CONSTRaining DARK ENERGY FROM THE SPLITTING ANGLE STATISTIC OF STRONG
GRAVITATIONAL LENSES

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ABSTRACT

Utilizing the CLASS statistical sample, we investigate the constraint of the splitting angle statistic of strong gravitational lenses (SGL) on the equation-of-state parameter \( w = p/\rho \) of the dark energy in the flat cold dark matter (CDM) cosmology. Through the concomiting number density of dark halos described by the Press–Schechter theory, dark energy affects the efficiency with which dark-matter concentrations produce strong lensing signals. The constraints on both constant \( w \) and time-varying \( w(z) = w_0 + w_z(z)/(1+z) \) from the SGL splitting angle statistic are consistently obtained by adopting a two-model combined mechanism of a dark halo density profile matched at the mass scale \( M_c \). Our main observations are that (1) the resulting model parameter \( M_c \) is found to be \( M_c \sim 1.4 \) for both constant \( w \) and time-varying \( w(z) \), which is larger than \( M_c \sim 1 \) obtained in literatures; (2) the fitting results for the constant \( w \) are found to be \( w = -0.89^{+0.09}_{-0.26} \) and \( w = -0.94^{+0.07}_{-0.16} \) for the source redshift distributions of the Gaussian models \( g(z_s) \) and \( g^*(z_s) \), respectively, which are consistent with the \( \Lambda \)CDM at 95\% C.L.; (3) the time-varying \( w(z) \) is found to be \( \sigma_8 = 0.74; (M_c; w_0, w_a) = (1.36; -0.92, -1.31) \) and \( (M_c; w_0, w_a) = (1.38; -0.89, -1.21) \) for \( g(z_s) \) and \( g^*(z_s) \), respectively; the influence of \( \sigma_8 \) is investigated and found to be sizable for \( \sigma_8 = 0.74-0.90 \). After marginalizing the likelihood functions over the cosmological parameters \( (\Omega_M, h, \sigma_8) \) and the model parameter \( M_c \), we find that the data of SGL splitting angle statistic lead to the best-fit results \( (w_0, w_a) = (-0.88^{+0.10}_{-1.03}, -1.55^{+1.77}_{-1.38}) \) and \( (w_0, w_a) = (-0.91^{+0.60}_{-1.46}, -1.60^{+1.60}_{-2.57}) \) for \( g(z_s) \) and \( g^*(z_s) \), respectively.

Key words: cosmological parameters – cosmology: observations – cosmology: theory – gravitational lensing – quasars: general

1. INTRODUCTION

Since the direct confirmation of the presence of dark energy by Type Ia supernovae (SNe Ia) observation (Riess et al. 1998), the investigation of its property is one of the most important object in cosmology. Many theoretical models have been developed to explain or describe dark energy, which is widely believed as the main component of the cosmological energy today. By now one key to the question seems to be the precise measurement of the equation-of-state parameter \( w = p/\rho \) of the dark energy (see a review paper, Peebles & Ratra 2003). Compared with the data, the limit to the value of parameter \( w \) is continuously improved by many experimental groups, including the SN Ia (Riess et al. 2004), the cosmic microwave background (CMB; Spergel et al. 2007), and the weak gravitational lenses (WLG; Weinberg & Kamionkowski 2003). As a complement to these observations, we shall utilize the splitting angle statistic of strong gravitational lenses (SGL) to quantificationally investigate its constraint on the parameter \( w \).

Light lines traversing the universe are attracted and refracted by the gravitational force of the galaxies on its path, which bring us the signal of the SGL effect, one of which is the multiple images of a single far galaxy. Through comparing the observed number of lenses with the theoretical expected result as a function of image separation and cosmological parameters, it enables us to determine the allowed range of the parameter \( w \). Linder (2004) demonstrated that with the addition of strong lensing image separation measurements, the estimates for time-varying \( w(z) = w_0 + w_z(z)/(1+z) \) from CMB, SNe Ia, and WGL could be improved modestly. To see that, in this paper we shall carefully investigate the analytic process and the power of SGL data only. There are numerous works on the study of the relation between DE and SGL splitting angle statistic (e.g., Porciani & Madau 2000; Li & Ostriker 2002; Kuhlen et al. 2004), but the exact mass density profile of dark halos is unknown yet, which produces the most theoretical uncertainty of the analytic process. The two most widely used density profiles are the singular isothermal sphere (SIS) profile and the Navarro–Frenk–White (NFW) profile. Some analyses were based solely on the SIS model (e.g., Chae et al. 2002). While from the analyses by Li & Ostriker (2002, 2003), and also by Sarbu et al. (2001), it gave a convective illumination that a combined mechanism of at least two models can effectively reproduce the observed curve of the lensing probability \( P(\theta) \) of the image splitting angle \( \theta \). To achieve this, an additional parameter \( M_c \) is introduced to divide the mass scale of dark halos into different parts as a certain density profile is thought to work only on each part.

The rate of structure growth, which determines the number density of dark halos as the SGL lenses, is very sensitive to the normalization parameter of the matter power spectrum, \( \sigma_8 \). The value of \( \sigma_8 \) is often related to the matter density \( \Omega_M \) and constrained by the CMB or cluster abundance observation. Several years ago, in the lambda cold dark matter (LCDM) universe with \( \Omega_M = 0.27 \), \( \sigma_8 \) was found to be larger than 0.9 (e.g., Spergel et al. 2003; Wang & Steinhardt 1998). Then, the three-year data of the Wilkinson Microwave Anisotropy Probe (WMAP; Spergel et al. 2007) provided a noticeably smaller value \( \sigma_8 = 0.74 \pm 0.05 \) in comparison with the first-year data \( \sigma_8 = 0.92 \pm 0.10 \). By utilizing the recent clustering results of XMM-Newton soft (0.5–2 keV) X-ray sources, the X-ray clustering, and SN Ia data, Basilakos & Plionis (2006) showed that \( \sigma_8 \approx 0.73 \) for \( \Omega_M = 0.26 \) and \( w = -0.90 \), which is consistent with the new result. A smaller \( \sigma_8 \) implies less structure growth at late times and less lensing probability for a single source.
In this paper, by using the CLASS statistical sample (Browne et al. 2003), we are going to show how the constraint on the dark energy equation-of-state parameters \( w(z) \) can reliably be obtained for the cases with the assumption of constant \( w \) and the time-varying parameterization \( w(z) = w_0 + w_1 z / (1 + z) \), respectively. We highlight three issues which have not previously been investigated. First, we investigate the influences of parameter \( w \) on every step of the lensing process to find the most important point where the change of parameter \( w \) shows its effect. Second, by comparing the results for different distributions of source redshifts which have been used in previous works (e.g., Chae et al. 2002; Li & Ostriker 2002), we illustrate the quantitative influence that is introduced by the uncertainty of the source distribution. Third, we focus on the constraint of the data on the time-varying parameterization \( w(z) \). Our paper is organized as follows. Section 2 outlines the cosmological model, the mass fluctuations, and the Press–Schechter (P–S) function used in our calculation. Section 3 describes the density profiles and the lensing probabilities. Section 4 gives our data analysis and numerical results. The conclusions are presented in the last section.

