A cryogenic heat exchanger with bypass and throttling and its thermodynamic analysis

X Tao¹,², D L Liu¹,², L Y Wang¹,², J Shen¹,², and Z H Gan¹,²,³

¹Institute of Refrigeration and Cryogenics, Zhejiang University, Hangzhou 310027, China
²Key Laboratory of Refrigeration and Cryogenic Technology of Zhejiang Province, Hangzhou 310027, China
³E-mail: gan_zhihua@zju.edu.cn

Abstract. A precooled Joule-Thomson (J-T) cooler refrigerates at liquid helium temperature. Its third stage heat exchanger works below 20 K. Hot fluid cannot be sufficiently cooled due to nonidealism of the heat exchanger and helium-4 properties. In a J-T cycle of low pressure ratio, the heat exchanger with bypass and throttling improves the refrigeration capacity. Bypass and throttling reduces the temperature difference and entropy generation within the heat exchanger.

1. Introduction
Some space detectors need to operate at liquid helium temperature. It is the working temperature of some superconducting materials, and is required as sink temperature in the millikelvin range. The precooled J-T cooler is composed of a regenerative cooler and a J-T cooler. Advantages of different coolers are combined to improve refrigeration efficiency. Precooled J-T coolers are widely utilized in space missions to achieve refrigeration capacity of milliwatts[1, 2].

Figure 1 shows the flow diagram of the precooled J-T cooler. The J-T cooler is composed of three recuperative heat exchangers, which are tube-in-tube helically coiled heat exchangers that run in the counter-flow mode. Hot fluid flows in the inner tube while cold fluid flows in the outer tube. The losses of the first and second stage heat exchangers are compensated by the Stirling cooler. The third stage heat exchanger is connected to a 4 K stage evaporator. The losses of this heat exchanger would reduce the refrigeration capacity, which is a small fraction of the heat transferred within this heat exchanger. So this heat exchanger ought to be approximately ideal. All in all, the third stage heat exchanger is the key component, whose efficiency affects the refrigeration capacity directly.

2. Analysis of the third stage heat exchanger
In this section, heat transfer and flow within the third stage heat exchanger is analyzed numerically. Details about building the model can be referred to [3]. Figure 2 illustrates the computational grid. Figure 3 illustrates the i⁰th element. The following assumptions are considered in modeling:

- The flow is steady and one-dimensional.
- The thermal-conduction resistance of the wall is negligible.
- No heat leaks from the environment.
**Nomenclature**

| Symbol | Description |
|--------|-------------|
| d      | tube diameter, m |
| D      | helical coil diameter, m |
| D_h    | hydraulic diameter, m |
| f      | friction factor |
| h      | specific enthalpy, J/kg |
| L      | heat exchanger length, m |
| m      | mass flow rate, kg/s |
| n      | number of heat exchanger elements |
| Nu     | Nusselt number |
| P      | pressure, Pa |
| Q_r    | refrigeration capacity, W |
| Re     | Reynold number |
| s      | specific entropy, J/(kg·K) |
| S     | entropy increase rate, W/K |
| S_gen  | entropy generation rate, W/K |
| T      | temperature, K |
| U      | heat transfer conductance, W/(m²·K) |
| u      | fluid velocity, m/s |
| α      | ratio of bypassed mass flow rate to overall mass flow rate |
| β      | length ratio of one section to overall heat exchanger |
| ΔA     | differential area for heat transfer of heat exchanger element, m² |
| ΔL     | differential length of heat exchanger element, m |
| ΔP     | pressure drop, Pa |
| ΔQ     | differential heat transfer rate within heat exchanger element, W |
| ε      | heat exchanger effectiveness |
| ρ      | density, kg/m³ |
| λ      | thermal conductivity, W/(m·K) |
| Nu     | Nusselt number |
| ρ      | density, kg/m³ |

