Quantum (in)stability of 2D charged dilaton black holes and 3D rotating black holes

SHIN’ICHI NOJIRI\textsuperscript{1} and SERGEI D. ODINTSOV\textsuperscript{2}

Department of Mathematics and Physics
National Defence Academy
Hashirimizu Yokosuka 239, JAPAN
\textsuperscript{♠} Tomsk Pedagogical University
634041 Tomsk, RUSSIA

ABSTRACT

The quantum properties of charged black holes (BHs) in 2D dilaton-Maxwell gravity (spontaneously compactified from heterotic string) with $N$ dilaton coupled scalars are studied. We first investigate 2D BHs found by McGuigan, Nappi and Yost. Kaluza-Klein reduction of 3D gravity with minimal scalars leads also to 2D dilaton-Maxwell gravity with dilaton coupled scalars and the rotating BH solution found by Bañados, Teitelboim and Zanelli (BTZ) which can be also described by 2D charged dilatonic BH. Evaluating the one-loop effective action for dilaton coupled scalars in large $N$ (and $s$-wave approximation for BTZ case), we show that quantum-corrected BHs may evaporate or else anti-evaporate similarly to 4D Nariai BH as is observed by Bousso and Hawking. Higher modes may cause the disintegration of BH in accordance with recent observation by Bousso.

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\textsuperscript{1} e-mail : nojiri@cc.nda.ac.jp
\textsuperscript{2} e-mail : odintsov@tspi.tomsk.su
1 Introduction

String theory opens new possibilities in the study of fundamental problems in black holes (BHs) physics, like string origin of BH entropy (for a review, see [1]). Moreover, string duality dictates various relations between higher dimensional BHs and lower dimensional BHs, as for example between 5D and 4D BHs (where D-branes methods have been applied to calculate the entropy [2]) from one side and 3D and 2D BHs [3, 4] from another side. This fact indicates that applying results of (quantum) calculations in 2D one can learn about properties of higher dimensional BHs.

Another indication that such approach may be very useful comes from the study of back-reaction of quantized matter to 4D BH when spherical reduction is applied. Here, the resulting effective action turns out to be the effective action of 2D dilatonic gravity with dilaton coupled matter.

Recently, some interesting effects have been discovered in this direction. Working in large $N$ and $s$-wave approximation (quantum minimal scalars) Bousso and Hawking [5] demonstrated that degenerate Schwarzschild-de Sitter (SdS) (Nariai) BH may not only evaporate but also anti-evaporate. The effect of anti-evaporation has been confirmed in refs. [6] for conformal matter where large $N$ approach and 4D anomaly induced effective action have been used. It has also been shown in refs. [1] that some of anti-evaporating SdS BHs may be initially stable when Hartle-Hawking no boundary condition [7] is used.

Further studying SdS BH perturbations due to quantum minimal scalars Bousso [8] found new effect: proliferation of de Sitter space, which means disintegration of this space into an infinite number of copies of itself (the instability of de Sitter space has been demonstrated in ref. [9]).

It could be really interesting to understand if all these developments may be realized in other types of BHs. The natural candidate to think about is charged BH. Working in large $N$ approach for dilaton coupled quantum scalars we study here the quantum properties of 2D charged BHs. The corresponding classical solutions have been found in [10] (see also ref. [11]). They may be considered as some compactifications of Type II string solutions. Moreover, very naturally they may be considered as 2D analog of Reissner-Nordström (RN) charged 4D BH. Note that some properties of 2D charged BHs [10] have been discussed in refs. [12, 13] (for the calculation of BH entropy in this case, see [14, 15]).

On the other hand, Kaluza-Klein reduction of 3D gravity with minimal
scalars leads to 2D dilaton-Maxwell gravity with dilaton coupled scalars and the rotating BH solution found by Bañados, Teitelboim and Zanelli (BTZ) can be also described by 2D charged dilatonic BH. BTZ black hole (BH)\cite{14} attracts a lot of attention due to various reasons. In particular, it is related via T-duality with a class of asymptotically flat black strings \cite{15} and via U-duality it is related \cite{8} with 4D and 5D stringy BHs \cite{4} which are asymptotically flat ones. Hence, microscopically computing the entropy of BTZ BH \cite{17} may be applied via duality relations for the computing of entropy for higher dimensional BHs. This fact is quite remarkable as above types of BHs look completely different from topological, dimensional or space-time points of view.

BTZ BH being locally AdS$^3$ without curvature singularity may be considered as a prototype for general CFT/AdS correspondence \cite{17}. Indeed, 3D gravity has no local dynamics but BH horizon induces an effective boundary (actually, 2D WZW model). Hence, quantum studies around BTZ BH may help in better understanding of above correspondence.

The paper is organised as follows. In the next section we consider 2D dilaton-Maxwell gravity which represents toy model for 4D or 5D Einstein-Maxwell theory and review its 2D charged BH solutions discussed by McGuigan, Nappi and Yost. The one-loop effective action for dilaton coupled 2D scalar which is obtained by spherical reduction from 4D or 5D minimal scalar is found. We work in large $N$ approximation. Quantum dynamics of 2D charged BHs is discussed in third section. In particular, quantum corrected 2D BH is constructed. The analytical and numerical study of its perturbations is presented. It shows the existence of stable (evaporating) and unstable (anti-evaporating) modes. Calculation of quantum corrections to mass, charge, temperature and BH entropy is done in section 4. Due to duality with 5D stringy BH last result gives also quantum corrected BH entropy for 5D BH. After that we study quantum dynamics of BTZ BH due to 3D minimal scalars. We work in large $N$ and $s$-wave approach where minimal scalars are described as 2D dilaton coupled scalars. The quantum corrected version of BTZ BH is found. It is shown that it can also evaporate or anti-evaporate. Higher modes perturbations in the quantum spectrum for these two kinds of charged BHs are also briefly discussed.
2 One-loop effective action

We start from the action which has been considered by McGuigan, Nappi and Yost in ref. [10]. As it has been shown in [10], this action follows from compactification of heterotic string theory:

\[ S = \frac{1}{16\pi G} \int d^2x \sqrt{g} e^{-2\phi} \left( R + 4(\nabla \phi)^2 + 4\lambda^2 - 4F_{\mu\nu}^2 \right). \] (1)

where \( \lambda^2 \) is cosmological constant, \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \) \( \phi \) is dilaton. Note that above action has the form typical for 4D or 5D Einstein-Maxwell theory spherically reduced to two dimensions. Hence, it can be considered as toy model to describe 4D or 5D BH with spherical reduction.

