Two-level system with noise: Blue’s function approach.

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Abstract

By using the random matrix approach and generalized Blue’s function representation we solve analytically the model of an effective two-level system coupled to a noise reservoir. We show that calculated spectral properties of the system are in agreement with the numerically simulated results. We outline possible applications of the model in the field of condensed phase reactions.


1 Introduction

The theory of open quantum systems has been found to play a key role in many areas of theoretical physics [1]. The growing interest focuses on quantum dynamics in condensed phase systems, to which the most widely used approach is the reduced-density-matrix theory [2, 3, 4, 5]. While a microscopic treatment of dissipative motion involves hermitean Hamiltonians, use of non-hermitean Hamiltonians serves as a convenient way to give a reduced description where the hermitean part of the Hamiltonian refers to free undamped dynamics and the remainder describes a damping imposed on the system by some external noise source (“heat bath”).

Statistical properties of complex systems can be successfully investigated within the framework of the random-matrix theory [6] which turned out to be quite general and a powerful phenomenological approach to a description of various phenomena such as quantum chaos [7], complex nuclei [8], chaotic scattering [5] and mesoscopic physics [4]. Aspects of vastly different physical situations such as electron localization phenomena in disordered conductors and semiconductors [9], disordered quantum wires [10] and quantum Hall effect [11] can be described in the language of the random matrix theory. In all the realms mentioned above, the Hamiltonian of the system is rather intricate to be handled or simply unknown. In such cases the integration of the exact equations is replaced by the study of the joint distribution function of the matrix elements of the Hamiltonian $P(H)$. If there is no preferential basis in the space of matrix elements, so that the system is as random as possible and equal weights are given to all kinds of interactions, one has to require the probability $P(H)dH$ to be invariant under similarity transformations $H \rightarrow R^{-1}HR$ with $R$ being orthogonal, unitary or a symplectic $N \times N$ matrix reflecting the fundamental symmetry of the underlying Hamiltonian.

The natural way of addressing the problems of randomness coupled to various sources is to use technique of the “free random variables” [12, 13, 14, 15]. This method provides an elegant way of “linearizing” the process of determining the average eigenvalue distributions for convolutions representing an analogue of the logarithm of Fourier transformation of the usual convolutions. Recently, the generalization of the “addition law” for hermitean random matrices to the non-hermitean case has been derived [15].

In section 2, we introduce, after Zee [13] and Janik et al. [15] basic concepts of the
“addition laws”, for both hermitean and non-hermitean ensembles.

In section 3, we apply the addition law to the dissipative system of a two-level deterministic Hamiltonian coupled to a random noise. The effective Hamiltonian is obtained by the standard Wigner–Weisskopf reduction [14, 17, 18] by partition. In this method the total Hilbert space is divided into two subspaces and the Hamiltonian is integrated over the elements of one of them, eventually mapping the eigenvalue problem of high dimension onto a lower one. Models of such type are widely used in e.g. condensed phase dynamics [19, 20] where one is typically interested in the detailed dynamics of only a small part of the overall system, so one partitions the total system into the subsystem of interest and a bath:

\[ H = H_{\text{sys}} + H_{\text{bath}} + V \]  (1.1)

Here \( H_{\text{sys}} \) and \( H_{\text{bath}} \) are the Hamiltonians of the isolated subsystem and the bath; \( V \) describes their interaction. The appropriate scheme for partitioning is not always obvious [1]; successful choice minimizes the coupling \( V \) so that its effects can be studied perturbatively. After the separation, the state of the system is described by its reduced density matrix which is obtained from the complete system-plus-bath density by taking the trace over the bath variables. When the relaxation of the bath is rapid compared to the evolution of the system, the bath can be assumed to be equilibrated at all times. Treating the interaction of the system and bath perturbatively to the second order, the evolution of the reduced density matrix can be then recasted in the form of the dissipative Liouville-von-Neumann equation [3].

Similar ideology is used e.g. to study the electron or wave passage in disordered medium. In the field of chemical physics the condensed phase reactions, such as electron and proton transfers are of primary interest and are commonly investigated in terms of a dissipative two-level system, often referred to as spin-boson problem [19, 21].

