Radio impulsive events in quiet solar corona and Axion Quark Nugget Dark Matter

Shuailiang Ge,∗ Md Shahriar Rahim Siddiqui,† Ludovic Van Waerbeke,‡ and Ariel Zhitnitsky§
Department of Physics and Astronomy, University of British Columbia, Vancouver, V6T 1Z1, BC, Canada

The Murchison Widefield Array (MWA) has recorded [1] impulsive radio events in the quiet solar corona at frequencies 98, 120, 132, and 160 MHz. We propose that these radio events represent the direct manifestation of the dark matter annihilation events within the so-called axion quark nugget (AQN) framework. It has been previously argued that the AQN annihilation events in the quiet solar corona [2, 3] can be identified with nanoflares originally conjectured by Parker long ago [4]. In the present work we further support this claim by demonstrating that the radio observations [1], including the frequency of appearance, temporal and spatial distributions, energetics, and other related observables are nicely matching the generic consequences of the AQN annihilation events in the quiet corona. We propose to test these ideas by analyzing the correlated clustering radio impulsive events in the different frequency bands. We also make generic predictions for low (80 and 89) MHz and high (179, 196, 217 and 240) MHz frequency bands which had been already recorded but not published by [1] yet. We also suggest to test this proposal by studying possible cross-correlation between MWA radio signals and Solar Orbiter recording of the extreme ultraviolet photons (coined as the “campfires”) to support or refute this proposal.

I. INTRODUCTION

In this work we discuss two naively unrelated stories. The first one is motivated by the recent studies [1] of the impulsive radio events observed in the quiet solar corona at frequencies 98, 120, 132, and 160 MHz, carried out by the Murchison Widefield Array (MWA). The second one is the study of a specific dark matter model, the so-called axion quark nugget (AQN) dark matter model [5] and its application to the solar corona heating [2, 3]. The basic reason how and why these two stories are linked one another represents the topic of the present work.

The short answer is that the impulsive radio events observed in [1] appear to have all the features normally attributed to the so-called nanoflares which were originally conjectured by Parker [4] as a possible resolution of the corona heating mystery [6]. On the other hand, the AQNs entering the Sun’s corona lead to impulsive energy injection events which can potentially provide the proper amount of energy needed to heat the corona. This numerical coincidence was one of the main reasons to identify the AQN annihilation events with the nanoflares [2, 3]. We will show that the annihilation events proposed in [2, 3] share many features with the impulsive radio signals observed by [1] in terms of rate of appearance, temporal and spatial distributions, energetics, and other related observables. Furthermore, most of the AQN-annihilation energy will be released in the so-called transition region at the altitude around 2000 km, which is known to be the most puzzling region of the solar corona where the temperature and the density experience a dramatic change across a thin layer of the order of 100 km thickness, or less.

We now elaborate on each side of these linked stories. We start with an overview of the solar corona heating puzzle: the photosphere is in thermal equilibrium at ∼ 5800 K, while the corona has a temperature of a few 10⁶ K [6]. Physically, this high temperature corresponds an energy excess of a few 10⁴⁷ erg s⁻¹ that is mostly observed in the extreme ultraviolet (EUV) and soft X-ray bands. The conventional view is that the corona excess heating is explained by nanoflares, a concept originally invented by Parker [4]. The short energy bursts associated with these nanoflares are significantly below detection limits and have not yet been observed in EUV or X-ray regime. In fact, all coronal heating models advocated so far seem to require the existence of an unobserved (i.e. unresolved with current instrumentation) source of energy distributed over the entire Sun [7]. Therefore, the “nanoflares” are modelled as invisible generic events, producing an impulsive energy release at small scale, where their cause and their nature are not specified, see the review papers [8, 9]. It is precisely this generic definition of nanoflares which has been adopted in [1] to explain the impulsive radio events observed in the quiet solar corona in terms of frequency of appearance, duration, and wait times distribution recorded at frequencies 98, 120, 132, and 160 MHz. The authors of ref. [1] have argued that the radio observations allow to probe much weaker energy with much better temporal and spatial resolutions in comparison with the current generation instrumentation in EUV and X-rays energy bands. In other words, the radio observations can potentially “see” nanoflares and their “internal structures” where much more energetic EUV and X-rays instruments cannot.

Now we are highlighting the basic features of the AQN model which represents the second part of our story, while deferring a more detailed overview to Section II. The axion quark nugget (AQN) dark matter model [5] was invented long ago with a single motivation, which was to explain “naturally” the observed similarity between the dark and the visible matter densities in the

∗ slge@phas.ubc.ca
† shahriar.na007@gmail.com
‡ waerbeke@phas.ubc.ca
§ arz@phas.ubc.ca
Our presentation is organized as follows. In section II, we overview the basic ideas of the AQN model paying special attention to the specific topics relevant for the present studies. In next section III we highlight some specific features related to the solar corona heating within the AQN framework. In sections IV, V we present our estimates supporting the main claim of this work that the observations [1], including the frequency of appearance, temporal and spatial distributions, energetics, and other related observables in radio frequency bands nicely match the generic consequences of the AQN annihilation events in corona.

II. THE AQN MODEL: THE BASICS

It is commonly assumed that the Universe began in a symmetric state with zero global baryonic charge and later (through some baryon number violating process, non-equilibrium dynamics, and $CP$ violation effects, realizing three famous Sakharov’s criteria) evolved into a state with a net positive baryon number, this is called “baryogenesis”.

The original motivation for this model comes from the possibility of an alternative to this scenario where the “baryogenesis” is replaced by a charge separation process (rather than charge generation) in which the global baryon number of the universe remains zero at all times. In this model the unobserved antibaryons come to comprise the dark matter in the form of dense nuggets of anti-quarks and gluons in colour superconducting (CS) phase. This “charge separation” process results in two populations of AQN carrying positive and negative baryon numbers. In other words, an AQN can be formed of either matter or antimatter. However, due to the global $CP$ violating processes associated with $\theta_0 \neq 0$ during the early formation stage, the number of nuggets and antimuggets will be different$^1$. This difference is always an order one effect irrespective of the parameters of the theory, the axion mass $m_a$ or the initial misalignment angle $\theta_0$. We refer to the original papers [28–31] devoted to the specific questions related to the nugget’s formation, generation of the baryon asymmetry, and survival pattern of the nuggets during the evolution in early Universe with its unfriendly environment.

Antimatter nuggets can interact with regular matter via annihilations, which leads to electromagnetic radiation, whose spectral characteristic and flux can be calculated in the AQN framework. The emission is sufficiently dim to not violate any known observational constraints, but it is strong enough to offer a possible solution to some unexplained astrophysical observations. For instance, it is known that the galactic spectrum contains several excesses of diffuse emission the origin of which is not well

---

$^1$ This source of strong $CP$ violation is no longer available at the present epoch as a result of the dynamics of the axion, which remains the most compelling resolution of the strong $CP$ problem, see original papers on the axion [12–18], and recent reviews [19–27].
established, and remains debated. The best known example is the strong galactic 511 keV line. If the nuggets have a baryon number in the \(< B > \sim 10^{25}\) range they could offer a potential explanation for several of these diffuse components, at radio, X-ray and \(\gamma\)-ray wavelengths. In all these cases, the photon emission originates from the outer layer of the nuggets known as the electrosphere, and all intensities in different frequency bands are expressed in terms of a single parameter \(< B >\) such that all the relative intensities are unambiguously fixed because they are determined by the Standard Model (SM) physics. It is very nontrivial consistency check of the model that the required \(< B >\) to explain these excesses of the galactic diffuse emission belongs to the same mass range. For further details see the original works \(32-37\) with explicit computations of the galactic radiation excesses for the various frequencies.

The AQNs may also offer a resolution to some seemingly unrelated puzzles such as “Primordial Lithium Puzzle” \(38\) and the longstanding puzzle with the DAMA/LIBRA observation of the annual modulation at \(9.5\sigma\) confidence level \(39\). Furthermore, it may resolve the observed \((\text{by XMM-Newton at 11 } \sigma\text{ confidence level }\) \(40\)) puzzling seasonal variation of the X-ray background in the near-Earth environment in the 2-6 keV energy range as suggested in \(41\). The AQN annihilation events in the Earth’s atmosphere could also lead to resolution of another mysterious type of explosions as discussed in \(42\) when the infrasonic and seismic acoustic waves have been recorded without any traces of accompanying meteor-like events. The AQN annihilation events which occur under the thunderstorm may also explain several puzzling events observed by Telescope Array collaboration as discussed in \(43\).

In the context of our present study, however, the most important application is a possible explanation of the “Solar Corona Mystery” \(2, 3\), which is reviewed in details in next subsection III.

The key parameter which essentially determines the intensity of the effects mentioned above is the average baryon charge \(< B >\) of the AQNs. It is expected that AQNs do not have all the same \(B\), but rather is given by a distribution function \(f(B)\). There is a number of constraints on this parameter which are reviewed below. The AQNs are macroscopically large objects with a typical size of \(R \sim 10^{-5}\) cm and roughly nuclear density resulting in masses roughly \(10^9\) g. For an AQN with baryonic charge \(B\), its mass is given by \(M_N \sim m_p|B|\). For the present work we adopt a typical nuclear density of order \(10^{40}\) g cm\(^{-3}\) such that a nugget with \(|B| \sim 10^{25}\) has a typical radius \(R \sim 10^{-5}\) cm. It should be contrasted with conventional meteors of mass \(10^9\) g which would have a typical size of order \(1\) cm, occupying a volume approximately 15 orders of magnitude larger than the AQN’s volume. One can view an AQN as a very small neutron star (NS) with nuclear density. The difference is that the NS is squeezed by the gravity, while the AQN is squeezed by the axion domain wall pressure.

We now overview the observational constraints on such kind of dense objects which plays a key role in our analysis. The strongest direct detection limit\(^2\) is set by the IceCube Observatory’s, see Appendix A in \(45\):

\(< B > > 3 \cdot 10^{24} \quad \text{[direct (nond)etection constraint], (1)}\)

The authors of \(46\) use the Apollo data to constrain the abundance of quark nuggets in the region of 10 kg to one ton. It has been argued that the contribution of such heavy nuggets must be at least an order of magnitude less than would saturate the dark matter in the solar neighbourhood \(46\). Assuming that the AQNs do saturate the dark matter, the constraint \(46\) can be reinterpreted that at least 90\% of the AQNs must have masses below 10 kg. This constraint can be approximately expressed in terms of the baryon charge:

\(< B > \lesssim 10^{28} \quad \text{[Apollo constraint]. (2)}\)

Therefore, indirect observational constraints \((1)\) and \((2)\) suggest that if the AQNs exist and saturate the dark matter density today, the dominant portion of them must reside in the window:

\(3 \cdot 10^{24} \lesssim < B > \lesssim 10^{28} \quad \text{[constraints from observations]. (3)}\)

The authors of ref.\(47\) considered a generic constraints for the nuggets made of antimatter (ignoring all essential specifics of the AQN model such as quark matter phase of the nugget’s core). Our constraints \((3)\) are consistent with their findings including the Cosmic Microwave Background (CMB), Big Bang Nucleosynthesis (BBN), and others, except the constraints derived from the so-called “Human Detectors”. We think that the corresponding estimates of ref.\(47\) are oversimplified and do not have the same status as those derived from CMB or BBN constraints\(^3\).

