Electromagnetic-Power-Based Modal Classification, Modal Expansion, and Modal Decomposition for Perfect Electric Conductors

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Traditionally, all working modes of a perfect electric conductor are classified into capacitive modes, resonant modes, and inductive modes, and the resonant modes are further classified into internal resonant modes and external resonant modes. In this paper, the capacitive modes are further classified into intrinsically capacitive modes and nonintrinsically capacitive modes; the resonant modes are alternatively classified into intrinsically resonant modes, which are further classified into nonradiative intrinsically resonant modes and radiative intrinsically resonant modes, and nonintrinsically resonant modes; the inductive modes are further classified into intrinsically inductive modes and nonintrinsically inductive modes. Based on the modal expansion corresponding to these new modal classifications, an alternative modal decomposition method is proposed. In addition, it is also proved that all intrinsically resonant modes and all nonradiative intrinsically resonant modes constitute linear spaces, respectively, but other kinds of resonant modes cannot constitute linear spaces; by including the mode 0 into the intrinsically capacitive mode set and the intrinsically inductive mode set, these two modal sets become linear spaces, respectively, but other kinds of capacitive modes and inductive modes cannot constitute linear spaces.

1. Introduction

Resonance is an important concept in electromagnetics. Based on whether the resonant modes radiate electromagnetic (EM) energy, they are classified into internal resonant modes and external resonant modes, and these two kinds of resonant modes are widely applied in EM cavities [1–5] and EM antennas [6–10], respectively.

The most commonly used mathematical method for researching internal resonant modes is the eigenmode theory (EMT) [1, 2, 11], and the EMT can construct the basis of the internal resonance space (which is constituted by all internal resonant modes [12]), and the basis is called eigenmodes. The most commonly used mathematical methods for researching external resonant modes are the singularity expansion method (SEM) [13–20] and characteristic mode theory (CMT) [21–30], and the modes constructed by SEM and CMT are, respectively, called natural modes and characteristic modes (CMs). Based on the results given in [29], it is easy to conclude that all the natural modes are resonant. Recently, [30] generalizes the traditional CMT to internal resonance problem, and it also proves that all nonradiative modes are resonant; all nonradiative modes constitute a linear space called nonradiation space, which is the same as the internal resonance space; all nonradiative CMs constitute the basis of the nonradiation space and internal resonance space, and then they are equivalent to the eigenmodes from the aspect of spanning whole space. Based on the above observations, the fundamental modes which are resonant can be classified into four categories internal resonant eigenmodes, external resonant natural modes, radiative resonant CMs, and nonradiative resonant CMs, and the relationships and differences among these fundamental resonant modes are analyzed in papers [27, 28, 30].

This paper alternatively classifies all resonant modes into three categories: nonradiative intrinsically resonant modes, radiative intrinsically resonant modes, and nonintrinsically...
resonant modes, and discusses the relationships and differences among them. Following this alternative modal classification for resonant modes, this paper further classifies all capacitive modes into two categories intrinsically capacitive modes and nonintrinsically capacitive modes and further classifies all inductive modes into two categories intrinsically inductive modes and nonintrinsically inductive modes. By employing the modal expansions corresponding to these new modal classifications, an alternative modal decomposition method is proposed in this paper, and at the same time, some further conclusions are obtained. In addition, some typical examples are also provided to verify the conclusions obtained in this paper.

2. Modal Classification

When the EM field $\{\vec{E}, \vec{H}\}$ is incident on a perfect electric conductor (PEC), an electric current $\vec{j}$ will be induced on the PEC. All possible working modes $\vec{j}$ constitute a linear space called modal space [12, 21, 22, 30].

If $\vec{j}$ is expanded in terms of independent and complete basis functions, there exists a one-to-one correspondence between $\vec{j}$ and its expansion vector $\vec{a}$ [12, 22, 30], and the linear space constituted by all possible $\vec{a}$ is called expansion vector space (where $\vec{a}$ is the vector constituted by all expansion coefficients). The following parts of this paper are discussed in the expansion vector space and frequency domain.

In the expansion vector space, the complex power $P$ done by $\vec{E}$ on $\vec{j}$ has the matrix form $P = \vec{a}^H \cdot \vec{P} \cdot \vec{a}$, and then the radiated power $P_{rad} = \text{Re} \{P\}$ and imaginary power $P_{imag} = \text{Im} \{P\}$ can be expressed in their matrix forms as $P_{rad} = \vec{a}^H \cdot \vec{P}_{rad} \cdot \vec{a}$ and $P_{imag} = \vec{a}^H \cdot \vec{P}_{imag} \cdot \vec{a}$ [29, 30]. Here, the superscript “$H$” represents the transpose conjugate of a matrix or vector, and the method to obtain the matrix $\vec{P}$ (which is identical to the impedance matrix $\vec{Z}$ derived from discretizing the electric field integral equation by employing Galerkin’s method, except a coefficient 1/2) can be found in [22, 25, 29, 30]; $\vec{P}_{rad} = (\vec{P} + \vec{P}^H)/2$ and $\vec{P}_{imag} = (\vec{P} - \vec{P}^H)/2j$ [29, 30].

2.1. Traditional Modal Classification. The matrix $\vec{P}_{rad}$ is positively semidefinite [30], so $\vec{a}^H \cdot \vec{P}_{rad} \cdot \vec{a} \geq 0$ for any $\vec{a}$, and the modes corresponding to $\vec{a}^H \cdot \vec{P}_{rad} \cdot \vec{a} = 0$ and $\vec{a}^H \cdot \vec{P}_{rad} \cdot \vec{a} > 0$ are called nonradiative modes and radiative modes, respectively. In addition, the semidefiniteness of the matrix $\vec{P}_{rad}$ implies that $\vec{a}^H \cdot \vec{P}_{rad} \cdot \vec{a} = 0$ if and only if $\vec{P}_{rad} \cdot \vec{a} = 0$ [31], i.e.,

$$\vec{P}_{rad} \cdot \vec{a} = 0 \iff \vec{a}^H \cdot \vec{P}_{rad} \cdot \vec{a} = 0 \iff \text{mode } \vec{a} \text{ is nonradiative.} \quad (1)$$

Thus, all nonradiative modes $\vec{a}_{\text{non-rad}}$ constitute a linear space (i.e., the null space of $\vec{P}_{rad}$ [31]) called nonradiation space (which is identical to the internal resonance space [30]), and any $\vec{a}_{\text{non-rad}}$ satisfies the following orthogonality:

$$\vec{a}^H \cdot \vec{P}_{rad} \cdot \vec{a}_{\text{non-rad}} = 0 = (\vec{a}_{\text{non-rad}})^H \cdot \vec{P}_{rad} \cdot \vec{a}, \quad (2)$$

for any working mode $\vec{a}$ (because $\vec{P}_{rad}$ is Hermitian [30]).