We solve the analogue of this problem by investigating analytically an effective two-level system coupled to a Gaussian dissipative term. We demonstrate the phase transition in such a system corresponding to the structural change in the average spectral distribution of the eigenvalues. We present also the results of the numerical simulations, confirming our analytical results. Finally, section 5 outlines the possible applications and generalizations of the considered model.
2 Addition laws for random ensembles.

The generic problem of finding the Green’s function of a sum of two independent hermitean matrices (here Hamiltonians) of dimension \( N \)

\[
\phi_1 + \phi_2
\]  

(2.2)

has been considered in the seminal work by Voiculescu [12], and recently popularized by Zee [13].

The main idea of Voiculescu was to find an elegant way of “linearizing” the process of determining the average eigenvalue distribution for the convolution of non-commuting operators, hence finding an analogue of the logarithm of Fourier transformation for the usual convolutions. As an example, let us consider two hermitean operators in matrix representation, (2.2), populated from the statistical ensemble \( P(\phi_1, \phi_2) \), provided that in the large \( N \) limit \( P(\phi_1, \phi_2) \) factorizes into \( P_1(\phi_1) \cdot P_2(\phi_2) \). Then the resolvent

\[
G(z) := \frac{1}{N} \int d\phi_1 d\phi_2 P(\phi_1, \phi_2) \text{Tr} \frac{1}{z - \phi_1 - \phi_2}
\]  

(2.3)

could be expressed (at least \textit{a priori}) knowing only the individual resolvents

\[
G_i(z) = \frac{1}{N} \int d\phi_i P_i(\phi_i) \text{Tr} \frac{1}{z - \phi_i} \quad i = 1, 2.
\]  

(2.4)

The physical interpretation of the addition law comes from the observation that

\[
G(z) = \frac{1}{N} \text{Tr} \left\langle \frac{1}{z - \phi} \right\rangle \equiv \frac{1}{z - \Sigma(z)}
\]  

(2.5)

where

\[
\langle \ldots \rangle = \int P(\phi) \ldots [d\phi]
\]  

(2.6)

and \( \Sigma(z) \) is nothing but the self-energy. Through his diagrammatic analysis, Zee [13] introduced the ”Blue’s function” that is just the functional inverse of the resolvent

\[
B(G(z)) = z
\]  

(2.7)

and satisfies the additivity law

\[
B_{1+2}(z) = B_1(z) + B_2(z) - \frac{1}{z}
\]  

(2.8)
The operational algorithm is extremely simple when using the Blue’s function approach: First, having both ensembles, construct the two resolvents \( G_1 \) and \( G_2 \). Second, invert them functionally, resulting in \( B_1, B_2 \). Third, add them using the “addition law” (2.8). Fourth, invert functionally the \( B_{1+2} \), getting the Green function for the sum \( G_{1+2} \equiv G \). Fifth, get the average spectral distribution from \( G \), using the standard construction
\[
\varrho(\epsilon) = -\frac{1}{\pi} \lim_{\lambda \to 0} \text{Im} G(z)|_{z=\epsilon+i\lambda}.
\] (2.9)
The generalization for the nonhermitean case amounts to considering the isospin Green functions which now form a 2 by 2 matrix in isospin space, indexed by \( z \) and \( \bar{z} \).

The generalized Blue’s function \[15\] is now a matrix valued function of a 2 \( \times \) 2 matrix variable defined by
\[
B(G) = Z = \begin{pmatrix} z & 0 \\ 0 & \bar{z} \end{pmatrix}
\] (2.10)
The Green’s function is found from the matrix version of (2.8):
\[
\begin{pmatrix} z & 0 \\ 0 & \bar{z} \end{pmatrix} = B_1(G) + B_2(G) - \frac{1}{G}
\] (2.11)
In general, two kinds of the solutions to this matrix equations are possible. The first, when \( G \) is diagonal, reproduces the hermitean construction modulo the anti-analytical copy in \( \bar{z} \), totally decoupled from the \( z \)-solution. Therefore this one reproduces the hermitean (or antihermitean case) only. The relevant solution is the one when the off-diagonal element of the Green function is non-zero. In this case, the resolvent is nonanalytic, being the function of both variables \( z, \bar{z} \). The support of the eigenvalues forms the two-dimensional “islands” of nonanalyticity, (opposite to the hermitean case, when only cuts constitute the support of real eigenvalues) and the density distribution could be read-out from the 2-dimensional “Gauss law” \[22\]
\[
\text{divE} = 4\pi \varrho(z, \bar{z})
\] (2.12)
In this two-dimensional electrostatics divergence means differentiation with respect to \( \bar{z} \), and electric field has two components, \( E_x = 2\text{Re} G_{zz}, E_y = -2\text{Im} G_{zz} \).

