Entropy generation and natural convection of nanofluids in a trapezoidal cavity having an inner solid cylinder

Muhamad S. Ishak\textsuperscript{1,2,*}, Ammar I. Alsabery\textsuperscript{1,3} and Ishak Hashim\textsuperscript{1}

\textsuperscript{1}Department of Mathematical Sciences, Faculty of Science & Technology, Universiti Kebangsaan Malaysia, 43600 Bangi Selangor, Malaysia
\textsuperscript{2}Department of Foundation and Diploma Studies, College of Engineering, Universiti Tenaga Nasional, 43000, Kajang, Selangor, Malaysia
\textsuperscript{3}Refrigeration & Air-conditioning Technical Engineering Department, College of Technical Engineering, The Islamic University, Najaf, Iraq

*Corresponding author:safwanukm@gmail.com

Abstract. A numerical analysis of entropy generation and natural convection in a trapezoidal cavity with an internal solid cylinder filled with Al\textsubscript{2}O\textsubscript{3}–water nanofluid has been investigated using finite difference method. The bottom wall is thermally insulated while the left and right walls were cooled isothermally. Remainder of these walls are kept adiabatic. Particular factors have been focused on the effects of Rayleigh number, dimensionless radius of the solid cylinder and nanoparticles of volume fraction on streamlines, isotherms, isentropic, local and average Nusselt number. Obtained results have demonstrated that the Rayleigh number and size of the solid cylinder are important control parameters for optimizing heat transfer and Bejan number.

1. Introduction

Heat transfer and fluid flow is very important in engineering applications such as solar collectors, heat exchangers, food processor, and electronic cooling. One of its major modes is natural convection. Many researchers had applied the natural convection in their researches such as Basak & Chamkha\textsuperscript{[1]}, De Vahl Davis G., \textsuperscript{[2]}, and Nawaf & Yusli\textsuperscript{[3]}. This current study, however, is not limited to natural convection; it also includes entropy generation. In engineering applications, calculating entropy production in systems with convective heat transfer is significant because it provides information on local and global energy losses. Thus, it helps us to design and optimize the engineering systems. In 2008, Rejane et al.\textsuperscript{[4]} investigated the effects of the Rayleigh number, aspect ratio and irreversibility coefficient to examine the generation of entropy in rectangular cavity. They found that the aspect ratio and irreversibility coefficients enhanced the second law of thermodynamics. Sheremet et al.\textsuperscript{[5]} also extended the same problem but with various temperature distributions at left vertical wall. They reported that the temperature distribution and the wave number increase the entropy generation. Moreover, Sheremet et al.\textsuperscript{[6]} proposed the heated square cavity and nanofluid-filled cavity. Then, Bouchoucha et al.\textsuperscript{[7]} also explored the effects of thickness of bottom surface enclosure on the heatlines and entropy generation.
The shape of the cavity also plays an important role in the heat convection and fluid flow. The researchers also believe that the shape of the cavity can enhance the convective heat transfer and minimize the irreversibility in the system. Different researchers have attempted using different shapes of the cavity for instance, Yasin et al.[8] in their study, applied the triangular enclosure, while Yonghua et al. [9] applied circular tubes and Mahmoodi and Sebdani [10] used square cavity in their experiment. In 2014, Ramakrishna et al.[11] used the porous trapezoidal cavity to analyse the heat flow and entropy generation with various Darcy and Prandtl numbers. Yasin et al.[12] also used the trapezoidal enclosure with various side wall inclination angles. The study by Yasin was further extended by Tanmay et al.[13] onto the study of the entropy generation. Saleh et al.[14] also used the trapezoidal cavity to investigate the heat transfer using nanofluid with various parameters while Moukalled & Marwan[15] investigated the first law of thermodynamics in trapezoidal enclosure that are partially divided.

