SUPERRESOLVING DISTANT GALAXIES WITH GRAVITATIONAL TELESCOPES: KECK LASER GUIDE STAR ADAPTIVE OPTICS AND HUBBLE SPACE TELESCOPE IMAGING OF THE LENS SYSTEM SDSS J0737+3216

PHILIP J. MARSHALL,1 TOMMASO TREU,1,2 JASON MELBOURNE,3,4 RAPHAËL GIAYAZZI,1 KEVIN BUNDY,5 S. MARK AMMONS,3,4 ADAM S. BOLTON,5 SCOTT BURLES,7 JAMES E. LARKIN,8 DAVID LE MIGNANT,2,9 DAVID C. KOO,3 LÉON V. E. KOOPMANS,10 CLAIRE E. MAX,3,4 LEONIDAS A. MOUSTAKAS,11 ERIC STEINBRING,12 AND SHELLEY A. WRIGHT18

Received 2007 May 22; accepted 2007 August 31

ABSTRACT

We combine high-resolution images in four optical/infrared bands, obtained with the laser guide star adaptive optics (LGSAO) system on the Keck telescope and with the Hubble Space Telescope (HST), to study the gravitational lens system SDSS J0737+3216 (lens redshift 0.3223, source redshift 0.5812). We show that (under favorable observing conditions) ground-based images are comparable to those obtained with HST in terms of precision in the determination of the parameters of both the lens mass distribution and the background source. We also quantify the systematic errors associated with both the incomplete knowledge of the PSF and the uncertain process of lens galaxy light removal and find that similar accuracy can be achieved with Keck LGSAO as with HST. We then exploit this well-calibrated combination of optical and gravitational telescopes to perform a multiwavelength study of the source galaxy at 0.01″ effective resolution. We find the Sérsic index to be indicative of a disklke object, but the measured half-light radius ($r_e = 0.59 \pm 0.007$ stat $\pm 0.1$ sys kpc) and stellar mass ($M^* = 2.0 \pm 1.0$ stat $\pm 0.8$ sys $\times 10^9 M_\odot$) place it more than 3σ away from the local disk size-mass relation. The SDSS J0737+3216 source has the characteristics of the most compact faint blue galaxies studied and comparable size and mass to dwarf early-type galaxies in the local universe. With the aid of gravitational telescopes to measure individual objects’ brightness profiles to 10% accuracy, the study of the high-redshift size-mass relation may be extended by an order of magnitude or more beyond existing surveys at the low-mass end, thus providing a new observational test of galaxy formation models.

Subject headings: galaxies: fundamental parameters — gravitational lensing — instrumentation: adaptive optics — methods: data analysis — techniques: high angular resolution

Online material: color figure

1. INTRODUCTION

Galaxies do not appear in arbitrary combinations of luminosity, mass, and shape but instead obey empirical scaling relations (such as the fundamental plane for early-type galaxies). Explaining the origin and cosmic evolution of the scaling relations is a fundamental goal of galaxy formation theories.

As far as disk galaxies are concerned, the hierarchical structure formation scenario predicts a correlation between size and stellar mass, with width depending on the distribution of the initial spin of the dark halos (Fall & Efstathiou 1980). At any given mass, the expected distribution of sizes is well-approximated by a lognormal distribution. Qualitatively, this prediction is quite robust, although the exact forms of the correlation and the distribution depend on the details of baryonic processes such as energy feedback from star formation and bulge instability (Mo et al. 1998; Shen et al. 2003; Tonini et al. 2006; Dutton et al. 2007; Stringer & Benson 2007). Therefore, measuring the shape and width of the correlation provides not only a test of the standard paradigm but also valuable information on the poorly understood baryonic processes happening at subgalactic scales.

From an empirical point of view, the relation between size, luminosity (or, equivalently, surface brightness), and stellar mass is well established for disk galaxies in the local universe (e.g., Shen et al. 2003; Driver et al. 2005). Analysis of suitable objects in the Sloan Digital Sky Survey (SDSS) shows that at any given mass (luminosity) the distribution of galaxies is indeed well-approximated as lognormal, although the scaling with mass of the characteristic size and the width of the distribution are non-trivial. Defining disk galaxies as those being well fit by a single Sérsic component with index $n < 2.5$, Shen et al. (2003) find that above a characteristic stellar mass ($M_{*,0} \sim 10^{10}$), size scales rapidly with stellar mass ($R \sim M^{0.39}$) and the scatter is relatively small ($\sigma_{R/M} \sim 0.34$). Below the characteristic stellar mass the correlation flattens ($R \sim M^{0.14}$) and the scatter increases significantly ($\sigma_{R/M} \sim 0.47$).

At intermediate redshift (0.1 $\leq z \leq 1$) the nature and interpretation of the size-luminosity or size-mass relation is more uncertain. Several authors (e.g., Ferguson et al. 2004; Barden et al. 2005;
Trujillo et al. 2006; Melbourne et al. 2007) have used Hubble Space Telescope (HST) images to determine the sizes of intermediate- and high- \( z \geq 1 \) redshift galaxies down to the resolution and completeness limits of HST (roughly equivalent to 1 kpc and \( 10^{10} M_{\odot} \)). Recent studies conclude, taking selection effects into account, that there is significant evolution in the size-luminosity relation (Barden et al. 2005; Trujillo et al. 2006; Melbourne et al. 2007). However, it is hard to disentangle luminosity evolution from size evolution, ensure that samples at different redshifts are directly comparable, and compare results from different studies, as the selection criteria are often similar but not identical (e.g., color vs. morphology; morphology determined via Sérsic index vs. bulge-to-disk decomposition vs. concentration parameter vs. visual classification). Overall, it appears that disk galaxy evolution cannot be explained by pure luminosity or pure size evolution but requires a combination of both. In contrast, the relation between size and explained by pure luminosity or pure size evolution but requires an account, that there is significant evolution in the size-luminosity relation (Barden et al. 2005; Trujillo et al. 2006; Melbourne et al. 2007).

In this paper we present multicolor high-resolution images of the gravitational lens system SDSS J0737+3216 (Bolton et al. 2006), obtained with both the HST and the laser guide star adaptive optics (LGSAO) system on Keck II. The scientific goal of the analysis of this case study is twofold. First, we perform a detailed comparison of the results of the lens modeling across bands, showing that, when a bright nearby star is available for tip-tilt correction and conditions are favorable, the most important parameters can be measured with comparable accuracy with HST and Keck LGSAO. Second, we exploit this particular cosmic telescope to achieve superresolution of the source galaxy. (See McKean et al. [2007] for Keck LGSAO observations of a lens with a pointlike source.) With a lens magnification of \( \mu \geq 10 \), the resolution of the HST and Keck images (\( \sim 0.1'' \) FWHM) corresponds to a physical scale of 0.66 kpc/\( \mu \approx 0.05 \) kpc at the redshift of the source \( z_s = 0.5812 \), comparable to the resolution attainable from the ground when studying galaxies in the Virgo Cluster in 1'' seeing. We derive the Sérsic index, size, and stellar mass of the source, and show that by using gravitational telescopes the size-mass relation may be extended by an order of magnitude in size with respect to current studies, thus allowing one to probe, for example, whether the change in slope and intrinsic scatter below the characteristic mass persists to higher redshifts.

This paper is organized as follows. After describing the observations in § 2, we outline in §§ 3 and 4 two sources of systematic error and our strategies for dealing with them, before explaining our modeling methodology in § 5. In §§ 6 and 7 we present our results, which are then discussed (§§ 8 and 9) before we draw conclusions in § 10. Throughout this paper magnitudes are given in the AB system. We assume a concordance cosmology with matter and dark energy density \( \Omega_m = 0.3, \Omega_{\Lambda} = 0.7 \), and Hubble constant \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \).

2. DESCRIPTION OF THE OBSERVATIONS

2.1. NIRC2 on Keck

On 2006 December 11 we imaged SDSS J0737+3216 with the LGSAO system on the Keck II telescope. The images were taken in the \( K' \) band with the near-infrared camera (NIRC2) in the wide (\( 40'' \times 40'' \)) field of view. The pixel scale for this configuration is 0.04'' pixel\(^{-1} \). A total of 3120 s of exposure was obtained. To avoid saturating stars in the field, individual exposures were 1 minute in duration (divided into two 30 s co-additions). A dither was executed after every set of two exposures to improve the sky sampling. Dithers were randomly chosen using a script with a circular dither pattern of radius 3''. The laser was positioned at the center of each frame, rather than fixed on the central galaxy. E. Steinbring et al. (2008, in preparation) demonstrate that this method provides a more uniform AO correction over a larger area, in comparison with the fixed laser method. Observing conditions during the run were good.