Note that the non-analytical solution is a consequence of the “spontaneous breakdown of analyticity” \[15\]. Naively, the resolvent (2.8) is the function of \( z \) variable only. But the true ground state is not respecting holomorphic separability into \( z \) and \( \bar{z} \).
copies, resulting in the spontaneous breakdown of this “symmetry”, by forming the mixed “condensates” $\mathcal{G}_{zz}$ and therefore breaking the analyticity (non-analyticity) of the solution. Below we demonstrate this scenario in the case of Wigner–Weisskopf reduction.

3 Dissipative two-level system.

We consider the Hamiltonian with the Hilbert space spanned by $N$ discrete states $|k\rangle$ and $M$ continua $|\chi_n(E)\rangle$:

$$
\mathcal{H} = \sum_{k=1}^{N} E_k |\psi_k\rangle\langle \psi_k| + \sum_{n=1}^{M} \int dE E |\chi_n\rangle\langle \chi_n| \\
+ \gamma^{1/2} \sum_{n=1}^{M} \sum_{k=1}^{N} \int dE (V^n_k(E) |\psi_k\rangle\langle \chi_n| + h.c.).
$$

(3.13)

The bound continuum coupling is characterized by the coupling constant $\gamma$ and the energy dependent matrix $V^n_k(E)$. Note that in this model there is no direct coupling (interactions) between continuum channels $n,n'$. After eliminating the continua $|\chi_n(E)\rangle$, the effective Hamiltonian $H_{eff}$ takes the form

$$
H_{eff} = \mathcal{H} - i\gamma V V^\dagger,
$$

(3.14)

where $\mathcal{H}$ is $N \times N$ deterministic Hamiltonian with the bound-state energies $\epsilon_1, \ldots, \epsilon_N$ and the dissipative part is obtained via so-called Markovian limit. The operators $V^n_k = \langle \psi_k | V |\chi_n\rangle$ stand for the coupling of the bound-state eigenvectors $\psi_k$ ($k=1, \ldots, N$) corresponding to eigenvalues $\epsilon_1, \ldots, \epsilon_N$ with the continuum states $\chi_n$ ($n=1, \ldots, M$) and account for the depletion of the bound state subspace due to the leakage of the probability mass into the continua. The eigenvalues $\epsilon = x - iy$ of the effective Hamiltonian $H_{eff}$ are in general complex, their imaginary parts must be positive for damped systems and give the lifetimes of the corresponding eigenvalues proportional to $1/y$.

*In other words, the free energy associated with the non-analytical solution lies below the one related to the analytical resolvent in the domain of the complex plane.

†The Markovian property assumes exponential relaxation of the response of the continuum when it acts on the test system and rephrased in thermodynamic terms, would correspond to the high temperature Debye relaxation bath. In the above model, Markovian approximation should be understood as the condition for almost instantaneous decay of the kernel in the evolution (Schrödinger) equation satisfied by the bound-space state vectors $|\psi_k\rangle$. The condition is met if the energy dependence of the matrix elements $V^n_k(E)$ is negligible.
The model Hamiltonian of that type (3.14) is widely used in quantum chaotic scattering problems [5] but has been also introduced to study unimolecular dissociation of selectively excited polyatomic molecules [23, 24]. For most dissociating molecules above the tunneling regime (when the excitation energy is well above the dissociation threshold) the coupling between the molecular complex and the continuum is strong and the density of strongly mixed molecular states is large, so that the unimolecular decay is characterized by overlapping resonance states.

We will solve the system in the regime when both $M$ and $N$ are large, but $m = M/N$ is fixed (which implies strongly overlapping resonances). To simulate the effect of the "noise" represented by statistical properties of the operators $V, V^\dagger$ we will assume that the N by M matrices $V$ are random. For simplicity we will choose randomness as Gaussian $\text{G}$, thus working within the Gaussian unitary ensemble (GUE). Furthermore, we will investigate a two-level system, with deterministic eigenvalues $\epsilon_1 = \ldots = \epsilon_{N/2} \equiv \epsilon$ and $\epsilon_{N/2} = \ldots = \epsilon_N \equiv -\epsilon$.

