Anthropic prediction for $\Lambda$ and the $Q$ catastrophe

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We discuss probability distributions for the cosmological constant $\Lambda$ and the amplitude of primordial density fluctuations $Q$ in models where they both are anthropic variables. With mild assumptions about the prior probabilities, the distribution $P(\Lambda, Q)$ factorizes into two independent distributions for the variables $Q$ and $y \propto \Lambda/Q^3$. The distribution for $y$ is largely model-independent and is in a good agreement with the observed value of $y$. The form of $P(Q)$ depends on the origin of density perturbations. If the perturbations are due to quantum fluctuations of the inflaton, then $P(Q)$ tends to have an exponential dependence on $Q$, due to the fact that in such models $Q$ is correlated with the amount of inflationary expansion. For simple models with a power-law potential, $P(Q)$ is peaked at very small values of $Q$, far smaller than the observed value of $10^{-5}$. This problem does not arise in curvaton-type models, where the inflationary expansion factor is not correlated with $Q$.

I. INTRODUCTION

The fundamental theory of nature may admit multiple vacua with different low-energy constants. This possibility has attracted much attention in the context of “string theory landscape” [1–3]. When combined with the theory of eternal inflation [4,5], it leads to the picture of a “multiverse”, where constants of nature take different values in different post-inflationary (thermalized) regions of spacetime. The key problem in this theoretical
framework is to calculate the probability distribution for the constants. Once we have the distribution, we can use the principle of mediocrity - the assumption that we are typical among the observers in the universe - and make predictions for the constants at a specified confidence level.

A major success of this program has been the prediction of a non-zero cosmological constant $\Lambda$ [6–9]. The observed value of $\Lambda$ is well within the $2\sigma$ range of the theoretical distribution. Recently, however, it has been argued that this successful prediction does not survive when other parameters, such as the amplitude of primordial density fluctuations $Q$, are also allowed to vary [10,11]. Here, we critically examine these claims and discuss some issues surrounding the calculation of prior probabilities. We start with a review of the original prediction for $\Lambda$.

After this work was completed, we noticed the paper [12] by Feldstein, Hall and Watari, which has a substantial overlap with our work.

**II. ANTHROPIC PREDICTION FOR $\Lambda$**

The key observation, due to Weinberg, is that structure formation effectively stops when $\Lambda$ comes to dominate the universe. If no galaxies are formed by that time, then in the future there will also be no galaxies, and therefore no observers. The corresponding anthropic bound on the cosmological constant is [6]

$$\Lambda \lesssim 500\rho_{m0}$$  \hspace{1cm} (1)

where $\rho_{m0}$ is the present matter density. A lower bound on $\Lambda$ can be obtained by requiring that the universe does not recollapse before life had a chance to develop. Assuming that the timescale for life evolution is comparable to the present cosmic time, one finds [13,14] $\Lambda \gtrsim -\rho_{m0}$.

The anthropic bound (1) specifies the value of $\Lambda$ which makes galaxy formation barely possible. However, if the effective $\Lambda$ varies in space, then most of the galaxies will not be
in regions characterized by this marginal value, but rather in regions teeming with galaxies, where $\Lambda$ dominates after a substantial fraction of matter has already clustered.

To make this quantitative, we define the probability distribution $P(\Lambda)d\Lambda$ as being proportional to the number of observers in the universe who will measure $\Lambda$ in the interval $d\Lambda$. This distribution can be represented as a product [7]

$$P(\Lambda)d\Lambda = n_{obs}(\Lambda)P_{\text{prior}}(\Lambda)d\Lambda. \quad (2)$$

Here, $P_{\text{prior}}(\Lambda)d\Lambda$ is the prior distribution, which is proportional to the volume of those parts of the universe where $\Lambda$ takes values in the interval $d\Lambda$, and $n_{obs}(\Lambda)$ is the number of observers that are going to evolve per unit volume. The distribution (2) gives the probability that a randomly selected observer is located in a region where the effective cosmological constant is in the interval $d\Lambda$.

