Abstract

The effects of correlations on the bulk properties of nuclei are investigated in large model spaces including up to 21 single-particle orbits. The evaluation of the single-particle Green function is made feasible by means of the BAGEL approximation. The spectral function for single-nucleon pick-up and removal is investigated for the nuclei $^{16}O$ and $^{40}Ca$. Special attention is paid to the effects produced by correlations on the calculated ground state properties of closed shell nuclei. It is observed that correlations beyond the Brueckner Hartree Fock approximation tend to improve the results obtained using realistic nucleon nucleon interactions.
1 Introduction

The nuclear shell-model, which describes the nucleus as a system of nucleons moving without correlations in a mean field, has been very successful in explaining a large number of basic nuclear phenomena at low energies. Nevertheless, it has become quite obvious that to derive nuclear properties from a realistic nucleon-nucleon (NN) interaction it is necessary to consider the correlations between the nucleons. This necessity arises from the fact that NN interactions, which are adjusted to describe NN scattering data, contain very strong short-range components. As a consequence, Hartree-Fock (HF) calculations employing such interactions predict bulk properties of nuclei which are far off the empirical data and, in fact, fail to account for the binding of nuclei.

Several methods have been developed to deal with the strong short-range components of the NN interaction. One of these methods is to consider that the wavefunctions contain short-range correlations, e.g. Jastrow correlation functions, which are optimized in a variational calculation [1]. An alternative approach is to introduce effective operators. A typical example of such an effective operator is the Brueckner G-matrix [2, 3]. This G-matrix is evaluated from the NN interaction by solving the Bethe-Goldstone equation, which accounts for the virtual excitation of two interacting nucleons into single-particle states above the Fermi energy, i.e. states unoccupied by other nucleons. In solving the Bethe-Goldstone equation for realistic NN interactions one has to consider 2-particle configurations up to an excitation energy of a few GeV [4, 5, 6].

The consideration of the short range, or high-energy, correlations in the Brueckner-Hartree-Fock (BHF) approach results in a major improvement on the prediction of nuclear properties, as compared to the independent-particle or HF approach. However, trying to evaluate the binding-energy and saturation density of nuclear matter, or the binding-energies and radii of finite nuclei within the BHF approximation, one still observes significant deviations from the corresponding data [7]. Motivated by the success of the Walecka model [8], attempts have been made to include relativistic effects within the so-called Dirac-BHF approach, leading to quite an improvement for the description of nuclear matter [9]. The corresponding calculation for finite nuclei, however, did not yield a satisfying agreement between theory and experimental data [10].

It is worth noting that quite different results are obtained in the studies of infinite nuclear matter and finite nuclei. This demonstrates that the structure of finite nuclei is more complicated and cannot be treated simply as a piece of nuclear matter of finite size. These differences cannot be attributed to the short-range correlations, as the configurations at high energies should be similar in both cases. It is, therefore, more likely that the different behaviour of nuclear matter and finite nuclei is due to the excitation modes of these systems at lower energies. At such energies the excitation spectrum for a finite system still reflects the shell structure, which is absent for the
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In this study, we investigate improvements to the BHF approach caused by a detailed treatment of low- and medium-energy excitations. Specifically, we investigate the effects produced by configurations with excitation energy up to about 100 MeV. These configurations are formed from single-particle states which all belong to a few major shells around the Fermi level. The effects produced on the nuclear wavefunction by the admixtures of these configurations will be described in the following as long-range correlations, in contrast to the short-range correlations discussed above.

For the study of these long-range correlations we employ a Green-function approach within a finite model space of discrete single-particle states \([4, 11, 12, 13]\). We describe the irreducible self-energy for the nucleons in terms of the HF contribution plus terms of higher order in the residual NN interaction, which are described by the diagrams shown in Fig. 1. For the residual interaction \(G\) we consider a G-matrix, which sums up all contributions originating from high-energy 2-particle excitations, i.e, excitations outside the space of our calculation (model-space). Thus all intermediate states displayed in Fig. 1 must be understood to belong to the model-space under consideration. Our definition of the nucleon self-energy is different from that employed in the BHF approximation, which includes diagram 1a, and those containing particle-particle ladder diagrams like 1c, but ignores contributions like those described by diagrams 1b, 1d and 1e. This non-symmetric treatment of particle-particle and hole-hole excitation is a typical feature of the Brueckner hole-line expansion. It may very well be justified for the study of short-range correlations, as the high-energy particle-particle configurations do not have any counterpart on the hole-hole side. The ordering of the hole-line expansion, however, may be questionable if low-energy excitations are to be considered. Therefore we would like to consider in our approximation for the self-energy, the contribution of all terms with intermediate 2 particle-1 hole and 2 hole-1 particle states (see Fig. 1) including all interactions between those states.

For the irreducible self-energy one may then solve the Dyson equation to determine the reducible self-energy or the corresponding single-particle Green-function. In the Lehmann representation this Green function is defined in terms of many poles below and above the Fermi-energy and the value of the spectral function at these poles. In ref. 13 the so-called BAGEL approach has been introduced to represent the complicated structure of the Green-function in terms of a few “characteristic” poles. Using this approximate representation, it becomes possible to evaluate a self-consistent Green-function in the sense that the irreducible self-energy is calculated using the Green-function, which, in turn, is obtained from the Dyson equation for this self-energy \([14]\).

One of the aims of the present study is to investigate the sensitivity of the results for the Green-function or the spectral function on the approximations chosen for the irreducible self-energy and on the choice of the model-space. As examples we consider the closed shell nuclei \(^{16}O\) and \(^{40}Ca\). The results obtained for the spectral function
can directly be compared with experimental data obtained in \((e, e')p\) experiments [13, 16, 17]. From the Green-function we can furthermore derive the occupation probabilities for the various single-particle orbits. These occupation probabilities are quite a conspicuous measure for the deviation from the independent particle model, i.e. a measure for the amount of correlation effects. Such occupation probabilities may be compared to results obtained for nuclear matter by using the Green-function approach [4, 18], within Brueckner theory [19, 20] or by means of the correlated basis function theory [21].

Finally, one may also use the Green-function to determine the ground-state energy and other observables like the radius. One can study the effects of correlations beyond the BHF approximation and estimate the importance of particle-particle and hole-hole ladder diagrams on the saturation properties of infinite nuclear matter [22, 23].

The Green-function approach is reviewed briefly in sect. 2. The presentation there helps to establish the nomenclature but also to define some details of the present investigation. Results on the spectral function, occupation probabilities, binding energies and radii for \(^{16}\text{O}\) and \(^{40}\text{Ca}\) obtained with the use of various approximations, are presented in sect. 3. The main conclusions are summarized in sect. 4.

