TURBULENT SHEAR ACCELERATION

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ABSTRACT

We consider particle acceleration by large-scale incompressible turbulence with a length scale larger than the particle mean free path. We derive an ensemble-averaged transport equation of energetic charged particles from an extended transport equation that contains the shear acceleration. The ensemble-averaged transport equation describes particle acceleration by incompressible turbulence (turbulent shear acceleration). We find that for Kolmogorov turbulence, the turbulent shear acceleration becomes important on small scales. Moreover, using Monte Carlo simulations, we confirm that the ensemble-averaged transport equation describes the turbulent shear acceleration.

Key words: acceleration of particles – cosmic rays – ISM: supernova remnants – plasmas – turbulence

Online-only material: color figures

1. INTRODUCTION

Charged particles are accelerated to relativistic energies in many astrophysical objects. In addition, turbulence is also expected. In fact, strong turbulence is observed in recent two- and three-dimensional simulations for supernova remnants (SNRs; Giacalone & Jokipii 2007; Inoue et al. 2009; Guo et al. 2012; Caprioli & Spitkovsky 2013), pulsar wind nebulae (PWN; Komissarov & Lyubarsky 2004; Del Zanna et al. 2004; Porth et al. 2013), astrophysical jets (Aloy et al. 1999; Mizuta et al. 2010; López-Cámara et al. 2013), etc.

There are mainly two acceleration mechanisms by turbulence. One is due to wave–particle interactions, where the particle mean free path is comparable to the wavelength of electromagnetic fluctuations (e.g., Skilling 1975; Schlickeiser & Miller 1998). The other is due to large-scale fluctuations of plasma flows, where the particle mean free path is smaller than that on turbulent scales (e.g., Bykov & Toptygin 1993). Turbulence is generally divided by compressible and incompressible modes. Particle acceleration by large-scale compressible turbulence has been discussed in many papers (e.g., Bykov & Toptygin 1982; Ptuskin 1988; Jokipii & Lee 2010). However, particle acceleration by large-scale incompressible turbulence (turbulent shear acceleration) has not been investigated in detail, although Bykov & Toptygin (1983) have briefly discussed the turbulent shear acceleration.

Particle acceleration by a simple incompressible flow (shear flow) has already been investigated in many papers (e.g., Berezhko & Krymskii 1981; Earl et al. 1988; Webb 1989; Ostrowski 1990; Rieger & Duffy 2006). However, shear flows are potentially unstable to the Kelvin–Helmholtz instability and produce turbulence. Therefore, turbulent shear acceleration is expected to be important. In this Letter, we investigate turbulent shear acceleration by considering the ensemble average of an extended transport equation that includes particle acceleration by shear flows.

We first derive an ensemble-averaged transport equation in Section 2, and provide analytical solutions for simple cases in Section 3. We then perform Monte Carlo simulations in Section 4. Section 5 is devoted to the discussion.

2. DERIVATION OF THE ENSEMBLE-AVERAGED TRANSPORT EQUATION

In this section, we derive the ensemble-averaged transport equation of energetic particles. Propagation and acceleration of energetically charged particles in a plasma flow are described by a transport equation. Parker (1965) derived the transport equation, which includes spatial diffusion, convection, and adiabatic acceleration, after which his work was extended by several authors. For isotropic diffusion and a nonrelativistic plasma flow, the extended transport equation is given by Equation (4.5) of Webb (1989) and Equation (9) of Williams et al. (1993):

\[
\frac{\partial F}{\partial t} + U_i \frac{\partial F}{\partial x_i} - \frac{\partial}{\partial x_i} \left( \kappa \frac{\partial F}{\partial x_i} \right) - p \frac{\partial U_i}{\partial p} \frac{\partial F}{\partial U_i} + \frac{1}{p^2} \frac{\partial}{\partial p} \left( \kappa \frac{\partial F}{\partial p} \right) - \frac{D_{U_i}}{\partial t} \frac{\partial F}{\partial U_i} + \frac{1}{p^2} \frac{\partial}{\partial p} \left( \kappa \frac{\partial F}{\partial p} \right) = 0,
\]

where \( \Gamma \) is defined by

\[
\Gamma = \frac{1}{5} \left( \frac{\partial U_i}{\partial x_j} \frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j} \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{15} \frac{\partial U_i}{\partial x_i} \frac{\partial U_j}{\partial x_j}.
\]

