Control analysis and simulation of multi-body systems using full state controllers

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Abstract. This paper simulates a dynamic model of a multi-body dynamic system with a Linear Flexible Joint cart with a single Inverted Pendulum (LFJ_IVP). The dynamic equation of motion for the nonlinear system was linearised utilising a suitable state space method with full state feedback and full state observer controllers that act as both servos and regulator systems. The results showed a significant improvement as compared with other researchers’ work using other controls, including an improvement in the response of both the cart position (the state output of the plant), which tracked the designed external input signal to within 20%, with similar results for spring deflection, and an improvement in the amount of deflection of the right angle around the vertical position by 37.24%. There was also a marked improvement in electrical energy consumption, by approximately 55.7% based on the application of the full state feedback controller. Comparing these results to those of other controllers, such as a full state observer controller, there was a significant improvement in the consumption of electrical energy of approximately 33%.

1. Introduction
The inverted pendulum system is considered one of the ideal systems, offering an experimental platform for the verification of control theory and its impact on practical applications. A focus on the control of the inverted pendulum with additional degrees of freedom offers a testing demonstration problem for learning control system experimentation [1]. A flexible joint can be used in addition to a rigid pole, though the cart of the pendulum system is considered a typical benchmark problem in the control field as a fully underactuated system with a highly nonlinear structure, having one control input for a two degrees-of-freedom (DOF) system. The simulation or physical control of an inverted pendulum is thus often used to test new control theories or methods with a wide range of realistic applications, including balance control in walking robots, verticality control in rocket launches, military applications such as rocket propellers and missiles, earthquake-resistant buildings, aerospace applications, and attitude control in satellite flights [2]. The goal of controlling an inverted pendulum is to ensure that when it initially begins to deviate from the vertical open-loop unstable position [3], a state-feedback controller can stabilise the pendulum [4]. It is thus important for the location of the roots to be known, as this determines the type of system, and whether it has an integrator or not, when using the pole-placement method.

Theoretically, it can be assumed that all states are available for feedback; however, in practice, this is not the case, as only certain states can be directly measured and thus made available for feedback [5]. A feedback linearization control law is made implementable by using a full state observer for trajectory tracking and control of a flexible-joint robotic system. In this case, a computer can be used to observe all states available for feedback, thus creating a full-state observer. When the dominant controller poles
in a regulator design are placed far to the left of the \( \omega j \) axis of the s-plane, large elements of the state feedback gain matrix take on large values of the actuator output, so that saturation may take place, and the designed system will not function as planned. Control of the observer may also be unstable when the observer poles are far away from the \( \omega j \) axis, despite the feedback system being stable. An observer controller must not be unstable, so moving of the observer poles toward the \( \omega j \) in the left-half of the s-plane until the observer controller becomes stable is required. The location of the desired closed-loop pole must also be modified, as placing observer poles far from the \( \omega j \) axis will cause an increase in noise and bandwidth. A full state observer may be used to minimise observer bandwidth, preventing the noise of the sensor from becoming a problem, however [6].

2. Linearization of dynamic system

Performance of a cart on a single inverted pendulum connected to a flexible joint cart is shown in figure 1. This is a two-dimensional case where the pendulum moves only in the plane of the page. Both carts are coupled together and attached to the linear servo of the inverted pendulum base unit by means of a linear spring with equivalent stiffness (k). The positive linear displacement is to the right of the observer. The positive sense of rotation of the pendulum is defined as counter clockwise when facing the cart pinions. Finally, the zero angles, \( \theta = 0 \), of the pendulum correspond to an inverted pendulum pointing perfectly vertically upwards.

