QUANTUM DARWINISM AND ENVARIENCE*

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Abstract

Effective classicality of a property of a quantum system can be defined using redundancy of its record in the environment. This allows quantum physics to approximate the situation encountered in the classical world: The information about a classical system can exist independently from its state. In quantum theory this is no longer possible: In an isolated quantum system the state and the information about it are inextricably linked, and any measurement may – and usually will – reset that state. However, when the information about the state of a quantum system is spread throughout the environment, it can be treated (almost) as in classical physics – as (in effect) independent from the state of the open quantum system of interest. This is a central idea that motivates the quantum Darwinism approach to the interpretation problem. Quantum Darwinism differs from the traditional approach suggested by the von Neumann model of quantum measurement and offers a new perspective on the emergence of the everyday classical reality that is complementary to the one suggested by decoherence: Selection of preferred states occurs as a result of the ‘selective advertising’, a proliferation of the information about the stable pointer states throughout the Universe. This view of the emergence of the classical can be regarded as (a Darwinian) natural selection of the preferred states. Thus, (evolutionary) fitness of the state is defined both by its ability to survive intact in spite of the immersion in the environment (i.e., environment-induced superselection is still important) but also by its propensity to create offspring – copies of the information describing the state of the system in that environment. I show that this ability to ‘survive and procreate’ is central to effective classicality of quantum states. Environment retains its decohering role, but it also becomes a “communication channel” through which the state of the system is found out by the observers. In this sense, indirect acquisition of the information about the system from its environment allows quantum theory to come close to what happens in the classical physics: The information about a classical system can be “dissociated” from its state. (In the case of an isolated quantum system this is impossible – what is known about it is inseparably tied to the state it is in.) I review key ideas of quantum Darwinism and investigate its connections with the environment – assisted invariance or envariance, a recently identified symmetry exhibited by pairs of entangled quantum systems that is responsible for the emergence of probability (allowing, in particular, a completely quantum derivation of the Born’s rule) within the wholly quantum Universe.

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INTRODUCTION

Quantum measurement problem is a technical euphemism for a much deeper and less well-defined question: How do we, ‘the observers’, fit within the physical Universe? This issue is especially apparent in quantum physics because, for the first time in the history of science a majority (but not all) physicists seriously entertain the possibility that the framework for the ultimate universal physical theory is provided by quantum mechanics.

Quantum physics relevant for this discussion is (contrary to the common prejudice) relatively simple. By this I mean that some of the key features of its predictions can be arrived at on the basis of the overarching principles of quantum theory and without reference to the minutiae of other specific ingredients (such as the details of the forces). Quantum superposition principle is such an overarching principle of quantum theory. It leads to predictions that seem difficult to reconcile with our perception of the familiar classical Universe of everyday experience. The aim of this paper is to show that the emergence of the classical reality can be viewed as a result of the emergence of the preferred states from within the quantum substrate thorough the Darwinian paradigm, once the survival of the fittest quantum states and selective proliferation of the information about them are properly taken into account.

Measurement problem has been the focus of discussions of the interpretation of quantum theory since its inception in its present form in the mid-1920’s. Two new ideas that are the focus of this paper – quantum Darwinism and envariance – were introduced very recently. Exploration of their consequences has only started. This presentation provides a somewhat premature (and, consequently, rather speculative) “preview” of their implications. We shall start with the von Neumann model of quantum measurements. It has provided the standard setting for the exploration of the role of observers and information transfer since it was introduced in 1932. We shall then go on and describe how von Neumann’s model is modified by the introduction of the environment in the more modern treatements, and briefly review consequences of decoherence and of the environment-induced superselection or einselection that settle some of the issues.

Quantum Darwinism and envariance rely on the presence of the environment. They explore a similar set of questions as the theory of decoherence and einselection, but from a very different vantage point: Rather than limit attention to the consequences of the immersion of the system $S$ or of the apparatus $A$ in the environment $E$ on the state of $SA$, the focus shifts to the effect of the state of $SA$ (or more precisely, the to-be-classical observables of that object, including in particular the apparatus pointer $A$) on the state of the environment.

The study of decoherence already calls for a modification of von Neumann’s model – for the addition of the environment. Quantum Darwinism is another radical change – a change of focus, of the subject of discourse. It is based on the realisation that almost without exception we – the observers – acquire information about “measured systems” or the state of the “apparatus pointers” indirectly – by monitoring the environment: It correlates with the system as a result of decoherence, which is caused by the environment
(in effect) monitoring $\mathcal{A}$ and / or $\mathcal{S}$.

Monitoring by the environment is responsible for the negative selection, for destabilisation of the vast majority of the states in the relevant Hilbert spaces of the open systems. What is left are the preferred pointer states. This, in essence, is the environment-induced superselection *einsel*ection. Quantum Darwinism is based on the observation that intercepting such ‘second hand’ information about the system by measuring fragments of the environment makes only some of the states of the system of interest accessible. These states happen to be the preferred pointer states of $\mathcal{S}$. The reason for their selection that is also ‘Darwinian’: Pointer states are not only best in surviving the hostile environment, but are also best in *proliferating* – throughout the rest of the Universe, using environment as the medium – the information about themselves. This allows many observers to simultaneously find out about the pointer states, and to do so indirectly, and therefore, without perturbing them any further than decoherence already did. Objective existence of pointer states of quantum systems can be accounted for in this way\textsuperscript{1–3}. Hence, quantum analogues of the Darwinian criterion of ‘fitness’ can be seen in the (ein-)selection of ‘the classical’.

