Electrical engineering unit for the reactive power control of the load bus at the voltage instability

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Abstract. For the purpose of reactive power control error reduction and decrease of the voltage sags in the electric power system caused by the asynchronous motors started the mathematical model of the load bus was developed. The model was built up of the sub-models of the following elements: a transformer, a transmission line, a synchronous and an asynchronous loads and a capacitor bank load, and represents the automatic reactive power control system taking into account electromagnetic processes of the asynchronous motors started and reactive power changing of the electric power system elements caused by the voltage fluctuation. The active power/time and reactive power/time characteristics based on the recommended procedure of the equivalent electric circuit parameters calculation were obtained. The derived automatic reactive power control system was shown to eliminate the voltage sags in the electric power system caused by the asynchronous motors started.

1. Introduction
The electric power system is intended to supply consumers timely with the power of the appropriate quality and in sufficient quantity provided that the transmission losses are minimized. Actually, these losses accounts for the substantial amount of the electric power as both bulk power systems and distribution systems of Russia operate at the low power factor.
The main reactive power consumers are transformers and asynchronous motors operating at the power factor that is equal to 0.75-0.9. The total reactive power losses account for over 50% of the total reactive power produced. One of the ways to reduce these losses is the reactive power compensation in the load buses.
In the presence of the synchronous motors in the load buses the reactive power compensation can be carried out by means of the automatic excitation control continuously monitoring such parameters as reactive power of the asynchronous load, voltage of the load bus, etc. [1, 2] and applying them as functions to build the control system.
Sufficient number of papers [3-10] covers mathematical models and excitation control algorithms of the synchronous machines operating as generators. Much fewer papers [2, 11-14] address the issue about the mathematical models of the synchronous motors. Moreover, the voltage fluctuation of the electric power system and transients during asynchronous motor breaking away, accelerating and re-accelerating are not taken into consideration in those models.
2. Models and methods

2.1. The mathematical model of the load bus

The single-line diagram of the load bus is shown in figure 1.

The elements of the single-line diagram are as follows: the transformer ($T_1$); the transmission line (TL); the asynchronous motors (AM) with the total rated power $P_r=3000$ kW; the synchronous motor (SM); the capacitor bank (CB).

The letter symbols of the diagram are following: $P$, $P_1$, $P_2$ – active powers of the transmission line, the synchronous and the asynchronous motors, respectively; $Q$, $Q_1$, $Q_2$ – reactive power of the transmission line, the synchronous and the asynchronous motors, respectively; $U_1'$, $U_1$, $U_2$ – primary and secondary voltages of the transformer and voltage at the input terminals of the motors and the capacitor bank, respectively.

The mathematical model of the load bus is represented by the operator equations (1) [15, 16]:

$$\Delta E_q(p) = \left[ \frac{1}{R_f} \Delta U_f(p) + T_1 p \Delta \theta(p) - T_1 p \Delta U_2(p) \right] W_2(p)$$

$$\Delta \theta(p) = \left[ \frac{k_1}{E_{q0}} \Delta E_q(p) + k_2 \Delta m(p) + \frac{k_1}{U_{o2}} \Delta U_2(p) \right] W_1(p)$$

$$\Delta Q(p) = k_2 \Delta U_1(p) + k_4 \Delta \theta(p) + k_6 \Delta E_q(p)$$

$$\Delta U_j(p) = [k_{i1} \Delta Q(p) + \Delta U_1(p) + (k_5 \Delta m(p) - \Delta P_2(p))k_9] k_{i1}$$

The parameters of the synchronous motor:

$$T_1 = \frac{3U_{o2} X_{ad} \sin \theta_0}{2\omega_0 R_f x_d}; \quad X_{ad} = \sqrt{3} x_{ad}; \quad T_q = \frac{x_{ad} \cos \theta_0}{\omega_0 R_f x_d}; \quad \omega_0 = 2\pi f$$

$$W_2(p) = \frac{1}{T_1 p + 1}; \quad T_2 = \frac{x_d}{x_g} T_1; \quad x_d = x_d - \frac{3X_{ad}^2}{2x_f} T_f = \frac{x_f}{\omega_0 R_f}$$

$$k_1 = -t g \theta_0; \quad k_2 = \frac{1}{\cos \theta_0}; \quad W_1(p) = \frac{1}{T_1^2 p^2 + 2\epsilon_1 T_1 p + 1}; \quad T_i = \sqrt[3]{\frac{J}{p_1 m_{em} M \cos \theta_0}}$$

$$\epsilon_1 = \frac{m_{bd}}{2\omega_0 M_{em} \sqrt{p_1 m_{em} M \cos \theta_0}}; \quad m_{bd} = \frac{M_{bd}}{M_{em}}; \quad m_{em} = \frac{M_{em}}{M}; \quad k_3 = \frac{E_{q0} \cos \theta_0}{x_d}$$

$$k_4 = -\frac{U_{o2} E_{q0}}{x_d} \sin \theta_0; \quad k_5 = \frac{U_{o2} \cos \theta_0}{x_d}; \quad k_9 = P_{en}$$

