OSCAR KLEIN AND GAUGE THEORY

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Abstract

In this talk, delivered at the Oscar Klein Centenary Symposium in Stockholm, I review the 1938 conference held in Warsaw devoted to “New Theories in Physics”. I review all of the talks presented at this meeting and discuss in detail Klein’s paper where he proposed a unified model of electromagnetism and the nuclear force that foreshadowed the later developments of non-Abelian gauge theories.

1 Introduction

Fifty-six years ago, in September 1938, there was a remarkable meeting in Warsaw devoted to “New Theories in Physics”[1]. This was the last scientific gathering which brought together many of the pioneers of quantum mechanics and the leading lights of theoretical physics before World War II brought an end to science as they knew it. It was organized by the International Union of Physics and the Polish Intellectual Cooperation Committee, an organization set up by the League of Nations to promote intellectual cooperation. The conference was held in Poland about a year before the war broke out and it was already clear that the intellectual cooperation was beginning to break down. Thus, for political reasons, there were no Germans, Italians, or Russians at this meeting. Six years ago a conference was held in Kazimierz, just outside of Warsaw, to commemorate the fiftieth anniversary of this meeting. I was asked to summarize the conference. As you know that is an awful job, and anyway I did not find the conference that interesting so I decided instead to summarize the 1938 conference, which I found quite fascinating.

The highlight of that conference, at least with the hindsight of history, was the remarkable paper by Oscar Klein in which he proposed a unified model of electromagnetism and the nuclear force based on Kaluza-Klein ideas. This paper stands out in its originality and its brilliance from the other contributions to the conference.

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and it foreshadowed the later developments of non-Abelian gauge theories that are the foundation of our present theory of particle physics. On this occasion of Klein centenary I thought appropriate to repeat my summary of the 1938 conference.

The Warsaw meeting attracted many distinguished scientists. The list of participants included N. Bohr, L. Brillouin, L. de Broglie, C. Darwin, A. Eddington, R. Fowler, G. Gamow, S. Goudsmit, Oscar Klein, H. Kramers, L. de Kronig, P. Langevin, C. Moeller, J. von Neumann, F. Perrin, L. Rosenfeld, and E. Wigner. There was a report from Heisenberg who did not attend, presumably he was not allowed to go to Poland, and from E. Milne. I noticed that the three participants who came from my home institution of Princeton (Goudsmit, Von Neumann and Wigner) were all part of the large gift of the Nazis to American science. The scientists sat around a round table, which stimulated discussions. One of the nice features of this meeting was that there were only nine talks but extensive discussions, recorded by industrious graduate students and postdocs. Thus the proceedings contain the discussions which were at times much more interesting than the talks.

I shall review the whole conference, which consisted of nine talks, discussing each of them in term and Klein’s contribution in detail. I will also try to draw some lessons from this remarkable episode. If I seem to emphasize the confusion and the errors made by many of the heroes of modern science at this meeting (including Klein), an easy task to do with the hindsight of fifty years, it is not out of disrespect for these giants of modern physics but rather for two reasons:

- To put Klein’s contribution in perspective. By discussing some of the other talks, sometimes in humorous tones, this will make Klein’s contribution look even more remarkable.

- I think it is very important in studying intellectual history not to indulge in hero worship. History is not just an account of the great triumphs and successes of the past but also of the false leads and errors and mistakes that our heroes made. Only if we learn about these can we truly appreciate their triumphs. Only by studying the false paths they sometimes followed can we begin to appreciate them as real human beings and not as gods.

I shall start by reviewing all of the other talks and then we will move on to Oscar Klein.

2 Review of the Conference

2.1 Bohr and von Neuman

After an opening address by Professor Bialobrzeski the first talk was given by Niels Bohr, followed by a contribution from John von Neuman. They discussed interpretational issues in quantum mechanics. The title of Bohr’s talk was “The Causality
Bohr presented a very clear and comprehensive review of his views of complementarity and the Copenhagen interpretation of quantum mechanics. In this paper he gave, for the first time, a precise definition of what a physical phenomenon meant, namely that

One should reserve the word phenomenon for the comprehension of the effects observed under given experimental conditions.

Bohr presented a very clear exposition of the Copenhagen interpretation of quantum mechanics but did not discuss any new theories. von Neuman, who spoke after Bohr, talked about two of his recent papers. One was a proof that there could be no hidden variable explanation of quantum mechanics. He presented the proof of this assertion. von Neuman’s hidden variable theorem was wrong, as was discovered many years later by David Bohm who constructed a consistent hidden variable theory and by John Bell who pointed out von Neuman’s mistake. The second of von Neuman’s contributions was a discussion of some work he had done with the mathematician Birkhoff in which they tried to understand quantum mechanics by changing the rules of ordinary logic, replacing the structure of the propositional calculus of logic based on Boolean algebra by one based on the properties of rays in a Hilbert’s space. This was not wrong, but Bohr objected strenuously. He remarked that

Personally, he compelled himself to keep the logical forms of daily life.

