Mathematic Modeling of Complex Hydraulic Machinery Systems When Evaluating Reliability Using Graph Theory

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Abstract. The main technological equipment of pipeline transport of hydrocarbons are hydraulic machines. During transportation of oil mainly used of centrifugal pumps, designed to work in the "pumping station-pipeline" system. Composition of a standard pumping station consists of several pumps, complex hydraulic piping. The authors have developed a set of models and algorithms for calculating system reliability of pumps. It is based on the theory of reliability. As an example, considered one of the estimation methods with the application of graph theory.

Introduction
The system of trunk pipeline transport is a complex set of engineering structures with the system of security and environmental protection. The analysis of the experience of exploitation of main facilities of transport showed that most of them happen because of failures and damage to the hardware. Using the developed methods and apparatus of semi-Markov models and the theory of construction of graph States, let us analyze the algorithm for calculating the persistence on a difficult technical systems of pipeline transport of oil.

1. Problem definition

Monitoring the reliability of hydraulic machines is a difficult task. In the practice of their operation in the oil and gas sector, such monitoring is carried out using a complex of diagnostic parameters such as bearing temperature, vibration, pressure, performance. In addition, the monitoring of leaks and the parameters determining the possibility of occurrence of cavitation regime. One such operational parameter is the coefficient of hydraulic reliability.

For the implementation of the proactive functions the evaluation of reliability is reduced to the problem of determining the probabilistic characteristics of the life of the system and determining availability according to the criterion of performance persistence, for example, hydraulic reliability.

Consider a typical scheme of a pumping station, a pump with a design capacity \( Q_1 \) and \( Q_2 \), the power \( N_1 = N_2 \). The reliability function of the pumps has the form \( F_1(t) = c_1e^{-\lambda_1t} \), \( F_2(t) = c_2e^{-\lambda_2t} \), the recovery time of each machine has an exponential distribution \( F_{1_{\text{rep}}}(t) = e^{-\mu_1t} \), \( F_{2_{\text{rep}}}(t) = e^{-\mu_2t} \), with
parameters \( \mu_1 \) and \( \mu_2 \), the required minimum acceptable level of reliability \( J_{ac} \), schedule actual hydraulic reliability \( J_{ac} \) defined in real time. On site there is a tank farm with the volume \( V \), the time for park empty \( t_r \), assessment of the temporary reserve \( D(t) \).

It is required to determine: probabilities of transitions to all possible conditions and their moments, an availability quotient of system in case of the set working schedule; a shelf life of system, the predicted maintenance time by criterion of hydraulic reliability of system[1-3].

2. Methods, results and discussion

Let at the time of the beginning of the analysis the system be in operating state. In the line No. 1 transfer of a product with a productivity is performed \( Q_1 \), on the second – with a productivity \( Q_2 \). Two options of functioning of system are possible. According to the first option in operation there are both pumps. Then let \( t_1 \) — time, during which both machines at the same time are in work according to the plan of transfer for 1 calendar year \( t_y \), i.e. both machines have no structural allowance. Then provided that reliability of one of objects decreases, reliability of system in general decreases, we accept probabilistic parameter \( \alpha \), characterizing probability of decrease in reliability of system is lower than reliability of one, admissible owing to loss, of elements, equal \( \alpha_1 = t_1 / t_y \). By the second option of operation in work there is only one hydraulic machine. Thus the second carries out function of not loaded allowance. Then

\[
\alpha_2 = t_2 / t_y,
\]

where \( t_2 \) - is time during which the element of system has a structural allowance.

It is necessary to construct a state graph for the considered system, accepting that \( E_0 \) hydraulic reliability is able \( J_{ac} \geq J_{sal} \) (figure 1).

![State graph example for an assessment of reliability of system.](image-url)
In case of decrease in hydraulic reliability of an element No.1 or No.2 to level \( J_{act} < J_{al} \) there is a transition of system to conditions \( E_1 \) and \( E_3 \) respectively. Decrease in reliability \( J_{act} < J_{al} \) in this case means that the system in general doesn't conform to the requirement of reliability and technical intervention is necessary. The system is in conditions \( E_1 \) and \( E_3 \) also technical intervention with intensity is required \( \mu_1 \) and \( \mu_2 \). In case of recovery of an element in time \( tr \) the system passes into a reliable condition \( E_0 \), otherwise – in refusal \( E_2 \) and \( E_4 \). During operation when there is a structural allowance, the system with intensity of \( \lambda_1 \alpha_2 \) and \( \lambda_2 \alpha_2 \) passes into the conditions of \( E_3 \) and \( E_5 \), similar to \( E_1 \) and \( E_3 \), however transfer can be performed by means of machines, carrying out function reserve while the system recovery is made [3-11].

