Enhanced Impurity Scattering due to Quantum Critical Fluctuations: 
Perturbational Approach

Kazumasa MIYAKE and Osamu NARIKIYO

Department of Physical Science, Graduate School of Engineering Science, Osaka University, Toyonaka, Osaka
560-8531;
a) Department of Physics, Graduate School of Science, Kyushu University, Fukuoka 812-8581

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It is shown on the basis of the lowest order perturbation expansion with respect to critical fluctuations that the critical fluctuations give rise to an enhancement of the potential scattering of non-magnetic impurities. This qualitatively accounts for the enhancement of the resistivity due to impurities which has been observed in variety of systems near the quantum critical point, while the higher order processes happen to give much larger enhancement as seen from the Ward identity arguments. The cases with dynamical critical exponent $z=2$ and $z=3$ are discussed explicitly.

KEYWORDS: enhanced impurity scattering, quantum critical phenomena, enhanced residual resistivity

§1. Introduction

Recently the so-called non-Fermi liquid behaviors near the quantum critical point (QCP) have attracted much attentions in heavy electron systems in which the tuning of the quantum parameter is relatively easy, while such phenomena have also been studied in $d$-band metals exhibiting ferromagnetic QCP. Indeed, pressures of the order of several GPa can convert the magnetic ground state into the paramagnetic one, or vice versa, through the magnetic QCP. The anomalous behaviors of physical quantities around metallic and magnetic QCP have been successfully analyzed by the so-called SCR theory (and its extensions), except for some cases, while the basis of SCR theory has also been established on the basis of perturbational renormalization group method at least up to the intermediate coupling regime. The reason why such an approach is successful even in the strongly correlated metals should be attributed to the validity of the Fermi liquid theory for the description of the normal state on which the magnetic phase transition can be discussed.

It has been well recognized that the impurity scattering gives rise to a drastic effect on the critical behaviors near the QCP. For example, the dynamical exponent $z$ is altered considerably owing
to the impurity scattering: \( z = 3 \) in pure system for the ferromagnetic QCP is changed to \( z = 4 \) in the system with non-magnetic impurities.\(^{21} \) The effect of impurity scattering on the critical exponent of the temperature dependence of the resistivity has recently been discussed and attracted much interest.\(^{22} \) However, the effect of critical fluctuations on the impurity potential has scarcely been discussed so far, while a general argument on the Ward identity was given in relation to the insulating behaviors in non-Fermi liquids subject to the impurity scattering.\(^{23,24} \) On the other hand, the enhancement of the residual resistivity \( \rho_0 \) at around QCP has been reported in MnSi under the pressure of \( P \approx 15 \text{GPa} \)\(^{15} \) where the ferromagnetic state disappears, and in CeCu\(_2\)Ge\(_2\) under the pressure of \( P \approx 17 \text{GPa} \) where the rapid valence change of Ce ion may occur.\(^{8,25} \) Similar behavior has recently been observed in UGe\(_2\) under pressures.\(^{26} \) It has also been reported that the resistivity of CeNi\(_2\)Ge\(_2\) under \( P \approx 1.6 \text{GPa} \) grows up when the temperature is decreased below \( T = 2 \text{K} \).\(^{10} \)

A purpose of this paper is to discuss the effect of quantum critical fluctuations on the potential of non-magnetic impurity. It is shown on the basis of the perturbational treatment that the fluctuations associated with QCP can give rise to an enhancement of impurity potential which is, in the limit of zero-momentum transfer, in proportional to the mass enhancement factor \( 1/z_{cr} \), due to the critical fluctuations. As a result, the residual resistivity \( \rho_0 \) is shown to exhibit an enhancement as a function of the parameter measuring quantum criticality. This consequence is quite general while the mass enhancement factor itself depends on the dynamical exponent, and can be compared to the experimental observations in a couple of systems exhibiting QCP.

