Dynamic confinement of jets by magneto-torsional oscillations

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Abstract

Many quasars and active galactic nuclei (AGN) appear in radio, optical, and X-ray maps, as bright nuclear sources from which emerge single or double long, thin jets (Thomson et al., 1993). When observed with high angular resolution these jets show structure with bright knots separated by relatively dark regions. High percentages of polarization, sometimes more than 50% in some objects, indicates the nonthermal nature of the radiation which is well explained as the synchrotron radiation of the relativistic electrons in an ordered magnetic field.

A strong collimation of jets is most probably connected with ordered magnetic fields. The mechanism of magnetic collimation, first suggested by Bisnovatyi-Kogan et al. (1969), was based on the initial charge separation, leading to creation of oscillating electrical current, which produces azimuthal magnetic field, preventing jet expansion and disappearance. Here we consider magnetic collimation, connected with torsional oscillations of a cylinder with elongated magnetic field. Instead of initial blobs with charge separation, we consider a cylinder with a periodically distributed initial rotation around the cylinder axis. The stabilizing azimuthal magnetic field is created here by torsional oscillations, where charge separation is not necessary. Approximate simplified model is developed. Ordinary differential equation is derived, and solved numerically, what gives a possibility to estimate quantitatively the range of parameters where jets may be stabilized by torsional oscillations.

Key words: magnetic fields; galaxies: jets.

1 Introduction

Objects of different scale and nature in the universe: from young and very old stars to galactic nuclei show existence of collimated outbursts - jets. Geometrical sizes of jets lay between parsecs and megaparsecs. The origin of jets is not well understood and only several qualitative mechanisms are proposed which are not justified by calculations. Theory of jets must give answers to three main...
questions: how jets are formed? how are they stabilized? how do they radiate? The last question is related to the problem of the origin if relativistic particles in outbursts from AGN, where synchrotron emission is observed. Relativistic particles, ejected from the central machine rapidly loose their energy so the problem arises of particle acceleration inside the jet, see review of Bisnovatyi-Kogan (1993).

It is convenient sometimes to investigate jets in a simple model of infinitely long circular cylinder, Chandrasekhar & Fermi (1953). The magnetic field in the collimated jets determines its direction, and the axial current may stabilize the jet’s elongated form at large distances from the source (e.g. in AGNs), (Bisnovatyi-Kogan et al., 1969), see also Istomin and Pariev (1996), Beskin (2005). When observed with high angular resolution these jets show a structure with bright knots separated by relatively dark regions (Thomson et al., 1993). High percentages of polarization, sometimes exceeding 50%, indicate the nonthermal nature of the radiation, which is well explained as synchrotron emission of the relativistic electrons in an ordered magnetic field. Estimates of the lifetime of these electrons, based on the observed luminosities and spectra, often give values much less than the kinematic ages $t_k = d/c$, where $d$ is the distance of the emitting point from the central source. There is a necessity of continuous re-acceleration of the electrons in the jets in order to explain the observations. The acceleration mechanism for electrons in extragalactic jets proposed by Bisnovatyi-Kogan & Lovelace (1995), considers that intense long-wavelength electromagnetic oscillations accompany a relativistic jet, and the electromagnetic wave amplitudes envisioned are sufficient to give in situ acceleration of electrons to the very high energies observed $> 10^{15}$ eV. It was assumed that jets are formed by a sequence of outbursts from the nucleus with considerable charge separation at the moment of the outburst. The direction of motion of the outbursts is determined by the large-scale magnetic field. When the emitted wave is strong enough it washes out the medium around and the density can become very small, consisting only of the accelerated particles. The action of the oscillating knot is similar to the action of a pulsar, considered as an inclined magnetic rotator. Both emit strong electromagnetic waves, which could effectively accelerate particles. The model of enhanced oscillations of the cylinder, and electromagnetic field around it, have been studied by Bisnovatyi-Kogan (2004).

Here we consider stabilization of a jet by pure magnetohydrodynamic mechanism, connected with torsional oscillations. Such type of oscillations in neutron stars had been considered by Bastrukov et al. (2002). We suggest that the matter in the jet is rotating, and different parts of the jet rotate in different directions. Such distribution of the rotational velocity produces azimuthal magnetic field, which prevents a disruption of the jet. The jet remains to be in a dynamical equilibrium, when it is representing a periodical, or quasiperiodical structure along the axis, and its radius is oscillating with time all along the axis. The space and time period of oscillations depend on the conditions at jet formation: the length scale, the amplitude of the rotational velocity, and the strength of the magnetic field. The time period of oscillations should be
obtained during construction of the dynamical model, what also should show at which input parameters may exist a long jet, stabilized by torsional oscillations. 2D nonstationary MHD calculations are needed, to solve the problem numerically. Here we construct a very simplified model of this phenomena, which, nonetheless, permits to confirm the reality of such stabilization, to estimate the range of parameters at which is takes place, and the connection between the time and space scales, magnetic field strength, and the amplitude of rotational velocity.

