BFV analysis of the $U_{EM}(1)$ gauged SU(3) WZW model and the Faddeev-Jackiw approach

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Abstract

The four dimensional SU(3) WZW model coupled to electromagnetism is treated as a constrained system in the context of Batalin-Fradkin-Vilkovisky formalism. It is shown that this treatment is equivalent to the Faddeev-Jackiw (FJ) approach. It is also shown that the field redefinitions that transform the fields of the model into BRST and $\sigma$ closed are actually the Darboux’s transformations used in the FJ formalism.

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1 Introduction

In [6] the SU(2) WZW model coupled to electromagnetism was treated as a constrained system in the context of the Batalin-Fradkin-Vilkovisky (BFV) formalism [1]. Comparison was made between this method and the Faddeev-Jackiw (FJ) approach to constrained systems. Common features were emphasized. In this letter we extend this analysis in the SU(3) case.

In the FJ [2] approach we start with a Lagrangian density first order in time derivatives. Then by using Darboux's theorem and Euler-Lagrange equations we transform it into an expression whose canonical one-form is diagonal and where the constraints occur in specific terms linear in coordinate variables. Next we solve the constraint equations and after substituting the solutions into the expression for the Lagrangian density we repeat the whole process again until we end up with an unconstrained Lagrangian density, with diagonal canonical one form and whose phase space is reduced. On the contrary in the BFV formalism the phase space of the theory is extended. This is done in two steps. First a canonical momentum conjugate to every Lagrange multiplier is introduced (which has to vanish) so increasing the number of constraints. Second a ghost field is introduced for every constraint so extending the phase space of the theory. The gauge fixing is done by properly choosing the gauge fermion.

2 The SU(3) WZW model coupled to electromagnetism

In [7] the SU(3) WZW model coupled to electromagnetism [4, 5] was treated as a constrained system in the context of the FJ formalism. This model describes the low energy interactions of the eight Goldstone bosons and photons including those related to the axial anomaly. The effective action of the model up to second order in the pion fields \( \theta_a, a = 1, ..., 8 \) is given by

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{EM}} + \mathcal{L}_{\sigma}^{(2)} + \mathcal{L}_{\text{WZW}}^{(2)} + \mathcal{O}(\theta^3),
\]

\[
\mathcal{L}_{\text{EM}} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu},
\]

\[
\mathcal{L}_{\sigma}^{(2)} = \frac{1}{2} (\partial_\mu \theta_a) (\partial^\mu \theta_a) + e A^\mu (\theta_2 \partial_\mu \theta_1 - \theta_1 \partial_\mu \theta_2 + \theta_5 \partial_\mu \theta_4 - \theta_4 \partial_\mu \theta_5) + \frac{e^2}{2} A_\mu A^{\mu} (\theta_1^2 + \theta_2^2 + \theta_3^2 + \theta_5^2),
\]

\[
\mathcal{L}_{\text{WZW}}^{(2)} = -\frac{Ne^2}{12\pi^2 f_\pi} e^{\mu \nu \alpha \beta} A_\mu (\partial_\nu A_\alpha) (\partial_\beta \theta_3) + \frac{1}{\sqrt{3}} \partial_\beta \theta_8.
\]

and as an expression first order in time derivatives of the fields is written as follows

\[
\mathcal{L}_{\text{eff}} = -\pi \cdot \dot{A} + p_a \dot{\theta}_a - H_0^{(2)} - A_0^{(2)} - \nabla \cdot \pi + \mathcal{O}(\theta^3),
\]

\[
H_0^{(2)} = \frac{1}{2} [\pi^2 + B^2 + (\nabla \theta_a)^2 + p_a^2] + \frac{e^2}{2} A^2 (\theta_1^2 + \theta_2^2 + \theta_3^2 + \theta_5^2)
\]
\[ + e A \cdot (\theta_1 \nabla \theta_2 - \theta_2 \nabla \theta_1 + \theta_4 \nabla \theta_5 - \theta_5 \nabla \theta_4) - \frac{N_c e^2}{6 \pi^2 f_\pi} (\pi \cdot B)(\theta_3 + \frac{1}{\sqrt{3}} \theta_8), \]

\[ \rho^{(2)} = e(p_2 \theta_1 - p_1 \theta_2 + p_5 \theta_4 - p_4 \theta_5). \]

See Appendix for notation.

