Origin of the Norton-type wave scattered by a sub-wavelength metallic slit

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(Dated: October 7, 2014)

We analytically and numerically clarify the physical origin and the behaviour of the Norton-wave scattered by a narrow slit, at optical frequencies. This apparently surface field, which comes in addition to the surface plasmon polariton and classical cylindrical lightwaves, features its own scattering lobe associated to oscillating induced currents extending within both horizontal metal parts forming the slit. Theory is given taking into account the finite size of the aperture.

A famous experiment [1] reporting an abnormal light transmission (a few percents) through sub-wavelength hole arrays sparked off a great surge of interest in the optical properties of nano-structured metallic surfaces. The authors initially advocated the unique role of surface plasmons. It is true that such modes are significantly present in the visible range, on noble metals. Nevertheless, sub-wavelength apertures, intrinsically, behave also as dipole antennas supposed to scatter a continuum of evanescent waves and quasi-omnidirectional space ones. In particular, these lightwaves may be preponderant at the close vicinity of the apertures, and can participate quantitatively to extraordinary transmission [2]. In other cases, taking narrow slits, dipolar interactions are responsible for some local enhancements and far-field modulations [3]. Following debates [4] about the actual mediation of extraordinary transmission at optical frequencies (plasmon, surface lightwaves, or a mixture of them, depending on the spectral window and geometry), efforts were done to finely describe analytically the electromagnetic field scattered by a slit, often reduced as a punctual source [5]-[8]. It is difficult to find a closed-form expression when the metal permittivity is finite, as known in the antennas context [9]. An intriguing result is the change of spatial damping of the non-plasmonic contribution faraway, at the metal level [3], reminiscent of a Norton-type wave [8] i.e. a ground radio wave [10]. One could believe that a kind of surface field with the wavevector of light is also launched along the metal together with the polariton as a companion-wave exchanging energy, but this terminology is not really appropriated, as we will see.

In most papers interested in this question, a purely mathematical approach of the scattering integral, strictly at the surface level, and for a punctual scatterer, is only partly satisfactory as we miss essential features to have a complete picture of the electromagnetic entity we actually consider. An analysis discriminating each contribution in the broad space and giving an explicit physical interpretation is still lacking. This paper aims to refresh the problem in the framework of the simplified and highly instructive modal method, by taking into account the fundamental waveguide mode inside the sub-λ slit, in TM polarization (see Fig.1(a)). In this paper, we do not consider the indentation as a punctual scatterer. Take a good metal with a strongly negative permittivity ε. The constant surface impedance boundary conditions are applicable while tangential wavevectors of the scattered waves are much smaller than k|ε| (i.e. \( \sqrt{k^2 - k^2_{||}} \approx k \sqrt{\varepsilon} \) in the metal), where \( k = 2\pi/\lambda \). This is the case for noble metals in the infrared and for reasonably sub-wavelength geometries. After lengthy algebra [3], and omitting the time dependence \( e^{-i\omega t} \), we get the sequel (for \( y > 0 \)):

\[
H_z(x, y) = e^{-ik_{||}x \cos \theta} \frac{e^{i k \sin \theta x} + Z e^{ik_{||}x \cos \theta}}{\cos \theta + Z} e^{i k \sin \theta x} \quad (1)
\]

\[
+ \alpha(k, Z) \int_{-\infty}^{\infty} \frac{S_{s, Z}(u)}{v + Z} e^{i k (ux + vy)} du \quad (2)
\]
where the vector \( \nu = \sqrt{1-u^2} \) (arg \( \nu \in [0; \pi] \)),

\[
S_{w,z}(u) = \frac{1}{2} \left[ \sec((\nu + u)\frac{kw}{2}) + \sec((\nu - u)\frac{kw}{2}) \right] (3)
\]

with \( \sec(x) = \sin(x)/x \) and \( \nu = [-2iZ/kw]^{1/2} \).

