ISOBAR ELECTROPRODUCTION AS A BACKGROUND FROM INTERACTION OF BEAMS WITH RESIDUAL GAS AT $\phi$-FACTORIES.

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(November 4, 2021)

Abstract

It is shown that when beams interact with a residual gas at $\phi$-factories the reaction of the electroproduction of the $\Delta(1232)$ isobar proceeds vigorously. The isobar decay gives $\sim 10^7$ pions during an effective year of $10^7$ s per meter of a residual gas. These pions are emitted largely across the beam axis and have a resonance energy distribution with a peak nearby 265 MeV of a width close to 120 MeV in the isobar rest system. There are presented formulae for the distributions of the four-momentum transfer square, the angles, the energies and the momentum of the decay products, that is all required for the simulation of the process under consideration.

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The aim of $\phi$-factories (DAΦNE in Frascati, which is to be launched in 1997 [1], and another one, which is building up in Novosibirsk [1,2]) is to carry out the precision measurements of the most important physical values, $\epsilon'/\epsilon$ first [3].

To realize this program it needs much to study facilities, all most essential backgrounds. A source of such backgrounds is an interaction of beams with a residual gas.

In the paper we show that the cross section of the electroproduction of the isobar $\Delta(1232)$ with $I(J^P) = \frac{3}{2}(\frac{3}{2}^+)$ at a nucleon for the electron (positron) energy 509.5 MeV (the $\phi$-factories energy) is equal to 3 $\mu$b. It results in the production of $\sim 10^7$ isobars during an effective year of $10^7$ s per meter of a residual gas at the total electric current of beams $1.3 \rightarrow 5.2$ A and the residual gas pressure $\sim 1$ nTorr (as projected at DAΦNE [4]).

From the isobar decay ($\Delta(1232) \rightarrow \pi N$ and $\gamma N$) there are produced the $\pi$-mesons, having the resonance energy distribution with the peak nearby 265 MeV of a width close to 120 MeV, emitted largely across the beam axis, and the photons, having the resonance energy distribution with the peak nearby 257 MeV of a width close to 120 MeV, emitted more isotropically.

For the process under discussion it is characteristic that protons and neutrons, emitted largely across the beam axis from isobar decay, have the very narrow energy resonance distribution (the width is 30 MeV) nearby 970 MeV.

We present formulae for distributions of the four-momentum transfer square of the electron (positron), for distributions of the angles of decay pions (nucleons) and decay photons (nucleons) in the isobar rest system, for distributions of the pion, nucleon and photon energies in the isobar rest system and also for distributions of the pion (nucleon) momentum in the isobar rest system, that is all required for the simulation and the separation of the background under discussion.

The phenomenological Lagrangian density describing the interaction of the isobar with the nucleon and the photon (the magnetic dipole transition) [5]

$$L_{em} = e \frac{\mu}{m_N} F^{\nu\rho}(x)(\bar{\psi}_\nu(x)\gamma_\rho\gamma_5\psi(x) + h.c.),$$

(1)
where \( e \) is the electron charge, \( \alpha = e^2/4\pi = 1/137 \), \( m_N = 0.94 \text{ GeV} \) is the nucleon mass, \( F^{\nu \rho}(x) = \partial^\nu A^\rho(x) - \partial^\rho A^\nu(x) \) is the electromagnetic field, \( \psi_\nu(x) \) is the spinor-vector isobar field and \( \psi(x) \) is the spinor nucleon field.

The width of the radiative decay \( \Delta \to \gamma N \)

\[
\Gamma(\Delta \to \gamma N, m_\Delta) = \alpha \left( \frac{\mu}{m_N} \right)^2 \omega^3(m_\Delta) \left( 1 + \frac{1}{3} \left( \frac{m_N}{m_\Delta} \right)^2 \right),
\]

where \( \omega(m_\Delta) = m_\Delta (1 - m_N^2/m_\Delta^2) / 2 = 0.257 \text{ GeV} \) is the photon energy, \( m_\Delta = 1.232 \text{ GeV} \).

