Research on the Dynamic Characteristics of the High Air Speed Nozzle in Isentropic Flow

Dong Xiang, Yingting Wang and Chenjing Li
School of Mechatronic Engineering, Harbin Institute of Technology, Harbin, Heilongjiang, 230103, China

Email: xiangdong@hit.edu.cn;

Abstract: The application of microsatellites has been development with the continuous expansion of space missions in resents years. The microsatellites have several characteristics such as light weight, small size and low cost. The platform flotation table is one of the simulation ways on the ground. With the continuous development of space technology, satellite ground simulation platform flotation table as a key technology which attracts lots of researchers. In this paper, the highspeed nozzle has been analyzed based on the research of Laval nozzle. The state of shock waves can be researched by the Laval nozzle and the air speed from the nozzle can be calculated. The flow field of the high speed nozzle has been simulation by the FLUENT software. The nozzle mode has been built by the GAMBIT and the model is meshed in structured grid. The simulation result shows that the air in the nozzle has many speeds in different place. The stable speeds are 390 m/s and 150m/s in the larynx and trailing end position. The maximum are reached into the nozzle and the speed is 578m/s, which is deviated from the theoretical analysis. At the constant pressure of 4 bar, the mass of nozzle flow is 0.000318758 kg/s. The mass of outlet flow is 0.000318798 kg/s, and the calculation error is 10^{-8}. The thrust of the nozzle is also stable at 3ms, and its stability value is 50.1 mN. The maximum force of the outlet is 176 mN before the nozzle reaches stability. The simulation result can guided for the nozzle design and its more significance for the high precision control and measurement of nozzles.

1. Introduction
The application of microsatellites has been development with the continuous expansion of space missions in resents years[1-4]. The microsatellites have several characteristics such as light weight, small size and low cost[5-6]. The platform flotation table is one of the simulation ways on the ground. With the continuous development of space technology, satellite ground simulation platform flotation table as a key technology which attracts lots of researchers[7-8]. The nozzle is one of the most parts of the platform. At present, the design and research of supersonic nozzle mainly depends on simulation optimization and experiment. The internal flow field of the nozzle should be simulated by the finite element analysis software. The study of supersonic nozzles focuses on the static characteristics. The pressure and velocity can be influenced by the optimizing structure. Some results shows that the flow field changes slightly in the tapering section and the change is much smaller[9]. The most drastic changing part is concentrated near the throat. The expanding section changing is relatively gentle, which generally satisfies the process of decompression, acceleration and air cooling. The wall friction has a great influence on the velocity and temperature. The outlet velocity of the nozzle has a great
relationship of the outlet diameter. With the increase of the outlet diameter, the outlet velocity increases obviously. This part of results can guide to the design the nozzle in this work.

It is very important in the microsatellite that research on low thrust measurement system of Laval nozzle. In 2016 year, researcher Lu used FLUENT calculation software to study the low thrust measurement system of Laval nozzle. It is analyzed on the influence of the baffle on the nozzle[10]. Three kinds of Laval nozzles with outlet diameters of 5.1 mm, 3.6 mm and 2.3 mm were numerically calculated, and their thrust values were 10N. The results show that changing pressure ratios of nozzle can influence the outlet pressure of nozzle when the baffle fixed. When the total inlet pressure is constant, each type of nozzle has a suitable baffle location area. There are many researchers has study on the nozzle. However, the research on supersonic nozzle is mostly focused on the static state of the nozzle. The finite element analysis and simulation software such as Fluent is used to analyze the changes of pressure, specific impulse and velocity of supersonic nozzle under different conditions, so as to get the different sizes of Laval nozzle under specific conditions. However, little research has been focus on the initial period of the nozzle impulse. The main problem is that the force produced by the micro nozzle is too small and the response time is too short to be measured and studied.

