Research article

The dynamics of surface wave propagation based on the Benjamin Bona Mahony equation

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1. Introduction

Extreme waves pose the greatest threat to ships and offshore structures when in operation [1, 2, 3, 4, 5, 6, 7, 8]. These waves can appear suddenly and unpredictably in a calm sea. Therefore, information on the appearance and presence of extreme waves is essential for activities involving marine objects. The emergence and existence of such extreme waves have been widely conveyed in various media. Nikolikina and Didenkulova [9] reported and analyzed the emergence of extreme waves in the period of 2006-2010. A wave is said to be extreme if its height exceeds 2.2 times the height of the surrounding waves [10, 11]. The event of extreme waves is always associated with the phenomena of phase singularity that begins with intense wave troughs, known as holes in the sea [5, 12]. To face the worst case, it is first necessary to conduct a laboratory test for ship models or other floating objects before their real form’s operation in the ocean. Therefore, extreme wave generation is a non-negotiable laboratory activity. Wave generation in accordance with the desired characteristics is not easy to do in the laboratory due to the physical limitations of the laboratory. During this time, trial and error attempts are always made in the laboratory, which consequently the effectiveness and efficiency of the wave generation process is still a problem.

Determining the parameters used in generating extreme waves in the laboratory requires a complete understanding of the characteristics of extreme waves, events, and propagation. To this end, various studies relating to extreme waves have been and continue to be carried out by researchers, both on the emergence of phenomena and generation processes in the laboratory. Extreme waves on deep water were observed by Waseda et al. [13] in the North West Pacific Ocean. Numerical study of rogue waves based on breather solutions under a finite water depth of nonlinear Schrödinger was proposed by Hu et al. [14]. Islas and Schober [15] analyzed the dissipation effects on the development of rogue waves by using the 1-D Dysthe equation. Kadomtsev-Petviashvili (2 + 1)-D equation for seeking rogue waves solution to nonlinear evolution equation (NEE) is proposed by Xu et al. [16]. Hu et al. [17] discussed the rogue wave and interaction phe-
nomenon to (1 + 1)-dimensional Ito equation. Internal rogue waves in stratified flows and the dynamics of wave packets were proposed by Liu et al. [18]. Wang et al. [19] investigated the dynamics of breathers and rogue waves in the higher-order nonlinear Schrodinger equation. The amplification of wave in the framework of forced nonlinear Schrodinger equation is investigated by Slunyaev et al. [20]. Multi-parameters perturbation method for dispersive wave equations is studied by Cahyno et al. [21]. The third-order approximated theory of Korteweg-de Vries equation and a quantity called Maximal Temporal Amplitude (MTA) for seeking nonlinear evolution of wave group with three frequencies is discussed in Ramli [22]. The modulational instability normal-dispersion regime for generating extreme waves is observed by Wabnitz et al. [23]. Modulational instability is considered as a way to group waves. The instability of long internal wave modulation has been studied with the extended Korteweg-de Vries equation and the extended Bona-Benjamin-Mahony equation [24]. The known effect of modulation instability can occur for long internal waves with medium amplitude [25]. Direct numerical simulations based on two-phase Navier-Stokes equations were regarded by Peric et al. [26]. Blackledge [27] obtained the explicit extreme waves can not be found by pure intuition or by simple calculations because of complicated. Onorato et al. [28] discussed rogue waves occurring from the Gaussian behavior for random waves in the ocean.

Stability analysis of extreme waves can be assessed based on several similar cases. In the modified Korteweg-de Vries-Zakharov-Kuznetsov (mKdV-ZK) equation, Zakharov-Kuznetsov-Burgers (ZKB), and the nonlinear Ginzbach-Landau equation, a traveling waveform solution is found [29, 30, 31, 32, 33, 34]. Traveling waves may experience modulational instability [35, 36, 37]. Modulation instability can occur, for example, in the case of the extended Korteweg-de Vries and the extended Benjamin-Bona-Mahony equation (see [25, 38, 39]). We obtain the non-linear coefficient of the extended high order non-linear KdV equation regarding the wave problem [40, 41, 42]. The problem of water waves also leads to the nonlinear Olver dynamic equation with a solitary wave solution from the nonlinear Olver dynamic equation [43]. In previous research, nonlinear wave behavior has been studied with two-dimensional nonlinear Kadomtsev-Petviashvili (KP) Equations and Camassa-Holm Equations [44, 45].