In case of decrease in reliability of a reserve thread is down to one level lower admissible after float time the system passes into conditions of \( E_7 \) and \( E_{10} \) with the level of reliability is lower than the admissible.

The shelf life of system is determined by the time spent in conditions \( E_0 - E_1 \), \( E_0 - E_3 \), \( E_0 - E_6 \), \( E_0 - E_9 \).

System of the equations in an integrated form for the count:

\[
\begin{align*}
Q_{0}(t, t_{rr}) &= \int_{0}^{t} Q_{1}(t - x, t_{rr}) dP_{0,1}(x) + \int_{0}^{t} Q_{3}(t - x, t_{rr}) dP_{0,3}(x) + \\
&+ \int_{0}^{t} Q_{5}(t - x, t_{rr}) dP_{0,5}(x) + \int_{0}^{t} Q_{8}(t - x, t_{rr}) dP_{0,8}(x); \\
Q_{1}(t, t_{rr}) &= \int_{0}^{t} Q_{0}(t - x, t_{rr}) dP_{1,0}(x) + P_{1,2}(t); \\
Q_{3}(t, t_{rr}) &= \int_{0}^{t} Q_{0}(t - x, t_{rr}) dP_{3,0}(x) + P_{3,4}(t); \\
Q_{5}(t, t_{rr}) &= \int_{0}^{t} Q_{0}(t - x, t_{rr}) dP_{5,0}(x) + \int_{0}^{t} Q_{6}(t - x, t_{rr}) dP_{5,6}(x); \\
Q_{6}(t, t_{rr}) &= \int_{0}^{t} Q_{6}(t - x, t_{rr}) dP_{6,5}(x) + P_{6,7}(t); \\
Q_{8}(t, t_{rr}) &= \int_{0}^{t} Q_{0}(t - x, t_{rr}) dP_{8,0}(x) + \int_{0}^{t} Q_{9}(t - x, t_{rr}) dP_{8,9}(x); \\
Q_{9}(t, t_{rr}) &= \int_{0}^{t} Q_{9}(t - x, t_{rr}) dP_{9,8}(x) + P_{9,10}(t)
\end{align*}
\]

System of the equations in a probabilistic form:
\[
\begin{aligned}
&P_0(t, t_r) = 1 - P_{01}(t) - P_{03}(t) + P_{05}(t) + P_{08}(t) - \int_0^t P(t - x, t_r) dP_{01}(x) + \\
&+ \int_0^t P(t - x, t_r) dP_{03,0}(x) + \int_0^t P(t - x, t_r) dP_{05,0}(x) + \int_0^t P(t - x, t_r) dP_{08,0}(x);
&P_1(t, t_r) = 1 - P_{10}(t) - P_{12}(t) + \int_0^t P(t - x, t_r) dP_{10,0}(x);
&P_2(t, t_r) = 1 - P_{20}(t) - P_{24}(t) + \int_0^t P(t - x, t_r) dP_{23,0}(x);
&P_3(t, t_r) = 1 - P_{30}(t) - P_{36}(t) + \int_0^t P(t - x, t_r) dP_{35,0}(x);
&P_4(t, t_r) = 1 - P_{45}(t) - P_{47}(t) + \int_0^t P(t - x, t_r) dP_{46,0}(x);
&P_5(t, t_r) = 1 - P_{56}(t) - P_{9,9}(t) + \int_0^t P(t - x, t_r) dP_{58,0}(x) + \int_0^t P(t - x, t_r) dP_{8,9}(x);
&P_6(t, t_r) = 1 - P_{9,8}(t) - P_{9,10}(t) + \int_0^t P(t - x, t_r) dP_{9,8}(x)
\end{aligned}
\]