The observations were made as part of the Center for Adaptive Optics Treasury Survey (CATS; J. E. Larkin et al. 2008, in
Four subexposures were obtained with a half-integer pixel offset within 10" after masking objects. Frames were then flat-fielded and sky flat frames were created from the individual science exposures. A sky frame and preparation, which aims to image an extragalactic AO observation. We picked a star pair that had a tip-tilt orientation and separations were made immediately following the lens observations. A further constraint on the PSF is provided by the multiple images produced by the lens, but the PSF varies over time, not only as a result of the "breathing" of the telescope over the course of an orbit, but also monotonically as the system ages; the Tiny Tim approximation is not always sufficient.

In order to accurately predict the data given a model lens image, we must convolve it with the PSF of the telescope. For the instruments on HST the PSF is calculable from the engineering parameters that characterize the optics and detectors, using the Tiny Tim package (Krist & Hook 1997). However, the PSF varies over time, not only as a result of the PSF of limited use. In principle, one could include some variable parameters to describe the model PSFs introduced above and then fit for them simultaneously with the lens model parameters. We show in §6 that there is indeed enough information in our data to constrain the PSF, thanks to the multiple images produced by the lens, but defer the investigation of model PSF parameters to further work. Here we take a pragmatic approach and use nearby unsaturated stars as estimates of the PSF at the position of the lens. For the case of SDSS J0737+3216 there are three suitable stars within 10" from the lens; we excluded small cutout images of these stars from the images from each instrument/filter combination. The properties of these stars (henceforth referred to as PSF1, PSF2, and PSF3) are given in Table 1. In addition, for the NIRC2 observations we used a fourth star as described in §2.1. The use of any one of these stellar model PSFs constitutes an assumption which we can test using the statistical model selection procedure we describe below.

This phenomenological model has the advantage that it takes into account the time variability of the PSF as well as possible, providing a simultaneous estimate of the PSF with the actual data. It also takes into account the details of the image combination procedure in a natural way; whatever was done to the pixels of the lens image was also done to the PSF. One disadvantage of our approach is the introduction of additional noise; however, the stars are significantly brighter than the lens system, and the

---

**TABLE 1**

| ID       | R.A. (J2000.0) | Decl. (J2000.0) | $\theta_h$ | $\theta_T$ | $m_K$ (AB) |
|----------|----------------|----------------|------------|------------|-------------|
| PSF0     | 07 03 11.84    | -08 20 51.8    | 0.0        | 17.8       | 15.0        |
| PSF1     | 07 37 28.54    | +32 16 10.2    | 8.5        | 14.7       | 18.2        |
| PSF2     | 07 37 28.46    | +32 16 12.3    | 6.3        | 16.4       | 18.1        |
| PSF3     | 07 37 28.51    | +32 16 24.2    | 5.6        | 28.4       | 18.3        |

**Notes:** $	heta_h$ is the angular separation between the PSF star and the lens system/laser spot chip position. $\theta_T$ is the angular separation between the PSF star and the tip-tilt star. PSF0 was observed only during the AO run. Units of right ascension are hours, minutes, and seconds, and units of declination are degrees, arcminutes, and arcseconds.

---

13 Prior to the taking of these deep images, shallow (420 s) integrations were obtained with ACS in both the F435W and F814W filters (Bolton et al. 2006) as part of the initial SLACS snapshot program. These data are not used here due to the low signal-to-noise ratio and significant cosmic-ray contamination, both of which prevent detailed study of the faint ring.

---

In order to accurately predict the data given a model lens image, we must convolve it with the PSF of the telescope. For the instruments on HST the PSF is calculable from the engineering parameters that characterize the optics and detectors, using the Tiny Tim package (Krist & Hook 1997). However, the PSF varies over time, not only as a result of the "breathing" of the telescope over the course of an orbit, but also monotonically as the system ages; the Tiny Tim approximation is not always sufficient.

Somewhat similarly, the PSF derived from first principles for an AO system is the sum of a Moffat profile for the seeing disk and the diffraction pattern due to the telescope itself. In practice, the seeing and the Strehl ratio vary over the course of a set of observations, making a priori predictions of the PSF of limited use.

In principle, one could include some variable parameters to describe the model PSFs introduced above and then fit for them simultaneously with the lens model parameters. We show in §6 that there is indeed enough information in our data to constrain the PSF, thanks to the multiple images produced by the lens, but defer the investigation of model PSF parameters to further work. Here we take a pragmatic approach and use nearby unsaturated stars as estimates of the PSF at the position of the lens. For the case of SDSS J0737+3216 there are three suitable stars within 10" from the lens; we excluded small cutout images of these stars from the images from each instrument/filter combination. The properties of these stars (henceforth referred to as PSF1, PSF2, and PSF3) are given in Table 1. In addition, for the NIRC2 observations we used a fourth star as described in §2.1. The use of any one of these stellar model PSFs constitutes an assumption which we can test using the statistical model selection procedure we describe below.

This phenomenological model has the advantage that it takes into account the time variability of the PSF as well as possible, providing a simultaneous estimate of the PSF with the actual data. It also takes into account the details of the image combination procedure in a natural way; whatever was done to the pixels of the lens image was also done to the PSF. One disadvantage of our approach is the introduction of additional noise; however, the stars are significantly brighter than the lens system, and the
pixel noise in the PSF images can, we believe, be safely neglected. Three other disadvantages of our approach are that the stellar spectra will not exactly match the spectra of the lens or source galaxies within a given filter, the position of the PSF stars within a pixel will not exactly match the intrapixel centroiding of the lens or source galaxies, and the PSF at the position of the stars will not exactly match that at the lens position. In the absence of a suitable interpolation scheme to solve these problems, we resign ourselves to having just three models to choose from, and we attempt to infer the most appropriate one of the three from the data. Following this procedure will give us an indication of the relative importance of accurately knowing the PSF. In other words, the variation of the results as a function of adopted PSF will give us an indication of the systematic error introduced by our approximate PSF. As we show in the next sections the parameters that we are interested in are fairly insensitive to the choice of PSF, and our ignorance of the PSF is not the dominant source of error in our analysis.

4. LENS GALAXY SUBTRACTION

As can be seen in Figure 1, the lens galaxy is much brighter than the (lensed) source galaxy and is a significant source of contamination at the arc positions. The usual approach to this profile is to subtract a smooth intensity distribution fitted to the lens galaxy light. Bolton et al. (2006) found it necessary to use a flexible B-spline model, combined with careful manual masking of the multiple images, in order to obtain a satisfactory removal of the lens light. The problem is that it is fundamentally very difficult to disentangle the light coming from the lens galaxy from that coming from the source. Moustakas et al. (2007) used the simpler elliptically symmetric Moffat profile; a Sérsic profile fit could also have been performed. To quantify this source of systematic uncertainty we investigate both lens galaxy subtraction methods found in the literature, test them as best we can using the data, and compare the results in terms of relevant lens and source parameters.

In the subtraction scheme \( sub \) we used a SExtractor segmentation map to mask out the detected pixels associated with the lensed images and then fitted an elliptically symmetric B-spline model with two angular modes (see the Appendix of Bolton et al. 2006 for details). In this scheme, there is a danger that the tangentially stretched images will be truncated, leading to an overly compact inferred source. The Moffat profile fit (subtraction scheme \( msub \)) was performed as in Moustakas et al. (2007), with no masking of the image. This model has the benefit of being somewhat more robust, but it must be expected to provide a much poorer quality of fit, leaving more lens galaxy flux in the residual image and leading to a brighter, larger inferred source. Based on these considerations, we expect that the two schemes will bracket the ideal solution and thus provide an estimate of the systematic uncertainty. A Sérsic profile fit may well provide a better fit to the lens galaxy light than the Moffat profile; we use the Moffat profile in order to make our systematic error estimate a conservative one.