Activities on wave generation in the wave tank of hydrodynamic laboratory, especially on extreme wave cases, have been deterministically, analytically and numerically observed. Fernandez et al. [46] studied a self-correcting method for extreme wave generation. The volume of fluid (VOF) method for the extreme wave generation is simulated in Xi-zeng et al. [47]. Groves and Abdussamie [48] studied the generation of rogue waves at the model scale. Huijsmans et al. [49] proposed an experiment on extreme wave generation using soliton on the finite background. Ramli [50] discussed an extreme wave generation based on soliton on finite background deterministically. Li et al. [51] discussed a generation mechanism of rogue waves for the discrete nonlinear Schrodinger equation. Xia et al. [52] simulated freak waves in random sea state numerically. In the wave generation process at the wave tank in the hydrodynamic laboratory, a time signal is given to a wave generator that determines the motion of the type of wave generator that pushes the water. Several types of wave generators in the hydrodynamic laboratory are discussed in [53, 54, 55, 56]. The waves generated propagate downstream along with the wave tank.

As nonlinearity effects, the original signal will have peaking and splitting phenomena during its propagation, (see [57, 58, 59, 60, 61]). This phenomenon may lead to amplitude amplification of the waves, as reported in [62]. Due to the wave generator physical limitations, these effects are not easy to study over the long-distance and time that are relevant for the laboratory [63].

Regarding deterministic extreme wave generation in hydrodynamic laboratories, Marwan et al. [64, 65] have found a mathematics solution for predicting the wave surges position during wave propagation in the laboratory. This formula is derived by utilizing third-order side band terms on the KdV equation and the Maximal Temporal Amplitude (MTA) magnitude. The accuracy of this position has been tested by comparing it with the results of Stansberg [59], Westhuis [60] experiments, and the results of HUBRIS numerical software calculations [61]. However, the wave heights at these extreme positions calculated using this approach do not match the experimental results and the results of the HUBRIS numerical software calculation. It is thought to be caused by cutting the KdV equation solution only to the third order. To understand these discrepancies, especially the bichromatic waves, Ramli et al. [66] conducted an investigation using the sideband approach of the five-order KdV and MTA equations. The results obtained that the increase in the maximum amplitude of order $O(q/x^2)$ bichromatic waves, where $q$ and $x$ represent the amplitude and envelope of the bichromatic wave. However, despite the increase in the wave peak amplitude, it is not yet following the results of the experiment, as reported by Afriadi et al. [67].

As stated earlier, extreme wave events are always associated with the phase singularity. The existence of wave dislocation characterizes the phase singularity. The wave dislocation phenomenon is indicated by wave merging or wave splitting. Previous studies on extreme waves have utilized wave envelope equations in analyzing the phase singularity. Karjanto et al. [68] use the wave group KdV equation that evolves following the NLS equation. The phase singularity is examined through the wave profile displayed in its changes in the space and time plane. Huijsmans et al. [49] mention that the phase singularity is a phenomenon that occurs when the real value of the envelope amplitude disappears. This can be seen from the wave casing curve that touches the horizontal axis (space and/or time) at several points periodically. A similar method was applied in [68] to study the phase singularity of the wave group BBM equation. By obtaining the NLS envelope, Halfiani et al. [69] show that the wave profile occurs separation and unification of waves around extreme waves.

