False vacuum transitions —Analytical solutions and decay rate values

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Abstract – In this work we show a class of oscillating configurations for the evolution of the domain walls in Euclidean space. The solutions are obtained analytically. Phase transitions are achieved from the associated fluctuation determinant, by the decay rates of the false vacuum.

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Introduction. – The interest in studying nonlinear systems has gradually grown since the 1950s, due to the fact that nonlinearity is present in many different areas of science, including particle physics, plasma physics, cosmology and field theory, among others. Nonlinear systems, particularly those having solitonic excitations, play a prominent role in the modelling of several physical, chemical and biological systems as well, e.g., in the electric conductivity of organic materials, for which polarons and other polymer chain solitons provide conducting polymers [1]. Moreover, solitons are also related to electric conduction through DNA molecules [2]. In particular, solitons play a fundamental role in field theory, encompassing applications from particle physics to condensed matter. Non-Abelian solitons are specially important in gauge theories of elementary particle physics [3]. Gravitating non-Abelian solitons were firstly discussed in the context of four-dimensional Einstein-Yang-Mills theory [4]. In the context of string theory, supersymmetric solitons further play an important role in the study of the non-perturbative sector and in understanding string dualities. In condensed matter, the homotopy classification of finite energy configurations can be realized by similar forms as can be accomplished in field theory, when the solitary-wave solutions of SU(2) gauge theory are classified likewise.

When a model with a field-dependent potential, which has two or more degenerate minima, is taken into account, different ground states arise at different portions of the space. For instance, the so-called domain walls can be described [5,6], connecting different portions of the space where the field has different values for the potential degenerate minima, even further in asymmetric scenarios [7]. In other words, the field configuration interpolates between the potential minima. Solitons can describe such field configurations presenting a localized and shape-invariant aspect, and have a finite energy density [8]. The presence of those configurations is well understood in a wide class of models, that encompass monopoles, textures, strings and kinks as well [9].

In a seminal work by Coleman, a classical field theory was presented for the study of the false vacuum decay in theories involving asymmetrical potentials by analysing a kind of \(\lambda \phi^4\) asymmetric model. In this theory, the relative minimum corresponds to the false vacuum, whereas the absolute minimum corresponds to the true one. After that, Callan and Coleman presented the associated quantum corrections for the theory [10]. These preliminary results paved the way to an increasing interest in cosmology, providing models where the scalar field potential driving inflation has low energy minima, also known as false vacua. The absolute minimum of the energy density corresponds to the so-called true vacuum state of the Universe.

An important application of this theory is the process that involves phase transitions in statistical mechanics. In this case, inside the false vacuum, there occurs the formation of a bubble of true vacuum which initiates the decay. From a quantum-mechanical point of view, it corresponds to a tunnelling probability from a false vacuum
to the true one. Moreover, the theory of vacuum decay has a considerable physical importance and can include the effects of gravity [11]. In fact, in a cosmological perspective, the early Universe had an extremely high-energy density in a region of false vacuum. As it expands and cools down, it passes through a phase transition towards the true vacuum [12].

A wide class of problems involving phase transitions are supported by the $\phi^4$ theory. However, due to the nonlinearity of this theory it is very rare to analytically approach the problem. Recently an interesting model [13] was proposed, named asymmetrical double-quadratic model. This model is similar to the asymmetric $\lambda \phi^4$ characterized by the following potential:

$$V(\phi) = \frac{1}{2} \phi^2 - |\phi| - c \phi + \frac{1}{2} (\epsilon - 1)^2.$$  \hspace{1cm} (1)

A similar model has been extensively studied in the literature in several contexts. As an example, oscillons configurations [14], which are time-dependent and long-living solutions, were obtained in the presence of the so-called signum-Gordon model [15].

On the other hand, some years ago the Asymmetrical Double-Quadratic (ADQ) model allowed the attainment of oscillating configurations in Minkowski space-time [16]. In fact, oscillating configurations were shown to be responsible for a delay in the transition from a false to a true vacuum. Besides, a connection with phase transitions, which occur in ferromagnetic materials, was presented. In addition, fermions bound states were proposed in a background of static solutions provided from the ADQ model [17].

