Cosmological radar ranging in an expanding universe

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ABSTRACT
While modern cosmology, founded in the language of general relativity, is almost a century old, the meaning of the expansion of space is still being debated. In this paper, the question of radar ranging in an expanding universe is examined, focusing upon light travel times during the ranging; it has recently been claimed that this proves that space physically expands. We generalize the problem into considering the return journey of an accelerating rocketeer, showing that while this agrees with expectations of special relativity for an empty universe, distinct differences occur when the universe contains matter. We conclude that this does not require the expansion of space to be a physical phenomenon, rather that we cannot neglect the influence of matter, seen through the laws of general relativity, when considering motions on cosmic scales.

Key words: cosmology: theory.

1 INTRODUCTION
The question ‘Is space really expanding?’ has recently (re)surfaced in the literature, with varying views on whether the expansion of space is a phenomenon which can be directly observed. Whiting (2004) entered the fray with a Newtonian analysis of particles detached from the Hubble flow, showing their motion does not agree to the simple viewpoint of expanding space–time as a form of force. Chodorowski (2007a) also considered the physical implications of the expansion of space, suggesting that the superluminal expansion of distant objects, often touted as the proof of expansion, can be removed with a transformation to conformal coordinates, and hence cannot be physical, although it has been shown that superluminal expansion does in fact remain (Lewis et al. 2007). Francis et al. (2007) assessed the situation in detail, showing that the view of expanding cosmologies as expanding space is a valid interpretation as long as the equations of relativity are used to guide common sense.

Recently, Abramowicz et al. (2007) considered radar ranging of a distant galaxy in expanding cosmologies and concluded that the fact that the radar and Hubble distance, from \( d = H_0 v \), differ in all but an empty universe, that space must really expand\(^1\). In a counter-argument, Chodorowski (2007b) again considers radar ranging in open cosmological models. Instead of examining distances, he focuses upon the transit time of light in usual cosmological coordinates and its conformal representation. With this he reveals that in the former coordinates, the paths are asymmetrical in transit time, taking longer on the return journey, whereas in conformal coordinates, the light travel times to and from the distant galaxy are equal. Hence, he concludes that the expansion of space is a coordinate-dependent effect, which can be made to disappear with the correct coordinate transform, and therefore the expansion of space is not a physical phenomenon.

In this contribution, we examine the recent debate on cosmological radar ranging of objects, clarifying some of the issues discussed by other authors and demonstrate that while expanding space remains a useful concept, such experiments in no way require expanding space as a physical effect. The issue of radar ranging in Friedmann–Lemaître–Robertson–Walker (FLRW) universes is addressed in Section 2, conformal coordinates and generalized to include the motion of an accelerating observer in Section 4. A comparison of our results in light of previous studies is presented in Section 5, where we also offer our conclusions on the issue of expanding space.

2 BACKGROUND

2.1 FLRW and conformal cosmology
Modern cosmological models are described by the relativistic equations for a homogeneous and isotropic distribution of matter and
energy. Many elementary textbooks show how, under these assumptions, the space–time of the universe is described by the FLRW metric, whose invariant interval is given by

\[ ds^2 = dr^2 - a^2(t') \left( dx^2 + R_o^2 x^2 (d\theta^2 + \sin^2 \theta \, d\phi^2) \right), \tag{1} \]

where \( S(x) = \sin x, \, x, \sin x \) for spatial curvatures of \( k = +1 \) (closed), \( k = 0 \) (flat) and \( k = -1 \) (open), respectively, with the curvature given by \( R_o^2 \); note, \( c = 1 \). The scalefactor, \( a(t') \), governs the dynamics of the expansion and is dependent upon the relative mix of matter and energy in the universe.

Conformal transformations preserve angles at a point and are important in geometry. For the purposes of this study, the space–times of interest are those which are ‘conformally flat’, such that the metric of a curved space–time is related to that of the flat space–time of special relativity via

\[ g = \Omega(x) g_{\text{flat}}, \tag{2} \]

where \( \Omega(x) \) is an arbitrary function. If we take a particular two-dimensional slice of the FLRW metric above (equation 1) then it is easy to construct a conformally flat transformation such that

\[ dx^2 = a^2(\eta)(d\eta^2 - dx^2) \tag{3} \]

defining the conformal time to be related to the universal time through \( d\tau = a(\eta) d\eta \). Throughout this paper, such a transformation will be referred to as a partial conformal transformation; clearly, in the \((x, \eta)\) coordinates light rays (\( dx = 0 \)) will trace out the undistorted light cones of special relativity [see fig. 1 of Davis & Lineweaver (2004)].

