Theoretical and experimental investigation of Lamb waves excited by partially debonded rectangular piezoelectric transducers

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Abstract

The paper proposes a new hybrid approach technique to simulate acousto-ultrasonic wave excitation and propagation due to operation of the partially debonded piezoelectric transducer attached to a plate-like structure. The semi-analytical boundary integral equation method is applied to calculate guided waves propagation in the unbounded structures and to separate different guided waves in the piezo-induced wave-fields. The obtained model is verified experimentally using the scanning laser Doppler vibrometry. Eigenfrequencies are calculated and analysed for various sizes of the transducer and for different bonding conditions between the transducer and the waveguide. The impact of the transducer’s height, size and debonding area on symmetric and antisymmetric Lamb waves excitation is analysed. The paper demonstrates that one-sided debonding of the transducer exerts intense influence on the distribution of the wave energy among the excited Lamb wave modes, while center debonding has a sizable impact only at relatively high frequencies.

Keywords: piezoelectric transducer, Lamb waves, debonding, wave motion, waveguide, resonance, experiment

(Some figures may appear in colour only in the online journal)

1. Introduction

Structural health monitoring (SHM) is known as the continuous and automated method for monitoring and evaluating the condition of a load-carrying structure based on data acquisition, post-processing and analysis of the measured data [1]. The reliable long-term attachment or integration of sensors on structures, which should be inspected, is a major prerequisite. The main goal of the SHM system is to provide adequate information on the state of the inspected structure to find structural damages before they reach critical levels and provide adequate information for condition-based maintenance [2, 3]. The acquired data is used to identify incipient damage, providing low-cost inspection compared to a conventional manual examination and reduction of the total system
downtime. Moreover, nowadays novel techniques employing data acquired by sensors are proposed to determine the damage location and size as well as to estimate the remaining life-time before the system failure due to the presence of the defect.

The health monitoring of structures using ultrasonic waves has been proved to be a reliable and cost-effective tool [4]. If plate-like structures are considered, Lamb waves are widely used for the inspection due to their capability to propagate over long distances without significant attenuation and with sensitivity to all types of structural defects [5]. Many model-based studies of guided wave propagation in elongated structures confirm the convenience and relevance of the use of guided waves for SHM and nondestructive testing (NDT) applications. It was demonstrated that Lamb waves’ sensitivity to notches depends on the defect depth to plate thickness ratio [6]. Cho and Lissenden [7] showed the potential of piezo-generated guided waves in the monitoring of fatigue crack growth based on the transmission coefficient depending on the crack size. Experimental and finite element-based studies of Lamb wave interaction with defects in an aluminum plate were provided [5, 8]. Recently, Shen and Cesnik [9] investigated nonlinear scattering and mode conversion of Lamb waves interacting with breathing cracks. Besides the above-mentioned papers, a large number of studies of guided waves interaction with defects has been published in the last decades (for more references see reviews on guided waves applications in SHM and NDT [10–12]).

Debonding between a piezoelectric wafer active sensor (PWAS) and a host structure could occur due to different factors such as bonding defects, impact loading, highest forces at the bonding interface, environment effects, fatigue effects and others [13–17]. The defective sensor can significantly influence the damage detecting algorithms output and lead to false diagnostic reports. Investigation of the effects of the PWAS debonding on vibration control of smart beams performed in [13] showed that a debonding located at the end of the piezoelectric transducer could significantly worsen the control of the first several modes of the beam. Kumar et al [14] analysed the performance of an active control system with healthy and unhealthy actuators. They ascertained that debonding in actuator reduces both its load-carrying ability and electro-mechanical potential and influences active damping and active stiffening effects. In [17], authors employed improved layerwise theory and the FEM to investigate sensor partial debonding influence on the active vibration control of a smart composite laminate. A finite element model for a piezoelectric plate with edge debonded actuators was developed in [18]. Based on this model, the authors showed that the edge debonding of actuators results in a considerable degradation in actuation authority and vibration control performance. The effects of a bonding layer including its possible degradation on the coupled electro-mechanical behaviour of piezoelectric actuators subjected to high-frequency electric loads were analysed in [19].

There have been developed several methods to detect sensors’ failure. In [20], a technique for sensor debonding monitoring based on the change in voltage relations between the segmented electrodes of a piezoelectric patch was suggested. It was numerically shown that the developed technique is applicable to detect one-sided debonding and also central debonding. In [21], the sensor error function was derived using the measurements from the health sensor, which was later on employed to detect and isolate the instants of sensor failure. It was also shown that debonding affects the electrical impedance of the piezoelectric sensor [22] and, therefore, it might be used for the inspection of piezoelectric transducers themselves [16].

To develop effective damage detection algorithms, a detailed understanding of Lamb waves excitation and interaction with various structural defects is required. Related phenomena can be studied in laboratory experiments, but their understanding can be achieved only with the help of a combination of experimental, numerical and analytical approaches. Thus, a reliable mathematical model simulating Lamb wave excitation, propagation and scattering in elongated plate-like structures is needed. To date, a variety of pure numerical [23–25] and semi-analytical [26–29] approaches have been applied to model Lamb waves excitation by a perfectly bonded piezoelectric transducer. Thus, the finite element method (FEM) was applied in [23] to simulate three-dimensional Lamb wave propagation excited by a phased array transducer. The spectral element method was employed in [24] to model wave propagation induced by a built-in piezoelectric actuator. In [26], the authors used a one-dimensional model to simulate the dynamic interaction of a piezoelectric actuator with an elastic half-plane. Additionally, a modelling approach based purely on the spectral element method was shown in [30]. The semi-analytical approach was suggested in [27] for PWAS-structure interaction, where the PWAS was simulated using a lamina model excluding vertical displacements and shear strains [27].

Another implementation of the semi-analytical approach was proposed in [28], where the boundary integral equation method (BIEM) was used to model perfectly bonded PWAS. For practical applications detailed understanding of the waves propagating in structure is essential, therefore, several hybrid methods for simulating piezo-induced ultrasonic guided waves were proposed. In [31], the FEM was employed to model piezoelectric actuator and afterwards the calculated displacement field was used in the local interaction simulation approach to model guided waves. Another hybrid approach was presented in [32], where the authors modelled a perfectly bonded PWAS using expansion via Chebyshev polynomials, while the solution in the host structure was obtained via the BIEM. Though this approach provides an analytical solution in the waveguide, it does not suit for faulty sensor modelling. In the semi-analytical hybrid approach (SAHA) [29], a piezoelectric actuator is simulated by the frequency domain spectral element method (FDEEM) [33], while the propagation of the excited Lamb waves in the laminate structure is modelled via the semi-analytical BIEM [34].

