Giant Hall effect in the ballistic transport of two-dimensional electrons

Yu. O. Alekseev 1 and A. P. Dmitriev 2

1 Lyceum "Physical-Technical High School", Hlopin 8-3A, St. Petersburg, 194021, Russia
2 Ioffe Institute, Politekhnicheskaya 26, 194021, St. Petersburg, Russia

We have studied magnetotransport of a degenerate two-dimensional electron gas in a Hall sample in the Knudsen regime, when the mean free paths of electrons with respect to their collisions with each other and with impurities are much larger than the width of the sample. In contrast to the usually considered symmetric sample, whose both its edges reflect electrons diffusely, we considered an asymmetric sample, one edge of which reflects them diffusely, while the other specularly. It is shown that in such structure in low magnetic fields the Hall coefficient is parametrically large in comparison with its standard value. Also the situation is discussed when all types of scattering can be neglected except for scattering at the edges of the sample.

INTRODUCTION

In recent years, in connection with the impressive progress in the creation of two-dimensional systems with a record mobility of carriers, interest in the theoretical study of the effect of interparticle interaction on transport phenomena has sharply increased [1-11], the role of which in dirty systems with low mobility is insignificant. At the same time, research is being conducted in two directions. On the one hand, the hydrodynamic regime of electron transport is being intensively studied, which is realized, apparently, in experiments on giant temperature-dependent magnetoresistance in ultrapure semiconductor and graphene samples [12-24]. On the other hand, ballistic and intermediate between ballistic and hydrodynamic regimes are examined extensively.

This, second direction, of research interest, is first of all, of considerable theoretical interest, since the conditions for the realization of the hydrodynamic regime in a degenerate Fermi gas differ from those in the case of an ordinary, non-degenerate gas and liquid. The effects caused by external fields are often more pronounced in the ballistic regime than in the case of the local equilibrium hydrodynamic regime. Finally, in most experiments in small magnetic fields, it is precisely the ballistic transport regime that is realized, since the mean free path relative to interparticle collisions turns out to be on the order of, or even larger than the characteristic spatial scales of the flow. As the magnetic field increases, the cyclotron radius begins to play the role of the path length, and the conditions for the applicability of the hydrodynamic description are satisfied.

In papers [3] and [11], the Knudsen regime of current flow in a long narrow two-dimensional sample with diffusely scattering boundaries was considered, where the mean free path relative to interparticle collisions is much larger than the sample width. The electron gas was considered to be degenerate. The limit of arbitrarily weak electric and magnetic fields was studied, and the magnetoresistance and Hall coefficient $R_H$ were found. It turned out, in particular, that $R_H$ in this limit is half the value usual for Ohmic transport.

Bearing in mind that, in experiments, the properties of the edges of the sample may differ, in this work we studied magnetotransport in ballistic regime in an asymmetric sample, one of the edges of which is smooth, i.e. reflects electrons specularly, while the other scatters them diffusely. It is shown that in this case the Hall coefficient is anomalously large as compared to its standard value. Finally, at a semi-quantitative level, the situation is discussed when all types of scattering can be neglected except for scattering at the edges of the sample. Note that the anomalously large value of the Hall coefficient in the structure studied by us is, apparently, among the so-called ballistic anomalies in magnetotransport, discussed in the scientific literature in 1980 - 1990s (see, for example, a review [25]).

PROBLEM STATEMENT AND BASIC EQUATIONS

We will study the electrical transport of a degenerate two-dimensional electron gas in a long narrow sample with a width $W$ to which a time-independent uniform longitudinal electric field $E_0$ and a magnetic field $B$ perpendicular to the sample plane are applied (see Fig. 1). The sample will be assumed to be sufficiently clean, and the temperature sufficiently low, so that the electron-phonon scattering can be neglected, and the mean free path relative to the scattering of electrons by each other and by impurities is much larger than its width. One edge of the sample is considered smooth, reflecting electrons "specularly", while the other is rough, scattering them diffusely. We direct the axis $x$ along the field $E_0$ and align it with the smooth edge of the sample, direct the axis $y$ into the sample, and the magnetic field along the axis $z$ (see Fig. 1). Bearing in mind to calculate the linear response of the system to electric and magnetic fields, we will consider them arbitrarily weak.

