NONPERTURBATIVE EFFECTS IN LOW ENERGY EFFECTIVE THEORIES OF QCD ‡

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ABSTRACT

In this talk I concentrate on the role of chiral symmetry realisation by spin-1 fields in the low energy QCD effective lagrangian. I assume that chiral symmetry is nonlinearly realised and that spin-1 fields transform homogeneously under chiral rotations, which is in sharp contrast to previous works where the \( \rho \) and the \( \alpha_1 \) mesons were treated as approximate gauge bosons of some chiral group. I emphasise the role played by four-meson couplings which in our scheme are essential for the theory to make sense. By requiring the kinetic energy density of the theory to be bounded from below we find inequalities relating three- and four-point meson couplings. It is finally shown how the combination of our analysis and of unitarity requirements naturally leads to a low-energy phenomenological lagrangian for the nonanomalous sector of \( \pi \rho \alpha_1 \) strong interactions.

1. Introduction

Strong interactions at low energies are quite well understood in terms of an effective meson lagrangian\(^1\). The starting point for such an effective lagrangian is the nonlinear sigma model of pseudoscalar pions with spontaneous breaking of chiral symmetry, a central feature of low energy QCD. The experimental discovery of meson resonances as well as some theoretical notions such as the large \( N_c \) expansion of QCD\(^2\), strongly support the idea of introducing mesons other than the pion into this model. There is a considerable amount of work in the literature treating the role played by massive spin-1 mesons (the \( \rho \)- and the \( \alpha_1 \)-mesons) in low-energy lagrangians. In most of these works isovector resonances are introduced as massive Yang-Mills particles\(^3\) or as gauge bosons of local chiral symmetry\(^4\), and low energy phenomena such as \( \rho \)-coupling universality, the KSRF relation are then nicely described.

It should be made clear however that there is neither experimental evidence nor theoretical prejudice from QCD to support an even approximate dynamical gauge boson character of the spin-1 resonances and therefore justify among other things the conspicuous emergence of Yang-Mills self-couplings of the \( \rho \) and the \( \alpha_1 \) fields. As we stated above, the nice feature of the “gauge” models is their natural compliance with the phenomenologically successful notion of vector meson dominance, but as other authors have shown\(^5\), this feature is not unique to the models

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of Refs. 3-4. It can also be obtained in cases where chiral symmetry is realised in a less exotic manner.

In our approach we construct a lagrangian consistent with general principles of quantum field theory and chiral symmetry. Vector meson dominance can be implemented later, if so desired. We assume therefore a homogeneous transformation law for isovector spin-1 fields. One of the purposes of my talk is to convince you that even in that case constraints relating three- and four-point coupling strengths do exist. These derive from demanding the hamiltonian to be bounded from below. I will extend our previous analysis of the \( \pi \rho \) system to the description of interacting pions, \( \rho \)- and \( a_1 \)-mesons. In section 2 I introduce you to the symmetry structure of our scheme and build the basic interaction lagrangian. Section 3 is devoted to the investigation of the energies of some nonperturbative field configurations. It is shown that these energies are unbounded from below at the three-meson coupling level. In section 4 I demonstrate how the inclusion of four-point effective couplings counterbalances the dangerous contributions of the three-point terms to these energies and derive inequalities between three- and four- meson couplings for the theory to make sense. We finally discuss in section 5 how unitarity arguments based on vector dominance could lead to saturation of these inequalities and suggest a novel low-energy \( \pi \rho a_1 \) effective lagrangian consistent with chiral symmetry and general field theoretical principles.

2. Nonlinear realisations of chiral symmetry

Let us start with the lagrangian of the nonlinear sigma model defined in terms of the \( SU(2) \) field \( U \) as:

\[
\mathcal{L}_{NL\sigma} = \frac{f^2}{4} < \partial^\mu U \partial_\mu U^\dagger >,
\]

\( f \) being the pion decay constant. We define \( U \) as \( U = \exp \left( i \vec{\tau}. \vec{F}(x) \right) \) with the pion field given by \( \vec{F} = \vec{F} \). Other parametrisations are perhaps more suitable for perturbative evaluations of Green’s functions, but are not as convenient for investigations of the large field region, which is of interest for our purposes.