2. BASIC CONSIDERATIONS

In this section, we shall describe some ingredients used in our calculations.

2.1. Cosmological Model and Mass Fluctuations

During the last two decades more and more observational evidence suggests that our universe at present is accelerated expanding and dominated by a spatially smooth component with negative pressure, the so-called dark energy. Besides the common cosmological constant supposition, an attractive alternative candidate for dark energy is the potential energy of a slow-varying scalar field, which is conveniently parameterized through \( w = p/\rho \). The conventional scalar field models, i.e., quintessence models, with a positive kinetic energy term in the field Lagrangian require \( w \gtrsim -1 \) (e.g., Ratra & Peebles 1988; Caldwell et al. 1998), and phantom dark energy, adopting alternatively a negative kinetic energy term, gives the parameter space \( w < -1 \) (e.g., Caldwell 2002; Carroll et al. 2003; Cline et al. 2004). To fit the data of the observation and give the allowed parameter space, we use two typical parameterizations, i.e., constant \( w \) and time-varying \( w(z) \), as follows:

Case I : \( w = \text{constant}, \)

Case II : \( w(z) = w_0 + w_1 z / (1 + z) \), \( w_0 = d w(z)/d z |_{z=0} = 0 \).

The simple form \( w(z) = w_0 + w_1 z \) is not favored because the value of \( w(z) \) runs to infinity when the redshift \( z \) goes to infinity. Throughout this paper, we assume a flat CDM universe with the present matter density relative to the critical density \( \Omega_M = 0.24 \) and the Hubble parameter \( h = 0.73 \) (Spergel et al. 2007).

Dark halos of galaxies and galaxy clusters are formed through linear growth and nonlinear collapse of primordial fluctuations of matter in the early universe. The time-varying state-of-equation parameter \( w(z) \) is usually realized by models with slow-varying scalar fields, while these scalar fields of dark energy begin to cluster gravitationally and contribute to the perturbation spectrum only at the very large spatial scale \( L > 100 \text{ Mpc} \), which corresponds to a very small wavenumber \( k < 0.01 \text{ Mpc}^{-1} \) (e.g., Ma et al. 1999). In our present considerations, the most concerned length scale \( l < 1 \text{ Mpc} \) is far less than the length scale \( L \) and the concerned fluctuations are obtained by integrating the mass power spectrum \( k^3 P(k) \) up to a larger value of the wavenumber \( k > 1.0 \text{ Mpc}^{-1} \). Noting the fact that the increasing function \( k^3 P(k) \propto k^3 \) when \( k \ll 1.0 \text{ Mpc}^{-1} \), the contributions from the integral region \( k < 0.01 \text{ Mpc}^{-1} \) are very small, namely, the contribution of the dark energy cluster to mass fluctuations can be neglected. Thus, we can directly utilize the mass power spectrum of ΛCDM cosmology. In our calculation, the linear CDM power spectrum is computed by adopting the fitting formulae given by (Eisenstein & Hu 1999)

\[
\Delta_\ell(k, z) \equiv \frac{k^3}{2\pi^2} P(k, z) = \delta_\ell^2 \left( \frac{c k}{H_0} \right)^{3+n} T^2(k) D^2(z). \tag{1}
\]

The initial power spectrum index \( n \) is fixed to be \( n = 1 \). \( T \) is the transfer function

\[
T = \frac{L}{L + C q_{\text{eff}}^2}, \tag{2}
\]

with

\[
L \equiv \ln (e + 1.84 q_{\text{eff}}), \quad q_{\text{eff}} \equiv \frac{k}{\Omega_M h^2 \text{ Mpc}^{-1}}, \quad C \equiv 14.4 + \frac{325}{1 + 60.5 q_{\text{eff}}}. \tag{3}
\]

The parameter \( \delta_H \) is the amplitude of perturbations on the horizon scale today and related to the rms density fluctuations in spheres of radius \( r_s = 8 h^{-1} \text{ Mpc} \), the so-called \( \sigma_8 \) by

\[
\delta_H = \frac{\sigma_8}{\left[ \int_0^{\infty} (dk/k) \Delta(k, 0) W(k r_s) \right]^{1/2}} = \frac{\sigma_8}{\left[ \int_0^{\infty} (dk/k)(ck/H_0)^{3+n} T^2(k) W^2(k r_s) \right]^{1/2}}, \tag{5}
\]

where \( W(k r_s) \) is the top-hat window function: \( W(k r_s) = 3(\sin(k r_s)/(k r_s)^3) - (\cos(k r_s)/(k r_s)^2) \). To show the power of SGL data only, we do not use any analytical fitted form of the parameter \( \sigma_8 \), expressed by \( \Omega_M \) and \( w \). Alternatively, unless special clarification we choose to normalize the power spectrum to \( \sigma_8 = 0.74 \), the best-fit value given by WMAP three-year data (Spergel et al. 2007).