We write the Boltzmann kinetic equation for the one-
particle distribution function \( f(\mathbf{r}, \mathbf{v}) \) in the form
\[
\mathbf{v} \frac{\partial f}{\partial \mathbf{r}} - e \mathbf{E} \frac{\partial f}{\partial \mathbf{p}} + \omega_c \frac{\partial f}{\partial \phi} = S_{ee}[f] + S_{imp}[f],
\]
where \( \mathbf{v} \) and \( \mathbf{p} = m \mathbf{v} \) are the speed and momentum of the electron, \( \varepsilon > 0 \) is the magnitude of its charge, \( \mathbf{E} \) is the electric field equal to the sum of longitudinal and Hall \( \mathbf{E}_H \) fields, \( \phi \) is the angle of the velocity vector measured from the ordinate axis, \( \omega_c = eB/mc \) is the cyclotron frequency, \( S_{imp}[f] = -\frac{(f - f_0)}{\tau_{imp}} \) is the integral of collisions with impurities, which we will assume to be short-range, \( f_0 \) is the symmetric part of the distribution function, and \( S_{ee}[f] \) is the integral of electron-electron collisions, as which we will use the model collision integral (see, for example, [26] and [3]), which in the simplest way takes into account the conservation of the number of particles, energy and momentum in electron-electron collisions:
\[
S_{ee}[f] = -\frac{1}{\tau_{ee}}(f - P_{01}[f]),
\]
where \( P_{01} \) is the operator of projecting the function of the angular variable \( \phi \) onto the zero and first harmonics.

Representing the distribution function in the form
\[
f = f_F + \frac{\partial f_F}{\partial \varepsilon} g,
\]
where \( f_F \) is the equilibrium Fermi function, \( \varepsilon \) is the electron energy, taking into account the smallness of the perturbation, the degeneracy of the electron gas, and the independence of the distribution function from \( x \) due to the homogeneity of the system along the ordinate axis, from (1) we obtain:
\[
\sin \phi \frac{\partial g}{\partial \phi} - eE_0 \cos \phi - eE_H \sin \phi + \frac{1}{\tau_{ee} \omega_c} \frac{\partial g}{\partial \phi} = 0.
\]
where \( \tau_{ee} = v_{ee}/\omega_c \) is the cyclotron radius, \( \gamma_{ee} = 1/\tau_{ee} \), \( \gamma_{imp} = 1/\tau_{imp} \), \( l_{ee}/\tau_{imp} = v_F \tau_{ee}/\tau_{imp} \), \( g_0 \) is the symmetric part of the function \( g \), \( g^\pm \) is the projection \( g \) onto the cosine, and \( g^\mp \) onto the sine.

The function \( g_0(y, \phi) \) is responsible for the change in the concentration of electrons at a given point, and through the functions \( g^\pm \) and \( g^\mp \) the densities of the longitudinal and transverse currents are expressed, respectively. The boundary conditions for the function for the run are written as:
\[
f^+(0, \phi) = f^-(0, -\phi), \quad 0 \leq \phi \leq \pi,
\]
\[
f^-(W) = C^- = \frac{1}{2} \int_0^\pi f^+(W') \sin \phi d\phi,
\]
\[
\sin \phi \frac{\partial g_0}{\partial \phi} - eE_0 \cos \phi = -\gamma_{ee} (g_0 - g_0^\pm) - \gamma_{imp} g_0.
\]

Finally, we will simplify it even further by omitting the function \( g_0^\pm \), which will be justified below.

The solution of the resulting equation that satisfies boundary conditions (5) and (6) has the form
\[
g_0^-(y, \phi) = \frac{eE_0 \cos \phi}{\gamma} \left[ 1 - \exp \left( -\gamma \frac{y \pm W}{\sin \phi} \right) \right],
\]
\[
\gamma = \gamma_{ee} + \gamma_{imp}.
\]

FIG. 1: Ballistic sample with a rough and a specular edges in external electric and magnetic fields.

