A frequency iterative algorithm for inversing the distribution of bubbles in seawater

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Abstract. It is of great significance for acoustics and oceanography to obtain the distribution of bubbles in seawater. Generally, resonance estimation is widely used in the distribution inversion. Based on the resonance estimation theory, the distribution of bubbles can be derived by neglecting the off-resonance effect. However, neglecting the off-resonance effect will create deviation of the distribution. In order to reduce inversion error caused by resonance estimation, this paper proposes a revised method named frequency iterative algorithm. Frequency bands are divided into several intervals. In the inversion process, the off-resonance effect is computed from relative low frequency band. When the high frequency band is inversed, the off-resonance effect has been removed. To test the performance of the new inversion algorithm, the distribution of bubbles is set to follow Gaussian distribution in the simulation environment. Simulation experiments show that frequency iterative algorithm is an efficient method to correct the inversion error in the bubbles of small radii. In the process of frequency iterative algorithm, the more iteration it does, the less off-resonance effect appears. It can reduce the impacts due to the off-resonance effect and improve accuracy of the inversion.

1. Introduction
When air comes into contact with seawater as waves break on the sea surface, a layer of bubbles is formed. When sound waves propagate in seawater through the bubbles, there is a significant attenuation in the adsorption of sound. Under the effect of sound waves, the bubbles are compressed and expanded, leading to internal temperature change, and thus exchange heat at the air-sea interface and translate sound energy into thermal energy. Moreover, as the resonant cavity, the bubbles can scatter incident sound wave, leading to a significant attenuation in the incident sound energy. Therefore, it is significant for research on acoustics and oceanography to measure bubble distribution in seawater.

From the perspectives of optics and acoustics, two methods can be used to detect the radii of the bubbles and their distribution in water. In the acoustic method, one of the two methods, bubble distribution is usually inverted by sound attenuation data obtained through experiments. For the first class of Fredholm’s integral formula for sound attenuation, Clay and Medwin proposed the resonance estimation method. As the resonance hypothesis simplifies the integral formula of sound attenuation, the resonance estimation method can better invert the bubble distribution in a simpler manner. It has been widely used in the inversion of bubble distribution in water using the acoustic method.

Kerry and Elan conducted detailed research on the resonance estimation method. In combining porosity with a sound attenuation coefficient, which is closely related to the inversion, they proposed an inversion algorithm for the resonance spectrum and explored the selection of a hypothesis.
concerning the distribution. Moreover, they analyzed the source of error in resonance spectroscopy and proposed a modified method of stepwise inversion for it [1]. Leighton proposed a nonlinear acoustical method for inversion of bubble distribution [2] [3] [4]. In the 1980s and 1990s, Zuwen conducted significant research on ocean bubbles that indicated that when bubble concentration is very high, interactions occur among them. The relevant parameters of bubbles can be analyzed by the inversion method in acoustics [5] [6] [7]. Wang and Zuwen’s derivation method is similar to that of the high concentration bubbles in the wake of ships [8]. After analyzing the scope of applicability of the resonance inversion method, in a combination of the inverse relationship between the radiation damping coefficient and the frequency of the sound signal, Lin Ju established the modified AW model to improve the resonance estimation method [9]. Xie proposed the correction of sound velocity for the error in resonance spectroscopy that improved the results of inversion. However, little similar research has been conducted on the off -resonance effect [10]. Fan Yuzhe proposed the concept of the equivalent spatial correlation function. The prevalent inversion method in acoustics can accurately inverse the distribution and trend of gathering of bubble clusters [11]. Zhang Yi developed an inversion method in acoustics where bubble radius and density are inversed by the scattering intensity of the bubble cluster at different sound frequencies [12]. In Zheng’s work, the wave equation of the displacement vector of porous media under the influence of bubble is deduced, and the prediction model of the acoustic velocity dispersion and attenuation of non -water saturated porous media is established. It is shown that the bubble content, size and frequency of driving sound field are the main factors that affect the propagation of sound waves in water saturated porous media with small bubbles [13]. Based on the bubble coupled vibration equation, Wang uses the successive approximation method to analyze the nonlinear acoustic response of bubbles. His analysis shows that the number of bubbles in the bubble group and the initial radius of the bubbles should have significant influence on the resonance sound. The relative position of bubbles in the bubble group has relatively little effect on the resonant sound [14].

