Fibre reinforced concrete in flexure and single fibre pull-out test: a correlation

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Abstract. The aim of the present work is to assess whether a single fibre pull-out test can be related to the behaviour of multiple fibres in fibre reinforced concrete under bending condition. A simple model based on the stress block theory is described and compared with experimental results on three point bending tests with aligned fibres.

1. Introduction

Fibre reinforced concrete (FRC) is adopted in many civil applications such as tunnelling and flooring with the main benefit being improved toughness when compared to plain concrete. Such improvement has drastically reduced or eliminated the use of conventional steel reinforcement. In Australia, macro synthetic fibres are widely used in the mining sector as fibre-reinforced shotcrete (FRS). This offers a cheap and fast solution in ground control systems.

Despite the advantages in the use of fibres, high variability in the post-crack performance of FRC and FRS remains an issue which is still the object of many studies. Engineers often resort to empirical methods for design. The design of shotcrete underground linings, for example, relies on the Barton’s Chart [1] and its modifications [2, 3] which relates the energy capacity of FRS based on empirical observations.

To fulfill this lack of understanding, there is a need to provide a simple tool to demystify the behaviour of FRC/FRS for design purposes that is accurate, can be replicated and can be easily implemented.

The concept of stress block is used for basic design in conventional flexure members in civil applications and offers a way to calculate the bending capacity of structural members. In order to model the tensile forces acting on cracked unreinforced concrete, a variety of stress profiles have been adopted in the literature, mostly using the fictitious crack model [4]. With the presence of fibres in the matrix, the choice of the stress profile dictates the accuracy of the model. Many researchers spread the effect of fibres throughout the entire cracked section.

A few studies have attempted to approach FRC with local forces induced by the bridging action of fibres, most notably [5] and [6]. Both researchers modelled the tensile forces imparted by the fibres with experimental results obtained from the pull-out test. To tackle the problem of fibres randomly distributed across the crack section, [5] assumed a pure random distribution of fibres (all orientations and embedment length have equal probability). In [6], the cracked section is divided in layers. For each layer an idealised most probable pull-out response was created such that it could describe the behaviour of all fibres in the beams for each embedment length.
and orientation and is based on the most likely orientation and embedment length measured in each layer of a specific beam. The outcomes are different, from one side the first study concludes that this approach is valid and it also explains how the variability of the results of third point beams comes from the variability in the pull-out response. In [6], one of the conclusions is that the single fibre pull-out test may not be representative of the behaviour of fibres in FRC under bending conditions. In both cases, however, the randomness of fibres, and therefore high variability of the results, gives a complicated picture of the relationship between single fibre pull-out and fibre-reinforced concrete in bending.

The question that is yet to be answered and validated experimentally is whether a single fibre pull-out test can be effectively used to describe the complex behaviour of multiple fibres bridging the crack in flexure.

In light of recent developments based on beams with non-random aligned fibres [7], this paper aims at establishing a link between the single fibre pull-out test and bending capacity of FRC/FRS beams using the stress block theory. If the relationship is found on a simple case with aligned fibres, it can open the way to a more general formulation to model randomly dispersed fibres in FRC and FRS.

2. Model

In order to focus on the relationship between the single fibre pull-out test and the three point bending test, a semi-analytical model has been implemented. The model should be able to output the post-peak response of beams with aligned fibres based solely on the results obtained from the single fibre pull-out test as well as information on the location and number of fibres in the beam. Since the behaviour of a fibre pulling out of concrete strongly depends on how the fibre fails during the process, the mode of failure is an extra input of the model. For this particular combination of fibres and mortar used in this research three distinct modes of failure called Pull-out mode, Mixed mode and Snap mode have been identified. Further details can be found in section 4.