Typically, conformal representations of FLRW cosmologies employ only the partial transform, although this flattens the radial part of the metric, and hence will not, in general, produce flat Special Relativistic-like light cones in fully four-dimensional space–time. For a general cosmology with spatial curvature, a full conformal transformation is required to make the metric conformally flat in all four dimensions, as explored in detail by Infeld & Schild (1945), in which the comoving radial coordinate must be transformed as well. For instance, in an open universe the fully conformal coordinates (see Chodorowski 2007b and Lewis et al. 2007 for the derivation and more details) are

\[ r = Ae^\chi \sinh \chi, \quad t = Ae^\chi \cosh \chi \tag{4} \]

where \( A \) is a constant with \( \chi = x/R_o \). We have referred to the fully conformal coordinates as \((r, t)\), matching those employed in previous studies. It is now expected that light rays follow the classic special relativistic light cones in these coordinates. Clearly, since the spatial part of a flat \((k = 0)\) FLRW metric is already flat, the partial and fully conformal transformations are equivalent. In this study, both the partial and fully conformal transformations will be considered, to provide a comparison to previous studies and allow a correspondence with the flat space–time of special relativity as \( \Omega_o \to 0 \).

### 3 RADAR RANGING

The principle of radar ranging is simple; to calculate the distance to a distant object, fire a radar pulse at it and time the interval \( \Delta \tau \) until the beam returns; here, \( \tau \) is the proper time as measured by an observer sending out and receiving the radar beam. From this, it is straightforward to define a radar distance

\[ D_{\text{rad}} = \frac{\Delta \tau}{2} \tag{5} \]

assuming \( c = 1 \). For cosmological cases, it is usual to measure the comoving distance to a galaxy whereas the time is measured by an observer at the origin. Clearly, this is a well-defined experiment as we are asking about the time ticked off along a world line, an observable quantity. Remember, however, that Chodorowski (2007b) asked a somewhat different question, namely how much time passes on the individual outwards and return journeys.

Fig. 1 presents the radar ranging experiment for an open universe in fully conformal coordinates; here, a radar pulse (green line) leaves the origin, and is reflected back from a distant, comoving (constant spatial FLRW coordinates) object, later being received back at the origin. Note that in this conformal picture, comoving observers move along paths which originate at the origin and move along a line of constant slope given by

\[ \frac{dr}{d\tau} = \tanh(\chi), \tag{6} \]

where \( \chi \) is the comoving coordinate of the fundamental observer, with light rays travelling at 45°. Given this picture, it is simple to see that the differing travel times noted by Abramowicz et al. (2007) are simply an issue of synchronicity.

While the two panels in Fig. 1 both represent a fully conformal representation of an \( \Omega < 1 \) universe, each display differing lines of simultaneity; in the left-hand panel, the dashed line represents constant \( \tau \), or proper time as measured by fundamental observers. In standard FLRW universes, these hyperbolae represent times (slices) of equal matter/energy density as seen by comoving observers. In the right-hand panel, the lines of simultaneity are represented by lines of constant comoving coordinate \( t \). An examination of the left-hand panel, whose lines of synchronicity represent constant cosmological time in the FLRW metric, reveals that both observers agree that the duration of the outward light ray (red paths of the observers) is shorter than the return journey (shown in blue). Both observers agree on the length of time each leg of the journey took. However, an examination of the right-hand panel, where lines of synchronicity are defined by slices of constant comoving coordinate \( t \), reveals a different picture. Now, both observers agree that the duration of the outward and return journey is equal, but they disagree on how much time the journey took in total.

How are we to interpret this picture? Clearly, the issue lies with the fact that measuring the journey time for any individual leg depends upon the comparison of clocks in differing inertial frames, a message stated by Chodorowski (2007b) who considered a Milne (special relativistic universe) and inertial observers. This paper shows that this is generally true in the relativistic interpretation of any FLRW universe and the ‘different space–time structure(s)’ (figs 3–4 of Abramowicz et al. (2007)) are purely due to the way they have chosen define synchronicity and slice up space–time.

### 4 CARRYING A CLOCK

Of course, a major problem with considering the path of a photon is that it is null and the affine parameter that describes its path has no physical significance. But what if the photon is replaced with an observer who can tick off their own proper time on the journey? To this end, we consider an observer who accelerates away from the origin in a rocket with a constant proper acceleration. The acceleration is continued for a fixed amount of proper time \( \Delta \tau \).
(for the rocketeer), followed by a coasting period where the rocket is turned off. The rocket is then swung around and the rocketeer accelerates back towards the origin for a time $2\Delta \tau$, before again entering a coasting period. The rocket is turned round again so that the acceleration is away the origin and fired for a time $\Delta \tau$. Within the flat space–time of special relativity, such a symmetric path will bring the rocketeer to rest at the origin at the conclusion of their journey.

