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Pitfalls in long memory research
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Abstract: This paper offers a multifaceted perspective of the literature on long memory. Although the research on long memory has played an instrumental role in elevating the level of scholarly discourse on market efficiency, the authors believe that the issue of the prevalence of long memory or lack thereof remains unsettled. While long memory models should be in the econometrician’s toolbox, their use should be governed by an initial exploratory analysis of the data being studied and the context of the research questions being addressed. Mere fixation on the presence/absence of long memory without taking due cognisance of other confounding factors would pave way for confirmation bias. Consequently, this paper pinpoints the possible pitfalls and potential trade-offs in modeling long memory in asset prices. While not a comprehensive meta-analysis of the literature on long memory, this paper offers a selective bibliography of prior works on long memory that is geared to nudge researchers to exercise caution and judgement while exploring long memory in asset prices.

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1. Introduction
While studies grounded in Long Memory constitute a notable strand of literature disputing market efficiency, such studies are not devoid of caveats. Long Memory gained traction in scholarly discourse due to Mandelbrot’s work on asset prices using rescaled range estimation techniques...
A lot of water has flown under the bridge since then. Meanwhile, the literature on long memory has become multi-dimensional in nature. While the initial works on re-examining market efficiency using methodologies that are theoretically grounded in long memory offered the much-needed contrast to a somewhat homogeneous literature on market efficiency; inferences from such studies on the prevalence of long memory or lack thereof have not been unequivocal.

Although there are a few review papers that discuss long memory (Baillie, 1996; Guégan, 2005; Lim & Brooks, 2011), literature seems wanting on bringing the various arguments for and against the observation of long memory together. In this backdrop, the authors believe a snapshot of prevailing literature on long memory in asset prices, without losing sight of the attendant contexts behind such studies, is the need of the hour. Such a snapshot would aid researchers to take stock of the various facets of the discourse on long memory, in a manner that would nudge them to exercise requisite caution before drawing any definitive inference on long memory in asset prices in their future research endeavours.

While prior studies on long memory have significantly broadened the literature landscape on market efficiency, a definitive takeaway from such literature on long memory that is oblivious to other confounding factors which can manifest as long memory would be short-sighted and self-fulfilling. In short, this is as much an attempt to sensitize researchers about the pitfalls in research on long memory, as highlighting the prominence of long memory in the context of revisiting market efficiency.

2. Definition, measures and methodologies

2.1. Definition

For a second-order stationary process $X_t$ with an auto-covariance function $\gamma_X(k)$, $X_t$ has

a) Short Memory, if

$$0 < \sum_{k=1}^{\infty} \gamma_X(k) < \infty$$

b) Anti-Persistence, if

$$\sum_{k=1}^{\infty} \gamma_X(k) = 0$$

c) Long Memory, if

$$\sum_{k=1}^{\infty} \gamma_X(k) \to \infty$$

Long memory (or persistence) implies that a positive or negative movement is more likely to be followed by another move in the same direction. On the other hand, for an anti-persistent process, a positive movement is more likely to be followed by a move in the opposite direction. In other words, a persistent process is trending whereas an anti-persistent process shows mean reversion.

Beran, Feng, Ghosh, and Kulik (2016) provides a detailed review of various definitions of long memory and the conditions in which these can be used interchangeably. The different measures of long memory are as follows.

2.2. Measures

2.2.1. Hurst exponent

The most popular measure for long memory is the “Hurst Exponent” (denoted as $H$). This measure gained traction owing to Mandelbrot and Wallis’s pioneering work on operational hydrology (Mandelbrot &
There are several methodologies that are used to calculate the Hurst Exponent. The classical-rescaled range ($R/S$) analysis proposed by Hurst (1951) and its subsequent variants, such as Modified $R/S$ analysis and Rescaled Variance ($V/S$) analysis, are the most prominent ones.

When $0.5 < H < 1$, the autocovariances are positive at all lags and the time series process is called persistent. When $0 < H < 0.5$, the autocovariances at all lags are negative and the time series process is called anti-persistent.

### 2.2.2. Fractional order of integration

Another popular approach to ascertain long memory or lack thereof is to measure the fractional order of integration (denoted as $d$) of a time series. This paved the way for ARFIMA-FIGARCH models, which were designed to explicitly model long memory in the first and second moments (Baillie, Bollerslev, & Mikkelsen, 1996; Granger & Joyeux, 1980; Hosking, 1981).

#### 2.2.3. Fractal dimension

Gneiting and Schlather (2004) describe the fractal dimension, $D$, of a surface as a roughness measure with $D \in [n, n + 1)$ for a surface in $\mathbb{R}^n$ where higher values can be interpreted as rougher surfaces. Technically, Fractal dimension($D$) and Hurst exponent($H$) are independent of each other. Fractal dimension is a local property, while Hurst exponent is a global property, which is used to characterize the long-memory dependence in a time series. For self-affine processes, local properties are reflected in global ones, which lead to the relationship $D + H = n + 1$ between $D$ and $H$ for a self-affine surface in $n$-dimensional space.

Table 1 offers a snapshot of the above stated measures of long memory.

### 2.3. Methodologies

Over the years, a number of methodologies have been proposed by researchers for measuring long memory. While in many cases, long memory in conditional mean and variance are studied independently, unified approaches to study long memory are also present (Teyssière, 1997).

Several popular heuristic methods to measure long memory in the first and second moments include the Rescaled range ($R/S$) method, Rescaled variance ($V/S$) method and Detrended Fluctuation Analysis (DFA). Prominent semi-parametric estimators of long memory include the Log Periodogram estimator (Geweke & Porter-Hudak, 1983), and Local Whittle estimator. These estimators were defined based on linear time series models and should be used to measure long memory in the first moment. It was found that the GPH estimator can be downward biased when used to measure long memory in volatility (Deo & Hurvich, 2001). The Whittle estimator is found to be more robust in measuring the long memory in volatility (Hurvich & Ray, 2003).

In terms of parametric modeling-based approaches, Mandelbrot and Van Ness (1968) introduced the fractional brownian motion. This was a generalisation of the standard brownian motion by incorporating a self-similar parameter $d \in (-0.5, 0.5)$ and provides the most basic framework for studying long memory. Another important approach of modeling long memory is the ARFIMA—FIGARCH class of models. While ARFIMA model is used to model long memory in the first moment (return time series), FIGARCH is used for modeling the long memory in volatility.