Of course, we have no idea how to calculate $n_{obs}$, but what comes to the rescue is the fact that the value of $\Lambda$ does not directly affect the physics and chemistry of life. As a rough approximation, we can then assume that $n_{obs}(\Lambda)$ is simply proportional to the fraction of matter $f$ clustered in giant galaxies like ours (with mass $M \gtrsim M_G = 10^{12}M_\odot$),

$$n_{obs}(\Lambda) \propto f_G(\Lambda). \quad (3)$$

The idea is that there is a certain number of stars per unit mass in a galaxy and certain number of observers per star. The same approximation can be used for other “life-neutral” parameters, like the amplitude of primordial density fluctuations $Q$ or the neutrino masses.

Now we have to decide what should be used for the prior probability in (2). At present, the details of the fundamental theory and of the inflationary dynamics are too uncertain for a definitive calculation of the prior. We shall instead rely on the following general argument [15,16].

Suppose some parameter $X$ varies in the range $\Delta X$ and is characterized by a prior distribution $P_{\text{prior}}(X)$. Suppose further that $X$ affects the number of observers in such a way that this number is non-negligible only in a very narrow range $\Delta X_{\text{obs}} \ll \Delta X$. Then one
can expect that the function $P_{\text{prior}}(X)$ with a large characteristic range of variation should be very nearly a constant in the tiny interval $\Delta X_{\text{obs}}$. In the case of $\Lambda$, the range $\Delta \Lambda$ is set by the Planck scale or by the supersymmetry breaking scale, and we have $(\Delta \Lambda)_{\text{obs}}/\Delta \Lambda \sim 10^{-60} - 10^{-120}$. Hence, we expect

$$P_{\text{prior}}(\Lambda) \approx \text{const.}$$  \hspace{1cm} (4)

We emphasize that the assumption here is that the value $\Lambda = 0$ is not in any way special, as far as the fundamental theory is concerned, and is, therefore, not a singular point of $P_{\text{prior}}(\Lambda)$.

Combining Eqs.(2),(3),(4), we obtain

$$P(\Lambda) \propto f_G(\Lambda).$$  \hspace{1cm} (5)

The fraction of clustered matter, $f_G(\Lambda)$, can be calculated using the Press-Schechter approximation. Restricting attention to positive values of $\Lambda$, one finds [9,17]

$$f_G(\Lambda) \propto \text{erfc}\left(0.8 \left(\frac{\Lambda}{\rho_m \sigma^3_G}\right)^{1/3}\right),$$  \hspace{1cm} (6)

where $\sigma_G$ is the density contrast on the galactic scale and $\rho_m$ is the matter density. The product $\rho_m \sigma^3_G$ is time-independent during the matter era, so it can be evaluated at any time.

Introducing a dimensionless variable

$$y \equiv \frac{\Lambda}{\rho_m \sigma^3_G},$$  \hspace{1cm} (7)

we can write the distribution as

$$dP(y) \propto \text{erfc}(0.8y^{1/3})y d(\ln y).$$  \hspace{1cm} (8)

This distribution is peaked at $y \sim 1$. Discarding 2.5% at both tails of (8) yields the range

$$0.043 < y < 16.$$  \hspace{1cm} (9)

This is the prediction for $y$ at 95% confidence level.
The product $\rho_m \sigma_G^3$ can be expressed in terms of the present-day values, $\rho_{m0}$ and $\sigma_{G0}$. Hence, $y$ is a measurable quantity, and the anthropic prediction can be tested. As already mentioned, the observed value of $y$ is well within the 95% range (9) (see, e.g., [18]).

