The approximate tree decays \( B^- \to \pi^- \pi^0/\rho^- \rho^0 \) may serve as benchmark channels for testing the various theoretical descriptions of the strong interaction dynamics in hadronic \( B \) meson decays. The ratios of hadronic and differential semileptonic \( B \to \pi \ell \nu/\rho \ell \nu \) decay rates at maximum recoil could help to clarify this point.

I. INTRODUCTION

A wealth of observables at current and future \( B \) physics experiments is related to exclusive hadronic decay modes. \( B \) decays into a pair of light (charmless) mesons are of particular phenomenological interest as they are mediated by rare flavour-changing \( b \to q \) \((q = u, d, s)\) quark transitions and the interference of several weak decay amplitudes may induce sizeable CP-violating effects.

The complicated strong interaction dynamics in hadronic decays poses a serious challenge for accurate theoretical predictions. In recent years systematic methods have been developed, which are based on the factorization of short- and long-distance effects in the heavy quark limit \( m_b \gg \Lambda_{QCD} \). The theoretical concepts are known as QCD factorization (QCDF) \([1]\), soft-collinear effective theory (SCET) \([2]\) and the pQCD approach \([3]\).

In this letter we consider the decays \( B^- \to \pi^- \pi^0/\rho^- \rho^0 \) within the QCDF framework, which is based on the statement that the hadronic matrix elements of the operators in the effective weak Hamiltonian simplify in the heavy quark limit according to \([1]\)

\[
(M_1 M_2 | Q_i | \hat{B}) \simeq F^{BM_i}(0) f_{M_i} \int du \ T^I_t(u) \phi_{M_2}(u) (1)
\]

\[
+ \hat{f}_B f_{M_i} f_{M_2} \int d\omega dv du \ T^{II}_{\omega}(\omega, v, u) \phi_{B}(\omega) \phi_{M_1}(v) \phi_M(u).
\]

The factorization formula implies, on the one hand, that the theoretical prediction requires non-trivial hadronic input parameters, such as decay constants \( f \), moments of light-cone distribution amplitudes \( \phi \) and form factors \( F \), which encode all long-distance effects in the limit \( m_b \to \infty \). The power of the decomposition in \([1]\) lies, on the other hand, in the fact that it provides the path to a systematic implementation of radiative corrections. The short-distance hard-scattering kernels \( T^I_t \) are perturbatively calculable and currently being worked out to next-to-next-to-leading order (NNLO) \([4, 5, 6]\).

The NNLO calculation is to date incomplete, but a subset of hard-scattering kernels, which specify the so-called topological tree amplitudes, has recently been determined to NNLO \([4, 6]\). This allows us to present the first complete NNLO prediction within the QCDF framework for the decays \( B^- \to \pi^- \pi^0/\rho^- \rho^0 \), which are pure tree decays in the excellent approximation that small electroweak penguin amplitudes are neglected \([28]\).

As the considered decays are likely to be dominated by their standard model contribution, they may serve as benchmark channels for testing the various theoretical descriptions of the strong interaction dynamics in hadronic \( B \) decays. By normalizing the hadronic decay rates to their semileptonic counterparts \( B \to \pi \ell \nu/\rho \ell \nu \) at maximum recoil, most of the theoretical uncertainties from hadronic input parameters and \( |V_{ub}| \) drop out and one obtains precision observables for testing the QCD dynamics of the topological tree amplitudes. We confront the NNLO prediction in QCDF with experimental data and find support for the factorization assumption. We also take a look at the other tree-dominated \( B \to \pi \pi/\pi \rho/\rho \rho \) decay modes and conclude that the colour-suppressed tree amplitudes seem in general to be somewhat enhanced, which may hint at a smaller value of the first inverse moment of the \( \rho \) meson light-cone distribution amplitude, \( \lambda_B \approx 250 \text{ MeV} \). Precise measurements of the \( B \to \rho \ell \nu \) spectrum could help to clarify this point.