2 Axially symmetric MHD equations

Axially symmetric MHD equations at $\frac{\partial}{\partial t}$, for the perfect gas with an infinite conductivity, are written, in cylindric coordinates $(r, \phi, z)$, in the form (Landau and Lifshits, 1982; Bisnovatyi-Kogan, 2001)

\[
\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} - \frac{v_z^2}{r} = - \frac{1}{\rho} \frac{\partial P}{\partial r} - \frac{\partial \varphi G}{\partial r} + \frac{1}{\rho c} (j_\varphi B_z - j_z B_\varphi), \tag{1}
\]

\[
\frac{\partial v_\varphi}{\partial t} + v_r \frac{\partial v_\varphi}{\partial r} + v_z \frac{\partial v_\varphi}{\partial z} + \frac{v_z v_\varphi}{r} = \frac{1}{\rho c} (j_z B_\varphi - j_\varphi B_z), \tag{2}
\]

\[
\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} = - \frac{1}{\rho} \frac{\partial P}{\partial z} - \frac{\partial \varphi G}{\partial z} + \frac{1}{\rho c} (j_\varphi B_r - j_r B_\varphi), \tag{3}
\]

\[
\frac{\partial \rho}{\partial t} + v_r \frac{\partial \rho}{\partial r} + v_z \frac{\partial \rho}{\partial z} + \rho \left[ \frac{1}{r} \frac{\partial (rv_r)}{\partial r} + \frac{\partial v_z}{\partial z} \right] = 0, \tag{4}
\]

\[
\frac{\partial B_z}{\partial t} = - \frac{\partial}{\partial z} (v_z B_r - v_r B_z), \tag{5}
\]

\[
\frac{\partial B_\varphi}{\partial t} = \frac{\partial}{\partial z} (v_\varphi B_z - v_z B_\varphi) - \frac{\partial}{\partial r} (v_\varphi B_r - v_r B_\varphi), \tag{6}
\]

\[
\frac{\partial B_r}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} [r (v_z B_r - v_r B_z)], \tag{7}
\]

\[
\frac{1}{r} \frac{\partial}{\partial r} (r B_r) + \frac{\partial B_z}{\partial z} = 0, \tag{8}
\]

\[
j_r = - \frac{c}{4\pi} \frac{\partial B_\varphi}{\partial z}, \tag{9}
\]

\[
j_\varphi = \frac{c}{4\pi} \left( \frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r} \right), \tag{10}
\]

\[
j_z = \frac{c}{4\pi r} \frac{\partial}{\partial r} (r B_\varphi), \tag{11}
\]
\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \varphi_G}{\partial r} \right) + \frac{\partial^2 \varphi_G}{\partial z^2} = 4\pi G \rho, \quad (12)
\]
\[
\frac{\partial E}{\partial t} + v_r \frac{\partial E}{\partial r} + v_z \frac{\partial E}{\partial z} + \frac{P}{\rho} \left[ \frac{1}{r} (rv_r) + \frac{\partial v_z}{\partial z} \right] = -f \quad (13)
\]
\[
P = P(\rho, T), \quad E = E(\rho, T), \quad f = f(\rho, T), \quad (14)
\]

Here \( \mathbf{v} = (v_r, v_\varphi, v_z) \) is the velocity vector, \( \mathbf{B} = (B_r, B_\varphi, B_z) \) is the magnetic field vector, \( \mathbf{j} = (j_r, j_\varphi, j_z) \) is the vector of the electrical current, \( \rho, P, E, \) are density, pressure, and internal energy, respectively, \( f \) is the cooling function, due to photons or neutrino.

We consider a long cylinder with a magnetic field directed along its axis. This cylinder will expand unlimitedly under the action of pressure and magnetic forces, so no confinement will be reached. The limitation of the radius of this cylinder could be possible in dynamic state, when the whole cylinder undergoes magneto-torsional oscillations. Such oscillations produce toroidal field, which prevent a radial expansion. There is a competition between the induced toroidal field, compressing the cylinder in radial direction, and gas pressure, together with the field along the cylinder axis (poloidal), tending to increase its radius. During magneto-torsional oscillations there are phases, when either compression or expansion forces prevail, and, depending on the input parameters, we may expect three kinds of a behavior of such cylinder.

