Calculation of the Higgs boson mass using the complementarity principle

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Abstract

We compute the Higgs mass in a model for the electroweak interactions based on a confining theory. This model is related to the standard model by the complementarity principle. A dynamical effect due to the large typical scale of the Higgs boson shifts its mass above that of the $W$-bosons. We obtain $m_H = 129.6$ GeV.

Recently we have proposed a model for the electroweak interactions based on a confining $SU(2)$ theory [1]. It was shown that, at least at low energies, this model is complementary or dual to the electroweak standard model [2]. The complementarity principle states that there is no phase transition between the Higgs and the confinement phase if there is a Higgs boson in the fundamental representation of the gauge group [3,4]. The Lagrangian of the theory under consideration is exactly that of the standard model before gauge symmetry breaking. However, the sign of the Higgs boson squared mass is changed, i.e., it is positive, and the gauge symmetry is thus unbroken. We have the following fundamental left-handed dual-quark doublets, which we denote as $D$-quarks:

leptonic $D$-quarks

\[
\bar{l}_i = \begin{pmatrix} l_1 \\ l_2 \end{pmatrix} \quad (\text{spin } 1/2, \text{ left-handed})
\]

hadronic $D$-quarks

\[
\bar{q}_i = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \quad (\text{spin } 1/2, \text{ left-handed}, SU(3)_c \text{ triplet})
\]

scalar $D$-quarks

\[
\bar{h}_i = \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \quad (\text{spin } 0)
\]

The right-handed particles are those of the standard model. As the gauge symmetry is unbroken, physical particles must be singlets under $SU(2)$ transformation, and we thus get the following particle spectrum

\[
\nu_L = \frac{1}{F} (\bar{h} l) = l_1 + \mathcal{O}\left(\frac{1}{F}\right) \approx l_1.
\]
\[ e_L = \frac{1}{F} (\epsilon_{ij} h_i l_j) = l_2 + O \left( \frac{1}{F} \right) \approx l_2, \]

\[ u_L = \frac{1}{F} (\bar{h} q) = q_1 + O \left( \frac{1}{F} \right) \approx q_1, \]

\[ d_L = \frac{1}{F} (\epsilon_{ij} h_i q_j) = q_2 + O \left( \frac{1}{F} \right) \approx q_2, \]

\[ H = \frac{1}{2F} (\bar{h} h) = h_1 + \frac{F}{2} + O \left( \frac{1}{2F} \right) \approx h_1 + \frac{F}{2}, \]

\[ W^2 = \frac{2i}{gF^2} (\bar{h} D_{\mu} h) = B^{3,2}_\mu + O \left( \frac{2}{F} \right) \approx B^{3,2}_\mu, \]

\[ W^- = \frac{\sqrt{2} i}{gF^2} (\epsilon^{ij} h_i D_{\mu} h_j) \approx B^{-}_\mu + O \left( \frac{2}{F} \right) \approx B^{-}_\mu, \]

\[ W^+ = \left( \frac{\sqrt{2} i}{gF^2} (\epsilon^{ij} h_i D_{\mu} h_j) \right)^\dagger \]

\[ = B^+_\mu + O \left( \frac{2}{F} \right) \approx B^+_\mu. \]

where \( g \) is the coupling constant of the gauge group SU(2)_L and \( D_{\mu} \) is the corresponding covariant derivative. We have used the unitary gauge

\[ h_i = \left( \begin{array}{cc} F + h_{1(1)} & \\
0 & \end{array} \right). \]

where \( F \) is the parameter appearing in the expansion of the bound states (1). Matching the expansion of the Higgs boson to the standard model we get \( F = 492 \) GeV [1]. Using this expansion, we can associate a certain scale, which is proportional to \( F \), to each particle. The scale of the W-bosons is then \( \Lambda_W = \sqrt{2} F/4 = 173.9 \) GeV. As can be seen from the expansion for the Higgs boson (1), there is a factor four between the expansion parameter of the Higgs boson and that of the W-bosons, thus one finds \( \Lambda_H = \sqrt{2} F = 695.8 \) GeV. This factor four is dictated by the algebraic structure of the underlying gauge theory.

