SINGLE SPIN ASYMMETRY FOR $p^+p \rightarrow \pi X$
IN PERTURBATIVE QCD

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Abstract:
Within the QCD-improved parton model and assuming the factorization theorem
to hold in the helicity basis and for higher twist contributions, we show how non
zero single spin asymmetries in hadron-hadron high energy and moderately large $p_T$
inclusive processes can be obtained, even in massless perturbative QCD, provided the
quark intrinsic motion is taken into account. A simple model is constructed which
reproduces the main features of the data on the single spin asymmetry observed in
inclusive pion production in $pp$ collisions.
1 Introduction

Single spin asymmetries in high energy and moderately large $p_T$ inclusive hadronic processes have recently received much attention, both experimentally [1]-[4] and theoretically [5]-[13]. Whereas they are expected to vanish at leading twist in massless perturbative QCD, higher twist effects might still be important in the kinematical region of the available data and may give origin to non zero values. Among the attempted explanations, quark-gluon correlations [5, 8, 13] and transverse $k_t$ effects in the quark distribution [6, 7] or fragmentation functions [11] have been considered.

We analyse here the large single spin asymmetries observed in the collision of transversely polarized protons off unpolarized protons, with the production of pions with $p_T$ values up to 2 GeV/c, $p^\uparrow p \rightarrow \pi X$. (1)

Several experimental results are available for such processes [2]-[4] and show clear patterns in the dependence of the spin asymmetry

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

on $x_F = 2p_L/\sqrt{s}$, where $p_L$ is the pion longitudinal momentum in the $pp$ c.m. system, and $\sqrt{s}$ is the total c.m. energy. The proton spin is perpendicular to the scattering plane and there exist data for $\pi^\pm$ and $\pi^0$; proton spin orientations parallel to the scattering plane would give, by parity invariance, zero single spin asymmetries. A striking dependence of $A_N$ on $p_T$ which was reported at $x_F \approx 0$ [3] is now believed, after a more careful analysis of the data [14], not to be observed anymore; $A_N(x_F \approx 0)$ shows no sign of dependence on $p_T$ and remains consistent with zero in the whole range of $p_T$ explored so far, $(0 < p_T < 4)$ GeV/c [14].

Some previous data [15]-[17] seem to be in disagreement with the findings of Refs. [1]-[4], [14]; in particular, they do not show sizeable values of $A_N$ for $\pi^-$ [16, 17], show strong $p_T$ dependences for $\pi^0$ [13] and $\pi^+$ [17] and have contradictory results on $x_F$ dependences [16, 17]. However, all these data come from experiments at much smaller energy (proton beams of 13.3, 18.5 or 24 GeV/c) than those of Refs. [1]-[4], [14] (200 GeV/c) and we do not consider them here because they might be outside the range of applicability of perturbative QCD.

The usual QCD description of large $p_T$ inclusive production is based on the factorization theorem, according to which the cross-section for the production, say, of a large $p_T$ pion in the collision of two protons is given by the convolution of an elementary cross-section with the number densities of quarks and gluons inside the protons (distribution function) and the number density of pions inside a quark or gluon (fragmentation function). The elementary cross-section contains all the dynamical and quantum-mechanical information on the constituent interaction, whereas the distribution and fragmentation functions are phenomenological ways of modeling the non-perturbative long distance physics: they can be measured in other processes and their large $Q^2$ evolution is given by perturbative QCD.
We know that the above hard scattering scheme works well for unpolarized processes and indeed it has been tested in many experiments. It has also been generalized to the polarized case [18], so that it may be applied to the description of several processes involving polarized hadrons [19]; however, the existing spin data do not allow yet a definite test of its validity.

