Search for a Stochastic Gravitational-wave Background with Torsion-bar Antennas

Ayaka Shoda\textsuperscript{1}, Masaki Ando\textsuperscript{2}, Kenshi Okada\textsuperscript{1}, Koji Ishidoshiro\textsuperscript{3}, Wataru Kokuyama\textsuperscript{1}, Yoichi Aso\textsuperscript{1}, Kimio Tsubono\textsuperscript{1}

\textsuperscript{1}Department of Physics, The University of Tokyo, Bunkyo-ku, Tokyo 113-0033, Japan
\textsuperscript{2}Department of Physics, Kyoto University, Kitashirakawa Oiwake-cho, Sakyo-ku, Kyoto 606-8502, Japan
\textsuperscript{3}Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan

E-mail: shoda@granite.phys.s.u-tokyo.ac.jp

Abstract. We performed a simultaneous observational run with prototypes of Torsion-bar Antenna (TOBA) and searched for a stochastic gravitational waves (GW) background. TOBA is a new type of GW detector which measures differential rotation of two test-mass bars caused by tidal force from GWs. It fundamentally has a good sensitivity at lower frequencies, such as 0.1 – 1.0 Hz. The prototype has a 20-cm test mass bar which is levitated by the pinning effect of a superconductor. The data was taken from 1:00 am to 10:00 am on March 11th 2011, at Tokyo and Kyoto in Japan. As a result, we did not detect a stochastic GW background with false alarm rate of 5 %, and set an upper limit on a stochastic GW background. Our 95 % confidence upper limit is $\Omega_{gw} h^2_0 < 1.2 \times 10^{19}$ at 0.06 – 0.9 Hz, where $\Omega_{gw}$ is the GW energy density per logarithmic frequency interval in units of the critical density and $h_0$ is the Hubble constant per 100 km/sec/Mpc. We had established the simultaneous observation and the analysis pipeline with two TOBAs, and set an upper limit at a wider frequency band.

1. Introduction

Recently, many gravitational wave (GW) detectors have been developed. Though GWs have not been detected yet, once detected, they will provide us new aspects of universe. Especially, a stochastic GW background is a cosmologically interesting target. It will provide us much information about phenomena occurred in the universe shortly after its birth, such as the inflation. Therefore, it has been eagerly searched for with a number of experiments.

The ground-based laser interferometer GW detectors, LIGO \cite{1} and Virgo \cite{2}, have set a 95 % confidence upper limit on a stochastic GW background of in the frequency band around 100 Hz \cite{3}. A pair of synchronous recycling interferometers set an upper limit at 100 MHz \cite{4}. The cryogenic bar detectors, Explorer and Nautilus, had done at 907 Hz \cite{5}. Also, the ALLEGRO resonant-bar detector and LIGO Livingston had set an upper limit at 917 Hz \cite{6}. At much lower frequency band, the Doppler tracking with the Cassini spacecraft at $10^{-6} - 10^{-3}$ Hz \cite{7}, the pulsar timing by PSR B1855+09 at $10^{-9} - 10^{-7}$ Hz \cite{8}, and measurement of cosmic microwave background has established an upper limit at $10^{-18} - 10^{-16}$ Hz \cite{8}\cite{9}.

So far, no upper limit had been set at 0.01 – 1 Hz. The space GW detectors, LISA \cite{10} and DECIGO \cite{11}, are planned to be launched and explore this frequency band. However, it is very
challenging to launch more than three satellites and construct laser interferometer between the satellites.

Then, we designed a novel GW detector, "Torsion-bar Antenna", called TOBA [12]. It is fundamentally sensitive to GWs below 1 Hz even on the ground. We have already developed prototypes of TOBA with one 20-cm long test mass bar. By a single prototype TOBA, the first upper limit on a stochastic GW background had been set at 0.2 Hz [13].

However, it is difficult to judge whether a stochastic GW background is present or not by a single detector. We can mention about the presence of a stochastic GW background only if the signal exceeds the expected noise level. Therefore, it is necessary to take cross-correlation of multiple detectors in order to search a stochastic GW background. In this paper, we report on the simultaneous observational run and the results obtained by the cross-correlation analysis with two prototypes of TOBA placed at Tokyo and Kyoto in Japan. Here we demonstrated the two-detector correlation with TOBA and evaluate the its capabilities for low-frequency GW observations.

