B-mode Detection with an Extended Planck Mission

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Abstract. The Planck satellite has a nominal mission lifetime of 14 months allowing two complete surveys of the sky. Here we investigate the potential of an extended Planck mission of four sky surveys to constrain primordial B-mode anisotropies in the presence of dominant Galactic polarized foreground emission. An extended Planck mission is capable of powerful constraints on primordial B-modes at low multipoles, which cannot be probed by ground based or sub-orbital experiments. A tensor-scalar ratio of $r = 0.05$ can be detected at a high significance level by an extended Planck mission and it should be possible to set a 95% upper limit of $r \lesssim 0.03$ if the tensor-scalar ratio is vanishingly small. Furthermore, extending the Planck mission to four sky surveys offers better control of polarized Galactic dust emission, since the 217 GHz frequency band can be used as an effective dust template in addition to the 353 GHz channel.

Keywords: CMBR experiment, CMBR theory, inflation

1. Introduction

High precision experiments, most notably WMAP, have demonstrated that the acoustic peak structure of the cosmic microwave background (CMB) agrees extremely well with the predictions of simple inflationary models [1]. Nevertheless, relatively little can be inferred from current observations about the detailed dynamics of inflation and some cosmologists have questioned whether inflation actually took place [2].

The detection of a primordial $B$-mode polarization pattern in the CMB would provide one of the strongest pieces of evidence that the Universe experienced an inflationary phase [3]. Tensor perturbations generated during inflation source $E$ and $B$-mode polarization anisotropies of roughly equal magnitude, whereas scalar perturbations produce only $E$-modes. A detection of a $B$-mode anisotropy would therefore confirm the existence of a background of stochastic gravitational waves generated during inflation and would fix the energy scale of inflation via

$$V^{1/4} \approx 3.3 \times 10^{16} r^{1/4} \text{ GeV}$$

[4], where $r$ is the relative amplitude of the tensor and scalar power spectra (defined as in [5]). At present, the limits on $r$ are relatively poor. Direct upper limits on the $B$-mode

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power spectrum from the WMAP 5-year data set a limit of $r \lesssim 10$ [1]. Indirect upper limits, derived by combining WMAP temperature and $E$-mode data with a variety of other astrophysical data sets, yield 95% limits of $r < 0.22$ if the scalar spectral index is set to a constant, and $r < 0.55$ is the scalar spectral index is allowed to ‘run’ ($dn_s/d\ln k \neq 0$).

To improve on these limits, a number of sensitive ground based/sub-orbital polarization experiments are either taking data or under construction [6]. The most sensitive of these experiments aim to detect (or set limits) on a $B$-mode polarization pattern if $r$ is as low as $\sim 10^{-2}$. In addition, groups in Europe and the USA have considered designs for a future $B$-mode optimised space satellite [7].

The Planck satellite is scheduled for launch in April 2009‡. Planck has polarization sensitivity in 7 channels over the frequency range 30-353 GHz. However, Planck was designed to make high resolution ($\sim 5'$) sensitive measurements of the temperature and $E$-mode anisotropies. It was not optimised to detect a large-angle $B$-mode polarization pattern. As a consequence, Planck lacks sensitivity compared to $B$-mode optimised experiments, particularly those that use large bolometer arrays as detectors. Nevertheless, there are several reasons to investigate Planck’s capability for $B$-mode detection in some detail:

[1] As a space experiment Planck will operate in a stable environment and covers a wide frequency range, offering control of polarized synchrotron and dust foregrounds which will dominate over any primordial $B$-mode signal.

[2] Planck will scan the entire sky and is the only experiment on the horizon which can probe polarization anisotropies at low multipoles $\ell \lesssim 10$. Planck therefore provides a useful complement to ground based/sub-orbital experiments that probe higher multipoles.

