Exotic hadrons in the decays of vector bottomonia

V Baru\textsuperscript{1,2,3}, A Filin\textsuperscript{4}, C Hanhart\textsuperscript{5}, A Nefediev\textsuperscript{3,6} and Q Wang\textsuperscript{1}

\textsuperscript{1} Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany
\textsuperscript{2} Institute for Theoretical and Experimental Physics, B. Cheremushkinskaya 25, 117218 Moscow, Russia
\textsuperscript{3} P.N. Lebedev Physical Institute of the Russian Academy of Sciences, 119991, Leninskiy Prospect 53, Moscow, Russia
\textsuperscript{4} Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany
\textsuperscript{5} Forschungszentrum Jülich, Institute for Advanced Simulation, Institut für Kernphysik and Jülich Center for Hadron Physics, D-52425 Jülich, Germany
\textsuperscript{6} National Research Nuclear University MEPhI, 115409, Kashirskoe highway 31, Moscow, Russia

E-mail: arseniy.filin@ruhr-uni-bochum.de

Abstract. Line shapes of the $Z_{b}^{\pm}(10610)$ and $Z_{b}^{\pm}(10650)$ bottomonium-like exotic states produced in the decays of the vector bottomonium $\Upsilon(10860)$ are analysed. A combined analysis of the existing experimental data in the elastic and inelastic decay channels of the $\Upsilon(10860)$ is performed within a nonperturbative coupled-channel approach which complies with the requirements of unitarity and analyticity of the multichannel amplitude. The nature of the $Z_{b}$ states is revealed and the parameters of the interaction are extracted from the fit to the data.

1. Introduction

Although the quark model in its simplest formulation, where, for example, mesons are described as quark-antiquark states, is known to be a successful phenomenological tool for understanding, explaining and predicting the spectrum and other properties of hadrons, it is also known to fail for various states. In the last decade, many such unusual, or exotic, hadrons have been observed in the spectrum of charmonium and bottomonium. Among those, the charged states $Z_{c}^{\pm}(3900)$ \cite{1,2}, $Z_{c}^{\pm}(4020)$ \cite{3}, $Z_{c}^{\pm}(4430)$ \cite{4,5,6} are of most interest since they cannot be conventional $QQ$ ($Q$ is a heavy quark) mesons as their minimal quark contents is four-quark. Similarly, in the spectrum of bottomonium, the charged $Z_{b}^{\pm}(10610)$ and $Z_{b}^{\pm}(10650)$ twin states with the quantum numbers $J^{PC} = 1^{+-}$ were observed by the Belle Collaboration as peaks in the invariant mass distributions of the $\Upsilon(nS)\pi^{\pm}$ ($n = 1, 2, 3$) and $h_{b}(mP)\pi^{\pm}$ ($m = 1, 2$) subsystems in the dipion transitions from the vector bottomonium $\Upsilon(10860)$ \cite{8,9}. Later, they were also discovered in the elastic $BB^{*}$ and $B^{*}B^{*}$ channels \cite{10,11}. At present, both a tetraquark structure \cite{12,13} and a hadronic molecule interpretation \cite{15,16,17,18,19,20,21,22,23,24} are claimed to be consistent with the data for these two exotic states. Meanwhile, their proximity to the $BB^{*}$ and $B^{*}B^{*}$ thresholds together with the fact that those are by far the most dominant decay channels of the $Z_{b}^{\pm}(10610)$ and $Z_{b}^{\pm}(10650)$, respectively, provides a strong support for their molecular interpretation in the sense of a large probability to observe them in the respective open-bottom hadronic channels.
A recent review of the theory of hadronic molecules can be found in Ref. [25]. Since both the \( Z_b^+(10610) \) and \( Z_b^0(10650) \) contain a heavy \( b \bar{b} \) pair, it is expected that the constraints from the heavy-quark spin symmetry\(^1\) (HQSS) should be very accurate for these systems. For instance, HQSS allows one to explain naturally the interference pattern in the inelastic channels \( \Upsilon(nS)\pi \) and \( h_b(mP)\pi \) [15]. It is used as one of the basic principles for constructing the effective field theory (EFT) approach to the \( Z_b \)'s developed in Ref. [26], in line with unitarity and analyticity of the multichannel amplitude.

