New Trajectory Control Directional MWD Accuracy Prediction and Wellbore Positioning Method

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Abstract: The deviation control is to restrict the drilling direction of the bit from time to time. The drilling direction is of course depending on the direction of the resultant forces acting on the bit. What is the relationship between these directions? Are there any other influential factors? Answers to such questions, different points of view were subjected to analysis.

Key words: Wellbore Trajectory, Bit Trajectory, Actual/Planned Path, Measurement While Drilling (MWD), Logging While Drilling (LWD), Position Uncertainty, Error Accuracy Prediction, Weighting Function

INTRODUCTION

In the rectangular coordinate system shown in Fig. 1, the side forces \( R_P \) and \( R_Q \) are acting along X-axis and Y-axis respectively. The resultant force \( R \) is combined by three mutually perpendicular components; they are \( R_P, R_Q \) and the weight on bit \( P_B \).

\[ V = \cos(I_P)\cos(A_P)(N_b-N*) + \sin(I_p)\cos(A_P)(E_b-E*) - \sin(A_P)(TVD_b-TVD*) \]
\[ H = \cos(I_P)(E_b-E*) - \sin(I_P)(N_b-N*) \]
\[ A = A_b - A* \]
\[ I = I_b - I* \]
\[ \Delta V = 100 \frac{V^n-V^{n+1}}{\Delta L^n} \]
\[ \Delta H = 100 \frac{H^n-H^{n+1}}{\Delta L^n} \]
\[ (A_p)_n = \Delta A_p^n \]
\[ (I_p)_n = \Delta I_p^n \]

The superscript \((n)\) in the definitions of each relative change is refer to the respective values during the prior computing of hole deviation; \((n-1)\) refers to values at planned hole drilled between the two foregoing hole deviation computations. The superscript \( (\cdot) \) defines the measured data and the subscript \( (b) \) refers to current well bore total depth. Thus \( \Delta L \) is \( (MD^*)^{(n)} \) which is preferably somewhat short. Performing two successive coordinate axis rotations derive the equations for \( V \) and \( H \) the first rotation is by the deviation angle \( \theta^* \) about the TVD axis. The aforementioned vector is orthogonal to the planned path at \( MD^* \), then the required \( \Delta TVD^* \) equals zero; i.e. Respective to hole deviation, a preferable method by which to
mathematically represent the entire planned drill path is to parametrically define each Cartesian coordinate and hole inclination and azimuth, in terms of measured depth. That is the planned path is designed and then represented as follows:

\[ N_{\text{MD}} = P_{\text{1,MD}}; \quad E_{\text{MD}} = P_{\text{2,MD}}; \quad \text{TVD}_{\text{MD}} = P_{\text{3,MD}}; \quad \Phi_{\text{MD}} = P_{\text{4,MD}}; \quad \Theta_{\text{MD}} = P_{\text{5,MD}} \]

The rate of change in lineal relationship between the planned and actual well paths is assumed to remain the same over small distances; this assumption is often completely valid. As the hole is drilled, it is necessary to determine where on the plan one would prefer the wellbore to exist. The linear distance between the current bottom hole location and a point on the planned path is computed with the 3D distance formulas. This is generally represented by Eq.1

\[ d_{\text{p}}(N_x,E_x,\text{TVD}_x,\Phi_x,\Theta_x) = \sqrt{(N_x - N_{\text{MD}}) + (E_x - E_{\text{MD}}) + (\text{TVD}_x - \text{TVD}_{\text{MD}})} \]  

Let \( \text{MD}^* \) represent the measured depth along the planned path, whose respective Cartesian coordinates minimize the distance computed with Eq.1. Therefore, \( \text{MD}^* \) found by taking the derivative of Eq.1 with respect to \( \text{MD} \) and setting the result equal zero.

\[ \frac{d \text{MD}}{d \text{MD}} = \frac{(N_{\text{MD}} - N_x) + (E_{\text{MD}} - E_x) + (\text{TVD}_{\text{MD}} - \text{TVD}_x)}{\sqrt{(N_x - N_{\text{MD}}) + (E_x - E_{\text{MD}}) + (\text{TVD}_x - \text{TVD}_{\text{MD}})}} \]  