It is remarkable that action (1) has the classical solutions which correspond to the 2D charged black hole (2D analogue of Reissner-Nordström black hole) with multiple horizon. In the explicit form its metric and dilaton look like [10, 11]

\[ ds^2 = -\left(1 - 2me^{-2\lambda x} + q^2e^{-4\lambda x}\right)dt^2 + \left(1 - 2me^{-2\lambda x} + q^2e^{-4\lambda x}\right)^{-1}dx^2 \]

\[ e^{-2(\phi - \phi_0)} = e^{2\lambda x}. \] (2)

Here \( q \) and \( m \) are parameters related to the charge and the mass of BH, respectively. The extremal solution is given by putting \( q^2 = m^2 \). If we define new coordinates \( r \) and \( \tau \) by

\[ e^{-2\lambda x} = \frac{m + \epsilon \tanh 2\lambda r}{q^2} \]

\[ t = \frac{q}{\epsilon} \tau \]

\[ \epsilon^2 \equiv m^2 - q^2, \]

\( r \to +\infty \) corresponds to outer horizon and \( r \to -\infty \) corresponds to inner horizon. Taking the limit \( \epsilon \to 0 \), we obtain

\[ ds^2 = \frac{1}{\cosh^2 \lambda r} \left( d\tau^2 - dr^2 \right) \]

\[ e^{-2(\phi - \phi_0)} = \frac{1}{q}. \] (4)
Note that the dilaton field becomes a constant in the limit.

We will discuss now the quantum corrections induced by $N$ free conformally invariant dilaton coupled scalars $f_i$ (no background scalars $f_i$):

$$S^f = -\frac{1}{2} \int d^2x \sqrt{-g} e^{-2\phi} \sum_{i=1}^{N} (\nabla f_i)^2 .$$

(5)

Note that the above action (5) appears as a result of spherical reduction from 4D or 5D minimal scalar.

We present now the two-dimensional metric as the following:

$$ds^2 = e^{2\sigma} \tilde{g}_{\mu\nu} dx^\mu dx^\nu$$

(6)

where in conformal gauge $\tilde{g}_{\mu\nu}$ is the flat metric. Then the total quantum action coming from quantum scalars is given as (we work in large $N$ approximation what justifies the neglecting of proper quantum gravitational corrections):

$$\Gamma = W + \Gamma[1, \tilde{g}_{\mu\nu}]$$

(7)

where $W$ is conformal anomaly induced effective action [18, 19] and $\Gamma[1, \tilde{g}_{\mu\nu}]$ is one-loop effective action for scalars (3) calculated on the metric (6) with $\sigma = 0$ (the general covariance should be restored after all). Note that general structure of conformal anomaly has been discussed in ref. [20] while quantum conformal field theory in general dimensions (bigger than two) has been discussed in ref. [21] (for earlier work, see [22]). It could be interesting to generalize above works for the case when external dilaton presents.

Following [19] (due to corresponding result for conformal anomaly of [13, 14, 19, 23]), we get

$$W = -\frac{1}{2} \int d^2x \sqrt{-g} \left[ \frac{N}{48\pi} R \frac{1}{\Delta} R - \frac{N}{4\pi} \nabla^\lambda \phi \nabla_\lambda \phi \frac{1}{\Delta} R + \frac{N}{4\pi} \phi R \right] .$$

(8)

We are left with the calculation of the conformally invariant part of the effective action $\Gamma[1, \tilde{g}_{\mu\nu}]$. This term has to be calculated on the flat space, after that the general covariance should be restored. It is impossible to do such calculation in closed form. One has to apply some kind of expansion. Using Schwinger-De Witt type expansion, we get this effective action as local curvature expansion.
Using the results of ref. [19], one can obtain:

\[ \Gamma[1, \tilde{g}_{\mu
u}] = \frac{N}{24\pi} \int d^2x \sqrt{-\tilde{g}} (-\theta \tilde{\nabla}_\mu \phi \tilde{\nabla}_\mu \phi) \ln \mu^2 + \cdots \]  

(9)

Here \( \mu \) is dimensional parameter.\(^3\) Now, one has to generalize \( \Gamma[1, \tilde{g}_{\mu
u}] \) in order to present it in general covariant form:

\[ \Gamma[1, g_{\mu
u}] = -\frac{N}{4\pi} \int d^2x \sqrt{-g} \tilde{\nabla}_\mu \phi \tilde{\nabla}_\mu \phi \ln \mu^2 + \cdots . \]  

(10)

This action is conformally invariant as it should be in accordance with Eq.(7) where \( W \) gives scale-dependent part of total one-loop effective action. Note that terms which are not written explicitly in Eq.(10) are conformally invariant, higher derivatives terms on \( \phi \). Moreover, as it is easy to see for minimal scalars \( \phi = 0 \), \( \Gamma[1, \tilde{g}_{\mu
u}] = 0 \). That corresponds to general result that in two dimensions, effective action (7) is defined completely by only anomaly induced action \( W \) (which is not the case in the presence of dilaton).

It is also interesting to note that if one uses some form of non-local expansion in the calculation of \( \Gamma[1, \tilde{g}_{\mu
u}] \), one would get the leading terms of this expansion similar to some terms in Eq.(8). In other words, such procedure would just decrease some of coefficients in Eq.(8) (apart from appearance of few local terms.)

### 3 Quantum dynamics of 2D charged BHs

Let us work in the conformal gauge

\[ g_{\pm \mp} = -\frac{1}{2} e^{2\rho} , \quad g_{\pm \pm} = 0 \]  

(11)

The equations of motion with account of quantum corrections are given by variations of sum \( S(1) + W(8) + \Gamma(10) \) over \( g^{\pm \pm}, \rho, \phi \) and \( A_\mu \):

\[
0 = \frac{1}{8G} e^{-2\phi} \left( 4\partial_\pm \rho \partial_\pm \phi - 2\partial_\pm^2 \phi \right) + \frac{N}{12} \left( \partial_\pm^2 \rho - \partial_\pm \rho \partial_\pm \rho \right) \\
+ \frac{N}{2} \left\{ (\partial_\pm \phi \partial_\pm \phi) \rho + \frac{1}{2} \partial_\pm \left( \partial_\pm \phi \partial_\pm \phi \right) \right\}
\]

\(^3\)Note that more exactly, one should write \( \ln \frac{L^2}{\mu^2} \) where \( L^2 \) is covariant cut-off parameter.
\[ + \frac{N}{4} \left\{ 2 \partial_\pm \rho \partial_\pm \phi - \partial_\pm^2 \phi \right\} + t^\pm (x^\pm) \]
\[ + \frac{N}{2} (\partial_\pm \phi \partial_\pm \phi) \ln \mu^2 \]  
(12)

0 = \frac{1}{8G} e^{-2\phi} \left( 2 \partial_+ \partial_- \phi - 4 \partial_+ \phi \partial_- \phi - \lambda^2 e^{2\rho} + e^{-2\rho} (F_{+-})^2 \right) 
- \frac{N}{12} \partial_+ \partial_- \rho - \frac{N}{4} \partial_+ \phi \partial_- \phi + \frac{N}{4} \partial_+ \partial_- \phi 
(13)

