On Local Observations in Quantum Gravity

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ABSTRACT

By taking into account both quantum mechanical and general relativistic effects, I derive an equation that describes limitations on the measurability of space-time distances as defined by a material reference system.
I Introduction

The construction of a quantum theory incorporating gravity will certainly require the development of several new tools, also to replace some of those we are accustomed to use within quantum theories in fixed (or classically dynamical) geometry.

Progress in the development of the new tools is possible even before having a fully consistent quantum gravity; most notably, as seen in the investigation of gedanken experiments for the quantum measurement of distances[1-5] and the (semi-classical) quantum analysis of black holes[6], one can try to develop intuition for the nature of this tools by analyzing problems in which the incompatibility between quantum mechanics and (classical) general relativity is more evident.

Some of the familiar tools that we need to relinquish or modify have also been identified; in particular, it is clear that local observables are not easily available in a diffeomorphism-invariant quantum theory. In order to have local observables one can introduce a material reference system (MRS) and include in the analysis the dynamics of the objects that form the reference system[7].

In this paper I combine the elements of two of the types of analyses mentioned above. I consider the quantum measurement of the distances defined by a MRS. For the measurement procedure I follow my analysis of Ref.[4], appropriately adapted to the context at hand, and as MRS I adopt one of those discussed by Rovelli in Ref.[7]. My objective is to provide evidence that the commutation relations satisfied by the local observables defined by such MRSs could be very different from the ones satisfied by the local observables of ordinary quantum theories in fixed geometry. This evidence shall be encoded in the structure of a lower bound on the uncertainty present in the quantum measurement under investigation.

II Nature of the Uncertainties

Let me start by reviewing the role of the MRSs. The difficulties in introducing local observables in quantum (and even classical) gravity originate from the fact that diffeomorphism invariance washes away[7] the individuality of the points of the universe manifold. This individuality can be regained by specifying points by means of some matter, i.e. introducing a MRS, but then the analysis has to take into account the dynamics of this matter.

MRSs have appeared in one form or another (such as dust, fluids, etc.) rather frequently in the literature. I follow Rovelli’s discussion[7]; specifically, for simplicity, I consider a MRS composed of identical bodies (defining space points) with some internal physical variables defining time instants, which is one of the MRSs considered in Ref.[7], and discuss the quantum measurement of the distance between (the respective centers of mass of) two of such bodies.

As discussed in Refs.[1, 3, 4], this type of measurement is naturally carried out by exchanging a light signal between the two bodies. For conceptual simplicity, let me take one of the two bodies to be a clock (after all, a clock is a body with some internal physical variables defining time instants). The measurement could then be performed by “attaching” non-rigidly a “light-gun” (i.e., a device capable of sending a light signal when triggered) and a detector to the clock, and “attaching” a mirror to the other body. The system could be set up so that a light signal be sent toward

*The reader can easily realize that rigid attachment is not consistent with the causal nature of the theory.
the mirror when the clock reads the time $T_i$, and to record the time $T_f$ shown by
the clock when the light signal is detected by the detector after being reflected by
the mirror. Clearly the time $T = T_f - T_i$ is related to the distance $L$; for example,
in Minkowski space and neglecting quantum effects one simply finds that $L = c\frac{T}{2}$,
with $c$ denoting the speed of light. I am interested in including quantum mechanical
and general relativistic effects in the analysis of this measurement procedure, and
therefore the relation between $T$ and $L$ is more complex. The “actual” distance $L$
and the outcome $T$ (time read by the clock) of a measurement procedure, are related
as follows\cite{4}

$$L = c\frac{T}{2} \pm \delta L + \Delta L \pm \delta g L , \label{eq1}$$

where $\delta L$ is the total quantum mechanical uncertainty due to the quantum mechanical
uncertainties in the position and velocity of the various agents in the measurement
procedure (as discussed in Refs.\cite{1,3}, contributions to $\delta L$ come, for example,
from the spread in the position of the various devices during the time $T$), $\Delta L$ is the
total correction due to the gravitational forces among the agents in the measurement
procedure (for example, one such correction results from the gravitational attraction
between the photons composing a light signal and the devices in the apparatus), $\delta g L$
is the total quantum mechanical uncertainty which results from the uncertainties in
the gravitational forces among the agents in the measurement procedure (for example,
as a result of the quantum mechanical spread in the mass of the clock, there is an
uncertainty in the strength of the gravitational attraction exerted by the clock on the
photons composing the light signal).

Concerning $\Delta L$, let me point out that, as observed in Ref.\cite{4}, this contribution to
Eq.\cite{1} does not play any role in my study because in a context in which both general
relativistic and quantum mechanical effects are taken into account, $\Delta L$ represents a
correction, not an uncertainty. In fact, $\Delta L$ can be calculated and taken into account
in the analysis of the outcome of the measurement, therefore leading to no additional
uncertainty.