1. The oscillation amplitude is low, so the cylinder suffers unlimited expansion (no confinement)
2. The oscillation amplitude is too high, so the pinch action of the toroidal field destroys the cylinder, and leads to formation of separated blobs.
3. The oscillation amplitude is moderate, so the cylinder survives for an unlimited time, and its parameters (radius, density, magnetic field etc.) change periodically, or quasi-periodically in time.

Solution of MHD equations \((1)-(14)\) could give, in principle, the answer about a correctness of the above scenario. It is reasonable nevertheless to try to find a simple approximate way for obtaining a qualitative answer, and to make a rough estimation of parameters leading to different regimes.

### 3 Profiling in axially symmetric MHD equations

We'll try to simplify the system of equations in such a way, that the resulting system should contain the most important property of the dynamical competition between different forces, to check the possibility of the dynamical confinement. We use for this purpose a profiling procedure. Let us neglect the gravity in the direction of the cylinder axis \( (z) \), and approximate density by a function \( \rho(t, z) \), suggesting a uniform density along the radius, cylinder density. The components of the velocity and magnetic field are approximated as
\[ v_r = r \alpha(t,z), \quad v_\varphi = r \Omega(t,z), \quad v_z = 0; \quad (15) \]

\[ B_r = r h_r(t,z), \quad B_\varphi = r h_\varphi(t,z), \quad B_z = B_z(t,z). \quad (16) \]

In this case the current components (9)-(11) are written as

\[ j_r = -cr \frac{\partial h_\varphi}{\partial z}, \quad j_\varphi = cr \frac{\partial h_r}{\partial z}, \quad j_z = \frac{ch_\varphi}{2\pi}. \quad (17) \]

After neglecting velocity \( v_z \) along the axis, we should omit the corresponding Euler equation (3), and the radial pressure gradient is approximated by the linear function

\[ \frac{\partial P}{\partial r} = \lambda \frac{P}{R^2}, \quad (18) \]

where the constant \( \lambda \sim 1 \) is connected with the equation of state, \( P(t,z) \) is the pressure, \( R(t,z) \) is the radius of the cylinder. In the subsequent consideration we consider an adiabatic case, where the polytropic equation of state \( P = K \rho^\gamma \) is considered instead of the energy equation (13). Neglecting the \( z \) derivatives in the Poisson equation (12), we obtain \( \varphi_G = \pi G \rho r^2 \). Substituting (15)-(18) into the original system of the equations, we obtain for the profiling functions the following equations

\[ \frac{\partial a}{\partial t} + a^2 - \Omega^2 = \lambda \frac{P}{\rho R^2} - 2\pi G \rho + \frac{1}{4\pi \rho} \left( B_z \frac{\partial h_r}{\partial z} - 2h_\varphi^2 \right), \quad (19) \]

\[ \frac{\partial \Omega}{\partial t} + 2a \Omega = \frac{1}{4\pi \rho} \left( B_z \frac{\partial h_\varphi}{\partial z} + 2h_r h_\varphi \right), \quad (20) \]

\[ \frac{\partial h_r}{\partial t} = \frac{\partial (a B_z)}{\partial z}, \quad (21) \]

\[ \frac{\partial h_\varphi}{\partial t} = \frac{\partial (\Omega B_z)}{\partial z} - 2(a h_\varphi - \Omega h_r), \quad (22) \]

\[ \frac{\partial B_z}{\partial t} = -2a B_z, \quad (23) \]

\[ \frac{\partial \rho}{\partial t} = -2a \rho, \quad (24) \]

\[ \frac{\partial R}{\partial t} = a R. \quad (25) \]

It follows from (23)-(25) relations, representing conservation of mass, and magnetic flux equivalent to freezing condition

\[ \rho R^2 = C_m(z), \quad B_z R^2 = C_b(z), \quad B_z = \frac{C_b(z)}{C_m(z)} \rho. \quad (26) \]
In our subsequent consideration the arbitrary functions will be taken as constants: $C_m(z) = C_m, C_b(z) = C_b$. The algebraic relations (20) may be used instead of any two equations from (23)-(25).

