Simulation of Bell states with incoherent thermal light

Hui Chen\(^1\), Tao Peng, Sanjit Karmakar and Yanhua Shih
Department of Physics, University of Maryland, Baltimore County, Baltimore, MD 21250, USA
E-mail: chenhui1@umbc.edu

*New Journal of Physics* 13 (2011) 083018 (12pp)
Received 19 January 2011
Published 12 August 2011
Online at [http://www.njp.org/](http://www.njp.org/)
doi:10.1088/1367-2630/13/8/083018

**Abstract.** This paper reports our experimental study of thermal light multi-photon qubits. Taking advantage of two-photon interference, we have successfully observed Bell-type correlation from mutually incoherent and orthogonally polarized thermal fields. The visibilities of the polarization correlation and the temporal anti-correlation both exceed 71\%, indicating the behavior of a two-photon Bell state or a two-digit qubit.

**Contents**

1. Introduction 1
2. The experiment 3
3. The theory 5
4. Summary 9
Acknowledgments 11
References 11

1. Introduction

One of the critical issues of quantum computing is the requirement for entangled states of a large number of particles, namely qubits. For instance, a qubit consists of coherent superposition among \(2^n\) binary bits of \(n\)-digits, representing all possible integers from 0 to \(2^n-1\):

\[
|\Psi\rangle = \frac{1}{\sqrt{2^n}} (|0, 0, \ldots, 0, 0\rangle + |0, 0, \ldots, 0, 1\rangle + |0, 0, \ldots, 1, 0\rangle + \cdots + |1, 1, \ldots, 1, 1\rangle).
\] (1)

\(^1\) Author to whom any correspondence should be addressed.
Although studies of entangled states have greatly advanced our understanding of multi-particle coherence, there are still issues in producing large numbers of multi-particle entangled states [1]. With regard to a large number of independent photons, thermal radiation is a promising source [2, 3] for quantum information processing if the thermal system could simulate the behavior of entangled states. In a series of studies, we found a mechanism for generating such multi-photon states. In this paper, we report one of our recent experiments on Bell-correlation measurement with incoherent thermal fields. This experiment demonstrated the working principle of thermal light multi-photon interference, which is important for constructing \( n \)- photon qubits.

The study of two-photon Einstein–Podolsky–Rosen (EPR)–Bohm–Bell states has greatly advanced our understanding of quantum entanglement [4–11]. Entangled photon pairs produced by spontaneous parametric down-conversion (SPDC) have been the most successful two-photon source for constructing Bell states experimentally since 1986 [9]:

\[
|\Psi_1\rangle = \frac{1}{\sqrt{2}}[(|x_1^R\rangle|y_2^R\rangle - |y_1^T\rangle|x_2^T\rangle)],
\]

which can be rewritten, in terms of qubits, as

\[
|\Psi\rangle = \frac{1}{\sqrt{2}}[(|0_1\rangle|1_2\rangle - |1_1\rangle|0_2\rangle)].
\]

Figure 1(a) shows a typical experimental setup. The interferometer introduces two different yet indistinguishable paths: (i) both signal and idler are reflected by the beamsplitter (BS) and (ii) both are transmitted. The superposition of these two amplitudes is usually expressed as the above Bell state.

The polarization correlation measurement on the Bell state gives the probability that a signal–idler photon pair triggers a joint photodetection event [13, 14]:

\[
G^{(2)}(\theta_1, \theta_2) = \langle \langle \Psi | \hat{E}^{(-)}(\theta_1) \hat{E}^{(-)}(\theta_2) \hat{E}^{(+)}(\theta_2) \hat{E}^{(+)}(\theta_1) |\Psi\rangle \rangle_{\text{Ensemble}} \propto \sin^2(\theta_1 - \theta_2),
\]

where \( \hat{E}^{(\pm)} \) is the positive or negative field operator. The biphoton state of SPDC is a pure state in which all measured signal–idler pairs are in the same state, and the ensemble average in equation (4) is trivial and ignorable. Choosing \( \theta_1 - \theta_2 = 0 \) or \( \theta_1 - \theta_2 = \pi/2 \), a two-photon anti-correlation ‘dip’ or correlation ‘peak’ is observable. In the language of EPR–Bohm, the two photons that produce a joint photodetection event may take any polarization before passing polarizers \( A_1 \) and \( A_2 \); however, if one of them is found to be polarized at a certain orientation, the other one must be polarized orthogonally to that orientation with certainty.

