Effective description of THz localized waveguide resonance through metal film with split ring resonator holes: zero refractive index

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Abstract: Using a periodic array of split ring resonator holes within a terahertz range, we numerically and experimentally confirmed a zero refractive index at localized waveguide resonant frequency of aluminum film. The effective index was directly calculated from the phase difference of electromagnetic waves passing through film and air. Thickness-independent resonant frequency, as well as spatially static hole resonant modes, clearly verified a zero refractive index. For experimentation, we fabricated samples by means of a femtosecond laser machining system and employed a terahertz time domain spectroscopy system to measure transmitted terahertz pulses. Further, the effective index of refraction extracted from phases and amplitude of measured transmitted pulses confirmed a zero refraction index at resonant frequency.

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1. Introduction

Structured perfect metal film has attracted much attention because of special electromagnetic properties not found in perfect metal film. For example, perfect metal film with periodic hole arrays can support surface bound waves, i.e., designer surface plasmons similar to metal film surface plasmons [1]. Moreover, metal film with periodic cut-through slit arrangements can act as dielectric layers, possessing high refractive indices [2].

One of the benefits of perfect metal film having hole arrays has been enhanced transmission. Since discovery of enhanced transmission via real metal film with periodic subwavelength hole arrays [3], researchers have intensively investigated physical origins leading to enhanced transmission [4], especially demonstrating that surface plasmon resonance has contributed to improved transmission.

Interestingly, Z. Ruan et al., have shown that a different type of resonance exists in perfect metal film with periodic hole arrays. Specifically, the electromagnetic field localized in each hole causes localized waveguide resonance, being both dependent and independent of hole shapes and period, respectively [5]. Recently, Y. Bao et al. have suggested that the phase difference between localized resonance modes and surface Bloch waves can affect transmission spectra [6].

Using the effective refractive index, many researchers have tried to elucidate the resonant transmission properties of perfect metal film with periodic hole arrays. In general, researchers have obtained the effective refractive index of structured perfect metal film by means of either the Drude model or direct retrieval from transmission and reflection coefficients [7–10]. Previous studies have revealed that perfect transmission occurs when the effective refractive index satisfies impedance matching condition $n_{\text{eff}} = 1.0$ [9] or $-2.0$ [10]. However, such an impedance matching condition fails to explain localized waveguide resonance properties. Thus, rigorous investigation into the effective refractive index is required to explain localized waveguide resonance properties.

By directly measuring the phase difference of electromagnetic waves passing through film and air, we accurately calculated the effective refractive indices of perfect metal film with periodic split ring resonator holes around localized waveguide resonant frequencies. We found that the effective indices were zero at resonant frequencies. The thickness-independent resonant frequency and spatially static behavior of hole resonant modes clearly confirmed the zero refractive index. Further, we examined whether the surface impedance of structured perfect metal film could match air impedance, despite the refractive index measuring zero. To investigate the zero refractive index experimentally, we fabricated free-standing aluminum (Al) films with periodic split ring resonator holes via a femtosecond laser machining technique. A terahertz time-domain spectroscopy (THz-TDS) system was employed to measure both transmitted wave phases and amplitude. The effective index of refraction extracted from transmitted THz pulses corroborated a zero refractive index at localized resonant frequency.

2. Results and discussion

We considered Al films with periodic air holes as suitable for exhibiting resonance transmission within the THz range, which was observed experimentally [10–12]. Circular holes are not suitable for studying localized waveguide resonance of different linearly
polarized waves. Thus, we chose split ring resonator holes, exhibiting localized waveguide resonances for two different polarized waves at different frequencies.

Figure 1 shows the simulated transmission spectra of polarized waves normally impinging on periodically patterned Al film with \( a = 250 \ \mu m, \ w = 10 \ \mu m, \ l = 110 \ \mu m, \ g = 20 \ \mu m, \) and \( d = 17.5 \ \mu m; \) \( a \) is hole period, \( w \) is hole width, \( l \) is side hole length, \( g \) is the split ring resonator gap, and \( d \) is Al film thickness. Further, the inset indicates the definition of structural parameters and polarization directions of incident light with respect to holes.

The finite-difference time-domain (FDTD) method with periodic boundary condition was employed within the simulation to address infinitely-large periodic structures [13]. The complex dielectric constant of aluminum was extracted from the optical constant of Al using the Drude equation, where \( \varepsilon_0 = 4320, \ \omega_p = 1.73 \times 10^{16} \text{ Hz}, \) and \( \gamma = 6.93 \times 10^{16} \text{ Hz} \) [14]. Actually, the dielectric constant of Al was large enough in the THz range for us to regard Al as a perfect metal.

