LOCAL SIMULATIONS OF THE MAGNETOROTATIONAL INSTABILITY IN CORE-COLLAPSE SUPERNOVAE

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ABSTRACT

Bearing in mind the application of core-collapse supernovae, we study the nonlinear properties of the magnetorotational instability (MRI) by means of three-dimensional simulations in the framework of a local shearing box approximation. By systematically changing the shear rates that symbolize the degree of differential rotation in nascent proto-neutron stars (PNSs), we derive a scaling relation between the turbulent stress sustained by the MRI and the shear-vorticity ratio. Our parametric survey shows a power-law scaling between the turbulent stress ($\langle w_{\text{tot}} \rangle$) and the shear-vorticity ratio ($g_q$) as $\langle w_{\text{tot}} \rangle \propto g_q^{0.36}$ with an index of $\delta \sim 0.5$. The MRI-amplified magnetic energy has a similar scaling relative to the turbulent stress, while the Maxwell stress has a slightly smaller power-law index ($\sim 0.36$). By modeling the effect of viscous heating rates from MRI turbulence, we show that the stronger magnetic fields, or the larger shear rates initially imposed, lead to higher dissipation rates. For a rapidly rotating PNS with a spin period in milliseconds and with strong magnetic fields of $10^{15} \text{ G}$, the energy dissipation rate is estimated to exceed $10^{51} \text{ erg s}^{-1}$. Our results suggest that the conventional magnetohydrodynamic (MHD) mechanism of core-collapse supernovae is likely to be affected by MRI-driven turbulence, which we speculate, on the one hand, could harm the MHD-driven explosions due to the dissipation of the shear rotational energy at the PNS surface; or, on the other hand, its energy deposition might be potentially favorable for the working of the neutrino-heating mechanism.

Key words: instabilities – magnetic fields – magnetohydrodynamics (MHD) – supernovae: general – turbulence

Online-only material: color figures

1. INTRODUCTION

Numerical simulations of magnetohydrodynamic (MHD) stellar explosions already started in the early 1970s shortly after the discovery of pulsars (LeBlanc & Wilson 1970; Bisnovatyi-Kogan et al. 1976; Müller & Hillebrandt 1979; Symbalisty 1984). However, it is only recently that MHD studies have returned to the forefront of supernova research after a number of extensive MHD simulations (e.g., Ardeljan et al. 2000; Yamada & Sawai 2004; Kotake et al. 2004a, 2004b; Obergaulinger et al. 2006a, 2006b; Burrows et al. 2007; Cerda-Durán et al. 2007; Takiwaki et al. 2009; Scheidegger et al. 2010; Takiwaki & Kotake 2011; Obergaulinger & Janka 2011; Kotake et al. 2006, 2012a, 2012b for recent reviews). The main reasons for this refocus are observations indicating very asymmetric explosions (Wang et al. 2001, 2002), and the interpretation of magnetars (Duncan & Thompson 1992; Lattimer & Prakash 2007) and gamma-ray bursts (e.g., Woosley & Heger 2006; Yoon & Langer 2005) as a possible outcome of the magnetorotational core-collapse of massive stars.

The MHD mechanism of stellar explosions relies on the extraction of rotational-free energy from the collapsing progenitor core via magnetic fields. Hence, a high angular momentum of the core is preconditioned for facilitating the mechanism (Meier et al. 1976). Given the (rapid) rotation of the pre-collapse core, there are at least two ways to amplify the initial magnetic fields to a dynamically important strength, namely, with field wrapping by means of differential rotation that naturally develops in the collapsing core, and by the so-called magnetorotational instability (MRI; see Balbus & Hawley 1998). Akiyama et al. (2003) were the first to point out that the interfaces surrounding the nascent proto-neutron stars (PNSs) generally satisfy the instability criteria for the MRI. Therefore, any seed magnetic fields can be amplified exponentially in the differentially rotating layers, much faster than the linear amplification due to field wrapping. After the MRI enters the saturated state, the field strength might reach $\sim 10^{15}-16 \text{ G}$, which is high enough to affect supernova dynamics. The MRI not only amplifies the magnetic fields, but also plays a crucial role in operating the MHD turbulence (see Hawley et al. 1995; Balbus & Hawley 1998; Masada et al. 2006). The turbulent viscosity sustained by the MRI can convert a fraction of the shear rotational energy to the thermal energy of the system. Thompson et al. (2005) suggested that the additional energy input from turbulent viscous heating can help the neutrino-driven supernova explosion. Followed by the exponential field amplification and the additional heating, a natural outcome of the magnetorotational core-collapse may be the formation of energetic bipolar explosions, which might be observed as so-called hypernovae (see Tanaka et al. 2009 and references therein).

Here it is noted that bipolar explosions obtained in the previous MHD supernova simulations mentioned above are not driven by the MRI, but predominantly by field wrapping, assuming very strong pre-collapse magnetic fields ($\gtrsim 10^{14} \text{ G}$) in general. The growth rate of MRI-unstable modes depends on the product of the initial field strength and the wavenumber of the mode. In the case of the canonical initial fields ($\sim 10^{9} \text{ G}$), as predicted by recent stellar evolution models (Heger et al. 2005), the fastest growing modes are estimated to be at most a few meters in the collapsing iron core (Obergaulinger et al. 2009). Unfortunately, however, it is still computationally too expensive to resolve those scales in global MHD simulations, which are typically more than two orders of magnitude smaller than the
typical finest grid size. To reveal the nature of the MRI, local simulations focusing on a small part of the MRI-unstable region are expected to be quite useful as traditionally studied in the context of accretion disks (see Balbus & Hawley 1998).

