Pairing symmetry signatures of $T_1$ in superconducting ferromagnets

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We study the nuclear relaxation rate $1/T_1$ as a function of temperature for a superconducting-ferromagnetic coexistent system using a p-wave triplet model for the superconducting pairing symmetry. This calculation is contrasted with a singlet s-wave one done previously, and we see for the s-wave case that there is a Hebel-Slichter peak, albeit reduced due to the magnetization, and no peak for the p-wave case. We then compare these results to a nuclear relaxation rate experiment on $UGe_2$ to determine the possible pairing symmetry signatures in that material. It is seen that the experimental data is inconclusive to rule out the possibility of s-wave pairing in $UGe_2$.

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In the BCS theory of superconductivity, the conduction electrons in a metal cannot be both ferromagnetically ordered and superconducting. Superconductors may expel small magnetic fields passing through them, but strong magnetic fields destroy the superconductivity (SC). Even small amounts of magnetic impurities are usually enough to eliminate SC. Much work has been done both theoretically and experimentally to understand this interplay, and to explore the possibility of coexistence between these two ordered states.

The archetypal material that does display this coexistence is $UGe_2$, and it is generally believed that the superconductivity is governed by triplet pairing due to the presence of the internal ferromagnetism. However, there have been some convincing calculations that instead consider s-wave superconductivity to be favored in the presence of this magnetization. For instance, Abrikosov and Suhl demonstrate that $UGe_2$ should have s-wave Cooper pairing which is mediated by localized ferromagnetic spins. Also, Blagoev et al. studied a weak ferromagnetic Fermi liquid and showed that s-wave superconductivity is possible and subsequently energetically favored in the ferromagnetic state.

The contributions of all of these theories and calculations have naturally led to a healthy debate on this specific property of this fascinating material, and thus the nature of the pairing is still an open question.

Motivated by this debate and the experimental data that should hold the answer, the present authors have recently studied various properties of a mean-field model of coexistent itinerant ferromagnetism and s-wave superconductivity. The electronic specific heat was calculated and quantitatively fit the experimental data quite well, as well as several other properties, including the nuclear relaxation rate.

In this paper we show the results of a nuclear relaxation rate $1/T_1$ calculation from a different starting model in which there is p-wave pairing in the superconducting channel. This result is then compared to the s-wave calculation and the experimental data to determine a signature of the much disputed pairing symmetry. We find that there is a small signature around $T_c$ in the observed data of a small Hebel-Slichter peak that resembles the s-wave curve, but that this comparison is inconclusive to verify either type of pairing.

A triplet state can be described by the order parameter $\Delta_{\alpha\beta}(k) = \langle c_{k,\alpha}^\dagger c_{k,\beta}\rangle$, where $\alpha, \beta = \uparrow, \downarrow$, and $\sigma = x, y$. The three-dimensional complex vector $\mathbf{d}(k)$ fully characterizes the triplet pairing state. For simplicity, the easy axis of magnetization is assumed to be in the z-direction. Because of the pair-breaking effect of the strong exchange field, only the Cooper pairs with equal spin states will survive.

In the case of equal spin pairing we can write vector $\mathbf{d}$ in the form $\mathbf{d} = (d_x, d_y, 0)$. Denoting $\Delta = d_x \pm d_y$, the superconducting order parameter (SCOP) becomes:

$$\Delta_{\alpha\beta}(k) = \begin{pmatrix} \Delta_-(k) & 0 \\ 0 & \Delta_+(k) \end{pmatrix}. \quad (1)$$

An attractive pair-forming interaction $V$ is assumed, and the coexistent Hamiltonian is then

$$H_{FM+SC} = \sum_{k,\alpha} c_{k,\alpha}^\dagger c_{k,\alpha} - I \int d\mathbf{r} s(\mathbf{r}) \cdot s(\mathbf{r}) + \frac{1}{2} \sum_{k, k'} V_{\alpha\beta,\lambda\mu}(k, k') c_{k,\alpha}^\dagger c_{k',\beta} c_{k',\lambda}^\dagger c_{k,\mu}. \quad (2)$$

The equations for the SCOP have the usual BCS-like form:

$$\Delta_-(k) = -\frac{1}{V} \sum_{k'} V(k, k') \left( \frac{1 - 2f(E_-(k'))}{2E_-(k)} \right) \Delta_-(k') \quad (3)$$

$$\Delta_+(k) = -\frac{1}{V} \sum_{k'} V(k, k') \left( \frac{1 - 2f(E_+(k'))}{2E_+(k)} \right) \Delta_+(k'), \quad (4)$$

where $f(E)$ is the Fermi-Dirac distribution function, and $\Delta_+$ and $\Delta_-$ are the SCOP for the up-up and the down-down spin pairing states respectively. The equation for the magnetic order parameter (MOP) is

$$M = \frac{1}{V} \sum_k \left\{ \frac{\epsilon_k^\dagger [1 - 2f(E_-)]}{2E_-(k)} - \frac{\epsilon_k^\dagger [1 - 2f(E_+)]}{2E_+(k)} \right\}, \quad (5)$$
where $\epsilon_{k^{\uparrow\downarrow}} \equiv \epsilon_{k} \pm M_{\pm}$. Eqs. (5) are coupled with each other via the quasiparticle spectrum $E_{\pm}(k) = \sqrt{(\epsilon_{k} \pm M_{\pm})^2 + |\Delta_{\pm}(k)|^2}$.

