Piecewise Linear map enabled Harris Hawk optimization algorithm

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Abstract. Chaotic maps were usually introduced to improve the original swarm-based nature-inspired algorithms. Due to their chaotic characteristics, the chaotic maps were introduced to replace the pseudo random numbers in computer engineering and consequently better performance would be achieved. In this paper, we introduce another chaotic improvement to the Harris hawk optimization (HHO) algorithm with Piecewise Linear map. Nevertheless, the chaos would be introduced to improve the randomness of the controlling parameter which was used to balance the ratio of exploration and exploitation. Monte Carlo simulation experiments were carried out and results confirmed this kind of improvements would significantly raise the capability in optimization.

1. Introduction

The nature-inspired algorithms have been a hotspot in computer engineering. Scientists and engineers all over the world are proposing new kind of algorithms, improvements, or applications. Due to the embedded characteristics of nature inspiration, almost all of the nature-inspired algorithms would embrace the randomness in mathematical mechanisms. And consequently the defect of pseudo random numbers in computer engineering might decrease their capability. Therefore, all of the other methods to replace the randomness would be tried to improve the capability.

Literally speaking, there are two kinds of improvements to replace the random numbers involved in algorithms. First of all, considering the pseudo random numbers were distributed in Gauss distribution, every value would have equal probability to be chosen, which means that the overall steps after several rounds of iterations might be the same. If all the individuals in the group are exploring and developing towards the target at an overall equal pace, some of them might be trapped in local optima, some of them might keep being away from the global optimum. Therefore, Levy flights might be a better choice because individuals with Levy flights might carry on several small steps with a big one. And consequently, the Levy flight was almost introduced to all of the nature-inspired algorithms, for instance, the Levy flight bat algorithm[1], Levy flight slime mould (SM) algorithm[2], the Levy flight grey wolf optimization (GWO) algorithm[3]. Better performance of the Levy flight improvement reasonably was achieved and verified literally. The second way to conquer the defects of pseudo randomness is the replacement of randomness with better methods. The famous chaos and chaotic maps were involved to do so.

In this paper, we proposed a new kind of improvements other than replacing the randomness by inspiration of the GWO algorithm[4]. Piecewise Linear map would be introduced to improve the Harris
hawk optimization (HHO) algorithm\cite{5} for better performance and a controlled parameter that balance the exploration and exploitation ratio would be improved with chaos.

2. The HHO algorithm

Inspired by the hunting behavior of Harris Hawks, the HHO algorithm opened the multiple updating discipline principle\cite{6} for individuals for the first time. The positions of individuals would be guided with various ways in exploration and exploitation.

2.1. The controlled parameter balancing exploration and exploitation

In the HHO algorithm, there was a controlled parameter called energy $E$ which was used to balance the exploration and exploitation. This parameter was defined declined linearly from its maximum to the minimum:

$$ E = 2E_0 \left(1 - \frac{t}{maxIter}\right) \tag{1} $$

Where $t$ represents the current iteration number and $maxIter$ represents the maximum iteration number allowed in this given problem.

According to the definition of the HHO algorithm, if energy is larger than 1.0, the rabbits have so much energy and the Harris hawks could only pursue them in a hurry, that is to say, in such circumstances, the Harris hawks would perform exploration for the prey. On the contrary, if energy is smaller than 1.0, the rabbits might be exhausted and now the Harris hawks would perform smart actions to frighten the rabbits to make more exhausted, even dead or caught at the end.

2.2. Exploration

During the exploration of individuals, their positions would be guided according to their current positions $x_i(t)$ in the current iteration $t$, and random selected candidates $x_r(t)$, furthermore, half of them would be reinitialized around the global best candidate $x_b(t)$ and the averaged candidate $x_m(t)$:

$$ x_i(t + 1) = \begin{cases} 
  x_r(t) - r_1|x_r(t) - 2r_2x_i(t)| & q \geq 0.5 \\
  x_b(t) - x_m(t) - r_3[lb + r_4(ub - lb)] & q < 0.5 
\end{cases} \tag{2} $$

Where $q$ and $r_1, r_2, r_3, r_4$ are all random numbers in Gauss distribution in interval of 0 and 1. The definitional domain of the given problem is $[lb, ub]$.

