A NEW METHOD FOR POINT-SPREAD FUNCTION CORRECTION USING THE ELLIPTICITY OF RE-SMEARED ARTIFICIAL IMAGES IN WEAK GRAVITATIONAL LENSING SHEAR ANALYSIS

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ABSTRACT

Highly accurate weak lensing analysis is urgently required for planned cosmic shear observations. For this purpose we have eliminated various systematic noises in the measurement. The point-spread function (PSF) effect is one of them. A perturbative approach for correcting the PSF effect on the observed image ellipticities has been previously employed. Here we propose a new non-perturbative approach for PSF correction that avoids the systematic error associated with the perturbative approach. The new method uses an artificial image for measuring shear which has the same ellipticity as the lensed image. This is done by re-smearing the observed galaxy images and observed star images (PSF) with an additional smearing function to obtain the original lensed galaxy images. We tested the new method with simple simulated objects that have Gaussian or Sérsic profiles smeared by a Gaussian PSF with sufficiently large size to neglect pixelization. Under the condition of no pixel noise, it is confirmed that the new method has no systematic error even if the PSF is large and has a high ellipticity.

Key word: gravitational lensing: weak

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1. INTRODUCTION

Weak gravitational lensing has been widely recognized as a unique and very powerful method for studying not only the mass distribution of the universe but also cosmological parameters (Mellier 1999; Schneider 2006; Munshi et al. 2008). One of the most interesting aspects of this field is to accurately measure the cosmic shear (the coherent distortion of background galaxies induced by the large-scale structure of the universe) because this effect depends on the evolution of the structure which is influenced by the nature of dark energy. There have been some detections of cosmic shear (Bacon et al. 2000, 2003; Maoli et al. 2001; Refregier et al. 2002; Hamana et al. 2003; Casertano et al. 2003; van Waerbeke et al. 2005; Massey et al. 2005; Hoekstra et al. 2006). However, since the distortion is very weak and there are many sources of noise, a very accurate measurement scheme is needed for the huge number of background galaxies as well as a sophisticated analysis scheme for the measured shear which avoids systematic errors as much as possible. At present, there are several plans for wide field surveys of a huge number of galaxies intended to reduce the statistical error. This means the systematic errors in shear analysis methods become larger than the statistical errors. In fact, current survey plans require systematic errors less than 1% (WFIRST-AFTA http://wfirst.gsc.nasa.gov/, KIDS http://kids.strw.leidenuniv.nl/, Hyper Supreme-Cam http://www.naoj.org/Projects/HSC/HSCProject.html, Dark Energy Survey http://www.darkenergysurvey.org/, Euclid http://sci.esa.int/euclid, etc.), and 0.1% is required for future planned surveys (lsst http://www.lsst.org). There is another important point in the analysis of such wide survey data. For example, in the plan for the HSC wide survey, a few hundreds of millions of galaxies will be measured; therefore, slow methods are not realistic even if the methods have a high accuracy. Thus it is essential to develop a fast shear analysis scheme that is free from systematic errors.

There have been many studies in this regard (Kaiser et al. 1995; Bernstein & Jarvis 2002; Hirata & Seljak 2003; Refregier 2003; Kuijken 2006; Miller et al. 2007; Kitching et al. 2008; Zhang 2008; Melchior 2011), which have been tested using simulated data (Heymans et al. 2006; Massey et al. 2007; Bridle et al. 2010; Kitching et al. 2012). We also have developed new analysis methods, including the E-HOLICs method (E-HOLICs part1, Okura & Futamase 2011; part2, Okura & Futamase 2012; part3, Okura & Futamase 2013). The E-HOLICs method avoids systematic error caused by the approximation in the weight function by adopting an appropriate elliptical weight function for shape measurement. However, the E-HOLICs method uses an approximation for point-spread function (PSF) correction similar to other approaches. Because of this, previous approaches including E-HOLICs cannot correct for the PSF effect in some conditions (e.g., large PSF or high elliptical PSF).

