Fierz-Pauli Mass From Torsion

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Abstract

A perturbative regime based on contorsion as a dynamical variable is performed in a particular massive extension of the frame bundle gauge formulation of gravity in 2+1 dimension. This construction preserves parity and the coincidence with the perturbative Hilbert-Einstein-Fierz-Pauli can be shown. As a result, this model is unitary.

Since very long time ago, there has been great interest in the study of gravity in 2+1 dimension. Recently, an unitary four order massive model of gravity in a Riemannian 2 + 1 dimensional space-time which preserves parity, has taken place in a very fundamental contribution[1]. In a perturbative regime and a subsequent order reduction of the former formulation, the matching between this one and Fierz-Pauli gravitons theory can be shown. The discussion about unitarity in these types of theories has been remarked in the study of some $R^2$-actions[2], where linearization can reproduce the Fierz-Pauli model with no ghost degrees of freedom.

The extension to a non-Riemannian context, considering the torsion (contorsion) as a dynamical field is exhaustively focussed by some authors. There, a study of the effects of torsion on massive terms and unitarity in a gravity model[4] and the consideration of a gauge formulation of gravity based on the Lorentz group, taking the torsion as a dynamical variable and metric as a fixed background[3], are some enterprises.

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The main purpose of this work is to explore the introduction of appropriate torsion quadratical Lagrangian terms \((T^2\text{-terms})\) in a pure Yang-Mills formulation of gravity (YM)\([5, 6, 7]\), based on a frame bundle with \(GL(3, R)\) as a gauge group, preserving parity and general covariance. Its linearization conduces to a theory of particles with helicity \(\pm 2\) and Fierz-Pauli mass, where neither auxiliary fields nor second order reduction are needed. The idea about considering \(T^2\text{-terms}\) in a dynamical theory of torsion has been considered in the past\([8]\).

This letter is organized as follows. We start with a brief review on notation of the cosmologically extended YM formulation\([6]\) in \(N\)-dimensions and its topologically massive version in \(2 + 1\)\([7]\). Next we present a massive \(T^2\)-model which preserves parity and the linearization which describes a spin 2 propagation of contorsion with a Fierz-Pauli mass. Although \(T^2\)-terms provide mass only to the spin 2 component of contorsion, the linearized theory loses the gauge invariance and there is no any residual invariance. This is clearly established through a standard procedure for the study of possible chains of gauge generators\([9]\). We end up with some remarks.

The brief review of a YM construction starts here. Let \(M\) be a \(N\)-dimensional manifold with a metric, \(g_{\mu\nu}\) provided. A (principal) fiber bundle is constructed with \(M\) and a 1-form connection is given, \((A_{\lambda})^\mu_{\nu}\), which will be non metric dependent. The affine connection transforms as \(A_{\lambda}' = UA_{\lambda}U^{-1} + U\partial_{\lambda}U^{-1}\) under \(U \in GL(N, R)\). Torsion and curvature tensors are \(T^\mu_{\lambda\nu} = (A_{\lambda})^\mu_{\nu} - (A_{\nu})^\mu_{\lambda}\) and \(F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]\), respectively. Components of the Riemann tensor are \(R^\sigma_{\alpha\mu\nu} \equiv (F^\rho_{\nu\mu})^\sigma_{\alpha}\). The gauge invariant action with cosmological contribution is\([6]\)

\[
S_0^{(N)} = \kappa^{2(4-N)} \left\langle -\frac{1}{4} \text{tr} \ F^{\alpha\beta} F_{\alpha\beta} + q(N)\lambda^2 \right\rangle ,
\]  

where \(\kappa^2\) is in length units, \(\left\langle \ldots \right\rangle \equiv \int d^N x \sqrt{-g} \ldots\), \(\lambda\) is the cosmologic constant and the parameter \(q(N) = 2(4-N)/(N-2)^2(N-1)\) depends on dimension. The shape of \(q(N)\) allows the recovering of (free) Einstein’s equations as a particular solution when the torsionless Lagrangian constraints are imposed and it changes its sign when \(N > 5\). Particularly, in \(2 + 1\) dimension take place the topologically massive version\([7]\), up to
a global minus sign

