**tbW vertex in the**

**Littlest Higgs Model with T-parity**

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A study of the effective \(tbW\) vertex is done in the littlest Higgs model with T parity that includes the one loop induced weak dipole coefficient \(f_{2R}\). The top’s width, the \(W\)-boson helicity in the \(t\to bW^+\) decay as well as the t-channel and the s-channel modes of single top quark production at the LHC are then obtained for the \(tbW\) coupling. Our calculation is done in the Feynman-’t Hooft gauge, and we provide details of the analysis, like exact formulas (to all orders of the expansion variable \(v/f\)) of masses and mixing angles of all the particles involved. Also, a complete and exact diagonalization (and normalization) of the scalar sector of the model is made.

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**I. INTRODUCTION**

The Top quark plays a major role in the research program of the LHC. The top is the only quark that decays Weakly before hadronization, therefore we have an opportunity to study *bare* quark properties like spin, mass and couplings\([1, 2]\). Recent measurements of the single top quark production as well as the W-helicity in the \(t\to bW^+\) decay have been made by the D0 and CDF groups at the Tevatron and these have (for the first time) set direct constraints on the effective \(tbW\) vertex\([3]\). On the other hand, the high production of top quarks at the LHC will make it possible to probe directly this vertex down to a few percent deviation level for the left handed coefficient \(f_{1L}\) and to set limits of order \(10^{-2}\) for \(f_{2R}\), and of order \(10^{-1}\) for the right handed \(f_{1R}\) and \(f_{1L}\)\([4]\). From the theoretical standpoint, observables that depend directly on the \(tbW\) coupling like single top production, the top’s
width and the W-helicity in top’s decay have been studied in models beyond the SM like the minimal supersymmetric standard model \cite{5} and the Topcolor assisted Technicolor (TC2) model \cite{6}.

In the Standard Model (SM) the Higgs boson receives large quadratic divergent corrections from the heavy gauge bosons and from this fermion. Models beyond the SM are studied that alleviate this problem, two important examples are Supersymmetry and Technicolor (and TC2) \cite{7}. Another possible solution is provided by the recently proposed Little Higgs Models \cite{8,9} (for a review see Ref. \cite{10}). In these models the quadratic divergent Higgs mass corrections get canceled at the one loop level via the contribution from certain (very heavy) partners of the gauge bosons and the top quark (i.e. the $W^\pm_H$, the $Z_H$, and the $T$ quark). One explicit model has become well known, and it is called the “Littlest Higgs Model” (LH) \cite{9}. The LH model is based on a non-linear sigma model of an $SU(5)/SO(5)$ global symmetry breaking. It consists of two $SU(2) \times U(1)$ gauge symmetries that break down to the SM gauge symmetry at a certain scale $f$. The phenomenology of the model deals with heavy partners of the SM gauge bosons, like $W^\pm_H$, $Z_H$ and a heavy photon $A_H$, as well as a heavy partner of the top quark $T$ \cite{11}. These heavy partners mix with the lighter SM gauge bosons and this gives rise to tree level contributions to precision electroweak observables. Therefore, strong constraints have greatly limited the parameter range of the model (for instance: $f \geq 4$ TeV) \cite{12}. A way out of this obstacle is given by implementing a new symmetry called T-parity, where T-parity even and T-parity odd particles do not mix \cite{13}. There is one model that is often studied in the literature; it is based on the previous LH model and is known as the Littlest Higgs Model with T-parity (LHT) \cite{14}. Electroweak precision constraints for the LHT model allow the scale $f$ to be as low as $\sim 500$ GeV \cite{15}. This model has therefore received more attention recently, with many phenomenological studies on production and decays of the new heavy particles \cite{16} as well as theoretical studies such as T-parity violation \cite{17}, top quark induced vacuum alignment \cite{18}, and two vacuum expectation value (VEV) scales $f$ in LH models \cite{19}.

In this paper we study the $tbW$ vertex in the context of the Littlest Higgs Model with T-parity (LHT). We will often refer to and will use the notation of Ref. \cite{20}. A detailed explanation of the model can be found in Refs. \cite{20,21}. In this work we focus on the interactions that are relevant to the study of the effective $tbW$ vertex.

In the literature an expansion in powers of $\epsilon = v/f$ is usually made for the masses
and mixing angles derived from the Lagrangian of the model. Here, we have obtained the exact (all powers in $\epsilon \equiv v/f$) formulas for masses and mixings. Similar expressions have already appeared in Ref. [22] and we have found agreement. Moreover, we provide in detail the diagonalization procedure of the scalar sector, including the Goldstone bosons that are eaten by the gauge bosons and that participate in the one-loop calculation as is done in the Feynman-t’Hooft gauge. We provide Feynman rules that are not found in previous studies of the model.

The next section has the brief presentation of the LHT Lagrangians (Kinetic and Yukawa) and the definition of mass eigenstate fields in terms of the original interaction eigenstates. Then, in the following section, we will discuss the effective $tbW$ vertex obtained from tree and one-loop level contributions. From this effective vertex we compute some of the observables associated to the top quark, like the top’s decay width, the $W$-boson helicity in the $t \rightarrow bW^+$ decay, the single top production process in the two most important modes: the t-channel and the s-channel.

II. THE LITTLEST HIGGS MODEL WITH T-PARITY.

The LHT model is based on a non-linear sigma model for an $SU(5)/SO(5)$ symmetry breaking. The non-linear $\Sigma$ field is given as

$$\Sigma = e^{2i\Pi/f_0}, \quad \text{with} \quad \Sigma_0 = \begin{pmatrix} 0_{2\times 2} & 0_{2\times 1} & 1_{2\times 2} \\ 0_{1\times 2} & 1 & 0_{1\times 2} \\ 1_{2\times 2} & 0_{2\times 1} & 0_{2\times 2} \end{pmatrix}. \quad (1)$$

where $f \sim O(1)$ TeV is the symmetry breaking scale known as the “pion decay constant”. The “pion matrix” contains a total of 14 pion fields [15]:

$$\Pi = \begin{pmatrix} -\omega^0/2 - \eta/\sqrt{20} & -\omega^+/\sqrt{2} & -i\pi^+/\sqrt{2} & -i\phi^{++} & -i\phi^+/\sqrt{2} \\ -\omega^-/\sqrt{2} & \omega^0/2 - \eta/\sqrt{20} & (v + h + i\pi^0)/2 & -i\phi^+ / \sqrt{2} & (-i\phi^0 + \phi^0_P)/\sqrt{2} \\ i\pi^-/\sqrt{2} & (v + h - i\pi^0)/2 & \sqrt{4/5}\eta & -i\pi^+ / \sqrt{2} & (v + h + i\pi^0)/2 \\ i\phi^- & i\phi^- / \sqrt{2} & i\pi^- / \sqrt{2} & -\omega^0/2 - \eta/\sqrt{20} & -\omega^- / \sqrt{2} \\ i\phi^-/\sqrt{2} & (i\phi^0 + \phi^0_P)/\sqrt{2} & (v + h - i\pi^0)/2 & -\omega^+ / \sqrt{2} & \omega^0/2 - \eta/\sqrt{20} \end{pmatrix} \quad (2)$$

Seven of these fields get eaten by the gauge bosons of the model. The other seven become physical, in particular the $h$ field becomes the (little) Higgs field whose mass is protected
from quadratic divergencies by the collective symmetry breaking mechanism of the Little Higgs model.\[9\]

An \[SU(2) \times U(1)]^2\] subgroup of the global \(SU(5)\) symmetry is gauged. The gauged generators have the form

\[
Q_1^a = \begin{pmatrix} \sigma^a/2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad Y_1 = \text{diag}(3, 3, -2, -2, -2)/10, \\
Q_2^a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\sigma^{a*}/2 \end{pmatrix}, \quad Y_2 = \text{diag}(2, 2, 2, -3, -3)/10.
\]

The kinetic term for the \(\Sigma\) field can be written as

\[
\mathcal{L}_{\text{kin}} = \frac{f^2}{8} \text{Tr} D_\mu \Sigma (D^\mu \Sigma)^\dagger,
\]

where

\[
D_\mu \Sigma = \partial_\mu \Sigma - i \sum_j \left[ g_j W^a_j (Q^a_j \Sigma + \Sigma Q^{a^*}_j) + g_j^1 B_j (Y_j \Sigma + \Sigma Y_j) \right],
\]

with \(j = 1, 2\). Here, \(B_j\) and \(W^a_j\) are the \(U(1)_j\) and \(SU(2)_j\) gauge fields, respectively, and \(g_j^1\) and \(g_j\) are the corresponding coupling constants. The vev \(\Sigma_0\) breaks the extended gauge group \([SU(2) \times U(1)]^2\) down to the diagonal subgroup, which is identified with the standard model electroweak group \(SU(2)_L \times U(1)_Y\).

The field \(H\) has the appropriate quantum numbers to be identified with the SM Higgs; after electroweak symmetry breaking (EWSB), it can be decomposed as \(H = (-i\pi^+, v + h + i\pi^0)/\sqrt{2}\), where \(v = 246\) GeV is the EWSB scale.

