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Pruning Deep Neural Networks Architectures with Evolution Strategy

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Abstract—Currently, Deep Neural Networks (DNNs) are used to solve all kinds of problems in the field of machine learning and artificial intelligence due to their learning and adaptation capabilities. However, most of the successful DNN models have a high computational complexity, which makes them difficult to deploy on mobile or embedded platforms. This has prompted many researchers to develop algorithms and approaches to help reduce the computational complexity of such models. One of them is called filter pruning where convolution filters are eliminated to reduce the number of parameters and, consequently, the computational complexity of the given model. In the present work, we propose a novel algorithm to perform filter pruning by using Multi-Objective Evolution Strategy (ES) algorithm, called DeepPruningES. Our approach avoids the need for using any knowledge during the pruning procedure and helps decision makers by returning three pruned DNN models with different trade-offs between performance and computational complexity. We show that DeepPruningES can significantly reduce a models computational complexity by testing it on three DNN architectures: Convolutional Neural Networks, Residual Neural Networks, and Densely Connected Neural Networks.

Index Terms—Deep Neural Networks, Filter Pruning, Multi-Objective Optimization, Evolution Strategies, Evolutionary Algorithm.

I. INTRODUCTION

Deep Neural Networks (DNNs) are among the most popular machine learning models used for computer vision applications at the moment. Although the basic theory behind DNNs was developed in the late 1980s [11], only recently they were able to surpass humans in such tasks [2], [3]. This is mainly due to recent advancements in big data analytics and general-purpose computing on graphics processing units (GPUs).

DNNs are data and model dependent. In order to achieve good classification results, DNNs need to be designed with millions of nodes, also called neurons, in its structure which requires a massive amount of computational power to use them. Furthermore, training such large DNNs with small amounts of data can lead to overfitting problems where the model learns the training data very well, but it is unable to generalize its knowledge to unseen data. For example, ImageNet, one of the most popular databases used for large-scale image classification, is composed of 14 million images distributed in 1,000 classes [4], [5]. A typical DNN capable of classifying such large databases may contain a few hundred million parameters. For instance, VGG-16, one of the top performer models in the ImageNet database in 2014, has 138 million parameters [6] and a computational complexity of $15.5 \times 10^9$ floating point operations (15.5 GFLOPS) [7].

In general, DNNs used for image recognition and classification tasks require powerful hardware such as GPU workstations and datacenters for training and inference. Consequently, these models are not suitable for use in consumer hardware, such as smartphones and portable appliances. Currently, a constant connection to the internet is required if a developer is interested in performing computer vision tasks in such devices. However, access to the internet may not always be available or it may not be reliable to perform such tasks seamlessly to users.

Thus, machine learning researchers are increasingly interested in finding novel ways of reducing the computational complexity of DNNs while maintaining their performance. There are mainly three approaches to achieve this goal: (1) designing DNNs architectures from the ground up taking into consideration the limitations of the target platform; (2) making use of specialized mobile hardware capable of running DNNs efficiently; and (3) modifying existing DNNs architectures by removing redundant parameters. Each one of these techniques is detailed in the following.

In the first approach, novel DNN operations and architectures can be created to take advantage of the particularities of mobile and embedded platforms. For instance, CondenseNets [8] and MobileNets [9] are two specialized DNN architectures created to be used on mobile platforms, such as smartphones. Both architectures make use of specialized convolutional operations, i.e., group and depth-wise convolutions, which decreases their computational complexity without degrading their performance. Likewise, Neural Architecture Search (NAS) can also be used to find DNN architectures suitable for mobile and embedded devices. The NetAdapt algorithm developed by Yang et al. is one classic example capable of generating suitable DNNs architectures given a certain resource budget. NetAdapt can find specific DNN architectures tailored to an individual device [10]. Another example is the FastDeepIoT algorithm developed by Yao et al. [11] where an initial highly accurate model is found, and, then the algorithm compresses the model by taking into consideration the execution time in a specific mobile and embedded device.

The second approach of dealing with the computational requirements of DNN models is to develop specialized mobile and embedded hardware. One of the most iconic examples is the Jetson embedded platforms developed by Nvidia which features a GPU capable of performing inference in real-
time using complex DNN models [12], [13]. Another area of intense interest is the use of Field-Programmable Gate Array (FPGA) for DNN model inference. Some researchers have adapted deep learning frameworks and libraries to work with FPGA platforms resulting in inference time comparable with the ones from applying GPUs [14], [15]. However, the use of such platforms does not address the problem of how to reduce the computational complexity of DNN models. In that sense, Ding et al. developed a method to reduce the computational complexity of Convolutional Neural Network (CNN) operations by using block-circulant weight matrices for implementation in FPGAs [16]. In their work, the weight matrices of each layer are converted to block-circulant matrices and inference operations are performed using Fast Fourier Transform multiplications resulting in reduced computational complexity for inference and memory use.

Pruning is considered a third approach to reduce the computational complexity of DNN models. In this case, the hypothesis is that many DNN models handcrafted by humans have too many unnecessary parameters, and the identification and elimination of such parameters can reduce the model complexity without degrading its performance. Ding et al. developed a technique called Auto-balanced Filter Pruning where entire feature maps can be removed from multiple layers based on their importance by using \( l2 \) regularization [17]. However, that technique requires that the initial DNN model be trained with strong regularization where weak feature maps are regularized with positive factors while strong ones are regularized with negative factors. This initial training phase makes it easier to identify the filters to be removed during the pruning process, but it requires the retraining of any DNN used for pruning which can take a long time. Luo et al. [18] also performed pruning of DNN models by removing entire feature maps from multiple layers. Their method is called ThiNet and it uses statistics information from the next layer instead of the information from the current layer being pruned. The method developed by Li et al. eliminates entire feature maps by computing the sum of the absolute values of the filters in a given layer, and removing the ones with the smallest sum [19].

It is also worth mentioning other approaches used to deal with the high computational demands of DNN models. For example, it is possible to perform weight quantization which reduces the number of bits needed to represent the weights [20]. An extreme form of weight quantization is used in binarized networks in which all weights are represented using a single bit [21], [22]. Another approach is to design DNN models that compute the residual mapping of a function, also known as shortcut or skip connections, this is used by Deep Residual Networks (ResNets) [23]. The residual mapping avoids the problem of the vanishing gradient when creating models with hundreds or thousands of layers. Residual mapping does not directly reduce the computational complexity of a DNN, but it helps in achieving high performance by using convolutional layers with smaller filter size. Densely Connected Convolutional Networks (DenseNets) [24] also use shortcut connections from previous layers, but, instead of using the output of only a single layer as in a ResNet, it uses the output from all previous layers as the input to the current one. In other words, each convolution layer in a DenseNet is connected to all subsequent ones in a fully-connected fashion. Similar to ResNets, the use of multiple shortcut connections between layers does not decrease the computational complexity of the network, but it allows the use of smaller networks with higher performance when compared with DNNs with no shortcut connections.

