Sensitivity and Linearity of Superconducting Radio-Frequency Single-Electron Transistors: Effects of Quantum Charge Fluctuations

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We have investigated the effects of quantum fluctuations of quasiparticles on the operation of superconducting radio-frequency single-electron transistors (RF-SETs) for large values of the quasiparticle cotunneling parameter \( \alpha > \frac{8 E_r}{E_c} \), where \( E_r \) and \( E_c \) are the Josephson and charging energies. We find that for \( \alpha > 1 \), subgap RF-SET operation is still feasible despite quantum fluctuations that renormalize the SET charging energy and wash out quasiparticle tunneling thresholds. Surprisingly, such RF-SETs show linearity and signal-to-noise ratio superior to those obtained when quantum fluctuations are weak, while still demonstrating excellent charge sensitivity.

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Most theoretical studies of RF-SET performance focus on normal metal SETs, either in the sequential tunneling or cotunneling regimes, while most experiments are performed using a superconducting SET (SSET) with a readout device for charge based qubits, and a sensor for real-time electron counting experiments. Linearity is a fundamental assumption of theoretical discussions of the quantum limits of amplifiers. Nonetheless, there has been no detailed investigation of the range of linear response for the RF-SET.

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FIG. 1: (a) Schematic diagram of the SET illustrating RF operation. A voltage \( v_{in} \) consisting of dc and RF biases \( V_{dc} \) and \( v_{RF} \) is incident on a tank circuit consisting of an inductor \( L \), a capacitor \( C_p \), and the SET, with tunnel junction resistances and capacitances \( R_{1(2)} \) and \( C_{1(2)} \). A small charge oscillation \( q_0 \cos \omega t \) modulates the reflection coefficient \( \Gamma \) of the tank circuit and therefore the reflected voltage \( v_r \). (b) Electron micrograph of S2 (taken after all measurements). Gates G1 and G2 were used vary the SET offset charge. (c) Power spectrum of \( v_r \) for \( q_0 = 0.063e \) rms and \( \omega / 2 \pi = 100 \) kHz. The measured sideband power and noise floor were used to find the charge sensitivity and SNR of the RF-SET.
TABLE I: Sample parameters. Resistances are in kΩ, energies in µeV, and areas in $10^{-1}$ µm$^2$.

| Sample | $R_n$ | $\Delta$ | $E_c$ | $E_J$ | $\alpha$ | $A_{tol}$ | $E^0_c$ | $E^0_J$ |
|--------|-------|----------|-------|-------|---------|----------|---------|---------|
| S1     | 58    | 200      | 230   | 22    | 0.78    | 4.1      | 254     | —       |
| S2     | 38    | 200      | 250   | 34    | 1.08    | 3.4      | 291     | 258     |
| S3     | 24    | 190      | 162   | 54    | 2.65    | 5.0      | 218     | 162     |

In Fig. 2 we show representative $I$-$V$ characteristics of the samples in the superconducting state, measured for different $Q_0$, with $q_0 = 0$. For S1, we observe clear above-gap ($V_{dc} > 800 \mu$eV) current modulation corresponding to Coulomb blockade of quasiparticle tunneling [Fig. 2(a)]. The main sub-gap features corresponding to the JQP cycles are sharp and clearly distinguished. As illustrated in Fig. 3, the simplest JQP cycle consists of resonant tunneling of a Cooper pair through one junction and dissipative tunneling of two quasiparticles through the other, transporting two electrons through the SET. The cycle can occur only when the transition $0 \rightarrow 1$ ($1 \rightarrow 0$) is allowed, i.e., for $eV_{dc} > E_c + 2\Delta$ where $E_c = e^2/2C_2$ is the charging energy of the SET and $C_2 = C_1 + C_2 + 2C_g$ its total capacitance. While the JQP cycle is forbidden at lower bias, at $Q_0/e = n_g \approx \frac{1}{2}$ and $eV_{dc} = 2E_c$ Cooper pair tunneling is resonant at both junctions and the double JQP (DJQP) cycle becomes possible. The fact that sequential tunneling cannot occur via either cycle for $2E_c \approx 2E_{dc} \lesssim E_c + 2\Delta$ is reflected in S1 by a sharp drop in current at $V_{dc} \approx 630 \mu$V just below the JQP feature.

