Stiffness weak link identification of cantilever beam based on vibration test data

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Abstract. The evaluation of the weak link of machine tool structural stiffness is the key link to improve the machining performance of the machine tool. It is proposed that the serial structure of the machine tool be equivalent to a cantilever beam structure and the static rigidity of the machine tool be studied by using the method of dynamic test. A theoretical method for evaluating the position of the weak link of cantilever beam stiffness and the magnitude of stiffness by using the curve angle index is proposed. This paper mainly studies the evaluation effect of indicators. Taking the MATLAB finite element numerical model of cantilever beam structure as a numerical example, the dynamic test data of each node of the numerical model are obtained by using the state space method. After Fourier transform, the angle index is studied by taking the amplitude of 0 Hz. The weakening of the elastic modulus of the element is used to simulate the weak link of stiffness, and the evaluation effect of the corner index is studied under the three conditions of weakening the stiffness of a single element, weakening the stiffness of two elements, weakening the stiffness of three elements and involving noise. The results of simulation and experiment show that the curve angle can be used to better evaluate the weak link of the cantilever beam, which has a certain practical value in engineering.

1. Introduction
Stiffness of machine tool is one of the key indexes of machine tool processing stability and efficiency. In order to improve the stiffness of the machine tool, it is necessary to know the stiffness of each link of the machine tool so as to achieve the best effect by increasing the stiffness in the weak link[1].

Static stiffness method and dynamic stiffness method are the main methods to identify the weak links of machine tool structure. Scholars at home and abroad have done a lot of research on the two methods. There are mainly two methods based on static stiffness. The first static stiffness test method is mainly to apply load pressure to key parts of the machine tool, measure the deformation of each structure of the machine tool with a measuring device, and calculate the structure with the largest deformation by selecting a conversion center [2], which is the weak link of the machine tool stiffness; The second is to calculate the maximum deformation position through static analysis of finite element [3-4]. In the static stiffness test, a static load is applied to key parts of the machine tool through a force application object,
and displacement is recorded by a dial indicator or an eddy current sensor. Both methods of measuring displacement need to select a test reference point and a clamping device for fixing the test device, but the clamping device cannot find a suitable position to be installed in the measurement process, and the selection of the reference point is easy to deform, affecting the accuracy of the test. Through the static analysis of the finite element model, the accuracy of the finite element model itself cannot be guaranteed, so the analysis result is greatly reduced. On the whole, the static stiffness method is not simple and accurate. The dynamic stiffness test method is a method combining finite element modal analysis with experimental modal analysis. First, the three-dimensional model of the machine tool is established, which is preliminarily analyzed for modal by finite element analysis software. Then, the machine tool is tested for vibration. According to the experimental modal analysis results, the parameters of the finite element model are modified, and the modified model is re-analyzed by finite element modal analysis. The analysis results are compared with the experimental modal results, and then the finite element model is modified again. Repeating the steps until the finite element modal analysis results are consistent with the experimental modal results, and use the final model to perform modal analysis of the finite element to calculate the most deformed machine tool structure, which is the weak link of the machine tool structure [5-7]. The dynamic stiffness test method mainly needs to build an accurate finite element model. The machine tool is a composite structure. In the model parameter correction, the joint surface parameter between the structures is the main target of the correction, followed by the geometric shape of the machine tool structure. However, there are too many correction methods to make the finite element modal analysis result consistent with the experimental modal analysis result. Therefore, the correction method has complexity and blindness, resulting in a low accuracy of the final analysis result.

In engineering application, a quick and simple method is needed to locate the weak link of machine tool stiffness. The machine tool is formed by assembling various structures. There are mainly two kinds of assembly routes. The first is the lathe bed, lead screw, guide rail, slide plate, workbench and workpiece. The second is the lathe bed, upright post, lead screw, guide rail, cross beam, lead screw, guide rail, slide saddle, connecting sleeve and cutter. Both of the two combined routes are the result of series connection of various structures. In order to solve the above problems, based on the study of the machine tool stiffness chain established by Uriate et al [8] and Liu qiwei et al [9] using the series stiffness method, this paper proposes that the machine tool structure be equivalent to a cantilever beam structure, and to study the evaluation method of the structural stiffness weakness of the machine tool by studying the evaluation method of cantilever beam weak weakness, thus greatly simplifying the evaluation method. The main objective of this method is to qualitatively evaluate the weak link of machine tool structural stiffness and provide an information basis for further positioning the weak link components. This simplified method fully meets the practical goal of the project. This method only uses hammering excitation method and acceleration sensor vibration pick-up test method, and does not need to accurately know the parameters of each joint surface, thus skipping the research of joint surface, and evaluating the weak link of machine tool stiffness by using curve angle index of cantilever beam under static characteristics under low frequency vibration state, and the evaluation method is completely based on the test data. Therefore, this method is simple, effective and accurate.

