Solving reliability redundancy allocation problem using grey wolf optimization algorithm

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Abstract. This work presents a metaheuristic approach of solving Reliability-Redundancy-Allocation-Problem (RRAP) of a system using Grey Wolf Optimization (GWO) algorithm. The RRAP is restructured here for different configurations of a system such as series, series-parallel, bridge, and a practical system of over-speed protection. The solution of RRAP provides the decision in selecting the optimal number of redundant components with the corresponding reliability level of each subsystem to maximize the overall reliability of a system subjected to non-linear resource constraints. The proposed approach using the GWO algorithm provides better results with higher exploration and exploitation capability of search space than the existing solutions in the literature. Further, with the computation of Maximum Possible Improvement (MPI) using other optimization methods, it is evident that GWO solves the RRAPs efficiently and delivers maximum reliability of the system with an optimal selection of components.

Keywords: Reliability; Redundancy; Meta-heuristic Optimization; Grey Wolf Algorithm

1. Introduction
A system is formed with the collection of components, units arranged in a specific manner in order to perform the required task with the motive to have reliable and satisfactory performance. Reliability refers to the probability with which a system functions uninterrupted in a specified period of time and operating conditions [1]. Advancement and fast growing technology in a system is committed for optimum maximum reliability ensuring minimum failure of the units in the system. This inspires the system engineers to pursue research to address the issue of reliability to meet this commitment in real engineering applications. Reliability can be improved either by replacing the existing components with the more reliable units or by adding redundant units/components in parallel. The former approach is known as Reliability Allocation. The later approach is Redundancy Allocation but results in costly and bulky system. All these approaches aims to achieve optimal reliability of the system under the constraints of available resources. The Reliability Optimization Problem (ROP) [2-3] involves selection of components with redundancy levels optimally subjected to various constraints. Conventional mathematical methods [4]-[7], heuristic methods and several intelligent meta-heuristics have been successfully developed and applied on number of redundancy optimization problems over the last two decades [8]–[12]. Besides improving the reliability using the aforementioned approaches, Reliability Redundancy Allocation Problem (RRAP) considers both aspects of allocation to resolve the issue of reliability of the system. Thus, this problem points to upgrade system reliability of with optimal selection of units or subsystems to form a complete working system under the constraints of available resources and formulated as continuous nonlinear constraint problem. The allocation of redundancy and apportionment of reliability to the components, defines the problem of Reliability
Redundancy Allocation (RRA) in systems [13]-[16]. Thus, RRAP points to upgrade the reliability of a system with an optimal selection of components or subsystems to form a complete working system under the constraints of available resources. These are known as constrained-nonlinear-mixed-integer type problems[17-18].

The computational complexity of RRAP has prompted the researchers to solve this problem using various approaches which are broadly categorized as heuristic approaches [19]-[23] and meta-heuristic approaches [24]-[38]. Solutions using heuristic approaches seek derivative of the function which is a complex nonlinear constraint and is a tedious task. Meta-heuristic approaches have received bright spotlight as they provide a simple, flexible, derivation-free solution to optimization problems in addition to avoid local optima [14]. In the case of a large problem, meta-heuristic algorithms have been combined with other heuristic methods to check time consumption and complexity of computation [24]-[26]. Recently, the subsequent work based on artificial bee colony (ABC) and cuckoo search (CS) algorithm has been used to solve four known reliability problems[31][32]. The work has claimed enhancement of the accuracy and convergence rate of the results.

In this paper, the work is inspired by the free lunch theorem [39] which has proved that there is no best meta-heuristic technique to solve all kinds of optimization problems. The relative improvement in system reliability can be indicated with the help of Maximum Possible Improvement index (MPI) [32]. Further, some methods have been studied by considering numerical examples of series system [21-22], [24], [27-29], [31-32], series-parallel system [27-29], [31-32], bridge system [27-28], [31-32]. In addition to these systems, the RRAP has been investigated for the over-speed protection system [38]. In the present study, these numerical examples are considered for the demonstration of the Grey Wolf Optimization based RRAP optimization. Also, it is considered that there is no existing component with fixed reliability i.e. all components are newly developed, unlike the case reported in [14].

Grey wolf optimization (GWO) is a meta-heuristic method to solve the optimization problems in the engineering field [40]. Recent works have compared GWO with other optimization techniques and found to have the potential to apply in many applications [40]. The present study has implemented the GWO technique to solve RRAP. The main objective of the work is to enhance the overall reliability of the system. For this, the number of redundant components and the reliability of each component in the subsystems are the decision variables to be optimized in the system.

