We suggest that an interplay between microscopic and macroscopic physics can give rise to dark matter (DM) whose interactions with the visible sector fundamentally undulate in time, independent of celestial dynamics. A concrete example is provided by fermionic DM with an electric dipole moment (EDM) sourced by an oscillating axion-like field, resulting in undulations in the scattering rate. The discovery potential of light DM searches can be enhanced by additionally searching for undulating scattering rates, especially in detection regions where background rates are large and difficult to estimate, such as for DM masses in the vicinity of 1 MeV where DM-electron scattering dominantly populates the single electron bin. An undulating signal could also reveal precious dark sector information after discovery. In this regard we emphasise that, if the recent XENON1T excess of events is due to light DM scattering exothermically off electrons, future analyses of the time-dependence of events could offer clues as to the microscopic origins of the putative signal.

I. INTRODUCTION

The evidence for dark matter is unequivocal, yet our ignorance of the dark sector remains as vast as the Universe it shapes. The experimental effort to detect it is proportionately extensive, pushing back frontiers on mass scales ranging from cosmologically light bosonic fields to astronomically massive objects. Direct detection experiments aimed at observing the scattering of galactic dark matter particles on matter within the laboratory have formed a significant component of this global effort. This strategy has for decades focussed on searching for weakly interacting massive particles (WIMPs) scattering off nuclei. However, in the last decade, due to a growing synergy between advancing experimental techniques and an evolving theoretical landscape, a new frontier below the GeV scale is opening, with a burgeoning number of experiments and analysis techniques planned and proposed. As a smoking gun for galactic dark matter scattering and also as a powerful background mitigation strategy, the modulation of dark matter scattering rates on diurnal and annual timescales has been a useful tool in direct detection strategies. Both timescales follow from the time-dependent angle of attack of the detector into the prevailing galactic dark matter wind. Even though the precise form of these modulations depends on the details of the local dark matter velocity distribution and on the dark matter scattering process (see e.g. [1] for the case of dark matter-electron scattering, which is most relevant to this paper), the existence of some modulating component on these timescales is certain. It is then natural to ask whether we should expect solar system or dark matter dynamics to be the sole origin of a modulation? In particular, might there be a fundamental microscopic mechanism that modulates the dark matter scattering rate? To that end, we here propose a model of dark matter interactions where the coupling between the visible and dark sectors, and therefore the dark matter scattering cross-section, undulates in time with a period which is unrelated to celestial dynamics. We refer to this class of models as Undulating Dark Matter (UDM).

We expect that looking for an undulation in the dark matter scattering rate would be an especially useful aide in searches for sub-GeV mass dark matter, for example by dark matter-electron scattering. Here, background rates in the single electron bin are typically high and challenging to model confidently, in contrast to the multi-electron bins. Even if the background rate were only poorly understood, we find that an undulating contribution to the signal can be constrained several orders of magnitude better than a constant contribution, for a reasonably wide window of frequencies. This strategy could boost discovery potential in high background search regions, such as the single-electron event bins of the upcoming SENSEI [2–4] and SuperCDMS experiments [5, 6], and the electronic recoil mode of the XENON1T experiment [7] (in which an excess of events has recently been observed).
In the particular model that we propose, which should be thought of as just one possible mechanism for UDM, the dark matter interacts with the visible sector via an undulating electric dipole moment (UEDM). Models in which DM interacts with the visible sector predominantly via an EDM operator comprise a viable and phenomenologically relevant class of scenarios which have been studied for some time \[9,17\]. Since it violates CP, the discovery of a DM EDM would carry with it not only the triumph of discovery, but also precious microscopic information about the dark sector.

A sizeable EDM would point towards a microscopic marriage between electromagnetic substructure and CP violation. The substructure of a neutral DM particle may be confined, as it is for the neutron (e.g. \[18,19\]), or it may be perturbative, simply implying some perturbative coupling of the dark matter to additional massive charged states (e.g. \[20,21\]). In the latter case these states may be within or beyond the Standard Model, but they should not be too decoupled if we want the DM EDM to be observable. Regarding CP violation, in either case we may take our cue from nature.

