LETTER

Deterministic photonic spatial-polarization hyper-controlled-not gate assisted by a quantum dot inside a one-side optical microcavity

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Abstract

To date, all work concerning the construction of quantum logic gates, an essential part of quantum computing, has focused on operating in one degree of freedom (DOF) for quantum systems. Here, we investigate the possibility of achieving scalable photonic quantum computing based on two DOFs for quantum systems. We construct a deterministic hyper-controlled-not (hyper-CNOT) gate operating in both the spatial mode and polarization DOFs for a photon pair simultaneously, using the giant optical Faraday rotation induced by a single-electron spin in a quantum dot inside a one-side optical microcavity as a result of cavity quantum electrodynamics. With this hyper-CNOT gate and linear optical elements, two-photon four-qubit cluster entangled states can be prepared and analyzed, which give an application to manipulate more information with less resources. We analyze the experimental feasibility of this hyper-CNOT gate and show that it can be implemented with current technology.

(Some figures may appear in colour only in the online journal)

1. Introduction

Quantum mechanics theory, the core of quantum information, can greatly improve the power of dealing with and transmitting information. Quantum computing [1] is required for the precise control and manipulation of the states of quantum systems in quantum information processes. It is proven that two-qubit controlled-not (CNOT) gates (or the equivalent two-qubit quantum gates) assisted by single-qubit gates are sufficient for universal quantum computing, so it is important to construct a two-qubit CNOT gate. So far, much work has been carried out on constructing a two-qubit CNOT gate or its equivalent in notably trapped ions [2], nuclear magnetic spins [3], free electrons [4] and polarized photons [5–10]. Pioneering work by Knill et al. [5] showed that a photonic CNOT gate could be created with a maximal success probability of 3/4, resorting to only single-photon sources, detectors and linear optical elements such as beam splitters. Since this original work, there has been significant progress [6–9] in constructing a CNOT gate with linear optics, which realizes probabilistically the nonlinear coupling between two photons using interference, at least two ancilla photons, single-photon detectors and conditioning. With nonlinear optics, a deterministic CNOT
gate can also be constructed. In 2004, Nemoto and Munro [10] constructed a near deterministic CNOT gate using several single-photon sources, linear optical elements, photon number resolving quantum nondemolition detectors and feed-forward. The key element of the CNOT gate in their proposal is the quantum nondemolition detector (QND) based on cross-Kerr nonlinearity.

Although there is much valuable work on the construction of quantum logical gates, especially CNOT gates, this is all focused on operating in one degree of freedom (DOF) for quantum systems. There are, so far, no quantum entangling gates operating in more than one DOF for quantum systems. There are many advantages in dealing with quantum information processes in a larger Hilbert space, especially with its robustness against noise [11] and the high channel capacity. High-dimensional entanglement has been realized in multipartite [12, 13] and multidimensional [14, 15] quantum systems. Although universal quantum computing can be realized with two-qubit CNOT gates and single-qubit operations, it would be convenient to have multi-qubit quantum logic gates in quantum computation. In 2006, Fiurášek [16] proposed some schemes for the probabilistic direct realization of fundamental Toffoli and Fredkin gates with linear optics, and he presented a scheme for a linear optical quantum Fredkin gate based on the combination of an experimentally demonstrated linear optical partial-SWAP gate and controlled-Z gates in 2008 [17]. Gong et al. [18] also discussed the realization of a quantum Fredkin gate using CNOT gates with only linear optics and single photons. Large Hilbert space with more than one DOF has also been discussed in some applications in quantum communication in recent years [19–24], especially for hyperentanglement which is defined as quantum systems entangled in more than one DOF. Besides the task in which hyperentanglement is used to assist quantum information processing in one DOF, a complete analyzer for hyperentangled Bell states has also been constructed [25] with cross-Kerr nonlinearity to increase the channel capacity of long-distance quantum communication in more than one DOF.

