Nonperturbative Decoupling and Effective Field Theory

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Abstract

We examine recent claims that nonperturbative effects can prevent the decoupling of a heavy fermion whose mass arises from a Yukawa coupling to a scalar field. We show that in weakly coupled, four dimensional models such as the standard model with heavy mirror fermions the effects of the heavy fermions can always be accounted for by local operators involving light fields. We contrast this with the case of the 1+1 dimensional Abelian Higgs model, in which there does not appear to be a local effective field theory describing the low energy physics when heavy fermions are integrated out.
1 Decoupling and heavy fermions

Perturbative considerations \[1\] lead us to believe that heavy particles can be decoupled from low energy degrees of freedom. We expect that the lagrangian describing the low energy degrees of freedom is affected by heavy particles only through renormalization effects and higher dimension operators which become negligible as the particle is made infinitely massive. These considerations are central to the idea of effective field theory \[2\] (for a recent review, see \[3\]). Briefly stated, the idea of effective field theory is that the S-matrix for light degrees of freedom can always be encoded in terms of an effective lagrangian containing all possible renormalizable plus higher dimension terms involving light fields, consistent with unbroken symmetries\[†\]. The mass scale suppressing the higher dimension operators indicates where the effective theory breaks down, and usually corresponds to a heavy particle mass threshold. In particular, nonlocal terms are not considered except insofar as the higher dimension terms can be considered the expansion of some heavy particle propagator.

It has been known for some time that there are some exceptions to the expectations of perturbative decoupling. In particular, if the heavy particle is a chiral fermion which participates in anomaly cancellation, its decoupling can leave behind a Wess-Zumino term which cancels the apparent anomalies of the remaining low energy degrees of freedom \[4, 5\]. However, the Wess-Zumino term is a local functional of the light fields and therefore does not violate the expectations of effective field theory. Recently, Banks and Dabholkar \[6\] have claimed to exhibit models in which heavy particles do not decouple from low energy physics, and in fact manifest themselves nonlocally in the low energy effective lagrangian. The models in question exhibit topologically nontrivial field configurations such as instantons. The otherwise heavy fermion fields have zero modes in the background of these field configurations, and this is the source of possible nondecoupling. While the models under consideration in \[6\] are admittedly fine-tuned and unnatural, the violation of decoupling is nonetheless interesting from a theoretical viewpoint as a test of the effective field theory ideas discussed above.

In this letter we will reexamine the claims of Banks and Dabholkar. Two types of models are discussed in \[6\]: the standard model plus mirror fermions (motivated by considerations of lattice fermion doubling) and the Abelian Higgs model in 1+1 dimensions. We find that it is only in the two dimensional model that an actual violation of decoupling occurs. This is due to the peculiar infinite range effects of instantons in this model, which restore the otherwise broken gauge symmetry and alter the asymptotic states. In the four dimensional case the effects of the heavy fermions on the S-matrix for light states can be reproduced by higher dimensional operators. However, we find that although the S-matrices for light states

\[†\]This point of view was emphasized by S. Weinberg in his 1993 Loeb Lectures.
can be made to agree in the full and effective theories, the Green’s functions or correlators, cannot. Since it is the S-matrix that is measurable and physical, this distinction, although curious, is unimportant from the viewpoint of effective field theory.

