Weak Forms of Soft Separation Axioms and Fixed Soft Points

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**ABSTRACT**

Realizing the importance of separation axioms in classifications of topological spaces and studying certain properties of fixed points, we formulate new soft separation axioms, namely \(tt\)-soft \(bT_i\) \((i = 0, 1, 2, 3, 4)\) and \(tt\)-soft \(b\)-regular spaces. Their definitions depend on three factors: soft \(b\)-open sets, total belong and total non-belong relations. In fact, they are genuine generalizations of \(p\)-soft \(T_i\)-spaces in the cases of \(i = 0, 1, 2\). With the help of examples, we study the relationships between them as well as with soft \(bT_i\) \((i = 0, 1, 2, 3, 4)\) and soft \(b\)-regular spaces. Some interesting properties of them are obtained under the conditions of soft hyperconnected and extended soft topological spaces. Also, we show that they are preserved under finite product soft spaces and soft \(b\)-homeomorphism mappings. Finally, we introduce a concept of \(b\)-fixed soft points and investigate its main properties.

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1. Introduction

Molodtsov’s soft set [1] was established in 1999 as a new technique for tackling real-life problems that suffer from imprecision and uncertainty. Molodtsov [1] investigated the merits of soft sets in comparison to probability theory and fuzzy set theory. The soft set-theoretic concepts were then introduced and investigated by a number of researchers, and many applications of soft sets were made in different disciplines such as decision-making [2], engineering [3] and medical science [2].

In 2011, Shabir and Naz [4] used soft sets defined over an initial universal set with a fixed set of parameters to introduce the concept of soft topological space. Researchers then studied several concepts of classical topological spaces through soft topological spaces and discussed the validity of some known topological results in soft topological spaces. Soft compactness was defined and studied by [27], in 2012. Hida [6] distinguished between two types of soft compactness depending on the belong relation. Al-shami et al. [7] studied almost soft compact and approximate soft compact spaces as extensions of a soft compact space. [8] utilized soft \(b\)-open sets to generalize soft compactness. The behavior of
soft closed sets in a soft Hausdorff was revised in [9] and many of the allegation results of soft separation axioms were corrected in [10] with the help of concrete examples. Al-shami and Kočinac [11] proved that the enriched and extended soft topologies are coincide. This result is very important in the study of the interrelationships between soft topological space and its parametric topological spaces.

The relations of belong and non-belong given in [4] were utilized in the studies of soft set and soft topology. However, the authors of [12], in 2018, came up new relations of belong and non-belong between an ordinary points and soft set, namely partial belong and total non-belong relations. In fact, these relations widely open the door to study and redefine many soft topological notions. This leads to obtain many fruitful properties and changes that can be seen significantly on the study of soft separation axioms as it was shown in [12–14]. These relations were studied in the contents of bipolar soft sets [15] and double framed soft sets [16].

Das and Samanta [17] studied the concept of a soft metric based on the soft real set and soft real numbers given in [18]. Wardowski [19] tackled the fixed point in the setup of soft topological spaces. Abbas et al. [20] presented soft contraction mappings and established a soft Banach fixed point theorem in the framework of soft metric spaces. Recently, many researchers explored fixed point findings in soft metric type spaces, see, for example, [21–24].

One of the main ideas that helps to prove some properties and eliminate some problems on soft topology is the concept of a soft point. It was first defined by Zorlutuna et al. [25] in order to study the interior points of a soft set and soft neighborhood systems. Then [18] and [26] redefined soft points concurrently to discuss soft metric spaces. In fact, the recent definition of a soft point shows similarity between many set-theoretic properties and their counterparts on soft setting. Two types of soft topologies, namely enriched soft topology and extended soft topology were introduced in [27] and [26], respectively. The equivalence between these two topologies have been recently proved in [11]. Recently, Al-shami [28, 29] has presented some practical applications of soft compact and soft separation axioms, and Kočinac et al. [30] have studied Menger spaces in soft setting.

We organized this paper as follows: Section 2 recalls the basic principles of soft sets and soft topologies. In Section 4, we introduce the concepts of \( tt \)-soft \( bT_i \) \((i = 0, 1, 2, 3, 4) \) and \( tt \)-soft \( b \)-regular spaces with respect to the ordinary points by using total belong and total non-belong relations. The relationships between them and their main properties are discussed with the help of interesting examples. In Section 5, we explore a \( b \)-fixed soft point theorem and study some main properties. In particular, we conclude under what conditions \( b \)-fixed soft points are preserved between a soft topological space and its parametric topological spaces. Section 6 concludes the paper.

2. Preliminaries

To well understand the results obtained in this study, we shall recall some basic concepts, definitions and properties from the literature.

2.1. Soft Sets

**Definition 2.1 ([1]):** Let \( X \) be the universal set and \( M \) be a set of parameters. A pair \((G, M)\) is said to be a soft set over \( X \) provided that \( G \) is a map of \( M \) into the power set \( 2^X \).
In this study, we use a symbol $G_M$ to refer to a soft set instead of $(G, M)$ and we identify it as ordered pairs $G_M = \{(m, G(m)) : m \in M \text{ and } G(m) \subseteq 2^X\}$. A family of all soft sets defined over $X$ with $M$ is denoted by $S(X_M)$.

**Definition 2.2 ([31]):** A soft set $G_M$ is said to be a subset of a soft set $H_M$, denoted by $G_M \subseteq H_M$, if $G(m) \subseteq H(m)$ for each $m \in M$.

The soft sets $G_M$ and $H_M$ are said to be soft equal if each one of them is a subset of the other.

**Definition 2.3 ([1, 12]):** Let $G_M$ be a soft set over $X$ and $x \in X$. We say that:

(i) $x \in G_M$, it is read: $x$ totally belongs to $G_M$, if $x \in G(m)$ for each $m \in M$.

(ii) $x \notin G_M$, it is read: $x$ does not partially belong to $G_M$, if $x \notin G(m)$ for some $m \in M$.

(iii) $x \in G_M$, it is read: $x$ partially belongs to $G_M$, if $x \in G(m)$ for some $m \in M$.

(iv) $x \notin G_M$, it is read: $x$ does not totally belong to $G_M$, if $x \notin G(m)$ for each $m \in M$.

**Remark 2.1:** Let $G_M$ be a soft set over $X$ and $x \in X$. We say that:

(i) $G_M$ totally contains $x$ if $x \in G_M$.

(ii) $G_M$ does not partially contain $x$ if $x \notin G_M$.

(iii) $G_M$ partially contains $x$ if $x \subseteq G_M$.

(iv) $G_M$ does not totally contain $x$ if $x \notin G_M$.

**Definition 2.4 ([32]):** The relative complement of a soft set $G_M$ is a soft set $G_M^c$, where $G^c : M \to 2^X$ is a mapping defined by $G^c(m) = X \setminus G(m)$ for all $m \in M$.

**Definition 2.5 ([12, 17, 26, 33]):** A soft set $(G, M)$ over $X$ is said to be:

(i) A null soft set, denoted by $\Phi$, if $G(m) = \emptyset$ for each $m \in M$.

(ii) An absolute soft set, denoted by $X$, if $G(m) = X$ for each $m \in M$.

(iii) A soft point $P_m^x$ if there are $m \in M$ and $x \in X$ such that $G(m) = \{x\}$ and $G(m') = \emptyset$ for each $m' \in M \setminus \{m\}$. We write that $P_m^x \in G_M$ if $x \in G(m)$.

(iv) A stable soft set if there is a subset $A$ of $X$ such that $G(m) = A$ for each $m \in M$ and it is denoted by $\hat{A}$. In particular, we denote by $x_M^A$ if $A = \{x\}$.

(v) A countable (resp. finite) soft set if $G(m)$ is countable (resp. finite) for each $m \in M$. Otherwise, it is said to be uncountable (resp. infinite).

**Definition 2.6 ([32, 33]):** Let $G_M$ and $H_M$ be two soft sets over $X$.

(i) Their intersection, denoted by $G_M \cap H_M$, is a soft set $U_M$, where a mapping $U : E \to 2^X$ is given by $U(m) = G(m) \cap H(m)$.

(ii) Their union, denoted by $G_M \cup H_M$, is a soft set $U_M$, where a mapping $U : E \to 2^X$ is given by $U(m) = G(m) \cup H(m)$.

**Definition 2.7 ([34]):** Let $G_M$ and $H_M$ be two soft sets over $X$ and $Y$, respectively. Then the Cartesian product of $G_M$ and $H_M$, denoted by $G \times H_M$, is defined as $(G \times H)(m, m') = G(m) \times H(m')$ for each $(m, m') \in M \times M$. 


The soft union and intersection operators were generalized for any number of soft sets in a similar way.

**Definition 2.8 ([35]):** A soft mapping between \( S(X_M) \) and \( S(Y_N) \) is a pair \((f, \phi)\), denoted also by \( f_\phi \), of mappings such that \( f : X \to Y, \phi : M \to N \). Let \( G_M \) and \( H_N \) be subsets of \( S(X_M) \) and \( S(Y_N) \), respectively. Then the image of \( G_M \) and pre-image of \( H_N \) are defined as follows.

