Computing the Subgame Perfect Nash Equilibriums in Parallel Allocation Indivisible Items

Wei Huang1,*, Dong Cai2, Yuge Lu2, Wen Huang2, Qian Tang2 and Youqiang Li2

1 Guangxi Key Laboratory of Trusted Software, Guilin University of Electronic Technology, Guilin City, Guangxi, China
2 School of Computer Science and Information Security, Guilin University of Electronic Technology, Guilin City, Guangxi, China
Email: huangwei@guet.edu.cn

Abstract. Multi-Agent resource allocation has also become a research hotspot in the field of artificial intelligence in recent years. A lot of related work aims to design a procedural resource allocation system with execution efficiency. This paper studies a parallel allocation method for allocating indivisible items between agents in which every agent asks for an object among those that remain in every round, and every reported item will be allocated randomly to an agent reporting it. In order to depict the entire distribution process, the authors propose a method of constructing a game tree of parallel allocation. Study the corresponding relationship between the total number of branches of the game tree and items. In the last part, prof the parallel allocation mechanism is dynamic game and show that the subgame perfect Nash equilibrium (SPNE) can be found by backward introduction the game trees. Provides a method to compute the unique subgame perfect Nash equilibrium in linear time.

Keywords. Parallel allocation; Nash equilibrium; game tree; backward induction.

1. Introduction

Resource allocation has always been a hot topic in economics and computer science [1]. In real life, there are plenty of examples, such as the allocation of courses and fixed classrooms, training opportunities [2] for the unemployed and so on. The resources available for allocation are limited, so how to allocate resources in consideration of “benefit”, “equity” and “computational problem” is the focus of scholars’ research [3-6].

In general, there are four types of resource allocation problems [1], (a) dividable [7, 8] or indivisible [9], (b) centralized [10] or decentralized [11], (c) economic benefit or social equality [11], (d) allow currency payments and transfers or not [8, 12].

Scholars proposed much mechanism to allocate the indivisible items, Brams et al. [13] proposed a cake-cutting protocol, but it only work for two agents. Chevaleyre et al. [14] proposed a negotiation framework. Tominaga et al. [15] proposed a Sequential Allocation with Random Sequences, Bouveret and Lang [9] studied a sequential allocation. Kalinowski [16] et al. proposed for any utility function, agent can compute the expected reward in linear time.

Huang define and study a parallel elicitation-free allocation protocol that is insensitive to agents’ identities [17]. In each round of parallel allocation, each Agent reports its favorite one from the remaining items according to its own preference. If a remaining item is only reported by a single agent, then the agent will get the item; otherwise, the ownership of the item will be determined by the drawing of all the agents reporting the item.
In this paper, we find a way to visually represent the parallel allocation process. The strategical issues of picking sequences have already been studied by Kohler and Chandraesekaran [18], who prove that the subgame perfect Nash equilibrium can be computed in by reversing the policy and preference orderings. Kalinowski et al. [2] extend this result to any two-agent picking sequence, and investigate the computational complexity of computing a subgame perfect Nash equilibrium for more than two agents. In this paper, we get the sub-game perfect Nash equilibrium (SPNE) in parallel allocation by backward induction the game tree.

This paper contributes as follows:
(1) Design a game tree construction method for parallel allocation in Two-Agents.
(2) Find the relationship between the number of items and branches of the game tree in Two-Agents.
(3) By backward induction the game tree of parallel allocation, we can get the subgame perfect Nash equilibrium (SPNE)
(4) Find a method to compute the unique sub-game perfect Nash equilibrium.

2. Model and Notions
Let an allocation represented by a triple \((O, N, >)\) \(O\) is an indivisible collection of items, \(N\) represent the collection of agents, and \(\triangleright\) represent a sequence of preferences (Each agent has a strict priority vector for the item set \(O\)) In the whole paper, we as assume \(2 \leq N \leq |O|\).

The parallel allocation is described as follows. In each round of parallel allocation, each Agent reports its favorite one from the remaining items according to its own preference. If a remaining item is only reported by a single agent, then the agent will get the item; otherwise, the ownership of the item will be determined by the drawing of all the agents reporting the item.

