Abstract—A concurrent dual-band self-oscillating mixer (SOM), based on a ring-shaped stepped-impedance resonator, is proposed and analyzed in detail. Taking advantage of the ring even and odd resonances, the circuit can operate in concurrent dual quasi-periodic mode and injection-locked mode. In the second case, it behaves as a dual-band zero-intermediate-frequency (IF) mixer. Initially, an analytical study of the SOM behavior in the two modes is presented. Then a variety of accurate numerical methods are used for an in-depth investigation of the main aspects of its performance, including stability, conversion gain, linearity, and phase noise. The recently proposed contour-intersection technique and the outer-tier perturbation analysis are suitably adapted to the SOM case. A method is also presented to distinguish the parameter intervals leading to heterodyne and to zero-IF operation at both the lower and upper frequency bands. In the zero-IF SOM, the possible instantaneous unlocking in the presence of modulated input signals is investigated and avoided. The methods have been applied to a dual mixer at the frequencies 2.4 and 4.1 GHz.

Index Terms—Harmonic balance (HB), oscillators, phase-noise analysis, stability analysis.

I. INTRODUCTION

T HE recent works [1]–[3] present a novel concept to obtain zero-intermediate-frequency (IF) conversion using an injection-locked oscillator, suitably designed and biased to enhance its mixing capabilities. The so-called zero-IF self-oscillating mixer (SOM) enables the implementation of compact transmitters and receivers. The concept is of general interest in systems requiring a small size and weight, such as the RFID tag proposed in [1]. Considering also the increasing demand of multiband wireless systems, Pontón et al. [4] proposed a concurrent dual-band zero-IF SOM, which extends the compact frequency conversion to simultaneous operation in two frequency bands. This requires a concurrent dual-frequency oscillator at two incommensurable fundamental frequencies $\omega_1$ and $\omega_2$ [5]–[7], which corresponds to a doubly autonomous quasi-periodic solution. In the concurrent dual-frequency zero-IF SOM, each oscillation gets locked to its corresponding RF signal [4], so the circuit behaves as a concurrent dual-frequency injection-locked oscillator.

A major challenge in the design of concurrent dual-frequency oscillators is the robustness of this concurrent operation mode [8], which mathematically coexists with two periodic solutions, at $\omega_1$ and $\omega_2$, respectively, and with the dc solution. For a reliable concurrent operation, the quasi-periodic solution at $\omega_1$ and $\omega_2$ must be the only stable one. Recently, a concurrent dual-frequency oscillator based on a stepped-impedance loop resonator [9]–[11] has been proposed [12]–[14]. This element enables a compact implementation of two orthogonal resonances at $\omega_1$ and $\omega_2$ with high quality factor and an excellent isolation. It has the advantage of a compact size, usually much smaller than the one resulting from two independent resonators with similar quality factors. With this element, low phase noise can be achieved, as shown in [15], where a ring-shaped step-impedance resonator was used to reduce the phase-noise spectral density of a single-frequency voltage-controlled oscillator (VCO).

Departing from a dual-frequency oscillator based on a ring-shaped resonator, Pontón et al. [4] presented a double functionality SOM, able to operate both as a zero-IF SOM and as a heterodyne SOM [16]–[18]. In heterodyne mode each of the two concurrent oscillations ($\omega_1$ and $\omega_2$) mixes with its corresponding input signal, that is, $\omega_{1\text{in}}$ mixes $\omega_1$ and $\omega_{2\text{in}}$ mixes with $\omega_2$, to provide two distinct IF outputs. This work expands [4] with a nonlinear analysis of the SOM in both heterodyne and zero-IF modes. Initially, an analytical investigation of the SOM behavior in the two modes is presented. It provides insight into the conversion gain, linearity, and oscillation extinction, in the heterodyne case, and the injection-locked operation and mechanisms for the amplitude and frequency demodulation, in the zero-IF case. Then, the transistor-based dual-band SOM is addressed with a variety of numerical methods based on harmonic balance (HB).

The first goal will be the detailed stability analysis of the ring-resonator dual-frequency oscillator, considering the dc solution, the distinct periodic oscillations at $\omega_1$ and $\omega_2$, and the concurrent quasi-periodic oscillation at $\omega_1$ and $\omega_2$. In heterodyne mode, the SOM will be analyzed through an extension of the contour-intersection method proposed in [19] and [20] to a quasi-periodic regime. The variation of the conversion gain is analyzed exporting a gain surface that must be interpolated at the solution points obtained through the
contour intersection. The phase noise is analyzed with a perturbation formulation based on an outer-tier admittance-type description of the SOM circuit. The input-generator values (in terms of frequency and power) for operation as a heterodyne SOM and as zero-IF SOM are distinguished through a description of the SOM circuit. The input-generator values are calculated with the transfer functions

\[ A_y(\omega) = - \frac{V(\omega)}{I} \bigg|_{E_{\text{in}}=0} \quad \text{and} \quad A_g(\omega) = - \frac{V(\omega)}{E_{\text{in}}} \bigg|_{V=0} \]  

where \( I \) is the current of the nonlinear source. In the absence of input power, \( |V_1| = 0 \), (2a) particularizes to the free-running oscillation at \( \omega_o \). In this simple model, the oscillation frequency is obtained from \( \text{Im}(A_y(\omega_o)) = 0 \). The capacitances \( C_1 \) and \( C_2 \) will give rise to a small shift with respect to the resonance frequency of \( Z_G(\omega) \). One should note that even when \( E_{\text{in}} \neq 0 \), system (2a) admits a solution with zero oscillation amplitude \( V_0 = 0 \), which should be unstable and avoided in the SOM analysis. In the numerical methods, this will be done with the aid of an auxiliary generator (AG) [26].

When \( E_{\text{in}} > 0 \), \( i(v) \) provides an additional term at the IF \( \omega_{\text{in}} - \omega_o \) that, in this simplified analysis, can be approached as \( I_{\text{IF}} = g_2 V_o V_1 \). With \( Z_G = Z_{\text{out}} = R_o \), and assuming a small \( C_2 \), the conversion gain is approximated as

\[ G_{\text{IF}} = \frac{4(g_2 V_o)^2 |V_1|^2 R_o^2}{E_{\text{in}}^2}. \]  

For small \( E_{\text{in}} \), one can neglect the second term in the nonlinear current of (2b), which provides the small-signal gain

\[ G_{\text{ss}} = 4(g_2 V_o)^2 \frac{|A_g(\omega_{\text{in}})|^2 R_o^2}{1 + A_y(\omega_{\text{in}})|g_m + \frac{3}{2}g_3 V_o^2|^2}. \]  

The higher \( V_o \), the smaller the second term in the denominator, due to the opposite signs of \( g_m \) and \( g_3 \). Thus, the small-signal gain increases with \( V_o \), \( g_2 \) and the transfer function \( |A_g| \).

Now we will consider the variation of \( G_{\text{IF}} \) in (4) versus \( P_{\text{in}} = E_{\text{in}}^2/(8R_o) \). This is shown in Fig. 2(a) where the results of system (2a) have been validated with an HB simulation of the circuit in Fig. 1, with the aid of an AG [8], [23]. Fig. 2(b) presents the corresponding variation of \( V_o \) and \( |V_1| \).

In agreement with (2a), when \( E_{\text{in}} \) increases, the larger \( |V_1| \) leads to a reduction of \( V_o \), due to the opposite signs of \( g_m \) and \( g_3 \). The analytical model also explains the gain expansion in Fig. 2(a). The relatively fast decrease of \( V_o \) may give rise to an increase in the slope of \( |V_1| \) versus \( P_{\text{in}} \) [Fig. 2(b)] since the counteracting effect of \( g_3 V_o^2 |V_1|/2 \) in (2b) becomes smaller when \( V_o \) decreases. Eventually, the gain decreases with \( V_o \) and tends to \(-\infty\) when the oscillation is extinguished.

