Neutrino Oscillations by Interaction with Moduli

K. Kobayakawa\(^1\), Y. Sato\(^2\) and S. Tanaka\(^3\)

\(^1\) Fukui University of Technology, Gakuen-cho, Fukui 910, Japan
\(^2\) Graduate School of Science and Technology, Kobe University, Nada, Kobe 657, Japan
\(^3\) Division of Natural Environment and High Energy Physics, Faculty of Human Development, Kobe University, Nada, Kobe 657, Japan

Abstract

We would like to point out the possibility to detect the low-energy signals of moduli in the superstring theory in the neutrino oscillation. The idea is based on the characteristics that the couplings of moduli are different in matter. We estimate the oscillation probability both in the long baseline and in the solar neutrino oscillations and examine the detectable region of the moduli effect.

\(^*\) On leave from Faculty of Human Development, Kobe University
Recent LEP data [1] suggest the evidence of Grand Unified Theories such as SU(5), SO(10), flipped SU(5) and so on. Furthermore the data fit better on including supersymmetry. On the theoretical side to solve the gauge hierarchy problem the idea of supersymmetry is very persuasive. However, SUSY GUT does not contain the interaction of gravity. At present it is conceived that the superstring theory alone may include all interactions consistently in the theory. Phenomenologically the heterotic superstring theory [2] is most attractive. There are several ways of compactification and after that there come out very many vacua [3]. They are parametrized, in general, by moduli [4] which are singlet superfields under the gauge group of the standard model, SU(3)C × SU(2)L × U(1)Y. For example, some of them describe the size and shape of the compactified space. But masses of moduli are not known, though their vacuum expectation values are supposed to be of the order of Planck scale. Their interactions with matter are also model-dependent. Even the number of moduli depend on the structure of the vacuum considered. Consequently it is very helpful to detect the moduli.

In this paper we would like to point out that moduli characteristic in the superstring theory may give new low-energy signals which could be tested in the neutrino oscillation experiments. Moduli generally couple to ordinary matter with nonrenormalizable interactions. Such couplings are expressed in the superpotential effectively as (in the lowest dimension)

\[ P_{\text{nonren}} = \frac{c_{ijk}^I}{M_S} \varphi_i \varphi_j \varphi_k M_I, \quad (I = 1, 2, 3, \ldots), \]

where \(\varphi_{i,j,k}\) are matter superfields, \(M_I\) are moduli superfields and \(M_S\) is the string scale (\(\sim 10^{18}\)GeV). Such terms at low energies induce Yukawa-type couplings between the ordinary matter and (real) scalar fields or pseudoscalar fields i.e. moduli:

\[ \mathcal{L}_Y = \frac{< H_2 >}{M_S} h_{ij}^{(u)} \bar{\nu}_R^{i} \nu_L^{j} M_I + \frac{< H_2 >}{M_S} h_{ij}^{(u)} \bar{u}_R^{i} u_L^{j} M_I \\
+ \frac{< H_1 >}{M_S} h_{ij}^{(d)} \bar{d}_R^{i} \bar{d}_L^{j} M_I + \frac{< H_1 >}{M_S} h_{ij}^{(e)} \bar{\ell}_R^{i} \bar{\ell}_L^{j} M_I + h.c., \]

where \(i\) and \(j\) are generation indices \((i = 1, 2, 3)\) and \(< H_{1,2} >\) are the vacuum expectation values of the Higgs doublets and \(\gamma\)-matrices are dropped. While dilaton \(S\) interacts with ordinary matter universally like graviton, moduli interact (or not interact) with various couplings. Moduli interact with ordinary matter as a coherent attractive force [4]. (We also consider the possibility
of moduli interaction generating a repulsive force.) Since the interaction strength is comparable to that of gravity force, this behaves as a kind of fifth force if the mass of the exchanged particle is small enough \[5\]. Ordinary moduli mass is expected to be of the order of the gravitino mass. Because moduli have a flat potential perturbatively and they must get mass by non-perturbative effects. It is usually said that it occurs after supersymmetry breaking. So it would be as heavy as other scalar sparticles. However, there are arguments that some moduli would have very tiny mass:

1. for real \( M_I \) the moduli mass (\( m_{M_I} \)) may be induced by radiative corrections (\( m_{M_I} \approx 10^{-18} \text{GeV} \) \[3\], or there may be a special cancellation of two terms in the mass equation \[7\]. In ref.\[8\], it is estimated that \( m_{M_I} \) can be about \( m_3^2 / \text{Re} M_I \), where \( m_3^2 \) is the gravitino mass.