2 BAGEL approximation for the one-body Green-function

2.1 Model Space and effective Hamiltonian

As already mentioned in the introduction, one of the main aims of the present investigation is to explore effects of long-range correlations by means of the Green-function approach within a certain truncated model-space. On the other hand, the effects of short-range correlations are taken into account by the introduction of an effective interaction, i.e a G-matrix appropriate for the model-space. Such a concept of a double-partitioned Hilbert space has been used before for finite nuclei [13, 24] and infinite nuclear matter [23, 25]. The G-matrix is determined as the solution of the Bethe-Goldstone equation

\[
\mathcal{G} = V + V \frac{Q_{\text{mod}}}{\omega - Q_{\text{mod}}TQ_{\text{mod}}} \mathcal{G}.
\]

In this equation \(T\) is the kinetic energy operator while \(V\) stands for the bare two-body interaction. For the latter we have chosen the One-Boson-Exchange potential \(C\) defined in table A.1 of [26]. The Pauli operator \(Q_{\text{mod}}\) is defined in terms of harmonic-oscillator single-particle states. Thus applying \(Q_{\text{mod}}\) to two-particle states
|αβ > one obtains

\[ Q_{\text{mod}}|αβ > = \begin{cases} 
0 & \text{if } α \text{ or } β \text{ below Fermi level} \\
0 & \text{if } α \text{ and } β \text{ in model space} \\
|αβ > & \text{else}
\end{cases} \] (2)

Here the Fermi level refers to the Fermi level of the independent particle model for the nucleus under consideration. The model-space in eq. (1) consists of all single-particle states up to and including the 0h1f2p shell of the oscillator potential, while a constant value of \( ω = -30 \text{ MeV} \) has been adopted for the starting energy. This choice of using a constant starting energy in the G-matrix is of course an approximation introduced to simplify the calculations. In sect. 3 we discuss the sensitivity of our results on the choice of the starting energy. There we will also consider the effects of using different model spaces.

A major problem in the use of the modern OBE versions of realistic NN interactions, which are defined e.g. in [26], is the fact that BHF calculations using these interactions yield predictions for the radii which are too small compared to the experimental data [27]. This has serious consequences on calculations beyond the mean field approximation. If one uses e.g. such BHF wave functions to evaluate the residual interaction to be used in shell-model calculations, one obtains results which are in poor agreement with the experimental data. On the other hand, evaluating the matrix elements for a set of harmonic oscillator wave functions which yield accurate radii, leads to matrix elements for the shell-model calculation, which provide a fair agreement with experiment [28]. Therefore one is tempted to modify the hamiltonian in such a way that a HF calculation using \( \mathcal{G} \) yields a self-consistent basis of oscillator wave functions which reproduce the experimental radius. This is achieved by subtracting from the \( \mathcal{G} \) defined in eq. (1) the non-diagonal terms of the HF single-particle hamiltonian in the oscillator basis

\[ Δ\mathcal{G} = \sum_{α≠β}(T_{αβ} + U_{αβ}) a_α^d a_β^\dagger , \] (3)

where \( T_{αβ} \) and \( U_{αβ} \) define the matrix elements of the kinetic energy and the HF single-particle potential , respectively, calculated in a harmonic oscillator basis. This basis is defined by an oscillator parameter of \( b = 1.76 \text{ fm} \) for all studies of \(^{16}O\) and \( b = 2.0 \text{ fm} \) in case of the nucleus \(^{40}Ca\).

### 2.2 Green function and matrix diagonalisation

As a first step towards the evaluation of the single-particle Green function we determine the contributions to the irreducible self-energy, which are of second order in the residual interaction:

\[ Σ^{(2)}_{αβ}(ω) = \frac{1}{2} \sum_{γδμ} \int \frac{dω_1}{2πi} \int \frac{dω_2}{2πi} <αμ|\mathcal{G}|γδ > <γδ|\mathcal{G}|βμ > g_γ(ω - ω_1 + ω_2)g_δ(ω_1)g_μ(ω_2) . \] (4)
Note that the summation on single-particle states $\gamma$, $\delta$, $\mu$ is restricted to states within the model-space. For discrete states $\alpha$, the $g_{\alpha}(\omega)$ denote the Hartree-Fock (HF) approximation for the Green function [13]. This equation can be rewritten into the form

$$\Sigma_{\alpha\beta}^{(2)}(\omega) = \frac{1}{2} \sum_{p_1,p_2,h} <\alpha|p_1,p_2,h|\beta> <\alpha|p_1,p_2,h|\gamma> <\gamma|h_1,h_2,p|\beta>$$

$$+ \frac{1}{2} \sum_{h_1,h_2,p} <\alpha|h_1,h_2,p|\beta> <\alpha|p_1,p_2,h|\gamma> <\gamma|\alpha|p_1,p_2,h|\beta>$$

where we have introduced the abbreviation

$$e(\alpha, \beta, \gamma) = \epsilon_{HF}^{\alpha} + \epsilon_{HF}^{\beta} - \epsilon_{HF}^{\gamma}$$

and the $\epsilon_{HF}^{\alpha}$ are the HF single-particle energies. In eq. (5) the summations on particle labels like $p_1,p_2$ and $p$ are restricted to those single-particle states within the model space, which are above the Fermi level, whereas the labels $h_1,h_2$ and $h$ refer to hole states. It is evident that the first term on the right hand side of eq. (5) refers to the 2 particle - 1 hole contribution to the self-energy (diagram a of Fig. 1) while the second term defines 2 hole - 1 particle contribution (diagram 1b). This self-energy can now be inserted into a Dyson equation for the Green function $G_{\alpha\beta}(\omega)$ taking into account the correlation effects contained in $\Sigma^{(2)}$.

$$G_{\alpha\beta}(\omega) = \delta_{\alpha\beta} g_{\alpha}(\omega) + \sum_{\gamma} g_{\alpha}(\omega) \Sigma_{\alpha\gamma}(\omega) G_{\gamma\beta}(\omega)$$

$$= \sum_{n} \frac{<\Psi_0^A|a_\alpha^{\dagger}\Psi_0^A+1^n>|<\Psi_0^A+1^n|a_\alpha|\Psi_0^A>|}{\omega - \omega_n^+ + i\eta}$$

$$+ \sum_{m} \frac{<\Psi_0^A|a_\beta^{\dagger}\Psi_0^A-1^m>|<\Psi_0^A-1^m|a_\alpha|\Psi_0^A>|}{\omega - \omega_m^- - i\eta}. \quad (8)$$