\( Z = e^{-1/2} \) is the surface impedance (small and essentially imaginary). \( n_g = \sqrt{1-u^2} \) is the effective index of the fundamental mode guided along the slit (built by the antisymmetric coupling of vertical wall plasmons). \( S(u) \) is the Fourier transform of its eigenfunction, and \( \alpha \) its excitation coefficient \( [3] (\alpha \propto E^\text{lit}_2(y) \) but it has no importance in the forthcoming discussion). The scattering integral \( [2] \), i.e. the field structure, is what interests us, and is independent of the slit reaction. It is worth recalling that \( E_x(x, y = 0) \propto Z \cdot H_z(x, y = 0) \) at air/metal interfaces. \( E_y \) is odd with respect to \( x \), \( E_x \) is even.

When the metal is perfect inside the slit (\( \varepsilon = -\infty \), \( n_g = 1 \)), we immediately get \( S(u) = \sec(kwu/2) \). And if \( Z = 0 \) everywhere, the scattered field may be exactly \( [2] \) turned into an integral of some zero order Hankel function of the first kind \( H_1^{(1)} \) over the slit width. Consequently, the field scattered in any direction is cylindrical:

\[
H_z(kr > 1) \approx \alpha(k) \sqrt{\frac{2\pi}{kr}} e^{i(kr - \pi/4)} (4)
\]

with \( r = \sqrt{x^2 + y^2} \) and \( kw < 1 \). This solution verifies the Sommerfeld radiation condition. It is possible to show that \( E_y \) has an almost identical expression, so that the power flux \( E_y \times H_z^* \) propagating along the perfect metal surface has a \( 1/x \) spatial damping, for \( kr > 1 \).

Let us come back to the real metal case and put aside the specular term \( [1] \). We know the scattered field \( [2] \) is the sum of two main contributions: a surface polariton (SP) mode (plasmon for a metal, phonon for an ionic crystal in the Reststrahlen band, Zenneck wave for other lossy materials \( [10] \ldots \)) and a "photonic" field resembling the dipolar-type field \( [4] \) of the perfect metal case, say:

\[
H_z(x, y > 0) = H_z^{SP}(x, y) + H_z^{Ph}(x, y). (5)
\]

The pole of the integrand corresponds to the transverse plasma oscillation generated by each metallic edge of the aperture. Applying the residue theorem, for \( y > 0 \),

\[
H_z^{SP}(x, y) = -\frac{2\pi Z}{n_{SP}} S_{w,z}(n_{SP}) e^{i(k(n_{SP}|x| - Zy)} (6)
\]

with \( n_{SP} = \sqrt{1-Z^2} (\Re(n_{SP}) > 1) \). For \( |x| < w/2 \), \( H_z^{SP} \) still has a plasmonic nature since it is supported by the guided mode of the cavity. Eq.\((6) \) (not really new) explains the trade-off on the ratio \( S(c)/\Re(\varepsilon) \) to generate a strong and long-range surface polariton mode. \( S(n_{SP}) \) also implies that when \( w \approx \lambda/\Re(n_{SP}) \), destructive interferences annihilate the polariton whatever the slit reaction is (see Fig.\([3\)b])). Besides, for \( w \ll \lambda \), the waveguide index \( n_g \) exhibits an increasing imaginary part (absorption) which may attenuate the \( \alpha \) coefficient, and then, the SP generation, as experimentally observed \([3] \). Before going ahead, Figure 2 gives a numerical example showing the weight of each magnetic component scattered at the surface. While the plasmon and the photonic field are all proportional to the coefficient \( \alpha \), \( H_z^{Ph} \) predominates in the neighbouring of the sub-\( \lambda \) aperture, with a damping similar to that of the perfect metal case. Due to different spatial dampings, the polariton is rapidly the majoritary mode over many wavelengths. This is general provided \( |Z| \) and/or \( |S| \) (Fig.[1]b)) are not too small.

