Estimation of Timing Offsets and Phase Shifts Between Packet Replicas in MARSALA Random Access

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Abstract — Multi-replicA decoding using corRelation baSed LocAlisAtion (MARSALA) is a recent random access technique designed for satellite return links. It follows the multiple transmission and interference cancellation scheme of Contention Resolution Diversity Slotted Aloha (CRDSA). In addition, at the receiver side, MARSALA uses autocorrelation to localise replicas of a same packet so as to coherently combine them. Previous work has shown good performance of MARSALA with an assumption of ideal channel state information and perfectly coherent combining of the different replicas of a given packet. However, in a real system, synchronisation errors such as timing offsets and phase shifts between the replicas on separate timeslots will result in less constructive combining of the received signals. This paper describes a method to estimate and compensate the timing and phase differences between the replicas, prior to their combination. Then, the impact of signal misalignment in terms of residual timing offsets and phase shifts, is modeled and evaluated analytically. Finally, the performance of MARSALA in realistic channel conditions is assessed through simulations, and compared to CRDSA in various scenarios.

I. INTRODUCTION

On satellite return links, Demand Assignment Multiple Access (DAMA) methods allow the user terminals to access the satellite resources using a set of carrier frequencies divided into timeslots. In a system with bursty internet traffic and relatively short packets, DAMA methods would induce resource request/allocation delays and might not be even practical for very large populations of terminals. In this context, the use of Random Access (RA) methods possibly in association with DAMA for data transmission, would be of interest.

Legacy synchronous RA protocols (i.e., Slotted Aloha (SA) [1], Diversity Slotted Aloha (DSA) [2]) offer rather poor performance due to the fact that packet collisions are often destructive. New protocols have arisen to improve the performance of synchronous RA. For this, these recent protocols apply the principle of Successive Interference Cancellation (SIC), besides redundancy transmission.

Among these techniques, Contention Resolution Diversity Slotted Aloha (CRDSA) [3] has been included as an option in the recent standard DVB-RCS2 [4], [5]. In CRDSA, each user sends multiple replicas of the same packet on different timeslots of the frame. Each copy contains pointers towards the next replicas. The receiver performs interference cancellation each time a packet is decoded successfully. In the first version of CRDSA, each user transmits only two copies per packet, and all the terminals are supposed to transmit at equal power. The performance of CRDSA has been enhanced with CRDSA++ [6] where more than two replicas per packet can be transmitted by each user. CRDSA++ also exploits the received packets power unbalance. Irregular Repetition Slotted Aloha (IRSA) has been proposed in [7] as a generalized version of CRDSA. In IRSA, the terminals can send different numbers of packet replicas, in order to enhance the diversity on a given frame.

Another RA technique proposed in [8] is Multi-Slot Coded Aloha (MuSCA). In MuSCA, instead of sending replicas of the same packet, the transmitter encodes the packet with a robust forward error correction (FEC) code of rate R, then divides the code word into multiple fragments and adds separately coded signalling fields to localise each fragment on the frame. The receiver first decodes the signalling information in order to localise the fragments associated to one packet, then combines the fragments together to reconstruct the codeword. MuSCA significantly improves the system performance, at the cost of increased signalling overhead and incompatibility with the DVB-RCS2 standard.

In order to further enhance the RA performance and to be compatible with the DVB-RCS2 standard, another RA method named Multi-ReplicA Decoding using corRelation baSed LocAlisAtion (MARSALA) has been introduced in [9]. MARSALA proposes a new decoding technique for CRDSA based on replicas localisation using autocorrelation. In particular, MARSALA takes advantage of correlation procedures between the signals received on different timeslots, in order to locate the replicas of one packet even when all of them are undergoing a collision and their pointers are not decodable. Furthermore, the coherent combination of the identified packet replicas allows to enhance the Signal to Noise plus Interference Ratio (SNIR), in order to possibly
enable decoding and trigger additional SIC steps. The transmitter side in MARSALA is the same as in CRDSA, the only modifications are at the receiver side. MARSALA does not require any implementation modifications to the DVB-RCS2 standard.