In order to illustrate the interplay between the MOP and the SCOP, we solve Eqs. (3)-(5) self-consistently for the simplest case of a spherical Fermi surface, assuming that the SC pairing strength is in the p-channel, $V(k, k') = g_{ij}(k, k') \sum_{m=-1}^{1} Y_{1m}(k)Y_{1m}^*(k')$, and has the BCS-like form, i.e., $g_{ij}(k, k')$ vanishes everywhere except the narrow region near the Fermi surface and does not depend on the exchange interaction $I$. We have used $\Delta_{\pm}(k) = \Delta_{0\pm} Y_{1\pm}(k)$. The SCOP will have point nodes at the poles of the Fermi surface.

The order parameters, $M$ and $\Delta$, have dependencies such as $M = M(g, I, T)$ and $\Delta = \Delta(g, I, T)$ at $T \neq 0$. After solving these three coupled integral equations numerically, we study the characteristic behavior of the order parameters.

The variation of the transition temperature of the ferromagnetic and the superconducting transitions as a function of $I$ has been studied previously, and here we mention the results. For $I \leq 1.0$, which corresponds to the paramagnetic phase $M = 0$, the superconducting transition temperature for the up-up, $T_{c+}$, and down-down, $T_{c-}$, spin pairing is equal, implying that $\Delta_{+} = \Delta_{-}$. This corresponds to the planer (unitary) phase of the triplet pairing. But for $I > 1.0$, there is a finite magnetization and the transition temperatures $T_{c+}$ and $T_{c-}$ will not be equal, implying that $\Delta_{+} \neq \Delta_{-}$. Upon further increase of $I$, and the subsequent increase of $M$, $T_{c+}$ increases and $T_{c-}$ decreases to zero. Therefore, the finite magnetization induces the non-unitary type of superconducting triplet state.

Here, we study the temperature dependence of $M$, $\Delta_{+}$, and $\Delta_{-}$ for fixed values of $I$ and $g$. For $I \leq 1.0$, in the paramagnetic phase, the variation of $\Delta_{+}$ and $\Delta_{-}$ is identical to that of the mean field result $\Delta \propto (T_{c} - T)^{1/2}$ as in the BCS case. Similarly, the temperature variation of $M$ in the absence of superconductivity, i.e., $\Delta_{+} = \Delta_{-} = 0$, follows the trend as is predicted by mean field theory (solid line in Fig. (1)). We show the variation of $M$, $\Delta_{+}$, and $\Delta_{-}$ as a function of $T$ for the chosen values of $I = 1.018$ and $g = 0.95$ (coexistence of magnetism and both equal spin pairing triplet state superconductivity) in Fig. (1). The temperature axis has been normalized with the ferromagnetic transition (Curie) temperature $T_{m}$ to show the relative magnitudes of the corresponding transition temperatures. We can tune values of $I$ and $g$ so that the transition temperatures will be comparable.

We see in Fig. (1) that the SCOP decrease with temperature. The MOP decreases as well for $T \geq T_{c+}$, but for $T \leq T_{c+}$ we see some new features. If $\Delta_{+} > \Delta_{-} \neq 0$, for $T \leq T_{c+}$, $M$ (dashed line) is always bigger than its value in the pure ferromagnetic state when $\Delta_{+} = \Delta_{-} = 0$ (solid line). When there is a finite $\Delta_{+}$ and $\Delta_{-}$, $M$ increases with the increase in $T$ from 0 to $T_{c+}$, and then decreases sharply in the range of temperature $T_{c-} < T < T_{c+}$ (dashed line). The increase in $M$ for $0 < T < T_{c-}$ is due to the decreasing $\Delta_{-}$ as the temperature increases. On the other hand, the sharp decrease in $M$ for $T_{c-} \leq T < T_{c+}$ is not only due to the temperature, but more strongly to the decrease of $\Delta_{+}$. Again we see that for $0 < T < T_{c-}$, $M$ is bigger if $\Delta_{-} = 0$ (dotted line) than when $\Delta_{-} \neq 0$ (dashed line). These results are in agreement with the general intuition that the magnetization has a cooperative relation with the up spin pairing state, and a competitive relation with the down spin pairing. Our ultimate goal is to study the variation of the relaxation rate $1/T_{1}$ with temperature for this p-wave state, so next we derive the equation for the density of states as a function of excitation energy.