\( F(p, x, t), x_i, U_i, \kappa(p) \) are the distribution function, position, plasma velocity, and spatial diffusion coefficient, respectively. \( v \) and \( p \) are the particle velocity and four momentum in the fluid rest frame, respectively. The spatial diffusion coefficient, \( \kappa \), is represented by \( \kappa(p) = \tau(p) v^2/3 \) for isotropic diffusion, where \( \tau(p) \) is the mean scattering time and \( v \) is the particle mean free path. The first four terms of Equation (1) are the same as the Parker equation and the others are additional terms. The fifth term describes the shear acceleration and the sixth term becomes important for \( v \sim U_i \).
In order to understand essential features of the turbulent shear acceleration, we consider incompressible turbulence \((\partial U_i/\partial x_i = 0)\) and do not take into account the spatial transport; that is, we consider the spatially averaged distribution function, \(V^{-1} \int_V F d^3x\), where \(V\) is the system volume that we consider. By integrating Equation (1), the extended transport equation can be reduced to

\[
\frac{\partial}{\partial t} \frac{1}{V} \int_V F d^3x - \frac{1}{p^2} \frac{\partial}{\partial p} \left( \frac{\tau p^4}{3} \frac{1}{V} \int_V \Gamma F d^3x \right) \]

\[-\frac{1}{p^2} \frac{\partial}{\partial p} \left( \frac{\tau p^4}{3} \frac{1}{V} \int_V DU_i DU_j \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j} F d^3x \right) + Q(p) = 0,
\]

where \(Q(p)\) is the particle flux passing through the surface of the integrated volume. As long as we consider a timescale smaller than \(V^{1/3}/\delta u_i\) and \(V^{2/3}/k\), we can neglect the particle flux, \(Q(p)\). In other words, we can neglect the escape of particles from the system once we consider a sufficiently large system.

In this Letter, we assume that the plasma velocity field, \(U_i(x, t)\), is static, random, statistically homogenous, and isotropic incompressible turbulence, that is, \(U_i = \delta u_i(x)\) and \(\langle \delta u_i \rangle = 0\), where \(\langle \ldots \rangle\) denotes ensemble average. The correlation function of the plasma velocity field is given by

\[
\langle \delta u_i(x) \delta u_j(x') \rangle = \int \frac{d^3k}{(2\pi)^3} K_{ij}(k) e^{i(k(x-x'))},
\]

and

\[
K_{ij}(k) = S(k) \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right),
\]

where \(k\) and \(S(k)\) are the wavenumber and spectrum of incompressible turbulence, respectively. As long as we consider only particle acceleration, we can assume the velocity field to be static when the fluctuating timescale, \(\tau\), is smaller than the variable timescale of fluid, \(T \approx (k \times \max(|\delta u, v_{ph}|))^{-1}\), where \(v_{ph}\) is a phase velocity. In this Letter, we consider \(\tau v < 1\) and \(v > \max(|\delta u, v_{ph}|)\), so that \(\tau/T \approx \tau k < 1\). Hence, we can assume a static velocity field in this Letter.

