A comprehensive study of the frequency dependence of the photon-magnon coupling for multiple magnetic samples is made possible with a tuneable 3D-printed re-entrant cavity. Strong coupling is achieved, with values ranging between 20–140 MHz. The reworked theory and thereafter experimental proof enables coupling to be calculated from simulations alone, enabling future experiments with exotic cavity designs to be precisely engineered, with no limitations on sample and cavity geometry. Finally, the requirements of the deep strong coupling regime are shown to be achievable in such experiments.

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I. INTRODUCTION

The past decade has seen rapid development in the field of cavity magnonics. As far as light–matter interactions go, magnetic materials are attractive given their exceptionally large density of spins and low losses, the former making them easy to couple light to and the latter giving long coherence times. In addition, the collective spin wave resonance (quantised as the magnon) is widely tuneable with an external magnetic field, therefore interactions can be generated for any desired frequency. As a result, the strong coupling regime, in which the magnon-photon coupling rate is greater than the cavity and magnon loss rates; essential for coherent transfer of quantum information, is easily achievable in these systems even at room temperature. For these reasons, a great deal of research has aimed at developing these systems with the application of quantum information processing and memory in mind [1–4]. A hybrid system combining the fast manipulation rates of superconducting qubits with the assets of magnonic systems through strong coupling with a microwave cavity presents as a promising solution for such applications [5–7].

Initially parallel to cavity magnonics, but now converging, is the field of spintronics, in which an electrical current pumping (readout) of spin waves is achieved through the (inverse) spin Hall effect. This effect relies on ferromagnet–normal metal heterostructures and has been demonstrated for a wide variety of material combinations in thin film form [8–11]. There now exists the real possibility of combining the two fields to enable the transmission and electrical read out of quantum states in ferromagnets using a hybrid architecture [12–14]. Furthermore, due to the possibility of coupling magnon modes to photons at optical frequencies [7, 15–18], magnon systems may be considered as a candidate for coherent conversion of microwave and optical photons. In addition, magnons interact with elastic waves [7, 19] permitting the combination of mechanical and magnetic systems. These systems therefore possesses great potential as an information transducer that mediates interconversion between information carriers of different physical nature.

Finally, on a more fundamental level, cavity–magnon systems have allowed the study of strong coupling physics [20], with a handful of experiments demonstrating the ultrastrong coupling (USC) regime [21–24], in which the coupling rate is an appreciable proportion of the system frequency (i.e. $g/\omega > 0.1$). New and interesting physics has been discovered in these regimes [20, 25–30], in which the rotating wave approximation breaks down. One regime still relatively unexplored experimentally however, is the deep strong coupling (DSC) regime [31], in which the coupling energy is larger than the system energy, i.e. $g > \omega$.

Moving towards the aforementioned goals, possessing the ability to design experiments with full knowledge of cavity mode frequencies and coupling strengths, regardless of cavity and sample geometry, would be an invaluable tool for the community. It would permit the placement of cavity and hybridised modes at exact frequency locations for resonant enhancement purposes or simplify the task of merging multiple forms of magnon couplings (optics/microwaves/acoustics/electrical) within the one system, prior to cavity construction. Having this predictive capacity for exotic cavity designs in which the RF field is not necessarily uniform over the magnetic sample is also desirable, as well as for thin film samples if one wishes to engage spintronic applications.

In this paper we demonstrate that a theoretical model for accurately predicting mode couplings does exist, when
treated in the correct fashion. Furthermore, it is demonstrated that this same theory is applicable for non-uniform excitation of magnetic samples whether they possess axisymmetric geometry or not. It has been previously stated that coupling rates for non-spherical geometries are difficult to calculate given magnon mode shapes in these samples are not known [21]. On the contrary, by utilising a widely tuneable microwave cavity and two different yttrium iron garnet (YIG) sample geometries, as well as comparing to a host of other experimental results taken from the literature, we demonstrate here that through correct use of the so-called “form factor” \( \eta \) and the correct value for spin density \( n_s \), the largest cavity–magnon coupling \( g_{cm} \) in any system can be accurately predicted. Finally, using this analysis, we demonstrate for the first time the requirements of cavity magnon experiments to reach the DSC regime.