2. for imaginary \( M_I \), in ref.\[6\] it is argued that \( m_{M_I} \) can be \( 2 \times 10^{-24} \text{GeV} \). However, in ref.\[8\] it is said that they are massless. In ref.\[9\], on the other hand, they are said to gain huge mass of the order of the SUSY-breaking scale.

There is no definite mass which can be calculated numerically, and also the form of the scalar potential is not known yet. So we do not get into the details of the models here and take a mass of a modulus (especially tiny one) as a parameter \( m_{M_I} \) and its relative interaction strength as parameters \( f_{ij} \), and explore the possibilities of finding the effects of moduli in the terrestrial experiments, not in cosmology.

First we discuss long baseline neutrino oscillations. The planning experiments such as from FNAL to SOUDAN2 \[10\] is illustrated schematically in Fig.1. The neutrino source from an accelerator is located at the point \( x_1 \), and the detector at the point \( x_2 \). The muon neutrino (\( \nu_\mu \)) beam with energy \( E \) (of the order from one GeV to a few ten GeV) propagates along the \( x \)-axis. We assume, for simplicity, that there is at least one modulus which interacts with \( \nu_\tau \) and/or \( \nu_\mu \) and u- or d-quark (or electron). For example, \( h^{(u)}_{22} \neq 0 \), \( h^{(u)}_{11} \neq 0 \) and others can be zero in eq.\( (2) \). Although the interaction strength is gravitational, it may be detectable in the neutrino oscillations when \( m_{M_I} \) is very tiny. We take it in this paper in the range of \( 10^{-24} - 10^{-15} \text{GeV} \).

We define the eigenstate of mass plus moduli interaction as \((\nu_2, \nu_3)\), and the flavor eigenstate as \((\nu_\mu, \nu_\tau)\). The latter eigenstate is expressed by the
former with a mixing angle $\zeta$ as

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos \zeta & \sin \zeta \\ -\sin \zeta & \cos \zeta \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}. \tag{3}$$

The Hamiltonian of mass eigenstate is given by

$$H = \begin{pmatrix} p + \frac{m_2^2}{2p} - f_{22}\phi & -f_{23}\phi \\ -f_{32}\phi & p + \frac{m_3^2}{2p} - f_{33}\phi \end{pmatrix}, \tag{4}$$

where $p$ is the momentum of a neutrino beam and $m_2$ and $m_3$ are the masses of mass eigenstates. $f_{ij}\phi$ in eq.(4) represent the potentials induced by moduli interaction. The difference of modulus coupling with matter is stuffed into $f_{ij}(|f_{ij}| \leq 1; i, j = 2, 3)$. We can take $f_{23} = f_{32}$. Since $f_{23}$ is not necessarily zero, this Hamiltonian is not always diagonal. In a relativistic case $\phi$ is represented as the product of energy of a neutrino beam and the potential per unit mass due to moduli interaction with matter

$$\phi = EV, \quad V = G_M \frac{M}{r} \exp(-\mu r). \tag{5}$$

Here $\mu$ can be regarded as almost same as the moduli mass and $G_M$ is a common coupling constant of moduli so that the maximum value among $|f_{ij}|$ is unity. In eq.(5), $M$ is the mass of the matter which is interacting with neutrino interchanging moduli.

To estimate $V$, the contribution to $V$ from the whole earth is added up. Then

$$\phi_{\text{global}} = -\frac{3G_MM_EE}{2\mu^2R_E^3} \frac{R_E + \mu^{-1}}{\xi_0} \times \left[\exp\{-\mu(R_E - \xi_0)\} - \exp\{-\mu(R_E + \xi_0)\}\right] + \frac{3G_MM_EE}{\mu^2R_E^3}, \tag{6}$$

where $M_E$ and $R_E$ denote the mass and the radius of the earth, respectively. $\xi_0$ is the distance between the beam and the center of the earth as shown in Fig.1. We can put $\xi_0 = R_E$ approximately. The value of eq.(5) is almost the same as that of the sphere with radius $\mu^{-1}$ because of the exponential decrement in eq.(5).

