Quantum cloning in coupled states of optical field and atomic ensemble by quasi-condensation of polaritons

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Abstract. We consider a new approach for storing quantum information by macroscopic atomic excitations of two level atomic system. We offer the original scheme of quantum cloning of optical field into the cavity polaritons containing the phase insensitive parametrical amplifier and atomic cell placed in the cavity. The high temperature quasi-condensation (and/or condensation) phenomenon for polaritons arising in the cavity under the certain conditions is proposed for the first time.

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1. Introduction

At present a significant progress in quantum information, communication and especially quantum cryptography requires the new methods and approaches in quantum information processing - see e.g. [1]. Important step in this direction can be taken with the help of novel optical devices operated with atoms for quantum storage, memory and transmission of information. As a rule, memory devices proposed now in atomic optics for those purposes explore various methods of entanglement of atoms with quantized electromagnetic field and the mapping of quantum state of light onto the atomic ensembles [2–6]. The principal point is to achieve the light-atom coherence for long. For instance, in Ref. [2] atomic ensemble has been prepared as an approximately coherent spin state with only one projection, say $J_x$, having non-zero average value. Within this limit atomic system can be described by collective magnetic momentum $J$ of the ground state. The fidelity of up to 70% and memory lifetime of up to 4 $msec$ have been achieved experimentally for Gaussian light pulses in the three step scheme of passing for light-atom interaction, and subsequent measurements in the system have been carried out by feedback introduced into the atoms.
Another attractive and practically important opportunity to create macroscopic coherence in atomic medium and to store coherently the optical information as a result, is given by effect of electromagnetically induced transparency (EIT) taking place in three level atomic systems with so called Λ-configuration of the energy levels. In this case strong (classical) coupling field creates transparency window in the medium, and second weak (quantum) probe field propagates through the resonant atomic system with a very small absorption \[4\]. The EIT effect is accompanied by significant reduction of observable group velocity of propagating signal pulse, as well. Such a macroscopic coherence as a result of the field-medium interaction has been observed for both hot and ultracold atomic ensembles. As an example, the authors of Refs. \[5\] propose to use such a “stopped” light for coherent (classical) storage of information in ultracold \((T \approx 450 \text{ nK})\) sodium atoms near the transition to the Bose-Einstein condensation (BEC) state. In fact, the phenomenon is demonstrated by switching signal pulse with delay time (i.e. storage time of light in the medium) \(\tau \approx 45 \mu\text{sec}\) and more. The Doppler broadening, being limitation for the process, can be suppressed for the case. In contrast, in Ref. \[7\] the energy transfer from light to atomic excitations for hot \((T \approx 360 \text{ K})\) atomic \(Rb\) vapor cell has been observed as a principal approach to storing information.

It is important to mention that quantum memory proposals based on EIT-type interaction in the medium leaves open principal question about criterion for storing and transmission of quantum information – cf. with \[6\]. Physically, quantum properties of atomic system coupled with e.m. field, can be described in terms of “bright” (optical branch) and “dark” (atomic branch) polaritons, being the two solutions of the problem in the terms of linear superposition of quantized optical field and atom excitation states – see Refs. \[4, 8\] for more details. But existence of polaritons is not sufficient especially for quantum storage and transfer of information by some quantum memory device, and specific properties of quantum state become more important in the case – see Ref. \[9\], cf. \[2\]. In particular, in Ref. \[9\] we propose a new quantum non-demolition (QND) technique to store the quantum information which is very close to the QND measurement procedure in quantum and atomic optics \[10\]. We have shown that atomic system under the BEC condition satisfies formulated criteria for reasonable efficiency of quantum information storage and transmission simultaneously, and therefore is perspective for creating a long lived memory. However, the atomic BEC medium is not feasible for wide practical exploring due to extremely low temperature (about few nanoKelvins) conditions. Therefore the study of the problem of high-temperature quasi-condensation phenomenon for cavity excitons and polaritons evoks a great interest \[11–13\]. Although polaritonic condensate is nonequilibrium principaly, the formation of single (macroscopic) coherent polariton state (ground state with momentum \(\vec{k} = 0\)) has been observed recently for quantum wells placed in semiconductor CdTe/CdMgTe-microcavity \[12\]. In Ref. \[13\] the authors experimentally measure the dispersion curve for low brunch polaritons in semiconductor (Ga-As)-microcavity. It has been shown that the statistical properties of excitons can be changed into the coherent ones.
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Speaking more precisely such a strong coupling regime for the cavity polaritons can be considered as a weakly interacting 2D Bose-gas for which the Kosterlitz-Thouless phase transition occurs at the temperatures high enough (room) due to extremely small mass of polaritons $m_{\text{eff}} \simeq 5 \times 10^{-33} \text{g}$ \cite{13, 14}. The last circumstance is very attractive and important for practical purposes and namely, for elaboration of quantum memory devices.

