On The New Symmetries in Electrodynamics and Quantum Theory

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ABSTRACT

The generalized definition of symmetry is formulated. Application of this definition for symmetric analysis of theoretical physics equations is considered. The version of electrodynamics is constructed permitting the faster-than-light motions of particles with real masses. Some elements of physical interpretation of the proposed theory are presented.

Keywords: Symmetries, Relativity Principle, Velocity of Light, Faster-Than-Light Motion

1. INTRODUCTION

The study on the symmetric properties of theoretical and mathematical physics equations is a standard and important task of scientific research. It is connected with the fact that the symmetric properties of theoretical physics equations contain fundamental information on the properties of the world around. As an illustration we shall dwell on three examples.

Let us consider the Newton dynamics equation (1687)

\[ ma = F, \]  

where \( m \) is the mass of a particle, \( a \) is the acceleration, \( F \) is the force acting on a particle. It is well-known that the equation is invariant with respect to the Galilei space-time transformations

\[ x' = x - V t, \quad y' = y, \quad z' = z, \quad t' = t, \]

\( \beta = V/c \), \( n = c/e \), \( (x, y, z) \) are the spatial variables, \( t \) is the time, \( V = (V, 0, 0) \) is the velocity of inertial motion of the reference frame \( K' \) relative to \( K \). It is follows from here that the space surrounding us is 3-dimensional and possesses the Euclid metric

\[ ds^2 = dx^2 + dy^2 + dz^2. \]

Time is universal at any point of space. Space is empty and serves as arena where events have been taking place. Space does not influence on these events. The 3-dimensional coordinate space and time exist independently of one another.

These concepts were kept until 1873 when Maxwell equations were discovered

\[ \nabla \times E + \frac{1}{c} \frac{\partial H}{\partial t} = 0, \quad \nabla \times H - \frac{1}{c} \frac{\partial E}{\partial t} = \frac{4\pi}{c} \rho, \quad \nabla \cdot E = 4\pi \rho, \quad \nabla \cdot H = 0. \]

\( (E, H) \) are the electric and magnetic fields respectively, \( \rho \) is the electrical charge density, \( v \) is the velocity of electrical charge, \( c \) is the speed of light. In 1904 to 1905 in the fundamental works of Lorentz, Einstein and Poincaré it was shown that the Maxwell equations are invariant with respect to the Lorentz transformations

\[ x' = \frac{x - V t}{\sqrt{1 - V^2/c^2}}, \quad t' = \frac{t - V x/c^2}{\sqrt{1 - V^2/c^2}}, \]

where \( y' = y, \quad z' = z, \quad c' = c \). The consequence from the found symmetry is a profound change in our ideas of space and time. Time has been ceased to be independent. Space has become 4-dimensional and is characterized by the pseudo-Euclid metric

\[ ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2, \]

into which the differentials of time and coordinates enter on equal rights. The constant of the speed of light plays a fundamental role. But space, as in the case of classic electrodynamics, is empty and does not exert any effect on dynamics of physical processes happening in it.

But in 1928 the Dirac equation was formulated

\[ (\gamma^a p_a - mc)\Psi(x) = \frac{e}{c} \gamma^a A_a \Psi(x), \]

where \( x = (x^0, x), \quad x^0 = ct, \quad e \) is the electric charge, \( A^a = (A^0, A) \) is the 4-potential, \( \gamma^a A_a = \gamma^0 A^0 - \gamma^1 A^1 - \gamma^2 A^2 - \gamma^3 A^3 \), \( \gamma^a \) are the Dirac matrices. It is invariant with respect to the transformation of the special form \( C \) known as the charge conjugation. This transformation is accompanied by the reversal of signs of electric charge.
charge, energy and momentum. The idea of antiparticle, positron, and Dirac electronic vacuum comes into being. Empty space, vacuum, ceased to be empty. Vacuum is filled with electrons at the states with negative energies. Vacuum influences on quantum-mechanical processes going on in the real world.