(i) \( f_\phi(G_M) = (f_\phi(G))_N \) is a subset of \( S(Y_N) \) such \( f_\phi(G)(n) = \bigcup_{m \in \phi^{-1}(n)} f(G(m)) \) for each \( n \in N \).

(ii) \( f_\phi^{-1}(H)_M = (f_\phi^{-1}(H))_M \) is a subset of \( S(X_M) \) such that \( f_\phi^{-1}(H)(m) = f^{-1}(H(\phi(m))) \) for each \( m \in M \).

**Definition 2.9 ([25]):** A soft map \( f_\phi : S(X_M) \to S(Y_N) \) is said to be injective (resp. surjective, bijective) if \( \phi \) and \( f \) are injective (resp. surjective, bijective).

### 2.2. Soft Topology

**Definition 2.10 ([4]):** A family \( \tau \) of soft sets over \( X \) under a fixed set of parameters \( M \) is said to be a soft topology on \( X \) if it satisfies the following.

(i) \( \tilde{X} \) and \( \tilde{\Phi} \) are members of \( \tau \).

(ii) The intersection of a finite number of soft sets in \( \tau \) is a member of \( \tau \).

(iii) The union of an arbitrary number of soft sets in \( \tau \) is a member of \( \tau \).

The triple \((X, \tau, M)\) is called a soft topological space. A member in \( \tau \) is called soft open and its relative complement is called soft closed.

**Proposition 2.1 ([4]):** In \((X, \tau, M)\), a family \( \tau_m = \{G_M : G_M \in \tau\} \) is a classical topology on \( X \) for each \( m \in M \).

\( \tau_m \) is called a parametric topology and \((X, \tau_m)\) is called a parametric topological space.

**Definition 2.11 ([4]):** Let \((X, \tau, M)\) be a soft topological space and \( \emptyset \neq Y \subseteq X \). A family \( \tau_Y = \{\tilde{Y} \cap G_M : G_M \in \tau\} \) is called a soft relative topology on \( Y \) and the triple \((Y, \tau_Y, M)\) is called a soft subspace of \((X, \tau, M)\).

**Definition 2.12:** \((X, \tau, M)\) is said to be:

(i) soft hyperconnected ([36]) if it does not contain disjoint soft open sets;

(ii) soft extremally disconnected ([37]) if the closure of every soft open set is soft open.

**Definition 2.13 ([38]):** A subset \( G_M \) of \((X, \tau, M)\) is called soft \( b \)-open if \( G_M \subseteq \text{int}(\text{cl}(G_M)) \cap \text{cl}(\text{int}(G_M)) \).

**Theorem 2.2 ([38]):**

(i) Every soft open set is soft \( b \)-open.

(ii) The arbitrary union of soft \( b \)-open sets is soft \( b \)-open.
Definition 2.14 ([38]): Let $G_M$ be a subset of $(X, \tau, M)$. Then $G_M^b$ is the intersection of all soft $b$-closed sets containing $G_M$.

It is clear that: $x \in G_M^b$ if and only if $G_M \cap U_M \neq \emptyset$ for each soft $b$-open set $U_M$ totally containing $x$; and $P_M^b \in G_M^b$ if and only if $G_M \cap U_M \neq \emptyset$ for each soft $b$-open set $U_M$ totally containing $P_M^b$.

Theorem 2.3 ([39]): The soft intersection of finite soft pre-open subsets of a soft hyperconnected space is soft pre-open.

Definition 2.15 ([40]): $(X, \tau, M)$ is said to be:

(i) soft $bT_0$ if for every $x \neq y \in X$, there is a soft $b$-open set $U_M$ such that $x \in U_M$ and $y \notin U_M$; or $y \in U_M$ and $x \notin U_M$;

(ii) soft $bT_1$ if for every $x \neq y \in X$, there are two soft $b$-open sets $U_M$ and $V_M$ such that $x \in U_M$ and $y \notin U_M$; and $y \in V_M$ and $x \notin V_M$;

(iii) soft $bT_2$ if for every $x \neq y \in X$, there are two disjoint soft $b$-open sets $U_M$ and $V_M$ such that $x \in G_M$ and $y \notin F_M$;

(iv) soft $b$-regular if for every soft $b$-closed set $H_M$ and $x \in X$ such that $x \notin H_M$, there are two disjoint soft $b$-open sets $U_M$ and $V_M$ such that $H_M \subseteq U_M$ and $x \in V_M$;

(v) soft $b$-normal if for every two disjoint soft $b$-closed sets $H_M$ and $F_M$, there are two disjoint soft $b$-open sets $U_M$ and $V_M$ such that $H_M \subseteq U_M$ and $F_M \subseteq V_M$;

(vi) soft $bT_3$ (resp. soft $bT_4$) if it is both soft $b$-regular (resp. soft $b$-normal) and soft $bT_1$-space.

Definition 2.16 ([8]): A family $\{G_M : i \in I\}$ of soft $b$-open subsets of $(X, \tau, M)$ is said to be a soft $b$-open cover of $\tilde{X}$ if $\tilde{X} = \bigcup_{i \in I} G_M$. $(X, \tau, M)$ is said to be soft $b$-compact if every soft $b$-open cover of $\tilde{X}$ has a finite subcover.

To study the properties that preserved under soft $b^*$-homeomorphism maps, the concept of a soft $b$-irresolute map will be presented in this work under the name of a soft $b^*$-continuous map.

Definition 2.17 ([41]): $g : (X, \tau, M) \rightarrow (Y, \tau, M)$ is called soft $b^*$-continuous if the inverse image of each soft $b$-open set is soft $b$-open.

Proposition 2.4 ([41]): The soft $b^*$-continuous image of a soft $b$-compact set is soft $b$-compact.

Definition 2.18 ([38]): A soft map $f : (X, \tau, A) \rightarrow (Y, \tau, B)$ is said to be:

(i) soft $b$-continuous if the inverse image of each soft open set is soft $b$-open;

(ii) soft $b$-open (resp. soft $b$-closed) if the image of each soft open (resp. soft closed) set is soft $b$-open (resp. soft $b$-closed);

(iii) a soft $b$-homeomorphism if it is bijective, soft $b$-continuous and soft $b$-open.
**Definition 2.19:** A soft topology $\mathcal{T}$ on $X$ is said to be:

(i) an enriched soft topology $[27]$ if all soft sets $G_M$ such that $G(m) = \emptyset$ or $X$ are members of $\mathcal{T}$;
(ii) an extended soft topology $[26]$ if $\mathcal{T} = \{G_M : G(m) \in \tau_m \text{ for each } m \in M\}$, where $\tau_m$ is a parametric topology on $X$.

Al-shami and Kočinac $[11]$ proved the equivalence of enriched and extended soft topologies and obtained many useful results that help to study the relationships between soft topological spaces and their parametric topological spaces.

**Theorem 2.5 ([11]):** A subset $(F, M)$ of an extended soft topological space $(X, \mathcal{T}, M)$ is soft $\beta$-open if and only if each $m$-approximate element of $(F, M)$ is $\beta$-open.

**Proposition 2.6 ([5]):** Let $\{ (X_i, \tau_i, M) : i \in I \}$ be a family of pairwise disjoint soft topological spaces and $X = \bigcup_{i \in I} X_i$. Then the collection

$$\mathcal{T} = \{ (G, M) \subseteq X : (G, M) \subseteq \bigcap_{i \in I} X_i \text{ is a soft open set in } (X_i, \tau_i, M) \text{ for every } i \in I \}$$

defines a soft topology on $X$ with a fixed set of parameters $M$.

**Definition 2.20 ([5]):** The soft topological space $(X, \mathcal{T}, M)$ given in the above proposition is said to be the sum of soft topological spaces and is denoted by $(\oplus_{i \in I} X_i, \mathcal{T}, M)$.

**Theorem 2.7 ([5]):** A soft set $(G, M) \subseteq \bigoplus_{i \in I} X_i$ is soft $\beta$-open (resp. soft $\beta$-closed) in $(\oplus_{i \in I} X_i, \mathcal{T}, M)$ if and only if all $(G, M) \subseteq \bigcap_{i \in I} X_i$ are soft $\beta$-open (resp. soft $\beta$-closed) in $(X_i, \tau_i, M)$.

**Proposition 2.8 ([19]):** Let $g_\varphi : (X, \mathcal{T}, M) \to (X, \mathcal{T}, M)$ be a soft map such that $\bigcap_{n \in \mathbb{N}} g_\varphi^n(X)$ is a soft point $P_m$. Then $P_m$ is a unique fixed point of $g_\varphi$.

**Theorem 2.9 ([25]):** Let $(X, \mathcal{T}, A)$ and $(Y, \mathcal{T}, B)$ be two soft topological spaces and $\Omega = \{G_A \times F_B : G_A \in \mathcal{T} \text{ and } F_B \in \mathcal{T}\}$. Then the family of all arbitrary union of elements of $\Omega$ is a soft topology over $X \times Y$ under a fixed set of parameters $A \times B$.

3. Further Properties of Soft $\beta$-open Sets

In the following results, we prove under what condition the family of soft $\beta$-open subsets of $(X, \mathcal{T}, M)$ forms a soft topology over $X$ that is finer than $\mathcal{T}$. In fact, it will help us to study some properties of soft $\beta$-separation axioms and soft $\beta$-compact spaces, see, for example, Theorem (4.7) and Proposition (4.13).