0 = \frac{1}{8G} e^{-2\phi} \left( -4 \partial_+ \partial_- \phi + 4 \partial_+ \phi \partial_- \phi + 2 \partial_+ \partial_- \rho + \lambda^2 e^{2\rho} + e^{-2\rho} (F_{+-})^2 \right) 
+ 2N \left\{ \frac{1}{8} \partial_+ (\rho \partial_- \phi) - \frac{1}{8} \partial_- (\rho \partial_+ \phi) - \frac{1}{8} \partial_+ \partial_- \rho \right\} 
- \frac{N}{2} \partial_+ \partial_- \phi \ln \mu^2 
(14)

0 = \partial_\pm (e^{-2\phi-2\rho} F_{+-}) . 
(15)

Eq. (13) can be integrated to give
\[ e^{-2\phi-2\rho} F_{+-} = B \text{ (constant)} . \]  
(16)

We first consider the static case and replace \( \partial_\pm \) by \( \pm \frac{1}{2} \partial_r \). Then, using (16), Eqs. (12), (13) and (14) are rewritten as follows:

0 = \frac{1}{8G} e^{-2\phi} \left( \partial_r \rho \partial_r \phi - \frac{1}{2} \partial_r^2 \phi \right) + \frac{N}{48} \left( \partial_r^2 \rho - \partial_r \rho \partial_r \rho \right) 
+ \frac{N}{8} \left( \rho + \frac{1}{2} \right) \partial_r \phi \partial_r \phi + \frac{N}{16} \left\{ 2 \partial_r \rho \partial_r \phi - \partial_r^2 \phi \right\} + t_0 
+ \frac{N}{8} \partial_r \phi \partial_r \phi \ln \mu^2 
(17)

0 = \frac{1}{8G} e^{-2\phi} \left( -\frac{1}{2} \partial_r^2 \phi - \partial_r \phi \partial_r \phi - \lambda^2 e^{2\rho} + B^2 e^{2\rho+4\phi} \right) 
+ \frac{N}{48} \partial_r^2 \rho + \frac{N}{16} \partial_r \phi \partial_r \phi - \frac{N}{16} \partial_r^2 \phi 
(18)

0 = \frac{1}{8G} e^{-2\phi} \left( \partial_r^2 \phi - \partial_r \phi \partial_r \phi - \frac{1}{2} \partial_r^2 \rho + \lambda^2 e^{2\rho} + B^2 e^{2\rho+4\phi} \right) 
+ \frac{N}{8} \partial_r (\rho \partial_r \phi) + \frac{N}{16} \partial_r^2 \rho + \frac{N}{8} \partial_r^2 \phi \ln \mu^2 . 
(19)

Since the classical extremal solution can be characterized by constant dilaton field \( \phi \), we assume \( \phi \) is constant even when we include the quantum
correction:

\[ \phi = \phi_0 \text{ (constant)} \]  \hspace{1cm} (20)

Then (18) and (14) have the following forms:

\[
0 = e^{-2\phi_0} \left( -\lambda^2 + B^2 e^{4\phi_0} \right) e^{2\rho} + \frac{GN}{6} \partial_r^2 \rho 
\]  \hspace{1cm} (21)

\[
0 = e^{-2\phi_0} \left( \lambda^2 + B^2 e^{4\phi_0} \right) e^{2\rho} + \left( -\frac{1}{2} e^{-2\phi_0} + \frac{GN}{2} \right) \partial_r^2 \rho 
\]  \hspace{1cm} (22)

The condition that Eqs. (21) and (22) are compatible with each other gives

\[
B^2 = \frac{\lambda^2 e^{-4\phi_0} \left( e^{-2\phi_0} - \frac{4GN}{3} \right)}{e^{-2\phi_0} - \frac{2GN}{3}} 
\]  \hspace{1cm} (23)

and we find the scalar curvature \( R \) is constant

\[
R = -2e^{2\rho} \partial_r^2 \rho = R_0 \equiv -\frac{8\lambda^2 e^{-2\phi_0}}{e^{-2\phi_0} - \frac{2GN}{3}} . 
\]  \hspace{1cm} (24)

We see that quantum corrections decrease the value of curvature if compare with the classical case. (24) can be integrated to give

\[
e^{2\rho} = e^{2\rho_0} \equiv \frac{2C}{R_0} \cdot \frac{1}{\cosh^2 \left( r \sqrt{C} \right)} . \hspace{1cm} (25)
\]

Here \( C \) is a constant of the integration. Finally (17) gives

\[
t_0 = \frac{GN}{6} C \hspace{1cm} (26)
\]

We now investigate the (in)stability of the above obtained extremal solution by the perturbation

\[
\rho = \rho_0 + \epsilon R \hspace{0.5cm} \text{,} \hspace{0.5cm} \phi = \phi_0 + \epsilon S \hspace{1cm} (27)
\]

Here \( \epsilon \) is an infinitesimally small constant. By following Bousso and Hawking’s argument \[ F \], we neglect the second term in \( \delta \). (Note that we keep
similar term with \( \ln \mu^2 \) as it could be large). Then (13) and (14) (and (16)) give the following equations:

\[
0 = \frac{1}{8G} e^{-2\phi_0} \left\{ 2\partial_\perp \partial_\perp S - 2\lambda^2 e^{2\rho_0}(R - S) + 2B^2 e^{2\rho_0 + 4\phi_0}(R + S) \right\} - \frac{N}{12} \partial_\perp \partial_- R + \frac{N}{4} \partial_\perp \partial_- S
\]

\[
0 = \frac{1}{8G} e^{-2\phi_0} \left\{ -4\partial_\perp \partial_- S + 2\partial_\perp \partial_- R - 4\partial_\perp \partial_- \rho_0 S + 2\lambda^2 e^{2\rho_0}(R - S) + 2B^2 e^{2\rho_0 + 4\phi_0}(R + S) \right\} - \frac{N}{4} \partial_\perp \partial_- R - \frac{N}{2} \ln \mu^2 \partial_\perp \partial_- S.
\]

We are going to study (in)stability of black holes in the same way as in four dimensions [3]. As in the previous works, we assume \( R \) and \( S \) have the following form

\[
R(t, r) = P \cosh t \sqrt{C} \cosh \alpha \sqrt{C} \cosh r \sqrt{C}.
\]

\[
S(t, r) = Q \cosh t \sqrt{C} \cosh \alpha \sqrt{C}.
\]

Then we obtain the following equations

\[
\cosh^2 \left( r \sqrt{C} \right) \partial_\perp \partial_- R = AR
\]

\[
\cosh^2 \left( r \sqrt{C} \right) \partial_\perp \partial_- S = AS
\]

\[
A \equiv \frac{\alpha(\alpha - 1)C}{4}.
\]

