Rest masses of elementary particles as effective masses at zero temperature

C. Quimbay and J. Morales

Departamento de Física, Universidad Nacional de Colombia
Ciudad Universitaria, Bogotá, Colombia

October 3, 2001

Abstract

We introduce a new approach to generate dynamically the masses of elementary particles in the $SU(3)_C \times SU(2)_L \times U(1)_Y$ Standard Model without Higgs Sector (SMWHS). We start from the assumption that rest masses correspond to the effective masses of particles in an elementary quantum fluid at zero temperature. These effective masses are obtained through radiative corrections, at one-loop order, in the context of the real time formalism of quantum field theory at finite temperature and density. The quantum fluid is described in structure and dynamics by the SMWHS and it is characterized by non-vanishing chemical potentials associated to the different fermion flavour species. Starting from the experimental mass values for quarks and leptons, taking the top quark mass as $m_t = 172.916$ GeV, we can compute, as an evidence of the consistency of our approach, the experimental central mass values for the $W^\pm$ and $Z^0$ gauge bosons. Subsequently we introduce in the SMWHS a massless scalar field leading to Yukawa coupling terms in the Lagrangian density. For this case we can also compute the experimental central mass values of the $W^\pm$ and $Z^0$ gauge bosons using a top quark mass value in the range $169.2$ GeV < $m_t$ < $178.6$ GeV; this range for the top quark mass implies that the scalar boson mass must be in the range $0 < M_H < 152$ GeV.
1 Introduction

The determination of the mechanism by which elementary particles acquire mass is an important step in the aim to understand nature at the microscopic level. In this determination the Higgs mechanism is one of the most attractive approaches known in particle physics. However this mechanism, which is based on the existence of a Higgs sector of scalar fields, is the most intriguing pillar of the Standard Model (SM). The spin-zero Higgs field is a doublet in the SU(2) space carrying non-zero hypercharge, and it is a singlet in the SU(3) space of color. The Higgs potential must be such that one of the neutral components of the Higgs field spontaneously acquires a non-vanishing vacuum expectation value, thereby giving masses to the $W^\pm$ and $Z^0$ gauge bosons. In this way the $SU(2)_L \times U(1)_Y$ symmetry is spontaneously broken into the $U(1)_{em}$ symmetry. Simultaneously the Yukawa couplings between the Higgs boson and fermion fields lead to the generation of masses for the elementary fermions. As a consequence, the Lagrangian is symmetric under $SU(2)_L \times U(1)_Y$ transformations but the vacuum is not. Bosonic gauge fields and fermionic matter fields acquire masses through their interactions with the Higgs field. The above mechanism implies the existence of a new particle in the physical spectrum, the neutral Higgs boson. The search for the Higgs boson is a focus of present and future experimental research. The possible existence of the Higgs boson with a mass $M_H \approx 115$ GeV, of which there is no conclusive evidence from LEP, must be confirmed or discarded at LHC in the future. On the other hand, it is important to show up that the couplings of quarks and leptons to the weak gauge bosons, $W^\pm$ and $Z^0$, are indeed precisely those prescribed by the electroweak gauge symmetry of the SM. The triple gauge vertices $\gamma W^+ W^-$ and $Z^0 W^+ W^-$ have also been found in agreement with the specific prediction of the SM at tree level. These facts have been proved with high accuracy through the experimental precision tests of the SM. This means that it has been verified that the electroweak gauge symmetry is indeed unbroken in the vertices of the theory, or in other words, the currents are indeed conserved.

It is well known that the Higgs mechanism is not the unique approach capable of generating the masses of elementary particles. In the following we mention some mechanisms of mass generation known in literature. One of these mechanisms is the dynamical breaking of the symmetry through radiative corrections at zero temperature developed by Coleman and Weinberg. Electroweak dynamical symmetry breaking, also called technicolor, provides an attractive mechanism for generating the gauge boson masses, while extended technicolor, walking technicolor, top condensation, and top-color assisted technicolor are mechanisms that have also been investigated. Another known mechanism responsible for the occurrence of mass terms, embedded in quantum field theory itself, is originated in the boundary conditions of
fields and related to the topology of the manifold [11]. Dynamical mass generation at finite temperature using thermal corrections to the self energy is another alternative [12]-[14]. However all these approaches seem to be unable to explain the spectrum of particle states as fully as the Higgs mechanism does.

In this paper we propose a new approach to generate the masses of elementary particles starting from the assumption that they correspond to the effective masses of the particles in an elementary quantum fluid at zero temperature. The physical source of mass generation in our approach is due to radiative corrections obtained in the framework of the quantum field theory at finite temperature and density. The elementary quantum fluid is constituted by massless quarks and massless leptons interacting through massless gluons, massless $W^\pm$ bosons, massless $Z^0$ bosons, massless photons and, possibly, massless scalar bosons. The fundamental effective model describing the structure and dynamics of this quantum fluid corresponds to the $SU(3)_C \times SU(2)_L \times U(1)_Y$ Standard Model without the Higgs Sector (SMWHS). A massless neutral scalar field can be included in the Lagrangian density of the SMWHS leading to Yukawa coupling terms between the massless fermions and the massless scalar boson with the same coupling constant ($Y$) for all the terms. This model describes the couplings of quark and leptons to the weak gauge bosons and the triple gauge boson vertices in accordance with the experimental observation that the electroweak currents are conserved [4]. In this model there are no Goldstone boson fields and the massless particle spectrum is symmetric under the gauge group. The elementary quantum fluid is characterized by non-vanishing fermionic chemical potentials ($\mu_{fi}$) associated to the different quark and lepton flavours. The values of the different $\mu_{fi}$ are free parameters in this quantum fluid.

We first calculate the fermionic effective masses starting from the one-loop self-energy at finite temperature ($T$) and non-vanishing fermionic chemical potentials ($\mu_f \neq 0$) and, in the rest frame, we identify these effective masses in the zero temperature limit with the fermion masses. Specifically, we obtain the fermion masses depending on the $\mu_{fi}$ values and the gauge coupling constants of the fundamental effective model. Afterwards, starting from the known experimental values of the fermion masses, we obtain values for each $\mu_{fi}$ and from these ones we calculate, as an evidence of the consistency of our approach, the correct values for the $W^\pm$ and $Z^0$ gauge bosons masses using an analogous procedure as the one performed for the fermions. Specifically the bosonic effective masses are calculated starting from the one-loop bosonic polarization tensor at $T \neq 0$ and $\mu_f \neq 0$. The mixing between the $SU(2)_L$ gauge boson ($W^3_\mu$) and the $U(1)_Y$ gauge boson ($B_\mu$) prevents the photon from acquiring mass. For the case where a massless neutral scalar field has been included in the Lagrangian density of the SMWHS, we also generate dynamically the mass of the neutral scalar boson starting from the scalar polarization tensor.

In Section 2 we mention some relevant theoretical aspects which support our pro-
posal. Before considering the real case of SMWHS we first calculate, in Section 3, the effective masses of the particles of an elementary quantum fluid at zero temperature that is described in structure and dynamics by a non-abelian gauge theory with Yukawa coupling terms and, we identify these effective masses with the rest masses of the particles in the vacuum. The effective masses are obtained at one-loop order and they are gauge invariant. In Section 4, using the SMWHS and following the same procedure as in Section 3, we dynamically generate the masses of the elementary fermions (quarks and leptons) and the electroweak bosons ($W^\pm$ and $Z^0$). These rest masses are obtained as functions of the $\mu_{f_i}$ values and the gauge coupling constants of the SMWHS. We use the experimental mass values for the fermions to obtain the $\mu_{f_i}$ values and we then calculate the $W^\pm$ and $Z^0$ gauge boson masses. The obtained gauge boson masses are in agreement with their experimental values. In Section 5, we add to the SMWHS Lagrangian density a massless neutral scalar field, including Yukawa coupling terms, and we then compute the $W^\pm$ and $Z^0$ gauge boson masses using an analogous manner as it was done in Section 4. Varying the top quark mass inside the experimental range imposes a range of possible values for the scalar boson mass. Our conclusions are summarized in Section 6.