A relevant nonlinear effect in an SOM is the oscillation extinction when increasing \( P_{\text{in}} \), which occurs at an inverse Hopf bifurcation [21]–[25], denoted with H in Fig. 2, and is expressed by setting \( V_o = 0 \) in (2a)

\[ 1 + A_y(\omega_o) \left( g_m + \frac{3}{2}g_3 |V_{\text{IF}}|^2 \right) = 0. \]
of oscillation point, $\omega$ into a low-amplitude curve of periodic solutions at $\omega$. Current through $\in$ Fig. 3(a), where the curves are traced in terms of the $P_n$. The corresponding $P_n$ is calculated replacing $|V_{11}|$ in (2b). For the same $g_m$, $P_n$ at the Hopf bifurcation will be higher for a smaller $|g_3|$ and a larger detuning of $\omega_m$ from the original free-running frequency.

B. Zero-IF SOM

In the zero-IF SOM, the oscillator is injection locked to the input source at $\omega_m$, which is the only fundamental frequency. The circuit behavior is described with the two following equations at dc and $\omega_m$:

$$V_{dc} + A_y(0) \left( \frac{V_2^2}{2} \right) = 0$$  \hspace{1cm} (7a)

$$V_o + A_y(\omega_m) \left( g_m V_o + \frac{3}{4} g_3 V_o^3 \right) + A_g(\omega_m) E_{in} e^{j\phi} = 0$$  \hspace{1cm} (7b)

where for better insight some terms have been neglected. The phase origin is set at the control voltage, $V_o$, and $\phi$ is the opposite of the phase shift with respect to the input source. In our simplified model, (7b) can be solved independently of (7a). For each $\omega_m$, it provides a system of two complex equations in $V_o$ and $\phi$. When also considering distinct values of $E_{in}$, one obtains a curve family versus $\omega$. This is shown in Fig. 3(a), where the curves are traced in terms of the current through $C_1$. To understand the family, one should note that for $E_{in} = 0$, (7a) particularizes to the free-running oscillation point, $\omega_o, V_o, V_o$, plus the trivial solution $V_o = 0$. For small $E_{in}$, a linearization about the free-running point, $\omega_o, V_o$, gives rise to an ellipse and the trivial solution evolves into a low-amplitude curve of periodic solutions at $\omega_m$. This, one obtains two isolated curves, though only the closed curve corresponds to injection-locked operation. As $E_{in}$ increases the closed curve deviates from a perfect ellipse [$E_{in} = 0.2$ V in Fig. 3(a)]. The boundaries of the locking band are given by the two turning points of the closed curve, though only its upper or lower section is stable [23], [24]. When further increasing $E_{in}$, the two curves merge into one that exhibits folding due to the presence of turning points [$E_{in} = 0.3$ V and $E_{in} = 0.4$ V in Fig. 3(a)].

The turning points in the curves obtained for the lower $E_{in}$ correspond to local-global bifurcations [21]–[23] at which an incommensurate oscillation arises with a zero value of the beat frequency. This beat frequency grows quickly when moving away from the turning points and agrees with the IF in heterodyne mode. In fact, the merged curves of Fig. 3(a) are stable only in a certain frequency band about $\omega_o$, delimited by Hopf bifurcations, at which an incommensurate oscillation arises from zero amplitude. The curves in Fig. 2(b) constitute an example of the generated quasi-periodic solutions, not traced in Fig. 3(a). In zero-IF mode, the frequency of the input source must belong to the stable locking bands, which are broader for higher $P_n$ [Fig. 3(a)]. In heterodyne mode, one must avoid these bands. More details on the frequency selection are given in Section V.

One can also represent the injection-locked curves versus $E_{in}$ at constant $\omega_m$, as done in Fig. 3(b), where two distinct $\omega_m$ values are considered. Roughly speaking, the low-amplitude section of these curves (from zero to the first turning point $T_1$) corresponds to solutions in which the self-oscillation is not excited. For low $E_{in}$, they are in the low-amplitude curves of Fig. 3(a), below the closed ones. In turn, the upper and middle sections correspond to the closed curves in Fig. 3(a). When increasing $E_{in}$ from zero, the circuit is initially unlocked and exhibits a quasi-periodic solution [not represented in Fig. 3(b)]. It becomes locked at the turning point $T_2$. The $E_{in}$ value at $T_2$ will be smaller for a smaller detuning with respect to the free-running frequency. A change of $E_{in}$ at a constant $\omega_m$ will give rise to a variation of $V_o$, and, in turn, to a variation of $V_{dc}$, as shown in (7a). Assuming an input amplitude modulation and neglecting initially any dynamic effects due to the modulation frequency, one will obtain a baseband signal, resulting from the time variation of the previous dc term. Under faster variations, one should consider time-varying variables $V_{dc}(t), V_o(t), \phi(t)$ at the scale of the modulation, so the frequency-dependent functions $A_y$ and $A_g$ will be expressed in a first-order Taylor series [28] about dc and $\omega_m$. The multiplication by the frequency increment implies a time differentiation [29], so one obtains the
envelope-domain system

\[
A_V(0)V_{dc}(t) - j \frac{dA_V(\omega)}{d\omega} \bigg|_{\omega=0} \dot{V}_{dc}(t) + A_{YN}(0) \left( g_2^2 \frac{V_o^2(t)}{2} \right) \\
- j \frac{dA_{YN}(\omega)}{d\omega} \bigg|_{\omega=0} g_2 V_o(t) \dot{V}_o(t) = 0 \\
A_V(\omega_n) V_o(t) - j \frac{dA_V(\omega)}{d\omega} \bigg|_{\omega=\omega_n} \dot{V}_o(t) + A_{YN}(\omega_n) \\
\times \left( g_m V_o(t) + \frac{3}{4} g_3 V_o^3(t) \right) \\
- j \frac{dA_{YN}(\omega_n)}{d\omega} \bigg|_{\omega=\omega_n} \left( g_m + \frac{9}{4} g_3 V_o^2(t) \right) \dot{V}_o(t) \\
+ A_{GN}(\omega_n) E_{in}(t) e^{j\phi(t)} \\
- j \frac{dA_{GN}(\omega_n)}{d\omega} \bigg|_{\omega=\omega_n} (\dot{E}_{in}(t) e^{j\phi(t)} + j\phi(t) E_{in}(t) e^{j\phi(t)}) = 0 
\]

(8)

where \(A_V\) and \(A_G\) have been redefined to avoid frequency-dependent denominators. Note it would be possible to consider higher order terms in the Taylor-series expansion. Fig. 3(c) presents the behavior under an input amplitude modulation with a rectangular waveform of 2 MHz and amplitude 0.05 V, at \(f_{in} = 4.1\) GHz and \(E_{in} = 0.025\) V. The injection-locked oscillator can also demodulate frequency modulation (FM) signals [30], [31]. Under a change of \(\omega_n\) in injection-locked conditions, there will be a variation of both \(\phi\) and \(V_o\), as gathered from (7b). From (7a), the latter will give to a change in \(V_{dc}\), so the circuit should be able to demodulate FM signals, as shown in Fig. 3(d), where a frequency-shift keying (FSK) modulation of 1 Mb/s has been introduced at \(f_{in} = 4.1\) GHz and \(E_{in} = 0.1\) V. The sensitivity to the modulation will increase with the slope of the oscillation amplitude versus \(\omega_n\).

III. DESIGN AND STABILITY ANALYSIS OF THE CONCURRENT DUAL-FREQUENCY OSCILLATOR

A. Circuit Topology

The circuit in Fig. 4(a) and (b) operates as a concurrent dual-band SOM, due to its capability to exhibit two simultaneous oscillations at the frequencies \(\omega_1\) and \(\omega_2\). Following [12], the circuit is based on a ring-shaped stepped-impedance resonator, which enables an implementation of two equivalent one-wavelength resonators in a single compact-size resonator. As in the single-frequency VCO of [15], two transistor devices are connected to two of the resonator ports. Our goal has been to obtain a demonstrator-independent resonance modes at the incommensurate frequencies \(f_1 = 2.4\) GHz and \(f_2 = 4.1\) GHz; these values enable a comparison of the phase-noise spectral density with previous single-frequency realizations [32], [33] using similar components. The resonator design departs from the structure sketched in Fig. 5(a), which contains four sections of \(\pi/2\) electrical length at \(f_0\). To achieve the two resonances at \(f_1\) and \(f_2\), the ratio \(R_Z\) between the characteristic impedances \(Z_2\) (of the higher-impedance section) and \(Z_1\) (of the lower-impedance section) must be

\[ R_Z = \tan^2 \left( \frac{\theta_0 f_2}{f_0} \right) \]

(9)

where \(\theta_0 = 2\pi/8\). In our case, \(R_Z = 2.32\). In these conditions, when exciting the resonator at the Port 1 (3) one obtains resonance frequency at \(f_1 = 2.4\) GHz \((f_2 = 4.1\) GHz).
The ring is introduced into the oscillator with the aid of coupled lines, like in [12]. After slightly optimizing the dimensions, the electromagnetic simulations provide the two independent resonances shown in Fig. 5(b) and (c), where the input susceptance has been represented versus frequency. The two unloaded quality factors are $Q_1 = 235$ at $f_1 = 2.4 \text{ GHz}$ and $Q_2 = 140$ at $f_2 = 4.1 \text{ GHz}$.

