Three-Dimensional Fourier Scattering Transform and Classification of Hyperspectral Images

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Abstract—Recent research has resulted in many new techniques that are able to capture the special properties of hyperspectral data for hyperspectral image analysis, with hyperspectral image classification as one of the most active tasks. Time-frequency methods decompose spectra into multi-spectral bands, while hierarchical methods like neural networks incorporate spatial information across scales and model multiple levels of dependencies between spectral features. The Fourier scattering transform is an amalgamation of time-frequency representations with neural network architectures, both of which have recently been proven to provide significant advances in spectral-spatial classification. We test the proposed three dimensional Fourier scattering method on four standard hyperspectral datasets, and present results that indicate that the Fourier scattering transform is highly effective at representing spectral data when compared with other state-of-the-art spectral-spatial classification methods.

Index Terms—Scattering transform, Fourier scattering transform, hyperspectral image (HSI), supervised classification, convolutional neural networks

I. INTRODUCTION

Hyperspectral image sensors routinely collect hundreds of bands of different wavelength channels of the surface of the Earth [1]. Due to the rapidly growing amount of available hyperspectral image (HSI) data [2], there is much interest in the development of algorithms that automatically classify the pixels of a hyperspectral image. However several characteristics of HSI data make this task challenging: the high-dimensionality of the data, the low spatial resolution which results in unfavorable signal-to-noise ratio, and that labeled data is scarce and typically not transferable across different datasets due to e.g., location and weather effects.

As the review by He et al. [3] noted, neural networks (NNs) and deep learning, having achieved breakthroughs in image classification, is starting to be applied to hyperspectral image processing [4], [5], [6], [7], [8], to some advantages and disadvantages. The hierarchical network for feature extraction at multiple levels has a potential to produce highly informative features for classification, but there is a great number of parameters to train which results in long training times. At the same time, time-frequency analysis for HSI has recently proven to provide both meaningful and high quality results [9], [3], [10], especially when the filters are specifically designed for HSI data [11]. These time-frequency tools are an underutilized tool when compared with the more popular wavelet based methods [12], [13], [14], [15], [16], [17].

The Fourier scattering transform (FST), introduced in [18], [19], can be viewed as a modern time-frequency approach to machine learning. It unifies deep learning architectures with time-frequency generated filters in order to capture higher order correlations between time-frequency representations. The FST uses fixed filters instead of adaptable or learned ones such as those in NNs, but the FST does not require computationally expensive training and enjoys theoretical guarantees that NNs lack. In particular, the FST automatically removes small diffeomorphic nonlinearities or perturbations such as noise, which are typically irrelevant in classification tasks. The three-dimensional Fourier scattering transform (3D FST) is a special case of the FST which is obtained by employing three dimensional time-frequency generated filters. When used on HSI data, it provides a multi-layer spatial-spectral decomposition. We argue that the spectra generated by standard material classes are more discriminable in the time-frequency domain compared to other representations. This argument was also made in [11], which showed that decomposing the signal using time-frequency filters provides informative features for HSI data. However, the 3D FST further refines this idea by integrating together the spectral and spatial information, and incorporating them into a multi-layer setting, where deeper layers capture more complex features.

We demonstrate that the 3D FST provides state-of-the-art performance on HSI data. We compare to results with various neural network, wavelet, and scattering based methods [12], [5], [20], [11], [21], [8], [7], [16], [15], [17], [22], [23], [17]. One notable method that we carefully compare with is the three-dimensional wavelet scattering transform (3D WST). The wavelet scattering transform (WST) was originally developed by Mallat [24] and the 3D WST was applied to HSI classification in [17]. Our results show that time-frequency Fourier features are more suitable than time-scale wavelet features for HSI discrimination and classification purposes. We obtain state of the art results on the Indian Pines, Pavia University, and Pavia Center datasets at 10%, 10%, and 1% of training data respectively. We also provide an open
source implementation of all code used for our algorithms and experiments.

The rest of this paper is organized as follows. Section II-A reviews related work on using neural networks, wavelets, and time-frequency bases for feature extraction and the classification of HSI data. Section II-B provides background information on scattering transforms. Section III defines the 3D FST and how it provides a joint spectral-spatial representation suited for HSI data. Section IV introduces the datasets that 3D FST is evaluated on, explains the parameter choices in the 3D FST, and discusses the results on these datasets while comparing them to other competing methods. Section V concludes the paper.

II. RELATED WORK

A. Previous Work

Many methods in the literature have concentrated solely on the spectrum for the classification of HSI data. More recently, to improve classification performance spectral-spatial techniques which better exploit the properties of HSI data have become popular. Our review of neural network, wavelet, and time-frequency methods can be roughly split into four categories: 1) purely spectral techniques 2) spectral methods that incorporate spatial pre/post processing, 3) purely spatial methods that may include spectral pre/post processing, and 4) those that integrate spectral-spatial information at once.

