O(α) Improvement of Nucleon Matrix Elements*

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We report on preliminary results of a high statistics quenched lattice QCD calculation of nucleon matrix elements within the Symanzik improvement programme. Using the recently determined renormalisation constants from the Alpha Collaboration we present a fully non-perturbative calculation of the forward nucleon axial matrix element with O(α) lattice artifacts completely removed. Runs are made at β = 6.0 and β = 6.2, in an attempt to check scaling and O(α^2) effects. We shall also briefly describe results for ⟨x⟩, the matrix element of a higher derivative operator.

1. INTRODUCTION

In this talk we shall describe results for nucleon matrix elements: ⟨N|O^R|N⟩ (at \(\vec{p} = 0\)) using O(α) Symanzik improved fermions for

- the vector current, ⟨N|V_μ^R|N⟩ (as a warm-up exercise)
- the axial current, ⟨N,s|A_μ^R|N,s⟩ = Δq_μ, where Δq is the fraction of the nucleon spin carried by the quark q
- ⟨x⟩(q), the fraction of the nucleon momentum carried by quark q

We have worked in the quenched approximation and generated O(500) configurations at \(β = 6.0\) on a 16^3 × 32 lattice and O(150) configurations at \(β = 6.2\) on a 24^3 × 48 lattice. In both cases we have used three \(κ\) values to enable us to extrapolate to the chiral limit. For the hadron spectrum see \([3]\). The method to determine the matrix elements is standard, see eg \([2]\); we only note that we are computing just the quark line connected term.

2. IMPROVED OPERATORS

Symanzik improvement is a systematic improvement of the action and operators to O(α^n) (here O(α^3)) by adding a basis of irrelevant operators to completely remove O(α^{n-1}) effects. Restricting improvement to on-shell matrix elements means that the equations of motion (EOM) can be used to reduce the set of operators. For the action we only need one additional operator – the clover term, \([3]\), with known coefficient \(c_{sw}(g^2)\), \([3]\). We write for the improved axial, vector currents:

\[ V_μ = V_μ + c_1 V_μ a \bar{q} \gamma_μ D_μ q + \ldots, \]
\[ A_μ = A_μ - c_1 A_μ a \bar{q} σ_μ γ_5 D_μ q + \ldots, \]

while \langle N|\mathcal{T}_{44} - \frac{1}{2} \sum_i |N_i⟩|N⟩ = -2m_N^2 \langle x⟩, with

\[ \mathcal{T}_{μν} = \bar{q} γ_μ \gamma_ν D_μ q + c_1 a \bar{q} σ_μ γ_5 \bar{D}_μ D_ν q \]
\[ - c_2 a \bar{q} \bar{D}_ν D_μ D_ν q + \ldots, \]
shown are two points (stars) from a method described in [4]. Again there is good agreement (this time at a non-zero value of $c_{1V}$).

For the axial current, again the Alpha Collaboration has non-perturbatively found $Z_A(c_{1A} = 0)$, [5]. This is sufficient for us as we are only interested in the results in the chiral limit. Performing these extrapolations for $\beta = 6.0$, 6.2 gives the scaling plot, Fig. 2 To attempt a comparison with Wilson data, we have also plotted the result at $\beta = 6.0$, [6] using the non-perturbative $Z_A$ as found in [7] of 0.78. This is close to the value (0.75) given from linearly interpolating the non-perturbative Ward Identity results in [7] to $\beta = 6.0$, which would indicate that $O(a)$ effects between these two methods are small. In Fig. 2 we expect to have to make a linear extrapolation in $a^2$ for the improved case.

For $\langle x \rangle$ there is no non-perturbative determination of the renormalisation constants yet available. First order perturbation theory gives, however, numbers rather close to 1, [1], and indeed using tadpole improved $(TJ)$ perturbation theory, we see that with one derivative operators the (large) tadpole terms cancel. This might indicate that perturbation theory is reasonably correct. From the definition of the improved operator we see that
we have parameters $b_0$, $c_1$, $c_2$. To remove $O(a)$ effects completely we must only have a residual one parameter degree of freedom (from the EOM), so that, e.g., $c_2 \equiv c_2(c_1)$ and $b_0 \equiv b_0(c_1)$. These functions are unknown at present. At tree level $c_2 = c_1$, $b_0 = 1 - c_1$, which we shall use here. In Fig. 3 we compare

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{A comparison between first order perturbation theory and TI perturbation theory, using the boosted coupling $\alpha_{\overline{MS}}(\mu_a)$ given in table I of [8]. Also used in the perturbative coefficient is $\tilde{c}_{sw} = c_{sw}(g^2 u_0^3)$, ($u_0^3 = \langle 1 \rangle_{tr}^{U_{plaq}}$).}
\end{figure}

one loop perturbative results, and TI results for $\langle x \rangle^{(u)}$ in the chiral limit. We know that upon using the true coefficients then the result must be independent of $c_1$. TI appears to achieve this somewhat better than 1-loop perturbation expansion, so we shall use this in our scaling plot shown in Fig. 3. This operator has an anomalous dimension, so we must scale the results to the same $\mu$; this has been performed with the scaling formula $\langle x \rangle|_\mu = (\alpha_{\overline{MS}}(\mu)/\alpha_{\overline{MS}}(\mu_0))^{32/99} \langle x \rangle|_{\mu_0}$. Scaling the $\beta = 6.2$ result to $\beta = 6.0$ where $\mu \sim 1.95$(GeV)$^{-1}$ gives a 4% increase. We compare with the phenomenological value, [1]. Our results seem rather constant (in $a^2$), suggesting that even in the continuum limit the lattice value is too high. We can only speculate on the discrepancy: most likely this is due to a lattice problem – quenching, chiral extrapolation, $Z$ not accurately enough known or perhaps there is a phenomenological prob-

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4}
\caption{Scaling plot of non-singlet $\langle x \rangle^{(u)} - \langle x \rangle^{(d)}$ against $a^2$, using $c_1 = 0$. Same notation as in Fig. 3.}
\end{figure}

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