Strong decays with the boost-corrected wave functions

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Abstract

Strong decay probabilities are calculated using the Lorentz contracted wave functions of decay products, determined in the arbitrary dynamical scheme with the instantaneous interaction. It is shown that the decay width obtains an additional factor defined by the contraction coefficient $C_m(s)$, which for two-body equal mass decays is $C_m^2(s) = 4m^2/s$, $s = E^2$. The resulting decay widths are compared to the experimental data, where in particular the $\rho(770), \rho(1450)$ decay data require an additional $1/s$ dependence of the width to fit the data. Important consequences for the dynamics of hadron decays and scattering are shortly discussed.

1 Introduction

The theory of a hadron decay into other hadrons or hadrons plus elementary objects include overlap integrals of hadron wave functions, which correspond to the instantaneous images of these hadrons, and the question arises whether one takes into account the Lorentz contraction of accelerated hadrons. For this purpose one needs to describe the motion and interaction of extended objects and for that one has to know behavior of the Green’s functions and the wave functions of extended objects under the applied boost, e.g. to know how the velocity $v$ of the system affects the hadron wave function.
As an example one can consider the hadron decay matrix element of the process \( h \to h_1 + h_2 \), e.g. \( \rho \to \pi + \pi \), where the pions move with high velocity and therefore their wave functions enter in the strong decay matrix element in the Lorentz transformed way.

It is a purpose of present paper to derive the behavior of the hadron wave functions in the moving system and calculate the resulting behavior of the hadron decay matrix element. As it is known, in the relativistic field theory the general formalism can be constructed in three different ways:

1) the instant form,

2) the point form and

3) the light front form.

In the instant form the wave function of any nonlocal object consisting of several elements can be defined at one moment of time and the frame (boost) dependence is dynamically generated in connection with Hamiltonian. In the literature different approaches have been developed for the practical realization of this problem, e.g. the canonical formalism in [2], analysis of the operator matrix elements between wave functions and form factors [3], [4]. As it is, the theory of the frame dependence of the Green’s functions of any nonlocal objects is closely related to the properties of the interaction terms in the Lagrangian, and one must envisage the instantaneous interaction for the first formalism, in particular confinement for the strong interaction and the Coulomb force in QED. The dynamical studies in this direction have been done recently, in Refs. [4, 5, 6] in several examples of systems. Later on, in [7] the properties of the spectrum and the wave functions in the moving system were studied in the framework of the relativistic path integral formalism [8, 9, 10, 11]. This method essentially exploits the universality and the Lorentz invariance of the Wilson-loop form of interaction, which produces both confinement and the gluon-exchange interaction in QCD. Moreover, in this formalism the Hamiltonian \( H \) with the instantaneous interaction between quarks in QCD (called the relativistic string Hamiltonian (RSH)) and charged particles in QED was derived and therefore the known defects of the Bethe-Salpeter approach are missing there. In [7] it was shown that the eigenvalues and the wave functions, defined by the RSH, transform in the moving system in accordance with the Lorentz rules. Indeed, using the invariance law under the Lorentz transformations [12, 13],

\[
\rho(x, t)dV = \text{invariant},
\] (1)
where $\rho(x, t)$ is the density, associated with the wave function $\psi_n(x, t)$,

$$
\rho_n(x, t) = \frac{1}{2i} \left( \psi_n \frac{\partial \psi_n^+}{\partial t} - \psi_n^+ \frac{\partial \psi_n}{\partial t} \right) = E_n |\psi_n(x, t)|^2,
$$

and $dV = d\mathbf{x}_\perp dx_\parallel$. One can use the standard transformations,

$$
L_P dx_\parallel \rightarrow dx_\parallel \sqrt{1 - v^2}, \quad L_P E_n \rightarrow \frac{E_n}{\sqrt{1 - v^2}};
$$