The Hamiltonian (4) can be diagonalized and we get

$$\frac{d}{dx} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} p + \frac{m_2^2}{2p} + \alpha & 0 \\ 0 & p + \frac{m_3^2}{2p} + \beta \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}, \tag{7}$$
where $\alpha$ and $\beta$ are parameters introduced for convenience sake:

$$\alpha = \lambda_2 - p - \frac{m_2^2}{2p}, \quad (8)$$

$$\beta = \lambda_3 - p - \frac{m_3^2}{2p}, \quad (9)$$

$$\lambda_{2,3} = p + \frac{m_2^2 + m_3^2}{4p} - \frac{f_{22} + f_{33}}{2} \phi$$

$$\pm \left[ \left( \frac{\Delta m^2}{4p} \right)^2 + \left( \frac{\Delta f \phi}{2} \right)^2 - \frac{\Delta m^2}{4p} \Delta f \phi + f_{23}^2 \phi^2 \right], \quad (10)$$

where $\Delta m^2 = m_3^2 - m_2^2$ and $\Delta f = f_{33} - f_{22} (|\Delta f| \leq 1)$ represents the difference of the coupling constants of the two neutrino species with matter.

Solving eq. (7) and using eqs. (3) and (8)-(10), we obtain the oscillation probability:

$$P(\nu_\mu \to \nu_\tau) = \sin^2 \zeta \times \sin^2 \left[ \frac{L}{c} \left\{ \left( \frac{\Delta m^2}{4E} \right)^2 + \left( \frac{\Delta f \phi}{2} \right)^2 - \frac{\Delta m^2}{4E} \Delta f \phi + f_{23}^2 \phi^2 \right\} \right]. \quad (11)$$

The first term inside the brace is due to the vacuum oscillation and the last three terms are due to moduli interaction. So comparing the two kinds of contributions, we can examine the effect of moduli interaction. The former is proportional to $E^{-1}$ and the latter is proportional to $E$. Therefore the higher the energy of neutrino is, the larger the effect of moduli is. The effect of moduli interaction may be detected experimentally, if it is at least about $10^{-3}$ of that of vacuum oscillation [12]. Here we neglect the term of $f_{23}^2 \phi^2$ so it might be underestimation of moduli effect.

We examine the detectable region of two parameters $\mu$ and $\Delta f$ as follows. The force induced by moduli interactions behaves like the fifth force which many experiments have tested and put restrictions. First fixing the value of $\mu$, which we set at a reciprocal of the force range $\lambda$, we take $G_M$ in eq.(5) at the maximum value of allowable $G_5$. In this way we get the limit value of $\Delta f$ at each $\mu$. Denoting $\alpha = \frac{G_M}{G_N}$, where $G_N$ is the gravitational constant, we use the limit values of $(\mu [\text{GeV}], \alpha)$ for the attractive force from ref. [13]: for example, $(2.0 \times 10^{-22}, 3.0 \times 10^{-6})$, $(2.0 \times 10^{-20}, 1.6 \times 10^{-4})$, (5)
Let us consider two versions of $\Delta m^2$. First, if $\nu_\tau$ is regarded as a candidate of dark matter, then $\Delta m^2$ is expected to be about 100eV$^2$. Second, according to Kamiokande atmospheric neutrino data, $\Delta m^2 \approx 10^{-2}$eV$^2$. Next, we take as the energy of neutrino $E$ the following three typical examples:

(i) KEK $\rightarrow$ Kamioka ($E = 1.4$GeV)
(ii) FNAL $\rightarrow$ SOUDAN2 ($E = 10$GeV)
(iii) $E = 1$TeV (such neutrinos are detectable in e.g.DUMAND)

Figs.2a and 2b correspond to $\Delta m^2 = 100$eV$^2$ and $\Delta m^2 = 10^{-2}$eV$^2$ in the case that moduli interaction is an attractive force. The observable region is the upper part of the dotted line in (i), the dashed line in (ii) and the solid line in (iii), respectively. Namely the lines show the limit $\Delta f \phi = 10^{-3} \Delta m^2 / 2E$. As the energy of the neutrino increases, the detectable region becomes wide. The effect of moduli interaction is more significant in the case of Fig.2b than that of Fig.2a. Similarly in the case of repulsive force, the observable region of $\Delta f$ is shown in Fig.3a ($\Delta m^2 = 100$eV$^2$) and in Fig.3b ($\Delta m^2 = 10^{-2}$eV$^2$).