In this paper, we concentrate on PSF correction and propose a new method that is free from the approximations used in previous methods and thus free from the systematic error associated with PSF correction. The idea is to produce an artificial image that has the same ellipticity as the lensed image. This is done by re-smearing the PSF smeared image of the lensed image in such a way that if the lensed image is smeared by a PSF that has same ellipticity as the lensed image, then the ellipticities of the smeared image, the PSF image, and the lensed image should be the same. This means that we can obtain the ellipticity of the lensed image (i.e., PSF-corrected ellipticity) by adding an additional PSF effect for the smeared image and PSF image to make two artificial images with the same ellipticity (see Section 3.2). We refer to this as the ellipticity of re-smeared artificial images (ERA) method.

This paper is organized as follows. In Section 2, we explain our notation and definitions used in this paper. There we also describe the zero plane used in the ERA method. We then present PSF smearing and PSF correction as used in the ERA method in Section 3. In Section 4, we test this method using simple test images. Finally, we summarize the method and give some comments.
2. BASICS

In this section we describe the notation, the definitions, and the basic ideas used in this paper. The basics of weak lensing can also be seen in Bartelmann & Schneider (2001).

2.1. Zero Plane and Zero Image

In this paper, we use the zero plane and the zero image instead of the source plane. The idea of the zero plane is that the intrinsic ellipticity of the source comes from an (imaginary) circular source (called the zero image) in the imaginary plane (the zero plane) (details can be seen in E-HOLICs part 2).

Suppose we have a reduced lensing shear and an intrinsic ellipticity of, respectively, \( g^L \) and \( g^I \). Then the relationship of the displacements between the zero plane \( \tilde{\beta} \), the source plane \( \beta \), and the lens plane \( \theta \) are described as

\[
\tilde{\beta} = \beta - g^I \beta^* \tag{1}
\]

\[
\tilde{\beta} = \theta - g^C \theta^* \tag{2}
\]

where \( g^C \) is the combined shear which is defined as

\[
g^C \equiv g^I + g^L \frac{1}{1 + g^I g^L} \tag{3}
\]

Figure 1 shows the relation between the zero, source, and image planes.

This combined shear includes the intrinsic ellipticity and the lensing reduced shear. Since the intrinsic ellipticities are random, the lensing reduced shear can be obtained by removing the intrinsic ellipticity as

\[
\langle g^C - g^L \rangle = (g^I) = 0. \tag{4}
\]

This shows that we can obtain the lensing reduced shear in two steps. The first step is to obtain the combined shear from each object (Equation (2)) and the second step is to obtain the lensing reduced shear by averaging (Equation (4)).

In this paper, we consider only the first step—the relationship between the zero plane and the lens plane—and we use \( \tilde{\beta} \) as \( \beta \) and \( g^C \) as \( g \) for notational simplicity.

2.2. Notation and Definitions

From this point, we use the complex coordinate \( \theta \equiv \theta_1 + i\theta_2 \) in the image plane and \( \beta \equiv \beta_1 + i\beta_2 \) in the zero plane. The reduced shear, which is defined by the gravitational convergence \( \kappa(\theta) \) and the gravitational shear \( \gamma(\theta) \), is also complex, \( g(\theta) \equiv g_1(\theta) + i g_2(\theta) \equiv \gamma(\theta)/(1 - \kappa(\theta)) \). The coordinate is associated with each image and set at the coordinate’s origin at the centroid of the image, which is defined by requiring that the dipole moment of the image vanishes.

We use \( Z^N_M(I, \epsilon_w) \) as the complex moment of image \( I(\theta) \) measured with the weight function \( W(\theta, \epsilon_w) \) which has an arbitrary profile and an ellipticity \( \epsilon_w \) defined as

\[
Z^N_M(I, \epsilon_w) \equiv \int d^2 \theta \theta^\ast \kappa^N_M I(\theta) W(\theta, \epsilon_w) \tag{5}
\]

\[
\theta^N_M \equiv \theta + \frac{\kappa^N_M}{\gamma^N_M}, \tag{6}
\]

where \( \theta^N_M \) is the higher order complex displacement from the centroid. Then we simplify the notation of combinations of the complex moments as

\[
\frac{Z^N_M(I, \epsilon) + Z^0_N(I', \epsilon)}{Z^0_R(I, \epsilon)} = \left[ \frac{Z^N_M + Z^0_P(I', \epsilon)}{Z^0_R} \right]_{(I, \epsilon)}. \tag{7}
\]