In fact, the Bell correlation is not restricted to entangled states. In a set of experiments that we carried out recently (some experiments are yet to be published), we have observed the polarization correlation of equation (4) as well as the temporal anti-correlation ‘dip’ and

\[ |\Psi^{(\pm)}\rangle = (1/\sqrt{2})[(|X_1\rangle|Y_2\rangle \pm |Y_1\rangle|X_2\rangle] \] and \[ |\Phi^{(\pm)}\rangle = (1/\sqrt{2})[(|X_1\rangle|X_2\rangle \pm |Y_1\rangle|Y_2\rangle] \], has been produced experimentally by using the signal–idler photon pair of SPDC with well-defined polarization.

Here, we adopted Glauber–Scully’s theory to separate quantum expectation from statistical ensemble average. The advantage can be seen immediately from the following analysis. We have better pictures on quantum interference and on ensemble statistics, respectively. The ensemble average on pure states is a trivial process, while the ensemble average on the thermal state should be ‘specially’ taken care in order to keep the quantum interference effect observable.

New Journal of Physics 13 (2011) 083018 (http://www.njp.org/)
Figure 1. Schematic representation of a typical two-photon polarization interferometer by using entangled photons (a) [9] or using two independent thermal radiations (b). (a) Produced from type-II SPDC, the signal and idler photons are orthogonally polarized and labeled as $\hat{x}$ and $\hat{y}$, respectively. (b) The source consists of two mutually incoherent and orthogonally polarized thermal radiations A and B. BS is a non-polarizing 50/50 BS. $A_1$ and $A_2$ are polarization analyzers oriented at $\theta_1$ and $\theta_2$, respectively. The double arrows represent manipulations of optical delay. The polarization correlation of $\sin^2(\theta_1 - \theta_2)$ can be observed in both sources.

correlation ‘peak’ using mutually incoherent and orthogonally polarized thermal fields A and B as shown in figure 1(b) [12].

2. The experiment

The experimental setup is illustrated in figure 2, which is similar to the Alley–Shih experiment of 1986 [9], except that the input entangled photon pair is replaced by two mutually incoherent thermal fields A and B (see figure 1(b)). The diameter of the laser beam is $D \sim 5$ mm, and the distance from the ground glass to the fiber tips is about $d_A \simeq d_B \sim 200$ mm. Therefore, the planes of the fiber tips satisfy the Fresnel near-field condition, $\Delta \theta = \frac{\delta}{d_A} > \frac{\lambda}{D}$. On the planes, the coherence area has a size of about $l_c = d_A \frac{\delta}{D} \sim 31 \, \mu$m. The $5 \, \mu$m diameter of the fiber tips ensures that the fibers are both smaller than the transverse coherence area of the field. On the other hand, since the fiber tips are scannable, one can move the fiber tip B outside the coherence area of the field at fiber tip A, namely $|\rho_A - \rho_B| > l_c$ ($\rho_A$ and $\rho_B$ are the transverse coordinates of the fiber tips), which ensures that the fields A and B are mutually incoherent. Thus, after the two orthogonally oriented polarizers $P_A$ and $P_B$, we now achieve two mutually incoherent and orthogonally polarized thermal radiations A and B.

The experiments in [12] have also found that anti-correlation and other effects are not restricted within a pair of entangled photons.

New Journal of Physics 13 (2011) 083018 (http://www.njp.org/)
Figure 2. Schematic setup of the Bell-correlation experiment. The pulsed laser is a continuous-wave (CW) mode-locked Ti:sapphire laser beam with a central frequency at $\lambda \sim 780$ nm, 150 fs pulse width and 78 MHz repetition rate. A rotating ground glass diffuser (GG) is employed to produce pseudo-thermal light [15]. IF is an interference filter. BS1 and BS2 are two 50/50 non-polarizing beamsplitters. After BS1, the light beams are coupled into two polarization-maintaining fibers (green cords) through their fiber tips. Then, at the other ends, the output beams are collimated via fiber collimators (FCs). Before the input ports of BS2, two polarizers $P_A$ and $P_B$ define two orthogonal polarizations $\hat{x}$ and $\hat{y}$. At the output ports of BS2, two polarization analyzers $A_1$ and $A_2$ are adopted for post-selection joint-detection measurement. The fiber tips are mounted on three-dimensionally scannable stages. By scanning the fiber tip $A$ longitudinally, an optical delay $\delta$ is introduced to perform temporal correlation measurements.