\section{Renormalization of impurity potential by critical fluctuations}

\textit{- A perturbational approach -}

In this section, we discuss the renormalization of impurity potential due to exchanging critical fluctuations by perturbation method. The lowest order correction of impurity potential is given by the Feynman diagram shown in Fig. 1. Its analytic expression for the vertex correction factor, corresponding to the scattering of quasiparticles from \( \vec{p} - \vec{k}/2 \) to \( \vec{p} + \vec{k}/2 \), is given as

\[
\Delta \Gamma_{\vec{k},\vec{p}}(i\epsilon_n; \eta) = \lambda^2 T \sum_{n'} \sum_{\vec{p'}} \chi(\vec{p} - \vec{p}', i\epsilon_n - i\epsilon_{n'}) \\
\times G(\vec{p'} - \frac{\vec{k}}{2}, i\epsilon_{n'}) G(\vec{p'} + \frac{\vec{k}}{2}, i\epsilon_{n'}),
\]

where \( \lambda \) is the coupling constant between the critical fluctuation modes and the quasiparticles. The Green function \( G \) of quasiparticles is expressed as

\[
G(\vec{p}, i\epsilon_n) = \int_{-\infty}^{\infty} dx \frac{A(\vec{p}, x)}{x - i\epsilon_n},
\]

and the propagator of fluctuation modes is expressed as

\[
\chi(\vec{q}, i\omega_n) = \int_{-\infty}^{\infty} dy \frac{B(\vec{q}, y)}{y - i\omega_n}.
\]
Here, we assume that the bare quasiparticles are well defined so that their spectral weight is approximated as

\[ A(\vec{p}, x) \simeq \bar{z} \delta(x - \xi_{\vec{p}}), \]  

where \( \bar{z} \) is the renormalization amplitude due to the effects other than the critical fluctuations; namely, it includes the effect of local correlation leading to heavy electron state. The spectral weight of fluctuation mode is given, by definition, as

\[ B(\vec{q}, y) \simeq \frac{1}{\pi} \text{Im} \chi(\vec{q}, y + i\delta), \]  

where the propagator of fluctuation modes is assumed to be parametrized as follows:

\[ \chi(q, \omega) \simeq \frac{\chi^{(0)}_Q}{\eta + A(\vec{q} - \vec{Q})^2 - iC_q\omega}, \]

where \( \chi^{(0)}_Q \) is of the order of \( \bar{N}_F \), the renormalized density of states (DOS) of bare quasiparticles at the Fermi level. The coefficient \( C_q \) in (2.6) depends on \( q \) in general and its \( q \)-dependence is dependent on the dynamical structure of QCP. In the case of the conventional antiferromagnetic (AF) fluctuations, \( C_q \) is essentially independent of \( q \) leading to the dynamical exponent \( z=2 \), while in the case of ferromagnetic fluctuations \( C_q = C/q \), \( C \) being a constant, leading to \( z=3 \). In the case of AF fluctuations where the even number of magnetic ions are contained in the unit cell and they are equivalent locally, and of uniform fluctuations which is not accompanied by conserved quantity such as valence fluctuations, \( C_q = C/\max\{q, \ell^{-1}\} \) in the limit \( T \to 0 \), \( \ell \) being the mean free path of the impurity scattering.

Substituting (2.2) and (2.3) into (2.1) and performing the summation with respect to \( n' \), the vertex correction factor for scattering potential is reduced to

\[ \Delta\Gamma_{\vec{k}, \vec{p}}(\epsilon + i\delta; \eta) = \frac{\lambda^2}{2} \sum_{\vec{q}} \int_{-\infty}^{\infty} dy B(\vec{q}, y) \frac{1}{\xi_{\vec{q}+\vec{k}/2-\vec{q}} - \xi_{\vec{q}+\vec{k}/2-\vec{q}}}, \]
\[ \times \left[ \text{coth} \frac{y}{2T} + \tanh \frac{\xi_{\bar{p}+\bar{k}/2-\bar{q}}}{2T} \right] - \text{coth} \frac{y}{2T} + \tanh \frac{\xi_{\bar{p}-\bar{k}/2-\bar{q}}}{2T} \right] \right], \quad (2.7) \]

where the analytic continuation \( \imath \epsilon_n \to \epsilon + \imath \delta \) has been performed, and \( \tilde{\lambda} \equiv \tilde{\varepsilon} \lambda \) is the effective coupling constant. In the limit of \( T \to 0 \), using the spectral function \( B \), (2.5), with \( \chi \) given by (2.6), the integration with respect to \( y \) in (2.7) can be easily performed obtaining

\[ \Delta \Gamma_{\bar{p},\bar{p}}(\epsilon; \eta) = \frac{\tilde{\lambda}^2}{2\pi} \sum_{\bar{q}} \chi(\bar{q}, 0) \frac{F(\tilde{E}_+, \xi_+) - F(\tilde{E}_-, \xi_-)}{\xi_- - \xi_+} \quad (2.8) \]

where \( \xi_\pm \equiv \xi_{\bar{p}+\bar{k}/2-\bar{q}} \), and the functions \( F \)'s are defined by

\[ F(\tilde{E}_\pm(\epsilon), \xi_\pm) \equiv -2\tilde{E}_\pm \ln |\tilde{E}_\pm| + \pi \text{sign}(\xi_\pm) \quad (2.9) \]

where

\[ \tilde{E}_\pm(\epsilon) \equiv \frac{C_q(\epsilon - \xi_\pm)}{\eta + Aq^2}. \quad (2.10) \]