Here, the two independent modes of the resonator are used to fulfill the oscillation conditions at $\omega_1$ and $\omega_2$, respectively. The ring resonator is connected to the source terminals of the two transistor devices as the only feedback element. The two circuit sections, denoted as Section 1 and Section 2 in Fig. 4(a), oscillate at the respective frequencies $\omega_1$ (lower) and $\omega_2$ (higher). The two RF input signals, at $\omega_{m_1}$ and $\omega_{m_2}$, are introduced through a transmission-line section used for matching purposes. For heterodyne SOM operation, each suboscillator includes an open-circuited $\lambda/4$ stub (at $\omega_1$ and $\omega_2$, respectively) connected to the drain terminal [Fig. 4(a)], as well as a low-pass filter. Under zero-IF operation, if there is a negligible spectrum content near dc, the output signal can still be extracted through the same output network. It can also be extracted from the drain-bias network with the aid of a buffer. At each frequency the circuit combines the oscillator and mixer functions and can provide conversion gain. The total consumption in concurrent operation is $48 \text{ mW}$. As seen in Fig. 4(b), the stepped-impedance resonator is the largest circuit component. It has a footprint of $15 \text{ mm} \times 15 \text{ mm}$, whereas that of the packaged transistors is $2 \text{ mm} \times 2.1 \text{ mm}$.

Prior to the introduction of the RF signals, the circuit must exhibit two concurrent oscillations at the frequencies $\omega_1$ and $\omega_2$. For a reliable operation, this doubly autonomous quasi-periodic solution must be the only stable one. However, due to the circuit autonomy, this solution coexists with a dc solution and two distinct periodic solutions at $\omega_1$ and $\omega_2$, which must be unstable. In the following, a detailed stability analysis of the three types of solution is presented.

**B. Stability of the DC Solution**

The stability analysis of the dual-band SOM in Fig. 4(a) through the pole–zero identification of a closed-loop transfer function [34]–[38] may suffer from insufficient observability, due to the high isolation between the two suboscillators. This identification relies on the fact that all the closed-loop transfer functions that can be defined in a linear system share the same denominator and therefore should exhibit the same poles [34]–[38], formally agreeing with the roots of the characteristic determinant that define the stability properties. Unlike the poles, the zeroes depend on the closed-loop transfer function, and cancellations/quasi-cancellations of poles and zeroes may take place in some of these functions [34]–[38].

If the canceled/quasi-cancelled poles are in the right-hand side (RHS) of the complex plane, one will miss instabilities. This is usually due to a low observability of some unstable loops, as expected in the circuit in Fig. 4(a). To account for the complete structure, instead of using closed-loop transfer functions, we will calculate the circuit characteristic determinant [39] in a way that ensures that it cannot exhibit any RHS poles. In comparison with the well-established methods [39], [40] (also based on the calculation of the characteristic determinant), the one used here [27] has the advantage of not requiring access to the active-device intrinsic terminals. The stability information is in the zeroes of the characteristic determinant and avoiding by construction the existence of RHS poles will prevent any cancellations or quasi-cancellations. This determinant is obtained by: 1) partitioning the circuit into open circuit (OC)-stable blocks and 2) calculating the total impedance matrix at the $N$ ports defined in the partition.

The verification of the DC stability of the blocks will be done inside the blocks through standard pole–zero identification [34]–[38], which due to the limited size of the blocks can be applied reliably. Note that an analogous procedure with short circuit (SC) stable blocks and a total admittance matrix is also possible [27].

In an initial test, the passive linear network considered in the circuit of Fig. 4(a) contains the ring-shaped stepped-impedance resonator only, and the active blocks are constituted by the remaining components of the two suboscillators. Thus, we start with $N = 2$ ports, corresponding to those denoted as Ref$_2$ and Ref$_3$ in Fig. 4(a). However, the defined active blocks are unstable when terminated in OC at Ref$_2$ and Ref$_3$, respectively. As a result, we have to consider additional ports in the two active blocks: the first (second) active block has now the ports Ref$_1$ and Ref$_2$ (Ref$_3$ and Ref$_4$). When terminating the first active block in OC at Ref$_1$ and Ref$_2$, it becomes stable for all the $V_{GS}$ values. This has been verified through pole–zero identification inside the block, at different observation nodes. Fig. 6(a) presents the variation of the real part of the dominant poles of the first active block versus $V_{GS1}$. All the poles have a negative real part so the block is OC stable. Likewise, when terminating the second active block in OC at Ref$_3$ and Ref$_4$, it becomes stable [Fig. 6(b)] for all the $V_{GS}$ values. If the active blocks had still been unstable, one would have needed more analysis ports. Thus, the number of ports only depends on

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**Fig. 6.** Stability analysis of the two-port active blocks considered in Fig. 4 when terminated in OCs, through standard pole–zero identification. (a) Active block with the two ports Ref$_1$ and Ref$_2$. Variation of the real part of the dominant poles versus $V_{GS1}$. (b) Active block with the two ports Ref$_3$ and Ref$_4$. Variation of the real part of the dominant poles versus $V_{GS3}$. 

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**Table 3.** Performance comparison between the ring- and stepped-impedance resonator based SOMs.

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**Ref.**

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**Note:**
the active blocks and their stability properties. The stability analysis of the whole structure (including the passive linear network) is addressed calculating the total impedance matrix at Re\(f_1\), Re\(f_2\), Re\(f_3\), and Re\(f_4\). The determinant of this total impedance matrix is

\[
\text{det}(s) = \text{det}
\begin{bmatrix}
Z_{1,11}(s) & Z_{1,12}(s) & 0 & 0 \\
Z_{1,21}(s) & Z_{1,22}(s) & 0 & 0 \\
0 & 0 & Z_{2,11}(s) & Z_{2,12}(s) \\
0 & 0 & Z_{2,21}(s) & Z_{2,22}(s)
\end{bmatrix}
\begin{cases}
z_{11}(s) & z_{12}(s) & z_{13}(s) & z_{14}(s) \\
z_{21}(s) & z_{22}(s) & z_{23}(s) & z_{24}(s) \\
z_{31}(s) & z_{32}(s) & z_{33}(s) & z_{34}(s) \\
z_{41}(s) & z_{42}(s) & z_{43}(s) & z_{44}(s)
\end{cases}
= 0
\]

(10)

where \(s\) is the complex frequency, the boxes in the left matrix correspond to the impedance matrices of the 2 \(\times\) 2 active blocks, and \([z_{ij}]\) is the impedance matrix of the entire remaining passive linear part (including the ring resonator). Provided that the two active subcircuits are stable under OC terminations, the determinant function (10) cannot exhibit any RHS poles. This is because none of the \(Z\)-parameters in the active-block impedance matrix can have poles on the RHS since each of these parameters constitutes a particular closed-loop transfer function, calculated under OC terminations. Obviously, the matrix describing the passive linear network cannot exhibit any RHS poles either. In practice, one directly calculates the total impedance matrix, with no need for the decomposition in (10), considered only for demonstration purposes. Note that avoiding, by construction, the coexistence of RHS zeroes and poles prevents the risk of missing instabilities.