In Section 2 we adapt the formalism of Refs. [9] and [18] to the case of the single spin asymmetry (2). In order not to obtain a zero result higher twist effects have to be introduced; this can be done at different stages and several suggestions or attempts have been proposed in the literature [9]-[11], [20, 21]. Single spin effects in the elementary reactions alone are bound to be proportional to \( \alpha_s m_q / \sqrt{s} \) [22], where \( m_q \) is the quark mass, and are then expected to be negligible at high energies, even taking constituent quark masses into account [1, 21]. Spin effects might then be present in the distribution or fragmentation functions: the former has been suggested by Sivers [6, 7] and the latter by Collins [9], whose idea has been further developed and applied in Ref. [11]. Qiu and Sterman [8] have used both higher order elementary interactions and higher twist distribution functions to predict a sizeable single spin asymmetry in large \( p_T \) direct photon inclusive production, \( p^\uparrow p \rightarrow \gamma X \).

The approach we describe here is equivalent, although derived in a different way, to the suggestion of Refs. [9, 18] and supports it; we then discuss (Section 3) a simple model which implements the idea and gives very good agreement with the data. The approach of Refs. [6, 7] has been criticized in Ref. [9] on the ground of violating the time reversal invariance of QCD. This is true only at leading twist order, if soft initial state interactions between the colliding protons are neglected, which need not be the case in Sivers or our model; we will further comment on this in Section 2. A short conclusion is given in Section 4.

2 The single spin asymmetry in the hard scattering scheme

Let us then consider the process (1), supposing the initial protons moving along the \( z \)-axis and choosing \( x \) as the production plane; the incoming proton is polarized parallel (\( \uparrow \)) or opposite (\( \downarrow \)) the \( \hat{y} \)-direction so that, in the helicity basis,

\[
| \uparrow \rangle = \frac{1}{\sqrt{2}}(|+\rangle + i|\rangle) \tag{3}
\]

\[
| \downarrow \rangle = \frac{-1}{\sqrt{2}}(|+\rangle - i|\rangle) \tag{4}
\]

According to the QCD factorization theorem the differential cross-section for the hard scattering of a polarized proton with spin \( \uparrow \) (and similarly for spin \( \downarrow \)) on an unpolarized target proton, resulting in the inclusive production of a pion with energy \( E_\pi \) and three-momentum \( p_\pi \), can be written as [9, 18, 19]

\[
\frac{E_\pi}{d^3 p_{\pi}} \sum_{a,b,c,d} \rho_{\lambda_a,\lambda_b,\lambda_c,\lambda_d} \int dx_a dx_b \frac{1}{z} \rho^{a/p_1}_{\lambda_a,\lambda_c} f_{a/p_1}(x_a) f_{b/p}(x_b) \frac{M_{\lambda_a,\lambda_c;\lambda_a,\lambda_b} \cdot M^{\lambda_c,\lambda_d;\lambda_a,\lambda_b} \cdot D_{\lambda_c,\lambda_d}(z)}{}
\]

\[
\sim \frac{1}{2} \sum_{a,b,c,d} \sum_{\lambda_a,\lambda_b,\lambda_c,\lambda_d} \frac{1}{z} \rho^{a/p_1}_{\lambda_a,\lambda_c} \rho_{\lambda_a,\lambda_c} f_{a/p_1}(x_a) f_{b/p}(x_b) \frac{M_{\lambda_a,\lambda_c;\lambda_a,\lambda_b} \cdot M^{\lambda_c,\lambda_d;\lambda_a,\lambda_b} \cdot D_{\lambda_c,\lambda_d}(z)}{}
\]

\[
\int dx_a dx_b \frac{1}{z} \rho^{a/p_1}_{\lambda_a,\lambda_c} f_{a/p_1}(x_a) f_{b/p}(x_b) \frac{M_{\lambda_a,\lambda_c;\lambda_a,\lambda_b} \cdot M^{\lambda_c,\lambda_d;\lambda_a,\lambda_b} \cdot D_{\lambda_c,\lambda_d}(z)}{}
\]
where \( f_{a/p}(x_a) \) is the number density of partons \( a \) with momentum fraction \( x_a \) inside the polarized proton \([\text{similarly for } f_{b/p}(x_b)]\) and \( \rho_{\lambda_a,\lambda_b}^{a/p}(x_a) \) is the helicity density matrix of parton \( a \) inside the polarized proton \( p^\uparrow \). The \( \hat{M}_{\lambda_a,\lambda_b;\lambda_a,\lambda_b} \)'s are the helicity amplitudes for the elementary process \( ab \to cd \); if one wishes to consider higher order (in \( \alpha_s \)) contributions also elementary processes involving more partons should be included. \( D_{\lambda_c,\lambda_c}^{\pi/c}(z) \) is the product of fragmentation amplitudes

