Exclusive charmonium decays

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The role of power corrections and higher Fock-state contributions to exclusive charmonium decays will be discussed. It will be argued that the $J/\psi (\psi')$ decays into baryon-antibaryon pairs are dominated by the valence Fock-state contributions. $P$-wave charmonium decays, on the other hand, receive strong contributions from the $c\bar{c}g$ Fock states since the valence Fock-state contributions are suppressed in these reactions. Numerical results for $J/\psi (\psi') \rightarrow BB$ decay widths will be also presented and compared to data.

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1. INTRODUCTION

Exclusive charmonium decays have been investigated within perturbative QCD by many authors, e.g. [1]. It has been argued that the dominant dynamical mechanism is $c\bar{c}$ annihilation into the minimal number of gluons allowed by colour conservation and charge conjugation, and subsequent creation of light quark-antiquark pairs forming the final state hadrons ($c\bar{c} \rightarrow n(g^*) \rightarrow m(q\bar{q})$). The dominance of annihilation through gluons is most strikingly reflected in the narrow widths of charmonium decays into hadronic channels in a mass region where strong decays typically have widths of hundreds of MeV. Since the $c$ and the $\sigma$ quarks only annihilate if their mutual distance is less than about $1/m_c$ (where $m_c$ is the $c$-quark mass) and since the average virtuality of the gluons is of the order of $1 - 2 \text{ GeV}^2$ one may indeed expect perturbative QCD to be at work although corrections are presumably substantial ($m_c$ is not very large).

In hard exclusive reactions higher Fock-state contributions are usually suppressed by inverse powers of the hard scale, $Q$, appearing in the process ($Q = 2m_c$ for exclusive charmonium decays), as compared to the valence Fock-state contribution. Hence, higher Fock-state contributions are expected to be negligible in most cases. Charmonium decays are particularly interesting because the valence Fock-state contributions are often suppressed for one or the other reason. In such a case higher Fock-state contributions or other peculiar contributions such as power corrections or small components of the hadronic wave functions may become important. In the following I present a few examples of charmonium decays with suppressed valence Fock-state contributions:

1. Hadronic helicity non-conserving processes (e.g. $J/\psi \rightarrow \rho \pi, \eta_c \rightarrow B\bar{B}, \chi_c \rightarrow B\bar{B}$). The standard method of calculating the valence Fock-State contributions leads to vanishing helicity non-conserving amplitudes. There are many attempts to understand the helicity non-conserving processes (e.g. intrinsic charm of the $\rho$ meson [2]; diquarks in baryons [3]) but a satisfactory explanation of all these processes is still lacking.

2. $G$ parity violating $J/\psi$ decays (e.g. $\pi^+\pi^-, \omega\pi^0, \eta\rho$). These reactions can proceed through the electromagnetic elementary process $c\bar{c} \rightarrow \gamma \rightarrow n(q\bar{q})$ and/or may receive contributions from the isospin violating part of QCD. In general, these contributions are rather small and thus other contributions may play an important role here.

3. Radiative $J/\psi$ decays into light pseudoscalar mesons. The purely electromag-
nomic process $c\bar{c} \to \gamma^* \to \gamma q\bar{q}$, a contribution of order $\alpha^3$ being proportional to the time-like $\pi\gamma$ transition form factor at $s = M^2_{J/\psi}$, together with a power correction provided by the VDM contribution $J/\psi \to \rho\pi, \rho \to \gamma$ [8], leads to $\Gamma(J/\psi \to \pi^0\gamma) = 2.86 \text{ eV}$ in agreement with experiment (3.43±1.2 eV [8]). Similar estimates of the $\eta\gamma$ and $\eta'\gamma$ widths fail. Agreement with experiment is here obtained from a twist-4 gluon component of the singlet-$\eta$ state [8], i.e. from a power correction. That gluon component can occur as a consequence of the $U(1)$ anomaly.

4. $\chi_{cJ}$ decays. For the $\chi_{cJ}$ mesons the $c\bar{c}$ pair forms a colour-singlet $P$-wave in the valence Fock state (notation: $c\bar{c}_1(3P_J)$). The next-higher Fock state is a $c\bar{c}_8$ $S$-wave where the quark-antiquark pair forms a $c\bar{c}_8(3S_1)$ state. For this reason the latter contribution is customarily referred to as the colour-octet contribution. The colour-singlet and octet contributions to the $\chi_{cJ} \to h\bar{h}$ decay amplitude behave as [8]

$$M^{(c)}_J \sim \frac{f^2}{f^2} f^{(c)}(3P_J) m_c^{-n_c}. \quad (1)$$

The singlet decay constant, $f^{(1)}(3P_J)$, which represents the derivative of the two-particle (non-relativistic) coordinate space wave function at the origin, and the octet decay constant, $f^{(8)}(3P_J)$, as a three-particle wave function at the origin, are of the same dimension, namely GeV$^2$. Hence, $n_1 = n_8$. In fact, $n_8 = 1 + 2n_h$ where $n_h$ is the dimension of the light hadron’s decay constant, $f_h$. It is, therefore, unjustified to neglect the colour-octet contributions in the $\chi_{cJ}$ decays.