The Lagrangian in Eq. (1) is invariant under T-parity provided that \(g_1 = g_2 (\equiv \sqrt{2}g)\) and \(g_1' = g_2' (\equiv \sqrt{2}g')\). The T-parity gauge boson eigenstates (before EWSB) have the simple form, \(W_\pm = (W_1 \pm W_2)/\sqrt{2}\), \(B_\pm = (B_1 \pm B_2)/\sqrt{2}\), where \(W_+\) and \(B_+\) are the standard model gauge bosons and are T-even, whereas \(W_-\) and \(B_-\) are the additional, heavy, T-odd states. From now on we will denote \(W_-\) and \(B_-\) as \(W_H\) and \(B_H\), whereas \(W_+\) and \(B_+\) will be written simply as \(W\) and \(B\). After EWSB, the T-even neutral states \(W^3\) and \(B\) mix to produce the SM \(Z\) boson and the photon. Since they do not mix with the heavy T-odd states, the Weinberg angle is given by the usual SM relation, \(\tan \theta_w = g'/g\), and
no corrections to precision electroweak observables occur at tree level. The mixing for the neutral gauge boson mass eigenstates is written as

\[
\begin{pmatrix}
Z \\
A
\end{pmatrix} = \begin{pmatrix}
c_w & -s_w \\
s_w & c_w
\end{pmatrix} \begin{pmatrix}
W^3 \\
B
\end{pmatrix}, \quad \begin{pmatrix}
Z_H \\
A_H
\end{pmatrix} = \begin{pmatrix}
c_H & s_H \\
-s_H & c_H
\end{pmatrix} \begin{pmatrix}
W_H^3 \\
B_H
\end{pmatrix},
\]

(6)

where \( s_w = \sin \theta_w \approx \sqrt{0.223} \) refers to the SM mixing angle, and \( s_H = \sin \theta_H \) refers to the heavy boson mixing angle. \( \tan \theta_H (\equiv t_H) \) must satisfy the equation:

\[
t_H^2 + 2a_H t_H - 1 = 0, \quad \text{with} \quad a_H = 4 \left( \frac{1}{t_w} - \frac{t_w}{5} \right) / s_v^2 - \left( \frac{1}{t_w} - t_w \right) / 2,
\]

(7)

where \( t_w = s_w / c_w \approx 0.536 \) and

\[
s_v \equiv \sin \sqrt{2 \epsilon}, \quad c_v \equiv \cos \sqrt{2 \epsilon} \quad \text{with} \quad \epsilon \equiv \frac{v}{f}.
\]

(8)

With this value of \( t_w \) we obtain

\[
\tan \theta_H = \sqrt{a_H^2 + 1 - a_H} \approx 0.142 \epsilon^2 - 0.068 \epsilon^4
\]

(9)

The gauge boson mass terms obtained from Eq. (4) are as follows:

\[
M_W^2 = \left( \frac{f g}{2} \right)^2 (1 - c_v)
\]

\[
M_Z^2 = \left( \frac{f g}{2 c_w} \right)^2 (1 - c_v)
\]

\[
M_{W_H}^2 = \left( \frac{f g}{2} \right)^2 (3 + c_v)
\]

\[
M_{Z_H}^2 = (f g)^2 \left( 1 - \frac{s_v^2}{8} + \delta_H \right)
\]

\[
M_{A_H}^2 = (f g)^2 \left( t_w \left( \frac{1}{5} - \frac{s_v^2}{8} \right) - \delta_H \right)
\]

\[
\delta_H \equiv \frac{t_H c_H^2}{8} \left( 2 s_v^2 t_w + t_H \left[ -8 + t_w \left( \frac{8}{5} - s_v^2 \right) + s_v^2 \right] \right)
\]

(10)

Similar expressions are given in Ref. [22]; our formulas are presented differently but agree with theirs (our mixing angle \( \theta_H \) differs in sign).

**A. The Yukawa couplings for quarks**

The Little Higgs model introduces a heavy partner of the top quark (the heavy Top quark) with the purpose of cancelling out the one loop quadratic divergent radiative corrections to
the Higgs mass \( \tilde{\sigma} \). When T-parity is implemented in the fermion sector of the model we require the existence of mirror partners for each of the original fermions. This means that for the third family we have, in addition to the usual bottom and top quarks, the mirror bottom, the mirror top; as well as the heavy Top quark with its own mirror quark.

The T-parity invariant Yukawa Lagrangian of the LHT model is separated into four parts that generate masses for mirror quarks, down type quarks, first two generations of up type quarks and finally the top quark and its heavy partner. It is the latter that is defined in such a way that the top quark quadratic radiative corrections to the Higgs mass are canceled. We are only interested in the Yukawa Lagrangian of the third family. A presentation that includes first two families and the corresponding Cabibbo-Kobayashi-Maskawa mixing can be found in Ref. [23]. For the purpose of our work we consider only the mirror and top quark Yukawa Lagrangians [20]:

\[
\mathcal{L}_\kappa = -\kappa f \left( \bar{\Psi}_2 \xi \Psi_c + \bar{\Psi}_1 \Omega \xi^\dagger \Omega \Psi_c \right) + \text{h.c.}
\]

\[
\mathcal{L}_{\text{down}} = \frac{i\lambda_d}{2\sqrt{2}} \epsilon_{ijk} \epsilon_{xy} \left[ \Psi_2^T \Sigma_{ijy} \Sigma_{jxz} X - \left( \bar{\Psi}_1^T \Sigma_0 \right)_x \Sigma_{iyz} \Sigma_{jxz} \bar{X} \right] d_R^+ 
\]

\[
\mathcal{L}_t = -\frac{\lambda_1 f}{2\sqrt{2}} \epsilon_{ijk} \epsilon_{xy} \left[ \left( \bar{Q}_1 \right)_i \Sigma_{jxz} \Sigma_{jky} - \left( \bar{Q}_2 \Sigma_0 \right)_i \Sigma_{jxz} \bar{\Sigma}_{jky} \right] u_R^+ 
= -\lambda_2 f \left( \bar{U}_L U_{R1} + \bar{U}_L U_{R2} \right) + \text{h.c.} ,
\]

where \( \xi \equiv e^{iM/f} \), \( \Omega \equiv \text{diag}\{1,1,-1,1,1\} \) and \( \bar{\Sigma} \equiv \Sigma_0 \Omega \Sigma^\dagger \Omega \Sigma_0 \). \( \epsilon_{ijk} \) and \( \epsilon_{xy} \) are antisymmetric tensors where \( ijk = 1,2,3 \) and \( xy = 4,5 \). The Lagrangian \( \mathcal{L}_{\text{down}} \) that gives mass to the bottom quark will be not be used in our calculation as we take \( \lambda_d \equiv 0 \). The details of this Lagrangian can be found in Ref. [20]. Nevertheless, we provide Feynman rules for \( \lambda_d \neq 0 \).

In order to obtain the (exact) expressions for masses and mixings we will use the vev of the field \( \Sigma \) as given in Eq. (A2), as well as the vev of the \( \xi \) field:

\[
\langle \xi \rangle_0 = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & \frac{1+c_v'}{2} & \frac{i}{\sqrt{2}} s_v' & 0 & \frac{c_v'-1}{2} \\
0 & \frac{i}{\sqrt{2}} s_v' & c_v' & 0 & \frac{i}{\sqrt{2}} s_v' \\
0 & 0 & 0 & 1 & 0 \\
0 & \frac{c_v'-1}{2} & \frac{i}{\sqrt{2}} s_v' & 0 & \frac{1+c_v'}{2} 
\end{pmatrix},
\]

where \( c_v' \equiv \cos (\epsilon/\sqrt{2}) \) and \( s_v' \equiv \sin (\epsilon/\sqrt{2}) \). The \( u_R^+, U_{R1} \) and \( U_{R2} \) quark fields are right handed \( SU(2) \) singlets. The upper plus sign in \( u_R^+ \) denotes that it is a T-even (T-parity
eigenstate) fermion. With the other two \((U_{R_1}^R, U_{R_2}^R)\) we can define \(T\)-even and \(T\)-odd linear combinations: \(U_R^\pm \equiv (U_{R_1}^R \mp U_{R_2}^R)/\sqrt{2}\).\(^{20}\)

On the other hand, the \(\Psi\) and \(Q\) fields are left handed \(SU(5)\) multiplets defined as:

\[
\Psi_1 = \begin{pmatrix}
  id_1 \\
  -iu_1 \\
  0 \\
  id_2 \\
  -iu_2
\end{pmatrix}, \quad 
\Psi_2 = \begin{pmatrix}
  0 \\
  0 \\
  id_1 \\
  0 \\
  id_2
\end{pmatrix}, \quad 
Q_1 = \begin{pmatrix}
  id_1 \\
  -iu_1 \\
  U_{L_1} \\
  0 \\
  0
\end{pmatrix}, \quad 
Q_2 = \begin{pmatrix}
  0 \\
  0 \\
  U_{L_2} \\
  id_2 \\
  -iu_2
\end{pmatrix} \tag{13}
\]

From these we define the \(T\)-parity eigenstates \(u_L^\pm \equiv (u_1 \mp u_2)/\sqrt{2}\), \(d_L^\pm \equiv (d_1 \mp d_2)/\sqrt{2}\), and \(U_L^\pm \equiv (U_{L_1} \mp U_{L_2})/\sqrt{2}\).

The \(\Psi_c\) multiplet is composed of 5 right handed \(T\)-odd quark fields:

\[
\Psi_c \equiv \begin{pmatrix}
  ib^- \\
  -ia^- \\
  \chi^- \\
  in^- \\
  -ip^-
\end{pmatrix}^T \tag{14}
\]

It turns out that two linear combinations of these become the right handed mass eigenstates of the mirror top and mirror bottom quarks. The other three linear combinations are extra \(T\)-odd fermions that are assumed to have very large Dirac masses, so that they decouple from the main theory\(^{14,21}\).

Below we write down the mass eigenstates that arise from the Lagrangian in Eq. (11) for the top and its heavy partner:

\[
\begin{pmatrix}
  t^+_{L(R)} \\
  T^+_{L(R)}
\end{pmatrix} = \begin{pmatrix}
  c_{L(R)} & -s_{L(R)} \\
  c_{L(R)} & s_{L(R)}
\end{pmatrix} \begin{pmatrix}
  u^+_{L(R)} \\
  U^+_{L(R)}
\end{pmatrix}, \tag{15}
\]

where \(c_{L(R)} = \cos \theta_{L(R)}\), \(s_{L(R)} = \sin \theta_{L(R)}\), and \(\theta_{L(R)}\) is the mixing angle of the left (right) top and heavy Top quarks.

The mixing angles must satisfy two equations, which we write in terms of \(\tan \theta_{L(R)}\):

\[
\begin{align*}
  r(1 + c_v) + \sqrt{2} r s_v \tan \theta_L - 2 \tan \theta_R &= 0 \\
  \sqrt{2} r s_v \tan \theta_R - 2 \tan \theta_L - r(1 + c_v) \tan \theta_L \tan \theta_R &= 0,
\end{align*} \tag{16}
\]

with \(r \equiv \lambda_1/\lambda_2\).