The lagrangian (1) is invariant under the linear \( SU(2)_L \otimes SU(2)_R \) group rotation \( U \rightarrow g_L U g_R^\dagger \). It is also invariant under the following nonlinear rotation:\n
\[
u(\vec{F}) \rightarrow g_L u(\vec{F}) h^\dagger(\vec{F}) = h(\vec{F}) u(\vec{F}) g_R^\dagger,
\]

where \( u \) is the square root of \( U \) and \( h(\vec{F}) \) is a compensating transformation ensuring that \( U \) transforms linearly. Consider now the following axial-vector and vector respective field gradients:

\[
u_\mu = i(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger)
\]

\[
\Gamma_\mu = \frac{1}{2} (u^\dagger \partial_\mu u + u \partial_\mu u^\dagger).
\]

These gradients transform as

\[
u_\mu \rightarrow h(\vec{F}) u_\mu h^\dagger(\vec{F})
\]

\[
\Gamma_\mu \rightarrow h(\vec{F}) \Gamma_\mu h^\dagger(\vec{F}) + h(\vec{F}) \partial_\mu h^\dagger(\vec{F}).
\]
It is seen that while \( u_\mu \) transforms homogeneously the transformation of \( \Gamma_\mu \) contains an inhomogeneous part as a result of the field dependence of \( h(\vec{F}) \). How should spin-1 fields transform in this framework? Basically there are two possibilities forming a group: homogeneous or inhomogeneous. The inhomogeneous group was chosen by Weinberg in its study of the \( \pi\rho \) system. This approach later acquired the name of “hidden gauge symmetry” approach, because the associated lowest order invariant lagrangian preserves not only chiral symmetry but also a certain sort of a gauge symmetry. It is one of my purposes to point out that this symmetry was never revealed by low energy \( \pi\pi \) scattering experiments. Furthermore in the inhomogeneous approach it is impossible to define similar transformations for both the \( \rho \) and the \( a_1 \) fields, simply because the associated particles have opposite parity. In contrast the homogeneous transformation is the simplest one consistent with chiral symmetry and it can be applied to both the \( \rho \) and the \( a_1 \) fields. We adopt this point of view and assume that the \( \rho \)- and the \( a_1 \)-mesons transform as

\[
V_\mu \rightarrow h(\vec{F}) V_\mu h^\dagger(\vec{F})
\]

\[
A_\mu \rightarrow h(\vec{F}) A_\mu h^\dagger(\vec{F})
\]

under the nonlinear group. In this expression \( V_\mu = \tau.\vec{V}_\mu \) and \( A_\mu = \tau.\vec{A}_\mu \). To build invariant couplings we define covariant derivatives of spin-1 fields transforming as in eq. (5)

\[
\nabla_\mu = \partial_\mu + [\Gamma_\mu, \ ],
\]

in such a way that \( \nabla_\mu V_\nu \) and \( \nabla_\mu A_\nu \) also transform homogeneously: \( \nabla_\mu V_\nu \rightarrow h \nabla_\mu V_\nu \ h^\dagger \) and similarly \( \nabla_\mu A_\nu \rightarrow h \nabla_\mu A_\nu \ h^\dagger \).

The invariant lagrangian at quadratic order in the fields is given by

\[
L^{(2)}_{\pi\rho a_1} = \frac{f^2}{4} < u_\mu u^\mu > - \frac{1}{4} < V_{\mu\nu} V^{\mu\nu} > - \frac{1}{4} < A_{\mu\nu} A^{\mu\nu} >
+ \frac{M_\rho^2}{2} < V_\mu V^\mu > + \frac{M_{a_1}^2}{2} < A_\mu A^\mu >,
\]

where \( V_{\mu\nu} = \nabla_\mu V_\nu - \nabla_\nu V_\mu \) and \( A_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu \) are the covariant field strengths of the spin-1 resonances. We introduce chirally invariant mass terms for the \( \rho \)- and the \( a_1 \)-mesons and we assume that the coupling \( c < A_\mu u^\mu > \) is not present. Actually with the choice \( c = 0 \) no diagonalisation of \( \pi\rho a_1 \) interactions is needed - obviously not a disadvantage of our framework. At the three-point level we shall consider some chirally invariant terms consistent with charge conjugation and parity invariance, containing at least one pion field gradient:

\[
L^{(3)}_{\pi\rho a_1} = - \frac{i}{2\sqrt{2}} \left\{ g_1 < V_{\mu\nu} [u^\mu, u^\nu] > + g_2 < A_{\mu\nu} ([V^\mu, u^\nu] - [V^\nu, u^\mu]) >
+ g_3 < V_{\mu\nu} ([A^\mu, u^\nu] - [A^\nu, u^\mu]) > \right\}.
\]

The lagrangian \( L^{(2)}_{\pi\rho a_1} + L^{(3)}_{\pi\rho a_1} \) has six free parameters that one would ideally like to determine from QCD. But this problem seems to be elusive since a sensible method
to perform such an extraction from QCD has not yet been discovered.

