Self-consistent quasi-linear modelling of Lower Hybrid Current Drive in ITER and DEMO

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Abstract. First pass absorption of the Lower Hybrid waves in thermonuclear devices like ITER and DEMO is modelled by coupling the ray tracing equations for the wave phase and amplitude with the quasi-linear evolution of the electron distribution function. A system of coupled ordinary differential equations for each Fourier component of the spectrum radiated by the LH antenna is derived and solved when considering both 1D/2D Fokker-Planck model for the electron distribution function. This allows to reconstruct and to evolve the quasi-linear diffusion coefficient consistently with the wave propagation, to calculate the power deposition profile and the amount of current driven by the wave. As usually assumed, the Lower Hybrid Current Drive is not effective in a plasma of a tokamak fusion reactor like ITER or DEMO, because the high electron temperature would enhance the wave absorption and then limit the RF power deposition to the very periphery of the plasma column (near the separatrix). In this work by extensively using this self-consistent modelling for the propagation and absorption of the LH wave, a parametric study on the wave spectrum (and consequently on the antenna design) as spectrum width, peak value, secondary lobes etc. has been performed very accurately. Such a careful investigation aims at controlling the power deposition layer possibly in the external half radius of the plasma, thus providing a valuable aid to the solution of how to control the plasma current profile in a toroidal magnetic configuration. This analysis is useful not only for exploring the possibility of profile control of a pulsed operation reactor, but also in order to reconsider the feasibility of steady state regime.

1. Introduction
The reactor–oriented research, now in progress, is based on Deuterium-Tritium plasmas in a Tokamak configuration, and is aimed at demonstrating the scientific, technological and economic feasibility of fusion power. Although a reactor should in principle operate in steady-state, a more conservative pulsed option is now receiving major consideration, because tools for active and fine control of the radial current density profile are unavailable. This control of the current density, necessary for preventing the $q$-profile to evolve towards conditions deleterious for stability [1], is mandatory for steady state operation, as well as for pulsed operations that require profile control by non-inductive current to be added to the self-generated bootstrap current. The lower hybrid current drive (LHCD) [2,3] is attractive for a reactor, owing to the non-inductive current produced with high efficiency by microwave power at a few GHz coupled to plasma by waveguide antennas. However, two problems have persisted for decades which prevented the LHCD tool to be considered of practical use for a reactor: i) the difficult extrapolation of the LHCD effect at reactor graded high densities, due to
parasitic effect of plasma edge [4] and, ii) the strong wave damping which would make the LH power deposition in hot plasmas too peripheral [5]. The first problem (i) was solved by a method assessed on FTU [4], and based on previous theoretical predictions that high plasma edge temperature in reducing parasitic effects as Parametric Instabilities (PI) at the edge, should allow the penetration of the LH power to the plasma core [6,7]. This favourable condition should be naturally met in reactor plasma, as shown later on. Concerning the second problem (ii), the strong dependence of the LH wave damping on the electron temperature is considered problematic for DEMO reactor, as the power should be deposited not deeper than the pedestal radial layer, where $T_e \approx 8$ keV [5]. In this paper, we propose a self-consistent modelling based on the simultaneous solution of the ray tracing equation, which involve all the components of the power spectrum, and coupled with the evolution in the velocity space of the electron distribution function (Quasi-Linear Fokker-Planck Equation). The ray tracing system that includes the evolution along the trajectory of the power associated to the single ray at the antenna, enables the reconstruction of the quasi-linear diffusion coefficient and consequently the determination of the electron distribution function due to LH. This method is particularly suitable for the first pass absorption where apart from a very short transient time $\tau_{ql}$, where the incoming wave is linearly absorbed, at the steady state the quasi-linear effects are prevailing and determine the establishment of an equilibrium distribution function (tail) which is responsible of the subsequent damping of the wave. In reactor plasma, full LH power deposition takes place in less than half radial pass and application of this self-consistent method appears well justified. This feature of the modelling clarifies better the crucial role of power spectrum (the nominal one delivered by the antenna) and that of the broadening (possibly produced by PI at the plasma edge) in determining the radial LH deposition profile [6,7,8,9]. Moreover, by controlling the shape of the spectrum, its width, the $n||$ peak and the electric field intensity (surface power density at the antenna), it is possible to control the deposition layer and to recover the analytical formula obtained by F Santini in Ref. [10]. This approach has been indeed routinely used in the “LHstar” code, which after calculating the effects of spectral broadening produced at the edge by PI, integrate the ray tracing equations simultaneously with the Quasi-Linear Fokker-Planck evaluation of the electron distribution function in 2D velocity space. This code was successfully employed earlier for producing LH deposition profiles in agreement with available data of important experiments of JET [6], and for assessing on FTU the method for producing LHCD effects at reactor relevant high plasma density [4]. In this work in order to elucidate the calculation mechanism we will use the quasi-analytical solution for the 1D QL Fokker-Planck equation, by considering the quasi-linear diffusion coefficient which stems out from ray-tracing and power damping equations and then we will give an example by calculating the LHCD for DEMO reactor plasma parameter as resulting from Ref. [11]. In this context it should be also useful to give some evaluation of LH damping on the non-negligible fraction of $\alpha$-particles present in DEMO, which results of about <5% of the total absorbed power.

