Position Resolution of a Double Junction Superconductive Detector Based on a Single Material

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Abstract. The Naples group from Istituto Nazionale di Fisica Nucleare presented the results of theoretical investigations of a new class of superconductive radiation detectors – double junction superconductive detector based on a single material [1]. In such detectors, the absorption of energy occurs in a long superconductive strip while two superconductive tunnel junctions positioned at the ends of the strip provide the readout of the signals. The main peculiarity of this type of detectors is that they are based on a single superconducting material, i.e., without trapping layers at the ends of the strip. In this paper, general approach to the position resolution of this type of detectors has been attempted. The formula for the position resolution is derived. It is shown that the application of the aluminium for the absorber may be the best possible way not only due to the small gap energy, but also mainly for STJ fabrication technology based on the aluminium oxide tunnel barrier.

1. Introduction
Superconducting single photon radiation detectors are attractive devices for energy-resolved detection of individual photons or particles over a broad energy range with high counting rates. In imaging detectors with a large detecting area, one way to reduce the number of channels that has to be readout is in using DROIDs (Distributed Read Out Imaging Device) [2]. DROIDs provide rather a good position and energy resolution. In a DROID radiation is absorbed in a long and narrow strip, which is as long as possible to cover the largest area. Originally, DROIDs were designed by using a higher gap superconductor for the absorber and a lower gap superconductor for the STJs electrodes to exploit the quasiparticle trapping and the multiplication effects. Excess quasiparticles produced in the absorber by an incident photon diffuse toward the opposite sides where they are trapped and counted by the two superconductive tunnel junctions (STJ). Two STJ signals provide possibility of determining the incident particle absorption point.

As, the presence of traps is not a fundamental ingredient of the DROID, the Naples group from Istituto Nazionale DI Fisica Nucleare proposed to investigate a new class of superconductive radiation detectors – double junction superconductive detector based on a single material [1]. This class of DROID is not affected by the proximity effect, and can be made of the material with a low energy gap. The Naples group proposed to use the aluminum for this type of DROIDs because of STJ fabrication technology based on the aluminium oxide tunnel barrier.
2. Relationship between position and energy resolution

In this paper, the factors affecting on the position resolution of DROID’s based on a single material is analyzed. In a DROID the total charge collected by the two STJs is given by the formula:

\[ Q = Q_1 + Q_2, \]

where \( Q_1 \) and \( Q_2 \) are the charges counted by the left and the right STJs. Thus, the total collected charge is proportional to the incident particle energy. It was analyzed in [3] that the best analytic expression for estimation of the incident particle coordinate was derived in [2]:

\[ x = \frac{L}{2\alpha} \ln \left( \frac{Q_1 \exp(-\alpha/2) + Q_2 \exp(\alpha/2)}{Q_1 \exp(\alpha/2) + Q_2 \exp(-\alpha/2)} \right), \]

where \( L \) is the length of the superconductive strip. The x-axis with the origin in the midst goes from the left to the right STJ. The dimensionless parameter \( \frac{\Lambda}{L} = \frac{D_{\tau_{\text{loss}}}}{\tau_{\text{loss}}} \) measures the absorber length relative to the quasiparticle diffusion length \( \Lambda = \sqrt{D\tau_{\text{loss}}} \); \( D \) and \( 1/\tau_{\text{loss}} \) are the quasiparticle diffusion constant and the loss rate. This formula is rather simple and account for the principal processes, which govern the signal of the STJs that are in our case diffusion and quasiparticle losses. It is also easy to use this formula in on-line data handling.

In [3], it was shown that from the equation (1) and (2) it follows that the mean square deviation of the incident particle coordinate, with accounting for correlation between fluctuations of STJs signals, has the form:

\[ \sigma_x = \frac{L}{\alpha} \sinh \alpha \left( \frac{p\sigma_1^2 + q\sigma_2^2 - pq\sigma_Q^2}{Q(p^2 + q^2 + 2pq \cosh \alpha)} \right)^{1/2}, \]

where \( p = Q_2/Q \), \( q = Q_1/Q \) are branching ratio parameters; \( \sigma_1^2 \), \( \sigma_2^2 \) and \( \sigma_Q^2 \) are the variances of the collected charges by the left and the right STJs, and the variance of the total collected charge.