In this letter, we investigate the possibility of achieving scalable photonic quantum computing based on two DOFs for quantum systems, which is different from all the existing work concerning the construction of quantum logic gates operating in one DOF for quantum systems. We construct a deterministic hyper-controlled-not (hyper-CNOT) gate which performs as a CNOT gate in both the spatial mode and the polarization DOFs for a two-photon system simultaneously, assisted by two quantum dots embodied in one-side optical microcavities (QD–cavity). Exploiting the giant optical Faraday rotation of the left-circularly and right-circularly polarized photons with, respectively, a singly charged QD (e.g., a self-assembled In(Ga)As QD or a GaAs interface QD) located in the distributed Bragg reflectors are partially reflective) to achieve maximal light–matter coupling. According to Pauli’s exclusion principle, a negatively charged exciton \( X^- \) of two electrons bound to one hole [31] can be optically excited when an excess electron is injected into the QD. The optical resonance of \( X^- \) with circularly polarized photons depends on the excess electron spin in the QD [32], as shown in figure 1. For the excess electron spin state \( | \uparrow \rangle \), the negatively charged exciton \( | \uparrow \downarrow \downarrow \rangle \) with two electron spins antiparallel is created by resonantly absorbing a left-circularly polarized photon \( | L \rangle \). Here \( | \uparrow \rangle \) describes the heavy-hole spin state \( | + \rangle \). For the excess electron spin \( | \downarrow \rangle \), the other negatively charged exciton \( | \downarrow \uparrow \downarrow \rangle \) can be created by resonantly absorbing a right-circularly polarized photon \( | R \rangle \). Here \( | \downarrow \rangle \) describes the heavy-hole spin state \( | - \rangle \). This

As an application of this hyper-CNOT gate, one can use it to prepare entangled two-photon four-qubit cluster states easily and to analyze the 16 hyperentangled cluster states simply, resorting to some linear optical elements. We analyze the experimental feasibility of this hyper-CNOT gate, and our results show that it can be implemented with current technology.

2. Construction of deterministic spatial-polarization hyper-controlled-not gate

In 2008, Hu et al. [26, 27] pointed out that the interaction of left-circularly and right-circularly polarized photons with a QD–cavity system can be used in quantum information processes. With this optical property of QD–cavity systems, a multi-qubit entangler [26–28] and photonic polarization Bell-state analyzer [29, 30] can be constructed. In 2010, Bonato et al. [30] constructed a CNOT gate operating on a hybrid quantum system composed of a photon and an electron spin using the interface between the photon and the electron spin in a double-sided QD–cavity system in the weak coupling regime.

The QD–cavity system used in our proposal is constructed with a singly charged QD (e.g., a self-assembled In(Ga)As QD or a GaAs interface QD) located in the center of a one-side optical resonant cavity (the bottom distributed Bragg reflectors are 100% reflective and the top distributed Bragg reflectors are partially reflective) to achieve maximal light–matter coupling. According to Pauli’s exclusion principle, a negatively charged exciton \( X^- \) consisting of two electrons bound to one hole [31] can be optically excited when an excess electron is injected into the QD. The optical resonance of \( X^- \) with circularly polarized photons depends on the excess electron spin in the QD [32], as shown in figure 1. For the excess electron spin state \( | \uparrow \rangle \), the negatively charged exciton \( | \uparrow \downarrow \downarrow \rangle \) with two electron spins antiparallel is created by resonantly absorbing a left-circularly polarized photon \( | L \rangle \). Here \( | \uparrow \rangle \) describes the heavy-hole spin state \( | + \rangle \). For the excess electron spin \( | \downarrow \rangle \), the other negatively charged exciton \( | \downarrow \uparrow \downarrow \rangle \) can be created by resonantly absorbing a right-circularly polarized photon \( | R \rangle \). Here \( | \downarrow \rangle \) describes the heavy-hole spin state \( | - \rangle \). This

![Figure 1](image_url)

**Figure 1.** \( X^- \) spin-dependent transitions with circularly polarized photons. (a) A charged QD inside a one-side micropillar microcavity interacting with circularly polarized photons. (b) \( X^- \) spin-dependent optical transition rules due to Pauli’s exclusion principle. \( L \) and \( R \) represent left-circularly and right-circularly polarized photons, respectively. \( \uparrow \) and \( \downarrow \) represent the spins of the excess electron. \( \uparrow \downarrow \uparrow \) and \( \downarrow \uparrow \downarrow \) represent the negatively charged exciton \( X^- \).