Envariance focuses on the origins of ignorance (and, hence, information) in the quantum Universe. It leads to the definition of probabilities – to the completely quantum derivation\textsuperscript{1,4} of the Born’s rule. Again, introduction of the environment is essential in this argument. In its presence one can delineate what aspects of the state of a system (that is correlated with the environment) cannot be known to the observer. In this way – by starting from a quantum definition of ignorance – the operational definition of probabilities can be obtained as a consequence of a quintessentially quantum sort of a correlation – quantum entanglement. It is interesting to note that analogous derivation cannot be repeated classically. This is because in classical physics information about the state can be “dissociated” from that state, while in quantum physics what is known about the state cannot be treated separately from the state. Consequently, in quantum physics it is possible to know precisely the joint state of two systems, but be provably ignorant about the states of the component subsystems.

Both of these themes – quantum natural selection and envariance – have benefited from the inspiration and support of John Archibald Wheeler. To begin with, one of the two portraits displayed prominently in John’s office in Austin, Texas, was of Charles Darwin (the other one was of Abraham Lincoln). This was symptomatic of the role theme of evolution played in John’s thinking about physics (see, e.g., Wheeler’s ideas on the evolutionary origin of physical laws\textsuperscript{10}). While I was always fond of looking at the ‘natural world’ in Darwinian terms, this tendency was very much encouraged by John’s influence. It seems quite natural to look at the emergence of the classical as a consequence of a quantum analogue of natural selection. Last not least (and on a lighter note) while my wife Anna and I were visiting John on his ‘High Island’ summer estate in Maine, we were put up in a cottage in which – I was told – James Watson wrote “The Double Helix”...

While quantum Darwinism benefited from Wheeler’s boldness and encouragement, *envariance* bears a more direct Wheeler *imprimitur*: Late in the year 1981 John and I
were putting finishing touches on *Quantum Theory and Measurement*¹¹, and that included writing a section on *Further Literature*. At that time I was fascinated with the idea that quantum states of entangled systems are in a sense relative – defined with respect to one another. Thus, John has caught me speculating ... Zurek notes that “Nothing can keep one from thinking about [the two spins in a singlet] as the measured system and ... a quantum apparatus. [In that language] ... spin-system always points in the direction which is opposite to the direction of the ... spin-apparatus. This is a definite, “coordinate-independent” statement.” John overcame my reluctance and included these musings about ‘the relativity of quantum observables’ (see p. 772 of Ref. 11). These very same ideas have recently – and after a long gestation period – begun to mature into a new way of looking at information and ignorance in the quantum context. Derivation of Born’s rule based on the symmetries anticipated in that twenty - years old passage is presented in this paper in the sections on envariance. I am sure that this result is just a “tip of the iceberg”, and I am convinced that envariance will prove to be a useful way of looking at various quantum issues of both fundamental and practical significance.

**QUANTUM MEASUREMENT: VON NEUMANN’S MODEL**

The traditional statement of the measurement problem goes back to von Neumann⁵, who has analysed unitary evolutions that take initial state $|\psi_S\rangle$ of the system and $|A_0\rangle$ of the apparatus into and entagled joint state $|\Psi_{SA}\rangle$:

$$
|\psi_S\rangle|A_0\rangle = (\sum_k a_k |s_k\rangle)|A_0\rangle \rightarrow \sum_k a_k |s_k\rangle|A_k\rangle = |\Psi_{SA}\rangle.
$$

(1)

Von Neumann has realised that while $|\Psi_{SA}\rangle$ exhibits the desired correlation between $S$ and $A$, the unitary pre-measurement (as the ‘conditional dynamics’ step described by Eq. (1) is often called) does not provide a satisfactory account of a ‘real world’ measurements. There are two reasons why Eq. (1) falls short of that goal: They are respectively identified as “basis ambiguity” and “collapse of the wavepacket”.

Basis ambiguity¹² is a direct consequence of the superposition principle: According to it, one can rewrite an entangled bipartite state such as $|\Psi_{SA}\rangle$ of Eq. (1) in an arbitrary basis of one of the two subsystems (say, $S$) and then identify the corresponding basis of the other (i.e., the apparatus $A$). That is:

$$
|\Psi_{SA}\rangle = \sum_k a_k |s_k\rangle|A_k\rangle = \sum_k b_k |r_k\rangle|B_k\rangle = \ldots,
$$

(2)

where $\{|s_k\rangle\}$ and $\{|r_k\rangle\}$ (as well as $\{|A_k\rangle\}$ and $\{|B_k\rangle\}$) span the same Hilbert space $H_S$ ($H_A$), while $a_k$ (and $b_k$) are complex coefficients.

Basis ambiguity can be also regarded as a consequence of entanglement. It is troubling, as it seems to imply that not just the outcome of the measurement, but also the set of states that describe the apparatus is arbitrary. Hence, any conceivable superposition (including the counterintuitive “Schrödinger cat” states¹³) should have an equal right to be a valid
description of a real apparatus (or a real cat) in a completely quantum Universe. This is
blatantly at odds with our experience of the macroscopic objects (including, for instance,
states of the pointers of measuring devices) which explore only a very limited subset of the
Hilbert space of the system restricted to the familiar, localized, effectively classical states.

