Cosmological Bound from the Neutron Star Merger GW170817 in scalar-tensor and $F(R)$ gravity theories

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(Dated: January 30, 2018)

We consider the evolution of cosmological gravitational waves in scalar-tensor theory and $F(R)$ gravity theory as typical models of the modified gravity. Although the propagation speed is not changed from the speed of light, the propagation phase changes when we compare the propagation in these modified gravity theories with the propagation in the $\Lambda$CDM model. The phase change might be detected in future observations.

PACS numbers: 04.30, 04.30.Nk, 04.50.+h, 98.70.Vc

I. INTRODUCTION

The early-time inflation and late-time acceleration are eventually most important problems in physics of this century. In order to solve these problems, number of models, including the theories modifying the Einstein gravity, have been proposed and investigated (for the review, see [1–5]). The fundamental problem is to identify the model which most naturally describes the universe. Although we have obtained some constraint from the cosmology [6], the recent discovery of the gravitational wave [7, 8], may also indicate the possibility to select the most realistic theory or give some constraints on the model by using the observational data [9–13]. Especially, in [14], it has been shown that the amplification of the amplitude of gravitational wave changes in the scalar-tensor theory and $F(R)$ gravity if compared with the Einstein gravity (for the modification in the brane world or the domain wall universe, see [15]).

In fact, the recent observation of GW170817 [16], where the gravitational wave was generated by the neutron stars merger, may put some constraints to different cosmological theories. In the observation of GW170817, not only the gravitational wave but also the signal of the gamma ray has been detected simultaneously. This gives the strong constraint on the ratio for the speeds of the gravitational wave and the light,

$$\left| \frac{c_{GW}}{c} - 1 \right| < 6 \times 10^{-15}.$$  

Here $c$ is the speed of the light and $c_{GW}$ is the propagating speed of the gravitational wave. Eq. (1) constrains the mass of the graviton and the parameters in the scalar-tensor theory of Horndeski or Galileon type model [17–21]. In case of the covariant Galileon model [22], whose Lagrangian density is given by

$$\mathcal{L} = X + G_4(X)R + G_{4,X} \left( (\nabla^2 \phi)^2 - \nabla_{\mu} \nabla_{\nu} \phi \nabla^\mu \nabla^\nu \phi \right), \quad X = -\frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi,$$

$$G_4(X) = \frac{M_{Pl}^2}{2} + \frac{2c_0}{M_{Pl}} \phi + \frac{2C_4}{\Lambda^6} X^2,$$  

(2)

(for the details of the notation, see [22]), the terms including $c_4$, where the derivative term of the scalar field couples with the curvature, induce the modification of the effective metric for the gravitational wave,

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + C \partial_{\mu} \phi \partial_{\nu} \phi.$$  

(3)

It gives the variation of the speed of the propagation of the gravitational wave,

$$\left| \frac{c_{GW}}{c} - 1 \right| = \frac{4c_4 x^2}{1 - 3c_4 x^2}, \quad x = \frac{\dot{\phi}}{HM_{Pl}}.$$  

(4)

Therefore Eq. (1) gives a strong constraint on the model. It is the purpose of our work to investigate whether even if the derivative term of the scalar field does not couple with the curvature, there appears the modification in the
propagation of the gravitational wave. Note that in case of Moffat’s modified gravity (MOG), that is, Scalar-Tensor-Vector-Gravity (STVG) [23] (see also [24, 25]), which is also a typical modified gravity model, the consistency with the GW170817 event [16] has been confirmed in [26].

In the next section, we give a general setup for the propagation of the gravitational wave. In Section III we consider the scalar-tensor theory. We show that the propagation phase of the gravitational wave changes in the background although the propagation speed is the same light speed. In Section IV, we consider the propagation of the gravitational wave and we show that the propagation is different from that in the scalar-tensor theory.