The specifications of components such as cost, weight, and volume form the nonlinear functions and considered as the constraints of the system. The main contributions of the proposed work are as follows:

- Implementation of GWO technique to solve the RRAPs for various benchmark systems.
- Statistical analysis of the performance with the comparisons of solutions obtained by the proposed and other reported works.

The article has been organized as follows: A brief discussion about the RRAP is given in section 2. In section 3, different RRAP models of benchmark problems are presented. In section 4, GWO is briefly reviewed along with its algorithm for its implication to a problem. A comparative study of numerical results considering different models is discussed in section 5. Finally conclusion is drawn in section 6 based on the results.

2. Reliability Redundancy Allocation Problem (RRAP)

Improvement in system reliability can be brought about by two ways:
(i) Enhancing the reliability of individual component in the system i.e. reliability allocation
(ii) Using the redundant components for less reliable components i.e. redundancy allocation

It turns out from the aforementioned methods that if a system is considered to be of many correlated sub-systems, then apportionment of the reliability and allocation of the redundant components in each subsystem is the decisive factor for the reliability of a complete system. This problem is considered as Reliability Redundancy Allocation Problem (RRAP). The objective function of RRAP considers the system configuration i.e. interconnection of subsystems (e.g. series, bridge etc.), reliability level of each component and number of parallel redundant component (available in case of failure of any
component). This function also accommodates constraints of resources and the problem is termed as nonlinear constraint optimization problem. The general problem of RRAP is expressed as:

Maximize the overall system reliability objective function, \( R_s(.) \) belongs to \( i^{th} \) subsystem of overall \( M \) subsystems which is under a set of constraint function, \( g(.) \) having \( b \) as vector of resource limitation. Also, \( r_i \) and \( n_i \) are reliability and number of redundant components respectively. The mathematical form of RRAP is presented as:

\[
\begin{align*}
\text{Maximize} & \quad R_s(r_1, r_2, \ldots, r_m; n_1, n_2, \ldots, n_m) \\
\text{subject to constraints} & \quad g(r_1, r_2, \ldots, r_m; n_1, n_2, \ldots, n_m) \leq b \\
& \quad 0 \leq n_i \leq n_{\text{imax}}; \quad n_i \in \mathbb{Z}^+, r_i \in [0, 1] \in \mathbb{R}^+
\end{align*}
\]

The objective of the problem is to maximize overall system reliability by deciding the reliability of every component and number of redundant components i.e. \( r_i \) and \( n_i \) in each of \( i^{th} \) subsystem. The following assumptions and notations are considered in this work.

2.1. Abbreviations and Acronyms

The following abbreviations and symbols used are given below in the Table 1.

| Notation | Details |
|----------|---------|
| \( R_s \) | System reliability |
| \( M \) | Number of subsystems in the system |
| \( M \) | Number of constraints |
| \( n_i \) | Number of components in the subsystem \( i \), \( 1 < i \leq M \) |
| \( N \) | Vector of redundancy allocation for the system |
| \( R \) | Reliability of each component of the subsystem \( i \), \( 1 < i \leq M \) |
| \( g_i \) | Vector of component reliabilities for the system |
| \( w_{ij} \) | \( j^{th} \) constraint function, \( j = 1, 2, \ldots, M \) |
| \( c_i \) | Weight of each component in subsystem \( i \), \( 1 < i \leq M \) |
| \( v_i \) | Cost of each component in subsystem \( i \), \( 1 < i \leq M \) |
| \( R_i \) | Volume of each component in subsystem \( i \), \( 1 < i \leq M \) |
| \( Q_i \) | Reliability of the \( i^{th} \) subsystem |
| \( n_{\text{imax}} \) | Unreliability of the \( i^{th} \) subsystem |
| \( r_i \) | Maximum number of components in subsystem \( i \), \( 1 < i \leq M \) |
| \( C, V, W \) | Upper limit of system cost, volume and weight |
| \( S \) | Set of feasible solution |
| \( MPI \) | Maximum possible improvement |
| \( RRAP \) | Reliability-redundancy-allocation-problem |
2.2. Assumptions

- Each component and system can either be working or fail.
- The failure of the overall system is independent of the failure of a component of any of the subsystems.
- All redundant components are active and need no repairing.
- The reliability and parameters (like cost, weight, and volume) of all components are same in a subsystem.

3. RRAP of systems with different configurations

The mathematical representation of RRAP of systems with different configurations along with the related diagrams (as shown in figure 1) are considered in the following subsections. In these problems, different components (say, \( n \)) having respective reliability \( r_1, r_2, \ldots, r_m \) with their redundant components (say, \( m \)) are connected in a different configuration to form a system. The reliability of the overall system depends upon the reliability of each unit in the subsystem. The optimal solution lies in designing the number of redundant components in each subsystem with a respective reliability level such that the overall reliability is achieved under the given constraints. This is carried out in the preceding section using the optimization algorithm.

