Qualitative analysis of wind-turbine wakes over hilly terrain

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Abstract. In this work, wind-turbine wakes are studied over flat and hilly terrains. Measurements made by using stereoscopic PIV are compared to data obtained from numerical simulations using RANS equations and an actuator-disc method. The numerical and experimental data show similar qualitative trends, indicating that the wind-turbine wake is perturbed by the presence of the hills. Additionally, a faster flow recovery at hub height is seen with the hilly terrain, indicating that the hills presence is beneficial for downstream turbines exposed to wake-interaction effects. The Jensen wake model is implemented over the hilly terrain and it is shown that this model cannot accurately capture the wake modulations induced by the hills. However, by superimposing a wind-turbine wake simulated over flat terrain on the hilly-terrain flow field, it is illustrated that the commonly-used wake-superposition technique can yield reasonable results if the used wake model has sufficient accuracy.

1. Introduction
Wind turbines operating at onshore sites are often found over hilly or mountainous landscapes. Terrain variations at such sites influence the wind-turbine performance and the downstream wake evolution. At specific wind-farm sites, wind-turbine data can be collected and compared with numerical simulations including site-specific terrain models [1]. In such simulations, it has been observed that complex terrain can enhance the turbulent activity in the flow, resulting in a faster wake recovery and higher power outputs from the turbines [2]. While studies of this kind give a good overview of the site-specific flow fields, the terrain complexity makes it difficult to draw conclusions applicable for a wider range of topographical cases. In this respect, it is advantageous to study more generalised terrain models covering fundamental flow situations. This was for instance done through wind-tunnel measurements by Tian et al. [3], who observed higher power performance for wind turbines placed on the crest and on the leeward side of an isolated hill. It has also been found that an upstream hill can increase the turbulence intensity in the inflow seen by a turbine, and quicken the wake recovery [4]. This effect is reduced with increased distance between the hill and the downstream turbine. Terrain features far upstream can nonetheless influence the flow field over onshore sites, as observed by Walmsley et al. [5].

The flow interaction over multiple hills is largely dependent of the shape of the hill and its slope, where steep hill slopes with sharp crests often result in flow separations that significantly reduce the available wind power over downstream hills [6]. Improvements in the available power that a wind turbine can potentially extract have been observed when the distance between two adjacent hills is small, due to an upward deflection of the flow [7]. For a larger number of
periodic hills, regions of correlated flow have been found to extend up to distances equal to the hill spacing [8], as expected. This makes the wake development behind a wind turbine placed over such terrain features intriguing to study, and that has been the focus of the current work.

In the present paper, the wake evolution behind a single wind turbine, operating over flat and hilly terrain, is studied under homogeneous inflow conditions. The used terrain model consisted of a series of streamwise periodic sinusoidal hills. Wind-tunnel measurements made using stereoscopic particle image velocimetry (PIV) were compared with numerical simulations performed using an actuator-disc approach and Reynolds averaged Navier-Stokes (RANS) equations in ANSYS CFX. The purpose of this was initially to investigate if the trends seen in the measurement data could be understood with a fairly simple numerical model. Additionally, since the PIV measurements had a limited field of view, the simulations provided additional insight concerning the studied flow conditions. Moreover, the simulations allowed to do extended comparisons with engineering wake-model approaches applied over hilly terrain.

The present paper is structured as follows: the wind-tunnel measurements and the numerical procedure are described in sections 2 and 3. Thereafter, the simulation results are compared with some of the experimental data in section 4. Finally, conclusions regarding the results are made in section 5.

