A Supersymmetry Anomaly

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A supersymmetry anomaly is found in the presence of non-perturbative fields. When the action is expressed in terms of the correct quantum variables, anomalous surface terms appear in its supersymmetric variation – one per each collective coordinate. The anomalous surface terms do not vanish in general when inserted in two- or higher-loop bubble diagrams, and generate a violation of the SUSY Ward identities.

1. The assumption that supersymmetry (SUSY) is not broken explicitly by non-perturbative quantum effects underlies many investigations of supersymmetric gauge theories. This assumption was tested in numerous semi-classical instanton calculations, but its validity beyond the semi-classical approximation has remained elusive.

No consistent UV regularization known today preserves SUSY. Also, one cannot define a SUSY theory in a finite box, because the SUSY variation involves the canonical fields as well as their conjugate momenta, and no choice of boundary conditions can ensure a vanishing SUSY current at the boundaries. These (not unrelated) facts mean that the possibility that quantum effects do violate SUSY merits a careful investigation.

In perturbation theory, conservation of the SUSY current can be enforced order by order. However, the significance of this observation is limited, since perturbation theory is an asymptotic expansion. Whether or not SUSY is an exact symmetry, is a question that must be settled by going beyond perturbation theory.

When spontaneous SUSY breaking does not take place, SUSY should be realized via exact Ward identities (WI) of the general form \( \langle \delta \mathcal{O} \rangle = 0 \). Here \( \mathcal{O} \) is a gauge invariant (multi)local operator, and \( \delta \) stands for the SUSY variation. In order to examine the validity of SUSY WIs, we have calculated \( \langle \delta \mathcal{O} \rangle \) in the most general continuum path-integral framework, using the standard rules of Quantum Field Theory. The result is the following equation

\[
\langle \delta \mathcal{O} \rangle = \sum_n \left\langle \mathcal{O} \delta \zeta_n \oint d\sigma_{\mu} J_{\mu}^n \right\rangle.
\]

The sum in eq. (1) runs over the collective coordinates pertaining to a given non-perturbative sector. \( \delta \zeta_n \) is the SUSY variation of the \( n \)-th collective coordinate. The current \( J_{\mu}^n \) involves a \( \zeta_n \)-derivative of the quantum field, and its explicit form is given below. The surface integration is at space-time infinity. The normal-ordering symbol means that the one-loop diagram obtained by contracting the two fields in \( J_{\mu}^n \) is discarded.

The SUSY anomaly is a sub-leading effect that arises from the interplay between two different physical scales.

Previous non-perturbative calculations were restricted to the semi-classical approximation, which is too crude to expose any anomalous breaking of SUSY. An exception is the two-loop result of ref. [3], which, however, has a limited scope, because it pertains to Super Yang-Mills, a theory that does not admit a systematic small-\( g \) expansion due to IR divergences. In contrast, our results are directly applicable to any non-perturbative sector of a weakly-coupled SUSY theory, and to any desired order.

In an operator language, we find a violation of the SUSY algebra at the non-perturbative level. A similar pattern exists in the case of the chiral anomaly – the axial charge \( Q_5 \) is conserved to all orders in perturbation theory, and only when non-perturbative effects are taken into account does one have \( Q_5 \neq 0 \).

The non-conservation of \( Q_5 \) is a consequence of the existence of zero modes in the eigenmode expansion of the fermion fields. Neither the UV regularization, nor the triangle graph, play a direct role. While the details are different, and more complicated, in the SUSY case, it remains true that the UV regularization plays little role in what follows.

Eq. (I) stems from a feature of the action principle, whose significance has not been appreciated before – the surface terms in the variation of the action depend in a non-trivial way on the nature of the independent field variables. Specifically, when the action is expressed in terms of the quantum variables pertaining to a non-perturbative sector, its SUSY variation contains the anomalous surface terms that ultimately appear in eq. (I).