Fig. 7(a) presents the results of the identification of \(\text{det}(jo)\) when the gate-bias voltages of the two transistors are \(V_{GS1} = -0.35\) V and \(V_{GS2} = -0.55\) V. Remember that when using the characteristic determinant the stability information is in the zeroes. It shows two pairs of complex-conjugate zeroes in the RHS, about the two expected oscillation frequencies \(f_1 = 2.49\) GHz and \(f_2 = 4.22\) GHz. The result is compared with two distinct analyses through pole–zero identification of the whole structure, in the two different sections of the circuit. Each identification [Fig. 7(b) and (c)] detects a single pair of RHS complex-conjugate poles at the corresponding oscillation frequency. Thus, the standard pole–zero identification is unable to capture the stability properties of the whole structure. Fig. 7(d) presents the evolution of the real part of the zeroes of \(\text{det}(s)\) when varying \(V_{GS1}\) at constant \(V_{GS2} = -0.55\) V. The second suboscillator [associated with Section 2 in Fig. 4(a)] is always unstable, so for all the values of \(V_{GS1}\), there is a pair of complex-conjugate zeroes in the RHS. The pole–zero identification is unable to detect this additional pair of RHS poles.

C. Stability of the Periodic Solutions

A common problem in concurrent dual-frequency oscillators is the coexistence [8] of the doubly autonomous quasi-periodic solution (having \(\omega_1\) and \(\omega_2\) as the two fundamental frequencies), with stable periodic solutions at \(\omega_1\) and/or \(\omega_2\) (and no concurrence). For the stability analysis of these periodic solutions, an AG [23] will be introduced in the circuit section (1 or 2) exhibiting the oscillation. The periodic solution at \(\omega_1\) (\(\omega_2\)) is analyzed by setting the AG frequency to this value, doing \(\omega_{AG} = \omega_1\) (\(\omega_{AG} = \omega_2\)). Then, the AG amplitude \(V_{AG}\) and \(\omega_{AG}\) will be optimized to fulfill \(Y_{AG} = 0\), where \(Y_{AG}\)is the ratio between the AG current and amplitude. In the particular case of the circuit in Fig. 4, the AG is connected in parallel at the source terminal. Once the free-running oscillation of Section 1 (2a) is obtained, the stability analysis of this solution is carried out through standard pole–zero identification [34]–[38], introducing a small-signal current generator in Section 2 (1), which is the one prone to oscillate at the frequency \(\omega_2\) (\(\omega_1\)). Fig. 8(a) shows the variation versus the gate-bias voltage \(V_{GS2}\) of the real part of the dominant poles (calculated through identification in Section 2) when Section 2 exhibits a steady-state periodic oscillation at \(f_1 \cong 2.4\) GHz. In turn, Fig. 8(b) shows the variation versus \(V_{GS1}\) of the real part of the dominant poles (calculated through identification in Section 2) when Section 1 exhibits a steady-state periodic oscillation at \(f_2 \cong 4.1\) GHz. The oscillation in each section is never stable if the other section has surpassed the oscillation threshold determined with the dc stability analysis described in Section III-B. Compare the crossing of the poles in Fig. 8(b), with Section 1 in an oscillatory state, with the crossing of the zeroes in Fig. 7(d), with Section 1 in a nonoscillatory state. The crossing to the RHS takes place for approximately the same value of \(V_{GS1}\). In fact, the Hopf bifurcation from
dc regime [21]–[25] takes place at $V_{GS1} = -0.791$ V and the (secondary) Hopf bifurcation from a periodic regime takes place at $V_{GS1} = -0.794$ V. In each case and due to the high isolation between Section 1 and Section 2, standard pole-zero identification is unable to detect the pair of poles in the imaginary axis, associated with the steady-state oscillation in the other section.

### D. Stability of the Quasi-Periodic Oscillation

To obtain the concurrent doubly autonomous quasi-periodic oscillation, two AGs are introduced into the circuit, one per suboscillator. They are connected at the source terminals of the two transistor devices and operate at the frequencies $\omega_{AG1} = \omega_o1$ and $\omega_{AG2} = \omega_o2$, with the respective amplitudes $V_{AG1}$ and $V_{AG2}$. In the presence of these two AGs, the circuit is analyzed with two-tone HB, at the two incommensurable oscillation frequencies $\omega_{AG1} = \omega_o1$ and $\omega_{AG2} = \omega_o2$. The two AG frequencies and amplitudes $\omega_{AG1}$, $\omega_{AG2}$, $V_{AG1}$, and $V_{AG2}$ are simultaneously optimized in order to fulfill $Y_{AG1} = 0$ and $Y_{AG2} = 0$, where $Y_{AGi}$ ($i = 1, 2$) is the current-to-voltage ratio at each of the two generators. The oscillation is ideally suppressed at the circuit output, containing an open-circuited $\lambda/4$ stub at the oscillation frequency. Thus, the spectrum in each suboscillator has been simulated and measured at the input terminal (Fig. 9). The two oscillations are concurrent, very robust, and exhibit a good isolation.

The stability of the doubly autonomous quasi-periodic solution is analyzed linearizing the two outer-tier admittance functions $Y_{AGi}$ and $Y_{AG2}$ about this quasi-periodic steady-state solution, as done in [8]. This provides four Lyapunov exponents [21], [22]. Two of them are zero due to the double oscillation at the upper band. Fig. 10(b) shows the variation of the doubly autonomous quasi-periodic solution, in terms of the oscillation amplitude at the source terminal in Section 1 and Section 2, versus $V_{GS1}$ for $V_{GS2} = -0.55$ V. The oscillation at the lower band has very low impact on the oscillation at the upper band. Fig. 10(b) shows the variation of $a_{12}$ and $a_{21}$ through the solution curve in Fig. 10(a). The larger values are comprised in the interval $V_{GS1} = -0.75$ V to $V_{GS1} = -0.8$ V, which agrees with the one in which the oscillation in the lower band has the strongest effect on the

![Graph](Image1)

![Graph](Image2)

Fig. 8. Stability analysis of the periodic oscillations at $\omega_1$ and $\omega_2$. The Hopf bifurcations $H_1$ and $H_2$ take place at $V_{GS1} = -0.791$ V and $V_{GS2} = -0.794$ V, respectively. (a) Pole-zero identification in Section 2. (b) Pole-zero identification in Section 1.

![Graph](Image3)

![Graph](Image4)

Fig. 9. Doubly autonomous quasi-periodic solution. Simulated and measured spectrum at each of the two suboscillators, biased at $V_{GS1} = -0.35$ V and $V_{GS2} = -0.55$ V, respectively. (a) Lower-frequency oscillator. (b) Upper-frequency oscillator.

where

$$a_{11} = \frac{\partial Y_{T1}/\partial Y_{o1}}{\partial Y_{T1}/\partial Y_{o1}}$$

$$a_{12} = \frac{\partial Y_{T1}/\partial Y_{o2}}{\partial Y_{T1}/\partial Y_{o2}}$$

$$a_{21} = \frac{\partial Y_{T2}/\partial Y_{o1}}{\partial Y_{T2}/\partial Y_{o1}}$$

$$a_{22} = \frac{\partial Y_{T2}/\partial Y_{o2}}{\partial Y_{T2}/\partial Y_{o2}}$$

All the derivatives of the admittance functions are calculated applying finite differences to the two AGs used to obtain the doubly autonomous quasi-periodic solution, as explained in [8]. The two circuit sections in Fig. 4(a) would be fully isolated for $a_{12} = 0$ and $a_{21} = 0$ and the two Lyapunov exponents would become $\lambda_1 = a_{11}$ and $\lambda_2 = a_{22}$. Thus, the coefficients $a_{12}$ and $a_{21}$ give a measure of the connectivity between the two circuit sections. Fig. 10(a) presents the variation of the doubly autonomous quasi-periodic solution, in terms of the oscillation amplitude at the source terminal in Section 1 and Section 2, versus $V_{GS1}$ for $V_{GS2} = -0.55$ V. The oscillation at the lower band has very low impact on the oscillation at the upper band. Fig. 10(b) shows the variation of $a_{12}$ and $a_{21}$ through the solution curve in Fig. 10(a). The larger values are comprised in the interval $V_{GS1} = -0.75$ V to $V_{GS1} = -0.8$ V, which agrees with the one in which the oscillation in the lower band has the strongest effect on the
upper-band oscillation. Fig. 10(c) shows the variation of the two exponents $\lambda_1$ and $\lambda_2$ versus $V_{\text{GS}1}$. (d) Experimental validation of the stable behavior versus $V_{\text{GS}1}$.