2. Torsion-bar Antenna

2.1. Principle of Torsion-bar Antenna

TOBA has two test mass bars suspended at their centers. (See figure 1.) When they are arranged orthogonal to each other, the bars rotates differentially by the tidal force caused by GWs. Their differential angular fluctuation is monitored by laser interferometric sensors.

![Figure 1. A conceptual drawing of TOBA.](image)

Let us consider a test mass bar is arranged along x axis, and suspended so that they rotate around the z axis. (Figure 1.) When the GW come along z axis, the bars are affected by the tidal force from GWs. The angular fluctuation $\theta$ obeys the equation of motion:

$$I \ddot{\theta} + \gamma \dot{\theta} + \kappa \theta = F_{gw}(t)$$

where $I$ is the moment of inertia of the test mass, $\gamma$ and $\kappa$ are the damping constant and the spring constant around z axis, and $F_{gw}$ is the torque caused by the GW. Assuming that the antenna is smaller enough than the wavelength of the GW, and that GW has $\times$-polarization, the angular fluctuation is simply represented as

$$\tilde{\theta}(f) = \frac{q_x}{2I} \tilde{h}_\times(f)$$

above the resonant frequency of the suspension $f_0 = 1/(2\pi)\sqrt{\kappa/I}$, where $q_x = q_{12} = q_{21}$ is the dynamic quadrupole moment of the test mass, and $h_\times$ is the amplitude of the $\times$-polarized GW [12].
TOBA and laser interferometric GW antennas has sensitivity to GWs above the resonant frequency. The point is that the resonant frequency in the rotational degree of freedom is on the order of a few mHz while the resonant frequency of the pendulum is around 1 Hz. This is the reason that a TOBA is fundamentally sensitive to GWs below 1 Hz even on the ground.

2.2. Prototype Torsion-bar Antenna
We have developed prototypes of TOBA. Pictures of the prototype TOBA is shown in figure 2.

![Prototype Torsion-bar Antenna](image)

Figure 2. Pictures of the prototype TOBA detector. The left is the picture of the all experimental system. The Laser beam comes from the left hand side of the vacuum tank and composes the interferometer in order to monitor the rotation of the test mass bar. At the top of the vacuum tank, a superconductor is set for the magnetic levitation. The right picture is of a test mass bar. It has a magnet whose magnetic field is pinned by the superconductor and mirrors for the interferometer.

It has a 20-cm test mass bar, which is levitated by the flux pinning effect of a superconductor. This magnetic suspension let the test mass free to rotate while it provide large suspension force. In this case, its rotational resonance frequency is about 5 mHz. The rotation of the test mass bar is read by a laser Michelson interferometer. A laser source has a wavelength of 1064 nm and an output power of 40 mW. The beam goes to the mirrors attached at the both ends of the test mass, thus the differential change in the two beam path lengths is proportional to the rotation angle. The test mass is controlled by coil-magnet actuators. To compose the actuators, the test mass has two magnets of $\phi 1$ mm at the each end of the bar.

We have almost the same prototype TOBAs at University of Tokyo and Kyoto University. We performed simultaneous observational run with them.

3. Observation
The observation is performed from 1:00 am – 10:00 am at March 11, 2011 at Tokyo and Kyoto in Japan. The latitude and longitude of Tokyo is $35.71^\circ$ N and $139.76^\circ$ E, and of Kyoto is $35.03^\circ$ N and $135.78^\circ$ E. The Tokyo site is about 370 km far from the Kyoto site. (See figure 3) The both test mass bars are oriented to the north-to-south direction. An overlap reduction function which represents the difference of the response to GWs between two detectors is shown in figure 4 [14]. Note that the overlap reduction function of two TOBAs is calculated in the same way as two interferometers since the antenna pattern of TOBA is the same. At lower frequency than 1 Hz, the overlap reduction function is almost one, which means that the two detectors respond to GWs almost equally.

We recorded pulse per second (PPS) signals from GPS as well as feedback and error signals during the observation. The time shift between the two cites are adjusted after the observation.
Figure 3. The location of Tokyo and Kyoto in Japan. Kyoto cite is about 370 km far west from Tokyo cite.