2. We first specify our notation. In a non-perturbative sector, each bosonic field \( B(x) \) is split into a classical and a quantum part \( B(x) = b(x) + \beta(x) \). Here \( b(x) \) is the classical field and \( \beta(x) \) is the quantum part. It is not assumed that \( b(x) \) is an exact solution of the classical field equations. Fermion fields are denoted \( \psi(x) \). Also, \( Q(x) \) denotes any field, and \( \hat{q}(x) \) is its quantum part. (If \( Q(x) = \psi(x) \) is a fermion field, then \( Q(x) = \hat{q}(x) \).) The background field \( b(x) \) depends explicitly on the collective coordinates \( \zeta_n \). In addition, an infinite number of gauge degrees of freedom \( \omega^a(x) \) should be fixed.
The quantum part of each field is expanded in terms of independent modes \( \hat{q}(x) = \int \, dp \, \chi_p(x) \hat{q}_p \). The amplitude of a quantum mode is denoted \( \hat{q}_p \). The corresponding eigenfunction is \( \chi_p(x) \), with eigenvalue \( \lambda_p \). The symbol \( \int \, dp \) stands for \( \int (dp/2\pi) \) where \( p = \sqrt{\lambda_p \mu_p} \) and \( \sum \) is over all other quantum numbers of the continuous spectrum, plus a sum over normalization states. In particular, fermionic zero modes are included in the eigenmode expansion of the fermion fields. The quantum part of the bosonic fields satisfies \( \mathcal{P}_B^\parallel \beta = 0 \), where
\begin{equation}
\mathcal{P}_B^\parallel = \sum_{IJ} [b_{IJ}(b_{IJ})^{-1}] (b_{IJ}),
\end{equation}
and \( I = (n, \omega) \) is a generic index. \( b_{IJ}(y) = \Omega_n(x) \delta(x-y) \), where \( \Omega_n(x) \) is the linear differential operator such that \( \Omega_n(x) \) is an infinitesimal local gauge transform of the classical field. \( b_{IJ}(y) = b_n + \Omega \omega_n \) where \( \omega_n \) is determined by the background gauge condition \( \Omega^I b_{n} = 0 \). The bosonic eigenfunctions are determined by the eigenvalue equation \( L_B^I \chi^\omega = \lambda^\omega \chi^\omega \). Here \( \lambda^\omega \) is an eigenvalue of \( p^2 + M^2 \) where \( M \) is the mass matrix. \( L_B^I = \mathcal{P}_B^\parallel L_B \mathcal{P}_B^\parallel \) where \( L_B \) is the differential operator that enters the quadratic part of the bosonic action, and \( \mathcal{P}_B^\parallel (x,y) = \delta(x-y) - \mathcal{P}_B^\parallel (x,y) \) is a transversal projector. Notice that all eigenfunctions and, hence, all quantum fields \( \hat{q}(x) \), are explicit functions of the collective coordinates \( \zeta_n \).

Consider an infinitesimal field transformation \( Q(x) \rightarrow Q(x) + \delta Q(x) \). To keep the discussion general, we only assume at the moment that \( \delta Q(x) \) is a local function of \( Q(x), Q_n(x), \) etc. In a functional integral, the field variation \( \delta Q(x) \) must be generated by variations of the independent variables, which include the collective coordinates \( \zeta_n \), the gauge degrees of freedom \( \omega^a(x) \), and the amplitudes of the quantum modes \( \hat{q}_p \). Explicitly,
\begin{equation}
\delta Q(x) = \delta \hat{q}(x) + \sum_n Q_n(x) \delta \zeta_n + Q_\omega(x) \delta \omega(x). \tag{3}
\end{equation}

Here \( \delta \hat{q}(x) = \int \, dp \, \chi_p(x) \delta \hat{q}_p \). Eq. (3) defines the independent variations \( \delta \zeta_n, \delta \omega^a(x) \) and \( \delta \hat{q}_p \) in terms of \( \delta Q(x) \). The field derivative \( \delta Q \) is given by \( b_{IJ} + \beta_{IJ} \) for bosons (\( \psi_I \) for fermions). For the quantum part of any field \( \hat{q}_n(x) = -i g T^a \hat{q}(x) \) and \( \hat{q}_{IJ}(x) = \int \, dp \, \chi_{p,n}(x) \hat{q}_p \) where \( \chi_{p,n} = \chi_{p} - i g T^a \omega^a \chi_p \). An explicit expression for \( \delta \zeta_{IJ} = (\delta \zeta_n, \delta \omega^a(x)) \) follows from eq. (3) by exploiting the constraint \( \mathcal{P}_B^\parallel \beta = 0 \). In the SUSY case one has \( \delta \zeta_{IJ} = \sum_n C_{IJ} \delta \zeta_n \). In this expression, \( \Gamma \) is a constant matrix (that matches the indices of bosons and fermions), \( \Gamma \psi = \delta_{IJ} \) is the SUSY variation of bosons, and \( C_{IJ} = (b_{IJ} + \beta_{IJ}) \).

