Massive-scalar Absorption by Extremal $p$-branes

M. Cvetič, H. Lü, J. F. Vázquez-Poritz

†Dept. of Phys. and Astro., University of Pennsylvania, Philadelphia, PA 19104

ABSTRACT

We study the absorption probability of minimally-coupled massive scalars by extremal $p$-branes. In particular, we find that the massive scalar wave equation under the self-dual string background has the same form as the massless scalar wave equation under the dyonic string background. Thus it can be cast into the form of a modified Mathieu equation and solved exactly. Another example that we can solve exactly is that of the $D = 4$ two-charge black hole with equal charges, for which we obtain the closed-form absorption probability. We also obtain the leading-order absorption probabilities for D3-, M2- and M5-branes.

---

1 Research supported in part by DOE grant DOE-FG02-95ER40893
1 Introduction

There has been considerable interest recently in studying absorption probabilities for fields propagating in various black hole and p-brane backgrounds [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. One of the motivations is the conjectured duality of supergravity on an AdS spacetime and the conformal field theory on the boundary of the AdS [19, 20, 21]. Previously, the study of absorption was mainly concentrated on the case of massless scalars. Some work has been done for the cases of the emission of BPS particles from five- and four-dimensional black holes [2]. These BPS particles can be viewed as pp-waves in a spacetime of one higher dimension. Hence they satisfy the higher-dimensional massless wave equations.

In this paper, we consider the absorption probability of minimally-coupled massive particles by extremal p-branes. The wave equation for such a scalar depends only on the metric of the p-brane, which has the form

$$ds^2 = \prod_{\alpha=1}^{N} H_{\alpha}^{-\frac{\tilde{d}}{D-2}} dx^\mu dx^n \eta_{\mu \nu} + \prod_{\alpha=1}^{N} H_{\alpha}^{\frac{d}{D-2}} dy^m dy^m, \quad (1.1)$$

where $d = p + 1$ is the dimension of the world volume of the p-brane, $\tilde{d} = D - d - 2$, and $H_{\alpha} = 1 + Q_{\alpha}/r^{\tilde{d}}$ are harmonic functions in the transverse space $y^m$, where $r^2 = y^m y^m$. (Note that the ADM mass density and the physical charges of the extreme p-brane solutions are proportional to $\sum_{i=1}^{N} Q_{i}$ and $Q_{i}$, respectively)

It follows that the wave equation, $\partial_M (\sqrt{g} g^{MN} \partial_N \Phi) = m^2 \Phi$, for the massive minimally-coupled scalar, with the ansatz, $\Phi(t, r, \theta_i) = \phi(r) Y(\theta_i) e^{-i\omega t}$, takes the following form:

$$\frac{d^2 \phi}{d\rho^2} + \frac{\tilde{d} + 1}{\rho} \frac{d \phi}{d \rho} + \left[ \prod_{\alpha=1}^{N} (1 + \frac{\lambda_{\alpha}}{\rho^{\tilde{d}}}) - \ell(\ell + \tilde{d}) \frac{\rho^2}{\omega^2} \prod_{\alpha=1}^{N} (1 + \frac{\lambda_{\alpha}}{\rho^{\tilde{d}}}) \right] \phi = 0, \quad (1.2)$$

where $\rho = \omega r$ and $\lambda_{\alpha} = \omega Q_{\alpha}^{1/\tilde{d}}$. Note that when $m = 0$, the wave equation depends on $\tilde{d}$, but is independent of the world-volume dimension $d$. This implies that the wave equation for minimally-coupled massless scalars is not invariant under the vertical-dimensional reduction, but is invariant under double-dimensional reduction of the corresponding p-brane [18]. However, for massive scalars, the wave equation (1.2) is not invariant under either double or vertical reductions.

The absorption probability of massless scalars is better understood. It was shown that for low frequency the cross-section/frequency relation for a generic extremal p-brane coincides with the entropy/temperature relation of the near extremal p-brane [18]. There are a few examples where the wave equations can be solved exactly in terms of special functions. Notably, the wave equations for the D3-brane [14] and the dyonic string [17] can be cast...
into modified Mathieu equations. Hence, the absorption probability can be obtained exactly, order by order, in terms of a certain small parameter. There are also examples where the absorption probabilities can be obtained in closed-form for all wave frequencies [18].