The corresponding BFV action is given by

\[ S_{BFV} = \int d^4x (\pi \cdot \dot{A} + \pi_0 \dot{A}_0 + p_a \dot{\theta}_a + \dot{\mathcal{C}} \mathcal{P} + \dot{\mathcal{C}} \mathcal{P} - H_0^{(2)}) + \int dt [\Psi, Q] \]  

(3)

The scalar potential has become a full dynamical variable and its conjugate momentum \( \pi_0 \) has to vanish. We have also introduced the canonical pair \((C, P)\) of a ghost field and its conjugate momentum corresponding to the constraint \( G_1 = \rho^{(2)} - \nabla \cdot \pi \), and the canonical pair \((\bar{C}, \bar{P})\) of an antighost field and its canonical momentum, corresponding to the constraint \( G_2 = \pi_0 \). \( \Psi \) is the gauge fermion and Q is the BRST charge. The two constraints are first class. The BRST charge is given by

\[ Q = \int d^3 x [C(\rho^{(2)} - \nabla \cdot \pi) + i \bar{P} \pi_0] \]  

(4)

The canonical Hamiltonian \( \int d^3x H_0^{(2)} \) and \( S_{BFV} \) are invariant under the BRST transformations

\[ sA = - \nabla C, \quad sC = 0, \]
\[ s\mathcal{P} = \nabla \cdot C - \rho^{(2)}, \quad s\pi = 0, \]
\[ sA_0 = i \bar{P}, \quad s\bar{P} = 0, \]
\[ s\bar{C} = - i \pi_0, \quad s\pi_0 = 0, \]
\[ s\theta_1 = - e \theta_2 C, \quad s\theta_2 = e \theta_1 C, \]
\[ sP_1 = - e P_2 C, \quad sP_2 = e P_1 C, \]
\[ s\theta_4 = - e \theta_5 C, \quad s\theta_5 = e \theta_4 C, \]
\[ sP_4 = - e P_5 C, \quad sP_5 = e P_4 C, \]
\[ s\theta_3 = s\theta_6 = s\theta_7 = s\theta_8 = 0, \]
\[ sp_3 = sp_6 = sp_7 = sp_8 = 0. \]

We decompose \( A \) and \( \pi \) into transverse \((A^T, \pi^T)\) and longitudinal \((A^L, \pi^L)\) components

\[ A^T = A - \nabla A^L, \quad A^L = \nabla A^L, \quad A^L = \frac{1}{\sqrt{2}} (\nabla \cdot A), \]
\[ \pi^T = \pi - \frac{\nabla}{\sqrt{2}} \pi^L, \quad \pi^L = \frac{\nabla}{\sqrt{2}} \pi^L, \quad \pi^L = \nabla \cdot \pi. \]

The following field redefinitions

\[ p_1 \rightarrow p_1 \cos \alpha + p_2 \sin \alpha, \quad \theta_1 \rightarrow \theta_1 \cos \alpha + \theta_2 \sin \alpha, \]
\[\begin{align*}
p_2 & \rightarrow p_2 \cos \alpha - p_1 \sin \alpha , \\
p_4 & \rightarrow p_4 \cos \alpha + p_5 \sin \alpha , \\
p_5 & \rightarrow p_5 \cos \alpha - p_4 \sin \alpha ,
\end{align*}\]

where \( \alpha = eA' \), transform \( \theta_i \) and \( p_i \) fields \((i = 1, 2, 4, 5)\) into BRST and \( \sigma \) closed (physical).

So we have

\[\begin{align*}
sA' & = -C , \\
s\mathcal{P} & = \pi' - \rho^{(2)} , \\
sA_0 & = i\mathcal{P} , \\
s\tilde{C} & = -i\pi_0 , \\
s\theta_a & = 0 , \\
s\sigma & = \frac{\nabla}{\nabla^2} (s\mathcal{P} + \rho^{(2)}) , \\
s\pi_0 & = is\tilde{C} ,
\end{align*}\]

and

\[\begin{align*}
\sigma(-C) & = A' , \\
\sigma\pi' & = \mathcal{P} , \\
\sigma(i\mathcal{P}) & = A_0 , \\
\sigma(-i\pi_0) & = \tilde{C} , \\
\sigma\theta_a & = 0 , \\
\sigma p_a & = 0 ,
\end{align*}\]

where \( \sigma \) is the contracting homotopy operator \([3]\). Note that there is no way that we can transform the fields \( A^L, A_0, C, \tilde{C}, \mathcal{P}, \mathcal{P} \) into BRST and \( \sigma \) closed.