3.1. Series System

Consider a series system having \( n = 5 \) subsystems with \( m \) redundant units and this system is shown in figure 1(a). The mathematical expression of RRAP is presented as:

\[
\text{Maximize } R_s(r, n) = \prod_{i=1}^{5} [1 - (1 - r_i)^{n_i}] \\
\text{subject to constraints } g_1(r, n), g_2(r, n), g_3(r, n) \tag{2}
\]

Figure 1(a). Series system

3.2. Series-Parallel system

This system is presented in figure 1(b) and the mathematical expression of RRAP is presented as:

\[
\text{Maximize } R_s(r, n) = 1 - (1 - R_1 R_2) [1 - (R_3 + R_4 - R_3 R_4) R_5] \\
\text{subject to constraints } g_1(r, n), g_2(r, n), g_3(r, n) \tag{3}
\]

Where, \( R_1 = 1 - (1 - r_i)^{n_i} \)

Figure 1(b). Series-Parallel system

3.3. Bridge System

This system is presented in figure 1(c) and the mathematical expression of RRAP is presented as:
Maximize \( R_s(r, n) \)
\[
\begin{align*}
&= R_s[(1 - Q_1 Q_3)(1 - Q_2 Q_4) + Q_s[1 - (1 - R_1 R_2)(1 - R_3 R_4)]] \\
& \text{subject to constraints } g_1(r, n), g_2(r, n), g_3(r, n) \\
& \text{Where, } Q_i = 1 - R_i = (1 - r_i)^{n_i}
\end{align*}
\] (4)

The constraints for the aforementioned problems given in subsections 3.1-3.3 are considered in terms of cost, weight, and volume of the system and expressed as:

\[
\begin{align*}
g_1(r, n) &= \sum_{i=1}^{n} v_i n_i^2 - V \leq 0 \\
g_2(r, n) &= \sum_{i=1}^{n} (\alpha_i (-1000/\ln r_i)^{\beta_i} \left[ n_i + \exp \left( \frac{n_i}{4} \right) \right] - C \leq 0 \\
g_3(r, n) &= \sum_{i=1}^{n} \omega_i n_i \exp \left( \frac{n_i}{4} \right) - W \leq 0 \\
0.5 \leq r_i \leq 1, r_i \in [0, 1] \subset R^+, 1 \leq n_i \leq 5, n_i \in Z^+; i = 1, 2, \ldots, m\end{align*}
\] (5)

The constraints considered in expression (5) are \( g_1(r, n) \) i.e. volume constraints, \( g_2(r, n) \) i.e. cost constraints and \( g_3(r, n) \) i.e. weight constraints respectively. Further, \( v_i \) and \( \omega_i \) are the volume and weight of each component at stage \( i \) respectively; parameter \( \alpha_i \) and \( \beta_i \) are the shaping and scaling factor respectively of the cost reliability curve of each component in the stage \( i \); \( C, V, W \) represents the upper limit on cost, volume and weight, respectively.

**Figure 1(c).** Bridge system  
**Figure 1(d).** Over-speed protection system

3.4. **Over-speed protection system**

The fourth problem is considered for the reliability redundancy allocation problem of the over-speed protection system for a gas turbine as shown in figure 1(d). Over-speed detection is continuously provided by the electrical and mechanical systems. When an over-speed occurs, it is necessary to cut off the fuel supply. For this purpose, 4 control valves \((V_1 - V_4)\) must close. The control system is modelled as a 4-stage series system. The formulation of RRAP for the above system is expressed in equation (6).
Maximize \( R_g(r,n) = \prod_{i=1}^{4} \left[ 1 - (1 - r_i)^{n_i} \right] \)
\( g_1(r,n) = \sum_{i=1}^{4} v_i n_i^2 - V \leq 0 \)
\( g_2(r,n) = \sum_{i=1}^{n} (\alpha_i (-1000/\ln r_i)^{\beta_i} [n_i + \exp \left( \frac{n_i}{4} \right)] - C \leq 0 \)
\( g_3(r,n) = \sum_{i=1}^{4} \omega_i n_i \exp \left( \frac{n_i}{4} \right) - W \leq 0 \)
\( 0.5 \leq r_i \leq 1, r_i \in [0,1] \subset R, 1 \leq n_i \leq 10, n_i \subset Z^+, i = 1, 2, 3, 4 \)

4. Solution of RRAP using metaheuristic optimizations

Metaheuristic approaches have been reported for RRA in different systems that have been mentioned in the preceding section. The description of two popular approaches viz. PSO and CS is given as follows and is then compared with the solution of RRAP algorithms with GWO (proposed approach) in Section 5.

