Estimation of Acoustic Channel Impulse Response at Low Frequencies Using Sparse Bayesian Learning for Nonuniform Noise Power

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ABSTRACT Sparse Bayesian learning (SBL) has been extended to estimate acoustic channel impulse responses (CIRs) at low frequencies, where matched filter (MF)-based CIR estimation suffers from low resolution due to a limited frequency band. In this study, the extended SBL was developed to account for nonuniform noise power in a signal model for the CIR via a formulation that considers inconsistent noise and multiple measurements, which cannot be handled in the conventional SBL. The extended SBL is applied to simulated and measured acoustic data and then compared with the MF and existing SBLs. With the advancement in the schemes, the time resolution and denoising are enhanced; especially, the results of the extended SBL on the simulated data show that it clearly distinguishes the two adjacent arrivals with moderate errors in the estimated time delays. Additionally, as a result of applying it to the measured array data, the extended SBL can achieve a high resolution in the CIR estimation, which retains assured arrivals with a single measurement, while some arrivals are weakened by the insufficient measurement for the extended SBL.

INDEX TERMS Acoustic channel impulse response, nonuniform noise distribution, sparse Bayesian learning, time-delay estimation.

I. INTRODUCTION

When a wave travels in a medium, the useful information of the medium between the source and receiver is embedded in channel impulse responses (CIRs) and transfer functions (TFs). Several applications, which include radar and sonar systems, exploit the CIR (or TF) for target-range estimation (time-delay estimation) and communication (channel parameter estimation) [1].

When a source waveform is known and its autocorrelation is compressed, a matched filter (MF), which correlates the source waveform with the received signal, is conventionally used for the CIR estimation. The MF improves the time resolution via the pulse compression and reduces noise that is uncorrelated with the source waveform, which is better for identifying the arrivals that compose of the CIR [2]. However, the time resolution is inversely proportional to the bandwidth of the source waveform, which restricts the application of the MF to high-frequency sources with broad bandwidths [3].

Compressive sensing (CS), which was expected to solve an underdetermined linear system by imposing a sparsity condition on the solution [4], [5], was used to improve the CIR estimation [6]. Ekanadham et al. [7] solved the linear system for CIRs using a modified CS, which is referred to as continuous basis pursuit (CBP), where time-shifted source waveforms are linearly combined to express the time-domain signal. The CBP that interpolates the time-shifted source waveforms by using the Taylor approximation or trigonometric spline outperforms the conventional CS which suffers from a basis mismatch caused by the arrivals with time delays.
that do not belong to the time-shifted source waveforms. Fyhn et al. [8] estimated the amplitudes and the time delays of arrivals in the CIRs using a CS with polar interpolation, which outperformed the CBP.

In underwater acoustics, the CS has been predominantly applied to estimate the direction of arrivals (DOAs) from underwater targets [9], [10], [11], [12], [13], [15], [15] and was introduced in the underwater acoustic channel parameter estimation by several researchers [6], [16], [17], [18], whereas the MF-based CIR estimation is a conventional approach that was employed for active sonar systems [1], [2]. The sparse solutions in the CS literature help estimate the parameters with super-resolution [12], [17], [19]. However, manually determined hyperparameters in the schemes that are associated with measurement noise are sensitive to sparsity. They affect the performance of the schemes, which hinder their practical applications.

Sparse Bayesian learning (SBL) has been introduced to address the DOA or the channel parameter-estimation problem in underwater acoustics. The SBL was originally devised for classification and regression in machine learning [20]. In the SBL framework, the probability models for unknowns and measurement are derived using Bayesian inference with hyperparameters that are related to noise and source variances (or powers), which are automatically evaluated during the iterative process to update the probability models and the variances.

The SBL can be applied to determine a solution for a linear system, which is similar to the CS. The SBL has been primarily used for the DOA estimation in underwater acoustics due to its well-established linear system that relates DOAs with measurements along multiple sensors in an array. Furthermore, a sparse solution that was obtained via the SBL provides a high-resolution DOA with suppressed noise [21], [22], [23], [24], [25]. In the DOA estimation using the SBL, the standard SBL has been extended for more accurate and reliable evaluation of the DOAs using multiple measurements, the interpolation between the discretized bases, and complex noise models with different variances along the measurements [21], [22], [23], [26], [27]. In the SBL-based DOA estimation using multiple measurements, the common supports of the unknowns along multiple measurements (i.e., the estimated DOAs) have been found due to the arrivals having consistent directions over multiple measurements [23], [27].

An inherent limitation of the SBL-based scheme arises from the discretized bases, which leads to a basis mismatch. A signal representation via interpolation using adjacent bases around the true arrival in the off-grid direction was adopted to mitigate the basis-mismatch problem, and a mismatch from the predefined bases was evaluated during the iterative process in the SBL. [21], [22]. Meanwhile, most studies that used the SBL operated on the basic assumption of uniform noise variance over the measurement. This assumption was relaxed by Gerstoft et al. [26] with an approximated updating rule for heteroscedastic noise, which enables the treatment of nonuniform noise with the SBL [27].

The CS has been previously used to estimate acoustic CIRs by utilizing the sparse arrivals in acoustic channels [6], [12], [16], [17], [19]. Meanwhile, the CS-based DOA estimations were recently advanced by the SBL, which can find the DOAs into sensor array without additional post-processing [21], [22], [23], [24], [25], [26], [27]. To the best of our knowledge, the SBL has not been used in estimating the CIRs. Thus, in this study, to extract the CIRs from a measurement (a signal emitted by a transducer, transmitted through the ocean waveguide, and received by a hydrophone), the SBL is adopted due to its properties that are exhibited in the DOA estimation as well as for free manual hyperparameters and the properties of SBL are demonstrated in the CIR estimation.

A linear system for the CIR estimation is established by expressing the frequency domain, which is the TF and the candidate arrivals have distinct time delays. Note that noise in the linear system is scaled by the source spectrum, which results in nonuniform noise powers over the measurements that are relative to the frequency response of the received signal. The conventional SBL should be modified to accommodate the nonuniform noise, which was attempted by several researchers [26], [27], [28]. In the present study, a rigorous mathematical expression that accounts for the nonuniform noise is derived, and the associated updating rule is obtained, which is different from the previous studies by using the heuristic approach [26], [27] or considering only the real values and a single measurement [28]. Particularly, in [26] and [27], the previously established stochastic likelihood function was exploited, which provided an asymptotically efficient estimate of noise. To the best of our knowledge, no studies have been conducted on the SBL where the basic expressions for noise are rigorously extended for nonuniform noise to estimate the amplitudes and the time delays of arrivals in the CIRs. In Section II, the linear system for the CIRs is derived in the frequency domain using the definition of the TF. The conventional SBL is extended to account for the nonuniform noise powers in the linear system, which is discussed in Section III. The novel SBL is examined in terms of the time resolution enhancement and the noise reduction with simulated and measured acoustic data, which are shown in Sections IV and V, and the results are compared with the results from the MF and the existing SBLs. To investigate the performances of the conventional and the extended SBLs for uniform and nonuniform noise cases, we performed numerical experiments and discuss the validation regions of the SBLs. Section VI summarizes the present study.