This paper is organized as follows: in the next section we present the asymmetrical model to be analysed and the classical field configurations. In the third section we obtain the analytical solutions for the scalar field. In the fourth section we calculate the respective decay rates. In the last section we present the conclusions.

**Classical field configurations.** The so-called Generalized Asymmetrical Doubly Quadratic Model (GADQM) was studied in [17], corresponding to a generalized version of the ADQ model. The advantage of the GADQM is that the vacua can be chosen to represent slow-roll potential, yielding inflaton fields which are important in cosmological inflationary scenarios. The authors in [17] considered the problem of fermion boundstates and zero modes in the background of kinks of the GADQM. Motivated by that work, Brito, Correa, and Dutra [18] showed an approach to construct nonlinear systems with analytical multikink profile configurations. A prominent consequence of this work is the possibility that the resulting field configurations can be applied to the study of problems of condensed matter, cosmology, and braneworld scenarios. Thus, in order to employ this model, that, moreover, is exactly solvable, we will study the model whose potential is given by [17,18]

$$V_i(\phi) = \frac{\lambda_i}{2} \left( \left( \phi + \frac{a_i}{2} \right)^2 + b_i \right), \quad -\infty < \phi \leq -a, \quad i = 1, 2, 3, \quad 0 \leq \phi \leq a, \quad i = 4.$$  \hspace{1cm} (2)

where $\lambda_{i+1} = \lambda_i (\frac{a_i^2 + 4b_i}{a_{i+1}^2 + 4b_{i+1}})$ and $i = 1, 2, 3$. Figure 1 depicts this potential, presenting four asymmetric vacua located at $\pm 3a/2, \pm a/2$. Moreover, we can see in fig. 1 that the potential has three false vacua and a true vacuum. Thus, according to the pioneering work of Coleman [12], the false vacua can decay, where phase transitions play an important role in various phenomena. For example, this occurs in the nucleation process of statistical physics [19], in cosmological contexts [20], and in the Weinberg-Salam model [21] as well.

In this case it is possible to have a phase transition between vacua. It is important to remark that a class of configurations in 1, 2 and 3 dimensions, in a similar potential, was presented in refs. [16,18], proposing the existence of kink configurations in the presence of the ADQ model. At this point, it is important to point out that in refs. [16,18] the space-time used by the authors is Minkowskian, and the field configurations correspond...
to the static case. Here we will work in a single-scalar-field theory in four-dimensional space-time, which has the following Euclidean action:

$$S_E[\phi] = \int d^4x \left[ \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi + V(\phi) \right],$$  

(3)

where $V(\phi)$ is given by the potential shown in eqs. (2a)–(2d). Note that the Euclidean theory is left invariant by $O(4)$ transformations. Moreover, the Euclidean system is not similar as the Minkowskian system, since $(x_0^2 - x^2)$ for Minkowski coordinates is invariant, whereas the Euclidean version is provided by $\sum_{i=1}^4(x^i)^2$. Hence, the two systems are different, although the tunnelling processes in field theory are described by the field equations in Euclidean space-time. Now, from the variation of the above equation, the corresponding Euclidean equation of motion is provided by

$$\left( \frac{\partial^2}{\partial \tau^2} + \sum_{i=1}^{\nu} \partial_i \partial_i \right) \phi = \frac{\partial V}{\partial \phi},$$  

(4)

where $N$ represents the number of dimensions. Moreover, as a consequence of the Euclidean theory, we can observe in the above equation an elliptic differential operator, which opens new possibilities to find a different set of real non-singular solutions. For the model to be analyzed, eq. (4) reads

$$\frac{d^2 \phi_j(\rho)}{d\rho^2} + \frac{N d\phi_j(\rho)}{\rho d\rho} - \lambda_j \phi_j(\rho) = \lambda_j A_j, \quad j = 1, \ldots, 4,$$  

(5)

with $\rho = \sqrt{\bar{x}^2 + \tau^2}$, wherein $\bar{x}$ represents the three spatial coordinates and $\tau$ the Euclidean time, $A_1 = 3a/2, A_2 = a/2, A_3 = -a/2$ and $A_4 = -3a/2$.