Obergaulinger et al. (2009) were the first to report their numerical simulations of the linear growth and nonlinear properties of the MRI in the supernova environment. To ease a drawback of the local shearing box simulation, they employed the shearing disk boundary conditions by which the global radial density in the vicinity of the equatorial region in the supernova core can be taken into account. By performing such a *semi-global* simulation systematically in two and three dimensions, they derived scaling laws for the termination of the linear growth of the MRI. As estimated in Akiyama et al. (2003), the MRI was shown to amplify the seed fields exceeding $10^{15}$ G. These important findings create several questions that motivate us to join in this effort, such as how nonlinear properties and scaling laws could be changed by other parameters yet are unexplored in the supernova context (which we will explain in the next paragraph), and whether the viscous heating maintained by the MRI turbulence could or could not affect the supernova mechanism.

A fundamental and long-lasting issue regarding the MRI itself is understanding the features of the MRI in the nonlinear phase and to specify which physical quantities determine the saturation levels of the MRI. So far, extensive efforts have been made to measure the parameter dependence of the MRI-sustained turbulence by means of numerical simulations. It was reported that the amplitude of the turbulent stress maintained by the MRI depends on the net value of the initial magnetic field and the gas pressure of the system; that is,

$$w_{\text{tot}} \propto B^\xi p^\zeta, \quad (1)$$

where $\xi$ and $\zeta$ are the power-law indices (see also Blackman et al. 2008 for the scaling relation between the turbulent stress and the plasma beta). Hawley et al. (1995) found that the saturation amplitude depends on the field strength when the system is penetrated initially by a uniform vertical field ($w_{\text{tot}} \propto B$), but is independent of the initial field strength if there is no net magnetic flux in the system (see also Sano et al. 2004). However, in the density stratified system, it was recently suggested that turbulent stress increases almost linearly with the magnetic energy of the net vertical field ($w_{\text{tot}} \propto B^2$; Suzuki & Inutsuka 2009; Okuzumi & Hirose 2011). A weak dependence of the gas pressure on the MRI turbulence was found by Sano et al. (2004): $\zeta \simeq 1/4$ for models with an initial net-zero magnetic flux and $\zeta \simeq 1/6$ in the system penetrated by a uniform magnetic field. The physical mechanisms responsible for these parameter dependences remain an issue under considerable debate.

In the stellar interior condition, such as in the supernova core, it is important to understand how the degree of differential rotation of the PNS could affect the nonlinear properties of the MRI. This is because the shear rate at the PNS surface ($q = -d \ln \Omega / d \ln r$ with $\Omega$ and $r$ representing the angular velocity and radius, respectively) is time and location dependent unlike accretion disks in which a force balance between the centrifugal force and gravity is generally maintained.

In this work, we study MRI-driven turbulence and its nonlinear properties by performing three-dimensional (3D) simulations in the framework of a local shearing box approximation. By systematically changing the shear rates that symbolize the degree of differential rotation in nascent PNSs, we specifically study how the nonlinear properties of the MRI-driven turbulence respond to it. Based on our numerical results, we estimate the energy deposition rate due to the viscous heating driven by the MRI and discuss its potential impacts on the supernova mechanism.

This paper opens with descriptions of the numerical methods and the initial settings in Section 2. The numerical results are presented in Section 3. Based on the numerical results, we then move on to address possible impacts of the MRI on the supernova mechanism in Section 4 followed by discussions in Section 5. We summarize our new findings in Section 6.

### 2. NUMERICAL METHODS AND INITIAL SETTINGS

In order to study the nonlinear properties of the MRI, we solve MHD equations with an MPI-parallelized finite-difference code that was originally developed by Sano et al. (1998). The hydrodynamic part is based on the second-order Godunov scheme (van Leer 1979), which consists of Lagrangian and remap steps. The exact Riemann solver is modified to account for the effect of tangential magnetic fields. The field evolution is calculated with a consistent MoC-CT method and can thus avoid the numerical explosive instability that appeared in the strong shear layer when the MoC method was adopted (Clarke 1996). The energy equation is solved in the conservative form. The advantages of our scheme are its robustness for strong shocks and the satisfaction of the divergence-free constraint of magnetic fields (Evans & Hawley 1988; Stone & Norman 1992).

We perform a series of 3D compressible MHD simulations by adopting the local shearing box model described in detail by Hawley et al. (1995). In the shearing box model, MHD equations are written in a local Cartesian frame of reference $(x, y, z)$, corotating with the local portion of the stellar interior, which rotates at the angular velocity $\Omega$ corresponding to a fiducial radius in the cylindrical coordinate $R$. Then the coordinates are presented as $x = r - R$, $y = R\phi - \Omega t$, and $z$. The fundamental equations are written in terms of these coordinates within a small region surrounding the fiducial radius, in $\Delta r \ll R$,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (2)$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla \left( P + \frac{|\mathbf{B}|^2}{8\pi} \right) + \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi \rho} - 2\Omega \times \mathbf{v} - 2q\Omega^2 x \mathbf{i}, \quad (3)$$

$$\frac{\partial \varepsilon}{\partial t} + (\mathbf{v} \cdot \nabla) \varepsilon = -\frac{P}{\rho} \nabla \cdot \varepsilon + \Phi, \quad (4)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \quad (5)$$

where $\varepsilon$ is the specific internal energy, and the other parameters have their usual meanings. The term $-2q\Omega^2 x$ in the momentum equation is the tidal expansion of the effective potential with a shear rate $q = -d \ln \Omega / d \ln r$. Assuming the ideal gas, the pressure is given by $P = (\gamma - 1) \varepsilon \rho$. A constant ratio for specific heat $\gamma$ is considered for simplicity. We choose adiabatic gas with $\gamma = 5/3$ in this paper.

Since we focus on the local properties of the instability, we employ, as is described schematically in Figure 1, a numerical grid representing a small portion of the convectively stable upper PNS below the neutrinosphere where the strong shear...
is naturally developed when the core collapses (Akiyama et al. 2003; Thompson et al. 2005). Adopting the differentially rotating matter as an unperturbed state, the azimuthal velocity is given by $v_\phi = q \Omega x$ in the frame corotating with the velocity $R \Omega$. The radial force balance at the initial state is thus achieved between the Coriolis force and the tidal force, which is the residual of the gravitational force that primarily balances with the background pressure gradient force. We discuss the applicability of our numerical results on the supernova environment and the effects of neutrino viscosity in Section 5.