**Subscripts**

- c: cold side
- h: hot side
- f: flow process
- i: node index
- in: inlet
- out: outlet
- str: straight tube
- t: heat transfer process
- w: wall

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**Figure 1.** Flow diagram of the precooled J-T cooler

**Figure 2.** Computational grid of the numerical model
As illustrated by figure 3, there are three heat flows within the $i$th element:

- $\Delta \dot{Q}_{c,i}$, which is the convection between the cold fluid and the wall.
- $\Delta \dot{Q}_{w,i}$, which is the convection between the wall and the hot fluid.
- $\Delta \dot{Q}_{w,i}$ and $\Delta \dot{Q}_{w,i+1}$, which are conductions within the wall in the longitude direction.

\[
\Delta \dot{Q}_{c,i} = U \Delta A (T_{c,i} - T_{c,i+1}) / 2 \\
\Delta \dot{Q}_{w,i} = U \Delta A (T_{w,i} + T_{w,i+1} - T_{c,i}) \\
\Delta \dot{Q}_{w,i} = \frac{\lambda}{2\Delta L} \Delta A (T_{w,i} - T_{w,i+1})
\]

Where $U$ is the heat convection coefficient, $\Delta A$ is the heat transfer area, $\lambda$ is the thermal conductivity of the wall, and $\Delta L$ is the length of the element.

The pressure drop is calculated according to equation (4). $\Delta P_i$ is the pressure drop within the $i$th element, $f_i$ is the friction factor, $D_h$ is the hydraulic diameter:

\[
\Delta P_i = \frac{f_i U L}{2D_h} \rho \Delta \dot{m}_i^2
\]

In figure 1, the refrigeration capacity ($\dot{Q}_R$) is the enthalpy flow crossing the cold end of the third stage heat exchanger. $\dot{m}$ is the mass flow rate:

\[
\dot{Q}_R = \dot{m}(h_{c,in} - h_{h,out})
\]

A specific example can be analyzed. Table 1 lists the inlet conditions and the mass flow rates. The ends are assumed to be adiabatic. Table 2 lists the material and geometry parameters. The numerical model is developed with EES, whose property data of helium are utilized [4].

### Table 1. Operating conditions of the example heat exchanger

| Description | Working fluid | Cold inlet temperature | Cold inlet pressure | Hot inlet temperature | Hot inlet pressure | Cold mass flow rate | Hot mass flow rate |
|-------------|---------------|------------------------|---------------------|-----------------------|-------------------|--------------------|--------------------|
| Symbol      | ---           | $T_{c,in}$             | $P_{c,in}$          | $T_{h,in}$           | $P_{h,in}$        | $\dot{m}_c$        | $\dot{m}_h$        |
| Value       | $^4$He        | 4.23 K                 | 102 kPa             | 17.3 K               | 1020 kPa          | 7.53 mg/s          | 7.53 mg/s          |

### Table 2.

| Description | Metal | Inner diameter of the inner tube | Outer diameter of the inner tube | Inner diameter of the outer tube | Helical coil diameter | Length of heat exchanger |
|-------------|-------|---------------------------------|---------------------------------|---------------------------------|-----------------------|--------------------------|
| Symbol      | ---   | $d_1$                           | $d_2$                           | $d_3$                           | $D$                   | $L$                      |
| Value       | SS304 | 1.753 mm                        | 3.175 mm                        | 4.572 mm                        | 0.15 m                | 2.1 m                    |