Two resonant frequencies (0.3571 and 1.0368 THz) existed for \( y \)-polarized light and one resonant frequency (0.7477 THz) for \( x \)-polarized light. Observably, resonant frequencies depended only on total hole length [8,9]. We also found that resonant transmission related to surface plasmon resonance depended on hole periodicity within a higher frequency range (not shown in Fig. 1) [5].

Figure 2 shows the simulated distributions of currents induced by resonant modes at 0.357 (a) and 0.747 THz (b). Arrows indicate current directions. One can see that currents concentrate nearby the split ring hole and circulate along the hole. However, no net current circulation was observed, resulting in the absence of magnetic resonance [8,9].

In order to obtain an effective refractive index at the resonant frequency, we rigorously and directly calculated the phase difference between electromagnetic waves passing through film and air [15]. A monochromatic \( y \)-polarized plane wave was employed at the first resonant frequency (0.3571 THz) in simulation.

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Fig. 1. Simulated transmission spectra for \( y \)-polarized wave (solid line) and \( x \)-polarized wave (dashed line) normal incident on Al film with an array of split ring resonator holes when \( a = 250 \ \mu m, \ w = 10 \ \mu m, \ l = 110 \ \mu m, \ g = 20 \ \mu m, \) and \( d = 17.5 \ \mu m. \) The inset depicts dimensions of a split ring resonator hole and the polarization directions of incident light for the hole.
Fig. 2. Simulated distributions of currents, $J$, induced by the resonant modes at 0.3571 (a) and 0.7477 THz (b). Arrows indicate both magnitude and directions of the currents. The distributions were drawn in a log-scale.

Fig. 3. (a) Transmitted electric fields of $y$-polarized plane waves with the first resonant frequency as a function of time during film presence (solid line) and absence (dashed line). The inverse of one oscillation period, 2.803 ps, corresponded to resonant frequency 0.3571 THz. The difference in film and air refractive indices gave rise to negative time delay $-55.55 \text{ fs} = (n_{\text{eff}} - 1) d / c$, resulting in $n_{\text{eff}} = 0.038$. (b) The real and imaginary parts of effective refractive indices, $n_{\text{eff}}$ (solid circles) and $\kappa_{\text{eff}}$ (open circles), as a function of frequency around 0.3571 THz.

Figure 3(a) shows transmitted electric fields as a function of time during film presence (solid line) and absence (dashed line). The inverse of one oscillation period, 2.803 ps, corresponded to resonant frequency. Further, the difference in film and air refractive indices caused a time delay between transmitted electric fields. The negative time delay indicated that
the refraction index of the film was smaller than that of air. Accordingly, the real refraction index of the film could then be calculated from the time delay, \(-55.55 \text{ fs} = (n_{\text{eff}} - 1) d / c\) [16]. The calculated \(n_{\text{eff}}\) was almost zero, i.e., 0.038. The real and imaginary parts of effective refractive indices, \(n_{\text{eff}}\) (solid circles) and \(\kappa_{\text{eff}}\) (open circles), around resonant frequency are shown in Fig. 3(b). The imaginary parts of refractive indices, \(\kappa_{\text{eff}}\), can be calculated from transmitted wave amplitudes, \(\sqrt{T} e^{-2\pi d/\lambda_0 \kappa_{\text{eff}}}\), around resonant frequency under an assumption of \(T \sim 1.0\), where \(T\) is transmittance. As expected, \(\kappa_{\text{eff}}\) has a minimum value at resonant frequency. The zero index of refraction was also observed at higher resonant frequencies of 0.7477 and 1.036 THz.

Figure 4 shows resonant frequencies of the \(x\)- and the \(y\)-polarized waves as a function of film thickness normalized to \(d_0 = 17.5 \ \mu\text{m}\). The dashed lines denote the positions of resonant frequencies when \(d = d_0\). Resonant frequencies were almost independent of film thickness. For the lowest resonant frequency, frequency deviation was about 0.08\%, even though the thickness increased as many as six times. Thickness-independent resonant frequency behavior strongly reinforced localized waveguide resonant frequency as being determined by the in-plane geometric parameters of the unit hole [5].

The zero refractive index led to an infinite resonant wavelength of \(\lambda_{\text{rec}} = \lambda_0 / n_{\text{eff}}\) in the resonator, where \(\lambda_0\) is a wavelength in free space. Therefore, the electromagnetic wave with infinite wavelength corresponded to the static electromagnetic field. In other words, the phase difference between the electromagnetic waves all throughout the film holes was zero. To demonstrate the static behavior of the resonant mode, we calculated the spatial distribution of the electric field at the resonant frequency. Accordingly, perfect metal film thickness was 100 \(\mu\text{m}\), and the frequency of the \(x\)-polarized input wave was 0.7276 THz.