Although the profile of the weight function is arbitrary, it should be a realistic profile, and we use the elliptical Gaussian weight function in the following simulations. If the weight function is an elliptical Gaussian such as

\[
W(\theta, \epsilon_w) = \exp \left( -\frac{\theta^2_0 - \Re \left[ \epsilon_w^2 \theta^2 \right]}{\sigma_w^2} \right), \tag{8}
\]

the Gaussian scale \( \sigma_w^2 \) should be fixed as the condition where a signal-to-noise ratio (S/N) of the monopole moment of the image has a maximum. In this paper, we refer to the scale of the weight function as the maximum S/N scale and define the maximum S/N radius \( R_w \) as

\[
2 R_w^2 = \sigma_{\epsilon_w}^2. \tag{9}
\]

Here S/N is defined as

\[
\text{SN} \equiv \frac{\int d^2 \theta I(\theta) W(\theta, \epsilon_w)}{\sqrt{\int d^2 \theta W^2(\theta, \epsilon_w)}} = \frac{Z^0_N(\theta, \epsilon_w)}{\sqrt{\int d^2 \theta W^2(\theta, \epsilon_w)}}. \tag{10}
\]

so the weight scale should be set as

\[
\sigma_w = \left( \frac{Z^2_{10} - \Re \left[ \epsilon_w^2 \theta^2 \right]}{Z^2_{00}} \right)^{\frac{1}{2}}_{(I, \epsilon_w)}. \tag{11}
\]

In the above, we require that the dipole moment vanishes, so

\[
Z^1_{10}(I, \epsilon_w) = 0. \tag{12}
\]

2.3. The Relationship between Ellipticity and Reduced Shear

The ellipticity of the image \( I(\theta) \) is defined by the quadrupole moments as

\[
\epsilon \equiv \frac{Z^2_{10}}{Z^2_{00}}_{(I, \epsilon_w)}, \tag{13}
\]

where the ellipticity of the weight function is set to the same value as the measured ellipticity. Displacements in the zero plane and the image plane are related as

\[
\beta = (1 - \kappa) (\theta - g \theta^*). \tag{14}
\]

The brightness distribution of the lensed image \( I^{\text{lensed}}(\theta) \) is made from the zero image \( I^0(\theta) \) by lensing, so \( I^0(\beta) = I^{\text{lensed}}(\theta) \).
The relationship between the ellipticity of the zero image $\epsilon^0$ and the lensed image $\epsilon^L$ is obtained as

$$0 = \epsilon^0 = \epsilon^L - 2g + g^2\epsilon^L_s \over (1 + g^2) - 2\text{Re}[g^2\epsilon^L],$$

where $g = |g|$. Here, because $g$ and $\epsilon^L$ have the same phase angle (so $g^2\epsilon^L_s = g\epsilon^L_s$), we obtain

$$0 = \epsilon^0 = \epsilon^L - \delta \over 1 - \text{Re}[\delta^2 \epsilon^L].$$

(16)

where $\delta \equiv 2g/(1 + g^2)$ is the complex distortion. Equation (16) can also be obtained by changing $\epsilon^0$ in Equation (15) to $(\epsilon^0 - g^2\epsilon^0_s)/(1 - g^2)$. Finally, we obtain

$$\epsilon^L = \delta.$$  

(17)

This result means that we obtain the complex distortion $\delta$ (and also reduced shear) from the $\epsilon^L$ ellipticity of the lensed image $I^{\text{Lnsd}}$.

3. PSF CORRECTION USING THE ERA METHOD

The observed image is not the lensed image but the result of various effects such as atmospheric turbulence, photon noise, and pixelization of the lensed image. These various effects smear the image and change the ellipticity. The PSF is supposed to express the smearing. Thus we need to correct the smearing effect to obtain the correct ellipticity. In this section, we explain the PSF effect and present the PSF correction using the ERA method.

3.1. Point-spread Function Effect

In the PSF correction, the smearing effect is supposed to be described as follows:

$$I^{\text{Smd}}(\theta) = \int d^2\theta' I^{\text{Lnsd}}(\theta - \theta') P(\theta'),$$

(18)

where $I^{\text{Smd}}(\theta)$ is the brightness distribution for the observed smeared image and $P(\theta)$ is the PSF. We denote the ellipticity of $I^{\text{Smd}}(\theta)$ as $\epsilon^{\text{Smd}}$. The PSF effect occurs not only for galaxies but also for stars. Since a star is a point source, the brightness distribution of the smeared image of a star gives the PSF at that position. Therefore, the PSF in the field can be obtained by an interpolation from their values at the positions of stars (we do not consider this interpolation here).