The second line (eq.(8)) exhibits the Lehmann representation of the single-particle Green function in terms of the spectroscopic amplitudes $<\Psi_0^A|a_\alpha^{\dagger}|\Psi_0^A+1^n>$ and $<\Psi_0^A+1^n|a_\beta^{\dagger}|\Psi_0^A>$, where $a_\alpha$ ($a_\beta^{\dagger}$) stands for the single-particle annihilation (creation) operator in the HF basis. The state $\Psi_0^A$ refers to the ground-state of the $A$-particle system, while $\Psi_0^A+1^n$ ($\Psi_0^A-1^m$) stands for the states of the $A+1$-particle ($A-1$) system as obtained in the present approach. Also the energy variables

$$\omega_n^+ = E_n^{A+1} - E_0^A,$$

$$\omega_m^- = E_0^A - E_m^{A-1}, \quad (9)$$

are defined in terms of energies obtained for the states of the nuclei with $A$, $A+1$ and $A-1$ nucleons. The substantial ingredients of the Lehmann representation of
2 BAGEL APPROXIMATION FOR THE ONE-BODY GREEN-FUNCTION

Eq. (8) can be obtained from a solution of an eigenvalue problem \[13\]

\[
\begin{pmatrix}
\varepsilon_{HF}^\alpha & 0 & a_1 & \ldots & a_K & A_1 & \ldots & A_L \\
0 & \varepsilon_{HF}^\beta & b_1 & \ldots & b_K & B_1 & \ldots & B_L \\
a_1 & b_1 & 0 & \ldots & 0 & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
a_K & b_K & 0 & \ldots & 0 & 0 & \ldots & 0 \\
A_1 & B_1 & 0 & \ldots & 0 & 0 & \ldots & 0 \\
A_L & B_L & 0 & \ldots & 0 & 0 & \ldots & 0
\end{pmatrix}
\begin{pmatrix}
X_{0,\alpha}^n \\
X_{0,\beta}^n \\
X_1^n \\
\vdots \\
X_K^n \\
Y_1^n \\
\vdots \\
Y_L^n
\end{pmatrix} = \omega_n
\begin{pmatrix}
X_{0,\alpha}^n \\
X_{0,\beta}^n \\
X_1^n \\
\vdots \\
X_K^n \\
Y_1^n \\
\vdots \\
Y_L^n
\end{pmatrix} . \tag{11}
\]

In writing this equation, we assume that for a given set of conserved quantum numbers (parity, isospin, angular momentum) we have in our model space two HF single-particle states (\(\alpha, \beta\)) and \(K\) 2p1h configurations with energies (see definition of eq. (6))

\[12\]

\[a_i = \langle \alpha h | G | p_1 p_2 \rangle , \quad b_i = \langle \beta h | G | p_1 p_2 \rangle . \tag{13}\]

Furthermore we consider \(L\) 2h1p configurations of the same symmetry as the single-particle states. These have energies

\[14\]

\[E_j = e(h_1, h_2, p) , \tag{14}\]

and connecting matrix elements

\[15\]

\[A_j = \langle \alpha p | G | h_1 h_2 \rangle , \quad B_j = \langle \beta p | G | h_1 h_2 \rangle . \tag{15}\]

Solving eq. (11) one obtains eigenvalues \(\omega_n\). These eigenvalues must be identified with \(\omega_n^+\) of the Lehmann representation of eq. (8), if \(\omega_n\) is an energy above the Fermi energy\(^1\). In this case we have

\[16\]

\[\langle \Psi_0^A | a_\alpha | \Psi_n^A \rangle = X_{0,\alpha}^n . \tag{16}\]

If \(\omega_n\) is below the Fermi energy we obtain \(\omega_n = \omega_n^-\) and

\[17\]

\[\langle \Psi_0^A | a_\alpha^\dagger | \Psi_n^A \rangle = X_{0,\alpha}^n . \tag{17}\]

In a straightforward way one can improve the approximation discussed so far and incorporate the effects of residual interactions between the 2p1h and 2h1p configurations, i.e. account for diagrams like those displayed in Fig. 1c, 1d and 1e in the

\(^1\)The definition of the Fermi energy is given below
definition of the self-energy. One simply has to modify the corresponding parts of the matrix in eq. (11) and replace
\[
\begin{pmatrix}
e_1 & \ldots & 0 \\
\vdots & \ddots & \vdots \\
0 & \ldots & e_K
\end{pmatrix} \mapsto \mathcal{H}_{2p1h}, \quad \text{and} \quad \begin{pmatrix}
E_1 & \ldots & 0 \\
\vdots & \ddots & \vdots \\
0 & \ldots & E_K
\end{pmatrix} \mapsto \mathcal{H}_{2h1p}, \quad (18)
\]
where \(\mathcal{H}_{2p1h}\) and \(\mathcal{H}_{2h1p}\) contain the residual interactions in the 2p1h and 2h1p subspaces.

The normalization of the eigenvectors of eq. (11) ensures that
\[
\sum_n |X_{0,\alpha}^n|^2 + \sum_m |X_{0,\alpha}^m|^2 = \sum_n |<\Psi_{A+1}^n|a^\dagger_\alpha|\Psi_0^A>|^2 + \sum_m |<\Psi_{A-1}^m|a_\alpha|\Psi_0^A>|^2 = 1, \quad (19)
\]
where the sum on \(n\) accounts for all solutions with \(\omega_n^+\) larger than the Fermi energy and the sum on \(m\) for all solutions with energies \(\omega_m^-\) below the Fermi energy. These spectroscopic factors can also be used to determine the occupation probabilities
\[
N_\alpha = \sum_m |<\Psi_{A-1}^m|a_\alpha|\Psi_0^A>|^2. \quad (20)
\]
The approach discussed up to now, however, is not number-conserving in the sense that it is not guaranteed that
\[
\sum_\alpha N_\alpha (2j_\alpha + 1) = A, \quad (21)
\]
with \((2j_\alpha + 1)\) representing the degeneracy of the states \(\alpha\). This equation, however, allows one to determine the Fermi energy in such a way that the particle number calculated according to the left hand side of eq. (21) is as close as possible to the mass number \(A\) of the nucleus under consideration. An approach which is strictly number-conserving is obtained only if the Green function used to calculate the self-energy (like in eq. (4)) is consistent with the Green function resulting from the solution of the Dyson equation (7) [29, 14].

