QUANTUM FIELD THEORY WITHOUT DIVERGENCE A

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ABSTRACT. On the basis a new conjecture, we present a new Lagrangian density and a new quantization method for QED, construct coupling operators and mass operators, derive scattering operators $S_f$ and $S_w$ which are dependent on each other and supplement new Feynman rules. $S_f$ and $S_w$ together determine a Feynman integral. Hence all Feynman integrals are convergent and it is unnecessary to introduce regularization and counterterms. That the energy of the vacuum state is equal to zero is naturally obtained. From this we can easily determine the cosmological constant according to data of astronomical observation, and it is possible to correct nonperturbational methods which depend on the energy of the ground state in quantum field theory. On the same basis as the new QED, we obtain naturally a new SU(2)xU(1) electroweak unified model whose $\mathcal{L} = \mathcal{L}_F + \mathcal{L}_W$, here $\mathcal{L}$ is left-right symmetric. Thus the world is left-right symmetric in principle, but the part observed by us is asymmetric because $\mathcal{L}_W$ and $\mathcal{L}_F$ are all asymmetric. This model do not contain any unknown particle with a massive mass. A conjecture that there is repulsion or gravitation between the W-particles and the F-particles is presented. If the new interaction is gravitation, W-matter is the candidate for dark matter. If the new interaction is repulsion, W-matter is the origin of universe expansion.

1. QUANTIZATION FOR FREE FIELDS

1.1. Introduction. There are the following five problems to satisfactorily solve in the conventional quantum field theory (QFT).

1. The issue of the cosmological constant.
2. The problem of divergence of Feynman integrals with loop diagrams.
3. The problem of the origin of asymmetry in the electroweak unified theory.
4. The problem of triviality of $\phi^4$–theory.
5. The problems of dark matter and the origin of some cosmic phenomena.

In brief, we present a new conjecture and a new quantization method, on the basis construct a self-consistent QFT without divergence, give a fully method evaluating Feynman integrals (see the second and third papers) and possible solutions to the five problems.

Divergence of Feynman integrals with loop diagrams seems to have been solved by introducing the bare mass and the bare charge or the concepts equivalent to them. But both bare mass and charge are divergent and unmeasured, thus QFT is still not perfect. In order to overcome the shortcomings, people have tried many methods. For example, G. Scharf attempted to solve the difficulty by the causal approach[1]. Feynman integrals with loop diagrams are not divergent in some supersymmetric theories. But the supersymmetry theory lacks experiment foundations.

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In fact, there should be no divergence and all physical quantities should be measurable in a self-consistent theory.

According to the given generalized electroweak unified models which are left-right symmetric before symmetry spontaneously breaking, asymmetry is caused by symmetry spontaneously breaking. In such models there must be many unknown particles with massive masses. Such models are troublesome and causes many new problems. Hence the origin of asymmetry in the electroweak unified theory should still be explored.

More than 90% of matter in cosmos is not composed of the conventional baryons. Thus the problem of dark matter comes into being. Asymmetry of left-right and existence of dark matter imply that there is an unknown world.

It is difficult on the basis of the conventional QFT to solve the problems above. In order to solve the problems it is necessary to present a new conjecture. The new conjecture of the present theory is the conjecture.

1. A new symmetry and a new Lagrangian density.

**Conjecture:** Any particle can exist in two sorts of states —— \( F \)-particle described by \( L_F \) and \( W \)-particle described by \( L_W \). \( L = L_F + L_W \), \( L_F \) and \( L_W \) are independent of each other before quantization and dependent on each other after quantization. That the particles described by \( L_F \) (\( F \)-particles) and the particles described by \( L_W \) (\( W \)-particles) are symmetric.

We explain the conjecture as follows.

That the \( F \)-particles and the \( W \)-particles are symmetric imply that every particle in \( L_F \) is accordant with a particle in \( L_W \) and the properties of the two particles are the same, e.g., there are two sorts of electrons, i.e., \( F \)-electron and \( W \)-electron. That \( L_F \) and \( L_W \) are independent of each other implies that there is no coupling between the fields in \( L_F \) and the fields in \( L_W \), hence the energy determined by \( L_F \) and the energy determined by \( L_W \) are respectively conservational and a real \( F \)-particle and a real \( W \)-particle cannot transform from one to another. But after quantization, \( L_F \) and \( L_W \) will be dependent on each other and the two sorts of virtual particles can transform from one to another. The relativistic theory is very perfect, and existence of negative energies is its essential character. We think that positive energies and negative energies depend on each other, not only are negative energies not a difficulty, but have profound physical meanings. Existence of antiparticles is only, in fact, a result of particle-inversion symmetry, and do not reveal the essence of negative energies. In frame of relativistic quantum mechanics, on the basis to reexplain the physical meanings of negative-energy states we can illuminate necessity and self-consistency of the conjecture in another paper.

We call the conjecture the F-W (fire-water) symmetry conjecture. We may also call conjecture the L-R (left-right) symmetry conjecture since \( L_F \) and \( L_W \) describe respectively the left-hand world (matter world) and right-hand world (dark-matter world) and \( L_F + L_W \) is left-right symmetry.

2. Transformation operators and a new method to quantize fields.

Because particles can exist in the two sorts of states, we can define transformation operators which transform a \( F \)-particle into a \( W \)-particle or a \( W \)-particle into a \( F \)-particle, and can quantize fields by the transformation operators replacing creation and annihilation operators in the conventional QFT. Thus it is necessary that \( g_f \) and \( m_{ef} \) respectively become operators \( G_F \) and \( M_F \) to be determined by \( S_w \), and \( g_w \) and \( m_{ew} \) respectively become operators \( G_W \) and \( M_W \) to be determined by \( S_f \),
here $S_w$ and $S_f$ are the scattering operators respectively determined by $L_W$ and $L_F$. $G_F$ and $M_F$ multiplied by field operators $\psi$ and $\Delta_f$ become the coupling coefficient $g_f(p_2, p_1)$ and mass $m_{efw}(p)$ determined by scattering amplitude $\langle W_f | S_w | W_i \rangle$, and $G_W$ and $M_W$ multiplied by field operators $\psi$ and $\Delta_w$ become $g_w(p_2, p_1)$ and $m_{ew}(p)$ determined by scattering amplitude $\langle F_f | S_f | F_i \rangle$. Thus after quantization, $L_F$ and $L_W$ will be dependent on each other.

3. Two sorts of corrections.

In the conventional QED, there are two sorts of parameters, e.g., the physical charge and the bare charge, and one of corrections originating $S$ equivalent to $S_f$. In contrast with the given QED, there is only one sort of parameters defined at so-called subtraction point $p_2$, $q_1$ and $q'$, i.e., $g_f(q_2, q_1) = g_w(q_2, q_1) = g_0$ and $m_{ef}(q) = m_{ew}(q) = m_{0}$, and there are two sorts of corrections originating from $S_w$ and $S_f$ to scattering amplitudes, $g_0$ and $m_{0}$. Thus $L_F$ and $L_W$ together determine the loop-diagram corrections. When $n$-loop corrections originating from $S_f$ and $S_w$ are simultaneously considered, the integrands causing divergence in $\langle F_f | S_f | F_i \rangle$ or $\langle W_f | S_w | W_i \rangle$ will cancel each other out, hence all Feynman integrals are convergent. For example (see third chapter), the one-loop correction to mass of a free $F$-electron is

$$\delta m = m_{ef}^{(1)}(q) = m_{eff}^{(1)}(q) + m_{efw}^{(1)}(q) = \frac{3m_0}{4\pi^3} \int d^4k \left\{ \frac{1}{(k^2 + m^2)^2} - \frac{1}{(k^2 + m^2)^2} \right\} = 0, \quad (1.1.1)$$

where $m_{eff}^{(1)}$ originates from $S_f$, $m_{efw}^{(1)}$ originates from $S_w$, and the superscript (1) denotes 1-loop correction. Thus it is unnecessary to introduce counterterms and regularization. We give a set of complete Feynman rules to evaluate Feynman integrals by the new concepts (see the second and third papers).

It should be pointed that in the meaning of perturbation theory, for initial states and final states with given momenta, e.g., the momenta at the subtraction point, we can give the absolutely precise coupling coefficients and masses, and from this give absolutely precise $L_F^{(0)}$, $L_W^{(0)}$, $H_F^{(0)}$, $H_W^{(0)}$, $S_f^{(0)}$ and $S_w^{(0)}$. But for arbitrary initial states and final states, we cannot give the absolutely precise coupling coefficients, masses, $L_F$, $L_W$, $H_F$, $H_W$, $S_f$ and $S_w$. In fact the coupling coefficients and masses will be corrected by $n$-loop diagrams. For arbitrary initial states and final states $S_f^{(0)}$ and $S_w^{(0)}$ etc., are only approximate. Of course, by $S_f^{(0)}$ and $S_w^{(0)}$ we can obtain scattering amplitudes approximate to arbitrary $n$-loop diagrams.

4. $\langle 0 | H | 0 \rangle \equiv E_0 = E_{0F} + E_{0W} = 0$ is naturally derived, thereby we can easily determine the cosmological constant according to data of astronomical observation, and it is possible to correct nonperturbational methods which depend on the energy of the ground state in QFT.

5. Generalizing the present theory to the electroweak unified theory, we will see a possible origin of symmetry breaking. According to this model, the world is symmetric on principle since $L = L_W + L_F$ is symmetric, but the world observed by us is asymmetric since $L_W$ or $L_F$ is asymmetric. In this model there is no unknown particle with a massive mass (see the third paper).

6. Because there is no interaction between the two sorts of matter by a given quantizable field, the only possibility is that there is repulsion or gravitation between the two sorts of matter. If the new interaction is gravitation, W-matter is the
candidate for dark matter\cite{3}. If the new interaction is repulsion, it is possible that the new interaction is the reason for the expansion of cosmos. It is also possible that there is new and more important relationship between the two sorts of matter.

By the conventional creation and annihilation operators in the conventional QFT we can also obtain the similar results, provided we suppose $\mathcal{L} = \mathcal{L}_F + \mathcal{L}_W$ and that $g_f$ and $m_f$ are determined by $S_w$ and $g_w$ and $m_w$ are determined by $S_f$ (of course, in this case this conjecture is not natural). It is also possible to obtain the same results, but that F-particles possess positive energies and W-particles possess negative energies, provided $\mathcal{L}_F$ and $\mathcal{L}_W$ are independent of each other in classical meanings.

The present theory contains four parts. The first part takes QED as an example to illuminate the method to reconstruct QFT, and give the solutions to the issue of the cosmological constant and the problem of divergent Feynman integrals in QED. The first part is the present paper. The second part discuss the problems on dark matter\cite{3}. The third part discusses the problem of the origin of asymmetry in the electroweak unified theory in detail. The fourth part discusses the problem of triviality of $\varphi^4$–theory.

The outline of this chapter is as follows. 2. Lagrangian density; 3. Quantization for free fields; 4. Energies and charges of particles; 5. Subsidiary condition; 6. The equations of motion; 7. The physical meanings of that the energy of the vacuum state is equal to zero; 8. Summary.

The contents of the rest are as follows.

Section 2. Coupling Operators and Feynman Rules
1. Introduction; 2. Interacting Lagrangian density, Hamiltonian density and equations of motion; 3. Coupling operators and mass operators; 4. Expansion of the Hamiltonian; 5. Scattering operators and Feynman rules; 6. Summary.

Section 3. One-loop Correction and Supplementary Feynman Rules
1. Introduction; 2. Two sorts of correction; 3. The first sort of 1-loop corrections; 4. The second sort of one-loop corrections and the total one-loop corrections; 5. $n$–loop corrections for the coupling constants and the masses;

Section 4. Generalize to the $SU(2) \times U(1)$ electroweak unified model and interaction between W-matter and F-matter
1. Generalize to the $SU(2) \times U(1)$ electroweak unified model; 2. Interaction between W-matter and F-matter.

Section 5. Conclusions and prospects.
Captions for figures.

Appendix A; Appendix B; Appendix C.

1.2. Lagrangian density. We suppose the Lagrangian density for the free Dirac fields and the Maxwell fields to be

\begin{align}
\mathcal{L}_0 &= \mathcal{L}_{F0} + \mathcal{L}_{W0}, \\
\mathcal{L}_{F0} &= -\bar{\psi}(x)(\gamma_\mu \partial_\mu + m)\psi(x) - \frac{1}{2} \partial_\mu A_\nu \partial_\mu A_\nu, \\
\mathcal{L}_{W0} &= \bar{\psi}(x)(\gamma_\mu \partial_\mu + m)\psi(x) - \frac{1}{2} \partial_\mu A_\nu \partial_\mu A_\nu,
\end{align}
(1.2.2) and (1.2.3) imply the Lorentz gauge to be already fixed. The difference between (1.2.1) and the corresponding Lagrangian density in the conventional QED is $L_{W_0}$ which describes motion of particles existing in the other form. We call $\psi, A_\mu, \bar{\psi}$, the F-electron field, the F-photron field, the W-electron field and the W-photron field, respectively. The signs of the fermion parts in $L_F$ and $L_W$ are opposite, the signs of the boson parts are the same. The difference comes from the difference of property of fermion and boson. This difference is not essential. The key of matter is that F-particles and W-particles are symmetric. In fact, redefining field operators by transformation operators, we can also have $L_F$ and $L_W$ to be symmetric.

The conjugate fields corresponding to the fields are respectively

$$\pi_\psi = \frac{\partial L}{\partial \dot{\psi}} = i\psi^+, \quad \pi_\mu = \frac{\partial L}{\partial \dot{A}_\mu} = \dot{A}_\mu, \quad (1.2.4)$$

$$\pi_\bar{\psi} = \frac{\partial L}{\partial \dot{\bar{\psi}}} = -i\bar{\psi}^+, \quad \pi_\bar{\mu} = \frac{\partial L}{\partial \dot{\bar{A}}_\mu} = \dot{\bar{A}}_\mu, \quad (1.2.5)$$

From the Noether's theorem, we obtain the conservational quantities

$$H_0 = H_{F_0} + H_{W_0}, \quad (1.2.6)$$

$$H_{F_0} = \int d^3x \left[ \psi^+ \dot{H}_0 \cdot \psi + \frac{1}{2} \left( \dot{A}_\mu \cdot \dot{A}_\mu + \partial_j A_\nu \cdot \partial_j A_\nu \right) \right], \quad (1.2.7)$$

$$H_{W_0} = -\int d^3x \left[ (\psi^+ \dot{H}_0 \cdot \psi - \frac{1}{2} (\dot{A}_\mu \cdot \dot{A}_\mu + \partial_j A_\nu \cdot \partial_j A_\nu) \right], \quad (1.2.8)$$

$$Q = Q_F + Q_W, \quad Q_F = \int d^3x \psi^+ \cdot \psi, \quad Q_W = \int d^3x \bar{\psi}^+ \cdot \bar{\psi}, \quad (1.2.9)$$

where $\dot{H}_0 = \gamma_4 (\gamma_j \partial_j + m)$. The global gauge transformations corresponding to (1.2.9) are

$$\psi \rightarrow \psi' = e^{i\alpha} \psi, \quad \psi \rightarrow \psi' = e^{-i\alpha} \psi. \quad (1.2.10)$$

The Euler-Lagrange equations of motion being derived from the Hamilton's variational principle are

$$i \frac{\partial}{\partial t} \psi = \hat{H}_0 \psi, \quad \Box A_\mu = 0, \quad (1.2.11)$$

$$i \frac{\partial}{\partial t} \bar{\psi} = \hat{H}_0 \bar{\psi}, \quad \Box \bar{A}_\mu = 0. \quad (1.2.12)$$

It is seen from (1.2.11)-(1.2.12) that although $L_{F_0} \neq L_{W_0}$, the equations satisfied by $\psi$ and $A_\mu$ are the same as those satisfied by $\bar{\psi}$ and $A_\mu$, respectively. This implies that for a relativistic physical system, only equations of motion are insufficient for corrective description of all properties of the system. A complete Lagrangian density is very necessary.
When $\psi$, etc., are regarded as the classical fields and

$$\partial_\mu A_\mu = \partial_\mu A_\mu = 0,$$

(1.2.13) $\psi$ and $\psi'$ can be expanded in terms of the complete set of plane-wave solutions

$$\frac{1}{\sqrt{V}} u_p e^{ipx}, \quad \frac{1}{\sqrt{V}} v_p e^{-ipx}, \quad s = 1, 2,$$

(1.2.14) where $px = pxt$, $E_p = \sqrt{p^2 + m^2}$, and $A_\mu$ and $A_\mu'$ can be expanded in terms of the complete set

$$\frac{1}{\sqrt{2\omega_k V}} e^\lambda_{k\mu} e^{\pm ikx},$$

(1.2.15) where $kx = kx - \omega_k t$, $\omega_k = |k|$, $\lambda = 1, 2$. To get a completeness relation, it is necessary to form a quartet of orthonormal 4-vectors[4].

$$e^1_k = (\varepsilon^1_k, 0), \quad e^2_k = (\varepsilon^2_k, 0), \quad e^3_k = - [k + \eta (k\eta)] / k\eta, \quad \eta = (0, 0, 0, i),$$

$$e^4_k = i\eta, \quad \varepsilon^{1,2}_k \cdot k = 0.$$ Moreover, all four vectors are normalized to 1, i.e.,

$$e^\lambda_e k e^{\lambda'}_k = \delta_{\lambda\lambda'}, \quad \sum_{\lambda=1}^4 e^\lambda_{k\mu} e^{\lambda\nu}_{k\mu} = \delta_{\mu\nu}.$$

1.3. Quantization for free fields.

1.3.1. Field operators and equations of motion. We now regard $\psi$ etc., as quantum fields. $\psi$, $A_\mu$, $\psi'$ and $A_\mu'$ as the solutions of the equations of the quantum fields (1.2.11) - (1.2.12) can also be expanded in terms of the complete sets (1.2.14) and (1.2.15), respectively, only the expanding coefficients are all operators. Thus we have

$$\psi_0 (x) = \frac{1}{\sqrt{V}} \sum_{ps} \left( | I_p \leq a_{ps}(t) | u_p e^{ipx} + | b_{ps}(t) \geq I_{(-p)} v_p e^{-ipx} \right),$$

(1.3.1) $A_{0\mu} (x) = \frac{1}{\sqrt{V}} \sum_k \frac{1}{\sqrt{2\omega_k V}}

$$\sum_{\lambda=1}^4 c^\lambda_k \left( \hat{c}_{k\lambda}(t) \leq e^{ikx} + | \hat{a}_{k\lambda}(t) \geq i(-k) e^{-ikx} \right),$$

(1.3.2) $\psi_0 (x) = \frac{1}{\sqrt{V}} \sum_{ps} \left( | b_{ps}(t) \geq I_p u_p e^{ipx} + I_{(-p)} \leq a_{ps}(t) \leq v_p e^{-ipx} \right),$

(1.3.3)
\[ \Delta_{\mu}(x) = \frac{1}{\sqrt{V}} \sum_{k} \frac{1}{\sqrt{2\omega_k}} \cdot \sum_{\lambda=1}^{4} c_{k\mu}^\lambda \left( | \pi_{k\lambda}(t) \gg j_{(-k)} e^{ikx} + j_{k} \ll \pi_{k\lambda}(t) e^{-ikx} \right), \quad (1.3.4) \]

\[ \pi_{0\psi} = i\psi_0^+(x) \]
\[ = \frac{i}{\sqrt{V}} \sum_{p^+} \left( | a_{p^+}(t) \gg I_{p^+} u_{p^+} e^{-ipx} + I_{(p^-)}^+ b_{p^+}(t) \ll v_{p^+} e^{ipx} \right), \quad (1.3.5) \]

\[ \pi_{0\mu} = \frac{i}{\sqrt{V}} \sum_{k} \sqrt{\frac{\omega_k}{2}} \cdot \sum_{\lambda=1}^{4} c_{k\mu}^\lambda \left( | \pi_{k\lambda}(t) \gg j_{(-k)} e^{ikx} - | \pi_{k\lambda}(t) \gg j_{(-k)} e^{-ikx} \right), \quad (1.3.6) \]

\[ \mathcal{W}_{0\mu} = -i\psi_0^+(x) = \frac{-i}{\sqrt{V}} \sum_{p^+} \left( | I_{p^+}^+ b_{p^+}(t) \ll u_{p^+} e^{-ipx} + | a_{p^+}(t) \gg I_{p^-}^+ v_{p^+} e^{ipx} \right), \]

\[ \mathcal{W}_{0\mu} = \frac{i}{\sqrt{V}} \sum_{k} \sqrt{\frac{\omega_k}{2}} \cdot \sum_{\lambda=1}^{4} c_{k\mu}^\lambda \left( | \pi_{k\lambda}(t) \gg j_{(-k)} e^{ikx} - j_{k} \ll \pi_{k\lambda}(t) \gg e^{-ikx} \right), \quad (1.3.7) \]

\[ | \pi_{k\lambda} \gg = \left\{ \begin{array}{l} | \pi_{k\lambda} \gg, \lambda = 1, 2, 3, \\ - | \pi_{k\lambda} \gg, \lambda = 4 \end{array} \right., \quad (1.3.9a) \]

\[ | \pi_{k\lambda} \gg = \left\{ \begin{array}{l} | \pi_{k\lambda} \gg, \lambda = 1, 2, 3, \\ - | \pi_{k\lambda} \gg, \lambda = 4 \end{array} \right., \quad (1.3.9b) \]

We call such operators as \( I_{p^+} \ll a_{p^+}(t) \) and \( | \pi_{k\lambda}(t) \gg j_{k} \) transformation operators. Such an operator as \( \ll a_{p^+}(t) \) changes as time, and \( I_{p} \) and \( j_{k} \) etc. do not change. In the Heisenberg picture the evolution of a quantum field system as time is carried by the field operators according to the equations of motion

\[ (1.3.11) \quad \dot{F} = -i [F, H_0] = -i [F, H_0 F], \]
(1.3.12) \[ \hat{W} = i [W, H_0] = i [W, H_{0W}], \]
where \( F = \psi_0(x), \psi_0^+(x), A_{00}(x), \pi_{00}(x), |a_{ps}(t)\rangle, |b_{ps}(t)\rangle, |c_{k\lambda}(t)\rangle, \leq a_{ps}(t)|, \leq b_{ps}(t)| \) and \( \leq c_{k\lambda}(t)| \); \( W = \bar{\psi}_0(x), \bar{\psi}_0^+(x), \bar{A}_{00}(x), \bar{\pi}_{00}(x), |\bar{a}_{ps}(t)\rangle, |\bar{b}_{ps}(t)\rangle, |\bar{c}_{k\lambda}(t)\rangle \). The equation (1.3.11) is well-known as the Heisenberg equation, while (1.3.12) is a new equation of motion. We will see

(1.3.13) \[ [H_{0W}, H_{0F}] = [H_{0F}, H_0] = [H_{0W}, H_0] = 0, \]
hence \( H_{0F} \) and \( H_{0W} \) are the constants of motion. Thus, from (1.3.11) – (1.3.13) we have

(1.3.14) \[ F(t) = e^{iH_{0F}t}F(0)e^{-iH_{0F}t} = e^{iH_{0W}t}F(0)e^{-iH_{0W}t}, \]

(1.3.15) \[ W(t) = e^{-iH_{0W}t}W(0)e^{iH_{0F}t} = e^{-iH_{0W}t}W(0)e^{iH_{0W}t}, \]

(1.3.11) – (1.3.12) are consistent with (1.2.11) – (1.2.12), and (1.3.14) – (1.3.15) are consistent with (1.3.10).

1.3.2. Properties and multiplication rules of the transformation operators. 1. A transformation operator as \( I_p \leq a_{ps} \) is regarded a whole, hence the order of its two parts cannot be reversed, though \( I_p \) and \( \leq a_{ps} \) can be be separated, say, \( I_p \leq a_{ps} \) and \( \leq c_{k\lambda} \) cannot be written as \( \leq a_{ps} | I_p \) and \( \leq c_{k\lambda} \), respectively. When \( I_p \) and \( \leq a_{ps} \) do not constitute a transformation operator, they can exchange their order. Hence this property should be regarded as a definition for transformation operators.

