A new method for calculating the exchange flux in discrete fracture model for two-phase flow in fractured porous media

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Abstract. The discrete fracture model (DFM) is widely used in order to simulate the flow state of two phase flow in fractured reservoirs. There is a strong capillary pressure contrast between the matrix and the fracture in low-permeability reservoirs with conductive fractures. The physical quantities of the matrix-fracture interface change drastically and the flow pattern near the interface depends on the flow direction which make it very difficult to calculate the matrix-fracture exchange flux. New interface conditions are applied in this paper to get a scheme for calculating the exchange flux. Based on new interface conditions the scheme for calculating the exchange flux is established. This scheme can improve the computational efficiency of the multi-phase flow in DFM since considering the discontinuous of physical quantities near the interface. The numerical test show that the proposed model has better performance than the traditional model.

1. Introduction

Low-permeability reservoirs account for a high proportion of petroleum geological reserves, but such reservoirs are difficult to extract using conventional methods due to their low permeability. Fracturing technology has been used widely as a means to increase production effectively. This technology can create several large-scale conductive fractures in the reservoirs, thereby accelerating oil inflow into production wells and improving oil recovery efficiency. In recent years, scholars have developed the discrete fracture model (DFM) [1-4], which assumes that the flow in the fracture is one-dimensional in a d-dimensional system to simulate the reserves with conductive fractures.

The DFM was initially introduced by Noorishad and Baca et al. to simulate the single phase flow [1, 2]. In their model matrix nodes and fracture nodes overlapped with each other, which governing equations were solved with superposition. This model could simulate the single-phase flow, but for multi-phase flow it was not accurate. Slough and Fard et al. improved the DFM to simulate the multiphase flow [3, 4]. They proposed to separate the physical quantities of the matrix and the fracture and couple the matrix and fracture equations by using the matrix-fracture exchange flux. The exchange flux is calculated by discrete Darcy’s law. Although this method is widely applied in lots of simulators, it still has problem when simulating the low permeability reserves with conductive fractures. In this kind of reserves, there is big contrast capillary pressure and thereby the physical quantity may vary drastically at the matrix-fracture interface, making it difficult to calculate the exchange flux [5]. In this case,
the traditional interface conditions may not be suitable, the mesh near the interface need to be sufficiently subdivided if using the classical numerical scheme.

The flow properties of two-phase flow at the matrix-fracture interface are analyzed in this paper, and new interface conditions are proposed. Based on this, the calculation format of exchange flux is established. This method in this paper can overcome the difficulty of the numerical dispersion due to the discontinuities of the variables near the \( m-f \) interface, and it is not necessary to subdivide the mesh, which can improve the computational efficiency of the multi-phase flow in the DFM.

2. Algorithm description

2.1. Mathematical model

In this article, we consider 2D incompressible two-phase flow. The \( \alpha \) (\( \alpha = \)oil or water) phase mass conservation equation in the matrix and fracture can be written as:

\[
\phi^m \frac{\partial S^m}{\partial t} + \nabla \cdot \mathbf{u}^m = 0
\]

(1)

\[
\phi^f \frac{\partial S^f}{\partial t} + \frac{\partial u^f}{\partial x_f} = q^f
\]

(2)

where the subscript \( m \) and \( f \) represents the variables in the matrix and the fracture; \( \phi \), \( \rho \) and \( \mathbf{u} \) are porosity, phase saturation and velocity, respectively. The source term \( q^f \) is the transfer flow between the matrix and fracture which can be calculated as:

\[
q^f = \frac{u^{m,+} \cdot n^m + u^{m,-} \cdot n^m}{d_f}
\]

(3)

where \( n^m \) is unit external normal vector of the matrix boundary; the superscripts ‘+’ and ‘-’ represent the two sides of fracture respectively; \( d_f \) represents the fracture width.

This paper focuses on the exchange item \( q^f \) and we will discuss the computing of it in discrete expression below.