Using experimental data [1] \( \Gamma(\Delta \to \gamma N, m_\Delta) = BR(\Delta \to \gamma N, m_\Delta) \cdot \Gamma(\Delta)(m_\Delta) = 0.58 \cdot 10^{-2} \cdot 0.12 \text{ GeV} = 0.7 \cdot 10^{-3} \text{ GeV} \), one gets \( \mu^2/m_N^2 = 4.7 \text{ GeV}^{-2} \).

Now one can calculate the amplitudes of the isobar electroproduction \( (e^-N \to e^-\Delta) \), see Fig. 1. We can conveniently use the helicity amplitudes in the reaction center mass system \( A^{\lambda_f \lambda_N \lambda_\Delta \lambda_e} \), where \( \lambda_N, \lambda_\Delta, \lambda_e \) are the helicities of the nucleon, of the isobar, of the initial and final electrons respectively. The isobar production amplitudes by the positron differ from the corresponding electroproduction amplitudes by sign only.

Let us write out the amplitudes essential to our consideration

\[
A^{\frac{1}{2} \frac{-1}{2} \frac{1}{2} \frac{1}{2}} = A^{\frac{-1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}} = e^2 \frac{\mu}{m_N} \sqrt{2(t - t_{\min}(m))} \left( s - m_N^2 \right) \frac{f(t)}{t},
\]

\[
A^{\frac{1}{2} \frac{-1}{2} \frac{-1}{2} \frac{1}{2}} = A^{\frac{-1}{2} \frac{1}{2} \frac{-1}{2} \frac{1}{2}} = e^2 \frac{\mu}{m_N} \sqrt{2(t - t_{\min}(m))} \left( s - m_N^2 \right) \left( 1 - \frac{m^2 - m_N^2}{s - m_N^2} \right) \frac{f(t)}{t},
\]

and

\[
A^{\frac{1}{2} \frac{-1}{2} - \frac{1}{2} \frac{1}{2}} = A^{\frac{-1}{2} \frac{1}{2} \frac{1}{2} - \frac{1}{2}} = e^2 \frac{\mu}{m} \sqrt{\frac{2}{3}(t - t_{\min}(m))} \left( s - m_N^2 \right) \frac{f(t)}{t},
\]

\[
A^{\frac{1}{2} \frac{-1}{2} \frac{1}{2} - \frac{1}{2}} = A^{\frac{-1}{2} \frac{1}{2} - \frac{1}{2} \frac{1}{2}} = e^2 \frac{\mu}{m} \sqrt{\frac{2}{3}(t - t_{\min}(m))} \left( s - m_N^2 \right) \left( 1 - \frac{m^2 - m_N^2}{s - m_N^2} \right) \frac{f(t)}{t},
\]

where \( m \) is the isobar mass (an invariant mass of \( \pi N \) or \( \gamma N \) in which the isobar decays), \( t = -(k - k')^2 = -(p' - p)^2 = (k + p)^2 = (k' + p')^2 \), \( p, p', k \) and \( k' \) are the nucleon, isobar, initial and final electron (positron) four-momenta respectively, see Fig. 1, \( f(t) = 1/(1 + 2t)^2 \) is the "dipole" formfactor of the electromagnetic transition \( N \to \Delta \), see, for example, [3,4], and references quoted there. Hereafter \( t \) comes in units of GeV^2.
We ignore the amplitudes $A^{l\lambda}_{\frac{1}{2}\frac{3}{2}}$, $A^{l\lambda}_{\frac{3}{2}\frac{1}{2}}$, $A^{l\lambda}_{\frac{1}{2}\frac{1}{2}}$, and $A^{l\lambda}_{\frac{3}{2}\frac{3}{2}}$, which are proportional to $t$, and contributions in the amplitudes of Eqs. (3) and (4), which are also proportional to $t$. The magnitude of all omitted contributions in the total cross section has the order of 1%.