These are the consequences of the physical interpretation of abstract mathematical symmetries in theoretical physics. It will not be overstated if we relate the D’Alembert and the Schrödinger equations to such equations. They have extensive applications in experiment, are as significant for physics as the ones already found. Thus, the purpose of the present work is the finding of additional symmetries inherent in the equations of D’Alembert, Dirac, Maxwell and Schrödinger.

2.DEFINITION OF SYMMETRY

In solving this problem, it is necessary to take into account a number of the rigorous theorems: the 13-dimensional Schrödinger group $Sch_{13}$ is the maximal group of symmetry of Schrödinger equation in the space of classical physics (Niederer, 1972, Hagen, 1972); the 15-dimensional conformal group $C_{15}$ is the maximal group of symmetry of the equations of D’Alembert and Maxwell in the Minkowski space (Pauli, 1921).

It is essential that all these symmetries relate to so named the Lie-type symmetries in the space of classical physics, or in the Minkowski space. It follows from here that for finding the new symmetries it is necessary either to be beyond the space of classical physics and the Minkowski space or to turn to the non-Lie generalized types of symmetry. It is in this way that the new results have been obtained. Let us turn first to the task of searching for the new symmetries on the base of the generalized understanding of symmetry.

Let some linear partial differential equation $L\phi(x) = 0$ be given. By symmetry of this equation we shall understand the set of $Q$-operators satisfying the permutation relations of the $p$-th order $[L...[L,Q]...]_{(p-fold)} \phi(x) = 0$ on the solutions $\phi(x)$. When $p = 1$, this definition coincides with the standard one (Malkin and Man’ko, 1965; Kyriakopoulos, 1968).

As an illustration, let us use this definition for the one component field $\phi(x)$ when $p = 2$, and the symmetry operators $Q$ belong to some Lie algebra: $[L[L,Q]] = \zeta(x)L$; $Q = \xi^a(x)\partial_a + \eta(x)$; $[Q_t,Q_m] = C_{lmn}Q_n$. By substituting the expression for the $Q$-operator into the operator equality and by equating the coefficients with the same derivatives on the left and on the right, a set of the determining differential equations for the unknown functions $\xi^a(x)$, $\eta(x)$ and $\zeta(x)$ may be obtained. Commutation properties of the operators enable one to find the structure constants and to identify the algebra. On the base of this algebra, the coordinate transformations $x' = x'(x)$ may be found by integrating the Lie equations $dx^a/\partial \theta = \xi^a(x')$, $x^a(\theta=0) = x^a$ (the is the group parameter). Let us determine the law of transforming the field function by the relation $\phi'(x') = \Phi(x)\phi(x)$. Here the weight function $\Phi(x)$ may be calculated from the set of engaging equations $A\Phi(x)\phi(x) = 0$, $L\phi(x) = 0$, where the former was obtained by replacing the variables in the initial primed equation $L\phi(x') = 0$. The compatibility of the set $A\Phi\phi = 0$, $L\phi = 0$ is, according to the definition, the condition of transforming the initial equation into itself $L\phi'(x') = 0 \rightarrow (x' = x'(x)\phi'(x') = \Phi(x)\phi(x)) \rightarrow A\Phi(x)\phi(x) = 0$, $L\phi(x) = 0$ [1].

3.GALILEI INVARIANCE OF MAXWELL EQUATIONS

Let us apply the algorithm to the equations chosen for analysis and consider first the D’Alembert equation. We start from the equation $L\phi = \Box \phi = 0$ for scalar field and the solution in the form of plane waves $\phi = e^{-i\omega(t-n \cdot x/c)}$. The equations and the solutions will be considered within the space of classical physics. Let us introduce the Galilei transformations $\Box$ and turn to the permutation relations of the Galilei transformations generator $H = t\partial_t$ with the D’Alembert operator $[\Box,H] = 0$. It can be seen from here that the Galilei transformations for the D’Alembert equation are symmetry transformations of the type $p = 2$. Let us do the replacement of variables taking into account the law of field transforma-
tion \( \phi' = \Phi \phi \) and find the weight function from the condition of compatibility of the equation

