Article

Propagation of Voltage Deviations in a Power System

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Abstract: The paper deals with voltage profiles in a power system. The analysis of these profiles is important due to the requirement that the Root-Mean-Squared (RMS) values of nodal voltages should be within certain ranges, as well as to ensure desired power flows in a power system. In both cases, it is desirable to indicate points in a power system where it is reasonable to apply remedial measures to meet the requirements for RMS values of nodal voltages, or to effectively control the power flows in a power system. In general, candidate nodes for remediation are established based on operational experience or measurement data from a certain time point (sometimes from several time points). The paper presents a method that provides a basis for determining the aforementioned candidate nodes based on the behavior of a system over a certain period of time, which is an unquestionable advantage of this proposal. In order to achieve the abovementioned goal, the method provides for the analysis of propagation of voltage RMS value deviations in a power system. The analysis of correlational relationships between the RMS values of nodal voltages is used for this. After presentation of the theoretical background, the new original method is described in the paper. Then, case studies showing the utilization of that method are presented. At the end of the paper, features of the proposed method are enumerated.

Keywords: correlation; power system; voltage RMS value deviation

1. Introduction

Important quantities characterizing the state of a power system (PS) are voltages at nodes of this system. Various electromagnetic phenomena can influence those voltages [1]. When power quality is taken into account, the effects of that impact can be assessed from different points of view. In this paper, voltage changes are considered. In [1], the term “voltage change” is defined as “a variation of the Root-Mean-Square (RMS) or peak value of a voltage between two consecutive levels sustained for definite but unspecified durations”. Further, “voltage change” is meant as “a variation of the RMS value of a voltage between two consecutive levels sustained for definite but unspecified durations”. In the paper, we use the term “voltage deviation”. That term is meant as the variation of the RMS value of a voltage which is determined with respect to a reference voltage, such as nominal voltage, a mean value of the operating voltage, or a declared supply voltage [2]. During power-system operation, the RMS value of Nodal Voltage (RMS_NV) in each node is kept within a certain range. In the event that RMS_NV could be outside such a range, steps are taken to maintain RMS_NV at the desired value. For this purpose, on-load tap changers, various types of compensators, and generator voltage regulation are used. If there is a need for additional voltage control measures in a PS, an analysis is performed to determine where in the PS these measures should be applied. In this case, it is necessary to find nodes in the PS where voltage changes have a significant impact on voltage changes at other nodes, or, in other words, to learn about the phenomenon of voltage-change propagation in the PS.

The problem of the nodal voltage profile in a PS plays an important role in solving the problem of locating additional reactive power sources (the problem of reactive power
planning) in order to ensure optimal reactive power flow. There are many papers in which locations of additional devices having influence on nodal voltages in a PS are considered, e.g., [3–8].

In [3], one objective of reactive power planning is the minimization of voltage deviation. The voltage deviation index takes into account the deviation of RMS of each nodal voltage in the PS from the reference value. One of the objectives of reactive power planning in [4] is also the minimization of the voltage deviation index. In that case, the reference voltage is equal to 1 pu.

The indices characterizing nodal voltages can be used for finding nodes for the placement of additional reactive power sources.

In [5], it is ascertained that, for a candidate node from the viewpoint of placement of an additional reactive power source (compensator), the voltage-reactive power index should be sufficiently large. The index defined in the paper characterizes the influence of nodal reactive power, at the node under consideration, on the voltages in a PS. It is the mean value of derivatives of nodal voltages in a PS, in respect to nodal reactive power.

In [6], to determine candidate nodes for the placement of additional reactive power sources, for each node, the sum of changes of all nodal voltages in PS is calculated when the nodal reactive power of this node increases by a certain amount. Load nodes with the large and mentioned sums are good candidates for additional reactive power sources sites.

In [7], it is found that, if the voltage deviation index for a node, defined as difference of 1 pu and RMS of the nodal voltage, is large enough, then the considered node is a candidate for the placement of a capacitor.

More complicated calculations of index qualifying nodes to locate additional reactive power sources are in [8]. For a given node, such an index is the square root of the mean value of the squared differences 1 pu and the RMS of the nodal voltage, which is calculated for the particular nodes, assuming that mentioned nodal voltages are determined when a capacitor with a size equal to 25% of the total system capacity is connected to the considered node. The smallest values of the index qualify the appropriate nodes for the location of the capacitors.

In papers [3–8], one of the objectives is to identify the location of additional reactive power sources which can be used to provide the required RMS_NVs in a PS. Determining the mentioned location should be carried out by taking into account all possible states of a PS, or, at least, taking into account the states representative in this system. That is a serious problem in the research on reactive power planning (as it can be seen in the cited papers, in which this problem is not solved). That problem requires further research. In this paper, an attempt is made to indicate nodes in a PS, which, based on the knowledge of the entire history to date, can be treated as candidates for locating additional reactive power sources.

In the initial part of this article, it has been noticed that the location of the voltage control measures in a PS, which is also referred to in reactive power planning, is associated with the recognition of voltage-change propagation in a PS. The paper presents an original approach to the investigation of the propagation of RMS_NV deviations in a PS. Considerations in the paper are associated with seeking answers to the following questions: (i) how strong is the impact of deviation of selected RMS_NV on RMS_NVs of other nodes; (ii) for which node is the impact of deviation of RMS_NV on RMS_NV of any other node the strongest; (iii) how large an area of a PS, in which the impact of a deviation of RMS_NV for one node on RMS_NVs, for other nodes can be observed; (iv) with which node is there associated the largest area of a PS in which the impact of deviation of RMS_NV of the considered node on RMS_NVs of other nodes is observable; and (v) what are the areas of the network in which the propagation of voltage deviations is possible. The mentioned questions deal with the propagation of RMS_NVs deviations in a PS. The paper solves the outlined problems with the use of an original approach, assuming an analysis of Correlational Relationships (CRs) between RMS_NVs.
Correlation analyses are used for solving different problems in PSs [9–14]. For instance, paper [9] deals with the study of the correlation between injected power and voltage quality at the node where a new energy source is planned for installation. Similarly, paper [10] describes a correlation analysis of the impact of nodal reactive powers on power flow in a PS. In [11], a correlation analysis is applied for solving the problem of grouping nodes. The identification of a low-frequency oscillation source is discussed in [12]. Likewise, paper [13] presents a correlation-analysis-based method for the localization of the deviation source by evaluating the correlation between the harmonic voltage of the node and the harmonic current of the deviation source. In [14], the sources of power quality disturbances are identified based on the analysis of aggregated data from a distributed measurement system with the use of Pearson’s correlation coefficient.

Analysis of the current state of the art indicates that, so far, the problems, which are the subject of this paper, formulated in the form of questions, have not been solved on the basis of correlation analysis.

The further parts of the paper are organized as follows. Section 2 presents the theoretical background of the proposed method for solving the considered problems. Section 3 outlines the developed method. In Section 4, there is a case study, in which the described method is used. The conclusion is in Section 5.

2. Theoretical Background

Generally, the behavior of different electricity customers is not exactly the same. One can assume that it is random. Moreover, events in a PS resulting from various reasons (e.g., short circuits, switching out) cannot be predicted in advance. Therefore, we can assume that any quantities distinguished in a PS can be considered as random. In this situation, various problems related to PSs can be solved using statistical analysis.

In this paper, the dependence of RMS_NVs is being researched to determine the propagation of changes of RMS_NVs in a PS. For the purposes of examining the dependence of RMS_NVs treated as stochastic variables, a correlation analysis is used. The methods of this analysis make it possible to determine the strength of CRs between RMS_NVs. CR between two quantities, \(X\) and \(Y\), will hereinafter be denoted by \(\rho_{XY}\).