The flow is confirmed to be laminar based on calculation. The Nusselt numbers and friction factors of straight tubes are related to geometry conditions, which are calculated according to equations (6) and (7). Helical configuration enhances the heat transfer and elevates the pressure drop. The effects are calculated according to equations (8) and (9) [5, 6]:

\[
Nu_{air} = \begin{cases} 5.66 & \text{, cold fluid in concentric circular tube} \\ 3.66 & \text{, hot fluid in circular tube} \end{cases}
\]
The effectiveness of the heat exchanger ($\varepsilon$) is 97.17%. Figure 4 illustrates the temperature distribution. The horizontal axis is nondimensional length. Temperature difference ($\Delta T$) increases from the hot end to the cold end. The pinch point is at the hot end. The temperature difference between the fluids at the hot end ($\Delta T_{\text{hot}}$) is 0.39 K, while that at the cold end ($\Delta T_{\text{cold}}$) is 2.83 K. $\varepsilon$ can be improved by increasing heat transfer area. When $\varepsilon$ is 100%, $\Delta T_{\text{cold}}$ is 2.55 K. The property of $^4$He accounts for that. The specific heat capacity of high pressure helium is larger than that of low pressure helium below 20 K. Even for the case where the heat exchanger is ideal, $\Delta T_{\text{cold}}$ cannot be eliminated.

$$f_{av} = \begin{cases} 
\frac{24}{Re_c}, & \text{cold fluid in concentric circular tube} \\
\frac{16}{Re_b}, & \text{hot fluid in circular tube}
\end{cases} \quad (7)$$

$$\frac{Nu_{\text{cold}}}{Nu_{av}} = 1 + 3.6(1 - \frac{D_h}{D})(\frac{D_b}{D})^{0.8} \quad (8)$$

$$f_{av} = 1 + 0.0823(1 + \frac{D_h}{D})(\frac{D_b}{D})^{0.53} Re_{0.25} \quad (9)$$

Figure 4. Temperature distribution for original structure

3. The third stage heat exchanger with bypass and throttling

In large scale helium liquefiers, Collins cycle with expanders is widely utilized to change the distribution of mass flow and precool the high pressure fluid. In order to reduce vibration and mitigate risks, turbo-expanders are adopted in most cases. In this paper, the refrigeration capacity is of the order of milliwatts; the mass flow of the J-T cycle is of the order of milligrams per second; the pressure ratio is 10. As a result, turbo-expanders are not practical. What affects the temperature distribution within heat exchangers is the heat capacity rate ($C_p$), which is the product of specific heat capacity ($c_p$) and mass flow rate ($m$). $c_p$ varies with pressure, and cannot be changed once the pressure of a cycle is given. But $m$ can be changed to make $C_p$ of both sides be approximate in balance. In order to change the temperature distribution within the heat exchanger and reduce $T_{h,\text{out}}$, in this section, a J-T valve is utilized as the bypass and throttling component. The distribution of $m$ is changed to make up for the unbalance of $c_p$.

Figure 5 illustrates the modified third stage heat exchanger. Part of the hot fluid is extracted and enters the cold side after throttling. Bypass and throttling divides the heat exchanger into three sections:

- The hot fluid is extracted at 9a, and mixes with the cold fluid at 9b. Bypass and throttling section is between 9a and 9b. The mass flow of the cold fluid is larger than that of the hot fluid. Length ratio of this section to the overall heat exchanger is $\beta_2$.

- Cold end section is between 9b and 10, which is close to the cold end of the heat exchanger. The mass flow of the cold fluid is the same as that of the hot fluid, which is the remainder mass flow after bypass. Length ratio of this section to the overall heat exchanger is $\beta_1$. 

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Hot end section is between 9 and 9a, which is close to the hot end of the heat exchanger. The mass flow of the cold fluid is the same as that of the hot fluid, which is the overall mass flow of the J-T cycle. Length ratio of this section to the overall heat exchanger is $\beta_3$.

The aim of bypass and throttling is to change the temperature distribution within the heat exchanger. $\Delta T$ reaches the maximum near the cold end. As a result, bypass and throttling section should be close to the cold end. Besides, mix of the bypass fluid and the cold fluid at 9b should be an isothermal process to reduce entropy generation. The ratio of bypass mass flow to overall mass flow is $\alpha$. The remainder mass flow, which accounts for $(1-\alpha)$ of the overall mass flow, refrigerates at liquid helium temperature. At low values of $\alpha$, the effect of changing the temperature distribution is limited. At high values, the remainder mass flow decreases and the refrigeration capacity decreases.