The measured depth that sets the right hand side of Eq. 2 equal zero is \( \text{MD}^* \); therefore, the denominator may be ignored and \( \text{MD}^* \) is found by solving Eq. 2.

\[ d_{\text{MD}} = (N_{\text{MD}} - N_x) + (E_{\text{MD}} - E_x) + (\text{TVD}_{\text{MD}} - \text{TVD}_x) \]  

**Well Bore Position Uncertainty:** In 3D, the confidence region is most often depicted as ellipsoids because ellipsoids are the constant value contours of the 3D Gaussian probability density function. The technique used is based on the generalization of the linear regression model: \( \bar{y} = X\hat{\beta} + \bar{\varepsilon} \); where: \( \bar{y} \) is an \( (m \times 1) \) vector by one vector of observations. \( \hat{\beta} \) is a \( (p \times 1) \) vector by one vector of model parameters. \( X \) is an \( (m \times p) \) matrix of regression variables, which establishes a linear relationship between the observations and the model parameters. \( \bar{\varepsilon} \) is an \( (m \times 1) \) vector by one vector of random errors that characterizes the uncertainty observation. \( (m \times 1) \) is the number of columns in the vector \( \bar{y} \). \( (n \times 1) \) is the north component of a position vector. \( (p \times 1) \) is the probability density. Assuming \( \bar{\varepsilon} \) is zero mean and has a Gaussian probability distribution, the probability density function for the random variable \( \bar{y} = X\hat{\beta} + \bar{\varepsilon} \) is:

\[ p(\bar{y}; \hat{\beta}) = \frac{\exp \left( -\frac{1}{2}(\bar{y} - X\hat{\beta})^T \Sigma^{-1} (\bar{y} - X\hat{\beta}) \right)}{2\pi^{n/2} \sqrt{\det(\Sigma)}} \]  

where: \( \Sigma \) is the covariance matrix for the random vector \( \bar{\varepsilon} \). Maximization of Eq.4 with respect to \( \hat{\beta} \) yields the following estimate \( \hat{\beta} \) and its covariance \( C_{\beta} \).

\[ \hat{\beta} = (X^T X)^{-1} X^T \bar{y} \]

\[ C_{\beta} = (X^T X)^{-1} \]

X is the transpose of X. Assume we have \( (k \times 1) \) measurement can be written in the following form:

\[ \bar{r}_i = \bar{r}_i + \delta \bar{r}_i \]

the position vector of the \( i \)th measurement is:

\[ \bar{r}_i = [n_i, e_i, d_i] \]

is: \( \delta \bar{r}_i = [\delta n_i, \delta e_i, \delta d_i] \) and the error in the \( i \)th measurement:

\[ \delta \bar{r}_i = [\delta n_i, \delta e_i, \delta d_i] \]

A sequence of these position measurements can be written in the following form:

\[ [\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_k] = [n_1, e_1, d_1; n_2, e_2, d_2; \ldots; n_k, e_k, d_k] \]

which each I is a \( (3 \times 3) \) identity matrix and \( 1 \leq j \leq k \).

The covariance matrix, \( C_{\beta} \), can be written as:

\[ C_{\beta} = \{ \delta \bar{r}_i \bar{r}_j \} = \begin{pmatrix} \langle \delta n_1 \delta n_1 \rangle & \langle \delta n_1 \delta e_1 \rangle & \langle \delta n_1 \delta d_1 \rangle \\ \langle \delta e_1 \delta n_1 \rangle & \langle \delta e_1 \delta e_1 \rangle & \langle \delta e_1 \delta d_1 \rangle \\ \langle \delta d_1 \delta n_1 \rangle & \langle \delta d_1 \delta e_1 \rangle & \langle \delta d_1 \delta d_1 \rangle \end{pmatrix} \]