We choose normalization to be $\rho_0 = 1$, $\Omega = 10^{-3}$, and the computational domain has a radial size $L_x = 4$, an azimuthal size $L_\phi = 4$, and a vertical size $L_z = 1$. All of the runs use a uniform grid of $256 \times 256 \times 64$ zones. The initial field geometry is a uniform vertical magnetic field $B = B_0 e_z$ (net non-zero flux). We assume, for all the models, that the initial gas pressure is $P_0 = 5 \times 10^{-7}$ and the initial ratio of the gas and magnetic pressures, that is the plasma beta, is $\beta_0 = 3200$. The pressure scale height is then $H_p = 1$ and is the same as the vertical box size. With this normalization, the initial field strength is $B_0 = 6.26 \times 10^{-5}$, yielding $v_{A0} = 1.77 \times 10^{-5}$, where $v_{A0} = B_0 / (4\pi \rho_0)^{1/2}$ is the Alfvén speed. Note that these non-dimensional parameters can be translated to a dimensional form suitable for supernova environments, such as $\rho = 10^{12}$ g cm$^{-3}$, $L = 10^2$ cm, $\Omega = 10^2$ rad s$^{-1}$, $P = 5 \times 10^{25}$ dyne cm$^{-2}$, and $B = 2.2 \times 10^{12}$ G, which are summarized in Figure 1.

The initial characteristic wavelength of the MRI is given as $\lambda_{MRI} = 4\pi \eta v_{A0} / \Omega$ for an ideal MHD case (Balbus & Hawley 1998), where $\eta \equiv 1/\sqrt{4q - q^2}$. Its ratio to the vertical box size thus varies because $\lambda_{MRI} / L_z = 0.22\eta$ in our models. It is empirically known that, to gain a saturation level correctly, the MRI wavelength must be resolved at the saturated state by at least six grid zones (see Sano et al. 2004). Here, we choose the initial setting that satisfies this empirical rule for all the models.

3. NUMERICAL RESULTS

3.1. A Fiducial Run

As a fiducial run, we examine temporal evolutions of the MRI in a model with a shear rate of $q = 0.5$. Initial perturbations are introduced by giving random velocity perturbations, which are taken to have a zero mean value with a maximum amplitude of $|\delta u| / \sqrt{\gamma P_0 / \rho_0} = 5 \times 10^{-3}$.

To characterize the properties of the MRI-driven turbulence, we pay particular attention to the magnetic energy, the Maxwell and Reynolds stresses, and total turbulent stress defined, respectively, by

$$E_{\text{mag}} \equiv |B|^2 / 8\pi,$$

$$w_M \equiv -B_\phi B_\phi / 4\pi,$$

$$w_R \equiv \rho v_\phi \delta v_\phi,$$

$$w_{\text{tot}} \equiv w_M + w_R,$$

where $\delta v_\phi$ is the perturbed azimuthal velocity. Note that the volume average of these quantities is represented by single brackets (like $\langle E_{\text{mag}} \rangle$) and a time and volume average is represented by double brackets (like $\langle \langle E_{\text{mag}} \rangle \rangle$).

The thick, dash-dotted, and dashed curves in Figure 2 show the temporal evolution of volume-averaged magnetic energy, and Maxwell and Reynolds stresses normalized by an initial gas pressure of $P_0$. Note that the horizontal axis is normalized by the rotation period $t_{\text{rot}} \equiv 2\pi / \Omega = 6.28 \times 10^3$ (62.8 [ms] in the dimensional form). In order to demonstrate the typical evolution of the MRI, four time snapshots of the 3D structure of the radial magnetic field are visualized by means of the volume rendering method in Figure 3. The color bar indicates the amplitude of the radial magnetic field—yellow for the positive values, and blue for the negative value. Panels (a)–(d) correspond to the snapshots at the times $t = 6t_{\text{rot}}, 8t_{\text{rot}}, 9t_{\text{rot}},$ and $11t_{\text{rot}}$, respectively.

As described in previous MRI studies of accretion disks with a Keplerian rotation of $q = 1.5$, there are three typical evolutionary stages observed in our fiducial run. These are (1) linear exponential growth stage, (2) transition stage, and (3) nonlinear turbulent stage (Hawley & Balbus 1992; Hawley et al. 1995). Each stage is denoted by white, dark gray, and light gray shaded regions in Figure 2.
In stage (1), the channel structure of the magnetic field exponentially evolves and inversely cascades to a larger spatial scale with small structures merging as the magnetic field is amplified. This occurs because the channel mode of the MRI is an exact solution even for the nonlinear MHD equation (Goodman & Xu 1994). The temporal evolution of channel structures in the radial magnetic field is shown in panel (a) of Figure 3.

Then, in stage (2), the channel structure of the magnetic field is disrupted via the parasitic instability and/or magnetic reconnection at the transition stage (Goodman & Xu 1994; Obergaulinger et al. 2009). The channel disruption induces a drastic phase shift from a coherent structure to a turbulent tangled structure of the field as illustrated by panels (b) and (c) of Figure 3. The magnetic energy stored in the amplified magnetic field is then converted to the system’s thermal energy.

Finally, in stage (3), the nonlinear turbulent state emerges after the channel disruption is maintained long enough by MRI-driven turbulence as is demonstrated in panel (d) of Figure 3.

In the shearing box model adopted by our study, such free energy is continuously injected from the radial boundaries. This is the reason why the turbulent stage powered by the MRI is maintained in our simulation. We discuss in Section 5 the applicability of our numerical model in comparison to the shearing disk model employed in Obergaulinger et al. (2009).