In previous work [10], [11], we have analyzed the effect of residual channel estimation errors on the performance of RA methods that use the SIC principle. In MARSALA, an additional critical task is to estimate and compensate for the synchronisation differences between the replicas on separate timeslots, so as to allow their coherent combination. This step is very important to maximize the combination gain. There are two main contributions in this paper: a method to estimate and compensate the timing offset, the phase shift, and the frequency offset relative to the useful signal to demodulate and decode. \( \tau_1 \), \( \phi_1 \), and \( \Delta f_i \) denote respectively, the timing offset, the phase shift, and the frequency offset relative to the useful signal on \( TS_{ref} \). The signal received on \( T_{S_{ref}} \) is expressed as follows

\[
 r_1(t) = y(t + \tau_1)e^{j(\phi_1 + 2\pi \Delta f_i t)} + n(t) + \zeta(t) \quad (1)
\]

with \( y \) being the useful signal to demodulate and decode, \( \tau_1 \), \( \phi_1 \), and \( \Delta f_i \) denote respectively, the timing offset, the phase shift, and the frequency offset relative to the useful signal on \( TS_{ref} \). \( n(t) \) is the AWGN term and \( \zeta(t) \) represents the total interferent signals on \( TS_{ref} \). The signal \( y(t) \) can be detailed as

\[
 y(t) = \sum_{k=0}^{L-1} a_k h(t - kT_s) \quad (2)
\]

with \( a_k \) being the \( k^{th} \) symbol corresponding to \( y \), and \( h \) denoting the shaping filter function. As for the remaining \( (N_b - 1) \) replicas of \( y \), their corresponding signal can be expressed as

\[
 r_i(t) = y(t - N_i T_s + \tau_i)e^{j(\phi_i + 2\pi \Delta f_i t)} + n_i(t) + \zeta_i(t) \quad (3)
\]

with \( i \) being the integer index identifying each replica, \( i \in \{2 : N_b\} \). \( N_i \) is the number of symbols separating the useful replica on \( TS_{ref} \) from its \( i^{th} \) replica. \( \tau_i \) and \( \phi_i \) refer respectively to the timing offset and the phase shift relative to the useful signal on \( TS_{ref} \). \( n_i \) and \( \zeta_i \) are respectively, the AWGN term and the total interferent signals on the timeslot containing the \( i^{th} \) replica.

III. ESTIMATION AND COMPENSATION OF THE TIMING OFFSET AND PHASE SHIFT BETWEEN REPLICAS

This section aims to describe a synchronisation scheme for the received packet replicas in MARSALA. The main objective is to be able to perform coherent replicas combination. In the following, we first recall the method for replicas localisation using a correlation based technique. Then, we describe the procedure to estimate and compensate the timing offsets and phase shifts between localised replicas.

A. Replicas localisation using a correlation based technique

To localise the replicas of a given packet, the receiver computes the cross-correlation between \( r_1 \) and the signals on the rest of the frame. The correlation peaks indicate the
locations of the replicas of a packet present on $TS_{ref}$. The signal $r_1$ described in Section II, can be expressed using $r_1$, $n_1$, $\zeta_1$, $n_t$ and $\zeta_t$ as shown below

$$r_1(t) = r_1(t - N_t T_s + \Delta \tau_{1,i}) e^{j\Delta \phi_{1,i}} + n_{tot}(t) \tag{4}$$

where $N_t T_s - \Delta \tau_{1,i} = N_t T_s - (\tau_i - \tau_j)$ is the timing offset between the packet on $TS_{ref}$ and its $i$th replica. $\Delta \phi_{1,i} = \phi_i - \phi_j + 2\pi \Delta f_1 (N_t T_s - \Delta \tau_{1,i})$ is the phase shift between the two replicas. $n_{tot}$ is the total noise plus interference term expressed as follows.

$$n_{tot}(t) = [n_1(t) + \zeta_1(t)] - [n_1(t - N_t T_s + \Delta \tau_{1,i}) + \zeta_1(t - N_t T_s + \Delta \tau_{1,i})] e^{j\Delta \phi_{1,i}} \tag{5}$$