where, \( (d, e, n) \) is the vertical component of a position vector. \( (e, n) \) is the east component of a position vector. \( (i, j) \) is an integer between 1 and \( k \) that designate the \( i \)th member of a set of \( (k \) measurement. \( (j, k) \) is an integer between 1 and \( k \) that designate the \( i \)th member of a set of \( k \) measurements. \( (k) \) is the number of position measurements included in the \( i \)th estimate. \( (i) \) is a tag used to designate the true bottom hole location. \( \bar{r}_i \) is the \( i \)th measurement of position vector. \( \delta \bar{r}_i \) is the uncertainty in the \( i \)th
measured position vector. Each term of the form
\[
\left( \delta \hat{r}_y \delta \hat{r}_y^T \right)
\]
is a (3*3) covariance matrix defines a 3D Gaussian distribution with a probability density function in the following form:
\[
p(\hat{r}) = \frac{1}{2\pi^{3/2}\sqrt{|C_{ij}|}} e^{-\frac{1}{2} (\hat{r} - \hat{r}_i)^T C_{ij}^{-1} (\hat{r} - \hat{r}_i)}
\]
and because the covariance matrices, C_{ij}, are diagonal, the probability density function reduces to:
\[
p(\hat{r}) = \frac{1}{2\pi^{3/2}\sqrt{|C_{ii}|}} e^{-\frac{1}{2} \left( \frac{(x_i - x)^2}{C_{xx}} + \frac{(y_i - y)^2}{C_{yy}} + \frac{(z_i - z)^2}{C_{zz}} \right)}
\]
where, \((x)\) is the element of the position covariance of matrix in the x-coordinates. \((y)\) is the element of the position covariance of matrix in the y-coordinates. \((z)\) is the element of the position covariance of matrix in the z-coordinates. The constant value contours of Eq.5 are family of ellipsoids defined by the equation of the quadratic expression in the exponent to a constant. For each ellipsoid, the length of the north, east and down semi-major axes are: \(s \sqrt{C_{xx}}, s \sqrt{C_{yy}}, s \sqrt{C_{zz}}\).

The mathematical basis of the HDC technique can be summarized by restating the basic formula in the following format:
\[
HDC = [I_n^T (C_y)_n I_n]^{-1} [I_n^T (C_x)_n I_n]^{-1} \times \hat{y}
\]
The covariance matrix of the HDC is given as:
\[
C_{HDC} = (I_n^T C_n^{1/2} I_n)^{-1}
\]

Error Accuracy Prediction: The central limit theorem\(^1\) ensures that the statistical distribution of each \(\delta \hat{r}_i\) will be approximately Gaussian and independent of the distribution of the individual error budget, Fig. 2 and 3. The following assumptions are implicit in the error models and mathematics presented:

* Errors in calculated well position are caused exclusively by the presence of measurement errors of well bore survey station.
* Wellbore survey station are three element measurement vectors, the elements being a longhole depth (D), inclination (I) and azimuth (A). The propagation mathematics also requires a tool angle (\(\alpha\)) at each station.
* Errors from different error sources are statistically independent.

* There is a linear relationship between the size of each measurement error and the corresponding change in calculated well position.
* The combined effect on calculated well position of any number of survey stations is equal to the vector sum of their individual effects.
* No restrictive assumptions are made about the statistical distribution of measurement errors.

Fig. 2: Vector Error at Point of Interest

Fig. 3: The Final Section of the Well Showing Planned/Actual Wellbore Position and the Tool Face Angle Error

or the best estimate of position uncertainty it is tempting to differentiate minutely among tools type and models, summing configurations, bottom hole assembly (BHA) design, geographical location and several other variables. While justifiable on technical ground, such an approach is impractical for the daily work of the well planner.

