Reduced order models for EHD controlled wake flow

Juan D’Adamo†, Roberto Sosa†, Ada Cammilleri‡, Guillermo Artana†
† CONICET, Laboratorio de Fluidodinámica, ‡ Departamento de Matemática, Facultad de Ingeniería, Universidad de Buenos Aires.
CP 1063 Avda. Paseo Colón 850, Buenos Aires, Argentina
E-mail: jdadamo@fi.uba.ar, rsosa@fi.uba.ar

Abstract. We present in this work a way to construct reduced order model (ROM) for wake flows at low Reynolds numbers, from experimental data of particle image velocimetry (PIV). The proper orthogonal decomposition method (POD) allows the extraction of a small number of functions or modes that describes the flow velocity field. Galerkin projection of Navier Stokes equations onto these modes leads to obtain a system of ordinary equations, the reduced order model.

We take interest particularly in the study of the modifications introduced by an electrohydrodynamic (EHD) actuator on the flow. EHD action is obtained from a gaseous discharge of dielectric barrier type (DBD), produced by electrodes flush mounted on a cylinder surface. The reduced order model presents as an effective tool in order to analyse the coupled electrical and hydrodynamic phenomena. We focus on the flow changes considering the voltage between the electrodes as a control parameter.

Keywords: POD, reduced order models, wake flow, electrohydrodynamics.

1. Introduction

Reduced order models are widely used in fluid mechanics in order to represent the main dynamical behaviour of a flow. Within this framework, a few number of modes can be enough to perform analysis, understanding and predictions for the different states of a given flow. Wake flows are particularly interesting in terms of academic and many industrial purposes and we will focus our study to this kind of flows. The flow around a cylinder has long been a benchmark problem concerning wake flows. In order to obtain such models, we focus on the POD technique which main aspects are described next.

Also known as Karhunen-Loève Decomposition, Singular Value Decomposition, Principal Components Analysis, POD has been first introduced in the context of turbulence by Lumley (1967)[10]. It has been used by different authors (see for instance a review by Holmes et al.(1996)[8]) as a method to obtain approximate descriptions of the large scale or coherent structures in laminar and turbulent flows. Without any a priori hypothesis on the flow, the POD method provides a flow representation in terms of a linear combination of basis functions, or modes, ordered decreasingly by their kinetic energy content. Indeed, once the basis is obtained, a Galerkin projection is performed to reduce the order of Navier-Stokes equations. It results a system of ordinary differential equations that should describe approximately the flow. However, the solution of such system is not straightforward. After the Galerkin projection, it cannot be assured that the error of the approximation is bounded. Additional hypothesis on the model
are required. Previous works on the flow around a cylinder started with Deane et al. (1991) [6] and since then, many efforts have been performed in order to construct a control system of the POD low order model. Some drawbacks of this technique are the lack of stability and the lack of robustness of the model away from the flow observations.

In order to improve these aspects, we had proposed in D’Adamo et al. (2007) [5] a scheme based on data assimilation theory. Another method was presented by Kalb & Deane (2007) [9] which performs a linear correction on the dynamical systems coefficients, and it was tested on direct numerical simulation data and on the Lorenz system. On the other hand, Noack et al. (2003) [12] had pointed out that the error sources start solving the system directly obtained by the Galerkin projection. It is not possible to assure, a priori, the system structural stability. Little perturbations on the equation system coefficients can produce qualitatively different solutions. For this reason, the authors proposed the POD-Galerkin reduction to an invariant, inertial, manifold towards a normal form, a non linear equation which has a prototypical structure.

On the other hand, in the context of wake flows study, POD modes can be interpreted as fluctuations that superpose to a stationary solution, a base flow, unstable, of the Navier Stokes equations. Noack et al. (2003) [12] constructed their model from this hypothesis. However, in recent articles (Thiria et al. (2007) [16], Barkley (2006) [2]) it has been emphasized the importance of the time averaged flow as basic state that can account for stability properties of wakes under forcing conditions. On these bases, we developed a POD-ROM constructed from experimental (PIV) data.

