Laboratory sources of gravitational waves

I V Fomin, V O Gladyshev, V S Gorelik, V L Kauts, A V Kaytenko and E A Sharandin
Physics Department, Bauman Moscow State Technical University, 2-nd Baumanskaya street 5, Moscow, 105005, Russia
E-mail: ingvor@inbox.ru

Abstract. We consider the possibility of generating and subsequently detecting artificial gravitational waves in a laboratory based on different approaches. As one of the methods for creating gravitational waves, we investigate oscillations of the ions of the crystal lattice of a dielectric under the influence of high-power laser radiation. The characteristics of gravitational waves obtained by this method are compared with the results of using the other approaches as well.

1. Introduction
The existence of gravitational waves is a consequence of the General theory of Relativity and any other metric gravity theories which leads to the need for their theoretical and experimental studies (for a review see, for example, [1-4]). The experimental discovery of gravitational waves from the black holes and neutron stars mergers in LIGO and VIRGO experiments [5, 6] based on the interferometric method [7] confirmed the validity of Einstein’s gravity theory.

Inflationary models of the early universe provide a theoretical justification for cosmological relic gravitational waves that affect the anisotropy and polarization of cosmic background radiation (CMB) [8, 9]. Nevertheless, this type of gravitational waves was not directly detected and, at the moment, observational data provide only the upper limit of the ratio of the squared amplitudes of relic gravitational waves and scalar cosmological perturbations [8].

One can also consider the possibility of creating artificial gravitational waves with characteristics sufficient for their further detection. Artificial (laboratory) sources of gravitational waves imply gravitational radiation of small amplitude and power, and thus, weak gravitational waves are considered as small perturbations $h_{\mu\nu}$ of Minkowski space-time $\eta_{\mu\nu}$. Thus, the metric of perturbed space-time is defined as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$.

Einstein's equations in this case can be written as [1]

$$\Box \bar{h}_{\mu\nu} = - \frac{16\pi G}{c^4} T_{\mu\nu},$$

(1)
where $T_{\mu\nu}$ is the energy-momentum tensor of matter, $G$ is the gravitational constant, $c$ is the speed of light in vacuum, $\Box$ is the d'Alembert operator, and tensor $h_{\mu\nu}$ satisfies the following conditions

$$\partial^\mu h_{\mu\nu} = 0, \quad h_{\mu\nu} = \frac{1}{2} \eta_{\mu\nu} h.$$ 

The solution of equation (1) can be written as follows [1]

$$h_{ij} = \frac{4G}{rc^5} A_{ijkl} (\hat{n}) \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} T_{ik} \left( \omega \frac{\omega n}{c} \right) e^{(-i\omega t - r/c)},$$ (2)

where $\omega$ is the frequency of gravitational waves, and tensor $A_{ijkl}$ is defined as

$$A_{ijkl} (\hat{n}) = P_{ik} P_{jl} - \frac{1}{2} P_{ij} P_{kl}, \quad P_{ij} (\hat{n}) = \delta_{ij} - n_i n_j,$$ (3)

where vector $n_i$ determines the direction of propagation of the gravitational wave.

The various sources of gravitational waves are considered, which are determined by a different type of energy-momentum tensor. Irradiation of matter with short pulses of high-power laser radiation is considered as a possible mechanism for generating high-frequency gravitational waves, which implies an increase in the characteristics of gravitational-wave radiation with an increase in the power of modern lasers [4, 10, 11].

An alternative mechanism is the parametric processes of the interaction of electromagnetic and gravitational waves in a constant magnetic field only [12-14] or in additional dielectric medium as a result of the large optical nonlinearity of the condensed dielectric and a sharp deceleration of the group velocity of light during the interaction of intense laser radiation with matter [15, 16].

In this paper, we firstly consider the model of shock waves proposed in the paper [11] that occur when a powerful laser pulse acts on a thin plate of the target material. Shock waves generate the corresponding gravitational waves and, thus, based on this model, their characteristics can be determined. Further, we will consider a model of an ionic crystal interacting with an electromagnetic wave. Also, we will find the characteristics of gravitational waves corresponding to this process and compare it with the results of the previous model. In conclusion, we will consider the prospects of detecting gravitational waves obtained by these methods.