We will comment on eq.(11) a little more. The formula in the brace can be written as

$$\frac{L}{c} \left( \frac{\Delta m^2}{4E} - \frac{\Delta f}{2} \phi \right),$$

for $f_{12} = 0$. In eq.(12) the first term is

$$\frac{L}{c} \frac{\Delta m^2}{4E} = 1.27 \left( \frac{\Delta m^2}{E \text{GeV}} \right) \left( \frac{L}{\text{km}} \right),$$

and the second term is approximately given from eq.(6) as

$$\frac{\Delta f}{2} \phi = \frac{3L}{4c} \Delta f \frac{G_M M L}{\mu^3 R_E^3}$$

$$= 2.54 \Delta f \left( \frac{\alpha}{10^{-4}} \right) \left( \frac{1}{10^{-6} \text{GeV}} \right)^2 \left( \frac{E}{\text{GeV}} \right) \left( \frac{L}{\text{km}} \right).$$

When $\Delta f = 1$ and all other physical quantities are $O(1)$ in the denoted units, both values of eqs.(13) and (14) are near $\pi$ which gives the maximum value of $P(\nu_\mu \rightarrow \nu_\tau)$. We touch upon the effect of moduli interaction to solar neutrino oscillations briefly. Taking MSW effect into account and using the number
density of electrons \( N_e \) which reads

\[
N_e(R) = 245N_A \exp \left( -10.54 \frac{R}{R_{\text{sun}}} \right)
\]  

We get the probability that \( \nu_e \) changes into \( \nu_\mu \) as

\[
P(\nu_e \rightarrow \nu_\mu) = \sin^2 \zeta_{m} \sin^2 \left[ \frac{1}{2c} \left\{ \frac{\Delta m^2}{2E}L_{\text{sun}} - \Delta f \Phi \right\} \right]
\]

\[
\Phi = \frac{93\pi G_M}{\mu^2} Em_p \left( \alpha_p + \alpha_n + \frac{m_e}{m_p} \right) N_A R_{\text{sun}}
\]

\[
\times \left[ \exp \left( -10.54 \frac{R_{\text{min}}}{R_{\text{sun}}} \right) - \exp( -10.54 ) \right]
\]

where \( \Delta m^2 = m_{\mu}^2 - m_{\tau}^2 \), \( \Delta f = f_{22} - f_{11} \), \( \alpha_p \) and \( \alpha_n \) correspond to contribution of protons and neutrons, respectively. \( \nu_e \)'s are supposed to be generated around the distance of \( R_{\text{min}} (\simeq 0.1R_{\text{sun}}) \) from the center and \( L_{\text{sun}} = R_{\text{sun}} - R_{\text{min}} \simeq 0.9R_{\text{sun}} \). \( \Phi \) is the integrated value of \( \phi \) with respect to \( R \) from \( R_{\text{min}} \) to \( R_{\text{sun}} \). The second term inside the brace of eq.(16) is the effect of moduli interaction. The ratio of \( |\Delta f \Phi| \) to \( \Delta m^2 L_{\text{sun}}/2E \) is estimated to be \( 8 \times 10^{-9} \) for a typical example of \( \mu = 10^{-22}\text{GeV} \), \( G_M = 10^{-6}G_N \), \( E = 10\text{eV} \) and \( \Delta m^2 = 10^{-5}\text{eV}^2 \). Thus we conclude that the moduli interaction affords no significant effect in the solar neutrino oscillation.

