Incorporating suction in to stability charts for unsaturated soil slopes

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ABSTRACT

This paper presents charts derived from stability analyses of curvilinear slopes in non-homogeneous unsaturated soils. The charts enable quick and inexpensive stability analyses to be conducted in practice. The soils are assumed to obey the Mohr-Coulomb failure criterion. Suction effects are captured using the effective stress concept. Cohesion and the contribution of suction to the effective stress are linear functions of depth. It is shown that a stable curvilinear slope profile is uniquely governed by dimensionless parameters. The charts are presented using these dimensionless parameters and may be used in the preliminary design of stable curvilinear slopes. In general, suction has a similar influence to cohesion, and as the contribution of suction to effective stress increases, steeper curvilinear slopes become stable. An example of application is included showing the design of a curvilinear slope.

Keywords: suction, effective stress, slope, stability chart

1 INTRODUCTION

Stability analyses of slopes are important in geotechnical engineering. For newly constructed slopes they enable safe surface profiles to be determined for certain soil properties. For existing slopes they enable margins of safety to be determined so needs for remediation and stabilisation can be identified and risks assessed. Furthermore, for failed slopes, they enable a strength property of the soil at the time of failure to be back-calculated.

Most detailed slope stability analyses involve computational tools, such as finite element methods e.g. Griffiths and Lane (1999) or software assisted limit equilibrium methods e.g. Cheng et al. (2007). However, for simple slope geometries and soil profiles, the results of stability analyses may be presented in stability charts. The charts are of interest in that they enable quick and inexpensive stability analyses to be conducted in practice. They are especially useful for preliminary analyses, checking detailed design analyses and making comparisons between design alternatives. In stability charts, soil and slope parameters feature inside dimensionless parameters.

Most stability charts are restricted to slope profiles which are planar. Some charts, for example those of Sokolovski (1954) and Jeldes et al. (2014b), may be applied to profiles which are convex or concave (collectively referred to as curvilinear in this paper), but only for the special case when the curvilinear surface has a vertical tangent at the top of the slope. Uriel Romero (1967) and Uriel Romero (1969) considered one type of curvilinear slope having a non-zero vertical tangent at the top of the slope, although the results cannot be generalised to other non-vertical tangent slope profile geometries.

Natural slopes subject to weathering are more likely to have a concave profile than a planar profile (Utili and Crosta, 2011). Some log-spiral concave surface may be more stable than a planar profile (Utili and Nova, 2007). Concave slopes may also have superior erosion resistance compared to planar slopes (Young and Mutchler, 1969a, Young and Mutchler, 1969b, Meyer and Kramer, 1969, Rieke-Zapp and Nearing, 2005, Schor and Gray, 2007) making them preferable within many embankments and cuttings (Jeldes et al., 2014a, Jeldes et al., 2014b). However, there is limited ability to address their stability without resorting to computational tools.

Another restriction of most existing stability charts is that they require soil strength to be defined by the Mohr-Coulomb failure criterion with the soil having a constant cohesion and or friction angle throughout the slope. Very few charts are relevant to when soil strength varies spatially within the soil (Booker and Davis, 1972, Griffiths and Yu, 2015), yet this is known to be the case in many practical situations.

This paper addresses these restrictions. Stability analyses of curvilinear slopes made of unsaturated and non-homogenous soils are conducted using slip line theory, building on the work of Sokolovski (1954). The

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soil is assumed to be a continuous body in static equilibrium at the onset of failure. The effective stress concept for unsaturated soils and the Mohr-Coulomb failure criterion are adopted. The slip line governing equations are of a hyperbolic type and define two families of stress characteristics (Sokolovski, 1954, Atkinson and Potts, 1975). Because the soil is assumed to have a non-zero frictional strength the slope profiles are logarithmic spirals (Sokolovski, 1954, Davis, 1968). Cohesion and the contribution of suction to the effective stress and strength are non-uniform and vary linearly with depth. The results of the analyses are presented in stability charts. Defining parameters appear in dimensionless form. In conducting the analyses, the work of Sokolovski (1954) is extended so curvilinear slope profiles other than those with a vertical tangent at the top of the slope are considered. An example of application is given showing how a curvilinear slope can be designed.

