Light-by-Light Scattering and Spacetime Noncommutativity

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In this letter, we determine the scale of noncommutativity $\Lambda_{NC}$ by comparing predictions of our fully-fledged (with respect to the NC parameter $\theta_{\mu\nu}$) NCQED model for the exclusive $\gamma\gamma \to \gamma\gamma$ process to the recent measurement by ATLAS of the PbPb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. Employing the fiducial cuts of the ATLAS analysis, together with sophisticated integral domain and the realistic luminosity function, by performing the eighthfold integration we first compute the full SM fiducial cross section to be $\sigma_{SM}(\text{PbPb} \to \text{Pb}^*\text{Pb}^*\gamma\gamma) \approx 57$ nb. Including interference, next we set a bound on the NC scale as $\Lambda_{NC} \gtrsim 0.1$ TeV. Not only this limit could be strengthened further by next generation collider data at least $\Lambda_{NC} \sim 0.5$ TeV, but a distinctive feature starts showing up in the distribution for larger diphoton masses. Such a feature is a genuine peculiarity of our $\theta$-exact model, which is not revealed in a nonlinear Born-Infeld extension of QED or making use of Lorentz-violating operators in electrodynamics.

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Spacetime noncommutativity and quantizations of the electromagnetic field in such noncommutative spacetime, was an original idea to cure UV divergences plaguing quantum field theory, suggested by Heisenberg to Pierls, followed to Pauli, and finally executed a bit later by Oppenheimer’s graduate student, Snyder [1][2]. Nowadays such models appear quite naturally in certain limits of string theory in the presence of a background $B^{\mu\nu}$ field. Specifically, a low-energy limit is identified where the entire boson-string dynamics in a Neveu-Schwartz condensate is described by a minimally coupled supersymmetric gauge theory on noncommutative (NC) space [3] such that the mathematical framework of NC geometry/field theory [4][5] does apply. In our model of the NC spacetime we consider coordinates $x^\mu$ as the hermitian operators $\hat{x}^\mu$ [5], were the coordinate-operator commutation relation $[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$, $|\theta^{\mu\nu}| \sim \Lambda_{NC}^{-2}$ is realized by the Moyal-Weyl star($\star$)-product, i.e. the $\star$-commutator of the usual coordinates, implying the spacetime uncertainty relations $\Delta x^\mu \Delta x^\nu \geq \frac{1}{2} |\theta^{\mu\nu}|$. Here $\theta^{\mu\nu}$ is a constant real antisymmetric matrix of dimension length$^2$, and $\Lambda_{NC}$ being the NC scale, to which we set a bound from experiments [6].

Serious efforts on formulating $\theta$-exact NCQFT models, strongly boosted by the Seiberg-Witten (SW) map [7] based enveloping algebra approach, enables a direct deformation of comprehensive models like the SM or GUTs. So it appear to study ordinary gauge theories with additional interactions generally induced by the $\star$-product and SW maps [7][9], respectively.

The $\theta$-exact-model of NCQFT [7][11], inspired by the string theories [3], due to the nice mathematical properties nowadays stands for its own, as higher, nonlocal, nonlinear spontaneously Lorentz-violating QFT. Thus there was a number of discussions about properties of NC theories, regarding general questions about causality and/or unitarity of the NCQFT’s, involving all NC types, that is time-like, space-like and light-like; see [14][17].

Noncommutativity from the string perspective and modifications of gravity–black holes–, the vacuum-birefringence, connections with Holography, see-saw neutrino masses and the Yukawa couplings, etc., were extensively discussed in [18][21], respectively. Also our theory very well apply to the ultra high energy (UHE) photons, electrons and neutrinos produced in the cosmic laboratories. Recently we have applied our $\theta$-exact model on the early universe right-handed neutrino physics considered in PTOLEMY experiment [12][13].