Recently in Ref. \cite{15} we have shown that a quasi-condensation phenomenon can be also achieved for polaritons in atomic system under the strong coupling condition for e.m. field and two-level atoms in the cavity, described by relation

$$\omega_c = \sqrt{\frac{2\pi d^2 \omega_0 n}{\hbar}} \gg 1/\tau_{\text{coh}},$$

where $\omega_c$ is so-called cooperative frequency, $\omega_0$ is the frequency of atomic transition, $d$ is the dipole momentum, $\tau_{\text{coh}}$ is the time of coherence of the medium, $n$ is the density of atomic cloud, $\hbar$ is the Planck constant. Within this limit the process of vacuum Rabi splitting for the cavity modes occurs, and such a splitting corresponds to the degeneracy of the 2D ideal polariton gas. In particular, this takes place in a real experimental situation with the hot Na atomic vapor cell placed into the resonator \cite{16}. Moreover is also possible to achieve a true room temperatures BEC for polaritons under the certain conditions for optical trapping of the cavity polaritons.

In this paper we propose a new method to store quantum information by means of quantum cloning procedure for continuous variables of optical field in a light-atom interaction in the cavity under condition of strong coupling regime (cf. \cite{13}). The high temperature quasi-condensation phenomenon for the cavity polaritons occurs in the case (Section 2). We consider simple model for interaction of two-level atoms with quantized e.m. field in the cavity. In Section 3 we examine a new scheme for optimal quantum cloning of information in such coherent atomic system. In Appendix we develop a Holstein-Primakoff approach to formulate the quasi-condensation condition for macroscopic coupled states being realized for excitations in two level bosonic system interacting with quantum e.m. field. In conclusion we briefly discuss an appropriate application of the scheme for the problem of quantum memory in continuous variables.

2. Quantum macroscopic excitations in Bose-gases and quasi-condensation phenomenon

To describe the interaction of two level atomic system with quantum e.m. field we consider a localized exciton model in the frames of the Dicke Hamiltonian for the interaction of the $N$ two level atoms with quantized e.m. field (cf. \cite{14}):

$$H = \sum_{\vec{k}} E_{\text{ph}}(\vec{k}) \psi_{\vec{k}}^\dagger \psi_{\vec{k}} + \sum_{j}^{N} \frac{E_{\text{at}}}{2} (b_j^\dagger b_j - a_j^\dagger a_j) + \sum_{\vec{k}} \sum_{j}^{N} \frac{g}{\sqrt{N}} \left( \psi_{\vec{k}}^\dagger a_j^\dagger b_j + b_j^\dagger a_j \psi_{\vec{k}} \right) \quad (1)$$

where $\psi_{\vec{k}}$ ($\psi_{\vec{k}}^\dagger$) is the annihilation (creation) operator for photon with momentum $\vec{k}$; $E_{\text{at}}$ is the energy of atomic transition between $|a\rangle$ and $|b\rangle$ levels (we neglect here moving
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the atoms in the cavity), $E_{ph}(k)$ defines dispersion relation for photons in the cavity, $g$ characterizes the atom-field coupling.