2. Coupled-channel problem

The effective potentials between two heavy \( B^{(*)} \) mesons, which enter the Lippmann-Schwinger equations, contain local contact terms and the contributions from the lightest pseudoscalar Goldstone boson octet. The former can be presented in the matrix form as

\[
v(p, p') = \begin{pmatrix}
C_d(1+\epsilon) + D_d(p^2 + p'^2) & D_{SD}p'^2 & C_f + D_f(p^2 + p'^2) & D_{SD}p'^2 \\
D_{SD}p'^2 & 0 & D_{SD}p'^2 & 0 \\
C_f + D_f(p^2 + p'^2) & D_{SD}p'^2 & C_d(1-\epsilon) + D_d(p^2 + p'^2) & D_{SD}p'^2 \\
D_{SD}p'^2 & 0 & D_{SD}p'^2 & 0
\end{pmatrix},
\]

where the low-energy constants (LEC’s) \( C_d \) and \( C_f \) (\( D_d \) and \( D_f \)) describe the diagonal and off-diagonal \( S \)-to-\( S \)-wave interactions of the order \( O(p^0) \) (\( O(p^2) \)) which represent the leading (next-to-leading) order of the effective theory under construction while the LEC \( D_{SD} \) describes the \( S-D \) transitions at NLO. The quantity \( \epsilon \) parameterises the HQSS violation; its effect is found to be negligible [26], so we set \( \epsilon = 0 \) from now onwards.

The potential matrix (1) is given in the basis of elastic channels (labelled by greek letters \( \alpha, \beta, ... \))

\[
\{B\bar{B}^*[S], B\bar{B}^+[D], B^+\bar{B}^*[S], B^+\bar{B}^+[D]\},
\]

with the letters \( S \) or \( D \) in square brackets standing for the orbital angular momentum \( L \) in the corresponding elastic channel. Further details of constructing the elastic potential consistent with HQSS at leading order \( O(p^0) \) (LO) can be found in Refs. [15,27–30] while the \( O(p^2) \) (NLO) terms were derived Ref. [26].

The inelastic channels are available are exhausted by the set (labelled by latin letters \( i, j, ... \))

\[
\{\Upsilon(1S)\pi, \Upsilon(2S)\pi, \Upsilon(3S)\pi, h_b(1P)\pi, h_b(2P)\pi\}.
\]

We assume that all inelastic channels only couple to the \( S \)-wave elastic ones as their couplings to the \( D \)-wave elastic channels are suppressed by the ratio of the typical momentum transfer (few hundred MeV) squared to the \( B \)-meson mass squared. Then the transition potentials between the \( S \)-wave \( \alpha \)-th elastic channel and the \( i \)-th inelastic channel can be parameterised through the coupling constants \( g_{i\alpha} \),

\[
v_{i\alpha}(k_i, p) = v_{\alpha i}(p, k_i) = g_{i\alpha}k_i^{l_i},
\]

where \( k_i \) and \( l_i \) are the momentum and the angular momentum in the \( i \)-th inelastic channel, respectively. Furthermore, since the coupling of the pions to hadrons containing only heavy quarks is strongly suppressed, we disregard all direct transitions between the inelastic channels. As a result, the inelastic channels can communicate with each other through the elastic channels only — see the discussions in Refs. [26,31–33].

\(^1\) This symmetry stems from the fact that the spin-dependent interactions for a heavy quark are suppressed by the quark mass \( m_Q \), so that, in the strict limit \( m_Q \to \infty \), the spin of the heavy quark decouples from the system.
The discussion of the role of the OPE in hadronic molecules has a long history. In a pioneering work [34] a vector meson exchange was proposed as a key ingredient of the potential between a heavy meson and a heavy antimeson. Then OPE was used in the deuteron in Refs. [35–38], however, such an approach was criticised in Refs. [39–41], where amongst other issues the potential importance of the three-body dynamics was stressed. A particular form of the OPE potential used in the equations and its partial wave decomposition can be found in Ref. [26]. It is only important to stress here that the OPE is included in the current scheme in the dynamical form and fully nonperturbatively.

3. Fit to the data
The line shapes of the two \( Z_b \) states in both elastic \((B^{(*)} \bar{B}^*)\) and inelastic \((h_b(mP)\pi, m = 1, 2)\) channels are fitted using the numerical solutions of the Lippmann–Schwinger equations. The line shapes in the inelastic \( \Upsilon(nS)\pi \) \((n = 1, 2, 3)\) channels cannot be included into the fit yet since the data contain a significant contribution driven by the two-pion final state interaction that we cannot include straightforwardly in the present approach. In addition, the analysis would have to be multidimensional. The parameters of the best fit are listed in tables 1 and 2. The quality of the fit can be assessed given that \( \chi^2/\text{d.o.f} \approx 0.87\). Finally, the fitted line shapes with the uncertainties which correspond to a 1\( \sigma \) deviation in the parameters are shown in figure 1.

