Magnon-driven dynamics of frustrated skyrmion in synthetic antiferromagnets: effect of skyrmion helicity oscillation

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Abstract

A theoretical study on the interplay of frustrated skyrmion and magnons should reveal new physics and future experiment designs. In this study, we investigate the magnon-driven dynamics of frustrated skyrmion in synthetic antiferromagnets based on micromagnetic simulations, focusing on the effect of skyrmion helicity oscillation. The oscillation speed and Hall angle of the frustrated skyrmion depending on the magnon intensity and damping constant are simulated, which demonstrates that the skyrmion helicity oscillation effectively suppresses Hall motion. The elastic scattering theory reveals that the helicity oscillation affects the scattering cross-section of injected magnons, which in turn effectively modulates the skyrmion Hall motion. This study provides a comprehensive understanding of magnon-skyrmion scattering in frustrated magnets, thus benefiting future spintronic and magnonic applications.

1. Introduction

Magnetic skyrmions [1, 2], which are topologically vortex-like spin configurations, have attracted extensive attention since their first discovery in B20 compounds [3]. This is due to their interesting physics and potential applications in skyrmion-based spintronic devices, such as race-track memory and logic units. Ferromagnetic skyrmions have been observed in series of chiral magnets [3–7] and heavy metal/ferromagnetic films [8–12] with inversion broken symmetry, which is attributed to the indispensable Dzyaloshinskii–Moriya interactions (DMI). Moreover, studies have proven that skyrmions can be effectively driven by various stimuli [13], including spin-polarized electric current [14–16], gradient magnetic field [17, 18], and gradient electric oscillating field [19, 20].

Most recently, studies have theoretically predicted [17, 21–25] and experimentally discovered [26–28] that skyrmions can be stabilized by competing interactions in frustrated magnets with inversion symmetry in the absence of DMI. Compared with skyrmions stabilized by DMI, those in frustrated magnets have a new degree of freedom associated with the rotation of helicity due to the fact that the skyrmion energy is independent of the direction of spin rotation [22]. Interestingly, the rotation of helicity couples to the usual translational motion of skyrmions driven by the spin Hall effect induced by the in-plane current, resulting in rotational skyrmion motion [22, 24, 29]. It has been suggested that the rotational motion of skyrmions can be harnessed to build nano-oscillators.

Compared with electric current, magnons as the quanta of spin waves, which drive skyrmion motion without Joule heating due to the absence of physical charge transport, are particularly attractive due to their advantage of low-energy consumption [30–40]. As a matter of fact, the magnon driven skyrmion dynamics in ferromagnets and antiferromagnets have been explored. For example, magnons in ferromagnets are only
right-circularly polarized and deflected by the effective field from skyrmions, which in turn drives the skyrmion toward the magnon source accompanied by Hall motion through momenta exchange [34, 37, 41]. Moreover, circularly polarized magnons are deflected by antiferromagnetic (AFM) skyrmions in the transverse direction, which drives skyrmion Hall motion even in antiferromagnets [38, 40].

The important works cited in the previous paragraphs have revealed interesting electric-driven skyrmion dynamics in frustrated magnets as well as magnon-driven skyrmion dynamics in ferromagnets and antiferromagnets; however, the dynamic properties of frustrated skyrmions driven by polarized magnons remain elusive. On the one hand, it is expected that the translational motion of frustrated skyrmion probably couples to the helicity oscillation mode as magnons pass through the skyrmion. Moreover, the rotation of helicity affects the AFM berry dynamic term, effectively modulating the skyrmion dynamics. As a matter of fact, a strong dependence of magnon-domain wall scattering on the precession speed of the AFM wall was recently revealed [42]. With the increase of the precession speed, the scattering potential well is replaced by the potential barrier, resulting in the reflection of magnons and the forward motion of the wall. To some extent, the skyrmion helicity oscillation could also affect the magnon-driven skyrmion dynamics in frustrated magnets, considering the high relevance between these magnetic structures.

