Order picking optimization with order assignment and multiple workstations in KIVA warehouses

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Abstract

We consider the problem of allocating orders and racks to multiple stations and sequencing their interlinked processing flows at each station in the robot-assisted KIVA warehouse. The various decisions involved in the problem, which are closely associated and must be solved in real time, are often tackled separately for ease of treatment. However, exploiting the synergy between order assignment and picking station scheduling benefits picking efficiency. We develop a comprehensive mathematical model that takes the synergy into consideration to minimize the total number of rack visits. To solve this intractable problem, we develop an efficient algorithm based on simulated annealing and dynamic programming. Computational studies show that the proposed approach outperforms the rule-based policies used in practice in terms of solution quality. Moreover, the results reveal that ignoring the order assignment policy leads to considerable optimality gaps for real-world-sized instances.

Keywords: parts-to-picker; order picking; order assignment; sequencing; scheduling

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1 Introduction

1.1 Background and motivation

At 00:26 on November 11 (Double 11 Shopping Festival), 2020, the peak of order creation in T-mall reached 583,000 orders per second (Sina, 2020). The rapid development of e-commerce brings new challenges to warehouse operations, among which order picking plays a crucial role and directly affects the overall order fulfillment efficiency (Lamballais et al., 2017; Shen et al., 2020). The Robotic Mobile Fulfillment System (RMFS) is invented to improve order picking efficiency and reduce labour costs by exploiting racking-moving mobile robots (Boysen et al., 2017). The cooperation between the robots and the movable racks eliminates the pickers’ unproductive movement in the picker-to-parts systems (Battini et al. 2017). The remarkable picking performance of RMFS is reported over 600 order-lines per hour per workstation, which is far better than that of traditional manual warehouses (Wulfrat, 2012; Banker, 2016). Nevertheless, the growing demand and increasingly tight delivery schedules brought by the prosperity of e-commerce urge to change the current extensive management in RMFS (Batt and Gallino, 2019). The operational optimization for order picking, which is the most time-consuming and asset-intensive process in such systems, has been an immediate priority (Azadeh et al., 2019; Zhuang et al., 2021).

In this paper we consider the order picking process in the context of the KIVA warehouse, which is the first representative application of RMFS (Enright and Wurman, 2011). The layout of KIVA warehouses is illustrated in Figure 1, in which orders, workbenches, and movable racks are the crucial elements that affect the picking efficiency (Weidinger et al., 2018). First, a batch of orders received by the warehousing management system (WMS) needs to be distributed to the individual picking stations. Then, each picking station receives a fixed set of orders, which are processed on the workbench. For an isolated workstation, order picking optimization is to optimize order sequencing and rack sequencing based on the determined order set and rack set, which are intertwined decisions. Specifically, order sequencing determines the SKUs to be
assembled simultaneously on the workbench, which needs to be synchronized with the racks assigned to the station satisfying the corresponding demands, i.e., the rack disposal sequence. An optimized order sequencing and rack sequencing can improve the picking efficiency, and halve the robot fleet (Boysen et al., 2017; Yang et al., 2021). However, the picking optimization of the KIVA warehouse is a challenging problem because, in practice, such a warehouse often uses multiple picking stations in parallel. With the growing number of machines, i.e., picking stations in our paper, the complexity usually grows exponentially, making the problem intractable (Zhuang et al., 2021).

**Figure 1.** The layout of a KIVA warehouse with three workstations

Motivated by the above observation, we study the joint optimization of order assignment and picking station scheduling in the KIVA warehouse, seeking to address the following questions with a view to reducing the total number of rack visits: First, which orders should be assigned to the same picking station? Second, how to sort the pending orders at each station? Third, which racks and when should they be selected to satisfy the orders synchronously on every single workbench at a time, i.e., which racks should be moved to which picker, and the sequence in which the racks are presented to the picker?
1.2 Contributions and paper organization

Nowadays, increasing numbers of similar robotic systems, which markedly affect the manual picking decision, have been installed to support modern B2C distribution centres. Despite the widespread use of this type of system in real life, it has not received comparable research attention. In this paper we consider joint optimization in the order picking process in the KIVA warehouse, which assigns orders to multiple stations while determining the processing sequence of the orders and synchronization of the racks at the pickers. We make three contributions in this paper:

First, we develop a mathematical model for the intractable problem under study, which can find the optimal solutions for small-sized instances. To the best of our knowledge, this is the first study to consider order assignment in order picking optimization under multi-workstation setting. Second, showing that the problem is NP-hard, we propose a hybrid search approach based on simulated annealing and dynamic programming, which we show through numerical studies provides good quality solutions for real-world-sized instances. Also, we numerically find that the proposed approach outperforms the well-established rules widely applied in real-life robotic warehouses. Third, we ascertain the potential benefits of joint optimization of order assignment and picking station scheduling. Specifically, we identify the beneficial effect of applying the proposed order assignment rules on the total number of rack visits. A single rack visit can satisfy nearly four times the number of orders when we apply the joint optimization, compared with the separate picking station scheduling under the fixed order assignment.

We organize the rest of the paper as follows: In Section 2 we review the related literature to identify the research gaps and position our paper. We discuss the practical workflow of the KIVA warehouse and introduce the problem in Section 3. In Section 4 we formulate the problem as a mathematical model to treat the general problem. We present the proposed heuristic procedure in Section 5. We report the results of numerical studies conducted to assess the proposed approach and discuss the managerial insights.
of the research findings in Section 6. Finally, in Section 7, we conclude the paper and suggest topics for future research.

2 Literature review

There has been an abundance of research on optimizing order processing in picker-to-parts warehouses, detailed summaries of which can be found in de Koster et al. (2007) and Gu et al. (2010). As the scale of e-commerce shipments expands, many B2C distribution centres have applied automatic systems involving a wide range of parts-to-picker technologies to release the manual labour (see, e.g., Zaerpour et al., 2015; Kumawat and Roy, 2021). The elementary decisions concerning the daily operations of a robotic warehousing system are essentially the same as those of the traditional manual warehouse. However, the autonomous rack delivery assisted by robots in the former system requires modifications of the decision-making process (Weidinger et al., 2018).

Specifically, in the traditional warehouse, the storage area is static, from which each picker carries a fixed order batch each time, walking or driving through the area to execute the task. The source of gaining efficiency often involves optimization of the zoning, order batching, batch sequencing, and picker routing decisions (Scholz et al., 2017). However, in the KIVA warehouse, thanks to the presence of mobile robots and movable racks, the storage assignment changes dynamically, orders are processed one after the other instead of in fixed batches, and the picker can reach any rack without moving, so picking efficiency will benefit from the synergetic interaction of the orders, racks, and picking stations. Accordingly, there exist significant and challenging decisions concerning parts-to-picker in the KIVA system, such as layout design, storage assignment, order picking, and robotic planning, which have not been adequately addressed (Azadeh et al., 2019; Boysen et al., 2019), yielding the following research challenges.

- Most studies on layout design can be categorized as systematic analysis, focusing on modelling techniques to estimate the performance of different system scenarios
without considering optimization. Lamballais et al. (2017) used queuing theory to analytically estimate the maximum order throughput, average order cycle time, and robot utilization in a robotic mobile fulfillment system. They verified the analytic estimates by simulation. They finally suggested the optimal configuration and reasonable operating policies for the warehouse manager based on their finding that the maximum order throughput is insensitive to the dimensional parameters of the storage area.

- Most studies on storage assignment consider two elementary decisions: one is which SKUs should be stored together on the same rack and the other is where the racks should be placed in the storage area. A typical storage choice in the KIVA warehouse is the mixed-shelves policy, in which the items of the same SKU are spread all over the warehouse on multiple racks (Bartholdi and Hackman, 2014). This scattered storage contributes to a greater probability that some racks holding the requested items are close by (Weidinger and Boysen, 2018). Moreover, all the racks are identical and can be re-located dynamically to any parking position. Thus, for the latter decision, Weidinger et al. (2018) addressed the problem of where to park the racks during order processing when they are consistently moved between the picking stations and the storage area.

- Most studies on robotic planning concern the task allocation and traffic planning decisions, which coordinate the mobile robots with all their different destinations and avoid deadlocks. The coordination of multiple agents has attracted most research on rack-moving mobile robots (see, e.g., Wurman et al., 2008; D’Andrea and Wurman, 2008; Roodbergen and Vis, 2009).

In this paper we focus on the order picking process, which is at the heart of any warehouse (Azadeh K et al., 2017). Van Gils et al. (2018) presented a comprehensive classification and review of picking systems. Winkelhaus et al. (2021) developed a framework for Order Picking 4.0 as a sociotechnical system, considering substitutive and supportive technologies. The KIVA picking system deploys mobile robots to bring
movable racks to stationary pickers so that each picker concentrates only on picking station scheduling. Note that the orders always far exceed the station capacity, which implies that a batching decision should be made to concurrently process some orders on the workbench. By synchronizing the batches of orders to be jointly handled on the bench and the racks visiting a station, fewer racks may be delivered to a station (Boysen et al., 2017). There has been plentiful of research on order batching in the traditional warehouse, which divides orders by certain rules to achieve specified objectives (Pan et al., 2015; Çeven and Gue, 2017; Ardjmand et al., 2018). In contrast, the batches in the KIVA setting change in real time because each order may have a different processing time, which introduces a new decision on the sorting of a given set of orders dynamically according to the actual processing of the orders concerned.

