A fuzzy behavioral portfolio decision model with trapezoidal fuzzy return and aspiration

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Abstract. This paper deals with the stock portfolio selection problem involved with trapezoidal fuzzy number returns and multiple mental accounts. A fuzzy behavioral portfolio decision model is proposed to maximize the possibilistic mean value of portfolio return and ensure the portfolio return of each mental account exceeding the given minimum fuzzy aspiration level with a given probability. Then, some programming models are designed to solve the optimal portfolio strategy. Finally, one numerical example is given to illustrate the effectiveness of the proposed fuzzy behavioral portfolio decision approach.

1. Introduction

In 2000 [1] Shefrin and Statman proposed behavioral portfolio theoretical framework for asset choice under uncertainty based on prospect theory [2-4]. In the behavioral portfolio process, each portfolio layer is associated with a particular aspiration level and resembles a separate mental account [5]. After that, Ma [6] proposed a practical decision-making method for behavioral portfolio choice. Mehlawat [7] and Amelia [8] developed multi-criteria behavioral portfolio decision models. Jin [9] developed multi-period and multi-objective behavioral portfolio selection approach. Recently, trapezoidal fuzzy numbers have been widely used in handling and describing imprecise and complex phenomena that often rise in financial and managerial systems. In uncertain portfolio decision environment, the returns of financial assets are conveniently evaluated by fuzzy numbers. Inspired by the idea of Markowitz’s M-V model, a lot of fuzzy portfolio model extensions have been proposed to deal with portfolio decision with fuzzy number return and risk under fuzzy uncertain environment. For example, Fang [10] and Gupta [11] studied the portfolio models based on fuzzy decision theory and fuzzy programming technique. Zhang [12,13] proposed some portfolio decision models based on possibilistic mean and variance. Li [14] presented the portfolio selection model based on fuzzy skewness. Mehlawat [15] and Yue [16] proposed fuzzy higher order moment portfolio models. Liagkouras [17], Liu [18-19] and Mehlawat [20] also discussed the fuzzy multi-period portfolio decision models. However, the above-mentioned fuzzy portfolio decision models have not considered the different psychological accounts and sentiments of investors. In traditional behavioral portfolio theory, the fuzzy returns and fuzzy aspiration levels of invested assets are not taken into account. As we know, in real market environment the future returns of the invested assets are not known and incomplete, and the portfolio return of each mental account can be easily evaluated by trapezoidal fuzzy number. Therefore, in this paper we propose a new fuzzy behavioral portfolio decision model to obtain the optimal asset allocation strategy between multiple mental accounts.
2. Preliminaries

Definition 1 [21,22] A fuzzy set of real line with a normal, fuzzy convex and continuous trapezoidal membership function of bounded support is called a trapezoidal fuzzy number $\tilde{R}=(a,b,c,d)$, where $\xi=(b-a), \eta=(d-c)$ are respectively called left and right spread of trapezoidal fuzzy number $\tilde{R}$.

Definition 2. Let $\tilde{R}=(a,b,c,d)$ be a trapezoidal fuzzy number, the $\lambda$-level cut set [12] of $\tilde{R}$ is defined as $\tilde{R}_\lambda=[b-\xi(1-\lambda), c+\eta(1-\lambda)]=[R^-(\lambda), R^+(\lambda)], \forall \lambda \in (0,1]$.

Definition 3. Let $\tilde{R}_1=(a_1,b_1,c_1,d_1), \tilde{R}_2=(a_2,b_2,c_2,d_2)$ be two trapezoidal fuzzy numbers, for any $x \geq 0$, the addition and scale multiplication of TFNs [22] are defined as

\begin{align*}
(1) \quad \tilde{R}_1+\tilde{R}_2 &= (a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2), \quad (2) \quad x\tilde{R}_1 = (xa_1,xb_1,xc_1,xd_1), \quad \text{for } i=1,2.
\end{align*}