Before examining the \( \eta \) dependence of (2.8), let us investigate the lowest order correction for the selfenergy of quasiparticles due to the critical fluctuations given by (2.6). Such a selfenergy is given by the Feynman diagram shown in Fig. 2 which corresponds to Fig. 1 for the lowest order correction to the impurity potential. Analytic expression of this selfenergy is given by

\[ \Sigma(\bar{p}, \imath \epsilon_n) = \lambda^2 T \sum_m \sum_{\bar{q}} \chi(\bar{q}, i\omega_m) G(\bar{p} - \bar{q}, \imath \epsilon_n - i\omega_m). \quad (2.11) \]

Performing the summation with respect to \( m \) and analytic continuation \( \imath \epsilon_n \to \epsilon + \imath \delta \), we obtain

\[ \tilde{\varepsilon} \Sigma(\bar{p}, \epsilon + \imath \delta) = -\frac{\tilde{\lambda}^2}{2\pi} \sum_{\bar{q}} \int_{-\infty}^{\infty} dy B(\bar{q}, y) \times \frac{\text{coth} \frac{y}{2T} + \tanh \frac{\xi_{\bar{p}-\bar{q}}}{2T}}{y + \xi_{\bar{p}-\bar{q}} - \epsilon - i\delta}. \quad (2.12) \]

In the limit of \( T \to 0 \), using (2.5) and (2.6), \( y \)-integration is easily performed obtaining

\[ \tilde{\varepsilon} \Sigma(\bar{p}, \epsilon + \imath \delta) = -\frac{\tilde{\lambda}^2}{2\pi} \sum_{\bar{q}} \chi(\bar{q}, 0) F_0(\tilde{E}_0, \xi_{\bar{p}-\bar{q}}), \quad (2.13) \]

where the function \( F_0 \) is defined as

\[ F_0(\tilde{E}_0(\epsilon), \xi_{\bar{p}-\bar{q}}) \equiv \frac{-2\tilde{E}_0 \ln |\tilde{E}_0| + \pi \text{sign}(\xi_{\bar{p}-\bar{q}})}{\tilde{E}_0^2 + 1}, \quad (2.14) \]

where

\[ \tilde{E}_0(\epsilon) \equiv \frac{C_q(\epsilon - \xi_{\bar{p}-\bar{q}})}{\eta + Aq^2}. \quad (2.15) \]
It is noted that the following relation holds:

$$\lim_{k \to 0} F(\tilde{E}_\pm(e), \xi_\pm) = F_0(\tilde{E}_\pm(e), \xi_{\vec{p}-\vec{q}}).$$  \hfill (2.16)$$

From Eqs. (2.13)-(2.15), we obtain the relation

$$-\bar{z} \frac{\partial \Sigma(\vec{p}, e)}{\partial e} = \frac{\tilde{\lambda}^2}{2\pi} \sum_{\vec{q}} \chi(\vec{q}, 0) \times \frac{C_q}{\eta + Aq^2} \frac{\partial F_0(\tilde{E}_0, \xi_{\vec{p}-\vec{q}})}{\partial \tilde{E}_0}.$$  \hfill (2.17)$$

In the limit of forward scattering, i.e., \( k \to 0 \), of the quasiparticles near the Fermi surface, the vertex correction factor (2.8) can be estimated with using (2.9) and (2.10) as follows:

$$\lim_{k \to 0} \Delta \Gamma_{\vec{k}, \vec{p}}(e; \eta) = -\bar{z} \frac{\partial \Sigma(\vec{p}, e)}{\partial e} \times \frac{\tilde{\lambda}^2}{2\pi} \sum_{\vec{q}} \chi(\vec{q}, 0) \lim_{k \to 0} \frac{\partial F(\tilde{E}_\pm, \xi_\pm)}{\partial \tilde{E}_\pm} + \frac{\tilde{\lambda}^2}{2\pi} \sum_{\vec{q}} \chi(\vec{q}, 0) \lim_{k \to 0} \frac{\pi}{\xi_+ - \xi_-} \left[ \text{sign}(\xi_+) - \text{sign}(\xi_-) \right].$$ \hfill (2.18)$$