Electro-hydrodynamic (EHD) actuators for flow control have been receiving special attention in the last years Artana et al. (2003) [1]. Through the ionization of flowing air close to the surface of the body, EHD devices produce a modification of the condition of the flow at the wall. We propose in this work, to identify such modifications in the flow around a cylinder by means of the POD based ROM.

2. POD-Galerkin Model

2.1. Introduction

The POD-Galerkin method has been extensively presented in many articles. The method foundations can be studied in Holmes et al. (1996) [8]. Let us review its main characteristics. A flow field \( u(x, t) \in L^2(\Omega) \), where \( \Omega \) is a physical domain, can be represented in terms of basis functions \( \phi_i(x) \) and their temporal coefficients \( a_i(t) \) such that:

\[
 u(x, t) = \sum_{i=0} a_i(t) \phi_i(x)
\]  

(1)

It is necessary to define an inner product as

\[
(u, v) = \int_{\Omega} u \cdot v \, dx
\]

The modes \( \phi(x) \), global and mutually orthogonal, are extracted from a set of observations \( u(x, t_i) \) obtained experimentally or numerically, and they are optimal with respect to the average kinetic energy representation of the flow contained in \( \Omega \). The spatial modes inherit by construction properties of the flow such as its boundary conditions and, when we work with incompressible flows, they are also divergence-free.

When working on finite dimension, the modes identification reduces to solve the singular value decomposition (SVD) of a \( V \) matrix that represents in its columns the different snapshots of the velocity fields \( u(x, t_i) \).
The Navier Stokes equations, and the incompressibility condition,

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\nabla p + \frac{1}{Re} \nabla^2 u \quad (2)$$

$$\nabla \cdot u = 0 \quad (3)$$

are then reduced by means of a Galerkin projection onto the subspace generated by the first $s$ vectors of the POD basis, $\{\phi_1 \ldots \phi_s\}$. The ROM has the form:

$$\frac{da}{dt} = \dot{a} = M(a) \quad (4)$$

where $M$ represents a nonlinear operator which is a polynomial that contains the projected Navier-Stokes terms and its boundary conditions. The projection produces the following terms:

$$\left( \frac{\partial u}{\partial t}, \phi_k \right) = \frac{da_k}{dt}$$

$$\left( \frac{1}{Re} \nabla^2 u, \phi_k \right) = \frac{1}{Re} \int_{\Omega} \nabla^2 u \phi_k dx = \frac{1}{Re} \sum_{i=1}^{s} a_i \nabla^2 \phi_i \cdot \phi_k dx$$

$$\left( \nabla p, \phi_k \right) = \int_{\Omega} \nabla p \phi_k dx = \int_{\delta\Omega} (p \phi_k) \cdot dl$$

$$\left( (u \cdot \nabla)u, \phi_k \right) = \sum_{i=1}^{s} \sum_{j=1}^{s} a_i a_j \int_{\Omega} (\phi_i \cdot \nabla) \phi_j \cdot \phi_k dx \quad (5)$$

These can be included in a polynomical form on the temporal coefficients $a_1 \ldots a_s$ for $M$.

$$\dot{a} = F(a) = \sum_{k=1}^{N_m} c_k P_k \quad (8)$$

where $P_k \quad k = 1 \ldots N_m$ is a vector of all the monomials of different degrees in variables $a = (a_1 \ldots a_s)$, and $c_k$ are their corresponding coefficients. The number $N_m$ depends on the number of degrees and on the number of modes considered, the resolved modes.

2.2. System identification

The solution of (8) is not straightforward. Equation (8) determines in general a dynamical system that in many cases may converge to erroneous states after a relative short time of integration. Indeed, different authors have observed that the predictions at long time may fail and that, for instance, models describing a cylinder wake flow, after a few cycles of vortex shedding may exhibit differences of phase and amplitudes with respect to empirical values, as pointed out by Kalb & Deane(2007) [9]. Holmes et al.(1996)[8] had mentioned as possible responsible for this behaviour the following points: Neglect of the incidence of boundary or pressure terms in the computation of the dynamical system coefficients; inaccurate estimation of derivatives of spatial modes that determine the dynamical system coefficients; and that the low-dimensional truncation produces loss of dissipation and energy transfer between the different scales of the flow and it only reproduces velocity fields which are close to the spatial structures averaged in the decomposition.

