TESTS OF CPT AND LORENTZ SYMMETRY IN PENNING-TRAP EXPERIMENTS

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INTRODUCTION

The CPT theorem \cite{1} states that local relativistic quantum field theories of point particles in flat spacetime must be invariant under the combined operations of charge conjugation C, parity reversal P, and time reversal T. As a result of this invariance, particles and antiparticles have equal masses, lifetimes, charge-to-mass ratios, and gyromagnetic ratios. The CPT theorem has been tested to great accuracy in a variety of experiments \cite{2}. The best bound is obtained in experiments with neutral mesons, where the figure of merit is

\[ r_K \equiv \frac{\left| (m_K - m_{\bar{K}}) / m_K \right|}{2 \times 10^{-18}}. \] (1)

Experiments in Penning traps have also yielded sharp bounds on CPT violation, including the best bounds on lepton and baryon systems. Two types of experimental tests are possible in Penning traps. Both involve making accurate measurements of cyclotron frequencies \( \omega_c \) and anomaly frequencies \( \omega_a \) of single isolated particles.
confined in the trap. The first compares the ratio \(2\omega_a/\omega_c\) for particles and antiparticles. In the context of conventional QED, this ratio equals \(g - 2\) for the particle or antiparticle. A second experiment compares values of \(\omega_c \sim q/m\), where \(q > 0\) is the magnitude of the charge and \(m\) is the mass, and is therefore a comparison of charge-to-mass ratios.

Experiments comparing \(g - 2\) for electrons and positrons yield the figure of merit\(^3\) \(^{,}\) \(^{,}\) \(^{,}\)

\[
r_g \equiv |(g_e^- - g_e^+)/g_{\text{avg}}| \lesssim 2 \times 10^{-12}, \tag{2}
\]

while the charge-to-mass-ratio experiments yield the bound \(^3\)

\[
r_{q/m}^e \equiv |[(q_e^-/m_e^-) - (q_e^+/m_e^+)]/(q/m)_{\text{avg}}| \lesssim 1.3 \times 10^{-7}. \tag{3}
\]

To date, no experiments measuring \(g - 2\) for protons or antiprotons have been performed in Penning traps because of the difficulty in obtaining sufficient cooling and an adequate signal for detection of the weaker magnetic moments. However, proposals have been put forward that might make these types of experiments feasible in the future\(^3\). The best current tests of CPT in proton and antiproton systems come from comparisons of the charge-to-mass ratios\(^3\), which yield the bound

\[
r_{q/m}^p \equiv |[(q_p/m_p) - (q_{\overline{p}}/m_{\overline{p}})]/(q/m)_{\text{avg}}| \lesssim 1.5 \times 10^{-9}. \tag{4}
\]

It is interesting to note that in the neutral meson experiments which yield the bound on \(r_K\) in (1), measurements are made with an experimental uncertainty of approximately one part in \(10^4\). In contrast, measurements of frequencies in Penning traps have experimental uncertainties of about one part in \(10^9\). This raises some intriguing questions about the Penning-trap experiments as to why they do not provide better tests of CPT when they have better experimental precision. In the context of conventional QED, which does not permit CPT breaking, it is not possible to pursue these types of questions. Instead, one would need to work in the context of a theoretical framework that allows CPT breaking, making possible an investigation of possible experimental signatures. Only recently has such a framework been developed\(^3\).

In this paper, we describe the application of this theoretical framework to experiments on electron-positron and proton-antiproton systems in Penning traps. Our results have been published in Refs.\(^{9,10}\).

**THEORETICAL FRAMEWORK**

The framework we use\(^8\) is an extension of the SU(3)\(\times\)SU(2)\(\times\)U(1) standard model originating from the idea of spontaneous CPT and Lorentz breaking in a
more fundamental model such as string theory \[11, 12\]. This framework preserves various desirable features of quantum field theory such as gauge invariance and power-counting renormalizability. It has two sectors, one that breaks CPT and one that preserves CPT, while both break Lorentz symmetry. The possible CPT and Lorentz violations are parametrized by quantities that can be bounded by experiments. Within this framework, the modified Dirac equation describing a fermion with charge $q$ and mass $m$ is given by

$$\left( i\gamma^\mu D_\mu - m - a_\mu \gamma^\mu - b_\mu \gamma^5 \gamma^\mu - \frac{i}{2} H_{\mu\nu} \sigma^{\mu\nu} + ic_{\mu\nu} \gamma^\mu D^\nu + id_{\mu\nu} \gamma^5 \gamma^\mu D^\nu \right) \psi = 0 .$$

