PARTICLES WITH NEGATIVE ENERGIES IN BLACK HOLES

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The problem of the existence of particles with negative energies inside and outside of Schwarzschild, charged and rotating black holes is investigated. Different definitions of the energy of the particle inside the Schwarzschild black hole are analyzed and it is shown in what cases this energy can be negative. A comparison is made for the cases of rotating black holes described by the Kerr metric when the energy of the particle can be negative in the ergosphere and the Reissner–Nordstrøm metric.

Keywords: Negative energy, black holes, Penrose process.

1. Introduction

The problem of time in cosmology, mainly in quantum cosmology, is one of the favorite topics of Prof. M. Castagnino. In this paper, being a tribute to his anniversary, we shall discuss a similar problem arising inside and outside black holes. It is the problem of the energy of the particle falling inside the black hole. It is well known that the definition of the energy depends on the definition of time, but time is different outside and inside of the black hole. It is often claimed that space and time are somehow changed going one to another after crossing the horizon of the black hole. Very popular is the idea that differently from the region outside of the static black hole particles inside the black holes can have negative energies. Usually it is discussed concerning the Hawking effect of the evaporation of black holes due to particle creation from vacuum by the gravitation of the hole. It is claimed that a pair of particles with total energy equal to zero is created by the static gravitational field of the black hole so that the particle with negative energy

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goest inside the black hole while the particle with positive energy is going outside
and is observed as Hawking radiation. The mass of the black hole in this process
due to receiving the negative energy becomes smaller so that one can speak about
the gradual annihilation of the black hole itself.

In Ref. [3] S. Hawking wrote: “Just outside the event horizon there will be
virtual pairs of particles, one with negative energy and one with positive energy.
The negative particle is in a region which is classically forbidden but it can tunnel
through the event horizon to the region inside the black hole where the Killing
vector which represents time translations is spacelike. In this region the particle
can exist as a real particle with a timelike momentum vector even though its energy
relative to infinity as measured by the time translation Killing vector is negative.
The other particle of the pair, having a positive energy, can escape to infinity where
it constitutes a part of the thermal emission ... The probability of the negative
energy particle tunneling through the horizon is governed by the surface gravity
since this quantity measures the gradient of the magnitude of the Killing vector or,
in other words, how fast the Killing vector is becoming spacelike. Instead of thinking
of negative energy particles tunneling through the horizon in the positive sense of
time one could regard them as positive energy particles crossing the horizon on
pastdirected world-lines and then being scattered on to future-directed world-lines
by the gravitational field.”

Surely one can obtain the Hawking effect without this appeal to negative energies
considering the collapse of the star and particle creation in this case when going to
the static limit of the black hole[4]. However let us try to investigate the problem:
in what sense one can speak about the existence of particles with negative energies
inside the Schwarzchild black hole? If this energy has a large absolute value can such
a black hole become a wormhole? It is known that negative energies are needed for
the wormhole formation and it is very attractive to think about the physical black
holes as wormholes.

2. The Energy Conserved Relative to Infinity

Let the spacetime have the timelike Killing vector \( \zeta^i \) orthogonal to some set of
spacelike hypersurfaces \( \{ \Sigma \} \). From the definition of the Killing vector one has
\[
\nabla^i \zeta^k + \nabla^k \zeta^i = 0 .
\]

(1)

Let \( T_{ik} \) be the metrical energy-momentum tensor of some field or covariantly con-
served energy-momentum tensor of some matter. The translational symmetry with
the generator \( \zeta^i \) leads to the conservation of the quantity
\[
E(\zeta) = \int_{\Sigma} T_{ik} \zeta^i \, d\sigma^k ,
\]

(2)

which follows from the covariant conservation of \( T_{ik} \) and Eq. (1) leading
to \( \nabla^i (T_{ik} \zeta^k) = 0 \). The quantity \( E(\zeta) \) plays the role of the energy.
If the vector $\zeta^i$ is not timelike but it is still the Killing vector, while $\{\Sigma\}$ is some set of hypersurfaces not necessarily spacelike then formula (2) is still defining some conserved quantity.

If $\zeta^i$ is not a Killing vector then the quantity defined by (2) generally is not conserved. In part 3 we shall consider (2) also for such situations.

