**KN** scattering amplitude revisited in chiral unitary approach and a possible broad resonance in $S = +1$ channel

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We revisit the $KN$ scattering amplitude in order to investigate the possibility for the existence of a broad resonance in the $I = 0$ $KN$ channel around the energy of 1620 MeV with 310 MeV width. We use the chiral unitary model to describe the $K^+N$ scattering amplitudes and determine the model parameters so as to reproduce the differential cross sections of the $K^+N$ scatterings and the $I = 0$ total cross section up to $p_{lab} = 800$ MeV/c, where inelastic contributions are significant. Performing analytic continuation of the determined amplitude to the complex energy plane, we find a pole for a broad resonance state. We point out that the rapid increase appearing in the $I = 0$ total cross section at $p_{lab} = 500$ MeV/c is a hint of the possible broad resonance of strangeness $S = +1$.

1. Introduction

The study of meson-baryon scattering is very important to understand the properties of hadron resonances. Because hadron resonances are short living and decay immediately by the strong interaction, they appear only in the scattering processes and one can deduce the properties of the hadron resonances only from the investigation of the scattering process. Description of scattering amplitude is one of the first steps to investigate the hadronic resonances. Once one obtains realistic scattering amplitudes reproducing the scattering cross sections in terms of analytic functions, one can perform analytic continuation to complex energy plane and obtain properties of the resonances, such as their masses, widths and coupling strengths. For the purpose of description of the scattering amplitude in an analytic way, one of the theoretical tools is the chiral effective theory in which the low energy theorems by chiral symmetry constrain the hadronic interactions. Chiral perturbation theory describes scattering amplitudes for the lowest channels, while some unitarization procedure is necessary when one encounters resonances and open channels, where hadronic dynamics plays an important role. For instance, chiral perturbation theory works well for the $\pi N$ scattering at low energies, while, for the $KN$ channel, since the $\Lambda(1405)$ resonance is located...
in the $I = 0$ channel below the threshold and the $\pi \Sigma$ and $\pi \Lambda$ channels are open, one needs unitarization of the amplitude and takes into account of coupled channels.

In this article, we reexamine the elastic scattering amplitude of $KN$ in low energies, $p_{\text{lab}} < 800$ MeV/$c$, based on the chiral unitary approach and study the possibility of an $S = +1$ exotic resonance in $I = 0$ channel. Baryons with strangeness $S = +1$ are so-called exotic hadrons, because their quantum numbers cannot be described by three constituent quarks. For the $S = +1$ baryon, one needs at least one anti-strange quark and more than three quarks to compensate the negative baryon number of the anti-strange quark to have baryon number +1 in total. Thus, the minimal quark contents are $uudd\bar{s}$ for charge $Q = +1$. Although there are no reasons to forbid the existence of such states in quantum chromodynamics, the experimental evidence for existence of the $S = +1$ baryons is not well confirmed.

The scattering amplitudes of the $K^+N$ scattering in low energies have been studied for a long time. A comprehensive review can be found in Ref. [1]. There are three $K^+N$ amplitudes, $K^+p \to K^+p$, $K^+n \to K^+n$ and $K^+n \to K^0p$ and isospin symmetry reduces two independent amplitudes for $I = 0$ and $I = 1$. The $K^+p$ amplitude can be observed directly from the $K^+p \to K^+p$ scattering experiment and provides the $I = 1$ amplitude, while for the $K^+n$ amplitudes one needs nuclear targets, such as deuterium, and obtains the $I = 0$ amplitude using the $I = 1$ amplitude. It is known that the $K^+N$ scatterings are almost elastic for $p_{\text{lab}} < 800$ MeV/$c$ and inelastic contributions are not significant [2]. For low energies, the $K^+p$ scattering is described by $S$-wave [3]. In addition, the differential cross section of the $K^+p$ scattering in low energies shows constructive interference between the Coulomb and strong interactions at very forward angles. This implies that the low energy $K^+p$ scattering is to be repulsive [3, 4]. In contrast, the $I = 0$ amplitude is more ambiguous. In Refs. [5, 6], it was shown that the scattering amplitude for $I = 0$ has $P$-wave contribution to reproduce the $K^+d$ scattering up to $p_{\text{lab}} < 500$ MeV/$c$. The phase shift analysis up to 1.5 GeV/$c$ by Ref. [7] found several solutions, in which one solution implies that the low energy scattering is described dominantly by $S$-wave, while another solution reproduces the amplitude mainly by $P$-wave. The phase shift analyses with new data performed by Refs. [8–10] supported the latter $P$-wave solution. The analysis carried out by Ref. [11] treated both $I = 0$ and $I = 1$ amplitudes at the same time, and found that the $P$-wave contribution was significant for the low energy $I = 0$ amplitude.

The search for $S = +1$ resonance has been carried out in the past. In the earlier studies, a possible $S = +1$ broad resonance $Z^*$ in the $KN$ scattering with $I = 0$ were discussed [7, 12–19]. In Ref. [15], it was pointed out that there the $K^+N$ total cross sections [12] and $K^{-}$-photoproduction [13] showed two bump structures which would have risen possible $KN$ resonances with $I = 0$. Although the phase shift analysis by Martin [11] found that there were no significant resonances in the partial wave amplitude, the Argand diagram suggested that there could be some broad resonances appearing in $P_{01}$ and $D_{03}$ [20–22]. These resonances were reported as broad resonances above the energies where the inelastic contributions are significant. The studies of the bump structure in cross sections and the behavior of the Argand diagrams are not sufficient to fix the existence of the resonance states. One of the promising ways is to analyze the scattering amplitude as a analytic function and to carry out analytic continuation of the amplitude into the complex energy plane. Resonance states are expressed as poles of the scattering amplitude. Another kind of the resonance with $S = +1$ was suggested by LEPS collaboration in photoproduction experiments [23]. They claimed
a narrow resonance, $\Theta^+$, with $S = +1$ and 1.5 GeV/$c^2$ mass [23, 24]. This experiment was motivated by a theoretical work [25] predicting a narrow resonance with $S = +1$ around 1540 MeV/$c^2$. The $\Theta^+$ resonance is obviously different from the previous $Z^*$ resonance. Here we revisit the possibility of the existence of a $Z^*$-type broad resonance with $S = +1$ and $I = 0$ at lower energies than where the inelastic contribution starts to be significant.