II. TREE AMPLITUDES

The decay amplitudes for hadronic \( B \) meson decays are conveniently parameterized by a set of topological amplitudes, which contain short-distance QCD and some electroweak effects. In the notation of \([2]\) they read

\[
A(B^- \to \pi^- \pi^0) = \left[ \lambda_u \left( \alpha_1 + \alpha_2 + \frac{3}{2} \alpha^\pi_{3,EW} + \frac{3}{2} \alpha^\rho_{4,EW} \right) \right] \frac{A_{2\pi}}{\sqrt{2}}
\]

\[
+ \lambda_d \left( \frac{3}{2} \alpha^\pi_{3,EW} + \frac{3}{2} \alpha^\rho_{4,EW} \right) \frac{A_{2\pi}}{\sqrt{2}}
\]

with \( \lambda_v = V_{ub} V^*_{ub} \) and \( A_{2\pi} = i G_F / \sqrt{2} m_B^2 f_{B} F_{B \pi}^{B \pi}(0) \) and similarly for \( B^- \to \rho^- \rho^0 \) with \( f_{\rho} \to f_\rho \), \( F_{B \pi}^{B \pi} \to A_0^{\rho \rho} \) and...
we deduced our default values for the hadronic parameters from recent lattice and sum rule calculations (where available). In general the parameters related to the pion (\( f_\pi, a_\pi^0, a_\pi^2 \)) are better determined than the ones related to the rho meson (\( f_\rho, a_\rho^0, a_\rho^2, A_0^{\rho B} \)). While there exists a large number of calculations for the B meson decay constant \( f_B \), less is known about the moments of the B meson wave function (\( \lambda_B, \sigma_1, \sigma_2 \)). Our value for \( \lambda_B \) is based on a QCD sum rule calculation and on estimates from the operator product expansion, accounting for recent claims that higher dimensional operators lower the value of \( \lambda_B \) (last paper of [13]). We estimate the size of higher order perturbative corrections by varying the factorization scales \( \mu_h \) and \( \mu_{hc} \) independently within the ranges specified in Table I. On the other hand we evaluate the non-factorizable power corrections at a fixed scale \( \mu_0 = 1.5 \) GeV. The latter introduce certain model parameters (\( \rho_H, \rho_B \)) and some additional hadronic parameters. We use \( (\bar{m}_u + \bar{m}_d)(2\text{MeV}) = 8 \) MeV, \( \bar{m}_b(\bar{m}_b) = 4.2 \) GeV and \( f_\rho^+(1\text{GeV}) = 165 \text{MeV}. \) This brings us to our NNLO prediction of the colour-allowed \((\alpha_1)\) and colour-suppressed \((\alpha_2)\) tree amplitudes. In the \( B \rightarrow \pi \pi / \pi \rho / \rho \rho \) channels we obtain

\[
\begin{align*}
\alpha_1(\pi \pi) &= 1.013 +0.017 +0.008 +0.014 + \ldots +0.027 +0.006 +0.020 +0.014 i = 1.013 +0.023 + \ldots +0.027 +0.025 i, \\
\alpha_2(\pi \pi) &= 0.195 +0.119 +0.255 i = 0.195 +0.134 + \ldots -0.101 +0.081 i, \\
\alpha_1(\rho \rho \rho) &= 1.017 +0.017 +0.001 +0.014 + \ldots +0.025 +0.007 +0.019 +0.014 i = 1.017 +0.024 + \ldots +0.025 +0.023 i, \\
\alpha_2(\rho \rho \rho) &= 0.177 +0.110 +0.025 +0.055 + \ldots -0.097 +0.021 +0.021 +0.055 i = 0.177 +0.126 + \ldots -0.097 +0.062 i,
\end{align*}
\]

where the uncertainties in the intermediate results stem from the variation of hadronic input parameters, higher order perturbative corrections and the considered model for power corrections, respectively, which have been added in quadrature for our final error estimate.

We see, on the one hand, that the colour-allowed tree amplitudes \( \alpha_1 \) can be computed precisely in the factorization framework. The colour-suppressed amplitudes \( \alpha_2 \) suffer, on the other hand, from substantial theoretical uncertainties. The problem is related to certain cancellations between various perturbative contributions, which make the real parts particularly sensitive to the spectator scattering mechanism which is proportional to the hadronic ratio \( f_M, f_B/\lambda_B F^{BM1}(0) \). Our poor knowledge of the B meson parameter \( \lambda_B \) in particular translates into the uncertainties \( +0.107 +0.094 \) and \( +0.096 -0.043 \) for the real parts of \( \alpha_2(\pi \pi) \) and \( \alpha_2(\rho \rho \rho) \), respectively.
III. BRANCHING RATIOS

The branching ratios of $B^+ \to \pi^- \pi^0 / \rho^- \rho^0$ depend in addition on electroweak penguin amplitudes, cf. [2]. These amplitudes have not yet been determined to NNLO [29], but their numerical values are rather small ($|\alpha_{3/4,EW}| \lesssim 0.01$). As they are not CKM-enhanced in tree-dominated decays, it is consistent to treat these amplitudes in the NLO approximation. The explicit NLO results can be found in [17,28] (they are formulated in a different operator basis of the effective Hamiltonian).