1) Pixelwise methods that extract wavelet, time-frequency, and neural network features solely in the spectral domain have been developed to address these challenges in HSI classification [25], [21], [26], [20], [27], [23]. The neural network (NN) family of methods iteratively composes layers of matrix multiplications or linear convolutions with a pointwise non-linear function. The weights in these matrices or convolution filters are adapted using backpropagation during an initial training phase. NNs consisting of 1D convolutions with spectra [21], and 2D convolutions ofreshaped 1D spectra vectors [26], as well as other Deep Belief Networks (DBN) [23] have been evaluated. Another family of methods are the wavelet and time-frequency methods which use a pre-determined basis to extract edge like features. 1D Morlet wavelet features with trainable scale and translation parameters have been extracted and train to a 2 layer NN [25], [28]. A compromise between the learned and potentially deep and complex neural networks and classical wavelet features are scattering transforms, which are particular types of operators introduced by Mallat [24] whose coefficients are computed with a hierarchical network structure that captures invariances in data. 1D Fourier scattering features coupled with an SVM have also proved to be effective, outperforming 1D wavelet scattering features [27]. These purely spectral methods improve upon the performance of simpler machine learning algorithms, like the application of SVMs on the 1D spectra of each pixel, but ignore the significant spatial structure present in HSI data.

2) A variety of methods have successfully used spatial information in pre-processing steps (more rarely post-processing as well) to improve classification performance while still focusing on the spectral aspects of the data [7], [29], [4], [30], [31]. Ashitha et al. [31] classify 1D wavelet scattering features with an SVM after smoothing each channel of the HSI with 2D Gaussian filters. Sandwiching a spectral NN between two 2D Gaussian blur layers with trainable variance greatly improves performance and remains one of the most competitive HSI methods [7]. Lee et al. followed up on this work a deep spectral NN with residual connections following a single 3D filter layer [29]. Acquarelli et al. made changes instead to the training process and included a spatial term in the regularizer of a purely spectral 1D convolutional neural network (CNN) [4].

3) Spatial methods that include some spectral preprocessing have also been applied to HSI data [22], [8], [5], [13], [27], [6]. Classical 2D CNNs and Recurrent CNNs (RCNN) have been used on each channel of HSI input independently [22]. Also popular has been using PCA or other dimensionality reduction methods to reduce the number of channels in the the HSI before using a 2D CNN [8], [5], or 2D wavelet scattering [13]. Our previous work performed competitively using 1D Fourier scattering preprocessing followed by 2D wavelet scattering [27]. Recently Deng et al. used a new CapsNet NN architecture [32] with 2D filters on each channel independently to achieve competitive results.

4) Integrated spectral-spatial methods combine information from spectral signatures and spatial neighborhoods simultaneously, and are also common. Some methods consider sequences of 1D spectra [20] (for example in variety of NN called Long Short Term Memory or LSTM) or flatten the HSI cube to a matrix and use 2D methods [33], but by far the most popular methods involve building 3D filters [22], [8], [15], [10], [14], [11], [10], [12], [17], [9]. 3D convolutional layers in NNs, CNNs, and RCNNs, both shallow and deep have been evaluated [5], [22]. But the lack of training data challenges models with many learnable parameters, and these networks struggled in comparison to methods with predetermined filters such as the 3D Gabor wavelets that Shen and Jia et al. used to extract features [15], [10], [14], and classify with a variety of algorithms, for instance a sparse representation based classification (3D WT+SRC) [15]. Bau et al. used the real part of 3D Gabor filters sampled densely in the time-frequency domain to get features used with a Mahalanobis distance classifier. He et al. decomposed the same filter into 8 subsifters, using only 3 to construct a discriminative low-rank Gabor mother filter (DLRGBF) used to extract features, a hand designed feature which proved to be very competitive with a least squares based classifier (3D DLRGF+LS). Qian and Cao et al. used a Haar 3D wavelet filter bank (3D DWT-FB) and discrete wavelet transform (3D DWT) with various classifiers. Tang et al. [17] proposed a 3D Gabor wavelet scat-
tering approach to extract features (3D WST), which decomposes the HSI across multiple wavelet scales and orientations and uses local averaging to keep class labels consistent in neighborhoods, and classified with a radial basis function SVM (3D WST+RBF-SVM).

The paradigm that we have ordered our review by has been the degree to which each method integrates spectral-spatial information, which greatly affects classification performance. Another characteristic to distinguish HSI methods worth mentioning is the amount of dependency or interaction between the spectral-spatial features that each technique models, as [3] points out.