The problem with the “collapse of the wavepacket” would persist even if one were to
somehow identify the preferred basis in the Hilbert space of the apparatus, so that prior
to observer’s contact with $A$ one could be at least certain of the ‘menu’ of the possible
outcome states of the apparatus, and the basis ambiguity would disappear. For, in the
end, we perceive only one of the possibilities, the actual outcome of the measurement.
“Collapse” is the (apparently random) selection of just one of the positions on the ‘menu’
of the potential outcomes with the probability given by Born’s rule.$^{14}$

Von Neumann discussed two processes that address the two aspects of the “quantum
measurement process” described above. While his investigation preceded the famous EPR
paper$^{15}$, and, hence, appreciation of the role of entanglement (which is behind the basis
ambiguity), he has nevertheless postulated *ad hoc* a non-unitary ‘reduction’ from a pure
state into a mixture:

$$
|\Psi_{SA}\rangle\langle\Psi_{SA}| \longrightarrow \sum_{k} |a_k|^2 |s_k\rangle\langle s_k| A_k\rangle\langle A_k| = \rho_{SA} .
$$

This process would have (obviously) selected the preferred basis. Moreover, von Neumann
has also speculated about the nature of the next step – the collapse, i.e., the perception, by
the observer, of a unique outcome. This could be represented by a non-unitary transition,
e.g. in a particular run of the experiment when state $\{|s_{17}\rangle\}$ is found:

$$
\sum_{k} |a_k|^2 |s_k\rangle\langle s_k| A_k\rangle\langle A_k| \longrightarrow |s_{17}\rangle\langle s_{17}| A_{17}\rangle\langle A_{17}| .
$$

In the collapse, the probability of any given outcome is given by Born’s rule, $p_k = |a_k|^2$.
Von Neumann has even considered the possibility that collapse may be precipitated by the
conscious observers. This ‘anthropic’ theme was later taken up by the others, including
London and Bauer$^{16}$ and Wigner$^{17}$.

The aim of this paper is to investigate – and where possible to settle – open questions
within the unitary quantum theory *per se*, without invoking any non-unitary or anthropic
deus ex machina such as Eqs. (3) or (4) above.

DECOHERENCE AND EINSELECTION

Contemporary view (dubbed even “the new orthodoxy”$^{18}$) is that the solution of the
measurement problem – and, in particular, the resolution of the issues described above in
the context of von Neumann’s original model – requires a more realistic account of what
actually happens during a measurement: While von Neumann has treated the $SA$ pair
as isolated from the rest of the Universe, the discussions over the past two decades have
paid a lot of attention to the consequences of the immersion of the apparatus (and, more
generally, of all the macroscopic objects) in their environments\textsuperscript{1–4,6–9,12,19–21}.

When the impossibility of perfect isolation of \( \mathcal{A} \) is recognised, the solution of the
basis ambiguity problem can be obtained\textsuperscript{1–3,12,19–21}. Preferred basis – candidate for the
classical basis in the Hilbert space of the system coupled to the environment – is induced
by the interaction with \( \mathcal{E} \), which can be regarded as monitoring by the environment of \( \mathcal{A} \).
The resulting transfer of information is selective. Thus, observer who is in turn monitoring
the environment to find out about the state of \( \mathcal{A} \) will obtain only censored information: He
will be able to readily find out about the preferred pointer states of the system, but but it
will be next to impossible to find out about their superpositions. Egalitarian principle of
superposition – the cornerstone of quantum mechanics – is grossly violated in such “open”
quantum systems. Different quantum states exhibit very different degree of resilience
in presence of the interaction with the outside. Thus, the question about effective classicality
is answered by the study of the relative stability. \textit{States that exist are the states that persist}
is one of the tenets of the \textit{existential interpretation}\textsuperscript{8,9}:

Preferred pointer states are – in contrast to arbitrary superpositions, which, in accord
with the superposition principle, have equal right to inhabit Hilbert space of an \textit{isolated}
system – resilient to the entangling interaction with the environment. Hence, they maintain
their identity – their ability to faithfully represent the system. Selection of the preferred set
of resilient pointer states in the Hilbert space is the essence of the environment - induced
superselection (einselection). It is caused by a (pre-)measurement - like unitary evolution
in which the environment \( \mathcal{E} \) becomes entangled with the apparatus:

\[
|\Psi_{SA}\rangle|e_0\rangle = (\sum_k a_k |s_k\rangle|A_k\rangle)|e_0\rangle \rightarrow \sum_k a_k |s_k\rangle|A_k\rangle|e_k\rangle = |\Phi_{SAE}\rangle. \tag{5}
\]

When the state of the environment contains an accurate record of the outcome, so that
\(|\langle e_k|e_l\rangle|^2 = \delta_{kl} \), the density matrix of the apparatus – system pair acquires the desired
form, as can be seen by tracing out the environment:

\[
\rho_{SA} = \text{Tr}_{\mathcal{E}} |\Phi_{SAE}\rangle\langle\Phi_{SAE}| = \sum_k |a_k|^2 |s_k\rangle\langle s_k||A_k\rangle\langle A_k|. \tag{6}
\]

This is clearly what is needed to solve the basis ambiguity problem (compare with Eq. (3)
above). Moreover, it has been by now confirmed in model calculations and corroborated
by experiments that the preferred pointer basis will habitually appear on the diagonal
of the density matrix describing \( \mathcal{A} \) after a decoherence time (which is very short for the
macroscopic systems). The question, however, can be raised about the justification of the
trace operation: The form of the density matrix relies on Born’s rule\textsuperscript{5}. Moreover, Eq. (6)
gets only half of the job done: Eq. (3) – the collapse – still needs to be understood.

