Reflection and Lesson Study in a Mathematics Didactics Course: A Case Study

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Teaching mathematics is complex. The changing role of the teacher and the increased demands and expectations placed upon mathematics teachers in the 21st century in South Africa significantly influence the types of knowledge and skills they require in their undergraduate education and professional development. Reflection, as an aspect of metacognitive awareness, enables teachers to think with what they know in learning about teaching and it is therefore an essential characteristic of professional development of mathematics student teachers. Adapted lesson study and literature regarding metacognitive awareness and reflection were implemented in a mathematics didactics course. In this qualitative interpretative case study, the levels of reflections (guided by reflective prompts) and metacognitive awareness in the reflections of one student teacher (Anne) in a mathematics didactics course were collected, analysed, and interpreted (as part of a bigger study). Anne gave evidence in her reflections of her personal insight on her task (teaching), the learners (person) in her classroom, and the teaching-learning strategies (strategies) she had to reflect on the planning, presenting, and evaluating her mathematics lessons. Using adapted lesson study and reflections seems to have facilitated Anne’s metacognitive awareness as well as her professional development as a mathematics teacher.

Keywords: adapted lesson study, student teachers, metacognitive awareness, reflection, professional development, case study

Introduction

Teaching has always been complex. Teaching mathematics in the 21st century is even more complex (Graham & Phelps, 2003). In the context of a teacher education programme (BEd) at a tertiary institution, adapted lesson study, metacognition, and reflection were implemented to develop student teachers’ metacognitive awareness as a means to prepare them to become lifelong active learners of teaching-learning who are responsible for their own growth and development (Lieberman & Miller, 1992).

Mathematics student teachers are learners on the one hand, learning mathematics, and learning to teach mathematics. And on the other hand, they are also future teachers of mathematics who will facilitate the learning of mathematics (Roux & Van der Walt, in process). According to Graham and Phelps (2003), student teachers are actively involved in the “practical know-how of teaching mathematics” whilst facilitating learning concerned with their professional development. These authors recommend that emphasis should be placed on metacognitive and reflective teaching-learning strategies which aim to professionally develop mathematics.
teachers (Graham & Phelps, 2003). Furthermore, Calderhead (1989) argues that reflection and metacognitive learning processes are constitutive of lifelong learning.

The tendency to think about a task—producing a metacognitive conceptualization of the (teaching or learning) task—and to use metacognitive knowledge and self-regulation is called reflection (Cornoldi, 2010). Cornoldi (2010) argues that a person develops reflection about the nature of his/her own cognitive activity and thinking about the possibility of extending and using this reflection. Reflection, as an aspect of metacognitive awareness, enables teachers to think with what they know in learning about teaching (Ertmer & Newby, 1996). Metacognitive knowledge includes knowledge about one’s own and others’ cognition, the cognitive demands of specific tasks, and available and appropriate teaching-learning strategies (Ertmer & Newby, 1996). Self-regulation entails planning, monitoring, and evaluation of action (e.g., teaching). Reflection is seen as the glue between metacognitive knowledge and self-regulation (Ertmer & Newby, 1996). Although one can distinguish between metacognitive knowledge, self-regulation, and reflection as dimensions of metacognitive awareness, one can never separate these dimensions.

Reflection emerges as a way of supporting students’ improved understanding of what and how they know and do as they develop their knowledge of teaching through reconsidering what, when, where, and how they learn during teaching, forming important aspects of metacognitive awareness. Reflection on knowledge facing towards the metacognitive knowledge (or experiences) or self-regulation that has been owned previously for its association with action (e.g., teaching) builds new and more comprehensive metacognitive knowledge (Muin, 2011).

The Problem Statement

Ambrose, Bridges, Lovett, DiPietro, and Norman (2010) assert that managing your own teaching is centred around metacognitive processes and is critical for effective teaching when complex pedagogical decisions have to be taken: namely to assess the cognitive demands of the teaching (-learning) task, evaluate their own and their learners’ knowledge, skills, and progress, plan their own appropriate approach, monitor their own and their learners’ progress, and adjust their teaching-learning strategies as needed. Future mathematics teachers can manage their own teaching that is centred around metacognitive processes and that is critical for effective teaching (Muin, 2011).

