Safe and energy-efficient ventilation in mines – application of the golden ratio method in designing forced air distribution for ventilation networks

Grzegorz Pach
Faculty of Mining and Geology, Silesian University of Technology, Akademicka 2, 44-100 Gliwice, Poland
grzegorz.pach@polsl.pl

Abstract. Mine ventilation networks are often designed with branches with dependent air currents. Among these, branches connecting the subnetworks of main ventilation fans may be distinguished. Their presence in the ventilation network allows to obtain an infinite number of air distribution variants fulfilling the imposed requirements of distribution. The distributions differ not only in terms of air flow rate in branches, but also in terms of main fan parameters, including their effective power. From the point of view of energy savings, it is beneficial to find such air distribution in which the effective power of the fans exhibits the lowest value. The paper presents an algorithm based on the golden ratio method, allowing to find the optimal air distribution with a certain degree of precision. The presented algorithm is applicable in all ventilation networks with two main fans with interconnected subnetworks. The paper includes an example of a ventilation network in which the method in concern was applied. The effective power of the fans in the network referred to above in the less advantageous scenario amounted to 379.5 kW, while after the optimization, an air distribution variant was found in which the power was limited to 149 kW.

1. Introduction
Underground exploitation of useful minerals is related to the construction and maintenance of workings that are interconnected forming the mine’s ventilation network. The structures of the ventilation networks differ from each other both in Poland and worldwide. Instances of mining plants are known where the ventilation is performed by means of one, two or multiple exhaust shafts with main ventilation fans [1]. The branches of the mine’s ventilation network may include ones that connect the subnetworks of the main fans [2]. Such branches, as suggested by the theory of mine ventilation, are classified as diagonal and the airflow in these branches is a dependent current of used air. Among other things, the process of restructuring of the Polish coal mining industry resulted in merging of mining plants and changes in the structure of their ventilation networks [3, 4] including the emergence of the ventilation branches referred to above.

The underground mining industry is characterized by the existence of numerous natural hazards. In case of a number of those, adequate ventilation of workings is a significant part of prevention practices [5]. Among these hazards one may distinguish: gas hazard, fire hazard, thermal hazard or coal dust explosion hazard. Due to the safety of the mining plant’s employees, it is important to ensure an amount of air and airflow rate that would be adequate to the magnitude of the above hazards in the workings that serve as the workplace for the miners [6, 7, 8, 9, 10]. This problem had been considered...
in professional literature [11, 12, 13] as the problem of calculating the forced air distribution or as the regulation of the ventilation network.

A considerable portion of the electric power used in mining plants serves ventilation purposes. According to Jesviet [14, 15], this consumption exceeds 25% of the total power consumption in a mining plant, while Nikolaev [16] provides that it reaches 50%. Jha [17] states that the mean cost of mine ventilation amounts to 1/3 of the total operating costs. The costs of the operation of the main ventilation fans are a significant component of the total costs of underground mine ventilation. Optimization of the main fan operation, along with the maintenance of adequate volume of air in the workings, may lead to a significant decrease in the amount of energy used by a mining plant and thus lead to the reduction of operating costs [18]. Studies over such an optimization were also conducted by Szlazak and Obracaj [19]. It should be underlined that an improvement of mine ventilation leading to a decrease in power consumption must most of all give consideration to the occupational safety of the miners (i.e. ensuring adequate amounts of air).

The paper presents a method of finding an optimal forced air distribution variant in a mining plant’s ventilation system. The presented method is applicable in ventilation networks containing a branch with air current connecting the subnetworks of the main fans. It allows to limit the power consumed by the main fans and thus to reduce their operating costs. In the presented example, the calculation was made under the assumption that the air density in the ventilation network is constant.

### 2. Forced air distribution in a mine ventilation network

The problem of forced air distribution encompasses the requirement of supplying specified amounts of air – with consideration given to the provisions of law and to natural hazards – to the workings constituting the workplace of miners. Such workings include i.e. the direct exploitation areas and functional chambers.

The known values concerning this issue are: the structure of the mine’s ventilation network, the known quantity of air in some of the ventilation branches and aerodynamic resistance of all the branches. The values to be determined are: the parameters related to the main ventilation fans – the pressure and air quantity, settings of airflow regulators and airflows in remaining branches [20, 21]. The regulators may be regulating air stoppings and/or auxiliary fans [11, 12].

