Pion valence quark PDF from lattice QCD

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We present lattice results on the valence-quark structure of the pion using a coordinate space method within the framework of Large Momentum Effective Theory (LaMET). In this method one relies on the matrix elements of a Euclidean correlator in boosted hadronic states, which have an operator product expansion at short distance that allows us to extract the moments of PDFs. We renormalize the Euclidean correlator by forming the reduced Ioffe-time distribution (rITD), and reconstruct the second and fourth moments of the pion PDF by taking into account of QCD evolution effects.

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1. Introduction

Parton distribution functions (PDF’s) describe the longitudinal momentum distribution of a parton within a hadron. We define the quark PDF as

$$f_j(x, \mu) = \frac{1}{2} \int \frac{d\xi}{2\pi} e^{-ixP^+\xi^-} \langle h(p)| [\bar{\psi}_j(\xi^-)\gamma^\nu W_L(\xi^-,0)\psi_j(0)]_\mu |h(p)\rangle$$

(1.1)

with $P = (P^+, \frac{m^2}{P^2}, 0_T)$ and $\xi = (0^+, \xi^-, 0_T)$. The parton longitudinal momentum fraction is $x$ and $\mu$ is the renormalization scale of the operator. Due to the dynamical nature of the operator, it is inaccessible from direct computation using lattice QCD. Recently, the LaMET approach was proposed to extract the PDF from a Euclidean correlator in highly boosted hadronic states [1, 2], which is related to the light-cone PDF or correlator in Eq.(1) through a factorization formula [3]. The factorization formula has large momentum or small distance expansion forms in the momentum or coordinate space, respectively. The latter has been exploited in the pseudo Ioffe-time distribution approach to extract the PDFs [4], which is also used by us in this proceeding.

1.1 Momentum space method

The quark qPDF is defined as [1]

$$q_j(x, \bar{\mu}, P_z) = \frac{1}{2} \int \frac{dz}{2\pi} e^{-ixP^z\tilde{z}} \langle h(p)| [\bar{\psi}_j(z)\Gamma W_L(z,0)\psi_j(0)]_{\tilde{\mu}} |h(p)\rangle.$$ 

(1.2)

Here quark-antiquark separation is in equal time, $\tilde{\mu} = (0,0,0,z)$. In addition, we have the freedom over what gamma matrix we wish to look at. In this study, we choose $\Gamma = \gamma_t$ to avoid operator mixing. We renormalized the qPDF matrix element non-perturbatively in the RI-MOM scheme, then match it perturbatively to an MS-scheme PDF using the matching kernel derived from LaMET under the form [5]

$$q_j(x, \bar{\mu}, P_z) = \int_{-1}^{+1} dy \frac{C(z, y, P_z, \bar{\mu})}{|y|} f(y, \mu) + O(m^2_P, \Lambda_{QCD}^2),$$

(1.3)

where $\bar{\mu}$ are the renormalization scales introduced in the RI-MOM scheme. It becomes evident that large value of $P_z$ is necessary for this matching to be reliable as it suppresses unwanted higher-twist terms. This approach was used to study valence PDF of the pion [6].

1.2 Coordinate space method

Alternatively, one can start from the Euclidean correlator itself,

$$\mathcal{M}(\nu, z^2; \mu^2) = \langle h(p)| [\bar{\psi}_j(z)\Gamma W_L(z,0)\psi_j(0)]_\mu |h(p)\rangle,$$

(1.4)

which has also been referred to as pseudo Ioffe-time distribution [4], as $\nu = z \cdot P$ is called the Ioffe-time. We define the reduced Ioffe-Time distribution (rITD) as

$$\mathcal{M}(\nu, z^2) = \mathcal{M}(\nu, z^2, \mu)/\mathcal{M}(0, z^2, \mu)$$

(1.5)

The above ratio is renormalization group invariant because the renormalization constants being $\nu$-independent cancel between the numerator and denominator. Recent progress has been made
in extracting information about the hadron PDF from this method [7, 8]. In this study we extract moments of the PDF using the following factorization formula [3]

\[
\mathcal{M}(\nu,z^2) = \sum_n C_n(\mu^2 z^2) (-i\nu)^n n! - a_{n+1}(\mu) + \mathcal{O}(z^2 m^2, z^2 \Lambda_{QCD}^2)
\]  

(1.6)

where \(a_{n+1}(\mu) = \int_{-1}^{1} dxx^n f(x;\mu)\). Wilson Coefficients \(C_n(\mu^2 z^2)\) are computed in Ref. [3]. To fit lattice matrix elements computed at multiple \(z^2\) values, we take into account the evolution effects in the strong coupling and PDF moments.