With the help of the single-particle Green function \(G_{\alpha\beta}\) one can evaluate the expectation value for an arbitrary single-particle operator \(\hat{O}\)
\[
<\Psi_0|\hat{O}|\Psi_0> = \int_C \frac{d\omega}{2\pi i} \sum_{\alpha\beta} <\alpha|\hat{O}|\beta> G_{\alpha\beta}(\omega), \quad (22)
\]
where the \(C\) below the integral sign denotes a contour integration counter-clockwise in the upper half plane including the real axis. Therefore only the contributions from the poles at energies below the Fermi energy have to be considered and using the
nomenclature of the matrix representation in eq. (11) this expectation value can be rewritten as
\[
< \Psi_0 | \hat{O} | \Psi_0 > = \sum_{\alpha \beta m} < \alpha | \hat{O} | \beta > X^m_{\alpha, \alpha} X^m_{\beta, \beta},
\]
where the sum on \(m\) is restricted to solutions with energies \(\omega_m\) below the Fermi energy. The Green function can furthermore be used to evaluate the binding energy of the \(A\)-nucleon system by
\[
E^A_0 = \frac{1}{2} \int_C \frac{d\omega}{2\pi i} \sum_{\alpha \beta} [ < \alpha | T | \beta > + \delta_{\alpha \beta} \omega ] G_{\alpha \beta} (\omega),
\]
where \(T\) denotes the kinetic energy.

2.3 The BAGEL approximation

After we have rewritten the solution of the Dyson equation in terms of an eigenvalue problem (eq. (11)), it is now straightforward to introduce the BAGEL approximation. The BAGEL approximation has originally been formulated \[30\] to define an appropriate truncation scheme for shell-model calculations in large model spaces. Since then this method has been employed in several calculations \[31\], including attempts to derive effective operators for truncated model spaces \[32\]. The manner in which the BAGEL approximation can be used to determine the single-particle Green function has already been discussed in \[13, 14\], so that only a brief outline of the method need to be given here.

For many applications of the single-particle Green function it is not necessary to know in detail all of its poles in the Lehmann representation. It should rather be sufficient to describe the energy distribution of the spectroscopic amplitudes in terms of a few “characteristic” states of the \(A \pm 1\) systems. For that purpose we consider the operator \(\hat{a}\) which corresponds to a part of the matrix in eqs. (11) and (18)
\[
\hat{a} = \begin{pmatrix}
\epsilon_{a1}^{HF} & a_1 & \cdots & a_K \\
a_1 & \cdots & \cdots & a_K \\
\vdots & \cdots & \cdots & \cdots \\
a_K & \cdots & \cdots & \cdots \\
\end{pmatrix} H_{2p1h},
\]
and apply this operator on the single-particle state \(|\alpha\rangle\), which in terms of the matrix representation of eq. (23) is described by the column vector \((1, 0 \ldots 0)^T\)
\[
\hat{a} |\alpha\rangle = \epsilon_{a\alpha}^{HF} |\alpha\rangle + \tilde{a}_1 |\alpha_1\rangle,
\]
where \(|\alpha_1\rangle\) is orthogonal to \(|\alpha\rangle\) and the coefficient \(\tilde{a}_1\) is chosen so that \(|\alpha_1\rangle\) is normalized. Following the Lanczos algorithm \[33\], one can subsequently construct additional states \(|\alpha_i\rangle\), which are all orthogonal to each other. Applying the Lanczos procedure \(N\) times one obtains \(N\) basis states of the 2p1h configuration space, which
have the same symmetry quantum numbers as the single-particle state $|\alpha>$.
If now the model-space contains another single-particle state $|\beta>$ of the same symmetry,
one may consider in an analogous way the operator $\hat{b}$ defined by
\[
\hat{b} = \begin{pmatrix}
\epsilon_{\beta}^{HF} & b_1 & \ldots & b_K \\
b_1 & \mathcal{H}_{2p1h} & \\
\vdots & & \\
b_K & & 
\end{pmatrix},
\]
and obtain by applying it initially to $|\beta>$ additional $N$ basis states $|\beta_i>$. In this
case special care should be taken that the states $|\beta_i>$ are not orthonormalized only
among themselves but also with respect to the $|\alpha_i>$. In a similar way we can furthermore construct for each single-particle state $|\alpha>$,
$M$ basis states of the 2h1p configuration space by considering the corresponding sub
matrices of eq. (11)
\[
\hat{A} = \begin{pmatrix}
\epsilon_{\alpha}^{HF} & A_1 & \ldots & A_L \\
A_1 & \mathcal{H}_{2h1p} & \\
\vdots & & \\
A_L & & 
\end{pmatrix}.
\]
In this way, the BAGEL(N,M) approximation reduces the matrix defined in eq (11) to a subspace of dimension $l \ast (N + M + 1)$, if $l$ is the number of single-particle states
of the same symmetry. The Green function of this BAGEL(N,M) approximation is
then defined in the manner discussed in the preceding section but now considering
only the eigenvalues and eigenvectors of the truncated matrix. It is obvious that the
BAGEL(0,0) corresponds to the HF approximation, while for large enough $N$ and $M$
the BAGEL(N,M) becomes identical to the exact solution of eq. (11).

3 Results and Discussion

3.1 The Spectral Function

The spectral function summarizes the spectroscopic factors for adding, at energies
above the Fermi energy $E_F$, or removing, at energies below $E_F$, nucleons from the
nucleus under consideration. For a model space with discrete single-particle orbits,
as we consider in the present investigation, the spectral function is given in terms of
$\delta$ functions at the various poles of the single-particle Green function (see eq. (8)).
For the sake of presentation, we will fold these $\delta$ functions with a gaussian distribution
assuming a with $\Gamma$. This width can be interpreted as a simple way to include the
escape width for the states of the residual nuclei, or as a tool to express the uncertainty
of the theoretical calculation. The second interpretation applies in particular to the
use of the BAGEL approximation. With this folding procedure the spectral function
is given by

\[ S_\alpha(\omega) = \frac{1}{\sqrt{\pi} \Gamma} \left[ \sum_n \exp{-\left(\frac{\omega - \omega_n^+}{\Gamma}\right)^2} \left< \Psi_{n+1}^A \left| a_\alpha^\dagger \Psi_0 \right> \right|^2 + \sum_m \exp{-\left(\frac{\omega - \omega_m^-}{\Gamma}\right)^2} \left< \Psi_{m-1}^A \left| a_\alpha \Psi_0 \right> \right|^2 \right] . \]  

(29)

In this equation, as in eq. (19), the summation over \( n \) is restricted to the poles \( \omega_n^+ \) above \( E_F \) and the summation over \( m \) to those below the Fermi energy.