2. Pertinent equations and modelling

The electron distribution function satisfies the quasi-linear Fokker-Planck equation and in 1D velocity space we can write

$$\frac{\partial \hat{f}_e(\tau,u)}{\partial \tau} = \frac{\partial}{\partial u} \left( D(u) \frac{\partial \hat{f}_e(\tau,u)}{\partial u} \right) + \alpha \frac{\partial}{\partial u} \left( \frac{1}{u^3} \frac{\partial \hat{f}_e(\tau,u)}{\partial u} + \frac{\hat{f}_e(\tau,u)}{u^3} \right)$$

(1)

where $\hat{f}_e(u)$, is the normalized to $n_0 u_{the}^3$ electron distribution function, $\tau = v_{ee} t$, is the normalized time (to the electron-electron collision frequency $v_{ee} = 2.91 \times 10^{-6} \ln \Lambda n_0 T_e^{-3/2}$), $u = \frac{v_i}{u_{the}}$ is the
normalized parallel velocity (to the thermal velocity), \( \alpha = \left( \frac{2 + Z_i}{2} \right) \), \( u_{the} = 4.2 \times 10^7 \sqrt{T_e} \). \( Z_i \) is the ion charge and

\[
D(u,x) = \frac{D_{gl}(u,x)}{u_{the}^2 v_e} = \left( \frac{1}{u_{the}^2 v_e} \right) \left( \frac{e^2}{m_e^2} \right) \omega \left| E_\| (x,k) \right|^2 \exp \left( - \frac{\omega}{\gamma_{||} u_{the}} \right) \]

is the normalized quasi-linear diffusion coefficient. Note that the dependence of the quasi-linear diffusion coefficient from the space coordinate is only a local dependence where the effects of the 2D wave propagation are accounted. An estimate of the quasi-linear diffusion time in which the fast electron tail will be restored in the layer where the LH is absorbed can be analytically evaluated. The following expression is obtained by considering the Fokker-Planck equation Eq. (1)

\[
\tau_{ql} = \tau_{e,\omega} \frac{\Delta u}{D(u)}
\]

