Contribution Indonesian Composite Index in PT Telekomunikasi Indonesia stock price model using 2-dimensional Geometric Brownian Motion

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Abstract. Theoretically, the movement of the Composite Stock Price Index (CSPI) is in line with the company's stock price movements. Hence, it would be appropriate to measure the CSPI contribution to the company's stock price regarding modeling the company's stock price. 2-dimensional Geometric Brownian Motion is believed to be the most appropriate model in this case. Therefore, this paper aims to project the share price of PT Telekomunikasi Indonesia in 2018 by considering the CSPI movement. The resulted mean absolute percentage error (MAPE) calculations lead to a conclusion that Prediction with 2-dimensional Geometric Brownian Motion (GBM) is more accurate than the individual modeling of stock prices of PT Telekomunikasi Indonesia. PT Telekomunikasi Indonesia Tbk's stock price modeling is more appropriately-used by taking heed on the movement of the Composite Stock Price Index. It is conclusive that the two dimensional geometric Brownian motion model provides an accurate prediction of PT Telekomunikasi Indonesia Tbk shares with MAPE is 1.980296%.

1. Introduction
The capital market is proven to be an essential part of economic growth in Indonesia. It primarily aims to support the sustainability of national development to increase equity, economic growth, and stability for the improvement of people's welfare. Transaction activities in the capital market as reflected in the increase in the stock price index are an indicator to see the growth of the economic activity. Stock is a sign of ownership of a company which is used to calculate the stock price index. Two kinds of indices are used as a parameter of the Jakarta Stock Exchange, namely the Individual Stock Price Index and the Composite Stock Price Index (CSPI). Individual stock price index (company) describes the development of the price of a stock while the CSPI describes the development of the overall price of shares traded on the stock, whether in the form of ordinary shares or preferred shares. The movement of the Composite Stock Price Index (CSPI) will theoretically be influenced by the movement of individual stock prices (companies). It occurs because the company's stock price is part of the calculation of the Composite Stock Price Index (CSPI). In general, CSPI is very much determined by the movement of stocks with a large market capitalization value. The market capitalization value is the value of all shares that are calculated based on the last price. This research aiming to conduct modeling and stock price Prediction is encouraged by the correlation between the two stock values. Generally, there have been many types of research addressing the close relationship between stock prices and the CSPI. The correlation of CSPI with ten leading stocks on the IDX and reveals that the ten stocks were positively correlated with the CSPI [1], [2] states that there is a correlation of about 0.872 between the CSPI and stock returns in the mining sector for the period of
2008-2012 [2], [3] investigated the correlation between stock index returns CSPI and the CSPI with the return of other stock price indexes. On this basis, he concludes that the movement of return stock price indices correlates positively with the JCI return.

However, it is obvious that the previous researches never address the prediction of the share prices based on the assumption that the stock prices incessantly move and affect each other. In this line, [4] analyzed stock prices on PT Ciputra Tbk's share data on a single basis. In addition, [5] and [6] conducted a stock price analysis of PT Aneka Tambang Tbk and PT Astra Agro Lestari Tbk by modeling the stock price on a single basis based on Geometric Brownian Motion with a leap.

On this basis, this research is expected to provide a better analysis based on the previous studies’ conclusion that there is a strong correlation between the company's stock price and the Composite Stock Price Index. Hence, it will be more appropriate to conduct modeling by paying attention to the joint movement between the two stock prices, and that Multidimensional Geometric Brownian Motion is the right method for this analysis.

2. Methodology

2.1. 2-Dimensional Geometric Brownian Motion

The dimensions of Lemma Ito n dimension as the resolution of stochastic of n-dimensional differential equations [7]. This theory is a one-dimensional development of Lemma Ito for the completion of a 1-dimensional Geometric Brownian Motion model. Furthermore [8] created a numerical solution for the multidimensional Geometric Brownian Motion model based on Lemma Ito n dimensions theory.

Moreover [9] applied a multidimensional model of Geometric Brownian Motion for the establishment of the optimal portfolio with the Markowitz method on three stocks that move multidimensionally. This study will lead to the right multidimensional model used in stocks with a strong correlation.

Furthermore, [10] who addressed the Multidimensional Geometric Brownian Motion in the stock portfolio of PT Matahari Department Store Tbk and PT Telekomunikasi Indonesia Tbk, concluded that stock price modeling based on multidimensional models is more accurate than the one-dimensional model.