II. THEORY

Cavity magnon polaritons are bosonic quasiparticles associated with the hybridisation of a photon and a magnon within a cavity resonator. In the strong coupling regime, this results in an anti–crossing in the dispersion spectrum as the magnon frequency is tuned close to the cavity frequency by application of an external DC magnetic field, \( H_0 \). The system can be modelled as two coupled harmonic oscillators, where the eigenfrequencies of the split mode are:

\[
\omega_{\pm} = \sqrt{\frac{\omega_c^2 + \omega_m^2}{2}} \pm \sqrt{\frac{(\omega_c^2 - \omega_m^2)}{2}} + 4\omega_c \omega_m g_{cm},
\]

where \( \omega_c \) is the cavity frequency in angular units, \( g_{cm} \) the coupling strength as a measure for the frequency of coherent information exchange, which determines the size of the mode splitting, and

\[
\omega_m^{\text{sphere}}(H_0) = \gamma H_0, \quad \omega_m^{\text{film}}(H_0) = \gamma \sqrt{H_0(H_0 + M_s)}
\]

are the magnon resonant frequencies as a function of \( H_0 \) and material dependent constants; saturation magnetisation \( M_s \) and gyromagnetic ratio \( \gamma \).

The Hamiltonian for a magnon/cavity photon system is described by the Tavis-Cummings model [32] and can be found elsewhere [23, 32, 35]. A succinct derivation of the formula for \( g_{cm} \) is given by Flower et al. [21], and appears as:

\[
g_{cm} = \frac{\gamma}{4\pi} \frac{\mu_0 \rho \omega_c}{V_m} = \eta \sqrt{\omega_c \gamma} \frac{\gamma}{4\pi} \frac{\mu}{g_{\mu B} \rho \mu_0 \hbar n_s},
\]

where \( S = \frac{\mu}{g_{\mu B}} N_s \) is the total spin number of the macrospin operator, \( \mu_B \) is the Bohr magneton, \( \mu = 5\mu_B \) is the magnetic moment for YIG, \( g = 2 \) is the Landé g-factor for an electron spin, \( N_s \) the number of spins in the sample, \( V_m \) the magnetic sample volume, \( \mu_0 \) is the vacuum permeability, \( \hbar \) the reduced Planck’s constant and \( n_s = N_s / V_m \) the density of spins. The so-called “form–factor”, \( \eta \) describes the proportion of the cavity mode’s magnetic field perpendicular to \( H_0 \) (\( \vec{H} \cdot \vec{e}_x \) and \( \vec{H} \cdot \vec{e}_r \)), assuming \( H_0 \) is applied along the z-axis) as well as the proportion of this perpendicular field located within \( V_m \) compared to the entire cavity volume, \( V_c \) [21]. It can be defined as:

\[
\eta = \sqrt{\frac{\int_{V_m} \vec{H} \cdot \vec{e}_x dV}{\sqrt{\int_{V_c} |\vec{H}|^2 dV}}}.
\]

It can be seen from the form of eq.(1) that the case of \( \eta = 1 \) would correspond to all of the applied RF cavity magnetic field being perpendicular to \( H_0 \) as well as being totally confined to \( V_m \). The latter condition represents a divergence from the definition of \( \eta \) given in previous publications [11, 21, 33] and in doing so extends the applicability of eq.(3) to systems in which RF fields are non-uniform over \( V_m \).

As highlighted in [21, 34], \( g_{cm} \) only depends on \( n_s, \mu, \eta \) and \( \omega_c \), and therefore an increase in \( N_s \) does not necessarily facilitate an increase in \( g_{cm} \). Take for example the case of a large cavity with localised field in a small volume \( V_f < V_c \), and assume \( V_f = V_m \): an increase in \( V_m \) and hence \( N_s \) will in fact reduce \( g_{cm} \) as the ratio of the integrals in eq.(1) remains unchanged but \( V_m \) has increased.