There are many advantages in dealing with quantum information processes in a larger Hilbert space, especially with its robustness against noise [11] and the high channel capacity. High-dimensional entanglement has been realized in multipartite [12, 13] and multidimensional [14, 15] quantum systems. Although universal quantum computing can be realized with two-qubit CNOT gates and single-qubit operations, it would be convenient to have multi-qubit quantum logic gates in quantum computation.
optical process can be described by Heisenberg equations for
the cavity field operator \( \hat{a} \) and \( X^- \) dipole operator \( \hat{\sigma}_- \) in the interaction picture [33],
\[
\frac{d\hat{a}}{dt} = -\left[ i(\omega_c - \omega) + \kappa \right] \hat{a} - g \hat{\sigma}_- - \sqrt{\kappa} \hat{a}_\text{in},
\]
\[
\frac{d\hat{\sigma}_-}{dt} = -\left[ i(\omega_{X^-} - \omega) + \frac{\gamma}{2} \right] \hat{\sigma}_- - g \hat{\sigma}_+ \hat{a},
\]
where \( g \) represents the coupling strength between the cavity mode and \( X^- \). \( \kappa/2 \) and \( \kappa_s/2 \) represent the decay rate and the side leakage rate of the cavity field, respectively. \( \gamma/2 \) represents the decay rate of \( X^- \). \( \omega_{X^-} \), \( \omega_c \), and \( \omega \) represent the frequencies of the \( X^- \) transition, the input probe light and the cavity mode, respectively.

If \( X^- \) stays in the ground state for most of the time (i.e., \( \langle \sigma_z \rangle \approx -1 \)), the reflection coefficient of a QD–cavity system for this weak excitation condition is [26]
\[
r(\omega) = 1 - \frac{\kappa \left[ i(\omega_{X^-} - \omega) + \frac{\gamma}{2} \right]}{\left[ i(\omega_{X^-} - \omega) + \frac{\gamma}{2} \right] \left[ i(\omega_c - \omega) + \frac{\kappa}{2} + \frac{\kappa_s}{2} \right] + g^2}.
\]
For the condition \( g = 0 \), the QD is uncoupled to the cavity (the cold cavity), and the reflection coefficient becomes
\[
r_0(\omega) = \frac{i(\omega_c - \omega) - \frac{\kappa}{2} + \frac{\kappa_s}{2}}{i(\omega_c - \omega) + \frac{\kappa}{2} + \frac{\kappa_s}{2}}.
\]

Figure 2. Schematic diagram of our spatial-polarization hyper-CNOT gate operating simultaneously in both the spatial mode and polarization degrees of freedom for a two-photon system. BS represents a 50:50 beam splitter which is used to perform a Hadamard operation on the spatial-mode states for a photon. CPBS represents a polarizing beam splitter on a circular basis, which transmits the photon in the right-circularly polarized state \( |R \rangle \), reflects the photon in the left-circularly polarized state \( |L \rangle \), and the wave plate \( WP_1 \) is used to perform a polarization phase-flip operation \( U_{1m} = -i[R \rangle \langle L | \] to achieve the relative phase shift \(-\pi/2\) between the two spatial modes for a photon \( (m = 2, 4) \). The interaction of a single photon with a QD–cavity system can be described as
\[
|R, \uparrow \rangle \rightarrow iL, \uparrow \rangle, \quad |L, \uparrow \rangle \rightarrow |L, \uparrow \rangle,
\]
|\( R, \downarrow \rangle \rightarrow |R, \downarrow \rangle, \quad |L, \downarrow \rangle \rightarrow -i|L, \downarrow \rangle\).

