Performance Evaluation of Hourly Solar Radiation Models on Inclined Surfaces in Longdong Area, China

Nana Yang¹, *, Haorui Liu¹, *a, Lei Zhao¹, b and Changji Xu², c

¹Department of Mechanical Engineering, LongDong University, Qingyang, China
²State Grid Gansu Electric Power Company Huan County Power Supply Company, Qingyang, China

*Corresponding author e-mail: 135098281@qq.com, a178212596@qq.com,
 b1390344186@qq.com, c328868149@qq.com

Abstract. Hourly global irradiations on tilted planes are required in various engineering calculations for solar systems which must be determined by converting the solar radiation intensities measured on a horizontal surface to the incident on the tilted surface of interest. There exist a large number of models designed to perform such a conversion. This paper presents the performance of five arithmetic models used to estimate solar irradiance on inclined surfaces. Utilizing solar radiation data from June to October measured in Longdong area of Gansu province, Model performance is assessed by an inter-comparison between the calculated and measured solar global radiation on a south-oriented surface tilted at 25°. Three statistical parameters (NMBE, NRMSE and R²) have been used to confirm the calculated values. The results show that the accuracy of the models is related to the sky conditions. Liu & Jordan model shows the best calculated performance under clear sky conditions, and Klucher model perform best under overcast and cloudy conditions. The other models included Perez, Hay and Skartveit & Olseth display an intermediary performance under all sky conditions. In order to estimate the tilt angle simply, two simple approximate equations (φ-15° model and Soulayman model) based on geographical latitude and solar declination are suggested for predicting daily optimum tilt angle in summertime.

1. Introduction
Solar conversion systems (flat plate solar collectors and PV cells) are tilted towards the sun in order to maximize the amount of solar radiation incident on the collector/cell surface, the tilt angle is one of the most important factors considered in solar energy system design. Many researchers have proposed many models to decide the optimum tilt angle for solar collectors. The majority of models to simply estimate the best tilt angle were carried out by considering the effects of the latitude, declination angle, clearing index which are based on maximizing the solar radiation falling on a sloped surface [1]. At weather stations, the global and diffuse solar radiation are generally measured on horizontal surfaces. Consequently, data on inclined surfaces are generally not available and solar radiation incident on a tilted surface must be determined by converting solar radiation intensities measured on a horizontal surface to that incident on the tilted surface of interest. Direct radiation on the horizontal plane can be directly converted to beam radiation on the inclined surface by the geometrical relationship between...
the two surfaces, but this is not the case for the diffuse component because diffuse radiation comes from all points of the sky except the sun [2]. Therefore, the key to estimate the total hourly radiation on the tilt surface is the estimation of diffuse radiation. There exist a relatively large number of models that correlate the diffuse hourly radiation on a tilted surface to that measured on a horizontal surface, but comparisons and modifications of these models have been done only for specific regions in the world [3]. The purpose of this work is to evaluate the applicability of 5 widely used models in Longdong area of Gansu province.

Generally, diffuse radiation models for inclined surfaces can be classified into two groups: isotropic and anisotropic models. The most common isotropic model is that developed by Liu and Jordan which assumes that the intensity of sky diffuse radiation is uniform over the sky dome. On the other hand, the anisotropic models assume the sky as anisotropic source of diffuse radiation. Many investigators have used the isotropic and anisotropic models for estimating the total solar radiation incident on tilted surfaces in different regions. Diez-Mediavilla et al. [4] compared 10 models used to estimate diffuse solar irradiance on inclined surface with actual data readings available on an hourly and a daily basis in Valladolid (Spain). Kambezidis et al. [5] presented a comparative assessment of tilted irradiation models, using hourly measurements of total solar irradiation on a surface tilted 50 degrees and oriented south in Athens. G. Notton et al. [6] tested 15 models to calculate the hourly global solar irradiation on a tilted surface, and the performance of models was evaluated against data (2 inclinations 45° and 60°) measured on the French Mediterranean site of Ajaccio. Evseev et al. [7] assessed 11 models that convert horizontal diffuse radiation to diffuse radiation incident on a tilted surface utilizing data measured in Beer Sheva, Israel. As the number of estimation models is many, generally researchers choose some models that have been found in previous studies to assess the accuracy in specific region or in special atmospheric conditions. So the lack of the assessment of models to calculate solar global radiation on inclined surfaces in Longdong area is the motivation of this study.

