Adequacy Criteria of Models of the Cargo Inspection System with Material Discrimination Option

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Abstract. Generalized adequacy criteria for mathematical models in order to discriminate materials in X-ray inspection systems by the dual-energy method were developed. Two main approaches of the examination systems to produce the adequacy criteria by the final and the intermediate parameters of the dual-energy method were analyzed. The criteria were specified in respect to the discrimination by the effective atomic number and by the method of level functions. Experimental and theoretical estimates of the discrimination parameters of the test object constituents scanned by fan beams of X-ray radiation with the maximal energies of 4.5 and 9 MeV are given.

1. Introduction

Digital radiography is widely used to examine luggage, vehicles, and containers [1–4]. In order to discriminate various materials in the inspected objects, the dual-energy method (DEM) is often used [4–6]. The discrimination is based on the correlation of X-ray parameters relevant for the object materials with those for known materials. An effective atomic number or a related parameter of a material is often used as a discrimination parameter [5, 7–9]. There are several approaches to generate primary radiographs based on the DEM, as well as its transformation to final DEM images or to images of the discrimination parameters as described in detail in [7]. Different physical factors affect the quality of the discrimination both for low [10] and high X-ray energies [11]. Until now, there is a lack of adequate mathematical models of digital radiography systems dealing with the discrimination of the test object materials and their constituents. Here, we show that adequate criteria for discrimination parameter can be obtained for the DEM method.

2. Generalised adequacy criteria for systems with discrimination by the dual energy method

Let A be a finite set, each element \( a \in A \) match one and only one group of the materials. It is most convenient for matching to use the effective atomic number which is the average atomic number for a mixture of materials in a given compound. In a radiographic inspection, it must be possible to identify the object’s constituents independent of their size, their mass density \( \rho \) and the transmission length \( H \) of the X-rays within the object. As an integral characteristic one may use the product \( \rho H \). From an application point of view, it is of interest to identify \( \rho H \) within \([\rho H_{\text{min}}, \rho H_{\text{max}}]\), in which the material
from the group \( a \in A \) is recognized with a given confidence probability. The object to be inspected consists of a finite number of constituents. Therefore it is logical to define a set

\[
R(a) = \{ \rho_H(a), i = 1..n(a), a \in A \},
\]

where \( n(a) \) is the number of constituents with \( \rho_H \) as the identifying parameter for a group of materials \( a \). Furthermore, let us introduce a set \( M_i \), which defines a region of interest in the following way:

\[
M_i = \{ (\rho_H, a): \rho_H \in R(a); a \in A \}.
\]

In processing the radiographs obtained by DEM for each test object constituent, a pair of intermediate parameters, a pair of DEM parameters, and a final identifying parameter may be computed. Any specified groups of parameters can be used to create adequacy criteria. The criteria parameters are represented in vectors \( p = (p_1, p_2, ..., p_k) \). To define the criteria two vectors are used: the theoretical vector \( p \) and the experimental vector \( \bar{p} \). The required criteria reduce in the most generalized form to

\[
M_i \subset M^* = \{ (\rho_H, a): |p_i(\rho_H, a) - \bar{p}_i(\rho_H, a)| \leq \Delta p_i(\rho_H, a); i = 1, k \}.
\]

where \( \Delta p_i(\rho_H, a) \) are the maximal deviations of the theoretical and the experimental parameters.

3. Discrimination by effective atomic number

The primary radiographs are obtained by scanning the test object with X-ray radiation at two energies \( E_1 \) and \( E_2 \). The primary radiographs are transformed in a first process into DEM images and in second stage to a final identity image, which represents the distribution of the effective atomic number \( Z \) overlaid on the primary radiograph. The algorithm to identify the test object material by DEM is described in detail in Ref. [7–13]. To estimate the adequacy of the corresponding mathematical model, it is necessary to compare the simulated images with the measured images of the test object. Ref. [7] describes a test object whose constituents have an equal quadratic cross-sections. Let us assign a set of elements \( a \in A \), as described in the first chapter, a value of the effective atomic number \( Z \). The test object constituents are characterized by a pair of numbers \((\rho_H, Z)\), where \( \rho_H \) is the ray thickness of the constituent in \( g/cm^2 \) and \( Z \) is the effective atomic number.