Before describing the measurement procedure of the Bell correlation, we would like to address an important issue. As we know, for thermal fields, a joint detection event of $D_1$ and $D_2$ may not be triggered solely by a pair of photons coming from $A$ and $B$, respectively (denoted as $G^{(2)}_{AB}$), i.e. it can also be triggered by two photons both coming from $A$ ($G^{(2)}_{AA}$) or both from $B$ ($G^{(2)}_{BB}$). This is one of the important differences between an entangled photon pair and incoherent thermal fields. To obtain $G^{(2)}_{AB}$ from the coincidence counting rate of $D_1$ and $D_2$, $G^{(2)}_{AA}$ and $G^{(2)}_{BB}$ must be excluded. Fortunately, both $G^{(2)}_{AA}$ and $G^{(2)}_{BB}$ can be easily measured by blocking the $B$-field or the $A$-field. $G^{(2)}_{AB}$ is then isolated by subtracting $G^{(2)}_{AA}$ and $G^{(2)}_{BB}$ from the overall second-order correlation ($G^{(2)}$), since $G^{(2)} = G^{(2)}_{AB} + G^{(2)}_{AA} + G^{(2)}_{BB}$.

In the Bell-correlation measurement, we have observed three types of correlations [9, 16]: (i) when $\theta_1 = \theta_2 = 45^\circ$, a temporal anti-correlation ‘dip’ was observed when we scanned the fiber tip $A$ longitudinally [16]; (ii) when $\theta_1 = 45^\circ$ and $\theta_2 = 135^\circ$, a correlation ‘peak’ was observed; (iii) fixing the fiber tip $A$ at $\delta = 0$, we observed a sinusoidal function of polarization correlation $G^{(2)}_{AB}(\theta_1, \theta_2) \propto \sin(\theta_1 - \theta_2)$ when manipulating the orientations of $A_1$ and $A_2$, where the minimum (i.e. $\theta_1 - \theta_2 = 0$) corresponds to the anti-correlation, and the maximum (i.e. $\theta_1 - \theta_2 = \pi/2$) corresponds to the correlation.

\footnote{In [16], the anti-correlation ‘dip’ was obtained by scanning the beamsplitter, which is different from the scanning of $\delta$. The difference is nontrivial.}

\textit{New Journal of Physics} 13 (2011) 083018 (http://www.njp.org/)
3. The theory

We start from \( G_{AB}^{(2)}(z_1, t_1; z_2, t_2) \), the probability of observing a joint photodetection event at \((z_1, t_1)\) and \((z_2, t_2)\), which is produced by two independent wavepackets, one excited from A and another from B,

\[
G_{AB}^{(2)}(z_1, t_1; z_2, t_2) \propto |\langle \hat{E}^{(+)}(z_2, t_2) \hat{E}^{(+)}(z_1, t_1)|\Psi_A \rangle|\Psi_B \rangle|^2_{\text{Ensemble}}
\]

\[
\equiv \langle |\Psi_{AB}(z_1, t_1; z_2, t_2)|^2 \rangle_{\text{Ensemble}}. \tag{5}
\]