### Table 1. Summary of measures of Long Memory

| Type of Persistence       | $H$  | $d = H - 0.5$ | $D = 2 - H$ |
|---------------------------|------|--------------|------------|
| Short Memory              | 0.5  | 0            | 1.5        |
| Anti-persistence (Mean Reversion) | (0,0.5) | (0,-0.5)    | (1.5,2)    |
| Long Memory (Persistence) | (0.5,1) | (0.5,0)      | (1,1.5)    |
The ARFIMA \((p, d, q)\) model, which was introduced by Granger (1980); Granger and Joyeux (1980) and Hosking (1981), is defined as
\[
\phi(L)(1 - L)^d y_t = \theta(L)\epsilon_t
\]
where \(\phi(L)\) and \(\theta(L)\) are lagged polynomials of orders \(p\) and \(q\) respectively, with \(\epsilon_t\) being white noise. For the ARFIMA model, the fractional parameter \(d\) lies between \(-0.5\) and \(0.5\). An ARFIMA process depicts long memory when \(0 < d < 0.5\), anti-persistence for \(-0.5 < d < 0\), short memory for \(d = 0\), and infinite memory (random walk) for \(d = 1\).

The incorporation of long memory in GARCH models was introduced by Robinson (1991) and built upon by Baillie et al. (1996); Ding and Granger (1996) and others. Among these approaches, the more popular FIGARCH\((p, d, q)\) model (introduced by Baillie et al. (1996)) is defined as
\[
\phi(L)(1 - L)^d \nu_t = \alpha_0 + \beta(L)\nu_t
\]
where \(0 < d < 1\), \(\nu_t = \epsilon_t^2 - h_t\) with \(h_t\) being the conditional variance. In ARFIMA model, the long memory operator is applied to unconditional mean \(\mu\) of \(y_t\), whereas in the case of FIGARCH model, it is applied to squared errors. However, the FIGARCH model has its own nuances that need to be remembered during application. The memory parameter of FIGARCH is actually \(-d\) and increases as \(d \to 0\). This happens due to the fact that the memory parameter acts on the squared errors in FIGARCH. Consequently, the Hyperbolic GARCH (HYGARCH) model was proposed by Davidson (2004).

Among other popular models on long memory include the Fractional Exponential (FEXP) model by Beran (1993) which is an extension of the ARFIMA model, long-memory stochastic volatility models (Breidt, Crato, & Lima, 1998; Harvey, 1998) and the Multi-Fractal model (MMAR model) by Mandelbrot, Fisher, and Calvet (1997).

The following Table 2 offers a snapshot of the notable methodologies pertaining to long memory along with their original sources. These popular long memory methodologies possess theoretical antecedents in diverse areas.

3. Evidence on the presence of long memory
Financial Literature is divided on the presence of long memory. Long memory has been observed in most economic variables (Hassler & Wolters, 1995; Koustas & Veloce, 1996), bonds (Backus & Zin, 1993), stocks (Hiemstra & Jones, 1997), indices (Bollerslev & Mikkelsen, 1996; Sadique & Silvapulle, 2001), currencies (Cheung, 1993), commodities (Cai, Cheung, & Wong, 2001; Elder & Serletis, 2008), derivatives (Barkoulas, Labys, & Onochie, 1999; Helms, Kaoen, & Rosenman, 1984) and other specialized instruments (Madhavan & Arrawatia, 2016).

However, this evidence of long memory varies across variables and markets. While asset returns have been shown to have weak to no evidence of long memory (especially in developed markets) (Floros, Jaffry, & Vale Lima, 2007; Henry, 2002), asset volatility has been found to show strong evidence of long memory (Bollerslev & Mikkelsen, 1996; Fleming & Kirby, 2011; Mighri, Mansouri, 2014). In contrast, recent studies with high-frequency data have shown the presence of anti-persistence in volatility (Gatheral, Jaisson, & Rosenbaum, 2018). Literature on trading volume seems to be consistently in support of long memory (Lillo & Farmer, 2004; Lux & Kaizoji, 2007). Studies on developing markets also show conflicting results. While prior studies showed stronger presence of long memory in developing markets (Hull & McGroarty, 2014; McMillan & Thupayagale, 2009), recent studies show that some developing markets have become more efficient than a few developed ones since the 2008 financial crisis (Mensi, Tiwari, & Al-Yahyaee, 2019; Sensoy & Tabak, 2016).
3.1. Plausible causes of true long memory

While the evidence on long memory is based on statistical and heuristic tests, the discussion remains incomplete without pointing out plausible causes of true long memory.

3.1.1. News flow and its interpretation

Long Memory can be a manifestation of the interaction between many diverse information processes and hence is inherent to the returns process (Andersen & Bollerslev, 1997). This goes against the argument of structural breaks leading to the hyperbolic decay of autocorrelations.

The arrival of news is seen as a driver for markets. Lillo and Farmer (2004) explain that the news could be classified as external or internal. Externals news are events outside of control of market participants (e.g., natural calamities). Such events are known to have power-law distributions. While internal news comes under the purview of market players, the ability to understand and act on them can be complicated due to the social dynamics such as herding behaviour. Moreover, limited attention and comprehension ability of humans coupled with their changing preferences towards fundamental and technical analysis can generate long memory in financial time series (Kirman & Teyssiére, 2002).

These aspects can be better understood in the framework of Adaptive Market Hypothesis (AMH) (Lo, 2004, 2005). Human decisions are usually made under incomplete information and are delayed due to other factors. Such time lags in responding to news arrival can lead to autocorrelations in order inflow.

| Sl. No. | Methodology                          | Reference                                                                 |
|--------|--------------------------------------|---------------------------------------------------------------------------|
| 1      | Classical Rescaled Range Estimation  | Hurst (1951); Mandelbrot and Wallis (1968)                               |
|        | (R/S Analysis)                       |                                                                           |
| 2      | Modified R/S Analysis                | Lo (1991)                                                                 |
| 3      | V/S Analysis                         | Giraitis, Kokoszka, Leipus, and Teyssière (2003)                          |
| 4      | Log Periodogram Estimator            | Geweke and Porter-Hudak (1983); Robinson (1994)                          |
| 5      | Gaussian Semi-parametric Estimator   | Robinson (1995)                                                           |
| 6      | Smoothed Periodogram Estimator       | Reisen (1994)                                                             |
| 7      | Detrended Fluctuation Analysis       | Peng et al. (1994); Kantelhardt et al. (2002)                             |
| 8      | Wavelet Analysis                     | Mallat (1989)                                                             |
| 9      | Whittle Estimation                   | Whittle (1951); Hou and Perron (2014)                                    |
| 10     | Fractional Brownian Motion           | Mandelbrot and Van Ness (1968)                                           |
| 11     | ARFIMA Model                         | Granger (1980); Granger and Joyeux (1980)                                 |
| 12     | Fractionally Integrated GARCH (FIGARCH) | Baillie et al. (1996)                                                |
| 13     | Hyperbolic GARCH (HYGARCH)           | Davidson (2004)                                                           |
| 14     | Fractional Exponential (FEXP) Model  | Beran (1993)                                                              |
| 15     | Long Memory Stochastic Volatility    | Breidt et al. (1998); Harvey (1998)                                      |
| 16     | Heterogeneous Long Memory (HAR)      | Müller et al. (1997a); Corsi (2009)                                      |
| 17     | Time-Varying Hidden Markov Model     | Bulla and Bulla (2006); Nystrup, Madsen, and Lindström (2017)             |
| 18     | Bayesian Long Memory                 | Koop, Ley, Osiewalski, and Steel (1997); Ravishanker and Ray (1997)      |
| 19     | Multi-Fractal Long Memory            | Mandelbrot et al. (1997); Colvet and Fisher (2002)                      |
| 20     | Detrended Cross-Correlation Analysis | Podobnik and Stanley (2008)                                              |
| 21     | Fractional Cointegration             | Granger (1986); Johansen (2008)                                          |
3.1.2. Market microstructure issues and other factors

