CONVERGENCE OF THE PERFECTLY MATCHED LAYER
METHOD FOR TRANSIENT ACOUSTIC-ELASTIC INTERACTION
ABOVE AN UNBOUNDED ROUGH SURFACE

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Abstract. This paper is concerned with the time-dependent acoustic-elastic interaction problem
associated with a bounded elastic body immersed in a homogeneous air or fluid above an unbounded
rough surface. The well-posedness and stability of the problem are first established by using the
Laplace transform and the energy method. A perfectly matched layer (PML) is then introduced
to truncate the interaction problem above a finite layer containing the elastic body, leading to a
PML problem in a finite strip domain. We further establish the existence, uniqueness and stability
estimate of solutions to the PML problem. Finally, we prove the exponential convergence of the
PML problem in terms of the thickness and parameter of the PML layer, based on establishing an
error estimate between the DtN operators of the original problem and the PML problem.

Key words. Acoustic wave equation, elastic wave equation, time domain, stability, perfectly
matched layer, exponential convergence, unbounded rough surface

AMS subject classifications. 78A46, 65C30

1. Introduction. Consider the problem of scattering of acoustic waves by an
elastic body immersed in a compressible, inviscid fluid (air or water) in a half-space
with an unbounded rough boundary. This problem is also referred to as a fluid-solid
interaction problem which can be mathematically formulated as an initial-boundary
transmission problem and has been widely studied (see, e.g. 1[21,23,26,30,32] and
the references quoted there). This problem can also be categorized into the class of
unbounded rough surface scattering problems, which is the subject of intensive studies
in the engineering and mathematics communities. For the rough surface scattering
problems, the usual Sommerfeld radiation condition and Silver-Müller radiation con-
dition is not valid anymore due to the unbounded structure. We refer to 5[6,8,9] for
the mathematical analysis of the time-harmonic case using both the integral equation
method and the variational method.

In most of real-world problems, the model setting not only depends on the space,
but also depends on the time. Recently, this class of problems has attracted much
attention due to their capability of capturing wide-band signals and modeling more
general material and nonlinearity (see, e.g. 1[10,28,36,37] and the references quoted
there). In particular, the analysis of time-dependent scattering problems can be found
in 1[11,36] for the acoustic case, in 1[12,19,20,29] for the electromagnetic case including
the cases with bounded obstacles, diffraction gratings and unbounded surfaces, and
in 1[21,26,38] for the time-dependent fluid-solid interaction problems including the
cases with bounded elastic bodies 1[26], locally rough surfaces 38 and unbounded
layered structures 21.

The perfectly matched layer (PML) method is a fast and effective method for
solving unbounded scattering problems which was originally proposed by Bérenger in 1994 for electromagnetic scattering problems [3]. A large amount of work have been done since then to construct various PML absorption layers [7, 11, 13, 27, 33, 35]. The key idea of the PML method is to surround the computational domain with a specially designed medium containing a finite thickness layer in which the scattered waves decay rapidly regardless of the frequencies and incident angles, thereby greatly reducing the computational complexity of the scattering problems. This makes the PML method a popular approach to solve a variety of wave scattering problems [2, 4, 18, 35].

The convergence of the PML method has always been a topic of interest to mathematicians. There are a lot of works on the convergence of the time-harmonic PML method, most of which focus on the exponential convergence of the PML method in terms of the thickness of the PML layer for the case of bounded scatterers (see, e.g., [2, 14, 16, 22, 27]). In 2009, Chandler-Wilde and Monk extend the PML method to time-harmonic scattering problems by unbounded rough surfaces in [7], where only the linear convergence of the PML method was established in terms of the PML layer thickness.

Compared with the time-harmonic case, only several results are available for the rigorous convergence analysis of the PML method for time-domain wave scattering problems. For the case of time-domain acoustic scattering by a bounded scatterer, the exponential convergence in terms of the thickness and parameter of the PML layer was proved in [11] for a circular PML method and in [15] for an uniaxial PML method. The method used in [11, 15] is based on the Laplace transform and complex coordinate stretching technique. For the case of time-domain electromagnetic scattering by bounded scatterers, the exponential convergence of a spherical PML method was recently shown in [39] in terms of the thickness and parameter of the PML layer, based on a real coordinate stretching technique associated with $[\text{Re}(s)]^{-1}$ in the Laplace domain, where $s \in \mathbb{C}^+$ is the Laplace transform variable. Recently, a time-domain PML method was studied in [1] for the transient acoustic-elastic interaction problem, where a bounded elastic body is immersed in a homogeneous, compressible, inviscid fluid (air or water) in $\mathbb{R}^2$. The well-posedness and stability estimate of the PML solution have been established, but no convergence analysis of the PML method is given in [1].

In this paper, we study the time-domain PML method for the transient acoustic-elastic interaction problem associated with a bounded elastic body immersed in a homogeneous, compressible, inviscid fluid (air or water) above an unbounded rough surface. Our purpose is to introduce a time-domain PML layer to truncate the unbounded domain of the interaction problem above a finite layer in the $x_3$ direction containing the elastic body, leading to a PML problem in a finite strip domain. The idea used in [11, 15] to construct the PML layer seems difficult to apply to the transient acoustic-elastic interaction problem considered in this paper. Motivated by [39], we make use of the real coordinate stretching technique associated with $[\text{Re}(s)]^{-1}$ in the Laplace domain with the Laplace transform variable $s \in \mathbb{C}^+$. The well-posedness and stability estimate of the PML problem are then established, by employing the Laplace transform and the energy method. Further, we establish the error estimate between the Dirichlet-to-Neumann (DtN) operators of the original problem and the PML problem, which is then used to prove the exponential convergence of the PML method in terms of the thickness and parameters of the PML layer.

The outline of this paper is as follows. In Section 2 we first formulate the transient interaction problem and then use the exact transparent boundary condition (TBC) to
reduce the unbounded interaction problem into an equivalent initial-boundary transmission problem in a finite strip domain. In addition, the well-posedness and stability are also studied for the reduced problem. In Section 3, we first propose the time-domain PML method for the acoustic-elastic interaction problem, based on the real coordinate stretching technique, and then establish its exponential convergence in terms of the thickness and parameters of the PML layer. Conclusions are given in Section 4.

2. The acoustic-elastic interaction problem. In this section, we formulate the mathematical formulation of the interaction problem for acoustic and elastic waves with appropriate transmission conditions on the interface between the elastic body and the acoustic medium. In addition, an exact time-domain transparent boundary condition (TBC) is proposed to reformulate the unbounded interaction problem into an initial-boundary value problem in a finite strip domain. We finally establish the well-posedness and stability of solutions to the reduced problem.

We first introduce some basic notions to be used in this paper. Throughout, let \( x = (\bar{x}^T, x_3)^T \), where \( \bar{x} = (x_1, x_2)^T \in \mathbb{R}^2 \). Denote by \( \Omega \) the bounded homogeneous, isotropic elastic body with a Lipschitz boundary \( \Gamma := \partial \Omega \) immersed in the unbounded domain \( \Omega_f^+ \), where \( \Omega_f^+ := \{ x \in \mathbb{R}^3 : x_3 > f(\bar{x}) \} \) with the boundary \( \Gamma_f := \partial \Omega_f^+ = \{ x \in \mathbb{R}^3 : x_3 = f(\bar{x}) \} \) described by the smooth function \( f \in C^2(\mathbb{R}^2) \). We assume that \( \Gamma_f \) lies between the planes \( x_3 = f_- \) and \( x_3 = f_+ \), where \( f_- := \inf_{\bar{x} \in \mathbb{R}^2} f(\bar{x}) \) and \( f_+ := \sup_{\bar{x} \in \mathbb{R}^2} f(\bar{x}) \) are two constants. Suppose the elastic body \( \Omega \) is described by a constant mass density \( \rho_e > 0 \). Let \( \Omega^c = \Omega_f^+ \setminus \Omega \) be connected and occupied by a compressible fluid with constant density \( \rho_0 > 0 \). Define \( \Gamma_h := \{ x \in \mathbb{R}^3 : x_3 = h \} \), where the positive constant \( h \) is assumed to be large enough such that \( \Gamma_h \) is over \( \Omega \), and let \( \Omega_h := \{ x \in \mathbb{R}^3 : f < x_3 < h \} \cap \Omega^c \). See Figure 2.1 for the geometric setting of the problem. Finally, define \( C_+ := \{ s = s_1 + is_2 \in \mathbb{C} : s_1, s_2 \in \mathbb{R} \text{ with } s_1 > 0 \} \).

\[ \Gamma_h \]

\[ \text{source} \]

\[ \Omega_h \]

\[ \Omega \]

\[ f \]

\[ f_- \]

\[ f_+ \]

\[ \Gamma \]

\[ r \]

\[ r_1 \]

\[ r_2 \]

\[ \text{Fig. 2.1. Geometric configuration of the interaction problem} \]