Now we solve for \( C, \mathcal{P}, \pi^L \) and \( \pi_0 \) from (7)

\[C = -sA' , \quad \mathcal{P} = -isA_0 , \quad \pi^L = \frac{\nabla}{\nabla^2} (s\mathcal{P} + \rho^{(2)}) , \quad \pi_0 = is\tilde{C},\]

and after performing the transformations (6) in (3) we substitute \( C, \mathcal{P}, \pi^L, \pi_0 \) from (9). We end up with the following expression for \( S_{BFV} \)

\[\begin{align*}
S_{BFV} & \rightarrow \int d^4x [\pi^T \cdot \dot{A}^T + p_a \dot{\theta}_a - H^{(2)} + s\mathcal{P}^{(2)}] + \int dt [\Psi, Q] , \\
H^{(2)} & = \frac{1}{2}[(\pi^T)^2 + B^2 - \rho^{(2)} \frac{1}{\nabla^2} \rho^{(2)} + (\nabla \theta_a)^2 + p_a^2] \\
& \quad + eA^T \cdot (\theta_1 \nabla \theta_2 - \theta_2 \nabla \theta_1 + \theta_4 \nabla \theta_5 - \theta_5 \nabla \theta_4) + \frac{2}{2}(A^T)^2 (\theta_1^2 + \theta_2^2 + \theta_3^2 + \theta_4^2) \\
& \quad - \frac{N e^2}{6\pi^2 f} [\pi^T + \frac{\nabla}{\nabla^2} \rho^{(2)}] \cdot B(\theta_3 + \frac{1}{\sqrt{3}} \theta_8) , \\
F^{(2)} & = i\tilde{C} \dot{A}_0 + \mathcal{P} \cdot A' + \frac{1}{2} (s\mathcal{P}) \frac{1}{\nabla^2} \mathcal{P} + \frac{1}{\nabla^2} \rho^{(2)} + \frac{N e^2}{6\pi^2 f} \frac{\nabla}{\nabla^2} \mathcal{P} \cdot B(\theta_3 + \frac{1}{\sqrt{3}} \theta_8) ,
\end{align*}\]
where $H^{(2)}_C$ is the Coulomb gauge Hamiltonian.

Finally we fix the gauge fermion as follows

$$\Psi = -\int d^3x F^{(2)}$$

and we get a Coulomb gauge expression for the effective action with the unphysical $A^L$ cancelled out. In [7] in the context of the FJ formalism we solved the constraint $\rho^{(2)} - \nabla \pi = 0$ for $\pi^L$ and we substituted back into the expression (2) for the effective Lagrangian density. We came up with an uncanonical expression which was diagonalized by performing the transformations (6). So it turns out that (6) are actually the Darboux’s transformations of the FJ formalism needed for this case.

### 3 Keeping next order terms

The effective Lagrangian density up to third order in $\theta_a$ is given by

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \mathcal{L}_(2) + \mathcal{L}_{WZW}^{(3)} + O(\theta^4) \ , \quad (11)$$

where $\mathcal{L}_{(2)}$ and $\mathcal{L}_{WZW}$ are given in (1) and

$$\mathcal{L}_{WZW}^{(3)} = -\frac{N ee}{3\pi^2 f_3^2} \epsilon^{\mu\nu\alpha\beta} (\partial_\mu A_\nu)(\theta_1 \partial_\alpha \theta_2 - \theta_2 \partial_\alpha \theta_1 + \theta_4 \partial_\alpha \theta_5 - \theta_5 \partial_\alpha \theta_4)(\partial_\beta \theta_3 + \frac{1}{\sqrt{3}} \partial_\beta \theta_8)$$

$$-\frac{N ee}{\sqrt{3\pi^2 f_3^2}} \epsilon^{\mu\nu\alpha\beta} (\partial_\mu A_\nu)(\theta_7 \partial_\alpha \theta_6 - \theta_6 \partial_\alpha \theta_7) \partial_\beta \theta_8$$

$$+ \frac{N ee^2}{18\pi^2 f_3^2} \epsilon^{\mu\nu\alpha\beta} (\partial_\mu A_\nu)(\partial_\alpha A_\beta)\{[4(\theta_1^2 + \theta_2^2) + 5(\theta_4^2 + \theta_5^2)]\theta_3$$

