Inflection-Point Inflation with Axion Dark Matter in light of Trans-Planckian Censorship Conjecture

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Motivated by the recently proposed Trans-Planckian Censorship Conjecture (TCC), we propose a gauged $B-L$ model of inflection-point inflation with axion dark matter. The Hubble scale during inflation ($H_{\text{inf}}$) satisfies the TCC bound of $H_{\text{inf}} \lesssim 1$ GeV, the axion dark matter scenario is free from the axion domain wall and isocurvature problems, and the axion decay constant can be larger than $10^{12}$ GeV. The seesaw mechanism is automatically incorporated in the model and the observed baryon asymmetry of the universe can be reproduced via resonant leptogenesis.

I. INTRODUCTION

The recently proposed Trans-Planckian Censorship Conjecture (TCC) [1] effectively states that sub-Planckian quantum fluctuations in an expanding universe should remain quantum and never get larger than the Hubble horizon. Applied to inflationary cosmology this conjecture yields an approximate upper bound of around $10^{10}$ GeV on the energy scale during inflation, or equivalently, the upper bound on the Hubble scale $H_{\text{inf}} \lesssim 1$ GeV during the (slow-roll) inflation [2]. The TCC offers a potential resolution of the so-called Trans-Planckian problem [3] for a low energy effective theory to be consistent with quantum gravity and avoid being relegated to the “swampland” [4] (see, also, Ref. [5]). Clearly, the TCC conjecture favors low scale inflation models, which motivates us here to implement inflection-point inflation [6] in a gauged $U(1)_{B-L}$ extension of the Standard Model (SM). (For recent work on inflation scenarios compatible to the TCC, see Ref. [7]). To account for the missing dark matter (DM) in the SM, we include a $U(1)$ Peccei-Quinn (PQ) symmetry [8], which resolves the strong CP problem of the SM and also provides a compelling DM candidate in the form of axion [9,10]. In order to cancel gauge anomalies due to the presence of the $B-L$ symmetry, three right-handed neutrinos have to be introduced. The presence of these latter neutrinos allows one to account for the observed solar and atmospheric neutrino oscillations. Furthermore, the observed baryon asymmetry can be explained via leptogenesis [11].

This brief note is organized as follows: In the next section, we propose a $U(1)_{B-L} \times U(1)_{\text{PQ}}$ extension of the SM, which appends $U(1)_{\text{PQ}}$ to the minimal $B-L$ model [12]. In Sec.III we discuss inflection-point inflation with $H_{\text{inf}} \lesssim 1$ GeV to satisfy the TCC bound, and in Sec.IV we discuss reheating after inflation and leptogenesis. An axion DM scenario with low scale $H_{\text{inf}}$ is discussed in Sec.V and our conclusions are presented in Sec.VI.

| $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_{B-L}$ | $U(1)_{\text{PQ}}$ |
|-----------|-----------|-----------|-------------|-----------------|
| $q_L^i$    | $3$       | $2$       | $1/6$       | $1/3$           |
| $u_R^i$    | $3$       | $1$       | $-2/3$      | $1/3$           |
| $d_R^i$    | $3$       | $1$       | $1/3$       | $1/3$           |
| $\ell_L^i$ | $1$       | $2$       | $-1/2$      | $-1$            |
| $\nu_R^i$  | $1$       | $1$       | $1$         | $-1$            |
| $N_R^i$    | $1$       | $1$       | $0$         | $1$             |
| $H_u$      | $1$       | $2$       | $1/2$       | $0$             |
| $H_d$      | $1$       | $2$       | $-1/2$      | $0$             |
| $S$        | $1$       | $1$       | $0$         | $0$             |
| $\Phi$     | $1$       | $1$       | $0$         | $-2$            |

**TABLE I.** Particle content of the model. In addition to the three generations of SM fermions ($i = 1, 2, 3$), three RHNs ($N_R^i$) are introduced. The scalar sector has two Higgs doublet fields ($H_u$ and $H_d$) and two SM singlet Higgs fields ($\Phi$ and $S$).