To derive the expressions for the density of states we use the usual relation, $N_{s}^{\uparrow\downarrow}(E) = \frac{1}{8\pi} \int_{0}^{\pi} d\theta \sin \theta \left(A^{\uparrow} + A^{\downarrow}\right)$, where $\sigma$ refers to the up spin or down spin fermions. Solving in the standard way using the property of the $\delta-$function, we get,

$$N_{s}^{\uparrow\downarrow}(E) = \frac{1}{8\pi} \int_{0}^{\pi} d\theta \sin \theta \left(A^{\uparrow} + A^{\downarrow}\right), \quad (6)$$

where

$$A^{\uparrow} = \frac{\sqrt{p_{F}^{2} + 2m^{*}(\pm IM_{\uparrow} + \sqrt{E^{2} - \Delta^{2}_{\uparrow}})}}{\sqrt{(E^{2} - \Delta^{2}_{\uparrow})}}, \quad (7)$$

$$A^{\downarrow} = \frac{\sqrt{p_{F}^{2} + 2m^{*}(\pm IM_{\downarrow} - \sqrt{E^{2} - \Delta^{2}_{\downarrow}})}}{\sqrt{(E^{2} - \Delta^{2}_{\downarrow})}}. \quad (7)$$

FIG. 1: Order parameters in this model as a function of $T$. $M$, $\Delta_{+}$, and $\Delta_{-}$ are labeled accordingly. For $M$, the solid curve refers to the pure ferromagnetic state, $\Delta_{+} = \Delta_{-} = 0$, the dashed line refers to $M$ when $\Delta_{+} > \Delta_{-} \neq 0$, and the dotted line when $\Delta_{-} = 0$ and $\Delta_{+} \neq 0$. The y-axis is a unitless measure and the x-axis is scaled by the Curie temperature ($T_{m}$).
The $+\,\text{sign}$ is for the $\uparrow$ spin fermions and the $-\,\text{sign}$ is for $\downarrow$ spin fermions on $\Delta_{\pm}$, respectively. These two expressions converge to the density of states of the normal state at the Fermi level, $N(0) = \frac{m^*}{\pi \hbar^2} p$, in the limit of all OP equal to zero. For $M = 0$, the expressions result in the density of states of the equal spin pairing triplet state superconductor, $N(E)/N(0) = E/2 \int d\theta \sin \theta/\sqrt{(E^2 - \Delta^2)}$. The important effect of the internal magnetization $M$ is to modify the value of the density of states. For increasing $M$, we observe an increase in $N_s^\uparrow(E)$ and a decrease in $N_s^\downarrow(E)$.

We plot the $T = 0$ density of states (DOS) scaled to the normal state value for positive excitation energies in Fig. 2. The inset A represents the density of states when $M = 0$, which is just a superconducting DOS and both spin components fall on the same line. The divergence in the density of states is at $E = \Delta_+ = \Delta_-$. The main body B of Fig. 2 shows the effect of the magnetization on the density of states when the magnitude of $M$ is such that $\Delta_- \neq 0$. We see that due to the finite magnetization, the density of states curve now splits for the up and the down spin fermions. The diverging density of states at the lower energy is at $E = \Delta_-$ and that at the higher value is at $E = \Delta_+$. For the value of $I$ at which $M$ is large enough to completely suppress the down spin pairing, the density of states for the down spin fermion will be constant as is shown in inset C. These modifications in the density of states due to the presence of the internal magnetization will manifest themselves in the nuclear relaxation rate of the system.

In general $1/T_1$ in the superconducting state is related to the (DOS) in the superconducting state, $N_s(E)$ as follows:

$$\frac{1}{T_1} \propto 2 \int_0^\infty (N_s(E))^2 f(E)(1 - f(E))dE,$$

where $f(E)$ is the Fermi-distribution function. In the presence of $M$ the functions will not be the same for the up and the down spin fermions as mentioned above, so we split the integral for different spin species. Hence,

$$\frac{1}{T_1} \propto \int_0^\infty (N_s^\uparrow(E))^2 f(E_+)(1 - f(E_+))dE + \int_0^\infty (N_s^\downarrow(E))^2 f(E_-)(1 - f(E_-))dE.$$

