Delayed Collapse of Protoneutron Stars
with Kaon Condensate
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Abstract

Equation of state with kaon condensate is derived for isentropic and neutrino-trapped matter. Both are important ingredients to study the delayed collapse of protoneutron stars. Solving the TOV equation, we discuss the static properties of protoneutron stars and implications for their delayed collapse.

1 Introduction

After supernova explosions, protoneutron stars (PNS) are formed with hot, dense and neutrino-trapped matter. They usually evolve to cold ($T \approx 0$) neutron stars through the deleptonization and cooling stages. But some of them may collapse to low-mass black holes during these stages by softening the equation of state (EOS) due to the hadronic phase transitions\textsuperscript{[1]}. This is called the delayed collapse; as a typical example neutrinos from SN1987A were observed at Kamiokande, but no pulsar yet, which suggests the possibility of the delayed collapse of SN1987A.

Kaon condensation, one of the candidates of the hadronic phase transitions, has been studied by many authors mainly at zero temperature since first suggested by Kaplan and Nelson\textsuperscript{[2]}. We know that the kaon condensation gives rise to the large softening of EOS. Recently there appear a few works about kaon condensation at finite temperature, but there was no consistent theory based on chiral symmetry. We have presented a new formulation to treat fluctuations around the condensate based on chiral symmetry\textsuperscript{[3],[4]}

Here we show the essential idea of our formulation and then discuss the properties of PNS and the possibility of the delayed collapse.
2 Formulation

We use the nonlinear sigma model (as a chiral Lagrangian) and treat fluctuation fields around the condensate by making full use of the idea of chiral rotation. Chiral Lagrangian on the chiral manifold \( G/H \simeq SU(3); G = SU(3)_L \times SU(3)_R, H = SU(3)_V \) is specified by the unitary matrix \( U(\phi_a) = \exp(2iT_a \phi_a/f) \) with Goldstone fields \( \phi_a \), generators of \( SU(3) \) Lie algebra: \( T_a \) and pion decay constant: \( f \).

Fluctuation around the condensate is written as \( U(\langle K^\pm \rangle, \tilde{\phi}_a) = \zeta \eta^2 \zeta \). \( \eta \) corresponds to a chirally rotated state from the meson vacuum: \( \eta = \exp\{i(V_+ K^- + V_- K^+)/\sqrt{2}f\} \) with \( V_\pm = T_3 \pm iT_5 \) and \( K^\pm = (\phi_4 \pm i\phi_5)/\sqrt{2} \). On the other hand, the classical condensate \( \zeta \) is also chirally rotated state: \( \zeta = \exp\{i(V_+ \langle K^- \rangle + V_- \langle K^+ \rangle)/\sqrt{2}f\} \). The most important field we are interested in is the fluctuation field around the condensate. In the usual approach the meson fields are separated directly: \( \phi_a = \langle \phi_a \rangle + \tilde{\phi}_a \). After putting this form into the Lagrangian, we find the integration measure is complicated and the Lee-Yang term appears in the path integral. We have therefore used another separation by the use of the successive chiral rotations; This can be regarded as a separation of zero-mode or setting of the local coordinates around the condensed point. Thus the Lee-Yang term disappears and we can take the flat curvature to one-loop order [4].

With this idea, we have performed the imaginary-time path integral to one-loop order and then derived the thermodynamic functions in a transparent way [4]. The \( KN \) interactions have been treated self-consistently within the Hartree approximation, and consequently the thermodynamic potential consists of an infinite series of many one-loop diagrams. There appear divergent integrals, but we can renormalize them properly. Then the dispersion relations for kaonic excitations, which are fundamental objects to study the thermal properties of the condensed phase, are derived. We have two modes in the condensed phase: a Goldstone-like soft mode and a very massive mode, which correspond to \( K^- \) and \( K^+ \)-mesonic excitations, respectively. Hence, the thermal loops due to the soft mode play an important role in the thermodynamic functions.
3 Numerical Results and Discussions

With thermodynamic potential, we can study the nature of kaon condensed state at finite temperature and then discuss some implications on the delayed collapse of PNS. We, hereafter, use the heavy-baryon limit for nucleons [3]. We show the phase diagram, EOS and then discuss the properties of PNS where thermal and neutrino-trapped effects are very important.