There followed a discussion, mostly of quantum logic. According to the rules of meeting only the invited speakers, were allowed to talk during the discussion periods. None of the younger members of the audience, in particular the many Poles who were present, were allowed to open their mouths. The only exception to this rule was a Monsieur Destouches from Paris who was allowed to talk, and in fact talked at great length after every talk except for Klein’s. Destouches was a protege of de Broglie and he occupied a position of some eminence in France. His contributions to physics were summarized by Abragam, who stated in his Memoirs that, as a student growing up in the French scientific environment, he felt it necessary to study Destouches’s papers.

I struggled very long to understand until I understood that there was nothing to understand.

Destouches contributed a long comment on von Neuman’s quantum logic and, since I did not struggle to understand, I will not summarize his remarks.

2.2 Louis de Broglie

The second talk was by de Broglie. Since de Broglie did not attend in person his contribution was read by E. Bauer and was supplemented by Destouches. de Broglie’s
talk was entitled “Links Between the Quantum Theory and Relativity”. He discussed the difficulties of reconciling quantum mechanics and relativity. What were these difficulties? de Broglie first remarked that Pierls and Landau have showed that one cannot define position to better than the Compton wave length of the electron. Spacetime, he said, is an idea drawn from large scale experience. And here we have a limitation on spacetime. Furthermore as he noted, the usual Hamiltonian quantization techniques treat time and space asymmetrically, which is in conflict with relativity and this bothered him. These concerns were then amplified by Monsieur Destouches, who described at length a totally bizarre relativistic particle dynamics of his invention in which each particle has its own time. He also described a theory of de Broglie which, as far as I can tell, had nothing to do with the previous issue, in which the photon was to be thought of as a composite of two spin-1/2 massless particles as if the photon was a composite of two neutrinos.

This paper provoked much criticism. Bohr clarified very clearly and concisely that: (1) the problem of localization, i.e., of measuring the position of a particle to better than its Compton wave length, disappears completely if you admit the reality of negative energy solutions as was explained, he says, by Klein; and (2) the problem of asymmetry of space and time is, as we all know today, merely a technical problem that could be dealt with; but in any case does not conflict with relativity.

2.3 Werner Heisenberg

The next talk was that of Oscar Klein, to which I shall return later. Following Klein was the contribution of Heisenberg. He too was not present, so his contribution was read by Kramers. Heisenberg discussed the limits of the applicability of the present system of theoretical physics. What were the problems as he saw them? He identified two major issues concerning theoretical physics at that time. The first was the existence of ultraviolet divergences in quantum field theory. What he meant at that time by ultraviolet divergences was the self energy of the electron—nothing more. The second problem that concerned him was the experimental observation of cosmic ray showers, in particular the occurrence of particle production in these high energy showers. Heisenberg concluded from the existence both of ultraviolet divergences and multi-particle production that there had to be a fundamental length of order the classical radius of the electron, below which the concept of length loses its significance and quantum mechanics breaks down. The classical electron radius, $e^2/mc^2$ is clearly associated with the divergent electron self-energy, but also happens to be the range of nuclear forces, so it has something to do with the second problem. Quantum mechanics itself, he said, should break down at these lengths. I have always been amazed at how willing the great inventors of quantum mechanics were to give it up all at the drop of a divergence or a new experimental discovery.

George Gamow was present as well and he gave a short presentation of an alternate explanation of cosmic ray showers that did not require giving up introducing
a fundamental length. His explanation was simply that nuclear forces were described by Fermi’s theory of beta-decay. He wrote down a formula for the cross section for the production rate of particles in Fermi’s theory which would go like the energy to the fifth power. This is wrong but we do know that the cross sections in Fermi’s theory of the weak interactions do increase with energy and he realized that. So maybe one could explain why the probability of producing many particles would increase at high energies and thus explain the multiparticle production in the cosmic ray showers. However, he noted that there is a slight problem; namely in order to account for the proton-neutron interaction as well as the showers one requires that the coupling be of order 1 instead of Fermi’s coupling, so that one is off by a factor of $10^{12}$. But he still presented the idea.

### 2.4 L. Brillouin

The next talk after was one of the most interesting of all. It was a talk by Brillouin called “The Individuality of Elementary Particles”. I suppose he was asked to talk about statistics. Instead he gave a long review of the present state of what we call today elementary particle physics. I find this talk, aside from Klein’s, to be the most interesting at the conference since it describes what people knew about particle physics in 1938 and what they regarded as the important problems.

