Polarized anisotropic spectral distortions of the CMB: galactic and extragalactic constraints on photon-axion conversion

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Abstract. We revisit the cosmological constraints on resonant and non-resonant conversion of photons to axions in the cosmological magnetic fields. We find that the constraints on photon-axion coupling and primordial magnetic fields are much weaker than previously claimed for low mass axion like particles with masses $m_a \lesssim 5 \times 10^{-13}$ eV. In particular we find that the axion mass range $10^{-14}$ eV $\leq m_a \leq 5 \times 10^{-13}$ eV is not excluded by the CMB data contrary to the previous claims. We also examine the photon-axion conversion in the Galactic magnetic fields. Resonant conversion in the large scale coherent Galactic magnetic field results in 100% polarized anisotropic spectral distortions of the CMB for the mass range $10^{-13}$ eV $\lesssim m_a \lesssim 10^{-11}$ eV. The polarization pattern traces the transverse to line of sight component of the Galactic magnetic field while both the anisotropy in the Galactic magnetic field and electron distribution imprint a characteristic anisotropy pattern in the spectral distortion. Our results apply to scalar as well as pseudoscalar particles. For conversion to scalar particles, the polarization is rotated by 90° allowing us to distinguish them from the pseudoscalars.
For $m_a \lesssim 10^{-14}$ eV we have non-resonant conversion in the small scale turbulent magnetic field of the Galaxy resulting in anisotropic but unpolarized spectral distortion in the CMB. These unique signatures are potential discriminants against the isotropic and non-polarized signals such as primary CMB, and $\mu$ and $\gamma$ distortions with the anisotropic nature making it accessible to experiments with only relative calibration like Planck, LiteBIRD, and CoRE. We forecast for PIXIE as well as for these experiments using Fisher matrix formalism.

**Keywords:** axions, CMBR theory

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1 Introduction

The Cosmic Microwave Background (CMB) was discovered by Penzias & Wilson [1] and later found to have an almost perfect blackbody spectrum by Far Infrared Absolute Spectrophotometer (FIRAS) experiment [2–5] with temperature $2.7255$ K (almost simultaneously confirmed by the rocket based experiment COBRA [6] with slightly less sensitivity.) Another instrument on board COBE, the differential microwave radiometer (DMR), discovered the nearly statistically isotropic fluctuations of order $10^{-5} - 10^{-4}$ K on top of the 2.7K background. The exquisite measurements of the CMB temperature and polarization angular fluctuations over the past few decades by several space-based (COBE [7], WMAP [8] and Planck [9]), ground-based (SPT [10], ACT [11], BICEP-KECK [12], POLARBEAR [13] etc.) and balloon-based (BOOMERANG [14], MAXIMA [15], etc.) missions are well explained by the 6 parameter $\Lambda$CDM (Lambda Cold Dark Matter) model and is one of the fundamental pillars of the standard cosmological model. Along with the angular fluctuations of the CMB field, we also expect deviations from the blackbody spectrum within the standard cosmological scenario [16–26] and measurement of these would deepen our understanding of both early and late time epoch of the Universe. Only one type of spectral distortion, the Sunyaev-Zeldovich effect or the $y$-type distortion [16], has so far been detected towards the clusters of galaxies [27–32]. Spectral distortions in CMB are also predicted by several high energy
physics scenarios which are important particularly in the pre-recombination epoch [33–39]. In brief, CMB spectral distortions provide an unexplored and extremely rich window to several astrophysical and cosmological phenomenon.

The current best constraints on the deviation of the sky-averaged CMB (monopole) from a blackbody spectrum come from FIRAS [2, 4] which gave an upper bound on spectral distortions of $\Delta I_\nu/I_\nu \lesssim 5 \times 10^{-5}$ at the peak of the blackbody spectrum. Recently, the constraints on the anisotropic spectral distortions, including the fluctuating contribution to the all sky average $y$ -distortion were obtained from the Planck and SPT data in [40, 41]. Upcoming proposed CMB missions like PIXIE [42] would have an instrumental noise of nearly 3 – 4 orders of magnitude better than FIRAS and hence would be able to measure the CMB spectral distortions with an unprecedented accuracy [43]. Measurement of the spectral distortions signal will also depend upon the successful cleaning of the foreground contaminations [41, 44–47]. Other CMB missions like CORE [48] and LiteBIRD [49] would be able to measure the spatially fluctuating part of the spectral distortions [50] at a much better precision compared to Planck. These experiments would be polarization sensitive in all frequency channels. These missions could pioneer a new era in cosmology by measuring several guaranteed but unexplored cosmological imprints on the CMB spectrum. Along with several well known sources of spectral distortions (such as $\mu$ [17, 18, 22], $y$ [19, 22, 25], dark matter annihilation [18]), coupling between photons and pseudoscalar axion like particles (ALPs) or light scalar particles (LSPs) in the presence of external magnetic field [51–58] is also a potential source of spectral distortion. ALPs are one of the promising candidates for dark matter and may be a solution to some of the anomalies in the standard $\Lambda$CDM model [59–64]. Regardless of whether the ALPs form the bulk of the dark matter, they are predicted almost ubiquitously in many beyond standard model theories of particle physics, including the string theory [65–67]. Indirect astrophysical searches of ALPs along with the growing ground-based experimental efforts [see 68, for a review of ground based experiments] like CAST [69], ALPS-II [70], MADMAX [71], ADMX [72], CASPER [73] are therefore very important.

The magnetic field is present in the Universe at different scales with a varying strength. The extragalactic magnetic field is expected to be of the order of $10^{-9}$ Gauss (nG) or smaller [74]. The extragalactic magnetic fields, particularly in voids and in the early Universe, if primordial in origin, must be stochastic (Gaussian random fields), described by a power spectrum (possibly scale invariant) [75], and therefore without a single coherence scale. Previous studies [57, 58] have considered Mpc scale extragalactic magnetic fields to study the imprints of the photon-ALP or photon-LSP conversion on CMB photons. The magnetic field is however known to be present at the Galactic (kiloparsec) scales (kpc scales), with much better understanding of its strength and morphology [77, 78].

In this paper, we focus on an unexplored scenario concerning the spectral distortions of CMB photons due to photon-ALP or photon LSP conversion in the presence of the local magnetic field from Milky Way. We will consider the ALPs from now on for definiteness but our results are applicable to LSPs also in a straightforward way as explained in section 3.1. The CMB photons passing through the Galactic halo to reach the earth can get converted to ALPs in the presence of the $10^{-6}$ Gauss ($\mu G$) magnetic field depending upon photon-ALP coupling $g_{\gamma a}$, ALP mass $m_a$, electron ($n_e$) and neutral hydrogen ($n_H$) number densities and strength of the magnetic field. As the Galactic magnetic field is not isotropic but exhibits large scale coherent structure and small-scale turbulent fluctuations [77–80], the spectral
distortions induced by the photon-ALP conversion must also exhibit large scale anisotropy and should be correlated with the large scale structure in the Galactic magnetic field. In particular, the regions of the sky with the stronger magnetic field can convert photons to ALPs more efficiently than the parts of the sky with the weaker magnetic field. Secondly, the presence of fluctuations in the spectral distortion makes this phenomenon measurable from CMB missions like Planck and LiteBIRD which have only relative calibration. The spectral distortion signal from the photon-ALP conversion exhibits a unique structure in both frequency and spatial domain, which makes it easier to distinguish from other cosmological (or astrophysical) sources and other systematics.

We review the physics of photon-axion conversion and re-evaluate the existing cosmological constraints on photon-ALP conversion in section 2. We discuss the signatures of spectral distortion due to photon-ALP conversion in the Milky Way in sections 3 and 4 and forecast the measurability of this phenomenon from several CMB missions like Planck, PIXIE, LiteBIRD, and CoRE in section 5 using Fisher Matrix. Finally in section 6, we conclude our study and discuss its future implications. We use natural units (with reduced Planck constant, speed of light and Boltzmann constant respectively set to unity $\hbar = c = k_B = 1$) when discussing physics but restore physical constants when discussing observations.

