Hybrid Active Contours Driven by Improved Laplace Adaptive Energy in Multiphase Level Set Framework

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Keywords: Image segmentation, Multiphase level set, Image Laplacian fitting energy, Adaptive segmentation.

Abstract. This paper presents a novel multiphase level set framework and its corresponding algorithms for image segmentation. The region fitting energies are described with the combination of the global and local distributions. An improved image Laplace term enhances the ability to locate the weak boundary. In order to better deal with the inhomogeneity, the adaptive segmentation energy is proposed to replace the fixed threshold and local Gaussian means and variances are added to get better precision of segmentation results. Experiment shows the proposed model is effective with application to MR images with intensity inhomogeneity.

Introduction

The traditional two-phase segmentation divides the image into the foreground and the background with the single level set function, the intensity distribution can be categorized into three classes: global statistical information [1,2], local statistical information [3] and others [4,5]. The multiphase segmentation usually uses multiple level set functions to co-evolve simultaneously. Later, some scholars combined the hybrid activity model with the multiphase segmentation methods to deal with complex images [6,7].

This paper presents a hybrid active contour model to get more accurate segmentation. We construct the hybrid model by incorporating statistical information: gradient, variances and means. Experimental results illustrate the good performance of the proposed model for MR images with intensity inhomogeneity.

Laplace Adaptive Hybrid Active Contour Driven by Local and Global Intensity Fitting Energy (LGLAF Model)

Image Preprocessing

We combined the gradient reciprocal weighting method and the median filtering algorithm, then took the average of the processing results as the preprocessing of the image.

\[ u_0 = \frac{1}{2}(I_{\text{gra}} + I_{\text{med}}). \] (1)

where \( I_{\text{gra}} \) and \( I_{\text{med}} \) are the resulting image processed by the gradient reciprocal weighted smoothing and two dimensional median filter, respectively.

Automatic Acquisition of the Initial Contour

The LGLAF model adopts the photometric invariance theory with tensor based features to detect the features of the image objects [8]. Eigen-value analysis of the tensor is defined by:

\[ \lambda = \frac{1}{2}(g_x \cdot g_x + \overline{g_y} \cdot \overline{g_y} + \sqrt{(g_x \cdot g_x - \overline{g_y} \cdot \overline{g_y})^2 + (2g_x \cdot \overline{g_y})^2}). \] (2)

where \( g_x \) and \( g_y \) are the features which derived from the color tensor. The direction of \( \lambda \)
indicates the prominent local orientation:

\[ \theta = \frac{1}{2} \arctan \left( \frac{2\hat{g}_x \hat{g}_y}{\hat{g}_x^2 - \hat{g}_y^2} \right). \]  

(3)

We use the above method to extract image features by setting the threshold as the initial contour.

**Multiphase Level Set Model**

**The Composition of the Framework.** In the LGLAF model, two level set functions are used to perform four phases:

\[
\begin{align*}
M_1(\Phi_1(x), \Phi_2(x)) &= H_{\epsilon}(\Phi_1(x)) \cdot H_{\epsilon}(\Phi_2(x)); \\
M_2(\Phi_1(x), \Phi_2(x)) &= H_{\epsilon}(\Phi_1(x)) \cdot [1 - H_{\epsilon}(\Phi_2(x))]; \\
M_3(\Phi_1(x), \Phi_2(x)) &= [1 - H_{\epsilon}(\Phi_1(x))] \cdot H_{\epsilon}(\Phi_2(x)); \\
M_4(\Phi_1(x), \Phi_2(x)) &= [1 - H_{\epsilon}(\Phi_1(x))] \cdot [1 - H_{\epsilon}(\Phi_2(x))];
\end{align*}
\]

(4)

The total energy functional is defined by:

\[
E(\Phi_1, \Phi_2) = \mu \sum_{i=1}^{2} \epsilon_{DRT}(\Phi_i) + \upsilon \sum_{i=1}^{2} \epsilon_{EDGE}(u_i) + \sum_{i=1}^{2} \epsilon_{region}(\Phi_i),
\]

(5)

where \(\mu\) and \(\upsilon\) are weight coefficients.

The regularized energy \(\epsilon_{DRT}(\Phi_i)\) is introduced to reserve the level set function regularity and avoid the reinitialize [10].

\[
\epsilon_{DRT}(\Phi_i) = \int p(|\nabla \Phi_i(x)|) dx \quad i = 1, 2.
\]

(6)

The edge stopping function \(g\) is incorporated in the length constrain term \(\epsilon_{EDGE}(u_i)\) that defined as:

\[
\epsilon_{EDGE}(\Phi_i) = \int g(|\nabla u_0(x)|)|\nabla \Phi_i(x)| dx \quad i = 1, 2.
\]

(7)

The region fitting energy \(\epsilon_{region}(\Phi_i)\) is consisting of global and local statistical information:

\[
\epsilon_{region}(\Phi_1, \Phi_2) = \int_{\Omega} \Phi_1(x) \cdot S_1(x) dx + \int_{\Omega} \Phi_2(x) \cdot S_2(x) dx,
\]

(8)

\[
\begin{align*}
S_1(x) &= (r_1 - r_3)H_{\epsilon}(\Phi_2) + (r_2 - r_4)(1 - H_{\epsilon}(\Phi_2)) \\
S_2(x) &= (r_1 - r_2)H_{\epsilon}(\Phi_1) + (r_3 - r_4)(1 - H_{\epsilon}(\Phi_1))
\end{align*}
\]