Proposition 1. Let $\tilde{R}_1=(a_1,b_1,c_1,d_1), \tilde{R}_2=(a_2,b_2,c_2,d_2)$ be two trapezoidal fuzzy numbers, for all $\lambda \in (0,1)$, and $x \geq 0$, by employing the fuzzy extension principle we can easily get

\begin{align*}
(1) \quad \tilde{R}_1+\tilde{R}_2 &= (a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2), \quad (2) \quad x\tilde{R}_1 = [xa_1,xb_1,xc_1,xd_1], \quad \text{for } i=1,2.
\end{align*}

Definition 4[21]. Let $\tilde{R}=(a,b,c,d)$ be a trapezoidal fuzzy number with $\lambda$-level set $\tilde{R}_\lambda$, the possibilistic mean value of fuzzy number $\tilde{R}$ is defined as

$$E(\tilde{R})=\int_0^1 \lambda [R^-(\lambda)+R^+(\lambda)]d\lambda = \int_0^1 \lambda [b+c+(\eta-\xi)(1-\lambda)]d\lambda = (b+c)/2+(\eta-\xi)/6.$$

Definition 5[23]. Let $\tilde{R}=(a,b,c,d)$ be a trapezoidal fuzzy number, $\tilde{R}_0$ be a fuzzy origin, the distance between $\tilde{R}$ and $\tilde{R}_0$ is defined as

$$D_p(\tilde{R},\tilde{R}_0)=\left\{ \left[R^-(\lambda)-0\right]^p + \left[R^+(\lambda)-0\right]^p \right\}^{1/p} = \left\{ \left[R^-(\lambda)^p + \left[R^+(\lambda)^p \right] \right]^{1/p}, \quad p \geq 1.\right\}$$

Definition 6. Let $\tilde{R}_1$ be a fuzzy origin, $\tilde{R}_1=(a_1,b_1,c_1,d_1), \tilde{R}_2=(a_2,b_2,c_2,d_2)$ be two positive trapezoidal fuzzy numbers, for any $x_1,x_2 \geq 0$, taking $p=1$, the distance between $(x_1\tilde{R}_1+x_2\tilde{R}_2)$ and $\tilde{R}_0$ can be defined as

$$D(x_1\tilde{R}_1+x_2\tilde{R}_2,\tilde{R}_0) = x_1(a_1+b_1+c_1+d_1)/2 + x_2(a_2+b_2+c_2+d_2)/2.$$

Definition 7 [23]. Let $\tilde{R}=(a,b,c,d)$ be any trapezoidal fuzzy number, the sign distance between $\tilde{R}$ and $\tilde{R}_0$ is defined as $\hat{D}(\tilde{R},\tilde{R}_0) = \gamma(\tilde{R})D(\tilde{R},\tilde{R}_0)$, where $\gamma(\tilde{R})$ is the sign function of $\tilde{R}$.

Proposition 2. Let $\tilde{R}_1=(a_1,b_1,c_1,d_1)$ be a series of positive trapezoidal fuzzy numbers, for any $x_i \geq 0$, $i=1,2,\cdots,n$, then we get the sign distance between $\sum_{i=1}^n x_i\tilde{R}_1$ and $\tilde{R}_0$ as follows:

\begin{align*}
\hat{D}(\sum_{i=1}^n x_i\tilde{R}_1,\tilde{R}_0) &= \gamma(\sum_{i=1}^n x_i\tilde{R}_1)D(\sum_{i=1}^n x_i\tilde{R}_1,\tilde{R}_0) = \frac{1}{2} \sum_{i=1}^n x_i(a_1+b_1+c_1+d_1).
\end{align*}

It can be easily verified by Definition 6 and $\sum_{i=1}^n x_iR^+_1(\lambda) \geq \sum_{i=1}^n x_iR^-_1(\lambda) \geq 0$.