In the confinement phase the Higgs boson is the s-wave of the SU(2) theory, whereas the W-bosons are the corresponding p-waves. Thus one expects the Higgs-boson to be lighter than the W-bosons. But, as we shall show, a dynamical effect shifts the Higgs-boson mass above that of the W-bosons mass. The reason for this phenomenon is the large Higgs-boson scale compared to that of the W-bosons.

The masses of the physical Higgs- and W-bosons, being bound states consist of a constituent mass \( m^0_H = m_W^0 = 2m_h \), where \( m_h \) is the mass of the scalar D-quark and of dynamical contributions. We have to consider two types of diagrams: the one-particle reducible diagrams (1PR) and the one-particle irreducible diagrams (1PI). For the Higgs-boson mass, we have to take the self-interaction and the contribution of the Z- and W+-bosons into account (see Fig. 1). The fermions couple via Yukawa coupling to the Higgs boson, and as this interaction is not confining, fermions cannot contribute to the dynamical mass of the Higgs boson.

The first task is to extract the constituent mass from the experimentally measured W-bosons mass. The fermions contribute to the dynamical mass of the W-bosons as they couple via SU(2) couplings to the electroweak bosons but the divergence is only logarithmic [5] and we shall only keep the quadratic divergences. We have considered the tadpoles and the one-particle-irreducible contributions at the one loop order (the diagrams contributing to the W-bosons mass are similar to those contributing to the Higgs-boson mass). Using the duality described in [1], these duality diagrams can be related to the Feynman graphs of Fig. 2. The Feynman graphs have been evaluated.

![Fig. 1. (a), (b) dual diagram: one loop 1PI contribution to \( m_H \); (c) dual diagram: one loop 1PR contribution to \( m_H \).](image-url)
in Ref. [5] as a function of a cut-off parameter and we will only keep the dominant contribution which is quadratically divergent. We obtain:

\[ m_W^2 = (m_0^W)^2 + \frac{3g^2 A_W^2}{32\pi^2 m_W^2} (m_H^2 + 2m_W^2 + m_Z^2). \] (3)

This equation can be solved for \( m_0^W \):

\[ (m_0^W)^2 = m_W^2 - \frac{3g^2 A_W^2}{32\pi^2 m_W^2} (m_H^2 + 2m_W^2 + m_Z^2). \] (4)

We can now compute the dynamical contribution to the Higgs-boson mass. The exact one loop, gauge invariant counterterm has been calculated in Refs. [5–7]. Using the results of Ref. [7], where this counterterm was calculated as a function of a cut-off, we obtain:

\[ m_H^2 = (m_0^H)^2 (m_H^2) + \frac{3g^2 A_H^2}{32\pi^2 m_W^2} (m_H^2 + 2m_W^2 + m_Z^2) \]

\[ + \frac{3g^2 m_H^2}{64\pi^2 m_W^2} \left( m_H^2 \ln \frac{A_H^2}{m_H^2} - 2m_W^2 \ln \frac{A_W^2}{m_W^2} \right) - m_Z^2 \ln \frac{m_Z^2}{m_W^2}. \] (5)

The unknown of this equation is the Higgs boson’s mass \( m_H \). This equation can be solved by numerical means. We obtain two positive solutions: \( m_{H1} = 14.1 \text{ GeV} \) and \( m_{H2} = 129.6 \text{ GeV} \). The first solution yields an imaginary constituent mass and is thus also discarded. The second solution is the physical Higgs-boson mass. We thus obtain \( m_H = 129.6 \text{ GeV} \) in the one loop approximation. The constituent mass is then \( m_0^W = 78.8 \text{ GeV} \).

As expected the dynamical contribution to the \( W \)-bosons masses is small and the Higgs-boson mass is shifted above that of that of the \( W \)-bosons mass because of the large Higgs-boson scale.

Note that our prediction \( m_H = 129.6 \text{ GeV} \) is in good agreement with the requirement of vacuum stability in the standard model which requires the mass of the Higgs boson to be in the range 130 to 180 GeV if the standard model is to be valid up to a high energy scale [8]. We can thus deduce that the duality we have described [1] must also be valid up to some high energy scale. Our result is also in good agreement with the expectation \( m_H = 98^{+58}_{-38} \) GeV based on electroweak fits [9].

This has also consequences for the model proposed in [10], where we assumed that the Higgs boson does not couple to \( b \)-quarks. Because of the dynamical effect we have discussed, the Higgs boson is relatively heavy and should decay predominantly into electroweak bosons.

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