Leptogenesis and baryon asymmetry in the early Universe for the case arbitrary hypermagnetic helicity

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Abstract. We study leptogenesis and baryon asymmetry generation in plasma of the early Universe before the electroweak phase transition (EWPT) accounting for chirality flip processes via inverse Higgs decays and sphaleron transitions which violate the left lepton number and wash out the baryon asymmetry of the Universe (BAU). The hypermagnetic helicity evolution proceeds in a self-consistent way with the lepton asymmetry growth. The hypermagnetic helicity plays a key role in lepto/baryogenesis in our scenario and the more hypermagnetic field is close to the maximum helical one the faster BAU grows up the observable value , \( B_{obs} \sim 10^{-10} \).

1. Introduction
The generation baryon asymmetry of Universe (BAU) still open problem in cosmology, it is necessary exist processes with baryon-number violation [1]. These processes appear in the Standard Model (SM) by quantum anomalies [2, 3]. In other hand there is a question origin seed magnetic field in early Universe, and cosmology approach for it: seed magnetic field may be originated from hypercharge field at electroweak phase transition (EWPT). Many papers have investigated the connection between a primordial magnetic field (PMF) and the baryon asymmetry of the universe (BAU) [4] - [21].

We study the hot universe plasma before EWPT at the stage \( T_{RL} > T > T_{EW} \), when left leptons in the SM doublet \( L = (\nu_{eL}, e_L)^T \) enter equilibrium with right electrons \( e_R \) due to inverse Higgs decays like \( e_R e_L \rightarrow \varphi(0), e_R \bar{\nu}_{eL} \rightarrow \varphi(-) \). This happens during universe cooling just at the temperature \( T_{RL} \sim 10 \) TeV when the rate of chirality flip \( \Gamma_{RL} \sim T \) becomes bigger than the Hubble expansion \( H \sim T^2, \Gamma_{RL} \geq H \). This leads to an additional polarization effect given by left lepton macroscopic currents in a seed hypermagnetic field \( B_Y \), \( J^{(e)}_{i5} = \prec \tilde{\psi}_{eL} \gamma_5 \gamma_3 \psi_{eL} > \sim \mu_{eL} B_Y^i \), \( J^{(\nu)}_{i5} = \prec \tilde{\nu}_{eL} \gamma_5 \gamma_3 \nu_{eL} > \sim \mu_{eL} B_Y^i \), where electron chemical potential \( \mu_{eL} \) coincides with the neutrino one, \( \mu_{eL} = \mu_{\nu_{eL}} \).

Accounting for the evolution of the left lepton asymmetry \( (n_{eL} - n_{\bar{e}L}) \sim \mu_{eL}(t) \) at temperatures \( T_{EW} < T < T_{RL} \) given by the left electron chemical potential \( \mu_{eL}(t) \neq 0 \) which evolves due to the Abelian anomaly and undergoes sphaleron influence.
The goal of the present work is a description of BAU evolution down to the EWPT time accounting for the hypermagnetic diffusion for a Kazantsev’s spectra of the energy density $\rho_Y(k,t)$.

2. Leptons asymmetry evolution

In the rest frame of the medium as a whole the Faraday induction equation governing hypermagnetic fields $B_Y = \nabla \times Y$ reads

$$\frac{\partial B_Y}{\partial t} = \nabla \times \alpha_Y B_Y + \eta_Y \nabla^2 B_Y,$$

where at the temperatures $T_{RL} > T > T_{EW}$ the hypermagnetic helicity coefficient $\alpha_Y$ is given by the right and left electron chemical potentials $\mu_{eR}, \mu_{eL}$

$$\alpha_Y(T) = +\frac{g^2(\mu_{eR} - \mu_{eL})}{4\pi^2\sigma_{cond}/2},$$

and $\eta_Y = (\sigma_{cond})^{-1}$ is the hypermagnetic diffusion coefficient, $\sigma_{cond}(T) \simeq 100T$ is the hot plasma conductivity. Let us stress that $\alpha_Y$ -effect in Faraday equation (1) arises due to Abelian anomalies for right and left electron (neutrino) currents both nonpersistent in the presence of a hypercharge field $Y_\mu$ at the temperatures $T < T_{RL}$. Multiplying equation (1) by the corresponding vector potential and adding the analogous construction produced by evolution equation governing the vector potential (multiplied by hypermagnetic field) after integration over space we get the evolution equation for the hypermagnetic helicity $H_Y = \int d^3x Y \cdot B_Y$:

$$\frac{dH_Y}{dt} = -2 \int_V (E_Y \cdot B_Y) d^3x - \oint [Y_0 B_Y + E_Y \times Y] d^2S =$$

$$= -2\eta_Y(t) \int d^3x (\nabla \times B_Y) \cdot B_Y + 2\alpha_Y(t) \int d^3x B_Y^2(t).$$

For the single symmetric phase before EWPT we have just omitted in the last line in equation (3) the surface integral $\oint (\ldots)$ since hypercharge fields vanish at infinity. However, such surface integral can be important at the boundaries of different phases during EWPT, $T \sim T_{EW}$.