5. LENS MODELING METHODOLOGY

Modeling of the images of extended sources lensed by galaxy-scale lenses has been the subject of some considerable research in the last few years (see, e.g., Warren & Dye 2003; Treu & Koopmans 2004; Dye & Warren 2005; Koopmans 2005; Suyu et al. 2006; Brewer & Lewis 2006; Barnabé & Koopmans 2007). The differences between these works revolve around the choice of regularization scheme for the reconstructed source plane image, while the lens models are largely consistent between the methods and reflect the simplicity and consistency observed in gravitational lens potentials (Koopmans et al. 2006). The regularization is important due to the very large numbers of parameters employed to describe the source plane intensity.

In this work and in a previous article (Moustakas et al. 2007), we choose to model the source galaxy using simply parameterized, elliptically symmetric Sérsic profile components. We pursue this approach for two reasons. First, images of intermediate- and high-redshift galaxies very often show morphologies representable by collections of simply parameterized components (bulges, disks, star-forming regions, etc.). The second reason is that we seek a quantitative understanding of galaxy luminosity, mass, size, and shape as a function of redshift, and this is best achieved by analyzing the image data within the context of a sensible
phenomenological model (the Sérsic profile). The resulting inferences will, of course, be model-dependent (by design), and we should expect the corresponding precision to be high as a result of the additional information used in the fit. Most importantly, our results will be directly comparable to other photometric and morphological studies. After all, a pixel-based reconstruction will have to be fit by a parameterized Sérsic model in order to derive shape and luminosity parameters that can be compared with the literature.

For our lens models we follow previous authors and use the singular isothermal ellipsoid (SIE) model (e.g., Kormann et al. 1994). A number of authors (e.g., Treu & Koopmans 2004; Rusin & Kochanek 2005; Koopmans et al. 2006) have shown the SIE model to provide a very good approximation of the lens potential on galaxy scales. The basic lens equations describing the deflection of light by this model can be found readily elsewhere (e.g., Kormann et al. 1994; Evans & Wilkinson 1998; Evans & Witt 2001; Schneider 2006) and are not repeated here. Suffice it to say that given the deflection angle as a function of lens plane position, the corresponding source plane position can be rapidly calculated using the formulae in, e.g., Evans & Witt (2001). The price we pay for this high computation speed is a significant systematic error in the source parameters as inferred through the lens. The intrinsic spread of logarithmic density slopes (where the SIE profile has slope $m = 1$) is approximately 0.12, based on the large sample of strong lenses analyzed by Koopmans et al. (2006); in the Appendix we show that this gives rise to a fractional uncertainty in source size of about 12% and an error in the inferred source magnitude of 0.26. Implementing a more flexible lens model would translate this systematic error into a statistical one; while this is beyond the scope of this paper we note that a reasonable goal is to reduce all other systematic errors to below the level set by the lens mass profile.

Since our source surface brightness distribution is the analytic Sérsic profile, we can compute the source intensity at each desired source plane position and assign it to the original image plane pixel value; we do this on a twice-subsampled grid to reduce rounding errors. (This simple but effective “poor man’s ray-tracing” is described further in Schneider et al. 1992.) In this way a predicted image can be calculated for any given set of lens and source parameters. Before comparison with the data image we convolve the model image with a PSF image (derived from the image of a nearby star, as described in §3). With the PSF image grid being much smaller than the data image grid, the speed of computation is greatly increased.

The $N$-pixel model image $d_p(x)$ and data image $d$ are compared via the likelihood function

$$\Pr(d|\mathbf{x}) = \frac{1}{Z_L} \exp \left( -\frac{\chi^2}{2} \right),$$

where $\chi^2 = \sum_i \left[ d_p(x_i) - d_i \right]^2 / \sigma_i^2$ and $Z_L = (2\pi)^{N/2} \prod_i \sigma_i$. (1)

This form contains an implicit assumption of uncorrelated Gaussian pixel noise, which is well justified for the background-limited Keck data. When using the *HST* images, we note that the counts are always such that the Gaussian approximation to the Poisson distribution is always valid, and we compute the uncertainties $\sigma_i$ from the square root of the image itself. We account for the correlated noise introduced by the drizzling procedure by computing the equivalent single-pixel noise (Casertano et al. 2000), essentially by reducing the uncertainties by a factor close to the fourth power of the ratio between the output and input pixel scales. This has the effect of making the reduced $\chi^2$ approximately equal to unity in the case of a good fit. In principle one could estimate the pixel covariance matrix and use that in the calculation of $\chi^2$, at greater computational expense. We leave this to future work and note that the correlated errors are unlikely to affect our statistical error bars by more than a factor of 2. As we shall see, systematic errors are of greater concern.

Our simple lens model has five parameters: position ($x$ and $y$), velocity dispersion $\sigma_{\text{SIE}}$, mass distribution ellipticity $\epsilon$, mass distribution axis ratio $q$, and source magnitude (where the logarithmic nature of this quantity captures our even greater prior ignorance). For the ellipticity we assume the standard weak-lensing intrinsic ellipticity distribution, a Rayleigh distribution of mean 0.25. (Note that the relation between the effective radius $\theta_e$, effective semimajor axis $a_e$, and axis ratio $q_e$ is $q_e = a_e \sqrt{1 - q}$. so that our effective radii may be compared directly with the “circularized” radii of, e.g., Shen et al. 2003.) We shall see in §§6.1 and 7.1 that our choices of prior PDF have very little influence on the posterior inferences. These are defined by the joint posterior PDF,

$$\Pr(\mathbf{x}|d, H) = \frac{\Pr(d|\mathbf{x}, H)\Pr(\mathbf{x}|H)}{\Pr(d|H)}, \quad (2)$$

where $\Pr(\mathbf{x}|H)$ is the product of the individual prior PDFs referred to above. We sample the unnormalized number of equation (2) using the multipurpose Markov chain Monte Carlo (MCMC) code *BayeSys*, a robust package used in a number of other cosmology and lensing analyses (e.g., Odman et al. 2004; Marshall 2006; Limousin et al. 2007; Jullo et al. 2007).

The symbol $H$ in equation (2) represents the set of assumptions that go into the inference of the parameters via the MCMC analysis. Such models can be compared quantitatively using the evidence, $\Pr(d|H)$. This statistic is calculated during the initial “burn-in” period of the sample and, while dominated by the goodness of fit, does take into account the different prior PDFs that might be employed. For further reading about evidence analysis, we recommend MacKay (2003).

In this work, the prior PDFs are kept fixed, while different PSF models and lens galaxy subtraction schemes are tried, an approach also followed by S. H. Suyu et al. (2008, in preparation). A simple ranking could be achieved by using some different monotonic function of the $\chi^2$ statistic; we note here, however, that the correct

14 While the strong lensing image separation is a direct measurement of the mass enclosed by the Einstein radius, when working with the SIE model the overall normalization is more conveniently described by the single parameter $\sigma_{\text{SIE}}$. This has the added benefit of being (more or less) straightforwardly connected to dynamical mass estimates from spectroscopic velocity dispersions (e.g., Treu & Koopmans 2002).

15 See http://www.maxent.co.uk.
Fig. 2.—Data (left panels), predicted data (middle panels), and residuals (right panels) for the best-fit lens models. Top row, NIRC2-LGS AO K-band data; second row, HST NICMOS data; third row, HST ACS F814W data; bottom row, HST ACS F555W data. The critical curve and asteroid caustic of the lens model are overlaid in each case. The optimal PSF model was used for each data set, and the lens galaxy subtraction scheme was sub. The pixel scale is 0.0397", all these cutout images are 2.81" on a side.
weights to use when combining parameter estimates from different analyses are exactly the evidence values (provided all models are deemed equally probable a priori). This can be seen by marginalizing the parameter posterior PDF over the models; each individual model’s posterior gets multiplied by its (renormalized) evidence during the summation:

$$Pr(x|d) = \sum_H Pr(x|d, H)Pr(H).$$  \hspace{1cm} (3)

In practice, one model often has much higher evidence than the others on offer, meaning that the sum can be approximated by this single term; this is model selection.

6. LENS MODELING RESULTS

Figure 2 shows the fits to the four imaging data sets introduced above. For the subtraction scheme sub (shown in the figure; see § 4) the residuals are close to zero, with little significant structure in the residual images (especially in the infrared filters). We show the results of the statistical model selection analysis in Table 2, for all data sets.

