On the holographic dark energy in chameleon scalar-tensor cosmology

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Abstract We study the holographic dark energy (HDE) model in generalized Brans-Dicke scenario with a non-minimal coupling between the scalar field and matter lagrangian namely Chameleon Brans Dicke (CBD) mechanism. In this study we consider the interacting and non-interacting cases for two different cut-offs. The physical quantities of the model such as, equation of state (EoS) parameter, deceleration parameter and the evolution equation of dimensionless parameter of dark energy are obtained. We shall show that this model can describe the dynamical evolution of fraction parameter of dark energy in all epochs. Also we find the EoS parameter can cross the phantom divide line by suitable choices of parameters without any mines kinetic energy term.

Keywords Generalized chameleon Brans Dicke mechanism, Holographic dark energy, Conservation equation.

1 Introduction

Cosmological and astrophysical observational data risen from supernovae type Ia (SNIa) (Riess et al. 1998; Perlmutter et al. 1999; Bean et al. 2001; Riess et al. 2004), Cosmic Microwave Background Radiation (CMBR) (Bennett et al. 2003) and Sloan Digital Sky Survey (SDSS) (Abazajian et al. 2003, 2004, 2005; Tegmark et al. 2004; Hao et al. 2010) indicate that the Universe is in accelerated expansion regime. There are two approaches to justify the source of accelerating phase of the Universe. Some people look for the source of this acceleration in the geometrical part of the Hilbert-Einstein action and have studied the modified gravity (Wands 1994; Nojiri and Odintsov 2007; Guranizo et al. 2010; Saaidi et al. 2012a; Saaidi and Aghamohammadi 2012; Saaidi et al. 2012b; Aghamohammadi et al. 2009, 2010). As a second way, some researchers propose an eccentric form of matter namely dark energy (DE) (Boisseau et al. 2000; Sahoo and Singh. 2002, 2003; Capozziello et al. 2003; Faraoni 2007; Jorge et al. 2010). Although the nature and origin of the DE are ambiguous for researchers up to now, but people proposed some useful candidates which could satisfy both theoretical and observational results (Nojiri and Odintsov 2004; Padmanabha 2003; Biswas et al. 2006; Sahni et al. 2003; Arkani-Hamed et al. 2004; Piazza and Tsujikawa 2004). Amongst these proposals cosmological constant model ,Λ, is the fundamental block. It is clear that this model suffers from two well known problems i.e. the "cosmological coincidence problem" and "the fine tuning problem", we refer the reader for more details to (Einstein et al. 1917; Peebles and Ratra 2003; Carroll 2001; Sahni and Starobinisky 2000; Padmanabha 2003; Steinhardt 1997).

Recently scalar field models attract more attentions to investigate the behavior and nature of the DE. Most of DE models treat scalar fields as DE component with a dynamical equation of state. The basic dynamical DE proposal which is called ”quintessence” model consider the slow-roll down of a scalar field and suggests an energy form with negative pressure (Peebles and Ra-
2 Field equations and conservation relation of energy

Generalized CBD theory in which scalar field has non-minimal coupling with both geometry and matter sectors is considered as

\[ A = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \left( \phi R - \omega(\phi) \nabla_\alpha \phi \nabla^\alpha \phi - V(\phi) \right) \right\} \]

\[ + f(\phi) L_m \], \tag{2} \]

where \( R \) is the Ricci scalar, \( \omega(\phi) \) is the CBD parameter (i.e., as a coupling function), and \( L_m \) is the lagrangian of the matter. \( \phi \) is CBD scalar field and \( V(\phi) \) is inverse power law potential which defined as \( V(\phi) = M^{4+\nu}/\phi^\nu \) with a positive constant \( \nu \), (Peebles and Ratra 1988; Wang et al. 2006; Binev 2000; Saaidi et al. 2011b). Note that the last term in the action indicates the interaction between the matter and some arbitrary function.
\[ f(\phi) \text{ of the CBD scalar field. One can obtain the gravitational and scalar field equations of motion by varying the action (2) with respect to (w.r.t) } g^{ab} \text{ and } \phi \text{ respectively. The gravitational field equation is} \]