There are discussions on direct phenomenological consequences of moduli in cosmology in ref.[7] and others. Here we point out that moduli may give signals even in accelerator experiments, that is, long baseline neutrino oscillations. In some cases, the effect might be seen in a short baseline neutrino oscillation. For example, in the CHORUS experiment[22] they try to detect \( \tau \) after \( \nu_\tau \)-nucleon charge current interactions. At \( E \simeq 30\text{GeV} \), \( L = 0.8\text{km} \), \( \Delta f \phi/2 = 61\Delta f \) for \( \mu \sim 10^{-20} - 10^{-24}\text{GeV} \). Then the experiment proves affirmative for moduli, if \( |\Delta f| \gtrsim 10^{-2} \).

In conclusion if neutrino oscillation experiments will be scrupulously performed with various conditions, a clue of the form of moduli interaction with matter and mass of a modulus might be obtained. When moduli effect is detected in neutrino oscillation, there must exist at least one modulus with tiny mass and the structure of the true vacuum or the mechanism of SUSY breaking would be restricted.
References

[1] See, for example, LEP collaborations, Phys.Lett. **B276** (1992) 247.

[2] D. Gross, J. Harvey, E. Martinec and R. Rohm, Nucl. Phys. **B256** (1985) 253; Nucl. Phys. **B267** (1986) 75.

[3] M. Green, J. Schwarz and E. Witten, Superstring Theory: 2 (Cambridge University Press, 1987); M. Kaku, Strings, Conformal Fields and Topology (Springer, 1991)

[4] for a review, S. Ferrara and S. Theisen, preprint CERN-TH.5652/90 (1990)

[5] M. Čvetic, Phys. Lett. **B229** (1989) 41

[6] A. de la Macorra, Phys. Lett. **B335** (1994) 35

[7] B. de Carlos, J. A. Casas, F. Quevedo and E. Roulet, Phys. Lett. **B318** (1993) 447

[8] S. Kelley, J. L. Lopez, D. V. Nanopoulos and A. Zichichi, preprint CERN-TH.7433/94 (1994) et al., hep-ph/9409223

[9] L. E. Ibañez and D. Lüst, Phys. Lett., **B267** (1991) 51

[10] Fermi-Lab p-875 MINOS Collab.

[11] M. Gasperini, Phys. Rev. **D39** (1989) 3606

[12] K. Iida, H. Minakata and O. Yasuda, Mod. Phys. Lett. **8A** (1993) 1037

[13] E. Fischbach and C. Talmadge, Nature, Vol.356 (1992) 207

[14] F. D. Stacey, G. J. Tuck and S. C. Holding et al., Rev. Mod. Phys. **59** (1987) 157

[15] H. Harari, Phys. Lett. **B216** (1989) 413; Phys. Lett. **B292** (1992) 189

[16] Kamiokande Collaboration, Y. Fukuda et al., Phys. Lett. **B335** (1994) 237

[17] KEK proposal E-362

[18] DUMAND Collaboration, C. M. Alexander et al., 23rd Int. Cosmic Ray Conf. (Calgary) **4** (1993) 515
[19] L. Wolfenstein, Phys. Rev. D17 (1978) 2369

[20] S.P. Mikheyev and A.Yu. Smirnov, Nuovo Cimento 9C (1986) 17

[21] J.N. Bahcall and R.K. Ulrich, Rev. Mod. Phys. 60 (1988) 297

[22] CERN WA-95 Experiments CHORUS Collaboration
Figure captions

Fig.1: Schematic of Long Baseline Neutrino Oscillations. The neutrino beam is injected from the accelerator at the point $x_1$ and detected at the point $x_2$.

Figs.2a, 2b: Observable region in the $(\mu, \Delta f)$ plane for the case of attractive force.

a: $\Delta m^2 = 100\text{eV}^2$; b: $\Delta m^2 = 10^{-2}\text{eV}^2$.

The observable regions are shown by the upper part of lines. The dotted, the dashed and the solid lines correspond to (i)$E = 1.4\text{GeV}$, (ii)$10\text{GeV}$, (iii)$1\text{TeV}$, respectively.

Figs.3a, 3b: Observable region in the $(\mu, \Delta f)$ plane for the case of repulsive force.

The observable regions and other notations are the same as Fig.2.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9504370v1
This figure "fig2-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9504370v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9504370v1
This figure "fig2-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9504370v1
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9504370v1