The distribution function of particles can also be divided by an ensemble-averaged component and a fluctuated one, that is, \(F = f + \delta f\) and \((F) = f\). The spatial average in Equation (3) can be interpreted as ensemble average because we consider a larger system size than the turbulent scale. Then, from Equation (3), the ensemble-averaged transport equation is represented by

\[
\frac{\partial f}{\partial t} - \frac{1}{p^2} \frac{\partial}{\partial p} \left( \frac{\tau p^4}{3} \left( \langle \Gamma \rangle + \langle U_j \delta u_i / \partial x_j \rangle \frac{\partial}{\partial x_i} \frac{1}{v^2} \frac{\partial}{\partial p} \right) \right) = 0,
\]

where we have assumed that distributions of \(\delta f\) and \(\delta u_i\) are symmetric about the mean values, \(f\) and 0, respectively, so that third moments are zero. From Equations (4)–(6), the ensemble-averaged transport equation can be represented by

\[
\frac{\partial f}{\partial t} - \frac{1}{p^2} \frac{\partial}{\partial p} \left( p^2 D_{TSA} \frac{\partial f}{\partial p} \right) = 0,
\]

where the momentum diffusion coefficient, \(D_{TSA}(p)\), is given by

\[
D_{TSA}(p) = \frac{2}{9} p^2 \tau(p) \int \frac{d^3k}{(2\pi)^3} S(k) k^2 \left( \frac{3}{5} + \frac{\langle |\delta u|^2|\rangle}{v^2} \right).
\]

Here we consider turbulence with a large length scale compared with the particle mean free path, \(\tau v\), so that the upper limit of the \(k\)-integral should be limited by \(\min(k_{\text{max}}, k_{\text{res}})\), where \(k_{\text{max}}\) is the maximum wavenumber of turbulence and \(k_{\text{res}} \approx (\tau v)^{-1}\). The momentum diffusion coefficient, \(D_{TSA}\), is dominated by small-scale turbulence when \(k^2 S(k)\) is an increasing function of \(k\). Therefore, the turbulent shear acceleration becomes important on the small scale for a Kolmogorov-like spectrum \((S(k) \propto k^{-11/3})\).

3. ANALYTICAL SOLUTION

In this section, we present specific expressions of the momentum diffusion coefficient and analytical solutions of the ensemble-averaged transport equation for simple velocity spectra. We especially focus on the turbulent shear acceleration of relativistic particles \((v \approx c)\) in nonrelativistic turbulence \((\langle \delta u^2 \rangle \ll c^2)\), so that we neglect the term \((\langle \delta u^2 \rangle/v^2)\) in Equation (8). Here we assume a functional form of the mean scattering time, \(\tau(p)\), to be \(\tau_0 p/\tau_0 p_0^2\), where \(p_0\) and \(\tau_0\) are the initial four momentum and the mean scattering time of particles with \(p_0\), respectively. To make the expression simple, hereafter the four momentum, time, and momentum diffusion coefficient are normalized by \(p_0\), \(\tau_0\), and \(\rho_0^2\tau_0^{-1}\), respectively. Normalized quantities are denoted with a tilde.

For a static monochromatic spectrum of incompressible turbulence, \(S(k)\) is given by

\[
S(k) = \frac{(\langle |\delta u|^2\rangle/2 \pi)^2}{4k_0^2} \delta(k - k_0).
\]

From Equations (8) and (9), the momentum diffusion coefficient is represented by

\[
\tilde{D}_{TSA} = \frac{(\tau_0 v k_0^2) (\langle |\delta u|^2\rangle/2 \pi)^2}{15 v^2 p^{2+\alpha}}.
\]

For a static Kolmogorov-like spectrum of incompressible turbulence, we assume that \(S(k)\) is given by

\[
S(k) = \frac{(\langle |\delta u|^2\rangle/2 \pi)^2}{6(k_{\text{max}}^{-2/3} - k^{-2/3})} k^{-11/3}\quad(\text{for } k_0 \leq k \leq k_{\text{max}}).
\]

Then, from Equations (8) and (11), the momentum diffusion coefficient is represented by

\[
\tilde{D}_{TSA} \approx \frac{(\tau_0 v k_0^2) (\langle |\delta u|^2\rangle/2 \pi)^2}{30 v^2 p^{2+\alpha}} \left( \frac{\min(k_{\text{max}}, k_{\text{res}})}{k_0} \right)^{4/3},
\]

where we have assumed \(k_0 \approx \min(k_{\text{max}}, k_{\text{res}})\). The factor, \(\min(k_{\text{max}}, k_{\text{res}})/k_0^{4/3}\), is expected to be large. Therefore, the Kolmogorov-like turbulent cascade enhances the turbulent shear acceleration. For \(k_{\text{res}} < k_{\text{max}}\), \(\tilde{D}_{TSA}\) is represented by

\[
\tilde{D}_{TSA} \approx \frac{(\tau_0 v k_0^2) (\langle |\delta u|^2\rangle/2 \pi)^2}{30 v^2 p^{2+\alpha/3}}.
\]