II. UNDERWATER ACOUSTIC CHANNEL IMPULSE RESPONSE

A. ACOUSTIC CHANNEL IMPULSE RESPONSE USING AN MF

When underwater sound that is emitted from a source is transmitted through the ocean, the signal that arrives at the receiver contains multiple signals that pass through different paths that arise from the waveguide from the interfaces
(sea bottom and surface) and the scatterers (submerged objects and fish schools). An impulse response of the acoustic channel (acoustic CIR) is approximated by assuming an insignificant dispersion of acoustic waves during sound propagation (particularly, the Doppler effect is ignored because our attention is limited to the low frequency region), as follows [2, 6, 12, 16, 18, 19, 30]:

\[ h(t) = \sum_{k=1}^{K} a_k \delta(t - t_k), \] (1)

where \( a_k \) and \( t_k \) are the amplitude and time delay of arrival, respectively, and \( K \) is the number of (dominant) arrivals. The received signal, \( y(t) \), is represented by the convolution between the source waveform \( s(t) \) and the acoustic CIR \( h(t) \) with ubiquitous noise \( n(t) \) as follows:

\[ y(t) = \sum_{k=1}^{K} a_k s(t - t_k) + n(t). \] (2)

A known source waveform is exploited to estimate the CIR when operating an active sonar system, which uses a transducer and a receiver that have almost flat frequency responses in the source bandwidth. By using the Fourier transform (FT), (2) is denoted in the frequency domain as follows:

\[ Y(\omega) = S(\omega) H(\omega) + N(\omega), \] (3)

where \( S(\omega) \), \( H(\omega) \), and \( Y(\omega) \) are the FTs of the source waveform, CIR, and received signal, respectively. For a received signal without noise, the CIRs can be obtained via the inverse FT of \( Y(\omega) / S(\omega) \). However, the CIR cannot be directly evaluated due to the ubiquitous noise, \( N(\omega) \). Alternatively, an MF using cross-correlation between the source and the received signals is applied to estimate the CIR. The MF exhibits both advantages in regards to distinguishing the arrivals and enhancing the SNR by exploiting the frequency band of the source signal (pulse compression), whereas its performance is degraded at low frequencies due to the utilization of a limited frequency band.

**B. ACOUSTIC CHANNEL ESTIMATION USING SBL**

To overcome the limitation of the MF, an SBL applicable to solve a linear system is used for a high-resolution CIR estimation at low frequencies that uses its sparse solution. The linear system in this study is derived with the approximation of \( H(\omega) \) in (3) as follows [2, 6, 12, 16, 18, 19, 30]:

\[ \hat{Y}(\omega) = H(\omega) + \hat{N}(\omega) \approx \sum_{n=1}^{N} x_n e^{-j\omega n \Delta t} + \hat{N}(\omega), \] (4)

where \( \hat{Y}(\omega) \) and \( \hat{N}(\omega) \) are \( Y(\omega) / S(\omega) \) and \( N(\omega) / S(\omega) \), respectively. For the approximation, the time delay in the continuous domain is discretized with the grid of \( \Delta t \), which is equal to \( 1/f_s \), where \( f_s \) is the sampling frequency and \( N \Delta t \) is the largest observation time of the received signal. When \( n \Delta t \) in (4) matches \( t_k \) in (1) with a sufficiently fine grid, the corresponding amplitude \( x_n \) is equivalent to \( a_k \); otherwise, \( x_n \) becomes zero. The number of non-zero elements is the number of arrivals \( K \). By arranging (4) according to the source frequency band, the linear system is established as follows:

\[
\begin{pmatrix}
\hat{Y}_1 \\
\vdots \\
\hat{Y}_M
\end{pmatrix} =
\begin{pmatrix}
e^{-j\omega_1 \Delta t} & \cdots & e^{-j\omega_M \Delta t} \\
\vdots & \ddots & \vdots \\
e^{-j\omega_{M-1} \Delta t} & \cdots & e^{-j\omega_M \Delta t}
\end{pmatrix}
\begin{pmatrix}
x_1 \\
\vdots \\
x_N
\end{pmatrix}
+ \begin{pmatrix}
\hat{N}_1 \\
\vdots \\
\hat{N}_M
\end{pmatrix}.
\] (5)

Equation (5) is rearranged by adopting the following vector and matrix notation: \( y = Ax + n \), where \( y, x, n \), and \( A \) are the measurement, unknown, noise, and transformation matrix, respectively. It is worth noting that the signal model cannot be in the form of a linear system without dividing \( S(\omega) \) in (3), because the corresponding unknown that has \( S(\omega) x_n \) as an element varies according to the equality conditions. The measurement element \( \hat{Y}_m \) and the noise element \( \hat{N}_m \) are \( \hat{Y}(\omega) \) and \( \hat{N}(\omega) \) at the angular frequency \( \omega_m \), respectively. \( \omega_1 \) and \( \omega_M \) correspond to the lowest and highest frequencies, respectively, which are used for the SBL-based CIR estimation and belong to the source frequency band. The number of equations, \( M \), is smaller than the number of unknowns, \( N \). The SBL is used to determine a solution of the underdetermined linear system (i.e., the amplitudes and the time delays of arrivals that constitute of the CIR).

In contrast, a linear system can be constructed in the time domain, where the transformation matrix comprises of the time-shifted source waveforms as its columns and \( M \) is equal to \( N \). In this study, the solution of the frequency domain linear system is preferred because less equality conditions cause a more refined CIR with a sparser solution by recovering less meaningful arrivals than the given equality conditions.

Note that the noise powers in (5) (i.e., variances of \( \hat{N}_m \)) are inconsistent due to the fluctuating ocean noise spectrum being scaled by the source spectrum [29], which violates the assumption of constant noise power over the measurements in the conventional SBL [20]. Therefore, the conventional SBL should be modified to treat the nonuniform noise powers, which is described in the following section. Hereafter, the conventional and expanded SBL are referred to as uniform noise SBL (UN-SBL) and nonuniform noise SBL (NN-SBL), respectively.

**III. SBL FOR NONUNIFORM NOISE POWER**

An SBL was first introduced by Tipping [20] for classification and regression in machine learning. It was later adopted for high-resolution beamforming in underwater acoustics, where noise powers over measurements (frequency domain signals at sensors in array) are assumed to be constant, as in the original paper, except for a few studies [26, 27], where the formulations in the SBL were modified approximately to treat the nonconstant noise. However, in this study, the SBL is rigorously expanded by deriving mathematical expressions for the probabilities in the SBL for inconsistent noise powers.
regarding complex value and multiple measurements. Afterwards, the results for the SBL using different approaches for the nonconstant noise are compared with each other.