\[ \phi G_{ab} \equiv \phi \left[ R_{ab} - \frac{1}{2} g_{ab} R \right] = f(\phi) T_{ab} + T_{ab}^\phi, \quad (3) \]

where \( G_{ab} \) is the Einstein tensor, \( R_{ab} \) is the Ricci tensor, \( T_{ab} \) is the energy-momentum tensor of the matter which is given by

\[ T_{ab} = -\frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{ab}} \mathcal{L}, \quad (4) \]

and \( T_{ab}^\phi \) is defined as

\[ T_{ab}^\phi = \frac{\omega}{\phi} \left[ \nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla_c \phi \nabla^c \phi \right] + \nabla_a \nabla_b \phi - g_{ab} \nabla^a \phi - \frac{1}{2} g_{ab} V(\phi). \quad (5) \]

We suppose that all components of matter (cold dark matter and DE) are perfect fluid and then we can introduce the stress-energy tensor of matter as

\[ T_{ab} = (\rho_t + p_t) u_a u_b + p_t g_{ab}, \quad (6) \]

where \( u^\mu \) is the four-vector velocity of the fluids and \( \rho_t \) and \( p_t \) indicate the total energy density and pressure respectively.

The scalar field equation of motion is obtained as

\[ \left[ 2 \omega(\phi) + 3 \right] \nabla^a \phi = f(\phi) T - 2 f' (\phi) L_m + \phi V'(\phi) - 2 V(\phi) - \omega(\phi) \nabla_a \phi \nabla^a \phi, \quad (7) \]

where \( T \) is the trace of \( T_{ab} \) and prime denotes derivative with respect to \( \phi \). Setting \( f(\phi) = 1 \) and \( V(\phi) = 0 \), the above equations reduce to those of Ref. (Kamenshchik et al. 2001; Bento et al. 2002; Saaidi et al. 2011a; Saaidi and Mohammadi 2012).

It is seen that for solving (7) we need an explicit form of matter Lagrangian, \( L_m \). The Bianchi identities, together with the identity \( (\nabla \nabla_a - \nabla_a \Box) V_c = R_{ab} \nabla^b V_c \), imply the non-(covariant) conservation law

\[ \nabla^a T_{ab} = [T^b_b - \delta^b_0 L_m] \nabla_a \ln(f), \quad (8) \]

and, as expected, in the limit \( f(\phi) = \text{constant} \), one recovers the conservation law \( \nabla_a T^{ab} = 0 \).\footnote{Eq. (8) is not recognized correctly in Phys. Lett. B 697, 285 (2011), therefore the results which obtained in their work is not correct.}

We consider, the homogeneous and isotropic FLRW background metric with line element

\[ ds^2 = -dt^2 + a^2(t) \left[ -1 + kr^2 + r^2 d\theta^2 + \sin^2 \theta d\phi^2 \right]. \quad (9) \]

Where \( a(t) \) is the scale factor and \( k = -1, 0, +1 \) indicate the open, flat and close Universe respectively. From Eqs. (3), (6) and (9), one can obtain the components of gravitational equation as

\[ 3 \left( H^2 + \frac{k}{a^2} \right) = f(\phi) \frac{\dot{\phi}}{\phi} - 3 \frac{\dot{\phi}}{a} \left( \frac{\phi}{2} \right) + \omega(\phi) \frac{\dot{\phi}^2}{\phi^2} + V(\phi), \quad (10) \]

\[ 2 \ddot{H} + 3 H^2 + \frac{k}{a^2} = - \frac{f(\phi)}{\phi} - \omega(\phi) \frac{\dot{\phi}^2}{\phi^2} - 2 \frac{\ddot{\phi}}{a} \left( \frac{\dot{\phi}}{\phi} \right) + V(\phi) \quad \ bif \quad (11) \]

Here \( H = \dot{a}/a \) is the Hubble parameter, and dot indicates differentiative w.r.t the cosmic time, \( t \).