The close relationship between stock prices and the CSPI has been widely addressed by several previous studies, including the one conducted by [1] examined the correlation between CSPI and ten leading stocks on the IDX and stated that the ten stocks were positively correlated with the CSPI. In addition, [2] stated that there is a correlation of 0.872 between the CSPI to stock returns in the mining sector for the period of 2008-2012. [3] also examined the correlation of stock index returns to the CSPI with the return of other stock price indexes. This study came up with a conclusion that the movement of return stock price indices correlates positively with the CSPI return.

Unfortunately, previous researches never take heed on the prediction of the prices of these shares based on the assumption that the prices of these stocks incessantly move and influence each other. The most closely related research is the one conducted by [4] which analyzed stock prices on PT Ciputra Tbk's share data on a single basis. Another study is by [5] and [8] concerning with a stock price analysis of PT Aneka Tambang Tbk and PT Astra Agro Lestari Tbk by modeling the stock price on a single basis based on Geometric Brownian Motion with a jump.

However, this research is expected to provide a better analysis based on previous studies which have concluded that there is a strong correlation between the company's stock price and the Composite Stock Price Index. Hence, it is projected that there will be more appropriate modeling if attention is paid to the joint movement between the two stock prices. In this case, Multidimensional Geometric Brownian Motion is the right method for this analysis.

Theorem 1. Suppose that \( \{X(t)\} \) is a solution of stochastic differential equations and \( g(t, x) \) is a function that is continuously differentiated in \( t \) and and continuously differentiated together with \( x \). Then, the stochastic process of \( \{g(t, X(t))\} \) is the solution of the following stochastic differential equations
\[ dg(t,X) = \left[ \frac{\partial g(t,X)}{\partial t} + \mu(t,X) \frac{\partial g(t,X)}{\partial x} + \frac{1}{2} \sigma^2(t,X) \frac{\partial^2 g(t,X)}{\partial x^2} \right] dt + \sigma(t,X) \frac{\partial g(t,X)}{\partial x} dW \]

The function of \( S(x) = \log X \) and \( X(0) \) is the initial value of \( X \). The calculation applying Lemma Ito results in

\[ \ln X(t) = \left( \mu - \frac{1}{2} \sigma^2 \right) t + \sigma \left( W(t) - W(0) \right) \]

which thus

\[ X(t) = X(0) \exp \left( \mu - \frac{1}{2} \sigma^2 \right) t + \sigma \left( W(t) - W(0) \right) \]

2.2. 2-Dimensional GBM Theorem

**Theorem 2.** Suppose that \( \{(X_1(t), X_2(t))\} \) is the solution of a stochastic differential equation and \( (t, x_1, x_2) \) is a function that is continuously differentiated in continuous and differentially differentiated against \( x_1 \) and \( x_2 \). Thus, \( g(t, X_1(t), X_2(t)) \) is the solution of the following stochastic differential equations

\[
 dg(t,X_1,X_2) = \frac{\partial g(t,X_1,X_2)}{\partial t} dt + \frac{\partial g(t,X_1,X_2)}{\partial x_1} dX_1 + \frac{\partial g(t,X_1,X_2)}{\partial x_2} dX_2 \\
+ \frac{1}{2} \left[ \left( \sigma_{11}^2 + \sigma_{12}^2 \right) \frac{\partial^2 g(t,X_1,X_2)}{\partial x_1^2} + \left( \sigma_{21}^2 + \sigma_{22}^2 \right) \frac{\partial^2 g(t,X_1,X_2)}{\partial x_2^2} \right] \\
+ 2(\sigma_{11} \sigma_{21} + \sigma_{12} \sigma_{22}) \frac{\partial^2 g(t,X_1,X_2)}{\partial x_1 \partial x_2} \]

Suppose the function of \( S(x) = \log X \) and \( X(0) \) is the starting point of \( X \). The calculation using Two Dimensional Ito’s Lemma result in

\[ X_1(t) = X_1(0) \exp \left( \mu_1 - \frac{1}{2} \sigma_{1j}^2 \right) t + \sum_{j=1}^{\sigma_{1j}} \sigma_{1j} \left( W(t) - W(0) \right) \]

\[ X_2(t) = X_2(0) \exp \left( \mu_2 - \frac{1}{2} \sigma_{2j}^2 \right) t + \sum_{j=1}^{\sigma_{2j}} \sigma_{2j} \left( W(t) - W(0) \right) \]