The solutions of these equations are:

\[
\tan \theta_L = \sqrt{a_m^2 + 1 - a_m},
\]

\[
\tan \theta_R = \sqrt{a_m^2 + 1 - a_m},
\]

\[
\tan \theta_L = \sqrt{a_m^2 + 1 - a_m},
\]

\[
\tan \theta_R = \sqrt{a_m^2 + 1 - a_m},
\]
\[
\tan \theta_R = \frac{r}{2} \left( 1 + c_v + \sqrt{2} s_v \tan \theta_L \right),
\]
\[
a_m \equiv \frac{1 + c_v - 3 s^2_v/2 + 2/r^2}{\sqrt{2} s_v (1 + c_v)}.
\]

The masses of the top and its heavy partner are:
\[
m_{t^+} = \frac{1}{2} f \lambda_2 c_L c_R \left[ 2 \tan \theta_L \tan \theta_R + r(\sqrt{2} s_v - (1 + c_v) \tan \theta_L) \right]
\]
\[
m_{T^+} = \frac{1}{2} f \lambda_2 c_L c_R \left[ 2 + r \tan \theta_R (1 + c_v + \sqrt{2} s_v \tan \theta_L) \right]
\]

Expanding in powers of \(\epsilon\):
\[
\tan \theta_L \simeq \frac{\beta^2}{1 + \beta^2} \epsilon + \frac{\beta^4 + 2 \beta^2 - 5 \beta^6}{6(1 + \beta^2)^3} \beta^2 \epsilon^3,
\]
\[
\tan \theta_R \simeq \beta \epsilon + \frac{\beta^2 - 1 \beta^2}{1 + \beta^2} \frac{1}{2} \epsilon^2.
\]
\[
m_{t^+} \simeq \frac{f \lambda_1}{\sqrt{1 + \beta^2}} \left( \epsilon - \frac{2 + \beta^2 + 2 \beta^4}{6(1 + \beta^2)^2} \epsilon^3 \right)
\]
\[
m_{T^+} \simeq f \lambda_2 \sqrt{1 + \beta^2} \left( 1 - \frac{\beta^2}{2(1 + \beta^2)^2} \epsilon^2 \right)
\]
\[
\cos \theta_L \equiv c_L \simeq 1 - \frac{\beta^4}{2(1 + \beta^2)^2} \epsilon^2.
\]

(17)

Our formulas for the mixing angles and masses are in agreement with those of Ref. [22], where \(\alpha \equiv \theta_R\) and \(\beta \equiv \theta_L\).

The T-odd top and heavy Top quarks are defined as:
\[
t_{L}^- = u_{L}^- , \quad T_{L}^- = U_{L}^- ,
\]
\[
t_{R}^- = p_{R}^- , \quad T_{R}^- = U_{R}^- .
\]

(18)

Where the \(p'\) field comes from the redefinition of the right handed T-odd fields of Eq. (14):
\[
\begin{pmatrix}
p_{R}^- \cr
a_{R}^- \cr
\chi_{R}^-
\end{pmatrix}
= \begin{pmatrix}
\frac{1+c'_v}{2} & \frac{c'_v-1}{2} & -\frac{1}{\sqrt{2}} s'_v \\
\frac{1+c'_v}{2} & \frac{1+c'_v}{2} & \frac{1}{\sqrt{2}} s'_v \\
\frac{1}{\sqrt{2}} s'_v & \frac{1}{\sqrt{2}} s'_v & c'_v
\end{pmatrix}
\begin{pmatrix}
p^- \cr
a^- \cr
\chi^-
\end{pmatrix} .
\]

(19)

Notice that the mass of \(t^-\) comes from the mirror fermion Lagrangian, whereas the mass of \(T^-\) comes from the top quark Lagrangian (see Eq. (11)):
\[
m_{t^-} = \sqrt{2} f \kappa
\]
\[
m_{T^-} = f \lambda_2 .
\]
For the calculations in this work we will set $\kappa = 1$ so that the masses of mirror fermions are just $\sqrt{2} f$. The presence of the LHT mirror fermions is vital for the good high energy behaviour of the model, in particular they play an essential role in the scattering process $u\bar{u} \rightarrow W^+_H W^-_H$ [20]. Our choice of $\kappa = 1$ and the corresponding values of the T-odd fermion mass respects the unitarity bounds of this process, as well as the limits coming from the contributions to the four fermion contact interaction $e^+ e^- \rightarrow q\bar{q}$ [21].

For completeness, let us write down the masses of the T-even and T-odd bottom quarks. The mass of $b^+$ is given by the down type Yukawa Lagrangian given in Eq. (11). (See Ref. [20]):

$$m_{b^+} = \frac{f}{\sqrt{2}} \lambda_d s_v c_v^{-1/4}$$
$$m_{b^-} = \sqrt{2} f \kappa.$$  

Notice that the formulas we have obtained are exact (to all orders in the $\epsilon$ expansion); in particular, the mirror fermion masses are equal for $t^-$ and $b^-$ quarks. We remind the reader that in our calculation we take the mass of $b^+$ as zero ($\lambda_d \equiv 0$). Feynman rules with the mass eigenstates can be found in Appendix B.

III. THE $\bar{b}bW^+$ COUPLING IN THE LHT MODEL.

Let us define the effective $\bar{b}bW^+$ coupling as follows:

$$\mathcal{L}_{\bar{b}bW} = \frac{g}{\sqrt{2}} W^-_\mu \bar{b} \gamma^\mu (f_{1L} P_L + f_{1R} P_R) t$$
$$- \frac{g}{\sqrt{2} M_W} \partial_\mu W^-_\mu \bar{b} \sigma^{\mu\nu} (f_{2L} P_L + f_{2R} P_R) t + h.c.,$$  \hspace{1cm} (20)

where we have used the mass scale $m_W$ that is also used in the literature [24, 25, 26].

In the SM the values of the form factors at tree level are $f_{1L} = V_{tb} \approx 1$, $f_{1R} = f_{2L} = f_{2R} = 0$. Radiative corrections to the factors $f_{1R}$ and $f_{2L}$ must be zero if we neglect the mass of the bottom quark. We take $m_b \equiv 0$ in this work, so we set $f_{1R} = f_{2L} \equiv 0$ for this study. These couplings can be probed by studying the top decay $t \rightarrow bW^+$ and the single top production processes [24, 27]. The dimension five coupling $f_{2R}$ is different from zero at one loop: we obtain $f_{2R}^{\text{SM}} = 0.00201 (0.00214)$ for $m_H = 120 (150)$ GeV. This value seems to be too small to be probed at the LHC [4]. In fact, the dominant radiative corrections for the top width or single top production comes from QCD [27]. We would like to know if the
FIG. 1: Some Feynman diagrams that give rise to the $f_{2R}$ coupling of the $tbW$ vertex in the LHT model.

coefficient $f_{2R}$ predicted by the LHT model could be large enough to be measured at the LHC.

In the LHT model, the coefficient $f_{1L}$ is modified at tree level by the $tT$ mixing angle $\theta_L$ ($f_{1L} = c_L$). The tree level $tbW$ vertex is reduced by the factor $c_L$ and this translates into a lower production at the LHC [28].

We have performed the one loop contribution to $f_{2R}$ in the LHT model. We have worked in the Feynman-t’Hooft gauge, where there are a total of 47 diagrams to compute if we take the bottom quark mass as zero. Some of the diagrams are shown in Figure 1. (We provide the Feynman rules necessary for such a calculation in Appendix B.) Notice that the Goldstone bosons in the original Lagrangian have to be diagonalized and normalized. We have done all this exactly (at all orders in powers of $\epsilon$) in Appendix A. The exponential expansion in the Lagrangians of Eqs. (4) and (11) generate vertices of dimension 4 and higher that contribute at one loop to $f_{2R}$. As it turns out, the contribution from the dimension 4 terms to the $tbW$ vertex are finite, whereas the contribution from the higher dimension terms is divergent. This is no surprise because the LHT model is a non-renormalizable effective low energy model with a cut-off scale ($\Lambda \sim 4\pi f$). In principle, all the operators that are consistent with the symmetries of the LHT model should be considered [15]. In our study,
we disregard effects from higher dimension terms and keep only the contribution from the dimension 4 couplings that render a finite result [29].

Concerning the specific numerical values used for the parameters of the model, we have chosen values from the allowed region of $f$ vs $\lambda_2$ that is shown in Fig. 1 of Ref. [22]. The mass of the top quark is taken $m_t = 173$ GeV, and this sets the value of $\lambda_1 \simeq 1$ (with a very small dependence on the value of $f$). The masses of the $t^-$ and $b^-$ mirror quarks is taken as $\sqrt{2}f$. The masses of the physical T-odd scalars $\phi'^\pm$, $\phi'^0$, and $\phi^0$ are taken as $\sqrt{2}m_H f/v$ ($= 0.69f$ for $m_H = 120$ GeV) as it is done in the literature [11, 21]. As it is well known, in the Feynman-t’Hooft gauge the masses of Goldstone bosons ($\pi'^\pm$, $\pi'^0$, $\omega'^\pm$, $\omega'^0$ and $\eta'$) are equal to the masses of their corresponding gauge bosons which are given in Eq. (10). We have chosen a range of the scale $f$ between 550 GeV and 1550 GeV. Smaller values of $f$ are prohibited by the low energy data [15]. Higher values are allowed but not interesting as the value of $f_{2R}$ remains essentially constant above the 1.5 TeV scale.

![Graph showing the variation of $f_{2R}$ for $m_H = 120$ GeV](image)

**FIG. 2:** The $f_{2R}$ variation for $m_H = 120$ GeV.

The variation of $f_{2R}$ from $f_{2R}^{SM}$ as predicted by the LHT model turns out to be of the order expected by a one loop correction. In fact, $\Delta f_{2R}/f_{2R}^{SM}$ is under 20% for the allowed values of the scale $f$ and the Yukawa coefficient $\lambda_2$. We show the variation of $f_{2R}$ as a function of the scale $f$ in Fig. 2. We also show in Fig. 3 how the variation in $f_{2R}$ diminishes with higher values of the Higgs mass. In contrast to the SM the LHT model allows for higher values of $m_H$ and is still compatible with the electroweak precision data [15], however we have found
that for bigger $m_H$ the deviation in $f_{2R}$ gets smaller and thus less interesting. (Observe in Fig. 1 that the Higgs field also appears in the contributions from the LHT heavy states.) From now on we will assume a fixed value $m_H = 120$ GeV.

It is possible to obtain the variation in the top width, the W-helicity in the $t \rightarrow bW^+$ decay, as well as the s and t channels of the single top production processes once we have the effective $tbW$ coupling. A general analysis of this coupling and the observables mentioned has been done in Ref. [24]. Let us apply this approach to the effective $tbW$ coupling as predicted by the LHT model. The total $t \rightarrow bW^+$ decay width of the top quark can be written as a sum of the contributions from each of the three polarizations of the $W^+$ boson:

$$\Gamma_t = \Gamma_0 + \Gamma_- + \Gamma_+$$

$$= \frac{g^2 m_t}{64 \pi} \frac{(a_t^2 - 1)^2}{a_t^4} \left( a_t^2 L_0^2 + 2T_m^2 + 2T_p^2 \right),$$

$$L_0^2 = (f_{1L} + f_{2R}/a_t)^2 + (f_{1R} + f_{2L}/a_t)^2,$$

$$T_m^2 = (f_{1L} + a_t f_{2R})^2,$$

$$T_p^2 = (f_{1R} + a_t f_{2L})^2,$$

$$a_t = \frac{m_t}{m_W}. \tag{21}$$

From this expression we define the W-helicity ratios

$$f_0 \equiv \frac{\Gamma_0}{\Gamma_t}, \quad f_- \equiv \frac{\Gamma_-}{\Gamma_t}, \quad \text{and} \quad f_+ \equiv \frac{\Gamma_+}{\Gamma_t}.$$
Notice that the $T_p$ coefficient is zero for $m_b = 0$. However, we are including it here for the sake of completeness. For $f_+ = 0$ we have that $f_0 + f_-$ must be equal to one. Therefore, it is only necessary to study one of them. In this work we show the deviation in $f_-$ predicted by the LHT model.