2 A theoretical overview

To understand why our approach is a plausible way to generate consistently the masses of elementary particles we first mention some relevant theoretical aspects which support it.

It has been suggested that the Universe and its ground state - the physical vacuum - may behave like a condensed matter system with a complicated and possibly degenerate ground-state \cite{15}-\cite{18}. This vacuum might be a richly structured medium, and an early indication of this feature could be the Dirac’s sea \cite{16}. As the physical vacuum is a complicated structure governed by locality and symmetry, one can learn how to analyse it by studying other systems with analogous properties like those found in condensed matter physics \cite{16}.

There are several phenomena in nature which have their origin in the properties of physical vacuum; one of them is the Casimir effect which describes an attractive force between two conducting plates \cite{13}. In quantum field theory the vacuum is a well-defined quantum state, specifically, the ground state of a system of fields. In this framework the Casimir effect is due to quantum fluctuations of the electromagnetic zero-point field in the intervening space. The same type of Casimir effect arises in condensed matter physics, particularly in correlated fluids, due to thermal and/or quantum fluctuations \cite{20}. In this case the thermal fluctuations are modified by boundaries (membranes) resulting in finite-size corrections. Both cases can be studied
for boundaries of arbitrary shape using the path integral formalism \cite{20}. With the inclusion of quantum fluctuations the electromagnetic vacuum behaves, essentially, as a complex quantum fluid and modifies the motion of objects through it. In particular, the effective mass of a plate depends on its shape and becomes anisotropic \cite{20}. When considering the analogy of the Casimir effect in condensed matter the following correspondence must be taken into account: the ground state of a quantum fluid corresponds to the vacuum of a quantum field theory \cite{20}. This ground state is the quantum state of the fluid at zero temperature. In some cases the analogy between the quantum vacuum and the quantum fluid becomes exact. For instance, the low-energy fermionic and bosonic collective modes can correspond to the chiral fermions and gauge fields in quantum field theory \cite{21}.

The advantage of the quantum fluid is that the structure of the ground state is known, at least, in principle. For quantum fluids it is possible to calculate the phenomenological relevant parameters starting from a first principle microscopic theory. This effective theory, called the Theory of Everything, is “a set of equations capable of describing all phenomena that have been observed” in these quantum systems \cite{22}. For instance, in the context of this theory, Volovik has reported the calculation of a set of phenomenological parameters for the liquid $^4$He at zero external pressure, obtaining a good agreement with experimental values \cite{23}.

A phenomenological fact observed in the physical vacuum is that the elementary fermions (quarks and leptons) and the electroweak bosons ($W^\pm$ and $Z^0$) are massive in the rest frame. We can think that the rest masses of these elementary particles reflect the physical properties of the vacuum. Our proposal is based on the two following ideas: (i) the ground state of a quantum fluid at zero temperature corresponds to the vacuum of quantum field theory \cite{20}, (ii) it is possible to reproduce some observed properties of the physical vacuum, for instance the rest masses of elementary particles, starting from a first principle microscopic theory that describes an elementary quantum fluid \cite{22}, for instance the MSWHS describing the fundamental interactions of quarks and leptons. Consequently, we can assume that the effective masses of the particles in the elementary quantum fluid at zero temperature correspond to the rest masses of the elementary particles in the physical vacuum.

\section{Dynamical mass generation in a non-abelian gauge theory}

In this section we calculate the effective masses of particles in an elementary quantum fluid at zero temperature described by a non-abelian gauge theory. This calculation is performed at one loop-order in the framework of the real time formalism of the quantum field theory at finite temperature and non-vanishing chemical potential.
The quantum fluid is characterized by non-vanishing fermionic chemical potentials \( \mu_{f_i} \neq 0 \) where \( f_i \) represents the different fermion species in the fluid. In this section we will take \( \mu_{f_1} = \mu_{f_2} = \ldots = \mu_f \). The elementary quantum fluid is constituted by massless fermions interacting through massless gauge bosons and massless scalar bosons. The fundamental effective theory describing the structure and dynamics of this quantum fluid is the non-abelian gauge theory with Lagrangian density given by \(^{[24]}\):

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^A F_{\mu\nu}^A + \bar{\psi}_m \gamma^\mu \left( \delta_{mn} i \partial_\mu + g L_{mn}^{A} A_\mu^A \right) \psi_n + \frac{1}{2} D^\mu \phi D_\mu \phi + Y \bar{\psi}_m \Gamma^i_{mn} \psi_n^R \phi + H.C.,
\]

where \( A \) runs over the generators of the group and \( m, n \) over the states of the fermion representation. The covariant derivative \( (D_\mu) \) is \( D_\mu = \delta_\mu + ig T_A A_\mu^A \), being \( T_A \) the generators of the SU\((N)\) gauge group and \( g \) the gauge coupling constant. The last term of \(^{[3.1]}\) represents the Yukawa interaction between the fermion fields and the scalar field \( \phi \). The representation matrices \( L_{mn}^{A} \) are normalized by \( \text{Tr} (L_A L_B) = T(R) \delta^{AB} \) where \( T(R) \) is the index of the representation. In the calculation of the fermionic self-energy appears \( (L^A L^A)_{mn} = C(R) \delta_{mn} \), where \( C(R) \) is the quadratic Casimir invariant of the representation \(^{[24]}\).

At finite temperature and density, the Feynman rules for vertices are the same as those at \( T = 0 \) and \( \mu_f = 0 \), while the propagators in the Feynman gauge for massless gauge bosons \( D_{\mu\nu}(p) \), massless scalars \( D(p) \) and massless fermions \( S(p) \) are \(^{[23]}\):

\[
\begin{align*}
D_{\mu\nu}(p) &= -g_{\mu\nu} \left[ \frac{1}{p^2 + i\epsilon} - i\Gamma_b(p) \right], \\
D(p) &= \frac{1}{p^2 + i\epsilon} - i\Gamma_b(p), \\
S(p) &= \frac{\not{p}}{p^2 + i\epsilon} + i\not{p} \Gamma_f(p),
\end{align*}
\]

where \( p \) is the particle four-momentum and the plasma temperature \( T \) is introduced through the functions \( \Gamma_b(p) \) and \( \Gamma_f(p) \), which are given by

\[
\begin{align*}
\Gamma_b(p) &= 2\pi \delta(p^2) n_b(p), \\
\Gamma_f(p) &= 2\pi \delta(p^2) n_f(p),
\end{align*}
\]

with

\[
\begin{align*}
n_b(p) &= \frac{1}{e^{(p \cdot u)/T} - 1}, \\
n_f(p) &= \theta(p \cdot u) n_{f_1}^\uparrow(p) + \theta(-p \cdot u) n_{f_1}^\downarrow(p).
\end{align*}
\]
\( n_b(p) \) being the Bose-Einstein distribution function. The Fermi-Dirac distribution functions for fermions \( n_f^-(p) \) and for anti-fermions \( n_f^+(p) \) are:

\[
 n_f^+(p) = \frac{1}{e^{(p-u+\mu)/T} + 1} \tag{3.9}
\]

In the distribution functions (3.7) and (3.8), \( u^\alpha \) is the four-velocity of the center-of-mass frame of the dense plasma, with \( u^\alpha u_\alpha = 1 \).