where \( \Delta u \) is the resonance width of the coupled LH spectrum in the velocity space normalized to the electron thermal speed, \( u_1 \) is the lower bound of the spectrum, i.e. \( u_1 \approx 3.5 u_{the} \) to satisfy the electron Landau absorption criterion, \( n_0 \sqrt{T_{keV}} \approx 5 - 7 \) [12]. \( D \) is the normalized quasi-linear diffusion coefficient Eq. (2) that is correlated with the absorbed power. In a typical DEMO LH scenario, we have that the absorbed lower hybrid power is 80MW for the case under consideration; thus, \( D \) is about \( 10^3 \) and the launched \( n\_peak \approx 1.8 \). Consequently, by inserting these values in Eq. (3) the quasi-linear time is \( \tau_{ql} \approx 2.8 ms \). This time is very short with respect to the RF pulse life-time, this means that a steady state in the quasi-linear LH absorption is suddenly formed just after the switch on of the RF generator. At the steady state the solution of Eq. (1) is

\[
\hat{f}_e(u) = \frac{1}{(2\pi)^{3/2}} \exp \left\{ -\int \frac{\alpha u}{(\alpha + u^3 D(u))} du \right\}
\]

where \( C \) is a constant which can be determined by the integral of the distribution function over the velocity space. The equation for the spatial damping rate can be written as [13]

\[
\frac{dP(x)}{dx} = -2 \Gamma_{ql}^{\omega} \left( \omega, k_\perp, k_\parallel, x, P(x) \right) P(x)
\]

where

\[
\Gamma_{ql}^{\omega} = \sqrt{\frac{\omega}{2 \left( k_\perp^2 + k_\parallel^2 \right) n_{the}^2 \omega} \left( \frac{\omega}{\alpha}\right) D(\omega/k_{\parallel n_{the}}) \exp \left\{ -\int \frac{\alpha u}{(\alpha + u^3 D(u))} du \right\} \}
\]

which in the case of Maxwellian plasma reduces to the well-known expression
\[ \Gamma_{\text{Lin}} = \sqrt{\frac{\pi}{2}} \frac{\omega_{\text{pe}}^2}{(k_\perp^2 + k_\parallel^2)u_\text{th}^2} \frac{\omega}{k_{\text{th}}} \exp \left\{ -\frac{1}{2} \left( \frac{\omega}{k_{\text{th}}} \right)^2 \right\} \] (7)

By using the power instead of the electric field we can write the quasi-linear diffusion coefficient in Eq. (6) as

\[ D(x,u) = \frac{4\pi^2 e^2}{\omega c m_e u_\text{th}^2(x)} \frac{1}{V_{\text{eff}}(x)} \left( \frac{\omega}{ck_\perp} \right) P(x,u) \Sigma_{\text{antenna}} \] (8)

Upon using Eq. (8) in (5), and giving the power spectrum at the edge

\[ P(x=1,k_n) = P_0 \exp \left( \frac{k_n - k_{\text{in}}}{\Delta k_n} \right) \], Eq. (5) transforms in a non-linear system of coupled differential equations, after changing the integral in a summation over all the “N” components in which we have decomposed the wave spectrum. Defining the integrand in Eq. (6) as

\[ G \left( P(x,u),u_\omega, x \right) = \frac{\alpha u}{\left( \alpha + u^3 D_{\text{coeff}}(x,u) P(x,u) \right)} \] (9)

we have

\[ \int G(P(u,x),u,x) du = \sum_{n=1,N} G(P_n(u_\omega,x),u_\omega,x) \] (10)

and

\[ \frac{dP_n(x)}{dx} = -2F(\omega, k_\perp, k_{\text{in}}, x) \exp \left\{ -\sum_{n=1,N} G \left( \frac{\omega}{k_{\text{in}}u_{\text{th}(x)}}, P_n(k_{\text{in}}(x)) \right) \right\} P_n(x) \] (11)

where

\[ F(\omega, k_\perp, k_{\text{in}}, x) = \frac{1}{2} \sqrt{\frac{\pi}{2}} \frac{\omega_{\text{pe}}^2}{k_{\text{in}}u_{\text{th}}^2} \left( \frac{\omega}{k_{\text{in}}u_{\text{th}}} \right) \left( 1 - \frac{\omega_{\text{pe}}^2}{\omega^2 - \Omega_n^2} - \frac{\omega_{\text{in}}^2}{\omega^2 - \Omega_n^2} \right) \left( \frac{\alpha}{\omega/(k_{\text{in}}u_{\text{th}})} \right)^3 \] (12)