\[
A \Phi(x) \phi(x) = [(\partial_0 + \beta \partial_x)^2 / \lambda^2 - \Delta] \Phi \phi = 0
\]

with the equation \( \Box \phi(x) = 0 \). The weight function takes the form

\[
\Phi(x) = e^{-i[(1-\lambda)\omega(t-n \cdot x/c) - \beta \omega(n_x t-x/c)]/\lambda},
\]

where \( n = c/c, \lambda = c'/c \). In this case the D’Alembert equation transforms into itself \( \Box \phi'(x') = 0 \). In accordance with the standard understanding of symmetry. To Galilei invariance of D’Alembert equation there corresponds the Galilei invariance of the light cone equation \( c^2 t^2 - x^2 = 0 \) (it can be proved by direct calculation).

Let us turn now to the Maxwell equations and consider the homogeneous equations and their solutions in the form of the plane waves \( (E, \mathbf{H}) = (l, m) \cdot e^{-i\omega(t-nx/c)}, \) where \( l, m \) are the polarization vectors. Here the field, as distinct from the previous case, is multi-component. Therefore, we shall search for the field transformation law in the more complicated form

\[
\begin{align*}
E'_x &= \Phi(x) E_x,
E'_y &= \Phi(x) k(E_y + h_{yz} H_z),
E'_z &= \Phi(x) k(E_z + h_{zy} H_y),
H'_x &= \Phi(x) H_x,
H'_y &= \Phi(x) k(H_y + e_{yz} E_z),
H'_z &= \Phi(x) k(H_z + e_{zy} E_y).
\end{align*}
\]

Here \( \Phi(x) \) is the weight function corresponding to the condition of the D’Alembert equation invariance; \( k, e_{yz}, e_{zy}, h_{yz}, h_{zy} \) the parameters of field transformations. By replacing the variables, taking into account the weight function \( \Phi(x) \), we find a set of 8 algebraic equations for the unknown values \( k, e_{yz}, e_{zy}, h_{yz}, h_{zy} \). The set is compatible and has the solutions:

\[
\begin{align*}
k &= + \frac{n_x (\beta - n_x) + \lambda}{1 - n_x^2},
e_{yz} &= + \frac{n_x (\beta - n_x) + \lambda}{n_x (\lambda - 1) + \beta},
h_{yz} &= - \frac{n_x (\beta - n_x) + \lambda}{n_x (\lambda - 1) + \beta},
e_{zy} &= - e_{yz} = h_{zy} = - h_{yz}.
\end{align*}
\]

where \( e_{yz} = - e_{zy} = h_{zy} = - h_{yz} \). In accordance with the generalized understanding of symmetry, it means that the Maxwell equations are invariant with respect to the Galilei transformations \( [1] \). In the limit of low velocities the formulae for field transformations take the form known under the name of the Galilean limit

\[
E' \approx E + \beta X H, \quad H' \approx H - \beta X E.
\]

These relationships form, in fact, the same limit both in the relativistic and in the Galilei case. Here the situation is similar to the situation concerning the transformations of space-time variables when \( v \ll c \)

\[
x' = x - V t, \quad y' = y, \quad z' = z, \quad t' = t, \quad c' = c.
\]

These transformations, being neither the Galilean nor the relativistic transformations, are the limiting relationships for both the cases.