In the SBL framework, the unknown \( \mathbf{x} \) and noise \( \mathbf{n} \) are treated as random vectors, which consist of independent random variables, and they are assumed to follow a zero-mean Gaussian distribution that has different variances (or different powers), which are hidden variables inferred using measurement \( \mathbf{y} \). A linear system in the SBL is solved by finding \( \mathbf{x} \), which maximizes the probability as follows [20]:

\[
\mathbf{x}_{est} = \arg\max_{\mathbf{x}} p(\mathbf{x}, \mathbf{y}_s, \mathbf{y}_n | \mathbf{y})
\]

\[
\mathbf{x}_{est} = \arg\max_{\mathbf{x}} p(\mathbf{x} | \mathbf{y}_s, \mathbf{y}_n) p(\mathbf{y}_s, \mathbf{y}_n | \mathbf{y}).
\] (6)

In this study, \( \mathbf{y}_s \) is a vector that comprises of the signal powers along the time delays, and \( \mathbf{y}_n \) is a vector that comprises of the noise powers along the frequencies that belong to the source frequency band. Unlike the previous studies [20], [21], [22], [23], a constant noise power in the UN-SBL is replaced with vector \( \mathbf{y}_n \) in the NN-SBL to account for the different noise variances over the source frequency band. As displayed in the second line in (6), the estimation is conducted in two phases. First, using a given measurement \( \mathbf{y} \), the variances are obtained by maximizing \( p(\mathbf{y}_s, \mathbf{y}_n | \mathbf{y}) \), which is equivalent to maximizing \( p(\mathbf{y} | \mathbf{y}_s, \mathbf{y}_n) \) when \( \mathbf{y}_s \) and \( \mathbf{y}_n \) are uniformly distributed, which is the situation in this study, and there are no preferences for specific components in \( \mathbf{y}_s \) and \( \mathbf{y}_n \) [20]. The solution \( \mathbf{x}_{est} \) is then obtained with the maximum a posteriori estimate of \( p(\mathbf{x} | \mathbf{y}, \mathbf{y}_s, \mathbf{y}_n) \) using the measurement with the variances from the previous phase.

A. PROBABILITY MODELS FOR NN-SBL

The NN-SBL starts by deriving an analytic expression for the posterior probability \( p(\mathbf{x} | \mathbf{y}, \mathbf{y}_s, \mathbf{y}_n) \), which is denoted below using Bayes’ theorem:

\[
p(\mathbf{x} | \mathbf{y}, \mathbf{y}_s, \mathbf{y}_n) = \frac{p(\mathbf{y} | \mathbf{x}, \mathbf{y}_s, \mathbf{y}_n) p(\mathbf{x} | \mathbf{y}_s, \mathbf{y}_n)}{p(\mathbf{y} | \mathbf{y}_s, \mathbf{y}_n)},
\] (7)

where \( p(\mathbf{y} | \mathbf{x}, \mathbf{y}_s, \mathbf{y}_n) \) is the likelihood function, and \( p(\mathbf{x} | \mathbf{y}_s, \mathbf{y}_n) \) is the prior function. Furthermore, \( p(\mathbf{y} | \mathbf{y}_s, \mathbf{y}_n) \) is the evidence (marginal likelihood) that is used to evaluate the hidden variables of the variances.

The noise is an independent random variable that follows a zero-mean circularly symmetric complex Gaussian distribution, so the likelihood function is expressed as follows using the relation between the measurement and the noise, which is based on (5):

\[
p(\mathbf{y} | \mathbf{x}, \mathbf{y}_n) = \frac{1}{\pi^M} \prod_{m=1}^{M} (\mathbf{y}_n)_m
\]

\[
\times \exp \left\{ - \sum_{m=1}^{M} \frac{|(\mathbf{y} - \mathbf{A}\mathbf{x})_m|^2}{(\mathbf{y}_n)_m} \right\}
\]

\[
= \frac{1}{\pi^M |\mathbf{\Gamma}_n|} \exp \left\{ - (\mathbf{y} - \mathbf{A}\mathbf{x})^H \mathbf{\Gamma}_n^{-1} (\mathbf{y} - \mathbf{A}\mathbf{x}) \right\}.
\] (8)

where \( (\mathbf{v})_m \) and \( (\mathbf{v})^H \) denote the \( m \)th element and the conjugate transpose (or Hermitian transpose) of vector \( \mathbf{v} \), respectively. \( \mathbf{\Gamma}_n \) is a diagonal matrix that results from the independent noise making off-diagonal components zeros, and its diagonal components are \( \mathbf{y}_n \) (i.e., \( \mathbf{\Gamma}_n= \text{diag}(\mathbf{y}_n) \)).

The prior corresponds to the distribution of the unknown comprising independent random variables as follows:

\[
p(\mathbf{x} | \mathbf{y}_s, \mathbf{y}_n) = \frac{1}{\pi^M |\mathbf{\Gamma}_n|} \exp \left\{ - \sum_{m=1}^{M} \frac{(\mathbf{x}_m)^2}{(\mathbf{y}_n)_m} \right\}
\]

\[
= \frac{1}{\pi^M |\mathbf{\Gamma}_s|} \exp \left( -\mathbf{x}^H \mathbf{\Gamma}_s^{-1} \mathbf{x} \right),
\] (9)

where \( \mathbf{\Gamma}_s \) is a diagonal matrix whose diagonal components equal \( \mathbf{y}_s \) (i.e., \( \mathbf{\Gamma}_s= \text{diag}(\mathbf{y}_s) \)).

The components of \( \mathbf{x} \) are activated when the corresponding components of \( \mathbf{y}_s \) have non-zero values. During the application of the SBL for solving the linear system, \( \mathbf{y}_s \) appears sparsely, which is advantageous to enhance the resolution and suppressing noise [21], [22], [23], [26], [27].

The evidence can be derived using \( \mathbf{y}_s \) and \( \mathbf{y}_n \) with a linear system. The Gaussian-distributed unknown and noise in the linear system render the evidence normally distributed as follows:

\[
p(\mathbf{y} | \mathbf{y}_s, \mathbf{y}_n) = \frac{1}{\pi^M |\mathbf{\Sigma}_y|} \exp \left( -\mathbf{y}^H \mathbf{\Sigma}_y^{-1} \mathbf{y} \right).
\] (10)

The mean of \( \mathbf{y} \) is equal to a zero vector because each mean of the unknown and noise is the zero vector, \( \langle \mathbf{y} \rangle = \mathbf{A}\mathbf{x} + \mathbf{n} = \mathbf{0} \). \( \mathbf{\Sigma}_y \) is calculated with the definition of the covariance matrix as follows:

\[
\mathbf{\Sigma}_y = \langle (\mathbf{A}\mathbf{x} + \mathbf{n})(\mathbf{A}\mathbf{x} + \mathbf{n})^H \rangle = \mathbf{A}\mathbf{\Sigma}_x\mathbf{A}^H + \mathbf{\Gamma}_n.
\] (11)

Note that the nonuniform noise power leads to different forms of the likelihood function and the evidence from them in the UN-SBL, which subsequently result in a distinct posterior probability.

By substituting (8)–(10) in (7), the posterior probability that considers the inconsistent noise variances is derived using the Woodbury matrix identity as follows:

\[
p(\mathbf{x} | \mathbf{y}, \mathbf{y}_s, \mathbf{y}_n) = \frac{1}{\pi^M |\mathbf{\Sigma}_x|}
\]

\[
\times \exp \left\{ - (\mathbf{x} - \mathbf{\mu}_x)^H \mathbf{\Sigma}_x^{-1} (\mathbf{x} - \mathbf{\mu}_x) \right\}.
\] (12)

where \( \mathbf{\Sigma}_x = (\mathbf{\Gamma}_s^{-1} + \mathbf{A}^H \mathbf{\Gamma}_n^{-1} \mathbf{A})^{-1} \) and \( \mathbf{\mu}_x = \mathbf{\Sigma}_x\mathbf{A}^H \mathbf{\Gamma}_n^{-1} \mathbf{y} \). As expected, the posterior probability follows a Gaussian distribution, and its mean and covariance matrix are calculated using the estimated \( \mathbf{y}_s \) and \( \mathbf{y}_n \) with measurement \( \mathbf{y} \). \( \mathbf{x}_{est} \) is equivalent to \( \mathbf{\mu}_x \), where the posterior probability is the maximum.