4.1. Particle Swarm Optimization

The algorithm focuses the communal behaviour of birds, flocking of fish schooling[41]. It is population based meta-heuristic optimization algorithm with its ease of implementation and few hyper parameters tuning is required to find minima or maxima of a given function. To start with algorithm, it works with an assumption that every particle has a position and a velocity, which may be the possible solution in a high dimensional search sphere. The position and velocity are assumed as real-time vector in a plane. Initially, the particles are randomly distributed in a search space and each particle has its personal best solution, \( p_{best} \) and the whole flock has a global best solution, \( g_{best} \) among all. The objective is that the other particles will follow the leader or the global best solution considering the reference of personal best solution, \( p_{best} \) to enhance exploration capability.

The velocity of each particle is updated after every iteration with the equation mentioned below:
\[
v_d^{t+1} = w_nv_d^t + c_1r_1(p_{best} - x_d^t) - c_2r_2(g_{best} - x_d^t) \quad (7)
\]
As the velocity of each particle is volatile so is the position and a global solution is evaluated. Thus, the equation for updating the position is mentioned below:
\[
x_n^{t+1} = x_n^t + \alpha v_n t \quad (8)
\]

The pseudo code of Particle swarm optimization is presented as following:

**PARTICLE SWARM OPTIMIZATION PSEUDO CODE**

Start
1. Initialize the parameters, \( w_1, c_1, \alpha, and c_2 \)
2. Initialize the random Population of the particles.
3. Evaluate the fitness function of each particle
While \( t < t_{max} \)
4. Update the velocity
5. Update position
6. Evaluate new particles and find fitness functions
7. Update \( g_{best} and p_{best} \)
End While
End
4.2. **Cuckoo search optimization**

This algorithm is inspired from the art of birth given procedure of cuckoos [42]. To start with basic understanding, cuckoo lays eggs in the nest of other host birds and not at all of same species. Once the cuckoo laid their egg, the host bird either destroy the eggs or abandon the nest leaving her all other eggs too. This nature inspired process follow certain idealized rules given below:

Cuckoo lays an egg at a time and leaves the egg to a random nest. The nest with high quality eggs will be moved for the next generation. The host nests are fixed in number and the host bird can identify the cuckoo or different egg based on characteristics with probability, \( P_a \in [0,1] \).

The third rule is approximated and leveraged as precision of \( P_a \) of ‘n’ nests replaced by new nests with new random solutions. In this algorithm, egg in nests will be the solution while the cuckoo’s egg represents the new solution. The aim is to have the best solution to replace the previous solution that is in the nests.

The new solution of the cuckoo search is updated using the cuckoo current location and the probability, \( P_a \) using the equation given below:

\[
x_d^{t+1} = x_d^t + \alpha \odot \text{Levy}(x) \tag{9}
\]

Where, \( \alpha \) is step size, \( t \) is iteration \( d \) is dimension and \( \odot \) represents multiplication based on entry like other meta-heuristic approach.

The Levey flights are considered as more useful and efficient with high exploration capability as it has larger area of coverage via step size. The step size can be calculated as:

\[
\text{Levey}(\alpha) \sim \frac{u}{|y|^\alpha} \tag{10}
\]

Where, \( u \) and \( v \) are randomly distributed given as:

\[
u \sim N(0, \sigma_u^2), v \sim N(0, \sigma_v^2)
\]

\[
s_{\alpha} = \left[ \frac{\Gamma(1+\alpha) \sin \left( \frac{\pi \alpha}{2} \right)}{\Gamma \left( \frac{1+\alpha}{2} \right) \alpha^2 \left( \frac{\alpha-1}{2} \right)^{\alpha/2}} \right]^{1/\alpha} , \sigma_v = 1
\]

\[
\alpha \in [0.3, 1.99]
\]

Where, \( \Gamma() \) is gamma function of the variable. In Harish et al [32], the constraints are handled using a penalty function.

\[
F(x) = \begin{cases} f(x) \text{if } x \in S \\ f_w + \sum_{j=1}^{M} b_j (n) \text{if } x \notin S \end{cases}
\tag{11}
\]

Where, \( x \) is the solution, \( f_w \) is the worst feasible solution in population and set it to 0 if no solution. The pseudo code of Cuckoo search optimization is presented as following:
The aforementioned benchmark problems are solved by wolf grey optimization in this work and the results are compared with some of the latest optimization techniques. A brief description along with pseudo code is presented in the following section.