The action for a classical pointlike particle with mass $m$ is equal to
\[
S = -mc \int ds ,
\]
where $c$ is the light velocity. Let us find its energy-momentum tensor. Due to the definition one has
\[
T_{ik} = \frac{2c}{\sqrt{|g|}} \frac{\delta S}{\delta g^{ik}} \iff T^{ik} = -\frac{2c}{\sqrt{|g|}} \frac{\delta S}{\delta g_{ik}} .
\]
Here $g_{ik}$ is a spacetime metric, $g = \det \{g_{ik}\}$ and we take into account the relation $g^{ik} \delta g_{ik} = -g_{ik} \delta g^{ik}$. From (1) by variation of the action (2) one obtains (see Ref. 5, Chap. 12, Sec. 2) the energy-momentum tensor of the classical pointlike particle of mass $m$, at the point with coordinates $x_p$ as
\[
T^{ik}(x) = \frac{mc^2}{\sqrt{|g|}} \int ds \frac{dx^i}{ds} \frac{dx^k}{ds} \delta^4(x - x_p) .
\]

The value of (2) for a pointlike particle in a general metric and for an arbitrary vector $\zeta^i$ is
\[
E^{(\zeta)} = mc^2 \int dx^i \delta g^{ik} \zeta_k = mc^2 (u, \zeta) = c(p, \zeta) ,
\]
where $u^i = dx^i/ds$ is the four velocity, $p^i = mc^2 dx^i/ds$ is the four momentum.

If $\zeta^i$ is the Killing vector, then $E^{(\zeta)}$ is conserved for a pointlike particle with the action (3), i.e. for a particle freely moving on a geodesic line. This also follows from (7), because the scalar product $(u, \zeta)$ is constant on the geodesic (see problem 10.10 in Ref. 6).

If $\zeta^i$ is the translation vector on $x^0$ then
\[
\zeta^i = (1, 0, 0, 0)
\]
and (2) for the pointlike particle is
\[
E^{(\zeta)} = mc^2 g_{0i} \frac{dx^i}{ds} = mc^2 u_0 .
\]

For a massless particle the energy at infinity can be defined by a formula analogous to (8). Let $\tau$ be some affine parameter on the geodesic (for a timelike geodesic the role of this parameter plays the proper time). Then for a massless particle (photon) define
\[
E^{(\zeta)} = E_\infty \frac{dx^i}{c d\tau} \delta g_{ik} \zeta^k = E_\infty (u, \zeta) ,
\]
where $u^i = dx^i/(c d\tau)$, $E_\infty$ is some constant with the dimension of energy ($E_\infty = h\nu$ for the photon, if the parameter $\tau = t$ at infinity, $\nu$ is the frequency at infinity).
The metric of the static uncharged black hole in Schwarzschild coordinates has the form

\[ ds^2 = \left(1 - \frac{r_g}{r}\right)c^2dt^2 - \frac{dr^2}{1 - \frac{r_g}{r}} - r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2). \]  
(10)

Here \( r_g = 2GM/c^2 \) is the gravitational radius of the black hole of mass \( M \). For a Schwarzschild metric (10) vector (7) is the timelike Killing vector and one gets from (8)

\[ E(\zeta) = mc^2 \left(1 - \frac{r_g}{r}\right) dx^0 ds = mc^2 \left(1 - \frac{r_g}{r}\right) \frac{dt}{d\tau}, \]  
(11)

where \( d\tau = ds/c \).

Outside of the horizon (11) has the meaning of the energy of the particle in this metric

\[ E = mc^2 \sqrt{\frac{1 - \frac{r_g}{r}}{1 - \frac{v^2}{c^2}}}, \]  
(12)

where \( v \) is the velocity measured by the observer at rest in the Schwarzschild coordinates. Inside the horizon \( r < r_g \) the expression (11) also gives the value of some conserved entity called the energy relative to infinity. However, in this case the Killing vector (7) is spacelike, the coordinate \( t \) is not timelike but spacelike and formula (12) is incorrect.