The objective of this study is to compare five widely used models on a south-facing surface tilted at 25° and compare the results to that measured using statistical methods. Then the implication for determination of the optimum tilt angle is discussed.

2. Theory and models

2.1. Hourly total radiation on tilted surfaces

The incident global solar radiation on an inclined surface can be divided into three components: the beam component from direct irradiation of the tilted surface, the diffuse component and a reflected component that quantifies the radiation reflected from the ground to the tilted surface [8].

\[ I_{T\beta} = I_{b\beta} + I_{d\beta} + I_{r\beta} \]  

(1)

The direct component \( I_{b\beta} \) can be written in the following form:

\[ I_{b\beta} = I_{b} R_{b} \]  

(2)

Where \( R_{b} \) is geometric factor:

\[ R_{b} = \frac{\sin \delta \sin (\varphi - \beta) + \cos \delta \cos (\varphi - \beta) \cos \omega}{\sin \delta \sin \varphi + \cos \delta \cos \varphi \cos \omega} \]  

(3)
If the reflection is considered as isotropic, the reflected component by the ground $I_{r\beta}$ can be expressed as

$$I_{r\beta} = \frac{1}{2} I_{\rho}(1 - \cos \beta)$$

(4)

Kambezidis et al. [5] used three albedo models (constant albedo, seasonally varying albedo and anisotropic albedo) and showed that using an anisotropic and seasonally varying albedo did not improve the performance of their estimation compared to utilization of an albedo fixed at 0.2. In this work, a constant value for the albedo is taken equal to 0.2.

2.2. Hourly diffuse radiation on tilted surfaces

In this study, 5 models commonly used in solar energy to predict the diffuse radiation on a tilted surface are compared. The diffuse component on a tilted surface $I_{d\beta}$ can be represented by developing the diffuse transposition factor $R_d$.

$$I_{d\beta} = I_d R_d$$

(5)

2.2.1. Isotropic model. This model is also known as the Liu&Jordan model [9]. It assumes that diffuse radiation intensity is distributed uniformly over the sky dome and approximates overcast sky conditions. It is often used in engineering calculations because it is easy to implement.

$$R_d = \frac{1}{2}(1 + \cos \beta)$$

(6)

2.2.2. Anisotropic models. Hay model

This model assumes that the origins of sky diffuse radiation come from two primary sources, namely the disc of the sun and the rest of the sky with isotropic diffuse radiation [10].

$$R_d = \frac{I - I_d}{I_0} R_\beta + \frac{1}{2}(1 + \cos \beta) \left(1 - \frac{I - I_d}{I_0}\right)$$

(7)

$$I_0 = I_{sc} E_0 (\sin \delta \sin \varphi + \cos \delta \cos \varphi \cos \omega)$$

(8)

$$E_0 = 1 + 0.033 \cos \left\{ \left[ \frac{2 \pi d_n}{365} \right]\right\}$$

(9)

2.2.3. Klucher model. Klucher modified the Temps and Coulson model by introducing a function of determining the degree of cloud cover. When the sky is overcast, the value of $F$ is zero and Eq. (10) becomes isotropic. When value of $F$ tends to one, Eq. (10) is the same as equation given by Temps and Coulson [11].

$$R_d = \frac{1}{2}(1 + \cos \beta) \left[1 + F \sin^3 \left(\frac{\beta}{2}\right)\right] \left(1 + F \cos^2 \theta \sin^3 \theta_c\right)$$

(10)
\[ F = 1 - \left( \frac{I_d}{I} \right)^2 \]  

(11)

2.2.4. Skartveit and Osleth model. Solar radiation measurements carried out by Skartveit and Olseth indicated the fact that a significant part of sky diffuse radiation under overcast sky conditions comes from the sky region around the zenith. This effect vanishes with the disappearance of cloud cover. A Z correction factor is introduced based on Hay model to overcome this effect [12].