2 Review of CMB-ALP conversion physics and current constraints from cosmology

Photon-ALP conversion and its cosmological consequences are well studied topics in the literature [52–55, 81–90]. ALPs and photons oscillate into each other in the presence of a magnetic field [52–55, 57]. The interaction is given by

$$\mathcal{L}_{\text{int}} = g_{\gamma a} E_{\gamma} B_{\text{ext}} a, \quad (2.1)$$

where $B_{\text{ext}}$ is the external magnetic field, $E_{\gamma}$ is the electric field of the photon, $a$ is the axion field and $g_{\gamma a}$ is the photon-ALP coupling. Thus only the polarization with its electric field aligned with external magnetic field couples to the axion. Obviously photons and axions will couple only if the magnetic field has a component $B_T = B_{\text{ext}} - (B_{\text{ext}} \hat{k}) \hat{k}$ transverse to the photon propagation direction $\hat{k}$. The evolution equation for the two state quantum system, assuming relativistic ALP, is given by [53, 56]

$$\left( \omega + \begin{pmatrix} \Delta e \\ \Delta \gamma a \\ \Delta a \end{pmatrix} + i \partial_z \right) \begin{pmatrix} A_{\parallel} \\ a \end{pmatrix} = 0 \quad (2.2)$$

Here we want to study the evolution of the system along a spatial direction $z$, $\omega$ is the temporal frequency, in Fourier space $i \partial_z \rightarrow k$ the spatial frequency or the momentum and $A_{\parallel}$ is the photon polarization that is parallel to the component of the magnetic field $B_T$ transverse to the propagation direction. The mixing matrix elements are defined below. In general $\Delta e$, which is a function of free electron, atomic and molecular densities, will vary along the photon geodesic as CMB photons travel cosmological distances. $\Delta \gamma a$ is a function of magnetic field which is also spatially varying. We will, at first, ignore these complications and look at the solutions in the presence of homogeneous medium and magnetic fields and return to the inhomogeneous case later. The equation (2.2) is solved by diagonalizing the $2 \times 2$ matrix on the left hand side through rotation by mixing angle $\theta$. The probability of conversion of a photon (with linear polarization parallel to the component of magnetic field
transverse to the propagation direction) to an ALP is then given in the homogeneous case by [57]

\[
P(\gamma \to a) = \frac{(\Delta_{\gamma a} s)^2}{(\Delta_{osc} s/2)^2} \sin^2(\Delta_{osc} s/2)
\equiv \sin^2(2\theta) \sin^2(\Delta_{osc} s/2)
\]

(2.3)

where \( B_T \) is the transverse (to photon momentum) component of the magnetic field, \( s \) is the distance travelled by the photons, \( \theta \) is the mixing angle defined so that \( 2\theta \to 0 \) at high electron densities,

\[
\cos(2\theta) = \frac{\Delta_a - \Delta_e}{\Delta_{osc}}
\]

(2.4)

\[
\Delta_{osc}^2 = (\Delta_a - \Delta_e)^2 + 4\Delta_{\gamma a}^2,
\]

(2.5)

\[
\Delta_a = (n - 1)\omega,
\]

(2.6)

and \( n \) is the refractive index for photon propagation through matter. As we see below, for high electron densities photons have real effective mass, \( \Delta_e \propto -m_e^2 < 0 \) and \( \Delta_a \propto -m_a^2 < 0 \) and in the limit \( |\Delta_e| \gg |\Delta_a|, |\Delta_{\gamma a}| \) mixing is suppressed (\( \sin 2\theta \to 0 \)) and we get \( \cos(2\theta) \to 1 \). Similarly in vacuum, when \( \Delta_a \to 0 \), \( |\Delta_a| \gg |\Delta_{\gamma a}| \), again the mixing is suppressed (\( \sin 2\theta \to 0 \)) but we have \( \cos 2\theta \to -1 \). Photon axion mixing is maximum at resonance, \( \Delta_a = \Delta_e \) giving \( 2\theta = \pi/2, \sin 2\theta = 1 \).

For astrophysical matter densities, \( |n - 1| \ll 1 \) and can be approximated (away from resonance frequencies \( \omega_i \) of atoms) by [91]

\[
n - 1 \approx \frac{-m_e^2}{2\omega^2} \approx 2\pi\alpha \left( -n_e + n_H \sum_i f_i^H \frac{\omega^2}{\omega_i^2 - \omega^2} + n_{He} \sum_i f_i^{He} \frac{\omega^2}{\omega_i^2 - \omega^2} \right),
\]

(2.7)

where \( m_\gamma \) is the effective mass of the photon, \( \alpha \) is the fine structure constant, \( \omega \) is the angular frequency, \( n_e, n_H \) and \( n_{He} \) are the number the density of free electrons, neutral hydrogen, and neutral helium respectively, \( f_i^\alpha \) is the oscillator strength of element \( \alpha \in \{H, He\} \) for transitions with energy \( \alpha \omega_i \) from the ground state. For free electrons, there is no resonant frequency and the effective mass squared is positive. For neutral atoms, such as hydrogen, the effective mass squared is negative below the resonant frequency. Neutral atoms exist only after recombination and in ground state with the minimum resonant frequency of 10.2 eV corresponding to Ly-\( \alpha \) transition of hydrogen. For CMB we will therefore always have \( \omega \ll \omega_i \) and we can approximate \( \Delta_e \) as (ignoring helium and heavier elements) [92]

\[
\Delta_e \approx \frac{\omega_p^2}{2\omega} \left[ \frac{1 - 7.3 \times 10^{-3}n_H}{n_e} \left( \frac{\omega}{eV} \right)^2 \right]
\]

\[
= -2.6 \times 10^6 \left( \frac{n_e}{10^{-5}cm^{-3}} \right) \left( \frac{100 \text{ GHz}}{\nu} \right) \left[ 1 - 7.3 \times 10^{-3} \frac{n_H}{n_e} \left( \frac{\omega}{eV} \right)^2 \right] \text{Mpc}^{-1}
\]

(2.8)

where \( \omega_p^2 = 4\pi\alpha n_e/(m_e) \) is the plasma frequency.

For the range of parameters of interest we also have

\[
\Delta_{\gamma a} = \frac{g_{\gamma a}[B_T]}{2} = 15.2 \left( \frac{g_{\gamma a}}{10^{-11}\text{GeV}^{-1}} \right) \left( \frac{B_T}{\mu G} \right) \text{Mpc}^{-1},
\]

(2.9)

\[
\Delta_a = \frac{-m_a^2}{2\omega} = -1.9 \times 10^4 \left( \frac{m_a}{10^{-11}\text{eV}} \right) \left( \frac{100 \text{ GHz}}{\nu} \right) \text{Mpc}^{-1}
\]

(2.10)
Figure 1. The left panel shows frequency dependence of the CMB spectral distortion with $n_e = 10^{-5} \text{cm}^{-3}$ and $g_\gamma B_{\mu G} = 1$. The right panel shows oscillation length $\ell_{\text{osc}}$ as a function of photon frequency for same parameters. The axion mass is assumed to be small compared to the effective photon mass.

$g_{\gamma a}$ is the photon-ALP coupling and $\omega = 2\pi \nu$. The photon polarization state orthogonal to $B_T$ is unaffected. Thus initially unpolarized light propagating through a magnetic field will become polarized as intensity in one of the linear polarizations is decreased due to photon-ALP oscillation. These results apply only if both the electron density and the magnetic field are homogeneous. The departure from BlackBody (BB) spectrum for the affected polarization can be quantified by

$$I^\gamma = \frac{\Delta I^\gamma}{I^\nu} \equiv \frac{I^\text{obs}_\nu - I_\nu}{I_\nu} = -P(\gamma \to a),$$

(2.11)

where, $I_\nu = (h\nu^3/c^2)/(e^{h\nu/k_B T_{\text{CMB}}} - 1)$ is the Planck spectral form for single polarization in physical units, $h$ is the Planck’s constant, $c$ is the speed of light and $k_B$ is Boltzmann constant. This is plotted in figure 1a. We see that it is a fast oscillating function of frequency (as well as distance $s$) due to the second factor in eq. (2.3). In any experiment with reasonable frequency and angular resolution we will only detect the result of average over many oscillations. We can therefore replace the oscillating factor with its average over an oscillation giving

$$P(\gamma \to a) = \frac{2\Delta^2_{\gamma a}}{\Delta^2_{e}},$$

(2.12)

if the magnetic field is nearly coherent over the scales of size $s \gg \ell_{\text{osc}} \equiv 2\pi/\Delta_{\text{osc}}$. The oscillation length, $\ell_{\text{osc}}$ is plotted in figure 1b as a function of frequency $\nu$.

In reality both the electron density and magnetic fields are inhomogeneous. In particular the primordial magnetic fields (because of stochastic initial conditions) as well as small scale Galactic magnetic fields (because of turbulence) are expected to be stochastic. In this case, as a toy model, we can approximate the magnetic fields as composed of independent domains of size $d_0$ such that the magnetic field and electron density are homogeneous inside each domain but in different domains the magnetic field has different random orientations but same strength for simplicity. In the limit of large number of domains we can obtain an analytical solution for the conversion probability of the total unpolarized intensity given by [57, 93]

$$P(\gamma \to a)(r) = \frac{1}{3}\left(1 - e^{(-3P(\gamma \to a)r/2d_0)}\right) \quad r \gg d_0,$$

(2.13)
where $r$ is the size of the turbulent region, $P(\gamma \rightarrow a)$ is the probability of conversion inside each domain and $\bar{P}$ is the average conversion probability over the whole turbulent region. In this case, we have a spectral distortion but no polarization. In the limit $r \rightarrow \infty$ we saturate the probability with $1/3$ of the photons getting converted into ALPs.