It is evident that investigation of the dynamic behaviour of the debonded PWAS as well as alteration of the excited guided waves due to debonding is needed for refinement of the existing ultrasonic NDT and SHM methods, the development of the reliable self-diagnosis methods and the estimation of the probability of detection, which is crucial for
SAHA proposed by the authors in debonded PWAS is strikingly scarce. For this purpose, the wave excitation and propagation in the waveguide due to bonded PWAS, and Lamb waves excitation. Based on the SAHA model, the interrelation between eigen-frequencies, which allows to verify the model. A good agreement between experimental and numerical methods while many studies are devoted to the sensor diagnosis successful implementation of the SHM systems. However, while many studies are devoted to the sensor diagnosis methods [16, 17, 35, 36], a detailed investigation of Lamb wave excitation and propagation in the waveguide due to debonded PWAS is strikingly scarce. For this purpose, the SAHA proposed by the authors in [29], is employed here to simulate and analyse the dynamic interaction of a debonded PWAS with a layered elastic waveguide.

The SAHA allows separating different guided waves in the wave-field excited by PWAS and to calculate complex-valued eigenfrequencies, which provides a tool for studying the influence of PWAS debonding on Lamb wave excitation. A good agreement between experimental and numerical results is demonstrated, which allows to verify the model. Based on the SAHA model, the interrelation between eigen-frequencies of the structure, i.e. a waveguide with a surface bonded PWAS, and Lamb waves excitation (A₀ and S₀) is investigated for perfectly bonded, one-side debonded and centrally debonded PWASs. It is demonstrated that debonding between a PWAS and a plate causes considerable changes in the amount of energy transferred into the waveguide and its distribution among Lamb waves. Of course, some other situations (asymmetric two-sided debondings, three or more contact spots etc) could be also studied. Though variations in contact/ debonded area may lead to essential changes in wave-fields and wave energy distribution, no additional physical phenomena are expected to be revealed in these cases [37]. Accordingly, only three cases mentioned above are considered here as the most common. However, the discrepancy between bonded and centrally debonded PWASs is relatively small even at higher frequencies and for moderate severity of the damage. For one-side debonded PWASs only, strong resonances are observed at the resonance frequencies calculated as the real part of a certain eigenfrequency of the plate with PWAS. It can be concluded that information about wave energy flux distribution between A₀ and S₀ modes can be used to identify debonding of a PWAS.

2. Mathematical model

In this study, debonding of a rectangular PWAS is considered, therefore, instead of a full three-dimensional model, a two-dimensional assumption is used. If the PWAS is accurately attached and the excited wave-front is plane then the assumption of plane state with no influence of x₃ according to figure 1 is valid and a two-dimensional model can be used for the analysis of the effects of debonding (see also [38] for the problem with surface-bonded rectangular block). Therefore, the two-dimensional implementation of the SAHA is discussed in this section, and as it is shown in the next section it provides an accuracy, which is good enough to be in a good agreement with experimental observations.

2.1. Statement of the boundary value problem

In order to model piezo-induced ultrasonic elastic waves, it is considered that a piezoelectric transducer of the width w and height h occupies a domain Ω₂ and is bonded to an infinite isotropic plate Ω₁ of thickness H, see figure 1. Boundary S = ∂Ω₂ of the transducer is composed of four lines S = S₁ ∪ S₂ ∪ S₃ ∪ S₄. The piezoelectric transducer and the plate share the common surface S₁ which consists of two areas: S₁ = S₅ ∪ S₆; S₅ is the perfect contact area between the layer and PWAS and S₆ is the debonded area where stress-free surfaces are assumed for the layer and the PWAS. Hereinafter, all the wave-fields related to the domain Ω₁ are denoted by the upper index 1, with i = 1 for the layer and i = 2 for the PWAS.

The electro-mechanical coupling for piezoelectric materials is described by

$$
\begin{align*}
\sigma_{ij} &= C_{ijkl}s_{kl} - e_{ijkl}E_k, \\
D_i &= e_{ijkl}s_{kl} + \varepsilon_{ijkl}E_j,
\end{align*}
$$

where $\sigma_{ij}$ is the stress, $D_i$ is the electric displacement, $s_{kl}$ is the strain and $C_{ijkl}$, $e_{ijkl}$, $\varepsilon_{ijkl}$ are elastic, piezoelectric and dielectric constants respectively. The components of the electric field vector

$$E_k = -\frac{\partial \phi}{\partial x_k}$$

are expressed in terms of the electric potential $\phi$.

The problem under consideration is to be treated within the limits of the plane theory of elasticity taking into consideration plain strain assumption. The two-component displacement vector $u^{(1)}(x, t)$ satisfies the Lame equations.
within the elastic layer:

\[ (\lambda + \mu) \nabla \text{div} u^{(1)}(x, t) + \mu \Delta u^{(1)}(x, t) - \rho^{(1)} \frac{\partial^2 u^{(1)}(x, t)}{\partial t^2} = 0. \]  

(3)

Material properties of the elastic isotropic layer are given by the Lame constants \( \lambda, \mu \) and mass density \( \rho^{(1)} \).

The equations of motion for the piezoelectric media \( \Omega_2 \) are written as follows:

\[ \frac{\partial \sigma^{(2)}(x, t)}{\partial x_j} = \rho^{(2)} \frac{\partial^2 u^{(2)}(x, t)}{\partial t^2}, \quad \frac{\partial D_i(x, t)}{\partial x_i} = 0. \]  

(4)

An electric input impulse \( p(t) \) is applied at the upper surface of the PWAS at the moment \( t_0 \), while the bottom surface is grounded:

\[ \phi(x, t \geq t_0) = 0, \quad x \in S_1, \]
\[ \phi(x, t > 0) = V_0 \cdot p(t), \quad x \in S_3. \]  

(5)

Stress-free boundary conditions are formulated at the surfaces of the PWAS \( S_2, S_3, S_4, S_6 \) with normal \( n \)

\[ \sigma^{(2)}(x, t) \cdot n = 0, \quad x \in S_2 \cup S_3 \cup S_4 \cup S_6. \]  

(6)

For convenience the vector of normal and tangential stresses \( \tau = \{\tau_{12}, \tau_{22}\} \) is introduced. The faces \( \pm H \) of the layer are free of stress except for the contact area \( S_c \):

\[ \tau^{(1)}(x, t) = 0, \quad \{x_2 = -H\} \cup \{x_2 = 0\} / S_c. \]  

