Parameterized Problems Complete for Nondeterministic FPT time and Logarithmic space

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This talk

- Parameterized Complexity
- Complexity of Bandwidth
- Complexity class XNLP
- XNLP-complete problems
- Corollaries of XNLP-completeness
Parameterized Complexity

1990s: Downey and Fellows:

- $f(k) \cdot \text{poly}(n)$ time
- $n^{f(k)}$ time

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**Bandwidth**

**Given:** Graph $G = (V, E)$ and $k \in \mathbb{N}$.

**Goal:** Permute the vertices ($p(v) \in \{1, \ldots, n\}$) such that $\forall \{v, w\} \in E$:

$$|p(v) - p(w)| \leq k$$

Reordering of **symmetric matrices**, such that:
nonzero elements around diagonal
Bandwidth

• Claim [B, Fellows, Hallett, 1994]: Bandwidth is $W[t]$-hard $\forall t \in \mathbb{N}$

• Conjecture [Hallett, ~1994]: Bandwidth not in $W[P]$

• Theorem [Dregi, Lokshtanov, 2014]: Bandwidth is $W[1]$-hard for trees

• Theorem [B, 2020]: Bandwidth is $W[t]$-hard $\forall t \in \mathbb{N}$ for caterpillars

\[ \text{N}[f \text{ poly}, f \log] \text{ in [Elberfeld et al. 2015]} \]
Complexity classes: \( \text{L, NL, XL, XNL, XNLP} \)

- **L**: \( \log(n) \) space
- **NL**: nondeterministic \( \log(n) \) space \( \Rightarrow \) \( \text{poly}(n) \) time
- **XL**: \( f(k) \cdot \log(n) \) space
- **XNL**: nondeterministic \( f(k) \cdot \log(n) \) space \( \not\Rightarrow f(k) \cdot \text{poly}(n) \) time!
- **XNLP**: nondeterministic \( f(k) \cdot \log(n) \) space and \( f(k) \cdot \text{poly}(n) \) time
**XNLP-complete problems, known results**

- Elberfeld, Stockhusen and Tentau in 2015
  - Non-deterministic Turing Machine,
    - Worktape of $O(k)$ cells,
    - Alphabet of size $\text{poly}(n)$,
    - Running time bounded by $f(k) \cdot \text{poly}(n)$.
  - Timed Non-deterministic Cellular Automaton
  - Longest Common Substring
XNLP-complete problems, our results
Chained Weighted CNF-Satisfiability

\( F_1(X_1, X_2) \)

\( F_2(X_2, X_3) \)

\( F_3(X_3, X_4) \)

\( F_4(X_4, X_5) \)

\( \ldots \)

\( F_{r-1}(X_{r-1}, X_r) \)

\( (x_{11} \lor x_{21}) \land (x_{13} \lor x_{22} \lor x_{23}) \land \ldots \)
Chained Weighted CNF-Satisfiability

\[ k = \text{#true assignments per set} \]

\[ F_1(X_1, X_2) \quad \checkmark \]
\[ F_2(X_2, X_3) \quad \checkmark \]
\[ F_3(X_3, X_4) \quad \checkmark \]
\[ F_4(X_4, X_5) \quad \checkmark \]
\[ \vdots \]
\[ F_{r-1}(X_{r-1}, X_r) \quad \checkmark \]
Corollaries of XNLP-completeness

Consequence:
If problem $\mathcal{P}$ is XNLP-hard, then $\mathcal{P}$ is $W[t]$-hard for all $t \in \mathbb{N}$.

• Strengthens some known results:
  • List Coloring (pw)
  • Scheduling with Precedence Constraints
• May give easier proofs (e.g. Bandwidth)
Corollaries of XNLP-completeness

**Conjecture** [Pilipczuk and Wrochna 2018]:
Longest Common Subsequence has no algorithm in $n^{f(k)}$ time and $f(k) \cdot n^c$ space.

**Corollary (if true):**

**Conjecture:**
No XNLP-complete problem with algorithm in $n^{f(k)}$ time and $f(k) \cdot n^c$ space.
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