Investigation of the emergence of study the phase singularity phenomena can also be seen through Argand diagrams of complex-shaped wave covers. Argand diagrams are geometrically related to representing complex numbers [70]. Phasor lines are drawn on the Argand diagram, where the length of those lines is achieved by dividing the modulus of amplitude. The phasor direction describes the wave direction with an angle to the real term of the complex number being the angle of phase. Phasors and the angle of phase denote the wave phase. The phase singularity is shown by the phasor, which passes through the origin at the Argand diagram. Karjanto et al. [68] apply the displaced-phase amplitude variable to transform the wave envelope in the form of Soliton on Finite Background (SFB) into a polar form to study the phase singularity through Argand diagrams. In that study, it was found that there was a match between the emergence of the phase singularity phenomenon in the analysis of the wave profile and the Argand diagram.

This article discusses the use of the Benjamin Bona Mahony (BBM) equation to generate extreme waves. The BBM equation, first discovered by Benjamin et al. [71], is an improvement of the KdV equation in modeling surface wave propagation. The BBM equation has a different dispersion relation than the KdV equation. The dispersion relation of the BBM equation, which states the relationship between wavenumbers and frequency, illustrates that the BBM equation can work for any given wave number. The dispersion relation provides physical properties that make more sense for any values of the wavenumber and better short wave properties. Therefore, the wave criteria for extreme wave generation in the laboratory are sufficient. Moreover, it is known that the waveform of BBM meets the Nonlinear Schrodinger equation [69]. This NLS equation has different nonlinear and dispersion coefficients with the NLS envelope of the KdV waveform. The BBM wave propagates at a relatively slower speed compared to the KdV wave [72]. BBM wave groups also experience the phase singularity during its propagation. In [72], the phase singularity is explained through a sighting from the profile and contour of the wave groups. This research intends to investigate further about the phenomenon. The phase singularity phenomenon will
be analyzed through Argand diagrams of complex-shaped wave covers. The methodology of this study aims to provide equations to enable the investigation.

Furthermore, research on extreme wave generation by applying the BBM equation is not much done when compared to the KdV equation. Using Argand diagrams to reveal phenomena in the study of extreme waves is also of an infrequent conduct. Therefore, the utilization of argand diagrams which can explain the phase singularity of the BBM equation is a new approach that can be obtained.

2. Benjamin Bona Mahony equation

The water wave propagation in a pond is characterized by a complete equation consisting of Laplace’s equations with several boundary conditions. There are bound at the surface and the bottom of the pond. To observe the behavior of water waves through this equation is not an easy thing. Because of this, several researchers have made various simplifications to the complete equation. For example, Boussinesq (1872) found the Boussinesq equation for the propagation of waves of water on the surface in two opposite directions. Then in 1895, Korteweg and de Vries (1895) discovered the Korteweg de Vries (KdV) equation that models the propagation of surface water waves in one direction of propagation. Then Kadomtsev Petviashvili (1970) found the Kadomtsev Petviashvili (KP) equation to model wave propagation on surface water in various propagation directions. Then in 1972, Brooke Benjamin, Jerry L. Bona, and John Joseph Mahony introduced the surface wave equation in one direction of propagation, known as the Benjamin Bona Mahony (BBM) equation. This equation is derived from overcoming the weaknesses of the KdV equation in modeling wave propagation on surface water in one direction of propagation, especially for waves with relatively small wavenumbers. In the dimensionless variable, the BBM equation is written in the form [71]:

\[ \eta + \eta_x + \eta \eta_x - \eta_{xxt} = 0, \]

with \( \eta, x \) and \( t \) are elevation, space, and time variable, respectively. Suppose that the solution of the BBM equation (1) is expressed as:

\[ \eta(x, t) = a(x, t)e^{i\theta} + c.c. \]

where \( \theta = (kx - \omega t) \). Here, \( \omega, k, a \) and \( c.c \) are frequency, wave number, envelope wave and complex counterpart. Halfiani et al. [69] showed that the wave envelope satisfies the spatial Non-Linear Schrodinger (NLS) equation written as