Definition 8. For arbitrary trapezoidal fuzzy numbers $\tilde{R}_1, \tilde{R}_2$, by using the sign distance we can define the ranking of fuzzy numbers $\tilde{R}_1$ and $\tilde{R}_2$ as

\begin{align*}
(1) \quad \hat{D}(\tilde{R}_1,\tilde{R}_0) \geq \hat{D}(\tilde{R}_2,\tilde{R}_0), \quad \text{if } \tilde{R}_1 \geq \tilde{R}_2; \quad (2) \quad \hat{D}(\tilde{R}_1,\tilde{R}_0) < \hat{D}(\tilde{R}_2,\tilde{R}_0), \quad \text{if } \tilde{R}_1 < \tilde{R}_2.
\end{align*}
3. The formulation of multi-account fuzzy behavioral portfolio decision model

In this section we consider the fuzzy behavioral portfolio selection problem with multiple mental accounts $MA_i$ ($i=1,2,\cdots,t$). Suppose the stock investor has $t$ mental accounts. Each mental account $MA_i$ has $m_i$ risky assets $A_{ij}$ ($1 \leq j \leq m_i$). The future return rates of the invested stocks can be easily evaluated by trapezoidal fuzzy numbers. The investor intends to allocate his/her wealth among all the risky assets in $t$ mental accounts. Suppose the initial invested wealth value in total is 1, i.e., 

$$\sum_{i=1}^{t} \sum_{j=1}^{m_i} x_{ij} = 1.$$ 

Assume $\sigma_i$ is the importance degree of the holding mental account $i$; \( x_{ij} \) is the investment proportion of risky asset $j$ in mental account $i$; \( l_{ij}, u_{ij} \) respectively represent the lower boundary and upper boundary of capital invested in risky asset $j$ at mental account $i$. At the terminal of investment, every asset $A_{ij}$ has $n_{ij}$ possible fuzzy return states $\tilde{R}_{ij1}, \tilde{R}_{ij2}, \cdots, \tilde{R}_{ijn_{ij}}$ due to the different market situations. For each $MA_i$, all the fuzzy return value vector of all assets in $MA_i$ forms a invest value space $R_i = R_{i1} \times R_{i2} \times \cdots \times R_{im_i}$, which is the Cartesian product of $m_i$ return value set $R_{i1}, R_{i2}, \cdots, R_{im_i}$, where $R_{ij} = \{\tilde{R}_{ij1}, \tilde{R}_{ij2}, \cdots, \tilde{R}_{ijn_{ij}}\}$, $j=1,2,\cdots,m_i$. One can see that there are $n_i = n_{i1} \times n_{i2} \times \cdots \times n_{im_i}$ fuzzy return value vectors in $R_i$. Since we choose the alternative $m_i$ securities for each $MA_i$. The terminal return vector of these $m_i$ assets invested in $MA_i$ form the return value space $S_i = \{S_{i1}, S_{i2}, \cdots, S_{ik}, \cdots, S_{im_i}\}$, $S_{ik} = \{\tilde{R}_{ik(1)}, \tilde{R}_{ik(2)}, \cdots, \tilde{R}_{ik(jk)}, \cdots, \tilde{R}_{ik(m_i)}\} \in R_i$ $= R_{i1} \times R_{i2} \times \cdots \times R_{im_i}$, $k = 1,2,\cdots,n_i$; $jk \in \{1,2,\cdots,n_{ij}\}$, $j=1,2,\cdots,m_i$; $\tilde{R}_{ik(jk)}$ is the $jk$-th fuzzy return state of asset $j$ in $MA_i$ which can be evaluated by a trapezoidal fuzzy number. Let $p(S_{ik}) = p_{ik}$ be the probability of fuzzy return vector $S_{ik}$, and $\sum_{i=1}^{t} \sum_{k=1}^{n_i} p(S_{ik}) = \sum_{i=1}^{t} \sum_{k=1}^{n_i} p_{ik} = 1$.