IV. HETERODYNE SOM

As shown in Section II, a heterodyne SOM mixes the input signal at the frequency $\omega_m$ with a self-generated incommensurable oscillation at the frequency $\omega_o$ to provide an intermediate signal at the frequency $\omega_m$. In the concurrent dual-band SOM, the mixing takes place simultaneously in the two suboscillators, so, in principle, the analysis of this regime requires two AGs at the respective frequencies $\omega_{\text{AG}1} = \omega_1$ and $\omega_{\text{AG}2} = \omega_2$. However, due to the good isolation, demonstrated in Section III, it will be possible to optimize each suboscillator independently.

A. Gain Optimization

To optimize the gain of the heterodyne SOM, linear operation with respect to the input source will be initially considered, so the circuit can be linearized about the free-running solution obtained with the HB system [24], [25]

$$
\bar{H}_k(\bar{V}, \bar{\mu}) = [A_{\text{V}}(k\omega_o)]\bar{V}_k + [A_{\text{GN}}(k\omega_o)]\bar{F}_k(\bar{V}) \quad (12a)
$$

$$
\bar{Y}_{\text{AG}}(V_{\text{AG}}, \omega_o, \bar{\mu}) = \frac{I_{\text{AG}}}{V_{\text{AG}}} = 0 \quad (12b)
$$

where $\bar{H}_k(\bar{V}, \bar{\mu})$ with $-NH \leq k \leq NH$ is the set of error equations, depending on the vector $\bar{\mu}$ of analysis parameters (bias voltages and particular element values, for instance) and the whole set of harmonic components $\bar{V}$ of the control voltages of the nonlinear elements $\bar{F}(\bar{V})$, and $\bar{G}$ is the vector of independent sources. The matrices $[A_{\text{V}}(k\omega_o)][A_{\text{GN}}(k\omega_o)]$ and $[A_{\text{GN}}(k\omega_o)]$ account for the passive linear elements and, respectively, relate the control voltages $\bar{V}$ with $\bar{F}(\bar{V})$ and $\bar{G}$; their role is identical to that of $A_1$ and $A_2$ in (2a). Because the aim is to obtain the free-running solution, the vector $\bar{G}$ of independent generators does not include the RF input source. It contains the dc sources and an AG, with amplitude $V_{\text{AG}}$, at $\omega_o$. By fulfilling (12b), this AG prevents the undesired convergence to the coexistent dc solution. Now we consider a small RF input signal at $\omega_o+\Omega$, where $\Omega$ is IF. In the presence of this small input signal, the HB system can be linearized about the free-running solution obtained through (12a) with the conversion-matrix approach [41], which provides

$$
[A_{\text{V}}(k\omega_o + \Omega)]\Delta \bar{V}_k(k\omega_o + \Omega) \sum_{l=-NH}^{l=NH} [A_{\text{GN}}(k\omega_o + \Omega)] \frac{\partial \bar{F}_k}{\partial \bar{V}_l} \Delta \bar{V}_l + [A_{\text{GN}}(k\omega_o + \Omega)] \bar{G}_{\text{in}}(k\omega_o + \Omega) = 0 \quad (13)
$$

where $-NH \leq k \leq NH$ and the small RF signal is included in the vector $\bar{G}_{\text{in}}$ that fulfills $\bar{G}_{\text{in}}(k\omega_o + \Omega) = 0$ for $k \neq 1, -1$. Under any variation of $\bar{\mu}$, one solves sequentially (12a) and (13) to obtain the conversion gain $G_{\text{IF}}$ as the ratio between the output power at $\Omega$ and the available power at $\omega_m + \Omega$.

Equations (12a) and (13) have been used to select the values of the IF filter elements, $L_i$, $C_i$, where $i = 1, 2$, that maximize $G_{\text{IF}}$. Thus, the vector $\bar{\mu}$ contains, in each suboscillator, the elements $L_i$, $C_i$. The analysis has been carried out keeping the gate-bias voltages at their original values $V_{\text{GS}1} = 0.35$ V and $V_{\text{GS}2} = 0.55$ V since the impact of the bias conditions on the conversion gain and linearity is studied later in this section. For the constant offset frequency $f_{\text{IF}} = \Omega/(2\pi) = 100$ MHz, agreeing with the desired IF, one obtains the constant-gain contours shown Fig. 11. The contours in Fig. 11(a) [Fig. 11(b)] correspond to the suboscillator at 2.4 GHz (4.1 GHz). As gathered from these contours, $G_{\text{IF}}$ strongly depends on the values of the IF output network, which affect both the filtering and impedance matching at the IF.
The operation points selected are indicated in Fig. 11. Note that the suboscillator at 2.4 GHz is able to exhibit higher gain at lower IF. At 2.4 GHz, the measured conversion gain for the values indicated in Fig. 7 are −2.7 dB at 100 MHz and 7 dB at 70 MHz. At the upper frequency, the measured conversion gain is 3.3 dB at 100 MHz.

B. Nonlinear Response Versus the Input Power

After the optimization of the linear conversion gain of the heterodyne SOM, this circuit will be analyzed in nonlinear regime with respect to the input source. This is done introducing the RF input source in the vector \( \vec{G} \) of (12a) and considering two fundamental frequencies: \( \omega_{in} \) and \( \omega_{AG} \). The AG is maintained for this analysis and must fulfill \( Y_{AG} = 0 \) in the presence of the input source, which avoids the undesired convergence to a trivial periodic solution at \( \omega_{in} \). The solution curve versus a given parameter \( \eta \) can be traced through AG optimization in two-tone HB, at \( \omega_{in} \) and \( \omega_{AG} = \omega_{in} \), which would be done sweeping \( \eta \) and \( V_{AG} \) at each step [42]. However, due to the usual complexity of the solution curves, exhibiting turning points [23] that demand several manual parameter switches [18], [26], an extension of the contour-intersection method [19], [20] to the SOM regime has been carried out. This method enables an exhaustive search of coexisting solutions, with no need for continuation procedures. It also relies on the use of an AG, though this AG is not optimized, but introduced to obtain a nonlinear admittance function, as described in the following.

For each \( \eta \), a double sweep is performed in \( V_{AG} \) and \( \omega_{AG} \) in two-tone HB, at \( \omega_{in} \) and \( \omega_{AG} \). For each pair of values \( V_{AG} \) and \( \omega_{AG} \) one calculates \( Y_{AG} \), as the ratio between the AG current and voltage, as well as the conversion gain \( G_{IF} \). Then, three surfaces are obtained: \( S_{Re} \) in the space \( V_{AG}, \omega_{AG}, \text{Re}(Y_{AG}) \), \( S_{Im} \) in the space \( V_{AG}, \omega_{AG}, \text{Im}(Y_{AG}) \) and \( S_{GIF} \), in the space \( V_{AG}, \omega_{AG}, G_{IF} \). The surfaces \( S_{Re} \) and \( S_{Im} \) are used to obtain the solution points, whereas the surface \( S_{GIF} \) is used to obtain the conversion gain at these solution points. The intersection of the surface \( S_{Re} \) with the plane \( \text{Re}(Y_{AG}) = 0 \) provides the curve \( C_{Re} \). Likewise, the intersection of the surface \( S_{Im} \) with the plane \( \text{Im}(Y_{AG}) = 0 \) provides the curve \( C_{Im} \). Fig. 11(a) presents several of these curves, obtained when \( \eta = P_{in} \) in the suboscillator at the upper frequency, for the selected IF 100 MHz. The curves \( C_{Re} \) and \( C_{Im} \) can be expressed as

\[
C_{Re}(V_{AG}, \omega_{AG}) = S_{Re}(V_{AG}, \omega_{AG}) \cap \{\text{Re}(Y_{AG}) = 0\}
\]

\[
C_{Im}(V_{AG}, \omega_{AG}) = S_{Im}(V_{AG}, \omega_{AG}) \cap \{\text{Im}(Y_{AG}) = 0\}. \tag{14}
\]

