Mass Gap Problem Solution
in the Superfluid Quantum Space Model

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Abstract
A given problem in physics can be solved if it is well formulated. Well formulated means that it has a bijective correspondence to physical reality. Mass Gap Problem has no bijective correspondence with the physical reality and is that’s why not solvable mathematically. It can be solved in the frame of quantum mechanics by the formulation of the photon’s mass accordingly to the Planck-Einstein relation.

Keywords: Mass Gap Problem, Planck-Einstein relation, time, superfluid quantum space, bijectivity.

1. Introduction
To solve the Yang-Mills Mass Gap Problem [1] we have to understand well what mass is. We propose a bijectivity solution for the mass-gap problem where every element in the model of physical reality has exactly one correspondent model in physical reality. The model of physical reality is set Y and physical reality is set X. Every element in set Y has exactly one element in the set X. For example, element mass \( m_x \) in the physical universe has correspondent element mass \( m_y \) in the model of the universe. They are related by the bijective function of set theory:

\[
f : m_x \rightarrow m_y \quad (1).
\]

Einstein equation \( E = mc^2 \) has bijective correspondence with the physical universe:

\[
E_x = m_x c_x^2
\]
\[ E_Y = m_Y c_Y^2 \] (2).

Every object with mass \( m \) is existing in space we call today a superfluid quantum space (SQS). SQS is not “empty”, it is the fundamental energy of the universe. Elementary particles are different structures of SQS [2]. In the space-time model of Special Relativity, the fourth coordinate \( X_4 \) is imaginary, \( X_1, X_2, X_3 \) are real coordinates. In the time-invariant space model, the 4th coordinate \( Z_4 \) is a complex coordinate as the other three coordinates \( Z_1, Z_2, Z_3 \): “Time-invariant superfluid quantum space (SQS) has a general \( n \)-dimensional complex structure \( \mathbb{C}^n \); every point of it has complex coordinates:

\[ z_i = x_i + i y_i \] (3).

\((x_i, y_i) (i = 1, \ldots, n)\) is an ordered \( n \)-uple of real numbers \((x_i, y_i) \in \mathbb{R}^n\); for the purpose of this paper, we consider its subset \( \mathbb{C}^4 \) where all elementary particles are different structures of \( \mathbb{C}^4 \). SQS and have four complex dimensions \( z \) “[2]. The model of the superfluid quantum space is close to the model of 4D superfluid quantum space-time, also named “superfluid ether” which uses quaternions as its mathematical background [3].

The idea of “empty” space as something real is harming physics for more than 100 years. It is time we demolish the idea of empty space and introduce ether back into physics. Michelson-Morley's experiment has given null results because it was carried out by the proposition that Earth is moving through a stationary ether. Ether around the physical object is moving and rotating with the physical object. Masanory research suggested that on the distance of 20000 km above the Earth ether drift could be measured: “The satellites in the higher orbit (in the yellow region) have a possibility to detect the ether-drift. The evidence of the ether-drift can be proven by the fact that the ECI coordinate system does not work well. Of course, these experiments have not been carried out yet. The discussions of the ether-drift and frame-dragging were carried out more than 100 years ago. I have not carried out any calculation of the height of the frame-dragging using the theory of general relativity. At this stage, I consider that the height of the ether-drift detected is more than 20,000 km from the ground level” [4].

Mass \( m \) is in physics an element with the attribute of energy \( E \). It is false to think that this element mass can exist in space that has no attribute of energy \( E \). The equation (4) below is false:

\[ f: empty\ space_x \rightarrow empty\ space_y \] (4).
2. Mass Gap Problem from the perspective of mass-energy principle extension on SQS and Planck-Einstein relation

Mass $m$ of a given physical object is related to the energy of SQS accordingly to the well-known physical law of homogenous distribution of energy. Every physical system tends that the energy of the system is distributed in a homogeneous way; because of this every physical object with mass $m$ is diminishing the energy density of SQS exactly for the amount of its energy $E$:

$$\frac{E}{c^2} = m = \frac{(\rho_{E\text{max}}-\rho_{E\text{min}})V}{c^2}$$ \hspace{1cm} (5) [2],

where $\rho_{E\text{max}}$ is the energy density of the SQS infinitely far away from the physical object surface, $\rho_{E\text{min}}$ is the energy density of the SQS in the centre of a given physical object and $V$ is the volume of the object. In this way energy density of SQS remains uniform. In the Newtonian perspective, the area of space with a higher density is asserting a given pressure towards the area of space with a lower density. From a quantum physics perspective, universal space has the vector orientation towards lower energy density, or in space are quantum fluctuations towards lover energy density of space.

