Effect of the Global Rotation of the Universe on the Formation of Galaxies

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ABSTRACT

The effect of the global rotation of the universe on the formation of galaxies is investigated. It is found that the global rotation provides a natural origin for the rotation of galaxies, and the morphology of the objects formed from gravitational instability in a rotating and expanding universe depends on the amplitude of the density fluctuation, different values of the amplitude of the fluctuation lead to the formation of elliptical galaxies, spiral galaxies, and walls. The global rotation gives a natural explanation of the empirical relation between the angular momentum and mass of galaxies: \( J \propto M^{5/3} \). The present angular velocity of the universe is estimated, which is \( \sim 10^{-13} \text{ rad yr}^{-1} \).

Subject headings: cosmology: theory — galaxies: formation — galaxies: general

In a homogeneous universe which is more general than the Friedmann model, the matter may not only expand but also rotate relative to local gyroscopes. The rotation of the matter in the universe as a whole is usually called the global rotation of the universe, which has been investigated by many scientists (Gamow 1946, Gödel 1949 & 1990, Ellis 1971, Obukhov 1992, Korotky & Obukhov 1996). People usually think that the observed isotropy of the cosmic microwave background (CMB) strongly restricts the possible value of the angular velocity of the universe (Collins & Hawking 1973, Hawking 1974). However, when more general cosmological models are considered, the restriction may be much looser (Matzner 1997). For the Bianchi IX models the more realistic limits are thought to be of the order \( 10^{-12} \text{ rad yr}^{-1} \) (Ciufolini & Wheeler 1995). And more, some recent investigations reveal that there are a wide class of viable cosmological models for which the global rotation does not influence the isotropy of CMB at all and it is the shear which may affect the isotropy of CMB (Obukhov 1990, Korotky & Obukhov 1991 & 1996, Pavelkin & Panov 1995). Therefore it is significant to investigate the cosmic effects of the global rotation further.

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Birch (1982) has discovered the asymmetric distribution of the angles of the rotation of polarization vector of 132 radio sources and has tried to explain it via the global rotation. Korotky and Obukhov (1994) have applied the global rotation to explain the observed periodic distribution of galaxies on the large scale (Broadhurst et al 1990). Though the result may be controversial (Phinney & Webster 1983, Birch 1983), it is sufficient to show that the global rotation may be relevant to some important observational phenomena.

Here I try to investigate the effect of the global rotation on the formation of galaxies. I show that in a rotating universe galaxies automatically get the angular momentum when they form due to the conservation of angular momentum, which gives a natural interpretation of the rotation of galaxies. Such an idea has been postulated previously by some people (Gamow 1946, Gödel 1990, Collins & Hawking 1973). However, detail analysis has not been made and it has been worried that this may lead to that the orientation of galaxies should be aligned in some direction which seems contrary with the observations. But the anisotropy in the distribution of the orientation of galaxies has been found at different levels (MacGillivray & Dodd 1985, Djorgovsk, Sugai & Iye 1995, Hu et al 1995) and a pronounced anisotropy has been found recently (Parnovsky et al 1994) though the origin of the anisotropy is still in argument (Flin 1995). (In the end of this paper I will show that due to the irregularity of the shape of the proto-galaxies the distribution of the orientation of galaxies may be somewhat random which makes it difficult to measure the correlation of the orientation of galaxies). In this paper I derive a correlation between the angular momentum $J$ and mass $M$ of galaxies, which is consistent with the empirical relation (Brosche 1963, Ozernoy 1967, Burbidge & Burbidge 1975, Trimble 1988)