\[
D_{\lambda_c,\lambda_c}^{\pi/c} = \sum_{X,\lambda_X} \mathcal{D}_{\lambda_X,\lambda_c} \mathcal{D}_{\lambda_c,\lambda_c}^\ast
\]

where the \( \sum_{X,\lambda_X} \) stays for a spin sum and phase space integration of the undetected particles, considered as a system \( X \). The usual unpolarized fragmentation function \( D_{\pi/c}(z) \), i.e., the density number of pions resulting from the fragmentation of an unpolarized parton \( c \) and carrying a fraction \( z \) of its momentum is given by

\[
D_{\pi/c}(z) = \frac{1}{2} \sum_{\lambda_c} D_{\pi/c,\lambda_c}(z).
\]

For simplicity of notations we have not indicated in Eq. (5) the \( Q^2 \) scale dependences in \( f \) and \( D \); the variable \( z \) is related to \( x_a \) and \( x_b \) by the usual imposition of energy momentum conservation in the elementary \( 2 \to 2 \) process \([2,3]\); we have skipped, for the moment, some spin independent kinematical factors \([4]\) but we explicitly kept the factor \( 1/2 \) to remind that an average has been taken over the helicities of the unpolarized parton \( b \) (quark or gluon).

Eq. (5) holds at leading twist and large \( p_T \) values of the produced pion; the intrinsic \( k_{\perp} \) of the partons have been integrated over and collinear configurations dominate both the distribution functions and the fragmentation processes; one can then see that, in this case, there cannot be any single spin asymmetry. In fact, total angular momentum conservation in the (forward) fragmentation process [see Eq. (5)] implies \( \lambda_c = \lambda_c' \); this, in turns, together with helicity conservation in the elementary processes, implies \( \lambda_a = \lambda_a' \). If we further notice that, by parity invariance, \( D_{\pi/c,\lambda_c}^{\lambda_a,\lambda_a} \) does not depend on \( \lambda_c \) and that \( \sum_{\lambda_c} |\hat{M}_{\lambda_c,\lambda_c;\lambda_a,\lambda_b}|^2 \) is independent of \( \lambda_a \) we remain with \( \sum_{\lambda_b} \rho_{\lambda_a,\lambda_b}^{a/p} = 1 \). Moreover, in the absence of intrinsic \( k_{\perp} \) and initial state interactions, the parton density numbers \( f_{a/p}(x_a) \) cannot depend on the proton spin and any spin dependence disappears from Eq. (5), so that

\[
d\sigma^{p^\uparrow p \to \pi X} - d\sigma^{p^\downarrow p \to \pi X} = 0. \tag{8}
\]

Eq. (5) can be generalized with the inclusion of intrinsic \( k_{\perp} \) \([9]\) and this can avoid the above conclusion; for example \([9]\), the observation of a non zero \( k_{\perp} \) of a
final particle $C$ with respect to the axis of the jet generated by parton $c$ does not imply any more $\lambda_c = \lambda'_c$ and allows a non zero value of the asymmetry

$$d\sigma_{p^+p\rightarrow \pi, k_\perp x} - d\sigma_{p^+p\rightarrow \pi, -k_\perp x}. \quad (9)$$

The above asymmetry (9) is related to the so called Collins [11, 24, 25] or sheared jet [26] effect; it requires the measurement of the azimuthal angle $\phi$ of the outgoing hadron around the jet axis, but, apart from a small $\sin \phi$ dependence, it is a leading twist effect and it depends on some non perturbative quark fragmentation analysing power. When integrating over the azimuthal angle the effect might not entirely disappear because of some $\phi$ dependence in the elementary parton interaction. This idea was exploited in Ref. [11] where, essentially, the parton is produced in the forward direction and the final hadron $p_T$ is due to its transverse $k_\perp$ inside the jet. One cannot expect such a model to work at large $p_T$.