However the underlying physics is far from being complete. Indeed, on the basis of Ref.\([4] \), one shows that the photonic field near the slit may be expressed thanks to a finite correction (numerically verified) of the field of the perfect metal case. While \( x |Z|/\lambda < 1 \), and for \( x > w/2 \):

\[
H_z^{Ph}(x, y = 0) \approx \alpha \sec\left(\frac{kw}{2}\right) \cdot \left(\frac{2\pi}{kx} e^{-i\pi Z} - i\pi Z \right) - Z^2 2i\pi kx Z^3) e^{ikx} (7)
\]

The correction \( \sim \pi Z \) seems related to a energy transfer to the surface polariton. The propagating terms \( \sim Z^{2,3} \) are stranger as they are not linked to absorptions but to small out-of-phase radiations. Actually, in the real metal case, lightwaves themselves come from at least two contributions, i.e. \( H_z^{Ph}(kx > 0) \approx \alpha \sqrt{2\pi/kx} F(kx) e^{ikx} \), where \( F(kx) \) is an envelope function hiding another radiating sub-field which is not cylindrical. To show that, not only at the surface level, but in the full space, a relevant approach is to study the asymptotical behaviour of the field far enough from the aperture. A way to rapidly find such expressions is to resort to a double second order Taylor expansion. Indeed, let us consider again the integral \([2] \). One may put the phase \( \phi(u) = k(wz + vy) \) and \( f(u) = S_{w,z}(u)/(v + Z) \). We will apply the stationary phase method in the case where \( (Z, y) \neq (0, 0). \)

It can be intuitively understood that, although the field results from the contribution of a whole continuum of wavevectors, the oscillations of the exponential become
extremely rapid at large distance with a destructive interference of the spectrum, except when the phase \( \phi(u) \) is nearly constant, close to an extremum. The condition \( \phi'(u) = 0 \) is indeed fulfilled for a unique wavevector \((u, v) = (u_0, v_0) = (x/r, y/r)\), which well corresponds to a (homogeneous) radiated field. Thus, around \( u_0 \):

\[
\phi \approx \phi(u_0) + \left(\frac{u - u_0}{2}\right) \phi''(u_0) = kr \left[1 - \frac{(\Delta u)^2}{2} \left(\frac{r}{y}\right)^2\right], \tag{9}
\]

A Taylor expansion of \( f(u) \) is also applied, noticing that the \( f(u) \) contribution will be null as it is an odd function.

We are then driven to calculate different Fresnel integrals. If we introduce polar coordinates, by naturally putting \((u_0, v_0) = (\sin \varphi, \cos \varphi)\), with \( \varphi \in [-\pi/2; \pi/2] \) is an angle defined with respect to the \( y \) axis, we finally get:

\[
H^D_{z}(kr \gg 1) = H^D_{z}(kr) + H^N_{z}(kr) + O(kr^{-5/2}), \tag{10}
\]

with

\[
H^D_{z}(kr, \varphi) = \alpha S_{w,z}(\sin \varphi) \cos \frac{\varphi}{\cos \varphi + Z} \frac{2\pi}{kr} e^{i(kr - \varphi)}, \tag{11}
\]

\[
H^N_{z}(kr, \varphi) = \alpha \Theta(\varphi, Z) \sqrt{\frac{2\pi}{kr}} e^{i(kr + \varphi)}, \tag{12}
\]

and

\[
\Theta(\varphi, Z) \approx S_{w,z}(\sin \varphi) \left(\frac{Z}{Z + \cos \varphi}\right)^2 \left[1 + \frac{2 \cos \varphi \sin^2 \varphi}{Z + \cos \varphi}\right]. \tag{13}
\]

High order or minor terms are neglected. Through several simulations, we find a threshold \( x/\lambda > 10/|Z| \) over which this asymptotical expression of \( H^N_{z} \) starts to fit with the numerically calculated field, at the surface.

Let us get insight into the two electromagnetic entities obtained. The first contribution \( H^D_{z} \) is the usual dipolar field, which is actually valid whatever \((Z, \varphi)\) is. We analytically see that when \( Z = 0 \), we retrieve the perfect metal case with a far-field persisting at the surface \((\varphi = \pi/2)\) with a \( 1/\sqrt{kr} \) damping. But when \( Z \neq 0 \), \( H^D_{z}(\varphi \sim \pi/2) = 0 \) : we have the appearance of a shadow zone \( [3] \) (adjacent to the surface) for the radiated power, leaving the place only to the surface polariton and \( H^N_{z} \), when we are sufficiently far from the cavity.