\(^2\) There is also an indirect constraint on the flux of dark matter nuggets with mass \(M < 55\) g (which corresponds approximately \(B \simeq 10^{25}\)) based on the non-detection of etching tracks in ancient mica \(44\). It slightly touches the lower bound of the allowed range \((1)\), but does not strongly constraint entire window \((3)\) because the dominant portion of the AQNs lies well above its lower limit \((1)\) assuming the mass distribution \((8)\) as discussed below.

\(^3\) In particular, the rate of energy deposition was estimated in \(47\) assuming that the annihilation processes between antimatter nuggets and baryons are similar to \(\bar{p}p\) annihilation process. It is known that it cannot be the case in general, and it is not the case in particular in the AQN model because the annihilating objects have drastically different structures. It has been also assumed \(47\) that a typical x-ray energy is around 1 keV, which is much lower than direct computations in the AQN model would suggest \(42\). Higher energy x-rays have much longer mean-free path, which implies that the dominant portion of the energy will be deposited outside the human body. Finally, the authors of ref. \(47\) assume that an antimatter nugget will result in “injury similar to a gunshot”. It is obviously a wrong picture as the size of a typical nugget is only \(R \sim 10^{-5}\) cm while the most of the energy is deposited in form of the x-rays on centimeter scales \(42\) without making a large hole similar to bullet as assumed in \(47\). In this case a human’s death may occur as a result of a large dose of radiation with a long time delay, which would make it hard to identify the cause of the death.
We emphasize that the AQN model within the window (3) is consistent with all presently available cosmological, astrophysical, satellite and ground-based constraints. Furthermore, it has been shown that these macroscopic objects can be formed, and the dominant portion of them will survive highly disruptive events (such as BBN, galaxy and star formation etc) during the long evolution of the Universe [28–31]. This model is very rigid and predictive as there is no flexibility nor freedom to modify any estimates [2, 3, 32–39, 41–43] which have been carried out in drastically different environments when densities and temperatures span many orders in magnitude.

III. THE AQN MODEL: APPLICATION TO THE SOLAR CORONA HEATING

In this section we overview the basic characteristics of the nanoflares from AQN viewpoint. The corresponding results will play a vital role in our studies later on. We start with few historical remarks. The solar corona is a very peculiar environment. Starting at an altitude of 1000 km above of the photosphere, the highly ionized iron lines show that the plasma temperature exceeds a few $10^6$ K. The total energy radiated away by the corona is of the order of $L_{\text{corona}} \sim 10^{27}$ erg s$^{-1}$, which is about $10^{-6} \sim 10^{-7}$ of the total energy radiated by the photosphere. Most of this energy is radiated at the extreme ultraviolet (EUV) and soft X-ray wavelengths. There is a very sharp transition region located in the upper chromosphere where the temperature suddenly jumps from a few thousand degrees to $10^6$ K. This transition layer is relatively thin, 200 km at most. This transition happens uniformly over the Sun, even in the quiet Sun, where the magnetic field is small, $(\sim 1$ G), away from active spots and coronal holes. The reason for this uniform heating of the corona is not yet understood.

A possible solution to the heating problem in the quiet Sun corona was proposed in 1983 by Parker [48], who postulated that a continuous and uniform sequence of miniature flares, which he called “nanoflares”, could happen in the corona. This became the convetional view. The term “nanoflare” has been used in a series of papers by Benz and coauthors [49–53], and many others, to advocate the idea that these small “micro-events” might be responsible for the heating of the quiet solar corona. We want to mention in this short overview few relatively recent studies [54–61] and reviews [8, 9] which support the basic claim of early works: that nanoflares play the dominant role in the heating of the solar corona.

In what follows we adapt the definition suggested in [53] and refer to nanoflares as the “micro-events” in quiet regions of the corona, to be contrasted with “micro flares” which are significantly larger in scale and observed in active regions. The term “micro-events” refers to a short enhancement of coronal emission in the energy range of about $(10^{24} \sim 10^{28})$ erg. One should emphasize that the lower limit gives the instrumental threshold observing quiet regions, while the upper limit refers to the smallest events observable in active regions. The list below shows the most important constraints on nanoflares from the observations of the EUV iron lines with SoHO/EIT:

1. The EUV emission is highly isotropic [50, 52], therefore the nanoflares have to be distributed very “uniformly in quiet regions”, in contrast with micro-flares and flares which are much more energetic and occur exclusively in active areas [53]. For instance, flares have a highly non-isotropic spatial distribution because they are associated with the active regions;

2. According to [51], in order to reproduce the measured EUV excess, the observed range of nanoflares needs to be extrapolated from the observed events interpolating between $(3.1 \cdot 10^{24} \sim 1.3 \cdot 10^{26})$ erg to sub-resolution events with much smaller energies, see item 3 below.

3. In order to reproduce the measured radiation loss, the observed range of nano flares (having a lower limit at about $3 \cdot 10^{22}$ erg is due to the instrumental threshold) needs to be extrapolated to energies as low as $10^{22}$ erg and in some models even to $10^{20}$ erg, see table 1 in ref. [51];

4. The nanoflares and microflares appear in a different range of temperature and emission measure (see Fig.3 in [53]). While the instrumental limits prohibit observations at intermediate temperatures, nevertheless the authors of [53] argue that “the occurrence rates of nanoflares and microflares are so different that they cannot originate from the same population”. We emphasize on this difference to argue that the flares originate at sunspot areas with locally large magnetic fields $B \sim (10^2 \sim 10^3)$ G, while the EUV emission (which is observed even in very quiet regions where the magnetic field is in the range $B \sim 1$ G) is isotropic and covers the entire solar surface;

5. Time measurements of many nanoflares demonstrate the Doppler shift with a typical velocities (250-310) km/s, see Fig.5 in ref. [49]. The observed line width in OV of $\pm 140$ km/s far exceeds the thermal ion velocity which is around 11 km/s [49];

6. The temporal evolution of flares and nanoflares also appears different. The typical ratio between the maximum and minimum EUV irradiance during the solar cycle does not exceed a factor of 3 or so between the maximum at year 2000 and minimum in 2009 (see Fig. 1 from ref. [62]), while the same ratio for flares and sunspots is much larger, of the order of $10^2$. If the magnetic reconnection was fully responsible for both the flares and nanoflares, then the variation during the solar cycles should be similar for these two phenomena. It is not what is observed: the modest variation of the EUV with the solar cycles in comparison to the flare fluctuations suggests that the EUV radiation does not directly follow the magnetic field activity, and that the EUV flu-
tuation is a secondary, not a primary effect of the magnetic activity.

The nanoflares are usually characterised by the following distribution:

\[ dN \propto W^{-\alpha} dW \quad 10^{21} \text{erg} \lesssim W \lesssim 10^{26} \text{erg} \quad (4) \]

where \( dN \) is the number of nanoflare events per unit time, with energy between \( W \) and \( W + dW \). In formula (4) we display the approximate energy window for \( W \) as expressed by items 2 and 3 including the sub-resolution events extrapolated to very low energies. The distribution \( dN/dW \) has been modelled via magnetic-hydrodynamics (MHD) simulations [54, 63] in such a way that the Solar observations match simulations. The parameter \( \alpha \) was fixed to fit observations [54, 63], see description of different models in next subsection.

\[ \rho \lesssim 0.3 \text{ GeV cm}^{-3} \quad \text{where \( dN \) is the number of nanoflare events per unit time, with energy between \( W \) and \( W + dW \). In formula (4) we display the approximate energy window for \( W \) as expressed by items 2 and 3 including the sub-resolution events extrapolated to very low energies. The distribution \( dN/dW \) has been modelled via magnetic-hydrodynamics (MHD) simulations [54, 63] in such a way that the Solar observations match simulations. The parameter \( \alpha \) was fixed to fit observations [54, 63], see description of different models in next subsection.} \]

\[ \text{where } dN \text{ is the number of nanoflare events per unit time, with energy between } W \text{ and } W + dW, \text{ which occur as a result of complete annihilation of the antimatter AQN carrying the baryon charges between } B \text{ and } B + dB. \]

An immediate self-consistency check of this conjecture is the observation that the allowed window (3) for the AQNs baryonic charge largely overlaps with the approximate energy window for nanoflares \( W \) expressed by (4). This is because the annihilation of a single baryon charge produces the energy about \( 2m_p c^2 \simeq 2 \text{ GeV} \) which is convenient to express in terms of the conventional units as follows,

\[ 1 \text{ GeV} = 1.6 \cdot 10^{-10} \text{ J} = 1.6 \cdot 10^{-3} \text{ erg}, \quad (6) \]

Encouraged by this self-consistency check and a highly nontrivial energetic consideration (the DM density \( \rho_{\text{DM}} \simeq 0.3 \text{ GeV cm}^{-3} \) unambiguously predicts that the extra energy deposited in corona will be \( 10^{27} \text{ erg s}^{-1} \) as observed in EUV energy band) the authors of ref. [3] used power-law index \( \alpha \) entering (4) to describe the baryon number distribution \( dN/dB \) for the nuggets, which represents the direct consequence\(^4\) of the conjecture (5). To be more specific, in Monte Carlo (MC) simulations performed in [3] the baryon number distribution of the AQNs as given by (8) is assumed to directly follow the nanoflare distribution \( dN/dW \) with the same index \( \alpha \) as the conjecture (5) states.