When the mass $m$ is non-zero, we find that there are two examples for which the wave function can be expressed in terms of special functions and, thus, the absorption probabilities can be obtained exactly. One example is the wave equation in the self-dual string background, which can be cast into a modified Mathieu equation. Therefore, we can obtain the exact absorption probability, order by order, in terms of a certain small parameter. We discuss this example in section 2. Another example is the wave equation for the $D = 4$ two-charge black hole with equal charges. The wave function can be expressed in terms of Kummer’s regular and irregular confluent hypergeometric functions. It follows that we can obtain the absorption probability in closed-form, which we present in section 3. In both of the above examples, the massive scalar wave equation has the same form as the massless scalar wave equation under the backgrounds where the two charges are generically non-equal.

However, in general, the massive scalar wave equation (1.2) cannot be solved analytically. For low-frequency absorption, the leading-order wave function can be obtained by matching wave functions in inner and outer regions. In section 4, we make use of this technique to obtain the leading-order absorption probability for D3-, M2- and M5-branes.

2 Massive scalar absorption for the self-dual string

For the self-dual string ($Q_1 = Q_2 \equiv Q$), we have $d = \tilde{d} = 2$ and $\lambda_1 = \lambda_2 \equiv \lambda = \omega \sqrt{Q}$. It follows that the wave equation (1.2) becomes

$$\frac{d^2 \phi}{d\tilde{\rho}^2} + \frac{3}{\tilde{\rho}} \frac{d\phi}{d\tilde{\rho}} + \left(1 + \frac{\tilde{\lambda}_1^2 + \tilde{\lambda}_2^2 - \ell(\ell + 2)}{\tilde{\rho}^2} + \frac{\tilde{\lambda}_1^2 \tilde{\lambda}_2^2}{\tilde{\rho}^4}\right) \phi = 0,$$

(2.1)

where

$$\tilde{\lambda}_1 = \lambda, \quad \tilde{\lambda}_2 = \lambda \sqrt{1 - (m/\omega)^2},$$

$$\tilde{\rho} = \rho \sqrt{1 - (m/\omega)^2},$$

(2.2)

Thus the wave equation of a minimally-coupled massive scalar on a self-dual string has precisely the same form as that of a minimally-coupled massless scalar on a dyonic string, where $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$ are associated with electric and magnetic charges. It was shown in [17] that the wave equation (2.1) can be cast into the form of a modified Mathieu equation, and
hence the equation can be solved exactly. To do so, one makes the following definitions
\[ \phi(\tilde{\rho}) = \frac{1}{\rho} \Psi(\rho) \quad \tilde{\rho} = \sqrt{\tilde{\lambda}_1 \tilde{\lambda}_2} e^{-z}. \] (2.3)

The wave equation (2.1) then becomes the modified Mathieu equation [17]
\[ \Psi'' + (8\Lambda^2 \cosh(2z) - 4\alpha^2) \Psi = 0, \] (2.4)

where
\[ \alpha^2 = \frac{1}{4}(\ell + 1)^2 - \Lambda^2 \Delta, \]
\[ \Lambda^2 = \frac{1}{4}\tilde{\lambda}_1 \tilde{\lambda}_2 = \frac{1}{4}\lambda^2 \sqrt{1 - (m/\omega)^2} = \frac{1}{4}\omega\sqrt{\omega^2 - m^2 Q}, \] (2.5)
\[ \Delta = \frac{1}{\lambda_2} + \frac{1}{\lambda_1} = \sqrt{1 - (m/\omega)^2} + \frac{1}{\sqrt{1 - (m/\omega)^2}}. \] (2.6)

The Mathieu equation can be solved, order by order, in terms of \( \Lambda^2 \). The result was obtained in [17], using the technique developed in [22]. (For an extremal D3-brane, which also reduces to the Mathieu equation, an analogous technique was employed in [14].) In our case there are two parameters, namely \( \omega R \) and \( m/\omega \). We present results for two scenarios:

2.1 Fixed mass/frequency ratio probing

In this case, we have \( m/\omega = \beta \) fixed. The requirement that \( \Lambda \) is small is achieved by considering low-frequency and, hence, small mass of the probing particles. In this case, \( \Delta \) is fixed, and the absorption probability has the form [17]
\[ P_\ell = \frac{4\pi^2 \Lambda^{4+4\ell}}{(\ell + 1)^2 \Gamma(\ell + 1)^4} \sum_{n\geq 0} \sum_{k=0} b_{n,k} \Lambda^{2n} (\log \tilde{\Lambda})^k, \] (2.7)
where \( \tilde{\lambda} = e^{\gamma} \lambda \), and \( \gamma \) is Euler’s constant. The prefactor is chosen so that \( b_{0,0} = 1 \). Our results for the coefficients \( b_{n,k} \) with \( k \leq n \leq 3 \) for the first four partial waves, \( \ell = 0, 1, 2, 3 \), were explicitly given in [17]. In particular the result up to the order of \( \Lambda^2 \) is given by [17]
\[ P_\ell = \frac{4\pi^2 \Lambda^{4+4\ell}}{(\ell + 1)^2 \Gamma(\ell + 1)^4} \left[ 1 - \frac{8\Delta}{\ell + 1} \Lambda^2 \log \Lambda + \frac{4\Delta \Lambda^2}{(\ell + 1)^2} \left( 1 + 2(\ell + 1) \psi(\ell + 1) \right) + \cdots \right], \] (2.8)
where \( \psi(x) \equiv \Gamma'(x)/\Gamma(x) \) is the digamma function.

2.2 Fixed mass probing

Now we consider the case where the mass of the test particle is fixed. In this case, it is ensured that \( \Lambda \) is small by considering the limiting frequency of the probing particle, namely
\( \omega \to m^+ \), i.e. the particle is non-relativistic. In this limit, the value of \( \Delta \) becomes large (while at the same time the expansion parameter \( \Lambda \) can still be ensured to remain small). Furthermore, we shall consider a special slice of the parameter space where \( \alpha^2 \), given in (2.3), is fixed. The absorption probability for fixed \( \alpha \) was obtained in [17]. It is of particular interest to present the absorption probability for \( \alpha \to 0 \), given by

\[
P = \frac{\pi^2}{\pi^2 + (2 \log \bar{\Lambda})^2} \left( 1 - \frac{22}{3} \Lambda^4 (\log \bar{\Lambda})^2 - \frac{16}{9} (4 \zeta(3) - 3) \frac{\Lambda^4 \log \bar{\Lambda}}{\pi^2 + (2 \log \Lambda)^2} + O(\Lambda^8) \right), \quad (2.9)
\]

where \( \bar{\Lambda} = e^\gamma \Lambda \). When \( \alpha^2 < 0 \), we define \( \alpha^2 = i \beta \), and find that the absorption probability becomes oscillatory as a function of \( \Lambda \), given by [17]

\[
P = \sinh^2 \frac{2\pi \beta}{\sinh^2 2\pi \beta + \sin^2 (\theta - 4\beta \log \Lambda)} + \cdots, \quad (2.10)
\]

where

\[
\theta = \arg \frac{\Gamma(2i\beta)}{\Gamma(-2i\beta)}. \quad (2.11)
\]

Note that the \( \alpha \to 0 \) limit is a dividing domain between the region where the absorption probability has power dependence of \( \Lambda \) (\( \alpha^2 > 0 \)) and the region with oscillating behavior on \( \Lambda \) (\( \alpha < 0 \)).

### 3 Closed-form absorption for the \( D = 4 \) two-charge black hole

For a \( D = 4 \) black hole, specified in general by four charges \( Q_1, Q_2, P_1 \) and \( P_2 \) [23], we have \( d = \tilde{d} = 1 \). We consider the special case of two equal non-zero charges (\( Q_1 = Q_2 \equiv Q \) with \( P_1 = P_2 = 0 \)) and therefore \( \lambda_1 = \lambda_2 \equiv \lambda = \omega Q \). It follows that the wave equation (1.2) becomes

\[
\frac{d^2 \phi}{d\bar{\rho}^2} + 2 \frac{d\phi}{d\bar{\rho}} + \left[ (1 + \frac{\tilde{\lambda}_1}{\bar{\rho}})(1 + \frac{\tilde{\lambda}_2}{\bar{\rho}}) - \frac{\ell(\ell + 1)}{\bar{\rho}^2} \right] \phi = 0 \quad (3.1)
\]

where

\[
\tilde{\lambda}_1 = \frac{\lambda}{\sqrt{1 - (m/\omega)^2}}, \quad \tilde{\lambda}_2 = \lambda \sqrt{1 - (m/\omega)^2}, \quad \tilde{\rho} = \rho \sqrt{1 - (m/\omega)^2}, \quad (3.2)
\]