Considering that orders and racks are processed in a synchronized manner, there exists a close and mutually restrictive connection among the orders, racks, and workstations. The picking station scheduling problem is called the “mobile-robots-based order picking problem” in the literature (Boysen et al., 2017), which concerns a single picker with a given set of orders to be picked from a given set of racks. Moreover, the order sequencing decision is closely coupled with rack scheduling, necessitating the choice of the most suited racks and determining their arrival sequence (Yang et al., 2021). Both studies treated the picking stations as isolated; however, in practice, multiple stations operate in parallel, and the racks assigned to each station are not known before order allocation. Therefore, how to schedule the allocation of both orders and racks to minimize the total number of rack visits becomes another important issue in the mobile-robots-based distribution warehouse (Valle and Beasley, 2021; Xie et al., 2020). Consequently, the KIVA picking process involves several interrelated decision problems, which are often addressed separately for ease of treatment.

In conclusion, there exist no studies on the problem of joint optimization of order assignment and picking station scheduling in the KIVA warehouse, i.e., simultaneously deciding the order sets and rack sets assigned to stations, and synchronizing their
processing sequence. Regarding our contribution to the literature, our work falls within the operations and control domain of the structure suggested by Azadeh et al. (2019) and addresses the first two most important decisions of the structure proposed by Weidinger et al. (2018).

3 Problem description

We consider joint optimization in the picking process of the KIVA system, which comprises decisions on order and rack allocation to pickers and disposing sequence at each picker. There is growing interest in this joint problem in view of the interrelated impacts of the corresponding subproblems in operating the KIVA warehouses, which are often tackled separately for ease of treatment. Weidinger et al. (2018) provided a four-level hierarchy for order picking, where order selection with assignment and picking station scheduling are the foremost two levels. Merschformann et al. (2017) sketched the crucial relationships between our decision problems, which facilitate the exploitation of their synergy or avoidance of sabotaging one another’s success. We present the problem under study after introducing the robot-assisted KIVA system.

3.1 The robot-based KIVA order picking system

A warehousing system is defined as a combination of hardware and processes regulating the workflow among the hardware elements applied (Boysen et al., 2017). The KIVA system considered in this paper consists of four basic elements to enable the picking function, namely movable storage racks, picking workstations, multiple pickers, and mobile robots (e.g., Boysen et al., 2017; Weidinger et al., 2018; Yang et al., 2021). The mobile robots are powered by electricity, and they perform the rotation and lifting mechanisms with flexible wheels. As for their moving directions, the warehouse floor is invisibly subdivided into grids, each of which is marked with a barcode. An integrated camera system is used to continuously read the barcodes and the robot positions. The rotation mechanism makes the robot move linearly on all sides and the lifting unit can support more than 1,000 kilograms, so that the robot can complete a robotic task of
moving under the rack, lifting it, and transporting it from the storage area to a workstation (D’Andrea et al., 2008). More details of the system are given in Enright and Wurman (2011).

The elementary picking workflow in the KIVA system is as follows: The warehousing management system receives the orders and divides them into several batches. A static picker operating a workstation receives a batch of fixed orders assigned by the system to be disposed of (1). The system simultaneously determines the racks be allocated to a certain picker. The selected racks must enable the picker to satisfy all the SKUs for the assigned orders (2). At each workstation, the picker identifies the order bins with barcode labels and places them on the bench in turn based on the defined sequence. Whenever an order is completed, the corresponding bin is packed and moved out of the station. The vacant position is replaced by the subsequent one (3). According to the active orders on the bench, the allocated racks arrive in line. The picker retrieves items from the current rack and puts them in the relevant bins to satisfy each order (4).

3.2 The order assignment and picking station scheduling problem (OAPSSP)

The above order picking process is treated in the KIVA system with a set of workstations \( P = \{1, \ldots, m\} \), each of which is operated by a static picker \( p \), i.e., \( p \in P \). We define \( S \) as the set of total SKUs that can be purchased by customers from the warehouse. First, the system receives \( n \) orders to be processed together, which are divided into \( m \) sets, and each set contains an approximate average number of orders. Then we need to determine the \( m \) sets of given racks. The set of all the orders is defined by \( O = O^1 \cup O^2 \ldots \cup O^m \), where \( O^{p'} \cap O^p = 0 \), \( 1 \leq p' < p \leq m \). Specifically, each picker \( p \) processes a set of orders \( O^p = \{o^p_1, \ldots, o^p_{|O^p|}\} \), where \( \sum_{p=1}^m |O^p| = n \). Each order \( o^p_i \subseteq S, i = 1, \ldots, |O^p| \) allocated to picker \( p \) is defined as a set of SKUs required by the customer. Note that the subscript \( i \) implies the processing sequence of the order at station \( p \). Then we have a given set of all the useable mobile racks \( R = R^1 \cup R^2 \ldots \cup R^m \), where \( R^p = \{r^p_1, \ldots, r^p_{|R^p|}\} \) is the rack set allocated to picker \( p \) to enable the picking
of his/her assigned orders. Each rack $r_j^p \subseteq S, j = 1, \ldots, |R^p|$ allocated to picker $p$ is defined by the set of SKUs it contains, where the subscript also implies a permutation. Note that a rack can be allocated to different stations, and rack re-visits are allowed. Furthermore, the processing capacity of each workbench is $C$ order bins in parallel, i.e., the number of active orders to be processed simultaneously per picker is limited to $C$.

Suppose that the number of orders contained in each order set assigned to its relevant picker is much larger than $C$. Then it is necessary to coordinate the displayed sequences of the bins on the workbench and of the rational racks serving them over time. Note that in our setting, after finishing an order, the successive order, which substitutes its predecessor in the same position of the workbench, is still able to pick items from the current rack. Each sub-solution for a single station covers two aspects of the crucial characteristic, i.e., the processing sequence of the orders and the arriving sequence of the racks.

**Figure 2.** A representation of the order assignment and picking station scheduling problem

*Example:* Consider a $S = \{A, B, C, D, E, F, G\}$ of different SKUs, which are included in $n = 25$ orders divided into five sets $\{O^1, O^2, O^3, O^4, O^5\}$. Each set thus covers five
orders and is assigned to a specific station. Although we address the following problem for all the stations simultaneously, we only consider picker 5 in detail as an illustration as follows: Picker 5 receives a given order set \( O^5 = \{o_5, o_2, o_4, o_1, o_3\} \), where \( o_1 = \{B, C\}, o_2 = \{C\}, o_3 = \{A, B\}, o_4 = \{A, G\} \), and \( o_5 = \{C, E\} \). Then three racks containing SKUs \( r_1 = \{A, B\}, r_2 = \{C, F\}, \) and \( r_4 = \{E, G\} \) are selected. The capacity of the workbench is limited to \( C = 3 \). Figure 2 depicts a solution based on rack synchronization \( R^5 = \{r_4, r_2, r_1\} \), in which all the orders are satisfied after three rack visits.

4 Model formulation

According to the above-defined OAPSSP, we propose a comprehensive mixed-integer programming (MIP) model to minimize the total number of rack visits, which is considered a well-suited and fundamental objective for such systems (Boysen et al., 2017). Before presenting the model formulation, we introduce the corresponding sets, indices, input parameters, and decision variables as follows:

**Set**

- \( P \) Set of all the workstations
- \( S \) Set of all the SKUs the warehouse holds
- \( R \) Set of all the usable racks
- \( R_i \) Set of racks that contain SKU \( i \) \((R_i \subseteq R)\)
- \( O \) Set of all the orders to be processed

**Index**

- \( p \) Workstation index, \( p \in P \)
- \( o \) Order index \((o \subseteq S), o \in O\)
- \( i \) SKU index, \( i \in S \)
- \( r \) Usable rack index \((r \subseteq S), r \in R\)

**Parameter**

- \( m \) Number of workstations, the same as that of the order sets and rack sets \((p = 1, ..., m)\)
- \( n \) Number of orders to be processed
- \( C \) Capacity of each workstation
- \( T \) Number of time slots \((t = 1, ..., T)\)
### Decision variable

| Symbol | Description |
|--------|-------------|
| $\alpha_t^p$ | Continuous variable: 1, if the racks visiting station $p$ in $t-1$ and $t$ differ |
| $x_o^p$ | Binary variable: 1, if order $o$ is assigned to station $p$ |
| $k_{ot}^p$ | Continuous variable: 1, if order $o$ is tackled at station $p$ in slot $t$ |
| $y_r^p$ | Binary variable: 1, if rack $r$ is assigned to station $p$ |
| $l_{rt}^p$ | Binary variable: 1, if rack $r$ arrives at station $p$ in slot $t$ |
| $\pi_{io}^p$ | Binary variable: 1, if SKU $i$ of order $o$ tackled at station $p$ is delivered in slot $t$ |