The nanoflare distribution models proposed in [54, 63] have been adapted by [3]. Three different choices for the power-law index \( \alpha \) have been considered in [54, 63]:

\[ \alpha = 2.5, \quad 2.0, \quad \text{ or } \begin{cases} 1.2 & W \lesssim 10^{24} \text{ erg} \leftrightarrow B \lesssim 3 \times 10^{26} \\ 2.5 & W \gtrsim 10^{24} \text{ erg} \leftrightarrow B \gtrsim 3 \times 10^{26}. \end{cases} \]

In addition to the power law index \( \alpha \) different models are also characterized by different choices of \( B_{\text{min}} \): \( 10^{23} \) and \( 3 \times 10^{24} \). Therefore, a total of 6 different models have been discussed in [54, 63] which we expressed in terms of the baryon charge \( B \) rather than in terms of the nanoflare energy \( W \). We also fix \( B_{\text{max}} = 10^{28} \) to be consistent with constraint (3).

In this work, we will only use simulations that give \( \langle B \rangle \gtrsim 10^{25} \) in order to be consistent with (3). This means that we are excluding two models considered in [54, 63]: the one with \( B_{\text{min}} \sim 10^{23} \) and the one with \( \alpha = 2.5 \) and \( \alpha = 2 \). We also exclude the model with \( B_{\text{min}} \sim 10^{23} \) and that with \( \alpha = 1.25 \) and \( \alpha = 2.5 \) to simplify things as it produces results very similar to another model. For convenience, we label the remaining three models as follows:

Group 1 : \( B_{\text{min}} = 3 \times 10^{23}, \alpha = 2.5 \) \quad (7)

Group 2 : \( B_{\text{min}} = 3 \times 10^{24}, \alpha = 2.0 \)

Group 3 : \( B_{\text{min}} = 3 \times 10^{24}, \alpha = \begin{cases} 1.2 & B \lesssim 3 \times 10^{26} \\ 2.5 & B \gtrsim 3 \times 10^{26} \end{cases} \)

while \( B_{\text{max}} = 10^{28} \) for all the models.

The average baryon number of the distribution is defined as

\[ \langle B \rangle = \int_{B_{\text{min}}}^{B_{\text{max}}} dB \langle B f(B) \rangle, \quad \frac{dN}{dB} \propto f(B) \propto B^{-\alpha} \quad (8) \]

\[ \text{One should note that it has been argued [31] that the algebraic scaling (5) is a generic feature of the AQN formation mechanism based on percolation theory. The phenomenological parameter } \alpha \text{ is determined by the properties of the domain wall formation during the QCD transition in the early Universe, but it cannot be theoretically computed in strongly coupled QCD. Instead, it will be constrained based on the observations as discussed in the text.} \]
where $f(B)$ is normalized and the power-law is taken to hold in the range from $B_{\text{min}}$ to $B_{\text{max}}$.

The above estimate suggests a “miracle” coincidence between the energy/mass windows (3) and (4) for AQNs and nanoflares respectively. This coincidence is a strong support of our proposal [2, 3] that the nanoflares and the AQN annihilation events are the same phenomena (see items 2 and 3 of Section III A).

We are now in position to present several additional arguments to support this proposal: item 1 (Section III A) is also naturally explained in the AQN framework as the DM is expected to be distributed very uniformly over the Sun making no distinction between quiet and active regions, in contrast with large flares. A similar argument applies to item 4 as the strength of the magnetic field and its localization is absolutely irrelevant for the nanoflare events in form of the AQNs, in contrast with conventional paradigm when the nanoflares are thought to be simply scaled down configurations of their larger cousin which are much more energetic and occur exclusively in active areas and cannot be uniformly distributed.

The presence of the large Doppler shift with a typical velocities (250-310) km/s, mentioned in item 5, can be understood within the AQN interpretation as the following: the typical velocities of the nuggets entering the solar corona is very high, around 700 km/s. The Mach number $M = v_{\text{AQN}}/c_s$ is also very large. A shock wave will be inevitably formed and will push the surrounding material to the velocities which are much higher than would normally present in the equilibrium.

Finally, as stated in item 6, the temporal modulation of the EUV irradiance over a solar cycle is very small and does not exceed a factor $\sim 3$, as opposed to the much dramatic changes in Solar activity with modulations on the level of $10^3$ over the same time scale. This suggests that the energy injection from the nanoflares is weakly related to the Solar activity, which is in contradiction with the picture where magnetic reconnection plays an essential role in the formation and dynamics of the nanoflares. This is, however, consistent with our interpretation of nanoflares being associated with AQN annihilation events representing the main source of the EUV irradiance.

### IV. THE AQN MODEL CONFRONTS THE RADIO OBSERVATIONS

We start in subsection IV A by describing the basic mechanism of the radio emission due to the AQN events in the solar corona. We estimate the event rate in subsection IV B. The role of non-thermal electrons in generation of the radio signal is discussed in subsection IV C. Finally, in subsection IV D we estimate the intensity of the radio signal events.

#### A. Mechanism of the radio emission in solar corona.

It is generally accepted that the radio emission from the corona results from the interaction of plasma oscillations (also known as Langmuir waves) with non-thermal electrons which must be injected into the plasma [64]. An important element for successful emission of radio waves is that the plasma instability must develop. It occurs when the injected electrons have a non-thermal high energy component with the momentum distribution function characterized by the positive derivative with respect to the electron’s velocity. In this case plasma instability develops and the radio waves can be emitted.

The frequency of emission $\nu$ is mostly determined by the plasma frequency $\omega_p$ in a given environment, i.e.

$$\omega^2 = \omega_p^2 + k^2 \frac{3T}{m_e}, \quad \omega_p^2 = \frac{4\pi \alpha n_e}{m_e}, \quad \nu = \frac{\omega}{2\pi} \quad (9)$$

where $n_e$ is the electron number density in the corona, while $T$ is the temperature at the same altitude and $k$ is the wavenumber. For example, the frequency $\nu = 160$ MHz considered in [1] will be emitted when $n_e \simeq 3.4 \cdot 10^9$ cm$^{-3}$. One should emphasize that the emission of radio waves generically occurs at an altitude which is distinct from the altitude where the AQN annihilation events occur and where the energy is injected into the plasma. This is because the mean-free path $\lambda$ of the non-thermal electrons being injected into the plasma is very long $\lambda \sim 10^4$ km. Therefore, these electrons can travel very long distance before they transfer their energy into the radio wave energy as we discuss in subsection IV C.

We propose in this work that the non-thermal electrons will be produced by the AQNs when they enter the solar corona and the annihilation processes start. It is known that the number density of the non-thermal (suprathermal in terminology [64]) electrons $n_s$ must be sufficiently large $n_s/n_e \gtrsim 10^{-7}$ for the plasma instability to develop, in which case the radio waves will be generated [64]. As the density $n_s/n_e$ approaches the so-called threshold values at some specific frequencies the intensity increases sharply which we identify with observed bursts. These threshold conditions may be satisfied randomly in space in time depending on properties of the injected electrons [64]. All these plasma properties are well beyond the scope of this paper and shall not be discussed in the present work. However, we shall demonstrate that the number density of the non-thermal electrons $n_s$ generated by the AQNs can easily be in proper range $n_s/n_e \gtrsim 10^{-7}$ for the plasma instability to develop. To be more specific, in next subsection IV C we shall argue that the ratio $n_s/n_e \sim 10^{-7}$ is always sufficiently large for the plasma instability to develop which eventually generate the radio waves.

5 If the derivative has a negative sign it will lead to the so-called Landau damping.
Therefore our proposal for the observed radio events in [1] is that the AQN annihilation events (identified with nanoflares as explained in Section III B) will produce a large number of non-thermal electrons which generate the radio waves as a result of plasma instability. We shall support our proposal by estimating a number of observables analyzed in [1] in next subsections. As we shall see below this proposal is consistent with all observed data, including the frequency of appearance, the intensity radiation, duration, spatial and wait time distributions, to be discussed in next subsections IV B, IV C, IV D as well as in Sections V.

B. The event rate

We are in position now to interpret the radio emission data from [1] in terms of the AQN events. First of all, the nuggets start to loose their baryon charge due to the annihilation in close vicinity of the transition region at the height around 2200 km, see Fig. 5 in [3] and also Fig. 3 below. However, the radio emission will happen at much higher altitudes as we explain in subsection IV C.

In this subsection we want to compare the maximum radio event rate (33,481 events observed in 132 MHz frequency band during 70 minutes) to the expected rate of AQN events which are identified with nanoflares and must be much more numerous (according to conventional solar physics modelling). Specific nanoflares models [54, 63] (expressed by eq.(7) in terms of the baryon charge \( B \)) correspond to events rate which is at least few orders of magnitude higher than the observed radio event rate, see Fig 8 in [3]. There is no contradiction here because it is likely that the dominant portion of the nanoflares events are too small to be resolved. This point has been mentioned in items 2 and 3 in section III A with a comment that all models must include small but frequent events which had been extrapolated to sub-resolution region. Therefore, we interpret the low event rate at radio frequencies as the manifestation that only the strongest events with very short time scale discussed in Section V B.

We can compute (in terms of \( B \)) the event rate for the energetic AQNs which are powerful enough to generate the resolved radio impulses as recorded in [1]. The corresponding impact rate can be computed in the same way as Fig 8 from [3], the only difference being that the lower bound is determined by \( B \) rather than \( B_{\text{min}} \), i.e.

\[
\left( \frac{dN}{dt} \right)_B \propto \int_{B}^{B_{\text{max}}} dB \ f(B). \tag{10}
\]

Since the maximum number of detected radio events in Ref. [1] is 33481 at the 132 MHZ band in 70 minutes, the event rate is

\[
\frac{dN_{\text{obs}}}{dt} \sim \frac{33481}{70 \text{ minutes} \times 1/2} \sim 16 \text{ s}^{-1}. \tag{11}
\]

The factor 1/2 accounts for the fact that only half of the Sun’s whole surface is visible.

By equalizing (11) and (10) we can estimate the parameter \( \bar{B} \) when sufficiently large radio events originate from large nuggets with \( B \gtrsim \bar{B} \). The results are presented on Fig. 1. It is the intersection of the black dashed line (11) and the simulated line of each group given by eq.(7). The intersections are shown in the right subfigure, and the corresponding \( \bar{B} \) are respectively \( 5.65 \times 10^{26}, 2.21 \times 10^{27} \), and \( 1.95 \times 10^{27} \) for the three groups. We expect that only AQNs with masses greater than \( \bar{B} \) are sufficiently energetic to generate the observable impulsive radio events.

The parameter \( \bar{B} \) obviously depends on the size distribution models listed in (7), it corresponds to a detection limit and should not be treated as a fundamental parameter of the theory. An instrument with different resolution and/or sensitivity will affect the radio events selection criteria and therefore change the value of \( \bar{B} \), in which case some events from the continuum spectrum would be considered as impulsive events\(^6\).