It is worthwhile clarifying the apparent confusion surrounding the value of \( n_s \) for YIG. To the authors’ knowledge, there appears three different values of \( n_s \) quoted throughout the literature: \( 4.22 \times 10^{27} \text{ m}^{-3} \) [24, 39, 40], \( 2.11 \times 10^{28} \text{ m}^{-3} \) [14, 21, 23, 41] and \( 2.11 \times 10^{29} \mu_B \text{ m}^{-3} \) [12, 33, 42], and despite the appearance of \( \mu_B \) in the latter value, some authors appear to simply neglect it when using their equivalent eq.(3) as if they were using the second quoted value. The correct value to use to calculate \( g_{cm} \) from eq.(3) is \( n_s = 4.22 \times 10^{27} \text{ m}^{-3} \). This can be derived from \( n_s = \left( \frac{a}{2} \right)^3 \) where \( a = 12.376 \text{ Å} \) is the lattice constant of the cubic unit cell and the \( \frac{1}{8} \) transforms to the primary cell [37, 43]. Note that multiplying by \( 5\mu_B \) would net a result of \( 2.11 \times 10^{28} \mu_B \text{ cm}^{-3} \), but is not necessary as it is already taken into account in eq.(3) by the appearance of \( \mu \) in the \( S \) term.

III. EXPERIMENT DESCRIPTION

The cavity used (Fig.1(a)–(b)) is a tuneable realisation of a double post re-entrant cavity, which extracts greater coupling values for a given sample than could be achieved from a standard waveguide due to a focusing of magnetic field. These cavities, an extension of the single–post form [43, 49], were initially developed by Goryachev et al. [23].
and have since become more commonplace throughout the literature.

The cavity was 3D-printed and metallised by ElliptikaTM using the technique described in [48]. The two posts are commercially purchased and inserted subsequent to printing, as is the thread in the cavity base into which the posts are screwed. There are two first-order resonant modes of such a cavity, termed the dark and bright modes (DM and BM: Fig. 1(c)–(d)). Both contain the mode’s electric field between the top of the post and the roof of the cavity. For the DM, the E-fields are in-phase, resulting in the circulating H-fields destructively interfering in the region between the posts (hence “dark”), whilst the opposite is true for the BM [23]. The distances between the posts and the cavity roof, \( d_1 \) and \( d_2 \) (as depicted in Fig. 1(a)), therefore define the capacitance of the cavity and hence determine the resonant frequencies of the modes.

We use two magnetic samples of YIG; a thin film sample and a sphere. For the film sample, 9 \( \mu \)m of YIG has been grown by liquid phase epitaxy on both sides of a GGG substrate (6.5 \( \times \) 3.8 \( \times \) 0.5) mm\(^3\). The sphere has a radius of 235 \( \mu \)m. They can be placed in either the central position (CP) of the cavity, between the two posts or in the off-centre position (OCP). The thin film sample is positioned using a 3D printed sample holder, seen in Fig. 1(b), placed around the tuneable posts.

The cavity has radius \( r_{cav} = 20 \) mm and height \( h = 4.6 \) mm, whilst the posts have radius \( r_{post} = 2.05 \) mm and are located at \( x_{post} = 3.4 \) mm from the cavity centre. These parameters were chosen, based on the commercially purchased post dimensions, to maximise geometric factor for a given operating frequency, whilst still maintaining a broad frequency tuning range (see Supp. Mat.).

The full tuning dynamics of the cavity are plotted in Fig. 2 which illustrates the exceptional range made possible by screwing the posts into or out of the cavity. The transmission spectra of the cavity for different \( d_1 = d_2 \) values is shown in Fig. 2(a) demonstrating an ideal, clean spectra for filtering applications, whilst the agreement between simulation and experiment for \( d_1 = d_2 \) tuning up to \( \sim 1 \) mm is shown in Fig. 2(b). It can be seen in Fig. 2(c)–(e) that depending on the respective values of \( \{d_1, d_2\} \), there is a large array of DM and BM resonant frequency pairings available. The versatility and predictability with which both mode frequencies can be positioned is a large advantage of this cavity design. \( Q \)-factors of the modes depend on the post heights (see Supp. Mat.) and range in values from 150 – 800.