2. Commutators or anticommutators of such operators as \( \leq a_{ps} \) and \( |a_{ps}\rangle \).

Such an operator in the form \( \leq a_{ps} \) is equivalent to an annihilation operator \( a_{ps} \), and such an operator in the form \( |a_{ps}\rangle \) is equivalent to an creation operator \( a_{ps}^+ \) in the conventional \( QED \). Thus, let \( \alpha = a, b, c \) and \( \bar{a}, \bar{b}, \bar{c} = \gamma = e \) and \( c \) we have

(1.3.16) \[ \{ \leq a_{ps}(t) |, | \alpha_{p's'}(t) \rangle \} = \delta_{\alpha\alpha'}\delta_{pp'}\delta_{ss'}, \]

(1.3.17) \[ \{ \leq \gamma_{k\lambda}(t) |, | \gamma_{k\lambda'}(t) \rangle \} = \{ \delta_{\gamma\gamma'}\delta_{kk'}\delta_{\lambda\lambda'}, \lambda = 1, 2, 3, \]

The other commutators or anticommutators are all zero.

3. States and inner products of states.

(1.3.16) and (1.3.17) are the same as the anticommutation relations and the commutation relations of the conventional \( QED \), respectively. As the conventional \( QED \), from (1.3.16)-(1.3.17) we see free fermion states and n-photon states to be

(1.3.18) \[ |a_{ps}\rangle = |a_{ps}\rangle \langle 0|, \quad \langle a_{ps} | = \langle 0 | \leq a_{ps} |,\]
In order to compare the new QFT with the conventional QFT, we write the same results as the present papers can be derived with more complicated discussion. By such transformation operators we can construct a new QFT from which the other sort of representation is in appendix A. Considering (1.3.16)-(1.3.17) and

\[
|n_{k\lambda}\rangle \equiv \frac{1}{\sqrt{n!}}(c_{k\lambda} |n\rangle |0\rangle),
\]

\[
\langle n_{k\lambda} | \equiv \langle 0 | (c_{k\lambda} |n\rangle |0\rangle \frac{1}{\sqrt{n!}},
\]

\[
|\lambda_{k\lambda}\rangle \equiv \frac{1}{\sqrt{n!}}(c_{k\lambda} |n\rangle |0\rangle),
\]

\[
\langle \lambda_{k\lambda} | \equiv \langle 0 | (c_{k\lambda} |n\rangle |0\rangle \frac{1}{\sqrt{n!}}.
\]

(1.3.19a)

(1.3.19b)

The other sort of representation is in appendix A. Considering (1.3.16)-(1.3.17) and

\[
|a_{ps}\rangle \equiv |\gamma_{k\lambda}\rangle |0\rangle = \langle 0 | |a_{ps}\rangle \rangle = \langle 0 | \gamma_{k\lambda} |0\rangle = 0,
\]

\[
(0 | 0) = 1,
\]

we obtain the inner products of states to be

\[
\langle a_{ps} | \cdot | a_{p's'}\rangle = \langle a_{ps} | a_{p's'}\rangle = \delta_{a'a'}\delta_{pp}\delta_{ss'},
\]

\[
(1.3.20)
\]

\[
\langle \gamma_{k\lambda} | \cdot | \gamma_{k'\lambda'}\rangle = \langle \gamma_{k\lambda} | \gamma_{k'\lambda'}\rangle = \left\{ \begin{array}{cc}
\delta_{\gamma'\gamma}\delta_{kk'}\delta_{\lambda'\lambda}, & \lambda = 1, 2, 3, \\
-\delta_{\gamma'\gamma}\delta_{kk'}\delta_{\lambda'\lambda'}, & \lambda = 4
\end{array} \right.
\]

\[
(1.3.21)
\]

\[
\langle \beta_{ps} | \cdot | \gamma_{k\lambda}\rangle = 0.
\]

\[
(1.3.22)
\]

\[
(1.3.23)
\]

4. Inner products of \(I_p\), \(J_p\), \(L_p\), and \(J_p\).

The original form of transformation operators is the same as \(|q_{ps}\rangle \leq |a_{ps}\rangle |0\rangle\), \(|b_{ps}\rangle \rangle = |q_{ps}\rangle |0\rangle\), and their essential meanings are

\[
|q_{ps}\rangle \leq |a_{ps}\rangle \leq |b_{ps}\rangle |0\rangle = |q_{ps}\rangle |0\rangle, \quad \langle b_{ps} | b_{ps} \rangle = \langle q_{ps} | q_{ps} \rangle = \langle q_{ps} | q_{ps} \rangle = |c_{k\lambda}\rangle = |\lambda_{k\lambda}\rangle = |c_{k\lambda}\rangle.
\]

(1.3.24)

By such transformation operators we can construct a new QFT from which the same results as the present papers can be derived with more complicated discussion. In order to compare the new QFT with the conventional QFT, we write the transformation operators as the following form.

\[
|q_{ps}(t)\rangle \equiv |a_{ps}(t)\rangle = |q_{ps}\rangle |L_p^+ \cdot L_p \in a_{ps}(t) |, \quad |b_{ps}(t)\rangle = \sqrt{a_{ps}(t)} |b_{ps}(t)\rangle = |L_-^p \cdot L^+_p \cdot b_{ps}(t) |, \quad |c_{k\lambda}\rangle = |\lambda_{k\lambda}\rangle |L^+_k \cdot L_k \in c_{k\lambda}(t) |, \quad |b_{ps}(t)\rangle = \sqrt{a_{ps}(t)} |b_{ps}(t)\rangle = |L^+_p \cdot I_\lambda \in b_{ps}(t) |, \quad |a_{ps}(t)\rangle \equiv |q_{ps}(t)\rangle = |a_{ps}(t)\rangle |I^+_p \cdot I^-_p \cdot b_{ps}(t) |, \quad |c_{k\lambda}(t)\rangle = |\lambda_{k\lambda}(t)\rangle |J^+_k \cdot J_k \in c_{k\lambda}(t) |.
\]

(1.3.25)

where \(L^+_p \cdot L_p = J^+_k \cdot J_k = I^+_p \cdot I_p = J^+_k \cdot J_k = 1\). For the cause of mathematics we introduce \(L_p\) etc. In order to easily deal with problems, now we divide such operators into two parts. For example, we divide \(|q_{ps}\rangle |L_p^+ \cdot L_p \in a_{ps}(t) |\) into \(|a_{ps}\rangle |L_p^+\rangle \) and \(|L_p \in a_{ps}(t) |\), leave such a part as \(|a_{ps}\rangle |L_p^+ | to belong a coupling operator or a mass operator (see below), and leave such a part as \(L_p \in a_{ps} |\) to belong a field operator and also call it a transformation operator.
Let $K = I_p, I^+_p, J_p, J^+_p, J_k, J^+_k$. Their properties and the multiplication rules are as follows.

\[(1.3.26)\]

\[J^-_k = J^+_k, \quad J^-_k = J^+_k\]

$I_p, I^+_p, J_p, J^+_p, J_k, J^+_k$ can be regarded as a base vector of $I_p$-space, · · · a base vector of $J_k$-space, respectively. Their inner products are defined as

\[(1.3.27a)\]

\[(I_p, I_p) = I^+_p \cdot I_p = I_p \cdot I^+_p = \delta_{pp}\]

Similarly, we have

\[J^+_p \cdot J_p = J_p \cdot J^+_p = \delta_{pp}\]

\[J^+_k \cdot J_k = J_k \cdot J^+_k = \delta_{kk}\]

The other inner products are all zero. \[(1.3.27b)\]

5. Multiplication rules and commutation or anticommutation relations for the transformation operators.

When multiplying a transformation operator containing a factor $K$ by another operator containing a factor $K$, we define the product to be such an operator obtained after achieving multiplication of the two $K$s, i.e.,

\[AK_A \cdot K_B B \equiv A(K_A \cdot K_B)B, \quad (1.3.28a)\]

\[K_B B \cdot A K_A \equiv \pm A(K_A \cdot K_B)B, \quad (1.3.28b)\]

\[K_B B \cdot K_A A = (K_B \cdot K_A)BA, B K_B \cdot A K_A = BA(K_B \cdot K_A)\] \[(1.3.28c)\]

\[[A, K] = 0. \quad (1.3.28d)\]

where $A, B = a_{ps}(t) | \text{etc.}, \text{or } b^{ps}_{ps} | \text{etc.},$ when both $A$ and $B$ are fermion operators, (1.3.28b) takes ‘-‘, and otherwise takes ‘+’. $K$ can be constructed by states (see appendix B), but this is not necessary. It is obvious that only transformation operators can not form a closed algebra. A product of two F-transformation operators (two W-transformation operators) is a F-operators which transform a F-state into other F-state (a W-operators which transform a W-state into other W-state ). The F-transformation operators and the F-operators form a closed algebra. The W-transformation operators and the W-operators form a closed algebra. It can be seen from (1.3.27) that the product of a F-transformation operator and a W-transformation operator must be equal to zero.

When many operators containing $K$ multiply, associative law does no longer hold water. Hence we should appoint the associative order of such the operators. But the complicacy cannot appear in fact, since after Lagrangian or Hamiltonian density is constructed by completing the products of two field operators and the products of field operators and coupling operators or mass operators (see second paper), thus Lagrangian or Hamiltonian density does no longer contain $K$ so that multiplication of many operators containing $K$ will not appear. In interaction parts of Lagrangian or Hamiltonian density that every field operator directly multiplies by a coupling operator or a mass operator will be defined.

When a transformation operator and a operator not containing $K$ multiply, the multiplication rule is the same as usual.

It is obvious that an essential difference between transformation operators and the creation or annihilation operators is the factor $K$. 
From (1.3.16)-(1.3.17), (1.3.27)-(1.3.28) and (1.3.1)-(1.3.8), we easily derive the commutation or anticommutation relations of the transformation operators and the field operators.

\[
\begin{align*}
\{L_p & \leq a_{ps}(t) \mid a_{ps'}(t) \geq L'_{ps'} \} \\
& = \{I^+_{ps} \leq b_{ps}(t) \mid b_{ps'}(t) \geq I^-_{ps'} \} \\
& = \{J_{k} \leq c_{k\lambda}(t) \mid c_{k\lambda'}(t) \geq J^+_{k\lambda'} \} = 0, \\
\{I^+_{ps} & \leq \zeta ps(t) \mid \zeta ps'(t) \geq I^-_{ps'} \} \\
& = \{J_{p} \leq \zeta ps(t) \mid \zeta ps'(t) \geq I^+_{ps'} \} \\
& = \{J_{k} \leq \zeta k\lambda(t) \mid \zeta k\lambda'(t) \geq J^+_{k\lambda'} \} = 0,
\end{align*}
\]

\[(1.3.29)\]

\[(1.3.30)\]

\[(1.3.31)\] \(\{\psi_\alpha(x, t), \psi^+_\beta(y, t)\} = \{\psi_\alpha(x, t), \psi_\beta(y, t)\} = \{\psi^+_\alpha(x, t), \psi_\beta(y, t)\} = 0,\]

\[(1.3.32)\] \([A_\mu(x, t), \pi_\nu(y, t)] = [A_\mu(x, t), A_\nu(y, t)] = [\pi_\mu(x, t), \pi_\nu(y, t)] = 0,\]

\[(1.3.33)\]

\[(1.3.34)\] \([\bar{A}_\mu(x, t), \bar{\pi}_\nu(y, t)] = [\bar{A}_\mu(x, t), \bar{A}_\nu(y, t)] = [\bar{\pi}_\mu(x, t), \bar{\pi}_\nu(y, t)] = 0.\]

The others are all zero as well. The commutation or anticommutation relations are different from those of the conventional QED.

The new QFT seems to be more complicated than the conventional QFT from the rules above, in fact it is not true since regularization and counterterms are no longer necessary in the new QFT.

### 1.4. The energies and charges of particles

From (1.2.7) – (1.2.9), (1.3.1) – (1.3.8) and (1.26) – (1.28) we obtain

\[
H_{F0} = \sum_{ps} E_p (|a_{ps} \geq a_{ps}| + |b_{ps} \geq b_{ps}|) \\
+ \sum_k \omega_k \left(\sum_{\lambda=1}^{3} |c_{k\lambda} \geq c_{k\lambda}| - |c_{k4} \geq c_{k4}|\right),
\]

\[(1.4.1)\]

\[
H_{W0} = \sum_{ps} E_p (|\bar{b}_{ps} \geq \bar{b}_{ps}| + |\bar{a}_{ps} \geq \bar{a}_{ps}|) \\
+ \sum_k \omega_k \left(\sum_{\lambda=1}^{3} |\bar{c}_{k\lambda} \geq \bar{c}_{k\lambda}| - |\bar{c}_{k4} \geq \bar{c}_{k4}|\right),
\]

\[(1.4.2)\]

\[
Q_F = \sum_{ps} (|a_{ps} \geq a_{ps}| - |b_{ps} \geq b_{ps}|),
\]

\[(1.4.3)\]

\[
Q_W = \sum_{ps} (-|\bar{b}_{ps} \geq \bar{b}_{ps}| + |\bar{a}_{ps} \geq \bar{a}_{ps}|).
\]

\[(1.4.4)\]
From (1.4.1) and (1.4.2) we see that energies are positive-definite and (1.3.11) and (1.3.12) are consistent with (1.3.10), respectively. It is easily seen from (1.4.1) – (1.4.4) that

\[(\sigma \mid H_0 \mid \sigma) = (\sigma \mid H_0 \mid \sigma),\]

\[(\sigma \mid Q \mid \sigma) = (\sigma \mid Q \mid \sigma),\]

where \(\sigma = a_{ps}, b_{ps}, c_{k\lambda}\).

The Hamiltonian and charge operators can also be written as

\[H_{F0} = \int d^3x : [\psi^{j+} H_0 \psi^j + \frac{1}{2} \left( \dot{A}_\mu A^\mu_\alpha + \partial_j A^j_\nu \partial_j A^\nu_\nu \right)] \cdot \]

\[H_{W0} = - \int d^3x : \left[ (\psi^{j+} H_0 \psi^j - \frac{1}{2} (\dot{A}_\mu A^\mu_\alpha + \partial_j A^j_\nu \partial_j A^\nu_\nu) \right] \cdot \]

\[Q_F = \int d^3x : \psi^{j+} \psi^j \mid , \quad Q_F = \int d^3x : \psi^{j+} \psi^j \mid \]

where the double-dot notation \(\cdots\) is known as normal ordering. An operator product is in normal ordered form if all operators as \(| \alpha \rangle \) stand to the left of all operators as \(\langle \alpha |\). In (1.4.7)-(1.4.9),

\[\psi^j_0 (x) = \frac{1}{\sqrt{V}} \sum_{ps} \left( \left\langle a_{ps} (t) \mid u_{ps} e^{ipx} + \left\vert b_{ps} (t) \right\rangle v_{ps} e^{-ipx} \right\rangle\right),\]

\[A^j_\mu (x) = \frac{1}{\sqrt{V}} \sum_{ps} \frac{1}{2\omega_{k\lambda}} \sum_{\lambda=1}^4 c_{k\mu}^\lambda (\left\langle \alpha_{k\lambda} (t) \mid e^{ikx} + \left\langle \alpha_{k\lambda} (t) \right\rangle e^{-ikx} \right\rangle),\]

\[\bar{\psi}_n^j (x) = \frac{1}{\sqrt{V}} \sum_{ps} \left( \left\langle \left\vert b_{ps} (t) \right\rangle u_{ps} e^{ipx} + \left\langle b_{ps} (t) \right\rangle v_{ps} e^{-ipx} \right\rangle\right),\]

\[A^j_\mu (x) = \frac{1}{\sqrt{V}} \sum_{ps} \frac{1}{2\omega_{k\lambda}} \sum_{\lambda=1}^4 c_{k\mu}^\lambda (\left\langle \alpha_{k\lambda} (t) \right\rangle e^{ikx} + \left\langle \alpha_{k\lambda} (t) \right\rangle e^{-ikx} \right\rangle),\]

\[\pi^j_0 = i \psi^j_0 (x) = \frac{i}{\sqrt{V}} \sum_{ps} \left( \left\langle a_{ps} (t) \right\rangle u_{ps} e^{-ipx} + \left\langle b_{ps} (t) \right\rangle v_{ps} e^{ipx} \left\rangle\right\rangle\right),\]

\[\pi_0^\nu = A^j_\mu (x) = - \frac{i}{\sqrt{V}} \sum_{k} \frac{1}{2\omega_{k\lambda}} \sum_{\lambda=1}^4 c_{k\mu}^\lambda (\left\langle \alpha_{k\lambda} (t) \right\rangle e^{ikx} + \left\langle \alpha_{k\lambda} (t) \right\rangle e^{-ikx} \right\rangle),\]

\[\pi^j_0 = -i \psi^j_0 (x) = - \frac{i}{\sqrt{V}} \sum_{ps} \left( \left\langle \left\vert b_{ps} (t) \right\rangle u_{ps} e^{-ipx} + \left\langle a_{ps} (t) \right\rangle v_{ps} e^{ipx} \right\rangle\right),\]
(1.4.17) \[ \pi'_\mu = A'_\mu = \frac{-i}{\sqrt{V}} \sum_k \frac{|k|}{\sqrt{2\omega_k}} \sum_{\lambda=1}^4 e_{k\mu}^\lambda \left( e^{ikx} - e^{-ikx} \right), \]

where the operators \( a_{\mu\alpha}(t) \) etc., are the same as (1.3.10). From (1.3.16)-(1.3.17) and (1.4.10)-(1.4.17) we have

\[
(1.4.18) \quad \{\psi'_{\alpha}(x,t), \psi'^{\dagger}_{\beta}(y,t)\} = \{\psi'_{\alpha}(x,t), \psi_{\beta}(y,t)\} = \delta(x-y)\delta_{\alpha\beta},
\]

\[
(1.4.19) \quad [A'_{\mu}(x,t), \pi'_{\nu}(y,t)] = [A'_{\mu}(x,t), \pi_{\nu}(y,t)] = i\delta(x-y)\delta_{\mu\nu}.
\]

All other anticommutators and commutators are zero. (1.4.18)-(1.4.19) are the same as those in the conventional QFT. In contrast with the conventional QFT, (1.4.1)-(1.4.4) or (1.4.7)-(1.4.9) are the deductions from the multiplication rules (1.3.28), and propagators can be deduced from (1.3.16)-(1.3.17) or (1.4.18)-(1.4.19) (see (2.5.9)-(2.5.12)). Hence in present theory, (1.3.28) and (1.3.29) or (1.4.18)-(1.4.19) are self-consistent. In the conventional QFT, both propagators and Hamiltonian are deduced from (1.4.18)-(1.4.19), hence it is inevitable that \( \langle 0 | H | 0 \rangle = 0 \) and to introduce the definition of normal products which is equivalent to

\[
\{\psi'_{\alpha}(x,t), \psi'^{\dagger}_{\beta}(y,t)\} = \{\psi'_{\alpha}(x,t), \psi_{\beta}(y,t)\} = 0,
\]

\[
[A'_{\mu}(x,t), \pi'_{\nu}(y,t)] = [A'_{\mu}(x,t), \pi_{\nu}(y,t)] = 0.
\]

Thus the conventional QFT is not self-consistent.

1.5. Subsidiary condition. After the Maxwell field is quantized, the Lorentz condition (1.2.13) is no longer applicable. From (1.4.11) and (1.4.13) we have

\[
(1.5.1) \quad (\partial_\mu A'^{+}_\mu) = \frac{i}{\sqrt{V}} \sum_k \frac{|k|}{\sqrt{2\omega_k}} \left( \leq c_{k3} | -i \leq c_{k4} | e^{ikx} \right),
\]

\[
(1.5.2) \quad (\partial_\mu A'^{-}_\mu) = -\frac{i}{\sqrt{V}} \sum_k \frac{|k|}{\sqrt{2\omega_k}} \left( \leq c_{k3} | -i \leq c_{k4} | e^{-ikx} \right),
\]

Thus we define the subsidiary condition to be

\[
(1.5.3) \quad (\partial_\mu A'^{+}_\mu) | c_p \rangle = 0,
\]

\[
(1.5.4) \quad (\partial_\mu A'^{-}_\mu) | c_p \rangle = 0.
\]

| \( c_p \rangle \) and | \( e_{\pm p} \rangle \) are known as F-physics state ket and W-physics state ket, respectively. From (1.5.1) - (1.5.4) we obtain

\[
(1.5.5) \quad | c_p \rangle = | c_T \rangle \{ 1 + \sum_{k} f(k) | c_{p_k} \rangle + \cdots + \sum_{k_1 \cdots k_n} f(k_1, \cdots, k_n) | c_{p_{k_1}} \cdots c_{p_{k_n}} \rangle \},
\]
| | $| c_p \rangle = | c_T \rangle \{ 1 + \sum_k f(k) | c_p^k \rangle + \cdots$
| | $+ \sum_{k_1 \cdots k_n} f(k_1 \cdots k_n) | c_{p k_1} \cdots c_{p k_n} \rangle \}$, \hfill (1.5.6)

where $| c_T \rangle$ and $| c_T \rangle$ are states containing only transverse photons, and

\begin{align*}
| c_{p k} \rangle &= | c_{k 1} \rangle + i | c_{k 4} \rangle, \quad \text{(1.5.7)} \\
| c_{p k} \rangle &= | c_{k 3} \rangle + i | c_{k 4} \rangle. \quad \text{(1.5.8)}
\end{align*}

From (1.4.1)-(1.4.2) and (1.5.5)-(1.5.8) we obtain

\begin{align*}
\langle c_p | c_{p'} \rangle &= \langle c_T | c_{T'} \rangle, \quad \langle c_p | c_{p'} \rangle = \langle c_T | c_{T'} \rangle, \\
\langle c_p | H_0 | c_{p} \rangle &= \langle c_T | H_0 | c_{T} \rangle, \quad \langle c_p | H_0 | c_{p} \rangle &= \langle c_T | H_0 | c_{T} \rangle. \quad \text{(1.5.9)}
\end{align*}

1.6. The equations of motion. From (1.3.1)-(1.3.4), (1.3.6), (1.3.8), (1.4.1) and (1.4.2), we have

\begin{align*}
\frac{i}{\hbar} \frac{\partial \psi_0}{\partial t} &= [\psi_0, H F_0] = \hat{H}_0 \psi_0, \quad \text{(1.6.1)} \\
\frac{i}{\hbar} \frac{\partial \overline{\psi}_0}{\partial t} &= -[\overline{\psi}_0, H W_0] = \hat{H}_0 \overline{\psi}_0, \quad \text{(1.6.2)} \\
\frac{\partial A_{0\mu}}{\partial t} &= -i [A_{0\mu}, H F_0], \quad \frac{\partial \hat{A}_{0\mu}}{\partial t} = -i [\hat{A}_{0\mu}, H F_0] = \nabla^2 A_{0\mu}, \quad \text{(1.6.3)} \\
\frac{\partial \Delta_{0\mu}}{\partial t} &= i [\Delta_{0\mu}, H W_0], \quad \frac{\partial \hat{\Delta}_{0\mu}}{\partial t} = i [\hat{\Delta}_{0\mu}, H W_0] = \nabla^2 \Delta_{0\mu}, \quad \text{(1.6.4)} \\
\frac{i}{\hbar} \frac{\partial \psi_0'}{\partial t} &= [\psi_0', H F_0] = \hat{H}_0 \psi_0', \quad \text{(1.6.5)} \\
\frac{i}{\hbar} \frac{\partial \overline{\psi}_0'}{\partial t} &= -[\overline{\psi}_0', H W_0] = \hat{H}_0 \overline{\psi}_0', \quad \text{(1.6.6)} \\
\frac{\partial A_{0\mu}'}{\partial t} &= -i [A_{0\mu}', H F_0], \quad \frac{\partial \hat{A}_{0\mu}'}{\partial t} = -i [\hat{A}_{0\mu}', H F_0] = \nabla^2 A_{0\mu}', \quad \text{(1.6.7)}
\end{align*}
\[ \frac{\partial A'_\mu}{\partial t} = i[A'_\mu, H_{W0}], \quad \frac{\partial A'_{\nu}}{\partial t} = i[A'_{\nu}, H_{W0}] = \nabla^2 A'_{\mu}. \]

Since \([H_0, H_{F0}] = [H_0, H_{W0}] = 0\), \(H_{F0}\) and \(H_{W0}\) are the constants of motion. Thus we obtain (1.3.11)-(1.3.12) and

\[ \psi'_0(x, t) = e^{iH_{F0}t}\psi'_0(x, 0)e^{-iH_{F0}t}, \]

\[ \psi'_0(x, t) = e^{-iH_{W0}t}\psi'_0(x, 0)e^{iH_{W0}t}, \]

\[ A'_{\mu}(x, t) = e^{iH_{F0}t}A'_{\mu}(x, 0)e^{-iH_{F0}t}, \]

\[ A'_{\mu}(x, t) = e^{-iH_{W0}t}A'_{\mu}(x, 0)e^{iH_{W0}t}, \]

As seen the equations (1.6.1) - (1.6.4) are consistent with (1.2.11) - (1.2.12), respectively.

