Can Quintessence Be The Rolling Tachyon?

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In light of the recent work by Sen and Gibbons, we present a phase-plane analysis on the cosmology containing a rolling tachyon field in a potential resulted from string theory. We show that there is no stable point on the phase-plane, which indicated that there is a coincidence problem if one consider tachyon as a candidate of quintessence. Furthermore, we also analyze the phase-plane of the cosmology containing a rolling tachyon field for an exactly solvable toy potential in which the critical point is stable. Therefore, it is possible for rolling tachyon to be quintessence if one give up the strict constraint on the potential or find a more appropriate effective potential for the tachyon from M/string theory.

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1. Introduction

Recently, a field theory that describes the tachyon on a brane-antibrane system near the minimum of its potential has been proposed by Sen. Later on, Gibbons considered the effective action of the tachyon coupling to gravitational field on the brane by adding an Einstein-Hilbert term. This further enticed the investigation of "tachyon cosmology", which deal with the cosmology driven by the tachyon rolling down to its ground state. Inflation as well as brane cosmology driven by the rolling tachyon were also studied. Especially, Kofman and Linde pointed out the difficult for the rolling tachyon to drive the inflation, i.e. it can not produce enough inflation that compatible with the observation. So, it seems more suitable if we consider the rolling tachyon as the dark energy that accelerates the current expansion of the universe. It is worth noting that the tachyon here is different from the supersymmetric tachyon we investigated several years ago.

On the other hand, the spectrum of CMB anisotropies have been quite nicely fixed by current measurements, at least for the initial region of small angles. Although these observations should be extended to even larger redshifts and smaller angles, as several missions under preparation will do in the near future, it is widely accepted that about 70 percent of the total energy in the universe should be hidden as dark energy. Observations of type Ia supernova (SNIa) shows that the expansion of universe is accelerating and therefore requires that the equation-of-state parameter \( w = p/\rho \) of the total fluid should be necessarily smaller than \(-\frac{1}{3}\), where \( p \) and \( \rho \) are the pressure and energy density of the fluid in universe respectively. Hitherto, two possible candidates for dark energy have been suggested. One is the existence of a cosmological constant and another is quintessence which is a dynamical, slowly evolving, spatially inhomogeneous component scalar field with negative pressure. It is widely accepted that a successful theory for dark energy must be able to solve the "coincidence problem" and "fine-tuning problem". Also, one would like to find a natural quintessence model that can arise from high energy physics, or eventually, resulted from string theory. So, it would be very interesting to consider quintessence as the rolling tachyon of string theory.

Now the question is that if tachyon field can be a candidate of quintessence? Tachyon field holds negative pressure. So the key point is that whether the rolling tachyon can solve the "coincidence problem" and "fine-tuning problem". With the aid of phase-plane analysis, we show that there is a saddle point on the phase-plane, which indicates that the tachyon field need to be fine tuned to account for the current cosmological observations. It is worth noting that in the following discussion, like other authors, we assume that there is no direct coupling between tachyon and ordinary matter except through gravitational means.

2. The Rolling Tachyon

The effective Lagrangian density of tachyon of string theory in a flat Robertson-Walker background is as following:

\[
L = -V(T)\sqrt{1 + g^{\mu\nu}\partial_\mu T \partial_\nu T}
\]
where
\[ ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2) \] (2)

is the flat Robertson-Walker metric and
\[ V(T) = V_0(1 + \frac{T}{T_0}) \exp(-\frac{T}{T_0}) \] (3)
is the potential resulted from string theory[24]. It is not difficult to obtain the equation of motion of the tachyon field as well as the gravitational field as following:
\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{\kappa^2}{3} \rho_T \] (4)
\[ \dot{H} = -\frac{\kappa^2}{2}(\rho_T + p_T) \] (5)
\[ \ddot{T} + 3H\dot{T}(1 - \dot{T}^2) + \frac{V'(T)}{V(T)}(1 - \dot{T}^2) = 0 \] (6)

where the overdot represents the differentiation with respect to \( t \) and the prime denotes the differentiation with respect to \( T \). \( \kappa^2 = 8\pi G \) where \( G \) is Newtonian gravitation constant. The density \( \rho_T \) and the pressure \( p_T \) are defined as following:
\[ \rho_T = \frac{V(T)}{\sqrt{1 - \dot{T}^2}} \] (7)
\[ p_T = -V(T)\sqrt{1 - \dot{T}^2} \] (8)
the equation-of-state parameter is
\[ w = \frac{p_T}{\rho_T} = \dot{T}^2 - 1 \] (9)

It is clear that if the tachyon field can accelerate the expansion of the universe, there must be \( \dot{T}^2 < \frac{2}{3} \) and \(-1 < w < -\frac{1}{3} \).