[3] For inflation with a power law potential, $V(\phi) \propto \phi^\alpha$, the scalar spectral index, $n_s$, and tensor-scalar ratio are approximately

$$n_s \approx 1 - \frac{2 + \alpha}{2N}, \quad r \approx \frac{4\alpha}{N}, \quad \text{i.e.} \quad r \approx 8(1 - n_s) \frac{\alpha}{\alpha + 2} \quad (2)$$

(see e.g. [9] for a review), where $N$ is the number of inflationary e-folds between the time that CMB scales crossed the ‘horizon’ and the time that inflation ends. There are indications from WMAP and CMB experiments probing higher multipoles for a small tilt in the scalar spectral index $n_s \sim 0.97$ [1], [10]. If we take this tilt seriously, then the last of these equations suggests $r \sim 0.1$ for any $\alpha$ of order unity. For example, for $N \approx 60$ [11], the quadratic potential gives $n_s \approx 0.97, r \approx 0.13$. This sets an interesting target for Planck and other $B$-mode experiments. ‘High field’ inflation models with substantially lower values of $r$ require special fine tuning (though phenomenological models of this sort can certainly be constructed [12]).

Measuring inflationary $B$-modes on large angular scales is a formidably difficult problem. For $r \sim 0.1$, the primordial $B$-mode has an $rms$ amplitude of of only $\sim 0.06 \mu K$ on $7^\circ$ scales. The signal is therefore small, but in addition, Galactic polarized emission is

‡ For a description of Planck and its science case, see [8].
expected to be 10 to 20 times higher over the high Galactic latitude sky. The detection of a large-angular scale $B$-mode signal therefore requires polarized foreground subtraction to an accuracy of a few percent or better. In a recent paper [13], we presented a simple template-fitting scheme for modelling the polarization likelihood from multifrequency data. The scheme was tested using the Planck Sky Model (PSM) [14] for polarized foregrounds and found to work well for the signal-to-noise expected for the nominal Planck mission which allows two full sky surveys over 14 months. The lifetime of the High Frequency Instrument (covering the most sensitive CMB frequencies) on Planck is determined by the capacity of the $^3$He and $^4$He storage tanks that feed the 0.1K dilution refrigerator. The precise lifetime will depend on the operating conditions in-flight, but should be sufficient to provide 4 full sky surveys with some considerable margin.

The purpose of this paper is to analyse carefully the constraints on primordial $B$-mode anisotropies that might be achieved with an extended Planck mission, taking into account Galactic polarized foregrounds. We will show that an extended Planck mission offers greater scope for testing the subtraction of polarized dust emission, and should be capable of detecting a tensor-scalar ratio of $r \approx 0.05$ at a high significance level. If the tensor-scalar ratio is very small ($r \ll 0.05$), an extended Planck mission should be able to set a direct 95% upper limit of $r \sim 0.03$. An extended Planck mission is therefore competitive with the most ambitious of the polarization optimised ground-based/sub-orbital experiments.

2. B-mode detection with Planck

2.1. Likelihood analysis of multifrequency data with foregrounds

We first outline the procedure described in [13]. We designate a subset of the frequency channels as ‘CMB sensitive’ channels and construct a data vector, $x_i$ (where $i$ is the index number of the $3N_p$ pixels defining the map) as the inverse noise variance weighted sum of the temperature $T$, and Stokes parameters $Q$ and $U$, over these channels. The covariance matrix of this vector is written as

$$\langle x_x \rangle = S_{ij} + \Phi_{ij} + N_{ij},$$  \hspace{1cm} (3)

where $S_{ij}$ is the ‘signal’ covariance matrix (primordial CMB), $\Phi_{ij}$ is the covariance matrix of the residual foreground contamination and $N_{ij}$ is the instrumental noise covariance matrix. To remove foregrounds, we designate $N_F$ channels as templates, $F_i^k$, where the superscript denotes frequency, and we construct the data vector

$$Y_i = x_i - F_i^k \beta_i^k, \quad (\beta_i^k = \beta_{(T,Q,U)}^k, \quad \text{if } i \equiv (T,Q,U)).$$  \hspace{1cm} (4)