Let us change physical variables to the conformal ones using the conformal time $\eta = M_0/T$, $M_0 = M_0/1.166\sqrt{g^*}$, where $M_0 = 1.2 \times 10^{19}$ GeV is the Plank mass, $g^* = 106.75$ is the effective number of relativistic degrees of freedom.

In FRW metric $ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2)$ using definitions $a = T^{-1}$, $a_0 = 1$ at the present temperature $T_{now}$, $d\eta = dt/a(t)$, we input the following notations: $k = ka = const$ is the conformal momentum (giving a red shift for the physical one, $k \sim T = T_{now}(1 + z)$); $\xi_a(\eta) = a\mu_a = \mu_a / T$ is the dimensionless fermion anomaly changing over time; $\tilde{B}_Y = a^2 B_Y$, $Y = aY$ are the conformal dimensionless counterparts of hypermagnetic field and hypercharge potential correspondingly.

It is suitable to rewrite (3) using the conformal coordinate $\tilde{x} = x/a$ for the Fourier components of the helicity density $^1$, $\tilde{h}_Y(\eta) \equiv \int (\tilde{Y} \cdot B_Y) d^3x/V = \int d\mathbf{k} h_Y(k,\eta)$, and the hypermagnetic energy density $\tilde{\rho}_{B_Y}(\eta) = \tilde{B}_Y^2(\eta)/2 = \int d\mathbf{k} \tilde{\rho}_{B_Y}(k,\eta)$ defined as their spectra:

$$\tilde{h}_Y(k,\eta) = \frac{\tilde{k}^2 a^3}{2\pi^2 V} \tilde{Y}(k,\eta) \cdot \tilde{B}_Y^*(k,\eta),$$

$$\tilde{\rho}_{B_Y}(k,\eta) = \frac{k^2 a^3}{4\pi^2 V} \tilde{B}(k,\eta) \cdot \tilde{B}_Y^*(\tilde{k},\eta).$$

$^1$ Note that exponents $e^{ikx} = e^{ik\tilde{x}}$ coincide in Fourier integrals both in usual variables and in the conformal ones.
We consider inverse Higgs decays only or we neglect the Higgs boson asymmetry, $\mu_0 = 0$. The system of kinetic equations for leptons accounting for Abelian anomalies for right electrons and left electrons (neutrinos), inverse Higgs decays and sphaleron transitions as well, takes the form:

\[
\frac{dL_{eR}}{dt} = \frac{g^2}{4\pi^2s}(E_Y \cdot B_Y) + 2\Gamma_{RL} \{L_{eL} - L_{eR}\},
\]

\[
\frac{dL_{eL}}{dt} = -\frac{g^2}{16\pi^2s}(E_Y \cdot B_Y) + \Gamma_{RL} \{L_{eR} - L_{eL}\} - \left(\frac{\Gamma_{sph} T}{2}\right)L_{eL}.
\]

Here $L_b = (n_b - \bar{n}_b)/s \approx T^3\zeta_b/6s$ is the lepton number, $b = e_R, e_L, \nu_L$, $s = 2\pi^2g^*T^3/45$ is the entropy density, and $g^* = 106.75$ is the number of relativistic degrees of freedom. The factor=2 in the first line takes into account the equivalent reaction branches, $e_R\bar{e}_L \rightarrow \tilde{\varphi}^{(0)}$ and $e_R\bar{e}_L \rightarrow \tilde{\varphi}^{(-)}$; $\Gamma_{RL}$ is the chirality flip rate. Of course, for the left doublet $L_b^e = (\nu_L, e_L)$ kinetic equation for neutrino number is excess because $L_{eL} = L_{eL}$. Then $\Gamma_{sph} = C\alpha_W^0 C(3.2 \times 10^{-8})$ is the dimensionless probability of sphaleron transitions decreasing the left lepton numbers and therefore washing out baryon asymmetry of universe (BAU). It is given by the $SU(2)_W$ constant $\alpha_W = g^2/4\pi = 1/137\sin^2\theta_W = 3.17 \times 10^{-2}$ where $g = e/\sin\theta_W$ is the gauge coupling in SM and the constant $C \approx 25$ is estimated through lattice calculations (see, e.g., the chapter 11 in the book [22]).

In conformal variables after integration of the system (5) over volume $\int d^3x(...)/V$, transferring to the Fourier components for hypercharge fields the kinetic equations (5) take the form

\[
\frac{d\xi_{eR}(\eta)}{d\eta} = -\frac{3a'}{\pi} \int \frac{d\tilde{h}Y(k, \eta)}{d\eta} - \Gamma\left[\xi_{eR}(\eta) - \xi_{eL}(\eta)\right],
\]

\[
\frac{d\xi_{eL}(\eta)}{d\eta} = \frac{3a'}{4\pi} \int \frac{d\tilde{h}Y(k, \eta)}{d\eta} - \frac{\Gamma(\eta)}{2}\left[\xi_{eL}(\eta) - \xi_{eR}(\eta)\right] - \frac{\Gamma_{sph}}{2}\xi_{eL}(\eta),
\]

where

\[
\Gamma(\eta) = \left(\frac{242}{\eta_{EW}}\right) \left[1 - \left(\frac{\eta}{\eta_{EW}}\right)^2\right], \quad \eta_{RL} < \eta < \eta_{EW}
\]

is the dimensionless chirality flip rate $\Gamma = 2a\Gamma_{RL}$ [20], $\eta_{EW} = M_0/T_{EW} = 7 \times 10^{15}$ is the EWPT time at $T_{EW} = 100$ GeV.

