Surface Curvature and Vortex Stability
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We examine the stability of a pinned superfluid helium vortex line by measuring its persistence at elevated temperatures. Each vortex terminates at the surface of the container, at either a rounded bump or a conical indentation. We find that pinning with the bump termination is much easier to overcome thermally. This behavior would not be expected from considerations of vortex line tension alone. We take the observations as evidence of an additional contribution to the pinning energetics arising from the interaction of the superfluid order parameter singularity with the curvature of the container’s surface. By favoring pinning at points of negative Gaussian curvature, the surface interaction makes the bump a less advantageous pin site.

Topological defects play important roles in a variety of systems. Cosmic strings, defects created in the early universe, may provide an experimentally accessible test of string theories [1]. Domain walls affect the behavior of magnetic recording heads and other magnetic sensors [2], and flux line motion in superconductors introduces dissipation. Topological defects may also govern protein folding mechanisms [4]. In some situations, such as liquid crystal coatings of small particles, defects must always exist [3].

Recent work has explored how topological defects interact with surface curvature [3,6], a problem relevant to systems involving coated particles or flexible membranes. The energetics governing the variation of the order parameter for systems involving coated particles or flexible membranes. The same arguments extend to situations involving coated particles or flexible membranes. The same arguments extend to a three-dimensional system with line defects that terminate along a curved boundary.

Vortices in superfluid helium are precisely such defects. Each vortex core must either close on itself, forming a vortex ring, or terminate at a surface of the helium. This surface can be a free surface, if one exists, or a wall of the container holding the superfluid. In the latter case, experimental and computational work shows that the end of the vortex can become pinned in place.

Early measurements of torsional oscillator damping found a complicated frequency dependence that can be explained by waves along pinned vortices [7]. Measurements of thermal counterflow [5] and rotational acceleration [9] also gave evidence of pinning of vortices on rough walls. A more direct experimental verification [10] tracked the motion of a single vortex along the cell wall, including occasional pinning events when the motion ceased. In some cases the vortex worked its way free, while in others it remained pinned until the helium left its superfluid phase.

On the computational side, Schwarz uses a single vortex terminating on a half-infinite plane and exposed to a constant external flow velocity parallel to the plane [11]. For sufficiently low flow rates, if the plane has a hemispherical bump near enough to the vortex’s path, the vortex will spiral onto the bump and remain pinned there. The simulations correspond well to a naive picture of vortex pinning: by terminating at the hemisphere, the vortex has less length and correspondingly less kinetic energy. In an alternative formulation, the fluid velocity is highest atop the bump, causing a reduction in pressure that attracts the vortex to this position.

The present experiment tests the pinning of a vortex by a bump. Since a roughly hemispherical bump on a flat background has positive Gaussian curvature everywhere except near the rim where the bump meets the flat surface, the predicted curvature interaction [3,6] would favor vortex pinning around the edge of the bump, rather than at its peak.

We use a straight vibrating wire to trap a single vortex in superfluid 4He. As described elsewhere [12], we detect the vortex through a change in the beat frequency of the wire’s lowest normal modes. The measurements are done on a pumped 3He cryostat, which we rotate to create vorticity. All measurements, however, take place with the cryostat stationary. If a vortex becomes trapped along the entire length of the wire, an especially stable configuration ensues. Unless we deliberately disturb the vortex in some way, it usually remains in place until the cryostat warms to near or above the superfluid transition temperature. This behavior was the basis for the original demonstrations of quantized circulation in superfluids 3He [13,14] and 4He [15]. With mechanical or thermal perturbation, the vortex can dislodge from the wire, leading to circulation values intermediate to the expected quantum levels. A common configuration after the vortex comes free is for one end of the vortex to leave the wire and progress through the cell, terminating on the cylindrical wall [15,16]. Not surprisingly, such a partially attached vortex has an intermediate effect on the normal modes. With a 50 mm wire, we can detect the position of the attachment point in this configuration to better than 10 μm precision for a vortex detaching near the middle of the wire.