The second term of (2.18) vanishes, because the phase space of \( \vec{q} \) satisfying the condition \( \text{sign}(\xi_+)\text{sign}(\xi_-) < 0 \) is restricted in a very narrow region as can be seen by geometrical consideration in the wave vector space. Indeed, its component \( \Delta q_\parallel \) parallel to \( \vec{p} \approx \vec{p}_F \) is restricted in the region \( |\Delta q_\parallel| < (k/p_F)^2 \) because the angle between \( \vec{p} - \vec{k}/2 \) and \( \vec{p} + \vec{k}/2 \) is proportional to \( (k/p_F)^2 \), so that the second term of (2.18) vanishes as \( \propto k \). Therefore, we obtain

$$\lim_{k \to 0} \Delta \Gamma_{\vec{k}, \vec{p}}(e; \eta) \simeq -\bar{z} \frac{\partial \Sigma(\vec{p}, e)}{\partial e},$$ \hfill (2.19)$$

which implies that the renormalized impurity potential \( \tilde{u}(\vec{k}) \) is given, in the zero momentum transfer limit, as

$$\tilde{u}(\vec{k} \to 0; \vec{p}) = \left[ 1 - \bar{z} \frac{\partial \Sigma(\vec{p}, e)}{\partial e} \right] u(\vec{k} \to 0),$$ \hfill (2.20)$$
where \( u(\vec{k}) \) is the bare impurity potential. Namely, \( \tilde{u} \) is enhanced by the mass enhancement factor \( 1/z_{cr}(\vec{p}, \epsilon) \equiv [1 - \tilde{z}\partial \Sigma(\vec{p}, \epsilon)/\partial \epsilon] \) which expresses the excess enhancement due to the critical fluctuations beyond the local correlations leading to the heavy electrons. This is consistent with the exact result obtained on the Ward identity argument by Betbeder-Matibet and Nozières, who showed on the Fermi liquid formalism that the renormalized impurity potential is given as

\[
\tilde{u}(\vec{k} \to 0) = \frac{1}{z(1 + F_0^a)} u(\vec{k} \to 0),
\]

where \( z \) is the renormalization amplitude including all the manybody effects and \( F_0^a \) the Landau parameter. Fermi liquid correction corresponds to that of higher order perturbation with respect to critical fluctuations which is beyond treatment in this paper. It is noted that renormalized impurity potential depends on the momentum \( \vec{p} \) of incoming quasiparticles in general, especially near the AF-QCP. Of the effects making \( z \) decrease, that arising from the local spin correlations should be cancelled by the factor \( (1 + F_0^a) \) in the heavy electrons as discussed by, e.g., in Ref. However, those beyond it, such as valence or magnetic fluctuations associated with quantum criticality, can give rise to excess reduction of \( z \).

The renormalization effect of impurity potential given here is in a close relation to the conventional renormalization effect in the Fermi liquid theory for the conserved quantities which are expressed in terms of quasiparticles by the same formula as the non-interacting particles, i.e., the weight \( z \) of quasiparticles in the one-particle spectral weight is cancelled by the vertex correction \( z^{-1} \) due to the incoherent processes. The physical reason of the enhancement of the impurity potential may be understood as follows: According to the expression (2.7), the vertex correction includes the factor \( B(\vec{q}, y)/(y + \xi_{\vec{p} \pm \vec{k} - \vec{q}} - \epsilon) \), where it is to be remembered that \( B(\vec{q}, y) \) is the spectral weight of spin fluctuations with the wavevector \( \vec{q} \) and the energy \( y \). So, the enhancement may be related to that of the intermediate states of critical magnetic fluctuations associated with QCP.

For an explicit calculation of the renormalization amplitude \( z_{cr} \), it is convenient to rewrite (2.17) directly from (2.12) in the form

\[
-z \frac{\partial \Sigma(\vec{p}, \epsilon)}{\partial \epsilon} = \frac{\lambda^2}{4\pi^3} \int_{FS} \frac{d^2 p'}{|\vec{v}_p'|} \text{Re} \chi(\vec{p} - \vec{p}', \epsilon),
\]

where \( \vec{v}_p \)'s are the velocity of bare quasiparticles, and FS indicates that the integration with respect to \( \vec{p}' \) is taken on the surface with \( \xi_{\vec{p}'} \approx \epsilon \). Using the explicit form (2.6) for the propagator of fluctuations with \( \vec{Q} = 0 \) and the dynamical critical exponent \( z=3 \), the left hand side of (2.22) is calculated resulting in

\[
-z \frac{\partial \Sigma(\vec{p}, \epsilon)}{\partial \epsilon} = \frac{\lambda^2 \chi_0^{(0)}}{8\pi^2 A(v_F)} \begin{cases} \ln \frac{A q_c^2 + \eta}{\eta}, & (\epsilon = 0); \\
\frac{2}{3} \ln \frac{A q_c^2 + C|\epsilon|}{C|\epsilon|}, & (\eta = 0), \end{cases}
\]