**Elastic domain.** In the elastic body \( \Omega \), the elastic displacement \( u = (u_1, u_2, u_3)^T \) is governed by the linear elastodynamic equation:

\[
\rho_e \frac{\partial^2 u}{\partial t^2} - \Delta^* u = 0 \quad \text{in} \quad \Omega \times (0, T) 
\]  
(2.1)
where $\Delta^*$ is the Lamé operator defined as
\[
\Delta^* u := \mu \Delta u + (\lambda + \mu) \nabla \text{div } u = \text{div } \sigma(u).
\]
Here, $\sigma(u)$ and $\varepsilon(u)$ are called the stress and strain tensors, respectively, given by
\[
\sigma(u) = (\lambda \text{div } u) I + 2\mu \varepsilon(u) \quad \text{and} \quad \varepsilon(u) = \frac{1}{2} (\nabla u + (\nabla u)^T),
\]
where $I$ is the identity matrix and $\nabla u$ denotes the displacement gradient tensor:
\[
\nabla u = \begin{bmatrix}
\frac{\partial}{\partial x_1} u_1 & \frac{\partial}{\partial x_2} u_1 & \frac{\partial}{\partial x_3} u_1 \\
\frac{\partial}{\partial x_1} u_2 & \frac{\partial}{\partial x_2} u_2 & \frac{\partial}{\partial x_3} u_2 \\
\frac{\partial}{\partial x_1} u_3 & \frac{\partial}{\partial x_2} u_3 & \frac{\partial}{\partial x_3} u_3 
\end{bmatrix}.
\]

Further, Lamé constants $\lambda$ and $\mu$ are assumed to satisfy the condition that $\mu \geq 0$ and $3\lambda + 2\mu \geq 0$.

Fluid domain. In the unbounded fluid domain $\Omega^c$, the pressure $p$ and the velocity $v$ are governed by the conservation and dynamic equations in the time-domain:
\[
\frac{\partial p}{\partial t} = -c^2 \rho_0 \text{div } v + g(x, t), \quad \frac{\partial v}{\partial t} = -\rho_0^{-1} \nabla p \quad \text{in } \Omega^c \times (0, T). \tag{2.2}
\]
Eliminating the velocity $v$ from (2.2), we get the wave equation for the pressure $p$:
\[
\frac{\partial^2 p}{\partial t^2} - c^2 \Delta p = \partial_t g \quad \text{in } \Omega^c \times (0, T), \tag{2.3}
\]
where $c$ is the sound speed and $g$ is the acoustic source which is assumed to be supported in $\Omega_h$ and $g|_{t=0} = 0$. We assume that $p$ satisfies the Dirichlet boundary condition on $\Gamma_f$:
\[
p = 0 \quad \text{on } \Gamma_f. \tag{2.4}
\]
In addition, we impose the Upward Angular Spectrum Representation (UASR) condition on $p$ proposed in [6]:
\[
p(x, t) = \mathcal{L}^{-1} \left\{ \frac{1}{(2\pi)^2} \int_{\mathbb{R}^2} \exp(i(x_3 - h)i\sqrt{s^2/c^2 + |\xi|^2 + \bar{x} \cdot \xi}) \hat{p}(\xi, h) d\xi \right\} \tag{2.5}
\]
for $x \in \Omega_h^+: = \{x \in \mathbb{R}^3 : x_3 > h\}$, where $\mathcal{L}^{-1}$ is the inverse Laplace transform, $\hat{p}(\xi, h) = \mathcal{F}(p)\big|_{\Gamma_h}$ denotes the Fourier transform of $\hat{p} = \mathcal{L}(p)$ (the Laplace transform of $p$ with respect to $t$) restricted on $\Gamma_h$ (the definition and relationship of the Fourier and Laplace transforms are given in Appendix A), $\beta(\xi) = \sqrt{s^2/c^2 + |\xi|^2}$ with $\text{Re}[\beta(\xi)] > 0$ and $s \in \mathbb{C}_+$. Further, we have the following transmission conditions on the interface between the elastic and fluid media (see [26]):
(i) The kinematic interface condition
\[
\partial_n p = -\rho_0 n \cdot \partial_t^2 u \quad \text{on } \Gamma, \tag{2.6}
\]
(ii) The dynamic interface condition
\[
-pn = \sigma(u)n \quad \text{on } \Gamma, \tag{2.7}
\]
where \( \mathbf{n} \) is the unit normal on \( \Gamma \) directed into the exterior of the domain \( \Omega \).

To be more precise, the acoustic-elastic interaction problem we consider is that a time-dependent acoustic wave propagates in a fluid domain above a rough surface in which a bounded elastic body is immersed. The problem is to determine the scattered pressure in the fluid domain and the displacement field in the elastic domain at any time. The time-dependent scattering problem can be now modelled by combining (2.6)-(2.7) for the elastic displacement field \( u \) together with the transmission conditions (2.4) for the pressure field \( p \) in the fluid domain and the Dirichlet boundary conditions (2.1) for the elastic displacement field \( u \) on \( \Gamma_f \) as well as the homogeneous initial conditions

\[
\begin{align*}
  u(x,0) = \partial_t u(x,0) &= 0, \quad x \in \Omega, \\
p(x,0) = \partial_t p(x,0) &= 0, \quad x \in \Omega^c,
\end{align*}
\]  

(2.8)

which can be formulated mathematically as follows:

\[
\begin{align*}
  \rho_c & \frac{\partial^2 u}{\partial t^2} - \nabla^2 u = 0 & \text{in } \Omega \times (0,T), \\
  \frac{\partial^2 p}{\partial t^2} - c^2 \Delta p &= \partial_t g & \text{in } \Omega^c \times (0,T), \\
  u(x,0) = \partial_t u(x,0) &= 0 & \text{in } \Omega, \\
  p(x,0) = \partial_t p(x,0) &= 0 & \text{in } \Omega^c, \\
  \partial_n p &= -\rho_0 \mathbf{n} \cdot \partial_t^2 \mathbf{u} & \text{on } \Gamma \times (0,T), \\
  -pn &= \sigma(\mathbf{u})\mathbf{n} & \text{on } \Gamma \times (0,T), \\
  p &= 0 & \text{on } \Gamma_f \times (0,T), \\
  p &\text{ satisfies the UASR condition (2.5)}. \tag{2.9}
\end{align*}
\]

To study the well-posedness of the scattering problem (2.8), we reformulate it into a transmission problem in the strip domain \( \Omega_0 \cup \Omega_f \) by using the transparent boundary condition (TBC) on the plane \( \Gamma_h \) proposed in [21]:

\[
\partial_n p = \mathcal{T}[p] \quad \text{on } \Gamma_h \times (0,T). \tag{2.10}
\]

Then (2.9) can be equivalently reduced to the transmission problem (TP) in \( \Omega_h \cup \Omega_f \):

\[
\begin{align*}
  \rho_c & \frac{\partial^2 u}{\partial t^2} - \nabla^2 u = 0 & \text{in } \Omega \times (0,T), \\
  \frac{\partial^2 p}{\partial t^2} - c^2 \Delta p &= \partial_t g & \text{in } \Omega_h \times (0,T), \\
  u(x,0) = \partial_t u(x,0) &= 0 & \text{in } \Omega, \\
  p(x,0) = \partial_t p(x,0) &= 0 & \text{in } \Omega_h, \\
  \partial_n p &= -\rho_0 \mathbf{n} \cdot \partial_t^2 \mathbf{u} & \text{on } \Gamma \times (0,T), \\
  -pn &= \sigma(\mathbf{u})\mathbf{n} & \text{on } \Gamma \times (0,T), \\
  p &= 0 & \text{on } \Gamma_f \times (0,T), \\
  \partial_n p &= \mathcal{T}[p] & \text{on } \Gamma_h \times (0,T). \tag{2.11}
\end{align*}
\]

In the remaining part of this section, we establish the well-posedness and stability of the reduced problem (2.11) by using the Laplace transform. The proof is similar to that used in [21], and so we only present the main results without detailed proofs. To this end, we take the Laplace transform of \( p(x,t) \) and \( u(x,t) \), respectively, in (2.11) with respect to \( t \) and write \( \hat{p}(x,s) = \mathcal{L}[p](x,s) \) \( \hat{u}(x,s) = \mathcal{L}[u](x,s) \). Then (2.11)
can be reduced to the problem in $s$-domain:

\[
\begin{align*}
\Delta^* \mathbf{u} - \rho_s s^2 \mathbf{u} &= 0 \quad \text{in } \Omega, \\
\Delta \mathbf{p} - \frac{s^2}{c^2} \mathbf{p} &= -s\mathbf{g}/c^2 \quad \text{in } \Omega_h, \\
\partial_n \mathbf{p} &= -\rho_0 s^2 \mathbf{n} \cdot \mathbf{u} \quad \text{on } \Gamma, \\
-\mathbf{p} \mathbf{n} &= \sigma(\mathbf{u}) \mathbf{n} \quad \text{on } \Gamma, \\
\partial_n \mathbf{p} &= \mathcal{B}[\mathbf{p}] \quad \text{on } \Gamma_h,
\end{align*}
\]

(2.12)

where $s \in \mathbb{C}_+$ and $\mathcal{B}$ is the Dirichlet-to-Neumann (DtN) operator in $s$-domain satisfying $\mathcal{J} = \mathcal{L}^{-1} \circ \mathcal{B} \circ \mathcal{L}$.