TRANSPORT IN THE ABSENCE OF A MAGNETIC FIELD

In the absence of a magnetic field, the contribution to the symmetric part of the function \( g(y, \phi) \), which is linear in perturbation, is also equal to zero, i.e. \( g^0 = 0 \). This follows from the fact that for \( B = 0 \), the distribution of electrons across the sample cannot depend on the direction of the electric field \( E_0 \) applied to the sample (this is no longer the case \( B \neq 0 \) because of the appearance of the Lorentz force). Therefore, at \( B = 0 \) equation (4) takes the form
\[
\sin \phi \frac{\partial g_0}{\partial \phi} - eE_0 \cos \phi = -\gamma_{ee} (g_0 - g_0^\pm) - \gamma_{imp} g_0.
\]
\[
\frac{1}{\pi} \int_0^{2\pi} g_0(y, \varphi) \cos \varphi d\varphi \text{ from this expression we obtain }
\]
\[
g_0^\pm(\varphi) \approx \frac{4eE_0W}{\pi} \ln \left( \frac{1}{\gamma W} \right) \cos \varphi. \tag{9}
\]

Bearing in mind the inequality \( \gamma |y \pm W| \ll 1 \), from (8) and (9) it is easy to see what \( g_0^\pm \) is greater \( g_0 \) for all \( \varphi \), except for narrow regions around the directions \( \varphi = 0 \) and \( \varphi = \pi \), where, on the contrary, \( g_0 \gg g_0^\pm \). In this regard, it may seem that the correction \( g_0^\pm \) in (7) was unjustified, however, firstly, it is these narrow regions that made the main contribution to (9) and, secondly, outside these regions, the entire right-hand side of (7) is small in parameter \( \gamma W \) and can be omitted. The correction \( h(y, \varphi) \) to function (8), caused by taking into account \( g_0 \) in equation (7), can be found by the perturbation method, writing \( y(y, \varphi) \) in the form \( g = g_0 + h \) and substituting in (7) as \( g_0 \) expression (9). From the resulting equation, we find,
\[
h_0^\pm(\varphi) = \frac{4eE_0W \cos \varphi}{\pi} \left[ 1 - \exp \left( - \frac{\gamma y \pm W}{\sin \varphi} \right) \right],
\]

which, due to the inequality \( \gamma W \ll 1 \) is small compared to (8). For current
\[
I = \frac{e n_0}{\pi p_F} \int_0^W dy \int_0^{2\pi} g_0(y, \varphi) \cos \varphi d\varphi
\]

and resistivity \( \rho \), from (8) we obtain
\[
I \approx \frac{4e^2n_0E_0W^2}{\pi p_F} \ln \left( \frac{1}{\gamma W} \right), \quad \rho \approx \frac{\pi p_F}{4e^2n_0W \ln \left( \frac{1}{\gamma} \right)} \tag{10}
\]

In the expression for the current density, we neglected the terms that depend on \( y \) and do not contain a large logarithm \( \ln[1/(\gamma W)] \). Note that expression (10) for \( \rho \) is half that obtained in [3], which is not surprising, since there was considered a problem with two diffusely reflecting edges.

Result (10) has a simple physical meaning. In the system under consideration, the electron gas momentum can relax either upon collisions of electrons with a diffusely scattering edge of the sample, or upon their collisions with impurities. Due to the condition \( \gamma W \ll 1 \) we have adopted, typical electrons move along broken paths, randomly changing their direction of motion after each collision with a diffusely scattering edge of the sample. The characteristic momentum relaxation time of such electrons is of the order of \( W/v_F \ll \tau_{ee, \text{imp}} \) and their contribution to the conductivity is relatively small. The main contribution to the conductivity is made by electrons moving at small angles to the axis \( x \).

If \( \tau_{\text{imp}} \ll \tau_{ee} \), then the relaxation length of the momentum of such electrons is of the order of \( \tau_{\text{imp}} \) or less, and they make a proportional contribution
\[
\int \frac{d\varphi}{\varphi} \sim \ln \left( \frac{1}{\gamma_{\text{imp}}W} \right)
\]
to the conductivity. In the opposite limiting case \( \tau_{ee} \ll \tau_{\text{imp}} \), an electron moving at a small angle, having passed a length of the order of \( \tau_{ee} \) or less, is scattered at a rough edge or collides with another electron, after which it becomes typical and after a short time of the order \( W/v_F \) is diffusely scattered at the rough edge of the sample. The corresponding contribution to the conductivity is proportional \( \ln[1/(\gamma_{\text{imp}}W)] \).

In the general case, formula (10) is obtained. It is also clear from the last reasoning why, at \( \tau_{\text{imp}} \gg \tau_{ee} \), the outflow processes described by the integral of interparticle collisions \( S_{ee}[f] \) play the main role in the formation of the current, while the role of the incoming processes associated with the function \( g_0^\pm(y, \varphi) \) is small.