2.1. The Statistical Viewpoint on Evaluation of Dependence of Considered Variables

CR simply says that two variables perform in a synchronized manner [15]. From a statistical viewpoint, dependence or association between different variables can be evaluated using measures such as the Pearson product-moment correlation coefficient (Pearson’s correlation coefficient), the Spearman’s rank correlation coefficient (Spearman’s rho), or the Kendall’s rank correlation coefficient [16]. The presented measures are often described in the literature.

Depending on the nature of the relationship between considered variables \(X\) and \(Y\), different correlation coefficients are used. For the purpose of the considered analyses, we distinguish the relationships as follows: (i) linear, (ii) monotonic, (iii) other than monotonic. In (i), we use the Pearson’s correlation coefficient (PCC), in (ii), we use the Spearman’s rank correlation coefficient (SRCC), and in (iii), the Kendall’s rank correlation coefficient (KRCC) is used.

PCC for variables \(X\) and \(Y\) is calculated as:

\[
\rho_P = \frac{\text{cov}(X, Y)}{ \sigma_X \sigma_Y }
\]  

where: \(\text{cov}(X, Y)\) is the covariance of variables \(X\) and \(Y\); \(\sigma_X\), \(\sigma_Y\) are the standard deviations of variables \(X\), \(Y\), respectively.

If, for each of the variables \(X\) and \(Y\), there is \(m\) measurement data, i.e., \(X_i\), \(i = 1, 2, \ldots, m\), \(Y_i\), \(i = 1, 2, \ldots, m\) for \(X\) and \(Y\), respectively, then [16]:

\[
\text{cov}(X, Y) = \frac{1}{m} \sum_{i=1}^{m} (X_i - \bar{X})(Y_i - \bar{Y})
\]
\[ \sigma_X = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (X_i - \bar{X})^2} \]  
\[ \sigma_Y = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (Y_i - \bar{Y})^2} \]  
\[ \bar{X} = \frac{1}{m} \sum_{i=1}^{m} X_i \quad \bar{Y} = \frac{1}{m} \sum_{i=1}^{m} Y_i \]  

SRCC is calculated as PCC, taking into consideration not values but the rank values of the considered variables. Values of variables \( X \) and \( Y \) are converted to ranks \( \text{rg} \ X_i \), \( i = 1, 2, \ldots, m \), \( \text{rg} \ Y_i \), \( i = 1, 2, \ldots, m \), respectively, and SRCC is calculated as follows [16]:

\[ r_s = r_p(\text{rg} \ X, \text{rg} \ Y) = \text{cov}(\text{rg} X, \text{rg} Y) / \left( \sigma_{\text{rg} X} \sigma_{\text{rg} Y} \right) \]  

2.2. Statistical Significance of Correlational Relationships

\( H_0 \) hypothesis says that variables \( X \) and \( Y \) are not correlated, i.e., there is no CR between them.

CR, for which hypothesis \( H_0 \) is rejected in the appropriate test, is statistically significant. The CR is further called as Statistically Significant Correlational Relationship (SSCR).

2.3. The Nature of Relationships between Considered Variables

From the statistical viewpoint, to properly measure dependence or association between considered variables, one ought to investigate the nature of the relationship between these variables. In the paper, as stated before, we take into account the RMS_NVs at nodes in a PS.

Let us consider the relationship between RMS_NVs for nodes \( i \) and \( j \), which are connected by power line \( i-j \) in the following form (see Appendix A):

\[ V_j = \sqrt{V_i^2 + r_{ij}(P_{ij} - P_{ji}) + x_{ij}(Q_{I_{ij}} - Q_{I_{ji}})} \]  

where:

\[ Q_{I_{ab}} = Q_{ab} - 0.5b_{ab}V_a^2 \]  

\( V_a \ a = i, j, i, j \ in \{1, 2, \ldots, n\} \) are RMS_NVs for nodes \( i \) and \( j \), respectively; \( n \) is a number of nodes in PS; \( b = i, j, i, j \ in \{1, 2, \ldots, n\} \) \( b \neq a \); \( P_{ab}, Q_{ab} \) are active and reactive power flows on power line \( a-b \) at node \( a \); \( r_{ab}, x_{ab} \) are series resistance, series inductive reactance and shunt capacitive susceptance of power line \( a-b \) (‘power line \( a-b' \) and ‘power line \( b-a \)’ mean the same power line) modeled with the \( \Pi \) model.

Relationship (7) between \( V_i \) and \( V_j \) is nonlinear. Voltage magnitude \( V_j \) is a monotonic function of \( V_i \). Derivative \( \partial V_j / \partial V_i \) is as follows

\[ \frac{\partial V_j}{\partial V_i} = V_i / \sqrt{V_i^2 + r_{ij}(P_{ij} - P_{ji}) + x_{ij}(Q_{I_{ij}} - Q_{I_{ji}})} \]  

The numerator and denominator of a fraction on the right-hand side of formula (9) are always positive, which means that derivative \( \partial V_j / \partial V_i \) is always positive and \( V_j \) is a monotonic function of \( V_i \).

Now, let us consider the relationship between \( V_k \) and \( V_j \), when there is power line \( k-j \) and power line \( j-i \). The relationship between \( V_i \) and \( V_j \) is presented by formula (7). The relationship between \( V_k \) and \( V_j \) can be formulated analogously:

\[ V_k = \sqrt{V_j^2 + r_{jk}(P_{jk} - P_{kj}) + x_{jk}(Q_{I_{kj}} - Q_{I_{kj}})} \]
and then
\[ V_k = \sqrt{V_i^2 + W_i + W_j} \]  (11)

where:
\[ W_i = r_{ij}(P_{ij} - P_{ji}) + x_{ij}(Q_{1,ij} - Q_{1,ji}) \]  (12)
\[ W_j = r_{jk}(P_{jk} - P_{kj}) + x_{jk}(Q_{1,jk} - Q_{1,kj}) \]  (13)

Derivative \( \frac{\partial V_k}{\partial V_i} \) is as follows:
\[ \frac{\partial V_k}{\partial V_i} = \frac{V_i}{\sqrt{V_i^2 + W_i + W_j}} \]  (14)

Furthermore, derivative \( \frac{\partial V_k}{\partial V_i} \) is always positive. \( V_i \) and \( \sqrt{V_i^2 + W_i + W_j} \), i.e., \( V_k \) are positive. Thus, it can be concluded that \( V_k \), as the function of \( V_i \), is monotonic.

In the same way, as it was before, it can be argued that, if one can pass along power lines between any node \( i \) and node \( i \), then \( V_i \) is a monotonic function of \( V_i \).

On the basis of our considerations, one can state that investigations of CRs between RMS_NVs at nodes in a PS can be realized using SRCCs.

Analyzing (7), we can see that \( V_j \) depends on \( V_i \) and active and reactive power flows at both ends of the considered branch in a PS, i.e., \( P_{ij}, P_{ji}, Q_{ij}, Q_{ji} \). The directions of changes of the mentioned quantities, on which \( V_j \) depends, may be the same or the opposite. Thus, the impact of changes in \( V_i, P_{ij}, P_{ji}, Q_{ij}, Q_{ji} \) on changes in \( V_j \) may strengthen or weaken. Changes in \( V_i, P_{ij}, P_{ji}, Q_{ij}, Q_{ji} \) depend on a PS state. In turn, that PS state depends on loads.

Let us consider cases other than the earlier mentioned one. In that case, the considered voltages are not at the end nodes of the same branch. Those voltages can be at the end nodes of different branches, which have the same mutual node (Equation (11)) or which have no mutual node, but there is a passage path between the nodes that are taken into account. In the presented case, in the relationship between considered RMS_NVs, there are more quantities than there were in the first case.