4 Equilibrium configuration and linear oscillations

To check the properties of the approximate system (19)-(25), we consider linear oscillations of the equilibrium, infinite, self-gravitating cylinder with uniform magnetic field and rotation along its axis: $\frac{dB_x}{dz} = 0$, $\frac{dh_r}{dz} = 0$. In equilibrium state (index "0") we have

$$ a_0 = h_{z0} = h_{r0} = 0, \quad \rho_0 = \text{const}, \quad \Omega_0 = \text{const}, \quad \Omega_0^2 = 2\pi G \rho_0 - \frac{\lambda P_0}{\rho R_0^2}. \quad (27) $$

Linearizing equations (19)- (22) around the equilibrium state (27), we obtain

$$ \frac{\partial a}{\partial t} - 2\Omega_0 \Omega = \lambda \frac{P_0}{\rho_0 R_0^2} \left( P - \frac{\rho}{\rho_0} - 2 \frac{R}{R_0} \right) - 2\pi G \rho + \frac{B_{z0}}{4\pi \rho_0} \frac{\partial h_r}{\partial z}, \quad (28) $$

$$ \frac{\partial \Omega}{\partial t} + 2a \Omega_0 = \frac{B_{z0}}{4\pi \rho_0} \frac{\partial h_x}{\partial z}, \quad (29) $$

$$ \frac{\partial h_r}{\partial t} = B_{z0} \frac{\partial (a)}{\partial z}, \quad (30) $$

$$ \frac{\partial h_x}{\partial t} = \Omega_0 \frac{\partial B_z}{\partial z} + B_{z0} \frac{\partial \Omega}{\partial z} + 2\Omega_0 h_r. \quad (31) $$

Linearizing of (23)-(25), using the polytropic equation of state $P = K \rho^\gamma$, and looking for a solution in the form $\sim \exp i(kz - \omega t)$ we obtain

$$ \frac{R}{R_0} = \frac{a}{\omega}, \quad \rho = -2 \frac{R}{R_0} = -2 \frac{a}{\omega}, \quad P = \gamma \rho \frac{B_z}{B_{z0}}, \quad \Omega = -2 \frac{R}{R_0} = -2i \frac{\omega}{a}. \quad (29) $$

Here small perturbation values are taken without "0". From the last two equations (28) we obtain using (20)

$$ h_r = -B_{z0} \frac{ak}{\omega}, \quad h_x = -B_{z0} \frac{\Omega k}{\omega}. \quad (30) $$

First two equations (28) give after using (29), (30)

$$ \left[ 1 - \frac{k^2 V_0^2}{\omega^2} - \frac{V_0^2}{R_0^2 \omega^2} \left( 1 - \frac{1}{\gamma} \right) + 2 \frac{\Omega_0^2}{\omega^2} \right] a - 2i \frac{\Omega_0}{\omega} \Omega = 0 $$

$$ 2i \frac{\Omega_0}{\omega} a + \left( 1 - \frac{k^2 V_0^2}{\omega^2} \right) \Omega = 0. \quad (31) $$
Here we have introduced unperturbed sound ($V_{s0}$), and Alfven ($V_{A0}$) velocities, determined as

$$V_{s0}^2 = \frac{\gamma P_0}{\rho_0}, \quad V_{A0}^2 = \frac{B_{z0}^2}{4\pi \rho_0}.$$  (32)

Equations (31) lead to following dispersion equation

$$\omega^4 - 2\left[k^2 V_{A0}^2 + \Omega_0^2 + \lambda \frac{V_{A0}^2}{R_0^2} \left(1 - \frac{1}{\gamma}\right)\right] \omega^2$$

$$+ k^2 V_{A0}^2 \left[k^2 V_{A0}^2 - 2\Omega_0^2 + 2\lambda \frac{V_{A0}^2}{R_0^2} \left(1 - \frac{1}{\gamma}\right)\right] = 0.$$  (33)

Let us consider several particular cases.

1. Non-rotating, non-magnetized cylinder, $V_{A0} = \Omega_0 = 0$. It follows from (33)

$$\omega^2 = 2\lambda \frac{V_{A0}^2}{R_0^2} \left(1 - \frac{1}{\gamma}\right).$$  (34)

Equation (34) describes the only possible mode, remaining after fixing $v_z = 0$, and uniform density over the radius. It corresponds to homologous mode of perturbation, where only radius and density (pressure) are oscillating. The frequency (34) becomes zero at $\gamma = 1$ (isotherm). This degeneration is connected with the property of self-gravitating cylinder, which equilibrium is neutral (takes place at any radius) for the isothermal equation of state. This degeneration is equivalent to the well-known case at $\gamma = 4/3$ for a spherical star (Chandrasekhar, 1939). While we are not interested in study of homologous oscillations, we’ll take $\gamma = 1$ in all father consideration. We have than from (33) the dispersion equation

$$\omega^4 - 2(k^2 V_{A0}^2 + \Omega_0^2) \omega^2 + k^2 V_{A0}^2 (k^2 V_{A0}^2 - 2\Omega_0^2) = 0,$$  (35)

which solution is written as

$$\omega^2 = k^2 V_{A0}^2 + \Omega_0^2 \pm \Omega_0 \sqrt{4k^2 V_{A0}^2 + \Omega_0^2}.$$  (36)