4. **Lorentz Invariance of Schrödinger Equation**

Let us next consider Schrödinger equation \( L_S \psi = (i \hbar \partial_t + h^2 \triangle / 2m_0) \psi(x) = 0 \). We start from the generators \( M_{ok} = x^0 \partial_k - x^k \partial_0, k = 1, 2, 3 \) of the Lorentz transformations. In view of the permutation relations \([L_S, [L_S, M_{ok}]] = 0\), the Lorentz transformations are the Schrödinger equation symmetry transformations of the type \( p = 2 \). Using it as the base, we shall extend the Shrödinger equation to the relativistic domain of motions by introducing the relativistic mass \( m = m_0 / \sqrt{1 - \beta^2} \) and the relativistic energy \( E = mc^2 \) of a particle

\[
\left( i \hbar \partial_t + c^2 \hbar^2 \frac{\triangle}{2E} \right) \psi^r (x) = 0.
\]

The equation has two solutions:

\[
\begin{align*}
\psi^r_1 (x) &= e^{-i(m \omega^2 / 2h)(t - 2s \cdot x / v)},
\psi^r_2 (x) &= e^{-i(mc^2 / h)(t - \sqrt{2s \cdot x / c)}},
\end{align*}
\]

where \( s = v / v, x = (x, y, z), v = (v_x, v_y, v_z) \) is the velocity of a particle. Besides, equation \([13]\) is invariant with respect to Lorentz transformations \([\Phi]\) if the weight function \( \Psi \) from the relation \( \psi^r = \Psi \psi^r \) satisfies the equation \( [i \hbar (\partial_t + V \partial_x) + (c^2 h^2 (1 - \beta^2) / 2E (1 - V \cdot V/c^2)) ([\partial_x + \beta \partial_t / c]^2 (1 - \beta^2) + \partial_{yy} + \partial_{zz}]) \Psi \psi^r = 0 \). \( V = (V_x, 0, 0) \). The weight functions corresponding to the solutions \([14]\) are \( \Psi^r_1 \) and \( \Psi^r_2 \), take a rather complicated
form, and will not be presented here. The solutions (14) in the nonrelativistic approximation take the form

\[
\begin{align*}
\psi^1(x) &\to \psi_1(x) = e^{-i(Et - p\cdot x)/\hbar}; \\
\psi^2(x) &\to \psi_2(x) = e^{-i(\xi_0 t - \sqrt{2}p\cdot x/\beta)/\hbar},
\end{align*}
\]

where \( E = m_0c^2/2, \xi_0 = m_0c^2, p = m_0v; \) \( \psi_1(x) \) is the known solution; \( \psi_2(x) \) is the new linear-independent solution that reveals itself in analyzing the Lorentz invariance in the Schrödinger equation. By comparing the results obtained with the well-known results published, one can conclude that the Galilei group is the symmetry group in the sense of Lie for the Schrödinger equation and the symmetry group in the generalized sense for the equations of D’Alembert and Maxwell. Analogously, the Lorentz group is the symmetry group in the sense of Lie for the equations of D’Alembert and Maxwell and the symmetry group in the generalized sense for the Schrödinger equation [1].

5. FIVE-DIMENSIONAL SPACE WITH NONINVARIANT VELOCITY OF LIGHT

Let us dwell on examples of the new symmetries associated with the change-over to the space of different dimensionality. We shall introduce 5-dimensional abstract space of events \( V^5(t, x, c) \) in which the velocity of light \( c \) plays the role of additional independent variable. Next, we shall consider commutation relationships of the generators \( g_n \) of the conformal group \( C_{15} \) and the generators \( X_n = c^n[\partial x - t\partial c + N(t\partial c + x\cdot \nabla)] \) of the Virasoro infinite algebra with the D’Alembert operator in this space

\[
\begin{align*}
[\Box, g_n] &= 0, \quad g_n = \{c^np_a, M_{ab}\}; \\
[\Box, g_p] &= 0, \quad g_p = \{D, c^{-N}K_a\}; \\
[g_n, g_p] &= C_{spq}g_q; \\
[X_n, X_m] &= (n - m)X_{m+n}; \\
[X_n, g_n] &= 0.
\end{align*}
\]

Here \( g_n \) belong to a set of the operators \( c^np_a, M_{ab}, D, c^{-N}K_a; N = 0, \pm 1, \pm 2; \ldots \): \( C_{spq} \) are the Lie algebra structure constants of the conformal group; \( |m|, |n| < \infty \). According to the generalized definition of symmetry, this algebra is the invariance algebra of the D’Alembert equation and the Maxwell free equations of the type \( p = 1 \). The algebra may be brought into correspondence with the coordinate transformations group with the noninvariant velocity of light. In the particular case \( N = 0 \) these transformations were considered by Romain (1963), the author of the present work (1970), Di Jorio (1974), Sjödin (1979).