A need for developing and undertaking a mathematics didactics course that focused on the development of mathematics student teachers’ capacities as learners and teachers (Graham & Phelps, 2003) as well as on meaning making and inquiry (Freire, 1997) has been/was identified.

The aim of this research (as well as the bigger project) was to explore one final year pre-service mathematics teacher’s (Anne’s) levels of reflection as well as aspects of her metacognitive awareness through her reflections on a mathematics didactics course and a three-week work integrated learning (WIL) period at a rural primary school. This paper is part of a bigger research project that aimed to develop and refine didactics curricula for mathematics teacher education programmes that facilitate metacognitive awareness.

Reflection as a teaching-learning approach requires learners’ active engagement in their own learning which activates their metacognitive awareness to manage, e.g., their own thinking, self-assessment, critical thinking, goal setting, and reading comprehension (Cornodi, 2010).
Lesson Study

According to Lewis, Perry, and Hurd (2009, p. 286), “Lesson study makes various types of knowledge more visible, such as colleagues’ ideas about pedagogy and students’ mathematical thinking, thereby enabling teachers to encounter new or different ideas, and to refine their knowledge”.

Lesson study was adapted and implemented in a fourth year mathematics didactics coursework as a way to get student teachers actively involved in their own process of investigating the practice of teaching-learning (Fernandes & Yoshida, 2004). The student teachers worked in collaborative groups in a cycle of activities to plan, present, observe, and refine a lesson. The lesson focused on learners’ understanding and learning, making use of a well-designed pedagogy.

The student teachers had to collaboratively decide on a topic for Grade 6 learners; they had to set goals, plan a lesson (according to the Curriculum Assessment Policy Statement (SA DBE, 2011)), and present and refine the lesson. During all the activities/steps the group had to reflect on planning: e.g., what and how did they do the planning?; monitoring: e.g., what progress did they make? Should they change some activity?; evaluation: e.g., did it work/not? Why did/didn’t it work? How can the lesson be refined? (Takahashi, 2006)

The lesson study process activated and developed student teachers’ thinking about specific aspects of teaching practice such as different teaching strategies implemented in different grades and different topics (Lewis, Perry, & Hurd, 2009).

Reflection

According to Cornoldi (2010), reflection is the medium through which we teach and learn by planning, monitoring, and evaluating teaching and learning (Cornoldi, 2010). Reflection on metacognitive knowledge (person, task, strategy) precedes (for action), accompanies (in action), and follows (on action) cognitive strategies (Livingston, 1997): learning content (subject matter) on the one hand, and metacognitive knowledge (person, task, and strategy variables) and self-regulation (planning, monitoring, and evaluation) on the other (Livingston, 1997). Through reflection, student teachers develop a better understanding of themselves as teachers and of their teaching task, and enable them to build future teaching on the lessons learned from previous experiences (Sanders, 2009). Therefore reflection influences teachers’ complex conceptualisations of their own and their peers’ teaching practices.

According to Vygotsky (1987), an individual’s mental activity (reflection in this case) founded and integrated in a mutually, cultural, educational, and historical context. Social activity is a crucial aspect of learning about teaching a starting point to improve the quality of teaching mathematics.

Research Methodology

In this qualitative interpretive case study, the focus is on one mathematics student teacher’s (Anne’s—in a final year BEd-programme) reflections on her didactics course work as well as teaching experiences during a three week period of Work Integrated Learning (WIL) at a rural primary school. Reflection is seen as a meaningful way of approaching learning about teaching mathematics and better understanding of teaching mathematics (Loughran, 2002).

Population

The bigger study included all the fourth year student teachers (N = 30) who enrolled for a mathematics
didactics course at an Afrikaans rural university. All these mathematics student teachers participated in all the activities included in the course.