Applying the notation summarized above, we formulate our MIP model, denoted as OAPSSP, which consists of the objective function (1) and constraints (2) to (19). We define $\alpha_t^p$ as a “period” in the model, each of which comprises the time slot in which a certain subset of the orders and a certain rack are concurrently processed at a workstation. The next time slot will not emerge until one or more of the changes mentioned before occurs, i.e., two successive slots differ in at least one order being processed or in the visiting rack. So we can easily derive a trivial upper bound on the number of time slots $T = \lceil \frac{n}{m} \rceil \cdot |o_1| \cdot |R|$, where $[x/y]$ means rounding the result of the division $\frac{x}{y}$ downwards (while $\lfloor x/y \rfloor$ means rounding the result of the division $\frac{x}{y}$ downwards).

\[
\text{(OAPSSP) Minimize } \Gamma = \sum_{p \in P} \sum_{t=2}^{T} \alpha_t^p \tag{1}
\]

subject to

\[
\sum_{o \in O} x_o^p = \begin{cases} \lceil \frac{n}{m} \rceil, & 1 \leq p \leq n\%m \\ \lceil \frac{n}{m} \rceil, & n\%m < p \leq m \end{cases}, \quad \forall p \in P
\]  
\tag{2}

\[
\sum_{p \in P} x_o^p \leq 1, \quad \forall o \in O
\]  
\tag{3}

\[
\sum_{t=1}^{T} k_{ot}^p \leq T \cdot x_o^p, \quad \forall o \in O, \forall p \in P
\]  
\tag{4}

\[
\sum_{t=1}^{T} k_{ot}^p \geq x_o^p, \quad \forall o \in O, \forall p \in P
\]  
\tag{5}
\[
\sum_{o \in O} k_{ot}^p \leq C, \quad \forall t = 1, \ldots, T, \forall p \in P \tag{6}
\]
\[
\sum_{r \in R} l_{rt}^p \leq 1, \quad \forall t = 1, \ldots, T, \forall p \in P \tag{7}
\]
\[
\sum_{t=1}^{T} l_{rt}^p \leq T \cdot y_r^p, \quad \forall r \in R, \forall p \in P \tag{8}
\]
\[
\sum_{t=1}^{T} l_{rt}^p \geq y_r^p, \quad \forall r \in R, \forall p \in P \tag{9}
\]
\[
\sum_{t=1}^{T} \pi_{lot}^p \geq x_o^p, \quad \forall i \in o, \forall o \in O, \forall p \in P \tag{10}
\]
\[
\sum_{r \in R_i} l_{rt}^p + k_{ot}^p \geq 2 \pi_{lot}^p, \quad \forall i \in o, \forall o \in O, \forall p \in P, \forall t = 1, \ldots, T \tag{11}
\]
\[
k_{ot}^p + k_{ot}^p \cdot (1 + \alpha_{t}^p), \quad \forall o \in O, 1 \leq t < t' < t'' \leq T, \forall p \in P \tag{12}
\]
\[
l_{rt}^p - l_{r(t-1)}^p \leq \alpha_{t}^p, \quad \forall r \in R, \forall p \in P, \forall t = 2, \ldots, T \tag{13}
\]
\[
0 \leq \alpha_{t}^p \leq 1, \quad \forall t = 1, \ldots, T, \forall p \in P \tag{14}
\]
\[
0 \leq k_{ot}^p \leq 1, \quad \forall o \in O, \forall t = 1, \ldots, T, \forall p \in P \tag{15}
\]
\[
x_o^p \in \{0,1\}, \quad \forall o \in O, \forall p \in P \tag{16}
\]
\[
y_r^p \in \{0,1\}, \quad \forall r \in R, \forall p \in P \tag{17}
\]
\[
l_{rt}^p \in \{0,1\}, \quad \forall r \in R, \forall t = 1, \ldots, T, \forall p \in P \tag{18}
\]
\[
\pi_{lot}^p \in \{0,1\}, \quad \forall i \in o, \forall o \in O, \forall t = 1, \ldots, T, \forall p \in P \tag{19}
\]

Objective (1) minimizes the total number of rack visits at all the workstations. Eq. (2) ensures that the workload assigned to each workstation is approximately balanced, where \(n \% m\) means the modulus operation. Eq. (3) guarantees that each order is allocated to only one workstation. Eqs (4) and (5) ensure that only if an order is allocated to a certain workstation, it will be processed at the station in some time slot(s) or it will not be processed at the station at all. Eqs (6) and (7) guarantee that in each time slot, at most \(C\) orders are processed or at most one rack is visiting. Eqs (8) and (9) ensure that only if a rack is assigned to a certain workstation, it will be processed at the station in some time slot(s) or it is not processed at the station at all. Eq. (10) states that
all the SKUs required by order $o_i$ allocated to workstation $p$ should be delivered at the station, which can happen only in a slot where both $o_i$ and a suitable rack are concurrently processed due to Eq. (11). Eq. (12) guarantees that an order must be processed in succession. Finally, (13) records the rack visiting changes. Eqs (14) to (19) are the integrality constraints, i.e., the domain restrictions of the decision variables. Note that $\alpha^p_i$ and $k^p_{o_i}$ are either forced to take the value 1 or 0 due to Eqs (12) and (13), and the binary nature of the other variables.

The complexity of the proposed MIP model depends on the numbers of customer orders, required SKUs, feasible racks, and workstations. Moreover, the model consists of $|O||P| + |R||P| + |R||T||P| + |O||O||P|$ binary variables, $|T||P| + |O||T||P|$ continuous variables, and $|P| + |O| + 3|O||P| + 3|T||P| + 2|R||P| + |o||O||P| + 2|o||O||P||T| + 2|R||P||T| + \{1(|T| - 2) + 2(|T| - 3) + 3(|T| - 4) + \cdots + (|T| - 2)(|T| - (|T| - 1))\}$ constraints. Note that the final term of the total constraint calculation is caused by constraint (12). The larger the instance size becomes, the larger the solution space and the greater the number of constraints are. Therefore, solving the MIP model by a commercial solver such as Gurobi is very difficult and time-consuming.

In addition, OAPSSP is not only a complicated MIP model but also an NP-hard problem because if the orders and racks allocated to each workstation are fixed, it reduces to the mobile robot-based order picking problem (MROP) (Boysen et al., 2017). They showed that MROP is NP-hard by reducing it to the set covering problem (Garey & Johnson, 1979). Consequently, efficient heuristic algorithms are necessary for tackling the problem, the development and implementation of which we discuss in the next section.

5 The proposed approach

We develop a metaheuristic algorithm to effectively find a good feasible solution for real-world-sized instances of OAPSSP. The general search framework is based on simulated annealing (SA) (see, e.g., Kirkpatrick et al., 1983). SA is an algorithmic approach to solve combinatorial optimization problems (Cerný, 1985; Aarts et al., 1997). It randomizes the local search procedure and accepts changes that worsen the
solution with some probability. Thus, SA constitutes an attempt to reduce the probability of getting trapped in a suboptimal solution. In recent years, the SA framework has shown excellent performance in solving problems of sequencing requests (Baardman et al., 2021), workforce scheduling (Rijal et al., 2021), resource replenishment (Hof and Schneider, 2021), etc. In the following we first outline the framework of the proposed approach along with recalling the primary SA principles. Then we present a model-based matheuristic method as a reduction rule to narrow the search space of each instance, i.e., eliminating some of the racks within the warehouse while maintaining the optimization direction. We next introduce a heuristic search procedure to construct complete candidate solutions and describe the neighbourhood operators.

5.1 The heuristic framework

The heuristic operates as follows: First, an initial feasible solution $x_0 = (\theta_0, \mu_0)$ is constructed. Each solution $x$ consists of both schedule $\theta = \{O^1, \ldots, O^p, \ldots, O^m\}$ and schedule $\mu = \{R^1, \ldots, R^p, \ldots, R^m\}$, where $\theta$ contains $m$ detailed schedules for $m$ picking stations, and so does $\mu$. Let the fitness value $f(x)$ reflect the objective value of the solution $x$, i.e., $\sum_{p=1,\ldots,m} |\mu^p|$, which is calculated by the beam search procedure based on dynamic programming. Then the heuristic keeps iterating before the terminating criterion is met. In each iteration, the algorithm tries to find a solution with a smaller objective value by changing a part of the solution, namely $\theta$, for which one of its neighbours has been selected at random. Then a new order schedule $\theta'$ is obtained from the neighbourhood. Once $\theta'$ is given, the problem to obtain a new feasible solution $x'$ reduces to deciding which racks should be delivered to each station and their representation sequences. The current solution is updated following the primary SA principles, which have several key parameters: a temperature $\tau$, a cooling rate $\alpha$ ($0 < \alpha < 1$), and the length of each iteration epoch.
Algorithm 1 (Pseudocode of the proposed heuristic)