(2.23)
where \( \langle v_F \rangle \) is the averaged velocity of quasiparticles on the Fermi surface, and \( q_c \) is a cut-off wavenumber of the order of inverse of the lattice constant.

Similarly, for a class of fluctuations with \( \vec{Q} \neq 0 \) and the dynamical critical exponent \( z=2 \), \(-\partial \Sigma(\vec{p}, \epsilon)/\partial \epsilon \) given by (2.22) depends crucially on the position of \( \vec{p} \). Namely, for the momentum \( \vec{p}_h \) on the so-called “hot line”, where \( \xi_{\vec{p}_h}+\xi_q=\xi_{\vec{p}_h} = 0 \) on the Fermi surface, essentially the same expression as (2.23) is obtained. Its \( p \)-dependence around the “hot line” is parameterized by replacing \( \eta \) by \( \eta + Aq_m^2 \) in (2.23), where \( q_m \) is a measure of distance from the “hot line” on the Fermi surface. After averaging over \( q_m \) on the Fermi surface, one obtains

\[
-\left\langle \frac{\partial \Sigma(\vec{p}, \epsilon)}{\partial \epsilon} \right\rangle_{FS} = \frac{\tilde{\lambda}^2 \chi_Q^{(0)}}{8\pi^2 A(v_F)} \begin{cases} 
 b_1 - b_2 \sqrt{\frac{\eta}{Aq_m^2 + \eta}}, & (\epsilon = 0); \\
 b_1 - b_2 \sqrt{\frac{C|\epsilon|}{2Aq_m^2 + C|\epsilon|}}, & (\epsilon = 0),
\end{cases}
\]

(2.24)

where \( b_1 \) and \( b_2 \) are positive constant of \( \mathcal{O}(1) \) depending on the details of \( \vec{Q} \) and shape of the Fermi surface. It is remarked that the results (2.23) and (2.24) are consistent with those for the specific heat anomaly of SCR theory.\(^{16,17} \) In any case, the impurity potential for the forward scattering is enhanced in proportional to \( 1/z_{cr}(\vec{p}, \epsilon) \) as given by (2.20).

