Anatomy of nuclear matter fundamentals

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Abstract – The bridge between finite and infinite nuclear system is analyzed for the fundamental quantities like binding energy, density, compressibility, giant monopole excitation energy and effective mass of both nuclear matter and finite nuclei systems. It is shown quantitatively that by knowing one of the fundamental property of one system one can estimate the same in its counter part, only approximately.

The empirical values of nuclear matter binding energy, compressibility and density are essential quantities for the estimation of nuclear observables. These are the center of attraction from the inception of nuclear physics. Recently, it is shown that the density of finite nucleus can be derived from the nuclear matter estimation [1]. In the present letter, we want to show that not only the density of finite nucleus is connected with its infinite counter part, but also the compressibility which is an indispensable ingredient for nuclear study is also connected with each other. It is already a settled issue that the neutron skin thickness is connected with the neutron star. Not only the neutron skin, recently a large number of observables, such as the stiffness parameter Q and the L coefficient are interconnected to one another in a very correlated way. These are not the only correlation between the fundamental nuclear properties with the properties of equation of state (EOS) and asymmetric energy coefficient, hence consequently with the neutron star. These correlations are anatomically shown by the advent with the degree of neutron-proton asymmetry in nuclei with the experimental data on pygmy dipole strength [2].

These correlations are anatomically shown by the advent of the relativistic mean field (RMF) formalism originally suggested by Johnson and Teller [3] and later on developed by many others [4–7].

The advantage of the relativistic mean field formalism [8] is its easy application to both finite and infinite nuclear matter and the term-by-term analysis of the Lagrangian connected to the physical observables of the nuclear system. Taking into account a few parameters and the masses of mesons and nucleons, one can reproduce the finite nuclei properties through out the periodic chart [10]. The inclusion of relativistic frame-work and the meson-nucleon interaction, the model predicts phenomena in a much fundamental levels. In the present paper, we would like to use the well known Lagrangian of Boguta and Bodmer [7], along with the cross coupling addition, as suggested by Todd-Rutel et al. [11], with a few parameter sets, which are well tested in most of the regions of the nuclear landscape. In this Lagrangian, the nonlinear couplings of the $\sigma$–meson is included, which generates analogous effect of the three body interaction due to it’s off-shell meson couplings, which is essential for the saturation properties [12-14]. These terms give the long range repulsion of nuclear force generated by the singlet-singlet and triplet-triplet $nn$–interaction. Therefore the two nonlinear terms ($\frac{1}{4}b\phi^3$ and $\frac{1}{4}c\phi^4$) are not only mere addition to the Lagrangian, rather it is essential to add in the Lagrangian to get a proper description of nuclear system. The other cross coupling $R^2V^2$ is important for EOS and neutron-rich matter. The nucleon-meson interacting Lagrangian for a many-body nucleonic system is [7,9,11]:

$$
\mathcal{H} = \sum_i \phi_i \left[ -i\tilde{\alpha} \cdot \nabla + \beta m^* + g_\rho V + \frac{1}{2} g_\rho R_3 \right] \phi_i + \frac{1}{2} \left[ (\nabla \phi)^2 + m_{\phi}^2 \phi^2 \right] + \frac{1}{3} b\phi^3 \\
+ \frac{1}{4} c\phi^4 - \frac{1}{2} \left[ (\nabla V)^2 + m_{\rho}^2 V^2 \right] + \Lambda_v (R^2V^2)^2 \\
- \frac{1}{2} \left[ (\nabla R)^2 + m_{\rho}^2 R^2 \right] - \frac{1}{2} \left( \nabla A \right)^2. 
$$

Here $m$, $m_\pi$, $m_\rho$ and $m_\phi$ are the masses for the nucleon (with $m^* = m - g_\rho \phi$ being the effective mass of the nu-
cleon), $\sigma$, $\omega$- and $\rho$-mesons, respectively and $\varphi$ is the Dirac spinor. The field for the $\sigma$-meson is denoted by $\phi$, for $\omega$-meson by $V$, for $\rho$-meson by $R$ ($r_3$ as the $3^{rd}$ component of the isospin) and for photon by $A$.

The quantities such as $b$ and $c$ are the non-linear coupling constants for $\sigma$ mesons, and $\lambda_{\nu}$ is the crossed coupling constant for $\rho$- and $\omega$-mesons.