The optical method captures bubbles through photography using high -speed cameras at a high resolution to directly obtain the bubble distribution, but has stringent requirements concerning the illumination of the background environment. For bubble distribution on the ocean surface and subsurface, Cao Ruixue designed a high -resolution CCD imaging system to improve the source of background light and video capture. He effectively increased the inversion accuracy [15]. Although the optical method has high precision, the observable area is too small to meet the requirement of large observation areas in practice.

Although the acoustic method is inferior to the optical method in terms of accuracy, it is cheaper, simpler, more convenient to operate, and has a large observation area. Therefore, it is important for research on the internal mechanism of air-sea interaction.

In this paper, for problems in classic resonance spectroscopy, the error in the off -resonant frequency was simulated and analyzed. Based on this, a frequency-division iterative inversion method was proposed to effectively solve the defects in classic resonance spectroscopy due to the off -resonance effect and improve inversion accuracy.

2. Inversion method of resonance spectrum and error analysis

Of the assumptions of the resonance estimation method, only the resonance effect is taken into account while the effect of sound attenuation in the off -resonant frequency band is neglected.

There are some preconditions for the application of resonance spectroscopy. It is assumed that the radius of the bubble is $a$ and wavenumber of incident sound is $K$, which is in the range $Ka \leq 1$. As the influence of the off -resonance effect on the results of inversion is the major factor considered in this paper, the circumstance where the resonance spectroscopy does not meet the precondition of $Ka \leq 1$ in the very -high -frequency range (such as over 10000 KHz), thus affecting the inversion effect in the high -frequency band, is not considered.

In the acoustic theory of bubbles, the relation between the frequency of incident sound waves and resonant bubble radius is given as:
where $\gamma$ is the air specific heat ratio, set to 1.4, $P_0$ is the static pressure, and $\rho$ is the density of air.

Under the resonance condition, attenuation is maximum. By deriving the relationship among bubble radius, incident frequency, resonant frequency, and the coefficient of sound attenuation, the latter can be calculated.

In the combination of the cross-sections of the scattering and the attenuation, the integral formula for the coefficient of sound attenuation can be derived as follows [1]:

$$\alpha(f) = \frac{4\pi a^2 n(a)c\delta / f a}{\int_{a_0}^{a_1} \left[ (f_0 / f)^2 - 1^2 + \delta^2 \right] da}$$  \hspace{1cm} (2)

where $\alpha(f)$ is the sound attenuation coefficient with units in $dB/m$, $n(a)$ is the distribution of the bubbles in $m^3/m$, $\delta$ is the total damping coefficient, $f$ is the incident frequency, $f_0$ is the frequency of the resonant bubble, $c$ is the velocity of sound, and $a_0$ and $a_3$ are the minimum and maximum radii, respectively.

Based on the resonance hypothesis, only bubbles with resonance radii corresponding to the incident frequency can cause signal attenuation, whereas bubbles with other radii have no effect. That is, the off-resonance effect is neglected, and the effects of the viscous resistance of seawater and the surface tension in the bubble tension are not considered. Moreover, the damping coefficient is assumed to be constant. Thus, the integral formula of the coefficient of sound attenuation can be simplified as:

$$\alpha(f) = \frac{4\pi r^2 n(a_0)c\delta / f a}{\int_{a_0}^{a_3} \left[ (f_0 / f)^2 - 1^2 + \delta^2 \right] da}$$  \hspace{1cm} (3)

After further integral simplification, the inversion of the resonance estimation for bubble distribution can be represented as:

$$n(a_0) = \alpha(f) \delta_{\omega} \left( \frac{2\pi^2 a_3}{2\pi} \right)^1$$  \hspace{1cm} (4)

where $\delta_{\omega}$ is radiation damping, which is related to the velocity of sound:

$$\delta_{\omega} = \frac{3\gamma P_0}{\rho \sqrt{2\pi c}}$$  \hspace{1cm} (5)

The above is the fundamental theory of classic resonance estimation. From the inversion of resonance estimation, it can be seen that when the coefficient of sound attenuation of the corresponding radius is measured, the distribution of the bubbles can be quickly obtained.