Figures 3 and 4 report the experimental results of the two measurements, where we show \( G_{AB}^{(2)} \) only. The overall correlations, \( G_{AB}^{(2)} + G_{AA}^{(2)} + G_{BB}^{(2)} \), will be shown in the next section with analysis, in which we will see that the experimental data agree well with the quantum predictions.
Here we are applying Glauber–Scully’s theory, in which we first take the expectation value of a pair of single photons A and B whose states are $|\Psi_A\rangle$ and $|\Psi_B\rangle$, respectively, and then calculate the ensemble average of a large number of pairs. $\Psi_{AB}(z_1, t_1; z_2, t_2)$ is the effective wavefunction of a photon pair. The field operators can be written as:

$$\hat{E}^{(+)\dagger}(z_1, t_1) = \frac{1}{\sqrt{2}}[\hat{x} \cdot \hat{\theta}_1 \hat{E}^{(+)\dagger}(\tau_{A1}^R) + \hat{y} \cdot \hat{\theta}_1 \hat{E}^{(+)\dagger}(\tau_{B1}^R)],$$

(6)

$$\hat{E}^{(+)\dagger}(z_2, t_2) = \frac{1}{\sqrt{2}}[\hat{x} \cdot \hat{\theta}_2 \hat{E}^{(+)\dagger}(\tau_{A2}^R) - \hat{y} \cdot \hat{\theta}_2 \hat{E}^{(+)\dagger}(\tau_{B2}^R)],$$

where $\tau_{Aj} \equiv (t_j - t_{0A}) - (z_j - z_A)/c$, $\tau_{Bj} \equiv (t_j - t_{0B}) - (z_j - z_B)/c$, with $j = 1, 2$ labeling the optical delay from the detector $D_j$ to the input planes $A$ and $B$, respectively; $R$ and $T$ label the reflected path and the transmitted path. $t_{0A}$ and $t_{0B}$ are initial times when wavepackets $A$ and $B$ are created, respectively. In general, the creation times are mutually random due to their mutual incoherence nature. The ‘−’ sign in $\hat{E}^{(+)\dagger}(z_2, t_2)$ is introduced by the beamsplitter BS2. Thus,

$$\Psi_{AB}(z_1, t_1; z_2, t_2) = (\hat{x}_1 \cdot \hat{\theta}_1)(\hat{y}_2 \cdot \hat{\theta}_2)A(\tau_{A1}^R, \tau_{B2}^R) - (\hat{y}_1 \cdot \hat{\theta}_1)(\hat{x}_2 \cdot \hat{\theta}_2)A(\tau_{B1}^T, \tau_{A2}^T),$$

(7)

where

$$A(\tau_{A1}^R, \tau_{B2}^R) \equiv \langle 0|\hat{E}^{(+)\dagger}(\tau_{A1}^R)|\Psi_A\rangle \langle 0|\hat{E}^{(+)\dagger}(\tau_{B2}^R)|\Psi_B\rangle,$$

(8)

$$A(\tau_{B1}^T, \tau_{A2}^T) \equiv \langle 0|\hat{E}^{(+)\dagger}(\tau_{B1}^T)|\Psi_B\rangle \langle 0|\hat{E}^{(+)\dagger}(\tau_{A2}^T)|\Psi_A\rangle$$

are the two different yet indistinguishable two-photon amplitudes that indicate two quantum paths to trigger a joint event—the photons A and B are both reflected and both transmitted, respectively.

It is easy to see from equation (7) that, if $A(\tau_{A1}^R, \tau_{B2}^R)$ and $A(\tau_{B1}^T, \tau_{A2}^T)$ ‘overlap’ completely (quantum mechanically indistinguishable) from one joint event to another in the ensemble average, Bell correlation is expected.

In contrast with entangled states, a joint event of thermal light is produced by two independent and randomly distributed photons that fall into the coincidence time window by chance only. To obtain observable two-photon interference, it is required that: (i) the two wavepackets $A(\tau_{A1}^R, \tau_{B2}^R)$ and $A(\tau_{B1}^T, \tau_{A2}^T)$ must overlap for each measured pair; (ii) the ensemble average would not average out the two-photon interference from one pair to another$^6$. How do we achieve ‘overlap’ between $A(\tau_{A1}^R, \tau_{B2}^R)$ and $A(\tau_{B1}^T, \tau_{A2}^T)$?