The PSF changes the ellipticity of the lensed image. For the measured moment, using a moment method such as KSB or the PSF effect and present the PSF correction using the ERA effect to obtain the correct ellipticity. In this section, we explain the smearing. Thus we need to correct the smearing and pixelization of the lensed image. These various effects smear the image, so we must correct them.

3.2. New Method for Obtaining the Ellipticity of the Lensed Image

The idea of this new PSF correction method is to produce an artificial image from the observed smeared image under the condition that the artificial image has the same ellipticity as the lensed image. Then we can obtain the ellipticity of the lensed image and weak lensing shear by measuring the ellipticity of the artificial image. This method is referred to as ERA (ellipticity of the re-smeared artificial image). In this section, we present
the ERA method and Figure 2 shows a summary of the PSF correction.

Let us consider a circular image $I^{0\text{Smd}}(\beta)$ that is made from the zero image $I^0(\beta)$ by smearing with an arbitrary but circular PSF $P^0(\beta)$ which is defined as

$$I^{0\text{Smd}}(k) \equiv \hat{I}^0(k) \hat{P}^0(k)$$  \hspace{1cm} (24)

and let us define $I^{\text{Smd}}(\theta)$ and $P^E(\theta)$ as brightness distributions made by the lensing effect from $I^{0\text{Smd}}(\beta)$ and $P^0(\beta)$ respectively, so

$$I^{0\text{Smd}}(\beta) = I^{\text{Smd}}(\theta)$$  \hspace{1cm} (25)

$$P^0(\beta) = P^E(\theta).$$  \hspace{1cm} (26)

Because ellipticities of $I^{0\text{Smd}}(\beta)$ and $P^0(\beta)$ are 0, their lensed image $I^{\text{Smd}}(\theta)$ and $P^E(\theta)$ have ellipticity $\epsilon^L$. This means that smearing the lensed image $I^{\text{Smd}}(\theta)$ by an artificial PSF $P^E(\theta)$ with ellipticity $\epsilon^L$ makes an artificial smeared image $I^{\text{Smd}}(\theta)$ with ellipticity $\epsilon^L$, and thus the PSF effect is corrected by transforming the real PSF $P(\theta)$ to $P^E(\theta)$ and making the artificial image $I^{\text{Smd}}(\theta)$ as follows

$$I^{\text{Smd}}(k) = I^{\text{Lnsd}}(k) \hat{P}^E(k) = \hat{I}^{\text{Lnsd}}(k) \hat{P}(k) \hat{P}^E(k) / \hat{P}(k) = I^{\text{Smd}}(k) \hat{P}^E(k) / \hat{P}(k).$$  \hspace{1cm} (27)

In this way, we obtain the PSF-corrected ellipticity $\epsilon^L$ by measuring the ellipticity of $I^{\text{Smd}}(\theta)$. This equation is satisfied only when the $I^{\text{Smd}}(\theta)$ and $\hat{P}(k)$ have the same ellipticity $\epsilon^L$, given the observed $\hat{P}(k)$ and $\hat{I}^{\text{Lnsd}}(k)$, and thus this equation should be solved.

There are two possible ways to solve this equation. One is to use $P^E(\theta)$ directly, which means that the PSF correction is the re-smearing deconvolved image. However, because this method cannot avoid the problem of 0 dividing, Equation (27) must be modified as follows

$$\hat{I}^{\text{ESmd}}(k) = \hat{I}^{\text{Smd}}(k) \hat{P}^E(k) / |\hat{P}(k)|^2 + C_{\text{Dec}}. \hspace{1cm} (28)$$

One solves this equation by starting from the initial ellipticity $\epsilon^{\text{Smd}}$ under the condition that the ellipticities of $I^{\text{ESmd}}$ and $P^E(\theta)$ have the same values (referred to as method A).