The differential in $m$ and $t$ cross section of the process $e^+N \rightarrow e^+\Delta \rightarrow e^+\pi N$

\[
\frac{d^2\sigma}{dmdt} = 4\alpha^2 \left( \frac{\mu}{m_N} \right)^2 t - t_{\min}(m) t^2 \frac{1}{t^2} (f(t))^2 \times
\]

\[
\times \left( 1 + \frac{m_N^2}{3m^2} \right) \left[ 1 - \frac{m^2 - m_N^2}{s - m_N^2} + \frac{(m^2 - m_N^2)^2}{2(s - m_N^2)^2} \right] \frac{m^2\Gamma(\Delta \rightarrow \pi N, m)}{|D_\Delta(m)|^2},
\]

(5)

where the isobar propagator and the mass dependent isobar width have the forms

\[
D_\Delta(m) = m^2 - m_\Delta^2 + im\Gamma(\Delta \rightarrow \pi N, m),
\]

\[
\Gamma(\Delta \rightarrow \pi N, m) = \Gamma(\Delta \rightarrow \pi N, m_\Delta) \frac{m_\Delta}{m} \left( \frac{1 + m_N/m}{1 + m_N/m_\Delta} \right)^2 \frac{2(q(m)/q(m_\Delta))^3}{1 + (q(m)/q(m_\Delta))^2},
\]

(6)

$m_\pi = 0.14$ GeV is the pion mass, $q(m_\Delta) = 0.225$ GeV. In Eq. (6) we put $\Gamma_\Delta(m) = \Gamma(\Delta \rightarrow \pi N, m)$, $\Gamma(\Delta \rightarrow \pi N, m_\Delta) = 0.12$ GeV. The $m$ distribution integrated over the interval $t_{\min}(m) \leq t \leq t_{\max}(m)$

\[
\frac{d\sigma}{dm} = \sigma(m) = 4\alpha^2 \left( \frac{\mu}{m_N} \right)^2 \left( \ln \frac{t_{\max}(m) (1 + 2t_{\min}(m))}{t_{\min}(m) (1 + 2t_{\max}(m))} \right) - 1 + \frac{t_{\min}(m)}{t_{\max}(m)} -
\]

\[
- \frac{11 + 24t_{min}(m) + 24t_{min}^2(m)}{6 (1 + 2t_{min}(m))^3} + \frac{11 + 24t_{max}(m) + 24t_{max}^2(m)}{6 (1 + 2t_{max}(m))^3} \times
\]

\[
\times \left( 1 + \frac{m_N^2}{3m^2} \right) \left[ 1 - \frac{m^2 - m_N^2}{s - m_N^2} + \frac{(m^2 - m_N^2)^2}{2(s - m_N^2)^2} \right] \frac{m^2\Gamma(\Delta \rightarrow \pi N, m)}{|D_\Delta(m)|^2},
\]

(7)

where

\[
t_{\max}(m) = -2m_e + \frac{1}{2s} \left\{ (s - m^2 + m_e^2) (s - m_N^2 + m_e^2) +
\]

\[
+ \sqrt{(s - (m - m_e)^2) (s - (m + m_e)^2) (s - (m_N - m_e)^2) (s - (m_N + m_e)^2)} \right\},
\]
\[ t_{\text{min}}(m) = -2m_e + \frac{1}{2s} \left[ (s - m^2 + m_e^2) \left( s - m_N^2 + m_e^2 \right) - \right. \]
\[ \left. -\sqrt{(s - (m - m_e)^2) (s - (m + m_e)^2) (s - (m_N - m_e)^2) (s - (m_N + m_e)^2)} \right] , \tag{8} \]

\( m_e = 0.51 \cdot 10^{-3} \text{ GeV} \) is the electron mass.

At \( s = 1.842 \text{ GeV}^2 \) (the electron energy is equal to \( m_\phi/2 \)) the integrated over all interval \( m_\pi + m_N \leq m \leq \sqrt{s} - m_e \) total cross section \( \sigma = 2.95 \mu \text{b} \).

