Gauged $B - L$ unification and cosmology

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Abstract. We discuss some cosmological implications of low energy gauged $B - L$ symmetry with and without supersymmetry. Generic possibility of leptogenesis from a domain wall driven first order phase transition is shown to be a characteristic of such models.

Keywords: Left-Right, supersymmetry, grand unified theory, domain wall

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INTRODUCTION

There are several motivations to search for a grand unified theory. The tantalising possibility of gauge coupling unification into a simple gauge group is the most attractive. A dramatic signature of this unification, viz., proton decay has not been observed, though the possibilities are still open [1]. The other motivations arise out of the need to understand the known conserved charges in a larger perspective. Quantization of electromagnetic charge is one goal. The other intriguing feature of low energy physics is separate conservation of baryon and lepton numbers, without any associated gauge fields. This puzzle is partly resolved by the fact that the number $B + L$ is anomalous in the Standard Model (SM) and hence not really conserved. This leaves us with the need to understand $B - L$ symmetry.

The astrophysical and atmospheric data on neutrinos show that the total lepton number $L$ is conserved and that at least two of the neutrinos have mass. The smallness of these masses $\sim 10^{-3} - 10^{-1}$ eV, relative to the electroweak scale 250 GeV needs an explanation. See-saw mechanism which provides an attractive explanation implies Majorana mass and violation of total lepton number $L$ in perturbation theory at a high scale.

A very attractive possibility for unification is that the remaining exact global symmetry $B - L$ is gauged. In this review we summarise the situation for potential signatures of unification with gauged $B - L$. Thermal leptogenesis presents a very attractive link between low energy neutrino data and gauged $B - L$ symmetry. At present this link seems to have run into problems of energy scales as will be discussed. We make a case for a low energy $10^4 - 10^6$ GeV gauged $B - L$ symmetry based on several of our joint previous works. It relies on leptogenesis from a first order phase transition and demonstrates its naturalness from the point of view of gauged $B - L$. Consistency of this scenario with other cosmological issues and possible ways of verifying it are discussed.
COSMOLOGY WITH GAUGED $B - L$

There is at least one cosmological reason why gauged $B - L$ would be an appealing feature of nature. This is the observed baryon asymmetry of the Universe. At present three independent sources, direct observations, WMAP observations and nucleosynthesis data place the value of this asymmetry, expressed as ratio of net baryon number density to entropy density in photons, at $\eta_B \equiv n_B/s = 6 \times 10^{-10}$. Since $B + L$ is anomalous, its equilibrium value would be zero in the early Universe above the electroweak temperature scale $\sim 100\text{GeV}$. Thus baryon asymmetry must be generated out of $B - L$ asymmetry. If the latter is only a global charge, the observed baryon asymmetry could have arisen out of accidental initial conditions present at the Planck scale. A more appealing possibility is that the $B - L$ is a gauged abelian charge. This could be intrinsically so or arising from breaking of a more fundamental compact non-abelian symmetry group. But an abelian charge will have natural value zero in the early Universe. Any model of breaking this gauge invariance can then give masses to heavy neutrinos and also permit a dynamical computation of the baryon asymmetry.

A traditional scenario envisages generation of $L$ asymmetry from out-of-equilibrium decay of the heavy majorana fermions [2]. This asymmetry is then converted to $B$ asymmetry by electroweak sphaleronic processes [3] whose rate at a high temperature $T$ is given by

$$\Gamma_{\text{sphaleron}} = A\alpha_w^5 T^4$$  \hspace{1cm} (1)

with $A$ a numerical constant of order unity and $\alpha_w$ is the weak interaction coupling constant $g^2/4\pi$. The status of thermal Leptogenesis proposal may be summarised as follows. The net baryon asymmetry is given by

$$Y_B = 0.55\epsilon Y_{N_1}d,$$  \hspace{1cm} (2)

where $\epsilon$ is the $CP$ violation parameter, $Y_{N_1}$ is the abundance of the lightest of the heavy neutrino species $N_1$ at the $L$-violating scale and $d$ is subsequent dilution. Using see-saw mechanism, the Low energy neutrino data constrain $\epsilon$ to remain smaller than [4]