For the stiffness evaluation of the cantilever beam's weak link, the most widely used method is the modal analysis of the structural dynamic test, and the natural frequency and modal shape in the modal parameters can be used as indicators to measure the stiffness change [10]. The identification based on the frequency change is mainly that the natural frequency of the structure will change after the damage, based on which to judge whether the local stiffness of the structure will change. Hearn and Agbabian study the location of structural damage through the square ratio of the change of the natural frequency of the structure [11], and ZHAO Jun etc. explore the sensitivity of the frequency to the damage [12]. However, the natural frequency is a representation of the overall dynamic characteristics, but the location function of the local damage cannot be realized. The damage identification method based on vibration mode identifies structural damage by analyzing the change of vibration mode after the change of vibration mode. Wan Xiaopeng [13] and others studied the sensitivity and damage location detection of cantilever beam damage before and after the change of vibration mode. The results show that the
vibration mode is not sensitive to the location and degree of local damage. Using modal parameters as indicators to evaluate stiffness will inevitably introduce structural quality factors, resulting in poor evaluation results.

In this paper, a new index of curve angle based on test data is proposed to evaluate the weak link of stiffness, which makes use of the static characteristics of the structure at low frequency and avoids the influence of structural quality factors. On the basis of the difference between the measured data and the deflection data constructed based on the measured data, the curve angle index is used to evaluate the weak link of stiffness. The theoretical method is based on the vibration experimental data of the dynamic test, and needs to obtain the response data of each measuring point at low frequency and the modal parameters of the system at various modes. The state space method in modern control theory is used to reconstruct the system to eliminate noise interference in the test process and obtain the low frequency signal of the system.

2. Theoretical basis

According to the definition of transfer function

\[ H = \frac{X}{F} = \frac{1}{k - w^2m + jwc} \]  

\[ H = \frac{1}{k} \]

Where \( H \) is the transfer function, \( x \) is the frequency domain data of the displacement response, \( F \) is the frequency domain signal of the excitation force, and \( w \) is the frequency of the excitation force.

When the value of \( k \) is equal to zero, the formula (1) is

\[ H = \frac{1}{k} \]

At this time, the motion of the mass block mainly depends on the stiffness of the spring and is independent of the mass. The dynamic characteristics of the system show pseudo-dynamic characteristics, that is, static stiffness characteristics.

When the excitation force is a pulse signal in a wide frequency range, the response of the system in a low frequency signal will show a static characteristic. When the cantilever beam vibrates transversely, it will reciprocate with the axis as the equilibrium position. If the cantilever beam system is equivalent to a single-degree-of-freedom system composed of springs, damping and mass, and the cantilever beam specimen to be measured is proportional damping, isotropic and homogeneous, obeying Hooke's law. Therefore, under low frequency signals, the vibration characteristics of the cantilever beam also show pseudo-dynamics, and its vibration curve will conform to the deflection curve under static load. If the local stiffness of the beam is weak, its vibration curve will not conform to the deflection curve under static force, and the bending angle will increase at the local location where the stiffness is weak relative to that where the stiffness is not weak. Therefore, it is theoretically feasible to evaluate the stiffness of the weak link by using the curve angle as an index.

The cantilever beam dynamic test passes through sensors arranged at each equidistant measuring point of the cantilever beam, and the first sensor is close to the fixed end to obtain vibration information at the measuring point. After the time domain signal is processed by fast Fourier transform, the vibration amplitude of each measuring point at each frequency can be obtained.