Figure 1. Single-line diagram of the load bus.
where $R_f$ – excitation winding resistance; $E_{q0}$ – quadrature-axis electromotive force of the synchronous motor; $U_{o2}$ – steady-state load bus voltage; $x_{ad}$ – mutual reactance; $J$ – moment of inertia of the synchronous drive; $p_p$ – number of pairs of poles of the synchronous motor; $M_r$, $M_{bd}$, $M_{em}$ – rated, breakdown and maximum electromagnetic torques, respectively; $s_{bd}$ – breakdown slip; $x'_d$ – direct-axis transient reactance; $\omega_r$ – rated angular frequency of the synchronous motor; $x_f$ – excitation winding reactance; $P_{em}$ – ceiling electromagnetic power.

Transfer coefficients of the following load bus elements:

Asynchronous motors:

$$k_7 = \frac{2.3}{U_{o2}}$$

(3)

Capacitor bank:

$$k_8 = \frac{2Q_{o3}}{U_{o2}}$$

(4)

Transmission line:

$$k_9 = -\frac{R_{TL} + R_T}{2U_{o2} - U_{o1}}; k_{10} = -\frac{x_{TL} + x_T}{2U_{o2} - U_{o1}}; k_{11} = -\frac{U_{o2}}{2U_{o2} - U_{o1}}$$

(5)

where $R_{TL}$, $x_{TL}$ – resistance and reactance of the transmission line; $R_T$, $x_T$ – resistance and reactance of the transformer.

The transfer coefficient $k_{12}$ of the element representing the negative feedback by means of $k_7$ and positive feedback by means of $k_8$ is equal to:

$$k_{12} = \frac{1}{1 + (k_7 - k_8)k_{10}}$$

(6)

The set of equations (1)-(5) is represented in the form of the block-diagram in figure 2 where the voltage feedback $\Delta U_2$ of the synchronous motor is expressed by the elements with the coefficients $k_3$, $k_\delta/U_{o2}$ and the element with the transfer function $T_{4p}$.

**Figure 2.** Block-diagram of the load bus.

The parameters of the above-mentioned block-diagram equal: $T_0=0.31$ s, $\varepsilon_1=0.1$, $R_f=0.18$ Ohms, $X_{ad}=52$ Ohms, $T_2=0.367$ s, $U_{o2}=6000$ V, $E_{q0}=11.800$ V, $T_1=162$ A·s, $T_4=0.034$ s/Ohms, $k_1=-0.75$, $k_2=1.25$, $k_3=-0.078$ kA, $k_\delta=-1.29$ kVA, $k_5=0.154$ kA, $k_6=2.500$ kW, $k_7=-0.075$ kA⁻¹, $k_9=-1.1$, $k_{10}=1.19$. 


2.2. Automatic reactive power control system model

Functional diagram of the automatic reactive power control system is shown in figure 3.

The letter symbols of the diagram are following: RPD1, RPD2 – reactive power definers; $W_{R}(p)$, $W_{EC}(p)$, $W_{TE}(p)$ – transfer functions of the reactive power, excitation current and thyristor excitation, respectively; $K_{CS}$, $K_{RPS1}$, $K_{RPS2}$ – transfer functions of the current sensor, reactive power sensor of the load bus and reactive power sensor of the asynchronous motors, respectively.

The set of equations includes both the negative feedbacks of the excitation current $\Delta I_f$ and reactive power $\Delta Q$ and the positive feedback of the reactive power $\Delta Q_2$ that is the most contributing disturbance.

The asynchronous motors are started with delay of $t=\tau_0$. During the same period of time $t=\tau_0$ the RPD2 in collaboration with RPD1 produces the maximum power over this time. Thus, the feedforward control is provided by means of the elements with the transfer functions $e^{-\tau_0}$ and $1-e^{-\tau_0}$.