1.7. The physical meanings of that the energy of the vacuum state is equal to zero. From (1.2.6), (1.4.1) and (1.4.2) we obtain that the energy of the vacuum state,

\[ E_0 = \langle 0 | H_0 | 0 \rangle = 0. \]

It is easily seen that (1.7.1) is not relative to the definition for multiplication of transformation operators. (1.7.1) holds water provided \(H_0\) is composed of transformation operators. The result is in contrast with the given QED. According to the given QED, before redefining \(H_0\) as normal-ordered products \(E_0 \neq 0\). After redefining \(H_0\) as normal-ordered products, \(E_0 = 0\). This definition is, in fact, equivalent to demand

\[ \{a_p, a^+_p\} = \{b_p, b^+_p\} = [c_k\lambda, c^+_k\lambda] = 0, \]

in the conventional \(QED\). But in fact these commutation relations are equal to 1, and in other cases, e.g. in propagators, they must also be 1. Thus the conventional \(QFT\) is not consistent. In fact, this definition only transfers the divergence difficulty of the energy of the ground state. We may arbitrarily choose the zero point of energy in quantum field theory. But in the theory of gravitation, if \(E_0 \neq 0\), \(E_0\) will have gravitational effect. Hence we are not at liberty to redefine \(E_0 = 0\). Thus the knotty problem of the cosmological constant arises in the conventional \(QFT\) and the relativistic theory of gravitation\(^5\). In the present theory \(E_0 = 0\), hence the density of the energy of the vacuum state \(\rho_{vac} = 0\). Thus, it is seem that from the equation of gravitation field

\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \lambda g_{\mu\nu} = -8\pi G(T_{\mu\nu} - \rho_{vac}g_{\mu\nu}), \]

and astronomical observation values we can easily determine the cosmological constant \(\lambda\). But when to generalize the new QED to the standard electroweak model, expectation values of Higgs fields are not equal to zero. Thus in order to solve the cosmological constant problem, we must consider contribution of Higgs fields.
Considering Higgs fields, in another form of the same idea we can also obtain $E_0 = E_{F0} + E_{W0} = 0$ (see hep-th/0203230). Thus the cosmological constant $\lambda$ can be determined according to astronomical observation values. We will discuss the cosmological constant problem in another paper.

We second simply discuss the correction originating from $E_0 = 0$ to a nonperturbation method in quantum field theory.

When one evaluates the energy of a system by a nonperturbation method, e.g., a Hartree-type approximation\cite{6}, it is necessary to subtract the zero-point energy $E_0$\cite{7}. According to the given quantum field theory $E_0 \neq 0$, while according to the present theory $E_0 = 0$, hence we will obtain different results in nature.

We will discuss the two knotty problems above in detail in other papers.

1.8. Summary. We have presented a new conjecture. According to the conjectures, a particle can exist in two forms which are symmetric. From this we have presented a new Lagrangian density and a new quantization method for QED. That the energy of the vacuum state is equal to zero is naturally obtained. From this the cosmological constant is easily determined by astronomical observation values and it is possible to correct nonperturbation methods which depend on the energy of the ground state in quantum field theory.

2. COUPLING OPERATORS AND FEYNMAN RULES

2.1. Introduction. In the preceding chapter, we have presented a new Lagrangian density $\mathcal{L} = \mathcal{L}_F + \mathcal{L}_W$, defined the transformation operators and have quantized free fields by the transformation operators. Thus, in order to quantize interacting fields, it is necessary to transform the coupling coefficient $g_f$ and the electromagnetic mass $m_{ef}$ in $\mathcal{L}_F$ respectively into a coupling operator $G_F$ and a mass operator $M_F$, and to transform $g_w$ and $m_{ew}$ in $\mathcal{L}_W$ respectively into operators $G_W$ and $M_W$.

In the present chapter we will construct the coupling operators and mass operators, derive scattering operators $S_w$, $S_f$ and Feynman rules. We will see that $G_F$ and $M_F$ are determined by $S_w$, and $G_W$ and $M_W$ are determined by $S_f$. $G_F$ and $M_F$ multiplied by field operators $\psi$ and $A_\mu$ become the coupling coefficient $g_f(p_2, p_1)$ and the mass $m_{ef} (p)$ determined by the scattering amplitude $\langle W_f | S_w | W_i \rangle$, and $G_W$ and $M_W$ multiplied by field operators $\psi$ and $A_\mu$ become $g_w(p_2, p_1)$ and $m_{ew} (p)$ determined by the scattering amplitude $\langle F_f | S_f | F_i \rangle$. It is seen that after quantization, $\mathcal{L}_F$ and $\mathcal{L}_W$ are dependent on each other.

We think that for a self-consistent theory, there should be only one sort of physical parameters which are measurable and finite, and there is no other sort of parameters which are unmeasurable and divergent as the bare mass and the bare charge in the conventional QED. The mass $m_{e0}$ and the coupling constant $g_0$ defined at the so-called subtraction point are the parameters of the sole sort. From this we can determine $\mathcal{L}_F^{(0)}$ and $\mathcal{L}_W^{(0)}$, thereby determine scattering operators $S_f^{(0)}$ and $S_w^{(0)}$. $S_f^{(0)}$ and $S_w^{(0)}$ determine together the one-loop corrections to the coupling constants and the masses, thereby determine the one-loop corrections of scattering amplitudes as well. Thus we can derive $S_f^{(1-loop)}$, $S_w^{(1-loop)}$, and $S_f^{(n-loop)}$ and $S_w^{(n-loop)}$ order by order. We will see that the integrands causing divergence in a total correction cancel each other out. Thus all Feynman integrals will be convergent, and it is unnecessary to introduce regularization and counterterms.
2.2. Interaction Lagrangian density, Hamiltonian density and equations of motion. From (1.2.1) – (1.2.3) we can construct a Lagrangian density

\( \mathcal{L} = \mathcal{L}_F^0 + \mathcal{L}_W^0 + \mathcal{L}_F^1 + \mathcal{L}_W^1 \),

\( \mathcal{L}_F^0 = -\bar{\psi}(x)(\gamma_{\mu}\partial_{\mu} + m)\psi(x) - \frac{1}{2}\partial_{\mu}A_{\nu}\partial_{\mu}A_{\nu}, \quad \mathcal{L}_F^1 = ig\bar{\psi}\gamma_{\mu}A_{\mu}\psi, \)

\( \mathcal{L}_W^0 = \bar{\psi}(x)(\gamma_{\mu}\partial_{\mu} + m)\psi(x) - \frac{1}{2}\partial_{\mu}A_{\nu}\partial_{\mu}A_{\nu}, \quad \mathcal{L}_W^1 = ig\bar{\psi}\gamma_{\mu}A_{\mu}\psi, \)

which is invariant under the following gauge transformation.

\( \psi \rightarrow \psi' = e^{i\theta}\psi, \quad \bar{\psi} \rightarrow \bar{\psi}' = \bar{\psi}e^{-i\theta}, \)

\( A_{\mu} \rightarrow A'_{\mu} = A_{\mu} + \partial_{\mu}\theta(x), \)

\( \bar{\psi} \rightarrow \bar{\psi}' = e^{-i\theta}\bar{\psi}, \quad \bar{\psi} \rightarrow \bar{\psi}' = \bar{\psi}e^{i\theta}, \)

\( \bar{A}_{\mu} \rightarrow \bar{A}'_{\mu} = \bar{A}_{\mu} + \partial_{\mu}\theta(x), \)

where \( \psi, A_{\mu}, \bar{\psi} \) and \( \bar{A}_{\mu} \) are regarded as the classical fields, and \( g \) and \( m \) are constants. After quantization, as mentioned in the first chapter, field operators are composed of transformation operators, hence \( g \) and \( m \) must become respectively a coupling operator and a mass operator, i.e., in (2.2.2),

\( m \rightarrow M_F = m + \left( M_F - m \right) \equiv m + M_F; \quad g \rightarrow G_F; \)

in (2.2.3),

\( m \rightarrow M_W = m + \left( M_W - m \right) \equiv m + M_W; \quad g \rightarrow G_W; \)

here all \( M_F, G_F, M_W \) and \( G_W \) are operators. \( G_F, G_W, M_F \) and \( M_W \) are determined on the basis of the gauge and Lorentz invariance of the Lagrangian density (2.2.1). Thus we have

\( \mathcal{L}_{FI} = i\bar{\psi}\gamma_{\mu}G_F A_{\mu}\psi - \bar{\psi}M_F \psi, \)

\( \mathcal{L}_{W1} = i\bar{\psi}\gamma_{\mu}G_W A_{\mu}\psi + \bar{\psi}M_W \psi. \)

In (2.2.8)-(2.2.9) every field operator directly multiplies by a coupling operator or a mass operator. From (2.2.1), (2.2.8) and (2.2.9) we obtain the Hamiltonian to be

\( H = H_F + H_W, \)

\( H_F = H_{F0} + H_{FI}, \quad H_W = H_{W0} + H_{W1}, \)
\[ H_{F0} = \int d^3 x [\psi^+ \gamma_4 (\gamma_j \partial_j + m) \cdot \psi + \frac{1}{2} \left( \dot{A}_\mu \cdot \dot{A}_\mu + \partial_j A_\mu \partial_j A_\nu \right)] , \]
\[ H_{W0} = \int d^3 x \left[ -(\psi^+ \gamma_4 (\gamma_j \partial_j + m) \cdot \psi + \frac{1}{2} (\dot{A}_\mu \cdot \dot{A}_\mu + \partial_j A_\mu \partial_j A_\nu) \right] , \]
\[ (2.2.12) \quad H_{FI} = \int d^3 x H_{FI} = -\int d^3 x \mathcal{L}_{FI} , \]
\[ (2.2.13) \quad H_{WI} = \int d^3 x H_{WI} = -\int d^3 x \mathcal{L}_{WI} . \]

The Euler-Lagrange equations of motion can be derived from (2.2.1), (2.2.8) and (2.2.9).

\[ (2.2.14) \quad i \frac{\partial}{\partial t} \psi = \gamma_4 \left( \gamma_j \partial_j - i \gamma_\mu G_F \cdot A_\mu + m + M_F \right) \psi , \]
\[ (2.2.15) \quad \Box A_\mu = -i \bar{\psi} \gamma_\mu \cdot G_F \cdot \psi , \]
\[ (2.2.16) \quad i \frac{\partial}{\partial t} \psi = \gamma_4 \left( \gamma_j \partial_j + i \gamma_\mu G_W \cdot \dot{A}_\mu + m + M_W \right) \psi , \]
\[ (2.2.17) \quad \Box \dot{A}_\mu = -i \bar{\psi} \gamma_\mu \cdot G_W \cdot \dot{\psi} . \]

In the Heisenberg picture the evolution of a quantum field system as time is carried by the field operators according to the equations of motion as well,

\[ (2.2.18) \quad \dot{F} = -i [F, H] = -i [F, H_F] , \]
\[ (2.2.19) \quad \dot{W} = i [W, H] = i [W, H_W] , \]

where \( F = \psi(x), \psi^+(x), A_\mu(x), \pi_\mu(x), |a_{ps}(t)\rangle, |b_{ps}(t)\rangle, |c_{k\lambda}(t)\rangle, |d_{p}(t)\rangle, |e_{k\lambda}(t)\rangle, |f(t)\rangle, \) and \( W = \psi(x), \psi^+(x), A_\mu(x), \pi_\mu(x), |a_{ps}(t)\rangle, |b_{ps}(t)\rangle, |c_{k\lambda}(t)\rangle, |d_{p}(t)\rangle, |e_{k\lambda}(t)\rangle, |f(t)\rangle, \). It can be proven that (2.2.18) and (2.2.19) are consistent with (2.2.14) – (2.2.17), respectively (see appendix C). The equal-time anticommutation and commutation relations are the same as (1.3.16)-(1.3.17) and (1.3.29)-(1.3.34), respectively. The equation (2.2.18) is well-known as the Heisenberg equation, while (2.2.19) is a new equation of motion. It is easily seen from (2.2.10) – (2.2.13) that

\[ (2.2.20) \quad [H_F, H] = [H_W, H] = 0 , \]

hence \( H_F \) and \( H_W \) are the constants of motion. Thus, from (2.2.18) and (2.2.19) we have

\[ (2.2.21) \quad F(x,t) = e^{iHt} F(x,0) e^{-iHt} = e^{iH_Ft} F(x,0) e^{-iH_Ft} , \]
\[ (2.2.22) \quad W(x,t) = e^{-iHt} W(x,0) e^{iHt} = e^{-iH_Wt} W(x,0) e^{iH_Wt} . \]
At a given time, say $t = 0$, we can still expand the field operators in terms of their Fourier components as (1.3.1) – (1.3.8), provided let $t = 0$ in (1.3.1) – (1.3.8). Of course, in this case we cannot interpret $\langle a_{\mathbf{p}}(t) \mid \mathbf{b}_{\mathbf{p}}(t) \rangle$, etc., as single-particle annihilation or creation operators. Passage from time $t = 0$ to $t$ involves the replacements (1.3.1) – (1.3.8) by (2.2.21) – (2.2.22). After completing the products of two field operators and the products of field operators and coupling operators or mass operators, Lagrangian and Hamiltonian densities does no longer contain $K$–factor. Thus, that many operators containing $K$ multiply will not appear.

2.3. **Coupling operators and mass operators.** The coupling operators $G_F$, $G_W$ and the mass operators $M_F$ and $M_W$ should satisfy the following demands.

1. After multiplying by field operators, a coupling operator and a mass operator should become a coupling coefficient and a mass operator, respectively.

2. The coupling operators and the mass operators possess the same symmetry as those satisfied by the Lagrangian density containing them, e.g., the Lorentz invariance, the symmetry between $F$-particles and $W$-particles, particle-antiparticle symmetry, etc..

In order to construct the coupling operators, we first define the coupling coefficients. Let $A_{gf} (\mid a_{p_1, c_k} \rangle \rightarrow \mid a_{p_2, s} \rangle)$ be a transition amplitude of an initial state $\mid a_{p_1, c_k} \rangle$ to a final state $\mid a_{p_2, s} \rangle$, here it is not necessary that $p_1, p_2$ and $k$ satisfy the mass shell restrictions. We define the F-coupling constant as

$$g_f (\mid a_{p_1, c_k} \rangle \rightarrow \mid a_{p_2, s} \rangle) = g_f (p_2, p_1) = \frac{A_{gf} (\mid a_{p_1, c_k} \rangle \rightarrow \mid a_{p_2, s} \rangle)}{A'_{gf} (\mid a_{p_1, c_k} \rangle \rightarrow \mid a_{p_2, s} \rangle)},$$

where $S_f$ is the scattering operator for $F$-states.

$$A'_{gf} (\mid a_{p_1, c_k} \rangle \rightarrow \mid a_{p_2, s} \rangle) = \langle a_{p_2, s} \mid \left( - \int d^4 x : \bar{\psi}(0) \gamma_\mu A'_\mu \psi'(0) : \right) \mid a_{p_1, c_k} \rangle = - (2\pi)^4 \delta^4 (p_2 - p_1 - k) \frac{1}{V} \pi_{p_2, s} \gamma_\mu u_{p_1, s} \frac{1}{\sqrt{2\omega_k}} c_{k_\mu}.$$  \hspace{1cm} (2.3.2)

where $k = p_2 - p_1$, $\psi'$ and $A'_\mu$ are the same as (1.4.10)-(1.4.11) in form, but do not satisfy the equations for free particle when $p_2, p_1$ and $k$ are not in mass shell. The act of $A'_{gf}$ is only to eliminate the external-line factors of $A_{gf}$. Considering the Lorentz invariance and that an outgoing $\mid b(-p) \rangle$ is equivalent to an ingoing $\mid a_p \rangle$, and an outgoing $\mid c(-k) \rangle$ is equivalent to an ingoing $\mid c_k \rangle$, we have

$$g_f (p_2, p_1) = g_f (\mid c_k \rangle \rightarrow \mid b(-p_1, s) a_{p_2, s} \rangle) = g_f (\mid a_{p_1, s} b(-p_2, s) \mid c_k \rangle \rightarrow \mid 0 \rangle) = g_f (\mid b(-p_2, s) c_k \rangle \rightarrow \mid b(-p_1, s) \rangle) = g_f (\mid a_{p_1, s} \rightarrow \mid c(-k) a_{p_2, s} \rangle) = g_f (\mid 0 \rangle \rightarrow \mid c(-k) b(-p_1, s) a_{p_2, s} \rangle) = g_f (\mid a_{p_1, s} b(-p_2, s) \rightarrow \mid c(-k) \rangle) = g_f (\mid b(-p_2, s) \rightarrow \mid c(-k) b(-p_1, s) \rangle) = g_f ((p_2 - p_1)^2) = g_f (-\mu^2).$$  \hspace{1cm} (2.3.3)
\[
g_{w}(p_2, p_1) = g_{w}\left(\left|a_{p_1s_1\lambda k}\right> \rightarrow \left|a_{p_2s_2}\right>\right) = \frac{A_{gw}\left(\left|a_{p_1s_1\lambda k}\right> \rightarrow \left|a_{p_2s_2}\right>\right)}{A'_{gw}\left(\left|a_{p_2s_2\lambda k}\right> \rightarrow \left|a_{p_1s_1}\right>\right)} = \frac{\langle a_{p_2s_2} | S_w | a_{p_1s_1\lambda k}\rangle}{A'_{gw}\left(\left|a_{p_2s_2\lambda k}\right> \rightarrow \left|a_{p_1s_1}\right>\right)} \quad (2.3.4)
\]

Considering the symmetry of the F-particles and the W-particles, we have

\[
g_{f}(p_2, p_1) = g_{w}(p_2, p_1) \equiv g(p_2, p_1) = g((p_2 - p_1)^2) = g(-\mu^2).
\]

What relation is there between \(g_{f}(p_2, p_1)\) and \(G_F\)? We analyse \(G_F\) as follows.

Replacing (1.3.10) or (1.3.14)-(1.3.15) by (2.2.21)-(2.2.22), we obtain the expansions of \(\psi, A_{\mu}, \) and \(\mathbf{A}_{\mu}\) which are the same as (1.3.1)-(1.3.4) in form. Substituting the expansions into (2.2.12), (2.2.13), we obtain the expansions of the coupling terms \(H_{FI}\) and \(H_{WI}\). Without loss of generality, we take a term in (2.2.12)

\[
\langle a_{p_2s_2}(t) \rangle I_{p_2}^+ G_F I_{p_1} J_k \langle c_{k\lambda}(t) \rangle
\]
as an example to illustrate the structure of \(G_F\). The c-number part relating to external lines in (2.3.7) is ignored. Recalling (1.3.25)-(1.3.28), from the factor \(I_{p_2}^+ G_F I_{p_1} J_k\), we can see that the corresponding part in \(G_F\) can be written as

\[
L_{p_2} \langle a_{p_2s_2} | G'_F | a_{p_1s_1\lambda k}\rangle I_{p_1} J_k^+.
\]

Because \(\langle a_{p_2s_2} |\) and \(\langle a_{p_1s_1\lambda k}\rangle\) can be regarded as a final W-state and an initial W-state, \(\langle a_{p_2s_2} | G'_F | a_{p_1s_1\lambda k}\rangle\) must be directly proportional to the scattering amplitude \(A_{gw}\left(\left|a_{p_2s_2\lambda k}\right> \rightarrow \left|a_{p_1s_1}\right>\right)\). Comparing with (2.3.4), we define

\[
\langle a_{p_2s_2} | G'_F | a_{p_1s_1\lambda k}\rangle \equiv g_f(p_2, p_1) = \frac{\langle a_{p_2s_2} | S_w | a_{p_1s_1\lambda k}\rangle}{A'_{gw}\left(\left|a_{p_2s_2\lambda k}\right> \rightarrow \left|a_{p_1s_1}\right>\right)} \quad (2.3.9)
\]
as a coupling coefficient. From (2.3.4) and (2.3.9) we can anew obtain (2.3.6). This implies (2.3.9) is consistent with (2.3.6). Thus (2.3.8) can be written as

\[
L_{p_2} g_{f}(p_2, p_1) L_{p_1} J_k^+.
\]

It is possible that in \(g_{f}(p_2, p_1)\) \(p_2 \neq m^2\) or \(p_2^2 \neq m^2\), i.e., \(p_0\) is not solely determined by \(p\). We determine \(p_{20}\) and \(p_{10}\) according to the following method. It is easily
seen that
\[ \langle a_{p_{s}^{'},s_{2}} | (-i \int d^{3}x \hat{H}_{F}) | a_{p_{s},c_{k}}, = \sum_{p_{1}} g_{f}(p_{0}, p_{1}) \{ L_{p_{1}}L_{p_{1}}^{+} + L_{p_{1}}L_{p_{1}}^{+} + L_{p_{1}}L_{p_{1}}^{+} + L_{p_{1}}L_{p_{1}}^{+} \} \]
\[ \leq \sum_{p_{2}} \sum_{p_{1}} g_{f}(p_{2}, p_{1}) L_{p_{2}}L_{p_{1}}^{+}L_{p_{1}}^{+} \]  
\[ (2.3.10) \]

For the initial W-state \( \langle a_{p_{s},c_{k_{s}}} \rangle \) and the final W-state \( \langle a_{p_{s},c_{k_{l}}} \rangle \), \( g_{f}(p_{s}, p_{l}) \) is precise. If for arbitrary \( p_{2} \) and \( p_{1} \) we can also determine \( g(p_{2}, p_{1}) \), \( (2.3.10) \) is the precise coupling operator. In fact it is impossible to determine all \( g_{f}(p_{2}, p_{1}) \). We can only accurately determine some \( g_{f}(p_{2}, p_{1}), \) e.g., \( g_{f}(q_{2}, q_{1}) \), where \( q_{2} \) and \( q_{1} \) are the momenta at the so-called subtraction point which satisfy the restrictions
\[ i\gamma q_{1} + m_{0} = i\gamma q_{2} + m_{0} = 0, \quad q' = q_{2} - q_{1}, \quad q'^{2} = 0, \]  
\[ (2.3.11) \]

We define
\[ g_{f}(q_{2}, q_{1}) = g_{f_{0}}. \]  
\[ (2.3.12) \]

Replacing \( g_{f}(p_{2}, p_{1}) \) by \( g_{f_{0}} \), we obtain a approximate coupling operator,
\[ G_{F}^{(0)} = g_{f_{0}} \sum_{p_{2}} \sum_{p_{1}} L_{p_{2}}L_{p_{1}}^{+}L_{p_{1}}^{+}. \]  
\[ (2.3.13) \]
Analogously to $G_F(p_2, p_1)$ and $G_F^{(0)}$, and considering the symmetry between $\mathcal{L}_F$ and $\mathcal{L}_W$, from (2.3.9) we have
\begin{align}
\langle a_{p_{2s2}} | G_W' | a_{p_{1s1}}, c_{k\lambda} \rangle &= \frac{\langle a_{p_{2s2}} | S_f | a_{p_{1s1}}, c_{k\lambda} \rangle}{A_f' (\langle a_{p_{1s1}}, c_{k\lambda} \rangle \rightarrow a_{p_{2s2}})} \\
&= g_w(p_2, p_1) = g_f(p_2, p_1) = \langle a_{p_{2s2}} | G_F' | a_{p_{1s1}}, c_{k\lambda} \rangle. 
\end{align}