Thus the wave equation of a minimally-coupled massive scalar on a \( D = 4 \) black hole with two equal charges has precisely the same form as that of a minimally-coupled massless scalar on a \( D = 4 \) black hole with two different charges. The closed-form absorption probability for the latter case was calculated in [18] (see also [24]). The absorption probability for the former case is, therefore, given by

\[
P^{(\ell)} = \frac{1 - e^{-2\pi \sqrt{4\lambda^2 - (2\ell + 1)^2}}}{1 + e^{-\pi(2\lambda + \sqrt{4\lambda^2 - (2\ell + 1)^2})e^{-\pi\delta}}} \quad \omega \geq \ell + \frac{1}{2}, \quad (3.3)
\]
where
\[ \delta \equiv \tilde{\lambda}_1 + \tilde{\lambda}_2 - 2\lambda = \lambda[(1 - (m/\omega)^2)^{1/4} - (1 - (m/\omega)^2)^{-1/4}]^2 \geq 0 , \quad (3.4) \]

with \( P^{(\ell)} = 0 \) if \( \lambda \leq \ell + \frac{1}{2} \). In the non-relativistic case \((\omega \to m^+)\), the absorption probability takes the (non-singular) form \( P^{(\ell)} = 1 - e^{-2\pi \sqrt{4\lambda^2 - (2\ell + 1)^2}} \), with \( \lambda \sim mQ \).

The total absorption cross-section is given by:
\[ \sigma^{(\text{abs})} = \sum_{\ell \leq \lambda - \frac{1}{2}} \frac{\pi \ell (\ell + 1)}{\omega^2 - m^2} P^{(\ell)} \quad (3.5) \]

It is oscillatory with respect to the dimensionless parameter, \( M\omega \sim Q\omega = \lambda \). (\( M \) is the ADM mass of the black hole.) This feature was noted in [23] for Schwarzschild black holes and conjectured to be a general property of black holes due to wave diffraction. Probing particles feel an effective finite potential barrier around black holes, inside of which is an effective potential well. Such particles inhabit a quasi-bound state once inside the barrier. Resonance in the partial-wave absorption cross-section occurs if the energy of the particle is equal to the effective energy of the potential barrier. Each partial wave contributes a 'spike' to the total absorption cross-section, which sums to yield the oscillatory pattern. As the mass of the probing particles increases, the amplitude of the oscillatory pattern of the total absorption cross-section decreases.

4 Leading-order absorption for D3, M2 and M5-branes

In the previous two sections, we considered two examples for which the massive scalar wave equations can be solved exactly. In general, the wave function (1.2) cannot be solved analytically. In the case of low frequency, one can adopt a solution-matching technique to obtain approximate solutions for the inner and outer regions of the wave equations. In this section, we shall use such a procedure to obtain the leading-order absorption cross-sections for the D3, M2 and M5-branes.

We now give a detailed discussion for the D3-brane, for which we have \( D = 10, d = \tilde{d} = 4 \) and \( N = 1 \). We define \( \lambda \equiv \omega R \). It follows that the wave equation (1.2) becomes
\[ \left( \frac{1}{\rho^5} \frac{\partial}{\partial \rho} \rho^5 \frac{\partial}{\partial \rho} + 1 + \frac{(\omega R)^4}{\rho^4} - \sqrt{1 + \frac{(\omega R)^4}{\rho^4}} \left( \frac{m}{\omega} \right)^2 - \frac{\ell (\ell + 4)}{\rho^2} \right) \phi(\rho) = 0. \quad (4.1) \]