The CP-averaged branching ratios become

$$10^6 \text{Br}(B^- \to \pi^- \pi^0) = 6.22^{+1.14}_{-1.05} + 2.03 + 0.16 + 0.43 - 1.65 - 0.18 - 0.42,$$

$$10^6 \text{Br}(B^- \to \rho^- \rho^0) = 21.0^{+3.9}_{-3.5} + 7.4 + 0.5 + 1.5 - 6.1 - 0.7 - 1.4,$$

$$\text{Br}(B^- \to \rho^- \rho^0) = 21.0^{+8.5}_{-7.3}.$$ (4)

where the uncertainties in the intermediate results are due to CKM parameters, hadronic parameters, higher order perturbative corrections and non-factorizable power corrections, respectively.

Our NNLO results are in good agreement with experimental data [13] [20],

$$10^6 \text{Br}(B^- \to \pi^- \pi^0)_{\text{exp}} = 5.59^{+0.41}_{-0.40},$$

$$10^6 \text{Br}(B^- \to \rho^- \rho^0)_{\text{exp}} = 22.5^{+1.9}_{-1.9},$$ (5)

i.e. the experimental values are reasonably well reproduced by the central values of our NNLO prediction, which is based on the input parameters from Table III. One should keep in mind, however, that we could also have obtained similar numbers for the branching ratios with rather different values of the tree amplitudes, the form factors and $|V_{ub}|$. As we discuss in the following section, a much stronger test of the factorization assumption can be obtained by considering the ratios of hadronic and differential semileptonic decay rates, where the dependence on the form factors and $|V_{ub}|$ drops out to a large extent.

We may also take a look at the other tree-dominated $B \to \pi \pi / \rho \rho$ decay modes. We emphasize that the NNLO calculation of these branching ratios is to date still incomplete, since the QCD penguin amplitudes have not yet been determined to NNLO (this is why we do not discuss CP asymmetries in this letter). These modes also differ conceptually from $B^- \to \pi^- \pi^0 / \rho^- \rho^0$ in the sense that they receive contributions from weak annihilation, which constitutes another class of non-factorizable power corrections. We again use the model from [1] to estimate their size.

Our results for the CP-averaged branching ratios are shown in Table III. Apart from some exceptions ($\pi^+ \pi^-, \pi^0 \pi^0, \pi^- \rho^-, \pi^- \rho^+ \pi^0 \rho^0, \pi^- \rho^+ \rho^0$) our default prediction (with central values) is again in reasonable agreement with the data. The agreement is, however, less pronounced than for the pure tree decays $B^- \to \pi^- \pi^0 / \rho^- \rho^0$. Moreover, we point out that the colour-suppressed modes $(\pi^0 \pi^0, \pi^0 \rho^0, \rho^0 \rho^0)$ are subject to sizeable theoretical uncertainties. This is partly related to the problem mentioned at the end of the previous section ($\lambda_B$) and in addition to the fact that these modes are more likely to be affected by $1/m_b$-corrections.

In order to illustrate the correlation of the theoretical uncertainties, we show in Table III the central values of some extreme scenarios (in the spirit of [7]):

In Scenario A we study the dependence on the weak phase $\gamma$ (we set $\gamma = 110^\circ$). Modes that show a strong dependence on this scenario ($\pi^+ \pi^-, \pi^0 \rho^0, \rho^0 \rho^0$) are not particularly suited for our purposes, as we focus on testing the QCD dynamics of the topological tree amplitudes in this work.