As mentioned in the above, it is important to understand the resonance properties by studying the scattering amplitude, especially in terms of an analytic function. There are several approaches to describe baryon resonance. We use chiral unitary model, in which scattering problem is solved in a simplified manner by considering elastic unitarity of the two-body scattering and the elementary interaction is given based on chiral perturbation theory, firstly suggested in Ref. [26] and developed in [27], for the $S = −1$ channel, and we can find recent progress in a review article [28]. Chiral unitary model impose unitary condition by infinite summation of the specific diagram and describes the scattering amplitude as an analytic function. Thus, it is easy to perform analytic continuation of the scattering amplitude and to pin down the position of the resonance state in the complex energy plane.

One of the most successful examples of this approach is the finding of the double pole structure of the $\Lambda(1405)$ and investigation of its physical significance [29, 30]. It was reported in Ref. [31, 32] that the Tomozawa-Weinberg interactions for the exotic channels do not provide enough attraction to make two-body bound states for a Nambu-Goldstone boson and a hadron. Actually the Tomozawa-Weinberg term vanishes for $I = 0$ and $S = +1$. Here the next-to-leading contributions are responsible for the attraction to provide a broad resonance.

This paper is organized as follows. In Sect. 2, we construct $KN$ scattering amplitude using chiral perturbation theory and chiral unitary model. In Sect. 3, we determine the parameters which reproduce $KN$ scattering data. The total cross section and differential cross section data are compared with our results. Using the constructed amplitude, we discuss the possibility of resonance with large width. In Sect. 4, we summarize the results of this paper.

2. Formulation

For our theoretical investigation, we would like to represent the $KN$ scattering amplitude in an analytic function of the center of mass energy $W$. Once we parametrize the scattering amplitude in an analytic function, analytic continuation allows us to extend the amplitude to the complex energy plane, where resonances are represented as poles, and extract the properties of resonances, such as mass, decay width and coupling strength. For this purpose, we describe the $KN$ elastic scattering amplitude based on the chiral unitary approach by solving Lippmann-Schwinger equation

$$T = V + VGT$$

in a simplified way. In the chiral unitary approach, the interaction kernel $V$ is given by chiral perturbation theory and we restrict the intermediate state to the elastic channel. The model parameters are determined so as to reproduce the observed $KN$ cross section.

2.1. Scattering amplitude

Let us call the momenta of the kaon and nucleon in the initial (final) state by $p_1$ and $p_2$ ($p_3$ and $p_4$), respectively. According to Lorentz invariance, the $T$-matrix of the $KN$ scattering
can be written in terms of two Lorentz invariant functions, \( A(s, t) \) and \( B(s, t) \), in general, as

\[
T(s, t) = \bar{u}(\vec{p}_4, s_4) \left[ A(s, t) + \frac{1}{2} (\vec{p}_1 + \vec{p}_3) B(s, t) \right] u(\vec{p}_2, s_2)
\] (2)

with the Mandelstam variables \( s = (p_1 + p_2)^2 \) and \( t = (p_1 - p_3)^2 \) and the on-shell Dirac spinor \( u(\vec{p}, s) \) for nucleon with momentum \( p \) and spin \( s \). The Dirac spinor \( u(\vec{p}, s) \) is normalized by \( \bar{u}(\vec{p}, s) u(\vec{p}, s') = 2M_N \delta_{ss'} \) with the nucleon mass \( M_N \). For partial wave decomposition, we write the \( T \)-matrix in the center of mass system with two functions \( f \) and \( g \) in terms of the spin-nonflip and spin-flip parts as

\[
T(s, t) = \chi^\dagger(\lambda_4) \left[ f(W, \theta) - i(\vec{\sigma} \cdot \hat{n}) g(W, \theta) \right] \chi(\lambda_2)
\] (3)

where \( W \) and \( \theta \) are the total energy and scattering angle (angle between \( \vec{p}_1 \) and \( \vec{p}_3 \)) in the center of mass system, respectively, \( \hat{n} \) is the normal vector of the scattering plane defined by \( \hat{n} = (\vec{p}_3 \times \vec{p}_1)/|\vec{p}_3 \times \vec{p}_1| \), and \( \chi(\lambda) \) is Pauli spinor of nucleon with helicity \( \lambda \). The relation between \( A, B \) and \( f, g \) in the \( KN \) elastic scattering is given by

\[
f(W, \theta) = (E_N + M_N)(A + \omega B) + k^2 B + \frac{(E_N + M_N + \omega)B - A}{E_N + M_N} k^2 \cos \theta
\] (4)

\[
g(W, \theta) = \frac{A - (E_N + M_N + \omega)B}{(E_N + M_N)} k^2 \sin \theta
\] (5)

with the kaon energy \( \omega \), the center of mass momentum \( k \) and the nucleon energy \( E_N \).

The amplitudes \( f(W, \theta) \) and \( g(W, \theta) \) can be decomposed into partial waves with Legendre polynomials \( P_\ell(x) \) as

\[
f(W, \theta) = \sum_{\ell=0}^{\infty} f_\ell(W) P_\ell(\cos \theta),
\] (6)

\[
g(W, \theta) = \sum_{\ell=1}^{\infty} g_\ell(W) \sin \theta \frac{dP_\ell(\cos \theta)}{d \cos \theta}.
\] (7)

It is convenient to introduce the amplitude \( T_{\ell\pm} \) having definite total angular momentum \( j = \ell \pm \frac{1}{2} \) by

\[
f_\ell(W) = (\ell + 1)T_{\ell+}(W) + \ell T_{\ell-}(W),
\] (8)

\[
g_\ell(W) = T_{\ell+}(W) - T_{\ell-}(W),
\] (9)

or equivalently

\[
T_{\ell+}(W) = \frac{1}{2\ell + 1} (f_\ell(W) + \ell g_\ell(W)),
\] (10)

\[
T_{\ell-}(W) = \frac{1}{2\ell + 1} (f_\ell(W) - (\ell + 1)g_\ell(W)).
\] (11)