The matrix $\vec{P}_{imag}$ is indefinite [22, 30], so $\vec{a}^H \cdot \vec{P}_{imag} \cdot \vec{a}$ can be negative, zero, or positive, and the modes corresponding to $\vec{a}^H \cdot \vec{P}_{imag} \cdot \vec{a} < 0$, $\vec{a}^H \cdot \vec{P}_{imag} \cdot \vec{a} = 0$, and $\vec{a}^H \cdot \vec{P}_{imag} \cdot \vec{a} > 0$ are called capacitive modes, resonant modes, and inductive modes, respectively [11, 22, 29, 30]. According to whether the resonant modes radiate EM energy, the resonant modes are further classified into internal resonant modes (which do not radiate, so this paper calls them nonradiative resonant modes) and external resonant modes (which radiate, so this paper calls them radiative resonant modes) [12, 27, 30]. As demonstrated in [30] and Section 5 of this paper, the nonradiative modes must be resonant, so all capacitive and inductive modes must be radiative, and then this paper calls them radiative capacitive and inductive modes, respectively.

2.2. New Modal Classification. Besides traditionally classifying all modes into radiative capacitive modes, resonant modes (including nonradiative resonant modes and radiative resonant modes), and radiative inductive modes, an alternative classification for the resonant modes is proposed in this section.

The matrix $\vec{P}_{imag}$ is indefinite, so $\vec{a}^H \cdot \vec{P}_{imag} \cdot \vec{a} = 0$ does not imply that $\vec{P}_{imag} \cdot \vec{a} = 0$ [31], though $\vec{P}_{imag} \cdot \vec{a} = 0$ always implies that $\vec{a}^H \cdot \vec{P}_{imag} \cdot \vec{a} = 0$. This is equivalent to

$$\vec{P}_{imag} \cdot \vec{a} = 0 \iff \vec{a}^H \cdot \vec{P}_{imag} \cdot \vec{a} = 0 \iff \text{mode } \vec{a} \text{ is resonant,} \quad (3)$$

i.e., the condition $\vec{P}_{imag} \cdot \vec{a} = 0$ is stronger than the condition $\vec{a}^H \cdot \vec{P}_{imag} \cdot \vec{a} = 0$ to guarantee resonance. Based on this, $\vec{P}_{imag} \cdot \vec{a} = 0$ can be particularly called intrinsic resonance condition, if $\vec{a}^H \cdot \vec{P}_{imag} \cdot \vec{a} = 0$ is viewed as resonance condition.

Correspondingly, the modes satisfying $\vec{P}_{imag} \cdot \vec{a} = 0$ are called intrinsically resonant modes, and the resonant modes not satisfying $\vec{P}_{imag} \cdot \vec{a} = 0$ are called nonintrinsically resonant modes. Obviously, all intrinsically resonant modes constitute a linear space, i.e., the null space of $\vec{P}_{imag}$, and this space is called intrinsic resonance space. Similar to (2), any intrinsically resonant mode $\vec{a}_{\text{int-rad}}$ satisfies (4) for any $\vec{a}$:

$$\vec{a}^H \cdot \vec{P}_{imag} \cdot \vec{a}_{\text{int-rad}} = 0 = (\vec{a}_{\text{int-rad}})^H \cdot \vec{P}_{imag} \cdot \vec{a}. \quad (4)$$

When the intrinsically resonant mode $\vec{a}_{\text{int-rad}}$ satisfies the condition $(\vec{a}_{\text{int-rad}})^H \cdot \vec{P}_{rad} \cdot \vec{a}_{\text{int-rad}} = 0$, it is called nonradiative intrinsically resonant mode and correspondingly denoted as $\vec{a}_{\text{int-rad}}$. When the intrinsically resonant mode $\vec{a}_{\text{int-rad}}$ satisfies the condition $(\vec{a}_{\text{int-rad}})^H \cdot \vec{P}_{rad} \cdot \vec{a}_{\text{int-rad}} > 0$, it
is called radiative intrinsically resonant mode and correspondingly denoted as \( \alpha_{\text{rad}} \). As demonstrated in [30], \( \mathbf{P}^{\text{imag}} \cdot \alpha = 0 \), if \( \mathbf{P}_{\text{rad}} \cdot \alpha = 0 \). This implies that the intrinsic resonance space contains the whole nonradiation space. Then, the set constituted by all \( \alpha_{\text{int}}^{\text{non-rad}} \) must be a linear space, and this space is just the nonradiation space; all nonintrinsically resonant modes \( \alpha_{\text{non-int}}^{\text{non-rad}} \) are radiative, and they are particularly denoted as \( \alpha_{\text{rad}}^{\text{non-int}} \), for any mode \( \alpha \), \( \alpha_{\text{int}}^{\text{non-rad}} \) satisfies orthogonality:

\[
\bar{\alpha}^H \cdot \mathbf{P}_{\text{rad}} \cdot \alpha_{\text{int}}^{\text{non-rad}} = 0 = (\bar{\alpha}^H \cdot \mathbf{P}_{\text{rad}}) \cdot \bar{\alpha},
\]

\[
\bar{\alpha}^H \cdot \mathbf{P}_{\text{imag}} \cdot \alpha_{\text{non-rad}} = 0 = (\bar{\alpha}^H \cdot \mathbf{P}_{\text{imag}}) \cdot \bar{\alpha}.
\] (5)

In summary, by introducing the concepts of intrinsic resonance and nonintrinsic resonance, this section alternatively classifies all resonant modes into nonradiative intrinsically resonant modes \( \alpha_{\text{int}}^{\text{non-rad}} \), radiative intrinsically resonant modes \( \alpha_{\text{rad}}^{\text{int}} \), and radiative nonintrinsically resonant modes \( \alpha_{\text{rad}}^{\text{non-int}} \). Because the nonradiative intrinsically resonant modes \( \alpha_{\text{int}}^{\text{non-rad}} \) are the just the traditional internal resonant modes, the introduction of the radiative intrinsically resonant modes \( \alpha_{\text{rad}}^{\text{int}} \) and the radiative nonintrinsically resonant modes \( \alpha_{\text{rad}}^{\text{non-int}} \) essentially a subdivision for the traditional external resonant modes.

In addition, a similar subdivision for the capacitive modes and inductive modes will be provided in Section 4.

2.3. Classification for Characteristic Modes. Because both the above traditional modal classification and new modal classification are suitable for the whole modal space, they are also valid for the CM set \{\( \alpha_{\text{int}} \)\}. Here, the symbol \( \{\alpha_{\text{int}}\} \) is used to represent the expansion vector of CM \( \mathbf{J}^{\text{int}} \) in order to be distinguished from the expansion vector \( \bar{\alpha} \) of the general mode \( \mathbf{J} \).

