Single-Station TDOA Passive Location Under Multipath Propagation Conditions

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Abstract. Aiming at the problem of limited information obtained by single-station location, a single-station time difference of arrival (TDOA) location algorithm based on scatterer-assisted single-station location under the condition of multipath propagation is proposed in this paper. The location of scatterers are known, and Chan algorithm of direct wave is applied to geolocation in non-direct wave environment. The simulation results show that the algorithm can achieve the goal of location with high accuracy and low complexity. It has a certain reference value for the engineering realization of target location in complex signal propagation environment.

1. Introduction

Passive location technology [1] is a set of technical means by which the observing station can estimate the space coordinate position of the target by acquiring the location parameters of the target signal under test. According to the number of observation stations, passive positioning technology can be divided into single station positioning technology [2] and multi-station positioning technology [3]. Single station positioning technology has many advantages, such as small equipment, low cost, flexibility and good maneuverability. It has attracted more and more attention, and has become one of the hot issues in current research. However, because the amount of information obtained by a single Observatory is less than that obtained by multiple observatories, it is difficult to realize the single observer positioning technology. Generally speaking, a single passive static observing platform cannot carry out the radiation source. In complex propagation environment, non-line-of-sight (NLOS) propagation [4] between target and Observatory is common, and it is also one of the most important factors affecting positioning accuracy. In this paper, based on scatterer-assisted, the target signal is reflected and transmitted to the observatory through the scatterer to form multipath propagation. In this paper, multi-path transmission of information is used to achieve the target location.

The location method based on scatterer information was put forward later. For the first time, the scatterer information was used in the target location method in document [5]. The location of the target was estimated by using various positioning parameters such as angles of arrival (AOA), time of arrival (TOA) and Doppler frequency shift obtained from the observation station. The basic principle of specular reflection was proposed in document [6], which was measured by AOA. Literature [7] proposes a new single-station multipath passive location technology, which extracts the original location information from both the direct-path signal from the emitter and the multipath signal scattered by the emitter in the environment, and then realizes the estimation of the target's motion state.
However, the location method is based on the existence of the direct-path signal. Therefore, it is impossible to locate in the environment without direct path signals. Document [8] adds clustering technology based on TOA information of scatterers, and uses three base stations to locate targets.

In this paper, the single-station positioning technology in NLOS environment is mainly studied, and the scatterer information is used to realize the single-station positioning of the target. The organization of the remaining papers is as follows: Section 2 introduces the problem formulation; Section 3 describes the location algorithm and discusses the situation; Section 4 simulates the above algorithm; Section 5 is a summary.

2. Problem Formulation

Suppose there is an observation station located in \( \mathbf{q} = (x_0, y_0, z_0) \) in three-dimensional space. \( N_s \) scatterers with known positions are involved in the location of the target. \( \mathbf{s}_i = (x_i, y_i, z_i) (i = 1, 2, \ldots, N_s) \) represents the position coordinate vector of the \( i \)th scatterer, and \( \mathbf{p} = (x, y, z) \) represents the position coordinate vector of the target. When the target signal is transmitted to the observatory through direct and non-direct paths, the number of multi-paths of the signal to the observatory is \( N_s + 1 \). When the target signal is transmitted to the observatory only through non-direct path, the number of paths is \( N_s \). The corresponding location scenario schematic diagram is shown in Figure 1, in which the solid line represents the direct path and the dotted line represents the non-direct path.