$$+ \sqrt{3}[2(\theta_1^2 + \theta_2^2) + \theta_4^2 + \theta_5^2]\theta_8 + 2[(\theta_1 \theta_5 - \theta_2 \theta_4)\theta_7 + (\theta_1 \theta_4 + \theta_2 \theta_5)\theta_6]\}$$

$$- \frac{N ee^2}{3\pi^2 f_3^2} \epsilon^{\mu\nu\alpha\beta} A_\mu(\partial_\alpha A_\nu)(\theta_3 + \frac{1}{\sqrt{3}} \theta_8) \partial_\beta (\theta_1^2 + \theta_2^2 + \theta_4^2 + \theta_5^2) \ .$$

In the non-covariant notation, as an expression first order in time derivatives of the fields, $\mathcal{L}_{\text{eff}}$ is given by

$$\mathcal{L}_{\text{eff}} = -\pi \cdot \dot{A} + p_a \theta_a - H^{(2)}_0 - H^{(3)}_0 - A_0(\rho^{(2)} + \rho^{(3)} - \nabla \cdot \pi) + O(\theta^4) \ , \quad (12)$$

where $H^{(2)}_0$ and $\rho^{(2)}$ are given in (2) and

$$H^{(3)}_0 = -\frac{N ee}{3\pi^2 f_3^2} [\pi \times \nabla (\theta_3 + \frac{1}{\sqrt{3}} \theta_8) - (p_3 + \frac{1}{\sqrt{3}} p_8) B] \cdot (\theta_1 \nabla \theta_2 - \theta_2 \nabla \theta_1 + \theta_4 \nabla \theta_5 - \theta_5 \nabla \theta_4)$$

$$- \frac{N ee}{\sqrt{3\pi^2 f_3^2}} (\pi \times \nabla \theta_8 - p_8 B) \cdot (\theta_7 \nabla \theta_6 - \theta_6 \nabla \theta_7)$$
+ \frac{N_c e^2}{9 \pi^2 f_\pi^3} (\pi \cdot B) \{4(\theta_1^2 + \theta_2^2) + 5(\theta_4^2 + \theta_5^2)\} \theta_3 \\
+ \sqrt{3} [2(\theta_1^2 + \theta_2^2) + \theta_4^2 + \theta_5^2] \theta_8 + 2(\theta_1 \theta_5 - \theta_2 \theta_4) \theta_7 + (\theta_1 \theta_4 + \theta_2 \theta_5) \theta_6 \} \\
- \frac{N_c e^2}{3 \pi^2 f_\pi^3} (\pi \times A) \cdot [\nabla (\theta_1^2 + \theta_2^2 + \theta_4^2 + \theta_5^2)](\theta_3 + \frac{1}{\sqrt{3}} \theta_8) \\
- \frac{2N_c e^2}{3 \pi^2 f_\pi^3} (A \cdot B)(p_1 \theta_1 + p_2 \theta_2 + p_4 \theta_4 + p_5 \theta_5)(\theta_3 + \frac{1}{\sqrt{3}} \theta_8) \\
- \frac{N_c e}{3 \pi^2 f_\pi^3} B \cdot \nabla (\theta_3 + \frac{1}{\sqrt{3}} \theta_8)](p_2 \theta_1 - p_1 \theta_2 + p_5 \theta_4 - p_4 \theta_5) \\
- \frac{N_c e}{\sqrt{3} \pi^2 f_\pi^3} (B \cdot \nabla \theta_8)(p_0 \theta_7 - p_7 \theta_6) \\

\rho^{(3)} = -\frac{N_c e^2}{3 \pi^2 f_\pi^3} \nabla \cdot [B(\theta_1^2 + \theta_2^2 + \theta_4^2 + \theta_5^2)(\theta_3 + \frac{1}{\sqrt{3}} \theta_8)] \\

The BFV effective action is given by

$$S_{BFV} = \int d^4 x (-\pi \cdot \dot{A} + \pi_0 A_0 + p_a \dot{\theta}_a + \dot{\bar{C}} \bar{P} + \bar{C} \bar{P} - H_0^{(2)} - H_0^{(3)}) + \int dt [\psi, Q] \quad (13)$$