### 3.2. Shear Rate Dependence

In the stellar interior, such as the supernova cores, the rotation profile should be time- and location-dependent, unlike the typical accretion disk, which maintains a quasi-Keplerian rotation with a shear rate of \( q \approx 1.5 \). This occurs because the force balance is primarily achieved between the gravity and the pressure gradient force in the stellar interior. It should be important to study how the nonlinear properties of the MRI would respond to the change of the shear rate in order to estimate its impact on supernova dynamics. The shear rates used in our parametric survey are listed in Table 1 with a few diagnostic quantities, the time- and volume-averaged Maxwell stress, the Reynolds stress, magnetic energy, and the ratio of Maxwell and Reynolds stresses, calculated from the numerical data.

Figure 4 shows the temporal evolution of the turbulent stress for models with different shear rates. The gray, red, blue, orange, green, and purple curves indicate the models with \( q = 0.2, 0.5, 0.8, 1.2, 1.5, \) and 1.8, respectively. Note that the other physical parameters are the same as those used in the fiducial run. In the model, we found that both the linear growth rate and saturation amplitude of the turbulent stress increased as the shear rate increased. This trend can also be recognized in the spatial structures of magnetic fields in a saturated state.
Figure 4. Temporal evolution of volume-averaged total stress \( \langle w_{\text{tot}} \rangle \) normalized by the initial gas pressure \( P_0 \) for the models with different shear rates of \( q = 0.2 \) (gray), 0.5 (red), 0.8 (blue), 1.2 (orange), 1.5 (green), and 1.8 (purple), respectively.

(A color version of this figure is available in the online journal.)

The volume-rendered vertical magnetic field is visualized for the models with (a) \( q = 0.5 \) and (b) \( q = 1.5 \) in Figure 5. The time snapshot at \( t = 100t_{\text{rot}} \) was chosen for both models. The red corresponds to the positive value of the vertical field component and the blue is its negative value. The larger structure of the magnetic field is generated for the model with the larger shear rate. The other components of the magnetic field have similar nonlinear properties.

Figure 6(a) exhibits the time- and volume-averaged Maxwell stress \( \langle E_{\text{mag}} \rangle \) (red circles), Reynolds stress \( \langle w_R \rangle \) (blue squares), total stress \( \langle w_{\text{tot}} \rangle \) (green diamonds), and magnetic energy \( \langle E_{\text{mag}} \rangle \) (yellow triangles) as a function of parameter \( \beta_q \), where \( \beta_q \equiv q/(2 - q) \) is the ratio of vorticity \( 2 - q \Omega \) to the shear \( q \Omega \) (hereafter called the “shear-vorticity ratio,” see Abramowicz et al. 1996). Note that at the saturated state, we take the temporal average of volume-averaged stresses and magnetic energy from \( 50t_{\text{rot}} \) to \( 150t_{\text{rot}} \) for models 1–3 and from \( 100t_{\text{rot}} \) to \( 150t_{\text{rot}} \) for models 4–7. The black line, which is proportional to \( \beta_q^{1/2} \), is plotted as a reference.

The Maxwell stress dominates the Reynolds stress for all the models we surveyed although the fraction of the Reynolds stress in the total stress increases with the shear-vorticity ratio \( \beta_q \). It should be noted that the pure hydrodynamic shear instability develops in the system with the stronger shear of \( q \geq 2 \) (the so-called Rayleigh criterion, e.g., Chandrasekhar 1960). According to Hawley et al. (1999), the hydrodynamic turbulence is shown to change the properties of MRI turbulence when the Rayleigh criterion is satisfied (see also Workman & Armitage 2008).

The Reynolds stress then exceeds the Maxwell stress. However, since \( q \lesssim 2 \) is generally satisfied in the vicinity of the PNSs (e.g., Kotake et al. 2004a; Obergaulinger et al. 2006a, 2009), the Maxwell stress is expected to dominate over the Reynolds

![Figure 5. Three-dimensional volume rendering of the radial magnetic field for the models with (a) \( q = 0.5 \) and (b) \( q = 1.5 \). The color bar indicates the amplitude of the radial magnetic field, that is, the red denotes the positive value and the blue is the negative.](image)

(A color version of this figure is available in the online journal.)

### Table 1

| Shear Rate \( q \) | \( \langle w_M \rangle \)/\( P_0 \) | \( \langle w_R \rangle \)/\( P_0 \) | \( \langle E_{\text{mag}} \rangle \)/\( P_0 \) | \( \langle w_M \rangle \)/\( \langle w_R \rangle \) |
|-------------------|-----------------|-----------------|-----------------|-----------------|
| Model 1           | 0.2             | \( 2.06 \times 10^{-2} \) | \( 1.96 \times 10^{-4} \) | \( 4.11 \times 10^{-2} \) | \( 1.05 \times 10^{2} \) |
| Model 2           | 0.5             | \( 2.86 \times 10^{-2} \) | \( 1.28 \times 10^{-3} \) | \( 6.57 \times 10^{-2} \) | \( 2.23 \times 10^{1} \) |
| Model 3           | 0.8             | \( 3.63 \times 10^{-2} \) | \( 3.09 \times 10^{-3} \) | \( 9.41 \times 10^{-2} \) | \( 1.18 \times 10^{1} \) |
| Model 4           | 1.0             | \( 3.99 \times 10^{-2} \) | \( 5.15 \times 10^{-3} \) | \( 1.09 \times 10^{-1} \) | \( 7.78 \times 10^{0} \) |
| Model 5           | 1.2             | \( 5.38 \times 10^{-2} \) | \( 8.49 \times 10^{-3} \) | \( 1.46 \times 10^{-1} \) | \( 6.33 \times 10^{0} \) |
| Model 6           | 1.5             | \( 6.64 \times 10^{-2} \) | \( 1.78 \times 10^{-2} \) | \( 1.94 \times 10^{-1} \) | \( 3.72 \times 10^{0} \) |
| Model 7           | 1.8             | \( 9.63 \times 10^{-2} \) | \( 4.27 \times 10^{-2} \) | \( 2.96 \times 10^{-1} \) | \( 2.26 \times 10^{0} \) |