In case of ventilation networks with a single main fan (figure 1a) the method of calculating the forced air distribution was presented in the works of Salustowicz [11, 12]. In that method, the author introduced a critical path at which the airflow regulators setting value is zero. Such an operation allows to determine the optimal parameters of the main fan (fan pressure and air quantity) at which its effective power is the lowest. In networks with a single main fan, the number of equations resulting from the equilibrium laws (12 in case of the network shown in figure 1a) is identical to the number of unknown values (unknown airflows in 8 branches, settings of 3 regulators at the intakes to the areas and the pressure of the main fan – figure 1a). Also in case of ventilation networks with a greater number of main fans (figure 1b) the method developed by Salustowicz [13] finds its application due to the increased number of critical paths (in the network exhibited in figure 1b, two separate critical paths exist leading to the individual main fans, and thus the two regulators would be set to zero). The subnetworks of these fans may not, however, be connected by means of a ventilation branch. In such ventilation networks, the number of equations resulting from equilibrium laws is equal to the number of unknown values.

For ventilation networks with two main fans where such a branch exists (figure 1c), the number of equations resulting from equilibrium laws (16 equations – figure 1c) is less by 1 than the number of unknown values (17 unknown values). Due to this, a new method is necessary that would allow for finding air distribution that would be optimal in view of the air distribution fans’ effective power minimization.
Figure 1. Canonical diagrams of ventilation networks: a) with a single main fan, b) with two main fans without air currents connecting their subnetworks, c) with two main fans with an air current connecting their subnetworks.

The branch 8-10 (based on node numbering) in figure 1c connects the subnetworks of the main fans. It allows to “direct” the airflow to each of the fans. The whole air flowing out from branches through direct exploitation areas and the functional chamber (branches 6-8, 6-7, 5-7, 5-10 and 4-10) may be distributed in different proportions among the fan in the branch 9-12 and the fan in the branch 11-12. Each of the distributions will be different not only in terms of airflow in branches, but also in the fan pressure differences and the settings of the regulators (e.g. the aerodynamic resistance of regulating stoppings). It is thus useful to find such air distribution that the total effective power of the main fans would be as small as possible. Such a distribution may be considered optimal in terms of the fan’s power consumption and thus in terms of costs of power borne by the mining plant.

3. The golden ratio method
In ventilation networks with two main fans in which the subnetworks are interconnected, the assumption of a given airflow in each of the branches with take-off points (such as direct exploitation areas and functional chambers) within the full section (green p-c line – figure 1c), allows to achieve an infinite number of air distributions fulfilling the equilibrium laws. As mentioned earlier, in such a case the number of unknown values exceeds the number of equations resulting from the equilibrium laws by one. In such networks, the selection of an anti-tree [22] is possible that – except for the branches of the full section (such as branches 6-8, 6-7, 5-7, 5-10 and 4-10 in figure 1c) – would also incorporate a branch with the main fan (such as branch 9-12 in figure 1c). In case of branches of the anti-tree selected in such a manner, an independent condition of airflow may be forced for these branches. The airflow forced in the branch with the fan \(V_w\) must, however, fulfil the condition specified by formula 1, due to the known direction of air flow in exhaust shafts.

\[
0 \leq V_w \leq V_{pc}
\]

where:
\(V_w\) – airflow at the main fan,
\(V_{pc}\) – sum of airflows in branches of the full section.

For different values of the forced airflow at the \(V_w\) fan, different air distributions are achieved, characterized by different parameters of the main fans – including the effective power. To determine the air distribution for which the effective power of the fans assumes the minimal value, the golden ratio method [23, 24] that is known in mathematics may be applied.

In the problem in concern, the objective function assumes the form presented in formula 2.
\[ N_u = \Delta p_{w1} \cdot V_{w1} + \Delta p_{w2} \cdot V_{w2} = \Delta p_{w1} \cdot V_{w1} + \Delta p_{w2} \cdot (V_{pc} - V_{w1}) = \min \]  

where:

- \( N_u \) – the total effective power of the main fans,
- \( V_{w1}, V_{w2} \) – airflow at fan 1 and fan 2,
- \( \Delta p_{w1}, \Delta p_{w2} \) – the required pressure difference of fan 1 and fan 2.