2 PROBLEM SET-UP, GOVERNING EQUATIONS, BOUNDARY CONDITIONS AND SOLUTION METHOD

2.1 Problem set-up

A non-homogeneous unsaturated soil slope is shown in Fig. 1 along with the coordinates system adopted, with y and x representing vertical and horizontal directions, respectively. The failed part of the slope is denoted as the “sliding soil mass” in Fig. 1 which intercepts the curvilinear surface at y=H and the horizontal surface OA at x=L. A stress q is applied at y=0 over the distance L extending from x=0 to L.

![Diagram](image)

Fig. 1. Geometry and parameters of the curvilinear slope problem

It is assumed that soil shear strength (τ) is governed by the Mohr-Coulomb failure criterion:

\[ \tau = c' + \sigma' \tan \phi' \] (1)

in which \(c'\) is the soil cohesion and \(\phi'\) is the soil friction angle. \(\sigma'\) is the effective stress and for an unsaturated soil is defined as (Bishop, 1959):

\[ \sigma' = \sigma - u_a + \chi (u_a - u_w) \] (2)

where \(\sigma\) is total stress, \(u_a\) is pore air pressure, \(u_w\) is pore water pressure and \(\chi\) is the effective stress parameter.

Bishop (1960) suggested that \(\chi\) and thus \(\sigma'\) depend on many factors including degree of saturation, soil structure, the drying-wetting history and the stress history. The dependencies of \(\chi\) and \(\sigma'\) on these factors have been studied by many authors e.g. Bolzon et al. (1996), Khalili and Khabbaz (1998), Gallipoli et al. (2003), Lu and Likos (2006), Khalili et al. (2008), Dowenzia et al. (2011) and Lu et al. (2014). In these studies slightly different terminologies have been adopted for \(\chi\) and \(\sigma'\).

When \(u_a\) is equal to atmospheric pressure and is taken as the pressure datum, Eq. 2 can be rewritten as:

\[ \sigma' = \sigma + \chi s \] (3)

where \(s = u_a - u_w\) is soil suction. \(\chi s\) is the contribution of suction to the effective stress. The analyses and results presented here apply to when the changes to \(u_a\) are negligible. There may be examples when this is not the case and fully coupled numerical analyses may be warranted e.g. Oka et al. (2010) and Xiong et al. (2014).

For simplicity it is assumed that \(\phi'\) is constant and that \(c'\) and \(\chi s\) vary linearly with depth according to the functions:

\[ c' = c'_0 + k_c y \] (4)

\[ \chi s = (\chi s)_0 + k_{\chi s} y \] (5)

where \(c'_0\) is cohesion at \(y=0\) (that is at the top of the slope), \((\chi s)_0\) is contribution of suction to the effective stress at \(y=0\) and \(k_c, k_{\chi s}\) are constants. Vo and Russell (2016) showed that \(c'\) and \(\chi s\) may have similar but independent effects on an unsaturated soil’s strength.

Different combinations of \(c'\) and \(\phi'\) may define peak strength, critical state strength and residual strength. There is experimental evidence that \(c'\) and \(\phi'\) are independent of \(s\) at the critical state, meaning the same values apply to saturated, dry and unsaturated conditions (Escario, 1980, Gan et al., 1988, Likos, 2010). When soils exhibit significant suction hardening the values of \(c'\) and \(\phi'\) which define the peak strength may have a secondary dependence on \(s\), and may be slightly larger for unsaturated conditions than saturated or dry conditions.

For an effective stress analysis in a saturated soil \(\chi = 1\) and \(-u_w\) appears in Eqs. 3 and 5 in place of \(\chi s\). For a total stress analysis in a saturated soil the contribution of \(-u_w\) is ignored and Eq. 4 can be used to express a linear variation of undrained shear strength with depth. A linear variation of undrained shear strength with depth is often assumed for normally consolidated clays and layered soils (Gibson and Morgenstern, 1962, Bishop, 1966, Lump and Holt, 1968, Hunter and Schuster, 1968).