The existence of solid in the cavity may also influence the fluid flow and heat transfer. There are many papers that had discussed on the effects of solid insertion in the enclosure. Shu et al.[16] proposed the convective heat transfer in square enclosure with inner cylinder. Kim et al.[17] also considered a heated inner solid cylinder in the cold outer square enclosure with different vertical locations. They reported that the first law of thermodynamics are significant affected by the change in the inner solid position. Furthermore, Yoon et al.[18] studied the effect of inner cylinder location on the convective heat transfer at Rayleigh number $10^7$. However, Park et al.[19] applied finite volume method to investigate the effects of the solid inner cylinder in the square cavity. In a study by Hashim et al. [20], they inserted an inner block in a wavy enclosure with alumina-water nanoparticles and found that the nanoparticles and number of oscillations enhanced the heat transfer in the cavity.

Based on the review of the literature, it can be seen that to date, no research has been carried out in investigating the convective heat transfer and entropy generation in the trapezoidal enclosure with alumina-water nanofluid having solid cylinder. Therefore, the aim of this research is to observe the influence of the various Rayleigh numbers, size of inner solid cylinder and nanofluid volume fraction towards streamlines, isotherm, isentropic, average Nusselt and Bejan number.

2. Mathematical Formulation

Figure 1 shows a two-dimensional geometric model of natural convective flow and heat transfer inside a trapezoidal enclosure with a bottom wall of length $L$ and a top wall of range $L/2$, as well as an internal solid cylinder with a dimensional radius $r$. The bottom surface is supposed to be heated into a constant temperature of $T_h$ and both sloping surfaces with length 0.65$L$ and inclination angle $\phi$ are cold temperature, $T_c$. Meanwhile, the top surface is saved in adiabatic rule. The fluid flow in the trapezoidal cavity is tested for stability, laminarity, and loaded with alumina-water nanofluids. The Navier stokes equation and energy equations are:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \nu_{nf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]

\[
u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \nu_{nf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \beta_{nf} g(T - T_o)
\]
The heat equation of the internal body remains as:

\[
\frac{\partial^2 T_x}{\partial x^2} + \frac{\partial^2 T_y}{\partial y^2} = 0,
\]

where \(x\) and \(y\) are in the horizontal and vertical directions for the Cartesian coordinates, \(g\) the acceleration due to gravity, \(\rho_{nf}\) the density of nanofluid and \(\nu_{nf}\) is the nanofluid kinematic viscosity.

The thermophysical characteristics of the nanoparticles are:

\[
(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_p,
\]

\[
\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_nf},
\]

\[
\rho_{nf} = (1-\phi)\rho_f + \phi\rho_p,
\]

\[
(\rho \beta)_{nf} = (1-\phi)(\rho \beta)_f + \phi(\rho \beta)_p.
\]
while the water-Al₂O₃ nanofluids’ dynamic viscosity ratio are as follows.

\[
\frac{\mu_{nf}}{\mu_f} = 1\left(1 - 34.87\left(\frac{d_p}{d_f}\right)^{0.3}\phi^{0.03}\right)
\]  

(9)

the thermal conductivity ratio of alumina-water is determined by Corcione [21] as the following:

\[
\frac{k_{nf}}{k_f} = 1 + 4.4 \text{Re}_f^{0.4} \text{Pr}^{0.66}\left(\frac{T}{T_f}\right)^{10}\left(\frac{k_p}{k_f}\right)^{0.03}\phi^{0.66}
\]

(10)

where \(\text{Re}_f\) is described as

\[
\text{Re}_f = \frac{\rho_f u_f d_p}{\mu_f}, \quad u_f = \frac{2k_p T}{\pi \mu_f d_f}
\]

(11)

where \(k_b = 1.380648 \times 10^{-23} (J / K)\) the Boltzman constant, \(l_f = 0.17 \text{ nm}\), the mean path of fluid particles, \(d_f\) is the molecular diameter of water given as Corcione [21]

\[
d_f = \frac{6M}{N \pi \rho_f}
\]

(12)

where \(M\) is the molecular weight of the base fluid, \(N\) is the Avogadro number and \(\rho_f\) is the density of the base fluid as standard temperature (310K).

