Flat Direction Inflation with Running Kinetic Term and Baryogenesis

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We consider a possibility that one of the flat directions in the minimal supersymmetric standard model plays the role of the inflaton field and realizes large-field inflation. This is achieved by introducing a generalized shift symmetry on the flat direction, which enables us to control the inflaton potential over large field values. After inflation, higher order terms allowed by the generalized shift symmetry automatically cause a helical motion of the field to create the baryon number of the universe, while baryonic isocurvature fluctuations are suppressed.

I. INTRODUCTION

Inflation solves various theoretical difficulties of the standard big bang cosmology\textsuperscript{[1, 1]}\textsuperscript{1}. The most plausible way to realize inflation with a graceful exit is to introduce a gauge singlet inflaton field with a relatively flat potential\textsuperscript{[6, 7]}. Indeed, the single-field slow-roll inflation is consistent with observations including the cosmic microwave background (CMB) and large-scale structure data\textsuperscript{[8]}.

The BICEP2 collaboration recently reported detection of the primordial B-mode polarization of CMB\textsuperscript{[9]}, which could be originated from gravitational waves generated during inflation\textsuperscript{[10]}. The tensor-to-scalar ratio and the Hubble parameter during inflation suggested by the BICEP2 results are given by \( r = 0.20^{+0.07}_{-0.05} \) and \( H_{\text{inf}} \simeq 10^{14} \text{ GeV} (r/0.16)^{1/2} \) respectively.

Taken at face value, the BICEP2 results strongly suggest large-field inflation such as chaotic inflation\textsuperscript{[11]}, where the inflaton field excursion exceeds the reduced Planck scale, \( M_P \simeq 2.4 \times 10^{18} \text{ GeV} \textsuperscript{[12]} \). One way to control the inflaton potential over such a broad field range is to introduce a shift symmetry, under which the inflaton \( \phi \) transforms as

\[
\phi \rightarrow \phi + C,
\]

where \( \phi \) is the inflaton and \( C \) is a real transformation parameter. Here and in what follows we adopt the Planck units where \( M_P \) is set to be unity unless explicitly shown for convenience. The natural inflation\textsuperscript{[13]} or the multi-natural inflation\textsuperscript{[14, 15]} is realized if the shift symmetry is explicitly broken by sinusoidal functions. A similar shift symmetry was also used in the chaotic inflation in supergravity\textsuperscript{[16]}.

Recently, a generalized shift symmetry was proposed\textsuperscript{[18, 19]}:

\[
\phi^n \rightarrow \phi^n + C,
\]

where \( n \) is a positive integer. In this case it is \( \phi^n \) instead of \( \phi \) that plays the role of the inflaton. Interestingly, this allows \( \phi \) to be charged under gauge symmetry, because the inflaton remains gauge-singlet as long as \( \phi^n \) is gauge invariant. As we shall see shortly, the generalized shift symmetry naturally leads to a field-dependent kinetic term, and as a result, the inflaton potential form changes at large field values.

With the running kinetic term, the standard model (SM) Higgs field (as well as its supersymmetric extension) can play the role of the inflaton with a simple quadratic (or fractional power) potential\textsuperscript{[20]}. Similarly, one of the flat directions in the minimal supersymmetric standard model (MSSM) could play the role of the inflaton field and various implications for phenomenology and cosmology were discussed in Ref.\textsuperscript{[19]}\textsuperscript{, 2}.\textsuperscript{2}

In this Letter we revisit a possibility that one of the MSSM flat directions plays the role of the inflaton field. To this end we identify \( \phi^n \) with the flat direction and impose the generalized shift symmetry\textsuperscript{[2]}. Intriguingly, as pointed out in Ref.\textsuperscript{[20]}, the baryon or lepton-number violating operator required by the generalized shift symmetry necessarily induces a helical motion of the flat direction, creating the baryon asymmetry of the universe \textit{`a la} Affleck-Dine mechanism\textsuperscript{[23]}. Thus, baryogenesis is a built-in feature of the flat direction inflation with a running kinetic term. The flat direction condensates subsequently transforms into \( Q \) balls\textsuperscript{[24, 26]} and the reheating proceeds through the \( Q \)-ball decay\textsuperscript{[27]}. Since the flat direction consists of the supersymmetric partner of quarks and/or leptons, the reheating of the SM degrees of freedom is straightforward. It is noteworthy that the baryonic isocurvature perturbations are suppressed in our inflation model, in contrast to the usual case\textsuperscript{[28]}. Thus we investigate the flat direction inflation with a running kinetic term in great detail, focusing on the baryogenesis due to the flat direction.