A strategy \(\delta\) over some \(O' \subseteq O\) is a finite sequence \(\delta(1), ..., \delta(|\delta|)\) such that \(\delta(1) \in O'\) and \(\delta(i) \neq \delta(j)\) for any \(1 \leq i \neq j \leq |\delta|\).

Example 1. Let \(O = \{o_1, o_2, o_3, o_4, o_5\}, N = \{A, B\}.\) Agent A has the preference order \(o_1 > o_2 > o_3 > o_4 > o_5\), Agent B has the preference order \(o_3 > o_4 > o_1 > o_5 > o_2\), if both agents are picking sincerely, allocation process induced by the parallel protocol as table 1.

| Table 1. Allocation process. |
|-----------------------------|
| \(k\) | \(A\) | \(B\) | \(O_k\) |
|-----|-----|-----|-----|
| 1   | \(o_1\) | \(o_3\) | \(\{o_1, o_3\}\) |
| 2   | \(o_2\) | \(o_4\) | \(\{o_2, o_4\}\) |
| 3   | \(o_5\) | \(o_5\) | \(\{o_5\}\) |

Symbolic:
\(O_k\) denote the sets of items that will be reported by agents in round \(k\).
\(X_a(\delta)\) and \(X_b(\delta)\) denote the sets of items that will be reported by \(A\) and \(B\) respectively.
\(g\) is called the scoring function, two commonly used scoring functions are as follows:

\[
(Lexicographic) \; g_L(k) = 2^{|O|-k} \quad (1)
\]

\[
(Borda) \; g_B(k) = |O|-k+1 \quad (2)
\]

Agents expected utility. Use the following formula:

\[
Eu_i = \sum_{1 \leq k \leq |\delta|} u(\delta_i(k)) p_k \quad (3)
\]

\(p_k:\) the probability of obtaining the report item in the \(k\)-th round is distributed in parallel.
\(u(o):\) the value of the item \(o\).
\(\delta_i(k):\) represents what agent \(i\) will report in round \(k\).
3. The Construction of Game Tree

Game tree means that the actions of the players in a dynamic game can be expanded into a tree. It is a visual representation of the extended form, and can give you almost all the information of a finite game. The basic building materials include knot, branch and information set. In this paper, we present the process of parallel allocation in a tree, so that we can draw a game tree that is distributed in parallel allocation.

The method of drawing a game tree (Two-Agent and n items) is as follows:

Step 1: First draw the root of the tree, denoted by A1.

Step 2: Starting from A1, connecting n children is unified by B1 (these n children indicate that agent A has n choice at this time).

Step 3: Starting from B1, connect to n children (indicating that agent B currently has n choices).

Step 4: Add the node subscript, increase the subscripts of A and B, and reduce the total number of items accordingly (if agent A and agent B report the same item in the same round, n=n-1, respectively n=n-2). Repeat the above steps, until n = 0.

Example 2. Let $O = \{o_1, o_2\}$, $N = \{A, B\}$. Figure 1 illustrates the execution of the method that draws a game tree. In plot a, the root A1 connecting two children cause the Agent A has two items to choose in the first round. In plot b, because agent B also can choose two items in the first round, so the left and right B1 has two children, The left B1 means items agent B can choose when agent A select item 1 in the first round (respectively, right corresponding report item 2). In plot c and d, We can see that if the items in the first round are selected, the corresponding A2 and B2 nodes will have no children, and if they are not allocated, the remaining items will be the children of A2 and B2.

![Game Tree Diagram](image)

**Figure 1.** Diagram of example 2.

4. The Computing of Game Tree

Since the number of items allocated in each round of parallel allocation is one (two agents report the same item) or two, the game tree structure is inconsistent with the serially allocated game tree, so the
number of branches of the game tree is The correspondence between the number of assigned items was studied.