II. SETUP

Let us consider the equation for the propagation of gravitational wave in the expanding universe. The gravitational wave is given by the perturbation from the background geometry,

\[ g_{\mu\nu} \rightarrow g_{\mu\nu} + h_{\mu\nu} , \]  

where \( |h_{\mu\nu}| \ll 1 \) is the perturbation with respect to a given background \( g_{\mu\nu} \). It is straightforward to obtain the perturbed Ricci tensor and scalar,

\[
\delta R_{\mu\nu} = \frac{1}{2} \left[ \nabla_\mu \nabla_\rho h_{\nu\rho} + \nabla_\nu \nabla_\rho h_{\mu\rho} - \nabla_\mu \nabla_\nu (g^{\rho\lambda} h_{\rho\lambda}) - 2R^\lambda_{\nu\mu} h_{\lambda\rho} + R^\rho_{\nu} h_{\mu\rho} + R^\rho_{\rho} h_{\mu\nu} \right],
\]

\[
\delta R = -h_{\mu\nu} R^{\mu\nu} + \nabla_\mu \nabla_\nu h_{\mu\nu} - \nabla^2 (g^{\mu\nu} h_{\mu\nu}).
\]

As long as we consider the propagation of the gravitational wave, we need not to consider the perturbation of scalar modes because the spin two field, i.e. the graviton corresponding to the gravitational wave, does not mix with the scalar field (spin zero) field. By imposing the gauge condition

\[ \nabla^\mu h_{\mu\nu} = g^{\mu\nu} h_{\mu\nu} = 0, \]

the Einstein field equations

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa^2 T_{\mu\nu}, \]

take the perturbed form as follows,

\[ \frac{1}{2} \left[ \nabla_\mu \nabla_\nu h_{\mu\nu} + 2R^\lambda_{\nu\mu} h_{\rho\lambda} + R^\rho_{\nu} h_{\mu\rho} + R^\rho_{\rho} h_{\mu\nu} - h_{\mu\nu} R + g_{\mu\nu} R^{\rho\lambda} h_{\rho\lambda} \right] = \kappa^2 \delta T_{\mu\nu}. \]

The equation (10) indicates that the energy-momentum tensor perturbations affect the propagation of gravitational wave.

III. SCALAR-TENSOR THEORY

In order to specify the explicit form of \( \delta T_{\mu\nu} \), we consider the scalar theory whose action is given by

\[ S_\phi = \int d^4 x \sqrt{-g} L_\phi, \quad L_\phi = -\frac{1}{2} \omega(\phi) \partial_\mu \phi \partial^\mu \phi - V(\phi). \]

One finds

\[ T_{\mu\nu} = -\omega(\phi) \partial_\mu \phi \partial_\nu \phi + g_{\mu\nu} L_\phi, \]

and therefore

\[ \delta T_{\mu\nu} = h_{\mu\nu} L_\phi + \frac{1}{2} g_{\mu\nu} \omega(\phi) \partial^\rho \phi \partial_\lambda \phi h_{\rho\lambda}, \]

up to first order in perturbation. We are interested in the evolution of tensor gravitational wave so we can consider only the spatial component of \( h_{\mu\nu} \), that is, \( h_{ij} \).
Assuming a FRW spatially flat metric

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1,2,3} (dx^i)^2,$$

and $\phi = t$ in (13), the FRW equations have the following form,

$$\frac{3}{\kappa^2} H^2 = \frac{\omega}{2} + V, \quad -\frac{1}{\kappa^2} \left(2 \dot{H} + 3H^2 \right) = \frac{\omega}{2} - V,$$

and one gets

$$\omega = -\frac{2}{\kappa^2} \ddot{H}, \quad V = \frac{1}{\kappa^2} \left(\dot{H} + 3H^2 \right),$$

for the kinetic term and the scalar field potential expressed as functions of the scale factor $a(t)$ and its derivatives.

By using (10), (13), (16), and $\phi = t$, we find the evolution equation of gravitational wave:

$$0 = \left(2 \ddot{H} + 6H^2 + H\partial_t - \partial_t^2 + \frac{\triangle}{a^2} \right) h_{ij},$$

which clearly depends on the cosmological background.