Apart from this successful prediction, two additional factors make the anthropic explanation of the observed value of $\Lambda$ particularly compelling. First, there are no plausible alternatives. Second, the anthropic approach also gives a natural resolution to the coincidence puzzle: Why do we happen to live at the very special epoch when $\Lambda \sim \rho_m$? If we denote the redshifts at the epochs of galaxy formation and of $\Lambda$-domination by $z_G$ and $z_\Lambda$, respectively, then most of the galaxies should be in regions where $z_\Lambda \lesssim z_G$. Regions with $z_\Lambda \gg z_G$ will have very few galaxies, while regions with $z_\Lambda \ll z_G$ will be rare simply because they correspond to a very narrow range of $\Lambda$ near zero (small “phase space”). Hence, we expect a typical galaxy to be located in a region where

$$z_\Lambda \sim z_G.$$  \hfill (10)

The galaxy formation epoch, $z_G \sim 1 - 3$, is close to the present cosmic time. This explains the coincidence [19,20].

### III. VARIABLE $Q$

Banks et. al. [10] and Graesser et. al. [11] have argued that the successful prediction for $\Lambda$ will be destroyed if the density fluctuation amplitude $Q$ is also allowed to vary. $Q$ is defined as the density contrast at the time of horizon crossing. It is approximately the same on all scales of astrophysical interest. The anthropically allowed range of $Q$ is [21]

$$10^{-6} \lesssim Q \lesssim 10^{-4}. \hfill (11)$$

The observed value, $Q \sim 10^{-5}$, is in the middle of this range.

For larger values of $Q$, galaxies form earlier, so $\Lambda$ can get larger without interfering with the galaxy formation process, $\Lambda_{\text{max}} \propto Q^3$. Since larger $\Lambda$ correspond to larger phase
space, this suggests that the probability is maximized for large \( Q \) and \( \Lambda \), e.g., \( Q \sim 10^{-4}, \Lambda \sim 10^3 \rho_m \).

To examine the situation more carefully, let us now consider the probability distribution for \( \Lambda \) and \( Q \). Since the galactic-scale density contrast \( \sigma_G \) is linearly related to \( Q \), \( \sigma_G \propto Q \), we can equivalently consider the distribution for \( \Lambda \) and \( \sigma_G \),

\[
dP(\Lambda, \sigma_G) \propto P_{\text{prior}}(\Lambda, \sigma_G) \text{erfc}(0.8y^{1/3})d\Lambda d\sigma_G,
\]

where, as before, \( y \) is given by Eq. (7). The narrow anthropic range of \( \Lambda \) suggests, also as before, that the prior probability is independent of \( \Lambda \) in the range of interest,

\[
P_{\text{prior}}(\Lambda, \sigma_G) \approx P_{\text{prior}}(\sigma_G).
\]

Substituting this in (12) and changing variables from \((\Lambda, \sigma_G)\) to \((y, \sigma_G)\), we obtain [19]

\[
dP(y, \sigma_G) \propto \sigma_G^3 P_{\text{prior}}(\sigma_G) d\sigma_G \cdot \text{erfc}(0.8y^{1/3})dy.
\]

Remarkably, the distribution for \( y \) and \( \sigma_G \) is factorized. Moreover, the \( y \)-part of the distribution is exactly the same as it was for the original model, where \( \sigma_G \) was not variable. This means that the successful prediction for \( y \) is completely unaffected by the variation of \( \sigma_G \) [17].

**IV. THE LARGE \( Q \) CATASTROPHE**

Let us now consider the distribution for \( \sigma_G \),

\[
dP(\sigma_G) \propto \sigma_G^3 P_{\text{prior}}(\sigma_G) d\sigma_G.
\]

The value of \( \sigma_G \) is determined by the horizon-crossing perturbation amplitude \( Q \), which is in turn determined by the inflaton potential \( V(\phi) \),

\[
\sigma_G \propto Q \propto \frac{V^{3/2}}{V'},
\]

\[6\]
where the right-hand side is evaluated at the value of \( \phi \) corresponding to horizon crossing for the comoving scale of interest. With a power-law potential

\[
V(\phi) = \lambda \phi^n, \quad (17)
\]

this gives, neglecting logarithmic factors (see, e.g., [22]),

\[
Q \propto \lambda^{1/2}. \quad (18)
\]