We find that the different PSF models are easily differentiated (top half of the table), with typical evidence ratios of a few tens. This is reflected in the $\chi^2$ statistic, which is not surprising given that the parameter space volumes are identical between the different PSF models. The relative evidence is determined almost entirely by the goodness of fit, which is significantly better for PSF0 in the case of the NIRC2 data.\(^\text{16}\) This may be due to the shape of the PSF at the lens being better matched by a stellar image at the same position relative to the laser spot (which PSF0 provides). For the HST data sets, the most appropriate PSF star to use varies between filters, as we might expect.

The situation with the lens galaxy subtraction schemes is less clear; here the goodness of fit is dominated by the lens galaxy model such that we cannot use the evidence straightforwardly to select the most appropriate model for the source galaxy. The limiting case would be a lens galaxy model so flexible that all the flux was subtracted, leaving a zero-flux inferred source and a $\chi^2$ of zero. What we can take from Table 2 is that the low goodness of fit associated with subtraction scheme $msub$ indicates that a significant amount of lens galaxy flux is being left unsubtracted, a conclusion vindicated by inspection of the residual images (not shown). The different schemes provide us with a rough estimate of the contribution of lens galaxy subtraction to our systematic error budget.

A side effect of the domination of the lens galaxy subtraction problem is that the reduced $\chi^2$ values from the lens modeling are often not close to unity. However, this need not affect our conclusions about the PSF model for the fixed subtraction scheme: a good PSF is required at all four image positions, but the galaxy subtraction residuals vary between these points.

Figure 3 shows the one-dimensional marginalized probability distributions for a selection of lens and source model parameters, given the NIRC2+LGSAO infrared imaging data set, in order to illustrate the effect of the different PSF models and the different lens galaxy subtraction schemes on the inferences. Similar results were obtained from the other filters’ data and are not shown here for the sake of clarity.

This figure shows that the choice of PSF model is not critical in determining the available accuracy of the model parameters: in all cases the parameter estimates agree within the statistical precision. The choice of lens galaxy subtraction scheme has a more significant effect on the model parameters; in particular, the two schemes investigated give rise to a difference of ~0.2 mag in source brightness.

To marginalize over the range of PSF models one would use the relative evidence values to weight the different posterior PDFs (as shown in eq. [3]); however, since the evidence ratios in the top half of Table 2 are typically significantly different from unity, we approximate this procedure by simply selecting the PSF model with the highest evidence. For the rest of this paper,

\(^{16}\) We put the reduced $\chi^2$ values in context by computing the number of $\sigma$, $N_e$, by which the unreduced $\chi^2$ deviates from the mean of its distribution. We do this using Fisher’s approximation, that $(2\chi^2)^{1/2}$ is Gaussian-distributed with mean $(2k - 1)^{1/2}$ and unit variance, where $k$ is the number of degrees of freedom, assumed to be large.
Fig. 3.—Marginalized posterior probability distributions for four of the model parameters, given the NIRC2+LGSAO data only. Top row, lens SIE velocity dispersion; second row, lens mass ellipticity; third row, source AB magnitude; bottom row, source effective radius. Left panels, comparing different PSF models; right panels, comparing different lens galaxy subtraction schemes (sub, solid line; msub dashed line). [See the electronic edition of the Journal for a color version of this figure.]
we use the optimal PSF models for each data set (from the maximum evidence values given in Table 2) and assert the SExtractor-detected object mask subtraction scheme \( \text{sub} \); the alternative \( \text{msub} \) distributions given in Figure 4 (and those for the other model parameters) provide estimates of the systematic errors we expect for each parameter. We now compare parameter estimates in the four different filters to compute the properties of the lens and the source.

6.1. Lens Properties

Figure 4 shows the inferred SIE velocity dispersion and mass distribution ellipticity for the SDSS J0737+3216 lens. These parameters (along with the mass orientation, not shown) agree reasonably well across the filters, as they should given the achromaticity of the lensing effect. The largest discrepancies come from the deeper HST ACS F814W image. The likelihood function for these data is steeper, making it harder both for an MCMC sampler to explore the parameter space and for a simple model to provide a good fit. In this case the inferred parameter uncertainties should be accepted with caution. Still, the inferred SIE velocity dispersion is in good agreement with that found by Koopmans et al. (2006) from their shallower HST ACS snapshot data.

We note that an offset of 0.5 km s\(^{-1}\) in the velocity dispersion is equivalent to one of 3.4 mas in the Einstein radius, a fractional error of 0.3%. We assume that the reported image plate scales are known to better than this, but this may not be the case. The truncation of the posterior PDF for lens ellipticity is a direct result of our assumption of a prior on this parameter that was uniform between 0.0 and 0.3. The lack of strong degeneracy between ellipticity and any other parameter indicates that this truncation is not a problem in this case, but it serves as a warning for future analyses.

7. SOURCE PROPERTIES

Having calibrated the optics of our cosmic telescope, we turn our attention to the target of the observation: the lensed source at redshift \( z_s \). Figure 5 shows the multicolor reconstruction of this object, which shows the presence of a red, compact core centered on a more extended blue light distribution. The ellipticity and orientation of the source are a good match with those found from shallower data by Koopmans et al. (2006). We note that the alignment of the different filters’ reconstructions is very good, and that qualitatively we seem to be recovering the large-scale stellar component rather than being dominated by any smaller scale features.

7.1. Source Photometry and Morphology Results

The top left panel of Figure 6 shows the two-dimensional marginalized probability distributions for two source model morphology parameters, the effective radius \( r_e \) and Sérsic index \( n \), given each of the data sets. We again note that the precision available for each parameter is much higher for the deep HST ACS F814W image and very similar across the other three data sets. Likewise, the bottom panels in this figure show the inferred source orientation and ellipticity, which are reasonably constant through the bandpasses.

We infer a small, compact source galaxy across the whole wavelength range. The differences in morphology between the filters are not large, but there is a suggestion that in the redder bands the profile is slightly more compact, approaching the Gaussian distribution (\( n \approx 0.7 \)). However, the degeneracy between \( r_e \) and \( n \) can be clearly seen, warning us not to overinterpret the inferences; a robust conclusion is that the inferred Sérsic index is low in all

\[
\text{Velocity dispersion } \sigma / \text{ km s}^{-1}
\]

Fig. 4.— Marginalized joint posterior probability distributions, given each data set, for the lens SIE velocity dispersion and mass distribution ellipticity. The contours enclose 68% and 95% of the integrated probability. The solid curves are for the preferred galaxy subtraction scheme \( \text{sub} \), while the dashed curves are for the alternative scheme \( \text{msub} \).

Fig. 5.— Multifilter reconstruction of the source behind SDSS J0737+3216. From left to right we plot the data, predicted data, residuals, and reconstructed source plane images for the best-fit lens models, assuming an optimal PSF model and lens galaxy subtraction scheme \( \text{sub} \). Note the resolution of a red, compact core centered on a more extended blue light distribution. The red, green, and blue image channels are given by the \( K' \)-band, F814W-band, and F555W-band images, respectively, and the relative scales were chosen (manually) to equilibrate the noise levels across the channels.
filters. Likewise, the PDFs (dotted and solid lines) also show the effects of the different lens galaxy subtraction schemes on the inferred source morphology. In particular, the deep HST ACS F814W data can be seen, as expected, to be generally more systematics-dominated than those of the other filters, with significant (if small) differences in inferred effective radius and magnitude between different analyses. It is in this filter that the sensitivity to the different model assumptions is highest, and the limitations of our simply parameterized model are made most clear.

The photometry is also (unsurprisingly) affected by the lens galaxy subtraction: the lens subtraction systematic error can be seen in the top right panel of Figure 6. In the next section we use the photometry from the subtraction scheme sub, and we return to the systematic error budget in § 9.

7.2. Spectral Energy Distribution and Stellar Mass of the Source Galaxy

Armed with photometry from HST ACS (F555W and F814W), HST NICMOS (F160W), and NIRC2+LGSAO (K'), we now reconstruct the spectral energy distribution (SED) of the source. To account for uncertainty in the zero points and filter transmission curves, we assert statistical errors of 0.1 and 0.05, respectively,
and add these in quadrature to the statistical errors from the MCMC inferences.

Given the known redshift, we estimate the stellar mass by fitting the observed colors to a variety of SED templates (Bundy et al. 2006). The best-fit model is obtained for an exponential star formation rate with a short characteristic timescale $\tau \sim 0.04$ Gyr and young age ($\leq 0.7$ Gyr) and corresponds to a stellar mass of $\log_{10} M^*/M_\odot = (9.3 \pm 0.2$ stat $\pm 0.17$ sys) assuming a Chabrier IMF, where the error bar is obtained by marginalizing the posterior over the parameters of the stellar populations. In other words, the system appears to have undergone a very recent burst of star formation, consistent with its selection via emission lines. This inferred star formation history is consistent with the SED fitting performed by Guzmán et al. (2003) on a sample of luminous compact blue galaxies taken from the sample of Phillips et al. (1997).