\[ S^{(3)}_{tm} = S^{(3)}_o + \frac{m\kappa^2}{2}\left\langle \varepsilon^{\mu\nu\lambda} \text{tr}\left(A_\mu \partial_\nu A_\lambda + \frac{2}{3} A_\mu A_\nu A_\lambda\right) \right\rangle, \quad (2) \]

which contains the cosmologically extended TMG\[10\] when the torsionless constraints are introduced through a suitable set of Lagrangian multipliers. Obviously, (2) does not preserve parity, however it can be possible to write down other massive version which respect parity, for example

\[ S^{(3)}_m = S^{(3)}_o - m^2 \kappa^2 \langle T^{\sigma\rho\nu} T^{\mu\lambda\sigma} - T^{\lambda\mu\nu} T^{\mu\lambda\sigma} - \frac{1}{2} T^{\lambda\mu\nu} T^{\lambda\mu\nu} \rangle. \quad (3) \]

If we allow independent variations on metric and connection two types of field equations can be obtained. On one hand, variations on metric give rise to the expression of the gravitational energy-momentum tensor, \( T_\alpha^\beta \equiv \kappa^2 \text{tr}[F_\alpha^\sigma F_\beta^\sigma - \frac{g_\alpha^\beta}{4} F_{\mu\nu} F^\mu_{\nu}], \) in other words

\[ T_\alpha^\beta = -T_t^\alpha^\beta - \kappa^2 g_\alpha^\beta \lambda^2, \quad (4) \]

where \( T_t^\alpha^\beta \equiv -m^2 \kappa^2 [3t^{\alpha\sigma} t^\beta_\sigma + 3t^\alpha^\sigma t^\beta_\sigma - t^\alpha^\sigma t^\beta_\sigma - t^\alpha^\sigma t^\beta_\sigma - (t^\alpha^\beta + t^\beta^\alpha) t^\sigma_\sigma - \frac{5g_\alpha^\beta}{2} t_{\mu\nu} t^{\mu\nu} + \frac{3g_\alpha^\beta}{2} t_{\mu\nu} t^{\mu\nu} + \frac{g_\alpha^\beta}{2} (t_\sigma^\sigma)^2] \) is the torsion contribution to the energy-momentum distribution and \( t^{\alpha^\beta} \equiv \frac{\varepsilon^{\nu\sigma\alpha}}{2} T^\beta_{\mu\nu}. \) This says, for example, that the quest of possible black hole solutions must reveal a dependence on parameters \( m^2 \) and \( \lambda^2. \)

On the other hand, variations on connection provide the following equations

\[ \frac{1}{\sqrt{-g}} \partial_\alpha (\sqrt{-g} F_\alpha^\lambda) + [A_\alpha, F^\alpha_\lambda] = J^\lambda, \quad (5) \]

where the current is \( (J^\lambda)^\nu_\sigma = m^2 (\delta^\lambda_\nu K^\rho_\sigma - \delta^\nu_\sigma K^\rho_\lambda + 2K^\nu_\lambda) \) and the contorsion \( K^\lambda_{\mu\nu} \equiv \frac{1}{2}(T^{\lambda}_{\mu\nu} + T^{\lambda}_{\nu\mu} + T^{\lambda}_{\mu\nu}). \) We can observe in (5) that contorsion and metric appear as sources of gravity, where the cosmological contribution is obviously hide in space-time metric. In a weak torsion regime, equation (5) takes a familiar shape: \( D_\alpha F_\alpha^\lambda = J^\lambda, \) where \( D_\alpha \) is the covariant derivative computed with the Christoffel’s symbols.

With a view on the performing of a perturbative study of the model (3), we wish to note some aspects of the variational analysis of free action (1) in 2 + 1 dimensions.
First, the connection shall be considered as a dynamical field whereas the space-time metric would be a fixed background, in order to explore (in some sense) the isolated behavior of torsion (contorsion) and avoid higher order terms in the field equations. For simplicity $\lambda = 0$ can be assumed and this condition allows us to choose a not necessarily conformally flat fixed background.