**Theorem 3.1:** The following two properties are equivalent:

(i) $(X, \mathcal{T}, M)$ is soft extremally disconnected.
(ii) A soft set is soft $\beta$-open iff it is soft pre-open.

**Proof:** Let $(G, M)$ be a soft $\beta$-open set. Then $(G, M) \subseteq cl(int(cl(G, M))) \subseteq cl(G, M)$. By hypothesis, $cl(int(cl(G, M)))$ is soft open. This implies that $(G, M)$ is soft pre-open.
Conversely, let \((G, M)\) be a soft open set. Then \(cl(G, M) \subseteq cl(int(cl(G, M)))\). This means that \(cl(G, M)\) is a soft semi-open set. Then it is soft \(\beta\)-open. By hypothesis, it is soft pre-open. Therefore \(cl(G, M) \subseteq int(cl(G, M))\). Since \(int(cl(G, M)) \subseteq cl(G, M)\) is true, we obtain \(cl(G, M) = int(cl(G, M))\). Thus \(cl(G, M)\) is a soft open set. Hence, \((X, \tau, M)\) is soft extremally disconnected.

**Corollary 3.2:** The families of soft pre-open, soft \(b\)-open and soft \(\beta\)-open subsets of a soft extremally disconnected space coincide.

**Proposition 3.3:** Every soft hyperconnected space is soft extremally disconnected.

**Proof:** The proof follows from the fact that every soft open subset of soft hyperconnected space is soft dense.

**Theorem 3.4:** The soft intersection of finite soft \(b\)-open subsets of a soft hyperconnected space is soft \(b\)-open.

**Proof:** Let \(G_M\) and \(H_M\) be two soft \(b\)-open subsets of a soft hyperconnected space \((X, \tau, M)\). Then they are \(\beta\)-open sets. It follows from Theorem (3.1) that they are pre-open sets. It follows from Theorem (2.3) that \(G_M \cap H_M\) is soft pre-open. Hence, \(G_M \cap H_M\) is soft \(b\)-open, as required.

**Definition 3.1:** \((X, \tau, M)\) is said to be soft \(bT_2\) if for every \(P_m^X \neq P_{m'}^Y \in \tilde{X}\), there are two disjoint soft \(b\)-open sets \(U_M\) and \(V_M\) containing \(P_m^X\) and \(P_{m'}^Y\), respectively.

**Proposition 3.5:** If \((H, M)\) is a soft \(b\)-compact subset of a soft hyperconnected soft \(bT_2\)-space \((X, \tau, M)\), then \((H, M)\) is soft \(b\)-closed.

**Proof:** Let the given conditions be satisfied and let \(P_m^X \in \text{int}(H, M)^c\). Then for each \(P_{m'}^Y \in (H, M), \) there are two disjoint soft \(b\)-open sets \((U_i, M)\) and \((V_i, M)\) such that \(P_m^X \in (U_i, M)\) and \(P_{m'}^Y \in (V_i, M)\). It follows that \([(V_i, M) : i \in I]\) forms a soft \(b\)-open cover of \((H, M)\). Therefore, \((H, M) \subseteq \bigcup_{i=1}^n (V_i, M)\). Since \((X, \tau, M)\) is soft hyperconnected, then \(\bigcap_{i=1}^n (U_i, M) = (U, M)\) is a soft \(b\)-open set and since \((U, M) \cap \bigcup_{i=1}^n (V_i, M) = \emptyset,\) then \((U, M) \subseteq \text{int}(H, M)^c\). Thus \((H, M)^c\) is a soft \(b\)-open set. Hence \((H, M)\) is soft \(b\)-closed.

**Corollary 3.6:** If \((H, M)\) is a soft pre-compact stable subset of a soft hyperconnected soft \(bT_2\)-space \((X, \tau, M)\), then \((H, M)\) is soft \(b\)-closed.

**Proof:** Since \((H, M)\) is stable, then \(P_m^X \in (H, M)\) if and only if \(x \in (H, M)\). So by using similar technique of the above proof, the corollary holds.

4. **\(b\)-soft Separation Axioms**

By making use of the relations of total belong and total non-belong, we define new type of soft separation axioms, namely \(tt\)-soft \(bT_i\) \(\ (i = 0, 1, 2, 3, 4)\). We provide some examples to elucidate the relationships between them and to show some of their properties. Furthermore, we study the interrelations of them and topological and additive properties.
First of all, we see that it is necessary to classify containment into several categories as it is shown in remark below. Factually, this classification will play a vital role in redefining many soft theoretic-set and soft topological concepts, in particular, the concepts of soft interior and closure operators, soft compactness and soft separation axioms.

**Definition 4.1:** $(X, \tau, M)$ is said to be

(i) tt-soft $bT_0$ if for every $x \neq y \in X$, there exists a soft $b$-open set $U_M$ such that $x \in U_M$ and $y \notin U_M$ or $y \in U_M$ and $x \notin U_M$

(ii) tt-soft $bT_1$ if for every $x \neq y \in X$, there exist soft $b$-open sets $U_M$ and $V_M$ such that $x \in U_M$ and $y \notin U_M$; and $y \in V_M$ and $x \notin V_M$

(iii) tt-soft $bT_2$ if for every $x \neq y \in X$, there exist two disjoint soft $b$-open sets $U_M$ and $V_M$ such that $x \in U_M$ and $y \notin U_M$; and $y \in V_M$ and $x \notin V_M$

(iv) tt-soft $b$-regular if for every soft $b$-closed set $H_M$ and $x \in X$ such that $x \notin H_M$, there exist disjoint soft $b$-open sets $U_M$ and $V_M$ such that $H_M \subseteq U_M$ and $x \in V_M$

(v) tt-soft $bT_3$ (resp. tt-soft $bT_4$) if it is both tt-soft $b$-regular (resp. soft $b$-normal) and tt-soft $bT_1$.

**Remark 4.1:** It can be noted that: if $F_M$ and $G_M$ are disjoint soft set, then $x \in F_M$ if and only if $x \notin G_M$. This implies that $(X, \tau, M)$ is a tt-soft $bT_2$-space if and only if is a soft $bT_2$-space. That is, the concepts of a tt-soft $bT_2$-space and a soft $bT_2$-space are equivalent.

We can say that: $(X, \tau, M)$ is tt-soft $bT_2$ if for every $x \neq y \in X$, there exist two disjoint soft $b$-open sets $U_M$ and $V_M$ totally contain $x$ and $y$, respectively.

**Remark 4.2:** The soft $b$-regular spaces imply a strict condition on the shape of soft $b$-open and soft $b$-closed subsets. To explain this matter, let $F_M$ be a soft $b$-closed set such that $x \notin H_M$. Then we have two cases:

(i) There are $m, m' \in M$ such that $x \notin H(m)$ and $x \in H(m')$. This case is impossible because there do not exist two disjoint soft sets $U_M$ and $V_M$ containing $x$ and $H_M$, respectively.

(ii) For each $m \in M, x \notin H(m)$. This implies that $H_M$ must be stable.

As a direct consequence, we infer that every soft $b$-closed and soft $b$-open subsets of a soft $b$-regular space must be stable. However, this matter does not hold on the tt-soft $b$-regular spaces because we replace a partial non-belong relation by a total non-belong relation. Therefore a tt-soft $b$-regular space need not be stable.

**Proposition 4.1:**

(i) Every tt-soft $bT_i$-space is soft $bT_i$ for $i = 0, 1, 4$.

(ii) Every soft $b$-regular space is tt-soft $b$-regular.

(iii) Every soft $bT_3$-space is tt-soft $bT_3$.

**Proof:** The proofs of (i) and (ii) follow from the fact that a total non-belong relation $\notin$ implies a partial non-belong relation $\notin$.

To prove (iii), it suffices to prove that a soft $bT_i$-space is tt-soft $bT_i$ when $(X, \tau, M)$ is soft $b$-regular. Suppose $x \neq y \in X$. Then there exist two soft $b$-open sets $U_M$ and $V_M$ such that $x \in U_M$ and $y \notin U_M$; and $y \in V_M$ and $x \notin V_M$. Since $U_M$ and $V_M$ are soft $b$-open subsets of a
soft $b$-regular space, then they are stable. So $y \not\in U_M$ and $x \not\in V_M$. Thus $(X, \tau, M)$ is $tt$-soft $bT_1$. Hence, we obtain the desired result.

The following examples clarify that the converse of the above proposition is not always true.

**Example 1:** Let $M = \{m_1, m_2\}$. A family $\tau = \{\tilde{\Phi}, \tilde{X}, \{(m_1, \{x\}), (m_2, \emptyset)\}\}$ is a soft topology on $X = \{x, y, z\}$. Note that an $m_1$-approximate of any soft $b$-open set totally containing $y$ or $z$ contains $x$ as well. Therefore $(X, \tau, M)$ is not $tt$-soft $bT_0$. On the other hand, one can examine that $(X, \tau, M)$ is a soft $bT_1$-space.