\[ A_x + i\beta A_{xx} + i\gamma |A|^2 A = 0, \]

with \( A \) is the envelope amplitude equation, \( \beta = -2\alpha \omega + \omega^2 / k^3 \), \( \gamma = \alpha \omega^2 / (k^4 - 2\omega^2 \tau) \), and \( \alpha = (1 + 2\omega \omega) / (1 + k^2) \). Equation (3) was obtained by applying the multiple scale method and the transformation of fast to slow variable on the variables of space \( x \) and time \( t \) through \( \zeta = e^2 x, \tau = \epsilon(t - x/\rho), \) and \( a(x, t) = e A(\zeta, \tau) \).

The NLS equation can be used for problems involving wave signals as initial signals on the wavemaker to describe propagation in space. The independent variables \( \zeta \) and \( \tau \) express different meanings for each different problem. In the case of dispersive waves, \( \zeta \) represents the spatial variable (space), and \( \tau \) represents the time variable. The solution of the spatial NLS equation that can describe extreme wave events in a wave pool is the Soliton on Finite Background (SFB).

The Soliton on Finite Background (SFB) solution is a nonlinear interaction of a monochromatic amplitude \( r_0 \) signal, \( A_0(\xi) = r_0 e^{-i\sigma(\xi)} \), which is disturbed by a modulation wave with a small interval of wavenumbers and results in instability that increases exponentially. Such a signal is called a Benjamin Feir (BF) signal [73]. The spatial SFB solution is expressed as follows.

\[ A(\xi, \tau) = A_{SFB}(\xi, \tau)A_0(\xi). \]

for the NLS-spatial Equation (4), \( A_{SFB}(\xi, \tau) \) has the form [71]:

\[ A_{SFB} = \frac{\left(\frac{\partial^2 - 1}{\partial \tau^2}\right)\cosh(\sigma(\xi)) + \sqrt{1 - \frac{\partial^2}{\tau^2}} \cos(\sigma(\xi)) - i\sqrt{2 - \partial^2} \sinh(\sigma(\xi))}{\cosh(\sigma(\xi)) - \sqrt{1 - \frac{\partial^2}{\tau^2}} \cos(\sigma(\xi))}. \]

where \( \nu \) is modulation frequency, \( \sigma(\nu) = \nu \sqrt{2\nu^2 \bar{r}^2 - \bar{r}^2 \nu^2} \), and \( \bar{r} = r_0 \bar{\nu} \) are the degree of instability that occurs in SFB wave propagation. This SFB wave reaches a maximum amplitude at \( \left(0, \frac{2\nu \pi}{\omega} \right), n \in \mathbb{Z} \) [49]. Karjanto et al. [74] have shown that the frequency value \( \nu \) causes amplitude to be in the interval \( 0 < \bar{r} < \sqrt{2} \). The degree of instability only occurs at \( 0 < \bar{r} < \sqrt{2} \); the choice of the value of \( \bar{r} \) in this interval causes instability of the wave modulated by a pair of monochromatic waves.

In the envelope of the BBM wave group in its propagation, there is a phenomenon of modulation stability where the amplitude rises and falls symmetrically and the phenomena of the phase singularity [69]. The phase singularity are phenomena that occur when the real value of the amplitude \( |A_{SFB}| \) disappear. The phase singularity was marked by the existence of wave dislocation, which is indicated by wave merging or wave splitting [49].