Now, in order to solve eq. (5), the transformations $\phi_j(\rho) = -A_j + \varphi_j(\rho)$ in the fields $\phi_j$ are performed. Thus, it is not difficult to conclude that

$$\frac{d^2 \varphi_j(\rho)}{d\rho^2} + \frac{N d\varphi_j(\rho)}{\rho d\rho} - \lambda_j \varphi_j(\rho) = 0.$$  

(6)

**Analytical solutions.**— There are many possible ways to solve the above equation, each of them with advantages and disadvantages. We will work with the one which makes possible to find real solutions. Thus, in order to solve the above equation, we choose

$$\varphi_j(\rho) = \rho^n \Phi_j(\rho).$$  

(7)

Hence, the remaining function $\Phi_j(\rho)$ satisfies the Bessel equation

$$\rho^2 \frac{d^2 \Phi_j(\rho)}{d\rho^2} + \rho \frac{d\Phi_j(\rho)}{d\rho} - (\nu^2 + \lambda_j \rho^2) \Phi_j(\rho) = 0,$$  

(8)

where $\nu \equiv (N - 1)/2$. If the variable $Z = \rho \sqrt{\lambda_j}$ is taken into account, it yields the well-known solution

$$\Phi_j(Z) = a_j K_\nu(Z) + b_j I_\nu(Z),$$  

(9)

where $K_\nu(Z)$ and $I_\nu(Z)$ are modified Bessel functions of order $\nu$. Therefore, the complete solution has the form

$$\phi_j(\rho) = -A_j + \rho^n [a_j K_\nu(\rho \sqrt{\lambda_j}) + b_j I_\nu(\rho \sqrt{\lambda_j})].$$  

(10)

Now we search for configurations where four different regions do exist, wherein the field is connected. At this point it is important to remark that the solutions in each region must be continuous through the $\rho$ axis, which leads to $\phi_1(\rho_1) = \phi_2(\rho_1), \phi_2(\rho_2) = \phi_3(\rho_2)$ and $\phi_3(\rho_3) = \phi_4(\rho_3)$. On the other hand, we are looking for the case where $\phi_1(\rho = 0) = -3a/2$ and $\phi_4(\rho \to \infty) = 3a/2$, which results in

$$\phi_1(\rho) = -\frac{3a}{2} + \rho^\nu [a_1 K_\nu(\rho \sqrt{\lambda_j})], \quad 0 \leq \rho \leq \rho_1$$  

(11)

$$\phi_2(\rho) = -\frac{a}{2} + \rho^\nu [a_2 K_\nu(\rho \sqrt{\lambda_j}) + b_2 I_\nu(\rho \sqrt{\lambda_j})], \quad \rho_1 \leq \rho \leq \rho_2$$  

$$\phi_3(\rho) = -\frac{a}{2} + \rho^\nu [a_3 K_\nu(\rho \sqrt{\lambda_j}) + b_3 I_\nu(\rho \sqrt{\lambda_j})], \quad \rho_2 \leq \rho \leq \rho_3$$  

$$\phi_4(\rho) = \frac{3a}{2} + \rho^\nu [a_4 K_\nu(\rho \sqrt{\lambda_j})], \quad \rho_3 \leq \rho \leq \infty.$$  

(12)

To determine the constants $a_j$ and $b_j$ we use the condition of continuity of the function $\phi_1$ and of its derivative, in each point of the region $\rho$. Here, it is worth remarking that these conditions generate a set of six equations which allows the determination of the values of these constants. For instance, we show the profile of the field configuration in fig. 2 for a given set of values of the parameters of the model. Moreover, it is necessary to impose that the first derivative of the field configuration is continuous at the intermediate points. Such a constraint produces transcendental equations which restrict the distances among the roots of the scalar field $\phi$.

