The Neutrino Magnetic Moment Induced by Leptoquarks

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(November, 1998)

Abstract

Allowing leptoquarks to interact with both right-handed and left-handed neutrinos (i.e., “non-chiral” leptoquarks), we show that a non-zero neutrino magnetic moment can arise naturally. Although the mass of the non-chiral vector leptoquark that couples to the first generation fermions is constrained severely by universality of the $\pi^+$ leptonic decays and is found to be greater than 50 TeV, the masses of the second and third generation non-chiral vector leptoquarks may evade such constraint and may in general be in the range of $1 \sim 100$ TeV. With reasonable input mass and coupling values, we find that the neutrino magnetic moment due to the second generation leptoquarks is of the order of $10^{-12} \sim 10^{-16} \mu_B$ while that caused by the third generation leptoquarks, being enhanced significantly by the large top quark mass, is in the range of $10^{-10} \sim 10^{-14} \mu_B$. 

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I. INTRODUCTION

Existence of a non-zero neutrino magnetic moment has long been a concern of great interest, since it can have an observable laboratory effect such as neutrino-charge lepton elastic scattering, $e^+e^- \rightarrow \nu\bar{\nu}\gamma$, and also some important astrophysics effect, such as cooling of SN 1987A, cooling of helium stars, etc. It is likely that neutrinos may have a small but nonvanishing mass; for various bounds on magnetic moments and masses, see [1] and [2]. Within the framework of the standard model, a nonzero neutrino mass usually imply a nonzero magnetic moment. It has been shown that [4], for a massive neutrino,

$$\mu^S_M = \frac{3eG_F m_\nu}{8\pi^2 \sqrt{2}} = 3.2 \times 10^{-19} m_\nu (eV) \mu_B,$$

where $\mu_B$ is the Bohr magneton.

In models beyond the standard model, right-handed neutrinos are often included in interactions (for a review e.g. [3]), so that we need not depend on a nonzero neutrino mass to generate a nonzero magnetic moment. In this article, we consider the possibility of using leptoquark interactions to generate a nonzero neutrino magnetic moment. In many unification models, such as SU(5), SO(10), etc., one often put quarks and leptons into the same multiplet, so that leptoquarks arise naturally for connecting different components within the same multiplet. What makes a leptoquark unique and interesting is that it couples simultaneously to both a lepton and a quark. This may help generating a nonzero neutrino magnetic interaction. Specifically, when a top quark involves in the loop diagram, its mass provide a large enhancement for the neutrino magnetic moment. (In such diagram, a massless neutrino needs some massive internal fermion to flip its chirality, giving rise to some nonzero magnetic moment.)

We add right-handed neutrinos in the general renormalizable lagrangian of leptoquarks. Owing to existence of lepton numbers which recognize the generation, we distinguish leptoquarks by its generation quantum number, but this may induce four-fermion interactions which may enhance some helicity suppressed process such as $\pi^+ \rightarrow e^+\nu$ to the extent
that leptonic universality may even be violated. This usually gives a tight constraint on
the leptoquark \[^{5}\]. For non-chiral vector leptoquarks of electromagnetic strength coupling,
this corresponds to having a mass heavier than \(50 \text{ TeV}\) for the first generation leptoquark.
With such a heavy leptoquark, we still find a nonzero neutrino magnetic moment \(\mu_\nu\) up to
\(10^{-18} \mu_B\). For the second and third generation leptoquarks, their masses are not severely
constrained by the above process. Assuming their mass lying somewhere between \(1 \sim 100\)
\(\text{TeV}\), we obtain \(\mu_\nu\) of order \(10^{-12} \sim 10^{-16} \mu_B\) (from the second generation leptoquark) and
\(10^{-10} \sim 10^{-14} \mu_B\) (from the third generation leptoquark), respectively. Such predictions may
already have some observable effects such as those mentioned earlier.

II. NEUTRINO MAGNETIC MOMENT IN MODELS WITH LEPTOQUARKS

Leptoquarks arise naturally in many unification models which attempt to put quarks and
leptons in the same multiplet. There are scalar and vector leptoquarks which may couple to
left- and right-handed neutrinos at the same time, but only vector leptoquarks can couple
to the upper component of the quark SU(2) doublet. The heaviness of the top quark may
enhance the neutrino magnetic moment once we use the vector leptoquark to connect to
the quark doublet. Of course, there are subtleties regarding renormalization of the vector
leptoquark which may be treated in a way similar to gauge bosons. In our calculation, we
adopt Feynman rules in the \(R_\xi\)-gauge and take \(\xi \rightarrow \infty\) at the end of the calculation while
neglecting all unphysical particles in the \(R_\xi\)-gauge. (This is a step which has often been
employed in nonabelian gauge theories.)

