The origin of peculiar jet-torus structure in the Crab nebula

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\textbf{ABSTRACT}

Recent discoveries of the intriguing "jet-torus" structure in the Crab Nebula and other pulsar nebulae prompted calls for re-examining of their theory. The most radical proposals involve abolishing of the MHD approximation altogether and developing of purely electromagnetic models. However, the classical MHD models of the Crab Nebula were hampered by the assumption of spherical symmetry made in order to render the flow equations easily integrable. The impressive progress in computational relativistic magnetohydrodynamics in recent years has made it possible to study the Crab nebula via numerical simulations without making such a drastic simplification of the problem. In this letter we present the results of the first study of such kind. They show that the jet-torus pattern can be explained within MHD approximation when anisotropy of pulsar winds is taken into account. They also indicate that the flow in the nebula is likely to be much more intricate than it has been widely believed.

\textbf{Key words:} pulsars:general – supernova remnants – ISM:individual:the Crab Nebula – ISM:jets and outflows – MHD — shock waves

\section{INTRODUCTION}

The Crab Nebula is a prototype compact synchrotron nebula continuously powered by ultra-relativistic, magnetized wind from young rapidly rotating pulsar. As this nebula is confined within a nonrelativistic surrounding, the wind must terminate at a shock wave. It is the wind plasma heated to relativistic temperatures at this shock that fills the nebula and
produces the observed nonthermal electromagnetic emission from the radio to the gamma-ray band (Rees & Gunn 1974, Kennel & Coroniti 1984). Indeed, early observations of the Crab Nebula revealed an apparent central hole in the nebula brightness distribution (Scargle 1969), which was identified with the termination shock in the theoretical models of the nebula. However, the assumption of spherical symmetry utilized in these models in order to simplify calculations was shattered when recent X-ray and optical observations (Hester et al. 1995, 2002; Weiskopf et al. 2000) revealed the inner structure of the nebula in much greater details. They discovered two jets aligned with the rotational axis of the pulsar and a luminous torus or disc in its equatorial plane. As similar structures have been found in other pulsar wind nebulae (Gaensler, Pivovaroff & Garmire 2001; Helfand, Gotthelf & Halpern 2001; Pavlov et al. 2001; Gaensler et al. 2002; Lu et al. 2002), we must be dealing with quite generic a phenomenon.

The interaction between a relativistic MHD wind from the pulsar and a dense, nonrelativistic surrounding is a much more challenging problem in the case of anisotropic wind and there is no much hope to find its analytical solution. Fortunately, recent progress in numerical methods for relativistic gas dynamics and MHD (Marti & Muller 1999; Komissarov 1999) has made possible to approach this problem numerically. The main results of the first attempt of such a study are briefly described in this letter. Full details will be presented elsewhere.

2 STRUCTURE OF THE PULSAR WIND IN THE FAR ZONE

Although a self-consistent solution to the problem of pulsar wind remains to be found, it is commonly accepted that far away from the pulsar the wind can be considered as almost radial supermagnetosonic outflow with purely azimuthal magnetic field and anisotropic angular distribution of the energy flux (Michel 1982; Beskin, Kuznetsova & Rafikov 1998; Chiueh, Li & Begelman 1998; Bogovalov & Tsinganos 1999). According to the simplified split-monopole models of pulsar magnetospheres (Michel 1973; Bogovalov 1999), the total energy flux density of the wind, \( f_w \), satisfies the following equation

\[
 f_w = \frac{f_0}{r^2} \left( \sin^2 \theta + 1/\sigma_0 \right),
 \]

where \( r \) and \( \theta \) are the spherical coordinates whose polar axis is aligned with the rotation axis of the pulsar. The first term in the brackets represents the Poynting flux, whereas the second one accounts for the small initial contribution of particles, \( \sigma_0 \gg 1 \). It was shown recently that
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The termination shock of such a wind is not at all spherical. In fact, it is located significantly closer to the pulsar along its rotational axis than in the equatorial plane (Lyubarsky 2002; Bogovalov & Khangoulyan 2002).