Another possibility is to introduce the correction for PSF in the following way:

$$\hat{P}^E(k) = \hat{P}(k) \Delta \hat{P}(k). \hspace{1cm} (29)$$

This cancels $\hat{P}(k)$ in the denominator and avoids the problem of 0 dividing, so Equation (27) becomes

$$\hat{I}^{\text{ESmd}}(k) = \hat{I}^{\text{Smd}}(k) \Delta \hat{P}(k). \hspace{1cm} (30)$$

This method solves Equations (29) and (30) simultaneously to find $\Delta \hat{P}^E$ under the condition that $I^{\text{ESmd}}$ and $\hat{P}^E$ have the same ellipticity (referred to as method B). Figure 3 shows the relationship between the images.

Comparing these two methods, method A has a deconvolution constant, so we must be careful about the effect from it, and method B creates and measures two images so it is expected that a longer time is needed to obtain the PSF-corrected ellipticity compared with method A.

### 4. SIMULATION TEST

In this section, we perform several tests of the ERA method using simulated images and show the results. In the previous section, we introduce the artificial PSF $P^E(\theta)$ and the artificial image $I^{\text{ESmd}}$ with the lensed ellipticity, but do not mention their profile and size. In fact, these are both very flexible. In this test, we choose a Gaussian profile and use several sizes. A Gaussian profile is a standard choice and is appropriate for this initial test. Investigations to identify a better profile and size of the artificial PSF will be part of future work to further develop the ERA method. Parameters used in this test are as described below.

#### 4.1. Simulation Parameters

Table 1

| $C_{\text{Dec}}$ | $10^{-6}$ | $10^{-5}$ | $10^{-4}$ | $10^{-3}$ | $10^{-2}$ | $10^{-1}$ | $10^0$ |
|-----------------|---------|---------|---------|---------|---------|---------|-------|
| $\epsilon^{\text{Dec}}$ | 0.300   | 0.299   | 0.294   | 0.279   | 0.241   | 0.180   | 0.127 |

We used the following parameters for the simulation test. The profile of the lensed image is assumed to be [Gaussian, Sérsic $n = 4$] with an ellipticity $\epsilon^L = (0.3, 0.0)$ and the size is set based on the condition that the maximum S/N radius is $R_W = 20$ pixels. This size is larger than the standard image used in real analysis because we would like to avoid the effect
of pixelization in this test. The ellipticity of the PSF $\epsilon^p = [(0.0, 0.0), (0.1, 0.1), (0.3, 0.0), (-0.6, -0.6)]$ and method [A; B] indicates a PSF correction with method [A; B, KSB and simple deconvolution].

The first step of the simulation is to make images. The lensed and PSF images are made with assigned parameters, and we then make the smeared image $I^{Smd}$ in Fourier space from the lensed and PSF images.

PSF corrections proceed as follows.

1. Deconvolution. Deconvolve the smeared image $I^{Smd}$ by the PSF image $P$ for making the deconvolved image $I^{Dec}$ and measure the ellipticity of the deconvolved image, where we use a sufficiently small deconvolution constant ($C^{Dec} \ll |\hat{P}(k)|^2$). (In this test we do not have noise and thus we can take an arbitrarily small deconvolution constant.)

2. Method A. Re-smear the deconvolved image $I^{Dec}$ by $P^E$ for making $I^{ESmd}$ and measure the ellipticity of $I^{ESmd}$ which has same ellipticity as $P^E$. To find the ellipticity of $P^E$, we use an iterative technique. The initial choice of ellipticity for $P^E$ is that of $I^{Smd}$, then $I^{Dec}$ is smeared to make $I^{ESmd}$. After this, the measured ellipticity of the $I^{ESmd}$ is used as the ellipticity of $P^E$ in the next iteration (i.e., $\epsilon_n^{P^E} = \epsilon_n^{ESmd}$ in the nth iteration).

3. Method B. Re-smear the smeared image $I^{Smd}$ and PSF image $P$ by $\Delta P$ to make $I^{ESmd}$ and $P^E$. To find the ellipticity of $\Delta P$, we use the following iterative technique. The initial choice of ellipticity for $\Delta P$ is again that of $I^{Smd}$ to make $I^{ESmd}$ and $P^E$. In general, these ellipticities do not coincide, but the difference is added for the initial choice of ellipticity for $\Delta P$ in the second step and so on (i.e., $\epsilon_n^{\Delta P} = \epsilon_{n-1}^{\Delta P} + (\epsilon_n^{ESmd} - \epsilon_{n-1}^{P^E})$ in the nth iteration).

As a comment on the iterative techniques used in methods A and B, we believe that the iterative methods used are standard, but it is important to find a faster and more robust method which will be the subject of future studies.