Negative value of the energy outside the horizon due to (11) is possible either for particles with negative mass or for particles with positive mass but moving backwards in time. Both cases do not have clear physical sense. However, if the energy of the particle with positive mass inside the horizon \( r < r_g \) is negative relative to infinity this means due to (11) that \( dt/d\tau > 0 \). Inside the horizon particles with \( m > 0 \) are moving to the right along the space coordinate \( t \) if their energy relative to infinity is negative and they are moving to the left if their energy is positive! The examples of movement of particles with positive and negative energies are represented in Fig. 1 in Kruskal-Szekeres coordinates.

The analogous situation exists for massless particles (photons). If their energy is positive relative to infinity, then inside the horizon \( dt/d\tau < 0 \) represents movement to the left along an angle of 45° on Kruskal-Szekeres diagram. If their energy is negative relative to infinity, then inside the horizon \( dt/d\tau > 0 \) represents movement to the right along an angle of 45°.

The analysis of the light cones shows that for particles crossing the event horizon movement to the left in \( t \) corresponds for the particle falling inside the black hole in Kruskal–Szekeres coordinates to movement (at the moment of crossing the horizon) in the region II in the first quadrant, while movement to the right in \( t \) (at the same moment) means movement in the region II but in the second quadrant. The Kruskal–Szekeres diagram for the star collapsing into a black hole is given in
Particles with Negative Energies in Black Holes 5

Fig. 1. The radial falling inside the black hole of the massive particles with positive \((B^+H^+F^+)\) and negative \((B^-H^-F^-)\) energy. Here \(t_1^+ = -0.5r_g/c\), \(t_1^- = +0.5r_g/c\), \(r_1^+ = r_1^- = 1.2r_g\), the energy is \(|E|/mc^2 = 0.41\) in the left figure and 0.7 in the right one.

Ref. 8, Fig. 32.1. In this case the region II in the second quadrant is absent. So the interpretation of the process of radiation as creation of particles with negative energy exactly on the horizon is impossible!

Expression (11) can be obtained in another way. As it is known the geodesics in spacetime with metric \(ds^2 = g_{ik}dx^idx^k\) can be obtained (see Ref. 7, Chap. 1, Sec. 6) as the Euler-Lagrange equations for the extremal problem for the functional

\[
S = \frac{1}{c} \int L \, ds , \quad L = \frac{mc^2}{2} g_{ik} \frac{dx^i}{ds} \frac{dx^k}{ds} .
\]  

(13)

If the Lagrangian (13) does not explicitly depend on the variable \(t\), then the corresponding momentum is conserved:

\[
p_t = mc^2 g_{0i} \frac{dx^i}{ds} = \text{const} ,
\]

(14)

which coincides due to the formula (8) with the energy of the particle \(E(^G)\).

For example, in Schwarzschild spacetime (10) the Lagrangian \(L\) has the form

\[
L = mc^2 \left[ \left(1 - \frac{r_g}{r}\right) \ddot{r}^2 - \frac{\dot{r}^2}{c^2 \left(1 - \frac{r_g}{r}\right)} - \frac{r^2}{c^2} \left(\dot{\theta}^2 + \sin^2 \varphi \dot{\varphi}^2\right) \right] ,
\]

(15)

where the dot denotes derivative with respect to the proper time \(\tau\). The momentum \(p_t\) is equal to (11) and it is conserved because the Lagrangian (15) does not depend explicitly on the variable \(t\). So the conserved momentum \(p_t\) is identical to the energy of the particle relative to infinity in Schwarzschild gravitational field.
3. The Nonconserved Energy Inside the Black Hole

3.1. The Schwarzschild coordinates inside the black hole

Using notations
\[ \eta = -\frac{r}{c}, \quad \eta \in \left(-\frac{r_g}{c}, 0\right); \quad l = ct, \quad l \in \mathbb{R}, \]
the metric inside the horizon is given by
\[ ds^2 = \frac{c^2 d\eta^2}{\left(\frac{r_g}{c} - c \eta\right)} - \left(\frac{r_g}{c} - c \eta - 1\right) dt^2 - (c \eta)^2 \left(d\theta^2 + \sin^2 \theta \, d\varphi^2\right). \tag{16} \]