2 EFT for the vector standard model

The investigations of Banks and Dabholkar were motivated by the problem of fermion doubling in lattice versions of the standard model. The issue is whether doublers can be decoupled from the low energy physics by giving them masses of order the cutoff (e.g. via a Wilson-Yukawa type interaction) [7]. In [6] the question is addressed in several continuum models, the first of which is the standard model plus mirror fermions. By mirror fermions we mean left handed Weyl fermions transforming in the complex conjugate representations of the standard model fermions. The resulting model is thus vectorlike, but a large mass hierarchy between the standard model fermions and the mirrors can be maintained if the standard model Yukawa couplings are scaled down by a factor $f$ (say, $f \sim 10^{-2}$), with the mirror Yukawa couplings kept $\sim 1$. In order to keep the gauge bosons and Higgs particle in the low energy sector, we also rescale the renormalized gauge and scalar self couplings by the factor $f$. By performing the above rescalings we retain the use of weak coupling perturbation theory at the cost of having to fine tune enormously to keep the Higgs boson light in the face of large radiative corrections from the mirror fermions. (If the physical Higgs boson is not kept in the low energy sector, there will be no topologically nontrivial configurations with finite action in the low energy sector because such configurations require a zero of the Higgs field [8].) Note that the fermions that are decoupled in this model (and in the Abelian Higgs models studied in the next section) do not leave behind an uncanceled gauge anomaly. We therefore do not expect to find a Wess-Zumino term in the effective theory, which remains completely gauge invariant. Each of the terms appearing in the effective Lagrangian must be gauge invariant, and hence the renormalizable terms (dimension 4 or less) are completely specified as the usual ones, up to possible renormalization of the couplings.

Now consider nonperturbative baryon number violation in this model. In particular, consider the baryon violating decay of the (rescaled) deuteron, originally studied by ’tHooft [9]. In the low energy sector there appear to be instanton solutions mediating such decays. On the other hand, since the difference between ordinary and mirror baryon number is exactly conserved, we know that in the full theory such a decay is strictly forbidden. Both heavy and light fermions have zero modes in the $SU(2)$ instanton background, and therefore the functional determinant is zero for any topologically nontrivial process that involves only light modes. There appears to be a conflict between the exact and low energy descriptions of the model!
Furthermore, we find other discrepancies between the full and low energy theories when we consider correlators or Green’s functions. Imagine computing a correlator which contributes to some baryon number conserving process in the low energy sector. In general there will be contributions to such processes from instanton-antiinstanton configurations. Because of the zero modes mentioned above, each (anti)instanton vertex must (destroy) create a fermion of each flavor. Since in the exact theory there are no perturbative interactions which violate baryon number, each fermion line must connect the instanton and antiinstanton. Thus, configurations in which the pair are widely separated are suppressed by exponential factors $\exp(-M_F|x-y|)$, where $M_F$ is the mass of the heavy fermions and $|x-y|$ is the separation. These factors arise from the coordinate space heavy fermion propagators which connect the instanton and antiinstanton. Now consider the low energy theory. Since there are no heavy fermion fields (and therefore no heavy fermion zero modes) in the effective theory, this “confinement” of instantons (or topological charge in general) is absent. Correlators in the zero topological charge sector will receive contributions from instanton-antiinstanton pairs which are widely separated compared to the heavy fermion Compton wavelength $M_F^{-1}$. If the only additional terms induced in the low energy theory by integrating out heavy fermions are local functions of the light fields, suppressed by powers of $1/M_F$, it is hard to see how these additional terms can lead to confinement of instantons. In particular, the instanton field configurations are smooth functions, and in the case of large instantons ($\rho_I >> M_F^{-1}$), the effect of the higher dimension operators on the action of the instanton can be considered a small perturbation.

Banks and Dabholkar claim that the resolution of these puzzles is that the heavy fermions do not truly decouple from the low energy physics (they become “phantom fields”) and that a correct description of low energy physics requires some nonlocality of the effective theory. They further speculate that this is a generic phenomena associated with decoupling of heavy particles whose mass arises from spontaneous symmetry breaking. Because the heavy particles can become massless (or at least light) in the background of a field configuration with vanishing scalar vacuum expectation value, the logic is that they do not decouple from low energy processes mediated by such configurations.

Here we argue that the opposite is true: the effects of the virtual heavy fermions can be reproduced by appropriately chosen local operators of the light fields. More specifically, the S-matrices of the full and effective theory can be matched by appropriate choice of these operators. While this resolves issues of the first type mentioned above, e.g. deuteron decay, we will see that it does not resolve discrepancies in correlators or Green’s functions between the full and effective theories. This is acceptable from the standpoint of quantum field theory, as the S-matrix is the fundamental object that is related to experimental measurement. On the other hand, from the statistical mechanical viewpoint it is the entire correlator that may
be of interest, not just its asymptotic behavior which determines the S-matrix in field theory.