Eqs. (11) is the relevant equation system to be solved in order to have the quasi-linear self-consistent power deposition profile, and current drive evaluation. We recall that the 2D effects in the wave propagation (mainly affecting the evolution of the parallel wavenumber, as well as the shape of the antenna image inside the plasma) are kept into account by analytical integration of the propagation as given in Ref. [14]. For LH application to tokamak devices of thermonuclear interest like ITER and DEMO where the LH is characterized by first pass absorption, this method is particularly effective because the wave spectrum behaves coherently during the propagation and the rays characterizing the antenna both in the shape and spectrum do not scatter too much. In the multipass regime (not interesting for reactor application) this type of approach does not work correctly at least for more than two passes, and other approach based on statistical ray-tracing must be used [15].
3. Numerical results for DEMO

DEMO can operate in pulsed and Steady State regime both characterized by flat or peaked kinetic profiles. We have applied the above theory to calculate the QL power deposition profile for DEMO in flat and peaked density profiles with the two kinds of electron temperature profiles, as established in Ref. [16]. The LH power deposition profiles has been accounted by studying the effect of the wave spectrum on the deposition layer, in particular the width of the spectrum (considering an analytical Gaussian shape) and the strength of the admissible electric field surface density expressed in kW/cm² (for the LH <10kW/cm²). The results proposed here refers to the 1D QL theory as shown in the previous section, but it has been confirmed also by a more complete (full numerical approach) which uses the ray-tracing equations in a realistic DEMO flux-magnetic configuration and a much more sophisticated 2D full numerical integration for the relativistic Fokker-Planck equation, showing the correctness of our semi-analytical 1D QL self-consistent approach. The plasma data we have used in our simulation for DEMO flat scenario are the following: plasma radius \( a=2.25 \) m, major radius \( R_0=9 \) m, magnetic field on axis \( B_0=6.8 \) T, \( f_{LH}=5 \) GHz, \( n_{e0}=8.8 \times 10^{19} \) m\(^{-3}\), \( n_{eS}=5.9 \times 10^{19} \) m\(^{-3}\) with a very rapid decrease (e-folding 22 cm), we considered also a model of the scrape off plasma (from the separatrix at the wall of about 20 cm) in this layer the density exponentially drops of two order of magnitude. Concerning the electron temperature -which can indeed plays a role in the LH absorption- we have that at the plasma centre reaches the value of 25 keV down to 2.4 keV at the separatrix, being of about 9 keV at the pedestal. From the separatrix to the wall in the scrape-off layer the temperature also drops of a couple of order of magnitudes reaching the value of 25 eV at the antenna mouth.

In Fig. (1), a plot which shows the power deposition profile in W/m³ for two wave spectra (Gaussian shape) characterized by a different width \( \Delta n_i = \frac{C}{\omega} \Delta k_i \), is depicted vs x (the normalized radius).

![Figure 1. Current Driven Profile in A/m² for two wave spectra: (red line) Dn||=0.05, (blue line) Dn||=0.5.](image)

![Figure 2. Power spectra in a.u. for the cases considered in Fig. 1.](image)

The red curve with a sufficiently on-axis deposition is related to the thin spectrum \( \Delta n_i = 0.05 \), with a peak value of \( n||=1.8 \); the blue curve with deposition very peripheral, is related to the same spectrum but much larger \( \Delta n_i = 0.5 \) than the previous. In Fig. 2 the power spectra for both cases are compared.