Fig. 1 displays the SFB-BBM wave profile for \( \bar{r} = 1 \). In Fig. 1(a), it can be seen that amplitude occurs in wave propagation over a specified period. Also, Fig. 1(b) shows the phase singularity phenomenon, which is shown by the existence of separate or unified wave segments at the position before or after topping. The phase singularity did not occur when \( \bar{r} > \sqrt{2} \). In Fig. 2(a) and 2(b), wave dislocation occurs on the surface of the wave profile for \( \bar{r} = 0.7 \), \( \bar{r} = 1 \), and \( \bar{r} = 1.1 \). It shows that there is a critical point that becomes the threshold of the phenomena of the phase singularity. In Karjanto, et al. [68], it has been shown that the
critical point is $\tilde{v} = \sqrt{3/2}$. Fig. 2(d), 2(e), and 2(f) show the wave profile surface for several values $\tilde{v} = \sqrt{3/2}$. It appears that no wave dislocation occurred. Fig. 2 displays the SFB-BBM wave profile for $\tilde{v} = 1.3$, and the wave profile does not indicate the phase singularity. The dynamics of SFB and the phenomena of the phase singularity will be explained in more detail in the next section.

3. Variables of displaced phase-amplitude

Transforming variable of SFB in complex amplitudes forms into a variable of displaced polar form phase-amplitude is intended to examine amplitude changes in complex number planes with positioned phases that can be expressed by polar form. The phase change to position corresponds to the change in carrier wavelength of the wave group and becomes a driving force towards extreme waves [68, 75].

$$A(\xi, \tau) = A_0(\xi)F(\xi, \tau).$$

Equation (6) is a form of SFB, and $F$ solutions will be determined later, then defined phases $\phi(\xi, \tau)$ and amplitude $G(\xi, \tau)$.

$$F(\xi, \tau) = G(\xi, \tau)e^{i\phi(\xi, \tau)} - 1.$$  

By using displaced phase-amplitude variables, the exact NLS solution is derived based on a variational formula with $(\xi, \tau) = (0, 0)$ as the maximum position and time, written as follows [68].

$$G(\xi, \tau) = \frac{P(\xi)}{Q(\xi) - \cos(\nu\tau)},$$

$$P(\xi) = \tilde{v} \sqrt{\frac{\tilde{v}^2 + \sinh^2 \sigma(\xi)}{1 - \frac{\tilde{v}^2}{\tilde{v}^2}}},$$

$$Q(\xi) = \frac{\cosh \sigma(\xi)}{\sqrt{1 - \frac{\tilde{v}^2}{\tilde{v}^2}}},$$

$$\phi(\xi) = \arctan \left( \frac{\tilde{v}}{\tilde{v}^2} \tanh \sigma(\xi) \right).$$

Equations (6)–(11) were introduced in [68] and Karjanto et al. [76]. In this research, a simple form of those equations were used with BBM wave parameter as a carrier wave.

Fig. 3 present an Argand diagram with several phase angle values. It is found that the phase angle at the same position for $\tilde{v} = 0.3$ in Fig. 3(a) is higher than the other phase angles. When $\xi > 0$, the line is above the real axis and below the real axis for $\xi < 0$. When $\xi = 0$, the line is
as for $\xi \to \pm \infty$, the length of the line shrinks to a point. Some lines are crossed twice for one period in a specific position. At position $\xi = 0$, the line passes through the origin twice for all modulation frequencies used. It indicates the phenomenon of the phase singularity.

Lines that cross the point of origin can be clearly seen using the real part graph (see Fig. 4). At the position $\xi = 0$ for $\tilde{\vartheta} = 0.7$, $\tilde{\vartheta} = 0.7$, and $\tilde{\vartheta} = 0.7$, the line goes through $Re(F) = 0$ as twice (a pair of singular points) for one period. This is not appropriate given that the phase singularity phenomenon for the BBM group enveloped exists in the interval $[0, \sqrt{3}/2]$ for modulation frequency [72]. In Fig. 5, the enveloped curve is presented for three periods with several modulation frequencies, but the results show a non-periodic curve shape for all modulation frequencies (Fig. 5(a)–5(d)).