2.3.1. Traditional Classification for CMs. Traditionally, the CM set \{\( \alpha_{\text{int}} \)\} is divided into four subsets [22–30]: radiative capacitive CM set \( \{\alpha_{\text{cap}}^{\text{res}}\} \), nonradiative resonant CM set \( \{\alpha_{\text{non-rad}}^{\text{res}}\} \), radiative resonant CM set \( \{\alpha_{\text{res}}^{\text{res}}\} \), and radiative inductive CM set \( \{\alpha_{\text{ind}}^{\text{res}}\} \). For the convenience of the following parts of this section, the nonradiative and radiative resonant CMs are collectively referred to as resonant CMs, and the union of sets \( \{\alpha_{\text{cap}}^{\text{res}}\} \) and \( \{\alpha_{\text{res}}^{\text{res}}\} \) is correspondingly denoted as \( \{\alpha_{\text{res}}^{\text{res}}\} \), i.e., \( \{\alpha_{\text{res}}^{\text{res}}\} = \{\alpha_{\text{cap}}^{\text{res}}\} \cup \{\alpha_{\text{res}}^{\text{res}}\} \).

2.3.2. An Alternative Classification for Resonant CMs. As demonstrated in [22–30], all \( \alpha_{\text{res}}^{\text{res}} \) satisfy the characteristic equation \( \mathbf{P}_{\text{imag}} \cdot \alpha_{\text{res}}^{\text{res}} = 0 \). In fact, this equation is just the intrinsic resonance condition introduced in Section 2.2, so all \( \alpha_{\text{res}}^{\text{res}} \) are intrinsically resonant, and then they are particularly denoted as \( \alpha_{\text{int}}^{\text{res}} \). Correspondingly, \( \alpha_{\text{non-rad}}^{\text{res}} \) and \( \alpha_{\text{rad}}^{\text{res}} \) are particularly denoted as \( \alpha_{\text{int}}^{\text{res}} \) and \( \alpha_{\text{rad}}^{\text{res}} \), respectively.

All \( \alpha_{\text{int}}^{\text{res}} \) are independent of each other [22–30], and the rank of the set \( \{\alpha_{\text{int}}^{\text{res}}\} \) equals to the rank of the null space of \( \mathbf{P}_{\text{imag}} \), so they constitute the basis of the intrinsic resonance space [31], i.e., any intrinsically resonant mode \( \alpha_{\text{int}}^{\text{res}} \) can be uniquely expanded in terms of \( \{\alpha_{\text{int}}^{\text{res}}\} \). In addition, \( \{\alpha_{\text{int}}^{\text{res}}\} \) constitute the basis of the nonradiation space [30], i.e., any nonradiative mode \( \alpha_{\text{non-rad}} \) can be uniquely expanded in terms of \( \{\alpha_{\text{int}}^{\text{res}}\} \).

3. Modal Expansion

In this section, a further discussion on the CM-based modal expansions for various modes is provided, based on the new modal classification proposed in Section 2.

3.1. Modal Expansion for General Modes. Based on the independence property and completeness of the CM set \( \{\alpha_{\text{int}}\} \) [22–30], any mode \( \bar{\alpha} \) can be uniquely expanded in terms of some radiative capacitive CMs \( \alpha_{\text{cap}}^{\text{res}} \), some nonradiative resonant CMs \( \alpha_{\text{non-rad}}^{\text{res}} \), some radiative resonant CMs \( \alpha_{\text{res}}^{\text{res}} \), and some radiative inductive CMs \( \alpha_{\text{ind}}^{\text{res}} \) as

\[
\bar{\alpha} \sim \sum \alpha_{\text{cap}}^{\text{res}} + \sum \alpha_{\text{res}}^{\text{res}} + \sum \alpha_{\text{ind}}^{\text{res}}
\]

where the reason to use “\(~\)”, instead of “\(=\)”, will be explained in Section 4. Based on the expansion (6), some valuable conclusions shown in Figure 1 can be derived, and they are proved as follows:

(i) The proof for “1” is obvious

(ii) The proof for “2” it is obvious that \( \mathbf{P}_{\text{imag}} \cdot \alpha = 0 \), so mode 0 is intrinsically resonant. Thus, if \( \sum \alpha_{\text{cap}}^{\text{res}} + \sum \alpha_{\text{res}}^{\text{res}} = 0 \), then \( \sum \alpha_{\text{cap}}^{\text{res}} + \sum \alpha_{\text{res}}^{\text{res}} \) is intrinsically resonant.

(iii) The proofs for “3” and “7”: it is obvious that the term \( \sum \alpha_{\text{cap}}^{\text{res}} + \sum \alpha_{\text{res}}^{\text{res}} \) is intrinsically resonant. Thus, the mode \( \bar{\alpha} \) is intrinsically resonant, and if only if the term \( \sum \alpha_{\text{cap}}^{\text{res}} + \sum \alpha_{\text{res}}^{\text{res}} \) is intrinsically resonant, based on the intrinsic resonance condition introduced in Section 2.2.

(iv) The proof for “4” is obvious, because of (3)

(v) The proofs for “5” and “6”: because the term \( \sum \alpha_{\text{cap}}^{\text{res}} + \sum \alpha_{\text{res}}^{\text{res}} \) is intrinsically resonant, the imaginary power of mode \( \bar{\alpha} \) equals to the imaginary power of the term \( \sum \alpha_{\text{cap}}^{\text{res}} + \sum \alpha_{\text{res}}^{\text{res}} \) due to the orthogonality (4). Thus, both the “5” and “6” hold.
As pointed out in Section 2.3 and [30], any nonradiative intrinsically resonant mode $\tilde{\alpha}_{\text{non-rad}}^{\text{int res}}$ can be expanded as follows:

$$\tilde{\alpha}_{\text{non-rad}}^{\text{int res}} \sim \sum_{\text{non-rad}}^{\text{int res}} \tilde{\alpha}_{\text{non-rad}}^{\text{int res}}.$$  \hfill (9)

However, it cannot be guaranteed that the nonradiative term $\sum_{\text{non-rad}}^{\text{int res}} \tilde{\alpha}_{\text{non-rad}}^{\text{int res}}$ in the modal expansion of radiative intrinsically resonant mode $\tilde{\alpha}_{\text{rad}}^{\text{int res}}$ is zero because of (5), i.e.,

$$\tilde{\alpha}_{\text{rad}}^{\text{int res}} \equiv \sum_{\text{non-rad}}^{\text{int res}} \tilde{\alpha}_{\text{non-rad}}^{\text{int res}} + \sum_{\text{rad}}^{\text{int res}} \tilde{\alpha}_{\text{rad}}^{\text{int res}},$$  \hfill (10)

where the reason to use “≡” instead of “=” will be explained in Section 4.

