Minimum bending moment distribution in skeletal structures

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Abstract. The research presents skeletal steel structures analysis method according to BIM-based feasibility study tool proposed by Jaakko Haapio. Also in this paper an optimal joint stiffness determination formulae that was obtained. A multi-storey frame with different joint types was analyzed. The frame was modelled with different beams, columns and joints of different finite stiffness including the optimal one (hypothetically the most cost-effective) calculated by the formula. Five models of the frame were compared basing on cost criterion (joint fabrication and overall material costs) with each other and the one with optimal joint stiffness. The method and formula tested is applicable to frames with any number of spans and storeys,

1. Introduction

One of the most prospective issue in the skeletal steel structures design is building informational model analysis for optimal finite joint stiffness determination. General design approach considers ideally rigid and frictionless pinned joints. Thus, it is necessary to implement structural optimization methods considering finite stiffness of joints. In the paper bending moment distribution in skeletal steel structures has been studied. The minimal material consumption, factory labour cost, cost of transportation and assembling were analysed in order to provide appropriate assessment of the proposed method.

The topic was studied in different researches in the field of structural mechanics [1-7, 19]. The essential theory of skeletal steel structures behaviour was studied in the researches of Nethercot and Gardner [16, 17], where special aspects of steel structures joints design have been fully investigated and described. Even in the recent times the topics of many international conferences were dedicated to the above-mentioned issue. In European and US current design codes methods of steel structures analysis and, especially, joint stiffness model calculation were described a lot. The relevance of design code development in Russia is on the extremely high level now.

The aim of research is to conduct analytical and numerical studies of steel frames and joints with different types of connections, the design approaches and methods development of joint stiffness determination and steel frames design accounting with semi-rigid joints that includes, also, the software engineering aspects for computational approaches development of structural analysis.

The algorithm of skeletal steel structures with semi-rigid joints, subjected to equally distributed load and horizontal forces has been obtained. The algorithm requires an iterative approach as changes in joint stiffness cause changes in bending moment distribution, and as a result new cross-sections are assigned to the structural members, which leads to another joint stiffness model [10-14].
The experimental studies of “moment - angle of rotation” curve for different types of joints were studied [1,3,5-6,8-18]. The correspondence between the geometrical and mechanical characteristics of different joint types and its joint stiffness model was studied.

Apart from the structural analysis, economical features of the structure should be assessed as well. In PhD thesis of Jaakko Haapio an economic analysis tool for skeletal steel structures was shown with the purpose of optimization of the design and construction processes. As a result of analysis a method was presented that allows to determine the manufacturing, transportation and installation costs of steel structures already at the tender phase while taking into account their details. The special macros of the modeling program Tekla allows to take into consideration all features of the joint fabrication process. The program is based on the calculation methods developed by Jaakko Haapio in [4], which use time-based cost functions for the whole process of steel structures manufacturing, transportation and installation. In the program, also, carbon dioxide emissions of the structure can be easily derived from the Building Information Model (BIM) of the steel structure.

Cost of the frame is estimated for each design variant in order to compare the results and find the cheapest solution. As J. Haapio’s algorithm allows to estimate both costs of a structure and CO₂ emissions caused by production, transportation and assembly, there are several parameters to optimize; however in this work only costs are considered. The algorithm contains initial precalculated data for steel profiles from the European standard, so use of the macros is limited in terms of available profiles.

2. Methods

2.1. Optimal stiffness for multi-storey steel frame

The following frame was chosen for analysis in order to provide an assessment of proposed optimal joint stiffness determination. 10-storey 3-bay frame was subjected to equally distributed vertical and horizontal loadings as it is presented at figure 1. A steel frame is under a horizontal wind load W applied to column joints and vertical distributed load on beams q. The frame has k storeys and m spans. The joint of column base is rigid.

For approximate optimal joint stiffness calculation 2 columns and one beam from the first and second storeys of multi-storey frame has been analysed in accordance with following assumptions:

1. Top of columns are restricted, except the horizontal and vertical degree of freedom.
2. The horizontal loading was applied to the columns in assumption of equal distribution between all columns in each storey of the frame. And the first storey was subjected to the whole value.

As result, the analytical model presented in figure 1 with equally distributed load q and equivalent horizontal load W was transformed as it is presented in figure 2.
The calculations of the H-shaped frame were analysed and optimal stiffness of the joint is reached when the bending moment in the middle of beam span is equal to the most loaded joint. Based on this equilibrium equation the optimal stiffness formulae (1) for the frame joints of multi-storey multi-span frame was obtained:

\[
S_{opt} = \frac{\sqrt{20 \cdot g^2 \cdot w \cdot A + 9 \cdot B^2} - 3 \cdot (B + 4 \cdot w \cdot g) \cdot 3 \cdot E \cdot O}{-g \cdot A + 6 \cdot (B - 4t)}
\]  

(1)

The coefficients A and B are determined by the following formulas:

\[
A = 5 \cdot w \cdot \frac{g^2}{z^2} + 9 \cdot w \cdot \frac{g}{z} - 90 \cdot \frac{t}{z}
\]  

(2)

\[
B = 10t + 3gw
\]  

(3)

where \( w = -q \cdot \frac{L}{W} \).
\[ g = \frac{L}{h}; \]
\[ t = 4k + 1; \]
\[ z = \frac{l}{\sigma} \]

- \( k \) - number of storeys divided by number of spans;
- \( L \) - length of span;
- \( h \) - height of storey;
- \( n \) - number of storeys;
- \( m \) - number of spans;
- \( W \) - horizontal wind load;
- \( q \) - vertical distributed load on beams;
- \( E \) – Young's of elasticity of steel;
- \( O \) – column moment of inertia;
- \( I \) – beam moment of inertia.