Mean Absolute Percentage Error (MAPE) to determine the accuracy of the data. The MAPE value is determined by the following equation:

\[ MAPE = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{P_i - A_i}{A_i} \right| \times 100\% \]

\( A_i \) is the actual value of the period to \( t \). \( P_i \) is the value of Prediction at time \( t \), while \( n \) indicates the number of observation data.

| MAPE Value | Prediction Accuracy |
|------------|---------------------|
| < 10%      | Prediction accuracy is very good. |
| 11% - 20% | Prediction accuracy is good. |
| 21% - 50% | Prediction accuracy is pretty standard. |
| >51%       | Prediction accuracy is not accurate |

2.3. Research Stages

This research was conducted in three main stages, namely literary study, model development and computation system creation, and analysis on stock data and Indonesian companies’ Composite Stock Price Index. It was conducted in the following stages described below.
1. Conducting a normality test on the stock data of PT Telekomunikasi Indonesia Tbk and the Composite Stock Price Index
2. Modeling stock price movements with the Geometric Brownian Motion model and two-dimensional Geometric Brownian Motion
3. Calculating price predictions in samples of PT Telekomunikasi Indonesia Tbk and Joint Stock Price Index based on Geometric Brownian Motion models and two-dimensional Geometric Brownian Motion
4. Calculating the MAPE value of the two produced models
5. Calculating the prediction of the price out of the stock sample of PT Telekomunikasi Indonesia Tbk

3. Research Variable
This study relies on secondary data, namely the Composite Stock Price Index and the share price of PT. Telekomunikasi Indonesia Tbk with 120 TLKM codes, from December 12th 2018 to June 1st 2018 as data in samples. Meanwhile, the data taken from June 4th 2018 to June 29th 2018 are used as a sample of out-data. The company was chosen because it was recorded as a company that raised the rate of the CSPI in early 2018. The data were primarily retrieved from the website http://finance.yahoo.com.

4. Result and Discussion
The data pattern is known by creating scatter plots between the two variables, namely the sample of PT Telekomunikasi Tbk shares and a sample of the Composite Stock Price Index. The results are as follows:

![Figure 1. Consecutive plot of PT Telekomunikasi Tbk stock data and sample of Composite Stock Price Index.](image)

The plot results in figure 1 indicate the direction of the data movement pattern between the sample of PT Telekomunikasi Indonesia Tbk and a sample of the Composite Stock Price Index which highlights a linear relationship. On the basis of the result, the two variables are more appropriate for modeling together.

| PT Telekomunikasi Indonesia Tbk | CSPI |
|---------------------------------|------|
| **Statistics**                  | **Statistics** | **Value** | **Value** |
| Average                         | Average     | 3928.92   | 6291.45   |
| Maximum                         | Maximum     | 4440.00   | 6689.29   |
| Minimum                         | Minimum     | 3420.00   | 5733.85   |
| Variance                        | Variance    | 65448.92  | 62343.03  |
| Deviation Standard              | Deviation   | 255.83    | 249.69    |
|                                 | Standard    |           |           |
Based on table 2, it can be seen that the share price data of PT Telekomunikasi Indonesia Tbk is below the Composite Stock Price Index with almost the same average, maximum, and minimum differences. Daily price volatility as indicated by the standard deviation value also indicates that the share price of PT Telekomunikasi Indonesia and the Stock Price Index fluctuate with almost the same value. The assumptions required for the geometric Brownian motion modeling and two-dimensional geometric Brownian motion are that the data of each variable must be normally distributed. The following test results use the Kolmogorov-Smirnov normality test.

Hypothesis:

$H_0$: stock data accords with normal distribution
$H_1$: stock data does not accord with normal distribution

Significance Level: $\alpha=5\%$

Statistical Test:

$$D = \sup|F_0(x) - F(x)|$$

with: $F_0(x)$ = Cumulative probability Normal distribution.

$F(x)$ = Cumulative probability of empirical distribution.

Test criteria:

$H_0$ is rejected if the value of D is greater than the 1- $\alpha$ quantile of the Kolmogorov Smirnov test table, or $p$-value is $<\alpha$.