For any function $\omega(x, h)$ defined on $\Gamma_h$, the DtN operator $\mathcal{B}$ is defined by

\[
(\mathcal{B} \omega)(x, h) = -\int_{\mathbb{R}^2} \beta(\xi)\hat{\omega}(\xi, h)e^{i\xi \cdot x} d\xi.
\]

Then the following lemma was proved in [21] (see [21] Lemmas 2.4 and 2.5), where, for $s \in \mathbb{R}$ the space $H^s(\Gamma_h)$ denotes the standard Sobolev space on $\Gamma_h$ with its norm being defined via the Fourier transform as

\[
\|\phi\|_{H^s(\Gamma_h)} = \int_{\mathbb{R}^2} (1 + |\xi|^2)^s |\hat{\phi}(\xi, h)|^2 d\xi, \quad \phi \in H^s(\Gamma_h), \; s \in \mathbb{R}.
\]

**Lemma 2.1.** (i) For $s = s_1 + s_2 \in \mathbb{C}_+$ with $s_1 \geq \sigma_0 > 0$ the DtN operator $\mathcal{B}(s)$ is bounded from $H^{1/2}(\Gamma_h)$ to $H^{-1/2}(\Gamma_h)$, that is,

\[
\|\mathcal{B}(s)w\|_{H^{-1/2}(\Gamma_h)} \leq C(\sigma_0)\|w\|_{H^{1/2}(\Gamma_h)} \quad \forall w \in H^{1/2}(\Gamma_h),
\]

where $C(\sigma_0)$ is a constant depending only on $c$ and $\sigma_0$.

(ii) For any $\omega \in H^{1/2}(\Gamma_h)$ we have

\[
-\text{Re}(s^{-1}\mathcal{B}\omega, \omega)_{\Gamma_h} \geq 0, \quad s \in \mathbb{C}_+,
\]

where $(\cdot, \cdot)_{\Gamma_h}$ denotes the dual product on $\Gamma_h$ between $H^{1/2}(\Gamma_h)$ and $H^{-1/2}(\Gamma_h)$.

To study the $s$-domain problem (2.12) we introduce the Hilbert space $H := H^1_{\Gamma_f}(\Omega_h) \times H^1(\Omega)^3$, where $H^1_{\Gamma_f}(\Omega_h) := \{u \in H^1(\Omega_h) : u = 0 \text{ on } \Gamma_f\}$. The norm of the product space $H$ is defined by

\[
\|(q, \mathbf{v})\|_H := \left[\|q\|_{H^1(\Omega_h)}^2 + \|\mathbf{v}\|_{H^1(\Omega)^3}^2\right]^{1/2} \quad \text{for } (q, \mathbf{v}) \in H, \quad (2.15)
\]

where $\|\cdot\|_{H^1(\Omega_h)}$ denotes the usual $H^1$-norm and $\|\cdot\|_{H^1(\Omega)^3}$ is defined by

\[
\|\mathbf{v}\|_{H^1(\Omega)^3} := \left(\|\mathbf{v}\|_{L^2(\Omega)^3}^2 + \|
abla \mathbf{v}\|_{F(\Omega)}^2\right)^{1/2}
\]

with the Frobenius norm

\[
\|\nabla \mathbf{v}\|_{F(\Omega)} := \left(\sum_{j=1}^3 \int_{\Omega} \left|\nabla v_j\right|^2 dx\right)^{1/2}.
\]
It is easy to verify that
\[ \| \nabla v \|_{F(\Omega)}^2 + \| \nabla \cdot v \|_{L^2(\Omega)}^2 \lesssim \| v \|_{H^1(\Omega)}^2. \]

Hereafter, the expression \( a \lesssim b \) or \( a \gtrsim b \) means \( a \leq Cb \) or \( a \geq Cb \), respectively, for a generic positive constant \( C \) which does not depend on any function and important parameters in our model.

The variational formulation of (2.12) can be obtained as follows: Find a solution \((\tilde{p}, \tilde{u}) \in H := H^1_1(\Omega_h) \times H^1(\Omega)^3\) such that
\[ a((\tilde{p}, \tilde{u}), (q, v)) = \int_{\Omega_h} \frac{\tilde{g}}{c^2} \cdot \nabla q \, dx, \quad \forall \ (q, v) \in H, \quad (2.16) \]

where the sesquilinear form \( a(\cdot, \cdot) \) is defined as
\[ a((\tilde{p}, \tilde{u}), (q, v)) = \int_{\Omega_h} \left( s^{-1} \nabla \tilde{p} \cdot \nabla \bar{q} + \frac{s}{c^2} \tilde{p} \cdot \bar{v} \right) dx \]
\[ + \int_{\Omega} \left[ \rho_0 s (\lambda (\nabla \cdot \bar{u}) (\nabla \cdot \bar{v}) + 2\mu \varepsilon(\bar{u}) : \varepsilon(\bar{v})) + \rho_0 \rho_e |s|^2 s \bar{u} \cdot \bar{v} \right] dx \]
\[ - \int_{\Gamma_h} s^{-1} B [\tilde{p}] \cdot n d \gamma - \rho_0 \int_{\Gamma} s n \cdot \tilde{u} \bar{d} d \gamma + \rho_0 \int_{\Gamma} s \tilde{p} n \cdot \bar{d} d \gamma \quad (2.17) \]

with \( A : B = tr(AB^T) \) denoting the Frobenius inner product of the square matrices \( A \) and \( B \).

Letting \((q, v) = (\tilde{p}, \tilde{u}) \) in (2.17) and setting \( \omega = (\tilde{p}, \tilde{u}) \), applying the famous Korn’s inequality [31] Chapter 10
\[ \| \varepsilon(v) \|_{F(\Omega)}^2 + \| v \|_{L^2(\Omega)^3}^2 \gtrsim C_{\Omega} \| v \|_{H^1(\Omega)}^2, \quad \forall \ v \in H^1(\Omega)^3 \]
we obtain that
\[ \text{Re}[a(\omega, \omega)] \gtrsim \frac{s_1}{|s|^2} \left( \| \nabla \tilde{p} \|_{L^2(\Omega_h)}^2 + \| s \tilde{p} \|_{L^2(\Omega_h)}^2 \right) + \rho_0 s_1 \| \varepsilon(\tilde{u}) \|_{F(\Omega)}^2 + \rho_e \| s \tilde{u} \|_{L^2(\Omega)^3}^2 \]
\[ \gtrsim \frac{s_1}{|s|^2} C_1 \| \tilde{p} \|_{H^1(\Omega_h)}^2 + \rho_0 s_1 C_2 \| \tilde{u} \|_{H^1(\Omega)^3}^2 \]
\[ \gtrsim C \| \omega \|_{H^2}^2, \quad (2.18) \]

where use has been made of Lemma 2.1 (ii) to get the first inequality, \( C = \min\{ s_1 C_1 / |s|^2, \rho_0 s_1 C_2 \}, \) \( C_1 = \min\{1, |s|^2 / c^3\} \) and \( C_2 = C_{\Omega} \min\{2\mu, \rho_e \min\{1, |s|^2\}\} \).

This means that the sesquilinear form \( a(\cdot, \cdot) \) is uniformly coercive in \( H \). By Lemma 2.1 (i), the trace theorem (see [21] Lemma 2.2) and the Lax-Milgram theorem, we can obtain the following result on the well-posedness of the \( s \)-domain problem (2.12) or equivalently its variational formulation (2.17).