So the derived formulae is applicable to frames with any number of spans and storeys.

2.2. Skeletal steel structures design approach

Design process proposed by Jaakko Haapio provides economic benefits from modelling the joints at the basic design stage. At the first step the analysis of frame is conducted considering absolutely rigid joints (extended end-plate connection) and the resistance checks in accordance with the Eurocodes can be made using commercial software (Tekla Structures, Autodesk Robot, CoP). The base joints are also absolutely rigid. At the next step rotational stiffness of the joints was reduced. The extended end plate joint was changed and the profiles of frame members were redesigned for another moment distribution. At each step ultimate moment and connection components checks were carried out to see if the solution is still feasible. Change of joint type will not only affect stiffness, but also the flexural buckling lengths of columns that should be defined at each step for buckling analysis. The best solution among the analysed alternatives should be found in some iterations and the rotational stiffness of joints reduces.

So, the procedure of skeletal steel structures design can be presented in the following steps:

1. Introduce the initial members and joints into a product model with joint spring factors/moment resistances.
2. Execute the structural analysis with initial member sections and optimal joints stiffness presented in this paper.
3. Check the manufacturer’s profile and joint databases.
4. Check stresses of the members and shear stresses of the joints (moment check of joint is not needed as the moment of a joint is limited to full resistance).
5. Increase or decrease member and/or joint sizes according to the structural analysis.
6. When sizes are adequate, estimate the cost of the frame.
7. Vary the joint type with new joint parameters.
8. Repeat steps 4-7.
9. Choose the optimal alternative.

At the 6th step the cost and emissions can be calculated by the method presented in [4]. It is based on the Feature-Based Costing Method, where feature is an attribute which affects the costs of structure during the project.

3. Result and Discussion

As an example 10-storey and 3-bay steel frame with rigid joints, hinged joints and semi-rigid joints was studied. Wind load is \( W = 24 \text{ kN} \), load on the beams is equal to \( 66 \text{ kN/m} \), height of storey \( h=3.66 \text{ m} \), length of span \( L=6.1 \text{ m} \).
After the structural analysis with initial profiles and joint stiffnesses the appropriate members profiles should be chosen and joints should be analysed in accordance with ultimate moment and finite stiffness calculations.

“Moment - angle of rotation” curves for all joint types were obtained by modelling in IDEA StatiCA. Double web angle connection was chosen as hinged joint. Stiffness of this joint is equal to 700 kN*m/rad and ultimate bending moment is equal to 27.4 kN*m.

Flush end plate connection was chosen as an optimal semi-rigid joint. In the IDEA StatiCA flush end-plate connection was designed with finite stiffness that is equal to the optimal one 10800 kN*m/rad. Ultimate bending moment for this joint is equal to 210 kN*m.

Extended end-plate connection was chosen as a rigid joint with finite stiffness that is equal to 250000 kN*m/rad and ultimate bending moment is equal to 325 kN*m.

Also, two semi-rigid joints were added in the analysis with finite stiffness 4700 and 55000 kN*m/rad and ultimate bending moments 269 and 325 kN*m, consequently.

In the table 1 result of calculating steel frame is presented, where M1 – maximum bending moment in the span of beam, M2 – maximum bending moment in the most loaded joint of beam, M3 – maximum bending moment in columns.

The table below shows the cost of profiles only in the 6th column. According to the presented results, joint with the stiffness obtained using the derived formula of optimal joint stiffness is more cost-effective than other joints.

### Table 1. Multi-storey steel frame design results

| Type of joint       | $S_{mi}$ (kN/rad) | $M_1$ (kN*m) | $M_2$ (kN*m) | $M_3$ (kN*m) | P, €   |
|---------------------|-------------------|--------------|--------------|--------------|--------|
| Hinged              | 700               | 302.3        | 25.2         | 1037.8       | 27074  |
| Semi-rigid          | 4700              | 249.03       | 133.8        | 785.5        | 23930  |
| Optimal Semi-rigid  | 10800             | 197.1        | 197.4        | 532.9        | 21986  |
| Semi-rigid          | 55000             | 144.9        | 257.2        | 386.2        | 23756  |
| Rigid               | 250000            | 116.1        | 320.8        | 282.6        | 25654  |

![Graph](image-url)
4. Conclusions
For an approximate estimation of frame joint stiffness the formulae (1), applicable to frames with any number of spans and storeys, was obtained and the algorithm for optimal design of a multi-storey, multi-bay frame was presented in this paper.

The results of 10-storey and 3-bay steel frame study accounting joint stiffness show that semi-rigid joint with the stiffness obtained by the formulae (1) is the most cost-effective among the other types of joints with different stiffness characteristics, which proves the effectiveness of using proposed design method and derived formulae.

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