B. ESTIMATE OF THE VARIANCES \( \mathbf{y}_s \) AND \( \mathbf{y}_n \)

As previously described, \( \mathbf{y}_s \) and \( \mathbf{y}_n \) must be estimated to obtain the solution via the SBL. The variances that best
describe the given measurement are used for the solution, and
they are calculated as follows:
\[
(\bar{y}_s, \bar{y}_n) = \arg \max_{y_s, y_n} p(y_s, y_n | y). \tag{13}
\]

\[
p(y_s, y_n | y) = p(y | y_s, y_n) p(y_s) p(y_n). \tag{14}
\]

When there are no preferences over the signal and noise, which is the
situation in this study (i.e., both \(p(y_s)\) and \(p(y_n)\) follow a
uniform distribution) \cite{20}, \ref{13} becomes the same problem
of finding \(y_s\) and \(y_n\) to maximize \(p(y | y_s, y_n)\) as follows:
\[
(\hat{y}_s, \hat{y}_n) = \arg \max_{y_s, y_n} p(y | y_s, y_n). \tag{14}
\]

Herein, the expectation-maximization (EM) algorithm is applied to estimate \(y_s\) and \(y_n\), where \(x\) is treated as a latent
variable. The formulation for the EM-based estimation is as
follows \cite{20, 21, 22}:
\[
Q(y_s, y_n) = E_{y_s, y_n} \left[ \ln \frac{p(y | y_s y_n)}{p(y)} \right]
= E_{y_s, y_n} \left[ \ln \frac{p(y) p(y_s) p(y_n)}{p(y)} \right]. \tag{15}
\]

Equation \ref{15} is equivalent to \ref{14}. The second line enables the
variances to be estimated separately, so the conditional probabilities
for \(x\) and \(y\) are used to evaluate \(y_s\) and \(y_n\), respectively.

The expectation of \(\ln p(x | y_s)\) in the perspective of the posterior
probability is stated as follows, which omits the terms that are
irrelevant to \(y_s\):
\[
Q_s(y_s) = -\ln |\Sigma_s| - \langle x | \Sigma_s^{-1} x \rangle
= -\ln |\Sigma_s| - \text{tr} \left[ \Sigma_s^{-1} (\Sigma_s + \mu_s \mu_s^H) \right], \tag{16}
\]
where \(\text{tr}(M)\) is the trace of matrix \(M\). \(y_s\) is achieved
determining the vector that corresponds to the maximum of \(Q_s(y_s)\) as follows:
\[
(\gamma_s)_n = (\Sigma_s)_{nn} + |(\mu_s)_n|^2. \tag{17}
\]

\(M_{nn}\) is the \(n\)th diagonal component of the matrix \(M\). Using
\ref{17}, \(y_s\) (or \(\Gamma_s\)) is computed element-wise.

The expectation of \(\ln p(y | y_n)\) from the perspective of the posterior
probability is denoted as follows, which the terms relevant to \(y_n\) are
retained:
\[
Q_n(y_n) = -\ln |\Gamma_n| - \langle y | \Gamma_n^{-1} y \rangle
= -\ln |\Gamma_n| - \langle y - \mu_n | \Gamma_n^{-1} (y - \mu_n) \rangle
- \text{tr} \left[ \Sigma_n \Gamma_n^{-1} \right]. \tag{18}
\]

The extremum of the function \(Q_n(y_n)\) is obtained as follows:
\[
(\gamma_n)_m = |(y - \mu_n)_m|^2 + A_m \Sigma_m A_m^H, \tag{19}
\]

where \(A_m\) is the \(m\)th row of matrix \(A\). The noise variance \((\gamma_n)_m\)
is the noise power in the \(m\)th measurement, and it comprises
of two terms. The first is about the difference between the
\(m\)th measurement and the associated estimation, and the other
is related to the remnant weighted by the \(m\)th row of the
transformation matrix. When the noise powers have the same
value as in the UN-SBL, the constant noise power \(\bar{\gamma}_n\)
is expressed as follows:
\[
\bar{\gamma}_n = \frac{1}{M} \left\{ \| y - A \mu_x \|^2 + \text{tr} (A \Sigma_x A^H) \right\} \tag{20}
\]

\(\| v \|_2\) is the Euclidean norm of the vector \(v\). This is the
average of the noise powers over the measurement (i.e.,
\(\bar{\gamma}_n = \frac{\sum_{m=1}^M (\gamma_m)_m}{M}\) and it equals the previous result
\ref{22} in Ref. \cite{21} for a single measurement). The noise
power was previously estimated with a representative value as
the average, whereas, the power was expressed individually
according to the measurement component in this study. When
the variation of the noise powers is noticeable, \ref{19} should
accommodate the probabilities of \(y, \) which was considered
in this study. This is achieved at the cost of the increased number
of variables (the noise variances), and it is proportional to the
equality conditions in \ref{5}.

To estimate \(y_s\) and \(y_n\), the posterior probability
\(p(x, y_s, y_n)\) (or \(\mu_x\) and \(\Sigma_x\)) is required, and it is expressed
using \(y_s\) and \(y_n\) with the measurement, which they are
entangled to each other. Thus, the first step in the EM
algorithm is to generate \(p(x, y_s, y_n)\) with initial small
values \(y_s\) and \(y_n\) (initial guesses for \(y_s\) and \(y_n\)),
which allows \(\Sigma_x\) to be non-singular. In the next step, using \(\mu_x\)
and \(\Sigma_x\) in the posterior probability, \(\bar{y}_s\) and \(\bar{y}_n\) are computed
using \ref{17} and \ref{19}, respectively, which are used to update
the posterior probability. The expectation and maximization
steps are iterated until convergence, and they correspond to
the first and second terms in the second line of \ref{6}. The final
\(\bar{y}_s\) and \(\bar{y}_n\) are used to evaluate \(x_{est}\), which is equivalent to
the final \(\mu_x\).

C. EXTENSION OF NN-SBL FOR MULTIPLE
MEASUREMENTS

Unlike the previous studies \cite{20, 21, 22, 23}, the noise
power is estimated using the corresponding element in the
measurement vector, which reduces the performance of the
NN-SBL at low SNRs. To alleviate this problem, multiple
measurements can be exploited when the underwater sounds
are repeatedly emitted from a transducer for a relatively short
duration or in a stationary ocean environment.

