Misconception of triangle concept through epistemological mathematics belief

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Abstract. Triangle topics have been studied since preschool. However, there are still pre-service teachers who do not understand the triangle concept. This descriptive qualitative study aims to trace the understanding of the base and the altitude of the triangle concept. A total of 30 pre-service teachers of elementary school were assigned to solve the problem on the triangle topic. Furthermore, an interview was conducted to explore the subject's understanding of the triangle concept. The results revealed that the base of the triangle was understood as the horizontal side of the triangle, located at the bottom or one of the longest sides of the triangle. The altitude of the triangle was an vertical side of the right triangle or a distance between the base of the triangle and the vertex before it. Misconception of triangle concept was due to the given examples of triangles limited to the right triangle, the equilateral triangle, and the isoceles of triangle. Varying samples which represent all types of triangles and varied problem will build the understanding concept of the base and altitude of the triangle correctly.

1. Introduction

Triangle is one of the most essential topics that students learn in school. The topic of triangle is considered essential because it is utilized as a basis for studying several other mathematical topics [1]. Therefore, triangle topics are studied by students from preschool [2] up to high school even to college. The preschoolers, students are expected to recognize the two dimensional (including triangle) construct in various orientations, shapes, and sizes [3,4]. At the elementary school level, the triangle concept learning includes the properties and the classification two-dimensional shape, departing from parallel or perpendicular lines and angle size [5]. Pre-service teachers of elementary school also study triangle topic to improve sharpen their understanding of previous acquired concepts. Even pre-service teachers of elementary school learn to solve problems and how to teach the triangle concept as a provision to teach to their students. Therefore, understanding the concept of triangle is of urgency for them.

The importance of triangle topic encourages some experts to conduct research on the triangle topics. The studies of children aged 5-6 years show that it is not the similar example to the concept of a triangle that causes a child to misidentify a triangle example [6]. Children more easily understand the concept if they are able to identify prototype of triangle, but they have difficulties in identifying the types of triangle, especially if the triangle are reversed or rotated [7]. Teachers on teacher development programs for early childhood are able to identify examples and non-examples of the triangle and are able to define triangle [2]. However, there are still many teachers who have difficulty in choosing a learning model due to a lack of understanding of mathematical concepts, including the concepts of...
triangle [1]. Therefore, some studies developed instructional designs that can help students to understand the concepts of triangle and enable students to reactivate from the objectivity of right triangle [1,8].

Several studies have examined the understanding of the triangle concepts. The results of the investigation for problem-solving performance by pre-service teachers of elementary school indicate that they have an misconception of the triangle concepts. For instance, pre-service teachers of elementary school understand the concept of the altitude of the triangle on certain types of triangle, but not on all types of triangles. According to [9], the current understanding of mathematical concepts is influenced by previous learning experiences, since they are familiar with the concept of triangle in preschool through college. Previous learning experiences are able to form the beliefs of mathematical epistemology. Some studies show that the epistemological beliefs, caused by the learning experiences, have a solid impact towards the students’ mathematical behavior and performance [10]. Therefore, the misconception of triangle concepts can be traced from previous learning experiences.

This study aimed to obtain a description, regarding the form of understanding of pre-service teachers of elementary school towards the concepts of triangle, especially on the concepts of the base and the altitude of triangle. Furthermore, their experiences of pre-service teachers of elementary school in learning the topic on triangle were traced. Thus, the result of this study was expected to be used as a consideration for making learning plans or the development of teaching materials, especially on triangle topic.

2. Method

This research was a descriptive qualitative research. The subjects of this study were pre-service teachers of elementary school who were joining the elementary school teacher education program in private universities in Malang. The subjects had studied triangle topics on the elements of triangles, kinds of triangles, area and circumferences of triangle. The subjects had learned to solve problems related to triangles and had taught triangle topic in elementary school. The utilized instruments were problem-solving tasks (PST) and interview guidelines. The problem was a non-routine problem-solving task, drawing three different triangles that had the same area as the ABC triangle. The research instruments are presented in Figure 1.

![Figure 1. Problem Solving Task](image_url)

The triangle base is the same as 1/3 rectangle length. Make other three triangles that have the same area as triangle ABC!