![Figure 1. Linear flexible joint cart and single inverted pendulum conventions.](image)

The cart location for the inverted pendulum is at linear position \( z_L \), while the cart pendulum of linear the flexible joint is located at \( z_f \). The cart assembly of the inverted pendulum has mass, \( M_L \), and works by an actuated force, \( F(t) \). The combined masses of the flexible joint cart and pendulum are \( M_T \). The inverted pendulum and the linear flexible joint of pendulum carts have equivalent damping terms, \( B_L \) and \( B_T \), respectively, while the mass of the pendulum is defined as \( m_p \), with friction damping, \( B_p \), located at the pendulum center of mass. The distance from the pendulum pivot to the center of mass of the pendulum is \( L_p \). The equations of motion for the inverted pendulum with its cart, and the linear flexible joint cart for the given servo motor voltage and dynamics are found by applying the Lagrange equation [11].

\[
\frac{\partial}{\partial t} \left( \frac{\partial L}{\partial q_i^\prime} \right) - \frac{\partial L}{\partial q_i} = \eta_i
\]  

(1)

The generalised coordinate variables are the \( q_i \), such that

\[
q(t) = [z_L(t), z_f(t), \theta(t)]
\]  

(2)

and the Lagrange equations for kinetic and potential energy give

\[
L = T - V
\]  

(3)
where $T$ is the total kinetic energy of the system and $V$ is the total potential energy of the system. Thus, the Lagrange defines the difference between the system's kinetic and potential energies. The generalised forces term, $\eta_i$, is used to describe the non-conservative forces such as friction applied to a system of generalised coordinates. In this case, the governing dynamic equations can thus be expressed as

$$
M_L\ddot{z}_L - k(z_f + z_L) = F - B_L\dot{z}_L \tag{4}
$$

$$
m_p l_p \sin(\theta)\dot{\theta}^2 - m_p l_p \cos(\theta)\ddot{\theta} + (M_1 + m_p)\ddot{z}_f + k(z_f - z_L) = -B_t\dot{z}_f \tag{5}
$$

$$
m_p l_p^2 \ddot{\theta} - m_p l_p \dot{z}_f \cos(\theta) - m_p l_p g \sin(\theta) = -B_p \dot{\theta} \tag{6}
$$

The generalised force acting on the inverted pendulum cart is thus

$$
\eta_1 = F - B_L\dot{z}_L \tag{7}
$$

The actuated force on the linear flexible joint cart is

$$
\eta_2 = -B_t\dot{z}_f \tag{8}
$$

and the force acting on the pendulum is

$$
\eta_3 = -B_p \dot{\theta} \tag{9}
$$

Solving the three Lagrange equations for the second-order time derivatives of the Lagrange coordinates produces the following non-linear equations:

$$
\ddot{z}_L = \frac{1}{M_L} \left( -B_L\dot{z}_L - k z_L + k z_f + F \right) \tag{10}
$$

$$
\ddot{z}_f = \frac{1}{M_m} \left\{ m_p l_p \sin(\theta)\dot{\theta}^2 + \frac{B_p}{l_p} \cos(\theta)\dot{\theta} + B_t\dot{z}_f \right\} - k z_f - m_p l_p g \cos(\theta) \sin(\theta) \tag{11}
$$

$$
\ddot{\theta} = \frac{1}{M_m} \left\{ m_p \cos(\theta) \sin(\theta) \dot{\theta}^2 + \frac{B_p}{m_p l_p^2} \left( m_p + M_L \right) \dot{\theta} + \frac{B_t}{l_p} \cos(\theta) z_f \right\} - \frac{\dot{k}}{l_p} \cos(\theta) z_L - \frac{\dot{\theta}}{l_p} \left( m_p + M_L \right) \sin(\theta) \tag{12}
$$