Next, we test the following seven cases,

1. deconvolution: standard deconvolution where we use a small deconvolution constant that is not realistic due to the effect of pixel noise,

2. method A1: the size of $P^E$ is the same as the PSF $R^P$,

3. method A2: the size of $P^E$ is twice as large as the PSF $R^P$,

4. method A3: the size of $P^E$ is three times as large as the PSF $R^P$,

5. method B1: the size of $\Delta P$ is the same size as $R^P$,

6. method B2: the size of $\Delta P$ is the same size as $I^{Smd}$, and

7. KSB method: PSF correction using the KSB method,

where we choose the KSB method to compare its accuracy with our new method. Here we used the size of the weight function of a smeared galaxy for the size of the weight function of PSF(KSB+).

### 4.2. Simulation Results

Figures 4–19 show the PSF-corrected ellipticities. The conditions of the lensed images and PSF are shown in each caption. Figures 4, 5, 12, and 13 show the results with an isotropic PSF, and Figures 6, 7, 14, and 15 show the results with a small elliptical PSF.

These results show that the standard deconvolution method has a systematic error in some situations, e.g., when there is a large PSF and the KSB method overestimates in all situations. In contrast, methods A and B correct the PSF with systematic errors of about 0.01% or lower. Figures 8, 9, 16, and 17 show that all methods have no systematic errors because the ellipticity of the lensed images and that of the PSF have the same value, so the PSF acts as $P^E$ and we do not need the PSF correction. However, because KSB corrects both anisotropic and isotropic PSF, the KSB method has a systematic error. Figures 10, 11, 18, and 19 show the results with a large elliptical PSF. In real lensing analysis, such a large elliptical PSF is rare so we should discard such data, however, this sort of test is valuable for understanding the details of this new method. These results show that the deconvolution and KSB methods are not able to sufficiently correct such a large ellipticity in the PSF. PSF corrections with a small re-smearing function in method A have a systematic error. Method A involves creating the re-smereed image from the deconvolved image, resulting in a large systematic error with the deconvolved method. Method B1 with a PSF size 0.5 cannot correct the PSF because a high ellipticity (nearly 1) is needed for the re-smearing function $\Delta P$. Method B2 has a larger re-smearing function than method B1, so method B2 can

### Table 2

**Summary of Parameters Used in This Simulation**

| Figure number | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---------------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Image type    | G | G | G | G | G | G | G | G | S | S | S | S | S | S | S | S |
| PSF ellipticity | 0 | 0 | 1 | 1 | 3 | 3 | 6 | 6 | 0 | 0 | 1 | 1 | 3 | 3 | 6 | 6 |
| Method         | A | B | A | A | B | A | A | B | A | A | B | A | A | B | A | A |

**Notes.** Image type [G; S] means that the profile types used for images are the Gaussian; Sérsic]; PSF ellipticity [0; 1; 3; 6] means that the ellipticity of the PSF is [(0.0, 0.0); (0.1, 0.1); (0.3, 0.0); (−0.6, −0.6)]; and method [A; B] indicates a PSF correction with method [A; B, KSB and simple deconvolution].

### Table 3

**Result of Gaussian Profile Image Where the Size Ratio and Ellipticity of the PSF are 1.0 and (0.0, 0.0) (Figures 4 and 5)**

| Method     | Corrected $\epsilon_1$ | Corrected $\epsilon_2$ | Iteration Number | Calculation Time |
|------------|------------------------|------------------------|------------------|-----------------|
| Deconvolution | 0.300                  | 0.000                  | 1                | 1.088           |
| KSB        | 0.308                  | 0.000                  | 1                | 1.000           |
| ERA A1     | 0.300                  | 0.000                  | 4                | 7.160           |
| ERA A2     | 0.300                  | 0.000                  | 4                | 7.558           |
| ERA A3     | 0.300                  | 0.000                  | 4                | 8.153           |
| ERA B1     | 0.300                  | 0.000                  | 6                | 10.740          |
| ERA B2     | 0.300                  | 0.000                  | 4                | 8.542           |