(3)

to insure the invariance of (1). In its turn the invariance law implies that in the wave function $\psi(x, t) = \exp(-iE_n t)\varphi_n(x)$ the function $\varphi_n(x)$ is deformed in the moving system,

$$
L_P \varphi_n(x_\perp, x_\parallel) = \varphi_n \left( x_\perp, \frac{x_\parallel}{\sqrt{1 - v^2}} \right),
$$

(4)

and can be normalized as

$$
\int E_n |\varphi_n^{(v)}(x)|^2 dV_v = 1 = \int M_0^{(0)} |\varphi_n^{(0)}(x)|^2 dV_0,
$$

(5)

where the subscripts $(v)$ and $(0)$ refer to the moving and the rest frames. One of the immediate consequences from the Eqs. (3) and (4) is the property of the boosted Fourier component of the wave function:

$$
\varphi_n^{(v)}(q) = \int \varphi_n^{(v)}(r) \exp(iqr) dr = C_0 \varphi_n^{(0)}(q_\perp, q_\parallel \sqrt{1 - v^2}),
$$

(6)

where $C_0 = \sqrt{1 - v^2} = \frac{M_0}{\sqrt{M_0^2 + P^2}}$.

The equations (1) – (6) and in particular (6), formulated in Ref. [7], have been the basic elements of the analysis of the meson form factors in [14], where it was shown that the Lorentz contraction of the hadron wave functions creates a basically different behavior of form factors as functions of $Q^2$, such that arguments of wave functions are never in the asymptotically large region of momenta. In the concrete examples of the pion and kaon form factors the agreement with data was obtained with simple Gaussian wave functions in the whole region of $Q^2$. A similar situation holds for the proton and neutron form factors [15].

It is the purpose of the present paper to study the behavior of the hadron strong decay matrix elements using the Lorentz contracted wave functions of
decay products. To this end we need explicit expressions of the decay matrix elements in terms of these wave functions.

In section 2 we shall write the expressions for the meson decay matrix elements in the rest frame of the decaying meson. For this purpose we are using the relativistic theory of string breaking \[17, 18\], which is an extension and the relativistic version of the well-elaborated strong decay formalism, based on the original \(3\)\(P_0\) model \[18\] and its flux-tube modification \[19\]. For the analysis of the model see \[20\] and the reviews in \[21\]. In what follows we shall need also modified forms of the string breaking matrix elements, see e.g. \[17\]. In the case of the chiral mesons as decay products we shall be using the formalism, called the Chiral Decay Mechanism (CDM) described in \[22, 23, 24\]. The technic of the Fock–Feynman–Schwinger representation (FFSR) \[9, 10, 11\] allows to represent the results in a simple form, which can be compared to experimental and lattice data in section 3. In section 4 we discuss the consequences and extrapolations of our results, as well as possible implications of the Lorentz contraction for other hadron decays. The concluding section contains a summary of results and discussion.

2 Definition of the decay matrix element through the hadron wave functions

We start with the simplest form of the \(3\)\(P_0\) model as a interaction Hamiltonian

\[
H_I = g \int (d^3 x \bar{\psi} \psi),
\]

where \(g = 2m_q \gamma\), and \(\gamma\) is a phenomenological parameter. The relativistic form obtained in \[17\] can be written as

\[
S_{\text{eff}} = \int d^4 x \bar{x} \psi(x) M(x) \psi(x),
\]

where \(M(x) = \sigma(|x - x_Q| + |x - x_{\bar{Q}}|)\). Here \(x\) is string breaking point between the quarks \(Q\) and \(\bar{Q}\). In the momentum space one obtains as in \[17, 18\] for the decay of the hadron 1 into hadrons 2, 3

\[
J_{123}(p) = y_{123} \int \frac{d^3 q}{2\pi^3} \Psi_1(p, q) M(q) \psi_2(q) \psi_3(q).
\]
Here \( p, -p \) are the momenta of decay products and \( q \) are the internal momenta inside decay products, which we assumed to be identical for simplicity.