$$J \propto M^{5/3}. \quad (1)$$

Such an empirical relation was usually explained via the virial theorem with the assumption that galaxies have constant density (Ozernoy 1967), but why such an assumption should hold was not explained. Here I show that the global rotation may give a natural explanation of this relation which does not require the assumption of constant density. The present value of the angular velocity of the global rotation is estimated from the statistical analysis of the correlation between the angular momentum and mass of galaxies. The result is just within the limits of CMB for the Bianchi IX models obtained by Matzner and cited by Ciufolini and Wheeler (1995). The value of the angular velocity obtained is of the same order as that obtained by Birch (1982) and consistent with that obtained by Obukhov (1992), Korotky and Obukhov (1994). The relation between the primordial density fluctuation and the formation of galaxies in a rotating and expanding universe is also discussed. It is found that the morphology of the objects formed depends on the amplitude of the density fluctuation, different values of the fluctuations lead to the formation of elliptical galaxies, spiral galaxies, and walls.

The motion of the fluid in the universe can be described by the volume expansion scalar $\Theta$, the rotation tensor $\omega_{ab}$, and the shear tensor $\sigma_{ab}$. The homogeneous rotation of the fluid as a whole is the global rotation of the universe. If the fluid is the perfect fluid with the stress-energy tensor $T_{ab} = (\rho + p)u_{a}u_{b} + pg_{ab}$ ($\rho$ is the mass density and $p$ is the pressure), with the Einstein
equation the Raychaudhuri equation describing the relation among $\Theta$, $\omega_{ab}$, and $\sigma_{ab}$ can be written as

$$- \nabla_a A^a + \dot{\Theta} + \frac{1}{3} \Theta^2 + 2(\sigma^2 - \omega^2) = -4\pi G(\rho + 3p),$$

where $A^a = u^b \nabla_b u^a$ is the acceleration vector, the dot denotes the derivative $u^a \nabla_a$, and $\omega^2 \equiv \omega_{ab} \omega^{ab}/2$, $\sigma^2 \equiv \sigma_{ab} \sigma^{ab}/2$ (Ciufolini & Wheeler 1995). $\omega$ is also called the scalar angular velocity. The most important cases for perfect fluid are dust and radiation for that the universe is dominated by dust when $z < z_{eq}$ and dominated by radiation when $z > z_{eq}$ where $z_{eq} \sim 10^4$ is the redshift ($z \equiv a_0/a - 1$ where $a$ is the scale function defined by $\Omega = 3\dot{a}/a$ and the index “0” denotes the value at the present epoch) when the mass densities of dust and radiation are equal. It has been shown that the spatially homogeneous, rotating, and expanding universes filled with perfect fluid must have a non-vanishing shear (King & Ellis 1973, Raychaudhuri 1979). However, it seems reasonable to assume that $\sigma$ is sufficiently small compared with $\omega$ since the shear falls off more rapidly than the rotation as the universe expands (Hawking 1969, Ellis 1973) and the isotropy of the CMB restricts the shear more strongly. The conservation of energy and angular momentum gives (Ellis 1973)

$$\dot{\rho} = - (\rho + p) \Theta, \quad \omega \rho a^5 = \text{const.}$$

Especially, for dust we have $\rho_d \propto a^{-3}$ and $\omega_d \propto a^{-2}$, for radiation we have $\rho_r \propto a^{-4}$ and $\omega_r \propto a^{-1}$ (while in general $\sigma$ falls as $\sigma \propto a^{-3}$ (Hawking 1969)). Before the decoupling epoch $z_{dec} \sim 10^3$, the dust and the radiation interact with each other strongly, they can be treated as one unique fluid and have one unique angular velocity. After the decoupling, the dust and the radiation evolve separately, they have their own angular velocities which evolve according to different laws: $\omega_d \propto a^{-2}$ and $\omega_r \propto a^{-1}$. Since today the universe is dominated by dust, we take $\omega_d$ as the angular velocity of the universe though the radiation may have a more large angular velocity $\omega_r \sim z_{dec} \omega_d$. For the dust fluid we have $A^a = 0$ because the dust flows along geodesics. Neglecting the shear term which is assumed to be sufficiently small, then the first integration of Eq. (2) for dust gives