The first thing Brillouin he showed was a table of the elementary particles as known at the time:

| Particle          | Mass at rest | Charge | Spin |
|-------------------|--------------|--------|------|
| Electron          | $m_0$        | $-e$   | 1/2  |
| Positron          | $m_0$        | $+e$   | 1/2  |
| Heavy electron    | $100m_0$     |        |      |
| Barytron, Mesotron| $200m_0$     | $\pm e$| 1    |
| Neutron           | $M_n$        | 0      | 1/2  |
| Proton            | $M_p$        | $+e$   | 1/2  |
| Photon            | 0            | 0      | 1 (or 0?) |
| Neutrino          | 0            | 0      | 1/2  |

The table consisted of the electron, as well as the positron which had already been discovered, the proton and the neutron, the photon, the neutrino (which Brillouin identified with the anti-neutrino since it had no charge), heavy electrons and mesotrons.

This list contains some strange entries. First there was much confusion as to the nature of the particle that had been recently observed in cosmic rays. Everyone assumed that it was the particle that Yukawa had proposed as mediating the nuclear force. However, as we now know there were two new particles in the cosmic ray events,
the pion as well as the muon. This confusion is evident in the list and Brillouin refers sometimes to a heavy electron and sometimes to as a mesotron. The mesotron, Yukawa’s particle, he writes has spin one. Why? Yukawa originally supposed, he says, that the spin was equal to zero but later “calculations determined the spin to be 1”. He does not explain what those calculations were. Presumably they were the fact that Proca had suggested a wave equation for a spin-1 particle. It was very unclear at that time which equation one should use to describe a given particle.

The most fascinating thing in this list is the treatment of the photon. Brillouin says that

*The photon represents a daring abstraction, for it does not possess charge or mass when at rest.*

Thus in 1938, 33 years after Einstein’s proposal of the photon, it was still a daring abstraction. As to its spin, it was formally supposed to be nil, he says. But if it obeys a linear wave equation, then the spin should be one. And again, it was unclear to him, although not to Kramers, who gave a very nice retort in the discussion period, how you describe the wave equation of the photon. So this was the list of elementary particles.

What were the outstanding problems of particle physics? The first problem was the stability of the electron. Why is the electron stable? One theory that might deal with this problem, according to Brillouin, was the nonlinear theory of Born and Infeld in which the Maxwell Lagrangian is replaced by \( L = b^2 \sqrt{1 + (B^2 - E^2)/b^2} \), which reduces to Maxwell’s Lagrangian when \( b \to 0 \). How this theory solves the stability of electron was not explained.

The second problem was which wave equation to use for each particle. One had available the Dirac equation, the Klein-Gordon equation (which he called the Gordon-Maxwell equation), Proca’s equation and so on.

Then there is a long discussion of super quantization, which nowadays we call second quantization. This discussion is a marvelous illustration of how confused people were about the new quantum field theory. It was unclear to Brillouin whether second quantization was something which went beyond quantum mechanics of the usual type or not or whether it was necessary. He states quite clearly that second quantization is only necessary for particles obeying Bose statistics, with spin zero or spin one. Particles obeying Fermi statistics with half integer spin, that obey linear wave equations, do not require second quantization but can be treated by the hole theory of Dirac. Clearly there was no understanding that these two approaches were equivalent.

Brillouin also discussed the nuclear force. This discussion, as well as Gamow’s earlier remarks, illustrates that at the time there was absolutely no understanding that there were two forces under discussion, that there was any difference between the interactions that gave rise to beta decay and the forces that held the nucleus together and gave rise to neutron proton scattering. There was an enormous amount of confu-
sion as to whether one should describe the nuclear force using the Fermi interaction or Yukawa’s idea of a meson induced force. Finally there was a long discussion of de Broglie’s idea that the photon should be thought of as a neutrino and by neutrino pair.

There was a lot of discussion after this contribution. Some of the confusion was, or should have been, dispelled by Kramers. One must say that Kramers was the most intelligent participant in the discussion sessions.

2.5 Arthur Eddington

Following Brillouin that there was a talk by Eddington. This is one of the most remarkable episodes in the whole meeting. Eddington was a famous English astronomer who had made Einstein famous by observing the predicted deflection of light by the sun. He was a great astrophysicist and a great popularizer of science. As I child I remember reading his books. There were very well written, wonderful popular science. But at some point he over reached himself and thought he had a theory of everything including a precise determination of the fine structure constant (his theory gave \( \alpha = 1/136 \) —good to 1%), the radius of the universe, the ratio of all masses, etc. Eddington’s talk was entitled “The Cosmological Applications of the Theory of Quanta.” He took it for granted that everyone accepted the fact that he could calculate the value of \( \alpha \). He presented his calculations of the number of particles in the universe and the radius of the universe and so on. Thus the number of particle in the universe is \( 3.14510^{79} \approx 2 \times 136 \times 2^{256} \) and the radius of the universe is \( 1.23410^{27} \) cm. The theory is totally incomprehensible.