After more than decade we have proven that the U(N) NCQFT’s on Moyal manifold, with and without SW maps, together with or without supersymmetry included, are all equivalent and/or dual to each other [22][24]. This way we

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1 Or one could say, it has been shown in [22] that SUSY cancelation mechanism works for all the two-point functions when we have
have finally shown that in the NCQFT theory exact with respect to the NC parameter the UV/IR mixing always pops up, and that happened on three different manifolds: on Moyal space [10] with/without \( \theta \)-exact SW maps [23, 24], on \( \kappa \)-Minkowski [25] and on Snyder [26] noncommutative spaces, respectively. Also, the very notion of UV/IR mixing was implemented into the idea of scalar fields weak gravity conjecture (WGC) [27], where it manifests itself as a form of hierarchical UV/IR mixing and is tied to the interaction between the WGC and nonlocal–noncommutative–gauge operators. Additionally, we have recently successfully constructed the 3D ABJM theory on the Moyal NCSUSY space and show that all UV and IR divergences in 2- and 3-point functions of such theory disappear [28].

In this Letter, we apply our full fledged minimal \( \theta \)-exact NCQED model [10, 11] which, via 3-, and 4-photon couplings, produces novel contributions to the \( \gamma \gamma \rightarrow \gamma \gamma \) at tree-level, a particular process known as the light-by-light (LbyL) scattering in vacuum. This is a nonlinear process forbidden at the classical tree level QED due to the Landau-Young theorem, but predicted to occur via radiative quantum corrections in QED, and/or in the SM. That process has been proposed as a useful test of the NC physics [29], and nowadays is extremely interesting due to recent ATLAS measurements of the ultraperipheral collisions of lead ions [30, 34] and it was already used in constraining certain models beyond SM [35, 36].

Under assumption that main deviations from the SM in the exclusive \( \gamma \gamma \rightarrow \gamma \gamma \) scattering measurements are originating from the NCQED, in this letter by using our \( \theta \)-exact minimal gauge sector action

\[
S_{\text{gauge}}^{\text{min}} = \int -\frac{1}{2} F_{\mu
u} \ast F_{\mu
u},
\]

given in terms of the NC gauge field \( A_{\mu} \), we continue to search for the scale \( \Lambda_{NC} \) of the NC spacetime. The connection between the noncommutative and commutative fields is given by the \( \theta \)-exact Seiberg-Witten maps.

Our starting point to compute the \( \gamma \gamma \rightarrow \gamma \gamma \) scattering amplitudes in the NCQED from (1) is the color – star-product equivalence between the ordinary U(N) and the Moyal NC U(N) gauge theories, respectively:

\[
\text{tr} \prod_{i=1}^{n} T^{c_{i}} \iff \exp \left( -\frac{i}{2} \sum_{i=1}^{n} k_{2i-1} \theta k_{2i} \right).
\]

The conversion between color-ordered QCD helicity amplitudes \( M_{h_{\text{gauge}}}^{\text{helicity}} \) and its NCQED counterpart \( (M_{h_{\text{NC}}}^{\text{helicity}})_{\text{NC}} \), after including coupling factors and up to a phase factor with help of the helicity prescriptions, is giving NC contributions to the \( \gamma(k_1)\gamma(k_2) \rightarrow \gamma(k_3)\gamma(k_4) \) helicity amplitudes. That was possible since we have explicitly proved in [37] that, tree-level equivalence between QCD and NCQED holds, and that the NC scattering amplitudes are invariant under the \( \theta \)-exact SW maps, i.e. all additional contributions induced in the action [1] by the SW maps actually cancel out, leaving thus only *-product generated terms to contribute to the noncommutative amplitudes \( M_{h_{\text{NC}}}^{h} \), with helicities \( h = (+ + + +, + - + +, + + - -, + + + -) \):