Almost all systems in real-life are nonlinear, making the development of linear state equations from nonlinear state equations for small ranges of control and state variables important. Linearized equations can be developed using the Taylor series expansion for a nonlinear set of dynamics given by $\dot{z} = f(z, u)$ about the equilibrium points $z_0$ and $u_0$, and for equilibrium point sets $\theta(t) = \hat{\theta}(t) = 0$, the perturbations of state and input variables are $z(t) = z_0 + \delta z(t)$ and $u(t) = u_0 + \delta u(t)$. The small deviations in $\delta z$ and $\delta u$ must be small enough to be ignored in the higher-order terms in the Taylor expansion of $f(z, u)$. The state matrix of the system can then be characterised by setting the state derivative to zero such that

$$
\begin{cases}
\dot{z}_1 = f_1(z, u) = 0; \\
\dot{z}_2 = f_2(z, u) = 0; \\
\dot{z}_3 = f_3(z, u) = 0; \\
\dot{z}_4 = f_4(z, u) = 0; \\
\dot{z}_5 = f_5(z, u) = 0; \\
\dot{z}_6 = f_6(z, u) = 0 \\
\end{cases} \tag{13}
$$

$$
f_1(z, u) = \dot{z}_L; \tag{14}
$$
\[
\begin{align*}
    f_2(z, u) &= \left( \frac{1}{M_L} \right)(-B_L \dot{z}_L + Kz_s + F); \\
    f_3(z, u) &= \dot{z}_s; \\
    f_4(z, u) &= -\left( \frac{1}{\lambda_m} \right) \left\{ L_p \cdot m_p \cdot \sin \theta \cdot \dot{\theta}^2 + \frac{B_p \cos \dot{\theta}}{L_p} + B_L \dot{z}_L \right\} - \left( F - B_L \dot{z}_L + Kz_s \right) / M_L; \\
    f_5(z, u) &= \dot{\theta}; \\
\end{align*}
\]

where \( \lambda_m = -m_p + m_p \cdot \cos^2(\theta) - M_L \), and \( z_s \) is assumed to be the deflection of the spring in the form \( z_s(t) = z_F(t) - z_L(t) \). The output of the system can be written as \( y = z_L \) and the state outputs are \([z_s; \dot{\theta}]\). The state matrix \( A \) is thus

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & a_{22} & a_{23} & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & a_{42} & a_{43} & a_{45} & a_{46} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & a_{63} & 0 & a_{65} & a_{66} & 0
\end{bmatrix}.
\]

Where

\[
\begin{align*}
a_{22} &= -\left( \frac{B_L}{M_L} \right), & a_{23} &= \left( \frac{K}{M_L} \right), & a_{42} &= \left( \frac{B_L}{M_L} \right), \\
a_{43} &= -\left( \frac{K}{M_L} \right) - \left( \frac{M_t}{M_L} \right), & a_{45} &= \left( \frac{g \cdot m_p}{M_L} \right), & a_{46} &= -\left( \frac{B_p}{L_p \cdot M_t} \right), \\
a_{63} &= -\left( \frac{K}{L_p \cdot M_t} \right), & a_{65} &= \left( \frac{g \cdot (M_t + m_p)}{(L_p \cdot M_t)} \right), & a_{66} &= -\left( \frac{B_p \cdot (M_t + m_p)}{(L_p \cdot M_t \cdot M_t)} \right).
\end{align*}
\]

and the input (B), output (C), and feedback (D) matrices are

\[
B = \begin{bmatrix} 0 & \left( \frac{1}{M_L} \right) & 0 & -\left( \frac{1}{M_L} \right) & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & 1.0638 & 0 & -1.0638 & 0 \end{bmatrix}^T;
\]

\[
C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \end{bmatrix}; \quad D = \begin{bmatrix} 0; & 0; & 0 \end{bmatrix}
\]

For the linear flexible joint cart and pendulum system, the state vectors are thus

\[
\begin{align*}
    z(t) &= \begin{bmatrix} z_L(t) \\
    \dot{z}_L(t) \\
    z_s(t) \\
    \dot{z}_s(t) \\
    \theta(t) \\
    \dot{\theta}(t) \end{bmatrix}; \\
    \dot{z}(t) &= \begin{bmatrix} \dot{z}_L(t) \\
    \dot{\theta}(t) \\
    \ddot{z}_L(t) \\
    \ddot{z}_s(t) \\
    \ddot{\theta}(t) \end{bmatrix} = \begin{bmatrix} z_1 \\
    z_2 \\
    z_3 \\
    z_4 \\
    z_5 \end{bmatrix}.
\end{align*}
\]