Note that there is one to one correspondence between \( A \) and \( \alpha \) if we restrict \( A > 0 \) and \( \alpha < 0 \). Then (28) and (29) become algebraic equations:

\[
0 = \left\{ e^{-2\phi_0} \left( \frac{R_0}{C} A + 2\lambda^2 + 2B^2 e^{4\phi_0} \right) + \frac{GNR_0}{C} A \right\} Q
\]

\[
+ \left\{ e^{-2\phi_0} \left( -2\lambda^2 + 2B^2 e^{4\phi_0} - \frac{GNNR_0}{3C} A \right) \right\} P
\]

\[
0 = \left\{ e^{-2\phi_0} \left( -\frac{2R_0}{C} A - \frac{R_0}{2} - 2\lambda^2 + 2B^2 e^{4\phi_0} \right) - 2GN(\ln \mu^2) \frac{R_0}{C} A \right\} Q
\]

\[
+ \left\{ e^{-2\phi_0} \left( \frac{R_0}{C} A + 2\lambda^2 + 2B^2 e^{4\phi_0} \right) - \frac{GR_0}{C} A \right\} P.
\]
In order that Eqs. (32) and (33) have a non-trivial solution for $Q$ and $P$, we find

$$F(A) \equiv \left\{ e^{-2\phi_0} \left( \frac{R_0}{C} A + 2\lambda^2 + 2B^2 e^{4\phi_0} \right) + \frac{G R_0}{C} A \right\} \times \left\{ e^{-2\phi_0} \left( \frac{R_0}{C} A + 2\lambda^2 + 2B^2 e^{4\phi_0} \right) - \frac{GNR_0}{C} A \right\}$$

$$- \left\{ e^{-2\phi_0} (-2\lambda^2 + 2B^2 e^{4\phi_0}) - \frac{GNR_0}{3C} A \right\} \times \left\{ e^{-2\phi_0} \left( -\frac{2R_0}{C} A - \frac{R_0}{2} - 2\lambda^2 + 2B^2 e^{4\phi_0} \right) - 2GN(\ln \mu^2)\frac{R_0}{C} A \right\} = 0 . \quad (34)$$

Eq. (34) should be solved with respect to $A$.

When we compare the model with the 4D one, $e^{-\phi}$ is identified with the radius coordinate. Since the (apparent) horizon is a null surface, the horizon is given by the condition

$$\nabla \phi \cdot \nabla \phi = 0 . \quad (35)$$

Substituting (30) into (35), we find the horizon is given by

$$r = \pm \alpha t . \quad (36)$$

Therefore on the horizon, we obtain

$$S(t, r(t)) = Q \cosh^{1+\alpha} \alpha t \sqrt{C} . \quad (37)$$

This tells that the system is unstable if there is a solution $0 > \alpha > -1$, i.e., $0 < A < \frac{C}{\alpha}$. On the other hand, the perturbation becomes stable if there is a solution where $\alpha < -1$, i.e., $A > \frac{C}{\alpha}$.

The radius of the horizon $r_h$ is given by

$$r_h = e^{-\phi} = e^{-(\phi_0 + eS(t, r(t)))} . \quad (38)$$

Let the initial perturbation is negative $Q > 0$. Then the radius shrinks monotonically, i.e., the black hole evaporates in case of $0 > \alpha > -1$. On the other hand, the radius increases in time and approaches to the extremal limit asymptotically

$$S(t, r(t)) \to Qe^{(1+\alpha)\alpha t \sqrt{C}} . \quad (39)$$
in case of $\alpha < -1$. The latter case corresponds to the anti-evaporation of Nariai black hole observed by Bousso and Hawking [5].

Eq. (34) can be solved with respect to $\frac{A}{C}$:

$$\frac{A}{C} = \frac{1}{2} \left( -2304 + 192g + 36g^2 + 24g^2 \ln \mu^2 \right)^{-1}$$

$$\times \left\{ -2304 + 144g - g^2 - 6g^2 m \pm \left( 313344g + 63744g^2 
- 8640g^3 + g^4 + 27648g \ln \mu^2 + 53568g^2 \ln \mu^2 
- 5556g^3 \ln \mu^2 + 36g^2 (\ln \mu^2)^2 \right) \right\}^{1/2} \right\}$$

$$\sim \frac{1}{2} \pm \frac{1}{8} g^{1/2} + O(g)$$

$$g \equiv 8GNe^{2\phi_0}$$

In the classical limit $g = 0$, $A = \frac{C}{4}$, what tells that there does not occur any kind of the radiation in the solution. Near the classical limit $g \sim 0$, there are solutions corresponding to both of stable and instable ones. It might be surprising that there is an instable mode since the extremal solution is usually believed to be stable.

The global behavior of $A$ as a function of $g \equiv 8GNe^{2\phi_0}$ and $\ln \mu^2$ is given in Figures. In Fig.1, the vertical line corresponds to $\frac{A}{C}$ and the horizontal one to $g$ when $\ln \mu^2 = 1$. There is a singularity near $g \sim 5$. In the wide range from $g = 0$ to near the singularity, there coexist the stable and instable modes. Near the singularity, both of the modes become stable ones. Beyond the singularity, $\frac{A}{C}$ becomes negative and there is no consistent solution. In Fig.2a and 2b, we show the global behavior of two modes as a function of $g \equiv 8GNe^{2\phi_0}$ and $\ln \mu^2$. Fig.2a corresponds to the instable mode and 2b to the stable one. In Fig.2a, the singularity appeared in Fig.1 disappears near $g = 3$.

4 Quantum corrections to BH parameters

We investigate now the quantum corrections to mass, charge, etc. by considering the limit of $\phi \to -\infty$. In the large $N$ limit, the leading order is proportional to $N$. Since the leading order is the contribution of the one-loop correction it is also proportional to the effective gravitational constant.
\[ e^{2\phi_0}. \] Therefore the quantum correction under consideration appears in the form of \( Ne^{2\phi_0} \). Since \( e^{-\phi_0} \) is the scale of the black hole radius in the corresponding 4 dimensional model, the above expansion also corresponds to the large radius expansion. Therefore we can assume that the quantum corrections for \( \rho \) and \( \phi \) appear in the following form

\[ \rho = \rho_{cl} + 8GN e^{2\phi_0} \rho_1, \quad \phi = \phi_{cl} + 8GN e^{2\phi_0} \phi_1 \]  

(41)

neglecting the second and higher power of \( Ne^{2\phi_0} \). Here \( \rho_{cl} \) and \( \phi_{cl} \) are the classical parts of \( \rho \) and \( \phi \), respectively, which are given by (see (2))

\[
e^{2\rho_{cl}} = 1 - 2me^{-2\lambda x} + q^2 e^{-4\lambda x}
\]

\[
e^{-2(\phi_{cl} - \phi_0)} = e^{2\lambda x}.
\]

(42)

Since the limit of \( \phi \to -\infty \) means the large radius of black hole, we expand \( \rho_{cl} \) and \( \phi_{cl} \) as power series of \( e^{-2\lambda x} \). Using the Eqs.(17), (18) and (19) and the boundary conditions

\[
\rho_1, \phi_1 \to 0 \quad \text{when} \quad x \to -\infty
\]