\textsuperscript{1} The exponentially expanding universe was also studied in Refs.\textsuperscript{[2, 5]}.\textsuperscript{2}

\textsuperscript{2} See e.g. Refs.\textsuperscript{[21, 22] for other realization of the MSSM flat direction inflation.
The structure of the Letter is as follows. In the next section, we review the running kinetic inflation in general. We see how the flat direction fits into the context of the running kinetic inflation in Sec.III, where the dynamics of the field is also investigated numerically. In Sec. IV, the baryon number of the universe is estimated, and we conclude in Sec. V. In Appendix, we show a list of possible flat directions together with the predicted values of the scalar spectral index and the tensor-to-scalar ratio.

II. RUNNING KINETIC INFLATION

Let us briefly review the running kinetic inflation in supergravity [18, 19]. We introduce a chiral superfield \( \phi \), whose Kähler potential is invariant under the generalized shifts symmetry \( \tilde{Q} \). The Kähler potential thus is a function of \( (\phi^n - \phi^\dagger n) \), and we expand it as

\[
K = \sum_{k=1} c_k (\phi^n - \phi^\dagger n)^k = c_1 (\phi^n - \phi^\dagger n) - \frac{1}{2} (\phi^n - \phi^\dagger n)^2 + \cdots,
\]

where \( c_k \) is a numerical coefficient of \( O(1) \). In particular \( c_1 \) is a pure imaginary number and we normalize \( c_2 = -1 \). During inflation, the field stays along the inflationary trajectory which minimizes the Kähler potential \( K \). For simplicity we neglect higher order terms with \( k \geq 3 \), as they do not change the following argument.

For successful inflation, we introduce shift-symmetry breaking terms in the superpotential and Kähler potential as

\[
W = \lambda X \phi^n, \quad \Delta K = \kappa |\phi|^2,
\]

where \( \lambda, \kappa \ll 1 \). As we shall see below, \( \lambda \) is fixed by the normalization of curvature perturbations. We have introduced \( X \) for successful inflation at large field values \( |\phi|^\sim 0 \), and it can be stabilized at the origin with a positive Hubble-induced mass during inflation by higher-order terms in the Kähler potential.

The total superpotential and Kähler potential are given respectively by

\[
K = \kappa |\phi|^2 + c_1 (\phi^n - \phi^\dagger n) - \frac{1}{2} (\phi^n - \phi^\dagger n)^2 + |X|^2 + \cdots, \quad W = \lambda X \phi^n,
\]

where the dots represent higher order terms. Then the effective Lagrangian relevant for the inflation becomes

\[
\mathcal{L} = (\kappa + n^2 |\phi|^{2n-2}) \partial_\mu \phi \partial^\mu \phi^\dagger - V, \quad V = e^{\kappa |\phi|^2 + c_1 (\phi^n - \phi^\dagger n) - \frac{1}{2} (\phi^n - \phi^\dagger n)^2} \lambda^2 |\phi|^{2m}.
\]

Here we omit the soft supersymmetry (SUSY) breaking terms for the inflaton. Notice that \( X \) is assumed to be stabilized at the origin, \( X = 0 \), during inflation. For \( |\phi| \gtrsim (\kappa / n^2)^{1/(2n-2)} \), the \( \kappa \)-term is negligible, and therefore, it is \( \phi^\dagger \equiv \phi^n \) that is the canonically normalized field. The real component of \( \phi \) can take a super-Planckian field value and therefore becomes the inflaton, as it does not appear in the Kähler potential because of the shift symmetry. On the other hand, the imaginary component of \( \phi \) acquires a mass of order Hubble parameter, and it is stabilized where the Kähler potential is minimized, \( \phi^\dagger = c_1 \) for \( \text{Re}[\phi] \gtrsim 1 \). This equation determines the inflationary trajectory for \( \text{Re}[\phi] \gtrsim 1 \). The fact that the imaginary component has a mass of order Hubble parameter will be important to suppress the isocurvature perturbations, when applied to the MSSM flat direction. Along the inflationary trajectory, the effective Lagrangian is given by

\[
\mathcal{L} = \frac{1}{2} (\partial_R \hat{\phi})^2 - \lambda^2 (\hat{\phi}_R)^{2m/n},
\]

Note that, since the shift symmetry is only approximate, there could be (infinitely) many shift-symmetry breaking terms that are consistent with gauge (or other) symmetries. If the shift symmetry is of high quality, however, the higher order terms can be suppressed by powers of the order parameters such as \( \kappa \) or \( \lambda \). For instance, the quantum gravity corrections to the inflaton potential are suppressed by certain powers of the order parameters, because the gravity is necessarily coupled to the energy momentum tensor, which is accompanied by the order parameters. It is in principle possible that both \( X \phi^m \) and \( X \phi^{2m} \) terms play an important role during the last 50 – 60 e-foldings. In this case, the inflaton dynamics will be similar to the so-called polynomial chaotic inflation [17], and various values of \( n_s \) and \( r \) can be realized.
where $\hat{\phi}_R \equiv \text{Re}[\hat{\phi}] / \sqrt{2}$, and $\hat{\lambda}^2 \equiv (e^{-|c_1|^2/2} \lambda^2) / 2^{m/n}$. This is nothing but the chaotic inflation with a monomial potential $\hat{\phi}^p$ with $p = 2m/n$, and one can express the coupling $\lambda$, the spectral index $n_s$, and the tensor-to-scalar ratio $r$ respectively as