If the templates remove the foregrounds to high accuracy, the average of (4) over noise realizations is

$$\langle Y_i \rangle = s_i (1 - \sum_k \beta_i^k),$$  \hspace{1cm} (5)
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since the ‘template’ channels contain primordial CMB signal. If the coefficients $\beta$ are independent of the signal, the covariance matrix $\langle Y_i Y_j \rangle$ is

$$\langle Y_i Y_j \rangle = S_{ij}(1 - \sum_k \beta_k^i)(1 - \sum_k \beta_k^j) + N_{ij} + N_{ij} \beta_k^i \beta_k^j.$$ (6)

The coefficients $\beta$ are found by solving

$$\beta = (F^T C_Y(\beta)^{-1} F)^{-1}(F^T C Y^{-1} x),$$ (7)

where $F$ is a ‘large’ $3N_p \times N_F$ matrix with elements constructed from the template maps $F_i^k$ (see equation (26) of [13]). Finally, the likelihood for estimating cosmological parameters is computed from

$$L \propto \frac{1}{\sqrt{|C_Y|}} \exp \left(-\frac{1}{2} Y^T C Y^{-1} Y \right).$$ (8)

The motivation for the above procedure is discussed in detail in [13]. It is equivalent to a Bayesian marginalization of foreground templates subtracted from multifrequency maps in the limit that errors in the foreground subtraction are small compared to the instrument noise and cosmic variance. As demonstrated in [13] this condition is amply satisfied for the PSM if the highest and lowest polarized Planck channels are used as templates.

2.2. Application to simulated Planck data

For the simulations described here, we generated Gaussian realizations of the primordial CMB at a Healpix [15] resolution of $NSIDE = 2048$ using the cosmological parameters for the concordance $\Lambda$-dominated cold dark matter model determined from the WMAP 3 year data [16]. A Gaussian $B$-mode was added with a specified value of $r$. Galactic polarized foregrounds from the PSM were added to the primordial CMB at each of the Planck polarized frequencies. Uniform white noise, based on the detector sensitivities given in [8] for either the nominal mission lifetime (two sky surveys) or an extended mission lifetime (four sky surveys) was added to the maps at each frequency. These maps were then smoothed by a Gaussian of $7^\circ$ FWHM and repixelised to a Healpix resolution of $NSIDE = 16$. An internal polarization mask, constructed from the 217 GHz PSM as described in [13] was applied to each of the low resolution maps (see Figure 2 below). The mask is relatively conservative and removes 37% of the sky.

The main results of this paper are shown in Figure 1. This shows the likelihood function plotted as a function of $r$ for various noise levels and choice of templates.

For the results shown in Figure 1, a ‘CMB sensitive’ map was constructed as an inverse noise variance weighted sum of the four Planck channels at 70, 100, 143 and 217 GHz. The 30 GHz and 353 GHz channels were used to define low and high frequency templates. Likelihood distributions are plotted for simulations generated with the same random numbers for three values of the tensor-scalar ratio, $r = 0, 0.05$ and 0.1. The red dotted

§ We assume that all cosmological parameters are fixed. This is a good approximation, since $r$ is weakly correlated with other cosmological parameters. We fix the tensor spectral index to $n_t = 1$. 