(9)

**Local and Global Terms in Region Fitting Energy**

The improved image Laplace energy can accurately determine the object boundary [9]:

\[
\frac{\partial \phi}{\partial t} = \alpha \Delta \phi - (\phi - \Delta u_0)
\]

(10)

For the global segmentation term, the adaptive segmentation term is improved and weighted from CV model [1], LAW model [2]. Set \(a\) and \(b\) as the weight coefficients, \(\omega\) is a weighted function according to the local intensity of the image. The global segmentation term is given by:

\[
\epsilon_{glo}(c_i) = e_m - e_n = 2\delta(\phi)(c_m - c_n) \left[ (a + b)\left(1 - \frac{a + b\omega}{2}\right)c_m - \frac{1}{2}a + b(1 - \omega)\right]c_n
\]

(11)

For the local Gaussian term, the construction energy is adopted as follows:

\[
\epsilon_{loc} = \left[ \int_{\Omega} K_\sigma(x - y)(\log \sigma_m + \frac{(u_m - f_m)^2}{2\sigma_m^2}) dy - \int_{\Omega} K_\sigma(x - y)(\log \sigma_n + \frac{(u_n - f_n)^2}{2\sigma_n^2}) dy \right] \quad i = 1, 2, m, n = 1, 2, 3, 4
\]

(12)

Then the region fitting term referred to Eq. 9 is rewritten as follows:
\[ r_m - r_n = (1 - \theta(x)) \cdot \left\{ 2 \cdot \left[ (a + b) I - \left( \frac{1}{2} a + b \omega(x) \right) c_m - \left( \frac{1}{2} a + b (1 - \omega(x)) \right) c_n \right] + \right. \]
\[ \alpha \Delta \Phi_i - \Phi_i - \Delta u_i \right\} + \theta(x) \cdot \left[ \int_\Omega K_\sigma(x-y)(\log \sigma_m + \frac{(u_0-u_m)^2}{2\sigma_m^2}) dy - \int_\Omega K_\sigma(x-y)(\log \sigma_n + \frac{(u_0-u_n)^2}{2\sigma_n^2}) dy \right] \]
\[ i = 1, 2, m, n = 1, 2, 3, 4. \] (13)

\( \theta(x) \) is a function that adjusts the contribution of the local and global region fitting energies [4]. The global means \( c_i \), the local means \( u_i(x) \) and variance \( \sigma_i(x) \) in Eq. 13 are formulated by:

\[ c_i = \frac{\int_\Omega u_i M_i dx}{\int_\Omega M_i dx} \] \( i = 1, 2, 3, 4. \)
\[ u_i = \frac{\int_\Omega K_\sigma(x-y)u_i M_i dy}{\int_\Omega K_\sigma(x-y)M_i dy} i = 1, 2, 3, 4, \] (14)

\[ \sigma_i = \frac{\int_\Omega K_\sigma(x-y)[(u_0-u_i)^2M_i] dy}{\int_\Omega K_\sigma(x-y)M_i dy} \] \( i = 1, 2, 3, 4. \) (15)

\( \omega(x) \) in Eq. 12 is obtained as follows:

\[ \omega(x) = \frac{I_{LAW} - c_n}{c_m - c_n} \forall x \in \Omega \] \( m, n = 1, 2, 3, 4. \) (16)

\( I_{LAW} \) is a smooth function defined as:

\[ I_{LAW} = \frac{1}{2} K_\sigma * (u_m + u_n) \] \( m, n = 1, 2, 3, 4. \) (17)

**The Minimization of Multiphase Level Set**

Minimizing the energy functional in Eq. 8 with respect to \( \Phi_1 \) and \( \Phi_2 \):

\[ \frac{\partial \Phi_1}{\partial t} = \upsilon \delta \varepsilon (\Phi_1) \text{div} \left( g \frac{\partial \Phi_1}{\partial \varepsilon} \right) + \mu \left[ \nabla^2 \Phi_1 - \text{div} \left( \frac{\partial \Phi_1}{\partial \varepsilon} \right) \right] - \delta \varepsilon (\Phi_1)[S_1], \] (18)

\[ \frac{\partial \Phi_2}{\partial t} = \upsilon \delta \varepsilon (\Phi_2) \text{div} \left( g \frac{\partial \Phi_2}{\partial \varepsilon} \right) + \mu \left[ \nabla^2 \Phi_2 - \text{div} \left( \frac{\partial \Phi_2}{\partial \varepsilon} \right) \right] - \delta \varepsilon (\Phi_2)[S_2]. \] (19)

**Experimental Results and Discussions**

The parameters are fixed as follows: \( \varepsilon = 1, \sigma = 3, \mu = 1, \Delta t = 0.1, \upsilon = 0.001 \times 255^2 \) and \( a, b \) are varied according to different images. The following experiments shows the comparison of the proposed method with multiphase hybrid models on MR images in Fig. 1. Table 1 shows the quantitative comparison to the results.
The results show that the proposed LGLAF model produces more accurate results. The improved Laplace energy has a good behavior in tracking the blur boundaries. In order to overcome the difficulty in handling the intensity inhomogeneity, the adaptive segmentation term and the local Gaussian term introduced different statistical information to obtain precise object boundaries.

**Conclusion**

The paper proposed a hybrid multiphase model that consisted of local Gaussian energy, global adaptive energy, image Laplace fitting energy. The experiment demonstrates the better performance of the proposed method in processing MR images with intensity inhomogeneity.

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