It is natural to expect that if we have electromagnetic substructure in tandem with order-one CP violation then we should expect order-one EMDs, in whatever the natural units are. Yet, for QCD, measurements contradict this expectation to an absurd degree, requiring the EDM to be many orders of magnitude below the naïve expectation. Unexpected fine-tuning of parameters, the Nelson–Barr mechanism \[22,24\], and the axion \[25,28\] all offer possible answers to this puzzle. If the latter were the option preferred by nature, then it is tempting to speculate that whenever an EDM is connected to an underlying microscopic details underlying this effective theory, which therefore sets the scale at which we expect to observe undulating dark matter EMDs. If the dark sector is strongly-coupled, for example, we would expect c to be order-one, while in the weak-coupling régime we would expect c to be suppressed by a loop factor. In Appendix A we comment on the ballpark magnitude of the overall CP violation.

In the absence of the pseudoscalar \(\phi\), other sources of CP-violation must be present in order for \(\chi\) to acquire an EDM. For example, if the DM were a dark baryon, the presence of a CP-violating phase in dark quark mixing (analogous to the CP-violating CKM phase in the SM) would lead to CP-violating four-Fermion operators, themselves suppressed by some factor \(\sim 1/\Lambda_{CP}^2\), where \(\Lambda_{CP} > \Lambda\). In this scenario, one would expect the effective EDM to scale like \(\Lambda/\Lambda_{CP}^2\) \[18,19\]. In our model, we emphasize that the CP-violating parameter is simply the pseudoscalar \((\phi)\), and the correct CP-suppression is automatically encoded in the powers of \(\phi/f\).

We consider extending the Standard Model (SM) by a stable neutral Dirac fermion \(\chi\) with mass \(m_\chi\), which we suppose constitutes the bulk of the dark matter, as well as a light pseudo-scalar dark axion field \(\phi\), with mass \(m_\phi\). Both are SM singlets. The dark axion could arise as the would-be Goldstone boson associated to the breaking of an anomalous global \(U(1)_A\) symmetry at some high energy scale \(f\), à la Peccei and Quinn (PQ) \[25\,26\]. It is reasonable to suppose that this pair of dark (or, perhaps more accurately, ‘faint’) particles interacts with the SM predominantly via electric and magnetic dipole moment (EDM and MDM respectively) couplings to the photon. When ordered by increasing powers of the CP-odd field \(\phi\), which we take to be the only source of CP violation in the dark sector, the leading interaction that involves both \(\chi\) and \(\phi\) comes from the couplings

\[
\mathcal{L}_{\text{DM}} = \mathcal{L}_\chi + \mathcal{L}_\phi - c \frac{e}{2\Lambda} \sin \left( \frac{\phi}{f} \right) \overline{\chi} \gamma^\mu i\gamma_5 \chi \, F_{\mu\nu} + \text{MDM} ,
\]

(1)

where \(c\) is the electron charge, \(\Lambda\) is the scale associated with the dark sector dynamics, and \(c\) is a model-dependent dimensionless parameter. We emphasize that the EDM couplings, being CP-conserving, must be even under \(\phi \rightarrow -\phi\) and so their dependence begins at order \((\phi/f)^2\), a contribution that we do not write explicitly and will henceforth neglect.\(^2\)

The coupling \(c\), as well as the relation between the scale \(\Lambda\) and the dark matter mass \(m_\chi\), depend on the microscopic details underlying this effective theory, which therefore sets the scale at which we expect to observe undulating dark matter EMDs. If the dark sector is strongly-coupled, for example, we would expect \(c\) to be order-one, while in the weak-coupling régime we would expect \(c\) to be suppressed by a loop factor. In Appendix A we comment on the ballpark magnitude of the overall CP violation.