**First-order phase transitions.**— As argued in the seminal work by Coleman [12], a field theory with false vacuum allows to understand a large number of phenomena from condensed-matter physics to cosmology. The key for this understanding comes from the decay probability per unit volume, which is named decay rate. In particular, in a cosmological context, we can suppose that when the
Universe was created, it was far from any vacuum state. As it has expanded and cooled down, it first evolved to a false vacuum instead of the true one. Thus, in such a scenario, when the cosmological time elapses, the Universe should finally be settled in the true vacuum state. Therefore, in this section we calculate the decay rate of each phase transition that occurs from the false vacuum to the true vacuum.

In fact, let us denote as usual the two stable static states by \( \phi = \phi_{fv} \) (the false vacuum). Since \( \phi_{fv} \) is unstable, its energy \( E(\phi_{fv}) \) must present an imaginary part responsible for its decay width, as \( \text{Im } E(\phi_{fv}) = -\Gamma(\phi_{fv})/2 \). In a weakly coupling framework, the width \( \Gamma(\phi_{fv}) \) is exponentially small. The energy of the false vacuum reads a non-trivial saddle point, named the bounce, to the decay rate, is determined by the contribution of a non-trivial saddle point, named the bounce \( \phi_B \), of the Euclidean action \( S_E[\phi] \) [10–12].

It is important to remark that the decay rate represents the tunnelling probability from the false vacuum to the true one, per unit time per unit volume \( U \), in such a way that in the semiclassical limit it assumes the form

\[
\Gamma/U = K \exp(-S_E[\phi_{cl}]),
\]

where the prefactor term \( K \), which corresponds to quantum fluctuations with respect to the classical solution at the bounce field, is a coefficient that depends on the detailed form of \( V(\phi) \). Indeed, Coleman and Callan proved that the contribution of the bounce to the energy of the false vacuum reads [10]

\[
E_{cl} = -\frac{U}{2} \left( \frac{S_E[\phi_{cl}]}{4\pi^2} \right) e^{S_E[\phi_{cl}] - \frac{1}{2} \left| \frac{\partial^2}{\partial \phi^2} V''(\phi_{cl}) \right|}.
\]

In the above equation, the prime denotes the derivative with respect to the argument, \( \phi_{fv} \) is the false vacuum, and \( \phi_{cl} \) is the classical solution associated to \( \phi_B \). Furthermore, the ringed determinant in eq. (15) denotes that the zero modes of the operator \( -\partial_\phi \partial^\phi + V'' \), corresponding to the translational invariance about the bounce location, are taken out.

More precisely, the contribution of \( \phi_{fv} \) is determined by a Gaussian integral over the perturbations about the bounce, namely \( \phi(x) = \phi_{fv} + \eta(x) \). Hence (3) yields the fluctuations \( S(\eta) = \int d^4x \frac{1}{2} \eta \left[ -\partial^2 + V''(\phi_{fv}) \right] \eta \). Reference [22] introduces the eigenfunctions \( \eta_\lambda \) as

\[
\left[ -\partial^2 + V''(\phi_{fv}) \right] \eta_\lambda = \lambda \eta_\lambda,
\]

by the splitting \( \eta(x) = \sum_\lambda \eta_\lambda(x) \). Hence the contribution of the false vacuum to the integral \( \int D\phi e^{-S[\phi]} \) reads \( I_0 = \int \prod_\lambda \lambda^{-1/2} \), or equivalently, to \( I_0 = \prod_\lambda \lambda^{-1/2} \), the quantity \( \prod_\lambda \lambda^{-1/2} \) can be interpreted as the determinant of the operator \( -\partial_\phi \partial^\phi + V''(\phi_{fv}) \), since it corresponds to the product of its eigenvalues. Thus, \( I_0 = \det[-\partial^2 + V''(\phi_{fv})] \). Moreover, eq. (14) can also present zero eigenvalues (modes). In fact, the center of the bounce may be located at any point in Euclidean spacetime, leading to the existence of four zero modes around the bounce, determined by \( \eta_\mu = S^{-1/2} \partial_\mu \phi_B(x), \mu = 0,1,2,3 \) [22]. By treating the zero modes in the same way as non-zero modes, then the corresponding integral will diverge [22], which imposes that the zero modes are dismissed from the calculations in the previous formula for \( E_{cl} \), accordingly.