Equation (5) is valid from the scale of the proton to the scale of AGN. It can describe the process of the formation of the jets in the AGNs. The energy density of SQS there is so low that atoms become unstable and form jets that are spreading into intergalactic space [5]. Equation (5) is the extension of the mass-energy equivalence principle on the universal space that has its origin in time-invariant superfluid quantum space SQS and can solve the mass gap problem. The variable energy density of SQS is giving the origin to the inertial mass $m_i$ and gravitational mass $m_g$ as follows in Eq. (6) below:

$$m_i = m_g = \frac{(\rho_{E\text{max}}-\rho_{E\text{min}})V}{c^2}$$ \hspace{1cm} (6),

where $\rho_{E\text{max}}$ is the energy density of the space in interstellar space; $\rho_{E\text{min}}$ is the energy density of the space in the centre of the proton, and $V$ is the volume of the proton.
Equation (6) shows that inertia and gravity are the results of the dynamics between given mass $m$ and a variable energy density of SQS. It is false to think that inertial mass $m_i$ is the same phenomenon as the rest mass $m_0$ as the amount of energy $E$. Equation (7) below is false:

$$\frac{E}{c^2} = m_0 = m_i \quad (7).$$

We can combine two fundamental equations $E = mc^2$ and equation $E = hv$ and we get:

$$E = mc^2 = hv \quad (8)$$

$$m = \frac{hv}{c^2} \quad (9).$$

Photon is massless in the sense that it has no inertial mass $m_i$. But photon has energy $E$ and so it has correspondent mass $m$. Equation (9) is showing the mass of the photon related to its frequency. Combining (5) and (9) we get:

$$m = \frac{(\rho_{E_{\text{max}}} - \rho_{E_{\text{min}}})V}{c^2} = \frac{hv}{c^2} \quad (10).$$

Out of (10) follows:

$$h = \frac{(\rho_{E_{\text{max}}} - \rho_{E_{\text{min}}})V}{\nu} \quad (11).$$
The space energy density difference $\rho_{E_{\text{max}}} - \rho_{E_{\text{min}}}$ we can express as $\Delta \rho_E = \rho_{E_{\text{max}}} - \rho_{E_{\text{min}}}$ and we get:

$$h = \frac{\Delta \rho_E V}{\nu} \quad (12).$$

Eq. (12) shows that the value $\frac{\Delta \rho_E}{\nu}$ is constant. When frequency $\nu$ is increasing delta energy density $\Delta \rho_E$ is also increasing:

![Diagram showing delta energy density](image)

**Figure 2.** An increase in frequency increases delta energy density and keeps the same volume.

The volume of the photon is introduced as an approximation to describe how much of the volume of the SQS is in excitation when we have a single photon. Frequency and wavelength are related by the formula $\nu = \frac{c}{\lambda}$, where $c$ is the light speed. When the frequency is growing, the wavelength is getting smaller. This ideal volume $V$ of the photon is independent of the photon frequency and its wavelength; see figure 2, where volume $V_1$ of the photon with the low frequency and the volume $V_2$ of the photon with high frequency are equal: $V_1 = V_2$. Eq. (12) is bijective with the physical reality. Bijectivity assures falsifiability, Eq. (12) is falsifiable.

For Yang-Mills Mass Gap Solution is required: “It must have a “mass gap;” namely there must be some constant $\Delta > 0$ such that every excitation of the space has energy at least $\Delta$ “ [1]. Eq. (12) shows this constant $\Delta > 0$ is Planck constant $h$. Namely, every particle can be seen as the excitation of the SQS and is defined by the difference of vacuum pressure $\Delta p_E$, volume $V$ and frequency $\nu$ of a given particle. These three elements are defining the Planck constant $h$. A given excitation of SQS is producing a given elementary particle with energy $E$. We know the origin of the Planck constant (Eq. 12) and we see that the minimal “mass gap” in the excitation of SQS is defined by the Planck constant $h$. In physical terms “mass gap” would
be better named as “mass-energy gap”, because Einstein told us already that mass and energy are made out of the same “stuff”. What's this “stuff” is being clearly explained by Erving Schrodinger who also regarded space as the fundamental energy of the universe: “What we observe as material bodies and forces are nothing but shapes and variations in the structure of space”. Reintroduction of the SQS model and of the mass-energy equivalence extension on the SQS are helpful tools to develop physics where physical objects are the energy structures of the medium in which they exist. The electric field is the excitation of the 4th layer of SQS along the dimensions $z_1, z_2, z_3$. The magnetic field is the excitation of the 4th layer of SQS along the dimensions $z_2, z_3, z_4$. Magnetic and electric fields have in common dimensions $z_2, z_3$. Photon is the four-dimensional wave of the 4th layer of SQS [2].