2. Experimental procedure
The experimental measurements were performed in the closed loop Boundary Layer (BL) wind tunnel at KTH, which is a high flow quality facility with a 4.2 m long, 0.5 m wide and 0.75 m high test section [9]. Hilly terrain was introduced in the second half of the test section, where the hills (which had a sinusoidal shape) were made of XPS styrofoam. Five hills were used and the hill periodicity was four rotor diameters (based on the size of the used wind-turbine model). The hill height, $H$, was 7 cm (0.625 rotor diameters). The wind-turbine model had a rotor diameter, $D$, of 112 mm, with thin slightly-chambered fixed-pitch rotor blades made of carbon fibre reinforced plastic. The rotor Reynolds number was about $Re \approx 6 \cdot 10^4$ based on the rotor diameter and the inflow velocity. Although this is considerably lower than that of a full-scale wind turbine, a study by Chamorro et al. [10] indicated that the mean velocity statistics in the wake become independent of the Reynolds number at $Re \approx 4.8 \cdot 10^4$. Under this assumption, the flow characteristics seen with the present model will be similar to the conditions obtained in a full-scale situation. The turbine rotor was attached to a Faulhaber DC micro motor (working as a generator), and the angular velocity of the turbine was adjusted by altering its electrical load. The turbine DC generator was fastened to a 16 cm tall tower made of aluminium, and the wind turbine was placed on the top of the first hill. The tunnel free-stream velocity $U_\infty$, which was kept constant at 7.5 m/s during the measurements, was measured by means of a Prandtl tube mounted upstream of the terrain model. An illustration of the experimental setup, including cameras and a laser used during the PIV measurements, can be seen in Figure 1.

To observe the wind-turbine wake development with and without the hilly terrain, measurements were made using stereoscopic PIV. The PIV measurements were made by utilising two high-speed 10-bit CMOS cameras of type Photron Fastcam APX RS (that captured image pairs at 50 Hz for a sampling period of 61.44 seconds per plane), a 600 mJ Litron Nano-PIV pulsed Nd:YAG laser that emitted light at a wavelength of 532 nm, and a LaVision PTU X high-speed controller unit. A spherical and a cylindrical lens were used to generate a 2 mm thick light sheet that illuminated particles of DXS oil that were injected into the tunnel free stream. The camera lenses had a focal length of 105 mm that, together with the camera resolution, made it possible to capture planes with a 10x12 cm$^2$ field of view during the measurements (corresponding to 0.9x1.1$D^2$). The cameras were angled so that they captured planes perpendicular to the free-stream direction, and an Isel traversing system was used to move both the cameras and the laser system (which were rigidly joined) in the streamwise direction.
Figure 1. Illustration of experimental setup including PIV equipments.

The LaVision Davis 8.3 software was used for data acquisition and post processing. From the acquired flow images, three-component instantaneous flow fields were re-created through stereoscopic cross-correlations (using multiple passes with decreasing size). These were then averaged to view the mean flow fields at different downstream locations behind the turbine.

3. Numerical procedure
Numerical simulations were made using steady-state RANS equations in ANSYS CFX, where a geometry with the same cross-section and hill dimensions as in the wind-tunnel experiments was used. Only half of the domain was simulated to improve the computational efficiency: a symmetry boundary condition was used at the vertical symmetry plane of the domain, while a free-slip condition was used at the sidewall far from the rotor (namely the wall shear stress and the wall-normal velocity was set to zero). The ceiling and floor surfaces were modelled with free-slip and no-slip boundary conditions, respectively. To study the wake diffusion behind the turbine over a longer distance, the numerical terrain model was extended (compared to the experimental terrain model) by introducing six periodic hills instead of five. The simulation domain was elongated 1 m upstream and downstream of the terrain model to avoid undesirable effects from the inlet and outlet boundaries. Thereby, the total length of the simulation domain was 4.7 m (42D). The inlet boundary condition of the simulation domain was set to a homogeneous inflow velocity $U_\infty = 7.5$ m/s (corresponding to the free-stream velocity of the experiments) and a turbulence intensity of 5%. The turbulence intensity at the inlet was chosen by considering the resulting wake diffusion rate (over flat terrain) in comparison with empirical wake-model predictions. The outlet of the simulation domain was modelled with a constant pressure boundary condition. The Menter SST $K-\omega$ model [11] was used for the turbulence modelling in the flow domain. This method, which combines the standard $K-\epsilon$ and $K-\omega$ models, has been observed to be more accurate in capturing the wake development behind a single rotor than the $K-\epsilon$ model [12]. A first-order upwind advection scheme was used for the turbulence modelling, ensuring that sufficient numerical stability was achieved during the simulations with hilly terrain.