For our purposes, equation (3) can be rewritten as follows:

\[ \sigma_x = \frac{L}{\alpha} \sinh \alpha \sqrt{pq} \left( \frac{(q\eta_1^2 + p\eta_2^2 - \eta_Q^2)}{(p^2 + q^2 + 2pq \cosh \alpha)} \right)^{1/2}, \]

where \( \eta_1^2 = \sigma_1^2/\sigma_Q^2 \), \( \eta_2^2 = \sigma_2^2/\sigma_Q^2 \) and \( \eta_Q^2 = \sigma_Q^2/\sigma_Q^2 \) are the relative variances of the collected charges.

3. Energy resolution of DROID’s STJs

As the process of signal formation at the output of the DROID’s STJs detectors represents a random branching cascade process, so the formalism of generating functions (GF) is the most adequate for its formulation. For the process of registration by DROID’s STJs monoenergetic particles incident at the coordinate \( x \) on the absorber, the GF of takes the form:

\[ f_Q(x, s_1, s_2) = f_N \left[ 1 - \sum_{i=1}^{2} \tau_i(x) + \sum_{i=1}^{2} \tau_i(x) f_{m_i}([s_i]) \right] \prod_{i=1}^{2} f_{\text{noise}}([s_i]), \]

where \( s_i \) \((i = 1,2)\) are the auxiliary variables of the generating function for two STJ signals; \( f_N[s] \) is the GF of the number of nonequilibrium quasiparticles produced by the incident particle; \( \tau_i(x) \) \((i = 1,2)\) is the probability that a quasielectron produced in the absorber at coordinate \( x \) after the process of diffusion reaches the left or the right STJ.

In the formula (5) \( f_{m_i}([s_i]) \) \((i = 1,2)\) is the GF of the quasiparticle multitunneling through the insulating barrier of the left or the right STJ with the mean value and the variance of the number of multitunneling [4]:
where \( t_1 \) and \( t_2 \) are the tunneling probabilities in \( S_1/S_2 \) structure through the insulating barrier \( I \) from \( S_1 \) layer to \( S_2 \) layer, and back. In particular, for the DROID based on a single material \( S_1 \) superconductor is the absorber. If the backtunneling from \( S_2 \) layer to \( S_1 \) layer is completely suppressed, i.e. \( t_2 = 0 \), then obviously \( < m >= t_1 \) and \( \eta_m^2 = (1 - t_1)/t_1 \).

In the formula (5) \( f_{\text{noise}_i} \) \( [\delta] \) \( (i = 1,2) \) are the GF of the electronic tract noise of each STJ reduced to the amplifier input with the zero mean value and the variance \( \sigma^2_{\text{noise}_i} \). Thus, the amplification factor of the electronic tract is equal to unit.

From the GF (5), the expressions for the mean values and the variances of STJ’s output signals and the expressions for the mean value and the relative variance of the total collected charge are given by:

\[
< Q_{i,2}(x) >= < N > \tau_{i,2}(x) < m_{i,2} > ,
\]

\[
\eta_{i,2}^2(x) = \eta_N^2 - \frac{1}{< N >} \frac{\tau_{i,2}(x) < m_{i,2} >^2 (\eta_{m1,2}^2 + 1) + \sigma^2_{\text{noise}_{i,2}}}{< N > (\tau_{i,2}(x) < m_{i,2} >)^2},
\]

\[
< Q(x) >= < N > \sum_{i=1}^{2} \tau_i(x) < m_i > ,
\]

\[
\eta_Q^2(x) = \eta_N^2 - \frac{1}{< N >} \frac{\sum_{i=1}^{2} \tau_i(x) < m_i >^2 (\eta_{m1}^2 + 1) + \sum_{i=1}^{2} \sigma^2_{\text{noise}_i}}{< N > (\sum_{i=1}^{2} \tau_i(x) < m_i >)^2},
\]

where \( < N > = E/\varepsilon_{\text{eff}} \) and \( \eta_N^2 = F/ < N > \) are the mean value and the relative variance of the number of the excess quasielectrons produced by the incident particle of energy \( E \); \( \varepsilon_{\text{eff}} \) is the effective energy of quasielectron creation; \( F \) is the Fano factor. All collected charges are in units of the electron charge.