As large as this magnitude of the cross section is caused by the "large" logarithm in Eq. (8): \( \ln \left( t_{\text{max}}(m_\Delta)/t_{\text{min}}(m_\Delta) \right) = 13.11 \), where \( t_{\text{max}}(m_\Delta) = 0.169 \), \( t_{\text{min}}(m_\Delta) = 1.29 \cdot m_e^2 = 0.336 \cdot 10^{-6} \). The isobar \( \Delta(1.232) \) has no rival in this energy region.

The distributions in the pion energy \( E_\pi \), in the nucleon energy \( E_N \) and in the pion (nucleon) momentum \( q \) have the forms in the isobar rest system

\[ \frac{d\sigma}{dE_\pi} = \sigma(E_\pi) = \frac{m(E_\pi)}{\sqrt{m_N^2 + E^2_\pi - m_\pi^2}} \sigma(m(E_\pi)) , \]

\[ \frac{d\sigma}{dE_N} = \sigma(E_N) = \frac{m(E_N)}{\sqrt{m_\pi^2 + E^2_N - m_N^2}} \sigma(m(E_N)) , \]

\[ \frac{d\sigma}{dq} = \sigma(q) = \frac{qm(q)}{\sqrt{(m_N^2 + q^2)(m_\pi^2 + q^2)}} \sigma(m(q)) , \tag{9} \]

where \( m(E_\pi) = E_\pi + \sqrt{m_N^2 + E^2_\pi - m_\pi^2} \), \( m(E_N) = E_N + \sqrt{m_\pi^2 + E^2_N - m_N^2} \), \( m(q) = \sqrt{m_\pi^2 + q^2} + \sqrt{m_N^2 + q^2} \), \( q(m(q)) = q \).

The distribution \( \sigma(E_\pi) \) is shown in Fig. 2.

The amplitudes from Eqs. (3) and (4) allow us to build up the isobar spin density matrix and the angle distributions of the decay products in the helicity system, that is in the isobar rest system with the quantization direction along the isobar three-momentum in the reaction center mass system.

\(^1\)Note, that the cross section under discussion is equal to 7.74 \( \mu \text{b} \) at the \( c-\tau \)-factories \( (s = 4.644 \text{ GeV}^2) \) and to 11.63 \( \mu \text{b} \) at the \( b \)-factories \( (s = 17.8 \text{ GeV}^2) \).
The integrated over azimuth angle distribution in the $\Delta \to \pi N$ decay

$$
\frac{dW^{\pi(N)}}{d \cos \theta} = \frac{1}{1 + \frac{m_N^2}{3m^2}} \left\{ \frac{3}{4} \left( 1 + \frac{m_N^2}{9m^2} \right) \sin^2 \theta + \frac{m_N^2}{3m^2} \cos^2 \theta \right\},
$$

where $\theta$ is the angle between the pion three-momentum direction (nucleon) and the quantization direction.

At $m = m_\Delta$

$$
\frac{dW^{\pi(N)}}{d \cos \theta} = 0.67 \sin^2 \theta + 0.16 \cos^2 \theta.
$$

Note, that the average of Eq. (10) over $m$ changes the coefficients in Eq. (11) less than by one percent.

In deriving Eq. (10), we used the phenomenological Lagrangian density describing the interaction of the isobar with the nucleon and the pion

$$
L = \frac{G}{m_N} (\bar{\psi}_\nu(x) \psi(x) \partial^\nu \phi(x) + h.c.),
$$

where $\phi(x)$ is the pion field.

In the reaction center mass system, 64% of the isobars are emitted at an angle less than $10^\circ$ with reference to the beam axis, that is why the quantization direction is close to the beam direction. If to take into account that the isobar momentum is not large (the order of 100 MeV) then it is clear that the pion (nucleon) emission largely across the quantization direction, see Eq. (10), causes their emission largely across the beam direction.

In the decay $\Delta^+(1.232) (\Delta^0(1.232))$, there are produced the $\pi^0$-mesons twice as large as the $\pi^+$ ($\pi^-$)-ones. This property can be used to analyze the residual gas composition.