As $R_n$ decreases, so does current modulation for $eV_{dc} > 4\Delta$, consistent with suppression of the Coulomb blockade by quasiparticle cotunneling (Fig. 2(b) and (c)). In contrast, features corresponding to the JQP cycles still exist but become progressively less sharp. Since these cycles involve both Cooper pair and quasiparticle tunneling, we hypothesize that subgap quantum fluctuations of quasiparticles are strong, while quantum fluctuations of Cooper pairs remain weak. Since to the best of our knowledge no theoretical description of subgap quantum charge fluctuations in the SSET exists, we provide simple arguments supporting our hypothesis.

We first compare with known results for above-gap transport. Following Ref. [17] we define a parameter $\alpha \equiv \frac{\Delta}{eV_{dc}} \left( R_1^{-1} + R_2^{-1} \right) = \frac{8\Delta}{E_J}$ characterizing the strength of quantum fluctuations for quasiparticles, assuming $R_{1(2)} = R_n/2$ and using the Ambegaokar-Baratoff relation for the Josephson coupling energy $E_J = \frac{\Delta}{e} \frac{R_K}{R_n}$ where $R_K = \frac{\Sigma}{C_0}$. Quantum fluctuations are negligible for $\alpha \ll 1$. Determining $E_c$ from the location of the DJQP peak and $E_J$ from the total junction resistance we calculate $\alpha$ as in Table I. None of our samples satisfies $\alpha \ll 1$, although for S1 ($\alpha = 0.78$) some above-gap Coulomb modulation survives. The progressively weakening modulation for S2 ($\alpha = 1.08$) and S3 ($\alpha = 2.65$), is consistent with previous results [17].

Cotunneling as described in Ref. [17] occurs only for $V_{dc} > 4\Delta/e$: it results in two quasiparticle excitations and transfers a single electron through the SET. Other virtual processes, however, remain important for $V_{dc} < 4\Delta/e$. For normal SETs, $E_c$ can be renormalized by quantum charge fluctuations: e.g.,
near \( n_g = 0 \), the effective charging energy \( E_c \approx E_c^0(1 - 4q) \) where \( q = R_K/\pi^2 R_n \) is the dimensionless parallel conductance of the tunnel junctions and \( E_c^0 \) the bare charging energy; similar renormalization occurs in the superconducting state. Calculating the first-order energy shift due to transitions \( n \rightarrow n \pm 1 \), we find the renormalized charging energy \( E_c^* = E_c^0(1 + g\Delta E_R / E_c^0(1 + 2n_g) + \Gamma_E(1 - 2n_g))) \) where \( \Gamma_E = \int_0^\infty K_1^2(u)e^{-u}du \) is a Bessel function.

Using the expression for \( E_c^* \), we find empirically that \( E_c^0 = 254 \mu \)eV gives the measured \( E_c \) for S1. We measure the total geometric junction area \( A_{tot} \) for the samples with an estimated accuracy of \( \pm 20\% \), obtaining the values in Table I. Setting \( E_c^0 = e^2/2C_2^0 \) where \( C_2^0 = C_1^0 + C_2^0 + 2C_g \) and using \( 2C_g \approx 80 \) aF, we obtain \( C_1^0 + C_2^0 = 195 \) aF as the total unrenormalized junction capacitance for S1. Scaling this result according to \( A_{tot} \) we find \( C_2^0 \), \( E_c^0 \) and finally \( E_c^* \) for S2 and S3 [Table II]; agreement is excellent given the uncertainties in \( A_{tot} \). The inset to Fig. 2 shows the relative difference between \( E_c \) and \( E_c^0 \) scaled by \( 1/g \). The results agree with theory to within our experimental accuracy, providing strong evidence that subgap quantum fluctuations of quasiparticles occur in our samples.

Virtual quasiparticle tunneling may also play a role in subgap transport, as suggested by the softening of the JQP cycle cutoff in S2 and S3. To illustrate such effects more clearly we show a plot of the \( I(V_{dc}, n_g) \) surface for S2 in Fig. 3(a). The JQP resonances along the \( 0 \leftrightarrow 2 \) and \( 1 \leftrightarrow -1 \) lines and the DJQP peak at their intersection are clearly visible, but there is no sharp cutoff of the JQP process below the \( 1 \rightarrow 0 \) \( (0 \rightarrow 1) \) thresholds.