Decision:

| PT. Telekomunikasi Indonesia Tbk | CSPI |
|---------------------------------|------|
| **Statistics**                  | **Value** | **Statistics** | **Value** |
| KS                              | 0.097125 | KS              | 0.064996 |
| $p$-value                       | 0.2076  | $p$-value       | 0.6912  |
| Decision                        | $H_0$ is accepted | Decision       | $H_0$ is accepted |

Based on the $p$-value $<\alpha$, it is concluded that the data of PT Telekomunikasi Indonesia and the Stock Price Index are normally distributed. The nature of normality is also indicated by the Q-Q plot in figure 2.

The Q-Q plot of Figure 2 reveals a pattern with a linear tendency, which indicates that the distribution of sample data is in accordance with the theoretical distribution that is normal. After the
assumption of normality is met, the researcher conducted the next step which is parameter estimation. Parameters in Geometric Brownian Motion models and two-dimensional geometric Brownian motion are average values (μ), value of variance (σ2), and volatility values (σ). The parameter estimated values are as follows:

| Stock Names                               | \( \hat{\mu} \) | \( \hat{\sigma}^2 \) | \( \hat{\sigma} \) |
|-------------------------------------------|----------------|-------------------|----------------|
| PT Telekomunikasi Indonesia Tbk           | 3928.9170      | 65448.40000       | 255.8288       |
| Composite Stock Price Index               | 6291.4550      | 62343.03000       | 249.6859       |

After the parameter estimation value is revealed, the researcher carried on with the prediction making in the following step. The prediction was done in two ways, based on the geometric Brownian motion model and based on the two-dimensional geometric Brownian motion model. The predicted value of the share price of PT Telekomunikasi Indonesia Tbk based on the geometric Brownian motion model is presented in table 5.

| Period        | Actual stock value | Predicted stock value |
|---------------|--------------------|-----------------------|
| Jun 04, 2018  | 3640               | 3479.71               |
| Jun 05, 2018  | 3830               | 3493.77               |
| Jun 06, 2018  | 3790               | 3575.17               |
| Jun 07, 2018  | 3770               | 3641.81               |
| Jun 08, 2018  | 3610               | 3608.96               |
| Jun 11, 2018  | 3610               | 3536.23               |
| Jun 12, 2018  | 3610               | 3510.80               |
| Jun 13, 2018  | 3610               | 3432.28               |
| Jun 14, 2018  | 3610               | 3384.45               |
| Jun 15, 2018  | 3610               | 3335.24               |
| Jun 18, 2018  | 3610               | 3409.52               |
| Jun 19, 2018  | 3610               | 3409.21               |
| Jun 20, 2018  | 3710               | 3378.71               |
| Jun 21, 2018  | 3610               | 3286.42               |
| Jun 22, 2018  | 3580               | 3326.29               |
| Jun 25, 2018  | 3660               | 3250.55               |
| Jun 26, 2018  | 3690               | 3199.99               |
| Jun 27, 2018  | 3710               | 3233.09               |
| Jun 28, 2018  | 3660               | 3142.93               |
| Jun 29, 2018  | 3750               | 3115.80               |

The predicted stock value of PT Telekomunikasi Indonesia Tbk based on two-dimensional geometric Brownian motion is shown in table 6.

| Period        | Actual stock value | Predicted stock value |
|---------------|--------------------|-----------------------|
| Jun 04, 2018  | 3640               | 3479.71               |
| Jun 05, 2018  | 3830               | 3493.77               |
| Jun 06, 2018  | 3790               | 3575.17               |
| Jun 07, 2018  | 3770               | 3641.81               |
| Jun 08, 2018  | 3610               | 3608.96               |
| Jun 11, 2018  | 3610               | 3536.23               |
| Jun 12, 2018  | 3610               | 3510.80               |
| Jun 13, 2018  | 3610               | 3432.28               |
| Jun 14, 2018  | 3610               | 3384.45               |
| Jun 15, 2018  | 3610               | 3335.24               |
| Jun 18, 2018  | 3610               | 3409.52               |
| Jun 19, 2018  | 3610               | 3409.21               |
| Jun 20, 2018  | 3710               | 3378.71               |
| Jun 21, 2018  | 3610               | 3286.42               |
| Jun 22, 2018  | 3580               | 3326.29               |
| Jun 25, 2018  | 3660               | 3250.55               |
| Jun 26, 2018  | 3690               | 3199.99               |
| Jun 27, 2018  | 3710               | 3233.09               |
| Jun 28, 2018  | 3660               | 3142.93               |
| Jun 29, 2018  | 3750               | 3115.80               |