Our next task is to estimate the total luminosity \( L_{\bar{B}} \) released as a result of the complete annihilation of the large nuggets with \( B \gtrsim \bar{B} \). The calculation is similar to the estimation done for Fig 10 of [3], the only difference is that the lower bound is determined by \( \bar{B} \) rather than \( B_{\text{min}} \), i.e.

\[
L_{\bar{B}} \propto \int_{\bar{B}}^{B_{\text{max}}} dB \ B^{2} f(B). \tag{12}
\]

The results for the models listed in (7) are presented on Fig. 2. The corresponding \( L_{\bar{B}}^{\odot} \) assume the following values: \( 6.17 \times 10^{25} \text{ erg} \cdot \text{s}^{-1}, 2.05 \times 10^{26} \text{ erg} \cdot \text{s}^{-1} \) and \( 1.70 \times 10^{26} \text{ erg} \cdot \text{s}^{-1} \), which are approximately an order of magnitude smaller than the luminosity released by all the AQNs annihilation. This implies that only about 10% of the total the AQN-induced luminosity comes from the large nuggets with \( B \gtrsim \bar{B} \), which are the same AQNs assumed to produce the resolved radio events in [1]. Our estimates show that while the strong events with \( B \gtrsim \bar{B} \) are very rare with an impact rate approximately 3 orders of magnitude smaller than all AQN events, their contribution to the luminosity is suppressed only by one order of magnitude. This is, of course, due to the factor \( B^{2} \) in the expression for the luminosity (12).

The energy flux \( \Phi_{\bar{B}}^{\odot} \), observed on Earth, coming from

\(^6\) This estimate does not include the possibility of “clustering” events with very short time scale discussed in Section V B.

\(^7\) It is known that the continuum contribution in the radio emissions is similar in magnitude to the impulses events as we discuss in subsection IV D. Some of the events from continuum could be treated in future as impulsive events if a better resolution instrument is available. However, this does not drastically modify our estimate for \( \bar{B} \).
these large nuggets with $B \gtrsim \bar{B}$ is estimated as

$$\Phi_{\bar{B}} \simeq \frac{L_{\bar{B}}}{4\pi (AU)^2} \simeq (1.8 - 6) \cdot 10^{-2} \frac{\text{erg}}{\text{cm}^2 \text{s}},$$

(13)

where we used the range of numerical values for $L_{\bar{B}}$ estimated above. In the following we will establish the physical connection between the energy flux (13) generated by large nuggets with $B \gtrsim \bar{B}$ and the flux observed in radio frequency bands observed in [1]. In order to make this connection we have to estimate what fraction of the huge amount of energy due to the AQN annihilation is transferred to the tiny portion in the form of radio waves. To compute this efficiency we need to estimate the relative density of the non-thermal electrons which will be produced as a result of the AQN annihilation events. The estimation of this efficiency is the topic of the next subsection.

C. Non-thermal electrons

The starting point for our analysis is the number of annihilation events per unit length while the AQN propagates through the ionized corona environment:

$$\frac{dN}{dt} \simeq \pi R_{\text{eff}}^2 n_p,$$

(14)

where $n_p$ is the baryon number density of the corona (mostly protons) and the effective radius $R_{\text{eff}}$ of the AQNs can be interpreted as the effective size of the nuggets due to the ionization characterized by the nugget’s charge $Q$ as explained in [3]. The enhancement of the interaction range $R_{\text{eff}}$ due to the long range Coulomb force is given by (see [3] for the details):

$$\left( \frac{R_{\text{eff}}}{R} \right) = \epsilon_1 \epsilon_2, \quad \epsilon_1 \equiv \sqrt{\frac{8(m_e T_P)R^2}{\pi}}, \quad \epsilon_2 \equiv \left( \frac{T_j}{T_P} \right)^{\frac{1}{2}},$$

(15)
where $T_\text{j}$ is the internal temperature of the AQN and $T_\text{p}$ is the plasma temperature in the corona. An estimation of the internal thermal temperature $T_\text{j}$ is a highly non-trivial and complicated problem which requires an understanding of how the heat, due to the friction and the annihilation events, will be transferred to the surrounding plasma from a body moving with supersonic speed with Mach number $M \equiv v/c_\text{s} > 1$.

It is known that the supersonic motion will generate shock waves and turbulence. It is also known that a shock wave leads to a discontinuity in velocity, density and temperature due to the large Mach numbers $M \gg 1$. It has been argued in [3, 65] that, for a normal shock, the jump in temperature is given by the Rankine-Hugoniot condition:

$$\frac{T_\text{j}}{T_\text{p}} \approx M^2 \cdot \frac{2\gamma(\gamma - 1)}{\gamma + 1} \gg 1, \quad \gamma \approx 5/3. \quad (16)$$

and, as a result, all the electrons from the plasma which are on the AQN path within distance $R_{\text{eff}}$ will be affected. To be more precise these electrons will experience the elastic scattering by receiving the extra kinetic energy $\Delta E$ which lies in the window $\Delta E \in (T_\text{p}, T_\text{j})$. It is precisely these non-thermal electrons which will subsequently interact with the plasma and be the source of the plasma instability. These non-thermal electrons will transfer their energy to the emission of radio waves with frequency $\nu$ as explained at the end of Section IV A.

We are now in position to estimate the parameter $\eta$ defined as the ratio between the energy transferred (per unit length $l$) to the radio waves and the total energy produced by a single AQN (per unit length $l$) as a result of the annihilation process:

$$\eta \approx \frac{\Delta E}{2m_\text{e}c^2} \cdot \frac{[\pi R_{\text{eff}}^2 n_\text{e}]}{[\pi R_{\text{eff}}^2 n_\text{p}]} \approx \Delta E \cdot \frac{2m_\text{e}c^2}{10^{-7}}, \quad (17)$$

where the denominator accounts for the total energy due to the annihilation events with rate (14) and the numerator accounts for the kinetic energy received by affected electrons. In our estimate of (17), we assume an approximate local neutrality such that $n_\text{e} \approx n_\text{p}$. Furthermore, to be on the conservative side, we also assume that $\Delta E \approx 2T_\text{p} \approx 2 \cdot 10^2$ eV, such that $\Delta E$ only slightly exceeds the surrounding thermal energy $\approx T_\text{p}$. Finally, we also assume that the dominant portion of the $\Delta E$ will be eventually released in the form of radio waves. It is very likely that there are few missing numerical factors of order one on the right hand side in eq. (17) as our assumptions formulated above are only approximations. However, we believe that (17) gives a correct order of magnitude estimate for the energy efficiency transfer ratio $\eta$. We provide a few numerical estimates in next subsection IV D suggesting that (17) is very reasonable and consistent with observed intensities in radio bands [1].

The next step is the estimation of $n_\text{s}/n_\text{e}$, which must be sufficiently large $n_\text{s}/n_\text{e} \gtrsim 10^{-7}$ for the plasma instability to develop [64] (see section IV A). As we shall see now, the proposed mechanism indeed satisfies this requirement. We start with the expression of the total number of electrons $\Delta N_\text{e}$ to be affected while the AQN travels over a distance $l$:

$$\Delta N_\text{e} \sim (\pi R_{\text{eff}}^2 l) \cdot n_\text{e}(h), \quad l \approx v_{\text{AQN}} \Delta t,$$  \quad (18)

where $n_\text{e}(h)$ is the electron number density at the altitude $h \approx 2000$ km where annihilation events become efficient [3]. These affected electrons will an receive extra energy $\Delta E$ and extra momentum $m_\text{e}v_\perp$ with very large velocity component $v_\perp$ perpendicular to the nugget’s path as the shock front due to $M \gg 1$ has a form of a cylinder along the AQN path. A large portion of the AQN’s trajectories can be viewed as an almost horizontal path with relatively small incident angles toward the Sun (skim trajectories). These non-thermal electrons will have a component $v_\perp$ perpendicular to the nugget’s path and travel unperturbed up to a distance of the order of the mean free path $\lambda \sim 10^4$ km (to be estimated below).

After a time $\Delta t$, the same non-thermal electrons $\Delta N_\text{e}$ will have spread over a distance $r$ from the AQN’s path, estimated as follows:

$$\Delta N_\text{e} \sim (2\pi r \Delta r) \cdot n_\text{e}(r), \quad (19)$$

where $\Delta r$ is the width of the shock front measured at distance $r$. For a non-thermal electron traveling away from the AQN path with perpendicular velocity $v_\perp$, the distance $r$ is given by:

$$r \sim v_\perp \Delta t, \quad v_\perp \simeq \sqrt{\frac{2\Delta E}{m_\text{e}}} \simeq 10^4 \sqrt{\frac{T}{10^2 \text{eV}}} \frac{\text{km}}{\text{s}}. \quad (20)$$

Equalizing (18) and (19) we arrive to the following estimate for the ratio $n_\text{s}/n_\text{e}$:

$$\left[\frac{n_\text{s}(r)}{n_\text{e}(h)}\right] \simeq \left(\frac{R_{\text{eff}}^2}{r\Delta r}\right), \quad r \lesssim \lambda. \quad (21)$$

The expression (21) holds as long as $r \lesssim \lambda$. For larger distances $r \gtrsim \lambda$ the non-thermal electrons will eventually thermalize and lose their ability to generate a plasma instability. One should emphasize that $n_\text{s}(r)$ entering (21) is taken at the distance $r$ from the AQN path, while $n_\text{e}(h)$ is taken in the vicinity of the path, i.e. at $r \approx 0$.

We are interested in this ratio when both components are computed at the same location and we now have to check if it is larger than $10^{-7}$, the requirement to generate the plasma instability. The relevant configuration for our study corresponds to non-thermal electrons moving upward\footnote{the radio waves emitted at altitudes below $h$ will have much higher frequencies than considered in the present work, and shall not be discussed here.}. In this case the relation (21) assumes the form

$$\left[\frac{n_\text{s}(r)}{n_\text{e}(r)}\right] \simeq \frac{1}{2} \left[\frac{n_\text{e}(r+h)}{n_\text{e}(r)}\right] \cdot \left(\frac{R_{\text{eff}}^2}{r\Delta r}\right), \quad r \lesssim \lambda. \quad (22)$$
where the factor $1/2$ accounts for upward moving electrons and $n_e(r) \equiv n_e(r + h)$ is the electron density computed at distance $r$ above the AQN’s path (which is localized at an altitude of $h \approx 2000 \text{ km}$).