A general discussion of the influence of expanding space on the motion of an accelerating observer will be presented in a future contribution (Kwan & Lewis, in preparation), but here we consider three specific cases. The top row of Fig. 2 presents the journey of two rocketeers in an expanding, open universe; in this case, $\Omega_0 = 0.001$ and the resultant space–time structure should be akin to the Milne (empty) universe. The first rocketeer leaves from the origin, and carries out the symmetric accelerations outlined above. The second undertakes the same journey, starting at the same cosmic time, $\tau'$, but at a different comoving spatial location. To emphasize the issue of synchronicity when making comparisons between different coordinate systems, both journeys have been represented twice, in partially conformal coordinates on the left-hand side and in fully conformal coordinates on the right-hand side. As expected, in either representation the rocketeers return to rest at the origin of their journey. However, an examination of the rocketeer’s path in the partial conformal coordinates reveals that it is not symmetric, with more time $\eta$ spent reaching the most distant point from the origin, than the return journey. Furthermore, the mid-point of the journey as seen by the rocketeer (where the path colour switches from red to blue) also does not correspond to the most distant point reached in the journey.

Moving to the fully conformal picture (right-hand panel), we would expect this path to be virtually the same as that seen in special relativity (see Chodorowski 2007b, for a discussion of the behaviour of the conformal transformation for an open universe as $\Omega_0 \to 0$); this is precisely what is seen, with the path of the rocketeer starting at the origin being symmetric in the conformal time $t$. Remember that in this representation, observers at fixed comoving distances are now seen on sloping lines and the rocket still reaches its greatest comoving distance during its deceleration (dark blue), although the rocketeer reaches the maximum coordinate distance $r$ at the mid-point of the journey. The path of the rocketeer who starts at the non-zero comoving coordinate is a little more complex, and quite different to that seen in the partial conformal coordinates, but it also returns to its origin after a symmetric flight.

The second row of Fig. 2 presents identical journeys in an open, matter-dominated universe with $\Omega_0 = 0.500$, again with the left-hand panel presenting the partial conformal coordinates and the right-hand panel presenting the fully conformal representation. While several aspects of the paths of the rocketeer are similar to those seen in Fig. 2, there is a very important difference, namely that even though the paths are symmetric in terms of acceleration and coasting time for the rocketeer, they do not come to rest at the origin of their journey. In fact, in this open case, the rocketeer over shoots and even when their rocket is turned off, they are still moving away from their origin. Exactly the same behaviour is seen in the fully conformal picture. Although, it may seem that this asymmetry, absent in the case of a static universe (see Fig. 1), is an indication that space is really expanding, this effect only occurs with the introduction of matter content to act on the motion of the rocketeer. In any case, we might naively expect that if space is expanding, then the journey is longer on the way back than it was on the way forwards [cf. fig. 4 of Abramowicz et al. (2007)] and hence we might predict undershooting, rather than overshooting, the origin. It must be emphasized that this over shoot is no different in the Newtonian limit of the FLRW metric without expansion; using Gauss’s law we can see that at a given radius $R$ from the origin, the rocketeer experiences a gravitational acceleration towards the origin due to the mass contained by a sphere of radius $R$. This imaginary sphere changes size throughout the journey but the acceleration from gravity remains pointed towards the origin during the entire

Figure 1. Radar ranging in a fully conformal representation of an open universe. In both, a light ray emitted from the origin is represented by a dotted path, while a comoving observer is represented by the solid sloping line and another observer sits at the origin ($r = 0$). The green line shows the null geodesic representing the laser ranging beam originating at $r = 0$, which is reflected back by the distant comoving observer. In the left-hand panel, the dashed lines represent curves of constant time in the FLRW metric, while on the right-hand side, the dashed lines represent intervals of constant conformal time. The red and the blue paths represent the times measured by the observers for the outward and returning rays, respectively.
Figure 2. Cosmological radar trips for rocketeers in partially conformal (left-hand panels) and fully conformal (right-hand panels) representations of three open universes with varying matter content. For each universe, we have considered the paths of two rocketeers, one leaving from the origin and another that starts out from an arbitrary comoving location. The colour sections of the path describe the state of the rocket engine of the rocketeer. The first green region shows the initial outwards acceleration, followed by the red period corresponding to a coasting phase with no acceleration. The rocket is then swung around to accelerate towards the origin and this is represented by the blue and then green region. The change from blue to green indicates the midway point in this acceleration phase as measured in the rocketeers time. Following this, there is another coasting period, shown in red and finally the rocket is swung around again to accelerate away from the origin and this corresponds to the final blue region. The black dashed lines are the paths taken by a comoving observers starting from the origin, while the pink dashed lines are for comoving observers starting from the same comoving location as the second rocketeer. For the fully conformal cases, we have used $A = 5.3 \times 10^{-5}, 8.3 \times 10^{-2}$ and 28.3 for $\Omega_0 = 0.001, 0.500$ and 0.999, respectively.