3. Proposed Method

3.1. General Characteristics of the Method

The goal of the method is the determination of (i) CRs between RMS_NVs; (ii) the strongest SSCR between RMS_NVs; (iii) areas of a PS in which CRs of RMS_NV for selected node and RMS_NVs for other nodes are statistically significant; (iv) the node for which there is the largest area of a PS from among the areas pointed out in (iii); (v) the node for which the area of a PS distinguished in (iii) is characterized by the largest average SRCC; (vi) areas in a PS in which the propagation of RMS_NV deviation has a place; (vii) areas for which a PS can be divided due to the strongest correlation relationships; and (viii) nodes whose RMS_NVs have the strongest impact on the RMS_NVs of other nodes in areas pointed out in (vii).

The effects of the achievement of the abovementioned goals are:

A. An indication of the nodes for which RMS_NV deviations are most closely associated to each other.

B. An indication of the area of a PS where the RMS_NV deviations are significantly associated to the deviations of an RMS_NV for the considered node. The largest such area is determined. Moreover, the area of a PS where the RMS_NV deviations are most strongly associated with the deviation of RMS_NV for the considered node is found.

C. An indication of the area of a PS where the deviation of an RMS_NV for any node is significantly associated with the deviations of RMS_NVs for the remaining nodes.

3.2. The Principle of the Method

The principle of the method is as follows:
1. Calculate SRCCs for RMS_NVs for nodes in a PS, taking into account all possible pairs of these nodes.

2. Perform a significance test for all calculated SRCCs.

3. Find two nodes of a PS for which RMS_NVs are in the strongest SSCR.

4. For each node \( i \), determine the area of the PS defined by nodes for which RMS_NVs are in SSCRs with RMS_NV for node \( i \), assuming that the SRCC for each of the earlier-mentioned SSCRs is not less than the defined level \( p \) (for considered area the acronym AoSSCRdL is used). The determined area of the PS contains also node \( i \). AoSSCRdL associated with node \( i \), where \( i \) is a number of the node, \( i \in \{1, 2, \ldots, n\} \), is further called as \( A_{S,p,i} \).

5. For each AoSSCRdL \( A_{S,p,i} \), determine a number of SSCRs between RMS_NV for node \( i \) and RMS_NVs for other nodes of AoSSCRdLs. \( n_{AS,p,i} \) stands for such a number for AoSSCRdL \( A_{S,p,i} \).

6. Determine a node in the PS for which AoSSCRdL is the largest, i.e., for which \( n_{AS,p,i} \) has the largest value. RMS_NV for the found node is in SSCRs with RMS_NVs for the largest number of nodes.

7. Determine the average strength of CRs for each AoSSCRdL \( A_{S,p,i} \), i.e., AoSSCRdL associated with node \( i \in \{1, 2, \ldots, n\} \), characterized by \( r_{S_{a,i}} \), which is defined as:

\[
r_{S_{a,i}} = \sum_{j \in I_{AS,p,i}} r_{S_{ij}} \frac{n_{AS,p,i}}{n_{AS,p,i}}
\]

where \( r_{S_{ij}} \) is SRCC for \( V_i \) and \( V_j \), \( i, j \in \{1, 2, \ldots, n\} \) \( i \neq j \); \( I_{AS,p,i} \) is a set of numbers of nodes, which are in \( A_{S,p,i} \).

8. Determine a number of the node (called as \( i_{aM,p} \)) for which AoSSCRdL is characterized by \( r_{S_{a,i}} \) being maximal of values found for AoSSCRdLs \( A_{S,p,i} \) \( i \in \{1, 2, \ldots, n\} \). From among nodes of the considered PS, node \( i_{aM,p} \) is most strongly related to its AoSSCRdL from the viewpoint of CRs between RMS_NVs.

9. Determine the consistent area of the PS defined as

\[
A_{CS,p,i} = \bigcup_{i \in I_{CS,p}} A_{S,p,i} \quad i \in I_{CS,p}
\]

where \( i \) is a number of the considered consistent area of the PS; \( I_{CS,p} \) is a set of numbers of consistent areas; \( I_{CS,p,i} \) is a set of numbers of nodes, with each of them, AoSSCRdL \( A_{S,p,i} \) is associated and these AoSSCRdLs meet the conditions:

C1: In each such area there are nodes for which RMS_NVs are in SSCRs characterized by SRCCs not less than \( p \) (\( p < 1 \)),

C2: Each of the considered AoSSCRdLs has at least one mutual node with another area.

10. Determine domination areas. Each domination area \( A_{D,l} \), where \( l \) is a number of the area, \( l \in I_{D}, I_{D} \) is a set of numbers of all domination areas for the considered PS, is a consistent area containing: (i) the nodes of type A or AA; (ii) the nodes of type BA associated with node A or AA belonging to \( A_{D,l} \); (iii) the nodes of type B associated with nodes A, AA or BA belonging to \( A_{D,l} \). The node of type A is defined as certain node \( j^* \) for which RMS_NV the following condition is fulfilled:

\[
r_{ij^*} = \max_j r_{ij} \quad i, j \in \{1, 2, \ldots, n\} \quad i \neq j
\]

Node \( i \) taken into account in (17) is called the node of type B associated with node \( j^* \). Condition (17) is called the dominance condition. If the following dominance condition takes place

\[
r_{j*i} = \max_i r_{j*i} \quad i, j \in \{1, 2, \ldots, n\} \quad i \neq j
\]
where \( j^* \) has the same value as in (17), and \( i^* \) is equal to \( i \) in (17), then nodes \( i^* \) and \( j^* \) are the node of type AA.

If node \( i \) is the node of type B associated with node \( j^* \) being the node of type A, and node \( i \) is the node of type A with which certain node \( k \) is associated, then node \( i \) is called the node of type BA. As defined earlier, node \( k \) is the node of type B.

All A or AA nodes belonging to all areas \( A_{D,j} \ i \in A_{D} \) constitute the \( S_D \) set. The \( S_D \) set is a set of candidate nodes that should be considered when looking for the best locations for nodal voltage control devices.

3.3. Features of Consistent Areas \( A_{CS,p} \ i \neq 1 \in I_{CS,p} \)

1. The minimal number of nodes of the consistent area of the PS defined by (16) can be more than two. If \( p \) (in (16)) is equal to \( r_{S\text{max}} \), which is defined as

\[
    r_{S\text{max}} = \max_{i,j \neq j} S_{ij}.
\]

then there is at least one pair of nodes in the PS for which RMS_NVs are in CR characterized by \( r_{S\text{max}} \). If there is one such pair of nodes in the PS, then, for the level of SRCC \( p = r_{S\text{max}} \) for each of nodes of the pair, we have AoSSCRdL associated with this node, which has two elements (two nodes). This means that the area of the PS defined by (16) for \( p = r_{S\text{max}} \) also has two nodes of the PS. In the considered case \( I_{CS,p} = \{1\} \).

2. The number of consistent areas of the power system defined by (16) can be more than one. If there are AoSSCRdLs which have no mutual nodes, then there is more than one area defined by (16).

3. If node \( k \) is in area \( A_{CS,p,1} \ i_1 \in I_{CS,p} \), then this node is also in certain area \( A_{CS,p,2} \ i_2 \in I_{CS,p} \) (perhaps \( i_1 = i_2 \) when \( p_1 > p_2 \)).

If node \( k \) is in \( A_{CS,p,1} \ i_1 \in I_{CS,p} \), then, according to the definition (16), this node is at least in one AoSSCRdL; particularly it is in AoSSCRdL associated with it. Each of AoSSCRdLs existing in the definition of \( A_{CS,p,1} \ i_1 \in I_{CS,p} \) meet condition C1, where \( p = p_1 \). So, each of those AoSSCRdLs meet condition C1, when \( p = p_2 \), as well. That means that each of the considered AoSSCRdLs, and also AoSSCRdL, associated with node \( k \) occur in the definition of certain \( A_{CS,p,2} \ i_2 \in I_{CS,p} \). A consequence of that fact is that node \( k \) as well as all other nodes being in area \( A_{CS,p,1} \ i_1 \in I_{CS,p} \), are in area \( A_{CS,p,2} \ i_2 \in I_{CS,p} \) (perhaps \( i_1 = i_2 \) when \( p_1 > p_2 \)).