**Notes.** The shear rates used in our parametric survey are listed in the first column. The second, third, and fourth columns show the time- and volume-averaged Maxwell stress \( \langle w_M \rangle \), Reynolds stress \( \langle w_R \rangle \), and magnetic energy \( \langle E_{\text{mag}} \rangle \) normalized by the initial gas pressure \( P_0 = 5 \times 10^{-7} \). The ratio of Maxwell and Reynolds stresses \( \langle w_M \rangle \)/\( \langle w_R \rangle \) is in the fifth column. The time average is taken over the saturated stage from \( 50t_{\text{rot}} \) to \( 150t_{\text{rot}} \) for models 1–3 and from \( 100t_{\text{rot}} \) to \( 150t_{\text{rot}} \) for models 4–7. Model 2 corresponds to the fiducial one.
stress and plays a major role in the turbulent heating process there.

A power-law relation between the shear-vorticity ratio and turbulent stress appears in this figure. The power-law index $\delta$ of a scaling relation defined by

$$\langle \langle \omega_{\text{tot}} \rangle \rangle \propto g_q^\delta \tag{11}$$

is about 1/2 and the best fit is $\delta = 0.44$ among all the models in Figure 6(a). It is interesting that the magnetic energy has a similar scaling relation to the turbulent stress, which is roughly $\langle \langle E_{\text{mag}} \rangle \rangle \propto g_q^{1/2}$ (the best fit is $\delta = 0.46$), although the Maxwell stress has a power-law index of 0.36 (the best-fit value), which is a bit smaller than the power-law index for the total stress. Since it is well known that the maximum growth rate of the MRI is linearly proportional to the shear rate $q$ (that is $\gamma_{\text{max}} = q\Omega/2$), the scaling law of the turbulent stress that we found here should be explained through the nonlinear properties of the MRI-driven turbulence.

### 3.3. Our Work versus Previous Studies

Thus far, the dependence of the nonlinear properties of the MRI on the shear rate has been investigated only in models whose net magnetic flux is set to zero (Brandenburg et al. 1996; Abramowicz et al. 1996; Hawley et al. 1999; Ziegler & Rüdiger 2001; Pessah & Chan 2008). These previous studies found that the turbulent stress had a stronger dependence on the shear-vorticity ratio $g_q$ (or rather shear rate $q$ itself), which is in contrast to the weak dependence ($\delta = 1/2$; see Equation (11)) obtained here in the case of the non-zero magnetic flux.

Abramowicz et al. (1996; hereafter ABL96) numerically evaluated the MRI-sustained turbulent stress for different values of the shear-vorticity ratio in the stratified system with zero net magnetic flux. They found that the turbulent stress is roughly represented by a linear dependence on the shear-vorticity ratio, that is, $\delta = 1.0$ of Equation (11). On the other hand, the numerical results obtained by Ziegler & Rüdiger (2001; hereafter ZR01), with similar numerical settings as was adopted in ABL96, suggested the existence of relation $\langle \langle \omega_{\text{tot}} \rangle \rangle \propto q$ (i.e., $\propto 2g_q/(g_q + 1)$) in contrast to the findings of ABL96.

Figure 6. (a) Volume- and time-averaged Maxwell (red circles), Reynolds (blue squares), and total stresses (green diamonds) as a function of the shear-vorticity ratio $g_q$. The yellow triangles represent time- and volume-averaged magnetic energy at the saturated state. (b) The ratio of Maxwell and Reynolds stresses for the models with different initial settings as a function of the shear-vorticity ratio $g_q$. The blue squares, green crosses, yellow triangles, and red circles demonstrate the results of our work, Hawley et al. (1999), Liljeström et al. (2009), and Ziegler & Rüdiger (2001), respectively. The dashed line traces the prediction from the local model developed by Pessah et al. (2006). The solid line gives a power-law relation of $\langle \langle \omega_{\text{tot}} \rangle \rangle /\langle \langle \omega_{\text{mag}} \rangle \rangle \propto g_q^{-3/4}$.

(A color version of this figure is available in the online journal.)

The dependence of the turbulent stress on the shear rate was studied comprehensively by Hawley et al. (1999; hereafter HBW99) using an unstratified shearing box model with net zero magnetic flux. They found that the vorticity $\langle 2 - q\Omega \rangle$ limits hydrodynamic turbulence and strongly reduces the Reynolds stress at around $q = 0$. In contrast, the shear $(q\Omega)$ promotes turbulence, and thus the Maxwell and Reynolds stresses both are enhanced by an increasing shear rate. Since the $q$-dependence of the Reynolds stress is much stronger than that of the Maxwell stress, the stress ratio diminishes with an increasing $q$. When extracting the data from Figure 10 of HBW99, the time- and volume-averaged turbulent stress seems to behave as $\langle \langle \omega_{\text{tot}} \rangle \rangle \propto g_q^{1/2}$ (the best fit is $\delta = 0.55$).

Liljeström et al. (2009; hereafter LKKBL09) studied the nonlinear properties of MRI-driven turbulence with a varying shear rate using the local shearing box model with net zero magnetic flux. By fitting the numerical data listed in Table 1 of LKKBL09, the turbulent stress has a strong $q$-dependence like ABL96 and behaves approximately as $\langle \langle \omega_{\text{tot}} \rangle \rangle \propto g_q$ (the best fit is $\delta = 0.9$).

A theoretical model for the saturation of the Maxwell and Reynolds stresses in MRI turbulence was developed by Pessah et al. (2006; hereafter PCP06) in light of the similarities exhibited by the linear regime of the MRI and the turbulent state. On the basis of linear theory, they formulated a predictor function on the ratio of the Maxwell and Reynolds stresses at the saturated state, that is $\langle \langle \omega_{\text{mag}} \rangle \rangle /\langle \langle \omega_{\text{tot}} \rangle \rangle = (2 + g_q)/g_q \equiv (4 - q)/q$ in the other form. LKKBL09 found that the relation for the stress ratio derived by PCP06 predicts similar behavior, but provides a ratio that is a few times smaller than what was computed from their numerical results. The local closure model developed by Ogilvie (2003), however, was able to reproduce the numerical results of LKKBL09 quite well.