We choose the following initial conditions at the time $\eta_0 = \eta_{RL} = 7 \times 10^{13}$ that corresponds to the temperature $T_{RL} = 10$ TeV:

\[
\xi_{eL}(\eta_0) = 0, \quad \xi_{eR}(\eta_0) = 10^{-20}.
\]

3. Numerical solution

Well, general system of equation in conformal variables for arbitrary initial initiality has form
\[
\frac{d\tilde{h}_Y(\tilde{k}, \eta)}{d\eta} = -\frac{2k^2}{\sigma_c} \tilde{h}_Y(\tilde{k}, \eta) + \left(\frac{4\alpha' (\xi_{eR} + \xi_{eL}/2)}{\pi \sigma_c}\right) \tilde{\rho}_{B_Y}(\tilde{k}, \eta),
\]
\[
\frac{d\tilde{\rho}_{B_Y}(\tilde{k}, \eta)}{d\eta} = -\frac{2k^2}{\sigma_c} \tilde{\rho}_{B_Y}(\tilde{k}, \eta) + \left(\frac{\alpha' (\xi_{eR} + \xi_{eL}/2)}{\pi^2 \sigma_c}\right) \tilde{k}^2 \tilde{h}_Y(\tilde{k}, \eta),
\]
\[
\frac{d\xi_{eL}(\eta)}{d\eta} = -\frac{3\alpha'}{\pi \sigma_c} \int d\tilde{k} \tilde{k}^2 \tilde{h}_Y(\tilde{k}, \eta) + \frac{3\alpha'^2}{\pi^2 \sigma_c} \tilde{\rho}_{B_Y}(\eta) \left(\xi_{eR} + \xi_{eL}/2\right) - \frac{\Gamma}{2} (\xi_{eL} - \xi_{eR}) - \frac{\Gamma_{sph}}{2} \xi_{eL}(\eta),
\]
\[
\frac{d\xi_{eR}(\eta)}{d\eta} = \frac{12\alpha'}{\pi \sigma_c} \int d\tilde{k} \tilde{k}^2 \tilde{h}_Y(\tilde{k}, \eta) - \frac{12\alpha'^2}{\pi^2 \sigma_c} \tilde{\rho}_{B_Y}(\eta) \left(\xi_{eR} + \xi_{eL}/2\right) - \Gamma (\xi_{eR} - \xi_{eL}).
\]

and initial conditions:
\[
\xi_{eR}(\eta_0) = 10^{-20}, \quad \xi_{eL}(\eta_0) = 0, \quad \tilde{\rho}_{B_Y}(\eta_0) = A \tilde{k}^{n_s}, \quad \tilde{h}_Y(\tilde{k}, \eta_0) = q \cdot 2 \tilde{\rho}_{B_Y}(\tilde{k}, \eta_0)/k, \quad \text{where} \quad A \quad \text{is defined by the relation} \quad \int_{k_{min}}^{k_{max}} \tilde{\rho}_{B_Y}(k, \eta_0) \cdot dk = B_0^2/2.
\]

This system will be investigated numerically. In our previous papers we studied monochromatic spectrum and Kolmogorov spectrum. It was shown that \(\xi_{eR}\) grows from small initial values \(\xi_{eR}(\eta_0) = 10^{-10}\), and BAU reaches observed value \(10^{-10}\).

3.1. Results

Kazantsev spectrum was considered in some papers (see, for example, [24]). Here we study both right leptons asymmetry evolution and BAU evolution before EWPT, when initial value \(\xi_{eR}\) is very small - \(\xi_{eR}(\eta_0) = 10^{-20}\), and initial hypermagnetic helicity changes from 0.01 to 1.

**Figure 1.** Right asymmetry and BAU evolution for a set of \(q\). Left panel: Right asymmetry evolution. Right panel: BAU evolution. Red line corresponds \(q=1\) (full initial helicity), green line - \(q = 0.1\), blue line - \(q = 0.01\).

In the left panel in figure 1, we show solutions of kinetic equations for the right electron asymmetry \(\xi_{eR}(\eta)\) (on a logarithmic scale), and in the right panel - \(\lg(\text{BAU})\).

In our model we accounted for the weak sphaleron only which mediates a vacuum-vacuum transition in the \(SU(2)_L\) sector and induces reactions among the weakly interacting particles (left electrons and left (neutrinos) with the rate \(\Gamma_{sph} \sim 25\alpha_W T\)). In this work, we checked the
Kazantsev spectrum to ensure that the right lepton asymmetry could grow down to the EWPT and BAU reaches the observed values $10^{-10}$. There is a common conclusion that for a more helical HMF the lepto/baryo-genesis proceeds more faster, and we demonstrated such issue here for a Kazantsev spectrum.

4. References
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