The non-dimensional variables

\[
X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{uL}{a_f}, \quad V = \frac{vL}{a_f}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad \theta_i = \frac{T_f - T_c}{T_h - T_c},
\]

\[
D = \frac{d_f}{L}, \quad \text{Pr} = \frac{v_f}{a_f}, \quad \text{Ra} = \frac{g \beta_f (T_h - T_c) L^3}{v_f a_f}, \quad \rho = \frac{pL^2}{\rho_f a_f^2}, \quad H = \frac{h}{L}
\]

(13)

The dimensionless governing equations are then generated:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0,
\]

(14)

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{\partial P}{\partial X} + \text{Pr} \frac{\rho_f}{\rho_{nf}} \frac{\mu_{nf}}{\mu_f} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right)
\]

(15)

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = - \frac{\partial P}{\partial Y} + \text{Pr} \frac{\rho_f}{\rho_{nf}} \frac{\mu_{nf}}{\mu_f} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \left(\frac{\rho \beta}{\rho_{nf}} \beta_f\right) \text{Ra} \theta
\]

(16)
\[
U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) 
\]  
(17)

\[
\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} = 0.
\]  
(18)

The dimensionless boundary conditions regarding:

On the hot bottom surface \((AB)\):

\[
U = V = 0, \ \theta = 1, \ 0 \leq X \leq 1, \ Y = 0,
\]  
(19)

On the adiabatic top surface \((DC)\):

\[
U = V = 0, \ \frac{\partial \theta}{\partial Y} = 0, \ 0 \leq X \leq 1, \ Y = 0.65,
\]  
(20)

On the cold both sloping surfaces \((AD\) and \(BC)\):

\[
U = V = 0, \ \theta = 0, \ \forall X, \ \forall Y,
\]  
(21)

\[\theta = \theta_s, \text{ at the outer solid cylinder surface,}\]

\[
U = V = 0, \ \frac{\partial \theta}{\partial n} = K_r \frac{\partial \theta}{\partial n},
\]  
(22)

where \(K_r = k_s / k_{nf}\) keeps the thermal conductivity ratio above the covering of the inner body. At the heated bottom surface, the local Nusselt number is:

\[
Nu_{nf} = -\left( \frac{\partial \theta}{\partial X} \right)_{Y=0}.
\]  
(23)

Then, the average Nusselt-number calculated at the hot bottom surface is defined by:

\[
\overline{Nu_{nf}} = \int_{A}^{B} Nu_{nf} \ dY,
\]  
(24)

The entropy generation relation is:

\[
S = \frac{k_{nf}}{T_0} \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu_{nf}}{T_0} \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} \right)^2 \right].
\]  
(25)

Local entropy generation can be described in dimensionless form as:
\[ S_{\text{GEN}} = \frac{k_{nf}}{k_f} \left[ \left( \frac{\partial \theta}{\partial x} \right)^2 + \left( \frac{\partial \theta}{\partial y} \right)^2 \right] + \frac{\mu_{nf}}{\mu_f} N_{\mu} \left\{ 2 \left[ \left( \frac{\partial U}{\partial x} \right)^2 + \left( \frac{\partial V}{\partial y} \right)^2 \right] + \left( \frac{\partial U}{\partial y^2} + \frac{\partial V}{\partial x^2} \right)^2 \right\} \] (26)

where, \( N_{\mu} = \frac{\mu_f T_0}{k_f} \left( \frac{\alpha_f}{L(\Delta T)} \right)^2 \) is the irreversibility distribution ratio and \( S_{\text{GEN}} = S_{\text{gen}} \frac{T^2_0 L^2}{k_f (\Delta T)^2} \).