(7)

Electric displacements \( D(x, t) \) are equal to zero on the side boundaries of the PWAS:

\[ D_i(x, t) = 0, \quad x \in S_2 \cup S_4. \]  

(8)

Displacements and stresses are continuous at the contact area \( S_c \):

\[ \tau^{(1)}(x, t) = \tau^{(2)}(x, t), \quad x \in S_c; \]
\[ u^{(1)}(x, t) = u^{(2)}(x, t), \quad x \in S_c. \]  

(9)

Normal and tangential stresses are equal zero in the debonded area \( S_{d} \):

\[ \tau^{(1)}(x, t) = \tau^{(2)}(x, t) = 0, \quad x \in S_{d}. \]  

(10)

Due to linearity of equations (3) and (4) the Laplace transform is used in order to exclude time derivatives.

In order to simulate coupling between the layer \( \Omega_1 \) and the PWAS \( \Omega_2 \) the unknown function \( q(x_2) \) is introduced at the contact area \( S_c \):

\[ q(x_2) = \tau^{(1)}(x_2, 0) = \tau^{(2)}(x_2, 0), \quad x \in S_1. \]  

(11)

2.2. Solution of the stated problem

The described boundary value problem is solved here using the SAHA [29]. Based on the SAHA, we show how to extract information about eigenfrequencies of the system of a PWAS and an elastic layer. Moreover, we show the calculation of energy fluxes for separate guided wave modes, which we will use later for the detailed description of effects, resulting from the debonding of a transducer.

According to this method, the solution strategy is firstly to consider two separated problems and secondly to model the coupling at the surface \( S_c \). The first problem corresponds to the description of the displacements induced in the layer due to dynamic load function applied on its upper surface. The second problem is to describe the mechanical and electrical state of the piezoelectric transducer under given boundary conditions. Displacements in the elastic layer with a given surface load can be constructed using the BIEM [34]. In accordance with this approach, harmonic wave-fields \( u^{(1)}(x) \) are represented as the inverse Fourier transform with respect to horizontal the coordinate \( x_1 \) as follows:

\[ u^{(1)}(x) = \frac{1}{2\pi} \int_{\Omega} K(\alpha, x_2) Q(\alpha) e^{-i\alpha x_1}d\alpha. \]  

(12)

Here, the integrand is a multiplication of the Fourier transform of the Green’s matrix \( K(\alpha, x_2) \) and \( Q(\alpha) \) which is the Fourier transform of \( q(x_2) \) introduced in (11). \( \alpha \) is the Fourier transform parameter. If the poles \( \pm \xi_k \) of the Green’s Matrix \( K(\xi) \) are assumed in the upper complex half-plane (are known), then the integral representation (12) can be evaluated in terms of the Cauchy’s residue theorem and Jordan lemma as follows [29]:

\[ u^{(1)}(x) = \sum_{k=1}^{\infty} a_k^+(x_2) e^{i\xi_k x_1}. \]  

(13)

The FDSEM based on a variational formulation is used to simulate time-harmonic motion of the PWAS. The variational formulation of the boundary value problem for the piezoelectric transducer occupying \( \Omega_2 \) is then rewritten:

\[ \int_{\Omega_2} \frac{\partial \sigma^{(2)}(x)}{\partial x_j} v_j(x) d\Omega_2 + \rho^{(2)} \omega^2 \int_{\Omega_2} u^{(2)}(x) v_i(x) d\Omega_2 = 0, \]
\[ \int_{\Omega_2} \frac{\partial D_i(x)}{\partial x_i} v_i(x) d\Omega_2 = 0. \]  

(14)

Here \( v = \{v_1, v_2, v_3\} \) are test functions, see more in [29].

The FDSEM requires the boundary of the body \( \Omega_2 \) to be discretised into finite elements. The displacements \( u^{(2)} \) and the electric potential \( \varphi \) are approximated by Lagrange interpolation polynomials in both coordinates \( x_1 \) and \( x_2 \) on each element. The solution vector \( \mathbf{y} = \{u_1, u_2, \varphi\} \) of the system (13) is written as follows:

\[ y_k(x) = \sum_{l} y_{kl} ^{l} C^{l}(x_1, x_2), \quad k = 1, 3. \]  

(15)

Here \( C^{l}(x_1, x_2) = C^{l}(x_1) C^{l}(x_2) \) are interpolation polynomials used in the FDSEM, \( R(x_1, x_2) \) is the specially determined index function, depending on the element and node number.

The SAHA is based on the coupling of the two different methods: the BIEM and the FDSEM in the contact area \( S_c \) with the boundary conditions (9)–(10). The unknown function \( q(x_2) \) is to be found in order to solve the coupled
The total amount of the wave energy transferred from the PWAS into the waveguide is calculated as follows [39]:

\[ P^0 = \frac{1}{2\pi} \int_{\Gamma} \mathbf{K}(\alpha, x_2) \mathbf{Q}(\alpha) \mathbf{Q}^*(\alpha) d\alpha. \]  

In this paper, we also use another well-known method for comparison purposes. The problem under consideration can be evaluated using the standard FEM simulation software COMSOL Multiphysics, including electro-mechanical coupling. The solution is constructed at the finite elements employing interpolation polynomials of the first and second order. This method is purely numerical and therefore demands great computational power. Although, it can be applied to a problem of any geometry, but is limited to a finite size.

One of the methods to simulate piezo-induced load is the use of a pin-force model (PFM) [40] where the contact stresses are replaced with the pin forces concentrated at the tips of the transducer. The load function in this way takes the form:

\[ \mathbf{q} = \left\{ \tau_A \left[ \delta \left(x_1 - \frac{w}{2} \right) - \delta \left(x_1 + \frac{w}{2} \right) \right], 0 \right\}. \]  

Here \( \delta \) is the Dirac delta function and the value of \( \tau_A \) is determined within the PFM.