II. $U(1)_{B-L} \times U(1)_{\text{PQ}}$ EXTENSION OF THE SM

The minimal $B-L$ model [12] is a well-motivated extension of the SM, where the global $B-L$ (baryon minus lepton number) symmetry of the SM is gauged. The introduction of three right-handed neutrinos (RHNs) is crucial to cancel all the gauge and mixed gauge-gravitational anomalies. The RHNs acquire Majorana masses from the $U(1)_{B-L}$ symmetry breaking, and after the electroweak symmetry breaking, the type-I seesaw mechanism [13] generates tiny masses for the observed neutrinos. The particle content of our $U(1)_{B-L} \times U(1)_{\text{PQ}}$ model is listed in Table I. The SM singlet scalar field $\Phi$ breaks the $B-L$ symmetry and is crucial for implementing the inflection-point inflation. The introduction of two SM Higgs doublets ($H_u,d$) and a $U(1)_{B-L}$ singlet scalar ($S$) is crucial to incorporate the PQ symmetry. One may regard our model as a $U(1)_{B-L}$ extension of the Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) model [14,15] with $S$ being the PQ field.

The gauge and $U(1)_{\text{PQ}}$ invariant Higgs potential is given...
by
\[ V = - \sum_{i=u,d} \mu_i^2 \left( H_i^\dagger H_i \right) + \sum_{i=u,d} \lambda_i \left( H_i^\dagger H_i \right)^2 \]
\[ + \left( \sqrt{2} \Lambda_H (H_u \cdot H_d) S + \text{h.c.} \right) \]
\[ + \lambda_\Phi \left( \Phi^\dagger \Phi - \frac{v_\Phi^2}{2} \right)^2 + \lambda_S \left( S^\dagger S - \frac{v_P^2}{2} \right)^2 \]
+ mixed quartic terms, \hspace{1cm} (1)
where \( \Lambda_H, v_{BL} \) and \( v_{PQ} \) are mass parameters, all couplings are chosen to be real and positive, the \( \cdot \) dot represents contraction of \( SU(2) \) indices by \( \varepsilon \) tensor, and the last term on the right-hand side indicates the mixed quartic scalar couplings such as \( (H_u H_d)(\Phi^\dagger \Phi), (H_u H_d)(S S') \), etc. For simplicity, we assume that the mixed quartic couplings involving \( \Phi \) and \( S \) are negligibly small, and the \( B-L \) and the PQ symmetry breaking scales are much higher than the electroweak scale and \( \Lambda_\Phi \).

With the vacuum expectation values (VEVs) of \( \Phi \) and \( S \) given by \( \langle \Phi \rangle = v_{BL}/\sqrt{2} \) and \( \langle S \rangle = v_{PQ}/\sqrt{2} \), respectively, the SM singlet Higgs fields can be parameterized as
\[ \Phi(x) = \frac{1}{\sqrt{2}} \left( \phi(x) + v_{\chi} \right) e^{i\chi(x)/v_{\chi}}, \]
\[ S(x) = \frac{1}{\sqrt{2}} \left( s(x) + v_{\eta} \right) e^{i\eta(x)/v_{\eta}}. \hspace{1cm} (2) \]
The field \( \chi \) is the would-be Nambu-Goldstone (NG) boson that is absorbed by the \( U(1)_{B-L} \) gauge boson \( (Z') \), and the field \( a(x) \) is the NG boson (axion) associated with the PQ symmetry breaking. The masses for \( \phi \) and \( s \), and the \( Z' \) boson are given by
\[ m_{\phi} = \sqrt{2} \lambda_{\Phi} v_{BL}, \hspace{0.5cm} m_s = \sqrt{2} \lambda_s v_{PQ}, \hspace{0.5cm} m_{Z'} = 2g v_{BL}, \hspace{1cm} (3) \]
respectively, where \( g \) is the \( B-L \) gauge coupling.

With our assumption of negligible mixed quartic couplings, we analyze the Higgs doublets sector separately from the \( \Phi \) and \( S \) sectors. The PQ symmetry breaking generates a mixing mass term for the two Higgs doublets, \( m^2(H_u \cdot H_d) \), where \( m^2 = \Lambda_{\Phi} v_{PQ} \). Note that with the PQ charge assignments listed in Table I, \( H_u \) \( (H_d) \) only couples with the up-type (down-type) SM fermions. Therefore, the low energy effective theory of our model after the \( B-L \) and PQ symmetry breakings is nothing but the type-II two Higgs doublet SM (+ axion). Since this two Higgs doublets model is well studied (see, for example, [14]), we skip the detailed phenomenology of the low energy effective theory.

