THE THERMAL REGULATION OF GRAVITATIONAL INSTABILITIES IN PROTOPLANETARY DISKS

BRIAN K. PICKETT
Department of Chemistry and Physics, Purdue University Calumet, 2200 169th Street, Gyte Building 257, Hammond, IN 46323

ANNIE C. MEJÍA AND RICHARD H. DURISEN
Department of Astronomy, Indiana University, Swain Hall West 319, 727 East 3rd Street, Bloomington, IN 47405

PATRICK M. CASSEN
SETI Institute, 2035 Landings Drive, Mountain View, CA 94043

DONALD K. BERRY
University Information Technology Services, Indiana University, Bloomington, IN 47408

AND

ROBERT P. LINK
Department of Astronomy, University of Virginia, P.O. Box 3818, Charlottesville, VA 22903

Received 2002 October 7; accepted 2003 March 1

ABSTRACT

We present a series of high-resolution, three-dimensional hydrodynamics simulations of a gravitationally unstable solar nebula model. The influences of both azimuthal grid resolution and the treatment of thermal processes on the origin and evolution of gravitational instabilities are investigated. In the first set of simulations, we vary the azimuthal resolution for a locally isothermal simulation, doubling and quadrupling the resolution used in a previous study; the largest number of grid points is $(256, 256, 64)$ in cylindrical coordinates $(r, \varphi, z)$. At this resolution, the disk breaks apart into a dozen short-lived condensations. Although our previous calculations underresolved the number and growth rate of clumps in the disk, the overall qualitative, but fundamental, conclusion remains: fragmentation under the locally isothermal condition in numerical simulations does not in itself lead to the survival of clumps to become gaseous giant protoplanets. Since local isothermality represents an extreme assumption about thermal processes in the disk, we also present several extended simulations in which heating from an artificial viscosity scheme and cooling from a simple volumetric cooling function are applied to two different models of the solar nebula. The models are differentiated primarily by disk temperature: a high-$Q$ model generated directly by our self-consistent field equilibrium code and a low-$Q$ model generated by cooling the high-$Q$ model in a two-dimensional version of our hydrodynamics code. Here, “high-$Q$” and “low-$Q$” refer to the minimum values of the Toomre stability parameter $Q$ in each disk, $Q_{\text{min}} = 1.8$ and 0.9, respectively. Previous simulations, by ourselves as well as others, have focused on initial states that are already gravitationally unstable, i.e., models similar to the low-$Q$ model. This paper presents for the first time the numerical evolution of an essentially stable initial equilibrium state (the high-$Q$ model) to a severely unstable one by cooling. The additional heating and cooling are applied to each model over the outer half of the disk or the entire disk. The models are subject to the rapid growth of a four-armed spiral instability; the subsequent evolution of the models depends on the thermal behavior of the disk. The cooling function tends to overwhelm the heating included in our artificial viscosity prescription, and as a result the spiral structure strengthens. The spiral disturbances transport mass at prodigious rates during the early nonlinear stages of development and significantly alter the disk’s vertical surface. Although dense condensations of material can appear, their character depends on the extent of the volumetric cooling in the disk. In the simulation of the high-$Q$ model with heating and cooling applied throughout the disk, thin, dense rings form at radii ranging from 1 to 3 AU and steadily increase in mass; later companion formation may occur in these rings as cooling drives them toward instability. When heating and cooling are applied only over the outer radial half of the disk, however, a succession of single condensations appears near 5 AU. Each clump has roughly the mass of Saturn, and some survive a complete orbit. Since the clumps form near the artificial boundary in the treatment of the disk gas physics, the production of a clump in this case is a numerical artifact. Nevertheless, radially abrupt transitions in disk gas characteristics, for example, in opacity, might mimic the artificial boundary effects in our simulations and favor the production of stable companions in actual protostellar and protoplanetary disks. The ultimate survival of condensations as eventual stellar or substellar companions to the central star is still largely an open question.

Subject headings: hydrodynamics — instabilities — planetary systems: formation — planets and satellites: formation — solar system: formation — stars: formation

1. INTRODUCTION

Gravitational instabilities (GIs) in protostellar and protoplanetary disks have been proposed as a formation mechanism for stellar and substellar companions (see, e.g., Kuiper 1951; Cameron 1978; Boss 1998). Several researchers have shown that such instabilities can induce vigorous mass and angular momentum transport, vertical alteration of the disk structure, expansion of the disk, and, under certain circumstances, the fragmentation of a disk into dense
concentrations of material, all on dynamical timescales (e.g., Tomley, Cassen, & Steinman-Cameron 1991; Tomley, Steinman-Cameron, & Cassen 1994; Laughlin & Bodenheimer 1994; Boss 1997, 1998, 2000, 2001, 2002; Nelson et al. 1998; Nelson, Benz, & Ruzmaikina 2000; Pickett et al. 1998, 2000a, 2000b; Mayer et al. 2002). In particular, much attention has been given to the possibility that the GI mechanism operating in a massive stage of the solar nebula could have produced Jupiter (Boss 1997, 1998, 2000, 2002), as well as the many dozens of massive, presumably Jovian exoplanets so far detected (see, e.g., Marcy, Cochran, & Mayor 2000). GIs may even be able to form entire gas giant planetary systems with properties similar to those observed (Mayer et al. 2002). The best argument in favor of the scenario is the short timescale under which planets may be formed in this way: hundreds to thousands of years, compared to the expected disk lifetime of, at most, perhaps several million years. The time required for the formation of gas giant planets may also be the best argument against the standard core accretion theory, which could require millions to hundreds of millions of years (Boss 2001). GIs in more massive disks could conceivably produce larger mass objects as well, such as brown dwarfs and binary companion stars.

On the other hand, the GI mechanism suffers from at least two serious problems itself. First, it is not known at what level instabilities ultimately saturate, and therefore it is not known whether GIs in nature actually induce fragmentation. Second, even if the instabilities reach high enough amplitudes to produce a fragmented disk, the eventual fate of condensations so formed is difficult to determine numerically because of the complex character of a fragmented disk and thermal, rotational, and tidal disruption (Pickett et al. 1998, 2000a, hereafter PCDLI and PCDLII, respectively). We might include a third problem, namely, the fact that most simulations, including our own previous calculations, start with initial states that are already gravitationally unstable, and so it would be difficult, perhaps impossible, for realistic disks to achieve the level of activity seen in numerical simulations.

It is not known, for example, whether or not protoplanetary disks are ever cool enough or massive enough for instabilities to grow. Even if an initially hot, stable disk cools because of radiation, it first must pass through a state of marginal instability. If GIs appear but then saturate at modest nonlinear levels, the resulting gentle redistribution of mass and angular momentum could bypass the violent evolution seen in initially unstable disks. These problems arise primarily from idealized simulation conditions that most researchers have used out of numerical necessity, in particular, limited treatments of the thermal behavior of the disk and low or limited spatial resolution.

We have attempted to address some of these issues in previous papers. In Pickett et al. (2000b), we conducted three-dimensional simulations of a solar nebula model in order to determine how the thermal behavior of the disk controls the outcome of GIs. The calculations were an extension of studies of a much smaller, but more massive, protostellar disk model (PCDLI; PCDLII). We have found in all previous simulations that the thermal energetics are critical to (1) the initiation of nonaxisymmetric GIs, (2) the ultimate strength at which such instabilities saturate, and (3) the ability of a disk to produce high-density condensations, perhaps the precursors to companion stars or planets. The most severe case considered was the “locally isothermal” assumption, in which the disk temperature was independent of both time and azimuth; radial and vertical temperature gradients were held fixed. Spiral instabilities grew quickly to extreme amplitude (|bp/ρ| \sim 1), and the destruction of the disk occurred as a result. Disk disruption in turn led to complex behavior that was not conducive to clump survival. For example, the locally isothermal simulations in PCDLI, PCDLII, and Pickett et al. (2000b) led to the rapid collapse of disk material into clumps because of the interaction of several spiral modes, followed by the expansion of the disk. Even when individual clumps were isolated and found to be stable in terms of energy and mass, tidal and thermal interaction of the clumps with the rest of the disk material as well as the central star doomed the condensations to destruction. In an adiabatic evolution that included artificial bulk viscosity to treat the effects of shocks (denoted ADIAV), irreversible gas heating limited the maximum amplitude of spirals. While no condensations at all formed in this case, the instabilities grew to sufficient strength (|bp/ρ| \sim a few \times 10^{-1}) that sustained mass and angular momentum transport occurred. Additionally, strong heating localized in the upper tenuous regions of our disk caused vertical expansion and significant alteration of the vertical structure.

It has become increasingly clear that thermal energetics will drive the evolution of a gravitationally unstable disk (Durisen 2001). However, the ultimate impact of heating and cooling on the formation of companion objects via the GI mechanism is still subject to debate, mostly because of the apparently contradictory results from different researchers working with similar initial disk models. On the one hand, GIs seem capable of producing strong condensations if the numerical resolution is high enough, cooling is strong enough, or heating is weak (Boss 2000, 2001, 2002; Mayer et al. 2002). Yet, efficient cooling such as occurs in the locally isothermal condition is not always sufficient by itself to initiate clumping (Boss 2000; PCDLI; PCDLII; Pickett et al. 2000b); instead, the disk breaks into slender, transient, high-density spirals and spiral fragments. Additional heating, whether due to the suppression of decompressional cooling (Boss 2001) or the inclusion of an artificial bulk viscosity (Nelson et al. 2000; Nelson 2000; PCDLII; Pickett et al. 2000b; Boss 2003), tends to stabilize the disk against the formation or maintenance of asymmetric structure, including condensations.