(2.3.15) 

\begin{equation}
G_W = \sum_{p_2} \sum_{p_1} g_w(p_2, p_1) I_{p_2} I_{p_1} I_{k}^+. 
\end{equation}

(2.3.16) 

\begin{equation}
G_W^{(0)} = g_{w0} \sum_{p_2} \sum_{p_1} I_{p_2} I_{p_1} I_{k}^+, \quad g_{w0}(q_2, q_1) = g_{f0} = g_0. 
\end{equation}

Let $m_{ef}$ be the mass originating from the electromagnetic interaction, $m_f'$ be the mass originating from the other interactions, $m_f = m_f' + m_{ef}$ is the total mass of a F-electron. Analogously to $G_F$, we define
\begin{align}
m_{ef} (| a_{p_{1s1}} \rangle \rightarrow | a_{p_{2s2}} \rangle) &= \frac{A_{mf} (| a_{p_{1s1}} \rangle \rightarrow | a_{p_{2s2}} \rangle)}{A_{mf}' (| a_{p_{1s1}} \rangle \rightarrow | a_{p_{2s2}} \rangle)} \\
&= \langle a_{p_{2s2}} | S_f | a_{p_{1s1}} \rangle \langle -i \int d^4x : \bar{\psi}' \psi : | a_{p_{1s1}} \rangle \rangle, 
\end{align}

(2.3.17) 

\begin{align}
A_{mf}' (| a_{p_{1s1}} \rangle \rightarrow | a_{p_{2s2}} \rangle) &= \langle a_{p_{2s2}} | -i \int d^4x : \bar{\psi}' \psi : | a_{p_{1s1}} \rangle \rangle \\
&= -i (2\pi)^4 \delta^4 (p_2 - p_1) \frac{1}{V} \pi_{p_{2s2} u_{p_{1s1}}}. 
\end{align}

(2.3.18) 

Because of the symmetry of $| a_{p_{1s1}} \rangle$ and $| b_{p_{1s1}} \rangle$, we have
\begin{equation}
m_{ef} (| a_{p_{1s1}} \rangle \rightarrow | a_{p_{2s2}} \rangle) = m_{ef} (| b_{p_{1s1}} \rangle \rightarrow | b_{p_{2s2}} \rangle) = m_{ef} (p_1). 
\end{equation}

(2.3.19) 

Similarly, we can define the electromagnetic mass of a W-electron as
\begin{align}
m_{ew} (p_1) &= \frac{A_{mw} (| a_{p_{1s1}} \rangle \rightarrow | a_{p_{2s2}} \rangle)}{A_{mw}' (| a_{p_{1s1}} \rangle \rightarrow | a_{p_{2s2}} \rangle)} \\
&= \langle a_{p_{2s2}} | S_w | a_{p_{1s1}} \rangle \langle -i \int d^4x : \bar{\psi}' \psi : | a_{p_{1s1}} \rangle \rangle, 
\end{align}

(2.3.20) 

\begin{align}
A_{mw}' (| a_{p_{1s1}} \rangle \rightarrow | a_{p_{2s2}} \rangle) &= \langle a_{p_{2s2}} | -i \int d^4x : \bar{\psi}' \psi : | a_{p_{1s1}} \rangle \rangle \\
&= -i (2\pi)^4 \delta^4 (p_1 - p_2) \frac{1}{V} \pi_{p_{1s1} u_{p_{1s1}}}. 
\end{align}

(2.3.21) 

Considering the symmetry of $\mathcal{L}_W$ and $\mathcal{L}_F$, we have
\begin{equation}
m_{ew} (p) = m_{ef} (p) = m_e (p). 
\end{equation}

(2.3.22)
Without loss of generality, we take a term in (2.2.12)

\[ |a_{p_{2s2}}(t) \rangle \geq I_{p_{2}}^+ \cdot M_{F} \cdot I_{p_{1}} \leq |a_{p_{1s1}}(t) \rangle \]

as an example to illuminate the structure of \( M_{F} \). Recalling (1.3.25)-(1.3.28), we can know that the part in \( M_{F} \) corresponding to \( I_{p_{2}}^+ \cdot M_{F} \cdot I_{p_{1}} \) should be written as

\[ I_{p_{2}} (a_{p_{2s2}} | M_{F} | a_{p_{1s1}}) I_{p_{1}}^+ \]

Because \( \langle a_{p_{2s2}} | \) and \( \langle a_{p_{1s1}} | \) may be regarded as an initial W-state and a final W-state, respectively, \( \langle a_{p_{2s2}} | M_{F} | a_{p_{1s1}} \rangle \) must be directly proportional to the scattering amplitude \( A_{m_w} (|a_{p_{1s1}}| \rightarrow |a_{p_{2s2}}|) \). Comparing \( \langle a_{p_{2s2}} | M_{F} | a_{p_{1s1}} \rangle \) with (2.3.20), we can define \( \langle a_{p_{2s2}} | M_{F} | a_{p_{1s1}} \rangle \) as the electromagnetic mass of a F-electron.

\[ \langle a_{p_{2s2}} | M_{F} | a_{p_{1s1}} \rangle = \frac{\langle a_{p_{2s2}} | S_{m} \cdot a_{p_{1s1}} \rangle}{\langle a_{p_{2s2}} | -i \int d^4x : \bar{\psi} \psi : \cdot a_{p_{1s1}} \rangle} \equiv m_{ef} (p_{1}) , \]

where \( p_{2} = p_{1} \). From (2.3.20) and (2.3.25) we can anew obtain (2.3.22). It is seen that (2.3.25) is consistent with (2.3.22). Substituting (2.3.25) into (2.3.24), and considering

\[ \langle a_{p_{2s2}} | M_{F} | a_{p_{1s1}} \rangle = \langle b_{p_{2s2}} | M_{F} | b_{p_{1s1}} \rangle = m_{ef} (p_{1}) \]

and the expansions of \( \bar{\psi} \) and \( \psi \), we have

\[ M_{F} = \sum_{p_{2}} \sum_{p_{1}} m_{ef} (p_{1}) L_{p_{2}} L_{p_{1}}^+. \]

For some \( p \), e.g., \( p = q \), we can determine \( m_{ef} (q) \). Replacing \( m_{ef} (p_{1}) \) in (2.3.26) by \( m_{ef} (q) \), we obtain a approximate mass operator

\[ M_{F}^{(0)} = m_{ef0} \sum_{p_{2}} \sum_{p_{1}} L_{p_{2}} L_{p_{1}}^+, \quad m_{ef0} \equiv m_{ef} (q) . \]

Analogously to (2.3.26) and (2.3.27) and considering the symmetry between \( L_{F} \) and \( L_{W} \), we have

\[ \langle a_{p_{2s2}} | M_{W} | a_{p_{1s1}} \rangle = \frac{\langle a_{p_{2s2}} | S_{f} \cdot a_{p_{1s1}} \rangle}{\langle a_{p_{2s2}} | -i \int d^4x : \bar{\psi} \psi : \cdot a_{p_{1s1}} \rangle} \]

\[ \equiv m_{ew} (p_{1}) = m_{ef} (p_{1}) = \langle a_{p_{2s2}} | M_{F} | a_{p_{1s1}} \rangle , \]

\[ M_{W} = \sum_{p_{2}} \sum_{p_{1}} m_{ew} (p_{1}) I_{p_{2}} I_{p_{1}}^+, \]

\[ M_{W}^{(0)} = m_{ew0} \sum_{p_{2}} \sum_{p_{1}} I_{p_{2}} I_{p_{1}}^+, \quad m_{ew0} \equiv m_{ew} (q) = m_{ef0} \equiv m_{e0} . \]

Because we cannot differentiate \( m_{ef0} \) from \( m_{f0} \) in \( m_{f0} = m_{f0} + m_{ef0} \), for convenience, we define

\[ m_{ef0} = m_{ew0} = m_{e0} = 0, \quad m_{f0} = m_{f0} = m_{e0} = m_{e0} = m_{0} . \]

Of course, for an interacting field operator \( p^{2} - p_{0}^{2} \neq -m^{2} \) and \( k^{2} \neq 0 \). When \( p_{2} = q_{2} \) and \( p_{1} = q_{1} \), both \( g_{0} \) and \( m_{e0} \) are accurate. When \( p_{2} \neq q_{2} \) or \( p_{1} \neq q_{1} \), both
$g_0$ and $m_{c0}$ are approximate. If the subtraction point is a nonphysical point for a
coupling constant, we assume that $g$ can analytically expand from the nonphysical
point into physical regions.

2.4. Expansion of the Hamiltonian. Because when many operators containing
$K$ multiply, associative law does no longer hold water. Hence it is necessary to
define the order of multiplication of a coupling or a mass operator and transformation
operators. When we construct the interaction Lagrangian or Hamiltonian density,
we define the order of multiplication of a coupling or a mass operator and t ransformation
operators. Thus substituting the expansions of $\psi, \overline{\psi}, A_\mu, \overline{\psi}, \overline{\psi}$ and $A_\mu$
and (2.3.10), (2.3.15), (2.3.26) and (2.3.29) into (2.2.11)-(2.2.13), we obtain

$$H_{F0} = \sum_{\mu} \{ | a_{p\mu} (t) \rangle \langle a_{p\mu} (t) | u_{p\mu}^+ \gamma_4 (i \gamma_j p_j + m) u_{p\mu}$$

$$+ \langle a_{\bar{p}\mu} (t) | \langle a_{\bar{p}\mu} (t) | v_{\bar{p}\mu}^+ \gamma_4 (i \gamma_j p_j + m) u_{\bar{p}\mu}$$

$$+ | a_{\bar{p}\mu} (t) \rangle \langle b_{p\mu} (t) \rangle \langle b_{p\mu} (t) \rangle u_{p\mu}^+ \gamma_4 (i \gamma_j p_j + m) v_{p\mu}$$

$$- | b_{\bar{p}\mu} (t) \rangle \langle c_{k\lambda} (t) \rangle \langle c_{k\lambda} (t) \rangle v_{p\mu}^+ \gamma_4 (i \gamma_j p_j + m) v_{p\mu}\}$$

$$+ \sum_k \frac{\omega_k^2 + k^2}{2 \omega_k} \left( \sum_{\lambda=1}^3 | c_{k\lambda} (t) \rangle \langle c_{k\lambda} (t) | - | c_{k\lambda} (t) \rangle \langle c_{k\lambda} (t) | \right)$$

$$\sum_k \frac{\omega_k^2 - k^2}{4 \omega_k} \left( | c_{k\lambda} (t) \rangle \langle c_{-k\lambda} (t) | + | c_{k\lambda} (t) \rangle \langle c_{-k\lambda} (t) | \right)$$

$(2.4.1)$

$$H_{FLM} = \sum_{\mu} m_{ef} (p) \{ | a_{p\mu} (t) \rangle \langle a_{p\mu} (t) | u_{p\mu}^+ \gamma_4 u_{p\mu}$$

$$+ | a_{\bar{p}\mu} (t) \rangle \langle b_{p\mu} (t) \rangle \langle b_{p\mu} (t) \rangle u_{p\mu}^+ \gamma_4 u_{p\mu}$$

$$- | b_{\bar{p}\mu} (t) \rangle \langle c_{k\lambda} (t) \rangle \langle c_{k\lambda} (t) \rangle v_{p\mu}^+ \gamma_4 v_{p\mu}\}$$

$(2.4.2)$

$$H_{FIG} = -\frac{i}{\sqrt{V}} \sum_{p_{2} \not=p_{1}} \sum_{s_{2} \not=s_{1}} \{ | a_{p_{2}s_{2}} (t) \rangle \langle a_{p_{1}s_{1}} (t) | \pi_{p_{2}s_{2}} \gamma_{\mu} u_{p_{1}s_{1}}$$

$$+ | a_{\bar{p}_{2}s_{2}} (t) \rangle \langle a_{\bar{p}_{1}s_{1}} (t) | \pi_{\bar{p}_{2}s_{2}} \gamma_{\mu} u_{p_{1}s_{1}}$$

$$+ | a_{p_{2}s_{2}} (t) \rangle \langle b_{p_{1}s_{1}} (t) \rangle \langle b_{p_{1}s_{1}} (t) \rangle \pi_{p_{2}s_{2}} \gamma_{\mu} v_{(-p_{1})s_{1}}$$

$$- | b_{\bar{p}_{1}s_{1}} (t) \rangle \langle b_{\bar{p}_{2}s_{2}} (t) \rangle \langle b_{\bar{p}_{2}s_{2}} (t) \rangle \pi_{\bar{p}_{2}s_{2}} \gamma_{\mu} v_{(-p_{1})s_{1}} \frac{1}{\sqrt{2 \omega_k}}$$

$$\sum_{\lambda=1}^4 \left( c_{k\mu} \langle c_{k\lambda} (t) + c_{(-k\mu)} \rangle \pi_{(-k\lambda)} (t) \right),$$

$(2.4.3)$
where \( k = p_2 - p_1 \).

\[
H_{W0} = \sum_{ps} \{ | b_{ps}^+(t) \rangle \langle b_{ps}(t) | u_{ps}^+ \gamma_4 (i \gamma_j p_j + m) u_{ps} \}
\]

\[
- \{ | b_{ps}^+(t) \rangle \langle b_{ps}(t) | v_{(-p)s}^+ \gamma_4 (i \gamma_j p_j + m) v_{ps} \}
\]

\[
- \{ | a_{ps}^+(t) \rangle \langle a_{ps}(t) | u_{(-p)s}^+ \gamma_4 (i \gamma_j p_j + m) v_{ps} \}
\]

\[
\sum_k \frac{\omega_k^2 + k^2}{2\omega_k} \sum_{\lambda=1}^3 \{ \langle \bar{\xi}_\lambda(t) | a_{ps}^+(t) \rangle a_{ps}(t) | \bar{\xi}_\lambda(t) \rangle \}
\]

\[
\sum_k \frac{\omega_k^2 - k^2}{4\omega_k} \sum_{\lambda=1}^3 \{ \langle \bar{\xi}_\lambda(t) | a_{ps}^+(t) \rangle a_{ps}(t) | \bar{\xi}_\lambda(t) \rangle \}
\]

\[
H_{WIM} = \sum_{ps} m_{cw} (p) \{ | b_{ps}^+(t) \rangle \langle b_{ps}(t) | u_{ps}^+ \gamma_4 u_{ps} \}
\]

\[
- \{ | b_{ps}^+(t) \rangle \langle b_{ps}(t) | v_{(-p)s}^+ \gamma_4 v_{ps} \}
\]

\[
- \{ | a_{ps}^+(t) \rangle \langle a_{ps}(t) | u_{(-p)s}^+ \gamma_4 v_{ps} \}
\]

\[
\sum_k \frac{\omega_k^2 + k^2}{2\omega_k} \sum_{p_2s_2 p_{1s_1}} \{ \langle \bar{\xi}_{(-p)s_2}(t) | a_{p_2 s_2}^+(t) \rangle a_{p_2 s_2}(t) | \bar{\xi}_{(-p)s_1}(t) \rangle \frac{1}{\sqrt{2\omega_k}}
\]

\[
\sum_{\lambda=1}^4 e^{\lambda^\dagger \mu(p)} \langle \bar{\xi}_{(-p)s}(t) | a_{p_2 s_2}^+(t) \rangle a_{p_2 s_2}(t) | \bar{\xi}_{(-p)s}(t) \rangle \}
\]

From (2.4.1)-(2.4.6) we can prove (2.2.18)-(2.2.19) to be consistent with (2.2.14)-(2.2.17). (see appendix C).

In fact, we can only obtain an approximate Hamiltonian in which the coupling constants and masses are approximately regarded as the same, i.e., \( g(p_2, p_1) = g_0 \) and \( m_e(p) = 0 \). In this case, we can write \( H_F \) and \( H_W \) as

\[
H_F^{(0-loop)}(g_0, m_e) = \int d^3x : [\psi'^+ \gamma_4 (\gamma_j \partial_j - ig_0 \gamma_\mu A_\mu' + m) \psi' + \frac{1}{2} (A_\mu' \partial_\mu' + \partial_\mu' A_\mu')^2]
\]

\[
H_W^{(0-loop)}(g_0, m_e) = \int d^3x : [-\psi'^+ \gamma_4 (\gamma_j \partial_j + ig_0 \gamma_\mu A_\mu' + m) \psi' + \frac{1}{2} (A_\mu' \partial_\mu' + \partial_\mu' A_\mu')^2]
\]
here the expansions of the field operators \( \psi'(x,t) \) etc. are the same as (1.4.10) – (1.4.17) in form, but the operators \( \lesssim a_{p\lambda}(t) \) etc. in (1.4.10) or (1.4.14)-(1.4.15) must be replaced by (2.2.21)-(2.2.22). When \( p_2 = q_2 \) or \( p_1 = q_1 \), both \( H_F^{(0-loop)}(g_0,m_{e0}) \) and \( H_W^{(0-loop)}(g_0,m_{e0}) \) are accurate. When \( p_2 \neq q_2 \) or \( p_1 \neq q_1 \), both \( H_F^{(0-loop)}(g_0,m_{e0}) \) and \( H_W^{(0-loop)}(g_0,m_{e0}) \) are approximate. The equal-time anticommutation and commutation relations for the operators \( \lesssim b_{p\lambda}(t) \) and \( \psi'_\mu(x,t) \), etc., are the same as (1.3.16) – (1.3.17) and (1.4.18) – (1.4.19), respectively. In this case, the Heisenberg equations are

\[
(2.4.9) \quad \dot{F} = -i \left[ F, H_F^{(0-loop)} \right] = -i \left[ F, H_W^{(0-loop)} \right],
\]

\[
(2.4.10) \quad \dot{W} = i\left[W, H_W^{(0-loop)}\right] = i\left[W, H_F^{(0-loop)}\right],
\]

where \( H^{(0-loop)} = H_F^{(0-loop)} + H_W^{(0-loop)} \), \( F = \psi', A'_\mu, \psi, A_\mu, \lesssim a_{p\lambda}(t) \), \( \lesssim b_{p\lambda}(t) \), \( \lesssim c_{k\lambda}(t) \); \( W = \psi', A'_\mu, \psi, A_\mu, \lesssim a_{p\lambda}(t) \), \( \lesssim b_{p\lambda}(t) \), \( \lesssim c_{k\lambda}(t) \). From (2.4.9) – (2.4.10) we obtain (2.2.14) – (2.2.17) (where \( M_F \to M_F^{(0)} \), \( G_F \to G_F^{(0)} \), \( M_W \to M_W^{(0)} \), \( G_W \to G_W^{(0)} \)) and

\[
(2.4.11) \quad i\frac{\partial}{\partial t}\psi' = \gamma_4 \left( \gamma_j \partial_j - i\gamma_\mu g_0 A_\mu + m \right) \psi',
\]

\[
(2.4.12) \quad \Box A'_\mu = -i\overline{\psi}' \gamma_\mu g_0 \psi',
\]

\[
(2.4.13) \quad i\frac{\partial}{\partial t}\psi' = \gamma_4 \left( \gamma_j \partial_j + i\gamma_\mu g_0 A'_\mu + m \right) \psi',
\]

\[
(2.4.14) \quad \Box A'_\mu = -i\overline{\psi}' \gamma_\mu g_0 \psi',
\]

\[
(2.4.15) \quad \psi'(x,t) = e^{iH^{(0-loop)}t} \psi'(x,0) e^{-iH^{(0-loop)}t} = e^{iH_F^{(0-loop)}t} \psi'(x,0) e^{-iH_F^{(0-loop)}t},
\]

\[
(2.4.16) \quad A'_\mu(x,t) = e^{iH^{(0-loop)}t} A'_\mu(x,0) e^{-iH^{(0-loop)}t} = e^{iH_F^{(0-loop)}t} A'_\mu(x,0) e^{-iH_F^{(0-loop)}t},
\]

\[
(2.4.17) \quad \overline{\psi}'(x,t) = e^{-iH^{(0-loop)}t} \overline{\psi}'(x,0) e^{iH^{(0-loop)}t} = e^{-iH_W^{(0-loop)}t} \overline{\psi}'(x,0) e^{iH_W^{(0-loop)}t},
\]

\[
(2.4.18) \quad A'_\mu(x,t) = e^{-iH^{(0-loop)}t} A'_\mu(x,0) e^{iH^{(0-loop)}t} = e^{-iH_W^{(0-loop)}t} A'_\mu(x,0) e^{iH_W^{(0-loop)}t},
\]

(2.4.11)-(2.4.12) and (2.4.15)-(2.4.16) are the same as those of the conventional QED. (2.4.13)-(2.4.14) and (2.4.17)-(2.4.18) are different from those of the conventional QED. It is seen from the \( \mathcal{L}_F \) we can also obtain the results of the conventional QED.
2.5. Scattering operators and Feynman rules. As the conventional QED, transforming the Heisenberg picture into the interaction picture, we can derive the scattering operator $S_f$ and $S_w$ from (2.4.2)-(2.4.3), (2.4.5)-(2.4.6), (2.4.7) and (2.4.8),

\begin{align}
S_f &= 1 + \sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} d^4x_1 \cdots \int_{-\infty}^{\infty} d^4x_n T : H_{FI}(g,m_c) : \mathcal{H}_{FI}(g,m_c) \\
S_w &= 1 + \sum_{n=1}^{\infty} \frac{i^n}{n!} \int_{-\infty}^{\infty} d^4x_1 \cdots \int_{-\infty}^{\infty} d^4x_n T : H_{WI}(g,m_c) : \mathcal{H}_{WI}(g,m_c) \\
S_f^{(0)}(g_0,m_{c0}) &= 1 + \sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} d^4x_1 \cdots \int_{-\infty}^{\infty} d^4x_n T : H_{FI}^{(0)} : H_{FI}^{(0)} \\
S_w^{(0)}(g_0,m_{c0}) &= 1 + \sum_{n=1}^{\infty} \frac{i^n}{n!} \int_{-\infty}^{\infty} d^4x_1 \cdots \int_{-\infty}^{\infty} d^4x_n T : H_{WI}^{(0)} : H_{WI}^{(0)} \\
H_{FI} &= -i\bar{\psi}\gamma_\mu : G_F : A_\mu \psi + \bar{\psi} : M_F : \psi, \\
H_{WI} &= -i\bar{\psi}\gamma_\mu : G_W : A_\mu \psi - \bar{\psi} : M_W : \psi, \\
H_{FI}^{(0)}(g_0,m_{c0}) &= -ig_0 : \psi^{\mu} + \gamma_4 \gamma_\mu A'_\mu \psi' : \\
H_{WI}^{(0)}(g_0,m_{c0}) &= -ig_0 : \psi^{\mu} + \gamma_4 \gamma_\mu A'_\mu \psi' :
\end{align}

where all field operators in (2.5.5)-(2.5.8) satisfy free-field equations as the conventional QED, $G_F$ and $M_F$ are respectively determined by (2.3.10) and (2.3.26), $G_W$ and $M_W$ are respectively determined by (2.3.15) and (2.3.29), the symbol $T$ denotes that the products are in time-ordered form. Replacing $g(p_1,p_2)$ and $m_c(p_1)$ in $H_{FI}$ and $H_{WI}$ by $g_0$ and $m_{c0} = 0$, respectively, we obtain $S_f^{(0)}(g_0,m_{c0})$ and $S_w^{(0)}(g_0,m_{c0})$ from $S_f(g,m_c)$ and $S_w(g,m_c)$. If for arbitrary momenta $p_1$ and $p_2$ we could determine the accurate coupling constants $g(p_1,p_2)$ and the masses $m_c(p)$, we could evaluate accurate scattering amplitudes by only tree diagrams determined by $S_f(g,m_c)$ or $S_w(g,m_c)$. But this is impossible. It should be pointed that in the meaning of perturbation theory, for initial states and final states with given momenta, e.g., the momenta at the subtraction point, we can give the absolutely precise coupling coefficients and masses, and from this give absolutely precise $\mathcal{L}_F$, $\mathcal{L}_W$, $H_{FI}^{(0)}$, $H_{WI}^{(0)}$, $S_f^{(0)}$ and $S_w^{(0)}$. But for arbitrary initial states and final states, we cannot give the absolutely precise coupling coefficients, masses, $\mathcal{L}_F$, $\mathcal{L}_W$, $H_F$, $H_W$, $S_f$ and $S_w$. In fact the coupling coefficients and masses will be corrected by n-loop diagrams, hence for arbitrary initial states and final states we can only give approximate $\mathcal{L}_F$, $\mathcal{L}_W$, $H_F$, $H_W$, $S_f$ and $S_w$. Of course, by such $S_f^{(0)}$ and $S_w^{(0)}$ we can obtain scattering amplitudes approximate to arbitrary n-loop diagrams. Thus, the two scattering operators $S_f$ and $S_w$ are not practical to evaluate scattering amplitudes. Therefore we can evaluate Feynman amplitudes by only $S_f^{(0)}$.
and $S_w^{(0)}$ and perturbation approximation. The scattering operator $S_f^{(0)} (g_0, m_{e0})$ is the same as the scattering operator of the conventional QED. $S_w^{(0)}$ differ from $S_f^{(0)}$ only in the sign before $i$ in form. Hence we can evaluate scattering amplitudes by $S_f^{(0)}$ and $S_w^{(0)}$ in the same method as is used by the conventional QED.