Alternatively, the bounce action \( S_E[\phi_{cl}] \) is eviced on the right-hand side of eq. (15). Hence, in general, the above-mentioned pre-factor reads [23]

\[
K = \frac{S_E[\phi_{cl}]^2}{4\pi^2} \left[ \frac{\det[-\partial^2 + V''(\phi_{cl})]}{\det[-\partial^2 + V''(\phi_{fv})]} \right].
\]

Now, in order to evaluate the decay rate per unit volume and unit time \( \Gamma/U \), firstly we need to find the classical solution \( \phi_{cl} \). Thus, from the Euclidean action (3), we have the following equation of motion:

\[
\left( \frac{d^2}{d \rho^2} + \frac{N}{4} \frac{d}{d \rho} \right) \phi_{cl} - V'(\phi_{cl}) = 0.
\]

Here, for the model here presented by eqs. (2a)–(2d), \( \phi_{cl} \) represents the classical solution for each region. Thus, using eqs. (11), (12) we can obtain the correct form of the Euclidean action, which is fundamental for the decay rate calculation. For simplicity, we can rewrite the Euclidean action in the following compact form:

\[
S_E[\phi_{cl}] = \sum_{j=1}^{4} \frac{\kappa^{(j)}}{j!},
\]

where

\[
S_E^{(j)} = \frac{2\pi^{p+1}}{\Gamma(p+1)} \int_{\rho_0}^{\rho_4} \rho^{p+2} \left[ \frac{1}{2} \left( \frac{d \phi}{d \rho} \right)^2 + V_j(\phi_j) \right] d\rho,
\]

with \( \rho_0 = 0 \) and \( \rho_4 = \infty \). In fact, both the coefficients \( K \) and the exponential factor explicitly depend upon the action. Hence, for the model here analysed, the decay rate is given by the transition from the false vacuum, located at \( \phi_{fv}^{(1)} = -3a/2 \), to the true vacuum, at \( \phi_{fv}^{(4)} = a/2 \). Note that before reaching the point \( \phi_{fv}^{(4)} \), the decay process rolls through the other two metastable potentials, located at \( \phi_{fv}^{(2)} = -a/2 \) and \( \phi_{fv}^{(3)} = a/2 \). Hence, the decay rate from the false vacuum to the true vacuum becomes

\[
\Gamma/U = \sum_{j=1}^{3} (\Gamma_j/U_j),
\]

with the following definition:

\[
\Gamma_j/U_j := \frac{1}{(2\pi)^2} \left( \sum_{n,m=1}^{4} S_E^{(n)} S_E^{(m)} \left[ \frac{\det(M_{ij})}{\det(M_{ij}^0)} \right] \right)^{-1/2} \times \prod_{q=1}^{4} \exp(-\frac{q}{4}),
\]
where we used the relation in eq. (18). Moreover, we employ the fluctuation operator \( M_{(j)} \) in the background of the classical solution and its counterpart \( M_{0}^{(j)} \) in the false vacuum, which are given by

\[
M_{(j)} = -\partial^\mu \partial_\mu + V_{j+1}(\phi^{(j+1)}), \quad M_{0}^{(j)} = -\partial^\mu \partial_\mu + V_{j}(\phi^{(j)}),
\]

(21)

(22)

where \( \phi^{(k)} \rightarrow \phi_{\mu}^{(k)} \), for \( k = 1, 2, 3 \), and \( \phi^{(4)} \rightarrow \phi_{4}^{(4)} \). Now, it is convenient to use the fact that we have the \( O(4) \) spherical symmetry, in such a way that the operators \( M_{(j)} \) and \( M_{0}^{(j)} \) can be decomposed with respect to \( O(4) \) angular momenta. Thus, we can separate the operators into partial waves, which can be written as radial operators in the following form:

\[
M_{(j,l)} = -\partial^2 \rho + \frac{l(l + 2)}{\rho^2} + V_j(\rho),
\]