Figure 4. The overlap reduction function of two prototypes of TOBA at Tokyo and Kyoto. It is almost 1 below 10 Hz.

so that the PPS signals rise at the same position in the time series data. We also used the on-line calibration signal which is 8.7 Hz sine wave in order to monitor the fluctuation of the control open-loop gain. The on-line calibration signal is injected into the feedback signal, and the real-time gain is calculated by comparing signal before the injection and signal after the injection.

The equivalent strain noise spectra of two detectors are shown in fig 5. Their sensitivity is limited by the seismic noise at higher frequency than 0.1 Hz, and by the magnetic coupling noise at lower frequency than 0.1 Hz [13] [15].

Figure 5. The equivalent strain noise spectra of two detectors. Blue line is the strain of the detector in Tokyo, and green line is the strain of the detector in Kyoto.
4. Analysis

4.1. A spectrum of a stochastic gravitational wave background

Our target, a stochastic GW background, is considered to be a superposition of GWs from astrophysical sources and from cosmological sources. Here, we assume that a stochastic GW background is stationary, unpolarized, and isotropic. Also, the amplitude of a stochastic GW background $h_{ij}$ is assumed to have random variables, and we characterize the amplitude by its ensemble average of power spectrum density (PSD) $S_{gw}(f)$ written as

$$S_{gw}(f) = \frac{3H_0^2}{10\pi^2} f^{-3} \Omega_{gw}(f).$$  \hspace{1cm} (3)

Here, $H_0$ is the Hubble constant [16], and $\Omega_{gw}(f)$ is defined as

$$\Omega_{gw}(f) = f \frac{d\rho_{gw}}{df},$$  \hspace{1cm} (4)

where $\rho_c = 3c^2H_0^2/8\pi G$ is the critical energy density of the universe and $\rho_{gw} = (c^2/32\pi G)(\dot{h}_{ij}\dot{h}^{ij})$ is the gravitational-wave energy density [17].

4.2. The principle of the cross correlation analysis

The equivalent strain output $s_i(t)$ is written as

$$s_i(t) = n_i(t) + h_i(t),$$  \hspace{1cm} (5)

where $i$ donates the index of the two detectors. $n_i(t)$ and $h_i(t)$ is the detector’s noise in equivalent strain and the signal due to a stochastic GW background. The cross correlation between the two outputs is

$$Y = \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' s_1(t)s_2(t')Q(t-t')$$

$$\sim \int_{-\infty}^{+\infty} df \tilde{s}_1^*(f)\tilde{Q}(f)\tilde{s}_2(f),$$  \hspace{1cm} (6)

where $T$ is the observation time and $Q(t)$ is a real filter function called an optimal filter, which is decided so that signal-to-noise ratio is maximized. Here we assume that $T$ is long enough to be approximated as $T \rightarrow +\infty$ and that $Q(t)$ converges on 0 when $t$ is large. Note that the integrated interval has to be restricted for the analysis with the actual discrete output data.

The signal-to-noise ratio of $Y$, which is equal to $\langle Y \rangle/\sigma_Y$, is maximized when

$$\tilde{Q}(f) = N \frac{\gamma(f)}{P_1(f)P_2(f)f^{3}},$$  \hspace{1cm} (7)

where $\gamma(f)$ is the normalized overlap reduction function and $P_i$ ($i = 1, 2$) is the power spectrum density of the $i$-th detector’s output. $N$ is a normalization factor set so that $\langle Y \rangle = \Omega_{gw}h_0^2T$:

$$N = \frac{20\pi^2}{3H_{100}^2} \left( \int_{-\infty}^{+\infty} df \frac{\gamma^2(|f|)}{f^6P_1(|f|)P_2(|f|)} \right)^{-1}.$$  \hspace{1cm} (8)

Here, $H_{100} = 100$ km/sec/Mpc and $h_0 = H_0/H_{100}$ is the normalized Hubble constant.

If you want to know more details, see [17].
4.3. Main analysis

The process of the main analysis is divided into four parts: data selection, calculation of the cross correlation, signal detection, and calculation of the upper limit. Before calculating the cross correlation, we have to remove the data which badly affects the result, such as glitches. Then, we decide whether a stochastic GW background is present or not according to the cross correlation value calculated with survived data. When it is judged that a stochastic GW background is absent, the upper limit on a stochastic GW background is calculated.

4.3.1. Data selection

Ideally, the noise levels of the two detectors should be stationary through the whole observation. However, there are some glitches or non-stationary noise in the time series data, which will affect the result. Therefore we remove the data where the noise level is bigger.