A careful examination of both numerical and physical issues is needed to address the primary question: can GIs in realistic disk models under realistic physical conditions produce and maintain the progenitors of giant planets? A key numerical concern is whether or not the resolution of both the hydrodynamical grid (or number of particles) and the gravitational potential solver (see, e.g., Boss 2000) affects the formation and survival of clumps (either adversely or advantageously). Resolution effects are in principle easy to identify and fix, although of course the solution may make numerical simulations impractical in terms of time and computing resources. Conversely, the implementation of sufficiently realistic physics is much more difficult. Ideally, all relevant processes that affect the thermal balance of the disk, whether they locally heat or cool the disk, must be treated. For example, Boss (2002) used an energy equation, diffusion approximation treatment for radiative transfer, and an appropriate nebular equation of state (EOS) in several disk simulations. He found that disks can break into clumps under these conditions, as what he identifies as
vertical convective cells evidently cool the midplane of the disk efficiently.

Since no treatment for shock heating was included in these simulations, it is not clear whether or not the clumps would still form and last if an artificial viscosity (AV) scheme were employed to include the heating due to irreversible processes. In fact, Boss's most recent simulations, which do include an artificial bulk viscosity scheme, suggest that clumping is reduced if the heat generated by the AV scheme is large enough (Boss 2003).

We investigate some of these numerical and physical issues in two sets of three-dimensional hydrodynamics simulations. First, we return to the model studied in Pickett et al. (2000b) using much higher azimuthal resolution in our cylindrical grid, increasing the number of azimuthal grid points from 64 to 128 and 256 (the radial and vertical grid sizes of 256 and 64 zones, respectively, remain unchanged). Since we wish to know whether disk fragmentation leads to the formation of companions under even the most optimistic thermal assumptions, the locally isothermal condition is used. We denote these evolutions the “ISO” cases. Higher resolution simulations are useful because it is possible that the somewhat coarse azimuthal resolution in previous calculations could artificially aid in the destruction of clumps.

We also present several simulations in which two new solar nebula models and more realistic thermal physics are used. The models are distinguished primarily by their minimum value of the Toomre stability parameter $Q$, where

$$Q = \frac{c_s \kappa}{\pi G \Sigma},$$

and $c_s$ is the local adiabatic sound speed, $\kappa$ is the epicyclic frequency, $G$ is the universal gravitation constant, and $\Sigma$ is the surface density (Toomre 1964). Our “high-$Q$ model” has a minimum value of $Q = 1.8$, and so we expect the model to be initially only marginally unstable to the development of nonaxisymmetric structure. Our simulations of the high-$Q$ model are the first three-dimensional hydrodynamics evolutions that begin from an essentially stable initial equilibrium, which is then driven to a gravitationally unstable state by cooling. On the other hand, the “low-$Q$ model” is generated from the high-$Q$ model by slowly cooling it in an axisymmetric version of our hydrodynamics code (see PCDLI), and with a minimum value of $Q = 0.9$, it is expected to be violently unstable to spiral disturbances.

The current low-$Q$ model is by design very similar to the solar nebula model in Pickett et al. (2000b) and Boss (1997, 1998) and the ISO models in this paper. The main difference between the two is the method of construction, which is detailed below. The three-dimensional hydrodynamics code used to evolve these models includes both the irreversible heating and dissipation from an AV scheme and a volumetric cooling function that can be adjusted to different vertical cooling rates. These additional heating and cooling effects are implemented through either the entire disk or the outer radial half of the disk, in order to study the effects of sharp thermal boundaries on the growth of nonaxisymmetric instabilities.

The paper is organized as follows. In § 2, we discuss our method for generating and interpreting our protoplanetary disks by use of a self-consistent field (SCF) method and axisymmetric cooling. The three-dimensional hydrodynamics code and simulation conditions are described in § 3. The results for our three-dimensional hydrodynamics simulations are presented in §§ 4 and 5. In § 6, we discuss the implications of these simulations and compare them to other evolutions of protostellar and protoplanetary disks. Conclusions and areas for future study are presented in § 7.

## 2. PROTOSTELLAR DISK MODELS

Axisymmetric (two-dimensional) equilibrium models of star/disk systems are required as initial conditions for our three-dimensional simulations. These models are produced using a grid-based SCF scheme (Ostriker & Mark 1968) that iterates between solutions of Poisson’s equation and the equilibrium force-balance equation (Hachisu 1986). A polytropic EOS is assumed, where the pressure $P$ and the mass density $\rho$ are related by $P = K \rho^n = K \rho^{n+1}/n$ for $n$ and $K$ constant. We typically use $n = 5/2 (\gamma = 5/3)$, a value justified in the warm inner disk by assuming that the gas is monatomic and in the cooler outer disk by assuming that the ortho/para states of hydrogen are not in local thermodynamic equilibrium (T. Hartquist 2002, private communication). The SCF scheme requires that the configuration be in pure axial rotation with the angular speed $\Omega$ constant on cylinders about the rotation axis. The rotation of the model is determined by giving the specific angular momentum $j$ as a function of cylindrical mass fraction $m_c$ (Bodenheimer & Ostriker 1973).

Until recently, we were only able to create one-parameter families of nondimensional equilibrium states by adopting a fixed functional form of $j(m_c)$ (Pickett, Durisen, & Davis 1996; Pickett, Durisen, & Link 1997). The parameter could be chosen to be the ratio of total rotational kinetic to total gravitational potential energy $T/W$, the ratio of equatorial to polar radius $R_{eq}/R_p$, or the ratio of disk mass to total mass $M_{disk}/M_{tot}$. While versatile, the previous version of the SCF code would not allow control over the final, converged form of the surface density distribution $\Sigma(r)$. We have now developed an iterative method of adjusting $j(m_c)$ that targets a disk $\Sigma(r)$ by assuming that the rotation in the disk is nearly Keplerian. We can therefore generate star/disk models with any choice of $M_{disk}/M_{tot}$, $R_{eq}/R_p$, and surface density power-law index $p$.

### 2.1. Model Generation

The radial profile of surface density $\Sigma(r)$ found in theoretical models of disks around young stars is usually well characterized by one or several power laws $r^{-p}$ in different parts of the disks, with typical power-law indices $p$ ranging from 0.5 to 1.5 (see, e.g., Boss 2000, 2001; D’Alessio, Calvet, & Hartmann 2001; Kamp & van Zadelhoff 2001). Some observations (see, e.g., Dutrey et al. 1998) also indicate that power-law disks, with indices $p \approx 1.6$ at large $r$, may be good approximations of real protostellar disks.

The new SCF code can create an equilibrium state in which the surface density is forced to follow a single power-law index in the disk region. Consider a power-law distribution of surface density $\Sigma$ as a function of cylindrical radius $r$ from the rotation axis, i.e.,

$$\Sigma(r) = \Sigma_0 r^{-p},$$

where $\Sigma_0$ and $p$ are constants. The mass of the disk inside
the cylinder of radius $r$ is

$$M_{\text{disk}}(r) = 2\pi \Sigma_0 \int_{R_*}^{r} r^{1-p} \, dr,$$  \hspace{1cm} (3)

where the integral runs from the assumed star/disk boundary $R_*$ to the radius $r$. Guaranteeing that the total system mass $M_{\text{tot}}$ equals the stellar mass $M_*$ plus the total disk mass $M_{\text{disk}}$ gives

$$M_{\text{disk}}(r) = (M_{\text{tot}} - M_*) \frac{(r^{2-p} - R_*)^{2-p}}{(r^{2-p} - R_*)^{2-p} - (R_*)^{2-p}}.$$  \hspace{1cm} (4)

Using equation (3), we can now express the radius as a function of $m_c(r) = (M_* + M_{\text{disk}})/M_{\text{tot}}$ as follows:

$$r(m_c) = \left[ \frac{(m_c - M_*)/M_{\text{tot}}}{(1 - M_*/M_{\text{tot}})} \right]^{1/(2-p)}.$$  \hspace{1cm} (5)

If we require the $j$ at this $r$ to be roughly Keplerian for material orbiting a mass $M_{\text{tot}}m_c(r)$, then

$$j(m_c) = \sqrt{\frac{G M_{\text{tot}} m_r(m_c)}{}}.$$  \hspace{1cm} (6)

At the star/disk boundary, $j(M_*/M_{\text{tot}})$ is simply $(GM_*/R_*)^{1/2}$. Inside $R_*$, the angular momentum distribution is chosen to be either that of a uniformly rotating (UR) $n = 3/2$ polytrope (Bodenheimer & Ostriker 1973) to approximate a UR star or zero for the case of a nonrotating (NR) star. The $j(m_c)$ in equations (5) and (6) varies from iteration to iteration because we use the $m_c$ from the previous step to determine the $j(m_c)$ for the current step.

To make the models, the target $M_{\text{disk}}/M_{\text{tot}}$, $R_{\text{eq}}/R_p$, and $p$ are specified, and $R_{\text{eq}}/R_p$ is set equal to a target value. Starting with $R_p = R_{\text{eq}}$, a series of provisional equilibria are converged with incrementally smaller $R_p$ values until $R_{\text{eq}}/R_p$ reaches the target value. The mass is then distributed such that it nearly follows the specified law, the rotation of the disk is approximately Keplerian, and the mass inside $R_*$ is close to the target $M_*$ (see Table 1). At each step in the reduction of $R_p$ to its target value, the models become more difficult to converge, especially for large target values of $R_{\text{eq}}/R_p$. So far, we have been able to create models with $R_{\text{eq}}/R_p$ as large as 40.