2. VELOCITY SCALING

Recently, the importance of higher Fock states in understanding the production and the inclusive decays of charmonium has been pointed out [8]. As has been shown the long-distance matrix elements can there be organized into a hierarchy according to their scaling with $v$, the typically velocity of the $c$ quark in the charmonium. One may apply the velocity expansion to exclusive charmonium decays as well [8]. The Fock-state expansions of the charmonia start as

$$|J/\psi\rangle = O(1)\langle c\bar{c}_1(3S_1)\rangle + O(v)\langle c\bar{c}_8(3P_J)gg\rangle + O(v^2),$$

$$|\chi_{cJ}\rangle = O(1)\langle c\bar{c}_1(3P_J)\rangle + O(v)\langle c\bar{c}_8(3S_1)gg\rangle + O(v^2). \quad (2)$$

Combining the fact that the hard scattering amplitude involving a $P$-wave $c\bar{c}$ pair is of order $v$, with the Fock-state expansion (2), one finds for the amplitudes of $\chi_{cJ}$ decays into, say, a pair of pseudoscalar mesons the behaviour

$$M(\chi_{cJ} \to P\bar{P}) \sim a_1 a_2 \beta + b_1 \beta^2 \sqrt{\alpha_s^{soft}} + O(v^2), \quad (3)$$

where $a$ and $b$ are constants and $\alpha_s^{soft}$ comes from the coupling of the Fock-state gluon to the hard process. For the decay reaction $J/\psi \to B\bar{B}$, on the other hand, one has

$$M(J/\psi \to B\bar{B}) \sim a_1 a_2 \beta + b_1 \beta^2 \sqrt{\alpha_s^{soft}} + O(v^3). \quad (4)$$

Thus, one sees that in the case of the $\chi_{cJ}$ the colour-octet contributions are not suppressed by powers of either $v$ or $1/m_c$ as compared to the contributions from the valence Fock states [8]. Hence, the colour-octet contributions have to be included for a consistent analysis of $P$-wave charmonium decays. Indeed, as an explicit analysis reveals [8], only if both the contributions are taken into account agreement between predictions and experiment is obtained for the $\chi_{cJ} \to P\bar{P}$ decay widths. For more details and numerical results for decay widths, see the talk by G. Schuler in these proceedings. The situation is different for $J/\psi$ decays into baryon-antibaryon pairs: Higher Fock state contributions first start at $O(v^2)$, see [8]. Moreover, there is no obvious enhancement of the corresponding hard scattering amplitudes, they appear with at least the same power of $\alpha_s$ as the valence Fock state contributions. Thus, despite of the fact that $m_c$ is not very large and
v not small ($v^2 \simeq 0.3$), it seems reasonable to expect small higher Fock-state contributions to the baryonic decays of the $J/\psi$.

In the following sections I will report on an analysis of the processes $J/\psi(p) \to B\bar{B}$ performed with regard to these observations [1].

3. THE MODIFIED PERTURBATIVE APPROACH

Recently, a modified perturbative approach has been proposed [1] in which transverse degrees of freedom as well as Sudakov suppressions are taken into account in contrast to the standard approach of Brodsky and Lepage [11]. The modified perturbative approach possesses the advantage of strongly suppressed end-point regions. In these regions perturbative QCD cannot be applied. Moreover, the running of $\alpha_s$ and the evolution of the hadronic wave function can be taken into account properly.

Within the modified perturbative approach an amplitude for a $2S+1L_J$ charmonium decay into two light hadrons, $h$ and $h'$, is written as a convolution with respect to the usual momentum fractions, $x_i, x_i'$ and the transverse separations scales, $b_i, b_i'$, canonically conjugated to the transverse momenta, of the light hadrons. Structurally, such an amplitude has the form

$$M^{(c)}(2S+1L_J \to hh') = f^{(c)}(2S+1L_J) \times \int [dx][dx'] \int \frac{d^2b}{(4\pi)^2} \frac{d^2b'}{(4\pi)^2} \Psi_h^T(x, b) \Psi_h'^T(x', b') \times \tilde{T}_H^{(c)}(x, x', b, b', t) \exp \left[-S(x, x', b, b', t)\right],$$