First, we choose the interaction parameters $I$ and $g$ such that there is both the up-up spin and the down-down spin pairing (the corresponding transition temperatures are not equal) and study the variation of $1/T_1T$ with respect to temperature. The results have been presented in Fig. 3, where the temperature has been normalized with the transition temperature for the up-up spin pairing. We choose $I$ and $g$ such that $T_{c_+/c_-} \approx 2.5$. In Fig. 3 we see that $1/T_1T$ decreases for $T \leq T_{c_+}$. This decrement characterizes the onset of the spin triplet pairing. With the increase in temperature the rate continues to decrease, but not as strongly as it would for the $M = 0$ case, as shown by the dotted line. The dotted line is just the p-wave superconducting case, and when

![FIG. 2: Density of states as a function of the excitation energy for each spin species. The dotted line corresponds to the normal state at the Fermi level. Inset A corresponds to $M = 0$, B to $M \neq 0$ for which down spin pairing is finite and C to $M \neq 0$ for which down spin pairing is completely suppressed. Note that the density of states of the up and the down spin fermions are not identical when $M \neq 0$.](image1)

![FIG. 3: $1/T_1T$ vs. normalized temperature with respect to the transition temperature of the up spin Cooper pairs, $T_{c_+}$. The dotted line is the p-wave superconducting case, with no magnetization. The top solid line is for the case when there is no pairing of down spins, where the only transition is at $T/T_{c_+} = 1$ when the system becomes an up spin paired superconductor, and saturates at $T = 0$. The bottom solid curve shows a second transition when the down spin Cooper pairs are formed and another gap $\Delta_-$ opens up, and is fully suppressed to zero. The values of $I$ and $g$ are chosen so that $T_{c_+/c_-} = 2.5$.](image2)
the gap opens up, the rate is suppressed quickly.

On the other hand, when we have a finite magnetization and only superconducting pairing of the up spins, we refer to the top solid curve of Fig.(3). In this case, the transition at $T = T_{c+}$ results in a decrease of the relaxation rate due to the presence of the gap $\Delta_{s}$, but the rate does not decrease as sharply as the unpolarized case because there is a finite magnetization. Furthermore, there is a normal fermionic component in this case since the down spins have not paired (due to our choice of the interaction parameters), and thus this finite density at the Fermi level allows for a nonzero relaxation rate at zero temperature. This follows from the fact that the entire Fermi surface is not gapped.

Finally, the bottom solid curve of Fig.(3) is calculated when we allow for the down spins to pair. This results in a second superconducting transition at $T_{c-}$ and the opening up of a second gap $\Delta_{s}$. The relaxation rate goes to zero when all the states are frozen out at zero temperature as expected. Note that there is no Hebel-Slichter peak for the p-wave case.

We now compare these results with the s-wave singlet nuclear relaxation rate and the experimental data. We have received data from the Kotegawa group on $UGe_2$. The calculation of the singlet has been carried out in a prior publication, where the details can be found. In Fig.(4) we calculate $1/T_1$ versus temperature and show all three curves. The experimental data was taken at 13kbar. We emphasize that at this pressure and temperatures, there is coexistence of superconductivity and magnetism. Also, the theoretical triplet curve is for the case of one single transition at $T = T_{c}$ corresponding to the up spins forming Cooper pairs. The down spins do not pair in this case and behave basically as normal fermions.

Several features are seen in Fig.(4). First of all, the data is not very consistent near the critical temperature, due to the difficulties in the measurement. However, a small peak below $T_{c}$ is noticeable, which follows the trend of the s-wave case of the solid line. This very well could be the reduced Hebel-Slichter peak that was predicted due to the presence of the ferromagnetism which produces a finite density of states at the Fermi level due to the existence of gapless fermions.

This p-wave curve is calculated without down spin pairing, and referring to Fig.(3) where $1/T_1T$ is constant at small temperatures, $1/T_1$ goes to zero with $T$ as expected. The low temperature behavior of the data seems to agree well within the triplet scenario. We do not observe in the experimental data a second transition and a second gap opening up from the pairing of the down spins. From our theoretical analysis, we would expect this to happen at a temperature of around $T = 0.018K$, however the experiments have only been carried out down to $T = 0.04K$.

In conclusion, we have studied a p-wave model of superconducting pairing in the background of itinerant ferromagnetism and calculated the nuclear relaxation rate $1/T_1$. Since there is still much debate as to the type of pairing in these systems, we feel a direct comparison between what is expected for different types of pairing and experimental data is the most important goal. We see that around the critical temperature, the s-wave model agrees well with the experiment, as it does in the specific heat, but at lower temperatures there are discrepancies with the theory. The model does make clear that the magnetization suppresses strongly the superconducting signatures, which is certainly being observed experimentally, and a closer look at other electronic properties in conjunction with careful experiments is what is needed to determine the nature of the superconducting pairing symmetry.

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**FIG. 4:** Comparison of theoretical and experimental data. The temperature axis is scaled by the SC transition temperature. The experimental data was taken at 13kbar: at this pressure there is an observed superconducting transition at $T \sim 0.5K$.
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