Cosmic ray driven instabilities in a plasma flow upstream of a supernova shock

S M Osipov\textsuperscript{1} and A M Bykov\textsuperscript{1,2}

\textsuperscript{1} Ioffe Institute, 26 Politekhnicheskaya st., St. Petersburg 194021, Russia
\textsuperscript{2} Peter The Great St. Petersburg Polytechnic University, 29 Politekhnicheskaya st., St. Petersburg 195251, Russia

E-mail: osm2004@mail.ru

Abstract. Forward shocks in supernova remnants are known as efficient cosmic ray (CR) accelerators. They may convert a sizeable fraction of the shock ram pressure into relativistic particles with highly non-equilibrium distribution functions. We discuss here a specific unstable mode driven by the CR-current and CR-pressure gradients which is producing the magnetic fluctuations in a cold plasma of the shock upstream. The growth rates of the new CR-driven mode propagating obliquely to the mean magnetic field are presented in the short-wavelength limit and compared to the known Bell’s CR-current and Drury-Dorfi-Falle acoustic instabilities.

1. Introduction

Energetic particle acceleration at collisionless shocks by the Fermi mechanism – the diffusive shock acceleration (DSA) is one of the most realistic way to produce bulk of the galactic cosmic rays (e.g. [1, 2]). The acceleration occurs due to the efficient scattering of the fast superthermal particles by supersonically moving magnetic field fluctuations providing the multiple crossings of the shock front with the jump of the MHD flow velocity (see e.g. [3, 4, 5, 6] for a review). The magnetic turbulence with a very broad dynamical range of the fluctuations providing energetic particle scattering in the shock upstream can be efficiently produced by the accelerated CRs [7, 8, 9, 10, 11, 12]. In the case of highly efficient CR acceleration by shocks a sizeable fraction of the shock ram pressure can be transferred into the highest energy end of accelerated CRs with a non-equilibrium distribution and CR pressure gradient (see e.g. [13]) which may be the source of free energy to produce the magnetic turbulence by a number of instabilities. The acoustic type instability due to CR pressure gradient in the shock precursors was found in [14, 15, 11] while a fast CR current driven instability was discovered by A.Bell [8, 9]. The presence of the CR pressure gradient may have an effect on the CR current driven instability. In this paper we present an analysis of a system with both the CR pressure gradient and the CR current and show that the new unstable modes can growth in this case which is typical for DSA models of the supernova shocks.

2. Dynamics of cold plasma interacting with cosmic rays

The equations of momentum conservation of a quasi-neutral system consisting from the background plasma with a relativistic component which is appropriate for the upstream flow in the collisionless shock waves is given by (see [12] for details):
\[ \rho \left( \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \nabla) \mathbf{u} \right) = -\nabla p_g + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B} - \frac{1}{c} (j^{cr} - en_{cr} \mathbf{u}) \times \mathbf{B} - \int \mathbf{p} I[f] d^3 p, \tag{1} \]

\[ \frac{\partial P_\alpha}{\partial t} + \nabla \alpha P_{cr} + \nabla \beta \Pi_{\alpha \beta} = \left[ \frac{1}{c} (j^{cr} - en_{cr} \mathbf{u}) \times \mathbf{B} + \int \mathbf{p} I[f] d^3 p \right] \alpha, \tag{2} \]

where \( \mathbf{B} \) is magnetic field which is frozen into plasma, \( \rho, p_g \) and \( \mathbf{u} \) are the density, pressure and bulk velocity of the background plasma respectively. While \( n_{cr} \) and \( j^{cr} \) are the number density and the electric current of the relativistic component (cosmic rays). The CR momentum density of CRs is \( P \), and the stress-energy tensor of CRs is \( \Pi_{\alpha \beta} \). CRs number density is assumed to be much less than that of background plasma while the CR pressure \( p_{cr} \) is typically larger than \( p_g \). The averaged distribution function of CRs \( f \) is determined by a kinetic equation (see [12]) with the collisional integral \( I[f] \) which is determined the interaction of CRs and the background plasma. We used here the simple relaxation time approximation for \( I[f] \) (see [10, 12]) which in the rest frame of the background plasma can be reduced to:

\[ I[f] = -\nu (f - f_{iso}), \tag{3} \]

where \( f_{iso} \) is the isotropic part of the CR distribution function and \( \nu \) is the collision rate. It is assumed that the magnetic turbulence which determines the collisional term Eq.(3) is statistically homogeneous and isotropic in the rest frame of the background plasma.

Furthermore, we assumed \( \nu = a \Omega \), where \( \Omega = \frac{e c B_0}{\mathcal{E}} \) is the gyro-frequency of a CR particle of the energy \( \mathcal{E} \) and \( a \) is the collisionality parameter. Then from Eq.(3) one can get

\[ \int \mathbf{p} I[f] d^3 p = -\frac{a B_0}{c} j^{cr}, \tag{4} \]

which is a ponderomotive force in Eq.(1).