On the other hand, for a helicity oscillating AFM skyrmion, which is a Newtonian particle rotating around an axis, the angular momentum is transferred from the injected magnons to the skyrmion, which definitely affects the whole scattering process. Thus, if frustrated skyrmion dynamics driven by polarized magnons can be clarified, this will assist in selecting potential materials for future experiments. Furthermore, such a clarification will help in the design of devices as well as benefit the development of spintronics and the rising field of magnonics.

In the present study, we investigate the skyrmion dynamics in antiferromagnetically coupled frustrated bilayers driven by circularly polarized magnons by numerical simulations based on the Landau–Lifshitz–Gilbert (LLG) equation. By numerically analyzing the skyrmion dynamics while changing the damping constant and magnon intensity, we find that the skyrmion helicity oscillation effectively suppresses the skyrmion Hall motion. Furthermore, the dependence of scattering cross-section of the magnons on the helicity oscillation speed is uncovered using the elastic scattering theory to better understand the simulated results. Thus, this work reveals the relation between the skyrmion helicity oscillation and Hall motion, which will be applicable to future spintronic and magnonic applications based on frustrated magnets.

The remaining part of this article is organized as follows: in section 2, we present the simulated results and discussion. Section 3 is devoted to the elastic scattering theory, and the conclusion is presented in section 4.

2. Numerical simulations of skyrmion dynamics driven by polarized magnons

In this section, we first present the atomistic spin model and simulation method, and then give the simulated results and discussion.

2.1. Model and methods

We consider two ferromagnetic layers with competing Heisenberg exchange interactions based on the $J_1$–$J_2$–$J_3$ model on a 200 × 200 square lattice [29]:

\[
H = H_{\text{top}} + H_{\text{bottom}} + H_{\text{inter}}
\]

\[
H_{\text{top, bottom}} = -J_1 \sum_{\langle i\delta \rangle} S_i^z \cdot S_\delta^z - J_2 \sum_{\langle\langle i\delta\rangle\rangle} S_i^{1,2} \cdot S_\delta^{1,2} - J_3 \sum_{\langle\langle i\delta\rangle\rangle} S_i^{1,2} \cdot S_\delta^{1,2}
\]

\[-K_0 \sum_i (S_i^z \cdot e_z)^2 \]

\[
H_{\text{inter}} = -J_{\text{inter}} \sum_i S_i^1 \cdot S_i^3,
\]

where $S_i^1$ ($S_i^3$) is the normalized atomic spin on site $i$ in the top/bottom ferromagnetic layer, $J_1 = J$ is the exchange interaction between the nearest neighbor spins, $J_2 = -0.1J$ and $J_3 = -0.15J$ are the interactions between the next-nearest neighbors and next-next-nearest neighbors, respectively, $K_0$ is the anisotropy constant, $e_z$ is the unit vector along the z-axis, and $J_{\text{inter}}$ is the AFM interface coupling between the nearest neighbors [43]. Without loss of generality, $K_0 = 0.02J$ and $J_{\text{inter}} = 0.7J$ are chosen in stabilizing skyrmion.
Figure 1 shows the evolution of spin configurations of the top ((a), (c) and (e)) and bottom ((b), (d) and (f)) layers. The black arrow depicts the rotation direction of spin in skyrmion.

The parameters are chosen to stabilize the skyrmion based on the simulated phase diagram (given in SM figure 1), and the main conclusion hardly be affected by the choice.

The dynamics of the skyrmion driven by magnons is investigated by solving the LLG equation:

\[
\frac{\partial S_i}{\partial t} = -\gamma S_i \times H_i + \alpha S_i \times \frac{\partial S_i}{\partial t},
\]

where \(\alpha\) is the damping constant and \(H_i = -\mu_s^{-1} \partial H/\partial S_i\) is the effective field with the saturation moment \(\mu_s\). Generally, we use the fourth-order Runge–Kutta method to solve the LLG equation, and the time step is \(0.01 \mu_s/\gamma J\).