Static Deformation Formula of Cantilever Beam Subjected to Free End Force

\[ V = -\frac{p}{6EI}a^2(3L - a) \]

Where \( V \) is the deflection of the beam, \( L \) is the length of the beam, \( a \) is the distance from the fixed end, \( P \) is the applied static load, \( E \) is the modulus of elasticity, and \( I \) is the moment of inertia of the cross section relative to the direction of vibration of the beam.
The specific value of the hammering force at one frequency cannot be accurately known under the hammering excitation during dynamic testing, but at one frequency, the hammering force can be regarded as a fixed value.

\[ A = \frac{P}{(6EI)} \]  

(4)

The first measuring point near the fixed end is selected as the standard point, and according to the data \( y_1 \) of the first measuring point, the \( a \) value under a low frequency signal can be calculated and recorded as \( A_1 \).

\[ A_1 = \frac{y_1}{(a_1(3L - a_1))} \]  

(5)

Taking \( A_1 \) as the coefficient of the new deflection equation, the deflection curve can be constructed according to the coordinates of other measuring points, and the constructed deflection data matrix elements can be obtained, namely:

\[ V_s(i) = A_1 a_i^2 (3L - a_i) \]  

(6)

Where \( V_s(i) \) is the data of the \( i \)-th measuring point in the structural deflection data, and \( a_i \) is the distance from the origin.

The element in the deflection \( v_{si} \) of the structure is represented by \( V_s \), namely,

\[ V_s = [v_{s1} v_{s2} v_{s3} \ldots v_{si}] \]  

(7)

\( v_{si} \) represents the data of the structure of the \( i \)-th measuring point.

The test data at 0Hz are expressed in a \( V_m \) matrix and the elements in \( V_m \) are expressed in \( v_{mi} \):

\[ V_m = [v_{m1} v_{m2} \ldots v_{mi}] \]  

(8)

\( v_{mi} \) Represents the actual data of the \( i \)-th measuring point.

According to equations (7) and (8), the curve angle index \( \theta_c \) is expressed as follows:

\[ \theta_c(i) = -\tan^{-1}\frac{v_{s1} - v_{m1} - v_{s(i+1)} + v_{mi}}{h} + \tan^{-1}\frac{v_{si} - v_{mi} - v_{s(i-1)} + v_{m(i-1)}}{h} \]  

(9)

Where \( \theta_c(i) \) is the curve corner representing the \( i \)-th measuring point, and \( h \) is the distance between the two measuring points.

The position where the abrupt change occurs in \( \theta_c \) corresponds to the position where the stiffness of the structure is weak, and its relative size indicates the magnitude of the stiffness weakness.

3. State space equation for system reconstruction

In the actual dynamic testing process, due to the problem of measuring range of vibration pick-up equipment, accuracy of acquisition system and interference of environmental noise, the acquisition of low-frequency signals becomes difficult to obtain, resulting in inaccurate stiffness evaluation results. Under the condition of not increasing the test cost, the test data are identified by singular value decomposition of power spectral density matrix [14], the system is reconstructed using the state space of modern cybernetics, and the low-frequency signal after system reconstruction is obtained again through simulation.
3.1. Differential equations of system vibration
A rectangular coordinate system is established with the fixed end where the axis of the beam is located as the origin and the axis of the beam as the horizontal axis. For the first order mode, each mode is directly independent of each other, and the vibration equation satisfies:

\[ \ddot{z}_i + 2\zeta_i w_i \dot{z}_i + w_i^2 z_i = F_i \]  \hfill (10)

Where \( \zeta_i \) is the i-th order damping ratio, \( w_i \) is the i-th order natural frequency, \( F_i \) is the modal force \[ 15 \] in the i-th order mode, of \( z_i \) is modal vibration displacement in the i-th order mode, \( \dot{z}_i \) is modal vibration velocity in the i-th order mode and the modal vibration acceleration in the i-th order mode of is \( \ddot{z}_i \).