### 2.2. Sample Star/Disk Models

Two examples of our two-dimensional star/disk models are shown in Figure 1. In each case, the top panel is a meridional density contour plot of the axisymmetric model. The bottom panel shows $m_c$ and the angular speed $\Omega$ as a function of radius, the latter normalized by the maximum value $\Omega_{\text{max}}$ Models with different target parameters have similar characteristics to the ones shown in Figure 1, namely, the star is roughly spherical, the star/disk boundary is well defined, and the disk flares.

Table 1 gives the target and output parameters of several different models for comparison. The output parameters are, on average, within 4% of the targets. The errors are largely due to the finite size of the grid cells; the converged value of $R_{\text{eq}}/R_p$ can only be approximated to within one cell’s width from the target. The power-law midplane temperature index $q$ is calculated assuming that the temperature $T \sim r^{1/n}$, as one would expect for a polytropic ideal gas with ratio of specific heats $\gamma = 1 + 1/n$. To give some feeling for the spatial resolution, Table 1 gives $R_{\text{eq}}$ and $K_{R_p}$, the radial and vertical grid cell numbers for the position of the equatorial and polar radii, respectively, for equal spacing in $r$ and $z$. $J_{\text{bdry}}$ is the radial cell at which $\Omega$ reaches a maximum, thus indicating the star/disk boundary. The ERP/MIRP is a ratio of initial orbital periods that expresses the dynamic range in disk rotation. The equatorial rotation period (ERP) is defined as the initial rotation period of material at the outer edge of the disk; the minimum initial rotation period (MIRP) is the rotation period determined at the inner disk edge, i.e., at the radial position of $\Omega_{\text{max}}$. The virial test VT is a measure of the accuracy of the model, and it is defined as $VT = |E_{\text{grav}} + 2E_{\text{rot}} + 3P|/|E_{\text{grav}}|$, where $P = \int P \, dV$. For an object in perfect equilibrium, VT = 0; most of our equilibrium states have virial tests VT ≈ a few × 10⁻³.

### 2.3. Disks with Central Holes

Our star/disk models are computed in dimensionless units and can be scaled to any relevant size from less than 1 to thousands of AU (PCDLI; Pickett et al. 2000b; Durisen, Mejia, & Pickett 2001). However, the star scales accordingly, reaching unphysical sizes when the disk is any larger than several AU in radius. To address this issue, we have developed a method to remove the star from a star/disk.

| Target Parameters | Output Parameters | Typical Models |
|-------------------|-------------------|----------------|
| $M_{\text{disk}}/M_{\text{tot}}$ | $R_{\text{eq}}/R_p$ | $p$ | $q$ | $J_{\text{req}}$ | $K_{R_p}$ | $J_{\text{bdry}}$ | ERP/MIRP | VT |
| UR |  |  |  |  |  |  |  |  |
| 0.091 | 20.0 | 1.0 | 0.096 | 19.08 | 1.01 | 1.21 | 250 | 15 | 15 | 57.3 | 2.4 (3) |
| 0.125 | 10.0 | 0.5 | 0.128 | 9.92 | 0.45 | 0.93 | 250 | 27 | 28 | 22.9 | 1.8 (3) |
| 0.125 | 20.0 | 0.5 | 0.130 | 20.00 | 0.50 | 0.92 | 242 | 14 | 14 | 60.2 | 2.6 (3) |
| 0.125 | 40.0 | 0.5 | 0.129 | 41.50 | 0.50 | 0.92 | 500 | 14 | 15 | 163.8 | 2.4 (3) |
| 0.125 | 10.0 | 1.0 | 0.125 | 9.92 | 1.00 | 1.20 | 250 | 27 | 29 | 23.3 | 1.2 (3) |
| 0.125 | 20.0 | 1.0 | 0.124 | 17.71 | 1.00 | 1.19 | 250 | 16 | 18 | 51.5 | 1.9 (3) |
| 0.125 | 40.0 | 1.0 | 0.125 | 35.57 | 1.00 | 1.18 | 500 | 16 | 19 | 133.0 | 1.8 (3) |
| 0.125 | 20.0 | 1.5 | 0.126 | 26.67 | 1.52 | 1.46 | 250 | 14 | 15 | 65.3 | 2.6 (3) |
| 0.125 | 10.0 | 1.5 | 0.135 | 9.92 | 1.57 | 1.52 | 250 | 27 | 27 | 23.0 | 5.4 (4) |
| 0.125 | 20.0 | 1.5 | 0.134 | 19.08 | 1.53 | 1.49 | 250 | 15 | 15 | 59.1 | 4.4 (3) |
| 0.250 | 40.0 | 1.5 | 0.261 | 41.50 | 1.50 | 1.44 | 500 | 14 | 14 | 168.5 | 4.5 (3) |
model and replace it with its fixed potential. The resulting equilibrium state has a central hole where the star used to be, with the disk inner radius roughly equal to the old star/disk boundary.

The central star is detached from the disk by evolving the axisymmetric star/disk model in a two-dimensional version of our three-dimensional hydrodynamics code (PCDLI). Strong localized cooling is applied over all \( z \) in five adjacent radial zones outside the star-disk boundary until the disk becomes only one cell high above the midplane in at least one of the zones. The functional form of the cooling is similar to that used in producing the cooled disk models in previous papers (PCDLI; PCDLII; Pickett et al. 2000b), as well as the low-\( Q \) model below, and has the effect of maintaining vertical isentropy at each radius. Consider the star/disk model used for the high-resolution simulations in this paper (Fig. 1a); target parameters are \( M_{\text{disk}}/M_{\text{tot}} = 0.125, R_{\text{eq}}/R_p = 20.0 \), and \( p = 0.5 \). When this model is cooled in axisymmetry from \( J = 15 \) to \( 19 \) inclusive, the “star” and “disk” detach at \( J = 16 \). The potential due to the mass inside the gap is then saved and added to the calculated potential of the evolving disk in subsequent two- or three-dimensional simulations. The actual density information is deleted from the grid, and no hydrodynamics are performed inside the central cavity.

2.4. High-\( Q \) versus Low-\( Q \) Disk Models

We denote the models discussed above, with or without detached stars and disks, as high-\( Q \) models, since their minimum values of \( Q \) are still somewhat large in the outer regions of the disks. With minimum values of \( Q \sim 2 \) (Fig. 2), the disks are expected to be at best marginally unstable to the growth of nonaxisymmetric disturbances (PCDLI).

A second class of models, which we denote low-\( Q \) models, can be constructed by cooling large radial regions of the disk until either a specified \( Q(r) \) or \( T(r) \) is reached. The procedure is similar to that used to carve out the inner hole, here

![Fig. 1a](image1.png)

**Fig. 1a** — Sample equilibrium star/disk states produced by the SCF method. The top panels show meridional structure of the two-dimensional models. The density contour values are normalized by the central density: \( \rho/\rho_c = 0.9, 0.7, 0.5, 0.3, 0.1, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, \) and \( 10^{-7} \). The bottom panels show the cylindrical mass fraction \( [m_c(r), \text{solid line}] \) and normalized angular speed \( [\Omega(1/\Omega_{\text{max}}), \text{dotted line}] \) as functions of radius. The model parameters are (a) \( M_{\text{star}}/M_{\text{tot}} = 0.5, \Sigma(r) \sim r^{-1/2}, \) and \( R_{\text{eq}}/R_p = 20 \), and (b) \( M_{\text{star}}/M_{\text{tot}} = 0.1, \Sigma(r) \sim r^{-1}, \) and \( R_{\text{eq}}/R_p = 40 \).

![Fig. 1b](image2.png)

**Fig. 1b** — Sample equilibrium star/disk states produced by the SCF method. The top panels show meridional structure of the two-dimensional models. The density contour values are normalized by the central density: \( \rho/\rho_c = 0.9, 0.7, 0.5, 0.3, 0.1, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, \) and \( 10^{-7} \). The bottom panels show the cylindrical mass fraction \( [m_c(r), \text{solid line}] \) and normalized angular speed \( [\Omega(1/\Omega_{\text{max}}), \text{dotted line}] \) as functions of radius. The model parameters are (a) \( M_{\text{star}}/M_{\text{tot}} = 0.5, \Sigma(r) \sim r^{-1/2}, \) and \( R_{\text{eq}}/R_p = 20 \), and (b) \( M_{\text{star}}/M_{\text{tot}} = 0.1, \Sigma(r) \sim r^{-1}, \) and \( R_{\text{eq}}/R_p = 40 \).

![Fig. 2](image3.png)

**Fig. 2** — Radial distribution of Toomre \( Q \) for selected uncooled models generated using the SCF scheme. Shown are the values of \( Q \) as a function of radius \( r \), normalized by the equatorial radius \( R_{\text{eq}} \). The curve types represent the \( M_{\text{star}}/M_{\text{tot}} = 0.5, \Sigma(r) \sim r^{-1/2}, \) and \( R_{\text{eq}}/R_p = 20 \) case (solid line); the \( M_{\text{star}}/M_{\text{tot}} = 0.5, \Sigma(r) \sim r^{-1}, \) and \( R_{\text{eq}}/R_p = 20 \) case (dotted line); the \( M_{\text{star}}/M_{\text{tot}} = 0.5, \Sigma(r) \sim r^{-1/2}, \) and \( R_{\text{eq}}/R_p = 40 \) case (dot-dashed line).
applied more gently and to a larger region in the outer half of the disk. Cooling proceeds at each radius until a prescribed value of \(Q\) is reached; the disk may also heat locally if \(Q(r)\) drops below the desired value. In the present study, we have chosen \(Q(r) \sim 1\) in the radial region \(0.8 < r/R_{eq} < 1\), in order to mimic the \(Q(r)\) distribution in Boss (1997, 1998). Given the low value of \(Q\), one expects these models to be violently unstable to axisymmetric and spiral modes. Figure 3 shows the structure of such a disk after cooling has been completed. As a result of the cooling, the disk shrinks radially, and a somewhat overdense ring of material forms in the outer disk. The model is largely unaffected inside the inner cooling radius.