$$
\lambda \simeq 5.1 \times 10^{-4} e^{\frac{|c_1|^2}{2} - \frac{m}{n} \left(\frac{m}{n}\right)^{\frac{n-m}{n}} N^{-\frac{m+n}{n}}} ,
$$

$$
n_s = 1 - \left(1 + \frac{m}{n}\right) \frac{1}{N} ,
$$

$$
r = \frac{8m}{n} \frac{1}{N} ,
$$

where $N$ is the e-folding number, and we have used in Eq.(11) the Planck normalization on the curvature perturbations, $\Delta_{R}^2 \simeq 2.2 \times 10^{-9}$ [8].

Inflation ends when the canonically normalized inflaton $\hat{\phi}_R$ becomes comparable to unity. Then the inflaton field leaves the inflationary trajectory, and starts rotation because of the $\phi$-number violating terms in Eq. (9). Note that such $\phi$-number violating terms are allowed by the shift symmetry, and indeed, one can see from Fig. (b) that, as long as $c_1 \neq 0$, the inflationary trajectory is off-set along the imaginary component of $\phi$, leading to the rotation after inflation. On the other hand, if $c_1 = 0$, the inflationary trajectory coincides with the real axis, and no rotation is induced after inflation. Therefore we expect that the resultant $\phi$-number is an increasing function of $c_1$. In fact, we verify this numerically, although the $c_1$-dependence is nontrivial.

After the amplitude of the field decreases below $|\phi| < (\kappa/n^2)^{1/(2n-2)}$, $\phi$ itself becomes the dynamical variable, and it rotates in the potential $|\phi|^{2m}$ until the soft SUSY breaking mass term $m^2_\phi|\phi|^2$ dominates the potential. Finally the inflaton decays into radiation, but this process is highly model-dependent.

### III. FLAT DIRECTION AS INFLATON

In MSSM, flat directions are composed of squarks and sleptons. The potential is flat in the SUSY limit at renormalizable level, but lifted by SUSY breaking effects and nonrenormalizable terms. The former leads to a soft mass term, while the latter gives rise to higher order terms in the potential.

In order to see how to fit the flat direction in the context of the running kinetic inflation, we first specify the direction. Flat directions are represented by gauge-invariant (GI) monomials and they are all classified in Ref. [29]. Therefore, the flat direction is a good candidate of the inflaton $\hat{\phi} = \phi^n$, where $\phi$ should be gauge-singlet. In the previous section we have not included the gauge interactions of $\phi$. In fact, the gauge bosons coupled to the flat direction acquire a heavy mass at large fields values of $\phi$. For the parameters of our interest, however, the physical gauge boson masses as well as the field value of $\phi$ do not exceed the Planck scale, even though the canonically normalized inflaton $\hat{\phi}$ does become super-Planckian. Also, $\hat{\phi}$ has only suppressed interactions with the gauge fields and other SM fields because of the running kinetic term. Therefore, the argument in the previous section can be applied to the MSSM flat directions.

For successful inflation in supergravity, as mentioned in the previous section, the superpotential must have the form of (7). This is the case for the $d$ directions in Ref. [29], where $d = m + 1$. We list some of these directions and the nonrenormalizable superpotential that lifts the corresponding direction in Table I. For these directions, the inflation takes place effectively for $V \sim \hat{\phi}^p$ with $p = 1.6 - 3.2$. A complete list of the flat directions and the predicted values of $n_s$ and $r$ are given in the Appendix.

Note that, although we do not necessarily impose the matter parity here, we can do so by considering the generalized shift symmetry on the gauge monomial squared. For instance, in the case of $LLe$, we can impose the generalized shift symmetry on $(LLe)^2$ instead of $LLe$ so that the matter parity is kept even for $c_1 \neq 0$ in Eq. (6). In this case, the actual value of $n$ is obtained by multiplying $n'$ in Table 1 or 2 by a factor of two.

| Direction | GI monomials | $W_{NR}$ |
|-----------|--------------|----------|
| $L, d, c$ | $LLe$, $n' = 3$ | $H_uLddd$, $m = 4$ |
| $L, d$ | $LLddd$, $n' = 5$ | $H_uLLLe$, $m = 4$ |
| $Q, u, c$ | $QuQuQu$, $n' = 5$ | $H_uQuQuQu$, $m = 8$ |

$^4$ The other degrees of freedom $Q_1$ can be stabilized at the origin, if there are $|X|^2|Q_1|^2$ terms in the Kähler potential.
Now we show numerical results of the dynamics of the inflaton field for \( n = 3 \) and \( m = 4 \), for example. In this case, the inflationary trajectory is represented as

\[
3\phi_1^2 \phi_2 - \phi_3^2 = \frac{|c_1|}{2} = \frac{1}{2},
\]

(14)

where \( \phi = \phi_1 + i\phi_2 \) (\( \phi_1, \phi_2 \) is real), and we take \( c_1 = i \) in the last equality.

