Fermionic Construction in the Supersymmetric Coset Model

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Abstract

It is known previously that the operator product expansion (OPE) between the first $\mathcal{N} = 3$ multiplet and itself contains the second $\mathcal{N} = 3$ multiplet in the supersymmetric coset model. In this paper, by using their realizations in terms of various fermions, we compute the four kinds of OPEs between the first and the second $\mathcal{N} = 3$ multiplets for fixed $N$ and $M$ where the group of the coset contains $SU(N+M)$. By supersymmetrizing the above OPEs in $\mathcal{N} = 3$ superspace and using the various Jacobi identities between the currents, we determine the $\mathcal{N} = 3$ supersymmetric OPE between the first and the second $\mathcal{N} = 3$ multiplets completely. The right hand side of this OPE contains the various $\mathcal{N} = 3$ multiplets: the $SO(3)$ singlet $\mathcal{N} = 3$ multiplets of superspin-$\frac{5}{2}, 2, 3, 4$ and the $SO(3)$ triplet $\mathcal{N} = 3$ multiplets of superspin-$\frac{7}{2}, 3, \frac{7}{2}$. The $\mathcal{N} = 2$ superspace description and the decoupling of the spin-$\frac{1}{2}$ current of the $\mathcal{N} = 3$ superconformal algebra are also described.
1 Introduction

The Virasoro zeromode acting on the primary state, which corresponds to the WZW primary field, is given by the quadratic Casimir operator of the finite Lie algebra with some coefficient. Then the spin of the primary state is the $\frac{1}{2}$ times the quadratic Casimir eigenvalues divided by the sum of the level and the dual Coxeter number of the finite Lie algebra \[1, 2\]. The adjoint representation at the level, which is equal to the dual Coxeter number, has the spin-$\frac{1}{2}$. For the diagonal bosonic coset model \[1\], the $\mathcal{N} = 1$ supersymmetry generator of spin-$\frac{3}{2}$ was initiated by \[3, 4, 5\] and see also the previous coset constructions in \[6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\] for the various supersymmetric models \[1\]. In the different type of $\mathcal{N} = 2$ supersymmetric coset model \[17, 18\], its $\mathcal{N} = 3$ version in \[19\] has been studied. Moreover, the lowest $\mathcal{N} = 3$ multiplet in terms of various fermions has been obtained in \[20\].

The coset model in \[17, 18\] is described as

$$\frac{SU(N + M)_k \times SO(2NM)_1}{SU(N)_{k+M} \times SU(M)_{k+N} \times U(1)_{NM(N+M)(k+N+M)}}.$$  (1.1)

\(^1\)The other way to generalize the bosonic theory to the supersymmetric theory in two dimensional conformal field theory is to introduce the complex fermions in the bosonic coset model.
Note that the level $k$ in (1.1) should be equal to the dual Coxeter number of $SU(N + M)$ for its $\mathcal{N} = 3$ version: $k = N + M$. The additional third supersymmetry generator of spin-$\frac{3}{2}$ is obtained from the adjoint fermions of $SU(N + M)$ and the fermions of $SO(2NM)$; [19].

In this paper, by using the explicit realizations for the $\mathcal{N} = 3$ multiplets in terms of various fermions, we calculate the four kinds of OPEs between the first and the second $\mathcal{N} = 3$ multiplets for fixed $N$ and $M$ ($N = 3$, $M = 2$) with the help of the Thiemelens package [21]. By supersymmetrizing the above OPEs in $\mathcal{N} = 3$ superspace and using the various Jacobi identities between the currents as done in [23, 24], we determine the $\mathcal{N} = 3$ supersymmetric OPE between the first and the second $\mathcal{N} = 3$ multiplets completely. The right hand side of this OPE contains the following $\mathcal{N} = 3$ multiplets:

\[
\Phi^{(2)}_i(Z) = \frac{i}{2} \bar{\psi}^{(3)}(z) + \theta^i \frac{i}{2} \bar{\phi}^{(2),i}(z) + \theta^{3-i} \frac{1}{2} \bar{\psi}^{(2)}(z) + \theta^{3-0} \bar{\phi}^{(3)}(z),
\]

\[
\Phi^{(2)}(Z) = \frac{i}{2} \bar{\psi}^{(2)}(z) + \theta^i \frac{i}{2} \bar{\phi}^{(2),i}(z) + \theta^{3-i} \frac{1}{2} \bar{\psi}^{(2)}(z) + \theta^{3-0} \bar{\phi}^{(2)}(z),
\]

\[
\Phi^{(3),i}(Z) = \frac{i}{2} \bar{\psi}^{(3),i}(z) + \theta^i \frac{i}{2} \bar{\phi}^{(3),i}(z) + \theta^{3-i} \frac{1}{2} \bar{\psi}^{(3),i}(z) + \theta^{3-0} \bar{\phi}^{(3),i}(z),
\]

\[
\Phi^{(3)}(Z) = \frac{i}{2} \bar{\psi}^{(3)}(z) + \theta^i \frac{i}{2} \bar{\phi}^{(3),i}(z) + \theta^{3-i} \frac{1}{2} \bar{\psi}^{(3),i}(z) + \theta^{3-0} \bar{\phi}^{(3)}(z),
\]

\[
\Phi^{(4),i}(Z) = \frac{i}{2} \bar{\psi}^{(4),i}(z) + \theta^i \frac{i}{2} \bar{\phi}^{(4),i}(z) + \theta^{3-i} \frac{1}{2} \bar{\psi}^{(4),i}(z) + \theta^{3-0} \bar{\phi}^{(4),i}(z),
\]

\[
\Phi^{(4)}(Z) = \frac{i}{2} \bar{\psi}^{(4)}(z) + \theta^i \frac{i}{2} \bar{\phi}^{(4),i}(z) + \theta^{3-i} \frac{1}{2} \bar{\psi}^{(4),i}(z) + \theta^{3-0} \bar{\phi}^{(4)}(z),
\]

These $\mathcal{N} = 3$ multiplets do not overlap with the ones in [20]. We do not present the coset field contents for these currents because we know only for fixed $(N, M)$ values and they are rather complicated.