**Lemma 2.2.** For each \( s \in C_+ \), the variational problem (2.16) has a unique solution \((\tilde{p}, \tilde{u}) \in H\) satisfying that
\[ \| \nabla \tilde{p} \|_{L^2(\Omega_h)^3} + \| s \tilde{p} \|_{L^2(\Omega_h)} \lesssim \frac{|s|}{s_1} \| \tilde{g} \|_{L^2(\Omega_h)}, \quad (2.19) \]
\[ \| \nabla \tilde{u} \|_{F(\Omega)} + \| \nabla \cdot \tilde{u} \|_{L^2(\Omega)} + \| s \tilde{u} \|_{L^2(\Omega)^3} \lesssim \frac{1}{s_1 \min\{1, s_1\}} \| \tilde{g} \|_{L^2(\Omega_h)}, \quad (2.20) \]
To prove the well-posedness of the reduced problem (2.11), and establish the convergence of the PML method, we need the following assumptions on the inhomogeneous term $g$:

$$g \in H^3(0, T; L^2(\Omega_h)), \quad \|g\|_{H^3(0, \infty; L^2(\Omega_h))} \lesssim \|g\|_{H^3(0, T; L^2(\Omega_h))}.$$  

(2.21)

Further, we always assume that $g$ can be extended to $\infty$ with respect to $t$ such that

$$g \in H^3(0, \infty; L^2(\Omega_h)), \quad \|g\|_{H^3(0, \infty; L^2(\Omega_h))} \lesssim \|g\|_{H^3(0, T; L^2(\Omega_h))}.$$  

(2.22)

By using Lemma 2.2 and a similar argument as in the proof of Theorem 3.2 in [21], the well-posedness and stability of the PML problem can be established based on a similar method as used in the proof of Theorem 3.2 in [21]. Finally, we prove the exponential convergence of the PML method via constructing a special PML layer in the $x_3$-direction, based on the real coordinate stretching technique.

3. The time-domain PML problem. In this section, we shall derive the time-domain PML formulation for the acoustic-elastic interaction problem (2.9). The well-posedness and stability of the PML problem can be established based on a similar method as used in the proof of Theorem 3.2 in [21]. Finally, we prove the exponential convergence of the time-domain PML method via constructing a special PML layer in the $x_3$-direction, based on the real coordinate stretching technique.

3.1. The PML problem and its well-posedness. Let us first introduce the PML geometry which is presented in Figure 3.1. Let $\Omega_{h+L} = \{x \in \mathbb{R}^3 : f < x_3 < h+L\} \cap \Omega^c$ denote the truncated PML domain and let $\Omega_h^L = \{x \in \mathbb{R}^3 : h < x_3 < h+L\}$ denote the PML layer with the exterior boundary $\Gamma_{h+L} := \{x \in \mathbb{R}^3 : x_3 = h + L\}$, where $L > 0$ is the thickness of the PML layer. Now, let $s_1 > 0$ be an arbitrarily fixed parameter and let us introduce the PML medium property $\sigma = \sigma(x_3)$:

$$\sigma(x_3) = \begin{cases} 
1 & \text{if } x_3 \leq h, \\
1 + s_1^{-1} \sigma_0 \left(\frac{x_3 - h}{L}\right)^m & \text{if } h < x_3 \leq h + L,
\end{cases}$$  

(3.1)

where $\sigma_0$ is a positive constant, $m \geq 1$ is a given integer. In what follows, we will take the real part of the Laplace transform variable $s \in \mathbb{C}_+$ to be $s_1$, that is, $\text{Re}(s) = s_1$.

We now derive the PML equation by the technique of change of variables, starting with the real stretched coordinate $\hat{x} = (\hat{x}_1, \hat{x}_2, \hat{x}_3)$ with

$$\hat{x}_1 = x_1, \quad \hat{x}_2 = x_2, \quad \hat{x}_3 = \int_{f_-}^{x_3} \sigma(\tau)d\tau + f_-.$$
Taking the Laplace transform of the wave equation (2.3) with respect to $t$ gives

$$
\Delta \tilde{p} - \frac{s^2}{c^2} \tilde{p} = 0 \quad \text{in } \Omega_h^L. \quad (3.2)
$$

Denote by $\tilde{p}_{pml}$ the PML extension of the pressure $\tilde{p}$ satisfying (3.2). Formally, the technique of change of variables requires $\tilde{p}_{pml}$ to satisfy

$$
\sum_{j=1}^{3} \frac{\partial^2 \tilde{p}_{pml}}{\partial x_j^2} - \frac{s^2}{c^2} \tilde{p}_{pml} = 0 \quad \text{in } \Omega_h^L.
$$

Then, by the chain rule and using the fact that $d\tilde{x}_3/dx_3 = \sigma$, we obtain the PML equation

$$
\Delta_p \tilde{p}_{pml} - \frac{s^2}{c^2} \tilde{p}_{pml} = 0 \quad \text{in } \Omega_h^L, \quad (3.3)
$$

where

$$
\Delta_p := \sum_{j=1}^{2} \frac{\partial}{\partial x_j} \left( \sigma \frac{\partial}{\partial x_j} \right) + \frac{\partial}{\partial x_3} \left( \frac{1}{\sigma} \frac{\partial}{\partial x_3} \right) = \nabla \cdot (\mathbb{D} \nabla)
$$

with the diagonal matrix $\mathbb{D} = \text{diag}(\sigma, \sigma, 1/\sigma)$.

Combining the elastic wave equation (2.1) and the interface conditions (2.6)-(2.7),

**Fig. 3.1. Geometric configuration of the truncated PML problem**
we obtain the truncated PML problem in $s$-domain:

\[
\begin{aligned}
\Delta^s \tilde{u}_{pml} - \rho_c s^2 \tilde{u}_{pml} &= 0 & \text{in } \Omega, \\
\Delta_p \tilde{p}_{pml} - \frac{s^2 \sigma}{c^2} \tilde{p}_{pml} &= -s \tilde{g}/c^2 & \text{in } \Omega_{h+L}, \\
\frac{\partial_n \tilde{p}_{pml}}{= -\rho_0 s^2 n \cdot \tilde{u}_{pml}} &= \text{on } \Gamma, \\
-\tilde{p}_{pml} n &= \sigma(\tilde{u}_{pml}) n & \text{on } \Gamma, \\
\tilde{p}_{pml} &= 0 & \text{on } \Gamma_f, \\
\tilde{p}_{pml} &= 0 & \text{on } \Gamma_{h+L},
\end{aligned}
\]

where the unbounded domain is truncated into the finite strip layer $\Omega_{h+L}$ by imposing the homogeneous Dirichlet boundary condition on $\Gamma_{h+L}$, in view of the exponential decay of the transformed pressure field $\tilde{p}$.

We now prove the well-posedness of the truncated PML problem (3.4a)-(3.4f) by the variational method in the Hilbert space $\tilde{H} := L^2(\Omega_{h+L})$ with $\Omega_{h+L} := \{u \in H^1(\Omega_{h+L}) : u = 0 \text{ on } \Gamma_f \cup \Gamma_{h+L}\}$ and the norm of $\tilde{H}$ is defined similarly as that of $H$ in (2.15) with $\Omega_h$ replaced by $\Omega_{h+L}$. To this end, use Green’s and Betti’s formulas as well as the transmission conditions (3.4c)-(3.4d) to obtain the following variational formulation of the PML problem (3.4a)-(3.4f); find a solution $(\tilde{p}_{pml}, \tilde{u}_{pml}) \in \tilde{H}$ such that

\[
a_{pml}((\tilde{p}_{pml}, \tilde{u}_{pml}), (q, v)) = \int_{\Omega_h} \frac{\tilde{g}}{c^2} \cdot \nabla q dx \quad \forall (q, v) \in \tilde{H},
\]

where the sesquilinear form $a_{pml}(\cdot, \cdot)$ is defined as

\[
a_{pml}((\tilde{p}_{pml}, \tilde{u}_{pml}), (q, v)) = \\
\int_{\Omega_{h+L}} (s^{-1} \nabla \tilde{p}_{pml} \cdot \nabla q + \frac{s \sigma}{c^2} \tilde{p}_{pml} \cdot \nabla q) dx \\
+ \int_{\Omega} [\rho_0 s |\nabla \cdot (\tilde{u}_{pml}) + 2 \mu \varepsilon(\tilde{u}_{pml}) : \varepsilon(\nabla v)] + \rho_0 s |\tilde{u}_{pml} \cdot \nabla v | \ dx \\
- \rho_0 \int_{\Gamma} s n \cdot \tilde{u}_{pml} d\gamma + \rho_0 \int_{\Gamma_f} \tilde{p}_{pml} n \cdot \nabla q d\gamma.
\]

Noting that $1 \leq \sigma \leq 1 + s_1^{-1} \sigma_0$ for $x \in \Omega_{h+L}$, combining the Korn’s inequality, we have

\[
\mathrm{Re} \left[ a_{pml}((\tilde{p}_{pml}, \tilde{u}_{pml}), (\tilde{p}_{pml}, \tilde{u}_{pml})) \right] \\
= \mathrm{Re} \left[ \int_{\Omega_{h+L}} s^{-1} \left( |\nabla \cdot \tilde{u}_{pml}|^2 + \sigma |\Delta \tilde{p}_{pml}|^2 + \frac{1}{\sigma} \frac{s \sigma}{c^2} |\tilde{p}_{pml}|^2 \right) dx + \int_{\Omega_{h+L}} \frac{s \sigma}{c^2} |\tilde{p}_{pml}|^2 dx \\
+ \rho_0 s_1 \left( \mu |\nabla \cdot \tilde{u}_{pml}|^2_{H^1(\Omega)} + 2 |\mu \varepsilon(\tilde{u}_{pml})|^2_{F(\Omega)} + \rho_0 \frac{s \sigma}{c^2} |\tilde{u}_{pml}|^2_{L^2(\Omega)^3} \right) \right] \\
\geq \frac{1}{1 + s_1^{-1} \sigma_0} \left( \frac{s_1}{|\nabla \cdot \tilde{u}_{pml}|^2_{H^1(\Omega)}} + \frac{s_1}{s |\nabla \cdot \tilde{u}_{pml}|^2_{L^2(\Omega)^3}} \right) \\
+ s_1 \min \{1, s_1^2 \} \left( |\nabla \tilde{u}_{pml}|^2_{F(\Omega)} + |\nabla \cdot \tilde{u}_{pml}|^2_{L^2(\Omega)^3} + |s \tilde{p}_{pml}|^2_{L^2(\Omega)^3} \right),
\]

which means that $a_{pml}(\cdot, \cdot)$ is uniformly coercive in $\tilde{H}$. 

