THE “SPIN” STRUCTURE OF THE NUCLEON*
– A LATTICE INVESTIGATION

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ABSTRACT

We will discuss here an indirect lattice evaluation of the baryon axial singlet current matrix element. This quantity may be related to the fraction of nucleon spin carried by the quarks. The appropriate structure function has recently been measured (EMC experiment). As in this experiment, we find that the quarks do not appear to carry a large component of the nucleon spin.

1. Introduction and Theoretical Discussion

Hadrons appear to be far more complicated than the (rather successful) constituent quark model would suggest. For example an old result is from the πN sigma term, which seems to give a rather large strange component to the nucleon mass. A more recent result is from the EMC experiment which suggests that constituent quarks are responsible for very little of the nucleon spin. These are non-perturbative effects and so is an area where lattice calculations may be of some help.

The EMC experiment measured deep inelastic scattering (DIS) using a µ beam on a proton target. The new element in the experiment was that both the initial µ and proton were longitudinally polarised. A measurement of the difference in the cross sections for parallel and anti-parallel polarised protons enabled the structure function $g_1(x, Q^2)$ to be found. Theoretically this is of interest as using the Wilson operator product expansion, moments of the structure function are related to certain matrix elements. In this case, defining $s^{\mu}Aq = \langle Ps|\bar{q}\gamma^{\mu}\gamma_5q|Ps\rangle$ for $q = u, d$ or $s$ quarks (i.e. the expectation value of the axial singlet current) the lowest moment is then given by

$$\int_0^1 dx g_1(x, Q^2) = \frac{1}{2} \left( \frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right)$$

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\[
Q_{EMC}^2 \approx 11 \text{ GeV}^2.
\]

Results from neutron and hyperon decays have been used to eliminate two of the unknowns on the RHS of this equation.

\[\Delta q\] can be given a physical interpretation, as the quark spin operator, \(\hat{S}_{\text{quark}}^i\), is local and gauge invariant (in distinction to the gluon spin operator or orbital angular momentum operator) and leads to

\[
2\langle Ps | \hat{S}_{\text{quark}}^i | Ps \rangle \equiv \Delta \Sigma = \Delta u + \Delta d + \Delta s 
\approx 0.68 + 3\Delta s.
\]

Before the EMC experiment, the Ellis-Jaffe sum rule was applied: the strange content of the proton was taken as zero, i.e. \(\Delta s = 0\). However, the EMC experiment gave for the LHS of eq. (1) \(\approx 0.126\) giving \(\Delta s \approx -0.2\), a large negative strange contribution to the proton and \(\Sigma \approx 0\), i.e. the total quark spin is zero. This situation is often referred to as “the spin crisis of the quark model”, although what was actually measured was the nucleon matrix element of the axial singlet current.

2. The lattice calculation

We now turn to our lattice calculation. By working in euclidean space on a lattice we can turn our problem into a statistical mechanics one, which can be approached numerically via the calculation of correlation functions using Monte Carlo techniques. We have generated configurations using dynamical staggered fermions \((\chi)\) on a \(16^3 \times 24\) lattice at \(\beta = 5.35\), \(m = 0.01\), which corresponds to a quark mass of about 35MeV. Staggered fermions describe 4 degenerate quark flavours, so we do not have quite the physical situation that we wish to describe (i.e. 2 light quarks and one heavier quark). Nevertheless, this discretisation of the fermions has some advantages – most notably good chiral properties (as \(m \to 0\)) when the chiral symmetry is spontaneously broken and there is a Goldstone boson, the \(\pi\).

The general procedure on the lattice to measure matrix elements is to consider 3-point correlation functions. For an arbitrary operator \(\Omega\), it can then be shown that \((B\) is the nucleon operator)\n
\[
C(t; \tau) = \langle B_{\bar{p},\bar{s}}(t) \Omega(\tau) B^\dagger_{p',\bar{s}'}(0) \rangle = \sum_{\alpha,\beta = N,\Lambda} A_{\alpha\beta} \langle a | \hat{\Omega} | b \rangle \mu_{\alpha}^{t-\tau} \mu_{\beta}',
\]

for \(\frac{1}{2}T \gg t \gg \tau \gg 0\), where \(T\) is the time box size = 24 here. \(\mu_N = + \exp(-E_N)\) and the parity partner (which always occurs when using staggered fermions) is denoted by \(\Lambda\), so that \(\mu_{\Lambda} = - \exp(-E_{\Lambda})\). The amplitudes \(A_{\alpha\beta}\) and energies \(E_{\alpha}\) are known from 2-point correlation functions. Thus from eq. (4) we can extract \(\langle Ps | \Omega | Ps \rangle\). Simply setting \(\Omega\) to be \(\bar{\chi}\gamma_i\gamma_5\chi\) has, however, a number of disadvantages:
the correlation function in eq. (3) has connected and disconnected parts, in the fit there are cross terms present, and the operator must be renormalised. Another approach is to consider the divergence of the current. This can be related to the QCD anomaly. In the chiral limit an equivalent formulation of the problem is thus \((n_f = 4)\)

\[
\Delta \Sigma = \lim_{\vec{p}' \to \vec{p}} \frac{n_f}{i(\vec{p} - \vec{p}') \cdot \vec{s}} \langle P(\vec{p}), \vec{s}|\hat{Q}(\vec{p} - \vec{p}')]|P(\vec{p}'), \vec{s}'\rangle,
\]

with topological charge

\[
\hat{Q}(\vec{p}) = \frac{1}{8\pi^2} \int d^3 \vec{x} F_{\mu\nu} \tilde{F}^{\mu\nu} e^{i\vec{p} \cdot \vec{x}}.
\]

(We shall take \(\vec{p}' = \vec{0}, \vec{p} = (0, 0, p)\).) On the lattice we have used the Lüscherg construction for this charge. Although technically complicated it is integer valued and hence has no renormalisation. In the 3-point correlation function there are no cross terms (\(Q\) has a definite parity). A disadvantage is that we need to take \(p \to 0\). On our lattice the smallest momentum available is the rather large \(p \approx 500\) MeV.

Nevertheless on attempting this measurement, we find a reasonable signal with \(\Delta \Sigma \approx 0.18(2)\). This is small and tends to support the EMC result, which would indicate a rather large sea contribution to the proton. The existence of the QCD anomaly also proved important. Further details of our calculation are given in 8.

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4. References

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