In Scenario B we pursue the question if the data are in accordance with a large colour-suppressed amplitude, which may be realized in the factorization framework by a very low value of $\lambda_B = 200$ MeV (we moreover decrease the form factors to $F^2_{B^0}(0) = 0.21$ and $A^2_{B^0}(0) = 0.27$). This scenario shows a satisfactory description of the data, in particular the "problematic" modes $\pi^+ \pi^-, \pi^- \rho^-$ and $\pi^- \rho^+$ are - by construction - in much better agreement with the data.

It is tempting to understand the large experimentally observed $\pi^0 \rho^0$ branching ratio as an indication for sizeable non-factorizable power corrections. It is hard to address this issue in a model-independent way. We would like to emphasize, however, that some observables are indeed more likely to be affected by $1/m_b$-corrections than others (cf. the column labelled "pow" in Table III). We in particular expect the branching ratios of the tree decays $B^- \to \pi^- \pi^0 / \rho^- \rho^0$ to be clean observables as they are free of weak annihilation contributions.

In order to quantify this question we study the influence of a large annihilation amplitude in Scenario C (within the BBNS model). It turns out that it is almost impossible to enhance the $\pi^0 \rho^0$ decay rate and to simultaneously decrease the $\pi^+ \pi^-$ rate without fine-tuning the model parameters [30]. Moreover, the overall pattern of branching ratios and in particular the rates of the other colour-suppressed modes seem to disfavour a generic scenario with large annihilation contributions.

This is illustrated in Scenario C, where we double the default value of the BBNS model for universal weak annihilation, i.e. we set $\rho_A = 1$ and $\phi_A = 0$. We conclude that we do not see any clear pattern of abnormally large power corrections in the data and prefer to be guided by clean observables rather than by the colour-suppressed and penguin-contaminated $\pi^0 \rho^0$ branching ratio, which cannot be predicted precisely in the factorization framework anyway. We admit that our conclusion is a model-dependent statement, which is, however, supported by a light-cone sum rule analysis, which finds even smaller annihilation contributions than the BBNS model with default parameters [21].
\[ \gamma = 90^\circ, \lambda_B = 250 \text{ MeV and } F^{\pm \pi}_B(0) = 0.23, \text{ which are within the ranges of our default parameters from Table II.} \]

These values are inspired by a fit to a set of particularly clean observables that we discuss below. We refrain from presenting the details of our fit and prefer to simply illustrate the effects of such a combined scenario [31].

\section*{IV. PRECISION OBSERVABLES}

Our predictions for the branching ratios from Table II typically have \textasciitilde 40\% uncertainties, which are largely related to an overall normalization from \( |V_{ub}| F^{\mp \pi}_B(0) \) and \( |V_{ub}| A^{B\rho}_B(0) \). This particular source of uncertainties can be eliminated by normalizing the hadronic decay rates to the differential semileptonic rates at maximum recoil,

\[ \frac{d\Gamma}{dq^2}(B^0 \to \pi^+ \ell^- \bar{\nu}_\ell) = \frac{G_F^2 (m_B^2 - m_{\pi}^2)^2}{192\pi^3 m_B^5} |V_{ub}|^2 F^{B\pi}_B(0)^2 \]

(6)

and similarly for \( B^0 \to \rho^+ \ell^- \bar{\nu}_\ell \) with \( F^{B\rho}_B(0) \) and \( m_{\pi} \to m_{\rho} \). The situation is, however, different for the colour-suppressed modes (\( \pi^0, \rho^0 \)) which are rather dominated by the uncertainties from \( A_B \) and power corrections than by form factor uncertainties and \( |V_{ub}| \). We therefore do not consider these modes in this section.

The BaBar collaboration has measured the semileptonic \( B^0 \to \pi^+ \ell^- \bar{\nu}_\ell \) decay spectrum to high accuracy [22]. The data has been investigated in detail under various types of form factor parameterizations in [23]. This analysis uses the HFAG average for the absolute branching ratio and finds \( |V_{ub}| F^{B\pi}_B(0) = (9.1 \pm 0.7) \cdot 10^{-4} \), which is to be compared with our default value \( 10.3 \cdot 10^{-4} \) and \( 8.3 \cdot 10^{-4} \) from Scenario B. The experimental value has been adopted in conjunction with our default value for \( |V_{ub}| \) to fix the form factor \( F^{B\pi}_B(0) = 0.23 \) in Scenario D.