$$|\epsilon| \leq 9.86 \times 10^{-8} \left( \frac{M_1}{10^9\text{GeV}} \right) \left( \frac{m_{(3)}}{0.05\text{eV}} \right)$$  \hspace{1cm} (3)

where $m_{(3)}$ is the mass of the heaviest of the light neutrinos. Combining with the observed bound on Baryon asymmetry from WMAP gives a lower bound on $M_1$ as [3, 6]

$$M_1 \geq O(10^9\text{GeV}) \left( \frac{2.5 \times 10^{-3}}{Y_{N_1} d} \right) \left( \frac{0.05\text{eV}}{m_{(3)}} \right).$$  \hspace{1cm} (4)

Since the last two quantities in brackets are expected to be order unity, this constrains $M_1$ to be bigger than $10^9\text{GeV}$.

The high scale means direct verification of this symmetry is impossible in foreseeable future. But more seriously, the bound does not agree with supersymmetry (SUSY) as a potential solution of the fundamental Higgs and hierarchy problems. Most supersymmetric models have a problem of overabundance of gravitinos if subsequent to any scale higher than $10^9\text{GeV}$, the Universe has had purely radiation and matter dominated epochs.
MODEL INDEPENDENT LIMIT ON HEAVY NEUTRINO MASS

The above constraint on the scale of heavy neutrino masses is too stringent due to the assumption of purely thermal leptogenesis. Leptogenesis from a first order phase transition provides a constraint which is significantly weaker. The only requirement is that the raw lepton asymmetry created by the first order phase transition should be erased only partially by the majorana neutrinos. The requisite baryon asymmetry is generated out of the remainder by the sphaleronic effects.

The dilution factor $d$ introduced in eq. (2) is determined by two kinds of processes. They are (i) scattering processes (S) among the SM fermions and (ii) Decay (D) and inverse decays (ID) of the heavy neutrinos. The dominant contributions to the two types of processes are governed by the temperature dependent rates

$$
\Gamma_D \sim \frac{h^2 M_1^2}{16 \pi (4T^2 + M_1^2)^{1/2}} \quad \text{and} \quad \Gamma_S \sim \frac{h^4}{13 \pi^3 (9T^2 + M_1^2)},
$$

where $h$ is typical Dirac Yukawa coupling of the neutrino.

We shall see in later discussion that the raw $B-L$ asymmetry generated can be $\sim O(1)$. Accordingly we parameterise the dilution factor in the exponential form $d = 10^{-d_B}$, so that natural value of $d_B$ ranges from 0 to 10. The integrated dilution factors resulting from soution of Boltzmann equations can be transformed into an upper limit on the light neutrino masses using the canonical seesaw relation. First consider the case $M_1 > T_{B-L}$, where $T_{B-L}$ is the temperature of the $B-L$ symmetry breaking phase transition. In this case the dilution processes are rather ineffective and the condition is that in fact $d_B$ should be vanishingly small. In this case the raw lepton asymmetry should be produced in the required range of values, $\sim 10^{-9}$. $M_1$ remains unconstrained.

In the opposite regime $M_1 < T_{B-L}$, both of the above types of processes could freely occur. The condition that complete erasure is prevented requires that the above processes are slower than the expansion scale of the Universe for all $T > M_1$. It turns out to be sufficient [7] to require $\Gamma_D < H$ which also ensures that $\Gamma_S < H$. This leads to a requirement on the lightest neutrino mass $m_{(1)}$,

$$
m_{(1)} < m_* \equiv 4\pi g_*^{1/2} \frac{G_N^{1/2}}{\sqrt{2} G_F} = 6.5 \times 10^{-4} \text{eV}
$$

where the parameter $m_*$ [7] contains only universal couplings and $g_*$, the effective number of thermodynamic degrees of freedom, and may be called the cosmological neutrino mass.

