Tests of Low Scale Quantum Gravity in $e^-e^-$ and $\gamma\gamma$ Collisions

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Arkani-Hamed, Dimopoulos and Dvali have recently proposed that gravity may become strong at energies near 1 TeV due to the existence of large extra dimensions thus ‘removing’ the hierarchy problem. In this talk we examine the exchange of towers of Kaluza-Klein gravitons and their influence on Møller scattering as well as the production of pairs of massive gauge bosons in $\gamma\gamma$ collisions. These tower exchanges lead to a set of new dimension-8 operators that can significant alter the Standard Model expectations for these processes. In the case of $\gamma\gamma$ collisions, the role of polarization for both the initial state photons and the final state gauge bosons in improving sensitivity to graviton exchange is emphasized.

Recently, Arkani-Hamed, Dimopoulos and Dvali (ADD) have proposed an interesting solution to the hierarchy problem. ADD hypothesize the existence of $n$ additional large spatial dimensions in which gravity (and perhaps Standard Model singlet fields) can live, called ‘the bulk’, whereas all of the fields of the Standard Model (SM) are constrained to lie on ‘the wall’, which is our conventional 4-dimensional world. In such a theory the hierarchy is removed by postulating that the string or effective Planck scale in the bulk, $M_s$, is not far above the weak scale, e.g., a few TeV. Gauss’ Law then provides a link between the values of $M_s$, the conventional Planck scale $M_{pl}$, and the size of the compactified extra dimensions, $R$: $M_{pl}^2 \sim R^n M_s^{n+2}$ where the constant of proportionality depends upon the geometry of the compactified dimensions. If $M_s$ is near a TeV then $R \sim 10^{30/n-19}$ meters; for separations between two masses less than $R$ the gravitational force law becomes $\sim 1/r^{2+n}$. For $n = 1$, $R \sim 10^{11}$ meters and is thus excluded, but, for $n = 2$ one obtains $R \sim 1$ mm, which is at the edge of the sensitivity for existing experiments.

Astrophysical arguments based on supernova cooling and cosmological arguments seem to require that $M_s > 100$ TeV for $n = 2$, but allow $M_s \sim 1$ TeV for $n > 2$.

The detailed phenomenology of the ADD model has begun to be explored for a wide ranging set of processes in a growing series of recent papers where it has been shown that the ADD scenario has two basic classes of collider tests. In the first class, a K-K tower of gravitons can be emitted during a decay or scattering process.

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leading to a final state with missing energy. The rate for such processes is strongly
dependent on the number of extra dimensions as well as the exact value of $M_s$. In
the second class, which we consider here, the exchange of a K-K graviton tower
between SM fields can lead to almost $n$-independent modifications to conventional
cross sections and distributions or they can possibly lead to new interactions. The
exchange of the graviton K-K tower leads to a set of effective color and flavor
singlet contact interaction operator of dimension-eight with the overall scale set
by the cut-off in the tower summation, $\Lambda$, which naively should be of comparable
magnitude to $M_s$. Thus one introduces a universal overall order one coefficient for
these operators, $\lambda$ (whose value is unknown but can be approximated by a constant
which has conventionally been set to $\pm 1$) with $\Lambda$ being replaced by
$M_s$.

Linear colliders will provide the opportunity to make precision measurements
of a number of elementary processes making small deviations from SM expecta-
tions due to new physics easily observable. It is well known that Moller scattering
is particularly sensitive to both contact interactions as well as new neutral gauge
bosons so we may expect reasonable sensitivity to graviton exchange. The shifts
in the angular distribution due to gravity are shown in a sample case in Fig. 1;
additional information can be obtained from the left-right polarization asymmetry,
$A_{LR}$. Following now-standard procedures we can estimate the search reach for
$M_s$ for a fixed sign of $\lambda$ assuming a given integrated luminosity and beam polar-
thization via a Monte Carlo approach. In particular the search reach is obtained by
fitting to the total number of events, the shape of the angular distribution and the
angle-dependent values of $A_{LR}$. The result of this approach is also shown in Fig.1
together with the corresponding reaches obtained from Bhabha scattering and from
combining multiple final states using the process $e^+e^-\rightarrow f\bar{f}$ [where $f = \mu, \tau, b, c, t$
etc.] as obtained by Hewett. While for typical luminosities the inclusive analysis
leads to an $M_s$ reach of $\sim (6 - 7)\sqrt{s}$, the corresponding reach for Moller scattering
is $\sim (5 - 6)\sqrt{s}$.