Thus, we are interested in absorption by the Coulomb potential in 6 spatial dimensions. For \( \omega R \ll 1 \) we can solve this problem by matching an approximate solution in the inner region to an approximate solution in the outer region. To obtain an approximate solution
in the inner region, we substitute $\phi = \rho^{-3/2} f$ and find that

$$\left( \frac{\partial^2}{\partial \rho^2} + \frac{2}{\rho} \frac{\partial}{\partial \rho} - \left( \frac{15}{4} + \ell (\ell + 4) \right) \frac{1}{\rho^2} + 1 + \frac{(\omega R)^4}{\rho^4} - \sqrt{1 + \frac{(\omega R)^4}{\rho^4} \left( \frac{m}{\omega} \right)^2} \right) f = 0. \tag{4.2}$$

In order to neglect $1$ in the presence of the $\frac{1}{\rho^2}$ term, we require that

$$\rho \ll 1. \tag{4.3}$$

In order for the scalar mass term to be negligible in the presence of the $\frac{1}{\rho^2}$ term, we require that

$$\rho \ll \left[ \left( \frac{\omega}{m} \right)^4 (\frac{15}{4} + \ell (\ell + 4)) - (\omega R)^4 \right]^{1/4}. \tag{4.4}$$

Physically we must have $m \leq \omega$. Imposing the low-energy condition $\omega R \ll 1$ causes (4.3) to be a stronger constraint on $\rho$ than is (4.4). Under the above conditions, (4.2) becomes

$$\left( \frac{\partial^2}{\partial \rho^2} + \frac{2}{\rho} \frac{\partial}{\partial \rho} - \left( \frac{15}{4} + \ell (\ell + 4) \right) \frac{1}{\rho^2} + \frac{(\omega R)^4}{\rho^4} \right) f = 0, \tag{4.5}$$

which can be solved in terms of cylinder functions. Since we are interested in the incoming wave for $\rho \ll 1$, the appropriate solution is

$$\phi_o = i \left( \frac{(\omega R)^4}{\rho^2} \left( J_{\ell+2}(\frac{\omega R)^2}{\rho}) + i N_{\ell+2}(\frac{\omega R)^2}{\rho} \right) \right), \quad \rho \ll 1, \tag{4.6}$$

where $J$ and $N$ are Bessel and Neumann functions. In order to obtain an approximate solution for the outer region, we substitute $\phi = \rho^{-5/2} \psi$ into (4.1) and obtain

$$\left( \frac{\partial^2}{\partial \rho^2} - \left( \frac{15}{4} + \ell (\ell + 4) \right) \frac{1}{\rho^2} + \frac{(\omega R)^4}{\rho^4} - \sqrt{1 + \frac{(\omega R)^4}{\rho^4} \left( \frac{m}{\omega} \right)^2} \right) \psi = 0. \tag{4.7}$$

In order to neglect $\frac{(\omega R)^4}{\rho^4}$ in the presence of the $\frac{1}{\rho^2}$ term, we require that

$$\rho \gg (\omega R)^2. \tag{4.8}$$

Within the scalar mass term, $\frac{(\omega R)^4}{\rho^4}$ can be neglected in the presence of $1$ provided that

$$\rho \gg \omega R. \tag{4.9}$$

Imposing the low-energy condition, $\omega R \ll 1$, causes (4.9) to be a stronger constraint on $\rho$ than (4.8). Under the above conditions, (4.7) becomes

$$\left( \frac{\partial^2}{\partial \rho^2} - \left( \frac{15}{4} + \ell (\ell + 4) \right) \frac{1}{\rho^2} + 1 - \left( \frac{m}{\omega} \right)^2 \right) \psi = 0. \tag{4.10}$$

Equation (4.10) is solved in terms of cylinder functions:

$$\phi_\infty = A \rho^{-2} J_{\ell+2}(\sqrt{1 - (m/\omega)^2} \rho) + B \rho^{-2} N_{\ell+2}(\sqrt{1 - (m/\omega)^2} \rho), \quad \rho \gg \omega R, \tag{4.11}$$
where \( A \) and \( B \) are constants to be determined.