\text{(2)}

System of the equations in Laplace-Carson's forms:

\[
\begin{aligned}
&Q^*_{0}(s, t_r) = Q^*_{3}(s) P^*_{0,3}(s) + Q^*_{1}(s, t_r) P^*_{0,1}(s) + Q^*_{5}(s, t_r) P^*_{0,5}(s) + Q^*_{8}(s, t_r) P^*_{0,8}(s);
&Q^*_{1}(s, t_r) = Q^*_{0}(s, t_r) \cdot P^*_{1,2}(s);
&Q^*_{3}(s, t_r) = Q^*_{0}(s, t_r) P^*_{3,4}(t);
&Q^*_{5}(s, t_r) = Q^*_{0}(s, t_r) P^*_{5,6}(s) + Q^*_{6}(s, t_r) P^*_{5,6}(s);
&Q^*_{6}(s, t_r) = Q^*_{6}(s, t_r) P^*_{6,7}(s);
&Q^*_{8}(s, t_r) = Q^*_{0}(s, t_r) P^*_{8,0}(s) + Q^*_{9}(s, t_r) P^*_{8,9}(s);
&Q^*_{9}(t, t_r) = Q^*_{9}(s, t_r) P^*_{9,10}(s)
\end{aligned}
\]

\text{(3)}

Using this model we set semi-Markov process in a general view as \(P(t) = \|P_{ij}(t)\|\). Then, if the system is in a status \(i\), the transition probabilities between statuses of system will have an appearance:

\[
P_{ij}(t) = \int_0^t \prod_{k=1}^n (1 - F_{ik}(x)) dF_{ij}(x)
\]

\text{(4)}

where \(F_{ij}(x)\) - the distribution function of time of stay of an element is able i upon the following transition to a status j;
\(k \in N, k \in (1; n)\) - numbers of statuses except a status of j in which one-step transition from a status is possible i;
\(F_{ij}(x)\) - the distribution function of time of stay of an element is able i upon the following transition to a status k.

For determination of a distribution function of transitions from a status to a status it is necessary to determine statistically function of change of a keeping of system by criterion of hydraulic reliability \(F_{ij}\)
, in particular $F(J_s,t)$. For example, for the transmission system the function of distribution of hydraulic reliability $J_s(t)$ will be constructed in real time with use of methods of statistical modeling and the least squares on the base to operation properties of the equipment and the dispatcher data since the beginning of the analyzable period (figure 4).

![Figure 2. Distribution function of $F_i(t)$ for the oil pipeline](image)

Further we will determine distribution parameters of a keeping of $F_i$ by the diagram of change of hydraulic reliability under the law of change of coefficient of hydraulic reliability for the interclearing period of 65 days from conditions:

$$J_{s,i} = J_s(t^*) \quad t^* = \frac{\ln(J_{s,i})}{C}, \quad \lambda = 1/t^*, \quad (5)$$

where $t^*$ - time to failure; $J_{s,i}$ - critical level of an index of hydraulic reliability.

Thus, based on empirical function of change of coefficient of hydraulic reliability, we will receive a distribution function of failures (a distribution function of frequencies of achievement by system of critical values of coefficient $J_s$ and failure density by criterion of $J_s$).

For determination of the transition probability of system from a status 0 in a status 1 we will use a graph of fig. 1, supposing that adjacent statuses are No. 3, 5, 8. Without the need for the accounting of a temporal reserve the formula (3) will assume an air:

$$P_{0i}(t) = \int_0^t (1-F_{03}(x)) \cdot (1-F_{05}(x)) \cdot (1-F_{08}(x))dF_{01}(x) \quad (6)$$

Time distribution functions for statuses can be defined as

$$\int_0^t f(u)du, \quad u = F(x), \quad (7)$$

where $f$ - density function.

- in particular:

$$F_{01}(x) = 1 - e^{\lambda_1\alpha_1(x)},$$

$$F_{08}(x) = 1 - e^{\lambda_2\alpha_2(x)},$$

$$F_{03}(x) = 1 - e^{\lambda_2(1-\alpha_2)(x)}$$

$$F_{05}(x) = 1 - e^{\lambda(1-\alpha)(x)}, \quad \alpha_1 = 1 - \alpha_2. \quad (8)$$
The transition probability of $P_{01}$ is calculated as follows:

$$P_{01}(t) = \int_0^t (1 - F_{00}(x)) \cdot (1 - F_{05}(x)) \cdot (1 - F_{03}(x)) dF_{01}(x) =$$

$$= \int_0^t (1 - (1 - e^{-\tilde{\lambda}^2 q_2(x)})) \cdot (1 - (1 - e^{-\tilde{\lambda}^2 (1-q_2)(x)})) \cdot (1 - 1 - e^{-\tilde{\lambda}^2 (1-q)(x)}) dF_{01}(x) =$$

$$= \int_0^t (e^{-\tilde{\lambda}^2 q_2(x)}) \cdot (e^{-\tilde{\lambda}^2 (1-q_2)(x)}) \cdot (e^{-\tilde{\lambda}^2 (1-q)(x)}) dF_{01}(x) =$$