3. Experimental investigation and verification

3.1. Experimental setup

To verify the obtained mathematical model and its numerical solution an experiment has been conducted. The scheme of the experiment is shown in figure 2. A rectangular PWAS with dimensions 70 mm × 10 mm × 0.2 mm has been glued on the surface \( z = 0 \) of an aluminium plate of the dimensions 500 mm by 500 mm and a thickness of \( H = 1 \) mm. An electric voltage signal \( p(t) \) is applied to the piezoelectric transducer and Lamb waves are excited at the surfaces of the plate (\( z = 0, z = -H \)). In this study, only \( N_c = 5 \) cycles of Hann window are used to determine the input voltage (20). As a measurement quantity, the velocity perpendicular to the plate surface is measured at several points according to figure 2 in the middle of the 70 mm direction of the transducer. These measurements have been conducted with a 1D
the course of two independent experiments with perfectly bonded experiments have been conducted twice. In experiments and especially the process of partial bonding, the back surface of the plate was accounted the effects of states in the experimental set-up were examined. To take into account the fluctuation within setting up the experiment and especially the process of partial bonding, the experiments have been conducted twice.

Due to the continuous sizes of the specimen, Lamb waves cannot be transmitted to the waveguide from the debonded side of the transducer and therefore greater amount of energy is dispatched from the glued side. It is evident, that a difference in contact conditions leads to a non-uniform distribution of the Lamb waves.

In order to demonstrate complete wave patterns excited with the central frequency $f_0 = 180$ kHz figures 4–5 are presented. These plots illustrate the surfaces of the velocities of the motion measured on the surface of the plate $u_z(x, 0, -H, t)$ and the corresponding calculated data with dependence on the spatial variable $x$ ($y = 0, z = -H$) and time $t$. Propagation of the two sustained modes $A_0$ and $S_0$ is observed on the plots. Due to the continuous sizes of the specimen, Lamb waves are reflected from the sides of the plate, what is visible on the figures 4(a)–5(a). As long as the simulation is provided for the infinite plate, there is no such reflections in figures 4(b)–5(b). Trapped energy effect is visible in the case of debonded

helium-neon laser Doppler vibrometer (LDV) from Polytec. The device is composed of a CLV700 measurement head with a CLV800 laser unit, connected with a controller CLV1000. This controller is equipped with modules CLVM002, CLVM030 and module CLVM200. This setup is limited in the frequency range up to $250$ kHz, already with reduced amplitudes in this frequency area. The laser is mounted on a two-axial measurement table to be able to measure at all positions on the plate. The data acquisition is realized via Matlab, see more details in section 6 in [16].

In order to obtain experimental data of two different bonding conditions (states), two experiments have been conducted. Firstly the PWAS has been glued only partly. For this state the contact area is $D_1 = \left[0, \frac{w}{2}, 0\right]$ and debonded in the area $D_2 = \left[0, \frac{w}{2}, 0\right]$. After the measurements of the velocities of motion excited by the partially debonded PWAS were performed, the transducer has been properly glued in the area $D_1 = \left[0, \frac{w}{2}, 0\right]$, $D_2 = [\emptyset]$. For the second state, velocities of the motion have been measured as well. In this way, two states in the experimental set-up were examined. To take into account the effects of fluctuation within setting up the experiment and especially the process of partial bonding, the experiments have been conducted twice.

Figure 3 shows velocities of the motion measured on the back surface of the plate $z = -H$ during two independent experiments. Registered data is almost identical in case of the perfectly glued PWAS. However, for the case of the half-debndoned transducer, experimental results differ. In the bonded direction ($x < -\frac{w}{2}$) there is a good agreement while in the opposite one, where the PWAS is debonded ($x > \frac{w}{2}$), measured signals virtually mismatch. Such an effect can be a result of certain difficulties during experimental specimen preparation. Since the transducer length ($70$ mm) to width ($10$ mm) ratio is equal $7$, i.e. large enough, a plane state, which can be described by 2D can be observed in the experiment. However, this results in a long boundary line between bonded and debonded part and complicates a proper check of bonding conditions over the length of the transducer after setting up the experiment.

One can observe that velocities of the motion measured on the surface of the plate with the half-debonded transducer are 1.5 times higher in the bonded direction $x < -\frac{w}{2}$ compared to the same values measured with the perfectly glued transducer. At the other hand, velocities of the motion measured from the debonded side of the transducer are slightly lower. It looks like in case of debonding excited wave energy cannot be transmitted to the waveguide from the debonded side of the transducer and therefore greater amount of energy is dispatched from the glued side. It is evident, that a difference in contact conditions leads to a non-uniform distribution of the Lamb waves.

In order to demonstrate complete wave patterns excited with the central frequency $f_0 = 180$ kHz figures 4–5 are presented. These plots illustrate the surfaces of the velocities of the motion measured on the surface of the plate $u_z(x, 0, -H, t)$ and the corresponding calculated data with dependence on the spatial variable $x$ ($y = 0, z = -H$) and time $t$. Propagation of the two sustained modes $A_0$ and $S_0$ is observed on the plots. Due to the continuous sizes of the specimen, Lamb waves are reflected from the sides of the plate, what is visible on the figures 4(a)–5(a). As long as the simulation is provided for the infinite plate, there is no such reflections in figures 4(b)–5(b). Trapped energy effect is visible in the case of debonded
transducer: it continues wave excitation for some time after the input signal (20) had been applied.

3.2. Comparison of the theoretical and experimental results

To verify the obtained mathematical model, a comparison between the experiment and calculated signals has been performed. For the simulation the following values have been used: $w = 10\, \text{mm}$, $h = 0.2\, \text{mm}$, $H = 1\, \text{mm}$, $V_2 = 12\, \text{V}$. For the first state when the transducer is perfectly glued, the contact area $S_c = [-5, 5] \, \text{mm}$ and debonded area $S_d = [\sigma]$. While for the second state when the PWAS is debonded $S_c = [-5, 0] \, \text{mm}$ and debonded area $S_d = [0, 5] \, \text{mm}$. A comparison has also been performed using FEM software COMSOL Multiphysics.

Figure 6 shows velocities of the motion measured on the surface $z = 0$ in the points $x = \pm 30\, \text{mm}$ and $x = \pm 95\, \text{mm}$ with the perfectly glued PWAS and calculated with three different models: the standard FEM model (COMSOL), the PFM [3] and with the help of the SAHA. The PFM has proven to be valid at rather low frequencies and when transducer’s to waveguide’s thicknesses ratio tends to be less than one. Therefore, one can see in figure 6 that the PFM allows getting acceptable results in case of accurate contact conditions, though local maxima do not fully coincide with the experimental signal. Wave-fields calculated with the FEM coincide with the SAHA model completely, though amplitudes of the signals differ from the experiment data with factor 1.2.