Suppose now that \( \lambda \) is a variable, which may be determined by some additional scalar field. It seems natural to assume that the range of variation for \( \lambda \) is \( \Delta \lambda \sim 1 \). The observed value of \( \sigma_G \) is obtained for \( \lambda \sim 10^{-14} \), and the anthropic range (11) corresponds to \( 10^{-16} \lesssim \lambda \lesssim 10^{-12} \). Since this range is so narrow, Graesser et. al. [11] argue that the same logic we used for the cosmological constant implies that the prior for \( \lambda \) should be nearly flat in the range of interest,

\[
dP_{\text{prior}}/d\lambda \approx \text{const}. \quad (19)
\]

Then it follows from (18) that \( dP_{\text{prior}}(Q) \propto QdQ \), and Eq. (15) gives

\[
dP(Q) \propto Q^4dQ. \quad (20)
\]

This distribution is strongly biased in favor of large values of \( Q \). If anthropic factors cut off the distribution above \( Q^{(\max)} \sim 10^{-4} \), then Eq. (20) suggests that this cutoff value is \( 10^5 \) times more probable than the observed value \( Q \sim 10^{-5} \). This is the large \( Q \) catastrophe.

We note, however, that there is an important difference between the cosmological constant and the inflaton self-coupling \( \lambda \), which may invalidate the argument for the flat prior (19). Unlike the small cosmological constant, the value of \( \lambda \) has a strong effect on the dynamics of inflation. As we shall see later, smaller values of \( \lambda \) give larger inflationary expansion factors. Hence, \( \lambda = 0 \) is a rather special value of the coupling, and the flat prior assumption is not justified.
V. THE PROBLEM OF THE PRIOR

The calculation of prior probabilities is a very challenging and important problem. The main difficulty is that the volume of thermalized regions with any given values of the parameters (such as $Q$ or $\Lambda$) is infinite, even for a region of a finite comoving size. To compare such infinite volumes, one has to introduce some sort of a cutoff. For example, one could include only regions that thermalized prior to some time $t_c$ and evaluate volume ratios in the limit $t_c \to \infty$. However, one finds that the results are highly sensitive to the choice of the cutoff procedure (in this case, to the choice of the time coordinate $t$) [23]. (For a recent discussion, see [24,25].)

An eternally inflating universe consists of isolated thermalized regions embedded into the inflating background of false vacuum. These thermalized islands are rapidly expanding into the inflating sea, but the gaps between them are also expanding, making room for more thermalized islands to form. The thermalization surfaces at the boundaries between inflating and thermalized spacetime domains are three-dimensional, infinite, spacelike hypersurfaces. The spacetime geometry of an individual thermalized domain is most naturally described by choosing the corresponding thermalization surface as the origin of time. The domain then appears as a self-contained, infinite universe, with the thermalization surface playing the role of the big bang. Following Alan Guth, we shall call such infinite domains “pocket universes”. All pocket universes are spacelike-separated and are, therefore, causally disconnected from one another.\(^1\)

The reason for the cutoff-dependence of probabilities is that the volume of an eternally inflating universe is growing exponentially with time. The volumes of regions with all possible values of the parameters are growing exponentially as well. At any time, a substantial part of the total thermalized volume is “new” and thus close to the cutoff surface. It is not

\(^1\)In models where false vacuum decays through bubble nucleation, the role of pocket universes is played by individual bubbles.
surprising, therefore, that the result depends on how that surface is drawn.

It has been suggested in [26] that the resolution of this difficulty may lie in the direction of switching from a global distribution, defined with the aid of some globally defined time coordinate, to a pocket-based distribution. In a particular case, when the variable parameters take all their values within each pocket universe, the problem appears to have been solved completely. In this case, all pocket universes are statistically equivalent, and we can pick any one of them to calculate the distribution. The prior probability for the constants can be identified with the volume fraction occupied by the corresponding regions of the pocket universe. We can find this fraction by first evaluating it within a sphere of large radius $R$ and then taking the limit $R \to \infty$. This is the so-called "spherical cutoff method".