We now turn to the derivation of eq. (4). We introduce the functional differentiation operator
\begin{equation}
\mathcal{T} = \int \, dp \, \delta \hat{q}_p \frac{\partial}{\partial \hat{q}_p} + \sum_n \delta \zeta_n \frac{\partial}{\partial \zeta_n}. \tag{4}
\end{equation}

We will also write \( \mathcal{T}_q \) and \( \mathcal{T}_\zeta \) to denote respectively the first and second terms on the r.h.s. of eq. (4). For any local gauge-invariant operator \( \mathcal{O} \), one has \( \mathcal{T} \mathcal{O} = \delta \mathcal{O} \), where by definition, \( \delta \mathcal{O} = \delta Q (\partial \mathcal{O} / \partial Q) + \delta Q_n (\partial \mathcal{O} / \partial Q_n) + \cdots \). The expectation value \( \langle \delta \mathcal{O} \rangle \) is given by the functional integral
\begin{equation}
\langle \delta \mathcal{O} \rangle = \int \prod_n d \zeta_n \mathcal{D} \hat{q} J e^{-S} \mathcal{T} \mathcal{O}, \tag{5}
\end{equation}
where \( J = \text{Det} C / \text{Det} (b_{IJ} + \beta_{IJ})^{1/2} \) is a generalized Fadeev-Popov jacobian \( \mathcal{F} \). \( C_{IJ} \) is the same matrix that enters the definition of \( \delta Q \) above. Integration by parts now leads to the fundamental WI
\begin{equation}
\langle \delta \mathcal{O} \rangle = \left\langle \mathcal{O} (\mathcal{T} S) \right\rangle + \left\langle \mathcal{O} \delta \mu \right\rangle. \tag{6}
\end{equation}

Here \( \delta \mu \) is (minus) the variation of the path integral measure
\begin{equation}
- \delta \mu \equiv \mathcal{T} \log J + \int \, dp \, \left( \frac{\partial \delta \hat{q}_p}{\partial q_p} + \frac{\partial \delta \zeta_n}{\partial \zeta_n} \right). \tag{7}
\end{equation}

We comment in passing that the above reasoning is completely general. It is instructive to see how the formalism works in the case of an infinitesimal translation. Let us define \( \delta \mu Q = Q_{\mu} \). In this case, all collective and quantum variables are invariant, except for the translation collective coordinates \( x^a_0 \), which transform according to \( \delta x^a_0 = -\delta \mu_{\mu} \). Applying eqs. (3) and (7), one easily verifies that \( \langle Q_{\mu} \rangle = 0 \).

We now have to compute \( \mathcal{T} S \), paying attention to the surface terms. It is convenient to start with the action principle
\begin{equation}
\int d^4 x \, \delta \mathcal{L} - \int d^4 x \, \delta \sigma (\Pi_{\mu} \delta Q) = \int d^4 x \, \delta \mathcal{L} (-\Pi_{\mu,\mu} + \partial \mathcal{L} / \partial Q) . \tag{8}
\end{equation}

As usual \( \Pi_{\mu} = \partial \mathcal{L} / \partial Q_{\mu} \) where the lagrangian \( \mathcal{L} = \mathcal{L}(Q, Q_{\mu}) \). Substituting eq. (3) into the r.h.s. of eq. (8), and integrating by parts leads to
\begin{align}
\int d^4 x \, \delta \mathcal{L} (-\Pi_{\mu,\mu} + \partial \mathcal{L} / \partial Q) = & \int d^4 x \, \delta \hat{q} (-\Pi_{\mu,\mu} + \partial \mathcal{L} / \partial Q) \\
+ \sum_n \delta \zeta_n \int d^4 x \, \left( \pi_{\mu} Q_{\mu,n} + (\partial \mathcal{L} / \partial Q) Q_{n} \right) \\
- \sum_n \delta \zeta_n \int d \sigma_{\mu} J^n_{\mu}. \tag{9c}
\end{align}

We have used the gauge invariance of the action. Here
\begin{equation}
J^n_{\mu} = \Pi_{\mu} Q_{n}. \tag{10}
\end{equation}