In the terminology of [3], the simplest dependency system is the case where features are extracted directly from the HSI data. This encompasses the majority of the methods we presented. However neural networks with multiple layers naturally model a hierarchical interaction of features, with as many degrees of interaction as number of layers in the network: [6], [20], [5], [4], [26], [23], [29], [22], [8], [21]. The same can be said for the layers of scattering networks: [17], [63], [13], [11], [27]. This distinguishes these two classes of approaches from purely wavelet [34], [12], [15], [16] or time-frequency methods [11], [9], [3], [10], and yields more sophisticated features that yield better classification results, as we show in Section IV.

B. Fourier Scattering Transform

We begin by formally defining the scattering transform. Fix a sequence $\Phi = \{\phi, \phi_\lambda\}_{\lambda \in \Lambda}$ of square integrable functions on $\mathbb{R}^d$, where $\Lambda$ is the index set of the sequence. Given an input function $f$ defined on $\mathbb{R}^d$, we iteratively convolve it with this sequence and take the modulus in the following way. For each index $\lambda \in \Lambda$, let

$$U[\lambda](f) = |f * \phi_\lambda|,$$

where $*$ is the convolution of functions on $\mathbb{R}^d$. We can extend this rule to multi-indices. For each $\lambda = (\lambda_1, \ldots, \lambda_k) \in \Lambda^k$, let

$$U[\lambda](f) = U[\lambda_k] \cdots U[\lambda_1](f).$$

The scattering transform $S_\Phi$ associated with $\Phi$ is formally defined as the sequence of functions

$$S_\Phi(f) = \{f * \phi, U[\lambda](f) * \phi\}_{\lambda \in \Lambda^k, k \geq 1}.$$

See Figure 1 for a visualization of the scattering transform as a convolutional neural network.

The mathematical properties of the scattering transform and the features that it generates greatly depend on the underlying sequence of functions. Mallat [24] and his collaborators [35] primarily considered the wavelet (time-scale) case, where $\phi$ is the father wavelet and $\{\phi_\lambda\}_{\lambda \in \Lambda}$ are dilations of the mother wavelet function. The resulting transform is called the wavelet scattering transform (WST) and it provides a powerful multi-scale representation [35]. In contrast, two authors of this paper studied the time-frequency analogue [13], [19], where $\phi$ is a band-limited function and $\{\phi_\lambda\}$ are modulations of $\phi$. The resulting transformation is called the Fourier scattering transform (FST) and it provides a hierarchical time-frequency representation of the data.

Although wavelet based techniques have recently dominated the field of HSI analysis due to their overall impact on image processing, see e.g., [25], [15], [10], time-frequency methods form a natural foundation for spectral data exploration. They were the basis for the early Fourier transform imaging spectroscopy methods [36], [37], as well as for recent attempts to analyze hyperspectral imagery [11].

The wavelet and Fourier scattering transforms provide entirely different representations: the WST computes localization and scale characteristics, whereas the FST provides frequency distribution information. Nonetheless, and perhaps surprisingly, both transformations satisfy similar properties: they are energy preserving, are non-expansive, and contract sufficiently small translations and diffeomorphisms, see the theorems in [24], [18] for precise estimates. These properties explain why they are effective feature extractors, since small perturbations typically have no effect on a data point’s label. In the context of hyper-spectral image classification, the spectra of similar components have common characteristics up to small deviations, which are then removed by either the FST or WST.

A scattering transform has an infinite number of layers and each node has infinitely many children. Thus, we can only compute a finite subset of the coefficients, but not all scattering transforms can be truncated in a faithful way. Thankfully, the FST can be truncated without destroying its properties. Indeed, theoretical results guarantee that the total energy contained in the $k$-th order FST coefficients is at most $\varepsilon^{k-1}$ of the original energy of $f$ for some small $\varepsilon \in (0, 1)$, see [18].

III. Methodology

The FST is a generic transformation that is suitable for many applications and purposes. To differentiate between the generic FST with the particular kind that we use in this paper for hyperspectral image classification, we shall call the latter as the three-dimensional Fourier scattering transform (3D FST), which we describe below.

In the context of HSI data, we have $d = 3$ and we view a HSI as a function $f$ defined on a rectangular subset of $\mathbb{Z}^3$. That
Hence, the first order 3D FST coefficients carry information about a spatially-averaged short-time Fourier transform of $f$. While naive local averaging improves stability to small deformations, it also removes a significant amount of high-frequency information because $g$ is a low-pass function. The lost components are included in the functions, $U_m(f) * g_n$. However, these functions suffer from the same instability properties as $U_m(f)$. The second order intermediate 3D FST coefficients are

$$U_{m,n}(f)(x,y,b) = [(U_m(f) * g'_n)(x,y,P'b)].$$

These intermediate coefficients are also unstable to small diffeomorphisms, so we perform a local averaging. The second order 3D FST coefficients are

$$S_{m,n}(f)(x,y,b) = (U_{m,n}(f) * g''b)(x,y,P''b).$$

We also note that theoretical results in [18] guarantee that $S_{m,n}(f)$ is small when $n \geq m$, so we can improve the computational efficiency of the algorithm by only computing the coefficients for which $n \neq m$.