2. The shock waves model

Shock waves arising from the irradiation of matter with short pulses of high-power laser radiation were considered as sources of high-frequency gravitational waves in [11]. The coordinate of the wave front of the shock wave can be written as follows $z_S = v_S t$, where the propagation velocity of the shock wave can be determined as

$$v_S = \sqrt{\frac{p_S}{\rho_0}}.$$ (4)

In this expression $p_S$ is the pressure of the shock wave and $\rho_0$ is the density of the medium.

The density of a shock wave in the direction of propagation $z$ is defined as

$$\rho_S = \begin{cases} 
4\rho_0, & 3z_S < z < z_S, \\
\frac{3}{4} \rho_0, & z_S < z < L, \\
\rho_0, & otherwise,
\end{cases}$$ (5)
where $L$ is the size of the material in the direction of propagation of the shock wave.

The pressure of the shock wave in the medium $p_S$ and the intensity of the laser radiation $I_L$ are related as follows

$$p_S = \left( \rho_c I_L^2 \right)^{1/3}. \quad (6)$$

The critical density $\rho_c$ can be defined as

$$\rho_c = \frac{\epsilon_0 m_e m_i (2\pi c)^2}{Ze^2 \lambda_L^2}, \quad (7)$$

where $\epsilon_0$ is permittivity of vacuum, $m_e$ is the rest mass of the electron, $m_i$ is the mass of the ion, $e$ is the charge of electron, $\lambda_L$ is the wavelength of the laser, $Z$ is the atomic number.

The amplitude of the gravitational wave $h_{zz}$ and the power of gravitational radiation $\mathcal{L}_g$ induced by the shock waves are determined as follows [11]

$$h_{zz} = \frac{7 GP_L \tau}{2 r \epsilon_0 c^4} \sqrt{\frac{\rho_c}{\rho_0}} = 2.89 \times 10^{-44} \times \sqrt{\frac{\rho_c}{\rho_0} \left( \frac{P_L \tau}{r} \right)}, \quad (8)$$

$$\mathcal{L}_g = \frac{147 GS^2 \rho_0^5 \rho_c^4}{160 c^5 \rho_0^4} = \frac{147 GP_L^2 \rho_c}{160 c^5 \rho_0} = 2.53 \times 10^{-53} \times \left( \frac{\rho_c}{\rho_0} \right) P_L^2, \quad (9)$$

where $P_L$ is the laser power and $\tau$ is the laser pulse duration.

Estimates for the case $Z = 6$ (carbon), $\lambda_L = 351$ nm, $P_L = 5 \times 10^{14}$ W, $\tau = 1$ ns, $r = 10$ m give the following values of the amplitude and power of gravitational waves [11]

$$h_{zz} = 5.1 \times 10^{-40}, \quad (10)$$

$$\mathcal{L}_g = 7.86 \times 10^{-25} W. \quad (11)$$

The obtained amplitude is almost 20 orders of magnitude smaller than the amplitude of gravitational waves from astrophysical sources detected in LIGO and VIRGO experiments [5, 6]. According to the presented estimates, the development of fundamentally new methods for detecting such gravitational waves is necessary. As the other direction, one can consider the possibility of increasing the amplitude of gravitational waves due to the material of the medium $\sqrt{\rho_c/\rho_0}$ (taking into account the wavelength $\lambda_L$) and the energy of laser radiation $W_L \tau$.

3. The dielectrics in the field of electromagnetic wave

Now, we consider the other approach to analyze the gravitational waves production by means of the interaction of matter and electromagnetic waves. For this purpose, we will study the mechanism of excitation of gravitational waves in an ionic crystal as a result of the passage of an intense electromagnetic wave through it. When a plane electromagnetic wave passes through the crystal, the ions in each cell will make forced oscillations around the equilibrium position, leading to the appearance of an additional, periodically changing in time, dipole moment in this cell. The total contribution of all such periodically changing dipole moments from all the cells of the crystal is manifested on a macro scale as the effect of polarization of the medium in the field of an external electromagnetic wave.