The NN-SBL that uses a single measurement has been
expanded to exploit the common time delays of arrivals over
the multiple measurements \cite{22, 23}. The first step for the
expansion is to modify the posterior probability to accommoda-
tate the multiple measurements as follows:
\[
p(X | Y, y_s, y_n) = \frac{p(Y | X, y_n) p(X | y_s)}{p(Y | y_s, y_n)}. \tag{21}
\]

where the measurement matrix \(Y\) comprises of the mea-
surement vectors as its columns and an unknown matrix \(X\)
comprises of the corresponding unknown vectors \(Y = [y_1, \ldots, y_L]\)
and \(X = [x_1, \ldots, x_L]\). \(L\) is the number of
measurements that are used for the estimation. The random
vectors of \(x_l\) and \(y_l\) are assumed to be independent of each

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other, and the likelihood function, which is the prior probability, and evidence for multiple measurements can then be stated as follows:

\[
p(Y | X, y_n) = \prod_{l=1}^{L} \frac{1}{\pi^M |\Gamma_n|} \exp \left\{ - (y_l - \bar{x}_l) \Gamma_n^{-1} (y_l - \bar{x}_l) \right\}.
\]

(22)

\[
p(X | y_s) = \prod_{l=1}^{L} \frac{1}{\pi^N |\Gamma_s|} \exp \left\{ - x_l \Gamma_s^{-1} x_l \right\}.
\]

(23)

\[
p(Y | y_s, y_n) = \prod_{l=1}^{L} \frac{1}{\pi^M |\Sigma_s|} \exp \left\{ - y_l \Sigma_s^{-1} y_l \right\}.
\]

(24)

The measurement \( y_j \) and the corresponding unknown \( x_i \) vary over the duration of the measurement due to noise and subtle variations in the ocean environment. However, \( \Gamma_n, \Gamma_s, \) and \( \Sigma_s \) (or \( y_n \) and \( y_s \)) remain constant (hereafter, subscript \( l \) indicates the dependence of the measurement). By substituting (22)–(24) in (21), the posterior probability for the multiple measurements is derived as follows:

\[
p(X | Y, y_s, y_n) = \prod_{l=1}^{L} \frac{1}{\pi^N |\Sigma_s|} \exp \left\{ - (x_l - \mu_x)_l \Sigma_s^{-1} (x_l - \mu_x)_l \right\}.
\]

(25)

\( \mu_x \) is the \( l \)th column of \( \mu_X = \Sigma_s A^H \Gamma_n^{-1} Y \): \( \mu_x = \Sigma_s A^H \Gamma_n^{-1} y_l \). As in the single-measurement SBL, the posterior probability is used for deriving \( \hat{y}_s \) and \( \hat{y}_n \).

Updating rules for \( \hat{y}_s \) and \( \hat{y}_n \) in the multiple measurement SBL are based on the expectations of \( \ln p(X | y_s) \) and \( \ln p(Y | X, y_n) \) from the perspective of the posterior probability, respectively, and their maxima are at \( \gamma_s \) and \( \gamma_n \), as follows:

\[
(\gamma_s)_n = (\Sigma s_m + \frac{1}{L} \| \mu_X \|_F^2)^{-1}.
\]

(26)

\[
(\gamma_n)_m = \frac{1}{L} || Y - A \mu_X ||_F^2 + \Lambda_m \Delta_s A^H.
\]

(27)

As shown in (26) and (27), the means of the signal and the noise powers over multiple measurements are used for the update, which benefits the estimation at low SNRs. In contrast, the average of the noise variances over frequency components and multiple measurements can be used to derive the updating rule for the uniform noise case as follows:

\[
\tilde{y}_n = \frac{1}{M} \{ \frac{1}{L} || Y - A \mu_X ||_F^2 + \text{tr} \left( \Delta_s A^H \right) \}.
\]

(28)

where \( ||M||_F \) is the Frobenius norm of matrix \( M \).

When multiple measurements are exploited, \( \mu_X \) obtained using \( \hat{y}_s \) and \( \hat{y}_n \) from the EM algorithm, which comprises columns that have non-zero elements at the same locations. In the SBL using multiple measurements, \( \bar{x}_{est} \) is computed with the average of \( \mu_X \) along the measurements (column axis), and the shared common supports improve the robustness of the NN-SBL.

### IV. PERFORMANCE OF THE CHANNEL IMPULSE RESPONSE ESTIMATOR USING NONUNIFORM NOISE SBL

#### A. IDEAL TF WITH UNIFORM AND NONUNIFORM NOISE POWER

The performance of the CIR estimator using the NN-SBL is examined using simulated data that considers two different situations with constant and nonconstant noise variances in the measurements. When generating synthetic data (measurement), an ideal TF using (5) neglecting acoustic wave distortion during propagation is applied, and the time delay of arrival on the grid of the observation time discretized with a sampling frequency. This experimental setup allows for the direct performance demonstration of the SBL-based CIR estimator according to the SNR, which excludes the pulse distortion and the basis-mismatch effects on the estimator, which is shown in the later numerical experiments.

A single arrival is presumably in a received signal for the convenient evaluation of the errors from the SBLs. The longest observation time was 200 ms, and the signal with a frequency band between 250 Hz and 750 Hz, which correspond to the lowest and highest frequencies in (5), respectively, randomly arrives between 20 ms and 180 ms in the simulation. The ideal TF without noise is produced with the transformation matrix multiplied by \( x \), which comprises of a single non-zero element (value of one) that corresponds to the random arrival time. The transformation matrix is formed with the discretized time delays (along columns) and the angular frequencies (along rows) using a sampling frequency of 3,000 Hz (four times the maximum frequency). There are 601 unknowns (\( N \)) and 100 equality conditions (\( M \)).

The following noise is added to the noise-free ideal TF to examine the robustness of the SBL-based estimator.

\[
\tilde{N}_{m,l} = \tilde{N}_P (\omega_m) \{ U (-0.5, +0.5) + i U (-0.5, +0.5) \},
\]

(29)

where \( U(a, b) \) is the uniform distribution between \( a \) and \( b \). The subscripts \( m \) and \( l \) denote the \( l \)th measurement at \( \omega_m \). \( \tilde{N}_P (\omega_m) \) determines the SNR of the measurement at \( \omega_m \). The noise variances are constant over multiple measurements due to the stationary ocean environment, so \( \tilde{N}_P (\omega_m) \) is independent of the measurement time (or measurement number \( l \)). For a simulation with uniform noise power, \( \tilde{N}_P (\omega_m) \) becomes

---

**TABLE 1. NN-SBL algorithm.**

| Step | Action |
|------|--------|
| 1    | Initialize: \( y_s = 0.001 \), \( y_n = 0.001 \), \( \epsilon_{\text{min}} = 10^{-5} \), \( j_{\text{max}} = 200 \) |
| 2    | \( j = j + 1 \) if \( y_n^j \neq y_n^{j-1} \) and \( y_s^j \neq y_s^{j-1} \) |
| 3    | \( \Sigma_s = (\Gamma_n^{-1} + A^H \Gamma_n^{-1} A)^{-1} \) |
| 4    | \( \mu_x = \Sigma_s A^H y \) |
| 5    | \( y_n^j = (y_s^j - \mu_x^j)^2 + \Lambda_m \Sigma_s A^H \) |
| 6    | \( y_s^j = \frac{1}{L} || Y - A \mu_X ||_F^2 + \text{tr}(\Delta_s A^H) \) |
| 7    | Output: \( \mu_x, y_s^j, y_n^j \) |

---

**TABLE 2. NOISE POWER.**

| Noise Power | Description |
|-------------|-------------|
| \( P_m \)   | Measured noise power at \( \omega_m \) |
| \( P_n \)   | Noise power at\( \omega_n \) |
| \( P_e \)   | Estimated noise power at \( \omega_e \) |
constant over the angular frequencies as well as the measurement time.