$\gamma\gamma$ collisions may be possible at future $e^+e^-$ linear colliders by the use of
Compton backscattering of low energy laser beams. The backscattered laser photon
spectrum, $f_\gamma(x = \frac{E_\gamma}{E_e})$, is far from being monoenergetic and is cut off above
$x_{max} \simeq 0.83$ implying that the photons are significantly softer than their parent
lepton beam energy. The shape of the spectrum as well as the energy dependence
of the polarization of the resulting hard photon depends upon the polarizations
of both the laser and initial electron beam. In what follows we will label the six in-
dependent polarization possibilities by the corresponding signs of the electron and
laser polarizations as $(P_{e1}, P_{l1}, P_{e2}, P_{l2})$. For example, the configuration $(+ + -)$
corresponds to $P_{e1} = -0.9$, $P_{l1} = +1$, $P_{e2} = 0.9$ and $P_{l2} = -1$. Clearly some of
these polarization combinations will be more sensitive to the effects of K-K towers
of gravitons than others.

$\gamma\gamma$ collisions offer a unique and distinct window on the possibility of new physics
in a particularly clean environment. Unlike particle production in $e^+e^-$ collisions,
however, $P, C$, plus the Bose symmetry of the initial state photons forbids the
existence of non-zero values, at the tree level, for either forward-backward angular
asymmetries or left-right forward-backward polarization asymmetries. These were
both found to be powerful tools in probing for K-K graviton tower exchanges in the
Figure 1: (Left) Deviation from the expectations of the SM (histogram) for Moller scattering at a 500 GeV $e^+e^-$ collider for the number of events per angular bin, $N$, as a function of $z = \cos \theta$ assuming $M_s = 1.5$ TeV. The two sets of data points correspond to the choices $\lambda = \pm 1$ and an assumed integrated luminosity of 75 fb$^{-1}$. (Right) Search reaches for $M_s$ at a 500 GeV $e^+e^-/e^-e^-$ collider as a function of the integrated luminosity for Bhabha (dashed) and Moller (dotted) scattering for either sign of the parameter $\lambda$ in comparison to the ‘usual’ search employing $e^+e^- \rightarrow f\bar{f}$, inclusively.

Figure 2: Differential cross section for $\gamma\gamma \rightarrow W^+W^-$ at a 1 TeV $e^+e^-$ collider for (left) the SM and with $M_s = 2.5$ TeV with (right)$\lambda = 1$. In the left panel, from top to bottom in the center of the figure the helicities are $(+++)$, $(+--)$, $(-++)$, $(+--)$, $(+-)$, and $(--)$; in the right panel they are $(-++)$, $(+--)$, $(+--)$, $(+-)$, $(++)$, and $(+-)$. 
$e^+e^-$ initiated channels\[1\]. By analogy with the $e^+e^- \rightarrow f\bar{f}$ analysis above one can examine the corresponding $\gamma\gamma \rightarrow f\bar{f}$ process. In the case the best reach is obtained by combining the three lepton flavors, $tt$ and $jj$ final states. This has been done in the case of unpolarized beams\[6\] and yields a search reach of $(4-5)\sqrt{s}$. For any given choice of the initial state laser and electron polarizations, labelled by $(a,b)$ below, we can immediately write down the appropriate cross section by folding in the corresponding photon fluxes and integrating:

$$
\frac{d\sigma^{ab}}{dz} = \int dx_1 \int dx_2 \left[ f_\gamma^a(x_1, \xi_1) f_\gamma^b(x_2, \xi_2) \left( \frac{1 + \xi_1 \xi_2}{2} \frac{d\hat{\sigma}_{++}}{dz} + \frac{1 - \xi_1 \xi_2}{2} \frac{d\hat{\sigma}_{+-}}{dz} \right) \right].
$$

The $++$ and $+-$ labels on the subprocess cross sections indicate the appropriate values of $\lambda_{1,2}$ to chose in their evaluation.

Figure 3: (Left) Fraction of LL(solid), TL+LT(dashed) and TT(dotted) $W^+W^-$ final states after angular cuts for the process $\gamma\gamma \rightarrow W^+W^-$ at a 1 TeV $e^+e^-$ collider as a function of $M_s$ for either sign of $\lambda$. The initial state polarization is ($-++$). (Right) Differential polarization asymmetries for $\gamma\gamma \rightarrow W^+W^-$ at a 1 TeV $e^+e^-$ collider for the SM(solid) as well with graviton tower exchange with $M_s=2.5$ TeV with $\lambda = \pm 1$(the dotted and dashed curves). We label the three cases shown by the first entry in the numerator in the definition of $A_{pol}$. Red represents an initial polarization of ($+++$), green is for the choice ($+++-$) and blue is for the case ($-++$).