Figure 7 shows the fluxes (and uncertainties) used in the fit plotted as a function of wavelength, with the best-fitting galaxy template normalized to the observed $K'$ luminosity overlaid. For reference, the absolute AB magnitude in the F555W band is $-19.66 \pm 0.05$. We note that choice of IMF is the single largest source of systematic uncertainty ($0.2\sim0.3$ dex; Bundy et al. 2006) in the absolute stellar mass. However, when comparing stellar masses with other surveys we must compute the same model-dependent masses. Both Shen et al. (2003) and Barden et al. (2005) assume a Kroupa IMF, which results in stellar masses different from those assuming a Chabrier IMF by just 0.05 dex.

Likewise, we note that the stellar masses of less well-resolved galaxies in the literature typically also come from a global modeling of the object photometry (rather than a joint morphological and photometric analysis), justifying our approach to modeling the SED here. The Sérsic indices measured in $\S$ 7.1 are also sufficiently similar to justify the assumption of a single stellar population when estimating the stellar mass. We do not, in any case, expect the systematic error in the absolute stellar mass introduced to be greater than that from the IMF uncertainty. Furthermore, the consistency between the filters (all the way out to the $K'$ band) suggests that we are not dominated by small-scale star-forming regions in either mass or size measurements.

8. SYSTEMATIC ERRORS

Photometry with AO imaging has the reputation of being at best difficult and at worst inaccurate. In this work we have looked carefully at several systematic errors associated with photometric and morphological measurements of small extended objects viewed through galaxy-scale gravitational lenses, and we now discuss these errors in a little more detail.

8.1. Model-dependent LGSAO Photometry

The basic problem of measuring the total flux of an object, and the radius within which half of this total flux is contained, is partially solved by the assumption of a sensible model intensity distribution, allowing the light profile to be extrapolated beyond the data region. This solution is, of course, only as good as the model assumption, but at least it leads to a set of well-defined quantities (e.g., “Sérsic magnitudes”). The underlying assumption is that high-resolution imaging data provide enough constraints on the inner profile of the object that the extrapolated quantities can be accurately inferred.

One could argue that imposing a model in this way “biases” the results; distant galaxies are not necessarily expected to have pure Sérsic profiles. The system studied here at least appears to be simple, in that a single image component provides a reasonable fit in the infrared, but there are suggestions in the bluer filters that the galaxy has a more complex morphology. This is, perhaps, to be expected, given that this system was selected for its emission-line spectrum, indicating ongoing star formation and consequent likely clumpy morphology. However, if we are to quantify galaxies like the source behind SDSS J0737+3216 in a way that permits comparison with other data sets and/or a physical theory, then the Sérsic profile appears to be the most natural choice, given its widespread use. The galaxy itself may not be well-fit by a Sérsic profile, but that does not mean that knowing its Sérsic parameters is not useful. The fitting of a lensed Sérsic profile is an appropriate way of measuring the average properties of the source light distribution, even in the blue filters. We note that the residual features in the bluer images are smaller still than the inferred Sérsic component, suggesting that we are measuring the principal stellar structure and not a smaller, brighter, peripheral star-forming region, even at shorter wavelengths.

8.2. PSF Model Selection and Truncation

Assuming a model galaxy profile and having four predictable copies of the same image means that the PSF structure can be inferred concurrently with the source itself. Indeed, we have shown that PSF selection via the Bayesian evidence is possible; there is information in all the imaging data analyzed on the most appropriate PSF. We have noted that, since the number of model parameters is unchanged between the different PSF models, the evidence is being dominated by the goodness of fit. However, the PSF suitability is related to the choice of source galaxy model and its parameter prior PDFs. This leads to the evidence being a sharper tool for PSF selection than the reduced $\chi^2$ values, as can be seen in Table 2. In the case studied here, the PSF selection is interesting but not critical, as we clearly see that there are larger systematic effects at play.

Our treatment of the PSF with a small cutout star image is cause for more concern. Our (internally normalized) PSF postage stamps, at 16 pixels in width, only span 1.5 times the seeing disk FWHM ($\approx0.4''$). To quantify the effect of this on the inferred model parameters, we simulated NIRC2+LGS AO observations of a gravitational lens having the same properties as SDSS J0737+3216 (i.e., the parameter values found in $\S$ 6.1 and 7.1).
For the PSF we used a concentric sum of two Gaussians (representing the seeing disk wings and the Airy pattern core, with relative weights given by the Strehl ratio), following Law et al. (2006). The simulated data were generated with a large 72 pixel PSF cutout, while the MCMC sampler was provided with a posterior PDF assuming a small, renormalized 16 pixel PSF cutout. Investigating input Strehl ratios of 0.2, 0.3, and 0.4 (and assuming FWHM values of 0.10" and 0.40" for the K' band Keck diffraction pattern core and seeing disk, respectively), we found that from a choice of model double Gaussian PSFs with the true seeing and core size and Strehl ratios of 0.2, 0.3, and 0.4 the evidence selected the correct (input) PSF each time, by about the same margin as seen with the real data. Using the maximal evidence PSF, we then found that the magnitude of the source was underestimated by 0.03 mag comparable to the statistical uncertainty. This is significantly smaller than the other estimated errors (that were used in the stellar mass calculation) but comparable to the error introduced by the lens galaxy subtraction. The effective radius was found to be overestimated by 0.005", a small but statistically significant increase; the Sérsic index was also overestimated by 0.1 or so. These shifts, while contributing to the overall systematic error budget, do not affect our conclusion about the unusual size of this source, which we discuss below.

We conclude that LGSAO photometry of faint extragalactic extended sources at the 0.05 mag accuracy level (not including zero-point and filter curve calibration) is perfectly possible using techniques such as those used in this work. However, we caution that the conditions of observations were exceptionally good, in terms of both seeing and stability of the PSF. This is supported by the fact that the specially observed star PSF0 gave the best results and by the consistency between the results obtained with this star and those obtained with the serendipitous stars observed in the object field itself. This consistency is not guaranteed in general, since the PSF can be expected to change significantly on timescales like the time interval between our observations of the lens field and of PSF0 (J. Graham 2007, private communication), and spatial variations of the PSF can be significant (E. Steinbring et al. 2008, in preparation). However, it bodes well for the future that our results would be essentially unchanged had we only used the stars in the field of the target.

### 8.3. Overall Systematic Error Budget

In §§ 6.1 and 7.1 we identified the lens galaxy subtraction as a serious issue leading to the dominant systematic error when inferring the model parameters from well-calibrated data and assuming an isothermal density profile lens. A better approach would be to fit the lens and source simultaneously, making use of the quadruple imaging to constrain the two intensity distributions with minimal degeneracy. From the Moffat profile fit residuals (scheme $m_{sub}$; Table 2) we see that such a procedure would require a flexible model (such as the B-splines used here) for the lens galaxy light in order to get a good fit. This is not computationally feasible within the current framework, but it should be possible in the semilinear formalisms of Warren & Dye (2003) and others.

Comparison of the parameter estimates between subtraction schemes does give a quantitative feel for the systematic errors introduced by the lens galaxy subtraction. These are compared with the other errors identified in this work in Table 3. We see that even the largest image analysis systematic error, that due to the lens galaxy subtraction, is still smaller than that introduced by the assumption of an isothermal density profile lens mass distribution. Conservatively combining all the systematic errors by simple addition, the resultant systematic errors on the size and stellar mass are approximately 0.1 kpc and $0.8 \times 10^{9} M_{\odot}$; these may be compared with the statistical uncertainties shown in Figure 7.