That is to say, the individuals in swarms are equally split into two groups, half of them would update their positions according to their current positions and a random weighted distance between them and random selected candidates. While half of them could update their positions according to the averaged candidates and random initializations.

2.3. Exploitation

When the rabbits got exhausted, the Harris hawk would perform some smart actions to make them more exhausted, dead, or caught at the end. Another random number $r$ was introduced:

$$ x_i(t + 1) = \begin{cases} 
  x_b(t) - x_i(t) - E|J \cdot x_g(t) - x_i(t)| & r \geq 0.5, |E| \geq 0.5 \\
  x_g(t) - E|x_g(t) - x_i(t)| & r \geq 0.5, |E| < 0.5 \\
  f_1(x) & r < 0.5, |E| \geq 0.5 \\
  f_2(x) & r < 0.5, |E| < 0.5 
\end{cases} \tag{3} $$

Where,

$$ f_1(x) = \begin{cases} 
  Y = x_b(t) - E|J \cdot x_b(t) - x_i(t)| & f(Y) < f(x_i) \\
  Z = Y + r_5 \times LF(D) & f(Z) < f(x_i) 
\end{cases} \tag{4} $$

$$ f_2(x) = \begin{cases} 
  Y = x_b(t) - E|J \cdot x_b(t) - x_m(t)| & f(Y) < f(x_i) \\
  Z = Y + r_5 \times LF(D) & f(Z) < f(x_i) 
\end{cases} \tag{5} $$

And, $LF(x)$ is the Levy flight function defined with a given parameter $\beta = 1.5$ in the following equation:
\[ LF(x) = 0.01 \times \frac{\mu \times \sigma}{|v|^\beta}, \quad \sigma = \left( \frac{\Gamma(1 + \beta) \times \sin\left(\frac{\pi \beta}{2}\right)}{\Gamma\left(\frac{1 + \beta}{2}\right) \times \beta \times 2^{(\beta - 1)/2}} \right)^1/\beta \]  

(6)

3. The Piecewise Linear map enabled HHO algorithm

There might be several classical chaotic maps introduced to improve the capability of optimizations. Piecewise Linear map might be also popular in improvements:

\[ x(n + 1) = \begin{cases} 
\frac{x_n}{1 - \lambda} & \text{if } 0 < x_n \leq 1 - \lambda \\
\frac{x_n - (1 - \lambda)}{\lambda} & \text{if } (d \equiv 1 - \lambda) < x_n < 1 
\end{cases} \]  

(7)

Chaotic improvements to the algorithms were usually introduced to replace the pseudo random numbers in the initialization[7], or in the iterations[8]. The most popular way might be carrying on extra rounds of iterations with chaos around the current global best candidate[9]. However, there might be another way for the chaos when we focused on the controlled parameter balancing the exploration and exploitation ratio. The controlled parameter declined linearly and it was also multiplied with pseudo random numbers during iterations, which meant that they might also be introduced to improve. Literal research[4] had already proved that if chaotic maps were introduced, the chaotic enabled algorithm might significantly increase the capability. In this paper, we proposed a Piecewise Linear map enabled HHO algorithm, whose controlled parameter would be a combination of the origin and the chaos formulated as follows:

\[ Ec = E + \omega_{\text{max}} - (\omega_{\text{max}} - \omega_{\text{min}}) \frac{t}{\text{maxIter}} \cdot x_n \]  

(8)

Where, \( \omega_{\text{max}}, \omega_{\text{min}} \) represent the maximum and minimum values. The new chaotic map enabled controlled parameter is a combination of linear declination and chaos. The new controlled parameter would increase the fluctuation and keep more individuals remain in exploration even at the end of iterations, as shown in Figure 1.

![Figure 1](image)

4. Simulation experiments

In this section, we would carry on some simulation experiments to find whether the chaotic enabled HHO algorithm could perform better than the original in optimization or not.
4.1. Benchmark functions
For simplicity, only several classical benchmark functions were involved in simulation experiments, as shown in Table 1. There are three unimodal (F1-F3) and three multimodal (F4-F6), and all of them have their global optima at the Origin in Descartes coordination system.