The expression (22) has a conventional form for a cylindrical geometry with the expected suppression factor $r^{-1}$ at large distances and constant value for $\Delta r$. However, it is known that the width of the shock $\Delta r$ induced energy flux which is transferred to the radio started, see Fig. 3.

$\Delta r$ is also growths with time$^9$ as $\Delta r \propto \sqrt{t} \propto \sqrt{tR_{e,f}}$. Therefore, we expect that a proper scaling at large $r$ assumes the form:

$$
\left[ \frac{n_s(r)}{n_e(r)} \right] \sim \frac{1}{2} \left[ \frac{n_e(h)}{n_e(r + h)} \right] \left( \frac{R_{e,f}}{r} \right)^{3/2}, \quad r \approx \lambda, \quad (23)
$$

We will calculate this ratio for large nuggets with $B \gtrsim B_{\text{th}}$ which are capable of generating the resolved radio signals. Using our previous parameters estimates for $\epsilon_1$ and $\epsilon_2$ from Section IV.C of [3] and using the electron number density in Table 26 of [66], we arrive at the estimate

$$
\left[ \frac{n_s(r)}{n_e(r)} \right] \gtrsim 10^{-7} \text{ for } r \sim 10^4 \text{ km.} \quad (24)
$$

The condition (24) implies that $n_s/n_e$ is indeed sufficiently large for the plasma instability to develop [64] on distances of order $r \sim 10^4 \text{ km}$ from the nugget’s path. This implies that the non-thermal electrons can propagate upward to very large distances before they transfer their energy to the radio waves at much higher altitudes, of order $(h + r)$. The scale $r \sim 10^4 \text{ km}$ assumes the same order of magnitude value as the mean free path $\lambda$, which at altitude $h \approx 2150 \text{ km}$ can be estimated as follows:

$$
\lambda^{-1} \approx \sigma n_p, \quad \sigma \approx \frac{\alpha^2}{T_p^2}, \quad \lambda \sim 10^4 \text{ km.} \quad (25)
$$

At the same time the density $n_p$ drastically increases (at least 4 times) at slightly lower altitudes $h \approx 2000 \text{ km}$, such that the mean-free path $\lambda$ decreases correspondingly, and the condition (24) breaks down. Therefore, our approximation (24) is marginally justified for the AQN induced non-thermal electrons produced from the altitudes $h \approx 2150 \text{ km}$ where the annihilation processes just started, see Fig. 3.

**D. Radio flux intensity**

In this subsection we estimate the portion of the AQN-induced energy flux which is transferred to the radio waves $\Phi_{\text{radio}}$. We express $\Phi_{\text{radio}}$ in terms of the energy flux emitted by the nuggets as radio waves:

$$
\Phi_{\text{radio}} \approx \Phi_{B}^{\gamma} \cdot \eta \left( \frac{\Delta B}{B} \right) \quad (26)
$$

$$
\approx (0.6 - 6) \cdot 10^{-10} \text{ erg cm}^{-2} \text{ s}^{-1}, \quad (\text{theoretical prediction})
$$

where the first factor $\Phi_{B}^{\gamma}$, given by (13), reflects the contribution of the large nuggets with $B \gtrsim B_{\text{th}}$ to the total AQN-induced luminosity. The factor $\eta$ is given by (17) and represents the portion of the energy transferred to the radio frequency bands through the non-thermal electrons leading to the plasma instability. Finally, the factor $\Delta B/B \sim (0.3 - 1) \cdot 10^{-4}$ describes a typical portion of the baryon charge annihilated in the altitude range $(2000, 2510) \text{ km}$. This is precisely the region where the AQN annihilation events effectively start and where the interaction of the AQNs with surrounding plasma produce the non-thermal electrons which eventually generate the radio waves. The Monte-Carlo simulations for $\Delta B/B$ are presented on Fig. 3. One can see that the dominant portion of the annihilation events occur at lower altitudes $h \lesssim 2000 \text{ km}$. However, the mean free path $\lambda$ at lower altitudes of the affected electrons is too short as our estimations (25) suggest. Therefore, the affected electrons from altitudes $h \lesssim 2000 \text{ km}$ cannot reach higher altitudes where the radio waves are generated. This is precisely the source of the suppression expressed in the ratio $\Delta B/B \ll 1$.

We can now compare our estimate (26) to the observed intensities measured in radio frequency bands by [1]:

$$
\frac{d\Phi_{\text{radio}}}{d\omega}(160 \text{ MHz}) \approx 6 \text{ SFU}, \quad \Delta \omega = 2.56 \text{ MHz}
$$

$$
\frac{d\Phi_{\text{radio}}}{d\omega}(120 \text{ MHz}) \approx 3 \text{ SFU}, \quad \Delta \omega = 2.56 \text{ MHz} \quad (27)
$$

where

$$
\text{SFU} \equiv 10^4 \text{ Jy} = 10^{-19} \text{ erg Hz cm}^{-2} \text{ s}^{-1}. \quad (28)
$$

The observations [1] were done in twelve frequency bands from $80 \text{ MHz}$ to $240 \text{ MHz}$ with $\Delta \omega = 2.56 \text{ MHz}$ bandwidth each. It is known [67, 68] that the radio emission occurs in the entire energy band $\sim (0 - 200) \text{ MHz}$, and not specifically in one of the 12 frequency narrow bands. It is also known [67, 68] that the contributions from continuum and impulsive fluxes are approximately the same in all frequency bands. Therefore we estimate the total intensity in radio bands by multiplying (27) with $\sim 200 \text{ MHz}$ to account for the entire radio emission associated with short impulsive events as well as the continuum:

$$
\Phi_{\text{total}}^{\text{radio}} \approx (0.6 - 1.2) \cdot 10^{-10} \text{ erg cm}^{-2} \text{ s}^{-1} \quad (\text{observation}), \quad (29)
$$

Despite the fact that our calculation involves various steps and approximations, the total measured flux (29) is consistent with our order of magnitude estimate (26).
We consider this as a highly non-trivial consistency check for our proposal as it includes a number of very different elements which were studied previously for a completely different purpose in a different context.

We conclude this section with few important remarks. The occurrence probability shown on Fig 4 in ref. [1] suggests that the power-law index \( \alpha \) is always large, with \( \alpha > 2 \). As explained in the text we cannot predict this index theoretically, but all the nanoflare models used in our studies as expressed by eq. (7) are consistent with the observed power-law index \( \alpha \). The same line of arguments may also explain the observed huge difference between the number of observed events (4748) at smallest frequency band (98 MHz) in comparison to the rate at larger frequency bands where the recorded number of events is almost one order of magnitude higher [1]. Indeed, the non-thermal electrons must pass a region with higher \( \nu \) before they can reach the region of relatively low \( \nu \). Some portion of these non-thermal electrons will transfer their energy to the radio emission in higher frequency bands before reaching the region with low \( \nu \). These arguments suggest that counting rate at even lower frequencies (such 80 and 89 MHz bands recorded by MWA) should be even lower than 4748 events recorded at 98 MHz [1].

V. THE AQN MODEL. WAIT TIME DISTRIBUTION.

The goal here is to understand the wait time distribution reported by [1]. The main observation was that the impulsive events are non-Poissonian in nature. This non-Poissonian feature is shown on Fig 7 of [1] where the occurrence probability at small wait times (below 10 seconds) is linearly growing instead of approaching a constant, which is what is expected for a Poissonian distribution.

FIG. 3. Left: Mass fraction \( 1 - \Delta B/B \) being annihilated as a function of the altitude. This is plotted by taking the average of the mass loss profiles of the AQNs above \( B \) (i.e. the AQNs that will generate radio emissions) where \( B \) has been determined by (11). We see that the AQNs start to annihilate at about 2150 km. Right: Luminosity per unit length as a function of the altitude where \( L \) is converted from the mass loss according to (6). This is plotted also by taking the average of the AQNs above \( B \).
We shall argue below that, in the AQN model, such a behaviour could be explained by the presence of “effective” clustering of events when one and the same AQN in flight may generate a cascade of seemingly independent events on short time scales. These events however, are not truly independent, as they result, in fact, from one and the same AQN when the typical mass loss time is measured in 10-20 seconds, see Fig. 6 in [3]. Few short radio pulses on scales of few seconds could be easily generated during this long flight time. Such “clustering” will violate the assumption of the Poissonian distribution of independent events.

In what follows we develop an approach which can incorporate such “clustering” at small time scales, while the distribution remains Poissonian at larger time scales, i.e. the time scale of distinct AQNs entering the Corona. The corresponding approach is known as a non-stationary Poissonian process which results in Bayesian statistics, which is the topic of the next subsection.

A. Non-Poissonian processes. Overview.

We start with an overview of the non-Poissonian processes and outline the conventional technique to describe them, as given in [69, 70]. In case of a conventional random stationary Poissonian process, the waiting time distribution \( P(\Delta t) \) is expressed as an exponential distribution:

\[
P(\Delta t) = \lambda e^{-\lambda \Delta t}, \quad \int P(\Delta t)d\Delta t = 1, \tag{30}
\]

where \( \lambda \) in this section is mean event occurrence rate. For a constant \( \lambda \), this distribution describes a stationary Poissonian process. When \( \lambda(t) \) depends on time, one can generalize (30) and introduce the probability function of waiting times which becomes itself a function of time [69]:

\[
P(t, \Delta t) = \lambda(t + \Delta t) \exp \left[ - \int_t^{t+\Delta t} \lambda(t')dt' \right]. \tag{31}
\]

If observations of a non-stationary Poisson process are made during a time interval \([0, T]\), then the distribution of waiting times \( P(\Delta t) \) will be, weighted by the number of events \( \lambda(t)dt \) in each time interval \((t, t+dt)\), given by:

\[
P(\Delta t) = \frac{1}{N} \int_0^T \lambda(t)P(t, \Delta t)dt, \quad N = \int_0^T \lambda(t)dt. \tag{32}
\]

If \( \lambda \) varies adiabatically one can subdivide non-stationary Poisson processes into piecewise stationary Poisson processes (Bayesian blocks), take the continuum limit and represent the distribution of waiting times as follows [69, 70]:

\[
P(\Delta t) = \frac{\int_0^T \lambda^2(t)e^{-[\lambda(t)\Delta t]}dt}{\int_0^T \lambda(t)dt}. \tag{33}
\]

One can check that the expression (33) reduces to its original Poissonian expression (30) when \( \lambda \) is time independent.