### 9. DISCUSSION: THE SIZE-MASS RELATION AT $z = 0.6$

From the analysis presented above we obtain the final result that $\log \rho_{r}/kpc = (-0.23 \pm 0.005 \text{ stat} \pm 0.07 \text{ syst})$, $\log M^{*}/M_{\odot} = (9.3 \pm 0.2 \text{ stat} \pm 0.17 \text{ syst})$, and $n = (1.0 \pm 0.1 \text{ stat} \pm 0.2 \text{ syst})$ for the size, stellar mass, and Sérsic index of the SDSS J0737+3216 source galaxy, where these numbers are global estimates based on all the filters’ data; the plots in Figure 6 and the systematic error analysis of the previous section indicate that the small differences between the different bands are not significant. We overlay these values on the local relation for “disk” galaxies (i.e., those with Sérsic index $n < 2.5$) derived from SDSS by Shen et al. (2003) and the corresponding $z \approx 0.6$ relation derived from GEMS by Barden et al. (2005) and interpreted by Somerville et al. (2008) in Figure 8. For its stellar mass (which is a factor of 5 smaller than the GEMS completeness limit), the source behind SDSS J0737+3216 appears to be about a factor of 3 smaller than the average local galaxy (Shen et al. 2003), putting it approximately 3 $\sigma$ below the local size-mass relation for galaxies with a low Sérsic index.

The distant GEMS data do not extend to small enough masses to allow for a direct comparison with our measurement. However, bringing our point into agreement with the typical $z = 0.6$ galaxy from GEMS would require a somewhat marked flattening of the size-magnitude relation at masses lower than $10^{10} M_{\odot}$, since redshift 0.6, which appears unlikely given the very modest evolution observed at masses above $10^{10} M_{\odot}$. Barden et al. (2005) find a constant size-mass relation; Trujillo et al. (2006) measure $r_{e} \sim (1+z)^{-0.4 \pm 0.06}$ for disklike galaxies, predicting that the mean object at $z \approx 0.58$ is only about 0.83 times the size of the mean local disklike galaxy. Even comparing with the incomplete GEMS data at $\sim 10^{9} M_{\odot}$ (Fig. 10 of Barden et al. 2005) we see that our object is unusually small. Thus, we conclude that the source galaxy behind SDSS J0737+3216 is, relative to existing surveys, somewhat extreme in terms of mass and size, if it is indeed a disk galaxy.

Such compact galaxies have, however, been well studied. Koo et al. (1994) identified a sample of compact, narrow emission line galaxies in the Hubble Deep Field at redshift $\approx 0.2$ with $17$ We note that the SDSS size estimates were made in the $r$ band, such that the effective rest-frame wavelength is about 5700 Å. At redshift 0.6, this falls in the $i$ band, as used (approximately) in GEMS. Our corresponding F814W band measurement is the one most affected by systematic errors; however, the global size estimate we use can be seen (Fig. 6) to be representative of the size in this filter.

### Table 3

| Description                               | $\delta r_e$ (kpc) | $\delta m_{AB}$ (mag) | $\delta \log M^{*}/M_{\odot}$ (dex) |
|-------------------------------------------|--------------------|-----------------------|-------------------------------------|
| Incorrect stellar PSF model              | 0.01               | 0.01                  | 0.005                               |
| Truncated PSF image                       | 0.005              | 0.003                 | 0.015                               |
| Lens galaxy subtraction                   | 0.01               | 0.10                  | 0.04                                |
| Stellar mass IMF choice                   | N/A                | N/A                   | 0.05                                |
| Lens model density slope                  | 0.07               | 0.26                  | 0.11                                |
| Total (approximate)                       | 0.10               | 0.40                  | 0.17                                |
luminosities $M_B \approx -19$ and sizes $r_e \approx 1$ kpc. These objects comprise a small fraction of the ubiquitous faint blue galaxies reviewed by Ellis (1997). Extending the sample to intermediate redshift and focusing on the higher luminosity members ($M_B \approx -21$), Koo et al. (1995) found using high-resolution spectroscopy that these objects appear more similar to local H II galaxies (dwarf galaxies showing violent star formation activity) and suggested that these systems would evolve into today’s dwarf spheroids. This conclusion was also reached by Phillips et al. (1997) in an extension of that work, although they note that the number densities are such that not all compact galaxies at intermediate redshift can be progenitors of spheroids. However, Hammer et al. (2001) argue that the observed narrow emission line widths may not represent the depth of the whole galaxy potential, and instead argue on the basis of the stellar masses they infer from their spectra and photometry that the more luminous, compact galaxies are more likely the progenitors of the bulges of present-day massive spiral galaxies. At the stellar mass scale inferred in this work ($10^{9.3}$), the source behind SDSS J0737+3216 appears closer to the low-luminosity end of the samples of Koo et al. (1995) and Phillips et al. (1997) but is almost a factor of 2 smaller than the limiting size of their sample ($0.16^\prime$) and is fully resolved. Indeed, our physical resolution is comparable to that reachable for galaxies in the Virgo Cluster with ground-based seeing; we find that the SDSS J0737+3216 source is comparable to the smallest dwarf ellipticals seen in the Virgo Cluster (Fig. 8; Capaccioli et al. 1992; Ferrarese et al. 2006) and is typical of the objects in the smallest size bin of the Sérsic-selected “elliptical” galaxies in the GEMS survey (McIntosh et al. 2005).

Can we interpret our superresolved source morphology as being that of a forming spheroid? The low Sérsic index measured would suggest not, placing it firmly in the disklike samples of the literature. However, at low masses there is evidence that elliptical galaxies can have Sérsic indices of 1 and below (e.g., Trujillo et al. 2004). The consistency in morphology between the observation filters is indicative of a regular spheroid (although the residual structure in the bluer filters would argue against a highly evolved, smooth stellar distribution). Perhaps the strongest indicator is the position of the source in the size-mass plane. In Figure 8 we show the local and $z = 0.6$ size-mass relations for “elliptical” ($n > 2.5$) galaxies from SDSS and GEMS (Shen et al. 2003; McIntosh et al. 2005) and can see that the source behind SDSS J0737+3216 sits rather more comfortably with these relations, albeit at significantly (a factor of 2–4) lower mass.

Our results demonstrate that, using a gravitational telescope to superresolve the source, it is possible to study in considerable detail atypical sources that may well be missed or excluded by nonlensed surveys. In fact, size-mass studies at redshift 0.5 and above have necessarily focused on the high-luminosity end and have inevitably included a size cut to remove stars from the catalogs, before a further completeness cut that discards the least massive galaxies. It is not clear just how many small galaxies are being overlooked in this way. The higher resolution afforded by our gravitational telescope allows us to study the structure and surface brightness profiles of the compact blue galaxies in much greater detail and with higher precision and to extend the investigation to smaller sizes still. For comparison, the total magnification provided by SDSS J0737+3216 is $\mu \approx 13$, indicating an angular resolution in the source plane of approximately 0.01". The 10% accuracy we obtain on our size measurement indicates that we are still some way from the limit imposed by the resolution of our optics. Indeed, we note that this accuracy can be improved by a further factor of 2 by simply using a more flexible lens model.

Having demonstrated the power of this method, a larger sample of objects is needed in order to infer statistically meaningful conclusions about the low-mass tail of the mass-size relation. As clearly discussed by Barden et al. (2005), to achieve this goal it is crucial to understand the selection function of the objects being used in the size-mass relation study. Due to a form of the
so-called magnification bias, gravitational lens surveys such as SLACS tend to favor compact sources. SLACS lenses were selected from (spectroscopic) observations where the system is essentially unresolved, meaning that lens systems with high total magnification are preferentially detected. This bias is strongest when the source is pointlike, i.e., much smaller than the size of the fiber and the Einstein radius (≈1′). Thus, it is not so surprising that the first source to be studied is compact. This realization implies that when performing statistical analyses of the size-mass relation of lensed galaxies it will be necessary to use Monte Carlo simulations to understand and quantify the selection function of multiple-image systems. Applying our methodology to other lens systems will, once the selection effects are quantified, extend this study to join the existing statistical analyses of higher mass disks to probe the small size (i.e., low angular momentum) regime.

The SLACS lenses are well-suited to this task: the efficiency of the survey is such that some 100 high-magnification systems like SDSS J0737+3216 are expected to be found by the end of the program. (The number of systems currently confirmed using high-resolution imaging is already close to this figure.) Extending the study to sources at even higher redshifts requires more lenses to be discovered at greater distances: the SL2S survey (Cabanac et al. 2007) is expected to discover ~100 suitable systems, with sources at redshifts of 1.0 and higher. However, the detection of these systems (via a ground-based imaging survey in the r′ and i′ bands) will lead to different selection effects than those present in the SLACS survey and will again require Monte Carlo simulations in order to understand them.