The aim of security investor is to maximize the weighted return $\sum_{i=1}^{t} \sigma_i E(X_i)$ of portfolio over the whole $t$ mental accounts and ensure that the probability of portfolio return $\tilde{R}_p = \tilde{R}_{p1}, \tilde{R}_{p2}, \cdots, \tilde{R}_{p(tm_i)}$ of $MA_i$ exceeding the given fuzzy aspiration level $\tilde{r}_i$ is greater than $\alpha_i$ for each $MA_i$. $E(X_i)$ is the expected return of $MA_i$; $X_i = (x_{i1}, x_{i2}, \cdots, x_{im_i})$ is the portfolio vector representing the allocation vector among all the $m_i$ assets in mental account $MA_i$.

Thus, we can formulate the multi-account fuzzy behavioral portfolio problem as following model.

$$\text{(M1) } \text{max } E(X) = \sum_{i=1}^{t} \sigma_i E(X_i) = \sum_{i=1}^{t} \sum_{k=1}^{n_i} p_{ik} E(\sum_{j=1}^{m_i} \tilde{R}_{ij(k)} x_{ij})$$

s.t. \[ \sum_{i=1}^{t} \sum_{j=1}^{m_i} x_{ij} = 1; \] \[ 0 \leq l_{ij} \leq x_{ij} \leq u_{ij} \leq 1. \]

where $X = (x_{11}, \cdots, x_{im_i}, x_{21}, \cdots, x_{2m_i}, \cdots, x_{t1}, \cdots, x_{tm_i})$ is the portfolio vector over all the mental accounts.

To solve the above fuzzy behavioral portfolio model we give the following procedure algorithm.

**Step 1.** Find the efficient subset $T_{\alpha_i}$, from fuzzy return space $R_i$ of each mental account $MA_i$ satisfying the following two conditions: (1) the probability summation of all the fuzzy return vector in the subset $T_{\alpha_i}$ is greater than the given safety level $\alpha_i$; (2) the probability summation of the remained
fuzzy return vector if removing any one fuzzy return vector from the subset \( T_{i_l} \) is less than \( \alpha_i \). If there exists \( \tau_i \) subsets of fuzzy return space \( R_i \) for each \( MA_i \) satisfying the above-mentioned two conditions, we can denote the family by \( T_i = \{ T_{i_1}, T_{i_2}, \ldots, T_{i_{\tau_i}} \} \) regarding each \( MA_i \).

**Step 2.** Construct the following models M2 based on each subset \( T = (T_{i_1}, T_{i_2}, \ldots, T_{i_{\tau_i}}) \) \( \in T_1 \times T_2 \times \cdots \times T_{\tau_i} \) regarding all the \( MA_i \), where \( T_{i_l} = \{ S_{ik}^{(q_l)}, S_{ik}^{(s_l)}, \ldots, S_{ik}^{(a_l)} \} \subset R_i \) \( , q_l = 1, 2, \ldots, \tau_i \). \( i \leq k_1 < k_2 < \cdots < k_i < n_i = n_1 \times n_2 \times \cdots \times n_{m_i} ; \delta_{ik}^{(q_i)} = \{ \tilde{R}_{ik_1}^{(q_i)}, \tilde{R}_{ik_2}^{(q_i)}, \ldots, \tilde{R}_{ik_{\tau_i}}^{(q_i)} \}, i \leq l \leq r \).

\[
\begin{align*}
\max E(X) &= \sum_{i=1}^{n'} \sigma_i (\sum_{k=1}^{k_{\alpha_i}} p(S_{ik}) E(\sum_{j=1}^{m_i} \tilde{R}_{ijk} x_{ij})) \\
\text{s.t.} \quad &\sum_{j=1}^{m_i} x_{ij} = 1; \\
&0 \leq l_i \leq x_{ij} \leq u_i \leq 1;
\end{align*}
\]

for any \( q_1 \in \{ 1, 2, \ldots, \tau_1 \} ; \quad q_2 \in \{ 1, 2, \ldots, \tau_2 \} ; \quad q_3 = \{ 1, 2, \ldots, \tau_3 \} ; \quad \ldots ; \quad q_r = \{ 1, 2, \ldots, \tau_r \} \).

where \( X = (x_{11}, \ldots, x_{1m_1}, x_{21}, \ldots, x_{2m_2}, \ldots, x_{1m_1}, x_{1m_2}, \ldots) \) is the optimal invest portfolio vector in all the mental accounts for the fuzzy return subset \( T = (T_{i_1}, T_{i_2}, \ldots, T_{i_{\tau_1}}, T_{i_{\tau_1}}) \).