(43)

\[
(\rho \to \rho_{cl}, \quad \phi \to \phi_{cl}),
\]

we find

\[
\rho_1 = - \left( \frac{7}{32} + \frac{1}{4} \ln \mu^2 \right) e^{-2\lambda x} - \left( \frac{5}{12} + \frac{1}{2} \ln \mu^2 \right) e^{-4\lambda x} + O(e^{-6\lambda x})
\]

\[
\phi_1 = - \frac{1}{32} e^{-2\lambda x} + \left( \frac{13}{96} + \frac{1}{16} \ln \mu^2 \right) me^{-4\lambda x} + O(e^{-6\lambda x})
\]

\[
t_0 = -\lambda^2 GN(1 + \ln \mu^2).
\]

(44)

Here we change the radial coordinate by

\[ dx = e^{2\rho} dr, \]

(45)

which gives the metric of the form

\[ ds^2 = -G(x)dt^2 + \frac{1}{G(x)} dx^2, \quad G(x) \equiv e^{2\rho(r(x))}. \]

(46)

Since the parameter \( m \) related to the black hole mass is classically given by

\[ \rho_{cl} = -me^{-2\lambda x} + O(e^{-4\lambda x}), \]

(47)
we can find the quantum correction $\delta m$ to $m$ as

$$
\delta m = \left(\frac{7}{32} + \frac{1}{4} \ln \mu^2\right) 8 G N e^{2\phi_0} \ .
$$

(48)

If we use the mass formula from [10]

$$
M = \frac{8}{\sqrt{\alpha}} m e^{-2\phi_0} \ ,
$$

(49)

$\delta m$ gives the quantum correction to mass,

$$
\delta M = \frac{8}{\sqrt{\alpha}} \left(\frac{7}{32} + \frac{1}{4} \ln \mu^2\right) 8 G N e^{2\phi_0} \ .
$$

(50)

On the other hand, (16) and (44) tell that the asymptotic behavior of the electric field is not changed by the quantum correction. This would restrict parameter $q$, which is related with $B$ by

$$
B = \lambda q e^{-2\phi_0} \ ,
$$

(51)

and the charge of black hole would not change.

Eq.(44) tells

$$
G(x) \equiv e^{2\rho} = 1 + 2 \left\{ - m - \left(\frac{7}{32} + \frac{1}{4} \ln \mu^2\right) 8 G N e^{2\phi_0} \right\} e^{-2\lambda x}
$$

$$
+ \left\{ q^2 - \left(\frac{5}{6} + \frac{1}{2} \ln \mu^2\right) 8 G N e^{2\phi_0} \right\} e^{-4\lambda x} + O(e^{-6\lambda x}) \ ,
$$

(52)

what gives the shift of the position of the horizons

$$
e^{2\lambda x^\pm} = m \pm \sqrt{m^2 - q^2}
$$

$$
+ 8 G N e^{2\phi_0} \left\{ - \left(\frac{7}{32} + \frac{1}{4} \ln \mu^2\right) + \frac{m}{\sqrt{m^2 - q^2}} \left(\frac{1}{96} + \frac{1}{4} \ln \mu^2\right) \right\} .
$$

(53)

When the metric has the form of (16) and $G(x) \to 1$ when $x \to +\infty$, the temperature $T$ of the black hole is given by,

$$
T = \frac{1}{4\pi} \left| \frac{dG}{dx} \right|_{x=x^+} .
$$

(54)
Using (52), we find

\[ 4\pi T = \frac{4\lambda}{q^2} \left\{ -m^2 + q^2 + m\sqrt{m^2 - q^2} \right\} \]

\[ + 32\lambda G N e^{2\phi_0} \left[ -\frac{7}{32} - \frac{1}{4} \ln \mu^2 + \frac{m^2}{q^2} \left( \frac{5}{6} + \frac{1}{2} \ln \mu^2 \right) \right. \]

\[ \left. + \frac{1}{q^2\sqrt{m^2 - q^2}} \left\{ - \left( \frac{5}{4} + \frac{3}{2} \ln \mu^2 \right) m^3 + \left( \frac{1}{48} + \frac{1}{2} \ln \mu^2 \right) q^2 m \right\} \right] \] \quad (55)

Recently Teo has found that the model of McGuigan, Nappi and Yost is related with five dimensional extremal black hole in Type II superstring theory [4] by the duality. The five dimensional black hole can be obtained as a solution of ten dimensional Type II supergravity when five space coordinates \((x_5, x_7, \cdots x_9)\) are compactified. The metric has the following form:

\[ ds^2 = -(H_1 K)^2 f dt^2 + H_1^{-1} K \left( dx_5 - (K' - 1) dt \right)^2 \]

\[ + H_5 \left( f^{-1} dr^2 + r^2 d\Omega_3^2 \right) + \sum_{i=6}^{9} dx_i^2 \] \quad (56)

Here \(d\Omega_3^2\) is the metric on the unit three-sphere and

\[ r^2 = \sum_{i=1}^{4} x_i^2 \], \quad f = 1 - \frac{r_0^2}{r^2} ; \]

\[ H_1 = 1 + \frac{r_0 \sinh^2 \alpha}{r^2} \], \quad H_1' = 1 - \frac{r_0 \sinh \alpha \cosh \alpha}{r^2} H_1^{-1} ; \]

\[ H_5 = 1 + \frac{r_0 \sinh^2 \beta}{r^2} \], \quad H_5' = 1 - \frac{r_0 \sinh \beta \cosh \beta}{r^2} H_5^{-1} ; \]

\[ K = 1 + \frac{r_0 \sinh^2 \gamma}{r^2} \], \quad K' = 1 - \frac{r_0 \sinh \gamma \cosh \gamma}{r^2} K^{-1} . \quad (57)

The entropy is, as usually, given by

\[ S = \frac{A_H}{4G_5} = \frac{2\pi^2 r_0^3 \cosh \alpha \cosh \beta \cosh \gamma}{4G_5} \] \quad (58)

Here \(A_H\) is the horizon area and \(G_5\) is Newton constant in five dimensions. The extremal limit is obtained by taking \(r_0 \rightarrow 0\) but keeping \(r_0 \sinh \alpha\), \(r_0 \sinh \beta\) and \(r_0 \sinh \gamma\) to be finite. When taking the extremal limit and
setting $\alpha = \gamma$, it was shown [4] that the metric (56) is dual to the following metric

$$ds^2 = -\left(1 + \frac{r_1^2}{r^2}\right)^{-2} dt^2 + \frac{r_5^2}{r^2} dr^2 + r_5^2 d\Omega_3^2 + \sum_{i=5}^9 dx_i^2.$$  \hspace{1cm} (59)

Here

$$r_1 = r_0 \sinh \alpha, \quad r_2 = r_0 \sinh \beta.$$  \hspace{1cm} (60)

The metric is direct product of 5 dimensional compact space corresponding to $(x_5, x_6, \cdots, x_9)$, three sphere of the radius $r_5$ and two-dimensional spacetime. Changing the coordinate

$$\sqrt{m^2 - q^2 e^{-\lambda x}} = \frac{r_0^2}{r^2 + r_1^2}, \quad r_0 = \frac{1}{\lambda},$$  \hspace{1cm} (61)

the metric of the two-dimensional spacetime becomes that of (2). It was also shown that the entropy $S$ in (58) is given in two dimensional language as follows

$$S = 4\pi e^{-2\phi_0} e^{\lambda x^+}.$$  \hspace{1cm} (62)

Using the quantum correction in (53), we find the quantum correction to the entropy

$$S = 4\pi e^{-2\phi_0} \left[m + \sqrt{m^2 - q^2}ight.$$ 

$$+ 8G N e^{2\phi_0} \left\{ - \left( \frac{7}{32} + \frac{1}{4} \ln \mu^2 \right) - \frac{m}{\sqrt{m^2 - q^2}} \left( \frac{1}{96} + \frac{1}{4} \ln \mu^2 \right) \right\} \right]$$  \hspace{1cm} (63)

Hence, we demonstrated that two-dimensional considerations may be useful to define quantum corrections not only to two-dimensional parameters but also to five-dimensional ones (like entropy).