(23)

\[
M_{0}^{(j,l)} = -\partial^2 \rho + \frac{l(l + 2)}{\rho^2} + V_j(\rho),
\]

(24)

where \( V_j(\rho) \equiv V_j^{(j)}(\phi^{(j)}) \). For simplicity, let us define

\[
G_{(j,l)} \equiv \frac{\det(M_{(j,l)})}{\det(M_{0}^{(j,l)})}.
\]

(25)

In this case, \( G_{(j,l)} \) can be computed by the so-called Gelfand-Yaglom (G-Y) method [24], which efficiently and allows to find the determinant of an ordinary differential operator without necessity of compute its eigenvalues. The G-Y method states that for the radial operators (23) and (24), \( G_{(j,l)} \) can be easily computed in the following form:

\[
G_{(j,l)} = \left[ \lim_{\rho \to \infty} \Psi_{(j,l)}^{(j,l)}(\rho) \right]^{(l+1)^2},
\]

(26)

where \( \Psi_{(j,l)}(\rho) \) and \( \Psi_{0}^{(j,l)}(\rho) \) are regular solutions of the equations

\[
M_{(j,l)} \Psi_{(j,l)}(\rho) = 0, \quad M_{0}^{(j,l)} \Psi_{0}^{(j,l)}(\rho) = 0.
\]

(27)

Here, it is important to highlight that there are three important types of eigenvalue associated to the fluctuation operator. The first is the \( l = 0 \) sector, which has a negative eigenvalue mode of the fluctuation operator. In this case, it is responsible for the instability of the configuration and leads to decay. The second is the \( l = 1 \) sector, where there is a fourfold degenerate zero eigenvalue (zero modes) of the fluctuation operator. From a physical viewpoint, these four zero modes are equivalent to the Goldstone modes, that exist due to breaking of translational invariance. Finally, the third type consists of the \( l \geq 2 \) sectors, with positive eigenvalues. However, we emphasize that in our analysis the zero modes are removed from the fluctuation determinant.

Our next step, in order to compute the decay rate, is to compute the explicit form of the Euclidean action. To accomplish it, it is necessary to determine the integral (18) for each region. Let us then write \( \rho = \zeta/\sqrt{\xi} \), in such a way that eq. (10) reads

\[
\phi_j(\zeta) = -a_j + \zeta^2 \bar{a}_j K_{\nu}(\zeta) + \bar{b}_j I_{\nu}(\zeta),
\]

(28)

where \( \bar{a}_j \equiv a_j/\sqrt{\xi} \), and \( \bar{b}_j \equiv b_j/\sqrt{\xi} \). In this case, eq. (18) can be rewritten as

\[
S_E^{(j)} = d_{(j,\nu)} \int_{\zeta_0/(\sqrt{\lambda_j})}^{\zeta_1/(\sqrt{\lambda_j})} \frac{1}{2} \left( \frac{d\phi_j}{d\zeta} \right)^2 + V_j(\phi_j(\zeta)) \, d\zeta,
\]

(29)

with the following redefinitions:

\[
d_{(j,\nu)} := \frac{2\pi \nu + 1}{\lambda_j^2(\nu + 1)},
\]

(30)

\[
V_j(\phi_j(\zeta)) := \frac{\lambda_j}{2} \left( \phi_j(\zeta) + G_j \right)^2 \left[ \phi_j(\zeta) + G_j \right],
\]

(31)

where \( G_j \) corresponds to the value in each vacuum. In addition, the derivative \( d\phi_j/d\zeta \) is crucial in our calculation, yielding

\[
\frac{d\phi_j}{d\zeta} = \zeta^2 [-\bar{a}_j K_{\nu-1}(\zeta) + \bar{b}_j I_{\nu-1}(\zeta)].
\]

(32)