### 3.4. Modal Expansion for Nonintrinsically Resonant Modes

If a nonintrinsically resonant mode $\tilde{\alpha}_{\text{rad}}^{\text{non-int res}}$ is expanded as follows:

$$\tilde{\alpha}_{\text{rad}}^{\text{non-int res}} \equiv \sum_{\text{cap}}^{\text{cap}} \tilde{\alpha}_{\text{cap}}^{\text{cap}} + \sum_{\text{cap}}^{\text{cap}} \sum_{\text{rad}}^{\text{cap}} \tilde{\alpha}_{\text{cap}}^{\text{cap}} + \sum_{\text{non-rad}}^{\text{non-rad}} \tilde{\alpha}_{\text{non-rad}}^{\text{non-rad}},$$  \hfill (11)

it can be concluded that

$$\sum_{\text{cap}}^{\text{cap}} \tilde{\alpha}_{\text{cap}}^{\text{cap}} + \sum_{\text{cap}}^{\text{cap}} \sum_{\text{rad}}^{\text{cap}} \tilde{\alpha}_{\text{cap}}^{\text{cap}} + \sum_{\text{non-rad}}^{\text{non-rad}} \tilde{\alpha}_{\text{non-rad}}^{\text{non-rad}} \neq 0,$$  \hfill (12)

based on Figure 1. In fact, it can be further concluded that

$$\sum_{\text{cap}}^{\text{cap}} \tilde{\alpha}_{\text{cap}}^{\text{cap}} \neq 0,$$  \hfill (13)

$$\sum_{\text{cap}}^{\text{cap}} \sum_{\text{rad}}^{\text{cap}} \tilde{\alpha}_{\text{cap}}^{\text{cap}} \neq 0,$$

because if the capacitive term is nonzero and the inductive term is zero, then the imaginary power of $\tilde{\alpha}_{\text{rad}}^{\text{non-int res}}$ is negative due to (4), which leads to a contradiction; if the capacitive term is zero and the inductive term is nonzero, then the imaginary power of $\tilde{\alpha}_{\text{rad}}^{\text{non-int res}}$ is positive due to (4), which leads to a contradiction; if both the capacitive and inductive terms are zero, then the term $\sum_{\text{cap}}^{\text{cap}} \tilde{\alpha}_{\text{cap}}^{\text{cap}} + \sum_{\text{cap}}^{\text{cap}} \sum_{\text{rad}}^{\text{cap}} \tilde{\alpha}_{\text{cap}}^{\text{cap}} + \sum_{\text{non-rad}}^{\text{non-rad}} \tilde{\alpha}_{\text{non-rad}}^{\text{non-rad}}$ must be zero, which leads to a contradiction with (12).

### 4. Modal Decomposition

If the terms $\sum_{\text{cap}}^{\text{cap}} \tilde{\alpha}_{\text{cap}}^{\text{cap}}$, $\sum_{\text{cap}}^{\text{cap}} \sum_{\text{rad}}^{\text{cap}} \tilde{\alpha}_{\text{cap}}^{\text{cap}}$, $\sum_{\text{int}}^{\text{res}} \tilde{\alpha}_{\text{rad}}^{\text{int res}}$, and $\sum_{\text{res}}^{\text{int}} \tilde{\alpha}_{\text{rad}}^{\text{int res}}$ in the above-mentioned modal expansion formulations are denoted as $\tilde{\alpha}_{\text{rad}}$, $\tilde{\alpha}_{\text{non-rad}}$, $\tilde{\alpha}_{\text{rad}}^{\text{int res}}$, and $\tilde{\alpha}_{\text{rad}}$, respectively, then the CM-based modal expansions (6), (7), (8), (9), (10), and (11) can be alternatively written as follows:

$$\tilde{\alpha} \sim \tilde{\alpha}_{\text{rad}} + \tilde{\alpha}_{\text{non-rad}} + \tilde{\alpha}_{\text{rad}}^{\text{int res}} + \tilde{\alpha}_{\text{rad}}^{\text{int res}}.$$  \hfill (14)

Figure 1: Some “equivalence” relationships related to various resonances.
and
\[ a_{\text{res}} = \beta_{\text{cap}}^{\text{res}} + \beta_{\text{int}}^{\text{res}} + \beta_{\text{int}}^{\text{res}} + \beta_{\text{ind}}^{\text{res}}, \tag{15} \]
\[ a_{\text{int}}^{\text{res}} \sim \beta_{\text{non-rad}}^{\text{res}} + \beta_{\text{rad}}^{\text{res}}, \tag{16} \]
\[ a_{\text{int}}^{\text{non-rad}} \sim \beta_{\text{non-rad}}^{\text{res}}, \tag{17} \]
\[ a_{\text{rad}}^{\text{int}} \equiv \beta_{\text{non-rad}}^{\text{res}} + \beta_{\text{rad}}^{\text{res}}, \tag{18} \]
\[ a_{\text{non-int}}^{\text{res}} \equiv \beta_{\text{cap}}^{\text{res}} + \beta_{\text{int}}^{\text{res}} + \beta_{\text{int}}^{\text{res}} + \beta_{\text{ind}}^{\text{res}}, \tag{19} \]

where to utilize the symbol “\( \beta \)” is to emphasize that these terms are the building block terms in CM-based modal expansions. The formulations (14), (15), (16), (17), (18), and (19) are, respectively, called the electromagnetic-power-based (EMP-based) modal decompositions for general modes and various resonant modes. In fact, the EMP-based modal decompositions for any radiative capacitive mode \( a_{\text{cap}}^{\text{rad}} \) and any radiative inductive mode \( a_{\text{ind}}^{\text{rad}} \) can be similarly expressed as follows:
\[ a_{\text{cap}}^{\text{rad}} \equiv \beta_{\text{cap}}^{\text{rad}} + \beta_{\text{non-rad}}^{\text{rad}} + \beta_{\text{rad}}^{\text{rad}} + \beta_{\text{ind}}^{\text{rad}}, \tag{20} \]
\[ a_{\text{ind}}^{\text{rad}} \equiv \beta_{\text{cap}}^{\text{rad}} + \beta_{\text{non-rad}}^{\text{rad}} + \beta_{\text{rad}}^{\text{rad}} + \beta_{\text{ind}}^{\text{rad}}. \tag{21} \]

As the continuation of the conclusions given in Sections 2 and 3, the following further conclusions can be derived from the above EMP-based modal decompositions (14), (15), (16), (17), (18), (19), (20), and (21).