$$= \int_0^t (e^{-\tilde{\lambda}^2 q_2 x - \tilde{\lambda}^2 (1-q_2)(x) x - \tilde{\lambda}^2 q(x) x - \tilde{\lambda}^2 x q(x)) dF_{01}(x) =$$

$$= \int_0^t (e^{-\tilde{\lambda}^2 q_2 x - \tilde{\lambda}^2 (1-q_2)(x) x - \tilde{\lambda}^2 q(x) x - \tilde{\lambda}^2 x q(x) x - \tilde{\lambda}^2 x q(x)) dF_{01}(x) =$$

$$= \int_0^t (1 - e^{-\tilde{\lambda}^2 \mu_0(x)}) dF_{01}(x) = \lambda q_1 \left(\frac{\tilde{\lambda}^2 q_1 + \lambda q_1}{\tilde{\lambda}^2 q_1 + \lambda q_1} \right) \left(1 - e^{-\tilde{\lambda}^2 \mu_0(x)}\right)$$

(9)

Remaining transition probabilities are calculated similarly.

For determination of the transition probability of system from a status 1 in a status 0 we will receive:

$$P_{10}(t) = \int_0^t (1 - F_{12}(x)) dF_{10}(x)$$

(10)

where $F_{10}(x) = 1 - e^{-\mu_1(x)}$ - repairing function.

For determination of the transition probability of system from a status 1 in a status 2 taking into account a temporal reserve the formula will assume an air:

$$P_{12}(t) = \int_0^t (1 - F_{12}(x)) \cdot (1 - D(x)) dF_{12}(x)$$

(11)

where $D(x)$ - distribution function of a temporal reserve, $\text{D}(\text{x}) = 1$ npu $t < t; \text{D}(\text{x}) = 0$ npu $t \geq t$;

$F_{12}(x) = 1 - e^{-\mu_2(x)}$ - repairing function.

By results of calculations, conversions and integration the following dependences for determination of the transition probabilities are received:

- from a status $E_0$:

$$P_{01}(x) = \lambda_1 \cdot \alpha_1 \left(1 - e^{-(\lambda_2 + \lambda_0) x}\right), \quad P_{03}(x) = \lambda_2 \cdot \alpha_2 \left(1 - e^{-(\lambda_2 + \lambda_0) x}\right),$$

$$P_{05}(x) = \lambda_1 \cdot \alpha_2 \left(1 - e^{-(\lambda_2 + \lambda_0) x}\right), \quad P_{0,8}(x) = \lambda_2 \cdot \alpha_2 \left(1 - e^{-(\lambda_2 + \lambda_0) x}\right).$$

(12)

- from statuses $E_5$, $E_6$: 

\[
\begin{align*}
P_{56}(x) &= \frac{\lambda_2 \cdot \alpha_2}{\lambda_2 + \mu_1} (1 - e^{-(\lambda_2 + \mu_1)x}), \\
P_{59}(x) &= \frac{\lambda_1 \cdot \alpha_2}{\lambda_1 + \mu_2} (1 - e^{-(\lambda_1 + \mu_2)x}), \\
P_{50}(x) &= \frac{\mu_1}{\lambda_2 \alpha_2 + \mu_1} (1 - e^{-(\lambda_2 \alpha_2 + \mu_1)x}), \\
P_{50}(x) &= \frac{\mu_2}{\lambda_1 \alpha_2 + \mu_2} (1 - e^{-(\lambda_1 \alpha_2 + \mu_2)x}).
\end{align*}
\]  
(13)

- from statuses \(E_1, E_3, E_6, E_9\):

\[
\begin{align*}
P_{3,0}(t) &= e^{-\mu_{3,0} t}, \quad t \geq \text{tr}, \\
P_{3,0}(t) &= e^{-\mu_{3,0} t}, \quad t < \text{tr}, \\
P_{1,0}(t) &= e^{-\mu_{1,0} t}, \quad t \geq \text{tr}, \\
P_{1,0}(t) &= e^{-\mu_{1,0} t}, \quad t < \text{tr}, \\
P_{6,5}(t) &= e^{-\mu_{6,5} t}, \quad t \geq \text{tr}, \\
P_{6,5}(t) &= e^{-\mu_{6,5} t}, \quad t < \text{tr}, \\
P_{9,8}(t) &= e^{-\mu_{9,8} t}, \quad t \geq \text{tr}, \\
P_{9,8}(t) &= e^{-\mu_{9,8} t}, \quad t < \text{tr}.
\end{align*}
\]  
(14)

From statuses \(E_1, E_3, E_6, E_9\):

- in case \(t < \text{tr}\), that probabilities \(P_{1,2}, P_{3,4}, P_{6,7}, P_{9,10}\) are equal 0, and \(t \geq \text{tr}\) - equal 1.