As a result of the primary processing of the original radiographs of the test objects \( I_1 \) and \( I_2 \), corresponding to the X-ray energies \( E_1 \) and \( E_2 \), the mean values of ray-thicknesses of all test object constituents can be estimated for energies \( E_1 \) and \( E_2 \). The constituent with the characteristics \((\rho_H, Z)\) is associated with its ray-thicknesses \( Y_1(\rho_H, Z) = Y(E_1, \rho_H, Z) \) and \( Y_2(\rho_H, Z) = Y(E_2, \rho_H, Z) \), as measured by the absorption length of the X-ray.

As a next step the DEM parameters \( A(\rho_H, Z) \approx \rho_H \) and \( B(\rho_H, Z) \approx \rho_H F(Z) \) must be found for all constituents \((\rho_H, Z)\), \( \rho_H \in R(a), a \in A \). For X-ray sources with energies up to 150 keV, the function \( F(Z) \approx Z^{1.6} \) [14], but for high-energy X-rays with energies above 1.022 MeV, the function \( F(Z) = Z \) [11]. The last stage of the algorithm reduces to the estimation of the effective atomic numbers of materials for all test object constituents, that is the set \( \bar{Z}(\rho_H, Z), \rho_H \in R(Z), Z \in A \). The corresponding formula has the form

\[
\bar{Z}(\rho_H, Z) = F^{-1}\left[ B(\rho_H, Z) / A(\rho_H, Z) \right]
\]

where \( F^{-1} \) is the inverse function of \( F \). The concretization of the adequacy criterion (3) in respect to the discrimination by the effective atomic number is in a formalized form given by

\[
M_i \subset M^* = \{ (\rho_H, Z): |\bar{Z}(\rho_H, Z) - Z_i(\rho_H, Z)| \leq \Delta Z(\rho_H, Z) \}
\]

where \( \Delta Z(\rho_H, Z) \) is the equality of the final DEM images.
Let us call the corresponding criterion the adequacy criterion for intermediate DEM parameters $Y_1(\rho_H, Z)$ and $Y_2(\rho_H, Z)$. Furthermore, we formalise the introduced criterion in respect to the test object. This formalization is implemented by analogy with the approach based on expression (3). The adequacy criterion with the intermediate DEM parameters has the form

$$M_i \subset M^* = \{ (\rho_H, Z) : |p_i(\rho_H, Z) - \bar{p}_i(\rho_H, Z)| \leq \Delta p_i(\rho_H, Z) ; i = 1, 2 \}.$$  (6)

As pairs of theoretical and experimental parameters $p_1, p_2$ and $\bar{p}_1, \bar{p}_2$ in expression (6), one can use corresponding values of ray thicknesses $Y(E_1, \rho_H, a)$ and $Y(E_2, \rho_H, a)$, or the final DEM parameters – $A(\rho_H, Z)$ and $B(\rho_H, Z)$.

4. Discrimination by the method of level functions

The procedure for the discrimination of test-object materials by level functions is described in detail in Ref. [7]. The method is based on the calculation of the discrimination parameter $q$ for each point $(x, y)$ of the original radiographs. The value of the discrimination parameter $q$ for point with coordinates $(x, y)$ is calculated by expression

$$q(x, y) = \frac{Y_1(x, y)}{Y_2(x, y)} = Q(Y_i(x, y)).$$  (7)

The ratio $q$ is a function of the ray thickness for an X-ray radiation source with the maximal energy $E_1$. Here, for maximal X-ray energies less than 1 MeV, we select $E_2 < E_1$, but for maximal energies larger than 1 MeV we select $E_2 > E_1$. The functions $Q(Y_i(x, y))$ are generated by special test objects, described in chapters 1 and 2, for all classes $a, a \in A$ of identifiable materials and serve to build a set of level functions $U(Y_i(x, y))$ and $U(Y_i(x, y))$.

The test object material along the beam connecting the source and the point $(x, y)$, refers to class $a, a \in A$, if the following relation is satisfied

$$U(Y_i(x, y), a) \leq q(x, y) < U(Y_i(x, y), a).$$  (8)

For the concretization of the adequacy criterion (3) for material discrimination by level functions, we make use of the fact that the set of level functions is defined uniquely by the set of the functions $Q(Y_i(x, y), a), a \in A$. Here, a material class $a$ is selected as the material most applicable for this class. We shall use polyethylene, aluminium, and iron as materials.