After Eddington’s talk there was a very long discussion session. Eddington was the Carl Sagan of his time, a very popular figure with the media. He published his theories of everything, but not in scientific journals. He never appeared at scientific meetings and all of the scientists resented him for his publicity seeking and lack of critical scientific attitude. This was the first time he had ever talked about these theories to a scientific audience. Many in the audience were waiting to ambush him. Everyone jumped on him, including Kramers, von Neumann, Rosenfeld, Wigner, Gamow, Fowler and Bohr. Everyone said, very politely, that the way he approaches all parts of physics, including quantum mechanics and relativity, is in contradiction with the ordinary theory of quantum mechanics and relativity.

Kramers was elected by the younger members, especially Gamow, to deal with Eddington and he gave the longest discussion in which he criticized Eddington’s views. When I was at the anniversary meeting in Kazimierz the organizers showed me an illustration, that came from their private files, in the form of a medal that Gamow presented to Kramers after he had performed this service to the community. The medal reads: “For the masterpiece of polite scolding.” For most of the the participants this talk and the following discussion was the highlight of the meeting.
2.6 A. E. Milne

Following Eddington came the contribution of the cosmologist E. Milne. Milne was not present so his contribution was read by Darwin—a very respectable physicist. Milne gave a talk on “A Possible Mode of Approach to Nuclear Dynamics”, which was even crazier than Eddington’s. He introduced some sort of absolute time based on Mach’s principle, gave up conservation of energy and momentum and then he deduced Coulomb’s law and the Bohr orbits and so on. When Darwin finished presenting the talk of his colleague, he stated that “having read Professor Milne’s paper, he wished to say that he did not agree with the conclusions of the paper or certain of the assumptions in it.” Since Milne was absent, there was no discussion of the paper.

2.7 Paul Langevin

The last talk, by P. Langevin, was entitled “On the Positivistic and Realistic Trends in the Philosophy of Physics.” It was philosophy and not physics—positivism versus realism. It is hard for me to read this kind of stuff. As far as I can tell, realism won.

Finally the meeting ended with a comment of the chairman, C. Bialobrzeski. After thanking the participants he remarked about the great contributions of modern science as indicated by the energy theory of Wilhelm Ostwald who showed that energy was the primordial substance. He stated that:

*The chief advantage of the energy theory is that this doctrine bridges the gulf that separates physical and psychic phenomena.*

This gives you some idea of the background to Oscar Klein’s contribution to the meeting.

3 Oscar Klein’s Theory

Oscar Klein gave the fourth talk entitled “On the Theory of Charged Fields.” He started by explaining the motivation for the theory. The primary motivation was Yukawa’s meson hypothesis, made in 1935 and recently confirmed by experiment. This proposal of Yukawa and its rapid experimental confirmation had an enormous impact on theoretical physics, certainly on Klein. Yukawa proposed that the force between protons and neutrons was mediated by a meson in the same way that the electromagnetic force is mediated by the photon, except that Yukawa’s meson was very massive. If the meson mass was of order 100 MEV then one could explain the short range nature of the nuclear force. The evidence in cosmic rays for a particle that might fit this role came very shortly after the proposal was made.

Klein stated that Yukawa’s idea and its confirmation implied *a considerable enlargement of the field concept*. What did he mean? The paradigm of a quantum field theory at that time was quantum electrodynamics. The developers of quantum
electrodynamics, including Klein, knew that the theory had severe ultraviolet divergences. The divergences that they focused on were the self energy divergences. They believed that these divergences meant that the theory must be altered at distances smaller than the Compton wave length of the electron \( \frac{h}{mc} \approx 10^{-11} \) cm. (Remember Heisenberg’s paper.) Mesotron dynamics, according to Yukawa, involved a particle that is about 100 times heavier than the electron. Therefore, Klein notes, mesotron dynamics can work down to a much smaller distance of order the Compton wave length of the pion. Thus if we incorporate the mesotron field we might extend the framework of quantum field theory by two orders of magnitude farther, from \( 10^{-11} \) cm., in the case of QED by itself, to \( 10^{-13} \) cm. in the case of the nuclear force. Furthermore, Klein noted that if we combine electromagnetism with the nuclear force we might somehow be able to understand the self energy problem; in fact we might be able to understand the rest mass of the electron. After all the mass of the electron might just be Coulombic in origin, if the characteristic distance scale is set by the heavy meson mass. Since the ratio of the electron mass, \( M_e \) to the meson mass \( M_m \) is of order \( \alpha \), the Coulomb potential at a distance of order the heavy meson Compton wave length is of order the rest mass of the electron, \( \alpha M_m \approx M_e \). That is what Klein meant by “a reasonable enlargement of the field concept.”