We present simulations of two centrally evacuated solar nebula models. The high-\(Q\) model is produced directly by the SCF scheme followed by inner-disk cooling and has target parameter values of \(M_{disk}/M_{tot} = 0.125\), \(R_{eq}/R_p = 20\), and \(p = \frac{1}{3}\) (the initial SCF model is depicted in Fig. 1a). The low-\(Q\) model is generated by cooling the outer region of the high-\(Q\) model’s disk in axisymmetry until a broad region has \(Q \sim 1\). The meridional structure of the models are compared in Figure 3. Note that much of the original structure of the disk is largely preserved in the steps from the SCF model to the detached high-\(Q\) model to the low-\(Q\) model.

The high-\(Q\) model extends to radial zone 240; the low-\(Q\) model extends to radial zone 210. The inner disk boundary is located at radial zone 16 for both models. Selecting a stellar mass and equatorial radius scales the rest of the model properties in physical units. For purposes of comparison with other work, we have chosen to set the equatorial radius of the low-\(Q\) model to \(R_{eq} = 10\) AU, an initial stellar mass of \(M_* = 1\) \(M_*\), and an initial disk mass of \(M_{disk} = 0.14\) \(M_*\).

The high-\(Q\) model has the same \(M_*\) and \(M_{disk}\) but a larger equatorial radius, \(R_{eq} \approx 11.4\) AU. Table 2 lists \(M_*\), \(M_{disk}\), the ratio of the equatorial radius to inner disk radius \(R_{eq}/R_{inner}\), the surface density \(\Sigma\), midplane temperature \(T_{mid}\), and the outer disk rotation period (ORP) for both the high-\(Q\) and low-\(Q\) models. The ORP for the low-\(Q\) model is determined at radial zone 200, i.e., \(r = 9.5\) AU, and is the fiducial dynamical time used in the simulations.

The low-\(Q\) model is similar in many respects to the models discussed in Boss (1997, 1998), in particular the values of \(M_*/M_{tot}\), \(R_{eq}/R_p\), and the initial distribution of \(Q(r)\). The surface density distribution power-law matches that in the Boss models out to \(r = 5\) AU; outside that radius, the Boss \(\Sigma(r) \sim r^{-1}\). As a consequence, our models contain more mass in the outer regions, and this may explain why the instabilities that do develop there occur on somewhat faster timescales. The models studied in Boss (2000, 2001, 2002) have a larger equatorial radius (20 AU) but also have an inner boundary located farther out in the disk (4 AU) and a lower disk mass (\(M_{disk} = 0.09\) \(M_*\)).

### 3. THREE-DIMENSIONAL HYDRODYNAMICS

#### 3.1. The Three-dimensional Hydrodynamics Code

The second-order three-dimensional hydrodynamics code with self-gravity is described in detail in PCDLII. The equations of hydrodynamics are solved in conservative fashion on a cylindrical grid with typical initial resolution of \((r, \phi, z) = (256, 128, 32)\). In some simulations, the radial and vertical grids are doubled, since the disk expands radially because of gravitational torques and vertically because of heating. We use an energy equation to evolve the internal energy explicitly unless the locally isothermal condition is specified. Bulk viscosity is modeled using a von Neumann & Richtmeyer AV scheme (Norman & Winkler 1986; PCDLII). Our AV is designed specifically to account for the real effect of irreversible heating and momentum dissipation that occurs in shocks. Simulations without an AV prescription thus lose energy if shocks develop, resulting in potentially significant cooling (see, e.g., PCDLII). The locally isothermal condition does not require an energy equation to close the set of hydrodynamics equations, and the AV is not implemented. The current

| Parameter | High-\(Q\) | Low-\(Q\) |
|------------|------------|------------|
| \(M_*/M_{tot}\) | 1.0 | 1.0 |
| \(R_{eq}/R_{inner}\) | 0.143 | 0.143 |
| Radial zone number for \(R_{eq}\) | 240 | 210 |
| \(\Sigma(5\) AU\) (g cm\(^{-2}\)) | 4700 | 4700 |
| \(\Sigma(9\) AU\) (g cm\(^{-2}\)) | 4000 | 5600 |
| \(T_{mid}(5\) AU\) (K) | 100 | 100 |
| \(T_{mid}(9\) AU\) (K) | 66 | 38 |
| \(Q(5\) AU\) | 3.5 | 3.5 |
| \(Q(9\) AU\) | 1.8 | 0.9 |
| ORP (yr) | 30 | 30 |
version of the hydrodynamics does not include an “alpha viscosity” prescription.

3.2. Simulation Conditions

Symmetry about the equatorial plane is assumed, and material is allowed to flow freely off the upper and outer grid boundaries. However, in most simulations, the grid is expanded to prevent significant flow of material off the grid. Material inside \( R_{\text{inner}} \) is hydrodynamically frozen, a condition equivalent to treating the star as a central point mass. Any gas that is transported inside this radius is added to the central star’s mass.

In the locally isothermal simulations only, all nonaxisymmetric structure is suppressed inside the radius \( R_{\text{esi}} = 1.9 \) AU. We force axisymmetry by setting the nonaxisymmetric Fourier components of the density to zero and performing the inverse Fourier transform to reclaim the axisymmetric density. The Fourier transforms of the density are a natural by-product of our potential solver (Pickett et al. 1996). The suppression occurs every time step. This simulation condition is admittedly artificial and was chosen for two reasons: (1) to match similar treatment of the inner disk in Boss (1998) in order to compare our three-dimensional hydrodynamics simulations with his and (2) to prevent nonaxisymmetric disturbances with fast growth times and pattern periods from infecting the disk, as was seen, for example in other simulations (Nelson et al. 1998; PCDLI; PCDLI1). We relax these conditions in the heating and cooling simulations. In the ISO simulations, we define an EOS radius, \( R_{\text{EOS}} = 3.8 \) AU. Inside \( R_{\text{EOS}} \), the disk is evolved using an adiabatic EOS, i.e., \( P = (\gamma - 1)\epsilon \), where \( \epsilon \) is the internal energy density and \( \gamma = 5/3 \). Outside this radius, the locally isothermal assumption is maintained. The heating and cooling evolutions use the same adiabatic EOS augmented by AV and/or volumetric cooling.

Each simulation begins with random cell-to-cell density perturbations. The ISO calculations use density perturbations in the range \( \delta \rho / \rho = \pm 4 \times 10^{-2} \) for \( r > 8 \) AU; the heating and cooling simulations use a maximum perturbation of \( \delta \rho / \rho = 10^{-4} \). We choose these initial conditions in order to stimulate all possible nonaxisymmetric modes in an unbiased fashion and to ensure that the outer disk drives the evolution and not the disk inner edge (see, e.g., PCDLI).

3.3. Volumetric Cooling

Our three-dimensional hydrodynamics code includes a form of volumetric cooling such that the cooling rate \( \Lambda \) guarantees the preservation of vertical isentropy. Instead of cooling the disk to achieve a target Toomre \( Q \) as in the two-dimensional code, cooling time is held constant, i.e.,

\[
t_{\text{cool}} = \frac{\epsilon}{\Lambda} = \text{constant} .
\]

In the present study, a \( t_{\text{cool}} \) equal to 2 ORPs was applied to every cell containing a density greater than \( 10^{-7} \rho_c \), where \( \rho_c \) is the initial central density. This value of \( t_{\text{cool}} \) does not necessarily reflect the actual rate at which a protoplanetary disk will lose energy because of radiation. Rather, \( t_{\text{cool}} = 2 \) ORPs represents a state in which the energy loss rate is large enough to be interesting dynamically. Nevertheless, recent preliminary simulations with a more realistic treatment of vertical radiation losses show that \( t_{\text{cool}} \sim a \) few ORPs or less (Mejia, Durisen, & Pickett 2003).

In some of the simulations, cooling is applied outside radial zone 120 (\( r = 5.7 \) AU) and tapers linearly to zero between 4.8 and 5.7 AU. The ratio of the cooling time to the local orbital period decreases with increasing \( r \), mimicking the radial dependence in opacity in protostellar disks (Laughlin & Bodenheimer 1994).

4. MODERATE- AND HIGH-RESOLUTION ISO SIMULATIONS

In the first set of simulations, we examine the effects of azimuthal grid resolution on the locally isothermal evolution of a solar nebula model, increasing the number of azimuthal zones from 64 to 128 and 256 zones. Results for the 64 azimuthal zone case are described in Pickett et al. (2000b). There the rapid growth of two- and three-armed spiral disturbances led to the ejection of much of the disk material from the original computational grid and the formation of multiple, transient, high-density arcs of material. The initial state for these simulations is similar to the low-\( Q \) model discussed above, although the method for its generation was slightly different; the outer region was first cooled to \( Q(r) \sim 1 \), followed by detachment of the central star and disk (Pickett et al. 2000b). Table 3 lists for each case the simulation length in computation steps, years, and ORPs; the symmetry of the dominant nonaxisymmetric mode \( m \) and the \( e \)-folding times \( \tau_m \) and pattern periods \( P_m \) of the dominant modes in years. Regardless of the resolution, multiple-armed spirals grow in the outer regions of the disk, with \( e \)-folding times less than one-third of an ORP. The detailed nonlinear structure of the disk does depend on the resolution used. Figure 4 compares the disk structure between 2 and 3 ORPs (60–90 yr) for each of the two resolutions used here and in Pickett et al. (2000b). The instabilities also induce copious mass and angular momentum transport, as can be seen in the initial and final distributions of cylindrical mass fraction \( m_c \). (Fig. 5).