3. The existence of imaginary time in Euclidean Green’s functions is questionable

In Green’s functions imaginary time $it$ is still understood as a dimension: “In a relativistic quantum field theory the indefinite metric of Minkowski space causes many problems which could be avoided by replacing the time $t$ by $it$ or the energy $E$ by $iE$, thereby passing from Minkowski space to Euclidean space. This idea was first used by Dyson in perturbation theory. He continued the Feynman integrands analytically to imaginary energies in order to move the paths of integration away from the mass shell singularities of the causal propagators. Schwinger studied the analytic continuation of time ordered Green's functions to imaginary times and their transformation properties under the Euclidean group. He determined the Euclidean Green's functions (Schwinger functions) as solutions of certain differential equations. In axiomatic field theory it followed from investigations by Wightman, by Hall and Wightman and by Jost that the Green's functions (Wightman distributions) are boundary values of functions (Wightman functions) which are analytic in the permuted extended tubes. The Euclidean Green's functions could then be defined as the restriction of the Wightman functions to points with imaginary time and real space components” [6]. The model of imaginary time does not respect the principle of bijectivity, it has no bijective correspondence with the physical world. Bijectivity assures falsifiability. Every model that is bijective is also falsifiable. The model of imaginary time is not falsifiable and so cannot represent a stable scientific basis for the solution of the mass gap problem. Our proposal is bijective, it is based on quantum mechanics and represents the bijective solution of the mass gap problem.
SQS model is time-invariant, there is no imaginary time \( it \). All coordinates are spatial. Changes run in the SQS that is time-invariant. Time as duration enters the existence when measured by the observer [7]. Computer simulations suggest the existence of a "mass gap" in the solution to the quantum versions of the Yang-Mills equations. But no mathematical proof of this property is known [1]. Our intent in this article is to find the solution for the “Yang-Mills Mass Gap problem” from the perspective of physics by using bijectivity. Euclidean Green's functions are abstract mathematical objects which do not have bijective correspondence with the physical world. Osterwalder and Schrader are introducing Euclidean vector \( \mathbb{R}^4 \) with “time direction” that is orthogonal to the “space directions”. There is no such a dimension in the universe that would have “time direction”; this misunderstanding we have in physics since Einstein that has interpreted the time \( t \) as the 4th coordinate \( X_4 \) of a Minkowski manifold. He wrote: “If we replace \( x, y, z, \sqrt{-1} ct \) by \( x_1, x_2, x_3, x_4 \), we also obtain the result that \( ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 \) is independent of the choice of the body of reference. We call the magnitude \( ds \) the “distance” apart of two events or four-dimensional points. Thus, if we choose as time variable the imaginary variable \( \sqrt{-1} ct \) instead of the real quantity \( t \), we can regard the continuum space-time, in accordance with the special theory of relativity, as an “Euclidean” four-dimensional continuum, a result following by the consideration of the preceding section” [8]. In the above citation, Einstein suggestion that we can choose the time variable \( t \) as the imaginary variable can be written as follows:

\[
 t = \sqrt{-1} ct \quad (13).
\]

Eq. (13) is false because on the left side of the equation we have \( t \) and on the left side we have \( \sqrt{-1} ct \). Combining Eq. (14) with equation well know equation \( X_4 = ict \) we get:

\[
 X_4 = ict^2\sqrt{-1} \quad (14).
\]

Equation (14) confirms Einstein did a mistake keeping and interpreting time as the dimension of a four-dimensional continuum. Physics is still today suffering this misinterpretation of time. Einstein told us: “People like us who believe in physics know that the distinction between past, present and future is only a stubbornly persistent illusion”. Einstein was aware that there is no physical time, however, he kept time as the 4th dimension of the space-time continuum which seems wrong. There is no imaginary time and there is no “time direction” in the universe; this
means Osterwalder’s and Schrader’s mathematical model on Euclidean Green functions [6] has no bijective correspondence with the physical world and cannot contribute to the solution of the “Mass Gap”. Osterwalder’s and Schrader’s model has internal inconsistency because it considers the existence of an imaginary time \( it \). The universe exists in time-invariant SQS, also mathematics that we humans developed exists in time-invariant SQS. Keeping time in mathematical models as a dimension will not give good results. We have to develop models of physical reality where time is the duration of changes running in a time-invariant SQS and is the result of the measurement from the side of the observer.

We proposed in this article the solution of the “mass gap” problem based on the Planck-Einstein relation that occurs in the time-invariant SQS. The result is that in the view of the mass-energy equivalence principle, the minimal change of “mass-energy” excitation and so the minimal mass gap in the superfluid quantum space and is defined by Planck constant \( h \).

4. Conclusions

The mathematical solution of the “Mass Gap Problem” might not be found because by keeping imaginary time, Euclidean green’s functions do not have bijective correspondence with physical reality. Our proposal for the solution of the “Mass Gap Problem” is based on the variable energy density of superfluid quantum space and on the Planck-Einstein relation.

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