\[
\begin{align*}
M_{\text{NC}}^{++++]_{32\pi\alpha}} &= \frac{s}{u} \sin \frac{k_1 \theta k_2}{2} \sin \frac{k_3 \theta k_4}{2} - s^2 \frac{tu}{u} \sin \frac{k_1 \theta k_4}{2} \sin \frac{k_2 \theta k_3}{2}, \\
M_{\text{NC}}^{[-++]_{32\pi\alpha}} &= \frac{t^2}{su} \sin \frac{k_1 \theta k_2}{2} \sin \frac{k_3 \theta k_4}{2} - t \frac{u}{u} \sin \frac{k_1 \theta k_4}{2} \sin \frac{k_2 \theta k_3}{2}, \\
M_{\text{NC}}^{[-+-+]} &= \frac{u}{s} \sin \frac{k_1 \theta k_2}{2} \sin \frac{k_3 \theta k_4}{2} - u \frac{t}{u} \sin \frac{k_1 \theta k_4}{2} \sin \frac{k_2 \theta k_3}{2}.
\end{align*}
\]

We apply our NCQED to the data for LbyL scattering obtained through precision measurements of the process PbPb→Pb*Pb*\( \gamma \gamma \) in ATLAS experiment [33, 34]. To do that we compute the full lepton, quark and W\( \pm \)-bosons one-loop SM diagrams, i.e. the 2-, 3-, and 4-point functions scalar integrals, like for example in [38, 39], which together with noncommutative amplitudes [4] give the following exclusive \( \gamma \gamma \rightarrow \gamma \gamma \) cross section:

\[
\frac{d\sigma(\omega_1, \omega_2, x, \varphi)}{d\Omega} = \frac{1}{(16\pi)^2} \left( \frac{1}{(\omega_1(1-x)+\omega_2(1+x))^2} \sum_{h} \left( |M_{h_{\text{SM}}}^{h}|^2 + M_{h_{\text{SM}}}^{h} M_{h_{\text{NC}}}^{h} + M_{h_{\text{SM}}}^{h} M_{h_{\text{NC}}}^{h} + |M_{h_{\text{NC}}}^{h}|^2 \right). \right.
\]

Photons from the Pb ions can be viewed as photon beams in the equivalent photon approximation [40, 41], where due to the coherent action of all protons in the nucleus, the electromagnetic field surrounding each fast moving nucleus
with the charge \( Ze \) is very strong, and it is approximated by a distribution of (almost) real photons moving along the beam direction. To take into account the peripheral nature of the process total cross section is constructed using a photon number function \( N(\omega, |\vec{b}|) \) including 2D "\( \vec{b} \)-impact parameter" [12] that marks the position of the ion from the position of impact on the plane perpendicular to the beam direction.

In the ultraperipheral scattering one considers the electromagnetic fields of the incoming ions as a spectrum of real photons whose energy space \((\omega_1, \omega_2)\) is usually re-parametrized by the di-photon invariant mass \( m = \sqrt{\omega_1 \omega_2} \) and rapidity \( Y = \frac{1}{2} \ln \frac{\omega_1}{\omega_2} \). Energy is maximal for symmetric systems \( \omega_1 = \omega_2 \approx \gamma/b_{\min} \) where rapidity vanishes, with \( b_{\min} \) being the minimum separation between the two equal charged nuclei of radius \( R_N \). We use for the lead ion spectrum the \( \vec{b} \)-dependent expression integrated from \( b_{\min} \) to infinity, with the requirement \( b_{\min} = R_N \) plus a correction equivalent to the geometrical condition \( |\vec{b}_1 - \vec{b}_2| > R_{\min} = 2R_N \) so that collisions happen without beams breaking or other problems [30]. Cross section of the Pb-Pb collision is then expressed in the fiducial phase space as the eightfold integral:

\[
\sigma = \frac{1}{2} \int dmdY \frac{d^2 L_{\gamma\gamma}}{dm dY} d\Omega \frac{d\sigma(\omega_1, \omega_2, \vartheta, \varphi)}{d\Omega},
\]

by convoluting [4] with luminosity function \( \frac{d^2 L_{\gamma\gamma}}{dm dY} \):