We believe that all the works done in building DNN models with the best performance possible should not be discarded when trying to find models with reduced computational complexity. In this paper, we leverage such works by proposing a novel pruning strategy to eliminate unnecessary parameters from existing top performing models. Specifically, we propose the elimination of unnecessary output filters from convolutional layers to reduce the models computational complexity. This approach is considered the most suitable one for most applications because it eliminates the need of having specialized hardware to run such models. It also gives us some insight in the structure of DNNs in general, and how most model could be created using a lot less parameters. In this sense, the pruning of a DNN model can be seen is a combinatorial optimization where we try to find the optimal combination of convolution filters in different layers in a given model. It can also be formulated as a multi-objective optimization problem where we have two conflicting objectives: (1) reduce the number of parameters in a given model as much as possible while (2) keeping the models performance as high accuracy as possible. We are also interested in prune the models without relying in any prior knowledge about them allowing us to prune many different DNN architectures with a single algorithm. The use of Evolutionary Computation (EC) algorithms is ideal for this case. Such algorithms are well suitable for combinatorial optimization with conflicting objectives. Specifically, Evolution Strategy (ES) [25] is one such algorithm where a population of candidate solutions is evolved by performing random perturbations in their parameter space.

Thus, the main objective of this paper is to propose the use of Evolution Strategy for pruning DNN models which we call DeepPruningES. We chose to use ES because of its simplicity, but powerful ability to find good solutions. We would like to highlight to the reader that we are interested in find good solutions instead of optimal ones. When pruning DNN models, we can only find solutions with some tradeoff between computational complexity and performance. The best solutions are in the so-called Pareto frontier where no solution is strictly better than the others. However, the task of finding the true Pareto frontier for a given model pruning task is not trivial. Hence, one of the contributions of our work is in providing a set of three solutions with different tradeoffs at the end of the pruning procedure. In this regard, a decision maker (DM) can choose the solutions that fits his/her needs the best. This is done by keeping track of the solutions in the population with the smallest training error, the smallest number of parameters, and the best one in between which we called the knee solution. Another contribution is that the proposed algorithm can prune Convolutional Neural Networks...
(CNN), Residual Neural Networks (ResNets), and Densely Connected Neural Networks (DenseNets) which, to the best of our knowledge, no other algorithm proposed in the literature is capable of pruning. The proposed algorithm uses two binary strings to encode ResNet and DenseNets. For ResNets, one string encodes the parameters in layers between the residual connections, and the other encodes the parameters of the residual connections. For DenseNets, one string encodes the parameters of each output layer from a bottleneck block, and the other encodes the parameters of the bottleneck blocks itself. Thus, we are able to prune the entirety of a ResNet or DenseNet model not just its intermediate layers.

The remaining sections of the present work is organized as follows. A detailed background about Deep Neural Networks models, such as CNN, ResNet, and DenseNet, DNN filter pruning, Evolution Strategies is given in Section II. The proposed algorithm is presented in Section III. The experimental design used to validate the algorithm is discussed in Section IV. The experimental results are analyzed in Section V followed by the conclusions and future work in Section VI.

II. BACKGROUND

A. Deep Neural Networks

A Deep Neural Network (DNN) is a computational model inspired by the way animals brains work. Its basic computational node is known as an artificial neuron which is composed of a set of weights and a transfer function. To compute the output of a neuron, first a weight operation is performed with its inputs and weights. Then, the result is passed through a non-linear function, known as activation function, to produce an output which can be used as inputs to subsequent neurons in a given DNN. In this sense, multiples neurons stacked together constitute a layer, and multiples layers stacked together constitute the architecture of a DNN. Mathematically, a DNN is defined as in Eq. 1 where \( i \) represents the layer number, \( X \) is a tensor representing the inputs of the entire DNN, \( O_i \) is a tensor representing the output of layer \( i \), \( f_i(\cdot) \) represents the activation function used in layer \( i \), and \( Z_i \) is the output of the weight operation, \( g_i(\cdot) \), between the output from the previous layer, \( O_{i-1} \), and the weights of the current layer, \( W_i \). The three most common used weight operations are defined in Eq. 2 and they are convolution, \( \odot \), max or average pooling, and matrix multiplication, used in fully-connected layers.

\[
\begin{align*}
O_1 &= X \\
O_i &= f_i(Z_i) \\
Z_i &= g_i(O_{i-1}, W_i)
\end{align*}
\]

The convolution layer was developed by LeCun et al. in 1989 [1], and allows DNNs to go deeper due to its parameter sharing nature which reduces the total number of parameters when compared with an equivalent network constructed using only fully-connected layers. The pooling layer further decreases the number of parameters in a DNN by subsampling its inputs with a max or average operation. Convolutional Neural Networks (CNNs) are constructed basically by stacking multiples convolution, pooling and fully-connected layers.

In the case of Residual Neural Networks (ResNets) [23], their architecture is composed of stacked blocks, called residual blocks, which is a combination of two convolution layers with one shortcut connection. If we define the output of the previous residual block as \( O_{i-1} \), we can define the output of the current block, \( O_i \), as in Eq. 3, where \( f(\cdot) \) is a rectified activation function, ReLU, i.e., \( f(x) = \max(0, x) \), \( W_{i1} \) are the weights of the first convolution layer, and \( W_{i2} \) are the weights of the second convolution layer. The visualization of such block can be seen in Fig. 1.

\[
O_i = f(W_{i2} \odot f(W_{i1} \odot O_{i-1}) + O_{i-1})
\]  

Densely Connected Neural Networks (DenseNets) [24] are also composed of blocks, called Dense Blocks, which consist of multiples convolution layers. Their main difference compared with ResNets is that the outputs of each convolution layer in a given Dense Block is used as input for all subsequent convolution layers from the same Dense Block. The number of output feature maps from any given convolution layer is called growth rate, \( k \). Because the outputs from previous layers are concatenate and used as the input of the current layer, the number of input feature maps in layer \( i \) is equal to \( k_0 + k \times (i - 1) \), where \( k_0 \) is the number of output feature maps from the first layer. Furthermore, in order to improve computational efficiency, each layer in a Dense Block is composed of a bottleneck layer followed by a convolution layer. A bottleneck layer is simply a convolution layer with \( 1 \times 1 \) convolution filters which receives all the inputs from previous layers. A transition layer is used between each Dense Block with the purpose of reducing the number of inputs to the next block. The transition layer is normally built by using a pooling layer followed by \( 1 \times 1 \) convolution layer. Fig. 2 illustrates the connections between layers in a Dense Block with three bottleneck layers.

B. Deep Neural Network Filter Pruning

The weights of a convolution layer, also known as filters or kernels, can be represented as a four dimensional tensor \( K \) with dimensions equal to \( (O \times D \times W \times H) \), where \( O \) is the total number of kernels with size \( (D \times W \times H) \), \( D \) is the depth of each kernel, \( W \) is the width of each kernel, and \( H \)
is the height of each kernel. Optionally, each one of the $O$ kernels can have a single parameter to represent its bias. If one would like to reduce the computational complexity of a single convolution layer, one approach would be to eliminate some of these $O$ kernels in their entirety. This approach is indeed one of the most common techniques used to reduce the computational complexity of DNN models, and it is known as filter pruning. This process is illustrated in Figs. 3 and 4 where a filter is first chosen to be eliminated, indicated by the dashed lines in Fig. 3, and, then, the entire DNN model is adjusted resulting in a pruned model, shown in Fig. 4. It is important to note that when filters are eliminated in layer $i$, the corresponding depths of all filters in the subsequent layer $i + 1$ need to be removed to produce a valid architecture.