The linear growth function \( D(z) \) is proportional to the linear density perturbation \( \delta = \rho_M/\rho_M \). The evolution of linear perturbation is

\[
\delta + 2 \frac{\dot{a}}{a} \delta = 4 \pi G \rho_M \delta, \tag{6}
\]

where \( a \) is the scale factor \( a = (1 + z)^{-1}, \) dot means derivative with respect to the physical time \( t \), the background matter density \( \rho_M = \rho_0 (1 + z)^3, \rho_0 = \Omega_M \rho_{\text{crit},0}, \) and \( \rho_{\text{crit},0} = 3H_0^2/(8\pi G) \) is today’s critical mass density in the universe. Then with the definition \( D(z) \equiv \delta(z)/\delta(z = 0) \), we can obtain the equation of \( D(a) \):

\[
\frac{d^2 D}{da^2} = \frac{3}{2a^2} D - \frac{3}{2a} \frac{dD}{da} \left[ 1 - w(a)(1 - \Omega) \right], \tag{7}
\]

where \( \Omega \) is the matter density parameter \( \Omega = \Omega_M (1 + z)^3/(H_0^2) \) and

\[
\frac{H}{H_0} = \frac{\dot{a}}{a H_0} = \sqrt{\Omega_M (1 + z)^3 + \Omega_D \exp\left( \int dz' \frac{3}{1 + w(z')} \right)/(1 + z')}. \tag{8}
\]
Here, $H_0$ is the present Hubble constant $H_0 = 100 \text{ km s}^{-1}\text{Mpc}^{-1}$ and $\Omega_{DE} = (1 - \Omega_M)$ for a flat universe. With two boundary conditions, $\mathcal{D}(a)|_{a = 0} = 0$ and $\mathcal{D}(a)|_{a = 1} = 1$, Equation (7) can be calculated numerically. The second boundary condition means $\mathcal{D}(a)$ is normalized to $\mathcal{D}(a = 1) = 1$.

### 2.2. Mass Function and Spherical Collapse Approximation

According to the P–S theory, the comoving number density of dark halos virialized by redshift $z$ with mass in the range $(M, M + dM)$ is given by

$$n(M, z) dM = \frac{\rho_0}{M} f(M, z) dM.$$  

$\mathcal{f}(M, z)$ is the P–S function. We utilize the modified form by Sheth & Tormen (1999):

$$f(M, z) = -\frac{0.383}{\sqrt{\pi}} \frac{\delta_c}{\Delta} \frac{d\Delta}{dM} \left[ 1 + \left( \frac{\Delta^2}{0.707\delta_c^2} \right)^{0.3} \right]^2 \times \exp \left[ -\frac{0.707}{2} \left( \frac{\delta_c}{\Delta} \right)^2 \right],$$

$$\Delta^2(M, z) = \int_0^\infty \frac{dk}{k} \Delta(k, z) W^2(kr),$$

where $\Delta$ is the variance of the fluctuations in a sphere containing a mean mass $M$, and $M$ is related to the length scale $r$ via

$$M = \frac{4\pi}{3} r^3 \rho_M.$$  

The parameter $\delta_c(z)$ is the linear overdensity threshold for a spherical collapse by the redshift $z$. The matter with the overdensity in a certain scale of the early universe would undergo the density growth and the spatial scale reducing. When its average matter density reaches $\delta_c(z)$, virialization starts and then a dark halo is formed. In this paper, we follow Wang & Steinhardt (1998) and Weinberg & Kamionkowski (2003) to calculate $\delta_c(z)$. Under the approximation of spherical top-hat collapse and labeling $R$ as the spatial length scale for a halo with a certain mass, the collapse process is determined by the Friedmann equation

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} (\rho_M + \rho_{DE}),$$

and the time–component of the Einstein equations

$$\frac{\ddot{r}}{r} = -4\pi G \left[ \left( w + \frac{1}{3} \right) \rho_{DE} + \frac{1}{3} \rho_{\text{halo}} \right],$$

where $\rho_{DE}$ is the energy density of dark energy and $\rho_{\text{halo}}$ is the uniform matter density in the scale $r$. When the scale factor $a(z)$ is very small, i.e., $a(z) \to 0$ or $z \to z_0 \to \infty$, the equivalent linear overdensity threshold $\delta_c(z)$ at $z_0$ can approximately be evaluated through the dark halo density $\rho_{\text{halo}}$ and the background matter density $\rho_M$, i.e., $\delta_c(z_0) \simeq (\rho_{\text{halo}}/\rho_M - 1)$. Then utilizing the two boundary conditions, $r(a)|_{a = 0} = 0$ and $dr/da|_{a = a_\text{vir}} = 0$ with $a_\text{vir}$ the scale factor at the turnaround time, the function $\delta_c(z)$ can be calculated numerically as follows:

$$\delta_c(z) = \delta_c(z_0) D(z)/D(z_0), \quad \delta_c(z_0) \simeq (\rho_{\text{halo}}/\rho_M - 1).$$

Note that the turnaround time $t_\text{vir}$ is determined through the virial time $t_\text{vir}$ when the overdensity matter starts to form dark halos, i.e., $t_\text{vir} = t_\text{vir}/2$, which is corresponding to the virial redshift $z$ according to the P–S theory. In this sense, the boundary condition $dr/da|_{a = a_\text{vir}} = 0$ is related to the virial redshift $z$, so the resulting scale $r(a)$ of dark halos will depend on the virial redshift $z$. As a consequence, the dark halo density $\rho_{\text{halo}}$ actually relies on the virial redshift $z$.

Denote $\Delta_{\text{vir}} \equiv \rho_{\text{halo}}/\rho_M$ as the ratio of the cluster to the background density. Then the nonlinear overdensity $\Delta_{\text{vir}}(z)$ can also be calculated directly from Equations (11) and (12) with their two boundary conditions and the $\delta_c(z_0)$ given above.

In Figure 1, we plot the P–S function $f$ against the mass $M(10^{15} h^{-1} M_\odot)$ of dark halos at redshift $z = 0.0, 1.5$, and $3.0$, respectively. The dashed, solid, dotted, and dash-dotted curves are for the cases of $w = -0.5$, $w = -1.0$, $w = -1.5$, and $w(z) = -1.0 - z/(1 + z)$.
for a small $M \sim 0.001$, the change of $w$ can bring somewhat significant shift of function $f$ in most of the ranges of the redshift $z$. This produces the most power of SGL data to constrain the parameter $w$.

3. DENSITY PROFILE AND LENSING PROBABILITY

The SGL lensing efficiency is very sensitive to the density profile of dark halos: under the same conditions, the efficiency of the SIS model is larger, at least by 1 order of magnitude, than that of the NFW model for the image separation angle $\delta \theta < 30^\circ$. In this section, we shall discuss the influence of different $w$ on the lensing probability $P(\Delta \theta_0)$ of the SIS profile and the NFW profile, respectively.