Concerning the pressure term, several authors have neglected the influence of the pressure term
in wake flows arguing that this effect could be disregarded when large wake domains were considered. However, depending on the flow geometry and truncation level, neglecting this term could add uncertainty in the ability of the computed dynamical system to faithfully represent the actual flow dynamics. Noack et al. (2005)[13] proposed an additional quadratic term which is estimated solving Poisson equation. The Poisson equation, determined applying the divergence operator on the Navier-Stokes equations, can be written as:

\[ \nabla^2 p = -\nabla \cdot (u \cdot \nabla u) \quad \text{in} \quad \Omega \]  

and allows to estimate pressure from the velocity field described as in Eq. (1).

Another way to correct the model (8) is to apply the intrinsic stabilization algorithm developed by Kalb & Deane (2007)[9], which corrects the linear terms and is effective in situations where the pressure is adequately expressed by a linear model, i.e. channel flow.

Using PIV data, the system coefficients may be poorly determined. To avoid the direct estimation of these coefficients, Perret et al. (2006)[14] proposed a polynomial identification procedure which consists in writing \textit{a priori} the ROM. Provided the temporal coefficients \( a_j \) and their time derivatives \( \dot{a}_j \), we can derive from (8) a linear system, which may be solved by least mean square or singular value decomposition. This is why we had not stated, intentionally, the degree of (8). The unresolved modes can be represented as functions, polynomials, of the resolved ones and therefore increase the system degree. We remark that the pressure field (9) is implicitly included in this formulation.

It must be pointed out that estimating \( \dot{a}(t) \) from traditional schemes of finite differences leads to important numerical errors, further amplified when working with experimental data. To deal with this problem, it is possible to perform a polynomial approximation for each \( a_j \) for every instant \( t_i \). Consecutive time intervals allow to calculate \( a_j(t_i) \approx p_j(t) \), where \( p_j \) is a polynomial of degree \( n \), and therefore the derivatives are obtained analytically deriving \( p_j(t) \). This idea was introduced by Maquet et al. (2004)[11] in a work on dynamical systems identification.

We had proposed in D’Adamo et al. (2007)[5] a method to work with experimental data improving the polynomial identification with an assimilation technique. Considering a noise \( \delta(t) \) as the difference between the model prediction and an observations set, \( \dot{a} = F(a) + \delta(t) \), it is minimized iteratively. The scheme does not deal with the system structural instability but it provides a way to work with noisy data.

A dynamical system is structural unstable when any little perturbation on its coefficients leads to solution qualitatively different. In our problem, the methods listed above don’t assure structural stability. In order to construct a robust system, once its coefficients are determined, we analyse it by means of dynamical system theory: particularly the center manifold and normal form theory. Indeed, a first reduction can be achieved by studying the evolution of the variables contained in the center manifold of (8) and considering that the remaining variables are \textit{slaved} to them. Noack et al. (2003)[12] studied the particular problem of POD models for wakes by means of the center manifold theory. On the other hand, once the problem dimension is reduced, we may simplify the nonlinear terms of the resulting equation, according to simmetry considerations. This nonlinear change of coordinates is called the normal form theory and it is applied particularly to this problem in D’Adamo et al. (2008a)(2008b)[3][4].

2.3. Dynamical system analysis.

We know that the physical model for the wake flow problem is represented by the Stuart-Landau equation (see, for instance [15]):

\[ \frac{dA}{dt} = (\gamma + i\omega)A + (c_r + ic_i)|A|^2A \]  

(10)
and the vortex onset is described as a Poincaré-Andronov-Hopf bifurcation (see [18]§3.1B for a definition) in agreement with experiments Provansal et al. (1987)[15]. These reasons sustain the hypothesis of an equation of degree three for the modes amplitudes that resumes the coherent structures behaviour.

On the other hand, in agreement with Barkley’s hypothesis, the fluctuations $a(t)$ are superposed to the mean flow, and the solution $a = 0$, a fixed point, corresponds to the base flow. A center manifold reduction on this point can simplify our problem.

The center manifold theory is a rigorous mathematical technique used to reduce dynamical systems dimension. As we pointed out, a Poincaré-Andronov-Hopf bifurcation must be described by our model for $a = 0$. We consider a bifurcation parameter $\mu = Re - Re_c$, where Reynolds critical number $Re_c$ corresponds to the value of $Re$ for the vortex shedding onset.