This semi-analytical model is based under the following assumptions:

- The problem can be simplified as a 2D problem (this assumption holds if the fibres are aligned).
- The cracked surface is plane and propagates at mid-span in the beam loaded in a three point bending configuration.
- The pin of the crack is located at a certain known height. It has been observed experimentally in [7] that cracks propagate some 90% of the beam’s height regardless of the number of fibres with only small variations during the test. The pin of the crack is where
the crack width is zero, the crack opens linearly to the bottom of the cross section where the crack mouth opening displacement is defined as $CMOD$ (as shown in figure 1).

- The post-peak response is governed by the fibres, the compressive force at the top of the cross section equates the tensile forces coming from the fibres for equilibrium. Furthermore, the process zone (the area of cracked concrete that still offers some bridging action) is considered negligible whereas the compressive stresses are assumed constant.

- There is no interaction between fibres.

- The bridging action of each fibre can be obtained from a single fibre pull-out test and it is the same for all fibres with the same mode of failure.

The output of the model is the load $P$ required to deflect the beam down to a certain amount of deflection $\delta$ that plays the role of independent variable. Other inputs are:

(i) Number of layers of fibres $n_l$ in the cross section (see figure 1).

(ii) Number of fibres categorised by their mode of failure (Pull-out mode, Mixed mode and Snap mode) $n_{P,i}$, $n_{M,i}$ and $n_{S,i}$ for each layer $i$.

(iii) The functions $P_f(w_{f,i})$, $M_f(w_{f,i})$ and $S_f(w_{f,i})$ which relate the magnitude of the bridging force of each fibre to the crack width ($w_f$) and are obtained experimentally from the single fibre pull-out test. Each function describes a certain mode of failure. The crack width is calculated at each layer $i$.

(iv) $\delta - CMOD$ relationship. For simply supported beams, the following relationship can be used:

$$CMOD = \frac{4h}{L}\delta$$

where $h$ is the height of the beam and $L$ is the distance between the supports. The relationship has been experimentally validated in [7] and is valid for all beams regardless of the number of fibres.

With reference to figure 1, the following expressions hold:

$$\tan \theta/2 = \left(\frac{CMOD/2}{d}\right)$$  \hspace{1cm} CMOD found using Equation 1  \hspace{1cm} (2)

$$w_{f,i} = 2 \cdot (y_i - y^*) \cdot \tan \theta/2$$  \hspace{1cm} (3)

$$C = \sum_{i=1}^{n_l} [P_f(w_{f,i}) \cdot n_{P,i} + M_f(w_{f,i}) \cdot n_{M,i} + S_f(w_{f,i}) \cdot n_{S,i}]$$  \hspace{1cm} Equilibrium  \hspace{1cm} (4)

$$M = \sum_{i=1}^{n_l} [P_f(w_{f,i}) \cdot n_{P,i} + M_f(w_{f,i}) \cdot n_{M,i} + S_f(w_{f,i}) \cdot n_{S,i}] \cdot (y_i - 0.5y^*)$$  \hspace{1cm} Equilibrium about $C$  \hspace{1cm} (5)

$$P = \frac{4M}{L}$$  \hspace{1cm} (6)
where \( C \) is the resultant of the compressive stresses acting in the uncracked section of the beam which span from the top until \( y^* \) which is assumed, for simplicity, constant at 90\% of the height of the beam (i.e. \( y^* = 0.1h \)). The internal moment \( M \) is obtained from the equilibrium about \( C \). Finally, the external load \( P \) (which is the main output of the model) is obtained equating the internal moment \( M \) with the external moment due to a load centrally located in a simply supported beam.

The model is admittedly very simple. Some assumptions (such as the position of the pin of the crack being constant) could be removed to improve the validity of the model. The aim of this preliminary study, however, is to investigate a possible relationship between the single fibre pull-out test and three point bending test with an approach as simple as possible. Further developments can take into account a more realistic model for improved accuracy.

3. Testing Procedure

3.1. Pull-out test

To complete the model described in section 2, one of the inputs is the load-displacement curve obtained from a fibre in a single pull-out test. For this purpose, a preliminary campaign with 10 specimens has been undertaken to quantify the variability of the results and to account for all possible modes of failures. The mortar and the fibres used in the pull-out tests are the same as reported in Reference [7].