Various market microstructure-based factors can lead to long memory. For example, iceberg orders wherein large orders are split into many smaller ones before being sent to the exchange, might lead to power-law based autocorrelations in order flow mechanism (Lillo, Mike, & Farmer, 2005). Similarly, simulation studies suggest that the observed long memory in order flow, volume and volatility can be attributed to the inherent imitative and adaptive behaviour of various market participants (LeBaron & Yamamoto, 2007, 2008).

Other notable explanatory factors pertaining to long memory include bid-ask spreads, non-synchronous trading (Campbell, Lo, & MacKinlay, 1997), influence of institutional investors (Gabaix, Gopikrishnan, Plerou, & Stanley, 2006), extent of market openness (Lim & Brooks, 2010) and speed of price adjustment (Zheng, Liu, & Li, 2018). Lastly, long memory has also been attributed to the economic and institutional differences between emerging and developed markets (Llaw, 2009).

4. Long memory: beware of false positives

Empirical research seems divided on the debate on differentiating true and spurious long memory in economic variables. While the prevalence of long memory runs contrary to market efficiency, this section discusses several known pitfalls that can cause false positives in long memory analysis.

4.1. Structural breaks

Potter (1979) argued that long memory may be an artifact of non-homogeneity in the data. He referred to several studies on precipitation and concluded that studies with homogeneous data did not support the presence of long memory. This was also proved in other studies. For example, introducing a trend to a stationary time series can create long memory bhattacharya1983hurst. (Bhattacharya, Gupta, & Waymire, 1983). Simulations based on incorporating breaks in a data generating process (DGP) provide evidence of spurious long memory (Diebold & Inoue, 2001). Various empirical studies using financial market data have also shown the confounding effect structural breaks can have on long memory in returns (Granger & Hyung, 2004) and volatility (Liu, 2000).

However, distinguishing between long memory and the structural break is mathematically difficult. This is similar to the confusion between unit root and structural breaks. For true long memory processes, several tests for structural change may show structural break when there should be none (Kuan & Hsu, 1998). On the other hand, long memory estimators will be biased towards long memory for stationary processes with level shifts (Perron & Qu, 2010). Literature provides several avenues to check for the confounding effect of structural breaks on long memory and also to incorporate both phenomena in models to measure their individual effects.

Quite a few statistical tests exist to check for this confounding effect. Most are hypothesis tests based on LM, LR, Wald and CUSUM tests for breaks in mean (Hidalgo & Robinson, 1996; Mayoral, 2012; Perron & Qu, 2010; Qu, 2011; Shao, 2011; Shimotsu, 2006; Wenger, Leschinski, & Sibbertsen, 2018b; Wright, 1998). Similarly, tests for differentiating structural break from long memory in volatility are also available (Hwang & Shin, 2015). Recently, Sibbertsen, Leschinski, and Busch (2018) proposed a multivariate approach to differentiate between structural breaks and long memory.

Model-specific studies are also available. For example, ARFIMA-based models that are robust to structural breaks have been proposed (Baillie & Morana, 2012; Shi & Ho, 2015). Similarly, attempts to capture and differentiate structural breaks from long memory in volatility have led to pertinent improvisations of FIGARCH (Baillie & Morana, 2009), Markov Switching GARCH (Charfeddine, 2014) and HAR-RV (Hwang & Shin, 2018) class of models. Volatility specific structural break tests like the ICSS test (Inclan & Tiao, 1994) and its variants can be used with FIGARCH models to differentiate between structural breaks and long memory (Walther, Klein, Thu, & Piontek, 2017). Lastly, Long Memory Estimators that are robust to structural breaks have also been proposed by Hou and Perron (2014).
For a review of the literature of tests that aid in differentiating structural breaks from long memory, the readers may refer to Sibbertsen (2004), Banerjee and Urga (2005) and Wenger, Leschinski, and Sibbertsen (2018a).

4.2. Temporal aggregation
Temporal Aggregation refers to the transformation of a time series data at a frequency lower than the original DGP. In some cases, data is also analyzed after aggregation over a longer duration to remove seasonal fluctuations. A typical example is data on industrial production where data is only available quarterly. While this aids in smoothing the data points as well as filtering out the high frequency noise, it can also manifest as spurious long memory. For example, LeBaron (2001) showed that a temporally aggregated series created by adding up just three short memory linear time series of different time scales can show spurious long memory.

Having said so, evidence in support of true long memory, notwithstanding temporal aggregation is also available. For instance, Andersen and Bollerslev (1997) showed that true volatility persistence can be attributed to temporal aggregation of heterogeneous inflow of news over time. Their work lends credence to long memory dependence being an inherent feature of the DGP and not a spurious manifestation of temporal aggregation (Mcmillan & Speight, 2008). A notable result in this school of thought was brought forward by Souza (2008) where it was shown that for true long memory series, temporal aggregation does not change the estimated memory parameter.

This association between temporal aggregation and long memory has also formed the basis of a specific class of volatility models called Heterogeneous Autoregressive models (Corsi, 2009; Müller et al., 1997b). These models draw motivation from the Heterogeneous Market Hypothesis (Müller et al., 1993) and also the “Mixture of Distribution Hypothesis” of Andersen and Bollerslev (1997).

To assist researchers, there are several tests to differentiate between true long memory and spurious long memory owing to temporal aggregation (Davidson & Rambaccussing, 2015; Frederiksen & Nielsen, 2013; Kuswanto, 2011; Ohanissian, Russell, & Tsay, 2008).

4.3. Cross-sectional aggregation
Just like temporal aggregation, cross-sectional aggregation can also lead to spurious observations of long memory in time series variables. A large number of AR(1) processes can be added to produce a time series that would show a long memory under certain assumptions (Granger, 1980).