Arguing similarly as in the proof of Lemma 3.2 (noting that the TBC in the s-domain is now replaced with the Dirichlet boundary condition), we can obtain the following theorem.

**Theorem 3.1.** The truncated PML variational problem (3.5) has a unique solution \((\tilde{u}_{\text{pml}}, u_{\text{pml}}) \in \tilde{H}\) for each \(s \in \mathbb{C}_+\) with \(\text{Re}(s) = s_1 > 0\). Further, we have the following estimates

\[
\|\nabla \tilde{p}_{\text{pml}}\|_{L^2(\Omega_h+L)^3} + \|s \tilde{p}_{\text{pml}}\|_{L^2(\Omega_h+L)^3} \lesssim \frac{(1 + s_1^{-1}\sigma_0)|s|}{s_1} \|\tilde{g}\|_{L^2(\Omega_h)},
\]

(3.6)

\[
\|\nabla u_{\text{pml}}\|_{\tilde{H}(\Omega)} + \|\nabla \cdot u_{\text{pml}}\|_{L^2(\Omega)} + \|s u_{\text{pml}}\|_{L^2(\Omega)^3} \lesssim \sqrt{1 + s_1^{-1}\sigma_0} \|\tilde{g}\|_{L^2(\Omega_h)}.\]

(3.7)

Taking the inverse Laplace transform of the system (3.4a)-(3.4f), we obtain the truncated PML problem in the time-domain:

\[
\begin{aligned}
\Delta^* u_{\text{pml}} - \rho \nabla^2 u_{\text{pml}} &= 0 & \text{in } \Omega \times (0, T), \\
\Delta p_{\text{pml}} - \frac{\sigma}{c^2} \nabla^2 p_{\text{pml}} &= -g/c^2 & \text{in } \Omega_{h+L} \times (0, T), \\
 u_{\text{pml}}(x, 0) &= \partial_t u_{\text{pml}}(x, 0) = 0 & \text{in } \Omega, \\
p_{\text{pml}}(x, 0) &= 0 & \text{in } \Omega_{h+L}, \\
\partial_n p_{\text{pml}} &= -\rho_0 n \cdot \nabla u_{\text{pml}} & \text{on } \Gamma \times (0, T), \\
-p_{\text{pml}} n &= \sigma(u_{\text{pml}}) n & \text{on } \Gamma \times (0, T), \\
p_{\text{pml}} &= 0 & \text{on } \Gamma_{h+L} \times (0, T), \\
u_{\text{pml}} &= 0 & \text{on } \Gamma_{h+L} \times (0, T).
\end{aligned}
\]

(3.8)

Note that \(s_1\) appearing in PML medium property \(\sigma\) is an arbitrarily fixed, positive parameter, as mentioned earlier at the beginning of this subsection. In the Laplace transform domain, the transform variable \(s \in \mathbb{C}_+\) is taken so that \(\text{Re}(s) = s_1 > 0\), and in the subsequent study of the well-posedness and convergence of the truncated PML problem (3.8), we take \(s_1 = 1/T\).

By using Theorem 3.1 and a similar argument as in the proof of Theorem 3.2 in [21], we can establish the following result on the well-posedness and stability of the PML problem (3.8).

**Theorem 3.2.** Let \(s_1 = 1/T\). Then the truncated PML problem (3.8) has a unique solution \((p_{\text{pml}}, u_{\text{pml}})\) with

\[
\begin{aligned}
p_{\text{pml}} &\in L^2(0, T; H^1_0(\Omega_{h+L}^1)) \cap H^1(0, T; L^2(\Omega_{h+L})), \\
u_{\text{pml}} &\in L^2(0, T; H^1(\Omega^3)) \cap H^1(0, T; L^2(\Omega^3))
\end{aligned}
\]

and satisfies the stability estimates

\[
\begin{aligned}
\max_{t \in [0, T]} \left(\|\partial_t p_{\text{pml}}\|_{L^2(\Omega_{h+L}^1)} + \|\nabla p_{\text{pml}}\|_{L^2(\Omega_{h+L}^1)^2}\right) &\lesssim (1 + \sigma_0 T) \|\partial_t g\|_{L^1(0, T; L^2(\Omega_h))},
\max_{t \in [0, T]} \left(\|\partial_t u_{\text{pml}}\|_{L^2(\Omega^3)} + \|\nabla \cdot u_{\text{pml}}\|_{L^2(\Omega)} + \|\nabla u_{\text{pml}}\|_{\tilde{H}(\Omega)}\right) \\
&\lesssim \sqrt{1 + \sigma_0 T} \|\partial_t g\|_{L^1(0, T; L^2(\Omega_h))}.
\end{aligned}
\]
3.2. A DtN operator for the PML problem. We now derive an error estimate between the DtN operators of the original scattering problem (2.9) and equivalently the problem (2.11) and the PML problem (3.8). We start by introducing the DtN operators of the original scattering problem (2.9) or equivalently the problem (2.11) and the PML problem (3.8). Given \( \Phi \in H^{1/2}(\Gamma_h) \), define \( \bar{R}_{pml} \Phi := \partial_{x_3} v \) on \( \Gamma_h \), where \( v \in H^1(\Omega_h^+) \) satisfies the following problem in the PML layer:

\[
\begin{cases}
\Delta_p v - \frac{s^2\sigma}{c^2} v = 0 & \text{in } \Omega_h^+,
\v = \phi & \text{on } \Gamma_h,
\v = 0 & \text{on } \Gamma_{h+L}.
\end{cases}
\]

Taking the Fourier transform of (3.9) with respect to \( \tilde{x} \), we get

\[
\begin{cases}
\frac{\partial}{\partial x_3} \left( \frac{1}{\sigma} \frac{\partial}{\partial x_3} \hat{\v}(\xi, x_3) \right) - \left( \frac{s^2\sigma}{c^2} + \sigma |\xi|^2 \right) \hat{\v}(\xi, x_3) = 0, & h < x_3 < h + L,
\hat{\v}(\xi, x_3) = \phi(\xi, h), & x_3 = h,
\hat{\v}(\xi, x_3) = 0, & x_3 = h + L.
\end{cases}
\]

Solving (3.10) gives

\[
\hat{\v}(\xi, x_3) = A e^{\beta(\xi)(x_3 - h)} + B e^{-\beta(\xi)(x_3 - h)}, \quad h < x_3 < h + L,
\]

where \( \beta(\xi) = \sqrt{s^2/c^2 + |\xi|^2} \) with \( \text{Re}[\beta(\xi)] > 0 \) and \( A, B \) are unknown functions of \( \xi \) to be determined.