In the literature, there are multiple ways of choosing which filters to eliminate in each convolution layer, a procedure known as filter selection. In general, some statistics from the current layer is needed to determine which filters to eliminate. The following are some of the most used decision criteria found in the literature [26]:

- **Mean activation**: It was used by Molchanov et al. in [27] where the mean activation of each filter from a given layer is computed, and only the filters with the highest mean activation are kept.
- **$l_1$-norm**: It was used by Li et al. in [19] where the importance of a filter is determined by its $l_1$-norm. Filters with smaller $l_1$-norm are considered unimportant compared with filters with large $l_1$-norm.
- **Average Percentage of Zeros (APoZ)**: It was proposed by Hu et al. in [28], and it eliminates the filters with a large number of zero activations during inference time.
- **Entropy**: In this case, the entropy of a single filter is computed by feeding input images to the network and grouping the corresponding outputs in different bins. The key idea is that unimportant filters will have very similar outputs with different inputs. It was proposed by Luo and Wu in [29].
- **Random**: It randomly select filters to be eliminated without using any prior knowledge of the DNN architecture or filters. In [26], Mittal et al. showed that eliminating filters at random produced comparable results with other selection criteria.

### C. Evolution Strategies

The work done by Mittal et al. in [26] shows that filter pruning with randomized heuristics produces results as good as when using heuristics based on prior knowledge. Moreover, the use of randomized heuristics was capable of producing excellent results even when used for automatic generation of DNN architectures [30]. Thus, the use of an algorithm that relies on random changes of candidate solutions is ideal for performing filter pruning.

The evolutionary computation (EC) family of algorithms possess such characteristic. Specifically, Evolution Strategies (ESs) [25] relies more heavily on the use of random changes than any other EC algorithms. ESs are meta-heuristic algorithms used to solve optimization problems. Initially, ESs were created to solve design problems automatically, but its creators quickly realized that they could be used to solve all sorts of optimization problems, such as binary optimization and combinatorial optimization. Similar to other EC algorithms, ESs are population-based algorithms where an individual in the population is a possible solution.

In general, ESs work in a loop, known as generations, where these two basic steps are performed [25]:

- In every generation, modify the individuals parameters by using recombination and/or mutation.
- Keep the best individuals for the next generation and repeat the process.

This family of algorithms uses the following standard notations: $(\mu/\rho, \lambda) - ES$ or $(\mu/\rho+\lambda) - ES$, where $\mu$ represents the number of individuals that will be kept and used as parents to the next generation, $\lambda$ represents the number of individuals that will be generated at every new generation from the $\mu$ parents selected in the previous generation, and $\rho$ is the number of individuals used for recombination. The $comma (,)$ and $plus (+)$ operators define how the selection operator of the algorithm will work. In the $comma$ version, the algorithm will select $\mu$ individuals only from $\lambda$ offspring, and the old parents will be completely discarded even if they are better than the offspring. In the $plus$ version, the algorithm will select $\lambda$ individuals from all $\mu + \lambda$ individuals, i.e., parents and offspring are used together during the selection procedure. Thus, the $plus$ version is an elitist algorithm where the best solutions are kept in the population indefinitely. Recombination is an optional step in ESs and it works by randomly selecting $\rho$ individuals from $\lambda$ parents. In ES, recombination always produces one new individual from two or more parents, and each one of its parameters are selected from one of the parents at random. Let $P_j$ be a vector representing the $N$ parameters of a parent individual $j$, then the parameters of a new offspring, $O$, will be equal to

$$O_k := rand_j(P_{jk}), \quad \text{with } 1 \leq k \leq N \text{ and } j = 1, ..., \rho. \quad (4)$$

In the present work, DNN models are pruned with the help of ES and without the use of any prior knowledge. In our case, the selection pressure produced by the evolutionary process will be responsible for finding a good combination of filters to be kept in a pruned DNN model.

### III. Proposed Algorithm

The general framework of the proposed DeepPruningES is shown in Algorithm [1]. It is comprised mainly of the standard ES $plus$ version adapted to perform DNN pruning. It receives as input the size of the offspring population $(\lambda_{size})$, the maximum number of generations that the algorithm will run $(gen)$, the original DNN model $(dnn)$ that will be pruned, the number of epochs to be used during an individual evaluation $(\epsilon_{eval})$, the learning rate used during an individual evaluation $(\alpha_{eval})$, and the number of epochs $(\epsilon_{fine})$ and learning rate $(\alpha_{fine})$ to be used during the fine-tuning of the best individuals found at the end. The final output of the proposed DeepPruningES will be three DNN models with different trade-offs called here as knee, boundary heavy, and boundary light solutions.
There are four important pieces in Algorithm 1: Population Initialization (line 1), Knee and Boundary Selection (line 4), Offspring Generation (line 5), and Fine-Tuning of the best solutions found (line 7). Each one of them will be explained in detail in the following subsections, as well as the genetic representation of an individual.

A. Genetic Representation of an Individual

Representation is one of the most important aspects of any population-based algorithm. In the proposed DeepPruningES, we use two binary strings to represent the filters of a DNN model. It is important to note that our proposed algorithm only prunes convolution layers because they have a much higher computational complexity compared to fully-connected ones [19]. In this representation, each bit represents one single filter. For example, if we want to represent two convolution layers, one with 32 filters and the other with 64 filters, we would need a binary string with 96 bits, and a bit assigned to zero means that the corresponding filter would be eliminated.

However, the representation depends on the type of DNN model being pruned. When pruning simple CNN models, only a single binary string is used, and each bit will represent a filter of a convolution layer.
Fig. 5. Binary representation of a CNN model. All layers are convolution layers.

Fig. 6. Binary representation of a CNN model after removing half of the filters from each layer.

Fig. 7. Binary representation of a ResNet with 20 layers.

B. Population Initialization

The first step of the proposed DeepPruningES is to initialize a population of $3 + \lambda_{size}$ individuals. First, all individuals are initialized as an identical copy of the original DNN model being pruned. Then, their genes are mutated with probability $p_m$ to generate a pruned version of the original model. Finally, the main for-loop, shown in Algorithm 1 line 2, can be executed.

The main idea of initializing the population in this way instead of a purely randomized initialization is that the population will resemble the original DNN model allowing us to achieve good results even when using smaller populations and fewer generations.