Then we solve the above \( \tau = \tau_1 \times \tau_2 \times \cdots \times \tau_r \) programming models and obtain the corresponding \( \tau \) optimized solutions \( X^{(1)}, X^{(2)}, \ldots, X^{(r)} \), where \( X^{(\theta)} = (x_{11}^{(\theta)}, \ldots, x_{1m_1}^{(\theta)}, x_{21}^{(\theta)}, \ldots, x_{2m_2}^{(\theta)}, \ldots, x_{1m_1}^{(\theta)}, x_{1m_2}^{(\theta)}, \ldots) \big|_{\theta=1,2,\ldots,\tau} \) is the optimal portfolio vector among \( m_1 + m_2 + \cdots + m_r \) alternative assets in all the mental accounts.

**Step 3.** Substitute the portfolio vector \( X^{(\theta)}, \theta = 1, 2, \ldots, \tau = \tau_1 \times \tau_2 \times \cdots \times \tau_r \) in objective function and compute \( E(X^{(\theta)}) = \max_{1 \leq \theta \leq \tau} E(X^{(\theta)}) \). \( \delta = \arg \max_{1 \leq \theta \leq \tau} E(X^{(\theta)}) \), then \( X^{(\delta)} \) is the final optimal portfolio strategy of the presented fuzzy behavioral portfolio model.

### 4. Illustrative example

**Example 1.** Consider an investor joining the stock investment in financial market with initial capital \( W=10,000 \) RMB. Assume the security market investor has two mental accounts \( MA_1, MA_2 \) for trading. All the financial securities in the above two mental accounts are selected from Shanghai Stock Exchange in China. Suppose mental account \( MA_1 \) includes two stocks \( A_{11}, A_{12} \) with relatively lower trapezoidal fuzzy return rates. And mental account \( MA_2 \) includes two stocks \( A_{21}, A_{22} \) with relatively higher trapezoidal fuzzy return rates. By analyzing the stock historical price data and the corporation financial information, we can employ fuzzy linguistic variable [22] to assess the possible fuzzy return states for each stock asset under bear and bull market situations. The corresponding trapezoidal fuzzy return assessments of the selected stock assets are listed in Table 1.

For each mental account \( MA_i \), the fuzzy return value vectors of all stock assets in \( MA_i \) forms an investment value space \( R_i = R_{i_1} \times R_{i_2} \), \( i = 1, 2 \). The fuzzy return value space of mental account \( MA_i \) is expressed by \( S_i = \{ S_{i_1}, S_{i_2}, S_{i_3}, S_{i_4} \} = \{ (\tilde{R}_{i_1}, \tilde{R}_{i_2}), (\tilde{R}_{i_1}, \tilde{R}_{i_2}), (\tilde{R}_{i_1}, \tilde{R}_{i_2}), (\tilde{R}_{i_1}, \tilde{R}_{i_2}) \} \).