### §3. Effect of Critical Fluctuations on Residual Resistivity

In order to see how this enhancement of impurity potential affects the behaviors of the resistivity, we need to know the \( k \)-dependence of \( \tilde{u}(k) \) for the scattering from \( \vec{p} - \vec{k}/2 \) to \( \vec{p} + \vec{k}/2 \) near the Fermi surface. At first sight, the equality (2.19) holds also for general values of momentum transfer \( \vec{k} \) because the factor \([F(\vec{E}_+, \xi_+) - F(\vec{E}_-, \xi_-)]/(\xi_- - \xi_+)\) in (2.8) could be approximated by \(-(\partial F/\partial \vec{E})_{\xi=\xi_{\pm}}\) for \( \vec{q} \sim \vec{Q} \) where \( \chi(\vec{q}, 0) \) is diverging as \( 1/\eta \) near the QCP. However, this is not the case as shown by explicit calculation of (2.8) without using such an approximation. In this sense, a part of the results of Ref.\(^{25} \) should be revised as below. Indeed, the \( k \)-dependence of \( \tilde{u}(k) \) is estimated as follows. The expansion of \([F(\vec{E}_+, \xi_+) - F(\vec{E}_-, \xi_-)]/(\xi_- - \xi_+)\) with respect to \( k \) in (2.8) is allowed so long as \( k \ll |\vec{p} - \vec{q}| \sim p_F \), so that the relation similar to (2.18) holds. However, the factor \( \lim_{k \to 0} \partial F(\vec{E}_\pm, \xi_{\pm})/\partial \vec{E}_\pm \) in (2.18) should be replaced by \( \partial F(\vec{E}_\pm, \xi_{\vec{p} - \vec{q}})/\partial \vec{E}_\pm |_{\vec{E}_\pm = \tilde{E}_0(\epsilon - sk^2)} = \partial F_0(\tilde{E}_0, \xi_{\vec{p} - \vec{q}})/\partial \tilde{E}_0 |_{\tilde{E}_0 = \tilde{E}_0(\epsilon - sk^2)} \), where \( s \) is a coefficient of \( \mathcal{O}(1/\tilde{m}) \) with \( \tilde{m} \) being the effective mass of bare quasiparticles. It is because the energy of bare quasiparticle with momentum \( \vec{p} \) is given by \( \xi_p \approx \xi_{\vec{p} + \vec{k}/2} - sk^2 \) and different from those of incoming and outgoing particles, i.e., \( \xi_{\vec{p} + \vec{k}/2} = \xi_{\vec{p} - \vec{k}/2} \), where we are considering the elastic scattering. Therefore, for \( k \ll p_F \), we obtain instead of (2.19)
the following relation
\[ \Delta \Gamma_{\vec{k}, \vec{p}}(\epsilon; \eta) \simeq - \frac{\partial \Sigma(\vec{p}, \epsilon)}{\partial \epsilon} \bigg|_{\epsilon = -sk^2} \].
\[ \text{(3.1)} \]
Namely, the impurity potential \( \tilde{u}(k) \), for \( k \ll p_F \), giving the scattering on the Fermi surface (i.e., \( \epsilon = 0 \)), is renormalized as
\[ \tilde{u}(\vec{k}; \vec{p}) = \left[ 1 - \bar{z} \frac{\partial \Sigma(\vec{p}, \epsilon)}{\partial \epsilon} \right]_{\epsilon = -sk^2} u(\vec{k}; \vec{p}), \]
\[ \text{(3.2)} \]
where \( \partial \Sigma(\vec{p}, \epsilon)/\partial \epsilon \) is given by (2.23) in the case of QCP with \( z = 3 \). In the case of QCP with \( z = 2 \), \( \partial \Sigma(\vec{p}, \epsilon)/\partial \epsilon \) has large dependence on \( \vec{p} \) as discussed above.

If the bare impurity potential causes essentially the Born scattering, the residual resistivity \( \rho_0 \) is enhanced by the critical fluctuations. Namely, \( \rho_0 \) is given as
\[ \rho_0 \propto \left\langle \left[ 1 + \Delta \Gamma_{\vec{k}, \vec{p}}(0; \eta) \right]^2 \right\rangle_{FS} \times 2\pi N_F \bar{c}_{\text{imp}} |u(\vec{k}; \vec{p})|^2 (1 - \cos \theta), \]
\[ \text{(3.3)} \]
where \( c_{\text{imp}} \) is a concentration of impurity, \( \theta \) is an angle between \( \vec{p} \pm \vec{k}/2 \), and the on-shell condition \( \epsilon = \xi_{\vec{p} \pm \vec{k}/2} = 0 \) has been used. Here it is noted that explicit dependence of renormalization amplitude \( z_{\text{cr}} \) does not appear due to cancellation between that for DOS and that for the damping rate of quasiparticles. We perform the calculation of (3.3) for the spherical Fermi surface with using a unit of wavenumber such that \( 2p_F \sqrt{s} = 1 \), and assuming that \( A_{\text{q}}^2 = 1 \). In the case of \( z = 2 \), the average over \( q_m \), the measure of distance from the “hot line”, is approximated by \( \int_0^\infty dq_m (\cdots) / \int_0^\infty dq_m \). The results of numerical calculations of \( \rho_0 \) as a function of \( \eta \) are shown in Fig. 3 in an arbitrary unit by normalizing at \( \eta = 0 \). Although the logarithmic divergence in \( \Delta \Gamma \) of (2.23) is smeared out due to the geometrical factor \( (1 - \cos \theta) \) in the case of \( z = 3 \), a rather sharp cusp structure still remains in \( \rho_0 \) as a function of \( \eta \), like \( \rho_0 \propto 1 - 8\eta \) for \( \eta \sim 0 \). Here it is noted that \( k \approx 2p_F \sin(\theta/2) \). In case of \( z = 2 \), the result is much more smeared by the extra average process over \( q_m \).

§4. Discussions

The result obtained in §3 explains qualitatively the anomaly of \( \rho_0 \) observed near the ferromagnetic QCP,\(^{15,26}\) because the main anomaly arises from the factor \( 1/z \) in (2.21). In the case of AF-QCP, the less pronounced anomalies are expected to be observed. This is also consistent with the experimental fact that no strong anomaly of \( \rho_0 \) is observed around AF-QCP with \( z = 2 \).