We also introduce the partial-wave decomposed interaction kernels \( V_{\ell+}(W) \) and \( V_{\ell-}(W) \) in the same way.
Here we show the $KN$ scattering amplitudes in the isospin channels, $T^I=0$ and $T^I=1$. The amplitudes in the particle basis can be obtained as

\begin{align}
T_{K^+p \rightarrow K^+p} &= T^{I=1} \\
T_{K^+n \rightarrow K^+n} &= \frac{1}{2}(T^{I=1} + T^{I=0}) \\
T_{K^+n \rightarrow K^0p} &= \frac{1}{2}(T^{I=1} - T^{I=0})
\end{align}

Taking spin average in the initial state and spin summation in the final state for nucleon, we calculate the differential cross section in the center of mass frame as

$$
\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2s} (|f(W, \theta)|^2 + |g(W, \theta)|^2)
$$

and the total cross section by integrating the differential cross section in terms of the scattering angle as

$$
\sigma = \frac{1}{32\pi s} \int_{-1}^{1} d\cos \theta (|f(W, \theta)|^2 + |g(W, \theta)|^2)
$$

2.2. Chiral Lagrangian

The leading order chiral Lagrangian for the baryon field $B$ reads

$$
\mathcal{L}^{(1)}_{MB} = \text{Tr} \left[ (B(i\partial - M_0)B) - \frac{D}{2} \text{Tr} \left( B\gamma_\mu \gamma_5 \{u^\mu, B\} \right) - \frac{F}{2} \text{Tr} \left( B\gamma_\mu \gamma_5 [u^\mu, B] \right) \right],
$$

where $M_0$ is the baryon mass at the chiral limit, the baryon and meson fields, $B$ and $\Phi$, are written in the SU(3) matrix form

\begin{align}
B &= \begin{pmatrix}
\Sigma^0/\sqrt{2} + \Lambda/\sqrt{6} & \Sigma^+ & p \\
\Sigma^- & -\Sigma^0/\sqrt{2} + \Lambda/\sqrt{6} & n \\
\Xi^- & \Xi^0/\sqrt{2} & -2\Lambda/\sqrt{6}
\end{pmatrix}, \\
\Phi &= \begin{pmatrix}
\pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\
\pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\
K^- & K^0/\sqrt{6} & -2\eta/\sqrt{6}
\end{pmatrix},
\end{align}

we parametrize the chiral field $U$ in the CCWZ form as

$$
U = \xi^2 = \exp \left( i \sqrt{\frac{2}{f}} \Phi \right)
$$

with a scale parameter $f$, which is turned to be identified as the meson decay constant in the leading order calculation of chiral perturbation theory, the covariant derivative for the baryon field is introduced as

$$
D_\mu B = \partial_\mu B + [\Gamma_\mu, B],
$$

with the mesonic vector current

$$
\Gamma_\mu = \frac{1}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger),
$$

and the meson-baryon coupling is given through the mesonic axial vector current

$$
u_\mu = i \left( \xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger \right)
$$

with low energy constants $D$ and $F$. The parameters $D$ and $F$ are to be determined by the axial couplings of the baryons at tree level.
The next-leading order of the chiral Lagrangian is composed of several terms

$$\mathcal{L}_{MB}^{(2)} = b_B \text{Tr} (\bar{B}\{\chi_+, B\}) + b_F \text{Tr} (\bar{B}\{\chi_+, B\}) + b_0 \text{Tr}(\bar{B}B)\text{Tr}(\chi_+) + d_1 \text{Tr} (\bar{B}\{u_\mu, [u^\mu, B]\})$$

$$+ d_2 \text{Tr} (\bar{B}[u_\mu, [u^\mu, B]]) + d_3 \text{Tr} (\bar{B}u_\mu)\text{Tr}(u^\mu B) + d_4 \text{Tr} (\bar{B}B)\text{Tr}(u^\mu u_\mu)$$

$$- \frac{g_1}{8M_N^2} \text{Tr} (\bar{B}\{u_\mu, [D^\mu, D^\nu]B\}) - \frac{g_2}{8M_N^2} \text{Tr} (\bar{B}[u_\mu, [D^\mu, D^\nu]B])$$

$$- \frac{g_3}{8M_N^2} \text{Tr}(\bar{B}u_\mu)\text{Tr}(u_\mu, [D^\mu, D^\nu]B) - \frac{g_4}{8M_N^2} \text{Tr}(\bar{B}[D^\mu, D^\nu]B)\text{Tr}(u_\mu u_\mu)$$

$$- \frac{h_1}{4} \text{Tr} (\bar{B}[\gamma^\mu, \gamma^\nu]Bu_\mu u_\nu) - \frac{h_2}{4} \text{Tr} (\bar{B}[\gamma^\mu, \gamma^\nu]u_\mu [u_\nu, B])$$

$$- \frac{h_3}{4} \text{Tr} (\bar{B}[\gamma^\mu, \gamma^\nu]u_\mu \{u_\nu, B\}) - \frac{h_4}{4} \text{Tr}(\bar{B}[\gamma^\mu, \gamma^\nu]u_\mu)\text{Tr}(u_\nu B) + \text{h.c.}$$

(24)

where \(b_i, d_i, g_i\) and \(h_i\) are low energy constants. The terms with \(b_i\) and \(d_i\) appear in the typical SU(3) chiral Lagrangians, while the terms with \(g_i\) and \(h_i\) are introduced as an extension of the SU(2) chiral Lagrangian \([33, 34]\). (See also Ref. \([35]\).) The scalar field \(\chi_+\) is given by

$$\chi_+ = 2B_0 \left( \xi M \xi + \xi^\dagger M \xi^\dagger \right),$$

(25)

with the quark mass matrix

$$\mathcal{M} = \text{diag} (m, \tilde{m}, m_s),$$

(26)

where \(\tilde{m}\) stands for the mass of the \(u\) and \(d\) quarks by assuming isospin symmetry, while \(m_s\) means the strange quark mass. Parameter \(B_0\) is a positive constant related to the meson mass and always appears together with the quark mass. The low energy constants some combinations of \(b_i, d_i, g_i\) and \(h_i\) are to be fixed by the \(K^+N\) scattering cross sections.