Another possible $k_\perp$ effect, suggested by Sivers [3, 7], may originate in the distribution functions. To see how this comes out from the general scheme we rewrite Eq. (9) taking into account the parton intrinsic momentum in the number density $f_{a/p}$:

$$\frac{E_\pi d\sigma_{p^+p\rightarrow \pi X}}{d^3 p_\pi} - \frac{E_\pi d\sigma_{p^+p\rightarrow \pi X}}{d^3 p_\pi} \sim \frac{1}{2} \sum_{a,b,c,d} \sum_{\lambda_a, \lambda'_b, \lambda'_c, \lambda_d} \int d^2 k_{\perp a} dx_a dx_b \frac{1}{z} \rho^{a/p\perp}_{\lambda_a, \lambda'_b} \hat{f}_{a/p}(x_a, k_{\perp a}) \hat{f}_{b/p}(x_b) \hat{M}_{\lambda_c, \lambda_d; \lambda_a, \lambda_b} \hat{M}^*_{\lambda'_c, \lambda'_d; \lambda_a, \lambda_b} D_{\pi/c}(z), \quad (10)$$

where $\hat{f}$ denotes the $k_\perp$ dependent number density.

We can now argue, as in the previous case when no $k_\perp$ was taken into account, that angular momentum, helicity and parity conservation eliminate all dependences on the parton helicities in Eq. (10); however, a dependence on the hadron spin may remain in $\hat{f}_{a/p\perp}(x_a, k_{\perp a})$, analogously to the Collins effect in the fragmentation process. Then one has [3, 7]:

$$\frac{E_\pi d\sigma_{p^+p\rightarrow \pi X}}{d^3 p_\pi} - \frac{E_\pi d\sigma_{p^+p\rightarrow \pi X}}{d^3 p_\pi} \sim \frac{1}{4} \sum_{a,b,c,d} \sum_{\lambda_a, \lambda'_b, \lambda_c, \lambda_d} \int d^2 k_{\perp a} dx_a dx_b \frac{1}{z}$$

$$\times \left[ \hat{f}_{a/p\perp}(x_a, k_{\perp a}) - \hat{f}_{a/p\perp}(x_a, k_{\perp a}) \right] f_{b/p}(x_b) \left| \hat{M}_{\lambda_c, \lambda_d; \lambda_a, \lambda_b} \right|^2 D_{\pi/c}(z). \quad (11)$$

Several comments are now in order.

There is a new quantity which appears in Eq. (11):

$$\Delta_N f_{a/p\perp}(x_a, k_{\perp a}) \equiv \sum_{\lambda_a} \left[ \hat{f}_{a, \lambda_a/2}(x_a, k_{\perp a}) - \hat{f}_{a/\lambda_a}(x_a, k_{\perp a}) \right] \quad (12)$$

$$= \sum_{\lambda_a} \left[ \hat{f}_{a, \lambda_a/2}(x_a, k_{\perp a}) - \hat{f}_{a/\lambda_a}(x_a, -k_{\perp a}) \right]. \quad (13)$$
where \( \hat{f}_{a, \lambda_a/p^\uparrow}(x_a, k_{\perp a}) \) is the number density of partons \( a \) with helicity \( \lambda_a \), momentum fraction \( x_a \) and intrinsic transverse momentum \( k_{\perp a} \) in a transversely polarized proton [with spin \( \uparrow \) or \( \downarrow \) according to Eq. (3) or (4)]. Eq. (13) follows from Eq. (12) by rotational invariance and explicitly shows that \( \Delta^N f_{a/p^\uparrow}(x, k_{\perp}) = 0 \) when \( k_{\perp} = 0 \).