3.2. State variables of the system
The state variables of the cantilever beam vibration system are as follows: \( x_{i1} = z_i \) is the vibration mode displacement in the i-th order mode, and \( x_{i2} = \dot{z}_i \) is the modal vibration velocity in the i-th order mode. State variables of cantilever beam vibration system:

\[ X = [x_{i1} \ x_{i2} \ x_{21} \ x_{22} \ ... \ x_{i1} \ x_{i2}] \]  \hfill (11)

3.3. Input variables of the system
The external input variable of the cantilever beam vibration system is the impulse excitation acting on the free end of the beam. Under each mode, the input variable of the system is related to the position where the excitation is applied

\[ U = [\phi_{i1q} \ \phi_{i2q} \ ... \ \phi_{iEq}] \]  \hfill (12)

Where \( \phi_{iq} \) is the i-th mode shape and \( q \) is the mode shape data related to the input signal position.

3.4. Output variables of the system
The output variable is determined by the research goal of the cantilever beam. Therefore, the output variable of the selected system is the displacement

\[ Y = [x_1 \ x_2 \ x_3 \ ... \ x_i] \]  \hfill (13)

\[ x_1 = \phi_{11} x_{11} + \phi_{12} x_{21} + \cdots + \phi_{1i} x_{1i} \]
\[ x_2 = \phi_{12} x_{11} + \phi_{22} x_{21} + \cdots + \phi_{2i} x_{1i} \]
\[ x_3 = \phi_{13} x_{11} + \phi_{23} x_{21} + \cdots + \phi_{3i} x_{1i} \]
\[ x_i = \phi_{1i} x_{11} + \phi_{2i} x_{21} + \cdots + \phi_{ii} x_{1i} \]

Where \( \phi_{ij} \) is the i-th element of the j-th mode shape and \( x_{1i} \) is the displacement of the i-th measuring point.

According to the differential equation of the cantilever beam vibration system described in equations \( 10 \) to \( 13 \), and according to the input variables, state variables and output variables of the established cantilever beam vibration system, the standard form of the state space equation of the cantilever beam vibration system is established:

\[ \dot{X} = AX + BU \]  \hfill (14)
\[ Y = CX + DU \]  \hfill (15)
For equations (14) to (15), \( X \) is state space vectors of the system; \( Y \) is an array of output variables for the system; \( U \) is an array of input variables for the system; \( A \) and \( B \) are coefficient matrix of the state space equation; \( C \) and \( D \) are coefficient matrix of the state space equation.

4. Example analysis

4.1. Model

Regardless of the shear deformation of the beam, the uniform mass element matrix of Euler-Bernoulli beam element is selected, and the mass matrix \( m \) and stiffness matrix \( k \) of the beam finite element model are assembled by MATLAB programming. For the dynamic equation established by finite element method [16]:

\[
M\ddot{\delta} + C\dot{\delta} + \delta = F
\]  

(16)

For that dam coefficient of the proportional damping structure

\[
C = \alpha M + \beta k
\]  

(17)

Where \( \alpha \) and \( \beta \) is any constant, \( \delta \) is node displacement, \( \dot{\delta} \) is node velocity, \( \ddot{\delta} \) is node acceleration.

The expression of equation (16) and equation (17) in the state space is

\[
\dot{X} = AX + B
\]  

(18)

Where \( X = \left[ \begin{array}{c} \delta \\ \dot{\delta} \end{array} \right] = \left[ \begin{array}{cc} 0 & I \\ -M^{-1}K & -M^{-1}K \end{array} \right] B = \left[ \begin{array}{c} 0 \\ M^{-1}F \end{array} \right] \)

According to the output equation of the system, the system equation can be expressed as:

\[
\dot{X} = AX + B
\]  

(19)

\[
Y = C_Y X
\]  

(20)

Where: \( Y \) is the output variable and \( C_Y \) is the output matrix.

As shown in Figure 1, the cantilever beam is long and divided into 20 segments of equal length (numbers in the circle are the cell numbers and numbers in the lower row are the node numbers). Section moment of inertia is \( I = 2.08 \times 10^{-5} \text{m}^4 \), area is \( A = 0.025 \text{m}^2 \), material elastic modulus is \( E = 2.1 \times 10^{11} \text{Pa} \), density is \( \rho = 7850 \text{Kg/m}^3 \).