**Sample**

Although the reflections for in- and on-action of the whole group (N = 30) (final year intermediate (Grade 4 to 6) and senior (Grade 7 to 9) phase student teachers) were collected, analysed, and interpreted to reveal student teachers’ levels of reflection and professional development, as well as their recommendations to enable the teacher educator (author) to revisit and improve course work in mathematics teacher education, this paper reports on a specific case: that of Anne.

The participant is female, who attended English medium primary and secondary schools but chose to enrol at a rural Afrikaans university. Although the university provides for simultaneous interpretations to English speaking students in all undergraduate studies, Anne chose not to make use of this service. However, she wrote tests, exams, did presentations, assignments, and participated in discussions using her home language (English). She is an average performer (between 65% and 72%) in both mathematics and mathematics didactics courses.

Anne was purposefully chosen from the population as a case study because she was able to verbalise her reflections on her experiences fluently; her own reflections (verbatim) (in English) could be used to support the validity and trustworthiness of the results and discussion thereof; and she gave evidence of profound professional development during the specific semester, and Anne was placed at a rural B B’s Primary School … a very low SES school which is sensitive towards the needs of each of its learners … during the three-week WIL-period.

**Data Collection Procedures**

**Outline of Course Work for Undergraduate (BEd) for Student Mathematics Teachers**

In Table 1 an outline of all the mathematics courses student teachers had to enrol for, is listed.

| 2nd year | 3rd year | 4th year |
|----------|----------|----------|
| 1st semester | 1st semester | 1st semester |
| Number and operations | Geometry | Didactics: Adapted lesson study |
| 2nd semester | 2nd semester | 2nd semester |
| Number theory | Statistics | Didactics |
| 1st semester | 2nd semester | Didactics |
| Didactics | Didactics | Didactics |

The author has developed and facilitates two (out of four) mathematics education didactics courses focusing on Grades 4 to 9—mathematics, as well as the two accompanying didactics courses. The author re-visits the content of the didactics courses on a yearly basis to ensure high quality education.

Adapted lesson study was used as a vehicle in a fourth year mathematics didactics course to facilitate pre-service mathematics teachers’ metacognitive awareness and more specifically their reflective practice during lesson planning (reflection before action), presenting the lesson (reflection during/in action) and evaluating (on) it afterwards. Adapted lesson study required from student teachers to set goals, to plan, present, and refine their lesson. Groups reported on a regular basis on their progress and challenges.
Furthermore, student teachers had to conduct a literature study on lesson study, metacognition, and mathematical knowledge for teaching. Their assignments included the collaborative planning of two lessons for learners in a Grade 6 mathematics class according to the Curriculum Assessment Policy Statement (SA DBE, 2011) requirements using an adapted lesson study approach.

During the semester (course work) regular feedback and support from the lecturer and peers were provided during whole class and group discussions. They regularly reflected on what they had done and learned—as a group and individually. All students were encouraged to teach the lessons their group prepared (in the didactics course work) during work integrated learning (WIL) at the school where they were placed. At the end of the three-week period (WIL) (halfway in the semester), they had to reflect individually on their teaching experiences during WIL, on the didactics module course work, and to pose recommendations on how the course work could be improved.

Some examples of reflective prompts that student teachers received are provided below:

- How and what did you apply and implement what you have learned and done in lesson studies in MATHD (course), during WIL?
- Describe your most positive experience—concerning teaching and/or learning—during WIL (reflection on action).
- What about teaching-learning in mathematics? Do you still not understand? (reflection on action)
- What, do you think, are the strengths and weaknesses for student teachers regarding learning about teaching in MATHD?