Circularly polarized magnons are excited in the region of \(x = [60, 64]\) by applying an AC magnetic field \(h_{RH} = h [\cos(\omega_0 t)e_x + \sin(\omega_0 t)e_y]\) with the magnitude \(h\), frequency \(\omega_0 = 0.6\gamma J/\mu_s\), and time \(t\), where \(e_x/e_y\) is the unit vector along the \(x/-y\)-axis, which drives the translational motion of the skyrmion in accompany of the constant helicity oscillation. Here, we intend to study the skyrmion dynamics in an infinite film where the injected magnons hardly be reflected at the boundary. Thus, the absorption boundary is used to exclude the effect of the magnon reflection [34, 40, 44]. The position of the skyrmion \(X_i\) is estimated by

\[
X_i = \int \frac{|\mathbf{n} \cdot (\partial_x \mathbf{n} \times \partial_y \mathbf{n})| \, dx \, dy}{|\mathbf{n} \cdot (\partial_x \mathbf{n} \times \partial_y \mathbf{n})| \, dx \, dy}, \quad i = x, y,
\]

where \(\mathbf{n}\) is the staggered Néel vector \(\mathbf{n} = (S^1 - S^2)/2\). Then, the skyrmion trajectory and Hall angle can be obtained numerically.

2.2. Skyrmion helicity oscillation

In general, frustrated skyrmion has an additional degree of freedom in helicity, which may induce a helicity oscillation of the skyrmion driven by various external stimuli. As a matter of fact, the helicity oscillation is observed in the simulated skyrmion dynamics driven by polarized magnons, as shown in the following diagram.

Figure 1 presents the evolution of the skyrmion configuration of the bilayer system with the damping constant \(\alpha = 0.001\) under \(h = 0.04\mu_s\), which consists of a top skyrmion (a), (c) and (e) and a bottom skyrmion (b), (d) and (f) with opposite topology charge \(Q\). The bilayer skyrmions driven by the injected magnons move simultaneously, and the skyrmion helicity \(\eta\) changes with time at a uniform speed, demonstrating a stable helicity oscillation of the skyrmions (see SM videos 1 and 2). Then, we investigate the dependence of the oscillation speed \(\Omega\) on several parameters, noting that \(\Omega\) is a crucial parameter in modulating the skyrmion dynamics.

Generally, the skyrmion helicity oscillation is determined by the momentum transfer from the magnons to the skyrmion which depends on the skyrmion scattering potential and the magnon density. Therefore,
several parameters including the damping constant \( \alpha \), AC magnetic field strength \( h \), spin-wave frequency \( \omega \), and interlayer interaction \( J_{\text{inter}} \) affect the helicity oscillation. However, \( \omega \) and \( J_{\text{inter}} \) affect directly the skyrmion Hall angle, and this effect is hardly eliminated to uncover the influence of the helicity oscillation on the Hall angle. Thus, we modulate the damping coefficient and excitation field strength to control the oscillation speed \( \Omega \).

Figure 2(a) presents the simulated \( \Omega \) as a function of \( h \) for \( \alpha = 0.001 \). One notes that the magnon density grows as \( h \) increases, resulting in an enhancement of the angular and linear momentum transfer between the magnons and skyrmion. Thus, the skyrmion moves at a high translational speed with a high oscillation speed. A similar phenomenon is observed in the simulated damping effect on the helicity oscillation, as shown in figure 2(b) where gives the simulated \( \Omega \) as a function of \( \alpha \) under \( h = 0.05 \) J/\( \mu \)s. With increasing \( \alpha \), both the translation and oscillation speeds of the skyrmion decrease due to the fact that an enhanced damping term always lowers the skyrmion’s mobility. Furthermore, the results show that the oscillation speed eventually tends to be saturated for \( h > 0.05 \) J/\( \mu \)s or \( \alpha < 0.001 \), and this phenomenon is similar to that of the domain wall precession driven by magnons [42]. One notes that the oscillation leads to an effective anisotropy field which is increased as the oscillation speed increases. As a result, the oscillation speed has an upper limit because that a strong anisotropy field will destroy the topological texture.

As a matter of fact, the helicity oscillation of skyrmion driven by spin–orbit torque has been revealed in earlier work [25], and the oscillation significantly affects the skyrmion dynamics and induces a rotation motion of the skyrmion. Since the helicity oscillation has been confirmed, its effect on the skyrmion dynamics driven by polarized magnons can be subsequently investigated.