This study attempts to provide alternative simplification of Equations (6)–(11) that meet the phase singularity requirements. Using these equations, another simplification form can be obtained as:

$$A(\xi, r) = A_0(\xi) \left[ \frac{\sqrt{\xi^2 + \beta^2} \cos(\xi) - \beta \sin(\xi)}{\sqrt{\xi^2 + \beta^2} \cos(\xi) + \beta \sin(\xi)} \right], \tag{12}$$

with $\eta$ is position and $r$ is time in period $\frac{2\pi}{\nu}$ for $n = 0, \pm 1, \pm 2, \ldots$. Value of $\nu = r_0 \sqrt{2}/\beta$, $0 < \beta < \sqrt{2}$ is modulation frequencies, $\sigma = \pi \nu^2 \beta$ and $\vartheta = \nu \sqrt{2} - \beta^2$ are the rate of instability of Benjamin Feir's modulation, and $A_0(\xi)$ is initial wave. The results obtained using Equation (12) will be discussed in the next section.

4. Results and discussion

Investigation on the dynamics of the BBM wave group was carried out using the SFB equation (Equation (12)) with the dispersive coefficient $\beta$ and the nonlinear coefficient $\gamma$ of the spatial NLS-BBM equation to obtain the modulation frequency value $\vartheta$. Representations of the SFB Equation are given through Argand diagrams to investigate changes in the wave phase concerning position. Argand diagram is used to represent complex numbers geometrically. Complex numbers are designated as points on the complex axis, with horizontal axes represents the real part of the number, and vertical axes represents the imaginary part.

It could be observed that the phase angle is higher for smaller modulation frequencies in the same position. The higher the angle of phase, the steeper the resulting waves. Vice versa, the angle of phase is small, and the resulting wave has a sloping shape. For frequencies $\tilde{\vartheta} = 0.7$, $\tilde{\vartheta} = 1$, $\tilde{\vartheta} = 1.3$, and $\tilde{\vartheta} = 1.4$, when $\xi > 0$, the line is above the real axis and is below the real axis for $\xi < 0$. When $\xi = 0$, the line is exactly on the real axis. As for $\xi \to \pm \infty$, the length of the line shrinks to a point. Some lines are crossed twice for one period in a specific position. The relationship of the magnitude of the phase angle and line position to the modulation frequency shows the same results as before. However, at position $\xi = 0$, the line passes through the origin as twice only for modulation frequencies $\tilde{\vartheta} = 0.7$ (Fig. 6(a)) and $\tilde{\vartheta} = 1$ (Fig. 6(b)), which indicates a phase singularity phenomenon for both the modulation frequencies. For modulation frequencies $\tilde{\vartheta} = 1.3$ (Fig. 6(c)) and $\tilde{\vartheta} = 1.4$ (Fig. 6(d)), the line at position $\xi = 0$, does not pass through the origin.

The phenomenon of the phase singularity arises as one of the effects of modulation instability and is the driving force for extreme waves. Phase singularity are interesting to investigate because they produce a significant difference in the amplitude of the wave envelope where
Fig. 4. Real part graphs for modulation frequencies: (a) $\tilde{\nu} = 0.7$, (b) $\tilde{\nu} = 1$, (c) $\tilde{\nu} = 1.3$, (d) $\tilde{\nu} = 1.4$.

Fig. 5. The enveloped curve for modulation frequencies: (a) $\tilde{\nu} = 0.7$, (b) $\tilde{\nu} = 1$, (c) $\tilde{\nu} = 1.3$, (d) $\tilde{\nu} = 1.4$. 
Fig. 6. SFB Argand diagram for modulation frequencies: (a) \( \bar{\epsilon} = 0.7 \), (b) \( \bar{\epsilon} = 1 \), (c) \( \bar{\epsilon} = 1.3 \), (d) \( \bar{\epsilon} = 1.4 \).

Fig. 7. SFB real part graphs for modulation frequencies: (a) \( \bar{\epsilon} = 0.7 \), (b) \( \bar{\epsilon} = 1 \), (c) \( \bar{\epsilon} = 1.3 \), (d) \( \bar{\epsilon} = 1.4 \).
Fig. 8. SFB enveloped curve for modulation frequencies: (a) $\tilde{v} = 0.7$, (b) $\tilde{v} = 1$, (c) $\tilde{v} = 1.3$, (d) $\tilde{v} = 1.4$.

