Duality in $Sp$ and $SO$ Gauge Groups from M Theory

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Abstract

We describe fivebrane configurations in M theory whose 4-d spacetime contains $N = 1$ supersymmetric $Sp$ or $SO$ gauge fields and fundamentals of these groups. We show how field-theory dualities for $Sp$ and $SO$ groups can be derived using these fivebrane configurations in M theory.
1 Introduction

It is difficult to understand the dynamics behind the electric-magnetic duality in supersymmetric field theory [1] in the context of field theory. String theory brane dynamics [2] turns out to provide new information about the phenomena of duality in supersymmetric field theories. Here, we will explore the M theory approach for learning about the low-energy limit of $N = 1$ supersymmetric field theories [3-9]. So far, many field-theory results have been rederived using various configurations of branes in string theory [10-19]. Depending on the geometry of the brane setup one obtains different gauge theories with varying amount of supersymmetry on the world-volume of branes. In the given examples continuous deformations of branes lead to theories with the same infrared dynamics, while the high-energy field content can change under deformations. Many examples of duality have been confirmed this way. Similar insights have been gained using F theory approach [20-27].

However, these brane configurations in the context of string theory involve a singularity at the points where the branes join. To avoid these type of singularities it has been suggested to consider the given brane setup embedded into M theory [28, 29]. The advantage of M theory is that it smoothes out many of the singularities encountered at the joining of branes. For example, D-4 branes can be thought of as 5-branes wrapped around the eleventh dimension. This way, the brane setup corresponding to interesting 4-d field theories will from a single 5-brane surface in M theory. For example, a common setup in type IIA string theory for studies of dualities is a series of parallel NS 5-branes connected by Dirichlet 4-branes. In M theory, D-4 branes become 5-branes wrapped around compact eleventh dimension. Therefore branes connect smoothly in such a setup. Using single brane configuration avoids singularities present in other approaches. With this approach, Witten computed the elliptic curves describing the Coulomb branch of $N = 2$ $SU(N)$ theory using M theory in Ref. [29]. Subsequently, the authors of Ref. [30, 31, 32] generalized these results to other classical groups. $N = 1$ theories can be studied on the world-volume of branes in M theory as well. Several authors obtained results about the confining phase of $N = 1$ supersymmetric QCD, dynamically generated superpotentials and also gaugino condensation [3-7, 9]. Recently, Schmaltz and Sundrum have pointed out that the embedding of the type IIA theory can also avoid the singularities that one encounters in string theory when moving branes across each other. They have shown how to derive electric-magnetic duality in $N = 1$ $SU(N)$ theories with fundamentals from M theory [3]. In their setup a single M theory 5-brane describes both the electric and the magnetic theories of Seiberg’s SUSY QCD, if non-vanishing masses for the quarks are assumed. This way they obtained a smooth interpolation between the electric and the magnetic descriptions of SUSY QCD. However, a non-vanishing mass term for this smooth interpolation was crucial, the M theory surface for the massless case is still singular.

In this paper we generalize the results of Ref. [8] and derive duality in $Sp$ and $SO$ gauge groups [33, 34]. We first describe brane configurations in type IIA string theory, which we
later interpret in the context of M theory. We will show that the same curve describes the
original $Sp(2N)$ [$SO(N)$] theories as well as the dual $Sp(2F - 2N - 4)$ [$SO(F - N + 4)$]
theories. For the $Sp$ theories we also extend our results to a setup with finite fourbranes,
where the antisymmetric meson field of the dual theory will emerge explicitly.

2 Semi-infinite Brane Configuration for $Sp(2N)$

We begin by considering brane configurations in type IIA string theory. We denote spacetime
coordinates by $x^0, x^1, \ldots, x^9$, where $x^0, \ldots, x^3$ denote the usual 4-d spacetime. For future
reference let us define $v = x^4 + ix^5$ and $w = x^7 + ix^8$. Once we move on to M theory we will
denote the eleventh dimension by $x^{10}$. The $x^{10}$ coordinate is periodic under $x^{10} \rightarrow x^{10} + R$,
where $R$ is the compactification radius. From the string theory point of view $R = g_s$, so
small radius limit is equivalent to weakly-coupled string theory. It will be later useful to
define $s = x^6 + ix^{10}$, $t = \exp(-\frac{s}{R})$. Also for convenience, we choose the units such that the
string scale is set to one, $m_s = 1$.