3. Nonperturbative pathologies

The issue I address here is rather different: assuming that \( g_1, g_2, g_3 \) are given by the underlying QCD dynamics, are there any relations between these parameters and higher order ones? Previous investigations suggest that this question should be addressed in a nonperturbative framework. In particular does the theory \((7, 8)\) yield a hamiltonian that is bounded from below? To find an answer let us investigate the hamiltonian associated with the lagrangian \( L^{(2)} \pi \rho a + L^{(3)} \pi \rho a \) in terms of the canonical degrees of freedom: the fields \( \vec{F}, \vec{V}_i, \vec{A}_i \) and their conjugate momenta, respectively \( \vec{\phi}, \vec{\pi}_i, \vec{\chi}_i \). The energy can be written as a sum of two terms \( H = H_T + H_V \), where the kinetic energy is \( H_T \) and the potential energy is \( H_V \). The potential part contains only space components and in the three-point case is given by

\[
H_V = \int d^3x \left\{ \frac{f_2^2}{2} (\partial_i F) \partial^i F + M_a^2 A_i A_i + \frac{1}{4} \left[ A_{ij} + i \sqrt{2} g_2 [V_i, u_j] - [V_j, u_i] \right] \right\} + M_\rho^2 (V_i)^2 + \frac{1}{2} \left[ V_i + i \sqrt{2} g_1 [u_i, u_j] + i \sqrt{2} g_3 ([A_i, u_j] - [A_j, u_i]) \right] \right\},
\]

(9)

The kinetic piece needs some work in order to eliminate the dependent variables \( \vec{V}_0, \vec{A}_0 \). It turns out that the generic expression for \( H_T \) in the case of a quadratic in momenta theory is:

\[
H_T = \int d^3x \left\{ \frac{1}{2} \vec{\Phi} A^{-1} \vec{\Phi} + \frac{\vec{\pi}_i^2}{4} + \frac{\vec{\chi}_i^2}{4} + \frac{1}{2} \vec{P} \vec{P}^{-1} \vec{P} \right\},
\]

(10)

where \( \vec{\phi}, \vec{\pi}, \vec{\chi} \) are linearly related to the momenta \( \vec{\phi}, \vec{\pi}_i, \vec{\chi}_i \) and \( A, P \) are isospin tensor functions of \( \vec{F}, \vec{V}_i, \vec{A}_i \), which depend on the detailed structure of the dynamics. Let us now investigate the energy of a classical meson mapping. The simplest such object one can imagine has an isospin content specified by a constant unit vector \( \vec{F} \):

\[
\vec{F}_0(\vec{x}) = F(\vec{x}) \vec{F},
\]

(11)

where \( F(\vec{x}) \) is a regular function of space. Such a configuration is topologically trivial. For the vector and the axial vector fields I will assume that they are parallel to the pion field:

\[
\vec{V}_i(\vec{x}) = V_i(\vec{x}) \vec{F},
\]

(12)

\[
\vec{A}_i(\vec{x}) = A_i(\vec{x}) \vec{F}.
\]

The special form of our ansatz result in a potential energy which is simply given by:

\[
H_V = \int d^3x \left\{ \frac{f_2^2}{2} (\partial_i F)^2 + M_a^2 A_i^2 + \frac{1}{2} (\partial_i A_j - \partial_j A_i)^2 + M_\rho^2 V_i^2 + \frac{1}{2} (\partial_i V_j - \partial_j V_i)^2 \right\},
\]

(13)

the potential energy of a free theory, an obviously positive quantity. We therefore concentrate in the kinetic energy of the theory as given by \( H_T \). Because of the particular isospin structure we consider here only momenta that point in a direction
perpendicular to that of the pion actually “see” the couplings to the vector mesons. We assume:

\[ \vec{\phi} = \phi(\vec{x}) \hat{\phi} \]
\[ \vec{\pi}_i = \pi_i(\vec{x}) \hat{F}_i \]
\[ \vec{\chi}_i = \chi_i(\vec{x}) \hat{F}_i \]