The limiting conditions are constituted by equations resulting from the first and second Kirchhoff’s laws. The studied objective function and the limiting conditions are convex functions, which is confirmed by the studies presented in works [2, 25]. It is thus possible to apply the golden ratio method to find the minimum of the function exhibited in formula 2. One should note that after forcing the airflow at fan 1, the forced air distribution may be determined with values of fan pressure differences \((\Delta p_{w1}, \Delta p_{w2})\) characteristic to the forced airflow. The objective function given in formula 2 is thus a function of one independent variable – the airflow at fan 1. The shape of this function has been shown in figure 2.

\[ d_i = k \cdot d_{i-1} \]  

Subsequently, the values of the forced airflows at the fan are calculated in line with the formula

\[ V_{w1,i,j} = V_{w1,p,i-1} - d_i \]  

\[ V_{w1,p,i} = V_{w1,i-1,j} + d_i \]

for airflows at the main fan forced in that manner, the air distribution in the fresh air and used air zones is calculated (e.g. using the Cross method). Next, the settings of the regulators and the
parameters of the main fans should be calculated (using the Sałustowicz methods), including their total effective power. Point C (figure 2) corresponds to the air distribution in which the forced airflow at the fan is calculated using function 4a, while point D corresponds to the air distribution resulting from condition 4b.

Symbols 1 and p found in the subscripts of formulas 4a, 4b and 5a, 5b correspond to the left boundary of the interval and the right boundary of the interval, respectively – due to the airflow at the fan.

Another step in the method is the comparison of the total values of effective powers of fans, obtained at the air distributions earlier determined (air distributions corresponding to points C and D in figure 2). Two following cases are possible in the comparison

\[ N_u f(V_{wl,i}) \geq N_u f(V_{wl,p}) \]  \hspace{1cm} (5a)

in such a case the interval is tightened on the left side. The newly formed interval in which the minimum effective power of the fans will occur will be between the projections of the points C and B. Such a situation has been exhibited in figure 2 (the new interval was marked with a thick line).

The second possible case is given by the formula 5b

\[ N_u f(V_{wl,i}) < N_u f(V_{wl,p}) \]  \hspace{1cm} (5b)

in such a case the interval is tightened on the right side. In this case the new interval will be located between the projections of the points A and D.

After the tightening of the interval, another iteration proceeds starting from the calculation of the interval length using formula 3. Further procedure is similar to the first iteration. The calculations are completed when the length of the uncertainty interval will be smaller than allowable (assumed before the calculations were commenced).

4. Example of calculations

Figure 3 presents a canonical diagram of a ventilation network in coal mine A for which calculations optimizing the forced air distribution using the golden ratio method was conducted. The ventilation network is comprised of 5 air off-take points, including 4 direct exploitation areas (in branches 21-8, 22-7, 23-7 and 25-10) and a functional chamber (in branch 24-10). In branches preceding the off-take points, airflow regulators are located in the form of regulation stoppings (branches 6-21, 6-22, 5-23, 5-24 and 4-25). The main ventilation fans are located in branches 12-14 (W1 fan) and 13-14 (W2 fan). The network contains branches with air currents connecting the W1 and W2 subnetworks, namely branches 9-10 and 9-11.

![Figure 3. Canonical diagram of the A mine ventilation network.](image-url)
The ventilation network of the mine A includes 19 nodes and 26 branches. The cyclomatic number for that network is thus 8. 26 Independent equations are thus possible, including 18 resulting from the first law of equilibrium and 8 resulting from the second law of equilibrium. The number of unknown values is 27, including: airflows in 21 branches, aerodynamic resistance of 4 regulating stoppings (the resistance for one of the stoppings is zero in line with Saustowicz’s method), the required pressure for 2 main fans. The obtained system of equations is thus overdetermined.

Table 1 presents the information regarding the branches with air off-takes.

| Start node | End node | Aerodynamic resistance [kg m⁻¹] | Imposed air quantity [m³ min⁻¹] |
|------------|----------|---------------------------------|---------------------------------|
| 21         | 8        | 0.450                           | 1000                            |
| 22         | 7        | 0.300                           | 1200                            |
| 23         | 7        | 0.600                           | 800                             |
| 24         | 10       | 0.200                           | 500                             |
| 25         | 10       | 0.700                           | 1500                            |

The total airflow through the off-take points \( V_p \) is 5000 m³ min⁻¹ and thus the forced airflow at the W1 fan \( V_{p0} \) must fit within the range from 0 m³ min⁻¹ and 5000 m³ min⁻¹.