It is convenient to define the following effective terms:

$$x_0 = L_0^2 - 1, \quad x_5 = a_t^2(f_{2R} + f_{2L})^2,$$

$$x_m = T_m^2 - 1, \quad x_p = T_p^2.$$  \hfill (22)

Then, the W-helicity ratios and the single top production cross section for the s and t channels are given by

$$f_0 = \frac{a_t^2(1 + x_0)}{a_t^2(1 + x_0) + 2(1 + x_m + x_p)},$$

$$f_+ = \frac{2x_p}{a_t^2(1 + x_0) + 2(1 + x_m + x_p)},$$

$$f_- = \frac{2(1 + x_m)}{a_t^2(1 + x_0) + 2(1 + x_m + x_p)},$$

$$\sigma_t = \sigma_{t}^{SM} + a_0 x_0 + a_m x_m + a_p x_p + a_5 x_5,$$

$$\sigma_s = \sigma_{s}^{SM} + b_0 x_0 + b_m x_m + b_p x_p + b_5 x_5.$$  \hfill (23)

The numerical values of the $a_j$ and $b_j$ coefficients are given in Ref. [24] for a mass of the top quark $m_t = 178$ GeV. In Table (I) we show their values for $m_t = 173$ GeV. We have used the CTEQ6L1 parton distribution function when integrating over the parton luminosities [30].

As mentioned above, the LHT model predicts a tree level reduction of the $tbW$ coupling. Therefore, an important feature of this model is that at tree level all single top production modes as well as the total decay width show the same proportional deviation from the SM prediction [28]. In our study, we want to consider the additional effect from the dimension-5 $f_{2R}$ coupling that arises at the one loop level in LHT.

The SM born level prediction of the $t \rightarrow bW^+$ width of the top quark is $\Gamma(t \rightarrow bW^+) = 1.5$ GeV for $m_t = 173$ GeV. There is a 10% decrease when QCD and electroweak corrections as well as non-zero $m_b$ and finite W-boson width effects are considered [27, 31]. In Fig. (4) we show the deviation in the total width of the top quark coming from the LHT model. The solid lines in Fig. (4) give the reduction in $\Gamma_t$ as a function of the scale $f$ and three different values of $\lambda_2$. For these lines both the effective $f_{1L}$ and $f_{2R}$ couplings are considered. Nonetheless, we
FIG. 4: The variation in the top width. Solid lines contain the contribution from both the $f_{1L}$ and $f_{2R}$ couplings. Dotted lines are for $f_{2R} = 0$.

FIG. 5: The variation in the W-boson helicity ratio $f_-$. It would require a deviation of $f_{2R}$ much bigger than 20% in order to have a significant change in $f_-$. Also show in dotted lines the same curves obtained when only the $f_{1L}$ coupling is considered. Dotted and solid lines almost overlap: as expected, the change in $\Gamma_t$ is driven mainly by the tree level mixing with the heavy top. We conclude that the small changes in $f_{2R}$ cannot be seen by measuring $\Gamma_t$. Also, notice that the (%) reduction in Fig. (4) is entirely due to the
cosine of the $tT$ mixing angle $c_L$, which according to formula (17) tends to 1 when either $\epsilon \to 0$ or $r = \lambda_1/\lambda_2 \to 0$.

As for the W-boson helicity ratios $f_0$ and $f_-$, in principle, these observables are more sensitive to the $f_{2R}$ coupling. Notice that $L_0$ and $T_m$ in equation (21) get exactly the same correction if only the $f_{1L}$ is modified (we have set $f_{1R} = f_{2L} = 0$). This means that the ratios $f_0$ and $f_-$ do not change at all from their SM values when we consider only the tree level $tbW$ coupling of the LHT model. However, when we consider the change in the $f_{2R}$ coupling we do observe a deviation that (unfortunately) turns out to be very small (of order less than 0.1%) as it is shown in Fig. (5). We conclude that the W-boson helicity ratios $f_0$ and $f_-$ require a substantial deviation in the dimension five coupling $f_{2R}$ in order to show significant changes from their SM values.

As mentioned above, the effective couplings $f_{1L}$ and $f_{2R}$ can also be probed with single top production. In comparison with the top decay width $\Gamma_t$ and the W-helicity ratio $f_-$, the cross section could be more sensitive to the $f_{2R}$ coupling. We show the deviation in the t-channel cross section in Fig. (6). Notice that the change when we go from considering the deviation in $f_{1L}$ only (dotted lines), to considering both the deviations in $f_{1L}$ and $f_{2R}$ (solid lines) is hardly visible. This change is slightly more pronounced for the s-channel cross section as shown in Fig. (7). We can observe from the values of the effective coefficients in Table (I) that for this channel the $f_{2R}$ coupling has a somewhat bigger effect through the $x_5$ and $x_0$ terms defined in equation (22). For instance, for a scale $f = 550$ GeV and $\lambda_2 = 1.5$

| t-channel: | $a_0$ | $a_m$ | $a_p$ | $a_5$ | $\sigma_t^{SM}$ |
|-----------|-------|-------|-------|-------|----------------|
| Tevatron  | 0.995 | -0.089| -0.181| 0.336 | 0.906          |
| LHC ($t$) | 174.2 | -22.2 | -38.1 | 78.2  | 151.99         |
| LHC ($\bar{t}$) | 111.9 | -23.4 | -14.6 | 48.7  | 88.46          |

| s-channel: | $b_0$ | $b_m$ | $b_p$ | $b_5$ | $\sigma_s^{SM}$ |
|------------|-------|-------|-------|-------|----------------|
| Tevatron   | -0.094| 0.040 | 0.040 | 0.263 | 0.306          |
| LHC ($t$)  | -1.58 | 6.30  | 6.30  | 7.02  | 4.716          |
| LHC ($\bar{t}$) | -0.944| 3.83  | 3.83  | 3.76  | 2.884          |

TABLE I: The single top production cross section coefficients of Eq. (23) for $m_t = 173$ GeV. Last column is the Born level production cross section in the SM. All in units of pb.
FIG. 6: The variation in the $\sigma_t$ cross section. Solid lines contain the contribution from both the $f_{1L}$ and $f_{2R}$ couplings. Dotted lines are for $f_{2R} = 0$.

the tree level $f_{1L}$ reduction expected in the LHT model brings about a 1.0% reduction in $\sigma_s$ (dotted line), whereas the combined $f_{1L}$ and $f_{2R}$ deviations bring a smaller 0.8% reduction in $\sigma_s$ (solid line). The reason for this can be seen in Fig. (2). The LHT contribution, on the one hand, decreases the value of $f_{1L}$, and on the other hand increases the (positive) value of $f_{2R}$ with respect to the SM. We thus have a small compensation in the value of the $x$ terms in Eq. (22).

**IV. CONCLUSIONS**

Besides the SM electroweak parameters, the LHT model adds three more free parameters: $\kappa$, $\lambda_2$ and the scale $f$ (which is associated with an estimated cut-off $\Lambda \sim 4\pi f$). We have chosen a value of the mirror fermion yukawa $\kappa = 1$ that for our range of $f$ gives T-odd fermion masses that are consistent with bounds from four fermion contact interactions $e^+e^-q\bar{q}$ and from unitarity in $u\bar{u} \to W^+W^-H$ scattering processes. Our study concentrates on the two other parameters: $\lambda_2$ (which drives $m_{T^+}$) and the scale $f$. As for the values of $\lambda_2$ and $f$ we have chosen $\lambda_2 = 1, 1.3$ and $1.5$ and $550 \leq f \leq 1550$ GeV as suggested by Ref. [22].

Because of the mixing between the top quark and its heavy partner $T^+$, the LHT model predicts a tree level reduction of the dim 4 $f_{1L}$ coupling that implies an expected proportional
FIG. 7: The variation in the $\sigma_s$ cross section. Solid lines contain the contribution from both the $f_{1L}$ and $f_{2R}$ couplings. Dotted lines are for $f_{2R} = 0$.

reduction in the top width and single top production. Changes in $f_{1L}$ by themselves do not vary the predicted W helicity fractions in the $t \rightarrow bW$ decay. However, the contribution from the $f_{2R}$ coupling could modify these fractions. The dim 5 weak dipole coupling $f_{2R}$ arises at the one loop level in the SM and in the LHT model. It is somewhat increased in size by the LHT model. However, the contribution to $f_m$ (and $f_0$) is negligible. The increase in $f_{2R}$ predicted by the LHT tends to counteract the effects of the reduction in $f_{1L}$. In any case, the effects from $f_{2R}$ are very small. We have found that the tree level analysis of the top’s width and the single top cross section remains valid for the LHT model. Top quark observables like total width, and single top production are sensitive to the mixing with the heavy $T^+$ partner. In particular, the LHC may probe deviations of the $f_{1L} \ tbW$ coupling down to a few percent \cite{4} and this would imply an indirect probe of the scale $f$ (and the yukawa $\lambda_2$) of the LHT model ( see Eq. (17) ). Of course, there are direct tests of the new heavy states at the LHC that will give more precise determination of these parameters. A recent study (see Ref. \cite{22}) has shown that signal events from $T^+$ and $T^-$ production can be distinguished from SM backgrounds so that the mass and mixings of the top partners can be obtained with relatively good accuracies. Furthermore, other studies have shown that, since the mass of the T-odd fermions cannot be too heavy to be consistent with low energy
data, they can be produced at high enough rates at the LHC\cite{20}.

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APPENDIX A: THE GOLDSTONE BOSON SECTOR IN THE 'T HOOFT-FEYNMAN GAUGE.

