Modeling and optimizing the bus operation based on the bus app

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Abstract—Bus congestion is a common phenomenon in the process of bus operation, which will worsen bus service to a certain extent and further affect residents’ travel choice. To cope with this problem, this paper proposed a new passenger arrival model taking into account the usage of bus app, which represents the dynamic passenger arrival distribution and where bus capacity constraints are explicitly considered. First, a bus propagation model was developed to carry out theoretical analysis of the bus bunching problem. Afterwards, the new passenger arrival rule, namely, triangular passenger arrival rule, was obtained by fitting the researching data of the passengers who used “bus is coming” app. Finally, under this law of arrival, a new enhanced bus propagation model was developed. By comparing the simulated results, we found that the occurrence of bus bunching was reduced dramatically and the operation of the bus was more stable, which indicated the superiority of the triangular passenger arrival rule.

1. INTRODUCTION

The influx of a large number of private cars makes urban traffic congestion more serious. The public transportation, as one of the effective ways to relieve traffic congestion, has been strongly advocated for its low price and efficient utilization of resources. However, the bus bunching is commonly observed in the propagation of the bus and it reduces its service reliability. Hickman (2001) mentioned that the passengers have always attached great importance to the on-time performance and schedule reliability of the bus [1]. Newell and Potts (1964) originally come up with the idea that one reason for the bus bunching is that the bus could be slowed down by an unforeseeable disruption relative to the following bus, more passengers would arrive at the stop and further delay it. On the contrary, the trailing bus tends to catch up [2]. Then fewer passengers need to be loaded by the subsequent bus at that stop and the departure time of the bus would be earlier than scheduled. This will lead to a longer running cycle and a poor reliability.

Generally speaking, the ways of counteracting the bus bunching can be divided into two groups, namely, the control based on the bus and the control based on the passenger behavior. The first approach has been widely used on many studies currently. Daganzo et al. (2009) described an approach which is based on the real-time headway information to eliminate bus bunching [3]. They showed that this method would need less slack time than the conventional, schedule-based approach and it would reduce the passengers waiting time and increase bus productivity. Xuan et al. (2011) presented a series...
of dynamic holding strategies based on the deviations between the bus arrival time and the virtual schedule at the control points [4]. Daganzo et al. (2011) established a self-coordinating control project, which plays a role in adjusting the bus travelling speed in real-time based on the space between its front and rear part [5]. Hernández et al. (2015) originally proposed an optimization model which has the ability of implementing a holding control for a corridor with numerous bus lines [6]. They showed that in the simulation, the overall waiting time of the passengers reduced 55% and they got a lower variability of the bus headways. Berrebi et al. (2015) built a new holding strategy based on the real-time information for the bus propagation, which can reduce the average waiting time and the total bus service time at the stop [7]. Besides the holding control strategies, He et al. (2019) presented a new strategy of adjusting the speed of the buses in the bus accommodation lanes [8]. And it is verified that this method can not only stabilize the propagation of bus line but also shorten the waiting and travelling time of passengers.

As for the passenger behavior-based way, there has been a significant body of literatures about it. Bowman et al. (1981) developed a model of passenger arrival behaviors, which owing the passenger arrival patterns to the characteristics of the bus operation [9]. Quarmby et al. (1967) already illustrated that, compared with the time on board travelling, passengers gave higher priority to the time of waiting for bus at the station [10]. Fonzone et al. (2015) assumed that passengers settle their arrival time at stops based on a continuous logit model in case of missing the bus [11]. Finally, they draw a conclusion that the process of bus bunching can be dramatically affected by the passengers’ non-uniform arrival patterns.

Therefore, we are inspired by the discussion above. Our primary objective in this paper is to identify possible measures that could help operators and decision makers to realize the full potential of bus app, such as “bus is coming”, more specifically by including the use of bus app and the passenger arrival rule in the design of the bus schemes.

The remainder of this paper is organized as follows. In Section 2, we process and analyze the experimental data. In Section 3, the bus propagation models are developed respectively based on the uniform arrival rule and triangle arrival rule. In Section 4, we verify the effectiveness of the proposed methods through an idealized bus line base on a real bus line in Xi’an, China. Finally, Section 5 draws conclusions of the study and discusses the practical implications on bus bunching.

2. METHODOLOGY AND DATA
We chose Xi’an 265 bus line for research, deciding the Chang’an university garden road stop as our research site, which is shown in figure 1.