We use a brane configuration similar to that of Ref. [8]. We consider Dirichlet 4-branes
stretched between NS 5-branes. We will take some 4-branes to be semi-infinite. Our config-
uration is illustrated in Fig. 1. All branes fill the 4-d spacetime and are placed at $x^9 = 0$.
There are two 5-branes: one at $x^6 = 0$ occupying $x^4$ and $x^5 (5_v)$, and another one at $x^6 = s_0$
occupying $x^7$ and $x^8 (5_w)$. There are $N$ 4-branes suspended between the 5-branes and also
$F$ 4-branes extending from the $5_v$ brane to minus infinity in the $x^6$ direction. In order to
obtain $Sp$ or $SO$ gauge groups we need to make an orientifold projection [35]. Orientifold
projection combines spacetime symmetry and a parity inversion on the world-sheet. Under
this projection the spacetime coordinates transform as

$$(x^4, x^5, x^7, x^8, x^9) \rightarrow (-x^4, -x^5, -x^7, -x^8, -x^9).$$

Modding out the spacetime by this transformation is indicated in Fig. 1 by an orientifold
4-plane. The orientifold 4-plane extends in $x^1$, $x^2$, $x^3$ and $x^6$ directions. 4- and 5-branes are
placed symmetrically with respect to the orientifold. The parity projection, $\Omega$, allows for
$\Omega^2 = \pm 1$. The 4-d gauge group is symplectic when $\Omega^2 = -1$, and it is orthogonal otherwise.
In order to generate non-vanishing masses for the flavors, the semi-infinite 4-branes are
assumed to be non-overlapping. The distances of these 4-branes from the orientifold plane
 correspond to masses of fundamental fields $m_i$. We will assume that all these masses are
different. Strings connecting the finite 4-branes correspond to massless vector fields of $Sp$
($SO$) gauge groups. The separation between the $5_v$ and $5_w$ branes in the $x^6$ direction is
related to the gauge coupling of the $4 - d$ theory:

$$\frac{8\pi^2}{g_4^2} \sim \frac{s_0}{g_s}.$$
Having described the topology of the brane configuration we lift the above setup into M theory, which also allows us to take the limit of large string coupling. The 4-branes gain an extra dimension, since they are compactified on $x^{10}$ in the M theory description, and connect smoothly with the $5_v$ and $5_w$ branes. We first consider the case of $Sp$ gauge group and determine the curve corresponding to the M theory configuration [29, 3]. The setup of Fig. 1 will correspond to an $Sp(2N)$ gauge theory with $2F$ fundamental fields which have a mass term, $mQQ$, in the superpotential. In M theory this setup will occupy $\mathbb{R}^4 \times \Sigma$, where $\Sigma$ is a one complex dimensional Riemann surface. In the case of $Sp(2N)$ this surface is described by the curve

$$t \prod_{i=1}^{F} (v^2 - m_i^2) = \xi v^{2N+2},$$

$$vw = \zeta.$$  \(1\)

It is helpful to identify the symmetries associated with rotations in the $v$ and $w$ planes. These symmetries are anomalous so the scale of 4-d theory, $\Lambda_{Sp}$, transforms under these symmetries. The mass terms, $mQ_iQ_j$, will be kept invariant under the transformations by assigning appropriate charges to $m$. The mesons, $M$, in the dual theory which we will describe later, always carry the charges of quark bilinears. The table of charges is as follows:

|   | $Q$ | $m$ | $\Lambda_{Sp}^{3(N+1)-F}$ | $v$ | $w$ |
|---|-----|-----|--------------------------|-----|-----|
| $R_v$ | 0   | 2   | $2N+2-2F$ | 2   | 0   |
| $R_w$ | 1   | 0   | $2N+2$   | 0   | 2   |

