Numerical simulation of slow deformation perturbations in fault zones

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Abstract. Over the past half a century, the concept of slow deformation waves of the Earth has been developed and widely discussed in the Earth sciences. The velocity of slow waves is considered to be 5–6 orders of magnitude less than the velocity of sound and 7–8 orders of magnitude greater than tectonic flows. Analyzing and classifying various manifestations of slow deformation waves in the geomedium accumulated over forty years V. G. Bykov identifies two types of autowaves in his review—inter-fault and intra-fault. Here, the process of generation and propagation of slow deformation disturbances between two faults in an elastoplastic medium is studied numerically. The faults were defined as narrow elongated soft areas inclined to the axis of the load application. Just in these faults plastic deformation was permitted to generate. The simulations proved that the fronts of deformation waves head towards each other at approximately the same velocities, their shapes being close to planar but slightly curved.

1. Introduction
The study of the spatiotemporal migration of modern geodynamic processes is, on the one hand, one of the most interesting problems of geodynamics, on the other, one of the most debatable problems. Explanation of migration of earthquake foci in zones of large seismically active faults by the trigger effect of unidirectional spatial movement of slow inelastic deformations proposed by C. Richter [1] and K. Mogi [2] initiated the problem of slow deformation waves in the lithosphere.

Slow deformation waves are not directly recorded. Their existence is established indirectly by registration of variations in geophysical fields, in particular, by directional migrations of earthquakes. Involvement in the description of slow deformation perturbations, for example, heat conduction equations (diffusion) or the sin-Gordon equations, in the case of their representation as solitons, is based only on a priori certitude that such wave perturbations exist and are, for example, solitons. These equations are not connected to the evolution of the stress-strain state, which means that their solutions can only characterize a possible qualitative picture of slow deformation perturbations in geomedia. The autowave or autosoliton interpretation of these phenomena as self-organization processes in active dissipative media seems to be more adequate [3].

From the recent reviews presented in the papers [4, 5], it follows that there are two main types of deformation waves. Waves of the first type—inter-fault waves—are basic, have a long length and can originate in subduction zones, or be generated by the “lithosphere – asthenosphere” system, or have a
different global nature. Waves of the second type—intra-fault—are generated first due to their transformation when passing through large inland fault zones. The velocities of spatial migration of inter-fault and intra-fault deformation waves differ and, according to different estimates, range from 10 to 100 km/year for the former and from 10 to 4 km/year or less for the latter [3].

Thus, an important task is to study the features of generation and interaction of slow deformation autowaves using the mathematical and numerical apparatus of deformable solid mechanics. A successful solution to this problem is possible on the basis of the fundamental theory of the evolution of the stress-strain state in a loaded solid medium, which covers the entire spectrum of key processes of deformation and fracture [6].

The aim of this work is to study the features of the formation of slow deformation perturbations in an elastoplastic medium using numerical simulation methods.

2. Mathematical model

To describe the processes of generation and propagation of slow deformation disturbances, in this work, we used a model that combines the approach of solid mechanics with cellular automata algorithm that provides a way for the plastic strain to advance into the elastic domain [7]. A model of an elastic-plastic media with the Drucker-Prager yield criterion was used. An algorithm of cellular automata with the von Neumann neighborhood was implemented for transmitting slow perturbation.

The complete mathematical model contains the fundamental equations of continuum mechanics (1)–(2), the decomposition of the stress tensor to the hydrostatic pressure and deviator stresses (3), constitutive evolutionary equations of an elastic-plastic medium (4)–(7) with unassociated plastic flow rule (8) and the Drucker-Prager plastic potential (9) and yield function (10).

\[
\frac{d\rho}{dt} + \rho \text{div} \vec{v} = 0 \tag{1}
\]

\[
\rho \frac{du_i}{dt} = \frac{\partial \sigma_{ij}}{\partial x^j} + \rho F_i \tag{2}
\]

\[
\sigma_{ij} = -P \delta_{ij} + S_{ij}; \quad P = -\frac{1}{3} \sigma_{ii} \tag{3}
\]

\[
\dot{P} = -K(\dot{\varepsilon}_{ij}^e) \tag{4}
\]

\[
\dot{S}_{ij} = 2\mu(\dot{\varepsilon}_{ij}^e - \frac{1}{2}\delta_{kk}\dot{\varepsilon}_{kk}^e) - S_{ik}\dot{\omega}_{kj} + S_{kj}\dot{\omega}_{ik} \tag{5}
\]