In Fig. 3 the power damping rate \( \frac{1}{P} \frac{dP}{dx} \) is reported vs x (normalized radius) for both spectra shown in
Fig. 2 (red (thin) and blue (large)), and also when quasi-linear effects are neglected (Maxwellian distribution function (green line)). In the case of linear damping (Maxwellian plasma) it is worth to note that the wave absorption is always peripheral independently of the spectrum shape. This leads to argue that the quasi-linear effects accounted by a formation of the tail in the distribution function as can be seen in Fig. (4) where the Maxwellian $f_e$ is compared to the tailored-LH distribution function, are as important as the spectrum is narrower.

The quasi-linear effect, in fact, by smoothing the distribution function, reduces the damping factor \[ \left( \Gamma - \frac{\partial f_e}{\partial u} \rightarrow 0 \right) \], at least for lower resonant velocities, allowing the wave to penetrate deeper. Using the linear Maxwellian damping the absorption is fulfilled when $T_{keV} = 7.7 \text{keV}$ (linear) to $T_{keV} = 15 \text{keV}$ (quasi-linear) [17], which is coherent with the numerical results which shows that for $n_{peaK} = 1.8$ and narrow spectrum the linear damping is located around $x=0.95$ while the quasi-linear around $x=0.6$. This fact can be quantitatively understood by analytical inspection of the Eq. (6), and (8); looking at Eq. (8), in fact, we can assume that $D(u)$ behaves like

\[ D(u) \approx D_0 \exp \left\{ -\left[ \left( u - u_c \right)/\Delta u \right]^2 \right\} \]: the Gaussian form of the wave spectrum in the velocity space.

When the quasi-linear diffusion coefficient is very strong $D_0 \gg 1$, and the spectrum is very narrow ($\Delta u/u_c \ll 1$), by expanding the exponential function in Taylor series we have that the integral in Eq. (6) reduces to

\[ \int \frac{\alpha u}{\alpha + u^3 D(u)} \, du \bigg|_{\omega = \omega_{kpeK}} \approx \frac{\alpha}{D_0 u^2} \frac{\Delta u}{\omega} \bigg|_{\omega = \omega_{kpeK}} + \ldots \ldots \ldots \]  \hspace{1cm} (13)

and consequently approximating the exponential function in Eq. (6) we have
increase this condition is no more fulfilled and it is worth to note that this analytical calculation at the \( \omega = \omega_0 \) by increasing \( D \) does not differ so much from the effects due to the reduction of \( D \) and increase of \( \Delta k_{||} \). In Fig. (5) we have evaluated and plotted, in equal condition of narrow spectrum \( \Delta = \Delta_{Lin} \), and absorbed power (50MW), the effects due to the reduction of \( D \) and increase of \( \Delta k_{||} \). As is possible to notice on the plot, where the peak of the power density profile \( x_p \) is plotted (bottom axis and red bullets) vs the surface power density (at the antenna in kW/cm²-2), by reducing \( P_d \) at the antenna (or equivalently \( D \)), the peak of the deposition tends to move toward the plasma edge. On the same plot (top axis and blue diamonds) \( x_p \) is plotted vs \( \Delta k_{||} \); by increasing \( \Delta k_{||} \) the peak of the power deposition moves towards the plasma edge. This behaviour is perfectly accounted by Eq. (15), which shows that the damping rate \( \Gamma_{QL}^{es} \) increases by decreasing \( D \) (or equivalently the surface power density at the antenna \( P_d \)), and by increase \( \Delta k_{||} \). It is worth to note that this analytical calculation explains the formulae derived by F Santini in Ref. [10]. In Fig. 5 the red bullets and the blue diamonds are the result of the numerical code while the black shaded line is the interpolation line which in the first case scales like \( \left( x_p - C/P_d^{0.1} \right) \) (where \( C=0.5 \)), while in the second case like a straight line as prescribed by Eq. (15). Finally in Fig. 6 the dynamical evolution of the power spectrum from the antenna to the absorption layer is depicted. It is possible to note, in addition to the wave absorption which reduces the size of the spectrum, also the propagation effects that lead to the increase of the parallel wavenumber and consequently the peak of the spectrum moves from 1.8 at the antenna to almost \( n||=2 \) near the radius of total wave absorption.