\(^2\)In the absence of the pseudoscalar \(\phi\), other sources of CP-violation must be present in order for \(\chi\) to acquire an EDM. For example, if the DM were a dark baryon, the presence of a CP-violating phase in dark quark mixing (analogous to the CP-violating CKM phase in the SM) would lead to CP-violating four-Fermion operators, themselves suppressed by some factor \(\sim 1/\Lambda_{CP}^2\), where \(\Lambda_{CP} > \Lambda\). In this scenario, one would expect the effective EDM to scale like \(\Lambda/\Lambda_{CP}^2\) \[18,19\]. In our model, we emphasize that the CP-violating parameter is simply the pseudoscalar \((\phi)\), and the correct CP-suppression is automatically encoded in the powers of \(\phi/f\).
Wilson coefficient

\[ d_\chi \equiv \frac{e c}{\Lambda} \quad (2) \]

in three simple possibilities for the underlying dark sector microscopics:

1. Strictly QCD-like dark sector dynamics, in which the DM is a neutral baryon akin to the neutron;

2. The DM is a neutral baryon of a QCD-like theory but where the number of colours \( N_c \) is very large;

3. The DM is an elementary fermion whose EDM arises perturbatively.

The point of these UV considerations is to highlight how the expected size of the Wilson coefficient \( d_\chi \) can depend strongly on the microscopic details of the theory. When we come to study UEDMs in direct detection in §IV, we will briefly discuss implications for the type of microscopic theory we might expect to underlie the EFT in Eq. (1).

Until then, we will take \( d_\chi \) as a purely phenomenological parameter which dictates the low-energy physics. Nonetheless, even in the context of the EFT we cannot take \( d_\chi \) to be arbitrarily large for any value of \( m_\chi \) and \( m_\phi \), because the interactions in Eq. (1) allow corrections to both the fermion and dark axion mass, dependent on the microscopics. As we will soon see, the mass of the dark axion sets the frequency of the undulating EDM signal. While this frequency is \textit{a priori} a free parameter, we will be most interested in cases where it is of the order of inverse months to minutes, corresponding to \( 10^{-20} \text{ eV} \lesssim m_\phi \lesssim 10^{-15} \text{ eV} \) or so. Because of this the dark axion must be far lighter than the fermion, and the stronger bound on \( d_\chi \) comes from the expected correction to \( m_\phi \). Based on \( \hbar \) counting we estimate

\[ \delta m^2_\phi \sim \frac{1}{4} \left( \frac{1}{16\pi^2} \right)^2 \frac{d^2_m m_\chi^6}{f^2} , \quad (3) \]

motivated by the cutoff dependence of the two-loop diagrams shown in Fig. 1 in which we assume the cutoff scale in the loop diagrams is of order \( m_\chi \), and have also considered the MDM contributions. Consequently, requiring any such corrections to be below the dark axion mass, we limit

\[ d_\chi \lesssim 32\pi^2 m_\phi f \frac{m_\chi^3}{m_\phi^2} . \quad (4) \]

We emphasize that this rough ‘bound’, while imprecise up to order-one factors, is a model-independent estimate of the upper limit on the Wilson coefficient \( d_\chi \) that follows from self-consistency of the couplings in the EFT alone in the absence of any additional protection mechanism, such as low-scale supersymmetry in the dark sector. In specific microscopic descriptions, such as the three listed above (and discussed in Appendix A), \( d_\chi \) will often be required to be smaller still.

### III. UNDULATING DARK EDMS

The lagrangian terms in Eq. (1) give rise to an effective EDM \( d_{\chi}^\text{eff}(t) \) for the dark matter which, due to \( \phi \), is time-dependent, where

\[ d_{\chi}^\text{eff}(t) \equiv d_\chi \sin \left( \frac{\phi(t)}{f} \right) . \quad (5) \]

Supposing the (light) dark axion field \( \phi \) is cosmologically present, its amplitude will be well-approximated by solving the classical equations of motion. In an FLRW background cosmology, one has

\[ \dot{\phi} + 3H \dot{\phi} + m_\phi^2 \phi = 0 . \quad (6) \]

Assuming \( m_\phi^2 \phi \gg 3H \dot{\phi} \), the dark axion undergoes weakly damped oscillations. On short enough time scales (with respect to \( H^{-1}(t_0) \)), we can assume that \( \phi(t) \) oscillates harmonically with frequency \( m_\phi \).