A linear change of coordinates (Real Jordan form) transforms the $s$-dimensional Eq. (8) into the form:

$$
\begin{align*}
\dot{x} &= Ax + f(x, y, \mu) \\
\dot{y} &= By + g(x, y, \mu)
\end{align*}
$$

where $x \in \mathbb{R}^c$ and $y \in \mathbb{R}^{u+e}$, for our problem the center manifold dimension $c = 2$ and $s = c + u + e$, being $u$ and $e$ the dimension of the unstable and stable manifold respectively. We consider then

$$
A = \begin{pmatrix}
\text{Real}(\lambda) & -\text{Imag}(\lambda) \\
\text{Imag}(\lambda) & \text{Real}(\lambda)
\end{pmatrix}
$$

where $\lambda = \lambda(\mu) = \alpha(\mu) + i\omega(\mu)$ represents the eigenvalues of $A$ and we suppose that $B$ does not contain eigenvalues of zero real part.

In agreement with the Center Manifold theory (see i.e. Wiggins(2003)[18]§2.1) we know that the center manifold can be represented locally by:

$$
\{(x, y)/y = h(x), h(0) = 0, Dh(0) = 0\}
$$

and that, in such a case, the dynamics of Eq. (11) is determined by:

$$
\dot{x} = Ax + f(x, h(x), \mu)
$$

for $\|x\| < \delta$, sufficiently small. In our applications we consider $h(x) = p(x)$, a polynomial of degree 3 in $x \in \mathbb{R}^c$. This is a practical choice: higher order polynomials do not assure a better reconstruction and, on the other hand, they amplify the coefficients’ noise.

However, we cannot assure yet that the solution of Eq. (11) has the characteristic behaviour of a stable limit cycle, convergent towards the trajectory defined by the set $\{a_j(t_i)\}$. The polynomial $f(x)$ can be simplified depending on $A$ eigenvalues. For this problem, when $\mu = 0$, these are a pair of conjugate complex of zero real part. When $\mu > 0$, the $A$ eigenvalues become of positive real part (i.e. $\alpha$) and, based on this fact, the resulting normal form (see i.e. Wiggins(2003)[18]) is

$$
\begin{align*}
\dot{x}_1 &= \alpha x_1 - \omega x_2 + (ax_1 - bx_2)(x_1^2 + x_2^2) \\
\dot{x}_2 &= \omega x_1 + \alpha x_2 + (bx_1 + ax_2)(x_1^2 + x_2^2)
\end{align*}
$$

without considering terms of degree equal or greater than 5. We verify that this equation is equivalent to Stuart-Landau (10), but we have obtained it from the nonlinear analysis of POD-Galerkin system. In polar coordinates, Eq. (14) has the form:

$$
\begin{align*}
\dot{r} &= \alpha r + ar^3 \\
\dot{\theta} &= \omega + br^2
\end{align*}
$$

The analysis of the dynamics of these equations, for the values of $\alpha$ and $\omega$ of our applications, confirms that the original system has in the origin an unstable focus and an asymptotically stable
It results for the cylinder wake flow in this regime, a third order system that describes the dynamics of a two dimensional manifold. The remaining variables have dynamics that collapse into the center manifold. For this reason, we restrict our analysis of the wake flow modification to the study of just 2 variables, that are constructed from POD modes, that describes the center manifold.

3. Experimental Setup

3.1. Wind tunnel and cylinder
The experimental configuration we settled for this work consists of a flow around a circular cylinder at low Reynolds number, $Re = 100$. The velocity field measurements were undertaken with the cylinder placed in a closed loop wind tunnel with a test section of 18x18 cm$^2$. In our study the EHD actuators were mounted on the surface of a polymethyl methacrylate cylinder that were 20 mm in diameter $D$. The cylinder was a hollow tube with a 4 mm thick wall.