The true situation will be in between the above two extreme limits and both the magnetic field and electron density would vary along the photon geodesic. In particular, when photons and ALPs propagate through the inhomogeneous medium, there is the possibility of resonance when the effective mass of photon becomes equal to the mass of the ALP,

$$\Delta_e = \Delta_a.$$ (2.14)

For inhomogeneous matter distribution and magnetic fields, relevant for considering the CMB - ALP conversions, the conversion probability is sensitive to how fast the matter/electron density and the magnetic fields, and therefore the mixing angle $2\theta$, change compared to the oscillation length, $\ell_{osc} = 2\pi/\Delta_{osc}$ [56]. We therefore define an adiabaticity parameter (with $\nabla$ denoting the spatial derivative with respect to the physical distance along the line of sight or proper time),

$$\gamma_{ad} = \frac{\pi}{\ell_{osc} \nabla \theta} = \frac{\Delta_{osc}}{\sin(2\theta) \cos(2\theta) \nabla (\ln \Delta_{\gamma a}) + \sin(2\theta) \Delta_e / \Delta_{osc} \nabla (\ln \Delta_e)}$$ (2.15)

with the adiabatic limit defined as $\gamma_{ad} \gg 1$. The propagation is adiabatic when the length scale over which the mixing angle changes, $1/\nabla \theta$ is much larger than the oscillation length $\ell_{osc}$. In the adiabatic limit, the final conversion probability depends only on the initial mixing angle when the photon is emitted, $\theta_0$, and the final mixing angle at the detector $\theta$ irrespective of whether there is a resonance [56]

$$P(\gamma \rightarrow a) = \frac{1}{2} (1 - \cos^2 \theta_0 \cos 2\theta)$$ (2.16)

The first term in the denominator in eq. (2.15) arises due to the inhomogenous magnetic fields and the second term due to inhomogeneity in the matter distribution and ionization fraction. At resonance $\sin(2\theta) = 1; \cos(2\theta) = 0$ and the first term in the denominator due to the inhomogeneous magnetic field vanishes. Therefore, for resonant conversion only the inhomogeneity in the matter is relevant and the adiabaticity parameter becomes

$$\gamma_{ad}(\text{resonance}) = 4\Delta_{\gamma a}^2 / |\nabla \Delta_e|.$$ (2.17)

A given cosmological average recombination and reionization history fixes the denominator. We can therefore plot the $g_{\gamma a} B_T$ for which $\gamma_{ad}(\text{resonance}) = 1$ as a function of redshift. This curve separates the adiabatic and non-adiabatic regions of the parameter space at each redshift. For values of $g_{\gamma a} B_T$ above this curve we will have adiabatic resonances for ALP mass which satisfies $m_a = m_\gamma$ at that redshift and below this curve we will have a non-adiabatic resonance. This is shown in figure 2. The comoving magnetic field is plotted which is related to the physical magnetic field by $B_T(\text{physical}) = (1 + z)^2 B_T(\text{comoving})$. 

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Figure 2. Transition condition between the adiabatic and non-adiabatic resonance is plotted as $g_{\gamma a}B_T$ (comoving) in units of $10^{-10}$ GeV$^{-1}$nG. The $g_{\gamma a}B_T$ much larger than the threshold curves will result in adiabatic resonances while much smaller values will result in non-adiabatic resonances.

For the non-trivial solutions of eq. (2.2) to exist, the determinant of the operator on the left hand side must vanish. This gives us two dispersion relations [56] defining the two eigenstates ($m_{\text{eff}}^2 = \omega^2 - k^2 \approx 2\omega(\omega - k)$) of the system,

$$2\omega(\omega - k) = -\omega(\Delta_a + \Delta_a) \pm \omega\Delta_{\text{osc}}$$

$$= m_a^2 + m_\gamma^2 \pm \left( \frac{m_a^2 - m_\gamma^2}{2} \right)^2 + \omega^2 g_{\gamma a}^2 B_T^2 \right]^{1/2}$$

(2.18)

The dispersion relations or the eigenstates of the Hamiltonian are shown in figure 3 as a function of the electron number density assuming fully ionized plasma. The resonance happens when $m_a = m_\gamma$. If the resonance is adiabatic, $\gamma_{\text{ad}} \gg 1$, the system stays in the instantaneous eigenstate on the same branch of the dispersion relation. In particular in this case a photon produced at high density away from the resonance would follow the upper branch as the density of the medium decreases and we will have a full conversion to ALPs at sufficiently low densities.\(^1\) This can also be seen from eq. (2.16) with $\cos 2\theta_0 \approx 1$, $\cos 2\theta \approx -1$ giving $P(\gamma \rightarrow a) \approx 1$. If the density of the medium changes rapidly compared to the oscillation length, $\ell_{\text{osc}}$, there is a non-zero probability that the quantum system would make a transition between the two eigenstates or branches of the dispersion relation. In the limit that the change in density near the resonance is linear, the transition probability is given by the Landau-Zener formula [56, 97–100]

$$p = e^{-\pi \gamma_{\text{ad}}/2}$$

(2.19)

and the conversion probability is

$$P(\gamma \rightarrow a) = \frac{1}{2} (1 - (1 - 2p) \cos 2\theta_0 \cos 2\theta)$$

(2.20)

\(^1\)This is similar to the MSW effect [94–96] in case of neutrino flavor oscillations.
Figure 3. Dispersion relations for the photon-ALP system as a function of the electron density. The electron density in the solar neighbourhood, in particular the local bubble, is also marked. For this plot the ALP mass is $m_a = 10^{-14}$ eV, and $g_{\gamma a}B_T = 10^{-6}$ GeV$^{-1}$ nG. Also shown are the trajectories along the dispersion relations for adiabatic and non-adiabatic cases when photons/ALP are propagating through an inhomogeneous plasma.

Figure 4. Cosmic evolution of the photon effective mass $m_\gamma$. We assume a standard ΛCDM recombination history with reionization happening at $z = 8$ calculated using CLASS [101] in HyREC mode [102–104].

The cosmic evolution of the photon effective mass $m_\gamma$ is shown in figure 4. For detection at earth, we should take into account the fact that solar system is inside a local hot bubble with electron density $\sim 5 \times 10^{-3}$ cm$^{-3}$ [105–107] with radius $\sim 100$ pc. The ionization fraction is high enough that we can ignore the neutral hydrogen contribution to the photon mass, yielding $m_{\gamma}^{\text{local}} \approx 2.6 \times 10^{-12}$ eV. This is also shown in figure 4. For ALP masses far from...
resonance, we have the oscillation length $\ell_{\text{osc}} \lesssim 7 \times 10^{-3}(\nu/150\text{GHz})$ pc. Therefore, for $m_a \gg m_{\gamma}^{\text{local}}$ we are detecting the photons in vacuum while for $m_a \ll m_{\gamma}^{\text{local}}$ we are detecting the photons at high electron density. Previous analysis of cosmological CMB constraints has assumed the cosmic average electron density of $n_e \approx 2 \times 10^{-7}$ cm$^{-3}$ for calculating $m_{\gamma}$ at the detection point [58, 108].

2.1 Cosmological constraints: resonant case

Before the epoch of electron-positron annihilation at $z \sim 10^8 - 10^9$, the high electron-positron number density results in large scattering rate of photons with electron and positrons, heavily damping the photon-ALP conversions [109, 110]. We can therefore take as the initial state pure photons at $z \sim 10^8$. Depending on the ALP mass, we may have one or more resonances between $z = 10^8$ and today. We can roughly estimate the effect of resonances on the final state from figure 3. As long as the resonances are non-adiabatic and the detection is done far from the resonance, the probability of conversion remains small. Also, if there are an even number of completely adiabatic resonances, we again end up with a photon. To get a large conversion probability from photon to ALP we must have an odd number of adiabatic resonances. From figure 4, we see that for $m_a \gtrsim 3 \times 10^{-12}$ there is only one resonance and the condition that this resonance should be non-adiabatic gives an upper limit on $g_{\gamma a}B_T$ (figure 2). For $8 \times 10^{-15} \lesssim m_a \lesssim 2 \times 10^{-12}$ multiple resonances happen. In this case, the most stringent constraints would come from the most adiabatic resonance which would occur between recombination and reionization from figure 2. These constraints have already been derived by [108] and we will not repeat them here. It was claimed in [58] that the occurrence of two non-adiabatic resonances places very stringent constraints and in particular rules out the mass range between $10^{-14} \leq m_a \leq 5 \times 10^{-13}$. However, we see from figure 3 that this cannot happen. In fact starting at high density and going through two non-adiabatic resonances, we will end up in the top dispersion relation in figure 3 on the right/high-density side of the resonance. In [58] however it was assumed that the photons are finally detected in vacuum, implying an additional adiabatic resonance which is not present in the formula (for two resonances) that they used making these constraints of [58] invalid. If there was indeed an additional adiabatic resonance, it would be this resonance which would provide the final constraint on $g_{\gamma a}B_T$ as were derived in [108].