As a first example, we present in Fig. 2 results for the spectral function of the \( p_{3/2} \) orbits in \( ^{40}Ca \) for energies below the Fermi energy. In this figure we compare the results obtained in the BAGEL(M=10, N=10) approach to those obtained using \( M=N=20 \). If we choose for the presentation a width of \( \Gamma = 2 \) MeV (lower part of the figure) differences between the two approximations are clearly visible. It is not only that the \( M=N=20 \) distribution displays more poles, as the corresponding Green function contains more poles, but also the position of the peaks are shifted if the accuracy of the BAGEL approximation is improved. If, however, we examine the same results with a reduced energy resolution, obtained by increasing the width in eq.(29) to \( \Gamma=5 \) MeV, the differences between the 2 BAGEL approximation become invisible. This is a demonstration of the fact that the BAGEL(M,M) approximation reproduces all energy-weighted moments of the exact calculated distribution from order \( n = 0 \) to the order \( n = 2M+1 \). In the following discussion, we will present results for the BAGEL(20,20) approximation and display the data assuming a width \( \Gamma=2\)MeV.

The spectral functions displayed in figures 3 and 4 have been obtained by multiplying the function \( S_\alpha \) above with the degeneracy factors \( 2j_\alpha + 1 \) and adding up the various contributions with the same orbital angular momentum. Fig. 3 shows the spectral functions for \( ^{16}O \) in the energy interval ranging from \( \omega = -50 \) MeV to 30 MeV. The results in this figure clearly reflect the shell-structure of nuclei and the limitations of the independent particle model (IPM). For orbital angular momentum \( l = 0 \) one observes a strong peak at -4 MeV, just above the Fermi energy, which can be identified with the \( 1s_{1/2} \) state of the IPM. The single-particle strength for the \( 0s_{1/2} \) and \( 2s_{1/2} \) states is distributed in energy intervals around -40 MeV and around +20 MeV, respectively.

Similar features are also observed for the \( l = 1 \) states. The strength below the Fermi energy is mainly located at an energy \( \omega \) around -18 MeV. This peak originates from two poles: one at \( \omega=-19.12 \) MeV \( (p_{3/2}) \), the other at -16.73 MeV \( (p_{1/2}) \). Each of these poles carries around 75% of the total strength below the Fermi energy in \( p_{3/2} \) and \( p_{1/2} \) states, respectively. With the energy resolution (\( \Gamma = 2 \) MeV) considered for the figures, the spin-orbit splitting is not resolved. It is worth noting that the contributions to the self-energy beyond HF reduce the calculated spin-orbit splitting for the \( l = 1 \) hole states from 3.6 MeV to 2.4 MeV. This last number is calculated by taking the energy difference between the two states carrying maximum strength. The decrease in the spin-orbit splitting is in agreement with the results of previous
calculations \[34\], which suggest that relativistic effects are mainly responsible for the spin-orbit splitting between hole states.

Additional \(l = 1\) single-particle strength is observed at lower energies, although the individual contributions are so small that they are barely visible in Fig. 3. It is worth noting that the total occupation probability (see eq. (20)) is 0.926 (0.901) for the \(p_{3/2}\) \((p_{1/2})\) state while the contribution of the main peak to this value is only 0.781 (0.790). The additional strength originates from states at lower energies, which implies larger excitation energies in the A-1 nucleus. The \(l = 1\) spectral function furthermore displays the single-particle strength at positive energies, which corresponds to the \(1p\) and \(2p\) states of the IPM.

In the top part of Fig. 3 the spectral function for the \(l = 2\) and \(l = 3\) states are displayed. In the IPM these states are unoccupied. Due to the correlations included in our approximation, we observe an occupation probability of 0.052 and 0.010 for the \(d\) and \(f\)-states, respectively. In the case of the \(l = 2\) spectral function one can observe two peaks originating from the \(d_{5/2}\) and \(d_{3/2}\) states. The calculated spin orbit splitting between these states is 4.92 MeV, slightly below the HF result of 5.41 MeV. For these particle states, the result with and without correlations, is in good agreement with the experimental value of 5.09 MeV. This is again in line with the arguments given in ref. \[34\], as the relativistic effects discussed there should not influence the spin-orbit splitting for the particle states.

In Fig. 4 we show the spectral function calculated for \(^{40}Ca\), considering the orbital angular momenta \(l = 0\) to \(l = 3\). The presentation is limited to energies below the Fermi energy, since for these energies a comparison with experimental data, deduced from \((e,e')p\) experiments is possible \[17\]. In the case of \(l = 0\), the spectral strength for the hole-state close to the Fermi energy is nicely reproduced. The broad distribution of \(l = 0\) strength originating from deep-lying hole states, which is predicted in the calculation down to energies of \(-100\) MeV, has not been resolved in the experimental data. Also for \(^{40}Ca\) the spin-orbit splitting calculated for the \(d\) hole states is smaller than the observed splitting. In the experimental data one finds a spectral strength below the Fermi-energy for \(l = 3\) (notice the modified scale in the lower right part of Fig. 4). Such a strength is completely absent in the IPM or HF approach. Including correlation effects we observe a strength of similar size as the experimental data. While, however, the experimental strength is located in essentially one state, the calculation yields a distribution over various states. The agreement between theory and experiment is reasonable, keeping in mind that the theoretical results have been derived from a realistic NN interaction without any adjustable parameter. Results on the occupation probabilities obtained for \(^{40}Ca\) considering various approximations for the nucleon self-energy are listed in table \[4\].
3.2 Correlations and Ground-State Properties

A major aim of the present investigation is to study the effects of correlations included in the definition of the nucleon self-energy on the bulk properties of closed shell nuclei. For that purpose we consider various approximations. The first of these is what we will call the Hartree-Fock (HF) approximation, i.e. the self-energy is calculated in the HF approximation. Note, however, that this HF approximation is something in between a conventional HF and BHF calculation. The G-matrix interaction (see section 2.1) accounts for intermediate 2-particle states, which are outside the considered model space. However, in contrast to a self-consistent BHF calculation it does not account for 2-particle states between the Fermi level and the limit of the model space. Such correlations are taken into account in the subsequent approximations. Furthermore the hamiltonian has been adjusted in such a way that this HF approximation leads to a value for the radius of the nuclei $^{16}O$ and $^{40}Ca$, which are close to the experimental values (eq.(3)). Therefore, in studying effects of correlations beyond HF, one should pay attention only to the effects in the calculated radii relative to the HF result and not expect any improvement compared to the experiment.

The second approximation considered in our study consists in adding the contribution of the second-order (2p1h) diagram (a) in Fig. 1 to the self-energy. This approximation will be labeled “bare 2p1h” approach in the tables and the discussion below. A part of the residual interaction in the space of 2p1h configurations is given by the interaction between the two particle lines. The approximation which accounts for this part of the residual interaction and includes the diagram of Fig. 1c plus all other particle-particle ladder diagrams in the definition of the self-energy, corresponds to the Brueckner - Hartree-Fock approach and will be identified in the following as “BHF”. On the other hand the term ”2p1h” describes the approximation in which the whole residual interaction is considered in the 2p1h subspace (diagram of Fig. 1d and others). Finally, the term “total” describes the calculation in the complete space of 2p1h and 2h1p configurations with all residual interactions included. This means that the complete matrix in eq. (11) extended by the modifications of eq. (18) is considered to construct the Green function.