As mentioned earlier, \( \hat{N}_{m,l} \) is defined by the FT of the time-domain noise divided by that of the source waveform. Thus, \( \hat{N}_{m,l} \) is used to compute the SNR in the simulation. A specific SNR at \( \omega_m \) and an average SNR over \( \omega_m \) are calculated using the random noise with the equations as follows (the SNR\(_s\) are referred as SNR\(_m\) and SNR\(_a\)):

\[
\text{SNR}_m = -10 \log_{10} \left( \frac{1}{L} \sum_{l=1}^{L} |\hat{N}_{m,l}|^2 \right),
\]

\[
\text{SNR}_a = -10 \log_{10} \left( \frac{1}{ML} \sum_{m=1}^{M} \sum_{l=1}^{L} |\hat{N}_{m,l}|^2 \right).
\]

SNR\(_m\) and SNR\(_a\) represent the variation in the SNR along the angular frequencies and the representative SNR for a trial in the simulation, respectively.

First, the NN-SBL is applied to the CIR estimation when uniform noise power exists along the frequency components of the measurement, and the result from the NN-SBL is compared with that from the UN-SBL. Twenty multiple measurements \( (L = 20) \) were used for the estimation of the CIRs. The received signal with a random arrival time was contaminated with uniform random noise for each trial, which \( \hat{N}_p \) is a constant. An error is evaluated with the absolute difference of the true delay from the estimated delay that corresponds to the maximum amplitude from the SBL. The errors are calculated according to the various values of \( \hat{N}_p \) to examine the robustness of the SBLs (Fig. 1(a)). For a fixed \( \hat{N}_p \), the trial is repeated 50 times; the means of SNR\(_a\) and estimation error are values on the x and y axes of Fig. 1(a), respectively. While the noise that follows uniform distribution in the simulation deviates from the assumption of SBL (Gaussian distribution), the SBLs show superior performances as in [26], [27], and [28]. Some estimation results at SNRs less than \(-11\) dB for both SBLs have the largest peaks at time delays that are different from the true values, which increase the estimation error. As expected, the error becomes insignificant with an increase in the SNR (or a decrease in \( \hat{N}_p \)). At the SNRs above \(-11\) dB, the SBLs accurately estimate the time delays, and the estimation error becomes almost zero, which the order of error is \(10^{-14}\) ms. Fig. 1(b) shows the CIR from the SBLs for a specific trial with an SNR\(_a\) of approximately \(-14\) dB. While minor peaks appear and the maximum amplitude is considerably less than the true value of 1 in the CIR, the arrival can be clearly distinguished from the noise, and the time delay that corresponds to the maximum amplitude matched the true value (vertical dash-dotted line in Fig. 1(b)).

Fig. 1(c) includes the corresponding noise variances of the CIRs in Fig. 1(b). They are calculated using (27) and (28) for the NN-SBL and the UN-SBL, respectively. They are then compared with the measured noise variances based on (30). \( \hat{N}_p \) is independent of the frequency, so the measured noise variances fluctuate slightly around \(-14\) dB, which is in good agreement with the results from the NN-SBL. The constant noise variances from the UN-SBL are located at the center of the fluctuation. While the NN-SBL requires more noise variances to be estimated using the EM algorithm with the same number of equality conditions, it delivers a performance that is similar to that the performance of the UN-SBL in the environment of uniform noise power due to the sufficient measurements, as shown in Fig. 1. The two SBLs show different trends in regards to estimating the CIRs under insufficient measurements. The NN-SBL underestimates the arrival amplitudes, which the opposite happens in the UN-SBL. Furthermore, the UN-SBL inclines to include the fake arrivals by noise due to the simplified uniform noise assumption. Each scheme has pros and cons when using insufficient measurements, which are shown in Fig. 6.

As described in (4), the noise with the TF is scaled by the FT of the source waveform, which incurs nonuniform noise variances over the measurement along the frequency.

**FIGURE 1.** The conventional and the extended sparse Bayesian learning (UN-SBL and NN-SBL) using simulated data, which included noise with constant variance over measurements along the frequency. An ideal transfer function is used for the simulation. (a) The mean estimation error evaluated by the difference between the true and the estimated time delays. (b) Channel impulse responses from the SBLs at an SNR of approximately \(-14\) dB. The vertical dash-dotted line indicates the true arrival. (c) The corresponding noise variances along the frequency, which are compared to the measured noise variances using (30).
A frequency-dependent $\hat{N}_P(\omega_m)$ is used to generate the random noise, which is given by

$$\hat{N}_P(\omega_m) = \hat{N}_C(0.8U(0,1)+0.2).$$

(32)

$\hat{N}_C$ determines the SNR of the trial. The constant, which is 0.2, ensures that $\text{SNR}_{m}$ is not too large.

From Fig. 2(a), the NN-SBL, which considers different noise variances, outperforms the UN-SBL, which has the largest estimation errors at SNRs less than $-11$ dB, which occurred in the previous case. The NN-SBL also exhibits an improved result for inconsistent noise power, when compared with Fig. 1(a), where all the measurements over the frequency are corrupted by significant noise. However, in Fig. 2(a), the nonuniform noise variances induce relatively high SNRs in parts of the measurement, which is also illustrated in Fig. 2(c), and the NN-SBL can impose more priority on the high SNR measurements when evaluating the signal powers using the EM algorithm, which leads to a better result. In contrast, the UN-SBL cannot utilize the fine measurement components by treating the complex noise with the simple noise model, and its performance is similar to the performance that is illustrated in Fig. 1(a).

While the UN-SBL has a maximum peak, which corresponds to the true arrival time discriminated from minor peaks, it underestimates the arrival amplitude at a low SNR of approximately $-14$ dB (Fig. 2(b)). This problem is alleviated by the NN-SBL exploiting the high SNR measurement components, which is shown in Fig. 2(c), where the noise powers oscillate significantly along the measurement. The noise variances from the NN-SBL are in good agreement with the measured noise variances, whereas they are simplified as a single value in the UN-SBL and deviate from the true values.

At this time, (2) is used for a more realistic simulation, where a chirp signal with a center frequency of 500 Hz (see Figs. 3(a) and 3(b)) randomly arrives at the receiver on the grids between 20 ms and 180 ms with additive white Gaussian noise (AWGN) and outliers (intensive impulse signals). Three percentages of the simulated signal are the randomly emerging outliers, and their amplitudes follow uniform distribution between five and ten times the largest value of the transmitted signal. The ideal TF with a flat response along the frequency is substituted with a TF with an uneven response that reflects the frequency-dependent attenuation [30]. The amplitudes of the TF in the frequency band that are used for the estimation are random numbers that are uniformly selected between 0.9 and 1.1. The FT of the received signal divided by that of the chirp signal is the measurement, and 20 measurements are applied for the estimation via the SBLs. As shown in Fig. 3(c), considerable peaks in the MF result arising from the noise with the outliers are significantly diminished by the NN-SBL, whereas the sparse solution from the UN-SBL also provides a clearer CIR compared to the MF result, which the noticeable peaks remain, and they are detrimental to the accurate evaluation of the true arrival. The inset displays the difference between the NN-SBL and the UN-SBL results near the true arrival.

As expected, the source spectrum causes the AWGN to have different SNRs along the frequency (Fig. 3(d)). A source frequency component with a larger value tends to yield a higher SNR and vice versa, which results in SNR having a similar shape to that of the source spectrum.