In this paper we perform an analysis of the linear stability of the shock upstream flow where both the CR pressure gradient and the CR electric current are present. The shock may be oblique, with an arbitrary angle between the local magnetic field \( \mathbf{B}_0 \) to the shock normal. Consider a small perturbation of magnetic field \( \delta \mathbf{B} \) propagating along an arbitrary direction to the local mean field (cf [9] where the CR current driven modes without the CR pressure gradient were studied). However, we limit ourselves to the short wavelength limit where the mode wavenumber \( k \) is \( k \frac{v}{\nu} \gg 1 \) and \( k \frac{v}{\Omega} \gg 1 \). The magnetic fluctuations within this range of the wavenumbers do not produce a sizeable response of the CR distribution function contrary to that is in the long-wavelength limit considered in [12]. We consider the fluctuations in the rest frame of the background plasma. While the CR distribution is not steady in this system the corrections to the values under consideration are of the order \( u_{sh}^2 v^2 \), where \( u_{sh} \) is the shock speed, and are small for non-relativistic shocks. The unperturbed values (indexed with 0) in Eq.(2) satisfy the following equation

\[ \nabla p_{cr0} = \frac{1}{c} j^{cr}_0 \times \mathbf{B}_0 + \int \mathbf{p} I[f_0] d^3 p. \tag{5} \]

We further consider the systems with \( p_{cr0} \gg p_g \) and \( p_{cr0} \gg \frac{B_0^2}{4\pi} \) and use the following notations: \( \mathbf{e}_x, \mathbf{e}_y \) and \( \mathbf{e}_z \) are the unit orthogonal orts, \( \mathbf{B}_0 = B_0 (\mathbf{e}_x \cos \varphi + \mathbf{e}_z \sin \varphi) \). Here the
unit vector $\mathbf{e}_x$ is directed along the shock normal. Then using Eq.(4) one can get from Eq.(5):

$$j_0^r = en_{cr}u_{sh} \left( e_x - \frac{1}{a} \frac{\sin \varphi}{1 + \sin^2 \frac{\varphi}{a^2}} e_y + \frac{1}{a^2} \frac{\sin \varphi \cos \varphi}{1 + \cos^2 \frac{\varphi}{a^2}} e_z \right),$$

and

$$-\nabla p_{cr} = \frac{en_{cr}u_{sh}B_0}{c} a \frac{1 + \frac{1}{a^2}}{1 + \cos^2 \frac{\varphi}{a^2}} \mathbf{e}_x.$$

3. CR-driven instabilities: growth of the short-wavelength modes

The dispersion equations for the CR-driven modes in the short wavelength limit can be obtained by the standard procedure by imposing the small perturbations of the background values. The analysis is similar to that was done in [9], but with the account for the CR pressure gradient. Therefore, following [9], we assumed $k_0 \gg k$, where $k_0 = \frac{4\pi e n_{cr} u_{sh}}{c B_0}$. The dispersion equations in this case can be reduced to

$$\omega^4 = \left( k_x' \cos \varphi + k_z' \sin \varphi \right)^2 \left( 1 + \frac{\sin^2 \varphi}{a^2} \left( 1 + \cos^2 \frac{\varphi}{a^2} \right)^2 \right) + \frac{1}{a^2} \frac{\sin^2 \varphi \cos^2 \varphi}{\left( 1 + \cos^2 \frac{\varphi}{a^2} \right)^2},$$

$$\omega^2 = -i \left( k_x' a - k_y' \frac{\sin \varphi}{1 + \cos^2 \frac{\varphi}{a^2}} + k_z' \frac{1}{a} \frac{\sin \varphi \cos \varphi}{1 + \cos^2 \varphi} \right),$$

where $\omega^2 = \frac{\omega^2}{v_a^2 k_0 \mu}$, $v_a = \frac{B_0}{\sqrt{4\pi \rho_0}}$ is the alfvenic speed, $k' = \frac{k}{k_0}$. The mode determined by Eq.(8) is the extension of the Bell’s mode [9] which has the growth rate $\gamma_B = \left( \frac{(k_B) \|j\|}{c \rho_0} \right)^2$. Note that in Fig. 1 we plotted the dimensionless growth rates as given above.

4. Summary

We studied the instabilities of cold plasma with cosmic rays as it is expected to be typical for the upstream flows of strong collisionless shocks in supernova remnants. The model simultaneously accounts for the presence of the CR pressure gradient as well as the CR current. A study of the full dispersion equation in the short wavelength limit revealed two growing modes. The mode which is determined by Eq.(8) has a growth rate $\gamma_B$. This mode is a modification of the Bell’s CR current driven mode [9] by the effect of the CR pressure gradient. Moreover, a new short wavelength mode was found with the growth rate $\gamma_D$ which is determined by the dispersion equation Eq.(9). This mode in the long wavelength limit is associated with the acoustic mode driven by CR pressure gradient [14, 16]. In Fig. 1 we compare the growth rates of the modes given by Eq.(8) and Eq.(9). The new short wavelength modes have the distinct angular dependence of their growth rates ($\gamma_D$ and $\gamma_B$) as it is clearly seen in Fig. 1. In particular, for $\varphi = 0$

$$\gamma_D \sim \sqrt{|k'_x|} \quad \text{and} \quad \gamma_B \sim \sqrt{|k'_y|}, \quad \text{for} \quad \varphi = \frac{\pi}{2} \quad \gamma_D \sim \sqrt{|k'_x a - k'_y|} \quad \text{and} \quad \gamma_B \sim \sqrt{|k'_z|} \sim \sqrt{|\mu|},$$

where $k'_x = \sqrt{1 - \mu^2} \cos \psi$, $k'_y = \sqrt{1 - \mu^2} \sin \psi$, $k'_z = \mu$.

The collisionality parameter $a$ control the ratio of the maximal growth rate of the new mode to the CR current driven mode by Bell [9]. The maximal growth rate of the new mode is increasing with the collisionality parameter $a$. 

\[3\]
Figure 1. The angular dependence of the growth rates of the CR-driven mode derived from Eq.(8) and Eq.(9). Left panel: the growth rate of the mode denoted $\gamma_D$ derived from Eq.(9). In the right panel we show $\gamma_B$ derived from Eq.(8). $k'_x = \sqrt{1 - \mu^2} \cos \psi$, $k'_y = \sqrt{1 - \mu^2} \sin \psi$, $k'_z = \mu$. The parameter $a=0.5$. At the top panels $\varphi = \frac{\pi}{3}$, while $\varphi = \frac{\pi}{6}$ at the bottom panels.

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