For a comparison between our work and previous studies, the ratio of the time- and volume-averaged Maxwell and Reynolds stresses is plotted as a function of the shear-vorticity ratio $g_q$ in Figure 6(b). The blue squares correspond to our result, the green crosses show the numerical data of HBW99, the orange triangles are the results of LKKBL09, and the red circles present the results of ZR01. A power-law relation of
\langle \langle w_{\text{M}} \rangle \rangle / \langle \langle w_{\text{R}} \rangle \rangle \propto \epsilon^{3/4}$ is shown by the solid line for reference. The theoretical model of the stress ratio developed by PCP06 is traced by the dashed line. Remember that the previous studies, as was summarized above, provided different $g_q$-dependencies of the turbulent stress. The non-zero magnetic flux is imposed only in our work, though the shearing box approximation is adopted in all the models. The stress ratio of HBW99 is calculated by extracting the data from Figure 10 of HBW99.

In all the models, the stress ratio decreases as the shear-vorticity ratio $g_q$ increases. This suggests that the Reynolds stress generally has a stronger $g_q$-dependence than the Maxwell stress. As LKKBL09 discussed, the model stress ratio of PCP06 predicts similar behavior, but gives a value that is a few times smaller than what was obtained by all the numerical models, especially in the range of $0.2 \lesssim g_q \lesssim 10$. It is a bit surprising that the stress ratio is similar among all the works except HBW99, even though they predict different power-law relations between the turbulent stress and the shear-vorticity ratio. The stress ratio is roughly represented by a power-law relation $\langle \langle w_{\text{M}} \rangle \rangle / \langle \langle w_{\text{R}} \rangle \rangle \propto g_q^{5/3}$ in the range of $0.5 \lesssim q \lesssim 1.8$. The similar nonlinear characteristics that are behind the MRI-driven turbulent flows suggest that the stress ratio might be one of key parameters for validating the nonlinear saturation mechanism of the MRI.

4. IMPACT ON THE SUPERNOVA EXPLOSION

As in Thompson et al. (2005), the $\alpha$-description (Shakura & Sunyaev 1973) conventionally used in the context of accretion disks is the simplest way to model the turbulent heating. However, it is quite uncertain whether the prescription is really applicable to the supernova problem. Based on our numerical results, we hope to reexamine whether the turbulent heating due to the MRI could impact the explosion mechanism.

Combining our result with turbulent stress scaling relations described by Equation (1), the heating rate maintained by MRI turbulence can be expressed as

$$
\epsilon_{\text{MRI}} = \langle \langle w_{\text{M}} \rangle \rangle q \Omega / B_0 \propto \left( \frac{B}{B_0} \right)^\xi \left( \frac{P}{P_0} \right)^\zeta \left( \frac{g_q}{g_{q_0}} \right)^\delta \left( \frac{q}{q_0} \right)^\zeta \left( \frac{\Omega}{\Omega_0} \right). \tag{12}
$$

where $\epsilon_{\text{MRI}}$ represents the turbulent heating rate at the reference state with $B_0$, $P_0$, $g_{q_0}$, $q_0$, and $\Omega_0$. The power-law indices are positive values depending on the reference of the stratification or net magnetic flux and are expected to be $\xi = 1 \sim 2$ (Hawley et al. 1995; Sano et al. 2004; Suzuki & Inutsuka 2009; Okuzumi & Hirose 2011), $\zeta = 1/4 \sim 1/6$ (Sano et al. 2004), and $\delta = 1/2 \sim 1$ (see Section 3.3).

When choosing the physical parameters adopted in the fiducial model as the reference state, that is, $B_0 = 2.2 \times 10^{12}$ G, $P_0 = 5 \times 10^{25}$ erg cm$^{-3}$, $\Omega_0 = 10^2$ rad s$^{-1}$, $g_{q_0} = 0.33$, and $q = 0.5$, we can estimate, from Equation (10), the reference heating rate as $\epsilon_0 = 0.03 P_{0} g_{q_0} h \Omega \approx 10^{26}$ erg cm$^{-3}$ s$^{-1}$ (see Section 3.1 for the fiducial run).

We make a crude estimate of the volume in which the MRI-driven turbulence becomes active as $V_{\text{MRI}} = 4\pi R^2 h \approx 10^{21}$ cm$^3 R^2 h_6$, where $R_7 = R/10^7$ cm is the typical radius of the PNS normalized by $10^7$ cm, and $h_6 = h/10^6$ cm is the typical radial thickness of the MRI-active layer normalized by $10^6$ cm. Then, the energy releasing rate $L_{\text{MRI}} \equiv \epsilon_{\text{MRI}} V_{\text{MRI}}$ becomes

$$
L_{\text{MRI}} \approx 10^{47} R_7^2 h_6 \text{ erg s}^{-1} \times \left( \frac{B}{B_0} \right)^\xi \left( \frac{P}{P_0} \right)^\zeta \left( \frac{g_q}{g_{q_0}} \right)^\delta \left( \frac{q}{q_0} \right)^\zeta \left( \frac{\Omega}{\Omega_0} \right). \tag{13}
$$

Since we are interested in the MRI in the post-bounce stage of core-collapse supernovae, the gas pressure in the neutrino opaque upper PNSs is a known parameter and should be $P = 10^{31}$ erg cm$^{-3}$. When applying the typical gas pressure at the post-bounce phase to Equation (13) with the remaining parameters, the energy releasing rate is slightly enhanced and becomes $L_{\text{MRI}} \approx 10^{48}$ erg s$^{-1}$, almost independent of $\xi$. The magnetic field strength and the rotation profile are completely unknown parameters in the supernova environment. These thus determine whether the MRI-sustained turbulent heating can assist the supernova explosion or not.