We begin our analysis by constructing a general renormalizable lagrangian for the
quark-lepton-leptoquark coupling. Following \[^{6}\], we demand such action to be an
SU(3)\(\times\)SU(2)\(\times\)U(1) invariant which conserves the baryon and lepton numbers but, in addition to \[^{3}\] we add terms which couple to right-handed neutrinos. For leptoquarks with the
fermion number \(F \equiv 3B + L = 0\),

\[
\mathcal{L}_{F=0} = (g_{1L} \bar{Q}_L \gamma^\mu L_L + g_{1R} \bar{D}_R \gamma^\mu l_R + g_{\nu R} \bar{U}_R \gamma^\mu \nu_R) V_{1\mu}^{(4)}
\]

3
\begin{align}
+ & (g_{2L}^d D_R^i L_L^i \tau_{2ij} + g_{2R} Q_L^j \nu_R^j) S_2^{(\frac{1}{2})} \\
+ & (g_{2L}^u U_R^i L_L^i \tau_{2ij} + g_{2R} Q_L^j \nu_R^j) S_2^{(\frac{3}{2})} \\
+ & + g_{3L} \bar{Q}_L \gamma^\mu L_L V_{3\mu}^{(\frac{3}{2})} + g_{1R} U_R^i \gamma^\mu \nu_R l_R V_{1\mu}^{(\frac{3}{2})} + g_{1L} D_R \gamma^\mu \nu_R V_{1\mu}^{(-\frac{1}{2})} \\
+ & + c.c.,
\end{align}

and, for \( F = \pm 2 \),

\begin{align}
\mathcal{L}_{F=2} = (h_{2L} \bar{u}_L^i \gamma^\mu \bar{L}_L^i \tau_{2ij} + h_{2R} \bar{Q}_L^j \gamma^\mu \nu_R^j) V_{2\mu}^{(-\frac{1}{2})} \\
+ & (h_{1L} \bar{Q}_L^j \gamma^\mu \bar{L}_L^i \tau_{2ij} + h_{1R} \bar{U}_R^j \gamma^\mu \nu_R^j) S_1^{(\frac{1}{2})} \\
+ & + (h_{2L} \bar{D}_R^i \gamma^\mu \bar{L}_L^i \tau_{2ij} + h_{2R} \bar{Q}_L^j \gamma^\mu \nu_R^j) V_{2\mu}^{(\frac{3}{2})} \\
+ & + h_{3L} \bar{Q}_L^j \gamma^\mu \nu_R^j L_L S_3^{(\frac{3}{2})} + h_{1R} \bar{D}_R^i \gamma^\mu \nu_R^j S_1^{(\frac{3}{2})} + h_{1\nu} \bar{U}_R^j \nu_R S_1^{(-\frac{1}{2})} \\
+ & + c.c..
\end{align}

The notation adopted above is self explanatory; for example, \( S, V \) denotes scalar and vector leptoquarks respectively, the superscript is its average electric charge or the hypercharge \( Y \), and the subscript of a leptoquark denotes which SU(2) multiplet it is in, and the generation index is suppressed. From here it is clear that among those leptoquarks that couple to neutrinos of both chiralities, a radiative \( \nu \nu \gamma \) diagram with the exchange of a virtual \( U \)-type quark can proceed only when accompanied by a vector leptoquark, namely \( V_{1\mu}^{(\frac{3}{2})} \) in \( \mathcal{L}_{F=0} \) or \( V_{2\mu}^{(-\frac{1}{2})} \) in \( \mathcal{L}_{F=2} \); on the other hand, the exchange of a virtual \( D \)-type quark can proceed only with the scalar leptoquark, namely \( S_2^{(\frac{3}{2})} \) in \( \mathcal{L}_{F=0} \) or \( S_1^{(\frac{3}{2})} \) in \( \mathcal{L}_{F=2} \). Note that we do not consider mixing between different leptoquarks due to Higgs interactions, which will introduce additional parameters. The diagram in question is shown explicitly in Fig. 1.

Given these couplings, it is straightforward to calculate induced neutrino magnetic moments via one loop diagrams. To see that the heavy top quark mass can enhance the prediction, we calculate the \( \nu \nu \gamma \) diagram with the exchange of up-type quark and \( V_{1\mu}^{(\frac{3}{2})} \) i.e. the first term in \( \mathcal{L}_{F=0} \). As one of the standard methods to treat loop diagrams involving massive vector particles, we use Feynman rules in the \( R_\xi \)-gauge and take \( \xi \to \infty \) at the end.
of calculation while neglecting any unphysical particle. In addition to minimum substitution, we add the term $e Q
u V^\dagger V_\nu F^{\mu\nu}$ in the lagrangian, such that the whole $VV\gamma$ coupling is in a form similar to the non-abelian $WW\gamma$ type coupling, and the procedure results in a finite limit under $\xi \to \infty$. We obtain, with all couplings chosen to be real,