4. Position resolution of DROIDs

Taking into account, that in our case charge branching parameters express through the introduced quantities as

\[
p = \tau_2(x) < m_2 > / \sum_{i=1}^{2} \tau_i(x) < m_i > ,
\]

\[
q = \tau_1(x) < m_1 > / \sum_{i=1}^{2} \tau_i(x) < m_i > ,
\]

the numerator of the equation (4) can be rewritten as follows:

\[
q \eta_1^2 + p \eta_2^2 - \eta_Q^2 =
\]

\[
\frac{[\tau_1(x) (\eta_{m2}^2 + 1) + \tau_2(x) (\eta_{m1}^2 + 1)] < m_1 > < m_2 >}{< N > (\tau_1(x) < m_1 > + \tau_2(x) < m_2 >)^2} +
\]

\[
\frac{\tau_1^2(x) < m_1 >^2 \sigma^2_{\text{noise}_2} + \tau_2^2(x) < m_2 >^2 \sigma^2_{\text{noise}_1}}{< N >^2 (\tau_1(x) < m_1 > + \tau_2(x) < m_2 >)^2 \tau_2(x) < m_2 > \tau_1(x) < m_1 >}.
\]
Finally, the dependence of the mean square deviation of the incident particle coordinate on the branching ratio parameter $p$ has the form:

$$\sigma_x(p) = L \sinh \alpha \frac{1}{\alpha \left[ 1 + 4 p(1 - p) \sinh(\alpha/2) \right]} \cdot \left\{ \frac{p(1 - p)}{Q} \left[ m_1 > \left( \eta_{m1}^2 + 1 \right) \right] + \frac{p^2 \sigma_{\text{noise}1}^2 + (1 - p)^2 \sigma_{\text{noise}2}^2}{Q^2} \right\}^{1/2}, (15)$$

It is necessary to emphasize that because of the correlation between fluctuations of the STJs signals the position resolution of a DROID does not depend on the fluctuations of the number of quasielectrons, i.e. on the Fano factor.

To facilitate the extraction of the position resolution dependence on absorber material characteristics, let us consider a symmetric DROID. In the case of symmetric DROID, $< m_1 >= < m_2 > = m$, $\eta_{m1}^2 = \eta_{m2}^2 = m^2$, $\sigma_{\text{noise}1}^2 = \sigma_{\text{noise}2}^2 = \sigma_{\text{noise}}^2$, and the mean square deviation of the incident particle coordinate can be rewritten as follows:

$$\sigma_x(p) = L \sinh \alpha \frac{1}{\alpha \left[ 1 + 4 p(1 - p) \sinh(\alpha/2) \right]} \cdot \left\{ \frac{p(1 - p)}{Q} \left[ m > \left( \eta_{m}^2 + 1 \right) \right] + \frac{\sigma_{\text{noise}}^2 (1 - 2 p(1 - p))}{Q^2} \right\}^{1/2}, (16)$$

If there is no multitunneling, i.e. $t_1 = 0$, then from (6) and (7) it follows that $< m > (\eta_{m}^2 + 1) = 1$.

In this case

$$\sigma_x(p) = L \sinh \alpha \frac{1}{\alpha \left[ 1 + 4 p(1 - p) \sinh(\alpha/2) \right]} \cdot \left\{ \frac{p(1 - p)}{Q} + \frac{\sigma_{\text{noise}}^2 (1 - 2 p(1 - p))}{Q^2} \right\}^{1/2}. (17)$$

When the quasiparticles absorption is negligible, i.e. $\alpha << 1$, the formula (2) reduces to

$$x = \frac{L Q_1 - Q_2}{2 Q_1 + Q_2}, (18)$$

and the formula (17) reduces to

$$\sigma_x(p) = L \left\{ \frac{p(1 - p)}{Q} + \frac{\sigma_{\text{noise}}^2 (1 - 2 p(1 - p))}{Q^2} \right\}^{1/2}. (19)$$

For a symmetric DROID with negligible absorption losses, the minimum of the mean square deviation of the incident particle coordinate is at $x = 0$, i.e. for $p = 1/2$. The minimum of the mean square deviation of the incident particle coordinate is

$$\sigma_x(1/2) = L \left\{ \frac{1}{4Q} + \frac{\sigma_{\text{noise}}^2}{2Q^2} \right\}^{1/2}. (20)$$

It is necessary to remind that in all formulae the total collected charge is in units of the electron charge. Therefore, the expression in brackets in (20) is dimensionless.

As $1/Q \propto \Delta/E$, where $\Delta$ is the gap energy of absorber and $E$ is the incident particle energy, so the absorber with the smallest gap energy must realize the best position resolution of DROIDs. Use the aluminum for the absorber may be the best possible way not only due to the small gap energy, but also mainly for STJ fabrication technology based on the aluminum oxide tunnel barrier.
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