2.2. Traditional computational method for exchange flux

The traditional computational method for exchange flux in discrete scheme is expressed as:

\[
q_{a,j} = K^{\alpha} \lambda_{\alpha,a,j} \frac{\Delta P_{\alpha,a,j}}{d_{ij}} \alpha = w, o
\]

(4)

where \( a \) and \( j \) represent the matrix block the fracture block; \( d_{ij} \) and \( d_{ji} \) represent the distances from the discrete pressure points \( P_i \) and \( P_j \) to the interface \( \Gamma_{ij} \) respectively; \( \lambda_{\alpha,a,j}^{up} \) is the mobility of upstream grid \( \Delta P_{\alpha,a,j} \) is the phase pressure difference.

This scheme is based on Taylor’s expression near the matrix-fracture interface which is not valid when simulating the problem with big capillary pressure in the matrix as the capillary pressure may not be continuous.
2.3. Proposed computational method for exchange flux

According to the relationship between the oil phase pressure difference $\Delta P_o$ and the capillary pressure difference $\Delta P_c$, the flow across the interface can be divided into three cases:

$$
\begin{cases}
S_i^m = 1 \ (\Delta P_o \geq \Delta P_c) \\
q_o = 0 \ (0 < \Delta P_o < \Delta P_c) \\
\theta = q_o / q_o = \lambda_o^i (S_j) / \lambda_o^j (S_j) \ (\Delta P_o \leq 0)
\end{cases}
$$

(5)

From (5) we can see when the pressure difference between the matrix and the fracture can overcome the capillary end effect, the two-phase fluids will flow from the matrix to the fracture and the water phase saturation at matrix side interface is 1 [6], this condition is widely applied in DFM [3-4]. While when the pressure difference can’t overcome the capillary end effect we use other conditions similar like the conditions proposed by Wang (2006) to get more accurate result [7].

According to D.D. Huang (1998)’s derivation [6], we have

$$
\Delta P = - \int_{S_i} \frac{\lambda^m}{\lambda^o - \lambda^w \theta} \frac{dP^m}{dS} dS
$$

(6)

Using (5) and (6) we can get $\theta$ and $S_i^m$. The exchange flux can be calculated by:

$$
q_o,ij = - \frac{K_{ij}^m}{d_{ij}} \int_{S_i} \frac{dP^m}{dS} \lambda^o \lambda^w \theta dS
$$

(7)

$$
q_o,ij = \theta q_o,ij
$$

(8)

3. Numerical test

Considering a two-phase flow case with two fractures embedded in a square reservoir. The sketch map of the simulation area is shown in Figure 1. Relevant parameters are presented in Table 1. The initial water saturation is 0.01 and the injection rate for each injection well is 1 m$^3$/day. In both the traditional DFM and the proposed one, 40×40 cells are used to calculate the flow in matrix. In the reference model, each fracture grid cell and the matrix grid cell adjacent to the fracture are refined into 10 sub-cells.

![Figure 1. The sketch map of the simulation area and the meshing in the example.](image-url)
Table 1. Relevant parameters for example

| Parameter                     | Value                  |
|-------------------------------|------------------------|
| Domain dimensions:            | 100 m×100 m            |
| Matrix properties             | $\phi^m = 0.3$, $K^m = 1$ mD |
| Fracture properties           | $\phi^f = 1.0$, $K^f = 1.0 \times 10^3$ D, $d_f = 1$ cm |
| Viscosity                     | $\mu_o = 0.001$ Pa·s, $\mu_w = 0.006$ Pa·s |
| Capillary pressure            | $P_c^f = -B \ln S_w$, $B = 1.0$ MPa |
| Relative permeabilities       | $k_{rw} = (1 - S_w)^2$, $k_{rw} = S_w^2$ |

In Figure 2, the water-cut curves of the production wells are plotted for all three models. It can be seen that the traditional DFM underestimates the water-cut and predicts a later water breakthrough time, while the proposed method can predict as accurate results as the reference one.

![Figure 2](image_url)

**Figure 2.** Water cut comparisons of (a) upper right corner well and (b) lower left corner well for different models

4. Conclusion

Based on new interface conditions the proposed model can better describe the flow characteristics near the m-f interface, and therefore more effectively calculate the m-f exchange flux. The numerical results show that the traditional DFM model overestimates the m-f exchange flux, which leads to underestimate the water-cut. The proposed model is accurate and the results are consistent with the single porosity model in which fracture grids and the matrix grids near the fracture are subdivided.

Acknowledgments

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