$$\mathcal{L}^{\text{eff}} = -\frac{e}{2m_e} \bar{\nu} \left( \frac{\sigma^{\mu\nu}}{2} \right) \nu F_{\mu\nu} F_2, \quad |\mu_\nu| = \frac{e}{2m_e} F_2$$

$$F_2 = \frac{1}{16\pi^2} \frac{2m_e}{M} \left\{ \frac{g_1^L g_2^\nu}{M_v} \left[ Q_q( f_1(a) + f_2(a) ) + Q_v( f_3(a) + f_4(a) ) \right] \\
+ (g_2^L + g_\nu^2) \frac{m_v}{M_v} \left[ Q_q( g_1(a) + g_2(a) ) + Q_v( g_3(a) + g_4(a) ) \right] \right\},$$

where $a = m_q^2/M_v^2$, $Q_q = -Q_v = 2/3$, and $e > 0$, while $f_i$ and $g_i$ are given by

$$f_1(a) = \frac{2(-1 + a^2 - 2a \log(a))}{(a - 1)^3},$$

$$f_2(a) = -\frac{a(3 - 4a + a^2 + 2\log(a))}{2(a - 1)^3},$$

$$f_3(a) = -\frac{3(-1 + 4a + 3a^2 - 2a^2 \log(a))}{2(a - 1)^3},$$

$$f_4(a) = -1/2,$$

$$g_1(a) = \frac{(-4 - 5a^3 + 9a + 6a(2a - 1) \log(a))}{6(a - 1)^4},$$

$$g_2(a) = \frac{a(3 - 4a + a^2 + 2\log(a))}{4(a - 1)^3},$$

$$g_3(a) = \frac{(7 - 33a + 57a^2 - 31a^3 + 6a^2(3a - 1) \log(a))}{12(a - 1)^4},$$

$$g_4(a) = \frac{(2 - 6a + 15a^2 - 14a^3 + 3a^4 + 6a^2 \log(a))}{12(a - 1)^4}.$$
III. CONSTRAINTS AND NUMERICAL RESULTS

Before working out numerical predictions, we need to consider the constraints arising from the leptonic decays of the pseudoscalar meson, such as $\pi^+ \rightarrow e^+\nu$. Integrating out $V^{(4)}$ and performing Fierz reordering, we obtain $L_{\text{eff}}$ relevant to the leptonic decay of a pseudoscalar meson,

$$L_{\text{eff}} = \frac{1}{M^2} (2g_{1L}^* g_{1R} \bar{D}_R U_L \bar{\nu}_L l_R + 2g_{1\nu}^* g_{1L} \bar{D}_L U_R \bar{\nu}_R l_L$$

$$- g_{1L}^* g_{1L} \bar{D}_L \gamma^\mu U_L \bar{\nu}_L \gamma_\mu l_L - g_{1\nu}^* g_{1R} \bar{D}_R \gamma^\mu U_R \bar{\nu}_R \gamma_\mu l_R + \text{c.c.})$$

We consider the universality constraint arising from the $\pi^+$ leptonic decay, and neglect the neutrino mass contribution. Define $R = Br(\pi^+ \rightarrow e^+\nu)/Br(\pi^+ \rightarrow \mu^+\nu)$. The first and third terms of $L_{\text{eff}}$ have interference with the standard model Fermi interaction. This is an order of $1/M^2$ correction to $R$, while the other term is a correction of order $1/M_\nu^4$. Furthermore, the first term which is scalar coupling is enhanced by a factor of $m_\pi^2/((m_u + m_d)m_e)$ so this is the dominant term to constrain the mass of the leptoquark. We assume $g_{1\nu}^* = g_{1L} = g_{1R} = g$ which is a natural assumption for the vector leptoquark. We obtain

$$R^{\text{exp}} = R^{\text{sm}}(1 + 2 \frac{m_\pi^2}{m_e(m_u + m_d)}(- \frac{g_{1L}^* g_{1R}}{\sqrt{2}M_\nu^2 G_F})), \quad (8)$$

where experimental average $R^{\text{exp}} = (1.230 \pm 0.004) \times 10^{-4}$ [9], and standard model calculation $R^{\text{sm}} = (1.2352 \pm 0.0005) \times 10^{-4}$ [7]. This correspond to

$$M_\nu > g m_\pi \sqrt{\frac{\sqrt{2}}{0.0075G_F m_e(m_u + m_d)}} \sim 50 \left(\frac{g}{e}\right) \text{TeV}. \quad (9)$$

For a coupling of the electromagnetic strength, this correspond to having the vector leptoquark with a mass greater than 50 TeV for the first generation. This constraint is in fact more servere than what we may obtain from the atomic parity vialation experiment, which we shall ignore in this paper. For the second and third generations leptoquarks, there is no direct restriction from the universality of the $\pi$ leptonic decay, nor from the atomic parity violation experiment. Nevertheless, one can find various lower bounds for the leptoquark.
mass [1], from direct searches at the HERA ep collider, the Tevatron p\bar{p} collider, and at the LEP e^+e^- collider. Typical bounds from direct searches is about few hundreds GeV, while the bounds from indirect searches are given in [8]. We shall consider a leptoquark mass in the general range of TeV’s.