The amplitude of these oscillations, call it \( \phi_0 \), is important to the phenomenology of our dark sector, since it sets the size of the effective dark matter EDM. If the dark axion constitutes some fraction \( r \) of the local dark matter energy density \( \rho_{DM} \approx 0.3 \text{ GeV/cm}^3 \), then

\[ \frac{\phi_0}{f} \approx \sqrt{\frac{2 \times 10^{-15} \text{ MeV}^2}{m_\phi f}} . \quad (7) \]

We will suppose that the dark fermion \( \chi \) (rather than the dark axion) constitutes most of the dark matter, so the reader can keep in mind a value of \( r \lesssim 0.1 \).

Expanding Eq. (5) to leading order in \( \phi/f \) and substituting in Eq. (7) one expects \( d_{\chi}^\text{eff}(t) \approx |d_{\chi}^\text{eff}| \cos(m_\phi t) \), with the modulation frequency equal to the mass of the dark axion, and with amplitude \( |d_{\chi}^\text{eff}| \approx d_\chi \phi_0/f \), which can be expressed as

\[ \frac{|d_{\chi}^\text{eff}|}{\mu_B} \approx \frac{e}{\Lambda} \frac{\sqrt{r}}{m_\phi f} 2 \times 10^{-15} \text{ MeV}^3 , \quad (8) \]

in units of the Bohr magneton \( \mu_B = e/2m_e \). The EFT bound (4) implies an upper limit

\[ \frac{|d_{\chi}^\text{eff}|}{\mu_B} \lesssim 2 \times 10^{-12} \left( \frac{\text{MeV}}{m_\chi} \right)^3 \sqrt{r} , \quad (9) \]

which turns out to be independent of \( f \) and \( m_\phi \), depending only on the dark matter mass \( m_\chi \). The line on which this bound is saturated is included (in blue) in Fig. 2.

\[ ^{3}\text{In fact, things are a little more subtle, because the virialization of any light scalar dark matter particle in the galactic halo leads to an effective quality factor of } 10^6 \text{ [14]} \text{ (see also e.g. [22][23] for further discussions of the coherence properties of a cosmologically-present axion-like particle), meaning that the assumption of harmonic oscillations is only appropriate on timescales shorter than } \tau \sim 10^6/m_\phi \text{. In the contexts of the direct detection experiments we consider in [15] this is a safe assumption.} \]
There are collider physics constraints which are flat in dance, at the same reheating temperature \([21]\). Finally, the mechanism \([37, 38]\) accounts for the observed DM relic abundance in the early Universe, thus affecting its expansion. This gives a more stringent bound \([21]\). We have also included a line (orange) that indicates where the freeze-in mechanism \([37, 38]\) gives 10 MeV (taking a higher reheating temperature will be 10 MeV (taking a higher reheating temperature will give a more stringent bound) \([21]\). We have also included a line (orange) that indicates where the freeze-in mechanism \([37, 38]\) accounts for the observed DM relic abundance, at the same reheating temperature \([21]\). Finally, there are collider physics constraints which are flat in \(m_\chi\) for sub-GeV dark matter masses, the strongest of which comes from the mono-photon channel at LEP \([39]\), giving a constraint \([17, 40]\) plotted in Fig. 2.

Our primary interest, however, lies in the constraints coming from direct detection experiments. As an example, in Fig. 2 we have plotted constraints (labeled SENSEI@MINOS) based on data recently released by the SENSEI collaboration \([4]\), obtained using a prototype Skipper-Charge-Coupled-Device (Skipper-CCD),\(^5\) as well as a future projection for a silicon 30 kg-year experiment (which we have taken from Ref. \([21]\)).

