Research on Orbit Optimization of Manned Spacecraft Based on Dynamics

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Abstract. Based on the dynamics angle, the paper finds a suitable launch window based on the pork-chop diagram, and then uses the conic splicing method to carry out the preliminary design of the orbit. The two-body model is used to design the orbit at each stage, and finally under the precise mechanical model, using differential correction method, accurate track design based on B plane parameters.

Keywords: Orbital dynamics, manned space engineering, orbit optimization, spacecraft orbit.

1. Introduction
The spacecraft's orbit is low and the spacecraft's solar wing is large, so the earth's non-spherical gravity and atmospheric drag are its main driving forces. At the same time, low orbit leads to a short observable arc at the ground measuring station. These factors have brought challenges to orbit determination. The manned spacecraft carries a GPS receiver. Although there is only pseudo-range positioning result information in the downlink data transmission, it provides continuous measurement data, which makes the orbit determination result improved to a certain extent than simply using ground station discontinuous observation data [1]. By using a purely dynamic orbit determination method and using GPS pseudo-range positioning result data, the spacecraft's orbit determination and extrapolation can reach an accuracy of 50 meters. However, during the depressurization of the spacecraft's orbital module, we found that there was a significant deviation between the forecast ephemeris and the GPS measured information. The depressurization process of the orbital cabin of the spacecraft produced a disturbing force on the spacecraft, which affected the orbit of the spacecraft. The spacecraft carries a microgravity accelerometer, which can monitor the forces on the spacecraft such as orbital control, attitude control, and unexpected vibration, and can visually display the disturbing force generated by the orbital cabin pressure relief process. Obviously, the influence of disturbance force in the pressure relief process should be considered in the process of orbit determination. In this paper, a semi-empirical dynamics model of the orbital module pressure relief disturbance is established, and the three model parameters are solved based on the pseudo-range positioning result data of the satellite GPS receiver.
2. Preliminary design of ground fire transfer track

2.1. Heliocentric orbit design and search for launch window
The hyperbola has a residual velocity when it reaches the edge of the earth's gravitational influence. This hyperbolic residual velocity $v_{\infty}$ is usually called hyperbolic overspeed. The calculation formula is

$$v_{\infty} = v_1 - v_{LE}$$

In the formula, $v_1$ is the velocity vector of the spacecraft, and $v_{LE}$ is the velocity vector of the earth orbiting the sun at the moment of launch. Launch energy is a key parameter that affects the initial design of the mission. When the mass of the spacecraft is constant, the greater the launch energy, the stronger the carrying capacity of the required launch vehicle [2]. Its size is the square of the hyperbolic overspeed during launch.

$$C_3 = \|v_1 - v_{LE}\|^2$$

When it is assumed that the spacecraft starts to transfer from the mooring orbit, the velocity increment $\Delta v_1$ imposed on the mooring orbit is

$$\Delta v_1 = v_{s1} - v_p = \sqrt{\frac{2\mu}{r}} + C_3 - \sqrt{\frac{u}{r}}$$

Similarly, it is possible to define the hyperbolic overspeed of the spacecraft reaching the target celestial body and the speed increment when reaching the target mooring orbit.

$$v_{s2} = v_2 - v_{TF}$$

$$\Delta v_2 = \sqrt{\frac{2\mu}{r_p}} + v_{s2}^2 - \sqrt{\frac{u}{r_p}}$$

Where $v_1$ is the velocity vector when the spacecraft arrives at the target planet, $v_{at}$ is the velocity vector of the target planet orbiting the sun at the moment of launch, and $r_p$ is the height of the pericentric point when the spacecraft captures the target celestial body. During the entire flight, the required total velocity increment $\Delta v_{total}$ is

$$\Delta v_{total} = \Delta v_1 + \Delta v_2$$

2.2. Determination of geocentric parameters
In the preliminary design of the parameters of the geocentric segment, it is assumed that the detector in the Earth's influence sphere is only affected by the gravity of the earth, so that the orbit parameters are calculated according to the two-body orbit characteristics, and the escape velocity increment is assumed to be pulse. Suppose the launch process of the probe is: after the probe is launched from the ground, it first enters a circular parking orbit; then after a period of taxiing, at a specific time and at a specific location, it enters a hyperbola after the final stage of rocket acceleration the orbit escapes, as shown in Figure 1. In order to avoid unnecessary fuel consumption as much as possible, it is assumed that the parking orbit and the hyperbolic orbit are coplanar, and the perigee radius of the hyperbola is the same as the radius of the circular orbit, and the speed is tangent [3]. Therefore, the content of the preliminary design of the geocentric parameters mainly includes two aspects: the characteristic analysis of the parking track parameters and the selection of the parking track parameters.
Figure 1. The geometric relationship of the in-plane parameters of the geocentric escape hyperbolic trajectory