![Figure 1 Sketch of 3-D localization scene with known scatterers location](#)

2.1. Direct and Non-direct Path Propagation Conditions

Where \( i = 0 \) represents the parameters of the direct path. The distance between the target and the observatory is expressed as:

\[
    r_0 = \| \mathbf{p} - \mathbf{q} \| \tag{1}
\]

The distance between the scatterer and the target emitter is expressed as:

\[
    r_i = \| \mathbf{s}_i - \mathbf{p} \| \tag{2}
\]

The distance between the scatterer and the observatory is expressed as:

\[
    h_i = \| \mathbf{s}_i - \mathbf{q} \| \tag{3}
\]
The time delays of the direct path and the \( i \) th non-direct path are expressed as follows:

\[
\tau_0 = \frac{r_0}{c},
\]
\[
\tau_i = \frac{h_i + r_i}{c},
\]
\[\text{(4)}\]

Assuming that the signal and noise are independent of each other and are all zero-mean Gaussian stationary random sequences, and the direct path is taken as the reference path, the TDOA value between the \( i \) th non-direct path and the direct path is:

\[
\tau_{i,0} = \tau_i - \tau_0
\]
\[\text{(5)}\]

If the propagation speed of the signal is \( c \), then,

\[
r_{i,0} = r_i - r_0 = c\tau_{i,0} - h_i
\]
\[\text{(6)}\]

In addition,

\[
r_i^2 = (x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2
\]
\[
= K_i - 2x_ix - 2y_iy - 2z_iz + x^2 + y^2 + z^2
\]
\[\text{(7)}\]

2.2. Non-direct path propagation

Compared with direct path and non-direct path, the information available is less than one direct path. If the first non-direct path is taken as the reference path, the TDOA value between the third non-direct path and the first non-direct path is:

\[
\tau_{i,1} = \tau_i - \tau_1
\]
\[\text{(8)}\]

Then,

\[
r_{i,1} = r_i - r_1 = c\tau_{i,1} + h_i - h_1
\]
\[\text{(9)}\]

In addition,

\[
r_i^2 = (x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2
\]
\[
= K_i - 2x_ix - 2y_iy - 2z_iz + x^2 + y^2 + z^2
\]
\[\text{(10)}\]

Among them, \( K_i = x_i^2 + y_i^2 + z_i^2 \). Formula (6) and formula (9) are a set of non-linear equations about the position of the target. The estimation of the position of the target can be obtained by solving them with appropriate algorithms.

3. Positioning Algorithm

Chan algorithm [9] is a non-recursive hyperbolic equation solution with analytic solution. The feature of the algorithm is that the computation is small and the positioning accuracy is high under the condition that the noise obeys the Gauss distribution. The algorithm uses twice weighted least squares method to get the final target position estimation. Preliminary position estimation can be obtained from the results of one calculation, and the results of one estimation can be improved by the constraints of additional variables. This algorithm is actually an approximation of the maximum likelihood estimation method, so it is also called the approximate maximum likelihood estimation (AMLE) method.
In this paper, Chan algorithm is applied to multipath propagation environment. Scattering body position is regarded as base station position, and it is substituted for the algorithm to estimate target position. For the case of direct wave and non-direct path propagation, Chan algorithm can be divided into two cases: the number of scatterer is two and more than two because of the existence of direct path information. In the case of non-direct path propagation, Chan algorithm can be divided into two cases: the number of scatterer is three and more than three. In this paper, the non-direct path propagation is taken as an example to derive the algorithm.

3.1. Three scatterers
In three-dimensional space, there are three scatterers, the target height is known, and the target is located on the ground or in the air. Then,

\[ \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x_{2,1} & 2y_{2,1} \\ 2x_{3,1} & 2y_{3,1} \end{bmatrix}^{-1} \times \begin{bmatrix} -2r_{2,1} \\ -2r_{3,1} \end{bmatrix} + \begin{bmatrix} -r_{2,1}^2 - 2z_{2,1}z + K_2 - K_1 \\ -r_{3,1}^2 - 2z_{3,1}z + K_3 - K_1 \end{bmatrix}, \]  

(11)

By substituting formula (11) for formula (10) and making \( R_1 \) is a second-order equation about \( \psi \). The positive root of equation (10) is substituted for formula (11), and the position estimation of the target is obtained. In some cases, two positive roots may be obtained, and the base station can be limited to a certain area according to prior knowledge to obtain the unique estimation of the target location.