**Example 2:** Let $M = \{m_1, m_2\}$. A family $\tau = \{\tilde{\Phi}, \tilde{X}, G_i_M : i = 1, 2\}$ is a soft topology over $X = \{x, y\}$, where

$G_{1M} = \{(m_1, \{x\}), (m_2, \{y\})\}$ and $G_{2M} = \{(m_1, \{y\}), (m_2, \{x\})\}$.

It can be checked that every subset of $(X, \tau, M)$ is soft $b$-open. This means that there exist unstable soft $b$-open subsets of $(X, \tau, M)$. Therefore $(X, \tau, M)$ is not soft $b$-regular and hence it is not soft $bT_3$. However, $(X, \tau, M)$ is a $tt$-soft $b$-regular and $tt$-soft $bT_1$. Hence, it is $tt$-soft $bT_3$.

Before we show the relationship between $tt$-soft $bT_i$-spaces, we need to prove the following useful lemma.

**Lemma 4.2:** $(X, \tau, M)$ is a $tt$-soft $bT_1$-space if and only if $x_M$ is soft $b$-closed for every $x \in X$.

**Proof:** *Necessity:* For each $y \in X \setminus \{x\}$, there is a soft $b$-open set $G_{iM}$ such that $y \in G_{iM}$ and $x \not\in G_{iM}$. Therefore $X \setminus \{x\} = \bigcup_{i \in I} G_i(m)$ and $x \not\in \bigcup_{i \in I} G_i(m)$ for each $m \in M$. Thus $\bigcup_{i \in I} G_{iM} = X \setminus \{x\}$ is soft $b$-open. Hence, $x_M$ is soft $b$-closed.

*Sufficiency:* Let $x \neq y$. By hypothesis, $x_M$ and $y_M$ are soft $b$-closed sets. Then $x_M^c$ and $y_M^c$ are soft $b$-open sets such that $x \in (y_M)^c$ and $y \in (x_M)^c$. Obviously, $y \not\in (y_M)^c$ and $x \not\in (x_M)^c$. Hence, $(X, \tau, M)$ is $tt$-soft $bT_1$.

**Proposition 4.3:** Every $tt$-soft $bT_i$-space is $tt$-soft $bT_{i-1}$ for $i = 1, 2, 3, 4$.

**Proof:** We prove the proposition in the cases of $i = 3, 4$. The other cases follow similar lines.

For $i = 3$, let $x \neq y$ in a $tt$-soft $bT_3$-space $(X, \tau, M)$. Then $x_M$ is soft $b$-closed. Since $y \not\in x_M$ and $(X, \tau, M)$ is $tt$-soft $b$-regular, then there are disjoint soft $b$-open sets $G_M$ and $F_M$ such that $x_M \subseteq G_M$ and $y \in F_M$. Therefore $(X, \tau, M)$ is $tt$-soft $bT_2$.

For $i = 4$, let $x \in X$ and $H_M$ be a soft $b$-closed set such that $x \not\in H_M$. Since $(X, \tau, M)$ is $tt$-soft $bT_1$, then $x_M$ is soft $b$-closed. Since $x_M \cap H_M = \emptyset$ and $(X, \tau, M)$ is soft $b$-normal, then there are disjoint soft $b$-open sets $G_M$ and $F_M$ such that $H_M \subseteq G_M$ and $x_M \subseteq F_M$. Hence, $(X, \tau, M)$ is $tt$-soft $bT_3$.

The following examples show that the converse of the above proposition is not always true.
Example 3: Let $M = \{m_1, m_2\}$. A family $\tau = \{\Phi, \bar{X}, ((m_1, \{x\}), (m_2, \{x\}))\}$ is a soft topology on $X = \{x, y\}$. The a family of all soft $b$-open subsets of $(X, \tau, M)$ is $S(X_M) \setminus \{G_i : i = 1, 2, 3\}$, where

- $G_{1M} = \{(m_1, \{y\}), (m_2, \{y\})\}$;
- $G_{2M} = \{(m_1, \{y\}), (m_2, \emptyset)\}$ and
- $G_{3M} = \{(m_1, \emptyset), (m_2, \{y\})\}$.

Now, $x \neq y$. We have $x \in \{(m_1, \{x\}), (m_2, \{x\})\}$ and $y \notin \{(m_1, \{x\}), (m_2, \{x\})\}$. Therefore, $(X, \tau, M)$ is tt-soft $bT_0$. However, there does not exist a soft $b$-open set such that $x$ does not totally belong to it. Hence, $(X, \tau, M)$ is not tt-soft $bT_1$.

Example 4: It is well known that a soft topological space is a classical topological space if $E$ is a singleton. Then it suffices to consider examples that satisfy an $bT_2$-space but not $bT_3$; satisfy an $bT_3$-space but not $bT_4$.

In what follows, we establish some properties of tt-soft $bT_i$ and tt-soft $b$-regular.

Lemma 4.4: Let $U_M$ be a subset of $(X, \tau, M)$ and $x \in X$. Then $x \notin U_M^b$ iff there exists a soft $b$-open set $V_M$ totally containing $x$ such that $U_M \cap V_M = \Phi$.

Proof: Let $x \notin U_M^b$. Then $x \in (U_M^b)^c = V_M$. So $U_M \cap V_M = \Phi$. Conversely, if there exists a soft $b$-open set $V_M$ totally containing $x$ such that $U_M \cap V_M = \Phi$, then $U_M \subseteq V_M$. Therefore $U_M^b \subseteq V_M^c$. Since $x \notin V_M$, then $x \notin U_M^b$. ■

Proposition 4.5: If $(X, \tau, M)$ is a tt-soft $bT_0$-space, then $x_M^b \neq y_M^b$ for every $x \neq y \in X$.

Proof: Let $x \neq y$ in a tt-soft $bT_0$-space. Then there is a soft $b$-open set $U_M$ such that $x \in U_M$ and $y \notin U_M$ or $y \in U_M$ and $x \notin U_M$. Say, $x \in U_M$ and $y \notin U_M$. Now, $y_M \cap U_M = \Phi$. So, by the above lemma, $x \notin y_M^b$. But $x \in x_M^b$. Hence, we obtain the desired result. ■

Corollary 4.6: If $(X, \tau, M)$ is a tt-soft $bT_0$-space, then $P_X^b \neq P_{X'}^b$ for all $x \neq y$ and $m, m' \in M$.

Theorem 4.7: Let $M$ be a finite set and $(X, \tau, M)$ be soft hyperconnected. Then $(X, \tau, M)$ is a tt-soft $bT_1$-space if and only if $x_M = \bigcap (U_M : x \in U_M \in \tau^b)$ for each $x \in X$.

Proof: To prove the ‘if’ part, let $y \in X$. Then for each $x \in X \setminus \{y\}$, we have a soft $b$-open set $U_M$ such that $x \in U_M$ and $y \notin U_M$. Therefore $y \notin \bigcap (U_M : x \in U_M \in \tau^b)$. Since $y$ is chosen arbitrary, then the desired result is proved.

To prove the ‘only if’ part, let the given conditions be satisfied and let $x \neq y$. Let $|M| = n$. Since $y \notin x_M$, then for each $j = 1, 2, \ldots, n$ there is a soft $b$-open set $U_{iM}$ such that $y \notin U_i$ and $x \in U_{iM}$. Since $(X, \tau, M)$ is soft hyperconnected, then it follows from Theorem (3.4) that $\bigcap_{i=1}^n U_{iM}$ is a soft $b$-open set such that $y \notin \bigcap_{i=1}^n U_{iM}$ and $x \in \bigcap_{i=1}^n U_{iM}$. Similarly, we can get a soft $b$-open set $V_M$ such that $y \in V_M$ and $x \notin V_M$. Thus $(X, \tau, M)$ is a tt-soft $bT_1$-space. ■

Theorem 4.8: If $(X, \tau, M)$ is an extended tt-soft $bT_1$-space, then $P_X^b$ is soft $b$-closed for all $P_X^b \in \bar{X}$. 
**Proof:** It follows from Lemma (4.2) that \(X \setminus \{x\}\) is a soft b-open set. Since \((X, \tau, M)\) is extended, then a soft set \(H_m\), where \(H(m) = \emptyset\) and \(H(m') = X\) for each \(m' \neq m\), is a soft b-open set. Therefore \(X \setminus \{x\}\) \(\cup H_m\) is soft b-open. Thus \((X \setminus \{x\}\) \(\cup H_m)\) is soft b-closed.

**Corollary 4.9:** If \((X, \tau, M)\) is an extended tt-soft b\(T_1\)-space, then the intersection of all soft b-open sets containing \(U_M\) is exactly \(U_M\) for each \(U_M \subseteq X\).

**Proof:** Let \(U_M\) be a soft subset of \(\widehat{X}\). Since \(P^b_M\) is a soft b-closed set for every \(P^b_M \in U^c_M\), then \(\widehat{X} \setminus P^b_M\) is a soft b-open set containing \(U_M\). Therefore \(U_M = \bigcap(\widehat{X} \setminus P^b_M : P^b_M \in U^c_M)\), as required.

**Theorem 4.10:** Let \((X, \tau, M)\) be finite soft hyperconnected. Then \((X, \tau, M)\) is tt-soft b\(T_2\) if and only if it is tt-soft b\(T_1\).

