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To cite this article: I B P A Pranidhana and P H Gunawan 2018 J. Phys.: Conf. Ser. 971 012032

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Computational parallel for shallow water-sediment concentration coupled model

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Abstract. The goal of this paper is to analyze the parallel performance using OpenMP platform on shallow water-sediment concentration coupled model. The sediment model is coupled with the shallow water model for generating the water flow. In this paper, convection-diffusion equation is used to describe the sediment movement. The numerical results using OpenMP platform is shown satisfying to reduce the computational time. Indeed, the parallel time is faster than the serial time even along the increasing of the discrete points. The result using number of grid sizes 1600, the speedup is obtained 1.583 times. Meanwhile the efficiency is observed 39.57%. Moreover, the average efficiency from 5 times experiments is found 38.9%.

1. Introduction

Suspended sediment model is a mathematical model of sediment transport over the water flow. For instance, the sandy bottom in coastal area will be driven by the water flow and will eventually fall down until it is suspended. This can produce the propagation of concentration of sediment from high to low regions. Generally, the high concentration of sediment is located at the bottom, meanwhile the low concentration is located near the water surface. This concentration profile can be modeled by the steady suspended sediment model. The steady model of suspended sediment can be found for instance in [1].

The main focus of sediment model in [1] is two-dimensional (2D) vertical steady state of sediment. The formula of this model is given as follows

\[
\frac{\partial \Phi}{\partial x} + \frac{\omega_s}{u(z)} \frac{\partial \Phi}{\partial z} = \frac{D}{u(z)} \frac{\partial^2 \Phi}{\partial z^2},
\]

where \( \Phi \) is the concentration of sand sediment, \( \omega_s \) the settling velocity of sand particles, \( u(z) \) the velocity function depend on \( z \) and \( D \) a constant diffusion coefficient of turbulent flow. Moreover, \( x \) and \( z \) are the horizontal and vertical coordinates respectively.

The numerical result of this steady model is shown in Figure 1. The detail of this numerical simulation will be given in Section 4.1. Here, the result by a constant water velocity depend on the vertical height \( u(z) \) is shown that the water flow disturbs the concentration of sand at surface. Additionally, Figure 1 shows the increasing of sand concentration along horizontal direction at steady state.
In this paper, the goal is to investigate suspended sediment change (1) where the water velocity is changing in time. This means the velocity function in (1) should be depend on spatial vertical coordinate \( z \) and time \( t \). In order to transform velocity function of sediment, the interaction of sediment and water dynamic should be considered since the velocity of water is depend on spatial and time variables. Thus in this paper, the steady state suspended sediment model will be coupled with shallow water model which can be connected through velocity function.

Another contribution of this paper is to analyze the computational time of parallel algorithm for water-sediment interaction. The increasing of number of grid sizes in horizontal and vertical directions can produce the increasing of computational cost. Thus, the parallel algorithm is required to minimize this cost. Several references such as in [2, 3, 4, 5, 6] are shown the parallel computing using OpenMP can accelerating the numerical computation. Therefore here, architecture shared parallel programming by OpenMP platform will be used for simplicity.

2. Coupled model of water-sediment interaction and its numerical scheme

2.1. Dynamical model of water flow

The well known water flow model for describing the shallow area is called shallow water equations [7]. This model consists of two equations, mass (2) and momentum (3) equations, which can be written with zero bottom as

\[
\frac{\partial h}{\partial t} + \frac{\partial (hv)}{\partial x} = 0, \tag{2}
\]

\[
\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + g \frac{\partial h}{\partial x} = -S_f, \tag{3}
\]

where \( h(x,t) \) denotes the water depth, \( v(x,t) \) horizontal velocity, \( g \) gravitational force, and \( S_f \) a friction term.

The approximation solutions (2 - 3) by some numerical methods can be found in some references. The one well known method is called the finite volume method (FVM), for instance see [8, 7, 9, 10, 11, 12, 13, 14] and [15]. This method is a robust and shock capturing method which is still widely used for approximating shallow water equations.
Here, the staggered grid scheme FVM [7] will be used. This scheme is shown straightforward and simple to implement. Moreover, in [8] the scheme is proved satisfying the entropy balance. Following the notation in [7], let the discrete properties of shallow water equations on domain $\Omega = [-L, L]$ are given as follows

$$
\Delta x = \frac{2L}{N_x}, \quad x_k = \Delta x \times k, \quad k \in \mathcal{M} = \{0, 1, 2, \ldots, N_x\},
$$

$$
t^n = \Delta t \times n, \quad n \in \mathcal{T} = \{0, 1, 2, \ldots\}.
$$

The discretization of mass balance (2) at full spatial discrete point $k \in \mathcal{M}$ is given as

$$
\frac{h_{k}^{n+1} - h_{k}^{n}}{\Delta t} + \frac{q_{k+\frac{1}{2}}^{n} - q_{k-\frac{1}{2}}^{n}}{\Delta x} = 0, \quad (4)
$$