In Fig. 1(a), the trajectory of the field \( \phi \) is shown. We set the initial conditions as \( \phi_1 = 2.3 \) and \( \phi_2 = 0.0315 \), from where the inflation lasts for \( N \simeq 58 \) e-foldings. The field first evolves along the inflationary trajectory (14), and leaves from there when \( \phi \sim 1 \). We also take the values of \( \lambda = 2.8 \times 10^{-6} \) (for \( N = 50 \)) and \( \kappa = 0.01 \), which is consistent with the Planck normalization on the curvature perturbations. In terms of the canonical normalized field for large amplitudes, the dynamics of the field \( \hat{\phi} (= \phi^3) \) looks more familiar, shown in Fig. 1(b).

![Fig. 1](image)

**Fig. 1:** Left: Trajectory of the field \( \phi \) for \( n = 3 \) and \( m = 4 \). We set \( \lambda = 2.8 \times 10^{-6} \) and \( \kappa = 0.01 \). Right: Trajectory of the field \( \hat{\phi} \) for the same dynamics.

After inflation, the field starts rotation due to the \( \phi \)-number violating terms in Kähler potential. Drawing-star-like behavior will end when the field amplitude decreases to \( \phi \sim \kappa^{1/(2n-2)} \). Afterward, the canonically normalized field is represented by \( \varphi \), where \( \phi = \varphi e^{i\theta} / \sqrt{2\kappa} \), and the Lagrangian is given by

\[
\mathcal{L} = \frac{1}{2}(\dot{\varphi}^2 + \varphi^2 \dot{\theta}^2) + \frac{1}{2}m_{\varphi}^2 \varphi^2 + \frac{\lambda^2}{(2\kappa)m} \varphi^{2m},
\]

(15)

where the second term is the soft SUSY breaking mass term. The field will rotate in the potential of the form \( V \sim \varphi^{2m} \), until the soft SUSY breaking mass term dominates the potential.

**IV. BARYOGENESIS**

The soft SUSY breaking mass term is actually given by

\[
V_{\text{soft}} = \frac{1}{2}m_{\varphi}^2 \varphi^2 \left(1 + K \log \frac{\varphi^2}{2M^2} \right),
\]

(16)

where \( K \) is a numerical coefficient of the one-loop radiative corrections and it becomes negative when the gaugino loop dominates [25], and \( M \) is the renormalization scale where \( m_{\varphi} \) is evaluated. In the gravity-mediation or anomaly mediation with a generic Kähler potential [30], we expect that the soft mass is comparable to the gravitino mass, \( m_{\varphi} \sim m_{3/2} \). We assume the gravity mediation in the following, but our results can be straightforwardly applied to the pure gravity mediation scenario [31], or the minimal split SUSY [32, 33]. The typical value of \( K \) is \( |K| = 0.01 - 0.1 \) for the gravity mediation, while \( |K| \) can be a few orders of magnitudes smaller if the gaugino masses are one-loop suppressed with respect to the sfermion masses [34].
Once the amplitude of the field gets smaller than \( \varphi_{\text{eq}} \), where

\[
\varphi_{\text{eq}} = \left( \frac{(2 \kappa)^m}{2 \lambda^2} \right)^{1/(2m-2)} m_{\varphi}^{1/(m-1)},
\]

the field experiences spatial instabilities due to the negative value of \( K \) and transforms into \( Q \) balls \([24, 26]\). The charge of the formed \( Q \) ball is estimated as \([26]\)

\[
Q = \beta \left( \frac{\varphi_{\text{eq}}}{m_{\varphi}} \right)^2,
\]

where \( \beta = 0.02 \) \([35]\).