**Proof:** **Necessity:** It is obtained from Proposition (4.3).

**Sufficiency:** For each \(x \neq y\), we have \(x_M\) and \(y_M\) are soft b-closed sets. Since \(X\) is finite, then \(\bigcup_{y \in X \setminus \{x\}} y_M\) and \(\bigcup_{x \in X \setminus \{x\}} x_M\) are soft b-closed sets. Since \((X, \tau, M)\) is soft hyperconnected, then \((\bigcup_{y \in X \setminus \{x\}} y_M)^c = x_M\) and \((\bigcup_{x \in X \setminus \{x\}} x_M)^c = y_M\) are soft b-open sets. The disjointness of \(x_M\) and \(y_M\) end the proof that \((X, \tau, M)\) is tt-soft b\(T_2\).

**Theorem 4.11:** \((X, \tau, M)\) is tt-soft b-regular iff for every soft b-open subset \(F_M\) of \((X, \tau, M)\) totally containing \(x\), there is a soft b-open set \(V_M\) such that \(x \in V_M \subseteq V^b_M \subseteq F_M\).

**Proof:** Let \(x \in X\) and \(F_M\) be a soft b-open set totally containing \(x\). Then \(F^c_M\) is b-closed and \(x_M \cap F^c_M = \emptyset\). Therefore there are disjoint soft b-open sets \(U_M\) and \(V_M\) such that \(F^c_M \subseteq U_M\) and \(x \in V_M\). Thus \(V_M \subseteq U_M \subseteq F_M\). Conversely, let \(F^c_M\) be a soft b-closed set. Then for each \(x \notin F^c_M\), we have \(x \in V_M\). By hypothesis, there is a soft b-open set \(V_M\) totally containing \(x\) such that \(V^b_M \subseteq F_M\). Therefore \(F^c_M \subseteq (V^b_M)^c\) and \(V^b_M \subseteq (V^b_M)^c\). Thus \((X, \tau, M)\) is tt-soft b-regular, as required.

**Theorem 4.12:** The following properties are equivalent if \((X, \tau, M)\) is a tt-soft b-regular space.

(i) a tt-soft b\(T_2\)-space,
(ii) a tt-soft b\(T_1\)-space,
(iii) a tt-soft b\(T_0\)-space.

**Proof:** The directions \((i) \rightarrow (ii)\) and \((ii) \rightarrow (iii)\) are obvious.

To prove \((iii) \rightarrow (i)\), let \(x \neq y\) in a tt-soft b\(T_0\)-space \((X, \tau, M)\). Then there exists a soft b-open set \(G_M\) such that \(x \in G_M\) and \(y \notin G_M\), or \(y \in G_M\) and \(x \notin G_M\). Say, \(x \in G_M\) and \(y \notin G_M\). Obviously, \(x \notin G^c_M\) and \(y \notin G^c_M\). Since \((X, \tau, M)\) is tt-soft b-regular, then there exist two disjoint soft b-open sets \(U_M\) and \(V_M\) such that \(x \in U_M\) and \(y \in G^c_M \subseteq V_M\). Hence, \((X, \tau, M)\) is tt-soft b\(T_2\).

**Proposition 4.13:** Let \((X, \tau, M)\) be finite soft hyperconnected. If \((X, \tau, M)\) is a tt-soft b\(T_2\)-space, then it is tt-soft b-regular.
Proof: Let $H_m$ be a soft $b$-closed set and $x \in X$ such that $x \notin H_m$. Then $x \neq y$ for each $y \in H_m$. By hypothesis, there are two disjoint soft $b$-open sets $U_m$ and $V_m$ such that $x \in U_m$ and $y \in V_m$. Since $(y : y \in X)$ is a finite set, then there is a finite number of soft $b$-open sets $V_{i,m}$ such that $H_m \subseteq \bigcup_{i=1}^{n} V_{i,m}$. Since $(X, \tau, M)$ is soft hyperconnected, then it follows from Theorem (3.4) that $\bigcap_{i=1}^{n} U_{i,m}$ is a soft $b$-open set containing $x$. Since $\bigcap_{i=1}^{n} U_{i,m} \bigcap_{i=1}^{n} V_{i,m} = \Phi$, then $(X, \tau, M)$ is tt-soft $b$-regular.

Corollary 4.14: The following properties are equivalent if $(X, \tau, M)$ is finite soft hyperconnected.

(i) a tt-soft $bT_3$-space,
(ii) a tt-soft $bT_2$-space,
(iii) a tt-soft $bT_1$-space.

Proof: The directions (i) $\rightarrow$ (ii) and (ii) $\rightarrow$ (iii) follow from Proposition (4.3).

The direction (iii) $\rightarrow$ (ii) follows from Theorem (4.10).

The direction (ii) $\rightarrow$ (i) follows from Proposition (4.13).

Theorem 4.15: Let $(X, \tau, M)$ be extended and $i = 0, 1, 2, 3, 4$. Then $(X, \tau, M)$ is tt-soft $bT_i$ iff $(X, \tau_m)$ is $bT_i$ for each $m \in M$.

Proof: We prove the theorem in the case of $i = 4$ and one can similarly prove the other cases.

Necessity: Let $x \neq y$ in $X$. Then there exist two soft $b$-open sets $U_m$ and $V_m$ such that $x \in U_m$ and $y \notin U_m$; and $y \in V_m$ and $x \notin V_m$. Obviously, $x \in U(m)$ and $y \notin U(m)$; and $y \in V(m)$ and $x \notin V(m)$. Since $(X, \tau, M)$ is extended, then it follows from Theorem (2.5) that $U(m)$ and $V(m)$ are $b$-open subsets of $(X, \tau_m)$ for each $m \in M$. Thus $(X, \tau_m)$ is a $bT_1$-space. To prove that $(X, \tau, M)$ is $b$-normal, let $F_m$ and $H_m$ be two disjoint $b$-closed subsets of $(X, \tau_m)$. Let $F_m$ and $H_m$ be two soft sets given by $F(m) = F_m, H(m) = H_m$ and $F(m') = H(m') = \emptyset$ for each $m' \neq m$. It follows, from Theorem (2.5) that $F_m$ and $H_m$ are two disjoint soft $b$-closed subsets of $(X, \tau, M)$. By hypothesis, there exist two disjoint soft $b$-open sets $G_m$ and $W_m$ such that $F_m \subseteq G_m$ and $H_m \subseteq W_m$. This implies that $F(m) = F_m \subseteq G(m)$ and $H(m) = H_m \subseteq W(m)$. Since $(X, \tau, M)$ is extended, then it follows from Theorem (2.5) that $G(m)$ and $W(m)$ are $b$-open subsets of $(X, \tau_m)$. Thus $(X, \tau_m)$ it is a $b$-normal space. Hence, it is a $bT_4$-space.

Sufficiency: Let $x \neq y$ in $X$. Then there exists two $b$-open subsets $U_m$ and $V_m$ of $(X, \tau_m)$ such that $x \in U_m$ and $y \notin U_m$; and $y \in V_m$ and $x \notin V_m$. Let $U_m$ and $V_m$ be two soft sets given by $U(m) = U_m, V(m) = V_m$ for each $m \in M$. Since $(X, \tau, M)$ is extended, then it follows from Theorem (2.5) that $U_m$ and $V_m$ are soft $b$-open subsets of $(X, \tau, M)$ such that $x \in U_m$ and $y \notin U_m$; and $y \in V_m$ and $x \notin V_m$. Thus $(X, \tau, M)$ is a tt-soft $bT_1$-space. To prove that $(X, \tau, M)$ is soft $b$-normal, let $F_m$ and $H_m$ be two disjoint soft $b$-closed subsets of $(X, \tau, M)$. Since $(X, \tau, M)$ is extended, then it follows from Theorem (2.5) that $F(m)$ and $H(m)$ are two disjoint $b$-closed subsets of $(X, \tau_m)$. By hypothesis, there exist two disjoint $b$-open subsets $G_m$ and $W_m$ of $(X, \tau_m)$ such that $F(m) \subseteq G_m$ and $H(m) \subseteq W_m$. Let $G_m$ and $W_m$ be two soft sets given by $G(m) = G_m$ and $W(m) = W_m$ for each $m \in M$. Since $(X, \tau, M)$ is extended, then it follows from Theorem (2.5) that $G_m$ and $W_m$ are two disjoint soft $b$-open subsets of $(X, \tau, M)$ such that $F_m \subseteq G_m$ and $H_m \subseteq W_m$. Thus $(X, \tau, M)$ is soft $b$-normal. Hence, it is a tt-soft $bT_4$-space.
In the following examples, we show that there is no a relationship between soft topological space and their parametric topological spaces in terms of separation axioms if a condition of an extended soft topological space given in the above theorem does not exist.

**Example 5:** Let \((X, \tau, M)\) be the same as in Example (1). We showed that \((X, \tau, M)\) is not \(tt\)-soft \(bT_0\). On the other hand, \(\tau_{m_1} = \{\emptyset, X, \{x\}\}\) is a parametric topologies on \(X\). It can be checked that \((X, \tau_{m_1})\) is \(bT_0\).