\[
\frac{d^2 L_{\gamma\gamma}}{dm dY} = \frac{4\pi}{m} \int d|\vec{b}_1|d|\vec{b}_2| \int_{0}^{2\pi} d\phi N(\omega_1, |\vec{b}_1|)N(\omega_2, |\vec{b}_2|) \Theta \left( \sqrt{|\vec{b}_1|^2 + |\vec{b}_2|^2 - 2|\vec{b}_1||\vec{b}_2| \cos \phi - R_{\min} \right),
\]

where \( R_{\min} = 2R_{\text{Pb}} = 14 \text{ fm} \approx 71 \text{ GeV}^{-1} \). Fourfold integration over impact parameters \( \vec{b}_1 \) and \( \vec{b}_2 \) has been performed with the monopole photon number function \( N(\omega, b) \), containing very important Lorentz-relativistic factor \( \gamma = \sqrt{\gamma_N/(2m_N)} \) for the \( N^{th} \) nucleus, expressed in terms of the 2\( ^{nd} \) kind modified Bessel function, and given explicitly in [12]. Remaining integration over \( dmdY d\Omega \) has to be performed over an domain defined according to the experimental selection rules, the ATLAS cuts. For a typical cut which requires transverse energy \( E_T \geq E_0 \) and both outgoing particles to bear absolute pseudorapidities \(|\eta_3,4| \leq \eta_0\), the full integral domain is defined as follows:

\[
\varphi \in [0, 2\pi], \quad |\cos \vartheta| \leq \frac{e^{2y_0} - 1}{e^{2y_0} + 1}, \quad m \geq 2E_0,
\]

\[
Y \in \left[ \frac{1}{2} \left( \frac{1}{2} \ln \frac{1 + x}{1 - x} - y_0 \right), \frac{1}{2} \left( y_0 - \frac{1}{2} \ln \frac{1 + x}{1 - x} \right) \right]
\cap \left[ \frac{1}{2} \ln \frac{1 + x}{1 - x} + \ln \left( \frac{m}{2E_0} - \sqrt{\frac{m^2}{4E_0^2} - 1} \right), \sqrt{\frac{m^2}{4E_0^2} - 1} \right]
\cup \left[ \frac{1}{2} \ln \frac{1 + x}{1 - x} + \ln \left( \frac{m}{2E_0} + \sqrt{\frac{m^2}{4E_0^2} - 1} \right), \sqrt{\frac{m^2}{4E_0^2} - 1} \right],
\]

with details being given in [37]. From experiment we have \( E_0 = 3 \text{ GeV} \) and \( y_0 = 2.37 \) to define the ATLAS cuts [33, 34]. Using the monopole \( N(\omega, b) \)'s [12] we estimate full SM one-loop contributions to the fiducial cross section to be \( \sigma_{\text{SM}}(\text{PbPb} \rightarrow \text{Pb}^*\text{Pb}^*\gamma) = 57 \text{ nb} \), which is comparable to the previously reported values [33, 34].

The ATLAS experiment with Pb ion center of mass energy \( \sqrt{s_{\text{NN}}} = 5.02 \text{ TeV} \) [33] is sensitive to most of the 4\( ^{\pi} \) solid angle, implying that the integral [3] ranges over the ATLAS cuts: \( m_{\text{Pb}} = 0.9315 \text{ GeV} \), \( \gamma = 2693 \) giving the data span as a function of the diphoton-mass (FIG. 1).