It is convenient to introduce \( f(\lambda) \) which describes the adiabatic changes of \( \lambda \) as follows:

\[
f(\lambda) \equiv \frac{1}{T} \frac{dt}{d\lambda}, \quad f(\lambda)d\lambda = \frac{dt}{T}, \quad \int d\lambda f(\lambda) = 1. \tag{34}
\]

In terms of \( f(\lambda) \) the distribution of waiting times (33) assumes the form

\[
P(\Delta t) = \frac{\int_0^\infty \lambda^2 f(\lambda)e^{-[\lambda \Delta t]}d\lambda}{\int_0^\infty \lambda f(\lambda)d\lambda}. \tag{35}
\]

The stationary Poissonian distribution corresponds to \( f(\lambda) = \delta(\lambda - \lambda_0) \) such that the distribution of waiting times (35) reduces to the original expression (30) with constant \( \lambda_0 \) as it should.

B. AQN induced clustering events

We are now in position to describe the physics of “effective” clustering events using non-stationary Poissonian distribution framework (35) as outlined above. As previously mentioned several, short radio pulses on scales of few seconds could be easily generated during a single AQN “relativly” long flight time of the order of 10-20 seconds (see Fig. 6 in [3]).

With this picture in mind, we introduce the following \( \lambda(t) \) dependence to describe non-stationary Poisson processes. At long time scales \( t > t_0 \) we keep the constant \( \lambda_0 \) corresponding to the stationary Poissonian distribution:

\[
\lambda = \lambda_0 \quad f(\lambda) \sim \delta(\lambda - \lambda_0), \quad \text{for} \quad t > t_0, \tag{36}
\]
while for shorter time scales $t < t_0$ we parameterize $f(\lambda)$ as follows:

$$f(\lambda) = c\lambda^\beta, \quad \lambda = \lambda_0 \left[ t \frac{t}{t_0} \right]^\frac{1}{\beta + 2} \quad \text{for} \quad t < t_0,$$  

(37)

where $\beta, \lambda_0$ and $t_0$ parameters should be fitted to match the observational signal distribution. One can combine equations (36) and (37) to represent $f(\lambda)$ as follows:

$$f(\lambda) = \left[ \left( \frac{T - t_0}{T} \right) \delta(\lambda - \lambda_0) \right]$$  

$$+ \left[ \frac{\beta + 1}{T} t_0 \left( \frac{\lambda}{\lambda_0} \right)^\beta \theta(\lambda_0 - \lambda) \right],$$  

(38)

where factor $(T - t_0)/T$ is inserted in front of delta function $\delta(\lambda - \lambda_0)$ to preserve the normalization (34).

One should emphasize that the $\lambda_0$ is not the mean event occurrence rate $\langle \lambda \rangle$ anymore. Instead, the proper value for $\langle \lambda \rangle$ reads:

$$\langle \lambda \rangle \equiv \int \lambda f(\lambda) d\lambda = \lambda_0 \left[ 1 - \frac{1}{\beta + 2} \left( t_0 \frac{t}{T} \right) \right].$$  

(39)

The physical meaning of parameter $(t_0/T)$ is obvious: it determines the portion of the clustering events. In case when $(t_0/T) \ll 1$ the clustering events play very minor role, while for $(t_0/T) \approx 1$ the clustering events become essential. In the limit $t_0 \to 0$ the mean value $\langle \lambda \rangle$ approaches its unperturbed magnitude $\lambda_0$ corresponding to the stationary Poisson distribution, as it should.

Now we are in position to compute $P(\Delta t)$ as defined by (35):

$$P(\Delta t) = \frac{1}{\langle \lambda \rangle} \int_0^\infty \lambda^2 f(\lambda) e^{-|\lambda \Delta t|} d\lambda,$$  

(40)

with $f(\lambda)$ as given by (38). The result can be represented as follows:

$$P(\Delta t) = \frac{\lambda_0^2}{\langle \lambda \rangle} e^{-|\lambda_0 \Delta t|} \cdot \left( \frac{T - t_0}{T} \right)$$  

$$+ \frac{(\beta + 1)\lambda_0^2}{\langle \lambda \rangle} \cdot \left( t_0 \frac{t}{T} \right) \cdot \left[ \int_0^{\lambda_0} d\lambda \frac{\lambda}{\lambda_0} \exp\left( \beta \lambda \right) e^{-|\lambda \Delta t|} \right],$$  

(41)

where the first term describes the stationary Poisson distribution while the second term describes the deviation from Poisson distribution at small time scales. The second term in distribution (41) can be expressed in terms of the lower incomplete $\gamma(s, x)$ function defined as follows:

$$\gamma(s, x) \equiv \int_0^x u^{s-1} e^{-u} du, \quad \gamma(s, x) = \Gamma(s) - \Gamma(s, x).$$  

(42)

where $\Gamma(s)$ is the gamma function and $\Gamma(s, x)$ is the upper incomplete gamma function. We identify the parameters from the integrand entering (41) as follows:

$$u = \lambda \Delta t, \quad x = \lambda_0 \Delta t, \quad s = \beta + 3$$  

(43)

to arrive to the following expression for $P(\Delta t)$ in terms of the lower incomplete $\gamma(s, x)$ function:

$$P(\Delta t) = \frac{\lambda_0^2}{\langle \lambda \rangle} e^{-|\lambda_0 \Delta t|} \cdot \left( \frac{T - t_0}{T} \right)$$  

$$+ \frac{\lambda_0^2(\beta + 1)}{\langle \lambda \rangle} \cdot \left( t_0 \frac{t}{T} \right) \cdot \left[ \frac{1}{\lambda_0 \Delta t} \right]^{\beta+3} \cdot \gamma(\beta + 3, \lambda_0 \Delta t).$$  

(44)

This expression is correct for any value of $t_0/T$. However, it is very instructive to see explicit dependence on $\Delta t$ when $t_0/T \ll 1$ is small, and the Poisson distribution is restored.

With this purpose in mind we simplify expression (44) by expanding the incomplete gamma function entering (44). Therefore, the expression (44) can be simplified as follows:

$$P(\Delta t) \approx \frac{\lambda_0^2}{\langle \lambda \rangle} e^{-|\lambda_0 \Delta t|} \cdot \left( \frac{T - t_0}{T} \right)$$  

$$+ \frac{\lambda_0^2(\beta + 1)}{\langle \lambda \rangle} \cdot \left( t_0 \frac{t}{T} \right) \cdot \left[ \frac{1}{\lambda_0 \Delta t} \right]^{\beta+3},$$  

(45)

where we use the identity (42) and ignored the exponentially small contribution coming from incomplete upper gamma function:

$$\Gamma(s, x) \to \infty \to x^{s-1} \exp(-x).$$  

(46)

In the limit $(t_0/T) \to 0$ we recover the conventional Poisson distribution, while $(t_0/T) \neq 0$ describes the deviation from Poisson statistics in this simplified setting.

We are now ready to analyze the non-Poisson distribution given by (45). Important point here is that this distribution is a superposition of two parts: The first term describes the Poisson distribution with small correction in normalization. Most important part for us is the second term which is parametrically small at $(t_0/T) \ll 1$. However, it could become the dominant part of the distribution $P(\Delta t)$ at small $\Delta t \to 0$ due to a high power $(\Delta t)^{-\beta+3}$ in the denominator (45).

It is interesting to note that the authors of ref. [1] noticed that their data can be fitted as a superposition of two terms which have precisely the form of two terms entering (45). However, the authors of ref. [1] fitted the observed signal to an expression which represents the product of two terms rather than in form of sum of two terms entering the eq. (45) with well-defined physical meaning of the relevant parameters such as $(t_0/T)$. In next subsection we fit that data from [1] using exact (41) and simplified (45) expressions for $P(\Delta t)$. Our main conclusion of this fit is that the clustering events play the dominant role in the distribution $P(\Delta t)$.

C. Wait time distribution. Theory confronts the observations.

We are now in position to compare the occurrence probability presented on Fig. 7 in ref. [1] with our theoretical formula (41) which deviates from Poisson distribution as it includes the clustering events.
First, we have to comment that the occurrence probability plotted on Fig. 7 in ref. [1] is different from the wait time distribution $P(\Delta t)$ defined in the previous subsection. It is convenient to explain the difference using the description in terms of the discrete bins $[\Delta t_i, \Delta t_{i+1}]$. In these terms, Fig. 7 of ref. [1] is a histogram, where the blue points represent the values $n_i/N$ where $n_i$ is the number of events with wait time located in the bin $[\Delta t_i, \Delta t_{i+1}]$ and $N$ is the total number of events. However, by definition, the wait time distribution $P(\Delta t_i)$ is obtained by dividing $n_i/N$ by the bin width $[\Delta t_{i+1} - \Delta t_i]$ for proper normalization of $P(\Delta t_i)$. Indeed,

$$P(\Delta t_i) = \frac{n_i}{N} \frac{1}{[\Delta t_{i+1} - \Delta t_i]}, \tag{47}$$

$$\sum_i P(\Delta t_i) [\Delta t_{i+1} - \Delta t_i] = \sum_i \frac{n_i}{N} = 1.$$

The authors of ref. [1] noticed that their data can be nicely fitted using the following function

$$P(\Delta t) = A(\Delta t)^{-\alpha} \exp(-\lambda \Delta t), \tag{48}$$

where the continuum limit is already assumed. We confirm that the good match can indeed be achieved, and the corresponding fit is shown by the red line on Fig. 5.

We already noticed that our formula (41) describing the clustering of events is given by a superposition of the same functions entering (48). The corresponding fit is shown on Fig. 6. One can see that the data from ref. [1] are nicely described by our expression (41) which was derived assuming the non-Poissonian processes due to the clustering events. It is important to emphasize that this nice fit is achieved for parameter $(t_0/T)^\alpha \lesssim 1$ being order of one, which implies that the clustering events play the dominant role in data presented in [1].

As the clustering events play the major role one may wonder if our estimate for parameter $B$ in section IV B may be modified as a result of these clustering events. We think that the corresponding variation is numerically relatively mild, and does not modify the picture advocated in this work. Therefore, we ignore the corresponding modifications in $B$ for the present studies.

We have discussed at length that the presence of such clustering events is a very generic feature of the mechanism for the radio wave emission as presented in this work. We interpret the nice fit shown on Fig. 6 of the data from ref. [1] with our expression (41) as an additional strong support for this mechanism when emissions in radio bands always accompany the nanoflare events which are identified with the AQN annihilation events as advocated in this framework.