The research instrument was given to 30 pre-service teachers of elementary school. The results of problem solving were grouped into two categories, namely P1 and P2 categories. P1 is a subject that provides problem solving even though it is not entirely true, while P2 is a subject that does not provide problem solving at all. The subjects, categorized as P1, were counted 24 people. Meanwhile, there were as many as 6 people who were categorized as P2. Furthermore, an in-depth interview was conducted on the subjects of P1 and P2 category. Interviews were subjected to six P1 subjects and three P2 subjects. Interview was intended to explore the concept of the base and altitude of the triangle.
departing from the result of PST completion and triangle material learning experiences. The excavated learning experience focused more on the learning process on the triangle material that the subjects had followed and the cause of the misconception of the concept of the base and altitude of the triangle.

3. Results and Discussion

3.1. P1 Data Category
P1 solved the problem on the PST by calculating the area of the ABC triangle, then drew three other triangles that have the same base size and altitude as the ABC triangle. The results of problem resolution P1 are presented in Figure 2.

![Figure 2. Problem Solving of P1](image)

P1 initially determined the base and altitude of the triangle ABC. The triangle ABC base was \( \frac{1}{3} \) length of rectangle = \( \frac{1}{3} \times 11 = 3.66 \) (rounded into 3.7 cm). The altitude of the triangle was the same as the width of the rectangle, which was 6 cm. After that, P1 drew two right triangles (Figure 2a and 2b) that had the same base and altitude as the ABC triangle. The triangle in Figure 2a had a base of 3.7 cm and a altitude of 6 cm. The triangle in Figure 2b had a base of 6 cm and a altitude of 3.7 cm. Furthermore, the area of the triangles Figure 2a and 2b was obtained by the formula:

\[
\text{The area of the triangle} = \frac{1}{2} \times \text{(base)} \times \text{(altitude)} \quad (1)
\]

so the area of triangles in Figures 2a and 2b was:

\[
\text{The area of the triangle in figure } 2a = \frac{1}{2} \times 3.7 \times 6 = 11 \quad (2)
\]

\[
\text{The area of the triangle in figure } 2b = \frac{1}{2} \times 6 \times 3.7 = 11 \quad (3)
\]

The triangles in Figures 2a and 2b were assumed as two distinct triangles, that have the same area. The two triangles have different the base and the altitude size. P1 saw the base of the triangle as one of the horizontal sides of the triangle that was located at the bottom, while the altitude of the triangle was one of vertical sides on the right triangle. Since they had an unequal base and the altitude, the two triangles (Figure 2a and 2b) were considered two distinct triangles.

The triangle in Figure 2c had a base of 6 cm and a altitude of 3.7 cm. It was discovered from the way P1 determined the area of the triangle in Figure 2c.

\[
\text{The area of the triangle in figure } 2c = \frac{1}{2} \times 6 \times 3.7 = 11 \quad (4)
\]

P1 considered the triangle in Figure 2c to be a right triangle. However, P1 could not show where the right angle was in the triangle in Figure 2c and began to hesitate with the result of the problem solving. This was shown in the following dialog.

R : “If the area of triangle in figure 2c = \( \frac{1}{2} \times 6 \times 3.7 = 11 \), so what kind of triangle is it?”

P1 : (silent)

R : “The right triangle or not?”

P1 : (silent any longer, then murmuring) “The right angled triangle.”

R : “Where is the position of the right angle?”
P1: (silent)

The investigation on the learning experiences that triggered P1 in completing the PST comprised three. First, the triangle examples that were found during the learning process or in the textbook were the right triangles, the equilateral triangles, and the isosceles triangles as shown in Figure 3.

![Figure 3. Triangle Image in Textbook](image)

Second, the problem of the area of the triangle was presented directly, i.e., providing by the needed information, such as the size of the base, the altitude, or the area of the triangle. Some examples of triangle area problems are presented in Figures 4. Solving the problem in Figure 4 could be done by substituting a known size into the formula (1).

![Figure 4. The Triangle Area Problems in Textbooks](image)
Third, the problem of triangle area that was oftenly come across by P1 was the area of the right triangle. Therefore, when they got the task to determine the area of a triangle, then the thoughts of P1 were the area of the right triangle.

3.2. P2 Data Category

P2 could not solve the problem on PST. This was due to P2’s lack of understanding towards the concept of the base and the altitude of the triangle as it is presented in Figure 5.