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho - \frac{2}{3} \omega^2 - \frac{\kappa}{a^2},$$

where $\omega = \omega_d$, $\kappa$ is the integral constant which can be made to be $+1$, 0, or $-1$ by rescaling. It should be remembered that though Eq. (4) describes the motion of the dust fluid, exactly $\rho$ is the total mass density of dust and radiation since the right hand side of Eq. (2) comes from $-R_{ab} u^a u^b$ and the Einstein equation (Ciufolini & Wheeler 1995). $\omega$ and $\rho$ can be written as $\omega = \omega_0 (1 + z)^2$, $\rho = \rho_{d0} (1 + z)^3 + \rho_{r0} (1 + z)^4$. Because the Einstein-de Sitter model has provided sufficiently good description of the universe since decoupling, we expect that $\omega_0^2$ should be sufficiently small compared with $G\rho_{d0}$.

Now consider the formation of galaxies in a rotating and expanding universe. At some early epoch there is some density fluctuation in a region, then the expansion of the matter inside
and around the region begins to be increasingly decelerated. Eventually the matter may stop expanding and begin to collapse and form a galaxy. For simplicity, we assume that the fluctuation is spherically symmetric. Then the region containing the proto-galaxy or the matter destined to form a galaxy should also be spherically symmetric. Suppose the original mass and angular momentum of the proto-galaxy are $M$ and $J_i$, the original radius of the proto-galaxy is $r_i$, then $J_i = 2Mr_i^2\omega_i/5$, where $\omega_i$ is the angular velocity of the universe at that epoch. This angular momentum is relative to gyroscopic frames. Another kind of useful local frames are galactic frames which our usual measurements are made relative to, by definition which co-rotate with the global rotation and whose origins are fixed at galactic centers. Certainly the original angular momentum relative to galactic frames is zero. After the galaxy has formed, the galaxy rotates relative to the galactic frames, which is caused by the Coriolis force or the conservation of angular momentum, just like the formation of cyclones on the ground of the earth. At any epoch after the galaxy has formed, the angular momentum of the galaxy relative to the gyroscopic frames is $J_f = J + \beta M r_f^2 \omega_f$, where $J$ is the angular momentum of the galaxy relative to the galactic frames, $r_f$ is the radius of the galaxy, $\omega_f$ is the angular velocity of the universe, and $\beta$ is a parameter determined by the distribution of the mass of the galaxy. Using $\omega \propto (1 + z)^2$, $\rho_d \propto (1 + z)^3$, and $M = 4\pi\rho_d r_i^3/3$ (it is usually assumed that galaxy formation takes place after the decoupling), the conservation of angular momentum $J_i = J_f$ leads to

$$J = \frac{2}{5} \left( \frac{3}{4\pi\rho_{d0}} \right)^{2/3} \omega_0 M^{5/3} - \beta r_f^2 (1 + z_f)^2 \omega_0 M.$$  \hspace{1cm} (5)

For $z_f$ not too larger than 1, the second term in the right hand side of Eq. (5) is usually sufficiently small compared with the first term, then we have

$$J \simeq k M^{5/3}, \quad k = \frac{2}{5} \left( \frac{3}{4\pi\rho_{d0}} \right)^{2/3} \omega_0,$$  \hspace{1cm} (6)

which is consistent with the empirical relation in Eq. (1). Thus the global rotation of the universe gives a natural explanation of the observed correlation between the angular momentum and mass of galaxies.