In the single-photon approximation, the states of wavepackets $A$ and $B$ can be modeled as $|\Psi\rangle$:

$$|\Psi_A\rangle \simeq \int d\omega \, f(\omega) \, e^{i\omega t} \hat{a}^\dagger(\omega) \, |0\rangle,$$

(9)

$$|\Psi_B\rangle \simeq \int d\omega \, f(\omega) \, e^{i\omega t} \hat{b}^\dagger(\omega) \, |0\rangle,$$

$^6$ Entangled states achieve these two conditions in nature. For instance, (i) the signal–idler photon pair of SPDC is generated simultaneously; (ii) the biphoton state is a pure state, i.e. all signal–idler pairs are in the same state and thus form a homogeneous ensemble.
Figure 5. Naturally, $A(\tau_{R1}, \tau_{R2})$ and $A(\tau_{T1}, \tau_{T2})$ can never be overlapping except when having $t_{0A} = t_{0B}$ by chance (upper part). What we have done in this experiment is to force $|t_{0A} - t_{0B}| \ll t_c$ by applying a short pulse to excite the wavepackets $A$ and $B$ (lower part). For simplicity, here $\delta = 0$.

where $f(\omega) = \frac{t_c}{\sqrt{2\pi}} e^{-(\omega - \omega_0)^2/t_c^2}$, the spectrum function of IF with a coherence time of $t_c$ and a central frequency at $\omega_0$. Therefore,

$$A(\tau_{R1}, \tau_{R2}) = e^{-\tau_{R1}^2/t_c^2} e^{-\tau_{R2}^2/t_c^2} e^{-i\omega_0(\tau_{R1} + \tau_{R2})},$$

$$A(\tau_{T1}, \tau_{T2}) = e^{-\tau_{T1}^2/t_c^2} e^{-\tau_{T2}^2/t_c^2} e^{-i\omega_0(\tau_{T1} + \tau_{T2})}.$$  

The above amplitudes contain the envelopes (the Gaussian functions) and the modulations ($e^{-i\omega_0(\tau_{T1} + \tau_{T2})}$ and $e^{-i\omega_0(\tau_{R1} + \tau_{R2})}$, respectively). Actually, the modulations will not affect the correlation in this experiment after taking normal square of the effective wavefunction in equation (5). Thus, we will only consider the envelopes in the calculations. Naturally, the envelopes can be rewritten in the axes of $t_1 + t_2$ and $t_1 - t_2$:

$$\tilde{A}(\tau_{R1}, \tau_{R2}) \equiv e^{-\tau_{R1}^2/t_c^2} e^{-\tau_{R2}^2/t_c^2}$$

$$= e^{-[(t_1 + t_2) - (t_{0A} + t_{0B}) - (z_1 + z_2) - (z_{A} + z_{B})]/c^2} e^{[-(t_1 - t_2) - (t_{0A} - t_{0B}) - \delta]/2t_c^2},$$

(10)

$$\tilde{A}(\tau_{T1}, \tau_{T2}) \equiv e^{-\tau_{T1}^2/t_c^2} e^{-\tau_{T2}^2/t_c^2}$$

$$= e^{-[(t_1 + t_2) - (t_{0A} + t_{0B}) - (z_1 + z_2) - (z_{A} + z_{B})]/c^2} e^{[-(t_1 - t_2) + (t_{0A} - t_{0B}) - \delta]/2t_c^2}.$$
where \( \delta = [(z_1 - z_2) - (z_A - z_B)]/c \) (note that \( z_1 = z_2 \) was chosen in the experiment). It is not too difficult to see that the two wavepackets overlap completely along the axis of \( t_1 + t_2 \). However, they never overlap completely along the axis of \( t_1 - t_2 \), unless \( t_{0A} = t_{0B} \) and \( \delta = 0 \) (i.e. \( z_A = z_B \)). It is easy to achieve \( z_A = z_B \) by scanning \( \delta \), while it is definitely nontrivial to achieve \( t_{0A} = t_{0B} \) for thermal light, see figure 5. What we have done in this experiment is to force \( |t_{0A} - t_{0B}| \ll t_0 \) by applying a short pulse (150 fs) to excite the wavepackets at points \( A \) and \( B \); see the lower part of figure 5. As is clearly shown in figure 5, the shorter the pulse width we chose, the greater the amount of overlap between \( \mathcal{A}(\tau^R_{A1}, \tau^R_{B2}) \) and \( \mathcal{A}(\tau^T_{B1}, \tau^T_{A2}) \) that is achievable and thus the higher the degree of second-order coherence we can obtain. In Dirac’s language, we have achieved a condition for randomly paired photons to interfere with the pair itself.