Whereas perfect fluid is an averaged properties of matter then it is not necessary to know an exact description of matter, therefore it is more common to work directly with energy-momentum tensor instead of Lagrangian. But in present model the Lagrangian, \( L_m \), is explicitly appeared in equation of motion of scalar field, (7), and we have to know what is it?. It was considered that Eq. (4) can give us a stress-energy tensor, (6), for a perfect fluid with a matter Lagrangian as \( L_m = p_t \) (Brown 1993; Haecking and Ellis 1973; Gibson and Hawking 1997; Saaidi 2012), where \( p_t \) is the pressure of the fluid. In fact the on-shell action, which is the proper volume integral of the pressure

\[ A(\text{on-shell}) = \int d^4 x \sqrt{-g} p_t(\mu, s), \quad (12) \]

give the stress-energy tensor, (4), by varying it w.r.t \( g^{ab} \). By adding some surface integral to the above action, the action will change its on-shell value without affecting the equation of motion. By considering this fact it is shown that, Lagrangian is not unique. It is found that the other choices can be \(-p_t(n, s)\) and \(-n_a(n, T)\) (Brown 1993; Haecking and Ellis 1973; Gibson and Hawking 1997) where \( p_t \) is the total density energy, \( n \) is the density of particles, \( a_t(n, T) = p_t/n - T s \), is the physical free energy, \( T \) is temperature of fluid and \( s \) is entropy. For a complete review see (Gibson and Hawk-
ing 1997; Brown et al. 1993). Based on the earlier discussion, Eq. (13) with together (4) give a stress-energy tensor of matter as (6).

In this work, we assume

\[
L_m = p_t, \tag{14}
\]

so Eq. (7) is reduced to

\[
\left[2\omega(\phi) + 3\right] \Upsilon = \left[\rho_t - 3p_t\right] f(\phi) - \frac{1}{2} \phi f'(\phi) p_t + 2V(\phi) - \phi V'(\phi), \tag{15}
\]

Where \( \Upsilon = \left[\frac{\gamma}{\rho} + 3H \dot{\phi} + \frac{\omega'(\phi)}{2\omega(\phi) + 3}\phi^2 \right] \). By using Eqs. (8) and (14) one can attain conservation equation as

\[
\dot{\rho}_t + 3H \rho_t (1 + \omega_t) = -\frac{\dot{f}}{f}(1 + \omega_t) \rho_t, \tag{16}
\]

where \( \rho_t = \rho_\Lambda + \rho_m \) and we have used \( p_t = \omega_t \rho_t \). Notice \( p_t = \rho_\Lambda + \rho_m \) yields \( p_t = \omega_t \rho_t = \omega_m \rho_m + \omega_\Lambda \rho_\Lambda \). So one can rewrite (16) as follows

\[
\dot{\rho}_\Lambda + 3H \rho_\Lambda (1 + \omega_\Lambda) = -\frac{\dot{f}}{f}(1 + \omega_\Lambda) \rho_\Lambda, \tag{17}
\]

\[
\dot{\rho}_m + 3H \rho_m (1 + \omega_m) = -\frac{\dot{f}}{f}(1 + \omega_m) \rho_m. \tag{18}
\]

So according to the original definition of HDE density, Eq. (1), the HDE density in the CBD scenario is defined as

\[
\rho_\Lambda = \frac{3c^2 M_P^2 \dot{\phi}}{L^2}, \tag{19}
\]

moreover, critical energy density, \( \rho_c \), and energy density of curvature, \( \rho_k \), in the generalized CBD model, are

\[
\rho_c = 3\phi H^2, \tag{20}
\]

\[
\rho_k = -\frac{3\phi k}{a^2}. \tag{21}
\]

\[\text{For more convenience we consider } M_P^2 = 1. \text{ Therefore energy density parameters are obtained as}
\]

\[
\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = \frac{c^2}{H^2 L^2}, \tag{22}
\]

\[
\Omega_k = \frac{\rho_k}{\rho_c} = \frac{k}{a^2 H^2}, \tag{23}
\]

\[
\Omega_m = \frac{\rho_m}{\rho_c} = \frac{\rho_m}{3H^2 \dot{\phi}}. \tag{24}
\]