### 3.1. SIS Profile as a Lens

The SIS profile has a simple spherically symmetric form (Schneider et al. 1992)

$$\rho(r) = \frac{\sigma_v^2}{2\pi G} \frac{1}{r^2}, \quad (13)$$

where $\sigma_v$ is the velocity dispersion, which can be related to the mass $M$ of a dark halo via $\sigma_v^2 = GM/2 \ r_{\text{vir}}$ after integrating the density function $\rho(r)$ from $r = 0$ to $r = r_{\text{vir}}$. Here, $r_{\text{vir}}$ is the virial radius of a dark halo, which is commonly defined by demanding that the mean density within the virial radius of the halo be a factor $\Delta_{\text{vir}}$ times larger than the background density, $\rho_M$, i.e.,

$$M = \frac{4\pi}{3} \Delta_{\text{vir}} \rho_c r_{\text{vir}}^3.$$

Eliminating the dependence on the $r_{\text{vir}}$, we get

$$M = \frac{\sigma_v^3}{G} \left( \frac{6}{\pi \Delta_{\text{vir}} \rho_c r_{\text{vir}}} \right)^{1/2}. \quad (14)$$

This profile is supported by the observed flat rotation curves of the spiral galaxies and is widely utilized in the gravitational lensing analysis. Due to its symmetry, the lensing analysis is quite easy. Integrate the density component along the line of sight (LOS) and then we get its surface mass density

$$\Sigma(\xi) = \frac{\sigma_v^2}{2G} \frac{1}{\xi}, \quad (15)$$

where $\xi \equiv |\xi|$ and $\xi$ is the position vector in the lens plane. The lensing equation is given by

$$\eta = \frac{D_A}{D_L} \xi - \frac{D_{A}^S}{D_{L}^S} \alpha(\xi), \quad (16)$$

where $\eta$ is the source position, $D_A^S$, $D_L^S$, and $D_{A}^L$ are the angular-diameter distances from the observer to the source, from the observer to the lens, and from the lens to the source. The angle $\alpha(\xi)$ is the gravitational deflection angle. For a circularly symmetric surface mass density, $\Sigma(\xi) = \Sigma(\xi)$, images appear on the plane defined by the observer point, the lens center, and the source position, and the angle $\alpha(\xi)$ is given by

$$\alpha(\xi) = \frac{8\pi G}{c^2 \xi} \int_0^\xi \xi' \Sigma(\xi') \, d\xi'. \quad (17)$$

To simplify the lensing equation, we define the length scales in the lens plane and the source plane as

$$\xi_0 = 4\pi \left( \frac{\sigma_v}{c} \right)^2 \frac{D_A^S D_{A}^L}{D_L^S}, \quad \eta_0 = \xi_0 \frac{D_{A}^S}{D_{L}^S}. \quad (18)$$

Then the position vector of a point in the lens plane or source plane is $\xi = x\xi_0$ or $\eta = y\eta_0$. After the reduction, the lensing equation for an SIS lens is given by

$$y = x - \frac{|x|}{x}. \quad (19)$$

It is easy to see that when $|y| \leq 1$, i.e., $|x| \leq 1$, a single source has double images with the separation $\Delta x \equiv 2$ and the splitting angle

$$\Delta \theta = \frac{\xi_0}{D_A} \Delta x = 8\pi \left( \frac{\sigma_v}{c} \right)^2 \frac{D_A^S}{D_L^S} \frac{\pi M^2 \rho_M \Delta_{\text{vir}}}{6} \quad (20)$$

Hence, the cross section for two images with a splitting angle $\Delta \theta > \Delta \theta_0$ is given by

$$\sigma = \pi \xi_0^2 \vartheta (\Delta \theta - \Delta \theta_0) = \pi \xi_0^2 \vartheta (M - M_0), \quad (21)$$

where $\vartheta$ is the step function, and $M_0$ related with $\Delta \theta_0$ can be solved from Equation (20).

The probability of a source at the redshift $z_s$ undergoing a lensing event on account of the galaxy distributions from
the source to the observer can be obtained by dividing the total lensing cross section by the area $A(z)$ of the lens plane (Schneider et al. 1992)

$$P = \int_0^{\infty} \int_0^{\infty} \frac{dD_L}{dz} dM \frac{(1+z)^3 n(M,z) \sigma(M,z) dM dz}{},$$

where $D_L$ is the proper distance from the observer to the lens.

Inserting Equations (8) and (21) into Equation (22), we have for the SIS case (Li & Ostriker 2002)

$$\frac{dP(\Delta \theta_0)}{d(\Delta \theta_0) dz} = 16\pi^3 \rho_{s,0} \Omega_M (1+z)^3 \frac{dD_L}{dz} \left( \frac{D_L^2 D_A^2}{D_S^2} \right)^2 \times \int_0^{\infty} f(M,z) \left( \frac{\sigma_s}{c} \right)^4 \theta(M-M_0) dM.$$  \hspace{1cm} (23)

Differentiating this expression with respect to $\Delta \theta_0$, we can obtain the probability density for a source to have a double image with a splitting angle $\Delta \theta = \Delta \theta_0$:

$$\frac{d^2 P(\Delta \theta_0)}{d(\Delta \theta_0) dz} = 16\pi^3 \rho_{s,0} \Omega_M (1+z)^3 \frac{dD_L}{dz} \left( \frac{D_L^2 D_A^2}{D_S^2} \right)^2 \times \int_0^{\infty} f(M,z) \left( \frac{\sigma_s(M_0)}{c} \right)^4 \frac{dM_0}{d\Delta \theta_0}.$$  \hspace{1cm} (24)

The proper distance $D^L$ and the angular-diameter distance $D^A$ from the redshift $z_1$ to $z_2$ are calculated via

$$D^L(z_1, z_2) = \int_{z_1}^{z_2} \frac{dz}{(1+z)H(z)},$$

$$D^A(z_1, z_2) = 1 + \int_{z_1}^{z_2} \frac{dz}{H(z)}.$$  \hspace{1cm} (25)

3.2. NFW Profile as a Lens

The NFW profile, based on the N-body numerical simulation of CDM, is a very important approach for understanding the formation of galaxies and clusters of galaxies (e.g., Zhao 1996; Hanya & Habe 2001). Its mass density is given by (Navarro et al. 1995, 1996, 1997)

$$\rho_{\text{NFW}}(r) = \frac{\rho_s r_s^3}{r (r + r_s)^2},$$  \hspace{1cm} (26)

where $\rho_s$ and $r_s$ are constants:

$$\rho_s = \frac{\rho_{M,\text{vir}}}{c^3 f(c_1)} , \hspace{1cm} r_s = \frac{c_1}{1 + c_1} \left( \frac{3M}{4\pi \rho_{M,\text{vir}}} \right)^{1/3} ,$$  \hspace{1cm} (27)

with $f(c_1) = \ln(1 + c_1) - c_1/(1 + c_1)$. For the concentration parameter $c_1$, we adopt the fitting formulae given by Bullock et al. (2001): $c_1 = 9(1+z)^{-1} (M/1.5 \times 10^{13} h^{-1} M_\odot)^{-0.13}$.