![Figure 1. Xi’an 265 bus line.](image)

We arranged the 30 passengers with daily travel demands to take the bus at around 9 a.m. every weekday and record the actual arrival time of both buses and researchers. The app interface is shown in figure 2.
Figure 2. The using interface of the bus app.

| Deviation (min) | -8 | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 0 |
|----------------|----|----|----|----|----|----|----|----|----|
| Arrival rate   | 0.6| 1.1| 1.9| 3  | 4.9| 5.9| 7  | 3.7| 1.9|

After processing the data from research, the passenger average arrival rate was obtained, which is shown in Table 1. It can be easily seen from the figure 3 that the function of passenger arrival rate and the difference between passenger arrival time and bus arrival time moves like a triangle. So, we named this rule as the triangular passenger arrival rule.

We assumed that the x-coordinate of the peak value of the function is $x_m$, and further formulated that the optimized function expression is:

$$y^* = \begin{cases} a_1 x(i) + b_1, & -8 < x(i) < x_m \\ a_2 x(i) + b_2, & x(i) > x_m \end{cases}$$

It subjects to

$$a_2 < 0, x_m < 0, b_1 > 0, b_2 > 0, \int (y - y^*)^2 = 0$$

Figure 3. The average arrival rate distribution.
The optimizing parameters are as follows: \( a_1 = 1.1143, a_2 = -2.550, b_1 = -2.0137, b_2 = 9.0287, b_3 = 1.6500 \), and drew the graph of the triangular arrival function in figure 4.

\[
y^* = \begin{cases} 
1.1143x + 9.0287, & -8 \leq x \leq -2.013 \\
-2.55x + 1.65, & -2.013 < x \leq 0 
\end{cases}
\]

Figure 4. The arrival rate optimized by the fmincon.

### 3. MODEL DESCRIPTION

The assumptions:

1. It is stipulated that overtaking is not allowed in the station.
2. Passengers get on and off at a fixed speed.
3. The boarding time is the main factor that influence the dwell time of the bus.

| Notation | Description                           | Unit |
|----------|---------------------------------------|------|
| \( i, j, k \) | The number of buses/stops             | -    |
| \( C \)    | The capacity of bus                   | pax/veh |
| \( \alpha_k/\beta_k \) | The passenger arrival/alighting rate at stop \( k \) | pax/min |
| \( r \)    | The average boarding rate             | Pax/min |
| \( h \)    | The departure interval of a bus route | min  |
| \( H_{i,k} \) | The inter-departure headway between the successive buses | min |
| \( A_{i,k}/D_{i,k} \) | The arrival/departure time of bus \( i \) at stop \( k \) | min |
| \( T_{i,k} \) | The link travel time                   | min  |
| \( W_{i,k}/P_{i,k} \) | The number of waiting/ alighting passengers | pax |
| \( \bar{W}_{i,k} \) | The number of passengers who are able to board the bus | pax |
| \( \bar{r}_{i,k} \) | The boarding time of the passengers   | min  |
| \( \bar{t}_{i,k} \) | The bus dwelling time                  | min  |
| \( \bar{L}_{i,k} \) | The number of on-board passengers     | pax  |

### 3.1. The general bus propagation model with capacity constraint

The arrival time of bus \( i \) at stop \( k \) is the sum of the departure time from stop \( k-1 \) and the random link travel time between stop \( k-1 \) and \( k \):

\[
A_{i,k} = D_{i,k-1} + T_{i,k-1}
\]

The bus departure time is determined by its arrival time and dwell time:

\[
D_{i,k} = A_{i,k} + \bar{t}_{i,k}
\]
On account of uncertainty in travel times, the order of buses arriving at a stop may have altered. Therefore, the preceding car of the subject bus $i$ may not be the bus $i-1$. The formula of inter-departure headway is:

$$H_{i,k}=D_{i,k}D_{i,k}$$ (7)

The actual boarding time is the specific value between the actual number of arriving passengers who are able to board and the passenger boarding rate:

$$t_{i,k} = \frac{B_{i,k}}{r}$$ (8)

The bus service time can be approximately equal to the boarding time of passengers:

$$\bar{t}_{i,k} = t_{i,k}$$ (9)

3.2. The uniform passenger arrival model

Many studies have suggested that because of the unaware of the real-time running time of buses, passengers could only arrive at the stop randomly. Sánchez-Martínez et al. (2016) validated that passenger arrives at bus stops follow uniform distribution [12]. Based on the analysis of the survey data, we assumed that the average arrival rate and alighting demand of passengers at each stop are approximately following table 3:

| S | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|---|----|----|----|----|----|----|----|----|----|----|
| $\alpha_k$ | 3% | 4% | 5% | 5% | 3% | 3% | 2% | 4% | 2% | 0  |
| $\beta_k$  | 0  | 0.1| 0.2| 0.3| 0.3| 0.3| 0.5| 0.6| 1  |

The total number of passengers waiting to board bus $i$ and stop $k$ consist of those who arrive during the inter-departure headway and those fail to board the previous bus $j$:

$$W_{i,k} = \alpha_k * H_{i,k} + t_{i,k}$$ (10)