For all studies beyond HF the single-particle Green function has been determined in the BAGEL(M=10,N=10) approximation. The results on the bulk properties of nuclei discussed below turned out to be extremely stable with respect to this approximation. Increasing the accuracy of the BAGEL to M=N=20, did not lead to any change in the results displayed in the tables.

For the resulting Green functions we have determined the occupation probabilities $N_{\alpha}$ according to eq. (20) and a mean value for the spectral distribution below the Fermi energy, defined by

$$\epsilon_{<,\alpha} = \frac{1}{N_{\alpha}} \int_C \frac{d\omega}{2\pi i} \omega G_{\alpha\alpha}(\omega) = \frac{1}{N_{\alpha}} \sum_{m} \omega_{m}^{-} |<\Psi_{A^{-1}}|a_{\alpha}|\Psi_{0}>|^2. \quad (30)$$

The contour integral in the first part of the equation is defined in the same way as in
eq. (22) and as before the summation in the second part of the equation is restricted to poles in the Green function with an energy $\omega_n$ below the Fermi energy. In the case of the HF approximation the definition in eq. (30) leads to the HF single-particle energy. The modification of these mean values due to the correlations give some insight into the effects of correlation on the calculated binding energy (compare eq. (24)).

Results for the occupation probabilities and the $\epsilon_\alpha^<$ are listed in table 1 for some single-particle orbits of $^{40}\text{Ca}$. One finds that the admixture of 2p1h configurations reduces the occupation probability by 3% for the deep-lying hole states and by around 5% for states closer to the Fermi level. Inclusion of the residual interaction between the 2p1h configurations slightly enhances these depletions for the occupation probabilities. The effects of the residual interaction are dominated by the pp ladder diagrams while the additional interaction terms can essentially be ignored. The removal of occupation strength from the state below the Fermi energy to 2p1h configurations above $E_F$ is accompanied by a lowering of the mean values $\epsilon_\alpha^<$ below the HF single-particle $\epsilon_{HF}$. This is obvious since one has the following relation [13]

$$\sum_\alpha N_\alpha \epsilon_\alpha^< + (1 - N_\alpha) \epsilon_\alpha^> = \sum_\alpha \epsilon_{HF}^\alpha ,$$

where the sum on $\alpha$ is a sum on all single-particle orbits of the same symmetry and the mean value for the spectral distribution above the Fermi energy is defined in analogy to eq. (30) by

$$\epsilon_\alpha^> = \frac{1}{1 - N_\alpha} \sum_n \omega_n^+ | < \Psi_n^{A+1} | a_\alpha^\dagger | \Psi_0^> |^2 .$$

It is this lowering of the mean values $\epsilon_\alpha^<$, which is the main source for the gain in energy per particle when we evaluate the energy according to eq. (24) using the “bare 2p1h”, “BHF” and “2p1h” approximations. The calculated values for the energy per particle for $^{16}\text{O}$ and $^{40}\text{Ca}$ are listed in table 2. As seen in this table, the inclusion of 2p1h configurations within our model space increases the calculated binding energies by 1.9 MeV and 1.6 MeV per nucleon for $^{16}\text{O}$ and $^{40}\text{Ca}$, respectively. As already discussed above, the effects of the residual interaction between 2p1h configurations, are small (0.3 MeV per nucleon) and are mainly due to the particle-particle interactions contained in the BHF approach. The effect of the 2p1h admixture on the calculated radii is essentially negligible.

It should be mentioned that the way we calculate the energy per nucleon in the BHF approximation is slightly different from the conventional form. In conventional BHF calculations one determines the total energy from the mean values $\epsilon_\alpha^<$, which, for the corresponding approximation in the self-energy, are identical to the BHF, $\epsilon_{BHF}^<$, single-particle energies and calculates the total energy from

$$E_{\text{conv}}^{BHF} = \frac{1}{2} \sum_\alpha [ < \alpha | T | \alpha > + \epsilon_{BHF}^\alpha ] n_\alpha ,$$

3. RESULTS AND DISCUSSION
with $n_\alpha$ the occupation probabilities of the HF or IPM approach. This expression essentially corresponds to eq. (24) except that the conventional BHF approach ignores the differences in the occupation probabilities for the various states. The calculated binding energy per nucleon according to eq. (24), using for the particle number the results of eq. (21) are very close to those obtained in the conventional way (differences are typically around 0.1 MeV).

The inclusion of 2h1p configurations in the “total” calculation leads to a fragmentation of the strength into many states below the Fermi energy. One finds, that occupation probabilities and mean values for the spectral distribution $\epsilon_\alpha$ are not very much affected by the inclusion of these additional correlations (see table I) for the states which are occupied in the IPM. The total approach, however, leads to a non-vanishing occupation also for configurations with quantum numbers $\alpha$, which are unoccupied in the IPM. The mean value $\epsilon_\alpha$ for these 2h1p configurations is typically quite attractive. Therefore one observes a non-negligible gain in the binding energy due to the hole-hole correlations. In our standard model space this gain is 0.4 MeV and 0.3 MeV per nucleon for $^{16}O$ and $^{40}Ca$, respectively.

Considering just the energies it is obvious, that the effect of 2p1h correlations is significantly larger than those obtained from 2h1p. This would support the BHF approximation, which includes the effects of the former kind but ignores 2h1p contributions. For the calculation of the nuclear radii, however, the 2h1p correlations seem to be more important than those deduced from particle-particle correlations. The inclusion of 2h1p correlations yields an increase for the calculated radii of around 0.1 fm. Alone this increase does not seem to be dramatic. It is worth noting, however, that the combination, an increase of calculated binding energy and radius, moves the result for these bulk properties of nuclei away from the so-called “Coester band” [27]. This is a similar feature as has also been observed for hole-hole correlations in infinite nuclear matter [22, 23].

The sensitivity of our results on the choice of the model space and on the parameters of the residual interaction is examined in table 3. This table contains results for $^{16}O$ for three choices of the model space: one with 10 active orbits, another with 15 and a third where 21 active orbits are considered. This table contains results for the energies per nucleon obtained in the HF approximation, for the “total” approach and the difference between these 2 values. Since the effective interaction depends on the choice of the model space, also the HF results change significantly. For the small model space ($N_{cut}=10$) the effects of 2p1h correlation with 2 particles in sdg-shell or above are contained already in the interaction. Therefore a larger binding energy is obtained in the HF approximation for this small model space than for the HF calculation using the interaction which is appropriate for a larger model space.