In the LHT model, the charged fields $\omega^\pm$ and $\phi^\pm$ as well as the neutral fields $\omega^0$, $\eta$ and $\phi^0_p$, mix at order $(\frac{v}{f})^2$. It is a linear combination of these that is eaten by the heavy gauge bosons when the extended gauge group is broken down to $SU(2)_L \times U(1)_Y$. On the other hand, the $\pi$ fields are T-parity even and do not mix with the other scalars. They are absorbed by the standard model $W/Z$ bosons as usual. The fields $h$, $\phi^0$, $\phi^0_p$ and $\phi^\pm$ remain in the spectrum (after diagonalization). The basis of the kinetic Lagrangian of Eq. (4) is an exponential matrix that is usually computed up to the first few leading terms. However, it is possible to obtain the exact expressions for the kinetic $(\partial^\mu \phi \partial_{\mu} \phi)$ scalar, the scalar-boson mixing $(W^\mu_\pi^\mu \pi^\nu)$ and boson mass terms. (It is from the latter that the boson masses of Eq. (10) were obtained.)

In obtaining the following formulas it is convenient to notice that the vev value of the field matrix $\Pi$ Eq. (2) is proportional to a matrix $M_0$:

$$M_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \quad (A1)$$

for which is easy to prove that $M_0^{2n+1} = 2^n M_0$ and $M_0^{2(n+1)} = 2^n M_0^2$ with $n = 0, 1, 2, 3, ...$

With these identities it can be shown that the vev of $\Sigma$ is:

$$\langle \Sigma \rangle_0 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & -\frac{1-c_\epsilon}{2} & \frac{i}{\sqrt{2}} s_\epsilon' & 0 & \frac{1+c_\epsilon}{2} \\ 0 & \frac{i}{\sqrt{2}} s_\epsilon' & c_\epsilon' & 0 & \frac{i}{\sqrt{2}} s_\epsilon' \\ 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1+c_\epsilon}{2} & \frac{i}{\sqrt{2}} s_\epsilon' & 0 & -\frac{1-c_\epsilon}{2} \end{pmatrix}, \quad (A2)$$

where $s_\epsilon = \sin \sqrt{2} \epsilon$ and $\epsilon \equiv v/f$ as defined in Eq. 8. This expression can be used in the kinetic Lagrangian Eq. (4) to obtain the (exact) mixing and mass terms of the LHT model. The diagonalization and normalization of Goldstone boson fields has been discussed at order
\( \epsilon^2 \) for the charged sector in Ref. [15] and for the neutral sector in Ref. [32]. Below, we will make the same analysis for charged and neutral bosons exactly (at all orders in \( \epsilon \)).

1. The charged \( W^\pm \) bosons.

Let us write down the part of the kinetic Lagrangian Eq. (4) that involves the charged bosons of the LHT model. It is convenient to put it in a matrixial form:

\[
\mathcal{L}_{\text{kin}} = \partial^\mu \phi^{+\mu} \partial_\nu \phi^{-\nu} + 2\tilde{\kappa} \partial^\mu \pi^{+\mu} \partial_\nu \pi^{-\nu} + f g \frac{1-c_v}{\sqrt{2} y} (W^+_\mu \partial^\mu \pi^- + W^-_\mu \partial^\mu \pi^+) \\
+ (\partial_\nu \omega^- - \partial_\nu \phi^-) B \left( \begin{array}{cc} \partial^\mu \omega^+ \\ \partial^\mu \phi^+ \end{array} \right) + f g W^+_{H\mu} \left( \frac{1 + \kappa_0}{2} - i \frac{1 - \kappa_0}{2} \right) \left( \begin{array}{c} \partial^\mu \omega^- \\ \partial^\mu \phi^- \end{array} \right) \\
+ f g W^-_{H\mu} \left( \frac{1 + \kappa_0}{2} - i \frac{1 - \kappa_0}{2} \right) \left( \begin{array}{c} \partial^\mu \omega^- \\ \partial^\mu \phi^- \end{array} \right)
\]

with

\[
B = \begin{pmatrix} \frac{1}{2} + \tilde{\kappa} & -i \left( \frac{1}{2} - \tilde{\kappa} \right) \\ i \left( \frac{1}{2} - \tilde{\kappa} \right) & \frac{1}{2} + \tilde{\kappa} \end{pmatrix}
\]

\[
\kappa_0 = \frac{s_v}{\sqrt{2} \epsilon} = 1 - 2 \epsilon^2 + \frac{4 \epsilon^4}{3!} + \frac{4 \epsilon^4}{5!} - \ldots
\]

\[
\tilde{\kappa} = \frac{1 - c_v}{2 \epsilon^2} = \frac{1}{2!} - \frac{2 \epsilon^2}{4!} + \frac{4 \epsilon^4}{6!} - \ldots
\]

We then redefine the T-even charged scalar \( \pi^\pm \) as well as the T-odd \( \omega^\pm \) and \( \phi^\pm \) to diagonalize the Lagrangian. The new T-even \( \pi'^\pm \) field is given by:

\[
\pi'^\pm \equiv \frac{\pm i}{\sqrt{2\tilde{\kappa}}} \pi'^\pm
\]

An extra phase \( i \) multiplies the \( \pi'^\pm \) field so that the \( W^\pm \pi'^\pm \) mixing and the \( SU(2) \) gauge-fixing terms become identical to the usual SM expressions[33]. The other two charged scalars are redefined as:

\[
\begin{pmatrix} \omega^+ \\ \phi^+ \end{pmatrix} \equiv T \begin{pmatrix} i \omega'^+ \\ \phi'^+ \end{pmatrix} \quad T = \frac{\hat{m}_{W_H}}{2\tilde{\kappa} + \kappa_0} \begin{pmatrix} 2\tilde{\kappa} + \kappa_0 & i(1 - \kappa_0)\sqrt{2\tilde{\kappa}} \\ i(2\tilde{\kappa} - \kappa_0) & (1 + \kappa_0)\sqrt{2\tilde{\kappa}} \end{pmatrix}
\]

where \( \hat{m}_{W_H} = \frac{M_{W_H}}{f g} = \frac{\sqrt{\lambda + \kappa_0}}{2} \). Notice that the new \( \omega'^\pm \) field has an extra phase factor \( \pm i \) that is convenient to use so that the Feynman rules of this T-odd Goldstone boson resemble
the rules of its T-even counterpart (the $\pi^\pm$ boson). On the other hand, for the physical heavy T-odd $\phi'^\pm$ boson we choose not to insert the phase factor. The Feynman rules in appendix B stand for the new $\pi'^\pm$, $\omega'^\pm$, $\phi'^\pm$, etc. fields, but we have dropped the $t$ symbol for simplicity.

In terms of the new (mass eigenstates) fields the Lagrangian becomes:

$$L_{\text{kin}} = \partial^\mu \phi'^+ \partial_\mu \phi'^- + \partial^\mu \pi'^+ \partial_\mu \pi'^- + iM_W (W^-_\mu \partial^\mu \pi'^+ - W^+_\mu \partial^\mu \pi'^-) + \partial^\mu \omega'^+ \partial_\mu \omega'^- + iM_{W_H} (W^-_{H\mu} \partial^\mu \omega'^+ - W^+_{H\mu} \partial^\mu \omega'^-)$$

(A7)

where the $W^\pm_{H\mu} \partial^\mu \omega'^-$ mixing term is canceled (after integration by parts) when we add the usual gauge-fixing term:

$$\Delta L = -\frac{1}{\xi_L} |\partial^\mu W^\pm_\mu - iM_W \xi_L \pi'^\pm| | - \frac{1}{\xi_H} |\partial^\mu W^\pm_{H\mu} - iM_{W_H} \xi_H \omega'^\pm|.$$  

(A8)

To obtain Feynman rules it is convenient to use an expansion in powers of $\epsilon = v/f$:

$$T = 1 + \begin{pmatrix} 1 & 4i \\ 2i & 1 \end{pmatrix} \frac{\epsilon^2}{24} + \begin{pmatrix} -\frac{61}{8} & 13i \\ \frac{19}{2}i & -1 \end{pmatrix} \frac{\epsilon^4}{720}$$

(A9)

2. The neutral bosons.

The neutral boson sector in the Lagrangian of Eq. (4) can also be written in the following matricial form (notice that $M_Z = \frac{v^2}{2c_w}\sqrt{2\kappa}$):

$$L_{\text{kin}} = \frac{1}{2} (\partial^\mu h)^2 + \tilde{\kappa} (\partial^\mu \phi^0)^2 + \tilde{\kappa} (\partial^\mu \pi^0)^2 + \frac{v\bar{g}}{c_w}\tilde{\kappa} Z^{\mu} \partial_\mu \pi^0 + \bar{x}^T A \bar{x} + A^\mu_H \bar{a}^T \bar{x} + Z^\mu_H \bar{b}^T \bar{x}$$

(A10)

with

$$\bar{x} = \begin{pmatrix} \partial_\mu \omega^0 \\ \partial_\mu \eta \\ \partial_\mu \phi^0_P \end{pmatrix} \quad \bar{a} = \frac{fgc_H}{8} \begin{pmatrix} 8t_w - (4\kappa_1 + \kappa_2)x_\omega \\ (8t_w - 5\kappa_1 x_\omega)/\sqrt{5} \\ \sqrt{2}\kappa_1 x_\omega \end{pmatrix} \quad \bar{b} = \frac{fgc_H}{8} \begin{pmatrix} 8 - \kappa_1 x_\eta \\ (8 - \kappa_2 x_\eta)/\sqrt{5} \\ -\sqrt{2}\kappa_1 x_\eta \end{pmatrix}$$

$$A = \frac{1}{16} \begin{pmatrix} 7 + \kappa_0^2 & \sqrt{5}(1 - \kappa_0^2) & -\sqrt{2}(1 - \kappa_0^2) \\ \sqrt{5}(1 - \kappa_0^2) & 3 + 5\kappa_0^2 & \sqrt{10}(1 - \kappa_0^2) \\ -\sqrt{2}(1 - \kappa_0^2) & \sqrt{10}(1 - \kappa_0^2) & 6 + 2\kappa_0^2 \end{pmatrix}$$
and

\[ \kappa_1 = 1 - \kappa_0 c_v \quad \kappa_2 = 3 + 5\kappa_0 c_v \]
\[ x_\omega = t_w + t_H \quad x_\eta = 1 - t_w t_H \]
\[ y_\omega = t_w - 5t_H \quad y_\eta = 5 + t_w t_H \]

(A11)

Where \( t_w \equiv s_w/c_w \) and \( t_H \) are \( \text{Tan}(\theta_w) \) and \( \text{Tan}(\theta_H) \) respectively (see Eq. (7)). To normalize the \( \pi^0 \) and \( \phi^0 \) fields we simply redefine \( \pi^0 \equiv \sqrt{2\kappa\pi^0} \) and \( \phi^0 \equiv \sqrt{2\kappa\phi^0} \). (The Higgs field \( h \) needs no redefinition.) We also redefine the other scalars to properly diagonalize the Lagrangian:

\[
\begin{pmatrix}
\omega^0 \\
\eta \\
\phi^0_P
\end{pmatrix} \equiv T
\begin{pmatrix}
\omega'^0 \\
\eta' \\
\phi'^0_P
\end{pmatrix} \quad \text{with} \quad T = \frac{1}{\kappa_0 \sqrt{2} \sqrt{1 + 3c_v^2}} D_1 \begin{pmatrix} t_{ij} \end{pmatrix} D_2
\]