Other important results to find the complete Euclidean action form are the integrals

\[
\int d\zeta \zeta^m K_{\nu}(\zeta) = 2^{-(\nu+2)} \pi^{m+1} \csc(\pi \nu)
\]

\[
\times \left[ \frac{\Gamma \left( \frac{m - \nu + 1}{2} \right) - \Gamma^2 \left( \frac{m + \nu + 1}{2} \right)}{\Gamma \left( \frac{m + \nu + 1}{2} \right)} \right] \sum_{l=1}^{2} \mathcal{H}_{RZ} \left( \{Q_l\}, \{R_l, T_l\}, \frac{\zeta^2}{4} \right),
\]

(33)

\[
\int d\zeta \zeta^m I_{\nu}(\zeta) = 2^{-(\nu+2)} \pi^{m+1} \Gamma \left( \frac{m + \nu + 1}{2} \right)
\]

\[
\times \mathcal{H}_{RZ} \left( \{Q_2\}, \{R_2, T_2\}, \frac{\zeta^2}{4} \right),
\]

(34)

where \( Q_l \equiv \frac{m + (-\nu)^l + 1}{2} \), \( R_l \equiv 1 + (-\nu)^l \), \( T_l = Q_l + 1 \).

Furthermore, the function \( \mathcal{H}_{RZ} \left( \{Q_l\}, \{R_l, T_l\}, \zeta^2/4 \right) \) is the regularized generalized hypergeometric function and \( \Gamma(X) \) is the gamma function. Thus, by using the above relations and the usual procedure presented in [23,25], we can obtain, after straightforward calculations, the value of the fluctuation determinant (20) in each region, and as a consequence the decay rates. In this case, the transition rates can be determined from a straightforward example. For \( \nu = 1 \), \( \lambda_1 = 1 \), \( a = 5 \), \( b_1 = 1 \), \( b_2 = 0.5 \), \( b_3 = 0.1 \), \( b_4 = 0 \), \( \rho_1 = 10^{-3} \), \( \rho_2 = 10^{-2} \) and \( \rho_3 = 10^{-1} \), we find that

\[
\Gamma_1/V_1 \sim 10^{-25},
\]

(35)

\[
\Gamma_2/V_2 \sim 10^{-14},
\]

(36)

\[
\Gamma_3/V_3 \sim 10^{-5}.
\]

(37)
At this point, it is important to remark that here the advantage is that in each region the model is exactly solvable. Hence the decay rate was obtained in an analytical form. Another important consequence is that the approach used can be applied to study the phase transitions in models where we have piecewise potentials.

Concluding remarks and outlook. — We investigated a class of oscillating configurations for the evolution of the domain walls in Euclidean space. We found in an analytical form the configurations of the field and the decay rate of the false vacuum.

Based on cosmological numerical values we can further incorporate a better description of the phase transitions, as follows. In the cosmological context, the Universe is known to have different dominant dynamical components since its origin, e.g., from radiation and relativistic matter at early time, to late-time dark-energy component, passing through the matter-dominated scenario. In further work we intend to physically interpret the vacuum decays of our model as the transition of different dynamical eras of the Universe, i.e., the transition from a radiation to a matter-dominated scenario will be analysed merely as a transition from a false to a true vacuum and the same will happen for the other dynamical transitions of the Universe. To accomplish it, we will need the values for $|\vec{x}|$ and $\tau$ for each transition era of the Universe.

From fig. 2, it is possible to perceive our argumentation. One can interpret each of the regions I, II, III and IV as the different dynamical eras of the Universe. While regions II, III and IV would stand for the radiation, matter and dark-energy–dominated eras, the region I could account for the inflationary era (check, for instance [26–29]). Note that the scalar-field evolution also makes one able to predict the Universe fate, through a deep analysis on region IV. Such an analysis may, for example, predict the Universe to expand forever, yielding a Big-Freeze model [30].

The transition rates obtained offer a further interpretation for the dynamical evolution of the Universe. In fact, even if the state of the early Universe was cold enough not to provide a thermal transition to the true vacuum state, a quantum decay from the false vacuum to the true vacuum may still be possible through the barrier mechanism.

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