For high-reflectivity mirrors of the cavity the normal (to the plane of mirrors) component of the photon wave vector $k_\perp$ is quantized, i.e. $k_\perp = \pi m / L_{cav}$, where $L_{cav}$ is the length of the cavity (distance between two mirrors), $m$ is the number of modes. At the same time we have the mode continuum in the direction being parallel to the mirrors plane. In paraxial approximation ($k_\parallel << k_\perp$) the dispersion relation for photon energy $E_{ph}(k)$ inside of the cavity can be represented as:

$$E_{ph}(k) = \hbar c |\vec{k}| = \hbar c \sqrt{k_\perp^2 + k_\parallel^2} \approx \hbar c \left( k_\perp + \frac{k_\parallel^2}{2k_\perp} \right).$$

The polariton quasi-condensation occurs in the plane being normal to wave vector component $k_\perp$. Physically it means that macroscopic occupation of the state with $k_\parallel = 0$ takes place in the case. At the same time within the limit of a single cavity mode approximation (for $k_\perp$) we can rewrite Hamiltonian (1) in the form - see Appendix:

$$H = E_{ph}(k) \psi^\dagger \psi + E_{at} \phi^\dagger \phi + g \left( \psi^\dagger \phi + \phi^\dagger \psi \right),$$

where $\psi \equiv \psi(\vec{k})$ is the annihilation operator for a single mode optical field with the $\vec{k}$ wave vector. In (3) we also introduce the collective exitation operators $\phi$, $\phi^\dagger$ (cf. (A.11)):

$$\phi = \sum_{j=1}^{N} a_j^\dagger b_j \sqrt{N}, \quad \phi^\dagger = \sum_{j=1}^{N} b_j^\dagger a_j \sqrt{N}.$$  

The Hamiltonian (3) can be diagonalized with the help of unitary transformations

$$\Phi_\psi = \mu \psi - \nu \phi,$$
$$\Phi_\phi = \mu \phi + \nu \psi,$$

for annihilation operators of macroscopically populated polaritonic modes $\Phi_\psi$ and $\Phi_\phi$ corresponding to the upper and low brunch polaritons in the medium respectively. The parameters $\mu$ and $\nu$ are the real Hopfield coefficients that are determined by expressions (cf. [16]):

$$\mu = \left( \frac{4g^2}{2\sqrt{\delta^2 + 4g^2} \left( \delta + \sqrt{\delta^2 + 4g^2} \right)} \right)^{1/2}, \quad \nu = - \left( \frac{\delta + \sqrt{\delta^2 + 4g^2}}{2\sqrt{\delta^2 + 4g^2}} \right)^{1/2}$$

and fulfill the condition $\mu^2 + \nu^2 = 1$; $\delta = E_{at} - E_{ph}(k)$ is the energy detuning. Last parameter determines the photonic and atomic contribution to polaritons.

Indeed, for $4g^2 \gg \delta^2$ from (6) we have $\mu^2 \to 1$ ($\nu^2 \to 0$), that corresponds to exciton-like polariton $\Phi_\phi$ in (5b). In another limit when $4g^2 \ll \delta^2$ we have $\mu^2 \to 0$ ($\nu^2 \to 1$), that corresponds to photon-like low brunch of polaritons. From Eqs. (6)
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shows that polariton represents a half-matter and a half-photon ($\mu^2 = \nu^2 = 1/2$) quasi-particle under the resonance condition for $\delta = 0$.

The dispersion relation, i.e. the energy for the cavity polariton of both upper brunch ($E_\psi (k)$) and low brunch ($E_\phi (k)$), is defined as (cf. [13])

$$E_{\psi,\phi} (k) = \frac{1}{2} \left[ E_{at} + E_{ph} (k) \pm \sqrt{(E_{at} - E_{ph} (k))^2 + 4g^2} \right],$$

(7)

where expression for $E_{ph} (k)$ is presented in [2].