We may compare the propagation of the gravitational wave with that of the light or photon. The equation for the vector field corresponding to the photon is given by

$$0 = \nabla_{\mu} F_{\nu} = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} F_{\rho\sigma} \right) = \nabla^2 A^\nu - \nabla^\nu \nabla^\mu A_\mu + R^{\mu\nu} A_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

In the FRW universe, Eq. (18) can be rewritten as follows,

$$0 = \sum_{i=1,2,3} \partial_i \left(\partial_i A_t - \partial_t A_i \right), \quad 0 = \left(\partial_t + H \right) \left(\partial_i A_t - \partial_t A_i \right) + a^{-2} \left(\triangle A_i - \partial_t \sum_{j=1,2,3} \partial_j A_j \right).$$

We may choose the Landau gauge

$$0 = \nabla^\mu A_\mu = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} A_\nu \right) = -\partial_t A_t + 3H A_t + a^{-2} \sum_{i=1,2,3} \partial_i A_i.$$

Then Eq. (18) reduces to

$$0 = \nabla^2 A^\nu + R^{\mu\nu} A_\mu.$$

Furthermore because we are interested in the propagation of photon, we may assume,

$$0 = A_t = \sum_{i=1,2,3} \partial_i A_i.$$

Then the first eq. (19) and the gauge condition (20) are satisfied and the second equation in (19) gives

$$0 = -\left(\partial_t^2 + H \partial_t \right) A_i + a^{-2} \triangle A_i.$$

First, we consider the propagation in the de Sitter space-time, where the Hubble rate $H$ is a constant, $H = H_0$, and the scale factor is given by $a = e^{H_0 t}$. Because the propagation of the light is simpler than that of the gravitational wave, first we consider Eq. (23). We now assume the plane wave and separate the variable as follows, $A_i \propto e^{ik \cdot x} A_i(t)$ with the wave number $k$, and replace $\triangle$ by $-k^2 \equiv -k \cdot k$. Further we redefine a new variable $s$ by

$$s \equiv e^{-H_0 t}.$$

Eq. (23) can be rewritten as

$$0 = \left(\frac{d^2}{ds^2} + \frac{k^2}{H_0^2} \right) \dot{A}_i,$$
whose solution is trivially given by
\[ \hat{A}_i = A_{i0} \cos \left( \frac{k}{H_0} s + \theta_0 \right). \] (26)

Here \( A_{i0} \) and \( \theta_0 \) are constants.

Let us now consider the gravitational wave (17) in the background of de Sitter space-time. We further define variable \( u \) and a function \( l_{ij} \) by
\[ u = \frac{k}{H_0} s, \quad h_{ij} = s^{-\frac{3}{2}} l_{ij}. \] (27)

Then Eq. (17) can be rewritten as
\[ 0 = \left( \frac{d^2}{du^2} + \frac{1}{u} + 1 - \frac{5}{2} \right) l_{ij}, \] (28)
which is nothing but Bessel’s differential equation, whose solution is given by the Bessel functions \( J_{\pm \frac{5}{2}}(u) \). As long as we consider the gravitational wave from the black hole merger [7, 8] or neutron star [16], the magnitude of the variable \( s \) in (24) is the order of unity. On the other hand, the wave number \( k \) should be much larger than the Hubble constant \( H_0 \), which is the present value of the Hubble rate \( H \) and therefore \( u \) in (27) should be very large. Using the asymptotic behavior of the Bessel function
\[ J_\alpha(x) \to \sqrt{\frac{2}{\pi x}} \cos \left( x - \frac{\alpha + 1}{4} \pi \right), \] (29)
for large \( x \), we find
\[ h_{ij} \sim \frac{1}{s} \cos \left( \frac{k}{H_0} s + \frac{\pm 5 + 1}{4} \pi \right). \] (30)

By comparing (30) with (26) for the propagation of the light, there is no difference between the propagating speed of the light and the gravitational wave.