I intend to investigate the limitations on the accuracy of quantum measurements
of $L$ resulting from the fact that it is not possible to render arbitrarily small the
uncertainty introduced in the measurement of $L$ by the presence of $\delta L$ and $\delta g L$.
$\delta g L$ has already been considered in Refs.\cite{2,5}, and it has been found that $\delta g L \geq L_p$,
$L_p$ being the Plank length.

Concerning $\delta L$ I shall not attempt to derive the absolute lower bound (whose rigorous
derivation surely involves an extremely complex analysis) on the uncertainty of
quantum measurements of $L$, instead I shall look for a lower bound (which may well
be quite lower than the absolute lower bound) for this uncertainty. Consistently with
this objective, the only contribution to $\delta L$ that I will consider is $\delta x^{rel}$, defined as the
uncertainty introduced by the spread in the relative position between the center of
mass of the clock and the center of mass of the system composed by light-gun and
detector during the time $T$. The other contributions to $\delta L$ (given, among several oth-
ers, by the spread in the position of the light-gun, and detector with respect to their
center of mass, and by the spread in the relative position between the mirror and the
second body) could obviously only increase the lower bound on the uncertainty that
I will present. As done in Ref.\cite{3,4}, I give an intuitive and simple discussion rather
than attempting to find a stronger bound. By looking for a lower bound $\min\{\delta x^{rel}\}$
for $\delta x^{rel}$, I shall get a lower bound for the uncertainty $\delta L^{tot}$ in the length measurement under consideration, as given by the relation

$$\delta L^{tot} \equiv \delta L + \delta g L \geq \delta x^{rel} + L_p \geq \min\{\delta x^{rel}\} + L_p .$$  \hspace{1cm} (2)

In the following, in order to be able to use spherical symmetry, I shall also assume that the bodies composing the material reference system (including our clock) are spherical with radius $s$ and homogeneously-distributed mass $M$.

## III Derivation of the Bound

Ignoring for the moment the gravitational effects, the evaluation of the spread in the relative position between the center of mass of the clock and the center of mass of the system composed by light-gun and detector during the time interval $[T_i, T_i + T]$ is a simple quantum mechanical problem. Following Ref.\[1, 4, 3\], one finds that

$$\delta x^{rel} \equiv \delta x^{rel}(t \in [T_i, T_i + T]) \geq \delta x^{rel}_{t=T_i} + \frac{\hbar T}{\mu \delta x^{rel}_{t=T_i}} \sim \delta x^{rel}_{t=T_i} + \frac{\hbar}{c \mu} \frac{2L}{\delta x^{rel}_{t=T_i}} ,$$  \hspace{1cm} (3)

where $\delta x^{rel}_{t=T_i}$ is the initial (i.e. at the time $t = T_i$ when the light signal is emitted) spread, and $\mu$ is the relative mass, related to $M$, the mass of the clock, and $M_a$, the total mass of the apparatus composed of light-gun and detector, by the usual relation

$$\mu \equiv \frac{MM_a}{M + M_a} .$$  \hspace{1cm} (4)

(Also note that on the right-hand-side of Eq.(3) I used the fact that in first approximation $T \sim 2L/c$.)

Eq.(3) can be understood as follows\[1, 3, 4\]. Initially, the wave packet has relative-position spread $\delta x^{rel}_{t=T_i}$ and relative-velocity spread $\delta v^{rel}_{t=T_i}$. During the time interval $[T_i, T_i + T]$ the uncertainty in the relative position is given by

$$\delta x^{rel} \sim \delta x^{rel}_{t=T_i} + \delta v^{rel}_{t=T_i} T \sim \delta x^{rel}_{t=T_i} + \delta v^{rel}_{t=T_i} \frac{2L}{c} .$$  \hspace{1cm} (5)

Eq.(5) reproduces Eq.(3) once one takes into account the uncertainty principle, which states that

$$\delta x^{rel}_{t=T_i} \delta v^{rel}_{t=T_i} \geq \frac{\hbar}{\mu} .$$  \hspace{1cm} (6)

The most important feature of Eq.(3) is that it indicates that, with fixed masses $M$ and $M_a$, there is no way to prepare the $t = T_i$-wave-packet so that $\delta x^{rel} = 0$. In fact, Eq.(3) indicates that quantum mechanics leads to the following minimum value of $\delta x^{rel}$

$$\min\{\delta x^{rel}\} \sim \sqrt{\frac{\hbar T}{\mu}} \sim \sqrt{\frac{\hbar L}{c \mu}} ,$$  \hspace{1cm} (7)
where, as I shall continue to do in the following, I neglected numerical factors of $O(1)$, which are essentially irrelevant for the discussion presented in this paper.

Up to this point I have only used quantum mechanics and therefore it is not surprising to discover that, as shown by Eq. (7), $\min \{ \delta x_{rel} \} \to 0$ as $\mu \to \infty$. Indeed, the uncertainty that I am considering originates from the uncertainty in the kinematics of the bodies involved in the measurement procedure, and clearly this uncertainty vanishes in the limit of infinite masses (the classical limit) because in this limit Eq. (6) is consistent with $\delta x_{rel} = \delta v_{rel} = 0$.

