Study of the external section shape of the static characteristic of the antivibration suspension with a quasi-zero stiffness section

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Abstract. The paper considers the vibration isolation problem of the construction and road vehicles operator's seat with prescribed sinusoidal displacements of the seat base. This problem is highly relevant, since the road construction vehicles operation with soil, loads, road surfaces, etc. is accompanied by the strong vibrations. The investigation was carried out in terms of the influence of the suspension static power characteristic shape on the maximum vibration acceleration of the seat. Many antivibration mechanisms have a similar static characteristic. Currently, one of the promising directions in the development of the antivibration systems is the implementation of the systems with a quasi-zero stiffness section of the static power characteristic. Such systems are preferred to dampen the low-frequency vibrations. In this paper, we study the systems having a three-segment static characteristic, which middle segment is a horizontal straight line corresponding to a section of quasi-zero stiffness. However, two extreme segments providing the braking and stopping of the mechanism when going beyond the boundaries of the quasi-zero stiffness section were described by Hermite splines with a maximum second-order derivative. The analytical expression of such a two-point spline makes it possible to set not only the function values at two boundary points, but also the values of the first two derivatives of this function at the same points, i.e. to vary the shape of such spline within a wide range. This has made it possible to consider several dozen combinations of such splines having different values of the first derivative at both boundary points, i.e. to study the differently shaped static power characteristics of the antivibration mechanism of the seat under the same sinusoidal influences. The sum of accelerations calculated for a sample of the specified seat base vibrations from the amplitude and time values combinations was used as the assessment and evaluation criterion of various static power characteristics. Furthermore, the amplitude and time values combinations, at which the maximum value of the antivibration mechanism displacements exceeded the limits of at least one of the static characteristics, were excluded from consideration and were not taken into account when calculating the criterion values. The criterion minimum value was found out to be reached at zero value of the first force derivative at the internal boundary points of the external characteristic segments, at the maximum value of the first force derivative at the external boundary points of the external characteristic segments. In this case, the first force derivative has no discontinuities on the entire static force characteristic curve. The given static
characteristic is optimal according to the accepted criterion. The value of the acceleration sum criterion can be decreased fourfold in the studied range of the first derivatives from zero to twenty thousand Newton/meter.

**Key-words:** vibrations, vibration protection, acceleration, quasi-zero stiffness, static characteristic

1. Introduction
The vibration isolation problem of mobile vehicles is highly relevant [1]. The use of vibration isolation systems is a prerequisite for protecting the equipment [2], especially high-precision machines [3] and different vehicles operators [4]. Road [5], construction [6] and mining machines [7], earth-moving and utility vehicles [8] are particularly exposed to the effects of vibrations. Vibrations cause occupational diseases among the machine operators [9]. Besides, vibrations reduce a human operator's concentration, response, weaken the perception of audio and visual information at the time of exposure [10]. In most cases vibrations not only cause discomfort and even low back pain among operators, but also significantly reduce the performance and accuracy of the work performed [11]. Passive [12] and active [13] vibration isolation systems, which differ on the basis of utilizing the external energy, may have various design features. For example, pneumatic rubber-cord and other shells [14], as well as electrodynamic drives [15] are widely used in active systems. Despite their advantages, they are quite complex and expensive systems. Vibration isolation systems and mechanisms with the effect of quasi-zero stiffness [16] are also widely used, since they are quite promising and more effectively dampen vibrations in a wide range of frequencies, particularly low ones [17]. Moreover, sets of springs and various levers are utilized in vibration isolation systems, typically in passive ones [18]. Regardless of the type of vibration isolation system and its design features, the static power characteristic has the greatest impact on the system performance. The research objective is to study the influence of the external sections shape of the antivibration suspension static characteristic with a middle section of quasi-zero stiffness on the vibration isolation effectiveness. The external sections of the characteristic are necessary elements, as they limit the displacements of the vibration-isolated object.

2. Problem statement
The initial data are the static force characteristic of the vibration isolation system with one translational degree of freedom, which in turn is the dependence of the static force $F$ produced by the antivibration mechanism on the linear displacement of this mechanism $z_1$. The experimental variables characterizing the external sections shape of the antivibration suspension static characteristic are the first derivatives of the force at the internal ($v_{F0}$) and external ($v_{FT}$) points of each section, respectively. To simplify the problem, the external sections of the static characteristic are assumed to be symmetrical to each other.