We can make the above statements precise as follows. Let us denote the two eigenstates after $a^{\text{th}}$ resonance (far from the resonance) plotted in figure 3 by normalized states $|\psi_1(a)\rangle, |\psi_2(a)\rangle$. Each of these eigenstates $\psi_i$ is a superposition of photon and axion as determined by the mixing angle $\theta$. The level crossing probability $p$ then denotes the probability of going from initial state $|\psi_i(0)\rangle$ to final state $|\psi_j(1)\rangle$ after crossing one resonance,

$$|\langle \psi_i(1)|\psi_j(0)\rangle|^2 = \begin{pmatrix} 1-p & p \\ p & 1-p \end{pmatrix}.$$  \hspace{1cm} (2.21)

In case of $N$ resonances we get [100, 111]

$$|\langle \psi_i(N)|\psi_j(0)\rangle|^2 = |\langle \psi_i(N)|\cdots|\psi_k(a)\rangle\langle \psi_k(a)|\psi_l(a-1)\rangle\langle \psi_l(a-1)|\cdots|\psi_j(0)\rangle|^2$$

$$= \prod_{a=1}^{N} \begin{pmatrix} 1-p_a & p_a \\ p_a & 1-p_a \end{pmatrix} \equiv \begin{pmatrix} 1-p & p \\ p & 1-p \end{pmatrix},$$  \hspace{1cm} (2.22)
where $p$ is the final level crossing probability after $N$ resonances which can be written as

$$p = \frac{1}{2} \left( 1 - \prod_{a=1}^{N} (1 - 2p_a) \right). \quad (2.23)$$

In the above calculation we have ignored the interference between different resonances and treated the level crossing probabilities as classical probabilities. This is justified if there is decoherence in the wavefunction evolution [112, 113], for example, due to propagation through the stochastic primordial magnetic fields in-between the resonances. Starting at high electron densities, in case of $N$ even, we will be back on the right side of the resonance in figure 3 while for $N$ odd we will end up on the left (or low electron density) side of resonance. The photon to axion probability is equal to the probability that starting with an approximately pure photon case on the upper eigenstate at high electron densities we end up on the axion line far from the resonance. It is therefore given by

$$P(\gamma \rightarrow a) = \begin{cases} p : N \text{ even} \\ 1 - p : N \text{ odd} \end{cases} \quad (2.24)$$

We note that we do not have a choice to independently choose $N$ even or odd and at the same time require final detection at high density or in vacuum. Specifying one condition automatically fixes the other.

For the frequency range $\nu \gtrsim 150$ GHz, we will always have two resonances during the dark ages for small axion masses on account of the effective mass of photon getting dominated by neutral gas and becoming imaginary and then real again once reionization starts. In this case of two resonances, starting at upper left curve in figure 3, the conversion to axion would occur if at one of the resonances we cross level (with probability $p_i$) but fail to do so at the other resonance (with probability $(1 - p_j)$), where $p_i$ is the level crossing probability for $i$th resonance. We therefore have the total conversion probability for two resonances

$$P(\gamma \rightarrow a) = p = p_1(1 - p_2) + p_2(1 - p_1) \quad (2.25)$$

in agreement with eq. (2.23) and similar situation in [58]. This result is true for all axion masses, including $10^{-14} \leq m_a \leq 5 \times 10^{-13}$. It is also clear that if one of resonances is more adiabatic than the other (larger $(1 - p_1)$), then that resonance will dominate the overall probability. Using $P(\gamma \rightarrow a) = p$ (instead of $P(\gamma \rightarrow a) = 1 - p$ used by [58]) for this mass range yields the correct constraint which is similar to the mass range just outside this interval and much weaker than what is claimed in [58].

### 2.2 Cosmological constraints: non-resonant case

For axion mass $m_a \leq 10^{-14}$ the only resonances are the ones during the dark ages when the effective mass of the photons becomes imaginary for $\nu \gtrsim 150$ GHz (observed frequency today). These constraints are considered in [58]. In this case we can also expect competitive constraints from non-resonant photon-axion oscillations in the stochastic primordial magnetic fields in voids [57]. Previous studies of non-resonant conversion have relied on the toy model of randomly oriented magnetic field domains leading to eq. (2.13). This is however an unrealistic oversimplification. More realistically we should expect the primordial magnetic fields to be Gaussian random vector fields with a power spectrum that depends on the production
mechanism [75]. In this case we cannot separate the voids into homogeneous regions across which the magnetic fields change abruptly. The variation in magnetic fields across the void would be smooth and the adiabaticity parameter, eq. (2.15) plays an important role in this case. For the adiabatic evolution eq. (2.16) applies and the conversion probability would be determined by the high density regions at the edge of void with small mixing angles rather the low density regions near the center of the void with large mixing angles. For adiabatic evolution therefore we expect the photon-axion conversion to be highly suppressed.

For low axion masses, $\Delta a \ll \Delta e$ and small mixing angle $\Delta \gamma_a \ll \Delta e$ we have $\cos 2\theta \approx 1$ and the expression for adiabaticity parameter can be simplified, taking a single Fourier mode $k_B, k_e$ for the magnetic field and electron distribution respectively,

$$\gamma_{ad} \approx \frac{\Delta_{osc}}{\sin(2\theta)(k_B + k_e)} \approx 2 \left( \frac{n_e}{10^{-9} \text{ cm}^{-3}} \right)^2 \left( \frac{10^{-10} \text{ GeV}^{-1}}{g_{\gamma a}} \right) \left( \frac{1 \text{ nG}}{B_T} \right) \left( \frac{0.1 \text{ pc}^{-1}}{k_B} \right),$$

where we have assumed that the magnetic field changes more rapidly than the electron density. We also need the magnetic field to change randomly in order for eq. (2.13) to be applicable, therefore the evolution should be non-adiabatic w.r.t. the changes in the magnetic field. We see from eq. (2.26) that we have adiabatic evolution on scales $\gg$ pc, in particular for Mpc scale magnetic fields considered by [57] rendering their calculations unrealistic. Most of the contribution to $P(\gamma \to a)$ would come from magnetic fields on parsec scales or smaller where the contributions from different domains can add incoherently. We are therefore in the regime where $\Delta_{osc}s \ll 1$ in each domain of size $s \sim 10$ pc. In this limit we get from eqs. (2.3) and (2.13) for a void of radius $R_V$

$$\bar{P}(\gamma \to a) \approx \frac{P(\gamma \to a)R_V}{2s} \approx \frac{\Delta_{\gamma a}^2 R_V s}{2} = 10^{-4} \left( \frac{g_{\gamma a}}{10^{-10} \text{ GeV}^{-1}} \right)^2 \left( \frac{B_T}{1 \text{ nG}} \right)^2 \left( \frac{R_V}{1 \text{ Gpc}} \right) \left( \frac{s}{10 \text{ pc}} \right).$$

We note that in this limit the conversion probability is independent of frequency. The COBE-FIRAS limit on change in the CMB frequency spectrum at the peak of blackbody [2, 4] of $\lesssim 5 \times 10^{-5}$ translates into $g_{\gamma a}B_T \lesssim 10^{-10}$ GeV$^{-1}$nG, which is a factor of $\sim 20$ weaker than the limits obtained in [57]. Our limit is still a very rough limit. To get precise constraints we must evolve the CMB photons through a realistic void profile with a realization of Gaussian random magnetic field which we leave for future work. We can however make the following important observations from above discussion. We see from eq. (2.26) that smaller scales are more non-adiabatic and should therefore contribute the most to the photon-axion conversion. However from eq. (2.27) we see that the conversion probability decreases for small scales. The conversion probability in a domain and the effect of adiabaticity therefore oppose each other. We therefore have a sweet spot or a range of scales (around 10 pc for $g_{\gamma a}B_T = 10^{-10}$ GeV$^{-1}$nG) for which the net conversion probability is maximized.

3 Photon-ALP conversion in the Milky Way halo: resonant case

The previous work on CMB-ALP conversion only considered the cosmological evolution of the mean electron density and primordial magnetic fields. We can extend the analysis by
(a) For axion mass $m_a = 5 \times 10^{-12} \text{eV}$.

(b) For axion mass $m_a = 5 \times 10^{-13} \text{eV}$.

Figure 5. Maps of the resonance conversion signal from photon-ALP at photon frequency $\nu = 150 \text{GHz}$ for $g_{11} = 10^{-11} \text{GeV}^{-1}$ with Galactic magnetic field and electron density is depicted in $\log_{10}$ scale in the Galactic coordinates with $nside=512$ using HEALPix subroutine [114]. The signal $\Delta I_\nu/I_\nu \geq 10^{-10}$ corresponds to a sky fraction of (a) $f_{\text{sky}} = 0.68$ and the mean signal of $8.34 \times 10^{-10}$ and (b) $f_{\text{sky}} = 0.41$ and the mean signal $1.06 \times 10^{-8}$ for the two cases.

considering the propagation of CMB photons in the local Universe through the Milky Way halo to the Solar System. On scales greater than $\sim 100 \text{Mpc}$, we expect to approach the homogeneous Universe [115–120] and the electron density to reach the cosmic mean value of $\approx 2 \times 10^{-7} \text{cm}^{-3}$. The electron density should increase to the Milky Way circumgalactic value
of \( \approx 10^{-5}\text{cm}^{-3} \) near the Micky Way halo at radius of about \( \approx 100\text{kpc} \) \cite{121} to \( \sim 10^{-1}\text{cm}^{-3} \) near the plane of the Galaxy \cite{122} before decreasing to \( \sim 5 \times 10^{-3}\text{cm}^{-3} \) in the local hot bubble surrounding the Solar system \cite{105–107}. Of course, we do not expect the density to vary smoothly from intergalactic medium to us but expect the distribution to be fractal and filamentary \cite{123–127} and we leave a more careful and precise analysis taking into account the inhomogeneities in the electron density for future work. We can still get rough constraints using the average electron density variation from the intergalactic medium to the Solar system. At \( n_e = \{2 \times 10^{-7}, 10^{-5}, 10^{-1}\} \text{ cm}^{-3} \), \( m_e = \{1.7 \times 10^{-14}, 1.2 \times 10^{-13}, 1.2 \times 10^{-11}\} \text{ eV} \) respectively. For \( 10^{-14} \lesssim m_a \lesssim 10^{-12} \) (the upper limit coming from the density in the local bubble of \( n_e \sim 5 \times 10^{-3}\text{ cm}^{-3} \)), there is only one resonance and we can assume production in vaccum (\( \cos \theta_0 = 1 \)) and detection at high densities (\( \cos \theta = -1 \)) giving the level crossing probability (eq. \( (2.20) \))