The numerical model of the modified structure is based on that of the original structure. The following assumptions are added to build the model:
- The mix process at 9b takes no time. The temperature here is uniform.
- The velocity of the fluid at 9a and 9b is uniform.

The effect of bypass and throttling is judged by the refrigeration capacity, which is calculated according to equation (10). Mass flow is sacrificed to reduce the enthalpy of the hot fluid at the outlet.

$$\dot{Q}_R = m(1-\alpha)(h_{c,in} - h_{h,out}) \quad (10)$$

Figure 7 illustrates the refrigeration capacity as a function of $\beta_2$ and $\alpha$. The vertical axis is the ratio of the refrigeration capacity for the modified structure to that for original structure. The original refrigeration capacity is 29.72 mW, which is calculated according to the conditions in tables 2 and 3. For the modified structure, inlet temperatures and pressures (9 and 12) remain the same. Mass flow decreases within some sections, which is illustrated by figure 5. The geometry of the heat exchanger remains the same. The only difference between (a) and (b) is the value of $\beta_1$.

As illustrated by figure 7, maximum refrigeration capacity exists. At low values of $\alpha$, $T_{h,out}$ is not sufficiently reduced. At high values of $\alpha$, within bypass and throttling section, excessive cooling capacity of the cold fluid cannot be transferred to the hot fluid. At low values of $\beta_2$, at 9b, the temperature of the bypass fluid is lower than that of the cold fluid before mix. At high values of $\beta_2$, at
the cold fluid is colder than the bypass fluid before mix. The optimum $\beta_2$ makes the mix process isothermal. The refrigeration capacity reaches the maximum when $\beta_1=24\%$, $\beta_2=7.5\%$, $\alpha=47.5\%$, which increases by 8.14\% above that of the original structure. It is the optimum point.

Figure 7. Refrigeration capacity as a function of $\beta_2$ and $\alpha$, (a): $\beta_1=24\%$, (b): $\beta_1=20\%$

Figure 8 illustrates the temperature distribution in the optimum point. $\Delta T$ increases from the hot end to the cold end within hot end section and cold end section, and decreases from the hot end to the cold end within bypass and throttling section. The overall temperature difference is smaller when compared with figure 4. $\Delta T_{\text{hot}}$ decreases to 0.33 K and $\Delta T_{\text{cold}}$ decreases to 2.13 K. $\dot{Q}_R$ is equal to the enthalpy flow crossing the hot end of the heat exchanger since the heat leak is zero. The increase of $\dot{Q}_R$ is reflected by the decrease of $\Delta T$ at the pinch point, where the mass flow rate and operating pressure remain the same.

Figure 8. Temperature distribution for modified structure

The heat exchanger illustrated by table 1 has been analyzed above and more general conditions can be analyzed. Figure 9 illustrates the refrigeration capacity as a function of the pressure ratio in the J-T cycle, which is the pressure ratio of the hot fluid to the cold fluid in the third stage heat exchanger. The operating conditions and geometry conditions remain the same, which are illustrated by tables 1 and 2. The only exception is the pressure of the hot fluid at the inlet (9). The solid line is the refrigeration capacity of the original structure, corresponding to the left vertical axis. The maximum refrigeration capacity of the modified structure is compared with that of the original structure. The dash line is the ratio, corresponding to the right vertical axis. The effects of pressure ratio on the refrigeration capacity have been analyzed elsewhere[7]. At high values of pressure ratio, the refrigeration capacity is large but the effect of bypass and throttling is limited. At low values of pressure ratio, the refrigeration capacity is small but the effect of bypass and throttling is remarkable. When the pressure ratio is 6, in the original structure, approximately no refrigeration capacity can be achieved, but refrigeration
capacity of 10.5 mW can be achieved for the modified structure. For cases where the pressure ratios of J-T coolers are limited, such as J-T coolers driven by linear compressors or sorption compressors, the configuration of bypass and throttling may be utilized.