1: Apply RSP, resulting in $\theta_0$
2: Apply IBS based on $\theta_0$, resulting in $\mu_0$
3: Construct a feasible initial solution $x_0 = (\theta_0, \mu_0)$
4: Set $T = -w \cdot f(x_0)/\ln 0.5$, $K = 10$
5: Set the current solution $x \leftarrow x_0$, $f(x) \leftarrow f(x_0)$
6: while the stop criterion is not met do
7: Initialize $x^{\text{min}}, x^{\text{max}}$, and $k = 1$
8: while $k < K$ do
9: Randomly choose a neighbourhood operator $n() \in N$
10: Apply $n()$ to $\theta$, resulting in $\theta'$
11: Calculating $f(x')$ through IBS
12: if $f(x') < f(x)$ then
13: Set $x \leftarrow x'$, $x^{\text{min}} \leftarrow x'$, $x^* \leftarrow x'$
14: else
15: Set $x \leftarrow x'$ with probability $p = \exp\left(\frac{f(x')-f(x)}{T}\right)$
16: Set $x^{\text{max}} \leftarrow x'$
17: end if
18: $k = k + 1$
19: end while
20: $K = K \cdot \left(1 - \exp\left(\frac{f(x^{\text{min}})-f(x^{\text{max}})}{f(x^{\text{max}})}\right)\right)$
21: $T = \alpha \ast T$
22: end while
23: return $x^*$

- Following Ropke and Pisinger (2006), and Masson et al. (2013), the temperature $\tau$ is initialized in such a way that $\tau := -w \cdot f(x_0)/\ln 0.5$, where $w$ is an arbitrary parameter and $x_0$ is the initial solution, i.e., a solution that is $w\%$ worse than $f(x_0)$ has a $50\%$ probability to be accepted (cf. Kovacs et al. 2012).
- Once the temperature value $\tau$ is given, an epoch of $K$ iterations is started. Then $\tau$ is lowered to $\alpha \tau$ when an epoch is finished, and the next one is started. Finally, we
set the terminating criterion such that at most 5,000 iterations are executed or $\tau$ falls below 0.01 or a given time frame is consumed.

- According to Cho et al. (2005), the length of the initial epoch is set as ten and modified as $K := K + [K \cdot (1 - \exp \left( \frac{f_{\text{min}} - f_{\text{max}}}{f_{\text{max}}} \right))]$ after each epoch. Note that $f_{\text{min}}$ is the smallest fitness value recorded in the past epoch and $f_{\text{max}}$ is the largest.

In each iteration, a neighbourhood solution $x'$ is reached. Then, the difference in the fitness values $\Delta f = f(x') - f(x)$ is calculated. The new feasible solution $x'$ is always accepted if its objective value is better than that of $x$, i.e., $\Delta f < 0$. Otherwise, $x'$ substitutes $x$ with probability $p = \exp (-\Delta f / \tau)$.

5.2 Reduction rule

Given that the type of warehouses under study often adopts the a mixed-shelves or scattered storage policy, i.e., the same products are stored in multiple racks (Weidinger and Boysen, 2018), there exist many interchangeable racks that can be removed to narrow the search space. In the following we first propose a MIP formulation derived from the OAPSSP model to reflect the above rack selection problem, denoted as RSP. Note that the objective of RSP is to minimize the total number of racks allocated to the picking stations, which contributes to less movement of the racks and encourages the picking of multiple products for different orders from the same rack as discussed in Boysen et al. (2017) and Hanson et al. (2018). Then we apply a heuristic method to solve it, which draws mainly on the specific mathematical formulation matheuristics (cf. Boschetti et al., 2009; Valley and Beasley, 2021) and makes direct use of a standard optimization solver. Experiments have shown that software like Gurobi can solve the RSP efficiently by exploiting the mentioned method.

There exists an added element $\delta_{io}^p$ of the RSP model, which is the transformation of $\pi_{i{o}}^p$ and represents that SKU $i$ of order $o$ is processed at station $p$. The objective (20) minimizes the total number of racks allocated to process all the pending orders. Eqs (21) and (22) are the same as Eqs (2) and (3), respectively, which are the constraints of
workload balance and order assignment, respectively. Eq. (23) states that all the SKUs required by order \( o_i \) allocated to workstation \( p \) should be delivered to the station, which only happens when both conditions hold, i.e., order \( o \) and some rack containing its required SKUs are assigned to the special workstation due to Eq. (24). Note that each SKU \( i \) of order \( o \) can be traversed in our setting.

\[
(RSP) \text{ Minimize } \Lambda = \sum_{p \in P} \sum_{r \in R} y_r^p
\]

subject to

\[
\sum_{o \in O} x_o^p = \begin{cases} 
\lfloor n/m \rfloor, & 1 \leq p \leq n\%m \\
\lfloor n/m \rfloor, & n\%m < p \leq m
\end{cases}, \quad \forall p \in P
\]

(21)

\[
\sum_{p \in P} x_o^p \leq 1, \quad \forall o \in O
\]

(22)

\[
\delta_{io}^p \geq x_o^p, \quad \forall i \in O, \forall o \in O, \forall p \in P
\]

(23)

\[
\sum_{r \in R_i} y_r^p + x_o^p \geq 2\delta_{io}^p, \quad \forall i \in O, \forall o \in O, \forall p \in P, \forall t = 1, \ldots, T
\]

(24)

\[
x_o^p \in \{0, 1\}, \quad \forall o \in O, \forall p \in P
\]

(25)

\[
y_r^p \in \{0, 1\}, \quad \forall r \in R, \forall p \in P
\]

(26)

\[
\delta_{io}^p \in \{0, 1\}, \quad \forall i \in o, \forall o \in O, \forall p \in P
\]

(27)

Specifically, our matheuristics method is executed as follows: The RSP model above is solved optimally for each single picking station in a sequential manner, until all the stations have been considered. As supposed above, the picking stations and their corresponding pickers are indexed in decreasing order, i.e., \( P = \{1, \ldots, m\}, p \in P \). Thus, there exists a natural ordering for them. The detailed procedure and pseudocode are as follows:

(a) Set \( p = 1 \);

(b) Use the standard solver to optimally solve the RSP model, which has the single workstation and picker \( p \);
(c) Remove the orders assigned to the picker from the given data set and record the chosen racks;

(d) Set $p = p + 1$ and if $p \leq m$ return to (b);

(e) Use the solver to simultaneously solve the RSP model for all the $m$ workstations and pickers, but only attending to those racks that are chosen when the workstations are calculated individually.

If we perform Steps (a)-(d) in turn for every picker, a heuristic solution will be generated for the problem. However, performing Step (e) is potentially possible to further improve the quality of the solution, which utilizes our proposed formulation to solve the RSP problem, but only focusing on the subset of movable racks that are chosen for each workstation. Obviously, the number of racks that determines the computational effort will become far smaller. Furthermore, the procedure needs to perform $m + 1$ optimization processes as the total computational effort.

5.3 Constructing candidate solutions

5.3.1 An initial feasible solution

As proposed above, a complete candidate solution $x$ consists of two elements $\theta$ and $\mu$. The initial solution is generated as follows: We obtain an order schedule $\theta = \{O^1, ... O^p, ... O^m\}$ from the solution of the RAP model, which concludes the orders assigned to each picking station. Accordingly, $\theta_0$ can be determined in the way that the orders assigned to each station are encoded as a random permutation. Moreover, the whole order sequence is represented by a tuple where the sets of orders processed at the same station are separated by a symbol, e.g., symbol zero (see Figure 3). Once the order sequence is given, we can apply the following search procedure to obtain a high-quality rack schedule $\mu$, so returning a complete initial feasible solution.

![Order schedule $\theta$ representation](image-url)
5.3.2 Neighbourhood operators

In this section we employ three types of neighbourhood structures \( n() \in N \). As mentioned before, when the SA algorithm attempts to move to a new solution, one of these types is selected randomly with equal probability. Then the algorithm moves to the new solution that is a feasible solution chosen stochastically among the neighbouring solutions of this type. Position-based neighbourhoods are commonly used for permutations that represent scheduling problems. Therefore, we select three position-based neighbourhood operators for the problem.

- **Swap**: Select two points at random and swap their positions;

  \[
  \theta = \begin{array}{cccccccc}
  1 & 2 & 3 & 4 & 0 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
  \end{array}
  \]

  \[
  \theta' = \begin{array}{cccccccc}
  1 & 2 & 9 & 4 & 0 & 5 & 6 & 7 & 8 & 0 & 3 & 10 & 11 & 12 \\
  \end{array}
  \]

  **Figure 4(a).** Neighbourhood operator \( n(1) \)

- **Shift**: Randomly select three points and shift the points between the first two points to after the third point;

  \[
  \theta = \begin{array}{cccccccc}
  1 & 2 & 3 & 4 & 0 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
  \end{array}
  \]

  \[
  \theta' = \begin{array}{cccccccc}
  1 & 2 & 8 & 9 & 0 & 10 & 3 & 4 & 5 & 0 & 6 & 7 & 11 & 12 \\
  \end{array}
  \]

  **Figure 4(b).** Neighbourhood operator \( n(2) \)

- **Inversion**: Randomly select two points and reverse the order between them completely.

  \[
  \theta = \begin{array}{cccccccc}
  1 & 2 & 3 & 4 & 0 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
  \end{array}
  \]

  \[
  \theta' = \begin{array}{cccccccc}
  1 & 2 & 3 & 8 & 0 & 7 & 6 & 5 & 4 & 0 & 3 & 10 & 11 & 12 \\
  \end{array}
  \]

  **Figure 4(c).** Neighbourhood operator \( n(3) \)
5.3.3 Fitness value

Given a feasible solution $x = (\theta, \mu)$ of OAPSSP, $x$ is evaluated by $f(x) = \sum_{p=1}^{m} |\mu^p|$, where $m$ is the number of picking stations and $|\mu^p|$ is the length of the rack schedule, i.e., the number of rack visits, at picker $p$. We derive $f(x)$ from a designed beam search (BS) procedure, which is based on dynamic programming to find the optimal solution.