3.2. EHD excitation
The DBD plasma actuator used in our configuration consists in set of couples of flat aluminium foils electrodes disposed symmetrically in the spanwise direction of the cylinder surface, in an arrangement as shown in Fig. 1. For each pair, one of the electrodes is exposed to the air while the other is fully covered by the dielectric material. In this case, the cylinder thickness is used as the dielectric layer. The covered electrode is grounded, and the a.c. voltage is applied to the exposed electrode. Both electrodes are slightly overlapped (of the order of 0.5 to 1 mm), in order to ensure a uniform plasma in the full spanwise direction. The a.c. voltage amplitude ranges from 3 to 5 kV, and the operating frequency of the a.c. voltage supplied to the electrodes is 7.5 kHz. The plasma actuator is operated in "steady" manner, i.e. the input driving a.c. frequency is well above the fluid response frequency (taking into account that the Strouhal number $St \sim 0.2$ and the characterisitic length D=0.16 m, the vortex shedding frequency is $\sim 1Hz$), and therefore the flow senses a constant body force.

3.3. Velocity field measurements
The velocity field was measured using a PIV technique. These experiments were conducted using a Pixelfly PCO VGA camera, with a resolution of 640x480 pixels, 1/50 s of time between images.
A green laser Intelite GM32-150IH, 150 mW, combined with a rotating polyhedral mirror, was used to provide illumination on the test section. The algorithms used in this work belong to GPIV software [17] which is under GNU General Public License. We have considered for our images a two step grid refinement, so the final interrogation size is 16x16 pixels with a 50% overlapping. The flow domain covers about $6 \times 8$ diameters and it allows to describe the large structures of the flow. The time resolution was suitable to recover the flow dynamics, in a way that the vortex shedding frequencies are lower than the acquisition frequency.

4. Results
The modifications introduced by the plasma actuator can be described firstly by the mean flow field. We observe that the near wake region behind the cylinder enlarges under increasing voltage. The resulting mean flow resembles more to the stationary solution, characterized by a long recirculation zone.

The modal decomposition allow us firstly to construct a ROM for each case. In Fig. 2, the first POD spatial mode shows the structure of the large vortex associated to the von Karman street. Indeed, superposing the second mode and modulating these modes with their temporal coefficients, the main characteristics of the flow are recovered. For this kind of actuation, these global structures are conserved under EHD actuation.

The temporal coefficients are plotted in Fig. 3. We distinguish no changes on their frequency, the Strouhal number $St = fD/U$ remains practically the same (for this flow, $St \sim 0.17$). From the observations, we note that temporal modes amplitudes decrease with increasing EHD voltage. The actuator achieves a stabilizing effect on the wake as it dumps progressively the fluctuations as it can be verified from Fig. 3a) to c). This result is in agreement to the considerations remarked on mean flow modifications.

On the other hand, the ROM solution captures the flow attractor for each case, a limit cycle. The model is obtained after applying the polynomial identification technique, the center manifold and the normal forms reduction. Fig. 3d) shows the phase diagram for $AC = 5kV$, where the solution of any initial condition (i.e. points 1 to 4) is conducted, by means of the model, into the limit cycle. Structural stability of the dynamical system is therefore assured: solutions starting from different initial conditions are always attracted to the limit cycle. Hence, the model can reproduce transient states and it results an attractive tool in the study of transient excitations and to solve optimization problems.

![Figure 2. First POD spatial mode for the flow field. Left: Free of EHD actuation. Right: $AC = 5kV$ Geometrical description by streamlines.](image)
5. Conclusions

From the benchmark of the flow around a cylinder problem, we proposed a tool in order to compare flow modifications introduced by an EHD actuator. A reduced order model is constructed from experimental data, PIV measures, and it allows to capture the essential dynamics of the flow. We applied the POD-ROM to a set of images corresponding to the flow around a cylinder at a low $Re$ number, and the resulting flow under EHD actuation. As experimental data is subjected to many error sources, we performed methods of dynamical system identification and reduction to guarantee the model reliability and stability.

The mean flow is modified as the recirculation bubble length is increased by the EHD action. The actuator has a stabilizing effect on the flow, confirmed by the decrease of the fluctuations amplitudes, represented, in the ROM by the temporal coefficients. The POD-ROM scheme results a convenient tool to identify the dynamical changes introduced by the EHD actuator. The system is robust enough to solve solutions starting far from the limit cycle, and it presents as an interesting tool to study transient states and optimization.