![Figure 5. Troubleshooting P2](image)

The triangle in Figure 5 had an AB base. Although there was a hint that the base of the triangle was \( \frac{1}{3} \) of the rectangle length, P2 was still considered the base of the triangle to be the longest side of the triangle, which was AB. The altitude of the triangle was the distance between the base of the triangle with the vertex before it, which was “t” (distance between point C and AB). Since it could not determine the base and the altitude size, P2 could not determine the area of ABC triangle. This caused P2 could not draw another triangle that had the same area as the ABC triangle.

Problem solving that was conducted by P2 was based on the learning experiences. First, the understanding of the base and the altitude concept was derived from the triangle examples which were given during elementary school and college learning as it was shown in Figure 3. An example of an scalene triangle was presented when P2 was describing the various triangles based on the length of the sides or searching for triangle circumferences. Second, the problem of triangle area was presented by giving the base and the altitude size of the triangle directly as in Figure 4. The problem could be solved by utilizing formula (1). Third, P2 never encountered context-dependent problem. The problem in PST required P2 to suss out the correlation among concepts, existing in rectangle and triangle. The base and altitude of the triangle corresponded to the length and width of the rectangle. This caused P2 to hardly identify the base and altitude of triangles in Figure 1. Fourth, P2 had never been dealing with an open-ended problem, so P2 believed that the mathematical problem had only one correct answer. Thus, P2 did not think that they were actually able to draw three other triangles.

The results of problem solving and tracing of P1 and P2 learning experiences revealed that pre-service teachers of elementary school understood the concept of the base and altitude of the triangle through the examples, presented during the learning process or textbook. Examples of the provided triangles were the right triangle, the isosceles triangle, and the equilateral triangle. It formed a limited concept or mental structure-the concept of the base and the altitude of the triangle, understood by the pre-service teachers of elementary school - only be applied to right triangle, isosceles triangle, and equilateral triangle. This is called "met before", means the mental structure that is owned now as a result of experience previously encountered [12]. Met before that was owned by the pre-service teachers of elementary school above is included in the "problematic met before", means the learning experiences that do not support or raise issues in the current learning. Therefore, the learning experiences that were provided to pre-service teachers of elementary school should be comprehensive or accommodate the studied concepts. Examples of various base and altitude of triangle should be
given so that the pre-service teachers of elementary school would understand that the triangle base was one side of the triangle, whereas the altitude of the triangle was the perpendicular distance from the base to the opposite vertex [13].

The triangle problem that generally elementary school teachers solved were too simple, did not require analysis, and were provided with the same pattern on an ongoing basis (routine problems). Routine problems could be solved by well-known methods [14]. By remembering and utilizing the formula, the problem could be solved. It was less supportive towards the attachment and development of the concept that had been studied since routine problems forced pre-service teacher of elementary school to solve problems mechanically or procedurally. In contrast, non-routine problem solving demanded the application of high-level inference skills and provided intensive learning support [15] as well as helpful in understanding the acquired concept and their correlation [16]. Therefore, the presentation of non-routine issues should be presented in lessons and in textbooks.

Pre-service teachers of elementary school also had not ever experienced solving open-ended problems. This emerged the epistemological mathematics belief that there was only one way of settlement or there was only one correct answer. This belief restricts activity and creativity in solving problems. The presentation of an open ended problems corresponded to a triangle material that was usually presented by giving formula without explaining or understanding how to obtain the formula.

4. Conclusion
The findings of this study indicate that pre-service teachers of elementary school have less understanding of the base and the altitude concept. The comprehended concept by the pre-service teachers of elementary school is only applied to the equilateral triangles, as well as the isosceles triangles and the right triangles because the concept is obtained through the acquired examples during the learning process and the textbooks that are utilized as a reference in learning. Less exhaustive and varied examples lead to the formation of imperfect or partial concepts. Therefore, the learning process must be carefully planned so that each component, being studied is exactly complete and meaningful. The results of this study can be employed as a preliminary information to explore the understanding of other triangle concepts, (for instance: area or circumference) and the analysis of misunderstanding of preservice teacher on the triangle concept. In addition, the results of this study can be employed to prepare a learning plan or the source of triangle topics.

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