By studying the correlation between the angular momentum and mass of galaxies, it should be able to find the angular velocity of the universe. The correlation for spiral galaxies has been investigated in detail \cite{Nordsiek1973, Dai1978, Carrasco1982, Abramyan1985}. It seems a suitable value for $k$ is $\sim 0.4$ (in CGS units). Taking $\rho_{d0} = 1.88 \times 10^{-29}\Omega h^2$ g cm$^{-3}$ ($\Omega$ is the density parameter of dust and $h$ is the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$) and choosing $h = 0.75$ and $\Omega = 0.01$ (This value is measured dynamically for the rich clusters of galaxies \cite{Peeble1993}), we have

$$\omega_0 \simeq 6 \times 10^{-21} \text{rad s}^{-1} \simeq 2 \times 10^{-13} \text{rad yr}^{-1},$$ \hspace{1cm} (7)

which is consistent with the value obtained by Birch (1982) and just within the CMB limits for the Bianchi IX models \cite{Ciufolini1995}. The result is also consistent with that of Obukhov
Let us turn to discuss the relation between the primordial density fluctuation and the formation of galaxies in a rotating and expanding universe. Consider a spherical shell with initial radius \( r_i \) containing the spherically symmetric primordial density fluctuation with contrast \( \delta_i = \delta \rho_i / \rho_i \) \((0 < \delta_i \ll 1)\), where \( \rho_i \) is the initial average mass density of the universe and \( \delta \rho_i \) is the fluctuation. When the density fluctuation appears, the shell and its interior (and the part of its exterior near the shell) decrease the speed of expansion and are gradually separated from the other parts of the universe to form an isolated system. The mass contained in the shell is \( M = 4 \pi \rho_i r_i^3 (1 + \delta_i) / 3 \), which is supposed to be constant during the evolution (it is a good approximation if the shell is not very near the center of the fluctuation). Consider a mass element on the equator of the shell. When the system becomes isolated, the motion of the mass element is equivalent to that of a particle with unit mass in the potential \( U^e(r) = -GM / r + \omega_i^2 r_i^4 / (2r^2) \) with the initial conditions \( r = r_i \) and \( \dot{r} = H_i r_i \) at \( t = t_i \), where \( t_i \) is the time when the fluctuation takes place. The total conserved energy of the particle is \( \varepsilon^e \sim -\vartheta \delta H_i^2 r_i^2 / (2(1 + \vartheta)) \) where Eq. (4) has been used and the \( \kappa \) term has been dropped as usual. The parameter \( \vartheta \) is defined by

\[
\vartheta \equiv \frac{3 \delta_i}{(\omega_0 / H_0)^2(1 + z_i)} - 1, \quad \vartheta > -1, \tag{8}
\]

which describes the strength of the density fluctuation that takes place at the redshift \( z_i \). The solution is bound and there exists the turn-around point where the mass element stops expanding and begins to collapse, if \( \varepsilon^e \) is negative or \( \vartheta > 0 \). Under this condition, the solution is

\[
\frac{r}{r_i} \simeq \frac{1 + \vartheta}{2 \vartheta \delta_i} (1 - e \cos \xi), \quad \frac{t}{t_i} \simeq \frac{3}{4} \left( \frac{1 + \vartheta}{\vartheta \delta_i} \right)^{3/2} (\xi - e \sin \xi), \tag{9}
\]

where \( H_i \simeq 2 / (3t_i) \), \( \delta_i \ll 1 \), and \( \omega_i^2 / H_i^2 \ll 1 \) (For \( z_i \sim 10^3 \) we have \( \omega_i^2 / H_i^2 \sim 10^{-3} \) have been used, and \( e = [1 - 12 \delta^2 / (1 + \vartheta)^2]^{1/2} \). The collapse time in the equatorial direction is

\[
t_c^e \equiv t(\xi = 2\pi) - t_i \simeq \left[ \frac{3 \pi}{2 \delta_i^{3/2}} \left( \frac{1 + \vartheta}{\vartheta \delta_i} \right)^{3/2} - 1 \right] t_i. \tag{10}
\]