C. Individual Evaluation

Before we can talk about how the knee and boundaries solutions are selected, it is important to explain how an individual is evaluated in the proposed algorithm. This process is achieved by, first, eliminating the filters indicated in the
individuals gene, and, then, fine tuning the resulting pruned model using Stochastic Gradient Descent (SGD) for \( e_{\text{eval}} \) epochs and \( \alpha_{\text{eval}} \) learning rate in a small sample of the dataset that was used to train the original model. This allows the individual to restore some of its performance before the selection operation is performed. More details about how the dataset is sampled and for how long the pruned model is fine-tuned are presented in Section IV.

The individuals are evaluated in two objectives: the training error in the chosen dataset, and the number of floating-point operations (FLOPs) for a single input. These are conflicting objectives because it is not possible to maintain a small training error with a small number of floating-point operations. We choose to use the number of FLOPs instead of the total number of parameters or the total number of filters because the time to compute the outputs of any given layer is also dependent on the size of its inputs not only on its number of parameters. Thus, the total number of FLOPs of a model is considered a better measurement of its computational complexity.

**D. Knee and Boundary Selection**

In many real-world problems, decision makers (DM) have to optimize multiple parameters at the same time to accomplish some desired behavior. These problems are called multi-criteria decision making (MCDM) problems, where there is no single desired or optimal output to a given problem. In many cases, these problems are multi-objective optimization problems where the modification of a single parameter from a candidate solution will change the output of different objective functions. However, not all possible solutions are of interest in MCDM problems. Only the solutions located in the so-called Pareto optimal front in the objective space is of interest. This region is where solutions that are no worse than others in all objective functions are located. The knee solution is always found in the Pareto optimal front. The ability to locate the knee solution is of extreme importance in MCDM problems because this is the solution that can improve the most in any of the objectives with small changes in its parameters [31].

In the case of DNNs, the parameters are the number of filters still present in the architecture, and the objective functions that we would like to optimize are the DNN’s computational complexity, and its accuracy in a given dataset. Furthermore, we would like to provide three DNN models to DMs. So, depending on what type of device the DNN is being deployed on, one model could be more suitable than the others.

Commonly, all the solutions located in Pareto optimal front need to be found before the knee can be identified. Chiu et al. in [31] showed that knee is the solution in Pareto optimal front with the smallest Manhattan distance to the origin point of the objective space. However, finding the whole Pareto optimal front is a non-trivial task. Thus, we propose a modification of the Minimum Manhattan Distance algorithm developed by Chiu et al. to locate the knee solution. Because we are not interested in finding the whole Pareto optimal front, we can use the geometric position of all candidate solutions in the objective space to facilitate the search of the knee solution. This is done by simply computing the Manhattan distances of all candidate solutions, and finding the one with the smallest distance.

Thus, the proposed algorithm to perform the Knee and Boundary Selection is shown in Algorithm 2, where three individuals, called here as knee, boundary heavy, and boundary light solutions, are always selected at every generation. First, we need to evaluate the individuals in the population, \( P \), and to obtain their training error \( f_1 \) and number of FLOPs \( f_2 \). Second, we need to find the minimum and maximum values of both objectives in the population (Algorithm 2 line 2). Third, the Boundary Heavy solution will be the individual with the smallest training error in the population (i.e., the individual with its \( f_1 \) equal to \( \min(f_1) \)). While the Boundary Light solution will be the individual with the smallest number of FLOPs in the population, the individual with its \( f_2 \) equal to \( \min(f_2) \). Finally, we compute the Manhattan Distance of all individuals in the population by following the equation in Algorithm 2 line 6, and select the individual with the Minimum Manhattan Distance (MMD) to be our knee solution. The selection procedure is illustrated in Fig. 9 and only these three individuals are returned at the end of selection.

**E. Offspring Generation**

After the three individuals from the knee and boundaries are selected, the algorithm will use them to generate new offspring. First, from these three individuals, one is randomly selected to be the parent of a single offspring. Then, each one of its gene elements is randomly flipped with probability equal to \( p_m \). This process is repeated until we have an offspring population equal to \( \lambda_{\text{size}} \). It is also important to point out that we are following the standard ES algorithm where only the knee and boundaries solutions are used to generate a new offspring population. Thus, a newly created offspring is never used to generate another offspring. This method ensures
Algorithm 2: Knee and Boundary Selection

| Input : Individuals in the Population (P), original DNN model (dnn), number of epochs for individual evaluation (e_{eval}), learning rate for individual evaluation (α_{eval}). |
| Output: Three individuals: knee solution (µ.knee), boundary heavy solution (µ.heavy) and boundary light solution (µ.light). |

1 Evaluate Population(P, dnn, e_{eval}, α_{eval});
2 Find min(f_1), min(f_2), max(f_1), max(f_2);
3 µ.heavy ← P_i, where f_1(P_i) = min(f_1);
4 µ.light ← P_j, where f_2(P_j) = min(f_2);
5 for k = 1 to len(P) do
6     dist(k) = \frac{f_1(P_k) - min(f_1)}{max(f_1) - min(f_1)} + \frac{f_2(P_k) - min(f_2)}{max(f_2) - min(f_2)};
7 end
8 µ.knee ← P_k, where P_k has the minimum dist(k);
9 return µ.knee, µ.heavy, µ.light;

Fig. 9. Example of the knee, boundary heavy, and boundary light solutions.

that new offspring are always generated closer to the knee or boundary solutions allowing the algorithm to exploit these regions of interest efficiently.

F. Fine Tuning

The last step of the proposed algorithm is to fine tune the knee and boundaries solutions found in the last generation. They are fine-tuned using Stochastic Gradient Descent (SGD) during e_{fine} epochs and learning rate equal to α_{fine}. Different from the Individual Evaluation, the fine-tuning process uses the entire training set in order to improve these three solutions as much as possible. The number of epochs during fine-tuning is also higher than the one used during individual evaluation. At the end of this process, the three solutions are saved on disk for late use.

IV. EXPERIMENTAL DESIGN

In this section, we discuss the experimental design used to evaluate the proposed DeepPruningES. We begin by presenting the DNN architectures tested with the proposed work. Then, we analyze the algorithm parameters that was used to prune those DNN architectures. Furthermore, all results presented in this work were obtained using a Laptop PC with a Core i7-8750H CPU, 16 GB of RAM, and a NVIDIA GTX 1060 6GB GPU running PyTorch on Windows 10 Pro 64-bits.

A. Proposed DNN Architectures for Pruning

The proposed DeepPruningES is evaluated on state-of-the-art DNN architectures. As stated in the introduction of this work, the proposed algorithm can prune three types of DNN architectures: Convolutional Neural Networks (CNN), Residual Neural Networks (ResNet), and Densely Connected Neural Networks (DenseNets). We choose to test the algorithm with DNNs from these architectures given a challenging dataset.

The CIFAR10 dataset created by Krizhevsky [32] was the chosen one to be used in our experiments because, besides being a challenging dataset, there are many state-of-the-art DNNs widely reported to allow us to stay focus on quantifying the quality of the pruning task. This dataset contains 50,000 training images and 10,000 test images distributed in a total of 10 classes. All images are in colors and have size equal to 32 × 32 pixels. Some examples of images found in the CIFAR10 dataset can be seen in Fig. 10. There is also the CIFAR100 dataset, which contains 100 classes instead of 10. However, there is a lack of DNN pruning results using it in the current literature. Thus, all tests were performed using only the CIFAR10 dataset.