The energy releasing rate due to the MRI-driven turbulence $L_{\text{MRI}}$ is mapped in Figure 7 as functions of the magnetic field strength $B$ and the shear rate $q$. Panels (a) and (b) show the models with the weakest parameter dependence of $\xi = 1$ and $\delta = 1/2$, while panels (c) and (d) are for the models with the strongest parameter dependence of $\xi = 2$ and $\delta = 1$. The slow rotation of $\Omega = \Omega_0$ is assumed in the left panels and the fast rotation of $\Omega = 10^4 \Omega_0$ is in the right panels. The green, blue, and red dashed curves correspond to the energy releasing rates $10^{50}$, $10^{51}$, and $10^{52}$ erg s$^{-1}$, respectively. The darker color gives the higher energy releasing rate. Note that the gas pressure is fixed at $10^{31}$ erg cm$^{-3}$ and the weak gas pressure dependence of the MRI-driven turbulence is neglected here for simplicity.

The dark shaded parameter space provides a larger energy releasing rate of about $10^{51}$ erg s$^{-1}$, which should assist the supernova explosion. The MRI-turbulent heating then has a minor effect on the supernova explosion in the unshaded parameter space, which provides the luminosity $L_{\text{MRI}} \ll 10^{51}$ erg s$^{-1}$.

It is fairly obvious that the stronger magnetic field or the larger shear rate yields a higher energy release due to the MRI, and thus makes a greater contribution to the supernova explosion. When considering the moderate shear of $q \approx 1$ and the angular velocity of $\Omega = O(10^3)$ rad s$^{-1}$, which are plausible for a supernova environment (Ott et al. 2006), a strong magnetic field of $B \gtrsim 10^{15}$ G is required for the MRI-assisted supernova explosion in the case with the weakest parameter dependence of $\xi = 1$ and $\delta = 1/2$ (panels (a) and (b)). A magnetic field weaker than $10^{15}$ G can make the MRI-driven turbulence strong enough to assist the explosion when the MRI turbulence depends strongly on the physical parameters $B$ and $q$ (see panels (c) and (d)).

The nonlinear MRI studies that employ the shearing box model seem to suggest that non-canonical post-bounce states, such as having a strong magnetic field and/or large spin rate, must be developed in order to prompt the MRI-assisted supernova explosion. It is tempting to add those phenomenological heating rates as a sub-grid model to the global MHD supernova simulations to see the outcomes. However, before that, it is more important to conduct a more extensive nonlinear study of the...
normalized by $10^{10}$ cm$^2$ s$^{-1}$ or use a more realistic numerical setting in order to precisely fix the scaling relations.

5. DISCUSSION

5.1. Effects of Neutrino Viscosity on the MRI

In the neutrino opaque upper PNSs, the area we examine in this paper, the neutrino viscosity was shown to suppress the growth of the MRI when the viscous dissipation timescale of a typical MRI mode becomes shorter than the typical evolution time of the MRI (see Masada et al. 2007; Masada & Sano 2008). This is equivalent to

$$R_{\text{MRI}} = \frac{v_A^2}{v \Omega} \lesssim 1,$$

where $R_{\text{MRI}}$ is the Reynolds number for the MRI, $v_A$ is the Alfvén velocity, $\Omega$ is the angular velocity, and $v$ is the neutrino viscosity. For the magnetic field, this can be translated as

$$B \lesssim B_{\text{crit}} \equiv (4\pi v \rho \Omega)^{1/2},$$

$$= 3.5 \times 10^{12} \rho_{12}^{1/2} \omega_{10}^{1/2} \nu_{10}^{1/2} \Omega_{4}^{1/2} \quad \text{(G)},$$

where $\rho_{12}$ is the density normalized by $10^{12}$ g cm$^{-3}$, $T_{11}$ is the temperature normalized by $10^{11}$ K, $\Omega_{4}$ is the angular velocity normalized by $10^{4}$ rad s$^{-1}$, and $\nu$ is the neutrino viscosity normalized by $10^{16}$ cm$^2$ s$^{-1}$ (see Masada et al. 2007 for the magnitude of the neutrino viscosity). Such an enormous neutrino viscosity is plausible in the region just below the neutrinosphere where the strong differential rotation is developed.

Equation (15) means that if the magnetic field is weaker than the critical value, even locally, the neutrino viscosity suppresses the linear growth of the MRI. Longaretti & Lesur (2010) found that, in the non-resistive limit (i.e., highly conducting MHD fluid), the saturation level of the MRI does not seem to be affected by the viscosity at the nonlinear stage despite the change of the linear growth rate. Their results seem to suggest that the effects of neutrino viscosity on MRI-driven turbulence are only secondary in the supernova problem.

On the one hand, when the magnetic field is weaker than the critical value $B_{\text{crit}}$, the linear growth of the MRI is suppressed, and the growth time of the fastest growing mode of the MRI $t_{\text{MRI}}$ becomes

$$t_{\text{MRI}} = \left[\frac{2(2-q)}{q^2}\right]^{1/4} R_{\text{MRI}}^{-1/2} \Omega^{-1} \quad \text{(s)},$$

(Masada & Sano 2008) in contrast to a case with a larger magnetic field than the critical value given by $t_{\text{MRI}} = 2/(q\Omega)$. Figure 8 depicts the growth time of the fastest growing mode of the MRI as a function of the shear rate for the cases where $B \geq B_{\text{crit}}$ (solid line), $B = 10^{-2} B_{\text{crit}}$ (dashed line), and $B = 10^{-4} B_{\text{crit}}$ (dash-dotted line). We adopt the spin rate of $\Omega = 100$ rad s$^{-1}$ in this figure.