4.1. The 128 Azimuthal Zone Case

The nonaxisymmetric structure is clearly better resolved when 128 azimuthal zones are used compared to the 64 azimuthal simulation in Pickett et al. (2000b). In the present case, somewhat higher order modes (five- and six-armed spirals) appear, grow faster, and disrupt the disk more quickly than in the 64 azimuthal case. The dominant mode is a well-defined five-armed spiral located primarily in the initially unstable region of the disk. The \( e \)-folding time and pattern period for this mode are \( \tau = 0.30 \) ORP and \( P = 0.71 \) ORP, respectively. The \( m = 6 \) disturbance grows nearly as fast (\( \tau = 0.33 \) ORP) and with a similar pattern period, but it is not as well organized. Here, \( m \) refers to the symmetry of the disturbance, in this case a six-armed spiral. The pattern

| Azimuthal Zones | Steps | ORPs | Years | \( m \) | \( \tau_m \) (yr) | \( P_m \) (yr) |
|----------------|------|-----|------|------|-------------|-------------|
| 64             | 40,000 | 6.3 | 190   | 3    | 13          | 15          |
| 128            | 41,000 | 2.8 | 81    | 5    | 9.0         | 21          |
| 256            | 52,000 | 2.5 | 75    | 5    | 8.4         | 22          |
period of 0.71 ORP corresponds to a corotation at $r = 7.8$ AU, near the inner edge of the initially low-$Q$ region.

Late in the evolution, multiple spirals interfere with each other and produce a complicated set of density concentrations. At most, seven or eight individual high-density arcs can be seen (Fig. 4). The logarithmic gray scale spans 4.5 orders of magnitude in density; the density here is normalized by $\rho_0 = 4 \times 10^{-2} \text{ g cm}^{-3}$.

Fig. 4.—Comparison of ISO disk structure. Shown are equatorial plane density gray scales of the 64 azimuthal zone simulation at $t = 90$ yr (top), the 128 azimuthal zone simulation at $t = 60$ yr (middle), and the 256 azimuthal zone simulation at $t = 61$ yr (bottom). The logarithmic gray scale spans 4.5 orders of magnitude in density; the density here is normalized by $\rho_0 = 4 \times 10^{-2} \text{ g cm}^{-3}$.

The interaction of these disturbances, as well as other lower order spirals, leads to a considerably more complicated and interesting disk structure by the end of the simulation. As Figure 4 demonstrates, there are now more than a dozen high-density fragments in the midplane mass densities at the end of this simulation at 2.8 ORPs ($\sim 81$ yr). As we saw when we extended the grid in the high-resolution ISO simulation in PCDLII, the ejected spirals do not condense into anything that could be readily identified as a potential, long-lived companion object.

**4.2. The 256 Azimuthal Zone Case**

The situation is even more complex when 256 azimuthal zones are used, although the growth of the underlying structure is similar to the evolution described above. Like the 128 azimuthal zone case, five- and six-armed disturbances dominate the disk structure, with the $m = 5$ mode barely edging out the $m = 6$ mode. The growth times and pattern periods are also similar to the 128 azimuthal zone case, suggesting numerical convergence at least during the linear growth regime: $\tau_{5,6} = 0.28$, 0.35 ORP and $P_5 = 0.73$ ORP.

The interaction of these disturbances, as well as other lower order spirals, leads to a considerably more complicated and interesting disk structure by the end of the simulation. As Figure 4 demonstrates, there are now more than a dozen high-density fragments in the disk by 2 ORPs, and the nonlinear disturbances penetrate nearly down to the EOS barrier. The maximum density of these blobs is $\rho_{\text{max, cl}} = 2.0 \times 10^{-3} \text{ g cm}^{-3}$, not significantly different from that seen in either of the lower resolution simulations. However, nothing survives even a quarter-orbit, as was the case in the PCDLII and Pickett et al. (2000b) simulations. Although the high-density clumps eventually shear into somewhat lower density arcs of material, the density is still quite enhanced in these arcs compared to the initial, unperturbed density in that location.
5. SIMULATIONS WITH HEATING AND COOLING

We next discuss a series of simulations of the high-\( Q \) and low-\( Q \) models, in which heating and cooling are applied either over the outer half of the model or throughout the entire disk. Additionally, we present results for a simulation in which only the AV heating is applied to the outer half of the low-\( Q \) model’s disk. This “heating only” case is not strictly equivalent to the ADIAV case in Pickett et al. (2000b), since viscous heating and dissipation are applied over less disk area in the present case. In the following simulations, the axisymmetric constraint on the inner disk has also been removed, allowing the entire disk to participate in the growth of spiral structure.

5.1. General Results

Table 4 summarizes the results for these simulations. The table lists the minimum value of \( Q \); the presence of heating and cooling; the simulation length in computation steps, years, and ORPs; the e-folding time \( \tau_4 \) and pattern period \( P_4 \) of the dominant four-armed spiral mode in years; and comments regarding the production of high-density clumps. Here, “heating” refers to the heat input of the AV scheme, and “cooling” refers to the volumetric cooling function. Each simulation includes cooling and/or heating throughout the entire disk (“full”), over the outer half of the disk (“half”), or not at all (“none”). Both the high-\( Q \) and low-\( Q \) models are unstable to several low-order spiral disturbances, primarily a four-armed spiral that grows with an e-folding time of \( \tau_4 \sim 0.3-0.4 \) ORP. Figures 6 and 7 show the initial and final equatorial plane mass density distributions as gray scales. Additionally, significant alteration of vertical structure occurs as a result of the spiral structure seen in the equatorial plane images. Figure 8 shows initial and final isodensity surfaces for each of the simulations and the 256 azimuthal zone ISO evolution for comparison. Although the original disk surface height increases linearly with radial distance, by the end of each simulation, the structure has been profoundly affected. Surface corrugations generally trace the underlying spiral features seen in the equatorial plane mass density plots.

In all four cases, the multiple spiral instabilities lead to rapid material transport, although the rate and the extent to which this occurs depend on the balance of heating and cooling in the disk. Figure 9 shows the initial and final disk cylindrical mass fraction distribution as a function of radius for the high-\( Q \) and low-\( Q \) model simulations. Note that material is transported both outward beyond the initial equatorial radius and inward inside about 5 AU. In the high-\( Q \) model simulations, the mass inside 5 AU increases by roughly 40\% when heating and cooling are applied over the entire disk. Although this case is carried further than the simulation in which only half the disk includes heating and cooling, much of the inward mass transport occurs by 150 yr. The inward mass transport for the low-\( Q \) model case with half-heating and cooling amounts to roughly a 40\% increase in 310 yr. The low-\( Q \) model simulation with no cooling exhibits a much smaller inward mass transport rate, leading to an increase of about 5\% by the end of the simulation. Nevertheless, all models expel roughly 10\%–15\% of the total disk mass beyond the initial equatorial radius. Given the mass of the disk, the outward mass transport rates are on the order of \( 10^{-4} \) \( M_\odot \) yr\(^{-1}\). Inward transport rates range from \( 10^{-4} \) to \( 10^{-5} \) \( M_\odot \) yr\(^{-1}\). The change in angular momentum profiles are similarly dramatic and feature the bidirectional transport seen with disk mass. Roughly 15\%–20\% of the disk angular momentum is moved outside the initial equatorial radius by the end of each evolution.

5.2. The High-\( Q \) Model

5.2.1. Heating and Cooling: Outer Half (HighQ-HC-Half)

We denote the simulation of the high-\( Q \) model in which the outer half of the disk is evolved with the additional heating and cooling terms “HighQ-HC-Half.” The simulation is identical to the one discussed in Durisen et al. (2001), although there the model was scaled to represent a massive star/disk system, with an equatorial radius of 1000 AU and a total mass of 22.8 \( M_\odot \). We include the simulation here in order to compare it to the same type of evolution for the low-\( Q \) model as well as the case in which heating and cooling are applied over the entire disk.

The evolution extends to 5.1 ORPs (~150 yr). The disk is rather quiescent during the first 2.6 ORPs of its evolution, although the growth of a four-armed mode begins from the initial noise at about 2 ORPs. At \( t = 2.9 \) ORP, a dense ring forms near the outer edge of the disk due to the strong cooling there. By 3.8 ORPs, the growing four-armed spiral visibly distorts the model outside 5 AU. The mode is extremely coherent, and once initiated grows quickly, with an e-folding time \( \tau_4 = 0.38 \) ORP (~11 yr) and a pattern period of \( P_4 = 0.99 \) ORP (~30 yr). The later evolution of the object includes the presence of multiple low-order disturbances, although the four-armed mode is still clearly dominant in the outer region of the disk (Fig. 6). A relatively large condensation forms near the end of the simulation at 5 AU. The maximum clump density is \( \rho_{\max, cl} = 5.0 \times 10^{-5} \) g cm\(^{-3}\), or about an order of magnitude larger than the initial density of the disk in the location of the clump. The simulation was terminated at 5.1 ORPs because of numerical difficulties with the computational time step, and so the future survival of the clump is uncertain.