Our measurements focus on the thermal stability of a trapped vortex line. We assume that as a vortex comes free from the wire, it moves along the end of the cell until it reaches the cylindrical cell wall. If the cell endcap is flat, as in the leftmost inset of Figure 1, then the vortex length increases linearly as it leaves the wire. Furthermore, since the wire is much larger than the free vortex core, the energy per length of the vortex is larger when it is not trapped on the wire. If the vortex end is a fixed distance from the wire, minimizing the total vortex energy shows that the detached portion makes an angle with the wire, where \( \cos \theta = \frac{E_F}{E_W} \). Here \( E_F \) and \( E_W \) are the energies per length of a free vortex and of a vortex trapped on the wire, respectively. Using a free vortex core radius of 1.3 Å, a wire radius of 8 μm, and a cell radius of 1.5 mm gives \( \theta = 71^\circ \). With this value, we can then calculate the change...
in energy as the vortex end moves away from the wire, shown as the solid black curve in Figure 1. Once this energy barrier is overcome and the vortex has reached the cylindrical wall, its motion down the wall decreases the length of the trapped vortex (and the corresponding energy) without further change to the average free vortex length.

Changing the endcap geometry affects the energy considerations. If the endcap is drilled out, it resembles the middle inset of Figure 1. The half-angle is $59^\circ$, the angle of the tip of a standard twist drill. This geometry reduces the energy price for the vortex to move away from the wire, since the vortex length increases less than for a flat endcap. The dotted blue curve in Figure 1 shows this new energy.

A third geometry for the cell end, sketched in the rightmost inset of Figure 1, is a roughly hemispherical bump. Here the geometry enhances the energy as the vortex leaves the wire. The dashed red curve in Figure 1 plots the energy for a hemispherical bump with radius 1 mm.

In a conventional picture of vortex pinning, considering only the line energy of the vortex, one would expect the energy barrier to the vortex leaving the wire to be largest when the vortex terminates on a bump, and smallest when it terminates on a conical surface. However, the predicted geometric contribution to vortex energy from surface curvature alters the situation for cells that end with a bump. If the geometric term is strong enough, then the pinned vortex will not follow the wire until it reaches the bump. Rather, the vortex will leave the wire and terminate in the negative Gaussian curvature region around the edge of the bump. From there, the vortex need only traverse the remaining flat portion of the endcap to reach the cylindrical wall. Since the distance involved is shorter than if the wire began in the center of the endcap, the additional energy required to reach the cylindrical wall is lower than for either of the other two end geometries. This corresponds in Figure 1 to considering the energy change for the dashed curve between 1 mm and 1.5 mm, rather than from 0 to 1.5 mm.

The bumps are formed from Stycast 1266. We begin with a stycast surface cut flat with an end mill. To get an aspect ratio near 1 for the bump, we add a droplet of Stycast 1266 that is already partly set and is fairly viscous. Once the bump has dried, we measure its dimensions and drill a small hole through it for our wire. After the wire is in place, we add additional stycast to cover the hole that it emerges from.

The measurements described here come from three cells. Cell 1, of radius 1.5 mm, has one conical and one flat end. Cells 2 and 3, of radius 3.5 mm, each have a bump at one end with the other end flat. The larger radius in cells 2 and 3, which was used to provide space for the bump, should also increase the energy barrier for a vortex in these cells to depin. In cell 2 the bump has height 1.9 mm and radius 1.9 mm. Its cross-section along the cell end is quite regular as well, making it close to hemispherical. The bump in cell 3 has height 1.2 mm and radius 1.8 mm, with an irregular cross-section. For both cells 2 and 3 the Stycast bumps are not perfectly centered. Their closest approach to the cell wall is between 0.5 and 1.0 mm. The wire of cell 1 is centered much better, entering the cell through a hole 0.44 mm in diameter at the tip of the conical surface. Thus part of each bump is closer to the cell wall than is the wire of cell 1.

To test stability, we first trap a vortex so that it appears to cover the entire wire. We then heat the superfluid to test at what temperature the vortex comes free. Since the damping on the wire is too high for reliable measurements above 600 mK, we cool down after a few minutes to check whether an end of the vortex has dislodged. If not, we raise the temperature again, usually to a slightly higher value, and repeat the cycling until the vortex does leave the wire. In some cases the vortex remains trapped until the cryostat exhausts its $^3$He supply and warms above $T$. Figure 2 shows a typical heating sequence.
sequence, along with the signal we observe from the vortex. The temperature is raised successively to 600 mK, 800 mK, 1 K, and 1.2 K. (The temperature briefly overshoots its target each time, since the cryostat’s temperature control is optimized for rapid settling at a new temperature. The overshoots are repeatable from one thermal cycle to the next.) On cooling back to 500 mK, the vortex clearly remains in place after the first three anneals, but not after the last. We often find vortex precession signatures once the circulation level falls below $h < 1$, which means that the vortex is dislodging from the wire at one end. The extra noise in the circulation data at 600 mK comes from the increased damping of the vibrating wire.

