Upper critical field of $p$-wave ferromagnetic superconductors with orthorhombic symmetry

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Abstract. We extended the Scharnberg-Klemm theory of $H_{c2}(T)$ in $p$-wave superconductors with broken symmetry to cases of partially broken symmetry in an orthorhombic crystal, as is appropriate for the more exotic ferromagnetic superconductor UCoGe in strong magnetic fields. For some partially broken symmetry cases, $H_{c2}(T)$ can mimic upward curvature in two crystal axis directions, and reasonably good fits to some of the UCoGe data are obtained.

1. Introduction
There has long been an interest in the possibility of superconductivity with the paired electrons having an order parameter consisting of a triplet spin configuration and the corresponding odd orbital symmetry [1, 2, 3, 4, 5, 6, 7, 8]. The simplest odd orbital symmetry has the $p$-wave form [1]. In a crystal with non-cubic structure, there can be a variety of different $p$-wave states [1, 2, 3, 4, 5]. Depending upon the temperature $T$, magnetic field $H$, and pressure $P$, there can be phases corresponding to different triplet spin states [6, 7, 8]. One of the easiest ways to characterize the $p$-wave states is by measurements of the $T$ dependence of the upper critical field $H_{c2}(T)$ [1, 2]. However, when multiple phases are present in the same crystal, as in UPt$_3$, a proper analysis requires a variety of experimental results [6, 7].

Recently, a new class of ferromagnetic superconductors has been of great interest. Presently this class consists of UGe$_2$ [9], UIr [10], URhGe [11], and UCoGe [12], which except for UIr have orthorhombic crystal structures. For URhGe, the superconductivity arises within the ferromagnetic phase. That is also true for UCoGe at ambient pressure, but when sufficient pressure is applied, the ferromagnetic phase appears to disappear, leaving the superconducting phase without any obvious additional ferromagnetism [13, 14]. In the cases of UGe$_2$ and UIr, applying pressure within the ferromagnetic phase induces the superconductivity [9, 10]. In addition, polarized neutron studies have been interpreted as providing evidence for a field-induced ferrimagnetic state in UCoGe, with local moments of different magnitudes in opposite directions on the U and Co sites [15]. For a ferromagnetic superconductor with orthorhombic symmetry, the possible order parameter symmetries were given by Mineev [16].

Hardy and Huxley measured $H_{c2}(T)$ of URhGe at ambient pressure in all three-crystal axis directions [17]. Using only one fitting parameter for each field direction, they found that the...
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a reentrant phase, one that is close in field strength to the low-field phase [22]

occurred in very pure, well-aligned samples. This behavior may also have something to do with

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exceeded the Pauli limit for all field directions measured, providing strong evidence of a parallel-spin

pair state.

The first attempts to describe upward \( H_{c2}(T) \) curvature in all crystal axis directions were

based either upon ferromagnetic fluctuations [23], or upon a crossover from one parallel-spin

state to another [24]. Meanwhile, a mean-field theory of the complementary effects of itinerant

ferromagnetism and parallel-spin superconductivity was developed [25, 26]. To date, the field

dependence of this mutual enhancement has not been investigated. Here, we study the case in

which the \( p \)-wave pairing interaction strength is anisotropic, but finite in all crystal directions.

Since \( H_{c2} \) is essentially isotropic in the \( ab \) plane for samples of UCoGe with medium purity [19],

we studied the partially broken symmetry (PBS) state as a function of the pairing interaction

anisotropy. This can give a kink in \( H_{c2}(T) \) in at least one field direction [27].

**2. Upper critical field anisotropy of the PBS state**

We assume a \( p \)-wave pairing interaction as in Eq. (1) of Ref. [2], where we take \( V_3 > V_2 \geq V_1 \).

Then, for \( H || \hat{e}_3 \), the polar and two axial PBS states are obtained from

\[
\langle n | \Delta_{10} \rangle \alpha_n^{(p)} = 0 ,
\]

\[
\langle n | \Delta_{11} \rangle \pm \langle n | \Delta_{1,-1} \rangle \alpha_n^{(c)} = \mp b_n - 2 \langle n - 2 \rangle \Delta_{11} - b_n \langle n + 2 \rangle \Delta_{1,-1}, \tag{2}
\]

\[
\frac{1}{T} \left( \frac{\mu_B}{\Delta_{11}} \right)_{\text{CBS}} = \frac{1}{T} \left( \frac{\mu_B}{\Delta_{11}} \right)_{\text{PBS}} - \frac{1}{T} \left( \frac{\mu_B}{\Delta_{11}} \right)_{\text{CBS}}.
\]
where $a_n^{(p)} = [N(0)V_3]^{1} - a_n^{(p)}$, $a_n^{(-)} = [N(0)V_2]^{1} - a_n^{(-)}$, $a_n^{(+)} = [N(0)V_1]^{1} - a_n^{(+)}$, and

$$a_n^{(\lambda)} = \pi T \sum_{\omega_n} \int_0^\infty \! d\theta \sin \theta \left( \frac{3 \cos^2 \theta}{2 \sin^2 \theta} \right) \int_0^\infty \! d\xi e^{-2|\omega_n|\xi} e^{-\frac{1}{2} \xi \zeta_{12}} L_n(\zeta_{12}),$$