In a specific texture model of Dirac mass matrix $m_D$ [8, 9] the see-saw relation reads,

$$
m_{(1)} \simeq (2 \times 10^{-2})^2 \left( \frac{10^8 \text{GeV}}{M_1} \right) \text{eV}.
$$

Then eq. (6) seems to be naturally satisfied for $M_1 \gtrsim 10^8 \text{GeV}$. However the front factor arises from the squared Dirac mass of the charged leptons. If we assume the Dirac mass for neutrinos be $10^{-2}$ smaller, this factor becomes $(2 \times 10^{-4})^2$. This results in the modest bound $M_1 > 10^4 \text{GeV}$, on the mass of the lightest of the heavy majorana neutrinos.
SUPERSYMMETRIC LEFT-RIGHT SYMMETRIC MODELS

While elegant, seesaw mechanism predicts a new high scale which gives rise to a hierarchy. Inclusion of SUSY improves the situation, stabilizing hierarchies of mass scales that lie above the SUSY breaking scale. We assume the most optimistic value for the SUSY breaking scale, being the TeV scale without disturbing the SM. We explore the possibility for SM to have descended from a Left-Right symmetric theory, with a low scale, a few orders of magnitude removed from the TeV scale. At a higher energy scale the model may turn out to be embedded in the supersymmetric $SO(10)$.

We have considered two possibilities for a minimal supersymmetric Left-Right symmetric model with the gauge group $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. Both need quark and lepton superfields, one set for each generation. The minimal set of Higgs superfields required is bidoublets $\Phi_1, \Phi_2$ and triplets $\Delta, \bar{\Delta}, \Delta_c, \bar{\Delta}_c$ as detailed in [10] and [11]. Under discrete parity symmetry the fields have two possible transformation rules,

\begin{align*}
Q &\leftrightarrow Q_c^*, \\
L &\leftrightarrow L_c^*, \\
\Phi_i &\leftrightarrow \Phi_i^*, \\
\Delta &\leftrightarrow \Delta_c^*, \\
\bar{\Delta} &\leftrightarrow \bar{\Delta}_c^*, \\
\Omega &\leftrightarrow \pm \Omega_c^*.
\end{align*}

We refer to the original model with $+$ sign as MSLRM. In this case there is a need for Gauge Mediated SUSY Breaking or gravity induced soft terms to lift the symmetry between Left and Right vacua. The alternative with the $-$ sign is along the lines of the non-supersymmetric model of Chang et al. [12]. Here spontaneous breaking of parity is implemented within the Higgs structure of the theory. We dub this model MSLRM$^-$.

In both these models, $SU(2)_R$ first breaks to its subgroup $U(1)_R$, at a scale $M_R$, without affecting the $U(1)_{B-L}$. At a lower scale $M_{B-L}$, $SU(3)_c \otimes SU(2)_L \otimes U(1)_R \otimes U(1)_{B-L}$ breaks to $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. Consistency of the scheme requires a "see-saw" relation between these two scales and the electroweak scale $M_W$,

\begin{equation}
M^2_{B-L} \simeq M_R M_W
\end{equation}

In this scheme parity is spontaneously broken while preserving electromagnetic charge invariance. However, due to parity invariance of the original theory, the phenomenologically unacceptable phase $SU(3)_c \otimes SU(2)_R \otimes U(1)'_Y$ is quasi-degenerate with $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. This quasi-degeneracy gives rise to domain walls (DW) at the scale $M_R$, resulting in a first order phase transition.

USES OF DOMAIN WALLS

It has been shown [13, 14, 15] that if DW dominate the evolution of the universe for a limited duration, the associated secondary inflation removes the gravitino and other dangerous relic fields like moduli typically regenerated after the primordial inflation [16]. It was shown in [15, 11] that this can be successfully implemented in above class of Left-Right models in the course of the DW dominated first order phase transition.

Turning to leptogenesis, at least two of the Higgs expectation values in L-R model are generically complex, thus providing natural $CP$ violation permitting all parameters in the
Higgs potential to be real. Within the thickness of the domain wall the CP violating phase becomes position dependent. Under these circumstances a formalism exists [17, 18, 19], wherein the chemical potential $\mu_L$ created for the Lepton number can be computed as a solution of the diffusion equation whose source term contains derivatives of the position dependent complex Dirac mass. In [20] the existence of such a position dependent phase was established on general grounds and verified in numerical simulations. The resulting value of the raw Lepton asymmetry to the entropy density ratio was shown to be

$$\eta_{L}^{\text{raw}} \simeq 0.01 \frac{v_w}{g_* T^5 \Delta_w}$$

with $\Delta_w$ standing for the wall width, and $v_w$ for the average wall velocity. A possible verification of DW based phase transition scenario can be sought in the upcoming space based gravitational wave detectors capable of detecting the stochastic background arising at such phase transitions [21].