The second one, noted \( H^N_{z} \), is what has a direct link with the so-called Norton wave. This lightfield is not a surface one, albeit spatially concentrated as it is rapidly decaying in radial amplitude, and has no contribution in the real power scattered far away. The radiation pattern \( \Theta(\varphi) \) is a meaningful result as it will indicate the physical origin of these additional lightwaves. For grazing angles, it is not null at the surface but proportional to \( 1/Z^2 = \varepsilon \), which could be high if it was not counterbalanced by the \( r^{-3/2} \) damping. Other calculation\([3, 6, 8, 9]\) made for a punctual scatterer seem to be consistent with this \( \varepsilon \) factor, but it has not been commented. Besides, \( H^N_{z} \) presents an intrinsic phase quadrature with respect to \( H^D_{z} \), hence possible destructive interference of both fields in polar directions where they have close amplitudes. Strictly keeping cartesian coordinates and taking \( r \approx x \) close to the surface, one gets a \( y/(x^{3/2}) \) behaviour\([7]\) for \( H^D_{z}(kr) \) but this is misleading: the true wave non-null at the surface with a \( 1/3/2 \) damping is \( H^N_{z}(kr) \).

Figure 3(a) shows the behaviour of the total radiated light-field and that of its inner components. Unsurprisingly, \( H^D_{z} \) is quasi-isotropic, typical of a Rayleigh scattering. \( H^N_{z} \) clearly presents two horizontal half-lobes at each side of the cavity, which are physically connected to the metallic surface. But the remarkable effect is that combination of the dipolar field and \( H^N_{z} \) strongly modifies the radiation pattern, and gives the impression that a residual surface wave slides along the metal \((y = 0)\). We understand here that the transition from a \( 1/\sqrt{x} \) damping (near the cavity) to a \( 1/\sqrt{x^3} \) (far away from it) at the surface, already reported\([4]\), is due to a change of the majoritary scattering contribution. Outside the shadow zone, we retrieve the preponderant dipolar space field. When \( w \to \lambda \) (see Fig 3(b)), \( H^N_{z} \) vanishes (the polariton vanishes even more, according Fig 3(b)) and the radiation lobe of \( H^D_{z} \) becomes more focused in the normal direction. The behaviour of \( \Theta(\varphi) \) for arbitrary values of \( Z \) or \( w/\lambda \) is also given in Fig 3(c). When \( Z \to 0 \), the semi-lobes remain flattened against the surface, with a polar angle of maximum amplitude tending to \( \pi/2 \). The space of validity of\([12]\) is also rejected to infinity: at end, only \( H^D_{z} \) becomes relevant (perfect metal case).

What is the physical source of \( H^N_{z} \)? Reminding the \( E_y \) arrows in Fig 1(a), one has an effective vertical electric dipole at the aperture level, responsible for the Norton wave, whereas the horizontal momentum \( E_x \) (at the
and behaves itself radiates. Thus, while \( \alpha \) and gold at here from \((2)-(6)-(11)\). Note that calculus of \( H \) taking \( \alpha \) close to the aperture. As \( H_{\text{total}} \approx 4 \) just above the aperture, max of scales are willingly limited to better highlight the details of scattering patterns. The white star refers to a spatial zone where the purely photonic field (numerically calculated) is (at large \( kr \)).

An open question is to know to which extent \((2)-(6)-(11)\) give. \( H_{\text{total}} \approx 1/(−k3(Z)) \) is the ordinate of a plane along which \( H_{sp} \) cancels, whatever \( x \) is (at large \( kr \)). The electromagnetic radiation \( H_{\alpha} \) is linked to induced surface currents occurring in the skin depth of both metallic parts forming the slit. Indeed, the cavity can be viewed as a capacity under illumination whereas horizontal metallic parts play the role of (dissipative) inductances \[ \text{[13]} \]. The guided current component

\[
J_x(kx \gg 1) = \int_{-\infty}^{0} H_z^N(x, y)dy = \frac{Z}{ik} H_z^N(x, y = 0) \tag{14}
\]

corresponds to an oscillating field having the wavevector of the free space. Thus, while \( \alpha \neq 0 \), the surface itself radiates (thermal waves in our case) and behaves as a uniform leaky wave antenna \[ \text{[14]} \]. The polariton has nothing to do with it, but simply superposes to it, without radiating. While \( H_{sp} \) is damped due to absorption, \( H^N \) is dissipated through emission, and has not the same \( Z \)-dependence. The \( z \)-amplitude of \( \Theta \) seems related to a limit of the surfacic charge spreading around the slit.