### 2.3. Interaction kernel

The tree level amplitude of the \(KN\) scattering up to the next-to-leading order is composed by three parts, the leading order contact term, the hyperon crossed Born term, and the next-to-leading contact term. The leading order contact term is called Tomozawa-Weinberg term, and is determined by the SU(3) group structure of hadrons without the low energy constants. It is known to be absent for the \(KN\) channel with \(I = 0\):

$$V_{WT}^{I=0} = 0, \quad V_{WT}^{I=1} = \frac{1}{2f_K^2} \bar{u}(\vec{p}_4, s_4)(\vec{p}_1 + \vec{p}_3)u(\vec{p}_2, s_2).$$

(27)

The corresponding invariant amplitudes read

$$A_{WT}^{I=0} = B_{WT}^{I=0} = A_{WT}^{I=1} = 0, \quad B_{WT}^{I=1} = \frac{1}{f_K}. \quad (28)$$

For the Born term, we do not consider explicit baryonic states with strangeness \(S = +1\). The pentaquark \(\Theta^+\) is a candidate for such a state, but it is known to have a narrow width and a very weak coupling to \(KN\). The hyperons with \(S = -1\), \(\Sigma\) and \(\Lambda\), contribute to the \(KN\) amplitude as crossed Born terms. With the chiral Lagrangian (17), we obtain the crossed
Born terms as

\[ V_{\text{Born}}^{l=0} = \frac{3}{4} \frac{(D - F)^2}{f_K^2} \eta(\vec{p}_4, s_4) \gamma_5 \frac{M_\Sigma + \eta(\vec{p}_2, \vec{p}_3)}{M_\Sigma^2 - (p_2 - p_3)^2 - i\epsilon} \gamma_5 u(\vec{p}_2, s_2) \]

\[ + \frac{1}{12} \frac{(3F + D)^2}{f_K^2} \eta(\vec{p}_4, s_4) \gamma_5 \frac{M_\Lambda + \eta(\vec{p}_2, \vec{p}_3)}{M_\Lambda^2 - (p_2 - p_3)^2 - i\epsilon} \gamma_5 u(\vec{p}_2, s_2) \]  

(29)

\[ V_{\text{Born}}^{l=1} = -\frac{1}{4} \frac{(D - F)^2}{f_K^2} \eta(\vec{p}_4, s_4) \gamma_5 \frac{M_\Sigma + \eta(\vec{p}_2, \vec{p}_3)}{M_\Sigma^2 - (p_2 - p_3)^2 - i\epsilon} \gamma_5 u(\vec{p}_2, s_2) \]

\[ - \frac{1}{12} \frac{(3F + D)^2}{f_K^2} \eta(\vec{p}_4, s_4) \gamma_5 \frac{M_\Lambda + \eta(\vec{p}_2, \vec{p}_3)}{M_\Lambda^2 - (p_2 - p_3)^2 - i\epsilon} \gamma_5 u(\vec{p}_2, s_2) \]  

(30)

with the Σ mass \( M_\Sigma \) and Λ masses \( M_\Lambda \). The invariant amplitudes are written as

\[ A_{\text{Born}}^{l=0} = \frac{3}{4} \frac{(D - F)^2}{f_K^2} \frac{(M_N + M_\Sigma)(M_\Sigma^2 - u)}{u - M_\Sigma^2} - \frac{1}{12} \frac{(3F + D)^2}{f_K^2} \frac{(M_N + M_\Lambda)(M_\Lambda^2 - u)}{u - M_\Lambda^2} \]  

(31)

\[ B_{\text{Born}}^{l=0} = -\frac{3}{4} \frac{(D - F)^2}{f_K^2} \frac{u + M_\Sigma^2 + 2M_\Sigma M_N}{u - M_\Sigma^2} + \frac{1}{12} \frac{(3F + D)^2}{f_K^2} \frac{u + M_\Lambda^2 + 2M_\Lambda M_N}{u - M_\Lambda^2} \]  

(32)

\[ A_{\text{Born}}^{l=1} = \frac{1}{4} \frac{(D - F)^2}{f_K^2} \frac{(M_N + M_\Sigma)(M_\Sigma^2 - u)}{u - M_\Sigma^2} + \frac{1}{12} \frac{(3F + D)^2}{f_K^2} \frac{(M_N + M_\Lambda)(M_\Lambda^2 - u)}{u - M_\Lambda^2} \]  

(33)

\[ B_{\text{Born}}^{l=1} = -\frac{1}{4} \frac{(D - F)^2}{f_K^2} \frac{u + M_\Sigma^2 + 2M_\Sigma M_N}{u - M_\Sigma^2} - \frac{1}{12} \frac{(3F + D)^2}{f_K^2} \frac{u + M_\Lambda^2 + 2M_\Lambda M_N}{u - M_\Lambda^2} \]  

(34)

with Mandelstam variable \( u = (p_1 - p_4)^2 = 2M_N^2 + 2M_K^2 - s - t \), the nucleon mass \( M_N \) and the kaon mass \( M_K \). The \( KN \) invariant amplitudes at the next-to-leading order chiral perturbation theory are calculated for each isospin channel as

\[ V_{\text{NLO}}^{l} = \left[ \frac{4B_0}{f_K^2} (\hat{m} + m_s) b^l + \frac{2}{f_K^2} (p_1 \cdot p_3) d^l \right. \]

\[ + \frac{(p_2 \cdot p_1)(p_2 \cdot p_3) + (p_4 \cdot p_1)(p_4 \cdot p_3)}{2M_N^2 f_K^2} g^{l'} \eta(\vec{p}_4, s_4) u(\vec{p}_2, s_2) \]

\[ \left. - \frac{h^l}{2f_K^2} p_3^l p_3^{l'} \eta(\vec{p}_4, s_4) [\gamma^\mu, \gamma^\nu] u(\vec{p}_2, s_2) \right] \]  