(1) Three-particle four-qubit hybrid controlled-Z gate. We assume the initial states of the excess electron spin in QD1 and photon \( a \) are \( \frac{1}{\sqrt{2}}(|\uparrow \rangle + |\downarrow \rangle)_{\text{e}} \) and \( |\Phi_{\text{a}}\rangle_0 = (|a_1| R \rangle + |\alpha_2| L \rangle)_{\text{a}} \otimes (|\gamma_1| a_1 \rangle + |\gamma_2| a_2 \rangle) \), respectively. Here \( |a_1\rangle \) and \( |a_2\rangle \) represent the two spatial modes for photon \( a \). After photon \( a \) passes through the circular polarizing beam splitter CPBS (CPBS1 and CPBS3) and the half-wave plate HWP1 \( (X_1 \) and \( X_3) \), interacts with the electron spin in QD1, and then passes through another HWP1 \( (X_2 \) and \( X_4) \). CPBS (CPBS2 and CPBS4) and the wave plate W1 (\( U_{12} \)) in figure 2, the state of the system composed of QD1 and photon \( a \) is changed

\[
\frac{1}{\sqrt{2}}(|\uparrow \rangle + |\downarrow \rangle)_{\text{e}} \otimes (|\gamma_1| a_1 \rangle + |\gamma_2| a_2 \rangle).
\]
from $|\Phi_{ae1}\rangle$ to $|\Phi_{ae2}\rangle$. Here

$$|\Phi_{ae1}\rangle = (\alpha_1 |R_i| + \alpha_2 |L_i)|a\rangle \otimes \gamma_1 |a1\rangle + \gamma_2 |a2\rangle)$$

$$\otimes \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)_{e_1},$$

$$|\Phi_{ae2}\rangle = \frac{1}{\sqrt{2}}(\alpha_1 |R_i| + \alpha_2 |L_i)|a\rangle \otimes \gamma_1 |a1\rangle + \gamma_2 |a2\rangle)$$

$$+ \frac{1}{\sqrt{2}}(|\uparrow\rangle \gamma_1 |a1\rangle - \gamma_2 |a2\rangle).$$

(6)

This is the result for a controlled-Z gate constructed using the electron spin $e_1$ as the control qubit and the spatial modes of photon $a$ as the target qubit.

The initial state of QD 2 in figure 2 is prepared as $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)_{e_2}$, and the frequencies of the input photon and the cavity mode are adjusted to be $\omega = \omega_e \approx \hbar / 2$, the same as those for QD 1. After photon $a$ interacts with QD 2 and passes through WP 2 (U$_{23}$ and U$_{24}$) as shown in figure 2, the state of the spatial-polarization hyper-CNOT gate can be constructed with the system composed of QD 1, QD 2, and photon $a$ is changed from $|\Phi_{ae1}\rangle$ to $|\Phi_{ae2}\rangle$. Here

$$|\Phi_{ae1}\rangle = |\Phi_{ae1}\rangle \otimes \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)_{e_2},$$

$$|\Phi_{ae2}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \gamma_1 |a1\rangle + \gamma_2 |a2\rangle)$$

$$\otimes \gamma_1 |a1\rangle + \gamma_2 |a2\rangle) + \frac{1}{\sqrt{2}}(|\uparrow\rangle \gamma_1 |a1\rangle - \gamma_2 |a2\rangle).$$

(7)

This is the result for the three-particle four-qubit hybrid controlled-Z gate constructed using two electron spins as the control qubits and the polarization and spatial modes of photon $a$ as the target qubit. That is, when the spin of electron $e_1$ is in the state $|\downarrow\rangle_{e_1}$, the spatial mode $|a2\rangle$ of photon $a$ undergoes a phase shift $\pi$, and when the spin of electron $e_2$ is in the state $|\downarrow\rangle_{e_2}$, the polarization mode $|L_i\rangle$ of photon $a$ undergoes a phase shift $\pi$.