Based on these dimensionless density parameters, we can rewrite Eq. (10) as

\[
f(\phi) \Omega_t + \Omega_k + \frac{1}{2} \Omega_V + \Omega_\phi = 1, \tag{25}
\]

where

\[
\Omega_\phi = \frac{1}{3H^2} \left[ \frac{\omega(\phi) \phi^2}{2} - 3 \frac{\dot{\phi}}{a} \right], \tag{26}
\]

\[
\Omega_V = \frac{V(\phi)}{3H^2 \phi}. \tag{27}
\]

3 Future event horizon as an IR cut off

In this Section we want to calculate the physical quantities for a special cut off namely future event horizon. Event horizon is defined as \( L = a r(t) \), where \( a \) is scale factor and \( r(t) \) is

\[
r(t) = \frac{1}{\sqrt{|k|}} \sin(n(\sqrt{|k|}|y) = \begin{cases} 
\sin(y) & k = +1 \\
\sinh(y) & k = -1.
\end{cases}
\tag{28}
\]

In this relation \( y = R_h/a(t) \) where \( R_h \) is future event horizon and \( \sin(n(\sqrt{|k|}|y) \) indicates elliptic functions. So taking derivative \( L \) w.r.t the cosmic time and using Eq. (28) yeilds

\[
\dot{L} = HL + a \ddot{r}(t) = \frac{c}{\sqrt{\Omega_\Lambda}} \cos(y) - \cos(n(y). \tag{29}
\]

where \( \ddot{y} = \sqrt{|k|} \) and

\[
\cos(n(y) = \begin{cases} 
\cos(y) & k = +1 \\
1 & k = 0 \\
\cosh(y) & k = -1.
\end{cases}
\tag{30}
\]

3.1 Non-interacting HDE in CBD model

Now from conservation equation which is defined in Eq. (17) and definition of \( \rho_\Lambda \), Eq. (19), we can attain \( \omega_\Lambda \). So taking derivative Eq. (19) w.r.t the time gives

\[
\dot{\rho}_\Lambda = \rho_\Lambda \left( \frac{\dot{\phi}}{\phi} - 2 \frac{\dot{L}}{L} \right). \tag{31}
\]
For calculating (31), we must to find out an explicit form for \( \phi \), which is the solution of Eq. (15), but since Eq. (15) is not an independent equation, therefore obtaining an explicit form for \( \phi \) is not possible. So we should remove the extra freedom, \( \phi \), from the equations of motion. Recently people have considered the CBD scalar field as a power of the scale factor which is in a good agreement with the results of recent observational and experimental data (Banerjee and Pavon 2007). Therefore according to (Banerjee and Pavon 2007), we accept the following ansatz for \( \phi \) and \( f(\phi) \)

\[
\phi = a^\sigma, \quad f(\phi) = \lambda \phi^\delta.
\]

(32)

In the BD model one can define \( G_{\text{eff}} \propto 1/\phi \), where \( G_{\text{eff}} \) is the effective Newtonian gravitational constant. Since the observational data requires a constraint on \( G_{\text{eff}} \) as \( |G_{\text{eff}}/G_{\text{eff}}| \leq 3.32 \times 10^{-20} s^{-1} \) (Will 1993; Acquaviva and Verde 2007), then from \( |\dot{\phi}/\phi| = |G_{\text{eff}}/G_{\text{eff}}| = \sigma \dot{H} \), one can restrict the value of \( \sigma \) which approximately is \( \sigma \leq 0.01 \). On the other hand there is no any constraint on \( \lambda \) and \( \xi \), and based on observational evidences we should find out some constraints on them. So according to this choice one can rewrite Eqs. (17) and (18) as

\[
\dot{\rho}_\Lambda + \eta H \rho_\Lambda (1 + \omega_\Lambda) = 0, \quad \dot{\rho}_m + \eta H \rho_m (1 + \omega_m) = 0,
\]