Take \( x_3 = h \). Then, by the definition of \( \sigma \) we have \( \hat{x}_3 = h \), and so (3.11) implies that

\[
A + B = \hat{\phi}(\xi, h).
\]

Now, take \( x_3 = h + L \). Then a direct calculation gives

\[
\hat{x}_3 - h = \int_{f_-}^{f_+} \sigma(\tau) d\tau + f_- - h = \int_{h}^{h+L} \sigma(\tau) d\tau = \left( 1 + \frac{s^{-1} \sigma_0}{m + 1} \right) L := \tilde{L}.
\]

This, together with (3.11), implies that

\[
A e^{\beta(\xi)\tilde{L}} + B e^{-\beta(\xi)\tilde{L}} = 0.
\]

Solving (3.12) and (3.13) for \( A \) and \( B \) gives

\[
A = -\frac{e^{-\beta(\xi)\tilde{L}} \phi(\xi, h)}{e^{\beta(\xi)\tilde{L}} - e^{-\beta(\xi)\tilde{L}}} , \quad B = \frac{e^{\beta(\xi)\tilde{L}} \phi(\xi, h)}{e^{\beta(\xi)\tilde{L}} - e^{-\beta(\xi)\tilde{L}}}.
\]

Then we obtain the following solution of (3.10):

\[
\hat{\v}(\xi, x_3) = \frac{e^{-\beta(\xi)(x_3 - h - \tilde{L})} - e^{\beta(\xi)(x_3 - h - \tilde{L})}}{e^{\beta(\xi)\tilde{L}} - e^{-\beta(\xi)\tilde{L}}} \phi(\xi, h), \quad h < x_3 < h + L.
\]

Taking the derivative of (3.14) with respect to \( x_3 \) and evaluating its value at \( x_3 = h \), we obtain

\[
\frac{\partial \hat{\v}(\xi, h)}{\partial x_3} = -\beta(\xi) \frac{e^{-\beta(\xi)\tilde{L}} + e^{\beta(\xi)\tilde{L}}}{e^{\beta(\xi)\tilde{L}} - e^{-\beta(\xi)\tilde{L}}} \phi(\xi, h),
\]
where we have used the fact that $\sigma(h) = 1$. Now define

$$\mathcal{B}_{\text{pml}} \phi = -\beta(\xi) e^{-\beta(\xi) L} + e^{\beta(\xi) L} \phi(\xi, h).$$

Then taking the inverse Fourier transform of the above equation leads to the DtN operator $\mathcal{B}_{\text{pml}}$ defined as follows:

$$\mathcal{B}_{\text{pml}} \phi(\vec{x}, h) = -\int_{\mathbb{R}^2} \beta(\xi) e^{-\beta(\xi) L} + e^{\beta(\xi) L} \phi(\xi, h) e^{i\xi \cdot \vec{x}} d\xi, \quad \phi \in H^{1/2}(\Gamma_h).$$

Hence, the truncated PML problem (3.4a)-(3.4f) can be equivalently reduced to the boundary value problem in $\Omega_h \cup \Omega$:

$$\begin{cases}
\Delta^* u_{\text{pml}} - \rho_c s^2 \ddot{u}_{\text{pml}} = 0 & \text{in } \Omega, \\
\Delta \ddot{p}_{\text{pml}} - \frac{s^2}{c^2} \ddot{p}_{\text{pml}} = -s g / c^2 & \text{in } \Omega_h, \\
\partial_n \dot{p}_{\text{pml}} = -\rho_0 s^2 n \cdot \dot{u}_{\text{pml}} & \text{on } \Gamma, \\
-\dot{p}_{\text{pml}} n = \sigma(\dot{u}_{\text{pml}}) n & \text{on } \Gamma, \\
\dot{p}_{\text{pml}} = 0 & \text{on } \Gamma_f, \\
\partial_{zz} \ddot{p}_{\text{pml}} = \mathcal{B}_{\text{pml}} [\dot{p}_{\text{pml}}] & \text{on } \Gamma_h.
\end{cases} \quad (3.15)$$

Similarly as in the derivation of the problems (2.16) and (3.5), we can obtain the variational formulation of the problem (3.15): find $(\ddot{p}_{\text{pml}}, \dot{u}_{\text{pml}}) \in H$ such that

$$a_p((\ddot{p}_{\text{pml}}, \dot{u}_{\text{pml}}), (q, v)) = \int_{\Omega_h} \frac{\ddot{q}}{c^2} \cdot \ddot{v} dx \quad \forall (q, v) \in H, \quad (3.16)$$

where the sesquilinear form $a_p(\cdot, \cdot)$ is defined as

$$a_p((\ddot{p}_{\text{pml}}, \dot{u}_{\text{pml}}), (q, v)) = \int_{\Omega_h} \left[ s^{-1} \nabla \ddot{p}_{\text{pml}} \cdot \nabla q + \frac{s}{c^2} \ddot{p}_{\text{pml}} \cdot \ddot{v} \right] dx$$
$$+ \int_{\Omega_h} \left[ \rho_0 \sigma(x, u_{\text{pml}}) (\nabla \cdot u_{\text{pml}}) + 2 \mu \epsilon(u_{\text{pml}}) : \epsilon(\nabla) \right] + \rho_0 \rho_c |s|^2 s \ddot{u}_{\text{pml}} \cdot \nabla q + \rho_0 \int_{\Gamma_h} s n \cdot \ddot{u}_{\text{pml}} q d\gamma + \rho_0 \int_{\Gamma_h} s \ddot{p}_{\text{pml}} n \cdot \ddot{v} d\gamma.$$

### 3.3. Exponential convergence of the time-domain PML solution

In this subsection, we derive an error estimate between the solutions $(p, u)$ of the original problem (2.9) and the solutions $(p_{\text{pml}}, u_{\text{pml}})$ of the PML problem (3.3). To this end, we introduce some notations and norms. Denote by $L(X, Y)$ the standard space of the bounded linear operators from the Hilbert space $X$ to the Hilbert space $Y$.

We now establish the following error estimate between the DtN operators $\mathcal{B}$ and $\mathcal{B}_{\text{pml}}$ which is essential for the convergence analysis of the PML method.

**Theorem 3.3.** Let $\mathcal{L} = \sigma_0 L/(m + 1)$, $s = s_1 + is_2$ with $s_1 > 0$. Then we have

$$\|\mathcal{B} - \mathcal{B}_{\text{pml}}\|_{L(H^{1/2}(\Gamma_h), H^{-1/2}(\Gamma_h))} \leq \max \left\{ 1, \left| \frac{s_1}{\sigma_0} \right| \right\} \frac{2e^{-2\mathcal{L}/c}}{1 - e^{-2\mathcal{L}/c}} := C_U(s, \mathcal{L}). \quad (3.17)$$
Proof. By the definition of the norm (2.11) it follows that for any \( \phi \in H^{1/2}(\Gamma_h) \),
\[
\| ( \mathcal{B} - \mathcal{B}_{\text{pml}} ) \phi \|_{H^{-1/2}(\Gamma_h)}^2 
= \int_{\mathbb{R}^2} |\beta(\xi)|^2 \left( 1 + |\xi|^2 \right)^{-1/2} \left| 1 - \frac{e^{-\beta(\xi)\tilde{L}} + e^{\beta(\xi)\tilde{L}}}{e^{\beta(\xi)\tilde{L}} - e^{-\beta(\xi)\tilde{L}}} \right|^2 |\phi(\xi)|^2 d\xi 
\leq \int_{\mathbb{R}^2} \left( \frac{|s|^2}{c^2} + |\xi|^2 \right) \left( 1 + |\xi|^2 \right)^{-1/2} \left| 1 - \frac{e^{-\beta(\xi)\tilde{L}} + e^{\beta(\xi)\tilde{L}}}{e^{\beta(\xi)\tilde{L}} - e^{-\beta(\xi)\tilde{L}}} \right|^2 |\phi(\xi)|^2 d\xi 
\leq \max \left\{ 1, \frac{|s|^2}{c^2} \right\} \sup_{\xi \in \mathbb{R}^2} \left| 1 - \frac{e^{-\beta(\xi)\tilde{L}} + e^{\beta(\xi)\tilde{L}}}{e^{\beta(\xi)\tilde{L}} - e^{-\beta(\xi)\tilde{L}}} \right|^2 \| \phi \|^2_{H^{1/2}(\Gamma_h)}.
\]

Thus
\[
\| \mathcal{B} - \mathcal{B}_{\text{pml}} \|_{L(H^{1/2}(\Gamma_h), H^{-1/2}(\Gamma_h))} \leq \max \left\{ 1, \frac{|s|}{c} \right\} \sup_{\xi \in \mathbb{R}^2} \left| 1 - \frac{e^{-\beta(\xi)\tilde{L}} + e^{\beta(\xi)\tilde{L}}}{e^{\beta(\xi)\tilde{L}} - e^{-\beta(\xi)\tilde{L}}} \right|
\]