This new quantity can be regarded as a single spin asymmetry or analysing power for the \( p^\uparrow \rightarrow a + X \) process; if we define the polarized number densities in terms of distribution amplitudes as

\[
\hat{f}_{a, \lambda_a/p^\uparrow}(x_a, k_{\perp a}) = \int_{X_{p, \lambda_{X_p}}} \left| G_{a/p^\uparrow; \lambda_a}(x_a, k_{\perp a}) \right|^2
\]

then we have, in the helicity basis,

\[
\Delta^N f_{a/p^\uparrow}(x_a, k_{\perp a}) = \int_{X_{p, \lambda_{X_p}}} \sum_{\lambda_a} 2 \text{Im} \left[ G^{a/p}_{\lambda_{X_p}, \lambda_a; \uparrow}(x_a, k_{\perp a}) G^{a/p^*}_{\lambda_{X_p}, \lambda_a; \downarrow}(x_a, k_{\perp a}) \right] = 2 I_{a/p}^+(x_a, k_{\perp a}).
\]

Eq. (15) simply follows from Eqs. (12) and (14) via Eqs. (3) and (4) and shows the non diagonal nature, in the helicity indices, of \( I_{a/p}^+(x_a, k_{\perp a}) \).

Collins [9] has argued that a non zero value of \( I_{a/p}^+(x, k_{\perp}) \) is forbidden by the time reversal invariance of QCD; his argument is based on the analysis of the non diagonal matrix elements of the leading twist quark operator \( \bar{\psi} \gamma^+ \psi \), whose diagonal matrix elements are related to the distribution functions \( f_{q/p}(x, k_{\perp}) \). For such operator his argument is correct and indeed time reversal invariance forces the matrix elements between proton states with different helicities to be zero. In a more physical language this amounts to say that single spin asymmetries for the process \( p^\uparrow \rightarrow qX \) are forbidden by parity and time reversal invariance, which is true. However, as we said, initial state interactions (like soft gluon exchanges) between the incoming protons must certainly occur, and, at least in cases in which their neglect gives a zero result, one should consider them and relate the distribution functions to the inclusive cross-section for the process \( p_1 p_2 \rightarrow qX \); single (transverse) spin asymmetries are then certainly allowed (as the problem we are studying here confirms) via time reversal invariant scalar quantities like \( \varepsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu q^\rho s_1^\sigma \).

These soft gluon and initial state interactions which correlate partons from different hadrons and allow a non zero value of \( I_{a/p}^+ \) can only survive at higher twist; it has been shown [18] that at leading twist-2 the proof of the factorization theorem for the unpolarized case – with the cancellation of all soft gluon contributions – holds for the polarized case as well. Eq. (11), as it will be shown in the next Section, only gives higher twist contributions; assuming a non zero value of \( I_{a/p}^+ \) in the factorized structure of Eq. (11) amounts to assume the validity of the factorization theorem beyond leading twist.

In the operator language this approach has been advocated by Qiu and Sterman [8] who use generalized factorization theorems valid at higher twist and relate non
zero single spin asymmetries in $pp$ collisions to the expectation value of a higher twist operator, a twist-3 parton distribution, which explicitly involves correlations between the two protons and combines quark fields with a gluonic field strength. However, they still consider only collinear partonic configurations so that, in order to obtain non zero results, they have to take into account the contributions of higher order elementary interactions.

Our function $I_{a/p}^{\perp}(x_a, k_{\perp a})$ introduced in Eqs. (15) and (12) can be considered as a new phenomenological quantity which takes into account the non perturbative long distance physics, including initial state interactions, and plays, for single spin asymmetries in $pp$ collisions, the same role plaid by the distribution functions $f_{a/p}(x_a)$ in unpolarized processes. This function is zero in the absence of parton intrinsic motion, but, for $k_{\perp} \neq 0$, allows non zero single spin asymmetries even taking into account only lowest order perturbative QCD interactions among the constituents. Measurements of the asymmetries supply information on $I_{a/p}^{\perp}$, like measurements of unpolarized cross-sections supply information on $f_{a/p}$.