The above Lagrangian is used to determine the fundamental quantities of nuclear matter (BE/A, $J$, $K$) at different density $\rho$ both in (i) quantal and (ii) semi-classical approximations. The mean field (Hartree) approach of meson field is assumed in the quantal case and in semi-classical approximation, the scalar density ($\rho_s$) and energy density ($E$) are calculated using relativistic Thomas-Fermi (RTF) and relativistic extended Thomas-Fermi (RETF) formalisms. The RETF is the $h^2$ correction to the RTF, where the gradient of density is taken care. This term of the density takes care of the variation of the density and involves more in the surface properties.

We calculate the nuclear matter compressibility $K$, effective mass $m^*$ and binding energy per particle as a function of density using the RMF models. The similar quantities are also evaluated for some specific finite nuclei in the frame-work of same Hartree approximation. Since the collective properties of nuclei, such as giant monopole, quadrupole and dipole resonances do not depend much on the internal structure of nuclei, we use the RTF and RETF techniques to calculate the values, whenever required. The recently developed scaling and constrained calculations will be used to evaluate the giant monopole resonances [13] and all other quantities will be estimated by the relativistic Hartree approximation [11][16][13].

The mean density of a given nucleus is written by the fitting formula $\rho_A = \rho_0 - \left(\frac{\rho_0}{\text{A}}\right)^\alpha$. The corresponding $K_A$, $J_A$ and BE/A for the finite nucleus are noted down in the Hartree or RETF formalisms. Again all these quantities are calculated from the equation of state and compared in Table I. The nuclear matter compressibility $K_\infty$ is not directly measured experimentally, actually, the energy $E_M$ of the GMR of finite nuclei is measured. It is convenient to write this energy in terms of the compressibility $K_A$ for a finite nucleus of mass number $A$ as

$$E_M = \left\langle \frac{h^2 K_A}{M < r^2 >} \right\rangle,$$

where $<r^2>$ is the rms matter radius and $M$ the mass of the nucleon. The finite nucleus compressibility $K_A$ usually parametrized by means of a leptodermous expansion that is similar to the liquid drop model mass formula [21]:

$$K_A = K_\infty + K_{sf}^{-1/3} + K_{vs} I^2 + K_{coul} Z^2 A^{-4/3} + ...,$$

where $I = (N - Z)/(N + Z)$ is the neutron excess. Thus, the compressibility of a finite nucleus is an admixture of its volume, surface, asymmetric and Coulomb parts. In the semiclassical calculations, all these four contributions estimated combinedly. However, we can separate these individual terms taking the help of TF and ETF formalisms with some additional working conditions. For example, a nucleus with $N=Z$ and switching off the Coulomb contribution, only contributions come from volume and surface. The surface part can be estimated by taking the difference of compressibility between the TF and ETF formalisms, as ETF gives the surface contribution on top of the TF formalism. The finite nucleus compressibility $K_A$ compared with the one obtained from the infinite nuclear matter in Table I. From the Table, it is clear that the compressibility calculated for finite nucleus at its density is almost equal to the compressibility at similar density of finite nucleus. Similarly the binding energy obtained for finite nucleus can be equated with the nuclear matter value at the particular density of finite nuclei. This is also compared in Table I. Analogous to the compressibility, the binding energy of a nucleus can also be expressed in terms of a leptodermous expansion [12]:

$$BE(A, Z) = a_v A - a_s A^{2/3} - a_{coul} \frac{Z(Z-1)}{A^{1/3}} - a_{coul} (N-Z)^2 A + ..., \quad (4)$$