Figure 4(a) gathers fully calculated maps of the total scattered field \((2)-(6)-(11)\) and its inner components. The first map clearly shows the usual dipole-type radiation, the surface polariton and the presence of a shadow zone. While \( H_{sp} \) reveals a non-negligible surface near-field, the existence of the radiating \( H^N \) is confirmed by subtracting the polariton \((0)\) and the dipolar field \((11)\) from the total field. The new map exhibits the scattering lobes with a butterfly shape predicted by the \( \Theta \) function. This physical picture has gone unnoticed in the literature devoted to optics of metallic nano-structures. Additional details of the photonic component near the surface, far from the slit, are displayed in Fig 4(b). Along a direction normal to the surface, it well presents a small amplitude at the metal level (same value given by \( 12 \) taking into account the finite size \( w \)), cancels rapidly at the nodal plane \( y = [−k3(Z)]^{-1} \), and increases again as we progressively enter the dipolar lobe, as expected from Fig 3(a).

On an experimental level, this non-cylindrical Norton-type wave will be really apparent provided the SP collapses (faraway from the source or for strongly absorbing materials). It might generally have an absolute amplitude too weak to be practically exploited in photonic-size systems, but depending on the chosen permittivity (other conducting materials and multilayers), it may become the relevant wave \[ \text{[15]} \] to convey information at interfaces (not necessarily flat). Although \( H^N \) is not given for small \( kr \), Eq. \((5)\) would be an indication that it is likely present and immersed in the preponderant dipole field. An open question is to know to which extent \((4)\) radiation from induced currents may be channeled through diffraction orders and participate to extraordinary optical transmission. One can also wonder if there is an equivalent of this radiance in acoustics \[ \text{[18]} \]. Thus, the deeper fundamental understanding of the canonical slit case (conceptually close to sub-\( \lambda \) holes), here exemplified at telecom frequency, may appeal to new investigations and inspire other ways of lightwave engineering.

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1. T.W. Ebbesen et al, Nature 391, 667-669 (1998).
2. H.T. Liu and P. Lalanne, Nature 452, 728 (2008).
3. J. Le Perchec et al, Phys. Rev. Lett. 97, 036405 (2006); J. Light. Tech. 26, 638 (2008); New J. Phys. 13, 083025 (2011);
4. T. Visser, Nature Physics 2, 509 (2006).
5. P. Lalanne et al, Surface Science Reports 64, 453 (2009).
6. W. Dai, C. M. Soukoulis, Phys. Rev. B 80, 155407 (2009).
A. Yu. Nikitin, F.J. Garcia-Vidal, and L. Martin-Moreno, New J. Physics 11, 123020 (2009).

R.E. Collin, IEEE Antennas Propag. Mag. 46, 64 (2004).

J.R. Wait, IEEE Antennas Propag. Mag. 40, 7-22 (1998).

E.D. Palik, Handbook of Optical Constants of Solids (Academic Press, New York, 1985).

A. Wirgin, Opt. Commun. 7, 70 (1973).

R. Yang et al, J. Appl. Phys. 109, 103107 (2011).

D.R Jackson et al, Proc. IEEE 100, 2194 (2012).

Ming-Hui Lu et al, Phys. Rev. Lett. 99, 174301 (2007).

F. Capolino, D.R Jackson, and R. Wilton, IEEE Trans. Antennas Propag. 53, 91 (2005).

We could convolute the integrand in (2) with a Dirac comb introducing a periodization but the way we get (10) does not hold for small kr. The rigorous steepest descent method in a periodic case [17] could be useful.