First, the time series data is divided into several segments. (See figure 6.) Then, we remove the segments in which the data is noisy, and calculate the cross correlation only with the survived segments. Here, it is important to distinguish the noise from a stochastic GW background, otherwise we may remove the segments where a stochastic GW background is happened to be bigger. In this case, the frequency band where the sensitivity is best is used for the calculation of the cross correlation, and at other frequencies, the sensitivity is assumed to be limited by the noises. Therefore, we judge whether the data is noisy or not according to the whitened RMS at the frequencies excluding the analyzed frequency band.

4.3.2. Calculating cross correlation

The cross correlation value $Y$ is calculated with each survived segments according to the equation 6.

Here we restrict the analyzed frequency band, since the frequency band where the sensitivity is worse does not produces a good result. In order to decide the analyzed frequency band, we used the optimal filter $\tilde{Q}(f)$. The optimal filter is larger when the sensitivity to a stochastic GW background is better. Therefore, the frequency band where the optimal filter is biggest is chosen for the analyzed frequency band. (See figure 7)
4.3.3. Signal detection  According to $Y$'s calculated at each segments, we decide whether a stochastic GW background is present or not by the Neyman-Pearson criterion:

- if $\langle Y \rangle \geq z_{\alpha}$, a stochastic GW background is present.
- if $\langle Y \rangle < z_{\alpha}$, a stochastic GW background is absent.

Here, $\langle Y \rangle$ is the average of $Y$'s calculated with the survived segments and $\alpha$ is the false alarm rate. This detection threshold $z_{\alpha}$ is set so that

$$\alpha = \int_{z_{\alpha}}^{+\infty} dp(y|0),$$

where $p(y|0)$ is the probability distribution of $\langle Y \rangle$ when a stochastic GW background is absent.

In this case, we calculate the distribution of $\langle Y \rangle$ with time shifted data, which is considered to be relative to the probability distribution of $\langle Y \rangle$ when a stochastic GW background is absent. The distribution is shown in figure 8.

Figure 8. The distribution of $\langle Y \rangle$ with time shifted data. The sky blue, red, and green lines are the detection threshold $z_{\alpha}$ where $\alpha = 0.01, 0.05,$ and $0.1$ respectively.
4.3.4. Setting an upper limit  When we cannot detect a stochastic GW background, we set an upper limit on a stochastic GW background. An upper limit means the amplitude of a stochastic GW background which we can detect if it would come to these detectors with the same noise levels.

In order to calculate it, we use the actual data set and mock signals. We make a mock signal of a stochastic GW background with a certain amplitude according to equation 3 (see figure 9), and inject it into the observational data. Then, we perform the same analysis described above with the injected data. This process is repeated many times and we compute the rate at which we detect the mock signal, which is called detection efficiency. Figure 10 shows the detection efficiency at variable amplitude of a injected signals. The detection efficiency is equal to the confidence level to the upper limit. Therefore, for example, 95 % confidence upper limit is the amplitude of a mock signal at which the detection efficiency is 95 %.

![Figure 9.](image1) The strain of the mock signal of a stochastic GW background where $\Omega^{gw}h_0^2 = 10^{18}$. It is basically a Gaussian noise, but it is multiplied by $f^{-3/2}$.

![Figure 10.](image2) The detection efficiency. The blue dots are calculated probability and the blue line is an error function fitted to the blue dots. This detection efficiency is equal to $1 - \beta$, where $\beta$ is the false dismissal rate.

4.4. Parameter tuning
In the process of the main analysis, there are some parameters whose optimal values depend on the data quality. Therefore, we performed pre-analysis and tuned these parameters. Here the time shifted data is used in order to prevent the result from being intentionally good.

The parameters to be tuned are the length of segments, the frequency band used as the indicator of the noise level at data selection, the amount of the segments which is removed, and the bandwidth of the analyzed frequencies.

Then, A flow chart of the whole analysis process is shown in figure 11.