In contrast with entangled states, the statistical ensemble average is critical to the measurement of thermal light. In general, the ensemble average would average out the two-photon interference. We may introduce a function of ‘degree of overlap’ as \( g_s(t_{0-}) = \frac{1}{T_v \sqrt{\pi}} e^{-\tau^2/T_v^2} \). \( T_v \) represents a time window, such as the 150 fs pulse width of the thermal field, within which wavepackets \( A \) and \( B \) can only be excited. The ensemble average of the quantum interference cross-term is approximated as

\[
\mathcal{N} \int dt_{0A} dt_{0B} g_s(t_{0A} - t_{0B}) \tilde{\mathcal{A}}(\tau^R_{A1}, \tau^R_{B2})\tilde{\mathcal{A}}^*(\tau^T_{B1}, \tau^T_{A2})
= \frac{\mathcal{N}}{2} \int dt_{0A} dt_{0-} g_s(t_{0-}) \tilde{\mathcal{A}}(\tau^R_{A1}, \tau^R_{B2})\tilde{\mathcal{A}}^*(\tau^T_{B1}, \tau^T_{A2})
= \frac{\mathcal{N} \sqrt{\pi}}{2} \frac{\tau^2 e^{-\tau^2/(\tau^2 + T_v^2)}}{\sqrt{\tau^2 + T_v^2}} e^{-\tau^2/T_v^2},
\]

(11)

where \( t_{0-} = t_1 - t_2, t_0 = t_1 + t_2, t_{0-} \equiv t_{0A} - t_{0B} \) and \( t_{0+} = t_{0A} + t_{0B} \). \( \mathcal{N} \) is a normalization factor. Considering the unavoidable time average of the photodetection detection process, i.e. the time integral of \( t_1 \) and \( t_2 \), the cross-term finally becomes

\[
\frac{\mathcal{N} \sqrt{\pi}}{2} \frac{\tau^2 e^{-\tau^2/(\tau^2 + T_v^2)}}{\sqrt{\tau^2 + T_v^2}}.
\]

Similarly,

\[
\mathcal{N} \int dt_1 dt_2 \int dt_{0A} dt_{0B} g_s(t_{0A} - t_{0B}) |\tilde{\mathcal{A}}(\tau^R_{A1}, \tau^R_{B2})|^2
= \frac{\mathcal{N}}{2} \int dt_1 dt_2 \int dt_{0A} dt_{0B} g_s(t_{0A} - t_{0B}) |\tilde{\mathcal{A}}(\tau^T_{B1}, \tau^T_{A2})|^2
= \frac{\mathcal{N} \sqrt{\pi}}{2} \tau^2.
\]

(12)

Therefore, the coincident counting rate becomes

\[
R_{AB} = \int dt_1 dt_2 G_{AB}^{(2)}(z_1, t_1; z_2, t_2) \propto 1 - \cos 2\theta_1 \cos 2\theta_2 - \sin 2\theta_1 \sin 2\theta_2 \frac{e^{-\tau^2/(\tau^2 + T_v^2)}}{\sqrt{1 + (T_v/\tau^2)^2}}.
\]

(13)

By setting \( \theta_1 = \theta_2 = 45^\circ \), we can evaluate the visibility of the anti-correlation, which is a function of \( T_v \), \( D_c(T_v) = (2\sqrt{1 + (T_v/\tau^2)^2} - 1)^{-1} \). When \( T_v \) gets bigger, the visibility will go down to zero. Figure 6 shows the curve of \( D_c(T_v) \).
When $T_v \sim 0$, the coincident counting rate becomes
\[ R_{AB} = \propto 1 - \cos 2 \theta_1 \cos 2 \theta_2 - \sin 2 \theta_1 \sin 2 \theta_2 e^{-\delta^2 / t_c^2}. \]
Thus, a ‘dip’ or a ‘peak’ is expected in the coincident counting rate while scanning $\delta$ when choosing $\theta_1 = \theta_2 = 45^\circ$ or $\theta_1 = 45^\circ$, $\theta_2 = 135^\circ$:
\[ R_{AB} \propto 1 \mp e^{-\delta^2 / t_c^2}. \]
(14)

What’s more, taking $\delta = 0$, $\mathcal{A}(\tau_{A1}^R, \tau_{B2}^R)$ and $\mathcal{A}(\tau_{B1}^T, \tau_{A2}^T)$ as completely ‘overlapping’, we shall observe the EPR–Bohm–Bell correlation of equation (4):
\[ R_{AB} \propto \sin^2(\theta_1 - \theta_2), \]
indicating the behavior of a Bell state or a qubit.