Our previously imposed low-energy condition, \( \omega R \ll 1 \), is sufficient for there to be an overlapping regime of validity for conditions (4.3) and (4.9), allowing the inner and outer solutions to be matched. Within the matching region, all cylinder functions involved have small arguments. We use the same asymptotic forms of the cylinder functions as used by [18]. We find that

\[
A = \frac{4\ell^2 \Gamma(\ell + 3) \Gamma(\ell + 2)}{\pi(1 - (\frac{m}{\omega})^2)^{\frac{\ell + 2}{2}} (\omega R)^{2\ell}} \tag{4.12}
\]

The absorption probability is most easily calculated in this approximation scheme as the ratio of the flux at the horizon to the incoming flux at infinity. In general, this flux may be defined as

\[
F = i\rho^{d+1} \left( \frac{\partial \phi}{\partial \rho} - \phi \frac{\partial \tilde{\phi}}{\partial \rho} \right), \tag{4.13}
\]

where \( \phi \) here is taken to be the in-going component of the wave. From the approximate solutions for \( \phi \) in the inner and outer regions, where the arguments of the cylinder functions are large, we find that the in-going fluxes at the horizon and at infinity are given by

\[
F_{\text{horizon}} = \frac{4}{\pi} \omega^4 R^8, \quad F_\infty = \frac{A^2}{\pi \omega^4}. \tag{4.14}
\]

Thus, to leading order, the absorption probability, \( P \equiv F_{\text{horizon}}/F_\infty \), is

\[
P(\ell) = \frac{\pi^2 (1 - (\frac{m}{\omega})^2)^{\ell+2} (\omega R)^{4\ell+8}}{4^{2\ell+3}(\ell + 2)^2 ([\ell + 1])^4} \tag{4.15}
\]

In general, the phase-space factor relating the absorption probability to the absorption cross-section can be obtained from the massless scalar case considered in [21] with the replacement \( \omega \to \sqrt{\omega^2 - m^2} \):

\[
\sigma(\ell) = 2^n n/2 - 1 \Gamma(n/2 - 1)(\ell + n/2 - 1) \left( \frac{\ell + n - 3}{\ell} \right) (\omega^2 - m^2)^{(1-n)/2} P(\ell) \tag{4.16}
\]

where \( n = D - d \) denotes the number of spatial dimensions. Thus, for the D3-brane we find

\[
\sigma_{3\text{-brane}}(\ell) = \frac{\pi^4 (\ell + 3)(\ell + 1) [1 - (m/\omega)^2]^{\ell-1/2}}{(3)^2 \omega^{4\ell+3} ([\ell + 1])^4} \omega^{4\ell+3} R^{4\ell+8}. \tag{4.17}
\]

As can be seen, within our approximation scheme, the effects of a nonzero scalar mass amount to an overall factor in the partial absorption cross-section. Also, the s-wave absorption cross-section is increased by \( m \) and the higher partial wave absorption cross-sections are diminished by \( m \). This is to be expected, since the scalar mass serves to increase gravitational attraction as well as rotational inertia.

7
The above approximation scheme can be applied to massive scalar particles in all \( N = 1 \) \( p \)-brane backgrounds except for the case of \( D = 11 \) \( p \)-branes with \( \tilde{d} = 4 \) and \( \tilde{d} = 5 \), in which cases the scalar mass term cannot be neglected in the inner region. For \( N > 1 \), we are unable to find solvable approximate equations which give an overlapping inner and outer region.

For the M2-brane, we have \( D = 11, d = 3, \tilde{d} = 6 \) and \( N = 1 \):

\[
\sigma^{(l)}_{\text{M2-brane}} = \frac{\pi^{5}(l + 5)(l + 4)[1 - (m/\omega)^2]^{1/2}}{(15)2^{d+2}l!(l + 2)!}\omega^{3l+2}R^{3l+9} \tag{4.18}
\]

For the M5-brane, we have \( D = 11, d = 6, \tilde{d} = 3 \) and \( N = 1 \):

\[
\sigma^{(l)}_{\text{M5-brane}} = \frac{2^{2\ell+5}\pi^{3}(l + 2)(l + 3/2)(l + 1)[(l + 1)!]^2[1 - (m/\omega)^2]^{1/2}}{(2\ell + 3)^2[(2\ell + 2)!]^4}\omega^{6\ell+5}R^{6\ell+9} \tag{4.19}
\]

In fact, for all \( N = 1 \) \( p \)-branes, other than the two for which the approximation scheme cannot be applied, the partial absorption cross-sections have the same additional factor due to the scalar mass:

\[
\sigma_{\text{massive}}^{l} = \sigma_{\text{massless}}^{l}[1 - (m/\omega)^2]^{1/2}, \tag{4.20}
\]

for \( m \leq \omega \), and \( \sigma_{\text{massless}}^{l} \) has the same form as the leading-order absorption for massless scalars. Note that the suppression [enhancement] of the partial cross-section for \( \ell \geq 1 \) [for \( \ell = 0 \)], when the non-relativistic limit is taken.