(5)

where $x^{(c)}$, $b^{(c)}$ stand for sets of momentum fractions and transverse separations characterizing the hadron $h^{(c)}$. Each Fock state (see [2]) provides such an amplitude (marked by the upper index $c$). $\Psi_h$ denotes the transverse configuration space (light-cone) wave function of a light hadron. The $f^{(c)}$ is the charmonium decay constant already introduced in Sect. 1. Because the annihilation radius this is, to a reasonable approximation, the only information on the charmonium wave function required. $\tilde{T}_H^{(c)}$ is the Fourier transform of the hard scattering amplitude to be calculated from a set of Feynman graphs relevant to the considered process. $t$ represents a set of renormalization scales at which the $\alpha_s$ appearing in $\tilde{T}_H^{(c)}$ are to be evaluated. The $t_i$ are chosen as the maximum scale of either the longitudinal momentum or the inverse transverse separation associated with each of the internal gluons. Finally, $\exp [-S]$ represents the Sudakov factor which takes into account gluonic corrections not accounted for in the QCD evolution of the wave functions as well as a renormalization group transformation from the factorization scale $\mu_F$ to the renormalization scales. The gliding factorization scale to be used in the evolution of the wave functions is chosen to be $\mu_F = 1/b$ where $b = \max\{b_i\}$. The $b$ space form of the Sudakov factor has been calculated in next-to-leading-log approximation by Botts and Sterman [1].

As mentioned before, exclusive charmonium decays have been analysed several times before, e.g. [1]. New refined analyses are however necessary for several reasons: in previous analyses the standard hard scattering approach has been used with the running of $\alpha_s$ and the evolution of the wave function ignored. In the case of the $cJ$ also the colour-octet contributions have been disregarded. Another very important disadvantage of some of the previous analyses is the use of light hadron distribution amplitudes (DAs), representing wave functions integrated over transverse momenta, that are strongly concentrated in the end-point regions. Despite of their frequent use, such DAs were always subject to severe criticism, see e.g. [12]. In the case of the pion, they lead to clear contradictions to the recent CLEO data [13] on the $\pi\gamma$ transition form factor, $F_{\pi\gamma}$. As detailed analyses unveiled, the $F_{\pi\gamma}$ data require a DA that is close to the asymptotic form $\propto x(1-x)$ [13,14]. Therefore, these end-point region concentrated DAs should be discarded for the pion and perhaps also for other hadrons like the nucleon [13].

\[\text{In higher Fock-state contributions additional integration variables may appear.}\]
4. RESULTS FOR $J/\psi (\psi') \to B \bar{B}$

According to the statements put forward in Sects. 1 and 2, higher Fock-state contributions are neglected in this case. The $J/\psi$ colour-singlet component of the orbital angular momentum is assumed to be zero. Since SU(3) component of the orbital angular momentum is it follows that there is only one independent scalar quarks have to be coupled in an isospin 1

$$|J/\psi; q, \lambda; e \bar{c}_1\rangle = \frac{\delta_{ab}}{\sqrt{3}} \frac{f_{J/\psi}}{2 \sqrt{6}} \times \frac{1}{\sqrt{2}} (\sigma + M_{J/\psi}) f(\lambda). \quad (6)$$

The $J/\psi$ decay constant $f_{J/\psi} (= f^{(1)}(3S_1))$ is obtained from the electronic $J/\psi$ decay width and found to be 409 MeV. Except in phase space factors, the baryon masses are ignored and $M_{J/\psi}$ is replaced by $2m_c$ for consistency since the binding energy is an $O(v^2)$ effect.

From the permutation symmetry between the two $u$ quarks and from the requirement that the three quarks have to be coupled in an isospin 1/2 state it follows that there is only one independent scalar wave function in the case of the nucleon if the 3-component of the orbital angular momentum is assumed to be zero. Since SU(3)$_F$ is a good symmetry, only mildly broken by quark mass effects, one may also assume that the other octet baryons are described by one scalar wave function. It is parameterized as

$$\Psi_{123}^B(x, k_\perp) = \frac{f_{B_B}}{8 \sqrt{3!}} \phi_{123}^B(x) \Omega_{B_B}(x, k_\perp) \quad (7)$$

in the transverse momentum space. The set of indices 123 refers to the quark configuration $u_+, u_-, d_+$; the wave functions for other quark configurations are to be obtained from (6) by permutations of the indices. The transverse momentum dependent part $\Omega$ is parameterized as a Gaussian in $k_{\perp i}/x_i$ ($i = 1, 2, 3$). The transverse size parameter, $a_{B_B}$, appearing in that Gaussian, as well as the octet-baryon decay constant, $f_{B_B}$, are assumed to have the same value for all octet baryons. In these two parameters as well as the nucleon DA have been determined from an analysis of the nucleon form factors and valence quark structure functions at large momentum transfer ($a_{B_B} = 0.75 \text{GeV}^{-1}$; $f_{B_B} = 6.64 \times 10^{-3} \text{GeV}^2$) at a scale of reference $\mu_0 = 1 \text{GeV}$). The DA has been found to have the simple form

$$\phi_{123}^N(x, \mu_0) = 60 x_1 x_2 x_3 [1 + 3 x_1]. \quad (8)$$

It behaves similar to the asymptotic form, only the position of the maximum is shifted slightly. For the hyperon and decuplet baryon DAs suitable generalizations of the nucleon DA are used. Calculating the hard scattering amplitude from the Feynman graphs for the elementary process $c \bar{c} \to 3g^* \to 3(q\bar{q})$ and working out the convolution (5), one obtains the widths for the $J/\psi$ decays into $B \bar{B}$ pairs listed and compared to experimental data in Table 1. As can be seen from that table a rather good agreement with the data is obtained.

