Chiral Symmetry in Charmonium - Pion Cross Section

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Abstract

We perform a non-perturbative calculation of the $J/\Psi - \pi$ cross section using a $SU(2) \times SU(2)$ effective Lagrangian. Our results differ from those of previous calculations, specially in the description of vertices involving pions.

PACS: 12.39.Fe 13.85.Fb 14.40.Lb

Reliable values for the charmonium - hadron cross sections are of crucial importance in the context of quark- gluon plasma physics. Part of these interactions happens in the early stages of the nucleus- nucleus collisions and therefore at high energies ($\sqrt{s} \approx 10 - 20$ GeV) and one may try to apply perturbative QCD. On the other hand, a significant part of the charmonium - hadron interactions occurs when other light particles have already been produced, forming a “fireball”. Interactions inside this fireball happen at much lower energies ($\sqrt{s} \leq 5$ GeV) and one has to apply non-perturbative methods.

One possible reaction mechanism is meson exchange, that can be studied by means of effective Lagrangians, constrained by flavor and chiral symmetries as well as gauge invariance. This approach has been pioneered by Matinyan and B. Müller \cite{1} and further developed by three other groups \cite{2–7}.

In the first work, only $D$ meson exchange was considered and the authors obtained a small cross section of order of 2.5 mb at $\sqrt{s} = 5$ GeV for the process $\pi + J/\psi \rightarrow D + D^*$. In Refs. \cite{3,4} $D^*$ meson exchange and four-point couplings were included, the latter in order to preserve gauge invariance. This led to a much larger cross section, of around 30 mb at $\sqrt{s} = 5$ GeV, mainly due to the inclusion of the $D^*$ meson exchange. The contribution of the four-point diagram to the cross section is of the same order as the process with the $D$ meson exchange. In Ref. \cite{2} anomalous parity interactions were included. In particular, anomalous term such as $D^*D^*\pi$ opens new absorption channels not included before. In

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the case of the channel $\pi + J/\psi \rightarrow D + D^*$, the inclusion of the $D^* D^* \pi$ vertex increases even more the cross section, that becomes about 100 mb at $\sqrt{s} = 5 \text{ GeV}$. The inclusion of new absorption channels was also the idea guiding the work presented in Ref. [4]. As an example, the anomalous process $\pi + J/\psi \rightarrow \eta_c \rho$ alone gives a cross section of order of 60 mb at $\sqrt{s} = 5 \text{ GeV}$.  

A common feature of all these works is that they take SU(4) $\times$ SU(4) symmetry as the point of departure for describing mesonic interactions. As this symmetry is badly broken in nature, at some stage of the calculation one is forced to use the empirical values of the relevant masses. In this work we show that this procedure may lead to inconsistencies which are numerically important.

Let us consider, for instance, the diagram for the process $\pi + J/\psi \rightarrow D + D^*$ with the $D^*$ meson exchange, given in Fig. 1. It is based on a vector interaction among three spin one mesons, and the axial coupling between two pseudoscalar mesons and the $D^*$. For the former, one uses the Lagrangian of Refs. [2,6], given by

$$\mathcal{L}_{\psi^* D^* D^*} = i g_{\psi^* D^* D^*} \left[ \psi^{*\mu} \left( \partial_{\mu} D^{\nu\mu} \bar{D}_\nu - D^{\nu\mu} \partial_{\mu} \bar{D}_\nu \right) + (\partial_{\mu} \psi^{*\nu} D^*_\nu - \psi^{\nu} \partial_{\mu} D^*_\nu) \bar{D}^{*\mu} + D^{*\nu} \left( \psi^{\nu} \partial_{\mu} D^*_\mu - \partial_{\mu} \psi^{*\nu} D^*_\mu \right) \right].$$

(1)

For the axial vertex one finds two alternative forms in the literature, namely [1,6]

$$\mathcal{L}^{(I)}_{\pi DD^*} = \frac{i}{2} g_{\pi DD^*} \left[ \left( \bar{D} \tau D^{*\mu} - \bar{D}^{*\mu} \tau D \right) \cdot \partial_{\mu} \bar{D} - \left( \partial_{\mu} \bar{D} \tau D^{*\mu} - \bar{D}^{*\mu} \tau \partial_{\mu} D \right) \cdot \bar{D} \right],$$