Data Analysis

The a-priori codes used to code data for interpretation were compiled from the working definition of metacognitive awareness and are summarized in Table 2 below. The four levels of reflection, according to Kember, McKay, Sinclair, and Wong (2008), were implemented to analyse and interpret the data collected (Table 3).

| Metacognitive knowledge (variables) | Code | Self-regulation Code | Levels of reflection (Kember et al., 2008) | Code |
|------------------------------------|------|----------------------|-------------------------------------------|------|
| Person                             | P    | Planning PL          | Non-reflection L1                         | L1   |
| Task                               | T    | Monitoring M         | Understanding L2                          | L2   |
| Strategy                           | S    | Evaluating E         | Reflection L3                             | L3   |
|                                    |      |                      | Critical reflection L4                     | L4   |

Table 3 presents the multi-dimensional framework for coding and assessment of mathematics student teachers’ reflection for, in, and on action (Kember et al., 2008).

| Level 1 (lowest) Non-reflection | Level 2 Understanding | Level 3 Reflection | Level 4 (highest) Critical reflection |
|---------------------------------|-----------------------|--------------------|--------------------------------------|
| Reflections show … -no evidence of student attempting to reach an | Reflections show … -evidence of understanding of a concept or topic | Reflections show … -that theory is applied to practical situations | Reflections show … -evidence of a change in perspective over a fundamental |
understanding of the concept or theory which underpins the topic
-material has been placed into an essay without the student thinking seriously about it, trying to interpret the material, or forming a view
-largely reproduction, with or without adaptation, of the work of others
-material is confined to theory
-reliance upon what was in the textbook or the lectures, note
-theory is not related to personal experiences or real-life applications or practical situations
-situations encountered in practice will be considered and successfully discussed in relationship to what has been taught
-that there are personal insights which go beyond book theory
-belief or the understanding of a key concept or phenomenon
-critical reflection is unlikely to occur frequently

Results, Findings, and Discussions

Although these mathematics student teachers’ perceived themselves as metacognitive aware and self-directed learners (students) in 2013 in a mathematics course (Van der Walt, 2014), analysis of their reflections on their own learning in the didactics module and the teaching experiences during WIL indicated that the group (N = 30) mostly reflected at the lowest 2 levels (non-reflection and understanding) of the four-category scheme for coding and assessing reflection in written work (Kember et al., 2008). It seems as if these student teachers are not as self-directed and metacognitive aware as they perceived themselves during the previous study. However, Anne’s reflections were mostly at Level 3 (reflection) and sometimes even close to Level 4 (critical reflection).

Anne’s Reflections on Her Teaching Experiences During WIL

I knew that the lesson which I would present to the learners would be difficult (P T PL M) for learners in the class; (L3) the learners (P) do not willingly raise their hands to participate in a class discussion (P T M), to ensure (PL M) that the learners felt confident in speaking to me as a group; I ensured (PL M) that learners were working in groups where they could support each other in their open discussion with me.

Anne gave evidence of her personal insight on her task (planning and conducting a mathematics lesson) (L3), the learners and the strategies she has to take into consideration when planning the lesson (reflection for action).

Both Reflection Levels 2 and 3 provided evidence of her understanding the persons, task variables at hand, and strategy variable to be applied and implemented; she applied theory to practical situations and showed personal insight into the teaching situation.

During the planning of my lesson (T PL), I thought of every possible response (T PL M) that the learners will likely give me; I used this as my guide (T PL M) in developing my lesson plan. I tried to keep my lesson plan as simple as possible, the learners which I taught had severe boundaries to learning, and … (T P PL M) (L3).

This may be evidence that she was monitoring her planned strategies—against the lesson she planned and the learners in her classroom—while planning the lesson. Furthermore, Anne shows evidence of personal insights which go beyond book theory.