### 3.1 Fermionic self-energy and fermion mass

For a non-abelian gauge theory with parity and chirality conservation the real part of the self-energy for a massless fermion is written as:

\[
\text{Re} \Sigma'(K) = -aK - b\eta, \tag{3.10}
\]

\( a \) and \( b \) being the Lorentz-invariant functions and \( K^\alpha \) the fermion momentum. These functions depend on the Lorentz scalars \( \omega \) and \( k \) defined by \( \omega \equiv (K \cdot u) \) and \( k \equiv [(K \cdot u)^2 - K^2]^{1/2} \). Taking by convenience \( u^\alpha = (1, 0, 0, 0) \) we have \( K^2 = \omega^2 - k^2 \) and then, \( \omega \) and \( k \) can be interpreted as the energy and three-momentum, respectively. Beginning with (3.10) it is possible to write:

\[
a(\omega, k) = \frac{1}{4k^2} [\text{Tr}(K\text{Re} \Sigma') - \omega \text{Tr}(\eta \text{Re} \Sigma')] , \tag{3.11}
\]

\[
b(\omega, k) = \frac{1}{4k^2} [(\omega^2 - k^2)\text{Tr}(\eta \text{Re} \Sigma') - \omega \text{Tr}(K\text{Re} \Sigma')] . \tag{3.12}
\]

The full fermion propagator, including only mass corrections, is given by [26]

\[
S(p) = \frac{1}{K - \text{Re} \Sigma'(K)} = \frac{1}{r} \gamma^0 n - \frac{\gamma_i k^i}{n^2 \omega^2 - k^2} , \tag{3.13}
\]

where \( n = 1 + b(\omega, k)/r \omega \) and \( r = 1 + a(\omega, k) \). The propagator poles can be found when:

\[
\left[ 1 + \frac{b(\omega, k)}{1 + a(\omega, k)} \right]^2 w^2 - k^2 = 0. \tag{3.14}
\]

We observe in (3.14) that \( n \) plays a role like the index of refraction in optics. To solve the equation (3.14), first it is required to calculate \( a(\omega, k) \) and \( b(\omega, k) \). These functions can be calculated from the relations (3.11) and (3.12) in terms of the real part of the
fermionic self-energy. The one-loop diagrams that contribute to the fermionic self-energy are shown in Fig.(1). The contribution to the fermionic self-energy from the gauge boson diagram shown in Fig.(1a) is given by

$$\Sigma(K) = ig^2 C(R) \int \frac{d^4p}{(2\pi)^4} D_{\mu\nu}(p) \gamma^\mu S(p+K) \gamma^\nu, \quad (3.15)$$

where $g$ is the interaction coupling constant and $C(R)$ is the quadratic Casimir invariant of the representation.

Substituting (3.2) and (3.4) into (3.15), the fermionic self-energy can be written as $\Sigma(K) = \Sigma(0) + \Sigma'(K)$, where $\Sigma(0)$ is the zero-temperature and zero-density contribution and $\Sigma'(K)$ is the finite-temperature and chemical potential contribution. It is easy to see that:

$$\Sigma(0) = -ig^2 C(R) \int \frac{d^4p}{(2\pi)^4} \frac{g_{\mu\nu} \gamma^\mu (p+K) \gamma^\nu}{p^2}, \quad (3.16)$$

$$\Sigma'(K) = 2g^2 C(R) \int \frac{d^4p}{(2\pi)^4} \left[ (p+K) \Gamma_b(p) + (p+K) \Gamma_f(p) \right] \frac{1}{(p+K)^2}. \quad (3.17)$$

Keeping only the real part (Re $\Sigma'(K)$) of the temperature and chemical potential contribution, we obtain:

$$\text{Re} \Sigma'(K) = 2g^2 C(R) \int \frac{d^4p}{(2\pi)^4} \left[ (p+K) \Gamma_b(p) + (p+K) \Gamma_f(p) \right] \frac{1}{(p+K)^2}. \quad (3.18)$$

If we multiply (3.18) by either $K$ or $\psi$, take the trace and perform the integrations over $p_0$ and the two angular variables, the functions (3.11) and (3.12) can be written in the notation given in [27] as:

$$a(\omega, k) = g^2 C(R) A(w, k, \mu_f), \quad (3.19)$$

$$b(\omega, k) = g^2 C(R) B(w, k, \mu_f), \quad (3.20)$$

where the integrals over the modulus of the three-momentum $p = |\vec{p}|$, $A(\omega, k, \mu_f)$ and $B(\omega, k, \mu_f)$, are:

$$A(\omega, k, \mu_f) = \frac{1}{k^2} \int_0^\infty \frac{dp}{8\pi^2} \frac{2p - \omega p}{k} \log \frac{\omega + k}{\omega - k} \left[ 2n_b(p) + n_f(p) + n_f^+(p) \right], \quad (3.21)$$

$$B(\omega, k, \mu_f) = \frac{1}{k^2} \int_0^\infty \frac{dp}{8\pi^2} \frac{p(\omega^2 - k^2)}{k} \log \frac{\omega + k}{\omega - k} - 2\omega p \left[ 2n_b(p) + n_f(p) + n_f^+(p) \right]. \quad (3.22)$$
The integrals (3.21) and (3.22) have been obtained in the high density limit, i.e. \( \mu_f \gg k \) and \( \mu_f \gg \omega \), and keeping the leading terms in temperature and chemical potential [28]. Evaluating these integrals we obtain that \( a(\omega, k) \) and \( b(\omega, k) \) are given by:

\[
a(\omega, k) = \frac{M_F^2}{k^2} \left[ 1 - \frac{\omega}{2k} \log \frac{\omega + k}{\omega - k} \right], \tag{3.23}
\]

\[
b(\omega, k) = \frac{M_F^2}{k^2} \left[ \frac{\omega^2 - k^2}{2k} \log \frac{\omega + k}{\omega - k} - \omega \right], \tag{3.24}
\]

where the fermionic effective mass \( M_F \) is:

\[
M_F^2(T, \mu_f) = \frac{g^2 C(R)}{8} \left( T^2 + \frac{\mu_f^2}{\pi^2} \right). \tag{3.25}
\]

The value of \( M_F \) given by (3.27) is in agreement with [29]-[32]. We are interested in the effective mass at \( T = 0 \). For this case:

\[
M_F^2(0, \mu_F) = M_{F,\mu}^2 = \frac{g^2 C(R) \mu_F^2}{8 \pi^2}. \tag{3.26}
\]

Substituting (3.23) and (3.24) into (3.14), we obtain for the limit \( k \ll M_{F,\mu} \) that:

\[
\omega^2(k) = M_{F,\mu}^2 \left[ 1 + \frac{2}{3} \frac{k}{M_{F,\mu}} + \frac{5}{9} \frac{k^2}{M_{F,\mu}^2} + \ldots \right]. \tag{3.27}
\]

Owing to we have made the calculation at leading order in temperature and chemical potential, the dispersion relation is gauge independent [28]. It is very well known that the relativistic energy in the vacuum for a massive fermion at rest is \( \omega^2(0) = m_f^2 \). At zero temperature the quantum fluid is in the ground state and it corresponds to the vacuum of quantum field theory [20]. It is clear from (3.27) that if \( k = 0 \) then \( \omega^2(0) = M_{F,\mu}^2 \) and it is possible to identify the fermionic effective mass at zero temperature as the rest mass of the fermion.

We note that the contribution to the real part of the fermionic self-energy from the generic scalar boson diagram shown in Fig.(1b) has the same form as the gauge boson contribution (3.18). The factor \( 2g^2 C(R) \) is replaced by \( Y^2 C' \), where \( Y \) is the Yukawa coupling constant and \( C' \) is given in terms of the matrices of Clebsch-Gordan coefficients \( \Gamma^i \) [24]. The scalar boson contribution is proportional to \((\Gamma^i \Gamma^i)_{mm'} \equiv C' \delta_{mm'}\).

Considering all contributions to the fermionic self-energy, we obtain that the fermion mass generated dynamically is:

\[
m_f^2 = \frac{(2g^2 C(R) + Y^2 C') \mu_F^2}{16 \pi^2}. \tag{3.28}
\]
The fermion mass has been generated through the computation of the fermionic self-energy at one-loop order. The expression (3.28) is gauge invariant and in this expression both $\mu_f$ and $Y$ are free parameters.