We briefly introduce the general BS mechanism. A graph search heuristic initially applied in the field of speech recognition, BS was first introduced to solve scheduling problems and compared with other well-known heuristics by Ow and Morton (1988). Since then, BS has been extended to a powerful metaheuristic applicable to many real-world optimization problems (Blum, 2005; Boysen and Zenker, 2013). More details of this heuristic and its extensions can be found in Sabuncuoglu et al. (2008). BS executes the searching procedure based on a tree representation of the solution. However, it does not apply the breadth-first approach (e.g., the branch-and-bound technique) but restricts the number of nodes per stage to be further branched to a promising subset. The size of the subset is determined by a given parameter, i.e., the beam width $BW$, and the nodes to be selected in the subset are evaluated by the filtering process. Thus, the search process can be illustrated as follows: Starting with the root node, all the nodes of stage 1 are built, among which the promising nodes are identified by filtering. Note that filtering can be obtained by a priority value based on a specific issue. Thus, the promising subset of stage 1 consists of the $BW$ best nodes found by filtering, which are further branched to construct the set of nodes in stage 2. Then again, filtering is applied to delete some poor nodes of stage 2. The above steps continue until the final stage is reached and the result of BS is returned.

Three components need to be predefined when applying BS for a specific problem, namely graph structure, parameter $BW$, and filtering. In the following we provide their specifications for our problem.
We introduce the DP procedure that can be directly used for BS. The procedure can be subdivided into no more than $m(T + 1)$ stages, where stage $s^p = 0, 1, ..., T$ ($1 \leq p \leq m$) determines the allocation of racks to each sequence position at station $p$. Note that $T$ corresponds to the trivial upper bound on the number of time slots mentioned in Section 4. Each stage contains states $(\partial^1_c, ..., \partial^C_c, \psi, s^P)$, where:

- $\partial^P_c$ represents the set of unsatisfied SKUs required by the order currently processed in space $c$ on the workbench;
- $\psi$ is the pointer for the next permutation to be processed of the order sequence $\theta^P$;
- $s_0^P$ is the initial state holding $(o^1_{\theta^P}, ..., o^C_{\theta^P}, C + 1, 0)$.

As mentioned before, once an order is processed, it need not wait for the arrival of the successive rack. Moreover, the SKUs in $o_i$ should be provided from the first available rack containing it when $o_i$ is active on the workbench. Then we specify the transitions as follows, which exist only between two coterminous stages $s^P$.

1. If $\partial^P_c \setminus r_j \neq \emptyset$ for each $c$, then the state changes to $(\partial^1_c \setminus r_j, ..., \partial^C_c \setminus r_j, \psi, s^P + 1)$;
2. If $\partial^P_c \setminus r_j = \emptyset$ for any $c$ (maybe more than one), i.e., there exist $\bar{C}$ ($\bar{C} \leq C$) positions on the workbench in which the orders can be processed with rack $j$, then the pending orders in $[\psi, ..., \psi + \bar{C} - 1]$ positions of $\theta^P$ substitute these orders. Consequently, the corresponding states, pointer, and stage change to $o^P_{\theta^P \setminus r_j, ..., o^P_{\theta^P + \bar{C} - 1} \setminus r_j, \psi + \bar{C}, s^P + 1}$, respectively.

Finally, the successor states will not terminate until the state $(\partial^1_c = \emptyset, ..., \partial^P_C = \emptyset, |O^P| + 1, s^P)$ is reached, which represents that the order picking process at workstation $p$ is finished. Furthermore, for each picking station, the optimal objective value is equal to $s^P$ and the optimal rack sequence $\mu^P$ can be simply obtained based on backward recursion.
• Parameter BW

There may exist a poor upper bound $UB$ when applying a comparatively large $BW$, which hurts the performance of BS. Thus, an iterated beam search ($IBS$) can be applied to make BS benefit from a tight $UB$. Specifically, we initialize an ordered list of increasing beam widths $BW$. First, BS is executed with a small $BW$ for quickly generating an initial $UB$, which is passed to the next iteration of BS executed with a larger $BW$ and so on.

\begin{algorithm}
1: Input: the ordered list $\gamma$ of increasing beam widths $BW$
2: Initialize $UB = \infty$ and $i = 1$
3: while $i$ does not reach the length of $\gamma$ do
4:  Solve BS with $UB$ and $BW = \gamma_i$
5:  Update $UB = \text{result of BS}$
6: end while
7: Calculate $f(x)$ based on the optimal sequence
8: return $f(x)$
\end{algorithm}

• Filtering process

BS restricts the number of states that are further explored in each stage to the $BW$ most promising ones. To select the $BW$ states out of the set of branched states per stage, we rank them according to the number of orders that have not been processed, i.e., $|OP| - \psi$, and apply the minimum number of currently remaining SKUs on the workbench as the tie-breaker, i.e., $|\cup_{c=1}^C \delta_c^P|$.

6 Computational studies

In this section we present the results of numerical studies conducted to assess the performance of our proposed metaheuristic approach. As there exists no established testbed for OAPSSP, we generate our instances. First, we discuss in Section 6.1 how we generate the test instances. Then, we present in Section 6.2 the performance of our heuristic solution approaches in comparison with that of Gurobi. Finally, we provide in
Section 6.3 managerial insights and suggestions concerning a real-life robotic mobile fulfillment system. Specifically, we introduce as a benchmark a traditional policy that is widely used for the type of warehouses under study. In addition we introduce another benchmark that executes picking station scheduling with random order assignment. We also conduct a series of sensitivity analyses to examine how different parameters impact the order picking process.

We conduct all the numerical studies on a 64-bit PC with an Intel Core i7-10510U (1.80GHz and 2.30GHz) and 16.0 GB main memory operating under Windows 10. We code the procedures in C++ (Visual Studio 2019) and use the off-the-shelf solver Gurobi (version 9.1.0) to solve all the test instances. We repeat the solution procedure ten times in each parameter setting.

6.1 Instance generation

The KIVA system is typically applied in intelligent distribution centres where many small SKUs are stored in a scattered manner on racks. Our instance generation follows the real-world operating rules adopted in Jing Dong Asia No. 1 Warehouse that uses a similar system (Neuhub, 2019). We subdivide the test instances into small and large sizes. The former can be solved by the standard solver while the latter represent real-world-sized instances.

The basic steps of instance generation are as follows: First, there are a total of \( n \) orders requiring to be processed by \( m \) parallel picking stations over the processing horizon. There are the SKU set \( S \) and the rack set \( R \) with \( \beta \) different SKUs contained in each rack. Then, the number of spaces on each workbench \( C \) is given. The number of covered SKUs within \([\theta; \theta']\) for each of the \( n \) orders is randomly picked according to a discrete uniform distribution. We provide the details in the following.

The average order comprises just 1.6 items (Boysen et al., 2019) and the vast number of orders contain only one or two items (Weidinger, 2018). Accordingly, we set \([\theta; \theta']\) as [1; 2]. The racks may have up to 50 storage locations (see CNN Business, 2018). Valle and Beasley (2021) assumed that each rack can store 25 different products
per rack. Accordingly, we vary $\beta$ from 10 to 30 for different sized instances. Moreover, it is obvious that different SKUs in the distribution centre have varying picking frequencies, e.g., some SKUs belong to the best-selling products. To consider this, we make the following rule to generate the SKUs required by each order, which are randomly selected via an exponential distribution with an exponent of 0.5 (see, e.g., Boysen et al., 2017). Each SKU contained on a rack is also selected according to the same exponential distribution, so the better-selling items are more likely to appear on several racks concurrently.

6.2 Algorithmic performance

In this section we test the small-sized instances and provide the results when they are solved by the standard solver Gurobi and our proposed heuristic algorithm. We aim to demonstrate the sensitivity of the solution quality and computation time to the following input characteristics: number of picking stations $m$, number of orders $n$, total number of SKUs $|S|$, capacity of each workbench $C$, and storage diversity of each rack $\beta$.

We use the relative difference $rd$ (%) as the evaluation metric (Bodnar et al., 2015). Specifically, given two objective function values $f_A(x)$ and $f_B(x)$ obtained by applying algorithms $A$ and $B$, respectively, to solve an instance $x$, we compute the $rd$ of $A$ as

$$\text{rd} = \left(\frac{f_A(x)}{f_B(x)} - 1\right) \cdot 100\%.$$ 

Note that Opt. means the number of instances for which Gurobi can find the optimal solutions within the given time frame of 600 CPU seconds; $rd$ refers to the percentage of the relative difference between the best solution found by SA and the solution found by Gurobi; CPU time is in seconds. Tables 1 and 2 present the results for a total of 72 instances. We draw the following conclusions from the test results.