1.3 Comparison of coordinate space and momentum space methods in context of lattice calculations

From the perspective of lattice calculations, the difference between momentum space and coordinate space method comes from whether one fixes the momentum of the hadron to be large and varies the separation \(z\) in the quark bilinear operator, or if one varies \(z\) and \(P_z\) simultaneously to cover as many values of \(\nu\) as possible. Highly boosted hadrons require special interpolating hadron operator, tuned to optimize the overlap between with the ground state of the hadron of a given momentum, \(P_z\) [9]. In this sense, computing with the momentum space method is less expensive, since one would only need to tune the interpolating hadron operator for one momentum. For the coordinate space method one needs to tune the smearing operator for multiple momenta, resulting in multiple inversions of the Dirac Operator.

An attractive feature of the coordinate space approach based on rITD is the simplicity of the matching. There is no need to use an intermediate RI-MOM scheme to match the lattice data to PDF defined in \(\overline{\text{MS}}\) scheme. This statement, however, only holds for non-singlet case. In the gluon case there will be mixing with the singlet quark sector, and the matrix element at \(P_z = 0\) includes the moments \(\langle xq, g \rangle\), which must be computed independently. Therefore, the gluon rITD and PDF are no longer related by a perturbative matching, as it should include these nonperturbative moments with uncorrelated errors.

In the momentum space method one has to perform an integral over all \(z\) values to obtain the qPDF, \(\tilde{q}(x)\), including large \(|z|\) values, where the perturbative matching does not hold. This is not a problem for sufficiently large \(P_z\) since the contribution from large \(|z|\) is suppressed is suppressed. However, for values of \(P_z\) accessible in present day lattice calculations are no large enough to ensure this. (see e.g. discussions in Ref. [10]). In the coordinate space method one can choose \(z\) such that the perturbative matching is reliable, say \(|z| < 0.3\) fm. However, for values of \(P_z\) that are available in present day lattice calculations this translates to small values of \(\nu\). As we will see in section 3 having only small \(\nu\) values does not constrain the PDF or its higher moments well. Therefore, also in the coordinate space method it is important to consider large values of \(P_z\).

2. Details of Lattice Calculation

In this study we calculate the valence (isovector) quark pion PDF, \(f_u(x) - f_d(x)\) using 2+1 flavor gauge configurations generated by HotQCD collaboration with the Highly Improved Staggered Quark (HISQ) [11]. The strange quark mass was set to its physical value, while the light quark masses correspond to the pion mass of 161 MeV, in the continuum limit. We used two lattice
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Pion Valence Quark PDF

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| $n_z$ | 0 | 1 | 2 | 3 | 4 | 5 |
|-------|---|---|---|---|---|---|
| $P_z$ GeV coarse | 0 | 0.43 | 0.86 | 1.29 | 1.72 | 2.15 |
| $P_z$ GeV fine | 0 | 0.48 | 0.97 | 1.45 | 1.93 | 2.42 |
| $\zeta$ | 0 | 0.66 | 0.75 | 0.6 | 0 | 0.6 |

Table 1: The pion momenta, $P_z = 2\pi/Ln_z$ and the boost parameter $\zeta = k_z/P_z$ used in the valence quark propagator.

spacings 0.06fm and 0.04fm, corresponding to $\beta = 10/g^2 = 7.373$ and 7.825. The lattice volumes corresponding to coarse and fine lattices are $48^3 \times 64$ and $64^4$, respectively. We use the Wilson-Clover action for the valence quarks on one HYP-smeared [12] gauge background. We also used one HYP smeared links in gauge links that enter the spatial Wilson line. Our valence pion mass was set to 300 MeV. For two and three point functions we incorporated All-Mode Averaging, computing 32 sloppy samples and one exact sample on each configuration [13]. Here, sloppy and exact inversions have a stopping criteria of $10^{-4}$ and $10^{-10}$ respectively. On the coarse lattice, we have computed 100 configurations for pion momentum 0 GeV and 0.43 GeV, and 320 configurations for momentum 0.86 GeV to 2.15 GeV. On the fine lattice, we computed 206 configurations for all of the momentum. In Table 1 we show the values of $P_z$ used in this study. In order to obtain a signal for the ground state of fast moving, we used Gaussian boosted sources, where a momentum $k_z$ is injected to the quark propagator [9]. The values of $k_z$ are shown in Table 1. The Gaussian sources have been implemented using Coulomb gauge fixing [6] and the width of the Gaussian was set to 5.2 in lattice units for both lattice spacings. For the coarse lattices our setup is identical to the one used in Ref. [6]. We extract ground state matrix elements using the summation method [14].