So, let us consider a Minkowskian space-time with a metric $\text{diag}(-1, 1, 1)$ provided and, obviously with no curvature nor torsion. The notation is

\begin{align*}
\mathcal{g}_{\alpha\beta} &= \eta_{\alpha\beta} , \\
\mathcal{F}^{\alpha\beta} &= 0 , \\
\mathcal{T}^{\lambda}_{\mu\nu} &= 0 .
\end{align*}

Thinking in variations

\[ A_\mu = \overline{A}_\mu + a_\mu , \quad |a_\mu| \ll 1 , \]

for this case $\overline{A}_\mu = 0$. Then, action (1) take the form

\[ S^{(3)\text{L}}_o = \kappa^2 \left\langle -\frac{1}{4} \text{tr} f^{\alpha\beta}(a) f_{\alpha\beta}(a) \right\rangle , \]

where $f_{\alpha\beta}(a) = \partial_\alpha a_\beta - \partial_\beta a_\alpha$ and (10) is gauge invariant under

\[ \delta a_\mu = \partial_\mu \omega , \]

with the gauge group $G = (U(1))^3$.

Let us suppose we take a Weitzenböck space-time instead a Minkowski one, as the fixed background. Then, the condition (8) must be relaxed (i.e.: $\overline{T}^{\lambda}_{\mu\nu} \neq 0$) and the linearized action would be $S^{(3)\text{L}}_{\text{Weitzenböck}} = S^{(3)\text{L}}_o - \kappa^2 \left\langle \text{tr} f^{\alpha\beta}(a) [\overline{A}_\alpha, a_\beta] + \frac{1}{2} \text{tr} [\overline{A}^\alpha, a^\beta] ([\overline{A}_\alpha, a_\beta] - [\overline{A}_\beta, a_\alpha]) \right\rangle$, which now is gauge variant under (11). In this context, the gauge invariance can be recovered through a similar technique of Stückelberg auxiliary fields for a Proca type model. This situation suggest we would be in
presence of possible massive contributions (maybe with or without ghosts and/or
tachyons) due just to the background.

In order to describe in detail the action (10), let us consider a symmetric-antisymmetric
decomposition of perturbed connection in the form
\[
(a_\mu)^\alpha_\beta = \epsilon^{\sigma\alpha}_\beta k_{\mu\sigma} + \delta^\alpha_\mu v_\beta - \eta_{\mu\beta} \psi^\alpha, \tag{12}
\]
where \(k_{\mu\nu} = k_{\nu\mu}\) and \(v_\mu\) are the symmetric and antisymmetric parts of the rank two
perturbed contorsion (i.e., the rank two contorsion is \(K_{\mu\nu} \equiv -\frac{1}{2} \epsilon^{\sigma\rho}_\nu K_{\sigma\mu\rho}\)), respectively. It can be noted that decomposition (12) is not were performed in irreducible
spin components and explicit writing down of the traceless part of \(k_{\mu\nu}\) would be
needed. This component will be considered when the study of reduced action be
performed. Using (12) in (10), we get
\[
S^{(3) L}_{0} = \kappa^2 \langle k_{\mu\nu} \Box k_{\mu\nu} + \partial_\mu k_{\mu\sigma} \partial_\nu k_{\nu\sigma} - 2 \epsilon^{\alpha\beta}_\mu \partial_\alpha v_\beta \partial_\nu k_{\nu\sigma} - v_\mu \Box v_\mu + (\partial_\mu v_\mu)^2 \rangle, \tag{13}
\]
which is gauge invariant under the following transformation rules (induced by (11))
\[
\delta k_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \tag{14}
\]
\[
\delta v_\mu = -\epsilon^{\sigma\rho}_\mu \partial_\sigma \xi_\rho, \tag{15}
\]
with \(\xi_\mu \equiv \frac{1}{4} \epsilon^{\beta}_\alpha \alpha_{\mu} w^{\alpha}_\beta\). These transformation rules clearly show that only the anti-
symmetric part of \(w\) is needed (i.e.: only three gauge fixation would be chosen).