**Example 6:** Let \(M = \{m_1, m_2\}\). A family \(\tau = \{\emptyset, \widetilde{X}, \{(m_1, \{x\}), (m_2, \{y\})\}\}\) is a soft topology on \(X = \{x, y\}\). The a family of all soft \(b\)-open subsets of \((X, \tau, M)\) is \(S(X_M) \setminus \{G_{m_i} : i = 1, 2, 3\}\), where

\[
\begin{align*}
G_{m_1} &= \{(m_1, \{y\}), (m_2, \{x\})\}; \\
G_{m_2} &= \{(m_1, \{y\}), (m_2, \emptyset)\} \text{ and} \\
G_{m_3} &= \{(m_1, \emptyset), (m_2, \{x\})\}.
\end{align*}
\]

It can be checked that \((X, \tau, M)\) is \(tt\)-soft \(bT_4\). On the other hand, the two parametric topological spaces \((X, \tau_{m_1})\) and \((X, \tau_{m_2})\) are not \(bT_1\).

**Lemma 4.16:** Let \((G, A)\) and \((H, B)\) be two subsets of \((X_1, \tau_1, A)\) and \((X_2, \tau_2, B)\), respectively. Then:

(i) \(cl(G, A) \times cl(H, B) = cl((G, A) \times (H, B))\).

(ii) \(int(G, A) \times int(H, B) = int((G, A) \times (H, B))\).

**Proof:** (i) **Necessity:** Suppose that \(P^{(x, y)}_{(a, b)} \notin cl((G, A) \times (H, B))\). Then there exists a soft open subset \(\widetilde{U} \times \widetilde{V}\) of \((X \times Y, \tau \times \tau, A \times B)\) containing \(P^{(x, y)}_{(a, b)}\) such that \((\widetilde{U} \times \widetilde{V}) \cap (G, A) = \emptyset\). Hence, \(cl((G, A) \times (H, B)) = \emptyset\). This means \(P^{(x, y)}_{(a, b)} \notin cl(G, A) \times cl(H, B)\).

**Sufficiency:** Suppose that \(P^{(x, y)}_{a} \notin cl(G, A)\) or \(P^{(x, y)}_{b} \notin cl(H, B)\). Without loss of generality, let \(P^{(x, y)}_{a} \notin cl(G, A)\). Then there exists a soft open subset \(\widetilde{U} \times \widetilde{V}\) of \((X \times Y, \tau \times \tau, A \times B)\) containing \(P^{(x, y)}_{(a, b)}\) such that \((\widetilde{U} \times \widetilde{V}) \cap (G, A) = \emptyset\). Hence, \(cl((G, A) \times (H, B)) = \emptyset\). This means \(P^{(x, y)}_{(a, b)} \notin cl((G, A) \times (H, B))\).

(ii) By using a similar way above, one can prove item (ii).

**Theorem 4.17:** The property of being a \(tt\)-soft \(bT_i\)-space \((i = 0, 1, 2)\) is preserved under a finite product soft spaces.

**Proof:** We prove the theorem in case of \(i = 2\). The other cases follow similar lines.

Let \((X_1, \tau_1, M_1)\) and \((X_2, \tau_2, M_2)\) be two \(tt\)-soft \(bT_2\)-spaces and let \((x_1, y_1) \neq (x_2, y_2)\) in \(X_1 \times X_2\). Then \(x_1 \neq x_2\) or \(y_1 \neq y_2\). Without loss of generality, let \(x_1 \neq x_2\). Then there exist two disjoint soft \(b\)-open subsets \(G_{E_1}\) and \(H_{E_1}\) of \((x_1, \tau_1, M_1)\) such that \(x_1 \in G_{E_1}\) and \(x_2 \notin G_{E_1}\); and \(x_2 \in H_{E_1}\) and \(x_1 \notin H_{E_1}\). Obviously, \(G_{E_1} \times \tilde{X}_2\) and \(H_{E_1} \times \tilde{X}_2\) are two disjoint soft \(b\)-open subsets \(X_1 \times X_2\) such that \((x_1, y_1) \in G_{E_1} \times \tilde{X}_2\) and \((x_2, y_2) \notin G_{E_1} \times \tilde{X}_2\) and \((x_2, y_2) \in H_{E_1} \times \tilde{X}_2\) and \((x_1, y_1) \notin H_{E_1} \times \tilde{X}_2\). Hence, \(X_1 \times X_2\) is a \(tt\)-soft \(bT_2\)-space.
**Theorem 4.18:** The property of being a tt-soft $bT_i$-space is an additive property for $i = 0, 1, 2, 3, 4$.

**Proof:** To prove the theorem in the cases of $i = 2$. Let $x \neq y \in \bigoplus_{i \in I} X_i$. Then we have the following two cases:

1. There exists $i_0 \in I$ such that $x, y \in X_{i_0}$. Since $(X_{i_0}, \tau_{i_0}, M)$ is tt-soft $bT_2$, then there exist two disjoint soft $b$-open subsets $G_M$ and $H_M$ of $(X_{i_0}, \tau_{i_0}, M)$ such that $x \in G_M$ and $y \in H_M$. It follows from Theorem (2.7) that $G_M$ and $H_M$ are disjoint soft $b$-open subsets of $(\bigoplus_{i \in I} X_i, \tau, M)$.

2. There exist $i_0 \neq j_0 \in I$ such that $x \in X_{i_0}$ and $y \in X_{j_0}$. Now, $\tilde{X}_{i_0}$ and $\tilde{X}_{j_0}$ are soft $b$-open subsets of $(X_{i_0}, \tau_{i_0}, M)$ and $(X_{j_0}, \tau_{j_0}, M)$, respectively. It follows from Theorem (2.7), that $\tilde{X}_{i_0}$ and $\tilde{X}_{j_0}$ are disjoint soft $b$-open subsets of $(\bigoplus_{i \in I} X_i, \tau, M)$.

It follows from the two cases above that $(\bigoplus_{i \in I} X_i, \tau, M)$ is a tt-soft $bT_2$-space. The theorem can be proved similarly in the cases of $i = 0, 1$.

To prove the theorem in the cases of $i = 3$ and $i = 4$, it suffices to prove the tt-soft $b$-regularity and soft $b$-normality, respectively.

First, we prove the tt-soft $b$-regularity property. Let $F_M$ be a soft $b$-closed subset of $(\bigoplus_{i \in I} X_i, \tau, M)$ such that $x \notin F_M$. It follows from Theorem (2.7) that $\bigcap_{i \in I} \tilde{X}_i$ is soft $b$-closed in $(X_i, \tau_i, M)$ for each $i \in I$. Since $x \in \bigoplus_{i \in I} X_i$, there is only $i_0 \in I$ such that $x \in X_{i_0}$. This implies that there are disjoint soft $b$-open subsets $G_M$ and $H_M$ of $(X_{i_0}, \tau_{i_0}, M)$ such that $\bigcap_{i \in I} \tilde{X}_i = G_M$ and $y \in H_M$. Now, $G_M \bigcup_{i \neq i_0} \tilde{X}_i$ is a soft $b$-open subset of $(\bigoplus_{i \in I} X_i, \tau, M)$ containing $F_M$. The disjointness of $G_M \bigcup_{i \neq i_0} \tilde{X}_i$ and $H_M$ ends the proof that $(\bigoplus_{i \in I} X_i, \tau, M)$ is a tt-soft $b$-regular space.

Second, we prove the soft $b$-normality property. Let $F_M$ and $H_M$ be two disjoint soft $b$-closed subsets of $(\bigoplus_{i \in I} X_i, \tau, M)$. It follows from Theorem (2.7) that $\bigcap_{i \in I} \tilde{X}_i$ and $H_M \bigcap \tilde{X}_i$ are soft $b$-closed in $(X_i, \tau_i, M)$ for each $i \in I$. Since $(X_i, \tau_i, M)$ is soft $b$-normal for each $i \in I$, then there exist two disjoint soft $b$-open subsets $U_{iM}$ and $V_{iM}$ of $(X_i, \tau_i, M)$ such that $\bigcap_{i \in I} \tilde{X}_i \subseteq U_{iM}$ and $H_M \bigcap \tilde{X}_i \subseteq V_{iM}$. This implies that $F_M \subseteq \bigcup_{i \in I} U_{iM}$, $H_M \subseteq \bigcup_{i \in I} V_{iM}$ and $\bigcup_{i \in I} U_{iM} \bigcap \bigcup_{i \in I} V_{iM} = \emptyset$. Hence, $(\bigoplus_{i \in I} X_i, \tau, M)$ is a soft $b$-normal space.

In the following, we probe the behaviors of tt-soft $bT_i$-spaces under some soft maps.

**Definition 4.2:** A map $f_\varphi : (X, \tau, A) \rightarrow (Y, \tau, B)$ is said to be:

1. soft $b^*$-continuous if the inverse image of soft $b$-open set is soft $b$-open;
2. soft $b^*$-open (resp. soft $b^*$-closed) if the image of soft $b$-open (resp. soft $b$-closed) set is soft $b$-open (resp. soft $b$-closed);
3. soft $b^*$-homeomorphism if it is bijective, soft $b^*$-continuous and soft $b^*$-open.

**Proposition 4.19:** Let $f_\varphi : (X, \tau, A) \rightarrow (Y, \tau, B)$ be a soft $b$-continuous map such that $f$ is injective. Then if $(Y, \tau, B)$ is a $p$-soft $T_i$-space, then $(X, \tau, A)$ is a tt-soft $bT_i$-space for $i = 0, 1, 2$.