Figure 7 shows the comparison of the calculated and measured signals over spatial value $x$ with the central frequency of $80\, \text{kHz}$ at the fixed time $t$. Three independent models have been used for calculation. Data from the first experiment measured at the front surface of the plate ($z = 0$) is taken for comparison. It should be marked that there is some constant time-shift due to the experimental setup. First of all, the results obtained with the SAHA model completely agree with the same values calculated with the standard two
dimensional FEM model. The measured signal has the same waveform, though its amplitudes differ from the calculated signals. One of the reasons for such effect is that the two-dimensional mathematical model is based on a series of assumptions, which can lead to a discrepancy in amplitudes of the measured and calculated signals. Moreover, this discrepancy is also frequency-dependent. It is clearly seen if the three-dimensional COMSOL model is calculated. In this case, the amplitudes of the obtained signal coincide with the experimental data.

The other reason for the difference in amplitudes lies in the scattering of the data due to imperfections in the experimental setup. Moreover, it appears that such an elongated transducer (70 mm × 10 mm) excites a wavefront propagating mainly along the x axis, therefore amplitudes of the measured signal heavily depend on the line of the measurements. Thus, figure 8 illustrates the difference in amplitudes measured within the perfectly glued transducer at different measurement lines $y = 0$ mm and $y = 10$ mm during first and second experiments. It is clear that signals measured with two experiments at the same line $y = 0$ mm differ by 10%–12%. At the same time, a 10 mm shift of the measurement line

Figure 6. Velocities of the motion on the surface of the plate $\hat{u}_z(x = \pm 30$ mm, 0, 0, $t$) (a) and $\hat{u}_z(x = \pm 95$ mm, 0, 0, $t$) (b), measured and calculated with the central frequency $f_0 = 80$ kHz.

Figure 7. Velocities of the motion on the surface of the plate $\hat{u}_z(x, 0, 0, t = 0.1$ ms), measured and calculated with the central frequency $f_0 = 80$ kHz.

Figure 8. Velocities of the motion on the surface of the plate $\hat{u}_z(x, y, -H, t = 0.1$ ms), measured with the central frequency $f_0 = 80$ kHz in the course of two independent experiments with perfectly bonded ($S_d = \phi$) PWAS.
results in a decrease of the amplitudes by 15%–18%. However, the waveforms of the excited signals remain similar; therefore it can be concluded that the simulated wavefields are trustworthy and the designed experimental setup can be used to verify a two-dimensional model.

Figure 9 illustrates the measured and calculated signals with a central frequency of \(f_0 = 180\, \text{kHz}\) over the spatial value \(x\) at the time \(t = 0.06\, \text{ms}\). Again, the SAHA and 2D COMSOL models provide similar signals, which waveforms coincide with the one measured during experiment conduction. Though the amplitudes of the measured signal are considerably lower, the correcting factor for simulation is 0.4. The signal, calculated with the 3D COMSOL model, has lower amplitudes, which are closer to those observed in the experiment. Nevertheless, even the results of the 3D COMSOL model are twice as high as the experimental results. The reason for this effect could be that amplitudes are reduced as the frequency \(f_0 = 180\, \text{kHz}\) is close to the limit of 250\, kHz up to which the laser vibrometer can be used. Also, if the line of measurements is taken \(y = \pm 15\, \text{mm}\) then amplitudes of the measured signal are higher. Nevertheless, the waveforms of the experimental and calculated signals agree very well.

In order to verify the mathematical model for the case of a debonded PWAS \((S_d = [0])\), a comparison with experimental data has been made with the central frequencies \(f_0 = 80\, \text{kHz}\) and \(f_0 = 180\, \text{kHz}\). Figure 10 illustrates measured and calculated velocities of the motion of the plate surface in two points: from the glued side of the PWAS \(x = -113\, \text{mm}\) and from the debonded side \(x = 113\, \text{mm}\). The comparison shows that waveforms of the calculated and measured signals agree very well, but while the time of signal arrival matches perfectly on the right (from the glued side) there is some difference in the time of arrival on the left (from the debonded side). Although, if the debonded area is changed a little \(S_d = [4.5, 10]\) mm, the resulting agreement in the time of signal arrival becomes better.

Figure 11 shows the comparison of the signals obtained during the experiment and calculated with the help of the SAHA and the standard FEM method over the spatial value \(x\) with a central frequency of 80\, kHz. One can see that the amplitudes of the velocities of the motion are higher from the glued side. This effect is completely predicted by the mathematical model. The agreement of experimental and calculated signals is becoming worse for higher frequencies, see
figure 12, where velocities of the motion are illustrated with a central frequency of $f_0 = 180$ kHz. $S_d = [0, w]$.

Figure 12. Velocities of the motion on the surface of the plate $\hat{u}_x(x, 0, -H, t = 0.1$ ms), measured and calculated with the central frequency $f_0 = 180$ kHz. $S_d = [0, w]$.

The comparison of transient signals, calculated with the help of the SAHA, with experimental data verifies that the obtained two-dimensional mathematical model based on a semi-analytical hybrid approach is an efficient and reliable tool to simulate the dynamic interaction of a piezoelectric transducer on an elastic layer with different contact conditions. With the increase of the central frequency, additional effects appear and for its simulation, the use of three-dimensional mathematical models is suggested. With the aim to experimentally realize the two-dimensional case, the results also show the difficulties, scattering and uncertainties of the experimental investigation focusing debonded transducers.

4. Numerical analysis of the wave phenomena

In this section, we provide a detailed parametric analysis of the influence of PWAS’s dimensions and contact conditions between the actuator and the layer on Lamb wave excitation and resonance frequencies. The power density vector and corresponding energy streamlines are examined for specific debonding scenarios at frequencies corresponding to resonances or minima/maxima of the power input transmitted into the waveguide. Although the power input $P^0$, calculated with (18), cannot be measured in the experiment, the results of the comparisons performed in section 3 verifies the obtained model and allows to assume the reliability of the calculated power input and corresponding energy distribution coefficients. For further analysis, let us introduce the following energy distribution coefficients

$$\eta^\pm_m = P^\pm_m / P^0, \quad \eta_m = \eta^+_m + \eta^-_m, \quad \kappa^\pm_m = P^\pm_m / P^0,$$