5.2.2. Heating and Cooling: Full (HighQ-HC-Full)

We evolved the high-\( Q \) model with heating and cooling applied everywhere in the disk above the cutoff density for 16.6 ORPs (~500 yr). The early evolution of the object is fairly reminiscent of the HighQ-HC-Half case: instabilities

| Simulation          | \( Q_{\text{min}} \) | Heating | Cooling | Steps  | ORPs | Years | \( \tau_4 \) (yr) | \( P_4 \) (yr) | Condensations     |
|---------------------|---------------------|---------|---------|--------|------|-------|-----------------|--------------|------------------|
| HighQ-HC-Half       | 1.8                 | Half    | Half    | 90,000 | 5.1  | 150   | 11              | 30           | Dense condensation at 5 AU |
| HighQ-HC-Full       | 1.8                 | Full    | Full    | 225,000| 16.6 | 500   | 10              | 31           | Dense rings at 1, 2, and 3 AU |
| LowQ-HC-Half        | 0.9                 | Half    | Half    | 135,000| 10.4 | 310   | 7.9             | 22           | Multiple condensations at 6 AU |
| LowQ-HC-Half        | 0.9                 | Half    | None    | 90,000 | 7.0  | 210   | 7.9             | 22           | No condensations   |
require about 2 ORPs before growth begins and 4 ORPs before they reach nonlinear amplitudes and visibly distort the surface. A similar four-armed mode is dominant here, with \( r_4 = 0.34 \) ORP and \( P_4 = 1.03 \) ORP. The high-\( Q \) model evolves more violently when the heating and cooling are applied everywhere. This can be seen in the mass and angular momentum distributions as well as the equatorial plane density plots (Fig. 6). Additionally, the GIs drive the disk \( \Sigma(r) \) from a single-valued power law to a distribution that is approximately two-valued. Figure 10 shows the evolution of the surface density distribution, and a clear break at about 5 AU can be seen. Outside 5 AU, the surface density profile steepens appreciably, from the initial \( \Sigma(r) \sim r^{-1/2} \) to \( \sim r^{-5/2} \).

Although the spiral structure is striking and extensive, with many individual spirals sharply defined against the lower density background, few clumps form. The maximum arclet density is similar to that seen in the HighQ-HC-Half case. By the end of the simulation, the spirals dominate disk structure outside about 3 AU. Inside 3 AU, several rings of material develop after 8 ORPs (~240 yr). The rings are located at \( r = 1.1, 2.0, \) and 3.0 AU and ultimately contain \( 2.5M_J \).
4.7\(M_J\), and 9.2\(M_J\) of material, respectively. As the simulation proceeds, the rings gain mass and become more narrow. Figure 11 shows the development of the rings in a series of equatorial plane density gray scales at different times.

5.3. The Low-Q Model

5.3.1. Heating and Cooling: Outer Half (LowQ-HC-Half)

The LowQ-HC-Half evolution is reminiscent of the previous ISO simulations in that a collection of dense arcs appear within about 1 ORP (~30 yr). However, the effect is much less pronounced than in the 128 azimuthal zone ISO simulation. We carried the simulation to 10.4 ORPs (~310 yr). The disk is initially unstable to an \(m = 4\) mode, with \(\tau_4 = 0.26\) ORP and \(P_4 = 0.74\) ORP. As before, the disk is disrupted into a collection of high-density arcs (Fig. 7).

Unlike the high-\(Q\) disk or ISO simulations, a series of similar-sized condensations appear and disappear in approximately the same region of the disk. The clumps each have a mass of about 0.3\(M_J\) and appear between 5.2 and 5.8 AU, inside the thermal transition region of 4.8–5.7 AU. The clump density ranges from a factor of 30 to 60 over the
Fig. 8.—Comparison of vertical structure. Shown are isodensity surfaces for all the models discussed in this paper. The surfaces represent a density level of ρ/ρ⊙ = 10^{-6}. Except for the HighQ-HC-Full image, the rectangular cube encloses the initial computational grid. The linear scale in AU is also shown. (a) Initial high-Q model. (b) HighQ-HC-Half simulation at t = 150 yr. (c) HighQ-HC-Full simulation at t = 500 yr (complete expanded grid shown). (d) Initial low-Q model. (e) the LowQ-HC-Half simulation at t = 310 yr. (f) LowQ-H-Half simulation at t = 210 yr. (g) ISO 256 azimuthal zone simulation at t = 61 yr.
original disk density in that region. Figure 12 shows the disk structure at several times. One of the clumps does complete one fairly circular orbit before it is smeared out upon interacting with another spiral of gas. Like the HighQ-HC-Half simulation, the primary $m=4$ spiral is dominant outside 5 AU; inside 5 AU, where the initial value of $Q$ is still quite high (Fig. 3), the spiral structure is weaker, but still present.

5.3.2. Heating Only: Outer Half (LowQ-H-Half)

We carried the LowQ-H-Half simulation to 7.0 ORPs ($\approx 210$ yr). As in the LowQ-HC-Half simulation, the model is initially unstable to an $m=4$ mode, with $t_\lambda = 0.26$ ORP and $P_\lambda = 0.74$ ORP. Mild radial material transport does occur. As expected, the model behaves in a manner similar to the ADIAV simulation described in Pickett et al. (2000b), although here the model is better resolved and the AV affects a slightly more radially limited region of the disk. We also note that, although we no longer hold the inner radii axisymmetric or have an artificial EOS boundary, much of the nonlinear structure is nevertheless severely reduced in the inner disk (Fig. 7). As we have seen previously, the AV heating limits the strength of the instability in the outer region and the final disk structure is considerably more axisymmetric compared to the LowQ-HC-Half evolution. In fact, this is the only simulation that does not produce clumps, transient or otherwise.

6. DISCUSSION

6.1. The ISO Simulations

6.1.1. Grid Resolution Effects

Boss (2000) returned to the ISO simulations presented in Boss (1997, 1998) with very high azimuthal resolution (up to 512 azimuthal zones) and an increase in the number of terms in the spherical harmonic expansion (up to $\ell = m = 48$) of the density used in his gravitational potential solver. Boss found that the maximum density of clumps increased as either the hydrodynamical grid or the gravitational potential resolution increased. In fact, Boss argued that because the maximum density had not yet converged with resolution, larger grids and more accurately determined potential solvers may be needed in future calculations. Nelson (2003) also returned to previous smoothed particle hydrodynamics (SPH) simulations with higher resolution (hundreds of thousands of particles) and found that disk fragmentation did not occur with the increased resolution.

We have seen in our own calculations a quantitative change in the behavior of the disks when the resolution is increased from 64 azimuthal zones to 128 or 256 zones. The primary differences between the ISO calculations presented here and in Pickett et al. (2000b) are the order of the most unstable mode and the resulting nonlinear structure of the disk after disruption (see, e.g., Fig. 4). Specifically, the ISO simulation in Pickett et al. (2000b) found a three-armed spiral to be the dominant mode, which produced three slender arclike condensations rather than the five- and six-armed spirals and the multiple, although temporary, condensations seen in the higher resolution evolutions. Clearly, 64 and perhaps 128 azimuthal zone grids are insufficient for an adequately resolved picture of this disk’s evolution, particularly in the nonlinear regime. It should be noted, however, that 256 or more azimuthal zone simulations, while desirable, may not be necessary for numerical convergence in every disk evolution (see, e.g., PCDLII).

Beyond assessing the global structure of a disk, grid resolution can play a critical role in accurately determining the disposition of any condensations that do form, by either a nonconvergence in clump mass (Boss 2001) or the artificial
induction of clump formation by an inadequately resolved grid (Truelove et al. 1997). For example, it is possible that the long slender arcs of material in Pickett et al. (2000b) could be an artifact of the coarse azimuthal grid, particularly near the edge of the radial grid, where the azimuthal grid size can be much larger than the radial grid dimension. We have compared the cell dimensions across the computational grid with the Truelove density criterion for stability against numerically induced condensation (Truelove et al. 1997); in all the simulations presented here, the grid is sufficiently resolved azimuthally where condensations appear. Nevertheless, the differences between the low-resolution simulation in Pickett et al. (2000b) and the ISO simulations presented here are indeed large. It is important to remember, however, that the ultimate impact of the instabilities in all three cases is the same. Moreover, the difference between the 128 and 256 azimuthal zone simulation is not as great as the leap from 64 to 128 and suggests at least a partial numerical convergence. This is particularly true in terms of the growth rates and pattern periods detected in the disk.

Fig. 11.—Development of inner rings in the HighQ-HC-Full simulation. The series of gray scales depicts the growth of three dense rings at approximately 1, 2, and 3 AU. Here, the gray scale logarithmically spans the density range $-5.5 < \log(\rho/\rho_0) < -3.5$. The bounding box is $20 \times 20$ AU. Top left: $t = 270$ yr. Top right: $t = 390$ yr. Bottom: $t = 500$ yr.
6.1.2. **Numerical Destruction of Condensations?**

The accuracy of the gravitational potential solver is a valid concern. Unlike the simulations in Boss (2001), the maximum density of our clumps is essentially insensitive to grid resolution; in the ISO evolutions presented here and in Pickett et al. (2000b), the maximum clump density is approximately $2 \times 10^{-3}$ g cm$^{-3}$. For the material in the hydrodynamics grid, we compute a Fourier transform of the density in the $\phi$-direction, a direct solution by cyclic reduction of the transform in $(r, z)$, and a transform back to real space. The solution of Poisson’s equation is therefore very dependent on azimuthal resolution.