## 5 Conclusions

In this paper we have addressed the absorption cross-section for minimally-coupled massive particles in the extreme \( p \)-brane backgrounds. In particular, we found exact absorption probabilities in the cases of the extreme self-dual dyonic string in \( D = 6 \) and two equal-charge extreme black hole in \( D = 4 \). Notably these two examples yield the same wave equations as that of the minimally coupled massless scalar in the \( D = 6 \) extreme dyonic string, and two charge \( D = 4 \) extreme black hole backgrounds, respectively. Namely, one of the two charge parameters in the latter (massless) case is traded for the scalar mass parameter in the former (massive) case. Thus, for these equal charge backgrounds, the scattering of minimally-coupled massive particles can be addressed explicitly, and the distinct behavior of the absorption cross-section on the energy \( \omega \) (or equivalently momentum \( p \equiv \sqrt{\omega^2 - m^2} \)) is studied. In particular, the non-relativistic limit of the particle motion gives rise to a distinct, resonant-like absorption behavior in the case of the self-dual dyonic string.
We have also found corrections due to the scalar mass for the leading-order absorption cross-sections for D3-, M2- and M5-branes. In particular, in the non-relativistic limit, there is the expected suppression [enhancement] in the absorption cross-section for partial waves $\ell \geq 1$ [$\ell = 0$].

The results obtained for the absorption cross-section of the minimally-coupled massive scalars, in particular those in the extreme self-dual dyonic string background, may prove useful in the study of AdS/CFT correspondence [13]. Namely, the near-horizon region of the extreme dyonic string background has the topology of $AdS_3 \times S^3$, with the $AdS_3$ cosmological constant $\Lambda$ and the radius $R$ of the three-sphere ($S^3$) related to the charge $Q$ of the self-dual dyonic string as $\Lambda = R^2 = \sqrt{Q}$ (see e.g., [27]). On the other hand, the scattering of the minimally-coupled massive fields (with mass $M$) in the $AdS_3$ background yields information [20, 21] on the correlation functions of the operators of the boundary $SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$ conformal field theory [28] with conformal dimensions $h_\pm = \frac{1}{2}(1 \pm \sqrt{1 + M^2 \Lambda^2})$ [29]. The scattering analyzed here corresponds to that of a minimally-coupled massive scalar in the the full self-dual string background, rather than in only the truncated $AdS_3$ background. These explicit supergravity results may, in turn, shed light on the pathologies of the conformal field theory of the dyonic string background [30].

References

[1] A. Dhar, G. Mandal and S.R. Wadia, Absorption vs decay of black holes in string theory and T-symmetry, Phys. Lett. B388 (1996) 51, [hep-th/9605234].

[2] S.S. Gubser and I.R. Klebanov, Emission of charged particles from four- and five-dimensional black holes, Nucl. Phys. B482 (1996) 173, [hep-th/9608108].

[3] J. Maldacena and A. Strominger, Black hole greybody factors and D-brane spectroscopy, Phys. Rev. D55 (1997) 861, [hep-th/9609026].

[4] C.G. Callan, S.S. Gubser, I.R. Klebanov, and A.A. Tseytlin Absorption of fixed scalars and the D-brane approach to black holes, Nucl. Phys. B489 (1997) 65, [hep-th/9610172].

[5] I.R. Klebanov and S.D. Mathur, Black hole greybody factors and absorption of scalars by effective strings, Nucl. Phys. B500 (1997) 115, [hep-th/9701187].

[6] I.R. Klebanov, World volume approach to absorption by non-dilatonic branes, Nucl. Phys. B496 (1997) 231, [hep-th/9702076].
[7] S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, *String theory and classical absorption by threebranes*, Nucl. Phys. **B499** (1997) 217, [hep-th/9703040](http://arxiv.org/abs/hep-th/9703040).

[8] M. Cvetič and F. Larsen, *General rotating black holes in string theory: grey body factors and event horizons*, Phys. Rev. **D56** (1997) 4994, [hep-th/9705192](http://arxiv.org/abs/hep-th/9705192).