In addition to the Yukawa interactions of the SM lepton and quark, the following new Yukawa interactions involving the RHNs are introduced:
\[ \mathcal{L}_Y = - \frac{3}{2} \sum_{i,j=1} Y_{ij} \overline{N_R^i} \tilde{N}_R \Phi (f_L \cdot H_u) - \frac{3}{2} \sum_{k=1} Y_k \Phi N_R \tilde{N}^k_R, \hspace{1cm} (4) \]
where \( Y_D \) (\( Y_k \)) is the Dirac (Majorana) neutrino Yukawa coupling, and we have chosen a flavor-diagonal basis for the Majorana Yukawa coupling without loss of generality. Through the \( B-L \) and the electroweak symmetry breakings, the Dirac and the Majorana masses for the neutrinos are generated:
\[ m_D^{ij} = \frac{Y_D^{ij}}{\sqrt{2}} v_u, \hspace{0.5cm} m_{N^i} = \frac{Y_i}{\sqrt{2}} v_{BL}, \hspace{1cm} (5) \]
where \( v_u \) is the VEV of \( H_u \).

**III. INFLECTION-POINT INFLATION**

The inflection-point inflation (IPI) scenario is a unique low scale inflation scenario driven by a single scalar field \( \phi \). We begin by highlighting the key results of the IPI scenario. [See Ref. [6] for details.] Assuming that the inflaton potential exhibits an (approximate) inflection point, we express the inflaton potential around \( \phi = M \) close to the inflection point as
\[ V(\phi) \simeq V_0 + \sum_{n=1}^3 \frac{1}{n!} V_n (\phi - M)^n, \hspace{1cm} (6) \]
where \( V_0 = V(M), \hspace{0.5cm} V_n \equiv d^n V/d\phi^n \big|_{\phi=M} \), and the point \( \phi = M \) is identified with the inflaton value at the horizon exit corresponding to the pivot scale \( k_0 = 0.05 \text{ Mpc}^{-1} \) of the Planck 2018 measurements [17].

With the inflaton potential in Eq. (6), the slow-roll parameters are expressed as
\[ \epsilon \simeq \frac{M_P^2}{2} \left( \frac{V_1}{V_0} \right)^2, \hspace{0.5cm} \eta \simeq \frac{M_P^2}{4} \left( \frac{V_2}{V_0} \right), \hspace{0.5cm} \zeta \simeq \frac{M_P^4}{V_0} \left( \frac{V_3}{V_0} \right), \hspace{0.5cm} (7) \]
where \( M_P = 2.43 \times 10^{18} \text{ GeV} \) is the reduced Planck mass. The inflationary predictions are given by
\[ n_s = 1 - 6\epsilon + 2\eta, \hspace{0.5cm} r = 16\epsilon, \hspace{0.5cm} \alpha = 16\epsilon \eta - 24\epsilon^2 - 2\zeta^2, \hspace{0.5cm} (8) \]
where \( n_s, r, \) and \( \alpha \) are the scalar spectral index, the tensor-to-scalar ratio, and the running of the spectral index, respectively. The amplitude of the curvature perturbation \( \Delta^2_\mathcal{R} \) is given by
\[ \Delta^2_\mathcal{R} \simeq \frac{V_0}{24\pi^2 M_P^4}. \hspace{1cm} (9) \]

To be consistent with the central values from the Planck 2018 results [17], \( \Delta^2_\mathcal{R} = 2.195 \times 10^{-9} \) and \( n_s = 0.9649, V_{1,2,3} \) are expressed as
\[ \frac{V_1}{M_P^3} \simeq 1.96 \times 10^3 \left( \frac{M}{M_P} \right)^3 \left( \frac{V_0}{M_P^4} \right)^{3/2}, \]
\[ \frac{V_2}{M_P^2} \simeq -1.76 \times 10^{-2} \left( \frac{M}{M_P} \right)^2 \left( \frac{V_0}{M_P^4} \right), \]
\[ \frac{V_3}{M_P} \simeq 6.99 \times 10^{-7} \left( \frac{60}{N} \right)^2 \left( \frac{M}{M_P} \right) \left( \frac{V_0}{M_P^4} \right)^{1/2}, \hspace{1cm} (10) \]
where \( N \) is the number of e-folds which is defined as
\[ N = \frac{1}{M_P} \int_{\phi_e}^{\phi_i} d\phi \left( \frac{V}{dV/d\phi} \right) \]
with \( \phi_e \) is the inflaton field value at the end of inflation determined by \( \epsilon(\phi_e) = 1 \).
In order to solve the horizon problem of big bang cosmology, the number of e-folds is expressed as