4. Summary

In summary, in the simulation of the behavior of a two-photon qubit, we have (i) prepared a pair of orthogonal wavepackets $|\Psi_A\rangle$ and $|\Psi_B\rangle$; (ii) prepared two different yet indistinguishable alternative ways for a pair of wavepackets $A$ and $B$ to produce a joint photodetection event; (iii) achieved an experimental condition in which the statistical ensemble average does not average out the interference of each random photon pair while taking into account all possible realizations of the fields from one joint photodetection event to another.

In addition, we have asked ourselves two questions: (1) What will happen if the $A$-field and $B$-field are in coherent states? (2) What will happen if we increase the intensities of the incoherent thermal fields $A$ and $B$ beyond the single photon level? With regard to question (1), we have experimentally studied the following cases: (a) when the $A$-field and $B$-field come from a non-thermalized laser beam; (b) when the $A$-field and $B$-field come from two independent CW lasers; (c) when the $A$-field and $B$-field come from incoherent laser pulses. There is no sign of any Bell correlation from (a) and (b). Case (c) is complicated, the results depend on different experimental conditions, and the theory involves a number of important aspects of physics. Question (2) involves two more important aspects of physics: (a) the relationship between two-photon interference and intensity–intensity correlation; (b) the simulation of single-photon qubits by means of creation and annihilation of wavepackets at a certain higher energy level. The study of (c) in question (1) and the study of question (2) will be reported separately.
The observed $G_{AB}^{(2)} + G_{AA}^{(2)} + G_{BB}^{(2)}$ as well as $G_{AA}^{(2)}$ and $G_{BB}^{(2)}$. Subtracting
$G_{AA}^{(2)} + G_{BB}^{(2)}$ from $G_{AB}^{(2)} + G_{AA}^{(2)} + G_{BB}^{(2)}$, anti-correlation is expected with 100%
visibility.

The observed $G_{AB}^{(2)} + G_{AA}^{(2)} + G_{BB}^{(2)}$ as well as $G_{AA}^{(2)} + G_{BB}^{(2)}$. Subtracting
$G_{AA}^{(2)} + G_{BB}^{(2)}$ from $G_{AB}^{(2)} + G_{AA}^{(2)} + G_{BB}^{(2)}$, polarization correlation is expected with 100%
visibility.

Although the measurement of $G_{AB}^{(2)}$ gives a Bell correlation for incoherent thermal fields
A and B, we cannot ignore the contributions of $G_{AA}^{(2)}$ and $G_{BB}^{(2)}$, corresponding to the probabilities for two photons that both excited from A or both from B to produce a joint photodetection event. Fortunately, $G_{AA}^{(2)}$ and $G_{BB}^{(2)}$ are individually measurable and thus distinguishable from the measurement of $G_{AB}^{(2)}$. Figure 7 reports a typical observed $G_{AB}^{(2)} + G_{AA}^{(2)} + G_{BB}^{(2)}$ as well as $G_{AA}^{(2)} + G_{BB}^{(2)}$ as well as $G_{AA}^{(2)}$ and $G_{BB}^{(2)}$ during anti-correlation ‘dip’ and ‘peak’ measurements. Figure 8 shows the measurements of $G_{AB}^{(2)} + G_{AA}^{(2)} + G_{BB}^{(2)}$ and $G_{AA}^{(2)} + G_{BB}^{(2)}$ in a polarization correlation measurement.