(33)

(34)

where \( \eta = (3 + \sigma \xi) \). Therefore, from Eqs. (29), (31) and (33) one can obtain \( \dot{\rho}_\Lambda \) as

\[
\dot{\rho}_\Lambda = \rho_\Lambda \bigg[ \sigma - 2 + \frac{2}{c} \sqrt{\Omega_\Lambda \cos(\tilde{y})} \bigg],
\]

(35)

so the EoS parameter in noninteracting case is

\[
\omega_\Lambda = -1 - \frac{1}{\eta} \bigg[ \sigma - 2 + \frac{2}{c} \sqrt{\Omega_\Lambda \cos(\tilde{y})} \bigg].
\]

(36)

Note that in comparison with EoS in the noninteracting case which is obtained in Phys. Lett. B 697, 285 (2011), one can see that our results contain the chameleon effect which appeared in the action. It is important to note that for \( f(\phi) = 1 \) (\( \xi = 0 \)) this model reduces to generalized BD model and Eq. (33) reduces to its respective expression in BD model.

From (36) it is seen that for \( 0 < c < \sqrt{\Omega_\Lambda \cos(\tilde{y})} \) and any arbitrary positive value of \( \xi \), the equation of state parameter, \( \omega_\Lambda \), is less than \(-1\). This means that in this case the EoS parameter of HDE crosses the phantom divide line \( \omega_\Lambda = -1 \). On the other hand, since \( \sigma \leq 0.01 \) and it is very small than other quantities in Eq. (36), then for \( c \geq 1 \) the EoS can not crosses the line \( \omega_\Lambda = -1 \) unless \( \xi < 0 \) and \( |\xi| > 3/\sigma \). This means that this model can crosses the phantom divide line with a suitable choice of parameters.

Another useful cosmological parameter is deceleration parameter which is defined as

\[
q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2}.
\]

(37)

So using Eqs. (11), (22), (24), (26), (27) and (32), one can attain \( q \) as

\[
q = \frac{1}{(\sigma + 2)} \left[ 3f(\phi)\Omega_\Lambda \omega - \frac{3}{2} \Omega_\nu + (\sigma + 1)^2 \right. \]

\[
\left. + \sigma \left\{ \frac{\omega(\phi)\sigma}{2} - 1 \right\} + \Omega_\kappa \right].
\]

(38)

In the present time \( f(\phi) = \lambda, \Omega_\Lambda = 0.74, \Omega_\kappa = 0.02, \sigma = 0.01 \) and we can approximate the function of \( \omega(\phi) \simeq \omega_0 = 40000 \). Then one can get to

\[
q \simeq -1.493 \left[ 0.74 |\lambda \omega_0| + \frac{1}{2} |\Omega_\nu| - 0.41 \right],
\]

(39)

it is obviously seen that, the deceleration parameter can be negative by a suitable choice of \( \lambda \).

In this stage we examine the evolution of \( \Omega_\Lambda \). Using Eqs. (22), (29) and (37) we have

\[
\frac{d \ln(\sqrt{\Omega_\Lambda})}{d \ln(a)} = q + \sqrt{\frac{\Omega_\Lambda}{c}} \cos(\tilde{y}).
\]

(40)

Moreover, for checking the evolution of the EoS, \( \omega_\Lambda \), we have to examine \( \dot{\omega}_\Lambda \). We calculate the time derivative of \( \omega_\Lambda \) and get

\[
\dot{\omega}_\Lambda = -\frac{2H \sqrt{\Omega_\Lambda}}{c\eta} \left[ \left\{ q + \frac{1}{c} \sqrt{\Omega_\Lambda \cos(\tilde{y})} \right\} \cos(\tilde{y}) \right. \]

\[
\left. + \frac{\sqrt{|\kappa|}}{aH} \sin(\tilde{y}) \right].
\]