## 2 The OPEs between the first and second $\mathcal{N} = 3$ multiplets in the component approach

Let us consider the four types of OPEs between the currents for fixed $(N, M) = (3, 2)$ where the central charge is $c = 9 \frac{1}{4}$. By using the $\mathcal{N} = 3$ supersymmetry, the remaining twelve types

\[\mathcal{N} = 3\text{ superspace coordinates}\] 

\[Z = (z, \theta)\] 

\[\bar{Z} = (\bar{z}, \bar{\theta})\] 

\[\text{with} SO(3)\text{-vector index}\ i = 1, 2, 3.\] 

\[\text{The left covariant spinor derivative is} D^i = \theta^i \frac{\partial}{\partial z} + \bar{\theta}^i \frac{\partial}{\partial \bar{z}}\] 

\[\text{satisfying the anticommutators}\ \{D^i, D^j\} = 2 \delta^{ij} \frac{\partial}{\partial z}.\] 

\[\text{The fermionic coordinate difference for given index} i \text{ is defined as} \theta_{12} = \theta_1 - \theta_2, \text{ and the bosonic coordinate difference is given by} z_{12} = z_1 - z_2 - \theta_1 \bar{\theta}_2.\] 

\[\text{We will follow the conventions used in [20].}\] 

\[\text{The independent terms of the spin-4 current} \bar{\psi}^{(4)} \text{ for} (N, M) = (3, 2) \text{ are more than 180000.}\] 

\[\text{The central charge is given by} c = \frac{9}{4} NM.\]
of OPEs can be determined. By using the Jacobi identity between these OPEs with arbitrary coefficients, we obtain the final structure constants in these OPEs in terms of the arbitrary central charge.

2.1 The OPE between the spin-$\frac{3}{2}$ current and the spin-2 current

In [20], the lowest components of the first and second $\mathcal{N} = 3$ multiplet in terms of the coset fields are known. Then we can calculate the following OPE

$$\psi^{(\frac{3}{2})}(z)\psi^{(2)}(w) = \frac{1}{(z-w)^2}\left[-2i\mathcal{C}^{(\frac{3}{2})}(\frac{3}{2})\psi^{(2)}(w)\right] + \frac{1}{(z-w)^2}\left[\frac{1}{3}\partial\text{pole-2}\right](w) + \cdots. \quad (2.1)$$

It turns out that there exists a new primary current $\tilde{\psi}^{(\frac{3}{2})}$ with the structure constant $\mathcal{C}^{(\frac{3}{2})}(\frac{3}{2})$ at the second order pole of (2.1) \footnote{Although the field contents of this spin-$\frac{3}{2}$ current are the same as the one spin-$\frac{3}{2}$ current appearing in the left hand side of (2.1), each coefficient appearing in the composite operators is different from each other. We have $6\sqrt{\frac{4}{90}}\sqrt{\frac{27}{130}}(\frac{4}{3}J^1_\mu + J^2_\mu + j^\alpha)\Psi^\alpha(z) + \frac{13}{10\sqrt{15}}(-J^\mu_\mu + J^\rho_\rho + j^\rho)\Psi^\rho(z)$ while $\psi^{(\frac{3}{2})}(z) = \frac{3\sqrt{15}}{5}(-J_\mu^\rho J^\mu_\rho + J^\rho_\rho + j^\rho)\Psi^\rho(z) + \sqrt{\frac{13}{2}}(-J_\mu^\rho J^\rho_\mu + j^\mu)\Psi^\mu(z)$ from [20] where the adjoint index $\alpha$ is for $SU(N)$ and the adjoint index $\rho$ is for $SU(M)$. The various spin-1 currents are defined in Appendix C of [20].}

At the first order pole its descendant term appears. It is straightforward to compute all the other components in the first $\mathcal{N} = 3$ multiplet \footnote{The upper index of the structure constant stands for the spin of the $\mathcal{N} = 3$ multiplet in [12]. In this paper, there are seven undetermined structure constants associated with the ones in [12] and they can be fixed by calculating the OPEs between each $\mathcal{N} = 3$ multiplet and itself.} by using Appendix B (the $\mathcal{N} = 3$ primary conditions) of [20].