From (17) it is seen that the coordinate \( \eta \) plays the role of time inside the black hole. The negative sign in the first equation of (16) is chosen so that the future direction inside the black hole corresponds to increasing \( \eta \). Choose coordinates \( x^0 = c \eta \), \( x^1 = l \), \( x^2 = \vartheta \), \( x^3 = \varphi \). Then the 4-vector (7) is the generator of translations in time \( \eta \) (here it is not a Killing vector). The energy relative to such translations defined by (2) surely is not conserved. From (8) one obtains
\[ E^{(\eta)} = mc \frac{d\eta}{d\tau} = - mc \frac{dr}{d\tau}, \tag{18} \]

For movement inside the black hole one has \( \frac{dr}{d\tau} < 0 \) and \( E^{(\eta)} > 0 \). Let us find more exact limitations on the possible values of \( E^{(\eta)} \) for a massive particle inside the black hole. From the definition of the four velocity one has \( u^i u_i = 1 \) and for the Schwarzschild metric one gets
\[ \left(1 - \frac{r_g}{r}\right) \left(\frac{c dt}{ds}\right)^2 - \frac{1}{1 - \frac{r_g}{r}} \left(\frac{dr}{ds}\right)^2 - r^2 \left(\frac{d\theta}{ds}\right)^2 + \sin^2 \theta \left(\frac{d\varphi}{ds}\right)^2 = 1. \tag{19} \]

Inside the horizon \( r < r_g \) all terms in the left side of (19) are negative except the term with \( dr/ds \), so this term is larger or equal to one and
\[ - \frac{dr}{d\tau} \geq c \sqrt{\frac{r_g}{r} - 1}, \quad r < r_g. \tag{20} \]

So possible values of the “energy” \( E^{(\eta)}(r) \) for a given \( r \) inside the black hole satisfy the inequality
\[ E^{(\eta)}(r) \geq \frac{mc^2}{\sqrt{\frac{r_g}{r} - 1}}, \quad r < r_g. \tag{21} \]

The geodesics (with \( \vartheta = \pi/2 \)) in Schwarzschild metric satisfy the equations
\[ \left(\frac{dr}{c d\tau}\right)^2 = \left(\frac{r_g}{r} - 1\right) \left(1 + \frac{L^2}{r^2}\right) + \varepsilon^2, \quad r^2 \frac{d\varphi}{c d\tau} = L = \text{const} \tag{22} \]
Particles with Negative Energies in Black Holes

(see Ref. [7] Sec. 19), where \( \varepsilon = \text{const} \) is the specific energy relative to infinity, \( L \) denotes the angular momentum about an axis normal to the invariant plane in units of \( mc \). So the energy \( E^{(n)} \) introduced in (18) for the freely falling particle with mass \( m \) is

\[
E^{(n)}(r) = \frac{mc^2}{r_g - 1} \sqrt{\varepsilon^2 + \left( \frac{r_g}{r} - 1 \right) \left( 1 + \frac{L^2}{r^2} \right)}
\]

(23)

and it changes from \(+\infty\) on the horizon to 0 in the singularity, if \( L = 0 \). For geodesics with angular momentum \( L \neq 0 \), the energy \( E^{(n)} \) inside the horizon changes from \(+\infty\) on the horizon, decreases to some finite positive value and then it is again increasing to \(+\infty\) in the singularity.

Outside the hole the value (18) is not an energy and can be positive or negative according to the sign of \( dr/d\tau \).

3.2. The interior of the black hole as anisotropic cosmology

The spacetime inside the Schwarzschild black hole is an example of homogeneous space but anisotropic cosmological model. Really, if one introduces the variable

\[
d\xi = d\eta \sqrt{\frac{-c\eta}{r_g + c\eta}} \quad \xi = \sqrt{-\eta \left( \frac{r_g}{c} + \eta \right) - \frac{r_g}{c} \tan^{-1} \sqrt{\frac{-c\eta}{r_g + c\eta}}} \quad \xi \in \left( -\frac{\pi r_g}{2c}, 0 \right)
\]

(24)

the metric inside the horizon can be written as

\[
d\sigma^2 = c^2 d\xi^2 - f_1(\xi) \, dl^2 - f_2(\xi) \left( d\vartheta^2 + \sin^2 \vartheta \, d\varphi^2 \right)
\]

(25)

where \( f_1(\xi) \) and \( f_2(\xi) \) are some functions which can be found from (17) and (24).