(14)

with \( \hat{\phi} \cdot \vec{F} = 0 \). The kinetic energy of this field configuration reads

\[ H_T = \int d^3x \left\{ \frac{\phi^2}{2f^2s^2I} + \frac{1}{4} \left[ \pi_i^2 + \chi_i^2 + \frac{(\partial_i \pi_i)^2}{M_\rho^2} + \frac{(\partial_i \chi_i)^2}{M_a^2} \right] \right\}, \]

(15)

where \( s \) is a shorthand notation for \( \sin F/F \). Observe now the structure of the dimensionless “inertial” parameter \( I \) which contains all the nontrivial effects due to the spin-1 fields:

\[ I = \frac{1}{f^4 M_1 M_2} \left( f^4 M_1 M_2 - 8(g_2^3 V_i^2 M_2 M_\rho^2 + (g_1 \partial_i F - g_3 A_i)^2 M_1 M_\rho^2) \right. \]
\[ \left. + 16 \left[ g_4^2 M_2 (V_i^2 (\partial_i F)^2 - (V_i \partial_i F)^2) + g_3^2 M_1 (A_i^2 (\partial_i F)^2 - (A_i \partial_i F)^2) \right] \right) \],

(16)

with \( M_1 = (1/f^2)[2M_\rho^2 - 4g_2^3(\partial_i F)^2] \) and \( M_2 = (1/f^2)[2M_a^2 - 4g_3^2(\partial_i F)^2] \). While in the case of the nonlinear sigma model with vanishing couplings \( I \) is simply equal to 1, here it acquires negative contributions. For small fields \( F, V_i, A_i \approx 0 \) appropriate to perturbation theory one has \( I \approx 1 \) so no problem seems to appear in perturbative expansions of scattering amplitudes. For nonperturbative configurations however the situation changes dramatically since then negative contributions proportional to quadratic powers of the couplings can drive \( I \) to zero or negative values. The hamiltonian density acquires poles and the energy is not bounded from below. Such troubles can arise at fairly low energy scales. Consider a localised meson wave carrying momentum \( k_i \) and of amplitude \( F \approx 1 \). Assume to simplify further that all classical fields vanish except \( F \) and \( \phi \). The gradient \( \partial_i F \) is roughly approximated by \( k_i \) and \( I \) inside the meson wave looks like:

\[ I \approx \frac{1 - 2 \left( \frac{g_3^2}{M_a^2} + 2 \frac{g_2^3}{f^2} \right) k^2}{1 - 2 \frac{g_3^2}{M_a^2} k^2}. \]

(17)

At small or very large momenta \( k \) the inertial parameter is positive but for \( k^2 \) in the intermediate range

\[ \left( 2 \frac{g_3^2}{M_a^2} + 4 \frac{g_1^2}{f^2} \right)^{-1} < k^2 < \frac{M_a^2}{2g_3^2} \]

(18)

\( I \) becomes negative and as a consequence the kinetic energy density is negative, making the theory ill defined in these regions. Taking reasonable numerical values for the coupling constants\(^8\), the region of dangerous momenta is found to be \( 0.4 \) GeV \( < k < 2.0 \) GeV, which includes the range of masses of the \( \rho \) and the \( a_1 \) resonances. And this is precisely the range that one would like to describe by extending the low-energy effective theories to include spin-1 mesons!
We conclude that the Hamiltonian associated with the simplest three-point \( \pi \rho a_1 \) interactions is not bounded from below which is of course unacceptable.

4. Constraints on four-meson coupling strengths

Let us now consider the effect of some four-meson couplings that are relevant to our analysis:

\[
\mathcal{L}_{\pi \rho a_1}^{(4)} = \frac{1}{8} \left\{ g_4 < [u_\mu, u_\nu]^2 > + 2 g_5 < [u_\mu, u_\nu][A_\mu, u_\nu] > + 2 g_6 < [V_\mu, u_\nu]^2 > \\
- < [V_\mu, u_\nu][V_\nu, u_\mu] > + 2 g_7 < [A_\mu, u_\nu]^2 > - < [A_\mu, u_\nu][A_\nu, u_\mu] > \right\}. 
\] (19)

We introduce four new coupling constants \( g_4, g_5, g_6, g_7 \). Amongst these terms one can recognise a local four-point pion vertex, the so-called “Skyrme term”, as well as a term contributing to the decay \( a_1 \to \pi \pi \pi \).