Figure 1: Brane configuration of the electric theory.
Using these symmetries we can identify the parameters $\xi$ and $\zeta$:

\[ t \prod_{i=1}^{F} (v^2 - m_i^2) = v^{2N+2} (Pf m)^{2F-N-1}, \]

\[ vw = \Lambda_{Sp}^{3(N+1)-F} (Pf m)^{\frac{1}{N+1}}. \quad (2) \]

There are several consistency checks one can perform on the above form of the curve. We will check how the curve behaves for large values of $v$ and $w$, in which case we expect to reproduce the perturbative limit of the gauge theory. The symmetries of the curve in such limits should correspond to symmetries of the microscopic gauge theory. First, we can examine the asymptotic behavior of the curve for large $v$:

\[ tv^{2F-2N-2} \sim (Pf m)^{2F-N-1}, \quad w \sim 0, \quad (3) \]

which is indeed symmetric under $R_v$ and $R_w$. For large $w$, the curve can be approximated as

\[ tw^{2N+2} \sim \Lambda_{Sp}^{2[3(N+1)-F]} (Pf m)^{2F-N-1}, \quad v \sim 0. \quad (4) \]

Here, one needs to take into account transformation properties of $\Lambda_{Sp}$ under anomalous symmetries $R_v$ and $R_w$. From field theory one expects the presence of non-anomalous discrete symmetries. In either limit, $R_v$ rotation by $\frac{\pi}{2N+2} - 2F$ and $R_w$ rotation by $\frac{\pi}{2N+2}$ leave the curve invariant. $R_v$ is explicitly broken by the mass term $m_i$, while $R_w$ is broken to its $Z_{4(N+1)}$ subgroup. As in Ref. [8], we can calculate the separation of the 5$_v$ and 5$_w$ branes in the $s$ direction. This can be done by going to large values of $v$ on the 5$_v$ brane and to large values of $w$ on the 5$_w$ brane. Then using the above equations for $t$ we obtain that

\[ e^{-s_0/R} = \Lambda_{Sp}^{2[3(N+1)-F]}, \quad (5) \]

where $s_0$ is the distance between the two branes. This equation just expresses the logarithmic bending of branes at large distances. Note, that just like in the $N = 2$ case explained in Ref. [31, 32], we get a factor of two in front of the beta function in Eq. (5), which corresponds to a rescaling of the gauge coupling constant and appears due to the non-conventional embedding of $Sp(2N)$ into $SU(2N)$ [30, 31].

Let us now express the curve in terms of $t$ and $w$:

\[ t(-1)^F \prod_{i=1}^{F} (w^2 - w_i^2) = w^{2F-2N-2} \left( \prod_{i=1}^{F} w_i \right)^{\frac{2N+1}{F}}, \]

where $w_i \propto (Pf m)^{\frac{3}{N+1}} \Lambda_{Sp}^{\frac{3(N+1)-F}{N+1}} \left( \frac{1}{m} \right)_{ij}$. This expression for $w_i$ looks exactly like the field-theory relation for the VEVs of the meson operators, $M$, in the presence of mass terms $mQQ$ [34]:

\[ \langle M_{ij} \rangle = [2^{N-1} (Pf m)]^{\frac{N+1}{N+1}} \Lambda_{Sp}^{\frac{3(N+1)-F}{N+1}} \left( \frac{1}{m} \right)_{ij}. \]
After reabsorbing suitable numerical factors into the definition of $t$, the curve written in terms of $t$ and $w$, where we also replaced $w_i$ by $\langle M \rangle_i$

$$ t \prod_{i=1}^{F} (w^2 - \langle M \rangle_i^2) = w^{2\tilde{N}+2} (\text{Pf}(M))^{\frac{2F-\tilde{N}-1}{F}}, $$