Figure 1. The likelihood plotted as a function of the tensor-scalar ratio, $r$. Figure (a) to the left shows likelihoods for realizations with three values of the tensor-scalar ratio, $r = 0, 0.05$ and $0.1$ using the Planck 30 and 353 GHz channels as templates. The solid blue lines show the likelihoods for the nominal Planck mission of two sky surveys. The magenta dashed lines show the likelihoods for an extended Planck mission of four sky surveys. The dotted red lines show the likelihoods for a mission with negligible noise. Figure (b) to the right shows likelihoods for the simulations with $r = 0.05$ and $r = 0.1$, but now using the Planck 30 and 217 GHz as templates. The colour coding is the same as in Figure 1a: solid blue lines show likelihoods for the nominal Planck mission and dashed magenta lines show likelihoods for an extended Planck mission. In all cases, the polarization mask shown in Figure 2 has been applied to the simulated data.

lines show results for nearly noise free simulations (diagonal noise of 0.10 µK is added to the $Q$ and $U$ maps to regularize the signal covariance matrix $S$) of the primordial CMB alone. The solid blue lines show the distributions after template subtraction for simulations of the nominal Planck mission, and the dashed magenta lines show equivalent results for an extended Planck mission. As expected, the distributions for the extended Planck mission are significantly narrower than those for the nominal distribution. With an extended mission a tensor-scalar ratio of $r = 0.05$ is detectable at high significance. If $r \ll 0.05$, the likelihood function for the extended mission drops to 0.05 of its peak value at $r \sim 0.028$. An extended Planck mission is therefore competitive with the most sensitive of the $B$-mode optimised ground based/sub-orbital experiments.

The improved signal-to-noise of an extended Planck mission is illustrated visually in Figure 2. The upper panel shows the noise-free realizations of the primordial $B$-mode contribution to the $Q$ and $U$ maps for $r = 0.1$. The middle panel shows the reconstructed $B$-mode maps for the nominal Planck mission using the 30 and 353 GHz channels as templates. The lowest panel shows $B$-mode reconstructions for an extended Planck mission. Figure 3 shows quadratic maximim likelihood estimates (QML) [17] of the $E$ and $B$-mode power spectra for the simulations with $r = 0.05$. As expected, the error
Figure 2. $Q$ and $U$ maps smoothed with a Gaussian of FWHM=7° and pixelised at a Healpix resolution of $NSIDE = 16$. Upper panel shows the noise-free $B$-mode contribution to the $Q$ and $U$ maps for a realization with $r = 0.1$ for the region outside an internally generated polarization mask. The middle panel shows the foreground subtracted $B$-mode contribution for the nominal Planck mission using the 30 GHz and 353 GHz channels as templates. Equivalent maps for an extended Planck mission are shown in the lower panel.

bars on the extended mission power spectra are almost half those of the nominal mission for the noise dominated multipoles at $\ell \gtrsim 15$, but the effects of improved signal-to-noise can be seen over the entire range of multipoles shown in the Figure.

An extended Planck mission offers more than just reduced errors on the polarization power spectra. The results described above were derived using the specific choices of 30 GHz and 353 GHz Planck channels as templates. WMAP has provided high quality maps of the polarized Galactic emission over the whole sky at low frequencies [18] (particularly the $K$ and $Ka$ bands at 23 GHz and 33 GHz respectively). In addition, the low-frequency instrument on Planck will provide all-sky maps at 30, 44 and 70
Given the low frequency maps, these will produce a wealth of data with which to constrain polarized synchrotron emission. In contrast, the constraints on polarized dust emission are substantially weaker and Planck will rely on data from its own high frequency polarization channels (217 and 353 GHz) to provide dust templates over the whole sky. The small number of polarized channels on Planck limits the scope for checking the accuracy of foreground separation, particularly at high frequencies. The results
described above and in [13] show that if polarized dust emission is as simple as assumed in the PSM, then the Planck 353 GHz channel can provide an adequate dust polarized template. However, even if this is the case, it will be necessary to use the 217 GHz channel as a template to demonstrate consistency.

Figure 1b shows what happens if the 30 and 217 GHz channels are used as templates. Here we have constructed inverse noise variance weighted maps from the 70, 100 and 143 GHz channels and subtracted the 30 and 217 GHz channels as described in Section 2.1. (The simulations at each frequency are identical to those used to construct Figure 1a). Figure 1b shows the likelihoods for the nominal and extended missions for the simulations with $r = 0.1$ and $r = 0.05$. For the nominal mission, the likelihood distributions are much broader than those shown in Figure 1a. Significantly, for the $r = 0.05$ simulation, the nominal mission sets only an upper limit on $r$. With an extended Planck mission, the distributions become much narrower and it is then possible to detect $r = 0.05$ at a high significance level.