2.3. Effect of helicity oscillation on skyrmion Hall motion

In this part, we investigate the effect of the helicity oscillation on the skyrmion dynamics through tuning the parameters \( \alpha \) and \( h \), and pay particular attention on the skyrmion Hall motion.

Figure 3(a) shows the simulated trajectory of the skyrmion for various \( h \) for \( \alpha = 0.001 \) and the corresponding values of \( \Omega \). When the magnons are excited and injected under \( h = 0.02 \) J/\( \mu \)s, the skyrmion is driven away from the spin wave source in accompany of an obvious Hall motion. Interestingly, with the increase of \( h \), the Hall motion are gradually suppressed and the Hall angle is notably decreased. Similar to classical particle collision, the injected magnons will be strongly deflected by the skyrmion with a high oscillation speed, resulting in the decrease of the skyrmion Hall angle [35] with the increasing \( h \) attributing to the momentum conservation. This property is also confirmed by the simulated skyrmion motion depending on \( \alpha \), as shown in figure 3(b) where presents the skyrmion trajectory for various \( \alpha \) under \( h = 0.05 \) J/\( \mu \)s. The oscillation speed is decreased as \( \alpha \) increases, and the Hall motion is enhanced.

To some extent, the effect of the skyrmion helicity oscillation on skyrmion dynamics is similar to that of the AFM domain wall precession on wall dynamics which has been reported in earlier works [42]. Specifically, the wall precession induces a Coriolis force in the rotating frame, which changes the scattering process. For example, with the increasing precession speed, the scattering potential well is replaced by the potential barrier. Thus, considering the high relevance between domain wall and skyrmion in antiferromagnets, the oscillation mode of the skyrmion may alter the scattering potential of the skyrmion, which in turn tunes the magnon deflection and skyrmion Hall motion, as revealed in the simulations.

On the contrary, in the absence of the helicity oscillation, the scattering potential of the skyrmion hardly be affected by the AC magnetic field and damping constant, and a constant Hall angle is expected. To check this statement, we study the skyrmion motion driven by polarized magnons in the system with an additional DMI.
Figure 3. The simulated skyrmion trajectory for (a) various $h$ for $\alpha = 0.001$ (the black, red and blue lines correspond to the oscillation velocity $0.0015$, $0.0027$, and $0.0041\gamma J/\mu_s$, and the Hall angle $0.1493$, $0.1304$, and $0.1217\pi$, respectively), and (b) $\alpha$ under $h = 0.05\gamma J/\mu_s$ (the blue, red and black lines correspond to the oscillation velocity $0.00181$, $0.0032$, and $0.0051\gamma J/\mu_s$, and the Hall angle $0.1452\pi$, $0.1309\pi$ and $0.1205\pi$, respectively).

Figure 4. Skyrmion trajectory in the absence of skyrmion helicity oscillation (a) for various $h$ for $\alpha = 0.001$, and (b) for various $\alpha$ under $h = 0.02\gamma J/\mu_s$.

$$H_{\text{DMI}} = -D_0 \sum_i (\mathbf{S}_i^{1}\times \mathbf{S}_i^{2}, \mathbf{e}_y - \mathbf{S}_i^{1}\times \mathbf{S}_i^{2}, \mathbf{e}_x),$$

with the magnitude $D_0 = 0.17J$. In this case, the other parameters are set to be $J_2 = J_3 = 0$, $J_{\text{inter}} = -0.7J$, and $K_0 = 0.05J$ to stabilize the skyrmion. Figure 4(a) gives the simulated skyrmion trajectory for various $h$ for $\alpha = 0.001$. All the trajectories coincide well with each other, demonstrating the independence of the Hall angle on $h$. Moreover, the Hall angle does not depend on $\alpha$, as shown in figure 4(b) where presents the skyrmion trajectory for various $\alpha$ under $h = 0.02\gamma J/\mu_s$.