$$ vw = \tilde{\Lambda}_{Sp}^{\frac{3\tilde{N}+1-F}{N+1}} (\text{Pf}(M))^{\frac{1}{N+1}}, \quad (6) $$

where $\tilde{N} = F - N - 2$, while $\tilde{\Lambda}_{Sp}$ is the scale of the dual gauge group, and the scale of the magnetic theory is related to the scale of the electric theory as $\tilde{\Lambda}_{Sp}^{3\tilde{N}+1-F} \Lambda_{Sp}^{3(N+1)-F} = 16(-1)^{F-N-1} \mu F$. The scale $\mu$ is arbitrary in field theory, however in this derivation $\mu$ is proportional to the string mass scale.

This curve looks exactly like the original one, except that it corresponds to the dual gauge group $Sp(2F - 2N - 4)$, whose brane configuration is illustrated in Fig. 2. The distances between 4-branes and the orientifold plane can now be identified with the expectation values of the mesons in the magnetic theory. By comparing with the original curve, we can see that the expectation values of the meson fields play now the role of the dual quark masses, which is exactly what we expect from field theory $[34]$. Taking the limit $\tilde{\Lambda}_{Sp} \to 0$ (which corresponds to taking $\Lambda_{Sp} \to \infty$) and then $R \to 0$ while keeping the meson VEV fixed will result in the brane configuration displayed in Fig. 2. Thus we can see the emergence of the dual $Sp(2F - 2N - 4)$ gauge group by looking at the same M theory fivebrane in two different limits.
3 Semi-infinite Brane Configuration for SO(2N) and SO(2N+1)

We now repeat the above analysis for SO(N) groups. There are two cases that need to be dealt with separately: when N is even and N is odd. The brane configuration we use is that of Fig. [4]. We first consider SO(2N) groups with 2F vectors. The charge assignment under $R_v$ and $R_w$ rotation is the following:

|      | Q    | m   | $\Lambda_{SO}^{3(2N-2)-2F}$ | v   | w   |
|------|------|-----|-----------------------------|-----|-----|
| $R_v$ | 0    | 2   | $2(2N - 2) - 4F$           | 2   | 0   |
| $R_w$ | 1    | 0   | $2(2N - 2)$                | 0   | 2   |

We obtain the following curve in this case

$$tv^2 \prod_{i=1}^{F} (v^2 - m_i^2) = v^{2N} (\det m)^{\frac{F-N+1}{2}},$$

$$vw = \Lambda_{SO}^{3(2N-2)-2F} (\det m)^{\frac{1}{2N-2}}.$$  \hspace{1cm} (7)

We can perform the same consistency checks we did in the case of Sp. In the large $v$ limit we get

$$tv^{2F-2N+2} \sim (\det m)^{\frac{F-N+1}{F}}, \quad w \sim 0,$$  \hspace{1cm} (8)

while in the large $w$ limit

$$tw^{2N-2} \sim \Lambda_{SO}^{3(2N-2)-2F} (\det m)^{\frac{F-N+1}{F}}, \quad v \sim 0.$$  \hspace{1cm} (9)

It is straightforward to check that the symmetries we expect from field theory are properly reproduced. Here, the separation of $5_v$ and $5_w$ branes in the $x^6$ direction gives due to brane bending

$$e^{-s_0/R} = \Lambda_{SO}^{3(2N-2)-2F}.$$  

The meson fields in SO(N) theories obtain the following VEVs when mass terms for the quark fields are present

$$\langle M_{ij} \rangle = [16 (\det m)]^{\frac{1}{2N-2}} \Lambda_{SO}^{3(2N-2)-2F} \left( \frac{1}{m} \right)_{ij}.\langle M \rangle.$$  