Within the context of decoherence and einselection both of these questions – basis
ambiguity and collapse – can be (albeit to a different degree) addressed. It is by now largely
accepted (as a result of extensive studies of specific models) that under a reasonable set of
realistic assumptions preferred basis of an apparatus pointer (or of selected observables of any open macroscopic system) does indeed emerge. Thus, quantum entanglement (present after the pre-measurement, Eq. 1) will give way to a classical correlation between $S$ and $A$, with the same preferred pointer basis $\{|A_k\rangle\}$ habitually appearing on the diagonal of $\rho_{SA}$. This takes care of the basis ambiguity.

This conclusion, however, crucially depends on the trace operation, which is justified by employing Born’s rule – an important part of the quantum foundations, that is often regarded as an independent axiom of quantum theory intimately tied with the process of measurement. One may (as many have) simply accept Born’s rule as one of the axioms. But it would be clearly much more satisfying to derive it. This will be our aim in the discussion of envariance.

The other outstanding issue is the apparent collapse and – in particular – the objectivity of effectively classical (but, presumably, ultimately quantum) states. That is, classical states can be simply ‘found out’ by an observer who is initially completely ignorant. This is not the case for quantum states: Ideal measurement always yields an eigenvalue of the measured observable. Hence, it selects its (possibly degenerate) eigenstate. When the system does not happen to be in one of the eigenstates of the observable selected by the observer its measurement will perturb the state of the system by resetting it to one of the eigenstates of what is being measured. Yet, in our everyday experience we never have to face this problem: Somehow, at the macroscopic level classical reality is a fact of life: We find out about the rest of the Universe at will, without having to worry about what does (and what does not) exist. We start by addressing this second issue of the emergence of objectivity.

QUANTUM DARWINISM

A part of the paradigm of “quantum measurements” that is shared not just by von Neumann’s model, but by most of the other approaches to the interpretation of quantum mechanics is the belief that we – the observers – acquire information about quantum systems directly, i.e., by interacting with them. As was pointed out some time ago\textsuperscript{8,9}, this is never the case. For instance, a vast majority of our information is acquired visually. The information we obtain in this way does not concern photons, although our eyes act as photon detectors: Rather, photons play the role of carriers of information about objects that emit or scatter them. Moreover, we obtain all the information by intercepting only a small fraction of photons emitted by or scattered from the object of interest with our eyes. Thus, many more copies of the same information must be carried away by such photon environment. Upon reflection one is led to conclude that essentially the same scheme (but involving different carriers of information) is the rule rather than exception. Measurements carried out on the macroscopic objects are invariably indirect, and carriers of information always “fan out” most of the copies of the ‘data’, spreading it throughout the Universe. Observers use a fraction of the same environment that causes decoherence as a channel, to obtain information about the system of interest.
This distinction between direct and indirect acquisition of information may seem inconsequential. After all, replacing a direct measurement with an indirect one only extends the ‘von Neumann chain’. The overall state has the form of Eq. (5) and is still pure, with all of the potential outcomes present, superficially with no evidence of either Eq. (3) or the “collapse” of Eq. (4). Still, we shall show that when this situation is analysed from the point of view of the observer, most (and perhaps all) of the symptoms of classicality emerge.

To investigate a simple model of this situation we consider obvious generalisation of Eq. (5) we have used to describe decoherence:

$$|\Psi_{SA}^{(n)}\rangle \otimes_{n=1}^{N} |e_0^{(n)}\rangle = (\sum_k a_k |s_k\rangle |A_k\rangle) \otimes_{n=1}^{N} |e_0^{(n)}\rangle \rightarrow \sum_k a_k |s_k\rangle |A_k\rangle \otimes_{n=1}^{N} |e_k^{(n)}\rangle = |\Phi_{SAE}^{(n)}\rangle.$$  

(7)

There are $N$ environment subsystems here. The assumption is that they exist, and that they can be (like photons) accessed one at a time.

We first note that enlarging this composite environment $E^N$ of Eq. (7) is absolutely irrelevant from the point of view of its effect on the density matrix of the ‘object of interest’, $\rho_{SA}$. For, when either a simple environment of Eq. (5) or the multiple environment of Eq. (7) are traced out, the same $\rho_{SA}$ of Eq. (6) will obtain. So what (if anything) have we gained by complicating the model? Whatever it is, obviously cannot be inferred from the state of $SA$ alone. Yet, in classical physics the state of “the object of interest” was all that mattered! So where should we look now?

The inability to appreciate the implications of the difference between these two situations is indeed firmly rooted in the ‘classical prejudice’ that the information about the system is synonymous with its state, but that the presence of that information is physically irrelevant for that state. This classical belief in the analogue of the ‘separation of church from state’ is untenable in the quantum setting. For starters, there is ‘no information without representation’!