In general, however, there are several distinct types of pocket universes, and we have to face the problem of comparing volumes in different pockets. It is not clear how this can be done, since the spherical cutoff method cannot be applied to disconnected spaces. The problem of defining probabilities in the general case still remains unresolved. A conjecture toward this goal will be discussed in the following section.

To get some idea of what the prior distribution for $Q$ may be like, we shall first focus on the case of identical pockets, where we know how to calculate probabilities. The spacetime structure of a pocket universe is illustrated in Fig. 1. The surface $\Sigma_*$ in the figure is the thermalization surface. It is the boundary between inflating and thermalized domains of spacetime, which marks the end of inflation and plays the role of the big bang in the pocket universe. The surface $\Sigma_q$ is the boundary between the stochastic domain, where the dynamics of the inflaton field is dominated by quantum diffusion, and the deterministic domain, where it is dominated by the deterministic slow roll. Thus, $\Sigma_q$ marks the onset of the slow roll.
The prior probability $P_{\text{prior}}(X)dX$ for a variable $X$ is defined in terms of the volume fraction on the thermalization hypersurface $\Sigma_*$. It can be expressed as

$$P_{\text{prior}}(X) \propto P_q(X)Z^3(X),$$

where $P_q(X)$ is the distribution (volume fraction) on $\Sigma_q$, and $Z(X)$ is the volume expansion factor during the slow roll. We assume for simplicity that the field responsible for the value of $X$ interacts very weakly with the inflaton, so that $X$ does not change appreciably during the slow roll. Otherwise, we would have to include an additional Jacobian factor in Eq. (21), as indicated in [26].

The expansion factor $Z(X)$ can be found from

$$Z(X) \approx \exp\left[4\pi \int_{\phi_*}^{\phi_X} \frac{H(\phi, X)}{H'(\phi, X)} d\phi \right],$$

where

$$H(\phi; X) = \left[8\pi V(\phi, X)/3\right]^{1/2}$$

FIG. 1. The spacetime structure of a pocket universe.
is the inflationary expansion rate, $V(\phi, X)$ is the inflaton potential, and prime stands for derivative with respect to $\phi$. $\phi_{qX}$ and $\phi_{sX}$ are the values of $\phi$ at the boundary surfaces $\Sigma_q$ and $\Sigma_s$, respectively. They are defined by the conditions

$$\frac{H'}{H^2}(\phi_{qX}, X) \sim 1$$

and

$$\frac{H'}{H}(\phi_{sX}, X) \sim 1.$$  

The distribution $P_q(X)$ can in principle be determined from numerical simulations of the quantum diffusion regime. Some useful techniques for this type of simulation have been developed in [23,27]. The analysis is greatly simplified in a class of models where the potential is independent of $X$ in the diffusion regime, and $P_q(X)$ can be determined from symmetry,

$$P_q(X) = \text{const}.$$  

An example is a “new” inflation type model with a complex inflaton, $\phi = |\phi| \exp(iX)$. Inflation occurs near the maximum of the potential at $\phi = 0$, and we assume in addition that the potential is symmetric near the top, $V = V(|\phi|)$. Eq. (26) follows if this property holds throughout the diffusion regime. In such models, the distribution (21) reduces to

$$P_{prior}(X) \propto Z^3(X).$$

Eq. (27) has a simple intuitive meaning: the prior probability is determined by the volume expansion factor during the slow roll.

VI. A CONJECTURE FOR A MORE GENERAL CASE

In more general models, the factor $P_q(X)$ may provide an additional dependence on $X$. It has been recently conjectured [28] that this factor is of the form
\[ P_q(X) \propto \exp[S(X)], \tag{28} \]

where

\[ S(X) = \frac{\pi}{H^2} \langle \phi_q X, X \rangle \tag{29} \]

is the Gibbons-Hawking entropy of de Sitter space.