$$N \approx 42.0 + \frac{2}{3} \ln \left( \frac{H_{\text{inf}}}{0.1 \text{ GeV}} \right) + \frac{1}{3} \ln \left( \frac{T_R}{10^9 \text{ GeV}} \right),$$  \hspace{1cm} (12)

where $T_R$ is the reheat temperature after inflation. Assuming instant reheating, the reheat temperature is given by $T_R \sim \sqrt{H_{\text{inf}} M_P}$. To satisfy the TCC bound, $H_{\text{inf}} \lesssim 0.1$ GeV and thus $T_R \lesssim 10^9$ GeV. As a benchmark, let us take $H_{\text{inf}} = 0.1$ GeV and $N = 40$. In this case, the running of the spectral index is predicted to be $\alpha \approx -0.0062$ [6]. This is consistent with the Planck 2018 result $\alpha = -0.0045 \pm 0.0067$ [17] and can be further tested in the future [18].

Next we identify the inflaton in the IPI scenario with the $B-L$ Higgs field, $\phi = \sqrt{2} \text{Re}[\Phi]$. In our inflation analysis, we employ the Renormalization Group (RG) improved effective potential given by

$$V(\phi) = \lambda_\phi(\phi) \left( \Phi^\dagger \Phi - \frac{v_{BL}^2}{2} \right)^2 \approx \frac{1}{4} \lambda_\phi(\phi) \phi^4.$$  \hspace{1cm} (13)

Here, we have used $\phi \gg v_{BL}$ during the inflation, and $\lambda_\phi(\phi)$ is the solution to the following RG equations:

$$d\phi \frac{dg}{d\phi} = \frac{12}{16\pi^2} g^3,$$

$$dY_i \bigg|_{\phi} = -\frac{1}{16\pi^2} \left. Y_i \right| + \frac{1}{2} \sum_{j=1}^{3} Y_j^2 - 6g^2 \right),$$

$$d\lambda_\phi \bigg|_{\phi} = \beta_{\lambda_\phi}.$$  \hspace{1cm} (14)

The beta-function of $\lambda_\phi$ is approximately given by

$$\beta_{\lambda_\phi} \approx \frac{1}{16\pi^2} \left( 96g^4 - \sum_{i=1}^{3} Y_i^2 \right)$$  \hspace{1cm} (15)

for $\lambda_\phi \ll g^4, Y_i^2$. We later justify the validity of this approximation.

The RG improved effective potential in Eq. (13) leads to

$$\frac{V_1}{M^4} = \frac{1}{4} \left( 4\lambda_\phi + \beta_{\lambda_\phi} \right) \bigg|_{\phi = M},$$

$$\frac{V_2}{M^2} = \frac{1}{4} \left( 12\lambda_\phi + 7\beta_{\lambda_\phi} + M^2\beta_{\lambda_\phi}^\prime \right) \bigg|_{\phi = M},$$

$$\frac{V_3}{M} = \frac{1}{4} \left( 24\lambda_\phi + 26\beta_{\lambda_\phi} + 10M\beta_{\lambda_\phi}^\prime + M^2\beta_{\lambda_\phi}^\prime \right) \bigg|_{\phi = M}.$$  \hspace{1cm} (16)

where the prime denotes a derivative with respect to the field $\phi$. Since $M$ is very close to the inflection point, $V_1/M^4 \approx 0$ and $V_2/M^2 \approx 0$, which lead to $\beta_{\lambda_\phi}(M) \approx -4\lambda_\phi(M)$ and $M^2\beta_{\lambda_\phi}^\prime(M) \approx 16\lambda_\phi(M)$. With the couplings $g, Y_i, \lambda_\phi \ll 1$, we can approximate $M^2\beta_{\lambda_\phi}^\prime(M) \approx -M\beta_{\lambda_\phi}(M) \approx -16\lambda_\phi(M)$. Hence, the last equation in Eq. (16) is simplified as $V_3/M \approx 16\lambda_\phi(M)$. Comparison with $V_3/M$ in Eq. (10) with $Y_0 \approx (1/4)\lambda_\phi(M)M^4$ yields

$$\lambda_\phi(M) \approx 2.41 \times 10^{-15} \left( \frac{M}{M_P} \right)^2.$$  \hspace{1cm} (17)