To test the inversion effect of resonance spectroscopy, it is hypothesized that the bubble distribution follows a Gaussian distribution:

$$n(a) = \phi_0 \exp\left( -\frac{(a - \mu)^2}{2\sigma^2} \right)$$  \hspace{1cm} (6)

where $\phi_0$ is porosity. The porosity, mathematical expectation $\mu$, and variance $\sigma$ were set to 0.00005, 0.00006, and 0.00006, respectively. The radius of the bubble $a$ was in the range 20-250 $\mu$m. To improve computational efficiency, the incident frequency was set in the range 1-200 KHz. The resultant inversion is shown in figure 1.
Figure 1. Distribution of the simulation environment (red line) and the inversion (green line).

Inversion errors appeared in the interval of small radii (figure 1). From Formula (4), it is known that the inversion distribution is closely related to the coefficient of sound attenuation. Accordingly, the analysis of the difference in sound attenuation coefficients between the distribution of the simulation environment and that of the inversion is conducive to determining the source of the inversion error. The coefficient of sound attenuation is calculated by Formula (2), and the approximate coefficient of resonant sound attenuation can be obtained by substituting the inversion distribution into Formula (4). The comparison between the actual coefficient of sound attenuation and the approximate coefficient of resonant sound attenuation is shown below.

Figure 2. Sound attenuation coefficient of simulation environment (red line) and inversion (green line).

In figure 2, in the frequency range, the difference between the coefficients of sound attenuation of the inversion and the simulation environment increases with frequency. From Formula (2), it is evident that the high-frequency range in figure 2 corresponds to the interval of small radii in figure 1. This indicates that the inversion error within the range of small radii was caused by the incorrect estimation of the attention coefficient at higher frequencies. In the resonance hypothesis, only bubbles with resonance radii corresponding to the incident frequency can attenuate the signal while other bubbles have no effect on attenuation. Thus, in the high-frequency range, the sound attenuation of inversion with higher values occurred owing to the off-resonance effect, which is ignored in the resonance hypothesis.

If the inversion accuracy needs to be improved, a modified method is needed to consider the influence of the off-resonance effect on inversion. Analysis is needed to determine whether the main source of the off-resonance effect is low-frequency or high-frequency off-resonant bubbles.

By classic resonance spectroscopy, the sound attenuation error mainly concentrated at higher frequencies, resulting in a larger error of inversion in the internal of small radii. In the internal of small
radii with a Gaussian distribution, the sound attenuation coefficient was analyzed by a three-section distribution.

![Figure 3](image-url)

**Figure 3.** Resonance effect in the range 26-50 μm (green line), and off-resonance effect in the range 1-25 μm (blue line) and 51-75 μm (red line).

The distribution of the resonance effect in the range 26-50 μm was caused by a greater off-resonance effect at lower frequencies (51-75 μm) than at higher frequencies (1-25 μm), and the distribution of the bubble distribution had an effect on the intensity of the off-resonance effect (figure 3). This suggests that an analysis of the impact of the off-resonance effect on inversion must consider sound attenuation and bubble distribution. Under Gaussian distribution, using the inversion distribution obtained by the classic resonance estimation method, the off-resonance effect is mainly influenced by off-resonant bubbles at lower frequencies.

3. Frequency-division iterative inversion algorithm and its effect

As the resonance hypothesis only considers resonant sound attenuation, a larger error obtains between the sound attenuation of the inversion and actual attenuation in the high-frequency range. The error leads to a mistaken inversion in the range of smaller radii, and thus cannot be used for practical observation. To mitigate the error, a frequency-division iterative inversion algorithm is proposed in this paper. The distribution is inversed at lower frequencies to obtain the coefficient of sound attenuation, which is then inversed at higher frequencies to eliminate the off-resonance effect at low frequencies. Finally, the difference is substituted into Formula (4) and the modified distribution is obtained.

In the simulation process, the internal of the bubble radii are reasonably divided into \( n - 1 \) sections \((a_1, a_2, ..., a_m, ..., a_n)\).