It is easy to see that
\[
\sup_{\xi \in \mathbb{R}^2} \left| 1 - \frac{e^{-\beta(\xi)\tilde{L}} + e^{\beta(\xi)\tilde{L}}}{e^{\beta(\xi)\tilde{L}} - e^{-\beta(\xi)\tilde{L}}} \right| = \sup_{\xi \in \mathbb{R}^2} \frac{2e^{-2\beta_r(\xi)\tilde{L}}}{1 - e^{-2\beta_r(\xi)\tilde{L}}} \leq \sup_{\xi \in \mathbb{R}^2} \frac{2e^{-2\beta_r(\xi)\tilde{L}}}{1 - e^{-2\beta_r(\xi)\tilde{L}}}, \tag{3.18}
\]

where \( \beta_r(\xi) = \text{Re}[\beta(\xi)] \) and \( \beta_i(\xi) = \text{Im}[\beta(\xi)] \). By the formulas
\[
z^{1/2} = \sqrt{|z| + \frac{z}{2} + i \text{sign}(z_2) \sqrt{|z| - \frac{z_1}{2}}}, \quad z = z_1 + iz_2, \quad \text{Re}[z^{1/2}] > 0,
\]
we have
\[
\beta_r(\xi) = \frac{1}{\sqrt{2}} \left( |\beta^2(\xi)| + \text{Re}[\beta^2(\xi)] \right)^{1/2}
= \frac{1}{\sqrt{2}} \left( \left( \frac{s_1^2 - s_2^2}{c^2} + |\xi|^2 \right)^2 + \frac{4s_1^2s_2^2}{c^2} \right)^{1/2} + \left( \frac{s_1^2 - s_2^2}{c^2} + |\xi|^2 \right)^{1/2}.
\]

Since \( 2e^{-2\beta_r(\xi)\tilde{L}}/[1 - e^{-2\beta_r(\xi)\tilde{L}}] \) is monotonically decreasing with respect to \( \beta_r(\xi) \), then we need to seek the minimum of \( \beta_r(\xi) \) in \( \mathbb{R}^2 \). A direct calculation yields that \( \xi = 0 \) is the unique minimum point of the function \( \beta_r(\xi) \), and thus
\[
\beta_r(0) = \frac{s_1}{c}, \quad \frac{2e^{-2\beta_r(\xi)\tilde{L}}}{1 - e^{-2\beta_r(\xi)\tilde{L}}} \bigg|_{\xi=0} \leq \frac{2e^{-2e^{-1}\tilde{L}}}{1 - e^{-2e^{-1}\tilde{L}}},
\]

In addition, \( \beta_r(\xi) \to +\infty \) as \( \xi \to \infty \), and so \( 2e^{-2\beta_r(\xi)\tilde{L}}/[1 - e^{-2\beta_r(\xi)\tilde{L}}] \to 0 \) as \( \xi \to \infty \).
It is then concluded that
\[
\sup_{\xi \in \mathbb{R}^2} \frac{2e^{-2\beta_r(\xi)\tilde{L}}}{1 - e^{-2\beta_r(\xi)\tilde{L}}} \leq \frac{2e^{-2e^{-1}\tilde{L}}}{1 - e^{-2e^{-1}\tilde{L}}}.
\]
This, together with (3.18), implies the required estimate (3.17). □

For \( \omega := (\bar{p}, \bar{u}) \in H \) and \( \theta := (q, \nu) \in H \) it easily follows by the definition of the sesquilinear forms \( a(\cdot, \cdot) \) and \( a_p(\cdot, \cdot) \) that

\[
|a(\omega, \theta) - a_p(\omega, \theta)| = \left| \int_{\Gamma_h} s^{-1} \eta (\mathcal{B} - \mathcal{B}_{pml}) \bar{p} d\gamma \right| \\
\leq |s|^{-1} \|q\|_{L^{1/2}(\Gamma_h)} \|\mathcal{B} - \mathcal{B}_{pml}\|_{L(H^{1/2}(\Gamma_h), H^{-1/2}(\Gamma_h))} \|\bar{p}\|_{H^{1/2}(\Gamma_h)} \\
\leq |s|^{-1} \left[ 1 + (h - f_+)^{-1} \right] \|q\|_{H^1(\Omega_h)} \|\mathcal{B} - \mathcal{B}_{pml}\|_{L(H^{1/2}(\Gamma_h), H^{-1/2}(\Gamma_h))} \|\bar{p}\|_{H^1(\Omega_h)} \\
\leq |s|^{-1} \left[ 1 + (h - f_+)^{-1} \right] \|\mathcal{B} - \mathcal{B}_{pml}\|_{L(H^{1/2}(\Gamma_h), H^{-1/2}(\Gamma_h))} \|\theta\|_H \|\omega\|_H. \tag{3.19}
\]

where we have used the trace theorem (see [21] Lemma 2.2) to get the second inequality. Using (3.19) and Theorem 3.3, we can now prove the exponential convergence of the PML method.

**Theorem 3.4.** Let \((p, u)\) be the solution of the problem (2.9) and let \((p_{pml}, u_{pml})\) be the solution of the truncated PML problem (3.8) with \(s_1 = 1/T\) in the time-domain. Then, under the assumptions (2.11) and (2.22), we have the error estimate

\[
\int_0^T \left( \|p - p_{pml}\|_{H^1(\Omega_h)}^2 + \|u - u_{pml}\|_{H^1(\Omega_h)}^2 \right) dt \\
\leq \max \{1, T^2\} (T^4 + T^2)(\gamma_1 + \gamma_2)(1 + \sigma_0 T)^2 \frac{e^{-4\gamma_0 T/c}}{(1 - e^{-2\gamma_0 T/c})^2} \|q\|_{H^1(0, T; L^2(\Omega_h))}^2 \tag{3.20}
\]

where \(\gamma_1\) and \(\gamma_2\) are positive constants which are independent of \((p, u)\) and \((p_{pml}, u_{pml})\) but may depend on \(T\).

**Proof.** First, let \(\omega = (\bar{p}, \bar{u})\) and \(\omega_p = (\bar{p}_{pml}, \bar{u}_{pml})\) be the solutions of the variational problems (2.10) and (3.16), respectively. Then, by (3.19) we have

\[
|a(\omega - \omega_p, \omega - \omega_p)| = |a(\omega, \omega - \omega_p) - a(\omega_p, \omega - \omega_p)| \\
= |a_p(\omega_p, \omega - \omega_p) - a(\omega_p, \omega - \omega_p)| \\
\leq |s|^{-1} \left[ 1 + (h - f_+)^{-1} \right] \|\mathcal{B}_{pml} - \mathcal{B}\|_{L(H^{1/2}(\Gamma_h), H^{-1/2}(\Gamma_h))} \|\omega_p\|_H \|\omega - \omega_p\|_H.
\]

This, together with Theorem 3.3 and the uniform coercivity of \(a(\cdot, \cdot)\) (see (2.18)), implies that

\[
\|\omega - \omega_p\|_H \leq C^{-1} \|s|^{-1} \left[ 1 + (h - f_+)^{-1} \right] C_U(s, \ell) \|\omega_p\|_H,
\]

where \(C\) is defined in (2.18). By the Parseval identity (A.3) and the definition of \(C_U(s, \ell)\) it is deduced that

\[
\int_0^\infty e^{-2s_1 t} \|\mathcal{L}^{-1}(\omega - \omega_p)\|_H^2 dt \\
= \frac{1}{2\pi} \int_{-\infty}^\infty \|\omega - \omega_p\|_H^2 ds_2 \\
\leq \frac{1}{\pi} \int_0^\infty \left[ 1 + (h - f_+)^{-1} \right]^2 \frac{\max \{1, (|s|/c)^2 \}}{C^2 \|s\|^2} \frac{4e^{-2\ell/c}}{(1 - e^{-2\ell/c})^2} \|\omega_p\|_H^2 ds_2.
\]
This gives

\[
\int_0^T \left( \| p - p_{\text{pm}} \|_{H^1(\Omega_t)}^2 + \| u - u_{\text{pm}} \|_{H^1(\Omega_t)}^2 \right) dt \\
\leq \int_0^T e^{-2s_1(t-T)} \left[ \| p - p_{\text{pm}} \|_{H^1(\Omega_t)}^2 + \| u - u_{\text{pm}} \|_{H^1(\Omega_t)}^2 \right] dt \\
\leq e^{2s_1T} \int_0^\infty e^{-2s_1t} \left[ \| p - p_{\text{pm}} \|_{H^1(\Omega_t)}^2 + \| u - u_{\text{pm}} \|_{H^1(\Omega_t)}^2 \right] dt \\
= e^{2s_1T} \int_0^\infty e^{-2s_1t} \| \mathcal{L}^{-1}(\omega - \omega_p) \|_{H^1(\Omega_t)}^2 dt \\
\leq \frac{e^{2s_1T}}{\pi} \frac{4e^{-4T/c}}{(1 - e^{-2L/c})^2} \int_0^\infty \frac{[1 + (h - f_+)^{-1}]^2 \max \{1, (|s|/c)^2 \}}{C^2 |s|^2} \| \omega_p \|_{H^1}^2 ds_2. \tag{3.21}
\]