It is straightforward to match S-matrix elements between the exact and effective theories. The idea is that given any S-matrix element in the exact theory, involving only light in- and out-states, one can choose higher dimension operators in the effective theory to achieve matching. In the vector standard model the S-matrix elements in question are those that violate baryon number without violating mirror baryon number. In the exact theory, these S-matrix elements are exactly zero. In the low energy theory there are contributions from two sources: instantons and explicit baryon number violating operators such as

$$
\mathcal{O}_{12} = \prod_{I=1}^{N_F} \epsilon_{ij} \epsilon_{kl} \epsilon_{\alpha \beta \gamma} (q_i^\alpha q_j^\beta q_k^\gamma l_I),
$$

where \( \{i,j,k,l\} \) are SU(2) indices, \( \{\alpha,\beta,\gamma\} \) are color indices and \( I \) is a flavor index. The instantons\(^\dagger\) also contribute to correlators such as

$$
\langle \prod_{I=1}^{N_F} q(w_I)q(x_I)q(y_I)l(z_I) \rangle
$$

which yield S-matrix elements when the LSZ (Lehmann-Symanzik-Zimmermann) projection \([11]\) is applied:

$$
\langle i | S | f \rangle = LSZ \{ \langle \prod_{I=1}^{N_F} q(w_I)q(x_I)q(y_I)l(z_I) \rangle \}
$$

Recall that the LSZ projection removes all but the coefficient of the multiple on-shell pole of the correlator. In momentum space,

$$
LSZ = C \prod_{i=1}^{N_F} (i\not{p}_i + m) \prod_{j=1}^{N_b} (p_j^2 + M^2)
$$

with appropriate combinatorial factor \( C \). In order to reproduce the exact result of zero (i.e. a stable deuteron) the operator \( \mathcal{O}_{12} \), and similar ones involving additional derivatives, must be adjusted to cancel the instanton contribution to (3).

Let us study this more explicitly. Let

$$
G_E(x_1, \ldots, x_N) = \langle \prod_{I=1}^{N_F} q(x_1)q(x_2)q(x_3)l(x_4) \rangle_{\text{Exact}}
$$

be evaluated in the full theory, while

$$
G_{NP}(x_1, \ldots, x_N) = \langle \prod_{I=1}^{N_F} q(x_1)q(x_2)q(x_3)l(x_4) \rangle_{\text{Nonperturbative}}
$$

\(^\dagger\)By instantons here we really mean constrained instantons \([9, 10]\) since there are no exact instantons in a spontaneously broken theory. The constrained instantons exist at all sizes \( \rho \), with those of size \( \rho \sim v^{-1} \) giving the dominant contribution to the path integral.
is the contribution from nonperturbative field configurations computed in the effective theory. In the effective theory, there will also be a direct perturbative contribution $G_O(x_1,\ldots,x_N)$ from higher dimension operators $O_n$. Now consider matching the contributions to the light S-matrix from $G_E$ and from $G_{NP}$ plus $G_O$ by appropriate choice of operators $O_n$ containing $N$ light fields and any number of derivatives. We want

$$\langle i|S_E|f \rangle = \langle i|S_{NP}|f \rangle + \langle i|S_O|f \rangle,$$

where

$$\langle i|S_{E,NP,O}|f \rangle = \langle k_1,\ldots,k_m|S_{E,NP,O}|k_{m+1},\ldots,k_N \rangle$$

$$= \text{LSZ} \left[ \prod_{i=1}^N \int d^4x_i e^{i k_i \cdot x_i} (G_{E,NP,O}(x_1,\ldots,x_N)) \right]$$

$$= f_{E,NP,O}(k_1,\ldots,k_N).$$

Here $f_{E,NP,O}(k_1,\ldots,k_N)$ are simply the coefficients of the multiple poles in the Fourier transforms of $G_{E,NP,O}$. For Euclidean momenta with imaginary parts less than the light particle masses these residues are completely analytic, so a power series representation is always possible. We can therefore expand each $f_{E,NP,O}(k_1,\ldots,k_N)$ in the variables $k_i^2, k_i \cdot k_j$.