Thus, it is revealed that skyrmion helicity oscillation suppresses Hall motion, and the skyrmion Hall angle can be effectively modulated by the intensity of injected magnons and the damping constant in frustrated magnets. These findings provide useful information for future spintronic and magnonic applications. The magnon-skyrmion scattering can be revealed theoretically based on the elastic scattering theory, which will be helpful in understanding the physical mechanism behind the simulated results. In the following section, the scattering theory is used to investigate the skyrmion dynamics, which has been successfully applied in ferromagnetic system [36].

3. Theory for skyrmion dynamics driven by polarized magnons

In this section, we first present the model in the continuum framework, and then derive the scattering potential and the Hall angle of the skyrmion.

3.1. Model in the continuum framework

Considering the normalized staggered Néel vector $\mathbf{n}$ and magnetization $\mathbf{m} = (\mathbf{S}^1 + \mathbf{S}^2)/2$, we can rewrite the Hamiltonian as follows:
\[
H = -2J_1 \sum_{\langle ij \rangle} \mathbf{n}_i \cdot \mathbf{n}_j - 2J_2 \sum_{\langle\langle ij \rangle\rangle} \mathbf{n}_i \cdot \mathbf{n}_j - 2J_3 \sum_{\langle\langle\langle i j \rangle \rangle \rangle} \mathbf{n}_i \cdot \mathbf{n}_j - 2K_0 \sum_i (\mathbf{n}_i \cdot \mathbf{e}_z)^2 + J_{\text{inter}} \sum_i \mathbf{m}_i^2. \tag{5}
\]

Expanding \( \mathbf{n}_i = n_i (\mathbf{a}) \) around the lattice constant \( \mathbf{a} \), the exchange energy expression \( H_{\text{ex}} \) treated within the lowest relevant order reads [45]:

\[
H_{\text{ex}} = \sum_i a_i \left\{ \sum_{j\neq i} \left[ -\frac{1}{2} (\mathbf{j} - \mathbf{j})^2 J_{ij} \cdot (\nabla \mathbf{n})^2 \right] \right\}. \tag{6}
\]

Here, we neglect the \( (\nabla^2 \mathbf{n})^2 \) term in the Hamiltonian considering the fact that this energy term is much less than \((\sim 1/6)\) the \( (\nabla \mathbf{n})^2 \) term in the studied system. Therefore, while the \( (\nabla^2 \mathbf{n})^2 \) term is included to stabilize the skyrmion, it can be safely neglected for further investigation of the interplay between magnon and skyrmion, as demonstrated in earlier work [46]. Using the continuity approximation, one can rewrite equation (5) as follows:

\[
H = \frac{A}{2} (\nabla \mathbf{n})^2 - \frac{K}{2} n_z^2 + \frac{A_0}{2} m_0^2, \tag{7}
\]

where the exchange coupling \( A = (J_1 - 2J_2 - 4J_3) \), \( A_0 = 2J_{\text{inter}} \), and the anisotropy constant \( K = 4K_0 \). Then, the continuum Lagrangian density reads:

\[
L = \frac{\rho^2}{2A_0} \dot{\mathbf{n}}^2 - \frac{A}{2} (\nabla \mathbf{n})^2 - \frac{K}{2} n_z^2, \tag{8}
\]

where \( \rho = \hbar S/a \) is the density of the staggered spin angular momentum per unit cell [47, 48] with the reduced Planck’s constant \( \hbar \). Here, the effect of parity-breaking term \( \mathbf{m} \cdot \nabla \mathbf{n} \) is considered through modulating the \( A \) value [47].

To describe the magnons, we use a global frame defined by three mutually orthogonal unit vectors \( (e_1, e_2, e_3) \) where \( e_3 = \mathbf{n}_0/|\mathbf{n}_0| = e_z \times \mathbf{e}_3 \) with the equilibrium configuration \( \mathbf{n}_0 \) [49, 50]. Thus, an excited state can be parametrized as \( \mathbf{n} = n_0 + \delta_1 e_1 + \delta_2 e_2 \), where \( \delta_1 \) and \( \delta_2 \) describe the amplitude components of the magnon. Then, two monochromatic solutions with the complex fields \( \psi = (\delta_1 \pm i \delta_2)/\sqrt{2} \) for right (+)/left (−) circularly polarized magnon modes are obtained, corresponding to the anti-clockwise/clockwise precession of \( \mathbf{n} \). Subsequently, the magnon-skyrmion scattering is analytically calculated using elastic scattering theory, which was successfully applied in studying the skyrmion dynamics in chiral magnets [36].

