HADRON INTERACTIONS–HADRON SIZES

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INTRODUCTION

Hadronic interactions are successfully parameterized in different models. Those most frequently applied are the Regge parameterization and the geometrical model. In the present paper we want to apply the geometrical picture to different domains of the hadronic interaction. The model used is a very economical model in that it contains only few parameters most of which have a direct physical interpretation.

We first want to apply the geometrical model to hadron–proton interactions. Then we want to extend the model to deep inelastic scattering. In deep inelastic scattering at small \( x \), the interaction can be viewed upon as an interaction of the hadronic fluctuation of the photon with the proton. The size of the hadronic fluctuation is \( \langle r^2 \rangle \approx \frac{1}{Q^2} \). Thus, deep inelastic scattering offers a nice extension of the applicability of the geometrical model for hadronic sizes from 1 fm down to \( 10^{-3} \) fm. However, the nuclear shadowing demonstrates that in addition to the “hard” interaction corresponding to a hadronic size of \( 1/Q^2 \), an additional, “soft” component (large size) of the interaction in deep inelastic scattering is present. We will elaborate this in the section on nuclear structure function. We will point out the importance of these results for the theoretical treatment of the heavy ion reactions at high energies.

HADRON–PROTON CROSS SECTION

When analyzing the differential cross sections for high-energy \( pp \) collisions available at that time, Wu and Yang\(^1\) as well as Chou and Yang\(^2\) observed that the \( t \) dependence of the differential elastic cross section is closely related to the charge form factor \( F_i(t) \) \( i = 1, 2 \) of colliding hadrons. For small values of \( t \) the form factors are related to the mean-squared charge radii \( \langle r_{ch}^2 \rangle \) via

\[
F_i(t) = 1 + \frac{1}{6} \langle r_{ch}^2 \rangle t + 0(t^2)
\]

which implies that the slope parameter \( b_{12} \) is related to the charge radii via
\[ b_{12} = \frac{1}{3} \left( \langle r_{ch}^2 \rangle_1 + \langle r_{ch}^2 \rangle_2 \right). \]  

(2)

In a series of papers J. H"ufner and I have shown additional regularities in the hadron–hadron cross sections. The experimental data are shown in Fig. [1]. At small energies, \( \sqrt{s} < 10 \text{ GeV} \) quark-antiquark (Regge) exchange is the dominant interaction mechanism. The cross section falls off with \( \sigma_{TOT} \propto 1/\sqrt{s} \) (see Fig. [2(a)]). At energies \( \sqrt{s} > 10 \text{ GeV} \), the hadronic cross sections increase logarithmically as well as the slope parameters do. In the Regge nomenclature this regime is called Pomeron exchange (see Fig. [2(b)]). At high energies the hadronic cross section is entirely absorptive! The elastic cross section is therefore fully determined by absorption. The total and the elastic cross section are connected via the optical theorem symbolically as shown in Fig. [3]. Therefore in the geometrical model we consider the absorption and treat elastic scattering as a shadow of it. The geometrical properties of the hadronic interaction become transparent.

Introducing an effective hadron radius \( R(s) \) the total cross section can be written as

\[ \sigma_{TOT}(s) = aR_1^2(s)R_2^2(s). \]  

(3)

Figure 1. Schematically shown the total cross sections of pions, kaons, protons/antiprotons and photons on the proton.

Figure 2. Quark–antiquark or Regge exchange is schematically shown in figure (a) while (b) shows the two gluon exchange or Pomeron exchange which is the dominant interaction at high energies.
The total cross section square is proportional to the imaginary part of the elastic scattering amplitude at zero degrees.

\[ \sigma_{TOT} \propto \sum \int^2 \propto \text{Im} A( ) \]

and the slope parameter

\[ b_{12}(s) = \frac{1}{3} \left( R_1^2(s) + R_2^2(s) \right) . \] (4)

The constant \( a \) is a universal constant, i.e. it is valid for all hadrons and energies. Formula (3) may look strange but we have to remember that hadrons are color dipoles. The energy dependence of the effective radius is given by

\[ R^2(s) = R^2(s_0) \left( 1 + \epsilon \ln \left( \frac{s}{s_0} \right) \right) . \] (5)

At small energies Regge exchange is the dominant mechanism and formula (5) cannot be checked in this region. Nevertheless, we assume that there is an effective hadron radius in the value close to the charge radius. The logarithmic increase with energy (eq. [5]) can be worked out perturbatively. We will, however, give only a qualitative explanation of it. Figure 3 shows the spreading of gluons beyond the border of the hadron core.

Figure 3. The hadron, a color dipole, is surrounded by a cloud of gluons. The spreading of the gluons is governed by the random walk.