In the vector standard model $f_E = 0$ while $f_{NP}$ is determined by the currently unknown behavior of anomalous baryon number violating amplitudes at arbitrary momenta $k_i$. As for $f_O$, each operator $O_n$ contributes a term to $f_O$ with a power of $k_i$ for each derivative $\partial_i$ in the operator. The coefficients of the operators are then determined by equating the terms in the power series expansions of $f_{NP}$ and $f_O$.

It is important to emphasize that this matching must be order by order in some small parameter. This is because the nonperturbative correlators in the effective theory depend on the coefficients of the operators $O_n$. However, these effects will be higher order in the small parameter. In a weakly coupled theory like the vector standard model, the small parameter is $e^{-S_0}$ (the instanton action) times powers of the ratio of light and heavy scales. In a strongly coupled model the exponential factor may be absent but the second factor will remain.

It should be clear that the construction considered above is quite general. As long as we know the asymptotic (in-, out-) states of the low energy sector, we can reproduce the exact S-matrix between light fields by suitable choice of higher dimension operators $O$. The point is that any S-matrix element between $N$ light fields will get a direct contribution from the corresponding operator $O_N$. The momentum dependence of the S-matrix is determined by operators with derivatives acting on $O_N$. In the case at hand, the higher derivative operators are chosen to reproduce the “form factors” of instanton amplitudes.

Now we turn to the matching of correlators between the exact and effective theories. In perturbative calculations one often defines the effective theory by requiring that Green’s
functions match. Technically, this is of course too stringent a requirement. We are familiar with the result that in ordinary gauge theory different choices of gauge lead to different Green’s functions that typically disagree, while the corresponding S-matrix elements agree. Clearly, Green’s functions are not by themselves physical objects. Here we will argue that, at least as far as the nonperturbative effects in the vector standard model are concerned, it is impossible to match Green’s functions.

Suppose that we were able to impose that all correlators of a finite number of fields match exactly between the two theories. Consider the following correlator, that of a functional delta function:

\[ \langle \delta[\phi(x) - \phi_0(x)] \rangle = N \int D\phi \delta[\phi(x) - \phi_0(x)] e^{-S[\phi]} \]

where here we use \( \phi(x) \) generically to represent all fields. Now, we can represent \( \delta[\phi(x) - \phi_0(x)] \) in terms of a finite number of standard delta functions if we discretize spacetime:

\[ \delta[\phi(x) - \phi_0(x)] = \prod_x \delta(\phi(x) - \phi_0(x)). \]

Furthermore, the standard delta functions can be expanded in a power series in \( (\phi(x) - \phi_0(x)) \). The sequence \( \delta_n(\phi(x) - \phi_0(x)) = \frac{n}{\sqrt{\pi}} e^{-n^2(\phi(x) - \phi_0(x))^2} \) converges to a delta function, and since \( e^{-z^2} \) is analytic everywhere except \( z = \infty \) it can be expanded in a uniformly convergent series in \( z^2 \). Therefore, to any desired accuracy the expectation of the delta functional can be represented in terms of a finite number of correlators of a finite number of fields. If these correlators agree, then (up to a field independent constant) the actions of any chosen field configuration \( \phi_0(x) \) must agree in the exact and effective theories.

As mentioned previously, in the exact theory there is a long range “interaction” (if we think in terms of statistical mechanics, with energy substituted for action) between widely separated lumps with nontrivial topological charge. The interaction grows with separation \( |x - y| \) and is therefore confining. Given the arguments of the previous paragraph it is clear that correlator matching requires that this effect be reproduced in the effective theory.

It seems clearly impossible to induce a confining, long range interaction between lumps via local operators without introducing new degrees of freedom. But, for the sake of clarity, let us further belabor the point. Consider the effective lagrangian constructed so as to match S-matrix elements

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{ren}} + \mathcal{L}_{\text{pert}} + \mathcal{L}_{\text{NP}}. \]

Here the term \( \mathcal{L}_{\text{ren}} \) consists of renormalizable terms while \( \mathcal{L}_{\text{pert}}, \mathcal{L}_{\text{NP}} \) contain the perturbative and nonperturbative higher dimensional operators required for matching. In particular, all the terms in \( \mathcal{L}_{\text{pert}} \) conserve \( (B+L) \) number, while all the terms in \( \mathcal{L}_{\text{NP}} \) violate it. Both types of
terms are suppressed by powers of the ratio of light to heavy scales, and the nonperturbative terms also carry a suppression of $e^{-S_0}$.