In terms of the modes, the bilinear part of the action is \( S^{(2)} = (1/2) \int \, dp \, \lambda_{\mu} \hat{q}_p^2 \). Therefore,
In going from (11a) to (11b), we must let $L$ act on $\tilde{q}(x)$, and not on $\tilde{\theta}(x)$. This implies that expression (12) is equal to $T_{\sigma}S$. Putting everything together, we arrive at the result

$$T_{\sigma}S^{(2)} = \int dp \, \lambda_{\nu} \tilde{\theta}_p \tilde{\theta}_p$$

$$= \int d^4x \, \tilde{q}(x) L \tilde{q}(x).$$

In going from (11a) to (11b), we must let $L$ act on $\tilde{q}(x)$, and not on $\tilde{\theta}(x)$. This implies that expression (12) is equal to $T_{\sigma}S$. Putting everything together, we arrive at the result

$$T_{\sigma}S = \int d^4x \, \delta L - \oint d\sigma_\mu (\Pi_\mu \delta Q) + \sum_n \delta \zeta_n \oint d\sigma_\mu J_\mu^n. \tag{12}$$

Eq. (12) is completely general. In the SUSY case, $\delta L$ is a total derivative, and the first two terms on the r.h.s. of eq. (12) are equal to $\oint d\sigma_\mu S_\mu$. The last term in eq. (12) is an anomalous surface term. ($\oint d\sigma_\mu S_\mu$ vanishes unless there are massless one-fermion states in the spectrum of the SUSY current $S_\mu$.) In the above discussion, one can neglect modifications of the lagrangian by a total derivative $K_{\mu,n}$. This results in adding the $\zeta_\mu$-derivative of a current $K_{\mu,n}$ to $J_\mu^n$, which leaves invariant our final expression for the anomaly (see eq. (18) below).

It remains to compute $\delta L$. Using eq. (3), the expressions for $\delta \zeta_n$ and $\delta \tilde{\theta}_p$, and some lengthy algebra we get

$$\langle \delta_\zeta \mathcal{O} \rangle = \sum_n \left\{ \mathcal{O} \delta \zeta_n \left( \text{STr}(\partial/\partial \zeta_n) + \oint d\sigma_\mu J_\mu^n \right) \right\}. \tag{13}$$

The spectral trace $\text{STr}(\partial/\partial \zeta_n) = \int' dp \, (\chi_p)_{\zeta} \mathcal{O}$ comes from $\delta L$. In a gauge theory it includes the contribution of the ghost fields.

Regardless of its precise definition, the spectral trace cannot cancel $\oint d\sigma_\mu J_\mu^n$, because the former is a c-number function of the collective coordinates, whereas the latter is an operator. In fact, a detailed diagrammatic analysis which we relegate to a separate publication [1], shows that subtracting $-\text{STr}(\partial/\partial \zeta_n)$ from $\oint d\sigma_\mu J_\mu^n$ yields the normal-ordering prescription defined in eq. (12).

In terms of diagrams (compare eq. (12)),

$$\langle \mathcal{O} \rangle = \prod_n d\zeta_n \, e^{-S_{cl} - W_1 - W_2} \langle \mathcal{O} \rangle_\zeta. \tag{14}$$

At fixed $\zeta_n$, exp($-W_1$) = Det $(b_j | b_j)^\dagger$ times the functional determinants of $L_{\bar{F}}$ and $L_F$. $W_2$ is the sum of all connected bubble diagrams, and $\langle \mathcal{O} \rangle_\zeta$ is the sum of all diagrams that together constitute an insertion of $\mathcal{O}$. Using this notation, the result of the diagrammatic calculation is

$$\langle \delta_\zeta \mathcal{O} \rangle_\zeta = \sum_n \left\{ \langle \mathcal{O} \delta \zeta_n \oint d\sigma_\mu J_\mu^n \rangle_\zeta + \frac{\partial}{\partial \zeta_n} \langle \mathcal{O} \delta \zeta_n \rangle_\zeta \right\}$$

$$- \langle \mathcal{O} \delta \zeta_n \rangle_\zeta \frac{\partial}{\partial \zeta_n} (S_{cl} + W_1 + W_2). \tag{15}$$

Eq. (13) is valid in the (background) Landau gauge. Multiplying eq. (13) by exp($-S_{cl} - W_1 - W_2$), integrating over $\zeta_n$, and dropping a total $\zeta_n$-derivative, one arrives at eq. (11). In the semi-classical approximation, this result is consistent with the instanton calculus of Novikov et al. [1].