Studies on inflation data have attributed the observed long memory to cross-sectional aggregation since inflation is measured via aggregating various sectoral sub-indices that possess only short memory (Altissimo et al., 2009; Balcilar, 2004). On the other hand, prior works such as Kang, Cheong, and Yoon (2010) uncover evidence of long memory in the stock index as well as the underlying constituent stocks.

Cross-sectional aggregation also requires the count of individual series ($N$) to be very large. Granger (1980) postulated this case for $N \rightarrow \infty$. However, this count also varies across studies depending on other assumptions used for the Monte Carlo simulations. While Zaffaroni (2004) used a dataset with $N > 1500$ to reproduce the theoretical results, Leccadito, Rachedi, and Urga (2015) simulated long memory with $N = 500$ components only. Another study by Haldrup and Valdés (2017) showed that $N$ seems to depend on the extent of long memory in the individual series. If the individual series have a high long memory, an aggregate of just 250 such series can mimic that inherent long memory. However, for individual series with low levels of persistence, even an aggregate of $N = 10,000$ such series did not have a similar level of long memory. In addition, it was proved that when such a composite series is fractionally differenced, the autocorrelation function of its residuals still exhibits hyperbolic decay. This inability of ARFIMA models to suitably capture the long memory of the true DGP caused by cross-sectional aggregation calls for better models.
Long Memory can also be observed when a number of linear and homogeneous subsystems with short memory are connected to create a network structure (Schennach, 2018). This provides another possible reason for long memory without non-linearity, heterogeneity, unit roots or structural breaks. Several economic examples that can be modeled using these approach are firms in an industry and supply chain time series.

4.4. Biases in the estimation process and related issues

Differentiating true and spurious long memory calls for researchers to exercise judgement while choosing the estimation method. Not all estimators are equally suitable in all cases. Various studies have commented on the properties of popular R/S statistic and its many variants in terms of size and power (Lo, 1991; Teverovsky, Taqqu, & Willinger, 1999). Similarly, notable prior works offer a critical review of small sample properties of other estimators, such as, but not limited to, GPH (Agiaiakloglou, Newbold, & Wohar, 1993), Local Whittle (Hurvich & Ray, 2003), Higuchi, Peng and Wavelet estimators (Rea, Oxley, Reale, & Brown, 2013).

Applying an AR-GARCH filter on returns would significantly reduce the spurious long memory effect, for such a filter would, to a larger extent obviate the confounding effect of short memory (Lo, 1991). Further, the use of low-frequency data can cause a downward bias on the long memory estimates (Bollerslev & Wright, 2000; Souza & Smith, 2002). Also, the choice of proxies to measure volatility has an effect on long memory estimates (Wright, 2002).

Various characteristics of data also need to be reviewed before choosing methodologies. For instance, emerging markets, having higher levels of volatility, maybe more suitable for wavelet-based estimation (Ozun & Cifter, 2008). In general, Local Whittle estimators are observed to be the most stable among other long memory estimators (Hassler, 2011; Taqqu, Teverovsky, & Willinger, 1995). In addition, cyclical and seasonal patterns in data can lead to the observation of long memory in the squared return series (Lobato, 1997). Many long memory tests assume unconditional homoscedasticity. Tests that allow for heteroscedasticity (Harris & Kew, 2017) should be used for financial time series. Similarly, the absence of higher-order moments also manifest as long memory (Lobato & Savin, 1998).

While actual datasets may not show these exact pathologies, issues such as the presence of heavy tails are very much real. Consequently, it is advised to ascertain the model assumptions before employing them. If required, more robust models whose assumptions are closer to the empirical properties of the time series should be used.

5. Impact of long memory on the quality of forecasts

The most practical benefit of using long memory models is their better forecasting ability. Literature supports this finding across all approaches of modeling long memory, such as, fractionally integrated models (Bhardwaj & Swanson, 2006), HAR models (Corsi, 2009; Wen, Gong, & Cai, 2016), Fractal theory (Liu, Demirer, Gupta, & Wohar, 2019) among others. In addition, modeling long memory along with other stylizations of the underlying data, such as periodicity (Franses & Ooms, 1997), heteroscedasticity and structural breaks (Ma, Lu, Yang, & Zhang, 2019) have shown to improve forecasting performance. Similarly, model generalization, such as FIGARCH to hyperbolic GARCH (Davidson, 2004; Li, Li, & Li, 2015) or incorporating a multivariate framework (Balcilar, Gupta, & Jooste, 2017; Harris & Nguyen, 2013), also significantly increase forecasting accuracy.

Since volatility estimation and forecasting are essential in the context of risk management of financial portfolios, Value-at-Risk (VaR) calculations also stand to benefit from long memory models (Batten, Kinateder, & Wagner, 2014; Kinateder & Wagner, 2014; Meng & Taylor, 2018). Allowing for long memory in the cross-section of asset returns can help create specific trading strategies that can generate significant gains (Nguyen, Prokopczuk, & Sibbertsen, 2019).

Notwithstanding the above stated developments on the modeling front, prior studies also nudge researchers to exercise caution while modeling long memory. Notable studies have shown that similar
forecasting results can also be approximately matched by using standard ARIMA models of very high orders (Ray, 1993). In addition, the forecasting error pertaining to over-differencing, is significantly lesser than the forecasting error pertaining to under-differencing (Smith & Yadav, 1994). There are more nuances to be considered here. If the ARIMA coefficients are negative (especially for low values of $d$), standard ARMA models will provide similar short-term predictability. ARFIMA models would be better only for time series with strong persistence ($d \approx 0.5$) or longer-term prediction (Andersson, 2000; Man, 2003). Moreover, Granger and Hyung (2004) found that modeling a time series with only structural breaks and separately with only long memory can provide similar predictive performance with long memory model having a slight edge. Similar findings were reported for many extensions of ARFIMA models. Hence, the model specifications would depend on the researcher’s choice between parsimonious fractional models and the over-parametrized standard ARIMA models.

Another primary motive for employing long memory models is to study the impact of shocks to volatility on asset prices. If the impact of such shocks are short lived and modest, it calls into question the significance of employing long memory models for examining the DGP as has been illustrated by Christensen and Nielsen (2007).

6. Conclusion
If true long memory can be established, many modeling exercises should change. For instance, CAPM can be modified to include fractional returns (Raei & Mohammadi, 2008) and a persistent error term (Amano, Kato, & Taniguchi, 2012). Similarly, modeling exercises involving endogenous variables should be geared towards adequately capturing long memory. This would call for refinement of popular multivariate frameworks such as Granger Causality tests (Chen, 2006, 2008), VAR-MGARCH (Dark, 2018; Zhao, Liu, Duan, & Li, 2019) and Cointegration methods (Granger, 1986; Johansen, 2008). In addition, implied volatility models based on options pricing would be incomplete without incorporating long memory (Cardinali, 2012).