2.2 Governing equations

The Mohr-Coulomb failure criterion can be written as:

\[ \sigma_{yy} = [(1 + \sin \phi' \cos 2\theta)\sigma'_m - c' \cot \phi'] - \chi s \] (6)
\[ \sigma_{xx} = [(1 - \sin \phi' \cos 2\theta)\sigma_m' - c' \cot \phi'] - \chi \]  
\[ \sigma_{xy} = \sin \phi' \sin 2\theta \sigma_m' \]

where \( \sigma_{yy}, \sigma_{xx} \) are normal stresses in the \( y \) and \( x \) directions, respectively, \( \sigma_m' \) is effective mean stress, \( \sigma_{xy} \) is shear stress and \( \theta \) is angle between the vertical axis and the major principal stress direction.

The stress components in Eqs. 6 to 8 are scaled by a constant stress quantity \( S \) and differentiated with respect to lengths which are scaled by \( D \). The resulting expressions are substituted into the static equilibrium equation to obtain:

\[ (\eta) \equiv \left\{ \frac{dx}{\sigma_m} + 2 \tan \phi' \sigma_m d\theta = F(d\tilde{y} - \tan \phi' dx) \right\} \]
\[ (\xi) \equiv \left\{ \frac{dx}{\sigma_m} - 2 \tan \phi' \sigma_m d\theta = F(d\tilde{y} + \tan \phi' dx) \right\} \]

in which

\[ F = \left\{ \frac{(\eta)}{\chi} + \left( \frac{\sigma}{\sigma_m} \right) \right\} \]  
\[ \chi = \frac{\sin \phi'}{1 + \sin \phi'} \]

and \( \gamma \) is total unit weight of soil, \( \mu = \pi/4 = \gamma/2 \) and an overbar indicates a scaled quantity. Eqs. 9 and 10 express two families of characteristic curves \( \eta, \xi \) which are inclined at angles \( \pm \mu \) to the major principal stress direction.

When the stress scale is set to \( S = q + (\gamma_s)k_c \cot \phi' \) and the length scale is set to \( D = L \) it follows that \( F = L(\gamma + k_c \cot \phi' + k_p)/q + (\gamma + k_c \cot \phi') \). For a dry soil, \( F \) reduces to the parameter defined by Martin (2004) when applying this theory to the analysis of the bearing capacity of shallow footings, where \( L \) is the footing width and acts as the length scale.

Whatever the choice of \( S \) and \( D \) the solution of Eqs. 9 and 10 depends on \( \phi \) and the non-dimensional quantity \( F \). When applied to a boundary value problem, the solution also depends the boundary conditions.

### 2.3 Boundary conditions

With reference to the coordinates system shown in Fig. 1, two boundary conditions can be derived at the soil surface OA:

\[ \theta = 0 \quad (12) \]
\[ \sigma_m' = \frac{1}{1 + \sin \phi'} \quad (13) \]

Two boundary conditions can be derived at the slope profile OB:

\[ \frac{dx}{\sigma_m} = \tan \theta_s \quad (14) \]
\[ \sigma_m' = \frac{\chi \sigma + c' \cot \phi'}{1 - \sin \phi' \sigma + (\gamma + k_c \cot \phi') \tau} \quad (15) \]

where \( \theta_s \) is the angle between the major principal stress direction and the vertical axis at a point on the slope profile OB. When \( \gamma + k_c \cot \phi' + k_p \neq 0 \), \( \theta_s \) varies along the slope profile resulting in a curvilinear shape (Eq. 14). In particular, when \( \theta_s \) reduces with depth along the slope profile, it has a concave shape and when \( \theta_s \) increases with depth along the slope profile, it has a convex shape.

In this paper the shape of curvilinear slopes are obtained for when \( \gamma + k_c \cot \phi' + k_p \neq 0 \), as \( \gamma + k_c \cot \phi' + k_p \) can be thought of as an equivalent total unit weight.

The governing equations and boundary conditions in subsections 2.2 and 2.3 are valid for any constant stress scale and length scale.