Using this expression of $\lambda_\phi(M)$, we find $H_{\text{inf}}$ as a function of $M$:

$$H_{\text{inf}} \approx \sqrt{\frac{V_0}{3M_P^2}} \approx 3.45 \times 10^{10} \text{ GeV} \left( \frac{M}{M_P} \right)^3.$$  \hspace{1cm} (18)

For our benchmark $H_{\text{inf}} = 0.1$ GeV to satisfy the TCC bound, we find $M = 3.47 \times 10^{14}$ GeV. In this case, the prediction for $r$ is tiny, namely, $r \sim 10^{-31}$.

Let us consider some low energy predictions of the IPI scenario. In the following analysis, let us set $Y_1 \approx Y_2 \ll Y_3 \equiv Y$. The inflection point condition, $\beta_{\lambda_\phi}(M) \approx 0$, leads to

$$Y(M) \approx 3.13 g(M).$$  \hspace{1cm} (19)

With this relation, the RG equations in Eq. (14), and another inflection point condition, $M\beta_{\lambda_\phi}^\prime(M) \approx 16\lambda_\phi(M)$, we obtain

$$\lambda_\phi(M) \approx 3.09 \times 10^{-3} g(M)^6.$$  \hspace{1cm} (20)

Comparing this with Eq. (17), the $B-L$ gauge coupling is expressed as a function of $M$:

$$g(M) \approx 9.60 \times 10^{-3} \left( \frac{M}{M_P} \right)^{1/3}.$$  \hspace{1cm} (21)

With the relations of Eqs. (17), (19), and (21), we approximately solve the RG equations to find [6]

$$\lambda_\phi(\phi) \approx 1.93 \times 10^{-14} \left( \frac{M}{M_P} \right)^2 \left( \ln \left( \frac{\phi}{M} \right) \right)^2.$$  \hspace{1cm} (22)

The particle mass spectrum (see Eq. (3)) is found to be

$$m_{\psi} \approx 1.97 \times 10^{-7} v_{BL} \left( \ln \left( \frac{M}{v_{BL}} \right) \right) \left( \frac{M}{M_P} \right),$$

$$m_{\psi} \approx 1.86 \times 10^{-2} v_{BL} \left( \frac{M}{M_P} \right)^{1/3},$$

$$m_{N^3} \approx 0.84 m_{\psi}.$$  \hspace{1cm} (23)

For our benchmark $H_{\text{inf}} = 0.1$ GeV ($M = 3.47 \times 10^{14}$ GeV), we fix $v_{BL} = 2.10 \times 10^{12}$ GeV for the rest of our analysis, so that $\lambda_\phi(M) \approx 400$ GeV, $m_{\psi} \approx 2.10 \times 10^8$ GeV, and $m_{N^3} \approx 1.71 \times 10^9$ GeV.

With $M = 3.47 \times 10^{14}$ GeV, we show in Fig. 1 the RG running of the inflaton quartic coupling (top), and the corresponding RG improved effective inflaton potential (bottom). The top panel shows that the RG running of the quartic coupling (solid curve) exhibits a minimum at $\phi \approx M$ with an almost vanishing value, or equivalently, $\lambda_\phi(M) \approx 0$ and $\beta_{\lambda_\phi}(M) \approx 0$. This behavior of the running of $\lambda_\phi$ is essential to realize an approximate inflection-point at $\phi = M$ as shown in the bottom panel.