The pulse signal is selected as the input signal, and the decrease of elastic modulus is used to simulate the decrease of stiffness, thus forming a weak link of stiffness. Using the state space method, the Simulink module of MATLAB is used to obtain the time domain signal of dynamic simulation with or without weak stiffness links, and then the frequency domain signal is obtained by Fast Fourier transform. White noise is added to the time domain signal with weak stiffness, and the effect of noise on stiffness evaluation is studied through parameter identification and system reconstruction of the signal added with noise.
Fig 1. Finite element model of cantilever beam

4.2. Simulation

According to the vibration mechanics [17], the theoretical formula of the first three natural frequencies of the cantilever beam is obtained

\[ w_1 = \frac{3.516}{2 \pi} \sqrt{\frac{EI}{\rho AL^4}} \]
\[ w_2 = \frac{22.034}{2 \pi} \sqrt{\frac{EI}{\rho AL^4}} \]
\[ w_3 = \frac{61.701}{2 \pi} \sqrt{\frac{EI}{\rho AL^4}} \]

Fig. 2(b) shows that reconstructing a cantilever beam system using the state space method can fully express the physical state of the original system. Fig. 2(b) shows that the vibration curve of the cantilever beam is similar to the deflection curve under static load at low frequency (0Hz), and the cantilever beam can exhibit static characteristics in dynamic test, thus verifying the correctness of the theory. Table 1 show that the dynamic modeling in this paper is reasonable.

|                  | First order/Hz | Second order/Hz | Third order/Hz |
|------------------|----------------|-----------------|---------------|
| Simulation results | 58             | 362.1           | 999.2         |
| Theoretical result | 57.6           | 361.8           | 999.2         |

4.2.1. *A single element injury and has or has not noise interference*. Considering that only one element of the cantilever beam has different damage degrees, the evaluation effect of the theoretical method is verified. Take three cases where the stiffness of unit 9 is reduced by 30 %, 50 % and 70 %, respectively.
No noise is represented by E9, noise is represented by EN9, and after system reconstruction, and it is represented by ES9. Because it is a rotation angle index, the rotation angle of the free end node cannot be calculated, but one element has two nodes, which does not hinder the evaluation of the weak stiffness link. The following examples can prove it. Figure 4 shows whether there is noise or not in three cases and the evaluation effect of the index after system reconstruction. The abscissa in the figure is the node number, and the ordinate is the index value, expressed in radians. Fig. 3 (a) shows that the result of the evaluation based on the rotation angle index is accurate, with obvious abrupt changes in the weak stiffness, and the larger the stiffness loss, the larger the index abrupt change value. Since one element has two nodes, when the stiffness of one element is weak, two abrupt changes occur. Therefore, it is possible to accurately locate a unit damage and quantify the relative stiffness. In fig. 3 (b), 30 % white noise is added to the signal, i.e. the signal-to-noise ratio is 10db. Due to noise interference, low-frequency signal acquisition is not accurate. Although there is a sudden change at nodes 8 and 9, the following nodes also fluctuate to different degrees, confusing identification and leading to poor evaluation results. However, fig. 3 (c) shows that the evaluation effect after system reconstruction using noise-doped signals is much higher than that of pure noise signals, the corner value changes abruptly in the stiffness damage position, and the position index value of other nodes approaches zero, so that the size and position identification can be accurately evaluated.

![Fig 3. Identification of damage index of single element with different stiffness](image)

4.2.2. Two elements injury and have or have not noise interference. (1) Consider the case where the stiffness of an intermediate unit and an edge unit, i.e. 10 units and 20 units, are reduced by 20 %, 40 %, 60 % % and 30 %, 50 % and 70 %, respectively, with no noise indicated by E7 and E9, with noise indicated by EN7 and EN10, and with ES7 and ES10 after system reconstruction. Fig. 4 (a) shows that the result evaluated by the angle index is accurate, with obvious changes in the weak stiffness, and the larger the stiffness loss, the larger the index mutation value. Since the 20th is an edge unit, the corner index of the 20th node cannot be calculated, so only the 19th node has a sudden change, and a total of three nodes have a sudden change of position. In fig. 4 (b), 30 % white noise is added to the signal, and the index value of the unit node before stiffness damage is close to zero, but the index value of the subsequent unit node fluctuates to different degrees, and the evaluation effect is not good. Fig. 4 (c) shows that after the system reconstruction, its evaluation effect is greatly improved compared with the pure noise signal, so the method of reconstructing the system is effective.
Fig 4. Identification of damage indices of different stiffness for two element (Interval)