For an operator $\int d^4x \mathcal{O}(x)$, which is the integral over all space-time of a local density, eqs. (12) and (13) are modified by an additional surface term. This is true in particular for the SUSY generators. The additional surface term represents an anomalous transformation law, and it arises for the same reasons as the anomalous surface term in the variation of the action. See ref. 1 for the details. (SUSY violations arising from the UV regularization are cancelled by tuning the counter-terms order by order. This mechanism works both in the vacuum sector and in non-perturbative sectors, and is taken into account in eq. (15). The only remaining effect of the UV regularization amounts to anomalous local terms in the transformation law of certain composite operators [1].)

3. We now consider diagrams with an insertion of $\oint d\sigma_\mu J_\mu^n$. Terms in $J_\mu^n$ involving the classical field, as well as trilinear terms (which arise in a gauge theory) vanish at infinity and do not contribute to $\langle \mathcal{O} \rangle_\zeta$. For the trilinear terms this is due to transversality of the bosonic quantum field.) Therefore, henceforth $J_\mu^n = \tilde{\pi}_\mu q_{\mu,n}$, where $\tilde{\pi}_\mu = \hat{\psi}_\mu$ for bosons and ghosts, and $\tilde{\pi}_\mu = \tilde{\psi}_\mu$ for fermions.

For an asymptotically large distance $R$, the matrix element of $J_\mu^n$ between eigenstates of momenta $p$ and $p'$ involves an oscillatory factor exp($\pm i R(p\pm p')$). A non-vanishing result is therefore possible only when $\oint d\sigma_\mu J_\mu^n$ is inserted into a bubble diagram, because the latter contains a piece proportional to $\delta(p-\rho')$.

The only remaining term in eq. (15) thus factorizes into

$$\langle \mathcal{O} \delta_\zeta \rangle_\zeta \langle \oint d\sigma_\mu J_\mu^n \rangle_\zeta.$$

A diagram with an insertion of $\oint d\sigma_\mu J_\mu^n$ contains one special line $G_n(x,y) = \langle \tilde{q}(x) \tilde{q}_n(y) \rangle$. Explicitly

$$G_n(x,y) = \int' dp \, \chi_p(x) \frac{1}{\chi_p} \chi_{p,n}(y). \tag{16}$$

All other lines in the diagram correspond to ordinary propagators $G(x,y)$ in the given non-perturbative sector. However, since the surface integration takes place at infinity, one can replace $G(x,y)$ by the corresponding free propagator. Observe that in general (except for collective coordinates related to exact symmetries such as global translations) $G_n(x,y)$ is not a linear combination of ordinary propagators and their derivatives even asymptotically.

As a representative example, we now write down an explicit expression for the sum of all bubble diagrams with an insertion of $\oint d\sigma_\mu J_\mu^n$, where $\rho$ is the scale collective coordinate of the one-instanton sector. Exploiting the spherical symmetry of the instanton field, and removing kinematical factors, the asymptotic behaviour of a out-
state is $\chi_p \sim \exp(-ipr) + S \exp(ipr + i\alpha_1)$, where $\alpha_1$ is a constant phase that depends only on the angular momentum $l$. $S = S(p; l; \rho)$ is the first-quantized S-matrix associated with the differential operator $L_B^\dagger$ for bosons, with $L_F L_F^\dagger$ for fermions (we consider vector-like fermions for simplicity), and with $\Omega^\dagger \Omega$ for ghosts. At fixed $\rho$,

$$\langle \oint dp \sigma \mu : J^{(\rho)}_{\mu} : \rangle \rho = -tr (-)^F \int \frac{dp dp'}{8\pi^2} \tilde{\Sigma}(p') \tilde{G}(p') \times \left[ \oint dp \sigma \mu J^{(\rho)}_{\mu}(p', p) G_0(p) (\chi_p \chi_p^0) + h.c. \right]. \quad (17)$$