With \( c' = c \) the formulae contain the Voigt transformations (1887), Ives (1937), Palacios-Gordon (1957, 1962), Dewan (1961), Podlaha (1969). With \( c' = \lambda c \) (\( \lambda \) is the group parameter) they contain the transformations of the author of the present work (1970), Hsu (1976) and Mamaev (1990) [1].

6. DISCRETE “-C” SYMMETRY

We shall next turn to the new symmetry of discrete type. As is known, the discrete symmetries play an important role in modern physics, for example, in particle physics, quantum field theory, nuclear physics. One can note, as an example, the space inversion \( P(x \to -x) \), the time inversion \( T(t \to -t) \), the charge conjugation \( C \). Let us introduce a new discrete transformation - inversion of the velocity of light \( Q: t \to t, x \to x, c \to -c \). The inversion \( Q \) is closed into a group that forms a direct product with the Lorentz group (it does not change the form of the Lorentz transformations) and is the new transformation of discrete symmetry for the equations of classic and quantum electrodynamics: the equations of D’Alembert and Maxwell, the equations for movement of a charged particle in electromagnetic field, the Klein-Gordon-Fock equation, the Dirac equation and the Schrödinger equations [1]. The consequence of \(-c \) symmetry of Dirac equation is the possibility to interpret the charge conjugation \( C \) in terms of the \( Q \)-conjugation \( [C, Q] \) \( \Psi(x) = 0 \), where \( \Psi(x) \) is the solution of Eq. [7] [1].

7. INTERNAL SYMMETRY OF MAXWELL EQUATIONS

Besides the space-time symmetries, in physics applications also find so named internal symmetries. The internal symmetries are not connected with the properties of space and time and characterize the very objects to be studied. As applied to electromagnetic field, the investigation of such symmetries was started by Heaviside (1893), Rainich (1925) and Larmor (1928). It was then carried out by Markhashov (1966), Danilov (1967) and Ibragimov (1968) within the framework of the Lie classic algorithm. Fushchich and Nikitin (1978) and then the author (1982) of the present work continued these investigations with the help of the non-Lie algorithm (proposed by Fushchich and Nikitin) on the base of the Fourier transformations. We shall use the elements of this algorithm, start from the Maxwell homogeneous equations and introduce the Fourier transformations

\[
E(x^0, x) = (1/2\pi)^{3/2} \int \hat{E}(x^0, p)e^{ip\cdot x}d^3p, \\
H(x^0, x) = (1/2\pi)^{3/2} \int \hat{H}(x^0, p)e^{ip\cdot x}d^3p
\]

for electric \( E \) and magnetic \( H \) field and turn to the Maxwell equations in the \( p \)-space