A rank test indicates that the system is fully controllable and observable, as the rank of its controllability and observability matrices equals 6.
3. Control Design of servo system

The present dynamic system of (LFJ_IVP) involves an integrator that is clear from pole locations of the characteristics equation of the plant.

\[
\begin{align*}
s_1 &= 0 & \text{integrator} \\
s_2 &= -15.8544 & \text{real pole} \\
s_3 &= 2.4282 + 5.4012i \\
s_4 &= 2.4282 - 5.4012i \\
s_5 &= 1.8400 + 2.1236i \\
s_6 &= 1.8400 - 2.1236i
\end{align*}
\]

The configuration of the sitting servo control (LFJ_IVP) system as shown in Fig.2, the plant of this linear system is considered as a type one servo system that has a scalar control signal \((u)\) and the output signal scalar \((y)\). The pole placement method can be used to design controller gains when placing closed-loop poles to meet the desired specification.

![Design of servo control (LFJ_IVP) system.](image)

The requirements for the existent system with the cart pendulum of flexible joint receiving a step or square wave inputs are settling time for \((z_L)\) and \((\theta)\) of less than 5 seconds, rise time for \(z_L\) of less than 1 second, Overshoot of theta less than 20 deg (0.35 radians), steady-state error within 2%.

That’s clear from the characteristic equation the plant of (LFJ-IVP), the current system is unstable due to the positive complex poles at right-half of the s-plane, then the exponential terms in the system response will not approach zero as time \((t)\) increases. Tray to approximate the desired poles of the characteristic equation of the current (LFJ-IVP) plant have two negative complex conjugates near the imaginary axis and the other poles that are located far from the imaginary axis with large negative real parts. The exponential terms that correspond to these poles decay very rapidly to zero. Knowing that the horizontal distance from a closed-loop pole to the imaginary axis determines the fast settling time of transients due to that pole, while that the behavior of transient response is dependent upon the two negative complex conjugates especially that are closed to the imaginary axis [5].

Knowing that any pole of characteristic equation in the frequency domain is \(s = \zeta \omega_n \mp \omega_n \sqrt{1 - \zeta^2} \). Therefore, the desire poles of the existing analysis due to previous requirements must satisfy the constraints which illustrated in Fig.3, the complex conjugates locations and the other poles should be on the far away at less 5 times of \((\zeta \omega_n)\) on the s-plane.
The following table 1 describes the conduct of the desired first two complex conjugate poles of the current system that satisfies the previous transient response specifications and steady-state bands as in Figure 3.

**Table 1.** The transient behavior of two first complex conjugate poles within a designed bounded area.

| $\sigma + \omega_d j$ in radian per second | Damping Coefficient ($\zeta$) | Natural frequency in radian per second ($\omega_n$) | Settling time in the second ($t_s$) | Rise time in second ($t_r$) | Over-shoot (O.S.) |
|------------------------------------------|-------------------------------|-----------------------------------------------|-----------------------------------|---------------------------|-----------------|
| $2 + 2j$                                 | 0.7                           | 2.86                                          | 2                                 | 0.63                      | 4.6 %           |
| $3 + 3j$                                 | 0.83                          | 3.6                                           | 1.34                              | 0.5                       | 0.93%           |
| $4 + 1j$                                 | 0.97                          | 4.12                                          | 1                                 | 0.43                      | 4.6%            |
| $4 + 2j$                                 | 0.89                          | 4.47                                          | 1                                 | 0.4                       | 0.21%           |
| $4 + 4j$                                 | 0.7                           | 5.7                                           | 1                                 | 0.315                     | 4.6%            |