- Table 1 shows the results for instances with two picking stations in total, where each picker processes no more than ten orders. The standard solver Gurobi finds feasible solutions for all the instances, among which 63.9% are optimal. Our SA
solutions are always within a 7.9% gap from the optimal solutions produced by Gurobi and within 2% on average, whereas the average CPU time of the SA algorithm is less than 60 seconds (1 minute).

Table 1. Numerical results with $m = 2$ picking stations.

| $n$ | 15 | 20 |
|-----|----|----|
| $C$ | 3  | 5  | 3  | 5  |
| $\beta$ | Gurobi | SA | Gurobi | SA | Gurobi | SA | Gurobi | SA |
| Opt. | CPU | rd | Opt. | CPU | rd | Opt. | CPU | rd | Opt. | CPU | rd |
| 6  | 10  | 62 | 2.0  | 10  | 25 | 2.5  | 10  | 2  | 7.9  | 60  | 0  | 600  | 0.0 | 58  |
| 8  | 10  | 6  | 2.5  | 10  | 45 | 2.5  | 10  | 20 | 2.0  | 48  | 0  | 600  | 0.0 | 39  |
| 10 | 10  | 7  | 0.0  | 10  | 5  | 0.0  | 10  | 2  | 592  | 0.0 | 31  |

(a) No. of total SKUs $|S| = 20$

(b) No. of total SKUs $|S| = 25$

(c) No. of total SKUs $|S| = 30$

| $n$ | 15 | 20 |
|-----|----|----|
| $C$ | 3  | 5  | 3  | 5  |
| $\beta$ | Gurobi | SA | Gurobi | SA | Gurobi | SA | Gurobi | SA |
| Opt. | CPU | rd | Opt. | CPU | rd | Opt. | CPU | rd | Opt. | CPU | rd |
| 6  | 0   | 601 | 1.7  | 0  | 601 | 1.7  | 0  | 601 | 3.7  | 0  | 600  | 0.9 | 101 |
| 8  | 10  | 16 | 2.5  | 8  | 544 | 0.0  | 10  | 600 | 4.0  | 10  | 35   | 5.0 | 66  |
| 10 | 10  | 9  | 1.5  | 10  | 38 | 0.0  | 10  | 285 | 4.5  | 6  | 401  | 2.5 | 47  |
| 6  | 0   | 600 | 0.3  | 0  | 600 | 0.0  | 0  | 600 | 0.0  | 0  | 600  | 0.3 | 109 |
| 8  | 8   | 498 | 2.0  | 0  | 601 | 0.0  | 0  | 600 | 2.3  | 0  | 600  | 2.5 | 32  |
| 10 | 10  | 10 | 0.0  | 10  | 4  | 0.5  | 10  | 29 | 10  | 0.0 | 36  |

- It is clear from Table 1 that the computational time to obtain an optimal solution with Gurobi depends on the number of orders allocated to each station, the total number of SKUs held in the warehouse, and the storage diversity per rack. Indeed, these parameters can be used to characterize the complexity of the order picking operations.

- We next consider the instances with the number of stations being extended to three and report the results in Table 2. We see that the standard solver struggles to find the optimal solution within the given time frame as the size of the instance increases. Specifically, Gurobi finds the optimal solution for only one-twelfth of the instances. It should be mentioned that even when we relax the time limit to 1,800 CPU seconds (30 minutes), the performance of the commercial software has hardly any
improvement. In contrast, our SA solutions are always within a 9.8% gap from the best-known solutions, and the average gap is 4.8% when only considering the instances for which the optimal solutions are found.

Table 2. Numerical results with $m = 3$ picking stations.

| $n$ | $C$ | 25       | 30       |
|-----|-----|----------|----------|
|     |     | 3        | 5        | 3        | 5        |
| $\beta$ | Gurobi | SA | Gurobi | SA | Gurobi | SA | Gurobi | SA |
| Opt. | CPU | rd | Opt. | CPU | rd | Opt. | CPU | rd |

(d) No. of total SKUs $|S| = 20$

|     | 6  | 0  | 601 | 2.7 | 55 | 0  | 600 | 5.3 | 79 | 0  | 601 | 9.8 | 73 | 0  | 601 | 2.9 | 64 |
|-----|----|----|-----|-----|----|----|-----|-----|----|----|-----|-----|----|----|-----|-----|----|
| 8   | 0  | 600| -0.7| 37  | 0  | 601| 0.0 | 50  | 0  | 600| 2.7 | 45  | 8  | 544| 0.0 | 30  |
| 10  | 0  | 600| 0.0 | 29  | 10 | 21 | 5.0 | 37  | 0  | 600| -0.8| 35  | 0  | 601| 4.0 | 37  |

(e) No. of total SKUs $|S| = 25$

|     | 6  | 0  | 601 | 2.9 | 97 | 0  | 601| 0.0 | 79 | 0  | 600| 9.3 | 116| 0  | 601| 2.5 | 152|
|-----|----|----|-----|-----|----|----|-----|-----|----|----|-----|-----|----|----|-----|-----|----|
| 8   | 0  | 600| 5.6 | 41  | 0  | 600| -2.1| 66  | 0  | 600| 5.7 | 80  | 0  | 601| 3.3 | 97  |
| 10  | 0  | 601| -1.5| 46  | 0  | 601| 0.0 | 64  | 0  | 601| 1.7 | 61  | 10 | 62 | 9.5 | 56  |

(f) No. of total SKUs $|S| = 30$

|     | 6  | 0  | 600| 2.9 | 99 | 0  | 600| 2.5 | 139| 0  | 600| 5.5 | 152| 0  | 601| 4.4 | 119|
|-----|----|----|-----|-----|----|----|-----|-----|----|----|-----|-----|----|----|-----|-----|----|
| 8   | 0  | 601| 0.0 | 65  | 0  | 600| 2.9 | 80  | 0  | 600| 2.5 | 101| 0  | 600| 2.5 | 122|
| 10  | 0  | 600| 6.7 | 42  | 0  | 601| 0.5 | 38  | 0  | 601| 5.3 | 66  | 0  | 601| 2.5 | 81  |

We conclude that Gurobi is unable to solve real-world-sized instances of OAPSSP. Moreover, while the proposed solution approach performs well for small-sized instances, further assessment of its performance in tackling real-world-sized instances is needed.

6.3 Managerial aspects

In this section we explore how SA performs for real-life-sized instances, for which applying Gurobi is out of the question for its lack of computing power. We thus introduce as a benchmark a representative rule-based method widely used in the real world. Furthermore, we present a less sophisticated method as an alternative benchmark, which performs random order assignment, to ascertain the extent to which joint
optimization improves performance.

6.3.1 Benchmark processes

- **Rule-based method (RB)**

Many realistic warehouses simply apply the first-come-first-served (FCFS) rule for processing orders (Yang et al., 2021). Specifically, according to their arriving times, orders in the order pool have a natural permutation. Suppose that the set of orders to be processed is fixed. Then the first order is assigned to station one, the second order assigned to station two, and so on until all the orders have been allocated. Each picker will individually receive a picking list in a given sequence. For each station, the orders are released to the workbench sequentially according to the given list. Then the selected racks are delivered successively, i.e., one by one, each of which contains mostly the items required by the currently active orders. Once an order is processed, it is substituted by the next order stipulated on the list, and additional racks will arrive until the whole picking process is completed.

- **Random order assignment (ROA)**

To illustrate that joint optimization of the interrelated picking links plays a critical role in enhancing efficiency, we compare our proposed solution method with an alternative approach that optimizes picking station scheduling separately without the use of any order assignment policy. Specifically, the alternative method assigns orders to stations randomly and evenly, and generates an arbitrary processing sequence for each order set. Then picking station scheduling follows the elementary procedure introduced in Section 5.1 in detail; however, the original neighbourhood operators are restricted to be applied at a single station.

6.3.2 Numerical results

In this section we further test moderate- and large-sized instances in realistic settings. We set a time frame of 1,800 CPU seconds for solving each instance, which is regarded as a reasonable cap on the elapsed time allowed in actual operations considering the
time interval for batch generation in the order pool. We show in Table 3 the parameters set by our generator. We test each one of the parameter combinations ten times. All the results obtained are applied in the following computation and sensitivity analyses.

**Table 3.** Varying the parameter values for the generation of real-world-sized instances.

| Parameter | Description                                | Moderate | Large   |
|-----------|--------------------------------------------|----------|---------|
| $n$       | Number of orders                           | 500, 750 | 1000, 1500 |
| $m$       | Number of picking stations                 | 5        | 10      |
| $C$       | Capacity per workbench                     | 10, 15, 20 |         |
| $\beta$   | Storage per rack                           | 15, 25, 30 |       |
| $[\theta; \theta']$ | Quantity range of SKUs per order            | [1; 2]   |         |
| $|S|$   | Total number of SKUs                      | 800      |         |

Tables 4 and 5 present the numerical results for instances with five and ten picking stations in total, respectively, where the number of orders assigned to each station is set as 100 and 150. The column “Sol.” means the average objective value for each instance, and “rd” refers to the percentage of the relative difference between the best solutions found by the ROA/RB method and by SA. We first present the algorithmic performance of the three methods. Then we generate some managerial insights and operating suggestions from the following computation and sensitivity analyses.