(2)

and [2]

$$\mathcal{L}^{(II)}_{\pi DD^*} = i g_{\pi DD^*} \left( \bar{D} \tau D^{*\mu} - \bar{D}^{*\mu} \tau D \right) \cdot \partial_{\mu} \bar{D}.$$  

(3)

In the above equations, $\tau$ are the Pauli matrices, and $\bar{D}$ denote the pion meson isospin triplet, while $D \equiv (D^0, D^+)$ and $D^* \equiv (D^{*0}, D^{*+})$ denote the pseudoscalar and vector charm meson doublets, respectively. The Lagrangians in Eqs. (2) and (3) may be related by performing an integration by parts into the last term in Eq.(2), which allows us to write

$$\mathcal{L}^{(I)}_{\pi DD^*} = \mathcal{L}^{(II)}_{\pi DD^*} + \frac{i}{2} g_{\pi DD^*} \left( \bar{D} \tau \partial_{\mu} D^{*\mu} - \partial_{\mu} \bar{D}^{*\mu} \tau D \right) \cdot \bar{D}.$$  

(4)

This result indicates that the two forms of the $\pi DD^*$ interaction would be equivalent if the condition $\partial_{\mu} D^{*\mu} = \partial_{\mu} \bar{D}^{*\mu} = 0$ hold, which is the case for on mass shell vector mesons. However, in the sequence, we show that this condition is not valid in the presence of interactions, and hence that Eqs. (2) and (3) correspond to different dynamical hypotheses.

We begin by considering the specific case of the process displayed in Fig. 1. Calling the four-momenta of the initial mesons $\pi$ and $J/\psi$ by $p_1$ and $p_2$ and those of the final mesons $D^*$ and $D$ by $p_3$ and $p_4$ respectively, the vector vertex between the three vector mesons is given by

$$i \Gamma^\beta = i g_{\psi D^* D^*} \epsilon_{2\mu} \epsilon_{3\nu}^* \left( -2 p_3^\mu g^{\beta\nu} - 2 p_2^\nu g^{\beta\mu} + (p_2 + p_3)^\beta g^{\mu\nu} \right),$$

(5)

where $\epsilon_i$ is the polarization vector of the vector meson with momentum $p_i$, and we have already used the orthogonality relation for vector mesons: $\epsilon_2^\mu p_{2\mu} = \epsilon_3^\mu p_{3\mu} = 0$. For the
remaining part of the diagram, which includes the $D^*$ propagator ($\Delta_{\alpha\beta}$) and the axial vertex ($A^\alpha$), we get, using Eq. (2):

\[ iA^{(I)}_\alpha i\Delta^{\alpha\beta} = \frac{i}{2} g_{\pi DD^*} \left( p_1 + p_4 \right)_\alpha i\Delta^{\alpha\beta} \]

\[ = g_{\pi DD^*} \left[ p_{1\alpha} \left( \frac{1}{q^2 - m_{D^*}^2} \right) \left( g^{\alpha\beta} - \frac{q^\alpha q^\beta}{m_{D^*}^2} \right) + \frac{q^\beta}{2m_{D^*}^2} \right], \quad (6) \]

where $q = p_1 - p_4$. It is worth noting that the term proportional to $q_\beta$ corresponds to a contact interaction. In the case of Eq. (3), we get

\[ iA^{(II)}_\alpha i\Delta^{\alpha\beta} = ig_{\pi DD^*} p_{1\alpha} i\Delta^{\alpha\beta} \]

\[ = g_{\pi DD^*} p_{1\alpha} \left( \frac{1}{q^2 - m_{D^*}^2} \right) \left( g^{\alpha\beta} - \frac{q^\alpha q^\beta}{m_{D^*}^2} \right). \quad (7) \]