Moreover \( y_{123} \) is the trace of normalized spin-tensors corresponding to spin-angular parts of meson states and \( M(q) \) for the \( S \)-wave decay is a constant, proportional to the string tension, \( M(q) = O(1 \, \text{GeV}) \) and for the \( L \)-wave resonance it is proportional to the \( p^L \). Finally for the width one can write

\[
\Gamma(E) = \text{const} \, p(E)^{2L+1} |J(p(E))|^2. \tag{10}
\]

Here \( L \) is the angular momentum of the decay products. So far we are in the realm of the standard hadron decay formalism. We now take into account that the decay product wave functions are moving with the velocity \( \sqrt{s - (m_1 + m_2)^2} \), and hence their wave functions in momentum space are Lorentz contracted as shown in (6). To this end we must write \( J(p(E)) \) in terms of the contracted wave functions, namely as in (6), the wave function moving with the velocity \( v \) can be written as \( \psi_n^{(v)}(q) = C_0 \psi_n(q_{\perp}, q_{\parallel} \sqrt{1 - v^2}) \). Denoting the total energy \( E \) which coincides with the resonance mass at the resonance center, as \( s = E^2 \), one can write \( C_0 = \sqrt{1 - v^2} = \frac{m_2 + m_3}{\sqrt{s}} \). Therefore the integral in (8) can be rewritten as

\[
J(p) = \text{const} \int (d^3 q \Psi_1^0(q, p) \psi_2^*(q) \psi_3^*(q)) =
\]

\[
= \text{const} \, C_0^2 \int (d^2 q_{\perp} dq_{\parallel} \Psi_1^0 \psi_2(q_{\perp}, q_{\parallel} \sqrt{1 - v^2}) \psi_3(q_{\perp}, q_{\parallel} \sqrt{1 - v^2})) =
\]

\[
= \text{const} \, C_0 \int (d^2 q_{\perp} d\kappa \Psi_1^0 \psi_2(q_{\perp}, \kappa) \psi_3(q_{\perp}, \kappa)). \tag{11}
\]

Here \( \kappa = q_{\parallel} \sqrt{1 - v^2} \). Therefore the decay matrix element is multiplied by \( C_0 \) and the decay width is multiplied by \( C_0^2 \). Summarizing one can write for the two-body decay width of a resonance with account of Lorentz contraction (LC), which we write first in the case of equal masses \( m_2 = m_3 = m \)

\[
\Gamma(LC) = C_0^2 \Gamma(0) = \frac{4m^2}{s} \Gamma(0). \tag{12}
\]

Here \( \Gamma(0) \) denotes the decay width without LC dynamics. In the next sections we shall study the effect of LC in the concrete resonances and compare it with data. We shall also discuss the case of unequal masses and many-body decays.
3 Theory of the $\rho$-meson decays with account of Lorentz contraction

In PDG\textsuperscript{[25]} it is written: "the determination of the parameters of the $\rho(770)$ is beset with many difficulties because of its large width. In physical region fits, the line shape does not correspond to a relativistic Breit–Wigner function with a P-wave width, but requires some additional shape parameter." Indeed in the standard theory with the Lagrangian $L_{eff} = g_{\rho\pi\pi}e_{ijk}\rho^{\mu}\pi_j\partial_\mu\pi_k$ one obtains the width

$$\Gamma_\rho = \frac{g_{\rho\pi\pi}^2p^2}{48\pi m_\rho^3}, p = \sqrt{s - 4m_\pi^2},$$ \hfill (13)

the result which contradicts experimental data. Therefore Gounaris and Sakurai\textsuperscript{[26]} have suggested to modify this result introducing some model dependence of the decay width on energy, which is now is used in most data analysis. The numerous accurate experimental data (see some examples in \textsuperscript{[27]-[29]}) exploit the corrected equation for the $\Gamma_\rho(s)$, namely in \textsuperscript{[27]}

$$\Gamma_V(s) = \frac{m_V^2 p(s)^3}{s m_\pi^2} \Gamma.$$ \hfill (14)