The Error Propagation Mathematical Model: The method of position uncertainty calculation admits a number of variations, in that selection of the same set of conventions which always yield the same results. Recall and evaluate the vector error due to the presence of
error source \( (i) \) at the station \( k \), which is the sum of the effect of the error on the preceding and following survey displacement yield:

\[
\Delta r_j = \frac{D_j - D_{j-1}}{2} \left[ \sin I_{j-1} \cos A_{j-1} + \sin I_j \cos A_j \sin I_{j-1} \sin A_{j-1} + \sin I_j \sin A_j \cos I_{j-1} + \cos I_j \right]
\]  
(8)

the two differentials in the parentheses in Eq.8 may then be expressed as:

\[
\frac{d\Delta r_j}{dp_k} = \left[ \frac{d\Delta r_j}{dD_k} + \frac{d\Delta r_j}{dI_k} + \frac{d\Delta r_j}{dA_k} \right]
\]  
(9)

\[
\frac{d\Delta r_j}{dD_k} = \frac{1}{2} \left[ \sin I_{j-1} \cos A_{j-1} + \sin I_j \cos A_j \sin I_{j-1} \sin A_{j-1} + \sin I_j \sin A_j \cos I_{j-1} + \cos I_j \right]
\]  
(10)

\[
\frac{d\Delta r_j}{dI_k} = \frac{1}{2} \left[ -\sin I_{j-1} \cos A_{j-1} + \sin I_j \cos A_j \sin I_{j-1} \sin A_{j-1} + \sin I_j \sin A_j \cos I_{j-1} - \cos I_j \right]
\]  
(11)

\[
\frac{d\Delta r_j}{dA_k} = \frac{1}{2} \left[ \frac{(D_j - D_{j-1}) \cos I_k \cos A_k}{(D_j - D_{j-1}) \sin I_k} \right]
\]  
(12)

for the purpose of computation the error summation terminated at the survey station of interest the vector errors at this station are therefore given by:

\[
\varepsilon_{i,k}^* = \sigma_{i,d} \cdot \frac{d\Delta r_k}{dp_k} \cdot \frac{d\varepsilon_i}{dE_i}
\]

where, \( (\varepsilon^*) \) is the 1s.d vector error of the station of interest.

Writing \( \Delta r_k \) for the displacement between survey station (k-1) and (k), it may express the 1s.d error due to the presence of the \( ith \) error at the \( kth \) survey station in the \( lth \) survey leg as the sum of the effect on preceding and following calculated displacement.

\[
\varepsilon_{i,k} = \sigma_{i,d} \left( \frac{d\Delta r_k}{dp_k} + \frac{d\Delta r_{k+1}}{dp_{k+1}} \right) \frac{d\varepsilon_i}{dE_i}
\]

where; \( (\varepsilon) \) is the 1s.d vector error at an intermediate station. \( \sigma \) is the standard deviation of error vector. \( (p) \) is the wellbore position vector. \( (p) \) is the survey measurement vector \( (D, I, A) \). \( E \) is the particular value of a survey error. \( \frac{d\varepsilon_i}{dE_i} \) describes how is the changes in the measurement vector affect the calculated well position.

**Weighting Functions for Sensor Errors:** The weighting functions for constant and \( B_h \)-dependent magnetic declination errors are:

\[
\frac{\partial \varepsilon}{\partial \varepsilon_{i,x}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \frac{\partial \varepsilon}{\partial \varepsilon_{i,y}} = \begin{bmatrix} 0 \\ 0 \\ 1 / B \cos \Theta \end{bmatrix}
\]

for BHA sag and direction-dependent axial magnetic interference they are:

\[
\frac{\partial \varepsilon}{\partial \varepsilon_{i,x}} = \begin{bmatrix} \sin \theta \\ 0 \end{bmatrix}, \quad \frac{\partial \varepsilon}{\partial \varepsilon_{i,y}} = \begin{bmatrix} 0 \\ \sin \theta \sin A \end{bmatrix}
\]

and for reference, scale and stretch type depth error they are:

\[
\frac{\partial \varepsilon}{\partial \varepsilon_{i,x}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \frac{\partial \varepsilon}{\partial \varepsilon_{i,y}} = \begin{bmatrix} D \\ 0 \end{bmatrix}, \quad \frac{\partial \varepsilon}{\partial \varepsilon_{i,z}} = \begin{bmatrix} D - D_i \\ 0 \end{bmatrix}
\]

where: \( (B) \) is the magnetic declination, nT. \( \Theta \) is the magnetic dip angle, deg.