where, $a_v$, $a_s$ and $a_{coul}$ have their usual meaning of volume, surface and Coulomb coefficients, respectively. When we switch off the Coulomb repulsion for a symmetric finite nucleus, like $^{40}$Ca, the binding energy comes out from the volume and surface contributions. For example, the total binding energy of $^{40}$Ca is $342.216$ MeV ($8.5554$ MeV per particle) in an extended Thomas-Fermi calculation. It is $319.388$ MeV in Thomas-Fermi level, i.e., without surface correction (7.9847 MeV per particle). The contribution comes from Coulomb repulsion due to the 20 protons is $83.714$ MeV, i.e., $4.1857$ MeV per proton. Then the binding energy per nucleon only from volume contribution is ($8.5554 + 0.5707 + 4.1857$) MeV = $13.3118$ MeV, of course the quanlal effect is neglected in the evaluation. These values are $(7.956812+0.5095155+3.277564) = 11.384$ and $(7.881395+0.5350075+3.468285) = 11.885$ MeV for $^{40}$P and $^{40}$S, respectively. We get different binding for $^{40}$P, $^{40}$S and $^{40}$Ca and do not coincide with the nuclear matter binding as shown in Table I. This means, the binding arises from the proton-neutron orientation is different than neutron-neutron configuration. That means, one may not get the binding energy of finite nucleus with the help of the leptodermous expansion of mass formula. Because, the singlet-singlet and triplet-triplet interaction is less attractive than the singlet-triplet nucleon-nucleon interaction. Hence, the binding energy, very much depends on the nucleon-nucleon configuration both in finite nuclei and infinite nuclear matter, i.e., it is not only a function of mass number $A$, but function of both proton...
Table 1: The binding energy per nucleon \( (\cos BE/A) \), compressibility modulus \( \cos K \), asymmetry coefficient \( \cos J \) obtained from nuclear matter equation of state (EOS) compared with values of finite nuclei. The results of nuclear matter are listed at the density of finite nuclei using FSUGold parameter set. \( K_A \) and \( \cos K_A \) are the compressibility of finite nucleus obtained from scaling and constrained calculations in MeV, respectively.

| Nucleus | \( \rho_A \) | \( S K_A \) | \( L K_A \) | \( \cos K_A \) | \( \cos J \) | BE/A | \( \cos\text{BE/A} \) |
|---------|-----------|----------|----------|-----------|----------|------|-----------------|
| \(^{40}\text{P}\) | 0.0780 | 123.400 | 100.36 | 102.53 | 21.88 | 8.933 | 12.997 |
| \(^{40}\text{S}\) | 0.0780 | 127.028 | 112.15 | 102.53 | 21.88 | 8.375 | 12.997 |
| \(^{40}\text{Ca}\) | 0.0780 | 130.93 | 123.15 | 102.53 | 21.88 | 10.589 | 12.997 |
| \(^{112}\text{Sn}\) | 0.0933 | 147.23 | 140.26 | 129.86 | 24.54 | 11.854 | 14.358 |
| \(^{116}\text{Sn}\) | 0.0920 | 147.11 | 139.71 | 127.64 | 24.33 | 11.723 | 14.263 |
| \(^{120}\text{Sn}\) | 0.0944 | 146.62 | 138.66 | 132.00 | 24.75 | 11.575 | 14.452 |
| \(^{124}\text{Sn}\) | 0.0950 | 145.83 | 137.14 | 134.35 | 24.95 | 11.397 | 14.452 |
| \(^{208}\text{Pb}\) | 0.0990 | 147.37 | 134.57 | 138.42 | 25.37 | 11.919 | 14.721 |

Table 2: The calculated sum rule weight-age \( \sqrt{m_1/m_{-1}} \) compared with the recently measured data using FSUGold parameter set. The results are also compared with the theoretical calculations of pairing plus MEM.

| Nucleus | \( \sqrt{m_1/m_{-1}} \) | \( \Gamma \) | pairing+MEM | RETF | Expt. | RETF | Expt. |
|---------|----------------|-------|----------|-------|------|-------|------|
| \(^{204}\text{Pb}\) | 13.4 | 13.6 | 13.7±0.1 | 2.02 | 3.3±0.2 |
| \(^{206}\text{Pb}\) | 13.4 | 13.51 | 13.6±0.1 | 2.03 | 2.8±0.2 |
| \(^{208}\text{Pb}\) | 13.4 | 13.44 | 13.5±0.1 | 2.03 | 3.3±0.2 |

and neutron separately even in the contribution of volume energy in the leptodermous expansion. Thus, the leptodermous expansion in the power of nuclear mass to obtain the binding energy is only an approximation. The failure of the mass formulæ to predict the properties of nuclei away from the \( \beta \)-stability line based on leptodermos is noticed by Mittig et al. and later on confirmed by many authors [20]. The root of disagreement between \( BE/A \) and \( \cos BE/A \) for a definite \( \rho \) may be the following:

- There is no direct relation between \( BE/A \) and \( \cos BE/A \) as their sources of origin are different.
- There is no route to go from \( BE/A \) to \( \cos BE/A \) and vice versa.