The analysis of the differential semileptonic \( B \to \rho \ell \nu \) decay spectrum is more complicated as three different form factors contribute in this case (which confine to \( |V_{ub}| A^{B\rho}_B(0) \) at maximum recoil). Recent measurements by BaBar, Belle and CLEO provide data in 3-4 \( q^2 \)-bins [24], which does not yet allow to extrapolate the decay spectrum in a model-independent way. In a recent analysis the data has been combined with (quenched) lattice calculations of the form factors in the high \( q^2 \) region and light-cone sum rule predictions for \( q^2 = 0 \) [22]. This analysis yields \( |V_{ub}| A^{B\rho}_B(0) = (5.5 \pm 2.6) \cdot 10^{-4} \), which illustrates that the data is still premature. We therefore do not include this number in our analysis.

Our predictions for the ratios

\[ R_{M_1}(M_1 M_2) = \frac{\Gamma(B \to M_1 \bar{M}_2)}{d\Gamma(0 \to M_1^- \ell^- \bar{\nu}_\ell)/dq^2|_{q^2=0}} \]

(7)

are shown in Table II. For the \( \pi \rho \)-modes we chose the normalization such that the dependence on the form factor multiplying the colour-allowed amplitude is most strongly eliminated. From Table II it can be seen that the theoretical uncertainties have been reduced considerably to the level of \( \sim 15\% \). Moreover, correlations among different sources of theoretical uncertainties have been resolved to a large extent.

The first two ratios in Table II provide particular clean probes of the QCD dynamics of the topological tree amplitudes [2, 20]. In the factorization framework we have

\[ R_\pi(\pi^- \pi^0) \simeq 3 \pi^2 F_{\pi}^2 |V_{ud}|^2 |\alpha_1 + \alpha_2|^2, \]

(8)

where small electroweak penguin amplitudes have been suppressed. Our NNLO prediction for this ratio

\[ R_\pi(\pi^- \pi^0) = (0.76^{+0.12}_{-0.08}) \text{ GeV}^2 \]

(9)

is in good agreement with experimental data

\[ R_\pi(\pi^- \pi^0)_{\exp} = (0.81^{+0.14}_{-0.14}) \text{ GeV}^2 \]

(10)
which strongly supports the factorization assumption. It is, however, interesting that the central experimental value is in between our default prediction and the value 0.95 GeV^2 from Scenario B, which may hint at a somewhat larger value of the colour-suppressed amplitude and hence a lower value of the parameter $\Delta_R \approx 250$ MeV (which we adopt in Scenario D). Experimental data for the ratio $R_{\rho\rho}(\rho^0_L\rho^0_L)$ could help to clarify this point.

We recall that all other ratios from Table III receive contributions from QCD penguin amplitudes that are not yet completely available to NNLO. Among these $R_{\pi^+\pi^-}$, $R_{\rho^0_L\rho^0_L}$ and $R_{\rho^+\rho^-}$ are particularly suited to test the dynamics of the colour-allowed amplitudes. Our prediction for $R_{\pi^+\pi^-}$ compares again well to the experimental value.

The fourth colour-allowed ratio $R_{\pi^+\pi^-}$ is special, since the interference of the colour-allowed amplitude with the QCD penguin amplitude is not negligible in this case. This ratio is thus particularly sensitive to the choice of the weak phase $\gamma$. One should keep in mind, however, that the power corrections from weak annihilation represent another important source of uncertainties for this ratio. It is interesting to replace the BBNS model for weak annihilation by the light-cone sum rule prediction from [21], which strongly reduces the uncertainties from weak annihilation and hence enhances the sensitivity to $\gamma$ (we then find $1.03^{+0.02}_{-0.02}$ GeV^2). The current experimental value may then be considered as a hint at a large value $\gamma > 90^\circ$. A smaller value of $\gamma$, on the other hand, may then imply the presence of an additional contribution to the QCD penguin amplitude or that power corrections, which are neither from weak annihilation nor from chirally enhanced wave functions, have been underestimated in our approach. The latter would be conceptually important, as it would increase the total uncertainty from power corrections in QCDF. We refrain, however, from drawing any conclusions concerning $R_{\pi^+\pi^-}$ and its implications for $\gamma$, as long as the penguin amplitudes have not been calculated to NNLO.