(41)

In a spatially flat FLRW Universe, Eq. (41) becomes

\[
\dot{\omega}_\Lambda = -\frac{2H \sqrt{\Omega_\Lambda}}{c\eta} \left[ q + \frac{1}{c} \sqrt{\Omega_\Lambda} \right].
\]

(42)

Note that in the accelerated expansion phase of the Universe, \( q < 0 \), so Eq. (41) implies \( \dot{\omega}_\Lambda < 0 \) only for \( |q| < \sqrt{\Omega_\Lambda}/c \). Therefore in this case the EoS parameter of HDE in generalized CBD scenario evolves to super-negative value. Indeed this means that if \( |q| < \sqrt{\Omega_\Lambda}/c \) (where \( |q| = |q_{\omega_\Lambda} = -1| \)), then the phase transition take place from quintessence phase to phantom phase and vice versa (i.e., if \( |q| > \sqrt{\Omega_\Lambda}/c \) then the phase transition is take place from phantom phase to quintessence phase.)
3.2 Interacting HDE in generalized CBD model

In this step we consider an interacting between dark energy candidate (HDE) and (dark) matter. Recently interacting between DE and dark matter attracts very attentions, because the problems which arise from non-interacting case have improved by it. For more conversancy we refer the reader to study the recent works (Jamil et al. 2011; Saaidi et al. 2012c) and references there in. In fact the study of this interacting model should be done in a quantum gravity mechanism, but unfortunately such a model is not completely composed up to now. Following recent researches an interacting term \( Q \) has been considered which it play a source-like behavior. Therefore in this case, we can write the modified conservation equations as follows

\[
\begin{align*}
\dot{\rho}_A + \eta H \rho_A (1 + \omega_A) &= -Q, \quad (43) \\
\dot{\rho}_m + \eta H \rho_m (1 + \omega_m) &= Q. \quad (44)
\end{align*}
\]

Where \( Q \) is direct interaction between (dark) matter and DE. Therefore using Eqs. (35), (43) one can obtain the EoS parameter in the interacting case as

\[
\omega_A = -1 - \frac{1}{\eta} \left[ \sigma - 2 + \frac{2}{c} \sqrt{\Omega_A \cosh(y) + Q / (\rho_A H)} \right]. \quad (45)
\]

People have considered some well known candidates for interactive term, which amongst them we consider \( Q = 3b^2 H \rho_A \). Here \( b \) is a real constant. So Eq. (45) becomes

\[
\omega_A = -1 - \frac{1}{\eta} \left[ \sigma - 2 + \frac{2}{c} \sqrt{\Omega_A \cosh(y) + 3b^2} \right]. \quad (46)
\]

By comparing Eq. (46) with Eq. (36), one can see that the direct interaction between (dark) matter and DE helps phantom divide line crossing in this model. Whereas the EoS parameter in this case differs from EoS in noninteracting case we should obtain deceleration parameter in this case. Therefore from Eqs. (25), (30) one can attain

\[
q = \frac{1}{(\sigma + 2)} \left[ 3f(\phi)\Omega_A \omega_A - \frac{3}{2}\Omega_V + (\sigma + 1)^2 \right. \\
+ \left. \sigma \frac{(\omega(\phi)\sigma)}{2} - 1 \right] + \Omega_k. \quad (47)
\]

Notice that the deceleration parameter in interacting case explicitly contain the CBD scalar field function as the same as noninteracting case. We should emphasize that the \( q \) which obtained in noninteracting case in the (Phys. Lett. B 697, 285 2011) is not a function of the \( f(\phi) \) and is mentioned as a minor mistake. Note that the form of the deceleration parameter in interacting case is similar to non-interacting case but the equations of state parameter against the EoS. Also the evolution equation of dimensionless parameter, \( \Omega_A \), is such as (40) versus new \( q \), (47), then it is not necessary to find them again.