Now consider a widely separated pair of lumps at coordinates $x$ and $y$. Let $B_x$ and $B_y$ be balls centered at $x$ and $y$ with radii much less than $|x - y|$. In order to reproduce the long range interaction we require that

$$\Delta S = \int_{R^4 - B_x - B_y} d^4x \left\{ [\mathcal{L}_{pert} + \mathcal{L}_{NP}]_{lumps} - [\mathcal{L}_{pert} + \mathcal{L}_{NP}]_{vacuum} \right\} \sim M_F |x - y| \quad (15)$$

However, we can arrange that the fields approach their vacuum values to exponential accuracy far from the centers of the lumps. Since the field configurations are smooth, and the terms in $\mathcal{L}_{pert} + \mathcal{L}_{NP}$ are by assumption simply polynomials in the fields, we can arrange for the left hand side of (15) to be arbitrarily small for large separation. We are thus clearly unable to reproduce the linear potential.

3 EFT in two dimensions

Banks and Dabholkar also consider nondecoupling in the Abelian Higgs model in two dimensions. In two dimensions and at large-N (where N is the number of fermion flavors) they were able to explicitly construct a model with an arbitrarily large hierarchy between the heavy fermion mass and the scale of the light degrees of freedom (scalars, gauge bosons and light fermions). Again in this model there are nonperturbative effects which are suppressed by the presence of the heavy fermions, regardless of their mass. Here we argue that there is indeed a breakdown of effective field theory in the two dimensional case. In contrast with the vector standard model, instantons in the Abelian Higgs (AH) model have infinite range effects which cannot be reproduced even in the S-matrix by the introduction of local operators.

Consider the following lagrangian,

$$\mathcal{L} = \frac{1}{4e^2} F_{\mu\nu}^2 + |D_\mu \phi|^2 - V(\phi^* \phi) + \bar{\psi} \left\{ i \gamma^\mu \left[ \partial_\mu - q \left( \frac{1 - \epsilon}{2} \right) A_\mu \right] + g(\phi^*) \frac{q}{2} \left( 1 - \epsilon \right) + g(\phi) \frac{q}{2} \left( 1 + \epsilon \right) \right\} \psi \quad (16)$$

where $q$ and $\epsilon$ are $3 \times 3$ matrices: $q = diag(q_1, q_2, q)$ and $\epsilon = diag(1, 1, -1)$. The charges are chosen to satisfy the anomaly cancellation condition $q^2 = q_1^2 + q_2^2$. Note that there are two left moving and one right moving fermion of charges $q_1, q_2, q$ which are paired with singlets by the Yukawa couplings. It is important for our purposes that at least some of the fermions carry fractional charges, so that they cannot be screened by the scalar, which carries unit charge. As noted in [8], the fermions $\psi$ exhibit zero modes in topologically nontrivial backgrounds. Therefore, instanton amplitudes must be accompanied by the creation or destruction of
fermion number. The large-N limit is obtained by including $N$ copies of the heavy fermion field and rescaling the couplings so that $e^2N = E^2, \eta^2N = G^2$ and $\lambda N = \kappa$ remain fixed as $N \to \infty$. The Euclidean AH lagrangian is identical to the Landau-Ginzburg Hamiltonian for a two dimensional superconductor. In this context the instantons are simply vortices and the dilute instanton gas represents the statistical mechanics of these vortices.