The energy (8) relative to translations in time \( \xi \) is equal to

\[
E^{(\xi)} = mc^2 \frac{d\xi}{d\tau} = -\frac{mc}{\sqrt{r_g}} \frac{dr}{d\tau}
\]

(26)

Inside the black hole one has the inequality (20) and so

\[
E^{(\xi)}(r) \geq mc^2, \quad r < r_g.
\]

(27)

Using (22) one can see that for a particle moving along a geodesic inside the black hole, the energy \( E^{(\xi)} \) thus defined is equal to

\[
E^{(\xi)}(r) = mc^2 \sqrt{\frac{\varepsilon^2 r}{r_g - r} + 1 + \frac{L^2}{r^2}}.
\]

(28)

For a radially falling particle \( E^{(\xi)} \) is decreasing from \(+\infty\) on the horizon to \( mc^2 \) at the singularity. For geodesics with nonzero angular momentum the energy \( E^{(\xi)} \) inside the horizon changes from \(+\infty\) on the horizon, decreases to some positive value and then again increases to \(+\infty\) at the singularity.

Outside the horizon the variable \( \xi \) and the energy \( E^{(\xi)} \) have imaginary values.
3.3. Energy inside the black hole for conformal time

If one introduces in (17) the variable
\[ d \tilde{\xi} = \frac{-c \eta}{r g + c \eta}, \quad \tilde{\xi} = -\eta + \frac{r g}{c} \ln \left(1 + \frac{c \eta}{r g}\right), \quad \tilde{\xi} \in (-\infty, 0), \]
the metric inside the horizon can be written as
\[ ds^2 = a^2(\tilde{\xi}) \left(c^2 d\tilde{\xi}^2 - dl^2 - f(\tilde{\xi}) (d\theta^2 + \sin^2 \theta d\phi^2) \right), \]
where \( a(\tilde{\xi}) \) and \( f(\tilde{\xi}) \) are some functions which can be obtained from (17) and (29) and \( \tilde{\xi} \) is the analog of the conformal time in homogeneous isotropic cosmological models.

The energy relative to translations in time \( \tilde{\xi} \) is equal to \( E(\tilde{\xi}) = -mc \frac{dr}{d\tau} \) and due to (20) it satisfies the limitation
\[ E(\tilde{\xi})(r) = -mc \frac{dr}{d\tau} \geq mc^2 \sqrt{\frac{r g}{r} - 1}, \quad r < r_g, \]
For the particle falling on the geodesic (22)
\[ E(\tilde{\xi}) = -mc \frac{dr}{d\tau} = mc^2 \left(\frac{r_g}{r} - 1\right) \left(1 + \frac{r^2}{r_g^2}\right)^2 + \varepsilon^2, \]
\( E(\tilde{\xi}) \) is growing from \( mc^2|\varepsilon| \) on the horizon to \( +\infty \) at the singularity.

So our examples show that for particles falling inside the black hole one can introduce differently some value analogous to the energy which is non negative but it is not conserved in time. This situation is analogous to that in nonstationary external field. The gravitational field inside the Schwarzschild black hole surely is not static.

4. Kerr’s Metric

Let us compare the situation with negative energy inside the static black hole with the well known case of the rotating black hole. Kerr’s metric in Boyer–Lindquist coordinates is
\[ ds^2 = dt^2 - (r^2 + a^2) \sin^2 \theta \, d\phi^2 - \frac{2Mr (dt - a \sin^2 \theta \, d\phi)^2}{r^2 + a^2 \cos^2 \theta} - (r^2 + a^2 \cos^2 \theta) \left(\frac{dr^2}{r^2 - 2Mr + a^2} + d\theta^2\right), \]
where \( M \) is the mass of the black hole, \( aM \) its angular momentum. Here we use units where \( G = c = 1 \). On the event horizon one has
\[ r = r_H \equiv M + \sqrt{M^2 - a^2}. \]
The surface called the static limit is defined by
\[ r = r_0 \equiv M + \sqrt{M^2 - a^2 \cos^2 \theta}. \]
The spacetime region between the horizon and the static limit is called ergosphere. It is in the ergosphere that one has the trajectories of particles with negative energy. Inside the ergosphere the Killing vector of translations in time \((1, 0, 0, 0)\) becomes spacelike similar to that discussed by S. Hawking for static black holes in the citation in the beginning of this paper. The energy of the particle relative to infinity can be negative.