Similar to the case of SIS, we define the position vector in the lens plane and the source plane as $\mathbf{x} = x_r$, and $\mathbf{y} = y_r D_s^2 / D_A^2$, respectively. The surface mass density for the NFW profile is given by (Li & Ostriker 2002)

$$\Sigma(x) = 2\rho_s r_s \int_0^{\infty} (x^2 + z^2)^{-1/2} \left( (x^2 + z^2)^{1/2} + 1 \right)^{-2} d\xi.$$  \hspace{1cm} (28)

Then the reduced lensing equation is

$$y = x - \mu_s \frac{s(x)}{x} ,$$  \hspace{1cm} (29)

where

$$\mu_s = 4\rho_s r_s / \Sigma_{\text{crit}},$$

$$\Sigma_{\text{crit}} = \frac{1}{4\pi G} \frac{D_A^4}{D_S^2 D_A^2} ,$$

$$s(x) = \int_0^x u du \int_0^{\infty} (u^2 + z^2)^{-1/2} \left( (u^2 + z^2)^{1/2} + 1 \right)^{-2} d\xi.$$  \hspace{1cm} (30)

The dimensionless parameter $\mu_s$ determines the size of the lensing cross section $\sigma$ for a NFW halo to produce multiple images: larger $\mu_s$, smaller $\sigma$. The curve of $y$ to $x$ runs through the origin and has an extremum point centrally-symmetric on each side, whose coordinates $(x_{\text{crit}}, y_{\text{crit}})$ are determined by $dy/dx|_{x=0} = 0$ and $y_{\text{crit}} = y(x_{\text{crit}})$. Thus, a single source with a certain $y$ has multiple images when $|y| \leq y_{\text{crit}}$. Once more than two images are formed, we shall only consider the splitting angle $\Delta \theta$ between the two outside images. According to Li & Ostriker (2002), we shall neglect the variety of $\Delta \theta$ caused only by the motion of $y$ and get $\Delta \theta(y) \approx \Delta \theta(y = 0) = 2x_0$, where $x_0$ is the positive root of $y(x) = 0$. Then the splitting angle $\Delta \theta$ is given by

$$\Delta \theta = \frac{r_s}{D_A^2} \Delta x \approx \frac{2x_0 r_s}{D_A^2} ,$$  \hspace{1cm} (31)

and the cross section for forming multiple images with $\Delta \theta > \Delta \theta_0$ is

$$\sigma(\Delta \theta_0, M, z) \approx \pi y_{\text{crit}}^2 r_s^2 \theta(\Delta \theta - \Delta \theta_0).$$  \hspace{1cm} (32)

Figure 3 shows the splitting angle $\Delta \theta$ as the function of $M(10^{15} h^{-1} M_\odot)$ in the SIS and NFW cases for $w = -0.5$, $w = -1.0$, $w = -1.5$, and $w(z) = -1.0 - z/(1 + z)$. The source object is at $z_s = 1.5$, and the lens object is at $z = 0.3$. In Figure 4, we plot $\Delta \theta$ against the redshift of lens $z$ for $M = 0.01, 1.0, \text{and } 100$, respectively. The source object is at $z_s = 1.5$. The curves of $w(z)$ and $w = -1.0$ almost overlap in both figures. We can see that the $\Delta \theta$ produced by an NFW lens is more sensitive to the parameter $w$ than that of an SIS lens, especially for small $M$. For the SIS case, there are only quite small changes of $\Delta \theta$ for our different selections of $w$ in both Figures 3 and 4.

Figure 5 gives the cross section $\sigma$ against the redshift of lens $z$ for $M(10^{15} h^{-1} M_\odot) = 0.01, 1.0, \text{and } 100$, respectively. The source object is at $z_s = 1.5$. For the SIS case, there are not visible changes of $\Delta \theta$ for our different selections of $w$. For the NFW case, the cross section is more sensitive to the parameter $w$ for smaller $M$ and somewhat smaller $z$.

Using Equations (8), (22), and (32), we then get the differential lensing probability for the NFW case (Li & Ostriker 2002)

$$\frac{dP(\Delta \theta_0)}{dz} = \pi \rho_{s,0} \Omega_M (1+z)^3 \frac{dD_L}{dz} \times \int_0^{\infty} f(M,z) \left( \frac{\sigma_s}{c} \right)^2 r_s^2 \theta(M-M_0) dM .$$  \hspace{1cm} (33)
respectively. The dashed, solid, dotted, and dash-dotted curves are for the cases of thin lines show the splitting angle produced by an NFW lens and an SIS lens, respectively. The dashed, solid, dotted, and dash-dotted curves are for the cases of thin lines show the splitting angle produced by an NFW lens and an SIS lens, respectively.

The source object is at \( z_s = 1.5 \) and the lens object is at \( z = 0.3 \). The thick and thin lines show the splitting angle produced by an NFW lens and an SIS lens, respectively. The dashed, solid, dotted, and dash-dotted curves are for the cases of \( w = -0.5, w = -1.0, w = -1.5, \) and \( w(z) = -1.0 - z/(1 + z) \), respectively.

\[
\frac{d^2 P(\Delta \theta_0)}{d \Delta \theta_0 dz} = \pi \rho_{\text{crit},0} \Omega_M (1 + z)^3 \frac{d D_L}{dz} f(M_0, z) \times y^2(M_0) r^2(M_0) dM_0 d\Delta \theta_0.
\]

\[
\frac{d M_0}{d \Delta \theta_0} = \frac{D_L^4}{2} \left( r_x(M_0) \frac{dx_0}{d M_0} + x_0(M_0)r_x(M_0) \right) \frac{1/3 + 0.13}{M_0},
\]

\[
\frac{d x_0}{d M_0} = \frac{g(x_0)/x_0}{1 - \mu_x(M_0)(g'(x_0)/x_0 - g(x_0)/x_0^2)},
\]

\[
\frac{d \mu_x}{d M_0} = \frac{4r_x \rho_s}{M_0 \Sigma_{\text{cr}}} \left( 0.07 + \frac{0.13 c_1^2}{(1 + c_1)^2} (c_1) \right),
\]

\[
g'(x_0) = x_0 \int_0^{\infty} \left( x_0^2 + z^2 \right)^{-1/2} \left[ \left( x_0^2 + z^2 \right)^{1/2} + 1 \right]^{-2} dz. \quad (34)
\]

**Figure 3.** Splitting angle \( \Delta \theta \) as the function of \( M(10^{15} h^{-1} M_\odot) \) in the SIS and NFW cases for \( w = -0.5, w = -1.0, w = -1.5, \) and \( w(z) = -1.0 - z/(1 + z) \). The source object is at \( z_s = 1.5 \) and the lens object is at \( z = 0.3 \). The thick and thin lines show the splitting angle produced by an NFW lens and an SIS lens, respectively. The dashed, solid, dotted, and dash-dotted curves are for the cases of \( w = -0.5, w = -1.0, w = -1.5, \) and \( w(z) = -1.0 - z/(1 + z) \), respectively.