The issue is whether or not our hydrodynamics code is capable of seeing the formation and survival of dense objects, or whether the grid and especially the potential resolution will instead smear out originally bound objects into arcs and arclets. To test our code, we have loaded 10$M_\odot$ into a roughly uniform volume in the equatorial plane of our equilibrium model with 256 azimuthal zones; the volume

![Fig. 12a](image1.png) ![Fig. 12b](image2.png) ![Fig. 12c](image3.png)

**Fig. 12.**—Development of clumps in the LowQ-HC-Half simulation. The series of gray scales shows the appearance and evolution of several high-density condensations in a narrow radial range between 5 and 6 AU. The density gray scale and radial range of the images are the same as in Fig. 10. (a) $t = 310$ yr. (b) $t = 313$ yr. (c) $t = 316$ yr. (d) $t = 319$ yr. (e) $t = 322$ yr. (f) $t = 325$ yr.
measures four radial zones cells by one azimuthal zone. The mass is about an order of magnitude larger than the Jeans mass appropriate for the initial conditions of the disk in that location and is given purely circular orbital motion. Although some smearing of the density does occur, the mass survives nearly a complete orbital period before it spirals inward toward the EOS boundary and is smeared into a high-density arc.

In a second test, we loaded a roughly spherical, uniform density concentration of matter into an otherwise empty computational grid in order to compare the code’s numerical solution to the acceleration across the blob to the known analytic solution. The blob was placed at the midplane with azimuth angle $\phi = 90^\circ$ and at radial positions $r/R_{eq} = 0.25$, 0.50, and 0.75. Figure 13 shows the numerical solutions to the gravitational acceleration across the radial and azimuthal grid directions. In most cases, the calculated accelerations near the center of the blob and outside the blob are quite close to the analytic solution. However, our code underestimates the maximum gravitational acceleration located at the surface of the sphere. The acceleration across the radial grid coordinate agrees with the analytic solution to within about 20%, regardless of position in the disk. However, the accelerations calculated across the azimuthal
coordinate are systematically and dramatically lower than the analytic result as the sphere’s distance from the center of the grid is increased. This can be understood in terms of the increasing width of computational cells \( \frac{r D}{C_30} \) with increasing radial distance, leading to reduced physical resolution. These tests probably indicate that the ability of our hydrodynamics code to simulate the long-term evolution of a density concentration is limited by the grid geometry.

However, we note that (1) several of the condensations in the simulations are equally well resolved but are disrupted on timescales much shorter than their orbital periods and (2) when the outward radial velocities were artificially restricted, the code was able to resolve a 34\,M_J clump for 3.5 orbits in a comparison study of one of Boss’s evolutions (Pickett et al. 2000b). Although the results of our simulations cast some doubt on the efficacy of the GI mechanism to produce protoplanets, the ability of our hydrodynamics code to accurately follow the long-term evolution of a dense clump is restricted.

### 6.2. The High-Q and Low-Q Model Simulations

#### 6.2.1. General Comments

In general, the models exhibit four distinct stages in evolution, regardless of initial thermal state or later thermal conditions. Broadly defined, these stages are (1) initial quiescence, in which the initial density perturbation organizes into axisymmetric and nonaxisymmetric disturbances, (2) growth of nonaxisymmetric structure, typically a four-armed spiral, after 1–2 ORPs, (3) visible distortion of the disk structure about 3 ORPs after the initial growth of the four-armed spiral, and (4) saturation or weakening of nonaxisymmetric structure. In the three simulations with cooling, a dense ring of material also forms beyond 8 AU between stages 2 and 3. In the high-Q disks, the initiation of the four-armed spiral coincides with the lowering of \( Q(r) < 1 \) in the outer disk (Fig. 14), which occurs after only 2 ORPs of cooling. At least with the cooling time used here, the high-Q disk bypasses a gentle marginal evolution in structure and becomes gravitationally unstable very quickly.

![Fig. 13a](image1.png)  
**Fig. 13a**—Test of the gravitational potential solver. The figures show the analytic (solid line) and numerical solutions for the acceleration across a sphere of uniform density placed at three different radial positions in an empty grid. The sphere was placed in the midplane, centered at radial grids 32, 64, and 96 in a computational grid with \((r, \varphi, z) = (128, 128, 16)\). The radius of the sphere \(r_{sphere}\) is 5 times the radial grid size \(dr\). Positions are measured across the (a) radial or (b) azimuthal grid direction; here \(r_{sphere}\) is the position of the sphere’s center and \(R_{grid}\) is the radial length of the computational grid. All numbers are in nondimensional code units.

![Fig. 13b](image2.png)  
**Fig. 13b**

Fig. 14.—Toomre stability parameter \( Q(r) \) at the initiation of nonaxisymmetric structure for the high-Q model simulations. Shown are the distributions of \( Q(r) \) at the time when the dominant \( m = 4 \) mode begins growing in the disk. **Solid line**: Initial high-Q model \( Q(r) \). **Dotted line**: HighQ-HC-Half at \( t = 67 \) yr. **Dashed line**: HighQ-HC-Full at \( t = 66 \) yr.
Another common feature of the simulations is the radical changes in vertical structure that result from the instabilities. The disk surfaces generally evolve from a smooth, nearly linear height profile with radius to complex structures with corrugations, peaks, and valleys. Although we observe vertical motion in our simulations, they are not thermally driven convection cells. The deformations can be quite striking, particularly in the outer part of the models (Fig. 15). The change in surface structure may have observational consequences, such as the periodic extinction of light from the central star or, in a somewhat different context, the production of linearly distributed methanol masers around massive stars (Durisen et al. 2001). Dynamically, if radiation from the central star is included in the simulation physics, regions that are shaded from the central star’s radiation might cool even more, further driving the instability.

6.2.2. The Role of Heating and Cooling

The violent nature of the spiral instabilities in our simulations seems to make the long-term survival of condensations problematic at best. Nevertheless, even disks with complex nonaxisymmetric structure can produce orbiting clumps, at least temporarily, as long as cooling is efficient (Boss 2001; Nelson et al. 1998, 2000; Mayer et al. 2002). For example, the locally isothermal SPH simulations in Nelson et al. (1998, 2000) display disk fragmentation and the production of a few bound clumps.

More recently, Mayer et al. (2002) have conducted locally isothermal SPH disk simulations with up to several million particles. They followed the evolution of the disk from fragmentation to the formation of a small planetary system, which superficially resembles the few extrasolar systems currently known; similar calculations by Nelson (2003), however, do not yield disk fragmentation, as discussed in § 6.1. A similar simulation in Mayer et al. (2002), in which the original locally isothermal EOS was changed to adiabatic (with AV) after fragmentation, yielded weaker nonaxisymmetric structure, but clumps still formed after about 350 yr. However, Mayer et al. (2002) do not report results from simulations in which the AV is active throughout the entire simulation, and so it is uncertain whether or not the evolution of the disk would be more quiescent under more realistic simulation conditions. In fact, a simulation of a warmer disk (higher minimum $Q$) led to mass and angular momentum transport and spiral features, but no clumping, similar to what we have seen in our own previous calculations. Boss (2001) conducted high-resolution simulations of the solar nebula studied in Boss (2000) in which the locally isothermal condition has been relaxed and radiative cooling is used. The resulting disk evolution is reminiscent of the locally isothermal evolutions, and extremely high density knots of gas do form as a result. The disks in Boss (2002) develop clumps as well, which he attributes to the apparent establishment of vertical convection cells that would cool the disk midplane efficiently.

Most of the simulations that lead to vigorous clumping do not include the important heating and dissipation effects treated by an artificial (bulk) viscosity scheme, which could counteract the destabilization due to cooling. For example, the addition of heating limits the strength of nonaxisymmetric structure in general and clumping in particular in the Nelson et al. (2000) and Nelson (2000) simulations.

![Fig. 15.—Development of surface features in the HighQ-HC-Full simulation. The density isosurfaces show the three-dimensional structure of the model in the HighQ-HC-Full evolution at selected times. The box encloses the total expanded grid. See Fig. 8.](image-url)
Irreversible heating is not included in Boss (2001, 2002). To approximate this effect, Boss performs one run where decompressional \( PdV \) cooling is suppressed. In this case, the disk heats to stability, and the nonaxisymmetric structure remains weak, similar to that displayed in our LowQ-H-Half simulation.

More recently, Boss (2003) conducted several simulations with artificial bulk viscosity included in the code physics. He found that, if the heat input from the AV scheme is high enough, clumping can be significantly suppressed.

Tomley et al. (1991, 1994) were the first to demonstrate this connection between the strength of spiral disturbances and the thermal processes in disks in two-dimensional particle simulations. Other numerical calculations support this idea (PCDLI; PCDLII; Pickett et al. 2000b; Nelson et al. 1998, 2000; Nelson 2000; Boss 2001). However, our simulations show that the situation is in fact more complicated than the global trumping of cooling over heating. Globally, the internal energy subtracted by our volumetric cooling scheme is larger than that added by viscous heating. Locally, however, artificial viscous heating can overwhelm the volumetric cooling function. Figure 16 shows the equatorial plane increase in internal energy due to AV heating (top) and the decrease in internal energy due to the cooling function (bottom) late in the HighQ-HC-Full simulation. The AV heating gray scale resolves the underlying spiral features nicely and is due to the difference between the spiral’s pattern period and the local orbital periods in the disk (PCDLII). In this case, the pattern period of the dominant four-armed spiral is about 3 times slower than the orbital period of material at 4 AU. Irreversible heating is highly localized and can exceed the local cooling strength. Note, for example, the heating along the large spiral in Figure 16. Heating occurs along the trailing edge of the spiral inside the corotation radius at 7.8 AU, where the disk material is moving more quickly than the spiral. Outside corotation, heating occurs on the leading side of the spiral, where now the spiral is traveling faster than material orbiting in the disk. Along the corotation radius, there is no heating. In contrast, the cooling applied by our volumetric cooling function is far less localized and generally follows the high-density regions. Cooling is clearly required for the spiral features to collapse into dense structures, as can be seen in the rather washed-out appearance of the LowQ-H-Half disk (Fig. 4). Despite a minimum value of \( Q \) designed to make the low-\( Q \) model extremely unstable, the addition of AV heating is enough to prevent disk disruption.