The state feedback control law can be expressed as

$$u = -K_{place} z + K_1 r$$

and thus, the equation of a dynamic system can be given by

$$\dot{z} = Az + Bu = (A - BK_{place})z + Bk_1 r$$

When the closed-loop poles are at the desired position, the servo control LFJ-IVP system becomes an asymptotically stable system, and the output $y(\infty)$ approaches the constant value $r(\infty)$ for the step input, while the scalar input $u(\infty)$ approaches zero. The steady-state equation is thus

$$z(\infty) = (A - BK_{place})z(\infty) + Bk_1 r(\infty)$$

The error between the input and the steady state can be defined as

$$z(t) - z(\infty) = e(t)$$
The error dynamic equation thus becomes
\[ \dot{e} = (A - BK_{place})e \]  
(27)

As the LFJ-IVP system is full state controllable, by choosing the desired poles for the closed-loop control system to meet the desired response specifications for the matrix \([A - BK_{place}]\), the state feedback gain \(K_{place}\) is obtained using the pole-placement technique with a command place in the MATLAB code. The matrix \([A - BK_{place}]\) is an asymptotically stable matrix, with the regulator poles, which represent the matrix \([A - BK_{place}]\) eigenvalues, located in the left half of the s-plane.

4. Observer controller

There are two stages in the current design of the observer controller. The first stage is obtaining the feedback gain matrix \([K_{place}]\) to yield the desired characteristic equation and the second stage is the calculation of the observer gain matrix \([K_L]\). Assuming the plant state equation of LFJ-IVP as seen in Figure 4 such that \(\dot{z} = Az + Bu\), and the observer state feedback control,

\[ u = \bar{N}r - K_{place}\hat{z} \]  
(28)

\[ \hat{z} = Az + B[\bar{N}r - K_{place}\hat{z}] \]  
(30)

where \(\bar{N}\) is a scalar value of the reference input. The designed observer’s gains, as seen in Figure 4, are such that the simulation and the correction mechanism taken together minimise the differences between the observer and the plant state. This can also be done by describing the dynamics of the observer in terms of \([K_L]\), as in the following equations:

\[ \hat{z} = A\hat{z} + Bu + K_L(y - \hat{y}) \]  
(31)

\[ \hat{y} = C\hat{z} \]  
(32)

The state error vector is the difference between the plant \((z)\) and the estimated states \((\hat{z})\) such that \(e = z - \hat{z}\); the derivative of state error is thus \(\dot{e} = \dot{z} - \dot{\hat{z}}\).

\[ \dot{e} = (Az + Bu) - (A\hat{z} + Bu + K_L(y - \hat{y})) \]  
(33)

\[ \dot{e} = (A(z - \hat{z}) - K_LC(z - \hat{z}) = A e - K_LC e \]  
(34)

\[ \dot{e} = (A - K_LC) e \]  
(35)

Substituting in the estimated error vector to equation (30) creates a state equation with a new state matrix:

\[ \dot{z} = (A - BK_{place})z + BK_{place}e + B\bar{N}r \]  
(35)

The dynamic observed-state feedback control system in matrix form is thus

\[ \begin{bmatrix} \dot{z} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} (A - BK_{place}) & BK_{place} \\ 0 & (A - K_LC) \end{bmatrix} \begin{bmatrix} z \\ e \end{bmatrix} + \begin{bmatrix} B\bar{N} \\ 0 \end{bmatrix} r \]  
(36)

The pole of the state observer can be obtained by setting up the characteristic equation for the observer with \(|sI + (A - K_LC)|\) equal to zero. These poles describe the state error of the differences between the
estimated and actual states. The observer gain matrix $[K_L]$ can thus be calculated by setting the estimator poles to about 5 to 10 times the slowest pole.