**Figure 4.** Splitting angle \( \Delta \theta \) as the function of the redshift of lens \( z \) for \( M = 0.01, M = 1.0, \) and \( M = 100 \), respectively. The source object is at \( z_s = 1.5 \).

Multiple image gravitational lenses have been discovered and all have image separations \( \Delta \theta < 3'' \) (Browne et al. 2003). The data information of the 13 observed lens systems is not entire: the source redshift \( z_s \) and the lens redshift \( z \) are both unknown for one-lens systems, only \( z_s \) unknown for four-lens systems, and only \( z \) unknown for one-lens system.

### 4.1. Basic Preparations

The CLASS statistical sample uses the flux density ratio \( q_r \) of the multiple lensing images as a selection criterion of a sample: \( q_r = |\mu_+ / \mu_-| < 10 \) (Chae et al. 2002; Chen 2003a, 2003b), where \( \mu_+ \) and \( \mu_- \) are the magnifications of two (outside) images, respectively. The magnification of an image is determined by \( \mu = \left( z / dz \right)^{-1} \). The parameter \( q_r \) reduces the lensing cross section \( \sigma \). For an NFW case, the influence of \( q_r \) on \( \sigma \) is very small and can be neglected. For an SIS lens, it needs to multiply the \( \sigma \) by a factor of \( (9/11)^2 \).

As the exact redshift distribution of the CLASS statistical sample is unknown, Chae et al. (2002) utilized a Gaussian model with mean redshift \( \langle z_s \rangle = 1.27 \) given by Marlow et al. (2000) to describe the redshift distribution for the unlensed sources of the CLASS statistical sample. Denoting this Gaussian model as \( g^*(z_s) \), its distribution was explicitly plotted in Figure 5 of Chae et al. (2003), which was obtained from describing the redshift distribution of the flat-spectrum sources as shown in that Figure 5. Note that as such a Gaussian model has a physical cut at the point \( z_s = 0 \), it no longer has the standard form. Unlike the Gaussian model \( g^*(z_s) \) by Chae et al. (2003), here we shall take an alternative Gaussian model by directly fitting the redshift
distribution of the subsample of CLASS statistical sample given by Marlow et al. (2000) instead of fitting the redshift distribution of the flat-spectrum sources in Chae et al. (2003). Taking the general form of the Gaussian model

$$g(z_s) = \frac{N_s}{\sqrt{2\pi\lambda}} \exp\left[\frac{-(z_s - a)^2}{2\lambda^2}\right]$$

(35)

with $N_s$ being the normalization parameter $\int_{0}^{\infty} f(x)dx \equiv 1$, and requiring the mean value $\int_{0}^{\infty} xf(x)dx \equiv 1.27$ given by Marlow et al. (2000), we then only need to fit the remaining one parameter. The best-fit results are found to be

$$N_s = 1.6125; \quad a = 0.4224; \quad \lambda = 1.3761.$$  

(36)

For comparison, we also present some results based on two treating methods appearing in literatures by using the CLASS statistical sample: the redshift distributions of sources are the average redshift value $d(z_s)$ (see, e.g., Li & Ostriker 2002) and $f(z_s) = 0.204 + 0.2979z_s - 0.1121z_s^2 + 0.001584z_s^3$ (see, e.g., Sarbu et al. 2001). Figure 6 gives curves of the four models as a function of $z_s$ and the histogram of 27 CLASS subsamples from Marlow et al. (2000).

Before comparing with the CLASS statistical sample, we should consider the effect of magnification bias $B$, which causes the overrepresentation of the lensed objects in a flux-limited survey. The flux distribution of the CLASS statistical sample is well described by $N(f) \propto (f/f_0)^{\eta}$ with $\eta = 2.07 \pm 0.02(1.97 \pm 0.14)$ for $f \geq f_0(f \leq f_0)$ and $f_0 = 30$ m Jy (Chae et al. 2002).
\[ P(w) \text{ by an integration} \]
\[
p(w) \equiv P_{\text{obs}}(> \Delta \theta) = \int \int B \frac{dP(> \Delta \theta)}{dz} \varphi(z_s) dz dz_s, \tag{39}
\]
and
\[
q(w) \equiv \frac{dP_{\text{obs}}(> \Delta \theta)}{d\Delta \theta} = \int \int B \frac{d^2P(> \Delta \theta)}{d\Delta \theta dz} \varphi(z_s) dz dz_s, \tag{40}
\]
with \( \varphi(z_s) \) being the redshift distribution of sources.

We shall compare the theoretical results of the SIS case and the NFW case with the CLASS statistical sample. Figure 7 shows the lensing probability \( P(> \Delta \theta) \) as a function of the splitting angle \( \Delta \theta \) for the source redshift distribution \( d(z_s) \), \( f(z_s) \), \( g(z_s) \), and \( g^*(z_s) \). The thick line in each panel is induced from the 13 observed lensing data. It is seen that the lensing probability \( P(> \Delta \theta) \) of the SIS case and the NFW case are both sensitive to the parameter \( w \), especially for the NFW case. When the parameter \( w \) increases, the values of \( P(> \Delta \theta) \) for both SIS and NFW cases clearly increase in the whole concerned region of \( \Delta \theta \). Thus, it is feasible to constrain the parameter \( w \) from the SGL splitting angle data. The figure shows that the SIS model can only reproduce the data curve at small \( \Delta \theta < 1.5 \). When considering the rapid decline of \( P(> \Delta \theta) \) from the data line at large \( \Delta \theta \), a combined mechanism of the SIS and NFW models is needed to explain the whole experimental curve. Define a new model parameter \( M_c \), as Li & Ostriker (2002): lenses with mass \( M < M_c \) have the SIS profile, while lenses with mass \( M > M_c \) have the NFW profile. Then the differential probability
\[
dP/dM = dP_{\text{SIS}}/dM \varphi(M_c - M) + dP_{\text{NFW}}/dM \varphi(M - M_c),
\]
where \( \varphi \) is the step function, \( \varphi(x-y) = 1 \), if \( x > y \) and 0 otherwise. Because the splitting angle \( \Delta \theta \) is directly proportional to the mass \( M \) of lens halos, the contribution to large \( \Delta \theta \) of the SIS profile is depressed by \( M_c \). The lens data require a mass threshold \( M_c \sim 10^{13} h^{-1} M_\odot \), which is consistent with the halo mass whose cooling time equals the age of the universe today. In this note, we shall use such a two-model combined mechanism to calculate lensing probabilities.