$$b_n = \pi T \sum_{\omega_n} \int_0^\infty \! d\theta \frac{3}{2} \sin^3 \theta \int_0^\infty \! d\xi e^{-2|\omega_n|\xi} e^{-\frac{1}{2} \xi \zeta_{12}} F_n(\zeta_{12}),$$

where the upper (lower) terms in the parenthesis of $a_n^{(\lambda)}$ are for the polar ($\lambda = p$) and axial ($\lambda = a$) states, respectively, $\zeta_{12} = eH\xi^2v_F^2\sin^2\theta(m/m_{12})$, $m_{12} = \sqrt{m_1m_2}$, $m = (m_1m_2m_3)^{1/3}$, $L_n(z)$ are the Laguerre polynomials, $F_n(z) = \sum_{p=0}^{n} \frac{(-z)^{p+1} \sqrt{(n+2)(n+1)!}}{p!(n+2)(n-p)!}$, $N(0)$ is the single-spin density of states, and we set $h = c = k_B = 1$. For the field along $\hat{e}_1$ or $\hat{e}_2$, one rotates the axes by $\pi/2$ about $\hat{e}_2$ or $\hat{e}_1$, respectively, and lets $m_{12}$ be replaced by $m_{23}$ or $m_{13}$, respectively.

Since the low-field $H_{c2}(T)$ data of Huy et al. for UCoGe suggest that it has uniaxial symmetry, with $H_{c2}||\hat{a} \approx H_{c2}||\hat{b}$, in the following we will restrict our consideration to the $V_1 = V_2$ case [19]. In order to fit the Aoki et al. data with the S-shaped $H_{c2}||\hat{b}(T)$ curve, it is necessary to use the full orthorhombic anisotropy in Eqs. (1)-(4), and to include the spontaneous and field-dependent magnetization. To do so for the two axial states, one may obtain a recursion relation for either one of the amplitudes, $\langle n|\Delta_{1,\pm 1}\rangle$, by eliminating the other in Eq. (2), and then solving the recursion relation in terms of a continued fraction. In Fig. 1(a), we plotted $h_{c2,||c} = 2eH_{c2}(m/m_{12})v_F^2/(2\pi T_c^c)^2$ versus $t = T/T_c^c$ for the polar state and for a variety of PBS states with $-0.25 \leq \delta < 0$, where $\delta = \ln(T_a/T_c^c)$. Note that these PBS states all have slight upward curvature, but since $T_c^c > T_a^c$, the polar state dominates for all $T \leq T_c^c$. In Fig. 1(b), we plotted $h_{c2,\perp c} = 2eH_{c2}(m/\sqrt{m_1m_3})v_F^2/(2\pi T_c^c)^2$ versus $t = T/T_c^c$ for the CBS state and for various PBS states with $-0.25 \leq \delta < 0$. In this case, the CBS state dominates near to $T_c^c$, but there is a crossover to a PBS state for $-0.179 \leq \delta < 0$, resulting in a single kink in $H_{c2,\perp c}(T)$.

3. Fits to the Huy et al. UCoGe $H_{c2}(T)$ data

As a starting point, to see if there is any possibility of fitting the least anomalous region of the $H_{c2}(T)$ curves obtained for UCoGe, we assume uniaxial anisotropy and fit the data of Huy et al. [19]. In Fig. 2(a), the best fit to the $H||\hat{c}$ data is for the polar state, as shown. In Fig. 2(b), the best fits to the $H||\hat{a}$ and $H||\hat{b}$ data are both for $\delta = -0.07$, which show a distinct crossover from the CBS to the PBS state. This $\delta$ value is also consistent with the polar state best fit to the $H_{c2}||\hat{c}$ data in Fig. 2(a). We remark that when the spontaneous magnetization along the $c$-axis direction is included, the fitting to the data in Fig. 2(a) would be altered.

4. Conclusions

We found that it is possible to fit the upward curvature of the $H_{c2}(T)$ data for $H||\hat{a}$ and $H||\hat{b}$ from medium-purity UCoGe using a crossover from the completely broken symmetry polar state to a PBS state. However, in the model studied, it is not possible to fit the observed upward curvature of $H_{c2}(T)$ for $H||\hat{c}$, as the polar state alone provides the best fit to the data. At the very least, the spontaneous and field-dependent magnetization should be included in future fits, using an anisotropic itinerant ferromagnetic superconductor model similar to that previously studied [25, 26].

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Figure 2. Best fits to the data of Huy et al. for $\mu_0 H_c^2(T)$ in medium purity UCoGe [19]. (a) $H \parallel \hat{c}$. Open black diamonds: data. The red solid curve is for the polar state. (b) Data for $H \parallel \hat{b}$ (red crosses) and $H \parallel \hat{a}$ (open black circles). The solid black and blue dashed curves are for the CBS state and the PBS state with $\delta = -0.07$, respectively. The slopes at $T_c$ were adjusted to fit the data.

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