The idea is that the evolution in the diffusion regime is \textit{ergodic}, in the sense that all quantum states are explored with equal weight. The conjecture is that this ergodic property extends to the diffusion boundary \( \Sigma_q \). The right-hand side of Eq.\,(28) is simply the number of quantum states for the specified values of the parameters (fields) \( X \) on the boundary. The conjecture (28) has been verified in some specific models, but the conditions of its validity are not presently clear.

In models with pockets of several different types, we need to introduce an additional factor \( p_j \), characterizing the relative probability of different pockets. The full expression for the prior probability is then given by

\[ P_{\text{prior}}(X; j) \propto p_j e^{S(X; j)} Z^3(X; j), \tag{30} \]

where the index \( j \) labels different types of pockets. A natural choice for the factor \( p_j \) is the so-called comoving probability, which can be defined as the probability for a test particle, starting near the top of the potential \( V(\phi, X) \), to end up in a pocket of type \( j \). This probability can be easily calculated using the Fokker-Planck equation.

One problem with this definition of \( p_j \) is that it has some dependence on the initial conditions assumed for the test particle. For example, if the potential has several peaks of comparable height, different values of \( p_j \) will be obtained starting from different peaks. An alternative definition of \( p_j \), which does not suffer from this problem, has been suggested in [28].

In the rest of this paper, we shall assume that all pockets are identical, so the prior is given by Eq.\,(27) for symmetric potentials and by Eq.\,(21) with \( P_q(X) \) from Eq.\,(28) in the more general case. We note that for power-law potentials like (17), inclusion of the factor
exp(S) does not change the qualitative character of the result. For such potentials, Eq. (22) gives

\[ Z \approx \exp\left(\frac{4\pi}{n} \phi_{qX}^2\right), \]  

(31)

where we have assumed that \( \phi_{qX} \gg \phi_* \) (which is always satisfied for \( \lambda \ll 1 \)). On the other hand, it follows from Eq. (24) that \( H(\phi_{qX}, X) \sim 1/\phi_{qX} \), and thus

\[ S(X) \sim \phi_{qX}^2. \]  

(32)

Hence, the exponents in the volume factor (27) and in the ergodic factor (28) are both proportional to \( \phi_{qX}^2 \), differing only by a numerical coefficient.

It also follows from Eqs. (24),(17) that \( \phi_q \sim \lambda^{-1/(n+2)} \). Hence, we can write

\[ P_{prior}(\lambda) \propto \exp(C\lambda^{-2/(n+2)}), \]  

(33)

where \( C \sim 1 \) is a constant.

**VII. THE SMALL-Q CATASTROPHE**

Let us now consider what Eq. (33) implies for the prior distribution \( P_{prior}(Q) \). Substituting the relation (18) between \( Q \) and \( \lambda \) in (33), we obtain

\[ P_{prior}(Q) \propto \exp(C'Q^{-4/(n+2)}) \]  

(34)

with \( C' \sim 1 \). Assuming that \( \lambda \) is bounded from above by \( \lambda \lesssim 1 \), this equation applies for \( Q \lesssim 1 \).

The distribution (34) has a very sharp peak at small \( Q \). The exponential decay towards larger \( Q \) overrides any power-law prefactors, and thus the large \( Q \) catastrophe is averted. But now there is an even bigger problem. According to the distribution (34), small values of \( Q \sim 10^{-6} \) at the low end of the anthropic range are many orders of magnitude more probable than the observed value of \( Q \sim 10^{-5} \). Thus, we have a full-blown small-\( Q \) catastrophe.
This situation is not specific to power-law potentials. Both the inflationary expansion factor and the ergodic factor depend exponentially on the parameters specifying $H(\phi)$, so the exponential form of $P_{\text{prior}}(Q)$ is generic.

**VIII. CURVATON-TYPE MODELS**

The conclusion appears to be that the density perturbations due to fluctuations of the inflaton are negligible, and thus the observed perturbations must have been generated by some other mechanism. This conclusion has been reached in [7], where it was argued that the fluctuations are probably due to topological defects. However, CMB observations have ruled out defects as the dominant source of perturbations.