 Applying protocol tested in the above subsections, we compute numerically combined contribution from both, the SM one-loop and NCQED tree-level \( \gamma \gamma \rightarrow \gamma \gamma \) amplitudes in the ATLAS PbPb \( \rightarrow \text{Pb}^*\text{Pb}^*\gamma \) experiment. We first consider matching the reported experimental total cross section of 78 nb [34] by such combination, which yields a very small noncommutative scale \( \Lambda_{\text{NC}} \) of about 72 GeV. After checking the distribution of cross section with respect to di-photon invariant mass \( m \) we conclude that such scenario does not explain the observation of ATLAS experiment, since cross section induced by NCQED amplitude comes from a wide band at \( m \gtrsim 100 \text{ GeV} \), while the experimentally measured excess is below 30 GeV (FIG. 1). Since the relevant NC scale is small in the ATLAS PbPb \( \rightarrow \text{Pb}^*\text{Pb}^*\gamma \) scenario, we notice that pure NC amplitude \( |M_{\text{NC}}|^2 \) gives dominant contribution to the peak value of \( \frac{d\sigma}{dm} \) (FIG. 2), which occurs moderately above \( \Lambda_{\text{NC}} \). So the high energy peak value of \( \frac{d\sigma}{dm} \) in FIG. 2 is the direct consequence of NCQED amplitudes and determines the possibility of observing excessive events due to NCQED. We find that a peak value of \( \sim 1 \text{ nb/GeV} \) exists for \( \Lambda_{\text{NC}} \lesssim 53 \text{ GeV} \), \( \sim 0.1 \text{ nb/GeV} \) exists for \( \Lambda_{\text{NC}} \lesssim 72 \text{ GeV} \), and \( \sim 0.01 \text{ nb/GeV} \) exists
FIG. 1: Cross section versus di-photon invariant mass distribution in the PbPb → Pb∗Pb∗γγ experiment with the ATLAS cuts for the pure SM (black) as well as the SM+NCQED with Λ_NC values 53 (blue), 72 (green) and 100 (red) GeV.

| Λ_NC (GeV) | σ_NC (nb) | σ_NC2 (nb) | σ_NCQED (nb) | (dσ/dm)_{max} (nb/GeV) |
|------------|-----------|------------|--------------|-------------------------|
| 53         | 12.1      | 125.5      | 137.6        | 1.09                    |
| 72         | 3.6       | 17.6       | 21.2         | 0.13                    |
| 100        | 1.0       | 1.8        | 2.8          | 0.011                   |

TABLE I: Summary of the predicted σ_NCQED = σ_NC + σ_NC2 contributions to PbPb → Pb∗Pb∗γγ fiducial cross sections for the ATLAS cuts at various Λ_NC values.

for Λ_NC ≳ 100 GeV. Given the current integrated luminosity accumulation speed (~ 1 nb⁻¹/yr) of current LHC and assuming that high luminosity LHC will reach about ten times of this value. Using (3-7) we obtain total cross section in the fiducial phase space of 59.8 nb, relatively close to the experimental mean value, however only at Λ_NC = 100 GeV. This shows that the current ATLAS PbPb → Pb∗Pb∗γγ experiment is not quite adequate for bounding Λ_NC at present LHC energies.

The future higher energy experiments could increase the experimental sensitivity to the NC effects drastically. Thus to complete our investigation we also estimate potential improvements one would expect from next collider generation, in particular upgraded CERN LHC to high luminosity LHC (HL-LHC) up to √s_{pp} = 14 TeV, the proposal for next generation hadron collider SppC with energy √s_{pp} = 70 TeV [43], and from the Future Circular Collider (FCC) proposal up to √s_{pp} = 100 TeV [44][45]. Assuming that all the ATLAS cuts kinematics remains the same, except the energy scale/Lorentz factor which scales up to 5, 7 or 20 times with respect to the current ATLAS value √s_{NN} = 5.02 TeV, and estimate the NC scale Λ_NC corresponding to a high energy (dσ/dm)_{max} with ~ 0.01 nb/GeV magnitude. Table II shows that Λ_NC are about 2.5, 3.1 and 5.2 times of ∼ 100 GeV for the ATLAS energy scale 5.02 TeV, respectively. Such improvements are considerable, yet still insufficient to make this kind of experiment(s) convenient for bounding Λ_NC when comparing to other known bounds, unless the integrated luminosity can be further improved by multiple scales of the next generation hadron colliders.