Specifically, we choose to evaluate the proposed algorithm with the following networks: VGG16, VGG19, ResNet56, ResNet110, DenseNet50, and DenseNet100. The VGG16 and VGG19 [6] are two examples of CNNs created in 2014 which achieved good results in challenging image classifications datasets, such as CIFAR10 and ImageNet. As the name
suggests, VGG16 has a total of 16 layers while VGG19 has a total of 19 layers. The VGG16 and VGG19 have computational complexities of $3.15 \times 10^8$ and $4.01 \times 10^8$ FLOPs, respectively. Their original and fully trained architectures can achieve a test error of 6.06% and 6.18% in the CIFAR10 dataset, respectively. The ResNet56 and ResNet110 are two examples of ResNets, and they are the ones first proposed by the authors in [23] to be used with the CIFAR10 dataset. Their computational complexities are of $1.27 \times 10^8$ and $2.57 \times 10^8$ FLOPs, respectively. When fully trained, ResNet56 and ResNet110 can achieve a test error of 6.63% and 6.2% on CIFAR10, respectively. The ResNet56 contains a total of 56 layers, while the ResNet110 contains a total of 110 layers. For last, the DenseNet50 and DenseNet100 are also some of the proposed architectures based on the DenseNets authors to classify the CIFAR10 dataset. They have 50 and 100 layers, respectively, computational complexity of $0.93 \times 10^8$ and $3.05 \times 10^8$ FLOPs, respectively, and test error of 6.92% and 5.66%, respectively. Table I sums up the details about the DNN architectures used to evaluate the proposed algorithm.

B. Algorithm Parameters

The parameters values used to evaluate the proposed DeepPruningES algorithm are shown in Table II. Throughout this study, all results shown related to the proposed algorithm are obtained by using the parameters from Table II. If some results are obtained by using a different set of parameters, the difference is highlighted in the discussions. The next paragraphs will discuss the influence of each parameter in the final output of the DeepPruningES.

The size of the offspring ($\lambda_{size}$) defines how many offspring is generated at each generation. More offspring helps the algorithm in covering more of the objective space and increases the probability of finding good solutions. Likewise, a large number of generations ($gen$) also increases the probability of finding good solutions. However, when the number of offspring and/or generation increases, the computational cost to run the algorithm also increases. In our tests, 20 offspring and 10 generations are considered enough to achieve good results given affordable computational complexity.

The mutation probability ($p_{mut}$) is the probability of a single bit of an individuals gene being inverted. A very small mutation probability would take too long to produce significant changes in the population, which would require more individuals and more generations to reach a satisfactory result; while a very large mutation probability will produce too many changes in the population resulting in no improvement at all.. In our experiments, a probability of 0.1 is regarded as the best value heuristically that makes the individuals change in a way that improves the overall quality of the population.

The number of epochs and learning rate used during an individual evaluation ($e_{eval}, \alpha_{eval}$) will control how long an individual will be fine-tuned before its training error and number of FLOPs is evaluated. The values for these two parameters depend on the computational power available to the user. Ideally, if an individual can be fine-tuned much longer, this will give us a better sense of its overall fitness. We chose to use 5 epochs, which is compatible with other EC based algorithms used for DNNs architecture generation and pruning [18], [29], [33], and a learning rate of 0.1. In our case, we are using a large learning rate to improve the accuracy of the solutions faster.

For last, the number of epochs and learning rate to fine-tune the final solutions ($e_{fine}, \alpha_{fine}$) will improve the training error of the three final solutions. We chose to fine-tune them for 50 epochs with a learning rate of 0.01, which is also compatible with others’ work [19], [17]. In this case, we use a smaller learning rate than that during an individual evaluation to avoid training instabilities due to excessive removed filters. It is possible to improve the solutions found by the algorithm even further if they are fine-tuned for longer than 50 epochs. However, our limited computational power did not allow us to experiment with very long fine-tuning sessions.

As stated before, the proposed DeepPruningES uses the ES plus version for performing DNN filter pruning, which is an elitist version of ES where the best individuals are never lost. In Section V, an experiment using the ES comma version is presented, and it does not yield good results because the algorithm eliminates too many filters even from the boundary heavy and knee solutions.

V. EXPERIMENTAL RESULTS AND DISCUSSION

In this section, our pruning results are first presented and compared with peer competitors. Then, a detailed discussion about the obtained results follows.

A. Experimental Results

To evaluate the proposed DeepPruningES, the DNN architectures shown in Table I were pruned 10 times. The final results are shown in Table III where, for each DNN architecture, we show the best and mean test error and number of FLOPs for the knee, boundary heavy, and boundary light

| **DNN** | # layers | # FLOPs | Test error on CIFAR10 |
|---------|----------|---------|-----------------------|
| VGG16   | 16       | $3.15 \times 10^8$ | 6.06% |
| VGG19   | 19       | $4.01 \times 10^8$ | 6.18% |
| ResNet56| 56       | $1.27 \times 10^8$ | 6.63% |
| ResNet110| 110     | $2.57 \times 10^8$ | 6.2%  |
| DenseNet50| 50      | $0.93 \times 10^8$ | 6.92% |
| DenseNet100| 100    | $3.05 \times 10^8$ | 5.66% |

| **Advanced Parameters** | **Value** |
|-------------------------|-----------|
| Offspring size ($\lambda_{size}$) | 20         |
| Maximum number of generations ($gen$) | 10         |
| Mutation probability ($p_{mut}$) | 0.1        |
| Number of epochs for individual evaluation ($e_{eval}$) | 5          |
| Learning rate for individual evaluation ($\alpha_{eval}$) | 0.1        |
| Number of epochs for fine-tuning ($e_{fine}$) | 50         |
| Learning rate for fine-tuning ($\alpha_{fine}$) | 0.01       |
solutions after fine-tuning for 50 epochs. In Table IV, we show the results reported by the peer competitors who used the same CIFAR10 dataset. Other works found in the literature are not shown here because they used the ImageNet dataset, which is too big for us to run in our equipment and does not produce comparable results with other datasets.

For the VGG16 and VGG19, the proposed pruning algorithm obtain similar results. The boundary heavy solutions have an average of 20% reduction in the number of FLOPs, the boundary light solutions an average of 70% reduction, and the knee solutions an average of 57% reduction. The average test errors of the boundary heavy, boundary light, and knee solutions are around 8.6% ∼ 8.77%, 11.51% ∼ 12.01%, and 9.58% ∼ 9.87%, respectively. These are the expected results where the boundary heavy solutions were the least pruned, and also the ones with the highest accuracy. On the other hand, the boundary light solutions were the most pruned, and the ones with the lowest accuracy. For last, the knee solutions obtained pruned results and accuracy in between.