### 2.4 Solution method

A standard finite difference method and an iterative procedure are used to solve Eqs. 9 and 10. The method is adapted from Sokolovski (1954). The solution procedures used here were validated by setting \( \chi_s = 0 \) and \( c' = \text{constant} \) and reproducing one set of Sokolovski’s (1954) results.

### 3 RESULTS

#### 3.1 The case when \( q = q_{\text{min}} \)

In the boundary value problem shown in Fig. 1, point O is a singular point because of the jump in \( \theta \) between OA (Eq. 12) and OB (Eq. 14). At O, \( \theta \) can be allowed to numerically transit from 0 at the soil surface to \( \theta_s \) at the slope profile. The tangent of angle of the slope profile \( \theta_s \) at O is denoted \( T \) where

\[ T = \frac{\cot \phi'}{2} \ln \left( \frac{[\gamma_s \phi + c_0 \cot \phi']}{(\gamma + k_c \cot \phi') \left(1 + \sin \phi' \right)} \right) \]

The condition of a non-overlapping stress field at O requires \( T \leq 0 \) therefore the surcharge \( q \) must be larger than or equal to a minimum value \( q_{\text{min}} \) where:

\[ q_{\text{min}} = \frac{2 \chi_s \phi + c_0 \cot \phi' \sin \phi'}{1 - \sin \phi'} \]

Curvilinear slope profiles were obtained by Sokolovski (1954) for the case \( q = q_{\text{min}} \). When \( q = q_{\text{min}} \), the curvilinear slope is tangential to the vertical axis at O, i.e. \( T = 0 \) at O. Indeed \( q_{\text{min}} \) can be related to the maximum height \( y_0 \) of a vertical slope in a homogeneous soil (Chen and Scawthorn, 1970) and the maximum depth of a vertical tension crack (Michalowski, 2013).

Following the approach of Chen and Scawthorn (1970), the lower bound of \( y_0 \) in unsaturated soil is found to be:

\[ y_0 = \frac{(\gamma k_s + k_c (k_s - 1) - 2 k_c k_p)}{\gamma k_s + k_c (k_s - 1) - 2 k_c k_p} \]

where \( k_c \) is the Rankine active earth pressure coefficient.

Sokolovski (1954) presented analysis results in a chart for the case when \( q = q_{\text{min}} \). \( c' \) is constant with depth, \( \chi_s = 0 \) and \( \phi' = 20^\circ, 30^\circ, 40^\circ \). He used \( S = c' \) and \( D = c'/\gamma \) as the stress and length scales, respectively, so that \( F = 1 \) in Eqs. 9 and 10. The chart produced by Sokolovski (1954) is reproduced here in Fig. 2. His data is plotted using dash lines. New data generated by the authors is plotted using continuous lines.
3.2 The general case

The general case, or more specifically the case when \( q > q_{\text{min}} \) has not been considered by Sokolovski (1954) or Jeldes et al. (2014b). It is considered here for when \( c' \) and \( \chi_s \) vary linearly with depth and \( T < 0 \). \( q_{\text{min}} \) may be thought of as the load applied to the top of a slope by a layer of a soil with a vertical face of height \( y_0 = q_{\text{min}}/\gamma \). The quantity \( q - q_{\text{min}} \) represents a uniform surcharge on top of that layer. The curvilinear slope profile commences at \( O \) located at the base of the vertical face.

Figs. 3a-f plot the slope geometries in the \( y/D \sim x/D \) plane for \( F = 0.01, 0.05, 0.1, 0.5, 1, 5 \) and for \( \phi' = 20^\circ, 30^\circ, 40^\circ \) and \( T = 0, -0.1 \pi/2, -0.2 \pi/2, \cdots, -0.9 \pi/2, -\pi/2 \). These charts apply to when the stress and length scales used were \( S = q + (\chi_s)y + c_0'\cot\phi' \) and \( D = L \), respectively.

In applying the charts in Figs. 3 it is important to note that \( L \) (which is used to define \( D \)) defines a specific sliding soil mass of a certain size. The uniform surcharge \( q - q_{\text{min}} \) applies over this \( L \) (Fig. 1).