Important feature of low brunch polaritons in the cavity is determined by the minimum of $E_\phi (k)$ for $k = 0$. In particular, we can fulfill both the resonance condition $|k_\perp| = E_{at} / (\hbar c)$ for the cavity mode and the condition $k_\parallel = 0$ simultaneously. The fact results in quasi-condensation of polaritons for the case.

3. Quantum cloning with polaritons

Now we focus our attention on the problem of cloning optical field $\psi$ onto the atomic ensemble with the help of stationary polariton modes (5). The principal set-up for the cloning procedure under consideration is shown in Fig.1.

The initial radiation propagates through the phase-insensitive linear amplifier (marked as (1) in Fig.1) with the gain that is equal to two. The linear transformation for photon annihilation operators of the signal ($\psi$) and ancilla ($c$) modes is represented in the form:

$$\psi = \sqrt{2} \psi_{in} + c_{in}^\dagger, \quad c_{out} = \sqrt{2} c_{in} + \psi_{in}^\dagger,$$

(8)

where $c_{in}$ ($c_{out}$) is the value of operator at the input (output) of the amplifier. For the cloning procedure we assume $c_{in}$ to be the vacuum state.

Then, according to the scheme in Fig.1, the signal optical field is stored in the atomic medium. The bright and dark polaritons at the output of medium represent two clones of initial field $\psi_{in}$ as a result. One of them reproduces the atomic excitations in the medium.

Taking into account the Eqs.(5) and Eqs.(6) with $\mu^2 = \nu^2 = 1/2$ we represent a total unitary transformation for annihilation operators of bright (optical) $\Phi_\psi$ and dark (the excitation in matter) $\Phi_\phi$ clones:

$$\Phi_\psi = \psi_{in} + \frac{1}{\sqrt{2}} \left( c_{in}^\dagger - \phi \right), \quad \Phi_\phi = \psi_{in} + \frac{1}{\sqrt{2}} \left( c_{in}^\dagger + \phi \right).$$

(9)

Let us define the quadrature components for polaritons:

$$Q_{j}^{out} = \Phi_j + \Phi_j^\dagger, \quad P_{j}^{out} = i \left( \Phi_j^\dagger - \Phi_j \right), \quad j = \psi, \phi$$

(10)

The quadrature components corresponding to Eqs.(9) evolve as:

$$Q_{\psi,\phi}^{out} = Q_{\psi}^{in} + \frac{1}{\sqrt{2}} \left( Q_{c}^{in} \pm Q_{\phi}^{in} \right), \quad P_{\psi,\phi}^{out} = P_{\psi}^{in} - \frac{1}{\sqrt{2}} \left( P_{c}^{in} \pm P_{\phi}^{in} \right),$$

(11)
where \( Q^c_i = c_i + c_i^\dagger, \ P^c_i = i (c_i^\dagger - c_i) \) are the Hermietian quadratures for ancilla vacuum mode at the input of the amplifier (see Fig.1).

The expressions (9), (11) represent the desired linear transformations for optimal cloning procedure of continuous variables in the Heisenberg formalism \([17]\). For mean values of the quadratures (11)

\[
\langle X^\text{out}_{\psi,\phi} \rangle = \langle X^\text{in}_{\psi} \rangle,
\]

where the variable \( X = \{Q, P\}, j = \psi, \phi \). For their variances \( V_X = \langle (\Delta X)^2 \rangle \) following expressions are given:

\[
V_X^\text{out} \equiv V_X^j = V_X^j + 1.
\]

The last term in Eq. (13) characterizes the impossibility to perfect cloning of quantum state.