Without loss of generality, we will only express the cross-correlation function between $r_1$ and $r_j$. Since all the signals are supposed to have finite power, the correlator output can be expressed as follows

$$R_{r_1, r_j}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} r_1(t) r_j^*(t - \tau) dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} r_1(t - N_t T_s + \Delta \tau_{1,i}) r_j^*(t - \tau) e^{j\Delta \phi_{1,i}} dt$$

$$+ \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} n_{tot}(t) r_j^*(t - \tau) dt$$

$$= R_{r_1}(\tau - (N_t T_s - \Delta \tau_{1,i})) e^{j\Delta \phi_{1,i}} + R_{n_{tot}, r_j}(\tau) \tag{6}$$

with $R_{r_1}$ denoting the autocorrelation function of $r_1$ and $R_{n_{tot}, r_j}$ being the cross-correlation function between $n_{tot}$ and $r_j$. The operator $(\cdot)^*$ denotes complex conjugate.

The absolute value of $R_{r_1, r_j}$ reaches its maximum at $\tau = \tau_{max} = N_t T_s - \Delta \tau_{1,i}$, which represents the timing offset between the first and the $i$th replica of the same packet. For $\tau = \tau_{max}$, $R_{r_1, r_j}(\tau_{max})$ is expressed as shown below, with $P_{r_1}$ being the power of the signal $r_1$.

$$R_{r_1, r_j}(\tau_{max}) = P_{r_1} e^{j\Delta \phi_{1,i}} + R_{n_{tot}, r_j}(\tau_{max}) \tag{7}$$

B. Replicas synchronisation procedure

In order to combine $r_1$ and $r_j$ coherently, proper timing and phase synchronisation between the replicas is required. Therefore, the receiver shall estimate the timing offset $\tau_{max}$ and the phase shift $\Delta \phi_{1,i}$ caused by carrier frequency and phase variations between $r_1$ and $r_j$. In a real system, the received signal is sampled with an oversampling factor $Q$. If $\tau_{max,i}$ is not an integer multiple of $\frac{T_s}{Q}$, the correlation peak would be detected with a timing error $\epsilon r_i$. The timing error can be modeled with a random variable uniformly distributed in $[-\frac{T_s}{2Q}, \frac{T_s}{2Q}]$. Then, the estimate of the timing offset is expressed as $\hat{\tau}_{max,i} = (N_t T_s - \Delta \tau_{1,i} + \epsilon r_i)$. The value of the cross-correlation function $R_{r_1, r_j}$ at $\hat{\tau}_{max}$, is

$$R_{r_1, r_j}(\hat{\tau}_{max}) = R_{r_1}(\epsilon r_i) e^{j\Delta \phi_{1,i}} + R_{n_{tot}, r_j}(\hat{\tau}_{max})$$

$$+ \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} y(t + \tau) n_i(t) e^{j(\phi_i + 2\pi \Delta f_1 t + \Delta \phi_{1,i})} dt$$

$$+ \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} y(t + \tau) \zeta_i(t) e^{j(\phi_i + 2\pi \Delta f_1 t + \Delta \phi_{1,i})} dt$$

$$+ R_{n_i, r_j}(\hat{\tau}_{max}) + R_{\zeta_i, r_j}(\hat{\tau}_{max}) \tag{8}$$

with $H$ being the raised cosine filter function.

Given that $\Delta f_1$ is supposed to be constant over the duration of one frame, then the phase shift between replicas $\Delta \phi_{1,i}$ is also constant. Therefore, $\Delta \phi_{1,i} = \text{angle}(R_{r_1, r_j}(\hat{\tau}_{max})) = (\Delta \phi_{1,i} + \phi_{err})$, with $\phi_{err}$ being a phase error that depends on $\Delta f_1 \epsilon r_i$. $H(\epsilon r_i)$ and the cross-correlation functions between noise, interference and the useful signal. We can conclude from Eq. (8), that in order to minimize the error $\phi_{err}$, we shall choose a reference signal $r_1$ containing the lowest interference level. Since all the replicas of a same packet are equi-powered, then among the signals corresponding to replicas of a same packet, the signal having the lowest total received power on one timeslot shall have the lowest interference power of $\zeta_i$. In other words, in order to estimate $\Delta \phi_{1,i}$ after replicas localisation, the receiver shall choose $r_1$ as the received signal on one localised timeslot with the lowest power level.