(A12)

where \( T \) is conveniently written as a product of 3 matrices: \( D_1, (t_{ij}) \) and \( D_2 \). Two of them are diagonal matrices defined as \( D_1 = \text{diag}(1, \sqrt{5}, \sqrt{2}) \), and \( D_2 = \text{diag}(1/d_\omega, 1/d_\eta, 1/2) \), with

\[ d_\omega = \sqrt{8(t_w^2 + 5t_H^2) - 5s_v^2 x_\omega^2}, \quad \text{and} \]
\[ d_\eta = \sqrt{8(5 + t_w^2 t_H^2) - 5s_v^2 x_\eta^2}. \]

The matrix elements \( t_{ij} \) are as follows:

\[ t_{11} = \kappa_0 (5c_v^2 x_\omega^2 + 2t_w) + c_v y_\omega \quad t_{12} = \kappa_0 (5c_v^2 x_\eta - 2t_w t_H) - c_v y_\eta \quad t_{13} = -\kappa_1 \]
\[ t_{21} = \kappa_0 (c_v^2 x_\omega^2 + 2t_H) - c_v y_\omega \quad t_{22} = \kappa_0 (c_v^2 x_\eta + 2) + c_v y_\eta \quad t_{23} = -\kappa_1 \]
\[ t_{31} = -(\kappa_0 - c_v) y_\omega \quad t_{32} = (\kappa_0 - c_v) y_\eta \quad t_{33} = 1 + 3\kappa_0 c_v \]

where the \( x \)'s, \( y \)'s and \( \kappa \)'s are defined in equations (A4) and (A11).

It is convenient to make an expansion in powers of \( \epsilon = v/f \):

\[ T = 1 + T_2 \frac{\epsilon^2}{12} + T_4 \frac{\epsilon^4}{6} + \cdots \]

\[
T_2 = \begin{pmatrix}
1/2 & -2.183\sqrt{5} & 2\sqrt{2} \\
\frac{2.183}{2}\sqrt{5} & 5/2 & -2\sqrt{10} \\
-\sqrt{2} & \sqrt{10} & 1
\end{pmatrix} \quad T_4 = \begin{pmatrix}
-0.5682 & 0.6200 & \frac{41}{30\sqrt{2}} \\
0.1315 & -0.6093 & \frac{41}{6\sqrt{10}} \\
-0.1288 & 1.6184 & \frac{17}{240}
\end{pmatrix}
\]

(A13)
where we have taken $t_w = 0.536$, thus $(10 + t_w^2)/(5 - t_w^2) = 2.183$.

In terms of the new (mass eigenstates) fields the neutral boson Lagrangian becomes:

$$
\mathcal{L}_{\text{kin}} = \frac{1}{2} (\partial^\mu h)^2 + \frac{1}{2} (\partial^\mu \pi^0)^2 + M_Z Z^\mu \partial_\mu \pi^0 \\
+ \frac{1}{2} (\partial^\mu \omega^0)^2 + \frac{1}{2} (\partial^\mu \eta')^2 + \frac{1}{2} (\partial^\mu \phi^0)^2 + \frac{1}{2} (\partial^\mu \phi')^2 \\
+ M_{Z_H} Z_H^\mu \partial_\mu \omega^0 + M_{A_H} A_H^\mu \partial_\mu \eta' 
$$

where the mixing terms like $Z^\mu \partial_\mu \pi^0$ are canceled (after integration by parts) when we add the usual gauge-fixing terms:

$$
\Delta \mathcal{L} = -\frac{1}{2 \xi_A} (\partial_\mu A_\mu) A^\mu - \frac{1}{2 \xi_Z} (\partial_\mu Z^\mu - M_Z \xi_Z \pi^0)^2 \\
- \frac{1}{2 \xi_{A_H}} (\partial_\mu A_H^\mu - M_{A_H} \xi_{A_H} \eta')^2 - \frac{1}{2 \xi_{Z_H}} (\partial_\mu Z_H^\mu - M_{Z_H} \xi_{Z_H} \omega^0)^2 
$$

**APPENDIX B: FEYNMAN RULES.**

We want to show the Feynman rules that we used. The scalar fields are not the original interaction eigenstates but the mass eigenstates that are written as $\pi^{t\pm}$, $\pi^0$, etc. We have dropped the $t$ symbol to simplify the notation. Table II shows bosonic vertices that involve one charged $W^-$ SM boson. Tables III, IV and V show vertices for fermions and charged, T-even neutral and T-odd neutral scalar bosons respectively. For the fermion-gauge boson interactions we refer the reader to tables V and VI of Ref. [20]. We have carefully verified that our rules agree with the ones there. We have written in Table VI some others that do not appear in [20]. Other types of interactions, like four-boson vertices or dimension-5 $\bar{f}f\phi\phi$ ($\phi$ any scalar) vertices can be found in Ref. [21].

Please notice the definitions ($\epsilon = v/f$, $r = \lambda_1/\lambda_2$):

$$
S_{\lambda_{\nu\mu}} \equiv (p_--p_+)\lambda g_{\mu\nu} + (p_+ - p_0)_{\mu} g_{\nu\lambda} + (p_0 - p_-)_{\nu} g_{\lambda\mu} \\
q_w \equiv \frac{10 + t_w^2}{5 - t_w^2} \\
A_R \equiv i \sqrt{2} \lambda d \left[ -1 + \frac{r^4}{2(1 + r^2)^2 \epsilon^2} \right] \\
A_L \equiv i \sqrt{2} \lambda_2 r \sqrt{1 + r^2} \left[ 1 - \frac{1 + 3r^4}{4(1 + r^2)^2 \epsilon^2} \right]
$$
| Interaction | Feynman rule | Interaction | Feynman rule |
|------------|-------------|------------|-------------|
| $W^- W^+ A$ | $ig s_w S_{w\nu\mu}$ | $W^- W^+ H_H P$ | $\sim \epsilon^4$ |
| $W^- W^+ Z$ | $ig c_w S_{w\nu\mu}$ | $W^- \phi^+ A_H \nu$ | $ig^2 f_{\nu} (\frac{5}{20 - 4n_w} - \frac{1}{4}) \epsilon^2 g_{\mu\nu}$ |
| $W^- \pi^+ A$ | $ig s_w M_W (1 - \frac{1}{4} \epsilon^2) g_{\mu\nu}$ | $W^- \phi^+ Z_H \nu$ | $-ig^2 f (1 - \frac{3}{4} \epsilon^2) g_{\mu\nu}$ |
| $W^- \pi^+ Z$ | $-ig s_w^2 M_Z (1 - \frac{1}{4} \epsilon^2) g_{\mu\nu}$ | $W^- \phi^+ A_H \nu$ | $g^2 f E \epsilon^2 g_{\mu\nu}$ |
| $W^- W^+ h$ | $ig M_W (1 - \frac{1}{3} \epsilon^2) g_{\mu\nu}$ | $W^- \phi^+ Z_H \nu$ | $ig^2 f \frac{1}{4} (\frac{2}{3} + \frac{5}{3} \epsilon^2) \epsilon^2 g_{\mu\nu}$ |
| $W^- W^+ \pi^0$ | $0$ | $W^- \phi^+ \pi^0$ | $g (1 - \frac{1}{4} \epsilon^2) (p_{\omega+\mu} - p_{0\mu})$ |
| $W^- \pi^+ h$ | $\frac{i g^2}{2} [p_{\pi+\mu} (1 - \frac{5}{12} \epsilon^2) - p_{h\mu} (1 - \frac{1}{12} \epsilon^2)]$ | $W^- \phi^+ \pi^0$ | $g \sqrt{\frac{5}{12}} \epsilon^2 [(p_{\omega+\mu} - (\frac{q_w}{2} - \frac{3}{2} p_{\pi+\mu} - (q_w + \frac{1}{2}) p_{\omega+\mu})]$ |
| $W^- \pi^+ \pi^0$ | $\frac{\varphi}{2} (p_{\pi+\mu} - p_{0\mu})$ | $W^- \phi^+ \nu^0$ | $-\frac{g}{12 \sqrt{2} \epsilon^2} [(1 + 2i) p_{\phi+\mu} - p_{\omega+\mu}]$ |
| $W^- H^+ A_H \lambda$ | $-ig s_h S_{w\nu\mu}$ | $W^- \phi^+ \nu^0$ | $ig \sqrt{\frac{7}{2}} \epsilon^2 [1 (p_{\phi+\mu} - (\frac{5}{4} + i) p_{\omega+\mu})$ |
| $W^- H^+ Z_H \lambda$ | $ig c_h S_{w\nu\mu}$ | $W^- \phi^+ \nu^0$ | $-\frac{g}{6} (1 + \frac{4}{3} i) (p_{\phi+\mu} - p_{\omega+\mu}) \epsilon^2$ |
| $W^- H^+ l$ | $\frac{\sqrt{2}}{2} g^2 f (1 + q_w) \epsilon^2 g_{\mu\nu}$ | $W^- \phi^+ \nu^0$ | $-ig \sqrt{\frac{7}{2} \epsilon^2} \epsilon (5 p_{\phi+\mu} - p_{\omega+\mu})$ |
| $W^- H^+ \phi^0$ | $0$ | $W^- \phi^+ \phi^0$ | $\frac{g}{\sqrt{2}} [p_{\phi+\mu} - p_{\phi+\mu} - (24 \epsilon^2 (5 p_{\phi+\mu} - p_{\phi+\mu})]$ |
| $W^- H^+ \omega^0$ | $g^2 f (1 - \frac{3}{4} \epsilon^2) g_{\mu\nu}$ | $W^- \phi^+ \phi^0$ | $-i \frac{g}{\sqrt{2}} [p_{\phi+\mu} - p_{\phi+\mu} - (24 \epsilon^2 (5 p_{\phi+\mu} - p_{\phi+\mu})$ |

**TABLE II**: Feynman rules for three boson $W^-$ vertices. Some of these rules also appear in Ref. [32].

Here, $M_W$ and $M_Z$ stand for the SM mass of the $W^\pm$ and $Z$ bosons.