The wind-turbine rotor was described by using a simple actuator-disc approach, where a momentum loss, corresponding to a certain turbine thrust, was prescribed on a disc-shaped region projected by the rotor area (using the same rotor dimensions as in the wind-tunnel experiments). This was done by introducing a momentum source term, $S_M$, in the RANS equations, expressed as

$$S_{M,i} = \rho \frac{\alpha}{2} |\mathbf{U}| U_i, \quad (1)$$
where $\alpha = C_T/[\delta(1-a)^2]$ is a quadratic loss coefficient, $\delta$ is the thickness of the actuator-disc domain (set to $0.04D$ during the simulations) and $a$ is the axial induction factor defined as

$$a = \frac{1 - \sqrt{1 - C_T^2}}{2},$$

using actuator-disc theory. The thrust coefficient was set to $C_T = 0.8$, which is common for wind turbines operating close to their optimal performance conditions. For simplicity, an isotropic loss was imposed over the rotor area and rotational effects were not accounted for.

In the performed simulations, a swept mesh with hexahedral elements was used for the rotor disc, whereas an unstructured grid was used in the rest of the domain. The element size in the rotor-disc domain was $0.018D$, while elements with a size of $0.036 - 0.089D$ were used in the vicinity of the actuator disc and over the hills. Further away (near the domain boundaries) the element size was gradually increased to $0.54D$, since these regions were of lesser importance. A mesh-convergence study was made to ensure that the major flow features in the domain were properly captured, regarding both the velocity gradients over the hills and the wake diffusion downstream of the actuator disc. To resolve the wall boundary layer at the surface of the hills, the mesh was refined so that the normalised wall distance was below $y^+ = 1$. This was done using 10 prismatic inflation layers with a growth rate of 1.8 in a region extending to $0.07H$ above the hill surfaces (where $H$ is the hill height).

To ensure that the wake behind the actuator disc was properly resolved, and that the wake diffusion in the far-wake region was reasonably captured, comparisons were made with the Jensen and Ainslie wake models [13, 14] and the model by Porté-Agel et al. [15, 16]. Additionally, the flow upstream of the actuator disc was compared with the theoretical expression

$$\frac{U}{U_\infty} = 1 - a \left( 1 + \frac{X}{\sqrt{D^2/4 + X^2}} \right),$$

derived along the rotor centreline using the Biot-Savart law [17], where $X$ is the streamwise coordinate. The centreline velocity upstream and downstream of the actuator disc, estimated by the numerical simulations and theoretical wake models, can be seen in Figure 2 (in this case, the simulations were made over flat terrain). With the Jensen model, the wake constant $k$, which determines the slope of the wake expansion and influences the wake diffusion rate, was set to 0.04. As seen in the figure, the results predicted by the RANS simulations are in fairly good agreement with the empirical models (considering the approximations made with the different approaches).

4. Results

Experimental results obtained through the PIV measurements can be seen in Figure 3, where the mean-velocity values measured in planes behind the turbine (with and without hills) are shown. The results have been normalised with the tunnel free-stream velocity and the coordinate system has its origin at the location of the rotor hub. As visible from the results, the wake recovery is initially similar for the two cases. However, as the wake propagates over the second hill, there is a stronger downward wake deflection compared to the case with flat terrain. This is probably due to the pressure gradients induced by the hilly terrain.