trip. Thus, in addition to the thrust provided by the rocket, the rocketeer receives at all times an additional push towards the origin. We should not, therefore, be surprised to find the rocketeer overshoots. Thinking very simply about the effects of gravity, rather than the more nebulous expansion of space, gives a much simpler intuitive view.

This asymmetry is even more apparent in the bottom row of Fig. 2 which again presents the rocketeers’ paths in partial and fully conformal coordinates, except now the matter density is $\Omega_0 = 0.999$; in this case, the universe is approaching the spatially flat $\Omega_0 = 1$ universe and hence the slope of the comoving observer in the fully conformal picture is approaching that of the partially conformal case (as noted previously, for spatially flat models the partial and fully conformal transforms are equal). The key difference between the three cases represented in Fig. 2 is that the matter content of each universe increases, which we are free to interpret as the cause of the increasing asymmetry in the paths, rather than the asymmetry being caused by the expansion of space. Again, this overshoot can be understood in the Newtonian limit of the FLRW metric without expansion; we would expect approximately the same behaviour to occur for a rocketeer travelling at non-relativistic speeds in a Newtonian potential to a destination close by. The implications of
this journey on the question of whether space really expands are presented in Section 5.

4.1 Synchronizing clocks

It is clear that the timing issues related to the radar experiment are related to the synchronization of clocks (as are many of the problems and apparent paradoxes in relativity). However, the universe itself provides a clock that can be employed to provide at least a working definition of synchronicity using the density of matter/energy; as the universe expands, the density of matter falls and dashed lines in the right-hand panel of Fig. 1 correspond to slices of cosmic time in the FLRW metric along which the density is equal. Clearly, such synchronization is not possible in the Milne universe, as, being empty, there is no density yardstick with which to tick off cosmic time. However, the situation is the same in a universe containing only a cosmological constant term (with equation of state $w = -1$) as the energy density remains a constant.

With either of these cases, or any other universe model, we can imagine a ‘test cosmic microwave background (CMB)’, a homogeneous and isotropic bath of photons of negligible energy fraction defining the Hubble rest frame and comoving coordinates. By measuring the temperature of the CMB, all observers can calibrate their clocks to each other. Interestingly, even in the Milne universe which contains no gravitating energy, this test CMB provides a universal clock giving an excellent demonstration of how we can observe an apparent expansion of space in a universe known to be completely empty and equivalent to special relativity. Clearly, the interpretation of expanding space ‘stretching’ photons causing them to redshift, while being a useful teaching aid, does not describe a causal physical phenomenon if we can observe this effect in Minkowski space.

5 CONCLUSIONS: SO, IS SPACE REALLY EXPANDING?

This work has grown out of a recent exchange in the literature on the question on whether space ‘really’ expands. In a previous contribution (Francis et al. 2007), we argued that while space is ‘completely and utterly empty’ (to quote Steve Weinberg), it is perfectly valid to interpret the equations of relativity in terms of an expanding space. The mistake is to push analogies too far and imbue space with physical properties that are not consistent with the equations of relativity.

In their recent work, Abramowicz et al. (2007) showed that, in all but an empty universe, distances derived from the Hubble law and radar ranging differ and hence ‘one must conclude that space is expanding’. But how is this difference occurring? Is the expansion of space acting on a light ray (or even a rocketeer) as they travel through the universe? We can think of space as a rubber sheet that stretches to wash out peculiar motions and drives everything back into the Hubble flow (see Barnes et al. 2006). However, it is the presence of matter that necessitates the inclusion of gravitational forces upon the motion of the rocketeers and it is – the changing gravitational influence of matter in the universe on the rocketeers – that causes the increasing asymmetry moving down the panels in Fig. 2, not that space physically expands.

In closing, we state that it is a fool’s errand to search for the truth of the existence of expanding space; not only because it is dependant upon a choice of coordinates, but also because general relativity is represented by Newtonian physics in the weak field limit and the global behaviour of the FLRW metric always reduces to Newtonian gravity in the limit of the local universe with no need for expanding space. While the expansion of space is a valid picture (albeit a dangerous one, when working with the equations of relativity), any attempts to obtain observations to address the question of whether galaxies are moving through static space or are carried away by the expansion of space are doomed to failure.

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