These effects of the 2p1h correlations and the interference with 2h1p correlations are contained in the “total” calculation. The convergence of these correlation effects is rather slow with increasing model space. Both the calculated binding energies as well as the results for the radii are enlarged if the number of shells included in the
model space is increased. Therefore the features discussed above may get enhanced if even larger model spaces are considered.

Table 3 displays results also for an alternative choice for the starting energy in eq. (1), using $\omega = -5\text{MeV}$ instead of our standard choice $\omega = -30\text{MeV}$. One observes that the larger value for $\omega$ yields larger energies already in the HF approximation and also for the “total” calculation. The correlation effects reflected in the energy differences between these two approximations are rather insensitive to the choice of $\omega$.

4 Conclusions

The effects of correlations beyond the Hartree-Fock approximation are investigated in large model spaces including up to 21 active single-particle states of the harmonic oscillator potential. The residual interaction for this model space is derived from a realistic meson-exchange potential [26], accounting for short-range correlation effects beyond the model space by solving an appropriate Bethe-Goldstone equation. The correlations within the model space are evaluated in terms of the single-particle Green function. The Green function can easily be evaluated even for a large model space by employing the BAGEL [13, 14] approximation.

The spectral functions evaluated from this Green function for $^{16}\text{O}$ and $^{40}\text{Ca}$ yield a distribution of single-particle strength over a broad range of energies. A comparison with an empirical spectral function for $^{40}\text{Ca}$, determined from $(e, e')p$ experiments [17] demonstrates that the spectral function, calculated without adjustable parameter, produces a reasonable agreement. However, the spin-orbit splitting between hole states close to the Fermi energy is underestimated even after the inclusion of correlation effects. It is argued that relativistic features, not considered in the present work, are mainly responsible for this splitting [34].

Special attention is paid to the effects of correlation, treated in various approximations, on the calculation of bulk properties of nuclei, like binding energy and radius. It turns out that the effects of correlation on the calculated energies are dominated by the particle-particle (2p1h) correlations taken into account in the BHF approximation. Some additional binding energy is obtained if also hole-hole (2h1p) correlations are considered. The 2h1p correlations furthermore give rise to an increase for the calculated radius. Therefore the properties evaluated for ground states of nuclei are “moved off the Coester band” towards the experimental data [27]. This effect of hole-hole correlations is similar to the one observed in nuclear matter [22, 23].

The effects obtained from inclusion of the hole-hole correlations are not very large. In the model space under investigation they yield an extra gain in binding energy of around 0.4 MeV per nucleon and an increase of the radii of around 0.1 fm. The effects may get larger, if larger model spaces are taken into account. This is possible with the techniques explored here. However, for a realistic investigation one should try to
REFERENCES

represent the particle states in terms of plane waves rather than harmonic oscillator states.

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References

[1] V.R. Pandharipande and R.B. Wiringa, Rev. Mod. Phys. 51 (1979) 821
[2] K.A. Brueckner and D.T. Goldman, Phys. Rev. 117 (1960) 207
[3] B.D. Day, Rev. Mod. Phys. 39 (1967) 719
[4] W.H. Dickhoff and H. Müther, Rep. Prog. Phys. 11 (1992) 1947
[5] A. Ramos, A. Polls, and W.H. Dickhoff, Nucl. Phys. A503 (1989) 1
[6] M. Borromeo, D. Bonatsos, H. Müther and A. Polls, Nucl. Phys. A539 (1992) 189
[7] H. Müther, Prog. Part. and Nucl. Phys. 14 (1985) 123
[8] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. 16 (1986) 1
[9] R. Brockmann and R. Machleidt, Phys. Rev. C 42 (1990) 1965
[10] R. Fritz, H. Müther and R. Machleidt, Phys. Rev. Lett. 71 (1993) 46
[11] M.G.E. Brand, G.A. Rijsdijk, F.A. Muller, K. Allaart and W.H. Dickhoff, Nucl. Phys. A531 (1991) 253
[12] D. Van Neck, M. Waroquier and J. Ryckebusch, Nucl. Phys. A530 (1991) 347
[13] H. Müther and L.D. Skouras, Nucl. Phys. A555 (1993) 541
[14] H. Müther and L.D. Skouras, Phys. Lett. B 306 (1993) 201
[15] P.K.A. de Witt Huberts, Nucl. Phys. A507 (1990) 189c
[16] P. Grabmayr, G.J. Wagner, H. Clement, and H. Röhm, Nucl. Phys. A494 (1989) 244
[17] G. Kramer, thesis Univ. of Amsterdam (1990)
[18] B.E. Vonderfecht, W.H. Dickhoff, A. Polls and A. Ramos, Phys. Rev. C44 (1991) R1265
[19] C. Mahaux and R. Sartor, Adv. Nucl. Phys. 20 (1991) 1
REFERENCES

[20] H.S. Köhler, *Nucl. Phys.* **A529** (1991) 209

[21] O. Benhar, A. Fabrocini and S. Fantoni, *nucl. Phys.* **A505** (1989) 267

[22] H.Q. Song, S.D. Yang and T.T.S. Kuo, *Nucl. Phys.* **A462** (1987) 491

[23] M.F. Jing, T.T.S. Kuo and H. Mühler, *Phys. Rev.* **C 38** (1988) 2408

[24] B.R. Barrett and M.W. Kirson, *Adv. in Nucl. Phys.* **6** (1973) 219

[25] T.T.S. Kuo, Z.Y. Ma and R. Vinh Mau, *Phys. Rev.* **C33** (1987) 717

[26] R. Machleidt, *Adv. in Nucl. Phys.* **19** (1989) 189

[27] K.W. Schmid, H. Mühler and R. Machleidt, *Nucl. Phys.* **A530** (1991) 14

[28] M. Hjorth-Jensen, E. Osnes, H. Mühler and K.W. Schmid, *Phys. Lett.* **B248** (1990) 243

[29] G. Baym and L.P. Kadanoff, *Phys. Rev.* **124** (1961) 287

[30] H. Mühler, T. Taigel and T.T.S. Kuo, *Nucl. Phys.* **A482** (1988) 601

[31] W. Leitner and H. Mühler, *Nucl. Phys.* **A469** (1987) 61

[32] L.D. Skouras and H. Mühler, *Nucl. Phys.* **A515** (1990) 93

[33] J.H. Wilkinson, “*The Algebraic Eigenvalue Problem*”, Oxford Univ. Press (1965)