2.2 The OPE between the spin-2 currents and the spin-2 current

Now we can move on the next component of the first $\mathcal{N} = 3$ multiplet and the lowest component of the second $\mathcal{N} = 3$ multiplet of [20] \footnote{We have the $\mathcal{N} = 3$ stress energy tensor as follows: $\mathbf{J}(Z) = \frac{3}{2} \Psi(z) + \theta^i J^i(z) + \theta^{3-i} J^{3-i}(z) + \theta^3 T(z)$.}

$$\phi^{(2),i}(z)\psi^{(2)}(w) = \frac{1}{(z-w)^2}\left[-\frac{2}{3}i\mathcal{C}^{(\frac{3}{2})}(\frac{3}{2})\mathcal{C}^{(\frac{3}{2})}(\frac{3}{2})\phi^{(2),i}(w)\right] + \frac{1}{(z-w)^2}\left[\frac{1}{2}\partial\text{pole-2}\right] + \cdots$$

The upper index of the structure constant stands for the spin of the $\mathcal{N} = 3$ multiplet in [12]. In this paper, there are seven undetermined structure constants associated with the ones in [12] and they can be fixed by calculating the OPEs between each $\mathcal{N} = 3$ multiplet and itself.
After subtracting the descendant term in the first order pole of (2.2), we are left with three primary operators and a new primary current of spin-3. First of all, it is not clear how we can consider the last two primary operators, $\bar{C}^{(2)}(3)(2)$ and $\bar{C}^{(3)}(\frac{5}{2})(2)$ terms, at the first order pole. The way we do here is to calculate the highest order poles appearing in next two subsections, identify the two new lowest components of the second and third $\mathcal{N} = 3$ multiplet and obtain all the other components in each $\mathcal{N} = 3$ multiplet explicitly. After that, we return to the first order pole of (2.2) by allowing us to have all the possible composite operator terms of spin-3 and then it turns out that we are left with the first order pole above.

Note that there appears the dependence on the $SO(3)$ generators $^8 T^i$ where $i = 1, 2, 3$ compared with the work of [20]. The two indices $j$ and $\beta$ coming from $(T^j)^{\alpha \beta}$ and $\bar{\phi}^{(3),j,\beta}$ are contracted with each other and the index $\alpha$ becomes the $SO(3)$ vector representation due to the free index $i$ in both sides. Although we have considered all the possible terms by including the nonlinear terms in the $\mathcal{N} = 3$ multiplets, the above results imply that the nonlinear terms come from the components of $\mathcal{N} = 3$ stress energy tensor and the components of $\mathcal{N} = 3$ multiplets. In other words, there are no nonlinear terms in the components of $\mathcal{N} = 3$ multiplets.

Therefore, we have obtained the new lowest spin-3 component of the fourth $\mathcal{N} = 3$ multiplet in (1.2) and the remaining components can be determined by using the $\mathcal{N} = 3$ supersymmetry currents of the $\mathcal{N} = 3$ stress tensor as before.

2.3 The OPE between the spin-$\frac{5}{2}$ currents and the spin-2 current

From the third component of the first $\mathcal{N} = 3$ multiplet and the lowest component of the second $\mathcal{N} = 3$ multiplet, we obtain the following OPE

$$\psi^{(\frac{5}{2}),i}(z)\psi^2(w) = \frac{1}{(z-w)^2} \left[ -\bar{C}^{(2)}(\frac{5}{2})(2) \left( \frac{i}{3} \frac{c+15}{c(c-3)} \phi^{(3),j,i} + \frac{i}{c} \phi^{(2),i} \right) - 3i \frac{5(c-3)}{c} (\nabla \phi^{(2)}) + \bar{C}^{(2)}(\frac{5}{2})(2) \phi^{(3),j,i} + 2 \bar{C}^{(2)}(\frac{5}{2})(2) \phi^{(2),i} \right] (w)$$

$$+ \frac{1}{(z-w)^2} \left[ \frac{3}{5} \nabla \phi^{(2)} + \bar{C}^{(3),i}(2) \left( \frac{27i(c+1)}{c(c+3)(c-3)} \phi^{(2)} + \frac{5}{c} \phi^{(3),i} \right) (w) \right]$$

---

8Explicitly, we have the nonzero components of these generators as follows [20]: $(T^1)^{23} = -(T^1)^{32} = -i$, $(T^2)^{13} = -(T^2)^{31} = i$ and $(T^3)^{12} = -(T^3)^{21} = -i$ satisfying the algebra $[T^i, T^j] = i \epsilon^{ijk} T^k$. 

4
As described in previous section, we have found the second order pole of the above OPE (2.3) by calculating the highest order pole of the next OPE in next subsection and identifying the second $N = 3$ multiplet. In other words, the second order pole consists of a primary operator, a primary spin-$\frac{5}{2}$ current which belongs to the second $N = 3$ multiplet and other kind of spin-$\frac{5}{2}$ currents which is the component of the third $N = 3$ multiplet in (1.2).

At the first order pole, there are four kinds of primary operators, a primary spin-$\frac{5}{2}$ current belonging to the fifth $N = 3$ multiplet in (1.2) and other primary spin-$\frac{7}{2}$ currents which are the lowest components of the sixth $N = 3$ multiplet in (1.2). Also note that in the term of $\bar{C}^{(3)}_{(2)}$ in (2.3), we observe the presence of the fourth $N = 3$ multiplet.