Tool axis and tool angle are defined in Fig. 2. There are 12 sensor error sources and each requires its own weight function. These are obtained by differentiating the standard navigation equations for inclination and azimuth:

\[
I = \cos^{-1} \left( \frac{G}{\sqrt{G_x^2 + G_y^2 + G_z^2}} \right)
\]  
(13)

\[
A_n = \tan^{-1} \left( \frac{G_y B_z - G_z B_y}{B_z G_x^2 + G_y G_z^2 + G_z^2} \right)
\]  
(14)

and making use of the inverse relations:

\[
G_x = -G \sin I \sin \alpha \quad G_y = -G \sin I \cos \alpha \quad G_z = G \cos I
\]

\[
B_z = B \cos \Theta \cos \phi \cos A \sin \alpha - B \sin \Theta \sin I \sin \alpha + B \cos \Theta \sin A \cos \alpha
\]

\[
B_y = B \cos \Theta \cos \phi \sin A \cos \alpha - B \sin \Theta \sin I \cos \alpha - B \cos \Theta \sin A \sin \alpha
\]

\[
B_z = B \cos \Theta \phi \cos A \cos \alpha + B \sin \Theta \sin I \sin \alpha
\]

\[
B_z = B \cos \Theta \phi \sin A \sin \alpha
\]

**Effect of Axial Interference Correction:** Detailed of the interference corrections differ from method to method, but it is reasonable to characterize them all. From Eq. 15 and ignoring \( B_z \) measurement; then
\( [B \cos \Theta - \hat{B} \cos \Theta] + [B \sin \Theta - \hat{B} \sin \Theta] = \text{MINIMUM} \)

where \( \hat{B} \) and \( \hat{\Theta} \) are the estimated values of total field strength and dip angle respectively. Solving these three equations for azimuth leads to:

\[
P \sin A_n + Q \cos A_n + R \sin A_n \cos A_n = 0 \quad (16)
\]

where, \( P = (B_1 \sin \alpha + B_2 \cos \alpha) \cos I + \hat{B} \sin \hat{\Theta} \sin I \cos I \)

\( Q = -(B_1 \cos \alpha - B_2 \sin \alpha) : R = \hat{B} \cos \hat{\Theta} \sin^2 I \). The sensitive of computed azimuth to error in the sensor measurement are found by differentiating Eq.16 with respect to \( \hat{B} \) and \( \hat{\Theta} \). The misalignment error modeled by William as two uncorrelated errors corresponding to the X-axis and Y-axis of the associated inclination and azimuth error lead directly to the following weighting function:

\[
\frac{\partial p}{\partial e_x} = \begin{bmatrix} 0 \\ \sin \alpha \\ -\cos \alpha / \sin I \end{bmatrix} \quad \frac{\partial p}{\partial e_y} = \begin{bmatrix} 0 \\ \cos \alpha \\ \sin \alpha / \sin I \end{bmatrix}
\]

**Summation of Errors:** The contribution to survey station uncertainty from randomly propagation error source \((i)\) over survey leg \((l)\) (not containing the station of interest is:

\[
[C]_{i,l}^{\text{rand}} = \sum_{k=1}^{k_i} (e_{i,l,k})^T (e_{i,l,k})
\]

and the total contribution over all survey legs is

\[
[C]_{i,l}^{\text{rand}} = \sum_{j=1}^{l-1} [C]_{j,l}^{\text{rand}} + \sum_{k=1}^{k_i} \sum_{j=1}^{l-1} (e_{i,l,k})^T (e_{i,l,k}) + (e_{i,l,k}^*)^T (e_{i,l,k}^*)
\]

The contribution to survey station uncertainty from a systematic propagation error \((i)\) over survey leg \((l)\) is:

\[
[C]_{i,l}^{\text{sys}} = \sum_{j=1}^{l-1} [C]_{j,l}^{\text{sys}} + \sum_{k=1}^{k_i} (e_{i,l,k} + e_{i,l,k}^*) \left( \sum_{k=1}^{k_i} (e_{i,l,k} + e_{i,l,k}^*) \right)^T
\]