(5)

Here, $\psi$ is a four-component spinor, $iD_\mu \equiv i\partial_\mu - qA_\mu$, $A^\mu$ is the electromagnetic potential in the trap, and $a_\mu, b_\mu, H_{\mu\nu}, c_{\mu\nu}, d_{\mu\nu}$ are the parameters describing possible violations of CPT and Lorentz symmetry. The transformation properties of $\psi$ imply that the terms involving $a_\mu, b_\mu$ break CPT while those involving $H_{\mu\nu}, c_{\mu\nu}, d_{\mu\nu}$ preserve it, and that Lorentz symmetry is broken by all five terms. Since no CPT or Lorentz breaking has been observed in experiments to date, the quantities $a_\mu, b_\mu, H_{\mu\nu}, c_{\mu\nu}, d_{\mu\nu}$ must all be small.

**PENNING-TRAP EXPERIMENTS**

We use this theoretical framework to analyze tests of CPT and Lorentz symmetry in Penning-trap experiments. To begin, we note that the time-derivative couplings in (5) alter the standard procedure for obtaining a hermitian quantum-mechanical hamiltonian operator. To overcome this, we first perform a field redefinition at the lagrangian level that eliminates the additional time derivatives. We also use charge conjugation to obtain a Dirac equation and hamiltonian for the antiparticle.

To test CPT, experiments compare the cyclotron and anomaly frequencies of particles and antiparticles. According to the CPT theorem, particles and antiparticles of opposite spin in a Penning trap with the same magnetic fields but opposite electric fields should have equal energies. The experimental relations $g - 2 = 2\omega_a/\omega_c$ and $\omega_c = qB/m$ provide connections to the quantities $g$ and $q/m$ used in defining the figures of merit $r_g$, $r_e^{q/m}$, and $r_p^{q/m}$. We perform calculations using Eq. (5) to obtain possible shifts in the energy levels due to either CPT-breaking or CPT-preserving Lorentz violation. In this way, we examine the effectiveness of Penning-trap experiments as tests of both CPT-breaking and CPT-preserving Lorentz violation. From the computed energy shifts we determine how the frequencies $\omega_c$ and $\omega_a$ are affected and if the conventional figures of merit are appropriate.

For experiments in Penning traps, the dominant contributions to the energy come from interactions of the particle or antiparticle with the constant magnetic
field of the trap. The quadrupole electric fields generate smaller effects. In a perturbative calculation, the dominant CPT- and Lorentz-violating effects can therefore be obtained by working with relativistic Landau levels as unperturbed states. Conventional perturbations, such as the anomaly, will lead to corrections that are the same for particles and antiparticles. CPT- and Lorentz-breaking effects will result in either differences between particles and antiparticles or in unconventional effects such as diurnal variations in the measured frequencies.

RESULTS

Our calculations for electrons and positrons in Penning traps [9] show that the leading-order effects due to CPT and Lorentz breaking cause corrections to the cyclotron and anomaly frequencies:

\[
\omega_c^- \approx \omega_c^+ \approx (1 - e_{00}^e - e_{11}^e - e_{22}^e)\omega_c ,
\]

\[
\omega_a^\pm \approx \omega_a \mp 2b_3^e + 2d_{30}^e m_e + 2H_{12}^e .
\]

Here, \(\omega_c\) and \(\omega_a\) represent the unperturbed frequencies, while \(\omega_c^\pm\) and \(\omega_a^\pm\) denote the frequencies including the corrections. Superscripts have also been added on the coefficients \(b_{\mu}\), etc., to denote that these are parameters of the electron-positron system. From these relations we find the electron-positron differences for the cyclotron and anomaly frequencies to be

\[
\Delta \omega_c^e \equiv \omega_c^- - \omega_c^+ \approx 0 , \quad \Delta \omega_a^e \equiv \omega_a^- - \omega_a^+ \approx -4b_3^e .
\]

Evidently, in the context of this framework comparisons of cyclotron frequencies to leading order do not provide a signal for CPT or Lorentz breaking, since the corrections to \(\omega_c\) for electrons and positrons are equal. On the other hand, comparisons of \(\omega_a\) provide unambiguous tests of CPT since only the CPT-violating term with \(b_3\) results in a nonzero value for the difference \(\Delta \omega_a^e\).