\[
P(\gamma \rightarrow a) = 1 - p \approx \frac{\pi g_{\text{rad}}}{2} \approx \frac{2\pi \Delta_{\gamma a}^2}{|\nabla \Delta_a|} \lesssim 10^{-4}
\]

where we assumed that \( p \approx 1 \) to satisfy COBE constraint \cite{2,4} that the fractional change in CMB spectrum should be \( \lesssim 10^{-4} \). For \( 10^{-14} \lesssim m_a \lesssim 10^{-13} \text{ eV} \), the resonance happens outside the Galactic halo with \( |\nabla \Delta_a| \approx 2.5 \times 10^4 \text{ Mpc}^{-2} \) at 100 GHz and for \( 10^{-13} \lesssim m_a \lesssim 10^{-11} \text{ eV} \) there will be a resonance inside the Galactic halo with \( |\nabla \Delta_a| \approx 2.6 \times 10^{11} \text{ Mpc}^{-2} \) at 100 GHz giving

\[
\begin{align*}
g_{\gamma a}B_T &< 4 \times 10^{-10} \text{ GeV}^{-1}\text{nG} & 10^{-14} \lesssim m_a \lesssim 10^{-13} \text{ eV} \\
g_{\gamma a}B_T &< 13 \times 10^{-10} \text{ GeV}^{-1}\mu\text{G} & 10^{-13} \lesssim m_a \lesssim 10^{-11} \text{ eV}
\end{align*}
\]

We should emphasize an important difference between the last constraint and the constraints we get on cosmological scales: we know that the Galactic magnetic field with strength of \( \mu\text{G} \) exists \cite{77,78}. The above constraints is therefore directly on the coupling constant \( g_{\gamma a} \) where as the constraints of \cite{108} are on the combination \( g_{\gamma a}B_T \).

For \( 10^{-12} \lesssim m_a \lesssim 10^{-11} \) there will be a second resonance as the photons propagate from ISM to the local hot bubble surrounding the solar system which would be less adiabatic and hence give weaker constraints.

For the resonant conversion in the Galactic halo we can use the model of Galactic magnetic field and electron number density derived from astrophysical observations including synchrotron radiation, Faraday rotation, dispersion of pulsar radiation, angular broadening of extragalactic sources and other effects associated with scattering of radiation by electrons \cite{77,78,122,128}. The details of the Galactic model are given in appendix A. Given a model of Galactic magnetic field and electron distribution, we can calculate the photon-to-axion resonant conversion probability for a given axion mass \( m_a \) along any direction using eq. \( (3.1) \) at the distance \( r \) from us where \( m_a = m_\gamma \). The results are shown in figure 5 for \( m_a = 5 \times 10^{-12}, 5 \times 10^{-13} \text{ eV} \) assuming \( g_{\gamma a} = 10^{-11} \text{ GeV}^{-1} \) at observed frequency \( \nu = 150 \text{ GHz} \). A given axion mass traces a complicated shell around the Galaxy where \( m_a = m_\gamma \) resulting in rich features in the CMB spectral distortion map. In particular, given a Galactic model, each mass \( m_a \) has its unique morphological signature in the CMB sky which is quite different from any known cosmic or Galactic foregrounds and backgrounds. The north-south asymmetry in figure 5 is a reflection of the north-south asymmetry in the Galactic magnetic field (see
appendix A). We should emphasize that in addition this distortion is 100% polarized and therefore can be easily distinguished from non-polarized cosmic and Galactic components. The lower axion masses come into resonance further out in the halo where the electron number density is smaller giving higher distortions.

3.1 Distinguishing between scalars and pseudoscalars using polarization

If we had new low mass scalar particles (LSPs) mixing with the photons [56], we would get a similar anisotropic distortion pattern as in the case of pseudoscalars such as ALPs. The interaction for scalars ($\phi$) is given by

$$L_{\text{int}} = g_{\gamma \phi} B_\gamma B_{\text{ext}} \phi$$

(3.4)

and should be compared to eq. (2.1). In eq. (3.4) $B_\gamma$ is the magnetic field of the photon and $B_{\text{ext}}$ is the external magnetic field. For the scalars therefore, in the presence of external magnetic field, the photon polarization with its magnetic field along the external magnetic field is coupled to the axions and therefore the polarization of the distortion is rotated by 90° compared to the pseudoscalars or ALPs. For equivalent couplings, we will therefore have the same anisotropy signal on the sky but orthogonal polarization. The polarized signal discussed in this section, if detected, will not only tell us whether there is a light particle that mixes with photons and the mass of this particle but also whether it is a scalar or a pseudoscalar.

4 Photon-ALP conversion in the Milky Way halo: non-resonant case

There is no resonance for $m_a \lesssim 10^{-14}$ eV except for the resonances expected when neutral gas is encountered with very low ionization fraction [108] and we will ignore these as they require a detailed multiphase model of the Galaxy. For $m_a > 10^{-11}$ eV$^{-1}$, the required values of $g_{\gamma a}$ to produce any observable spectral distortions are ruled out by CAST [69]. We will consider non-resonant conversion for $m_a \lesssim 10^{-14}$ eV in this section.

For Milky way halo, the typical length of the coherent magnetic field is of kpc scale with the strength of $\mu G$ [77, 78]. In addition, there is a turbulent component to the magnetic field in the Milky Way confined mostly in the Galactic plane with coherent lengths of 100 pc or less [129–131]. Galactic magnetic field and electron density distribution in the Galactic halo is not yet known very accurately, but the situation is expected to improve with the upcoming missions like SKA [132] in the future. The recent measurement of the Galactic magnetic field from Faraday rotation map [79] and Planck dust map [80] also indicates fluctuations in the magnetic field dominant at large angular scales. As a result, we can also expect large-scale fluctuations in the component of Galactic magnetic field transverse to the direction of propagation. For the purpose of photon-axion conversion, what is important is not the fluctuations in electron density but rather the fluctuations in photon effective mass which can transition from real to being imaginary in the highly neutral gas. Fluctuations in the electron ionization fraction can thus make the propagation of photons/axions highly non-adiabatic and should be important for non-resonant conversion.

4.1 Coherent magnetic fields and electron distribution

A model of the coherent component of the Galactic magnetic field in the disk and halo of Milky way was developed by Jansson et al. [78]. The Magnetic field in the Galactic halo can be written as a superposition of a toroidal and poloidal components. In both these components the magnetic field changes on scales of $\sim$ kpc (see appendix A for details).
Figure 6. A map of the maximum probability of conversion from photon-ALP at photon frequency \( \nu = 500 \text{ GHz} \) in the Galactic coordinates with nside=1024 using HEALPix subroutine [114] are depicted in \( \log_{10} \) scale.

The electron density also decreases exponentially with increasing distance from the Galactic plane [121, 122, 128] with a scale height again of order \( \sim \) kpc. The photon-ALPs conversion probability, eq. (2.3), is proportional to \( B_T^2/n_e^2 \) for \( m_a \lesssim 10^{-14} \) eV. Since both \( B_T \) and \( n_e \) decrease with increasing distance from the Galactic center and the Galactic plane, there will be a maximum conversion probability at some distance \( s \) for each direction in the sky and hence we can have a map of this effect. This map provides an upper limit to the axion spectral distortion we can expect in the CMB. Using the model of the Galactic magnetic field and electron density mentioned above, we obtain map of the maximum photon-ALPs conversion probability \( P(\gamma \to a)/\langle \hat{n} \rangle \) as a function of the direction of the sky in figure 6 using HEALPix [114] with nside=1024.

From eq. (2.26) we see that for the Galactic parameters we will have adiabatic evolution for scales \( \gtrsim 10^{-4} \) pc,

\[
\gamma_{ad} \approx 2 \left( \frac{n_e}{10^{-5} \text{ cm}^{-3}} \right)^2 \left( \frac{10^{-10} \text{ GeV}^{-1}}{g_{\gamma a}} \right) \left( \frac{1 \mu \text{G}}{B_T} \right) \left( \frac{10^4 \text{ pc}^{-1}}{k_B} \right) \tag{4.1}
\]

a condition easily satisfied by the large scale average magnetic field and electron distribution. For adiabatic evolution (eq. (2.16)) the final conversion probability in the large scale coherent magnetic field of the Galaxy would be decided by the mixing angles very close to us in the high density region of the Galaxy. In figure 7 we show the evolution of the \( P(\gamma \to a) \) from outskirts of the Galactic halo to solar neighborhood. The final conversion probability is given by the value near the observer which is many orders of magnitude smaller than the maximum value at around 8 kpc (see also figure 6). Thus for all practical purposes we CMB spectral distortion contribution from the large scale morphology of the Galactic magnetic field can be neglected if we do not take into account the turbulence in the interstellar medium.
4.2 Random magnetic field and turbulent gas

We can see from eq. (2.15) that there are two ways to avoid adiabatic evolution: large gradients in either the magnetic field $B_T$ or photon effective mass $m_{\text{eff}}$. There is observational evidence for turbulence in the interstellar electrons from 100 pc scales down to sub-parsec scales [133] with Kolmogorov like power law. If the source of gas turbulence is magneto-hydrodynamics (MHD), we should expect stochastic magnetic fields on sub-parsec scales also. For turbulence on scales $s = 10^{-4}$ pc in regions of size $R \sim 1000$ pc we get

$$\bar{P}(\gamma \rightarrow a) \approx \frac{P(\gamma \rightarrow a)R}{2s} \approx \frac{\Delta_{\gamma a}^2 R s}{2} = 10^{-9} \left( \frac{g_{\gamma a}}{10^{-10} \text{ GeV}^{-1}} \right)^2 \left( \frac{B_T}{1 \mu G} \right)^2 \left( \frac{R}{1000 \text{pc}} \right) \left( \frac{s}{10^{-4} \text{ pc}} \right) \quad (4.2)$$

These distortions as well as the cosmological distortions from random magnetic fields in voids would be unpolarized.