Set the parking orbit radius (i.e., the radius of the hyperbolic perigee) to be 1, and it is easy to calculate the semi-major axis of the hyperbolic orbit, the eccentricity, the moment of momentum, and the perigee velocity according to the relevant formula of the hyperbolic orbit parameters:

\[
a_1 = -\frac{\mu}{v_{\infty 1}^2}, h_1 = r_{p1}\sqrt{\frac{2\mu}{v_{\infty 1}^2} + \frac{2\mu}{r_{p1}}}, e_1 = 1 + \frac{r_{p1}v_{\infty 1}^2}{\mu}, v_{p1} = \frac{h_1}{r_{p1}}
\] (7)

Among them, \(a_1\) is the semi-major axis of the orbit, \(e_1\) is the eccentricity, \(h_1\) is the gravity constant of the earth, \(v_{\infty 1}\) is the residual velocity of the geocentric hyperbola, \(h_1\) is the momentum of the hyperbolic orbit, and \(v_{p1}\) is the perigee velocity of the hyperbola [4]. According to the residual velocity vector \(v_{\infty 1}\), the right ascension \(\alpha_{\infty 1}\) and the declination \(\delta_{\infty 1}\) corresponding to the asymptotes of the transfer hyperbola can be calculated

\[
\alpha_{\infty 1} = \tan^{-1}(v_{xy1}, v_{xz1})
\] (8)

\[
\delta_{\infty 1} = 90^\circ - \cos^{-1}(v_{xz1})
\] (9)

Furthermore, the unit direction vector of the asymptote in the B plane can be determined

\[
\hat{S} = \begin{bmatrix}
\cos \delta_{\infty 1} \cdot \cos \alpha_{\infty 1} \\
\cos \delta_{\infty 1} \cdot \sin \alpha_{\infty 1} \\
\sin \delta_{\infty 1}
\end{bmatrix}
\] (10)

The other two coordinate systems of the B plane can be expressed as

\[
\hat{T} = \hat{S} \times \hat{u}
\] (11)

\[
\hat{R} = \hat{S} \times \hat{T}
\] (12)

In the formula, the angle between the plane \(\hat{u} = [0 \ 0 \ 1]^T\) and B can be determined by the declination of the asymptote and the orbital inclination
\[ \cos \theta = \cos \frac{i}{\cos \delta} \]

The unit angular momentum vector of the hyperbola can be expressed as
\[ \hat{h} = \hat{T} \sin \theta - \hat{R} \cos \theta \]

When the speed tends to infinity, the sine and cosine of the true anomaly can be determined as
\[ \cos \theta_\infty = -\frac{\mu}{r \nu^2} + \frac{\mu}{r} \quad \sin \theta_\infty = \sqrt{1 - \cos^2 \theta_\infty} \]

It is further possible to determine the perigee vector of the transfer hyperbolic orbit
\[ \hat{r}_p = \hat{S} \cos \theta_\infty - \left( \hat{h} \times \hat{S} \right) \sin \theta_\infty \]

It can be determined that the position vector of the spacecraft at the perigee of the hyperbolic transfer orbit is \( \hat{r}_p = r_p \hat{r}_p \), and the unit vector of the velocity is
\[ \hat{v}_p = \hat{h} \times \hat{r}_p \]