From the considered feature of consistent areas \( A_{CS,p} \ i \neq 1 \in I_{CS,p} \), it follows that, when \( p_1 > p_2 \), then area \( A_{CS,p,1} \ i_1 \in I_{CS,p} \) is included in area \( A_{CS,p,2} \ i_2 \in I_{CS,p} \).

4. When \( p_1 > p_2 \), area \( A_{CS,p,2} \ i_2 \in I_{CS,p} \) is identical to area \( A_{CS,p,1} \ i_1 \in I_{CS,p} \) or includes more nodes than area \( A_{CS,p,1} \ i_1 \in I_{CS,p} \) if all nodes of area \( A_{CS,p,1} \) are in area \( A_{CS,p,2} \).

If there is no node in the PS meeting the condition: this node is different from any node of area \( A_{CS,p,1} \ i_1 \in I_{CS,p} \), and there is AoSSCRdL for the level of values of SRCCs equal to \( p_2 \), which is associated with the mentioned node, then area \( A_{CS,p,2} \) is identical to area \( A_{CS,p,1} \).

If there is a node in the PS meeting the previously given condition, then the number of nodes in area \( A_{CS,p,2} \) is larger than the number of nodes in area \( A_{CS,p,1} \).

5. If, under assumption \( p_1 > p_2 \) area \( A_{CS,p,1} \ i_1 \in I_{CS,p} \) is included in area \( A_{CS,p,2} \ i_2 \in I_{CS,p} \) and area \( A_{CS,p,1} \) is not identical to area \( A_{CS,p,2} \), then, in this last area, there is at least one node (let us call it node \( l \)) for which RMS_NV is in CRs with RMS_NVs for other nodes being in area \( A_{CS,p,2} \). and these CRs are characterized by SRCCs less than \( p_1 \).

The given feature of consistent areas \( A_{CS,p} \ i \neq 1 \in I_{CS,p} \) can be justified in this way, that if the mentioned SRCCs were greater than \( p_1 \), then node \( l \) would be in area \( A_{CS,p,1} \).
4. Case Study 1

The analysis of the propagation of RMS_NV deviations is carried out under assumptions:

- IEEE 14-node test system is used [17] (Figure 1).
- We took into account 500 cases of power flow in the test system. The considered cases were determined by nodal active power for P-Q nodes and P-V nodes, nodal reactive power for P-Q nodes, and nodal-voltage magnitudes for P-V nodes, according to the formula:

  \[ W = a W_p + b \cdot W_p \]  \( \text{(20)} \)

where: \( W, W_p \) are the taken into consideration and base values of the mentioned quantity, respectively; \( a \) is a constant; \( b \) is a random value; \( a = 0.5 \) and \( b \in [0, 1] \) when the considered quantity is nodal active or reactive power; \( a = 0.9 \) and \( b \in [0, 0.2] \) when the considered quantity is nodal-voltage magnitude. Only such cases of power flow are analyzed for which RMS_NVs for all nodes of the test system are in range \([0.9; 1.1] \) pu.

![IEEE 14-node test system](image)

**Figure 1.** The IEEE 14-node test system.

Each case of power flow in the test system (the test-system operating state) is characterized by system active-power losses. For the considered test-system operating states, these losses are in the range: \([0.060, 0.532]\).

4.1. Results

The results of calculations of SRCCs for the test system are given in Table 1. In that table, there are SRCCs for different pairs of RMS_NVs. In Table 2, there are results of the significance test for particular SRCCs shown in Table 1, when \( \alpha = 0.01 \). The shaded array element (Table 2 and also Table 1) shows that, for a given pair of RMS_NVs, the \( H_0 \) hypothesis must be rejected (\( \text{rej } H_0 \) in Table 2), i.e., CR between RMS_NVs of the mentioned pair is statistically significant. Further, were considered only SSCRs.
Table 1. Spearman’s rank correlation coefficient (SRCC) for different pairs of root-mean-square values of nodal voltages (RMS_NVs) in the test system.

|     | V1   | V2   | V3   | V4   | V5   | V6   | V7   | V8   | V9   | V10  | V11  | V12  | V13  | V14  |
|-----|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| V1  | 1    | 0.648| 0.358| 0.668| 0.755| 0.227| 0.295| 0.067| 0.171| 0.197| 0.247| 0.235| 0.241| 0.231|
| V2  | 0.648| 1    | 0.556| 0.910| 0.922| 0.331| 0.498| 0.074| 0.396| 0.421| 0.432| 0.351| 0.368| 0.437|
| V3  | 0.358| 0.556| 1    | 0.736| 0.660| 0.284| 0.432| 0.138| 0.318| 0.340| 0.355| 0.299| 0.309| 0.346|
| V4  | 0.668| 0.910| 0.736| 1    | 0.985| 0.508| 0.705| 0.242| 0.589| 0.626| 0.642| 0.535| 0.560| 0.648|
| V5  | 0.755| 0.922| 0.660| 0.985| 1    | 0.535| 0.643| 0.191| 0.530| 0.577| 0.631| 0.557| 0.577| 0.618|
| V6  | 0.227| 0.331| 0.284| 0.508| 0.535| 1    | 0.456| 0.120| 0.477| 0.614| 0.897| 0.996| 0.988| 0.766|
| V7  | 0.295| 0.498| 0.432| 0.705| 0.643| 0.456| 1    | 0.686| 0.874| 0.866| 0.734| 0.506| 0.553| 0.817|
| V8  | 0.067| 0.074| 0.138| 0.242| 0.191| 0.120| 0.686| 1    | 0.328| 0.307| 0.233| 0.142| 0.164| 0.274|
| V9  | 0.171| 0.396| 0.318| 0.589| 0.530| 0.477| 0.874| 0.328| 1    | 0.984| 0.807| 0.534| 0.587| 0.915|
| V10 | 0.197| 0.421| 0.340| 0.626| 0.577| 0.614| 0.866| 0.307| 0.984| 1    | 0.894| 0.664| 0.711| 0.966|
| V11 | 0.247| 0.432| 0.355| 0.642| 0.631| 0.897| 0.734| 0.233| 0.807| 0.894| 1    | 0.924| 0.947| 0.964|
| V12 | 0.235| 0.351| 0.299| 0.535| 0.557| 0.996| 0.506| 0.142| 0.534| 0.664| 0.924| 1    | 0.996| 0.807|
| V13 | 0.241| 0.368| 0.309| 0.560| 0.577| 0.988| 0.553| 0.164| 0.587| 0.711| 0.947| 0.996| 1    | 0.846|
| V14 | 0.231| 0.437| 0.346| 0.648| 0.618| 0.766| 0.817| 0.274| 0.915| 0.966| 0.964| 0.807| 0.846| 1    |

f to r H₀—false to reject H₀, rej H₀—reject H₀

Table 2. Results of the significance test for SRCCs for different pairs of RMS_NVs in the test system.

|     | V1   | V2   | V3   | V4   | V5   | V6   | V7   | V8   | V9   | V10  | V11  | V12  | V13  | V14  |
|-----|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| V1  | rej H₀| rej H₀| rej f₀ | rej H₀| rej H₀| rej f₀ | rej H₀| rej f₀ | rej H₀| rej f₀ | rej H₀| rej f₀ | rej H₀| rej f₀ |
| V2  | rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀|
| V3  | rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀|
| V4  | rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀|
| V5  | rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀|
| V6  | rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀|
| V7  | rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀|
| V8  | rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀|
| V9  | rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀|
| V10 | rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀|
| V11 | rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀|
| V12 | rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀|
| V13 | rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀|
| V14 | rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀| rej H₀|

A characteristic of Table 1 is symmetry with respect to the main diagonal. Therefore, the analysis can be carried out taking into account the rows of the table or taking into account the columns of this table. Further consideration is made using rows of the table. In
each row, the SRCC with the highest value is distinguished (bold font), assuming that this coefficient characterizes SSCR.