5 Forecasts for CORE, LiteBIRD and PIXIE

The detectability of the temperature and polarization spectral distortions in the CMB would depend on sensitivity as well as frequency coverage and number of channels of a CMB experiment. The frequency coverage (i.e. channels covering the full CMB spectrum from Rayleigh-Jeans to Wien region) and sufficient number of frequency channels are essential if we are to distinguish between the axion spectral distortions, primary CMB anisotropies, $y$-type and $\mu$-type distortions and Galactic and extragalactic foregrounds. In the case of anisotropic
axion distortions coming from the photon-ALP conversion in the Galactic magnetic field, we can also use the morphology of the predicted signal to distinguish it from other components.

To estimate the detectability of the signal from CMB missions using the spectrum information alone, we do a Fisher matrix analysis \cite{134}. In reality we will also have spatial anisotropy information from the Galactic model or looking towards known voids. Our estimates from Fisher analysis should therefore be considered conservative. We model the observed intensity difference from the Planck spectrum with temperature $T_{\text{CMB}} = 2.7255$ K, $\Delta I_\nu$, as

$$\Delta I_\nu = \Delta T_{\text{CMB}} s_\nu^{\text{CMB}} + A_{\gamma a} s_\nu^{\gamma a} + y s_\nu^y + A_{\text{sync}} s_\nu^{\text{sync}} + A_{\text{Dust}} s_\nu^{\text{Dust}}, \quad (5.1)$$

here we have defined \cite{135, 136}

$$s_\nu^{\text{sync}} = \left( \frac{2k_B\nu^2}{c^2} \right) \left( \frac{\nu_s}{\nu} \right)^\alpha, \quad \nu_s = 30 \text{ GHz},$$

$$s_\nu^{\text{CMB}} = \frac{2k_B\nu^2}{c^2} \frac{x^2e^x}{(e^x - 1)^2}, \quad x = h\nu/(k_B T_{\text{CMB}}), \quad T_{\text{CMB}} = 2.7255 \text{ K},$$

$$s_\nu^{\text{Dust}} = \frac{2k_B\nu^2}{c^2} \left( \frac{\nu}{\nu_0} \right)^{\beta_{\text{dust}}+1} \left( \frac{e^{h\nu/(k_B T_d)} - 1}{e^{h\nu/(k_B T_d)} - 1} \right), \quad T_d = 18\text{K}, \quad \nu_0 = 545\text{GHz},$$

$$s_\nu^y = \frac{2\nu^3}{c^2} \left( \frac{xe^x}{(e^x - 1)^2} \right) \left( \frac{x(e^x + 1)}{e^x - 1} - 4 \right), \quad (5.2)$$

where $\Delta T_{\text{CMB}}$ is the CMB temperature anisotropy in CMB temperature units of $K_{\text{CMB}}$. $A_{\text{Dust}}$ is the brightness temperature of dust at $\nu_0 = 545$ GHz and $y$ is the dimensionless amplitude of the $y$-type distortion or the thermal Sunyaev-Zeldovich (tSZ) effect.

For the resonant conversion case, we can write for the 100% polarized signal,

$$s_\nu^{\gamma a} = \frac{h\nu^3}{c^2} \left( \frac{\nu}{\nu_0} \right) \frac{I^{\gamma a}(\nu_0, m_a)}{(e^x - 1)}, \quad (5.3)$$

where $-I^{\gamma a}(\nu_0, m_a)$ is the probability of conversion at $\nu_0 = 150$ GHz for axion mass $m_a$ for the fiducial Galactic model (see eq. (3.1)) with coupling $g_{\gamma a} = 10^{-10}$ GeV$^{-1}$ and the dimensionless amplitude is defined as

$$A_{\gamma a} \equiv \left( \frac{g_{\gamma a}}{10^{-10} \text{ GeV}^{-1}} \right)^2 \quad (5.4)$$

This polarized distortion is not degenerate with the Sunyaev-Zeldovich effect or the $y$-type distortion \cite{16} and we can ignore the $y$ component while fitting for it. The polarization pattern of this distortion will also be very different from the CMB primary and lensing polarization. In particular polarized axion distortion will have both E and B modes. We will therefore assume that the morphological information will separate the polarized axion distortion from the CMB primary and lensing polarization signals. The only serious contamination is therefore expected from the Galactic dust emission and also synchrotron emission if the low frequency channels (below 100 GHz) are also used. We use the complete frequency range and include synchrotron as well as dust contaminations in the Fisher analysis. We also do an analysis with only dust and frequency channels higher than 100 GHz and compare the Fisher matrix forecasts in table 2. We see that the presence of synchrotron radiation degrades constraints
by a factor of $\sim 2$. Future experiments like CBASS \cite{137} and NEXTBASS\textsuperscript{2} can make improvements in understanding the synchrotron emissions at low frequency. Use of these experiments jointly with LiteBIRD and CORE will improve measurability of the signal. By using the unique spatial structure of the photon-axion conversion signal, one can perform a spatial template base search in the data. This will enable further improvements in the Fisher estimates. So, our estimates presented here are very conservative and expected to improve in the future.

For the non-resonant conversion we have,

$$s_\nu^{\gamma a} = \left( \frac{2h \nu^3}{c^2} \right) \frac{I^{\gamma a}(R, s)}{(e^x - 1)}, \quad (5.5)$$

where $-I(R, s)$ is the frequency independent probability of conversion of unpolarized intensity (eqs. (2.13), (2.27), (4.2)) for turbulent magnetic fields of coherence length $s$ for photons traversing a distance $R$ for coupling $g_{\gamma a} = 10^{-10}$ GeV$^{-1}$ and the dimensionless amplitude in this case is defined as

$$A_{\gamma a} \equiv \left( \frac{g_{\gamma a} B_{\text{rms}}}{10^{-10} \text{ GeV}^{-1} \text{nG}} \right)^2 : \text{voids}, \quad (5.6)$$

$$A_{\gamma a} \equiv \left( \frac{g_{\gamma a} B_{\text{rms}}}{10^{-10} \text{ GeV}^{-1} \mu G} \right)^2 : \text{Galaxy}, \quad (5.7)$$

where $B_{\text{rms}}$ is the magnetic field strength on scales $s$. In this case the distortion is unpolarized and we must marginalize over the $y$-type distortion and CMB anisotropies.

The spectrum of each of the signal is plotted in figure 8 with the amplitudes chosen so that the intensities are of similar amplitude allowing us to compare the shapes of the spectra. The detectability of photon-ALP conversion depends upon the error budget of a particular mission, its frequency coverage and number of frequency channels available. The measurability of a non-degenerate parameter ($p_i$) depends upon the covariance matrix ($C_{ii}$), which in turn depends upon the inverse of the Fisher matrix elements ($C_{ii} = 1/F_{ii}$) \cite{134}. The elements of Fisher matrix for a set of parameters $P = (p_1, p_2, \ldots, p_n)$ are defined as \cite{134}

$$F_{ij} = \sum_{\alpha=1}^{n} \frac{\partial \Delta s(\nu_\alpha)}{\partial p_i} \frac{1}{(\Delta s^p_\nu)^2} \frac{\partial \Delta s(\nu_\alpha)}{\partial p_j}, \quad (5.8)$$

where, the sum is made over all frequency channels and $\Delta s^p_\nu$ denotes the value of instrumental noise specific to a particular mission. For the degenerate parameters the Fisher matrix is not diagonal. The elements of covariance matrix in this case are computed from the inverse of the Fisher Matrix ($C_{ij} = (F^{-1})_{ij}$). We use the model given in eq. (5.1), with the parameter vector for resonant polarized distortion given by $P = (A_{\gamma a}, A_{\text{Dust}}, \beta_d, \alpha, A_{\text{sync}})$ and for non-resonant unpolarized distortion given by $P = (\Delta T_{\text{CMB}}, A_{\gamma a}, A_{\text{Dust}}, \beta_d, y, \alpha, A_{\text{sync}})$. We calculate the Fisher matrix at following fiducial values of foreground model, $A_d = 100 \mu K$ at 545 GHz, $\beta_d = 1.5, A_{\text{sync}} = 150 \mu K$ at 30 GHz, $\alpha = 2.8$ \cite{135}.