We now turn to the prospect of detecting a signal with an undulating component. In order to extract limits on such a component from the real direct detection data of the XENON, SENSEI, SuperCDMS, and DAMIC collaborations, one requires more detailed information concerning the data acquisition (in particular, its binning in time) than is publicly available, and so we are not in a position to venture realistic limits in this paper.

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\(^5\) At the time of writing, these constraints are currently world-leading for DM masses \(m_\chi \in [0.6, 5] \text{ MeV}\), as well as in the low mass window \(m_\chi \in [1.2, 12.8] \text{ eV}\).

\(^6\) For these calculations we assume the local dark matter density is \(\rho_{DM} = 0.3 \text{GeV/cm}^3\), the dark matter escape velocity is 600 km/s, the mean local velocity of dark matter is 230 km/s and the average Earth velocity is 240 km/s.
FIG. 2: Constraints on a constant EDM signal from: stellar cooling of red-giant stars (RG), horizontal branch stars (HB), and the Sun; cooling of the proto-neutron star of SN1987A; Big Bang nucleosynthesis (BBN), assuming a reheating temperature of 10 MeV [21]; collider constraints using the mono-photon channel (LEP) [17]; and recent direct detection constraints from the SENSEI collaboration, showing the constraint coming from each individual electron bin [4], along with the projected sensitivity of a silicon 30 kg-yr experiment [21]. Analogous constraints and projections could be added for the XENON experiments, SuperCDMS, and DAMIC. Also shown is the line defined by Eq. (9), above which our effective field theory description likely requires a degree of fine-tuning to keep the dark axion light enough, as well as a line that indicates where freeze-in provides the full DM relic abundance, again assuming a reheating temperature of 10 MeV [21].

Instead, to get an idea of the exclusion or discovery power that an undulating signal offers, we consider a pseudo-experiment in which we ‘read out’ data from a flat distribution in 365 randomly chosen bins over a period $T_{Data}$. The time period could be any timescale relevant to a detection strategy. To estimate the sensitivity to oscillations we consider the sensitivity to the distribution $R(t) \propto q_1 + q_2 \sin(2\pi \nu t)$, which we assume models the combined background plus potential signal rate one could observe in a detector. We emphasise that the constant component $q_1$ may contain both signal and background contributions, which we might not be able to separate, while the undulating component $q_2$ is assumed to be pure signal. This distribution is characterized by two parameters, the frequency $\nu = 1/T_{Mod}$ and the undulation fraction $q_1/q_2$. Then, given such a pseudo-dataset which is not oscillating, one can ask what undulation fraction $q_1/q_2$ would be excluded at a given confidence level. In Fig. 3 we plot the corresponding 10% exclusion contours one would find for $q_1/q_2$ if a flat distribution were observed, where the label on each contour indicates the total number of signal plus background events distributed over the entire readout.

When the background component is small, or known precisely, the search for an unexpected undulation could be used to further characterise any dark matter discovery which has arisen through standard searches for an excess of events above background. However, the real strength of a search for oscillations is found in experimental regions where the background is large or poorly known. Backgrounds of this sort hamper the standard search for dark matter since the sensitivity can only be as strong as the systematic uncertainties on the background estimate allow. This is the case for the SENSEI@MINOS lines in Fig. 2 where one can see that the strongest search limits are for di- and tri-electron events due to the very low background rates. As one goes to lower masses, however, one is inevitably pushed towards relying on single electron events. It is precisely for this mass range that searches for undulating signals could enhance discovery potential, by factors comparable to the enhancement factors shown in Fig. 3. As light dark matter searches progress to lower cross sections the interplay between signal and background will evolve. If at lower cross sections additional backgrounds arise, or if they become more difficult to estimate at a level commensurate with the integrated exposure, then the enhanced sensitivity to undulating signatures will become of greater utility.