Table 6. PT Telekomunikasi Indonesia Tbk Share Price Prediction Based on Two Dimensional Geometric Brownian Motion Model
The evaluation of stock market predicted was done by using MAPE values. The MAPE values of the two models are illustrated in Table 7.

| Period    | Actual stock value | Predicted stock value |
|-----------|--------------------|-----------------------|
| Jun 04, 2018 | 3640              | 3521.20               |
| Jun 05, 2018 | 3830              | 3626.40               |
| Jun 06, 2018 | 3790              | 3590.45               |
| Jun 07, 2018 | 3770              | 3594.06               |
| Jun 08, 2018 | 3610              | 3605.84               |
| Jun 11, 2018 | 3610              | 3633.89               |
| Jun 12, 2018 | 3610              | 3565.27               |
| Jun 13, 2018 | 3610              | 3514.84               |
| Jun 14, 2018 | 3610              | 3541.12               |
| Jun 15, 2018 | 3610              | 3547.39               |
| Jun 18, 2018 | 3610              | 3556.43               |
| Jun 19, 2018 | 3610              | 3609.08               |
| Jun 20, 2018 | 3710              | 3605.63               |
| Jun 21, 2018 | 3610              | 3580.54               |
| Jun 22, 2018 | 3580              | 3560.76               |
| Jun 25, 2018 | 3660              | 3726.15               |
| Jun 26, 2018 | 3690              | 3700.78               |
| Jun 27, 2018 | 3710              | 3735.71               |
| Jun 28, 2018 | 3660              | 3681.95               |
| Jun 29, 2018 | 3750              | 3610.07               |

Based on the MAPE value in Table 7, it is revealed that both models have very good accuracy because the MAPE value is <10%. Two-dimensional Geometric Brownian Motion model has a smaller MAPE value than the MAPE Geometric Brownian Motion model. This result concludes that the two-dimensional geometric Brownian motion model is better than the geometric Brownian motion model. The two-dimensional geometric Brownian motion models pay attention to other variables, namely the CSPI value in predicting stock prices. This is in line with the theory asserting that stock price movements are influenced by price movements of the Composite Stock Price Index. This is in contrast to geometric Brownian motion models in stock price modeling that take no heed on other variables.

A comparison chart of actual stock prices with predicted shares can be seen in figure 3 and 4.
Comparison Chart Between Actual and Prediction Stock Prices of PT Telekomunikasi Indonesia Tbk with Geometric Brownian Motion

\[ \text{red = Prediction, black = Actual} \]

Price (Rupiah)

0 2000 5000

5 10 15 20

Figure 3. Plot comparison of PT Telekomunikasi Indonesia Tbk's actual and predicted share prices with geometric Brownian motion

Comparison Chart Between Actual and Prediction Stock Prices of PT Telekomunikasi Indonesia Tbk with Multidimensional Geometric Brownian Motion

\[ \text{green = Prediction, black = Actual} \]

Price (Rupiah)

0 2000 5000

5 10 15 20

Figure 4. Plot comparison of PT Telekomunikasi Indonesia Tbk's actual and predicted share prices with multidimensional geometric Brownian motion

Based on the two graphs, it is prominent that the stock prediction price chart on the two-dimensional geometric Brownian motion models accords with the actual stock price chart pattern. The trend of the green line which has better accordance with the black lines than that of the black lines and the red lines indicate that the prediction with the two-dimensional geometric Brownian motion method is more accurate than the geometric Brownian motion method.

5. Conclusion
To sum up, it is more appropriate to have the stock modeling of PT Telekomunikasi Indonesia Tbk by paying attention to the movement of the Composite Stock Price Index. The two-dimensional geometric Brownian motion model provides accurate prediction of PT Telekomunikasi Indonesia Tbk shares with MAPE = 1.980296%. This shows that the economic theory analysis which states that the Stock Price Index influences the stock price movements of public companies is empirically proven with the data presented in this study.

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