## 5 Kaluza-Klein reduction of 3d gravity

Let us apply above technique to BTZ BHs. Three dimensional Einstein gravity with the cosmological term has the exact rotating black hole solution [14] and the solution can be regarded as an exact solution of string theory [15]. In this section, we first consider the circle reduction of three dimensional Einstein gravity to two dimensional one. Now we identify the coordinates
as \( x^1 = t, \ x^2 = r, \ x^3 = \phi \). Here \( t, r \) and \( \phi \) are the coordinates of time, radius, and angle. We now assume that all the fields do not depend on \( x^3 \). Then, under the following infinitesimal coordinate transformation \( x^3 \to x^3 + \epsilon(x^1, x^2) \), the metric tensors transform as follows;

\[
\delta g^{(3)3} = \frac{\partial g^{(3)3}}{\partial \epsilon}, \quad \delta g^{(3)3\alpha} = 2g^{(3)3\alpha} \partial_{\epsilon}, \quad \delta g^{(3)\alpha\beta} = 0 \tag{64}
\]

Here \( \alpha, \beta = 1, 2 \). Eq.(64) tells that we can identify the gauge vector field \( A_\alpha \) with \( g^{(3)3\alpha} = g^{(3)\alpha\beta}A_\beta \). Eq.(64) also tells that the metric tensor \( g^{(3)\mu\nu} \) \((\mu, \nu = 1, 2, 3)\) can be parametrized a la Kaluza-Klein as

\[
g^{(3)\mu\nu} = \left( \begin{array}{cc}
g^{\alpha\beta} & g^{\alpha\gamma}A_\gamma \\
g^{\beta\gamma}A_\gamma & e^{2\phi} + g^{\gamma\delta}A_\gamma A_\delta
\end{array} \right) \tag{65}
\]

Then we find

\[
g^{(3)} \equiv \frac{1}{\det g^{(3)\mu\nu}} = ge^{-2\phi} \quad (g \equiv \frac{1}{\det g^{\mu\nu}}) \tag{66}
\]

\[
g^{(3)}_{\mu\nu} = \left( \begin{array}{cc}
g_{\alpha\beta} + e^{-2\phi}A_\mu A_\nu - A_\gamma & -A_\gamma \\
-A_\gamma & e^{-2\phi}
\end{array} \right) \tag{67}
\]

In the following, the quantities in three dimensions are denoted by the suffix “(3)” and the quantities without the suffix are those in two dimensions unless we mention. In the parametrization (65), curvature has the following form

\[
R^{(3)} = R + \Box \psi - \partial_\alpha \phi \partial^\alpha \phi - \frac{1}{4} e^{-2\phi} F_{\alpha\beta} F^{\alpha\beta}. \tag{68}
\]

Here \( F_{\alpha\beta} \) is the field strength: \( F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha \). Then the action \( S \) of the gravity with cosmological term and with \( N \) free scalars \( f_i \) \((i = 1, \cdots, N)\) in three dimensions is reduced as

\[
S = \frac{1}{4\pi^2} \int d^3x \sqrt{g^{(3)}} \left\{ \frac{1}{8G} (R^{(3)} + \Lambda) + \frac{1}{2} \sum_{i=1}^N \partial_\mu f_i \partial^\mu f^i \right\}
\]

\[
\sim \frac{1}{2\pi} \int d^2x \sqrt{g} e^{-\phi} \left\{ \frac{1}{8G} \left( R - \frac{1}{4} e^{-2\phi} F_{\alpha\beta} F^{\alpha\beta} + \Lambda \right) + \frac{1}{2} \sum_{i=1}^N \partial_\alpha f_i \partial^\alpha f^i \right\} \tag{69}
\]

Note that the contribution from the second and third terms in (68) becomes the total derivative which maybe neglected. One BH solution for the classical
action is given by 14

\[ ds^2 = -e^{2\rho_{cl}} dt^2 + e^{-2\rho_{cl}} dx^2, \quad e^{2\rho_{cl}} = -M + \frac{x^2}{l^2} + \frac{J^2}{4x^2} \]

\[ e^{-\phi} = e^{-\phi_{cl}} = e^{-\phi_0} x \]

(70)

Here \( \frac{1}{l^2} = \frac{\Lambda}{2} \). \( M \) is the parameter related to the black hole mass and \( J \) is that of the angular momentum in 3D model or the electromagnetic charge in the 2D model. The extremal limit is given by \( J^2 = l^2 M^2 \). In order to consider this limit, we change the coordinates as follows

\[ x^2 = \frac{l^2 M (1 + \epsilon \tanh r)}{2}, \quad t = \frac{l}{\epsilon \sqrt{2}} r. \]

(71)

Then \( r \to +\infty \) corresponds to outer horizon and \( r \to -\infty \) corresponds to inner horizon. Taking the limit of \( \epsilon \to 0 \), we obtain

\[ ds^2 = \frac{l^2 M}{4 \cosh^2 r} \left( dt^2 - dr^2 \right), \quad e^{-\phi} = le^{-\phi_0} \sqrt{\frac{M}{2}}. \]

(72)

Note that \( \phi \) becomes a constant in this limit.

Let us now discuss the quantum corrections induced by \( N \) free conformally invariant dilaton coupled scalars \( f_i \). We use (8) and (10) by replacing \( 2\phi \to \phi \). In the study of quantum corrected BH we work in the conformal gauge (11). We are going to study (in)stability of black holes in the same way as in four dimensions [6].