(35)

and the corresponding invariant amplitudes \( A \) and \( B \) read

\[ A_{\text{NLO}}^{l} = \frac{4B_0}{f_K^2} (\hat{m} + m_s) b^l + \frac{2}{f_K^2} (p_1 \cdot p_3) d^l \]

\[ + \frac{(p_2 \cdot p_1)(p_2 \cdot p_3) + (p_4 \cdot p_1)(p_4 \cdot p_3)}{2M_N^2 f_K^2} g^{l'} + \frac{p_1 \cdot (p_2 + p_4)}{f_K^2} h^{l'} \]  

(36)

\[ B_{\text{NLO}}^{l} = -\frac{2M_N}{f_K^2} h^{l'}. \]  

(37)

In these equations, the parameters \( b^l, d^l, g^l \) and \( h^l \) are defined by

\[ b^{l=0} = b_0 - b_F, \quad b^{l=1} = b_0 + b_D, \]

(38)

\[ d^{l=0} = 2d_1 + d_3 - 2d_4, \quad d^{l=1} = -2d_2 - d_3 - 2d_4, \]

(39)

\[ g^{l=0} = 2g_1 + g_3 - 2g_4, \quad g^{l=1} = -2g_2 - g_3 - 2g_4, \]

(40)

\[ h^{l=0} = h_1 + h_2 + h_3 + h_4, \quad h^{l=1} = h_1 - h_2 - h_3 - h_4, \]  

(41)
in terms of the low energy constants appearing in Lagrangian (24). We treat these combinations of the low energy constants as free parameters to be adjusted to reproduce observed $K N$ cross sections.

2.4. Unitarization

Unitarization is performed in each partial wave amplitude [36]. Because the total angular momentum is a good quantum number, Lippmann-Schwinger equation is also decomposed into partial waves as

$$T^{I}_{\ell\pm} = V^{I}_{\ell\pm} + V^{I}_{\ell\pm} GT^{I}_{\ell\pm}$$

(42)

where we have assume that we use a non-relativistic Green function and do not consider so-called zig-zag diagrams which mix the large and small components of the Dirac spinor. Supposing that we take only the on-shell contribution of the interaction kernel in the loop integral, we can solve Lippmann-Schwinger equation algebraically

$$T^{I}_{\ell\pm} = (1 - V^{I}_{\ell\pm} G)^{-1} V^{I}_{\ell\pm}$$

(43)

where the loop contribution $G$ for the $K N$ channel is given as a function of the center of mass energy, $W$, by

$$G(W) = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{(P - q)^2 - M_N^2 + i\epsilon} \frac{1}{q^2 - M_K^2 + i\epsilon}. \tag{44}$$

This integral can be performed by the dimensional regularization as

$$G(W) = \frac{1}{(4\pi)^2} \left\{ a(\mu) + \ln \frac{M_N^2}{\mu^2} + \frac{M_N^2 - M_K^2 + s}{2s} \ln \frac{M_K^2}{M_N^2} \right.$$  

$$+ \frac{\bar{q}}{\sqrt{s}} \left[ \ln (s - (M_N^2 - M_K^2) + 2\sqrt{s\bar{q}}) + \ln (s + (M_N^2 - M_K^2) + 2\sqrt{s\bar{q}}) \right.$$  

$$\left. - \ln (-s + (M_N^2 - M_K^2) + 2\sqrt{s\bar{q}}) - \ln (-s - (M_N^2 - M_K^2) + 2\sqrt{s\bar{q}}) \right] \right\}, \tag{45}$$

where $\mu$ is the scale parameter of the dimensional regularization and $a(\mu)$ is the subtraction constant depending on $\mu$. We take away the infinite part as renormalization procedure, and the subtraction constant is determined so as to reproduce experiments. For the interaction kernel, here we take the chiral perturbation amplitudes calculated in the previous section up to the next-to-leading order as

$$V' = V'_{\text{WT}} + V'_{\text{Born}} + V'_{\text{NLO}}$$

(46)

and perform partial wave decomposition in the way explained in Sec. 2.1.

2.5. Coulomb correction

For the $K^+p$ amplitude, we introduce the Coulomb correction as done in Ref. [37]. To the strong interaction part of the $K^+p$ scattering amplitude calculated in the center of mass
Table 1: The values of the fixed parameters. We take the isospin averaged masses.

| $M_N$     | $M_K$      | $M_\Lambda$ | $M_\Sigma$ | $f_K$ | $D$  | $F$  |
|----------|-----------|-------------|------------|-------|------|------|
| 938.9 MeV | 495.6 MeV | 1115.7 MeV | 1193.2 MeV | 110.0 MeV | 0.80 | 0.46 |

frame, we add the Coulomb amplitude

$$f_C = \frac{\alpha}{2k v \sin^2(\theta/2)} \exp \left[ -i \frac{\alpha}{v} \ln \left( \frac{\sin^2 \theta}{2} \right) \right] \tag{47}$$

with the scattering angle $\theta$, the fine structure constant $\alpha$ and the $KN$ relative velocity $v$ defined by

$$v = \frac{k(E_K + E_p)}{E_K E_p}, \tag{48}$$

and multiply the Coulomb phase shift factor $e^{2i\Phi_\ell}$ with

$$\Phi_\ell = \sum_{n=1}^\ell \tan^{-1} \frac{\alpha}{nv} \tag{49}$$

for $\ell > 0$ ($\Phi_0 = 0$) as

$$f^{K+p} = \sum_{\ell=0}^\infty \left[ (\ell + 1) T^{I=1}_{\ell+} + \ell T^{I=1}_{\ell-} \right] e^{2i\Phi_\ell} P_\ell(\cos \theta) - 8\pi \sqrt{s} f_C, \tag{50}$$

$$g^{K+p} = \sum_{\ell=1}^\infty \left[ T^{I=1}_{\ell+} - T^{I=1}_{\ell-} \right] e^{2i\Phi_\ell} \sin \theta \frac{dP_\ell(\cos \theta)}{d \cos \theta}. \tag{51}$$