The zero and first order 3D FST coefficients can be interpreted as spatially-smoothed versions of classical spectral-spatial representations. It is not as obvious what the second order coefficients represent. At first glance, the second order 3D FST coefficients appear similar to the Mel-frequency cepstral coefficients (MFCCs), but there is an important distinction. The MFCCs are calculated by fixing the spatial coordinate and then further decomposing the spectrogram along the frequency axis in log scale. MFCCs play an important role in audio analysis because more global characteristics, which are not captured by the spectrogram, contain important information.

In contrast to the MFCCs, the second order 3D FST coefficients are calculated by fixing the spectral variable and then further decomposing along the spatial coordinate. That is, $U_{m,n}(f)$ describes whether the $m$-th frequency of $f$ over intervals of length $M$ (a local property captured by the first order coefficients) varies at frequency $n$ over intervals of length $MM'$ (a more global property).

In summary, given a hyperspectral image $f$, the features generated by the 3D FST at location $(x,y)$ are the collection of vectors

- **Zero order:** $S_0(f)(x,y,\cdot)$
- **First order:** $\{S_0(f)(x,y,\cdot)\}_{m \in \Lambda_M}$
- **Second order:** $\{S_{m,n}(f)(x,y,\cdot)\}_{m \in \Lambda_M, n \in \Lambda_M'}$.

These vectors are concatenated to form a feature vector for each pixel $(x,y)$ of the hyperspectral image $f$.

### IV. Experiments and Results

#### A. Data Sets

We test the performance of these feature extractors on the following hyper-spectral databases:

- **Indian Pines (IP)** acquired over the Indian Pines test site in Northwestern Indiana in 1992 by the Airborne Visible / Infrared Imaging Spectrometer (AVIRIS) sensor.
- **Pavia University (PaviaU)** acquired during a 2001 flight campaign over Pavia, northern Italy, using the reflective optics system imaging spectrometer (ROSIS) sensor.
Botswana acquired over the Okavango Delta, Botswana in 2001, by the Hyperion sensor on the NASA EO-1 satellite [42].

Pavia Center (PaviaC-R) acquired in 2001 over Pavia, northern Italy, using the ROSIS sensor [43]. We use only the right half of the Pavia Center dataset as it contains the majority of the labeled pixels.

Smith Island captured by a Airborne Hyperspectral Scanner (HyMAP) on in 2000 over Smith Island, VA, a barrier island in the Virginia Coast Reserve. [44]

Table I shows additional information on all five datasets. The ground truth and labels for the five datasets are in Figs. 2 to 6. Datasets 1-5 can be downloaded from the webpage [45].

B. Experimental Setup

When using the Fourier scattering transform on HSI data, there are several parameter choices that impact the effectiveness of the method. Recall that \( g, g', g'' \) are the window functions used in each layer of the 3D FST, where \( M, M', M'' \) are the size of their supports, and \( P, P', P'' \) are the down-sampling parameters. For simplicity, in most of our experiments, we set \( M = M' = M'' = (11, 11, 3) \) and downsample by 3 in the spectral dimension.

PaviaU and PaviaC-R. We set \( M = M' = M'' = (7, 7, 7) \) and downsample by 3 in the spectral dimension.

Botswana. For the first layer we set \( M = (11, 11, 3) \) and we do not downsample. Otherwise we set \( M' = M'' = (11, 11, 8) \) and we downsample by 8 in the spectral dimension.

Smith Island. We set \( M = M' = M'' = (9, 9, 5) \) and downsample by 3 in the spectral dimension.

To make a fair and consistent comparison, we use the same linear support vector machine (SVM) as the classifier for each feature extractor and dataset. We train a SVM on a fraction of the labeled data and use the remainder for testing purposes. SVM aims to optimally separate the labeled data points into two disjoint classes and then classify the remaining ones according to this boundary. The SVM is implemented in Scikit-learn with default parameters.

We also evaluate using filters determined by the 3D WST by Tang et al. [17]. We implement the same feature extraction method and use the parameters recommended by Tang et al. There, the optimal parameters for 3D WST were determined to be filters of size \( 7 \times 7 \times 7 \) for two layers, with 9 orientations and 3 scales per layer. Their work also uses scale increasing paths analogous to frequency decreasing paths.

