) terms dominate, but \( O(p^2) \) and \( O(p^3) \) are both important and comparable corrections. The values \( b'_{21} = -8.2 \pm 0.7 \) and \( b'_{22} = 9.2 \pm 0.6 \) so obtained are stable and quite reasonable. The constant \( b_{10} \) is double valued, based only on the empirical fit to the data. However comparison to dispersion relation calculations ⨄, strongly favors one of the fits and results in a best value \( b_{10} = 13.7 \pm 4.5 \).

From the amplitudes we can obtain expressions for the s-wave and p-wave multipoles, in the energy region near threshold and exactly at threshold, in the two relevant isospin channels, namely, \( E^{0_{-}}_0, m^{0_{-}}_0, m^{0_{-}}_1, e^{0_{-}}_1 \). These rather complicated formulas are given explicitly in Ref. ⨅. Then, using the values of the \( b_i \) obtained above and in our earlier calculation of muon capture ⨆ we can evaluate these multipoles. The results for the s-wave multipoles agree well with previous HBChPT calculations ⨇ and with dispersion analyses ⨈. The p-wave multipoles are also in reasonable agreement with
the dispersion results for one of the two solutions. The other solution, which has a very large value of $b_{10}$ also gives multipole amplitudes quite different than those obtained in the dispersion relation calculation, and it is on that basis that we rule it out. In general the convergence is good for the electric multipoles, whereas for the magnetic multipoles the $O(p^3)$ terms are comparable to or only a bit smaller than the $O(p^2)$, thus suggesting that it would be interesting to extend this calculation to $O(p^4)$, which can still be done within the context of a one loop calculation.

4. CONCLUSIONS

To summarize, we have investigated the radiative capture of a charged pion by a nucleon using heavy baryon chiral perturbation theory and have obtained explicit expressions for the amplitude and for the s- and p-wave multipoles. Fits to the available cross section data allowed us to obtain the three necessary low energy constants. Using the values so obtained, the eight s- and p-wave multipoles (four for the $\pi^+$ case and four for the $\pi^-$ case) were calculated and compared with results previously obtained from dispersion theory [11]. In general the agreement was good, though the convergence of the results for the magnetic multipoles was not as fast as for the electric ones, thus suggesting that it might be valuable to consider extending the present work to $O(p^4)$ or to include explicit $\Delta(1232)$ fields in the chiral Lagrangian.

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