LHCb $P_c^+$ Resonances as Molecular States.

Abstract.
Recent experiments at the LHCb detector have once again raised the possibility that quark bags may exist containing more than three quarks. Specifically the LHCb collaboration points to evidence for hadronic resonances decaying into $J/\psi$ and proton: the $P_c(4450)^+$ and $P_c(4380)^+$. Here we put forth a case that a reasonable description of these states is possible as molecular resonances of a $p$ and a $J/\psi$. Our model seems to accommodate the observed states and their measured widths, both the lower lying, broader, negative parity state and the higher lying, narrow positive parity state. If these resonances do indeed exist one might envision a rich spectroscopy of such pentaquark states waiting to be discovered, though this would deepen the mystery of their absence in earlier hadron spectroscopy. Perhaps the presence of the heavy charmed quarks, as well as the lighter u,d and s quarks, is the determining factor.

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1. Introduction

We proposed previously [1], a description of the experimental observations [2, 3] that were made about a decade ago of a possible $\Theta^+$ pentaquark, as a molecule formed of $K^+$ and $n$ in a high relative angular momentum resonant state, as the most likely description for the narrow experimentally observed structure pointing to the existence of the state. However, we warned that verification of the data pointing to the existence of the $\Theta^+$ was essential and indeed the evidence did not hold up under further scrutiny [4]. The present experiment at LHCb [5] suggesting the observation of 5-quark structure appears to be of much higher quality and hence we now re-examine the possibility of describing the current observations in the same manner, modeling the $P_{c^+}$ as a molecular resonance having atomic components that are $p$ and $J/\psi$.

The recent data taken at LHCb [5] demonstrating the, albeit rare, production of a five quark resonance and its decay into proton and $J/\psi$ motivates a theoretical search for a possible molecular or shape resonance. The large mass of one of the decay products favours the construction of such a picture using a non-relativistic potential model [1]. This approach is quite successful, and easily can produce two states in reasonable agreement with the observed excitation energies, widths and parities suggested by the experimenters. In our description, the states possess orbital angular momentum $l = 3, 4$ with the higher angular momentum state being considerably narrower and higher in excitation energy. This of course would also accounts for the change in parity between the two observed states. The narrowness of the higher state is assured mostly by the higher centrifugal barrier that exists for the larger angular momentum state.

The experimental data, Figs(2a,2b), in the LHCb submission [5], provide strong evidence for a narrow feature and apparently equal statistical justification for a lower mass but considerably broader resonance. The two states seem inextricably linked in the molecular framework.

In the dynamical approach of Hasenfratz and Kuti [6], referred to in our earlier discussion of pentaquarks as molecular states [1], the surface of each component particle is deformable and susceptible to surface oscillations [7] which can be expanded in spherical harmonics, each labeled by an orbital angular momentum $l$.

The Hasenfratz-Kuti model employs a surface potential [7] with strength proportional to

$$V_S(l) = -(l - 1)(l + 2)\rho_0^2\sigma,$$  \(1\)

where $\rho_0$ is an average bag radius and $\sigma$ the surface tension. Such a coupling might arise from a comparable dependence in the overall effective $J/\psi$-$p$ interaction via particle-vibrational coupling. One outcome of our search is an $l$-dependence for the $l = 3$ and $l = 4$ potential depths whose ratio, $V_S(3)/V_S(4) \sim 1/2$, is close to that anticipated in Eq(1), given equal surface tensions and radii.

We explored a variety of molecular potentials, both volume and surface forms, employing the code GAMOW [8] to find the resonant states. For the volume potential
we tried a Woods-Saxon and an explicit surface forms. The strong centripetal potential for these relatively high orbital angular momentum states, however, renders both volume and surface forms effectively surface-peaked; so the choice of attractive potential is not actually that important.

We also explored the possibility that an $l = 2$ resonance exists and found a possible feature in the range of excitation energies $E \sim 100 - 200$ MeV with variable width, perhaps be less easily observable. Even ignoring the putative explicit $l$ dependence of the potential above, $l = 0$ and $l = 1$ resonances are absent a consequence of the lower centrifugal barriers.

Purely bag-like or soliton-like pentaquark models face a common theoretical problem. Their lowest lying states will have low relative angular momentum between the constituent quarks and so will be connected to low angular momentum $l = 0, 1$ outgoing waves \textsuperscript{9} thus acquiring very large widths.

2. Results and Comments

Since the various choices of potential give essentially equivalent results we present results only for a particular form of surface potential. These are displayed in Figs(1,2). The relative parities are experimentally opposite in sign and are reproduced in our modeling by the differing orbital angular momentum states $l = 3$ and $l = 4$ with correspondingly assigned total angular momenta $J = 3/2$ and $J = 5/2$, and appropriate $L - S$ coupling. For simplicity, spin dependent forces were omitted in our treatment.

To create a surface potential permitting differing interior and exterior diffusivities we choose a surface potential in the form:

\[
V_S(l) = V_s \ x^{\alpha-1}(1 - x)^{\beta-1},
\]  

(2) 

a simple finite range potential formed from a product of two power laws, with exponents $\alpha - 1$, and $\beta - 1$. The radial scale of the potential is given by the parameter $r_0$, with $x = r/r_0$. For a small inner diffusivity and an even steeper exterior rise we take $\alpha = 9$, $\beta = 5$ and for the scale parameter $r_0 = 0.85$ fm.