\[
B_R \equiv \frac{i \sqrt{2} \lambda_d r^2}{1 + r^2} \left[ -\epsilon + \frac{5 - 2 r^2 + 2 r^4}{6 (1 + r^2)^2} \epsilon^3 \right]
\]

\[
B_L \equiv i \sqrt{2} \lambda_2 \frac{r^2}{\sqrt{1 + r^2}} \left[ 1 - \frac{\epsilon^2}{4} \frac{3 + r^4}{(1 + r^2)^2} \right]
\]

\[
D_R \equiv (i + 4) \kappa \frac{\epsilon^2}{24}
\]

\[
D_L \equiv i \frac{r \lambda_2}{\sqrt{2 \sqrt{1 + r^2}}} \left[ -\epsilon + \frac{3 - 7 i + (2 - 8 i) r^2 + (19 - 4 i) r^4}{24} \right]
\]

\[
E_L \equiv i \frac{r^2 \lambda_2}{\sqrt{2 (1 + r^2)}} \left[ -\epsilon + \frac{19 + 4 i + (2 + 8 i) r^2 + (7 + 4 i) r^4}{24 (1 + r^2)^2} \right]
\]

\[
a_R \equiv -\kappa \left[ 1 - \frac{r^4 \epsilon^2}{2 (1 + r^2)^2} \right]
\]

\[
a_L \equiv \frac{r \lambda_2}{\sqrt{2 \sqrt{1 + r^2}}} \left( -\epsilon + \frac{i \epsilon^3 [2 - 7 i + (4 - 2 i) r^2 + (2 - 19 i) r^4]}{24 (1 + r^2)^2} \right)
\]

\[
b_R \equiv \frac{r^2 \kappa}{1 + r^2} \left[ -\epsilon + \frac{5 - 2 r^2 + 2 r^4}{6 (1 + r^2)^2} \epsilon^3 \right]
\]
\[b_L \equiv \frac{r^2 \lambda_2}{\sqrt{2(1 + r^2)}} \left[ \epsilon - \frac{19 + 2i + (2 + 4i)r^2 + (7 + 2i)r^4}{24(1 + r^2)^2} \epsilon^3 \right] \]

| Interaction |
|--------------|
| Feynman rule |
|----------------|
| \( \bar{t}^+ b^- \pi^+ \) | \( A_R P_R + A_L P_L \) |
| \( T^+ b^- \pi^+ \) | \( B_R P_R + B_L P_L \) |
| \( \bar{t}^+ b^- \pi^- \) | \( -i \frac{r}{2} \left[ e^3(1 + \frac{1}{24}r^3) \right] \) |
| \( T^- b^- \pi^- \) | \( 0 \) |
| \( \bar{t}^+ b^- \omega^+ \) | \( i \frac{\lambda_4}{\sqrt{2}} \left[ -\epsilon + (1 - 2i)\frac{1}{24} \epsilon^3 \right] P_R + i \kappa P_L \) |
| \( T^- b^- \omega^+ \) | \( i \frac{\lambda_4}{\sqrt{2}} \left[ -\epsilon + (1 - 2i)\frac{1}{24} \epsilon^3 \right] P_R + i \kappa P_L \) |
| \( \bar{t}^+ b^- \phi^+ \) | \( D_R P_R + D_L P_L \) |
| \( T^- b^- \phi^+ \) | \( 0 \) |
| \( \bar{t}^+ b^- \omega^- \) | \( i[a_R P_R + a_L P_L] \) |
| \( T^- b^- \omega^- \) | \( i[b_R P_R + b_L P_L] \) |
| \( \bar{t}^+ b^- \phi^- \) | \( i \frac{\lambda_4}{\sqrt{2}} \left[ -\epsilon + (1 + 4i)\frac{1}{24} \epsilon^3 \right] P_R - i(4i + 1)\frac{b_R P_L}{24} \) |
| \( T^- b^- \phi^- \) | \( 0 \) |

**TABLE III**: Feynman rules for fermion-charged scalar vertices.

\[G_R \equiv \frac{1}{r(1 + r^2)} \left[ \epsilon + \frac{3r^6 - 2r^4 + 8r^2 - 2}{6(1 + r^2)^2} \epsilon^3 \right] \]

\[G_L \equiv -1 + \frac{3 + r^2 + r^4}{2(1 + r^2)^2} \epsilon^2 \]

\[J_R \equiv \frac{r}{1 + r^2} \left[ \epsilon - \frac{\epsilon^3(13 - 4r^2 + 13r^4)}{12(1 + r^2)^2} \right] \]

\[J_L \equiv \frac{3\epsilon^2(1 + r^4)}{4(1 + r^2)^2} - 1 \]

\[K_R \equiv -\frac{\kappa}{\sqrt{2}} \left[ 1 - \left( 1 + q_w + \frac{12r^4}{(1 + r^2)^2} \right) \epsilon^2 \right] \]

\[K_L \equiv \frac{r \lambda_2}{2\sqrt{1 + r^2}} \left[ \epsilon - \left( q_w + \frac{3 - 6r^2 + 15r^4}{(1 + r^2)^2} \right) \epsilon^3 \right] \]

\[M_R \equiv \frac{r^2 \kappa}{\sqrt{2}(1 + r^2)} \left( -\epsilon + \left[ q_w + \frac{21 - 6r^2 + 9r^4}{(1 + r^2)^2} \right] \frac{1}{24} \epsilon^3 \right) \]

\[M_L \equiv \frac{r^2 \lambda_2}{2\sqrt{1 + r^2}} \left[ \epsilon - \left[ q_w + \frac{15 - 6r^2 + 3r^4}{(1 + r^2)^2} \right] \frac{1}{24} \epsilon^3 \right] \]

\[N_R \equiv \frac{\kappa}{\sqrt{10}} \left[ 1 + \left( 10 q_w - 5 - \frac{12r^4}{(1 + r^2)^2} \right) \epsilon^2 \right] \]

\[N_L \equiv \frac{r \lambda_2}{2\sqrt{5}(1 + r^2)} \left[ \epsilon + \left( 10 q_w + \frac{17 + 46r^2 + 5r^4}{(1 + r^2)^2} \right) \epsilon^3 \right] \]
\[
\begin{array}{c|c|c|c|c}
\text{Interaction} & \text{Feynman rule} & \text{Interaction} & \text{Feynman rule} \\
\hline
\tilde{t}^+ t^+ h & \frac{r \lambda_2}{\sqrt{1 + r^2}} \left[ -1 + \frac{2 + r^2 + 2 r^4}{2(1 + r^2)^2} \epsilon^2 \right] & \tilde{t}^+ t^+ \pi^0 & -\frac{r \lambda_2}{\sqrt{1 + r^2}} \left[ 1 - \frac{(1 + 5 r^4)}{4(1 + r^2)^2} \epsilon^2 \right] \gamma_5 \\
\tilde{T}^+ T^+ h & \frac{r^2}{(r^2 + 1)^{3/2}} \epsilon + \frac{r^2 (3 r^6 + r^4 + 8 r^3 - 5)}{6 (r^2 + 1)^{7/2}} \epsilon^3 & \tilde{T}^+ T^+ \pi^0 & \frac{r^4 \lambda_2}{(1 + r^2)^2} \left[ -\epsilon + \frac{19 - 4 r^2 + 7 r^4}{12 (1 + r^2)^2} \epsilon^3 \right] \gamma_5 \\
\tilde{t}^+ t^+ h & \frac{i r^2 \lambda_2}{\sqrt{1 + r^2}} (G_R P_R + G_L P_L) & \tilde{T}^+ t^+ \pi^0 & -\frac{r^2 \lambda_2}{\sqrt{1 + r^2}} (J_R P_R + J_L P_L) \\
\tilde{t}^- t^- h & i \lambda_d (-1 + \frac{4}{3} \epsilon^2) & \tilde{t}^- t^- \pi^0 & -\frac{\kappa}{2 \sqrt{2}} (\epsilon + \frac{1}{24} \epsilon^3) \gamma_5 \\
\tilde{T}^- T^- h & 0 & \tilde{T}^- T^- \pi^0 & 0 \\
\tilde{t}^- t^- h & 0 & \tilde{T}^- t^- \pi^0 & 0 \\
\tilde{b}^- b^- h & 0 & \tilde{b}^- b^- \pi^0 & 0 \\
\end{array}
\]

\text{TABLE IV: Feynman rules for fermion-neutral T-even scalar vertices.}

\[
\begin{align*}
V_L & \equiv -\frac{r \lambda_2}{\sqrt{2 (1 + r^2)}} \left[ \epsilon - \frac{\epsilon^3 (7 + 8 r^2 + 13 r^4)}{12 (1 + r^2)^2} \right] \\
\Xi_R & \equiv \frac{r^2 \kappa}{\sqrt{10 (1 + r^2)}} \left[ \epsilon + \left( 10 q_w - \frac{25 + 2 r^2 + 13 r^4}{(1 + r^2)^2} \right) \epsilon^3 \right] \\
\Xi_L & \equiv -\frac{r^2 \lambda_2}{2 \sqrt{5 (1 + r^2)}} \left[ \epsilon + \left( 10 q_w + \frac{5 + 46 r^2 + 17 r^4}{(1 + r^2)^2} \right) \epsilon^3 \right] \\
\beta_R & \equiv -\frac{r^2 \kappa}{4 (1 + r^2)^2} \epsilon^3 \\
\beta_L & \equiv \frac{r^2 \lambda_2}{\sqrt{2 (1 + r^2)}} \left[ -\epsilon + \frac{13 + 8 r^2 + 7 r^4}{12 (1 + r^2)^2} \epsilon^3 \right] \\
C_t & \equiv e_L^2 \left( e_w^2 - \frac{1}{3} s_w^2 \right) - \frac{4}{3} s_L^2 s_w^2 \\
C_T & \equiv s_L^2 \left( e_w^2 - \frac{1}{3} s_w^2 \right) - \frac{4}{3} e_L^2 s_w^2
\end{align*}
\]

[1] T. Han, The 'Top Priority' at the LHC, arXiv:0804.3178 [hep-ph]; T. M. P. Tait, Nucl. Phys. Proc. Suppl. 177-178, 11 (2008); A. Quadt, \textit{Top quark physics at hadron colliders}, Euro. Phys. J. C48 (2006) 835; D. Chakraborty, J. Konigsberg, D. Rainwater, Annu. Rev. Part. Nucl. Sci. 53 (2003) 301; M. Beneke, et.al., \textit{Top Quark Physics: 1999 CERN Workshop on the SM Physics (and more) at the LHC} (hep-ph/0003033); F. Larios, R. Martinez and M. A. Perez,
Ref. [20] and in Ref. [32].