It can be observed that as \( T \) transitions from zero to a negative value, the slope profile transitions from a concave shape to a convex shape. It can also be observed that as \( F \rightarrow \infty \), for a given \( \phi' \), the results become independent of \( T \) and the surface profiles become unique and planar.

These results are valid for any value of \( c_0', k_c, (\chi_s)_0, k_s \), \( \gamma \), \( L \) provided that \( q > q_{\text{min}} \) and \( \gamma + k_c\cot\phi' + k_s > 0 \).

Fig. 3. The general case for unsaturated soil where \( T < 0 \) at \( O \) (stress scale is \( q + (\chi_s)y + c_0'\cot\phi' \), length scale is \( L \)): a. \( F = 0.01 \), b. \( F = 0.05 \), c. \( F = 0.1 \), d. \( F = 0.5 \), e. \( F = 1 \), f. \( F = 5 \)
4 AN EXAMPLE OF APPLICATION

A hillside slope, 53 m in height, is to be landscaped to have a curvilinear profile to limit erosion and sediment runoff. The following three cases are considered in this example.

Case 1: $\phi'=30^\circ$, $\gamma=15$ kN/m$^3$ and $c'=30$ kPa at all depths. The slope is dry. No surcharge acts on the ground surface at the top of the hill meaning $q=q_{\text{min}}$ applies. This is an extended version of the example considered by Jeldes et al. (2014b).

Case 2: $\phi', \gamma, c'$ of case 1. The slope is dry. A uniform surcharge of 30 kPa is applied across the ground surface at the top of the hill extending a distance of 6.2 m from the slope face.

Case 3: $\phi', \gamma, c'$ of case 1. The slope is unsaturated. The contribution of suction to the effective stress can be expressed as $\chi_s=20-0.1y$ (kPa). A uniform surcharge is applied as in case 2.

For simplicity, the curvilinear slopes at limiting condition will be obtained for cases 1 to 3. Design for non-limiting conditions will be discussed in section 5.

4.1 Design for case 1

The soil properties correspond to $q=q_{\text{min}}=2c'\cos\phi'/(1-\sin\phi')=104$ kPa and $y_0=q_{\text{min}}/\gamma=6.93$ m.

When a pressure scale $S=c'/30$ kPa and a length scale $D=c'/\gamma=2$ m are chosen, this curvilinear profile for $\phi'=30^\circ$ in Fig. 2 can be adopted. This profile is dimensionalised and shown in Fig. 4a. As expected, this profile is identical to the one obtained by Jeldes et al. (2014b). This slope profile corresponds to many possible sliding soil mass which may be a state of limiting equilibrium. The one which passes through the bottom of the hillside extends along the horizontal surface OA by a distance $L=15.2$ m and is shown in Fig. 4b.

4.2 Design for case 2

The soil properties and surcharge condition correspond to $q=q_{\text{min}}+30=2c'\cos\phi'/(1-\sin\phi')+30=134$ kPa and $y_0=q_{\text{min}}/\gamma=6.93$ m.

The pressure scale $S=q+c'\cot\phi'=186$ kPa and length scale $D=6.2$ m correspond to $F=D\gamma/(q+c'\cot\phi')=0.5$. Also, $T=-0.153=-0.1\pi/2$ according to Eq. 16.

The slope profile corresponding to $F=0.5$, $\phi'=30^\circ$, $T=-0.1\pi/2$ can be obtained from Figs. 3. It is dimensionalised and shown in Fig. 5a. When the 30 kPa surcharge is applied on top of the hill between the edge and a point at a distance $L=6.2$ m from the edge, the failure plane will intersect the face of the slope rather than extend all the way to the toe. The extent of soil mass at limiting equilibrium for case 2 is shown in Fig. 5b.

For a failure plane to intersect the toe the surcharge of 30 kPa must extend a distance $L=14.6$ m from the edge. This is also indicated in Figs. 5a and 5b. The extent of the surcharge and $L$ control the size of the sliding mass and the depth of the failure plane.
4.3 Design for case 3

The soil properties and surcharge condition correspond to $q = q_{\text{min}} + 30 = 174$ kPa and $y_0 = 9.47$ m according to Eq. 18.