We now consider the case that the scale factor \( a(t) \) behaves as a power-law function,
\[ a(t) = \left( \frac{t}{t_0} \right)^\alpha, \] (31)
with \( t_0 \) and \( \alpha \) real constants. Depending on the value of \( \alpha \), Eq. (31) involves a power law (superluminal) inflation \( (\alpha \geq 1) \), a Friedmannian (subluminal) evolution \( (0 < \alpha < 1) \), and a pole-like (phantom [27–29]) behavior \( (\alpha < 0) \). The universe described by (31) can be realized by the scalar-tensor model in (11) by substituting Eq. (31) into the expressions for \( \omega(\phi) \) and \( V(\phi) \) (16), that is
\[ \omega(\phi) = \frac{2\alpha}{\kappa^2 t_0^2 \dot{\phi}^2}, \quad V(\phi) = \frac{3\alpha^2 - \alpha}{\kappa^2 t_0^2 \dot{\phi}^2}. \] (32)

In case that the expansion of the universe is generated by the perfect fluid with a constant equation of state parameter \( w \), the power-law expansion (31) is also realized by
\[ \alpha = \frac{2}{3 (1 + w)}. \] (33)

For the power-law case (31), the Hubble rate \( H \) and \( \dot{H} \) are given by
\[ H = \frac{\alpha}{t}, \quad \dot{H} = -\frac{\alpha}{t^2}. \] (34)

If we consider the gravitational wave from the black hole merger [7, 8] or neutron star merger [16], we may consider \( H \) to be a constant \( H \sim H_0 \). Because Eq. (34) shows \( H^2 \sim \dot{H} \) as functions of \( t \), one may take \( \dot{H} \) to be a constant, \( \dot{H} = H_1 \), as long as we can take \( H^2 \) to be a constant. Note the term including \( \dot{H} \) appears in Eq. (17) in addition to
the term of $H^2$. Then by regarding the terms $2\dot{H} + 6H^2$ with constants, $2\dot{H} + 6H^2 \sim 2H_1 + 6H_2^2$. Then by using (27) with the variable $s$ in (24), which is defined by using the constant $H_0$, that is, the Hubble constant in the present universe, instead of (28), we rewrite (17), as follows,

\[
0 = \left( \frac{d^2}{du^2} + 1 + \frac{(2)^2 - 2\frac{H_1}{H_0}}{u^2} \right) l_{ij},
\]

(35)

Then the solution is given by $J^{\pm \frac{5}{2}\sqrt{1 - \frac{4H_1}{25H_0^2}}}$, instead of (30), we find

\[
h_{ij} \sim \frac{1}{s} \cos \left( \frac{k}{H_0} s + \frac{\pm 5\sqrt{1 + \beta + 1}}{4} \pi \right), \quad \beta \equiv -\frac{4H_1}{25H_0^2},
\]

(36)

Hence, it does not appear the term including $\dot{H}$ in Eq. (23), which describes the propagation of the light. Thus, the ratio of the propagating light speed $c$ is identical with the propagating speed $c_{GW}$ of the gravitational wave. We should note that there appears the shift of the phase in (36) compared with (30) but it could be difficult to detect the phase. Because

\[
\beta = -\frac{4}{25\alpha} = -\frac{6(1 + w)}{25},
\]

(37)

if one can find the phase by the observations, we may find the constraint on the equation of state parameter $w$ in the scalar-tensor theory. In the radiation dominance epoch, $w = \frac{1}{3}$ and in the matter dominance epoch, $w = 0$. If current accelerating universe corresponds to the asymptotic de Sitter space-time, $w$ should be $-1$. From Eq. (37), we see that actually there is no difference of phase at the inflation and current universe but it may be some visible difference in radiation/matter dominated eras.