C. Analysis of Computational Cost

In Table II is the runtime performance of 3D FST and 3D WST. In our implementation we classified one pixel at a time, so the feature extraction performance is simply the number of pixels our scattering network could process at one time. In the classification setting, after all the labelled pixels were processed, a linear SVM was trained, and then the test samples were classified. The SVM performance numbers in Table III are computed from the test SVM, and the training was always faster than the testing of the SVM. The major factor in performance was not the filter size directly, but the number of filters used in total. In our experiments the number of filters per layer was the product of the size of the supports of the filters in each dimension, for example in the first layer the number of filters was \( M_1 \times M_2 \times M_3 \). For example after the first layer, our Botswana Fourier scattering network had 3 times as many filters as our Pavia network, and we see that it took 3 times as long to compute the features per pixel. Since the SVM did not take much time total, we saw no need to run PCA or any other form of dimension reduction reduction between feature extraction and classification, though it could have reduced memory usage by throwing away some coefficients. All our networks were designed so that the features for all the labelled pixels of each dataset could fit on a NVidia Titan X GPU with 12 GB of memory.

D. Discussion of Results

The accuracy tables for the feature extractors and datasets are displayed in Tables III to VII. The numbers were obtained by averaging 10 trials and the numbers in parentheses indicate the standard deviation of the results for these trials. We choose approximately 10%, 10%, 5%, 1%, and 5% of the labeled pixels, uniformly at random, in Indian Pines, Pavia University, Botswana, Pavia Center, and Smith Island, respectively, as training data for the linear SVM. Since we compare our method to the 3D WST method introduced in [17], we use their same dataset for PaviaU and Botswana, and for Indian Pines we use a dataset from [7] which has a slightly fewer number of training samples for every class. 3D WST with a linear SVM performs within a margin of error with 3D WST with a RBF-SVM, as reported by Tang et al. [17]: 95.98 (±0.46) vs 94.46 (±0.79) OA on Indian Pines at 10%, 99.22 (±0.10) vs 99.30 (±0.12) OA on PaviaU at 10%, and 96.98 (±1.06) vs 97.57 (±1.25) OA on Botswana at 5% for 3D WST vs 3D WST+RBF-SVM. We use a less powerful linear SVM with no hyperparameters tuned throughout when we report and spend no time on classifier specific model validation, for brevity we abbreviate with 3D FST and 3D WST instead of 3D FST+SV and 3D WST+SV. The classification maps for 3D FST and 3D WST are in Figs. 2 to 6. A comparison of our methods to the results in the literature for Indian Pines and PaviaU are in Tables VIII and IX.

The relatively uniform and dense geometric distribution of the ground truth for Indian Pines, PaviaU, and PaviaC, makes it easy to see the tradeoffs between time-scale and time-frequency scattering when looking at the classification maps, in addition to the per class accuracies. On these three datasets, the only disagreements appear on the edges of class shapes. In particular, on PaviaU 3D FST improves upon the accuracy of every class. It smooths and corrects sensible mistakes that
TABLE I

| Name        | Satellite | No. Bands | Bandwidth        | Meters per Pixel | Dimensions HxW | No. Labeled Pixels | No. Classes |
|-------------|-----------|-----------|------------------|------------------|----------------|-------------------|-------------|
| PaviaU      | ROSIS     | 103       | 430-860 nm       | 1.3 m            | 610x340        | 42776             | 9           |
| PaviaC-R    | ROSIS     | 102       | 430-860 nm       | 1.3 m            | 1096x492       | 103539            | 9           |
| IP          | AVIRIS    | 200       | 400-2500 nm      | 3.7 m            | 145x145        | 10249             | 16          |
| Smith Island| HyMAP     | 117       | 445-2486 nm      | 4.5 m            | 679x944        | 2743              | 22          |
| Botswana    | NASA EO-1 | 145       | 400-2500 nm      | 30 m             | 1476x256       | 3248              | 14          |

TABLE 1

ATTRIBUTES OF THE DATASETS USED.

Fig. 2. Indian Pines (a) False color image and (b) Ground Truth Labels. Classification results for (c) 3D WST and (d) 3D FST. (e) Class Labels.

Fig. 3. Pavia University (a) False color image and (b) Ground Truth Labels. Classification results for (c) 3D WST and (d) 3D FST. (e) Class Labels.

On the other hand, because of sparse and compact structure of the ground truth it is difficult to gain valuable geometric insight on the per class accuracies of Botswana and Smith Island. For Smith Island, both 3D FST and 3D WST give reasonable seeming estimates with shapes from the false color image being recognizable in the classification maps. For Botswana the structure of the ground truth besides the water is especially hard to tell from the false color image, and both 3D FST and 3D WST give abstract results that look appealing but have a different amount of smoothness.