Then, all the coexisting solution points for the particular parameter value \( \eta \) are given by the intersection points of \( C_{Re} \) and \( C_{Im} \), that is,

\[
P = C_{Re} \cap C_{Im} \tag{15}
\]

where \( P \) is a discrete set containing the solution points. Fig. 12(a) presents the collection of intersection points obtained when varying the input power, \( \eta = P_{in} \), in the suboscillator at the upper frequency. Representing the points \( P \) in (15) versus \( P_{in} \) one obtains the solution curve [Fig. 12(a)] in terms of \( V_{AG} \) (agreeing with the voltage amplitude at the node where the AG is connected) and \( \omega_{AG} \) (agreeing with the oscillation frequency). For low \( P_{in} \), there is a negligible variation of \( \omega_{AG} \) and \( V_{AG} \), and the solutions accumulate at the point surrounded with a circle.
To obtain the variation of the conversion gain $G_{IF}$ versus $\eta$, one should interpolate the surface $\gamma_{GIF}(\eta)$ at the solution points $P(\eta)$. In our case, this interpolation must be carried out at the points resulting from the intersections in Fig. 12(a). For $f_m = 100$ MHz, the curve providing $G_{IF}$ versus $P_m$ is shown in Fig. 12(b). Experimental measurements are also represented with good agreement. Fig. 12(c) presents the variation of $G_{IF}$ in the lower-frequency suboscillator. Two IFs are considered: 70 and 100 MHz (compared with the simulation results).

As seen in Fig. 12(b) and (c), the gain remains constant at the small-signal value up to certain $P_m$, then there is a small interval of gain expansion and an abrupt decay as the oscillation amplitude tends to zero at the Hopf bifurcation $H$. This form of gain variation is fully explained by the analytical model in (2a)–(6) of Section II. As also understood from this model, the device bias point should be suitably chosen since, besides a sufficient quadratic response, one must ensure enough oscillation amplitude. We have analyzed the conversion gain versus $V_{GS}$ for $P_m = -36$ dBm and $P_m = -16$ dBm with the results shown in Fig. 13(a), where experimental points are superimposed. When reducing $V_{GS}$ toward pinch-off, there is an interval of irregular behavior until the oscillation is eventually extinguished at the Hopf bifurcation $H$. Thus, one should avoid too low $V_{GS}$. Fig. 13(b) shows a more detailed analysis of the conversion gain in a narrower $V_{GS}$ interval, considering multiple $P_m$ values comprised between $P_m = -36$ dBm and $P_m = -16$ dBm. Higher linearity is expected for the $V_{GS}$ at which all the gain points are (nearly) overlapped. For comparison, the oscillation amplitude $V_o$ is also represented in Fig. 13(b).

The previous analyses have been carried out considering only the oscillation occurring in each particular circuit section:

\begin{align*}
[A_y(k_{ab} + l_{o_m})] \dot{V}_{k,l}(t) & - \int \frac{\partial [A_y(k_{ab} + l_{o_m})]}{\partial \omega} \dot{V}_{k,l}(t) \\
+ [A_{YN}(k_{ab} + l_{o_m}) \dot{F}_{k,l}(\ddot{V}(t)) & + [A_{YN}(k_{ab} + l_{o_m}) \dot{G}_{k,l}(t) \\
- \int \frac{\partial [A_{YN}(k_{ab} + l_{o_m})]}{\partial \omega} \dot{G}_{k,l}(t) & = 0
\end{align*}

where $-NH \leq k \leq NH$. As in (8), it would be possible to consider higher order terms in the Taylor expansion.

C. Intermodulation

The intermodulation distortion of the SOM will be analyzed considering two closely spaced input tones about $\omega_n$. The analysis is carried out with the envelope-transient method [18], representing the two input tones as a modulation of the carrier $\omega_n$. Two fundamental frequencies are considered in the Fourier series representation of the circuit variables: $\omega_m$ and $\omega_0$, where $\omega_0$ is the self-oscillation frequency in the presence of the nonmodulated input source, obtained through HB with the aid of an AG (optimized to fulfill $V_{AG} = 0$). When the modulation due to the two input tones is introduced, the control voltages become time varying $\ddot{V}(t)$ and, provided that the frequency spacing is sufficiently small in comparison with the carrier frequencies, one can expand the matrices $[A_y]$ and $[A_z]$ in a Taylor series about $(k_{ab} + l_{o_m})$, in a manner similar to what was done in (8). Taking into account that the frequency increments act like time differentiators, one obtains

Fig. 14. Concurrent operation of the heterodyne SOM under an identical IF of 100 MHz in the two suboscillators. Evolution of the IF output in these two suboscillators when increasing the input power at the upper band from $-36$ to $-16$ dBm, while holding the input power at the lower band constant at $P_m = -36$ dBm.
The oscillation is initialized connecting the AG to the circuit at the initial time $t_0$ only. This is done introducing a resistor $R_{AG}(t)$ in series with the AG, as shown in Fig. 4(a). This resistor has zero value at the initial time $t = t_0$ and very large value, ideally infinite, at $t > t_0$, which disconnects the AG from the circuit. For the analysis of the intermodulation spectrum, the time interval in steady-state regime must be long enough to ensure a sufficient accuracy in the Fourier transform. Fig. 15(a) compares the simulated and measured output spectrum in the upper band when the power of the input signal is $30$ dBm, the gain is $7$ dB and the phase noise $=-22$ A/Hz, for the first and second suboscillator, respectively. In the lower band $P_o$ = $-30$ dBm, the gain is $7$ dB and the phase noise is $-23$ A/Hz, for the first and second suboscillator, respectively.

**D. Noise Analysis**

The SOMs are generally intended for compact low-cost applications in which the oscillation is not locked to a reference, so one should minimize the phase noise of this oscillation. The IF of the heterodyne SOM is $\omega_o - \omega_n$, so the phase noise at the oscillation frequency $\omega_o$ will have a direct impact on the IF signal. The noise analysis of the SOM will be carried out with an outer-tier procedure, connecting two AGs at the transistor gate terminal, each one of the two fundamental frequencies $\omega_n$ and $\omega_o$. The AG at $\omega_n$ has the amplitude $V_n$ and phase shift with respect to the input source $\phi_n$. The AG at $\omega_o$ has the amplitude $V_o$. For a given input amplitude $E_{in}$ and frequency $\omega_{in}$, the two AGs are simultaneously optimized to fulfill

$$Y_o(V_o, \omega_o, V_i, \phi) = 0$$

$$Y_i(V_o, \omega_o, V_i, \phi) = 0$$

where the dependence on $E_{in}$ is implicit. In fact, the above two equations play a role similar to those in (2a), though in (17) the rest of circuit variables are considered in the HBM system that constitutes an inner tier. Now the effect of the noise sources will be considered. The phase perturbation of the input source at $\omega_{in}$ is $\psi(t)$ and the equivalent noise sources at $\omega_{in}$ and $\omega_o$ are $I_{Ni}$ (including the input amplitude noise) and $I_{No}$. Because the noise perturbations are small, one can expand the nonlinear equations play a role similar to those in (2a), though in (17) the finite differences [43] to the two AGs. After suppressing the steady-state terms, one obtains a perturbation system in the four increments $\delta V_i, \delta \phi_i, \delta V_o, \delta \phi_o$ of the respective variables $V_i, \phi_i, V_o, \phi_o$

$$\frac{\partial Y_i}{\partial V_i} \delta V_i + \frac{\partial Y_i}{\partial \phi_i} \delta \phi_i + \frac{\partial Y_i}{\partial \omega_o} \left(\delta \phi_i - j \frac{\dot{\psi}}{V_o} + j \frac{\dot{\phi}}{V_o} \right) + \frac{\partial Y_i}{\partial \omega_o} \delta \phi_o = \frac{I_{Ni}(t)}{V_o}$$

$$\frac{\partial Y_o}{\partial V_i} \delta V_i + \frac{\partial Y_o}{\partial \phi_o} \delta \phi_o + \frac{\partial Y_o}{\partial \omega_o} \left(\delta \phi_o - j \frac{\dot{\psi}}{V_o} + j \frac{\dot{\phi}}{V_o} \right) + \frac{\partial Y_o}{\partial \omega_o} \delta \phi_o = \frac{I_{No}(t)}{V_o}$$

