Signals of confinement in Green functions of SU(2) Yang-Mills theory

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The vortex picture of confinement is employed to explore the signals of confinement in Yang-Mills Green functions. By using SU(2) lattice gauge theory, it has been well established that the removal of the center vortices from the lattice configurations results in the loss of confinement. The running coupling constant, the gluon and the ghost form factors are studied in Landau gauge for both cases, the full and the vortex removed theory. In the latter case, a strong suppression of the running coupling constant and the gluon form factor at low momenta is observed. At the same time, the singularity of the ghost form factor at vanishing momentum disappears. This observation establishes an intimate correlation between the ghost singularity and confinement. The result also shows that a removal of the vortices generates a theory for which Zwanziger’s horizon condition for confinement is no longer satisfied.

Since Quantum Chromo Dynamics (QCD) has been recognized as the theory of strong interactions, a major challenge has been to explain confinement of quarks from first principles.

More than twenty years ago, Mandelstam and ’t Hooft conjectured that topological obstructions of the gauge field, such as monopoles and vortices, might play a key role for understanding confinement. By means of lattice simulations evidence has been accumulated that center vortices are responsible for confinement (see for a recent review): The quark antiquark potential calculated with vortex configurations reproduces the linear rising potential. On the other hand, a removal of the vortices from the lattice configurations “by hand” results in the loss of confinement. Secondly, it was observed that the properties of the confining vortices nicely extrapolate to the continuum limit for SU(2) and SU(3). Furthermore, in the vortex picture the finite temperature deconfinement phase transition appears in SU(2) as a vortex depercolation transition. Indeed, vortex projected ensembles nicely reflect the correct universality class of the transition in SU(2). In addition, models based upon the center vortex picture nicely reproduce the order of the deconfinement phase transition for both SU(2) and SU(3).

A different confinement picture was proposed by Gribov and further elaborated by Zwanziger. This picture makes use of QCD Green functions in Landau gauge, the IR properties of which presumably encode the information on confinement. It is argued that the gauge field configurations which are relevant in the thermodynamic limit are concentrated on the Gribov horizon. In this case, the ghost form factor in Landau gauge would diverge at zero momentum transfer showing that the above “horizon condition” is satisfied. Suman and Schilling obtained first indications using lattice simulations, that the ghost propagator in Landau gauged SU(2) theory is indeed more singular in the infrared than the free ghost propagator.

Both pictures of confinement, the center vortex picture and the Gribov-Zwanziger picture, are compatible given the fact that center vortex configurations lie on the Gribov horizon. The argument presented in can be extended to Landau gauge.

In this letter, we explicitly establish a relation between both confinement pictures. We will show that the removing of the center vortices eliminates the confinement signals from the Green functions: the singularity of the ghost form factor disappears. As a byproduct, we will also show that the running coupling constant is largely suppressed in the IR regime when the center vortices are removed. These findings are in accordance with the results in where a suppression of the gluon form factor was found upon vortex removal. These results underline the importance of the vortices as IR effective degrees of freedom. Preliminary results obtained by one of us have been presented in.

The investigation of the IR properties of Yang-Mills Green functions necessarily involves non-perturbative techniques. Besides lattice simulations, Dyson-Schwinger equations (DSE) variational techniques and flow equations have been applied. In particular, Dyson-Schwinger equations have attracted much interest over the last decade due to their usefulness for the description of the physics of hadrons. Let us focus onto the results from the DSEs in Landau gauge. Let \( F_R(p^2, \mu^2) \) and \( J_R(p^2, \mu^2) \) denote the form factors (for a renormalization point \( \mu \)) of the gluon and the ghost propagator, respectively. These form factors have been recently obtained from the coupled set of continuum DSEs in Landau gauge by several groups. In, it was firstly pointed out that the gluon and ghost form factors might satisfy simple scaling laws in the IR momentum range \( p \ll 1 \text{ GeV} \)

\[
F_R(p^2, \mu^2) \propto [p^2]^{\alpha}, \quad J_R(p^2, \mu^2) \propto [p^2]^{\beta} \tag{1}
\]
where the following remarkable sum rule holds for the IR exponents $\alpha$ and $\beta$:

$$\alpha + 2\beta = 0 .$$

(2)

It turns out that this result is rather independent of the truncation scheme under consideration although the precise values for $\alpha$ and $\beta$ strongly depends on the truncation scheme (see, e.g., [31, 37]). Disregarding the Gribov ambiguity, Taylor has shown that the ghost-ghost-gluon-vertex renormalization constant is finite in Landau gauge [38] at least to all orders perturbation theory. If this comes true at the genuine non-perturbative level, the running coupling can be defined from the form factors by

$$\alpha_R(p^2) = \alpha_R(\mu^2) F_R(p^2, \mu^2) J_R^2(p^2, \mu^2) .$$

(3)

The IR sum rule (2) also implies that the running coupling develops a fixed point in the IR limit

$$\lim_{p \to 0} \alpha_R(p^2) = \alpha_c = \text{constant} .$$

(4)

Note that this result is independent of the values of $\alpha$, $\beta$ as long as the IR sum rule (2) is satisfied.