In order to simplify $S_f^{(0)}$ and $S_w^{(0)}$, we must decompose the time-ordered products appearing in $S_f^{(0)}$ and $S_w^{(0)}$ into normal-ordered products. Considering the expansions of $\psi'(x)$ and $A'_\mu(x)$ to be the same as those of the free fields of the conventional QED and $A'_\mu(x)$ and $\psi'(x)$ to have the similar expansions, as the conventional QED, we have

\[
\langle 0 \mid T \psi^\alpha_\alpha (x_1) \bar{\psi}^\beta_\beta (x_2) \mid 0 \rangle = -i \int \frac{d^4 p}{(2\pi)^4} \frac{(m - ip\gamma)_{\alpha\beta}}{p^2 + m^2 - i\varepsilon} e^{ip(x_1 - x_2)} \equiv S_{Ff} (x_1 - x_2), \quad (2.5.9)
\]

\[
\langle 0 \mid T A'_\mu (x_1) A'_\nu (x_2) \mid 0 \rangle = \delta_{\mu\nu} \frac{-i}{(2\pi)^4} \int d^4 k \frac{1}{k^2 - i\varepsilon} e^{ik(x_1 - x_2)} \equiv \delta_{\mu\nu} D_{Ff} (x_1 - x_2), \quad (2.5.10)
\]

\[
\langle 0 \mid T \psi'^\alpha_\alpha (x_1) \bar{\psi}'^\beta_\beta (x_2) \mid 0 \rangle = -i \int \frac{d^4 p}{(2\pi)^4} \frac{(m - ip\gamma')_{\alpha\beta}}{p^2 + m^2 + i\varepsilon} e^{ip(x_1 - x_2)} \equiv S_{Fw} (x_1 - x_2), \quad (2.5.11)
\]

\[
\langle 0 \mid T A'^\mu_\mu (x_1) A'^\nu_\nu (x_2) \mid 0 \rangle = \delta_{\mu\nu} \frac{i}{(2\pi)^4} \int d^4 k \frac{1}{k^2 + i\varepsilon} e^{ik(x_1 - x_2)} \equiv \delta_{\mu\nu} D_{Fw} (x_1 - x_2), \quad (2.5.12)
\]

where $\varepsilon \to 0^+$, the integral contours are shown in Fig.2.1. The other contractions are all equal to zero. Comparing (2.5.11)-(2.5.12) with (2.5.9)-(2.5.10), we find the differences and the common properties among the four propagators as follows.

1. The places of the polar points of $D_{Fw}$ and $S_{Fw}$ are opposite to those of $D_{Fw}$ and $S_{Fw}$, respectively. The polar points of $D_{Fw}$ and $S_{Fw}$ are in the first and third quadrants, the polar points of $D_{Ff}$ and $S_{Ff}$ are in the second and the fourth quadrants (see Fig.2.1.1 and Fig.2.1.2).

2. Neglecting $\varepsilon$, $D_{Ff} = -D_{Fw}$ and $S_{Ff} = S_{Fw}$.

It can be seen that the Wick theorem still applicable to the present theory. Decomposing the time-ordered products appearing in (2.5.3) and (2.5.4) into normal-ordered products, we can evaluate S-matrix elements approximate to tree-diagrams. It is readily proven the S-matrix elements $\langle F_f \mid S_f^{(0)} (g_0, m_{e0}) \mid F_f \rangle$ approximate to tree-diagrams are the same as those obtained by the conventional QED. From the S-matrix elements $\langle F_f \mid S_f (g_0, m_{e0}) \mid F_f \rangle$ and $\langle W_f \mid S_w (g_0, m_{e0}) \mid W_i \rangle$ approximate to tree-diagrams we can obtain the Feynman rules in the momentum space (see Fig. 2.2.1 and Fig.2.2.2).

In addition to the rules given by Fig. 2.1 and Fig.2.2.1-2, there is an overall minus sign corresponding to a closed fermion loop and three supplementary Feynman rules (see the following chapter). The rules fully determine the method evaluating a F-amplitude or a W-amplitude.
2.6. **Summary.** On the basis of the first chapter we discuss quantization for interacting fields, derive Feynman rules and scattering operators. Since we quantize fields by the transformation operators replacing creation and annihilation operators in the conventional QED, it is necessary to replace coupling constants and masses by coupling operators and mass operators in the present theory. In contrast with the conventional QED, in the present theory, there is only one sort of parameters which are all finite and measurable and the two sorts of corrections originating \( S_f \) and \( S_w \). We will see that the integrands causing divergence in the two sorts of correction cancel each other out in the following.

3. **One-loop Correction and Supplementary Feynman Rules**

3.1. **Introduction.** We have constructed a new Lagrangian density of free fields and quantized the free field by the transformation operators in the first chapter, have discussed quantization for interacting fields and have derived the scattering operators \( S_f \), \( S_w \) and the Feynman rules in second chapter. In present paper we evaluate one-loop correction to the coupling coefficients and the masses in detail, derive the method evaluating \( n \)-loop correction.

If \( g_0 \) and \( m_{e0} \) are regarded constants, \( \mathcal{L}_F \) and \( \mathcal{L}_W \) were fully independent. In this case we can only obtain one new result i.e., \( \langle 0 | H | 0 \rangle = 0 \). But in fact, after quantization, though there is no coupling between the field operators \( \psi \) and \( A_\mu \) in \( \mathcal{L}_F \) and the field operators \( \psi \) and \( A_\mu \) in \( \mathcal{L}_W \), \( \mathcal{L}_F \) and \( \mathcal{L}_W \) are dependent on each other, since \( g_f(p_2,p_1) \) and \( m_{e1}(p_2,p_1) \) are determined by \( \mathcal{L}_W \) and \( g_w(p_2,p_1) \) and \( m_{w1}(p_2,p_1) \) are determined by \( \mathcal{L}_F \). \( S_f \) and \( S_w \) together determine scattering amplitudes. Thus, all scattering amplitudes will be convergent and it is unnecessary to introduce regularization and counterterms.

3.2. **Two sorts of correction.** When the momenta meeting at a single vertex \( p_2 = q_2, p_1 = q_1 \), scattering amplitudes \( A^{(0\text{-loop})}_{gf}(q_2,q_1), A^{(0\text{-loop})}_{gw}(q_2,q_1) \) and \( A^{(0\text{-loop})}_{m}(q_1) \) evaluated by \( S_f^{(0)}(g_0,m_{e0}) \) and \( S_w^{(0)}(g_0,m_{w0}) \) with tree diagrams are all accurate (it is possible that there is no real process for a single vertex). \( g_0 \) and \( m_{e0} \) determined on the basis of experiments and are accurate. When \( p_2 \neq q_2, p_1 \neq q_1 \) or for such a Feynman diagram with many vertices, \( g_0, m_{e0}, m_{w0} \) and the scattering amplitudes obtained by \( S_f^{(0)} \) and \( S_w^{(0)} \) with tree diagrams are no longer accurate and must be corrected. According to the perturbation theory, Feynman diagrams with the same initial state \( | a_{p_1}, s_1 \rangle | c_{K1} \rangle \) and final state \( | a_{p_2}, s_2 \rangle \), no matter whether or not \( p_1 = q_1 \) and \( p_2 = q_2 \), \( (q_1 \) and \( q_2 \) are the momenta at the subtraction point), the tree diagrams and all diagrams with \( n \)-loop must exist and must simultaneously be considered. It is obvious that corrections must begin with one-loop diagrams. The coupling constants, the masses and scattering amplitudes corrected to \( n \)-loop diagrams can be written as

\[
Z^{(n\text{-loop})}(p_2,p_1,k) = Z_0 + \sum_{i=1}^{n} Z^{(i)}(p_2,p_1,k),
\]

where \( Z = g_f, g_w, m_{e1}, m_{w1}, A_{gf}, A_{gw}, A_{mf}, A_{mw}, \) the superscript \( i \) denotes only the \( i \)th \( - \) loop diagrams to be evaluated. Because of the gauge invariance of the Lagrangian density, there is no correction of the mass of a free F-photon or W-photon.
We consider that a self-consistent theory should satisfy the following demands:

1. There is only one sort of physical parameters which are measurable and finite, e.g., there are only $g_0$ and $m_{e0}$ in the present theory, and there is no other sort of parameters as the bare mass and the bare charge in the conventional $QED$ which are unmeasured and divergent.

2. $Z^{(n\text{-loop})}(p_2, p_1, k)$ and $Z^{(1)}(p_2, p_1)$ must be finite and must tend to zero as $p_2 \rightarrow q_2$, $p_1 \rightarrow q_1$ and $k \rightarrow q'$, i.e.,

$$Z^{(1)}(q_2, q_1, q') = 0, \quad Z^{(n\text{-loop})}(q_2, q_1, q') = Z_0.$$  \hspace{1cm} (3.2.2)

3. $Z^{(n\text{-loop})}(p_2, p_1, k)$ should be consistent with the results obtained by the given $QED$.

According to the conventional $QED$, the 1-loop corrections to $g_{f0}(q_2, q_1)$ and $m_{e\text{e}}(q)$ are all divergent. This is because there is only one sort of corrections originating $S_f$. In order to remove divergence, it is necessary to introduce regularization and counterterms. For instance, in order to remove divergence of the one-loop correction of the mass of a free electron

$$\delta m = m_{\text{eff}}^{(1)}(q) = \frac{3\alpha}{4\pi^2} m \left( \frac{1}{i\pi^2} \int \frac{d^4 k}{(k^2 + m^2)^2} + \frac{3}{2} \right) \equiv A,$$

the counterterm $A\bar{\psi}\psi$ must be introduced, here $A$ is a divergent constant. Thus, there are two sorts of parameters in the conventional $QED$, i.e., the physical mass and charge and the so-called bare mass and bare charge. Both bare mass and bare charge are divergent and unmeasured. It is noteworthy that in order to determine $A$ we must firstly carry out integrantion over $k$. Thus regularization must be introduced.

As the conventional $QED$, it can be proven from the Feynman rules that the one-loop corrections only originating from $S_f^{(0)}(g_0, m_{e0})$ or $S_w^{(0)}(g_0, m_{e0})$ are all divergent and do not tend to zero as $p_2 \rightarrow q_2$ and $p_1 \rightarrow q_1$. This implies that there must be another sort of corrections which and the first sort of corrections must cancel each other out as $p_2 \rightarrow q_2$, $p_1 \rightarrow q_1$ and $k \rightarrow q' = q_2 - q_1$ and the sum of the two corrections must be finite. In contrast with the conventional $QED$, in the present theory, there is only one sort of parameters ($g_0$ and $m_{e0}$) which are finite and measurable and two sorts of correction originating from $S_f$ and $S_w$. $S_f$ and $S_w$ together determine $Z^{(n\text{-loop})}(p_2, p_1, k)$ and $Z^{(1)}(p_2, p_1, k)$ which satisfy the demands above.

We first take one-loop correction as an example to discuss the first sort of correction. As the conventional $QED$, there are one-loop corrections $A_{gf}^{(1)}(p_2, p_1)$ to $A_{gf}^{(0\text{-loop})}(p_2, p_1)$ and $A_{mff}^{(1)}(p_2, p_1)$ to $A_{mff}^{(0\text{-loop})}(p_2, p_1)$ originating from $S_f^{(0)}(g_0, m_{e0})$, and one-loop corrections $A_{gw}^{(1)}(p_2, p_1)$ to $A_{gw}^{(0\text{-loop})}(p_2, p_1)$ and $A_{miew}^{(1)}(p_2, p_1)$ to $A_{miew}^{(0\text{-loop})}(p_2, p_1)$ originating from $S_w^{(0)}(g_0, m_{e0})$. From the definition (2.3.1) we see that $A_{gf}^{(1)}$ firstly corrects the amplitude $A_{gf}^{(0)}(p_2, p_1)$ and thereby corrects $g_{f0}$, hence the momenta of $A_{gf}^{(1)}(p_2, p_1)$ and $g_{f0}^{(1)}(p_2, p_1)$ must be the same as those of $A_{gf}^{(0)}(p_2, p_1)$. We can evaluate $g_{f0}^{(1)}(p_2, p_1)$ by $S_f^{(1)}(g_0, m_{e0})$ as the conventional $QED$. 

Thus from (2.3.1) we have

\[ g_{ff}^{(1)} = g_{ff}^{(1)}(p_2, p_1) = \frac{\langle a_{p_2s_2} | S_f^{(1)}(g_0, m_{e0}) | a_{p_1s_1} \rangle}{\langle a_{p_2s_2} | S_f^{(0)}(g_0 = 1, m_{e0} = 0) | a_{p_1s_1} \rangle}, \]

where \( p \) may be a momentum of an internal line and does not satisfy the mass shell restriction, \( S_f^{(1)} \) is the scattering operator approximate to one-loop. Analogously to \( g_{ff}^{(1)} \), from (2.3.17), (2.3.4) and (2.3.20) we have

\[ m_{eff}^{(1)}(p_1) = \frac{\langle a_{p_2s_2} | S_f^{(1)}(g_0, m_{e0}) | a_{p_1s_1} \rangle}{\langle a_{p_2s_2} | S_f^{(0)}(g_0 = 1, m_{e0} = 0) | a_{p_1s_1} \rangle}, \]

\[ g_{wu}^{(1)}(p_2, p_1) = \frac{\langle a_{p_2s_2} | S_w^{(1)}(g_0, m_{e0}) | a_{p_1s_1} \rangle}{\langle a_{p_2s_2} | S_w^{(0)}(g_0 = 1, m_{e0} = 0) | a_{p_1s_1} \rangle}, \]

\[ m_{ewu}^{(1)}(p_1) = \frac{\langle a_{p_2s_2} | S_w^{(1)}(g_0, m_{e0}) | a_{p_1s_1} \rangle}{\langle a_{p_2s_2} | S_w^{(0)}(g_0 = 1, m_{e0} = 0) | a_{p_1s_1} \rangle}, \]

where the denominators of (3.2.3)-(3.2.6) are the same as (2.3.2), (2.3.18), (2.3.5) and (2.3.21), respectively. (3.2.3)-(3.2.6) are the first sort of corrections.

Secondly, we take one-loop correction as an example to consider the second sort of one-loop corrections. It is seen from (2.3.9) that \( A_{g_{wu}}^{(1)} \) originating from \( S_w \) not only corrects \( A_{g_{wu}}^{(0-loop)} \) and \( g_{w0} \), but also corrects \( g_{f0} \) and \( m_{ef0} \). We denote the sort of one-loop correction to \( g_{f0} \) originating from \( S_w \) by \( g_{f_w}^{(1)} \). In contrast with the first sort of corrections, the corrections of the second sort are to directly correct the coupling constants and the masses. Because \( g_0(q_2, q_1) \) and \( m_{e0}(q) \) are defined at the subtraction point, the second sort of corrections must also be defined at the subtraction point, i.e. \( g_{f_w}^{(1)} = g_{f_w}^{(1)}(q_2, q_1) \).

Considering

\[ g_{f}^{(1)}(p_2, p_1) = g_{f}^{(1)}(p_2, p_1) + g_{f}^{(1)}(q_2, q_1) \]

should be finite, from (2.3.9) and (2.3.5) we define

\[ g_{f}^{(1)}(q_2, q_1) = -g_{f_w}^{(1)}(q_2, q_1), \]

Analogously to \( g_{f_w}^{(1)} \), from (2.3.25) and (2.3.6) we define

\[ m_{e_{f_w}}^{(1)} = -m_{e_{f_w}}^{(1)}(q), \quad m_{e_f}^{(1)}(p) = m_{e_f}^{(1)}(p) + m_{e_{f_w}}^{(1)}(q). \]

Analogously to \( g_{f_w}^{(1)} \) and \( m_{e_{f_w}}^{(1)} \), there are the corrections \( g_{w_f}^{(1)} \) and \( m_{e_{w_f}}^{(1)} \) originating from \( S_f \) to \( g_{w0} \) and \( m_{e0} \). From (2.3.14), (2.3.3), (2.3.28) and (2.3.4) we define

\[ g_{w_f}^{(1)} = g_{w_f}^{(1)}(q_2, q_1) \]

\[ g_{w}^{(1)}(p_2, p_1) = g_{w}^{(1)}(p_2, p_1) + g_{w_f}^{(1)}(q_2, q_1), \]

\[ m_{e_{w_f}}^{(1)} = -m_{e_{w_f}}^{(1)}(q), \quad m_{e_{w}}^{(1)}(p) = m_{e_{w}}^{(1)}(p) + m_{e_{w_f}}^{(1)}(q). \]

(3.2.8)-(3.2.11) are the second sort of 1-loop corrections. (3.2.3)-(3.2.11) can be generalized to n-loop diagrams. Since there must be the two sorts of corrections simultaneously, both must simultaneously be considered and be approximated to the
same loop-number, i.e., after their integrands are added up, the corresponding integral is completed. Thus the integrands causing divergences will cancel each other out, consequently \( \langle F_f | S_f | F_i \rangle \) and \( \langle W_f | S_w | W_i \rangle \) will be all convergent, and it is unnecessary to introduce regularization and counterterms. For example, in the new QED, we have (1.1.1). From (3.2.7)-(3.2.11) we see that if let \( g_{fw}^{(1)} = g_{fw}^{(1)}(p_2, p_1) \), the total sum of corrections of the first and second sorts must be zero. This is equivalent to redefine the point with the momenta \( p_2 \) and \( p_1 \) as the subtraction point and \( g_f(p_2, p_1) \) is regarded as an accurate value. In fact, \( g_f(p_2, p_1) \) is unknown, and our aim is to evaluate an approximate \( g_f(p_2, p_1) \) from the accurate \( g_0(q_2, q_1) \) and \( m_{\text{eff}}(q_1) \). In meaning of perturbation theory, \( g_0(q_2, q_1) \) as a accurate value already contains \( g_{ff}^{(1)}(q_2, q_1) \) and \( g_{fw}^{(1)}(q_2, q_1) \), i.e., \( g_f^{(1)}(q_2, q_1) = g_{ff}^{(1)}(q_2, q_1) + g_{fw}^{(1)}(q_2, q_1) \).

From the F-W symmetry and (3.2.7)-(3.2.11) we can prove \( g_{ff}^{(1)}(q_2, q_1) = 0 \). Thus when \( g_{ff}^{(1)}(q_2, q_1) \) become \( g_{ff}^{(1)}(p_2, p_1) \), \( g_{fw}^{(1)}(q_2, q_1) \) appears. Analogous analysis holds water for \( m_{\text{eff}}^{(1)}(q) \), \( g_{\text{ew}}^{(1)}(q, q_1) \) and \( m_{\text{ew}}^{(1)}(q) \) and

\[
(3.2.12) \quad g_{\text{ew}}^{(1)}(q, q_1) = m_{\text{ew}}^{(1)}(q_1) = m_{\text{eff}}^{(1)}(q_1) = 0.
\]

For a free particle, \( p = q \) or \( k = q' \) and \( m_{\text{ew}}^{(1)}(q) = m_{\text{eff}}^{(1)}(q) = 0 \). When \( p \neq q \), \( m_{\text{eff}}^{(1)}(p) = m_{\text{ew}}^{(1)}(p) \neq 0 \). In this case \( m_{\text{eff}}^{(1)}(p) \) or \( m_{\text{ew}}^{(1)}(p) \) must join one or two vertexes, hence we can regard \( m_{\text{eff}}^{(1)}(p) \) or \( m_{\text{ew}}^{(1)}(p) \) as a part of corrections of the vertexes. We regard specially a correction to a propagator as the sum of partial corrections to the two vertexes joining the propagator. The correction to every vertex joining the propagator is the same, hence this correction is half of the correction to the propagator. On the other hand, we can also regard correction to an external line as a part correction to a vertex. We can prove that these results hold water for n-loop diagrams (see section 3.5). Thus, in present theory there is no correction to an external line and a propagator and there is only correction to a vertex.

As mentioned above, we see that \( \psi' \) and \( A'_\mu \) are the same as \( \psi \) and \( A_\mu \) in the conventional QED. Formulating the new QED by \( \psi' \), \( A'_\mu \), \( \psi' \) and \( A'_\mu \), the new QED will be the same as the conventional QED in form. The essential differences between the new QED and the conventional QED are that the Lagrangian density of the conventional QED is replaced by \( \mathcal{L} = \mathcal{L}_F + \mathcal{L}_W \) and \( \mathcal{L}_F \) and \( \mathcal{L}_W \) together determine correction to a scattering amplitude \( \langle F_f | S_f | F_i \rangle \) or \( \langle W_f | S_w | W_i \rangle \). Thus, though \( \mathcal{L}_F \) and \( \mathcal{L}_W \) are independent of each other in classical meanings, after quantization both are dependent on each other.