The equations of motion with account of quantum corrections are given by

\[ 0 = \frac{1}{8G} e^{-\phi} \left( -\partial_\pm \rho \partial_{\pm} \phi - \frac{1}{2} \partial^2_\pm \phi + \frac{1}{2} (\partial_\pm \phi)^2 \right) + \frac{N}{12} \left( \partial^2_\pm \rho - \partial_{\pm} \rho \partial_{\pm} \rho \right) \]

\[ + \frac{N}{8} \left\{ (\partial_\pm \phi \partial_{\pm} \phi) \rho + \frac{1}{2} \partial_\pm (\partial_\pm \phi \partial_{\pm} \phi) \right\} \]

\[ + \frac{N}{8} \left\{ 2 \partial_\pm \rho \partial_{\pm} \phi - \partial^2_{\pm} \phi \right\} + t^\pm (x^\pm) + \frac{N}{8} (\partial_\pm \phi \partial_{\pm} \phi) \ln \mu^2 \]

(73)

\[ 0 = \frac{1}{8G} e^{-\phi} \left( 4 \partial_{\pm} \partial_{\pm} \phi - 4 \partial_{\pm} \phi \partial_{\pm} \phi - \Lambda e^{2\rho} + 2e^{-2\phi - 2\rho} (F_{++})^2 \right) \]

\[ ^4\text{Quantum corrections due to matter near BTZ black hole have been also studied in [24].} \]
\[-\frac{N}{12} \partial_+ \partial_- \rho - \frac{N}{16} \partial_+ \phi \partial_- \phi + \frac{N}{8} \partial_+ \partial_- \phi \]

\[0 = \frac{1}{8G} e^{-\phi} \left( 4 \partial_+ \partial_- \rho + \frac{\Lambda}{2} e^{2\rho} + 3 e^{-2\phi-2\rho} (F_+^2) \right) \]

\[-\frac{N}{16} \partial_+ (\rho \partial_- \phi) - \frac{N}{16} \partial_- (\rho \partial_+ \phi) - \frac{N}{8} \partial_+ \partial_- \rho - \frac{N}{4} \partial_+ \partial_- \phi \ln \mu^2 \]

\[0 = \partial_\pm (e^{-3\phi-2\rho} F_{+\pm}) \, . \]

Eq. (76) can be integrated to give

\[e^{-3\phi-2\rho} F_{+\pm} = B \, (\text{constant}) \, . \]

We now assume \( \phi \) is constant as in the classical extremal solution (72) even when we include the quantum correction: \( \phi = \phi_0 \, (\text{constant}) \). Then we obtain

\[B^2 = \frac{\Lambda e^{-4\phi_0}}{2} \, , \quad R = R_0 \equiv - \frac{4 \Lambda e^{-\phi_0}}{e^{-\phi_0} - \frac{GN}{3}} \, . \]

Quantum corrected extremal (static) solution corresponding to the classical one (72) can be written as in (25) \((r = \frac{e^{x_+}-e^{-x_-}}{\sqrt{2}})\).

In order to investigate the (in)stability in the above solution, we consider the perturbation around the extremal solution as in (27). We neglect the second term in (8) again. Then (74) and (75) (and (77)) lead to the following equations:

\[0 = \frac{1}{8G} e^{-\phi_0} \left\{ 4 \partial_+ \partial_- S - \Lambda e^{2\phi_0} (2R - S) + 2B^2 e^{2\phi_0+4\phi_0} (2R + 3S) \right\} \]

\[-\frac{N}{12} \partial_+ \partial_- R + \frac{N}{8} \partial_+ \partial_- S \]

\[0 = \frac{1}{8G} e^{-\phi_0} \left\{ 4 \partial_+ \partial_- R - 4 \partial_+ \partial_- \rho_0 S + \frac{\Lambda}{2} e^{2\rho_0} (2R - S) \right. \]

\[+ 3B^2 e^{2\phi_0+4\phi_0} (2R + 3S) \] \[- \frac{N}{8} \partial_+ \partial_- R - \frac{N}{4} \ln \mu^2 \partial_+ \partial_- S \] .

By assuming \( R \) and \( S \) have the form of (30), we can rewrite (79) and (80) as algebraic equations and the condition that there is non-trivial solution for \( Q \) and \( P \) is given by

\[F^{BTZ}(A) \equiv e^{-2\phi_0} \left( \frac{2R_0}{C} A + 4\Lambda \right)^2 - \left( \frac{GNR_0}{C} A \right)^2 \]

\[+ \frac{GNR_0}{3C} A \left\{ e^{-\phi_0} \left( \frac{R_0}{2} + 4\Lambda \right) - GN(\ln \mu^2) \frac{R_0}{C} A \right\} = 0 \, (81) \]
Here $A = \frac{\alpha(\alpha-1)C}{4}$. $e^{-\phi}$ is identified with the radius in 3D model, so horizon is given by the condition of (35).

Eq. (81) can be solved with respect to $A$,

$$\frac{A}{C} = \left[-4096 - 2176g + 48g^2 \pm \left\{18939904g + 4321280g^2 - 208832g^3 + 2304g^4 + (786432g - 49152g^2 + 768g^3)\ln \mu^2\right\}^{\frac{1}{2}}\right] \times \left\{2 \left(-4096 + 16g^2 + 192g^2m\right)\right\}^{-1} \sim \frac{1}{2} \pm \frac{3\sqrt{2}}{4}g^{\frac{3}{2}} + \mathcal{O}(g)$$

(82)

Here $g \equiv 8\text{GN}e^{\phi_0}$. Note that there are two solutions near the classical limit $g \to 0$, which correspond to stable and instable modes, respectively. It might be surprising that there is an instable mode since the extremal solution is usually believed to be stable. The global behavior of $\frac{A}{C}$ is given in Figures. In Fig.3, the vertical line corresponds to $\frac{A}{C}$ and the horizontal one to $g$ when $\ln \mu^2 = 1$. There is a singularity when $g \sim 0.4376$. The singularity occurs when the denominator in (82) vanishes. In the range from $g = 0$ to near the singularity, there coexist the stable and instable modes. In Fig.4a and 4b, we present the global behavior of two modes as a function of $g \equiv 8\text{GN}e^{\phi_0}$ and $\ln \mu^2$. Fig.4a corresponds to the stable mode and 2b to the instable one.

6 Higher order perturbations and multiple BHs

Recently Bousso has shown the possibility that de Sitter space disintegrates into an infinite number of copies of de Sitter space. In the scenario, Schwarzschild de Sitter black hole becomes an extremal one which is known to be the Nariai solution. The topology of the Nariai space is $S^1 \times S^2$, therefore the solution can be regarded to express the topology of a handle. If there is a perturbation, multiple pair of the cosmological and black hole horizons are formed. After that the black holes evaporate and the black hole horizons shrink to vanish. Therefore the handle is separated into several pieces, which are copies of the de Sitter spaces.
The metric (1) or (25) in the extremal limit or its quantum analogue (23) has the same spacetime structure as the Nariai solution except its signature. Therefore there is a possibility of the multiple production of universes from the extremal solution.

If we define new coordinate $\theta$ by

$$\sin \theta = \tanh (r \sqrt{C})$$

the metric corresponding to (25) becomes

$$ds^2 = \frac{2C}{R_0} \left( -dt^2 + \frac{1}{C} d\theta^2 \right).$$

Since $\theta$ has the period $2\pi$, the topology of the space with the metric (84) can be $S_1$. Note that Eq.(83) tells there is one to two correspondence between $\theta$ and $r$.