- The straightforward rule-based (RB) method always requires almost negligible computing times to generate feasible solutions, even for cases involving 1,500 orders with ten picking stations. However, it creates considerable and unacceptable gaps relative to the solution values of ROA and SA, increasing with the number of orders to be processed per station. Under the worst-case scenario, the relative difference between RB and ROA reaches 94.3%, and that between RB and SA is even up to 1,077.4%.
- For ROA, while it does optimize the solution value derived from RB to some extent, it is dwarfed by SA. The capacity of ROA falls behind by an average of over a 100 per cent gap compared with SA. In addition, the advantage of ROA over SA in
terms of computing time shrinks as the size of the instance increases. Indeed, as the number of picking stations doubles, the computation of ROA tends to require twice as much time. However, the CPU time for the SA heuristic is not affected by this factor. In other words, ROA takes considerable time, yet achieving little improvement, which means that the time spent on picking station scheduling is close to futile if a reasonable order allocation policy is not adopted.

**Table 4** Numerical results with \( m = 5 \) picking stations.

| \( n \) | 500 | 750 |
|--------|-----|-----|
| \( \beta \) | SA | ROA | RB | SA | ROA | RB |
| Sol. | CPU | \( rd \) | CPU | \( rd \) | CPU | Sol. | CPU | \( rd \) | CPU | \( rd \) | CPU |
| (a) Capacity per workbench \( C = 10 \) |
| 15 | 350 | 1121 | 44.6 | 542 | 52.3 | 6 | 173 | 958 | 336.4 | 596 | 397.1 | 3 |
| 25 | 317 | 1011 | 35.3 | 557 | 67.8 | 3 | 118 | 957 | 440.7 | 566 | 653.9 | 2 |
| 30 | 302 | 1029 | 34.8 | 543 | 76.5 | 2 | 142 | 968 | 307.8 | 553 | 520.9 | 2 |
| (b) Capacity per workbench \( C = 15 \) |
| 15 | 231 | 1188 | 95.7 | 567 | 117.3 | 3 | 157 | 1075 | 324.8 | 556 | 424.2 | 2 |
| 25 | 210 | 975 | 69.1 | 553 | 143.8 | 3 | 174 | 973 | 187.9 | 549 | 373.6 | 2 |
| 30 | 140 | 965 | 133.6 | 542 | 264.3 | 2 | 114 | 916 | 321.9 | 563 | 620.2 | 2 |
| (c) Capacity per workbench \( C = 20 \) |
| 15 | 253 | 1091 | 63.2 | 584 | 77.9 | 3 | 223 | 1220 | 170.9 | 651 | 248.4 | 4 |
| 25 | 216 | 959 | 41.2 | 562 | 109.7 | 2 | 191 | 940 | 134.6 | 579 | 310.5 | 3 |
| 30 | 168 | 921 | 66.7 | 553 | 163.7 | 2 | 161 | 926 | 150.9 | 550 | 234.0 | 2 |

- Finally, our proposed SA algorithm can always provide satisfactory solutions within the given time frame even for large-sized instances. Moreover, the results show that the solution quality of SA is sensitive to the number of orders assigned to each station. When all the other parameters are fixed, the improvement of SA over the other two methods is always better when the average workload per station is 150 orders rather than 100 orders.

Furthermore, we focus on the results of the large-sized instances \( (m = 10) \) and further explore the impacts of different operating settings on picking efficiency. We define a new evaluation metric, i.e., order fulfillment per unit rack visit (of.), which indicates
the number of orders fulfilled during a rack visit, i.e., \( o_f = \frac{n}{\text{sol.}} \).

Table 5 Numerical results with \( m = 10 \) picking stations.

| \( \beta \) | \( n \) | 1,000 | 1,500 |
|---|---|---|---|
| | SA | ROA | RB | SA | ROA | RB |
| | Sol. | CPU | \( rd \) | CPU | \( rd \) | CPU | Sol. | CPU | \( rd \) | CPU | \( rd \) | CPU |
| \( (a) \) Capacity per workbench \( C = 10 \) | | | | | | | | | | | |
| 15 | 440 | 1532 | 130.9 | 1149 | 169.8 | 4 | 326 | 1798 | 371.8 | 1255 | 430.4 | 4 |
| 25 | 227 | 1378 | 270.9 | 1148 | 369.6 | 6 | 161 | 1277 | 686.3 | 1151 | 977.0 | 3 |
| 30 | 203 | 1377 | 287.7 | 1167 | 409.4 | 3 | 146 | 1227 | 728.1 | 1194 | 1077.4 | 2 |
| \( (b) \) Capacity per workbench \( C = 15 \) | | | | | | | | | | | |
| 15 | 475 | 1628 | 89.3 | 1331 | 104.0 | 5 | 358 | 1447 | 284.4 | 1380 | 357.3 | 3 |
| 25 | 382 | 1442 | 81.2 | 1131 | 156.8 | 3 | 231 | 1342 | 348.1 | 1210 | 608.7 | 3 |
| 30 | 369 | 1298 | 78.6 | 1117 | 169.9 | 4 | 174 | 1548 | 453.5 | 1169 | 844.8 | 2 |
| \( (c) \) Capacity per workbench \( C = 20 \) | | | | | | | | | | | |
| 15 | 415 | 1745 | 99.3 | 1331 | 108.9 | 4 | 445 | 1667 | 171.7 | 1430 | 250.6 | 3 |
| 25 | 283 | 1508 | 111.7 | 1133 | 208.5 | 4 | 280 | 1451 | 207.5 | 1288 | 454.3 | 3 |
| 30 | 264 | 1298 | 116.7 | 1116 | 234.9 | 3 | 242 | 1527 | 236.4 | 1236 | 541.3 | 3 |

Figure 5 visually confirms our previously proposed conclusion that SA outperforms the other two methods by a wide margin, and that ignoring the order assignment policy leads to considerable optimality gaps. At the same time, we gain some managerial insights.

- First, SA optimizes more effectively when the number of orders processed by a single picker is larger, while the other two approaches have the opposite characteristics. The reason may be that SA creates a scale effect through the joint optimization of order assignment, which implies that the closely associated relationships between our decision problems can facilitate either exploitation of their synergy or avoidance of sabotaging one another’s success.

- For the presented SA results, there is no evidence that the handling capacity per workbench \( C \) necessarily increases \( o_f \) for a given amount of rack storage \( \beta \), while expanding the storage density always increases \( o_f \) for a given \( C \). Therefore, a
A proper setting combination needs to be found when designing the warehouse. Specifically, in our experimental environment, the optimal combination setting is \( \frac{n}{m} = 150, C = 10, \beta = 30, \) and \( of. = 6.8. \) In other words, compared with the best case of ROA, a single rack visit can satisfy nearly four times the number of orders.

![Sensitivity analysis](image)

**Figure 5.** Sensitivity analysis.

### 7 Conclusions

We consider the joint optimization of order assignment and picking station scheduling in the robot-assisted KIVA warehouse. Contrary to picker-to-parts warehouses, the KIVA system handles the order picking process with static pickers and movable racks delivered to stations by mobile robots. Therefore, the order assignment policy has a direct impact on the subsequent rack selection, whereas picking station scheduling deals with synchronization of the processing sequencing of the assigned orders and arriving racks. These two interrelated decisions collectively determine the total number of robotic tasks. We formulate the resulting decision problem as an MIP model and ascertain its computational complexity. Our numerical studies show that Gurobi cannot solve large-sized instances in a reasonable time. Therefore, we propose a heuristic algorithm, which adapts the basic logic of simulated annealing, and applies the beam
search framework to construct the fitness value. The numerical studies show that our solution approach is effective in tackling large-sized instances with thousands of orders and ten picking stations. Furthermore, we compare our approach with a simple rule-based method applied widely in practice and a separate optimization process without the order assignment policy. We show that our proposed approach always achieves outstanding performance and joint optimization of the interlinked processes is necessary. Therefore, SA is a potent approach to address the questions we set out to answer in our study.

Regarding the robot-assisted KIVA warehouse, future research should focus more on the stochastic nature of the problem. In addition, more holistic problem settings should be tested in future research, where OAPSSP is coupled with rack storage assignment and/or robotic task allocation and traffic planning. The findings of such research will shed light on how best to organize the complicated operations in real-world parts-to-picker warehouses.

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References

Ardjmand, E., Shakeri, H., Singh, M., & Bajgiran, O.S. (2018) Minimizing order picking makespan with multiple pickers in a wave picking warehouse. *International Journal of Production Economics*, 206, 169-183.

Azadeh, K., De Koster, R., & Roy, D. (2017) Robotized warehouse systems: Developments and research opportunities. ERIM Report Series Research in Management, Erasmus Research Institute of Management (ERS-2017-009-LIS).
Baardman, L., Roodbergen, K.J., Carlo, H.J., Schotenoer, A.H. (2021) A Special Case of the Multiple Traveling Salesmen Problem in End-of-Aisle Picking Systems. *Transportation Science*, https://doi.org/10.1287/trsc.2021.1075.