The cavity is placed inside a DC magnet with the field, \( H_0 \) parallel to the posts of the cavity and therefore in the plane of the thin film sample (Fig. 1(a)-(b)). The magnetic field is swept such that the magnon resonance passes over both modes (by viewing the scattering parameters of the cavity on a Vector Network Analyser), then the mode frequencies are changed by tuning the post heights equally and the field swept again, and so on. In this way a multi-frequency readout of magnon-photon couplings can be achieved using a single cavity, without need for disassembly or re-calibration of instruments.
IV. RESULTS

The position of the centre of each anti-crossing can be used to determine $\gamma = 2\pi \times 28$ GHz/T and $M_s = 0.1775$ T in eq. (2), and hence values of $g_{cm}$ can be extracted from each of the individual density plots for a given $\{d_1, d_2\}$ (an example given in Fig. 3(a)) by fitting eq. (1). Finally a comprehensive survey of coupling as a function of frequency as well as $\eta$ for both thin film and spherical samples can be conducted, the results of which are presented in Fig. 3(b)–(d).

The form factor $\eta$ is different depending on whether the DM or BM are used, as well as the values of $\{d_1, d_2\}$ and the sample geometry. The dependence of $\eta$ on gap size is calculated using Finite Element Modelling (FEM) software COMSOL™ for each of the four unique cases: DM and BM for the thin film in the OCP, BM for the thin film in the CP (coupling with DM was too small to measure) and BM for the sphere in the CP. Specific $\eta$ values for each individual $\{d_1, d_2\}$ value can then be interpolated from the four resulting functions.

If one normalises the $g_{cm}$ values and both sides of eq. (3) by $\eta$ we observe agreement between experiment and theory, as shown in Fig. 3(d) for all samples in both positions CP and OCP. Experimental errors here are due to mesh and dimension errors in the FEM process used to determine $\eta$, which have a larger impact for small $\eta$ values.

![FIG. 3. (a) S21 spectra of the cavity at a fixed $\{d_1, d_2\}$ setting for the thin film sample located in the CP. Plotted in red is the $\omega_n(H_0)/2\pi$ curve for a thin film sample. (b) A collection of CP thin film and sphere coupling to the BM S21 spectra as a function of magnetic field. Each spectra is a separate experimental run with a unique $\{d_1, d_2\}$ setting. (c) $g_{cm}/2\pi$ as a function of mode frequency extracted from the spectra in (b). (d) $g_{cm}/2\pi\eta$ as a function of mode frequency with the fitted line given by eq. (3).](image-url)

![FIG. 4. Cavity–magnon coupling strength as a function of $g_{cm}/2\pi\eta$ for the thin film and sphere experimental results presented in this paper as well as a host of other experimental results taken from the literature. Eq. (3) is plotted as the dot–dashed line. The grey shaded regions represent the DSC regime for a given $\eta$ value, with the $g_{cm}$ value of the intercept point with eq. (3) (shown on the right hand axis) representing the minimum cavity frequency for the given $\eta$ required to obtain $g_{cm} > \omega_c$.](image-url)

V. DISCUSSION

Compared to majority of the literature’s expressions for $g_{cm}$, the uniqueness of eq. (3) is that there is no explicit dependence on the number of spins, $N_s$, as we work in density, $n_s$, nor is there dependence on the volume of the cavity or “mode volume”, as is often seen. Most of this information is handled and or cancelled out by our form of $\eta$. This means that the only experimentally dependent variables in eq. (3) are $\eta$ and $\omega_c$, when you assume consistent material values are used. Therefore any cavity–magnon experiment using YIG can be compared to the results here using this form of the equation by plotting $g_{cm}/2\pi$ against $\eta\sqrt{\omega_c}$, as demonstrated in Fig. 4.

Included in Fig. 4 are both spherical and thin film geometries, where the $\eta$ values have been calculated using FEM from the quoted sample and cavity dimensions in each of the referenced papers. There is exceptional agreement between the theory and the experimental results, demonstrating that regardless of cavity or sample geometry, mode splittings can be accurately predicted. This result provides experimentalists with 100% control and predictability of cavity–magnon experiments in the planing stage, prior to cavity construction. Combine this with the fine tuning ability of $g_{cm}$ using spintronic techniques [48], and hybridised mode eigenfrequencies can be engineered with excellent precision for any desired value. This is of great value if one wishes to interact with other, less manipulatable, resonant systems.