3.2 Bosonic polarization tensor and gauge boson mass

The most general form of the polarization tensor which preserves the invariance under rotations, translations and gauge transformations is [33]:

$$\Pi_{\mu\nu}(K) = P_{\mu\nu}\Pi_T(K) + Q_{\mu\nu}\Pi_L(K), \quad (3.29)$$

where the Lorentz-invariant functions $\Pi_L$ and $\Pi_T$, which characterize the longitudinal and transverse modes respectively, are obtained by contraction:

$$\Pi_L(K) = -\frac{K^2}{k^2}u^\mu u^\nu\Pi_{\mu\nu}, \quad (3.30)$$

$$\Pi_T(K) = -\frac{1}{2}\Pi_L + \frac{1}{2}g^{\mu\nu}\Pi_{\mu\nu}. \quad (3.31)$$

The bosonic dispersion relations are obtained by looking at the poles of the full propagator which is obtained by summing all the vacuum polarization insertions. The full bosonic propagator is [33]:

$$D_{\mu\nu}(K) = \frac{Q_{\mu\nu}}{K^2 - \Pi_L(K)} + \frac{P_{\mu\nu}}{K^2 - \Pi_T(K)} - (\xi - 1)\frac{K_\mu K_\nu}{K^4}, \quad (3.32)$$

where $\xi$ is a gauge parameter. The gauge invariant dispersion relations, describing the two propagation modes, are found for:

$$K^2 - \Pi_L(K) = 0, \quad (3.33)$$

$$K^2 - \Pi_T(K) = 0. \quad (3.34)$$

The one-loop fermion contribution to the vacuum polarization from the diagram shown in Fig.(2a) is given by

$$\Pi_{\mu\nu}(K) = ig^2C(R)N_f \int \frac{d^4p}{(2\pi)^4} Tr [\gamma_\mu S(p)\gamma_\nu S(p + K)], \quad (3.35)$$

where $S$ are the propagators (3.4) and $N_f$ is the number of fermions in the fundamental representation. Substituting (3.4) into (3.33) the polarization tensor can be written as $\Pi_{\mu\nu}(K) = \Pi_{\mu\nu}(0) + \Pi'_{\mu\nu}(K)$, where $\Pi_{\mu\nu}(0)$ is the zero-temperature and zero-density contribution and $\Pi'_{\mu\nu}(K)$ is the finite-temperature and chemical potential contribution.
It is easy to see that the real part of the finite-temperature and chemical potential contribution \( \text{Re} \Pi'_{\mu\nu}(K) \) is given by

\[
\text{Re} \Pi'_{\mu\nu}(K) = \frac{g^2 C(R) N_f}{2} \int \frac{d^4 p}{\pi^4} \frac{(p^2 + p \cdot K) g^{\mu\nu} - 2 p^\mu p^\nu - p^\mu K^\nu - p^\nu K^\mu}{(p + K)^2} \Gamma_f(p).
\]

(3.36)

Substituting (3.36) in (3.30) and (3.31) and keeping the leading terms in temperature and chemical potential, we obtain for the high density limit (\( \mu_f >> k \) and \( \mu_f >> \omega \)) that:

\[
\text{Re} \Pi'_L(K) = 3 M_B^2 \left[ 1 - \frac{\omega}{2k} \log \frac{\omega + k}{\omega - k} \right],
\]

(3.37)

\[
\text{Re} \Pi'_T(K) = 3 M_B^2 \left[ \frac{\omega^2}{k^2} + \left( 1 - \frac{\omega^2}{k^2} \right) \frac{\omega}{2k} \log \frac{\omega + k}{\omega - k} \right],
\]

(3.38)

where the bosonic effective mass \( M_B \) is:

\[
M_B^2(T, \mu_f) = \frac{1}{6} N g^2 T^2 + \frac{1}{2} g^2 C(R) N_f \left[ \frac{T^2}{6} + \frac{\mu_f^2}{2 \pi^2} \right],
\]

(3.39)

\( N \) being the gauge group dimension. The effective mass (3.39) is in agreement with [32, 34]. The bosonic effective mass at \( T = 0 \) is:

\[
M_B^2(0, \mu_f) = M_{B^\mu} = \frac{g^2 C(R) N_f \mu_f^2}{2 \pi^2},
\]

(3.40)

in agreement with the result obtained at finite density and \( T = 0 \) theory [32]. Substituting (3.37) and (3.38) into (3.33) and (3.34) for the limit \( k << M_{B^\mu} \), we obtain for the two propagation modes that:

\[
\omega_L^2 = M_{B^\mu}^2 + \frac{3}{5} k_L^2
\]

(3.41)

\[
\omega_T^2 = M_{B^\mu}^2 + \frac{6}{5} k_T^2.
\]

(3.42)

We note that (3.41) and (3.42) have the same value when the three-momentum goes to zero. It is clear from (3.41) and (3.42) that for \( k = 0 \) then \( \omega^2(0) = M_{B^\mu}^2 \) and it is possible to recognize the bosonic effective mass at zero temperature as the rest mass of the gauge boson:

\[
m_b^2 = M_{B^\mu}^2 = \frac{g^2 C(R) N_f \mu_f^2}{2 \pi^2}.
\]

(3.43)
3.3 Scalar polarization tensor and scalar boson mass

The full scalar boson propagator is:

\[ D(K) = \frac{1}{K^2 - \Pi(K)} \quad (3.44) \]

where the dispersion relations are obtained for:

\[ K^2 - \Pi(K) = 0, \quad (3.45) \]

The one-loop fermion contribution to the scalar polarization tensor from the diagram shown in Fig.(2b) is given by

\[ \Pi(K) = iY^2 N_f \int \frac{d^4p}{(2\pi)^4} Tr [S(p)S(p+K)], \quad (3.46) \]

where \( S \) are the propagators (3.34) and \( N_f \) is the number of fermions in the fundamental representation. Substituting (3.34) into (3.46), the polarization tensor can be written as \( \Pi(K) = \Pi(0) + \Pi'(K) \), where \( \Pi(0) \) is the zero-temperature and zero-density contribution and \( \Pi'(K) \) is the finite-temperature and chemical potential contribution.

The real part of the finite-temperature and chemical potential contribution \( \text{Re} \, \Pi'(K) \) is given by

\[ \text{Re} \, \Pi'(K) = \frac{Y^2 N_f}{2} \int \frac{d^4p}{\pi^4} \left( \frac{p^2 + p \cdot K}{(p + K)^2} \right) \Gamma_f(p). \quad (3.47) \]

Substituting (3.46) in (3.47) and keeping the leading terms in temperature and chemical potential, we obtain for the high density limit \( (\mu_f >> k \text{ and } \mu_f >> \omega) \) that:

\[ \text{Re} \, \Pi'(K) = Y^2 N_f \left[ \frac{T^2}{6} + \frac{\mu_f^2}{2\pi^2} \right] = M_S^2(T, \mu_f), \quad (3.48) \]

\( M_S \) being the scalar effective mass. For \( T = 0 \), the scalar effective mass is:

\[ M_S^2(0, \mu_f) = M_{S\mu}^2 = \frac{Y^2 N_f \mu_f^2}{2 \pi^2}. \quad (3.49) \]

The dispersion relation at \( T = 0 \) is obtained from (3.43) being:

\[ w^2 = k^2 + M_{S\mu}^2 \quad (3.50) \]

If we put \( k = 0 \) in (3.50) then \( w^2(0) = M_{S\mu}^2 \) and it is possible to identify the scalar effective mass at zero temperature as the rest mass of the scalar boson:

\[ m_s^2 = M_{S\mu}^2 = \frac{Y^2 N_f \mu_f^2}{2 \pi^2}. \quad (3.51) \]
4 Dynamical mass generation in the SMWHS