IV. DEFINITION OF AN ANALYTICAL MODEL FOR REPLICAS COMBINATION WITH SYNCHRONISATION ERRORS

After estimation and compensation of timing and phase according to the scheme described above, replicas combination is performed. However, residual timing offsets and phase shifts are added to the combined replicas. Therefore, we define an analytical model for the impact of imperfect replicas combination on the performance of MARSALA.

A. Replicas Combination Scheme

The signal $r_1$ on $TS_{ref}$ is sampled at time instants $t = n \frac{T_s}{Q}$, with $n$ being an integer varying from 0 to $(Q * L) - 1$. The resulting discrete signal is expressed as follows.

$$r_1(n) = y \left( n \frac{T_s}{Q} + \tau_i \right) e^{j(\phi_i + 2\pi \Delta f_1 n \frac{T_s}{Q})}$$

$$+ n_1 \left( n \frac{T_s}{Q} \right) + \zeta_1 \left( n \frac{T_s}{Q} \right) \tag{9}$$

The signal $r_j$ is sampled at instants $t = n \frac{T_s}{Q} + \hat{\tau}_{max,i}$. Then, the phase shift is corrected with respect to the phase and frequency offsets of $r_1$, by multiplying the resulting samples
with $e^{-j\Delta \phi_{1,1}}$. The corresponding discrete signal is expressed as shown below.

$$
ri(n) = y \left( n \frac{T_s}{Q} + \tau_1 + err_i \right) e^{j (\phi_1 + \phi_{err_i} + 2\pi \Delta f \kappa \frac{T_s}{Q})} + 
\left[ \eta_i \left( n \frac{T_s}{Q} + \tilde{\tau}_{max} \right) + \zeta_i \left( n \frac{T_s}{Q} + \tilde{\tau}_{max} \right) \right] e^{-j\Delta \phi_{1,1}} \tag{10}
$$

Replicas combination consists in the summation of $r_i$ and $r_j$ for $i = [2, \ldots, N_b]$, then matched filtering and downsampling the resulting signal at instants $k'T_s$, with $k'$ being an integer varying from 0 to $L - 1$. In the following, we will denote by $z_1$ and $z_2$ the noise plus interference term in signals $r_1$ and $r_i$, respectively. The resulting discrete signal is expressed as follows

$$
r_{sum}(k'T_s) = y \left( k'T_s + \tau_1 \right) + \sum_{i=2}^{N_b} \tilde{y} \left( k'T_s + \tilde{\tau}_{max} \right) e^{j\Delta \phi_{1,1}} \tag{11}
$$

where $\tilde{y}$ denotes the signal at the output of the matched filter, and $y_{sum}$ is detailed as shown below.

$$
y_{sum}(k'T_s) = \tilde{y} \left( k'T_s + \tau_1 \right) + \sum_{i=2}^{N_b} \tilde{y} \left( k'T_s + \tilde{\tau}_{max} + err_i \right) e^{j\phi_{err_i}} \tag{12}
$$

Since $y_{sum}$ is the sum of signals sampled at imperfect sampling times, it can be divided into two terms: a desired signal denoted by $y_{sum, des}$ and an Inter-Symbol Interference (ISI) term denoted by $y_{sum, isi}$. For the ISI term, we consider only the first two side lobes on either side of the raised cosine filter, since the interferent symbol would be attenuated by more than 10 dB beyond these lobes. Both terms are detailed below

$$
y_{sum, des}(k'T_s) = a_{k'} H(\tau_1) + \sum_{i=2}^{N_b} a_{k'} H(\tau_{err_i}) e^{j\phi_{err_i}} \tag{13}
$$

$$
y_{sum, isi}(k'T_s) = \sum_{l=k'-3}^{k'+3} a_l H \left( \left( k' - l \right) T_s + \tau_1 \right) + \sum_{i=2}^{N_b} \sum_{l=k'-3}^{k'+3} a_l H \left( \left( k' - l \right) T_s + \tau_{err_i} \right) \tag{14}
$$

with $a_{k'}$ and $a_l$ referring to the $k'$th desired symbol and the $l$th ISI symbol, respectively. $H$ is the raised cosine filter function. $\tau_{err_i}$ is equal to $(\tau_1 + err_i)$ and follows a triangular distribution $\Delta$ between $\frac{\pi}{2Q}$ and $\frac{\pi}{Q}$.