### 3.2. Scattering potential of skyrmion and total scattering cross-section

Considering a uniform helicity oscillation of the skyrmion, the skyrmion profile can be described by \( \varphi = \varphi_0 + \Omega t \) with the angular velocity \( \Omega = \Omega e_z \) along the \( z \)-axis. In this case, \( \dot{\mathbf{n}} \) in the rotating frame is updated to \( \dot{\mathbf{n}} + \Omega \times \mathbf{n} \) in the lab frame [42], and the Lagrangian density equation (8) changes to

\[
L = \frac{\rho^2}{2A_0} (\dot{\mathbf{n}})^2 + \frac{\rho^2}{A_0} n_0 (\Omega \times \mathbf{n}) + \frac{\rho^2}{2A_0} (\Omega^2 e_z \times \mathbf{n})^2 + \frac{A}{2} (\nabla \mathbf{n})^2 - \frac{K}{2} (n_z)^2. \tag{9}
\]

Thus, the Berry-phase term \( \mathbf{n} \cdot (\Omega \times \mathbf{n}) \) encodes the Coriolis force in the rotating frame and the energy term \( \Omega^2 (e_z \times \mathbf{n})^2 \) enhances the easy-axis anisotropy. For the parameterization of the magnons, the local orthogonal frame \( (e_1, e_2, e_3) \) is represented by the polar angle \( \theta \) and azimuthal angle \( \varphi \) of the local spin, where \( e_1 = (-\sin \varphi, \cos \varphi, 0) \), \( e_2 = (-\cos \theta \cos \varphi, -\cos \theta \sin \varphi, \sin \theta) \), and \( e_3 = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \), and \( \theta = \theta(r) \) is related to the position vector \( r \) with length \( r \) and azimuthal angle \( \chi \). In this frame, the remaining massive fluctuation modes are represented by the dimensionless complex field \( \psi \) and corresponding spinor notation \( \tilde{\psi} = (\tilde{\psi}, \tilde{\psi}^*) \), and the Néel vector is expressed as follows:

\[
\begin{align*}
n_x &= \sqrt{1 - 2|\psi|^2} \sin \theta \cos \varphi + \frac{1}{\sqrt{2}} (-\sin \varphi - i \cos \theta \cos \varphi) \psi + \frac{1}{\sqrt{2}} (-\sin \varphi + i \cos \theta \cos \varphi) \psi^* \\
n_y &= \sqrt{1 - 2|\psi|^2} \sin \theta \sin \varphi + \frac{1}{\sqrt{2}} (\cos \varphi - i \cos \theta \sin \varphi) \psi + \frac{1}{\sqrt{2}} (\cos \varphi + i \cos \theta \sin \varphi) \psi^* \\
n_z &= \sqrt{1 - 2|\psi|^2} \cos \theta + \frac{i}{\sqrt{2}} \sin \theta \psi - \frac{i}{\sqrt{2}} \sin \theta \psi^*.
\end{align*}
\]
For weak fluctuations, we expand the Lagrangian in the vicinity of the skyrmion solution to the second order. Then, the magnon dynamics are obtained as the eigenstates of the Klein–Gordon equation $H\psi = -\rho^2 I\partial_t^2 \psi/A_0$ with $H = H_0 + H_1$ and the identity matrix $I$. Here, the ground-state Hamiltonian $H_0$ describes the magnons in the absence of the skyrmion:

$$H_0 = A\left\{ -\partial^2_t - \frac{\partial^2}{r^2} + \frac{\partial^2}{r^2} + \frac{(K + \rho^2\Omega^2/A_0)}{A} + \frac{r^2}{2}(\frac{2\rho^2\Omega}{A_{\Omega}}\Omega \partial_\ell + \frac{2i\partial_\ell}{r^2}) \right\}. \quad (11)$$