Finally, this analysis reveals the limiting parameters
for achieving the DSC regime in cavity–magnon experiments using YIG. Given that \( g_{cm} \) will always sit along the dot–dashed line in Fig. 3 for a given value of \( \eta \) we can determine what the maximum cavity frequency will be such that \( g_{cm} > \omega_c^* \) by setting eq. (3) equal to \( \omega_c^*/2\pi \). We find that for the optimal scenario of \( \eta = 1 \), a frequency of \( \omega_c < 1.72 \text{ GHz} \) would achieve DSC, and therefore is not prohibited. Of course, achieving \( \eta = 1 \) is a difficult task; the largest value achieved in the experiments conducted here is \( \sim 0.04 \) for the thin film sample in the CP coupled to the BM. As demonstrated by Fig. 4 as \( \eta \) is reduced, the maximum frequency permitted for DSC becomes more and more prohibitive; \( \omega_c < 2.75 \text{ MHz} \) for \( \eta = 0.04 \), which is outside the range of what is classified as microwave.

Nonetheless, \( \eta \) values of 0.18 \[23\], 0.39 \[38\] and 0.82 \[21\] have been achieved previously and with clever cavity engineering, there is nothing preventing the DSC regime being achieved in such systems. Furthermore, by increasing the density of spins by using an alternative magnetic material, the dot-dashed line in Fig. 4 will move up and the frequency restrictions of DSC become relaxed. For example, lithium ferrite has a spin density 2.13 times larger than YIG and the same magnetic moment. For the optimal scenario of \( \eta = 1 \), a frequency \( \omega_c^*/2\pi \) would require \( \eta = 0.52 \) to achieve DSC, and an experiment with \( \eta = 1 \) would require \( \omega_c < 3.67 \text{ GHz} \).

VI. CONCLUSION

By utilising a novel cavity design, cavity–magnon polariton coupling strengths are analysed over a broad range of frequencies from 1–8 GHz. The experimental data enabled the appropriate application of the theoretical framework for calculating coupling rates, which was then also applied to a multitude of previous experimental results taken from the literature. Through an alternative definition of the form factor \( \eta \), all of these experiments can be directly compared. Using this universal characterisation of coupling strength, it is shown that any cavity–magnon experiment can be correctly predicted from simulation alone, regardless of cavity or sample geometry. The theoretical equations and physical constants described in this paper should be used to define and categorise any future cavity–magnon experiment.

The results of this research allow future experiments to be precisely engineered for exact positioning of hybridised mode frequencies for such purposes as interacting cavity–magnon polaritons with other systems. The pathway to achieving the DSC regime in these systems, something not yet achieved, is also laid out.

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Cavity Q factors

An important parameter of any microwave cavity is the so-called Geometric Factor, which can be calculated as

\[ GF = \mu_0 \omega c \frac{\int_V |H|^2 dV}{\int_{S_c} |H|^2 dS} \]  

(5)

This can be used to estimate cavity Q factors as \( Q = \frac{GF}{R_s} \), where \( R_s \) is the surface resistance of the cavity walls. It represents the ratio of the magnetic field within the volume of the cavity over the field on the metallic surface, where there exists some small resistance and hence microwave loss. Measured cavity Q factors are plotted in Fig. S1 with simulated GF factors shown as well, which have been scaled to fit by \( R_s = 0.09 \) Ω. We observe that the BM is unaffected by the presence of the sample holder in the cavity, whereas it becomes the dominant loss mechanism for the DM for Q factors above \( \sim 350 \), which is due to a higher electrical filling in the sample holder for the DM hence a higher impact of its dielectric loss.

FIG. S1. Unloaded Q factor as a function of mode frequency for the DM and BM, with and without the sample holder (SH) in the cavity. Simulated values are obtained from calculating the GF of the cavity and then scaling by \( 1/R_s \) to fit the data.