B. BASIS-MISMATCH EFFECTS ON THE PERFORMANCE OF NONUNIFORM NOISE SBL

When a sound wave propagates in the ocean, it arrives at a receiver with a time delay in the continuous domain. Furthermore, the predefined on-grid bases (or columns) of the transformation matrix induce inevitable errors in the linear system. To mitigate the problem due to the basis mismatch, a finer grid with a higher sampling frequency can be adopted, but it increases the computational burden as well as the similarity of the adjacent bases in the transformation matrix, which leads
to a very inefficient way to solve the linear system. Alternatively, the first-order Taylor expansion for the linear system can be used [21], [22], where an additional unknown vector (N dimension) for the difference between a true off-grid time delay and an estimated on-grid time delay is evaluated.

While the basis mismatch hinders the exact estimation of the CIRs, an estimated arrival from the SBL appears near a true arrival due to the orthogonal-like bases of the linear system for the CIR estimation. Thus, an on-grid SBL is adopted in this study, which uses a sampling frequency that is associated with the desired time resolution at the cost of the estimation error.

The similarity of two bases calculated via their inner product after normalization is given by

$$S(\Delta n) = \left| \frac{1}{M} \sum_{m=0}^{M-1} e^{-j2\pi \frac{m}{N} \Delta n} \right| \approx \left| \text{sinc} \left( \frac{M}{N} \Delta n \right) \right|,$$

where $\Delta n$ is the index difference between the time delays that corresponds to the bases. $\text{sinc}(x)$ is the normalized sinc function, which is defined as $\text{sinc}(x) = \sin(\pi x) / \pi x$. The exact expression for similarity is approximated with the assumption of $\Delta n \ll N$ for a convenient analysis. $M/N$ is directly proportional to the ratio of the source frequency band to the largest frequency for the discretized signal, so the sinc function becomes sharp for a broadband source signal, which forms more orthogonal-like bases. The broadband source is better for an SBL-based CIR estimation, which is similar to an MF.

The similarity decreases with an increase in $\Delta n$, which is shown in Fig. 4(a), and it includes similarities for all pairs of two columns in the transformation matrix, which is used for Figs. 1–3. Thus, the arrival with an off-grid time delay is represented by the bases near the arrival.

To demonstrate the orthogonal-like property of the bases in the CIRs, the estimation errors are computed using two different sampling frequencies of 3,000 Hz and 6,000 Hz, as shown in Fig. 4(b), where the chirp signal, as shown in Fig. 3, is used for the source waveform, and AWGN is added to the received signal that is computed from the convolution of the source waveform with the CIRs that have a time delay in the continuous domain as well as an uneven frequency response, as shown in Fig. 3. The overall estimation errors increase due to the basis mismatch as opposed to Fig. 2(a), where an ideal TF with an on-grid time delay is used for the simulation. However, the largest estimation error of 0.16 ms occurs at an SNR of $-16$ dB with a low sampling frequency. It is approximately half of the time difference between the two consecutive samples (sampling interval $1/f_s$) discretized with a sampling frequency of 3,000 Hz. This indicates that the major arrivals that were estimated from the SBL are located near the true arrival even at the lowest SNR of the simulation. As the SNR increases, both estimation errors for the different sampling frequencies decrease and converge to 0.08 ms and 0.04 ms, which are equal to a
quarter of the sampling intervals that corresponds to the sampling frequencies of 3,000 Hz and 6,000 Hz. As previously mentioned, a time delay in the simulation is random in the continuous domain, and its absolute time difference from the closest discrete time delay is between 0 and 0.5Δτ. When the estimated arrivals from the SBL are on the nearest grids of true arrivals at a specific SNR, the average of their time delays becomes a quarter of the sampling interval due to the absolute time difference between the true and the estimated arrivals following a uniform distribution of $U(0, 0.5\Delta\tau)$. While the sampling frequency determines the time resolution with respect to the ability to separate the adjacent arrivals in the SBL, a low sampling frequency (coarse bases) does not result in the incorrect detection of a signal far from the true arrival. The SBL can determine the best solution of the linear system at a sufficient SNR, and it is irrespective of the sampling frequency, as shown in Fig. 4(b). It also suffers less from the basis mismatch, which is unlike the sparse signal reconstruction-based beamforming [9], [13], [14], [15], [21], [22], [23], [24], [25], [26], [27].

**C. Examining Time Resolution of Nonuniform Noise SBL at Low Frequency**

When transmitting and receiving sound waves in a water tank, a low-frequency source waveform with a limited frequency band is not preferred due to the insufficient pulse compression that prevents the separation of the arrivals that are in close proximity. However, to investigate the low-frequency acoustic impedance of a submerged object in a water tank, a low-frequency source is indispensable, and a scheme to overcome the problem of the MF is required, which is the motivation for the present study.

To examine the time resolution of the NN-SBL at low frequencies, a numerical experiment to distinguish two adjacent arrivals is conducted. In the numerical experiment, a transducer and a hydrophone are 2 m and 1 m away from the center of the rigid target (thin steel plate with 1.5 m × 1.5 m), respectively. Also, the sound speed of the surrounding target is constant at 1500 m/s, and the high- and the low-frequency Hanning-weighted four period sine signals that have the same frequency bands as their center frequencies are used as the source waveforms, in which is similar to the study that was conducted by Choo and Song [31] that enabled a convenient comparison of the time resolutions at two different source frequencies (1 kHz and 10 kHz). To simulate a scattered signal from the target, the Helmholtz integral was used with the Kirchhoff approximation for both frequencies, which was valid in the high-frequency region. Equation (4) in Ref. [31] was modified to account for the rigid boundary condition of the target.

AWGN is added to the noise-free time-domain simulated signals. Fifty trials were repeated with randomly generated noise for each source signal, which twenty multiple measurements were used for each trial for the estimations, and the estimation means from the MF and SBLs are presented in Fig. 5. They are normalized with their maximum values for a convenient comparison, where both SNR, high and low frequencies are approximately −4.5 dB. In particular, the existing SBLs [26], [27], which can treat heteroscedastic noise, are applied to the same data for a comparison, which are hereinafter referred to as heteroscedastic-noise SBL (HN-SBL). The vertical dash-dotted lines indicate the true arrival time delays that correspond to the direct (the first) and the scattered (the second) signals.

At high frequencies (sampling frequency of 80 kHz), the MF can separate two arrivals due to the broadband signal whereas the overall offsets are induced by the noise. The noise also weakened the second arrival from the UN-SBL,
is mitigated by the NN-SBL, which estimates the arrivals peak emerges ahead of the scattered signal. The deterioration one, as displayed in the inset of Fig. 5(b), and the fictitious estimates the first arrival that is the farthest from the true sparse solutions. However, the UN-SBL results are inconsis-
more consistently along the trials and displays the higher time resolution with the sharpened arrival evaluations. The estimations are within one sample from the closest samples to the true arrivals and provide the most useful information among the schemes.

It is worth noting that the conventional SBL is corrected to treat the heteroscedastic noise heuristically in Refs. [26] and [27], and the performance degradation might be attributed to the heuristic derivation. In this study, the rigorous expansion of the conventional SBL is derived to consider the nonuniform noise exactly, which leads to the clearest CIR estimation among the SBLs.