This should be compared with our result in \textsuperscript{[12]} for the decay of the $\rho \to \pi\pi$, where $\Gamma(0)$ refers to the width without LC, which is proportional to $p(s)^3$ and hence two equations coincide up to the replacement of $4m_\pi^2$ by $m_V^2$, which is unimportant since $\Gamma_V$ is a variable numeric parameter. At this point one should take into account that analysis of the pion form factor in \textsuperscript{[14]} with the help of LC has shown that the effective pion masses in the $C_0$ should be taken in the nonchiral limit, i.e. around 350 MeV, yielding the numerical agreement of our \textsuperscript{[12]} with \textsuperscript{[14]}. Note, that the effect of the LC gives the exact reduction coefficient which can be compared to data and decay theory, provided such an exact decay theory without arbitrary parameters exists, which is not yet the case for the theory of strong hadron decays. We now turn to another example of hadron decay – the $\pi\pi$ decay of $\rho(1450)$, studied experimentally in \textsuperscript{[29]}, where the authors following \textsuperscript{[26]} have used a slightly different from \textsuperscript{[14]} parametrization of the width

$$\Gamma_V(s) = \frac{s \beta_\pi(s)^3}{m^2 \beta_\pi(m)^3} \Gamma = \frac{m^3 p(s)^3}{\sqrt{s} p^3(m)} \Gamma,$$ \hfill (15)

6
where \( \beta_\pi(s) = \sqrt{1 - 4m_n^2/s} \). One can see the factor \( \sqrt{s/m^2} \) difference between (14) and (15), but from the point of view of our theory the result of (14) is preferable, however the numerical difference between two results is not large and therefore all experimental results on both \( \rho(770) \) and \( \rho(1450) \) well fitted with modified width equations can be taken as a support of our LC formalism.

4 Extensions and discussion

We shall discuss below 3 possible extensions of the above formalism: (A) decays to two unequal mass hadrons, (B) decays to one hadron and an elementary object (e.g. \( \gamma \)), (C) decays to 3 or more hadrons.

(A) Till now we discussed strong hadron decay to two equal mass mesons. This definition presupposes the strong interaction decay matrix element containing an integral of decay product wave functions as in (11). We now turn to the case of unequal masses \( m_2, m_3 \), where the particles 2, 3 move with velocities \( v_2, v_3 \), where \( v_i = \frac{m_i}{\sqrt{m_i^2 + p^2}} \). As it is easy to see in (11) one obtains in the decay matrix element the factor \( K(LC) = C_0(v_1)C_0(v_2)I_{23} \), where \( I_{23} = f(dq_\parallel \psi_2(q_\parallel)\psi_3(q_\parallel)) \). To proceed we assume for the decay product wave functions the Gaussian form so that for the longitudinal part of wave functions one has \( \psi_i(q_\parallel) = N_i \exp\left(-\frac{q_\parallel^2}{\chi^2_i}\right) \), which yields for the integral

\[
I_{23} = \sqrt{\frac{m_3^2\chi_3}{(p^2+m_2^2)\chi_3} + \frac{m_2^2\chi_2}{(p^2+m_3^2)\chi_2}}.
\]

As a result one can define the LC coefficient \( K(LC) \) which is equal to

\[
K(LC) = \sqrt{\frac{C_0(v_2)C_0(v_3)}{\sqrt{C_0(v_2)^2\chi_3/\chi_2 + C_0(v_3)^2\chi_2/\chi_3}}}.
\]