Initially, in line with this method, the air distribution variants were determined in case of which the airflow at the W1 fan amounted to: 0 m³ min⁻¹ (point A, figure 2) and 5000 m³ min⁻¹ (point D, figure 2). For such distributions, the main fan parameters were determined along with their effective power. At the forced airflow at the W1 fan, amounting to 0 m³ min⁻¹ (point A, figure 2), the required pressure difference of the W1 fan amounted to 1244.326 Pa while the required pressure difference of the W2 fan was 3346.86 Pa. Their total effective power (determined using formula 2) was 278.905 kW. If, on the other hand, the forced airflow was 5000 m³ min⁻¹ (point D, figure 2) the following values were obtained: W1 pressure difference 4554.163 Pa, W2 pressure difference 1245.969 Pa and effective power of 379.513 kW.

The length of the uncertainty interval in the first iteration determined based on formula 3 was 3090.17 m³ min⁻¹. The values of the forced airflow in case of the W1 fan, determined in line with formulas 4a and 4b, amounted to: 1909.83 m³ min⁻¹ on the left side (point C, figure 2) and 3090.17 m³ min⁻¹ on the right side (point D, figure 2). In case of the airflow of 1909.83 m³ min⁻¹, the following fan parameters were obtained: W1 fan pressure difference of 1688.001 Pa, W2 fan pressure difference of 1894.456 Pa and the total effective power of the W1 and W2 fans amounting to 151.299 kW. In case of the airflow of 3090.17 m³ min⁻¹, these values amounted to: W1 pressure difference of 2374.719 Pa, W2 pressure difference of 1457.182 Pa and the effective power of 168.687 kW. Because the total effective power of the W1 and W2 fans at the forced airflow of 3090.17 m³ min⁻¹ at the W1 fan is higher than in the case of the air distribution variant with the forced airflow of 1909.83 m³ min⁻¹ (there is the dependence expressed by formula 5b), the uncertainty interval is tightened on the right side. The newly formed interval fits within the W1 airflow range of \(<0 \text{ m}^3\text{ min}^{-1} – 3090.17 \text{ m}^3\text{ min}^{-1}\). That range incorporates the air distribution at which the effective power of the main fans is minimal. An example of results of calculation of forced air distribution at a forced airflow of 3090.17 m³ min⁻¹ at the W1 fan has been presented in table 2 (printout from a computer program “WK”). The parameters of the branches including the W1 and W2 fans are in bold.

Results from the first 10 iterations were listed in table 3. “L” and “R” symbols in the first column of table 3 denote the information regarding the tightening of the uncertainty interval on the left and right side, respectively. A similar information is contained in table cells with fields in bold.
Table 2. Forced air distribution in the ventilation network of mine A.

