Optimization Design Method for Hydrostatic Guideway Based on PM Restrictor

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Abstract. Stiffness and flow rate are always used to evaluate the performance of the hydrostatic slides and well-designed parameters, such as flow rate ratio, initial flow rate, pump pressure and oil film thickness, allow the slides achieving nanoscale precision. In this study, the relationship between the design parameters and the stiffness and flow rate is reveal by an analytical model. Then a multi-objective optimization problem is solved using Non-dominated Sorting Genetic Algorithm II. The optimization objectives are maximize stiffness and minimize flow rate. At last, the optimized design parameters are calculated and corresponding maximum stiffness and minimum flow rate of the hydrostatic slides are acquired.

1. Introduction

Nowadays hydrostatic guideways are widely employed in ultra-precision machines as high stiffness, high load carrying capacity, and low frictional losses at low-to-moderate speeds, minimal wear, good accuracy and excellent damping. Many scholars studied the method of design and optimization of hydrostatic guideways to improve their performance [1]. For hydrostatic bearing, several objectives are required to be considered, such as minimum consumption of power [2], maximum stiffness [3], maximum load carrying capacity [4] and maximum dynamic stiffness [4]. The minimization of the consumption of power is the objective most commonly considered. In the meanwhile, the optimization design principle of hydrostatic bearings is maximum rigidity and minimum flow. The improper selection of the design parameters may result in low stiffness, poor geometrical accuracy, poor surface finish and poor stabilization. This may affect the quality of the machine tool. Therefore, selection of optimal combination of design parameters for hydrostatic guideway is a matter of concern for designers and researchers. In most of the cases, designers analysed these design parameters by using the finite element method (FEM) [5] and selected optimum combination of design parameters through experience and traditional design method, which is costly, time consuming and tedious. Thus, these factors have steered the designers towards applying optimization algorithm techniques for parameter design. Researchers had applied various classic optimization algorithms. However, the optimization problems of parameters design of hydrostatic guideway may not get the results using traditional methods. Therefore, an intelligent method is required for solving constrained design problems effectively.

In this work, a novel optimal design model of closed hydrostatic slide, which is controlled by PM (Progressive Mengen) restrictors, is proposed. Maximum stiffness and minimum flow rate are considered as the optimal objective, while the pump pressure, initial flow rate, flow rate ratio and oil film thickness are the design parameters. This multi-objective optimization problem is a constrained, non-linear and parameter bounded problem, which can be carried out by evolutionary algorithms.
Finally, the optimized design parameters and corresponding maximum stiffness and minimum flow rate are obtained.

2. Optimization design method of high-precision hydrostatic guideways

Based on the force balance equation, the load of the slide, \( F \), which is original slides design from structure parameters, can be calculated. Figure 1 represents stress analysis of a couple of aspectant bearing pads. The stresses of the pad 1 and pad 3 are \( pr_1 \) and \( pr_3 \), respectively.

By mechanical analysis, the equation of a couple of aspectant bearing pads can be presents as the Equation 1.

\[ F = pr_i A_i - pr_i A_i \]  \hspace{0.5cm} (1)

\[ A_i = (L_i - l_i)(B_i - b_i) \]  \hspace{0.5cm} (2)

According to the calculation method of the flow rate in the pocket of the rectangular oil pad, the flow rate can be obtained as follow:

\[ Q_{pi} = \frac{pr_i}{R_i} \]  \hspace{0.5cm} (3)

where \( R \) is defined as liquid resistance and can be calculated by

\[ R_i = \frac{6\rho \gamma}{h_i^3 \left( \frac{L_i - l_i}{b_i} + \frac{B_i - b_i}{l_i} \right)} = \frac{c_{ri}}{h_i^3} \cdot \rho \]

\( \rho \) is oil density, \( \gamma \) is viscosity, and \( c_R \) is a constant depending on structure parameters of pads, oil density and viscosity.

For the hydrostatic slide which is using PM controller, the parameters of the controller to control the pressure of the bearing pads are as follows: flow rate ratio \( c_r \), initial flow rate \( q_0 \), and pump pressure \( p_s \). Flow rate ratio is calculated by

\[ c_r = q_p / q_0 \]

where \( q_p \) represents the flow rate when the bearing pad pressure \( pr \) equals pump pressure \( p_s \) and \( q_0 \) represents the flow rate when the bearing pad pressure \( pr \) equals 0. The flow rate of bearing pads with PM controller can calculate using equation (4) in [7].

\[ Q_{PM} = q_0 \left[ 1 + (c_r - 1) \frac{pr}{p_s} \right] \]  \hspace{0.5cm} (4)

Assuming that there is no flow rate loss from the exit of PM restrictors to the pockets, the flow rate in the \( i \)th pocket should be equal to the flow rate out of the \( i \)th PM restrictor, which means \( Q_{PM} = Q_{pM} \). Thus, the pad pressure and flow rate of the \( i \)th pocket can be computed by:

\[ pr_i = \frac{p_s q_0 c_{ri} h_i^3}{p_i h_i^3 - (c_r - 1) q_0 c_{ri}} \]  \hspace{0.5cm} (5)

\[ Q_i = \frac{p_i q_0 h_i^3}{p_i h_i^3 - (c_r - 1) c_{ri} q_0} \]  \hspace{0.5cm} (6)
Film stiffness, an important indicator to evaluate performances of hydrostatic slides, is the rigidity of the oil film, which shows the ability to resist changes from the external loads and deformation in response to an applied force. The stiffness is defined as:

$$ j_\mu = \sum_{i=1,II} j_{\mu i} = \sum_{i=1,II} \frac{dF_i}{dh_i} = \sum_{i=1,II} \frac{-3q_{0i}c_R A_i}{h_i^4 \left( 1 - q_{0i}c_R (c_{ii - 1}) \right)^2} $$

where the negative sign indicates that the changing trend of the load and the film thickness is the opposite.