\[ R_d = \frac{I - I_d}{I_0} r_b + Z \cos \beta + \frac{1}{2} \left( 1 + \cos \beta \right) \left( 1 - \frac{I - I_d}{I_0} Z \right) \]  

(12)

\[ Z = \max \left[ 0, \left( 0.3 - 2 \frac{I - I_d}{I_0} \right) \right] \]  

(13)

2.2.5. Perez model. The model is based on a three-component treatment of a geometrical representation of the sky dome, a parametric representation of the insolation conditions and a statistical component linking the two. Perez et al. divided the sky diffuse component into the following three parts: background, circumsolar, and horizon zones. The contribution to diffuse radiation from the three parts is determined by two empirically derived coefficients \( F_1 \) and \( F_2 \), called “reduced brightness coefficients” [13]. The governing equation is

\[ R_d = \frac{1 + \cos \beta}{2} \left( 1 - F_1 \right) + F_1 \left( \frac{a_1}{a_2} \right) + F_2 \sin \beta \]  

(14)

\[ a_1 = \max(0, \cos \theta) \]  

(15)

\[ a_2 = \max(\cos 85^\circ, \cos \theta_z) \]  

(16)

\[ F_1 = \max \left[ 0, \left( F_{11} + F_{12} \cdot \Delta + F_{13} \cdot \theta_z \left( \frac{\pi}{180} \right) \right) \right] \]  

(17)

\[ F_2 = F_{21} + F_{22} \cdot \Delta + F_{23} \cdot \theta_z \left( \frac{\pi}{180} \right) \]  

(18)

\[ \Delta = m \frac{I_{\text{rei}}}{I_{\text{out}}} \]  

(19)

Where \( a_1, a_2 \) are the solid angles occupied by the circumsolar region weighted by its average incidence on the slope and the horizontal, respectively. \( F_1, F_2 \) are the coefficients of circumsolar and horizon brightness, respectively. \( \Delta \) is the sky brightness parameter. The required coefficients \( F_0 \) can be obtained from Perez et al. [13].
3. Experimental data
The measuring station is located at the energy building of Longdong university, Qingyang (Latitude 35.7°N, Longitude 107.7°E, Altitude 1380 meter above sea level), Gansu province, China. Four data sets of global solar irradiance have been collected from June 5, 2017 to October 10, 2017 covering sunny days, cloudy days, and overcast days: global, diffuse and direct radiation on the horizontal plane, global radiation on 25° tilted plane south-oriented. Other parameters such as temperature, pressure, relative humidity, wind speed and direction are also recorded. An adjustment of Pyranometer is made after a few days by changing the declination, and all measuring instruments are cleaned every 2 days.

The measuring data are recorded at every 2 min from morning till evening. The hourly radiation data are obtained from a numerical integration of the 2-min mean values. In order to ensure the accuracy of the measurement data used in model calculation, strict quality control is carried out on the hourly radiation data.

4. Models evaluation
4.1. Methods of statistical comparison
To evaluate the accuracy of the estimated data from the five models described above, three statistical indicators are used: normalized mean bias error (NMBE), normalized root mean square error (NRMSE) and coefficient of determination (R²). These parameters are defined as:

\[
NMBE = \frac{1}{\bar{H}} \left\{ \frac{1}{n} \sum_{i=1}^{n} (H_{c,i} - H_{m,i}) \right\}
\]

(20)

\[
NRMSE = \frac{1}{\bar{H}} \sqrt{\frac{1}{n} \sum_{i=1}^{n} (H_{c,i} - H_{m,i})^2}
\]

(21)

\[
R^2 = 1 - \frac{\sum_{i=1}^{n} (H_{c,i} - H_{m,i})^2}{\sum_{i=1}^{n} (H_{m,i} - \bar{H})^2}
\]

(22)

The NMBE provides information with respect to over or under estimation of the estimated data. A NMBE > 0 means that the predicted values tend to overestimate the observed values, whereas a NMBE < 0 corresponds to an underestimation of the observed values, and a low NMBE value is desired. NRMSE provides information on the short term performance of the model and is a measure of the variation of the estimated values around the measured data. R² is a number that indicates how well data fit a statistical model [14].