Really in the case of the spacelike Killing vector \((k_0, \mathbf{k})\) in some reference frame (see problem 10.15 in Ref. 6) one can write

\[ E^{(k)} = m\gamma(k_0 - \mathbf{v} \cdot \mathbf{k}), \]  

(36)

where \(\gamma = 1/\sqrt{1 - v^2}\), \(\mathbf{v}\) is the three-velocity of a particle, \(k_0 < |\mathbf{k}|\) and \(E^{(k)} < 0\) is possible.

Due to formula \(^{[3]}\) one gets

\[ E^{(c)} = m \left[ 1 - \frac{2Mr}{r^2 + a^2 \cos^2 \theta} \right] \frac{dt}{d\tau} + \frac{2Mra \sin^2 \theta}{r^2 + a^2 \cos^2 \theta} \frac{d\phi}{d\tau}. \]  

(37)

\(E^{(c)} < 0\) is possible for \(r < 2M\) and movement of particles rotating in the direction opposite to rotation of the black hole (see Ref. 7, Sec. 65). Existence of states with negative energy in the ergosphere leads to the possibility of getting energy from the rotating black hole.\(^{[10]}\)

In all the cases considered, the states with negative relative to infinity energy exist for the spacelike Killing vector either in ergosphere of the Kerr’s black hole or inside the horizon of the Schwarzschild’s black hole.

If one has other than gravitational interaction of the pointlike particle then states with conserved energy are possible for the timelike Killing vector. Let us consider this situation for the example of the charged nonrotating black hole.

5. Reissner–Nordstrøm Black Holes

The Reissner–Nordstrøm solution for the metric of the static charged black hole has the form

\[ ds^2 = \frac{\Delta}{r^2} dt^2 - \frac{r^2}{\Delta} dr^2 - r^2 \left( d\phi^2 + \sin^2 \theta \, d\theta^2 \right), \quad \Delta = r^2 - 2Mr + Q^2, \]  

(38)

where \(Q\) is the charge of the black hole with mass \(M\) (here we used the system of units \(G = c = 1\)). The roots of equation \(\Delta = 0\),

\[ r_H = M + \sqrt{M^2 - Q^2}, \quad r_C = M - \sqrt{M^2 - Q^2}, \]  

(39)

define the surfaces which are the event horizon \(r = r_H\) and the Cauchy horizon \(r = r_C\) for the charged black hole.
Equations of movement of the particle with specific (divided by mass) charge $q$ in the metric (38) are (see Ref. 7, Sec. 40)

$$\frac{\Delta}{r^2} \frac{dt}{d\tau} + \frac{qQ}{r} = \varepsilon = \text{const}, \quad r^2 \frac{d\varphi}{d\tau} = L = \text{const}, \quad (40)$$

$$\left( \frac{dr}{d\tau} \right)^2 + \frac{\Delta}{r^2} \left( 1 + \frac{L^2}{r^2} \right) = \left( \varepsilon - \frac{qQ}{r} \right)^2. \quad (41)$$

As one can see from (40), (41) for $qQ < 0$ one has states of the charged particle with negative value of the specific energy $\varepsilon \approx qQ/r_H$ in the vicinity of the event horizon $r_H$. Existence of states with negative energy leads to the possibility of getting energy from the charged black hole. 11, 12

For noncharged particles (with positive mass) outside the event horizon states with negative energy don’t exist. From (40) one comes to the conclusion that if only movement to the future in time is existing, i.e. $dt/d\tau > 0$ then $q = 0$ leads to $\varepsilon > 0$.

So negative relative to infinity energies are possible also for the timelike Killing vector but for the nongeodesic particle movement which occurs for the electromagnetic interaction of the charged particle in the field of the Reissner–Nordstrøm black hole. Negative relative to infinity energies in the given metric in all cases lead to the possibility of extracting energy from the given object. If these negative energies exist inside the event horizon the extraction of energy is possible due to the quantum process — the Hawking effect. If negative energies exist outside the event horizon extraction of energy occurs due to the classical process (Penrose process in the ergosphere of the rotating black hole).

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