The depth of the well is adjusted in the code GAMOW to obtain resonances at the known and observed excitation energies of the resonant states. The potential well depths and the widths of the states are then predictions of the model. Fixing the $l = 4$ state excitation energy at $E = 410$ MeV corresponding to an experimental total mass $M = 4450$ MeV we find, Fig(1), a width $\Gamma \simeq 50$ MeV and a maximum depth $V_S \simeq 2360$ MeV. Correspondingly we obtain a width of $\Gamma \simeq 142$ MeV and depth $V_S = 1410$ MeV for the appreciably broader $l = 3$ resonance at $E = 340$ MeV excitation energy, as illustrated in Fig(2). The ratio of these well depths is then close to that which follows from Eq(1). The wave functions of the resonant states, both the real and imaginary parts thereof, are also presented in these figures.

The wave functions for the two orbital states allow the atomic molecular components, $J/\psi$ and $p$ respectively some 0.3 and 0.8 fm in radius, to exist comfortably
as independent particles in the molecular state. One can nevertheless expect some distortion of the two bag surfaces. The outer radius of a resonant state is of course arbitrary, since the inside wavefunctions are matched by GAMOW smoothly onto outgoing spherical waves to solve the resonant condition on the scattering matrix. Clearly the effective inner region extends to more than some 1.5 fm. Averaging the potential over this limited region of its effectiveness leads to a considerably reduced mean well depth, and the dynamics can be safely described within a non-relativistic framework. This is evident in the smooth behaviour of the wavefunctions seen in both Fig(1) and Fig(2), indicating a rather small kinetic energy is present in either the \( l = 3 \) or the \( l = 4 \) state.

The production of the quarks involved in the initial \( P_c^+ \), in Fig(1b) of the experimental publication [5], is a short-time, hard QCD process, while the formation of the final observed hadrons involves much longer times \( \tau \sim 1 \text{ fm/c} \), \textit{i.e.} it is a soft process. Of course, as indicated by the LHCb Collaboration, in Fig(1a) and Fig(1b) the formation of the \( P_c^+ \) likely takes place in two stages, first through the production of a five quark \( \Lambda_b^0 \) state followed by the signature splitting, after the generation of a \( u\bar{u} \) pair from the vacuum, into final observed hadrons \( K^- \) and the accompanying “5-quark” state \( P_c^+ \). The latter subsequently decays into the identifying \( J/\psi \) and \( p \) hadrons. Perturbative QCD may be able to describe the production of the initial quarks but calculating wave functions for the final states is clearly, at present, possible only in a phenomenological model such as that employed here and like that used successfully to describe the \( J/\psi \) itself [10].

Given the much longer formation time of the molecular state, which necessarily requires the involvement of low momentum transfer processes, the initial quark production details aside from, of course, the number and type of quarks present should have little effect on the eventual molecular state that results.

As to related 5-quark resonances that one might expect exist, given the LHCb observations, certainly states with \( b\bar{b} \) replacing \( c\bar{c} \) in the \( J/\psi \) could be searched for; or, for that matter, states obtained by replacing the \( J/\psi \) by a higher bound charmonium state, for example the \( \psi' \). As we previously indicated, it is also possible that an \( l = 2 \) resonance could be present in the existing experiment. But this would be seen at substantially lower excitation energy and therefore would be distinct from the \( l = 3 \) and \( l = 4 \) states. It remains a puzzle why more than 3-quark hadrons are not evident at lower masses in the large pre-existing literature on hadron spectroscopy; this may, as pointed out above, be related to the presence in the \( P_c^+ \) of more massive charm or, possibly, if such additional states are found, bottom quarks.
Figure 1. \( l = 4 \) Resonance: Surface Potential and Wave Function. In our scheme this positive parity state corresponds to the narrow feature seen in the data from LHCb. We display the summed centripetal and quark-surface vibrational terms. The maximum depth of the inter-hadronic potential is searched on to yield the desired excitation energy. The resultant maximum depth \( V_s \simeq 2360 \text{ MeV} \) and width \( \Gamma \simeq 50 \text{ MeV} \) are then results of this search. The predicted width \( \Gamma \simeq 50 \text{ MeV} \) for the \( l = 4 \) state is consistent with the experimental value. Also shown are the real and imaginary wave functions yielded in GAMOW \[^8\]. Their smooth variation inside a radius of say 1.5–2.0 fm confirms the validity of the non-relativistic calculation. The real wave function which effectively extends uniformly to at least 1.5 fm indicates the atomic components of the two hadrons fit comfortably inside the proposed molecule.
Figure 2. A similar plot for the 340 MeV $l = 3$ resonance, clearly the negative parity partner of the state in Fig(1). The derived width for this state $\Gamma \simeq 142$ MeV is in reasonable agreement with that of this much broader state observed in the LHCb data. Also, the maximum depth of the surface potential derived, $V_s = 1410$ MeV, is close in ratio to that obtained for the $l = 4$ state predicted by the $l$-dependence in Eq(1), i.e. $\sim 1/2$. 
Acknowledgments

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