Concerning now the energy of the charge-zero meson configuration defined in the previous section, we note first that its potential energy is unaffected by the new couplings and is still given by eq. (13). The kinetic piece has the same form as in eq. (15) but with a new inertial function, \( \tilde{I} \). After a tedious but straightforward calculation one finds:

\[
\tilde{I} = \frac{1}{f^4 M_1 M_2} \left\{ f^4 \tilde{M}_1 \tilde{M}_2 - 8(g_2^2 - g_6)V_\rho^2 M_\rho^2 \tilde{M}_2 \\
+ 16 \left[ (g_2^2 - g_6)^2 (V_\rho^2 (\partial_i F)^2 - (\partial_i F V_i)^2) \right] \tilde{M}_2 \\
- 8 \left[ (g_1^2 - g_4)(\partial_i F)^2 + (g_3^2 - g_7)A_i^2 - 2 \left( g_3 g_1 - \frac{g_5}{2} \right) (\partial_i F A_i) \right] M_a^2 \tilde{M}_1 \\
+ 16 \left[ \left( (g_1^2 - g_4)(g_3^2 - g_7) - \left( g_3 g_1 - \frac{g_5}{2} \right)^2 \right) (\partial_i F)^4 \right. \\
\left. + (g_3^2 - g_7)^2 (A_i^2 (\partial_i F)^2 - (A_i \partial_i F)^2) \right] \tilde{M}_1 \right\}, 
\]

with \( \tilde{M}_1 = (1/f^2)[2M_\rho^2 - 4(g_2^2 - g_6)(\partial_i F)^2] \) and \( \tilde{M}_2 = (1/f^2)[2M_\rho^2 - 4(g_2^2 - g_6)(\partial_i F)^2] \). By requiring that for any value of the classical profiles \( \partial_i F, V_i, A_i \) the function \( \tilde{I} \) is non-negative, we find constraints on the couplings. Consider the following simplifying cases:

a) \( \partial_i F = A_i = 0 \quad \Rightarrow \quad \tilde{I}_a = 1 - \frac{4}{f^2}(g_2^2 - g_6)V_\rho^2 \)

b) \( \partial_i F = V_i = 0 \quad \Rightarrow \quad \tilde{I}_b = 1 - \frac{4}{f^2}(g_2^2 - g_7)A_i^2 \) (21)

c) \( V_i = A_i = 0 \quad \Rightarrow \quad \tilde{I}_c = \frac{1}{f^4 M_2} \left\{ 2M_\rho^2 - [4(g_2^2 - g_7) + 8(g_1^2 - g_4)M_\rho^2/f^2](\partial_i F)^2 \\
+ \frac{16}{f^2} \left[ (g_1^2 - g_4)(g_3^2 - g_7) - (g_3 g_1 - \frac{g_5}{2})^2 \right](\partial_i F)^4 \right\}. \)
Requiring positiveness of \( \tilde{x}_{a,b,c} \) for all possible values of the fields leads to:

\[
\begin{align*}
g_4 & \geq g_1^2 \\
g_6 & \geq g_2^2 \\
g_7 & \geq g_3^2 \\
(g_1^2 - g_4)(g_3^2 - g_7) & \geq (g_3g_1 - \frac{g_5}{2})^2.
\end{align*}
\] (22)

These conditions show that the Skyrme term and other four-point interactions are essential if the hamiltonian is to be bounded from below. In an effective theory where the spin-1 fields transform homogeneously they arise as counterterms for the bad behaviour of the vector-meson contributions. This is in sharp contrast to other approaches\(^3\),\(^4\), where the same Skyrme term emerges from the exchange of a very heavy \( \rho \)-meson.

5. Discussion

Our investigation of classical nonperturbative effects in low-energy chiral theories shows that the constraints (22), relating three- and four-point couplings, must be satisfied for a consistent description of the interactions between pions and spin-1 isovector mesons. We stress that chiral symmetry is implemented nonlinearly in this approach and the vector mesons are naturally assumed to transform homogeneously under chiral rotations. The constraints arise from demanding the hamiltonian to be bounded from below. They do not depend on phenomenological ideas such as vector dominance.