For the ResNet56 and ResNet110, the boundary heavy solutions see an average of 15% reduction in the number of FLOPs, the boundary light solutions an average of 77% reduction in the number of FLOPs, and the knee solutions an average of 59% reduction in the number of FLOPs. After fine-tuning, the boundary heavy, boundary light, and knee solutions obtain test errors on average of 7.93% ∼ 8.77%, 12.9% ∼ 13.46%, and 9.42% ∼ 9.98%, respectively. Even though ResNets are smaller than conventional CNNs, such as VGG16 and VGG19, by nature, these results show that they can still be pruned without losing too much accuracy.

For the DenseNet50 and DenseNet100, the average reduction in the number of FLOPs for the boundary heavy, boundary light and knee solutions are around 16%, 73%, and 50%, respectively. On the other hand, the average test errors for the boundary heavy, boundary light and knee solutions are 8.39% ∼ 9.26%, 11.9% ∼ 14.8%, and 9.37% ∼ 10.43%, respectively. These results are similar to the ones obtained with the pruning of ResNets which are impressive considering that DenseNets are even smaller than ResNets.

B. Results Discussion

As shown by the results in Table III, the proposed pruning algorithm can reduce the number of FLOPs in DNN architectures significantly while preserving good test errors. When comparing our results with the ones obtained by Li et al. in [19], the proposed DeepPruningES obtains higher pruning ratios with slight worse test errors on CIFAR10. Ding et al. in [17] were able to obtained better pruning ratios and test errors than the ones from the proposed algorithm. However, in their algorithm, the DNNs needs to be trained with strong and specialized regularization. The proposed DeepPruningES uses only the information about the DNN classification performance and computational complexity to prune it. Thus, without using any regularization or extra knowledge about the DNN being pruned, DeepPruningES is still capable of achieving quality results comparable with the ones from Ding et al. in [17].

The test errors showed in Table III can still be further improved given more computational resources are available. To demonstrate this, we choose one pruned solution from VGG16 to be retrained from scratch which means that all weights are reinitialized to random values and the network is trained from there. More specifically, a knee solution with 58.64% pruning ratio and test error of 9.94% is chosen to be retrained from scratch for 200 more epochs. The training curves for this knee solution is shown in Fig. 11. Its final test error is equal to 8.23% which is considerably better than the test error obtained after 50 fine-tuning epochs.

Our results are more detailed than the ones from peer competitors because six different DNN models with three different architectures and number of layers are evaluated. For example, to the best of our knowledge, no work in the literature has attempted to prune DenseNet architectures. Although DenseNets already have smaller computational complexity with better classification performance than standard CNNs, our results showed that they can still be pruned and still maintain competitive classification performance.

For last, we compared the proposed DeepPruningES, which is based on the elitist ES plus version, with a non-elitist comma version. The evolution of the population for 10 generation for the plus version is shown in Fig. 12 and for the comma version is shown in Fig. 13. In the proposed plus version, it is easier to keep the individual with the best training error throughout the generations than in the comma version. Moreover, the final solutions found when using the comma version are very close to each other in terms of error and number of FLOPs which defeats our objective of getting multiple solutions with different trade-offs. Thus, if one is going to use the proposed DeepPruningES for pruning DNN architectures, the plus versions of the algorithm is the suggested one to use.

VI. CONCLUSIONS AND FUTURE WORK

In this work, we propose an algorithm for pruning Deep Neural Networks architectures using Evolution Strategies, called here as DeepPruningES. The pruning is performed by
TABLE III
PRUNING RESULTS OBTAINED WITH THE PROPOSED DEEPPRUNINGES.

| DNN Model       | DeepPruningES |
|-----------------|----------------|
|                 | Solution Knee | Test error (best) | Test error (mean) | # FLOPs (best) | # FLOPs (mean) | % Pruned (best) | % Pruned (mean) |
| VGG16           | Boundary Heavy| 9.04%            | 9.58%            | 1.09 × 10⁸     | 1.22 × 10⁸     | 65.40%         | 61.58%         |
|                 | Boundary Light| 8.21%            | 8.6%             | 2.15 × 10⁸     | 2.49 × 10⁸     | 32.01%         | 20.88%         |
|                 |                | 10.51%           | 11.41%           | 0.88 × 10⁸     | 0.9 × 10⁸      | 72.17%         | 71.36%         |

|                 | Solution Knee | Test error (best) | Test error (mean) | # FLOPs (best) | # FLOPs (mean) | % Pruned (best) | % Pruned (mean) |
| VGG19           | Boundary Heavy| 9.04%            | 9.58%            | 1.53 × 10⁸     | 1.72 × 10⁸     | 61.86%         | 57.15%         |
|                 | Boundary Light| 8.21%            | 8.6%             | 2.7 × 10⁸      | 3.12 × 10⁸     | 32.56%         | 22.28%         |
|                 |                | 10.53%           | 12.03%           | 1.13 × 10⁸     | 1.18 × 10⁸     | 71.74%         | 70.69%         |

|                 | Solution Knee | Test error (best) | Test error (mean) | # FLOPs (best) | # FLOPs (mean) | % Pruned (best) | % Pruned (mean) |
| ResNet56        | Boundary Heavy| 9.28%            | 9.98%            | 0.432 × 10⁸    | 0.523 × 10⁸    | 66.23%         | 59.15%         |
|                 | Boundary Light| 8.11%            | 8.77%            | 1.01 × 10⁸     | 1.08 × 10⁸     | 21.31%         | 15.23%         |
|                 |                | 11.42%           | 13.36%           | 0.244 × 10⁸    | 0.286 × 10⁸    | 80.80%         | 77.67%         |

|                 | Solution Knee | Test error (best) | Test error (mean) | # FLOPs (best) | # FLOPs (mean) | % Pruned (best) | % Pruned (mean) |
| ResNet110       | Boundary Heavy| 8.66%            | 9.42%            | 0.905 × 10⁸    | 1.03 × 10⁸     | 64.84%         | 59.89%         |
|                 | Boundary Light| 7.43%            | 7.93%            | 2.14 × 10⁸     | 2.21 × 10⁸     | 16.72%         | 14.14%         |
|                 |                | 10.27%           | 12.9%            | 0.43 × 10⁸     | 0.56 × 10⁸     | 83.29%         | 77.86%         |

|                 | Solution Knee | Test error (best) | Test error (mean) | # FLOPs (best) | # FLOPs (mean) | % Pruned (best) | % Pruned (mean) |
| DenseNet50      | Boundary Heavy| 8.91%            | 9.26%            | 0.756 × 10⁸    | 0.779 × 10⁸    | 19.16%         | 16.59%         |
|                 | Boundary Light| 13.04%           | 14.8%            | 0.229 × 10⁸    | 0.244 × 10⁸    | 75.53%         | 73.91%         |
|                 |                |                  |                  |                |                |                 |                 |
|                 | Solution Knee | Test error (best) | Test error (mean) | # FLOPs (best) | # FLOPs (mean) | % Pruned (best) | % Pruned (mean) |
| DenseNet1100    | Boundary Heavy| 9.04%            | 9.47%            | 1.11 × 10⁸     | 1.21 × 10⁸     | 63.04%         | 60.31%         |
|                 | Boundary Light| 8.34%            | 8.39%            | 2.46 × 10⁸     | 2.49 × 10⁸     | 19.33%         | 18.24%         |
|                 |                | 10.47%           | 11.90%           | 0.82 × 10⁸     | 0.879 × 10⁸    | 73.09%         | 71.16%         |

Fig. 12. Evolution of the population when pruning VGG16 using the plus version of the proposed DeepPruningES.