[9] M. Cvetič and F. Larsen, *Grey body factors for rotating black holes in four-dimensions*, Nucl. Phys. **B506** (1997) 107, [hep-th/9706071](http://arxiv.org/abs/hep-th/9706071).

[10] R. Emparan, *Absorption of scalars by extended objects*, Nucl. Phys. **B516** (1998) 297, [hep-th/9706204](http://arxiv.org/abs/hep-th/9706204).

[11] S.S. Gubser, A. Hashimoto, I.R. Klebanov and M. Krasnitz, *Scalar Absorption and the breaking of the world-volume conformal invariance*, Nucl. Phys. **B526** (1998) 393, [hep-th/9803023](http://arxiv.org/abs/hep-th/9803023).

[12] H.W. Lee and Y.S. Myung, *Greybody factor for the BTZ black hole and a 5D black hole*, [hep-th/9804095](http://arxiv.org/abs/hep-th/9804095).

[13] S.D. Mathur and A. Matusis, *Absorption of partial waves by three-branes*, [hep-th/9805064](http://arxiv.org/abs/hep-th/9805064).

[14] S.S. Gubser and A. Hashimoto, *Exact absorption probabilities for the D3-brane*, [hep-th/9805140](http://arxiv.org/abs/hep-th/9805140).

[15] M. Taylor-Robinson, *The D1-D5 brane system in six dimensions*, [hep-th/9806132](http://arxiv.org/abs/hep-th/9806132).

[16] H. Awata and S. Hirano, *AdS7/CFT6 correspondence and matrix models of M5-branes*, [hep-th/9812218](http://arxiv.org/abs/hep-th/9812218).

[17] M. Cvetič, H. Lü, C.N. Pope and T.A. Tran, *Exact absorption probability in the extremal six-dimensional dyonic string background*, [hep-th/9901002](http://arxiv.org/abs/hep-th/9901002).

[18] M. Cvetič, H. Lü, C.N. Pope and T.A. Tran, *Closed-form absorption probability of certain D = 5 and D = 4 black holes and leading-order cross-section of generic extremal p-branes*, [hep-th/9901115](http://arxiv.org/abs/hep-th/9901115).

[19] J. Maldacena, *The large N limit of superconformal field theories and supergravity*, [hep-th/9711200](http://arxiv.org/abs/hep-th/9711200).

[20] S.S Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlations from non-critical string theory*, Phys. Lett. **B428** (1998) 105, [hep-th/9802109](http://arxiv.org/abs/hep-th/9802109).
[21] E. Witten, *Anti-de Sitter space and holography*, Adv. Theor. Math. Phys. 2 (1998) 253, hep-th/9802150.

[22] J. Dougall, *The solutions to Mathieu’s differential equation*, Proceedings of the Edinburgh Mathematical Society, **XXXIV** (1916) 176.

[23] M. Cvetič and D. Youm, *Dyonic BPS saturated black holes of heterotic string on a six torus*, Phys. Rev. **D53** (1996) 584, hep-th/9507090.

[24] V. Balasubramanian and F. Larsen, *Extremal branes as elementary particles*, Nucl. Phys. **B495** (1997) 206, hep-th/9610077.

[25] N. Sanchez, *The black hole: scatterer, absorber and emitter of particles*, In: String Theory in Curved Space Times (1998), hep-th/9711068.

[26] W.G. Unruh, *Absorption cross section of small black holes*, Phys. Rev. **D14** (1976) 3251.

[27] M. Cvetič and F. Larsen, *Near horizon geometry of rotating black holes in five-dimensions*, Nucl. Phys. **B531** (1998) 239, hep-th/9805097.

[28] J.D. Brown and M. Henneaux, *Central Charges In The Canonical Realization Of Asymptotic Symmetries: An Example From Three-Dimensional Gravity*, Commun. Math. Phys. **104** (1986) 207.

[29] V. Balasubramanian, P. Kraus and A. Lawrence, *Bulk vs. boundary dynamics in anti-de Sitter space-time*, Phys. Rev. **D59** (1999) 046003, hep-th/9805171.

[30] N. Seiberg and E. Witten, *The D1 / D5 system and singular CFT*, hep-th/9903224.