(2) Spatial-polarization hyper-CNOT gate. The second photon is prepared initially in the state $|\Phi_{b0}\rangle = (\beta_1 |R_i| + \beta_2 |L_i)|a\rangle$ (by making photon $b$ pass through BS$_1$, H$_1$, and H$_2$). After Hadamard operations are performed on both the spatial mode and the polarization DOFs for photon $b$ (by making photon $b$ pass through BS$_1$, H$_1$, and H$_2$), the state $|\Phi_{b0}\rangle$ is changed to $|\Phi_{b0}\rangle = (\beta_1 |R_i| + \beta_2 |L_i)|a\rangle$ (by making photon $b$ pass through BS$_1$, H$_1$, and H$_2$). Here $\beta_1 = \frac{1}{\sqrt{2}}(\beta_1 + \beta_2)$, $\beta_2 = \frac{1}{\sqrt{2}}(\beta_1 - \beta_2)$, $\beta_1 = \frac{1}{\sqrt{2}}(\beta_1 + \beta_2)$, and $\beta_2 = \frac{1}{\sqrt{2}}(\beta_1 - \beta_2)$. We then perform unitary operations on the two electron spins in QD 1 and QD 2, which transform the states $|\uparrow\rangle$ and $|\downarrow\rangle$ to $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$ and $\frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$, respectively. Subsequently, we let photon $b$ pass through CPBS ($CPS_2$ and $CPBS_7$), WP 1 ($X_5$ and $X_2$), QD 1, WP 1 ($X_4$ and $X_3$), CPBS ($CPS_5$ and $CPBS_8$), WP 2 ($U_{14}$), QD 2, and WP 2 ($U_{23}$ and $U_{24}$). After the interaction between photon $b$ and the two QDs, the state of the spatial-polarization hyper-CNOT gate can be constructed with the system composed of QD 1, QD 2, and photons $a$ and $b$ is changed from $|\Psi_{c}\rangle$ to $|\Psi_{b}\rangle$. Here

$$|\Psi_{c}\rangle = |\Phi_{ae2}\rangle \otimes (\beta_1 |R_i| + \beta_2 |L_i)|a\rangle$$(8)

$$|\Psi_{b}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \gamma_1 |a1\rangle(\delta_1 |b1\rangle + \delta_2 |b2\rangle)$$

$$+ \frac{1}{\sqrt{2}}(|\uparrow\rangle \gamma_2 |a2\rangle(\delta_1 |b1\rangle - \delta_2 |b2\rangle)) \otimes |\uparrow\rangle \gamma_1 |a1\rangle$$

$$\times (\beta_1 |R_i| + \beta_2 |L_i)|a\rangle$$

$$+ (\beta_1 |R_i| - \beta_2 |L_i)|b\rangle.$$
After performing Hadamard operations on both the polarization (HWP) and spatial-mode (BS) DOFs for photon a, the hyper-CNOT gate operating on the two-photon system in both its polarization and its spatial-mode DOFs. Moreover, in principle, this hyper-CNOT gate works in a deterministic way.

3. Preparation of two-photon four-qubit cluster state

As an application of our hyper-CNOT gate, we discuss the generation and the complete analysis of two-photon four-qubit cluster entangled states below. One can see that these tasks can be accomplished easily and simply with a hyper-CNOT gate and some linear optical elements.

With our hyper-CNOT gate operating in the spatial mode and the polarization DOFs for a photon pair in the initial state $\frac{1}{\sqrt{2}}(|R\rangle + |L\rangle)_a(|a_1\rangle + |a_2\rangle) \otimes |R\rangle_b|b_1\rangle$, it is, in principle, easy to prepare a hyperentangled two-photon four-qubit state

$$|\Phi_{ab}\rangle_1 = \frac{1}{\sqrt{2}}(|R\rangle_a|R\rangle_b + |L\rangle_a|L\rangle_b) \otimes (|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle).$$  

(13)

With linear optical elements, this hyperentangled state can be transformed into a two-photon four-qubit cluster state.