Fig. 13. Evolution of the population when pruning VGG16 using the comma version of the proposed DeepPruningES.

eliminating convolution filters from multiples layers throughout the network architecture. While other works use the current layers statistics to decide which filter to eliminate, the proposed algorithm eliminates filters by exploiting the objective space of each solution in the population. Moreover, to the best of our knowledge, the current work is one of the firsts to frame the pruning problem as a multi-objective optimization problem. In this sense, the proposed algorithm can find three different solutions with different trade-offs between training/test error and computational complexity without relying on any prior knowledge from the architecture being pruned.

The proposed DeepPruningES can achieve competitive results with only 20 particles and 10 generations when compared with state-of-the-art peer competitors work. The algorithm is also one of the firsts to pruning multiples DNN architectures,
such as CNNs, ResNets, and DenseNets architectures. Our results are limited only by our available computational power. With more computational resources available, the proposed algorithm could achieve even better results.

For future works, we will develop an algorithm for searching neural network architectures automatically and integrate it with the proposed pruning algorithm. This will create an algorithm capable of finding new DNN architectures with good performance while maintaining a small computational complexity.

REFERENCES

[1] Y. LeCun, B. Boser, J. S. Denker, D. Henderson, R. E. Howard, W. Hubbard, and L. D. Jackel, “Backpropagation Applied to Handwritten Zip Code Recognition,” Neural Computation, vol. 1, no. 4, pp. 541–551, Dec. 1989. [Online]. Available: http://www.mitpressjournals.org/doi/pdf/10.1162/neco.1989.1.4.541

[2] K. He, X. Zhang, S. Ren, and J. Sun, “Delving Deep into Rectifiers: Surpassing Human-Level Performance on ImageNet Classification,” in 2015 IEEE International Conference on Computer Vision (ICCV). Santiago, Chile: IEEE, Dec. 2015, pp. 1026–1034. [Online]. Available: http://ieeexplore.ieee.org/document/7410480/

[3] S. Ioffe and C. Szegedy, “Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift,” in Proceedings of the 32nd International Conference on Machine Learning, ser. Proceedings of Machine Learning Research, vol. 37. Lille, France: PMLR, Jul. 2015, pp. 448–456. [Online]. Available: http://proceedings.mlr.press/v37/ioffe15.html

[4] J. Deng, W. Dong, R. Socher, L.-J. Li, Kai Li, and Li Fei-Fei, “ImageNet: A large-scale hierarchical image database,” in 2009 IEEE Conference on Computer Vision and Pattern Recognition. Miami, FL: IEEE, Jun. 2009, pp. 248–255. [Online]. Available: http://ieeexplore.ieee.org/document/5206848/

[5] O. Russakovsky, J. Deng, H. Su, J. Krause, S. Satheesh, S. Ma, Z. Huang, A. Karpathy, A. Khosla, M. Bernstein, A. C. Berg, and L. Fei-Fei, “ImageNet Large Scale Visual Recognition Challenge,” International Journal of Computer Vision, vol. 115, no. 3, pp. 211–252, Dec. 2015. [Online]. Available: http://link.springer.com/10.1007/s11263-015-0816-y

[6] K. Simonyan and A. Zisserman, “Very Deep Convolutional Networks for Large-Scale Image Recognition,” arXiv:1409.1556, Sep. 2014. [Online]. Available: http://arxiv.org/abs/1409.1556

[7] W. Li, X. Zha, and S. Gong, “Person Re-Identification by Deep Joint Learning of Multi-Loss Classification,” in Proceedings of the Twenty-Sixth International Joint Conference on Artificial Intelligence. Melbourne, Australia: International Joint Conferences on Artificial Intelligence Organization, Aug. 2017, pp. 2194–2200. [Online]. Available: https://www.ijcai.org/proceedings/2017/305

[8] G. Huang, S. Liu, L. d. Maaten, and K. Weinberger, “CondenseNet: An Efficient DenseNet Using Learned Group Convolutions,” in 2018 IEEE/CVF Conference on Computer Vision and Pattern Recognition. Salt Lake City, UT: IEEE, Jun. 2018, pp. 2752–2761. [Online]. Available: https://ieeexplore.ieee.org/document/8573839/

[9] A. G. Howard, M. Zhu, B. Chen, D. Kalenichenko, W. Wang, T. Weyand, M. Andreetto, and H. Adam, “MobileNets: Efficient Convolutional Neural Networks for Mobile Vision Applications,” arXiv:1704.04861, Apr. 2017, arXiv: 1704.04861. [Online]. Available: http://arxiv.org/abs/1704.04861

[10] T.-J. Yang, A. Howard, B. Chen, X. Zhang, A. Go, M. Sandler, V. Sze, and H. Adam, “NetAdapt: Platform-Aware Neural Network Adaptation for Mobile Applications,” in Computer Vision ECCV 2018, V. Ferrari, M. Hebert, C. Sminchisescu, and Y. Weiss, Eds. Cham: Springer International Publishing, 2018, vol. 11214, pp. 289–304. [Online]. Available: http://link.springer.com/10.1007/978-3-030-02490-4_18

[11] S. Yao, Y. Zhao, H. Sun, L. Li, L. Su, and T. Abdelzaher, “FastDeepIoT: Towards Understanding and Optimizing Neural Network Execution Time on Mobile and Embedded Devices,” in Proceedings of the 16th ACM Conference on Embedded Networked Sensor Systems - SenSys ’18. Shenzhen, China: ACM Press, 2018, pp. 278–291. [Online]. Available: http://dl.acm.org/citation.cfm?doid=3275783.3275787

[12] H. Halawa, H. A. Abdelhafez, A. Boktor, and M. Ripeanu, “NVIDIA Jetson Platform Characterization,” in Euro-Par 2017: Parallel Processing, F. F. Rivera, T. F. Pena, and J. C. Cabaleiro, Eds. Cham: Springer International Publishing, 2017, vol. 10417, pp. 92–105. [Online]. Available: http://link.springer.com/10.1007/978-3-319-64203-1_7

[13] M. Su, J. Tan, C.-Y. Lin, J. Ye, C.-H. Wang, and C.-L. Hung, “Constructing a Mobility and Acceleration Computing Platform with NVIDIA Jetson TK1,” in 2015 IEEE 17th International Conference on High Performance Computing and Communications, 2015 IEEE 7th International Symposium on Cybersecurity and Safety, and 2015 IEEE 12th International Conference on Embedded Software and Systems. New York, NY: IEEE, Aug. 2015, p. 1853. [Online]. Available: https://ieeexplore.ieee.org/document/7336422/