Table 1 shows that, for different RMS_NVs, the number of SSCRs is different.

AoSSCRdLs are determined on the base of Tables 1 and 2 under the assumption that $p = p_{cr}$, where $p_{cr} = 0.4674$, 0.4674 is the critical value of SRCC in significance test for the significance level equal to 0.01. Elements of $i$-th AoSSCRdL are numbers of nodes for which there are RMS_NVs in SSCRs with RMS_NV for $i$-th node. It should be noted that $i$-th row of Table 1 (also Table 2) is associated with RMS_NV for $i$-th node. It should be added that an element of $A_{S_{pcr}}$, $i \in \{1, 2, \ldots, 14\}$ is the number of $i$-th node, as well.

AoSSCRdLs associated with particular nodes of the test system (i.e., $A_{S_{pcr}}, i \in \{1, 2, \ldots, 14\}$) are characterized in Table 3. In that table, nodes being in those areas and numbers $n_{AS_{pcr}}, i = 1, 2, \ldots, 14$ are given. One can state that each of numbers $n_{AS_{pcr}}, i = 1, 2, \ldots, 14$ is 1 greater than the number of nodes at which effect of propagation of deviation of $V_i$ is significant. Among the nodes belonging to each set $A_{S_{pcr}}, i = 1, 2, \ldots, 14$, there is one node whose number is marked in bold. RMS_NV for each of these nodes is in the strongest CRs with RMS_NV for the node with which AoSSCRdL is associated.

Table 3. Areas $A_{S_{pcr}}, i \in \{1, 2, \ldots, 14\}$ associated with particular nodes of the test system.

| $i$ | $n_{AS_{pcr}}, i$ | $r_{Sa_{pcr}, i}$ | Numbers of Nodes |
|-----|-------------------|-------------------|------------------|
| 1   | 4                 | 0.691             | 1, 2, 4, 5       |
| 2   | 6                 | 0.707             | 1, 2, 3, 4, 5, 7 |
| 3   | 4                 | 0.651             | 2, 3, 4, 5       |
| 4   | 13                | 0.676             | 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14 |
| 5   | 13                | 0.666             | 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14 |
| 6   | 9                 | 0.723             | 4, 5, 6, 9, 10, 11, 12, 13, 14 |
| 7   | 11                | 0.688             | 2, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14 |
| 8   | 2                 | 0.686             | 7, 8             |
| 9   | 10                | 0.700             | 4, 5, 6, 7, 9, 10, 11, 12, 13, 14 |
| 10  | 10                | 0.767             | 4, 5, 6, 7, 9, 10, 11, 12, 13, 14 |
| 11  | 10                | 0.827             | 4, 5, 6, 7, 9, 10, 11, 12, 13, 14 |
| 12  | 10                | 0.724             | 4, 5, 6, 7, 9, 10, 11, 12, 13, 14 |
| 13  | 10                | 0.752             | 4, 5, 6, 7, 9, 10, 11, 12, 13, 14 |
| 14  | 10                | 0.816             | 4, 5, 6, 7, 9, 10, 11, 12, 13, 14 |

Table 3 shows that nodes 4 and 5 have the largest AoSSCRdL ($n_{AS_{pcr}}, 4 = n_{AS_{pcr}}, 5 = 13$). Node 8 has the smallest AoSSCRdL ($n_{AS_{pcr}}, 8 = 2$).

In Table 4, there are rates $r_{Sa_{pcr}, i}$ $i = 1, 2, \ldots, 14$ for the nodes of the test system.

Table 4. Rates of strength of CRs for particular AoSSCRdLs $A_{S_{pcr}}, i = 1, 2, \ldots, 14$ in the order of decreasing.

| $i$ | 11 | 14 | 10 | 13 | 12 | 6 | 2 | 9 | 1 | 7 | 8 | 4 | 5 | 3 |
|-----|----|----|----|----|----|---|---|---|---|---|---|---|---|---|
| $r_{Sa_{pcr}, i}$ | 0.83 | 0.82 | 0.77 | 0.75 | 0.72 | 0.72 | 0.72 | 0.71 | 0.70 | 0.69 | 0.69 | 0.69 | 0.68 | 0.67 | 0.65 |

Rates $r_{Sa_{pcr}, i}$ $i = 1, 2, \ldots, 14$ in Table 4 are sorted in descending order. Rates $r_{Sa_{pcr}, 11}$ and $r_{Sa_{pcr}, 14}$ are the largest ones, rate $r_{Sa_{pcr}, 3}$ has the smallest value. The six largest values of rate $r_{Sa_{pcr}, i}$ is for the lower-voltage part of the test system (LVP_TS). LVT_TS contains nodes with the numbers 6, 9–14. The higher-voltage part of the test system (HVP_TS) contains the remaining nodes, i.e., nodes numbered 1–5, 7, and 8.

Depending on the level of SRCC, one or more consistent areas of a PS defined by (16) can be distinguished. These areas are shown in Tables 5 and 6.
Table 5. Consistent areas of the considered power system defined by (16), and characteristics of these areas.

| $p$         | $I_{CS,p,i}$ | Numbers of Nodes |
|------------|-------------|-----------------|
| 0.99603    | 12          | 6, 12           |
| 0.99597    | 12          | 6, 12, 13       |
| 0.9848     | 4           | 4, 5            |
| 0.9842     | 9           | 9, 10           |
| 0.9657     | 10          | 9, 10, 14       |
| 0.9639     | 10, 11      | 9, 10, 11, 14   |
| 0.9465     | 10, 11, 13  | 6, 9, 10, 11, 12, 13, 14 |
| 0.9100     | 4           | 2, 4, 5         |
| 0.8744     | 10, 11, 9   | 6, 7, 9, 10, 11, 12, 13, 14 |
| 0.7555     | 5           | 1, 2, 4, 5      |
| 0.7360     | 5, 4        | 1, 2, 3, 4, 5   |
| 0.7050     | 5, 4, 10, 11| 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14 |
| 0.6859     | 5, 4, 10, 11, 7| 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 |

Table 6. Consistent areas of the considered power system defined by (16) for different ranges of SRCC values.

| Ranges of SRCC Values | $i$   | Numbers of Nodes |
|-----------------------|-------|-----------------|
| [0.99597, 0.9960]     | 1     | 12, 13          |
| (0.9848, 0.99597]     | 1     | 6, 12, 13       |
| (0.9842, 0.9848]      | 1     | 4, 5            |
|                       | 2     | 6, 12, 13       |
| (0.9657, 0.9842]      | 1     | 4, 5            |
|                       | 2     | 6, 12, 13       |
|                       | 3     | 9, 10           |
| (0.9639, 0.9657]      | 1     | 4, 5            |
|                       | 2     | 6, 12, 13       |
|                       | 3     | 9, 10, 14       |
| (0.9465, 0.9639]      | 1     | 4, 5            |
|                       | 2     | 6, 12, 13       |
|                       | 3     | 9, 10, 11, 14   |
| (0.9100, 0.9465]      | 1     | 4, 5            |
|                       | 2     | 6, 9, 10, 11, 12, 13, 14 |
| (0.8744, 0.9100]      | 1     | 2, 4, 5         |
|                       | 2     | 6, 9, 10, 11, 12, 13, 14 |
| (0.7555, 0.8744]      | 1     | 2, 4, 5         |
|                       | 2     | 6, 7, 9, 10, 11, 12, 13, 14 |
Table 6. Cont.

| Ranges of SRCC Values | i   | Numbers of Nodes |
|-----------------------|-----|------------------|
| (0.736, 0.7555]      | 1   | 1, 2, 4, 5       |
|                       | 2   | 6, 7, 9, 10, 11, 12, 13, 14 |
| (0.7050, 0.736]      | 1   | 1, 2, 3, 4, 5    |
|                       | 2   | 6, 7, 9, 10, 11, 12, 13, 14 |
| (0.6859, 0.7050]     | 1   | 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 |
| [0.4674, 0.6859]     | 1   | 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 |

*i* is an element of set $I_{CS,p}$

Table 5 shows consistent areas of the considered PS defined by (16), maximal levels of SRCC for this areas and numbers of the nodes, AoSSCRdLs of which define these consistent areas. In turn, Table 6 presents consistent areas of the considered power system defined by (16) for different ranges of SRCC. In Table 5 and in Table 6, nodes where RMS_NVs are in the strongest SSCRs are shown in bold.