We remark that our study was exposed for the center manifold reduction, scheme valid for Reynolds numbers close to the threshold (for the cylinder wake $Re_c \sim 50$. Nevertheless, this
can be extended to the so called inertial manifold (see i.e. Noack et al. (2003) [12]), which is constructed by identifying the real positive eigenvalues of the jacobian $\partial F_i/\partial a_j$ in (8), and referring to the corresponding eigenmodes the remaining ones.

Further studies may be conducted after this work, as the introduction of a control parameter to produce only one dynamical model to represent each actuated case to allow close loop control schemes.

Acknowledgement This work has been undertaken with ANPCYT grant PICT 38070 and UBACYT grant number 1017

References
[1] G. Artana, R. Sosa, E. Moreau, and G. Touchard. Control of the near wake flow around a circular cylinder with electrohydrodynamic actuators. *Experiments in Fluids*, 36(6):580–588, 2003.
[2] D. Barkley. Linear analysis of the cylinder wake mean flow. *EPL (Europhysics Letters)*, 75(5):750–756, 2006.
[3] J. D’Adamo, A. Cammillieri, and G. Artana. Modelos reducidos para flujos de estela a bajos números de reynolds. In *INMAT*, Buenos Aires, Argentina, 2008. Facultad de Ingeniería (UBA).
[4] J. D’Adamo, A. Cammillieri, E. Mémin, and G. Artana. Reduced order models for wake flows at low reynolds numbers. In *Escola de transico e turbulencia*, San Carlos, Brasil, 2008. ABCM.
[5] J. D’Adamo, N. Papadakis, E. Mémin, and G. Artana. Variational assimilation of pod low-order dynamical systems. *Journal of Turbulence*, 8(9):1–22, 2007.
[6] A.E. Deane, G. Karniadakis, and S. Orszag. Low-dimensional models for complex geometry ows: Application to grooved channels and circular cylinders. *Physics of Fluids A*, 3:2337–2354, 1991.
[7] J. Guckenheimer and P. Holmes. *Nonlinear Oscillations, Dynamical Systems, and Bifurcation of Vector Fields*. Springer, 1986.
[8] P. Holmes, G. Berkooz, and J.L. Lumley. *Turbulence, Coherent Structures, Dynamical Systems and Symmetry*. Cambridge University Press, 1996.
[9] V.L. Kalb and A.E. Deane. An intrinsic stabilization scheme for proper orthogonal decomposition based low-dimensional models. *Physics of Fluids*, 19:18, May 2007.
[10] J.L. Lumley. The structure of inhomogeneous turbulence. *Atmospheric Turbulence and Radio Wave Propagation*, pages 166–178, 1967.
[11] J. Maquet, C. Letellier, and L.A. Aguirre. Scalar modeling and analysis of a 3d biochemical reaction model. *Journal of Theoretical Biology*, 228(3):421–430, 2004.
[12] B. Noack, K. Afanasiev, M. Morzynski, Gilead Tadmor, and Frank Thiele. A hierarchy of low-dimensional models for the transient and post-transient cylinder wake. *Journal of Fluid Mechanics*, 497(-1):335–363, 2003.
[13] B. Noack, Paul Papas, and Peter A. Monkewitz. The need for a pressure-term representation in empirical galerkin models of incompressible shear flows. *Journal of Fluid Mechanics*, 523(-1):339–365, 2005.
[14] L. Perret, E. Collin, and J. Delville. Polynomial identification of pod based low-order dynamical system. *Journal of Turbulence*, 7(17):1–15, 2006.
[15] M. Provansal, C. Mathis, and L. Boyer. Bénard-von kármán instability: transient and forced regimes. *Journal of Fluid Mechanics Digital Archive*, 182(-1):1–22, 2006.
[16] B. Thiria and J.E. Wesfreid. Stability properties of forced wakes. *Journal of Fluid Mechanics*, 579(-1):137–161, 2007.
[17] Gerber van der Graaf. Gpiv, open source software for particle image velocimetry. http://gpiv.sourceforge.net, 2008.
[18] Stephen Wiggins. *Introduction to Applied Nonlinear Dynamical Systems and Chaos*. Springer, New York, 2003.