2. Rotating non-magnetized cylinder, $V_{A0} = 0$, $\Omega_0 \neq 0$. Here the solution of dispersion equation $\omega^2 = \Omega_0^2 \pm \Omega_0^2$ describes the trivial mode $\omega^2 = 0$, connected with pure rotational perturbations, and radial oscillations due to an action of the centrifugal force $\omega^2 = 2\Omega_0^2$ (Fridman, Polyachenko, 1985).

3. Non-rotating magnetized cylinder, $V_{A0} \neq 0$, $\Omega_0 = 0$. The solution of the dispersion equation $\omega^2 = k^2 V_{A0}^2$ describes two different types of waves, propagating with Alfven velocity. To find out the nature of these oscillations note that (31) at $\gamma = 1$, $\Omega_0 = 0$ reduces to

$$\left(1 - \frac{k^2 V_{A0}^2}{\omega^2}\right) a = 0, \quad \left(1 - \frac{k^2 V_{A0}^2}{\omega^2}\right) \Omega = 0.$$  (37)
The first type of wave corresponds to perturbation only of the radial velocity, \(a \neq 0, \Omega = 0\). It follows that from (3), that \(h_r \neq 0, h_\varphi = 0\). This solution corresponds to the Alfvén wave along the axis where only radius is perturbed, and no rotation appears. The second type of wave describes perturbations of angular velocity at constant radius, \(a = 0, \Omega \neq 0\), where \(h_r = 0, h_\varphi \neq 0\). This wave corresponds to a pure torsional Alfvén wave along the cylinder axis. To obtain a standing wave (\(e_s\)) we need a combination of two waves running in opposite directions (\(e_{r\pm}\)).

\[
\begin{align*}
a[e_{r+} \cos (kz - \omega t) + e_{r-} \cos (kr + \omega t)] &= 2a e_s \cos kr \cos \omega t, \\
h_r[e_{r+} \cos (kz - \omega t) + e_{r-} \cos (kr + \omega t)] &= -Bz_0 \frac{ak}{\omega} [e_{r+} \cos (kz - \omega t) - e_{r-} \cos (kr + \omega t)] \\
&= -Bz_0 \frac{ak}{\omega} 2e_s \sin kr \sin \omega t = 2h_r e_s \sin kr \sin \omega t.
\end{align*}
\]

(38)

It is clear that radial velocity \(a\) and radial magnetic field \(h_r\) are oscillating in opposite phases. For the torsional wave we have similar relations:

\[
\begin{align*}
\Omega[e_{r+} \cos (kz - \omega t) + e_{r-} \cos (kr + \omega t)] &= 2\Omega e_s \cos kr \cos \omega t, \\
h_\varphi[e_{r+} \cos (kz - \omega t) + e_{r-} \cos (kr + \omega t)] &= -Bz_0 \frac{\Omega k}{\omega} [e_{r+} \cos (kz - \omega t) - e_{r-} \cos (kr + \omega t)] \\
&= -Bz_0 \frac{\Omega k}{\omega} 2e_s \sin kr \sin \omega t = 2h_\varphi e_s \sin kr \sin \omega t.
\end{align*}
\]

(39)

The rotational velocity and azimuthal magnetic field are also oscillating in opposite phases. In reality we may have a mixture of these two degenerate modes where all perturbations are not zero, what is always takes place in nonlinear case. We see here that the approximate system of equations describes correctly small perturbations, connected with radial and torsional modes, so we expect that these modes will be described correctly in general nonlinear case.

5 Farther simplification: reducing the problem to ordinary differential equation

While in the relativistic jet the self-gravitating force is expected to be much less than the magnetic and pressure forces, we neglect gravity in the subsequent consideration. Without gravity the equilibrium static state of the cylinder does not exist. We need to solve numerically the system of nonlinear equations (19)-(22), (25), (26) to check the possibility of the existence of a cylinder, which radius remains to be finite due to torsional oscillations (dynamic confinement).