Guided by our previous considerations, we shift attention from the state of the object of interest (the $SA$ pair) to the record of its state in the environment. Now there is our difference! Instead of a single (fragile) record of the state of the system we now have many identical copies. How many? The preferred states $\{|A_k\}\}$ of the apparatus have left $N$ imprints on the environment. This is easily seen in the example above, and can be quantified by one of the versions of the redundancy ratio, which in effect count the number of copies of the information about the object of interest spread throughout the environment.

One definition of the redundancy ratio starts with redundancy defined in terms of the familiar mutual information – a measure of correlation between the fragment of the environment $E^{(n)}$ and the object of interest $^{1,2}$. This leads to:

$$I(S : E^{(n)}) = H(S) + H(E^{(n)}) - H(S,E^{(n)}) ,$$  

(8)

where $H(S)$, $H(E^{(n)})$, and $H(S,E^{(n)})$ are the relevant individual and joint entropies. Above, we have also replaced $SA$ of Eq. (5) by a single object to simplify notation,
and to emphasize that this approach applies in general – and not just in measurement situations. Various entropies can be defined in several ways using obvious reduced density matrices of the relevant subsystems of the whole\textsuperscript{1,2,22}. Redundancy can be then estimated as:

\[ I^{(N)} = \sum_{k=1}^{N} I(SA : E^{(k)}) . \]  \hspace{1cm} (9)

The physical significance of redundancy in the context of our discussion is similar to its import in the classical information theory\textsuperscript{23}: Redundancy protects information about the object of interest. From the point of view of the interpretation of quantum theory, this implies, for example, that many different observers can find out the state of the object of interest independently – by measuring different fragments of the environment. This is how – I believe – states of the ultimately quantum but macroscopic objects in the world of our everyday experience acquire their \textit{objective existence}\textsuperscript{1–3}.

However, viewed in a Darwinian fashion, redundancy ratio has also a different significance: It provides, in effect, a measure of the number of “offspring” of the state in question. Thus, in the ideal case we have considered above proliferation of information has led to \( N \) descendants of the original state of the apparatus. The redundancy ratio in the example given above is:

\[ R = I^{(N)}/H(S) = N . \]  \hspace{1cm} (10)

Both the prerequisites for, and the consequences of high redundancy have significance that is best appreciated by invoking analogies with the “survival of the fittest”. To begin with, a state that manages to spread many imprints of its ‘genetic information’ throughout the environment must survive long enough – must be resistant to the perturbations caused by the environment. This points immediately to the connection with the pointer states\textsuperscript{12} – they remain unperturbed by decoherence. But this is in a sense just a different view of selection of the preferred states, which does not capitalise on the measure of their fecundity we have introduced above.

Darwinian analogy recognizes that proliferation of certain information throughout the environment makes its further proliferation more likely. This is best seen in a still more realistic extension of the models of the environments we have considered so far: Suppose that in addition to the immediate environments \( E^{(k)} \) there are also distant environments \( \varepsilon^{(l)} \), which do not interact directly with \( S \) but interact with the immediate environments through interaction that is local – i.e., that allows individual subsystems of the immediate environment to become correlated with individual subsystems of the distant environment. Then it is easy argue that the only information about \( S \) that can be passed along from \( E \)’s to \( \varepsilon \)’s will have to do with the preferred pointer states: Only locally accessible information\textsuperscript{22} can be passed along by such local interactions. Indeed, this connection between the selection of the preferred basis and redundancy was noted already some time ago\textsuperscript{19,24}.

We note in passing that there is an intimate relation between this necessity to make a selection of preferred states in the setting that involves “fan-out” of the information and the
no-cloning theorem\textsuperscript{25}, which, in effect, says that copying implies a selection of a preferred set of states that are copied. We also note that all of the above considerations depend on the ability to split the Universe as a whole into subsystems. This – as was already noted in the past — is a prerequisite of decoherence. Moreover, problems of interpretation of quantum physics do not arise in a Universe that does not consist of subsystems\textsuperscript{1,8,9}.

ENVIRONMENT - ASSISTED INVARIANCE

Envariance is an abbreviation for environment - assisted invariance, the peculiarly quantum symmetry exhibited by the states of entangled quantum systems. To explain it we consider a state vector describing system $S$ entangled (but no longer interacting) with the environment $E$. The joint state can be always written in the Schmidt basis:

$$|\psi_{SE}\rangle = \sum_{k}^{N} \alpha_{k}|s_{k}\rangle|\varepsilon_{k}\rangle.$$  \hspace{1cm}(11a)

For, even when the initial joint state is mixed, one can always imagine purifying it by enlarging the environment. As the environment no longer interacts with the system, probabilities of various states of the system cannot be – on physical grounds – influenced by such purification. In writing Eq. (11) we assumed that such purification was either unnecessary or was already carried out.

Environment - assisted invariance refers to the fact that there is a family of unitary quantum transformations $U_{S}$ that act on a system alone, and are non-trivial, so that $U_{S}|\psi_{SE}\rangle \neq |\psi_{SE}\rangle$, but their effect can be undone by acting solely on $E$. Thus, for any $U_{S}$ that has Schmidt states as eigenstates one can always find $U_{E}$ such that:

$$U_{E}(U_{S}|\psi_{SE}\rangle) = |\psi_{SE}\rangle$$  \hspace{1cm}(12)

This is evident, as unitaries with Schmidt eigenstates acting on $S$ will only rotate the phases of the coefficients $\psi_{SE}$. But these phases can be also rotated by acting on $E$ alone. Hence, transformations of this kind are envariant. It turns out that envariant transformations always have Schmidt eigenstates\textsuperscript{1}.