Figure 9. Refrigeration capacity as a function of pressure ratio

4. Entropy analysis of heat transfer and flow

This section analyzes the effects of bypass and throttling. The entropy generation ($\dot{S}_{gen}$) is divided into two parts within the heat exchanger, which are contributed by heat transfer and pressure drop respectively. The former part dominates. Equation (11) is the entropy expression of the $i$th element illustrated by figure 3. The first four terms represent the contributions from heat transfer. The last two terms is contributed by pressure drop [8, 9].

$$\Delta\dot{S}_{gen} = \Delta\dot{Q}_{c,i}(\frac{2}{T_{w,i} + T_{w,i+1}} - \frac{1}{T_{w,i}}) + \Delta\dot{Q}_{h,i}(\frac{1}{T_{w,i} - T_{h,i+1}} - \frac{2}{T_{w,i} + T_{h,i+1}}) + \frac{2\Delta\dot{Q}_{h,i+1}}{T_{w,i} + T_{h,i+1}} - \frac{2\Delta\dot{Q}_{c,i+1}}{T_{w,i} + T_{h,i+1}} + mRg \frac{\Delta P_{c,i}}{P_{c,i}} + mRg \frac{\Delta P_{h,i}}{P_{h,i}}$$ (11)

Figure 10 illustrates the entropy generation distribution within the heat exchangers, corresponding to figures 4 and 8. Within the original structure, $\dot{S}_{gen}$ increases sharply from the hot end to the cold end. Within the modified structure, $\dot{S}_{gen}$ decreases near the cold end.

Figure 10. Entropy generation distribution

A system is marked by dash lines in figure 1, which is composed of the third stage heat exchanger, the main J-T valve and the 4 K stage evaporator. Equation (12) is the entropy equation. $m(s_{out} - s_{in})$ is the entropy flowing out of the system. $\dot{S}_{b}$ is the entropy increase contributed by heat absorption within the evaporator. $\dot{S}_{gen}$ and $\dot{S}_{gen}$ are the entropy generations contributed by heat transfer and flow within the heat exchanger. $\dot{S}_{gen_{th}}$ is contributed by throttling within the main J-T
valve. A bypass J-T valve is added to the modified structure, which is illustrated in figure 5. Equation (13) describes the system. $\dot{S}_{gen_{th2}}$ is contributed by throttling within the bypass J-T valve. The other terms remain the same. Figure 11 illustrates the portion of every term. The portion of $\dot{S}_R$ increases from 18.79% to 20.24%, corresponding to the 8.14% increase of the net refrigeration. $\dot{S}_{gen_t}$, $\dot{S}_{gen_f}$ and $\dot{S}_{gen_{th1}}$ decrease substantially. $\dot{S}_{gen_{th2}}$ dominates the entropy generation.

\[
\dot{m}(s_{c, out} - s_{h, in}) = \dot{S}_R + \dot{S}_{gen_t} + \dot{S}_{gen_f} + \dot{S}_{gen_{th1}} \quad (12)
\]
\[
\dot{m}(s_{c, out} - s_{h, in}) = \dot{S}_R + \dot{S}_{gen_t} + \dot{S}_{gen_f} + \dot{S}_{gen_{th1}} + \dot{S}_{gen_{th2}} \quad (13)
\]

Figure 11. Comparison of net refrigeration and losses

5. Conclusion
A cryogenic heat exchanger with bypass and throttling is introduced in this paper, which takes effect especially at low pressure ratio. When the pressure ratio of the J-T cycle is 10, the modified structure increases the refrigeration capacity by 8.14% above that of the original structure.

Entropy generation minimization is used to analyze the heat exchanger. In the modified structure, the entropy generations of heat transfer and flow fall substantially, and the entropy increase of net refrigeration rises.

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