To apply the addition law (2.11), we need only the functional form of the corresponding Blue’s functions. For the deterministic part, the Green function is by definition

$$G_D(Z) = \frac{1}{2} \left( \frac{1}{Z - \epsilon} + \frac{1}{Z + \epsilon} \right).$$

(3.15)

where $z$ has been replaced by the matrix $Z = \text{diag}(z, \bar{z})$. Therefore the relevant Blue’s function is given by the equation

$$Z = \frac{1}{2} \left( (B_D(Z) - \epsilon)^{-1} + (B_D(Z) + \epsilon)^{-1} \right).$$

(3.16)

The Blue’s function for the noise term $i\gamma VV^\dagger$ can be determined by use of diagrammatic techniques and has been first derived in [13]:

$$B_{VV^\dagger} = m (1 - \Gamma Z)^{-1} \Gamma + \frac{1}{Z}.$$  

(3.17)

where $\Gamma$ is the coupling matrix

$$\Gamma = \begin{pmatrix} -i\gamma & 0 \\ 0 & i\gamma \end{pmatrix}.$$  

(3.18)

\footnote{By use of the central limit theorem, Gaussianity is a legitimate choice for the infinite normalized sum of independent fluctuating contributions. It can be shown also that the distribution for normalized sum of matrices $\phi_{\text{sum}} = \frac{1}{\sqrt{k}} \sum_{i}^{k} \phi_i$, each sampled from a distribution $P_i(\phi_i)$ follows [13, 14] the Gauss theorem of probability. Using measures different from the Gaussian one do not change the addition law discussed in this section.}
Note that all above equations have been written in the matrix form, in agreement with the discussion of the previous section.

We can further use the generalized addition formula (2.11) for the total system, resulting in

\[ B(Z) = B_D(Z) + m(1 - \Gamma Z)^{-1}\Gamma. \]  

(3.19)

Inverting that relation (i.e. replacing \( Z \) by \( G \)), we get

\[ Z = B_D(G) + m(1 - \Gamma G)^{-1}\Gamma. \]  

(3.20)

By similar replacement in equation (3.16) we get

\[ G = \frac{1}{2} \left( (B_D(G) - \epsilon)^{-1} + (B_D(G) + \epsilon)^{-1} \right). \]  

(3.21)

Calculating \( B_D(G) \) from (3.20) and plugging into (3.21) we arrive at the final equation for the Green function

\[ G = \frac{1}{2} \left( (Z - m(1 - \Gamma G)^{-1}\Gamma - \epsilon)^{-1} + (Z - m(1 - \Gamma G)^{-1}\Gamma + \epsilon)^{-1} \right). \]  

(3.22)

The matrix equation (3.22) is a central formula to the paper from which all further discussion of spectral properties of the system will be derived. We solve this equation using the explicit parameterization of the matrix \( G \) with the unknown entries

\[ G = \begin{pmatrix} a & b \\ d & c \end{pmatrix}. \]  

(3.23)

As already mentioned in the previous section, the solutions belong to two classes:

- Holomorphic (analytical) case.

This yields a trivial solution to the problem. In this case, we seek solutions with an imposed condition \( b = d = 0 \). The above matrix equation splits then into two mirror copies of algebraic equations for \( z \) and \( \bar{z} \), respectively. The equation for \( a = G_{zz} \) is of Cardano type

\[ a^3(\epsilon^2\delta^2 - \delta^2z^2) + a^2(\delta^2z + 2\delta z^2 - 2m\delta^2z - 2\epsilon^2\delta) + a(m\delta^2 + 2m\delta z - m^2\delta^2 - 2\delta z - z^2 + \epsilon^2) + z - m\delta = 0 \]

\[ \delta = -i\gamma \]  

(3.24)
with the Blue’s function

\[ B(z) = \frac{m \delta}{1 - \delta z} + \frac{1}{2z} + \left( \frac{1}{4z^2} + \epsilon^2 \right)^{1/2}. \tag{3.25} \]

Solution to those equations constitute nothing but a complement to the dashed domains of density \( \varrho(z, \bar{z}) \) pictured in Fig. 1. We reject it as a non-physical one. The proper solution to our problem follows a non-analytical case which we are going to discuss now.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{The support of the eigenvalues of the two-state system for different coupling at fixed filling ratio \( m = 0.25 \). The dots correspond to the numerical simulation of the Hamiltonian (3.14), while the solid lines come from the analytical result (3.29). Numerical simulations have been performed for 100 matrices of size \( N = 100 \) at \( \gamma = 1, \gamma = 3 \) (the upper row of drawings), and for 30 matrices with \( N = 400 \) at \( \gamma = 6 \) and \( \gamma = 10 \) (the lower row). The density of the dots is related to the the spectral density (3.28).}
\end{figure}

- Non-holomorphic (non-analytical) case.