Where $q_{k+\frac{1}{2}}^{n} = h_{k+\frac{1}{2}}^{n+1} v_{k+\frac{1}{2}}^{n+1}$, $h_{k+\frac{1}{2}}^{n+1} = \begin{cases} h_{k}^{n} & \text{if } v_{k+\frac{1}{2}}^{n} > 0, \\ h_{k+1}^{n+1} & \text{otherwise}. \end{cases}$

Meanwhile, the discrete form of momentum balance (3) at half spatial discrete point $k + \frac{1}{2} \in \mathcal{M} - \{0, N_x\}$ is given as

$$
\frac{v_{k+\frac{1}{2}}^{n+1} - v_{k+\frac{1}{2}}^{n}}{\Delta t} + \frac{1}{h_{k+\frac{1}{2}}^{n+1}} \left( \frac{q_{k+1}^{n+1} v_{k+1}^{n+1} - q_{k}^{n+1} v_{k}^{n+1}}{\Delta x} - v_{k+\frac{1}{2}}^{n} q_{k+\frac{1}{2}}^{n} - q_{k-\frac{1}{2}}^{n} - \frac{h_{k+1}^{n+1} - h_{k}^{n+1}}{\Delta x} \right) + g \frac{h_{k+1}^{n+1} - h_{k}^{n+1}}{\Delta x} = -S_{f,k}^{n}, \quad (5)
$$

Note that, Equations (4-5) are used full and half grid notations for mass and momentum variables respectively. Authors encourage the readers to see references [7, 8] for more detail about staggered grid scheme.

2.2. Suspended sediment model

Suppose $\Phi$ denotes concentration of sand sediment. When sand tends to move downwards due to gravity force, then sand particle has fall velocity $\omega_s > 0$. Directly, this gravitational effect produces the influx $-\omega_s \Phi$. Moreover, diffusion mechanism for sand particles occurs such that the concentration profile is flat (outflux). Thus, the relation of two fluxes can be given as

$$
Q = -\omega_s \Phi - D \frac{\partial \Phi}{\partial z}. \quad (6)
$$

In steady state situation the mass balance for suspended sand is given by the following equation:

$$
u(z) \frac{\partial \Phi}{\partial x} + \frac{\partial Q}{\partial z} = 0. \quad (7)
$$

Finally, substituting (6) to (7), the suspended sediment model in Equation (1) can be achieved. Here, the velocity is depend on the vertical direction and in parabolic shape

$$
u(z) = u_0 \frac{(-z^2 + 2az)}{a^2}. \quad (8)$$
This velocity function describes the velocity of water at bottom will be less than the surface in sediment area.

In this paper, a coupled model of SWE (2-3) and suspended sediment (1) will be elaborated. The idea is to give a small modification of (8) such that the average velocity from SWE can be transformed to the velocity with parabolic shape depend on $z$. The illustration of this coupled model can be seen in Figure 2. Then here, velocity of suspended sediment model will becomes

$$u(z, t) = \frac{v(x_0, t)}{a^2} \left(-z^2 + 2az\right). \quad (9)$$

This velocity will be acting on the interface between SWE model and sediment model.

Figure 2: The illustration of coupled model using SWE and suspended sediment model. The influx of water velocity for sediment change at $x_0$ will be given from SWE.

In order to approximate the solution of sediment equation (1), Crank-Nicolson method can be used. The discrete properties of $z$ direction on domain $\Omega_z [0, L_z]$ are given as follows

$$\Delta z = \frac{L_z}{N_z}, \quad z_i = \Delta z \times i, \quad i \in \{0, 1, \cdots, N_z\}.$$ 