\[
\begin{align*}
\hat{p}\cdot \hat{E} &+ \partial_t \hat{H} = 0, \\
\hat{p}\cdot \hat{H} - \partial_t \hat{E} &= 0, \\
\hat{H} &- \partial_t \hat{E} = 0.
\end{align*}
\]
From the condition of invariance of these equations with respect to the transformations of fields \( \mathcal{E} \to \mathcal{E}' \) and \( \mathcal{H} \to \mathcal{H}' \) we find a set of the sixteen 6x6 matrices \( Y_{LMN}(p) \), \( Z_{LMN}(p) \). \( p = (p_1, p_2, p_3) \), \( (L, M, N) = 0, 1 \) transforming the equations into themselves. The matrices satisfy the permutation relations of the Lie algebra, the Grassman algebra and super algebra \( [Y, Y] = Y; \ [Y, Z] = Z; \ [Z, Z] = Y \). Out of them the Lie 16-dimensional algebra is isomorphic to the Lie algebra of the unitary transformations group \( U(2)XU(2)XU(2)XU(2) \), which contains the 8-dimensional group \( U(2)XU(2) \) of Fushchich and Nikitin and the 2-dimensional group \( U(1)XU(1) \) of Danilov and Ibragimov. The latter includes the Larmor-Rainich transformations \( \mathbf{E}' = \mathbf{E} \cos \theta + \mathbf{H} \sin \theta \), \( \mathbf{H}' = -\mathbf{E} \sin \theta + \mathbf{H} \cos \theta \) (\( \theta \) is the group parameter) and the discreet Heaviside transformations \( \mathbf{E}' = \mathbf{H}, \ \mathbf{H}' = -\mathbf{E} \) at \( \theta = \pi/2 \). All the transformations in the x-space, except for the Danilov-Ibragimov transformations, are nonlocal (integral) [1].

8.NONLINEAR MAXWELL EQUATIONS

In all the cases considered we dealt with linear equations. But in contemporary physics nonlinear equations has received wide acceptance as well. Nonlinear equations are used, for example, in the catastrophe theory, in the theory of control, in quantum theory. They possess a number of such exclusive properties as the absence of superposition principle, the existence of nonlinear interaction of fields, the existence of soliton solutions. The first work on nonlinear equations in electrodynamics was done by Mie in 1912 on the base of introducing functions of field invariants into the theory. The concrete version of nonlinear equations was proposed by Born and then by Born and Infeld (1934). The nonlinear electrodynamics equations in the present work

\[
\nabla \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{H}}{\partial t} = 0,
\]

\[
\nabla \mathbf{H} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi}{c} F(I_1, I_2) \rho \mathbf{v},
\]

\[
\nabla \cdot \mathbf{E} = 4\pi F(I_1, I_2) \rho,
\]

\[
\nabla \cdot \mathbf{H} = 0.
\]

differ from the Born-Infeld equations and are of self-sufficient importance. (Here \( F \) is the arbitrary function of the Lorentz-invariants of the field \( I_1 = 2(\mathbf{E}^2 - \mathbf{H}^2), \ I_2 = (\mathbf{EH})^2 \)). With \( \rho = 0 \) they contain the Maxwell free equations; with \( F(I_1, I_2) = 1 \) they go into the Maxwell equations \( \nabla \cdot \mathbf{E} = 0, \ \nabla \cdot \mathbf{H} = 0 \); the nonlinearity is caused by the presence of currents and charges. The equations realize the principle of self-action with the result that the electric charge within the framework of these equations partially has a field nature. In view of the relativistic invariance the equations are of potential interest for physics [1].

9.APPLICATION TO PHYSICS

Next we shall turn to the question of physical interpretation of the mathematical results, in particular, from Section 5. As an example we shall consider the symmetry connected with violation of the Special Relativity (SR) second postulate - postulate of the invariance of the speed of light. The questions on the possibility of its violation and on the existence of the faster-than-light motions invariably attracts the scientific community’s attention. Abraham (1908), Ritz (1908), Blokhintsev (1946), Kirzhnits (1954), Terletsy (1960), Rapier (1962), Feinberg (1967), Loiseau (1968), Bolshakov and Sudarshan (1969), the author of the present work (1970), Marinov (1975), Hsu (1976), Recami (1982), Logunov (1982), Chubykalo and Smirnov-Rueda (1996), Russo (1998), Glashow (1999) and other authors investigated this problem from different points of view. Below we shall consider the author’s work [2] which starts from the invariance of 4-interval of space-time

\[
ds^2 = (c^2_0 + v^2)dt^2 - dx^2 - dy^2 - dz^2,
\]