4.2. Models accuracy in different conditions
The relative ability of the models to predict the global radiation on a tilted surface varies according to the sky conditions. Since all the models make assumptions regarding the isotropy/anisotropy of the sky conditions. This study classified sky conditions into 3 categories: overcast (\(K_I \leq 0.3\)), cloudy (\(0.3 < K_I \leq 0.6\)) and clear (\(K_I > 0.6\)). The hourly sky conditions clearness index is defined as:

\[
K_I = \frac{I}{I_0}
\]

(23)
The values of these parameters for each correlation in three sky conditions are given in Fig. 1(a-c). It can be observed from Fig. 1(a) that the values of NMBE calculated with five models exhibit generally small differences. For clear day, all the models over predict the radiation on an inclined surface at 25°, and for overcast and cloudy day, the models under predict the radiation on an inclined surface at 25°.

From Fig. 1(a) and (b) it is clear that the absolute values of NMBE and NRMSE are higher under overcast sky conditions than under cloudy and clear sky conditions, which indicates that the five models are more accurate for cloudy and clear sky conditions. The inter-comparison of the 5 models shows that Liu&Jordan model provides the smallest absolute NMBE and NRSME under clear sky conditions, and Klucher model provides the smallest absolute NMBE and NRSME under cloudy and overcast sky conditions. Consequently, Liu&Jordan model gives the best prediction for clear sky conditions, whereas Klucher model performs best for cloudy and overcast sky conditions. Hay model and Skartveit & Olseth model give similar results, and display an intermediary performance with Perez model.

The coefficient of determination (R^2) of the five models is shown through Fig.1(c). From Fig.1(c) it is clear that R2 of cloudy day is negative, which shows that there is no correlation between the calculated value and the measured value. The calculated value and measured value of Liu&Jordan model and Klucher model have best correlation for clear sky conditions and overcast sky conditions, respectively.
4.3. Optimum tilt angle based on seasonally average solar radiation

A number of simple models have been proposed to determine, or at least estimate the best tilt angle for solar collectors. Part of models for fixed optimum tilt angle were proposed based on latitude as follows: $\phi$, $\phi+10^\circ$, $\phi+(10^\circ\rightarrow30^\circ)$, $\phi-10^\circ$ or $\phi\pm10^\circ$, $\phi\pm15^\circ$, where the plus sign indicates the winter season and the minus sign refers to the summer season [15-16]. Others are conducted by latitude and declination. Stanciu et al. [17] found that when using the Hottel & Woertz model for estimating the incident solar radiation density, the optimum tilt angle for a flat plate collector should be computed as simplest as $\beta_{opt}=\phi-\delta$ function on the latitude and declination. Soulayman [18] proposed a general algorithm for calculating $\beta_{opt}$ for south facing collector. However, although in the past few years, researchers have made efforts for estimation of local optimum tilt angles, there is no definite value, or method can be recommended. One of the most effective methods to determine the optimum tilt angle is by maximizing solar radiation or energy collected on the surface. Fig. 2 provides information on choosing the optimum tilt angle for catching maximum solar radiation by comparison of calculated value and measured value against tilt angles, during the summer months.

It is obvious from the results of Fig. 2 that the calculated values of the five models to estimate radiation on inclined surfaces are consistent with the measured values against tilt angles, which show a single peak type change. It is seen that the optimum tilt angle calculated by Hay, Perez and Skartveit & Olseth models equals $20^\circ$ which are consistent with the measured value, and the optimum tilt angle is $15^\circ$ for Liu&Jordon and Klucher model. Although the models of Hay, Perez and Skartveit & Olseth overestimate the average daily total radiation, the optimum angle can be predicted accurately. One may notice that the optimum tilt angle is approximately equal to $\phi-15^\circ$ for latitude during the summer months. In this work, the models proposed by Soulayman and Stanciu et al. also used to estimate the optimum tilt which is $23.5^\circ$ and $27.5^\circ$, respectively. The value calculated by Soulayman model exhibits less difference compared with the experimental optimum tilt angle.
5. Conclusion
Irradiation data recorded for south-facing titled surface at inclination angle 25° during the daylight hours from June to October 2017 have been compared with the estimated solar radiation from inclined surface models. Based on the study, the main conclusions are as follows:

According to commonly used statistical test results namely NRMSE, NMBE and $R^2$, it can be concluded that the relative ability of 5 models to predict solar radiation on inclined surfaces widely varies according to the sky conditions. The estimation accuracy of all the models for clear sky conditions was best, and for cloudy sky conditions, the models gave the best result. By comparison, the Klucher model was found to perform best under overcast and cloudy sky conditions, whereas the Liu&Jordan model gave the best result for clear sky conditions.