One should emphasize that nanoflares are introduced as generic events, producing an impulsive energy release at small scale, see the review papers [8, 9]. It is a highly nontrivial consistency check of the entire framework that the nanoflares being realized as the AQN annihilation

---

10 Indeed, the observation time $T = 4.2 \cdot 10^3 s$, while good fit is achieved for $t_0 \approx (3 - 4) \cdot 10^3 s$ according to Fig. 6.
VI. CONCLUSION AND FUTURE DEVELOPMENT

The main results of the present work can be summarized as follow. The AQN annihilation events (identified with nanoflares) will inevitably be accompanied by radio events. This proposal is consistent with all observations reported by [1], including the frequency of appearance, temporal and spatial distributions, intensity, and other related observables. There are several direct consequences of this idea, which future observations will be able to support or refute:

1. The proposed mechanism suggests that the considerable portion of the radio events recorded at different frequencies might be emitted by one and the same AQN continuously generating radio signals at different frequency bands (as a result of different plasma frequencies at different altitudes). This picture suggests that there must be also a spacial correlation between (naively distinct) radio events in the same local patch with size $\sim 10^5$km in the different frequency bands with time delays measured in seconds.

The observation of the correlated clustering events as discussed in subsections VB, VC is the direct manifestation of these correlations observed in the same frequency band. We advocate the idea that similar correlations from different frequency bands must also exist. In principle this prediction of our proposal can be directly tested by MWA.

2. The same picture also suggests that the lower frequency waves are emitted from higher altitudes. The important point here is that the dependence of the intensity of the emission as a function of the altitude is a highly nontrivial function due to a number of reasons. First, there is a scaling law $r^{-3/2}$ suggesting that the intensity of non-thermal electrons is lower at higher altitudes. Secondly, some portion of the non-thermal electrons will release their energy during passage of the lower altitudes regions before they reach the higher altitudes with lower frequency $\nu$.

As a result of these suppression factors we expect that the low frequency emissions should be, in general, suppressed. Of course the radio emission is related to random processes and highly sensitive to some specific local features of the plasma and non-thermal electrons as discussed in subsection IV A. Therefore, our prediction on suppression is the subject of possible fluctuations within small frequency bands.

This tendency has been indeed observed for 98 MHz band when the recorded number of resolved events is at least one order of magnitude smaller than for three other bands with slightly higher frequencies. We predict that the emission rate with 80 and 89 MHz which had been recorded, but not yet published by [1], should demonstrate even lower rate of resolved events (even in comparison with 98 MHz emission). It is a highly nontrivial specific feature of our proposal as it is very hard to understand this observed property of the radio emission using any alternative models as the electron density in corona (and corresponding $\nu$) is very smooth function in this region, see Fig. 4. In principle this prediction of our proposal can be directly tested by MWA, as the observations according to [1] were done in 12 frequency bands, including low frequency 80 and 89 MHz bands.

3. In contrast with low frequency bands, the event rate for higher frequency bands should be higher than for recorded 160 MHz band due to the same reasons explained in item 2. This prediction can be directly tested in future analysis by studying the higher frequency emissions with $\nu > 160$ MHz as the observations according to [1] were done in 12 frequency bands, including high frequency 179, 196, 217 and 240 MHz bands. One should comment here that even higher frequency radiation with $\nu > 240$ MHz is the subject of strong absorption where the observed intensity shows some signs of suppression [68]. This strong absorption of high frequency bands obviously limits our perspectives to study the higher frequency emissions within proposed mechanism.

4. There was a recent announcement by the Solar Orbiter probe of observations of so-called “campfires” in the extreme ultraviolet frequency bands. It is naturally to identify such objects with large sized AQNs. These large AQNs represent the topic of the present work as they are capable to generate sufficiently strong radio signals which can be resolved. We suggest that the same events will have sufficient intensity to be visible in the extreme ultraviolet and can be recorded by the Solar Orbiter. Therefore, we suggest to study a possible cross correlation between MWA radio signals and Solar Orbiter recording of the extreme ultraviolet photons to support or refute this proposal.

ACKNOWLEDGEMENTS

This research was supported in part by the Natural Sciences and Engineering Research Council of Canada.

Appendix A: simulations

The appendix shows the details of the MC simulation implemented in this work.

1. simulation setup

The setup of the simulation in present work basically follows that in Ref. [3], which can be divided into three
steps. The first step is to generate a large number of dark matter particles in the solar neighborhood by MC method and collect those that will eventually impact the Sun. The second step is to identify the particles as AQNs and assign masses to them. We will use different valid models of the AQN mass distribution that are shown in (7). The third step is to solve the multiple differential equations that dominate the annihilation process of AQNs in the solar atmosphere.

Step 1. In this step, we simulate a MC sampling of dark matter particles in the solar neighborhood with the positions and velocities randomly assigned from the known distributions. It’s known that the velocity of the dark matter particles with respect to the solar system follows a Maxwellian distribution:

\[
    f_v(d\vec{v})d^3\vec{v} = \frac{d^3\vec{v}}{(2\pi\sigma^2)^{3/2}} \exp\left(-\frac{v_x^2 + v_y^2 + (v_z - v_0)^2}{2\sigma^2}\right) \tag{A1}
\]

where the velocity dispersion is \(\sigma \approx 110 \text{ km/s}\), and the velocity shift \(v_0 \approx 220 \text{ km/s}\) is due to the relative motion between the Sun and the dark matter halo.

The positions of particles are generated in such a way that they initially uniformly populate in a spherical shell around the Sun. The inner and outer boundaries of the spherical shell are respectively \(R_{\text{min}} = 1 \text{ AU}\) and \(R_{\text{max}} = 10 \text{ AU}\). Note that our choice of \(R_{\text{min}}\) is different from Ref. [3] where \(R_{\text{min}} = R_\odot\) there. Choosing a larger \(R_{\text{min}}\) is to reduce the effect of the Sun’s gravity on the initial velocity distribution (A1). The solar escape velocity at 1 AU is \(v_e \approx 42 \text{ km/s}\), so when a particle moves from infinity with the typical velocity \(v_0 = 220 \text{ km/s}\) to this distance, the velocity increment due to the Sun’s gravity is \(\Delta v = \sqrt{v_0^2 + v_e^2} - v_0 \approx 4 \text{ km/s}\) which is very small compared with \(v_0\).

Now, we can generate the MC sampled 3D positions (i.e. an uniform distribution inside the given spherical shell) and velocities (i.e. (A1)) for each particle. Same as Ref. [3], we generated \(N_{\text{sample}} = 2 \times 10^{10}\) such particles. These particles then move following the Newton’s gravity law attracted by the Sun. Since we know the initial position and velocity of each particle, we can calculate the trajectory of each particle using the classical two-body orbit dynamics. The criteria to determine whether or not a particle will impact the Sun are also the same as Ref. [3]. For a given particle, if the perihelion of the hyperbolic trajectory is smaller than \(R_\odot\) (and also if the velocity direction is inward), then it will impact the Sun. It turns out that from the sample of \(2 \times 10^{10}\), the number of particles that will impact the Sun is \(N_{\text{imp}} = 30457\). The trajectory and impact properties of these impacting particles are shown in Fig. 7.

The expression of the impact parameter \(b\) is

\[
    b = r_p \sqrt{1 + \frac{2GM_\odot}{r_p v_0^2}} \tag{A2}
\]

where \(r_p\) is the perihelion distance, \(v_0\) is the particle velocity at infinity that can be extrapolated from the initial velocity and position simulated, i.e. \(v_0 = \sqrt{v_i^2 - 2GM_\odot/r_i}\). The impacting requires \(0 \leq r_p \leq R_\odot\). If we take \(r_p = R_\odot\), we get the maximum impact parameter \(b_{\text{max}}\). The distribution of the impact parameter (in the form of \(b/b_{\text{max}}\)) is shown in the subplot (b) of Fig. 7.

From the distribution of impact time as shown in the subplot (c) of Fig. 7, we can calculate the impact rate. Following the logic in Ref. [3], we should only use the time window where the rate is constant. We choose it as \(t_{\text{imp}} \in [0.5, 1.5]\) months where the boundaries are denoted as two vertical lines in the plot. The rate in the time window is constant because the dominant part of particles impacting the Sun are the particles from the spherical shell between \(R_{\text{min}}\) and \(R_{\text{max}}\). Outside the time window, we see the rate drops because we did not simulate the particles outside the spherical shell. The impact rate is \(N(\Delta t_{\text{imp}})/\Delta t_{\text{imp}}\) where \(N(\Delta t_{\text{imp}})\) is the number of particles impacting the Sun in the time window chosen above. Note that this impact rate is not the true impact rate because the number of AQNs simulated, \(N_{\text{sample}} = 2 \times 10^{10}\), is not the true number of AQNs inside the spherical shell.

To convert the impact rate to the true impact rate, we need to multiply it by the scaling factor \(f_s\) which is the ratio of the true number of AQNs in the spherical shell to \(N_{\text{sample}}\):

\[
    f_S = \frac{\frac{4}{3}\pi(R_{\text{max}}^3 - R_{\text{min}}^3) \cdot n_{\text{AQN}}}{N_{\text{sample}}} \tag{A3}
\]

where \(n_{\text{AQN}}\) is the number density of antimatter AQNs in the solar system:

\[
    n_{\text{AQN}} = \left(\frac{2}{3} \cdot \frac{3}{5} \cdot \rho_{\text{DM}}\right) \cdot \frac{1}{m_p(B)} \tag{A4}
\]

\(\rho_{\text{DM}} \approx 0.3 \text{ GeV cm}^{-3}\) is the dark matter density in the solar system. \(3/5\) of the dark matter is in the form of antimatter AQN; \(2/3\) mass of an AQN is in the form of baryons (the remaining \(1/3\) is in the form of axions). \(m_p\) is the proton mass. \(B\) is the average baryon number carried by an AQN. It depends on different models of AQN mass distribution that will be discussed in Step 2. Thus, the true rate of (antimatter) AQNs impacting the sun is

\[
    \frac{dN}{dt} = \frac{N(\Delta t_{\text{imp}})}{\Delta t_{\text{imp}}} \cdot f_S, \quad t_{\text{imp}} \in [0.5, 1.5] \text{ months.} \tag{A5}
\]

In comparison, the number obtained in Ref. [3] is 36123. The difference is beyond the statistical fluctuation. This difference occurs not only due to our choice of a larger \(R_{\text{min}}\), the inner boundary of the initially simulated region, but also a technical detail that a different method (more appropriate) is chosen in determining the perihelion. However, all of these changes have no significant effects on the results as we can see in Fig. 7 by comparing it with Ref. [3].
Step 2. Now, we are ready to assign masses (baryon numbers) to the AQNs collected in Step 1 that will impact the Sun. For each AQN, its mass is assigned randomly with the probability following one of the three models of power-law distribution, (7). Thus, we have three copies of all the impacting AQNs with different mass distributions.