| Branch number | Start node | End node | Resistance kg·m⁻¹ | Air quantity m³·min⁻¹ | Head loss Pa | Fan press. Pa |
|---------------|------------|----------|-------------------|-----------------------|-------------|--------------|
| 1             | 1          | 2        | 0.0500            | 5000.00               | 83.3334     | 347.222      |
| 2             | 2          | 3        | 0.2500            | 2564.78               | 42.7463     | 456.811      |
| 3             | 2          | 4        | 0.2800            | 2435.22               | 40.5871     | 461.247      |
| 4             | 3          | 4        | 0.1200            | 364.78                | 6.0796      | 4.435        |
| 5             | 3          | 6        | 0.2000            | 2200.00               | 36.6667     | 268.889      |
| 6             | 4          | 5        | 0.1500            | 1300.00               | 21.6667     | 70.417       |
| 7             | 6          | 21       | 0.1151            | 1000.00               | 16.6667     | 31.972       |
| 8             | 21         | 8        | 0.4500            | 1000.00               | 16.6667     | 125.000      |
| 9             | 6          | 22       | 0.0000            | 1200.00               | 20.0000     | 0.000        |
| 10            | 22         | 7        | 0.3000            | 1200.00               | 20.0000     | 120.000      |
| 11            | 5          | 23       | 1.1664            | 800.00                | 13.3333     | 207.370      |
| 12            | 23         | 7        | 0.6000            | 800.00                | 13.3333     | 106.666      |
| 13            | 5          | 24       | 5.2095            | 500.00                | 8.3333      | 361.771      |
| 14            | 24         | 10       | 0.2000            | 500.00                | 8.3333      | 13.889       |
| 15            | 4          | 25       | 0.0137            | 1500.00               | 25.0000     | 8.575        |
| 16            | 25         | 10       | 0.7000            | 1500.00               | 25.0000     | 437.501      |
| 17            | 7          | 8        | 0.1500            | 941.99                | 15.6998     | 36.973       |
| 18            | 7          | 9        | 0.2000            | 1058.01               | 17.6335     | 62.188       |
| 19            | 8          | 11       | 0.0800            | 1941.99               | 32.3665     | 83.807       |
| 20            | 9          | 10       | 0.2500            | -90.17                | -1.5028     | -0.565       |
| 21            | 9          | 11       | 0.1600            | 1148.18               | 19.1363     | 58.952       |
| 22            | 10         | 13       | 0.2000            | 1909.83               | 31.8305     | 202.636      |
| 23            | 11         | 12       | 0.4000            | 3090.17               | 51.5028     | 1061.017     |
| 24            | 12         | 14       | 0.0000            | 3090.17               | 51.5028     | 0.000        |
| 25            | 13         | 14       | 0.0000            | 1909.83               | 31.8305     | 0.000        |
| 26            | 14         | 1        | 0.0000            | 5000.00               | 83.3334     | 0.000        |

Table 3. Optimization results of forced air distribution in the ventilation network of mine A.

| Number of the step | Airflow on the fan W1 [m³·min⁻¹] | Fan W1 pressure Δp₁ [Pa] | Fan W2 pressure Δp₂ [Pa] | Total effective power Nₐ [W] | The range of uncertainty δₘ [m³·min⁻¹] | Border points |
|--------------------|----------------------------------|--------------------------|--------------------------|-----------------------------|----------------------------------------|--------------|
| 0                  | 0                               | 1244.326                 | 3346.86                  | 278905                      | 5000                                   | <0-5000>     |
| I (R)              | 1909.83                         | 1688.001                 | 1894.456                 | 151299                      | 3090.17                                | <0-3090.17>  |
| II (L)             | 1180.34                         | 1431.229                 | 2328.528                 | 176392                      | 1909.83                                | <1180.34-    |
| III (L)            | 1909.83                         | 1688.001                 | 1894.456                 | 151299                      | 1180.34                                | <1909.83-    |
| IV (R)             | 2639.32                         | 2070.92                  | 1586.857                 | 153531                      | 729.49                                 | <1909.83-    |
| V (R)              | 2188.47                         | 1818.284                 | 1762.935                 | 148930                      | 450.85                                 | <1909.83-    |
| VI (L)             | 2360.68                         | 1908.718                 | 1690.213                 | 149448                      | 2360.68                                | <2360.68-    |
| VII (R)            | 2082.04                         | 1766.166                 | 1811.177                 | 149369                      | 278.64                                 | <2082.04-    |
| VIII (L)           | 2147.82                         | 1798.035                 | 1781.067                 | 149029                      | 172.21                                 | <2147.82-    |
After eight iterations, the uncertainty interval was 106.43 m³·min⁻¹ and the difference of the total effective power of fans for boundary air distributions was 81 W.

Figure 4 presents a graph exhibiting the dependence between the total effective power of W1 and W2 fans and the forced airflow at the W1 fan.

![Graph](image)

**Figure 4.** The dependence between the total effective power of fans and the forced airflow at the W1 fan.

### 5. Conclusions
1. In ventilation networks with two main ventilation fans with subnetworks interconnected by a branch there is an infinite number of solutions fulfilling the requirements of forced air distribution.
2. In case of each of these solutions a characteristic air distribution in the branches of the ventilation network is obtained different in terms of main fan parameter values including their effective power.
3. To determine the air distribution variant that would be characterized by the lowest value of the total effective power of the main fans it is possible to apply an algorithm based on the golden ratio method referred to in this paper.
4. In the example brought forward, the effective power of the fans in the network in the least advantageous scenario amounted to 379.5 kW while after the optimization an air distribution variant was found in which the power was limited to 149 kW.

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