The first version of the criterion can be named the criterion of identifying functions. Using the approach outlined in the first chapter, we define the adequacy criterion

$$M_i \subset M^* = \{ (\rho_H, a) : \|Q(Y_i(\rho_H, a)) - \bar{Q}(Y_i(\rho_H, a))\| \leq \Delta Q(Y_i(\rho_H, a)) \}.$$  (9)

The adequacy criterion based on expression (9) is much less dependent on the change of mutual geometric positions of the test object and the inspection system than the adequacy criterion by the final DEM parameters mentioned in the first chapter. The second concretization of the criterion (3) is similar to the adequacy criterion of the intermediate DEM parameters described in the previous chapter.

5. Computational formulae

5.1. Discrimination by effective atomic number

The theoretical values of the ray thicknesses $(\rho_H, Z)$ of the constituents for X-rays with the maximal energies $E_1, E_2$ are computed from the expressions [7]:

$$Y_i(\rho_H, Z) = -\ln \left[ \int_{N_{\text{in}}} \left( \frac{N_{\text{in}}}{N_{\text{in}}} \left( \frac{E_a(E)f(E, E, \chi(E, \chi(E, \chi))e^{-\chi(E, \chi)^2/kT}}{\Delta E} \right) \right) \right],$$

$$Y_i(\rho_H, Z) = -\ln \left[ \int_{N_{\text{in}}} \left( \frac{N_{\text{in}}}{N_{\text{in}}} \left( \frac{E_a(E)f(E, E, \chi(E, \chi))e^{-\chi(E, \chi)^2/kT}}{\Delta E} \right) \right) \right].$$  (10)
Here, \( N_{01} \) and \( N_{02} \) are the number of the X-ray quanta with energies \( E_1 \) and \( E_2 \), which are recorded by the detector in the absence of a test object. Furthermore, \( E_{\text{ab}}(E) \) is the mean value of the absorbed energy of the recorded photons with the energy \( E, f(E,E) \) is the energy spectrum of the X-ray radiation with maximal energy \( E, \eta(E,h) \) is the detection efficiency of the scintillator of thickness \( h \) for photons with energy \( E, m(E,Z) \) is the mass attenuation coefficient of the photon radiation with energy \( E, \text{int}(x) \) represents the integer part of number \( x, \) and \( \Delta I \) is the sampling rate.

In references [7, 13] the integral equations to find the DEM parameters \( A \) and \( B \) are given. In order to continue with the purpose of our work, we shall use this system of equations to find \( A \) and \( B \), similar to Eq. (10):

\[
\begin{align*}
\ln \left( \frac{\int E \cdot g(E)f(E,E)\,dE}{\int E \cdot g(E)f(E,E)\,dE} \right) &= -2Y_{1}\{pH,Z,\}
\end{align*}
\]

Here, \( g_1(E), g_2(E) \) describe the energy dependencies of the x-ray interaction with the material constituents. Index «1» corresponds to the Compton effect. Index «2» corresponds to the photo-effect for X-ray energies less than 1.022 MeV, and to the effect of pair creation for energies larger than 1.022 MeV. It should be mentioned that Eqs. (11) were derived under the condition \( \Delta I \approx 0 \), that is for an analog-to-digital converter (ADC) with an infinite number of bits. In Ref. [10], the influence of the ADC capacity on the accuracy of the effective atomic number definition was examined in detail. As a result of the solution of the Eqs. (11), there are defined DEM parameters \( A(pH,Z) \) and \( B(pH,Z) \) for each constituent \((pH,Z)\) of the test object.

It was emphasized above that in the final stage of the algorithm for the constituent \((pH,Z)\), an estimation of the effective atomic number can be found:

\[
Z_t = F^{-1}\left( \frac{B}{A} \right)
\]

The function \( F^{-1} \) has a different form for low-energy and high-energy realizations of the DEM. For the low-energies \( F^{-1}\left( \frac{B}{A} \right) = \sqrt{\frac{B}{A}} \), but for the high-energies \( F^{-1}\left( \frac{B}{A} \right) = \frac{B}{A} \). Evidently, there is a difference between the estimation of the effective atomic number \( Z_t \) and its true value \( Z \). The deviation is caused by some inaccuracies and computational errors.

Each element of primary radiographs of the test objects \( I_1 \) and \( I_2 \) presents itself an integer number ranging from 0 to \( 2^m-1 \), where \( m \) is the ADC capacity. The primary images are calibrated by use «black level» and «white level» images, and are then transformed by the logarithm operator. For the test object constituents with the characteristics \((pH,Z)\), there are computed experimental estimations of the ray thicknesses \( Y_{1}(pH,Z) \) and \( Y_{2}(pH,Z) \). The values \( Y_{1}(pH,Z) \) and \( Y_{2}(pH,Z) \) are used as the right part of the system (11) to get the experimental estimations of the DEM parameters \( \tilde{A}(pH,Z) \) and \( \tilde{B}(pH,Z) \), and then the experimental estimation of the effective atomic number \( \tilde{Z} \), which is obtained by Eq. (12).