2.3. The procedure of the reactive power/time and active power/time characteristics obtaining during the squirrel cage asynchronous motor starting

Active and reactive powers of the asynchronous motor started are calculated by equations:

$$P_2(s) = \frac{U_s^2 R(s)}{R^2(s) + x^2(s)}$$

$$Q_2(s) = \frac{U_s^2 x(s)}{R^2(s) + x^2(s)}$$

Slip ‘s’ can be found applying the next formula:

$$\frac{ds(t)}{dt} + \frac{2M_{ad}s(t)}{J(s^2(t) - s_{ad}^2)} = \frac{M_L}{J}$$

where $M_L$ – load torque.

Slip-resistance and slip-reactance relationships can be derived from the equivalent electric circuit [17]:

![Functional diagram of the automatic reactive power control system.](image-url)
\[ R(s) = R_i + \frac{x_2^2 R_2(s)}{sA(s)}; \quad x(s) = \frac{x_2(s)x_\mu + x_\mu \left( \frac{R^2(s)}{s} + x_2^2(s) \right)}{A(s)} \]

\[ A(s) = (x_\mu + x_2(s))^2 + \frac{R^2(s)}{s^2} \]  \hspace{1cm} (9)

where \(x_1, R_1\) – reactance and resistance of the stator winding; \(x_2, R_2\) – reactance and resistance of the rotor winding referred to the stator; \(x_\mu\) – magnetizing reactance (that represents the reactance of the air gap).

The reactances \(x_1(s), x_2(s)\) and resistance \(R_2(s)\) are not constant and depend on the slip ‘s’ as when the rotor accelerates the skin effect and rotor teeth saturation effect appear to a greater extent and result in decrease of \(x_1(s), x_2(s)\) and increase of \(R_2(s)\).

The relationships between \(R_2(s), x_K(s)\) and \(s\) are better to be represented by the approximated nonlinear function of the form [17]:

\[
R_2(s) = \begin{cases} 
R_2(s_r), \forall 0 \leq s \leq s_{bd} ; \\
R_2(0) + (R_2(1) - R_2(0))\sqrt{s}, \forall s_{bd} < s \leq 1 
\end{cases} \\
R_2(0) = \frac{R_2(s_r) - R_2(1)\sqrt{s_{bd}}}{1 - \sqrt{s_{bd}}} \hspace{1cm} (10)
\]

\[
x_K(s) = \begin{cases} 
x_K(s_r), \forall 0 \leq s \leq s_{bd} ; \\
x_K(s_r) - \frac{(x_K(s_r) - x_K(1))}{0.25 - s_{bd}}(s - s_{bd}), \forall s_{bd} < s \leq 0.25; \\
x_K(1), \forall 0.25 < s \leq 1 
\end{cases}
\]

where \(x_K(s)\) – total reactance of the stator and the rotor.

The stator reactance and the rotor reactance are set to as in [17] and calculated by the formulas:

\[
x_1(s) = 0.42x_K(s) \hspace{1cm} x_1(s) = 0.58x_K(s) \hspace{1cm} (11)
\]

In the existing papers the unknown values \(R_1, R_2(s_r), x_K(s_r)\) mentioned above are typically obtained from the solution of two equations derived from the electromagnetic power equation at \(s=s_r\) and \(s=s_{bd}\). But as long as there are three unknown values some authors [18, 19] predetermine unjustifiably one of them to solve the equation set, and the others [20, 21] apply different iterative algorithms to find a solution. In the first case the result is unambiguous and has unacceptable error if the unknown value is predefined incorrectly, in the second one, the calculation is extremely complicated and hardly applicable to the practice application.

In this paper the third equation [22]:

\[ R_2 = 2a_3s_{bd}x_K \]  \hspace{1cm} (12)

is added to the existing two ones:

\[ R^2_r + \frac{R^2_2}{s_r} - \frac{a_1}{s_r}R_2 + \frac{2}{s_r}R_2R_2 + x_K^2 = 0 \]  \hspace{1cm} (13)

\[ R_1 = a_2 - a_3x_K^2 \]  \hspace{1cm} (14)
where \( a_1 = \frac{U_r^2}{a_i P_r}, \quad a_0 = \frac{1.02}{1-s_r}, \quad r_a = \frac{a_0 a_i}{4b_{rd}}, \quad a_i = \frac{b_{rd}}{a_0 a_i}, \quad R_2 = R_c(s_r), \quad x_K = x_K(s_r), \quad b_{rd} \) – torque ratio at the breakdown slip.