What was Klein’s goal? His goal was very ambitious. It was nothing less than a theory of everything, but in a much more realistic sense than Eddington. He wanted to construct a field theory that described all the matter that that was known to exist, namely the neutron, the proton, the electron and the neutrino, interacting with the fields he thought are necessary to give all the forces that were known—electromagnetism and the nuclear force. Thus he wanted a theory of

\[
\text{Matter} : (n \ p) + (\nu \ e), \text{ Interacting with } (\text{Electromagnetic Field } \gamma) (\text{Mesotron Field } M^\pm) (3.1)
\]

Like everyone else he did not distinguish between the weak and the strong interactions, both were to be described by the mesotron field of Yukawa. Thus his goal was a complete and unified theory of electromagnetism plus the nuclear forces, and since the theory was based on a gravitational context, gravity as well. This was perhaps the first respectable attempt to construct a theory of everything.

How did Klein go about constructing this theory? The method he followed, not surprisingly given the history of Klein’s involvement with the unified theory of electromagnetism and gravity, was to use what he called the five dimensional representation. This of course was the Kaluza-Klein theory which explained electromagnetism in terms of a five dimensional theory of gravity, where the fifth dimension was compactified on a small circle. The main advantage of this approach, according to Klein, was that it automatically preserved energy-momentum conservation, charge conservation, and gauge invariance. Since he wanted to construct a new theory he decided to use this formalism which automatically preserved the symmetries. But the five dimensional theory was already constructed, so what was new? The new ingredient—which
explains the title of his talk—was that he wanted to describe charged gauge mesons and therefore he included in the theory, for the first time, $x^5$-dependent fields. ($x^5$ denotes the fifth dimension, a little circle of radius the Planck length $\approx 10^{-33}$ cm.) A field which has a non-trivial dependence on $x^5$ will carry quantized five-momentum. The fifth component of the momentum is quantized in units of the inverse radius and couples to to the long range five dimensional gravitational field. At low energies this appears like a charged particle coupled to the electromagnetic field. Thus by making the fields $x^5$-dependent one can describe charged particles. Klein needed to describe charged particles, both matter fields, such as the proton and the electron, and force fields, such as the mesotrons. In particular the five dimensional metric tensor field, which to a low energy observer looks like a four dimensional metric tensor field plus a vector meson field and a scalar meson field, will now have $x^5$-dependence. He ignores the charged graviton and the dilatons but identifies the charged gauge bosons with Yukawa’s mesotrons. The Dirac spinors that he introduces to describe the matter will also contain $x^5$-dependent pieces that will be used to describe describe the proton and the electron. Note that Klein was not trying to increase the symmetry of the world. There is no discussion of a new $SU(2)$ symmetry or of enlarging the notion of gauge invariance.

Let me reiterate. Klein’s goal was a theory of everything—a five dimensional theory of gravity plus electromagnetism plus the nuclear force—all the forces known at the time interacting with all of the matter known at the time. The matter he puts into two families, the the proton and neutron multiplet and the electron and neutrino multiplet. This is the first time that families are introduced. Klein notes the fact these multiplets are repetitious, much like the quark-lepton families of the standard model and he adjusts the mass (as we do today) to account for their mass differences. The parameters of his unified theory consist of the electric charge and the mass of the proton and neutron. The mass of the electron and the neutrino he takes to be zero. He imagines that electron mass will come emerge dynamically. He also adds a mass term for the mesotrons. Such a mass term violates the gauge invariance of the nonabelian Yang-Mills theory of these gauge bosons. But he was not trying to construct an $SU(2)$ gauge invariant theory and in fact he did not. It clearly bothered him to have to introduce a mesotron mass term by hand and he states that

It is not impossible that a further development of the theory will make this somewhat arbitrary addition superfluous, the mass appearing as some sort of self energy determined by the other lengths entering in the theory.

That of course is the way it works in the real world as we understand it now—the masses of the $W$ and the $Z$ mesons are not introduced by hand but generated dynamically. But Klein was not aware that these explicit mass terms violated gauge invariance. To the contrary, he states

As to the rest mass of the new particle, which does not appear in the ordinary field equations, it might be introduced by the addition of a term
in the Lagrangian without disturbing the invariance.

The reason was that he was not thinking of $SU(2)$ gauge invariance at all.