$\tilde{G}(p') = G_0^{-1}(p') + \tilde{\Sigma}(p)$, where $G_0(p)$ is the free propagator and $\Sigma(p)$ is the self-energy in the vacuum sector. $J^{(\rho)}_{\mu}(p', p)$ is the matrix element of $J^{(\rho)}_{\mu}$ between the singular part, $\chi_p \chi_p^0 \approx \pi \delta(p - p')(I + S^I)$ while $G_0(p) \oint dp \sigma \mu J^{(\rho)}_{\mu}(p, p) = ip \lambda^{-2} \partial \Sigma / \partial \rho$. (Recall that $\lambda^2$ diagonalizes $p^2 + M^2$.) We thus arrive at

$$\langle \oint dp \sigma \mu : J^{(\rho)}_{\mu} : \rangle \rho = -i \tr (-)^F \int \frac{d\lambda}{8\pi^2} \tilde{\Sigma} G \times \left[ (\partial \Sigma / \partial \rho)(I + S^I) - h.c. \right]. \quad (18)$$

This result generalizes to background fields lacking a spherical symmetry, and constitutes an explicit expression for the anomaly. Bubble diagrams with an insertion of $\oint dp \sigma \mu J^{(\rho)}_{\mu}$ have no overall UV divergence, because $\partial \Sigma / \partial \rho_n$ always vanishes rapidly for large $p$.

An geometric relation between the small-fluctuations operators follows from eq. (5). It reads

$$\Gamma L_B = L_F L_F^\dagger \Gamma \equiv V \cdot \delta \psi(b). \quad (19)$$

The symbol $\equiv$ means that eq. (19) holds when applied to functions $\chi^B(x)$ obeying the background gauge condition $\Omega^\dagger \chi^B = 0$, $\delta \psi(b)$ is the classical part of the SUSY variation of the fermions, $\Gamma$ has been defined below eq. (3), and $V$ is the three-index coupling defined by the fermion interaction lagrangian $L^{\phi \theta}_B = \phi \theta \psi \beta \bar{\psi}$. Eq. (19) implies that the scattering potentials that enter the bosonic and fermionic small-fluctuations equations are different. Consequently, $S_B \neq S_F$. (In four dimensions, the only exceptions are pure YM instantons with no Higgs field, when, due to the self-duality of exact solutions, the r.h.s. of eq. (13) involves a Dirac projection operator on a single chirality.) Since $S_B \neq S_F$, and since the all-orders expansion of $\Sigma G$ constitutes an infinite set of linearly independent functions of $p$, the sum of all bubble diagrams $\langle \oint dp \sigma \mu : J^{(\rho)}_{\mu} : \rangle \rho$ cannot vanish. This, in turn, implies that $\langle O \delta O \rangle$ is non-zero in general, because $O$ and the $\rho$-dependence of $\langle O \delta O \rangle$ are arbitrary.

In practice, $\langle \oint dp \sigma \mu : J^{(\rho)}_{\mu} : \rangle \rho$ should be $O(g^2)$. First, $\Sigma$ is manifestly $O(g^2)$. An additional $O(g^2)$ suppression factor appears because the fluctuations spectrum is approximately supersymmetric in the instanton’s core. The $\rho$-integration is dominated by $\rho \sim v^{-1}$, where $v$ is the Higgs VEV $\frac{v}{2}$. On the other hand, effects of the position-dependent background Higgs field, which are responsible for the discrepancy between $S_B$ and $S_F$, arise from the mass scale $m = gv$. As a result, the relevant form-factors depend on the dimensionless variable $m \rho \sim m/v = g$.

4. In this Letter we have derived a general expression for the SUSY anomaly. Our results prove that SUSY violations arise generically in asymptotically-free and super-renormalizable theories, when the full non-perturbative effects of the physical IR cutoff are taken into account. The essential ingredients of the violation are the existence of collective coordinates which transform non-trivially under SUSY, and the discrepancy between the dependence of the first-quantized S-matrices of bosons and fermions on the collective coordinates.

We now summarize the situation in various euclidean dimensions. In one dimension (SUSY q.m.) $S_B$ is always equal to $S_F$ on the relevant backgrounds. For two- and three-dimensional single instantons, i.e. fluxons and monopoles respectively, $S_B \neq S_F$, but both do not depend on the collective coordinates, which correspond only to exact symmetries. The single four-dimensional instanton was discussed above. Finally, multi instanton-antistanton configurations in more than one dimension lead to a non-trivial dependence of $S_B - S_F$ on some collective coordinates (such as relative positions).

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