IV. $F(R)$ GRAVITY

Let us now investigate $F(R)$ gravity in the similar fashion. Just for the simplicity, we assume that $F(R)$ is given by the power of the scalar curvature $R$,

\[
F(R) \sim R^m,
\]

(38)

which gives the power-law scale factor $\frac{1}{2} \sqrt{1 - \frac{4H_1}{25H_0^2}}$, where the exponent $\alpha$ is related with $m$ as follows,

\[
\alpha = -\frac{(m - 1)(2m - 1)}{m - 2}.
\]

(39)

By using the conformal transformation

\[
g_{\mu\nu} = \frac{1}{F'(R)} g_{E\mu\nu},
\]

(40)

the Jordan frame is mapped into the Einstein frame. In [14], it has been shown that the gauge conditions [8] do not change as long as we consider the propagation of the gravitational wave. For the power-law scale factor $\frac{1}{2} \sqrt{1 - \frac{4H_1}{25H_0^2}}$, the scalar curvature is proportional to $\frac{1}{t}$ and $F'(R)$ can be written as

\[
F'(R) \sim F_0 \left( \frac{t}{t_0} \right)^{-2(m-1)}.
\]

(41)

Then the metric [40] in the Einstein frame is given by

\[
ds_E^2 = F_0 \left( \frac{t}{t_0} \right)^{-2(m-1)} \left[ -dt^2 + \left( \frac{t}{t_0} \right)^{2(m-1)/m-2} \sum_{i=1,2,3} (dx^i)^2 \right].
\]

(42)

One may define the time coordinate $t_E$ in the Einstein frame as follows,

\[
t_E = t_0 \left( \frac{t}{t_0} \right)^{2-m}.
\]

(43)
Here $t_{E0}$ is a constant defined by

$$t_{E0} \equiv \frac{\sqrt{T_0 t_0}}{2 - m}. \quad (44)$$

By using $t_E$, the metric in the Einstein frame can be rewritten as

$$ds^2_E = -dt_E^2 + \left(\frac{t_E}{t_{E0}}\right)^{\frac{6(m-1)^2}{(m-2)^2}} \sum_{i=1,2,3} (dx^i)^2. \quad (45)$$

Then $\tilde{\alpha} \equiv \frac{2(m-1)^2}{(m-2)^2}$ can be identified with $\alpha$ in (31) in the Einstein frame, what is different from $\alpha$ in (39) in the Jordan frame.

The speed of the gravitational wave should not be changed when transition to different frame is made. The exponent $\tilde{\alpha}$ is related with $w$ by (33), the equation of state parameter $\tilde{w}$ in the Einstein frame is given by

$$\tilde{w} + 1 = \frac{2}{3\tilde{\alpha}} = \frac{2(m-2)^2}{9(m-1)^2}. \quad (46)$$

Then the parameter $\beta$ in the phase is now given by

$$\beta = -\frac{4}{25\tilde{\alpha}} = \frac{12(m-2)^2}{25(m-1)^2}, \quad (47)$$

If $w \sim -1$ as in the the Planck satellite data [30],

$$w = -1.019^{+0.075}_{-0.080}, \quad (48)$$

we find $m \sim 2$ and $w + 1 \sim \frac{2(2-m)}{9}$, and therefore

$$\beta \sim \frac{243(w + 1)^2}{25}, \quad (49)$$

which is rather different from the Eq.(37). Therefore even if $w$ is the same, there appears the difference in the phase. If one can detect the gravitational wave phase, we can distinguish the $F(R)$ gravity from the scalar-tensor theory. It also follows from (49) that the phase vanishes for the exact de Sitter space-time but even for the asymptotically de Sitter space-time, the phase does not vanish although it is, of course, very small.

V. SUMMARY

In this paper, we have investigated the propagation of the gravitational wave in the scalar-tensor theory and $F(R)$ gravity. Although we do not include the derivative coupling with the curvature in the scalar-tensor theory, we have shown that the propagation of the gravitational wave depends on the modified gravity as the scalar-tensor theory and $F(R)$ gravity theory. Some difference in the gravitational wave propagation phase if compare with the light one is observed.