While it cannot be denied that long memory models should be in the econometrician’s toolbox, their use should be governed by an initial exploratory analysis of the data and the context of the research questions. The researcher should keep in mind that to a man with a hammer, everything looks like a nail. Multiple confounding factors, such as, but not limited to, structural breaks and aggregation, can manifest as spurious long memory. This review hopes to nudge researchers to exercise judgement while choosing appropriate long memory models, for inferences derived from misspecified models can lead to misleading policy recommendations.

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Note
1. Over-differencing refers to first differencing a time series whose DGP is closer to $I(1)$, Under-differencing refers to fractionally differentiating a time series who DGP is closer to random walk ($f(1)$).

References
Agiokloglou, C., Newbold, P., & Wohar, M. (1993). Bias in an estimator of the fractional difference parameter. *Journal of Time Series Analysis*, 14(3), 235–246. doi:10.1111/j.1467-9892.1993.tb00141.x
Altissimo, F., Mojon, B., & Zaffaroni, P. (2009). Can aggregation explain the persistence of inflation? *Journal of Monetary Economics*, 56(2), 231–241. doi:10.1016/j.jmoneco.2008.12.013
Amano, T., Kato, T., & Taniguchi, M. (2012). Statistical estimation for cpm with long-memory dependence. *Advances in Decision Sciences*, 2012, 1–12. doi:10.1155/2012/571034
Andersson, T. G., & Bollerslev, T. (1997). Heterogeneous information arrivals and return volatility dynamics: Uncovering the long-run in high frequency returns. *The Journal of Finance*, 52(3), 975–1005. doi:10.1111/j.1540-6261.1997.tb02722.x
Andersson, M. K. (2000). Do long-memory models have long memory? *International Journal of Forecasting*, 16(1), 121–124. doi:10.1016/S0169-2070(99)00040-0
Backus, D. K., & Zin, S. E. (1993). Long-memory inflation uncertainty: Evidence from the term structure of
interest rates. *Journal of Money, Credit and Banking*, 25(3), 681–700. doi:10.1037/2077735

Boillie, R. T. (1996). Long memory processes and fractional integration in econometrics. *Journal of Econometrics*, 73(1), 5–59. doi:10.1016/0304-4076(95)01732-1

Boillie, R. T., Bollerslev, T., & Mikkelsen, H. O. (1996). Fractionally integrated generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 74(1), 3–30. doi:10.1016/0304-4076(95)01749-6

Boillie, R. T., & Morana, C. (2009). Modelling long memory and structural breaks in conditional vari- ances: An adaptive figarch approach. *Journal of Economic Dynamics and Control*, 33(8), 1577–1592. doi:10.1016/j.jedc.2009.02.009

Boillie, R. T., & Morana, C. (2012). Adaptive arfima models with applications to inflation. *Economic Modelling*, 29(6), 2451–2459. doi:10.1016/j.econmod.2012.07.011

Bollilar, M. (2004). Persistence in inflation: Does aggre- gate cause long memory? *Emerging Markets Finance and Trade*, 40(5), 25–56. doi:10.1080/1540496X.2004.11052583

Bollilar, M., Gupta, R., & Jooste, C. (2017). Long memory, economic policy uncertainty and forecasting us inflation: A bayesian varfima approach. *Applied Economics*, 49(11), 1047–1054. doi:10.1080/00036846.2016.1210777

Bonnee, A., & Urba, G. (2005). Modelling structural breaks, long memory and stock market volatility: An overview. *Journal of Econometrics*, 129(1–2), 1–34. doi:10.1016/j.jeconom.2004.09.001

Barkoulas, J. T., Labys, W. C., & Onochie, J. I. (1999). Long memory in futures prices. *Financial Review*, 34(1), 91–100. doi:10.1111/j.1540-6288.1999.tb00446.x

Batten, J. A., Kinateder, H., & Wagner, N. (2016). Multifractality and value-at-risk forecasting of exchange rates. *Physica A: Statistical Mechanics and Its Applications*, 401, 71–81. doi:10.1016/j.physa.2014.01.024

Beran, J. (1993). Fitting long-memory models by general- ized linear regression. *Biometrika*, 80(4), 817–822. doi:10.1093/biomet/80.4.817

Beran, J., Feng, Y., Ghosh, S., & Kulik, R. (2016). Long- memory processes. Berlin, Heidelberg: Springer. https://link.springer.com/book/10.1007%2F978-3-642-35512-7

Bhardwaj, G., & Swanson, N. R. (2006). An empirical investigation of the usefulness of arfima models for predicting macroeconomic and financial time series. *Journal of Econometrics*, 131(1–2), 539–578. doi:10.1016/j.jeconom.2005.01.016

Bhattacharyya, R. N., Gupta, V. K., & Waymire, E. (1983). The hurst effect under trends. *Journal of Applied Probability*, 20(3), 649–662. doi:10.2307/3213900

Bollerslev, T., & Mikkelsen, H. O. (1996). Modeling and pricing long memory in stock market volatility. *Journal of Econometrics*, 73(1), 151–184. doi:10.1016/0304-4076(95)01736-4

Bollerslev, T., & Wright, J. H. (2000). Semiparametric estima- tion of long-memory volatility dependen- cies: The role of high-frequency data. *Journal of Econometrics*, 98(1), 81–106. doi:10.1016/S0304-4076(99)00079-2

Breidt, F., Crato, N., & Lima, P. D. (1998). The detection and estimation of long memory in stochastic volatility. *Journal of Econometrics*, 83(1–2), 325–348. doi:10.1016/S0304-4076(97)00072-9

Bulba, J., & Bulba, I. (2006). Stylized facts of financial time series and hidden semi-markov models. *Computational Statistics & Data Analysis*, 51(6), 2192–2209. doi:10.1016/j.csda.2006.07.021

Cai, J., Cheung, Y.-L., & Wong, M. C. (2001). What moves the gold market? *Journal of Fu- tures Markets: Futures, Options, and Other Derivative Products*, 21(3), 257–278. doi:10.1002/1099-9939(200103)21:3<257::AID-FUT4>3.0.CO;2-W

Calvet, L., & Fisher, A. (2002). Multifractality in asset returns: Theory and evidence. *Review of Economics and Statistics*, 84(3), 381–406. doi:10.1162/003465302259420

Campbell, J. Y., Lo, A. W., & MacKinnon, A. C. (1997). The econometrics of financial markets. Princeton, New Jersey: Princeton University Press.