5. Simulation
The analysis and simulation of the linear LFJ-IVP system can be used to represent both regulator systems and control systems. For regulator systems, the reference input is constant, such as step input that is used in the current study with a constant value of 0.02 during simulation, while control systems have reference inputs that are time-varying, such as that used in [11], which adopted a square wave input with an amplitude of 0.02 mm, at a frequency about 0.5 Hz. In this study, the disturbance effect was also taken into consideration.

![Figure 4. Full state compensator for the LFJ-IVP system.](image)

6. Results
In the simulation, using the values in Table 1, the cart positions of the servo LFJ-IVP system, type 1 with state feedback controller working as a regulator, was tested using an external input signal with a magnitude of about 0.02 step, as seen in Figure 5. The response curves in this figure show the significant effect on the emergent behaviours of the transient response when using different complex conjugates obeying the design requirements under the bounded area for different tuning paths. As noted in Figure 3, the fastest response with a high over-shoot for complex conjugates $(4 \pm 4j)$ and the slowest response $(2 \pm 2j)$ were in between the required response $(4 \pm 2j)$; these responses combine to approximate a small rise, settling time, and minimum over-shoot.
The results shown in Figure 6 represented the responses of the output system (cart position \(z_L\) of the flexible joint) and the system states such as spring deflection \(z_s\) and pendulum angle \(\theta\), as well as the energy consumed by the servo motor \(u\).

**Figure 5.** Simulation response of cart position to 0.02 step input for different complex conjugate poles.

**Figure 6.** Response to 0.02 steps in a regulator (LFJ-IVP) system with a state-feedback controller.
These simulated responses thus achieve the required design requirements (see Table 1) at the lowest cost of energy (less than ± 1.3 volt) when using a state feedback controller to control a servo type 1 (LFJ-IVP) control system, as shown in Figure 3. Previous systems have been tested with a square signal generator acting as a disturbance signal input to the system with 0.02 m amplitude at a frequency of 50 Hz. The responses for that system are shown in Figure 7, and these indicate that the performance in tracking the input signal was acceptable despite the delay in the transient responses, such that the output system of the cart position ($z_1$) of the flexible joint oscillates back-and-forth, approximating the signal input generator required to balance the pendulum. The pendulum angle does not exceed 0.015 radians when balanced, and the spring deflection does not exceed ± 2 mm due to the integration of the system ultimately returning to the equilibrium (0 mm) position. The results obtained, as shown in Figures 8 and 9 are thus identical to those in Figures 6 and 7, respectively except for the power consumption curve in the electric servo motor. There, the maximum power consumption was reduced by using the observer controller from 3.9 V to approximately 2 V, as shown clearly in Figure 10 [11].

![Figure 7](image_url)

**Figure 7.** Simulated response of control (LFJ-IVP) system with a state-feedback controller.
**Figure 8.** Response to 0.02 step for regulator (LFJ-IVP) system with observer state-feedback controller.

**Figure 9.** Simulated response of control (LFJ-IVP) system with observed state-feedback controller.
7. Conclusions
In this paper, a non-linear system was approximated to a linear system using State-Space technology to facilitate the design of a controller subject to design conditions imposed by a beneficiary party. Based on this, the following conclusions emerged:

- It is possible to control the transient response of a linear system by assuming that the roots of the characteristic high-order equation contain a pair of complex-conjugate roots close to the imaginary coordinates that satisfy the design conditions (see Table 1 and Figure 3).
- The performance of the present system, a servo type 1 LFJ-IVP, was improved by using both existing state-feedback and observer controllers, as evidenced by matching the simulation responses for cart position, spring deflection, and pendulum angle with those of previous researchers [11]. This also showed that it is possible to obtain an improvement in the following performance responses:
  - Improvement in the cart position, and spring deflection by about 20%
  - Angle of the pendulum improved by about 37.24%.
  - Improvement (reduction) in the maximum power of electrical energy consumed by the servo motor by about 52.6%.
- Further improvement, of about 33%, occurred with regard to the maximum electrical energy when using an observed controller as compared to the state feedback.

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