This spatial smoothing is largely influenced by the choices of \( M, M', M'' \). The choice of these frequency/support parameters is what determines the size of the spatial neighborhood from which spectral-spatial features will be extracted for a single pixel. For example, for PaviaU the window input into the scattering network was 19, since each level of filters were size 7. This is not dissimilar to other 3D methods in the

3D WST makes along the edges: confusing gravel and bricks, and asphalt for both. For PaviaC, 3D FST and 3D WST differ slightly on classification of bricks and soil and asphalt and bitumen. On Indian Pines in general 3D FST corrects the edges that 3D WST makes mistakes on. 3D WST also makes a large portion of soybean notill for corn notill that 3D FST corrects. There is spatial smoothing present in both 3D WST and 3D FST.
literature. The choice is the same in 3D WST [17], and 3D CNNs used input windows of spatial size 13 [22], 27 and 29 [5].

In Indian Pines the median number of samples per class when 10% of data is used for training is 47. In this dearth of training data, 3D methods that require no training for feature extraction by far outperform multiple 3D CNN methods which require much more training data to reach the same OA [5]. [22]. However as seen in Table VIII there are still some trainable methods that are very competitive at 10% training data, though they are less competitive at 5% [1], [8]. The 3D methods in the table outperformed less spectral-spatial methods [4], [13], [21], [29]. Overall, the most powerful methods are the two time-frequency methods. Our method and He et al. ’s method [11] are within a margin of error of each other. Though [11] reports that 3D DLRGF+LS is their best method, they also include a classification performance for 3D DLRGF+SVM of 96.29 (± 0.96) which is higher than that for 3D DLRGF+LS but still within a margin or error of our result. When an SVM is used for classification 3D FST begins to outperform He et al. ’s 3D DLRGF method at low training set sizes: At 1% training data 3D FST has 80.44 (± 1.63) OA while 3D DLRGF+SVM has 77.96 (± 0.81). However using a more powerful LS-based collaborative classifier He et al. was able to achieve a more competitive OA of 83.59 (± 0.81) (this edge is not as apparent at larger training set sizes as seen in Table VIII). We leave the pairing of our feature extraction method with more sophisticated classification techniques to
future work.

On PaviaU the median number of samples per class when 10% of data is used for training is 305, leading to a slightly more favorable situation than in Indian Pines for a trainable feature extractor. In Table IX we see a 3D CNN network performs just slightly below 3D FST, but is our closest competitor at 10% training data. At 200 samples 3D FST is by far SoA. 3D FST outperforms all 3D wavelet methods on PaviaU, the methods that stand out as our greatest competitors for PaviaU are all neural network based methods. Some results in the literature are also reported at a variety of training set sizes.

### TABLE III
**Classification accuracy for Indian Pines across 10 trials on 10% of the data (the training set from [1]) with a linear SVM.**

| Class | Train | Test | 3D WST | 3D FST |
|-------|-------|------|--------|--------|
| 1     | 5     | 41   | 83.90 (±20.05) | 94.39 (±10.54) |
| 2     | 143   | 1285 | 96.30 (±1.73)   | 98.42 (±0.83)   |
| 3     | 83    | 747  | 95.31 (±2.08)   | 96.61 (±0.89)   |
| 4     | 23    | 214  | 94.02 (±3.40)   | 98.41 (±2.37)   |
| 5     | 50    | 433  | 95.52 (±3.01)   | 96.88 (±2.89)   |
| 6     | 75    | 655  | 98.75 (±0.61)   | 98.79 (±0.66)   |
| 7     | 3     | 25   | 94.00 (±6.04)   | 95.60 (±5.80)   |
| 8     | 49    | 429  | 98.41 (±1.25)   | 99.91 (±0.16)   |
| 9     | 2     | 18   | 77.78 (±21.44)  | 87.78 (±12.78)  |
| 10    | 97    | 875  | 92.27 (±2.14)   | 96.70 (±1.73)   |
| 11    | 247   | 2208 | 96.11 (±1.08)   | 98.90 (±0.75)   |
| 12    | 61    | 532  | 93.53 (±2.55)   | 97.29 (±1.09)   |
| 13    | 21    | 184  | 98.70 (±1.78)   | 97.77 (±2.68)   |
| 14    | 129   | 1136 | 98.35 (±0.54)   | 99.52 (±0.53)   |
| 15    | 38    | 348  | 94.97 (±2.51)   | 97.76 (±1.64)   |
| 16    | 10    | 83   | 96.99 (±2.80)   | 95.54 (±3.55)   |
| OA    |       |      | 95.98 (±0.46)   | 98.37 (±0.28)   |
| AA    |       |      | 94.06 (±2.01)   | 97.02 (±1.24)   |
| K     |       |      | 95.41 (±0.52)   | 98.14 (±0.32)   |