### B. Impact of imperfect replicas combination on the SNIR

In order to evaluate the impact of imperfect replicas combination on the performance of MARSALA, we compute the average power of the desired signal $P(y_{sum, des})$ as shown in the following.

$$
P(y_{sum, des}) = \frac{1}{L} \sum_{k'=0}^{L-1} |y_{sum, des}(k'T_s)|^2
$$

$$
= H^2(\tau_1) + \sum_{i=2}^{N_b} H^2(\tau_{err_i}) \left[ \cos^2(\phi_{err_i}) + \sin^2(\phi_{err_i}) \right] + 2 \sum_{i,j \neq i} H(\tau_{err_i}) H(\tau_{err_j}) \cos(\phi_{err_i}) \cos(\phi_{err_j}) + 2 \sum_{i,j \neq i} H(\tau_{err_i}) H(\tau_{err_j}) \sin(\phi_{err_i}) \sin(\phi_{err_j}) + 2H(\tau_1) \sum_{i=2}^{N_b} H(\tau_{err_i}) \cos(\phi_{err_i}) \tag{15}
$$

Given that $\tau_{err_i}$ and $\phi_{err_i}$ are random variables, we compute the average power of the desired signal $E[P(y_{sum, des})]$. Since the raised cosine filter function can be approximated with a sine cardinal (sinc) function on the interval $[-T_s, T_s]$, the mean values of the various terms in (15) are derived as detailed below.

$$
E[H(\tau_1)] = \frac{2Q}{T_s} \int_0^T \frac{\tau_1}{T_s} \sin \left( \frac{\tau_1}{T_s} \right) d\tau_1 = 2Q \text{Si} \left( \frac{\pi}{2Q} \right) \tag{16}
$$

With $Si$ being the sine integral function.

$$
E[H^2(\tau_1)] = \frac{2Q}{T_s} \int_0^T \sin^2 \left( \frac{\tau_1}{T_s} \right) d\tau_1
$$

$$
= \frac{2Q}{\pi} \text{Si} \left( \frac{\tau_1}{T_s} \right) - \frac{\sin^2 \left( \frac{\tau_1}{T_s} \right)}{\tau_1} \tag{17}
$$

Assuming that $\tau_1$, $\tau_{err_i}$, and $\tau_{err_j}$ are independent random variables and given that $\tau_{err_i} \sim \Delta \left( \frac{\pi}{2Q}, \frac{\pi}{Q} \right)$, then we can derive $E[H(\tau_{err_i})]$, $E[H^2(\tau_{err_i})]$, and $E[H(\tau_{err_i}) H(\tau_{err_j})]$. 

as shown in (18), (19) and (20) respectively.

\[
E[H(\tau_{err})] = \\
\frac{2Q}{Ts} \int_0^{\frac{Q}{\tau_{err}} + 1} \frac{Q}{T_s} + 1 \sin(\frac{\pi}{T_s}) d\tau_{err} \\
= \frac{2Q}{\pi} \left( S_i \left( \frac{2\pi}{Q} \right) + \frac{Q}{\pi} \left( \cos \left( \frac{\pi}{Q} \right) - 1 \right) \right) \tag{18}
\]

\[
E[H^2(\tau_{err})] = \\
\frac{2Q}{Ts} \int_0^{\frac{Q}{\tau_{err}} + 1} \sin^2(\frac{\pi}{T_s}) d\tau_{err} \\
= \frac{2Q}{\pi} \left( S_i \left( \frac{2\pi}{Q} \right) - \frac{\sin^2 \left( \frac{\pi}{Q} \right)}{2Q} \right) \\
+ \frac{Q^2}{\pi^2} \left( Ci \left( \frac{2\pi}{Q} \right) - \gamma + \log \left( \frac{Q}{2\pi} \right) \right) \tag{19}
\]

With \( Ci \) being the cosine integral function, \( log \) the natural logarithm and \( \gamma \) the Euler-Mascheroni constant, \( \gamma = 0.577216 \).