The operator

$$\Delta \equiv C \cosh^2 \left( r \sqrt{C} \right) \partial_+ \partial_-$$

in Eq.(31) can be regarded as the Laplacian on two-dimensional Lorentzian hyperboloid, i.e., the Casimir operator of $SL(2, R)$. The raising and lowering operators $L_{\pm}$ of $SL(2, R)$ are given by

$$L_{\pm} = e^{\pm t \sqrt{C}} \left( \sinh \left( r \sqrt{C} \right) \frac{\partial}{\partial t} \pm \cosh \left( r \sqrt{C} \right) \frac{\partial}{\partial r} \right).$$

The eigenfunction of $\Delta$ in $S$ and $R$ (30) is the sum of highest and lowest weight representation $e^{\pm t\alpha \sqrt{C}} \cosh^\alpha r\sqrt{C}$. We now consider the following eigenfunction

$$S(t, r) = \frac{Q}{4} \left( L_+ e^{t\alpha \sqrt{C}} \cosh^\alpha r\sqrt{C} + L_- e^{-t\alpha \sqrt{C}} \cosh^\alpha r\sqrt{C} \right)$$

$$= Q \sinh \left( t(\alpha + 1) \sqrt{C} \right) \cosh^\alpha r\sqrt{C} \sinh r\sqrt{C}. \quad (87)$$

Since the evaporation of the black hole in [8] corresponds to the instable mode, we restrict $\alpha$ to be $0 > \alpha > -1$. Then the condition of horizon (35) gives

$$\tanh \left( r \sqrt{C} \right) = \pm \frac{\alpha + 1 - \sqrt{(\alpha + 1)^2 - 4\alpha \tanh^2 \left( t(\alpha + 1) \sqrt{C} \right)}}{2\alpha \tanh \left( t(\alpha + 1) \sqrt{C} \right)}. \quad (88)$$
We find the asymptotic behaviors of $S$ are given by

$$S(t, r(t)) \xrightarrow{t \to 0} \pm (\alpha + 1)CQt^2$$

$$t \to +\infty \quad \pm \frac{Q}{2^{\alpha+2}} \left( \frac{1 - \alpha}{1 + \alpha} \right)^{\frac{\alpha+1}{2}} e^{t(\alpha+1)(\alpha+2)\sqrt{C}}$$

(89)

Since the radius of the horizon is given by (38), the horizon corresponding to $+$ sign in (89) grows up to infinity and that to $-$ sign shrinks to vanish when $t \to +\infty$ although the perturbation does not work when $S$ is large. The horizon of $+$ sign would correspond to outer horizon and that of minus sign to inner horizon. Since there is one to two correspondence between the radial coordinates $r$ and $\theta$, there are two outer horizons and two inner ones. When we regard the model as the model reduced from four (three) dimensions, the handle with the topology of $S_1 \times S_2$ ($S_1 \times S_1$) is separated to two pieces when the two inner horizons shrink to vanish. This result would be generalized if we use higher order eigenfunctions as perturbation

$$S(t, r) = \frac{Q}{4} \left( (L_+)^n e^{t\alpha \sqrt{C}} \cosh^\alpha r \sqrt{C} + (L_-)^n e^{-t\alpha \sqrt{C}} \cosh^\alpha r \sqrt{C} \right).$$

(90)

From the correspondence with Bousso’s work [8], there would appear $2n$ outer horizons and $2n$ inner horizons and the spacetime could be disintegrated into $2n$ pieces when the inner horizons shrink to vanish. Of course, the confirmation of this effect should be checked with some other methods. It is also interesting to note that proliferation of de Sitter space is somehow different from baby universes creation based also on the topological changes (for a review, see ref. [24]) as it occurs at large scales [3]. Note that horizon equation for the higher mode is $2n$-th order polynomial with respect to $\tanh r \sqrt{C}$, which is the reason why it is expected that there can be $2 \times 2n$ horizons. However, it is difficult to confirm that all solutions are real solutions.

7 Discussion

We studied quantum properties of 2D charged BHs and BTZ BH. The quantum effects of dilaton coupled scalars in large $N$ approximation have been taken into account. Quantum corrected solution for 2D charged BH has
been found and quantum corrections to mass, charge, Hawking temperature and BH entropy have been evaluated. As 2D dilaton-Maxwell gravity with non-minimal scalars under discussion may be also considered as toy (or spherically reduced) version for 4D or 5D Einstein-Maxwell-minimal scalar theory then higher dimensional interpretation of results may be done. (It is also possible due to duality of 2D BH with 5D BH). Quantum (in)stabilities of 2D charged BH and BTZ BH are discussed and it is shown the presence in the spectrum of the mode corresponding to anti-evaporation like in the case of 4D SdS BH. Hence, the important qualitative result of our work is that we show: quantum anti-evaporation of BHs (even BTZ BHs) is quite general effect which should be expected for different multiple BHs (and not only for SdS BH). Numerical calculation shows that evaporation is “more” stable effect, nevertheless. The interpretation of higher modes perturbations as the reason of disintegration of the space itself to copies is given. It is clear that in order to understand if the above effects may really happen in early universe one should select boundary conditions corresponding to BH formation and analyse above processes subject to such boundary conditions. Depending on the choice of boundary conditions they may be compatible with anti-evaporation as it happens for Nariai anti-evaporating BH [6].

It is not difficult to generalize the results of this work to other higher dimensional BHs where metric may be presented as the product of 2D charged BH with D-2-dimensional sphere. Then large \(N\) approximation and spherical reduction may be applied as well (as it was done in 3D case) and there are no principal problems to study quantum properties of such BHs (at least in large \(N\) approximation). From another point, duality may help in the study of connections between low and higher dimensional BHs on quantum level. Number of new effects are expected to be found in such studies.

It could be very interesting also to consider supersymmetric generalization of above results. It seems to be possible to do as anomaly induced effective action for dilaton coupled supersymmetric 2D or 4D theories is known [26].

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Figure Captions

Fig.1 $\frac{A}{C}$ (vertical line) versus $g \equiv 8GNe^{2\phi_0}$ (horizontal line) when $\ln \mu^2 = 1$.

Fig.2a,b Two branches of $\frac{A}{C}$ (vertical line) versus $g \equiv 8GNe^{2\phi_0}$ in $[0, 10]$ and $\ln \mu^2$ in $[0, 3]$. Fig.2a corresponds to the instable mode and 2b to the stable one.

Fig.3 $\frac{A}{C}$ (vertical line) versus $g \equiv 8GNe^{\phi_0}$ (horizontal line) when $\ln \mu^2 = 1$.

Fig.4a,b Two branches of $\frac{A}{C}$ (vertical line) versus $g \equiv 8GNe^{\phi_0}$ in $[0, 1]$ and $\ln \mu^2$ in $[0, 3]$. Fig.4a corresponds to the stable mode and 4b to the instable one.