5.2. Discrimination by the method of level functions

The theoretical value of the parameter \( Q \) for a constituent with the initial characteristics \((pH,Z)\) and the corresponding ray thicknesses \( Y_{1}(pH,Z) \) and \( Y_{2}(pH,Z) \) are founded on the equation

\[
Q(\tilde{Y}_{1}(pH,Z)) = \frac{Y_{1}(pH,Z)}{Y_{2}(pH,Z)}
\]

(13)

To estimate the ray thicknesses \( Y_{1}(pH,Z) \) and \( Y_{2}(pH,Z) \) the expressions (10) are used. The corresponding experimental values of the discrimination parameter \( \tilde{Q} \) for a constituent with the
parameters \((\rho H, Z)\) are defined by substituting in Eq. (13) the experimental values of the ray thicknesses \(Y_1(\rho H, Z)\) and \(Y_2(\rho H, Z)\).

6. Experimental validation of the adequacy of discrimination system-models by the dual energy method

The correspondence of the theoretical and the experimental discrimination parameters was carried out on the cargo inspection system of the National Research Tomsk Polytechnic University (TPU). The region of the consumer interest was limited by a set

\[
M = \{(\rho H, a) : \rho H \in \mathbb{R}(a) = [20, 40, 60, 80, 100]; a \in \mathbb{A} = [6, 13, 26]\}.
\]

(14)

Any element \(a\) from the set \(\mathbb{A}\) is associated with the material’s effective atomic number \(Z\). In the high-energy realization of the DEM, the primary radiographs were produced for a pair of maximal X-ray energies \(E_1 = 4.5\) MeV, \(E_2 = 9\) MeV. To describe the energy spectrum of X-ray radiation of betatron Shiff’s formula \([15]\) was used.

The dependencies of the mass coefficients of the radiation attenuation of energy \(m(E)\) were obtained from data libraries \([16, 17]\) together with the energy dependencies \(g_1(E), g_2(E)\) for the Compton effect and for the pair production \([16, 17]\). To record the X-ray radiation in the TPU cargo inspection system, CdWO₄ scintillator detectors were used having a length of 35 mm. The ADC capacity was \(m = 16\). Two adequacy criteria of the DEM-discrimination system models were verified experimentally. Figure 1 shows the theoretical, the computational, and the experimental dependencies \(Z_{\text{eff}}(\rho H)\). From the analysis of the data shown in figure 1, one can verify the adequacy of the discrimination model by an effective atomic number for the considered region of interest (14), and the maximal deviation \(\Delta Z_{\text{eff}} \approx 5\) of the effective atomic number.

![Figure 1](image1.png)

**Figure 1.** Theoretical and experimental dependencies \(Z_{\text{eff}}(\rho H)\) as a function of \(\rho H\). The solid lines indicate the theoretical expectation, the dashed lines the computational result and the symbols represent the experimental result: full squares for Fe; dots for Al, and triangles for \((CH_2)^n\).

Figure 2 shows the computational and the experimental dependencies of the discrimination parameters \(Q(Y_1)\). From the analysis of the data given in figure 2, one can conclude that for maximal deviations \(\Delta Q \approx 0.012\) of the discrimination parameter, the adequacy of the discrimination model by the method of level functions for the considered region of interest (14) is warranted.

![Figure 2](image2.png)

**Figure 2.** Theoretical and experimental dependencies \(Q(Y_1)\). Again as in Figure 1, the solid lines indicate the theoretical expectation, the dashed lines the computational result and the symbols the experimental results for Fe (full squares), for Al (dots) and for \((CH_2)_n\) (triangles).

7. Summary

The proposed adequacy criteria of the examination system models with the material discrimination option for the test objects and their constituents by the dual energy method can be used to design and to compare quantitatively different cargo inspection systems. Two main approaches to build the adequacy criteria by the final and the intermediate DEM parameters were analyzed. The criteria are selected in order to discriminate by the effective atomic number and by the method of level functions. The results of the experimental and the theoretical estimations of the discrimination parameters of the test object constituents, scanned by an X-ray fan-beam with the maximal energies 4.5 and 9 MeV, confirm the efficiency of the proposed adequacy criteria.
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