### 5.1 Future constraints from the unpolarized axion distortion (non-resonant conversion)

The results of the Fisher analysis are shown in figure 9 for future CMB missions like PIXIE \cite{42}, CORE \cite{48} and LiteBIRD \cite{49} and also for a mission with 10 times better sensi-
Figure 8. The spectra of the components mentioned in eq. (5.1) are compared with $A_{\text{CMB}} = \pm 10 \mu K$ for the CMB anisotropies, $A_{\text{dust}} = 1 \mu K$ at 545 GHz for thermal dust emission, $A_y = 2 \times 10^{-6}$ for thermal Sunyaev-Zeldovich (tSZ) effect, $\beta_d = 1.4$ [135], for synchrotron $A_{\text{sync}} = 150 \mu K$ at 30 GHz, $\alpha = 2.8$ [136], $A_{\gamma a} = 20, \tilde{\mathcal{I}}(m_a = 5 \times 10^{-13} \text{ eV}) = -10^{-6}$ for polarized axion distortion and $A_{\gamma a} \tilde{\mathcal{I}} = -10^{-5}$ for the unpolarized axion distortion.

The signal and thus our constraints depend not only on the quantity the coupling $g_{\gamma a}$ and magnetic field strength $B_{\text{rms}}^2$ which make up the amplitude $A_{\gamma a}$ (eq. (5.6), (5.7)) but also the void or Galaxy model as shown in eqs. (2.27) and (4.2) respectively. We can rescale the constraints for different void or Galactic model using eqs. (2.27) and (4.2). The constraints in figure 9 for the voids are for the parameters mentioned in eq. (2.27) with $R_v = 1 \text{ Gpc and } ...$
| Mission   | Frequency Channels (GHz) | Instrumental noise | All sky sensitivity (10^{-27} W m^{-2} sr^{-1} Hz^{-1}) | Duration (Months) |
|-----------|--------------------------|--------------------|--------------------------------------------------------|------------------|
| PIXIE     | 30-600 \(\Delta \nu = 15\) | \((\Delta I_I^I = 4 \times 10^{-24} \& \Delta I_P^I = 6 \times 10^{-25}) W m^{-2} Hz^{-1} sr^{-1}\) per pixel. Total number of Pixels \((N) = 49152\) | \(\Delta I_I^I = 18\) \(\Delta I_P^I = 2.7\) | 48 |
| LiteBIRD  | 40, 50, 60, 68, 78, 89, 100, 119, 140, 166, 195, 235, 280, 338, 402 | \(w_T^{-1/2} = (26.1, 16.7, 13.8, 11.2, 9.4, 8.1, 6.4, 5.3, 4.1, 4.5, 4.0, 5.3, 9.2, 13.5, 26.1) \mu K arcmin\) | \(\Delta I_I^I = (1, 0.98, 1.14, 1.15, 1.23, 1.32, 1.25, 1.3, 1.25, 1.6, 1.55, 2.1, 3.2, 3.6, 4.5)\) \(\Delta I_P^I = (1.42, 1.4, 1.6, 1.64, 1.74, 1.88, 1.8, 1.9, 1.8, 2.2, 2.2, 2.9, 4.8, 5.1, 6.3)\) | 36 |
| CORE      | 60, 70, 80, 90, 100, 115, 130, 145, 160, 175, 195, 220, 255, 295, 340, 390, 450, 520, 600 | \(w_T^{-1/2} = (7.5, 7.1, 6.8, 5.1, 5.0, 5.0, 3.9, 3.6, 3.7, 3.6, 3.5, 3.8, 5.6, 7.4, 11.1, 22.0, 45.9, 116.6, 358.3)\) \(\mu K arcmin\) | \(\Delta I_I^I = (0.62, 0.77, 0.93, 0.84, 1.0, 1.2, 1.1, 1.1, 1.3, 1.3, 1.4, 1.5, 2.1, 2.5, 2.9, 4.1, 5.3, 7.0, 9.3)\) \(\Delta I_P^I = (0.88, 1.09, 1.31, 1.21, 1.4, 1.7, 1.5, 1.6, 1.8, 1.9, 1.9, 2.1, 3.0, 3.5, 4.1, 5.9, 7.5, 9.9, 13)\) | 36 |

Table 1. Instrumental noise for different missions.

\(s_v = 10\) pc. For the Galaxy, the forecasts are obtained for \(R_g = 1\) kpc and \(s_g = 10^{-4}\) pc. Our results show that the turbulent component of the voids can impose stronger constraints than the Galaxy. This is because of larger scale \((R)\) of the voids as well as the larger turbulence scale \((s)\) when the propagation becomes non-adiabatic compared to the Galaxy. The larger non-adiabaticity scale \((s)\) is in turn the result of small electron densities in the voids (eq. (2.26)). We should therefore expect strongest constraints from the emptiest voids for the same magnetic field strength. Stacking the known voids from other cosmological probes could also improve the SNR and we leave a detailed study with more realistic void profile and magnetic field spectrum for future work.

5.2 Future constraints from the polarized anisotropic axion distortion (resonant conversion)

The 100% polarized anisotropic spectral distortion from the resonant conversion in the Galactic magnetic field can evade the contamination from statistically isotropic \(y\)-distortion and CMB anisotropies due to its characteristic polarization pattern in the sky. We can therefore assume that these components would be separated using the morphological and polarization
Figure 9. 68% contour between $y$ distortion and $A_{\gamma a}$ $\propto (g_{\gamma a} B_{\text{rms}}^{\gamma a})^2$ (defined in eq. (5.7) for the Galaxy and eq. (5.6) for the voids) for different future missions. In green (the smallest contour), we plot for a case with instrumental noise better than PIXIE by a factor of 10.

information and ignore them for the Fisher analysis. The only significant contamination we must distinguish (above 100 GHz) is then the Galactic dust contamination. In figure 10, we plot the possible constraints which can be obtained from the polarized signal in presence of synchrotron and dust after marginalizing over $\alpha$, $A_{\text{sync}}$, $\beta_d$. We use the mean signal calculated in section 3 and figure 5 from the parts of the sky with signal $\Delta I_\nu/I_\nu > 10^{-10}$. This selects a fraction of sky $f_{\text{sky}} = 0.68$ for axion mass $m_a = 5 \times 10^{-12}$ eV with average distortion $\bar{I}(\nu = 150 \text{ GHz}) = 8.3 \times 10^{-8}$ and $f_{\text{sky}} = 0.4$ for axion mass $m_a = 5 \times 10^{-13}$ eV with average distortion in this fraction of sky $\bar{I}(\nu = 150 \text{ GHz}) = 1.1 \times 10^{-6}$ for $g_{\gamma a} = 10^{-10} \text{ GeV}^{-1}$. We do the Fisher analysis for the two axion masses using these mean distortions. The signal for
Table 2. Fisher forecast for two different combinations of foregrounds Dust (D) and Synchrotron (S). For dust (D) only case, we used channels only above 100 GHz. For dust and synchrotron (S+D) case, we used all the frequency channels shown in table 1. The constraints we get for S+D case are around 2 times weaker compared to the case when only dust foreground is present. For CORE, the constraints degrade by around 5 times due to the absence of low frequency channels. These estimates are conservative and can be improved by using the unique spatial structure of the photon-axion conservation signal.

| Probe          | \([F^{-1}]_{ii}\) with D only | \([F^{-1}]_{ii}\) with S+D |
|----------------|---------------------------------|---------------------------|
| \(g_{10}^2\) for \(m_a = 5 \times 10^{-13}\) eV(\(\times 10^{-3}\)) | \[0.62, 1.92, 0.83\] | \[1.29, 3.5, 2.28\] |
| \(A_{\gamma a}\) for void (\(\times 10^{-4}\)) | \[1.63, 0.48, 0.23\] | \[3.9, 1.1, 1.12\] |
| \(A_{\gamma a}\) for galaxy | \[16.3, 4.83, 2.25\] | \[39, 11.1, 11.2\] |

low mass axions is stronger and hence can be better constrained at high latitudes in comparison to the signal from high mass axions (see eq. (3.1) and section 3) because the resonance happens further out in the Galaxy where the electron density is smaller. Our knowledge of electron distribution and magnetic fields in the Galaxy should improve considerably in not far future, on the similar timescales as the future CMB missions. We should therefore expect strong direct constraints on the photon-axion coupling from the polarized anisotropic distortions of the CMB in the future in the axion mass range where resonances can occur in the Galactic halo. A Fisher forecast for different combinations of the foreground is presented in table 2. The second column of table 2 is obtained with only dust as the foreground contaminations and we used frequency channels above 100 GHz. The Fisher forecasts for the more realistic situation including both synchrotron and dust and using all frequency channels are shown in the second column in table 2. The presence of synchrotron degrades the constraints by little more than a factor of 2.

5.3 Comparison with other experiments

We compare CMB forecasts with the current bounds from CAST experiment [69], SN1987A [138] and X-ray bounds from Coma cluster [139] in figure 11. The 95% upper limits are shown. The future CMB missions can therefore provide competitive constraints compared with the lab experiments such as CAST. Other bounds [57] available in the literature have looked at the extragalactic scenario with a much lower value of \(n_e\) and magnetic field. The bounds on \(g_{\gamma a} B_T\) by Tashiro et al. [58] are obtained using the primordial magnetic field with value today of the order \(nG\). Other astrophysical constraints are from the measurement of gamma ray signal from SN 1987A \((g_{\gamma a} \leq 5.3 \times 10^{-12}\) GeV\(^{-1}\)) [138] and from X-ray observations of the Coma cluster \((g_{\gamma a} \leq 1.4 \times 10^{-12}\) GeV\(^{-1}\)) [139].