5. Result
As a result of the parameter tuning, the length of the segment is set as 100 sec, i.e., the frequency resolution $df$ is 0.01 Hz. We calculate RMS as the indicator of noise level at 0.01 – 0.05 and 1.0 – 10.0 Hz. Each 50 % of the segments are removed by the data selection, which results in about 14,000-second effective observation time. The analyzed frequency band is adjusted to 0.06 – 0.9 Hz. The width of searched frequency band is 0.85 Hz.
The flow chart of the analysis process. The process is roughly divided into two parts: the pre-analysis for the parameter tuning and the main analysis. We use the time-shifted data at pre-analysis and decide optimal parameters. Then, we perform the main analysis with time-tuned data and detect a GW signal or set an upper limit on a stochastic GW background.

When the data is analyzed with these parameters, the detection threshold with false alarm rate 1 % for $\langle Y \rangle / T$ is $z_{0.01} = 1.2 \times 10^{19}$. And calculated $\langle Y \rangle / T$ with time adjusted data is $-1.4 \times 10^{17}$, which is smaller than $z_{0.01}$. Therefore, we concluded that a stochastic GW background is absent. Moreover, we calculated that 95 % confidence upper limit with false alarm rate 1 % is $\Omega_{gw} h_0^2 \leq 1.2 \times 10^{19}$.

This result is worse than the upper limit calculated with a single TOBA by K. Ishidoshiro [13]. It is considered to be because of a large seismic noise due to the earthquake occurred at May 11th, 2011. This observational run was started about 13 hours before the earthquake, and there were some foreshocks during the observation. Now we are planning to perform the simultaneous observational run again before long.

6. Summary and future plan
We searched a stochastic GW background with the prototypes of TOBA by the cross correlation method. TOBA is the novel type of GW detector with good sensitivity below 1 Hz unlike other ground-based detectors such as large interferometers. The prototype has 20-cm test mass bar, and levitated by the magnetic force of the superconductor and the magnet. Its sensitivity is

$h \sim 10^{-8} [1/\sqrt{\text{Hz}}]$ at 0.3 Hz.

We demonstrated the simultaneous observational run and cross correlation analysis with two prototype TOBAs at Tokyo and Kyoto, and established the cross correlation analysis scheme with TOBAs. As a result of the analysis for detect a stochastic GW background, we cannot detect a stochastic GW background. Our 95% confidence upper limit is $\Omega_{gw} h_0^2 \leq 1.2 \times 10^{19}$.

While we set a new upper limit at wider frequency band than previous result, this result is not good because the seismic noise is bigger than before. This is considered to be due to the fore-shocks due to the big earthquake occurred in Japan at May 11th, 2011. Therefore, we will perform the simultaneous observational run again. It is expected that we can derive a better result.
Figure 12. Upper limits established by TOBAs and other previous observations. This result does not exceed the previous result because of the seismic noise. However, we extended the analyzed frequency band.

References
[1] Abbott B P et al. 2009 Reports on Progress in Physics 72 076901
[2] Acernese F et al. 2008 Journal of Optics A: Pure and Applied Optics 10 064009
[3] Abbott B et al. 2009 Nature 460 990–994
[4] Akutsu T et al. 2008 Phys. Rev. Lett. 101 101101
[5] Astone P et al. 1999 Astron. Astrophys. 351 811–814
[6] Abbott B et al. (LIGO Scientific Collaboration and ALLEGRO Collaboration) 2007 Phys. Rev. D 76(2) 022001 URL http://link.aps.org/doi/10.1103/PhysRevD.76.022001
[7] Armstrong J, Iess L, Tortora P and Bertotti B 2003 Astrophys. J. 599 806
[8] Maggiore M 2000 Phys. Rep. 331 283–367
[9] Smith T, Pierpaoli E and Kamionkowski M 2006 Phys. Rev. Lett. 97 21301
[10] Danzmann K 1996 Class. Quantum Grav. 13 A247–A250
[11] Kawamura S et al. 2011 Class. Quantum Grav. 28 ISSN 0264-9381
[12] Ando M et al. 2010 Phys. Rev. Lett. 105 ISSN 0031-9007
[13] Ishidohiro K et al. 2011 Phys. Rev. Lett. 106 ISSN 0031-9007
[14] Allen B and Romano J 1999 Phys. Rev. D 59 102001
[15] Ishidohiro K 2000 Search for low-frequency gravitational waves using a superconducting magnetically-levitated torsion antenna Ph.D. thesis University of Tokyo
[16] Spergel D et al. 2003 Astrophys. J. 148 175–194 ISSN 0067-0049
[17] Maggiore M 2008 Gravitational Waves Volume I Theory and Experiments (Oxford University Press)