Further insights, regarding how the hills influence the wake development downstream of the turbine, can be gained from the numerical simulations. Results from simulations with flat and hilly terrain can be seen in Figure 4: it is evident that the numerical results show the same qualitative trends as the experimental data, namely that the wake developing over the hilly terrain is deformed due to speed-up effects as it passes over the hill tops, and it generally follows the curvature of the hills. This is particularly clear in the lower part of the wake, while the top
of the wake becomes flatter in comparison. Interestingly, the wake asymmetry introduced by the hills is also seen in the results viewed at the horizontal plane located at hub height, albeit to a lesser extent.

If an industrial wake model is used to study the wake propagation over complex terrain, the wake velocity deficit is usually superimposed on the velocity field computed over the terrain. To study the implications of using this methodology, simulations were made where the Jensen wake model [13] was superimposed on the flow field calculated over the hilly terrain (with no actuator disc). Figure 5 shows the resulting flow fields obtained by using this approach. The wake coefficient $k$ in the Jensen model was set to 0.04 and 0.075, to study the result sensitivity to that
Figure 4. Normalised streamwise velocity, $U/U_\infty$, estimated by the RANS simulations. Results are shown at the vertical symmetry plane with flat terrain (a) and with hilly terrain (b). Additionally, for the flow case with hills, the wake is viewed at a horizontal plane located at the rotor hub height (c). The results are viewed in planes parallel to the free-stream direction and velocity contours in the range of $U/U_\infty \in [0, 0.9]$ are included to better visualise the downstream wake evolution.

Figure 5. Normalised streamwise velocity, $U/U_\infty$, obtained at the vertical symmetry plane when applying the Jensen wake model over hilly terrain. The Jensen wake constant $k$ was set to 0.04 (a) and 0.075 (b). White lines are included along the wake edges for better visualisation.

constant (these $k$ values are typically used for offshore and onshore wind farms, respectively). As seen when comparing the wakes in this figure to the results in Figure 4b, the wake expansion differs significantly when using this method. While the wake interior in Figure 5 is modulated by the surrounding velocity field, the effects from the hills on the wake expansion is completely absent. When the wake coefficient is $k = 0.075$ (Figure 5b), the wake expansion is exaggerated compared to the actuator-disc RANS simulations. This was expected since the inlet turbulence intensity level during the RANS simulations was fairly low.

Returning to the RANS simulations with an actuator-disc model, a closer view of the velocity along the rotor centreline is visible in Figure 6a. Results from the PIV measurements are also shown. The simulation results over flat and hilly terrains show a reasonable agreement with the experimental data (considering the modelling assumptions made). Over flat terrain, the
Figure 6. (a) Normalised streamwise velocity, $U/U_\infty$, along the rotor centreline, estimated by RANS simulations with and without hills. Measurement results over flat and hilly terrains are also included. (b) Normalised streamwise velocity over hilly terrain, along the rotor centreline, estimated from RANS simulations, the Jensen wake model and the RANS-based wake-reconstruction technique. In the figure legends the actuator disc method is denoted by AD while the Jensen model is denoted by JM (with $k$ as the wake constant of the Jensen model) and WR denotes wake reconstruction.

experimentally-measured velocity is seen to asymptotically approach the free-stream value in a similar manner as predicted by the RANS simulations. Meanwhile, the experimental data recorded with hills are seen to agree with the corresponding actuator-disc simulations in a qualitative sense: evidently perturbations induced by the hills occur at similar downstream distances in both cases. Quantitatively, deviations are noticed when comparing the simulation results with the experimental data. This is particularly true closer to the rotor, since the used actuator-disc model could not capture the tip-vortex characteristics and the rotational effects that are significant in this region, and the variation in the load distribution over the rotor blades was not introduced in the simplified model. Despite these approximations, the qualitative agreement seen further downstream of the rotor indicates that the simulations can be used to gain additional insight about the wake development in the far-wake region with the studied flow cases.