Therefore, the momentum diffusion coefficient can be represented by \(\tilde{D}_{TSA} = \tilde{D}_0 p^{2+\beta}\) for the above simple cases, where \(\beta = \alpha\) for the monochromatic spectrum and the Kolmogorov spectrum of the case \(k_{\text{res}} > k_{\text{max}}\) and \(\beta = -\alpha/3\) for the Kolmogorov spectrum of the case \(k_{\text{res}} < k_{\text{max}}\).

We next discuss analytical solutions of the ensemble-averaged transport equation. We assume that particles are uniformly
distributed in the three-dimensional space and injected at time, \( t = 0 \), with the four momentum, \( \vec{p} = 1 \). Then, the ensemble-averaged transport equation is represented by

\[
\frac{\partial f}{\partial t} - \frac{1}{\beta^2} \frac{\partial}{\partial \vec{p}} \left( \vec{p}^2 \frac{\partial f}{\partial \vec{p}} \right) = \frac{N}{4\pi} \delta(\tilde{r}) \delta(\vec{p} - 1),
\]

where \( N \) is the number of injected particles. If the momentum diffusion coefficient is represented by \( D_{\text{TSA}} = \tilde{D}_0 \vec{p}^{2-\beta} \), for \( \beta \neq 0 \), the solution is given by (Berezhko 1982; Rieger & Duffy 2006)

\[
f(\vec{p}, \tilde{r}) = \frac{N}{4\pi |\beta| \tilde{D}_0 \tilde{r}} \tilde{p}^{-(3+\beta)/2} \exp \left( -\frac{1 + \tilde{p}^\beta}{\beta^2 \tilde{D}_0 \tilde{r}} \right) \times I_{1+3/\beta} \left( \frac{\tilde{p}^{-\beta/2}}{\beta^2 \tilde{D}_0 \tilde{r}} \right),
\]

where \( I_n \) is the modified Bessel function of the first kind. The solution approaches \( \tilde{p}^{-3} \propto \tilde{p}^{-\beta} \) for \( \tilde{p} \gg 1 \). For \( \beta = 0 \), the solution is given by (Rieger & Duffy 2006)

\[
f(\vec{p}, \tilde{r}) = \frac{N}{(4\pi)^{3/2} \tilde{D}_0 \tilde{r}} \exp \left( -\frac{\ln(\tilde{p} + 3 \tilde{D}_0 \tilde{r})^2}{4 \tilde{D}_0 \tilde{r}} \right),
\]

and the evolution of the mean momentum, \( \tilde{p}_m(\tilde{r}) = N^{-1} \int f(\vec{p}, \tilde{r}) 4\pi \vec{p}^2 d\vec{p} \), is given by

\[
\tilde{p}_m(\tilde{r}) = \exp(4\tilde{D}_0 \tilde{r}).
\]

Note that solutions of Equations (15) and (16) are not valid for \( \tilde{r} \ll 1 \) and \( \tilde{p} \gg 1 \) because of causality.

4. MONTE CARLO SIMULATION

In order to confirm analytical solutions presented in the previous section, we perform test particle Monte Carlo simulations. Here we focus on static, statistically homogeneous, and isotropic incompressible turbulence, that is, the velocity field, \( \delta u_i(x) \), is divergence-free. Such a vector field is numerically constructed by a summation of many transverse waves (Giacalone & Jokipii 1999). Simulation particles are isotropically and elastically scattered in the local fluid frame and move in a straight line between each scattering. The mean scattering time is given by \( \tau = t_0(p/p_0)^2 \). We use \( 10^4 \) simulation particles with the initial four momentum \( p_0 = 10mc \) and 100 transverse waves in order to construct velocity fields, where \( m \) and \( c \) are the particle mass and the speed of light. The mean amplitude of velocity fluctuations is taken to be \( \langle \delta u^2 \rangle = (0.05c)^2 \). We set the maximum wavenumber to be \( t_0ck_{\text{max}} = 10^{-1/3} \) for the Kolmogorov spectrum.