In this section, following the same procedure as in Section 3, we generate dynamically the masses of the elementary particles of the SMWHS. To do it, we first calculate the effective masses of the particles in an elementary quantum fluid described in structure and dynamics by the SMWHS. The elementary quantum fluid is constituted by massless quarks and massless leptons interacting through massless gluons, massless $W^{\pm}$ bosons, massless $Z^0$ bosons and massless photons. The quantum fluid is characterized by non-vanishing chemical potentials associated to the different fermion flavour species. We consider for quarks $\mu_u \neq \mu_d \neq \mu_c \neq \ldots \neq 0$, for charged leptons $\mu_e \neq \mu_{\mu} \neq \mu_{\tau} \neq 0$ and for neutrinos $\mu_{\nu_e} \neq \mu_{\nu_{\mu}} \neq \mu_{\nu_{\tau}} \neq 0$. The $SU(2)_L \times U(1)_Y$ Standard Model without Higgs Sector is described by the Lagrangian density:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{YM} + \mathcal{L}_{FB} + \mathcal{L}_{GF} + \mathcal{L}_{FP},$$

(4.1)

where $\mathcal{L}_{YM}$ is the Yang-Mills Lagrangian density, $\mathcal{L}_{FB}$ is the fermionic-bosonic Lagrangian density, $\mathcal{L}_{GF}$ is the gauge fixing Lagrangian density and $\mathcal{L}_{FP}$ is the Fadeev-Popov Lagrangian density. The $\mathcal{L}_{YM}$ is given by

$$\mathcal{L}_{YM} = -\frac{1}{4} W^A_{\mu\nu} W^A_{\mu\nu} - \frac{1}{4} F_{\mu\nu} F_{\mu\nu},$$

(4.2)

where $W^A_{\mu\nu} = \partial_\mu W^A_\nu - \partial_\nu W^A_\mu + g_w F^{ABC} W^B_\mu W^C_\nu$ is the energy-momentum tensor associated to the $SU(2)_L$ group and $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ is the one associated to the $U(1)_Y$ group. The $\mathcal{L}_{FB}$ is written as:

$$\mathcal{L}_{FB} = i \bar{\psi}_L \gamma^\mu D_\mu L + i \bar{\psi}_R \gamma^\mu D_\mu R + i \bar{\psi}_R^I \gamma^\mu D_\mu \psi_R^I,$$

(4.3)

where $D_\mu L = (\partial_\mu + ig Y_L B_\mu / 2 + i g T_i W^i_\mu) L$ and $D_\mu R = (\partial_\mu + ig Y_R B_\mu / 2) R$, being $g_w$ the gauge coupling constant associated to the $SU(2)_L$ group, $g_e$ the one associated to the $U(1)_Y$ group, $Y_L = -1$, $Y_R = -2$ and $T_i = \sigma_i / 2$. The $SU(2)_L$ left-handed doublet (L) is given by

$$L = \begin{pmatrix} \psi^i_L \\ \psi^I_L \end{pmatrix}.$$

(4.4)

For a non-abelian gauge theory with parity violation and quirality conservation like the SMWHS, the real part of the self-energy for a massless fermion is:

$$\text{Re} \Sigma'(K) = -\mathfrak{K} (a_L P_L + a_R P_R) - \mathfrak{K} (b_L P_L + b_R P_R),$$

(4.5)

where $P_L \equiv \frac{1}{2}(1 - \gamma_5)$ and $P_R \equiv \frac{1}{2}(1 + \gamma_5)$ are the left- and right-handed chiral projectors respectively. The functions $a_L$, $a_R$, $b_L$ and $b_R$ are the chiral projections of the Lorentz-invariant functions $a$, $b$ and they are defined in the following way:

$$a = a_L P_L + a_R P_R,$$

(4.6)

$$b = b_L P_L + b_R P_R,$$

(4.7)
The inverse fermion propagator is given by

\[ S^{-1}(K) = \mathcal{L}P_L + \mathcal{R}P_R \quad (4.8) \]

where:

\[ \mathcal{L}^\mu = (1 + a_L)K^\mu + b_Lu^\mu \quad (4.9) \]
\[ \mathcal{R}^\mu = (1 + a_R)K^\mu + b_Ru^\mu \quad (4.10) \]

The fermion propagator follows from the inversion of (4.8):

\[ S = \frac{1}{D} \left[ (\mathcal{L}^2\mathcal{R}) P_L + (\mathcal{R}^2\mathcal{L}) P_R \right] . \quad (4.11) \]

being \( D(\omega, k) = \mathcal{L}^2\mathcal{R}^2 \). The poles of the propagator correspond to values \( \omega \) and \( k \) for which the determinant \( D \) in (4.11) vanishes:

\[ \mathcal{L}^2\mathcal{R}^2 = 0. \quad (4.12) \]

In the rest frame of the dense plasma \( u = (1, \vec{0}) \), Eq.(4.12) leads to the fermionic dispersion relations for a chirally invariant gauge theory with parity violation, as the case of the SMWHS. Thus, the fermionic dispersion relations for this case are given by

\[ [\omega(1 + a_L) + b_L]^2 - k^2 [1 + a_L]^2 = 0, \quad (4.13) \]
\[ [\omega(1 + a_R) + b_R]^2 - k^2 [1 + a_R]^2 = 0. \quad (4.14) \]

Left- and right-handed components of the fermionic dispersion relations obey decoupled relations. The Lorentz invariant functions \( a(\omega, k) \) and \( b(\omega, k) \) are calculated from the expressions \((3.11)\) and \((3.12)\) through the real part of the fermionic self-energy. This self-energy is obtained adding all the posibles gauge boson contributions admited by the Feynman rules of the SMWHS.

We will work in the basis of gauge bosons given by \( B_\mu, W_3^\mu, W_\pm^\mu \), where the charged electroweak gauge boson is \( W_\pm^\mu = (W_1^\mu \mp iW_2^\mu) / \sqrt{2} \). The diagrams with an exchange of \( W_\pm^\mu \) gauge bosons induce a flavour change in the incoming fermion \( i \) to a different outgoing fermion \( j \).

For the quark sector, in the case of the flavour change contributions mentioned, the flavor \( i \) (\( I \)) of the internal quark (inside the loop) runs over the up (\( i \)) or down (\( I \)) quarks flavours according to the type of the external quark (outside the loop). Owing to each contribution to the quark self-energy is proportional to \((3.21)-(3.22)\),
the functions \( a_L, a_R, b_L \) and \( b_R \) are given by

\[
a_L(\omega, k)_{ij} = \left[ f_S + f_B + f_{W^3} \right] A(\omega, k, \mu_i) + \sum_I f_{W^\pm} A(\omega, k, \mu_I), \tag{4. 15}
\]

\[
b_L(\omega, k)_{ij} = \left[ f_S + f_B + f_{W^3} \right] B(\omega, k, \mu_i) + \sum_I f_{W^\pm} B(\omega, k, \mu_I), \tag{4. 16}
\]

\[
a_R(\omega, k)_{ij} = \left[ f_S + f_B \right] A(\omega, k, \mu_i), \tag{4. 17}
\]

\[
b_R(\omega, k)_{ij} = \left[ f_S + f_B \right] B(\omega, k, \mu_i). \tag{4. 18}
\]

The coefficients \( f \) are:

\[
f_S = \frac{4}{3} g_s^2 \delta_{ij}, \tag{4. 19}
\]

\[
f_B = \frac{1}{4} g_w^2 \delta_{ij}, \tag{4. 20}
\]

\[
f_{W^3} = \frac{1}{4} g_e^2 \delta_{ij}, \tag{4. 21}
\]

\[
f_{W^\pm} = \frac{1}{2} g_w K_{ii} K_{lj}, \tag{4. 22}
\]

where \( K \) represents the CKM matrix and \( g_s \) is the strong coupling constant. The integrals \( A(\omega, k, \mu_f) \) and \( B(\omega, k, \mu_f) \) have been obtained in the high density limit \((\mu_f > > k \text{ and } \mu_f > > \omega)\) and keeping the leading terms in temperature and chemical potential. They are given by

\[
A(\omega, k, \mu_f) = \frac{1}{8k^2} \left( T^2 + \frac{\mu_f^2}{\pi^2} \right) \left[ 1 - \frac{\omega}{2k} \log \frac{\omega + k}{\omega - k} \right], \tag{4. 23}
\]

\[
B(\omega, k, \mu_f) = \frac{1}{8k^2} \left( T^2 + \frac{\mu_f^2}{\pi^2} \right) \left[ \frac{\omega^2 - k^2}{2k} \log \frac{\omega + k}{\omega - k} - \omega \right]. \tag{4. 24}
\]