This case corresponds to a situation when, despite the resolvent naively is the function of the \( z \) variable only, the support of the spectrum depends on both \( z \) and \( \bar{z} \), forming therefore two-dimensional islands (in contrast to one-dimensional cuts of the analytical
solution) in the complex plane. The solution of the matrix equation now is given by

\[ a = \frac{x}{\gamma y} - \frac{x}{2(x^2 - \epsilon^2)} + i \left( \frac{1}{\gamma} + \frac{1}{y^2} \right) \left( \frac{x^2 - \epsilon^2}{y^2} \right) \]

\[ c = \bar{a} \]  \hspace{1cm} (3.26)

and \( b = d \), where \( b \) satisfies the equation

\[ b^2 = \frac{m}{\gamma y} + |a|^2 + \frac{1}{\gamma^2} + \frac{i}{\gamma}(a - \bar{a}) \].  \hspace{1cm} (3.27)

The spectral density obtained from Gauss law (2.12) reads

\[ \varrho = \frac{1}{4\pi} \left[ \frac{2}{\gamma y} \left( 1 + \frac{x^2 - \epsilon^2}{y^2} \right) + \frac{m-1}{y^2} + \frac{x^2 + \epsilon^2}{(x^2 - \epsilon^2)^2} \right] \]  \hspace{1cm} (3.28)

The shape of the envelopes (i.e. boundaries of 2-dimensional regions of spectral densities in the complex plane) is given by the condition \( b = G_{\bar{z}z} = 0 \) which is equivalent to

\[ \frac{x^2 - \epsilon^2 - \gamma y}{\gamma^2 y^2} = \frac{x^2}{4} \left( \frac{2}{\gamma y} - \frac{1}{x^2 - \epsilon^2} \right)^2 + \frac{1}{4} \left[ \frac{1}{\gamma} \left( 1 + \frac{x^2 - \epsilon^2}{y^2} \right) + \frac{m-1}{y} \right]^2 \].  \hspace{1cm} (3.29)

The spectrum undergoes two various phase transitions (cf. Fig. 1). For very low coupling, the energy islands do not connect the two states and only after reaching the critical value of the coupling \( \gamma \), a single island of the eigenvalues is being created (see the upper row of drawings in Fig. 1). We note, that the global energy scale \( \epsilon \) may be eliminated in (3.29) by introducing the dimensionless variables \( x' = x/\epsilon \), \( y' = y/\epsilon \) and \( \gamma' = \gamma/\epsilon \).

The second critical point (see the lower row of drawings in Fig. 1) corresponds to the situation when one island of non-analyticity splits into two, a low lying broad island and a narrow island of short-living states. The critical values of the coupling constant can be determined from the study of the discriminant of the boundary equation (3.29) at \( x = 0 \). The zero value of discriminant appears if distinct roots of the equation coincide, thus it is suggestive of occurrence of phase transitions discussed in the text. The condition is described by a second order equation in \( \gamma^2 \),

\[ (1 - m)^3 \gamma_*^4 + (m^2 - 20m - 8) \epsilon^2 \gamma_*^2 + 16 \epsilon^4 = 0 \].  \hspace{1cm} (3.30)

The solution \( \gamma_* \) scales with the energy scale \( \epsilon \), and the critical dimensionless coupling \( \gamma_*/\epsilon \) is universal. Figure 3 shows the two critical couplings as the function of the
filling ratio $m$. The lower value $\gamma_c$, corresponding to the connection of the two energy states, starts at value 2 and goes to $4/3\sqrt{3}$ as $m \to 1$. The other branch of critical coupling parameter $\gamma_s$, describing the splitting of the broad low lying island and the narrow, short-living region, starts at the same value as $\gamma_c$ and goes to infinity as $m \to 1$. For small values of $m$ ($m \to 0$), appearance of bridging states between the two "deterministic" states results in the immediate splitting of the cloud of the resonance states. Distribution of points within the clouds follows the ratio $(1 - m) : m$ between the low lying broad cloud and the short-living upper cloud, respectively.