However, it needs careful consideration in the case of AF-QCP in compounds which have even number of magnetically equivalent ions in the primitive cell such as CeCu\(_{6-x}\)Au\(_x\), containing magnetically almost equivalent four Ce\(^{+3}\) ions in the primitive cell. Namely, the universality class of
critical fluctuations of such systems belongs to that with the dynamical exponent $z=3$ as discussed previously in Ref.,$^{27}$ so that anomaly of $\rho_0$ near QCP is expected to become much sharper than that of the conventional AF-QCP with the dynamical exponent $z=2$. This conclusion is consistent with a strong pressure dependence of $\rho_0$ observed in CeCu$_6$,$^{34}$ which is considered to be located near AF-QCP because CeCu$_{5.9}$Au$_{0.1}$ exhibits a non-Fermi liquid behavior characteristic of QCP although its universality class has not been identified yet. The sharp decrease of $\rho_0$ as a function of the pressure, as shown in Fig. 2 of Ref.,$^{34}$ does not seem to be understood as the same mechanism as canonical behaviors of gradual decrease of $\rho_0$ under pressure which are observed in a series of heavy electron systems, such as CeInCu$_2$,$^{35}$ CeAl$_3$,$^{36}$ and so on.

In the case of QCP associated with a valence transition as observed in CeCu$_2$Ge$_2$,$^{37}$ the corresponding Fermi liquid effect in (2.21) may give dominant contribution and lead to much more pronounced enhancement of $\rho_0$ as will be discussed elsewhere. Such a Fermi liquid correction is related to the higher order perturbations with respect to the critical fluctuations. Analysis of these higher order terms is left for future study. The Fermi liquid correction is also important for the enhancement of exchange potential of magnetic impruity near the ferromagnetic QCP, and leads to a non-trivial effect.$^{38}$

The resistivity $\rho_{\text{imp}}$ due to impurity scattering is expected to have prominent $T$-dependence, in general, arising from renormalization of impurity potential by the critical fluctuations at around QCP.$^{10}$ Therefore, one has to be careful when $T$-dependence of the observed resistivity is compared to existing theories.$^{17,22}$

The enhancement of $\rho_0$ near the ferromagnetic QCP should work to suppress the anisotropic superconductivity which is expected to appear around there.$^{39}$ One has to remember this effect

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**Fig. 3.** Residual resistivity $\rho_0$ due to non-magnetic impurity as a function of inverse susceptibility $\eta$. 
when discussing the superconductivity induced by critical ferromagnetic fluctuations in real metals.