Crab’s jets, as well as jets of other pulsars, appear to originate from the pulsar (Weisskopf et al. 2000; Helfand, Gotthelf & Halpern 2001; Pavlov et al. 2001; 2003; Gaensler et al. 2002; Hester et al. 2002;). This seems to indicate that they are formed within the pulsar wind and the collimation by magnetic hoop stress suggests itself. However, a closer look reveals a number of problems with this explanation. First of all, such a collimation is found to be extremely ineffective in ultra-relativistic flows (Beskin et al. 1998; Chiuheh & Begelman 1998; Bogovalov & Tsinganos 1999; Lyubarsky & Eichler 2001). Moreover, the direct observations of proper motions in Crab’s and Vela’s jets indicate rather moderate velocities of only $0.3 \div 0.7c$ (Hester et al. 2002; Pavlov et al. 2003). In order to overcome these problems, Lyubarsky (2002) proposed that the jets are formed downstream of the termination shock where velocities are no longer ultra-relativistic and the magnetic collimation is much more effective. Because the termination shock is much closer to the pulsar along its rotational axis, this could give an impression of jets being produced by the pulsar itself.

Properties of the pulsar wind nebula strongly depend on the wind magnetization. It is widely accepted that a typical pulsar wind is launched as a Poynting-dominated flow but most of the electromagnetic energy is transferred to particle along the way to the termination shock (see, however, Begelman 1998). Although the problem of energy conversion remains a subject of intensive debate (e.g. Melatos 2002), a number of mechanisms have been proposed in recent years. Since the pulsar magnetic axis is inclined with respect to its rotational axis, a significant fraction of the Poynting flux is carried out by the component of electromagnetic field oscillating with the rotational period of the pulsar. These small-scale waves can decay via various dissipation processes (Lyubarsky & Kirk 2001; Melatos 2002; Lyubarsky 2003a; Kirk & Skjæraasen 2003). Even if the dissipation time-scale is larger than the time of travel from the pulsar to the termination shock, these waves rapidly decay at the termination shock. In this case, the post-shock parameters are still the same as if the alternating fields had already annihilated in the upstream flow (Lyubarsky 2003b). For this reasons, we assume in our model of the pulsar wind that all these waves have already decayed and transferred their energy to particles. The rest of the Poynting flux is transported by the large-scale azimuthal magnetic field. The exact latitudinal distribution of this field remains to be found but it must vanish both along the rotational axis, as any axisymmetrical magnetic field does, and in the...
equatorial plane, because the average magnetic field in the obliquely rotating magnetosphere is zero at the equator. We take the residual magnetic field in the wind in the form
\[ B = \sqrt{\frac{4\pi f_0}{c}} \frac{\xi}{r} \sin \theta \left( 1 - \frac{2\theta}{\pi} \right). \]
(2)
The free parameter \( \xi \leq 1 \) controls the wind magnetization.

3 NUMERICAL SIMULATIONS

The initial solution includes 1) a spherically symmetric shell of cold dense gas expanding radially with velocity of 5000 km/s and 2) a radial ultra-relativistic wind with the energy flux density given by equation (1) with \( \sigma_0 = 100 \), and the magnetic field described by equation (2). Provided the wind is ultra-relativistic, the solution is not sensitive to the exact value of the wind Lorentz factor, which we set to \( \gamma_w = 10 \). To carry out these axisymmetric simulations we used the Godunov-type scheme for relativistic magnetohydrodynamics constructed recently by Komissarov (1999).

The results for \( \xi = 0.3 \) are presented in Figure 1 (overall structure of the flow) and Figures 2, 3 (its central part). One can see that instead of a single termination shock, the numerical solution displays a whole complex of shocks. The equatorial, weakly magnetized part of the pulsar wind terminates at almost cylindrical shock crossing the equator, “the Mach belt”. At higher latitudes, the flow passes first through a highly oblique ”arch shock”. Downstream of this shock the flow remains supermagnetosonic with the typical velocity of 0.8 ÷ 0.9c. Then it passes through another shock, “the rim shock”, that originates from the edge of the Mach belt. In contrast to the spherically symmetric expansion assumed in the current theories of pulsar nebulae, most of the downstream flow is confined to the equatorial plane. Its typical velocity, \( v \approx 0.6c \), agrees with the measurements of proper motions in Crab’s torus (Hester et al. 2002). This equatorial outflow is eventually pushed back by the pressure force and the magnetic hoop stress and forms a backflow.

This backflow reaches the central region of the nebula and forms large-scale vortices at intermediate latitudes.