(i) In (14), (16), and (17), all the terms in the right-hand sides of these expansions can be zero or nonzero. In (15), \( \beta_{\text{non-rad}}^{\text{res}} \) and \( \beta_{\text{rad}}^{\text{res}} \) can be zero or nonzero, and \( \beta_{\text{cap}}^{\text{rad}} \) and \( \beta_{\text{rad}}^{\text{rad}} \) marked by single underlines can be simultaneously zero or simultaneously nonzero. In (19), \( \beta_{\text{cap}}^{\text{rad}} \) and \( \beta_{\text{rad}}^{\text{rad}} \) can be zero or nonzero, and \( \beta_{\text{int}}^{\text{res}} \) and \( \beta_{\text{ind}}^{\text{res}} \) marked by double underlines must be simultaneously nonzero. In (18), (20), and (21), the terms marked by double underlines must be nonzero. These are the just reasons to use “\( \sim \)”, “\( = \)”, and “\( \equiv \)” in the modal expansions (6), (7), (8), (9), (10), and (11) and the modal decompositions (14), (15), (16), (17), (18), (19), (20), and (21).

(ii) Because the term \( \beta_{\text{non-rad}}^{\text{res}} \) in (18) can be nonzero, then the set constituted by all radiative intrinsically resonant modes is not closed for addition, so all radiative intrinsically resonant modes cannot constitute a linear space [31]. Obviously, similar conclusions hold for the set constituted by all nonintrinsically resonant modes, the set constituted by all radiative capacitive modes, and the set constituted by all radiative inductive modes, because of the modal decompositions (19), (20), and (21). In addition, all resonant modes also cannot constitute a linear space. For example, if the imaginary powers of CMs \( \beta_{\text{cap}}^{\text{rad}} \) and \( \beta_{\text{rad}}^{\text{rad}} \) are normalized to \(-1 \) and \( 1 \), respectively, then the modes \( a = \beta_{\text{cap}}^{\text{rad}} + \beta_{\text{ind}}^{\text{rad}} \) and \( a = \beta_{\text{rad}} + e^{i\alpha} \beta_{\text{rad}}^{\text{rad}} \) must be resonant for any \( \alpha \in \mathbb{R} \), due to the orthogonality of CMs [22, 30]. However, the mode \( a + a \) might be nonresonant, because of the arbitrariness of \( \phi \). This implies that the set constituted by all resonant modes is not closed for addition.

(iii) Equation (15) implies that \( a_{\text{res}}^{\text{res}} \) might contain the \( \beta_{\text{rad}}^{\text{res}} \), \( \beta_{\text{non-rad}}^{\text{res}} \), and \( \beta_{\text{ind}}^{\text{res}} \) terms. Equations (16) and (18) imply that \( a_{\text{int}}^{\text{res}} \) and \( a_{\text{ind}}^{\text{res}} \) might contain the \( \beta_{\text{rad}}^{\text{res}} \) term. Equation (19) implies that \( a_{\text{non-int}}^{\text{res}} \) must contain the \( \beta_{\text{cap}}^{\text{rad}} \) and \( \beta_{\text{ind}}^{\text{rad}} \) terms. In fact, this is one of the reasons why the resonant modes cannot guarantee the most efficient radiation as observed in [28].

(iv) Equations (16), (17), and (18) imply that the modal decompositions for various intrinsically resonant modes only include the intrinsically resonant building block terms. In fact, this also implies that the modal compositions of various intrinsically resonant modes are more pure than the modal compositions of general resonant modes and nonintrinsically resonant modes, and these are just the reasons to call the equation \( \beta_{\text{mag}}^{\text{mag}} \cdot a = 0 \) an intrinsically resonant condition and to call the modes satisfying \( \beta_{\text{mag}}^{\text{mag}} \cdot a = 0 \) intrinsically resonant modes. Similarly, the radiative capacitive mode is particularly called radiative intrinsically capacitive mode and correspondingly denoted as \( a_{\text{cap}}^{\text{rad}} \), if the terms \( \beta_{\text{cap}}^{\text{rad}} \) and \( \beta_{\text{ind}}^{\text{rad}} \), and \( \beta_{\text{rad}}^{\text{rad}} \) in its modal decomposition are zero; the radiative inductive mode is particularly called radiative intrinsically inductive mode and correspondingly denoted as \( a_{\text{ind}}^{\text{rad}} \), if the terms \( \beta_{\text{cap}}^{\text{rad}} \) and \( \beta_{\text{ind}}^{\text{rad}} \), and \( \beta_{\text{rad}}^{\text{rad}} \) in its modal decomposition are zero. According to this, it is obvious that there are intrinsically capacitive and intrinsically inductive, respectively, so they are correspondingly denoted as \( \beta_{\text{cap}}^{\text{rad}} \beta_{\text{ind}}^{\text{rad}} \) and \( \beta_{\text{cap}}^{\text{rad}} \beta_{\text{ind}}^{\text{rad}} \) themselves are intrinsically capacitive and intrinsically inductive, respectively, so they are correspondingly denoted as \( \beta_{\text{cap}}^{\text{rad}} \beta_{\text{rad}}^{\text{rad}} \) and \( \beta_{\text{cap}}^{\text{rad}} \beta_{\text{rad}}^{\text{rad}} \). In addition, the radiative capacitive modes which are not intrinsically capacitive are particularly called radiative nonintrinsically capacitive modes and correspondingly denoted as \( a_{\text{rad}}^{\text{non-int cap}} \); the radiative inductive modes which are not intrinsically inductive are particularly called radiative nonintrinsically inductive modes and correspondingly denoted as \( a_{\text{rad}}^{\text{non-int ind}} \). Based on the above, the modal
decompositions (14), (15), (16), (17), (18), (19), (20), and (21) can be further rewritten as follows:

\[
\bar{a} = \bar{\rho}^{\text{int\ cap}} + \bar{\rho}^{\text{int\ res}} + \bar{\rho}^{\text{in\ res}} + \bar{\rho}^{\text{in\ ind}},
\]

and

\[
\bar{\alpha} = \bar{\rho}^{\text{int\ cap}} + \bar{\rho}^{\text{int\ res}} + \bar{\rho}^{\text{in\ res}} + \bar{\rho}^{\text{in\ ind}},
\]

\[
\bar{\alpha} = \bar{\rho}^{\text{int\ cap}} + \bar{\rho}^{\text{int\ res}} + \bar{\rho}^{\text{in\ res}} + \bar{\rho}^{\text{in\ ind}},
\]

\[
\bar{\alpha} = \bar{\rho}^{\text{int\ cap}} + \bar{\rho}^{\text{int\ res}} + \bar{\rho}^{\text{in\ res}} + \bar{\rho}^{\text{in\ ind}},
\]

and

\[
\bar{\alpha} = \bar{\rho}^{\text{int\ cap}} + \bar{\rho}^{\text{int\ res}} + \bar{\rho}^{\text{in\ res}} + \bar{\rho}^{\text{in\ ind}},
\]

\[
\bar{\alpha} = \bar{\rho}^{\text{int\ cap}} + \bar{\rho}^{\text{int\ res}} + \bar{\rho}^{\text{in\ res}} + \bar{\rho}^{\text{in\ ind}},
\]