[34] L. Zamick, D.C. Zheng and H. Mühler, *Phys. Rev.* **C45** (1992) 2763
Table 1: Occupation probabilities and mean values for the energy distribution below the Fermi energy, calculated for different single-particle states in $^{40}\text{Ca}$. The different columns correspond to various approximations used in the calculation for the self-energy of the nucleons. The column labeled “HF” stands for the Hartree-Fock approximation, while in the second column also the second-order diagram of Fig.1a has been taken into account (“bare 2p1h”). All pp ladder diagrams (like e.g. the diagram of Fig.1c) are included in the Brueckner-HF (“BHF”) approximation. If all residual interactions between 2p1h configurations (also Fig.1d) are taken into account, one obtains column 4 (“2p1h”). The results of the complete calculation, also including the 2h1p configurations with residual interactions are given in column 5 (“total”). The single-particle energies are given in MeV.

|               | HF     | bare 2p1h | BHF    | 2p1h   | total   |
|---------------|--------|-----------|--------|--------|---------|
| $0s_{1/2}$, $\epsilon_<$ | -58.211 | -61.098   | -61.189 | -61.126 | -61.368 |
| $N_\alpha$    | 1.0000 | 0.9722    | 0.9704  | 0.9714  | 0.9689  |
| $0p_{3/2}$, $\epsilon_<$ | -38.796 | -41.552   | -41.695 | -41.649 | -41.876 |
| $N_\alpha$    | 1.0000 | 0.9672    | 0.9637  | 0.9642  | 0.9610  |
| $0p_{1/2}$, $\epsilon_<$ | -35.901 | -39.074   | -39.236 | -39.165 | -39.432 |
| $N_\alpha$    | 1.0000 | 0.9618    | 0.9577  | 0.9589  | 0.9553  |
| $0d_{5/2}$, $\epsilon_<$ | -20.747 | -23.597   | -23.905 | -23.941 | -24.195 |
| $N_\alpha$    | 1.0000 | 0.9511    | 0.9383  | 0.9335  | 0.9267  |
| $0d_{3/2}$, $\epsilon_<$ | -19.446 | -19.861   | -20.231 | -20.170 | -20.509 |
| $N_\alpha$    | 1.0000 | 0.9363    | 0.9206  | 0.9199  | 0.9112  |
| $1s_{1/2}$, $\epsilon_<$ | -16.350 | -22.661   | -23.003 | -23.009 | -23.281 |
| $N_\alpha$    | 1.0000 | 0.9439    | 0.9298  | 0.9259  | 0.9187  |
| $0f_{7/2}$, $\epsilon_<$ | \(0.0\)  | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0459\) |
| $N_\alpha$    | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0444\) |
| $0f_{5/2}$, $\epsilon_<$ | \(0.0\)  | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0381\) |
| $N_\alpha$    | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0391\) |
| $1p_{3/2}$, $\epsilon_<$ | \(0.0\)  | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0391\) |
| $N_\alpha$    | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0\) | \(0.0459\) |
Table 2: Results for binding energy per nucleon (E/A), radius of the nucleon distribution and particle number A obtained in various approximations (see caption of table 1). The effects of the Coulomb energy and the charge radius of the proton have been removed from the experimental data to allow a direct comparison to the calculated values. Results are given for $^{16}O$ and $^{40}Ca$. For the calculation of $^{16}O$ particle states up to and including the sdg shell are taken into account, while in the case of $^{40}Ca$ all states up to the hfp shell are considered. All energies are listed in MeV and the radii in fm.

|        | $^{16}O$ |        | $^{40}Ca$ |
|--------|----------|--------|----------|
|        | E/A      | Radius | A        | E/A      | Radius | A        |
| HF     | -4.202   | 2.633  | 16.000   | -6.491   | 3.455  | 40.000   |
| bare 2p1h | -6.099   | 2.626  | 15.159   | -8.104   | 3.451  | 38.152   |
| BHF    | -6.373   | 2.625  | 14.894   | -8.275   | 3.449  | 37.765   |
| 2p1h   | -6.370   | 2.625  | 14.868   | -8.278   | 3.448  | 37.700   |
| total  | -6.747   | 2.720  | 16.262   | -8.587   | 3.533  | 40.379   |
| Exp    | -9.118   | 2.580  | 16.000   | -10.597  | 3.410  | 40.000   |
Table 3: Results for the energy per nucleon in the HF approximation, in the “total” approximation and the gain in energy $\Delta E$ relative to HF, obtained in various model spaces for the nucleus $^{16}$O. Furthermore the radius of the nucleon distribution and the particle number $A$ are listed calculated in the “total” approximation (see tables 1 and 2). The various model spaces are identified in terms of $N_{\text{Cut}}$ the number of active single-particle orbits.

| $N_{\text{Cut}}$ | $E_{HF}/A$ | $E_{\text{tot}}/A$ | $\Delta E/A$ | Radius | $A$ |
|-----------------|------------|-------------------|-------------|--------|-----|
| $\omega=30$ MeV |            |                   |             |        |     |
| 10              | -4.791     | -6.388            | -1.597      | 2.683  | 16.155 |
| 15              | -4.202     | -6.747            | -2.545      | 2.720  | 16.262 |
| 21              | -3.628     | -7.052            | -3.424      | 2.754  | 16.379 |
| $\omega=-5$ MeV |            |                   |             |        |     |
| 10              | -6.455     | -8.171            | -1.716      | 2.686  | 16.162 |
| 15              | -5.556     | -8.214            | -2.658      | 2.723  | 16.279 |
| 21              | -4.863     | -8.398            | -3.535      | 2.756  | 16.384 |
5 Figure Captions

Figure 1: Diagrams representing various contributions to the self-energy of second and higher order in the residual interaction $G$.

Figure 2: The spectral function defined in eq. (29) obtained for the $p_{3/2}$ states in $^{40}Ca$, employing two different BAGEL approximations. The curves in the upper part are obtained assuming a width of $\Gamma=5$ MeV, whereas in the presentation displayed in the lower part $\Gamma=2$ MeV has been used.

Figure 3: Spectral functions for the nucleus $^{16}O$ in a region of energies between $\omega=-50$ MeV to $\omega=30$ MeV. The spectral functions for the various orbital angular momenta ($l=0,1,2,3$) have been obtained from eq.(29) (width $\Gamma=2$ MeV) by multiplying the contributions from the different total angular momenta $j_\alpha = l_\alpha \pm 1/2$ by the degeneracy factors $2j_\alpha + 1$.

Figure 4: Spectral functions obtained for the nucleus $^{40}Ca$ for the various orbital angular momenta in an energy interval $[-50\text{ MeV}, -10\text{ MeV}]$. The experimental data are from ref. [17]. Note the different scales for the spectral function in the various parts of the figure.
This figure "fig1-1.png" is available in "png" format from:

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