Cardinali, A. (2012). Estimating volatility from atm options with long-nominal stochastic variance and long memory. *Applied Financial Economics*, 22(9), 733–748. doi:10.1080/09603107.2011.624082

Charfeddine, L. (2014). True or spurious long memory in volatility: Further evidence on the energy futures markets. *Energy Policy*, 71, 76–93. doi:10.1016/j. enpol.2014.04.027

Chen, W. D. (2000). Estimating the long memory granger causality effect with a spectrum estimator. *Journal of Forecasting*, 25(3), 193–200. doi:10.1002/for.981

Chen, W. D. (2008). Is it a short-memory, long-memory, or permanently granger-causation influence? *Journal of Forecasting*, 27(7), 607–620. doi:10.1002/for.1075

Cheung, Y. W. (1993). Long memory in foreign-exchange rates. *Journal of Business & Economic Statistics*, 11(1), 93–101. doi:10.2307/1391309

Christensen, B. J., & Nielsen, M. Ø. (2007). The effect of long memory in volatility on stock market fluctuations. *The Review of Economics and Statistics*, 89(4), 684–700. doi:10.1162/rest.89.4.684

Corsi, F. (2009). A simple approximate long-memory model of realized volatility. *Journal of Financial Econometrics*, 7(2), 174–196. doi:10.1093/jjfinec/nbp001

Dark, J. (2018). Multivariate models with long memory dependence in conditional correlation and volatility. *Journal of Empirical Finance*, 48, 162–180. doi:10.1016/j.jempfin.2018.06.011

Davidson, J. (2004). Moment and memory properties of linear conditional heteroscedasticity models, and a new model. *Journal of Business & Economic Statistics*, 22(1), 16–29. doi:10.1198/073500103288619359

Davidson, J., & Rambaccussing, D. (2015). A test of the long memory hypothesis based on self-similarity. *Journal of Time Series Econometrics*, 7(2), 115–141. doi:10.1515/jtse-2013-0036

Deo, R. S., & Hurvich, C. M. (2001). On the log periodogram regression estimator of the mem- ory parameter in long memory stochastic volatility models. *Econometric Theory*, 17(4), 688–710. doi:10.1017/S026646660174025

Diebold, F. X., & Inoue, A. (2001). Long memory and regime switching. *Journal of Econometrics*, 105(1), 131–159. doi:10.1016/S0304-4076(01)00073-2

Ding, Z., & Granger, C. W. (1996). Modeling volatility persis- tence of speculative returns: A new approach. *Journal of Econometrics*, 73(1), 185–215. doi:10.1016/0304-4076(95)01737-2

Elder, J., & Serletis, A. (2008). Long memory in energy futures prices. *Review of Financial Economics*, 17(2), 146–155. doi:10.1016/j.rfe.2006.10.002

Fleming, J., & Kirby, C. (2011). Long memory in volatility and trading volume. *Journal of Banking & Finance*, 35(7), 1714–1726. doi:10.1016/j.jbankfin.2010.11.007

Flóras, C., Jaffry, S., & Valle Lima, G. (2007). Long memory in the portuguese stock market. *Studies in Economics*
Kirman, A., & Teysiere, G. (2002). Microeconomic models for long memory in the volatility of financial time series. *Studies in Nonlinear Dynamics & Econometrics, 6*(4). doi:10.2202/1558-3708.1083

Koop, G., Ley, E., Osiewalski, J., & Steel, M. F. (1997). Bayesian analysis of long memory and persistence using arfima models. *Journal of Econometrics, 76* (1–2), 149–169. doi:10.1016/S0304-4076(97)01787-9

Koutris, Z., & Veloce, W. (1996). Unemployment hysteresis in Canada: An approach based on long-memory time series models. *Applied Economics, 28*(7), 823–831. doi:10.1080/00036849632826263

Kuan, C.-M., & Hsu, C.-C. (1992). Change-point estimation of fractionally integrated processes. *Journal of Time Series Analysis, 13*(6), 693–708. doi:10.1111/j.1467-9892.1992.tb00117.x

Kuswanto, H. (2001). A new simple test against spurious long memory using temporal aggregation. *Journal of Statistical Computation and Simulation, 81*(10), 1297–1311. doi:10.1080/000368406100034715

LeBaron, B. (2001). Stochastic volatility as a simple generator of apparent financial power laws and long memory. *Quantitative Finance, 16*(6), 621–631. doi:10.1080/14697680601150699

LeBaron, B., & Yamamoto, R. (2007). Long-memory in an order-driven market. *Physica A: Statistical Mechanics and Its Applications, 383*(1), 85–89. doi:10.1016/j.physa.2007.04.090

LeBaron, B., & Yamamoto, R. (2008). The impact of imitation on long memory in an order-driven market. *Eastern Economic Journal, 34*(4), 504–517. doi:10.1057/ejj.2008.32

Leccadito, A., Rachedi, O., & Urge, G. (2015). True versus spurious long memory: Some theoretical results and a monte carlo comparison. *Econometric Reviews, 34*(4), 452–479. doi:10.1080/07474938.2013.808462

Li, M., Li, W. K., & Li, G. (2015). A new hyperbolic garch model. *Journal of Econometrics, 189*(2), 428–436. doi:10.1016/j.jeconom.2015.03.034

Lillo, F., & Farmer, J. D. (2004). The long memory of the efficient market. *Studies in Nonlinear Dynamics & Econometrics, 8*(3). doi:10.2202/1558-3708.1226

Lillo, F., Mike, S., & Farmer, J. D. (2005). Theory for long memory in supply and demand. *Physical Review E, 71*(6), 066122. doi:10.1103/PhysRevE.71.066122

Lim, K.-P., & Brooks, R. D. (2010). Why do emerging stock markets experience more persistent price deviations from a random walk over time? A country-level analysis. *Macroeconomic Dynamics, 14*(S1), 3–41. doi:10.1017/S1365100509003979

Lim, K.-P., & Brooks, R. D. (2011). The evolution of stock market efficiency over time: A survey of the empirical literature. *Journal of Economic Surveys, 25*(1), 69–108. doi:10.1111/j.1467-6419.2009.00611.x

Lioy, K. H. (2009). Long-term memory in volatility: Some evidence from international securitized real estate markets. *The Journal of Real Estate Finance and Economics, 39*(4), 415. doi:10.1007/s11146-008-9120-8

Liu, M. (2000). Modeling long memory in stock market volatility. *Journal of Econometrics, 99*(3), 159–171. doi:10.1016/S0304-4076(00)00033-6

Liu, R., Demirer, R., Gupta, R., & Wohar, M. (2019). Volatility forecasting with bivariate multifractal models. *Journal of Forecasting*. doi:10.1002/jof.2169

Lo, A. W. (1991). Long-term memory in stock market prices. *Econometrica, 59*(5), 1279–1313. doi:10.2307/2938368

Lo, A. W. (2004). The adaptive markets hypothesis. *The Journal of Portfolio Management, 30*(5), 15–29. doi:10.3905/jpm.2004.442611

Lo, A. W. (2005). Reconciling efficient markets with behavioral finance: The adaptive markets hypothesis. *Journal of Investment Consulting, 7*(2), 21–44.