The alighting demand is proportional to the number of passengers on-board:

$$P_{i,k} = L_{i,k-1} * \beta_k$$ (11)

The total number of people who can able to board the bus should not exceed the remaining capacity of the bus.

$$\bar{W}_{i,k} = \min\{W_{i,k}, C - (1 - \beta_k)\}$$ (12)

The number of passengers failing to board is the difference between waiting passengers and that of boarding ones:

$$l_{i,k} = W_{i,k} - \bar{W}_{i,k}$$ (13)

The number of on-board passengers of bus $i$ when it departs from stop $k$ is the original passengers plus the new boarding passengers, and then subtract the alighting passengers:

$$L_{i,k} = L_{i,k-1} + \bar{W}_{i,k} - P_{i,k}$$ (14)

3.3. The triangular passenger arrival models

The total number of passengers who are waiting for the bus $i$ at the stop $k$, which is equal to the number of new arriving passengers plus those fail to board the previous bus $j$:

$$W_{i,k} = \int_{j_{i,k}}^{(A_{i,k} + f_{i,k})/60} y' * dt + l_{i,k}$$ (15)

As for the alighting demand, it is similar with that of condition of the uniform passenger arrival model:

$$P_{i,k} = L_{i,k-1} * \beta_k$$ (16)

The actual number of passengers boarding the bus becomes:

$$\bar{W}_{i,k} = r * \bar{t}_{i,k}$$ (17)
Therefore, when either of the following two situations occurs, the bus will choose to depart from the stop. We assume that $\varepsilon_1$ and $\varepsilon_2$ are sufficiently small values.

Case 1: the bus $i$ can take in all of the waiting passengers.

$$|W_{i,k} - \bar{W}_{i,k}| = |W_{i,k} - r \times \bar{t}_{i,k}| < \varepsilon_1 \tag{18}$$

Case 2: the bus $i$ has no more room to accommodate extra passengers.

$$|W_{i,k} - (C - (\bar{t}_{i,k-1} \times (1 - \beta_0))))| = |r \times \bar{t}_{i,k} - (C - (\bar{t}_{i,k-1} \times (1 - \beta_0))))| < \varepsilon_2 \tag{19}$$

Then we can get the value of $\bar{t}_{i,k}$ from the equations above.

The number of passengers failing to board is the difference between waiting passengers and the actual boarding ones:

$$l_{i,k} = W_{i,k} - \bar{W}_{i,k} \tag{20}$$

Similarly, the number of on-board passengers of bus $i$ when it departs from stop $j$ comes to:

$$L_{i,k} = L_{i,k-1} + \bar{W}_{i,k} - P_{i,k} \tag{21}$$

4. SIMULATION AND RESULTS

A simple numerical simulation based on the data from the Xi’an 265 bus line was conducted in this section. We set a general bus route with 15 buses and 10 stops. The boarding rate and the bus capacity are set at $r = 15$ pax/min and $C = 50$ pax/veh respectively with the departure headway at $h = 3.5$ min. The link travel times $\bar{T}_{i,j}$ are drawn from a log-normal distribution with the natural logarithmic mean and standard deviation of 5.0 min and 0.5 min, i.e., $\ln (t) = N (5.0, 0.5^2)$.

**Figure 5.** Bus trajectories for cases of uniform passenger arrival model

**Figure 6.** Bus trajectories for cases of triangular passenger arrival model

Compared to figure 5, it is clear from figure 6 that the problem of bus bunching has been largely improved under the law of triangular passenger arrival, but there still exists a few buses bunching at the end of the bus route, such as bus 8 and bus 9, bus 11 and bus 12. Bartholdi III et al. (2012) mentioned that the more passengers waiting for the bus, the longer the bus would dwell at the stop, and this would result in the long headway between the successive buses [13]. So, it is reasonable that the excessive boarding and alighting passengers lengthen the dwell time of the bus 8 and bus 11 and cause the occurrence of bus bunching. Koppisetti et al. (2018) had also encountered this problem, and they concluded that the bus is easy to be caught up by trailing one if it dwells for too long at the stop [14].
And it is depicted in the figure 7 that, under the triangular passenger arrival model, the average passenger waiting time decreased by 24.4% on average. It is helpful for improving the reliability of bus.

5. CONCLUSIONS
This paper developed a bus propagation model and triangular passenger arrival model by explicitly taking the use of bus app into account to counteract the bus bunching. Classifying and analyzing the data from the research helped us come up with the triangular passenger arrival rule. Then a simple numerical simulation based on the data from the Xi’an 265 bus line was conducted. We tested the performance of different combinations of the bus operation model and passenger arrival model under the same settings in a Matlab simulation environment. We found that applying the triangular passenger arrival model among buses improved service regularity, and reduced the occurrence of the bus bunching, which would be useful to help the bus agencies design the whole bus system.

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