3. Results

In the previous section, we have constructed the unitarized $KN$ amplitudes using the chiral unitary model. In this section, we carry out the $\chi^2$ fitting of the unitarized amplitude to the experimental data and determine the low-energy constants appearing in the next-to-leading order. We assume to fix the subtraction constants for $I = 0$ and 1 at a natural value $a^{I=0,1} = -1.150$ defined in Ref. [38] to suppress the number of the parameters in the situation that the experimental data have somewhat large error and disagreement in different experiments. Moderate change of the subtraction constants can be absorbed into the low-energy constants. We assume isospin symmetry and use the isospin averaged masses for the hadrons. We take the kaon decay constant as $f_K = (1.19 \pm 0.01) f_\pi = 110.0$ MeV where $f_\pi = 92.4$ MeV. The low-energy constants at the leading-order chiral Lagrangian $D$ and $F$ are already determined by the semileptonic hyperon beta decay reported in [39] as

$$D = 0.80, \ F = 0.46. \tag{52}$$

The values of these parameters are summarized in Table 1.
Table 2: The determined parameters for the $I = 1$ and $I = 0$ amplitudes. There are three solutions for $I = 0$. Solution 1, 2 and 3 are characteristic in reproduction of the experimental data. (See text in details.) The values of the parameters $b^I$, $d^I$, $g^I$ and $h^I$ are shown in unit of $10^{-3}$ MeV$^{-1}$. The subtraction constants for $I = 0$ and $I = 1$ are fixed as $a^{I=0,1} = -1.150$.

|        | Solution 1 | Solution 2 | Solution 3 |
|--------|------------|------------|------------|
| $b^I=1$ | 0.54       | 0.30       | 0.30       |
| $d^I=1$ | -0.29      | -0.24      | -0.24      |
| $I = 1$ | $g^I=1$ | 0.05       | 0.72       | 0.72       |
|        | $h^I=1$ | 0.03       | 1.05       | 1.05       |
|        | $\chi^2/N$ | 1.02      | 0.91      | 0.91       |
| $b^I=0$ | -0.05      | -0.51      | 0.38       |
| $d^I=0$ | 0.69       | 1.09       | 0.01       |
| $I = 0$ | $g^I=0$ | -0.59      | 0.01      | -1.35      |
|        | $h^I=0$ | 0.90       | -2.60     | -0.11      |
|        | $\chi^2/N$ | 14.3      | 14.3     | 22.4       |

3.1. Determining amplitude

We determine the $KN$ amplitudes for $I = 1$ and $I = 0$ reproducing the experimental data using the $\chi^2$ fitting of the low-energy constants to minimize the $\chi^2$ function

$$\chi^2 = \sum_i^N \left( \frac{y_i - f(x_i)}{\sigma_i} \right)^2,$$

where $y_i$, $f(x_i)$, $\sigma_i$ and $N$ are the experimental data, the theoretical calculations with the parameters, the errors of the data and the number of the data, respectively. In our analysis, we consider the partial waves up to the $D$-waves ($l = 2$). We restrict the energy region up to $p_{lab} = 800$ MeV/c, where the inelastic contribution such as pion production starts to be significant.

First of all, we determine the $I = 1$ scattering amplitude from the $K^+p$ elastic scattering data, which were observed well with small errors and certainly constrain the $I = 1$ parameters. To fix the $I = 1$ low-energy constants $b^{I=1}, d^{I=1}, g^{I=1}$ and $h^{I=1}$, we use the $K^+p$ differential cross section between $p_{lab} = 145$ to 726 MeV/c [4]. The fitted values of the parameters are summarized in the Table 2. We find two solutions for $I = 1$, which equivalently reproduce the cross sections. In Fig. 1, we present the total cross sections of the $I = 1$ $KN$ elastic scattering obtained with the fitted parameters and compare with the experimental data [4, 14, 19, 40–42]. The calculated amplitude gives a good reproduction of the data up to $p_{lab} = 800$ MeV/c. It shows that the $S$-wave contribution is dominated and the contributions from the partial waves higher than the $P$-wave are negligibly small. This is consistent with the old observation. In Fig. 2, we show the calculated differential cross sections and make a comparison with the experimental data. The figure shows that the obtained amplitudes reproduce the experimental data well for all the energies which we consider here.

Next, we determine the $I = 0$ low-energy constants $b^{I=0}, d^{I=0}, g^{I=0}$ and $h^{I=0}$ using the data of the $K^+n \rightarrow K^+n$ and $K^+n \rightarrow K^0p$ differential cross sections between $p_{lab} = 434$ to 780 MeV/c [16, 18, 43] and the $I = 0$ total cross section between $p_{lab} = 366$ to 799 MeV/c.
Fig. 1: The calculated $I = 1$ total cross sections using the Solution 1 and 2 in comparison with the experimental data [4, 14, 19, 40–42]. The partial wave components are also described by the dashed line. The horizontal axis means the $K^+$ meson incident momentum in the lab frame $p_{\text{lab}}$ in the unit of MeV/c and the vertical axis is the total cross section $\sigma$ in the unit of mb.

The $K^+n$ scattering amplitudes are linear combinations of the $I = 0$ and $I = 1$ amplitudes as shown in Eqs. (13) and (14). The $I = 1$ amplitude is already determined with the $K^+p$ elastic scattering. The $I = 1$ parameters are fixed, when the $I = 0$ parameters are determined. The fitted results for the $I = 0$ parameters are summarized in the Table 2. Here we propose three solutions which have reproduce the experimental data equivalently well and different character in the $I = 0$ total cross section, as we will discuss in details later. The $\chi^2/N$ values for $I = 0$ are not as good as the ones for $I = 1$. This is because the experimental data of the $K^+n$ scattering have relatively large errors and some different experiments are inconsistent each other.
Fig. 2: The calculated differential cross sections of the $K^+p$ elastic scattering using the Solution 1 and 2 in comparison with the experimental data of Ref. [4].