Let us assume that at the radius \( R_0 \) a gluon has a Bjorken \( x = x_0 \). After \( n \) splittings, for simplicity we assume in two equal parts,

\[ x = \left( \frac{1}{2} \right)^n x_0 . \] (6)

The average distance of gluon traveling between splitting is \( \lambda \). Gluons spread according to the random walk, so

\[ R^2 - R_0^2 = n \lambda^2 c \ln \frac{x_0}{x} . \] (7)

The interaction is totally absorptive, so the inelastic event is possible only if

\[ x_1 x_2 s \geq M_0^2 \] (8)
where \( x_1 \) and \( x_2 \) are Bjorken \( x \) of hadron 1 and 2, respectively. Inserting the condition (5) into formula (6) gives (4).

**DEEP INELASTIC SCATTERING**

Deep inelastic scattering at small Bjorken \( x \) can be viewed upon as an interaction of a hadron fluctuation of the photon with the proton (Fig. 5). The fluctuation length of the \( q\bar{q} \) pair is given by the offshellness \( \Delta E \) of the quark–antiquark system as compared to the photon

\[
\ell = \frac{1}{\Delta E} \approx \frac{1}{m x} \quad (9)
\]

where \( m \) is the proton mass and \( x = Q^2/2m\nu \). One sees that already for \( x = 0.1 \) the fluctuation length is \( \ell \approx 1 \text{ fm} \) and the approximation of the interaction as a hadronic one is quite good.

In deep inelastic scattering one measures the following cross section:

\[
\frac{d\sigma}{dxdQ^2} = \text{Photonflux} \cdot \sigma_{TOT}^{\gamma^*p}(x, Q^2) \quad (10)
\]

where the photon flux is given by the probability of the electron to emit a virtual photon which is believed to be well calculable within the QED. The second factor is the total cross section for the virtual photon \( \gamma^* \) with the proton. The structure function \( F_2^p(x, Q^2) \), the main objective of deep inelastic scattering, is related to \( \sigma_{TOT}^{\gamma^*p}(x, Q^2) \) via

\[
F_2(x, Q^2) \approx Q^2 \sigma_{TOT}^{\gamma^*p}(x, Q^2). \quad (11)
\]

As pointed out above, for low \( x \), the \( \sigma_{TOT}^{\gamma^*p} \) can be expressed in terms of the hadron fluctuation (light cone representation)

\[
\sigma_{TOT}^{\gamma^*p}(x, Q^2) = \sum_h W_h(x, Q^2)\sigma_{TOT}(h, p). \quad (12)
\]

The sum goes over all possible \( q\bar{q} \) pairs. One could try to calculate \( W_h(x, Q^2) \) using some model of hadronic fluctuation. We will just assume this to be taken into account by the normalization factor in the geometrical cross section of the \( q\bar{q} \) pair

\[
\sigma_{TOT}(h, p) \alpha \langle r^2_{q\bar{q}} \rangle = \frac{1}{m_q^2 + \frac{Q^2}{4}}. \quad (13)
\]
Here we assumed that for large $Q^2$ the transverse size of $q\bar{q}$ pair is given by $1/Q^2$. For $Q^2\to 0$ confinement determines the size of the $q\bar{q}$ pair and the radius of the hadronic fluctuation is just the radius of the $\rho$, the well-known vector dominance regime. We see that eqs. (13) and (12) give automatically the Bjorken scaling if inserted in eq. (11).

Conventionally, the total cross sections for virtual photon–photon interaction are plotted as the structure function (see eq. [11]). The measured structure functions as published by the H1 collaboration\(^4\) are shown in Fig. 6.

\[ F_2(x, Q^2). \]  

Figure 6. Proton structure function $F_2(x, Q^2)$. Data are taken from Ref. 4.

The Bjorken $x$ is given by $x = Q^2/2m_\nu$ where $\nu$ is the energy of the virtual photon. The nominator $2m_\nu$ then is the square of the center of mass energy of the photon–proton system

\[ W^2 = 2m_\nu. \]  

Thus, for a fixed $Q^2$, $W^2 \propto 1/x$. If we limit ourselves to the $x$ domain in which the hadronic interpretation is meaningful, the total $\gamma^*p$ cross sections are shown in Fig. 7. The behavior of $\sigma_{TOT}(\gamma^*p)$ with the center of mass energy square $W^2$ resembles strongly the one of the real hadrons $\sigma_{TOT}(h, p)$. However, the increase of the cross section with energy is $Q^2$ dependent. For $Q^2 \to 0$ it is exactly that of the $pp$ and $p\bar{p}$.

We have explained the increase of the cross section with energy by the increasing importance of the gluon halo in inelastic processes. It seems, however, that we missed the $Q^2$ dependence in this process. In fact, for the real hadrons, we considered in the first section, the $Q^2 \approx 0$ assumption is good enough. By virtual hadrons with sizes...
Figure 7. Total $\gamma^*p$ cross sections deduced from the structure functions of Fig. 6. The cross sections are plotted only for the energies where the geometrical model is applicable.