By results of the carried-out transformations and integration, the following dependences for definition of time of stay in statuses \((T_i)\) according to the count are received:

- for status \(E_0\):

\[
T_0 = \int_0^\infty (1 - F_{ij}(t))dt = \frac{1}{\Lambda_1} = \frac{1}{\sum_{i=1}^{m} \alpha_i \cdot \alpha_i}, \\
\Lambda_1 = \sum_{i=1}^{m} \alpha_i \cdot \alpha_i, \\
T_0 = \frac{1}{\lambda_1 + \lambda_2},
\]  
(19)

- for statuses \(E_1, E_3, E_6, E_9\):

\[
\begin{align*}
T_i &= \frac{1}{\mu_i} (1 - e^{-\mu_i t}); \\
T_1 &= \frac{1}{\mu_1} (1 - e^{-\mu_1 t}); \\
T_3 &= \frac{1}{\mu_2} (1 - e^{-\mu_2 t}); \\
T_5 &= \frac{1}{\mu_1} (1 - e^{-\mu_1 t}); \\
T_8 &= \frac{1}{\mu_2} (1 - e^{-\mu_2 t})
\end{align*}
\]  
(20)

- for statuses \(E_2, E_4, E_{10}, E_7\):

\[
\begin{align*}
T_i &= \frac{1}{\mu_i}; \\
T_2 &= \frac{1}{\mu_1}; \\
T_4 &= \frac{1}{\mu_2}; \\
T_7 &= \frac{1}{\mu_2}; \\
T_{10} &= \frac{1}{\mu_1};
\end{align*}
\]  
(21)

- for statuses \(E_5, E_8\):

\[
\begin{align*}
T_5 &= \frac{1}{\lambda_2 + \mu_1}; \\
T_8 &= \frac{1}{\lambda_1 + \mu_2}.
\end{align*}
\]  
(22)

On a formula of conversion of Laplace-Carson for function \(F(S)\):

\[
F^*(S) = S \int_0^\infty e^{-st} \cdot F(t) \cdot dt
\]  
(23)
we will receive formulas for determination of images for system of equations of an analysable graph.

The readiness quotient determined by criterion of a keeping from dependences and the equations for nested Markov «chains»:

\[ K_{\alpha}(J_{S},t) = \sum_{i_{i}E}^{\sum T_{i}} \pi(t) = \frac{\sum_{i_{i}E} T_{pi} \pi(t)}{\sum_{i_{E}} T_{pi} \pi(t)} \cdot \pi(t) = \pi(t) \cdot \sum_{i_{i}E} T_{pi} \pi(t) \ . \]

For \( \mu_{1} = \mu_{2}, \lambda_{1} = \lambda_{2} \) we will receive dependences for determination of an availability quotient:

\[ K_{\alpha}(J_{S},t) = \sum_{i_{s}E}^{\sum T_{i}} \pi(t) = \frac{\sum_{i_{E}} T_{pi} \pi(t)}{\sum_{i_{E}} T_{pi} \pi(t)} \cdot \pi(t) = \pi(t) \cdot \sum_{i_{i}E} T_{pi} \pi(t) \ . \]

For \( 2 \leq 1 \mu_{1}, 2 \leq 1 \lambda_{1} \) we will receive dependences for determination of an availability quotient:

\[ K_{\alpha}(J_{S},t) = \sum_{i_{s}E}^{\sum T_{i}} \pi(t) = \frac{\sum_{i_{E}} T_{pi} \pi(t)}{\sum_{i_{E}} T_{pi} \pi(t)} \cdot \pi(t) = \pi(t) \cdot \sum_{i_{i}E} T_{pi} \pi(t) \ . \]

Thus, with use of the graph theory analytical dependences for the description of a set of statuses of system of oil pipelines and determination of probable characteristics are received.

The developed system of equations of Laplace-Carson allows to determine stationary probabilities by numerical methods using packets of mathematical computation.

3. Summary

The received dependences allow to characterize a set of statuses of systems of hydraulic machines which in a general view consist of the elements described by different communications, connections, state graphs and intensity of failures and restoration. In general the technique can be applied by preparation of information to acceptance of administrative decisions in tasks of monitoring of reliability, faultlessness, a keeping and safety in case of reliability assessment of hydraulic machines.

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