“I had to identify specific student needs and form curricular goals” so as to “adapt my lessons in such a way as to ensure that the learning content will be relevant towards the experiences of the learners” (L2); she investigated/researched the learners’ real-life situation and described it as follows: “The learners at my school are from extremely poor backgrounds; these learners (most of them) attend school because they receive one warm meal—which is in some cases the only meal that they will have for that day…”

Anne first explained with understanding, what she learned about teaching mathematics (her task), the
learners (persons in her classroom), and how she considered appropriate (teaching- and learning-) strategies. She related this to task, person, and strategy variables in practice (work integrated learning) in a real-life situation; she turned to the application and implementation of what she learned (L3) to planning her lesson (practical situation/WIL). While doing so, there is evidence that she constantly monitored person, task, and strategy variables. In this regard she recalled that she had to identify specific goals for her lessons…

Lastly, during the debriefing stage I analysed the data which I have collected (learner achievement and progression). This served as the reflection of my lesson (T M E), where I allowed myself time to revisit the events of the lesson (T PL M E) and to ensure that I apply new improved strategies (S PL M) to the next lesson that I planned (L3). …The lesson study process has taught me to implement constant reflection (L2). I have also implemented what I have learned by making relevant and meaningful adjustments (L3).

Anne recalled that while she was planning the lesson she both monitored and evaluated the strategies she planned in her lesson in relation to the learners in her class. Her personal insights from theory and practice became clear when she explained what she did: preparing two lessons to accommodate all learners’ capabilities/learning styles.

Furthermore, Anne invited her mentor teacher at the school to evaluate (to observe and to give feedback, (L3)) her trial-lesson and afterwards she “revisit the events of the lesson to ensure that I apply new improved strategies to the next lesson that I planned”.

One could argue that the reflection on her side showed evidence of a slight change in her perspective over an understanding of planning and revisiting the lesson to ensure “best practice” (a key concept of lesson study) (close to L4, critical reflection). There is evidence that she applied theory (course work) to practical situations (planning a lesson—implementing aspects of adapted lesson study for a lesson to be presented during WIL).

My class mentor told me on the day of our arrival that the learners (P) often to not master outcomes (T) and those very simple and specially designed lessons (T) need to be presented to the learners. I did not present the learners with any specialised and simple work (T), … the outcomes as stated in the CAPS is what I expected from the learners. It was very meaningful to me when the assessment of the learners’ work showed positive marks (M E). Learners grasped the concepts and showed sufficient understanding of the work.

Anne challenged what her class mentor recommended to her when she warned her that the learners are not capable of learning more difficult work. She gave evidence of her monitoring and evaluating of the learners’ performance after teaching her lesson (reflection on action).

Although Anne listed (L1) what she learned, she also explained her understanding (L2) when she reflected on teaching a mathematics lesson to a Grade 6 class; she also recapped her reflection for teaching. She gave evidence of turning theory (course work) into practice (real-life professional work of a teacher).

Anne continues to reflect on (after action) the real-life situation (WIL):

It was a very positive learning and teaching experience (T) to me when the learners responded (P T M E) well to my teaching-style (L3). The learners were always actively involved during my lesson (P T S M); they worked well together and helped each other master the concepts.

While doing so, there is evidence that she constantly monitored person, task, and strategy variables. In this regard she recalled that she had to identify specific goals for her lessons.

“I had to identify specific student needs and form curricular goals” so as to “adapt my lessons in such a way as to ensure that the learning content will be relevant towards the experiences of the learners” (L2).
investigated the learners’ real-life situation: “The learners at my school are from extremely poor backgrounds,...”

**Anne’s Reflections on the Didactics Course Work**

Referring to the work of Deborah Ball on knowledge for teaching mathematics (Ball & Bass, 2002), Anne said that “(T)he course work richly enlightens mathematics teachers’ knowledge” (about teaching mathematics); … and “allows for meaningful insight into mathematical concepts…”

Anne also provided relevant recommendations for future refinement of the work integrated learning experience and the course work:

“The subject (course) MATHD richly enlightens mathematics teachers’ knowledge” (referring to the work of Deborah Ball on knowledge for teaching mathematics (Ball & Bass, 2002)), …MATHD allows for meaningful insight into mathematical concepts… the opportunity to work in a group (S) with other students, all with different perspectives (P). … taught me so much in working with other student teachers and … Students are able to share their views and insights with each other, (… accepting their point of view and challenging the ideas which they set forth) … are introduced into a learning context where meaningful content such as the lesson study cycle is taught, … to challenge all aspects of my thinking. … how to recognise when a learner develops a misunderstanding, I am able to immediately recognise its origin and how to alleviate the confusion. …

I believe the subject (course work) is challenging and forces students to examine their understandings of concepts and to be open towards change. (Although the course work) … is focused mainly on group work which is very beneficial … there should be more opportunity for students to work and be assessed on an individual basis.