\[
\bar{\alpha} = \bar{\rho}^{\text{int\ cap}} + \bar{\rho}^{\text{int\ res}} + \bar{\rho}^{\text{in\ res}} + \bar{\rho}^{\text{in\ ind}},
\]

\[
\bar{a} = \bar{\rho}^{\text{int\ cap}} + \bar{\rho}^{\text{int\ res}} + \bar{\rho}^{\text{in\ res}} + \bar{\rho}^{\text{in\ ind}},
\]

and

\[
\bar{a} = \bar{\rho}^{\text{int\ cap}} + \bar{\rho}^{\text{int\ res}} + \bar{\rho}^{\text{in\ res}} + \bar{\rho}^{\text{in\ ind}},
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\bar{a} = \bar{\rho}^{\text{int\ cap}} + \bar{\rho}^{\text{int\ res}} + \bar{\rho}^{\text{in\ res}} + \bar{\rho}^{\text{in\ ind}},
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\bar{a} = \bar{\rho}^{\text{int\ cap}} + \bar{\rho}^{\text{int\ res}} + \bar{\rho}^{\text{in\ res}} + \bar{\rho}^{\text{in\ ind}},
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\bar{a} = \bar{\rho}^{\text{int\ cap}} + \bar{\rho}^{\text{int\ res}} + \bar{\rho}^{\text{in\ res}} + \bar{\rho}^{\text{in\ ind}},
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\bar{a} = \bar{\rho}^{\text{int\ cap}} + \bar{\rho}^{\text{int\ res}} + \bar{\rho}^{\text{in\ res}} + \bar{\rho}^{\text{in\ ind}},
\]

\[
\bar{a} = \bar{\rho}^{\text{int\ cap}} + \bar{\rho}^{\text{int\ res}} + \bar{\rho}^{\text{in\ res}} + \bar{\rho}^{\text{in\ ind}},
\]

In (25) and (33), the terms \(\bar{\rho}^{\text{int\ res}} + \bar{\rho}^{\text{in\ res}} + \bar{\rho}^{\text{in\ ind}}\) and \(\bar{\rho}^{\text{int\ cap}} + \bar{\rho}^{\text{int\ res}} + \bar{\rho}^{\text{in\ res}}\) marked by double underlines must be nonzero, and this is just the essential characteristic of the nonintrinsically capacitive and inductive modes

(v) As concluded in Section 2, all CMs \(\{\bar{\alpha}_\xi\}\) constitute the basis of the whole modal space, and all intrinsically resonant CMs \(\{\bar{\alpha}_\xi^\text{res}\}\) constitute the basis of the whole intrinsic resonance space, and all nonradiative intrinsically resonant CMs \(\{\bar{\alpha}_\xi^\text{res\ non\ rad}\}\) constitute the basis of the whole nonradiation space. In fact, based on the conclusions obtained in the above parts of Section 4, it can be further concluded that the union of the mode 0 and all radiative intrinsically capacitive modes \(\bar{\alpha}_\xi^\text{cap\ rad}\) (i.e., \(\{0\} \cup \{\bar{\alpha}_\xi^\text{cap\ rad}\}\)) is a linear space called intrinsic capacitance space, which is spanned by all radiative intrinsically capacitive CMs \(\{\bar{\alpha}_\xi^\text{cap\ rad}\}\), and the union of the mode 0 and all radiative intrinsically inductive modes \(\bar{\alpha}_\xi^\text{ind\ rad}\) (i.e., \(\{0\} \cup \{\bar{\alpha}_\xi^\text{ind\ rad}\}\)) is a linear space called intrinsic inductance space, which is spanned by all radiative intrinsically inductive CMs \(\{\bar{\alpha}_\xi^\text{ind\ rad}\}\).

The relationships of various modes are illustrated in Figure 2. The modal classes in the solid-line boxes are linear spaces. The modal classes in the dotted-line boxes will become linear spaces, if the mode 0 is added to the classes.

5. Examples

In this section, some typical examples are provided for verifying the conclusions given in this paper.

5.1. PEC Sphere. In this section, a PEC sphere whose radius is 32 mm is considered and its triangular mesh is shown in Figure 3. The CMs of the PEC sphere shown in Figure 3 are obtained by using the methods provided in [22, 30], and they are normalized by using the method proposed in [30]. In [30], the normalized modal powers are alternatively called modal impedances, because their dimensions are ohm as explained in the appendix of this paper.

The normalized radiated powers corresponding to some typical CMs are shown in Figure 4, and the normalized imaginary powers corresponding to the CMs in Figure 4 are shown in Figure 5.

From Figures 4 and 5, it is obvious that the CMs are resonant when they are nonradiative [30], such as CM 1 at 4.10 GHz and 9.15 GHz, CM 2 at 5.80 GHz, CM 3 at 6.75 GHz, CM 4 at 7.45 GHz, CM 5 at 8.65 GHz, and CM 6 at 9.05 GHz. The working frequencies and the orders of degeneracy corresponding to these nonradiative CMs are listed in the first column of Table 1, and the working frequencies and the orders of degeneracy corresponding to the internal resonant eigenmodes derived from the analytical
method of EMT [1, 2] are listed in the second column of Table 1. The distributions of the modal currents corresponding to all degenerate nonradiative CMs at 4.10 GHz are shown in Figure 6. Taking the first modal current shown in Figure 6 as a typical example, the amplitude distribution of its tangential modal electric field is shown in Figure 7. From Figure 7, it is obvious that the first mode shown in Figure 6 is an internal resonant mode, because its tangential modal electric field is zero on the whole PEC boundary. From Table 1 and Figures 6 and 7, it is obvious that all nonradiative CMs at 4.10 GHz constitute the basis of the nonradiation space (i.e., the internal resonance space) at 4.10 GHz. Thus, the nonradiative CMs are equivalent to the internal resonant eigenmodes from the aspect of spanning whole nonradiation space (i.e., the internal resonance space).

But not all of the resonant CMs are nonradiative as shown in Figures 4 and 5, such as CM 3 at 4.10 GHz and 9.15 GHz, CM 5 at 5.80 GHz, CM 1 at 6.70 GHz, and CM 2 at 8.65 GHz. In fact, these radiative resonant CMs are just radiative intrinsically resonant modes. Taking all of the degenerate radiative resonant CMs at 4.10 GHz
as examples, their modal current distributions are shown in Figure 8.

In addition, at any working frequency, there may be some radiative intrinsically and nonintrinsically capacitive modes and some radiative intrinsically and nonintrinsically inductive modes. For example, at 4.10 GHz, CMs 2, 4, and 6 are intrinsically capacitive as illustrated in Figure 5, and at the same time, they are radiative as illustrated in Figure 4; at 4.10 GHz, the summation of CM 3 and CM 2 is a radiative nonintrinsically capacitive mode. At 4.10 GHz, CM 5 is intrinsically inductive as illustrated in Figure 5, and at the same time, it is radiative as illustrated in Figure 4; at 4.10 GHz, the summation of CM 3 and CM 5 is a radiative nonintrinsically inductive mode.