Identical expression emerges when we express the curve in terms of $w$ and $t$, which is a strong indication that the curve properly reproduces mesons VEVs in the magnetic theory. We obtain the following form for the curve:

$$tw^2 \prod_{i=1}^{F} (w^2 - \langle M_i \rangle^2) = w^{2N} (\det \langle M \rangle)^{\frac{F-N+1}{F}},$$

$$vw = \Lambda_{SO}^{3(2N-2)-2F} (\det \langle M \rangle)^{\frac{1}{2N-2}}.$$  \hspace{1cm} (10)
where $\tilde{N} = F - N + 2$, while $\Lambda_{SO}^{3(2N-2)-2F} - \tilde{\Lambda}_{SO}^{3(2\tilde{N}-2)-2F} = \mu^{2F}$. The fundamental result is again confirmed. The dual gauge group is $SO(2F - 2N + 4)$, which one obtains in the $\Lambda_{SO} \to 0, R \to 0$ limit while keeping the meson VEV fixed.

Let us now briefly summarize the same derivation for $SO(2N + 1)$. As before, we begin with the table of charge assignments.

| $Q$ | $m$ | $\Lambda_{SO}^{3(2N-1)-2F}$ | $v$ | $w$ |
|-----|-----|-----------------------------|-----|-----|
| $R_v$ | 0 | 2 | $2(2N - 1) - 4F$ | 2 | 0 |
| $R_w$ | 1 | 0 | $2(2N - 1)$ | 0 | 2 |

The curve describing the brane configuration for $SO(2N + 1)$ is

$$tv \prod_{i=1}^{F} (v^2 - m_i^2) = v^{2N} (\det m)^{2F - 2N + 1}$$

$$vw = \Lambda_{SO}^{3(2N-1)-2F} (\det m)^{1/2N-1}.$$  \hspace{1cm} (11)

We have checked the perturbative limits

$$v \to \infty, \quad tw^{2F-2N+1} \sim (\det m)^{2F-2N+1\over 2F}, \quad w \sim 0,$$

$$w \to \infty, \quad tw^{2N-1} \sim \Lambda_{SO}^{3(2N-1)-2F} (\det m)^{2F-2N+1\over 2F}, \quad v \sim 0,$$

and they indeed have the correct symmetry properties. Expressing the curve in the $t - w$ variables yields:

$$tw \prod_{i=1}^{F} (w^2 - \langle M_i \rangle^2) = w^{2\tilde{N}} (\det \langle M \rangle)^{2F-2\tilde{N}+1\over 2F},$$

$$vw = \tilde{\Lambda}_{SO}^{3(2N-1)-2F} (\det \langle M \rangle)^{1/2N-1}.$$  \hspace{1cm} (12)

Again, duality is reproduced properly. In the above considerations we did see the emergence of the expectation values of the meson fields, however the meson field itself corresponds to five dimensional excitations and their coupling to the four dimensional theory is very weak. We need to consider finite fourbranes instead of the semi-infinite ones to overcome this problem. This will be considered in the next section for the case of $Sp$ theories.

Note, that in the case of $SO(N)$ groups we had to restrict ourself to theories with even number of vectors, even though there is no field theory reason to do so. The string theory reason behind this is that we wanted to give a mass to every vector. In the orientifold construction however one vector is necessarily massless in the case of odd number of vectors, since one vector has to lie on the orientifold plane.
4 Finite Brane Configuration for $Sp(2N)$

Above, we have considered the case where the fourbranes giving rise to the fundamentals of the $SO$ and $Sp$ groups are semi-infinite. We saw in this picture how the dual gauge groups are emerging from one and the same M theory fivebrane configuration, but the massless meson fields required for Seiberg’s duality were missing. In order to get these fields, we consider the finite brane construction analogous to the one presented in Ref. [8]. We will consider only the $Sp(2N)$ case, while the other groups can be worked out similarly.

![Diagram of finite brane configuration](image)

Figure 3: Finite brane configuration of the electric theory.