**TABLE VI: Some Feynman rules for fermion-gauge boson vertices.** All the other rules appear in Ref. [2] T. M. P. Tait, Phys. Rev. D 61, 3473 (2006); R. Martinez, M. A. Perez and N. Poveda, Eur. Phys. J. C 53, 221 (2008); J. L. Diaz-Cruz, M. A. Perez and J. J. Toscano, Phys. Lett. B 398, 347 (1997).

[2] T. M. P. Tait, Phys. Rev. D 61, 034001 (1999); T. M. P. Tait and C. P. Yuan, Phys. Rev. D 63, 014018 (2000); Q. H. Cao, J. Wudka and C. P. Yuan, Phys. Lett. B 658, 50 (2007); Q. H. Cao and J. Wudka, Phys. Rev. D 74, 094015 (2006); F. Larios, M. A. Perez and C. P. Yuan,
Phys. Lett. B 457, 334 (1999); U. Baur, A. Juste, L. H. Orr and D. Rainwater, Nucl. Phys. Proc. Suppl. 160, 17 (2006); B. Grzadkowski and M. Misiak, Phys. Rev. D 78, 077501 (2008); arXiv:0802.1413 [hep-ph]; B. Sahin and I. Sahin, Eur. Phys. J. C 54, 435 (2008).

[3] R. Schwienhorst [D0 Collaboration and CDF Collaboration], arXiv:0805.2175 [hep-ex]; V. M. Abazov et al. [D0 Collaboration], arXiv:0807.1692 [hep-ex]; V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 100, 062004 (2008); C. I. Ciobanu [CDF Collaboration and D0 Collaboration], arXiv:0809.2173 [hep-ex].

[4] J. A. Aguilar-Saavedra, J. Carvalho, N. Castro, A. Onofre and F. Veloso, Eur. Phys. J. C 53, 689 (2008); J. A. Aguilar-Saavedra, Nucl. Phys. B 804, 160 (2008); F. Hubaut, E. Monnier, P. Pralavorio, K. Smolek and V. Simak, Eur. Phys. J. C 44S2, 13 (2005).

[5] J. Cao, R.J. Oakes, F. Wang and J.M. Yang, Phys. Rev. D68 (2003) 054019; M. Beccaria, C. M. Carloni Calame, G. Macorini, E. Mirabella, F. Piccinini, F. M. Renard and C. Verzegnassi, Phys. Rev. D 77, 113018 (2008).

[6] X. Wang, Q. Zhang and Q. Qiao, Phys. Rev. D71 (2005) 014035; X. Wang, Y. Xi, Y. Zhang and H. Jin, Phys. Rev. D 77, 115006 (2008).

[7] S. Chivukula, Models of Electroweak Symmetry Breaking, hep-ph/9803219.

[8] N. Arkani-Hamed, A. G. Cohen, and H. Georgi, Phys. Lett. B513 (2001) 232; N. Arkani-Hamed, A. G. Cohen, T. Gregoire and J. G. Wacker, JHEP 0208, 020 (2002); N. Arkani-Hamed, A. G. Cohen, E. Katz, A. E. Nelson, T. Gregoire and J. G. Wacker, JHEP 0208, 021 (2002); W. Skiba and J. Terning, Phys. Rev. D 68, 075001 (2003).

[9] N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP 0207, 034 (2002).

[10] M. Perelstein, Prog. Part. Nucl. Phys. 58, 247 (2007) M. Schmaltz and D. Tucker-Smith, Ann. Rev. Nucl. Part. Sci. 55, 229 (2005)

[11] T. Han, H. E. Logan, B. McElrath and L. T. Wang, Phys. Rev. D67, 095004 (2003); G. Burdman, M. Perelstein and A. Pierce, Phys. Rev. Lett. 90, 241802 (2003) [Erratum-ibid. 92, 049903 (2004)]; T. Han, H. E. Logan and L. T. Wang, JHEP 0601, 099 (2006); M. Perelstein, M. E. Peskin and A. Pierce, Phys. Rev. D 69, 075002 (2004); D. E. Kaplan, M. Schmaltz and W. Skiba, Phys. Rev. D 70, 075009 (2004); G. Azuelos et al., Eur. Phys. J. C 39S2, 13 (2005); J. A. Conley, J. L. Hewett and M. P. Le, Phys. Rev. D 72, 115014 (2005); C. X. Yue, L. Zhou and S. Yang, Eur. Phys. J. C 48, 243 (2006); C. X. Yue, S. Yang and L. H. Wang, Europhys. Lett. 76, 381 (2006); Y. B. Liu, J. F. Shen and X. L. Wang, arXiv:hep-ph/0610350; L. Wang,
29

W. Wang, J. M. Yang and H. Zhang, Phys. Rev. D 75, 074006 (2007); S. K. Kang, C. S. Kim and J. Park, Phys. Lett. B 666, 38 (2008); J. Boersma, Phys. Rev. D 74, 115008 (2006); G. A. Gonzalez-Sprinberg, R. Martinez and J. A. Rodriguez, Phys. Rev. D 71, 035003 (2005); W. Kilian, D. Rainwater and J. Reuter, Phys. Rev. D 74, 095003 (2006) [Erratum-ibid. D 74, 099905 (2006)]; K. Cheung, C. S. Kim, K. Y. Lee and J. Song, Phys. Rev. D 74, 115013 (2006); C. S. Chen, K. Cheung and T. C. Yuan, Phys. Lett. B 644, 158 (2007); K. Cheung and J. Song, Phys. Rev. D 76, 035007 (2007); K. Cheung, J. Song, P. Tseng and Q. S. Yan, arXiv:0806.4411 [hep-ph].

[12] C. Csaki, J. Hubisz, G. D. Kribs, P. Meade and J. Terning, Phys. Rev. D 67, 115002 (2003); C. Csaki, J. Hubisz, G. D. Kribs, P. Meade and J. Terning, Phys. Rev. D 68, 035009 (2003); J. L. Hewett, F. J. Petriello and T. G. Rizzo, JHEP 0310, 062 (2003); W. Kilian and J. Reuter, Phys. Rev. D 70, 015004 (2004); Z. Han and W. Skiba, Phys. Rev. D 72, 035005 (2005); M. C. Chen and S. Dawson, Phys. Rev. D 70, 015003 (2004); C. x. Yue and W. Wang, Nucl. Phys. B 683, 48 (2004); M. C. Chen, Mod. Phys. Lett. A 21, 621 (2006).

[13] H. C. Cheng and I. Low, JHEP 0408, 061 (2004); H. C. Cheng and I. Low, JHEP 0309, 051 (2003); H. C. Cheng, arXiv:0710.3407 [hep-ph]; H. C. Cheng, I. Low and L. T. Wang, Phys. Rev. D 74, 055001 (2006); M. Perelstein, Pramana 67, 813 (2006) W. Kilian, D. Rainwater and J. Reuter, Phys. Rev. D 71, 015008 (2005).

[14] I. Low, JHEP 0410, 067 (2004).

[15] J. Hubisz, P. Meade, A. Noble and M. Perelstein, JHEP 0601, 135 (2006).

[16] C. F. Berger, M. Perelstein and F. Petriello, arXiv:hep-ph/0512053; M. S. Carena, J. Hubisz, M. Perelstein and P. Verdier, Phys. Rev. D 75, 091701 (2007); A. Freitas and D. Wyler, JHEP 0611, 061 (2006); M. M. Nojiri and M. Takeuchi, JHEP 0810, 025 (2008); Y. B. Liu, J. F. Shen and X. L. Wang, arXiv:hep-ph/0610350; Q. H. Cao, C. R. Chen, F. Larios and C. P. Yuan, arXiv:0801.2998 [hep-ph]; C. R. Chen, K. Tobe and C. P. Yuan, Phys. Lett. B 640, 263 (2006); Q. H. Cao and C. R. Chen, Phys. Rev. D 76, 075007 (2007); K. Hsieh and C. P. Yuan, Phys. Rev. D 78, 053006 (2008); V. Barger, W. Y. Keung and Y. Gao, Phys. Lett. B 655, 228 (2007); A. Datta, P. Dey, S. K. Gupta, B. Mukhopadhyaya and A. Nyffeler, Phys. Lett. B 659, 308 (2008).

[17] C. T. Hill and R. J. Hill, Phys. Rev. D 76, 115014 (2007); ibid. Phys. Rev. D 75, 115009 (2007).
[18] B. Grinstein and M. Trott, arXiv:0808.2814 [hep-ph].
[19] R. Barcelo, M. Masip and M. Moreno-Torres, Nucl. Phys. B 782, 159 (2007).
[20] A. Belyaev, Chuan-Ren Chen, K. Tobe and C.-P. Yuan, Phys. Rev. D 74, 115020 (2006).
[21] J. Hubisz and P. Meade, Phys. Rev. D 71, 035016 (2005).
[22] S. Matsumoto, T. Moroi and K. Tobe, Phys. Rev. D 78, 055018 (2008).
[23] J. Hubisz, S. J. Lee and G. Paz, JHEP 0606, 041 (2006); M. Blanke, A. Buras, A. Poschienrieder, C. Tarantino, S. Uhlig and A. Weiler. JHEP 12, 003 (2006).
[24] C. R. Chen, F. Larios and C. P. Yuan, Phys. Lett. B 631, 126 (2005).
[25] G.L. Kane, G.A. Ladinsky and C.-P. Yuan. Phys. Rev. D 45 (1992) 124.
[26] F. del Aguila and J.A. Aguilar-Saavedra, Phys. Rev. D 67 (2003) 014009.
[27] W. Bernreuther, J. Phys. G 35, 083001 (2008) arXiv:0805.1333 [hep-ph].
[28] Q. H. Cao, C. S. Li and C. P. Yuan, Phys. Lett. B 668, 24 (2008).
[29] We would like to refer the reader to the discussion in Ref. [15] concerning the electroweak precision constraints on LHT and the effects of higher dimesion operators.
[30] J. Pumplin, D.R. Stump, J. Huston, H.L. Lai, P. Nadolsky and W.K. Tung, JHEP 0207 (2002) 012.
[31] H.S. Do, S. Groote, J.G. Korner and M.C. Mauser, Phys. Rev. D 67 (2003) 091501, and references therein. G. Calderon and G. Lopez Castro, Int. Journal Mod. Phys. A23, 3525 (2008).
[32] M. Blanke, A. Buras, A. Poschienrieder, C. Tarantino, S. Recksiegel, S. Uhlig and A. Weiler. JHEP 01, 066 (2007).
[33] W. Hollik, “Renormalization of the Standard Model.” MPI-PH-93-21, BI-TP-93-16, Apr 1993. 79pp. See also, T.-P. Cheng and L.-F. Li, “Gauge Theory of elementary particle physics”, Clarendon Press, Oxford 1984.