Theorem 1. With two agent and n items, the branches of game tree is $n! \left[ \left( \frac{5+\sqrt{5}}{10} \right)^n \left( \frac{1+\sqrt{5}}{2} \right)^n \right] + \left( \frac{5-\sqrt{5}}{10} \right)^n \left( \frac{1-\sqrt{5}}{2} \right)^n$.

Proof with two agents, according to the parallel allocation game tree that has been drawn, the correspondence between items and branches of the game tree is as follows in table 2:

| Items | 1 | 2 | 3 | 4 |
|-------|---|---|---|---|
| Branches | 1 | 4 | 18 | 120 |

We build a model based on observing the compositional rules of these game trees:

$-a_n$ means to the number of game tree branches in n items.

$-a_3 = (a_2 + 2 \times a_3) \times 3$

$-a_4 = (a_3 + 3 \times a_2) \times 4$

$\ldots$

$-a_n = (a_{n-1} + (n-1) \times a_{n-2}) \times n$

$a_1 = 1, a_2 = 4$

We started to solve this sequence problem:

$a_n = (a_{n-1} + (n-1) \times a_{n-2}) \times n$

$a_n = n(a_{n-1} + n(n-1) \times a_{n-2})$

$\frac{a_n}{n!} = \frac{a_{n-1}}{(n-1)!} + \frac{a_{n-2}}{(n-2)!}$

After calculation, we get $a_n = n! \left[ \left( \frac{5+\sqrt{5}}{10} \right)^n \left( \frac{1+\sqrt{5}}{2} \right)^n \right] + \left( \frac{5-\sqrt{5}}{10} \right)^n \left( \frac{1-\sqrt{5}}{2} \right)^n$.

5. Game Tree and Backward Induction

The basic idea of the backward induction method is to start from the last stage of the dynamic game. Players in the game follow the principle of maximizing utility to choose actions, and then gradually reverse to the previous stage, until the game starts to choose actions of the players. This method can effectively determine the Nash equilibrium of a game problem. In this section, we will use the backward induction method on the parallel distribution game tree to find the sub-game perfect Nash equilibrium of the parallel distribution game.

Theorem 2. With two agents, By backward induction the game tree of parallel allocation we can get a Subgame Nash equilibrium outcome.

Proof Let parallel allocation have a total of k rounds, according to the definition of parallel allocation and assumptions of this paper that $2 \leq N \leq |O|$. When $k > 1$, both Agent can observe the reporting strategy of the previous round, so the allocation belongs to dynamic game, then we can also backward induction its game tree to find the subgame perfect Nash equilibrium.

Example 3. Consider an allocation problem, let $O = \{o_1, o_2, o_3\}$, $N = \{A, B\}$, A has the preference order $o_1 > o_2 > o_3$ while B has the preference order $o_3 > o_1 > o_2$. Then picking sincerely, $X_A(\delta) = \{o_1, o_2\}, X_B(\delta) = \{o_3, o_2\}$, in scoring function Borda, $EU(X_B(\delta)) = 3.5$. But in the actual situation, the above choices will not appear, and the two self-interested agents will choose better items as much as possible. In the above example, Agent B has the opportunity to get his second favorite item $o_4$, so the real $X_A(\delta)$ should be $\{o_1, o_3\}$, so $EU(X_B(\delta)) = 4. A$ has to maintain his own strategy or he may loss his favorite item. This is a subgame perfect Nash equilibrium for both agent. We can get the same SPNE by backward induction the game tree of this allocation problem.

As we can see in figure 2. According to the agent’s preference in Example 3, let’s star backward
induction from layer B2. This layer is dominated by B, so (2), (4), (5), (6), (7), (9), (11), (12), (13), (14), (15), (18) win; In layer A2, this layer is dominated by A, so (2), (6), (9), (12), (14), (15), (18) win; Layer B1, (2), (6), (15), (18) are left; In A1, we only got (2). It means (2) is an subgame perfect Nash equilibrium. By observing the (2), we got \( X_A(\delta) = \{ o_1, o_2 \} \), \( X_B(\delta) = \{ o_1, o_3 \} \), the same reporting strategy as example 3.