In the spirit of decoherence we now focus on the system alone. Clearly, for an observer with no access to $E$, system must be completely characterised by the set of pairs $\{ |\alpha_{k}|, |s_{k}\rangle \}$: Only the absolute values of the coefficients can matter since phases of $\alpha_{k}$ can be altered by acting on $E$ alone, and $E$ is causally disconnected from $S$. Thus, in the case when all $|\alpha_{k}|$ are equal;

$$|\tilde{\psi}_{SE}\rangle = \sum_{k}^{N} |\alpha|e^{-i\varphi_{k}}|s_{k}\rangle|\varepsilon_{k}\rangle,$$  \hspace{1cm}(11b)

any orthonormal basis is obviously Schmidt, and we can use envariance to re-assign the coefficients to different states $\alpha_{k} \rightarrow \alpha_{l}$, $\alpha_{l} \rightarrow \alpha_{k}$, etc. Such swapping leaves the description of the system invariant: The coefficients can differ only by the phase, and we have proved
above that phases of the Schmidt coefficients cannot influence probabilities of the system alone\(^1\). (Indeed, if this was possible, faster than light communication would be also possible, as the reader can easily establish by extending the above argument.)

It is now evident that the probabilities of all \(k\)’s must be equal. Hence, assuming the obvious normalisation, they are given by:

\[
p_k = \frac{1}{N} . \quad (13a)
\]

Moreover, a collection of a subset of \(n\) amongst \(N\) mutually exclusive events (orthogonal states) has the probability:

\[
p_{k_1 \lor k_2 \lor \ldots \lor k_n} = \frac{n}{N} . \quad (13b)
\]

These results were easy to arrive at, but we have started with very strong assumption about the coefficients.

The case when \(|\alpha_k|\) are not equal is of course of interest. We shall reduce it to the case of equal coefficients by extending the Hilbert space of the environment. In the process we shall recover Born’s rule \(p_k = |\alpha_k|^2\). This will also provide a firmer foundation for the decoherence approach which until now uses Born’s rule to justify its reliance on reduced density matrices. We note that we have, in a sense, already gone half way in that direction: Phases in the Schmidt decomposition have been already shown to be irrelevant, so the probabilities must depend on the absolute values of the coefficients. We still do not know in what specific function is this dependence embodied.

To illustrate the general strategy we start with an example involving a two-dimensional Hilbert space of the system spanned by states \(\{|0\rangle, |2\rangle\}\) and (at least) a three-dimensional Hilbert space of the environment:

\[
|\psi_{SE} \rangle = (\sqrt{2}|0\rangle|+\rangle + |2\rangle|2\rangle)/\sqrt{3} . \quad (15a)
\]

The state of the system is on the left, and \(|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}\) exists in the (at least two-dimensional) subspace of \(E\) orthogonal to the environment state \(|2\rangle\), so that \(\langle 0|1\rangle = \langle 0|2\rangle = \langle 1|2\rangle = \langle +|2\rangle = 0\). To reduce this case to the case of Eq. (11b) we extend \(|\psi_{SE}\rangle\) above to a state \(|\phi_{SEE'}\rangle\) with equal coefficients by acting only on the causally disconnected \(E\) (which implies that probabilities we shall infer for \(S\) could not have changed). This can be done by allowing a c-shift act between \(E\) and \(E'\) so that (in the obvious notation) \(|k\rangle|0'\rangle \Rightarrow |k\rangle|k'\rangle\), and;

\[
|\psi_{SE}\rangle|0\rangle = \frac{\sqrt{2}(|0\rangle|+\rangle|0'\rangle + |2\rangle|2\rangle|0'\rangle)}{\sqrt{3}} \Rightarrow (\sqrt{2}|0\rangle|0'\rangle + |1\rangle|1'\rangle + |2\rangle|2\rangle)/\sqrt{3} \quad (16a)
\]

The cancellation of \(\sqrt{2}\) leads to:

\[
|\phi_{SEE'}\rangle = (|0\rangle|0\rangle|0'\rangle + |0\rangle|1\rangle|1'\rangle + |2\rangle|2\rangle|2'\rangle)/\sqrt{3} \quad (17a)
\]
The phases are again irrelevant as they can be altered by manipulating $E'$ alone. Clearly, for
the bipartite combination of $S$ and $E$ the three orthonormal product states have coefficients
with same absolute value and can be swapped. Hence, all of them must have the same
probability. Thus, by Eq. (13a), probabilities of $|0\rangle|0\rangle$, $|0\rangle|1\rangle$, and $|2\rangle|2\rangle$ are all equal
to 1/3. Moreover, two of them involve state $|0\rangle$ of the system. So, by Eq. (13b), the
probability of $|0\rangle$ state of the system is twice the probability of $|2\rangle$. Consequently:

$$p_0 = 2/3; \quad p_2 = 1/3.$$  \hspace{1cm} (18a)

Hence, in this special case – but using ideas that are generally applicable – I have derived
Born’s rule, i.e., demonstrated that entanglement leads to envariance and this implies
$p_k = |\alpha_k|^2$. It is straightforward (if a bit notationally cumbersome) to generalise this
derivation, and we shall do so in a moment. But the basic idea is already apparent and
worth contemplating before we proceed with a general case (where the main point is
somewhat obscured by notation).