3.3. The first sort of 1-loop corrections.

3.3.1. The first sort of one-loop corrections originating from \( S_f(g_0, m_{\text{eff}}) \). By \( S_f(g_0, m_{\text{eff}}) \) \((2.5.3)\) we can evaluate the 1-loop correction to the scattering amplitude and the electromagnetic mass \( m_{\text{eff}} \) of a free F-electron corresponding to figure 3.1.1-2[8],

\[
A_{\text{mff}}^{(1)}(q_1) = \langle a_{q_2s_2} | S_f^{(1)}(g_0, m_{\text{eff}}) | a_{q_1s_1} \rangle = \delta^4(q_2 - q_1) \frac{1}{V} \pi_{q_2s_2} \Sigma_{ff}^{(1)}(q_1) u_{q_1s_1}, \tag{3.3.1}
\]
\[
\Sigma^{(1)}_{ff}(q_1) = g_0^2 \int \frac{d^4k}{i\varepsilon} \frac{-i}{k^2 - \varepsilon} r_\mu \frac{-i [m - i(q_1 - k) \gamma]}{(q_1 - k)^2 + m^2 - i\varepsilon} r_\mu
\]
\[
= -i (2\pi)^4 A + BS^{-1}_{Ff}(q_1) + S^{-1}_{Ff}(q_1) \Sigma^{(1)}_{fc}(q_1) S^{-1}_{Ff}(q_1), \quad (3.3.2)
\]
\[
A = \frac{\alpha}{2\pi} m \left( \frac{3}{2} D + \frac{9}{4} \right), \quad D = \frac{1}{i\pi^2} \int d^4k \frac{1}{(k^2 + m^2)^2}.
\]
\[
B = -\frac{\alpha}{4\pi} \left( D - 4 \int_0^1 \frac{dx}{x} + \frac{11}{2} \right),
\]
\[
\Pi_{q_1s_1} S^{-1}_{Ff}(q_1) = S^{-1}_{Ff}(q_1) u_{q_1s_1} = \Sigma^{(1)}_{fc}(q_1) = 0.
\]
From (3.2.4) we have
\[
m^{(1)}_{ef}(q) = \frac{\langle a_{q_1s_2} | S_f^{(1)}(g_0, m_0) | a_{q_1s_1} \rangle}{\langle a_{q_1s_2} | -i \int d^4x :\bar{\psi}\psi : | a_{q_1s_1} \rangle} = A
\]
where \( D \) is divergent and \( \alpha = g_0^2/4\pi \). The 1-loop correction of the F-electron propagator with its two vertices shown in figure 3.2.1 is
\[
(3.3.7) \quad (-g_0\gamma_\mu) S^{(1)}_{Ff}(p)(-g_0\gamma_\nu) = g_0^2 \gamma_\mu S_F(p) \Sigma^{(1)}_{f}(p) S_F(p) \gamma_\nu,
\]
where the factors \((2\pi)^4 \delta(p_2 - p + k_2)\) and \((2\pi)^4 \delta(p - p_1 - k_1)\) are ignored, \( \Sigma^{(1)}_{fc}(p) \) is finite, \( B \) is divergent, and \( S^{-1}_{Ff}(p) = i(2\pi)^4 (i\gamma + m) \). But it is not necessary to evaluate the integral, because the integrand of \( D \) will be cancelled out by the other integrand in the total scattering amplitude. Because a propagator must join its two vertices, (3.3.7) may also be regarded as the sum of corrections to the two vertices shown in figure 3.2.1. The contributions of the two diagrams are the same. Thus, we obtain the first sort of one-loop corrections \( \Lambda^{(1)}_{f1} \) to a F-vertex to be
\[
(3.3.8) \quad \Lambda^{(1)L}_{f1\mu}(p) \equiv -\frac{1}{2} g_0\gamma_\mu S_F(p) \Sigma^{(1)}_{f}(p),
\]
\[
(3.3.9) \quad \Lambda^{(1)R}_{f1\nu}(p) \equiv -\frac{1}{2} \Sigma^{(1)}_{f}(p) S_F(p) g_0\gamma_\nu.
\]
The diagrams corresponding to (3.3.8) and (3.3.9) are shown in figure 3.2.1L and 3.2.1R. We always regard \( \Lambda^{(1)L}_{f1\mu}(p) \) and \( \Lambda^{(1)R}_{f1\nu}(p) \) as a whole. Thus, in addition to the Feynman rules in the section 2.5, from (3.3.8), (3.3.9), figure 3.2.1L and figure 3.2.1R we obtain the first supplementary Feynman rule.

The first supplementary Feynman rule: There is no correction to propagator, there are only correction to mass of a free particle and to a vertex. The sort of corrections analogous to (3.3.8)-(3.3.9) is the first sort of corrections to a vertex. A correction to a propagator can be regarded as a sum of the corrections to two vertexes joining the propagator.

We will see the total correction to the mass of a free particle to be zero.
Operating $\Lambda^{(1)\mu}_{f\mu} (q)$ on $u_{q\mu}$ and $\Lambda^{(1)\nu}_{f\nu} (q)$ on $\overline{u}_{q\nu}$, considering (3.3.5), from (3.3.8) and (3.3.9) we have

\begin{equation}
\Lambda^{(1)\mu}_{f\mu} u_{q\mu} = -\frac{1}{2} g_0 \gamma_\mu \left[ -i (2\pi)^4 A S_{Ff} (q) + B \right] u_{q\mu},
\end{equation}

\begin{equation}
\overline{u}_{q\mu} \Lambda^{(1)\nu}_{f\nu} = -\overline{u}_{q\nu} \frac{1}{2} \left[ -i (2\pi)^4 A S_{Ff} (q) + B \right] g_0 \gamma_\nu.
\end{equation}

From (3.3.10) or (3.3.11) we see that the rule above naturally eliminates the ambiguity of corrections to external lines in the conventional QED\cite{8}.

Because of the gauge invariance of the Lagrangian density, there is no correction to the mass of a free F-photon or a W-photon. The one-loop contribution of the scattering operator $S_f (q_0, m_0)$ to the F-photon propagator with its two vertices can be represented by the Feynman diagram shown in figure 3.3.1-2 Considering the current to be consensual, by the Feynman rules we have\cite{8}

\begin{equation}
(-g_0 \gamma_\mu) D^{(1)}_{Ff\mu\nu} (k) (-g_0 \gamma_\nu) = g_0 \gamma_\mu D_{Ff} (k) \Pi^{(1)}_{f\mu\nu} (k) D_{Ff} (k) g_0 \gamma_\nu,
\end{equation}

\begin{equation}
\Pi^{(1)}_{f\mu\nu} (k) = g_0^2 \int d^4p Tr \frac{m - ip_\mu}{p^2 + m^2 - i \varepsilon} \frac{m - i(p - k) \gamma}{(p - k)^2 + m^2 - i \varepsilon} \left[ \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right] \left[ C D^{-1}_{Ff} (k) + \Pi^{(1)}_{fc} (k) D^{-2}_{Ff} (k) \right],
\end{equation}

where $D^{-1}_{Ff} (k) = i (2\pi)^4 k^2$,

\begin{equation}
C = -\frac{\alpha}{3\pi} \left( D + \frac{5}{6} \right).
\end{equation}

$C$ is logarithmically divergent, $\Pi^{(1)}_{fc} (k)$ is finite and $\Pi^{(1)}_{fc} (q') = 0$.

Analogously to discussion for (3.3.8) and (3.3.9), from (3.3.12) and (3.3.13) we obtain the second sort of correction $\Lambda^{(1)}_{f2\nu}$ to a F-vertex to be

\begin{equation}
\Lambda^{(1)}_{f2\nu} (k) = \frac{1}{2} g_0 \gamma_\mu D_{f2} (k) \Pi^{(1)}_{f\mu\nu} (k)
\end{equation}

\begin{equation}
= \frac{1}{2} g_0 \gamma_\mu \left[ \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right] \left[ C + \Pi^{(1)}_{fc} (k) D^{-1}_{Ff} (k) \right].
\end{equation}

Considering $q'^2 = 0$ for external photon lines, from (3.3.15) we obtain

\begin{equation}
\Lambda^{(1)}_{f2\nu} (q') \frac{1}{\sqrt{2m_q V}} e^\lambda_{q\mu} = \frac{1}{2} g_0 \gamma_\mu \delta_{\mu\nu} C \frac{1}{\sqrt{2m_q V}} e^\lambda_{q\nu}.
\end{equation}
From (3.3.16) and (3.2.3) we obtain the second sort of the one-loop corrections originating from $S_f^{(1)} (g_0, m_{c_0})$ to the coupling constant to be

$$g_{ff2}^{(1)} (p_2, p_1) = \frac{(2\pi)^4 \delta^4 (p_2 - k_1 - p_1) V^{-3/2} \mathbf{p}_{p_2} \Lambda_{ff2}^{(1)} (k) u_{p_1, s_1, c k_1, \lambda} (2\omega_k)^{-1/2}}{\langle a_{p_2} | \int d^4x : \bar{\psi} \gamma_\mu A_\mu \psi : | a_{p_1, s_1, c k_1, \lambda} \rangle} = \frac{1}{2} g_0 \left\{ C + \Pi_{ff}^{(1)} (k) D_{ff}^{-1} (k) \right\}, \quad (3.3.17)$$

where the numerator in (3.3.17) corresponds to Fig. 3.4.3A to be the one part of the scattering amplitude $\langle a_{p_2} | S_f^{(1)} (g_0, m_{c_0}) | a_{p_1, s_1, c k_1, \lambda} \rangle$. The first supplementary Feynman rule still applies to this case.

The third sort of one-loop contribution $A_{ff3}^{(1)}$ of the scattering operator $S_f (g_0, m_{c_0})$ to the scattering amplitude $A_{gf} (p_2, p_1)$ is represented by the Feynman diagram shown in figure 3.4.4.A. By the Feynman rules we have:

$$A_{gf}^{(1)} (p_2, p_1) = \langle a_{p_2} | S_f^{(1)} (g_0, m_{c_0}) | a_{p_1, s_1, c k_1, \lambda} \rangle = \frac{1}{2} g_0 \left\{ C + \Pi_{gf}^{(1)} (k) D_{gf}^{-1} (k) \right\}, \quad (3.3.19)$$

where $L = -B$, is logarithmically divergent, $\Lambda_{ff3}^{(1)} (p_2, p_1)$ is finite and $\Lambda_{ff3}^{(1)} (q_2, q_1) = 0$. From (3.3.19) and (3.2.3) we obtain the third sort of the one-loop corrections originating from $S_f^{(1)} (g_0, m_{c_0})$ to the coupling constant $g_{f0}$ to be

$$g_{ff3}^{(1)} (p_2, p_1) = \frac{ig_0^2}{(2\pi)^4} \int \frac{d^4k}{k^2 - i\varepsilon} \frac{\gamma_\lambda (p_2 - k)^\lambda}{(2\omega_k)^2 + m^2 - i\varepsilon} \gamma_\mu (p_1 - k)^\mu = L \gamma_\mu + \Lambda_{ff3c}^{(1)} (p_2, p_1), \quad (3.3.20)$$

where $L = -B$, is logarithmically divergent, $\Lambda_{ff3}^{(1)} (p_2, p_1)$ is finite and $\Lambda_{ff3c}^{(1)} (q_2, q_1) = 0$. From (3.3.19) and (3.2.3) we obtain the third sort of the one-loop corrections originating from $S_f^{(1)} (g_0, m_{c_0})$ to the coupling constant $g_{f0}$ to be

$$g_{ff3}^{(1)} (p_2, p_1) = g_0 \mathbf{p}_{p_2} \Lambda_{ff3c}^{(1)} (p_2, p_1) \gamma_\mu u_{p_1, s_1, c k_1, \lambda} + g_0 L, \quad (3.3.21)$$

$$g_{f3}^{(1)} (q_2, q_1) = g_0 L, \quad (3.3.22)$$

(3.3.1)-(3.3.7), (3.3.12)-(3.3.14) and (3.3.19)-(3.3.20) are the same as those of the conventional QED, respectively.

3.3.2. The first sort of one-loop corrections to $g_w$ and $m_w$. We can obtain the Feynman diagrams determined by $S_w (g_0, m_{c_0})$ corresponding to those determined by $S_f (g_0, m_{c_0})$, provided we transform real lines into the dotted lines corresponding to them. Analogously to computation of (3.3.1) -(3.3.22), and paying attention to the differences between the vertex factor, the propagators and the external line
Substituting from (3.3.26) photon propagator with its two vertices is

\[ A^{(1)}_{www} (q) = \langle a_{q_{2}q_{3}} | S^{(1)}_{w} (g_{0}, m_{e}) | a_{q_{1}q_{1}} \rangle = \delta^{4} (q_{2} - q_{1}) \frac{1}{V} \pi_{q_{2}q_{3}}^{(1)} \Sigma^{(1)}_{ww} (p) v_{q_{1}q_{1}}, \quad (3.3.23) \]

\[ \Sigma^{(1)}_{ww} (q_{1}) = \int d^{4}k \frac{i}{k^{2} + i \varepsilon} r_{\mu} \left( \frac{-i [m - i (q_{1} - k) \gamma]}{(q_{1} - k)^{2} + m^{2} + i \varepsilon} r_{\mu} \right) \]

\[ = -i (2\pi)^{4} A + BS^{-1}_{fw} (q_{1}) + S^{-1}_{fw} (q_{1}) \Sigma^{(1)}_{ww} (q_{1}) S^{-1}_{fw} (q_{1}) \]

\[ = \Sigma^{(1)}_{ff} (q_{1}), \quad (3.3.24) \]

\[ (3.3.25) \]

\[ \Sigma^{(1)}_{ww} (q_{1}) = \Sigma^{(1)}_{fc} (q_{1}) = 0, \quad S^{-1}_{fw} (q_{1}) = S^{-1}_{F} (q_{1}). \]

Substituting \( A^{(1)}_{www} (p_{1}) \) into (3.2.6), we obtain the one-loop correction originating from \( S_{w} (g_{0}, m_{e}) \) to the same of a free W-electron to be

\[ m^{(1)}_{ww} (q) = A. \]

The one-loop contribution for the W-electron propagator with its two vertices originating \( S_{w} (g_{0}, m_{e}) \) is

\[ g_{0} \gamma^{\mu} S^{(1)}_{fww} (p) g_{0} \gamma^{\nu} = g_{0} \gamma^{\mu} S^{(1)}_{fw} (p) \Sigma^{(1)}_{ww} (p) S^{(1)}_{fw} (p) \gamma^{\nu}. \]

From (3.3.27) we obtain the first sort of one-loop corrections to a W-vertex to be

\[ \Lambda^{(1)L}_{ww1\mu} = \frac{1}{2} g_{0} \gamma^{\mu} S^{(1)}_{fw} (p) \Sigma^{(1)}_{ww} (p) = -\Lambda^{(1)L}_{ff1\mu}, \quad (3.3.28) \]

\[ \Lambda^{(1)R}_{ww1\nu} = \frac{1}{2} \Sigma^{(1)}_{ww} (p) S^{(1)}_{fw} (p) g_{0} \gamma^{\nu} = -\Lambda^{(1)R}_{ff1\nu}. \quad (3.3.29) \]

From (3.3.28) – (3.3.29) we have

\[ (3.3.30) \]

\[ \Lambda^{(1)L}_{ww1\mu} (q) v_{q^{*}} \frac{1}{\sqrt{V}} = \frac{1}{2} g_{0} \gamma^{\mu} [-i (2\pi)^{4} A S^{(1)}_{fw} (p) + B] v_{q^{*}} \frac{1}{\sqrt{V}}, \]

\[ (3.3.31) \]

\[ \frac{1}{\sqrt{V}} \pi_{q^{*}f} \Lambda^{(1)R}_{ww1\nu} (q) = \frac{1}{\sqrt{V}} \pi_{q^{*}f} \frac{1}{2} [-i (2\pi)^{4} A S^{(1)}_{fw} (p) + B] g_{0} \gamma^{\nu}. \]

The one-loop contribution of the scattering operator \( S_{w} (g_{0}, m_{e}) \) for the W-photon propagator with its two vertices is

\[ (3.3.32) \]

\[ g_{0} \gamma^{\mu} D^{(1)}_{FW\mu} (k) g_{0} \gamma^{\nu} = g_{0} \gamma^{\mu} D^{(1)}_{FW} (k) \Pi^{(1)}_{w\mu} (k) D^{(1)}_{FW} (k) g_{0} \gamma^{\nu}, \]
\[
\Pi_{\omega \mu \nu}^{(1)} (k) = g_0^2 \int d^4 p Tr \left\{ r_\mu \frac{m - ip}{p^2 + m^2 + i \varepsilon} r_\nu \frac{m - i(p - k)}{(p - k)^2 + m^2 + i \varepsilon} \right\} \\
= \left( \delta_{\mu \nu} - \frac{k_\mu k_\nu}{k^2} \right) \left[ C D_{Fw}^{-1} (k) + \Pi_{\omega c}^{(1)} (k) D_{Fw}^{-2} (k) \right] \\
= -\Pi_{\mu \nu}^{(1)} (k),
\]

(3.3.33)

Analogously to (3.3.15), from (3.3.32) we obtain the second sort of corrections \( \Lambda_{\omega w 2}^{(1)} \) for a W-vertex to be

\[
\Lambda_{\omega w 2}^{(1)} (k) = \frac{1}{2} g_0 \gamma \mu D_{Fw} (k) \Pi_{\omega \mu \nu}^{(1)} (k) \\
= \frac{1}{2} g_0 \gamma \mu \left( \delta_{\mu \nu} - \frac{k_\mu k_\nu}{k^2} \right) \left[ C + \Pi_{\omega c}^{(1)} (k) D_{Fw}^{-1} (k) \right].
\]

(3.3.35)

Considering \( q^2 = 0 \) for external photon lines, from (3.3.35) we have

\[
\Lambda_{\omega 2 \mu}^{(1)} (q') = \frac{1}{\sqrt{2 q' q}} q'^\mu = \frac{1}{2} g_0 \gamma \mu \left( \delta_{\mu \nu} - \frac{q_\mu q_\nu}{q^2} \right) C \frac{1}{\sqrt{2 q' q}} q'^\nu.
\]

From (3.3.35) and (3.2.5) we obtain the second sort of the one-loop corrections originating from \( S_w^{(1)} (g_0, m_\omega) \) to the coupling constant to be

\[
g_{\omega w 2}^{(1)} (p_2, p_1) = \frac{(2 \pi)^4 \delta^4 (p_2 - k_1 - p_1) V^{-3/2} \mathcal{F}_{p_2 s_2} \Lambda_{\omega w 2}^{(1)} (k) \psi_{p_1 s_1} r_{s_1} (2 \omega k_1)^{-1/2}}{\gamma \mu \omega \mu \left\{ \mathcal{F}_{p_2 s_2} \Lambda_{\omega 2 \mu}^{(1)} (k) D_{Fw}^{-1} (k) \right\}} \\
= \frac{1}{2} g_0 \left\{ C + \Pi_{\omega c}^{(1)} (k) D_{Fw}^{-1} (k) \right\}.
\]

(3.3.37)

\[
g_{\omega w 2}^{(1)} (q_2, q_1) = \frac{1}{2} g_0 C.
\]

Let \( A_{\omega w 3}^{(1)} (p_2, p_1) \) be the third sort of one-loop contributions originating from \( S_w (g_0, m_\omega) \) for the scattering amplitude \( A_{\omega w 3}^{(1)} (p_2, p_1) \). Analogously to \( A_{\omega f 3}^{(1)} (p_2, p_1) \), by the Feynman rules we have

\[
A_{\omega w 3}^{(1)} (p_2, p_1) = \left\langle \mathcal{F}_{p_2 s_2} \left| S_w^{(1)} (g_0, m_\omega) \right| \mathcal{F}_{p_1 s_1} \right\rangle \\
= g_0 \frac{(2 \pi)^4 \delta^4 (p_2 - p_1 - k)}{V} \frac{1}{\sqrt{2 p_2 \cdot k}} r_{s_2} (p_2, p_1) \psi_{p_1 s_1} \frac{1}{\sqrt{2 k_1 \cdot k}} r_{s_1} \mathcal{F}_{k_1 \lambda}.
\]

(3.3.39)
From (3.3.41) we can derive the Feynman rules for a circle as follows.

The second sort of corrections is easily evaluated by the first sort of corrections. For example, from (3.2.8)-(3.2.11), we have

\[ g^{(1)}_{w_3w_1}(p_2,p_1) = g_0 L_{\mu,w_3w_1}^{(1)}(p_2,p_1) = g_0 \left( \gamma_\mu \right)_{p_2p_1} + g_0 L, \]

(3.4.2)

\[ g^{(1)}_{w_3w_1}(q_2,q_1) = g_0 L. \]

3. The second sort of corrections directly corrects the coupling constants and the masses, hence it is equivalent corrections to a vertex. We represent it by a circle. From (3.2.8), (3.2.10), (3.3.18), (3.3.22), (3.3.38) and (3.3.42), we can derive

\[ g^{(1)}_{w_2}(q_2,q_1) = -g_{w_2}(q_2,q_1) = -g^{(1)}_{w_2}(q_2,q_1), \]

(3.4.3)

\[ g^{(1)}_{w_3}(q_2,q_1) = -g^{(1)}_{w_3}(q_2,q_1) = -g^{(1)}_{w_3}(q_2,q_1), \]

\[ g^{(1)}_{w_4}(q_2,q_1) = -g^{(1)}_{w_4}(q_2,q_1) = -g^{(1)}_{w_4}(q_2,q_1). \]

The second supplementary Feynman rule:

A. A circle is equivalent to a vertex for the part outside the circle.

B. The momenta of the external lines of the Feynman diagram inside the circle are the momenta \( q_2, q_1 \) and \( q' \) at the subtraction point.

C. The dotted-line circle with two speckles denotes the coefficient

\[ A^{-1}_{\mu,v_1} \left( \bar{w}_{\mu_{1v_1}} \right) = \left( \bar{w}_{\mu_{2v_2}} \right) \quad -i \int d^4x : \bar{w}_a w'_a : \left( \bar{w}_{\mu_{1v_1}} \right)^{-1}. \]
the transition amplitude inside this dotted-line circle is

\[
(3.4.5) \quad \langle \Psi_{q_2s_2} | S_w^{(1)}(g_0, m_0) | \Psi_{q_1s_1} \rangle.
\]

From (3.4.4) and (3.4.5) we derive the factor \( m_{cfw}^{(1)}(q) \) corresponding to the dotted-line in figure 3.1.2, figure 3.2.2, figure 3.4.1.B and figure 3.4.2.B to be \(-A\) (see (3.4.1)).

The real-line circle with two speckles denotes the coefficient

\[
(3.4.6) \quad A'_{mf}^{-1}(\Psi_{q_{1s_1}} \rightarrow \Psi_{q_{2s_2}}) = \langle \Psi_{q_2s_2} | -i \int d^4x : \overline{\psi'} \gamma^\mu A'_{\mu} \psi : | \Psi_{q_{1s_1}} \rangle^{-1},
\]

the transition amplitude inside this real-line circle is

\[
(3.4.7) \quad \langle \Psi_{q_2s_2} | S_f^{(1)}(g_0, m_0) | \Psi_{q_{1s_1}} \rangle.
\]

From (3.4.6) and (3.4.7) we derive the factor \( m_{vw}^{(1)}(q) \) corresponding to the real-line circle in figure 3.2.4, figure 3.4.1.D and figure 3.4.2.D to be \(-A\) (see (3.4.1)).

The dotted-line circle with two speckles denotes the coefficient

\[
A'_{yw}^{-1}(\Psi_{q_{1s_1}, c_{q'\lambda}} \rightarrow \Psi_{q_2s_2})
\]

\[
= \{ \langle \Psi_{q_2s_2} | \int d^4x : \overline{\psi} \gamma^\mu A'_{\mu} \psi : | \Psi_{q_{1s_1}, c_{q'\lambda}} \rangle \}^{-1},
\]

the transition amplitude inside this dotted-line circle is

\[
(3.4.9) \quad \langle \Psi_{q_2s_2} | S_w^{(1)}(g_0, m_0) | \Psi_{q_{1s_1}, c_{q'\lambda}} \rangle.
\]

From (3.4.8) and (3.4.9) we derive the factor \( g_{fw,2}^{(1)}(q_2, q_1) \) corresponding to the dotted-line in figure 3.4.3.B to be \(-\frac{1}{2}g_0C\) (see (3.4.2)) and the factor \( g_{fw,3}^{(1)}(q_2, q_1) \) corresponding to the dotted-line in figure 3.4.4.B to be \(-g_0L\) (see (3.4.3)).

The real-line circle with three speckles denotes the coefficient

\[
A'_{gf}^{-1}(\Psi_{q_{1s_1}, c_{q'\lambda}} \rightarrow \Psi_{q_{2s_2}})
\]

\[
= \{ \langle \Psi_{q_{2s_2}} | \int d^4x : -\overline{\psi} \gamma^\mu A'_{\mu} \psi : | \Psi_{q_{1s_1}, c_{q'\lambda}} \rangle \}^{-1},
\]

the transition amplitude inside this real-line circle is

\[
(3.4.10) \quad \langle \Psi_{q_{2s_2}} | S_f^{(1)}(g_0, m_0) | \Psi_{q_{1s_1}, c_{q'\lambda}} \rangle.
\]

From (3.4.10) and (3.4.11) we derive the factor \( g_{w,2}^{(1)}(q_2, q_1) \) to be \(-\frac{1}{2}g_0C\) (see (3.4.2)) and the factor \( g_{w,3}^{(1)}(q_2, q_1) \) to be \(-g_0L\) (see (3.4.3)).

4. The first supplementary Feynman rule still applies to the second sort of corrections. In this case, an explanation for the rule is shown by the figures 3.2.2 and 3.3.2. The examples are the figures 3.4.1.B, 3.4.1.D and 3.2.4.

Because the two sorts of corrections must simultaneously exist, both must simultaneously be considered, and both should be corrected to the same loop-number. For the reason, we define ‘whole Feynman diagram group with n-loop’.