Table 6 shows that area $A_{CS,p,1,p} \in [0.4674, 0.686]$ covers the entire power system. It means that a deviation of RMS_NV for one node causes deviations of RMS_NV at any other node. The strongest CRs are for area $A_{CS,p,1,p} \in [0.99597, 0.9960)$. When $p \in [0.9842, 0.9848]$ as well as $p \in [0.705, 0.9465)$, we have two areas: $A_{CS,p,1}$ and $A_{CS,p,2}$. Those areas are in the different parts of the test system separated by transformers. When $p \in [0.9465, 0.9842)$, there are three areas: $A_{CS,p,i} \quad i = 1, 2, 3$.

The data in Table 7 refer to the domination areas in the considered test system. There are three domination areas. The smallest such area covers 3 nodes, the largest—6 nodes. In particular domination areas, the nodes for which RMS_NVs have the strongest impact on RMS_NVs for other nodes are the nodes numbered 6 and 12, 4 and 5, 9 and 10, i.e., $S_D = \{4, 5, 6, 9, 10, 12\}$.

Table 7. Domination areas in the considered power system.

|   | N_{AD,I} | AA          | BA          | B  |
|---|----------|-------------|-------------|----|
| 1 | 3        | 6, 12       |             | 13 |
| 2 | 5        | 4, 5        |             | 1, 2, 3 |
| 3 | 6        | 9, 10       | 7, 14       | 8, 11 |

4.2. Discussion

The stronger the CR between RMS_NVs for two nodes, the stronger the impact of RMS_NV for one node on RMS_NV for second node, and the propagation of RMS_NV deviation between distinguished nodes is more visible. Considering individual AoSSCRdLs, one can note that the strongest CRs (the largest SRCCs) are between RMS_NVs for neighboring nodes. It is easy to explain. An exception is in the case of the pairs of nodes 10, 14 and 11, 14. In the case of the first pair of nodes, strength of CRs is considered from the viewpoint of node 14 (area $A_{S,p,14,p} \in [0.4674, 0.9657]$); in the case of the second pair of nodes, this strength is analyzed from the viewpoint of node 11 (area $A_{S,p,11,p} \in [0.4674, 0.9639]$). Each of the mentioned pairs of nodes is in one mesh of the network. The nodes, for which RMS_NVs are in the strongest CRs, exist in LVP_TS. This fact can be explained by smaller power flows in LVP_TS (see (7)).

Nodes 4 and 5 have the largest AoSSCRdLs, i.e., areas, each comprising 13 nodes. Nodes 4 and 5 are in the important points of the PS. Through these nodes, HVP_TS is connected with LVP_TS. These nodes have the largest number of connections to other nodes. Nodes in LVP_TS have slightly smaller AoSSCRs. Nodes: 1 and 3 in HVP_TS have the smallest AoSSCRdLs, if we do not take into account node 8 ($n_{AS,p,8} = 1 \quad p \in [0.4674, 0.686]$). Node 8 is a terminal node of one power line. The presented differences in sizes of
AoSSCRdLs are also a consequence of smaller power flows in LVP_Ts. This way we can also explain that RMS_NVs for nodes of LVP_Ts have greater impact on RMS_NVs for the nodes of AoSSCRdL associated with the previously mentioned nodes than it is in HVP_Ts.

Analyzing areas $A_{CS,p,i} \subseteq \{0.4674, 0.99603\}, \ i \in I_{CS,p}$ for different $p$, one can note that, for the smallest considered values of $p$, i.e., for $p \in [0.4674, 0.705]$, there is only one element of set $I_{CS,p}$. That area contains all nodes of the test system, when $p \in [0.4674, 0.6859]$. When $p \in [0.6859, 0.705]$, area $A_{CS,p,1}$ contains all nodes of the test system, with the exception of one node, i.e., node 8. For $p \in [0.705, 0.9848]$, there is more than one element of set $I_{CS,p}$.

For the given values of $p$, there is one area $A_{CS,p,1}$ in HVP_Ts and one area $A_{CS,p,2}$ or two areas $A_{CS,p,2}$ and $A_{CS,p,3}$ (for $p \in [0.9465, 0.9842]$) in LVP_Ts. Area $A_{CS,p,1}$ is separated from area $A_{CS,p,2}$ or areas $A_{CS,p,2}$ and $A_{CS,p,3}$ by the transformers, which are between HVP_Ts and LVP_Ts. For $p \in [0.9848, 0.9960]$, set $I_{CS,p}$ again contains only one element. That is area $A_{CS,p,1}$ in LVP_Ts. It contains nodes 6, 12 and 13 for $p \in (0.9848, 0.99597]$, and nodes 12 and 13 for $p \in (0.99597, 0.9960]$. It means that the CR between magnitudes of voltages at the respective nodes in area $A_{CS,p,1}$ has the strongest one. In the time period for which SRCCs for considered CR is calculated, a consequence of the mentioned fact is that changes of RMS_NV for one node of area $A_{CS,p,1}$ have the strongest impact on RMS_NV for other node of this area. The considered area $A_{CS,p,1}$ has the fewest possible number of nodes in the test system.

The method proposed in the paper allows the whole system to be divided into domination areas. In the domination areas, there are nodes for which RMS_NVs have a dominant influence on RMS_NVs for other nodes. For each domination area, the nodes for which RMS_NV have the greatest impact on RMS_NV for other nodes are distinguished. These are the nodes of type A or AA. Those nodes should be taken into account when seeking for the best locations for nodal voltage control devices.

5. Case Study 2

The purpose of this section is to determine the domination areas and candidate nodes for the location of additional nodal-voltage control devices on the basis of selected subsets of the set of the test-system operating states, which is considered in Section 4. $S$ stands for that set. The following assumptions are adopted:

- Three sets of operating states of the test system ($S_1$, $S_2$, $S_3$) are taken into account. Those are subsets of the aforementioned $S$ set.
- The test-system operating states are characterized by system active power losses.
- The number of operating states in each of the sets $S_1$, $S_2$ and $S_3$ is the same, and equals 75.

5.1. Results

Characteristics of the domination areas, which are determined in the considered case study, is in Table 8. Those areas are determined for the significance level $\alpha = 0.01$. 
Table 8. Dominance areas in the considered power system, determined when sets $S_1$, $S_2$, and $S_3$ are taken into account.

| Range of System Active Power Losses, pu | $l$ | $N_{AD,l}$ | AA | BA | B |
|----------------------------------------|-----|------------|----|----|---|
| [0.086, 0.102]                         | 1   | 4          | 4, 5 | 10, 11, 13, 14 | 9 |
|                                        | 2   | 7          | 6, 12 | 13 |
|                                        | 3   | 2          | 7, 8 | 13 |
| [0.174, 0.205]                         | 1   | 2          | 1, 2 | 13 |
|                                        | 2   | 2          | 4, 5 | 13 |
|                                        | 3   | 3          | 6, 12 | 14 |
| [0.335, 0.413]                         | 1   | 5          | 4, 5 | 14 |
|                                        | 2   | 9          | 6, 11 | 7, 13 |
|                                        |     |            |      | 8, 9, 10, 12, 14 | 13 |

Table 8 shows that, for operating states in set $S_1$, system active power losses are the lowest, and, for the operating states in set $S_3$, they are the largest.