In Fig. 3, we show the $I = 0$ total cross sections calculated with Solution 1, 2, 3 and find that there three solutions reproduce well the observed total cross section. As one can see, the $I = 0$ total cross section rapidly increases around $p_{lab} = 500$ MeV/c. In the three solutions, the partial wave which is responsible for the rapid increase of the cross section is different. Actually, as we shall see later, this feature links to the property of a possible resonance appearing in $I = 0$ with a large width. In Solution 1, the $P_{01}$ amplitude$^1$ dominantly contributes, and thus the rapid increases is caused by the $P$-wave amplitude. In Solution 2, both $P_{01}$ and $P_{03}$ amplitudes provide the contribution for the $I = 0$ total cross section and the $P_{03}$ amplitude is responsible for the rapid increase. In Solution 3, the $S_{01}$ amplitude substantially contributes and the rapid increase of the cross section stems from the $S$-wave amplitude. In this way, these three solutions have their own characteristic features in the $I = 0$ total cross section. In summary, we would say that Solution 1 is “$P_{01}$ dominant solution”, Solution 2 is “$P_{03}$ dominant solution”, and Solution 3 is “$S_{01}$ dominant solution”. Figure 4 shows the $K^+n$ elastic differential cross section. The Solution 1, 2 and 3 are mostly consistent with the experimental data. It should be pointed out that Solution 3 underestimates the differential cross sections in lower energies, $p_{lab} = 434, 526$ and 604 MeV/c. Figure 5 shows the $K^+n$ charge exchange differential cross section. Solution 1 and 2 show relatively good reproduction.

$^1$We use the partial wave convention $L_{I,J}$ with orbital angular momentum $L$, isospin $I$ and total angular momentum $J = L \pm 1/2$. 
Table 3: The resonance states of the Solution 1, 2 and 3.

| amplitude ($J^P$) | mass [MeV] | width [MeV] |
|-------------------|------------|-------------|
| Solution 1 $P_{01} (\frac{1}{2}^-)$ | 1621 | 312 |
| Solution 2 $P_{03} (\frac{3}{2}^-)$ | 1686 | 445 |
| Solution 3 $S_{01} (\frac{1}{2}^-)$ | 1624 | 264 |

except for the forward and backward scattering, while Solution 3 shows less agreement than the other. We cannot find sizable contradictions for Solution 1 and 2 with the experimental data. Solution 3 cannot reproduce the angular dependence of the differential cross section of the charge exchange especially for lower energies, because the amplitude of Solution 3 is composed of $S$-wave contribution. Thus, Solution 3 could be ruled out. Nevertheless, here we discuss also Solution 3, because we want to point out the relation between the rapid increase of the $I = 0$ total cross section and the existence of the possible broad resonance.

3.2. Possible broad resonances

We have constructed the $KN$ amplitudes which reproduce the experimental data well. In the following, we concentrate on the $KN$ partial wave amplitudes with $I = 0$ and discuss the outcome from the obtained amplitude. First of all, we look for poles of the scattering amplitude in the complex energy plane. Having the $KN$ scattering amplitude in an analytic form, we can perform analytic continuation of the scattering amplitude into the complex energy plane. We find a pole in the $P_{01}$ amplitude of Solution 1 at $z = 1621 - 156i$ MeV, which corresponds to a resonance state with mass 1621 MeV/$c^2$, width 312 MeV and $J^P = (1/2)^+$. The resonance has a quite large width and it could be hard to pin down the resonance in production experiments. Similarly, we find a pole of the $P_{03}$ amplitude of Solution 2 in the complex energy plane at $z = 1686 - 223i$ MeV corresponding to a resonance state with mass 1686 MeV/$c^2$, width 445 MeV and $J^P = (3/2)^+$. Since this resonance state is located far from the real axis, it is not constrained well by experimental observation appearing in the real axis and theoretical uncertainty should be large and this solution could be unstable against small deviation of experimental data. We also find a pole in the $S_{01}$ amplitude of Solution 3 at $z = 1624 - 132i$ MeV, which corresponds to a resonance state with mass 1624 MeV/$c^2$ and width 264 MeV.

Even though we find the resonance state as a pole of the scattering amplitude, there are no peak structure in the total scattering amplitude around the resonance energy. One usually expects that resonance states should appear as a peak in the cross section. It is not necessarily true when the resonance has a large width and substantial coupling to non-resonance background. We demonstrate this situation by using a simple amplitude in which a resonance pole is embedded in a constant background with a relative phase $\delta$:

$$f(E) = \frac{i}{E - M + i\Gamma/2} + b e^{i\delta}. \tag{54}$$

In Fig. 6, we show the cross sections of the amplitudes (54) with $\delta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ for $M = 1600$ MeV, $\Gamma = 300$ MeV and $b = 0.01$ MeV$^{-1}$. As one can see in the figure, the resonance shape depends on the relative phase. For $\delta = 0$, the resonance and background contributions are
Fig. 3: The $I = 0$ total cross sections calculated using the Solution 1, 2 and 3 in comparison with the experimental data [19, 40, 42].
Fig. 4: The differential cross sections of $K^+n$ elastic scattering using the Solution 1, 2 and 3 in comparison with the experimental data of Ref. [18, 43]. The momenta at $p_{lab}=640, 720$ and $780$ MeV/c are the data from Ref. [18]. The others are the data from Ref. [43].
Fig. 5: The differential cross sections of $K^+n$ charge exchange scattering using the Solution 1, 2 and 3 in comparison with the experimental data of Ref. [16, 43]. The momenta at the $p_{lab}=640$, 720 and 780 MeV/c are the data from Ref. [16]. The others are the data from Ref. [43].
Fig. 6: Fano resonance. Cross sections of amplitudes composed of a resonance and a continuum background with relative phase $\delta$ are shown. The resonance is assumed to have 1600 MeV mass and 300 MeV width. The resonance shape is dependent on the relative phase $\delta$. The vertical axis is in arbitrary unit.

interfered constructively a resonance peak appears in the cross section, while for $\delta = \pi$, the resonance and background contribute deconstructively and the resonance is seen as a dip. It is very interesting to see that, for the case of $\delta = \pi/2$, a rapid increase takes place at the resonance energy. This is also one of the resonance shapes. These kinds of resonances are known as Fano resonance [44].