Of course, if there are modulating backgrounds then they must be accounted for. They could be vetoed by omitting frequency ranges corresponding to experimental effects relating to power sources, or terrestrial effects,
FIG. 3: Exclusion power for a modulation fraction for $n$ total events distributed evenly over a time interval with 365 bins. When the frequency is greater than the inverse of the total data taking period, $T_{\text{Data}}/T_{\text{Mod}} \sim 1$, and smaller than the inverse bin width, $T_{\text{Data}}/T_{\text{Mod}} \sim 365$, there is strong additional sensitivity to any modulations present, reflected in strong expected exclusion limits, with the enhanced sensitivity improving statistically as $1/\sqrt{n}$.

with the efficacy of the veto depending on how well the background is understood. In this vein, the discovery of an unexpected modulating signal may reveal previously unknown background sources which may aid in background subtraction.

Implications for the microscopic theory

We now discuss whether the regions of parameter space probed by these direct detection contours can be realistically populated by the dark axion-induced UEDM couplings described in [11] given the consistency bound [4]. We then speculate on what this might imply about the underlying microscopic theory, anchoring our (limited) discussion in the three scenarios sketched in Appendix A, namely, that $\chi$ is (i) a strict QCD-like dark baryon, (ii) a large-$N_c$ dark baryon, or (iii) a fundamental fermion with perturbatively generated EDM couplings.

Recall that the dark axion mass $m_\phi$ is identified with the frequency of the undulating signal. For the upcoming SENSEI experiment, one would expect sensitivity to undulation frequencies of order $m_\phi \sim (1 \text{ week})^{-1}$ or slower, while SuperCDMS would likely be sensitive to faster undulation frequencies thanks to more frequent data readout, say of order $m_\phi \sim (1 \text{ min})^{-1}$. For the case of a strictly QCD-like dark sector, as shown in Appendix A, we expect the parametric dependence

$$d_\chi \approx 4\pi^2 e \frac{m_\phi^2 f^2}{m_\chi^2} ,$$

which would require severely trans-Planckian values of the symmetry breaking scale $f$ in order to generate large enough Wilson coefficients $d_\chi$ for $|d_\chi|$ to be realistically detectable by SENSEI or SuperCDMS, even with the aid of an undulating signal.

Arguably, this seems to point towards UV models for the dark sector which deviate from the strict QCD analogy. Firstly, if $\chi$ were a dark baryon for a QCD-like dark sector but with a large number of colours $N_c$, the Wilson coefficient $d_\chi$ would be expected to receive a large-$N_c$ enhancement. The SENSEI experiment could then be sensitive to UEDM signals with $f$ brought down to the Planck scale, but only for an exponentially large number of colours, and notwithstanding the problem that, if there is an MeV scale dark baryon, it would likely be accompanied by charged baryons at similar masses which would have been observed.

Much more palatable, then, is the scenario that $\chi$ is simply an elementary dark-charged fermion. Then the Wilson coefficient $d_\chi$ is no longer necessarily tied to the mass of the dark axion. We find that the interesting regions of parameter space in Fig. 2 can be probed for safely sub-Planckian values of $f$. For more theoretical details, we refer the reader to Appendix A.

Detecting the dark axion

The dark axion $\phi$, whose oscillations are responsible for the undulating EDM of the fermionic DM $\chi$, is itself cosmologically abundant, constituting a fraction $r \lesssim 0.1$ of the local DM energy density. One might therefore hope to probe the UEDM scenario that we have proposed by hunting for the dark axion.

Recall that the mass $m_\phi$ of the dark axion sets the undulation frequency $\nu = m_\phi/2\pi$ of the UEDM and thus of the DM scattering rate of electrons, discussed in the preceding subsections. Thus, if an undulation scattering rate were observed in a direct detection experiment, our model would make a striking prediction of the precise mass required of the dark axion.