For set $S_1$, there is no statistically significant relationship between RMS_NV at node 3 and RMS_NVs at any other node. This is the reason why the total number of nodes in all domination areas for set $S_1$ is equal to 13. For set $S_1$, area $A_{D,2}$ is the largest domination area. The number of nodes in that area is equal to 7. For set $S_1$, area $A_{D,3}$ with two nodes is the smallest domination area.

For set $S_2$, the number of the domination areas is equal to four. There are two domination areas, each with two nodes. The largest domination area is area $A_{D,4}$, having five nodes. In the case of set $S_2$, there is no statistically significant relationship between RMS_NV at node 3 and between RMS_NV at node 8, and RMS_NVs at any other node.

When the set $S_3$ is taken into account, the number of domination areas is two. The largest domination area consists of 9 nodes, and the smallest domination area contains 5 nodes.

Based on the analysis of domination areas, it can be concluded that the most important are RMS_NVs at the nodes:

- 4, 5, 6, 12, 7, 8 for the $S_1$ set,
- 1, 2, 4, 5, 6, 12, 9, 10 for the $S_2$ set,
- 4, 5, 6, 11 for the $S_3$ set.

Assuming that $S_{D,i}$ stands for $S_D$ determined for the $S_i$ set, where $i$ is a number of the power-system operating state set, we can write: $S_{D,1} = \{4, 5, 6, 12, 7, 8\}$, $S_{D,2} = \{1, 2, 4, 5, 6, 12, 9, 10\}$, $S_{D,2} = \{4, 5, 6, 11\}$. Still $S_D$ is understood as it was previously defined when set $S$ is taken into account.

5.2. Discussion

Table 8 shows that the number of dominance areas is different for different sets $S_1$, $S_2$ and $S_3$. For set $S_1$, the number of the dominance areas is equal to three and it is the same for set $S$ (Table 7). However, the set of nodes at which RMS_NVs have a dominating impact on RMS_NVs at other nodes is different for sets $S$ and $S_1$, i.e., set $S_D$ is different from set $S_{D,1}$. It is easy to notice that sets $S_{D,1}$, $S_{D,2}$ and $S_{D,3}$ differ in content. Moreover, $S_{D,2}$ and $S_{D,3}$ differ in their content from set $S_D$. Some nodes appear in all sets of $S_{D,1}$, $S_{D,2}$ and $S_{D,3}$, and also in set $S_D$. These are nodes with numbers 4, 5 and 6.

In this section, the individual operating states are characterized by system active power losses. These losses are a function of active and reactive power flows on the power network branches. In turn, these flows depend on the nodal voltages, their modules and arguments.
The greater the power flows on the power-network branches, the greater the voltage drops on these branches. With the existing voltage phase shifts at the ends of the test-system branches (a few to a dozen or so degrees), increasing these shifts is accompanied by a relatively fast increase in active power flows and a relatively slower increase in reactive power flows. Relatively speaking, there is a rapid increase in voltage phase shifts at the ends of the test-system branches, and much smaller increases in the modulus differences of the nodal voltages. Focusing on one branch, it is the effect of compensation of the components, depending on the power flows on the branch and the component depending on the square of the branch-current modulus in the formula, for the squares of voltage modules at the ends of the branch (formula (A6)). Consequently, with the considered increase in system active power losses, there is an increase in the number of statistically significant relationships between RMS NVs.

Table 8 clearly shows that changes in the system active power losses significantly affect the obtained investigation results. The domination areas, as well as the sets of candidate nodes for the location of additional RMS_NV control devices, are noticeably changing.

When analyzing sets $S_{D,1}$, $S_{D,2}$, $S_{D,3}$, and also $S_D$, a question can be asked regarding which of these sets should be taken into account when determining candidate nodes for a process of looking for the best locations for nodal-voltage control devices. The answer is the $S_D$ set. The rationale is that this set is established by taking into account all possible operating states of the power system. It is rational to do so when an investment decision is made.

Taking into account the previous considerations, it can be concluded that, due to the purpose of set $S_D$, when determining it, Condition C must be met. That condition is formulated as follows:

**Condition C:** All possible operating states in the system should be taken into account.

Condition C means that all states that result from the entire variety of combinations of values of nodal active and reactive powers of generation and loads should be taken into account.

The modules and phase angles of the nodal voltages are shaped by the generations and loads existing in the power system. The question arises whether there is an influence of generation and loads (active and reactive powers) on the most important effect of the method described in the paper, i.e., on set $S_D$.

Since there is an influence of nodal active and reactive powers on the nodal voltages, and also on the active and reactive power flow in the PS, when analyzing the relationships between the voltages that are given in the paper, it can be concluded that, as a result, there is a change in the strength of the relationships between RMS NVs.

We do not analyze the influence of nodal active and reactive powers of generation and loads on set $S_D$, due to the fact that they do not affect the principle of the method. They also do not affect the accuracy of the method. It is important that set $S_D$ includes those nodes that will actually be the best locations for nodal voltage control devices. This is ensured by Condition C.

### 6. Conclusions

The paper presents a new method which solves the problems related to the analysis of the propagation of RMS_NV deviations in a PS. As a consequence, the presented method provides a solution to the problem of the determination of candidate nodes for the location of additional reactive power sources in a PS, from the viewpoint of nodal voltages.

The method, which is described in the paper, assumes investigations of CRs between RMS_NVs for the nodes of a PS. For the aim of evaluation of the mentioned CRs, the idea of SRCC is utilized. Taking into account RMS_NVs, the most important results of applying the method are: (i) determination of consistent areas of PS, i.e., determination of areas of a PS, where the impact of RMS_NV for the considered node is significant, or, in other words, the propagation of deviations of the considered RMS_NV is essential; (ii) seeking nodes for which the aforementioned areas are the largest; (iii) seeking nodes for which RMS_NVs
have the strongest impact on RMS_NVs for other nodes; and (iv) defining domination areas where the influence of RMS NVs for certain nodes is dominant compared to RMS_NVs for other nodes.

In general, the developed method provides (i) an expansion of the knowledge of phenomena in a PS, and (ii) a finding of points in a PS where the application of preventive measures can be considered to limit the propagation of RMS_NV deviations in the system, and affect the power flow in a PS as required. The method is relatively simple. It requires measurement data of considered RMS_NVs for nodes of a PS. It does not require calculations of power flows. The method provides the basis for making decisions in the reactive power planning process based on the behavior of a PS over a certain period of time, and not taking into account just one moment. This is a very important advantage of the discussed method. This advantage is all the more valuable, as the papers on reactive power planning generally ignore the problem of studying a system’s behavior over a longer period of time. This can be explained by the high difficulty of solving the problem.

It should be emphasized that the analysis of system behavior in a sufficiently long period of time is the appropriate basis for making investment decisions that take place in reactive power planning. A sufficiently long period of time is a period of time for which the probability of skipping some system operating state, which has a significant impact on investment decisions, is minimized. This is not achieved by the approach that takes into account power system behavior at only one moment in time.

The described method was developed for steady states. Apart from that, there are no other restrictions on the use of the method.

The paper does not consider dynamic states. Considerations regarding dynamic states are the next stage of investigation presented in the paper.

**Author Contributions:** Conceptualization, K.W.; data curation, T.O.; formal analysis, K.W.; investigation, T.O.; methodology, K.W.; resources, T.O.; software, T.O.; supervision, K.W.; validation, K.W.; visualization, T.O.; writing—review and editing, T.O. and K.W. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research received no external funding.

**Conflicts of Interest:** The authors declare no conflict of interest.

**Appendix A**

Deriving the formula (7)

A model for power line \(i-j\) is presented in Figure A1.