Since \( s_1 > 0 \) is arbitrarily fixed, and by the definition of \( C \) (see (2.18)), there exists a sufficiently large positive constant \( M \) such that

\[
\frac{[1 + (h - f_+)^{-1}]^2 \max \{1, (|s|/c)^2 \}}{C^2 |s|^2} \leq \gamma_1 |s|^4 \tag{3.22}
\]

for \( s_2 \geq M \), where \( \gamma_1 \) is a constant independent of \( s_2 \). On the other hand, it is easy to see that

\[
\frac{[1 + (h - f_+)^{-1}]^2 \max \{1, (|s|/c)^2 \}}{C^2 |s|^2} \leq \gamma_2 \tag{3.23}
\]

for \( 0 \leq s_2 \leq M \), where \( \gamma_2 \) is a constant independent of \( s_2 \). By (3.22), (3.23), assumption (2.21), Theorem 3.1 and the Parseval identity (A.5) we obtain that

\[
\int_0^\infty \frac{[1 + (h - f_+)^{-1}]^2 \max \{1, (|s|/c)^2 \}}{C^2 |s|^2} \| \omega_p \|_{H^1}^2 ds_2 \\
\leq \int_0^M \gamma_2 |\omega_p|_{H^1(\Omega_t)}^2 ds_2 + \int_M^\infty \gamma_1 |s|^4 |\omega_p|_{H^1(\Omega_t)}^2 ds_2 \\
\leq \int_0^M \gamma_2 (1 + s^{-1}\sigma_0)^2 \left[ \frac{1 + 2s^2}{s^2 \min \{1, s_1^2 \}} \| \tilde{g} \|_{L^2(\Omega_t)}^2 + s_1^{-2} \| s\tilde{g} \|_{L^2(\Omega_t)}^2 \right] ds_2 \\
+ \int_M^\infty \gamma_1 (1 + s^{-1}\sigma_0)^2 \left[ \frac{1 + 2s^2}{s^2 \min \{1, s_1^2 \}} \| s^2\tilde{g} \|_{L^2(\Omega_t)}^2 + s_1^{-2} \| s^2\tilde{g} \|_{L^2(\Omega_t)}^2 \right] ds_2 \\
\leq C_0 \int_0^\infty \left[ \| \tilde{g} \|_{L^2(\Omega_t)}^2 + \| s\tilde{g} \|_{L^2(\Omega_t)}^2 + s_1^{-2} \| s\tilde{g} \|_{L^2(\Omega_t)}^2 \right] ds_2 \\
= \pi C_0 \int_0^\infty e^{-2s_1t} \left[ \| \tilde{g} \|_{L^2(\Omega_t)}^2 + \| \partial_t \tilde{g} \|_{L^2(\Omega_t)}^2 + \| \partial_t^2 \tilde{g} \|_{L^2(\Omega_t)}^2 \right] dt,
\]

where

\[
C_0 = (1 + s^{-1}\sigma_0)^2 \frac{1 + 2s^2}{s^2 \min \{1, s_1^2 \}} (\gamma_1 + \gamma_2).
\]

By this inequality and (3.21) the required estimate (3.20) follows easily on taking \( s_1 = T^{-1} \) and using the assumption (2.22), where integer \( m \geq 1 \) should be chosen.
small enough to ensure the rapid convergence (thus we need to take \( m = 1 \)) noting the definition of \( \mathcal{T} = \sigma_0 L / (m + 1) \). The proof is thus complete. \( \square \)

**Remark 3.5.** Theorem 3.4 implies that, for large \( T \) the exponential convergence of the PML method can be achieved by enlarging the thickness \( L \) or the PML absorbing parameter \( \sigma_0 \) which increases as \( \ln T \).

**4. Conclusion.** This paper studied the time-dependent acoustic-elastic interaction problem associated with a bounded elastic body immersed in a homogeneous air or fluid above a rough surface. A time-domain perfectly matched layer (PML) is introduced to truncate the unbounded domain of the interaction problem above a finite layer in the \( x_3 \) direction containing the elastic body, leading to a PML problem in a finite strip domain. The PML layer is constructed by the real coordinate stretching technique associated with \( \text{Re}(s)^{-1} \) in the Laplace domain, where \( s \in \mathbb{C}_+ \) is the Laplace transform variable. The well-posedness and stability estimate of the PML problem are established, based on the Laplace transform and a variational method. Moreover, the exponential convergence of the PML method has also been proved in terms of the thickness and parameters of the PML layer.

In practical computation, the PML problem obtained in this paper and defined in a strip domain must be truncated as well in the \( x_1 \) and \( x_2 \) directions, which may be achieved by constructing a rectangular or cylindrical PML layer in the strip domain. Further, our method can be extended to other time-dependent scattering problems, such as diffraction gratings, time-domain elastic scattering by rough surfaces in \( \mathbb{R}^2 \) and even more complicated electromagnetic-elastic interaction problems in unbounded layered structures. We hope to report such results in the future.

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**Appendix A. Laplace transform.**

For each \( s \in \mathbb{C}_+ \), the Laplace transform of the vector function \( \mathbf{u}(t) \) is defined as

\[
\mathcal{L}(\mathbf{u})(s) = \int_0^\infty e^{-st} \mathbf{u}(t) dt.
\]

The Fourier transform of \( \phi(\mathbf{x}, x_3) \) is defined by

\[
\hat{\phi}(\mathbf{\xi}, x_3) = \mathcal{F}(\phi)(\mathbf{\xi}, x_3) = \int_{\mathbb{R}^2} e^{-i\mathbf{\xi} \cdot \mathbf{x}} \phi(\mathbf{x}, x_3) d\mathbf{x}, \quad \mathbf{\xi} \in \mathbb{R}^2
\]

and the inverse Fourier transform of \( \hat{\phi}(\mathbf{\xi}) \) is

\[
\phi(\mathbf{x}, x_3) = \mathcal{F}^{-1}(\hat{\phi})(\mathbf{x}, x_3) = \frac{1}{(2\pi)^2} \int_{\mathbb{R}^2} e^{i\mathbf{\xi} \cdot \mathbf{x}} \hat{\phi}(\mathbf{\xi}, x_3) d\mathbf{\xi}.
\]

The Laplace transform has the following properties:

\[
\mathcal{L}(\mathbf{u}_t)(s) = s \mathcal{L}(\mathbf{u})(s) - \mathbf{u}(0), \quad (A.1)
\]

\[
\mathcal{L}(\mathbf{u}_{tt})(s) = s^2 \mathcal{L}(\mathbf{u})(s) - s \mathbf{u}(0) - \frac{d\mathbf{u}}{dt}(0), \quad (A.2)
\]

\[
\int_0^t \mathbf{u}(\tau) d\tau = \mathcal{L}^{-1}(s^{-1} \mathcal{L}(\mathbf{u})(s))(t), \quad (A.3)
\]
where $L^{-1}$ denotes the inverse Laplace transform.

By the definition of the Fourier transform we have that for any $s_1 > 0$,

$$\mathcal{F}(u(\cdot)e^{-s_1 \cdot})(s_2) = \int_{-\infty}^{+\infty} u(t)e^{-s_1 t}e^{-is_2 t}dt = \int_{0}^{\infty} u(t)e^{-(s_1 + is_2)t}dt = L(u)(s_1 + is_2), \quad s_2 \in \mathbb{R}.$$ 

From the formula of the inverse Fourier transform it is easy to verify that

$$u(t)e^{-s_1 t} = \mathcal{F}^{-1}\{\mathcal{F}(u(\cdot)e^{-s_1 \cdot})\} = \mathcal{F}^{-1}\left(L(u(s_1 + is_2))\right),$$

which implies that

$$u(t) = \mathcal{F}^{-1}\left(e^{s_1 t}L(u(s_1 + is_2))\right), \quad (A.4)$$

where $\mathcal{F}^{-1}$ denotes the inverse Fourier transform with respect to $s_2$.

By (A.4), the Plancherel or Parseval identity for the Laplace transform can be obtained (see [17, (2.46)]).

**Lemma A.1.** (Parseval identity) If $\tilde{u} = L(u)$ and $v = L(v)$, then

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{u}(s) \cdot v(s)ds_2 = \int_{0}^{\infty} e^{-2s_1 t}u(t) \cdot v(t)dt.$$

for all $s_1 > \lambda$, where $\lambda$ is the abscissa of convergence for the Laplace transform of $u$ and $v$.

**Lemma A.2.** [34 Theorem 43.1]. Let $\tilde{\omega}(s)$ denote the holomorphic function in the half plane $s_1 > \sigma_0$, valued in the Banach space $E$. The following statements are equivalent:

1) there is a distribution $\omega \in D'_+(E)$ whose Laplace transform is equal to $\tilde{\omega}(s)$, where $D'_+(E)$ is the space of distributions on the real line which vanish identically in the open negative half line;

2) there is a $\sigma_1$ with $\sigma_0 \leq \sigma_1 < \infty$ and an integer $m \geq 0$ such that for all complex numbers $s$ with $s_1 = \text{Re}(s) > \sigma_1$, it holds that $\|\tilde{\omega}(s)\|_E \lesssim (1 + |s|)^m$.

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