By classic resonance spectroscopy, the sound attenuation coefficient calculated is:

\[
\alpha_{0(f)} = \int_{a_1}^{a_n} \frac{4\pi a^2 n(a)(c\delta / fa)}{(f_0 / f)^2 - 1^2 + \delta^2} \, da
\]  

(7)

The inversion starts from the lower frequencies: that is, the first section spans from \(a_1\) to \(a_{n-1}\). It is assumed that the mth section is modified; then, the resonant sound attenuation in this section is:

\[
\Delta\alpha_{m(f)} = \alpha_{0(f)} - \alpha_{m-1(f)}
\]  

(8)

where

\[
\alpha_{m-1(f)} = \int_{a_{m-1}}^{a_{m-1}} \frac{4\pi a^2 n_{m-1}(a)(c\delta / fa)}{(f_0 / f)^2 - 1^2 + \delta^2} \, da
\]  

(9)
After the substitution of resonant sound attenuation into Formula (4), the modified distribution in this section is obtained. Furthermore, in combining the modified distributions of each section, the total modified bubble distribution is:

\[
\begin{align*}
\hat{M} &= \Delta\alpha_{\text{m}(f)j} (2\pi^2\lambda_1)^{-1} (a_0 \leq a_0 \leq a_{n-3}) \\
\hat{M} &= \Delta\alpha_{\text{m}(f)j} (2\pi^2\lambda_1)^{-1} (a_{n-2} \leq a_0 \leq a_{n-1}) \\
\hat{M} &= \Delta\alpha_{\text{m}(f)j} (2\pi^2\lambda_1)^{-1} (a\leq a_0 \leq a_n)
\end{align*}
\]

The distribution following the modification is shown in figure 4.

**Figure 4.** Distribution of the simulation environment (red line), inversion distribution of resonance spectroscopy (green line), and inversion distribution of the frequency-division iterative algorithm (blue line).

The level of coincidence between the modified distribution and the actual distribution in the range 20-100 μm was better (figure 4). This indicates that the effect of the frequency-division iterative inversion algorithm was superior to that of the original inversion algorithm, and error was smaller. At higher frequencies, the distributions of the coefficient of sound attenuation before and after the modification are shown in figure 5.

The sound attenuation coefficients are inversed by the modified distribution (figure 5). It is evident that the sound attenuation coefficient inversed by the frequency-division iterative algorithm was highly consistent with actual attenuation, and the modified effect was apparent. However, in figure 4, inversion errors still occur, which indicates that the source of error is not solely the off-resonance effect.

The foregoing simulation experiment shows that the modified frequency-division iterative inversion method can effectively reduce inversion error and improve inversion accuracy. The error in bubble distribution inversed by resonance spectroscopy occurs not only due to the off-resonance effect, but is also influenced by other factors.
Figure 5. Sound attenuation coefficient of simulation environment (red line), sound attenuation coefficient inversely by resonance spectroscopy (green line), and the frequency-division iterative algorithm (blue line).

4. Conclusions
The classic resonance estimation method can quickly inverse the distribution of bubbles in water, with a larger inversion error in the internal of small radii. The error is significantly affected by the high-frequency off-resonance effect. Thus, the classic resonance estimation method cannot be applied in the high-frequency range.

To eliminate the off-resonance effect, its source needs to be analyzed. In the analysis process, bubble distribution cannot be ignored. Thus, under a Gaussian distribution, the off-resonance effect was found to be mainly caused by off-resonant bubbles at lower frequencies.

For the inversion error at higher frequencies due to the off-resonance effect in the classic resonance estimation method, a frequency-division iterative inversion algorithm was proposed in this paper. The modified method can effectively eliminate the off-resonance effect on inversion at higher frequencies, and can improve inversion accuracy.

A comparison of the sound attenuations inversely by the classic resonance estimation method and the frequency-division iterative inversion algorithm showed that the latter effectively eliminates the off-resonance effect. The results were highly consistent with the actual distribution of sound attenuation, and error still occurred in the inversion. This indicated that the inversion error in the classic resonance estimation method is not only due to the off-resonance effect. At higher frequencies, a frequency dispersion was observed for the velocity of sound. This will be researched in future work.

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