The Eq. (26) can be rewritten in the following form:

\[ S_{\text{GEN}} = S_\theta + S_\nu, \] (27)

where \( S_\theta \), the entropy generation due to heat transfer irreversibility (HTI) while \( S_\nu \), the entropy generation due to nanofluid friction irreversibility (NFI),

\[ S_\theta = \frac{k_{nf}}{k_f} \left[ \left( \frac{\partial \theta}{\partial x} \right)^2 + \left( \frac{\partial \theta}{\partial y} \right)^2 \right], \] (28)

\[ S_\nu = \frac{\mu_{nf}}{\mu_f} N_{\mu} \left\{ 2 \left[ \left( \frac{\partial U}{\partial x} \right)^2 + \left( \frac{\partial V}{\partial y} \right)^2 \right] + \left( \frac{\partial U}{\partial y^2} + \frac{\partial V}{\partial x^2} \right)^2 \right\}. \] (29)

Then, Eq. (29) is integrated and the global entropy generation (GEG) is:

\[ \text{GEG} = \int S_{\text{GEN}} dX dY = \int S_\theta dX dY + \int S_\nu dX dY. \] (30)

The ratio between the entropy generation due to heat transfer by the total entropy generation is called as Bejan number, Be,

\[ Be = \frac{\int S_\theta dX dY}{\int S_{\text{GEN}} dX dY} \] (31)

The system is dominant with heat transfer irreversibility if \( Be \) approaches to 1 (\( Be > 0.5 \)), while the system is opposite (nanofluid friction irreversibility dominant) if \( Be < 0.5 \).

3. Numerical Method

The Galerkin weighted residual are employed to investigate the control equations (14) – (18) subject to the boundary conditions in Eqs. (19) – (23). The following procedure needs to be carried out to solve the finite element analysis for Equations (15) and (16);

Primarily, the penalty finite element method is applied by excluding the pressure (\( P \)) and including a penalty-parameter (\( \lambda \)) as the following:
\[ P = -\lambda \left( \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \right). \]

Then, the momentum equations for \( X \) and \( Y \)-directions are:
\[ \begin{align*}
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} &= -\lambda \left( \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \right) + \text{Pr} \frac{\rho_f}{\rho_{nf}} \frac{\mu_f}{\mu_j} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right), \\
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} &= -\lambda \left( \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \right) + \text{Pr} \frac{\rho_f}{\rho_{nf}} \frac{\mu_f}{\mu_j} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{(\rho \beta)_n}{\rho_{nf} \beta_j} R\alpha \text{Pr} \theta,
\end{align*} \]

As shown in Figure 2, the formulation is weighted for momentum equations by multiplying with an interval domain \( \Phi \) and integrating it across the computational domain discretised toward small triangular components. The weak formulations are:
\[ \begin{align*}
\int_{\Omega} \left( \Phi U^k \frac{\partial U^k}{\partial X} + \Phi V^k \frac{\partial U^k}{\partial Y} \right) dX dY &= -\lambda \int_{\Omega} \nabla \Phi \left( \frac{\partial U^k}{\partial X} + \frac{\partial V^k}{\partial Y} \right) dX dY \\
+ \text{Pr} \frac{\rho_f}{\rho_{nf}} \frac{\mu_f}{\mu_j} \int_{\Omega} \Phi \left( \frac{\partial^2 U^k}{\partial X^2} + \frac{\partial^2 U^k}{\partial Y^2} \right) dX dY, \\
\int_{\Omega} \left( \Phi U^k \frac{\partial V^k}{\partial X} + \Phi V^k \frac{\partial V^k}{\partial Y} \right) dX dY &= -\lambda \int_{\Omega} \nabla \Phi \left( \frac{\partial U^k}{\partial X} + \frac{\partial V^k}{\partial Y} \right) dX dY \\
+ \text{Pr} \frac{\rho_f}{\rho_{nf}} \frac{\mu_f}{\mu_j} \int_{\Omega} \Phi \left( \frac{\partial^2 V^k}{\partial X^2} + \frac{\partial^2 V^k}{\partial Y^2} \right) dX dY + \frac{(\rho \beta)_n}{\rho_{nf} \beta_j} R\alpha \text{Pr} \int_{\Omega} \phi \theta dX dY.
\end{align*} \]

Interpolation functions are chosen as a technique of taking an estimate of the velocity and temperature distribution as:
\[ \begin{align*}
U &\approx \sum_{j=1}^{m} U_j \phi_j (X,Y) \\
V &\approx \sum_{j=1}^{m} V_j \phi_j (X,Y) \\
\theta &\approx \sum_{j=1}^{m} \theta_j \phi_j (X,Y).
\end{align*} \]