4 Conformal-age-like length as an IR cut off

In this section we take a conformal age like parameter as the characteristic length scale \( L \), which is defined in the flat FLRW Universe as follows

\[
L = \frac{1}{a^3(t)} \int_0^t dt' a^3(t'). \quad (48)
\]

where \( a(t) \) is the scale factor (Huang and Wu 2011). So taking derivative with respect to the cosmic time from \( L \) and using Eqs. (19) and (28) we find

\[
\dot{L} = -4LH + \frac{1}{a}, \quad (49)
\]

4.1 Non-interacting HDE in CBD model

Now from conservation equation, Eq. (17), and definition of \( \rho_A \), Eq. (19), we can attain \( \omega_A \).

\[
\omega_A = -1 - \frac{1}{\eta} \left[ \sigma + 8 - \frac{2}{ca(t)} \sqrt{\Omega_A} \right]. \quad (50)
\]

It is seen that at present time and for \( c \geq 1 \), \( \omega_A < -1 \) i.e., this model can cross the phantom divide line. Also the deceleration parameter is

\[
q = \frac{1}{(\sigma + 2)} \left[ 3f(\phi)\Omega_A \left\{ -1 - \frac{1}{\eta} \left[ \sigma + 8 - \frac{2}{ca(t)} \sqrt{\Omega_A} \right]\right) \\
- \frac{3}{2}\Omega_V + (\sigma + 1)^2 + \sigma \frac{(\omega(\phi)\sigma)}{2} - 1 \right]. \quad (51)
\]

In order to gain better insight we focus on the situation which \( \omega(\phi) = \omega_0 = 40000 \). So in this case one can obtain the equation of motion for dimensionless parameter of dark energy density as

\[
\theta \Omega_A' = \Omega_A \left\{ \eta(1 + \omega_m) + \sigma + 8 - \frac{2}{ac} \sqrt{\Omega_A} \right\} (\theta - \Omega_A) \\
+ \frac{1}{2} (\xi + \nu) \sigma \Omega_V, \quad (52)
\]

where \( \theta = 1 - \sigma (\omega_0 / 6 - 1) \). Accepting that the Universe had the inflation epoch with \( \omega_m = -1 \), the radiation epoch with \( \omega_m = \omega_r = 1/3 \), and the matter dominant epoch with \( \omega_m = 0 \) before entering into the accelerating expansion phase, we want have an approximately investigation of \( L \) and \( \Omega_A \) in these three epochs.
In fact we want to obtain the fractional density of dark energy in the early universe from directly calculation. In this part of our work we assume the EoS parameter, $\omega_m$ is constant in all epochs. So according to Eq. (34) we have $\rho_m = \rho_0 a^{-\eta(1+\omega_m)}$ and from Eq. (10) we have

$$H^2 \simeq \left(\frac{\lambda \rho_0}{3\theta}\right) a^{-2\zeta} + \left(\frac{M_5^5}{6\theta}\right) a^{-2\sigma},$$

(53)

where the density of dark energy is ignored in early time. And since

$$\zeta = \frac{1}{2}(3 + \sigma + \eta \omega_m) \gg \sigma,$$

(54)

then the second term in the right hand side of Eq. (53) is negligible with respect to the first one and this relation reduced to

$$H^2 \simeq \left(\frac{\lambda \rho_0}{3\theta}\right) a^{-2\zeta}.$$ 

(55)

Therefore, from (48) we have

$$L = \left(\frac{a_i}{a}\right)^4 L_i + \frac{1}{3 + \zeta} \left[ \frac{1}{H a} - \frac{1}{H a_i} \left(\frac{a_i}{a}\right)^4 \right],$$

(56)

where $L_i = (1/a_i^4) \int_0^{t_i} dt a^3(t)$ and subscript "i" indicates the value of corresponding quantity at the beginning of the epoch. Also $a$ is the scale factor out of matter dominant epoch. This means $(a_i/a)^4 \ll 1$, so we have

$$L \sim \left(\frac{1}{3 + \zeta}\right) \frac{1}{H a}.$$ 

(57)

and from (22) we have

$$\Omega_\Lambda \sim \lambda(3 + \zeta)^2 a^2 (2 + \sigma \xi),$$

(58)