$\approx 1/Q^2$ the gluon halo starts with gluons of higher and higher transverse momenta. We have to consider the fact that the strong coupling constant $\alpha_s$ is $Q^2$ dependent.

This effect has been taken into account, for example in Ref. 5. In the $Q^2$ region accessible to measurement now, the $Q^2$ dependence of the gluon halo can be approximated by

$$
\sigma_{TOT}(Q^2, W^2) = aR_h^2(Q^2, W^2)R_p^2(Q^2 = 0, W^2)
= \frac{A}{m_q^2 + Q^2/4} \left(1 + \left(\varepsilon + \varepsilon' \ln \left(1 + \frac{Q^2}{Q_0^2}\right)\right) \ln \frac{x_0}{x}\right). \quad (15)
$$

Here $R_h^2$ is the radius square of the hadronic fluctuation, $R_p^2$ that of the proton and $a$ the coupling constant introduced in eq. (3). The curves in Fig. 6 are obtained by applying eq. (15).

Summarizing this section, one can conclude that the hadronic interaction is well simulated by the interaction of two color dipoles surrounded by a gluon cloud.

NUCLEAR STRUCTURE FUNCTION

The nuclear structure function is of interest because eventually heavy ion reactions will be treated from “first principles”. This means that the quark and gluon structure
functions have to be used as input in the calculation of heavy ion reactions at high energies. The collision is then described by elastic parton scattering (see e.g. Ref. 9). The nuclear structure function is not just the sum of $A$ nucleon structure functions! As we show below, nuclear shadowing distort the nuclear structure functions for $x < 0.1$. In order to obtain the gluon structure function of a nucleus the evolution has to be applied to the measured quark structure function. A direct determination of the quark nuclear structure function is required. In this paper, however, we will consider the nuclear structure function as a tool to improve our geometrical picture of the virtual photon interaction.

**Nuclear shadowing**

Nuclear structure functions have extensively been measured by the NMC collaboration at CERN in the eighties. Results of these measurements are shown schematically in Fig. 8. We are interested in the ratio of the nuclear structure function to the nucleon structure function. At $x < 0.1$ one observes shadowing, this means, the cross section per nucleon is reduced, the reduction depending on the mass number $A$. The surprising fact is that shadowing does not depend strongly on $Q^2$.

![Figure 8. Shadowing in nuclear structure functions shown schematically.](image)

Thus, the geometrical model we used above cannot be the full story. If the cross section of the hadronic fluctuation goes down like $1/Q^2$, then shadowing would very fast disappear according to

$$
\frac{F_2^A(x, Q^2)}{AF_2^N(x, Q^2)} = 1 - \frac{\varepsilon}{Q^2}.
$$

As this is not the case, there must be a soft component of the interaction. In fact, we have taken only a part of the geometrical object presenting the $q\bar{q}$ pair. In our consideration, eq. (3), we assumed that the photon momentum is taken by quark and antiquark equally. But this is not true.
In Figure 9 the quark takes over the ratio $\alpha$ of the photon momentum and the antiquark $1 - \alpha$ or vice versa. Considering an asymmetric pair the radius square of the $q\bar{q}$ pair is (see Ref. 6)

$$\langle r^2 \rangle \propto \frac{1}{m_q^2 + \alpha(1 - \alpha)Q^2}. \quad (17)$$

For $\alpha = \frac{1}{2}$ we obtain the expression (13) for the radius square. For asymmetrical pairs $\alpha < 1/Q^2$, the radius of the $q\bar{q}$ is of $\rho$-meson size and the cross section amounts to $\approx 20\, \text{mb}$. Certainly, the probability of having an asymmetric pair with $\alpha < 1/Q^2$ is small. The probability is proportional to $1/Q^2$.

Now, dividing the interaction in a hard (small size probe), corresponding to the approximately symmetric pairs, and a soft component, corresponding to the pairs with $\alpha < 1/Q^2$, we see that both scale with $1/Q^2$ and cannot be distinguished in the inclusive measurements of deep inelastic scattering on a proton. On the nucleus, however, the contribution of the soft component is strongly absorbed, dies off fast and is responsible for the shadowing. For a quantitative treatment of shadowing see for example Ref. 8.

CONCLUSIONS

A simple picture of a hadron and its interaction at high energies emerges: The hadron is a color dipole of size $R$ surrounded by a cloud of gluons. The interaction with the hadron core has to be treated “non-perturbatively” with only parameters being the hadronic size and the “universal” hadronic coupling constant.

The halo interaction can be understood within the perturbative approach. This simple picture of the hadronic interaction is supported by the results with hadrons of sizes $R \approx 1\, \text{fm}$ down to $R = 0.01\, \text{fm}$ corresponding to $Q^2 \approx 2000\, \text{GeV}^2$ as measured in deep inelastic scattering.

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