By applying the Galerkin weighted residual finite element method, we yielded the nonlinear residual equations for the momentum equations:
\[ \begin{align*}
R(i) &= \sum_{j=1}^{m} U_j \int_{\Omega} \left[ \sum_{j=1}^{m} U_j \phi_j \frac{\partial \phi_j}{\partial X} + \sum_{j=1}^{m} V_j \phi_j \frac{\partial \phi_j}{\partial Y} \right] \phi_j dX dY \\
&+ \lambda \left[ \sum_{j=1}^{m} U_j \int_{\Omega} \frac{\partial \phi_j}{\partial X} \frac{\partial \phi_j}{\partial X} dX dY + \sum_{j=1}^{m} V_j \int_{\Omega} \frac{\partial \phi_j}{\partial X} \frac{\partial \phi_j}{\partial Y} dX dY \right] \\
&+ \text{Pr} \frac{\rho_f}{\rho_{nf}} \frac{\mu_f}{\mu_j} \sum_{j=1}^{m} U_j \left[ \frac{\partial \phi_j}{\partial X} \frac{\partial \phi_j}{\partial X} + \frac{\partial \phi_j}{\partial Y} \frac{\partial \phi_j}{\partial Y} \right] dX dY,
\end{align*} \]
\[
R(2)_i = \sum_{j=1}^{m} V_j \int_{\Omega} \left[ \sum_{j=1}^{m} U_j \Phi_j \right] \frac{\partial \Phi_j}{\partial X} + \left( \sum_{j=1}^{m} V_j \Phi_j \right) \frac{\partial \Phi_j}{\partial Y} \right] \Phi_i \, dXdY \\
+ \lambda \left[ \sum_{j=1}^{m} U_j \frac{\partial \Phi_i}{\partial Y} dXdY + \sum_{j=1}^{m} V_j \frac{\partial \Phi_i}{\partial X} dXdY \right] + Pr \frac{\rho_f}{\rho_{nf}} \mu_f \left[ \frac{\partial \Phi_i}{\partial X} \frac{\partial \Phi_i}{\partial Y} \right] dXdY \\
+ \frac{(\rho \beta)}{\rho_{nf} \beta_f} \text{Ra} \int_{\Omega} \left[ \sum_{j=1}^{m} (\theta_j \Phi_j) \right] dXdY.
\]

to interpret \( k \) in the equations as related index, subscripts \( i \), residual number, \( j \), node number and \( m \), iteration number are used.

**Figure 2.** Grid-points distribution for size of 9846 elements.

A Newton-Raphson iteration approach was used to elucidate the nonlinear elements into the momentum equations. The solution’s convergence is allowed if any of the variables’ relative error meets the resulting convergence condition:

\[
\frac{|\Gamma^{m+1} - \Gamma^m|}{\Gamma^{m+1}} \leq 10^{-5}
\]
4. Results and Discussion

Figure 3. Variation of the streamlines (left), isotherms (middle), and isentropic (right) evolution by Rayleigh number \( (Ra) \) for \( \phi = 0.02 \) and \( R = 0.15 \).

Figure 3 represents the effects of several Rayleigh numbers on the streamline, isotherm, and isentropic for \( \phi = 0.02 \) and \( R = 0.15 \). It can be seen that the flow behaviour is affected as the Rayleigh number increases. At \( Ra = 10^3 \), 2 streamlines rotating cell appeared at the bottom of the cavity. Since only natural convection exists as a result of the force of gravity therefore the rotating cell appeared at the bottom of the cavity. The negative sign indicates that the cell is rotating clockwise. As \( Ra \) increases, the intensity of cells and cell size also increase. It occurs because the buoyancy forces exceed the inertia forces. The strength of the rotating cell also increases as Rayleigh number increased based on the value of the sign. It can be seen that the lines curve like a plume for heat transfer. The heat transfer moves up because the bottom surface is heated and the left and right walls are cooled. The heat transfer lines decreased as Rayleigh number increases. As it is isentropic, the lines are very small at the base of the wall cavity. At higher Rayleigh, the isentropic lines expand and cover the whole cavity.
Figure 4. Variation of the streamlines (left), isotherms (middle), and isentropic (right) evolution by size of solid cylinder for $Ra = 10^3$ and $\phi = 0.02$.