The displacement vector of crystal ions in the field of an electromagnetic wave can be defined as follows [17]
The quadrupole moment is defined as follows \[Q^\alpha(t) = \frac{1}{c^4} \int T^{00}(t,x^0,x^0,0) dx^0 dx^1 dx^2 dx^3,\] where \[T^{00}(t,x^0)\] is the first component of the energy-momentum tensor of the system.

In this case, we write the nonzero component of the quadrupole moment as

\[
Q^{\alpha\beta} = \int m_i \delta(x-x_i) x^\alpha \delta'(z) dy \delta'(z) dz + \int m_i \delta(x-x_i) x^\alpha \delta'(y) dy \delta'(z) dz = m_i x_i^\alpha + m_i x_i^\alpha,
\]

where we note that \[Q^{\alpha\beta} = Q_{\alpha\beta}.\]

The nonzero components of the tensor \(P_{ij}\) for the propagation direction \(z\) or for vector \(n_i = (0,0,1)\) of the gravitational wave are \(P_{11} = P_{22} = 1.\)

The components of the tensor of the gravitational wave in the transverse-traceless (TT) calibration are written as follows \[h_0^{TT} (t,z) = 0,\]
\[
R^{TT}_{ij}(t,z) = \frac{2G}{zc^4} \left[ \frac{d^2}{dt^2} Q^{TT}_{ij} \left( t - \frac{z}{c} \right) \right],
\]
where
\[
Q^{TT}_{ij} = \left( P_{ik} P_{jm} - \frac{1}{2} P_{im} P_{jn} \right) Q_{jm}.
\]

Thus, nonzero components of the tensor of the gravitational wave are defined as
\[
\bar{h}^{TT}_{xx} = -\bar{h}^{TT}_{yy} = \frac{G}{zc^4} \left[ \frac{d^2}{dt^2} Q^{TT}_{xx} \left( t - \frac{z}{c} \right) \right].
\]

After considering the frequency of the electromagnetic wave as
\[
\omega = \omega_r + \Delta \omega, \Delta \omega \ll \omega_r
\]
one has the approximate expression
\[
\frac{\omega_r}{\omega_r^2 - \omega^2} \approx -\frac{1}{2\Delta \omega}.
\]

Assuming the ion masses to be approximately equal \( m_i \approx m_c = m \) and neglecting small terms in equation (21) for (13)-(14) and (17) we obtain
\[
\bar{h}^{TT}_{xx} = -\bar{h}^{TT}_{yy} = \frac{G m_i (e_{m} - e_0)}{2 \pi z c^4} \left( \frac{\omega_r}{\Delta \omega} \right)^2 E_0^2 \times \Re \left\{ \frac{2i}{c} \left[ c k z - \omega_r (ct - z) \right] \right\}.
\]

For the \( N \)-ions of the crystal, taking into account that \( \sum m_i = N m_i \) and considering \( \gamma_a = \frac{e_{m} - e_0}{2\pi} = \text{const} \) as the dispersion factor, we obtain
\[
\bar{h}^{TT}_{xx} = -\bar{h}^{TT}_{yy} = \frac{G}{zc^4} \left( \frac{\omega_r}{\Delta \omega} \right)^2 \gamma_a E_0^2 V \times \Re \left\{ \frac{2i}{c} \left[ c k z - \omega_r (ct - z) \right] \right\},
\]
where \( V \) is the dielectric volume affected by electromagnetic radiation.

These gravitational-wave fluctuations of the space-time metric can be written as
\[
\bar{R}^{TT}_{ij} = \begin{pmatrix} -h_i & 0 & 0 \\ 0 & h_i & 0 \\ 0 & 0 & 0 \end{pmatrix}_{\alpha \beta} \times \Re \left\{ \frac{2i}{c} \left[ c k z - \omega_r (ct - z) \right] \right\},
\]
where \( \alpha, \beta = 1, 2, 3 \) and
\[
h_i = \frac{G}{zc^4} \left( \frac{\omega_r}{\Delta \omega} \right)^2 \gamma_a E_0^2 V = \frac{G m_i}{z c^4} \left( \frac{\omega_r}{\Delta \omega} \right)^2 \gamma_a E_0^2 N,
\]
is the amplitude of “plus” polarization of gravitational wave.