Another mechanism, which is consistent with the data, is the curvaton model [29,30]. In the simplest version, it involves a light scalar field $\chi$ (the curvaton) of mass $m \ll H$. Quantum fluctuations during inflation randomize the values of $\chi$, with a Gaussian distribution of width $\sim H^2/m$. This distribution is nearly flat, $dP(\chi) \propto d\chi$, in the range $|\chi| \ll H^2/m$. The energy density of the curvaton is negligible during inflation, but at a later stage it comes to dominate the universe. The curvaton density perturbations are of the order

$$Q \sim \delta\chi/\chi,$$

where $\delta\chi \sim H$ is the quantum fluctuation of $\chi$ during inflation, at the epoch when the relevant scale crossed the horizon. Eventually, the curvaton decays, and the density perturbation (35) is transferred to the radiation.

An alternative class of models [31,32] assumes that $\chi$ is a modulus, whose value determines the masses and/or couplings of some other particles. The resulting perturbation amplitude is still given by (35). In both cases, we can write

$$Q \sim H/\chi.$$  

To evaluate the prior distribution for $Q$, we shall consider the simplest model, where the inflaton potential is fixed. If, for example, the potential is given by Eq.(17), then we assume
that $\lambda$ has the same value everywhere in the universe. The variation of $Q$ is then entirely due to the long-wavelength fluctuations of the field $\chi$.

The crucial difference from the case of inflaton-induced perturbations is that the energy of $\chi$ is subdominant during inflation, and thus the inflationary expansion factor $Z$ is not correlated with $Q$. The prior distribution for $Q$ is

$$dP_{\text{prior}}(Q) \propto d\chi \propto dQ/Q^2,$$

and Eq.(15) gives

$$dP(Q) \propto QdQ.$$

This appears to be an improvement over the models that we considered so far. The exponential dependence on $Q$ has disappeared, and the growth of the probability towards larger $Q$ is considerably milder than before. However, the value of $Q \sim 10^{-4}$ is still 100 times more probable than the value we observe. We shall now discuss some ways to address this milder version of the large-$Q$ catastrophe.

**IX. EVADING THE LARGE-$Q$ CATASTROPHE**

A possible attitude is simply to accept that the observed value of $Q$ has probability of about 1%. This is somewhat uncomfortable, but perhaps not unreasonably small.

Alternatively, we can consider a multi-component curvaton model with the potential

$$V(\chi) = \frac{1}{2}m^2\chi^2,$$

(39)

where

$$\chi^2 \equiv \sum_{a=1}^{N} \chi_a^2.$$

(40)

In this case, the distributions (37) and (38) are replaced by

$$dP_{\text{prior}}(Q) \propto \chi^{N-1}d\chi \propto Q^{-(N+1)}dQ$$

(41)
and

\[ dP(Q) \propto Q^{2-N} dQ. \]  

(42)

For \( N = 3 \), the distribution is flat on the logarithmic scale.

Finally, suppose the distribution indeed rises towards larger \( Q \). Then we expect to have \( Q \sim Q_{\text{max}} \), where \( Q_{\text{max}} \) is the upper boundary of the anthropic range, while the observed value is \( Q \ll Q_{\text{max}} \). It is possible, however, that \( Q_{\text{max}} \sim 10^{-4} \) is an overestimate, and the actual bound is not so far from the observed value of \( Q \sim 10^{-5} \).

The original argument by Tegmark and Rees [21] is that, as \( Q \) gets larger, galaxies form earlier, so typical galaxies are smaller and denser. As a result, the rate of stellar encounters that can disrupt planetary orbits gets larger, and becomes unacceptably large for \( Q > \sim 10^{-4} \).