The chiral projections of the Lorentz-invariant functions are:

\[
a_L(\omega, k)_{ij} = \frac{1}{8k^2} \left[ 1 - F(\frac{\omega}{k}) \right] \left[ l_{ij} \left( T^2 + \frac{\mu^2}{\pi^2} \right) + c_{ij} \left( T^2 + \frac{\mu_i^2}{\pi^2} \right) \right], \tag{4. 25}
\]

\[
b_L(\omega, k)_{ij} = -\frac{1}{8k^2} \left[ \frac{\omega}{k} + \left( \frac{k}{\omega} - \frac{\omega}{k} \right) F(\frac{\omega}{k}) \right] \left[ l_{ij} \left( T^2 + \frac{\mu^2}{\pi^2} \right) + c_{ij} \left( T^2 + \frac{\mu_i^2}{\pi^2} \right) \right], \tag{4. 26}
\]

\[
a_R(\omega, k)_{ij} = \frac{1}{8k^2} \left[ 1 - F(\frac{\omega}{k}) \right] \left[ r_{ij} \left( T^2 + \frac{\mu^2}{\pi^2} \right) \right], \tag{4. 27}
\]

\[
b_R(\omega, k)_{ij} = -\frac{1}{8k^2} \left[ \frac{\omega}{k} + \left( \frac{k}{\omega} - \frac{\omega}{k} \right) F(\frac{\omega}{k}) \right] \left[ r_{ij} \left( T^2 + \frac{\mu^2}{\pi^2} \right) \right]. \tag{4. 28}
\]
where $F(x)$ is

$$F(x) = \frac{x}{2} \log \left( \frac{x + 1}{x - 1} \right)$$

(4.29)

and the coefficients are given by

$$l_{ij} = \left( \frac{4}{3} g_s^2 + \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2 \right) \delta_{ij},$$

(4.30)

$$c_{ij} = \sum_{l} \left( \frac{g_w^2}{2} \right) K^+_l K_{ij},$$

(4.31)

$$r_{ij} = \left( \frac{4}{3} g_s^2 + \frac{1}{4} g_e^2 \right) \delta_{ij}.$$

(4.32)

Substituting (4.25)-(4.26) into (4.13), and (4.27)-(4.28) into (4.14), we obtain for the limit $k << M_{(i,I)_{L,R}}$ that:

$$\omega^2(k) = M^2_{(i,I)_{L,R}} \left[ 1 + \frac{2}{3} \frac{k}{M_{(i,I)_{L,R}}} + \frac{5}{9} \frac{k^2}{M_{(i,I)_{L,R}}^2} + \ldots \right],$$

(4.33)

where:

$$M^2_{(i,I)_{L}}(T, \mu_f) = (l_{ij} + c_{ij}) \frac{T^2}{8} + \frac{\mu^2_{(i,I)_{L}}}{8\pi^2} + \frac{\mu^2_{(i,I)_{L}}}{8\pi^2},$$

(4.34)

$$M^2_{(i,I)_{R}}(T, \mu_f) = (r_{ij} + d_{ij}) \frac{T^2}{8} + \frac{\mu^2_{(i,I)_{R}}}{8\pi^2}. $$

(4.35)

We are interested in the effective masses at $T = 0$. For this case:

$$M^2_{(i,I)_{L}}(0, \mu_f) = l_{ij} \frac{\mu^2_{(i,I)_{L}}}{8\pi^2} + c_{ij} \frac{\mu^2_{(i,I)_{L}}}{8\pi^2},$$

(4.36)

$$M^2_{(i,I)_{R}}(0, \mu_f) = r_{ij} \frac{\mu^2_{(i,I)_{R}}}{8\pi^2}. $$

(4.37)

The fermionic dispersion relation for the lepton sector are similar to the relations (4.33), even though the effective masses changing. For this sector $g_s = 0$ and non-exist mixing between the charge lepton flavours. Then, for the lepton sector, the coefficients $l, c$ and $r$ in (4.36) and (4.37) are:

$$l = \left( \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2 \right),$$

(4.38)

$$c = \left( \frac{1}{2} g_w^2 \right),$$

(4.39)

$$r = \left( \frac{1}{4} g_e^2 \right).$$

(4.40)
For the neutrino sector the coefficient $r$ is zero since there are not right-handed neutrinos in the physical spectrum.

Following the same argument as in Section 3, we can identify the effective masses of the particles in the elementary quantum fluid at zero temperature as the masses of the elementary particles at rest. Coming from the left-handed and right-handed representations, respectively, we find for the quark sector that the rest masses are:

$$m_i^2 = \left[\frac{4}{3} g_s^2 + \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2\right] \frac{\mu_{iL}^2}{8\pi^2} + \left[\frac{1}{2} g_w^2\right] \frac{\mu_{iR}^2}{8\pi^2}, \quad (4.41)$$

$$m_I^2 = \left[\frac{4}{3} g_s^2 + \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2\right] \frac{\mu_{IL}^2}{8\pi^2} + \left[\frac{1}{2} g_w^2\right] \frac{\mu_{IR}^2}{8\pi^2}, \quad (4.42)$$

and

$$m_i^2 = \left[\frac{4}{3} g_s^2 + \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2\right] \frac{\mu_{iR}^2}{8\pi^2}, \quad (4.43)$$

$$m_I^2 = \left[\frac{4}{3} g_s^2 + \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2\right] \frac{\mu_{IL}^2}{8\pi^2}, \quad (4.44)$$

where the couple of index $(i, I)$ running over the quarks $(u, d), (c, s)$ and $(t, b)$.

For the lepton sector we find that the rest masses are:

$$m_i^2 = \left[\frac{1}{4} g_w^2 + \frac{1}{4} g_e^2\right] \frac{\mu_{iL}^2}{8\pi^2} + \left[\frac{1}{2} g_w^2\right] \frac{\mu_{iR}^2}{8\pi^2}, \quad (4.45)$$

$$m_I^2 = \left[\frac{1}{4} g_w^2 + \frac{1}{4} g_e^2\right] \frac{\mu_{IL}^2}{8\pi^2} + \left[\frac{1}{2} g_w^2\right] \frac{\mu_{IR}^2}{8\pi^2}, \quad (4.46)$$

and

$$m_i^2 = \left[\frac{1}{4} g_e^2\right] \frac{\mu_{iR}^2}{8\pi^2}, \quad (4.47)$$

where the couple of index $(i, I)$ running over the leptons $(\nu_e, e), (\nu_{\mu}, \mu)$ and $(\nu_{\tau}, \tau)$.

We have generated dynamically the masses of the fermionic spectrum. These masses depended on the $\mu_f$ values and the gauge coupling constants of the SMWHS. Because the $\mu_f$ values are free parameters, we can compute these starting from the known experimental values for the fermion masses.

To evaluate the bosonic polarization tensor associated with the $W^\pm, W^3_\mu, B_\mu$ gauge boson propagators, we follow the same procedure as in Section 3. We find that the gauge boson masses generated dynamically are:

$$M_{W^\pm}^2 = \frac{g_w^2}{4} \sum_{f=1}^{12} \left|\frac{\mu_{fL}^2}{2\pi^2}\right|, \quad (4.48)$$
\[ M_{W^3}^2 = \frac{g_w^2}{4} \sum_{f=1}^{12} \frac{|\mu_{fL}|^2}{2\pi^2}, \]  
\[ M_B^2 = \frac{g_w^2}{4} \sum_{f=1}^{12} \frac{|\mu_{fL}|^2}{2\pi^2}, \]

where the sum running over the twelve \( \mu_{fL} \) values associated with the elementary fermions of the physical spectrum.