In addition to the three-gluon contribution there is also an isospin-violating electromagnetic one generated by the subprocess $c \bar{c} \to \gamma^* \to 3(q\bar{q})$. According to this contribution seems to be small.

The extension of the perturbative approach to the baryonic decays of the $\psi'$ is now a straightforward matter. One simply has to rescale the $J/\psi$ widths by the ratio of the corresponding electron widths

$$\Gamma(\psi' \to B \bar{B}) = \frac{\rho_{p.s.}(m_B/M_{\psi'})}{\rho_{p.s.}(m_{B_B}/M_{J/\psi})} \times \frac{\Gamma(\psi' \to e^+ e^-)}{\Gamma(J/\psi \to e^+ e^-)} \Gamma(J/\psi \to B \bar{B}), \quad (9)$$

where $\rho_{p.s.}(z) = \sqrt{1 - 4z^2}$ is the phase space factor. Contrary to previous calculations the $\psi'$ and the $J/\psi$ widths do not scale as $(M_{J/\psi}/M_{\psi'})^8$ since the hard scattering amplitude is evaluated with $2m_c$ instead of the bound-state mass. Results for the baronic decay widths of the $\psi'$ are presented in Table 2. Again good agreement between theory and experiment is to be seen with perhaps the exception of the $3\Sigma^{0}_{0}$ channel. An additional factor of $(M_{J/\psi}/M_{\psi'})^8$ ($= 0.25$) in (9) would clearly lead to disagreement with the data.

5. CONCLUSIONS

Exclusive charmonium decays constitute an interesting laboratory for investigating power corrections and higher Fock-state contributions. In
Table 1
Results for $J/\psi \rightarrow B\bar{B}$ decay widths (in eV) taken from [9]. The three-gluon contributions are evaluated with $m_c = 1.5$ GeV, $\Lambda_{QCD} = 210$ MeV and $a_{B_{10}} = 0.85$ GeV$^{-1}$.

| channel | $p\bar{p}$ | $\Sigma^0\Sigma^0$ | $\Lambda\bar{\Lambda}$ | $\Xi^-\Xi^+$ | $\Delta^{++}\Delta^{-}$ | $\Sigma^*\Sigma^{*+}$ |
|---------|-----------|----------------|-----------------|-------------|----------------|----------------|
| $\Gamma_{3g}$ | 174 | 113 | 117 | 62.5 | 65.1 | 40.8 |
| $\Gamma_{exp}$ [9] | 188 $\pm$ 14 | 110 $\pm$ 15 | 117 $\pm$ 14 | 78 $\pm$ 18 | 96 $\pm$ 26 | 45 $\pm$ 6 |

Table 2
The three-gluon contributions to the $\psi' \rightarrow B\bar{B}$ decay widths (in eV) taken from [9].

| channel | $p\bar{p}$ | $\Sigma^0\Sigma^0$ | $\Lambda\bar{\Lambda}$ | $\Xi^-\Xi^+$ | $\Delta^{++}\Delta^{-}$ | $\Sigma^*\Sigma^{*+}$ |
|---------|-----------|----------------|-----------------|-------------|----------------|----------------|
| $\Gamma_{3g}$ | 76.8 | 55.0 | 54.6 | 33.9 | 32.1 | 24.4 |
| $\Gamma_{exp}$ [7] | 76 $\pm$ 14 | 26 $\pm$ 14 | 58 $\pm$ 12 | 23 $\pm$ 9 | 25 $\pm$ 8 | 16 $\pm$ 8 |

particular one can show that in the decays of the $\chi_{cJ}$ the contributions from the next-higher charmonium Fock state, $c\bar{c}g$, are not suppressed by powers of $m_c$ or $v$ as compared to the $c\bar{c}$ Fock state and therefore have to be included for a consistent analysis of these decays. For $J/\psi$ ($\psi'$) decays into $B\bar{B}$ pairs the situation is different: Higher Fock-state contributions are suppressed by powers of $1/m_c$ and $v$. Indeed, as an explicit analysis reveals, with plausible baryon wave functions a reasonable description of the baryonic $J/\psi$ ($\psi'$) decay widths can be obtained alone from the $c\bar{c}$ Fock state.

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