We first discuss the results of the Monte Carlo simulations for the momentum-independent scattering, that is, \( \alpha = \beta = 0 \). Figure 1 shows the evolution of the mean four momentum for \( \alpha = 0 \) and \( t_0ck_0 = 10^{-1} \). Particles are accelerated and simulation results are in good agreement with the analytical solutions of Equations (10), (12), and (17). By comparing the growth rate of the mean momentum of simulation particles with Equation (17), we can obtain the momentum diffusion coefficient of Monte Carlo simulations.

Figure 2 shows the wavenumber dependence of the momentum diffusion coefficient, \( D_0 = D_{\text{TSA}}/p^2 \) for \( \alpha = 0 \). Simulation results are in good agreement with Equations (10) and (12) as long as \( t_0ck_0 < 1 \), but simulation results for the monochromatic spectrum deviate from Equation (10) at \( t_0ck_0 \gg 1 \). As already mentioned in Section 2, this is because our treatment is not valid when the particle mean free path is larger than the turbulent scale. Furthermore, we have confirmed that the Kolmogorov-like turbulent cascade (blue) enhances the turbulent shear acceleration.

Figure 3 shows the distribution function, \( \tilde{p} dN/d\tilde{p} \propto p^3 f(p, t) \), at \( t/t_0 = 10^5 \) for \( \alpha = 0 \) and \( t_0ck_0 = 10^{-1} \). The simulation results (histograms) are in excellent agreement with the analytical solutions of Equation (16) (solid lines) for the monochromatic (red) and Kolmogorov (blue) spectra.

Figure 4 shows the distribution function at \( t/t_0 = 5 \times 10^6 \) and \( 10^7 \) for the monochromatic spectrum with \( t_0ck_0 = 10^{-2} \) and the Bohm-like diffusion, that is, \( \alpha = \beta = 1 \). The simulation results (histograms) are in excellent agreement with the analytical solutions of Equation (15) (solid lines) except for above \( p/p_0 \sim 10^2 \). As mentioned above, the disagreement is due to \( \tau(p)uk_0 > 1 \) at \( p/p_0 > 10^2 \).
5. DISCUSSION

We first discuss another important effect of turbulence on the particle transport. Bykov & Toptygin (1993) show that turbulence enhances spatial diffusion. For strong turbulence, \( \kappa_{\text{turb}} \) becomes of the order of \( L_0 \sqrt{\langle \delta u^2 \rangle} \) (Bykov & Toptygin, 1993), where \( L_0 \) is the injection length scale of turbulence, so that spatial diffusion of particles with a small mean free path is dominated by turbulent diffusion and an energy-independent diffusion is realized. The ratio of the turbulent diffusion and the Bohm diffusion, \( \kappa_{\text{Bohm}} \), is given by

\[
\frac{\kappa_{\text{turb}}}{\kappa_{\text{Bohm}}} \sim 3 \times 10^6 \frac{p}{m_p c} \left( \frac{\sqrt{\langle \delta u^2 \rangle}}{c} \right) \left( \frac{B}{1 \mu G} \right) \left( \frac{L_0}{1 \text{ pc}} \right),
\]

where \( m_p \) and \( B \) are the proton mass and magnetic field, respectively. Therefore, turbulent diffusion of energetic particles could be important in SNRs, PWNe, astrophysical jets, etc.