Assume the probability of the possible fuzzy return value vector can be evaluated as follows.

\[
p(S_{i_1} = (\tilde{R}_{i_1}, \tilde{R}_{i_2})) = 0.012, \quad p(S_{i_2} = (\tilde{R}_{i_1}, \tilde{R}_{i_2})) = 0.125,
\]
\[ p(S_{13} = (\tilde{R}_{112}, \tilde{R}_{122})) = 0.835, \quad p(S_{14} = (\tilde{R}_{112}, \tilde{R}_{122})) = 0.028. \]

Similarly, the fuzzy return value space of mental account \( MA_1 \) is expressed by
\[
S_2 = \{ \tilde{S}_{21}, \tilde{S}_{22}, \tilde{S}_{23}, \tilde{S}_{24} \} = \{(\tilde{R}_{211}, \tilde{R}_{212}), (\tilde{R}_{212}, \tilde{R}_{222}), (\tilde{R}_{212}, \tilde{R}_{222}), (\tilde{R}_{212}, \tilde{R}_{222})\},
\]
and
\[ p(\tilde{S}_{21} = (\tilde{R}_{211}, \tilde{R}_{221})) = 0.015, \quad p(\tilde{S}_{22} = (\tilde{R}_{211}, \tilde{R}_{222})) = 0.148, \]
\[ p(\tilde{S}_{23} = (\tilde{R}_{212}, \tilde{R}_{222})) = 0.812, \quad p(\tilde{S}_{24} = (\tilde{R}_{212}, \tilde{R}_{222})) = 0.025. \]

**Table 1.** The assessed trapezoidal fuzzy returns for the selected assets from two mental accounts.

| Mental account 1 | Trapezoidal fuzzy return | Mental account 2 | Trapezoidal fuzzy return |
|------------------|--------------------------|------------------|--------------------------|
| Asset \( A_{i1} \) | \( \tilde{R}_{111} = VL = (0.0, 0.0, 0.02, 0.07) \) | Asset \( A_{i2} \) | \( \tilde{R}_{211} = M = (0.32, 0.41, 0.58, 0.65) \) |
| \( \tilde{R}_{121} = M = (0.32, 0.41, 0.58, 0.65) \) | \( \tilde{R}_{212} = FH = (0.58, 0.63, 0.8, 0.86) \) |
| Asset \( A_{i2} \) | \( \tilde{R}_{121} = L = (0.04, 0.1, 0.18, 0.23) \) | Asset \( A_{i2} \) | \( \tilde{R}_{221} = FL = (0.17, 0.22, 0.36, 0.42) \) |
| \( \tilde{R}_{121} = FH = (0.58, 0.63, 0.8, 0.86) \) | \( \tilde{R}_{222} = H = (0.72, 0.78, 0.92, 0.97) \) |

Suppose the minimum fuzzy aspiration return of \( MA_1 \) and \( MA_2 \) are \( \tilde{r}_1 = (0.04, 0.1, 0.15, 0.28) \) and \( \tilde{r}_2 = (0.17, 0.3, 0.4, 0.6) \), respectively. And \( \alpha_1 = 0.96 \) is the minimum probability of the fuzzy return of portfolio in \( MA_1 \) exceeding the fuzzy aspiration return level \( \tilde{r}_1 \), \( \alpha_2 = 0.02 \) is the minimum probability of the fuzzy return of portfolio in \( MA_2 \) exceeding the fuzzy aspiration return level \( \tilde{r}_2 \). Assume the lower and upper proportion of capital invested in each asset are 0.1 and 0.5, respectively.

According to the previous procedure algorithm for solving multi-account fuzzy behavioral portfolio decision model, we take the following steps to get the optimal behavioral portfolio strategy.

1. For \( MA_1 \) we find that there exists one efficient subset \( T_{11} = \{ S_{12}, S_{13} \} = \{(\tilde{R}_{111}, \tilde{R}_{122}),(\tilde{R}_{112}, \tilde{R}_{121})\} \) of fuzzy return value space \( R_1 = R_{11} \times R_{12} \) satisfying all the two conditions in step 1 of previous procedure algorithm. For mental account \( MA_2 \) there exist the following three efficient subsets \( T_{21} = \{ S_{22} \} = \{(\tilde{R}_{211}, \tilde{R}_{222})\}, \) \( T_{22} = \{ S_{23} \} = \{(\tilde{R}_{212}, \tilde{R}_{222})\}, \) \( T_{23} = \{ S_{24} \} = \{(\tilde{R}_{212}, \tilde{R}_{222})\} \) of fuzzy return value space \( R_2 = R_{21} \times R_{22} \) satisfying all the two conditions in step 1 of previous procedure algorithm.