where the invariant \( c_0 = 3 \cdot 10^{10} \) cm/sec is the proper value of the velocity of light. As a result we may introduce on the path of moving the particle some universal time \( t = t_0 \) like Newtonian time in classical physics. Then the velocity of light \( c \) will depend on the emitter velocity according to the law \( c = c_0(1 + v^2/c_0^2)^{1/2} \). The space-time transformations retaining the invariance of the 4-interval \( ds^2 \) take the form

\[
x' = \frac{x - Vt}{\sqrt{1 - V^2/c^2}}, \ y' = y, \ z' = z, \ t' = t,
\]

where \( c' = c(1 - Vv_z/c^2)/(1 - V^2/c^2)^{1/2} \). \( v_z = x/t \). In the Minkowski space \( M^4(x^0, x) \) they may be classified as the transformations of space-time-velocity of light which belong to the direct product of the Lorentz group \( L_6 \) and the group of the scale transformations of the velocity of light \( \Delta_1(c) : c' = \lambda c \) (\( \lambda \) is the group parameter). The corresponding integral of action, Lagrangian, expressions for 4-momentum and the energy of a particle in electromagnetic field are constructed. In particular, the expressions for 3-momentum \( \mathbf{p} \) and energy \( \mathcal{E} \) take the form

\[
\mathbf{p} = m\mathbf{v}, \ \mathcal{E} = mc_0^2 c = mc_0^2 \sqrt{1 + v^2/c_0^2}.
\]

As a result the theory possesses a number of signs common to both SR and classical physics. For example, as in classic mechanics, the particle mass \( m \) is independent of its velocity \( v \). The particle energy \( \mathcal{E} = mc_0 = mc_0^2 (1 + v^2/2 + \ldots) \) in \( v < c_0 \) approximation coincides with the expression for kinetic energy in Newton mechanics with an accuracy.
of the rest energy $E_0 = mc_0^2$ (as in SR). The momentum $p$ and the energy $E$ are related by the relationship coinciding formally with the relativistic expression $E^2 - c_0^2p^2 = m^2c_0^4$ - invariant. The distinction consists in the fact that here the energy and the momentum are $E = mc_0^2(1 + v^2/c_0^2)^{1/2}$ and $p = mv$, respectively, but not $E = mc_0^2/(1 - v^2/c_0^2)^{1/2}$ and $p = mv/(1 - v^2/c_0^2)^{1/2}$ as in SR. The equations of electrodynamics are nonlinear and take the form

$$\nabla \times E - \frac{1}{c} \frac{\partial H}{\partial t} = 0,$$

$$\nabla \times H - \frac{1}{c} \frac{\partial E}{\partial t} = \frac{4\pi}{c} \rho,$$

$$\nabla \cdot E = 4\pi \rho, \quad \nabla \cdot H = 0,$$

$$\frac{dp}{dt} = \frac{c}{c_0} E + \frac{c}{c_0} v \times H, \quad \frac{dc}{dt} = \frac{c}{c_0} v \cdot E,$$

where $\nabla c = 0$. The equations engage through the velocity of light $c$. In $v^2/c_0^2 \ll 1$ approximation the equations (22) go into the standard equations of classical electrodynamics (1) with $c = c_0$.

Many experiments hitherto interpreted in terms of SR, may be explained within the framework of the constructed theory. For example, the experiments of Michelson and Fizeau, the aberration of light, the appearance of atmospheric $\mu$-mesons near the Earth’s surface, the Doppler-effect, a number of the well-known experiments to prove independence of the velocity of light from the velocity of light source, the decay of unstable particles, the generation of new particles from nuclear reactions, the Compton-effect, photo-effect. Let us consider some of them.