Figure 4 demonstrates the impacts of different sizes of solid cylinder on the streamlines, isotherms, and isentropic for $Ra = 10^3$ and $\phi = 0.02$. At small radius of solid cylinder, 2 circular rotating cells appeared at both right and left side of the cavity. As the radius of solid cylinder increases, the strength of the fluid and size of cell decrease due to the increase in buoyancy force. The heat transfer lines in the cavity are plume-shaped, and protrude in the cylinder. Increasing the radius of solid cylinder clearly affected the heat transfer in the cavity. The isentropic lines covered the whole cavity at smaller radius of solid cylinder. As the radius of the solid cylinder increases, the intensity of the isentropic lines decreases.

Figure 5 depicts the various of volume fractions of nanofluid with different Rayleigh numbers on the average Nusselt number at $R = 0.15$. Rayleigh number determines the ratio between the buoyant force and shear force. As Rayleigh increases, the buoyant force is dominant instead of the shear force. The result shows that the average of Nusselt-numbers remain for all volume fractions of nanofluid at smaller Rayleigh number. However, the average Nusselt number increases after the Rayleigh number reaches $10^5$ due to the dominant buoyant force. Meanwhile, the higher volume fraction of nanofluid
yields higher average Nusselt number compared to smaller nanofluid volume fraction due to the increase in thermal conductivity ratio as the nanoparticle volume fraction increases.

![Graph](image)

**Figure 5.** Variation of average Nusselt number with $Ra$ for various values of $\phi$ at $R = 0.15$.

The various Bejan numbers with $R = 0.15$ as functions of Rayleigh number with various nanofluid volume fraction are portrayed in Figure 6. Heat transfer irreversibility dominates the system for all nanoparticle volume fractions at smaller Rayleigh numbers. However, as Rayleigh number grows, the Bejan number declines for all volume fractions of nanofluid. This suggests that the nanofluid flow irreversibility has dominated the system as the number of Rayleigh increases.

Figure 7 illustrates the effects of different radius of solid cylinder on the average Nusselt-number at $\phi = 0.02$. We can see that for smaller Rayleigh numbers, the average Nusselt number is low and stays the same for all radius solid cylinders until $10^4$. However, the average Nusselt number increases after $10^4$. The best result for average Nusselt-number is at smaller radius of solid cylinder.

Figure 8 demonstrates the impact on the number of Bejan of the various size of the solid cylinder at $\phi = 0.02$. At smaller Rayleigh number, the Bejan number is equal to 1 for all radius of solid cylinder. It appears that the system is heat transfer irreversibility dominate, while the system is changed to nanofluid irreversibility as Rayleigh number increase.
Figure 6. Variation of number of Bejan with $Ra$ for different values of $\phi$ at $R = 0.15$.

Figure 7. Variation of average Nusselt number with $Ra$ for different values of $R$ at $\phi = 0.02$.
5. Conclusions

The convective generation of entropy and convection problem of an alumina-water nanofluid in a trapezoidal cavity with a solid cylinder in the cavity have been numerically explored. The various effects Rayleigh numbers, radius of solid cylinder and nanoparticle volume fraction streamlines, isotherm, isentropic, average Nusselt and number of Bejan have been calculated. The results are as follows:

- The increase in number of Rayleigh and solid cylinder radius has a significant impact on heat transfer and fluid flow.
- The average number of Nusselt rises with increasing numbers of Rayleigh.
- Heat transfer irreversibility dominates the system at smaller Rayleigh numbers, however nanofluid flow irreversibility dominates the system as the number of Rayleigh increases.
- At smaller radius of solid cylinder and higher nanoparticle volume fraction, the average Nusselt number reaches a maximum.

Figure 8. Variation of Bejan number with Ra for different values of R at $\phi = 0.02$. 
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