\[
E[H(\tau_{err}), H(\tau_{err})] = \\
\frac{4Q^2}{\pi^2} \left( S_i \left( \frac{2\pi}{Q} \right) + \frac{Q}{\pi} \left( \cos \left( \frac{\pi}{Q} \right) - 1 \right) \right)^2 \tag{20}
\]

The phase difference error \( \phi_{err} \) can be approximated to a Gaussian variable of mean zero and variance \( \sigma^2_{\phi_{err}} \) (see Section V-A). Then, the average of \( e^{j\phi_{err}} \) is equal to \( e^{-\sigma^2_{\phi_{err}}} \), and the variance of \( e^{j\phi_{err}} \) is \( e^{-\sigma^2_{\phi_{err}}} - 1 \) \( e^{-\sigma^2_{\phi_{err}}} \).

To analyze the ISI term after signal combination, we choose to proceed as done in [12] for cooperative MISO systems with time synchronization errors. The raised cosine pulse is approximated to a piecewise linear function with slopes \( m_i \). The upper bound of the worst case inter-symbol interference is obtained by considering \( a_i H ((k' - 1)T_s + \tau) = |m_i| |\tau| \). Thus the ISI term can be written as shown below.

\[
y_{sum,isi} = \sum_{l=0}^{m+3} \sum_{l=m-3, l \neq m}^{m+3} |m_l| \left| \frac{\tau_1}{T_s} \right| \\
+ \sum_{l=0}^{m+3} \sum_{l=m-3, l \neq m}^{m+3} |m_l| \left| \frac{\tau_{err}}{T_s} \right| e^{j\phi_{err}} \tag{21}
\]

Fig. 2: Normalized PDF of \( \phi_{err} \), for various numbers of interferents per slot

Since \( \tau_1 \sim U \left( \frac{-T_s}{2Q}, \frac{T_s}{2Q} \right) \) and \( \tau_{err} \sim \Lambda \left( \frac{-T_s}{2Q}, \frac{T_s}{2Q} \right) \), then \( |\tau_1| \sim U \left( 0, \frac{T_s}{2Q} \right) \) and \( |\tau_{err}| \sim \Lambda \left( 0, \frac{T_s}{2Q} \right) \). The mean and variance values of \( y_{sum,isi} \) are given in the following equations respectively.

\[
E[y_{sum,isi}] = \frac{\beta}{4Q} + \sum_{i=0}^{N_0} \frac{\beta}{3Q} e^{-\frac{\sigma^2_{\tau_{err}}}{2Q}} \tag{22}
\]

\[
\text{var}[y_{sum,isi}] = \frac{\beta^2}{48Q^2} + \sum_{i=0}^{N_0} \frac{\beta^2}{18Q^2} (e^{-\sigma^2_{\tau_{err}} - 1}) e^{-\sigma^2_{\tau_{err}}} \tag{23}
\]

The average power of the worst case ISI term is given by

\[
E[P(y_{sum,isi})] = E \left[ |y_{sum,isi}|^2 \right] = \text{var}[y_{sum,isi}] + E^2[y_{sum,isi}] \tag{24}
\]

Thus the average equivalent SNIR after imperfect signal combination (SC), due to imperfect synchronisation correction, can be expressed as follows

\[
E[SNIReq] = \frac{E[P(y_{sum,des})]}{E[P(y_{sum,isi})] + (I_1 + N_0) + \sum_{i=0}^{N_0} (I_1 + N_0)} \tag{25}
\]

with \( I_1 \) and \( I_f \) being the power of the total interference term on \( TS_{rej} \) and the timeslot containing the \( i^{th} \) replica, respectively.