Another backflow is launched from the surface of the equatorial disc as the result of strong magnetic braking. This inner backflow eventually forms magnetically collimated polar jets. The typical velocity of these jets is \( v_j \approx 0.5c \), which is well in agreement with observed velocities of Crab’s and Vela’s jets (Hester et al. 2002; Pavlov et al. 2003). Similar jets are produced in the model with \( \xi = 0.5 \) but not found in the solution with \( \xi = 0.2 \). The pulsar
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Figure 1. Fig.1. The ratio of magnetic pressure to the gas pressure and velocity field for the model with the magnetization parameter $\xi = 0.3$. The equatorial outflow is seen as the region of a particularly high magnetic pressure.

Figure 2. The gas pressure distribution in the central part of the solution shown in figure 1. The white circle in the centre shows the inner boundary of the computational domain whereas the adjacent region of dark blue color shows the pulsar wind zone. The “Mach belt” shock runs across the equatorial plane at the distance of $\approx 0.8$ from the symmetry axis; the “arch shock” is seen as the upper and the lower boundaries of the wind zone; the “rim shocks” originate from the points of intersection of the Mach belt and the arch shock.
Figure 3. The velocity magnitude, represented in color, and the flow direction, represented by arrows, in the central part of the solution shown in figure 1. Just above the equatorial outflow there seen a layer of backflow converging toward the symmetry axis. This backflow provides plasma for two transonic jets propagating in the vertical direction.

Wind magnetization is traditionally measured by the parameter $\sigma$, defined as the ratio of the Poynting flux to the kinetic energy flux of the wind (Kennel & Coroniti 1984). It turns out that in our model the mean $\sigma \approx 0.1\xi^2$. Thus, the polar jets are produced for $\sigma \geq 0.01$, which is somewhat higher than the previous estimates for the Crab Nebula based on simplified theoretical models (Kennel & Coroniti 1984; Emmering & Chevalier 1987; Begelman & Li 1992).

Figure 4 shows the simulated synchrotron X-ray emission from the inner part of the nebula obtained for the same orientation of the wind relative to the observer as in the Crab nebula. In order to create this image, we assumed that electrons (and positrons) are injected at the termination shock and then suffer synchrotron energy losses at a rate determined by the typical value of magnetic field in the numerical solution. Because the magnetic field in our simulations is purely azimuthal, it vanishes on the symmetry axis and so does the synchrotron emissivity. In a real jet, the strong velocity shear would generate the poloidal component of the magnetic field, which does not have to vanish on the axis. To take this into account, we added, only inside the jet, the poloidal field aligned with the flow velocity at the level of 30% of the gas pressure.
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Figure 4. Synchrotron X-ray image for the solution shown in figures 1-3. The nebula is tilted to the plane of the sky at the angle of 30°, just like the Crab Nebula. The brightness distribution is shown in the logarithmic scale.

4 DISCUSSION

The obtained image is excitingly similar to the X-ray image of the Crab nebula (Weisskopf et al. 2000). In addition to the polar jets, one can see a system of rings which makes an impression of a disc-like or even a toroidal structure. Well in agreement with the observations these rings are brighter on the counter-jet side, which is entirely due to the relativistic beaming effect. The bright spot within the inner ring may correspond to the so-called “sprite” of the Crab nebula (Hester et al. 2002). This emission originates in the high velocity plasma flowing just above the arch shock where its velocity vector points directly toward the observer.

On the whole, our model captures the main properties of the Crab nebula remarkably well. There are, however, some qualitative and quantitative differences which demand further investigation. Crab’s jet is not so straight and well collimated and it cannot be traced that far away from the pulsar. It bends, spreads, and eventually merges into the surrounding plasma (Weisskopf et al. 2000). The most likely reason for such a behavior is the development of the kink instability (Begelman 1998), which is suppressed in our simulations by the condition of axisymmetry. Full 3-dimensional simulations are needed to overcome this restriction.

The brightness contrast between the jet side and the counter-jet side of the rings in the
simulated maps is too high. Since this asymmetry is entirely due to the relativistic beaming, it strongly depends on the velocity field very close to the termination shock and hardly depends on anything else. The main factor determining this velocity field is the angular structure of the pulsar wind. Thus, the brightness asymmetry of Crab’s “torus” imposes strong observational constraints on the pulsar wind models. Future simulations will be used to determine the model parameters providing the best fit to the observational data.

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