**ENVARIENCE, IGNORANCE, AND INFORMATION**

The above derivation of probabilities in quantum physics is very much in the spirit
of the “ignorance interpretation”, but in the quantum context it can be carried out with
an important advantage: In the classical case observers assume that an unknown state
they are about to discover exists objectively prior to the measurement, and that the igno-
rance allowing for various swappings reflects their “subjective lack of knowledge”. Indeed,
the clash between this subjectivity of information on one hand and its obvious physical
significance on the other has been a source of a long - standing friction distilled into the
Maxwell’s demon paradox. *In quantum theory ignorance can be demonstrated in an objec-
tive fashion, as a consequence of envariance of a state perfectly known as a whole.* Above,
$SSE'$ is pure. Quantum complementarity enforces ignorance of the states of the parts as
the price that must be paid for the perfect knowledge of the state of the whole.

It seems ironic that a natural (and a very powerful) strategy to justify probabilities
rests – in quantum physics – on a more objective and secure foundation of perfectly known
entangled pure states than in the deterministic classical physics: When the state of the
observers memory $|\mu\rangle$ is not correlated with the system,

$$|\Psi_{\mu SE}\rangle \sim |\mu\rangle \sum_k |s_k\rangle|\epsilon_k\rangle$$  \hspace{1cm} (20)

and the absolute values of the coefficients in the Schmidt decomposition of the entangled
state describing $SE$ are all equal, and $E$ cannot be accessed, the resulting state of $S$ is
*objectively invariant* under all local measure - preserving transformations. Thus, with no
need for further excuses, probabilities of events $\{|s_k\}\}$ must be – prior to measurement –
equal.

By contrast, after observer (pre)measures the system, the overall state;

$$|\Phi_{\mu SE}\rangle \sim \sum_k |\mu_k\rangle|s_k\rangle|\epsilon_k\rangle$$  \hspace{1cm} (21)
obtains, with the correlation between his record $|\mu_k\rangle$ and the system state $|s_k\rangle$ allowing him to infer the state of the system from his record state. The invariance we have appealed to before is substantially restricted: Correlated pairs $|\mu_k\rangle|s_k\rangle$ can be no longer separated and have to be permuted together. Thus, to a friend of the observer, all outcomes remain equiprobable, but to the ‘owner of the memory $\mu$’ his state is in part described by what he has found out about the system. Consequently, $|\mu_k\rangle$ implies $|s_k\rangle$ and the probability conditioned on observer’s own state in the wake of the perfect measurement is simply $p_{s_k|\mu_k} = \delta_{lk}$. Conditional probability in quantum theory emerges as an objective consequence of the relationship between the state of the observer and the rest of the Universe, as the combined state under consideration (and not just the ill-defined and dangerously subjective “state of observers knowledge” about a “definite but unknown classical state”) is invariant in a manner that allows one to deduce equality of probabilities much more rigorously, directly and without the copious apologies required in the classical setting.

We note that the above discussion of the acquisition of information owes a great deal to Everett\textsuperscript{26}. The collapse occurs on the way from Eq. (20) to Eq. (21). Envariance has given us a new insight into the nature of collapse: It is the extent of the correlations – the proliferation of information – that is essential in determining what states of quantum systems can be perceived by observers. When an envariant swap can be carried out on the $\mathcal{SE}$ pair, without involving the state of the observer (see Eq. (20)), he is obviously ignorant of the state of $\mathcal{S}$. By contrast, a swap in Eq. (21) would have to involve the state of the observer. This is because the information he has acquired is inscribed in the state of his own memory. (There is no information without representation\textsuperscript{21}.) In this sense, envariance extends the existential interpretation\textsuperscript{8,9} introduced some time ago to deal with the issue of collapse.

**BORN’S RULE FROM ENVARIANCE – GENERAL CASE**

To discuss the general case we start with the state:

$$|\Psi_{\mathcal{SE}}\rangle = \sum_{k=1}^{N} \sqrt{\frac{m_k}{M}} |s_k\rangle |\epsilon_k\rangle,$$  \hspace{1cm} (15b)

where $M = \sum_{k=1}^{N} m_k$ assures normalisation. As the coefficients are commensurate, and as we assume that the Hilbert subspaces of $\mathcal{E}$ corresponding to different $k$ are at least $m_k$ dimensional, appropriate c-shift\textsuperscript{1,2}:

$$|\epsilon_k\rangle|\epsilon'\rangle = \left( \frac{1}{\sqrt{m_k}} \sum_{l_k=1}^{m_k} |\epsilon_{l_k}\rangle \right) |\epsilon'\rangle \implies \frac{1}{\sqrt{m_k}} \sum_{l_k=1}^{m_k} |\epsilon_{l_k}\rangle |\epsilon'_{l_k}\rangle$$

that couples $\mathcal{E}$ with at least as large $\mathcal{E'}$ yields:

$$|\Psi_{\mathcal{SE}}\rangle|\epsilon\rangle \implies M^{-1} \sum_{k=1}^{N} |s_k\rangle \left( \sum_{l_k}^{m_k} |\epsilon_{l_k}\rangle |\epsilon'_{l_k}\rangle \right)$$  \hspace{1cm} (16b)
Here, in contrast to (16a), we have immediately carried out the obvious cancellation, 
\( \frac{\sqrt{mk}}{\epsilon_j} = \sum_{l_k}^{m_k} |\epsilon_{l_k}\rangle \). It follows as a direct consequence of the relation between the states \( |\epsilon_k\rangle \) and their Fourier-Hadamard transforms \( |\epsilon_k\rangle \).