V. PERFORMANCE EVALUATION

A. Numerical evaluation of the equivalent SNIR with imperfect replicas combination

Based on the model presented in the previous section, we compute the Probability Density Function (PDF) of \( \phi_{err} \) through simulations. The numerical results are plotted in Fig. 2. Various scenarios are considered with various numbers of interferent packets on each replica. It is observed that the variable \( \phi_{err} \) follows a Gaussian distribution of mean \( \mu = 0 \) and variance \( \sigma^2_{\phi_{err}} \). The worst case estimation error is obtained in the case of three interferers per slot and
We suppose that the oversampling factor \( Q = 4, N_s = 2 \) replicas and \( E_s/N_0 = 7 \) dB. To have a reference curve, we plot \( SNIR_{eq,ref} \), the equivalent \( SNIR \) with no error model (i.e. perfect synchronisation correction) obtained with various interference configurations. Then, for the same interference configurations, we plot the average \( SNIR_{eq} \) with phase shift and timing offset errors. Both cases with \( \sigma_{\phi_{err}} = 0 \) and \( \sigma_{\phi_{err}} = 0.0125 \) (worst case scenario) are shown in Fig.3. We can observe that both curves are approximately the same. We conclude that the impact of \( \phi_{err} \) is not significant on the equivalent \( SNIR \) degradation and the degradation is mainly caused by the timing offsets. We can also observe that the degradation of the equivalent \( SNIR \) increases from 0.3 dB to 0.5 dB when \( SNIR_{eq,i} \) varies from \(-1.3\) dB to 2.3 dB. The results obtained with \( N_b = 3 \) replicas are approximately similar.

### B. Throughput and PLR simulation results

In the following, we use the numerical results of the analytical model proposed, in order to evaluate the impact of synchronisation errors on the performance of MARSALA, in terms of throughput and Packet Loss Ratio (PLR). All the packets are considered equi-powered. The number of terminals transmitting over one frame duration is \( \lambda \). Each frame is composed of \( N_s = 100 \) timeslots. The simulations environment is provided by a satellite communications simulator developed by Thales Alenia Space and CNES. The traffic profile tested is a Constant Bit Rate (CBR) profile. Residual channel estimation errors caused by imperfect interference cancellation have been taken into account. QPSK modulation and DVB-RCS2 turbocode for linear modulation of rate 1/3 are used in the simulation scenarios. We suppose that the Packet Error Rate (PER) obtained at an SNIR higher than the modcod decoding point is equal to \( 10^{-5} \); otherwise it is supposed equal to 1. The results obtained for CRDSA have been validated based on the numerical values found in the DVB-RCS2 guidelines. It is worth noting that CRDSA throughput and PLR performance are enhanced when 3GPP turbo code is used instead of DVB-RCS 2 turbo code, however this study will only evaluate the DVB-RCS 2 scheme.

Once a frame is received, the receiver tries to decode a packet using CRDSA. If the decoding is not successful, the receiver applies MARSALA to localise and combine the replicas of this packet. This procedure is repeated until no more packets can be decoded successfully. In order to compare several modcods, the normalized load (\( G \)) is expressed in bits per symbol and computed as shown below

\[
G = R \times \log_2(M) \times \frac{\lambda}{N_s} \tag{26}
\]

with \( R \) being the code rate and \( M \) the modulation order. The normalized throughput (\( T \)) is given by

\[
T = G \left(1 - PLR(G)\right) \tag{27}
\]

where \( PLR(G) \) is the probability that a packet is not decoded for a given \( G \) and a given \( SNIR \). We denote by MARSALA-2 and CRDSA-2, the MARSALA and CRDSA systems where each terminal transmits 2 replicas of a given packet. The same notation is taken for MARSALA-3 and CRDSA-3.

Following the numerical results of the theoretical study in Section IV-B, Fig.4 and Fig.5 trace the throughput and the PLR obtained with MARSALA-2 and MARSALA-3 in...
In this paper, we have designed a method to estimate and compensate the synchronisation errors between the replicas of a given packet, prior to signal combination in MARSALA. Based on the proposed estimation method, we have presented a detailed analytical model to evaluate the impact of signal misalignment on the combination performance in MARSALA. We have shown that the correction of the timing offsets differences and phase shifts between the replicas of a given packet prior to their combination is a critical task in MARSALA, and it can degrade the performance, if not done accurately. In order to further evaluate MARSALA combined with CRDSA, ongoing work is testing MARSALA with 3GPP turbo-code and in the case of unbalanced packets power. Finally, an interesting future work would be to combine MARSALA with Asynchronous Contention Resolution Diversity Aloha (ACRDA) [13], given the overall advantages of the asynchronous access mode.

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