**Anne’s Reflections on What She Still Does Not Understand or Know**

The last reflective prompt required student teachers to reflect on what they know they still do not know (understand) about teaching and/or learning mathematics. Aspects listed by Anne referred to reflections on situations encountered during WIL (action/practice) to be discussed in relation to what has been taught, e.g., “best” practice strategies (theory) taking the persons (specific learners) into account when planning and presenting a task (lesson) in real-life contexts (teaching at school) included:

How to (S) bring the mathematical concepts (T) into a real-world context based on the environment of the learners (P);

How to (S) effectively introduce (T) new mathematical concepts to the learners (P) where their interest in the lesson (T) is activated;

Forming a correct pertinent question (S) which is also based on real world problems (T); and

Different manners (S) in which to link learners’ (P) prior and existent knowledge with new mathematical concepts (T).

**Conclusion**

The opportunities offered to pre-service teachers to learn to teach mathematics (as learners) included experiences not only in unpacking mathematical concepts (Ball & Bass, 2002), the lesson study process, but also in learning to teach with and for metacognition (Van der Walt & Maree, 2007). Professional reflection tended to transform Anne’s teaching in some way (Ryan & Ryan, 2012; Hargrove & Netfeld, 2014) when she (e.g., recounts, describes, explains, and discusses her own lesson(s)), revisits past experiences, re-examination, and evaluation of experiences, beliefs, and knowledge (Kember et al., 2008).

Metacognition is fostered through reflection: i.e., people learn how to be critical in their thinking so they
are able to see what needs to be done to improve themselves as teachers and the quality of their teaching to facilitate more effective learning in mathematics (Ertmer & Newby, 1996). Mathematics teachers, being learners themselves, should first know how to reflect on their own teaching and learning before they can be responsible to facilitate and integrate metacognition in their own mathematics classrooms (Van der Walt, 2014).

To become lifelong learners, student teachers must learn to evaluate the demands of the task, their own knowledge and skills, plan their approach, monitor their progress, and adapt their teaching-learning strategies as needed (Shannon, 2008). These aspects are characteristic of metacognitive awareness. Better thinking is a means of education improving knowing and understanding and enabling one to think with what you know (Perkins, 1993). Social activity is an essential part of reflection and a crucial aspect of learning about teaching (Loughran, 2002) and a starting point to improve the quality of teaching mathematics (Kaune, 2006). Mathematics teachers, being learners themselves, should first know how to reflect on their own learning and teaching of mathematics before they can be responsible to develop, facilitate, and integrate metacognitive awareness in their own mathematics classes (Van der Walt, 2014). This implies that pre-service mathematics teachers should control their own teaching activities (cognition and metacognition) (Wirth & Perkins, 2008), and develop the ability to creatively and flexibly apply knowledge and skills in a variety of teaching-learning contexts (De Corte, Mason, De Paepe, & Verschaffel, 2011).

Reflection emerges as a way of supporting student teachers’ improved understanding of what they know and do as they develop their knowledge of teaching through reconsidering what and how they learn during teaching, forming important aspects of metacognitive awareness (Cornoldi, 2010). Reflection on knowledge facing towards the metacognitive knowledge (or experiences) or self-regulation that has been owned previously for its association with action (e.g., learning) builds new and more comprehensive metacognitive knowledge (Muin, 2011). Systematic reflection is seen as essential for continuing professional development and lifelong learning, and further studies are necessary for 21st century teacher training programmes that will facilitate mathematics teachers’ metacognitive awareness and their professional development.

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