Each of these error types is systematic among all stations in a well. The individual errors therefore are summed to give a total vector error from slot to station:

\[
E_{i,l,k} = \sum_{j=1}^{l-1} \left( \sum_{k=1}^{k_i} (e_{i,l,k} + e_{i,l,k}^*) \right) + \sum_{j=1}^{l-1} \left( \sum_{k=1}^{k_i} (e_{i,l,k} + e_{i,l,k}^*) \right)
\]

the total contribution to the uncertainty at survey station \(K\) is:

\[
[C]_{i,l,k}^{\text{well}} = E_{i,l,k} \cdot E_{i,l,k}^T
\]

where: \((E)\) is the sum of vector errors from slot to station of interest.

The total position covariance at survey station \((K)\) is the sum of the contributions from all the types of error source:

\[
[C]_{K}^{\text{sur}} = \sum_{i=1}^{N} [C]_{i,l,K}^{\text{rand}} + \sum_{i=1}^{N} [C]_{i,l,K}^{\text{sys}} + \sum_{i=1}^{N} [C]_{i,l,K}^{\text{well}}
\]

where the superscript \((\text{sur})\) indicates the uncertainty is defined at a survey station.

Error vectors due to bias error are given by:

\[
m_{i,l,k} = \mu_i \left( \frac{d\Delta r_i}{dp_l} + \frac{d\Delta r_{s,k}}{dp_l} \right) \frac{dp_l}{\partial e_i} \quad m_{i,l,k}^{*} = \mu_i \left( \frac{d\Delta r_i}{dp_l} \frac{dp_l}{\partial e_i} \right)
\]

where, \((m)\) is the bias vector error at an intermediate station. \((m^*)\) is the bias vector error at the station of interest.

The total survey position bias at survey station \((K, M_K^{\text{sur}})\) is the sum of individual bias vectors taken over all error source \((i)\), legs \((l)\) and station \((k)\):

\[
M_K^{\text{sur}} = \sum_{i=1}^{N} \left( \sum_{l=1}^{L} \sum_{k=1}^{K_i} m_{i,l,k}^{*} + \sum_{l=1}^{L} \sum_{k=1}^{K_i} m_{i,l,k} \right)
\]

where, \((W_{i,l,k})\) is the factor relating error magnitude to depth measurement uncertainty. \((v_k)\) is the along-hole unit vector at station \(K\).

Fig. 4 illustrates these results.
When calculating the uncertainty in the relative position between two surveys stations \((K_A, K_B)\), the uncertainty is given by:

\[
\begin{align*}
[C_k]_{ij} &= \sum_{n=1}^{12}\sum_{k=1}^{12}\sum_{k=1}^{12}\left(\rho\left(e_{n,k}^{T}e_{n,k}^{T}\right)\sigma_{k}\right),
\end{align*}
\]

the relative survey bias is simply:

\[
\begin{align*}
M_{K_A}^\text{sur} - M_{K_B}^\text{sur}
\end{align*}
\]

The uncertainty in this position error is expressed in the form of a covariance matrix:

\[
[C_k] = \begin{pmatrix}
\delta \hat{r}_h^T & \delta \hat{r}_h^T
\end{pmatrix}
\]

The results derived above are in an Earth-Referenced frame (north, east, vertical, subscript \((\text{nev})\)). The transformation of the covariance matrices and bias vector into the more intuitive borehole referenced frame (high side, lateral hole, subscript \((\text{hla})\)) is straightforward: 

\[
\begin{align*}
[b_h] = M_{\text{hla}}[T] = M_{\text{nev}}[T] = [T][C_k][C_k^T]\quad \text{etc.}
\end{align*}
\]

[T] is a rotation matrix. The uncertainties and correlations in the principal borehole directions are obtained from:

\[
\sigma_{H} = \sqrt{[C_{\text{hla}}][I,I]} \quad \text{etc.} \quad \rho_{H} = \frac{[C_{\text{hla}}][I,G]}{\sigma_{H}\sigma_{L}} \quad \text{etc.}
\]

The error models for basic interference-correction MWD have been applied to the standard well profiles to generate position uncertainties in each well. The results of several combinations are tabulated in Table 1 and 2.