The universe becomes dominated by the \( Q \) balls, and the reheating is induced by the \( Q \)-ball decay. The \( Q \) ball decays through its surface, and the rate has an upper bound called the saturated rate, which stems from the Pauli’s blocking of the produced fermions \([36]\). The decay rate is estimated as \([37, 38]\)

\[
\Gamma_Q = \frac{N_q}{Q} \frac{\omega_Q^3}{12 \pi^2} 4 \pi R_Q^2,
\]

where \( N_q \) is the number of the produced quarks by the decay, \( \omega_Q \) is the effective mass of the \( \varphi \) particle inside the \( Q \) ball, and \( R_Q \) is the size of the \( Q \) ball. They are given respectively by \([25]\)

\[
\omega_Q \simeq m_{\varphi}, \quad R_Q \simeq |K|^{-1/2} m_{\varphi}^{-1}.
\]

Then the reheating temperature is obtained as

\[
T_R = \left( \frac{90}{4 \pi^2 g_*} \right)^{1/4} \sqrt{\Gamma_Q M_P} \simeq 370 \text{ GeV} \left( \frac{g_*}{96.25} \right)^{-1/4} \left( \frac{m_{\varphi}}{10^4 \text{ GeV}} \right)^{7/6} \left( \frac{N_q}{18} \right)^{1/2} \left( \frac{|K|}{0.01} \right)^{-1/2} \left( \frac{\beta}{0.02} \right)^{1/4} \left( \frac{\lambda}{4.1 \times 10^{-6}} \right)^{1/3} \left( \frac{\kappa}{0.01} \right)^{-2/3},
\]

where \( g_* \) counts the relativistic degrees of freedom and we set \( m = 4 \) in the last equality. Applying this reheating temperature, we can estimate the baryon number of the universe as

\[
Y_b = \frac{n_b}{s} = \frac{3}{4} T_R \frac{n_b}{\rho_{\text{crit}}} = 3 \frac{3}{4} T_R \frac{n_b}{\rho_{\text{crit}} \varphi_{\text{eq}}} \simeq 3 \frac{3}{4} T_R \frac{1}{m_{\varphi}},
\]

\[
\simeq 2.8 \times 10^{-2} \left( \frac{g_*}{96.25} \right)^{-1/4} \left( \frac{m_{\varphi}}{10^4 \text{ GeV}} \right)^{1/6} \left( \frac{N_q}{18} \right)^{1/2} \left( \frac{|K|}{0.01} \right)^{-1/2} \left( \frac{\beta}{0.02} \right)^{-1/4} \left( \frac{\lambda}{4.1 \times 10^{-6}} \right)^{1/3} \left( \frac{\kappa}{0.01} \right)^{-2/3}. \]

Therefore, we can generate a sufficiently large amount of the baryon asymmetry from the inflaton dynamics. Note that, since the imaginary component of \( \hat{\phi} \) has a mass of order Hubble parameter during inflation, there is little degree of freedom other than the inflaton, which acquires quantum fluctuations during inflation. Therefore, there is no baryonic isocurvature perturbations, in contrast to the usual Affleck-Dine mechanism \([28]\).

The baryon asymmetry estimated above is much larger than the observed value of \( Y_b \sim 10^{-10} \), but the resultant baryon asymmetry can be suppressed in either of two ways. As mentioned before, the final baryon asymmetry can be suppressed if \( |c_1| \) is sufficiently small. The smallness of \( |c_1| \) can be understood if \( \hat{\phi}^n \) has an odd matter parity, as it would represent the small breaking of the matter parity. We have indeed confirmed numerically that the final baryon asymmetry can be suppressed for a sufficiently small value of \( |c_1| \), although it is nontrivial to derive its analytic dependence because of the complicated dynamics during oscillations (see Fig. 1). On the other hand, there may be late-time entropy production by a modulus decay of thermal inflation \([39]\), which dilutes the final baryon asymmetry. For example, if the axion is the main component of the dark matter, the Peccei-Quinn symmetry should be restored during inflation to avoid the isocurvature bounds \([40, 41]\). In this case, thermal inflation may naturally takes place by the saxion \([42]\).

Lastly let us comment that the reheating temperature is rather low than the reheating through large \( Q \) balls \([27]\). If the reheating process had taken place through perturbative decay with the rate \( \Gamma = \frac{f^2}{\pi m_{\varphi}} \), where \( f \) generally denotes a coupling constant, the reheating temperature would be estimated as

\[
T_R = 7 \times 10^9 \text{ GeV} \left( \frac{g_*}{200} \right)^{-1/4} \left( \frac{m_{\varphi}}{10^4 \text{ GeV}} \right)^{1/6} \left( \frac{f}{0.1} \right).
\]
Similarly, if the decay proceeds through nonperturbative particle production such as preheating and/or thermal dissipation, the reheating temperature will be high. If the reheating temperature is high, the resultant baryon asymmetry tends to be large, which would require late-time entropy production. Note that the dependence of the final baryon asymmetry on $|c_1|$ could be involved, because, for small values of $c_1$, nonperturbative effects are expected to be more efficient, whereas the baryon-number violating operator is suppressed. In any case, it will be possible to generate the right amount of the baryon asymmetry for sufficiently small values of $c_1$ since no baryon asymmetry is generated for $c_1 = 0$.