We’ll try instead to reduce the system to ordinary equations, making additional simplifications. Let us consider axially symmetric jet moving along z-axis.
with a constant bulk motion velocity, in the comoving coordinate frame. In the jet which confinement is reached due to standing magneto-torsional oscillations, there are points along \( z \)-axis where rotational velocity always remains zero in this frame. Let us take \( \Omega = 0 \) in the plane \( z = 0 \). Let us consider standing wave torsional oscillations with the space period along \( z \) axis equal to \( z_0 \). Then nodes with \( \Omega = 0 \) are situated at \( z = \pm n \frac{z_0}{2} \), \( n = 0, 1, 2, \ldots \). Let us write the equations, describing the cylinder behavior in the plane \( z = 0 \), where \( \Omega = 0 \). We have than equations in the plane \( z = 0 \) as

\[
\frac{d\tilde{\rho}}{dt} + \tilde{\rho}^2 = \frac{K}{R^2} + \frac{C_b}{4\pi C_m} \left( \frac{\partial \tilde{h}_r}{\partial z} \right)_{z=0} - \frac{\tilde{h}_r^2 R^2}{2\pi C_m}, \tag{40}
\]

from the equation (19);

\[
\frac{C_b}{4\pi C_m} \left( \frac{\partial \tilde{h}_r}{\partial z} \right)_{z=0} + \frac{\tilde{h}_r \tilde{h}_r R^2}{2\pi C_m} = 0, \tag{41}
\]

from the equation (20);

\[
\frac{d\tilde{h}_r}{dt} = C_b \left( \frac{\partial (a/R^2)}{\partial z} \right)_{z=0}, \tag{42}
\]

from the equation (21);

\[
\frac{d\tilde{h}_\varphi}{dt} = C_b \left( \frac{\partial (\Omega/R^2)}{\partial z} \right)_{z=0} - 2\tilde{a}\tilde{h}_\varphi, \tag{43}
\]

from the equation (22);

\[
\frac{d\tilde{R}}{dt} = \tilde{a}\tilde{R}. \tag{44}
\]

from the equation (25). The integrals of motion (26) in the plane \( z = 0 \) are written as

\[
\tilde{\rho} \tilde{R}^2 = C_m, \quad \tilde{B}_z \tilde{R}^2 = C_b, \quad \tilde{B}_z = \frac{C_b}{C_m} \tilde{\rho}.
\]

Initial conditions for the system (40) - (45) are

\[
\tilde{R} = R_0, \quad \tilde{\rho} = \rho_0 = \frac{C_m}{R_0^2}, \quad \tilde{B}_z = \frac{C_b}{R_0^2}, \quad \tilde{a} = \tilde{h}_r = \tilde{h}_\varphi = 0 \text{ at } t = 0.
\]

In (40) - (45) we have used relations

\[
\tilde{\rho} = \rho_0 \frac{R_0^2}{R^2}, \quad \tilde{B}_z = \rho_0 \frac{C_b R_0^2}{C_m R^2}, \tag{47}
\]

valid for any time. If the cylinder rotational velocity is antisymmetric relative to the plane \( z = 0, \Omega = 0 \), and cylinder density distribution is symmetric relative to this plane, then we have extremum (maximum) of the azimuthal magnetic field
\( h_0 \), with \( \left( \frac{\partial h_\varphi}{\partial z} \right)_{z=0} = 0 \), and zero value of \( \tilde{h}_\varphi = 0 \), which reaches an extremum (minimum) in this plane with \( \left( \frac{\partial h_\varphi}{\partial z} \right)_{z=0} = 0 \). The product \( a\rho \) also reaches an extremum in the plane \( z = 0 \), so that \( \left( \frac{\partial (a/R^2)}{\partial z} \right)_{z=0} = 0 \). The term with \( z \) derivative in the equation (43) is not equal to zero, and changes periodically during the torsional oscillations. We substitute approximately the derivative \( d/dz \) by the ratio \( 1/z_0 \), where \( z_0 \) is the space period of the torsional oscillations along \( z \) axis. While \( \Omega = 0 \) in the plane \( z = 0 \), its derivative along \( z \) is changing periodically with an amplitude \( \Omega_0 \), and frequency \( \omega \), which should be found from the solution of the problem. We approximate therefore

\[
\left( \frac{\partial (\Omega/R^2)}{\partial z} \right)_{z=0} = \frac{\Omega_0}{z_0R^2} \cos \omega t. \quad (48)
\]

Finally, we have from (41), (43), (44) the following approximate system of equations, describing the non-linear torsional oscillations of the cylinder at given \( z_0 \) and \( \Omega_0 \).