In the next Section we introduce a simple parametrization of $I_{a/p}^{\perp}$, based on our knowledge of the distribution functions, and show that it can reproduce with good accuracy the data on the single spin asymmetry $A_N^c = \frac{d\sigma^{p\uparrow\rightarrow\pi X} - d\sigma^{p\downarrow\rightarrow\pi X}}{d\sigma^{p\rightarrow\pi X} + d\sigma^{p\rightarrow\pi X}} = \frac{d\sigma^{p\uparrow\rightarrow\pi X} - d\sigma^{p\downarrow\rightarrow\pi X}}{2 d\sigma^{unp}}$. (16)

3 A simple phenomenological model

Inserting Eqs. (11) and (15), with the proper kinematical factors [23], into Eq. (16) yields

$$A_N = \frac{\sum_{a,b,c,d} \int dx_a dx_b d^2 k_{\perp a} I_{a/p}^{\perp}(x_a, k_{\perp a}) f_{b/p}(x_b) [d\hat{\sigma}/d\hat{t}(k_{\perp a})] D_{\pi/c}(z)/z}{\sum_{a,b,c,d} \int dx_a dx_b f_{a/p}(x_a) f_{b/p}(x_b) (d\hat{\sigma}/d\hat{t}) D_{\pi/c}(z)/z}, \quad (17)$$

where $d\hat{\sigma}/d\hat{t}$ is the unpolarized cross-section for the elementary constituent process $ab \rightarrow cd$; notice that the dependence on $k_{\perp a}$ has to be kept into account in such a quantity, otherwise the numerator of Eq. (17) would vanish upon integration over $k_{\perp a}$ due to the fact that $I_{a/p}^{\perp}(x_a, k_{\perp a})$ is an odd function of $k_{\perp}$, see Eq. (13). The constituent momentum fraction $z$ carried by the pion can be expressed in terms of $x_a$ and $x_b$ by requiring energy-momentum conservation in the elementary scattering.

All quantities appearing in Eq. (17) are either theoretically or experimentally known, with the exception of the new quantity $I_{a/p}^{\perp}(x_a, k_{\perp a})$, which, in principle, is measurable via the single spin asymmetry $A_N$. However, in order to give an estimate of $A_N$ and see if we can obtain reasonable values within our approach, we parametrize here $I_{a/p}^{\perp}$ in a most simple way.

We assume that the dependence of $I_{a/p}^{\perp}$ on $k_{\perp a}$ is sharply peaked around an average value $k_{0 a}^2 = (k_{\perp a}^2)^{1/2}$, value which may depend on $x_a$; the $x_a$ dependence of
\[ I_{\perp}^{a/p} \] which does not originate from the \( k_{\perp a} \) dependence is taken to be of the simple form
\[ N_a x_a^{\alpha_a} (1 - x_a)^{\beta_a}, \] (18)
so that we approximately have
\[
\int d^2 k_{\perp a} I_{\perp}^{a/p}(x_a, k_{\perp a}) \frac{d\hat{\sigma}}{dt}(k_{\perp a})
\]
\[
= \int_{(k_{\perp a})_x > 0} d^2 k_{\perp a} I_{\perp}^{a/p}(x_a, k_{\perp a}) \left[ \frac{d\hat{\sigma}}{dt}(+k_{\perp a}) - \frac{d\hat{\sigma}}{dt}(-k_{\perp a}) \right]
\]
\[ \simeq \frac{k_{\perp a}^0}{M} N_a x_a^{\alpha_a} (1 - x_a)^{\beta_a} \left[ \frac{d\hat{\sigma}}{dt}(+k_{\perp a}^0) - \frac{d\hat{\sigma}}{dt}(-k_{\perp a}^0) \right] , \] (19)
where \( M \) is a hadronic mass scale, \( M \simeq 1 \) GeV/c. A numerical estimate of \( k_{\perp a}^0/M \) can be found in Ref. [27] and can be accurately reproduced by the expression
\[ \frac{k_{\perp a}^0}{M} = 0.47 x_a^{0.68} (1 - x_a)^{0.48} , \] (20)
which we adopt in our calculations. Such value of \( k_{\perp a}^0 \) enters in the computation of \( [d\hat{\sigma}/dt(k_{\perp a}) - d\hat{\sigma}/dt(-k_{\perp a})] \); notice that such a quantity is \( O(k_{\perp}/p_T) \) [6], hence the contribution of Eq. (19) is a higher twist one.