[14] D. Danopoulos, K. Zachris, and D. Soudris, “Acceleration of image classification with Caffe framework using FPGA,” in 2018 7th International Conference on Modern Circuits and Systems Technologies (MOCAST). Thessaloniki: IEEE, May 2018, pp. 1–4. [Online]. Available: https://ieeexplore.ieee.org/document/8376580/

[15] J. Bottleson, S. Kim, J. Andrews, P. Bindu, D. N. Murthy, and J. Jin, “cICaffe: OpenCL Accelerated Caffe for Convolutional Neural Networks,” in 2016 IEEE International Parallel and Distributed Processing Symposium Workshops (IPDPSW). Chicago, IL, USA: IEEE, May 2016, pp. 50–57. [Online]. Available: http://ieeexplore.ieee.org/document/7529850/

[16] C. Ding, G. Yuan, X. Ma, Y. Zhang, J. Tang, Q. Qiu, X. Lin, B. Yuan, S. Liao, Y. Wang, Z. Li, N. Liu, Y. Zhao, C. Wang, X. Qian, and Y. Bai, “CIRCNN: accelerating and compressing deep neural networks using block-circuit weights matrices,” in the 50th Annual IEEE/ACM International Symposium on Microarchitecture - MICRO-50 ’17. Cambridge, Massachusetts: ACM Press, 2017, pp. 395–408. [Online]. Available: http://dl.acm.org/citation.cfm?doid=3123939.3124552

[17] X. Ding, G. Ding, J. Han, and S. Tang, “Auto-Balanced Filter Pruning for Efficient Convolutional Neural Networks,” in AAAI Conference on Artificial Intelligence, New Orleans, Louisiana, USA, 2018. [Online]. Available: https://aaai.org/ocs/index.php/AAAI/AAAI18/paper/view/16982/16417

[18] J.-H. Luo, J. Wu, and W. Lin, “ThiNet: A Filter Level Pruning Method for Deep Neural Network Compression,” in 2017 IEEE International Conference on Computer Vision (ICCV). Venice: IEEE, Oct. 2017, pp. 5068–5076. [Online]. Available: http://ieeexplore.ieee.org/document/8237803/

[19] H. Li, A. Kadar, I. Durdanovic, H. Samet, and H. P. Graf, “Pruning Filters for Efficient ConvNets,” in International Conference on Learning Representations, Toulon, France, 2017.

[20] S. Han, H. Mao, and W. J. Dally, “Deep Compression: Compressing Deep Neural Networks with Pruning, Trained Quantization and Huffman Coding,” in International Conference on Learning Representations, San Juan, Puerto Rico, USA, 2016, arXiv: 1510.00149. [Online]. Available: http://arxiv.org/abs/1510.00149

[21] M. Courbariaux, Y. Bengio, and J.-P. David, “BinaryConnect: Training Deep Neural Networks with binary weights during propagations,” in Advances in Neural Information Processing Systems 28. Curran Associates, Inc., 2015, pp. 3123–3131.

[22] I. Hubara, M. Courbariaux, D. Soudry, R. El-Yaniv, and Y. Bengio, “Binarized Neural Networks,” in 2016 IEEE Conference on Neural Information Processing Systems (ICCP). Barcelona, Spain, 2016. [Online]. Available: https://papers.nips.cc/paper/6573-binarized-neural-networks.pdf

[23] K. He, X. Zhang, S. Ren, and J. Sun, “Deep Residual Learning for Image Recognition,” in 2016 IEEE Conference on Computer Vision and Pattern Recognition (CVPR). Las Vegas, NV, USA: IEEE, Jun. 2016, pp. 770–777. [Online]. Available: http://arxiv.org/abs/1512.03385

[24] G. Huang, Z. Liu, L. V. d. Maaten, and K. Q. Weinberger, “Densely Connected Convolutional Networks,” in 2017 IEEE Conference on Computer Vision and Pattern Recognition (CVPR). Honolulu,
[25] H.-G. Beyer and H.-P. Schwefel, “Evolution strategies: A comprehensive introduction,” *Natural Computing*, vol. 1, no. 1, pp. 3–52, 2002. [Online]. Available: [http://link.springer.com/10.1023/A:1015059928466](http://link.springer.com/10.1023/A:1015059928466)

[26] D. Mittal, S. Bhardwaj, M. M. Khapra, and B. Ravindran, “Studying the plasticity in deep convolutional neural networks using random pruning,” *Machine Vision and Applications*, vol. 30, no. 2, pp. 203–216, Mar. 2019. [Online]. Available: [http://link.springer.com/10.1007/00138-018-01001-9](http://link.springer.com/10.1007/00138-018-01001-9)

[27] P. Molchanov, S. Tyree, T. Karras, T. Aila, and J. Kautz, “Pruning convolutional neural networks for resource efficient inference,” *arXiv:1611.06440*, Nov. 2016, arXiv: 1611.06440. [Online]. Available: [http://arxiv.org/abs/1611.06440](http://arxiv.org/abs/1611.06440)

[28] H. Hu, R. Peng, Y.-W. Tai, and C.-K. Tang, “Network trimming: A data-driven neuron pruning approach towards efficient deep architectures,” *arXiv:1607.03250*, Jul. 2016, arXiv: 1607.03250. [Online]. Available: [http://arxiv.org/abs/1607.03250](http://arxiv.org/abs/1607.03250)

[29] J.-H. Luo and J. Wu, “An entropy-based pruning method for CNN compression,” *arXiv:1706.05791*, Jun. 2017, arXiv: 1706.05791. [Online]. Available: [http://arxiv.org/abs/1706.05791](http://arxiv.org/abs/1706.05791)

[30] H. Liu, K. Simonyan, O. Vinyals, C. Fernando, and K. Kavukcuoglu, “Hierarchical representations for efficient architecture search,” *arXiv:1711.00436*, Nov. 2017, arXiv: 1711.00436. [Online]. Available: [http://arxiv.org/abs/1711.00436](http://arxiv.org/abs/1711.00436)

[31] W.-Y. Chiu, G. G. Yen, and T.-K. Juan, “Minimum Manhattan distance approach to multiple criteria decision making in multiobjective optimization problems,” *IEEE Transactions on Evolutionary Computation*, vol. 20, no. 6, pp. 972–985, Dec. 2016. [Online]. Available: [http://ieeexplore.ieee.org/document/7465803/](http://ieeexplore.ieee.org/document/7465803/)

[32] A. Krizhevsky, “Learning Multiple Layers of Features from Tiny Images,” University of Toronto, Technical Report, Apr. 2009. [Online]. Available: [https://www.cs.toronto.edu/~kriz/learning-features-2009-TR.pdf](https://www.cs.toronto.edu/~kriz/learning-features-2009-TR.pdf)

[33] F. E. Fernandes Junior and G. G. Yen, “Particle swarm optimization of deep neural networks architectures for image classification,” *Swarm and Evolutionary Computation*, vol. 49, pp. 62–74, Sep. 2019. [Online]. Available: [https://linkinghub.elsevier.com/retrieve/pii/S2210650218309246](https://linkinghub.elsevier.com/retrieve/pii/S2210650218309246)