5. Proposed Solution of RRAP using Grey Wolf Optimization

In the present work, the problem of reliability redundancy allocation (RRA) for the mentioned systems (discussed in Section 3) by employing a recent metaheuristic Grey Wolf Optimization (GWO) method. To best of our knowledge, GWO has not been used for the problem of RRA for any system configuration. The following steps are carried out for the proposed methodology:

5.1. Problem Formulation

In this the problem of RRA is defined for general systems. As presented in equation (1) Maximize the entire system reliability objective function, \(R_g\) \((.\) belongs to \(i^{th}\) subsystem of overall \(m\) subsystems which is under a set of constraint function, \(g(.\) having vector of resource limitation as \(b\).

The mathematical form of RRAP is presented as:

Maximize:

\[
R_g(r_1, r_2, \ldots, r_m; n_1, n_2, \ldots, n_m)
\]

subject to constraints:

\[
g(r_1, r_2, \ldots, r_m; n_1, n_2, \ldots, n_m) \leq b
\]

\[
0 \leq n_i \leq n_{\text{max}}, n_i \in Z^+, r_i \in [0, 1] \in R^+
\]

This problem is solved using the following GWO technique for series system, series-parallel system, complex system and over-speed protection system as shown in figure 1.

5.2. Proposed Methodology: GWO technique

In this subsection, fundamentals of the technique along with the pseudocode has been brought out. Grey Wolf Optimization (GWO) is an advanced Meta-heuristic optimization algorithm developed by Mirjalili et al. [40] in the year 2014. As the name depicts, mimic the natural behaviour of grey wolf generally found wild. This algorithm is primarily based on the hunting and social behaviour with hierarchical practices focus on leadership. For simulation purpose, the grey wolf pecking order is as follows-
The most prominent ones are the Alpha wolves, are the leader of the pack and are the decision makers. Second come the Beta wolves which the advisors of the Alpha group and sub-ordinate of the decision are making process. The whole process of finding the prey is bifurcated into three steps:

- Tracking, chasing and advancing towards prey
- Pursuing, surrounding and harassing the prey, until it stops its motion.
- Attacking the prey

The equation for surrounding is mention below:

\[
\overrightarrow{D} = C. \overrightarrow{X}_p(t) - \overrightarrow{X}(t) \quad \{12\}
\]

\[
\overrightarrow{X}(t + 1) = \overrightarrow{X}_p(t) - A.\overrightarrow{D} \quad \{13\}
\]

Where, \(t\), \(\overrightarrow{A}\), \(\overrightarrow{C}\), \(\overrightarrow{X}_p\) and \(\overrightarrow{X}\) denote current iteration, coefficient 1, coefficient 2, prey position and Grey wolf position respectively.

\[
\overrightarrow{A} = 2. \overrightarrow{a}. \overrightarrow{r}_1 \quad \{14\}
\]

\[
\overrightarrow{C} = 2. \overrightarrow{r}_2 \quad \{15\}
\]

Where, 
\[
\overrightarrow{a} = 2\left(1 - \frac{t}{t_{\max}}\right) \quad \{16\}
\]

\[
\overrightarrow{r}_1, \overrightarrow{r}_2 \in [0,1]
\]

The position of the grey wolves are updated after every iteration as they are moving towards the prey. The location update is based on the highest three obtained locations or the three best locations which are representative as follows:

\[
\begin{align*}
\overrightarrow{X}_1(t) &= \overrightarrow{X}_0 - \overrightarrow{A}_1.\overrightarrow{D}_a \\
\overrightarrow{X}_2(t) &= \overrightarrow{X}_0 - \overrightarrow{A}_2.\overrightarrow{D}_b \\
\overrightarrow{X}_3(t) &= \overrightarrow{X}_0 - \overrightarrow{A}_3.\overrightarrow{D}_\delta
\end{align*}
\]

\[
\overrightarrow{X}(t + 1) = \frac{\overrightarrow{X}_1(t) + \overrightarrow{X}_2(t) + \overrightarrow{X}_3(t)}{3} \quad \{17\}
\]

Where, \(\overrightarrow{D}_a\), \(\overrightarrow{D}_b\) and \(\overrightarrow{D}_\delta\) are vector distances represented as:

\[
\begin{align*}
\overrightarrow{D}_a &= \overrightarrow{C}_1.\overrightarrow{X}_0 - \overrightarrow{X} \\
\overrightarrow{D}_b &= \overrightarrow{C}_2.\overrightarrow{X}_0 - \overrightarrow{X} \\
\overrightarrow{D}_\delta &= \overrightarrow{C}_3.\overrightarrow{X}_0 - \overrightarrow{X}
\end{align*}
\]