If one group of Feynman diagrams corresponding to the same initial state and final state contain complete diagrams with n-loop coming from \( S_w^{(1)}(g_0, m_{e0}) \) and \( S_f^{(1)}(g_0, m_{e0}) \), the group of Feynman diagrams is called ‘whole Feynman diagram group with n-loop’ (WFDGNL).
For example, the figures 3.1.1-2; the figures 3.4.1.A-D, the figures 3.4.2.A-D, the figures 3.4.3.A-B and the figures 3.4.4.A-B; or the figures 3.2.1-4, the figures 3.3.1-2 are respectively a WFDG1L.

Taking the figures 3.4.1.A-D as an example, we explain construct of a WFDG1L. Because there is the contribution $g^{(1)}_1$ for $g^{(1)}_f$, there must be the contribution $g^{(1)}_{fw1}$ of the figure 3.4.1 C. Because the contribution $g^{(1)}_f$ originates from $m^{(1)}_{eff}$, there must be the contribution originates from $m^{(1)}_{ewfw}$ which is shown in the figure 3.4.1.B. Because the contribution $g^{(1)}_{fw}$ originates from $m^{(1)}_{ewfw}$, there must be the contribution originates from $m^{(1)}_{eff}$ which is shown in the figure 3.4.1D. By such method we easily construct a WFDG1L. Figure 3.4.1A-D contain the complete 1-loop corrections to a F-vertex derived from figure 3.4.1.A, hence the figures 3.4.1.A-D form a WFDG1L. Similarly, we can obtain the WFDG1Ls as shown by the figures 3.4.2, 3.4.3 and 3.4.4. Thus, we see that because $S_w(g_0, m_{e0})$ and $S_f(g_0, m_{e0})$ together determine 1-loop corrections to a F- or W-amplitude, for an arbitrary scattering amplitude, there must be a WFDGNL which determines the total n-loop correction to the scattering amplitude. From this we obtain the following Feynman rule.

The third supplementary Feynman rule:

For an arbitrary scattering amplitude, we should firstly determine a WFDGNL corresponding to the amplitude, then sum up the integrands of the WFDGNL, last carry out the integral.

The Feynman rules in the second chapter and the three supplementary Feynman rules fully determine the method evaluating a scattering amplitude by $S_w$ and $S_f$.

The properties of a WFDGNL are as follows.

1. The integrands causing divergence in a WFDGNL must cancel each other, thereby the Feynman integral of the WFDGNL must convergent. For example, the correction to $m_{e0}$ coming from the WFDG1L composed of figure 3.1.1-2 is (1.1.1); from the Feynman rules, (3.3.17), (3.4.2) we obtain the second sort of correction $g^{(1)}_f$ to $g^{(0)}_f$ coming from the WFDG1L Fig. 3.4.3.A-B is

$$g^{(1)}_f = g^{(1)}_{f1}(p_2, p_1) + g^{(1)}_{fw2}(q_2, q_1)$$

$$= \frac{1}{2} g_0 \left[ C + \Pi^{(1)}_{wc}(k) D_{fw}^{-1}(k) \right] - \frac{1}{2} g_0 C$$

$$= \frac{1}{2} g_0 \Pi^{(1)}_{wc}(k) D_{fw}^{-1}(k), \quad k = p_2 - p_1, \quad (3.4.12)$$

from (3.3.21) and (3.4.3) we obtain the third sort of correction $g^{(1)}_f$ to $g^{(0)}_f$ coming from the WFDG1L figure 3.4.4.A-B is

$$g^{(1)}_{f3}(p_2, p_1) = g^{(1)}_{f3}(p_2, p_1) + g^{(1)}_{fw3}(q_2, q_1)$$

$$= \frac{\Pi^{(1)}_{p_{2s2} A_{\nu,ff3c}} p_{2s2}}{\Pi^{(1)}_{p_{2s2} A_{\nu,ff3c}} p_{2s2}} (p_2, p_1) \gamma^\nu u_{p_{1s1}} e^{A_{\nu,ff3c}}_{k_{1\mu}} + g_0 L - g_0 L$$

$$= \frac{\Pi^{(1)}_{p_{2s2} A_{\nu,ff3c}} p_{2s2}}{\Pi^{(1)}_{p_{2s2} A_{\nu,ff3c}} p_{2s2}} (p_2, p_1) \gamma^\nu u_{p_{1s1}} e^{A_{\nu,ff3c}}_{k_{1\mu}}, \quad (3.4.13)$$
3.4.2. The total one-loop corrections to $g_{f0}$. By the Feynman rules we easily obtain the corrections to $g_{f0}$. From (3.4.1) and (3.3.2) we obtain the sum of the two amplitudes $A_{g_{fAB}}^{(1)}$ corresponding to figure 3.4.1-A-B is

$$
A_{g_{fAB}}^{(1)} = (2\pi)^4 \delta^4(p_2 - p_1 - k) \frac{1}{\sqrt{2\omega_k}} \bar{\pi}_{p_{2}p_{2}^{*}} \cdot \left\{ \frac{1}{2} \Sigma^{(1)}_f (p_2) S_{F_f} (p_2) (-g_0) \gamma_{\nu} \right\} u_{p_1,s_1} \frac{1}{\sqrt{2\omega_k}} \epsilon^\lambda_{\nu \nu},
$$

and

$$
= (2\pi)^4 \delta^4(p_2 - p_1 - k) \frac{1}{\sqrt{2\omega_k}} \bar{\pi}_{p_{2}p_{2}^{*}} \cdot \left\{ \frac{1}{2} \left( B + S_{F_f}^{-1} (p_2) \Sigma^{(1)}_{f_c} (p_2) \right)(-g_0) \gamma_{\nu} u_{p_1,s_1} \frac{1}{\sqrt{2\omega_k}} \epsilon^\lambda_{\nu \nu}. \right. \tag{3.14}
$$

From (3.4.1), (3.3.31) and the Feynman rules (3.4.6)-(3.4.9) we obtain the sum of the two amplitudes $A_{g_{fCD}}^{(1)}$ corresponding to figure 3.4.1-C-D is

$$
A_{g_{fCD}}^{(1)} = (p_{2}^{*}p_{2}) \left| -g_{f_{w1}}^{(1)}(q_2, q_1) \right| \int d^4x : \bar{\psi} \gamma_{\nu} A_{\nu} \psi' : \left| a_{p_1,s_1} c_{kl} \right) \right.

= (2\pi)^4 \delta^4(p_2 - p_1 - k) \frac{1}{\sqrt{2\omega_k}} \bar{\pi}_{p_{2}p_{2}^{*}} \cdot \left\{ \frac{1}{2} B g_0 \gamma_{\nu} u_{p_1,s_1} \frac{1}{\sqrt{2\omega_k}} \epsilon^\lambda_{\nu \nu}. \right. \tag{3.15}
$$

where

$$
g_{f_{w1}}^{(1)}(q_2, q_1) = -\left\{ \left| a_{q_{2}s_2} \right| \left| S_{w1}^{(1)} (g_0, m_{e0}) \right. \right.

+ \int d^4x : \bar{\psi} m_{w_f}^{(1)}(q_2) S_{F_w}(q_2) \gamma_{\nu} g_0 A_{\nu} \psi' : \left| a_{q_{1}s_1} l_{q_1} \right) \right\}

\cdot \left\{ \left| a_{q_{2}s_2} \right| \int d^4x : \bar{\psi} \gamma_{\nu} A_{\nu} \psi' : \left| a_{q_{1}s_1} l_{q_1} \right) \right\}^{-1}

= \frac{-1}{2} B \tag{3.16}
$$

From (3.14) and (3.15) we obtain the transition amplitude $A_{g_{1R}}^{(1)}$ corresponding to the WFDG1L figure 3.4.1-A-D to be

$$
A_{g_{1R}}^{(1)} = A_{g_{f1AB}}^{(1)} + A_{g_{f1CD}}^{(1)}

= \frac{1}{2} (2\pi)^4 \delta^4(p_2 - p_1 - k) \frac{1}{\sqrt{2\omega_k}} \bar{\pi}_{p_{2}p_{2}^{*}} S_{F_f}^{-1} (p_2) \Sigma^{(1)}_{f_c} (p_2) g_0 \gamma_{\nu} u_{p_1,s_1} \frac{1}{\sqrt{2\omega_k}} \epsilon_{\nu \nu}. \tag{3.17}
$$
\( A^{(1)}_{g_{f1L}} \) is convergent. Similarly, for the WFDG1L composed of figure 3.4.2.A-D we have

\[
A^{(1)}_{g_{f2AB}} = A^{(1)}_{g_{f2CD}} = \frac{1}{2} (2\pi)^4 \delta^4 (p_2 - p_1 - k) \\
\cdot \frac{1}{V} \pi p_{2s} g \gamma_{\nu} \Sigma^{(1)}_{f c} (p_1) S^{-1}_{f f} (p_1) u_{p_1, s_1} \frac{1}{\sqrt{2} e_{f, k \nu}}.
\] (3.4.18)

From (3.4.17), (3.4.18) we obtain the first sort of corrections \( g^{(1)}_{f1} (p_2, p_1) \) to \( g_f (q_2, q_1) \) to be

\[
g^{(1)}_{f1} (p_2, p_1) = \frac{A^{(1)}_{g_{f1R}} + A^{(1)}_{g_{f1L}}}{\{ a_{p_2, s_2} | - \int d^4 x : \tilde{\psi} \gamma_{\nu} \psi' : | a_{p_1, s_1} e_{f, k \nu} \}} \\
= \frac{g_0}{2} \pi p_{2s} \{ \gamma_{\nu} \Sigma^{(1)}_{f c} (p_1) S^{-1}_{f f} (p_1) + S^{-1}_{f f} (p_2) \gamma_{\nu} \} u_{p_1, s_1} e_{f, k \nu} \\
\cdot (\pi p_{2s} \gamma_{\nu} u_{p_1, s_1} e_{f, k \nu})^{-1}.
\] (3.4.19)

Feynman diagrams corresponding to any process must be composed of some WFDGNLs. Sum of WFDGNLs must be convergent. The sum of WFDGNLs 3.4.1-4 is the total one-loop correction \( g^{(1)}_f (p_2, p_1) \) to \( g_f (q_2, q_1) \).

\[
g^{(1)}_f (p_2, p_1) = g^{(1)}_{f1} (p_2, p_1) + g^{(1)}_{f2} (p_2, p_1) + g^{(1)}_{f3} (p_2, p_1).
\] (3.4.20)

From (3.4.12), (3.4.13) and (3.4.19) we see

\[
g^{(1)}_f (p_2, p_1) \text{ is finite}, \quad g^{(1)}_f (q_2, q_1) = 0.
\] (3.4.21)

Substituting (3.4.20) into (3.2.1), we have

\[
g^{(1)-\text{loop}}_f (p_2, p_1) = g^{(1)}_0 + g^{(1)}_f (p_2, p_1), \quad g^{(1)-\text{loop}}_f (q_2, q_1) = g_0.
\] (3.4.22)

Analogously (3.4.22), we can prove

\[
g^{(1)-\text{loop}}_w (p_2, p_1) = g^{(1)}_0 + g^{(1)}_w (p_2, p_1) = g^{(1)-\text{loop}}_f (p_2, p_1) \equiv g^{(1)-\text{loop}}_f (p_2, p_1),
\]

\[
g^{(1)-\text{loop}}_w (q_2, q_1) = g_0.
\] (3.4.23)

From (3.4.1) and (3.2.1) we obtain the total 1-loop correction to \( m_{e0} \) to be

\[
m^{(1)}_e (q) = m^{(1)}_{e f} (q) + m^{(1)}_{e w} (q) = m^{(1)}_{e w} (q) + m^{(1)}_{e w} (q) = m^{(1)}_{e w} (q) \equiv m^{(1)}_{e w} (q) = 0,
\]

\[
m^{(1)-\text{loop}}_e (q) = m_{e0} + m^{(1)}_{e f} (q) = m_{e0} + m^{(1)}_{e w} (q) = m^{(1)-\text{loop}}_e (q) \equiv m^{(1)-\text{loop}}_e (q) = 0.
\] (3.4.24)

Replacing \( g_0 \) by \( g^{(1)-\text{loop}}_f (p_2, p_1) \) and \( m_{e0} \) by \( m^{(1)-\text{loop}}_e (q) \) in \( S_f (g_0, m_{e0}) \) and \( S_w (g_0, m_{e0}) \), we obtain

\[
S^{(1)-\text{loop}}_f \equiv S_f \left( g^{(1)-\text{loop}}_f (p_2, p_1), m^{(1)-\text{loop}}_e (q) \right),
\] (3.4.26)

\[
S^{(1)-\text{loop}}_w \equiv S_w \left( g^{(1)-\text{loop}}_f (p_2, p_1), m^{(1)-\text{loop}}_e (q) \right).
\] (3.4.27)
Because only when \( p = q \), i.e. for a free F- or W-electron, the definitions of the masses (2.3.17) and (2.3.20) are meaningful, we take \( p = q \) in \( m^{(1\text{-loop})} \). We can prove

\[
m^{(n\text{-loop})}_e = m^{(n\text{-loop})}_e(q) = 0
\]

and when \( p \neq q \), \( m_e(p) \) must be attributed to correction of a vertex, in the perturbation theory, we may consider only corrections to a vertex in \( S^{(1\text{-loop})}_f \) or \( S^{(1\text{-loop})}_w \).

All scattering amplitudes corrected to 1-loop diagrams evaluated by (3.4.26) or (3.4.27) are finite. From the deductive process above we see that in contrast with the conventional QED, there is only one sort of parameters (mass and charge at the subtraction point) which are finite and measurable, there are the two sorts of corrections originating \( S_f \) and \( S_w \), all Feynman integrals are covergent and it is unnecessary to introduce regularization and counterterms in the present theory. It is easily seen that the one-loop corrections derived by the present theory are the same as those derived by the given QED.

According to the present theory, there is no correction to any external line, corrections to the mass of a free particle must be zero, and corrections to a propagator can be attributed to correction to the two vertexes joining it, hence we may consider only one sort of corrections, i.e., the sort of correction to a vertex.

### 3.5. \( n\text{-loop} \) corrections for the coupling constants and the masses.

The method deriving

\[
S^{(1\text{-loop})}_f = S_f \left( g^{(1\text{-loop})}_f, m^{(1\text{-loop})}_f \right) \quad \text{and} \quad S^{(1\text{-loop})}_w = S_w \left( g^{(1\text{-loop})}_f, m^{(1\text{-loop})}_w \right)
\]

from \( S_f \) \((g_0, m_{e0})\) and \( S_w \) \((g_0, m_{e0})\) can be generalized to derive \( S_f^{(n\text{-loop})} \) and \( S_w^{(n\text{-loop})} \) from \( S_f^{(1\text{-loop})} \) and \( S_w^{(1\text{-loop})} \). The recursion method is as follows.

Assume \( S_f \left( g^{(n\text{-loop})}_f, m^{(n\text{-loop})}_e \right) \) and \( S_w \left( g^{(n\text{-loop})}_f, m^{(n\text{-loop})}_e \right) \) to be given and

\[
g^{(n\text{-loop})}_f(p_2, p_1) = g^{(n\text{-loop})}_w(p_2, p_1) = g^{(n\text{-loop})}_f(p_2, p_1) = g^{(n\text{-loop})}_w(p_2, p_1) = 0 + \sum_{i=1}^{n} g^{(i)}(p_2, p_1),
\]

\[
g^{(i)}_f(p_2, p_1) = g^{(i)}_w(p_2, p_1), \quad g^{(n\text{-loop})}_f(q_2, q_1) = 0,
\]

\[
m^{(n\text{-loop})}_e = m^{(n\text{-loop})}_e(q) = \sum_{i=1}^{n} m^{(i)}_e(q) = 0,
\]

\( g^{(i)} \) and \( m^{(i)}_e \) denote \( i\text{-loop} \) corrections to \( g_0 \) and \( m_{e0} \), respectively. We can evaluate \( m^{(n+1)}_e(p) \) and \( m^{(n+1)}_e(q) \) by virtue of \( S_f^{(n\text{-loop})} \) and \( m^{(n+1)}_e(p) \) and \( m^{(n+1)}_e(q) \) by virtue of \( S_w^{(n\text{-loop})} \). It can be proven

\[
m^{(n+1)}_e(p) + m^{(n+1)}_e(q) = m^{(n+1)}_e(p) + m^{(n+1)}_e(q) = m^{(n+1)}_e(p) + m^{(n+1)}_e(q),
\]

\[
m^{(n+1)}_e(q) = 0, \quad m^{(n+1)}_e(q) = 0.
\]
Replacing \( m^{(n-loop)}_e \) in \( S_f^{(n-loop)} \) and \( S_w^{(n-loop)} \) by \( m_e^{(n+1-loop)} \), we obtain

\[
S_f^{(n+1-loop)} = S_f \left( g^{(n-loop)}, m_e^{(n+1-loop)} \right),
\]

(3.5.2)

\[
S_w^{(n+1-loop)} = S_w \left( g^{(n-loop)}, m_e^{(n+1-loop)} \right).
\]

(3.5.3)

We can evaluate \( g_{ff}^{(n+1)}(p_2,p_1) \) and \( g_{ww}^{(n+1)}(q_2,q_1) \) by \( S_f^{(n+1-loop)} \), and \( g_{ww}^{(n+1)}(p_2,p_1) \) and \( g_{fw}^{(n+1)}(q_2,q_1) \) by \( S_w^{(n+1-loop)} \). It can be proven

\[
g_{ff}^{(n+1)}(p_2,p_1) = g_{ff}^{(n+1)} \left( p_2,p_1 \right) + g_{ww}^{(n+1)} \left( p_2,p_1 \right)
\]

\[
g_{ww}^{(n+1)}(p_2,p_1) = g_{ww}^{(n+1)} \left( p_2,p_1 \right) + g_{fw}^{(n+1)} \left( p_2,p_1 \right)
\]

\[
g_{fw}^{(n+1)}(q_2,q_1) = 0,
\]

(3.5.4)

Replacing \( g^{(n-loop)} \) and \( m_e^{(n-loop)} \) in \( S_f^{(n-loop)} \) and \( S_w^{(n-loop)} \) by \( g^{(n+1-loop)} \) and \( m_e^{(n+1-loop)} \), we obtain

\[
S_f^{(n+1-loop)} = S_f \left( g^{(n+1-loop)}, m_e^{(n+1-loop)} \right),
\]

(3.5.5)

\[
S_w^{(n+1-loop)} = S_w \left( g^{(n+1-loop)}, m_e^{(n+1-loop)} \right).
\]

(3.5.6)

\( g^{(n+1)}(p_2,p_1) \) is finite.

It must be noted that all \( m_{fw}^{(n+1)} \), \( m_{ww}^{(n+1)} \), \( g_{ww}^{(n+1)} \), \( g_{fw}^{(n+1)} \) are independent of momenta \( p_2, p_1 \) and \( k \), since they are defined at the subtraction point. All scattering amplitudes approximate to \((n+1)\)-loop diagrams evaluated by virtue of \( S_f^{((n+1)-loop)} \) and \( S_w^{((n+1)-loop)} \) are finite. Detailed discussion will be given in another paper.

4. **Generalize to the SU(2) × U(1) electroweak unified model and interaction between W-matter and F-matter**

4.1. **Generalize to the SU(2)×U(1) electroweak unified model.** On the basis of the idea we can construct a left-right symmetric electroweak unified model. Taking the lepton part of the G-W-S model as example, the corresponding Lagrangian density will become

\[
\mathcal{L} = \mathcal{L}_F + \mathcal{L}_W - V_F - V_W,
\]

(4.1.1)
\[ L_F = -\bar{\psi}_L \gamma_\mu (\partial_\mu - i \cdot G_F \cdot \tau^j_{\mu} - \frac{i}{2} \cdot G'_F \cdot B_\mu) \cdot \psi_L \\
-\frac{1}{4} F^j_{\mu\nu} \cdot F^j_{\mu\nu} - \frac{1}{4} F_{\mu\nu} \cdot F_{\mu\nu} \\
- (\partial_\mu \phi^+ + i \phi^+ \cdot G_F \cdot \tau^j_{\mu} A^j_{\mu} + \frac{i}{2} \phi^+ \cdot G'_F \cdot B_\mu) \\
\cdot (\partial_\mu - i \cdot G_F \cdot \tau^j_{\mu} A^j_{\mu} - \frac{i}{2} \cdot G'_F \cdot B_\mu) \phi \\
- (\tau_R \cdot M_F \cdot \phi^+ \bar{\psi}_L + \bar{\psi}_L \phi \cdot M_F \cdot e_R), \quad (4.1.2) \]

\[ V_F = -\phi^+ \cdot \Omega_F \cdot \phi + \frac{1}{4} \phi^+ \cdot \Lambda_F \cdot \phi^+ \phi, \]

\[ L_W = -\bar{\psi}_R \gamma_\mu (\partial_\mu + i \cdot G_W \cdot \tau^j_{\mu} - \frac{i}{2} \cdot G'_W \cdot B_\mu) \cdot \psi_R \\
-\frac{1}{4} F^j_{\mu\nu} \cdot F^j_{\mu\nu} - \frac{1}{4} F_{\mu\nu} \cdot F_{\mu\nu} \\
- (\partial_\mu \phi^+ + i \phi^+ \cdot G_W \cdot \tau^j_{\mu} A^j_{\mu} + \frac{i}{2} \phi^+ \cdot G'_W \cdot B_\mu) \\
\cdot (\partial_\mu - i \cdot G_W \cdot \tau^j_{\mu} A^j_{\mu} - \frac{i}{2} \cdot G'_W \cdot B_\mu) \phi \\
+ (\tau_L \cdot M_W \cdot \phi^+ \bar{\psi}_R + \bar{\psi}_R \phi \cdot M_W \cdot e_L), \quad (4.1.4) \]

\[ V_W = -\phi^+ \cdot \Omega_W \cdot \phi + \frac{1}{4} \phi^+ \cdot \Lambda_W \cdot \phi^+ \phi \]

where \( G_F, G'_F, M_F, \Omega_F, A_F, G_W, G'_W, M_W, \Omega_W \) and \( \Lambda_W \) are all operators,

\[ \psi_L = \begin{pmatrix} \psi_e \\ e_R \end{pmatrix}, \quad \phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}, \quad (4.1.6) \]

\[ F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + \varepsilon_{ijk} \cdot G_F \cdot A^j_\mu A^k_\nu, \quad (4.1.7) \]

\[ F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \quad (4.1.8) \]

\[ \psi_R = \begin{pmatrix} \psi_e \\ e_L \end{pmatrix}, \quad \phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}, \quad (4.1.9) \]

\[ F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + \varepsilon_{ijk} \cdot G_W \cdot A^j_\mu A^k_\nu, \quad (4.1.10) \]

\[ F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \quad (4.1.11) \]
4.1.12 \[ \langle \phi \rangle_0 = \left( \begin{array}{c} 0 \\ \mu \end{array} \right), \quad \langle \phi \rangle_0 = \left( \begin{array}{c} 0 \\ \nu \end{array} \right). \]

Analogously to the new QED, we can prove for the electroweak unified model that all Feynman integrals are convergent, it is not necessary to introduce counterterms and regularization, there is only one sort of parameters which are all finite and measurable (at the subtraction point), there is no such parameters as a bare mass and a bare charge, and there is no triangle anomaly. The energy of the ground state of all fields except Higgs fields is still equal to zero. As the standard electroweak unified model, the expectation values of the Higgs fields are not equal to zero. In this case in order to solve the cosmological constant problem we must consider contribution of Higgs fields. In contrast with the other left-right electroweak unified models, there is no unknown massive particles in the present model. A new origin of left-right asymmetry is presented. From the model we see that the world is left-right symmetrical on principle since \( L_F + L_W \) is symmetric, but because both \( L_F \) and \( L_W \) are asymmetric, the world in which we exist is left-right asymmetrical in fact.