$$m = \{A_0, S_0\}$$

indicating the contribution of the $m$th Lamb wave in the total amount of power $P^0$ (18) transferred into the waveguide or into a given direction $P^\pm$ (for asymmetrically debonded PWASs). Here $P^\pm$ is sum of the $P^\pm_m$ obtained with (17) for all $m$. $A_0$ or $S_0$ mode excitation is considered and frequency ranges, where one Lamb wave is dominating for the particular PWAS’s size, are revealed. Three different bonding scenarios are investigated: perfectly glued PWAS, damaged contact in the centre of the PWAS (the edges are still properly glued) and the one-side debonded PWAS.
First, the influence of the thickness $h$ of a perfectly bonded PWAS on Lamb wave excitation is studied. Figure 13(a) illustrates the power $P(f,h)$ transferred into an elastic waveguide of thickness $H=1\,\text{mm}$ due to the action of the PWAS of width $w=10\,\text{mm}$. The real parts of complex-valued eigenfrequencies $\Re f_n(h)$ are also drawn in figure 13(a), while the imaginary parts are depicted in figure 13(b). One can conclude that the PWAS’s height has a small influence on the power at low frequencies (up to 300 kHz). With further frequency growth, narrow bands of high power occur, therefore, one can conclude that the PWAS thickness influence on the power $P^0$ increases. It should be noted that resonance frequencies, which are real and correspond to local extreme points, should be distinguished from complex-valued eigenfrequencies $f_n$. The analysis shows that the smaller absolute value of the imaginary part of the eigenfrequency, the closer eigenfrequency to a certain real resonance is and, therefore, the greater is the influence of the eigenfrequency. Thus, the first five eigenfrequencies exhibited in figure 13 have relatively large absolute values of the imaginary parts ($|\Im f_n| > 100\,\text{kHz}$) and, therefore, the real parts do not affect the location of maxima and minima of the power $P^0$. The imaginary part of the sixth eigenfrequency has a very small absolute value ($|\Im f_6| < 20\,\text{kHz}$) if $h > 1\,\text{mm}$, and its real part coincides with the local maximum of the power $P^0$. In addition, a global minimum of the power surface $P^0(f,h)$ is observed at $f \approx 430\,\text{kHz}$ and $h = 0.9\,\text{mm}$. The nature of the occurrence of this minimum is analysed below.

In order to study the influence of the PWAS’s width $w$ on wave excitation, surfaces $P^0(f,w)$ and eigenfrequencies $f_n$ for transducers of height $h = 0.2\,\text{mm}$ and $h = 1\,\text{mm}$ are shown in figures 14(a), (b) and 14(c), (d) respectively. Again, imaginary parts of eigenfrequencies $f_n$ are drawn in the bottom of figure 14. For thicker PWASs, the number of the eigenfrequencies in a certain range increases and they are situated closer to the real axis $\Im f = 0$ in the complex plane $f$. In the case of thinner transducer ($h = 0.2\,\text{mm}$), the surface $P^0(f,w)$ demonstrating energy flux dependence on the PWAS’s width is substantially smoother compared with $h = 1\,\text{mm}$ (see figures 14(a) and (c)). Naturally, the number of local maxima in the same frequency range is larger for thicker PWAS of height $h = 1\,\text{mm}$ than for $h = 0.2\,\text{mm}$.

Nevertheless, figures 13 and 14 do not provide explanation of the fact that some eigenfrequencies with relatively small $|\Im f_n|$ are revealed in $P^0(f,w)$ and $P^0(f,h)$ plots. For instance, eigenfrequencies $f_6$ and $f_8$ exhibit themselves in energy flux plots $P^0(f,w)$ and $P^0(f,h)$, while $f_7$ is not visible. To study this issue, eigenforms of these three eigenfrequencies have been calculated. Figure 15 demonstrate eigenforms (amplitudes of the displacement vector $|u| (x_1, x_2)$ normalized by maximum value) for three eigenfrequencies $f_6$ (figure 15(a)), $f_7$ (figure 15(b)) and $f_8$ (figure 15(c)). It is clearly seen that wave motion for $f_6$ and $f_8$ is dominantly localized in the PWAS and in the layer below the PWAS, i.e. in the region $-H < x_2 \leq h, |x_1| < w/2$. Whereas wave maxima values of the displacement vector for eigenfrequency $f_7$, which is not pronounced via peaks in energy flux plots, are in the regions situated in the waveguide, but to the right and to the left from the PWAS (not below it). Therefore, it can be concluded that two kinds of eigenfrequencies with relatively small values $|\Im f_n|$ can be separated: localized in the vicinity of the PWAS (they cause sufficient changes in $P^0$) and localized outside the area below the PWAS (they are not visible in $P^0$ plot as peaks).

Energy flux $P^0(f)$ and energy distribution coefficient $\eta_{lb}$ in dependence on frequency is exhibited in figures 16(a), (b) for PWASs of height $h = 0.2\,\text{mm}$ and $h = 1\,\text{mm}$ computed with the SAHA [29] and calculated applying the PFM. In figure 16(c), squares ($h = 0.2\,\text{mm}$) and circles ($h = 1\,\text{mm}$) show eigenfrequencies in $-150 \leq \Im f_n \leq 0\,\text{kHz}$ of the complex plane $f$. At frequencies up to 300 kHz, the PFM gives results quite similar to the results calculated by the SAHA for the PWAS of height $h = 0.2\,\text{mm}$, which is in a good agreement with theoretical predictions. The energy flux for the thicker PWAS ($h = 1\,\text{mm}$) discriminates significantly from the flux, actuated by the thinner PWAS ($h = 0.2\,\text{mm}$), see figure 16(a). This conclusion is also valid for the wave energy distribution among $A_0$ and $S_0$ modes. It can be seen in

![Figure 13. Energy flux $P^0(f,h)$ (a), real (a) and imaginary parts (b) of the eigenfrequencies $f_n(h)$ for bonded PWAS, $w = 10\,\text{mm}$.](image-url)
that Lamb wave excitation by the PWAS of height $h = 1$ mm leads to wider frequency ranges, where excitation of $A_0$ mode prevails. At the same time, the oscillation of the thinner PWAS ($h = 0.2$ mm) results in a smoother contribution of $A_0$ mode in the spectrum of excited transient signals. Therefore, it is essential to take into account PWAS’s height in theoretical and experimental studies.