I shall now describe the theory that Klein constructed. He starts with the matter sector much as Yang and Mills did in deriving Yang-Mills theory. Yang and Mills started with an isotopic spin doublet and tried to render that theory of isotopic spin doublets gauge invariant, inventing the Yang-Mills gauge bosons to do so. Klein starts with the neutron and the proton which he puts together in an isodoublet, following Heisenberg, although he never refers to isotopic spin symmetry. He puts the neutrino and the electron in a second isodoublet, which he remarks is just a repetition of the first. Both isodoublets are described as five-dimensional Dirac spinors. The proton and the electron acquire their charge through the $x^5$-dependence of the fields. Since $x^5$ is canonically conjugate to the fifth component of the momentum which is identified with electric charge, the $x^5$ derivative of $\Psi$ vanishes for the neutron and yields $e$ for the proton, and the same for the electron-neutrino multiplet. Thus the matter fields are:

$$\Psi_1 = \begin{pmatrix} \psi_n \\ \psi_p \end{pmatrix}, \quad \Psi_2 = \begin{pmatrix} \psi_n \\ \psi_e \end{pmatrix}; \quad \frac{\partial}{\partial x^5} \Psi_1 = \frac{i e}{\hbar c} \begin{pmatrix} 0 \\ \psi_p \end{pmatrix}, \quad \frac{\partial}{\partial x^5} \Psi_2 = -\frac{i e}{\hbar c} \begin{pmatrix} 0 \\ \psi_e \end{pmatrix}, \quad (3.2)$$

and Klein took the Lagrangian to be the relativistically covariant Dirac Lagrangian in five dimensions. (Five dimensional spinors were introduced previously by Schrodinger.) He adds a mass term by hand and arranges the mass of the proton and the neutron to be identical and for the mass of the electron and neutrino to be zero.

$$L = \bar{\Psi} i \gamma^\mu \partial^\mu \Psi + \bar{\Psi} M \Psi = \bar{\Psi} (i \gamma^\mu \partial^\mu + \sqrt{\kappa} \chi^\mu \gamma^\mu \frac{\partial}{\partial x^5}) \Psi + \bar{\Psi} M \Psi + \text{gravitational pieces} \quad (3.3)$$

Following the usual Kaluza Klein philosophy, when the five dimensional Dirac Lagrangian is written in four dimensions one gets, in addition to the usual Dirac Lagrangian, a term that looks like the minimal coupling of a gauge field $\chi^\mu$ to the electric current. In this case, since Klein starts with isodoublets, the gauge field is as a $2 \times 2$ matrix with diagonal components $A^\mu$, proportional to the neutral components of $g_{\mu 5}$ and off diagonal components $B^\mu$, a complex field with an $x^5$ dependence on corresponding to the $\pm 1$ charged components of $g_{\mu 5}$.

$$\chi^\mu = \begin{pmatrix} A^\mu & B^\mu \\ B^\mu & A^\mu \end{pmatrix}; \quad \frac{\partial \chi^\mu}{\partial x^5} = \frac{i e}{\hbar c} \begin{pmatrix} 0 & -B^\mu \\ B^\mu & 0 \end{pmatrix}. \quad (3.4)$$

The $A^\mu$ fields are identified with the electromagnetic field and the $B^\mu$ fields with the mesotrons that mediate the charge exchange forces between members of the isodoublet, the neutron and proton and the neutrino and the electron. In making this step Klein discards the $\bar{\Psi} \gamma^5 \partial^\mu \Psi$ term in the kinetic energy that would give generate a large mass term (of order the Planck mass) for the charged fermion fields. He remarks that this is consistent with the symmetries. Actually it is not consistent with the five
dimensional general covariance. But he did not care about that. He was only trying
to preserve the four dimensional symmetries. So he throws away the mass term of
the charged fermions coming from their kinetic energy in the fifth dimension in order
to keep them degenerate with their neutral partners.

Thus Klein has a wave equation for matter coupled to charged gauge fields
plus neutral gauge fields. Thus, in addition to electromagnetism mediated by the
photon, he gets other forces that identifies as the nuclear forces consisting of charge
exchange between protons and neutrons and charge exchange between protons and
neutrons and between electrons and neutrinos accounting for both the strong and the
weak nuclear forces.

Next Klein turns to the field equations for the gauge bosons? The method
was well known to Klein. He simply took the Einstein action in five dimensio ns,
$$\kappa \int d^5 x \sqrt{g} R_5,$$
and reduced it to four dimensional form. As in the standard Kaluza-
Klein theory the Lagrangian reduces to the four-dimensional Einstein Lagrangian
plus the square of the gauge field strength. The gauge field is given by the usual
commutator of the covariant derivatives that that appeared in the Dirac equation
$$\nabla_\mu = \partial_\mu - \sqrt{\kappa} \chi_\mu \partial_5.$$ This lead him to the Lagrangian for the gauge bosons, neutral
and charged

$$L_{\text{gauge}} = -\frac{1}{4} \left( A_{\mu\nu} A^{\mu\nu} + B_{\mu\nu} B^{\mu\nu} \right); \quad B_{\mu\nu} = \left( \partial_\mu - \frac{ie}{\hbar c} A_\mu \right) B_\nu - \left( \partial_\nu - \frac{ie}{\hbar c} A_\nu \right) B_\mu$$