3. Conclusions

In conclusion, an analytical inspection of the quasi-linear damping for the LH wave in conjunction with a self-consistent approach in the integration of the ray-trajectories, in particular the power damping equation system, has shown that, by properly considering QL effect of a strong LH power coupled to high temperature plasma, the lower hybrid current drive should be indeed used for active profile control in a reactor like DEMO and/or ITER by overcoming the difficulties connected to a peripheral absorption of the wave. The active control is operated by the antenna design, involving a suitable power spectrum and wavenumber control. The semi-analytical considerations have also been supported by full 2D numerical approach (not reported here but in Ref. [8]). The above results could

\[
\exp \left[ - \int_{u_m}^{\infty} \frac{\alpha u}{(\alpha + u^2 D(u))} \, du \right]_{u_m} = \exp \left[ \alpha D_0 \left( \frac{u_{\text{th}} k_{\text{peak}}}{\omega} \right)^2 \Delta k_{||} + \ldots \right] = 1 + \frac{\alpha}{D_0} \left( \frac{u_{\text{th}} k_{\text{peak}}}{\omega} \right)^2 \Delta k_{||} + \ldots \quad (14)
\]

This means that the quasi-linear damping Eq. (6) at the lowest order has the following form

\[
\Gamma_{QL}^{es} = 2 \sqrt{\pi} \left( \frac{\omega_{pe}^2}{k_{\perp}^2 + k_\parallel^2} \right) v_{th}^2 \left( \frac{\alpha}{\alpha + \left( \omega/(k_{\mu_{\text{the}}}) \right)^2 D \left( \omega/(k_{\mu_{\text{the}}}) \right)} \right) \left[ 1 + \frac{\alpha}{D_0} \left( \frac{u_{\text{th}} k_{\text{peak}}}{\omega} \right)^2 \Delta k_{||} \right] \quad (15)
\]

It is obvious that \( \Gamma_{QL}^{es} \) appears much weaker with respect to the linear damping \( \Gamma_{Lin}^{es} \) (given in Eq. (7)) in which the exponential reduction dominates:

\[
\frac{\Gamma_{QL}^{es}}{\Gamma_{Lin}^{es}} = \left( \frac{\alpha}{\alpha + \left( \omega/(k_{\mu_{\text{the}}}) \right)^2 D \left( \omega/(k_{\mu_{\text{the}}}) \right)} \right) \left[ 1 + \frac{\alpha}{D_0} \left( \frac{u_{\text{th}} k_{\text{peak}}}{\omega} \right)^2 \Delta k_{||} \right] \quad (16)
\]

when \( \Delta k_{||} \) increases this condition is no more fulfilled and \( \Gamma_{QL}^{es} \) does not differ so much from the linear \( \Gamma_{Lin}^{es} \). In Fig. (5) we have evaluated and plotted, in equal condition of narrow spectrum \( \Delta = \Delta_{Lin} = 0.05 \), and absorbed power (50MW), the effects due to the reduction of \( D \) and increase of \( \Delta k_{||} \).
be tested in an experiment using a small $\Delta k_|| (\approx 0.05)$ but relatively large $n_{\parallel0} (\approx 2.8)$ to compensate the lower electron temperatures of the today’s tokamaks, e.g., EAST [18], ranging at around 5 keV in the core. Furthermore, a small value of $\Delta k_||$ should be achieved in reactor devices since the large toroidal dimensions allows to have a grill with a large number of elementary waveguides.

**Figure 5.** Location of the peak of the deposition profile vs the antenna-surface power density (bottom), and the width of power spectrum (top). The dashed lines represent the interpolation of the numerical data (bullets).

**Figure 6.** Power spectrum (a.u.) vs $n_{\parallel}$ in dynamical evolution inside the plasma. The red curve corresponds to $x=1.1$ (antenna location). The absorption in the internal layers is related to the reduction of Gaussian size.

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