From (17) one can see that in the case when the meson 2 is more light \( (m_2 << p \) and/or more narrow \( \chi_2 >> \chi_3 \), only his contraction coefficient enters in the final answer \( K(LC) = C_0(v_2)\sqrt{\chi_3/\chi_2} \). To understand better the situation with unequal masses we can compare two decays \( \rho(1450) \rightarrow \omega \pi \) which we define as the decay (1) and \( \rho(1450) \rightarrow \pi \pi \) as decay (2). Assuming decay constants equal and all difference only due \( P \)-wave momenta and LC coefficients one obtains
\[ \Gamma_1 = \frac{p(\omega \pi) K_1(LC)^2}{p(\pi \pi) K_2(LC)^2} = 1.2, \quad (18) \]

which agrees with almost equal widths of decays (1) and (2).

(B) Consider e.g. the process \( h(1) \to h(2) + \gamma \) where \( \gamma \) has no internal structure and the corresponding wave function. It is clear that the decay matrix element has the structure in the c.m. of \( h(1) \) can be written in full analogy with the form factor written in (20) of [14]

\[ J(123) = \text{const} \int (d^3q)(\psi^0_1(q)\psi^Q_2(q + Q - \omega')\omega' + \omega''), \quad (19) \]

where \( \omega', \omega'' \) are stationary values of relativistic energies of two particles in the path integral (see [9, 10, 11]) and in this matrix element \( \gamma \) is emitted by particle'. In (19) the upper index \( (Q) \) in \( \psi \) denotes the momentum \( Q \) of the hadron and as in (11) one obtains an extra factor \( C_0(Q) \), but introducing \( \kappa \) this factor is cancelled, and one is left with the standard expression except that \( Q \) in the argument of \( \psi_2 \) is multiplied by the factor \( \sqrt{1 - v^2_Q} \), which strongly reduces the \( Q \) dependence at large \( Q \). These properties of the \( \gamma \) transitions are applicable to all decays including elementary objects without internal structure.

(C) We now consider strong decays to 3 and more hadrons, e.g. \( h(1) \to h(2, 3, 4) \) and assuming again the string-breaking mechanism the decay matrix element as in [17] will be proportional to

\[ J(1234) = \int (d^3q)\Psi_1(q, p_2, p_3, p_4)\psi^p_2(q)\psi^p_3(q)\psi^p_4(q), \quad (20) \]

where \( p_i \) in \( \psi^p_i \) is the momentum of hadron \( i \), \( p_1 + p_2 + p_3 = 0 \), so that the hadron wave functions acquire the factors \( C_0(v_i) \) and the longitudinal momenta of \( q \) in \( \psi_i(q) \) are multiplied by the same factors \( C_0(v_i) \), different in general for all \( i \). This creates a rather unusual distribution in the Dalitz plane which will be studied elsewhere.

5 Conclusion and an outlook

So far we have investigated only the most general and simple consequences of the LC for the wave functions in the case of hadron decays. We should stress that our analysis refers to the strong decays of hadrons, when the decay
is assumed to proceed in the nonperturbative way as in the string decay mechanism, so that in the decay matrix element participate the hadron wave functions as the whole objects, consisting of quarks and gluons connected by instantaneous strong interaction producing a string. Therefore in the decay of the string one obtains immediately again two string objects with their full wave functions and not separate quark and gluons, as it would be in the perturbative approach. From this point of view the use of the LC mechanism for the strong hadron decays seems to be well founded and the good agreement of the reduction coefficient $C_0^2$ with the well and long proved experimentally in $\rho(770)$ and $\rho(1450)$ decays gives additional support for it. We have not discussed all consequences of this LC formalism, which works also reasonably well in form factors of mesons [15] and baryons [16] and can in principle be used in all reactions where hadron wave functions enter explicitly. To proceed further one needs to develop an ”anatomy” of the decay and in general of the hadron exchange and creation processes, which is in progress. The author is grateful to A.M.Badalian for useful discussions and advices. This work is supported by the Russian Science Foundation (RSF) in the framework of the scientific project,Grant 16-12-10414.

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