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \frac{ie}{\hbar c} \left( B_\mu B_\nu - B_\nu B_\mu \right),$$

from which he derived the field equation for the gauge bosons. The action for the
charged mesons $B_\mu$ involves the minimal coupling of these charged bosons to elec-
tromagnetism. A new feature, which he points out, was the typical Yang-Mills term
$$\left( B_\mu B_\nu - B_\nu B_\mu \right)$$
in the electromagnetic field strength coming from the contribution of the charged vector bosons to the electric current. This looks a lot like Yang-Mills
theory. But actually it is not.
At this point Klein adds a mass term, $M^2 \bar{B}_\mu B^\mu$ as well, in order that the $B$ mesons can be identified with the massive mesotrons of Yukawa. The mass term he adds is consistent, he says, with gauge invariance. It is of course not consistent with non-Abelian gauge invariance but he was not thinking about non-Abelian gauge invariance. The mass term is consistent with electromagnetic gauge invariance.

Klein was not trying to construct an $SU(2)$ gauge theory. He was just trying to construct the $U(1)$ gauge theory of charged mesotrons, so the mass term was allowed. In fact Klein almost did construct an $SU(2)$ gauge theory; but he not quite. The reason had to do with hypercharge. If we write the $2 \times 2$ vector meson matrix in more conventional form, so that the coupling to the nucleon iso-doublet is $\bar{\Psi} \gamma_\mu W^\mu \Psi$, we see that

$$W_\mu = B_\mu^1 \sigma_1 + B_\mu^2 \sigma_2 + A_\mu \frac{1 - \sigma_3}{2} = \begin{pmatrix} 0 & B_\mu \\ B_\mu & A_\mu \end{pmatrix}, \quad (3.6)$$

These generators: $\sigma_1, \sigma_2$ and $\frac{1 - \sigma_3}{2}$ are not the generators of $SU(2)$. Correspondingly the action he writes down for the gauge bosons is not the $SU(2)$ Yang-Mills action, there is a factor of 2 wrong. This of course is well known to us today. To construct a gauge theory with one iso-doublet requires a gauge group of $SU(2) \times U(1)$, as in the standard model of Glashow, Weinberg and Salam. If you want a gauge theory with only one neutral gauge boson, the photon field, than one must put the matter into triplets of $SU(2)$, as in the Georgi–Glashow model. Klein followed neither of these approaches, since he was not trying to construct a non-Abelian gauge theory. So he almost invented $SU(2)$ gauge theory but not exactly. However, he was very close.

Klein ends his discussion by making a few remarks about the quantization of his theory. He notes that one can quantize this theory in the same way as electromagnetism. There is the usual problem of a singular Lagrangian but he says that Rosenfeld has solved that problem for for QED and one can do the same for his theory. Actually, we know that it is much more complicated to quantize such theories, but he did not know that.

Unlike all the other talks, the discussion following Klein’s talk was very short. There was only one remark by Moeller. Moeller noted that there was recent experimental evidence for a neutral component of the nuclear force. The exchange of a neutral heavy Yukawa meson does not seem to appear in your theory, Mr Klein, so what are you going to do about that? Klein answered that the cure is simple enough; he will just add to the $2 \times 2$ gauge field a new diagonal component,

$$\chi_\mu = \begin{pmatrix} A_\mu \\ B_\mu \\ A_\mu \end{pmatrix} \to \begin{pmatrix} A_\mu - C_\mu \\ B_\mu \\ A_\mu + C_\mu \end{pmatrix}. \quad (3.7)$$

The extra neutral gauge field $C_\mu$, he says should have no $x^5$ dependence and can be given any mass you want. The exchange of this new vector boson might explain the neutral nuclear force but, as he honestly remarks, being a vector particle it will be repulsive and not attractive. I regard this on-the-spot answer as quite remarkable. is it is sort of a generalization of $SU(2)$ to $SU(2) \times U(1)$, which, as you all know, was the step made 30 years later which gave rise to the modern electroweak theory.
There was no further discussion. Clearly the talk was over everyone’s head and might have been regarded, even in comparison to Eddington’s theory, as totally outlandish. From our point of view, over fifty years later, it seems remarkable how reasonable were the assumptions that he made and it seems amazing how close he came to the truth.

4 Conclusions

Why did Klein’s theory have no impact on the development of physics? There are many possible reasons. First, is that it is clear that Klein did not completely understand what he had done, a common phenomenon among pioneers who often make great leaps of imagination but do not appreciate the revolutionary aspects of their creations (a good case is Planck and the quantum theory.) Klein’s goal was to construct a theory of all the forces based on a $U(1)$ gauge theory of iso-spinors. He almost constructed an $SU(2)$ gauge theory, but not exactly. I do not think he really understood that he even came close to it; that was not his concern. Second, Klein never published a paper on this theory. As we have learned from Professor Pais he wrote to Bohr for his advice on publication. There is no evidence of a response from Bohr and for some reason Klein did not go ahead and publish. So his new ideas were buried in the rather obscure proceedings of the Warsaw conference. Finally, the second world war broke out and Klein was isolated from the community of physicists who were was off doing other things. By the time the war ended and people got back to doing this kind of physics he had probably forgotten what he had done.