2.4 The OPE between the spin-3 current and the spin-2 current

The final most complicated OPE can be summarized by

$$\phi^{(3)}(z)\psi^{(2)}(w) = \frac{1}{(z - w)^2} \left[ -\frac{6i}{c} \bar{C}^{(3)}(\frac{\phi}{z})(2) + \bar{C}^{(2)}_{(3)(2)}\bar{\psi}^{(2)}(z) \right] (w)$$

$$+ \frac{1}{(z - w)^2} \left[ 3 \frac{\partial}{\partial \text{pole-3}} - \bar{C}^{(3)}(\frac{\phi}{z})(2) \left( \frac{2i(7c + 15)}{(c - 3)(5c + 6)} \bar{J}^i\bar{\phi}^{(2),i} \right) 
+ \frac{8i(8c + 3)}{(c - 3)(5c + 6)} (\partial\bar{\psi}^{(\frac{7}{2})} - \frac{1}{4} \partial(\bar{\psi}^{(\frac{7}{2})})) + \frac{i(c^2 - 25c - 42)}{(c - 3)(5c + 6)} \bar{\phi}^{(3)} \right] \right] (w)$$

(2.3)
As explained before, the third order pole of (2.4) contains the lowest component of the second $\mathcal{N} = 3$ multiplet in (1.2). The component of fifth $\mathcal{N} = 3$ multiplet appears at the second order pole. We arrive at the lowest components of the sixth and seventh $\mathcal{N} = 3$ multiplets in the first order pole of (2.4). The two quasiprimary operators with $\bar{C}^{(3)}(\tilde{\mathbf{2}})(2)$ and $\bar{C}^{(2)}(\tilde{\mathbf{2}})(2)$ at the first order pole appear while the next two operators are primary.

Therefore, we have obtained the new seven $\mathcal{N} = 3$ multiplets in (1.2) by checking the presence of lowest components of those multiplets.
2.5 The $\mathcal{N} = 3$ superspace and the Jacobi identity

Because the necessary four kinds of OPEs in the previous subsections are found explicitly, we can use $\mathcal{N} = 3$ supersymmetry in the coset model we are describing to determine the remaining OPEs between the first and the second $\mathcal{N} = 3$ multiplets. That is, we simply generalize the above four kinds of OPEs to the complete OPEs in $\mathcal{N} = 3$ superspace by the following replacement

$$
\psi^{(\Delta),\alpha} \rightarrow -2i \Phi^{(\Delta),\alpha},
$$
$$
\phi^{(\Delta + \frac{1}{2}),i,\alpha} \rightarrow -2i D^i \Phi^{(\Delta),\alpha},
$$
$$
\psi^{(\Delta + 1),i,\alpha} \rightarrow -2D^{3-i} \Phi^{(\Delta),\alpha},
$$
$$
\phi^{(\Delta + \frac{3}{2}),\alpha} \rightarrow -D^{3-0} \Phi^{(\Delta),\alpha},
$$

(2.5)

together with the appropriate fermionic coordinates and the distance between the point $Z_1$ and the point $Z_2$.\footnote{Similarly, we have, for the $\mathcal{N} = 3$ multiplets in (1.2), $\bar{\psi}^{(\Delta),\alpha} \rightarrow -2i \bar{\Phi}^{(\Delta),\alpha}$, $\bar{\phi}^{(\Delta + \frac{1}{2}),i,\alpha} \rightarrow -2i D^i \bar{\Phi}^{(\Delta),\alpha}$, $\bar{\psi}^{(\Delta + 1),i,\alpha} \rightarrow -2D^{3-i} \bar{\Phi}^{(\Delta),\alpha}$ and $\bar{\phi}^{(\Delta + \frac{3}{2}),\alpha} \rightarrow -D^{3-0} \bar{\Phi}^{(\Delta),\alpha}$.}

Then we can read off the remaining OPEs by taking the $\mathcal{N} = 3$ superderivatives and putting the fermionic coordinates to zero. Because the complete OPEs are known in terms of the $\mathcal{N} = 3$ stress energy tensor and the various $\mathcal{N} = 3$ multiplets explicitly, we can put the arbitrary coefficients in the right hand sides of the complete OPEs and the number of unknown coefficients is given by sixty seven. By using the Jacobi identity between the currents of the $\mathcal{N} = 3$ stress energy tensor and the currents of the $\mathcal{N} = 3$ multiplets, all the structure constants are fixed in terms of the central charge. Some of them appear in the previous OPEs in (2.1), (2.2), (2.3) and (2.4).

Then by writing down these OPEs in $\mathcal{N} = 3$ superspace, we have the final form in next section. In Appendix A, we present the $\mathcal{N} = 3$ multiplets in (1.2) by using its $\mathcal{N} = 2$ superspace approach. In Appendix B, by decoupling the spin-$\frac{1}{2}$ current of the $\mathcal{N} = 3$ superconformal algebra, the corresponding components of $\mathcal{N} = 3$ multiplets are described.

3 The $\mathcal{N} = 3$ supersymmetric OPE

Therefore, by reexpressing all the component OPEs found in previous section in $\mathcal{N} = 3$ supersymmetric way, we arrive at the final $\mathcal{N} = 3$ supersymmetric OPE can be described as

$$
\Phi^{(\frac{1}{2})}(Z_1) \Phi^{(2)}(Z_2) = \frac{1}{z_{12}^{\frac{3}{2}}} \left[ \tilde{C}^{(\frac{1}{2})}(2) \Phi^{(\frac{1}{2})} \right](Z_2) + \frac{\theta_{12}^{3-0}}{z_{12}^{\frac{3}{2}}} \left[ -\frac{12}{c} \tilde{C}^{(\frac{1}{2})}(2) J \Phi^{(\frac{1}{2})} + \tilde{C}^{(1)}(\Phi^{(\frac{1}{2})}(2) \Phi^{(\frac{1}{2})} \right](Z_2)
$$