The frequency of natural transverse oscillations of ionic crystals \( \omega_r \approx 10^{15} \text{sec}^{-1} \) [17] close to the optical frequency range \( \omega_{opt} \approx 10^{14} - 10^{15} \text{sec}^{-1} \), therefore, the condition \( \Delta \omega \ll \omega_r \) is satisfied for laser radiation and one can use equation (27) to find the amplitude of gravitational waves for this case.
Now, let us estimate the maximum value of the amplitude of gravitational waves $h_+$, taking into account the critical value of the displacement of ions $\xi_{\text{max}}$ close to the decay of the crystal lattice.

From equation (12), taking into account (23), we obtain

$$
(E_{0})_{\text{max}} = -2\xi_{\text{max}}\Delta\omega = \mp 2\left(\frac{l_{\text{max}}}{2}\right)\sqrt{\rho\Delta\omega}.
$$

(28)

After substituting (28) into (27) one has

$$
h_+ = \frac{G}{2c^2}\gamma_{0}\xi_{\text{max}}^2 \omega_0^2 m_i N.
$$

(29)

For usual ionic crystals the dispersion factor $\gamma_{0} \approx 1$ [17], also, we consider following values of the model’s parameters $\omega_0 = 10^{9} \text{ sec}^{-1}$, $l_{\text{max}} = 10^{-9} \text{ m}$, $z = 10 \text{ m}$ and $m_i = 10^{-26} \text{ kg}$. For these values, from expression (29) we obtain

$$
h_+ = 10^{-63} \times N.
$$

(30)

For $N = 10^{23}$ one has $h_+ = 10^{-40}$ that corresponds to the result (10) which was obtained from the model of shock waves for the same distance $z = 10 \text{ m}$.

Now, we estimate the maximum value of the power of gravitational radiation as [1]

$$
\mathcal{L}_g = -\frac{dE_g}{dt} = \frac{G}{5c^5} \left(\ddot{O}_{ij}\ddot{O}_{ij}\right) = \frac{G}{40c^5} \frac{m_i^2 \gamma_{0}^2 \omega_0^6}{\rho^2 (\Delta\omega)^3} E_{0\text{max}}^4 N^2 = \frac{G}{40c^5} \frac{m_i^2 \gamma_{0}^2 \omega_0^6 l_{\text{max}}^4}{E_{0\text{max}}^4} N^2,
$$

(31)

where we neglect the small terms.

For the same parameters of ionic crystal, from equation (31) one has

$$
\mathcal{L}_g = 10^{-64} \times N^2 = 10^{-18} \text{ W} = 10^{-11} \text{ erg/sec}.
$$

(32)

Thus, for the model of interaction between dielectrics and electromagnetic wave field one has much higher power of gravitational wave radiation than in the model of shock waves (11) for the same amplitude.

We also note that the estimates obtained depend on the parameters of the crystals, and thus it is possible to increase the characteristics of gravitational waves due to the choice of specific dielectric media.

**Conclusion**

In this paper, we considered the methods for generating the gravitational waves by means of the interaction of laser radiation and matter. In the general case, to calculate the characteristics of gravitational waves, it is sufficient to use the model of shock waves. Nevertheless, for the case of the interaction of an electromagnetic wave and ionic crystals, it is necessary to consider another model, which is confirmed by a significant discrepancy in the estimation of the power of gravitational waves between these two approaches.

It should also be noted that the amplitude of gravitational waves induced by these methods is too small to use the standard interferometric method of their observation [5-7], in contrast to low-frequency gravitational waves from astrophysical sources. Thus, the possibility of observing artificial gravitational waves lies in the development of alternative detection methods [18-24], the sensitivity of which, at the moment, is also insufficient.
The development of new methods for detecting gravitational waves induced by interaction of electromagnetic wave and dielectrics with corresponding amplitude (29) and power (31) is the direction of the further work on the study of generation and detection of gravitational waves in the laboratory.

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