Nowadays, the most widely measurement technology of micro force is the micro-electronic systems (MEMS). It has been already development the micro sized nozzles with μN thrust force[11-14]. Due to the force of the micro high speed nozzle is too small, it can be stabilized from initial period in a very short time. At present, there are mainly two kinds of test methods for micro high speed nozzle which can be call the static thrust and impulse measurement. Impulse measurement is an indirect method to measure the impulse size. This may not be accurate enough for small satellites and air flotation platforms with stricter force requirements. When the time scale is large, the dynamic characteristics of the nozzle can be neglected. If the time scale reaches ms level, it is difficult to evaluate the accuracy of the experimental results because there has less theoretical calculation. Therefore, it is very important to study the dynamic characteristics of the nozzle, especially the change rules of the nozzle force before the flow field reaches stability.

In this paper, the high speed nozzle has been analyzed based on the research of Laval nozzle. The state of shock waves can be researched by the Laval nozzle and the air speed from the nozzle can be calculated. The flow field of the high speed nozzle has been simulation by the FLUENT software. The nozzle mode has been built by the GAMBIT and the model is meshed in structured grid. The simulation result shows that the air in the nozzle has many speeds in different place. The stable speeds are 390 m/s and 150 m/s in the larynx and trailing end position. The maximum are reached into the nozzle and the speed is 578 m/s, which is deviated from the theoretical analysis. At the constant pressure of 4 bar, the mass of nozzle flow is 0.000318758 kg/s. The mass of outlet flow is 0.000318798 kg/s, and the calculation error is 10^{-8}. The thrust of the nozzle is also stable at 3ms, and its stability value is 50.1 mN. The maximum force of the outlet is 176 mN before the nozzle reaches stability. The simulation result can guided for the nozzle design and it is more significance for the high precision control and measurement of nozzles.

2. The Engineering Theoretical Calculation of Laval Nozzle

The steady state theoretical calculation of the nozzle is based on the pressure, density, temperature and velocity at one interface. The parameters of different sections can be calculated. The basic equations are as follows:
The basic equations of pipe flow with variable cross section can be obtained by using the flow characteristics.

\[
\frac{dV}{V} = \frac{1}{1 - M_a^2} \frac{dA}{A}
\]

\[
dM_a = \frac{2 + (\gamma - 1)M_a^2}{2(1 - M_a^2)} \frac{dA}{A}
\]

\[
dp = \frac{\gamma M_a^2}{p} \frac{dA}{A}
\]

\[
d\rho = \frac{M_a^2}{p} \frac{dA}{A}
\]

\[
dT = \frac{(\gamma - 1)M_a^2}{T} \frac{dA}{A}
\]

(2)

It can be concluded the some parameters relationship on the cross section area such as velocity, Mach number, density and temperature. For supersonic flow, the speed increases when the area increase, and the pressure density and temperature are all getting lower. For subsonic flow, the speed decreases when the area increase, and the pressure density and temperature are all getting much higher. The energy equation is

\[
h + \frac{1}{2} V^2 = \frac{a_0^2}{\gamma - 1}
\]

(3)

The isentropic equation is

\[
\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{\frac{\gamma}{\gamma - 1}} = (1 + \frac{\gamma - 1}{2} M_e^2)^{\frac{\gamma}{\gamma - 1}}
\]

(4)

The outlet velocity can be calculated through the equation (3) and (4).

\[
V_e = \sqrt{2 \frac{R}{\gamma - 1} T_0 \left[1 - \left(\frac{p_e}{p_0}\right)^{\frac{\gamma - 1}{\gamma}}\right]}
\]

(5)

\[
M_e = \frac{V_e}{a_e} = \frac{2}{\gamma - 1} \left[\left(\frac{p_0}{p_e}\right)^{\frac{\gamma - 1}{\gamma}} - 1\right]
\]

(6)

The mass flow in the isentropic condition is

\[
m = \rho V_e A_e = A_e \rho_0 a_0 \sqrt{2 \left[\left(\frac{p_e}{p_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p_e}{p_0}\right)^{\frac{\gamma + 1}{\gamma}}\right]^2}
\]

(7)