In summary, at a frequency of 4.10 GHz, there exist some radiative intrinsically and nonintrinsically capacitive modes, some nonradiative intrinsically resonant modes, some radiative intrinsically and nonintrinsically resonant modes, and some radiative intrinsically and nonintrinsically inductive modes. All of the nonradiative intrinsically resonant modes constitute a linear space (i.e., the nonradiation space), and all of the nonradiative intrinsically resonant CMs shown in Figure 6 constitute the basis of this space. All of the intrinsically resonant modes (including both all of the nonradiative intrinsically resonant modes and all of the radiative intrinsically resonant modes) constitute a linear space (i.e., the intrinsic resonance space), and all of the intrinsically resonant CMs (including both all of the nonradiative intrinsically resonant CMs shown in Figure 6 and all of the radiative intrinsically resonant CMs shown in Figure 8) constitute the basis of this space.

The union of the mode 0 and all of the intrinsically capacitive modes constitute a linear space (i.e., the intrinsic capacitance space), and all of the intrinsically capacitive CMs constitute the basis of this space. The union of the mode 0 and all of the intrinsically inductive modes constitute a linear space (i.e., the intrinsic inductance space), and all of the intrinsically inductive CMs constitute the basis of this space.

### Table 1: The working frequencies (GHz) and the orders of degeneracy corresponding to the first several nonradiative CMs and internal resonant eigenmodes of the PEC sphere in Figure 3.

| Nonradiative CMs | Internal resonant eigenmodes [1, 2] |
|------------------|-----------------------------------|
| 4.10 (3)         | 4.09 (3)                          |
| 5.80 (5)         | 5.77 (5)                          |
| 6.75 (3)         | 6.70 (3)                          |
| 7.45 (7)         | 7.42 (7)                          |
| 8.65 (5)         | 8.60 (5)                          |
| 9.05 (9)         | 9.05 (9)                          |
| 9.15 (3)         | 9.13 (3)                          |

5.2. PEC Circular Disk. In this section, a PEC circular disk whose radius is 20 mm and height is 20 mm is considered and its triangular mesh is shown in Figure 9. The CMs of the PEC circular disk shown in Figure 9 are obtained by using the methods provided in [22, 30], and they are normalized by

**Figure 6:** The distributions of the modal currents corresponding to all degenerate nonradiative resonant CMs at 4.10 GHz.
using the method proposed in [30]. In [30], the normalized modal powers are alternatively called modal impedances, because their dimensions are ohm as explained in the appendix of this paper.

The normalized radiated powers corresponding to some typical CMs are shown in Figures 10(a) and 10(b), respectively, and the normalized imaginary powers corresponding to the CMs shown in Figures 10(a) and 10(b) are simultaneously shown in Figure 11.

From Figures 10(a), 10(b), and 11, it is obvious that the CMs are resonant when they are nonradiative, such as CM 1 at 5.75 GHz, CM 2 at 8.69 GHz, CM 3 at 9.15 GHz, and CM 4 at 9.45 GHz. The working frequencies and the orders of degeneracy corresponding to these nonradiative CMs are listed in the first column of Table 2, and the working frequencies and the orders of degeneracy corresponding to the internal resonant eigenmodes derived from the analytical method of EMT [1, 2] are listed in the second column of Table 2.

There is one and only one nonradiative CM at 5.75 GHz, and the distribution of its modal current is shown in Figure 12 and the amplitude distribution of its tangential modal electric field is shown in Figure 13. From Figure 13, it is obvious that the nonradiative CM at 5.75 GHz (shown in Figure 12) is an internal resonant mode, because its tangential modal electric field is zero on the whole PEC boundary. In fact, the nonradiative CM at 5.75 GHz (shown in Figure 12) is just the TE111 mode of the PEC circular disk, and this mode is just the dominant TE mode of the PEC circular disk shown in Figure 9.

In addition, there exist two degenerate nonradiative CMs at 9.15 GHz, and their current distributions are shown in Figure 14. Obviously, the modes in Figure 14 are just the TM110 modes of the PEC circular disk. From Table 2 and Figure 14, it is obvious that all nonradiative CMs at 9.15 GHz constitute the basis of the nonradiation space (i.e., the internal resonance space). Thus, the nonradiative CMs are equivalent to the internal resonant eigenmodes from the aspect of spanning whole nonradiation space.

But not all of the resonant CMs are nonradiative as shown in Figures 10(a), 10(b), and 11, such as CM 5 at 6.54 GHz, CM 6 at 8.60 GHz, and CM 7 at 9.62 GHz. In fact, these radiative resonant CMs are just the radiative intrinsically resonant modes. Taking the radiative resonant CM at 6.54 GHz as an example, its modal current distribution is shown in Figure 15.

In addition, at any working frequency, there may be some radiative capacitive CMs and some radiative inductive CMs. For example, at 5.75 GHz, CMs 2, 3, and 4 are capacitive as illustrated in Figure 11, and at the same time, they are radiative as illustrated in Figure 10(a); CMs 5, 6, and 7 are inductive as illustrated in Figure 11, and at the same time, they are radiative as illustrated in Figure 10(b).

In summary, for the PEC circular disk (shown in Figure 9) working at 5.75 GHz, there exist some intrinsically capacitive modes (such as CMs 2, 3, and 4), some nonintrinsically capacitive modes (such as the summation of CM 1 and CM 2), some nonradiative intrinsically resonant modes (such as CM 1), some radiative nonintrinsically resonant modes, some intrinsically inductive modes (such as CMs 5, 6, and 7), and some nonintrinsically inductive modes (such as the summation of CM 1 and CM 5); but there does not exist any radiative intrinsically resonant mode. Thus, the nonradiative CM shown in Figure 12 constitutes the basis of the whole nonradiation space at 5.75 GHz; and this mode is just the dominant TE mode of the PEC circular disk shown in Figure 9.
resonance space is identical to the nonradiation space at 5.75 GHz.

For the PEC circular disk (shown in Figure 9) working at 6.54 GHz, there exist some intrinsically capacitive modes (such as CMs 2, 3, and 4), some nonintrinsically capacitive modes (such as the summation of CM 5 and CM 2), some radiative intrinsically resonant modes (such as CM 5), some radiative nonintrinsically resonant modes, some intrinsically inductive modes (such as CMs 1, 6, and 7), and some nonintrinsically inductive modes (such as the summation of CM 5 and CM 1); but there does not exist any nonzero nonradiative mode, i.e., the only one nonradiative mode is the mode 0. Thus, the radiative resonant CM shown in Figure 15 constitutes the basis of the whole intrinsic resonance space at 6.54 GHz; the nonradiation space at 6.54 GHz is just the \( \{0\} \).