The similar situation is also aroused for compressibility \( K_A \). The \( K_A \) values for \(^{40}\text{P}\), \(^{40}\text{S}\) and \(^{40}\text{Ca}\) are listed in Table I. The difference in compressibility between ETF and TF calculations for \(^{40}\text{P}\), \(^{40}\text{S}\) and \(^{40}\text{Ca}\) are 1.987, 2.048 and 1.976 MeV, respectively. This is clear that the surface correction for these nuclei is less than 2%. The giant monopole excitation energy \( E_M = \sqrt{C_m/B_m} \) where \( C_m \) is the restoring force and \( B_m \) is the mass parameter. From this excitation energy, we have evaluated the compressibility modulus using equation (2). On the other hand the compressibility for nuclear matter \( K(\rho) \) as a function of density \( \rho \) is obtained from the formula [21],

\[
K(\rho) = 9\rho^2 \frac{\partial^2 (\mathcal{E}(\rho))}{\partial \rho^2},
\]

with \( \mathcal{E} \) is the nuclear matter energy density [22]. This may be due to the similar reasons as it is highlighted for binding energies about their different origin and unconnected relations of \( K_A \) and \( K(\rho) \). Now come to the description of giant monopole resonances and their link with the nuclear matter compressibility. Recently, hot discussions are going on to settle the issue of compressibility as well as...
its relation on giant monopole resonances (GMR). As it is stated earlier, the compressibility is not a measurable quantity, but is estimated using equation (2), where the GMR energy is an input. The monopole excitation energy is measured experimentally, which is a dynamical quantity. However, the compressibility \( K_A \) or \( K_\infty \) is a static quantity. Hence, it is always a challenge to estimate a static quantity from a dynamical source and we apprehense to get an accurate compressibility from the monopole excitation energy.

To verify this, we have plotted the sum rule weight factor \( \pi \alpha^2 m^3 \), which is the monopole excitation of the giant resonances in Figure 1 for Mo and Sn isotopes. Different force parameters having a wide range of nuclear matter compressibility from \( K_\infty = 210 \text{ MeV} \) to \( 400 \text{ MeV} \) are deployed in the calculations. For example, \( K_\infty \approx 210 \text{ MeV} \) for NL1 [23] and \( K_\infty \approx 400 \text{ MeV} \) for NL2 force [24]. The compressibility, represented in the parenthesis, lies in between these two extremes for all other [FSUGold (230 MeV), NL3 (271 MeV) [18], NL-SH (355 MeV) [25] and NL3* (258 MeV) [26]] parameter sets. We have also displayed the experimental results for comparisons. Although, the calculated results are obtained from a wide range of parameter sets having variety of \( K_\infty \), non of the parametrization could reproduce the experimental data for the whole isotopic series. The FSUGold parameter set is able to reproduce the data for Pb isotopes (See Table II) at an excellent agreement with the experimental values, as the set is designed to reproduce the GMR data for Pb and few other nuclei, however it fails to reproduce the data for Sn and Mo isotopes (see Fig. 1). For Sn series (lower part of the figure), as this set is designed to reproduce the GMR for Sn nuclei, however, fails to predict the data of Mo isotopes (upper part of the figure). That means, the FSUGold set reproduce the GMR values of Pb in an excellent agreement with data, but deviate by ~ 1 MeV for Sn series and drastically differs for Mo isotopes as shown in Fig. 1 and Table II.

In summary, in the present letter, we have shown the link of microscopic calculations with their classical counter parts including the leptodermous expansion for various physical observables. We have shown that the binding energy and compressibility modulus deduced from infinite nuclear matter can be approximated to the finite nucleus observables to some extend. There is a large discontinuity in the bridge between these two quantities and not possible to reach from one end to the other. That means, one can not get the finite nucleus binding energy and compressibility knowing the physical quantities in nuclear matter condition. Thus, the leptodermous expansion for compressibility and mass model are merely formulae and have no physical merit for prediction to unknown territory.

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