The pseudo code of Grey wolf optimization is presented as following:

\[
\text{GREY WOLF OPTIMIZATION PSEUDO CODE}
\]

\[
\begin{align*}
\text{Start} & \\
1. & \text{Initialize the hyper-parameters} \\
2. & \text{Initialize the random population of the grey wolves} \\
3. & \text{Evaluate the fitness function and find } \alpha, \beta, \delta \text{ wolves} \\
\quad & \text{While } t_r < t_{\max} \\
\quad & \text{For each wolf} \\
4. & \text{Update the position of search agents} \\
\quad & \text{End For} \\
5. & \text{Update } a, A \text{ and } C \\
6. & \text{Calculate the fitness of the search agents} \\
7. & \text{Update } X_\alpha, X_\beta \text{ and } X_\delta \\
\text{End While} & \\
\text{End}
\end{align*}
\]
5.3. GWO Methodology for RRAP

This section demonstrates the employment of GWO for the problem of RRA. For the constraint problem such as RRAP as given in section 3, the GWO approach, it is needed to remain in the search space as well as satisfy the three constraints namely, $g_1(r,n)$, $g_2(r,n)$, and $g_3(r,n)$ cost, volume and weight. For a system design, these constraints are very important and have a limit on them i.e. $C$, $V$ and $W$ respectively. While evaluating the solution, the maximum values of velocity is one and minimum is zero. If the constraints of the systems are not satisfied, it will have the system reliability value as zero as given in (17).

$$R_s = \begin{cases} R_s; & \text{if } \left\{ \begin{array}{l} g_1(n,r) \leq V \\ g_2(n,r) \leq C \\ g_3(n,r) \leq W \end{array} \right. \\ 0; & \text{else} \end{cases}$$  \hspace{1cm} (17)$$

As reliability cannot be negative so zero is considered as minimum value, and such as system is non-reliable system. The metaheuristic optimization starts as per the algorithm mentioned and random allotted values for subsystem reliability, ‘r’ and number of subsystems,’n’. The number of subsystems randomly allocated will depend upon population size. One random situation will be $(r_1, r_2, \ldots, r_n; n_1, n_2, \ldots, n_m)$. Now, the solution will be updated at every iteration. Hence, at end of solution, the maximum value of system reliability($R_s$) is taken.

The parametric values of the system constraints are pre-defined but can be adjusted depending upon the specifications of the system. Different systems may have different usage either on macroscopic or microscopic level, with different numbers of same systems but there is a need to have algorithm which is optimally reliable in the long run as well as the solution will remain in the limits. If the optimization of solution crosses the search space boundary, it is set to boundary depending upon the optimization which has been used. As per different searches, random algorithm is better approach as compared to limiting solution to boundary. The solution of all will remain in the limits of search space if any solution crosses the search space. The optimal solution of RRAP using GWO is explained with the help of flow chart as shown in Figure 2.

The research work of [43], a comparative study has been published comparing different optimization algorithms including GWO and CS to solve complex design optimization problems with enormous number of design variables and test the performance as well as robustness of these optimization algorithms. It is evident that GWO is one of the algorithms which has been found most capable in providing an optimal solution as compared to other algorithms including CS and PSO. The foremost benefits of the GWO method are fast convergence, better accuracy, extremely robust, and sturdy exploration capacity over the search space with an efficiency of 86%. The computation time for GWO and CS is comparable but CS is unable to provide satisfactory results. GWO proposed for RRAP of different systems which have been considered in section 3. Similar to other algorithm, common parameters of algorithm such as population, number of iterations, etc. are set.

| TABLE 2. PARAMETER VALUES FOR SERIES AND BRIDGE SYSTEM. |
|----------------------------------------------------------|
| $i$ | $10^5 \alpha_i$ | $\beta_i$ | $v_i$ | $\omega_i$ | $C$ | $V$ | $W$ |
|-----|-----------------|-----------|-------|-----------|-----|-----|-----|
| 1   | 2.330           | 1.5       | 1     | 7         | 175 | 110 | 200 |
| 2   | 1.450           | 1.5       | 2     | 8         |     |     |     |
| 3   | 0.541           | 1.5       | 3     | 8         |     |     |     |
| 4   | 8.050           | 1.5       | 4     | 6         |     |     |     |
| 5   | 1.950           | 1.5       | 2     | 9         |     |     |     |
### Table 3. Parameter Values for Series – Parallel System

| i | $10^5 \alpha_i$ | $\beta_i$ | $v_i$ | $\omega_i$ | C  | V  | W  |
|---|----------------|----------|-------|-----------|----|----|----|
| 1 | 2.550          | 1.5      | 2     | 3.5       | 175| 180| 100|
| 2 | 1.450          | 1.5      | 4     | 4.0       |    |    |    |
| 3 | 0.541          | 1.5      | 5     | 4.0       |    |    |    |
| 4 | 0.541          | 1.5      | 8     | 3.5       |    |    |    |
| 5 | 2.100          | 1.5      | 4     | 3.5       |    |    |    |