It is again clear from arguments similar to those of the previous section that effects like the confinement of instantons cannot be reproduced by a local effective lagrangian. However, in this model confining instantons does more than just prevent matching of correlators. In the AH model the instantons are crucial to determining the realization of the gauge symmetry as well as the asymptotic states of the model. In the absence of fermion zero modes, it is easy to see that in a sufficiently large box the effect of instantons is to restore the otherwise spontaneously broken $U(1)$ symmetry. Consider the vacuum expectation value (for simplicity we assume that the vacuum angle $\theta = 0$)

$$\langle \phi(x) \rangle = N \int D\phi \phi(x)e^{-S[\phi]}$$

(17)

In perturbation theory, we would find that $\langle \phi(x) \rangle \neq 0$, therefore signalling spontaneous symmetry breaking. In two dimensions, however, instantons can restore the symmetry. A simple way to see this is to consider the analogous Landau-Ginzburg model. In the Landau-Ginzburg model this correlator is equivalent to the average value of $\phi(x)$ in the presence of vortices at low temperature. A vortex centered at $y$ very far from $x$ exerts an effect on $\phi(x) = ve^{i\theta_{xy}}$, regardless of the separation $|x - y|$. Here $\theta_{xy}$ is the angle between the line $xy$ and an arbitrary line emanating from $y$. An anti-vortex has a similar effect except with opposite angle. Now, although vortices are extremely rare at low temperature, occurring with exponentially small density, in an infinite volume there are nevertheless an infinite number of vortices and anti-vortices. Each of these, centered at arbitrary points $y_i$, completely disorder the expectation value $\langle \phi(x) \rangle$ yielding zero in the infinite volume limit. In the dilute vortex approximation, we find

$$\langle \phi(x) \rangle = v \exp\left[ -k \int d^2R \left( 2 - \frac{\phi^+(x-R)}{v} - \frac{\phi^-(x-R)}{v} \right) \right],$$

(18)

where $k \sim v^2e^{-S_0}$ is the density of vortices, and $\phi_{+, -}$ is the vortex or antivortex solution. Since $\int d^2R [\phi_{+, -}(x - R)] = 0$, (18) goes to zero in the infinite volume limit.

This restoration of symmetry at sufficiently large distances suggests that a Coulomb interaction is present at large distances between charged particles. But in one spatial dimension a Coulomb interaction is confining, and therefore leads to the requirement that asymptotic states in the AH model with instantons be neutral, rather than charged, as would be expected from the perturbative Higgs mechanism. Indeed, one can verify that the Wilson loop for fractional charge $q$

$$\langle W \rangle = \langle e^q \int d^2x A_\mu \rangle = \langle e^q \int d^2x \epsilon_{\mu\nu} F^{\mu\nu} \rangle$$

(19)
displays area law behavior in the presence of instantons \[13\], thus signalling charge confinement.

The point of the previous comments is that the instantons in this model actually alter the asymptotic states of the model. When heavy fermions are introduced which exhibit zero modes in the instanton background, they suppress the instantons and therefore also prevent the restoration of the $U(1)$ symmetry. There are clearly no local operators which can mimic the effect of the heavy fermions: they would either have to induce a confining interaction between instantons (already ruled out by previous arguments), or eliminate topologically nontrivial configurations entirely from the low energy sector.

The existence of nontrivial configurations such as instantons in the low energy theory can be guaranteed on topological grounds as long as the Euclidean effective lagrangian remains positive definite. In that case, we continue to require that terms in the action like $|D_{\mu}\phi|^2$ and $F_{\mu\nu}^2$ approach zero at infinity. This condition is sufficient to guarantee the topological classification of configurations that leads to instantons or vortices. It is easy to see that the effect of exactly integrating out the heavy fermions leads to corrections to the effective action which leave it positive definite. The heavy fermion contribution to the exact effective action is

$$\Gamma_\psi[\phi] = -\ln\left(\int D\psi\ e^{-S[\phi,\psi]}\right),$$

where $\phi$ is meant to represent all of the light fields, and $C$ is a $\phi$ independent constant. $C$ is chosen by regulating the path integral in (21), and requiring

$$\left\| \int D\psi\ e^{-S[\phi,\psi]} \right\| < C, \ \forall \phi.$$  

With this choice of $C$, $\Gamma_\psi[\phi]$ is positive definite up to a $\phi$ independent constant. If a convergent expansion of the effective action in local operators exists, it must also be positive definite. Hence we expect the effective theory to contain instantons.