Perhaps the most interesting channels are $\gamma\gamma \rightarrow VV$ where $V$ is either $Z$ or $W$. While the first process only occurs at loop level in the SM (though it can now be mediated by tree level graviton exchange), the second has a very large tree level cross section. In either case many observables which may be sensitive to graviton tower exchange can be constructed in addition to angular distributions due to (i) the existence of the six distinct initial $\gamma\gamma$ polarization states allowing us to construct three different polarization asymmetries and (ii) the fact that the polarizations of the final state vector bosons can be measured through angular correlations of their decay products. The shifts in the $WW$ angular distributions are shown in Fig. 2. In the SM, the $\gamma\gamma \rightarrow W^+W^-$ reaction takes place through $t$- and $u$-channel $W$ exchanges as well as a $\gamma\gamma W^+W^-$ four-point interaction. The $t$- and $u$-channel exchanges thus lead to a sharply rising cross section in both the forward and backward directions. Note that in the SM there is no dramatic sensitivity to the initial state polarizations and all of the curves have roughly the same shape. In all
cases the total cross section, even after generous angular cuts, is quite enormous, of order \( \sim 100 \) pb, providing huge statistics to look for new physics influences. When the graviton terms are turned on there are several effects. First, all of differential cross section distributions become somewhat more shallow, but there is little change in the forward and backward directions due to the dominance of the SM poles. Second, there is now a clear and distinct sensitivity to the initial state polarization selections. In some cases, particularly for the \((-++-\)) and \((+-++-\)) helicity choices, the differential cross section increases significantly for angles near 90° taking on an m-like shape. This shape is, in fact, symptomatic of the spin-2 nature of the K-K graviton tower exchange since a spin-0 exchange leads only to a flattened distribution. In addition to a significant modification to the angular distribution, the K-K graviton tower exchange influences on the polarizations of the two W’s in the final state. In the SM, independent of the initial electron and laser polarizations, the final state W’s are dominantly transversely polarized. Due to the nature of the spin-2 graviton exchange, the K-K tower leads to a final state where both W’s are completely longitudinally polarized. To see this, we show in Fig. 3 the polarization fractions of the two W’s as a function of \( M_s \) at a 1 TeV collider. Here we see that the fraction of final states where both W’s are longitudinal, denoted by \( LL \), starts out near unity but falls significantly in the \( M_s = 2.5 - 3 \) TeV region giving essentially the SM results above \( M_s \approx 5.5-6 \) TeV. The reverse situation is observed for the case where both W’s are transversely polarized, denoted by \( TT \).

The three polarization asymmetries are also shown in Fig. 3. Note that as \( z \to \pm 1 \) the SM dominates due to the large magnitude of the \( u \)- and \( t \)-channel poles. Away from the poles the three asymmetries all show a significant sensitivity to the K-K tower of graviton exchange. Although these asymmetries are not very big the large statistics of the data samples obtainable for this channel indicate that they will be very well determined since many systematic errors will also cancel in forming the cross section ratios. It is clear from the discussion above that there are a large number of observables that can be combined into a global fit to probe very high values of \( M_s \) in comparison to the collider energy. It should be noted however that due to the large statistics available the eventually determined discovery reach for \( M_s \) using the \( \gamma\gamma \to W^+W^- \) process will strongly depend on the size and variety of the experimental systematic errors. The results of performing this fit for \( \lambda = 1 \) and for the six possible initial state polarizations are displayed in Fig. 4 which displays the reach as a function of the integrated luminosity. (The results for \( \lambda = -1 \) are almost identical.) Note both the strong sensitivity of the reach to the initial electron and laser polarizations as well as the large values obtainable particularly for the \((-++-\)) choice. In this particular case with 100 \( fb^{-1} \) of integrated luminosity the discovery reach is almost \( 11\sqrt{s} \) for either sign of \( \lambda \), which is greater than any other K-K graviton exchange process so far examined.

The process \( \gamma\gamma \to ZZ \) does not occur at the tree level in the SM or MSSM. At the one loop level in the SM the dominant contribution arises from W and fermion box diagrams and triangle graphs with s-channel Higgs boson exchange. This would seem to imply that this channel is particularly suitable for looking for new physics effects since the SM and MSSM rates will be so small due to the loop suppression. The SM cross section (which peaks in the forward and backward directions), after
a cut of $|z| < 0.8$, is found to be $\sim 80$ fb and almost purely transverse away from Higgs boson resonance peaks. In the case of the ADD scenario the tree level K-K graviton tower contribution is now also present. Neglecting the loop order SM contributions for the moment we obtain the polarization-dependent differential cross sections shown in Fig. 4. Note that since this is the pure K-K graviton tower term there is no dependence here on the sign of $\lambda$. This cross section is found to scale with $s$ and $M_s$ as $\sim s^3/M_s^8$ and in contrast to the SM case is observed to peak at $90^\circ$. A short analysis shows that essentially all of the $Z$’s in the final state are completely longitudinal with the $LL$ fraction being $\sim 99\%$ for the six possible initial state polarizations. Although a detailed study of the loop-induced SM-graviton tower exchange interference terms have not yet been performed it is difficult to see how the search reach in this channel can exceed $\sim 5$ TeV given the small magnitudes of the cross sections involved. Thus this process is not competitive with $\gamma\gamma \rightarrow WW$.

Signals for an exchange of a Kaluza-Klein tower of gravitons in the ADD scenario of low scale quantum gravity appear in many complementary channels simultaneously at various colliders. Such signatures for new physics are rather unique and will not be easily missed.

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