The construction of [8] for SUSY QCD started with a brane setup similar to the one presented above but the semi-infinite branes terminated in additional fivebranes filling out the w-plane. Just like in [8] we consider the case with a common mass $m$ for the $Sp$ fundamentals only. Since we are interested in having non-vanishing masses for every fundamental of $Sp(2N)$ the $F$ fourbranes must be placed symmetrically above and below the O4 orientifold. Because we are considering the case when these fourbranes terminate on additional fivebranes, part of the flavor symmetry will be gauged. The full flavor symmetry is $SU(2F)$, however it is broken by the mass term $m$. Thus the gauged part of the flavor symmetry will be only $SU(F)$. This brane setup is depicted in Fig. [8], while the spurious symmetries of the field theory are summarized in the table below.
There is a superpotential term $mQ\bar{Q}$ present in the field theory. In field theory this superpotential results in gaugino condensation with expectation value

$$\text{Tr} \langle Q\bar{Q} \rangle = (N + 1)2^{N-1} \left[ m^{F-N-1} \Lambda_{Sp}^{3(N+1)-F} \right]^{1 \over 2F} + F \left[ m^{2N-F} \Lambda_{SU}^3 \right]^{1 \over 2F}.$$ 

In M theory, the curve describing the $\Sigma$ Riemann surface for this brane setup is given by

$$t(v^2 - m^2)^F = v^{2N+2} m^{2F-2N-2}$$
$$w = {m \over v} \left[ m^{F-N-1} \Lambda_{Sp}^{3(N+1)-F} \right]^{1 \over 2F} + \left[ m^{2N-F} \Lambda_{SU}^3 \right]^{1 \over 2F}.$$ 

One can check that this curve obeys the spurious $R_v$ and $R_w$ symmetries as well as reproduces the correct classical limits:

$$v \to m, \ t \to \infty, \ w \to \infty, \ t \sim w^{2F-2N} m^{F-2N},$$
$$v \to 0, \ t \to 0, \ w \to \infty, \ t \sim \Lambda_{Sp}^{2[3(N+1)-F]} m^{2F-2N-2} \w^{2(N+1)}$$
$$v \to \infty, \ t \to 0, \ w \to 0, \ t \sim \left( {m \over v} \right)^{2F-2N-2}.$$ 

The weakly coupled string theory is reached by taking $R \to 0, \Lambda_{SU}, \Lambda_{Sp} \to 0$. Taking

$$\Lambda_{Sp}^{3(N+1)-F} = e^{-S_{Sp}/R}, \ \Lambda_{SU}^{3F-2N} = e^{S_{SU}/R},$$

where $S_{Sp} > 0, S_{SU} < 0$, and $m$ is fixed reproduces the setup of Fig. 3, with the fivebranes positioned at $x_6 = S_{SU}, 0, S_{Sp}$. Duality can be obtained by taking the $\Lambda_{Sp} \gg 1$ limit, which will now imply $S_{Sp} < 0$. Taking also the $R \to 0$ limit while keeping $S_{Sp}, S_{SU}$ and the meson VEV fixed gives the brane setup depicted in Fig. 4. The required scaling of the mass is given by

$$m^{F-N-1} \sim e^{S_{Sp}/R}.$$

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Thus one can see that in the classical limit one also needs to take $m \to 0$. We can read off the low-energy degrees of freedom as the string excitations connecting the D4 branes in Fig. 4. The $Sp(2F - 2N - 4)$ gauge bosons arise from the strings between the color branes on the right in Fig. 4, while the $F$ flavors of $Sp$ are the strings connecting color and flavor branes.

The fivebranes on the left are now parallel and thus the flavor branes can slide freely along the $w$ direction, giving rise to the meson field $M$. To understand the properties of this meson field we note that the left hand side of Fig. 4 is $N = 2$ supersymmetric, so there must be an adjoint field of the $SU(F)$ gauge group present. This is however only part of the antisymmetric meson field needed for the $Sp$-duality. Note however, that there are additional (massive) four dimensional excitations, which correspond to strings connecting the fourbranes above and below the orientifold. These together with the adjoint of $SU(F)$ exactly make up an antisymmetric tensor of $SU(2F)$, which then can be identified with the meson field in the $Sp$ duality. Thus in this case there are no missing components of the meson field, rather in the $m \neq 0$ some components of the meson field are massive, which is in agreement with the field theory results.

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