Field equations from (13), are
\[
2 \Box k_{\mu\nu} - \partial_\mu \partial_\sigma k^{\sigma}_\nu - \partial_\nu \partial_\sigma k^{\sigma}_\mu + \epsilon^{\rho\beta}_\mu \partial_\rho v_\beta + \epsilon^{\rho\beta}_\nu \partial_\rho v_\beta = 0, \tag{16}
\]
\[
\epsilon^{\rho\beta}_\sigma \partial_\rho k^{\mu}_\beta + \Box v^\beta + \partial^\beta v^\mu = 0, \tag{17}
\]
and note that (17) satisfies the consistence condition \(\Box k - \partial_\mu \partial_\nu k^{\mu\nu} = 0\).

Divergence of (17) says that \(\partial_\mu v^\mu\) is a harmonic 0-form then, if we define \(\tilde{\partial}_\sigma \equiv \Box^{-\frac{1}{2}} \partial_\sigma\), the following relation can be written
\[
v^\beta = -\epsilon^{\rho\beta}_\sigma \tilde{\partial}_\rho k^{\mu}_\sigma, \tag{18}
\]
up to a harmonic transverse 1-form. Using (18) in (16), gives rise to

$$\Box k_{\mu\nu} - \partial_\alpha k^\alpha_{\mu} - \partial_\sigma k^\sigma_{\mu \nu} + \partial_\mu \partial_\nu k = 0 ,$$

(19)

up to a harmonic 0-form. In close analogy with 2 + 1 dimensional Einstein gravity, this means the equivalence of equation (19) with the Hilbert-Einstein theory in a Riemannian space-time suggests the expected fact: free gravity in 2 + 1 does not propagate degrees of freedom.

Now we explore the perturbation of the massive case given at (3) and with the help of (12), the linearized action is

$$S^{(3)L}_m = \kappa^2 \langle k^\mu_{\nu} \Box k^\mu_{\nu} + \partial_\alpha k^\alpha_{\mu\nu} \partial_\nu k^\nu_{\sigma} - 2 \epsilon^{\sigma\alpha\beta} \partial_\alpha v_\beta k^\nu_{\sigma} - v_\mu \Box v^\mu + (\partial_\mu v^\mu)^2 - m^2 (k^\mu_{\nu} k^\mu_{\nu} - k^2) \rangle ,$$

(20)

Using a transverse (T) and traceless (t) decomposition of the fields, given by

$$k_{\mu\nu} = k_{T \mu \nu} + \hat{\partial}_\mu \theta^T_{\nu} + \hat{\partial}_\nu \theta^T_{\mu} + \hat{\partial}_\mu \hat{\partial}_\nu \psi + \eta_{\mu\nu} \phi ,$$

(21)

$$v_\mu = v^T_\mu + \hat{\partial}_\mu v ,$$

(22)

with the subsidiary conditions

$$k_{T \mu \nu} = 0 , \ \partial_\mu k_{T \mu \nu} = 0 , \ \partial^\mu \theta^T_{\mu} = 0 , \ \partial^\mu v^T_{\mu} = 0 ,$$

(23)

the reduced action is

$$S^{(3)Ls}_m = \kappa^2 \langle k^T_{\mu \nu} (\Box - m^2) k^{T \mu \nu} \rangle ,$$

(24)

saying that the contorsion propagates two massive helicities ±2 in close analogy with the Fierz-Pauli model. Moreover, from (24) a positive definite Hamiltonian can be obtained, so $S^{(3)Ls}_m$ is unitary.

The quadratical Lagrangian density dependent in torsion and presented in (3), has been constructed without free parameters, with the exception of $m^2$, of course. It has a particular shape which only gives mass to the spin 2 component of the contorsion, as we see in the perturbative regime. There might be some reason for the doubt
about what happens with a possible “residual” gauge invariance of the model in the sense that, only one (spin 2) massive component does propagates. The answer is that the model lost its gauge invariance and it can be shown performing the study of symmetries through computation of the gauge generator chains. For this purpose, a 2 + 1 decomposition of (20) is performed, this means