Fig. 9. Contour of SFB for (a) $\tilde{v} = 0.7$, (b) $\tilde{v} = 1$, (c) $\tilde{v} = 1.3$, (d) $\tilde{v} = 1.4$. 
the amplitude can increase considerably or disappear and occur after the peak. The phase singularity exists when the real value of the SFB amplitude disappears, and only a singular point occurs, the point when the amplitude is 0 so that the phase becomes undefined [68].

For $\theta = 0.7$ in Fig. 7(a) and $\theta = 1$ in (Fig. 7(b), the real part curve at position $\xi = 0$ passes $\text{Re}(F)$ = 0 as twice (pair of singular points) for one period thus causing an amplitude of 0 at that position. It is appropriate given that the phase singularity phenomenon for the BBM group envelope occurs when the modulation frequency is at interval $[0, 1/\sqrt{3}]$ [72]. The same phenomenon does not occur at $\theta = 1.3$ (Fig. 7(c)) and $\theta = 1.4$ (Fig. 7(d)). When the value of $\theta$ is beyond the interval, the curve is always positive, so there is no phase singularity.

Fig. 8 present the SFB wave envelope curve. The SFB envelope presented is for three periods with modulation frequencies $\theta = 0.7, \theta = 1, \theta = 1.3$ and $\theta = 1.4$. The whole picture shows the shape of a periodic curve for three periods, in stark contrast to the results obtained previously (Fig. 5).

Investigation of the phenomena of phase singularity from the SFB surface wave contour using displaced phase amplitude variable (Fig. 9) also shows the appropriate results (as shown in Fig. 2). Fig. 9(a) and 9(b) shows the phase singularity phenomenon, which is shown by the existence of separate and merge wave segments at the position before or after topping. In Fig. 9(c) the instability of the modulation still occurs as seen from the increase in amplitude but the phenomena of phase singularity no longer occur. While in Fig. 9(d) where the modulation frequency is outside the instability interval, there is no increase in the amplitude.

5. Conclusions

BBM wave dynamics are investigated to meet the need to generate extreme waves in controlled condition, as laboratory. The BBM wave group in its propagation is known to experience modulation instability that occurs at frequency intervals $0 < \theta < \sqrt{2}$. The instability is well known as the Benjamin-Feir Instability (BFI). Modulation instability increases wave amplitude so that it can be a driving force towards extreme waves. The instability of the modulation also results in the emergence of other phenomena, which also cause a significant increase in wave amplitude, namely the phase singularity.

The BBM wave group envelope meets the Nonlinear Schrodinger (NLS) equation. NLS can be construed as Soliton on Finite Background (SFB). SFB can explain the instability of Benjamin-Feir. Investigation of the dynamics of BBM wave and the propagation begin with the transformation of the SFB variable in complex amplitudes form into variables of displaced phase-amplitude in polar coordinate. It is found that the frequency modulation parameter affects the phase. The smaller the modulation frequency and the higher the amplitude of the phase angle of the wave. In vice versa, the higher the modulation frequency, the smaller the phase angle of the wave amplitude. It applies to the frequency interval $0 < \theta < \sqrt{2}$. The phase singularity phenomenon occurs when the modulation frequency is in the range $0 < \theta < \sqrt{3/2}$. The changes in phase concerning position also change in wavelength.

Declarations

Author contribution statement

D. Fadhiliani, V. Halfiani, S. Munzir, M. Ramli: Conceived and designed the analysis; Performed the experiments; Analyzed and interpreted the data; Wrote the paper.

M. Ikhwani, H. Quasar, S. Rizal, M. Safwan: Contributed analysis tools or data; Analyzed and interpreted the data; Wrote the paper.

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Competing interest statement

The authors declare no conflict of interest.

Additional information

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