It does not seem possible that the interactions induced by heavy fermions render the action of all topologically nontrivial configurations infinite, since for smooth configurations the effect of the higher dimension operators is suppressed by powers of the ratio of the light to heavy scale, which can be made arbitrarily small at large $N$. For any finite value of the vortex energy (or instanton action), however large, our previous arguments for symmetry restoration at infinite volume remain valid. Thus the spectrum of the low energy theory will always consist only of charge singlet states, as opposed to the spectrum of the exact theory.

\[\text{Note that this object requires careful definition } \[14\]. \text{ It does not contain a Wess-Zumino term because the gauge anomalies of the } \psi \text{'s have been chosen to cancel.}\]
with massive fermions, which exhibits the Higgs mechanism. Therefore in the AH model we have a bona fide counterexample to the idea of effective field theory: no local effective lagrangian can reproduce the exact S-matrix of this model in the light sector.

4 Final Comments

We have seen that, at least as far as nonperturbative effects are concerned, there is an important distinction to be made between matching the Green’s functions and matching the S-matrices of exact and effective low energy theories. We find that in the vector standard model studied by Banks and Dabholkar, the latter is possible while the former is not. Because the exact S-matrix can be reproduced, this model does not present a breakdown in effective field theory.

However, we do find, in agreement with Banks and Dabholkar, that in the two dimensional Abelian Higgs model it is impossible to reproduce the exact S-matrix in the low energy sector via a local effective lagrangian. This seems to be a result of the long range effects of topological defects in this class of models, and is clearly not a generic phenomena. It would appear that the necessity of “phantom fields” is limited to models of this sort, and does not extend to all models in which fermions receive their masses from Yukawa couplings.

We comment briefly on the relevance of these conclusions to the original lattice motivations for [7]. The point of Banks’ original arguments was that attempting to decouple fermion doublers in gauged chiral models by giving them masses of order the cutoff was doomed to failure insofar as the resulting model would not exhibit low energy baryon number violation. This conclusion remains valid, but it is instructive to note that from the low energy viewpoint the absence of baryon number violation is due to the presence of certain higher dimension operators which explicitly violate baryon number and which cancel the low energy nonperturbative effects exactly. In other words the low energy limit of the model with heavy doublers is essentially the conventional standard model plus some very small higher dimension operators. If the purpose of the hypothetical lattice simulation is to determine nonperturbative quantities such as the size and energy dependence of baryon number violation, then it is true that the Wilson-Yukawa method is insufficient. On the other hand, other dynamical properties of the system (such as baryon number conserving correlators) which may be relevant to issues such as the phase structure of the chiral model, will only suffer small corrections due to the induced operators.

Finally, we would like to mention another point of view advanced by Georgi, Kaplan and Morrin (GKM) [16]. These authors argue that the full and effective theories can be reconciled if it can be shown that there are no instantons in the low energy theory - in other words

*For a discussion of other problems inherent in Wilson-Yukawa models, see [15].
that some of the induced higher dimension operators render the instanton action infinite. This possibility was mentioned above, but discounted. Since even a ‘large’ instanton must have a zero of the Higgs field (due to topological considerations), GKM argue that it is possible that some of the induced higher dimension operators are singular when evaluated on such a zero. This possibility in itself seems only to render the action of topologically nontrivial configurations incalculable in the low energy theory, not necessarily infinite. It is further disturbing that in such a picture the low energy physics is sensitive to the behavior of the fields at arbitrarily short distances (i.e. - near the Higgs zero). However, if this picture is correct, it would imply that mirror fermions can be accounted for in any number of dimensions without nonlocality, or even anomalous higher dimension operators.

The author would like to acknowledge Steven Weinberg, whose 1993 Loeb Lectures on Effective Field Theory stimulated this investigation. He would also like to thank E. Farhi, H. Georgi, S. Hughes, L. Kaplan, D. Morrin and S. Osofsky for useful discussions. SDH acknowledges support from the National Science Foundation under grant NSF-PHY-92-18167, the state of Texas under grant TNRLC-RGY93-278B, and from the Harvard Society of Fellows.
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