Let us now examine optimal cloning procedure determined by Eqs.(9), (12) for polaritons in the cavity. The quantum state of the cavity polaritons being under condensation can be described by the state vector (cf. \([14]\)):

\[
|\Psi\rangle_{\text{pol}} = e^{\gamma \psi \dagger} \frac{1}{\sqrt{N!}} (\alpha a^\dagger + \beta b^\dagger)^N |\text{vac}\rangle,
\]

where \(|\text{vac}\rangle = |0\rangle_\text{at} |0\rangle_\text{ph} \) denotes the total vacuum state for the cavity photons \(|0\rangle_\text{ph} \) and for the two-level atomic system \(|0\rangle_\text{at} \); \( \gamma \) is the complex coherent amplitude of the \( \psi \) field. Parameters \(|\alpha|^2 = n_a/N \) and \(|\beta|^2 = n_b/N \) determine a relative population imbalance for atomic levels \(|a\rangle \) and \(|b\rangle \), respectively and obey the normalization condition \(|\alpha|^2 + |\beta|^2 = 1 \).

Using the polariton BEC state (14) for mean values \( \langle X^\text{out}_{\psi,\phi} \rangle \) and variances \( V_X^\text{out} \) of the output polariton quadratures (clones) we obtain:

\[
\langle Q^\text{out}_{\psi,\phi} \rangle = \langle Q^\text{in}_{\psi} \rangle \mp \sqrt{2N} |\alpha||\beta| \cos \varphi, \quad \langle P^\text{out}_{\psi,\phi} \rangle = \langle P^\text{in}_{\psi} \rangle \mp \sqrt{2N} |\alpha||\beta| \sin \varphi,
\]

\[
V_{Q,\psi}^\text{out} = V_{Q,\phi}^\text{out} = V_{Q,\psi}^\text{in} + 1 - 2|\alpha|^2 |\beta|^2 \cos^2 \varphi, \quad V_{P,\psi}^\text{out} = V_{P,\phi}^\text{out} = V_{P,\psi}^\text{in} + 1 - 2|\alpha|^2 |\beta|^2 \sin^2 \varphi,
\]

where \( \varphi = \varphi_\alpha - \varphi_\beta \) is the relative atomic phase \( (\alpha = |\alpha| e^{i\varphi_\alpha}, \beta = |\beta| e^{i\varphi_\beta}) \).

The expressions (14) demonstrate the deviations from “usual” optimal cloning procedure - cf. Eqs. (12)–(14).

To describe the quantum information processing, the fidelity criterion is used - see e.g. \([1,17]\). For optimal cloning procedure of coherent optical field the fidelity can be evaluated as:

\[
F = \frac{2}{\sqrt{\left(1 + V_{Q}^\text{out}\right) \left(1 + V_{P}^\text{out}\right)}}.
\]

Formally using the expressions (16) for relevant fidelity parameter \( F \) in Eq. (17) one can obtain the extreme value \( F = 0.8 \) that is determined by spin-squeezing for atomic system for \(|\alpha|^2 = |\beta|^2 = 1/2 \) and \( \varphi = \pi/4 \).
However to perfect quantum cloning procedure we should require the similarity for output and input quadrature components for the scheme in Fig. 1 which is possible in low density approximation only – see Eq. (A.12). In particular, within the limit $|\beta| \to 0$ and $|\alpha| \simeq 1$ respectively, for fidelity $F$ one can obtain a magnitude $2/3$ that corresponds to optimal quantum cloning process, i.e. optimal transfer of quantum information, embodied in the field $\psi_{in}$, and to coherent excitations in atomic medium - cf. [1, 17].

4. Conclusion

In the paper an opportunity to store quantum information in the bright and dark polaritons arising in quasi-condensation process in the cavity is given. We propose a special scheme for quantum cloning of signal field into the polariton states. The relevant fidelity criterion demonstrates the essentially nonclassical properties of storing quantum information. Such a cloning procedure can be useful for elaborating quantum memory devices. However, at least two additional conditions are obviously necessary to have for those purposes. First, it is necessary to have subsequent procedure of read-out of stored information. Second, we should have an opportunity to retrieve or verify stored optical field.