### Table 1: Standard Well Profile

| MD (m) | Inc (deg) | Azi (deg) | North (m) | East (m) | TVD (m) | VS (m) | DLS °/30m |
|-------|-----------|-----------|-----------|----------|---------|--------|-----------|
| 0     | 0         | 0         | 0         | 0        | 0       | 0      | 0         |
| 1200  | 0         | 0         | 0         | 0        | 0       | 0      | 0         |
| 2100  | 60        | 75        | 111.22    | 415.08   | 1944.29 | 429.79 | 2         |
| 5100  | 60        | 75        | 783.65    | 2924.62  | 3444.29 | 3027.79| 0         |
| 5400  | 90        | 75        | 857.8     | 3201.34  | 3521.06 | 3134.27| 3         |
| 5850  | 90        | 75        | 1530.73   | 5712.75  | 3521.06 | 5914.27| 0         |
| 609.6 | 0         | 0         | 0         | 0        | 0       | 0      | 0         |
| 1079.28 | 32    | 2         | 435.4     | 519.12   | 1072.32 | 411.75 | 2         |
| 1524 | 32        | 2         | 1176.48   | 41.08    | 1434.2  | 1193.06| 0         |
| 1684.185 | 32     | 2         | 1435.37   | 20.23    | 1570.91 | 1383.12| 3         |
| 1844.37 | 32    | 62        | 1619.99   | 318.22   | 1707.62 | 1626.43| 3         |
| 2004.554 | 32   | 92        | 1680.89   | 582      | 2013.23 | 1777.82| 0         |
| 2164.74 | 32   | 122       | 1601.74   | 840.88   | 2062.06 | 1796.7 | 3         |
| 2862.0263 | 62  | 220       | 364.88    | 700      | 2519.25 | 591.63 | 3         |
| 3810 | 62        | 220       | -1692.7   | -1026.15 | 2991.923 | -1948.01| 0         |
| 609.6 | 0         | 0         | 0         | 0        | 0       | 0      | 0         |
| 1100  | 50        | 0         | 245.6     | 0        | 1026.69 | 198.7  | 2.5       |
| 1700  | 50        | 0         | 705.23    | 0        | 1412.37 | 570.54 | 0         |
| 2450  | 0         | 0         | 1012.23   | 0        | 2070.73 | 818.91 | 2         |
| 2850  | 0         | 0         | 1012.23   | 0        | 2470.73 | 818.91 | 0         |
| 3030  | 90        | 283       | 1038.01   | -111.65  | 2585.32 | 905.39 | 15        |
| 3430  | 90        | 283       | 1127.99   | -501.4   | 2585.32 | 1207.28| 0         |
| 3730  | 110       | 193       | 996.08    | -727.87  | 2520    | 1197.85| 9         |
| 4030  | 110       | 193       | 721.4     | -791.28  | 2417.4  | 1069.86| 0         |
Table 2: Calculated position uncertainties (1 s.d) Uncertainties A Long-Borehole Axes

| Well No. | Depth interval (m) | Model | Option  | $\sigma_H$ (m) | $\sigma_L$ (m) | $\sigma_A$ (m) |
|----------|-------------------|-------|---------|---------------|---------------|---------------|
| 1        | 0 to 2500         | Basic | S, sym  | 20.116        | 84.342        | 8.626         |
| 2        | 0 to 2500         | Ax-int| S, sym  | 20.116        | 196.390       | 8.626         |
| 3        | 0 to 3800         | Basic | S, sym  | 16.185        | 29.551        | 10.057        |
| 4        | 0 to 3800         | Basic | D, sym  | 16.185        | 29.551        | 9.080         |
| 5        | 0 to 3800         | Basic | S, bias | 15.710        | 27.288        | 8.526         |
| 6        | 0 to 3800         | Basic | D, bias | 15.710        | 27.288        | 8.419         |
| 7        | (1) 0 to 1380     | Basic | S, sym  | 2.013         | 3.703         | 0.919         |
|          |                   |       |         |               |               |               |
|          | (2) 1410 to 3000  | Ax-ani| S, sym  | 3.239         | 3.646         | 7.890         |
|          | (3) 3030 to 4030  | basic | S, sym  | 5.604         | 9.594         | 9.594         |