### Table 4. Parameter Values for Overspeed Detection System

| i | $10^5 \alpha_i$ | $\beta_i$ | $v_i$ | $\omega_i$ | C  | V  | W  |
|---|----------------|----------|-------|-----------|----|----|----|
| 1 | 1.0            | 1.5      | 1     | 6         | 400| 250| 500|
| 2 | 2.3            | 1.5      | 2     | 6         |    |    |    |
| 3 | 0.3            | 1.5      | 3     | 8         |    |    |    |
| 4 | 2.3            | 1.5      | 4     | 7         |    |    |    |

**Figure 2.** Flowchart of GWO operation on RRAP
6. Results and Discussion

The simulation for the proposed work is implemented on MATLAB version 2018a. The algorithm is run 25 times with a fixed number of iterations and the population in each run. The mean value and the standard deviation are calculated which indicate the effectiveness and stability of the algorithm. The MPI index is computed to show the relative improvement after implementing the GWO as:

$$\text{MPI} = \frac{R_{GWO} - R_{Other}}{1 - R_{Other}}$$ (18)

With the help of values of parameters [24][31-32] given in Table 2 to Table 4, the RRAP for different benchmarks are solved and results are presented in the following section. The study is made by comparing the results of the existing approaches and the proposed approach. The comparative study is conclusively presented in Table 5 to Table 8.

The following observations are recorded below.

Refer to Table 5 and figure 3 which provides solutions for the RRAP in context to the series system, system reliability using GWO is noted to 0.938548768439 which signify an improvement ranging between 10.051% - 10.188% using the MPI index compared with previous work. The low value of standard deviation confirms the effectiveness and stability in results over 25 runs.

Refer to Table 6, indicating the improvement of results for the RRAP problem in the case of series-parallel system using GWO approach as compared to other techniques for evaluation the system reliability with MPI index ranging 45% to 60%. The system reliability in proposed work is highest i.e. 0.999987268820. Further, MPI of 25.48% is computed with respect to the latest work.

### Table 5. RRAP Solutions for Series System

| Method | Proposed Work |
|--------|---------------|
| $N$    | (3,2,2,3)     |
| $R$    | 0.779427      |
|        | 0.869402      |
|        | 0.902674      |
|        | 0.714038      |
|        | 0.786896      |
| $MIP$  | 10.188%       |
|        | 0.83819295    |
|        | 0.785452      |
|        | 0.711945      |
|        | 0.766955      |
|        | 0.931578      |
|        | 0.930580      |
|        | 0.8550652     |
|        | 0.87085933    |
|        | 0.91140223    |
|        | 0.8505522     |
|        | 0.870318      |
|        | 0.99996875    |
|        | 0.5926%       |
|        | 0.930850      |
|        | 0.85819295    |
|        | 0.885333      |
|        | 0.917958      |
|        | 0.8842998     |
|        | 0.807318      |
|        | 0.999997418   |
|        | 0.5926%       |
|        | 0.930850      |
|        | 0.85819295    |
|        | 0.885333      |
|        | 0.917958      |
|        | 0.8842998     |
|        | 0.807318      |
|        | 0.999997418   |
|        | 0.5926%       |

### Table 6. RRAP Solutions for Series-Parallel System

| Method | Proposed Work |
|--------|---------------|
| $n$    | (3.3,2,2,3)   |
| $r$    | 0.83819295    |
|        | 0.8550652     |
|        | 0.87085933    |
|        | 0.91140223    |
|        | 0.8505522     |
| $R_{G}$| 0.99996875    |
| $MIP$  | 0.5926%       |
|        | 0.930850      |
|        | 0.85819295    |
|        | 0.885333      |
|        | 0.917958      |
|        | 0.8842998     |
|        | 0.807318      |
|        | 0.999997418   |
|        | 0.5926%       |

Refer to Table 6, indicating the improvement of results for RRAP problem in the case of series-parallel system using GWO approach as compared to other techniques for evaluation the system reliability with MPI index ranging 45% to 60%. The system reliability in proposed work is highest i.e. 0.999987268820. Further, MPI of 25.48% is computed with respect to the latest work.
Similarly, in Table 7, it can be seen a 65.57% improvement compared to cuckoo search and more compared to the rest of the techniques in the series-parallel arrangement. It is noticeable that a small improvement in the system reliability is enough to hike up the MPI and used in different sectors. An improvement of 16.87% can be seen in the overall reliability of bridge system configurations. The value of the system reliability is presented in the Table 8.