As mentioned above, in principle $m_\phi$ is a free parameter of our theory. However, our greatest interest lies in masses in the ballpark range of $10^{-20}$ eV $\lesssim m_\phi \lesssim 10^{-15}$ eV or so, for which present and future DM-electron scattering experiments (including SENSEI, SuperCDMS,

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7 In the recent preliminary study of the silicon Skipper-CCDs to be used in the SENSEI instrument [11], the Skipper-CCDs were exposed for 20 hour periods for each read out.

8 While a huge `number of dark colours' might sound ludicrous, one could instead employ a holographic perspective [46] and loosely interpret this large-$N_c$ limit rather as an indication of a different description of the microscopic theory that enjoys nearly-conformal dynamics.
The marriage of these two elements is suggestive that the effective description of DM interacting via an UEDM. Of the three, the scenario in which the DM is an elementary fermion that acquires its UEDM from couplings to new physics, potentially enabling discovery when canonical time-independent analyses cannot disentangle signal from background. Pragmatically, this is arguably the best motivation to search for undulating DM.

Recently XENON1T has reported an excess in just such a region \(^8\). While this excess most likely has its origins in more mundane effects, it is interesting to speculate if it could be the harbinger of a DM discovery. For DM scattering, rather than absorption, vanilla scenarios are disfavoured due to the observed energy spectrum of the excess. However, exotic DM distribution subcomponents with high velocity components \(^{49–52}\), exothermic DM scattering \(^{53–58}\), or scattering which produces photons \(^{59–60}\), can provide possible DM scattering explanations. In these cases, if the DM coupling undulates then, irrespective of kinematic details, the scattering rate will undulate, although at present with low statistics the events are consistent with both constant and annual modulations. Nonetheless, in future a search for time-dependent signatures such as oscillations are warranted.

A lesson learned from nascent ventures into light DM direct detection is that existing approaches, and even apparatus, can often be repurposed to search for DM in previously unexplored kinematic regimes. This demonstrates the extraordinary versatility of technologies and detection strategies which have been developed over many years. Furthermore, these relatively recent developments highlight the importance of theoretical and experimental perspectives which evolve rapidly in response to advances on either side. This work demonstrates that a detection effort which also includes searches for DM undulations, in addition to annual and diurnal modulations, to directional signatures, and to standard constant isotropic scattering, could enhance the discovery and/or signal characterisation power of light DM direct detection efforts.

Acknowledgments

We are grateful to Rouven Essig and Tien-Tien Yu for help with QEdark, and to the Cambridge pheno working group, Malcolm Fairbairn, and Raffaele Tito DAgno for discussions. We are particularly grateful to Simon Knapen for numerous discussions through the later development of this work and to Simon Knapen and Tien-Tien Yu for detailed comments on this draft. This work has been supported by STFC consolidated grants ST/P000681/1 and ST/S505316/1.

Appendix A: Three microscopic candidates

In this Appendix, we give more details concerning three possible microscopic theories that could match onto the undulating DM effective field theory, commenting in
particular on the expected size of the EFT Wilson coefficient $d_\chi = ce/\Lambda$ in each scenario.

1. QCD-like dark baryon

Firstly, consider a scenario in which the dark sector is very closely analogous to QCD plus an axion, *i.e.* an SU(3) gauge theory at strong coupling. In that case, the quantity $d_\chi = ce/\Lambda$ is precisely analogous to the EDM of the neutron in QCD, in units of the effective $\theta$ angle (see *e.g.* [61] for a review). We would expect

$$\begin{align*}
d_\chi &\approx \frac{e}{4\pi^2} \frac{m_{q_{\text{dark}}}^2}{m_\chi^2}, \quad (A1)
\end{align*}$$

where $m_{q_{\text{dark}}}$ is a typical mass for the fundamental dark quarks. Note that $d_\chi$ scales linearly with $m_{q_{\text{dark}}}$, because in the limit that $m_{q_{\text{dark}}}^2$ vanishes one can rotate away the effective theta angle responsible for the dark EDM by a chiral transformation. Importantly, the dark axion mass squared is also expected to scale linearly with $m_{q_{\text{dark}}}$, assuming that the explicit breaking of the PQ-like $U(1)_A$ symmetry is dominated by the chiral anomaly contribution:

$$\begin{align*}
m_{\phi}^2 &\approx \frac{m_{q_{\text{dark}}}^2 (\bar{q}q)_{\text{d}}}{f^2}, \quad (A2)
\end{align*}$$

where $(\bar{q}q)_{\text{d}}$ is the quark condensate that spontaneously breaks chiral symmetry in the QCD-like dark sector. Supposing moreover that the Dirac fermion $\chi$ is, like the neutron of QCD, a composite state that is bound due to the strong dark dynamics, then its mass is bound to the quark condensate, $m_\chi^2 \approx 8\pi^2 (\bar{q}q)_{\text{d}}$ [62]. The upshot of these relations is that the dark matter EDM coefficient scales like

$$\begin{align*}
d_\chi &\approx \frac{e}{4\pi^2} \frac{m_{\phi}^2 f^2}{m_\chi^2}, \quad (A3)
\end{align*}$$

varying with the fifth inverse power of $m_\chi$.

Note that in this scenario one would invariably expect charged baryons. As a result, it is only a viable possibility for DM with mass at or above the electroweak scale, and not for DM masses in the MeV régime.

2. Large-$N_c$ dark baryon

There is no good reason, however, to suppose that the dynamics in the dark sector should be so strictly analogous to real-world QCD. A simple variant to consider is a QCD-like theory with gauge group $SU(N_c)$, but with a large number of colours $N_c$. In this scenario, which is under better theoretical control than QCD thanks to the $1/N_c$ expansion, the dark matter EDM is expected to receive a large-$N_c$ enhancement by a factor $\sim N_c \ln N_c$ [63] [64]. Again, in this scenario one would invariably expect charged baryons, thus it is also only a viable possibility for DM with mass at or above the electroweak scale.

3. Dark elementary fermion

As a third (and simpler) possibility, one could imagine that the dark fermion $\chi$ is not a composite state at all, but is rather an elementary SM singlet fermion. The EDM-like couplings in Eq. (1) could then be generated, for example, by integrating out heavy charged particles (such as a fermion and a scalar, following Refs. [20] [21]), with the factor of $\sin(\phi/f)$ in (1) resulting from the CP structure of the EDM interaction. Unlike in the previous two scenarios, this does not mandate the existence of any further charged states, and so the DM mass $m_\chi$ can safely reside in the MeV range of most interest to the electron-scattering detection experiments discussed in the main text.

In this case, one expects the Wilson coefficient to be [20]

$$\begin{align*}
d_\chi &\approx \frac{e g^2}{8\pi^2 M}, \quad (A4)
\end{align*}$$

if both the charged scalar and fermion have mass $M$ and couplings $g$ to the dark matter particle $\chi$. We emphasize that $d_\chi$ is essentially decoupled from the dark axion mass $m_\phi$, provided only that the EFT bound (4) is satisfied. Thus, $d_\chi$ and $m_\chi$ can, in this scenario, be treated as almost independent phenomenological parameters that map onto the magnitude and the frequency of the UEDM. There is, however, now an additional ‘naturalness bound’ coming from the expected correction to the mass of $\chi$ in the presence of the heavy charged states to which it couples. Based on the cutoff dependence of the appropriate one-loop diagrams, one expects [20] [21]

$$\begin{align*}
\delta m_\chi &\sim \frac{M^2}{2e} d_\chi \lesssim m_\chi, \quad (A5)
\end{align*}$$

resulting on an upper limit on $d_\chi$ that scales linearly with $m_\chi$. The dependence on $M$ means that the naturalness bound admits larger UEDMs for lighter $M$. Charged scalars as light as 100 GeV are still compatible with collider bounds [65] [66], in which case there is a region satisfying both naturalness bounds that will be probed by, say, a silicon 30 kg-year direct detection experiment (see Fig. 2).

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