The integrated over azimuth angle distribution in the $\Delta \to \gamma N$ decay

$$
\frac{dW^{\gamma(N)}}{d \cos \vartheta} = \frac{1}{\left( 1 + \frac{m_N^2}{3m^2} \right)^2} \left\{ \frac{1}{4} \left( 1 + \frac{2m_N^2}{m^2} + \frac{m_N^4}{9m^4} \right) \sin^2 \vartheta + \left( 1 + \frac{m_N^4}{9m^4} \right) \cos^2 \vartheta \right\} =
$$

$$
= 0.386 \sin^2 \vartheta + 0.728 \cos^2 \vartheta,
$$

where $\vartheta$ is the angle between the photon (nucleon) three-momentum direction and the quantization direction.
To get the energy and momentum distributions in the process $e^\pm N \rightarrow e^\pm \Delta \rightarrow e^\pm \gamma N$ it needs to substitute

$$
\Gamma(\Delta \rightarrow \pi N, m) \rightarrow \Gamma(\Delta \rightarrow \gamma N, m) =
$$

$$
BR(\Delta \rightarrow \gamma N, m_\Delta) \cdot \Gamma_\Delta(m_\Delta) \frac{m_\Delta}{m} \left( \frac{1 + m_\pi^2/3m^2}{1 + m_\pi^2/3m_\Delta^2} \right) \frac{2\omega(m)^3/\omega(m_\Delta)^3}{1 + \omega(m)^2/\omega(m_\Delta)^2}
$$

(14)

in Eqs. (5) and (7) and $E_\pi \rightarrow \omega$, $q \rightarrow \omega$ and $m_\pi \rightarrow 0$ in Eq. (9), where $\omega$ is the photon energy in the isobar rest system, $\omega(m) = m (1 - m_\pi^2/m^2)/2$.

The expected number of the produced $\Delta$-isobars per unit time and per unit length of a vacuum chamber

$$
N = \frac{2I}{e} n \sigma \left[ \frac{1}{m \cdot s} \right],
$$

where $n$ is a density of nucleons participating in the interaction in the vacuum chamber, $I$ is the electric current in a single beam.

The nucleon density $n$ is determined by the pressure of the residual gas and by its partial composition. The residual gas pressure $p \simeq 3 \cdot 10^{-9}$Torr\footnote{Residual gas pressure}. The partial composition of the residual gas, typically, is 50% of H$_2$, 30% of CO and 20% of CO$_2$\footnote{Partial composition of residual gas}. The molecule density can be evaluated by the formula

$$
p = n_M k T,
$$

where $k$ is the Boltzmann constant, $T = 300$K is the residual gas temperature which is determined by the temperature of the accelerator walls. Then one gets $n_M \simeq 10^{14}\nu{m}^{-3}$. An effective number of the nucleus nucleons, with which the electron (positron) interacts in this process, is equal to $A^{2/3}$, where $A$ is the total nucleon number in a nucleus. With consideration for the partial composition of the residual gas the effective density of the nucleons $n = 7 \cdot 10^{14}\nu{m}^{-3}$.\footnote{Effective density of nucleons.}

So, for the current $I = 1$A and the cross section $\sigma = 3\mu$b, the number of the isobars produced per unit time and per unit length
\[ N = 2 \left[ \frac{\text{events}}{\text{m} \cdot \text{s}} \right]. \]

Although this frequency of the counting is small in comparison with the frequency of the counting from the \( \phi \)-meson resonance \( \sim 1 \text{kHz} \), nevertheless, the events of this process can make the extraction of the rare decays at the \( \phi \)-factories difficult.

At present the study of the process of the \( \Delta \)-isobar electroproduction is beginning in the experiments with the SND detector at the accelerator complex VEPP-2M in Novosibirsk.

This work was partly supported by grants RFBR, 94-02-05 188, 96-02-00 548, and INTAS-94-3986.
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FIGURES

FIG. 1. The diagram describing the isobar electroproduction.

FIG. 2. The pion energy distribution $\sigma (E_\pi)$ in the isobar rest system.
Fig. 1