V. APPLYING NONUNIFORM NOISE SBL TO THE IN-SITU DATA
A. REAL MEASUREMENT DATA IN THE OCEAN: SAVEX15
The shallow-water acoustic variability experiment 2015 (SAVEX15) was conducted in the northeastern East China Sea [32], and the sound waves that were traveling within a shallow acoustic waveguide were measured at high and low frequencies. In this study, the low-frequency signals, which were recorded with a sampling frequency of 100 kHz, were used to examine the NN-SBL by comparing its results with the results from the MF and the existing SBLs, such as the UN-SBL and the HN-SBL.

When measuring the low-frequency signal, a transducer at a depth of 50 m transmitted a 100 ms-length linear frequency modulated (LFM) pulse waveform with a frequency band in the range of 0.5–2 kHz, and a vertical line array (VLA) that was comprised of evenly spaced 16 hydrophones (3.75 m)
was separated from the transducer at a distance of approximately 1.8 km for the recording. Different waveforms for underwater communications were subsequently transmitted from the transducer, so a single measurement for the LFM signal is available to estimate the CIRs of the SAVEX15.

B. ESTIMATED CHANNEL IMPULSE RESPONSE OF SAVEX15

To demonstrate the usability and feasibility of the NN-SBL in the CIR estimation, it was applied to the real measurement data, and the results were compared with the results from the other schemes. The MF was applied to the VLA data, whereas the UN-SBL, NN-SBL, and HN-SBL were applied to the VLA data after the downsampling to avoid the computational burden with a small $N$ in (5). A sampling interval of 10 in the original VLA data was used for the SBLs, and the sampling frequency is reduced to 10 kHz, which is sufficient to prevent aliasing at a low frequency. The CIRs along the hydrophone depths produced acoustic structures by the waveguide, which included the ‘x’ shapes that were formed by the incoming arrivals to the VLA from the upward and downward directions. They appeared more clearly with the improvement of the scheme (Fig. 6). The source waveform is cross-correlated with the VLA data in the MF, and the corresponding envelopes, which were based on the Hilbert transform, were used for the CIRs. This process leads to pulse compression for the LFM signal and allows for better observation of the acoustic structures with higher time resolutions as well as noise reduction. In contrast, the SBLs inherently derive a high-resolution CIR with less noise by solving the linear system of (5). For the visualization in this study, the SBL results were purposely convolved with an autocorrelation of the LFM signal, and the corresponding envelopes were used, which is illustrated in Fig. 6. The direct results for the SBLs were subsequently compared with the results from the MF in Fig. 7.

While the arrivals around 80 ms and 105 ms have high intensities due to the refractive sound speed profile in the SAVEX15, the arrivals that form the ‘x’ shapes are more focused on the performance comparison. The major arrivals were found to use the MF, but they entailed noise overall. The noise around the ‘x’ shapes caused the arrivals to appear stretched over the relative time delay. The noise near the lower left part of the second ‘x’ shape masked the arrivals. These are detrimental to the clear detection of the arrival directions to the VLA when time-domain beamforming is performed. The SBLs tend to select bases of the transformation matrix that correspond to the actual arrivals, so the noise in the measurement is suppressed during the iterative estimation of the signal powers in the SBLs. Thus, in the UN-SBL result, the noise is remarkably removed, and the masked arrivals around the second ‘x’ shape are more clearly observed. The performance in terms of the noise reduction is improved by the extended SBL, which is due to the more complex approach to noise in the measurement. Most of the noise in the MF result is diminished with the more advanced scheme, as shown in Figs. 6(a) and 6(c). This is achieved at the cost of the weakened arrivals. The NN-SBL estimates multiple
noise powers according to the frequencies here using a single measurement. This should not be sufficient for an accurate estimation, and it induces the weak arrivals. The HN-SBL estimates the CIRs too sparsely and filters out almost all the true arrivals except for the intensive arrivals in the middle of the water column. The upward and downward arrivals that form the ‘x’ shapes are either absent or weakly present.

As mentioned before, the ‘x’ shapes are formed by the upward and downward incoming arrivals getting closer at first and farther apart afterwards, which are merged at the VLA center. The measured signal at the sensor just above the center is used to examine the performance of the schemes in terms of the time resolution. Figure 7 shows parts of the CIRs from the MF and the SBLs that correspond to the dotted lines in Fig. 6. The MF result and the direct estimates from the SBLs are used here for the comparison after the normalization.

The triangles indicate the arrivals that are relevant to the ‘x’ shapes. The MF cannot distinguish the closest two arrivals near 85 ms, which is due to the lower time resolution by an insufficient frequency band at a low frequency, whereas the sparse estimations of the SBLs enable the arrivals to be separated, which included the other arrivals. The noise in the MF result, which has an intensity that is comparable to that the intensity of the arrivals, is noticeably reduced by the SBL-based CIR estimators (more by the NN-SBL). On the other hand, the arrivals are too diminished in the HN-SBL result. In particular, the arrival near 84 ms that comprise of the acoustic structure of ‘x’ shape is absent.

VI. SUMMARY

The MF that generally uses the cross-correlation of the source waveform with the measured signal is used to estimate a CIR that display the arrivals via the pulse compression and the suppression of uncorrelated noise. However, its application is limited by the insufficient frequency band of the source waveform, which leads to the broadening of the arrivals, which makes it difficult to distinguish the adjacent arrivals. The SBL, which was originally developed for classification and regression, was extended to estimate a high-resolution CIR with a limited frequency band.

In this study, the SBL was used to solve a linear system of CIRs, which were established by the physics of underwater sound propagation. The omnipresent noise was scaled using the source spectrum, which resulted in nonuniform noise powers in the measurement violating a basic assumption in the conventional SBL. To treat the inconsistent noise, the SBL was expanded mathematically to accommodate the noise, which was examined using simulated data that included a single arrival. The expanded SBL (referred to as the NN-SBL) delivered a superior performance with a more apparent arrival estimation than the existing SBLs (referred to as the UN-SBL and the HN-SBL) at a low SNR.

An inherent problem of the SBL is the basis mismatch that was caused by the discretized columns corresponding to the on-grid arrival times that did not precisely match the true off-grid arrival times. The orthogonal-like bases of the transformation matrix in the linear system fortunately ensure that the estimated arrival is close to the true value. The synthetic data, which was sampled with two different sampling frequencies, was used to support the properties of the bases. The NN-SBL has an arrival around the true one even at a low sampling rate with less time resolution.

A numerical experiment was conducted to demonstrate the high-resolution CIR via the SBLs, where a scattered signal from a finite submerged target was simulated with the direct arrival. When a low-frequency source was used with a frequency band of 1 kHz, the MF could not distinguish between the two arrivals with a travel distance difference of 2 m, which was due to its low time resolution being proportional to the source frequency band. On the contrary, the SBLs separated the scattered signal from the direct signal via the sparse estimation of the CIR linear system. The NN-SBL exhibited a better noise-reduction performance when the multiple measurements that were obtained over time were available.

The extended SBL was applied to the SAVEX15 data that was measured using a VLA at low frequencies (0.5–2 kHz) with the other schemes (MF, UN-SBL, and
HN-SBL). The noise smearing actual arrivals in the MF result was remarkably diminished by improving the schemes. While the arrivals are weakened by the insufficient measurements in the CIRs from the NN-SBL, it retained the highest possible arrivals and provided the most apparent estimation. Furthermore, the time resolution via the SBL was examined with the received signal that was measured by the sensor just above the VLA center, which had close arrivals from the upward and downward directions before the merging. As expected, all the close arrivals were separated with less noise using the sparse estimation.

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