In Fig. 7, we show the real and imaginary parts of the $I = 0$ scattering amplitudes of $P_{01}$ for Solution 1, $P_{03}$ for Solution 2 and $S_{01}$ for Solution 3, where the resonances are found. As seen in figure, a typical resonance structure is seen in the amplitudes, but the role of the real and imaginary parts is interchanged. (Usually the imaginary part has a peak structure, while the real part increases around the resonance point.) This is due to strong coupling of the resonance to the continuum background with some relative phase. In order to confirm whether the structure in the amplitude comes from the resonance state, we subtract the resonance contribution from the amplitude. We express the resonance contribution as the Breit-Wigner form, of which the numerator is obtained by calculating the residue of the amplitude at the resonance pole. The subtracted amplitudes are shown as dashed lines in Fig. 7. It implies that the subtracted amplitudes are almost constant without significant structure. Thus, the structure appearing in the amplitudes is caused by the resonance state.

As we have mentioned above, the imaginary part of the amplitude rapidly increases around the resonance energy. According to the optical theorem, the total cross section is proportional to the imaginary part. Therefore, we conclude that the rapid increase seen in the $I = 0$ total cross section around $p_{\text{lab}} = 500$ MeV/c can be a sign of the possible existence of a resonance with a large width. In addition, the spin-parity of the resonance can be learned by knowing which partial wave is responsible for the rapid increase of the $I = 0$ total cross section. Currently, we do not have enough data to pin down the spin-parity of the resonance state. Here we have proposed three solutions; in Solution 1, the rapid increase appears in the $P_{01}$-wave and the resonance should have $J^p = (1/2)^+$. In Solution 2, it does in the $P_{03}$-wave and the resonance should have $J^p = (3/2)^+$. In Solution 3, the $S_{01}$-wave contributes the rapid increase and the resonance should have $J^p = (1/2)^-$. It would be very interesting if one could understand the feature of the $I = 0$ total cross section around $p_{\text{lab}} = 500$ MeV/c with more accurate experimental data.
Fig. 7: The real and imaginary parts of the $I = 0$ amplitudes of $P_{01}$ for Solution 1, $P_{03}$ for Solution 2 and $S_{01}$ for Solution 1 around the resonance energy. The solid lines stand for the original amplitudes, while the dotted lines stand for the amplitudes obtained by subtracting the resonance pole.
Fig. 8: The Argand diagrams of the $S$ and $P$-waves in the $I = 0$ states up to the momentum $p_{\text{lab}} = 800$ MeV/c. Solutions 1, 2 and 3 are compared with the Martin’s amplitude [11] shown in the dotted line.

Finally in Fig. 8 we show the Argand diagrams of the $I = 0$ scattering amplitudes for the $S$ and $P$ waves up to the $p_{\text{lab}} = 800$ MeV/c using Solution 1, 2 and 3 and compare with Martin’s amplitude [11]. We find that the partial wave amplitudes for Solution 1 are very similar to the Martin’s amplitude. Thus, one could find a pole for a broad resonance also in the Martin’s amplitude. It is also interesting to point out that, for Solution 2, the $P_{03}$ channel has an attraction interaction and actually hold a broad resonance, while the $P_{01}$ channel is repulsive even though some contribution of $P_{01}$ is seen in the total cross section.

4. Conclusion
We have investigated the $K\Lambda$ elastic scattering below the energy where the inelastic contributions become significant, that is, $p_{\text{lab}} < 800$ MeV/c, by describing the scattering amplitude
in the chiral unitary approach as an analytic function. We utilize a next-to-leading chiral Lagrangian for the kernel interaction of the unitarized amplitude, and the low energy constants in the amplitude are determined to reproduce the differential cross sections of $K^+p \to K^+p$, $K^+n \to K^+n$ and $K^+n \to K^0p$. We have obtained good scattering amplitudes which reproduce the observed scattering cross section very well. Particularly, the $I = 1$ scattering amplitude, namely $K^+p$ elastic amplitude, has been determined well thanks to less ambiguous experimental data with small errors, and we have found that the $I = 1$ scattering amplitude at $p_{\text{lab}} < 800 \text{ MeV}/c$ is essentially described by $S$-wave contribution, which is consistent with our conventional knowledge. For the $I = 0$ amplitude, we have proposed three possible parameter sets, which reproduce the $I = 0$ scattering cross sections similarly and have different nature for the rapid increase appearing in the $I = 0$ total cross section around $p_{\text{lab}} = 500 \text{ MeV}/c$. In Solution 1, the rapid increase appears in $P_{01}$-wave contribution, in Solution 2 it stems from the $P_{03}$ amplitude, while in Solution 3 the $S_{01}$ contribution makes the rapid increase.

Having performed analytic continuation of the obtained $I = 0$ scattering amplitudes to the complex energy plane, we have found a pole corresponding to a broad resonance state around $E_{\text{c.m.}} = 1620 \text{ MeV}$ with 310 MeV width in each scattering amplitude. We would like to emphasize strongly that the existence of a broad resonance is responsible for the rapid increase of the $I = 0$ total cross section around $p_{\text{lab}} = 500 \text{ MeV}/c$. Thus, further investigation of the nature of the rapid increase of the $I = 0$ total cross section reveals directly the existence of the $S = +1$ exotic resonance state. Usually resonance states, especially narrow resonances, appear as a bump in the total cross section. Nevertheless, for broad resonances, because they strongly couple to the non-resonance background, their resonance shape seen in the cross section can be modified. This is known as Fano resonance.

In order to pin down the existence of the $S = +1$ broad resonance, one needs further detailed investigation. First of all, the resonance found in this work has a broad width and is located far from the real axis in the complex energy plane. The experimental informations are in the real axis and constrain the scattering amplitude well close to the real axis. To make the scattering amplitude, or the position of the pole, constrained more, more accurate experimental data are necessary. In addition, one also needs more reliable theoretical description. For instance, it could be necessary to introduce more terms into the interaction kernel. It is also important to describe $K^+d$ scattering with the deuteron wavefunction theoretically. This makes us to perform direct comparison of the theoretical calculation to the experimental observation.

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