As in the previous case $\pi_0$ is the canonical momentum conjugate to $A_0$ and has to vanish and we have also introduced the canonical pair $(C, P)$ corresponding to the constraint $G_1 = \rho^{(2)} + \rho^{(3)} - \nabla \cdot \pi$, and the canonical pair $(\bar{C}, \bar{P})$ corresponding to the constraint $G_2 = \pi_0$. Also by $\Psi$ and $Q$ we denote the gauge fermion and the BRST charge respectively.

$$Q = \int d^3 x [C(\rho^{(2)} + \rho^{(3)} - \nabla \cdot \pi) + i\bar{P} \pi_0] \quad (14)$$

The canonical Hamiltonian $\int d^3 x (H_0^{(2)} + H_0^{(3)})$ and $S_{BFV}$ are invariant under the following BRST transformations

$$sA = -\nabla C \ , \quad sC = 0 \\

sP = \nabla \cdot \pi - \rho^{(2)} - \rho^{(3)}, \quad s\pi = \frac{N_c e^2}{3 \pi^2 f_\pi^3} \nabla[(\theta_1^2 + \theta_2^2 + \theta_4^2 + \theta_5^2)(\theta_3 + \frac{1}{\sqrt{3}} \theta_8)] \times \nabla C, \quad \text{if} \quad \theta_1 = -e \theta_2 C, \quad sp_1 = -e p_2 C - \frac{2N_c e^2}{3 \pi^2 f_\pi^3} (B \cdot \nabla C) \theta_1 (\theta_3 + \frac{1}{\sqrt{3}} \theta_8),$$

$$s\theta_2 = e \theta_1 C, \quad sp_2 = e p_1 C - \frac{2N_c e^2}{3 \pi^2 f_\pi^3} (B \cdot \nabla C) \theta_2 (\theta_3 + \frac{1}{\sqrt{3}} \theta_8),$$

$$s\theta_3 = 0, \quad sp_3 = -\frac{N_c e^2}{3 \pi^2 f_\pi^3} (B \cdot \nabla C)(\theta_1^2 + \theta_2^2 + \theta_4^2 + \theta_5^2) \ ,$$

5
\[ s\theta_4 = -e\theta_5 C \ , \quad sp_4 = -e p_5 C - \frac{2N_c e^2}{3\pi^2 f_\pi^2} (B \cdot \nabla C) \theta_4 (\theta_3 + \frac{1}{\sqrt{3}} \theta_8) \ , \]
\[ s\theta_5 = e\theta_4 C \ , \quad sp_5 = e p_4 C - \frac{2N_c e^2}{3\pi^2 f_\pi^2} (B \cdot \nabla C) \theta_5 (\theta_3 + \frac{1}{\sqrt{3}} \theta_8) \ , \]
\[ s\theta_8 = 0 \ , \quad sp_8 = -\frac{N_c e^2}{3\sqrt{3} \pi^2 f_\pi^2} (B \cdot \nabla C) (\theta_1^2 + \theta_2^2 + \theta_3^2 + \theta_5^2) \ , \]
\[ s\theta_6 = s\theta_7 = sp_6 = sp_7 = 0 \ . \]