Each specimen consist of a cylinder with a diameter of 50 mm and 700 mm in length made of mortar with a single fibre located in the middle of the cylinder. Each cylinder was prepared in two stages. In the first stage, the lower half of the cylinder (up to 350 mm) was cast and a fibre was placed according to the fibre configuration desired (figure 2). The second stage takes place two days later; the second half of the cylinder is cast to complete the specimen resulting in a single fibre fully embedded in mortar. Being the length the fibre 65 mm, 32.5 mm are embedded in the lower cylinder and the other half in the upper cylinder. This two-stage procedure ensures that the bond between the two halves of the cylinder is near zero in order to measure solely
3.2. Three point bending test
As part of a wider study investigating the effects of the location and orientation of macro-synthetic fibres on the flexural response of FRC and FRS, a methodology has been developed to align and control the position of the fibres. A series of beams 600x150x150 mm have been tested in three point bending test with different number of fibres and different positions. In all beams, the fibres are aligned to the direction of the major stress and parallel to each other. The beams have been pre-notched at mid-span such that the crack propagates straight from the tip of the notch to the top of the cross section where the beam is loaded. The fibres have been centred in order to have an embedment length equal to half the length of the fibres (i.e. 32.5 mm). Further details and results on this experimental campaign can be found in [7]. After testing the beams, the cracked surface has been analysed to count the number of fibres and to assess their mode of failure. An example of this visual assessment is depicted in figure 3.

4. Results
The single fibre pull-out results showed, in general, low variability provided that the mode of failure and the embedment length are the same. The response of fibre does strongly depend on the way the fibre either breaks or pulls-out. At least three different modes of failure have been
Figure 4. Pull-out test results for different modes of failure. The pictures show how the fibre looks after the test for each mode of failure.

identified:

(i) The fibre breaks in a brittle fashion (Snap mode)
(ii) The fibre remains intact and pulls-out or slips from one side (Pull-out mode)
(iii) The fibre breaks but only after it has partially slipped from one side; this mode of failure will be hereafter referred to as Mixed mode.

Example of these modes of failure are illustrated in figure 4.

After testing the beams in a three point bending configuration, both crack surfaces have been visually inspected to assess the mode of failure of the fibres as depicted in figure 3. The magnitude of the bridging force of each fibre described by the functions $P_f(w_f)$, $M_f(w_f)$ and $S_f(w_f)$ is taken directly from the pull-out results of figure 4 according to the mode of failure. The results of the model proposed in section 2 for various beams is plotted in figure 5 and 6.

5. Summary and Conclusions

There is, in general, good agreement between the model that links the pull-out behaviour and the experimental results obtained from the three bending point test. The model can reasonably describe all the beam series included in this study.

A possible source of error can result from the assumption that the crack surface is a plane which in turn affects the assumption that the pull-out length is exactly half the length of the fibre. The mortar was designed to minimise this effect and the beams have been pre-notched to promote the formation of the crack to be aligned with the point load. Nonetheless, in some specimens the pulled-out lengths of some fibres deviated slightly (within few millimetres) from the ideal figure of 32.5 mm. The accuracy of the model is also affected by the difficulty to distinguish fibres that snapped from those that failed in a mixed mode and the pull-out curve attributed to those fibres could be inappropriate.

The model, given its simplicity, works well in an ideal configuration where fibres are all aligned but cannot be applied yet to conventional FRC/FRS where the fibres are uncontrollably dispersed within the volume of the material. However, as a preliminary study on the behaviour of fibres in concrete and to answer the question of whether a single fibre pull-out test can be used
to describe the behaviour of FRC/FRS, from the results in figure 5 and 6, it seems that pull-out test under direct tension represents the behaviour of multiple fibres embedded in mortar under bending conditions providing that the mode of failure of each fibre is assessed carefully.

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