Many efforts were devoted to determine the gluon and ghost form factors by lattice simulations [13], [39–43]. A recent combined study of the gluon/ghost formfactor and the running coupling constant was presented in [44]. There, it was found for the first time using lattice simulations that the ghost-ghost-gluon-vertex renormalization constant is indeed finite. This paves the path to the definition of the running coupling constant [44] via 2-point Green functions only. Moreover, the lattice data confirm the sum rule (2) and are consistent with an IR fixed point value of $\alpha_c = 5(1)$ for two colors. However, further lattice studies are desirable to determine the IR scaling exponents $\alpha$ and $\beta$ in a reliable way.

Here, we will work out the signature of confinement in the ghost form factor and the running coupling constant, and will confirm our earlier findings on the IR suppression of the gluon form factor when vortices are removed [15]. The present lattice simulations were carried out on a $16^3 \times 32$ lattice using Wilson action with $\beta$ values ranging from 2.15 to 2.5. Physical units are obtained by eliminating the lattice spacing $a$ using the measured values for $\sigma a^2$. A string tension $\sigma = 440 \text{MeV}$ was used as reference scale. The simulation parameters are listed in table I.

| $\beta$ | 2.15 | 2.2 | 2.3 | 2.375 | 2.45 | 2.525 |
|---------|------|-----|-----|-------|------|------|
| $\sigma a^2$ | 0.28(1) | 0.220(9) | 0.136(2) | 0.083(2) | 0.0507(8) | 0.0307(5) |

TABLE I: Lattice spacing $a$ in units of the string tension $\sigma$ for the values $\beta$ used in the present simulation.

In a first step, we identify the $\mathbb{Z}_2$ vortex configurations for a particular SU(2) lattice configuration by the method of center projection after maximal center gauge fixing. This procedure, described in detail in reference [3], generates a vortex structure which nicely extrapolates to the continuum limit. Once the vortices are identified, we used the Forcrand D’Elia method [45] to remove the vortices from the lattice configurations. The quark antiquark potential obtained in this way is shown in figure II in comparison with the full potential. Obviously, configurations from which the vortices were removed lack the capability to confine quarks.
Subsequently, Landau gauge is implemented for both theories, the full SU(2) Yang-Mills theory and its vortex removed counterpart, and the gluon and ghost form factors are extracted from both lattice ensembles. Landau gauge fixing and the calculation of the form factors are described in detail in [44]. At high momenta, the momentum dependence is well described by

\[ F_R(p^2, \mu^2) \approx d_2(\mu) \left( \frac{p^2}{\Lambda_{2}\text{--loop}^2} \right)^\gamma \]

where \( \mu \) is the renormalization point, \( \gamma \) is the leading-order anomalous dimension of the gluon (respectively ghost) propagator, given by \( \gamma = 13/22 \) (respectively \( \gamma = 9/44 \)). The running coupling constant at 2-loop level is independent of the renormalization scheme and for SU(2) given by

\[ \alpha_{2\text{--loop}}(x = \frac{p^2}{\Lambda_{2}\text{--loop}^2}) = \frac{6\pi}{11 \ln x} \left( 1 - \frac{102 \ln(\ln x)}{121 \ln x} \right) \].

It turns out that the choice \( \Lambda_{2}\text{--loop} \approx 950 \text{ MeV} \) well describes the lattice data (see [44] for details).

Here, we confirm our earlier findings [15] for the gluon form factor (see figure 2): the gluon form factor of the vortex removed theory is suppressed in the intermediate momentum range, while the UV regime is unchanged to a large extent.

The central results of the present paper are the ghost form factor and the running coupling constant of the vortex removed theory. Our numerical results are summarized in figures 3 and 4. While the ghost form factor of full SU(2) gauge theory diverges in the IR limit (see e.g. [13, 44]), our results suggest that it approaches a constant only slightly above 1 in the IR limit for the non-confining theory. Since the IR divergence is related to the proximity of the gauge configurations to the first Gribov horizon, our findings strongly support the Gribov–Zwanziger confinement picture. We here point out that Kugo and Ojima proposed a confinement criterion based upon the framework of the BRST quantization: this criterion signals that the physical subspace only consists of color singlet states [47]. In Landau gauge, this criterion is fulfilled if the ghost form factor is singular at zero momentum [48]. It therefore coincides with Zwanziger’s horizon condition. We stress, however, that for the derivation of the Kugo-Ojima criterion, one assumes that a BRST charge operator is uniquely defined for the whole configuration space within the first Gribov horizon. At the present stage of investigations, this assumption is unjustified due to the presence of Gribov ambiguities [49].

Finally, let us focus onto the imprint of the confining vortices on the running coupling strength \( \alpha_R(p) \). Figure 4 shows the running coupling for both, the full SU(2) theory and the one with the center vortices removed. Since the ghost formfactor approaches a constant in the vortex removed case and the gluon form factor is suppressed in the IR regime compared with the free one, it does not come as a surprise that the running coupling strength vanishes in the IR limit.

In conclusion, SU(2) Yang-Mill theory looses its capability to confine quarks when the confining vortices were removed by the Forcrand D’Elia procedure [45]. At the same time, the divergence of the ghost formfactor
at vanishing momentum disappears. Our findings therefore establish a connection between the vortex picture of confinement and the Gribov-Zwanziger confinement criterion. We furthermore find that the strength of the running coupling constant is drastically reduced in the intermediate momentum region. This also indicates a tight relation between the vortex picture and the spontaneous chiral symmetry breaking. The latter to happen, the integrated strength at intermediate momenta must exceed a critical value.

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