Could it have been different? Looked at from afar Klein’s attempt at a unified theory of the forces of nature in 1938 looks very similar to the successful theory of elementary particles that was completed in 1973, a non-Abelian gauge theory of the electro-weak and strong interactions, based on the gauge group $SU(2) \times U(1) \times SU(3)$. Could the route from Klein’s outline of a gauge theory of nuclear forces to the standard model been more direct? Is it possible that if Klein had published his paper or gone on the lecture circuit, people would have found these ideas fascinating and started to really understand gauge theories and developed the standard model earlier? Probably not. It seems inconceivable that one could have arrived at the standard model without going through the long succession of experiments of the 1950’s and 1960’s, accompanied by the many attempts at theoretical model building. The actual path to the standard model was indirect and based on trial and error. The experiments were crucial to this development. They revealed the small deviations from Dirac’s relativistic atom that stimulated the development of quantum field theory and the understanding of renormalization; the existence of a whole series of hadronic resonances that suggested the composite nature of hadrons; the elucidation of the symmetries, good and bad, of the weak interactions and the V-A nature of weak currents; the discovery of Yang-Mills theory; the approximate $SU(3) \times SU(3)$ symmetry of the strong interactions which; led to the hypothesis of quarks and color;
the understanding of chiral symmetry and spontaneous symmetry breaking and the Higgs mechanism that led to the electro-weak theory; the discovery of scaling in deep-inelastic scattering which led to the discovery of asymptotic freedom and the proposal of QCD. As should have been clear from my summary of the rest of the conference, the knowledge of particle physics in 1938 was incredibly primitive and the knowledge of quantum field theory was equally primitive. The experiments that were being carried out in 1938 were at energies of a few MeV at best. It was simply premature to attempt to develop a theory of the nuclear force when the characteristic scale of the strong interactions is a 100 MEV to 1GeV and the characteristic scale of the weak interactions is a 100 GeV. One required detailed experimental exploration at energies well above the characteristic mass scale of the the relevant interactions before things became clear.

What is the lesson of all of this for us now? Today we have a theory of all the forces of nature that we observe, just as Klein wanted, that agrees with all experiments up to energies of a TeV or so with impressive accuracy. Many theorists are trying, as Klein did to extrapolate to territory unexplored by experiment. Do we have any more chance of succeeding than did Klein?

Theorists are pretty good at extrapolating from what they already know to guess where new physics might arise, where problems will appear, where new thresholds will show up; even if they are bad at guessing the new physics at these thresholds. Thus, after the Fermi theory of the weak interactions one could easily guess that there had to be new physics at 100 GeV, even though one had no good idea as to what the new physics would be. As far as we can tell, if we use the standard model to extrapolate the known forces, we find that new physics—fundamentally new thresholds—will only appear at extraordinarily high energies, 17 orders of magnitude removed from present day experiment.

Of course there are likely to be many new experimental discoveries in between the TeV region and the Planck energy. All unified theories, certainly string theory, predict that there will be much new stuff in this region. But the truly new phenomenon that might indicate a fundamental modification of the laws of physics might not be seen until the unification or Planck scale.

Can we succeed in making this extrapolation. One can easily give arguments both pro and con. The arguments against success are easy—history teaches us that without direct experimental clues and tests theorists tend to go wrong. Klein’s example is a good case of how one can be so close to the truth, yet so far from true understanding. However there are some differences between the situation today and that in 1938. One is that we have, unlike Klein, an extremely solid spring board. Klein did not have a theory that explained everything that was observed at his time from which he was trying to extrapolate to higher energy. It is not easy to extend such a theory without contradiction, so consistency is a guide. Also, the extrapolation, when measured not in terms of energies but in terms of inverse couplings (theoretically the correct way to measure energies) is not such a big extrapolation. On an
inverse coupling scale, going from 1 MeV physics to energies of order the $W$ mass is the same as going from the $W$ mass to the Planck scale. This is a big extrapolation but not unprecedented. And finally we have the incredible luck of knowing a bit of Planck scale physics—namely gravity. We are therefore presented with the obvious challenge to understand that part of Planck scale mass physics, together with trying to unify the electro-weak and strong interactions.

In any case, in my opinion, we have no choice but to try. We must emulate Klein and be daring.

References

[1] New Theories in Physics. International Institute of Intellectual Cooperation, Paris (1939)