The velocity vector of the spacecraft at the perigee can be expressed as \( \hat{v} = v_p \hat{v}_p \). According to the position vector and velocity vector of the perigee, the six numbers of the hyperbolic orbit can be calculated. Considering that the interplanetary distance is much larger than that of the Earth's influence sphere, the position of the intersection of the geocentric hyperbolic orbit and the Earth's influence sphere has almost no influence on the orbit, and the magnitude and direction of the \( v_\infty \) vector completely determines the range of the detector's departure from the Earth. Therefore, it is very important to control the accuracy of the \( v_\infty \) vector size and direction [5]. In fact, there are countless escape hyperbolas that satisfy the constraints of the magnitude and direction of the remaining speed. These hyperbolas are located on the curved surface composed of the \( v_\infty \) vector (that is, the direction of the asymptote) that rotates the hyperbolic orbit in Figure 1 around the centre of the earth and the circular trajectory composed of all hyperbolic perigees is called the orbital launch circle, so the parking orbit must pass through the OL point, wait for the detector to run to the position of the orbital launch circle, apply pulses along the velocity direction, and enter the hyperbolic orbit, as shown in the Figure 2 shown.

Figure 2. Schematic diagram of orbital launch circle
Suppose the $v_{\infty 1}$ vector can be expressed as:

$$
V_{\infty 1} = v_{\infty 1} \begin{bmatrix} \cos \delta_{\infty 1} \cdot \cos \alpha_{\infty 1} & \cos \delta_{\infty 1} \cdot \sin \alpha_{\infty 1} & \sin \alpha_{\infty 1} \end{bmatrix}
$$

Then the right ascension of the vector from the center of the earth to the OL point is $(\pi + \alpha_{\infty 1})$, and the declination is $(-\delta_{\infty 1})$, expressed as:

$$
R_L = -r_p \begin{bmatrix} \cos \delta_{\infty 1} \cdot \cos \alpha_{\infty 1} & \cos \delta_{\infty 1} \cdot \sin \alpha_{\infty 1} & \sin \alpha_{\infty 1} \end{bmatrix}
$$

### 2.3. Determination of the parameters of the fire core segment

In the preliminary design of the fire core parameters, it can be considered that the probe in the Mars influence sphere is only affected by the central gravity of Mars, so the orbit parameters are calculated according to the two-body orbit characteristics, and the capture velocity increment is assumed to be pulse. The properties of the hyperbolic trajectory of the fire core section are completely similar to those of the geocentric section [6]. The constraint of the residual velocity of the hyperbolic orbit in the center of the earth is the design goal, and the constraint of the residual velocity of the hyperbola in the Mars segment is the initial condition, and its size and direction are fixed.

The main content of the selection of the orbital parameters of the Mars segment is to determine the near-fire radius and orbital inclination of the hyperbola. It is assumed that the orbit entering the hyperbola and the working orbit of the detector are coplanar. In order to realize the full coverage of Mars by the Mars rover, the orbit of Mars exploration is selected as the polar orbit, the inclination is $i_2 = 90^\circ$, and the hyperbolic near-fire point height is $r_{p2}$.

When designing the track, the parameters of the target hyperbola are usually described by B plane parameters. When the probe enters the Mars influence sphere, the nominal B-plane parameters $B_T$ and $B_R$ can be calculated according to the inclination angle and near-center distance $r_{p2}$ of the fire center hyperbolic entry speed $v_{\infty 2}$ and the target fire center orbit.

$$
[B] = \left[ \begin{array}{c} B_T \\ B_R \end{array} \right] = \left[ \begin{array}{c} \cos \phi \\ \sin \phi \end{array} \right] = \left[ \begin{array}{c} \cos i_2 \\ \cos \delta_{\infty 2} \end{array} \right] \left[ \begin{array}{c} \cos \phi \\ \sin \phi \end{array} \right] = \left[ \begin{array}{c} \cos \phi \\ \sin \phi \end{array} \right] \left[ \begin{array}{c} \cos i_2 \\ \cos \delta_{\infty 2} \end{array} \right]
$$

Where $\mu_2$ is the gravitational constant of Mars, $\phi$ is the angle between the B vector and the T vector, as shown in Figure 3, $i_2$ is the orbital inclination of the target fire core orbit, and the velocity $v_{\infty 2}$ is the entry velocity of the fire core hyperbola, $\delta_{\infty 2}$ is the Martian declination of the vector.