Critical section is defined as \(A_e\). If this section appeared, the mass flow rate of the nozzle reaches the maximum value, and the equation as follows:

\[
m_{max} = \rho_0 A_e V_e = \left(\frac{\gamma}{R}\right) \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma - 1}{\gamma}} \frac{p_0}{\sqrt{T_0}} A_e
\]

(8)

At this time, the maximum throat flow is
\[ m_{\text{max}} = \rho A \sqrt{\frac{k}{\gamma+1}} \left( \frac{2}{\gamma+1} \right)^{\gamma+1} \left( \frac{p_0}{\sqrt{T_0}} \right) A_{\text{n}} = 0.0003598 \text{kg/s} \] (9)

It can be calculated the speed is
\[ a_e = \sqrt{\gamma RT} = \sqrt{1.4 \times 286 \times 107.1} = 207.08 \text{m/s} \] (10)
\[ V_e = M_e a_e = 3 \times 207.08 = 621.24 \text{m/s} \] (11)

3. The Simulation of the High Air Speed Nozzle

The Gambit software is used to build the nozzle model. Then the actual flow field of the simulated nozzle is selected in FLUENT software. The cylindrical injection area was added to the tail of the nozzle in order to simulate the actual working conditions. The monitoring surfaces were set up to detect the internal flow field. The model is meshed. Figure 1 is the simulation model of the high air speed nozzle.

![Simulation model of the high air speed nozzle](image)

Figure 1 the simulation model of the high air speed nozzle.

There are 7 monitoring surfaces on the model. The surfaces 3, 5 and 7 are retract, larynx and trailing end position. The flow type of the inlet and outlet are the pressure. The mesh has been checked and there is no negative volume and area. The pressure based mode has been select. The viscous model is k-epsilon. The inlet condition has been set up as 4 bar. The outlet is 1bar and the air has been defined as ideal gas PISO arithmetic has been selected.

![Pressure-time curve of the Laval nozzle](image)

Figure 2 pressure-time curve of the Laval nozzle

The surfaces 3, 5 and 7 have been selected to present the different flow characteristic. Figure 2 shows the pressure-time curve of the Laval nozzle. The retract position has much higher pressure. The steady pressure is 4.0 bar from 0.5ms. When the air flow across the larynx position the pressure are
getting down quickly as shown in surfaces 5. The steady pressure is 1.5 bar. After this time the air flow in the trailing end, the pressure steady at 0.975 bar. With the pressure changing, the velocity changes obviously. Figure 3 shows the velocity-time curve of the Laval nozzle. In surfaces 3, the velocity is stable at 0.5 ms and the speed is 5 m/s. The velocity is fluctuated considerably before reaching stability. In surfaces 5, the velocity is stable at 2.5 ms and the steady speed is 390 m/s. In surfaces 7, the velocity reaches stability at 3 ms and the steady speed is 150 m/s. Pressure and velocity is fluctuated considerably before reaching stability.

![Figure 3](image)

**Figure 3** the velocity-time curve of the Laval nozzle

Through the FLUENT software, the dynamic velocity and pressure nephograms are also made in Figure 4 in different times. The changing pressure and velocity of nozzle can be seen intuitively from whole stages. The time 1 ms and 3 ms are selected in the Figure. Velocity does not maximize at the outlet. The maximum are reached into the nozzle and the speed is 578 m/s, which is deviated from the theoretical analysis.

![Figure 4](image)

**Figure 4** the dynamic velocity and pressure nephograms in different time.

The reason for the error may be that the engineering calculation considers the ideal isentropic flow in the nozzle, and there is deviation between the setting and the actual flow in the simulation. The mass of nozzle flow is calculated. At the constant pressure of 4 bar, the mass of nozzle flow is 0.000318758 kg/s. The mass of outlet flow is 0.000318798 kg/s, and the calculation error is 10^{-8}. It is basically in agreement with the theoretical calculation. The thrust of the nozzle is also stable at 3 ms, and its stability value is 50.1 mN. The maximum force of the outlet is 176 mN before the nozzle
reaches stability.