In Table III we also show some ratios of hadronic decay rates defined by

$$R(M_1M_2/M_3M_4) = \frac{\Gamma(\bar{B} \to M_1M_2)}{\Gamma(\bar{B}^\prime \to M_3M_4)}$$

The ratio $R(\rho^0_L\rho^0_L/\rho^+_L\rho^+_L)$ yields complementary information on the tree amplitudes from the $\rho$-sector, where the contamination from the QCD penguin amplitudes is known to be less important [8, 27]. We consider the experimental value for this ratio as another important evidence in favour of an enhanced colour-suppressed amplitude (Scenario B or D).

The ratios $R(\rho^0_L\rho^0_L/\rho^+_L\rho^+_L)$ and $R(\pi^+\pi^-/\pi^+\pi^-)$ of colour-allowed modes can be predicted precisely in the factorization framework. Whereas the second ratio is in nice agreement with the data, the first one seems to somewhat disfavour a scenario with a large weak phase $\gamma$.

The last two ratios from Table III finally refer to what is known as the $B \to \pi\pi$ puzzle. Whereas the ratio $R(\pi^0\pi^0/\pi^+\pi^-)$ is by construction in Scenarios A, B and D in better agreement with experimental data than our default prediction, the ratio $R(\pi^+\pi^-/\pi^0\pi^0)$ illustrates what we mentioned at the beginning of this section, i.e., the bulk of theoretical uncertainties does not drop out in ratios that involve colour-suppressed modes. The uncertainties of our default prediction

$$R(\pi^+\pi^-/\pi^0\pi^0) = 25.7^{+26.0}_{-18.7}$$

are thus extremely large and the central value is in vast disagreement with the data. This ratio may be brought down by a factor of ~3 in Scenarios A, B and D, which may be seen as an independent evidence in favour of these scenarios. The fact, however, that these predictions still

| Observable | Theory | CKM | had | $\mu$ | pow | A | B | C | D | Experiment |
|------------|--------|-----|-----|------|-----|---|---|---|---|-----------|
| $R_{\rho\rho}(\pi\pi)$ | 0.70±0.12 | +0.01 | +0.11 | +0.02 | +0.05 | 0.68 | 0.95 | 0.70 | 0.82 | 0.81±0.14 |
| $R_{\rho\rho}(\rho^0_L\rho^0_L)$ | 1.91±0.32 | +0.03 | +0.28 | +0.05 | +0.13 | 1.83 | 2.38 | 1.91 | 2.09 | n.a. |
| $R_{\rho\rho}(\pi\rho^0)$ | 0.85±0.22 | +0.08 | +0.17 | +0.03 | +0.11 | 1.01 | 1.16 | 0.93 | 1.07 | n.a. |
| $R_{\rho\rho}(\pi\rho^0)$ | 1.71±0.27 | +0.16 | +0.28 | +0.03 | +0.11 | 1.35 | 2.07 | 1.79 | 1.71 | 1.57±0.32 |
| $R_{\rho\rho}(\pi\rho^0)$ | 1.09±0.22 | +0.15 | +0.03 | +0.02 | +0.16 | 0.75 | 0.97 | 1.24 | 0.86 | 0.80±0.13 |
| $R_{\rho\rho}(\pi\rho^0)$ | 2.77±0.32 | +0.15 | +0.15 | +0.05 | +0.23 | 2.44 | 2.46 | 2.99 | 2.44 | 2.43±0.47 |
| $R_{\rho\rho}(\pi\rho^0)$ | 1.12±0.20 | +0.07 | +0.03 | +0.02 | +0.18 | 1.27 | 1.01 | 1.29 | 1.13 | n.a. |
| $R_{\rho\rho}(\rho^0_L\rho^0_L)$ | 2.95±0.37 | +0.15 | +0.16 | +0.06 | +0.28 | 2.61 | 2.68 | 3.22 | 2.64 | n.a. |
| $R_{\rho\rho}(\rho^0_L\rho^0_L)$ | 0.65±0.16 | +0.03 | +0.13 | +0.03 | +0.08 | 0.70 | 0.89 | 0.59 | 0.79 | 0.89±0.14 |
| $R_{\rho\rho}(\rho^0_L\rho^0_L)$ | 2.64±0.34 | +0.31 | +0.13 | +0.00 | +0.06 | 2.06 | 2.65 | 2.49 | 2.33 | 3.23±0.69 |
| $R_{\rho\rho}(\pi\pi)$ | 0.39±0.05 | +0.04 | +0.02 | +0.00 | 0.02 | 0.31 | 0.39 | 0.42 | 0.35 | 0.33±0.04 |
| $R(\rho^0_L\rho^0_L)$ | 0.65±0.19 | +0.10 | +0.14 | +0.03 | +0.08 | 0.90 | 0.98 | 0.57 | 0.95 | 1.01±0.09 |
| $R(\pi^+\pi^-/\pi^0\pi^0)$ | 25.7±26.0 | +2.72 | +7.0 | +2.6 | +10.2 | 9.33 | 8.32 | 17.3 | 8.13 | 3.33±0.43 |
suffer from $\sim 60\%$ uncertainties related mainly to the power corrections, $\alpha_s^2$, $\mu_{\text{nc}}$ and $f_B$ (in decreasing order of importance), shows that we cannot expect to predict this ratio precisely. We would like to add that there is no such puzzle in the $\pi\pi/\rho\rho$ channels, i.e. there is no general failure of QCDF to describe colour-suppressed modes.