Discrete form of (1) by Crank-Nicolson method can be seen as the following

$$\frac{\phi_{i+1}^k - \phi_i^k}{\Delta x} + \Theta \left\{ \begin{array}{l} \frac{\omega_s}{u} \left( \frac{\phi_{i+1}^k - \phi_i^{k+1}}{2\Delta z} \right) - \frac{D}{u} \left( \frac{\phi_{i+1}^k - 2\phi_i^{k+1} + \phi_{i-1}^k}{\Delta z^2} \right) \\ (1 - \Theta) \left( \frac{\omega_s}{u} \left( \frac{\phi_i^k - \phi_{i-1}^{k+1}}{2\Delta z} \right) - \frac{D}{u} \left( \frac{\phi_i^k - 2\phi_i^{k+1} + \phi_{i-1}^k}{\Delta z^2} \right) \right) \end{array} \right\} = 0. \quad (10)$$

where $\Theta \in [0, 1]$. Here, by using $\Theta = 0$ then the numerical scheme (10) is called explicit method. Meanwhile, it called implicit method if $\Theta = 1$. Generally, Crank-Nicolson use $\Theta = 0.5$ to gained the second order approximation which can produces system of equations. Further, this system of equations can be solved numerically by Thomas algorithm [1].

3. Parallel architecture using OpenMP

The application of parallel programming in this paper uses OpenMP platform. According to [6, 16, 17, 18, 19, 20], OpenMP is an application programming interface (API) for parallel programming which is designed for shared memory multiprocessors architecture. In computing process of coupled model SWE and suspended sediment, there are 9 stages of the processes. Where, 4 processes can be done in parallel and other processes can done without parallel (serial). The flow of this stages can be seen in Figure 3 for more detail.

The computation process start from serial part which are allocating size of variables. The goal of this stage is to create the type and structure of data and to initialize the variables. Next, program will compile the parallel section trough several stages. First step is to compute
Figure 3: The serial and parallel computation processes. Several tasks and procedures which can be parallelized are grouped in parallel part. Meanwhile, the rest of them are grouped in serial part.

the mass equation to obtain the solution of water height $h$ by (4). Second step is to compute momentum equation to obtain the average velocity $v$ by (5). Third step is to compute sediment concentration in order to obtain the solution of sediment profile $\Phi/\Phi_0$ by (10). And the forth step is to update the variables for next iterations. Finally, the code leads back to serial section for entering the visualization stage.

4. Numerical results

4.1. Simulation of steady sediment change

In this section, the simulation of steady flow of sediment change is elaborated. The simulation only use Equation (1) with (8) and unassisted SWE. In this simulation, the discrete properties are given as:

- The discrete domain: $\Delta x = 0.05$, $\Delta z = 0.1$, and $\Delta t = 0.0001$.
- The coefficients: $\Phi_0 = 1$, $u_0 = 1$, $D = 1$, $\omega_s = 0.3$ and $\Theta = 0.5$.
- At boundaries:

$$
\begin{cases}
\Psi = \Psi_0, & \text{if } z = 0, \text{ [Equilibrium concentration at bottom]}, \\
\omega_s \Phi - D \frac{\partial \Phi}{\partial z} = 0, & \text{if } z = a, \text{ [No sediment enters and leaves at water surface]},
\end{cases}
$$

At initial condition:

$$\Phi(z) = \Phi_0 \exp\left(\frac{-\omega_s z}{D}\right).$$

The result of this simulation can be seen in Figure 1. The initial condition gives the linear profile of concentration to the $z$ direction at $xD/a^2u = 0$. Afterward, form the result, it can seen clearly that the horizontal current $u(z)$ shift the contour plot to the right. The profile of sediment concentration ($\Phi/\Phi_0$) is in a good agreement with the adaptation length of this simulation. The profile is shown in parabolic shape at domain $[0 : 1]$ which agrees with the velocity profile (8).

4.2. Unsteady sediment

Here, the unsteady sediment change due to the change of water velocity will be elaborated. The dynamical system of water is governed by SWE (2-3). Meanwhile, the suspended sediment model is governed by (1) with velocity function (9).

The properties of initial sediment can be found in Section 4.1. However, several additional coefficients are given such as $g$ and $\mu$ are 9.8 and 40 respectively. Moreover note that here, the velocity function (9) is used, thus $u_0$ is no longer a coefficient.

Here, the domain in $z$- and $x$-direction of simulation are given as $0 \leq z/a \leq 1$ and $0 \leq xD/a^2u \leq 2$ respectively. Similar to the steady case, Crank-Nicolson method ($\Theta = 0.5$) is used to approximate (1). The initial condition of SWE is given as follows

$$h(xD/a^2u, 0) = \begin{cases} 15 & \text{if} \ -4.5 \leq xD/a^2u \leq -2.5 \\ 1 & \text{otherwise} \end{cases}$$

$$v(xD/a^2u, 0) = 0$$

**Figure 4:** Simulation results at time 0.0098 s. (a) Water height profile. (b) Water velocity profile. (c). Suspended sediment concentration profile.
The results of this simulation are shown in Figures 4-7. Several sequential of final time are recorded at $t = 98 \times 10^{-4}$ s, $t = 19 \times 10^{-1}$ s, $t = 99 \times 10^{-1}$ s and $t = 11.9$ s. At time $t = 98 \times 10^{-4}$ s, the initial dam break propagates the water level to the left and right side 4(a), thus increases the velocity at the discontinue points 4(b). Since at the interface ($xD/a^2u = 0$), the velocity is zero, then no sediment movement is detected in Figure 4(c).