However, the central observation of the present paper is that this scenario is significantly modified when the general relativistic effects relevant to our experimental set up are taken into account.

It is important to realize that, with fixed radius $s$ for our spherical clock, large values of the mass $M$ necessarily lead to great distorsions of the geometry, and well before the $M \to \infty$ limit (which is desirable for reducing the uncertainty given by Eq. (7), since $\mu \to \infty$ requires $M \to \infty$) our measurement procedure can no longer be followed. In particular, if $M \geq \frac{\hbar}{c} \frac{s}{L_p}$ an horizon forms around the center of mass of the clock and it is not possible† to have a light signal emitted at $t = T_i$ reaching the mirror positioned on the other body whose distance from the clock is being measured; therefore the condition

$$M \leq \frac{\hbar}{c} \frac{s}{L_p},$$

is necessary to perform the measurement.

Since $\mu \leq M/2$ (see Eq. (4)), Eqs. (7) and (8) combine to give

$$\min \{ \delta x_{rel} \} \sim \sqrt{cT L_p^2} \sim \sqrt{LL_p^2}.$$  

In turn, this can be combined with Eq. (2) to finally obtain a lower bound on the uncertainty in the measurement of the distance $L$

$$\delta L_{tot} \geq \min \{ \delta x_{rel} \} + L_p \sim \sqrt{\frac{cT L_p^2}{s}} + L_p \sim \sqrt{\frac{LL_p^2}{s}} + L_p.$$  

Notably $\min \{ \delta x_{rel} \}$ has introduced an $s$- and $L$- dependent contribution to the lower bound, and actually this contribution is larger‡ than the one previously identified; in

†If the light-gun is within the clock’s horizon then its light signals cannot reach the mirror. Its light signals can reach the mirror if instead the light-gun is outside the clock’s horizon; however, in that case it is still impossible to perform the measurement since the light-gun could not “read” the clock, and therefore could not be triggered by the clock.

‡Notice that in this context the $s \to \infty$ limit is meaningless, since it is not possible (not even conceptually) to set up the network of bodies necessary to form the MRS if each one of the bodies occupies all of space. Actually, within a MRS formed by bodies of size $s$ it only makes sense to consider distances $L$ greater than $s$, and, in order to have as fine as possible a network of bodies, one would like $s$ to be small, leading to large uncertainty in length measurements. Moreover, Rovelli’s observables are defined only with respect to a given MRS, i.e. in the case I considered they are characterized by a specific value of $s$, and therefore increasing $s$ one would not reach better and better accuracy in the measurement of the same observable, one would instead span a class of different observables which “live” in different MRSs, and are measurable with different accuracy depending on their specific value of $s$ (the value of $s$ of the MRS they “live” in).
fact, by construction of the MRS here considered, the distance between the centers of mass of two of the bodies is always larger than $s$, so that $\min\{\delta x^{\text{rel}}\} > L_p$.

IV Discussion and Outlook

In this paper, upon appropriate redefinitions and adaptations, I succeeded in following, within the specific framework of one of the MRSs considered in Ref. [7], all the steps of the measurement analysis presented in the more general/intuitive discussion of Ref. [4]. This has led to the $s$- and $L$- dependent lower bound (10) on the uncertainty in the measurement of a distance $L$ defined by a MRS characterized by the scale $s$. This has obvious physical interest on its own, and can be interpreted as evidence for the need of nonconventional commutation relations to be enforced on the quantum gravity local observables defined by MRSs.

There is one viewpoint from which part of this result is not surprising; in fact, it is nearly always the case that physics depends nontrivially on all the available scales, and therefore, one could have considered the possibility that the physics “seen” by a MRS might be dependent on the scales characterizing the MRS even without the analysis I presented. My analysis provides some intuition on how such dependence might come about.

I also hope that my result encourages (and gives hints to) future work on nonconventional commutation relations for local observables of quantum gravity. As a first step toward the introduction of bounds of the type (10) in candidate quantum gravity theories, I expect that it could be useful to consider the special case $s \sim L_p$ in which Eq. (10) takes the form

$$\delta L_{\text{tot}} \geq \min\{\delta x^{\text{rel}}\} + L_p \sim \sqrt{cT L_p} + L_p \sim \sqrt{L L_p} + L_p.$$  \hspace{1cm} (11)

Work is in progress investigating the possibility that relations of the type (11) might naturally arise in certain formulations of non-critical string theory.

A $L$-dependent (i.e. dependent on the time required to complete the measurement procedure) lower bound on the uncertainty in the measurement of the distance $L$ could also be important, as observed in Refs. [3, 8], in relations to possible mechanisms for (de)coherence in quantum gravity. I leave further investigation of this issue for future studies.

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