\footnote{Similarly, we have, for the $\mathcal{N} = 3$ multiplets in (1.2), $\bar{\psi}^{(\Delta),\alpha} \rightarrow -2i \bar{\Phi}^{(\Delta),\alpha}$, $\bar{\phi}^{(\Delta + \frac{1}{2}),i,\alpha} \rightarrow -2i D^i \bar{\Phi}^{(\Delta),\alpha}$, $\bar{\psi}^{(\Delta + 1),i,\alpha} \rightarrow -2D^{3-i} \bar{\Phi}^{(\Delta),\alpha}$ and $\bar{\phi}^{(\Delta + \frac{3}{2}),\alpha} \rightarrow -D^{3-0} \bar{\Phi}^{(\Delta),\alpha}$.}
\begin{align*}
&+ \frac{\theta^2_{12}}{z^2_{12}} \left[ \frac{1}{3} \bar{C}^{(2)}(\frac{\cdot}{2}) D^i \Phi^{(\frac{\cdot}{2})}\right] (Z_2) + \frac{\theta^2_{12}}{z^2_{12}} \left[ \bar{C}^{(2)}(\frac{\cdot}{2}) \left( \frac{(c + 9)}{c(c - 3)} J D^i \Phi^{(\frac{\cdot}{2})}\right) \right] (Z_2) \\
&- \frac{3(5c - 3)}{c(c - 3)} \bar{C}^{(2)}(\frac{\cdot}{2}) D^i \Phi^{(\frac{\cdot}{2})} + \frac{1}{4} \bar{C}^{(2)}(\frac{\cdot}{2}) D^i \Phi^{(\frac{\cdot}{2})} + \bar{C}^{(1)}(\frac{\cdot}{2}) \delta^{i\alpha} \Phi^{(\frac{\cdot}{2}), \alpha}\right] (Z_2) \\
&+ \frac{\theta^2_{12}}{z^2_{12}} \left[ \frac{3}{4} \partial(\theta^{\cdot}) - \text{term} \right) - \bar{C}^{(2)}(\frac{\cdot}{2}) \left( \frac{4(7c + 15)}{(c - 3)(5c + 6)} J D^i D^j \Phi^{(\frac{\cdot}{2})}\right) \\
&+ \frac{16(8c + 3)}{(c - 3)(5c + 6)} \left[ \bar{C}^{(2)}(\frac{\cdot}{2}) - \frac{1}{4} \partial(\Phi^{(\frac{\cdot}{2})}) \right] + \frac{(c - 25c - 42)}{(c - 3)(5c + 6)} D^{3-i} \Phi^{(\frac{\cdot}{2})}\right) \\
&+ \frac{3}{7} \bar{C}^{(2)}(\frac{\cdot}{2}) \delta^{i\alpha} D^i \Phi^{(\frac{\cdot}{2})} + \bar{C}^{(3)}(\frac{\cdot}{2}) \Phi^{(\frac{\cdot}{2})}\right] (Z_2) + \frac{1}{3} \left[ \frac{1}{z^2_{12}} \partial \left( \frac{1}{z^2_{12}} - \text{term} \right) \right] (Z_2) \\
&+ \frac{\theta^2_{12}}{z^2_{12}} \left[ \frac{1}{2} \partial(\theta^{\cdot}) - \text{term} \right) - \bar{C}^{(2)}(\frac{\cdot}{2}) \left( \frac{54(c + 1)}{c(c + 3)(c - 3)} J D^i \Phi^{(\frac{\cdot}{2})} - \frac{9(c + 1)}{(c + 3)(c - 3)} J D^i \Phi^{(\frac{\cdot}{2})}\right) \\
&- \frac{(c - 15)}{(c + 3)(c - 3)} \epsilon^{i j k} D^i J D^k \Phi^{(\frac{\cdot}{2})} - \frac{(5c - 3)}{2(c + 3)(c - 3)} \partial D^i \Phi^{(\frac{\cdot}{2})}\right) \\
&- \bar{C}^{(2)}(\frac{\cdot}{2}) \left( \frac{c}{20(c - 3)} D^{3-i} \Phi^{(\frac{\cdot}{2})} + \frac{3}{10(c - 3)} J D^i \Phi^{(\frac{\cdot}{2})} + \frac{6}{5(c - 3)} J D^i \Phi^{(\frac{\cdot}{2})}\right) \\
&- \bar{C}^{(2)}(\frac{\cdot}{2}) \left( \frac{i c}{6(c - 1)} \delta^{i \alpha} (T^{\cdot})^{\alpha \beta} D^i \Phi^{(\frac{\cdot}{2}), \beta} + \frac{2}{(c - 1)} \delta^{i \alpha} J \Phi^{(\frac{\cdot}{2})} + \bar{C}^{(3)}(\frac{\cdot}{2}) \delta^{i \alpha} \Phi^{(\frac{\cdot}{2})}\right) (Z_2) \\
&+ \theta^2_{12} \left[ \frac{3}{5} \partial(\theta^{\cdot}) - \text{term} \right) + \bar{C}^{(2)}(\frac{\cdot}{2}) \left( \frac{54(c + 1)}{c(c + 3)(c - 3)} J D^i \Phi^{(\frac{\cdot}{2})}\right) \\
&+ \frac{1}{(c + 3)(c - 3)(5c + 6)} \epsilon^{i j k} \partial D^i \Phi^{(\frac{\cdot}{2})} + \frac{2(c^2 - 18c - 27)}{(c + 3)(c - 3)(5c + 6)} \epsilon^{i j k} D^i J D^k \Phi^{(\frac{\cdot}{2})}\right) \\
&- \frac{1}{(c + 3)(c - 3)(5c + 6)} (c(c - 3) \partial J D^i \Phi^{(\frac{\cdot}{2})} - \frac{5(c^3 + 153c^2 + 396c + 216)}{5c} \partial(J D^i \Phi^{(\frac{\cdot}{2})})\right) \\
&- \frac{12(5c^3 - 18c - 9)}{c(c + 3)(c - 3)(5c + 6)} \left( \partial D^i J D^j \Phi^{(\frac{\cdot}{2})} - \frac{2}{5} \partial(D^i J \Phi^{(\frac{\cdot}{2})})\right) - \frac{18(c + 1)}{c(c + 3)(c - 3)} \epsilon^{i j k} J D^i J D^k \Phi^{(\frac{\cdot}{2})}\right) \\
&+ \bar{C}^{(2)}(\frac{\cdot}{2}) \left( \frac{36}{5(c - 3)(2c + 9)} J D^i \Phi^{(\frac{\cdot}{2})} - \frac{3}{10(c - 3)} J D^i \Phi^{(\frac{\cdot}{2})}\right) \\
&- \frac{1}{5(2c + 9)(c - 3)} (-6(c + 9) D^{3-i} \Phi^{(\frac{\cdot}{2})} - 9 \partial D^i \Phi^{(\frac{\cdot}{2})} - \frac{3(2c + 3)}{5(2c + 9)(c - 3)} \epsilon^{i j k} J D^i J D^k \Phi^{(\frac{\cdot}{2})}\right) \\
&+ \bar{C}^{(2)}(\frac{\cdot}{2}) \left( \frac{i}{(c - 1)} \delta^{i \alpha} (T^{\cdot})^{\alpha \beta} D^i \Phi^{(\frac{\cdot}{2}), \beta} - \frac{i}{14} \delta^{i \alpha} (T^{\cdot})^{\alpha \beta} (D^{3-i} \Phi^{(\frac{\cdot}{2}), \beta} - \frac{i}{5} (T^{\cdot})^{\beta \gamma} \partial \Phi^{(\frac{\cdot}{2}), \gamma})\right) \\
&+ \bar{C}^{(3)}(\frac{\cdot}{2}) \left( \frac{3i(c - 1)}{(7c - 6)} \delta^{i \alpha} (T^{\cdot})^{\alpha \beta} D^i \Phi^{(\frac{\cdot}{2}), \beta} - \frac{6}{(7c - 6)} \delta^{i \alpha} \Phi^{(\frac{\cdot}{2}), \alpha}\right)
\end{align*}
It is straightforward to obtain all the component OPEs from (3.1) by applying the above superderivatives to both sides and putting the fermionic coordinates to zero (2.5). The previous nine pole terms in (2.1), (2.2), (2.3) and (2.4) are distributed in (3.1). Compared with the
result in [20], the presence of the $SO(3)$ generators arise nontrivially. Under the large $c$ limit, most of the terms in (3.1) survive and this classical algebra should provide the asymptotic symmetry algebra of the matrix extension of $AdS_3$ bulk theory [19, 30, 31].