**Proof:** We only prove the proposition for $i = 2$. 

Let $f : (X, \tau, A) \rightarrow (Y, \tau, B)$ be a soft $b$-continuous map and $v \neq w \in X$. Since $f$ is injective, then there are two distinct points $x$ and $y$ in $Y$ such that $f(v) = x$ and $f(w) = y$. Since $(Y, \tau, B)$ is a $p$-soft $T_2$-space, then there are two disjoint soft open sets $G_B$ and $F_B$ such that $x \in G_B$ and $y \in F_B$. Now, $f^{-1}(G_B)$ and $f^{-1}(F_B)$ are two disjoint soft $b$-open subsets of $(X, \tau, A)$ such that $v \in f^{-1}(G_B)$ and $w \in f^{-1}(F_B)$. Thus $(X, \tau, A)$ is a $tt$-soft $bT_2$-space.

In a similar way, one can prove the following result.

**Proposition 4.20:** Let $f : (X, \tau, A) \rightarrow (Y, \tau, B)$ be a soft $b^*$-continuous map such that $f$ is injective. Then if $(Y, \tau, B)$ is a $tt$-soft $bT_i$-space, then $(X, \tau, A)$ is a $tt$-soft $bT_i$-space for $i = 0, 1, 2$.

**Proposition 4.21:** Let $f : (X, \tau, A) \rightarrow (Y, \tau, B)$ be a bijective soft $b$-open map. Then if $(X, \tau, A)$ is a $p$-soft $T_i$-space, then $(Y, \tau, B)$ is a $tt$-soft $bT_i$-space for $i = 0, 1, 2$.

**Proof:** We only prove the proposition for $i = 2$.

Let $f : (X, \tau, A) \rightarrow (Y, \tau, B)$ be a soft $b$-open map and $x \neq y \in Y$. Since $f$ is bijective, then there are two distinct points $v$ and $w$ in $X$ such that $v = f^{-1}(x)$ and $w = f^{-1}(y)$. Since $(X, \tau, A)$ is a $p$-soft $T_2$-space, then there are two disjoint soft open sets $U_A$ and $V_A$ such that $x \in U_A$ and $y \in V_A$. Now, $f_A(U_A)$ and $f_A(V_A)$ are two disjoint soft $b$-open subsets of $(Y, \tau, B)$ such that $x \in f_A(U_A)$ and $y \in f_A(V_A)$. Thus $(Y, \tau, B)$ is a $tt$-soft $bT_2$-space.

In a similar way, one can prove the following result.

**Proposition 4.22:** Let $f : (X, \tau, A) \rightarrow (Y, \tau, B)$ be a bijective soft $b^*$-open map. Then if $(X, \tau, A)$ is a $tt$-soft $bT_i$-space, then $(Y, \tau, B)$ is a $tt$-soft $bT_i$-space for $i = 0, 1, 2$.

**Proposition 4.23:** The property of being $tt$-soft $bTi$ $(i = 0, 1, 2, 3, 4)$ is preserved under a soft $b^*$-homeomorphism map.

We complete this section by discussing some interrelations between $tt$-soft $bT_i$-spaces $(i = 2, 3, 4)$ and soft $b$-compact spaces.

**Proposition 4.24:** A stable soft $b$-compact subset of a $tt$-soft $bT_2$-space is soft $b$-closed.

**Proof:** It follows from Proposition (3.5) and Remark (4.1).

**Theorem 4.25:** Let $H_M$ be a soft $b$-compact subset of a soft hyperconnected $tt$-soft $bT_2$-space. If $x \notin H_M$, then there are disjoint soft $b$-open sets $U_M$ and $V_M$ such that $x \in U_M$ and $H_M \subseteq V_M$.

**Proof:** Let $x \notin H_M$. Then $x \neq y$ for each $y \in H_M$. Since $(X, \tau, M)$ is a $tt$-soft $bT_2$-space, then there exist disjoint soft $b$-open sets $U_m$ and $V_m$ such that $x \in U_{im}$ and $y \in V_{im}$. Therefore $(V_{im})$ forms a soft $b$-open cover of $H_M$. Since $H_M$ is soft $b$-compact, then $H_M \subseteq \bigcup_{i=1}^{n} V_{im}$. By the soft hyperconnectedness of $(X, \tau, M)$, we obtain $\bigcap_{i=1}^{n} U_{im} = U_M$ is a soft $b$-open set. Hence, we obtain the desired result.
Theorem 4.26: Every soft hyperconnected, soft b-compact and tt-soft bT₂-space is tt-soft b-regular.

Proof: Let \( H_M \) be a soft b-closed subset of soft b-compact and tt-soft bT₂-space \((X, τ, M)\) such that \( x \notin H_M \). Then \( H_M \) is soft b-compact. By Theorem(4.25), there exist disjoint soft b-open sets \( U_M \) and \( V_M \) such that \( x \in U_M \) and \( H_M \subseteq V_M \). Thus \((X, τ, M)\) is tt-soft b-regular. ■

Corollary 4.27: Every soft hyperconnected, soft b-compact and tt-soft bT₂-space is tt-soft bT₃.

Lemma 4.28: Let \( F_M \) be a soft b-open subset of a soft b-regular space. Then for each \( P_x^m \in F_M \), there exists a soft b-open set \( G_M \) such that \( P_x^m \in G_M \subseteq F_M \).

Proof: Let \( F_M \) be a soft b-open set such that \( P_x^m \in F_M \). Then \( x \notin F_M^c \). Since \((X, τ, M)\) is soft b-regular, then there exist two disjoint soft b-open sets \( G_M \) and \( W_M \) totally containing \( x \) and \( F_M^c \), respectively. Thus \( x \in G_M \subseteq W_M \subseteq F_M \). Hence, \( P_x^m \in G_M \subseteq F_M \). ■

Theorem 4.29: Let \( H_M \) be a soft b-compact subset of a soft b-regular space and \( F_M \) be a soft b-open set containing \( H_M \). Then there exists a soft b-open set \( G_M \) such that \( H_M \subseteq G_M \subseteq F_M \).

Proof: Let the given conditions be satisfied. Then for each \( P_x^m \in H_M \), we have \( P_x^m \in F_M \). Therefore there is a soft b-open set \( W_{x M} \) such that \( P_x^m \in W_{x M} \subseteq W_{x M}^b \subseteq F_M \). Now, \( \{W_{x M} : P_x^m \in F_M\} \) is a soft b-open cover of \( H_M \). Since \( H_M \) is soft b-compact, then \( H_M \subseteq \bigcup_{j=1}^{n} W_{x M} \).

Putting \( G_M = \bigcup_{j=1}^{n} W_{x M} \). Thus \( H_M \subseteq G_M \subseteq F_M \). ■

Corollary 4.30: If \((X, τ, M)\) is soft b-compact and soft bT₃, then it is tt-soft bT₄.

Proof: Suppose that \( F_{1 M} \) and \( F_{2 M} \) are two disjoint soft b-closed sets. Then \( F_{2 M} \subseteq F_{1 M}^c \). Since \((X, τ, M)\) is soft b-compact, then \( F_{2 M} \) is soft b-compact and since \((X, τ, M)\) is soft b-regular, then there is a soft b-open set \( G_M \) such that \( F_{2 M} \subseteq G_M \subseteq F_{1 M}^c \). Obviously, \( F_{2 M} \subseteq G_M, F_{1 M} \subseteq (G_M^c) \) and \( G_M \cap (G_M^c) = \emptyset \). Thus \((X, τ, M)\) is soft b-normal. Since \((X, τ, M)\) is soft bT₃, then it is tt-soft bT₁. Hence, it is tt-soft bT₄. ■

5. b-fixed Soft Points of Soft Mappings

In this section, we introduce a b-fixed soft point property and investigate some main features, in particular, those are related to parametric topological spaces.

Theorem 5.1: Let \( \{B_n : n \in \mathbb{N}\} \) be a collection of soft subsets of a soft b-compact space \((X, τ, M)\) satisfying:

(i) \( B_n \neq \emptyset \) for each \( n \in \mathbb{N} \);
(ii) \( B_n \) is a soft b-closed set for each \( n \in \mathbb{N} \);
(iii) \( B_{n+1} \subseteq B_n \) for each \( n \in \mathbb{N} \).

Then \( \bigcap_{n \in \mathbb{N}} B_n \neq \emptyset \).
Proof: Suppose that \( \bigcap_{n \in \mathbb{N}} B_n = \emptyset \). Then \( \bigcap_{n \in \mathbb{N}} B_n^c = \bar{X} \). It follows from (ii) that \( \{ B_n^c : n \in \mathbb{N} \} \) is a soft \( b \)-open cover of \( \bar{X} \). By hypothesis of soft \( b \)-compactness, there exist \( i_1, i_2, \ldots, i_k \in \mathbb{N}, i_1 < i_2 < \cdots < i_k \) such that \( \bar{X} = B_{i_1}^c \cup B_{i_2}^c \cup \cdots \cup B_{i_k}^c \). It follows from (iii) that \( B_k \subset \bar{X} = B_{i_1}^c \cup B_{i_2}^c \cup \cdots \cup B_{i_k}^c \). This yields a contradiction. Thus we obtain the proof that \( \bigcap_{n \in \mathbb{N}} B_n \neq \emptyset \).