Figure 1 shows the examples of the studied static force characteristics of the vibration isolation system with one translational degree of freedom. Taking into account the gravity of the vibration isolated mass $m=200$ kg in the mathematical model, the static characteristics of the system will have a horizontal section of quasi-zero stiffness in the middle part with a length of $z_{qz}=0.1$ m ($F_0=1962$ N at $z$ $\in [0.05; 0.05]$ m, figure 1a). Two external sections of the force characteristic can be described by the additional force relative to a quasi-zero value (±1000 N), with the sections length of $z_{out}=0.1$ m and the studied force derivatives at the internal ($v_{F0}$) and external ($v_{FT}$) points of the section, correspondingly.

Figure 1a shows two static characteristics with the derivatives values combinations of $v_{F0}=0$ N/m; $v_{FT}=20000$ N/m (characteristic No. 1) and $v_{F0}=20000$ N/m; $v_{FT}=0$ N/m (characteristic No. 2).
If gravity is excluded from the mathematical model, the shift of the static force characteristic curve in the vertical direction down by the mass gravitational force value (figure 1b) gives the results completely identical to the model with gravity force [12].

![Figure 1](image-url)

**Figure 1.** Examples of the static force characteristics of the antivibration system with one degree of freedom: (a) is taking into account the gravity, (b), (c) are without taking into account the gravity.

External influences on the vibration isolated object were prescribed cyclic base movements characterized by the amplitude $A_{mp}$ and angular oscillation frequency $\omega$ and were calculated by the following formula:

$$Z_{op} = A_{mp} \sin(\omega t)$$

(1)

where $z_{op}$ is the vertical coordinate of the seat base (vehicle cab) in a fixed coordinate system; $\omega = 2\pi(T)^{-1}$ is the angular oscillations frequency of the seat base; $T$ is the oscillations period of the seat base; $t$ is the current time.

Furthermore, the initial data of the problem included the total simulation time $T_{kon}$ and measurement time of the model output parameters at the end of the process $T_{izm}$.

The maximum mass acceleration in a fixed coordinate system $a_{0\text{max}}$ achieved in a finite interval $T_{izm}$ of the transition time was used as a parameter characterizing a separate dynamic process of oscillations of the vibration isolated mass on a movable base. In this case, $T_{izm}$ represented 20% of the total simulation time $T_{kon}$.

The research objective was to define the dependencies of the integral criterion, i.e. the sum of the maximum seat accelerations $\Sigma a_{0\text{max}}$ (from a sample of $n$ combinations of the amplitude and angular frequency of the base vibrations), on the values of the first force derivatives at the internal ($v_{F0}$) and external ($v_{FT}$) points of the external sections of the static characteristic.

### 3. Theory

Forced vibrations of the antivibration suspended seat with a movable base were described by a well-known system of differential equations [12]:

$$\text{Equations...}$$
\[
\begin{align*}
\dot{v} &= (-b(v - A_{mp}\omega \cos(\omega t)) - F)(m)^{-1} \\
\dot{z} &= v 
\end{align*}
\]  

(2)

where \( b \) is the damping coefficient; \( F(t) = f[z_1(t)] \) is the value of the static force produced by the antivibration mechanism depending on its own displacement \( z_1 \); \( z_1(t) = z(t) - z_{op}(t) \).

The static force characteristic was described as three separate segments set by the analytical expressions of a constant (a horizontal straight line, a middle section of quasi-zero stiffness) and two-point Hermite splines with a maximum second-order derivative (two external sections of the characteristic) [19]. A well-known analytical expression of a Hermite spline [12] not provided here due to its length, includes the values of the zero, first and second derivative values at the internal and external points of the section. The system of differential equations (2) was solved by a well-known numerical Runge-Kutta forth-order method, using the MATLAB function ode45 [12].

4. Experimental results

The parameters of the first force derivatives \( v_{F0} \) and \( v_{FT} \), which varied in the range from 0 to 20.000 N/m (figure 1b) with a step of 5000 N/m, have been studied. Figure 1b shows the graphs of the static force characteristics obtained at 9 combinations of \( v_{F0} \) and \( v_{FT} \) (0; 10000; 20000 N/m) from 25 studied combinations (0; 5000; 10000; 15000; 20000 N/m).

The maximum seat acceleration sum \( \Sigma a_{0\max} \) in a sample of 25 combinations of the base vertical displacements amplitudes \( A_{mp} \) (0.01; 0.05; 0.09; 0.13; 0.17 m) and base oscillation period \( T \) (0.5; 2; 3.5; 5; 6.5 s) has been defined for each of the 25 combinations of the values \( v_{F0} \) and \( v_{FT} \).