5.3. PEC Plate. The CMs of the PEC plate (shown in Figure 16) whose size is 24 mm \( \times \) 32 mm are obtained by using the method provided in [22], and they are normalized by using the method proposed in [30]. The normalized radiated powers corresponding to some typical CMs are shown in Figure 17, and the normalized imaginary powers corresponding to the CMs in Figure 17 are shown in Figure 18.

From Figures 17 and 18, it is obvious that the PEC plate does not have any nonradiative CM in the whole frequency band 3–10 GHz, so the nonradiation space (i.e., the internal resonance space) of the PEC plate is \( \{0\} \) in band 3–10 GHz. CM 1 is radiative and resonant at 4.45 GHz, and its current distribution is shown in Figure 19(a). CM 2 is radiative and resonant at 7.01 GHz, and its current distribution is shown in Figure 19(b). CM 3 is radiative and resonant at 8.50 GHz, and its current distribution is shown in Figure 19(c). In addition, CM 4 is intrinsically capacitive at the whole band 3–10 GHz, and its current distribution is not explicitly shown here; CM 5 is intrinsically inductive at the whole band 3–10 GHz, and its current distribution is not explicitly shown here.
6. Conclusions

This paper alternatively proposes an EMP-based modal classification for all working modes of PECs, i.e., all modes are classified into radiative intrinsically and nonintrinsically capacitive modes, nonradiative intrinsically resonant modes, radiative intrinsically and nonintrinsically resonant modes, and radiative intrinsically and nonintrinsically inductive modes. Based on the new modal classification and the corresponding CM-based modal expansion, an alternative modal decomposition method is obtained, i.e., any working mode can be expressed as the superposition of a radiative intrinsically capacitive mode, a nonradiative intrinsically resonant mode, a radiative intrinsically resonant mode, and a radiative intrinsically inductive mode. In addition, some further conclusions are also obtained, for example, all intrinsically resonant modes and all nonradiative modes constitute linear spaces, respectively, and the resonant CMs and the nonradiative CMs constitute the basis of these two spaces, respectively, but other kinds of resonant modes cannot constitute linear spaces; by including the mode 0 into the intrinsically capacitive mode set and the intrinsically inductive mode set, these two modal sets become linear spaces, respectively, and the capacitive CMs and the inductive CMs constitute the basis of these two spaces, respectively, but other kinds of capacitive and inductive modes cannot constitute linear spaces. The conclusions given in this paper are verified by some typical examples (such as the modal classifications and

![Diagram](image_url)

Figure 10: (a) The normalized radiated powers corresponding to some typical CMs of the PEC circular disk shown in Figure 9; (b) the normalized radiated powers corresponding to another several typical CMs of the PEC circular disk shown in Figure 9.

![Diagram](image_url)

Figure 11: The normalized imaginary powers corresponding to the CMs shown in Figures 10(a) and 10(b).

Table 2: The working frequencies (GHz) and the orders of degeneracy corresponding to the first several nonradiative CMs and internal resonant eigenmodes of the PEC circular disk in Figure 9.

| Nonradiative CMs | Internal resonant eigenmodes [1, 2] |
|------------------|------------------------------------|
| 5.75 (1)         | 5.74 (1)                           |
| 8.69 (2)         | 8.69 (2)                           |
| 9.15 (2)         | 9.15 (2)                           |
| 9.45 (1)         | 9.45 (1)                           |
the modal decompositions for the PEC sphere, PEC circular disk, and PEC plate).

Appendix

Normalized Modal Power and Its Dimension Analysis

There exist some different kinds of normalization ways for modal power, and the most classical one may be the normalization way proposed by Prof. Harrington and Dr. Mautz [22, 23], i.e., to normalize modal radiated power to 1. In this paper, Prof. Harrington’s classical normalization way is not utilized, because in Prof. Harrington’s seminal paper [22], all nonradiative modes were not considered; in this paper, the nonradiative modes are emphasized; for the nonradiative modes, the modal radiated powers (which equal to 0) cannot be normalized to 1. Based on the above observations, another normalization way proposed in [30] is utilized in this paper, and this normalization way is suitable to any nonzero mode. In this appendix, the dimension analysis for the normalized modal power proposed in [30] is provided.

The modal power $P_\xi$ corresponding to the $\xi$th modal electric current $\mathbf{J}_\xi$ distributing on a two-dimensional surface $S$ is as follows [23, 29, 30]:

$$P_\xi = \frac{1}{2} \left\langle \mathbf{J}_\xi, \mathbf{E}_\xi \right\rangle_S,$$  

(A.1)
Figure 14: The distributions of the modal currents corresponding to all degenerate nonradiative CMs at 9.15 GHz.

Figure 15: The distribution of the modal current corresponding to the only radiative resonant CM at 6.54 GHz.
Figure 16: The triangular mesh of a PEC plate whose size is 24 mm × 32 mm.

Figure 17: The normalized radiated powers corresponding to some typical CMs of the PEC plate shown in Figure 16.

Figure 18: The normalized imaginary powers corresponding to the CMs shown in Figure 17.
where the inner product is defined as $\langle f, g \rangle_S = \iint_S f^* g \, dS$, the integral domain $S$ is the boundary of PEC, and the field $\vec{E}_\xi$ is the modal incident electric field corresponding to $\vec{J}_\xi$ [29]. Following the normalization way proposed in [30], the modal power $P_\xi$ is normalized as follows:

$$ P_\xi = \frac{1}{2} \frac{\langle \vec{J}_\xi^*, \vec{E}_\xi \rangle_S}{\langle \vec{J}_\xi^*, \vec{J}_\xi \rangle_S}. \quad (A.2) $$

For a physical or geometrical quantity $Q$, its dimension can be denoted as $[Q]$. Therefore, the dimension analysis for the normalized modal power $\bar{P}_\xi$ in (A.2) is as follows:

$$ [\bar{P}_\xi] = \frac{[\vec{J}_\xi^*] \cdot [\vec{E}_\xi] \cdot [S]}{[\vec{J}_\xi] \cdot [\vec{J}_\xi] \cdot [S]} = \frac{[\vec{E}_\xi]}{[\vec{J}_\xi]} $$

$$ = \frac{\text{volt/meter}}{\text{ampere/meter}} = \frac{\text{volt}}{\text{ampere}} = \text{ohm}, \quad (A.3) $$

because the dimension of modal incident electric field $\vec{E}_\xi$ is volt/meter [1, 11] and the dimension of modal surface electric current $\vec{J}_\xi$ is ampere/meter [1, 11]. Based on this
observation, [30] called the normalized modal power $\tilde{P}_k$ in (A.2) modal impedance, if the modal current $\tilde{J}_k$ distributes on a two-dimensional surface.

Data Availability

The figure data and table data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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