\[
\begin{align*}
\frac{d\tilde{a}}{dt} + \tilde{a}^2 &= \frac{K}{R^2} - \frac{\tilde{h}_\varphi^2 R^2}{2\pi C_m}, \\
\frac{d\tilde{h}_\varphi}{dt} &= C_b \frac{\Omega_0}{z_0 R^2} \cos \omega t - 2\tilde{a} \tilde{h}_\varphi, \\
\frac{d\tilde{R}}{dt} &= \tilde{a} \tilde{R}.
\end{align*}
\] (49)

The combination of last two equations gives

\[
\frac{d(\tilde{h}_\varphi R^2)}{dt} = C_b \frac{\Omega_0}{z_0 \omega} \cos \omega t, \quad (50)
\]

with the solution, satisfying initial condition (46), in the form

\[
\tilde{h}_\varphi R^2 = C_b \frac{\Omega_0}{z_0 \omega} \sin \omega t. \quad (51)
\]

With account of (51) the first and third equations in (49) give the equation

\[
R \frac{d(\tilde{a} R)}{dt} = K - \left( \frac{C_b \Omega_0}{z_0 \omega} \right)^2 \frac{\sin^2 \omega t}{2\pi C_m}. \quad (52)
\]

Two differential equations, (52) and the third equation (49) determine the behavior of the cylinder during magneto-torsional oscillations. Solutions where the radius does not go to infinity with time, determine a dynamically confined cylinder. Formation of blobs occurs, when the radius tends to zero. Long dynamically confined cylinder exist when its radius is changing with time between two finite values.
6 Numerical solution

Introduce non-dimensional variables

\[ \tau = \omega t, \quad y = \frac{\tilde{R}}{R_0}, \quad z = \frac{a \tilde{R}}{a_0 R_0}, \quad a_0 = \frac{K}{\omega R_0^2} = \omega, \quad R_0 = \frac{\sqrt{K}}{\omega}, \quad (53) \]

in which differential equations have a form

\[ \frac{dy}{d\tau} = z, \quad \frac{dz}{d\tau} = \frac{1}{y}(1 - D \sin^2 \tau); \quad y(0) = 1, \quad z = 0 \text{ at } \tau = 0. \quad (54) \]

Therefore, the problem is reduced to a system with only two non-dimensional parameters \( D = \frac{1}{2\pi KC_m} \left( \frac{C_0 \Omega_0}{2\omega} \right)^2 \), and \( y(0) \), and the second one is taken equal to unity in farther consideration. All qualitatively different solutions are reproduced inside this restricted set of parameters. Solution of this nonlinear system changes qualitatively with changing of the parameter \( D \).

The solution of this system was obtained numerically for \( D = 2, 2.1, 2.11, 2.15, 2.2, 2.25, 2.28, 2.4, 2.5, 2.6, 2.9, 3.0 \). Roughly the solutions may be divided into 3 groups.

1. At \( D \leq 2 \) there is no confinement, and radius grows to infinity after several low-amplitude oscillations (see Fig.1).

2. With growing of \( D \) the amplitude of oscillations increase, and at \( D = 2.1 \) radius is not growing to infinity, but is oscillating around some average value, forming rather complicated curve (Figs. 2-4).

3. At \( D = 2.28 \) and larger the radius finally goes to zero with time, but with different behavior, depending on \( D \). At \( D \) between 2.28 and 2.9 the dependence of the radius \( y \) with time may be very complicated, consisting of low-amplitude and large-amplitude oscillations, which finally lead to zero. The time at which radius becomes zero depends on \( D \) in rather peculiar way, and may happen at \( \tau \leq 100 \), like at \( D = 2.4, 2.6 \) (Figs. 7,10); or goes trough very large radius, and returned back to zero value at very large time \( \tau \sim 10^7 \text{ at } D = 2.5 \) (Fig. 9). Starting from \( D = 3 \) and larger the solution becomes very simple, and radius goes to zero at \( \tau < 2.5 \) (Fig. 13), before the right side of the second equation returned to the positive value. The results of numerical solution are represented in Figs. 1-13.

7 Discussion

Let us consider, a jet which has an equation of state in the form \( P = K \rho = v_s^2 \rho \), \( v_s^2 \leq c^2/3 \), \( v_s \) is the sound speed in the matter. For ultrarelativistic pair-plasma we have \( P = c^2/3 \). The non-dimensional parameter \( D \), as a function of the characteristic radius \( R_0 \), periodic length \( z_0 \) along \( z \) axis, initial density \( \rho_0 \), and magnetic field \( B_{z0}, \Omega_0 \) and \( \omega \) is written in the form

\[ D = \frac{1}{2\pi \rho_0} \frac{B_{z0}^2 R_0^2 \Omega_0^2}{z_0^2 \omega^2 v_s^2}. \quad (55) \]
Figure 1: Time dependence of non-dimensional radius $y$ (upper curve), and non-dimensional velocity $z$ (lower curve), for $D = 2.0$.