Table 1  Benchmark functions

| No. | Formulations | Domain | $f_{min}$ |
|-----|--------------|--------|-----------|
| F1  | $f(x) = \sum_{i=1}^{d} x_i^2$ | [-100, 100] | 0         |
| F2  | $f(x) = \sum_{i=1}^{d} |x_i| + \prod_{i=1}^{d} |x_i|$ | [-10, 10] | 0         |
| F3  | $f(x) = \sum_{i=1}^{d} \left( \sum_{j=1}^{i} x_j \right)^2$ | [-100, 100] | 0         |
| F4  | $f(x) = \frac{d}{10} + \sum_{i=1}^{d} x_i^2 - \frac{1}{10} \sum_{i=1}^{d} \cos (5\pi x_i)$ | [-100, 100] | 0         |
| F5  | $f(x) = \sum_{i=1}^{d} \frac{x_i^2}{4000} - \cos \left( \frac{x_i}{\sqrt{i}} \right) + 1$ | [-100, 100] | 0         |
| F6  | $f(x) = 0.5 + \frac{\sin^2 \sqrt{x_i^2 + x_{i+1}^2} - 0.5}{\left[ 1 + 0.001(x_i^2 + x_{i+1}^2) \right]}$ | [-100, 100] | 0         |

4.2. Experimental setup
For simplicity, all of the following experiments would be carried out in a same way setup in the original version of the HHO algorithm. Regardless the difference of computers and numbers of equations, the population size of swarms and maximum iterations numbers are set to 30 and 500, and 100 Monte Carlo experiments would be run and compared in a same way as done in the original simulation experiments [5].

4.3. Qualitative analysis
Qualitative analysis showed the results in figures and the comparison could be directly shown. In the simulation experiments of the HHO algorithm, the authors introduced seven unimodal and six multimodal benchmark functions. For simplicity, we would carry on several classical experiments and comparison would be made between the chaotic map enabled improved HHO and the original version, as shown in Figure 2 to Figure 5.

We can find that the Piecewise Linear map enabled HHO algorithm would perform dramatically better than the origin HHO algorithm, either in optimization of unimodal or multimodal benchmark functions.

4.4. Statistical analysis
In this experiments, we would run the simulation separately for 30 times and the dimensionality of each problems was also defined in 30, comparison might be made to the original version of the HHO algorithm, results were shown in Table 2.

Table 2  Comparison results of benchmark functions, with 30 dimensions

| Benchmark | HHO Mean | HHO Std. | Chaotic HHO Mean | Chaotic HHO Std. |
|-----------|----------|----------|-----------------|------------------|
| F1        | 6.996E-13| 3.668E-12| 4.889E-13      | 2.633E-12        |
| F2        | 0.015    | 0.082    | 1.508E-10       | 8.101E-10        |
| Benchmark | HHO | Chaotic HHO |
|-----------|-----|-------------|
| F3        | Mean: 7.634E-6, Std.: 4.1002E-5 | Mean: 5.250E-26, Std.: 2.275E-25 |
| F4        | Mean: 1.790E-6, Std.: 9.639E-6 | Mean: 1.469E-8, Std.: 7.910E-8 |
| F5        | Mean: 1.447E-15, Std.: 5.703E-15 | Mean: 0, Std.: 0 |
| F6        | Mean: 9.122E-16, Std.: 4.913E-15 | Mean: 0, Std.: 0 |

Figure 2  3D Profile of Sphere function  
Figure 3  Best fitness values versus iterations  
Figure 4  3D Profile of Griewank function  
Figure 5  Best fitness values versus iterations

Results in Table 2 did verify and confirm the better performance of the chaotic map enabled HHO algorithm. Based on the qualitative and statistical analysis, we can conclude the improvement.

5. Discussions and conclusions
In this paper, we proposed a new kind of chaotic improvement for the HHO algorithm which was called the chaotic map enabled HHO algorithm with Piecewise map. Unlike other kinds of chaotic
improvements, the chaotic map enabled improvements would be used to involve additional chaos in the controlled parameter balancing the exploration and exploitation.

Qualitative and statistical analysis were introduced to verify and confirm the capability of the improvement. Much better results were obtained and the results were promising, and application in real engineering problems might be taken for further researches.

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