The following field redefinitions
\[
\theta_1 \rightarrow \theta_1 \cos \alpha + \theta_2 \sin \alpha ,
\]
\[
\theta_2 \rightarrow \theta_2 \cos \alpha - \theta_1 \sin \alpha ,
\]
\[
\theta_4 \rightarrow \theta_4 \cos \alpha + \theta_5 \sin \alpha ,
\]
\[
\theta_5 \rightarrow \theta_5 \cos \alpha - \theta_4 \sin \alpha ,
\]
\[
\pi^T \rightarrow \pi^T - \frac{N_c e^2}{3\pi^2 f_\pi^2} \nabla [(\theta_1^2 + \theta_2^2 + \theta_3^2 + \theta_5^2) (\theta_3 + \frac{1}{\sqrt{3}} \theta_8)] \times A^L ,
\]
\[
p_1 \rightarrow p_1 \cos \alpha + p_2 \sin \alpha + \frac{2N_c e^2}{3\pi^2 f_\pi^2} (B \cdot A^L) (\theta_1 \cos \alpha + \theta_2 \sin \alpha) (\theta_3 + \frac{1}{\sqrt{3}} \theta_8) ,
\]
\[
p_2 \rightarrow p_2 \cos \alpha - p_1 \sin \alpha + \frac{2N_c e^2}{3\pi^2 f_\pi^2} (B \cdot A^L) (\theta_2 \cos \alpha - \theta_1 \sin \alpha) (\theta_3 + \frac{1}{\sqrt{3}} \theta_8) ,
\]
\[
p_3 \rightarrow p_3 + \frac{N_c e^2}{3\pi^2 f_\pi^2} (B \cdot A^L) (\theta_1^2 + \theta_2^2 + \theta_3^2 + \theta_5^2) ,
\]
\[
p_4 \rightarrow p_4 \cos \alpha + p_5 \sin \alpha + \frac{2N_c e^2}{3\pi^2 f_\pi^2} (B \cdot A^L) (\theta_4 \cos \alpha + \theta_5 \sin \alpha) (\theta_3 + \frac{1}{\sqrt{3}} \theta_8) ,
\]
\[
p_5 \rightarrow p_5 \cos \alpha - p_4 \sin \alpha + \frac{2N_c e^2}{3\pi^2 f_\pi^2} (B \cdot A^L) (\theta_4 \cos \alpha - \theta_5 \sin \alpha) (\theta_3 + \frac{1}{\sqrt{3}} \theta_8) ,
\]
\[
p_8 \rightarrow p_8 + \frac{N_c e^2}{3\sqrt{3} \pi^2 f_\pi^2} (B \cdot A^L) (\theta_1^2 + \theta_2^2 + \theta_3^2 + \theta_5^2) .
\]

transform the fields \( \theta_i (i = 1, 2, 4, 5) \), \( p_i (i = 1, 2, 3, 4, 5, 8) \) and \( \pi^T \) into BRST and \( \sigma \) closed. ( The same notation as in the previous case is kept ).

\[
\begin{align*}
& sA^{L'} = -C \ , \quad sC = 0 \ , \\
& s\mathcal{P} = \pi^{L'} - \rho^{(2)} - \rho^{(3)} \ , \quad s\pi^{L'} = 0 , \\
& sA_0 = i\mathcal{P} \ , \quad s\mathcal{P} = 0 , \\
& s\bar{C} = -i\pi_0 \ , \quad s\pi_0 = 0 \ , \\
& s\theta_a = 0 \ , \quad sp_a = 0 \ , \quad a = 1, 2, \ldots, 8 \\
& s\mathbf{A}^T = 0 \ , \quad s\pi^T = 0 ,
\end{align*}
\]

and
\[
\begin{align*}
& \sigma(-C) = A^{L'} \ , \quad \sigma A^{L'} = 0 , \\
& \sigma\pi^{L'} = \mathcal{P} \ , \quad \sigma\mathcal{P} = 0 ,
\end{align*}
\]
\[\sigma(i\bar{\mathcal{P}}) = A_0, \quad \sigma A_0 = 0,\]
\[\sigma(-i\pi_0) = \bar{C}, \quad \sigma(\bar{C}) = 0,\]
\[\sigma\theta_a = 0, \quad \sigma p_a = 0, \quad a = 1, 2, \ldots, 8\]
\[\sigma A^T = 0, \quad \sigma \pi^T = 0.\]

Now we proceed as in the previous case and solve for \(C, \bar{\mathcal{P}}, \pi^L\) and \(\pi_0\) from (17)
\[C = -sA^L, \quad \bar{\mathcal{P}} = -isA_0, \quad \pi^L = \frac{\nabla}{\sqrt{2}}(s\mathcal{P} + \rho^{(2)} + \rho^{(3)}), \quad \pi_0 = is\bar{C}.\]

After performing the transformations (16) in (13) and substituting the expression of \(C, \bar{\mathcal{P}}, \pi^L\) and \(\pi_0\) from (19) we obtain the following expression for \(S_{BFV}\)
\[S_{BFV} \rightarrow \int d^4x [-\pi^T \cdot \hat{A}^T + p_a\theta_a - H^{(2)} - H^{(3)} + s(F^{(2)} + F^{(3)})] + \int dt[\Psi, Q]\]

where \(H^{(2)}\) and \(F^{(2)}\) are given in (10),
\[H^{(3)} = -\frac{N_e}{3\pi^2 f_5^3} \pi^T \times \nabla(\theta_3 + \frac{1}{\sqrt{3}}\theta_8) - (p_3 + \frac{1}{\sqrt{3}}p_8\mathbf{B}) \cdot (\theta_1\nabla\theta_2 - \theta_2\nabla\theta_1 + \theta_4\nabla\theta_5 - \theta_5\nabla\theta_4)\]