$$S^{(3)\mathcal{L}}_m = \kappa^2 \left\{ \left[ -\dot{k}_{0i} + 2\partial_{n}k_{00} - 2\partial_{n}k_{ni} - 2\epsilon_{in}\dot{v}_{n} + 2\epsilon_{in}\partial_{n}v_{0} \right] k_{0i} + k_{ij} \dot{k}_{ij} \\
+ [2\epsilon_{nj}\partial_{n}k_{00} + 2\epsilon_{nj}\partial_{m}k_{nm} - \dot{v}_{j} - 2\partial_{j}v_{0}] \dot{v}_{j} + 2(\dot{v}_{0})^2 + k_{00}\Delta k_{00} \\
- 2k_{0i}\Delta k_{0i} + k_{ij}\Delta k_{ij} - (\partial_{i}k_{ii})^2 + \partial_{n}k_{ni}\partial_{m}k_{mi} - 2\epsilon_{ij}\partial_{i}v_{j}\partial_{n}k_{00} \\
- 2\epsilon_{im}\partial_{m}v_{0}\partial_{n}k_{nl} + v_{0}\Delta v_{0} - v_{i}\Delta v_{i} + (\partial_{n}v_{0})^2 \\
+ m^2[2k_{0i}k_{0i} - k_{ij}k_{ij} - 2k_{00}k_{ii} + (k_{ii})^2] \right\}, \quad (25)$$

where $\epsilon_{ij} \equiv \epsilon^{0}_{ij}$ and $\Delta \equiv \partial_{i}\partial_{i}$.

Next, the momenta are

$$\Pi \equiv \frac{\partial \mathcal{L}}{\partial \dot{k}_{00}} = 0, \quad (26)$$

$$\Pi^{i} \equiv \frac{\partial \mathcal{L}}{\partial \dot{k}_{0i}} = -2\dot{k}_{0i} - 2\epsilon_{in}\dot{v}_{n} + 2\partial_{i}k_{00} - 2\partial_{n}k_{ni} + 2\epsilon_{in}\partial_{n}v_{0}, \quad (27)$$

$$\Pi^{ij} \equiv \frac{\partial \mathcal{L}}{\partial \dot{k}_{ij}} = 2\dot{k}_{ij}, \quad (28)$$

$$P \equiv \frac{\partial \mathcal{L}}{\partial \dot{v}_{0}} = 4\dot{v}_{0}, \quad (29)$$

$$P^{j} \equiv \frac{\partial \mathcal{L}}{\partial \dot{v}_{j}} = -2\epsilon_{nj}\dot{k}_{0n} - 2\dot{v}_{j} + 2\epsilon_{nj}\partial_{n}k_{00} + 2\epsilon_{nj}\partial_{m}k_{mn} - 2\partial_{j}v_{0}, \quad (30)$$

and we establish the following commutation rules

$$\{k_{00}(x), \Pi(y)\} = \{v_{0}(x), P(y)\} = \delta^{2}(x - y), \quad (31)$$

$$\{k_{0i}(x), \Pi^{j}(y)\} = \{v_{i}(x), P^{j}(y)\} = \delta^{j}_{i}\delta^{2}(x - y), \quad (32)$$

$$\{k_{ij}(x), \Pi^{nm}(y)\} = \frac{1}{2}(\delta^{n}_{i}\delta^{m}_{j} + \delta^{m}_{i}\delta^{n}_{j})\delta^{2}(x - y). \quad (33)$$

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It can be noted that (26) is a primary constraint that we name

\[ G^{(K)} \equiv \Pi , \]  

where \( K \) means the initial index corresponding to a possible gauge generator chain, provided by the algorithm developed in reference[9]. Moreover, manipulating (28) and (30), other primary constraints appear

\[ G_i^{(K)} \equiv \partial_n k_{ni} - \epsilon_{in} \partial_n v_0 - \frac{\epsilon_{in}}{4} P^n + \frac{1}{4} \Pi^n , \]

and we observe that \( G^{(K)} \) and \( G_i^{(K)} \) are first class.

The preservation of constraints requires to obtain the Hamiltonian of the model. First at all, the Hamiltonian density can be written as

\[ H_0 = \Pi^i \dot{h}_0^i + \Pi^{ij} \dot{h}_{ij} + P \dot{v}_0 + P^i \dot{v}_i - \mathcal{L}, \]