(2) Consider the case where the stiffness of two adjacent intermediate units, i.e. 10 units and 11 units, is reduced by 20 %, 40 %, 60 % and 30 %, 50 % and 70 %, respectively, the non-noise is indicated by E10 and E11, the noise is indicated by EN10 and EN11, and with ES10 and ES11 after system reconstruction. Fig. 5 (a) shows that the evaluation results are accurate by the angle index. Although two units have been damaged, only three sudden changes occur, and the intermediate node, i.e. node 10, has a common node. This node has a sudden change when both units have suffered stiffness damage. In fig. 5 (b), 30 % white noise is added to the signal to evaluate the index, but fig. 5 (c) shows that after system reconstruction using noise-doped signals, the index fluctuates at the unit nodes where no stiffness damage occurs, but tends to zero overall, and the corner index can correctly evaluate the result.

Fig 5. Identification of damage indices of different stiffness for two element (Adjacent)

4.2.3. Three elements injury and have or have not noise interference. (1) Consider the case where the stiffness of the three spacing units, namely, 5 unit, 8 unit and 12 unit, is reduced by 20 %, 40 % and 60 %, respectively. With no noise indicated by E5,E8 and E12, with noise indicated by EN5,EN8 and EN12, and with EN5,EN8 and EN12 after system reconstruction. Fig. 6 (a) shows that the result of the evaluation based on the rotation angle index is accurate, and there are obvious changes in the weak stiffness, and the larger the stiffness loss, the larger the index mutation value, and the stiffness damage occurs in the three units, so there are six changes, so accurate positioning and quantification of the relative stiffness can be realized. In fig. 6 (b), the evaluation result of 30 % white noise is added to the signal, and the evaluation effect of the index is not ideal. In fig.6 (c), the evaluation result of the corner index after the system reconstruction using the noise-doped signal is much higher than that of the pure noise signal, but the effect is not obvious.

(2) Consider the case where the stiffness of three adjacent units, namely, namely, unit 6 unit 7 and unit 8, is reduced by 30 %, 50 % and 70 %, respectively. With no noise indicated by E6, E7 and E8, with noise indicated by EN6, EN7 and EN8, and with EN6, EN7 and EN8 after system reconstruction. Fig. 7 (a) shows that the results evaluated by the angle index are accurate, and there are obvious changes in the weak stiffness, the greater the stiffness loss, the greater the index mutation value and the stiffness loss of the three nodes. However, only four nodes have mutations because the two nodes have common
nodes, namely nodes 6 and 7. Since the stiffness damage of units 7 and 8 is greater, the index value of nodes 7 has the largest mutation, and nodes 5, 6 and 8 all have different degrees of mutation, the greater the stiffness damage, the greater the mutation value, and the corner index value of other nodes. Therefore, the damage of the three elements can be accurately located and the relative stiffness can be quantified.

In fig. 7 (b), 30 % white noise is added to the signal, i.e. the signal-to-noise ratio is 70 %. Due to noise interference, low-frequency signal acquisition is not accurate and index evaluation effect is not good. However, fig. 7 (c) shows that after the system reconstruction using noise-doped signals, the damage assessment effect of neighboring cells is better, the node with the weakest stiffness, i.e. node 7, can be accurately assessed, and the corner index of other damaged cells also has a sudden change. The assessment effect of nodes 5 and 8 is easily confused with that of neighboring nodes.

![Fig 6. Identification of stiffness damage indices of three element in different degrees (Adjacent and Interval)](image)

5. Conclusion

Based on the test data, a new index of curve angle is proposed to measure the stiffness change of cantilever beam, and the state space method is used to reconstruct the system to eliminate noise and obtain the high-precision angle index at low frequencies. The main conclusions are as follows:

(1) The simulation example shows that if the obtained low frequency signal (0Hz) is accurate and free of noise pollution, the position of the link with thin stiffness can be accurately identified and the size of the thin stiffness can be evaluated through the new index of curve angle.

(2) In order to avoid noise interference and inaccurate acquisition of low frequency signals during testing, the state space method can greatly improve the stiffness identification results under low frequency conditions, thus improving the evaluation effect of indexes.

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