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1) H. von Löhneysen : J. Phys. C 8, 9689 (1996); H. von Löhneysen, T. Pietrus, G. Portisch, H. G. Schlager, A. Schröder, M. Sieck, and T. Trappmann: Phys. Rev. Lett. 72, 3262 (1994).
2) K. Umeo, H. Kadomatsu and Takabatake : Phys. Rev. B 54, 1194 (1996).
3) C. Sekine, H. Sakamoto, S. Murayama, K. Hoshi and T. Sakakibara: Physica B 206&207 (1995) 291; T. Taniguchi, Y. Tabata, H. Tanabe and Y. Miyako: Physica B 230-232 (1997) 123.
4) S. Kambe, J. Flouquet and T. E. Hargreaves: J. Low Temp. Phys. 108 (1997) 383.
5) N. D. Mathur, F. M. Grosche, S. R. Julian, I. R. Walker, D. M. Freye, R. K. W. Haselwimmer and G. G. Lonzarich: Nature 394, 39 (1998); S. Raymond and D. Jaccard : Physica B 281&282, 1 (2000).
6) F. Grosche, M.J. Steiner, P. Agarwal, I. R. Walker, D. M. Freye, S. R. Julian and G. G. Lonzarich: Physica B 281&282, 3 (2000).
7) T. C. Kobayashi, T. Muramatsu, M. Takimoto, K. Hanazono, K. Shimizu, K. Amaya, S. Araki, R. Settai and Y. Onuki: Physica B 281&282, 7 (2000).
8) D. Jaccard, E. Vargoz, K. Alami-Yadri and H. Wilhelm: Rev. High Pressure Sci. Technol. 7 (1998) 412; D. Jaccard, H. Wilhelm, K. Alami-Yadri and E. Vargoz: Physica B 259-261 (1999) 1.
9) F. Steglich, B. Buschinger, P. Gegenwart, M. Lohmann, R. Helfrich, C. Langhammer, P. Hellmann, L. Donnevert, S. Thomas, A. Link, C. Geibel, M. Lang, G. Sparn and W. Assmus: J. Phys. C 8, 9909 (1996); F. Steglich: Z. Phys. B 103, 235 (1997); S. J. Lister, F. M. Grosche, F. V. Carter, R. K. W. Haselwimmer, S. S. Saxena, N. D. Mathur, S. R. Julian and G. G. Lonzarich: Z. Phys. B 103 (1997) 263; P. Gegenwart, P. Hinze, C. Geibel, M. Lang and F. Steglich: Physica B 281&282, 5 (2000).
10) T. Fukuhara and K. Maezawa: private communications.
11) O. Trovarelli, C. Geibel, S. Mederle, C. Langhammer, F. M. Grosche, P. Gegenwart, M. Lang, G. Sparn and F. Steglich: Phys. Rev. Lett. 85 (2000) 026.
12) G. Knebel, D. Braithwaite, G. Lapertot, J. Frouquet and P. C. Canfield: preprint, June 2001.
13) T. F. Smith, J. A. Mydosh and E. P. Wohlfarth: Phys. Rev. Lett. 27 (1971) 1732.
14) C. Pfleiderer, G. J. MacMullan and G. G. Lonzarich: Physica B 206&207 (1995) 847.
15) C. Thessieu, J. Frouquet, G. Lapertot, A. N. Stepnan and D. Jaccard: Solid State Commun. 95 (1995) 707.
16) T. Moriya: Spin Fluctuations in Itinerant Electron Magnetism (Springer-Verlag, Berlin, 1985)
17) T. Moriya and T. Takimoto: J. Phys. Soc. Jpn. 64 (1995) 960.
18) J. A. Hertz: Phys. Rev. B 14 (1976) 1165.
19) A. J. Millis: Phys. Rev. B 48 (1993) 7183.
20) L. D. Landau: Sov. Phys. JETP 3 (1957) 920; ibid. 5 (1957) 101.
21) P. Fulde and A. Luther: Phys. Rev. 170 (1968) 570.
22) A. Rosch: Phys. Rev. Lett. 82 (1999) 4280, and references therein.
23) G. Kotliar, E. Abrahams, A. E. Ruckenstein, C. M. Varma, P. B. Littlewood and S. Schmitt-Rink: Europhys. Lett. 15 (1991) 655.
24) C. M. Varma: Phys. Rev. Lett. 97 (1997) 1535.
25) K. Miyake, O. Narikiyo and Y. Onishi: Physica B 259-261 (1999) 676.
26) G. Oomi, T. Kagayama, F. Honda, Y. Onuki and E.V. Sampathkumaran: Physica B 281 & 282 (2000), 393.
27) M. Hatatani, O. Narikiyo and K. Miyake: J. Phys. Soc. Jpn. 67 (1998) 4002.
28) C. M. Varma: Phys. Rev. Lett. 75 (1995) 898.
29) O. Betbeder-Matibet and P. Nozières, Ann. Phys. 37 (1966) 17.
30) C. M. Varma, K. Miyake and S. Schmitt-Rink: Phys. Rev. Lett. 57 (1986) 626.
31) A. A. Abrikosov, L. P. Gorkov and I. Ye. Dzyaloshinskii: Quantum Field Theoretical Methods in Statistical Physics, 2nd editions (Pergamon Press, Oxford, 1965) §19.3.
32) A. J. Leggett: Phys. Rev. 140 (1965) A1869.
33) H. Maebashi: private communications.
34) S. Raymond and D. Jaccard: J. Low Temp. Phys. 120 (2000) 107.
35) T. Kagayama, G. Oomi, H. Takahashi, N. Mori, Y. Onuki and T. Komatsubara: Phys. Rev. B 44 (1991) 7690.
36) J. Flouquet, P. Haen, P. Lejay, P. Morin, D. Jaccard, J. Schweizer, C. Vettier, R. A. Fisher and N. E. Phillips: J. Magn. Magn. Mater. 90 & 91 (1990) 377.
37) Y. Onishi and K. Miyake: J. Phys. Soc. Jpn. 69 (2000) 3955.
38) H. Maebashi, K. Miyake and C. M. Varma: preprint, cond-mat/0109276.
39) Monthoux and G. G. Lonzarich: Phys. Rev. B 59 (1999) 14598, and references therein.