**Figure 2.** Diagram of example 3.

6. Two Agents with Completed Information

Kohler and Chandraesekaran [18] found the sub-game perfect Nash equilibrium calculation method of sequence allocation. Walsh and Xia [19] made an important progress: for any policy, they still can found the SPNE in polynomial time. we prove than, in parallel allocation, the subgame perfect Nash equilibrium computed also follow the same rules.

**Theorem 3.** With two agents and additive utilities, in Parallel method, allocate \( \text{(rev}(P_2), \text{rev}(P_1)) \) is an SPNE.

Proof we sum up to the number of items \( m \). Obviously \( m = 1 \) is trivial. Let the theorem holds for \( m - 1 \) items. We will prove it holds for \( m \). Let \( O = \{ 1, 2, \ldots, m \} \), and \( (Q_1, Q_2) = ((x_1, \ldots, x_{m_1}), (y_1, \ldots, y_{m_2})) \) denote the item of agent A and B report respectively. Then by induction hypothesis, for any \( i \leq m_1 \) (resp., \( j \leq m_2 \) ), \( x_i \in O \) (resp., \( y_j \in O \) is the \( (m_1 + 1 - i) \) th (resp., \( (m_2 + 1 - j) \) th) item allocated to Agent A (resp., Agent B).

W.l.o.g. We observe any SPNE of the game \((P_1, P_2)\) where agent A report an item in the first round. If he report \( x_1 \), we have the following two situation:

(a) A and B report the same item, i.e. \( x_1 \neq y_1 \) then by induction hypothesis, \((\{ x_2, \ldots, x_{m_1} \}, \{ y_2, \ldots, y_{m_2} \})\) is the SPNE allocation for \( O \setminus \{ x_1 \} \) which prove the case for \( m \).

(b) A and B report the different item, i.e. \( x_1 \neq y_1 \), then by induction hypothesis, \((\{ x_2, \ldots, x_{m_1} \}, \{ y_1, y_2, \ldots, y_{m_2} \})\) is the SPNE allocation for \( O \setminus \{ x_1 \} \). Same as above, we prove the case for \( m \).

Next, we will give reasons for not choosing other items in the first round of \((P_1, P_2)\). In order to draw a contradiction, suppose Agent A get a higher utility in an SPNE \((x_1^*, \ldots, x_{m_1}^*), (y_1^*, \ldots, y_{m_2}^*)\) where for any \( i \leq m_1 \) (resp., \( i \leq m_2 \) ), \( x_i \in O \) (resp., \( y_i \in O \) is the \( i \) th (resp., \( i \) th) item
reported by Agent A (respectively, Agent B), by proving that \( \text{Eu}_A(\{x_1, \ldots, x_m \}) \geq \text{Eu}_A(\{x_1', \ldots, x_m' \}) \) and the equality holds if and only if \( \{x_1, \ldots, x_m\} = \{x_1', \ldots, x_m'\} \). The definition of utility is above.

Let \( ((x_2', \ldots, x_m'), (y_2', \ldots, y_m')) \) denote the report strategy of allocate \( (\text{rev}(P_2 \setminus \{x_1'\}), \text{rev}(P_1 \setminus \{x_1'\})) \). By induction hypothesis, \( ((x_2', \ldots, x_m'), (y_2', \ldots, y_m')) \) is the unique SPNE of the sub-game where report \( x_1' \) was reported in the first stage of \( (P_1, P_2) \). It follows that \( \{x_2', \ldots, x_m'\} = \{x_2, \ldots, x_m\} \).

Now, suppose \( x_1' \) was allocated in the \( k \)-th round of game \( (\text{rev}(P_2), \text{rev}(P_1)) \) (Since \( x_1' \neq x_1 \), we have \( k \neq m_1 \)). We have the following two cases.