Starting from the expressions (4.41), (4.42), (4.45), (4.46), using the experimental central mass values for the fermions \[ m_u = 0.003, m_d = 0.006, m_c = 1.25, m_t = 0.122, m_b = 4.2, m_{\nu_e} = 0, m_e = 0.0005, m_{\nu_\mu} = 0, m_{\mu} = 0.1056, m_{\nu_\tau} = 0, m_{\tau} = 1.777, m_t = 174.3 \pm 5.1 \text{ GeV}, \]
and \[ g_s = 1.22029, g_w = 0.642343, g_e = 0.117906, \]
we obtain the \( \mu_{fL} \) values for all the fermion flavour species. Substituting these \( \mu_{fL} \) values in the expressions (4.48), (4.49) and (5.15), we obtain
\[ M_{W^\pm} = M_{W^3} = 81.0613 \pm 2.3667 \text{ GeV}, \]  
\[ M_B = 43.3326 \pm 1.2652 \text{ GeV}, \]
where the \( M_W \) value computed is in agreement with the experimental value.

By physical reasons \( W^3_\mu \) and \( B_\mu \) gauge bosons are mixed. After diagonalization of mass matrix, we get the physical fields \( A_\mu \) and \( Z_\mu \) corresponding to the massless photon and the neutral \( Z^0 \) boson of mass \( M_Z \) respectively, with the relations \[ M_{Z}^2 = M_{W^\pm}^2 + M_{B}^2, \]  
\[ \cos \theta_w = \frac{M_{W}}{M_{Z}}, \quad \sin \theta_w = \frac{M_{B}}{M_{Z}}, \]
where \( \theta_w \) is the weak mixing angle:
\[ Z^\mu_\mu = B_\mu \sin \theta_w - W^3_\mu \cos \theta_w, \]  
\[ A_\mu = B_\mu \cos \theta_w + W^3_\mu \sin \theta_w. \]
Substituting (4.51) and (4.52) into (4.53) we obtain that:
\[ M_Z = 91.9166 \pm 2.6837 \text{ GeV}, \]
in agreement with the experimental value. We observe that using a top mass value of 172.9159 GeV in the expressions (4.41), (4.42), (4.45) and (4.46), we obtain for the \( W^\pm \) and \( Z^0 \) boson masses the experimental central values: \( M_W = 80.419 \) and \( M_Z = 91.1882 \) GeV.
5 SMWHS plus a massless neutral scalar boson

In this section, we add to SMWHS Lagrangian density a massless neutral scalar field with Yukawa coupling terms between the massless fermions and the massless scalar boson. The Yukawa coupling terms are taken equals for all the fermions, \( i.e. \) the coupling constant (\( Y \)) is the same for all the couplings. Following the same procedure as in section 4, we generate dynamically the masses of the elementary particles. In this case the elementary quantum fluid is constituted by massless quarks and massless leptons interacting through massless gluons, massless \( W^\pm \) bosons, massless \( Z^0 \) bosons, massless photons and massless scalar bosons. The quantum fluid is characterized by non-vanishing chemical potentials associated to the different fermion flavour species \( \mu_{f_i} \neq 0 \). The elementary effective model is described by the Lagrangian density:

\[
\mathcal{L}_{\text{eff}}' = \mathcal{L}_{\text{eff}} + \mathcal{L}_{SB},
\]

where \( \mathcal{L}_{\text{eff}} \) is the SMWHS Lagrangian density (4.1) and \( \mathcal{L}_{SB} \) is the Scalar Boson Lagrangian density given by

\[
\mathcal{L}_{SB} = (D^\mu \phi)^\dagger (D_\mu \phi) + \mathcal{L}_Y,
\]

where \( D_\mu = \partial_\mu - ig'B_\mu/2 + ig\sigma_i W^i_\mu/2 \) and the Yukawa Lagrangian density \( \mathcal{L}_Y \) is:

\[
\mathcal{L}_Y = Y \left[ \bar{\psi}^f_R (\phi^\dagger L) + (\bar{\psi}^f_c L) \psi^f_R + \bar{\psi}^f_L (\phi^\dagger c L) + (\bar{\psi}^f c L) \psi^f_L \right],
\]

being \( Y \) the Yukawa coupling constant. The doublets \( \phi \) and \( \phi^\dagger \) are given by

\[
\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}_L,
\]

\[
\phi^\dagger = i\sigma_2 \phi^* = \begin{pmatrix} \phi^0 \\ -\phi^- \end{pmatrix}_L.
\]

We can also introduce in the Lagrangian density (5.2) a scalar potential leading to auto-interacting scalar terms, but these vertices will not affect the results that we obtain in this section. The Lagrangian density (5.3) can be written as:

\[
\mathcal{L}_Y = Y \sum_{e,\mu,d,u}^3 \left[ \bar{e}_R \phi^0 e_L + \bar{\nu}_e R \phi^0 \nu_e L + \bar{e}_L \phi^0 e_R + \bar{\nu}_e L \phi^0 \nu_e R \\
+ \bar{d}_R \phi^0 d_L + \bar{u}_R \phi^0 u_L + \bar{d}_L \phi^0 d_R + \bar{u}_L \phi^0 u_R \right],
\]

where the sum running over the three families of fermions and we have chosen a particular doublet \( \phi \) of the form:

\[
\phi = \begin{pmatrix} 0 \\ \phi^0 \end{pmatrix}_L.
\]
With this selection, we have taken the charged scalar boson as zero. Following the same procedure as in Section 4, we generate dynamically the masses of the elementary particles. Coming from the left-handed and right-handed representations, respectively, we find for the quark sector that the rest masses are:

\[ m_i^2 = \left[ \frac{4}{3} g_s^2 + \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2 + \frac{Y^2}{2} \right] \frac{\mu_i^2}{8\pi^2} + \left[ \frac{1}{2} g_w^2 \right] \frac{\mu_{iL}^2}{8\pi^2}, \quad (5.8) \]

\[ m_I^2 = \left[ \frac{4}{3} g_s^2 + \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2 + \frac{Y^2}{2} \right] \frac{\mu_I^2}{8\pi^2} + \left[ \frac{1}{2} g_w^2 \right] \frac{\mu_{IL}^2}{8\pi^2}, \quad (5.9) \]

and

\[ m_i^2 = \left[ \frac{4}{3} g_s^2 + \frac{1}{4} g_e^2 + \frac{Y^2}{2} \right] \frac{\mu_{iR}^2}{8\pi^2}, \quad (5.10) \]

\[ m_I^2 = \left[ \frac{4}{3} g_s^2 + \frac{1}{4} g_e^2 + \frac{Y^2}{2} \right] \frac{\mu_{IR}^2}{8\pi^2}, \quad (5.11) \]

where the couple of index \((i,I)\) running over the quarks \((u,d), (c,s)\) and \((t,b)\).

For the lepton sector we find that the rest masses are:

\[ m_i^2 = \left[ \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2 + \frac{Y^2}{2} \right] \frac{\mu_{iL}^2}{8\pi^2} + \left[ \frac{1}{2} g_w^2 \right] \frac{\mu_{iL}^2}{8\pi^2}, \quad (5.12) \]

\[ m_I^2 = \left[ \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2 + \frac{Y^2}{2} \right] \frac{\mu_{IL}^2}{8\pi^2} + \left[ \frac{1}{2} g_w^2 \right] \frac{\mu_{IL}^2}{8\pi^2}, \quad (5.13) \]

and

\[ m_i^2 = \left[ \frac{1}{4} g_e^2 + \frac{Y^2}{2} \right] \frac{\mu_{iR}^2}{8\pi^2}, \quad (5.14) \]

where the couple of index \((i,I)\) running over the leptons \((\nu_e, e), (\nu_\mu, \mu)\) and \((\nu_\tau, \tau)\).