2. Based on fuzzy value space subset \( (T_{11}, T_{21}) \), we set the following programming Model P1.

\[
(P1) \quad \max E(X) = \sum_{i=1}^{2} \sigma_i \sum_{k=1}^{4} p(S_{ik}) E(\sum_{j=1}^{2} \tilde{R}_{ij}(j) x_j )
\]
\[
\text{s.t.} \quad \begin{align*}
\tilde{R}_{111} x_{11} + \tilde{R}_{122} x_{12} & \geq \tilde{r}_1, \\
\tilde{R}_{211} x_{21} + \tilde{R}_{222} x_{22} & \geq \tilde{r}_2, \\
x_{11} + x_{12} + x_{21} + x_{22} & = 1, \\
0.1 \leq x_{11}, x_{12}, x_{21}, x_{22} & \leq 0.5
\end{align*}
\]

Then, by using Definition 4, 8, Proposition 1, 2 and substituting fuzzy return data in Table 1, Model (P1) can be transformed to the following equivalent form.

\[
\max E(X) = \sigma_1 [0.426844 x_{11} + 0.226795 x_{12}] + \sigma_2 [0.680025 x_{21} + 0.387992 x_{22}]
\]
\[
\begin{aligned}
0.045x_{11} + 1.435x_{12} &\geq 0.285, 0.98x_{11} + 0.275x_{12} \geq 0.285; \\
0.98x_{21} + 1.695x_{22} &\geq 0.735; \\
x_{11} + x_{12} + x_{21} + x_{22} &= 1; \\
0.1 \leq x_{11}, x_{12}, x_{21}, x_{22} &\leq 0.5.
\end{aligned}
\]

Based on fuzzy value space subset \((T_{11}, T_{22})\), we similarly establish the following programming Model P2.

\[(P2)\] \[
\begin{aligned}
\max \ E(X) &= \sum_{k=1}^{2} \sigma_k \sum_{k=1}^{4} P(S_{ik})E(\sum_{j=1}^{2} \tilde{R}_{ik(j)}x_{ij}) \\
\text{s. t.} \quad & \begin{cases} 
\tilde{R}_{11}x_{11} + \tilde{R}_{12}x_{12} \geq \tilde{r}_{1}, \tilde{R}_{112}x_{11} + \tilde{R}_{121}x_{12} \geq \tilde{r}_{1}; \\
\tilde{R}_{212}x_{21} + \tilde{R}_{222}x_{22} \geq \tilde{r}_{2}; \\
x_{11} + x_{12} + x_{21} + x_{22} &= 1; \\
0.1 \leq x_{11}, x_{12}, x_{21}, x_{22} &\leq 0.5.
\end{cases}
\end{aligned}
\]

That is, \[
\begin{aligned}
\max \ E(X) &= \sigma_1 [0.426844x_{11} + 0.226795x_{12}] + \sigma_2 [0.680025x_{21} + 0.387992x_{22}]
\text{s. t.} \quad & \begin{cases} 
0.045x_{11} + 1.435x_{12} \geq 0.285, 0.98x_{11} + 0.275x_{12} \geq 0.285; \\
1.435x_{21} + 0.585x_{22} \geq 0.735; \\
x_{11} + x_{12} + x_{21} + x_{22} &= 1; \\
0.1 \leq x_{11}, x_{12}, x_{21}, x_{22} &\leq 0.5.
\end{cases}
\end{aligned}
\]

Based on fuzzy value space subset \((T_{11}, T_{22})\), we establish the following programming Model P3.