In order to give numerical estimates of the asymmetry (17) we still need explicit expressions of the unpolarized distribution functions, \( f_{a,b/p} \), and the fragmentation functions, \( D_{\pi/c} \). At this stage we have only considered contributions from \( u \) and \( d \) quarks inside the polarized proton, which certainly dominate at large \( x_F \) values, that is \( a = u,d \) in the numerator of Eq. (17). Instead, we have considered all possible constituents in the unpolarized protons, with \( k_{\perp} = 0 \), and all possible constituent fragmentation functions. We have taken \( f_{q,\bar{q},g/p} \) from Ref. [28], \( D_{\pi/q,\bar{q}} \) from Ref. [29] and \( D_{\pi/g} \) from Ref. [30]. Given the very limited \( p_T \) range of the data we have neglected the QCD \( Q^2 \) dependence of the distribution and fragmentation functions.

By using Eqs. (19) and (20) into Eq. (17), together with the unpolarized \( f_{a,b/p} \) and \( D_{\pi/c} \) functions, we remain with an expression of \( A_N \) still dependent on a set of 6 free parameters, namely \( N_a, \alpha_a \) and \( \beta_a \) (\( a = u,d \)), defined in Eq. (18). We have obtained a best fit to the data [3], shown in Fig. 1, with the following values of the parameters:

\[
\begin{array}{ccc}
N_a & \alpha_a & \beta_a \\
\hline
u & 5.19 & 2.79 & 4.15 \\
d & -2.29 & 2.11 & 4.70 \\
\end{array}
\] (21)
As the experimental data [2] cover a $p_T$ range between 0.7 and 2.0 GeV/c we have computed $A_N$ at a fixed value $p_T = 1.5$ GeV/c. Our asymmetry decreases with increasing $p_T$ and increases at smaller $p_T$; however, we do not expect our approach to be valid at $p_T$ smaller than, say, 1 GeV/c. We will further comment on the $p_T$ range of our computation in the conclusions.

Notice that the above values (21) are very reasonable indeed; actually, apart from an overall normalization constant, they might even have been approximately guessed. The exponents $\alpha_{u,d}$ and $\beta_{u,d}$ are not far from the very naïve values one can obtain by assuming, as somehow suggested by Eqs. (15) and (14), that $I_{\pm}^{a/p}(x_a) \sim \sqrt{f_{\pm}(x_a)f_{\pm}(x_a)}$, where $f_{\pm}(x_a)$ denotes, as usual, the number density of quarks with the same (opposite) helicity as the parent proton. Also the relative sign and strength of the normalization constants $N_u$ and $N_d$ turn out not to be surprising if one assumes that there might be a correlation between the number of quarks at a fixed value of $\mathbf{k}_\perp$ and their polarization: remember that, according to SU(6), inside a proton polarized along the $\hat{y}$ direction, $P_y = 1$, one has for valence quarks $P_{u,y} = 2/3$ and $P_{d,y} = -1/3$.

It might appear surprising to have approximately opposite values for the $\pi^+$ and $\pi^-$ asymmetries, as the data indicate, and a large positive value for the $\pi^0$; one might rather expect $A_N \simeq 0$ for a $\pi^0$. However, this can easily be understood from Eq. (17) which we simply rewrite, for a pion $\pi^i$, as $A_{\pi^i} = N_i/D$, if one remembers that from isospin symmetry one has:

$$D_{\pi^0/c} = \frac{1}{2} \left( D_{\pi^+/c} + D_{\pi^-/c} \right).$$

Eq. (17) show that the relation (22) also holds for $N^0$ and $D^0$, so that

$$A_N^0 = \frac{N^+ + N^-}{D^+ + D^-} = A_N^1 \frac{1 + \frac{A_N^-}{A_N^+}}{1 + \frac{D^-}{D^+}}.$$  \hspace{1cm} (23)