\[
\dot{\varepsilon}_{ij}^p = \dot{\varepsilon}_{ij} - \dot{\varepsilon}_{ij}^e \tag{6}
\]

\[
\dot{\varepsilon}_{ij}^e = \frac{1}{2} \left( \frac{\partial u_i}{\partial x^j} + \frac{\partial u_j}{\partial x^i} \right); \quad \dot{\omega}_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x^j} - \frac{\partial u_j}{\partial x^i} \right) \tag{7}
\]

\[
\dot{\varepsilon}_{ij}^p = \dot{\lambda} \frac{\partial g(\sigma_{ij})}{\partial \varepsilon_{ij}}, \text{ if } f(\sigma_{ij}) \geq 0; \text{ otherwise } \dot{\varepsilon}_{ij}^p = 0 \tag{8}
\]

\[
g(\sigma_{ij}) = \Lambda P + \frac{1}{\sqrt{2}} S_{ij} S_{ij} + \text{const} \tag{9}
\]

\[
f(\sigma_{ij}) = \alpha P + \frac{1}{\sqrt{2}} S_{ij} S_{ij} - Y \tag{10}
\]

Here \( \rho \) is the density, \( u_i \) are the components of the velocity vector, \( \sigma_{ij} \) are the components of the stress tensor, \( F_i \) are the components of the vector of the bulk forces, \( P \) is the pressure, \( S_{ij} \) are the...
components of the deviatoric stress tensor, $\delta_{ij}$ is the Kronecker delta. $K$ is the bulk modulus of elasticity, $\mu$ is the shear modulus, $\dot{\varepsilon}^e_{ij}$ and $\dot{\varepsilon}^p_{ij}$ are the elastic and plastic components of the total strain rate $\varepsilon_{ij}$, respectively, the dot above the symbol means the material time derivative, $S_{ik}&\dot{\omega}_{kj} + S_{kj}\dot{\omega}_{ik}$ is the correction for the rotation that occurs when using corotational Jaumann derivative of the stress tensor, $f(\sigma_{ij})$ is the yield function, $g(\sigma_{ij})$ is the plastic potential function, $\Lambda$ is the dilatancy coefficient, $\alpha$ is the internal friction coefficient, $\lambda$ is the plastic multiplier.

To perform numerical simulation, the Wilkins [8] finite difference scheme was used. Simulations were fulfilled using a computer code written by the authors. The computer code has been used previously for simulating various ductile and brittle materials, rocks, geomedia [9–13].

3. Results and discussion

The behavior of a region of the medium with two faults in the conditions of uniaxial compression along the vertical axis was studied. A structural map of the model media with faults was constructed to study the generation and propagation of slow strain waves between the faults. The model map has two narrow elongated areas specified as faults inclined to the axis of load application, their ends being placed near the boundary of the calculation region (figure 1a). Just the faults are the place where the plastic deformation can arise according to the idea of cellular automata.

The simulations show that plastic strains generated in the faults and started to stem from the faults in opposite directions perpendicular to the faults (figure 1b). The fronts propagating toward the other fault have an almost planar slightly curved shape and approximately the same speed.

Figure 2 shows the distributions of increments of plastic strain that reflect the propagation of slow deformation perturbations in an elastic-plastic medium as a kind of solitary dissipative autowave. A detailed study showed that the speed of propagation of slow deformation perturbations exceeds the rate of loading of the boundaries of the calculated region by about 300 times. Taking into account the time compression [6], the average speed of propagation of slow deformation autowave number 1 (figure 1, b) was 27 km/year, and autowave number two (figure 1, a) equaled 23 km/year.
Figure 2. Distributions of strain increments of slow deformation waves.

4. Conclusions
As a result of the research, we can conclude that the approach based on the combined model of solid mechanics with the cellular automaton method allows one to describe slow deformation fronts of inelastic nature in nonlinear elastic-plastic media. The features of the propagation of deformation perturbations waves are investigated for a media with two elongated faults. The deformation perturbation fronts have an almost planar slightly curved shape and approach each other at approximately the same speed. The distributions of increments of plastic strain reflect the propagation of slow deformation perturbations in an elastic-plastic medium as a kind of solitary dissipative autowave. Variation of these distributions with time shows that the autowaves do not move steadily, their speed and width vary in time.

Acknowledgments
The study was carried out with the financial support of the Russian Science Foundation (project No. 19-17-00122).

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