3. Constraints on pion PDF from rITD

3.1 Studying rITD using Phenomenological Results

To demonstrate how the rITD can constrain the valence quark pion PDF we use the results from phenomenological analysis of Drell-Yan data by JAM collaboration [15]. First we notice that in the isospin symmetric limit, the valence quark distribution is symmetric around $x = 0$ (see discussions in Ref. [6]). Therefore, all the odd moments of the valence quark PDF will vanish. The JAM result for the pion valence quark PDF at $\mu = 3.2$ GeV and $x > 0$ can be parameterized using a simple form $f(x) \sim x^a(1-x)^b$, $a = -0.407$ and $b = 1.12$. Using this parameterization of PDF it is straightforward to calculate the moments and reconstruct the rITD corresponding to JAM result according to Eq. (1.6). Since in the lattice calculations we can only access a limited range of Ioffe-time $\nu$ the series in Eq. (1.6) can be truncated to relatively low order. This is shown in Figs. 1 and 2. If $\nu$ is smaller than 5 rITD can be described by 8th order polynomial. This implies that present day lattice calculations cannot constrain moments beyond $\langle x^8 \rangle$. From the above figures we can also see that $\langle x^4 \rangle$ terms begin to contribute at $\nu \sim 2$ and $\langle x^6 \rangle$ terms begin to contribute at $\nu \sim 3$.

Next we explore the $z^2$ evolution of the rITD using the JAM results for the valence pion PDF. This is shown in Fig. 3. From this figure we see that at large Ioffe time the $z^2$ dependence of rITD could be significant. However, in lattice calculations only certain values of $\nu$ can be probed because both $P_z$ and $z$ can be varied in certain steps. In the right panel of Fig. 3 we show then $z^2$ dependence
Figure 1: Truncated rITD from JAM result. Different curves follow different truncation in $v$.

Figure 2: Relative error between different truncation’s in $v$ of rITD from JAM result. The black curve shows 1 % relative error.

Figure 3: The $z^2$ dependence of Ioffe time for $v \in [0, 5]$ obtained from JAM result on valence quark pion PDF. The left panel shows the general case, while the right panel shows the $z$-dependence for the set of the $z^-$ values and momenta, $P_z$ available in lattice calculations with $a = 0.06$ fm, c.f. Table 1.

3.2 rITD from Lattice Data

We calculated the valence quark rITD of the pion for two lattice spacings, $a = 0.06$ fm and $a = 0.04$ fm. To reduce the errors of the lattice calculations we consider the following quantity

$$M^{\text{imp}}(v, z^2) = \frac{M(v, z^2)}{V(P)} \frac{V(0)}{M(0, z^2)},$$

with $V(P) = \langle P | \bar{\psi}(0) \gamma_0 \psi | P \rangle$ as suggested in Ref. [16]. Since the renormalization of the electric charge does not depend on the hadron momentum, multiplying the rITD by $V(0)$ and dividing by $V(P)$ does not change anything. However, the matrix elements calculated for the same hadron momentum are strongly correlated and therefore, using the above ratio strongly reduces the errors.
for the lattice estimate of rITD. The corresponding estimate for rITD are shown in Fig. 4. This figure shows that rITD calculated at two lattice spacings agree within errors, and thus cutoff effects are small even at small values of $z$. The $z$-dependence of rITD at fixed $\nu$ is small for the considered range of $z$ values in agreement with the expectations based on the analysis of rITD obtained from the JAM valence quark pion PDF.

To determine the moments of the PDF, we perform a combined fit across all of the lattice data up to some $z_o$ to a truncated polynomial in $\nu$ of the form (1.6) at renormalization scale $\mu = 3.2$ GeV, shown in Figs 5 and 6. Currently, more statistics is needed to quote a value for the moments.

However, one can see that the fit appears to stabilize if one includes larger $z^2$ values. This however introduces systematic uncertainties coming from non-perturbative physics and target-mass effects. We will try to estimate these effects in the future.
4. Conclusions

We have presented results of pion PDF using the OPE of a Euclidean correlator in boosted pion states. In addition we studied the $z^2$ evolution of the rITD as well as the behavior due to truncation as a function of $\nu$. We find large dependence in $z^2$. We also find how truncated fits are sensitive to the Ioffe-Time available. We present our rITD at two different lattice spacings. Finally we fit our rITD to quadratic and quartic polynomials in order to extract the second and fourth moments of valence quark pion PDF.

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