![Figure 3. Solution exploration at each iteration in series system](image1)

![Figure 4. Solution exploration at each iteration in series-parallel system](image2)
The exploration capability is less even if algorithm has a stable line solution in every iteration but due to this the solution is limited. The performance of Cuckoo search (CS) and GWO is quite comparable. Although, in GWO, after every iteration the solution finds a better solution compared the CS. But, GWO can exploit the algorithm and explore the search space in a better manner. At the end of the iterations, GWO has a better solution which is already stated in Table 5 - 8.

Further, for series-parallel in figure 4, bridge system as in figure 5 and over speed protection system in figure 6, similar behaviour is observed and, in the figures, zoomed towards the end of iterations, it is depicted clearly that the solution of GWO outperforms other algorithms. In Figure 4, due to less complexity of the system, all the optimization algorithms have a similar performance, but in case of PSO, the solution is stuck in local maxima, so whole unable to explore more solution. CS algorithm also has a good performance in terms of search capability, but unable to give a better solution compared to GWO. The zoomed portions will be able to give a clear presentation of figures as solutions are close at the end of the iterations.

| TABLE 7. RRAP SOLUTIONS FOR BRIDGE SYSTEM |
| Method | [21] | [24] | [29] | [31] | [28] | [32] | Proposed work |
|--------|------|------|------|------|------|------|---------------|
| $n$    | (3,3,3,2) | (3,3,3,1) | (3,3,3,1) | (3,3,2,4,1) | (3,3,2,4,1) | (3,3,2,4,1) | (5,5,2,1) |
| $r$    | 0.814483 | 0.814090 | 0.812485 | 0.828087 | 0.8286361 | 0.827855652338 | 0.834357661746 |
| $\gamma$ | 0.821383 | 0.864614 | 0.867661 | 0.857055 | 0.85802567 | 0.857626105413 | 0.866398306936 |
| $R_s$  | 0.896151 | 0.890291 | 0.861221 | 0.704163 | 0.91364616 | 0.914752916604 | 0.683506248872 |
| MPI    | 0.713091 | 0.701190 | 0.713852 | 0.640146 | 0.64803407 | 0.648217208595 | 0.65282 |
| Slacks of | 0.81604 | 0.965593 | 0.903800 | 0.901614 | 0.90159807702 | 0.90354635081 |
| Mean   | 0.81604 | 0.965593 | 0.903800 | 0.901614 | 0.90159807702 | 0.90354635081 |
| SD     | 1.03E-05 | 4.02E-05 | 7.04E-07 | 3.37E-04 | 0.99993862 | 0.9999827056775 | 0.9999863819915 |

| TABLE 8. RRAP SOLUTIONS FOR OVERSPEED PROTECTION SYSTEM |
| Method | [11] | [25] | [29] | [31] | [28] | [32] | Proposed Work |
|--------|------|------|------|------|------|------|---------------|
| $n$    | (6,6,3,5) | (3,6,3,5) | (5,5,5,5) | (5,6,4,5) | (5,6,4,5) | (5,5,4,6) | (5,5,4,7) |
| $r$    | 0.81604 | 0.960593 | 0.903800 | 0.901614 | 0.90159807702 | 0.90354635081 |
| $\gamma$ | 0.80109 | 0.760592 | 0.874992 | 0.849920 | 0.84997020 | 0.88822618417 | 0.88675106099 |
| $R_s$  | 0.98364 | 0.999468 | 0.999468 | 0.9995467458 | 0.999962144012 |
| MPI    | 0.9219% | 92.8% | 65.4% | 65.28% | 65.28% | 65.28% | 65.28% |
| Slacks of | 27 | 18 | 18 | 5 | 5 | 5 | 5 |
| Mean   | 0.9999487 | 0.9999546706268 | 0.999794394 |
| SD     | 0.924E-06 | 1.39E-05 | 6.98E-09 | 3.11E-04 |
Figure 5. Solution exploration at each iteration in bridge system

Figure 6. Solution exploration at each iteration in over-speed protection system

7. Conclusion
This paper investigates the reliability redundancy allocation problem considering systems with different benchmark configurations. The problem is optimally solved for a system by deciding the level of reliability and allocating the redundant components in each of its subsystems. Grey Wolf Optimization algorithm is applied successfully to get the optimal design of the available structure aiming for maximum system reliability under the defined nonlinear constraints. The superiority of the result of the present work is verified on the mentioned nonlinear problems indicating enhancement in the overall reliability of the system and MPI is calculated with respect to existing approaches to show percentage improvement in the results. Proposed approach of GWO algorithm can be applied to solve many other application like space capsule, satellite design, life support system etc. and comparative results with the existing approaches can be done.

In this study, it is assumed that components are non-repairable. For the further research, optimal solution of RRAP can be achieved by violating this assumption to tackle the more realistic situation.
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