in other words

\[ H_0 = \frac{\Pi^{ij} \Pi^{ij}}{4} + \frac{P^2}{8} - \frac{P^i P^i}{4} + \epsilon_{nj} \partial_n k_{nm} P^j + v_0 [\partial_i P^i + 4 \epsilon_{ml} \partial_m \partial_n k_{nl}] 
+ k_{00} [2 \partial_m \partial_n k_{mn} - \epsilon_{nm} \partial_n P^m + 2 k_{0i} \Delta k_{0i} - k_{ij} \Delta k_{ij} 
+ (\partial_i k_{i0})^2 - 2 \partial_n k_{ni} \partial_m k_{mi} + 2 \epsilon_{ij} \partial_i v_j \partial_n k_{n0} + v_i \Delta v_i - (\partial_n v_n)^2 
- m^2 [2 k_{0i} \dot{k}_{0i} - k_{ij} \dot{k}_{ij} + (k_{ii})^2] . \]

Then, the Hamiltonian is

\[ H_0 = \int dy^2 \mathcal{H}_0(y) \equiv \langle \mathcal{H}_0 \rangle_y \]

and the preservation of \( G^{(K)} \), defined in (34) is

\[ \{ G^{(K)}(x), H_0 \} = -2 \partial_m \partial_n k_{mn}(x) + \epsilon_{nm} \partial_n P^m(x) - 2 m^2 k_{ii}(x) . \]

The possible generators chain is given by the rule: \( "G^{(K-1)} + \{ G^{(K)}(x), H_0 \} = \text{combination of primary constraints}" \), then

\[ G^{(K-1)}(x) = 2 \partial_m \partial_n k_{mn}(x) - \epsilon_{nm} \partial_n P^m(x) + 2 m^2 k_{ii}(x) 
+ \langle a(x, y) G^{(K)}(y) + b^i(x, y) G_i^{(K)}(y) \rangle_y . \]
The preservation of $G_i^{(K)}$, defined in (35), is

$$\{G_i^{(K)}(x), H_0\} = \frac{\partial_n \Pi^{ni}(x)}{2} - \frac{\epsilon_i}{4} \partial_n P(x) + \frac{\epsilon_i}{2} \Delta v_n(x) + \frac{\epsilon_i}{2} \partial_n \partial_m v_m(x)$$

$$+ \frac{\epsilon_{nm}}{2} \partial_i \partial_n v_m(x) - (\Delta - m^2) k_0(x) , \quad (39)$$

then

$$G_i^{(K-1)}(x) = - \frac{\partial_n \Pi^{ni}(x)}{2} + \frac{\epsilon_i}{4} \partial_n P(x) - \frac{\epsilon_i}{2} \Delta v_n(x) - \frac{\epsilon_i}{2} \partial_n \partial_m v_m(x)$$

$$- \frac{\epsilon_{nm}}{2} \partial_i \partial_n v_m(x) + (\Delta - m^2) k_0(x)$$

$$+ \{a^i(x,y) G^{(K)}(y) + b^i_{ij}(x,y) G_j^{(K)}(y)\}_y . \quad (40)$$

The undefined objects $a(x,y)$, $b^i(x,y)$, $a^i(x,y)$ and $b^i_{ij}(x,y)$ in expressions (38) and (40), are functions or distributions. If it is possible, they can be fixed in a way that the preservation of $G^{(K-1)}(x)$ and $G_i^{(K-1)}(x)$ would be combinations of primary constraints. With this, the generator chains could be interrupted and we simply take $K = 1$. Of course, the order $K - 1 = 0$ generators must be first class, as every one. Next we can see that all these statements depend on the massive or non-massive character of the theory.

Taking a chain with $K = 1$, the candidates to generators of gauge transformation are (34), (35), (38) and (40). But, the only non null commutators are

$$\{G_i^{(1)}(x), G_j^{(0)}(y)\} = \frac{m^2}{4} \eta_{ij} \delta^2(x - y) , \quad (41)$$

$$\{G^{(0)}(x), G_i^{(0)}(y)\} = m^2 (\partial_i \delta^2(x - y) + \frac{b^i(x,y)}{4}) , \quad (42)$$

saying that the system of ”generators” is not first class. Moreover, the unsuccessful conditions (in the $m^2 \neq 0$ case) to interrupt the chains, are

$$\{G^{(0)}(x), H_0\} = m^2 (\Pi^{mn}(x) - 2 \partial_n k_0 m(x)) , \quad (43)$$

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\[ \{G^{(0)}_i(x), H_0\} = m^2(\partial_n k_{in}(x) + \partial_i k_{00}(x) - \partial_i k_{nn}(x)) , \]  

(44)

where we have fixed

\[ a(x, y) = 0 \]  

(45)

\[ b^i(x, y) = -2\partial^i \delta^2(x - y) \]  

(46)

\[ a^i(x, y) = 0 \]  

(47)

\[ b^i_j(x, y) = 0 \]  

(48)

All this indicates that in the case where \( m^2 \neq 0 \) there is not a first class consistent chain of generators and, then there is no gauge symmetry.