For the three cases considered in figure 16, the first maxima of the energy flux $P^0(f)$ are very close (around 220 kHz), even for the thicker transducer of height $h = 1$ mm. To compare and exhibit more detailed wave processes related to the interaction between the waveguide and PWASs of various thicknesses, wave energy distribution is considered at frequencies corresponding to local maxima of $P^0(f)$. The power density vector $e$ and related energy streamlines are shown in figure 17 for the PFM (a), PWAS’s height $h = 0.2$ mm (b) and $h = 1$ mm (c) at the frequencies corresponding to the first maximum of the $P^0$ for each case presented in figure 16(a). If the PWAS is substituted by point forces, a certain amount of wave energy is trapped in two large vortices located in the rectangular domain of the waveguide below the PWAS. Two large vortices are also observed for $h = 0.2$ mm.
Figure 16. Energy flux $P^0(f)$ (a) and the energy distribution coefficient $\eta_A(f)$ (b) for $A_0$ mode in the energy flux, eigenfrequencies $f_n$ with $-150 \leq \text{Im} f_n \leq 0$ kHz (c) for bonded PWAS of width $w = 10$ mm.

Figure 17. The power density vector $|e|$ and related energy streamlines. PFM (a), SAHA $h = 0.2$ mm (b), SAHA $h = 1$ mm (c).
though they are circulating through the transducer and the energy trapped in these vortices is much smaller compared to the PFM case. At last, if the PWAS is thick \( h = 1 \text{ mm} \) large vortices disintegrate and the energy flux is transferred into the layer. In each case, vortices obstruct the waveguide below the transducer, and it can be concluded that the latter leads to maximum energy flux transmittance.

For a complete analysis of \( P^0(f) \) behaviour, maxima and minima should be compared. Figure 18 illustrates the power density vector \( |e| \) and related energy streamlines excited in the layer by a thick PWAS \( h = 1 \text{ mm} \) at three frequencies, where the global minimum \( f = 430 \text{ kHz} \) and global maximum \( f = 486 \text{ kHz} \) of \( P^0 \) are achieved and for regular transmission \( f = 594 \text{ kHz} \), at \( f = \text{Ref}_7 \). Four huge vortices circulating in the transducer and the layer are visible in the case where a minimum of \( P^0 \) is achieved (see figure 18(a)). These vortices accumulate part of the energy excited by the PWAS and, therefore, the total amount of the energy flux \( P^0 \) is substantially smaller. On the contrary, only two tiny vortices are visible in figure 18(b) for the frequency with maximum power at 486 kHz and the amplitudes \( |e| \) in the far-field zone are greater than below the PWAS. It can be observed, that amplitudes of the power density vector \( |e| \) are almost \( 10^4 \) larger compared to the case, where minimum energy flux from PWAS is considered. A regular wave excitation is observed in figure 18(c). Though the considered frequency is obtained as a real part of the eigenfrequency \( f_7 \), the values of \( |e| \) are medium compared to two extremal cases discussed above.

4.2. One-side debonded PWAS

The parameter \( w_1 \) is introduced here to describe a one-side debonded PWAS with a debonding area \( S_d = [w_1, w] \). First, let us consider the particular debonding area \( S_d = [1.4, 5] \text{ mm} \). Figure 19(a) depicts the power \( P^0(f) \) and \( P_{\text{PS}}(f) \), while figure 19(b) illustrates the distribution of the energy flux between Lamb waves propagating to the right \( (P_{\text{PA}}^0, P_{\text{PA}}^0) \) and to the left \( (P_{\text{PA}}^0, P_{\text{PA}}^0) \) from the PWAS by means of the wave energy distribution coefficients \( \eta_{\text{PS}}^0(f) \) defined by relation (21).

Except for three relatively narrow frequency ranges, the amount of the wave energy flux transferred to the left (from the glued side of the PWAS) is greater. A very deep local minimum at 420 kHz, and a substantial number of sharp peaks are observed in \( P^0(f) \) plot in figure 19(a). In order to reveal their resonance nature, complex-valued eigenfrequencies \( f_n \) are also shown in figure 19(c) as circles. It should be mentioned that the computation of eigenfrequencies of a debonded PWAS is more complicated: several eigenfrequencies in a certain domain of the complex frequency plane \( f \) arises with the increase of the debonding area \( S_d \). Analysis shows that a particular eigenfrequency can be matched approximately with each sharp peak in \( P_{\text{PS}}^0(f) \) plot shown in figure 19(a). The majority of eigenfrequencies with \( |\text{Im} f_n| < 50 \text{ kHz} \) invoke peaks in \( P_{\text{PS}}^0(f) \) plots. As in the case of perfectly bonded PWAS, eigenfrequencies with relatively large \( |\text{Im} f_n| > 50 \text{ kHz} \) are not well pronounced, and no influence on resonance properties of the structure is revealed.

Figure 18. The power density vector \( |e| \) and related energy streamlines for the minimum of \( P^0 \) at \( f = 430 \text{ kHz} \) (a), the maximum of \( P^0 \) at \( f = 486 \text{ kHz} \) (b) and for regular excitation, but at \( f = \text{Ref}_7 = 594 \text{ kHz} \) (c).
In order to investigate the effect of the size $w_1$ of the debonded area $S_d$ on Lamb wave excitation, $P^0(f, w_1)$ and the relative energy flux $\kappa^0_{ab}(f, w_1)$ for one-sided debonded PWAS ($S_d = [w_1, w]$) with dimensions $h = 0.2$ mm and $w = 10$ mm are analysed. Figure 20(a) depicts the energy flux $P^0(f, w_1)$ transferred into the layer. Two kinds of trajectories revealing as local maxima can be distinguished by the thickness of dark zones, some of them have flat peaks,
whereas others have narrow peaks. They are located denser if the debonding area increases.

The share of the energy flux $\kappa = P / P_0$ transferred along the $x_1$ axis in $x \rightarrow -\infty$ (the glued side) in dependence on the frequency and the size of debonding is illustrated in figure 20(b). It should be mentioned that only trajectories with narrower peaks visible in $P(f, w_1)$ can be observed in $\kappa(f, w_1)$. Especially for the combinations of frequency $f$ and debonding tip $w_1$ laying on or in the vicinity of these traces, the amount of wave energy, which is transferred to the left, is much larger than the amount of wave energy, which is transferred to the right.

Figure 21 illustrates the influence of the debonding size on the wave energy distribution $\kappa(f, w_1)$ among Lamb wave modes propagating in negative ($x \rightarrow -\infty$) (figure 21(a)) and positive ($x \rightarrow \infty$) (figure 21(b)) directions. Here red and blue zones correspond to the situations, where $A_0$ or $S_0$ Lamb waves are dominating respectively. Wave energy distribution changes sufficiently in the vicinity of the trajectories revealed as narrow peaks in $\kappa(f, w_1)$ surface (see figure 20(b)). Here, $A_0$ mode excitation usually increases at the debonded edge ($\kappa_A(f, w_1)$) and decreases at the bonded edge ($\kappa_A(f, w_1)$) respectively.