3. Phase-Plane analysis

Phase-plane analysis has been proved to be a very powerful tool for us to investigate the behaviors of the field while not necessarily solving the equation of motion. To obtain the autonomous system corresponding to the equations of motion for tachyon, we can substitute Eq.(4) into Eq.(6) and obtain:
\[ \ddot{T} + \sqrt{3\kappa} \left( \frac{V(T)}{\sqrt{1 - \dot{T}^2}} \right)^{1/4} \dot{T}(1 - \dot{T}^2) + \frac{V'(T)}{V(T)} \dot{T} \left( 1 - \dot{T}^2 \right) = 0 \] (10)

Now, introduce the new variables \( X = T \) and \( Y = \dot{T} \) and then the autonomous system is as following:
\[ \frac{dX}{dt} = Y \] (11)
\[ \frac{dY}{dt} = \frac{X(1 - Y^2)}{T_0^2 + T_0X} - \sqrt{3V_0\kappa Y(1 - Y^2)^{3/4}} \left( 1 + \frac{X}{T_0} \right)^{1/2} \exp(-\frac{X}{2T_0}) \] (12)
It is not difficult to find the critical point of the phase-plane as \((X,Y) = (0,0)\). Expand the above equations Eq.(11) and Eq.(12) about its critical point \((0,0)\), we have

\[
\begin{align*}
\frac{dX}{dt} &= Y \quad (13) \\
\frac{dY}{dt} &= \frac{X}{T_0^2} - \sqrt{3V_0\kappa}Y \quad (14)
\end{align*}
\]

To determine the type of the critical points, we write down the eigen-equation of the above system as:

\[
\lambda^2 + \alpha\lambda + \beta = 0 \quad (15)
\]

where \(\alpha = \sqrt{3V_0\kappa}\) and \(\beta = -\frac{1}{T_0^2}\).

One can easily found that the two eigenvalues are:

\[
\begin{align*}
\lambda_1 &= \frac{-\sqrt{3V_0\kappa} - \sqrt{3\kappa^2V_0^2 + \frac{1}{T_0^2}}}{2} \quad (16) \\
\lambda_2 &= \frac{-\sqrt{3V_0\kappa} + \sqrt{3\kappa^2V_0^2 + \frac{1}{T_0^2}}}{2} \quad (17)
\end{align*}
\]

Clearly, we have

\[
\lambda_1 < 0 < \lambda_2 \quad (18)
\]

This shows that the critical point of the autonomous system is a saddle point and is unstable. That is to say, the evolution of the tachyon field is very sensitive to the initial condition and if we consider tachyon field as quintessence, we must carefully fine-tune it to match the current observations.

4. Phase-Plane Analysis for An Exactly Solvable Toy Model

A toy model of tachyon has been introduced by Feinstein\[4\] to show how the so-called power-law inflation solution can be constructed. To do so, we must rewrite the equations (Eq.(4) to Eq.(6)) as:

\[
\dot{T} = \frac{2}{3} H(T)^2 \quad (19)
\]

and

\[
H'^2 - \frac{9}{4} H^4(T) + \frac{1}{4} V^2(T) = 0 \quad (20)
\]

If one chooses the toy model potential

\[
V(T) = AT^{\frac{4(n-1)}{2n}} \sqrt{B + CT^{\frac{2n}{2n}}}
\]

where \(A > 0, B > 0\) and \(C < 0\) are constants, and \(0 < n < 1\) for the reality condition. From Eqs. (19) and (20), one has

\[
a(t) = \exp(mt^n)
\]

and
\[ T = \gamma t^{\frac{2-n}{2}} \]  

(23)

where \( m \) and \( \gamma \) are positive constants expressed in terms of \( A, B \) and \( C \).

In the following, we will present a phase-plane analysis to show that rolling tachyon with toy model potential Eq. (21). To do so, we replace \( V(T) \) in Eq. (10) by Eq. (21), let \( x = T \) and \( y = \dot{T} \) and rewrite Eq. (10) into the form of an autonomous system

\[
\frac{dx}{dt} = y
\]

\[
\frac{dy}{dt} = (1 - y^2) A_{n-1} B + (3n - 4) C x^{\frac{2n}{2n-2}} - \sqrt{3} \kappa y (1 - y^2)^{3/4} x^{\frac{2n-n}{2n-2}} (B + C x^{\frac{2n}{2n-2}})^{1/4}
\]

(25)

It is easy to find that the only critical point of this system is \( (2^{n-2} B^{1-n-1}, 0) \). It is worth noting that judging the stability of the critical point in general cases is a very complicated algebraic problem. Here we only consider the special case for \( n = \frac{2}{3} \). We find that when \( B > \frac{3}{6} A^2 \kappa C^2 \), the critical point is a stable focus; when \( B = \frac{3}{6} A^2 \kappa C^2 \), the critical point is a stable degenerated node; and when \( B < \frac{3}{6} A^2 \kappa C^2 \), the critical point is a stable node.

This shows that the critical point of the autonomous system is always stable. That is to say, the evolution of the tachyon field in this toy potential is not sensitive to its initial condition and so there isn’t so-called ”coincidence problem” which exists in the model with potential Eq. (3).

Finally, let’s briefly summarize the key aspects of this letter. In this letter, we have investigated the possibility of considering the rolling tachyon field as quintessence. We show, by using the phase-plane analysis, that in the potential from string theory there is no stable point in the phase-plane of the tachyon’s evolution equation. This indicates that if one considers tachyon of string theory as the quintessence, he will encounter the ”coincidence problem”. While, for the toy model we find that there is a stable point to which the field will evolve. Although the later one is only a toy model, it need not to particularly choose the initial conditions of the tachyon field. In this respect, the exactly solvable potential may be favorable.

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