Phase-Field Simulations of Cracks under Dynamic Loading

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Phase-field fracture simulations have been established to simulate crack propagation in fracture mechanics. This contribution sets the focus on different driving forces for crack growth and on the simulation of waves propagating along the fractured interfaces.

Typically, phase-field methods for brittle fracture employ a variational framework which has been proven to converge to Griffith's classical model. This approach, however, has limits in the pressure dominated regime. For that reason we propose ad-hoc driving forces which are motivated physically using general fracture mechanic concepts. Additionally we investigate the effect of the phase-field modeled cracks and interfaces on the propagation of the arising waves. The accuracy and the robustness of the simulation method will be demonstrated by numerical examples.

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1 Introduction

Phase-field models became very popular recently to predict the location of crack initialization and the crack path itself, cf. [7–9]. The main idea is to characterize the state of the material by an additional parameter z(x, t) : Ω0 × T → [0, 1] within the reference domain Ω0 and the time interval T = [0, T], T ∈ R+. In particular, the intact material is characterized by z = 0 and the fully broken state by z = 1. The total energy Etot consists of the free Helmholtz-energy density Ψ(u, z) and of surface energy contributions where the latter term has to be regularized because of the moving boundaries Γ0(t):

$$E^{tot}(u, z) = \int_{\Omega_0} \Psi(u, z) d\Omega + \int_{\Gamma_0(t)} G_c dA = \int_{\Omega_0} \Psi(u, z) + G_c \gamma(z) d\Omega$$

(1)

The choice of the crack density functional γ is not unique; in this work we apply a second order phase-field approach of the form: γ(z, ∇z) = \frac{1}{2} z^2 + \frac{1}{2} ∇z · ∇z. The length-scale parameter \( l_c \) is a measure for the width of the diffuse interface zone and acts two-fold, as a mesh dependent and as a material dependent parameter. Detailed investigations about its influence can be found in the literature, cf. [1, 7, 8].

The energy minimization problem of (1) corresponds to the coupling of the balance of linear momentum and the evolution equation of the phase-field z,

$$\tau \ddot{z} = Y$$

where Y summarizes the generalized thermodynamic forces Y = Ye + Yf = Ye − δz\gamma with the crack driving force Ye and \( \tau \) is a numerically determined retardation time.

2 Phase-field model

Commonly the crack driving force is deduced from an energy optimization, Ye = δzΨ. Since crack growth requires a state of tension and does not grow under compression with same energy this asymmetry has to be incorporated in the model. This applies for both linear and finite elasticity. Exemplarily, we outline the common decomposition in linear elasticity which is based on the decomposition of the principal strains such that the strain energy density results in

$$\Psi^+ = \frac{1}{2} \lambda (\text{tr} e)^2 \pm \mu e^+ : e^+$$

(2)

with e+ = \sum_{a=1}^{3}(e_a)_+ n_a ⊗ n_a and (●)± = \frac{1}{2}(● ± | ● ) , ● ∈ \{ e_a, \text{tr} e \} being the positive part of the eigen strains, cf. [8]. In finite elasticity we choose an additive decomposition of the invariants as proposed in [4] for which the strain energy density remains polyconvex. Please note that these decompositions are modeling assumptions and can be varied in different ways. In any case the assumption of decomposition results in different crack driving forces and also influences the crack propagation.

This situation motivated us to investigate the choice of crack driving force in more detail. In addition to variational crack driving forces also ad-hoc assumptions for Ye motivated by established failure criteria are introduced. Exemplarily, we focus here on the maximum principal stress criterion which can be traced back to the Rankine failure criterion. The driving force is

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given by \( Y^e = \left( \frac{\sigma}{\sigma_c} - 1 \right)_+ \) with the maximum stress \( \sigma_I = \max(\sigma_a) \) with the eigenvalues \( \sigma_a, a \in \{1, 2, 3\} \). This means that the crack starts propagating as soon as a critical stress \( \sigma_c \) is exceeded. This driving force can be replaced by a series of failure criteria like the maximum shear stress, the maximum principal strain or the Mohr-Coulomb failure criterion, cf. [1].

### 3 Wave propagation and Brazilian test

For purpose of illustration we focus on two aspects of our simulations and study the wave propagation and a dynamic Brazilian test in compression. The corresponding experiments are performed in a Split-Hopkinson Pressure Bar setup. The wave propagation in the bars is simulated by the linear wave equation \( \rho \ddot{u} - \nabla \cdot (\mathbf{C} : \dot{\epsilon}) = f \). This second order equation is reformulated in a first order Friedrich system which is then solved by a discontinuous Galerkin method for hyperbolic conservation laws in space and the midpoint rule in time. This discrete system conserves energy; the corresponding unidirectional wave propagation through the bar is depicted in Fig. 1.

The Brazilian test acts as a classical test to identify the tensile resistance of brittle material. A cylindrical specimen is compressed from two opposite sides until the tension within the specimen results in failure. The maximum stress occurs in the middle of the specimen and is given by Frocht [3] as

\[
\sigma_{\text{max}} = \frac{2F_{\text{max}}}{\pi LD}
\]

(3)

with the specimen diameter \( D \) and the length \( L \). The experimental setup is depicted in the left picture of Fig. 2 for a specimen of concrete \((E = 50\,000 \text{ N/mm}^2, \nu = 0.2)\). A detailed investigation can be found in [2]. The final results based on the phase-field method are demonstrated in the right plots of Fig. 2. The crack initialization and also the crack path itself can be observed in the numerical simulation and coincide nicely with the experiments, cf. [6]. The choice of the crack driving force is important because the standard variational ansatz does not lead to physically meaningful results here, cf. [2].

![Fig. 2: Dynamic Brazilian test (left) and numerical fracture simulations of the Brazilian test specimen (right).](image-url)