For the reason of comparisons, let us recall briefly some of the upper limit obtained from the leptonic scattering such as elastic \( \nu(\bar{\nu}) \) with \( l^+(l^-) \), \( e^+e^- \rightarrow \nu\bar{\nu}\gamma \), etc., and also from the astrophysical processes such as cooling of helium stars, red giant luminosity and so on [1]. As a reference point, we recall the standard model formula on the neutrino magnetic moment arising from a nonzero neutrino mass [4],

\[
\mu_{\nu}^{sm} = 3.2 \times 10^{-19} m_\nu (\text{eV}) \mu_B
\]

(referred to as “the extended standard electroweak theory”). Accordingly, the upper limit of \( \mu_\nu \) for the first generation neutrino is \( \mu_\nu^{sm} \leq 2.3 \times 10^{-18} \mu_B \) with \( m_\nu \leq 7.3 \text{ eV} \). The upper limit may also be obtained from leptonic scatterings, which is typically \( 10^{-10} \mu_B \), or from astrophysics studies with a more stringent upper limit of \( 10^{-11} \mu_B \). Our numerical results for the first generation are summarized in Fig. 2, where the neutrino magnetic moment \( \mu_\nu \) in units of \( \mu_B \) is shown as a function of the leptoquark mass. We note that, for the leptoquark mass \( V_{ij\mu}^{(2)} \) of 50 to 100 TeV, \( \mu_\nu \) is of order \( 10^{-18} \mu_B \), a value compatible with the extended standard electroweak theory.

The upper limit of \( \mu_\nu \) for the second generation neutrino is \( 0.51 \times 10^{-13} \mu_B \) (with \( m_\nu \leq 0.17 \text{ MeV} \)) in the extended standard electroweak theory [1], or in the range of \( 10^{-10} \mu_B \) from leptonic scatterings, while from astrophysics the typical value is \( 10^{-11} \mu_B \). In Fig. 3, we describe our prediction on the neutrino magnetic moment \( \mu_\nu \) in units of \( \mu_B \) as a function of the leptoquark mass of 1 to 100 TeV. We obtain \( \mu_\nu \) around \( 10^{-12} \sim 10^{-16} \mu_B \), a value very close to being observable.

The upper limit of \( \mu_\nu \) for the third generation neutrino is \( 1.1 \times 10^{-11} \mu_B \) (with \( m_\nu \leq 35 \text{ MeV} \)) in the extended standard electroweak theory [1], or in the range of \( 10^{-6} \sim 10^{-7} \mu_B \) from leptonic scatterings, while from astrophysics studies the upper limit is \( 10^{-12} \sim 10^{-11} \mu_B \). In Fig. 4, we plot the third generation neutrino magnetic moment \( \mu_\nu \) in units \( \mu_B \) as a function of the leptoquark mass in the range of 1 to 100 TeV. We find that \( \mu_\nu \) is of order
$10^{-10} \sim 10^{-14}\mu_B$.

IV. CONCLUSION

Vector leptoquarks in the TeV mass range, when couple to both left- and right-handed neutrinos, offer an alternative mechanism for generating a nonvanishing neutrino magnetic moment, which in some cases is by no means negligible. This alternative mechanism (which does not require a nonzero neutrino mass) makes use of the special feature that leptoquarks couple simultaneously to leptons and quarks. For the third generation neutrino, there is a potential enhancement from the very large top quark mass making the corresponding predicted neutrino magnetic moments fairly sizable.

V. ACKNOWLEDGMENTS

We would like to acknowledge Dr. C.-T. Chan for valuable discussions. This work was supported in part by a grant from National Science Council of Republic of China (NSC88-2112-M002-001Y).
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FIG. 1. One loop diagrams which give rise to a nonzero neutrino magnetic moment.

FIG. 2. The first generation neutrino magnetic moment $\mu_\nu$ in units of $\mu_B$ plotted as a function of the vector leptoquark mass in the range of 50 to 100 TeV.
FIG. 3. The second generation neutrino magnetic moment $\mu_\nu$ in units of $\mu_B$ plotted as a function of the vector leptoquark mass in the range of 1 to 100 TeV.

FIG. 4. The third generation neutrino magnetic moment $\mu_\nu$ in units of $\mu_B$ plotted as a function of the vector leptoquark mass in the range of 1 to 100 TeV.