where it has been considered that the complex-frequency increments give rise to a time differentiation of the amplitude and phase increments [44]. In the usual case of a low-noise input source, one can simplify (17) considering that $\psi(t)$ will be much smaller than the oscillation phase noise and may also disregard the derivatives $\dot{\psi}/V_o$. Splitting the two complex equations into real and imaginary parts and applying the Fourier transform, one obtains

$$S(\Omega) = M^{-1}(\Omega) N(\Omega) (M^{-1}(\Omega))^T$$

where $S(\Omega)$ is the correlation matrix of the noise perturbations, $N(\Omega)$ is that of the noise sources and the matrix $M(\Omega)$ is given by

$$M(\Omega) = \begin{bmatrix}
\frac{\partial Y_{i}}{\partial V_{i}} & \frac{\partial Y_{i}}{\partial \phi_{i}} & \frac{\partial Y_{i}}{\partial \omega_{o}} \\
\frac{\partial Y_{o}}{\partial V_{i}} & \frac{\partial Y_{o}}{\partial \phi_{i}} & \frac{\partial Y_{o}}{\partial \omega_{o}} \\
\frac{\partial Y_{i}}{\partial V_{o}} & \frac{\partial Y_{i}}{\partial \phi_{o}} & \frac{\partial Y_{i}}{\partial \omega_{o}} \\
\frac{\partial Y_{o}}{\partial V_{o}} & \frac{\partial Y_{o}}{\partial \phi_{o}} & \frac{\partial Y_{o}}{\partial \omega_{o}}
\end{bmatrix}$$

where the superscripts indicate real and imaginary parts. The matrix $N(\Omega)$ contains the spectral densities of $I_{Ni}$ and $I_{No}$. The spectral density of $I_{Ni}$ is approached as $4G_kT_o$, where $k$ is the Boltzmann constant, $G_o$ the real part of the source admittance seen from the gate terminal, and $T_o = 290^\circ$ K. The values are $|I_{Ni}|^2 = 8 \times 10^{-22}$ A$^2$/Hz, $|I_{No}|^2 = 1.1 \times 10^{-22}$ A$^2$/Hz, for the first and second suboscillator, respectively. On the other hand, the spectral density of $I_{No}$ is obtained by fitting [43] the phase-noise spectrum obtained through circuit-level simulations [45] with the one resulting from the noise.
p perturbation of \( Y_d(V_o, \omega_o) = 0 \) (in the absence of the input source). The spectral density of \( I_{N_o} \) contains both white noise and upconverted flicker-noise contributions. In the first suboscillator, it is \( |I_{N_{o1}}|^2 = (0.27 \times 10^{-20} + 3 \times 10^{-13}/f) \) A^2/Hz. In the second suboscillator it is \( |I_{N_{o2}}|^2 = (0.3 \times 10^{-20} + 5 \times 10^{-13}/f) \) A^2/Hz.

Solving (19), one obtains the phase and amplitude noise at the two fundamental frequencies \( \omega_o \) and \( \omega_{in} \) of the quasiperiodic solution. The analysis has been carried out at both the lower band [Fig. 16(a)] and upper band [Fig. 16(b)]. The phase noise \( |\delta \phi_o|^2 \) at the oscillation frequency has been represented with black dots. Of all the rest of noise contributions resulting from (18a) to (20), the dominant one is the amplitude noise associated with \( \delta V_i \), which has also been traced with black dots. This dominance of the noise associated with \( \delta V_i \) can be understood with the formulation (19) and (20). When solving for \( \delta V_i \) using the matrix in (20), the derivatives of the admittance functions with respect to \( V_o \) appear in the denominator only, unlike what happens with the rest of perturbed variables. Under rather small input amplitude \( E_{in} \), these derivatives will be much smaller than the ones with respect to \( V_o \) and \( \phi_o \), which explains the relatively large value of the amplitude noise at the input frequency. The results have been compared with the experimental measurements using the Rohde & Schwarz FSWP8-phase noise analyzer. The black and blue traces correspond to the experimental measurements of the phase-noise spectrum at the oscillation frequency \( \omega_o \) and the IF \( \omega_{in} - \omega_o \), respectively. At the lower offset frequencies, the two traces are nearly coincident. The phase noise predicted through (19) at \( \omega_o \) (represented with black dots) exhibits a very good agreement. Thus, at low offset frequencies the phase noise at IF basically agrees with that of the self-oscillation. At larger offset frequency, the two phase-noise spectra exhibit a different form of variation. The phase-noise at the oscillation frequency keeps decreasing with the same \(-20\)-dB/dec slope. In contrast, the phase noise at IF approaches the amplitude noise associated with \( \delta V_i \), which, in turn, is very similar to the amplitude noise measured at IF. The relevance of this amplitude noise at the IF is attributed to the down-conversion action. The corner from which the phase-noise spectrum flattens depends on the intensity of this amplitude noise. For completeness, the spectrum at both the lower and upper bands in injection-locked conditions has also been represented.

V. Zero-IF SOM

For operation as a zero-IF SOM, the circuit self-oscillation must be injection-locked by an input signal, as shown in Section II-B. The investigation here will focus on three aspects of the transistor-based implementation of Fig. 4: the operation bands as a zero-IF mixer, the linear gain analysis, and the behavior under modulation signals [1].

A. Operation Bands

Initially, the impact of one oscillator on the other will be neglected. The analysis of each injection-locked suboscillator will be carried out with one-tone HB at the corresponding input frequency \( \omega_{in} \). The solution curves, which should be analogous to those in Fig. 3 of Section II, are obtained through the contour-intersection method in [19], based on a two-stage procedure. The first step is a separate simulation of the passive-linear input network defined between the input source and the analysis node (gate terminal), to obtain its Norton equivalent current \( I_{N}(\omega) \) at each input frequency. This analysis should provide the scattering matrix [S] of the input network. The second step is the extraction of a nonlinear admittance function \( Y_{AG}(V_{AG}, \omega_{AG}) \) at the analysis terminal using an AG connected to this terminal. The purpose of this function is conceptually similar to that of the function \( Y_{AG} \) in Section IV-B. The function is calculated by performing a double sweep in the AG frequency \( \omega_{AG} \) and amplitude \( V_{AG} \) and calculating, at each step, the ratio between the AG current \( I_{AG} \) and \( V_{AG} \). Once the scattering matrix and the nonlinear admittance function have been calculated, the following relationships are used:

\[
Y_{AG}(V_{AG}, \omega_{AG})V = I_{N}(\omega_{AG})
\]

\[
I_{N}(\omega_{AG}) = \frac{2S_{21}E_{in}}{1 - S_{11} - (\Delta - S_{22})} R_o \tag{21}
\]

where \( S_{ij} \) are the scattering parameters of the input network \( \Delta \) is the determinant of the scattering matrix and \( R_o \) is the input generator resistance. The two equations in (21) can be combined in a single one. Then the family of solution curves versus the input frequency is obtained defining several contour levels in terms of \( E_{in} \) [19] in the plane \( \omega_{AG}, V_{AG} \).

At the lower and upper bands, one obtains the curves in Fig. 17, traced in terms of the oscillation amplitude \( V \) at...
Fig. 17. Zero-IF SOM. Family of solution curves versus $\omega$ in for different $E_{in}$ values. The injection-locked curve at $E_{in} = 0.1$ V obtained through contour intersections with one AG is compared with the one obtained through a concurrent-oscillation analysis, with two AGs. (a) Lower band. (b) Upper band.

As stated, the turning points in the curves obtained for the lower $E_{in}$ values correspond to an unlocking phenomenon that leads to operation as a heterodyne SOM. Thus, the representation of these points in the plane defined by the input frequency and input amplitude will provide the boundaries between the regions in which the circuit operates as a heterodyne mixer and as zero-IF SOM. In Fig. 18(a), this representation has been carried out at the lower operation band, comparing the results with the experimental measurements. The turning-point locus is traced with blue circles. As indicated in Section II-B, for the higher input amplitudes, the onset of quasi-periodic or mixer-like solutions is due to Hopf bifurcations, at which the oscillation arises from zero amplitude, as seen in Fig. 12. The Hopf-bifurcation locus has been represented with dotted lines in Fig. 18(a). This locus can be obtained by introducing an AG into the circuit at a small-signal amplitude $A_{AG} = \varepsilon$ and frequency $\omega_{AG} = \omega$, incommensurate with the input frequency $\omega_{in}$. Then, $\omega_{in}$ is swept, optimizing the input amplitude $E_{in}$ and the oscillation frequency $\omega$ at each step in order to fulfill $Y_{AG}(\omega, E_{in}) = 0$, as described in [23]. The circuit operates in a periodic regime inside the turning point locus and above the Hopf locus. Note that the upper section of the turning-point locus has no physical effect. Optimum operation as a zero-IF SOM would be obtained for the lower input voltages, where maximum advantage is taken of the oscillation amplitude. For illustration, Fig. 18(b) presents an experimental trace of the locking band obtained for $P_{in} = -22$ dBm.