For a mass element in the polar direction (the direction of the rotation), when the system becomes isolated, the motion is equivalent to that of a particle with unit mass in the potential \( U^{(p)}(r) = -GM / r \) with the initial conditions \( r = r_i \) and \( \dot{r} = H_i r_i \) at \( t = t_i \). The total energy is \( \varepsilon^{(p)} \simeq -(3 + \vartheta) \delta H_i^2 r_i^2 / (2(1 + \vartheta)) \) which is always negative. The solution is always bound and the turn-around point always exists. The solution is

\[
\frac{r}{r_i} \simeq \frac{1}{2 (3 + \vartheta) \delta_i} (1 - \cos \eta), \quad \frac{t}{t_i} \simeq \frac{3}{4} \left[ \frac{(3 + \vartheta) \delta_i}{(3 + \vartheta) \delta_i} \right]^{3/2} (\eta - \sin \eta). \tag{11}
\]
The collapse time in the polar direction is

\[ t_c^{(p)} \equiv t(\eta = 2\pi) - t_i \simeq \left[ \frac{3\pi}{2\delta_i^{3/2}} \left( \frac{1 + \vartheta}{3 + \vartheta} \right)^{3/2} - 1 \right] t_i. \] (12)

We find that \( t_c^{(e)} \gg t_c^{(p)} \) if \( 0 < \vartheta < 1 \).

There are three possible evolution results depending on the parameter \( \vartheta \):

- \( \vartheta > 1 \) or \( \vartheta \sim 1 \). Then \( t_c^{(e)} \sim t_c^{(p)} \), the matter in the equatorial and polar directions collapses and reaches dynamical equilibrium almost simultaneously. The objects so formed are in complete equilibrium in both the equatorial and the polar directions, which should have compact shapes. Such objects are just like elliptical galaxies, the formation of elliptical galaxies may therefore belong to such a case.

- \( 0 < \vartheta < 1 \). Then \( t_c^{(e)} \gg t_c^{(p)} \), the matter in the equatorial direction collapses and reaches dynamical equilibrium sufficiently later than that in the polar direction. When the matter in the polar direction has stopped collapsing and has reached the equilibrium, the matter in the equatorial direction is still flowing into the core and is rotating around the core. The matter in the polar direction is in complete equilibrium, while the matter in the equatorial direction is in quasi-equilibrium. The objects so formed are not as compact as that in the first case and are just like spiral galaxies, the formation of spiral galaxies may therefore belong to such a case.

These two cases provide a natural mechanism accounting for the formation of spiral galaxies and elliptical galaxies. A direct corollary is that the distribution of the average mass density of spiral galaxies should concentrate within a narrow range, the average mass density of elliptical galaxies should scatter in a more wide range and should be more large. This is consistent with the observations.

- \( -1 < \vartheta \leq 0 \). In such a case there is no bound solution in the equatorial direction, the matter in this direction will expand forever though the expanding speed decreases with time, even when the matter in the polar direction has collapsed and has reached equilibrium. As the results, only two dimensional bound structures can be formed, which can be regarded as proto-walls and provide natural seeds for the formation of wall structures in the universe. The surrounding matter and galaxies are drawn towards a proto-wall to form a wall structure. The scale \( L \) of the wall is approximately equal to the diameter \( D^{(w)} \) of the proto-wall, which can be estimated by \( L \sim D^{(w)} \sim D_i^{(w)}(1 + z_i) \), because the proto-wall can be approximately regarded as expanding with the universe in the equatorial direction. For a typical spiral galaxy, its original diameter can be estimated by \( D_i^{(s)} \sim D_0^{(s)}(\omega_i^{(s)}/\omega_0)/(1 + z_i) \) due to the conservation of vorticity, where \( D_0^{(s)} \) is the present diameter of the spiral galaxy and \( \omega^{(s)} \) is the angular velocity at \( r \sim D_0^{(s)}/2 \). Then \( L \sim 10D_0^{(s)}(\omega_i^{(s)}/\omega_0)^{1/2} \sim 10^2 \) Mpc, if
we take $D_i^{(w)} \sim 10D_i^{(s)}$, $\omega^{(s)} \sim 10^{-16}$ rad s$^{-1}$, $\omega_0 \sim 10^{-20}$ rad s$^{-1}$, and $D_0^{(s)} \sim 10^2$ kpc. This scale has the same order as that of the Great Wall.