In Figure 7, one can also find the influences of different source distributions on the lensing probability \( P(> \Delta \theta) \): the patterns of the function curves are hardly changed, but the function values for the same \( \Delta \theta \) slightly increase from \( d(z_s) \) to \( f(z_s) \).

4.2. Constraint on \( w \)

We now come to our main purpose, i.e., utilizing the SGL splitting angle statistic data from strong gravitational lenses to constrain the equation-of-state parameter \( w \) of dark energy. For that, we define the likelihood function as
\[
L(w) = (1 - p(w))^{N_l} \prod_{i=1}^{N_l} q_i(w). \tag{41}
\]
p\( (w) \) represents the model-predicted lensing probabilities \( P(> 0.3) \) of a source with the redshift distribution \( \varphi(z_s) \) and can be calculated by using Equations (23), (33), and (39). Here, \( \Delta \theta \geq 0.3 \) is an observational selection criterion.
\( g_i(w) \) is the model-predicted differential lensing probabilities \( d P(> \Delta \theta)/d \Delta \theta \) of the \( i \)th observed lens system with the splitting angle \( \Delta \theta \) and can be calculated by using Equations (24), (34), and (40). To utilize the data information adequately, we multiply Equation (39) by \( \delta(z - z') \) or/and \( \delta(z_{s} - z'_{s}) \) (replace \( \varphi(z_{s}) \)) for the \( i \)th observed lens system whose lens redshift \( z' \) or/and source redshift \( z'_{s} \) is/are known. For the unknown \( z' \) or \( z'_{s} \), we just integrate it out. Compared with utilizing the curve of lensing probability \( P(> \Delta \theta) \) as a function of the splitting angle \( \Delta \theta \) in the last subsection, which is introduced from lensed signals only, by using the likelihood function Equation (41), the unlensed signals are utilized and their influences are quite important due to the large exponential number \((N - N_0) = 8945\).

First, we discuss the possible constraints on the model parameter \( M_s \) and the constant \( w \). Under the given cosmological parameters \((\Omega_M, h, \sigma_8) = (0.24, 0.73, 0.74)\), Figure 8 shows the 68% C.L. and 95% C.L. allowed regions from the CLASS statistical sample for the source redshift distribution \( g(z_s) \), \( g'(z_s) \), \( d(z_s) \), and \( f(z_s) \), respectively. The crosshairs in the three panels mark the best-fit points \((w, M_s) = (-0.89, 1.37), (-0.94, 1.36), (-1.4, 1.68), \) and \((-0.73, 1.27)\) from left to right, where the unit of \( M_s \) is \(10^{15} h^{-1} M_\odot\). The source redshift distributions \( g(z_s) \) and \( g'(z_s) \) give nice constraints, while \( d(z_s) \), which is not a proper redshift distribution, provides somewhat strange unexpected results. The redshift distribution \( f(z_s) \), which has a larger part of galaxies at high redshift, prefers a larger \( w \) and smaller \( M_s \). From now on we will only consider the \( g(z_s) \) case and the \( g'(z_s) \) case, which are extracted from the subsample of the CLASS statistical sample (Marlow et al. 2000). Our best-fit result of \( M_s \approx 1.40 \) for both \( g(z_s) \) case and \( g'(z_s) \) case is larger than the value \( M_s \approx 1.0 \) obtained by Li & Ostriker (2002). The 95% C.L. allowed regions of the parameter \( w \) for the \( g(z_s) \) case and the \( g'(z_s) \) case are from \(-0.18 \) to \(-1.45 \) and from \(-0.11 \) to \(-1.85 \), which are consistent with the \( \Lambda \)CDM cosmology. Figure 2 in Chae (2007) shows a much negative result of the parameter \( w \). Our consideration here is based on the specially selected cosmological parameters, and also we have used the two-model combined mechanism, namely, the utilization of model parameter \( M_s \). As a consequence, our results avoid the large absolute value of parameter \( w \).

In Figure 9, we show the constraint for the parameters \((w_0, \sigma_8)\) appearing in a time-varying equation-of-state \( w(z) = w_0 + \sigma_8 z/(1 + z) \) under the given cosmological parameters \((\Omega_M, h) = (0.24, 0.73)\) for both \( g(z_s) \) and \( g'(z_s) \) cases, and with two different values of \( \sigma_8 = 0.74 \) and \( \sigma_8 = 0.90 \). The crosshairs mark the best-fit points: \((M_s; w_0, \sigma_8) = (1.36; -0.92, -1.31)\) for the \( g(z_s) \) case and \((M_s; w_0, \sigma_8) = (1.38; -0.89, -1.21)\) for the \( g'(z_s) \) case when \( \sigma_8 = 0.74 \) and \((M_s; w_0, \sigma_8) = (1.54; -0.83, -2.22)\) for the \( g'(z_s) \) case when \( \sigma_8 = 0.90 \). The parameter \( \sigma_8 \) has significant influences on the mass power
spectrum and the number density of dark halos, and our results show that the best fit \((w_0, w_a)\) are changed for the different selections of the parameter \(\sigma_8\); when \(\sigma_8\) increases from \(\sigma_8 = 0.74\) to \(\sigma_8 = 0.9\), the best-fit parameters \(w_0\) increase moderately and the best-fit parameters \(w_a\) have a sizable decrease for both the \(g(z)\) and the \(g'(z)\) cases.