![Figure A1. The assumed Π model of power line \(i-j\).](image)

For the assumed model of the power line, we have:

\[
V_j = V_i + I_j Z_{ij} \\
V_j^* = V_i^* + I_j^* Z_{ij}^*
\]  

(A1)

Multiplying \(V_j\) by its conjugate, we have:

\[
V_j^2 = V_i^2 + I_j^2 Z_{ij}^2 + V_i I_j Z_{ij}^* + V_i^* I_j^* Z_{ij}
\]  

(A2)

We can note, that:

\[
V_i I_j Z_{ij}^* + V_i^* I_j^* Z_{ij} = (P_{ij} + j Q_{ij}) Z_{ij}^* + (P_{ij} - j Q_{ij}) Z_{ij}
\]  

(A3)
and

\[ V_i^* Z_{ij} + V_j^* I_i Z_{ij} = P_{ij} (Z_{ij}^* + Z_{ij}) + Q_{ij}(Z_{ij}^* - Z_{ij}) \]  
(A4)

\[ V_i^* I_i^* Z_{ij} + V_i^* I_i Z_{jj} = 2r_{ij} P_{ij} + 2x_{ij} Q_{ij} \]  
(A5)

Inserting (A5) into (A2): 

\[ V_j^2 = V_i^2 + I_j^2 Z_{ij}^2 + 2r_{ij} P_{ij} + 2x_{ij} Q_{ij} \]  
(A6)

Now, let’s note, that:

\[ I_j^2 Z_{ij}^2 + 2r_{ij} P_{ij} + 2x_{ij} Q_{ij} = r_{ij}^2 + x_{ij}^2 I_j^2 + 2r_{ij} P_{ij} + 2x_{ij} Q_{ij} \]  
(A7)

and

\[ I_j^2 Z_{ij}^2 + 2r_{ij} P_{ij} + 2x_{ij} Q_{ij} = r_{ij}^2 + r_{ij}^2 + r_{ij}^2 + x_{ij} Q_{ij} + x_{ij} Q_{ij} + x_{ij} I_j^2 \]  
(A8)

Finally:

\[ I_j^2 Z_{ij}^2 + 2r_{ij} P_{ij} + 2x_{ij} Q_{ij} = r_{ij}^2 + r_{ij}^2 + x_{ij} Q_{ij} - x_{ij} Q_{ij} \]  
(A9)

Inserting (A9) into (A6), we have:

\[ V_j^2 = V_i^2 + r_{ij} (P_{ij} - P_{ji}) + x_{ij} (Q_{ij} - Q_{ji}) \]  
(A10)

and, as a result, derived formula (7):

\[ V_j = \sqrt{V_i^2 + r_{ij} (P_{ij} - P_{ji}) + x_{ij} (Q_{ij} - Q_{ji})} \]  
(A11)

To obtain (A9), we have used

\[ P_{ij} + P_{ji} + r_{ij} I_j^2 = 0 \quad Q_{ij} + Q_{ji} + x_{ij} I_j^2 = 0 \]  
(A12)

Relationships (A12) can be easily obtained considering power flows on power line \( i-j \).

References

1. Smith, J.C.; Hensley, G.; Ray, L. IEEE Recommended Practice for Monitoring Electric Power Quality; IEEE Std 1159-2019 (Revision of IEEE Std 1159-2009); IEEE: New York, NY, USA, 2019.
2. Energy Related Term Voltage Deviation. Available online: https://www.energy.eu/dictionary/data/1999.html (accessed on 26 February 2021).
3. Ghodrati, M.; Piri, M.; Sadr, S.M. Probabilistic Multi-Objective Reactive Power Planning Considering Large-Scale Wind Integration. In Proceedings of the 2019 International Power System Conference (PSC), Tehran, Iran, 9–11 December 2019; pp. 510–515.
4. Hashemi, Y.; Abdolrezaei, H.; Hashemi, B. Coordination of Generation and Reactive Power Expansion Planning Considering APFC Units. In Proceedings of the 2019 International Power System Conference (PSC), Tehran, Iran, 9–11 December 2019; pp. 490–496.
5. Chen, Y.; Ye, M.; Fu, C.; Li, J.; Xie, Y.; Zhao, H. Optimal Allocation of Dynamic Var Sources in Power Systems Using Two-Stage Strategy. In Proceedings of the 2020 IEEE 3rd Student Conference on Electrical Machines and Systems (SCEMS), Jinan, China, 4–6 December 2020; pp. 574–579.
6. Khan, S.; Bahadoorsingh, S.; Rampersad, R.; Sharma, C.; Powell, C. Reactive power planning combining the reduced jacobian V-Q and voltage sensitivity indices on the sub-transmission network of a caribbean island power system. In Proceedings of the 2018 IEEE Texas Power and Energy Conference (TPEC), College Station, TX, USA, 8–9 February 2018; pp. 1–6.
7. Bhatt, H.; Bhatt, D.V.; Pakka, V.H. Voltage Stability Index and Voltage Deviation Improvements using Intelligent Algorithms for Capacitor Sizing and Placement. In Proceedings of the 2018 IEEE PES Asia-Pacific Power and Energy Engineering Conference (APPEEC), Kota Kinabalu, Malaysia, 7–10 October 2018; pp. 422–427.
8. Mahfoud, R.J.; Alkayem, N.F.; Sun, Y.; Haes Alhelou, H.; Siano, P.; Parente, M. Improved Hybridization of Evolutionary Algorithms with a Sensitivity-Based Decision-Making Technique for the Optimal Planning of Shunt Capacitors in Radial Distribution Systems. Appl. Sci. 2020, 10, 1384. [CrossRef]
9. Tao, S.; Xu, Q.; Peng, Y.; Xiao, X.; Tang, N. Correlation between injected power and voltage deviation at the integrating node of new energy source. In Proceedings of the 2011 2nd International Conference on Artificial Intelligence, Management Science and Electronic Commerce (AIMSEC), Dengfeng, China, 8–10 August 2011; pp. 7031–7034.

10. Okon, T.; Wilkosz, K. Diagnostics of Reactive Power Flow in a Power Network. In Proceedings of the 2018 International Conference on Diagnostics in Electrical Engineering (Diagnostika), Pilsen, Czech Republic, 4–7 September 2018; pp. 1–4. [CrossRef]

11. Yang, X.; Song, D.; Liu, D.; Wang, F. Node grouping for low frequency oscillation based on Pearson correlation coefficient and its application. In Proceedings of the 2016 IEEE Int. Conf. on Power System Technology (POWERCON), Wollongong, NSW, Australia, 28 September–1 October 2016; pp. 1–5.

12. Zuo, J.; Xiang, M.; Zhang, B.; Hen, D.C.; Guo, H. Correlation data analysis for low-frequency oscillation source identification. In Proceedings of the 2017 4th International Conference on Systems and Informatics (ICSAI), Hangzhou, China, 11–13 November 2017; pp. 1466–1470.

13. Feng, D.; Zhou, S.; Wang, T.; Li, Y.; Liu, Y. A Method for Identifying Major Deviation Sources in a Regional Grid. In Proceedings of the 2018 IEEE International Power Electronics and Application Conference and Exposition (PEAC), Shenzhen, China, 4–7 November 2018; pp. 1–6.

14. Chmielowiec, K.; Wiczyński, G.; Rodziewicz, T.; Firlit, A.; Dutka, M.; Piątek, K. Location of power quality disturbances sources using aggregated data from energy meters. In Proceedings of the 2020 12th International Conference and Exhibition on Electrical Power Quality and Utilisation—(EPQU), Cracow, Poland, 14–15 September 2020; pp. 1–5.

15. Trochim, W.M.; Donnelly, J.P.; Arora, K. Research Methods: The Essential Knowledge Base; Cengage Learning, Inc.: Boston, MA, USA, 2014.

16. Wayne, D.W. Applied Nonparametric Statistics, 2nd ed.; PWS-Kent: Boston, MA, USA, 1990.

17. Power Systems Test Case Archive. Available online: http://www.ee.washington.edu/research/pstca/ (accessed on 26 February 2021).