As shown in figure 3, after Hadamard operations are performed on both the polarization (HWP) and the spatial-mode (BS) DOFs for photon a, the hyperentangled Bell state becomes

$$|\Phi_{ab}\rangle_2 = \frac{1}{\sqrt{2}}(|R\rangle_a + |L\rangle_a)(|a_1\rangle_b + (|R\rangle_b + |L\rangle_b))(\otimes (|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)).$$  

(14)

We then perform the spatial-mode controlled polarization phase-flip gate (HWP) on photon a, and the two-photon state $|\Phi_{ab}\rangle_2$ is transformed into $|\Phi_{ab}\rangle_3$

$$|\Phi_{ab}\rangle_3 = \frac{1}{\sqrt{4}}(|R\rangle_a + |L\rangle_a)(|a_1\rangle_b - (|R\rangle_b + |L\rangle_b))(\otimes (|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)).$$  

(15)

After performing Hadamard operations on both the polarization (HWP) and the spatial-mode (BS) DOFs for photon b, we finally achieve the two-photon four-qubit cluster state

$$|\Phi_{ab}\rangle_4 = \frac{1}{2}(|a_1\rangle|b_1\rangle(|R\rangle_a|R\rangle_b + |L\rangle_a|L\rangle_b)$$

$$- |a_2\rangle|b_2\rangle(|R\rangle_a|L\rangle_b - |L\rangle_a|R\rangle_b)).$$  

(16)

From the preparation process of a two-photon four-qubit cluster state, one can see that this state can be disentangled to the hyperentangled Bell state $|\Phi_{ab}\rangle_1$ with some Hadamard operations and a spatial-mode controlled polarization phase-flip gate using linear optical elements. With a hyper-CNOT gate, the 16 hyperentangled Bell states can be transformed into 16 two-photon four-qubit product states which can be distinguished simply with linear optical elements and single-photon detectors. That is, the 16 hyperentangled Bell states can be analyzed easily with our spatial-polarization hyper-CNOT gate.

4. Discussion and conclusion

A spatial-polarization hyper-CNOT gate is constructed with the interaction of circularly polarized photons and one-side QD–cavity systems. According to Pauli’s exclusion principle, this optical property is caused by the different reflection phase shifts for the left- and right-circularly polarized photons. By adjusting the frequencies as $\omega_r = \omega_c = \omega_0$, $\omega - \omega_c \approx \kappa/2$ and the cavity side leakage rate as $\kappa_s < 1.3\kappa$ [29], the relative phase shift for circularly polarized photons can achieve $\Delta\phi = -\pi/2$. Young et al [34] investigated the quantum-dot-induced phase shift experimentally in 2011, and showed that a QD-induced phase shift of 0.2 rad between an (effectively) empty cavity ($Q \approx 51\,000$, $d = 2.5$ μm) and a cavity with a resonantly coupled QD can be deduced using a single-photon level probe. The Hadamard operation and unitary rotation operation of an electron spin can be completed by single-spin rotations using nanosecond electron spin resonance microwave pulses [35]. The electron spin coherence time can be extended to μs using spin echo techniques [29] to protect the electron spin coherence with microwave pulses. The optical coherence time of an exciton is ten times longer than the cavity photon lifetime [36], with which optical dephasing only reduces the fidelity a few per cent. Hole spin dephasing is dominant in the spin dephasing of $X^-$ and this can be safely neglected with a hole spin coherence time three orders of magnitude longer than the cavity photon lifetime [37]. Heavy–light hole mixing, which causes optical selection rule imperfection, could be reduced by engineering the shape, size and type of the charged exciton [29].
very sensitive to side leakage when \( \kappa_s < \kappa \) [26]. Therefore, the fidelity \( F \) and the efficiency \( \eta \) of our spatial-polarization hyper-CNOT gate are