![Figure 2: The dimensionless critical couplings of the two-state model as functions of the filling ratio $m$. The left figure presents the critical coupling $\gamma_c$ at which the corridor between the two state is built up, the right figure displays the critical coupling $\gamma_s$ at which the cloud of resonance states splits.](image)

By integrating $ \varrho(x,y) $ over the support of $x$ we obtain the distribution of resonance widths in the model.

Using the symmetry of the spectra we arrive at

$$
\varrho(y) = \int_{\text{support}} dx \ \varrho(x,y) = R(x_u, y) - R(x_l, y),
$$

(3.31)

$$
R(x, y) = \frac{1}{2\pi} \left\{ \frac{2x}{\gamma y} + \frac{2x}{\gamma y^3} \left( \frac{x^2}{3} - c^2 \right) - \frac{(1 - m)x}{y^2} + \frac{x}{c^2 - x^2} \right\},
$$

where $x_u$ and $x_l$ are the two positive end points of the support (for a continuous support $x_l = 0$). The resulting density can be used to determine average decay rate in the system, and is shown in Fig. 3. The bound state component $|\psi\rangle$ evolves in time according to

$$
|\psi(t)\rangle = e^{-iH_{\text{eff}}t/\hbar}|\psi(0)\rangle
$$

(3.32)
Figure 3: The integrated spectral density, $\rho(y)$ describing the density of the resonance widths in the system governed by $H_{eff}$. Solid line stands for the analytical prediction, the dashed line represents numerical estimate obtained by use of an ensembles of 30 random matrices with size $N = 400$.

and its effective decay rate can be defined as the logarithm derivative of its norm

$$k_{eff} = -\frac{d}{dt} \ln \langle \psi(t) | \psi(t) \rangle . \tag{3.33}$$

If the initial state has nonzero overlap with only one eigenstate of $H_{eff}$, the above definition simplifies to $k_{eff} = 2y_i/\bar{h}$. In general case when resonances overlap, the average survival probability over the ensemble is given by

$$P(t) = \int_{0}^{\infty} \rho(y) \exp(-yt)dy \tag{3.34}$$

and is related to the distribution of kinetic rates $\rho(y)$ by the Laplace transform. The result is displayed in Fig. 4 showing the decay with some average decay rate for low couplings and the appearance of roughly two distinct decay rates for large couplings. The time scale is of the order of a femtosecond for $\epsilon = 1eV$, and scales linearly with energy. (Note, that within the paper the mean-square dispersion of elements of $V$ matrix has been put equal 1.)

By integrating the spectral density (3.28) along the width coordinates one can obtain
Figure 4: The decay of the survival probability (3.34) for various values of the coupling constant \( \gamma \). Nonexponential decay becomes transparent for increasing values of \( \gamma \). The solid line is the Laplace transform of the analytical width distribution (3.32). The dots correspond to the Laplace transform of the density obtained numerically with a set of 30 random matrices of size \( N = 400 \).

The total width assigned to a given energy. The resulting density reads

\[
\rho(x) = \int_{\text{support}} dy \rho(x, y) = \sum_{i=1,2} R_x(x, y_{2i+1}) - R_x(x, y_{2i+2}),
\]

(3.35)

\[
R(x, y) = \frac{1}{4\pi} \left\{ \frac{\epsilon^2 - x^2}{\gamma y^2} + \frac{1 - m}{y} + y \frac{\epsilon^2 + x^2}{(e^2 - x^2)^2} + 2 \frac{\ln |y|}{g} \right\},
\]

with \((y_1-y_2)\) and \((y_3-y_4)\) the supports of the integration. The inverse of that quantity gives information about the decay time of the state, labeled with the given energy. The distribution of the total width as the function of the energy at different couplings is presented in Fig. 4. The situation depicted in Fig. 4 strongly resembles “noise induced transition” when coupling to a fluctuating source induces new order in the system. For low values of \( \gamma \), distribution \( \rho(x) \) has maxima around eigenvalues of the deterministic Hamiltonian; that bimodality of \( \rho(x) \) changes with increasing value of \( \gamma \) which introduces new maximum around \( x = 0 \), strongly populated for high values of the coupling constant.
Figure 5: The integrated spectral density, $\rho(x)$ describing the total width of a given energy state. Solid line stands for the analytical prediction, the dashed line represents numerical estimate obtained by use of an ensembles of 30 random matrices of size $N = 400$.

