Sensitivity analysis for filtering with preliminary processing of measurements

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Abstract. The sensitivity of estimation algorithms to the parameters of the preliminary filter is analyzed, using the problem of gravity anomaly estimation using a damped gravimeter as an example. Two models for preliminary filter are considered, which corresponds the models for gravimeter sensitive element in the example. First is a physically substantiated elastic pendulum model. Second is its simplified version in the form of a first-order system model. The results of the sensitivity analysis of gravity anomaly estimation accuracy to the models are presented.

1. Introduction

Problem statements when the measurements of the estimated signal are subjected to pre-processing before being fed to the input of the estimation algorithm are often encountered in practice. In particular, one has to deal with such a problem when measurements, which contain a useful signal and additive noise, are preliminarily passed through a linear filter. Thus, the problem of optimal estimation of the useful signal is solved using pre-filtered measurements. A practical example of such a problem taken in this paper is the problem of gravity anomaly (GA) estimation using a damped marine gravimeter. The sensing element (SE) of which can be represented as a pendulum immersed in a viscous liquid [1-5]. It is noted in the previous work of the authors [6], that the accuracy of the optimal estimation does not depend on the presence of a preliminary filter in principle. However, this is true if the parameters of the preliminary filter are known exactly. In this paper the sensitivity of estimation algorithms to the parameters of the preliminary filter is analysed, using the above-mentioned problem of GA estimation using a damped gravimeter as an example. Two models for preliminary filter are considered, which corresponds the models for gravimeter SE in the example. First is a physically substantiated elastic pendulum model. Second is its simplified version in the form of a first-order system model. The results of the sensitivity analysis of GA estimation accuracy to the models are presented.

2. Sensitivity analysis for estimation problem using preliminary filtering

Consider preliminary filtering problem statement using the example of GA estimating. Following [3,4], we can write the simplest measurement model as:

\[ y = \Delta g + a_v + v_{gr}, \]

where \( \Delta g \) – GA, \( a_v \) – accelerations, defined by vertical object movement, \( v_{gr} \) – instrumental white noise component of gravimeter error. Solving the GA estimating problem from measurements (1) within the stochastic approach requires a model for GA and the vertical object acceleration. Model (1) assumes corrections for the Eötvös effect, normal gravity acceleration, zero point drift of the gravimeter, etc.
taken into account. In this work, as in [3, 4], Jordan’s model [7] for GA and the third-order model from [8] are used to describe vertical accelerations. In contrast to [3, 4], we introduce a preliminary filter that takes into account the dynamics of gravimeter SE and in the form of elastic pendulum damped by a viscous fluid:

\[
\begin{bmatrix}
  z_1 \\
  z_2
\end{bmatrix} = \begin{bmatrix}
  0 & \frac{1}{m} \\
  -\frac{c}{m} & -\frac{b}{m}
\end{bmatrix} \begin{bmatrix}
  z_1 \\
  z_2
\end{bmatrix} + \begin{bmatrix}
  0 \\
  1
\end{bmatrix} g ,
\]

(2)

\[\ddot{\tilde{y}} = \begin{bmatrix}
  \frac{c}{m} & 0
\end{bmatrix} \begin{bmatrix}
  z_1 \\
  z_2
\end{bmatrix} \]

(3)

where \( z_1, z_2 \) – state variables, \( m \) – mass, \( b \) – damping coefficient, \( c \) – spring resistance coefficient, \( g = \Delta g + a_v \) – apparent vertical acceleration, \( \ddot{\tilde{y}} \) – preliminary processed measurements entering the filtering problem. Along with (3), consider a simplified SE model in the form of a first-order plant [1]:

\[
\dot{z}_i = -\frac{1}{T} z_i + g ,
\]

(4)

\[\ddot{\tilde{y}} = M \dddot{z}_i ,
\]

(5)

where \( z_i \) – state variable of SE model, \( T \) – time constant, \( M \) – scale factor. For comparison, Bode magnitude plot and Bode diagrams of the models used, as well as the corresponding transient processes, are shown in Figure 1.

![Figure 1. (a) – Bode magnitude and phase responses, (b) – step responses for Model 1 – 1-st order plant, and Model 2 – elastic pendulum model.](image)

As can be seen from the figure, the transient processes of the systems are almost identical, the frequency characteristics differ only at relatively high frequencies. Note that, since the problem under consideration is similar to the problem of constructing a filter in the presence of an unchanged part of the system, then, provided that the errors \( v_{gr} \) are small, as noted above, the accuracy of GA estimation with the introduction of SE model will practically not change in comparison with the original problem of estimating GA without the SE model, [6, 9, 10].

Let us discuss the loss in accuracy when using the simplified model (4), (5) instead of (2), (3). This problem can be interpreted as the problem of assessing the sensitivity of the estimation algorithms, while the elastic pendulum model (2), (3) acts as the true one, and the SE model in the form of a first-order plant (4), (5) acts as a simplified (suboptimal) one. This problem can be solved using the appropriate software [11]. The actual suboptimal, calculated suboptimal and optimal root mean square error (RMSE)
of GA estimation for Rauch–Tung–Striebel non-stationary smoother, obtained by modeling the problem under consideration using the program [11], are shown in Figure 2a.

![Figure 2. RMSE of GA estimation (smoothing) for the optimal and suboptimal (simplified) SE model, with the exact – (a), and inaccurate – (b) determining the time constant of the time constant in the suboptimal SE model.](image)

It can be noted that in the case of using the simplified model (4), (5), the loss in accuracy compared to the optimal algorithm is about 1%. This is expected because the SE models have the difference in the Bode diagrams of only at high frequencies, where the amplitude of the useful signal is small, and the errors are filtered. The calculated GA estimation RMSE of suboptimal algorithm, which use the simplified model, is very close to the real one.

Let us also discuss the required accuracy of knowing the time constant for the preliminary filter (4), (5) when using the suboptimal model. Figure 2b shows the graphs of the standard deviation of the estimation, for conditions when the time constant in model (4), (5) is 5 s, (which corresponds to about 5%) differs from that specified in Figure 2a. It can be seen that with the wrong choice of the time constant in the simplified model, noticeable losses in accuracy will be only during the transient process, the accuracy in the steady state remains unchanged.

3. Conclusions
The sensitivity analysis for the estimation algorithms using pre-processed measurements is carried out. Using the GA estimation problem as an example, the results of comparing the GA estimation accuracy using the SE model in the form of an elastic pendulum and its approximation by an first-order plant are obtained. It is shown that, in the problem under consideration, the accuracy of GA estimation is not very sensitive to an incorrectly specified model of the preliminary filter.

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