Step 3. The evolution of an AQN in the solar atmosphere can be described by multiple differential equations, including the kinetic energy loss due to friction and the mass loss due to the annihilation events of the antibaryons carried by AQN with the baryons in the atmosphere. All of these differential equations are explicitly illustrated and listed in Ref. [3], and we do not need to bother to repeat them here. Step 3 is to solve these differential equations numerically for the $N_{\text{imp}}$ AQNs. To do this, in addition to the initial conditions and AQN masses that have been generated in the above two steps, we also need to know the density and temperature profiles of the solar atmosphere above the photosphere. In this work, we adopt the profiles presented in Ref. [66] which are more updated compared with those used in the previous work, Ref. [3].

Finally, we get the numerical solution which is about the mass loss varying with time (or equivalently, height above the solar photosphere) for the $N_{\text{imp}}$ AQNs. By analyzing these data, we obtain the information needed for the present work.

2. Results

The results obtained from the numerical simulations are presented in the main text. Here, we elaborate more on the details about the plots.

Fig. 1 shows the rate of AQNs in the mass range $[B, B_{\text{max}}]$ impacting the Sun. The rate is calculated as (A5) but with only large AQNs $B \geq \bar{B}$ taken into account. Varying the value of the cutoff $\bar{B}$ from $B_{\text{min}}$ to $B_{\text{max}}$, we then get how the impact rate depends on $\bar{B}$ as shown in the figure. The rate at $\bar{B} = B_{\text{min}}$ is the total impact rate. For the three groups, the total impact rates are respectively $4.17 \times 10^4 \text{ s}^{-1}$, $1.52 \times 10^4 \text{ s}^{-1}$ and $3.52 \times 10^3 \text{ s}^{-1}$ which match well Ref. [3] (see Figure 8 there).

In addition, the luminosity $L^\odot$ can be calculated as $L^\odot = 2 \langle \Delta m \rangle c^2 \cdot dN/dt$ where $\Delta m$ is an AQN’s mass loss in its trajectory through the solar atmosphere before entering the dense region, the photosphere. Similarly, we can compute the luminosity $L^\odot_{\bar{B}}$ by counting large AQNs ($B \geq \bar{B}$) only, and the result is shown in Fig. 2 for different groups of mass distribution. The total luminosity is obtained at $\bar{B} = B_{\text{min}}$. For the three groups, the total luminosity are respectively $1.05 \times 10^{27} \text{ erg} \cdot \text{s}^{-1}$, $1.07 \times 10^{27} \text{ erg} \cdot \text{s}^{-1}$ and $1.06 \times 10^{27} \text{ erg} \cdot \text{s}^{-1}$ which match well Ref. [3] (see Figure 10 there).

One may notice that in the two left subfigure of Fig. 1 and Fig. 2, the simulated lines become zigzag at large baryon numbers. This is because the proportion of large
AQN's is actually very small. Despite the number of all the impacting AQNs is as large as $30457$, the power-law index $\alpha \sim (2 - 2.5)$ makes the hit rate with large $B$ is very tiny when assigning masses to AQNs randomly in Step 2. For example, our statistical result shows that in Group 1, the number of AQNs with $B \geq 5 \times 10^{26}$ is only 12, and the number of AQNs with $B \geq 10^{27}$ is only 3. Such tiny number causes large statistical fluctuation, so we see the zigzags in the two left subfigures. We have to generate enough large AQNs to remove the large statistical fluctuation.

We resolve this technical problem as follows. We simulate another $10^{10}$ AQNs by redoing the three steps in the setup as described above. We call this procedure as the second-round simulation. We get 15019 AQNs that will impact the Sun out of the total $10^{10}$ AQNs. The masses (baryon numbers) assigned to these 15019 impacting AQNs are constrained in the range $B \in [B_L, B_{\text{max}}]$. $B_L$ for each group should be chosen well above $B_{\text{min}}$ to ensure that enough large AQNs can be generated, but $B_L$ should not exceed the start of the zigzags. Although we did not simulate all AQNs in this second-round simulation, we can extrapolate the “number” of impacting AQNs in the full mass range by looking at the proportion of large AQNs ($B \in [B_L, B_{\text{max}}]$) in the full mass range. Furthermore, we can calculate the extrapolated $N_{\text{sample}}$ and the extrapolated scaling factor $f_s$. Finally, we obtain the true impact rate of these large AQNs simulated in the second-round simulation. Similarly, we obtain the luminosity. The results are shown in the two right subfigures of Fig. 1 and Fig. 2, where we see that the large statistical fluctuation disappears.

---

[1] S. Mondal, D. Oberoi, and A. Mohan, The Astrophysical Journal 895, L39 (2020).
[2] A. Zhitnitsky, JCAP 10, 050 (2017), arXiv:1707.03400 [astro-ph.SR].
[3] A. Zhitnitsky, Sov. J. Nucl. Phys. 25 (1979).
[4] S. Weinberg, Physics Today 59, 30 (2006).
[5] J. Madsen, in Hadrons in Dense Matter and Hadrosynthesis, Lecture Notes in Physics, Berlin Springer Verlag, Vol. 516, edited by J. Cleymans, H. B. Geyer, and F. G. Scholtz (1999) p. 162, astro-ph/9809032.
[6] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Physical Review Letters 43, 103 (1979).

---

$13^\text{rd}$ The advantage of assigning AQNs masses only in the range of $[B_L, B_{\text{max}}]$ is that we don’t repeat generating a huge amount of small AQNs which are far more than needed and only to make the simulations extremely time-consuming.
[39] A. Zhitnitsky, Phys. Rev. D 101, 083020 (2020), arXiv:1909.05320 [hep-ph].
[40] G. W. Fraser, A. M. Read, S. Sembay, J. A. Carter, and E. Schyns, Mon. Not. Roy. Astron. Soc. 445, 2146 (2014), arXiv:1403.2436 [astro-ph.HE].
[41] S. Ge, H. Rachmat, M. S. R. Siddiqui, L. Van Waerbeke, and A. Zhitnitsky, (2020), arXiv:2004.00632 [astro-ph.HE].
[42] D. Budker, V. V. Flambaum, and A. Zhitnitsky, (2020), arXiv:2003.07363 [hep-ph].
[43] A. Zhitnitsky, (2020), arXiv:2008.04325 [astro-ph.HE].
[44] D. M. Jacobs, G. D. Starkman, and B. W. Lynn, Mon. Not. R. Astron. Soc. 450, 3418 (2015), arXiv:1410.2236.
[45] K. Lawson, X. Liang, A. Mead, M. S. R. Siddiqui, L. Van Waerbeke, and A. Zhitnitsky, Phys. Rev. D 100, 043531 (2019), arXiv:1905.00022 [astro-ph.CO].
[46] E. T. Herrin, D. C. Rosenbaum, and V. L. Teplitz, Phys. Rev. D 73, 043511 (2006), arXiv:astro-ph/0505584.
[47] J. Singh Sidhu, R. J. Scherrer, and G. Starkman, (2020), arXiv:2006.01200 [astro-ph.CO].
[48] E. N. Parker, Astrophys. J. 330, 474 (1988).
[49] S. Krucker and A. O. Benz, Sol. Phys. 191, 341 (2000), astro-ph/9912501.
[50] A. O. Benz and S. Krucker, in Recent Insights into the Physics of the Sun and Heliosphere: Highlights from SOHO and Other Space Missions, IAU Symposium, Vol. 203, edited by P. Brekke, B. Fleck, and J. B. Gurman (2001) p. 471. astro-ph/0012106.
[51] U. Mitra-Kraev and A. O. Benz, Astronomy & Astrophysics 373, 318 (2001), astro-ph/0104218.
[52] A. O. Benz and S. Krunker, Astrophys. J. 568, 413 (2002), astro-ph/0109027.
[53] A. O. Benz and P. C. Grigis, Advances in Space Research 32, 1035 (2003), astro-ph/0308323.
[54] A. Pauluhn and S. K. Solanki, Astronomy & Astrophysics 462, 311 (2007), astro-ph/0612585.
[55] I. G. Hannah, S. Christe, S. Krucker, G. J. Hurford, H. S. Hudson, and R. P. Lin, Astrophys. J. 677, 704 (2008), arXiv:0712.2544.
[56] S. Bingert and H. Peter, Astronomy & Astrophysics 550, A30 (2013), arXiv:1211.6417 [astro-ph.SR].
[57] S. Terzo, F. Reale, M. Miceli, J. A. Klimchuk, R. Kano, and S. Tsuneta, Astrophys. J. 736, 111 (2011), arXiv:1105.2506 [astro-ph.SR].
[58] S. J. Bradshaw, J. A. Klimchuk, and J. W. Reep, Astrophys. J. 758, 53 (2012), arXiv:1209.0737 [astro-ph.SR].
[59] D. B. Jess, M. Mathioudakis, and P. H. Keys, Astrophys. J. 795, 172 (2014), arXiv:1409.6726 [astro-ph.SR].
[60] A. S. Kirichenko and S. A. Bogachev, Astrophys. J. 840, 45 (2017), arXiv:1706.05852 [astro-ph.SR].
[61] C. Mac Cormack, A. M. Vásquez, M. López Fuentes, F. A. Nuevo, E. Landi, and R. A. Frazin, Astrophys. J. 843, 70 (2017), arXiv:1706.00365 [astro-ph.SR].
[62] S. Bertolucci, K. Zioutas, S. Hofmann, and M. Maroudas, Physics of the Dark Universe 17, 13 (2017), arXiv:1602.03666 [astro-ph.SR].
[63] Bingert, S. and Peter, H., A&A 550, A30 (2013).
[64] G. Thejappa, Solar Physics 132, 173 (1991).
[65] A. Zhitnitsky, Phys. Dark Univ. 22, 1 (2018), arXiv:1801.01509 [astro-ph.SR].
[66] E. H. Avrett and R. Loeser, Astrophys. J. Supp. 175, 229 (2008).
[67] R. Sharma, D. Oberoi, and M. Arjunwadkar, The Astrophysical Journal 852, 69 (2018).
[68] D. Oberoi, R. Sharma, and A. E. E. Rogers, Solar Physics 292, 1 (2017).
[69] M. S. Wheatland, P. A. Sturrock, and J. M. McTiernan, The Astrophysical Journal 509, 448 (1998).
[70] M. J. Aschwanden and J. M. McTiernan, The Astrophysical Journal 717, 683 (2010).