Lobato, I. N. (1997). Semiparametric estimation of seasonal long memory models: Theory and an application to the modeling of exchange rates. *Investigaciones Economicas, 21*(2), 273–295.

Lobato, I. N., & Savin, N. E. (1998). Real and spurious long-memory properties of stock-market data. *Journal of Business & Economic Statistics, 16*(3), 261–268. doi:10.1080/07350015908524479

Lux, T., & Kauzlaric, T. (2007). Forecasting volatility and volume in the Tokyo stock market: Long memory, fractality and regime switching. *Journal of Economic Dynamics and Control, 31*(6), 1808–1843. doi:10.1016/j.jedc.2007.01.010

Ma, F., Li, X., Yang, K., & Zhang, Y. (2019). Volatility forecasting: Long memory, regime switching and heteroscedasticity. *Applied Economics, 51*(38), 4151–4163. doi:10.1080/00036846.2019.1589645

Maddaloni, V., & Arrowatcha, R. (2016). Relative efficiency of g8 sovereign credit default swaps and bond scrips: An adaptive market hypothesis perspective. *Studies in Microeconomics, 4*(2), 127–150. doi:10.1177/232102216649479

Mallot, S. G. (1989). A theory for multisolution signal decomposition: The wavelet representation. *IEEE Transactions on Pattern Analysis & Machine Intelligence, 11*(7), 674–693. doi:10.1109/34.292463

Man, K. S. (2003). Long memory time series and short term forecasts. *International Journal of Forecasting, 19*(3), 477–491. doi:10.1016/S0169-2070(02)00060-2

Mandelbrot, B. B., Fisher, A. J., & Calvet, L. E. (1997). A multifractal model of asset returns (*Cowles Foundation Discussion Paper*, 1164). Retrieved from https://users.math.yale.edu/~bbm3/web_pdfs/Cowles1164.pdf

Mandelbrot, B. B., & Van Ness, J. W. (1968). Fractional brownian motions, fractional noises and applications. *SIAM Review, 10*(4), 422–437. doi:10.1137/1010093

Mandelbrot, B. B., & Wallis, J. R. (1968). Noah, joseph, and operational hydrology. *Water Resources Research, 4*(5), 909–918. doi:10.1029/WR004i005p00909

Mayoral, L. (2012). Testing for fractional integration versus short memory with structural breaks. *Oxford Bulletin of Economic and Statistics, 74*(2), 278–305. doi:10.1111/j.1468-0084.2011.00665.x

McMillan, D. G., & Speight, A. E. (2008). Long-memory in high-frequency exchange rate volatility under temporal aggregation. *Quantitative Finance, 8*(3), 251–261. doi:10.1080/14697680601150699

McMillan, D. G., & Thapayagole, P. (2009). The efficiency of African equity markets. *Studies in Economics and Finance, 26*(4), 275–292. doi:10.1108/10866730910957864

Meng, X., & Taylor, J. W. (2018). An approximate long-memory range-based approach for value at risk estimation. *International Journal of Forecasting, 34*(3), 377–388. doi:10.1016/j.ijforecast.2017.11.007

Mensi, W., Tiwari, A. K., & Al-Yahyaei, K. H. (2019). An analysis of the weak form efficiency, multi-fractality and long memory of global, regional and European stock markets. *The Quarterly Review of Economics and Finance, 72*, 168–177. doi:10.1016/j.qref.2018.12.001

Migliori, Z., Mansourii, F. (2014). Modeling international stock market contagion using multi–fractionally integrated arapch approach. *Cogent Economics & Finance, 2*(1), 963362. doi:10.1080/23322039.2014.963362
Müller, U. A., Dacorogna, M. M., Davé, R. D., Olsen, R. B., Pictet, O. V., & Von Weizsäcker, J. E. (1997a). Volatilities of different time resolutions—analyzing the dynamics of market components. Journal of Empirical Finance, 4(2–3), 213–239. doi:10.1016/S0927-5398(97)00007-8

Müller, U. A., Dacorogna, M. M., Davé, R. D., Olsen, R. B., Pictet, O. V., & Von Weizsäcker, J. E. (1997b). Volatilities of different time resolutions—analyzing the dynamics of market components. Journal of Empirical Finance, 4(2–3), 213–239. doi:10.1016/S0927-5398(97)00007-8

Müller, U. A., Dacorogna, M. M., Davé, R. D., Pictet, O. V., Olsen, R. B., & Ward, J. R. (1993). Fractals and intrinsic time—A challenge to econometricians. In 39th International AEA Conference on Real Time Econometrics. Retrieved from http://finance.martinsewell.com/stylized-facts/scaling/Muller-etal1993.pdf

Nguyen, D. B. B., Prokopczuk, M., & Sibbertsen, P. (2019). The memory of stock return volatility: Asset pricing implications. Journal of Financial Markets. doi:10.1016/j.finmar.2019.01.002

Nystrup, P., Madsen, H., & Lindström, E. (2017). Long memory of financial time series and hidden markov models with time-varying parameters. Journal of Forecasting, 36(8), 989–1002. doi:10.1002/for.2447

Ohanissian, A., Russell, J. R., & Tsay, R. S. (2008). True or spurious long memory? A new test. Journal of Business & Economic Statistics, 26(2), 161–175. doi:10.1198/073500107000000340

Ozun, A., & Cifter, A. (2008). Modeling long-term memory effect in stock prices: A comparative analysis with gph test and daubechies wavelets. Studies in Economics and Finance, 25(1), 38–48. doi:10.1108/1086730810857559

Peng, C.-K., Buldyrev, S. V., Havlin, S., Simons, M., Stanley, H. E., & Goldberger, A. L. (1994). Mosaic organization of DNA nucleotides. Physical Review E, 49(2), 1685. doi:10.1103/PhysRevE.49.1685

Perron, P., & Qu, Z. (2010). Long-memory and level shifts in the volatility of stock market return indices. Journal of Business & Economic Statistics, 28(2), 275–290. doi:10.1198/jbes.2009.06171

Podobnik, B., & Stanley, H. E. (2008). Detrended cross-correlation analysis: A new method for analyzing two nonstationary time series. Physical Review Letters, 100(8), 084102. doi:10.1103/PhysRevLett.100.084102

Potter, K. W. (1979). Annual precipitation in the northeast united states: Long memory, short memory, or no memory? Water Resources Research, 15(2), 340–346. doi:10.1029/WR015i002p00340