The 217 GHz channel therefore becomes a useful dust template for an extended Planck mission, and as Figure 1b demonstrates the improved constraints on $r$ from an extended mission are greater than one might naively imagine from a $\sqrt{2}$ improvement in the detector noise. In both Figures 1a and 1b most of the information on the primordial CMB anisotropies comes from the 100 and 143 GHz Planck bands, since these have high sensitivity. To remove polarized dust emission, we need to subtract only about 5% of the 353 GHz maps, whereas we must subtract about 26% of the 217 GHz maps. Although the 353 GHz channel has higher detector noise than the 217 GHz channel, it has a higher signal-to-noise for monitoring dust emission. Furthermore, by using the 217 GHz channel as a dust template, one can see from equations (5) and (6) that a significant fraction of the primordial signal is subtracted from the data vector $Y$, accentuating the impact of detector noise. For the nominal Planck mission these effects significantly degrade Planck’s sensitivity to inflationary $B$-modes if the 217 GHz channel is used as a dust template. However, with an extended Planck mission, the situation is substantially improved and it is then feasible to use both the 217 and 353 GHz channels to subtract polarized foregrounds thereby increasing the scope for redundancy checks. The ability to perform such checks will be especially important should Planck reveal any evidence for a primordial $B$-mode anisotropy.

3. Conclusions

Until a ‘CMBpol’ satellite [7] is launched, Planck is the only experiment capable of measuring primordial $B$-modes at low multipoles. It is therefore important to assess carefully Planck’s capability for detecting $B$-modes and to consider how the constraints on $B$-mode anisotropies improve if the Planck mission is extended to at least four sky surveys. This has been the goal of this paper.

Any small primordial $B$-mode signal must be extracted from the dominant polarized Galactic emission. Nevertheless, using the Planck Sky Model, we have shown that an
extended Planck mission can set strong constraints. An extended Planck mission can set a $\sim 95\%$ upper limit of $r \approx 0.03$ and can detect a $B$-mode with $r = 0.05$ at a high significance level. Furthermore, with an extended Planck mission it is feasible to use both the 353 and 217 GHz frequency bands to form dust templates suitable for probing tensor-scalar ratios of $r \sim 0.05$. This increases the scope for testing the accuracy of Galactic foreground subtraction using Planck data, and also improves the prospects for dealing with polarized foregrounds (particular at high frequencies) should they turn out to be more complicated than assumed in the PSM. We conclude that an extended Planck mission has comparable potential for primordial $B$-mode detection as the most sensitive of the ground based/sub-orbital experiments [6] planned for the next few years. Furthermore, Planck is complementary to such experiments because it is the only experiment capable of detecting inflationary $B$-modes at low multipoles. If a $B$-mode with $r \sim 0.1$ exists, as predicted in ‘high field’ inflation models with a power-law potential [19], there is a realistic prospect that within a few years we might be able to measure the $B$-mode power spectrum over a wide multipole range $2 \leq \ell \lesssim 200$.

Although we have focussed on $B$-mode detection in this paper, it is worth mentioning that measurements of the $E$-mode power spectrum with an extended Planck mission will lead to interesting new science. Accurate measurements of the $E$-mode spectrum will help to improve our knowledge of the reionization history [20]. Furthermore, there are indications from various analyses of the WMAP temperature data for curious ‘anomalies’ [21], e.g. alignments of the low multipoles, departures from statistical isotropy, and an unusual ‘cold’ spot. Many of the proposed physical explanations of these apparent anomalies have polarization signatures at low multipoles [22] that are potentially observable by Planck.

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