**IV. REHEATING AND RESONANT LEPTOGENESIS**

After inflation, the universe is thermalized by the SM particles created from the inflaton decay. To evaluate the reheat
temperature, we consider mixed quartic couplings of the inflaton with the SM Higgs doublets:
\[
V \supset 4 \lambda' (\Phi^4) \left( H_u^3 H_u + H_d^3 H_d \right) \supset \lambda' v_{BL} \phi h^2,
\]  
where we have introduce a common coupling \(\lambda'\), and \(h\) is the SM-like Higgs boson. The decay width of the inflaton is given by
\[
\Gamma(\phi \to hh) \simeq \frac{\lambda'^2 v_{BL}^2}{4\pi m_\phi},
\]
and reheat temperature is estimated to be
\[
T_R \simeq \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{\frac{\Gamma M_P}{M_P}},
\]
\[
\simeq 10^8 \text{ GeV} \left( \frac{\lambda'}{3.61 \times 10^{-12}} \right),
\]
where \(g_* \simeq 100\). In order for this \(T_R\) value not to exceed the maximum reheat temperature evaluated from instant reheating, \(T_R \sim 10^8\) GeV, we require \(\lambda' < 3.61 \times 10^{-12}\). We can easily verify that the \(\lambda'\) value is sufficiently small to be neglected in our previous analysis.

\footnote{For a resolution of the domain wall problem without inflation, see Ref. [24].}

In models with the type-I seesaw mechanism, leptogenesis [11] is the simplest mechanism for generating the observed baryon asymmetry in the universe. For thermal leptogenesis with hierarchical RHN mass spectrum, there is a lower bound on the lightest RHN mass of around \(10^{9-10}\) GeV [13] and thus the reheat temperature must be higher than this value. Since the maximum \(T_R \sim 10^8\) GeV is lower than this value, successful leptogenesis requires an enhancement of the CP-asymmetric parameter through a degenerate RHN mass spectrum, namely, resonant leptogenesis [21,22]. For the resonant leptogenesis, we set \(Y_1 \simeq Y_2\) for the Majorana Yukawa coupling of two RHNs.

However, since the RHNs have \(B-L\) gauge interaction, they can stay in thermal equilibrium with the plasma of the SM particles. As a result, the generation of lepton asymmetry is suppressed until the \(B-L\) interaction is frozen [22]. We now derive the condition for successful leptogenesis. Let us consider the process \(N_R \leftrightarrow Z' \leftrightarrow f_{SM}\), where \(f_{SM}\) are the SM fermions. We require this process to decouple before the temperature of the universe drops to \(T \sim m_{N^1} \simeq m_{N^2} = M_N\). Since \(M_N \ll m_{Z'}\), our setup, the \(Z'\) mediated process is effectively described as a four-Fermi interaction, and its thermal-averaged cross section can be approximated as [6]
\[
\langle \sigma v \rangle \simeq \frac{13}{192 \pi} \frac{T^2}{v_{BL}^2},
\]
for temperature \(T \gtrsim M_N\). In order for this process to be decoupled at \(T \sim M_N\), we impose \(\Gamma/H(T) < 1\), where \(\Gamma = n_{eq} (T) \langle \sigma v \rangle\) is the annihilation/creation rate of the RHNs with equilibrium number density \(n_{eq} (T) \simeq 2T^3/\pi^2\), and the Hubble parameter \(H(T) \sim T^2/M_P\). This leads to a lower bound on \(v_{BL}\):
\[
v_{BL} > 10^9 \text{ GeV} \left( \frac{M_N}{1.1 \times 10^7 \text{ GeV}} \right)^{3/4}.
\]
This is consistent with our choice of \(v_{BL} = 2.1 \times 10^{12}\) GeV and \(M_N \ll m_{Z'}, m_{N^2} \simeq 2.10 \times 10^8\) GeV.