3. Multi-parameter and multi-objective optimization algorithm

The principle of the optimization design of the hydrostatic bearing is well known as maximum stiffness and minimum flow rate. The optimization objectives are changed with the major parameters, such as, film thickness, pump pressure, initial flow rate and flow rate ratio. Mathematical programming method always has low efficiency and high sensitive in weights or order of targets when the objective and constraint function of the multi-objective optimization problem are non-linear, non-differentiable, or discontinuity. At present, evolutionary algorithms are commonly applied to resolve multi-objective optimization problem [8].

In this paper, Non-Dominated Sorting Genetic Algorithm II (NSGA-II) [9] is used to solve the multi-objective optimization problems of the hydrostatic slides design. This algorithm maintains an elitist mechanism for maintaining good individual and improving the population evolution to keep population diversity, therefore, the algorithm is still widely used in the multi-objective optimization. The mathematical model are as follows:

$$ \begin{align*}
\min \ F(x) &= [f_1(x), f_2(x)] \\
&\text{s.t.} \ h_{01} < x(1) < h_{02} \\
&\quad \text{pr}_{1} < x(2) < \text{pr}_{2} \\
&\quad c_{11} < x(3) < c_{12} \\
&\quad q_{01} < x(4) < q_{02}
\end{align*} $$

where $ f_1(x) = -j_\mu, f_2(x) = Q, x(1) = h_0, x(2) = \text{pr}, x(3) = c_r, x(4) = q_0.$

4. Case study

4.1. Boundary

According to the equation 4 and 5, the relationship between stiffness and flow rate and parameter $p_s$, $q_0$, $h_0$, and $c_r$ are shown in Figure 2 to Figure 5. In Figure 2, when the film thickness is less than 17μm, the flow rate increases rapidly then decreases. When the film thickness is 20μm, the change of flow rate remains stable. The flow rate changes rapidly that is not dependent on the motion stability of the slide.

In Figure 3, the stiffness reaches around 1400N/μm when pump pressure is 25bar and decreases slightly with pressure. The flow rate increases rapidly and reaches the maximum at when pump pressure is 18 bar. When the pressure keeps increasing, the flow rate decreases and later remains at 25 bar. In general, pump pressure is supposed to be higher than 25 bar, because the changes of stiffness and flow rate are unstable when the pressure is lower than 25 bar.

It can be observed in Figure 4 that when the flow ratio is more than 3.5, stiffness and flow ratio changes dramatically. Thus, the flow ratio should be less than 3.5.
In Figure 5, when the initial flow is more than 0.015 l/min, the flow increases rapidly. The flow changed rapidly that is not dependable for the motion stability of the slide, therefore, the initial flow should be lower than 0.15 l/min.

As a result, the range of the design parameters are:

\[
\begin{align*}
18 \mu m &< h_0 < 30 \mu m \\
28 \text{bar} &< p_r < 35 \text{bar} \\
2 &< c_r < 5 \\
0.01 \text{l/min} &< q_0 < 0.015 \text{l/min}
\end{align*}
\]  

(9)

4.2. Calculation and analysis

According to the optimal design model, the Pareto-front is shown in Figure 6. Ultimately, the selected design parameters of the hydrostatic slide are \( h_0 = 19 \mu m, \) \( p_r = 3.2 \text{ MPa}, \) \( c_r = 2.5, \) and \( q_0 = 0.014 \) l/min. The optimal results are the stiffness \( j_u = 1237 \text{ N/\mu m} \) and flow rate \( q = 0.029 \text{ l/min}. \) The stiffness \( j_u \) is 1056N/\mu m and low rate \( q \) is 0.037l/min, respectively, while the original design parameters are oil film thickness \( h = 20 \mu m, \) pump pressure \( p_r = 3.2 \text{MPa}, \) initial low rate \( q_0 = 0.018 \text{ l/min}, \) and flow rate ratio \( c_{rs} = 2.75. \) Comparing the performance of the hydrostatic slides which were designed in different design parameters, it is observed that the performance of hydrostatic slides which was designed by the optimization design parameters is much better than the performance of hydrostatic slides which was
designed by the original design parameters. The stiffness increases by 17% and the flow rate decreases by 22%.

Figure 6. Pareto front of the optimized results.

5. Conclusions
In this study, a novel optimal design model is established for designing a high-performance hydrostatic slide. The effects of slide, oil supply system and restrictors on stiffness and flow rate are taken account into the proposed optimization model. To achieve the best performance of the hydrostatic slide, the objectives of the optimization problem are the maximum stiffness and the minimum flow rate, while the pump pressure, initial flow rate, flow rate ratio and oil film thickness are the design parameters. After optimization, the maximum stiffness is 1237 N/μm and minimum flow rate is 0.029 l/min when flow rate ratio, initial flow rate, pump pressure and oil film thickness are 2.5, 0.014 l/min, 32 bar and 19 μm, respectively. Compared with the hydrostatic slides optimally designed by traditional design method, the comprehensive performance of the hydrostatic slides designed by the proposed method shows overwhelming advantages. The stiffness increases by 17% and the flow rate decreases by 22%.

In the further research, the more effective and accurate evaluation method and intelligent optimization algorithm of hydrostatic slide should be proposed to provide better theoretical basis and design method for the design of ultra-precision hydrostatic slide.

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