At higher temperatures, the damping is so large that we cannot extract the circulation at all, so the circulation curve has gaps during the high-temperature regions.

We repeated this procedure with many trapped vortices on the three wires. The results are summarized in the histograms of Figure 3. The filled diamonds represent cell 1, the open circles cell 2, and the $\times$’s cell 3. The upper graph shows the highest temperature reached without dislodging the vortex, while the lower graph shows the temperatures at which vortices did come free. The two graphs do not represent exactly the same vortices. Some vortices on the upper graph never dislodged before the cryostat warmed up; in this case there is no corresponding point on the lower graph, and the temperature in the upper graph may be artificially low. Conversely, some vortices on the lower graph dislodged on their first annealing. That anneal may have been above the minimum temperature needed to dislodge the vortex, so some points on the lower graph may be artificially high. The key observation is that in the absence of the bump no vortex ever depinned below 1.6 K (lower graph), while with a bump no vortex ever remained pinned above 1 K (upper graph). This dramatic difference clearly indicates that vortices along the wire are less stable in the presence of a bump.

In one respect, the histograms do not adequately display the contrast between the cells. Each trapped vortex figures only once in each graph of Figure 3. In cell 1, a vortex is often extremely stable even at the highest annealing temperatures, lasting through not just one thermal cycle to 1.8 K but twenty or more. Thus the difference in the number of thermal cycles survived by vortices is far greater than suggested by Figure 3.

In the present experiment, we cannot directly detect which end of the vortex dislodges. We expect each cell to have a less stable end at which the vortex generally works free. Table I shows the different possibilities for where the vortex depins in each cell. Our measurement shows that the vortex depins more easily in cells 2 and 3 than in cell 1; the implication for the relative energy barriers is given in the third column of Table I. Note that in the first two cases, the measurements suggest that a vortex depins more easily from a bump than from a conical end. The second two cases give no information on the relative stability of the bump and cone; but both lead to the unphysical conclusion that a vortex depins from a flat surface more easily in a larger cell, where it must travel farther. Thus we conclude that in cells 2 and 3 the vortex does depin

![Graph showing highest stable temperatures and depinning temperatures](image)

**TABLE I:** Possible relationships among energy barriers to dislodging a vortex. The first two columns compare the energy barriers at the two ends of each cell. The third column incorporates the experimental result that the barrier is highest in cell 1. Terms in parentheses in the third column simply repeat an inequality from one of the other columns.

| Cell 1 | Cells 2 and 3 | Measurement implication |
|--------|--------------|-------------------------|
| cone < flat | bump < flat | bump < cone |
| flat < cone | bump < flat | bump < flat (< cone) |
| cone < flat | flat < bump | flat < cone < flat |
| flat < cone | flat < bump | flat < flat |
from the bump, and that the energy barrier for this process is smaller than for depinning from the conical end of cell 1.

Our stability result is the opposite of what would be expected from the simple energy considerations of Figure 11. As suggested above, an interaction between the vortex and the surface curvature may provide the explanation. Although the existing calculations consider a two-dimensional system, we note that the interaction energy \( V \) is at least comparable in magnitude to the line energies in our geometry. Here \( s \) and \( \kappa \) are the superfluid density and quantum of circulation, respectively, and \( V \) is a curvature-dependent factor of order 1.