### 4 Conclusions and outlook

We have obtained the nontrivial $\mathcal{N} = 3$ supersymmetric OPE described in (3.1) and the right hand side of this OPE contain the various $\mathcal{N} = 3$ multiplets summarized by (1.2) where we have the explicit forms for the coset fields with fixed $(N, M)$.

Although the vacuum character in the $\mathcal{N} = 3$ Kazama-Suzuki model will be found explicitly for generic $N$ and $M$, it will not be easy to observe the extra higher spin currents completely as we add them in the right hand side of (3.1). In order to determine the structure constants appearing in these extra higher spin currents, we should use the other Jacobi identities between them. If the extra higher spin currents are present in the right hand side of (3.1) in the large $(N, M)$ values, we expect that they should appear linearly. The structure constants in them should contain the factors $(c - 6)$ corresponding to $(N, M) = (2, 2)$ case, $(c - 9)$ corresponding to $(N, M) = (3, 2)$ case and so on. We may try to calculate the OPEs manually explicitly with arbitrary $(N, M)$ dependence but this is beyond the scope of this paper.

The immediate question is that in order to see the full structure of this algebra we should compute the OPEs between the two lowest $\mathcal{N} = 3$ multiplets, the one in [20] and the one in this paper. In other words, we should calculate the OPEs $\Phi^{{\frac{3}{2}}}(Z_1) \bar{\Phi}^{{\frac{3}{2}}}(Z_2)$ and $\Phi^{{\frac{3}{2}}}(Z_1) \bar{\Phi}^{{\frac{3}{2}}}(Z_2)$ further. We expect that the $\mathcal{N} = 3$ multiplets found in [20] will appear in the right hand sides of above OPEs.

We can apply the fermionic construction of this paper to other coset model. If we do not have the $SO(2MN)_1$ and $SU(M)_{k+N}$ in the (1.1) with some change in the levels of the remaining factors, then we have the bosonic coset model. Recently, some of the nontrivial OPE in this model has been found in [32]. It would be interesting to study whether we can construct the $\mathcal{N} = 1$ supersymmetric coset model at the level $k = N + M$.

According to the classification of [17, 18], there is an orthogonal type of coset model. It is an open problem to describe whether we can observe its supersymmetric enhancement by taking the critical level as we do in this paper.