It is evident that relevant dynamical properties of the cavity polaritons are necessary to be included in the problem under consideration. Indeed, it is possible to show that solution of the problem described by Hamiltonian in Eq. (3) under the (A.12) condition can also be also represented by a linear transformation that looks like $\psi(t) \simeq \mu(t) \psi - \nu(t) \Phi_\phi$, $\phi(t) \simeq \mu(t) \Phi_\phi + \nu(t) \psi$, where $\mu(t)$, $\nu(t)$ are the time dependent transformation parameters, $\psi$ is the initial value of optical field at the read-out stage and $\Phi_\phi$ characterizes the dark polariton being one of the clones for the storing stage. In particular, we can infer initial information from atomic excitations, i.e. $\psi_{out} \equiv \psi(t = \tau) \simeq \Phi_\phi$ at fixed time $\tau$ when $\mu = 0$, $\nu = 1$.

In the paper we do not consider the problem of decoherence of atomic and/or optical system that becomes important especially for the purpose of a long lived quantum memory. Recently various multi-pass protocols have been proposed for retrieving quantum optical state, for verifying quantum storage state and for increasing the fidelity, respectively [2]. Indeed, the retrieval procedure for continuous variables consists of mapping measured light quadrature back into the atomic quadrature variables in the medium with certain feedback gain. In some sense such a technique is similar to the method proposed earlier to achieve a good QND-measurement for light quadrature component and/or photon number – see e.g. [10]. The problem of retrieval procedure for the scheme proposed by us in Fig.1 requires detailed analysis that is not the subject of study in the paper.

The physical scheme under discussion can be useful in problem of quantum cryptography with continuous variables as well. In particular, quantum cloning procedure may be used by eavesdropper for individual attacks of communication channel – cf. [17]. The asymmetric cloning procedure is also principal in the case. Necessary
asymmetry can be introduced by manipulation of detuning $\delta$ to achieve $\mu \neq \nu$ (see. Eqs. (5)).

Finally, we would like to note that cloning procedure discussed in the paper can be also realized in a similar way with the help of the cavity polaritons in quantum wells on the basis of modern semiconductor technology. This circumstance makes reasonable the method of storing quantum information from the practical point of view.

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Appendix. Coherent excitations for two-level oscillators

Here we consider the quantum properties of macroscopic excitations for a two level atomic system interacting with e.m. field. We use the general Holstein-Primakoff approach for that – cf. [15].

In Schwinger representation the pseudospin operators for a two level system can be written in the form

\begin{align}
S_x &= \frac{1}{2} (a^\dagger b + b^\dagger a), \\
S_y &= \frac{i}{2} (a^\dagger b - b^\dagger a), \\
S_z &= \frac{1}{2} (b^\dagger b - a^\dagger a), \\
S_0 &= \frac{1}{2} (a^\dagger a + b^\dagger b),
\end{align}

(A.1)

where $a \ (a^\dagger)$ and $b \ (b^\dagger)$ are the annihilation (creation) operators for the oscillators (atoms) at the lower $|a\rangle$ and upper $|b\rangle$ levels, respectively. These operators obey the usual Bose commutation relations; $S_0$ is the operator of the total particle number. Physically expressions (A.1) mean that in the case of BEC we can consider two macroscopically occupied quantum modes $a$ and $b$ for internal states $|a\rangle$ and $|b\rangle$ levels, respectively. These operators obey the usual Bose commutation relations; $S_0$ is the operator of the total particle number. Physically expressions (A.1) mean that in the case of BEC we can consider two macroscopically occupied quantum modes $a$ and $b$ for internal states $|a\rangle$ and $|b\rangle$ of atomic system instead of collective operators (4) – cf. [2, 9]. In other words the condensation (and/or quasi-condensation) phenomenon leads to the interaction of a single quantum cavity mode and atomic ensemble coherently similar to a single two level atom described by operators $a \ (a^\dagger)$ and $b \ (b^\dagger)$.