The skyrmion matrix scattering potential $H_s$ is given by

$$H_s = A(U_zr^2 + U_0I + U_xr^4), \quad (12)$$

with

$$U_z = 2i\left(\frac{\cos \theta - 1}{r^2}\right)\partial_\chi + \frac{\rho^2}{A_0}\Omega(\cos \theta - 1)\partial_\ell,$$

$$U_0 = \frac{3(\cos 2\theta - 1)}{2}\frac{\partial^2\theta^2}{2} - \frac{(K + \rho^2\Omega^2/A_0)(\sin^2 \theta - 2\cos^2 \theta + 2)}{A},$$

$$U_x = \frac{\sin^2 \theta}{2r^2} + \frac{(K + \rho^2\Omega^2/A_0)}{2A}\frac{\sin^2 \theta}{2A}. \quad (13)$$

From equation (10), the helicity oscillation is noted to enhance the easy-axis anisotropy to $K + \rho^2\Omega^2/A_0$ and to induce an additional effective Néel field $2i\Omega\tau_2\partial_\ell/A_0$ which is equivalent to the magnetic field in a ferromagnetic system [33]. Thus, the enhanced anisotropy and Néel field set an upper limit to the oscillation frequency, above which the skyrmion cannot be stabilized anymore. For high energy $\omega^2 \gg K$, the $U_z$ term dominates in the scattering potential and is only considered in solving the eigen equation. Moreover, the effective Néel field is rather larger than the effective anisotropy due to $\Omega^2 \ll 2\omega\Omega$, and the latter can be safely ignored.

Subsequently, we introduce the angular momentum $l$ with $\psi = \psi(r)e^{i(l\chi - \omega t)}$ to classify the eigenmodes [36, 51] and the eigen equation. Then, the eigen equation, $H_0$, and $U_z$ are updated as follows:

$$(U_z + H_0)\psi(r) = \rho^2\omega^2\psi(r)/A_0, \quad (14)$$

$$H_0 = A\left\{ -\partial^2_t - \frac{\partial^2}{r^2} + \frac{l^2}{2r^2} + \frac{(K + \rho^2\Omega^2/A_0)}{A} + \frac{2\rho^2}{A_{\Omega}}\Omega \omega - \frac{2l}{r^2} \right\}, \quad (15)$$

$$U_z = -2Al\left(\frac{\cos \theta - 1}{r^2}\right) + \frac{\rho^2}{A_0}\Omega \omega(\cos \theta - 1). \quad (16)$$

Figure 5(a) presents the calculated scattering potential $U_z$ as a function of $r$ for various $\Omega$ for $\omega^2/A_0 = 10$ K. It is clearly demonstrated that a local energy well in the skyrmion potential is induced by a finite $\Omega$, whose position and depth are determined by $\Omega$ and material parameters such as the anisotropy $K$. With the increase of $\Omega$, the energy well deepens and shifts slightly toward the skyrmion center. Thus, it is expected that more magnons will be trapped in the local well when the helicity oscillation is enhanced, resulting in a decrease of the scattering cross-section:

$$\delta_s = \frac{4}{K}\sum_{l=-\infty}^{+\infty} \sin^2 \delta_{l+1}, \quad (17)$$

where $\delta_{m+1}$ represents the phase shift. To calculate the scattering cross-section, we use the WKB approximation to estimate the phase shift $\delta_{l+1}$ and obtain [33]:

$$\delta_{l}^{\text{WKB}} = \int_{r_0}^{\infty} (\sqrt{\omega^2/A_0 - U_{\text{eff}} - k})dr + \pi|l - 1|/2 - r_0k, \quad (18)$$

where the distance $r_0$ corresponds to the first classical turning point approaching the potential, and $U_{\text{eff}}$ reads:

$$U_{\text{eff}} = \frac{Al(l-1)^2}{r^2} + \left(K + \frac{\Omega^2\rho^2}{A_0}\right) + \frac{2\omega\Omega\rho^2}{A_0}U_z. \quad (19)$$

Moreover, the scattering cross-section $\delta_s$ is calculated and the corresponding results are presented in figure 5(b). It is clearly indicated that $\delta_s$ decreases monotonously with the increase of $\Omega$, demonstrating that more magnons are captured by the deepened potential well.
3.3. Differential cross-section of magnons and skyrmion Hall angle