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rea(NRF) grant funded by the Korea government(MSIT)(No. 2020R1F1A1066893). MHK was supported by an appointment to the YST Program at the APCTP through the Science and Technology Promotion Fund and Lottery Fund of the Korean Government. MHK was also supported by the Korean Local Governments - Gyeongsangbuk-do Province and Pohang City.
A The $\mathcal{N} = 2$ superspace description of $\mathcal{N} = 3$ multiplets

We present each $\mathcal{N} = 3$ multiplet \( \mathcal{N} = 3 \) in terms of two $\mathcal{N} = 2$ multiplets as follows:\footnote{We have $\theta = \theta^1 + i\theta^2$ and $\bar{\theta} = -\theta^1 - i\theta^2$. By combining the two $\mathcal{N} = 2$ multiplet and the third fermionic coordinate $\theta^3$, we can reexpress them in terms of the $\mathcal{N} = 3$ multiplet as in [20].}

\[
\begin{align*}
\mathbf{W}(2) (Z) &= 2\psi(\frac{3}{2}) (z) + \theta(\phi(\frac{1}{2},1) + \bar{i}\phi(\frac{1}{2},2)) (z) + \bar{\theta}(\bar{\phi}(\frac{1}{2},1) + i\phi(\frac{1}{2},2)) (z) + \theta \bar{\theta} \psi(\frac{3}{2},3) (z), \\
\mathbf{W}(2') (Z) &= 2\phi(\frac{1}{2},3) (z) + \theta(\bar{\psi}(\frac{1}{2},1) - i\bar{\psi}(\frac{1}{2},2)) (z) + \bar{\theta}(\bar{\psi}(\frac{1}{2},1) + i\bar{\psi}(\frac{1}{2},2)) (z) + \theta \bar{\theta} 2\bar{\phi}(\frac{1}{2},3) (z), \\
\mathbf{W}(\bar{2}) (Z) &= 2\bar{\psi}(\frac{3}{2}) (z) + \bar{\theta}(\bar{\phi}(\frac{1}{2},1) + i\phi(\frac{1}{2},2)) (z) + \theta(\bar{\phi}(\frac{1}{2},1) + i\phi(\frac{1}{2},2)) (z) + \theta \bar{\theta} \bar{\psi}(\frac{3}{2},3) (z), \\
\mathbf{W}(\bar{2}') (Z) &= 2\bar{\phi}(\frac{1}{2},3) (z) + \theta(\bar{\psi}(\frac{1}{2},1) - i\bar{\psi}(\frac{1}{2},2)) (z) + \bar{\theta}(\bar{\psi}(\frac{1}{2},1) + i\bar{\psi}(\frac{1}{2},2)) (z) + \theta \bar{\theta} 2\bar{\phi}(\frac{1}{2},3) (z), \\
\mathbf{W}(\bar{3}) (Z) &= 2\bar{\psi}(\frac{3}{2}) , (z) + \theta(\phi(\frac{1}{2},3,1) + i\phi(\frac{1}{2},3,2)) (z) + \bar{\theta}(\bar{\psi}(\frac{1}{2},3,1) + i\bar{\psi}(\frac{1}{2},3,2)) (z) + \theta \bar{\theta} \bar{\psi}(\frac{3}{2},3) (z), \\
\mathbf{W}(\bar{3}') (Z) &= 2\bar{\phi}(\frac{1}{2},3) , (z) + \theta(\bar{\psi}(\frac{1}{2},3,1) - i\bar{\psi}(\frac{1}{2},3,2)) (z) + \bar{\theta}(\bar{\psi}(\frac{1}{2},3,1) + i\bar{\psi}(\frac{1}{2},3,2)) (z) + \theta \bar{\theta} 2\bar{\phi}(\frac{1}{2},3) (z).
\end{align*}
\]

The various OPEs in $\mathcal{N} = 2$ superspace can be obtained by using the package of [33] from the component results. Or we can express \( 3, 1 \) in terms of four OPEs in $\mathcal{N} = 2$ superspace.
B The $\mathcal{N} = 3$ multiplets after the decoupling of the spin-$\frac{1}{2}$ current of $\mathcal{N} = 3$ superconformal algebra

We can decouple the spin-$\frac{1}{2}$ current $\Psi$ of $\mathcal{N} = 3$ superconformal algebra and by using the following transformation of the currents we obtain the corresponding algebra explicitly