### Table 4

**Result of Sérsic Profile Image Where the Size Ratio and Ellipticity of the PSF are 1.5 and (0.1, 0.1) (Figures 14 and 15)**

| Method     | Corrected $\epsilon_1$ | Corrected $\epsilon_2$ | Iteration Number | Calculation Time |
|------------|------------------------|------------------------|------------------|-----------------|
| Deconvolution | 0.266                  | 0.017                  | 1                | 1.322           |
| KSB        | 0.309                  | −0.001                 | 1                | 1.000           |
| ERA A1     | 0.300                  | 0.000                  | 4                | 7.873           |
| ERA A2     | 0.300                  | 0.000                  | 4                | 8.655           |
| ERA A3     | 0.300                  | 0.000                  | 4                | 10.023          |
| ERA B1     | 0.300                  | 0.000                  | 6                | 11.746          |
| ERA B2     | 0.300                  | 0.000                  | 4                | 9.6061          |
correct the PSF with a smaller ellipticity for $\Delta P$ than method B1. This indicates we have to choose the size of the re-smearing function carefully when the PSF has both a large size and a large ellipticity. Indeed, in some cases, methods A and B give the same results; they show the arbitrariness for parameters of the ERA method. Tables 3 and 4 are samples of the results in each method where the iteration number changes with the
Figure 8. This figure is the same as Figure 4 except the ellipticity of the PSF is (0.3, 0.0).
(A color version of this figure is available in the online journal.)

Figure 9. This figure is the same as Figure 5 except the ellipticity of the PSF is (0.3, 0.0).
(A color version of this figure is available in the online journal.)

Figure 10. This figure is the same as Figure 4 except the ellipticity of the PSF is (−0.6, −0.6).
(A color version of this figure is available in the online journal.)

Figure 11. This figure is the same as Figure 5 except the ellipticity of the PSF is (−0.6, −0.6).
(A color version of this figure is available in the online journal.)

required precision of convergence and the calculation time is normalized by the required time in the KSB method because the calculation times change with machine speed.

We obtained similar results using different ellipticities for the lensed image of $\epsilon^L = (0.1, 0.0)$ and (0.5, 0.0). All figures show that methods A3 and B2 can correct the PSF in all situations, meaning that the ERA method with a large re-smearing function can correct the PSF. However, we note that re-smearing by means of a re-smearing function that is too large makes a large artificial image and more time is needed to measure the moments of this large artificial image.
However, it is important to find more appropriate profiles and sizes for $P^2$ to have faster analyses, and it is also necessary to find a more effective numerical iteration scheme. These problems will be studied in future research.

5. SUMMARY AND FUTURE WORK

We have developed an ERA method with a possible new PSF correction method for weak gravitational lensing shear analysis. The idea is to construct an artificial image with the same ellipticity as the lensed image by re-smearing the
observed image. This approach avoids the approximations in PSF corrections used in the KSB and E-HOLICs methods (i.e., Equation (20)), therefore there is no systematic error from PSF correction if we choose an appropriate function for re-smearing. Then, we tested this method with simple simulated images. The results of the simulation are as follows. The deconvolution method cannot estimate ellipticity correctly because deconvolution is not perfect when smearing using a large or high elliptical PSF. The KSB method has systematic errors that cause overestimation in the standard situation (in the simulation, it is about 2–3%, but that depends on the ellipticity of the image, the size of the PSF, etc.) and is not able to correct the smearing effect with a high elliptical PSF. However, methods A and B are both able to estimate the ellipticity correctly if the PSF
has a standard ellipticity. We also confirmed that re-smearing the deconvolved image (method A3) and re-smearing the smeared image (method B2) have no systematic error in PSF correction even if the size and the ellipticity of the PSF are large.

Although the tests performed here are not entirely realistic, the results are very encouraging. Further studies of systematic errors based on realistic data, (e.g., pixel noise, pixelization) are needed. In particular, pixel noise has the potential to make large systematic errors (see E-HOLICs part3). For example, in measuring the cosmic shear, galaxies in high redshift bins are relatively faint and have lower signal-to-noise ratios, thus they suffer from more systematic bias due to pixel noise than those at lower redshifts. Thus, the measured shear for high-$z$ galaxies would be underestimated. It is also important to find a more appropriate profile for the re-smearing function and to develop more effective iteration schemes which will result in faster analyses. Future planned surveys such as EUCLID and LSST will treat an enormous number of galaxies and thus not only low systematic errors but also fast analyses are essential. These problems will be addressed in future studies.

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