### 6.2.3. The Formation of Companion Objects

If the amount of overall cooling determines whether or not spiral structure can evolve into spiral fragments, then where and how that cooling is applied determines the nature of the dense structures so formed. When the additional heating and cooling terms are restricted to the outer half of the disk (HighQ-HC-Half and LowQ-HC-Half), individual subcondensations are at least produced. On the other hand, when the thermal effects are felt over the entire disk, it is apparently difficult for the spiral structure to form even short-lived knots of material. Instead, dense rings form in the inner part of the disk, where the spiral structure is weakest.

Of course, the appearance of dense clumps in a protostellar disk simulation does not guarantee survival on even
dynamical timescales. Even though the cooling function overwhelms the stabilizing influence of heating in general and the AV heating in particular, it is evidently not enough to produce bound condensations. As we have seen in previous calculations, extreme cooling is not in itself a sufficient condition for the collapse of structures into protoplanetary or protostellar companions.

The evolution of the HighQ-HC-Full case exhibits a phenomenon not previously reported in simulations of GIs. As noted above, the model develops three dense, but thin, rings nearly equally spaced at 1, 2, and 3 AU. The material in each ring contains several Jupiter masses. Given that the material is trapped in rings, one would expect compressional heating to be unimportant, and so the cooling has a chance to drive the local values of $Q$ to quite small values (Fig. 17). The inner rings are located in a region of the disk in which the value of $Q$ is still high. This may in fact protect the developing rings from the interference of violently evolving spiral structure. It is tempting to speculate that the continued evolution of this model will lead to further concentration of material in the rings and that at some point the rings will become gravitationally unstable themselves. If this is true, it could be that Jupiter-sized companions are formed directly in fairly circular orbits within a few AU of the central star. Of course, the nonlinear evolution and interaction of the unstable rings are almost certainly complicating factors in the survival of any clumps that do form. Additionally, the innermost ring forms near the inner edge of the disk, and it is not clear whether or not this ring is an artifact caused by the boundary. We are continuing this calculation and others like it and plan a more detailed analysis of the cause and ultimate consequences of the dense rings.

![Fig. 17.—Evolution of $Q(r)$ for the HighQ-HC-Full simulation. Shown is the distribution of $Q(r)$ at selected times. Note the development of a largely constant $Q \sim 1.3$. The thick solid lines mark the locations of the three rings at 1, 2, and 3 AU.](image-url)

7. SUMMARY

We conducted several high-resolution, three-dimensional hydrodynamics simulations of a solar nebula model. In all cases presented, multiple-armed spirals grew on extremely rapid timescales and induced rapid transport of mass and angular momentum. We have found that while strong cooling enhances the ability of a disk to produce dense condensations, whether the disk instability mechanism can produce long-lived companions is still an open question. High-resolution isothermal comparison simulations of the model studied show that, while the nonaxisymmetric structure in Pickett et al. (2000b) was underresolved, the increased resolution does not mean that the numerous clumps and arclets of material survive. In fact, the increase in resolution resulted in a decidedly more complex set of condensations, the survival of which depended on interactions with the remaining disk material, other condensations, and the central star.

Simulations in which both heating and cooling are included do show that the competition of both processes is vital to the production of widespread spiral structure. Cooling increases the likelihood of spiral arm collapse, while additional heating from shocks acts to disperse spiral structure. However, even when the local cooling effects overwhelm irreversible heating, as is the case in the simulations presented here, it is difficult to construct a situation in which a single clump or a small set of long-lived clumps forms on a stable orbit. The most promising situations are the simulations in which heating and cooling conspire to confine mass in a narrow radial region. We have seen this happen in our disk in two ways. One simulation with a sharp boundary in thermal properties, although numerically imposed on the simulation, was conducive to the formation of several small clumps, some of which survived a complete orbit. In the simulation with no such boundaries, dense rings form inside 3 AU, and it is possible that the rings themselves will become gravitationally unstable and fragment. However, we stress that the fragmentation of a disk, even under violent and extreme conditions, does not guarantee that companions will result from the debris.

The reliable determination of the final strength and effects of nonaxisymmetric disturbances in gravitationally unstable protostellar disks will require highly resolved, multidimensional hydrodynamics simulations in which a full suite of thermal physics, including heating and cooling, is employed.

For example, it is clear from the tests of our Poisson solver that extremely high azimuthal resolution or adaptive mesh techniques will be needed to treat dense clumps over many orbits. In terms of code physics, an AV scheme is in fact required to conserve energy in hydrodynamic simulations in which shocks develop. As we showed previously in PCDLII, simulations conducted under "adiabatic" conditions, but without AV, lost enough energy to affect the dynamics of the disk. A simulation that does not include a treatment for the heating and dissipation produced by irreversible processes introduces potentially significant cooling by design and so prevents a realistic assessment of whether a clump would have formed in the first place, let alone survive many orbits.

We thank A. Cameron, A. Boss, C. Chancey, S. Falle, T. Hartquist, G. Laughlin, A. Nelson, E. Rutha, S. Shore, and
D. Woolum for useful discussions and support during the writing of this paper. The comments of an anonymous referee and the scientific editor improved the manuscript. Portions of this work were conducted while B. K. P. enjoyed the hospitality of the Department of Physics and Astronomy at the University of Leeds as a Visiting Scholar. We are also grateful for the use of computing resources at Indiana University’s University Information Technology Services and the National Center for Supercomputing Applications. This work is supported by grants from the NASA Planetary Geology and Geophysics and Origins of Solar Systems programs and the SETI Institute.

REFERENCES

Bodenheimer, P., & Ostriker, J. 1973, ApJ, 180, 159
Boss, A. P. 1997, Science, 276, 1836
———. 1998, ApJ, 503, 923
———. 2000, ApJ, 545, L61
———. 2001, ApJ, 563, 367
———. 2002, ApJ, 576, 462
———. 2003, LPSC abstract
Cameron, A. G. W. 1978, Moon Planets, 18, 5
D'Alessio, P., Calvet, N., & Hartmann, L. 2001, ApJ, 553, 321
Durisen, R. H. 2001, in IAU Symp. 200, The Formation of Binary Stars, ed. H. Zinnecker & R. D. Mathieu (San Francisco: ASP), 381
Durisen, R. H., Mejia, A. C., & Pickett, B. K. 2001, ApJ, 563, L157
Dutrey, A., Guilloteau, S., Prato, L., Simon, M., Daudert, G., Schuster, K., & Menard, F. 1998, A&A, 338, L63
Hachisu, I. 1986, ApJS, 61, 479
Kamp, I., & van Zadelhoff, G.-J. 2001, A&A, 373, 641
Kuiper, G. P. 1951, in Astrophysics: A Topical Symposium, ed. J. A. Hynek (New York: McGraw-Hill), 357
Laughlin, G., & Bodenheimer, P. 1994, ApJ, 436, 335
Marcy, G. W., Cochran, W. D., & Mayor, M. 2000, in Protostars and Planets IV, ed. V. Mannings, A. Boss, & S. S. Russell (Tucson: Univ. Arizona Press), 1285
Mayer, L., Quinn, T., Wadsley, J., & Stadel, J. 2002, Science, 298, 1756
Mejia, A. C., Durisen, R. H., & Pickett, B. K. 2003, in ASP Conf. Ser. 294, Scientific Frontiers in Research on Extrasolar Planets, ed. D. Deming (San Francisco: ASP), in press
Nelson, A. F. 2000, ApJ, 537, L65
———. 2003, in ASP Conf. Ser. 294, Scientific Frontiers in Research on Extrasolar Planets, ed. D. Deming (San Francisco: ASP), in press
Nelson, A. F., Benz, W., Adams, F. C., & Arnett, D. 1998, ApJ, 502, 342
Nelson, A. F., Benz, W., & Ruzmaikina, T. V. 2000, ApJ, 529, 357
Norman, M. L., & Winkler, K.-H. 1986, in Astrophysical Radiation Hydrodynamics, ed. K.-H. Winkler & M. L. Norman (Dordecht: Reidel), 187
Ostriker, J. P., & Mark, J. W.-K. 1968, ApJ, 151, 1075
Pickett, B. K., Cassen, P., Durisen, R. H., & Link, R. 1998, ApJ, 504, 468 (PCDLI)
———. 2000a, ApJ, 529, 1034 (PCDLII)
Pickett, B. K., Durisen, R. H., Cassen, P., & Mejia, A. C. 2000b, ApJ, 540, L95
Pickett, B. K., Durisen, R. H., & Davis, G. A. 1996, ApJ, 458, 714
Pickett, B. K., Durisen, R. H., & Link, R. 1997, Icarus, 126, 243
Tomley, L., Cassen, P., & Steinman-Cameron, T. 1991, ApJ, 382, 530
Tomley, L., Steinman-Cameron, T., & Cassen, P. 1994, ApJ, 422, 850
Toomre, A. 1964, ApJ, 139, 1217
Truelove, J. K., Klein, R. I., McKee, C. F., Holliman, J. H., Howell, L. H., & Greenough, J. A. 1997, ApJ, 489, L179

1080 Pickett et al.