V. CONCLUSIONS

We have studied a possibility that one of the MSSM flat directions plays the role of the inflaton and realizes the large-field inflation, indicated by the recent detection of the primordial $B$-mode polarization by BICEP2. To this end we have imposed a generalized shift symmetry on the flat direction, which enables us to control the inflaton potential over super-Planckian field values. The flat direction inflation is realized in the context of the running kinetic inflation, where the kinetic term gets larger as the field amplitude increases. It renders the potential effectively flatter to be consistent with the CMB observations. We have found that the inflaton could be the flat directions such as $LLe$, $LLddd$, or $QuQue$, which are lifted by the nonrenormalizable superpotential of the form $W = X \hat{\phi}^m$. A complete list of the possible flat directions as well as the predicted values of $n_s$ and $r$ is given in the Appendix.

The high-scale inflation, in general, predicts large baryonic/CDM isocurvature fluctuations for those mechanisms of baryogenesis and/or dark matter creation by light scalar fields. Our model, however, does not generate any baryonic isocurvature fluctuations, since the orthogonal direction to the inflationary trajectory becomes heavy during inflation.

The helical motion of the flat direction after inflation automatically takes place due to the same $\phi$-number breaking terms in the Kähler potential, which are allowed by the generalized shift symmetry. When the soft SUSY breaking mass term dominates the potential, the field naturally transforms into $Q$ balls. Thus the reheating proceeds through the $Q$-ball decay, and the reheating temperature would be rather low. Importantly, it is straightforward to reheat the standard model particles since the inflaton is one of the MSSM flat directions. We have found that a sufficient amount of the baryon asymmetry can be created. For unsuppressed $\phi$-number violating operators, the resultant baryon number of the universe is much larger than the observed value, but it would be a remedy of baryogenesis for those scenarios that need late-time entropy production. Alternatively, the baryon asymmetry can be suppressed for a sufficiently small $|c_1|$.

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Appendix A: Flat directions for the running kinetic inflation

Here we show the complete list of the flat directions which can be the candidate for the inflaton in our model. In Table II we display the gauge-invariant (GI) monomials $(\phi^{n'})$ representing the corresponding flat directions, the nonrenormalizable superpotentials of the form of $X \phi^n$, which lift all the degrees of freedom in that direction, and the powers of the field $\hat{\phi} (= \phi^n)$ of the effective potential $V \sim \hat{\phi}^p$ in the cases for $n = n'$ and $n = 2n'$. For both cases, the predicted values of the scalar spectral index $n_s$ and the tensor-to-scalar ratio $r$ are shown in Table II together with $\lambda$ in Eq.(7), which is necessary for the Planck normalization on the curvature perturbations. See Eqs.(11) - (13).

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[3] D. Kazanas, Astrophys. J. 241, L59 (1980).