It is then clear that $A_N^0 \simeq -A_N^1$ implies $A_N^0 \simeq 0$ only if $D^- \simeq D^+$, i.e. if the unpolarized cross-sections for the production of a $\pi^-$ and a $\pi^+$ are approximately equal. This is true only at $x_F \simeq 0$. At large $x_F$ the minimum value of $x_a$ kinematically allowed increases and the dominant contribution to the production of $\pi^+$ and $\pi^-$ comes respectively from $f_{u/p}(x_a)$ and $f_{d/p}(x_a)$ [see the denominator of Eq. (17)]. It is known that $f_{d/p}(x_a)/f_{u/p}(x_a) \rightarrow 0$ when $x_a \rightarrow 1$; this implies that $D^-/D^+$ decreases with increasing $x_F$, so that at large $x_F$ we have $D^-/D^+ \ll 1$ and $A_N^0 \simeq A_N^1(1 - 2D^-/D^+) \simeq A_N^1$. Such a trend emerges both from the experimental data and our computations.

4 Conclusions

We have applied the QCD hard scattering approach, based on the factorization theorem in the helicity basis, to the description of single spin asymmetries in inclusive production of pions, with $p_T$ values in the 1 to 2 GeV/c range, in the scattering
of polarized protons off unpolarized ones. Such region of $p_T$ is a delicate one; even if we assume, somewhat optimistically, that the hard scattering scheme is suitable for the description of these processes, we expect that, due to the moderate $p_T$ values, higher twist contributions may still be sizeable and important. Indeed, the leading twist contribution to the single spin asymmetries is zero and the large experimental data should be explained via non leading terms.

In evaluating them one has to assume that the factorization theorem still holds at higher twist level; moreover, one can only take into account a few out of the many higher twist contributions and corrections to the simple hard scattering formulae. Here, we have considered the role of the parton intrinsic $k_{\perp}$ in the initial polarized proton $[3, 7]$, while neglecting other possible effects due, e.g., to transverse $k_{\perp}$ in the fragmentation process. In this our model can only be regarded as a phenomenological approach to the description of the otherwise mysterious large single spin asymmetries. Further application of the same model should test its validity.

We have shown how single spin asymmetries can be different from zero and originate from lowest order perturbative QCD interactions, provided some non perturbative and intrinsic $k_{\perp}$ effects are properly taken into account. In the helicity basis, suitable for the use of the factorization theorem, this non perturbative long distance information is contained in the function $I_{a/p}^{+/−}(x, k_{\perp})$, which has a simple parton model interpretation in the transverse spin basis, Eqs. (12)-(15).

Detailed data on single spin asymmetries in $pp$ inclusive interactions would yield information on $I_{a/p}^{+/−}(x, k_{\perp})$, similar to the information gathered on the unpolarized structure functions; a knowledge of $I_{a/p}^{+/−}$ from some process could then be used to predict spin effects in other processes. Here we have shown how a simple and realistic parametrization of $I_{a/p}^{+/−}$ can easily explain the data on single spin asymmetries for the process $p^{↑}p \rightarrow πX$ at large energy and $p_T ≃ 2$ GeV/c. Our model clearly exhibits the observed increase with $x_F$, at fixed $p_T$ values, of the magnitude of the asymmetries. A more detailed application of our approach to other processes, like $\bar{p}^{↑}p \rightarrow πX$, $p^{↑}p \rightarrow γX$ and $πp^{↑} \rightarrow πX$ is in progress [31]: this should help in assessing the relevance and importance of our estimates.

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Figure caption

Fig. 1 Fit of the data on $A_N$ [2], with the parameters given in Eq. (21); the upper, middle and lower sets of data and curves refer respectively to $\pi^+, \pi^0$ and $\pi^-$. 
This figure "fig1-1.png" is available in "png" format from:

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