The fermion masses depended on the \(\mu_{fi}\) values and the coupling constants. We can compute the \(\mu_{fi}\) values from the known experimental values of the fermion masses. Starting from the expressions \((5.8), (5.9), (5.12), (5.13)\), using the experimental central mass values for the fermions, substituting the obtained \(\mu_{fi}\) values in the expressions \((4.48), (4.49)\) and \((5.13)\), and using \((4.53)\), we find that the experimental mass central values for the \(W^\pm\) and \(Z^0\) gauge bosons can be obtained using a mass value for the top quark mass in the range \(169.2\ \text{GeV} < m_t < 178.6\ \text{GeV}\) and a Yukawa coupling constant in the range \(0 < Y < 0.607\). This range for the top quark mass implies that the scalar boson mass must be in the range \(0 < M_H < 152\ \text{GeV}\) as it is shown in the Fig.(3).
The same mechanism of mass dynamical generation acts on the scalar boson given a mass $M_H$ which can be calculated from:

\[
M_H^2 = Y^2 \sum_{f=1}^{12} \frac{|\mu_{fL}|}{2\pi^2},
\]

where the sum running over the $\mu_{fL}$ values that we have computed before.

6 Conclusions

We have introduced a new approach to generate dynamically the masses of elementary particles in the $SU(3)_C \times SU(2)_L \times U(1)_Y$ Standard Model without Higgs Sector (SMWHS). Our proposal have been founded on the two following ideas: (i) the ground state of a quantum fluid at zero temperature corresponds to the vacuum of quantum field theory \cite{20}, (ii) it is possible to reproduce some observed properties of the physical vacuum starting from a first principle microscopic theory that describes an elementary quantum fluid \cite{22}. Consequently, we have assumed that the effective masses of the particles in the elementary quantum fluid at zero temperature correspond to the rest masses of the elementary particles in the physical vacuum. These effective masses were obtained through radiative corrections, at one-loop order, in the context of the real time formalism of quantum field theory at finite temperature and density. The elementary quantum fluid was described in structure and dynamics by the SMWHS and it was characterized by non-vanishing chemical potentials associated to the different fermion flavour species ($\mu_{fi} \neq 0$). Subsequently we have introduce in the SMWHS a massless neutral scalar field leading to Yukawa coupling terms in the Lagrangian density. The effective masses have been obtained as function of the coupling constants and the unknown fermionic chemical potentials. Starting from the experimental mass values for quarks and leptons we have computed, as an evidence of the consistency of our approach, the values for the $W^\pm$ and $Z^0$ boson masses in agreement with the experimental values. We have found that if we use the $\mu_{fi}$ values obtained from the experimental fermion masses, the gauge boson masses are $M_W = 81.061 \pm 2.367$ GeV and $M_Z = 91.917 \pm 2.684$ GeV. For the case $Y = 0$, in which the scalar boson is absent in the effective model, we have obtained from $m_t = 172.916$ GeV the experimental central values $M_W = 80.419$ GeV and $M_Z = 91.188$ GeV. For the case $0 < Y < 0.607$, we have calculated the experimental mass central values for the $W^\pm$ and $Z^0$ gauge bosons using a mass value for the top quark mass in the range $169.2$ GeV < $m_t$ < 178.6 GeV. This range for the top quark mass implies that the scalar boson mass must be in the range $0 < M_H < 152$ GeV.
Acknowledgments

This work was supported by COLCIENCIAS (Colombia) and by Universidad Nacional de Colombia under research grant DIB 803629. We thank Rafael Hurtado, Yeinzon Rodríguez, Marta Losada, Maurizio De Sanctis, Carlos Avila and Germán Sinuco by stimulating discussions.

References

[1] J.F. Gunion, “Hunting the Higgs Boson(s)”, UCD-01-05, June 14, 2001 [hep-ph/0106154].

[2] B.A. Kniehl and A. Sirlin, Eur. Phys. J. C16 (2000) 635.

[3] C. Quigg, Acta Phys. Polon. B30 (1999) 2145.

[4] G. Altarelli, “The crucial problem: the electroweak symmetry breaking”, CERN-TH/99-365, November, 1999 [hep-ph/9912291].

[5] G. Degrassi, “Where is the Higgs”, DFPD-01/TH/05, February, 2001 [hep-ph/0102137].

[6] A.N. Okpara, “LEP-wide combination of the search for the standard model Higgs boson”, hep-ph/0105151, May, 2001.

[7] G. Ridolfi, “Search for the Higgs boson: theoretical perspectives”, GeF/TH/10-01, Juny, 2001. [hep-ph/0106300]

[8] S. Coleman and E. Weinberg, Phys. Rev. D 7 (1973) 1888.

[9] E. Farhi and L. Susskind, Phys. Rep. 74 (1981) 277.

[10] E. Farhi and R. Jackiw, eds., Dynamical gauge symmetry breaking, World Scientific (1982).

[11] J. A. Nogueira and P. L. Barbieri, “Boundary conditions as mass generation mechanism for real scalar fields”, hep-th/0108019, August, 2001.

[12] K. Babu, Phys. Lett. B115b (1982) 252.

[13] T.F. Treml, Phys. Rev D43 (1991) 1424.

[14] A. Romeo, J. Math. Phys. 34 (1993) 2206.
[15] B.L. Hu, 3rd Asia-Pacific Conf. Proceedings, Physics 1 (1988) 301.

[16] F. Wilczek, Int. J. Mod. Phys. A13 (1998) 863; Phys. Today 11 (1998) 11.

[17] F. Jegerlehner, Proceedings of 31st International Ahrenshoop Symposium on the Theory of Elementary Particles, Buckow, Germany, september 1997 [hep-th/9803021].

[18] R. Jackiw, Proc. Natl. Acad. Sc. USA 95 (1998) 12776.

[19] H.B.G. Casimir, Proc. K. Ned. Akad. Wet. 51 (1948) 793.

[20] R. Golestanian and M. Kardar, Phys. Rev. Lett. 78 (1997) 3421; Rev. Mod. Phys. 71 (1999) 1233.

[21] G.E. Volovik, Pisma Zh. Eksp. Teor. Fiz. 73 (2001) 419.

[22] R.B. Laughlin and D. Pine, Proc. Natl. Acad. Sc. USA 97 (2000) 28; C. Chaplin, E. Hohlfeld, R.B. Laughlin and D.I. Santiago, Phil. Mag. B 81 (2001) 235 [gr-qc/0012094].

[23] G.E. Volovik, “Vacuum in quantum liquids and in general relativity”, gr-qc/0104046.

[24] H.A. Weldon, Phys. Rev. D26 (1982) 2789.

[25] R. L. Kobes, G. W. Semenoff and N. Neiss, Z. Phys C29 (1985) 371.

[26] H.A. Weldon, Physica A 158 (1989) 169.

[27] C. Quimbay and S. Vargas-Castrillon, Nucl. Phys. B451 (1995) 265.

[28] J. Morales, C. Quimbay and F. Fonseca, Nucl. Phys. B560 (1999) 601.

[29] K. Kajantie and P.V. Ruuskanen, Phys. Lett. B121 (1983) 352.

[30] E.J. Levinson and D.H. Boal, Phys. Rev. D31 (1985) 3280.

[31] J.P. Blaizot and J.Y. Ollitrault, Phys. Rev. D48 (1993) 1390.

[32] M. Le Bellac, Thermal Field Theory. Cambridge monographs on mathematical physics, Cambridge University Press, 1996.

[33] H.A. Weldon, Phys. Rev. D26 (1982) 1394.

[34] E. Braaten and D. Segel, Phys. Rev. D48 (1993) 1478.
[35] T. Altherr and U. Kraemmer, Astropart. Phys. 1 (1992) 133.

[36] S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264.

[37] J. Pestieau and P. Roy, Phys. Rev. Lett. 23 (1969) 349; Lett. Nuovo Cimento 31 (1981) 625.

[38] D. E. Groom et al., Particle Data Group, Eur. Phys. J. C15 (2000) 1.
Figure 1: One-loop diagram contributions to the fermionic self-energy

Figure 2: One-loop diagram contributions to the polarization tensors

Figure 3: Dependence between the scalar boson and top quark masses