V. AXION DARK MATTER WITH \(H_{int} \lesssim 0.1\) GeV

The axion scenario not only offers an elegant solution to the strong CP problem but also provides us with a compelling DM candidate in the form of axion. However, this scenario in general suffers from cosmological problems, such as the domain wall and the isocurvature problems. For a review, see, for example, Ref. [25]. The domain wall problem arises because topological defects (strings and domain walls) associated with PQ symmetry breaking evolve to dominate the energy density of the universe which is inconsistent with cosmological observation. It can be solved if inflation takes places after the PQ symmetry breaking\footnote{For a resolution of the domain wall problem without inflation, see Ref. [24].}, namely, \(H_{int} < F_a = v_{PQ}/N_{DM}\), where \(F_a\) is the axion decay constant and \(N_{DM}\) (\(N_{DW} \neq 1\) in general) is the domain wall number. In our case, \(N_{DW} = 12\).
The measurement of supernova SN1987A pulse duration provides a model-independent constraint on the axion decay constant \( F_a \gtrsim 4 \times 10^8 \) GeV \([25]\). On the other hand, if inflation takes place after the PQ symmetry breaking, the axion obtains large fluctuations that generates isocurvature density perturbations, which are severely constrained by the Planck measurements \([17]\). With a natural assumption \( \theta_a = O(1) \) for the initial displacement of the axion field (misalignment angle) from the potential minimum at the onset of oscillations, we obtain an upper bound on \( H_{\text{inf}} \) \([23]\):

\[
H_{\text{inf}} < 2.08 \times 10^7 \text{ GeV} \left( \frac{F_a}{7.11 \times 10^{11}\text{ GeV}} \right)^{0.405}.
\] (29)

The Hubble scale to satisfy the TCC bound (\( H_{\text{inf}} \lesssim 0.1 \) GeV) is fully compatible with this bound, and our model is therefore free from the axion cosmological problems.

After the QCD phase transition, the coherently oscillating axion field behaves like cold DM whose abundance is given by \([23]\):

\[
\Omega_a h^2 \simeq 0.12 \left[ \left( \frac{H_{\text{inf}}}{2\pi F_a} \right)^2 \right] \left( \frac{F_a}{7.04 \times 10^{11} \text{ GeV}} \right)^{1.19} \simeq 0.12 \theta_a^2 \left( \frac{F_a}{7.04 \times 10^{11} \text{ GeV}} \right)^{1.19},
\] (30)

where we have used \( H_{\text{inf}}/(2\pi F_a) \ll 1 \) to obtain the final expression. To reproduce the observed DM abundance of \( \Omega_a h^2 = 0.12 \) \([26]\), we set \( F_a = 7.04 \times 10^{11} \) GeV for a natural choice of \( \theta_a \simeq 1 \). We can regard this \( F_a \) value as an upper bound to avoid the axion overabundance.

However, recently it has been pointed out in Refs. \([23, 28]\) that for \( H_{\text{inf}} \lesssim \Lambda_{\text{QCD}} \gtrsim 0.1 \) GeV there is an upper bound on the misalignment angle \([28]\):

\[
\theta_a \lesssim 0.34 \left( \frac{H_{\text{inf}}}{0.1 \text{ GeV}} \right)^2.
\] (31)

In Ref. \([28]\), the axion abundance is numerically evaluated for the maximum value of \( \theta_a \) to satisfy Eq. (31), and the relation between \( F_a \) and \( H_{\text{inf}} \) to reproduce the observed DM abundance \( \Omega_a h^2 = 0.12 \) has been obtained. For \( H_{\text{inf}} \lesssim 172 \) MeV, their result is well approximated using Eqs. (30) and (31):

\[
F_a \simeq 4.26 \times 10^{12} \text{ GeV} \left( \frac{0.1 \text{ GeV}}{H_{\text{inf}}} \right)^{3.36}.
\] (32)

Therefore, the upper bound on \( F_a \) is significantly relaxed for low-scale inflation. Satisfying the TCC bound on \( H_{\text{inf}} \lesssim 0.1 \) GeV, the axion DM scenario in our model remains viable with \( F_a \) larger than \( 10^{12} \) GeV.

VI. CONCLUSIONS

Inspired by the Trans-Planckian Censorship Conjecture, we have presented a well motivated \( U(1)_{B-L} \times U(1)_{PQ} \) extension of the Standard Model. The \( B - L \) component allows us to incorporate inflection-point inflation scenario at low scale that is compatible with the TCC bound on \( H_{\text{inf}} \lesssim 1 \) GeV. Its inflationary predictions are consistent with the Planck 2018 measurements, and the prediction for the running of spectral index of \( \alpha \approx -0.0062 \) which can be tested in the future. Because of the low scale inflation, the axion dark matter scenario is free from the domain wall and isocurvature problems, and the axion decay constant can be larger than \( 10^{12} \) GeV. The seesaw mechanism is automatically incorporated in the model and the observed baryon asymmetry of the universe can be reproduced via resonant leptogenesis.

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