However, it is conceivable that factors other than planetary orbit disruption play the key role in anthropic selection. One possibility is that perturbation of comets could be such a factor [17]. Comets move around the Sun, forming the Oort cloud of radius \( \sim 0.1 \) pc. Whenever the cloud is significantly perturbed by a passing star, a rain of comets is sent to the interior of the Solar system. Occasionally the comets hit the Earth, causing mass extinctions. Another threat is posed by nearby supernova explosions (see, e.g., [33] and references therein). Without attempting to estimate the rate of mass extinctions due to these effects, we note that the time it took to evolve intelligent beings after the last major extinction is comparable to the typical time interval between extinctions (\( \sim 10^8 \) years). This suggests that we are indeed close to the boundary of the anthropic range. A substantial increase in the rate of extinctions might interfere with the evolution of observers.

X. CONCLUSIONS

We studied the probability distribution for the cosmological constant \( \Lambda \) and the density contrast \( Q \) in models where both of these parameters are variable. With only mild assumptions about the prior probabilities, the distribution \( P(\Lambda, Q) \) factorizes into two independent
distributions for the variables $Q$ and $y = \Lambda/Q^2\rho_m$. (Here, $\rho_m$ is the density of nonrelativistic matter.) The distribution for $y$ is rather model-independent and is in a good agreement with the observed value of $y$. Thus, despite recent claims to the contrary, the successful anthropic prediction for $y$ is not destroyed by allowing $Q$ to vary.

The form of the probability distribution $P(Q)$ depends on the assumed model for the generation of density perturbations and on the mechanism responsible for the variation of $Q$ from one part of the universe to another. We first considered models where the perturbations are due to quantum fluctuations of the inflaton field $\phi$, and their magnitude varies due to the variation of the inflaton self-coupling $\lambda$. (The variation of $\lambda$ may be induced by fluctuations of some other light field. Alternatively, $\lambda$ may take a dense discretuum of values in different vacua of the landscape.) In this case, $P(Q)$ tends to have an exponential dependence on $Q$. The reason is that the same inflaton coupling $\lambda$ determines both the magnitude of $Q$ and the amount of inflationary expansion. The probability $P(Q)$ is proportional to the expansion factor, which depends exponentially on $\lambda$, and therefore on $Q$. For power-law, “chaotic” inflation models, we found that $P(Q)$ has an extremely sharp peak at small $Q$. Models of this kind cannot account for the observed density perturbations.

We next considered curvaton-type models, where the density perturbations are generated by one or several light fields $\chi_a$, other than the inflaton. In this case, the magnitude of $Q$ varies due to the fluctuations in the long-wavelength component of $\chi_a$, so there is no need to assume any variation of the couplings. The crucial difference from the case of inflaton-induced perturbations is that the energy of $\chi_a$ is subdominant during inflation, and thus the inflationary expansion factor is not correlated with $Q$. We find that, as a result, the distribution $P(Q)$ has a rather mild dependence on $Q$ and can even be flat in some models, in which case there is no conflict at all with observations. Both large and small-$Q$ catastrophes can thus be averted.

The models we have discussed are the simplest ones, with only two variable parameters. On the other hand, in the context of string theory landscape, we expect to have a multitude of variables, and perhaps none or very few parameters fixed. This multi-parameter space is
awaiting to be explored. Here, we shall only briefly comment on some issues relevant to the variation of $Q$.

A simple extension of the parameter space is achieved by combining the two models that we have discussed: the inflaton potential has a variable coupling $\lambda$, and there are, in addition, some light curvaton fields. As before, the probability distribution will grow exponentially towards small $\lambda$. This growth will eventually be cut off, due to constraints related to thermalization and baryogenesis. The resulting distribution will have a sharp peak at a value of $\lambda \ll 10^{-14}$, so most observers will find $\lambda$ to be close to that value. The density perturbations due to the inflaton will therefore be negligible. With $\lambda$ now fixed, the probability distribution for $Q$ will be the same as in curvaton-type models that we discussed earlier. An extension to more complicated inflaton potentials, specified by several variable parameters, requires further investigation.

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