and
\[F^{(3)} = \mathcal{P} \frac{1}{\sqrt{2}} \rho^{(3)} + \frac{N_e}{3\pi^2 f_5^3} [[\nabla \mathcal{P} \times \nabla(\theta_3 + \frac{1}{\sqrt{3}}\theta_8) \cdot (\theta_1\nabla\theta_2 - \theta_2\nabla\theta_1 + \theta_4\nabla\theta_5 - \theta_5\nabla\theta_4)\]

\[\quad - \frac{N_e^2}{9\pi^2 f_5^3} (\nabla \mathcal{P} \cdot \mathbf{B}) \cdot \{[4(\theta_1^2 + \theta_2^2) + 5(\theta_4^2 + \theta_5^2)]\theta_3\]

\[\quad + \sqrt{3}[2(\theta_1^2 + \theta_2^2) + \theta_4^2 + \theta_5^2]\theta_8 + 2[(\theta_1\theta_5 - \theta_2\theta_4)\theta_7 + (\theta_1\theta_4 + \theta_2\theta_5)\theta_6]\}

\[\quad - \frac{N_e^2}{3\pi^2 f_5^3} \hat{A}^T \cdot \nabla(\theta_3 + \frac{1}{\sqrt{3}}\theta_8)(p_2\theta_1 + p_3\theta_2 + p_4\theta_4 + p_5\theta_5)(\theta_3 + \frac{1}{\sqrt{3}}\theta_8)\]

\[\quad - \frac{N_e^2}{3\pi^2 f_5^3} \mathbf{B} \cdot \nabla(\theta_3 + \frac{1}{\sqrt{3}}\theta_8)(p_2\theta_1 - p_3\theta_2 + p_5\theta_4 - p_4\theta_5)\]

\[\quad - \frac{N_e^2}{\sqrt{3\pi^2 f_5^3}} (\mathbf{B} \cdot \nabla\theta_8)(p_6\theta_7 - p_7\theta_6)\].
Finally by fixing the gauge fermion as follows
\[ \Psi = - \int d^3x (F^{(2)} + F^{(3)}) \]
we end up with a Coulomb-gauge expression for the effective action. In [7] in the context of the FJ formalism we solved the constraint \( \rho^{(2)} + \rho^{(3)} - \nabla \pi = 0 \) for \( \pi^L \) and we substituted into the expression (12) for the effective Lagrangian density. The resulting uncanonical expression was diagonalized by performing the field transformations (16). So it is shown that also in this case the field transformations that transform the fields of the model into BRST and \( \sigma \) closed are the Darboux’s transformations of the FJ approach.

4 Conclusion

The four dimensional SU(3) WZW model coupled to electromagnetism was treated in the context of the BFV formalism for constraint systems. Comparison was made with the FJ approach and common features were stressed. It was shown that the field redefinitions that transform the fields of the model into BRST and \( \sigma \) closed are actually the Darboux’s transformations of the FJ approach.

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5 Appendix

Our metric is \( g_{\mu\nu} = \text{diag}(1, -1, -1, -1) \). We choose \( \epsilon > 0 \). We define \( \epsilon^{0123} = 1 \). By \( \pi \) we denote the electric field \( \mathbf{E} \) so that \( (\pi_\mu, A^\mu) \mu = 0, 1, 2, 3 \) is a canonical pair. We made use of the following Poisson brackets
\[
\begin{align*}
[A^\mu(x, t), \pi^\nu(y, t)] &= g^{\mu\nu}\delta(x - y), \\
[\theta_a(x, t), p_b(y, t)] &= \delta_{ab}\delta(x - y), \\
[C(x, t), P(y, t)] &= -\delta(x - y), \\
[\bar{C}(x, t), \bar{P}(y, t)] &= -\delta(x - y).
\end{align*}
\]
The Grassmann parities of the fields are given by \( \epsilon_{A_\mu} = \epsilon_{\pi_\mu} = \epsilon_{\theta_a} = \epsilon_{p_a} = 0 \), \( \epsilon_C = \epsilon_P = 1 \) and their ghost number \( gh(C) = -gh(P) = 1 \), \( gh(\bar{C}) = gh(\bar{P}) = -1 \), \( gh(A_\mu) = gh(\pi_\mu) = gh(\theta_a) = gh(p_a) = 0 \).

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