For perturbative or nonperturbative decay, the relativistic degrees of freedom $g_*$ might be altered due to the large effective mass of the particles coupled to the inflaton.
| Direction | GI monomials ($\phi(n')$) | $W_{NR} = X\phi^n$ | $p = 2m/n'$ | $p = 2m/2n'$ |
|-----------|--------------------------|-----------------------|--------------|--------------|
| $L, d$    | LLddd ($n' = 5$)         | $H_dLLddd$ ($m = 6$)  | $12/5 = 2.4$ | $12/10 = 1.2$|
| $Q, u$    | QQQQu ($n' = 5$)         | $\{ \begin{array}{ll} \QQQL (m = 3) \\ \QuQd (m = 3) \end{array}$ | $6/5 = 1.2$ | $6/10 = 0.6$ |
| $Q, u, e$ | QuQue ($n' = 5$)         | $H_dQuQue$ ($m = 8$)  | $16/5 = 3.2$ | $16/10 = 1.6$|
| $L, u, d$ | $\{ udd (n' = 3) \\ LLddd (n' = 5) \} | uddQdL (m = 5) | $10/3 = 3.33$ | $10/6 = 1.67$ |
| $L, d, e$ | $\{ LLe (n' = 3) \\ LLddd (n' = 5) \} | H_dLddd (m = 4) | $8/3 = 2.67$ | $8/6 = 1.33$ |
| $L, u, e$ | $\{ LLe (n' = 3) \\ uuuue (n' = 5) \} | QuLe (m = 3) | $6/3 = 2$ | $6/6 = 1$ |
| $Q, L, e$ | $\{ LLLe (n' = 3) \\ QQQL (n' = 5) \} | QuLe (m = 3) | $6/4 = 1.5$ | $6/8 = 0.75$ |
| $Q, L, d$ | $\{ QLd (n' = 3) \\ dddLL (n' = 5) \} | QQdL (m = 3) | $6/3 = 2$ | $6/6 = 1$ |
| $Q, L, u$ | $\{ QQQL (n' = 4) \\ QQQu (n' = 5) \} | QuLe (m = 3) | $6/4 = 1.5$ | $6/8 = 0.75$ |
| $Q, u, d$ | $\{ udd (n' = 3) \\ uved (n' = 4) \} | QQQL (m = 3) | $6/4 = 1.5$ | $6/8 = 0.75$ |
| $L, u, d, e$ | $\{ udd (n' = 3) \\ LLe (n' = 3) \} | QuLe (m = 3) | $6/3 = 2$ | $6/6 = 1$ |
| $Q, L, d, e$ | $\{ QLd (n' = 3) \\ dddLL (n' = 5) \} | QuLe (m = 3) | $6/4 = 1.5$ | $6/8 = 0.75$ |
| $Q, L, u, e$ | $\{ QQQL (n' = 4) \\ QQQu (n' = 5) \} | QuLe (m = 3) | $6/4 = 1.5$ | $6/8 = 0.75$ |
| $Q, Q, d, e$ | $\{ udd (n' = 3) \\ QuQd (n' = 4) \} | QQQL (m = 3) | $6/5 = 1.2$ | $6/10 = 0.6$ |
| $L, u, d, e$ | $\{ udd (n' = 3) \\ uved (n' = 4) \} | QuLe (m = 3) | $6/3 = 2$ | $6/6 = 1$ |
| $Q, L, u, e$ | $\{ QQQL (n' = 4) \\ QQQu (n' = 5) \} | QuLe (m = 3) | $6/4 = 1.5$ | $6/8 = 0.75$ |
| $Q, u, d, e$ | $\{ QuQd (n' = 4) \\ uved (n' = 4) \} | QQQL (m = 3) | $6/5 = 1.2$ | $6/10 = 0.6$ |
| $Q, L, u, d$ | $\{ QLd (n' = 3) \\ dddLL (n' = 5) \} | QuLe (m = 3) | $6/3 = 2$ | $6/6 = 1$ |
| $Q, Q, d, e$ | $\{ udd (n' = 3) \\ QuQd (n' = 4) \} | QQQL (m = 3) | $6/5 = 1.2$ | $6/10 = 0.6$ |
| $Q, L, u, e$ | $\{ QQQL (n' = 4) \\ QQQu (n' = 5) \} | QuLe (m = 3) | $6/4 = 1.5$ | $6/8 = 0.75$ |
| $Q, u, d, e$ | $\{ QLd (n' = 3) \\ dddLL (n' = 5) \} | QuLe (m = 3) | $6/3 = 2$ | $6/6 = 1$ |
| $Q, L, u, d$ | $\{ QQQL (n' = 4) \\ QQQu (n' = 5) \} | QuLe (m = 3) | $6/4 = 1.5$ | $6/8 = 0.75$ |
| $Q, u, d, e$ | $\{ udd (n' = 3) \\ QuQd (n' = 4) \} | QQQL (m = 3) | $6/5 = 1.2$ | $6/10 = 0.6$ |
| $Q, L, u, d$ | $\{ QLd (n' = 3) \\ dddLL (n' = 5) \} | QuLe (m = 3) | $6/3 = 2$ | $6/6 = 1$ |
TABLE III: Scalar spectral index $n_s$, tensor-to-scalar ratio $r$, and the coupling $\lambda$.