Correlation Between Borehole Axes

| Well No. | Depth interval (m) | Model | Option  | $\rho_{HH}$ | $\rho_{HL}$ | $\rho_{LA}$ |
|----------|-------------------|-------|---------|-------------|-------------|-------------|
| 1        | 0 to 2500         | Basic | S, sym  | -0.016      | +0.676      | -0.004      |
| 2        | 0 to 2500         | Ax-int| S, sym  | -0.005      | +0.676      | -0.005      |
| 3        | 0 to 3800         | Basic | S, sym  | +0.030      | -0.613      | +0.049      |
| 4        | 0 to 3800         | Basic | D, sym  | +0.030      | -0.429      | +0.073      |
| 5        | 0 to 3500         | Basic | S, bias | +0.050      | -0.607      | +0.145      |
| 6        | 0 to 3800         | Basic | D, bias | +0.050      | -0.574      | +0.148      |
| 7        | (1) 0 to 1380     | Basic | S, sym  | -0.007      | 0.633       | -0.006      |
|          |                   |       |         |             |             |             |
|          | (2) 1410 to 3000  | Ax-ani| S, sym  | -0.172      | 0.633       | -0.665      |
|          | (3) 3030 to 4030  | basic | S, sym  | -0.180      | -0.590      | +0.302      |

Survey Bias A Long- Borehole Axis

| Well No. | Depth interval (m) | Model | Option  | $b_H$ (m) | $b_L$ (m) | $b_A$ (m) |
|----------|-------------------|-------|---------|-----------|-----------|-----------|
| 1        | 0 to 2500         | Basic | S, sym  |           |           |           |
| 2        | 0 to 2500         | Ax-int| S, sym  |           |           |           |
| 3        | 0 to 3800         | Basic | S, sym  |           |           |           |
| 4        | 0 to 3800         | Basic | D, sym  |           |           |           |
| 5        | 0 to 3800         | Basic | S, bias | -6.788    | -12.4117  | +11.698   |
| 6        | 0 to 3800         | Basic | D, bias | -6.788    | -12.411   | -4.758    |
| 7        | (1) 0 to 1380     | Basic | S, sym  |           |           |           |
|          |                   |       |         |           |           |           |
|          | (2) 1410 to 3000  | Ax-ani| S, sym  |           |           |           |
|          | (3) 3030 to 4030  | basic | S, sym  |           |           |           |

Key to error model
- basic: Basic MWD
- Ax-int: Basic MWD with Axial interference correction

Key to calculation options
- S, sym: Uncertainty at survey station, all errors symmetric (i.e., no bias).
- S, bias: Uncertainty at survey station, selected errors symmetric modeled as bias.
- D, sym: Uncertainty at assigned depth, all errors symmetric (i.e., no bias)
- D, bias: Uncertainty at assigned depth, selected errors symmetric modeled as bias.

Example 1 and 2 (Table 2) compare the basic and interference in well Unity#30. Being a high inclination well running an approximately, the interference correction actually degrades the accuracy. The results are plotted in Fig. 5. Example 3 to 6 all represent the basic MWD error model applied to well RenMen#95. They differ in that each uses a different permutation of the survey station/assigned depth and symmetric error/survey bias calculation options. The variation of lateral uncertainty and ellipsoid semi-major axis, characteristics is shown in Fig. 6.
Example 7 breaks well Quan#95 into three depths intervals, with the basic and interference-correction models being applied alternately. This example is included as a test of error propagation (Fig. 7 and 8).

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