\[\bar{\psi}^{(\frac{1}{2})} \rightarrow \bar{\psi}^{(\frac{1}{2})}, \quad \bar{\phi}^{(2),i} \rightarrow \bar{\phi}^{(2),i},\]
\[\bar{\psi}^{(\frac{3}{2}),i} \rightarrow \bar{\psi}^{(\frac{3}{2}),i} - \frac{3}{c}\Psi \bar{\phi}^{(2),i}, \quad \bar{\phi}^{(3)} \rightarrow \bar{\phi}^{(3)} + \frac{3}{2c}\Psi \partial \bar{\psi}^{(\frac{1}{2})} - \frac{9}{2c}\partial \Psi \bar{\psi}^{(\frac{1}{2})},\]
\[\bar{\psi}^{(2)} \rightarrow \bar{\psi}^{(2)}, \quad \bar{\phi}^{(\frac{1}{2}),i} \rightarrow \bar{\phi}^{(\frac{1}{2}),i},\]
\[\bar{\psi}^{(3),i} \rightarrow \bar{\psi}^{(3),i} - \frac{3}{c}\Psi \bar{\phi}^{(3),i}, \quad \bar{\phi}^{(\frac{3}{2})} \rightarrow \bar{\phi}^{(\frac{3}{2})} + \frac{3}{2c}\Psi \partial \bar{\psi}^{(2)} - \frac{6}{c}\partial \Psi \bar{\psi}^{(2)},\]
\[\bar{\psi}^{(\frac{3}{2}),\alpha} \rightarrow \bar{\psi}^{(\frac{3}{2}),\alpha}, \quad \bar{\phi}^{(3),i,\alpha=j} \rightarrow \bar{\phi}^{(3),i,\alpha=j} - \frac{3}{c}i\varepsilon^{ijk}\Psi \bar{\phi}^{(\frac{1}{2}),\alpha=k},\]
\[\bar{\psi}^{(\frac{1}{2}),i,\alpha=j} \rightarrow \bar{\psi}^{(\frac{1}{2}),i,\alpha=j} - \frac{3}{c}\Psi \left(\bar{\phi}^{(3),i,\alpha=j} + \bar{\phi}^{(3),j,\alpha=i} - \delta^{ij}\bar{\phi}^{(3),k,\alpha=k}\right),\]
\[\bar{\phi}^{(4),\alpha=i} \rightarrow \bar{\phi}^{(4),\alpha=i} + \frac{3}{2c}i\varepsilon^{ijk}\Psi \bar{\phi}^{(\frac{1}{2}),j,\alpha=k} + \frac{3}{2c}\Psi \partial \bar{\psi}^{(3),\alpha=i} - \frac{15}{2c}\partial \Psi \bar{\psi}^{(\frac{1}{2}),\alpha=i},\]
\[\bar{\psi}^{(3),\alpha} \rightarrow \bar{\psi}^{(3),\alpha}, \quad \bar{\phi}^{(\frac{3}{2}),i,\alpha=j} \rightarrow \bar{\phi}^{(\frac{3}{2}),i,\alpha=j} - \frac{3}{c}i\varepsilon^{ijk}\Psi \bar{\phi}^{(3),\alpha=k},\]
\[\bar{\psi}^{(4),i,\alpha=j} \rightarrow \bar{\psi}^{(4),i,\alpha=j} - \frac{3}{c}\Psi \left(\bar{\phi}^{(3),i,\alpha=j} + \bar{\phi}^{(3),j,\alpha=i} - \delta^{ij}\bar{\phi}^{(3),k,\alpha=k}\right),\]
\[\bar{\phi}^{(\frac{3}{2}),\alpha=i} \rightarrow \bar{\phi}^{(\frac{3}{2}),\alpha=i} + \frac{3}{2c}i\varepsilon^{ijk}\Psi \bar{\phi}^{(4),j,\alpha=k} + \frac{3}{2c}\Psi \partial \bar{\psi}^{(3),\alpha=i} - \frac{9}{c}\partial \Psi \bar{\psi}^{(3),\alpha=i},\]
\[\bar{\psi}^{(3)} \rightarrow \bar{\psi}^{(3)}, \quad \bar{\phi}^{(\frac{3}{2}),i} \rightarrow \bar{\phi}^{(\frac{3}{2}),i},\]
\[\bar{\psi}^{(4),i} \rightarrow \bar{\psi}^{(4),i} - \frac{3}{c}\Psi \bar{\phi}^{(3),i}, \quad \bar{\phi}^{(\frac{3}{2})} \rightarrow \bar{\phi}^{(\frac{3}{2})} + \frac{3}{2c}\Psi \partial \bar{\psi}^{(3)} - \frac{9}{c}\partial \Psi \bar{\psi}^{(3)},\]
\[\bar{\psi}^{(\frac{3}{2}),\alpha} \rightarrow \bar{\psi}^{(\frac{3}{2}),\alpha}, \quad \bar{\phi}^{(4),i,\alpha=j} \rightarrow \bar{\phi}^{(4),i,\alpha=j} - \frac{3}{c}i\varepsilon^{ijk}\Psi \bar{\phi}^{(\frac{3}{2}),\alpha=k},\]
\[\bar{\psi}^{(4),i,\alpha=j} \rightarrow \bar{\psi}^{(4),i,\alpha=j} - \frac{3}{c}\Psi \left(\bar{\phi}^{(3),i,\alpha=j} + \bar{\phi}^{(3),j,\alpha=i} - \delta^{ij}\bar{\phi}^{(3),k,\alpha=k}\right),\]
\[\bar{\phi}^{(5),\alpha=i} \rightarrow \bar{\phi}^{(5),\alpha=i} + \frac{3}{2c}i\varepsilon^{ijk}\Psi \bar{\phi}^{(4),j,\alpha=k} + \frac{3}{2c}\Psi \partial \bar{\psi}^{(3),\alpha=i} - \frac{21}{2c}\partial \Psi \bar{\psi}^{(3),\alpha=i},\]
\[\bar{\psi}^{(4)} \rightarrow \bar{\psi}^{(4)}, \quad \bar{\phi}^{(\frac{3}{2}),i} \rightarrow \bar{\phi}^{(\frac{3}{2}),i},\]
\[\bar{\psi}^{(5),i} \rightarrow \bar{\psi}^{(5),i} - \frac{3}{c}\Psi \bar{\phi}^{(3),i}, \quad \bar{\phi}^{(\frac{3}{2})} \rightarrow \bar{\phi}^{(\frac{3}{2})} + \frac{3}{2c}\Psi \partial \bar{\psi}^{(3)}(z) - \frac{12}{c}\partial \Psi \bar{\psi}^{(3)}(z).

We expect that the result of this paper with [20] will produce the extension of the previous work in [34, 35].
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