The negative result of the Michelson experiment for an observer with a terrestrial source of light (the speed of light $c_0$) may be explained by the isotropy of space. Since the speed of light $c_0$ is the same in all directions, a shift of the interference pattern should be absent with the interferometer’s rotation. Analogously, in the case of an extraterrestrial source (the star moving inertial with a velocity $v$ relatively to the Earth), the velocity of light from the star $c = c_0(1 + v^2/c_0^2)^{1/2}$ does not depend on the direction of its propagation and is thus the same for an observer on the Earth. As a result, the interference pattern does not change with the interferometer’s rotation.

A free particle with the mass $m$ will move faster than light at the velocity $v = c_0((\sqrt{E/E_0})^2 - 1)^{1/2} > c_0$ if the particle energy satisfies the inequality $E > 2^{1/2}E_0$, where $E_0 = mc_0^2$ is the particle rest energy. As an illustration, for the electron faster-than-light motion begins with the energy $E \sim 723$ keV. The 1 MeV-electron velocity is $\sim 1.68c_0$; the 1 GeV-electron velocity is $\sim 2000c_0$.

The angle of aberration of the quasar Q1158+4635 with the parameter of red shift $z_\alpha = 4.73$ (Carswell and Hewett, 1990) is $\alpha \sim 11.6$ arc seconds; $z_\omega \sim 2.23$, the velocity of light is $c = c_0(1 + z_\lambda)/(1 + z_\omega) \sim 1.77c_0$ instead of $\alpha \sim 20.5$ arc seconds and $z_\lambda = z_\omega, c = c_0$ in SR.

The velocity of atmospheric $\mu$-mesons at the Earth’s surface is $\sim 100c_0$, the energy is $\sim 10.6$ GeV.

Independence of the velocity of light from the velocity of a source of light was considered in rather numerous experiments. As an example, we turn to the Filippas and Fox experiment (1964) in which the velocities of $\gamma$-quanta from the decay $\pi^0 \rightarrow \gamma + \gamma$ of fast $\pi^0$-mesons were compared. Because of independence of the velocity $c = c_0(1 + v^2/c_0^2)^{1/2}$ from the direction of the $\gamma$-emission, the velocities of $\gamma$-quanta, as in SR, should be the same. It means a negative result, in the standard sense, of this experiment and other experiments of such a type (Bonch-Bruevich and Molevovanov 1956, Sadeh 1963).

The velocity of Compton forward-scattered electron $v = c_0(h\omega/E_0)[1 - E_0/(2h\omega + E_0)]$ exceeds the speed of light $c_0$ if the energy of an incident gamma quantum $h\omega$ is more than 698 keV. The angular distribution of scattered gamma quanta coincides with the one obtained from SR.

The velocity of photoelectron $v = c_0[(h\omega + mc_0^2 - U)/m_c c_0^2] - 1)^{1/2}$ exceeds the speed of light $c_0$ if the photon energy satisfies the inequality $h\omega > 211$ keV + $U$, where $U$ is the ionization energy, $m_c c_0^2$ is the electron rest energy.

In sum, the $L_\pi X_\gamma \Delta_1(c)$-invariant theory has been constructed. The faster-than-light motions are possible in this theory. According to the construction, the proposed theory is close to the SR theory and based (as SR) on the principles of a symmetry approach. It has been found that this theory permits one to interpret a set of the well-known experiments. In the case of the Michelson and Fizeau experiments the results of the interpretation coincide with the accepted results. On the other hand, there is a number of the predictions which differ from SR predictions. These are, for example, faster-than-light motions of nuclear reactions products, faster-than-light motions in astrophysics. These predictions may be the subject of experimental investigation. The postulation $c' = c$ leads to SR.

10.REFERENCES

[1] G.A. Kotel’nikov. New Symmetries in Electrodynamics and Quantum Theory. Synopsis of thesis for a Doctor’s degree, RRC Kurchatov Institute, Moscow, 1999, 41 p.

[2] G.A. Kotel’nikov. On the Faster-Than-Light Motions in Electrodynamics. Proc. XII Int. Conf. on Selected Problems of Modern Physics, Dubna, June 8-11, 2003, D1, 2-2003-219, 143-147; physics/0311041