The pressure scale $S = q + (\chi_s)\theta + c'\cot\phi' = 246$ kPa and length scale $D = 6.2$ m correspond to $F = D(y + k_y) / (q + (\chi_s)\theta + c'\cot\phi') = 0.376$. Also, $\theta = -0.113 - 0.0717\pi/2$ according to Eq. 16.

The slope profile corresponding to $F = 0.376$, $\phi' = 30^\circ$, $\theta = -0.0717\pi/2$ can be obtained from Figs. 3. It is dimensionalised and shown in Fig. 6a. The influence of $\chi_s$ is evident through the formation of a steeper curvilinear slope for case 3 than case 2 (Fig. 5a). If the surcharge is applied on top of the hill between the edge and a distance $L= 6.2$ m from the edge, the failure plane intersects the face of the slope. The extent of soil mass at limiting equilibrium for case 3 is shown in Fig. 6b. Also, it has been determined that the failure plane would intersect the toe of the slope if the surcharge extended across the top of the slope by a distance $L = 15.9$ m. The extent of the sliding mass for this condition is also indicated in Figs. 6a and 6b.

5 NON-LIMITING CONDITIONS

In practice, it is often necessary that preliminary designs be carried out for non-limiting conditions. This may be done by reducing soil strength parameters by a factor of safety (FS). The reduced soil strength parameters are denoted $c'_m$, $\phi'_m$ and defined as:

$$FS = \frac{c'_m}{c_m} = \frac{\tan\phi'_m}{\tan\phi_m} \quad (19)$$

where FS is a constant greater than 1. Eq. 19 is the definition of Bishop (1955) and is widely used (Matsui and San, 1992, Dawson et al., 1999, Griffiths and Lane, 1999). Adopting the reduced strength parameters, a curvilinear slope design may be obtained which is not at a limiting condition of failure and may have superior resistance compared to planar slopes.

It is also possible to design for non-limiting conditions by increasing body forces and external forces by a factor in a way that drives failure in a particular problem. This approach has been employed in many limit equilibrium types of analyses although it is dependent on the boundary conditions of the problem (Sloan, 2013, Tschuchnigg et al., 2015).

It is important to note that the above two approaches are distinct. Which is adopted needs to be stated in a design unambiguously.

6 CONCLUSIONS

Stability analyses may be completed for slopes comprising unsaturated soils with convex or concave surface profiles, with soil having a strength defined by the Mohr-Coulomb failure criterion, a constant friction angle ($\phi'$), and profiles of cohesion ($c'$) and the contribution of suction to the effective stress ($\chi_s$) that vary linearly with depth. Once the constants defining the linearly varying $c'$ and $\chi_s$ profiles have been included in the stress characteristic curves the solution procedure needed to analyse the problem is well established.

The analysis results have been presented in a series of stability charts, which show surface profiles at the
onset of instability depend only on $\phi'$ and the dimensionless parameters $F$, $T$. Of these parameters $F=L(y^+k_c\cot\phi^*+k_d)/(q^+\chi^+c_0\cot\phi^*)$ contains a characteristic length for the slope ($L$), an equivalent total unit weight of the soil (contained in the parenthesis on the numerator, being the total unit weight adjusted to account of the linear variations of $c'$ and $\chi\phi$ profiles) and an equivalent cohesion (the denominator, being the actual cohesion at the top of the slope plus the effect of the $\chi\phi$ profile and a surcharge $q$ at the top of the slope). $T$ captures the inclination of the slope profile at the top of the slope.

$\chi\tan\phi'$ has a very similar influence to $c'$ on slope stability, and as $\chi\phi$ increases, steeper curvilinear slopes become stable. As $F \to \infty$ the results become independent of $T$ and the surface profiles become unique and planar.

The inclusion of $T$ in the governing dimensionless parameters has not been previously considered. Its inclusion enables charts to be produced of wide practical applicability to assist with preliminary design. A design example has been given on how to use the stability charts.

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