V. CONCLUSIONS

We presented the NNLO QCDF prediction for the approximate tree decays $B^- \rightarrow \pi^- \pi^0/\rho^-\rho^0$ and updated the global analysis of the other tree-dominated $B \rightarrow \pi\pi/\rho\rho$ decay modes. Our analysis from Section IV showed that QCDF yields precise theoretical predictions for particular ratios of decay rates. We find in general support for the factorization assumption and uncovered some hints for enhanced colour-suppressed amplitudes, which translate in QCDF into a small value of the $B$ meson parameter $\lambda_B$. Theoretical progress from non-perturbative methods on the hadronic ratio $f_M f_B / \lambda_B F^{BM}(0)$ as well as experimental measurements of the semileptonic $B \rightarrow \rho \ell \nu$ decay spectrum may shed further light on this issue.

Acknowledgments

We are grateful to Gerhard Buchalla for interesting discussions and comments on the manuscript. We would like to thank Andreas Hocker for helpful correspondence. The work from G.B. was supported by the DFG Sonderforschungsbereich/Transregio 9. The work from V.P. is partially supported by the Swiss National Foundation as well as EC-Contract MRTNCT-2006-035482 (FLAVIAnet). The Albert Einstein Center for Fundamental Physics is supported by the "Innovations- und Kooperationsprojekt C-13 of the Schweizerische Universitatskonferenz SUK/CRUS".