In sufficiently narrow samples, a different situation is possible. Due to the uncertainty principle, the minimum transverse momentum of electrons is of the order \( h/\gamma W \), so that the maximum time of motion without scattering before collision with the edge is of the order of \( mW^2/h \), and the minimum angle between the electron velocity vector and the ordinate axis is of the order of the diffraction angle \( \varphi \sim h/\gamma W \), i.e. \( W \ll \sqrt{\lambda_F} \), where \( l = 1/\gamma \), we again get formulas (10), which will enter \( \ln(k_FW) \) instead \( \ln[1/(\gamma W)] \).

**Hall Effect**

In this section of the article, we will find the Hall coefficient for our system in an arbitrarily weak magnetic field. For this, it is necessary to take into account in the kinetic equation (4) the terms with the Lorentz force and the Hall electric field. The function \( q^0(y) \) is now nonzero, since the action of the Lorentz force leads to a redistribution of electrons across the sample. Let us write it in the form \( g_0 + g_1 \), where \( g_1 \) is the correction caused by the magnetic field, which will be considered arbitrarily small and taken into account as a disturbance. We are interested in the contribution to \( g_1 \), linear in the magnetic field; therefore, in the term \( R_{ee} \partial g / \partial \varphi \) in equation (4), we can use function (8) as a function \( g \). In addition, since the influence of the magnetic field on the current appears only in the second order in \( B \), we will not take it into account by setting \( g^0 = 0 \). Then from (4) we obtain the equation
\[
\sin \varphi \frac{\partial g_1}{\partial y} = - \sin \varphi E_H + \gamma (g_1 - g_0^1) = - \frac{1}{R_e} \frac{\partial g_1}{\partial \varphi}.
\]
\[
\sin \varphi \frac{\partial \tilde{g}_1}{\partial y} + \gamma (\tilde{g}_1 - \tilde{g}_0) = -\frac{1}{R_c} \frac{\partial \tilde{g}_1}{\partial \varphi}
\]
(11)

where the function \( \tilde{g}_1 = g_1 + e\Phi \) is introduced, \( \Phi \) is the potential of the electric field.

The procedure for solving this equation is completely similar to the procedure for solving equation (7): having omitted the function \( \tilde{g}_1(y) \) in (11), we obtain the following expressions:

\[
\tilde{g}_1 \approx \frac{eE_0}{R_c} \left\{ \frac{\sin \varphi}{\gamma^2} \left[ 2 \exp \left( \frac{\gamma W}{\sin \varphi} \right) - 1 \right] + \frac{y - W}{\gamma} - \frac{\cos^2 \varphi}{2 \sin^3 \varphi} (y^2 + 2yW - W^2) - \frac{1}{2\pi \gamma^2} \right\}
\]

and

\[
\tilde{g}_1 \approx \frac{eE_0}{R_c} \left\{ \frac{\sin \varphi}{\gamma^2} \left[ \exp \left( -\frac{\gamma y + W}{\sin \varphi} \right) - 1 \right] + \frac{y - W}{\gamma} - \frac{\cos^2 \varphi}{2 \sin^3 \varphi} (y - W)^2 - \frac{1}{2\pi \gamma^2} \right\}
\]

Then we show that the correction arising from the account \( \tilde{g}_1(y) \) is parametrically small (see Appendix A). Substituting these expressions in

\[
2\pi \tilde{g}_1^0(y) = \int_0^{2\pi} g_1(y, \varphi) d\varphi
\]

and keeping only the main contribution, we find

\[
\tilde{g}_1^0 \approx -\frac{E_0}{\pi R_c} \left[ \frac{W^2}{\gamma^2(y + W)^2} - \frac{1}{2\gamma^2} \right].
\]

The function \( \tilde{g}_1^0/e \) is the electrochemical potential \( \Psi(y) \), its minus derivative is equal to the Hall field, and the difference in values at the edges of the sample is measured with a voltmeter and is equal to the Hall voltage, \( U_H = \Psi(W) - \Psi(0) \). Therefore, we have

\[
E_H \approx -\frac{2E_0W^2}{\pi R_c \gamma (y + W)}; \quad U_H \approx -\frac{3E_0}{4\pi R_c \gamma^2}.
\]

From here and from (10) for the Hall coefficient we obtain

\[
R_H = \frac{U_H}{BI} = \frac{3}{16\gamma^2 W^2 \ln(1/\gamma W) \epsilon_{\text{vac}}} \geq \frac{1}{\epsilon_{\text{vac}}},
\]