After marginalizing the cosmological parameters \((\Omega_M, h, \sigma_8)\) and the critical mass parameters \(M_c\) by the Monte Carlo method, we obtain the constraint on \((w_0, w_a)\) in Figure 10. For the three cosmological parameters, we assume the Gaussian prior distributions induced from the results of WMAP three-year data: \((\Omega_M \pm \sigma_{\Omega_M}, h \pm \sigma_h, \sigma_8 \pm \sigma_{\sigma_8}) = (0.238 \pm 0.019, 0.73 \pm 0.03, 0.74 \pm 0.06)\). For the model parameter \(M_c\), we integrate it from 1.0 to 4.0. The crosshairs mark the best-fit point \((w_0, w_a) = (-0.88, -1.55)\) for the \(g(z)\) case and \((w_0, w_a) = (-0.91, -1.60)\) for the \(g'(z)\) case. At the 95\% C.L., our fitting results are consistent with that of Barger et al. (2006), and the SGL splitting angle statistic with the source redshift distributions \(g(z)\) and \(g'(z)\) gives somewhat more negative values for the parameter \(w_a\).

5. CONCLUSIONS

From the above analyses, we have shown how the SGL splitting angle statistic can be used to quantitatively constrain the equation-of-state parameter \(w\) of dark energy. Though due to the limited spacetime, the difference of the parameter \(w\) has few influences on the splitting angle \(\Delta\phi\) and the lensing cross section \(\sigma\), while through the comoving number density of dark halos as sources and lenses described by P–S theory, dark energy can affect the efficiency with which dark-matter concentrations produce strong lensing signals. With a twomodel combined mechanism of the dark halo density profile, which introduces a model parameter \(M_c\), we have carefully investigated the constraints on the constant \(w\) and time-varying \(w(z) = w_0 + w_a z/(1 + z)\). We find the best-fit value \(M_c \approx 1.4\) for both \(w\) and \(w(z)\), such a value is larger than the value \(M_c \approx 1\) obtained by Li & Ostriker (2002). This is mainly because in our analyses both the lensed and unlensed signals have been utilized in the likelihood function Equation (41), while in the analyses by Li & Ostriker (2002), only the lensed signals were used. The transition from SIS to NFW characterized by the parameter \(M_c\) is also motivated by the process of baryonic cooling (e.g., Kochanek & White 2001), where the parameter \(M_c\) was introduced to divide the cooled (SIS) and uncooled (NFW) halos. The estimated value by Kochanek & White (2001) for \(h = 0.67\) is \(M_c \approx 1 \times 10^{13} M_{\odot}\) for the model without a bulge, which is also smaller than our fitting result. There are several differences between our calculations and theirs. First, we use a larger Hubble constant \(h = 0.73 > h = 0.67\) and in our Figure 9, we show that for a larger \(h\), we have a larger fit \(M_c\). Second, the SIS model used by us has a larger relative lensing cross section than the exponential disk used by Kochanek & White (2001), which gives a larger \(M_c\). Third, the ratio of the cluster to the background density \(\Delta_{vir} = \rho_{halo} / \rho_{M} > 150\) used by us is much larger than the value \(\Delta \approx 100\) used by Kochanek & White (2001), which could also have some influences on the results. Nevertheless, all the fitting results for \(M_c\) are consistent with appropriate considerations.

With the given cosmological parameters \((\Omega_M, h, \sigma_8) = (0.24, 0.73, 0.74)\), we have compared the results of constant \(w\) corresponding to the four kinds of source redshift distributions. It has been shown that \(d(z)\) is not suitable for the SGL data analysis. For the redshift distributions of the normalized Gaussian-type model \(g(z)\) and the Gaussian model \(g'(z)\), the fitting results are \((w, M_c) = (-0.89^{+0.49}_{-0.26}, 1.37^{+0.47}_{-0.33})\) and \((w, M_c) = (-0.94^{+0.57}_{-0.16}, 1.36^{+0.47}_{-0.33})\), respectively, and the fitting results for the constant \(w\) are consistent with the \(\Lambda\)CDM at 95\% C.L. For the time-varying \(w(z)\), we have first investigated the influence of \(\sigma_8\) with the redshift distributions \(g(z)\) and \(g'(z)\) and found that the fitting results of the double parameters \((w_0, w_a)\) are changed when \(\sigma_8\) increases from \(\sigma_8 = 0.74\) to \(\sigma_8 = 0.9\). With the above given cosmological parameters, the best-fitting results for the \(g(z)\) case are \((M_c; w_0, w_a) = (1.36; -0.92, -1.31)\) for \(\sigma_8 = 0.74\) and \((M_c; w_0, w_a) = (1.56; -0.81, -2.5)\) for \(\sigma_8 = 0.9\); and for \(g'(z)\) case, the best-fit results are \((M_c; w_0, w_a) = (1.38; -0.89, -1.21)\) for \(\sigma_8 = 0.74\) and \((M_c; w_0, w_a) = (1.54; -0.83, -2.22)\) for \(\sigma_8 = 0.9\).

After marginalizing our likelihood functions over the cosmological parameters \((\Omega_M, h, \sigma_8)\) (by using the prior probabilities induced from the WMAP three-year data) and the model parameter \(M_c\), we have obtained a reliable constraint on the parameters \((w_0, w_a)\). Within the allowed uncertainties, the results for \(w_0\) are consistent with the constraints obtained from the SNe Ia data (Barger et al. 2006). Our fit results are \((w_0, w_a) = (-0.88^{+0.49}_{-0.26}, -1.55^{+0.27}_{-0.88})\) for the \(g(z)\) case and \((w_0, w_a) = (-0.91^{+0.60}_{-0.54}, -1.60^{+1.28}_{-2.42})\) for the \(g'(z)\) case. It is noted that the best-fitting results based on the SGL splitting angle statistic favor negative values for the parameter \(w_a\), which differs from the best-fitting values obtained based on the SNe Ia data, where the best-fitting results favor positive values for the parameter \(w_a\) (Barger et al. 2006). A combining constraint is interesting and will be investigated elsewhere.

In conclusion, the quantitative investigation has shown that the SGL splitting angle statistic can lead to a consistent constraint on the constant \(w\) and the double parameters \((w_0, w_a)\) of the time-varying dark energy equation-of-state \(w(z) = w_0 + w_a z/(1 + z)\). Especially for the allowed range of parameters \((w_0, w_a)\), the SGL splitting angle statistic does give an interesting bound. It can be seen in Figures 8–10 that the normalized Gaussian-type source redshift distribution \(g(z)\) leads to the most stringent constraints. Though it does not yet allow to obtain a more accurate constraint, it can provide a complementarity to other constraints from SNe, CMB, and weak lensing.

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