It could be interesting to apply the above formulation to other modified gravities like $f(G)$ gravity whose action is given by

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} R + f(G) + L_{\text{matter}}\right). \quad (50)$$

Here $f(G)$ is a function of the Gauss-Bonnet invariant $G$, which is defined by

$$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\xi\sigma}R^{\mu\nu\xi\sigma}. \quad (51)$$

As the Gauss-Bonnet invariant $G$ is given by the squares of the curvatures, the correction to the propagation of the gravitational wave could be small. The question is if the gravitational wave speed can exceed the light speed in the $f(G)$ gravity model or only change of gravitational wave propagation phase occurs. This will be studied elsewhere.
In the case of the real matter and radiation, we know the operators corresponding to $\delta T_{\mu \nu}$ in (10) because the Lagrangian densities for the matter and the radiation are known. Then by using the statistical physics etc, we may estimate the average of the operators. The obtained expression for $\delta T_{\mu \nu}$ could be different from those in the scalar-tensor theory or $F(R)$ gravity theory.

Finally we consider the possibility to detect the shift of the phase which we found in this paper by cosmological observations. One way could be to observe the difference between the phases of two kinds of gravitational wave: one gravitational wave arrives at the earth directly from the source and other gravitational wave goes through nearby galaxies or any massive object before arriving at the earth. If the EoS parameter $w$ is in the range $-1 < w < 0$, the density of the dark energy decreases near the galaxy or any massive object due to negative pressure. Therefore there occurs the difference in the phases for these two kinds of the gravitational waves. As an example, we may consider the density of the dark energy around the spherically symmetric massive object and we may assume the Schwarzschild type metric,

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{-2\nu(r)}dr^2 + r^2d\Omega^2.$$  \hfill (52)

Here $d\Omega^2$ expresses the metric of the two-dimensional unit sphere. By assuming the energy density $\rho$ and the pressure $p$ of the dark energy only depend on the radial coordinate $r$, the $r$ component of the conservation law $\nabla^\mu T_{\mu r} = 0$ for the energy-momentum tensor gives,

$$\frac{dp}{dr} + \nu' (p + \rho) = 0.$$ \hfill (53)

Then if $p = w\rho$, we find

$$\rho = \rho_0 e^{-(1+\frac{1}{w})\nu}.$$ \hfill (54)

In the case of the Schwarzschild metric $e^{2\nu} = 1 - \frac{2M}{r}$, $e^{2\nu}$ is a monotonically increasing function with respect to $r$ outside of the horizon $r > 2M$. Then Eq. (54) tells that if $-1 < w < 0$, $\rho$ is an increasing function of $r$, that is, $\rho$ becomes smaller near the massive object. In case of the cosmological constant where $w = -1$, there does not occur the change of the density, which is clear from Eq. (54). In order to find the shift of the phase, we may observe the interference in the gravitational waves. For example, we may consider the case that there is a massive object between the earth and the source of the gravitational wave. Then there occurs the interference between the gravitational wave which passed near the massive object and the wave which passed the region far from the massive object. This interference changes the waveform in addition to the possible gravitational lensing effect. When we obtain Eq. (54), we have assumed that the dark energy is given by the perfect fluid whose EoS parameter is a constant $w$. The assumption could be changed if we consider the scalar-tensor theory or $F(R)$ gravity as the dark energy, which could be also an interesting future work.

**Acknowledgments.**

This work is supported (in part) by MEXT KAKENHI Grant-in-Aid for Scientific Research on Innovative Areas “Cosmic Acceleration” (No. 15H05890) (SN), by MINECO (Spain), project FIS2016-76363-P and JSPS S17116 short-term fellowship (SDO) and by CSIC I-LINK1019 Project(SN and SDO). This research was started while SN was visiting the Institute for Space Sciences, ICE-CSIC in Bellaterra, Spain (SN thanks Emilio Elizalde and SDO, and the rest of the members of the ICE for very kind hospitality).

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