The scattering amplitude $f$ is defined in terms of the long-distance asymptotic behavior of the magnon wave function, which can be defined by:

$$f(\chi) = \frac{e^{-i\pi/4}}{\sqrt{2\pi k}} \sum_{l=-\infty}^{\infty} e^{il\chi}(e^{i2l+1} - 1).$$

(20)

In figure 6(a), we present the scattering amplitude $|f(\chi)|^2$ as a function of $\Omega$. For every $\Omega$, the singularity of $U_z$ leads to the multiple peaks in the differential cross-section, which is the so-called 'rainbow scattering'. Interestingly, with the increase of $\Omega$, both the peaks shift toward the large $\chi$ side and the height of the left peak obviously decreases. Then, the scattering around the large $\chi$ region ($\sim 0.45\pi$) plays a more important role in modulating the skyrmion dynamics. As a result, the skyrmion Hall motion can be partially suppressed due to the momentum transfer between the skyrmion and magnons.

Unlike the case in ferromagnets where the skyrmion is a massless particle and the scattering process is governed by the topology term [37], the AFM skyrmion is a rigid Newtonian particle and its motion is mainly controlled by inertia [38]. Thus, we consider a classic Newtonian scattering process where the skyrmion momentum is transferred from the scattered magnons, and we obtain the skyrmion Hall angle depending on the scattering amplitude:

$$\frac{v_y}{v_x} = \int_{-\pi}^{\pi} \frac{\sin(\chi)|f(\chi)|^2 d\chi}{\int_{-\pi}^{\pi} (1 - \cos(\chi))|f(\chi)|^2 d\chi},$$

(21)

where $v_y$ and $v_x$ are the skyrmion speeds along the $y$-direction and $x$-direction, respectively.

The calculated Hall angle as a function of $\Omega$ is presented in figure 6(b), which illustrates the decrease of the angle with the increase of $\Omega$. Thus, it is theoretically demonstrated that skyrmion helicity oscillation affects the scattering magnitude of the injected magnons and suppresses the skyrmion Hall motion, well consistent with the numerical simulations.

In addition, our conclusion regarding the AFM bilayer system can naturally be extended to frustrated ferromagnets and ferrimagnets where the berry phases are replaced by $\cos \theta \delta t \varphi$ and $\delta t \cos \theta \delta t \varphi + \rho \delta \eta^2 / 2$. 

Figure 5. (a) The calculated $U_z$ as a function of $r$ for various $\Omega$, and (b) the total scattering cross-section as a function of $\Omega$.

Figure 6. (a) The calculated differential cross-section as a function of $\chi$ for various $\Omega$, and (b) the skyrmion Hall angle as a function of $\Omega$. The inset is the zoom of two peaks.
respectively. Specially, the oscillation is counterclockwise ($\Omega < 0$) for the right-handed magnons ($\omega < 0$), while it is clockwise ($\Omega > 0$) for the left-handed magnons ($\omega > 0$), which has been verified in our simulations (see SM videos 3 and 4). However, the effect of the helicity oscillation on the skyrmion dynamics hardly depends on the magnon handedness. Furthermore, for the two-sublattice systems where linearly polarized magnons can be generated, the direction of skyrmion helicity oscillation and the eigen-function should be updated, while the essential physics are similar.

4. Conclusion

In conclusion, we have investigated numerically and theoretically the dynamics of frustrated skyrmion in synthetic antiferromagnets driven by circularly polarized magnons. The helicity oscillation of the skyrmion is revealed to effectively suppress the skyrmion Hall motion, and the Hall angle can be modulated by the magnon intensity and damping constant. The scattering theory demonstrates that the scattering cross-section of magnons depends on the helicity oscillations, resulting in the decrease of the Hall angle. Recently, the coupling between the skyrmion intrinsic mode and magnons attracted wide attention in spintronics. In this work, we reveal that the skyrmion’s helicity oscillation affects the total scattering, which in turn tunes the skyrmion Hall motion. Therefore, this work provides a potential method in choosing materials for future skyrmion-based spintronic devices.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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