![Water height profile](image1)

![Water velocity profile](image2)

![Sedimentation profile](image3)

**Figure 5:** Simulation results at time 1.9998 s. (a) Water height profile (b) Water velocity profile. (c) Suspended sediment concentration profile.

Figure 5 shows the propagation of water height (a), velocity (b) and sediment movement (c) at final time $t = 19 \times 10^{-1}$ s. Since the velocity at $xD/a^2u = 0$ slightly increasing, sediment movement in the top layer can be detected.

Meanwhile, in Figures 6 and 7, sediment movement (c) can be easily observed since the propagation of water height and velocity is increasing at interface. This means velocity $u_0$ increasing and sediment profile will be close to the solution of steady (Figure 1).

Indeed from this results, the coupled model SWE and suspended sediment model is successfully implemented. The simulations show that the increasing velocity of shallow water at the interface of sediment domain, then sediment change can be observed. Here, since the velocity of SWE in interface close to 1, then the solution of sediment is observed in a good agreement with the steady solution.

### 4.3. Parallel performance

In order to analyze the parallel computing performance using OpenMP, several simulations in unsteady sediment model are given. The specifications of computer in this experiment is shown in Table 1.

The time result of simulation for several number of discrete sizes $N_x$ in shallow water-sediment concentration coupled model is shown in Table 2. The resulting time is obtained from the simulation with different discrete points $N_x$, which are 100, 200, 400, 800 and 160. Table 2 shows the CPU time in parallel code is faster than the serial code along the increasing of the discrete points. By using $N_x = 1600$ points, the execution time in parallel and serial are obtained 1431.259 s and 2265.401 s respectively. This means, by using 4 processors, the speedup of
Figure 6: Simulation results at time 9.9998 s. (a) Water height profile. (b) Water velocity profile. (c) Suspended sediment concentration profile.

Figure 7: Simulation results at time 11.9998 s. (a) Water height profile. (b) Water velocity profile. (c) Suspended sediment concentration profile.
Table 1: The specifications of computer in experiment of parallel computing.

| Name                  | Type                     |
|-----------------------|--------------------------|
| Processors            | Intel (R) Core (TM) i5-2500 |
| Number Of Processors  | 4                        |
| RAM Size              | 8 GB                     |
| Operating System      | Ubuntu 12.04.3           |

parallel performance is obtained 1.583 times and the efficiency is given 39.57%. Overall, the average of efficiency for all experiments is observed 38.9%. Indeed here, parallel programming can reduce the time computation successfully.

Table 2: The results of CPU time, speedup and efficiency in parallel computing.

| Discrete Points | Execution Time (s) | Speedup | Efficiency(%) | Average Efficiency(%) |
|-----------------|--------------------|---------|---------------|------------------------|
|                 | Serial | Parallel |                   |                        |
| 100             | 186.159 | 120.896 | 1.539 | 38.49 |
| 200             | 331.429 | 214.156 | 1.548 | 38.69 |
| 400             | 681.832 | 438.632 | 1.554 | 38.86 |
| 800             | 1132.156 | 725.012 | 1.562 | 39.05 |
| 1600            | 2265.401 | 1431.259 | 1.583 | 39.57 |

5. Conclusion
Here, the simulations of shallow water-sediments concentration coupled model has been successfully implemented. The solution of unsteady sediment model is in a good agreement with the steady solution when the velocity at the interface water-sediment tends to 1. The results of parallel programming with OpenMP is shown satisfying. The execution time in serial can be reduced by parallel programming. For simulation with 1600 grids, the computational time of simulation with parallel is 1431.259 s and serial is 2265.401 s. Those results show that OpenMP is successfully reduced the time computation. The maximum speedup and efficiency are observed 1.583 and 39.57% are respectively. Moreover, the average of efficiency in overall experiments is obtained 38.9%.

Acknowledgment
Second author would like to say thank you very much for Prof. Sri Redjeki P. for the great discussion in the beginning of sediment modeling.

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