Considering the spin rate and shear rate plausible for the nascent PNSs ($\Omega \sim O(10^2)$ rad s$^{-1}$ and $q \sim 1.0$), the growth time of the MRI becomes longer than 100 ms, which is only
marginal compared to the typical timescale of neutrino-driven explosions observed in recent supernova simulations (e.g., Marek & Janka 2009; Suwa et al. 2010; Takiwaki et al. 2001). We speculate that the activity of the MRI might be mainly driven by the shear rate and the angular momentum transport processes, such as the meridional circulation and the convective motion (e.g., Ruediger 1989), are considered to sustain the differential rotation even if the magnetic tension force acts to smear out the differential rotation. In the post-bounce supernova core, convectively stable regions, in which the neutrino cooling dominates over the neutrino heating, are formed between the nascent neutron star and the stalled bounce shock (Janka 2001). We speculate that the activity of the MRI might be maintained there in the presence of the secular differential rotation, which is close to the situation assumed in the shearing box simulation.

To draw a robust conclusion, one would naturally need to include the density stratification as well as the effects of neutrino heating/cooling, which determines the convective stability. This study is only a step toward improving our modeling, according to long to-do lists, to understand the role of MRI on the supernova mechanism.

5.3. Magnetic Field Structure during Saturation

All our numerical models yield a turbulent, highly tangled, magnetic field structure as the final saturated state. The turbulent flow and magnetic field persist during the saturation, and coherent channel structures appear only transiently. On the contrary, in Obergaulinger et al. (2009), large-scale coherent fields with efficient angular momentum transport emerge after the turbulent state and are maintained for some models that have a uniform initial magnetic field that uses simulation boxes with small radial and azimuthal aspect ratios $L_\phi/L_z$ and $L_\phi/L_x$.

The magnetic field structure during saturation is determined by whether flow-driven and current-driven parasitic instabilities, which are responsible for the destruction of channel solutions of the MRI, grow or not. Goodman & Xu (1994) analytically predicted that these parasitic modes require radial and azimuthal wavelengths larger than the vertical wavelength of the channel solution for being unstable (see also Latter et al. 2009; Pessah & Goodman 2009). The coherent channel structure emerging from the developed turbulence is thus expected to evolve into the large scale without destruction by the parasitic in the small simulation box.

Obergaulinger et al. (2009) numerically confirmed that small radial and azimuthal aspect ratios are required to maintain the large-scale structure of coherent magnetic fields at the saturated state (see Figures 23 and 24 in their paper). A stationary turbulent state with tangled magnetic fields even appears in the shearing disk model when the simulation box is large enough. The same trend regarding the magnetic field structure during saturation was also reported in the shearing box simulation by Bodo et al. (2008; see also Lesaffre et al. 2009).

As presented in Section 2, we chose the computational domain with large aspect ratios of $L_x/L_z = L_y/L_z = 4$ for all the models. The parasitic instabilities can thus evolve without being affected by the box geometry, which destroys coherent channels, yielding a continuously less violent turbulent state during the saturation. It should be stressed that, as studied by Sano & Inutsuka (2001), the recurrent formation of large-scale coherent fields can be observed in the saturated stage of our shearing box model when we reduce the simulation domain to that with small aspect ratios.

When the magnetorotational core collapses, the MRI-active region should be confined in the upper PNS with small radial and latitudinal extents of $\mathcal{O}(10^5 - 10^6)$ cm (see Section 4). The azimuthal thickness of the MRI-unstable region is expected to be significantly larger than the radial and latitudinal ones. This geometry would inhibit the development of the coherent structure of magnetic fields by prompting parasitic instabilities and lead persistently to a less violent MRI-driven turbulent state (cf., Obergaulinger et al. 2009). The turbulent, highly tangled magnetic field structure would be suitable for describing the MRI-active layer in the supernova cores.
6. SUMMARY

We performed a series of 3D compressible MHD simulations by adopting the local shearing box model. By systematically changing the magnitudes of the shear rate, we specifically studied how the nonlinear properties of the MRI-driven turbulence are controlled by the shear in the system. Applying our numerical results to the supernova environment, we examined the impact of the MRI on the supernova explosion mechanism. Our main findings are summarized as follows.

1. In our fiducial run with \( q = 0.5 \), we observed three typical evolutionary stages (1) the linear exponential growth stage, (2) the transition stage, and (3) the nonlinear turbulent stage), which are analogous to those found in previous works modeling accretion disks. This validates the theory that a variation in the shear rate does not change linear and nonlinear properties of the MRI qualitatively. The turbulent stress was saturated at the level of \( \langle (w_{tot}) \rangle \approx 0.03 P_0 \) in our fiducial model.

2. Our parameter survey resulted in a power-law relation between the shear-vorticity ratio and turbulent stress. The power-law index \( \delta = g_q \) is about \( 1/2 \). The MRI-amplified magnetic energy has a similar scaling relation to the turbulent stress, although the Maxwell stress has a power-law index of 0.36.

3. We found that the stress ratio, defined by \( \langle (u_M) \rangle/\langle (u_R) \rangle \), decreases as the shear-vorticity ratio increases. In addition, the stress ratio calculated from our numerical results has a similar magnitude and \( q \)-dependence as those obtained by previous works (LKKBL09; ZR01) despite the different computational settings. In the range \( 0.5 \lesssim q \lesssim 1.8 \), the stress ratio is roughly fitted by a power-law relation of \( \langle (u_M) \rangle/\langle (u_R) \rangle \propto g_q \). A stronger magnetic field or a larger shear rate provides a higher energy release due to the MRI-driven turbulence. For a rapidly rotating PNS with a spin period in milliseconds and with strong magnetic fields of \( 10^{15} \) G, the energy dissipation rate is estimated to exceed \( 10^{51} \) erg s\(^{-1} \). Our results suggest that the conventional MHD mechanisms of core-collapse supernovae are likely to be affected by MRI-driven turbulence, which we speculate, on the one hand, could ham MHD-driven explosions due to the dissipation of the shear rotational energy at the PNS surface; on the other hand, the energy deposition there might be potentially favorable for the working of the neutrino-heating mechanism.

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