Qu, Z. (2011). A test against spurious long memory. Journal of Business & Economic Statistics, 29(3), 433–438. doi:10.1198/jbes.2010.09153

Raei, R., & Mohammad, S. (2008). Fractional return and fractional cointegration. Applied Financial Economics Letters, 4(4), 269–275. doi:10.1080/17464540701720527

Ravishanker, N., & Ray, B. K. (1997). Bayesian analysis of vector arima processes. Australian Journal of Statistics, 39(3), 295–311. doi:10.1111/j.1467-842X.1997.tb00693.x

Ray, B. K. (1993). Modeling long-memory processes for optimal long-range prediction. Journal of Time Series Analysis, 14(5), 511–525. doi:10.1111/j.1467-9892.1993.tb00161.x

Rea, W., Oxley, L., Reale, M., & Brown, J. (2013). Not all estimators are born equal: The empirical properties of some estimators of long memory. Mathematics and computers in Simulation, 93, 29–42. doi:10.1016/j.matcom.2012.08.005

Reisen, V. A. (1994). Estimation of the fractional difference parameter in the arma (p,q,d) model using the smoothed periodogram. Journal of Time Series Analysis, 15(3), 335–350. doi:10.1111/j.1467-9892.1994.tb00198.x

Robinson, P. M. (1991). Testing for strong serial correlation and dynamic conditional heteroskedasticity in multiple regression. Journal of Econometrics, 47(1), 67–84. doi:10.1016/0304-4076(91)90078-R

Robinson, P. M. (1994). Semiparametric analysis of long-memory time series. The Annals of Statistics, 22(1), 515–539. doi:10.1214/aos/1176325382

Robinson, P. M. (1995). Gaussian semiparametric estimation of long range dependence. The Annals of Statistics, 23(5), 1630–1661. doi:10.1214/aos/1176324317

Saidie, S., & Silvapulle, P. (2001). Long-term memory in stock market returns: International evidence. International Journal of Finance & Economics, 6(1), 59–67. doi:10.1002/1jfe.143

Schennach, S. M. (2016). Long memory via networking. Econometrica, 86(6), 2221–2248. doi:10.3982/ECTA11930

Sensoy, A., & Tobak, B. M. (2016). Dynamic efficiency of stock markets and exchange rates. International Review of Financial Analysis, 47, 353–371. doi:10.1016/j.irfa.2016.06.001

Shao, X. (2011). A simple test of changes in mean in the possible presence of long-range dependence. Journal of Time Series Analysis, 32(6), 598–606. doi:10.1111/j.1467-9892.2010.00717.x

Shi, Y., & Ho, K.-Y. (2015). Long memory and regime switching: A simulation study on the markov-regime-switching arma model. Journal of Banking & Finance, 61, S189–S204. doi:10.1016/j.jbankfin.2015.08.025

Shimotsu, K. (2006). Simple (but effective) tests of long memory versus structural breaks. (Queen’s Economics Department Working Paper). Retrieved from https://www.econstor.eu/bitstream/10419/118937/1/qed_wp_1101.pdf

Sibbertsen, P. (2004). Long memory versus structural breaks: An overview. Statistical Papers, 45(4), 465–515. doi:10.1007/BF02760564

Sibbertsen, P., Leschinski, C., & Busch, M. (2018). A multivariate test against spurious long memory. Journal of Econometrics, 203(1), 33–49. doi:10.1016/j.jeconom.2017.07.005

Smith, J., & Yadav, S. (1994). Forecasting costs incurred from unit differencing fractionally integrated processes. International Journal of Forecasting, 10(4), 507–514. doi:10.1016/0169-2070(94)90019-1

Souza, L. R. (2008). Why aggregate long memory time series? Econometric Reviews, 27(1–3), 298–316. doi:10.1080/07474930701783408

Souza, L. R., & Smith, J. (2002). Bias in the memory parameter for different sampling rates. Journal of Econometrics, 109(2), 299–313. doi:10.1016/S0304-4076(01)00160-1

Taqqu, M. S., Teverovsky, V., & Willinger, W. (1995). Estimators for long-range dependence: An empirical study. Fractals, 3(4), 785–798. doi:10.1142/S0218348X95000692

Teverovsky, V., Taqqu, M. S., & Willinger, W. (1999). A critical look at lo’s modified r/s statistic. Journal of Statistical Planning and Inference, 80(1–2), 211–227. doi:10.1016/S0378-3758(98)00250-X
Teyssiére, G. (1997). Double long-memory financial time series. Queen Mary and Westfield College: Department of Economics.

Walter, T., Klein, T., Thu, H. P., & Plontek, K. (2017). True or spurious long memory in European non-EMU currencies. Research in International Business and Finance, 40, 217–230. doi:10.1016/j.ribaf.2017.01.003

Wen, F., Gong, X., & Cai, S. (2016). Forecasting the volatility of crude oil futures using HAR-type models with structural breaks. Energy Economics, 59, 400–413. doi:10.1016/j.eneco.2016.07.014

Wenger, K., Leschinski, C., & Sibbertsen, P. (2018a). Change-in-mean tests in long-memory time series: A review of recent developments. Advances in Statistical Analysis, 103(2), 237–256. doi:10.1007/s10182-018-0328-5

Wenger, K., Leschinski, C., & Sibbertsen, P. (2018b). A simple test on structural change in long-memory time series. Economics Letters, 163, 90–94. doi:10.1016/j.econlet.2017.12.007

Whittle, P. (1951). Hypothesis testing in time series analysis. Uppsala, Sweden: Almqvist & Wiksells.

Wright, J. H. (1998). Testing for a structural break at unknown date with long-memory disturbances. Journal of Time Series Analysis, 19(3), 369–376. doi:10.1111/1467-9892.00097

Wright, J. H. (2002). Log-periodogram estimation of long memory volatility dependencies with conditionally heavy tailed returns. Econometric Reviews, 21(4), 397–417. doi:10.1081/ETC-120015382

Zaffaroni, P. (2004). Contemporaneous aggregation of linear dynamic models in large economies. Journal of Econometrics, 120(1), 75–102. doi:10.1016/S0304-4076(03)00207-0

Zhao, L.-T., Liu, K., Duan, X.-L., & Li, M.-F. (2019). Oil price risk evaluation using a novel hybrid model based on time-varying long memory. Energy Economics, 81, 70–78. doi:10.1016/j.eneco.2019.03.019

Zheng, M., Liu, R., & Li, Y. (2018). Long memory in financial markets: A heterogeneous agent model perspective. International Review of Financial Analysis, 58, 38–51. doi:10.1016/j.irfa.2018.04.001