3.2. Four or more scatterers
The results of one estimate are as follows:

\[ z_a = \arg \min \{ (h - G_a z_a)^T \psi^{-1} (h - G_a z_a) \} = (G_a^T \psi^{-1} G_a)^{-1} G_a^T \psi^{-1} h \]  

(12)

Among them, \( z_a = [x, y, r_1]^T, \psi = c^2 B Q B, B = \text{diag}\{r_2^0, r_3^0, ..., r_{N_s}^0\} \). \( Q \) represents the covariance matrix of the error vector of TDOA measurements. And,

\[ h = \frac{1}{2} \begin{bmatrix} r_{2,1}^2 - K_2 + K_1 + 2z_{2,1}z \\ r_{3,1}^2 - K_3 + K_1 + 2z_{3,1}z \\ \vdots \\ r_{N_s,1}^2 - K_{N_s} + K_1 + 2z_{N_s,1}z \end{bmatrix} \].

In one estimate, Chan method considers that the variables \( r_i \) are not related to \( x \) and \( y \). In fact, \( r_i \) is a function of \( x \) and \( y \), and the result of one estimate has a great error.

The results of the second estimation are as follows:

\[ z_a = (G_a^T B^{-1} G_a Q^{-1} G_a B^{-1} G_a)^{-1} (G_a^T B^{-1} G_a Q^{-1} G_a B^{-1} G_a^T B^{-1} G_a^T h) \]  

(13)

Among them, \( h = \begin{bmatrix} (z_{a,1} - x_i)^2 \\ (z_{a,2} - y_i)^2 \\ z_{a,4}^2 \end{bmatrix}, G_a = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, B = \text{diag}\{x^0 - x_i, y^0 - y_i, r_i^0\} \), The position coordinates of the target are:

\[ \begin{bmatrix} x \\ y \end{bmatrix} = \sqrt{z_a} + \begin{bmatrix} x_i \\ y_i \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} x \\ y \end{bmatrix} = -\sqrt{z_a} + \begin{bmatrix} x_i \\ y_i \end{bmatrix} \]  

(14)
4. Simulation
In order to verify the positioning effect of Chan algorithm under the condition of multipath propagation, simulation analysis is carried out on the platform of MATLAB. Assume that the position of the target is \((50, 30, 40)\) (km), the location of the observatory is \((-5, -10, 10)\) (km), and the number of scatterers is three and their positions are: \((-10, 0, 10)\) (km), \((10, 10, -10)\) (km), \((-10, 10, 0)\) (km). The plane schematic diagram of the target radiation source, observation station and scatterer is shown in Figure 2.

![Figure 2](image)

**Figure 2** The plane schematic diagram of the target radiation source, observation station and scatterers.

The time difference error of 100ns ~ 1000 ns, the position error of 50 m, 100 m and 200 m observation stations and scatterers are added to the measurement data. The result of positioning error under non-direct path propagation is shown in Figure 3. When the number of scatterers is two under the conditions of direct and non-direct propagation, the algorithm can obtain the target position estimation by solving the second-order equation. The error figure of positioning error results is shown in Figure 4.

![Figure 3](image)

**Figure 3** Location Error Curve under Non-direct Path Propagation.

![Figure 4](image)

**Figure 4** Location Error Curve under Direct and Non-Direct Path Propagation Conditions.
From Figure 3 and Figure 4, it can be seen that the Chan algorithm can be applied to single station TDOA location under multipath propagation conditions to achieve the goal of target location. Moreover, with the increase of time difference error, the positioning error becomes larger and larger. When the position errors of scatterers and observation stations are added to the measured data, the positioning errors become larger and larger.

5. Conclusion
In this paper, single-station passive location under multipath propagation is studied. When both the target and the observation station are stationary, single-station cannot locate. With the help of scatterer, more parameter information can be obtained. Chan algorithm is applied to multipath environment. The simulation results show that this method can locate the target. The algorithm has the characteristics of high positioning accuracy and low computational complexity, but in the non-direct path environment, the positioning accuracy of the algorithm has a downward trend.

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