\[(P3)\] \[
\begin{aligned}
\max \ E(X) &= \sum_{k=1}^{2} \sigma_k \sum_{k=1}^{4} P(S_{ik})E(\sum_{j=1}^{2} \tilde{R}_{ik(j)}x_{ij}) \\
\text{s. t.} \quad & \begin{cases} 
\tilde{R}_{11}x_{11} + \tilde{R}_{12}x_{12} \geq \tilde{r}_{1}, \tilde{R}_{112}x_{11} + \tilde{R}_{121}x_{12} \geq \tilde{r}_{1}; \\
\tilde{R}_{212}x_{21} + \tilde{R}_{222}x_{22} \geq \tilde{r}_{2}; \\
x_{11} + x_{12} + x_{21} + x_{22} &= 1; \\
0.1 \leq x_{11}, x_{12}, x_{21}, x_{22} &\leq 0.5.
\end{cases}
\end{aligned}
\]

That is, \[
\begin{aligned}
\max \ E(X) &= \sigma_1 [0.426844x_{11} + 0.226795x_{12}] + \sigma_2 [0.680025x_{21} + 0.387992x_{22}]
\text{s. t.} \quad & \begin{cases} 
0.045x_{11} + 1.435x_{12} \geq 0.285, 0.98x_{11} + 0.275x_{12} \geq 0.285; \\
1.435x_{21} + 1.695x_{22} \geq 0.735; \\
x_{11} + x_{12} + x_{21} + x_{22} &= 1; \\
0.1 \leq x_{11}, x_{12}, x_{21}, x_{22} &\leq 0.5.
\end{cases}
\end{aligned}
\]

(3) By applying Matlab optimization toolbox we can easily solve the above three linear programming models \((P1)-(P3)\) regarding different weight vectors \(\sigma\) of two mental accounts. Finally, we obtain the optimal behavioral portfolio strategy \(X^* = (x_{11}^*, x_{12}^*, x_{21}^*, x_{22}^*)\), which is the optimal solver corresponding to the optimal model with maximum objective function value of the above three behavioral portfolio decision models. The optimal portfolio results are listed in the following Table 2.

From the Table 2, one can see that the optimal portfolio strategy changes with the different weight vector assigned to the mental accounts based on the investor’s risk attitude.
Table 2. Optimal behavioral portfolio strategy regarding different weight vector of two accounts.

| Weight vector of mental accounts | Maximum mean of fuzzy return for portfolio | $x_{11}^*$ | $x_{12}^*$ | $x_{21}^*$ | $x_{22}^*$ |
|----------------------------------|-------------------------------------------|------------|------------|------------|------------|
| $(0.1, 0.9)$                     | 0.3380                                     | 0.2372     | 0.1912     | 0.4716     | 0.1        |
| $(0.2, 0.8)$                     | 0.3165                                     | 0.2372     | 0.1912     | 0.4716     | 0.1        |
| $(0.3, 0.7)$                     | 0.2951                                     | 0.2372     | 0.1912     | 0.4716     | 0.1        |
| $(0.5, 0.5)$                     | 0.2521                                     | 0.2372     | 0.1912     | 0.4716     | 0.1        |
| $(0.7, 0.3)$                     | 0.2168                                     | 0.3173     | 0.1886     | 0.3941     | 0.1        |
| $(0.8, 0.2)$                     | 0.2039                                     | 0.3173     | 0.1886     | 0.3941     | 0.1        |
| $(0.9, 0.1)$                     | 0.1983                                     | 0.3638     | 0.1872     | 0.349      |            |

5. Conclusion

In this article, we consider the multi-account behavioral portfolio decision problem in fuzzy environment. We use the possibilistic mean to measure trapezoidal fuzzy number return of behavioral portfolio with multiple mental accounts. Furthermore, we present a new behavioral portfolio model with trapezoidal fuzzy number return which can maximize the expected return of the portfolio and ensure the fuzzy return of each mental account exceeding the given minimum fuzzy aspiration with a certain probability.

Acknowledgement

This work was supported by the Guangdong Basic and Applied Basic Research Foundation under Grant Nos. 2018A030313996 and 2018B030311004, 2021A1515011974.

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