First we show the phase diagram and the isothermal EOS in Figs. 1, 2. In the neutrino-trapped case we set $Y_{le} = Y_e + Y_{\nu_e} = 0.4$ where $Y_e(Y_{\nu_e})$ is the electron(electron-neutrino) number per baryon, while $Y_{\nu_e} = 0$ in the neutrino-free case. Both of the thermal and neutrino-trapped effects largely suppress

\begin{align*}
\mu_{\nu_e} &> 0 \quad \text{in the neutrino-trapped case} \quad \text{while} \quad \mu_{\nu_e} = 0 \quad \text{in the neutrino-free case, which means kaons should wait to condense until its energy further decreases. Both effects also stiffen the EOS in the condensed phase as well as the normal phase. These effects are more pronounced in the condensed state (see Fig. 2). Kaon condensation is the first order phase transition and thereby EOS includes thermodynamically unstable region. We applied the Maxwell construction to obtain the equilibrium curve for simplicity though.}
\end{align*}

![Figure 1: Phase diagram](image1.png)

![Figure 2: EOS for isothermal case](image2.png)
restrictly speaking, we need to take the Gibbs condition\textsuperscript{6}.

Solving the TOV equation with the EOS, we can study the nature of PNS for which the isentropic situation is relevant. In Fig.3 we show the mass-radius relation for the neutrino-trapped and -free cases with entropy per baryon $S = 0, 1$ or $2$. Once kaon condensation occurs in the core of the star, radius becomes smaller and gravitationally unstable branch appears because of the large softening of EOS. Both of the thermal and neutrino-trapped effects make the radius larger.

To discuss the possibility of delayed collapse of PNS, the total baryon number $N_B$ should be fixed as a conserved quantity during the evolution. In Fig.4 we show the mass versus total baryon number for gravitationaly stable PNS. Each terminal point represents maximum mass and maximum total baryon number in each configuration. If initial mass exceeds the terminal point, the star should collapse into a black hole (not a delayed collapse but an usual formation of a black hole). We have shown the neutrino-trapped and -free cases; the former case might be relevant for the deleptonization era, while the latter for the initial cooling era. It is interesting to see the difference between the neutrino-trapped and -free cases: the curve is shortened as the entropy increases in the former case, while elongated in the latter case.

Figure 3: Mass-radius curve for PNS in $\nu$-trapped($Y_{le} = 0.4$) and $\nu$-free cases.

Figure 4: Total baryon number versus mass for PNS.
These two features are essential for the following argument about the delayed collapse and maximum mass of the cold neutron stars.

The delayed collapse is possible if the initially stable star on a curve finds no end point on other curves as a result of the evolution through deleptonization or cooling with the baryon number fixed. Consider a typical case for example: A PNS has initially $Y_{le} = 0.4$ and $S = 2$ after supernova explosion and evolves through deleptonization to neutrino-free and $S = 2$ stage. We can clearly see the PNS with large enough mass can exist as a stable star at the beginning but cannot find any point on the neutrino-free and $S = 2$ curve. Therefore they must collapse to the low-mass black hole by deleptonization. It is to be noted that because the $Y_{le} = 0.4$ and $S = 2$ star never includes kaon condensate, its collapse is largely due to the appearance of kaon condensate in the core. Thus we may conclude that kaon condensation is very plausible to cause the delayed collapse.

Furthermore we can determine the maximum mass of cold neutron stars by examining the evolution of PNS at the initial cooling stage, where neutrinos are never trapped. Usually we assign the maximum mass of the cold neutron stars from the mass-radius curve by the use of the EOS at $T = 0$ (see the $S = 0$ and neutrino-free case in Fig.3). However, it is wrong when we take into account the evolution of neutron stars, especially in the initial cooling stage[7]. As already mentioned, the entropy dependence of the curve for the neutrino-free case is opposite to the neutrino-trapped case. Hence, once a PNS resides on the large-entropy curve, it necessarily evolves into the corresponding point on the smaller-entropy curve through initial cooling. As an example, consider the evolution of a PNS with $S = 2$. Because only the stars with $N_B \leq 2.14 \times 10^{57}$ at the beginning can evolve to the neutrino-free and cold $(S = 0)$ star, the maximum mass of cold neutron stars can be determined as $1.54M_\odot$.

In order to study the mechanism of delayed collapse and mass region which should collapse in more detail, we had better study the dynamical evolution beyond the static configurations. This work is in progress. As another remaining issue, we will refine the EOS to include the effects of thermal kaon loops on the nucleon propagator and the zero-point energy.

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