The operators $S_j$ satisfy the commutation relation of the $SU(2)$-algebra:

\begin{align}
[S_x; S_y] &= iS_z, \\
[S_z; S_x] &= iS_y, \\
[S_y; S_z] &= iS_x, \\
[S_j; S_0] &= 0, \quad j = x, y, z.
\end{align}

(A.2)

It is useful to introduce the ladder operators:

\begin{align}
S_+ &= S_x + iS_y = b^\dagger a, \\
S_- &= S_x - iS_y = a^\dagger b,
\end{align}

(A.3)
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which obey the commutation relations:

\[ [S_+; S_-] = 2S_z; \quad [S_z; S_\pm] = \pm S_\pm. \quad (A.4) \]

For the basis of two-mode Fock state, i.e. for

\[ |s_z\rangle = |n_a, n_b\rangle, \quad (A.5) \]

the \( S_z \) and \( S_0 \) operators are diagonal with the mean values

\[ s_z \equiv \langle S_z \rangle = \frac{1}{2} (n_b - n_a), \quad s \equiv s = \langle S_0 \rangle = \frac{1}{2} (n_b + n_a), \quad (A.6) \]

where \( n_a = \langle a^\dagger a \rangle \), \( n_b = \langle b^\dagger b \rangle \) are the average number of particles at the \( |a\rangle \) and \( |b\rangle \) levels. Using the expressions (A.6) it is easy to show that

\[ S_+ |s_z\rangle = \sqrt{(s + s_z + 1)(s - s_z)} |s_z + 1\rangle, \]
\[ S_- |s_z\rangle = \sqrt{(s - s_z + 1)(s_z + s)} |s_z - 1\rangle. \quad (A.7) \]

Let us introduce the spin excitation operators \( \phi (\phi^\dagger) \) according to the Holstein-Primakoff transformation as:

\[ S_- = \sqrt{2s - \phi^\dagger \phi} \phi, \quad S_+ = \phi^\dagger \sqrt{2s - \phi^\dagger \phi}, \]
\[ S_z = \phi^\dagger \phi - s. \quad (A.8) \]

The commutation relations (A.4) are satisfied when the operators \( \phi (\phi^\dagger) \) obey the following relations for Bose-system :

\[ [\phi; \phi^\dagger] = 1. \quad (A.9) \]

In the paper we consider the limit of small number of excitations \( \langle \phi^\dagger \phi \rangle \), i.e.:

\[ \langle \phi^\dagger \phi \rangle << s, \quad (A.10) \]

that corresponds to low density excitons approach - cf.\[13\]. In this case the operators \( \phi^\dagger (\phi) \) can be evaluated as (cf. Eqs. (4) ):

\[ \phi = \frac{S_-}{\sqrt{2s}} = \frac{a^\dagger b}{\sqrt{N}}, \quad \phi^\dagger = \frac{S_+}{\sqrt{2s}} = \frac{b^\dagger a}{\sqrt{N}} \quad (A.11) \]

Using the expressions (A.6), (A.11) it easy to present the relation (A.10) in the form:

\[ n_a >> n_b. \quad (A.12) \]

Thus, within this limit the boson-like excitations (polaritons) in the medium exhibit the quasi-condensation properties.
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Figure A1. Scheme of quantum cloning of light onto the polaritons. Here $\psi_{in}$ ($c_{in}$) is the annihilation operator for signal (ancilla) optical field at the input of a cloning device; we denote 1 as a linear amplifier, 2 is the atomic cell placed in the cavity, $\Phi_\psi$ and $\Phi_\phi$ are the polariton modes after the storage procedure has occurred.