Case 1: \( k \geq 2 \). Obviously, the first \( k-1 \) round allocated in game \( (\text{rev}(P_2), \text{rev}(P_1)) \) and in allocate \( (\text{rev}(P_2 \setminus \{x_1'\}), \text{rev}(P_1 \setminus \{x_1'\})) \) are the same. Let \( ((x_{k_1}, \ldots, x_{m_1}), (y_{k_1}, \ldots, y_{m_1})) \) denote these items allocated to Agent A and B respectively in first \( k-1 \) round. Let \( G' = ((x_{k_1}, \ldots, x_{m_1}), \{y_{k_2}, \ldots, y_{m_1}\}) \). We have the following two cases.

Case 1.1: Items allocated to agent A and B in \( k \)-th round are the same, which is \( x_1' \). We note that \( ((x_2', \ldots, x_{k_1-1}), (y_2', \ldots, y_{k_1-1})) \) is the outcome of allocate \( (\text{rev}(P_2 \setminus \{x_1'\}), \text{rev}(P_1 \setminus \{x_1'\})) \), round \( k \rightarrow m_1 - 1 \). Hence, by the induction hypothesis \( ((x_2', \ldots, x_{k_1-1}), (y_2', \ldots, y_{k_1-1})) \) is the only SPNE of game \( (\text{rev}(P_2 \setminus \{x_1'\}), \text{rev}(P_1 \setminus \{x_1'\})) \), round \( k \rightarrow m_1 - 1 \). By the induction hypothesis \( (x_1, \ldots, x_{k_1-1}, y_1, \ldots, y_{k_1-1}) \) which is the output of allocate \( (\text{rev}(P_2 \setminus \{G'\}), \text{rev}(P_1 \setminus \{G'\})) \), round \( k \rightarrow m_1 \), is the unique SPNE allocation of the game \( (\text{rev}(P_2 \setminus \{G'\}), \text{rev}(P_1 \setminus \{G'\})) \), round \( k \rightarrow m_1 \). Therefore, in the first stage of the game \( (P_2 \setminus \{x_1'\}, P_1 \setminus \{x_1'\}) \), round \( k \rightarrow m_1 \) if Agent A report \( x_1' \) rather than \( x_1 \), then he will not be strictly better off. which means \( \text{Eu}_A(\{x_1, \ldots, x_{m_1}\}) \geq \text{Eu}_A(\{x_1', x_2, \ldots, x_{m_1}\}) \) and the equality holds if and only if \( \{x_1, \ldots, x_{k_1-1}\} = \{x_1', x_2, \ldots, x_{k_1-1}\} \) (which is equivalent to \( \{x_1, \ldots, x_{m_1}\} = \{x_1', x_2, \ldots, x_{m_1}\} \)). This is a contradiction.

Case 1.2: Items allocated to agent A and B in \( k \)-th round are not the same, which Agent A is \( x_1' \) and Agent B is \( y_1' \). We note that \( ((x_2', \ldots, x_{k_1-1}), (y_2', \ldots, y_{k_1-1})) \) is the outcome of allocate \( (\text{rev}(P_2 \setminus \{x_1'\}), \text{rev}(P_1 \setminus \{x_1'\})) \), round \( k \rightarrow m_1 - 1 \). Hence, by the induction hypothesis \( ((x_2', \ldots, x_{k_1-1}), (y_2', \ldots, y_{k_1-1})) \) is the only SPNE of game \( (\text{rev}(P_2 \setminus \{x_1'\}), \text{rev}(P_1 \setminus \{x_1'\})) \), round \( k \rightarrow m_1 - 1 \). By the induction hypothesis \( (x_1, \ldots, x_{k_1-1}, y_1, \ldots, y_{k_1-1}) \) which is the output of allocate \( (\text{rev}(P_2 \setminus \{G'\}), \text{rev}(P_1 \setminus \{G'\})) \), round \( k \rightarrow m_1 \), is the unique SPNE allocation of the game \( (\text{rev}(P_2 \setminus \{G'\}), \text{rev}(P_1 \setminus \{G'\})) \), round \( k \rightarrow m_1 \). Same as above, this is also a contradiction.