Since our proposed scenario has the vortex pinning to the edge rather than the top of the bump, we address the possibility of a more direct measurement of vortex pinning near a bump. If a vortex trapped along the wire terminates at the edge of the bump, then the vortex must leave the wire shortly before the wire enters the top of the bump. The short length of wire with no circulation around it will slightly alter the observed beat frequency. However, the accuracy of our measurement is much lower than its precision, since an absolute calibration depends on the mass per length of the wire. Although we know this density approximately, it varies by up to 10\% among wires. Each wire is a single strand of NbTi, approximately 16 m in diameter, cut from multifilamentary superconducting magnet wire. Furthermore, the sensitivity is much lower near the end of the wire, since the influence of the vortex on the wire disappears at a vibrational node. In typical measurements one value of the beat frequency near where we expect \( N=1 \) is far more stable than any others, and we identify this value with \( N=1 \). We cannot distinguish whether this value corresponds to a vortex covering the entire wire or only 98\% of it. We would find a direct signal only if the vortex sometimes covered the entire wire and sometimes detached a short distance from the end. We would then observe more than one fairly stable circulation level in the vicinity of \( N=1 \). An appropriately shaped bump, with several metastable positions for a vortex to terminate, may allow this.

For future measurements, we can determine at which end the vortex comes free by using a cell with a diameter change in the middle. After a vortex works free, its subsequent precession, which depends strongly on the local cell diameter, will identify which half of the cell contains the detached portion. With this information we can better compare the pinning strength of different endcap geometries. Another plan for further work is to position a bump on the cylindrical wall midway along the cell, where the measurement sensitivity is highest. We have previously seen oscillation signatures for a vortex pinning on wall roughness, and it would be interesting to see how a vortex behaves on encountering a larger and better-characterized obstruction. Quantitative measurements of the pin strength may also be easier with the improved sensitivity. Our present results already demonstrate the need for an additional contribution to the energetics of a pinned vortex, such as an interaction potential between the vortex and the surface curvature.

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[1] T.W.B. Kibble, “Cosmic Strings Reborn,” astro-ph/0410073.
[2] N. Spaldin, Magnetic Materials–Fundamentals and Device Applications, (Cambridge University Press, Cambridge, 2003).
[3] M. Kléman, “Defects in liquid crystals,” Rep. Prog. Phys. 52, 555 (1989).
[4] E.D. Nelson and N.V. Grishin, “Efficient expansion, folding, and unfolding of proteins,” Phys. Rev. E70, 051906 (2004).
[5] V. Vitelli and A.M. Turner, “Anomalous coupling between topological defects and curvature,” Phys.Rev.Lett. 93, 215301 (2004) and cond-mat/0406329.
[6] V. Vitelli and D.R. Nelson, “Defect generation and deconfinement on corrugated topographies,” Phys. Rev. E70, 051105 (2004) and cond-mat/0406328.
[7] H.E. Hall, “An experimental and theoretical study of torsional oscillations in uniformly rotating liquid helium II,” Proc. Royal Soc. A245, 546 (1958).
[8] S.G. Hegde and W.I. Glaberson, “Pinning of superfluid vortices to surfaces,” Phys. Rev. Lett. 45, 190 (1980).
[9] P.W. Adams, M. Cieplak, and W.I. Glaberson, “Spin-up problem in superfluid \(^3\)He,” Phys. Rev. B32, 171 (1985).
[10] L.A.K. Donev, L. Hough, and R.J. Zieve, “Depinning of a superfluid vortex line by Kelvin waves,” Phys. Rev. B 64 180512 (2001) and cond-mat/0010240.
[11] K.W. Schwarz, “Three-dimensional vortex dynamics in superfluid \(^3\)He: Line-line and line-boundary interactions,” Phys. Rev. B 31 5782 (1985).
[12] L. Hough, L.A.K. Donev, and R.J. Zieve, “Smooth vortex precession in superfluid \(^4\)He,” Phys. Rev. B 65 024511 (2002) and cond-mat/0104525.
[13] W.F. Vinen, “The detection of a single quantum of circulation in liquid helium II,” Proc. Roy. Soc. London A260, 218 (1961).
[14] S.C. Whitmore and W. Zimmermann, Jr., “Observation of quantized circulation of superfluid helium,” Phys. Rev. 166, 181 (1968).
[15] J.C. Davis, J.D. Close, R. Zieve, and R.E. Packard, “Observation of quantized circulation in superfluid \(^3\)He-B,” Phys. Rev. Lett. 66, 329 (1991).
[16] R.J. Zieve et al., “Precession of a single vortex line in superfluid \(^3\)He,” Phys. Rev. Lett. 68, 1327 (1992).