Near the points \( \varphi = 0 \) and \( \varphi = \pi \) functions (12) have singularities of the form \( 1/\varphi^3 \) and \( 1/(\varphi - \pi)^3 \), making the main contribution (13) to the function \( \tilde{g}_1^0(y) \), the divergences arising in this case are "cut off" by exponential factors \( \exp[-\gamma(y \pm W)/\sin \varphi] \). Note that in the case of a symmetric structure, which was studied in papers [3] and [11], the main contributions to \( \tilde{g}_1^0(y) \) from functions \( \tilde{g}_1^+ \) and \( \tilde{g}_1^- \) cancel each other, and the Hall coefficient turns out to be equal \( 1/2\epsilon_{\text{vac}} \). In narrow samples, \( W \ll \sqrt{\lambda R} \), the divergences are "cut off" due to the principle of uncertainty at angles \( \varphi \) and \( \varphi - \pi \) order \( 1/R \), and as a result, the following expression is obtained for the Hall coefficient

\[
R_H = A (k_F W)^2 \frac{1}{\ln(k_F W) \epsilon_{\text{vac}}},
\]

where \( A \) is a numerical coefficient of the order of unity, which remains undefined within the framework of our consideration.

In conclusion of this section, we will explain the anomaly of large value of the Hall coefficient in the asymmetric structure we studied. To do this, let’s approach the problem from a slightly different point of view. We represent the distribution function in the form

\[
\int f_0(\varepsilon, \mathbf{r}, \varphi) \sim \tilde{f}(\varepsilon, \mathbf{r}, \varphi), \quad \int f_0^2 \tilde{f} d\varphi = 0 \]

by \( p_y \) and integrate over 2\( d^2p/(2\pi \hbar)^2 \). It will turn out

\[
\frac{\partial \tilde{\sigma}}{\partial y} + enE_y + \frac{\partial \Pi_{ix}}{\partial x} + \frac{\partial \Pi_{iy}}{\partial y} = m\omega_n V_y + \frac{mn}{\tau_{\text{imp}}} V_y,
\]

where the notation

\[
\Pi_{ik} \equiv m \int \tilde{f}_i \tilde{v}_k \frac{2d^2p}{(2\pi \hbar)^2},
\]

is introduced. The equation

\[
\frac{\partial \tilde{\sigma}}{\partial x} + enE_x + \frac{\partial \Pi_{ix}}{\partial x} + \frac{\partial \Pi_{iy}}{\partial y} + m\omega_n V_x + \frac{mn}{\tau_{\text{imp}}} V_x
\]

is obtained similarly. Equations (17) and (19) are exact. It is convenient to write them in the form

\[
enE_x + \frac{\partial \tilde{\sigma}}{\partial x} + \nabla \cdot \Pi_x + m\omega_n V_y + \frac{mn}{\tau_{\text{imp}}} V_x = 0,
\]

\[
enE_y + \frac{\partial \tilde{\sigma}}{\partial y} + \nabla \cdot \Pi_y - m\omega_n V_x + \frac{mn}{\tau_{\text{imp}}} V_y = 0,
\]

In equilibrium, the electric field and average velocity \( \mathbf{V}(r) \) are equal to zero; therefore, in the linear approximation, these equations take the form

\[
enE_x + \frac{\partial \tilde{\sigma}}{\partial x} + \nabla \cdot \Pi_x + m\omega_n n_0 V_y + \frac{mn_0}{\tau_{\text{imp}}} V_x = 0,
\]

\[
enE_y + \frac{\partial \tilde{\sigma}}{\partial y} + \nabla \cdot \Pi_y - m\omega_n n_0 V_x + \frac{mn_0}{\tau_{\text{imp}}} V_y = 0.
\]
Let us introduce the electric $\Phi$ and electrochemical potentials $\Psi = \Phi - \gamma / e n_0$ and rewrite the equations (21) through them

$$
\begin{align*}
\frac{e n_0}{c} \frac{\partial \Psi}{\partial x} - \frac{n_0 e B}{c} V_y + \frac{m n_0}{\tau_{\text{imp}}} V_x &= \nabla \cdot \Pi_x, \\
\frac{e n_0}{c} \frac{\partial \Psi}{\partial y} + \frac{n_0 e B}{c} V_x + \frac{m n_0}{\tau_{\text{imp}}} V_y &= \nabla \cdot \Pi_y.
\end{align*}
$$

The function $\Psi = \Phi - \gamma / e n_0$ is the same as the function $\Psi = \tilde{g}_1^0 / e = \Phi + g_0^0 / e$ we introduced above. Really:

$$
\theta = \int f_0 \nu \exp \left( \int_0^x \frac{\partial f_x}{\partial x} g_0^0 \nu \exp \right) = -\nu \exp g_0^0 = -n_0 g_0^0
$$

where $\nu = m / \pi h^2$ is the density of states.