The different identification schemes of the SLACS and SL2S surveys also introduce a different selection effect in terms of stellar population, which will have to be modeled and taken into account when interpreting the results. SLACS sources are emission-line-selected and will therefore be representative of actively star-forming galaxies, directly comparable with galaxies selected in narrowband surveys. SL2S sources are continuum-selected and contain a mix of actively star-forming, poststarburst, and quiescent stellar populations, directly comparable with the galaxy population studied by wide-field HST surveys in similar broad bandpasses.

An additional implication of the lensing selection effect is that the magnification bias in some sense increases the power of a galaxy survey by picking out the smallest objects and then making them measurable. With current technology, gravitational telescopes are the only way of accurately measuring such tiny objects.

10. CONCLUSIONS

We find that high-quality images from NIRC2+LGSAO are capable of providing very similar precision on simple lens and source model parameters to typical data sets from HST ACS and HST NICMOS. The data themselves contain information about the most appropriate PSF model to use, to the extent that a set of nearby unsaturated stars can be fruitfully compared using suitable statistics that are sensitive to the goodness of fit. We estimate that even for the LGSAO imaging this way of modeling the PSF allows a photometric precision of 0.05 mag. However, the calibration of isothermal galaxy-scale gravitational lenses as cosmic telescopes is very likely limited by the subtraction of the lens galaxy light. We estimate that this procedure introduces up to 0.1 mag of systematic error into the source galaxy photometry. However, this is still smaller than the error introduced by the assumption of an isothermal density profile for the lens itself.

With this in mind we draw the following conclusions about the source behind SDSS J0737+3216:

1. Our photometry is robust enough to permit a reconstruction of the SED, and we find a stellar mass of $2.0 \pm 1.0 \text{ stat } \pm 0.8 \text{ sys } \times 10^{9} M_{\odot}$. This is a factor of 5 smaller than the completeness limit of the GEMS disk galaxy analysis of Barden et al. (2005) and also smaller than the least massive spheroid at this redshift studied by McIntosh et al. (2005).

2. The Sérsic profile parameters of the source can be measured to high accuracy. We find an effective radius of $0.59 \pm 0.007 \text{ stat } \pm 0.1 \text{ sys } kpc (\approx 0.09'' \text{ with } \approx 10\% \text{ accuracy})$ and a Sérsic index of $1.0 \pm 0.1 \text{ stat } \pm 0.2 \text{ sys }$ in the F814W band (approximately rest-frame $B$), and these values change little over the rest-frame optical range.

3. This very small galaxy lies approximately 3 $\sigma$ below the local size-mass relation for disks. However, it shares the properties of the smallest of the compact narrow emission line galaxies of Koo et al. (1994) and, despite its low Sérsic index, is more typical of the dwarf early-type galaxies observed in the Virgo Cluster (Ferrarese et al. 2006) and the “elliptical” galaxies studied by McIntosh et al. (2005) at high redshift.

While the planned statistical analysis of a large sample of lensed galaxies will rely on the detailed understanding of the selection function, it is clear that the magnifying effect of gravitational lenses allows us to extend current size-mass studies to smaller sizes and lower masses than would otherwise be available, posing fresh challenges to models of galaxy formation and evolution.

We thank Laura Melling and Sherry Suyu for useful discussions when developing the lens modeling code, and we are grateful to the anonymous referee for insightful comments that led to some improvement of the paper. P. J. M. was given support by the TABASGO foundation in the form of a research fellowship. T. T. acknowledges support from the NSF through CAREER award NSF-0642621 and from the Sloan Foundation through a Sloan Research Fellowship. The work of L. A. M. was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with NASA. L. V. E. K. is supported in part through an NWO-VIDI program subsidy (project No. 639.042.505). T. T. and P. J. M. thank the Center for Adaptive Optics for organizing the 2007 spring retreat, during which part of this work was carried out. This work was supported in part by the National Science Foundation Science and Technology Center for Adaptive Optics, managed by the University of California at Santa Cruz under cooperative agreement AST 98-76783. This research is supported by NASA through Hubble Space Telescope programs SNAP-10174, GO-10494, SNAP-10587, GO-10798, and GO-10886, and in part by the National Science Foundation under grant No. PHY99-07949, and is based on observations made with the NASA/ESA Hubble Space Telescope and obtained at the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy, Inc., under NASA contract NAS 5-26555, and at the W. M. Keck Observatory, which is operated as a scientific partnership among the California Institute of Technology, the University of California, and the National Aeronautics and Space Administration. The Observatory was made possible by the generous financial support of the W. M. Keck Foundation. The authors wish to recognize and acknowledge the very significant cultural role and reverence that the summit of Mauna Kea has always had within the indigenous Hawaiian community. We are most fortunate to have the opportunity to conduct observations from this mountain.
APPENDIX

THE EFFECT OF THE LENS MASS DENSITY SLOPE ON THE INFERRED SOURCE SIZE AND MAGNITUDE

The local magnifying and distorting effect of a gravitational lens (see, e.g., Schneider 2006) can be summarized by the (inverse) amplification matrix, $A^{-1}$:

$$A^{-1} = \begin{pmatrix} 1 - \kappa + \gamma & 0 \\ 0 & 1 - \kappa - \gamma \end{pmatrix},$$  \hspace{1cm} (A1)

where $\kappa$ and $\gamma$ are two combinations of the spatial second derivatives of the projected gravitational potential—$\kappa$ is proportional to the projected (surface) mass density—in a Cartesian coordinate system aligned with the radial and tangential directions. To first order, a source of width $dx$ and length $dy$ (also aligned with these axes) is distorted into an image of width $dx_i$ and $dy_i$ according to

$$A \begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} dx_i \\ dy_i \end{pmatrix}.$$  \hspace{1cm} (A2)

The $A_{11}$ component describes the radial stretching of the source, while the $A_{22}$ component describes the tangential stretching. The factor by which the solid angle subtended is increased due to the lensing effect is the magnification $\mu = |A| = 1/|A^{-1}|$.

In terms of the Einstein radius ($\theta_E$; the radius at which the magnification is formally infinite), the quantities $\kappa$ and $\gamma$ are given by

$$\kappa = \frac{2 - m}{2} \left( \frac{\theta_E}{\theta} \right)^m, \hspace{0.5cm} \gamma = \frac{m}{2} \left( \frac{\theta_E}{\theta} \right)^m,$$

for a simple, spherically symmetric, power-law density profile lens (with logarithmic slope $m$). Two images form at positions $\theta_{\pm}$ that solve the lens equation,

$$\beta = \theta_{\pm} - \alpha(\theta_{\pm}), \hspace{0.5cm} \text{where, in this case,} \hspace{0.5cm} \alpha(\theta_{\pm}) = [\kappa(\theta_{\pm}) + \gamma(\theta_{\pm})]\theta_{\pm}.$$  \hspace{1cm} (A4)

If the source position $\beta \ll \theta_{\pm}$, as is the case when the images are highly magnified and are close to forming an Einstein ring, we find the images at

$$\theta_{\pm} \approx \theta_E(1 \pm \epsilon), \hspace{0.5cm} \text{where} \hspace{0.5cm} \epsilon = \frac{\beta}{m\theta_E},$$  \hspace{1cm} (A5)

The offset $\epsilon$ is well-constrained by the data, and so we proceed treating $\epsilon$ as a small ($\ll 1$) constant. At this point we note that the image positions and distortions do contain some information on the density slope $m$, allowing this parameter to be fitted. What we are working toward here is a quantification of the effect of perturbing the slope $m$ away from the isothermal value ($m = 1$).

Evaluating $\kappa$ and $\gamma$ at the image positions, substituting into equation (A1), and expanding to first order in $\epsilon$, we find that

$$A^{-1} \approx \begin{pmatrix} m(1 + \epsilon \mp me) & 0 \\ 0 & \pm me \end{pmatrix},$$  \hspace{1cm} (A6)

and that the inverse magnification is (also to first order) $\mu^{-1} \approx \pm m^2 \epsilon$.