Figure 2: Time dependence of non-dimensional radius $y$ (upper curve), and non-dimensional velocity $z$ (lower curve), for $D = 2.1$. 

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Figure 3: Time dependence of non-dimensional radius $y$ (upper curve), and non-dimensional velocity $z$ (lower curve), for $D_1 = 2.1$ during a long time period.

Figure 4: Time dependence of non-dimensional radius $y$ (upper curve), and non-dimensional velocity $z$ (lower curve), for $D_2 = 2.25$, during a long time period.
Figure 5: Time dependence of non-dimensional radius $y$ (upper curve), and non-dimensional velocity $z$ (lower curve), for $D = 2.28$.

Figure 6: Time dependence of non-dimensional radius $y$ (upper curve), and non-dimensional velocity $z$ (lower curve), for $D = 2.28$, during a long time period.
The amplitude of oscillations $\Omega$, and $\omega$ should be found from the solution of the nonlinear system (19)-(25), together with determination of the interval of values of $D$ at which confinement happens. In the approximate system (54) only one parameter $D$ characterizes different regimes, what for given values of $R_0$, $z_0$, $\rho_0$, and $B_{z0}$ determines a function $\Omega_0(D, \omega)$ for the collimated jet. To find approximately a self-consistent model with $\Omega_0, \omega(D)$ we may use the solution of linearized equations (19)-(25) with $\omega = kV_A$ from (37). The frequency of non-linear oscillations is smaller, and we may write

$$\omega = \alpha_n k V_A, \quad \alpha_n < 1, \quad k = \frac{2\pi}{z_0},$$

so that

$$\omega^2 = \alpha_n^2 k^2 V_A^2 = \frac{\alpha_n^2 \pi B_{z0}^2}{\rho_0 z_0^2}.$$  \hspace{1cm} (56)

Using it in (55), with account of (53), we obtain

$$\Omega^2 R_0^2 = 2\pi^2 D \alpha_n^2 v_s^2 < c^2, \quad R_0^2 = \frac{K}{\omega^2} = \frac{z_0^2 \rho_0 v_s^2}{\alpha_n^2 \pi B_{z0}^2}. \hspace{1cm} (57)$$

On the edge of the cylinder the rotational velocity cannot exceed the light velocity, so the solution with initial conditions in (54), corresponding to $y_0 = 1$, has a physical sense only at $v_s^2 < \frac{\alpha_n^2}{2\pi^2 D \alpha_n^2} \approx \frac{\alpha_n^2}{4\pi^2 D \alpha_n^2}$. Taking $\alpha_n^2 = 0.1$ for a strongly non-linear oscillations we obtain a very moderate restriction $v_s^2 < \frac{c^2}{4}$. While in the intermediate collimation regime the outer tangential velocity is not changing significantly, this restriction would be enough also for the whole period of the time. To have the sound velocity not exceeding $c/2$, the jet should contain baryons, which density $\rho_0$ cannot be very small, and its input in the total density in the jet should be larger than about 30%.

The confinement by torsional oscillations starts at $D = 2.1$, and at $D \geq 2.28$ the jet is divided into separate blobs according to Figs.9-17. So, the confinement by magneto-torsional oscillations can be realized in the physically available situation, what is the main conclusion of this paper.

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Figure 7: Time dependence of non-dimensional radius $y$ (upper curve), and non-dimensional velocity $z$ (lower curve), for $D = 2.4$.

Figure 8: Time dependence of non-dimensional radius $y$ (upper curve), and non-dimensional velocity $z$ (lower curve), for $D = 2.5$. 

$z' \lambda$
Figure 9: Time dependence of non-dimensional radius $y$ (upper curve), and non-dimensional velocity $z$ (lower curve), for $D = 2.5$, during a long time period.

Figure 10: Time dependence of non-dimensional radius $y$ (upper curve), and non-dimensional velocity $z$ (lower curve), for $D = 2.6$. 
Figure 11: Time dependence of non-dimensional radius $y$ (upper curve), and non-dimensional velocity $z$ (lower curve), for $D = 2.9$.

Figure 12: Time dependence of non-dimensional radius $y$ (upper curve), and non-dimensional velocity $z$ (lower curve), for $D = 2.9$ during a long time period.
Figure 13: Time dependence of non-dimensional radius $y$ (upper curve), and non-dimensional velocity $z$ (lower curve), for $D = 3.0$. 