4.2. Interaction between W-matter and F-matter. There is no interaction between W-particles and F-particles by a given quantizable field, hence only possibility is that there is repulsion or gravitation of the two sorts of particles. If the new interaction is gravitation, it is possible that W-matter is the candidate for dark matter\(^5\).

Because there are the two sorts of particles corresponding to \( L_W \) and \( L_F \), the energy-momentum tensor \( T_{\mu\nu} \) should be written as

\[ T_{\mu\nu} = T_{F\mu\nu} + T_{W\mu\nu}. \]

Correspondingly, the Einstein’s equation should also be written as

\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \lambda g_{\mu\nu} = -8\pi G \left( T_{F\mu\nu} + T_{W\mu\nu} \right). \]

Because there is no other interaction between the F-particles and the W-particles except the gravitation, we existing in F-world cannot detect the W-particles by other methods except the gravitation. Thus, if W-particles exist, they must be the dark matter for the F-world. Because the F-world and the W-world are symmetric, it is possible that the W-matter is 50 per cent of all matter in the cosmos. Other components of dark matter for the F-world may possibly be other undetected F-matter. The world in which we exist, i.e., the F-world, is left-hand world, then the W-world is the right-hand world. Thus the right-hand world is the dark matter world for the left-hand world, and vice versa. Detail discussion is given in [3].

If the new interaction is repulsion, it is possible that W-matter is the origin of universe expansion. A. Einstein presented the concept ‘dark energy’. If there is repulsion between W-matter and F-matter, we identify W-matter with ‘dark energy’. It is also possible that there is new and more important relationship between W-particles and F-particles. We will discuss the problem in another paper in detail.
5. Conclusions and prospects

We have presented a new conjecture. According to the conjectures, a particle can exist in two forms which are symmetric. From this we have presented a new Lagrangian density and a new quantization method for QED. That the energy of the vacuum state is equal to zero is naturally obtained. From this the cosmological constant is easily determined by astronomical observation values and it is possible to correct nonperturbational methods which depend on the energy of the ground state in quantum field theory.

We discuss quantization for interacting fields, derive Feynman rules and scattering operators. Since we quantize fields by the transformation operators replacing creation and annihilation operators in the conventional QED, it is necessary to replace coupling constants and masses by coupling operators and mass operators in the present theory. In contrast with the conventional QED, in the present theory, there is only one sort of parameters which are all finite and measurable and the two sorts of corrections originating $S_f$ and $S_w$.

We have evaluated the one-loop corrections to the coupling constants and the masses of an electron which are the same as those derived by the conventional QED, and have supplemented three new Feynman rules and one concept of WFDGNNL. A complete method to evaluate correction with n-loop diagrams is given. Feynman diagrams corresponding to any process must be composed of some WFDGNNLS. The integrands causing divergence in a WFDGNNL will cancel each other out, hence correction coming from a WFDGNNL must be convergent and it is unnecessary to introduce regularization and counterterms.

On the same basis as the new QED, we obtain naturally a new $SU(2) \times U(1)$ electroweak unified model whose $\mathcal{L} = \mathcal{L}_F + \mathcal{L}_W$, here $\mathcal{L}$ is left-right symmetric. Thus the world is left-right symmetric in principle, but the part in which we exist is asymmetric because both $\mathcal{L}_W$ and $\mathcal{L}_F$ are asymmetric. This model do not contain any unknown particle with a massive mass.

A conjecture that there is gravitation between the W-matter and the F-matter is presented and the W-matter is identified as ‘dark matter’. It is possible that the new interaction is the origin of some cosmic phenomena.

It is seen that the concepts of the new QED can also be generalized to QCD.

Caption for figures:

Fig.2.1. Contours for propagators. $p_0 = p_0, \omega = \sqrt{m^2 + p^2}, |k|$. Fig.2.1.1. Contours for $S_{Ff}$ and $D_{Ff}$. Fig.2.1.2. Contours for $S_{Fw}$ and $D_{Fw}$.

Fig. 2.1-2. The Feynman rules. In the table $P$ is an algebraic sum of the three 4-momenta meeting at a vertex. Moreover, there is an overall minus sign corresponding to a closed fermion loop.

Fig.2.2.1A

$$-\frac{1}{(2\pi)^4} \frac{g_\mu (2\pi)^4 \delta^4 (P)}{\delta^4 (p_i^2 + m^2 - i\varepsilon)}$$

Fig.2.2.1B

$$-\frac{i}{(2\pi)^4} \frac{(m - ip\gamma)_{\alpha\beta} e^{ip(x_1 - x_2)}}{p^2 + m^2 - i\varepsilon} \text{ with } \int d^4 p$$

Fig.2.2.1C

$$\frac{1}{(2\pi)^4} \frac{\delta_{\mu\nu} e^{ik(x_1 - x_2)}}{k^2 - i\varepsilon} \text{ with } \int d^4 k$$
\[ \frac{1}{\sqrt{\gamma_p u_s}} (\text{ingoing}) \quad \text{or} \quad \frac{1}{\sqrt{\gamma_p v_s}} (\text{outgoing}) \]

\[ \frac{1}{\sqrt{\gamma_p v_s}} (\text{outgoing}) \quad \text{or} \quad \frac{1}{\sqrt{\gamma_p u_s}} (\text{ingoing}) \]

\[ \frac{1}{\sqrt{\gamma_p}} e_{k\mu}^\lambda, \lambda = 1, 2 \]

\[ g_0 \gamma_\mu (2\pi)^4 \delta^4 (P) \]

\[ -i \left( m - i\gamma_\nu \right)_{\alpha\beta} e^{ip(x_1 - x_2)} \text{ with } \int d^4p \]

\[ \delta_{\mu\nu} \frac{i}{(2\pi)^4} \frac{1}{k^2 + i\epsilon} e^{ik(x_1 - x_2)} \text{ with } \int d^4k \]

\[ \frac{1}{\sqrt{\gamma_p u_s}} (\text{ingoing}) \quad \text{or} \quad \frac{1}{\sqrt{\gamma_p v_s}} (\text{outgoing}) \]

\[ \frac{1}{\sqrt{\gamma_p}} e_{k\mu}^\lambda, \lambda = 1, 2. \]

Fig. 3.1.1-2. A WFDGIL representing the two sorts of corrections to the scattering amplitude \( A_m (|a_{q_1} \rangle \rightarrow |a_{q_2} \rangle) \). Fig. 3.1.1 represents the contribution of \( m_{\text{eff}}^{(1)} (q) \); Fig. 3.1.2 represents the contribution of \( m_{\text{eff}}^{(1)} (q) \), and the dotted-line circle in Fig. 3.1.2 represents \( m_{\text{eff}}^{(1)} (q) \).

Fig. 3.2.1-4. A WFDGIL representing the one-loop corrections to \( S_{Ff} (p) \) with its two vertices. Fig. 3.2.1L represents the contribution of \( \Lambda^{(1)L}_{f f 1 \mu} \) (corresponding to \( g_{f,ff1}^{(1)} (p) \)). Fig. 3.2.1R represents the contribution of \( \Lambda^{(1)R}_{f f 1 \mu} \) (corresponding to \( g_{f,ff1}^{(1)} (p) \)).

Fig. 3.2.2 represents the contribution of \( g_{f,ffw}^{(1)} (p) \) (equivalent to \( m_{\text{eff}}^{(1)} (q) \)), and the dotted-line circle in Fig. 3.2.2 represents \( m_{\text{eff}}^{(1)} (q) \).

Fig. 3.2.3 represents the contribution of \( g_{f,ww1}^{(1)} (q_2, q_1) \), the dotted-line circle in Fig. 3.2.3 represents \( g_{f,ww1}^{(1)} (q_2, q_1) \).

Fig. 3.2.4 represents the contribution of \( g_{f,ww1}^{(1)} (q_2, q_1) \), the dotted-line circle in Fig. 3.2.4 represents \( g_{f,ww1}^{(1)} (q_2, q_1) \).
Fig. 3.3.1-2. A WFDGIL representing the one-loop corrections to $D_{ff}(k)$ with its two vertices. Fig. 3.3.1 represents the contribution of $g_{ff2}^{(1)}(k)$.

Fig. 3.3.2 represents the contribution of $g_{fw2}^{(1)}(q_2, q_1)$, and the dotted- line circle in Fig. 3.3.2 represents $g_{fw2}^{(1)}(q_2, q_1)$.

Fig. 3.4.1-4. The total one-loop corrections to the scattering amplitude $A_g (| a_{p_1s}, c_{k\lambda} \rangle \rightarrow | a_{p_2s_2} \rangle)$. Fig. 3.4.1 is a WFDGIL representing the contributions of the first sort. Fig. 3.4.1A-D represent the contribution of $g_{f,ff1}^{(1)}(p_2)$, $g_{f,fw1}^{(1)}(p_2)$, $g_{fiw1}^{(1)}(q_2, q_1)$ and $g_{fiw1}^{(1)}(q_2, q_1)$ in turn, the dotted- line circle in Fig. 3.4.1B, C, D represents $g_{fiw1}^{(1)}(q_2, q_1)$ and $g_{fiw1}^{(1)}(q_2, q_1)$ in turn.

Fig. 3.4.2 is a WFDGIL representing the contributions of the first sort. Fig. 3.4.2A-D represent $g_{f,ff1}^{(1)}(p_1)$, $g_{f,fw1}^{(1)}(p_1)$, $g_{f,fiw1}^{(1)}(q_2, q_1)$ and $g_{f,fiw1}^{(1)}(q_2, q_1)$ in turn, the dotted- line circle in Fig. 3.4.2B, C, D represents $g_{f,fiw1}^{(1)}(q_2, q_1)$ and $g_{f,fiw1}^{(1)}(q_2, q_1)$ in turn.

Fig. 3.4.3 is a WFDGIL representing the contributions of the second sort. Fig. 3.4.3A-B represent the contributions of $g_{ff2}^{(1)}(k)$ and $g_{fw2}^{(1)}(q_2, q_1)$ in turn, the dotted-line circle in Fig. 3.4.3B represents $g_{fw2}^{(1)}(q_2, q_1)$.

Fig. 3.4.4 is a WFDGIL representing the contributions of the third sort. Fig. 3.4.4A-B represent the contributions of $g_{f,ff3}^{(1)}(p_2, p_1)$ and $g_{fw3}^{(1)}(q_2, q_1)$ in turn, the dotted-line circle in Fig. 3.4.4B represents $g_{fw3}^{(1)}(q_2, q_1)$.

Appendix A:

All physics quantities of the vacuum state are zero, hence we have

$$
| \alpha_{ps} \rangle = | 0 \rangle, \quad | \alpha_{ps} \rangle = | \alpha_{ps} \rangle | 0 \rangle, \quad \langle \alpha_{ps} | = \langle \alpha_{ps} | | 0 \rangle = \langle 0 | \langle \alpha_{ps} |, \quad \langle 0 | = \langle 0 | 0 \rangle. \tag{A.1a,b,c}
$$

Thus $| n_{k\lambda} \rangle$ and $| \nu_{k\lambda} \rangle$ can also be represented by

$$
| n_{k\lambda} \rangle = \frac{1}{\sqrt{n}} (| c_{k\lambda} \rangle \otimes | 0 \rangle \cdots | c_{k\lambda} \rangle \otimes | 0 \rangle), \tag{A.2a}
$$

$$
| \nu_{k\lambda} \rangle = \frac{1}{\sqrt{n}} (| c_{k\lambda} \rangle \otimes | 0 \rangle \cdots | c_{k\lambda} \rangle \otimes | 0 \rangle). \tag{A.2b}
$$

The inner product of two states must be defined as a number. According to (A.2), only when the number of single-particle bras of a state is the same as the number of single-particle kets of other state, is it possible that the inner product of the two states is not equal to zero. It can be seen from (A.1) that we always have the two numbers to be the same by suitably supplement the number of $| 0 \rangle$ or $| 0 \rangle$. Hence in order to carry out the inner of two states, we should first have the two numbers are equal to each other. We define the inner product of two states to be such a product obtained by the way. For example,

$$
\langle \alpha_{ps} | \cdot | \alpha_{ps} \rangle \equiv \langle \alpha_{ps} | \langle \alpha_{ps} | \cdot | \alpha_{ps} \rangle | 0 \rangle = \langle \alpha_{ps} | \cdot | 0 \rangle = 0. \tag{A.3}
$$
It is obvious according to the definition that for an orthonormal set, the inner product of two different states must be zero. This result is the same as those obtained by (1.3.19).

**Appendix B:**

A possible definitions $L_p$ etc. are as follows.

$$
L_p = \frac{1}{\sqrt{2}}(|a_p^\sigma\xi + \eta^+(b_{-p}^\sigma)|),
$$

$$
I_p = \frac{1}{\sqrt{2}}(\xi^+|a_p^\sigma| + |b_{-p}^\sigma\eta|),
$$

(\text{B.1})

$$
J_k = \frac{1}{\sqrt{2}}(|c_k^\sigma|e^{i\varphi} + e^{-i\varphi}\xi_{-k}^\sigma|)
$$

(\text{B.2})

$$
J_k^+ = \frac{1}{\sqrt{2}}(|c_k^\sigma|e^{-i\varphi} + e^{i\varphi}\xi_{-k}^\sigma|)
$$

(\text{B.3})

$$
I_p = \frac{1}{\sqrt{2}}(|b_p^\sigma\eta + \xi^+(a_{-p}^\sigma)|),
$$

$$
I_p^+ = \frac{1}{\sqrt{2}}(\eta^+(b_p^\sigma| + |a_{-p}^\sigma\xi|),
$$

(\text{B.4})

$$
J_k = \frac{1}{\sqrt{2}}(|c_k^\sigma|e^{i\varphi} + e^{-i\varphi}\xi_{-k}^\sigma|),
$$

(\text{B.5})

$$
J_k^+ = \frac{1}{\sqrt{2}}(|c_k^\sigma|e^{-i\varphi} + e^{i\varphi}\xi_{-k}^\sigma|),
$$

(\text{B.6})

where all $\xi, \xi^+, \eta$ and $\eta^+$ are Grassman numbers, and $2\pi \geq \varphi \geq 0$. We define the inner products of the base vectors as follows

$$
Z_p \cdot Z_p' = \text{Tr} \int (d\xi d\xi'^+ + d\eta d\eta'^+) Z_p Z_p' = Z_p' \cdot Z_p,
$$

(\text{B.7})

$$
Y_k \cdot Y_k' = \frac{1}{2\pi} \text{Tr} \int_0^{2\pi} d\varphi Y_k Y_k' = Y_k' \cdot Y_k,
$$

(\text{B.8})

where $Z_p, Z_p', I_p, I_p^+, L_p$ and $L_p^+, Y_k, Y_k'$, $J_k, J_k^+, J_k^-$ and $J_k^-$. We define

$$
\text{Tr} \langle \alpha_p | \cdot | \alpha_p' \rangle = -\text{Tr} | \alpha_p' \rangle \langle \alpha_p | = \delta_{\alpha\alpha'} \delta_{pp'},
$$

(\text{B.9})

$$
\text{Tr} \langle \gamma_k | \cdot | \gamma_k' \rangle = \text{Tr} | \gamma_k' \rangle \langle \gamma_k | = \delta_{\gamma\gamma'} \delta_{kk'}.
$$

(\text{B.10})
From (B.1)-(B.10) we can obtain (1.3.26)-(1.3.28), e.g.,

\begin{align*}
I_p^+ \cdot I_{p'} &= Tr \int (d\xi d\eta^+ + d\eta d\eta^+)
\end{align*}

\begin{align*}
\frac{1}{2} \{ \xi^+ \xi |_{\Delta_p} &+ \eta^+ \eta |_{\Delta_p} + \xi^+ \eta |_{\Delta_p} + \eta^+ \eta |_{\Delta_p} \\
+ &\frac{1}{2} Tr \{ |_{\Delta_{p'}} \} \}
\end{align*}

\begin{align*}
= \frac{1}{2} Tr \{ |_{\Delta_p} \} - |_{\Delta_{p'}} \}
= \delta_{pp'} = I_{p'} \cdot I_p^+.
\end{align*}

(B.11)

\begin{align*}
\mathbf{J}_k \cdot \mathbf{J}_{k'} &= \frac{1}{2\pi} Tr \int_0^{2\pi} d\varphi \frac{1}{2} \{ \langle c_k | c_{k'} \rangle + e^{i2\varphi} | c_{-k} \rangle \langle c_{-k} | c_{k'} \rangle \\
+ &\langle c_k | \langle c_{-k'} | e^{-i2\varphi} + | c_{-k} \rangle \langle c_{-k} \rangle \}
\end{align*}

\begin{align*}
= \frac{1}{2} Tr \{ \langle c_k | c_{k'} \rangle + \langle c_{-k} | c_{-k'} \rangle \} = \delta_{kk'}.
\end{align*}

(B.12)

\[Z_p \text{ and } Y_k \text{ are always regarded as a whole, hence when an operator or a state is multiplied by } Z_p \text{ or } Y_k, \text{ we must first accomplish third inner, e.g.,}
\]

(B.13) \[Z_{p'} \vartriangleleft \sigma \rangle = Tr \int (d\xi d\eta^+ + d\eta d\eta^+)Z_p \vartriangleleft \sigma \rangle = 0.
\]

Analogously to (B.13), we have

\begin{align*}
\vartriangleleft \sigma \rangle \cdot Z_p &= \langle \sigma | Z_p = Z_p \cdot \langle \sigma | \\
Y_k \vartriangleleft \sigma \rangle &= \langle \sigma | \cdot Y_k = \langle \sigma | \cdot Y_k = \langle \sigma | \langle \sigma | = 0.
\end{align*}

(B.14)

Thus we have

(B.15) \[[\vartriangleleft \sigma \rangle, Z_p] = 0, \quad [Y_k, \vartriangleleft \sigma \rangle = 0.\]

According to the definitions (B.1)-(B.10), \(\mathbf{J}_k\) and \(\mathbf{J}_k^+\) etc. form respectively vector spaces. We call such a space which takes \(\mathbf{J}_k\) as a base vector the \(\mathbf{J}_k\) - space, etc.

**Appendix C:**

The equal-time anticommutation or commutation relations of such operators as \(\vartriangleleft a_{p_{s}}(t) \rangle \) are the same as (1.316)-(1.317). From (1.316)-(1.317) and (1.328) or (B.15) we have

\begin{align*}
\mathbf{L}_p \vartriangleleft a_{p_{s}}(t) \rangle &\vartriangleleft a_{p_{s} s_{2}}(t) \rangle \vartriangleleft a_{p_{s} s_{2} s_{3}}(t) \rangle \]
= \mathbf{L}_p \vartriangleleft a_{p_{s} s_{2}}(t) \rangle \delta_{p s_{2} s_{3}},
\end{align*}

(C.1)

\begin{align*}
\mathbf{L}_{(p)} \left| b_{p_{s}}(t) \right| &\mathbf{L}_{(-p)} \vartriangleleft a_{(-p) s_{2}}(t) \rangle \vartriangleleft a_{(-p) s_{2} s_{3}}(t) \rangle \\
= -\delta_{p(-p)} \delta_{s_{2} s_{3}} | b_{(-p) s_{2}}(t) \rangle \mathbf{L}_{(-p)};
\end{align*}

(C.2)

\begin{align*}
\mathbf{L}_{(-p)} \left| b_{p_{s}}(t) \right| &\mathbf{L}_{(-p)} \vartriangleleft a_{(-p) s_{2}}(t) \rangle \vartriangleleft a_{(-p) s_{2} s_{3}}(t) \rangle \\
= \mathbf{L}_{(-p)} \vartriangleleft a_{(-p) s_{2}}(t) \rangle \delta_{p(-p)} \delta_{s_{2} s_{3}},
\end{align*}

(C.3)

\begin{align*}
\mathbf{L}_p \vartriangleleft a_{p_{s}}(t) \rangle &\vartriangleleft a_{p_{s} s_{2}}(t) \rangle \vartriangleleft b_{(-p) s_{2}}(t) \rangle \\
= \delta_{p (-p)} \delta_{s_{2} s_{3}} | b_{(-p) s_{2}}(t) \rangle \mathbf{L}_p,
\end{align*}

(C.4)
\[ L_p \lesssim a_{p_1}(t), b_{(-p_1)s_1}(t) \lesssim b_{(-p_2)s_2}(t) \]
\[ = [ b_{p_2}(t) \geq L_{(-p)}, a_{p_2s_2}(t) \lesssim a_{p_1s_1}(t) ] \]
\[ = [ b_{p_1}(t) \leq a_{p_s}(t), b_{(-p_2)s_2}(t) \leq b_{(-p_1)s_1}(t) ] \]
\[ = [ b_{p_2}(t) \geq L_{(-p)}, a_{p_2s_2}(t) \geq b_{(-p_1)s_1}(t) \geq 0. \) (C.5)\

(C.6)
\[ \sum_s (u_{p_s} u_{p_s}^* + u_{(-p)s} u_{(-p)s}^*) = 1 \]

Substituting (2.4.1)-(2.4.3), (C.1)-(C.6) and
\[ \psi(x) = \frac{1}{\sqrt{V}} \sum_{p_s} \left( L_p \lesssim a_{p_s}(t), u_{p_s} e^{ipx} + b_{p_s}(t) \geq L_{(-p)} u_{p_s} e^{-ipx} \right), \]

into (2.2.18), i.e.,
\[ \dot{\psi}(x) = -i[\psi(x), H] = -i[\psi(x), H_F], \]

we obtain (2.2.14).

From (2.2.20) we have
\[ j_k \lesssim c_k \lambda(t), \quad | \tau_{(-k')\lambda'}(t) \rangle = \hat{j}_k \delta_{k(-k')} \delta_{\lambda\lambda'}, \]

(C.9)
\[ [ j_k \lesssim c_k \lambda(t), \tau_{(-k')\lambda'}(t) \rangle = \hat{j}_k \delta_{k(-k')} \delta_{\lambda\lambda'}, \]

\[ [ \tau_{k\lambda}(t) \geq \hat{j}_{(-k')}, c_k \lambda' \rangle \geq | \tau_{(-k')\lambda'}(t) \rangle \geq 0. \] (C.11)
\[ [ j_k \lesssim c_k \lambda(t), \tau_{k'\lambda'}(t) \geq c_k \lambda' \rangle \]
\[ = \hat{j}_k \delta_{kk'} \delta_{\lambda\lambda'} \]
(C.12)
\[ \tau_{k\lambda}(t) \geq \hat{j}_{(-k')}, \tau_{kk'}(t) \geq c_k \lambda' \rangle \]
\[ = -\hat{j}_{(-k')} \tau_{kk'} \lambda' \rangle \delta_{kk'} \delta_{\lambda\lambda'} \] (C.13)

(C.14)
\[ \sum_{\lambda=1}^{4} e_k^\lambda e_k^\lambda = \delta_{\mu\nu} \]

Substituting (2.4.1), (2.4.3), (C.9)-(C.13) and
\[ A_\mu(x) = \frac{1}{\sqrt{V}} \sum_k \frac{1}{\sqrt{2\omega_k}} \sum_{\lambda=1}^{4} (e_{k\mu}^\lambda \hat{j}_k \leq c_k \lambda(t) | e^{ikx} \]
\[ + | \tau_{k\lambda}(t) \geq \hat{j}_{(-k)} e^{-ikx} \), \]
\[ \pi_\mu = A_\mu(x) = -\frac{i}{\sqrt{V}} \sum_k \sqrt{2\omega_k} \sum_{\lambda=4}^{4} e_{k\mu}^\lambda (\hat{j}_k \leq c_k \lambda(t) | e^{ikx} \]
\[ - | \tau_{k\lambda}(t) \geq \hat{j}_{(-k)} e^{-ikx} \), \]
(C.15)
into (2.2.18), i.e.,

\[
\dot{A}_\mu (x) = -i[A_\mu (x), H_F], \quad \dot{\pi}_\mu = \dot{A}_\mu = -i[\pi_\mu (x), H_F],
\]

we obtain (2.2.15). It is seen that (2.2.18) is consistent with (2.2.14)-(2.2.15). Analogously, from (2.4.4)-(2.4.6) we can prove (2.2.19) to be consistent with (2.2.16)-(2.2.17).

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