For comparison, in Fig. 4(b) we show a simulation of the current in S2 based on sequential tunneling at an elevated temperature and including photon-assisted tunneling due to an electromagnetic environment. Despite the extreme conditions the quasiparticle tunneling thresholds are clearly visible, and the SET current drops nearly to zero between the JQP and DJQP features. The absence of quasiparticle thresholds in Fig. 4(a) calls for an explanation outside of the sequential tunneling picture.

A candidate process that could allow transport along the Cooper pair resonance lines between the JQP and DJQP features is illustrated schematically in Fig. 4(c). If below threshold the transition \( 1 \rightarrow 0 \) \( (0 \rightarrow 1) \) occurs virtually, the transitions \( 0 \leftrightarrow 2 \) and \( 2 \rightarrow 1 \) \( (-1 \leftrightarrow 1 \) and \( 1 \rightarrow 0 \)) are allowed, completing what we call the virtual JQP (VJQP) cycle. Two quasiparticle excitations are created, but two electrons are transferred through the SET, so that the process should be allowed for \( eV > 2\Delta \).

The energy barrier \( E_b \) for \( 1 \rightarrow 0 \) \( (0 \rightarrow 1) \) vanishes at threshold and climbs to \( E_b \approx E_c + 2\Delta \) at the DJQP peak. The process can be neglected if the allowed quasiparticle tunneling rate \( \Gamma_{qp} \) is small compared to the inverse dwell time of the virtual quasiparticle: \( \Gamma_{qp} \leq E_b/h \). Using \( \Gamma_{qp} = 4\Delta/e^2 R_n \), this becomes \( R_n \gg \frac{R_n 2\Delta}{\pi} \), which is violated for a range of voltages between the DJQP and JQP features. A detailed theoretical analysis is required to determine the contribution of the VJQP cycle to transport.

In contrast to the quasiparticle thresholds, features associated with Cooper pair tunneling are visible in both the data and the simulation, suggesting that the number of Cooper pairs is well defined. For the JQP process at resonance, the Cooper pair tunneling rate is \( \Gamma_{cp} \approx E_c^2/\hbar \Gamma_{qp} = \frac{\pi}{3} \frac{E_c^2}{4\Delta} \). Demanding that energy broadening due to Cooper pair tunneling be small compared to the typical energy barrier \( 4\Delta \) for virtual tunneling gives \( 2\hbar \Gamma_{cp}/4\Delta > \frac{16}{10\ E_c^2} \approx 1 \), which is easily satisfied even for S3. For S2 and S3, then, quantum fluctuations are significant for quasiparticles but small for Cooper pairs.

We now turn to RF operation. Optimal operating conditions were selected as follows: a small charge oscillation \( q_0 \approx 0.0066 \) rms was applied and the SNR determined from the power spectrum of \( v_c \), as in Fig. 3(c). Subgap operation (all samples) and above-gap operation (S1) were optimized over dc bias \( V_{dc} \), rf bias \( v_c \), and offset charge \( Q_0 \). We measured SNR versus input amplitude \( q_0 \) for each optimization and determined the charge sensitivity \( \delta q \) using \( \delta q = \frac{q_0}{V_{BW}}10^{-SNR/20} \) where the resolution bandwidth \( BW = 1 \) kHz and SNR is in dB.

The optimized biases for S1 and S3 are indicated in Fig. 3 and the results of the \( \delta q \) and SNR measurements in Fig. 4. For S1 the best \( \delta q = 9 \times 10^{-6} \ e/\sqrt{\text{Hz}} \) was found for \( V_{dc} = 860 \mu \)V, consistent with previous results. Linearly, however, was poor: as \( q_0 \) increases, the measured SNR rapidly becomes sublinear, and \( \delta q \).