Other process parameters accepted the fixed values, such as: \( m=200 \) kg, \( T_{kon}=500 \) s, \( T_{izm}=100 \) s, \( b=20 \) N/(m/s). The second force derivatives at the boundary points of the external section of the static force characteristic were assumed to be equal to zero. The zero derivatives at the boundary points of the external characteristic sections accepted the constant values of ±1000 N (figure 1).

Figure 2 shows the examples of a separate static characteristic of the antivibration mechanism (at \( v_{F0}=0 \) N/m, \( v_{FT}=10000 \) N/m, figure 2a) and time dependence of the seat acceleration obtained for the given characteristic in the fixed coordinate system (at \( A_{mp}=0.13 \) m; \( T=6.5 \) s, figure 2b).

**Figure 2.** Examples of the separate static characteristic of the antivibration mechanism (a) and of the time dependence of the seat acceleration (b).
Figure 3. The dependences of the maximum accelerations on the amplitude and oscillations period (a) and of the maximum accelerations sum on the first force derivatives $v_{F0}$ and $v_{FT}$ at the boundary points of the external sections of the static force characteristic (b).

An example of the maximum accelerations dependence on the amplitude and oscillations period obtained for the derivatives values $v_{F0}=0$ N/m; $v_{FT}=0$ N/m is presented in figure 3a. A graphical representation of the dependence of the maximum accelerations sum on the first force derivatives $v_{F0}$ and $v_{FT}$ at the boundary points of the external sections of the static force characteristic is shown in figure 3b.

5. Results discussion
During the results processing, for the combinations of $A_{mp}$ and $T$, under which the following condition was met:
the maximum load acceleration was assumed to be zero immediately for all combinations of $v_{F0}$ and $v_{FT}$:

$$\forall[v_{F0} \in (0; 5000; 10000; 15000; 20000) \land v_{FT} \in (0; 5000; 10000; 15000; 20000)],$$

$$a_{\text{omax}}(v_{F0}, v_{FT}, A_{\text{mp}}, T) = 0.$$  \hspace{1cm} (4)

This made it possible to exclude from consideration all combinations of $A_{\text{mp}}$ and $T$, under which the displacements of the vibration protection system mechanism $z_1$ went beyond the specified static power characteristics, i.e. to make a correct comparison of various static characteristics.

Consequently, a dependence graph $\sum a_{\text{omax}} = f(v_{F0}, v_{FT})$ (figure 3b) was obtained from a single subset of the values $A_{\text{mp}}$ and $T$ included in set $[A_{\text{mp}} \in (0.01; 0.05; 0.09; 0.13; 0.17 \text{ m}) \land T \in (0.5; 2; 3.5; 5; 6.5 \text{ s})]$

The analysis of this graph shows that the range of changing the criterion values of the seat accelerations sum is 400 %, i.e. the criterion values change fourfold.

6. Conclusions

The oscillations have been studied on the developed mathematical model of the antivibration suspended operator's seat oscillations at the defined sinusoidal seat vibrations. The model has made it possible to define the shape of the static power characteristic of the suspension as a three-segment spline. The middle segment was represented by a horizontal straight line and described a section of quasi-zero stiffness of the vibration protection system. Two external sections of the static power characteristic were set using the Hermite splines and provided a smooth stop of the suspension displacements when going beyond the boundaries of the quasi-zero stiffness section.

The influence of the external sections shape of the static power characteristic on the proposed criterion has been studied. The maximum load accelerations sum in the steady-state mode, which was calculated for a combination of several trajectories with different amplitudes and periods of the specified seat base vibrations, was used as a criterion.

The combinations, at which the maximum displacements of the antivibration mechanism went beyond the limits of the static power characteristic, were not taken into consideration when calculating the criterion value.

A dependence graph of the criterion value on the first two derivatives values at the boundary points of the external sections of the static characteristic was obtained for 25 combinations of the first derivatives of the static power characteristic.

The minimum value of the accepted criterion was found out to be achieved by combining the values of the first force derivatives $v_{F0}=0 \text{ N/m}$; $v_{FT}=20000 \text{ N/m}$ (characteristic No. 1 in figure 1b). In this case, the first force derivative has no discontinuities on the entire static power characteristic. The value of the acceleration sum criterion decreases by 4 times in the studied range of the first derivatives.

The results obtained may be of interest to developers and researchers of vibration protection systems of operator's seats of construction, road and other vehicles.

7. References

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