[1] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. 83 (1999) 1914, Nucl. Phys. B 591 (2000) 313, Nucl. Phys. B 606 (2001) 245.
[2] C. W. Bauer, S. Fleming, D. Pirjol and I. W. Stewart, Phys. Rev. D 63 (2001) 114020; C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. D 65 (2002) 054022.
[3] Y. Y. Keum, H. N. Li and A. I. Sanda, Phys. Rev. D 63 (2001) 054008.
[4] M. Beneke and S. Jäger, Nucl. Phys. B 751 (2006) 160; N. Kivel, JHEP 0705 (2007) 019; V. Pilipp, PhD thesis, LMU München, 2007, arXiv:0709.0497 [hep-ph]; Nucl. Phys. B 794 (2008) 154.
[5] M. Beneke and S. Jäger, Nucl. Phys. B 768 (2007) 51; A. Jain, I. Z. Rothstein and I. W. Stewart, arXiv:0706.3399 [hep-ph].
[6] G. Bell, Nucl. Phys. B 795 (2008) 1; PhD thesis, LMU München, 2006, arXiv:0705.3133 [hep-ph]; arXiv:0902.1915 [hep-ph].
[7] M. Beneke and M. Neubert, Nucl. Phys. B 675 (2003) 333.
[8] M. Beneke, J. Rohrer and D. Yang, Nucl. Phys. B 774 (2007) 64; M. Bartsch, G. Buchalla and C. Kraus, arXiv:0810.0249 [hep-ph].
[9] C. Bobeth, M. Misiak and J. Urban, Nucl. Phys. B 574 (2000) 291; M. Gorbahn and U. Haisch, Nucl. Phys. B 713 (2005) 291.
[10] K. G. Chetyrkin, M. Misiak and M. Munz, Nucl. Phys. B 520 (1998) 279.
[11] V. M. Braun et al., Phys. Rev. D 74 (2006) 074501; J. Bijnen and A. Kohldajirian, Eur. Phys. J. C 26 (2002) 67; A. B. Bakuleev, S. V. Mikhaiov and N. G. Stefanis, Phys. Lett. B 578 (2004) 91; P. Ball and R. Zwicky, Phys. Lett. B 625 (2005) 225; P. Ball, V. M. Braun and A. Lenz, JHEP 0605 (2006) 004.
[12] P. A. Boyle, D. Brommel, M. A. Donnellan, J. M. Flynn, A. Juttner and C. T. Sachrajda [RBC Collaboration and UKQCD Collaboration], arXiv:0810.1660 [hep-lat].
[13] M. Okamoto et al., Nucl. Phys. Proc. Suppl. 140 (2005) 461; E. Dalgic, A. Gray, M. Wingate, C. T. H. Davies, G. P. Lepage and J. Shigemitsu, Phys. Rev. D 73 (2006) 074502 [Erratum-ibid. D 75 (2007) 119906]; P. Ball and R. Zwicky, Phys. Rev. D 71 (2005) 014015; G. Dunlan, A. Khodjamirian, T. Mannel, B. Melic and N. Offen, JHEP 0804 (2008) 014.
[14] P. Ball, G. W. Jones and R. Zwicky, Phys. Rev. D 75 (2007) 054004.
[15] P. Ball and R. Zwicky, JHEP 0604 (2006) 046.
[16] P. Ball and R. Zwicky, Phys. Rev. D 71 (2005) 014029.
[17] E. Gamiz, C. T. H. Davies, G. P. Lepage, J. Shigemitsu and M. Wingate [HPQCD Collaboration], arXiv:0902.1815 [hep-lat]; C. Bernard et al., PoS LATTICE2008 (2008) 278; A. A. Penin and M. Steinhauser, Phys. Rev. D 65 (2002) 054006; M. Jamin and B. O. Lange, Phys. Rev. D 65 (2002) 056005.
[18] V. M. Braun, D. Y. Ivanov and G. P. Korchemsky, Phys. Rev. D 69 (2004) 034014; S. J. Lee and M. Neubert, Phys. Rev. D 72 (2005) 094028; H. Kawamura and K. Tanaka, Phys. Lett. B 673 (2009) 201.
[19] E. Barberio et al. [Heavy Flavor Averaging Group], arXiv:0808.1297 [hep-ex] and updates at http://www.slac.stanford.edu/xorg/hfag/index.html.
[20] B. Aubert et al. [BaBar Collaboration], Phys. Rev. Lett. 102 (2009) 141802; B. Aubert et al. [BaBar Collaboration], Phys. Rev. D 76 (2007) 052007; A. Somov et al., Phys. Rev. Lett. 96 (2006) 171801; B. Aubert et al. [BaBar Collaboration], Phys. Rev. D 78 (2008) 071104.
[21] A. Khodjamirian, T. Mannel, M. Melcher and B. Melic, Phys. Rev. D 72 (2005) 094012.
[22] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 98 (2007) 091801.
[23] P. Ball, Phys. Lett. B 644 (2007) 38.
[24] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 72 (2005) 051102; T. Hokuue et al. [Belle Collaboration], Phys. Lett. B 648 (2007) 139; N. E. Adam et al. [CLEO Collaboration], Phys. Rev. Lett. 99 (2007) 041802.
[25] J. M. Flynn, Y. Nakagawa, J. Nieves and H. Toki,
[26] J. D. Bjorken, Nucl. Phys. Proc. Suppl. 11 (1989) 325.
[27] R. Aleksan, F. Buccella, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Lett. B 356 (1995) 95.
[28] In our numerical analysis the electroweak penguin amplitudes will be included in the NLO approximation.
[29] Partial NNLO results of the penguin amplitudes from spectator scattering can be found in [5].
[30] It should be noticed that these decay rates depend on the same combination of annihilation amplitudes.
[31] We think that a sophisticated fit to the observables from Table III should only be considered when the semi-leptonic $B \to \rho \ell \nu$ spectrum has been measured precisely.