From Equation (13), the acceleration timescale, \( \tau_{\text{acc}} = p^2 / D_{\text{TSA}} \), of the turbulent shear acceleration for the Kolmogorov spectrum of the case \( k_{\text{res}} < k_{\text{max}} \) is represented by

\[
\tau_{\text{acc}} = \frac{30}{(\tau(p) e k_0)^{2/3} (\langle \delta u^2 \rangle)^{-1}} \frac{v_0}{(p/m_p c)} \left( \frac{\langle \delta u^2 \rangle}{c^2} \right)^{-1} \left( \frac{B}{1 \mu G} \right)^{-1/3} \left( \frac{L_0}{1 \text{ pc}} \right)^{2/3},
\]

where we have assumed \( L_0 = 2\pi / k_0 \) and the Bohm diffusion, \( \tau(p) = p/(e B) \), in the last equation. Therefore, particles can be accelerated to relativistic energies by large-scale turbulence in many astrophysical objects. In addition, if particles are initially accelerated at the shock, large-scale turbulence can change energy spectra of the accelerated particles in the shock downstream region.

Next, we compare the turbulent shear acceleration and particle acceleration by small-scale incompressible turbulence, that is, the second-order acceleration by Alfvén waves (Skilling, 1975). The momentum diffusion coefficient of the second-order acceleration by Alfvén waves is given by \( D_{\lambda} \sim p^2 v_A^2/(9e) \), where \( v_A \) is the Alfvén velocity. The ratio of the turbulent shear acceleration and the second-order acceleration by Alfvén waves is given by

\[
\frac{D_{\text{TSA}}}{D_{\lambda}} \sim \frac{\langle \delta u^2 \rangle}{v_A^2} \left( \frac{\tau v}{L_0} \right)^{2/3},
\]

where we have adopted Equation (13) as \( D_{\text{TSA}} \). Therefore, the turbulent shear acceleration could be more efficient than the second-order acceleration by Alfvén waves for super-Alfvénic turbulence \( (\sqrt{\langle \delta u^2 \rangle} > v_A (\tau v/L_0)^{1/3}) \). In other words, the turbulent shear acceleration becomes important when there are strong magnetic field fluctuations \( (\delta B / B_0 > 1) \) because the plasma velocity fluctuation by Alfvén waves, \( \delta u \), is represented by \( \delta u = v_A (\delta B / B_0) \), where \( \delta B \) and \( B_0 \) are the fluctuated and mean magnetic fields, respectively. Such a situation is expected to be realized in the downstream region of a high Alfvén Mach number shock (Giacalone & Jokipii, 2007; Inoue et al., 2009).

We have considered only isotropic diffusion and nonrelativistic turbulence in this Letter. Spatial diffusion is generally anisotropic because of the magnetic field. Isotropic diffusion is realized when magnetic field fluctuations with lengthscales comparable to the particle mean free path are large \( (\delta B / B_0 > 1) \) (e.g., Giacalone & Jokipii, 1999). Therefore, as discussed above, the turbulent shear acceleration is important when isotropic diffusion is realized. Simple extensions to anisotropic diffusion and relativistic turbulence are straightforward because extensions of Equation (1) have already been provided by Webb (1989) and Williams et al. (1993). This calculation will be addressed in future work.

6. SUMMARY

In this Letter, we have derived a particle transport equation averaged over random plasma flows in order to understand particle acceleration in incompressible turbulence with a larger length scale than the particle mean free path. We have considered the ensemble average of the extended transport equation provided by Webb (1989) and Williams et al. (1993). This is a simple extension of previous work that considered the ensemble average of the transport equation provided by Parker (1965). We have found that the turbulent shear acceleration by
incompressible turbulence becomes important on small scales for Kolmogorov-like turbulence. Moreover, we have performed Monte Carlo simulations and confirmed the turbulent shear acceleration. Recent simulations show that turbulence is produced in many astrophysical objects, so that turbulent diffusion and turbulent acceleration are expected to be important.

We thank T. Inoue and R. Yamazaki for useful comments about turbulence and simulation. This work is supported in part by a grant-in-aid from the Ministry of Education, Culture, Sports, Science, and Technology (MEXT) of Japan, No. 24·8344.

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