4 Collectivization of the spectra

At large coupling constant the spectrum shows the structural change. Two groups of distinct eigenvalues appear. The first, saturating the most of the total width corresponds to the direct resonances. The second group corresponds to long-living states and appears only, if the coupling is sufficiently strong. Such a ”reorganization” of the spectrum is usually observed \cite{27, 28, 29, 30} in systems with $N$ degenerate bound states coupled to $M$ continua. The system develops then $M$ decaying states and $N - M$ stable states whose population becomes trapped. The similar phenomenon of formation of a short-lived coherently decaying state in a system with almost degenerate levels is known in quantum optics (Dicke superradiance \cite{29}), nuclear physics (giant dipole resonances, collective isobar-hole states in scattering at intermediate energies \cite{30}) and dissipative spin systems \cite{31}. An interesting aspect of the phenomenon brings also a semiclassical description of collectivization discussed in papers of Gaspard and Rice \cite{32}. Multiple hoppings lead to very long trajectories (the delay time is large) saturating the Gutzwiller formula and causing breakdown of the ergodicity. The physical
mechanism responsible for the appearance of the above mentioned structural change in the spectra is caused by the presence of zero modes in the matrix $VV^\dagger$. If the coupling is strong, the zero modes are becoming delocalized, corresponding to the appearance of the coherent, collective state. In other words, for increasing coupling $\gamma$, the probability of hopping from one zero mode to another (or effective ”overlap” between the zero-mode states) is increasing, and the long strings due to multiple hopping can appear. This mechanism is somewhat analogous to the Mott’s conductor-insulator phase transition, when the conductance appears due to non-zero probability of the hopping of the electron between the various Fermi levels, so that the electrons are no longer localized. Another physical example corresponds to the picture of spontaneous breakdown of the chiral symmetry in quantum chromodynamics [33], when chiral condensate appears as a coherent effect of the delocalization of the zero modes of gluonic excitations.

5 Conclusions

Our analysis of the two-level system coupled to a random noise shows the advantage of using the Blue’s function approach [13] to study spectral properties of dynamical quantum systems. By use of the generalized Blue’s function [13], we are able to construct easily the Green function relevant for the system and from the last one we can read off spectral properties of the effective Hamiltonian.

Characteristic features of the distribution function of the eigenvalues related to the effective Hamiltonian depend strongly on the intensity of the coupling between the deterministic and random parts of the Hamiltonian. At low values of coupling constant, we observe some dispersion of eigenvalues density around deterministic energies of the two-level system. Beyond the first critical value of $\gamma = \gamma_c$, a common region connecting ”dispersed deterministic eigenvalues” is created and builds up with further increasing $\gamma$. At very high values of coupling, dissipativity entering the system through the term $VV^\dagger$ leads to reorganization of eigenvalues. We observe ”collectivization” of widths to a bridging, narrow region which is populated by low energy states. Its presence suggests then formation of long-living states, well separated from the cloud of resonances responsible for a rapid decay. Despite its simplicity, the model presented here can cover vast number of various applications. We believe that one of direct
use of the analysis is modeling unimolecular decay processes in polyatomic molecules and electron tunneling processes in biological media. Studies of an effective tunneling matrix in biomolecules require knowledge of the structure of off-diagonal elements of the generalized Green function discussed above. Analysis of the kinetic rate in such models is based on the form of two-particle (two-time) correlation function which, for the model Hamiltonian of ”deterministic plus random” dynamics, will be discussed in the forthcoming paper.

Acknowledgments

The authors acknowledge enlightening and instructive discussions with R. Janik, M. A. Nowak and I. Zahed on diagrammatic approach in the theory of random matrices. E.G-N. thanks U. Schmitt for many conversations on charge transfer models in condensed media and S. Kast for bringing to our attention relevant papers [23, 24]. G.P. acknowledges a partial support by the Hungarian Research Foundation OTKA.

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