These two equations are obtained from the electromagnetic power equation at \( s=s_r \) and \( s=s_{bd} \):

\[
\frac{a_0 P_r}{b_{rd} P_r} = \frac{U_r^2 R_c x_K^{-1}}{(R_i + R_c s_r^{-1})^2 + x_K^2}
\]

This third equation is derived from the reactive power rotor dissipation equation:

\[
\frac{P_r}{2b_{rd}} = \frac{U_r x_K}{(R_i + R_c s_r^{-1})^2 + x_K^2}
\]

The equations (12) and (14) plugged in the equation (13) give the following one-variable equation:

\[
\sum_{j=0}^{4} c_j x_K^j = 0
\]

where \( c_0 = \left( \frac{a_0 a_i}{2b_{rd}} \right)^2, \quad c_1 = \frac{2a_i - a_0}{s_r a_i^2}, \quad c_2 = \frac{1-2a_0 a_i + a_i^2 s_r^{-2}}{a_i^2}, \quad c_3 = -\frac{2d_a}{a_i s_r}, \quad c_4 = 1. \)

Magnetizing reactance [22]:

\[
x_m = \frac{U_r^2}{P_r(\eta_r^{-1} t^2 \varphi_r - 0.5 b_r^2)} - x_i
\]

where \( \eta_r \) – rated efficiency of the motor; \( \varphi_r \) – phase difference between the stator current and the voltage under rated operating conditions.

The locked-rotor resistance and reactance (at \( s=1 \)):

\[
R_c(1) = \frac{K_i a_i P_r}{3K_r I_r^2}
\]

\[
x_K(1) = \frac{U_r^2}{K_i a_i P_r R_c(1) - (R_i + R_c(1))^2}
\]

where \( K_i, K_r \) – locked-rotor current ratio and locked-rotor torque ratio; \( I_r \) – rated current of the motor.

### 3. Experiment results

The reactive power \( q_1(t) = Q_2(t)/P_2r \) against time and the active power \( p_1(t) = p_2(t)/P_2r \) against time characteristics are obtained under the conditions of the 125 kW asynchronous motor starting at the load torque of \( M_L = 1.1 M_r \) applying the formulas (6)-(16) and shown in figure 4. The derived characteristics have closely corresponded to the analogous ones \( q_3, p_2 \) calculated by the procedure [23] taking into consideration electromagnetic processes.

The error of the resistances and reactances calculation does not exceed 4% (the calculation algorithm can be found in [22]).

The reactive power \( \Delta Q(t) \) against time and the voltage \( \Delta U_2(t) \) against time characteristics are shown in figure 5. First characteristic is obtained during the 125 kW asynchronous motor starting, but without any control; second one – with control system (non-feedforward); third one – with feedforward control system at \( \tau_0 = 0.22 \) s; fourth one – with feedforward control system at \( \tau_0 = 0.4 \) s.
Figure 4. The reactive power and the active power against time characteristics: 1 – $q_1(t)$; 2 – $p_1(t)$, 3 – $q_2(t)$; 4 – $p_2(t)$.

Figure 5. The characteristics $\Delta Q(t)$, $\Delta U_2(t)$ during the 125 kW asynchronous motor starting: 1 – without any control system; 2 – with control system (non-feedforward); 3 – with feedforward control system at $\tau_0=0.22$ s; 3 – with feedforward control system at $\tau_0=0.4$ s.

4. Conclusion
The automatic reactive power control system of the load bus was developed. The main parameters of the load bus elements were taken into consideration in the mathematical model of this system, in contrast to the counterparts.

The reactive and active power calculation of the squirrel cage asynchronous motor started was carried out in accordance with the proposed procedure that has smaller error than other ones.

The proposed automatic reactive power control system with the feedforward control during the asynchronous motors starting improved the control quality substantially.

The reactive power recovery time at the load bus during the motor starting was reduced from 2.8 s to 0.3 s under non-feedforward control. The voltage sag duration was reduced in half under feedforward
control at $\tau_0=0.22$ s, but at $\tau_0=0.4$ s the voltage sag was eliminated almost completely. However, the amplitude of the controlled values increased, but still remained within the acceptable limits (5-10% of the rated value).

The calculated results were checked and proved on the laboratory test bench with the 5 kW synchronous motor and asynchronous motor with the total power 6 kW.

The obtained control system can be applied in the electric power systems of the oil fields, oil refining, chemical, machining and other plants, where the synchronous and asynchronous motors are used.

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