### TABLE IV
**Classification accuracy for PaviaU across 10 trials on 10% of the data (the training set from [1]) with a linear SVM.**

| Class | Train | Test | 3D WST | 3D FST |
|-------|-------|------|--------|--------|
| 1     | 658   | 5973 | 99.31 (±0.26) | 99.52 (±0.24) |
| 2     | 1828  | 1682 | 99.83 (±0.13) | 99.87 (±0.10) |
| 3     | 208   | 1891 | 96.49 (±0.96) | 99.13 (±0.77) |
| 4     | 305   | 2759 | 98.50 (±0.53) | 98.63 (±0.59) |
| 5     | 135   | 1210 | 99.65 (±0.46) | 99.68 (±0.39) |
| 6     | 503   | 4526 | 99.92 (±0.12) | 99.98 (±0.06) |
| 7     | 133   | 1197 | 98.44 (±0.66) | 99.54 (±0.35) |
| 8     | 368   | 3314 | 97.76 (±0.68) | 99.30 (±0.34) |
| 9     | 95    | 852  | 97.58 (±1.76) | 98.80 (±1.01) |
| OA    |       |      | 99.22 (±0.10) | 99.61 (±0.09) |
| AA    |       |      | 98.61 (±0.24) | 99.38 (±0.17) |
| K     |       |      | 98.97 (±0.14) | 99.49 (±0.12) |

### TABLE V
**Classification accuracy for Pavia Centre across 10 trials on 1% of the data with a linear SVM.**

| Class | Train | Test | 3D WST | 3D FST |
|-------|-------|------|--------|--------|
| 1     | 652   | 64626| 99.84 (±0.08) | 99.83 (±0.07) |
| 2     | 65    | 6443 | 96.73 (±1.46) | 97.42 (±1.09) |
| 3     | 29    | 2876 | 92.90 (±1.74) | 93.32 (±1.71) |
| 4     | 21    | 2119 | 96.06 (±2.74) | 95.34 (±3.16) |
| 5     | 65    | 6484 | 98.06 (±0.91) | 98.54 (±1.21) |
| 6     | 75    | 7510 | 98.01 (±1.10) | 98.42 (±0.72) |
| 7     | 72    | 7215 | 96.92 (±1.34) | 95.93 (±1.28) |
| 8     | 31    | 3091 | 99.66 (±0.30) | 99.37 (±0.78) |
| 9     | 21    | 2144 | 94.32 (±2.24) | 97.08 (±1.30) |
| OA    |       |      | 98.80 (±0.12) | 98.87 (±0.17) |
| AA    |       |      | 96.95 (±0.43) | 97.25 (±0.51) |
| K     |       |      | 97.94 (±0.21) | 98.07 (±0.30) |
TABLE VI

| Class | Train | Test | 3D WST | 3D FST |
|-------|-------|------|--------|--------|
| 1     | 14    | 256  | 99.96 ±0.12 | 99.06 ±0.21 |
| 2     | 6     | 95   | 98.11 ±0.56 | 96.42 ±0.16 |
| 3     | 13    | 238  | 97.27 ±1.92 | 95.97 ±2.99 |
| 4     | 11    | 204  | 99.61 ±0.83 | 99.95 ±0.16 |
| 5     | 14    | 255  | 86.94 ±7.82 | 88.67 ±8.86 |
| 6     | 14    | 255  | 93.69 ±8.05 | 97.65 ±9.28 |
| 7     | 13    | 246  | 99.80 ±0.44 | 100.00 ±0.00 |
| 8     | 11    | 192  | 99.43 ±1.46 | 99.43 ±1.81 |
| 9     | 16    | 298  | 97.55 ±2.21 | 98.89 ±1.81 |
| 10    | 13    | 235  | 99.32 ±0.73 | 99.00 ±0.00 |
| 11    | 16    | 289  | 97.58 ±1.85 | 97.85 ±4.92 |
| 12    | 10    | 171  | 98.13 ±1.98 | 97.60 ±3.17 |
| 13    | 14    | 254  | 99.33 ±1.64 | 99.92 ±0.25 |
| 14    | 5     | 90   | 86.78 ±7.83 | 88.78 ±9.24 |
| OA    |       |      | 96.98 ±1.06 | 97.55 ±0.92 |
| AA    |       |      | 96.68 ±1.06 | 97.16 ±1.05 |
| K     |       |      | 96.73 ±1.15 | 97.35 ±1.00 |