We have also found that to leading order there are no corrections to the \(g\) factors for either electrons or positrons. This leads to some interesting and unexpected results concerning the figure of merit \(r_g\) in Eq. (2). With \(g_{c^-} \approx g_{c^+}\) to leading order, we find that \(r_g\) vanishes, which would seem to indicate the absence of CPT violation. However, this cannot be true since the model contains explicit CPT violation. Furthermore, our calculations show that with \(\tilde{b} \neq 0\) the experimental ratio \(2\omega_a/\omega_c\) is field dependent and is undefined in the limit of vanishing magnetic field. Thus, the usual relation \(g - 2 = 2\omega_a/\omega_c\) does not hold in the presence of CPT breaking. For these reasons, the figure of merit \(r_g\) in Eq. (2) is misleading, and an alternative is suggested. Since the CPT theorem predicts that states of opposite spin
in the same magnetic field have equal energies, we propose as a model-independent figure of merit,

\[ r_{\omega_a}^e \equiv \frac{|E_{n,s}^e - E_{n,-s}^e|}{E_{n,s}^e} , \tag{9} \]

where \( E_{n,s}^e \) are the energies of the relativistic states labeled by their Landau-level numbers \( n \) and spin \( s \). Our calculations show \( r_{\omega_a}^e \approx |\Delta \omega_a^e|/2m_e \approx |2b_3^e|/m_e \), and we estimate as a bound on this figure of merit,

\[ r_{\omega_a}^e \lesssim 10^{-20} . \tag{10} \]

In Ref. [10], we describe additional possible signatures of CPT and Lorentz breaking. These include possible diurnal variations in the anomaly and cyclotron frequencies. Tests for these effects would provide bounds on various components of the parameters \( c_{\mu\nu}^e, d_{\mu\nu}^e, \) and \( H_{\mu\nu}^e \) at a level of about one part in \( 10^{18} \).

A similar analysis can also be performed on proton-antiproton experiments in Penning traps. In this context, it suffices to work at the level of an effective theory in which the protons and antiprotons are regarded as basic objects described by a Dirac equation. The coefficients \( a_{\mu}^p, b_{\mu}^p, H_{\mu\nu}^p, c_{\mu\nu}^p, d_{\mu\nu}^p \) represent effective parameters, which at a more fundamental level depend on the underlying quark interactions. Comparisons of protons and antiprotons in the context of this model yield the results for the proton-antiproton frequency differences,

\[ \Delta \omega_c^p \equiv \omega_c^p - \omega_c^\bar{p} = 0 , \quad \Delta \omega_a^p \equiv \omega_a^p - \omega_a^\bar{p} = 4b_3^p . \tag{11} \]

Assuming an experiment could be made sensitive enough to measure \( \omega_a^p \) and \( \omega_a^\bar{p} \) with a precision similar to that of electron \( g - 2 \) experiments, then the appropriate figure of merit would be

\[ r_{\omega_a}^p \equiv \frac{|E_{n,s}^p - E_{n,-s}^\bar{p}|}{E_{n,s}^p} . \tag{12} \]

A bound on this can be estimated as

\[ r_{\omega_a}^p \lesssim 10^{-23} . \tag{13} \]

It is apparent that an experiment comparing anomaly frequencies of protons and antiprotons in a Penning trap has the potential to provide a particularly tight CPT bound. Other signatures of CPT and Lorentz breaking involving diurnal variations in \( \omega_a \) and \( \omega_c \) are described in Ref. [10]. These additional signatures provide bounds on various components of \( c_{\mu\nu}^p, d_{\mu\nu}^p, \) and \( H_{\mu\nu}^p \) estimated at about one part in \( 10^{21} \).

CONCLUSIONS

We find that the use of a general theoretical framework incorporating CPT and Lorentz breaking permits a detailed investigation of possible experimental signatures
in Penning-trap experiments. Our results indicate that the sharpest tests of CPT symmetry emerge from comparisons of anomaly frequencies in \(g-2\) experiments. Our estimates of appropriate figures of merit provide bounds of approximately \(10^{-20}\) in electron-positron experiments and of \(10^{-23}\) for a plausible proton-antiproton experiment. Other signals involving possible diurnal variations provide additional bounds at the level of \(10^{-18}\) in the electron-positron system and \(10^{-21}\) in the proton-antiproton system. A table showing all our estimated bounds is presented in Ref. [11].

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