**B. Conversion Gain**

One can evaluate the conversion gain of the zero-IF SOM by applying the conversion matrix approach [25], [41] along the closed-synchronization curves. This is done linearizing the HB system about the injection-locked solution obtained for each input frequency $\omega_{in}$, in the presence of an additional input tone at a small offset frequency $\Omega$ with respect to $\omega_{in}$. This way,
we will be able to estimate the gain when the input signal is composed of a locking tone at $\omega_{\text{in}}$ (with respect to which the circuit may behave nonlinearly) and an additional small signal with a certain low-power spectrum versus $\Omega$. For this analysis, the value $\Omega/(2\pi) = 1$ MHz will be considered. The gain is defined as the ratio between the output power at $\omega_{\text{in}} + \Omega$ and the available power at $\omega_{\text{in}}$. The formulation is analogous to the one in (12a), though now the HB system is linearized about the periodic solution injection locked to the input tone, instead of the free-running oscillation.

The output signal can be extracted with the aid of a buffer connected to the resistors $R_i$, where $i = 1, 2$, in Fig. 4(a). The extraction through the IF filter will be possible provided that the modulation signal has a negligible spectral component near dc. This should enable a simplification of the circuit, avoiding additional active devices. Fig. 19(a) compares the simulated and measured gain through the closed synchronization curve obtained for $P_{\text{in}} = -16$ dBm when extracting the signal through the IF filter. As gathered from Fig. 19(a), there is a significant dependence of the gain value on the frequency of the locking tone, and it is possible to obtain a gain higher than 5 dB. This value is smaller at the upper band [Fig. 17(b)]. Both with the IF filter and the buffer, the gain increases when approaching the boundaries of the injection-locked band and would tend to infinity for $\Omega \rightarrow 0$. Despite the much larger gain values, operation near the band boundaries is not advised due to the small stability margins.

**C. Behavior Under Modulated Input Signals**

To optimize the behavior of a zero-IF SOM with an amplitude-modulated input signal at the carrier frequency $\omega_{\text{in}}$ it is advisable to trace the synchronized-solution curve versus $E_{\text{in}}$. As shown in Section II-B, this solution curve will be multivalued, so here it is obtained with the same two-stage procedure in (21). The results are shown in Fig. 20 and they present the variation of the oscillation amplitude at the gate node $V_{\text{gate}}$ versus $E_{\text{in}}$ at the lower and upper operation bands. In each case, three curves have been calculated, at distinct values of input frequency $\omega_{\text{in}}$. Note that the “S” shape of the curves was explained in the analytical study of Section II-B. In Fig. 20, for $E_{\text{in}}$ values lower than the one corresponding to the turning point $T_2$, the oscillation is unlocked and mixes with the input signal at $\omega_{\text{in}}$, so the circuit behaves in a quasi-periodic regime. The point $T_2$ is a local-global bifurcation at which the oscillation gets locked when increasing $E_{\text{in}}$ [22]. In agreement with the discussions of Section II, the locking takes place for lower $E_{\text{in}}$ when reducing the detuning with respect to the free-running frequency (compare Figs. 17 and 20).

Now an AM modulation with a rectangular signal will be considered, as done in [1]. The circuit is analyzed with envelope transient, considering a single fundamental frequency, given by that of the input source $\omega_{\text{in}}$. In a manner similar to what was done in Section IV-C, an AG is used to initialize the oscillation. The AG frequency $\omega_{\text{AG}}$, amplitude $V_{\text{AG}}$, and phase shift $\phi$ with respect to the input source are the ones resulting from the fulfillment of $Y_{\text{AG}} = 0$ in the absence of modulations. As in Section IV-C, the AG is connected to the circuit at the initial time only. If the operation conditions are such that the modulated input amplitude goes below the turning point $T_2$ in Fig. 20, the circuit undergoes an instantaneous unlocking, which may give rise to strong distortion effects. This can be seen in the envelope transient analyses of Fig. 21, where an amplitude modulation with a rectangular waveform of 2 MHz, centered about $E_{\text{in}} = 0.07$ V with two different amplitudes, has been considered.
Fig. 21. Instantaneous unlocking under an amplitude modulation. (a) Input-modulation waveforms superimposed on the static curve providing the locked-oscillation amplitude versus the input amplitude ($E_{in}$–$V_{gate}$). (b) Input and output waveforms for $\Delta V = 0.025$ V. Correct behavior. (c) Input and output waveforms for $\Delta V = 0.04$ V. Instantaneous unlocking. (d) Measured waveform under correct operation. (e) Measured waveform exhibiting instantaneous unlocking.

Fig. 22. Operation of the zero-IF SOM as an FSK and FM demodulator. (a) Simulation results as an FSK demodulator. (b) Experimental validation of the FM demodulation capability for an FM with a square wave of modulation rate of 5 kHz and FM deviation of 5 MHz.

To facilitate the comparison of the amplitude excursion with the static curve $E_{in}$–$V_{gate}$ in Fig. 20 (providing the oscillation amplitude versus the input amplitude), the two rectangular waveforms are superimposed on this curve in Fig. 21(a). If the lower limit of the rectangular signal is above the turning point $T_2$ of the static curve for the whole amplitude excursion, there will be no instantaneous unlocking, as in the case of Fig. 21(b). If the lower level is below $T_2$, an oscillation is observed at the beat frequency, or difference between the input and oscillation frequencies. This is the case of Fig. 21(c). More cycles of the beat frequency will be observed for a lower modulation frequency. The experimental waveforms for the same conditions of Fig. 21(b) and (c) are shown in Fig. 21(d) and (e). Because the input spectrum exhibits negligible content near dc, these signals could be extracted through the IF filter, which has the benefit of suppressing the dc levels. As gathered from Fig. 20, to avoid the instantaneous unlocking, one should increase the carrier power or choose a carrier frequency closer to the free-running one. Note that under an input modulation there will also be dynamic effects due to the nonnegligible values of the time derivatives of the state variables, as discussed in Section II. These effects will be more relevant near the turning point $T_2$ and can lead to unlocking before reaching this point.

As demonstrated in Section II-B, the zero-IF SOM can also demodulate an FSK signal. The circuit in Fig. 4(a) has been tested for this function, and the simulated results when considering an FSK of 1 Mb/s at $f_{in} = 4.1$ GHz and $E_{in} = 0.1$ V are shown in Fig. 22(a). Due to laboratory equipment limitations, measurements [Fig. 22(b)] have been performed for an FM with a square wave of modulation rate of 5 kHz and FM deviation of 5 MHz.

VI. CONCLUSION

A concurrent dual SOM with double functionality has been presented and investigated in detail. It is based on a concurrent dual-frequency oscillator implemented with a ring-shaped stepped-impedance resonator that exhibits two independent resonances at the frequencies 2.4 and 4.1 GHz with an excellent isolation. The circuit can behave as a concurrent heterodyne mixer and as a zero-IF mixer. The circuit operation has been exhaustively investigated in both linear and nonlinear conditions with respect to the input source. In heterodyne mode, the variation of the conversion gain versus the input power has been analyzed through a contour-intersection method that, in addition to a nonlinear admittance function at the oscillation frequency, requires the extraction and interpolation of a gain function. The input frequency and input power values for operation as a zero-IF SOM have been obtained through a bifurcation detection technique, able to predict the injection-locking bands. The small-signal gain in the presence of the locking tone has been analyzed applying the conversion-matrix approach along the closed injection-locked solution curve. The instantaneous unlocking under a modulated input signal has been investigated and a simple criterion to avoid this undesired phenomenon has been provided.

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