The resulting state can be rewritten in a simpler and more obviously invariant form:

\[
|\Phi_{SEE'}\rangle = M^{-1} \sum_{j=1}^{M} |s_{k(j)}\rangle |\epsilon_j\rangle |\epsilon_j'\rangle
\]

where the environmental states are orthonormal, and the system state is the same within different \( m_k \)-sized blocks (so that the same state \( |s_{k(j)}\rangle \) appears for \( m_k \) different values of \( j \), and \( \sum_{k=1}^{N} m_k = M \)).

As before (see Eq. (17a)) phases are irrelevant because of envariance. Hence, terms corresponding to different values of \( j \) can be swapped, and – by Eq. (13) – their probabilities are all equal to \( 1/M \). It follows that:

\[
p_k = p(|s_k\rangle) = m_k/M = |\alpha_k|^2
\]

in obvious notation. This, as promised, is Born’s rule. When \( |\alpha_k|^2 \) are not commensurate, one can easily produce sequences of states that set up convergent bounds on \( p_{s_k} \) so that – when the probabilities are assumed to be continuous in the amplitudes – the interval containing \( p_k \) shrinks in proportion to \( 1/M \) for large \( M \).

We emphasize again that one could not carry out the basic step of our argument – the proof of the independence of the probabilities from the phases of the Schmidt expansion coefficients, Eq. (12) and below – for an equal amplitude pure state of a single, isolated system. The problem with:

\[
|\psi\rangle = \sum_k \exp(i\phi_k)|k\rangle
\]

is the accessibility of the phases. Consider, for instance; \( |\psi\rangle = (|0\rangle + |1\rangle - |2\rangle)/\sqrt{3} \) and \( |\psi'\rangle = (|2\rangle + |1\rangle - |0\rangle)/\sqrt{3} \). In the absence of entanglement there is no envariance and swapping of states corresponding to various \( k \)'s is detectable: Interference measurements (i.e., measurements of the observables with (phase-dependent) Hadamard eigenstates \( |1\rangle + |2\rangle; |1\rangle - |2\rangle, \text{etc.} \) would have revealed the difference between \( |\psi\rangle \) and \( |\psi'\rangle \). Indeed, given an ensemble of identical pure states a skilled observer should be able to confirm that they are pure and find out what they are. Loss of phase coherence is needed to allow for the shuffling of the states and coefficients.

Note that in our derivation environment and einselection play an additional, more subtle role: Once a measurement has taken place – i.e., a correlation with the apparatus or with the memory of the observer was established (e.g., Eqs. (21) and (22)) – one would hope that records will retain validity over a long time, well beyond the decoherence.
timescale. Thus, a “collapse” from a multitude of possibilities to a single reality (implied by Eq. (22) above) can be confirmed by subsequent measurements only in the einselected pointer basis.

With this in mind, it is easy to see that – especially on the macroscopic level – the einselected states are the only sensible choice as outcomes: Other sets of states lose correlation with the apparatus (or with the memory of the observer) far too rapidly – on the decoherence timescale – to serve as candidate events in the sample space.

We close this part of our discussion by calling reader’s attention to the fact that the above derivation did not rely on – or even invoke – reduced density matrices, which are at the very foundation of the decoherence program. Indeed, we have used envariance to derive Born’s rule, and, hence, in a sense, to justify the form and the uses of the reduced density matrices. More extensive discussion of this point shall be given elsewhere (Zurek, in preparation).

SUMMARY AND CONCLUSIONS: QUANTUM FACTS

In spite of the preliminary nature of much of the above (which would seem to make “Conclusions” premature) we point out that if one were forced to attach a single label to the topics explored above, quantum facts would be a possible choice. Quantum Darwinism approaches this theme directly: Quantum states, by their very nature, share epistemological and ontological role – they are simultaneously a description of the state, and ‘the dream stuff is made of’. One might say that they are epiontic. These two aspects may seem contradictory, but, at least in quantum setting, there is a union of these two functions.

Quantum Darwinism puts forward a specific theory of how the ontic aspect – reliable classical existence states – can emerge from the quantum substrate. We shall not repeat the arguments already given in detail. But one might sum up the key idea by pointing to the role of the redundancy: Tenuous quantum facts acquire objective existence when the information they about them is widely spread (and therefore becomes easily accessible). Approximate (exact) classicality obtains in the limit of a large (infinite) redundancy. Redundancy is a measure of classicality.

Envariance is, by contrast, a way to capture the most tenuous aspect of the quantum – the ignorance (and, hence, the essence of what is epistemic: the information). Quantum facts are the opposite of envariant properties. Quantum fact are invariant under envariance. Thus, in a sense, what we have accomplished is to “corral” the problem of the emergence of the classical from quantum states between two extremes: The case – exploited by quantum Darwinism – where quantum facts become solid and reliable, and the opposite, when some properties of these states are envariant, and, therefore, demonstrably inconsequential. Investigation, in terms of envariance and quantum Darwinism, of what lies inbetween these two extremes is still in its early stages.

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