| $m$ | $n = n'$ | $N$ | $\lambda$       | $n_s$   | $r$    | $m$ | $n = 2n'$ | $N$ | $\lambda$       | $n_s$   | $r$    |
|-----|----------|-----|----------------|--------|--------|-----|----------|-----|----------------|--------|--------|
| 3   | 3        | 40  | $1.16 \times 10^{-5}$ | 0.950  | 0.200  | 3   | 6        | 40  | $2.16 \times 10^{-5}$ | 0.963  | 0.100  |
|     |          | 50  | $9.26 \times 10^{-6}$ | 0.960  | 0.160  |     |          | 50  | $2.46 \times 10^{-6}$ | 0.970  | 0.080  |
|     |          | 60  | $7.72 \times 10^{-6}$ | 0.967  | 0.133  |     |          | 60  | $2.15 \times 10^{-5}$ | 0.975  | 0.067  |
| 3   | 4        | 40  | $1.93 \times 10^{-5}$ | 0.956  | 0.150  | 3   | 8        | 40  | $3.35 \times 10^{-5}$ | 0.966  | 0.075  |
|     |          | 50  | $1.59 \times 10^{-5}$ | 0.965  | 0.120  |     |          | 50  | $2.87 \times 10^{-5}$ | 0.973  | 0.060  |
|     |          | 60  | $1.35 \times 10^{-5}$ | 0.971  | 0.100  |     |          | 60  | $2.54 \times 10^{-5}$ | 0.977  | 0.050  |
| 3   | 5        | 40  | $2.51 \times 10^{-5}$ | 0.960  | 0.120  | 3   | 10       | 40  | $3.52 \times 10^{-5}$ | 0.968  | 0.060  |
|     |          | 50  | $2.10 \times 10^{-5}$ | 0.968  | 0.096  |     |          | 50  | $3.05 \times 10^{-5}$ | 0.974  | 0.048  |
|     |          | 60  | $1.82 \times 10^{-5}$ | 0.973  | 0.080  |     |          | 60  | $2.71 \times 10^{-5}$ | 0.978  | 0.040  |
| 3   | 7        | 40  | $3.18 \times 10^{-5}$ | 0.964  | 0.086  | 3   | 14       | 40  | $3.53 \times 10^{-5}$ | 0.970  | 0.043  |
|     |          | 50  | $2.71 \times 10^{-5}$ | 0.9771 | 0.069  |     |          | 50  | $3.09 \times 10^{-5}$ | 0.9776 | 0.034  |
|     |          | 60  | $2.38 \times 10^{-5}$ | 0.976  | 0.057  |     |          | 60  | $2.76 \times 10^{-5}$ | 0.980  | 0.029  |
| 4   | 3        | 40  | $5.32 \times 10^{-6}$ | 0.942  | 0.267  | 4   | 6        | 40  | $2.25 \times 10^{-5}$ | 0.958  | 0.133  |
|     |          | 50  | $4.10 \times 10^{-6}$ | 0.953  | 0.213  |     |          | 50  | $1.86 \times 10^{-5}$ | 0.967  | 0.107  |
|     |          | 60  | $3.31 \times 10^{-6}$ | 0.961  | 0.178  |     |          | 60  | $1.60 \times 10^{-5}$ | 0.961  | 0.178  |
| 4   | 5        | 40  | $1.75 \times 10^{-5}$ | 0.955  | 0.160  | 4   | 10       | 40  | $3.27 \times 10^{-5}$ | 0.965  | 0.080  |
|     |          | 50  | $1.44 \times 10^{-5}$ | 0.964  | 0.128  |     |          | 50  | $2.80 \times 10^{-5}$ | 0.972  | 0.064  |
|     |          | 60  | $1.22 \times 10^{-5}$ | 0.970  | 0.107  |     |          | 60  | $2.47 \times 10^{-5}$ | 0.977  | 0.053  |
| 5   | 3        | 40  | $2.27 \times 10^{-6}$ | 0.933  | 0.333  | 5   | 6        | 40  | $1.64 \times 10^{-5}$ | 0.954  | 0.167  |
|     |          | 50  | $1.68 \times 10^{-6}$ | 0.947  | 0.267  |     |          | 50  | $1.34 \times 10^{-5}$ | 0.963  | 0.133  |
|     |          | 60  | $1.32 \times 10^{-6}$ | 0.956  | 0.222  |     |          | 60  | $1.13 \times 10^{-6}$ | 0.970  | 0.111  |
| 5   | 5        | 40  | $1.16 \times 10^{-5}$ | 0.950  | 0.200  | 5   | 10       | 40  | $2.91 \times 10^{-5}$ | 0.963  | 0.100  |
|     |          | 50  | $9.26 \times 10^{-6}$ | 0.960  | 0.160  |     |          | 50  | $2.46 \times 10^{-5}$ | 0.970  | 0.080  |
|     |          | 60  | $7.72 \times 10^{-6}$ | 0.967  | 0.133  |     |          | 60  | $2.15 \times 10^{-5}$ | 0.975  | 0.067  |
| 6   | 5        | 40  | $7.33 \times 10^{-6}$ | 0.945  | 0.240  | 6   | 10       | 40  | $2.51 \times 10^{-5}$ | 0.960  | 0.120  |
|     |          | 50  | $5.74 \times 10^{-6}$ | 0.956  | 0.192  |     |          | 50  | $2.10 \times 10^{-5}$ | 0.968  | 0.096  |
|     |          | 60  | $4.70 \times 10^{-6}$ | 0.963  | 0.160  |     |          | 60  | $1.82 \times 10^{-5}$ | 0.973  | 0.080  |
| 8   | 5        | 40  | $2.70 \times 10^{-6}$ | 0.935  | 0.320  | 8   | 10       | 40  | $1.75 \times 10^{-5}$ | 0.955  | 0.160  |
|     |          | 50  | $2.02 \times 10^{-6}$ | 0.948  | 0.256  |     |          | 50  | $1.44 \times 10^{-5}$ | 0.964  | 0.128  |
|     |          | 60  | $1.59 \times 10^{-6}$ | 0.957  | 0.213  |     |          | 60  | $1.22 \times 10^{-5}$ | 0.970  | 0.107  |