The optimum tilt angle of solar collectors located in Longdong area of Gansu province follow the general rule applied by many researchers that the optimum tilt is about $\phi$-15°in summertime. Soulayman model also could be employed to estimate the optimum tilt angle approximately. The estimation of the optimum tilt angle during other seasons need to be researched furthermore.

Acknowledgments
This work was financially supported by Key Research and Development Programs of Gansu Province (No. 17YF1GM046) and Science and Technology Support Project of Qingyang City (No. KZ2015-17).

References
[1] Yadav, et al. "Tilt angle optimization to maximize incident solar radiation: A review." Renewable & Sustainable Energy Reviews23.23(2013):503-513.
[2] Demain, Colienne , Michel Journée, and Cédric Bertrand. "Evaluation of different models to estimate the global solar radiation on inclined surfaces." Renewable Energy50.3(2013):710-721.
[3] Elminir, Hamdy K.,et al. "Optimum solar flat-plate collector slope: Case study for Helwan, Egypt." Energy Conversion and Management 47.5(2006):624-637.
[4] DIEZMEDIAVILLA, D. E. Miguel, and BILBAO. "Measurement and comparison of diffuse solar irradiance models on inclined surfaces in Valladolid (Spain)." Energy Conversion & Management 46.13(2005):2075-2092.
[5] Kambezidis, H. D, B. E. Psiloglou, and C. Gueymard. "Measurements and models for total solar irradiance on inclined surface in Athens, Greece." Solar Energy53.2(1994):177-185.
[6] Notton, G., C. Cristofari, and P. Poggi. "Performance evaluation of various hourly slope irradiation models using Mediterranean experimental data of Ajaccio." Energy Conversion and Management 47.2(2006):147-173.

[7] Evseev, Efim G., and A. I. Kudish. "The assessment of different models to predict the global solar radiation on a surface tilted to the south." Solar Energy 83.3(2009):377-388.

[8] Iqbal, M. "An Introduction to Solar Radiation." Space Science Reviews 39(1983):387–390.

[9] Liu, B., and Jordan, R. "Daily insolation on surfaces tilted towards equator." Trans ASHRAE, 67(1962):526-541.

[10] Hay, John E. "Calculation of monthly mean solar radiation for horizontal and inclined surfaces." Solar Energy 23.4(1979):301-307.

[11] Klucher, T.M. "Evaluation of models to predict insolation on tilted surfaces." Solar Energy 23.2(1979):111-114.

[12] Skartveit, Arvid, and J. A. Olseth. "Modeling slope irradiance at high latitudes." Solar Energy 36.4(1986):333-344.

[13] Perez, Richard, et al. "Modeling daylight availability and irradiance components from direct and global irradiance." Solar Energy 44.5(1990):271-289.

[14] Sobri, Sobrina, S. Koohi-Kamali, and N. A. Rahim. "Solar photovoltaic generation forecasting methods: A review." Energy Conversion and Management 156(2018):459-497.

[15] Elminir, Hamdy K., et al. "Optimum solar flat-plate collector slope: Case study for Helwan, Egypt." Energy Conversion and Management 47.5(2006):624-637.

[16] Skeiker, Kamal. "Optimum tilt angle and orientation for solar collectors in Syria." Energy Conversion & Management50.9(2009):2439-2448.

[17] Stanciu, Camelia, and D. Stanciu. "Optimum tilt angle for flat plate collectors all over the World – A declination dependence formula and comparisons of three solar radiation models." Energy Conversion and Management 81(2014):133-143

[18] Soulayman, S. Sh.. "On the optimum tilt of solar absorber plates." Renewable Energy 1.3–4(1991):551-554.