In FIG.3 the first left dashed/solid lines peaks, up to m ~ 100 GeV, corresponds to the σ_SM and σ-total di-photon cross section distributions, while the second solid lines bumps corresponds to the sum of interference and the pure NCQED terms, respectively. Such an outstanding feature in the distributions in the di-photon mass is absent in the Born-Infeld modification of the QED Lagrangian [35], and also in the nonminimal sector of Lorentz-violating operators that contribute to the LbyL cross section [36]. This outstanding second bump reflects the evolution of NC factors with respect to energy scales: When energy scales are much smaller than Λ_NC, the NC factors are increasing as monomials with high power. Once the energy scales become larger than Λ_NC the NC factors become oscillatory and bounded.
Consequently the NC amplitudes are very small at very low energies where SM contribution dominates, and then increase quickly and compete with the exponentially decreasing luminosity factor \( \frac{d\sigma}{dm} \) to give the rising side of the NC bump. Once the energy scale goes beyond \( \Lambda_{NC} \) the NC amplitudes start to deviate from monomial increase, so that \( \frac{d\sigma}{dm} \) falls down quickly. The subsequent oscillatory behaviors of the NC factors are fully suppressed by the luminosity function and can not be seen in this process. Qualitatively \( \frac{d\sigma}{dm} \) curves have two peaks, the SM at low energy, and the NCQED at high energy, between two peaks there is obviously always a minimum. Note that FCC and SppC energies are not so different for a log-log plot so their minimums are at similar position.

Finally in FIG’s. 2 and 3 that there are small dents between 100 and 200 GeV arising from the W-loop contributions to the pure SM and the SM×NCQED interference terms, respectively. Namely W contribution starts to show up by inducing a sharp drop/turn between 100 and 200 GeV in Fig.1 of Ref. [39]. Also the NC peaks at \( m \sim 700 \) GeV in FIG.3 could be a bit misleading because of the log-log plot. Actually it is a quite wide band if the di-photon mass scale is linear. The hardest problem is still the absolute value of the peak, which will require having totally 1000 nb\(^{-1}\) integrated luminosity to be absolutely clear, that is about 10 events at the peak. According to further planes of upgrading LHC to HL-LHC, project aims to crank up the performance of the LHC in order to increase the potential for discoveries after 2027. The objective is to increase luminosity by at least a factor of 10 beyond the LHC, and SppC’s design values, as well as beyond FCC proposals, [43–45, 48]. In that case 10 events at the peak are in fact accessible. We would urge our experimental colleagues to go for it.

| \( \sqrt{s_{NN}} \) (TeV) | \( \gamma \) | \( \Lambda_{NC} \) (GeV) | \( \sigma_{SM} \) (nb) | \( \sigma_{NCQED} \) (nb) |
|--------------------------|-----------|-----------------|-----------------|-----------------|
| 5.02                     | 2693      | 100             | 57              | 2.8             |
| 25.10                    | 13465     | 257             | 178             | 6.6             |
| 35.14                    | 18851     | 311             | 211             | 7.9             |
| 100.40                   | 53860     | 523             | 336             | 16.9            |

TABLE II: Estimations for ATLAS PbPb \( \rightarrow Pb^*Pb^*\gamma\gamma \) like experiments at higher energies. Here we made adjustments of the NC scale in a way to get \( \frac{d\sigma}{dm} \)\( \max \) \( \simeq \) 0.01 nb/GeV.
FIG. 3: Cross section versus di-photon invariant mass distribution in the future higher energy ATLAS PbPb → Pb*Pb*γγ like experiments. Dashed curves are for the SM, while solid correspond to the SM+NCQED, respectively. Red curves are for $\sqrt{s_{NN}} = 25.10$ TeV, $\Lambda_{NC} = 257$ GeV, while the blue curves are for $\sqrt{s_{NN}} = 35.14$ TeV, $\Lambda_{NC} = 311$ GeV.

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