The scattering amplitude is proportional to $\Gamma_\alpha A^\alpha$ and hence, the equivalence between these two calculations requires $\Gamma^\beta q_\beta = 0$. However, using Eq. (3) we find

\[ \Gamma^\beta q_\beta = g_{\psi D^* D^*} \epsilon_2^\mu \epsilon_3^\nu \left( m_\psi^2 - m_{D^*}^2 \right), \quad (8) \]

which vanishes only in the case of exact $SU(4)$, but is different from zero in the case of realistic masses. The full amplitude for the process in Fig. 1 is given by

\[ M^{(I)} = g_{\psi D^* D^*} g_{\pi DD^*} \epsilon_2^\mu \epsilon_3^\nu \frac{1}{(p_1 - p_4)^2 - m_{D^*}^2} \left[ g_{\mu\nu} \left( p_2 + p_3 \right)_\alpha + \left( \frac{m_\psi^2}{m_{D^*}^2} - 1 \right) (p_3 - p_2)_\alpha \right] - 2p_{3\mu} g_{\alpha\nu} - 2p_{2\nu} g_{\alpha\mu} \right] p_1^\alpha, \quad (9) \]

and

\[ M^{(II)} = M^{(I)} + \frac{1}{2} g_{\psi D^* D^*} g_{\pi DD^*} \epsilon_2^\mu \epsilon_3^\nu g_{\mu\nu} \left( \frac{m_\psi^2}{m_{D^*}^2} - 1 \right). \quad (10) \]

The difference between $M^{(I)}$ and $M^{(II)}$ is due to terms proportional to $\partial_\rho D^{*\mu}$ and $\partial_\mu D^{*\nu}$ in Eq. (2). Indeed, the interaction Lagrangian (1) gives rise to the following equation of motion for the $D^*$ meson:

\[ -\partial_\rho \left( \partial^\rho D^{*\lambda} - \partial^\lambda D^{*\rho} \right) - m_{D^*}^2 D^{*\lambda} = \frac{i g_{\psi D^* D^*}}{2} \left( \partial_\rho \psi^\rho \right) D^{*\lambda} - \psi^\lambda \left( \partial_\rho D^{*\rho} \right) \]

\[ + 2 \psi^\lambda \left( \partial_\rho D^{*\lambda} \right) - D^{*\rho} \left( \partial_\rho \psi^\rho \right) - \psi^\rho \left( \partial_\rho \left( \partial_\mu D^\nu \right) \right) + D^*_\rho \left( \partial_\lambda \psi^\rho \right) \]. \quad (11) \]

The equation of motion for the mesons $D^*$ and $J/\psi$ are totally analogous. Applying $\partial_\lambda$ into Eq. (11), we get

\[ -m_{D^*}^2 \partial_\lambda D^{*\lambda} = \frac{i g_{\psi D^* D^*}}{2} \left[ -\psi_\mu \partial_\lambda \left( \partial_\rho D^{*\rho} - \partial^\rho D^{*\lambda} \right) + \partial_\lambda \left( \partial_\mu \psi^\rho - \partial^\rho \psi^\mu \right) D^*_\rho \right]. \quad (12) \]

Thus, using the equations of motion for $D^*$ and $J/\psi$, and neglecting terms proportional to $g_{\psi D^* D^*}$ we have
\[ \partial_\lambda D^{*\lambda} = i \ g_{\psi D^* D^*} \left( \frac{m_\psi^2}{m_{D^*}^2} - 1 \right) \psi^{\lambda} D^{\ast}_\lambda. \] (13)

Using the above result in Eq. (4) we get, to order \( g^2 \):

\[ \mathcal{L}_{\pi DD^*}^{(I)} = \mathcal{L}_{\pi DD^*}^{(II)} - \frac{1}{2} g_{\pi DD^*} g_{\psi D^* D^*} \left( \frac{m_\psi^2}{m_{D^*}^2} - 1 \right) \psi^{\mu} \left( \bar{D} \tau^\mu D^* - \bar{D}^* \tau^\mu D \right) \cdot \bar{\pi}. \] (14)

The second term in this equation is precisely the effective four-leg interaction that gives rise to the difference between \( \mathcal{M}^{(I)} \) and \( \mathcal{M}^{(II)} \) as in Eq. (10). The important feature of the last term in Eq. (14) is that it breaks \( SU(2) \times SU(2) \) chiral symmetry when \( SU(4) \). As demonstrated long ago by Weinberg [8], the construction of chiral non-linear Lagrangians in the \( SU(2) \) sector requires necessarily gradient couplings for the pion. In the present problem, this means that \( \mathcal{L}_{\pi DD^*}^{(II)} \) is chiral symmetric whereas \( \mathcal{L}_{\pi DD^*}^{(I)} \) is not. Consistently the former yields the amplitude \( \mathcal{M}^{(II)} \), Eq. (9), which is linear in the pion momentum and would vanish if it were soft. The second term in Eq. (14), on the other hand, breaks chiral symmetry and gives rise to the large amplitude \( \mathcal{M}^{(I)} \).