\[
F = \frac{[2(|r_0| + |r_h|^2)]^4}{[2(|r_0| + |r_h|^2)^2 + 1]|r_0| - |r_h|(|r_0| + |r_h|^2)]^2} \\
\times \left( |r_0|^2 + |r_h|^2 \right)^2,
\]

\[
\eta = \left[ \frac{1}{4} (|r_0|^2 + |r_h|^2)^2 \right]^4.
\]

The fidelity and the efficiency of our spatial-polarization hyper-CNOT gate are shown in figure 4. One can see that this hyper-CNOT gate works efficiently in a strong coupling regime (\( g \geq \frac{1}{2} (\kappa + \kappa_s) \)). It is challenging to achieve strong coupling in QD–cavity systems experimentally. The coupling strength can be improved from \( g \geq 0.5 (\kappa + \kappa_s) \) (with quality factor \( Q = 8800 \)) [38] to \( g \geq 2.4 (\kappa + \kappa_s) \) (\( Q \sim 40 000 \)) [39] by engineering the sample design, growth and fabrication in \( d = 1.5 \mu m \) micropillar microcavities. If the coupling strength is \( g \geq 0.5 (\kappa + \kappa_s) \), we can achieve a fidelity \( F = 94.3\% \) and an efficiency \( \eta = 48.9\% \) for \( \kappa_s/\kappa = 0 \). If the coupling strength is \( g \geq 2.4 (\kappa + \kappa_s) \), we can achieve a fidelity \( F = 100\% \) and an efficiency \( \eta = 96.3\% \) for \( \kappa_s/\kappa = 0 \), and \( F = 94.7\% \) and \( \eta = 47.3\% \) for \( \kappa_s/\kappa = 0.2 \). Both the fidelity and the efficiency attain high values with a strong coupling strength, but they are reduced by the side leakage from the cavity.

In experiment, the quality factor \( Q \) is dominated by the side leakage and cavity loss rate in a micropillar, while \( g \) is dominated by the trion oscillator strength and the cavity modal volume. Hu and Rarity [29] pointed out that the \( Q \) factor can be reduced by thinning down the top mirror, which can increase \( \kappa \) and keep \( \kappa_s \) constant. Therefore, the coupling strength can be reduced to \( g \geq 1.3 (\kappa + \kappa_s) \) in a high-\( Q \) micropillar (\( Q \sim 18 900 \)) with the side leakage rate set to \( \kappa_s/\kappa \sim 0.2 \), and the corresponding fidelity \( F = 96\% \) and efficiency is \( \eta = 42.3\% \). It is expected for high efficiency operations to observe a small \( \kappa_s/\kappa \) in a strong coupling regime.

In conclusion, the construction of a deterministic spatial-polarization hyper-CNOT gate can be achieved with the giant optical Faraday rotation induced by a single-electron spin in a quantum dot inside a one-side optical microcavity as a result of cavity quantum electrodynamics. In order to obtain giant optical Faraday rotation, the frequencies of the input light and the cavity mode should be adjusted to \( \omega_o - \omega_c \approx \kappa/2 \), and the side leakage and the cavity loss rate \( \kappa_s/\kappa \) should be controlled to be as small as possible. With this hyper-CNOT gate, the preparation of two-photon four-qubit cluster states is easy, in principle, and a complete analysis of hyperentangled Bell states for two-photon systems is simple. We have analyzed the experimental feasibility of this hyper-CNOT gate, concluding that it can be implemented with current technology. This hyper-CNOT gate could give a powerful capability for quantum computing and quantum communication.

We have only discussed the construction of the spatial-polarization hyper-CNOT gate by exploiting the nonlinear optical properties of one-side quantum dot–cavity systems.

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