Figure 5 shows the evolution of the electric field in time. The metal film is centered on the origin, and the dotted lines denote the end faces. The different colors represent various times. Incident interference and reflected waves caused the ripples on the left side. Clearly, the electric field inside the metal film was almost static in space, but varied in time.
Ziolkowski has suggested that an electromagnetic field in a medium with zero permittivity and permeability should exhibit spatially static behavior because a zero refraction index—due to zero permittivity and permeability—requires zero phase variation within the medium [17]. Even though film permeability was not zero, our results strongly supported Ziolkowski’s suggestion. In particular, the medium with zero permittivity and permeability provided impedance matching conditions, facilitating comprehension of perfect medium transmission.

Although our structured metal films displayed zero permittivity, permeability was at 1.0 because the metal film did not exhibit magnetic response [8,9]. Hence, it should have perfectly reflected an electromagnetic wave with a zero refractive index frequency because its impedance, $Z = \sqrt{\mu / \varepsilon}$, was infinite. Such a contradiction could be addressed by the surface impedance concept of a structured metallic medium. In particular, surface impedance was provided by $Z = Z_0 \sqrt{1 / (1 + \varepsilon_{\text{eff}})}$, where $Z_0$ is the vacuum impedance, and $\varepsilon_{\text{eff}}$ is the effective dielectric constant of the metallic structure [10]. Hence, the zero effective dielectric constant resulted in impedance matching and resonance transmission.

Experimentally, to demonstrate a zero refraction index, free-standing samples were fabricated by drilling a periodic array of split ring resonator holes into commercial Al foil with a femtosecond laser machining system. Central wavelength and input pulse width were 785 nm and 184 fs, respectively; average power and repetition rate were 1.0 watt and 1 KHz, respectively [18]. The patterned sample size was 8 x 8 mm$^2$. Figure 6(a) shows a partial microscopic image of the fabricated sample, insets indicating geometrical parameters. Hole periods, $a$, were about 260 μm; hole side length, $l$, was about 110 μm; hole line width, $w$, was about 10 μm; the gap size, $g$, was about 15 μm; and Al foil thickness, $d$, was 25 μm. We also fabricated samples with a different period of 150 μm ($l = 110 \mu m$, $w = 25 \mu m$, $g = 10 \mu m$, and $d = 25 \mu m$).

A THz-TDS system was employed to measure transmitted THz pulses. The schematic view of the experimental setup is represented in Fig. 6(b). For THz pulse generation, we used a $p$-type (100) InAs wafer with about $10^{16}$ cm$^{-3}$ carrier concentration, as well as a femtosecond laser with a 76 MHz repetition rate and 190 fs pulse width. Average input laser power was 1.3 watts, while beam InAs wafer diameter was 4 mm. The 5-μm gap dipole antenna on the low temperature-grown GaAs wafer was used to detect THz pulses [19].
Figure 7 shows normalized transmission spectra of two Al film samples with $a = 150 \text{ μm}$ (red) and $a = 260 \text{ μm}$ (black) for $x$-polarized waves. One can see that resonance frequency was almost independent of the hole period, 0.874 THz ($a = 260 \text{ μm}$) and 0.908 THz ($a = 150 \text{ μm}$), even though the observed resonant frequencies were higher than the simulated ones. Al film THz transmittance with $a = 150 \text{ μm}$ was higher than that of Al film with $a = 260 \text{ μm}$ because air volume fraction of the former (~48%) was much larger than that of the latter case (~6.2%).

From phase and amplitude of time domain data, we were able to obtain an effective refraction index, $n_{\text{eff}}$ and $\kappa_{\text{eff}}$, of Al film, assuming periodic holes under structured Al film conditions to be a homogenous. The theoretical scheme for obtaining an effective refraction index from the measured wave form in a time domain is described in Ref. [20]. Using a numerical method, we extracted an effective refraction index from the Al film with periodic holes.

Figure 8(a) and 8(b) show an effective refraction index of samples with periods of 260 μm and 150 μm, respectively, around resonant frequencies. One can see that the real effective refraction index in the case of 150 μm clearly represents zero at the resonant frequency of 0.908 THz, while the index in the case of 260 μm is zero at 0.833 THz, a little below the resonant frequency of 0.874 THz ($n_{\text{eff}} \approx 0.5$). Notably, only a 5% discrepancy existed in
frequency. As expected, imaginary effective refraction indices in the case of 150 μm are smaller than those of 260 μm, as shown from transmission spectra in Fig. 7. Therefore, we could confirm that the extracted refraction index corresponded to the zero refraction index at resonant frequency.

![Graph](image)

Fig. 8. Extracted effective refractive index of Al films with $a = 260$ μm (a) and $a = 150$ μm (b).

3. Conclusion

In conclusion, we numerically and experimentally demonstrated that the effective refraction index of perfect metal film with a periodic split ring resonator array is zero at localized waveguide resonant frequency within a THz frequency range. Accordingly, the zero refraction index effectively accounts for localized waveguide resonant mode properties, thickness independent resonant frequency and spatially static resonant modes.

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