How galaxies get their angular momentum during their formation is an interesting and challenging problem in cosmology. Some people have found and discussed the similarity of spiral galaxies to turbulent eddies, and suggested that the primordial turbulence may lead to the formation of galaxies and may be the origin of the rotation of galaxies (von Weizsäcker 1951, Gamow 1952). But detail investigations have revealed that the primordial turbulence picture should fail since the turbulence could not have been kept for a long time against the dissipation (Jones & Peebles 1972, Jones 1973). Other people have suggested that galaxies acquire their angular momentum as they form by the tidal torques of neighboring proto-galaxies (Hoyle 1951, Peebles 1969 & 1971; Barnes & Efstathiou 1987), but it seems difficult to explain the empirical relation in Eq. (1) in this picture. In the scenario of the global rotation, the Coriolis force in the galactic frames makes galaxies to rotate automatically when they form, galaxies get their angular momentum from the global rotation of the universe due to the conservation of angular momentum. Galaxies rotate because the universe rotates. Differing from the primordial turbulence, this rotation can be kept due to the conservation of angular momentum and the dissipation cannot make the rotation to stop. In such a scenario, the empirical relation in Eq. (1) can be explained naturally. One may expect that in such a scenario the spins of galaxies should not distribute in the sky randomly, there should be a dipole anisotropy along the direction of the global rotation. As mentioned in the beginning of this paper, such kind of anisotropy in the distribution of the spin of galaxies has been found at different levels. Here I point out that the derivation from spherical symmetry of proto-galaxies before they collapse may weaken the alignment of the spin of galaxies which makes it very difficult to observe the correlation of the orientation of galaxies. One can imagine that a proto-galaxy may be highly asymmetric, the surface containing the matter destined to end up in a single galaxy may have a very irregular shape (Peebles 1980). The moment of inertia tensor of such an object is usually very complex compared with that of a sphere, then in general the angular momentum of the proto-galaxy should not take the same direction as the angular velocity. When the proto-galaxy rotates and expands together with the universe, its angular velocity is equal to that of the global rotation in both magnitude and direction. The rotation of the universe makes the angular momentum of the proto-galaxy, which is not aligned with the angular velocity with the fixed direction, to precess about the axis of the rotation. The magnitude of the angular momentum is constant during the precession. When the proto-galaxy becomes separated from the global rotation and expansion of the universe, and begins to collapse to form a galaxy, the interaction with its surroundings should become more and more weak and eventually negligible. Its angular momentum gradually becomes constant in both magnitude and direction. In general the direction of the angular momentum is not aligned with the global rotation. It should be determined by the shape of the proto-galaxy and the time when the proto-galaxy becomes an isolated system. Its distribution in space can be expected to be almost random, instead of a strong dipole distribution. As the galaxy evolves, the dissipation processes inside it cause that the component of its angular velocity perpendicular
to the angular momentum gradually vanishes, eventually the galaxy rotates about the direction of its angular momentum. The influence of the shape of the proto-galaxy on the formulae in Eq. (6) can be estimated dimensionally: Let $l_i$ be some linear scale of the proto-galaxy, then the mass $M \sim \rho_d l_i^3$, the moment of inertia $I \sim M l_i^2 \sim M^{5/3} \rho_d^{-2/3}$, and the angular momentum $J \sim I \omega_i \sim M^{5/3} \rho_d^{-2/3} \omega_0$. Therefore $J \sim k M^{5/3}$ still holds, with $k \sim \rho_d^{-2/3} \omega_0$. The initial shape of the proto-galaxy only affects the numerical factor in $k$. However, these do not seem to strongly influence the estimation of the order of magnitude of the angular velocity of the universe. And the scattering of the observational data around Eq. (6) may just reflect the effect of the original shape of proto-galaxies.

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