We can evaluate numerically the difference between \( \mathcal{M}^{(I)} \) and \( \mathcal{M}^{(II)} \) by calculating the differential cross section. After including isospin factors, the differential cross section is given by

\[ \frac{d\sigma}{dt} = \frac{1}{96\pi s^2 p_{i,cm}^2} \sum_{\text{spin}} |\mathcal{M}|^2, \] (15)

where \( p_{i,cm} \) is the three-momentum of \( p_1 \) (or \( p_2 \)) in the center of mass frame:

\[ p_{i,cm}^2 = \frac{s^2 + m_1^4 + m_2^4 - 2s m_1^2 - 2s m_2^2 - 2m_1^2 m_2^2}{4s}, \] (16)

with \( s = (p_1 + p_2)^2 \) and \( t = (p_1 - p_2)^2 \).

In order to compare our results with the previous ones, we use the same coupling constants as in refs. [4,6], namely \( g_{\pi DD^*} = 8.8 \) and \( g_{\psi D^* D^*} = 7.64 \), although in a recent paper we find a rather smaller value: \( g_{\pi DD^*} = 4.0 \pm 0.3 \) [9]. In Fig. 2 we show the cross section for the \( \pi + J/\psi \to D + D^* \) process obtained from Fig. 1 with the Lagrangian, \( \mathcal{L}_{\pi DD^*}^{(II)} \), in Eq. (3) (dashed line), and with the Lagrangian, \( \mathcal{L}_{\pi DD^*}^{(I)} \), in Eq. (2) (dash-dotted line). As expected, the cross section obtained with \( \mathcal{L}_{\pi DD^*}^{(I)} \), which breaks chiral \( SU(2) \times SU(2) \), is much bigger than that obtained with the chiral Lagrangian \( \mathcal{L}_{\pi DD^*}^{(II)} \). The difference between both results is even more important near threshold where the cross section obtained with \( \mathcal{L}_{\pi DD^*}^{(I)} \) grows very rapidly. For instance, at \( \sqrt{s} = 4 \) GeV the chiral Lagrangian gives \( \sigma \sim 3.5 \text{ mb} \), while \( \mathcal{L}_{\pi DD^*}^{(I)} \) gives \( \sigma \sim 11.5 \text{ mb} \). In Fig. 2 we also show, for completeness, the cross section for the \( \pi + J/\psi \to D + D^* \) process obtained with a \( D \) meson exchange (solid line). In this case the

\[ ^1 \text{The definition of } g_{\pi DD^*} \text{ here differs from the one used in ref. [3] by a factor } 1/\sqrt{2}. \]
result is the same with both Lagrangians since the external vector mesons are free and the conditions $\partial_{\mu} D^{*\mu} = \partial_{\mu} \bar{D}^{*\mu} = 0$ hold.

In conclusion, we have shown that the two alternative forms of the $\pi D D^*$ interaction Lagrangian found in the literature are not equivalent for processes involving a virtual $D^*$ meson exchange. In particular the form which breaks chiral $SU(2) \times SU(2)$ symmetry gives a bigger cross section, this effect being even more important near threshold. Since $SU(4)$ symmetry is strongly broken in nature, we believe that the interaction Lagrangians involving the pion should be constructed respecting chiral symmetry in the $SU(2)$ sector, which requires necessarily gradient couplings for the pion.

Acknowledgements: We would like to thank Che Ming Ko for useful discussions. This work has been supported by CNPq and FAPESP under contract number 1999/12987-5.
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FIGURES

FIG. 1. Diagram for the process $\pi + J/\psi \rightarrow D + D^*$ with the $D^*$ meson exchange.

FIG. 2. Cross sections for the process $\pi + J/\psi \rightarrow D + D^*$ with: $D$ meson exchange (solid line), $D^*$ meson exchange with the Lagrangian Eq. (3) (dashed line), and with the Lagrangian Eq. (2) (dash-dotted line).