It is well known, that PWAS excites waves mostly at its edges [3, 40]. If the PWAS is one-side debonded, Lamb waves are generated at the edges of the contact area, i.e. in the middle part of one-side debonded PWAS, where it is still glued. Figure 22 showing power density vector and the corresponding energy streamlines for the debonded PWAS $S_d = [1.4, 5]$ mm at the frequency $f = 49$ kHz, where a local maxima in $P(f)$ is achieved, illustrates this fact.

4.3. Centrally debonded PWAS

The influence of the central debonding of the PWAS on the waves excitation is analysed in this section. Parameter $\Delta w$ is introduced so that debonding of the PWAS is $S_d = [-w + \Delta w, w + \Delta w]$. Thus, if $\Delta w = 0.5$ mm, only 0.25 mm is bonded at each edge. Figure 23(a) shows the dependence of energy flux $P(f, \Delta w)$ excited in the structure by centrally debonded PWAS on the severity of the debonding and frequency. It is evident from figure 23(a) that central debonding has a modest influence on the excited energy flux $P(f)$ apart from several resonance frequencies. However, the number of such frequencies is significantly

Figure 21. Wave energy distribution coefficients $\kappa_A(f, w_1)$ (a) and $\kappa_S(f, w_1)$ (b) for one-side debonded PWAS ($w = 10$ mm, $h = 0.2$ mm, $S_d = [w_1, w]$).

Figure 22. The energy density vector $|e|$ and related energy streamlines at $f = 49$ kHz ($w = 10$ mm, $h = 0.2$ mm, $S_d = [1.4, 5]$ mm).
reduced in comparison to one-sided debonding. Since excitation is symmetrical in the considered case, only $\eta_{A_0}$ is demonstrated in figure 23(b). One can see that $A_0$ is most excited in the following frequency ranges: 0–70 kHz, 400–550 kHz. Besides, if the PWAS is debonded less than 50% then the influence of debonding area on wave distribution among Lamb waves is relatively low even at higher frequencies.
A particular case of centrally debonded PWAS with \( w = 10 \text{ mm}, h = 0.2 \text{ mm}, S_d = [-2.65, 2.65] \text{ mm} \) is illustrated in figure 24 in the same manner as for one-side debonded PWAS above (circles in figure 24(c) correspond to eigenfrequencies). To simplify comparison with perfectly bonded PWAS, dashed lines corresponding to PWAS with \( w = 10 \text{ mm}, h = 0.2 \text{ mm} \) and \( S_d = \emptyset \) are added in figures 24(a) and (b). A mismatch between centrally debonded and bonded PWASs is quite small at lower frequencies below 230 kHz, whereas the effect of debonding becomes more sufficient at higher frequencies.

A very deep non-resonant (eigenfrequencies are absent in its vicinity) minimum of a smooth wide arch in \( P^p(f) \) plot is reached at \( f = 605 \text{ kHz} \); around \( f = 605 \text{ kHz} A_0 \) dominates in the excited wave-fields. Three sharp peaks are revealed in \( P^p(f), P^p_+ (f) \) and \( \eta^p_k (f) \) plots, but only two of them are situated closely to \( \text{Re} \ f_0 \) (25 and 125 kHz) and can be reported as resonant ones. In figure 25, the energy density vector and the corresponding energy streamlines are depicted for two frequencies: \( f = 528 \text{ kHz} \) corresponding to the third local minimum (figure 25(a)) and \( f = 533 \text{ kHz} \) (figure 25(b)) corresponding to the local maximum next to the minimum at \( f = 528 \text{ kHz} \). The amplitudes of the excited wave-field in the far-field zone differ approximately 4 times, whereas the difference in frequency is 5 kHz only. The reason of the low excitation at the frequency \( f = 528 \text{ kHz} \) is that a considerable part of the wave energy is accumulated in the two huge vortices under the PWAS and it does not flow into the waveguide. At the same time, more wave energy is excited by the edges of the PWAS at the frequency \( f = 533 \text{ kHz} \), which is not captured under the PWAS and is radiated into the waveguide.

5. Conclusion

The semi-analytical hybrid approach combining the BIEM and the FDSEM is applied here to analyse Lamb wave excitation by a debonded PWAS. This method allows calculating efficiently eigenfrequencies and Lamb wave distribution in an unbounded layer with a debonded piezoelectric transducer. The results obtained during the experiments with perfectly bonded and partially debonded PWASs glued to an aluminium plate are presented and compared with simulations. The measured signals using laser Doppler vibrometer are employed to verify the mathematical model in case of various bonding conditions between the PWAS and the waveguide.

It is shown, that only eigenfrequencies with a small absolute value of the imaginary part \( \text{Im} f_0 \) \( < 50 \text{ kHz} \) correspond to the resonances for perfectly bonded PWAS and might influence the total output from the PWAS. The analysis shows that two different kinds of eigenfrequencies can be distinguished. Eigenvibrations corresponding to the first kind of eigenfrequencies are strongly localized in the PWAS and its vicinity. On the contrary, eigenforms corresponding to the second kind of eigenfrequencies reach their maxima values in the regions situated to the right and to the left from the PWAS. It should be noted that eigenfrequencies of the second kind are not related to real resonances, even if the absolute value \( \text{Im} f_0 \) is small.

Energy flux and excitation of Lamb waves \( A_0 \) and \( S_0 \) are analysed in dependence on the PWAS sizes and the debonding area between the PWAS and the waveguide. It is shown, that distribution between the \( A_0 \) and \( S_0 \) modes alters significantly if the transducer is thick, therefore, for deep analysis, one needs to take into account the transducer’s height. A one-sided debonding deeply influences the wavefields excited in the plate; the distribution of the power between Lamb waves alters significantly with minor debonding area sweep and becomes even more frequency-dependent.

On the contrary, if central debonding occurs, wave energy distribution among Lamb waves almost does not change for frequencies up to 600 kHz. Moreover, in particular cases, central debonding might lead to an increase in energy flux compared to the perfectly bonded PWAS. The provided analysis might be useful for the development of the
self-diagnosis methods for the PWASs and to improve the existing NDE and SHM algorithms employing piezoelectric transducers. It should be noted, that the presence of delamination in the vicinity of PWAS significantly affects the wavefields and eigenfrequencies, and the latter should be kept in mind if PWAS is tested.

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