A comparison between the simulations made over flat and hilly-terrain (with an inlet velocity $U_{in} = U_{\infty} = 7.5$ m/s), shows that the velocity increment over the first hill leads to a higher velocity at the rotor disc located at $X/D = 0$. When comparing the two cases where $U_{in} = U_{\infty}$, the results in Figure 6a indicate that the flow recovery at hub height is enhanced when introducing hills, in agreement with the experimental results. The higher flow recovery is beneficial when introducing multiple turbines exposed to wake-interaction effects. Over flat terrain, a velocity speed-up naturally leads to a quicker wake recovery. To isolate the effect induced by the hills from this, a simulation was made over flat terrain, where $U_{in}$ was set to the inflow velocity, $U_{ref}$, seen by the rotor over the hilly terrain. The turbine inflow velocity,
$U_{ref} = 8.3 \text{ m/s}$, was taken from a simulation that included hills but no turbine. As seen from the results (shown together with the other data in Figure 6a), the disc velocity at $X/D = 0$ was approximately the same with this inlet velocity as in the case with hilly terrain. With hilly terrain, the wake velocity is perturbed by local gradients, introducing wiggles in the results (footprints of these are also seen in the experimental results). Moreover, the wake velocity deficit is initially larger than in the flat-terrain cases. Further downstream, the hilly-terrain velocity deficit gradually approaches the flat-terrain results with $U_{in} = U_{ref}$, but the wiggles caused by the presence of the hills remain.

In Figure 6b, the centreline velocity from the actuator-disc simulation with hills, is compared to results obtained with the Jensen wake model applied over hilly terrain. As seen in Figure 6b, the velocity deficit is underestimated by the Jensen model, particularly close to the rotor disc. When the Jensen wake coefficient $k = 0.075$, the results are generally in poor agreement with the actuator-disc simulations over hilly terrain, while the agreement is somewhat better with $k = 0.04$, as expected from the low turbulence intensity in the simulations.

From the results in Figure 5, it was clear that the hill-induced perturbations at the edges of the wake could not be captured with the linear wake expansion introduced in the Jensen model. However, the complexity of the wake-development description differs between different engineering wake models. To investigate the feasibility of using the wake-superposition technique over hilly terrain (with less errors introduced by the wake-expansion model), a RANS-based wake reconstruction was made by combining the simulation data over flat terrain with the flow field computed over the hills. In this case, the reconstructed velocity field (denoted $\tilde{U}_s$) was obtained from

$$\tilde{U}_s = \tilde{U} - U_{ref} + \bar{U}_{AD}|_{(U_{in}=U_{ref})},$$

(4)

where $\tilde{U}$ is the velocity field obtained from RANS simulations over hilly terrain (without an actuator disc), and $\bar{U}_{AD}$ is the velocity field resulting from the actuator-disc RANS simulations over flat terrain with an inlet velocity $U_{in} = U_{ref}$. The reference velocity $U_{ref}$ is the velocity at the position of the rotor-disc centre, taken from the velocity field over the hills, $\tilde{U}$. The method reconstructs the wake over the hilly terrain by superimposing the wake deficit obtained over flat terrain (with an adjusted inflow velocity) on the velocity field obtained over the hilly terrain, in agreement with what done by typical engineering wake models. The resulting flow field was compared with the flow field obtained from the actuator-disc RANS simulations over hilly terrain (denoted $\tilde{U}_{AD}$). The results using the RANS-based wake reconstruction are shown together with the other data in Figure 6b. Close to the rotor disc, the RANS-based reconstruction significantly improves the results compared with the Jensen model: the results come closer to the velocities observed in the actuator disc simulation over hilly terrain. This becomes even clearer when looking at the entire velocity field in the vertical symmetry plane of the simulation domain. As seen in Figure 7b, the wake modulations induced by the hilly terrain are captured reasonably well with the RANS-based reconstruction, especially when compared to the actuator-disc simulations over hilly terrain (Figure 7a). In Figure 7c the difference between the velocity fields in Figures 7a and 7b, defined as