However, if we revisit the case \( m^2 = 0 \), conditions (43) and (44) are zero and the chains are interrupted. Now, the generators \( G^{(1)}, G^{(1)}_i, G^{(0)} \) and \( G^{(0)}_i \) are first class. Using (45)-(46), the generators are rewritten again

\[ G^{(1)} \equiv \Pi , \]  

(49)

\[ G^{(1)}_i \equiv \partial_n k_{ni} - \epsilon_{in} \partial_n v_0 - \frac{\epsilon_{in}}{4} P^n + \frac{1}{4} \Pi^i , \]  

(50)

\[ G^{(0)} = -\frac{\epsilon_{nm}}{2} \partial_n P^m - \frac{\partial_n \Pi^m}{2} , \]  

(51)

\[ G^{(0)}_i = -\frac{\partial_n \Pi^{ni}}{2} + \frac{\epsilon_{in}}{4} \partial_n P - \frac{\epsilon_{nm}}{2} \Delta v_n - \frac{\epsilon_{in}}{2} \partial_n \partial_m v_m - \frac{\epsilon_{nm}}{2} \partial_i \partial_n v_m + \Delta k_{0i} . \]  

(52)

Introducing the parameters \( \varepsilon(x) \) and \( \varepsilon^i(x) \), a combination of (49)-(52) is taken into account in the way that the gauge generator is

\[ G(\varepsilon, \varepsilon^i, \varepsilon, \varepsilon^i) = \langle \hat{\varepsilon}(x) G^{(1)}(x) + \hat{\varepsilon}(x) G^{(1)}_i(x) + \varepsilon(x) G^{(0)}(x) + \hat{\varepsilon}(x) G^{(0)}_i(x) \rangle , \]  

(53)
and with this, for example the field transformation rules (this means, \( \delta(...) = \{(...), G\})\) are written as

\[
\delta k_{00} = \dot{\varepsilon},
\]

\[
\delta k_{0i} = \frac{\dot{\varepsilon}^i}{4} + \frac{\partial_i \varepsilon}{2},
\]

\[
\delta k_{ij} = \frac{1}{4}(\partial_i \varepsilon_j + \partial_j \varepsilon_i),
\]

\[
\delta v_0 = \frac{\epsilon_{nm}}{4} \partial_n \varepsilon_m,
\]

\[
\delta v_i = \frac{\epsilon_{in}}{4} \dot{\varepsilon}_n - \frac{\epsilon_{in}}{2} \partial_n \varepsilon,
\]

and, redefining parameters as follows: \( \varepsilon \equiv 2\xi_0 \) and \( \varepsilon^i = 4\xi^i \), it is very easy to see that these rules match with (14) and (15), as we expected.

Finally, we underline that the massive model discussed, constructed in a similar way of a Proca theory (i.e.: quadratical in affine connection) but, with a general relativistic covariant behavior, preserves parity and it is unitary, at least in \( 2+1 \) dimension. Taking the metric as a fixed background and thinking about a non-Riemannian space-time, we have shown that contorsion propagates in an identical way as the Hilbert-Einsten-Fierz-Pauli model does. The lost gauge invariance does conduce to the appearance of two massive spin 2 degrees of freedom.

As we have given a glimpse, without introduction of explicit massive \( T^2 \)-terms in the action, it is possible to breakdown the residual gauge invariance (i.e.: reduction of general covariance which survives after a perturbative procedure) as a consequence rising from the choice of a particular fixed non-Riemannian background. The question of the existence of the alleged geometrical mechanism which gives mass to the fields and the existence of possible solutions of the non-perturbative theory, will be explored elsewhere.
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