Proposition 5.2: Let \((X, \tau, M)\) be a soft \( b \)-compact and soft \( bT_2\)-space and \( g_\varphi : (X, \tau, M) \to (X, \tau, M)\) be a soft \( b^*\)-continuous map. Then there exists a unique soft point \( P^x_m \in \bar{X}\) of \( g_\varphi \).

Proof: Let \( \{ B_1 = g_\varphi(\bar{X}) \text{ and } B_n = g_\varphi(B_{n-1}) = g_\varphi^n(\bar{X}) \text{ for each } n \in \mathbb{N} \} \) be a family of soft subsets of \((X, \tau, M)\). It is clear that \( B_{n+1} \subset B_n \) for each \( n \in \mathbb{N} \). Since \( g_\varphi \) is soft \( b^*\)-continuous, then \( B_n \) is a soft \( b \)-compact set for each \( n \in \mathbb{N} \) and since \((X, \tau, M)\) is soft \( bT_2\), then \( B_n \) is also a soft \( b \)-closed set for each \( n \in \mathbb{N} \). It follows from Theorem (5.1) that \((H, M) = \bigcap_{n \in \mathbb{N}} B_n \) is a non null soft set. Note that \( g_\varphi(H, M) = g_\varphi(\bigcap_{n \in \mathbb{N}} g_\varphi^n(\bar{X})) \subset \bigcap_{n \in \mathbb{N}} g_\varphi^n(\bar{X}) = (H, M) \). To show that \((H, M) \subset g_\varphi(H, M)\), suppose that there is a \( P^x_m \in (H, M) \) such that \( P^x_m \notin g_\varphi(H, M) \). Let \( C_n = g_\varphi^{-1}(P^x_m) \cap B_n \). Obviously, \( C_n \neq \emptyset \) and \( C_n \subset C_{n+1} \) for each \( n \in \mathbb{N} \). By Theorem 5.1, \( C_n \) is a soft \( b \)-closed set for each \( n \in \mathbb{N} \); and by Theorem (5.1), there exists a soft point \( P^x_m \) such that \( P^x_m = g_\varphi(P^x_m) \in g_\varphi(H, M) \). This is a contradiction. Thus \( g_\varphi(H, M) = (H, M) \). Hence, the proof is complete.

Definition 5.1: (i) \((X, \tau, M)\) is said to have a \( b \)-fixed soft point property if every soft \( b^*\)-continuous map \( g_\varphi : (X, \tau, M) \to (X, \tau, M) \) has a fixed soft point.

(ii) A property is said to be an \( b^*\)-soft topological property if the property is preserved by soft \( b^*\)-homeomorphism maps.

Proposition 5.3: The property of being a \( b \)-fixed soft point is a \( b^*\)-soft topological property.

Proof: Let \((X, \tau, M)\) and \((Y, \tau, M)\) be a soft \( b^*\)-homeomorphic. Then there is a bijective soft map \( f_\varphi : (X, \tau, M) \to (Y, \tau, M) \) such that \( f_\varphi \) and \( f_\varphi^{-1} \) are soft \( b^*\)-continuous. Since \((X, \tau, M)\) has an \( b \)-fixed soft point property, then every soft \( b^*\)-continuous map \( g_\varphi : (X, \tau, M) \to (X, \tau, M) \) has an \( b \)-fixed soft point. Now, let \( h_\varphi : (Y, \tau, M) \to (Y, \tau, M) \) be a soft \( b^*\)-continuous. Obviously, \( h_\varphi \circ f_\varphi : (X, \tau, M) \to (Y, \tau, M) \) is a soft \( b^*\)-continuous. Also, \( f_\varphi^{-1} \circ h_\varphi \circ f_\varphi : (X, \tau, M) \to (X, \tau, M) \) is a soft \( b^*\)-continuous. Since \((X, \tau, M)\) has an \( b \)-fixed soft point property, then \( f_\varphi^{-1}(h_\varphi(f_\varphi(P^x_m))) = P^x_m \) for some \( P^x_m \in \bar{X} \). Consequently, \( f_\varphi(f_\varphi^{-1}(h_\varphi(f_\varphi(P^x_m)))) = f_\varphi(P^x_m) \). This implies that \( h_\varphi(f_\varphi(P^x_m)) = f_\varphi(P^x_m) \). Thus \( f_\varphi(P^x_m) \) is a \( b \)-fixed soft point of \( h_\varphi \). Hence, \((Y, \tau, M)\) has an \( b \)-fixed soft point property, as required.

Before we investigate a relationship between soft topological space and their parametric topological spaces in terms of possessing a fixed (soft) point, we need to prove the following result.

Theorem 5.4: Let \( \tau \) be an extended soft topology on \( X \). Then a soft map \( g_\varphi : (X, \tau, M) \to (Y, \tau, M) \) is soft \( b^*\)-continuous if and only if a map \( g : (X, \tau_m) \to (Y, \tau_{\varphi(m)}) \) is \( b^*\)-continuous.

Proof: Necessity: Let \( U \) be an \( b \)-open subset of \((Y, \tau_{\varphi(m)})\). Then there exists a soft \( b \)-open subset \( G_M \) of \((Y, \tau, M)\) such that \( G(\varphi(m)) = U \). Since \( g_\varphi \) is a soft \( b^*\)-continuous map, then
$g_\phi^{-1}(G_M)$ is a soft $b$-open set. From Definition (2.8), it follows that a soft subset $g_\phi^{-1}(G_M) = (g_\phi^{-1}(G))_M$ of $(X, \tau, M)$ is given by $g_\phi^{-1}(G)(m) = g^{-1}(G(\phi(m)))$ for each $m \in M$. By hypothesis, $\tau$ is an extended soft topology on $X$, we obtain from Theorem (2.5) that a subset $g^{-1}(G(\phi(m))) = g^{-1}(U)$ of $(X, \tau_m)$ is $b$-open. Hence, a map $g$ is $b^*$-continuous.

**Sufficiency:** Let $G_M$ be a soft $b$-open subset of $(Y, \tau, M)$. Then from Definition (2.8), it follows that a soft subset $g_\phi^{-1}(G_M) = (g_\phi^{-1}(G))_M$ of $(X, \tau, M)$ is given by $g_\phi^{-1}(G)(m) = g^{-1}(G(\phi(m)))$ for each $m \in M$. Since a map $g$ is $b^*$-continuous, then a subset $g^{-1}(G(\phi(m)))$ of $(X, \tau_m)$ is $b$-open. By hypothesis, $\tau$ is an extended soft topology on $X$, we obtain from Theorem (2.5) that $g_\phi^{-1}(G_M)$ is a soft $b$-open subset of $(X, \tau, M)$. Hence, a soft map $g_\phi$ is soft $b^*$-continuous.

**Definition 5.2:** $(X, \tau)$ is said to have an $b$-fixed point property if every $b^*$-continuous map $g : (X, \tau) \rightarrow (X, \tau)$ has a fixed point.

**Proposition 5.5:** $(X, \tau, M)$ has the property of an $b$-fixed soft point iff $(X, \tau_m)$ has the property of an $b$-fixed point for each $m \in M$.

**Proof:** **Necessity:** Let $(X, \tau, M)$ has the property of an $b$-fixed soft point. Then every soft $b^*$-continuous map $g_\phi : (X, \tau, M) \rightarrow (X, \tau, M)$ has a fixed soft point. Say, $P_m^\phi$. It follows from the above theorem that $g_m : (X, \tau_m) \rightarrow (X, \tau_{\phi(m)})$ is $b^*$-continuous. Since $P_m^\phi$ is a fixed soft point of $g_\phi$, then it must be that $g_m(x) = x$. Thus $g_m$ has a fixed point. Hence, we obtain the desired result.

**Sufficiency:** Let $(X, \tau_m)$ has the property of an $b$-fixed point for each $m \in M$. Then every $b^*$-continuous map $g_m : (X, \tau_m) \rightarrow (X, \tau_{\phi(m)})$ has a fixed point. Say, $x$. It follows from the above theorem that $g_\phi : (X, \tau, M) \rightarrow (X, \tau, M)$ is soft $b^*$-continuous. Since $x$ is a fixed point of $g_m$, then it must be that $g_\phi(P_m^\phi) = P_m^\phi$. Thus, $g_\phi$ has a fixed soft point. Hence, we obtain the desired result.

6. Conclusion

One of the reasons of diversity of soft separation axioms is the variety of belong and non-belong relations between ordinary points and soft set. This article is devoted to studying separation axioms and fixed points in soft setting. First, we have introduced new soft separation axioms with respect to ordinary points by using total belong and total non-belong relations. This way of definition helps us to generalize existing comparable properties via general topology and to remove a strict condition of the shape of soft open and closed subsets of soft $b$-regular spaces. In general, we have studied their main properties and showed the interrelations between them with help of interesting examples. Second, we have defined $b$-fixed soft point theorem and investigated its basic properties. Finally, we hope that the concepts initiated herein will find their applications in many fields soon.

**Human participants**

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