With the full multichannel amplitude at hand we are in a position to extract the position of the poles corresponding to the \( Z_b \) states. For simplicity, the inelastic channels in the energy plane are interpreted as a single additional effective remote channel with the momentum \( k_{in} \) and, in order to find all relevant poles, both possibilities with \( \text{Im } k_{in} > 0 \) and \( \text{Im } k_{in} < 0 \) are considered. The resulting energies evaluated relative to the relevant elastic threshold,

\[
E_{Z_b} = M_p^{\text{pole}} - m_B - m_{B^*}, \quad E_{Z_b'} = M_{Z_b'}^{\text{pole}} - 2m_{B^*},
\]

**Table 1.** The \( \mathcal{O}(p^0) \) \((C_d \text{ and } C_f)\) and \( \mathcal{O}(p^2) \) \((D_d \text{ and } D_{SD})\) contact terms extracted from the fit. The contact term \( D_f \) is set to 0 because it is strongly correlated with \( C_f \), does not affect the quality of the fit and is, therefore, redundant.

| \( C_d, \text{ GeV}^{-2} \) | \( C_f, \text{ GeV}^{-2} \) | \( D_d, \text{ GeV}^{-4} \) | \( D_{SD}, \text{ GeV}^{-4} \) |
|------------------------|----------------|----------------|----------------|
| 1.34 ± 0.40           | −3.95 ± 0.27   | −3.38 ± 0.54   | −3.13 ± 0.61   |
Table 2. The fitted values of the coupling constants defined in equation (4). Only the absolute values are presented since the physical quantities are not sensitive to the couplings’ signs.

| $|g_{Y(1S)}|$, GeV$^{-2}$ | $|g_{Y(2S)}|$, GeV$^{-2}$ | $|g_{Y(3S)}|$, GeV$^{-2}$ | $|g_{h_b(1P)}|$, GeV$^{-3}$ | $|g_{h_b(2P)}|$, GeV$^{-3}$ |
|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 0.25 ± 0.06              | 0.88 ± 0.17              | 1.15 ± 0.29              | 1.92 ± 0.22              | 7.07 ± 0.84              |

Table 3. The energies $E_{Z_b}$ and $E_{Z_b'}$ defined in equation (7). The errors correspond to a 1σ deviation in the fitted parameters.

| $E_{Z_b}$(MeV) | $E_{Z_b'}$(MeV) |
|----------------|-----------------|
| $(−2.3 ± 0.5) − i(1.1 ± 0.1)$ | $(1.8 ± 2.0) − i(13.6 ± 3.1)$ |

are listed in table 3. The errors in the poles position correspond to a 1σ deviation in the parameters of the fit. Thus, the $Z_b(10610)$ is a shallow virtual state located just below the $B\bar{B}^*$ threshold while the $Z_b(10650)$ is consistent with an above-threshold resonance.

4. Conclusions
The main results obtained in this work may be summarised as follows.  

- By comparing the distributions obtained using the parameterisation derived in Refs. [31–33] with the results of a direct numerical solution of the Lippmann–Schwinger equations used in this work we confirm the validity of the aforementioned parameterisation for the data analysis for exotic near-threshold states.
- Meanwhile, we still observe a sizeable effect from pions on the line shapes which cannot be incorporated directly in the parameterisation of Refs. [31–33] because of its non-separable form.
- We find that the existing experimental data are consistent with HQSS.
- We build a nearly perfect fit to the data ($\chi^2$/d.o.f. $\simeq 1$) and fix all free parameters of the model this way.
- We extract the position of the poles responsible for the $Z_b^+(10610)$ and $Z_b^+(10650)$ and find them to reside on the unphysical Riemann sheets just below ($Z_b$) or just above ($Z_b'$) the corresponding elastic threshold.

We conclude stating that the approach developed in this work is well suited for making predictions for the other states in the spectrum of bottomonium which differ from the $Z_b^+(10610)$ and $Z_b^+(10650)$ by the heavy-quark spin orientation. Such states are known as spin partners of the $Z_b$’s. They have the $J^{PC}$ quantum numbers $J^{++}$ ($J = 0, 1, 2$) and are conventionally denoted as $W_{bJ}$. With the parameters of the model fixed from the data on the $Z_b$’s it is a straightforward application of our approach to predict in a parameter-free way the line shapes of the spin partners in all elastic and inelastic channels and to extract the positions of the poles responsible for them. Such partner states are expected to be copiously produced in the Belle-II experiment in the radiative decays of the vector bottomonium $\Upsilon(10860)$.
Figure 1. The fitted line shapes with the uncertainties corresponding to a 1σ deviation in the parameters of the fits. The experimental data are taken from [8, 11].

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