In the left-hand sides of equations (22), there are forces acting on a unit volume of the electron gas in $x$ and $y$ out of directions, and on the right, divergence of the momentum flow in the same directions. We are interested in the second equation. Let us rewrite it, taking into account that in our problem there is no dependence on the coordinate $x$ and the average velocity along the axis $y$ is zero

$$
\frac{e n_0}{c} \frac{\partial \Psi}{\partial y} + \frac{n_0 e B}{c} V_x = \frac{\partial \Pi_{yy}}{\partial y},
$$

and integrate across the sample. It turns out:

$$
U_H = \frac{1}{en_0} \left[ \Pi_{yy}(W) - \Pi_{yy}(0) \right] + \frac{BI}{en_0 c}
$$

Substituting now the function (12) multiplied $\partial f_x / \partial x$ into the definition $\Pi_{yy}$ and performing the integration, we again obtain expression (14) for the Hall voltage (note that the contribution to the integral of the symmetric part of the function $\tilde{g}_1$ is zero). It can be seen from the foregoing that the Hall voltage arising in our asymmetric sample is primarily intended to compensate for the gradient of the transverse-pulse flux density across the sample, and not the Lorentz force, as is usually the case in systems with an ohmic current flow. A nonzero gradient of the momentum flux density in the transverse direction also arises in a symmetric structure, but it is of the same order of magnitude as the Lorentz force and leads to a halving of the Hall coefficient compared to its standard value.

## CONCLUSION

In this work, we studied the Knudsen regime of a degenerate electron gas flow in a Hall sample, one edge of which reflects electrons diffusely, and the other specularly. The collisions of electrons with each other and their collisions with impurities were taken into account. It is shown that, in such an asymmetric sample, the Hall coefficient is parametrically large in comparison with its standard value $\frac{R^0_H}{en_0} = \frac{1}{en_0 c} R_H / R^0_H \sim l^2 / W^2 \ln(l/W)$, where $l$ is the effective momentum relaxation length $l \gg W$. In addition, the situation is discussed at a semiquantitative level when all types of scattering can be neglected except for scattering at the edges of the sample.

Finally, we note that, when deriving formula (15) for the Hall resistance, the role of “skipping orbits” near the rough edge was not thoroughly analyzed, which can lead to a change in the numerical factor in the expression for the Hall resistance [28].

We thank P. S. Alekseev for valuable discussions. Yu.A. also thanks to his teachers M. E. Kompan, N. M. Khimin, M. G. Ivanov. This work was supported by the Russian Science Foundation (grant No. 17-12-01182-c).

## APPENDIX A

Having performed the calculations described in the text, we obtain for corrections to functions (12)

$$
\begin{align*}
\frac{\gamma}{\sin \varphi} &= \int_0^y \exp \left( -\gamma \frac{y-y'}{\sin \varphi} \right) g_0^0(y') dy' \\
&+ \int_0^W \exp \left( -\gamma \frac{y+y'}{\sin \varphi} \right) g_0^0(y') dy',
\end{align*}
$$

where $\tilde{g}_0 = \tilde{g}_0^0(y')$ is the symmetric part of function (12). Having integrated it over and over, we obtain for the correction to the Hall coefficient

$$
\delta R_H \approx -\frac{1}{8\pi \gamma W} \cdot \frac{1}{en_0 c},
$$

which is parametrically less than expression (15).

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[27] Note that the condition of equality of the concentrations of particles flying up to a rough boundary and reflected from it, encountered in the literature, is erroneous; in particular, it leads to a nonzero particle flux through this boundary.
[28] P. S. Alekseev (private communication). If, however, when measuring the Hall voltage, the contact at the diffusely scattering edge is slightly shifted deep into the sample, then the numerical coefficient in (15) will be correct.