One might ask now whether there are any other constraints on the couplings from first principles. For instance another nonperturbative notion that one could invoke in this context is the unitarity of the scattering matrix. This was previously studied\(^9\) in the special case of the lagrangian (7, 8) without the \( a_1 \) \((g_2 = g_3 = 0)\). Working at tree-level it was found that further local pion interactions must be added by hand if the forward elastic \( \pi \pi \) scattering amplitude is to obey the Froissart bound\(^10\). These local interactions compensate for the most divergent contribution produced by \( \rho \)-exchange. The result, in the SU(2) sector is:

\[
L^{SU(2)}_{\text{local}} = \frac{g_1^2}{8} < [u_\mu, u_\nu]^2 > .
\] (23)

This is just the Skyrme term, with a coefficient that is fixed by the three-point coupling \( g_1 \). If one works at tree level, the \( a_1 \) does not contribute to \( \pi \pi \) scattering. Imposing unitarity therefore leads to saturation of the lower bound on \( g_4 \) in (22):

\[
g_4 - g_1^2 = 0.
\] (24)

Combining this with the final constraint in (22), we obtain a relation expressing the implications of unitarity for the couplings of the \textit{axial} meson:

\[
g_5 = 2g_1g_3.
\] (25)
This is nontrivially relating the strength of the $a_1 \rightarrow \pi\pi\pi$ decay to those of the processes $\rho \rightarrow \pi\pi$ and $a_1 \rightarrow \rho\pi$.

The saturation of two of our constraints follows from the assumption that an extreme version of vector dominance holds for strong interactions. In fact the value for $g_4$ determined assuming $\rho$-meson dominance agrees well with that from chiral perturbation theory\textsuperscript{11}, suggesting that at least at low energies, vector dominance is really making sense. We speculate that dominance of a single resonance may also hold in the axial-vector channel, leading to saturation of the remaining constraints in (24). In this case our lagrangian would simplify to:

$$
\mathcal{L}_{\pi\rho a_1} = \frac{f^2}{4} < u_\mu u^\mu > + \frac{M_\rho^2}{2} < V_\mu V^\mu > + \frac{M_{a_1}^2}{2} < A_\mu A^\mu > - \frac{1}{4} < \left( V_{\mu\nu} + \frac{i}{\sqrt{2}} (g_1 [u_\mu, u_\nu] + g_3 ([A_\mu, u_\nu] - [A_\nu, u_\mu])) \right)^2 > \\
- \frac{1}{4} < \left( A_{\mu\nu} + \frac{i}{\sqrt{2}} g_2 ([V_\mu, u_\nu] - [V_\nu, u_\mu])) \right)^2 > .
$$

(26)

This constitutes an effective lagrangian describing the strong interactions of $\pi\rho a_1$ mesons with a minimal number of free coupling constants. It is the simplest one compatible with chiral symmetry and leading to a hamiltonian which is free of pathologies.

To summarise: in our framework theory chiral symmetry is implemented in the simplest possible way and no speculative gauge symmetry assumption is made. Constraints between couplings are there to ensure that the hamiltonian is bounded from below, and vector meson dominance can be implemented by specific choices of parameters. It is therefore most natural to regard our lagrangian (26) as the starting point for any extension of chiral perturbation theory of pseudoscalar pions to the resonance region.

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7. References

1. S. Weinberg, *Phys. Rev.* **166** (1968) 1568; I. Gerstein, R. Jackiw, B. Lee and S. Weinberg, *Phys. Rev.* **D3** (1971) 2486; A. Salam and J. Strathdee, *Phys. Rev.* **D2** (1970) 2869
2. G. ’t Hooft, *Nucl. Phys.* **B79** (1974) 276
3. J. J. Sakurai, *Currents and Mesons*, University of Chicago Press, Chicago 1969; J. Schwinger, *Particles and Sources*, Clarendon Press, Oxford, UK, 1969
4. M. Bando, T. Kugo and K. Yamawaki, *Phys. Rep.* **164** (1988) 217
5. G. Ecker, J. Gasser, A. Pich and E. de Rafael, *Nucl. Phys.* **B321** (1989) 311
6. D. Kalafatis, Phys. Lett. B313 (1993) 115
7. S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2239; C. G. Callan, S. Coleman, J. Wess and B. Zumino, ibid 2247
8. B. Moussallam, private communication
9. G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, Phys. Lett. B223 (1989) 425
10. M. Froissart, Phys. Rev. 123 (1961) 1053; A. Martin, Phys. Rev. 129 (1963) 1432
11. J. Gasser and H. Leutwyler, Ann. Phys. (NY) 158 (1984) 142