Our success in simulating a two-photon qubit is a great step and has brought us closer to the capability of simulating the behavior of n-photon qubits for computation purposes as
shown in equation (1). To achieve the goal, we need to (i) prepare a set of $n$ incoherent sub-fields for the $n$th-order correlation measurement; (ii) perform Hadamard transform to change the state of each sub-field into $|0\rangle$ and $|1\rangle$; (iii) prepare $2^n$ different yet indistinguishable alternative ways for a group of $n$ photons to produce an $n$-fold joint photodetection event; (iv) achieve an ensemble average that maintains the effect of multi-photon interference from one $n$-photon joint photodetection event to another. The set of $n$ incoherent sub-fields can be easily prepared from a large angular sized chaotic-thermal source by utilizing the transverse coherence property of thermal fields, similar to the setup we have demonstrated in this experiment. A polarization BS can transfer each sub-field into binary states $|x\rangle$ ($|0\rangle$) and $|y\rangle$ ($|1\rangle$). By following a similar procedure to the one we have demonstrated in this experiment, in an $n$th-fold joint photodetection event, we are able to observe the signature of the required qubits.

In conclusion, we have successfully simulated two-photon Bell states with incoherent thermal light. The demonstrated working principle for achieving multi-photon interference of thermal light is fundamentally important and practically useful. At least, this experiment should be helpful in constructing $n$-photon qubits from incoherent thermal radiation, such as $n = 64$, $n = 128$, $n = 256$, . . . , for computational purposes.

Acknowledgments

The authors thank T B Pittman, M H Rubin, G Scarcelli, J P Simon, V Tamma and Y Zhou for helpful discussions. This work was partially supported by the AFOSR and the ARO-MURI program.

References

[1] Hübel H et al 2010 Nature 466 601
[2] Liu J B and Shih Y H 2009 Phys. Rev. A 79 023819
[3] Zhou Y, Simon J, Liu J and Shih Y H 2010 Phys. Rev. A 81 043831
[4] Einstein A, Podolsky B and Rosen N 1935 Phys. Rev. 47 777
[5] Bohm D 1989 Quantum Theory (New York: Dover)
[6] Bell J S 2004 Speakable and Unspeakable in Quantum Mechanics 2nd edn (Cambridge: Cambridge University Press)
[7] Clauser J F and Shimony A 1978 Rep. Prog. Phys. 41 1881
[8] Reid M D and Walls D F 1986 Phys. Rev. A 34 1260
[9] Alley C O and Shih Y H 1986 Foundations of Quantum Mechanics in the Light of New Technology ed M Namiki et al (Tokyo: Physical Society of Japan)

Shih Y H and Alley C O 1988 Phys. Rev. Lett. 61 2921
Ou Y and Mandel L 1988 Phys. Rev. Lett. 61 50
[10] Kwiat P G, Mattle K, Weinfurter H, Zeilinger A, Sergienko A V and Shih Y H 1995 Phys. Rev. Lett. 75 4337
[11] Fedrizzi A et al 2009 New J. Phys. 11 103052
[12] Kaltenbaek R, Blauensteiner B, Zukowski M, Aspelmeyer M and Zeilinger A 2006 Phys. Rev. Lett. 96 240502
Fulconis J, Alibart O, Wadsworth W J and Rarity J G 2007 New J. Phys. 9 276
Li X et al 2008 Opt. Express 16 12505
Pittman T B and Franson J D 2003 Phys. Rev. Lett. 90 240401

8 In the CW thermal field, the probability of having a large number of wavepackets within a short time window is small. However, the use of a short pulsed thermal field will greatly enhance the chances of creating high-order $n$ wavepackets for an $n$-fold joint detection.

New Journal of Physics 13 (2011) 083018 (http://www.njp.org/)
[13] Glauber R J 1963 Phys. Rev. Lett. 84 10
Glauber R J 1963 Phys. Rev. 30 2529

[14] Scully M O and Zubairy M S 1997 Quantum Optic 1st edn (Cambridge: Cambridge University Press)

[15] Martienssen W and Spiller E 1964 Am. J. Phys. 32 919

[16] Hong C, Ou Z and Mandel L 1987 Phys. Rev. Lett. 59 2044

[17] Shih Y H 2011 An Introduction to Quantum Optics: Photon and Biphotoh Physics 1st edn (London: Taylor and Francis)