We can now use this result to estimate the uncertainty on the inferred source size (denoted by $\sigma_e$) given by a systematic error in the model slope $m$. We first note that the inferred source plane solid angle is given by

$$\Omega = dx \, dy = \Omega_{\pm} \mu^{-1}(m),$$  \hspace{1cm} (A7)

where $\Omega_{\pm}$ is the solid angle subtended by each image and $\Omega \sim r_e^2$. A small change in the density slope away from a fiducial value of 1 gives rise to an error in source area $\Omega$ according to

$$\sigma_\Omega = \left. \frac{\partial \mu^{-1}}{\partial m} \right|_{m=1} \sigma_m, \hspace{0.5cm} \text{such that} \hspace{0.5cm} \frac{\sigma_\Omega}{\Omega} = \left. \frac{1}{\mu^{-1}} \frac{\partial \mu^{-1}}{\partial m} \right|_{m=1} \sigma_m.$$  \hspace{1cm} (A8)

From this and the result above we get

$$\frac{\sigma_\Omega}{\Omega} \approx 2\sigma_m, \hspace{0.5cm} \text{and so} \hspace{0.5cm} \frac{\sigma_e}{r_e} \approx \sigma_m.$$  \hspace{1cm} (A9)
Since gravitational lensing conserves surface brightness, the inferred source flux is simply proportional to the inferred source solid angle $\Omega$; consequently, the error in the AB magnitude due to uncertainty in the density profile slope is $\sigma_{m_{AB}} = (2.5/\log 10)(\sigma(\Omega)/\Omega) \approx 2.2\sigma_{m}$.

Koopmans et al. (2006) give $\sigma_{m} = 0.12$ for the intrinsic spread of the power-law indices, where the profile is constrained at two radii, the Einstein radius and the (smaller) effective radius. (Note that this small scatter was not appreciated by, e.g., Knudsen et al. [2001] in their analysis of magnification errors.) While the power-law index they quote is not quite the local slope at the Einstein radius that we require here, the range of radii they consider brackets the Einstein radius of SDSS J0737+3216, and therefore their value for $\sigma_{m}$ provides an approximate quantification of the size of the density slope systematic error.

REFERENCES

Barden, M., et al. 2005, ApJ, 635, 959
Barnabé, M., & Koopmans, L. V. E. 2007, ApJ, 666, 726
Bolton, A. S., Burles, S., Koopmans, L. V. E., Treu, T., & Moustakas, L. A. 2006, ApJ, 638, 703
Bolton, A. S., Burles, S., Treu, T., Koopmans, L. V. E., & Moustakas, L. A. 2007, ApJ, 665, L105
Brewer, B. J., & Lewis, G. F. 2006, ApJ, 637, 608
Bundy, K., et al. 2006, ApJ, 651, 120
Capaccioli, M., Caon, N., & D’Onofrio, M. 1992, MNRAS, 259, 323
Cabanac, R. A., et al. 2007, A&A, 461, 813
Cassano, R., et al. 2000, AJ, 120, 2747
Driver, S. P., Liske, J., Cross, N. J. G., De Propris, R., & Allen, P. D. 2005, MNRAS, 360, 81
Dutton, A. A., van den Bosch, F. C., Dekel, A., & Courteau, S. 2007, ApJ, 654, 277
Dye, S., & Warren, S. J. 2003, ApJ, 590, 673
Ellis, R. S. 1997, ARA&A, 35, 389
Evans, N. W., & Wilkinson, M. I. 1998, MNRAS, 296, 800
Evans, N. W., & Witt, H. J. 2001, MNRAS, 327, 1260
Fall, S. M., & Efstathiou, G. 1980, MNRAS, 193, 189
Ferguson, H. C., et al. 2004, ApJ, 600, L107
Ferrarese, L., et al. 2006, ApJS, 164, 334
Fruchter, A. S., & Hook, R. N. 2002, PASP, 114, 144
Gavazzi, R., et al. 2000, AJ, 120, 2747
Gavazzi, R., Treu, T., Rhodes, J. D., Koopmans, L. V. E., Bolton, A. S., Burles, S., Massey, R., & Moustakas, L. A. 2007, ApJ, 667, 176
Guzmán, R., Ostlin, G., Kunth, D., Bershady, M. A., & Koo, D. C. 2003, ApJ, 586, L45
Hammer, F., Gruel, N., Thuan, T. X., Flores, H., & Infante, L. 2001, ApJ, 550, 570
Jullo, E., Enei, J-P., Limousin, M., Eliasdottir, A., Marshall, P., & Verdugo, T. 2007, New J. Phys., submitted (arXiv: 0706.0048)
Knudsen, A., Ratnatunga, K. U., & Griffiths, R. E. 2001, AJ, 122, 103
Koo, D. C., Bershady, M. A., & Moustakas, L. A. 1999, ApJ, 519, 22
Koekemoer, A. M., Fruchter, A. S., Hook, R. N., & Hack, W. 2002, in Hubble after the Installation of the ACS and the NICMOS Cooling System, ed. S. Arribas, A. Koekemoer, & B. Whitmore (Baltimore: STScI), 337
Koo, D. C., Bershady, M. A., Wirth, G. D., Stanford, S. A., & Majewski, S. R. 1994, ApJ, 427, 19
Koo, D. C., Kron, R. G. 1988, ApJ, 325, 92
Koopmans, L. V. E. 2005, MNRAS, 363, 1136
Koopmans, L. V. E., Treu, T., Bolton, A. S., Burles, S., & Moustakas, L. A. 2006, ApJ, 649, 599
Kormann, R., Schneider, P., & Bartelmann, M. 1994, A&A, 284, 285
Krist, J. E., & Hook, R. N. 1997, in 1997 HST Calibration Workshop with a New Generation of Instruments, ed. S. Casertano (Baltimore: STScI), 192
Law, D. R., Steidel, C. C., & Erb, D. K. 2006, AJ, 131, 70
Limousin, M., et al. 2007, ApJ, 668, 643
MacKay, D. J. C. 2003, Information Theory, Inference and Learning Algorithms (Cambridge: Cambridge Univ. Press)
Marshall, P. 2006, MNRAS, 372, 1289
McIntosh, D. H., et al. 2005, ApJ, 632, 191
McKean, J. P., et al. 2007, MNRAS, 378, 109
Melbourne, J., Phillips, A. C., Harker, J., Novak, G., Koo, D. C., & Faber, S. M. 2007, ApJ, 660, 81
Melbourne, J., et al. 2005, ApJ, 625, L27
Moustakas, L. A., et al. 2007, ApJ, 660, L31
Noeske, K. G., Koo, D. C., Phillips, A. C., Willmer, C. A., Melbourne, J., Gil de Paz, A., & Papaderos, P. 2006, ApJ, 640, L143
Odman, C. J., Hobson, M., Lasenby, A., & Melchiorri, A. 2004, Int. J. Mod. Phys. D, 13, 1661
Phillips, A. C., Guzman, R., Gallego, J., Koo, D. C., Lownenthal, J. D., Vogt, N. P., Faber, S. M., & Illingworth, G. D. 1997, ApJ, 489, 543
Rawat, A., Kembhavi, A. K., Hammer, F., Flores, H., & Barway, S. 2007, A&A, 469, 483
Rusin, D., & Kochanek, C. S. 2005, ApJ, 623, 666
Schneider, P. 2006, in Gravitational Lensing: Strong, Weak and Micro, ed. G. Meylan, P. Jetzer, & P. North (Berlin: Springer), 1
Schneider, P., Ehlers, J., & Falco, E. 1992, Gravitational Lenses (Berlin: Springer)
Shen, S., Mo, H. J., White, S. D. M., Blanton, M. R., Kauffmann, G., Voges, W., Brinkmann, J., & Csabai, I. 2003, MNRAS, 343, 978
Somerville, R. S., et al. 2008, ApJ, in press (astro-ph/0612428)
Stringer, M. J., & Benson, A. J. 2007, MNRAS, submitted (astro-ph/0703380)
Suyu, S. H., Marshall, P. J., Hobson, M. P., & Blundford, R. D. 2006, MNRAS, 371, 983
Tonini, C., Lapi, A., Shankar, F., & Salucci, P. 2006, ApJ, 638, L13
Treu, T. 2007, in IAU Symp. 235, Galaxy Evolution Across the Hubble Time, ed. F. Combes & J. Palous (Cambridge: Cambridge Univ. Press), 12
Treu, T., & Koopmans, L. V. E. 2002, ApJ, 575, 87
Trujillo, I., Burkert, A., & Bell, E. F. 2004, ApJ, 600, L39
Trujillo, I., et al. 2006, ApJ, 650, 18
Tully, R. B., & Fisher, J. R. 1977, A&A, 54, 661
Warren, S. J., & Dye, S. 2003, ApJ, 590, 673