4. Conclusion
In this paper, the high speed nozzle has been analyzed based on the research of Laval nozzle. The state of shock waves can be researched by the Laval nozzle and the air speed from the nozzle can be calculated. The simulation result can guide for the design. After the acceleration, the air speed can reach the isentropic flow. The flow field of the high speed nozzle has been simulation by the FLUENT software. The nozzle mode has been built by the GAMBIT and the model is meshed in structured grid. The surfaces 3, 5 and 7 are retract, larynx and trailing end position. The flow type of the inlet and outlet are the pressure. The mesh has been checked and there is no negative volume and area. The simulation result shows that the air in the nozzle has many speeds in different place. The stable speeds are 390 m/s and 150m/s in the larynx and trailing end position. The maximum are reached into the nozzle and the speed is 578m/s, which is deviated from the theoretical analysis. At the constant pressure of 4 bar, the mass of nozzle flow is 0.000318758 kg/s. The mass of outlet flow is 0.000318798 kg/s, and the calculation error is $10^{-8}$. The thrust of the nozzle is also stable at 3ms, and its stability value is 50.1 mN. The maximum force of the outlet is 176 mN before the nozzle reaches stability. The simulation result can guided for the nozzle design and it is more significance for the high precision control and measurement of nozzles.

References
[1] Lee J K and Kwon S J. Mixing efficiency of amultilation micro mixer with consecutive recirculation zones[C] Chemical Engineering Science 64 (2009) 1223-1231.
[2] Lee J K, Kim K and Kwon S J. Design, fabrication, and testing of MEMS solid propellant thruster array chip on glass wafer[C]. Sensors and Actuators A 157 (2010) 126-134.
[3] Janson S W and Helvajian H. MEMS Microengineering and Aerospace Systems[C] Aiaa Paper, 1999
[4] Larangot B, Conédéra V, Dubreuil P, Conto T D and RossiC. Solid Propellant Micro Thruster : an alternative propulsion device for nanosatellite[J]. Journal of Propulsion & Power, 2002, 22(1):56-63.
[5] NagaïR H, NaraokaK, Sawadaaand K Asai. Pressure-Sensitive Paint Measurement of Pressure Distribution in a Supersonic Micronozzle[J]. AIAAJourna. 2008, 46: 1.
[6] TanakaaS, Kondoa K and Habub H. Test of B/T multilayer reactive igniters for a micro solid rocket array thruster[J]. Sensors and Actuators A,2008, 144: 361–366.
[7] Daniel W. Son Y. Lu T. MEMS Mega-pixel Micro Thruster Arrays for Small Satellite Stationkeeping[J]. MEMS Mega-Pixel Micro-Thruster Array, 2000.
[8] Gamero M and Torsional A. Balance that Resolves Sub-micro-Newton Forces[J]. Electric Rocket Propulsion Society,2001(01).
[9] Gao Q J, Tang H J, Wang Zhao H, Yong H E. Numerical simulation and structure optimization of supersonic nozzle based on Fluent[J]. Manufacturing Automation,2015,(04):88-90.
[10] Lu Y H, Wei W Q, Shao F X and Sun Y X. Numerical calculation for the small thrust measurement system of Laval nozzle[J]. Journal of Mechanical & Electrical Engineering,2016, (10):1188-1192.
[11] Eccardt P, lofink A, Garssen H. Analysis of crosstalk between fluid coupled CMUT membranes[J]. Ultrasonics Symposium, 2005: 593-596.
[12] Na S, Chen A I H and Wong L P P. Capacitive micromachined ultrasonic transducers based on annular cell geometry for aircoupled applications[J]. Journal of Ultrasonics, 2016, 71: 152–160.
[13] Merkowitz S M, Maghani P G and Sharma A. A Newton thrust-stand for LISA[J]. Classical & Quantum Gravity, 2002, 19(7):1745-1750.
[14] Pierre Pennarun, Carole Rossi, Daniel Estève, VéroniqueConédéra, Development of MEMS based safe electro-thermal pyrotechnic igniter for a new generation of microfuze[J] Microtechnologies for the New Millennium, 2005, 5836