An important constraint on these scenarios is that the DW must be metastable, with a decay temperature $T_D$ which must be larger than $\sim 10$ MeV in order to not interfere with Big Bang Nucleosynthesis (BBN). It has been observed in [22] that the free energy density difference $\delta \rho$ between the vacua, which determines the pressure difference across a domain wall should be of the order

$$\delta \rho \sim T_D^4$$

in order for the DW structure to disappear at the scale $T_D$. We use this constraint to determine the differences between the relevant soft parameters for a range of permissible values of $T_D$. The soft terms in the superpotential for the two models considered here have the form

$$\mathcal{L}_{\text{soft}} = \alpha_1 \text{Tr}(\Delta \Omega \Delta^\dagger) - \alpha_2 \text{Tr}(\bar{\Delta} \bar{\Omega} \bar{\Delta}^\dagger) + \alpha_3 \text{Tr}(\Delta_c \Omega_c \Delta_c^\dagger) + \alpha_4 \text{Tr}(\bar{\Delta}_c \bar{\Omega}_c \bar{\Delta}_c^\dagger)$$

$$+ m_1^2 \text{Tr}(\Delta \Delta^\dagger) + m_2^2 \text{Tr}(\bar{\Delta} \bar{\Delta}^\dagger) + m_3^2 \text{Tr}(\Delta_c \Delta_c^\dagger) + m_4^2 \text{Tr}(\bar{\Delta}_c \bar{\Delta}_c^\dagger)$$

$$+ \beta_1 \text{Tr}(\Omega \Omega^\dagger) + \beta_2 \text{Tr}((\Omega_c \bar{\Omega}_c)^\dagger).$$

In the case of MSLRM the $\alpha$ terms do not contribute and the other terms are constrained as listed in the table where we have considered $m_1^2 \simeq m_2^2 \equiv m^2$, $m_3^2 \simeq m_4^2 \equiv m^2$, $\langle \Omega \rangle \simeq M_R \sim 10^6$ GeV, $\langle \Delta \rangle \simeq M_{B-L} \sim 10^4$ GeV, and $T_D$ in the range 100 MeV – 10 GeV [14].

| $T_D$          | 10 GeV | $10^2$ GeV | $10^3$ GeV |
|---------------|--------|------------|------------|
| $(m^2 - m'^2)$ | $10^{-4}$ GeV$^2$ | 1 GeV$^2$ | $10^4$ GeV$^2$ |
| $(\beta_1 - \beta_2)$ | $10^{-8}$ GeV$^2$ | $10^{-4}$ GeV$^2$ | 1 GeV$^2$ |

The MSLRP $P$ is more restrictive, with $\alpha_3 = -\alpha_1$, $\alpha_4 = -\alpha_2$, $m_3^2 = m_1^2$, $m_4^2 = m_2^2$, $\beta_1 = \beta_2$. Here only the $\alpha$ terms contribute, and for a range of temperatures $T_D \sim 10^2$ GeV – $10^4$ GeV, the constraint reads

$$(\alpha_1 + \alpha_2) \sim 10^{-6} - 10^{-2}$$.
CONCLUSION

A new gauge force of nature, $U(1)_{B-L}$ may well be accessible in low energy accelerator data and in observable cosmological effects. The requirement of thermal leptogenesis would put its energy scale too high to be verified. However generic occurrence in such models of a first order phase transition driven by domain walls makes low energy baryogenesis possible. If the hierarchy with other energy scales is protected by supersymmetry, the same domain walls allow for a secondary inflation to sweep away unwanted relics. Supersymmetry breaking soft parameters of such a model can be constrained from cosmological considerations.

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