Since $c$ is at order of unity and $\sigma$ is very small, then Eq. (58) shows that the value of dimensionless parameter of dark energy before the accelerating expansion phase is very small. In fact Eq. (58) is an approximated solution for early time until matter dominant, so we can not match it with the present value of fractional parameter of dark energy. Therefore to obtain the reasonable value of parameters, we should find the exact solution of Eq. (52) and matching it with the present value of dark energy.

4.2 Interacting HDE with conformal age like length

In this section we consider an interaction between DE and the matter similar to Subsection.B in pervious Section. In this case the modified conservation equations are Eqs. (43) and (44). Then using Eqs. (31), (43) and (49) one obtains the EoS parameter as

$$\omega_\Lambda = -1 - \frac{1}{\eta} \left[ \sigma + 8 - \frac{2}{ca(t)} \sqrt{\Omega_\Lambda + Q/\rho_\Lambda H} \right],$$

(59)

and for $Q = 3b^2 H \rho_\Lambda$, Eq. (59) becomes

$$\omega_\Lambda = -1 - \frac{1}{\eta} \left[ \sigma + 8 - \frac{2}{ca(t)} \sqrt{\Omega_\Lambda + 3b^2} \right].$$

(60)

Eq. (60) shows that the direct interaction between (dark) matter and dark energy helps to crossing the phantom divide line.

Finally, the deceleration parameter of HDE model in CBD scenario for conformal age like length is attained as

$$q = \frac{1}{(\sigma + 2)} \left[ 3f(\phi)\Omega_\Lambda \left\{ -1 - \frac{1}{\eta} (\sigma + 8 - \frac{2}{ca(t)} \sqrt{\Omega_\Lambda} \right) 
+ Q/\rho_\Lambda H \right\} - \frac{3}{2} \Omega_V + (\sigma + 1)^2 + \sigma (\omega_0 \sigma - 1) \right] \right].$$

(61)

In the present time $f(\phi) = \lambda$, $\Omega_\Lambda = 0.74$, $\Omega_k = 0.02$, $\sigma = 0.01$ and we can approximate the function of $\omega(\phi) \simeq \omega_0 = 40000$. Then one can get to

$$q \approx -1.493 \left[ 0.74 |\omega_\Lambda| + \frac{1}{2} |\Omega_V| - 0.41 \right],$$

(62)

it is obviously seen that, the deceleration parameter can be negative by a suitable choice for $\lambda$. The differences between Eq. (62) and Eq. (40) is $|\omega_\Lambda|$ in which the value of $|\omega_\Lambda|$ in Eq. (62) is bigger than it in Eq. (40).

5 Conclusion and discussion

In this paper, we have considered holographic dark energy (HDE) in the Brans-Dicke cosmological model with a non-minimal coupling between the chameleon scalar field and matter. This model has a dynamical time dependent scalar field which behave like dark energy and then it might produce cosmic acceleration. However, we suppose the matter sector consists of matter and dark energy, so this means that we have assumed an interaction between the scalar field and dark energy too. This fact is clearly seen in Eqs. (10) and (11). Actually, non-minimal coupling between the scalar field and matter field generalizes the conservation equation of energy density which is diagnosed wrong in (Phys. Lett. B 697, 285 2011).

We studied the interacting and non-interacting cases of this model for two different infrared cutoffs; future
event horizon and conformal-age-like length. We obtained the EoS parameter, deceleration parameter and fraction parameter of dark energy for two mentioned cutoffs and we found that phantom crossing is possible in both of cutoffs by tuning the free parameters of the model. Note that in almost every cosmological model the fine tuning of parameters is necessary and our model also is not exception. At the end, comparing the obtained results for two mentioned cutoffs, show that the phantom crossing with conformal-age-like length is more possible than future event horizon cutoff.

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