Proton radii and two-photon exchange

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We investigate the effect of two-photon exchange processes upon the $\text{rms}$ and Zemach radii extracted from electron-proton scattering. We find that the changes are small and do not help to explain the discrepancy between experimental and calculated HFS in the hydrogen atom.

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I. INTRODUCTION

The recent progress in measurements of transition energies in the Hydrogen atom has been remarkable. The $2p-1s$ transition energy and the $1s$ hyperfine structure interval (HFS) are now known to 14 and 12 significant digits, respectively [1, 2]. The interpretation of these energies agree within the error bars. The proton moment relative to the HFS is the Zemach moment, derived from a convolution of charge- and magnetization densities. This correction depends on the particular choice of hadronic form factors [3] to $0.895 \pm 0.018$ fm. With this radius as input, calculated and experimental $2p-1s$ transition energies agree within the error bars. The proton moment relative to the HFS is the Zemach moment, derived from a convolution of charge- and magnetization densities. This correction depends on the particular choice of hadronic form factors [3] to $0.895 \pm 0.018$ fm. With this radius as input, calculated and experimental $2p-1s$ transition energies agree within the error bars.

These finite-size corrections can be determined from elastic electron-proton scattering at low momentum transfer $q$. The proton moment relevant for the $2p-1s$ energy (and the Lamb shift) is the charge $\text{rms}$-radius $r_{\text{rms}}$ which has been extracted from the world data on e-p scattering [2] to $0.895 \pm 0.018$ fm. With this radius as input, calculated and experimental $2p-1s$ transition energies agree within the error bars. The proton moment relevant for the HFS is the Zemach moment, derived from a convolution of charge- and magnetization densities. This correction depends on the particular choice of hadronic form factors [3] to $0.895 \pm 0.018$ fm. With this radius as input, calculated and experimental $2p-1s$ transition energies agree within the error bars.

This disagreement is partly explained by nuclear polarization effects in the hydrogen atom, processes involving virtual excitation of the proton to intermediary continuum states. Faustov et al. [4] have calculated this correction and find a contribution of 1.6 ppm [5], where the uncertainty is dominated by the uncertainty in the $r_{\text{rms}}$.

In order to give an impression of the calculated results, we show in Fig. 1 the two-photon correction for a typical electron energy. The two-photon exchange correction to elastic electron-proton scattering are described elsewhere [6-10]. We consider the contribution to the two-photon exchange box and crossed-box amplitudes with an intermediate nucleon. At the low momentum transfers of interest here (up to $q = 4 \text{ fm}^{-1}$) the contribution of an intermediate $\Delta$ or higher resonances) to the two-photon exchange amplitude is negligible [11].

Hadronic form factors consistent with the experimentally measured charge and magnetic form factors $G_{E}(q^2)$ and $G_{M}(q^2)$ are introduced at the photon-nucleon vertices. The two-photon exchange correction is relatively insensitive to the particular choice of hadronic form factors [11]. For very low $q$, the two-photon exchange correction behaves the same as that found for scattering from a point particle (e.g. electron-muon scattering), and is therefore completely independent of the hadronic form factors in this limit. This is a useful check on our calculation.

In order to give an impression of the calculated results, we show in Fig. 1 the two-photon correction for a typical electron energy. The two-photon exchange correction is compared to the contribution involving only the piece from a second soft photon (Coulomb distortion of the electron wave), calculated according to Ref. [6] in second Born approximation. The same contribution, but for a point-nucleus, is given for comparison. Fig. 1 shows that at forward angles the two-photon contribution is entirely dominated by Coulomb distortion, while at backward angles the exchange of two hard photons
FIG. 1: Relative contribution of two-photon exchange to elastic e-p scattering at E=160 MeV. The results in second Born-approximation account for the Coulomb distortion (exchange of second soft photon) only.

III. ANALYSIS OF WORLD E-P DATA

It has been previously shown in Ref. [3] that for an optimal determination of the proton moments a parameterization in terms of a Continued Fraction (CF) expansion should be used. For this parameterization, the contribution of model-dependence has been investigated in detail. As in Ref. [3] we use the cross sections up to a maximum momentum transfer of $4 \text{fm}^{-1}$.

We use the world cross sections on e-p scattering (for references see [3]). These data have been corrected for the contribution of two-photon effects (omitting the second Born Coulomb corrections, which are already included in the two-photon contribution), and then fitted with 5-parameter CF expressions for both the charge and the magnetic form factors $G_E(q^2)$ and $G_M(q^2)$. The two-photon corrected longitudinal/transverse separation is thus done implicitly during the fit. The charge-rms radius is obtained from the slope at $q^2=0$ of $G_E(q^2)$, the Zemach moment is obtained via

$$
\langle r \rangle_{(2)} = -\frac{4}{\pi} \int_0^\infty \frac{dq}{q^2}(G_E(q^2)G_M(q^2) - 1).
$$

The statistical errors have been determined using the error matrix, the systematic errors have been obtained by changing the data sets individually by their systematic errors, refitting the data and adding quadratically all the resulting changes of the moments. The contribution of the model dependence is accounted for as well.

IV. RESULTS AND CONCLUSION

The change of $\langle r \rangle_{(2)}$ found when removing the contribution of two-photon exchange (beyond Coulomb distortion) amounts to $+0.0052$ fm. This change is small ($\sim 40\%$ of the error bar) and goes in the wrong direction in terms of helping to explain the HFS discrepancy. The Zemach moment with two-photon effects corrected for amounts to $1.091 \pm 0.012$ fm. The change in the charge-rms radius, $+0.0015$ fm, is also small. The radius after two-photon correction amounts to $0.897 \pm 0.018$ fm.

From these results we conclude that the discrepancy between calculated and experimental HFS in the hydrogen atom cannot be attributed to two-photon exchange contributions to (e,e) that could have falsified the Zemach moment from e-p scattering. The origin of this discrepancy presumably has to be sought in the uncertain nuclear polarization correction to HFS, or potentially not yet calculated higher-order contributions to the HFS.

Acknowledgments

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