TABLE VII

| Class | Train | Test | 3D WST | 3D FST |
|-------|-------|------|--------|--------|
| 1     | 9     | 187  | 90.64 ±3.99 | 97.33 ±1.75 |
| 2     | 12    | 234  | 89.44 ±8.03 | 96.45 ±4.16 |
| 3     | 9     | 175  | 90.91 ±4.84 | 94.86 ±5.28 |
| 4     | 3     | 63   | 91.27 ±9.62 | 96.03 ±7.96 |
| 5     | 4     | 93   | 91.51 ±3.94 | 92.37 ±4.13 |
| 6     | 2     | 55   | 85.09 ±13.73 | 90.00 ±9.97 |
| 7     | 1     | 31   | 67.10 ±29.67 | 91.94 ±18.95 |
| 8     | 3     | 67   | 93.28 ±5.23 | 93.28 ±5.33 |
| 9     | 10    | 190  | 97.95 ±2.55 | 97.68 ±2.84 |
| 10    | 8     | 86   | 87.56 ±14.71 | 90.58 ±12.74 |
| 11    | 3     | 73   | 67.95 ±14.51 | 79.04 ±10.69 |
| 12    | 2     | 56   | 79.11 ±19.20 | 94.64 ±6.24 |
| 13    | 8     | 158  | 99.75 ±0.44 | 100.00 ±0.00 |
| 14    | 16    | 312  | 92.44 ±4.00 | 97.66 ±2.42 |
| 15    | 5     | 100  | 90.80 ±5.96 | 95.10 ±4.25 |
| 16    | 7     | 152  | 100.00 ±0.00 | 100.00 ±0.00 |
| 17    | 7     | 137  | 96.42 ±4.64 | 97.59 ±2.46 |
| 18    | 8     | 159  | 95.60 ±2.58 | 98.99 ±1.97 |
| 19    | 1     | 17   | 98.24 ±3.97 | 100.00 ±0.00 |
| 20    | 2     | 42   | 77.38 ±15.40 | 87.86 ±12.82 |
| 21    | 10    | 196  | 100.00 ±0.00 | 100.00 ±0.00 |
| 22    | 1     | 33   | 100.00 ±0.00 | 100.00 ±0.00 |
| OA    |       |      | 92.39 ±1.19 | 96.28 ±0.84 |
| AA    |       |      | 90.11 ±1.84 | 95.06 ±1.30 |
| K     |       |      | 91.88 ±1.27 | 96.02 ±0.89 |

TABLE VIII

| Method | 5% | 10% |
|--------|----|-----|
| 3D FST | 96.17 ±0.96 | 98.37 ±0.28 |
| 3D DLRGF+SVM | 96.29 ±0.96 | - |
| 3D DLRGF+Ls | 96.16 ±0.96 | - |
| 3D WST+RBF-SVM | - | 94.46 |
| 3D WST | 91.60 ±0.91 | 95.98 ±0.46 |
| 3D DWT-FB | - | 95.43 |
| 3D DWT | 94.5 | - |
| 3D WT+SRC | 96.04 | - |
| NN/CNN | 94.9 | 98.23 |
| 2D CNN | 90.01 | 97.45 |
| 1D LSTM | - | 90.93 |

TABLE IX

| Method | 200 samp | 10% |
|--------|-----------|-----|
| 3D FST | 98.55 ±0.21 | 99.61 ±0.09 |
| 3D CNN | - | 99.54 |
| 3D RCNN | - | 62 |
| 3D WST+RBF-SVM | - | 99.3 |
| 3D WST | 98.16 ±0.11 | 99.22 ±0.10 |
| 3D DWT-FB | - | 95.45 |
| 2D CNN | 95.97 | - |
| 1D DBN | 93.11 | - |
| 1D CNN | 92.56 | - |

per class which performs within a margin of error of 3D FST (95.9 ±1.38 vs 95.35 ±0.53). At 9% of training data Ma et al. ’s NN+CNN method [6] performs well though they only report 1 trial (99.86 vs 99.55 ±0.07). And at 3 samples, 3D WT [13] performs within a margin or error of our method also (64.32 vs 64.09 ±5.71). Overall, across all training set sizes, 3D FST performs at the SoA level on PaviaU.

The other datasets are less ubiquitous in the literature so we briefly summarize their performance versus the literature here. For Botswana 3D FST with a linear SVM greatly outperforms CNN methods [22], [20], and a 3D wavelet method [10]. On PaviaC, using less training data 3D FST outperforms a 3D CNN method [22], and at 1% training data and 30 samples per class our method beats spatial regularized spectral CNN method [4] and a manifold learning approach [46].

V. CONCLUSION

In this paper we have proposed a three-dimensional Fourier scattering transform for HSI classification. This method has the neural network like benefits of heirarchical feature extraction while bypassing the training process which is computationally expensive in both the amount of required training data and training time. Our three dimensional time-frequency features are well suited for HSI data since they decompose the HSI into multi-frequency bands and remove small perturbations such as noise. The 3D FST is particularly effective when
there is limited training data. As supported by the experimental results, 3D FST achieved SoA performance on four benchmark datasets, all while executing within a few minutes on a conventional GPU, and using a typical SVM for classification. An advantage of our method is its compatibility with conventional deep learning implementations. This readily allows for a shift from the pre-processing based classification with a linear SVM we presented to an end-to-end feature extraction and classification deep network. Our future work investigates this hybridization of scattering transforms with deep learning where the classification is performed with a neural network following a tunable scattering transform that serves as a feature extractor, and both are trained jointly. This has the potential to improve classification performance further as both the classification and feature filters will be learned for each the dataset.

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