6 Conclusions

We have studied a new avenue of spectral distortion of CMB photons due to photon-ALP and photon-LSP conversion in presence of our local magnetic field of Milky way. We consider both resonant (section 3) and non-resonant (section 4) Photon-ALP and photon-LSP conversions. Even though we have done our calculations specifically for pseudoscalars such as axions, our results apply, with trivial correspondence between the coupling constants and rotation of the polarization by 90°, almost unchanged to the scalar particles. The only observable difference
Figure 10. 68% contour between dust contamination $A_{Dust}$ and $g_{\gamma a}^2 \times \left( \frac{m_a}{5 \times 10^{-12} \text{eV}} \right) \times 10^{-2}$ for the proposed specifications of different future missions from the polarized anisotropic spectral distortions of the CMB. In green, (the smallest contour) we plot the case with 10 times better sensitivity than PIXIE. The magnitude of $g_{\gamma a}$ is expressed in units of $g_{10} = 10^{-10} \text{GeV}^{-1}$ with the average signal $\bar{I} = 8.3 \times 10^{-8}$ and $1.1 \times 10^{-6}$ for $m_a = 5 \times 10^{-12} \text{eV}$ and $5 \times 10^{-13} \text{eV}$ respectively.

between the scalar and pseudoscalars is that in the case where we have a polarized signal, the polarization of the distortion (photons which disappear due to conversion to scalars or pseudoscalars) is along the direction of the transverse magnetic field in case of pseudoscalars (eq. (2.1)) and in the orthogonal direction in the case of scalar particles. (eq. (3.4)).

The resonant conversions can happen in the Galactic halo for $10^{-14} \text{eV} \lesssim m_a \lesssim 10^{-11} \text{ eV}$. The probability of conversion depends on the electron distribution as well as the large scale magnetic field structure of the Galaxy imparting a characteristic anisotropy to the spectral distortion. In addition this distortion is 100% polarized. The polarized anisotropic spectral distortion provides an ideal target for future CMB missions which would focus on the polarized signals. The anisotropic nature of the signal means that it is accessible by the CMB experiments without absolute calibration. The polarization of the signal for scalars and pseudoscalars is orthogonal to each other. In this very interesting case, we therefore get the mass of the particle from the anisotropy pattern, which varies with the particle mass, and from the polarization we can tell whether the particle coupled to photons is a scalar or a pseudoscalar particle.

For axion masses $m_a \lesssim 10^{-14} \text{eV}$ we consider non-resonant conversion in the small scale turbulent Galactic magnetic field as well as the primordial stochastic magnetic fields in the voids. This distortion is unpolarized if it is the average over large number of random magnetic field configurations. If the small scale turbulent magnetic fields in the Galaxy are correlated with the large scale magnetic field structure [77], then we would expect an anisotropy similar to that shown in figure 6. The CMB spectral distortions from non-resonant conversion
depends sensitively on the model of turbulent magnetic fields in the Galaxy and in the voids as well as the electron density profiles. We have used a simplified model for Fisher matrix analysis to estimate the level of distortion and the constraints on photon-axion coupling accessible by these distortions. Our results are encouraging and motivate a more detailed analysis with realistic models of voids and Galaxy in the future.

**Figure 11.** Assuming a Gaussian probability distribution function, 2σ upper limit achievable on (a) $g_{10}$ for resonant conversion in the Galactic magnetic field for $m_a = 5 \times 10^{-13}$ eV and (b) on $A_{\gamma a} \propto (g_{\gamma a} B_{\text{rms}})^2$ for non-resonant conversion in voids are shown for different CMB missions with conservative marginalization over both dust and synchrotron. We also plot the 95% upper bound only on $g_{10}^2$ (cyan) from ground based experiment CAST [69], the gamma ray flux of SN1987A [138] and from X-ray observations of the Coma cluster [139].
We have also shown that the strong cosmological constraints for $10^{-14} \lesssim m_a \lesssim 5 \times 10^{-13}$ eV claimed by [58] are invalid. For lower ALP masses constraints were obtained on non-resonant photon-axion conversion in stochastic magnetic fields using a toy model with magnetic field abruptly changing direction on Mpc scales in [57]. We have shown that the constraints from a more realistic primordial magnetic field model are much weaker thus illustrating the sensitivity of the photon-axion conversion on the assumptions about the intergalactic magnetic fields.

We have used the mean signal for the Fisher matrix analysis in the case of resonant conversion in the Galactic halo. However as we can see from the maps in figure 5, there is large anisotropy in the signal with the signal varying by many orders of magnitude over the sky. In particular there are regions in the sky with much higher signal than the mean. We have also not used all channels available in the CMB experiments (like PIXIE) to simplify the analysis. Using higher frequency channels would require a more sophisticated model of dust emission than our 2-parameter model. Using additional channels would help improve the sensitivity and thus our constraints. Our results however rely on the knowledge of Galactic electron distribution and magnetic fields which we expect to improve significantly with the future radio surveys, in particular with the Square Kilometer Array [132] on time scales similar to the proposed CMB space missions.

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A Galactic magnetic field and electron density model

A model of the coherent component of the Galactic magnetic field in the disk and halo of Milky way was developed by Jansson et al. [78]. Magnetic field in the Galactic halo at a radius $r$ and height $l$ can be written into toroidal ($B^{\text{tor}}$) and poloidal ($B^{\text{pol}}$) component in terms of step function $L(l, h, w) = \left(1 + e^{-2(|l| - h)/w}\right)^{-1}$. The toroidal component is separated into the north ($B_n$) and south ($B_s$) component as [78]

$$B^{\text{tor}}(r, l) = e^{-|l|/l_0}L(l, h_{\text{disk}}, w_{\text{disk}}) \times \begin{cases} B_n (1 - L(r, r_n, w_h)) & l > 0, \\ B_s (1 - L(r, r_s, w_h)) & l < 0, \end{cases}$$

(A.1)
The step function \( L(l, h, w) \) goes to zero for \( l \to 0, h \gg w \) and unity at \( l \gg h \). The first factor of \( L \) means that we restrict the toroidal component to outside the disk of height \( h_{\text{disk}} \) and the second factor of \( L \) makes the field diminish outside a radius of \( r_n, r_s \) for the northern and southern regions of the Galaxy respectively.

\[
B^{\text{pol}}(r, l) = B_X e^{-r_p/r_X} \times \begin{cases}
\left( \frac{r_p}{r} \right), & \text{with } r_p = r - |l|/\tan(\Theta^0_X)r > r_X, \\
\left( \frac{r_p}{r} \right)^2, & \text{with } r_p = \frac{r r_X^C}{r_X + |l|/\tan(\Theta^0_X)} r < r_X
\end{cases}
\]

\[\Theta_X(r, l) = \tan^{-1} \left( \frac{|l|}{r - r_p} \right).\]  

The following best fit parameters for the toroidal and poloidal components of the magnetic field are fitted by [78]: \( l_0 = 5.3 \pm 1.6 \) kpc, \( r_n = 9.22 \pm 0.08 \) kpc, \( r_s > 16.7 \) kpc, \( w_h = 0.2 \pm 0.02 \) kpc, \( h_{\text{disk}} = 0.4 \pm 0.03 \) kpc, \( w_{\text{disk}} = 0.27 \pm 0.08 \) kpc, \( B_n = 1.4 \pm 0.1 \) \( \mu \)G, \( B_s = -1.1 \pm 0.1 \) \( \mu \)G, \( B_X = 4.6 \) \( \mu \)G, \( \Theta^0_X = 49 \pm 1^\circ \), \( r^c_X = 4.8 \pm 0.2 \) kpc, \( r_X = 2.9 \pm 0.1 \) kpc.

The electron density decreases exponentially with increasing distance from the Galactic plane [121, 122, 128]. The electron density in the Galactic halo can be modeled as \( \text{sech}^2(|l|/H) [122] \).

\[
n_e(r, l) = n_1 \left[ \frac{\cos(\pi r/2A_1)}{\cos(\pi R/2A_1)} \right] \text{sech}^2(|l|/H)U(r - A_1),
\]

where, \( U(x) \) is a step function. The value of vertical scale height of \( H = 0.95 \) kpc was used by Cordes et al. [122], which was later modified to \( H = 1.8 \) kpc by Gaensler et al. [128] and \( A_1 = 17 \) kpc, \( n_1 = 0.035 \text{cm}^{-3} \). This indicates a much higher electron density at high latitudes than the previous analysis [122]. Similar to the paper by Jansson et al. [78] (which provides the model for the magnetic field), we use the model of electron density given by Cordes et al. [122] with the improved model parameters from Gaensler et al. [128].

The estimation of photon-ALP conversion depends on the model of the magnetic field and electron density in the high latitudes. In particular, for the toroidal component, the magnetic field strength drops exponentially with a scale height of \( l = 5.3 \) kpc. The electron density also decreases exponentially with increasing distance from the Galactic plane [121, 122, 128]. Current observations do not provide a good probe for the electron density in the Galactic halo. However, from the upcoming mission SKA [132] an accurate observation of electron density can improve the constraints on the value of \( n_e \) and \( B \). In this paper, we will assume the electron density model of Cordes et al. [122] with the parameter values according to Gaensler et al. [128].

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