FIG. 3: Various JQP cycles. Here J2(1) is on the left (right) and \( V_{dc} > 0 \). Solid (empty) circles indicate quasielectrons (quasiholes) created during a cycle. (a) JQP cycle. Beginning in the state \( n = 0 \) \( (n = 1) \), where \( n \) is the number of excess electrons on the SET, the transition \( 0 \leftrightarrow 1 \ (1 \rightarrow 0) \) is allowed, bringing Josephson tunneling through J1(2) into resonance. Cooper pair tunneling \( 1 \leftrightarrow -1 \ (0 \leftrightarrow 2) \) is interrupted by quasiparticle tunneling through the opposite junction \(-1 \rightarrow 0 \ (2 \rightarrow 1) \), completing the cycle. (b) DJQP cycle. When Josephson tunneling is simultaneously resonant through both J1 and J2, transport occurs via the sequence \( 0 \leftrightarrow 2 \ 2 \rightarrow 1 \ 1 \leftrightarrow -1 \ -1 \rightarrow 0 \). (c) Proposed VJQP cycle. If the transition \( 0 \rightarrow 1 \ (1 \rightarrow 0) \) is forbidden, it may still occur virtually. The remaining JQP transitions are allowed for relevant \( V_{dc} \).
FIG. 4: False color images of $I(V_{dc}, n_g)$ for (a) S2 at $T = 20$ mK (b) a simulation at $T = 200$ mK assuming an electromagnetic environment with impedance $R_{env} = 50$ Ω and temperature $T_{env} = 1$ K. Cooper pair resonance lines $0 \leftrightarrow 2$ ($-1 \leftrightarrow 1$) and quasiparticle tunneling thresholds $1 \rightarrow 0$ ($0 \rightarrow 1$) are indicated by the dashed and solid lines.

worsens [Fig. 5(a)]. Since $\delta q$ apparently does not saturate even for $q_0 = 4.5 \times 10^{-3} e$ rms it is unclear how small $q_0$ must be to achieve linear response. For subgap operation ($V_{dc} = 600 \mu V$) of S1 [Fig. 5(b)], we find $\delta q \approx 1.3 \times 10^{-5} e/\sqrt{Hz}$, with SNR nearly linear to $q_0 \lesssim 0.01 e$ rms. Since $\delta q$ appears close to saturation at $q_0 = 3.1 \times 10^{-3} e$ rms, we may have approached linear response.

For S3 the best operating point occurred at $V_{dc} = 440 \mu V$ [Fig. 5(c)], between the DJQP and JQP features with $\delta q \approx 1.2 \times 10^{-5} e/\sqrt{Hz}$, better than that for subgap operation of S1. Moreover, linearity was vastly improved: the SNR remains linear and $\delta q$ nearly flat to $q_0 = 0.03 e$ rms indicating that we have achieved linear response in this sample. For S2 (data not shown) the best $\delta q \approx 1.2 \times 10^{-5} e/\sqrt{Hz}$ also occurred subgap, and the SNR was linear to $q_0 \approx 0.02 e$ rms.

We can now make some general statements about the effects of quantum fluctuations on RF-SET operation. For samples with smaller $\alpha$ such as S1, transport is fairly well described by the sequential tunneling picture: $I$-$V$ characteristics are sharp and vary strongly with $Q_0$ giving rise to excellent charge sensitivity. The same sharpness, however, prevents good linearity, since a large $q_0$ necessarily moves the SET far from optimal operation. For samples with larger $\alpha$ such as S3, quantum fluctuations cause at least two important effects. First, the subgap features are smoothed and broadened, improving linearity: e. g., in S3 there is no “dead spot” between the DJQP and JQP features for which the SSET current is roughly independent of $Q_0$. Second, renormalization of $E_c$ moves the DJQP feature to lower bias, so that the optimal rf amplitude of about $(2\Delta - E_c)/e$ increases with $\alpha$. Finally, the smaller $R_n$ simplifies impedance matching between the RF-SET and the 50 Ω coaxial line.

In conclusion, we have investigated the influence of quantum charge fluctuations on the charge sensitivity and SNR of RF-SETs. We find that RF-SETs with $\alpha > 1$ (strong quantum fluctuations) show both good linearity and good charge sensitivity. In contrast, RF-SETs with $\alpha < 1$ (weak quantum fluctuations) show poor linearity and only modestly better charge sensitivity. These findings assume particular importance given interest in the RF-SET as a potentially quantum-limited...
linear amplifier. We have achieved linear response only for subgap operation in samples with $\alpha \gtrsim 1$ for which quantum fluctuations of quasiparticles are substantial.

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