Figure 3. Schematic diagram of the fire core entering the hyperbolic orbit and the B plane

### 3. Simulation analysis

Based on the accurate design of the B-plane parameters, it can be determined that the launch time is 12-May-2018, 00:15:33.137, the arrival time is 10-Dec-2018, 00:12:47.156, and the spacecraft’s transfer time is 211.981 days. The entire transfer track is shown in Figure 4:
Figure 4. The transfer orbit of the detector in the J2000 inertial coordinate system

Through calculation, the orbit parameters of the anchored orbit on the earth and the hyperbolic orbit can be obtained, as shown in Table 1:

| Track type                      | Mooring track | Transfer track |
|---------------------------------|---------------|----------------|
| Semi-major axis a (km)          | 6578.137      | -54028.16      |
| Eccentricity e                  | 0             | 1.121753       |
| Orbital inclination i (rad)     | 0.497419      | 0.497419       |
| Argument of Perigee o (rad)     | 0             | 2.170167       |
| Ascending node right ascension Ω (rad) | 0.897177 | 0.897177 |
| True anomaly angle θ (rad)      | 2.170167      | 0              |

When launching, in the orbit of the earth, the pulse applied by the detector is $dv_1 = [-1.7842058339426, -2.70264143209095, -2.16701775917320] \text{km/s}$, and the pulse size is $\Delta V_1 = 3.8966 \text{km/s}$; from the moment of launch, when $t=18057600s$, the probe reaches the Martian influence ball, the pulse applied at this time is $dv_2 = [0.0901724915339645, 3.86006626112751, 0.0383287816881011] \text{km/s}$, the pulse size is $\Delta V_2 = 3.8613 \text{km/s}$; when $t=18316634.0185162s$, the probe reaches Mars and stops Orbit, the pulse applied at this time is $dv_3 = [0.898193335636322, -0.561658888501794, 1.62920970970374] \text{km/s}$, and the pulse size is $\Delta V_3 = 1.9433 \text{km/s}$. $dv_1$, $dv_2$ and $dv_3$ are all vectors in J200 inertial system. Eventually reach the orbit of the Mars target. The target orbit parameters are shown in Table 2:

| Track type                      | Mooring track |
|---------------------------------|---------------|
| Semi-major axis a (km)          | 3896          |
| Eccentricity e                  | 0             |
| Orbital inclination i (rad)     | 1.570796      |
| Argument of Perigee o (rad)     | 0             |
| Ascending node right ascension Ω (rad) | 2.954076       |
| True anomaly angle θ (rad)      | 2.775437      |
4. Conclusion
Calculation and analysis show that the semi-empirical model of orbital cabin pressure relief disturbance effectively solves the influence of pressure relief disturbance power on the spacecraft's orbit, and improves the orbit prediction accuracy by an order of magnitude (from more than 3km to less than 100m). Using a more detailed atmospheric damping force model may further solve the problem of non-white noise in the observation residuals.

References
[1] Yin, S., Li, J., & Cheng, L. Low-thrust spacecraft trajectory optimization via a DNN-based method. Advances in Space Research, 66(7) (2020) 1635-1646.
[2] Izzo, D., Märtens, M., & Pan, B. A survey on artificial intelligence trends in spacecraft guidance dynamics and control. Astrodynamics, 3(4) (2019) 287-299.
[3] Aziz, J. D., Parker, J. S., Scheeres, D. J., & Englander, J. A. Low-thrust many-revolution trajectory optimization via differential dynamic programming and a sundman transformation. The Journal of the Astronautical Sciences, 65(2) (2018) 205-228.
[4] Chai, R., Savvaris, A., Tsourdos, A., Chai, S., & Xia, Y. Stochastic spacecraft trajectory optimization with the consideration of chance constraints. IEEE Transactions on Control Systems Technology, 28(4) (2019) 1550-1559.
[5] Taheri, E., & Junkins, J. L. Generic smoothing for optimal bang-off-bang spacecraft maneuvers. Journal of Guidance, Control, and Dynamics, 41(11) (2018) 2470-2475.
[6] Chai, R., Savvaris, A., Tsourdos, A., Xia, Y., & Chai, S. Solving multiobjective constrained trajectory optimization problem by an extended evolutionary algorithm. IEEE transactions on cybernetics, 50(4) (2018) 1630-1643.