Banker, S. (2016) Robots in the warehouse: It’s not just Amazon. *Forbes*.

Bartholdi, J.J. III, Hackman, S.T. (2014) Warehouse and distribution science. Release 0.96. Supply Chain and Logistics Institute, https://www.warehouse-science.com/book/.

Batt, R. J., & Gallino, S. (2019). Finding a needle in a haystack: The effects of searching and learning on pick-worker performance. *Management Science*, 65(6), 2624-2645.

Battini, D., Calzavara, M.,Persona, A., & Sgarbossa, F. (2017). Additional effort estimation due to ergonomic conditions in order picking systems. *International Journal of Production Research*, 55(10), 2764-2774.

Blum, C. (2005) Beam-ACO—Hybridizing ant colony optimization with beam search: An application to open shop scheduling. *Computers & Operations Research*, 32(6), 1565-1591.

Bodnar, P., De Koster, R., & Azadeh, K. (2017) Scheduling trucks in a cross-dock with mixed service mode dock doors. *Transportation Science*, 51(1), 112-131.

Boschetti, M.A., Maniezzo, V., Roffilli, M., & Röhler, A.B. (2009) Matheuristics: Optimization, simulation and control. *Proceedings of International Workshop on Hybrid Metaheuristics* (pp. 171-177). Springer, Berlin, Heidelberg.

Boysen, N., Briskorn, D., & Emde, S. (2017) Parts-to-picker based order processing in a rack-moving mobile robots environment. *European Journal of Operational Research*, 262(2), 550-562.

Boysen, N., De Koster, R., & Weidinger, F. (2019) Warehousing in the e-commerce era: A survey. *European Journal of Operational Research*, 277(2), 396-411.

Boysen, N., & Zenker, M. (2013) A decomposition approach for the car resequencing problem with selectivity banks. *Computers & Operations Research*, 40(1), 98-108.
Çeven, E., & Gue, K.R. (2017) Optimal wave release times for order fulfillment systems with deadlines. *Transportation Science, 51*(1), 52-66.

Cho, H.S., Paik, C.H., Yoon, H.M., & Kim, H.G. (2005) A robust design of simulated annealing approach for mixed-model sequencing. *Computers & Industrial Engineering, 48*(4), 753-764.

CNN Business (2018) Life inside an Amazon fulfillment center. Available from [https://www.youtube.com/watch?v=iXxPabWb9nI](https://www.youtube.com/watch?v=iXxPabWb9nI) last accessed July 12, 2020.

D’Andrea, R., & Wurman, P. (2008) Future challenges of coordinating hundreds of autonomous vehicles in distribution facilities. *Proceedings of 2008 IEEE International Conference on Technologies for Practical Robot Applications* (pp. 80-83), IEEE.

De Koster, R., Le-Duc, T., & Roodbergen, K.J. (2007) Design and control of warehouse order picking: A literature review. *European Journal of Operational Research, 182*(2), 481-501.

Enright, J.J., & Wurman, P.R. (2011) Optimization and coordinated autonomy in mobile fulfillment systems. *Proceedings of the 25th AAAI Conference on Artificial Intelligence*.

Garey, M.R., & Johnson, D.S. (1979) *Computers and Intractability*. San Francisco: Freeman.

Gu, J., Goetschalckx, M., & McGinnis, L.F. (2010) Research on warehouse design and performance evaluation: A comprehensive review. *European Journal of Operational Research, 203*(3), 539-549.

Hanson, R., Medbo, L., & Johansson, M.I. (2018) Performance characteristics of robotic mobile fulfillment systems in order picking applications. *IFAC-PapersOnLine, 51*(11), 1493-1498.

Hof, J., & Schneider, M. (2021). Intraroute Resource Replenishment with Mobile Depots. *Transportation Science, 55*(3), 660-686.

Kovacs, A.A., Parragh, S.N., Doerner, K.F., & Hartl, R.F. (2012) Adaptive large
neighborhood search for service technician routing and scheduling problems. *Journal of Scheduling, 15*(5), 579-600.

Kumawat, G.L., & Roy, D. (2021) A new solution approach for multi-stage semi-open queuing networks: An application in shuttle-based compact storage systems. *Computers & Operations Research, 125*, 105086.

Lamballais, T., Roy, D., & De Koster, M.B.M. (2017) Estimating performance in a robotic mobile fulfillment system. *European Journal of Operational Research, 256*(3), 976-990.

Masson, R., Lehuédé, F., & Péton, O. (2013) An adaptive large neighborhood search for the pickup and delivery problem with transfers. *Transportation Science, 47*(3), 344-355.

Merschformann, M., Xie, L., & Li, H. (2017) RAWSim-O: A simulation framework for robotic mobile fulfillment systems. *arXiv preprint arXiv:1710.04726*.

Neuhub. (2019) From: [https://neuhub.jd.com/innovation/type/AGV](https://neuhub.jd.com/innovation/type/AGV) last accessed July 12, 2021.

Ow, P.S., & Morton, T.E. (1988) Filtered beam search in scheduling. *International Journal of Production Research, 26*(1), 35-62.

Rijal, A., Bijvank, M., Goel, A., & de Koster, R. (2021). Workforce Scheduling with Order-Picking Assignments in Distribution Facilities. *Transportation Science, 55*(3), 725-746.

Roodbergen, K.J., & Vis, I.F. (2009) A survey of literature on automated storage and retrieval systems. *European Journal of Operational Research, 194*(2), 343-362.

Ropke, S., & Pisinger, D. (2006) An adaptive large neighborhood search heuristic for the pickup and delivery problem with time windows. *Transportation Science, 40*(4), 455-472.

Pan, J.C.H., Shih, P.H., & Wu, M.H. (2015) Order batching in a pick-and-pass warehousing system with group genetic algorithm. *Omega, 57*, 238-248.

Sabuncuoglu, İ., Gocgun, Y., & Erel, E. (2008) Backtracking and exchange of
information: Methods to enhance a beam search algorithm for assembly line scheduling. European Journal of Operational Research, 186(3), 915-930.

Scholz, A., Schubert, D., & Wäscher, G. (2017) Order picking with multiple pickers and due dates—Simultaneous solution of order batching, batch assignment and sequencing, and picker routing problems. European Journal of Operational Research, 263(2), 461-478.

Shen, M., Tang, C. S., Wu, D., Yuan, R., & Zhou, W. (2020). Jd.com: Transaction-level data for the 2020 msom data driven research challenge. Manufacturing & Service Operations Management.

Sina, From: https://finance.sina.com.cn/chanjing/gsnews/2020-11-12/doc-iiznezxs1358721.shtml last accessed July 7, 2021.

Valle, C.A., & Beasley, J.E. (2021) Order allocation, rack allocation and rack sequencing for pickers in a mobile rack environment. Computers & Operations Research, 125, 105090.

Van Gils, T., Ramaekers, K., Caris, A., & De Koster, R.B. (2018) Designing efficient order picking systems by combining planning problems: State-of-the-art classification and review. European Journal of Operational Research, 267(1), 1-15.

Weidinger, F. (2018) A precious mess: on the scattered storage assignment problem. Operations Research Proceedings 2016 (pp. 31-36). Springer, Cham.

Weidinger, F., Boysen, N., & Briskorn, D. (2018) Storage assignment with rack-moving mobile robots in KIVA warehouses. Transportation Science, 52(6), 1479-1495.

Weidinger, F., & Boysen, N. (2018) Scattered storage: How to distribute stock keeping units all around a mixed-shelves warehouse. Transportation Science, 52(6), 1412-1427.

Winkelhaus, S., Grosse, E.H., & Morana, S. (2021) Towards a conceptualisation of Order Picking 4.0. Computers & Industrial Engineering, 159, 107511.

Wulfraat, M. (2012) Is the Kiva system a good fit for your distribution center? An
unbiased distribution consultant evaluation. *MWPVL International White Paper.*

Wurman, P.R., D’Andrea, R., & Mountz, M. (2008) Coordinating hundreds of cooperative, autonomous vehicles in warehouses. *AI Magazine, 29*(1), 9-9.

Xie, L., Thieme, N., Krenzler, R., & Li, H. (2021) Introducing split orders and optimizing operational policies in robotic mobile fulfillment systems. *European Journal of Operational Research, 288*(1), 80-97.

Yang, X., Hua, G., Hu, L., Cheng, T.C.E., & Huang, A. (2021) Joint optimization of order sequencing and rack scheduling in the robotic mobile fulfilment system. *Computers & Operations Research, 135*, 105467.

Zaerpour, N., Yu, Y., & De Koster, R. (2017) Small is beautiful: A framework for evaluating and optimizing live-cube compact storage systems. *Transportation Science, 51*(1), 34-51.

Zhuang, Y., Zhou, Y., Yuan, Y., Hu, X., & Hassini, E. (2021). Order Picking Optimization with Rack-Moving Mobile Robots and Multiple Workstations. *European Journal of Operational Research.*