Case 2: \( k = 1 \) In this case, \( x_1' \) is on the top of the game \( (P_2) \). Let \( ((x_2', \ldots, x_{m_1'}), (y_1', \ldots, y_{m_1'}) \) denote the outcome of allocate \( (\text{rev}(P_2 \setminus \{x_1'\}), \text{rev}(P_1 \setminus \{x_1'\})) \), round \( 2 \rightarrow m_1 \), we make compared between \( ((x_2', x_2, \ldots, x_1'), (y_2', \ldots, y_{m_1'}) \) and \( (x_1, \ldots, x_{m_1}, y_1, \ldots, y_{m_1} \) we can find, \( (x_2', \ldots, x_{m_1'}) \) and \( (x_2, \ldots, x_{m_1}) \) are the same, and \( \text{Eu}_A(\{x_1, x_{m_1}\}) > \text{Eu}_A(\{x_1', x_{m_1'}\}) \), \( \text{so \ Eu}_A(\{x_1, ..., x_{m_1}\}) > \text{Eu}_A(\{x_1', x_2', ..., x_{m_1'}\}) \), which contradicts the assumption.

**Algorithm 1:** Finding a Nash Equilibrium

**Input:** Agent A and Agent B’s preference order \( >_A, >_B \);

**Output:** A Nash Equilibrium Strategy for agent A and B, \( X_A(\delta) \) and \( X_B(\delta) \);

1. \( \delta > A \) - let \( o_1 > o_2 > \cdots > o_{l(0)} \) */
2. \( \delta > B \) - let \( o_1' > o_2' > \cdots > o_{l(0')} */
3. \( S = \{o_1, o_2, \ldots , o_{l(0)}\} \);
4. \( R = \{o_1', o_2', \ldots , o_{l(0')}\} \);
5. \( k \leftarrow 0; \)
6. \( j \leftarrow 0; \)
7. \( |O_l| \leftarrow 0; \)
8. \( |O_l| \leftarrow 0; \)
9. for i ← 1 to j do
10.   only if;
11.   if |O| = ∅ then
12.     break;
13.   end;
14.   O_1' ← O_1' ∪ {R_k};
15.   O_2' ← O_2' ∪ {S_k};
16.   O ← O \ ({R_k} ∪ {S_k});
17.   k ← 0;
18. end;
19. X_A(δ) ← rev(O_1')
20. X_B(δ) ← rev(O_2')
21. return X_A(δ), X_B(δ);

Example 4. Consider 2 items and the Agent A has the preference order:
A: o_1 > o_2
B: o_1 > o_2
According to theorem 1, O_1' = \{1,2\} and O_2' = \{1,2\}.

Consider 3 items and the Agent A has the preference order:
A: o_1 > o_2 > o_3
B: o_3 > o_1 > o_2
According to theorem 1, O_1' = \{1,2\} and O_2' = \{1,3\}.

Consider 4 items and the Agent A has the preference order:
A: o_1 > o_2 > o_3 > o_4
B: o_4 > o_2 > o_3 > o_1
According to theorem 1, O_1' = \{2,3,1\} and O_2' = \{2,3,4\}.

Consider 5 items and the Agent A has the preference order:
A: o_1 > o_2 > o_3 > o_4 > o_5
B: o_4 > o_1 > o_3 > o_5 > o_2
According to theorem 1, O_1' = \{1,3,2\} and O_2' = \{1,4,5\}.

7. Conclusions and Future Work
In order to further study the parallel distribution system, this paper proposes a construction method of game tree, which can construct a process to prove the distribution of the whole inseparable items under the parallel distribution system. At the same time, the properties of the game tree are also studied, and the corresponding relationship between the total number of branches of the game tree and the number of items allocated is found. Finally, the method of obtaining the sub-game perfect Nash equilibrium by inversely inductively distributing the game tree is introduced. We also find a way to compute the sub-game perfect Nash equilibrium.

However, when the number of two agents or the number of assigned items increases, the number of branches of the game tree becomes very large. Under the serial allocation, when n individuals allocate 2n items, the number of branches of the game tree will reach nn. Therefore, to find out the construction method of the game tree in which three or more agents are distributed in parallel, and the three and more agent subgame perfect Nash equilibrium algorithm [20] is our next work focus.

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