$$\Delta\tilde{U}/U_\infty = (\tilde{U}_s - \tilde{U}_{AD})/U_\infty,$$

(5)

is shown. As seen in Figure 7c, some differences between the two velocity fields remain in the lower part of the wake (particularly over the valley behind the first hill at $X/D \approx 2$). However, the differences between the velocity fields decrease with downstream distance from the rotor disc. The results from the RANS-based wake reconstruction indicate that the wake-superposition technique can generate reasonable predictions of the wake development over hilly terrain, given that the wake-expansion model has sufficient accuracy. While some modelling simplifications were made during the simulations, the comparative analysis (with and without
Figure 7. Normalised streamwise velocity at the vertical symmetry plane: (a) actuator-disc simulations over hilly terrain, \( \tilde{U}_{AD}/U_\infty \), (b) velocity field obtained using the RANS-based wake-reconstruction method, \( \tilde{U}_*/U_\infty \), (c) difference between the two velocity fields, \( \Delta \tilde{U}/U_\infty \). The velocity fields are zoomed in over the wake region and velocity contours are added for better wake visualisation.

the wake-superposition technique) ensured that the same errors were introduced for all simulated flow cases.

5. Conclusions
In the present paper, the wake development behind a single wind turbine placed over flat and hilly terrain has been studied. This was done both experimentally using stereoscopic PIV, and numerically by means of steady-state RANS simulations and an actuator-disc technique. The results clearly showed that the wind-turbine wake development can be significantly influenced by local terrain features, which are of great importance when planning for wind farms over complex terrain. The wake was deflected downward on the leeward side of a downstream hill, and generally followed the curvature over the hills. With hilly terrain, a faster flow recovery was seen at hub height, indicating that wind farms built over complex terrain (of similar appearance) could potentially produce more power than their flat-terrain counterparts.

The numerical model used in this work was somewhat simplified, but similar qualitative trends as in the experimental data could nevertheless be seen. Some engineering models used in industry to model wind-turbine wakes cannot account for perturbations induced by local pressure gradients. This was illustrated by using the Jensen wake model, which could not capture modulations of the wake induced by the hilly terrain. To perform a wake superposition with reduced errors due to the prescribed wake expansion, the wake-velocity deficit from an actuator-disc RANS simulation over flat terrain (with an adjusted inflow velocity) was superimposed on a flow field simulated over hilly terrain. The RANS-based wake reconstruction was seen to improve the results compared to the Jensen wake model, indicating that the wake-superposition technique can be used over hilly terrain if the wake expansion is properly modelled. The main
advantage with engineering wake models (such as the Jensen model) is their computational efficiency, which is why they are often used in industrial applications. However, the present work shows that additional care must be taken when studying wind turbines over complex terrain to ensure that the hill-induced wake modulations are captured. Additional analyses are required to further validate the wake-superposition technique over hilly terrains, including studies with different terrain features and multiple turbines. A more accurate modelling of the wind-turbine rotor is recommended to improve the results accuracy and the performance of different engineering wake models should be investigated. Nevertheless, the performed wake reconstruction showed promising results regarding the wake development behind a single wind turbine operating over hilly terrain.

6. Acknowledgments
The work presented in the present paper has been funded by The Swedish Research Council (VR) under a framework grant for strategic energy research. The authors wish to thank Jan-Åke Dahlberg at Vattenfall Vindkraft AB for manufacturing the wind-turbine rotors and for lending equipment to the experimental measurements. Rune Lindfors and Jonas Vikström at KTH are acknowledged for manufacturing items used in the wind-tunnel setup. Finally, Prof. P. Henrik Alfredsson at KTH and Jari Hyvärinen at ANKER-ZEMER Engineering AB are acknowledged for giving useful advices during the work.

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