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Abstract.

An optimal Bayesian update strategy that implements the subjective opinions of several experts are introduced for preventive maintenance of wind turbines while single expert opinion has been introduced by the author in the previous studies. This work is introducing the opinion of the wind farm manager or technician via subjective opinions based on a Bayesian adaptive update strategies for optimal preventive maintenance. Subjective opinion will be implemented to Bayesian cycles while experts can impact the distribution parameters with no knowledge of statistics but just by presenting their opinion as belief, disbelief or uncertainty. Statistical parameters such as minimal time of maintenance and cost of new strategy will be impacted by the interaction of wind farm manager and technician that interact with quantitative data with their opinions. The approach employs and complements the quantitative data from turbine Supervisory control and data acquisition (SCADA).

1. Introduction

Operation and maintenance strategies for wind farms are essential for economical energy production while the wind turbine fleet is increasing in capacity. Maintenance services for wind turbines are usually provided by manufacturer for normal operations within guarantee agreements. Wind managers who look for better warranty and maintenance contracts and wind turbine manufacturer can collaborate in a win-win situation in approaches like the approach presented in this paper. Herein Bayesian adaptive preventive maintenance strategy is employed with subjective opinion. Sharing of information between owner and manufacturer can be managed at different levels of complexity. In this context one approach that can solve this conflict of interest can be adaptive Bayesian preventive maintenance managed by the manufacturer while subjective opinion strategy managed by wind turbine owner, manager or technicians where the information exchange is managed via the probability distributions presenting some level of confidentiality.
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A modern wind turbine has complex electro-mechanical generating machine which is dependent on epistemic uncertainty affecting the structural reliability, the electro-mechanical reliability, the control system reliability and the reliability of the software contained within the control system. The wind turbine is also subject to aleatory uncertainty due to the weather itself, the combined effects of wind and waves [1]. The maintenance action can be at different levels of complexity, it might be replacement of the whole sub-system/system or replacement of part of the sub-system/system by minor repairs [2]. The block replacement policy with minimal repairs will be investigated here. To address the issues of access, logistics, transportation and weather in an efficient way, in wind turbine maintenance, the bundling of activities will to help to minimize the visits to the wind farm and to minimize the hazards. The block replacement used in this study is employed to exploit the strengths of this approach for wind farms as in previous studies [3].

For epistemic uncertainty modeling, Dempster-Shafer(DS) Belief Theory, also known as evidence theory provides a very general model for expressing beliefs to denote the set of exclusive possible states. This approach has also been researched in operation and maintenance literature for chaotic maintenance models [4] and for the cost optimization with belief functions [5]. However, DS theory does not require the additivity principle of probability theory, i.e. that the sum of probabilities on all pairwise exclusive possibilities sums up to one. DS theory enhance the notion of probability with highly expressive basic belief assignments. DS Belief Theory defines a model for expressing beliefs, uses beliefs to denote the set of exclusive possible states, which corresponds to a domain in subjective logic. Opinions of Subjective Logic are linked to the belief functions of DS evidence theory [6] [7] [8]. In Subjective Logic that is studied herein, it is possible to define a direct bijective mapping between the basic belief assignments of DS theory and the belief mass distribution and uncertainty mass of subjective opinions as a result the belief/uncertainty representation of subjective opinions are equivalent to DS theory belief functions but has different interpretation [6].

In view of the attractive nature of subjective logic, particularly relevant to operation and maintenance, one of the objectives of the present study is to investigate the probability density function outputs once more data is available from SCADA for Bayesian update strategies [9] [10] [11] [12] [13]. For the Bayesian adaptive update strategy, the methodology of references [12] [13] [14] is implemented. To implement expert opinions to parameters subjective opinion approach is employed [15]. While Bayes theorem provides a method for conditional probabilities in aleatory uncertainty. Subjective logic extends probability calculus whereby arguments are represented as opinions that can contain degrees of epistemic uncertainty of the wind farm expert. Finally, this article employs Bayes theorem in the formalism of subjective logic for operation and maintenance of wind turbines to address both aleatory uncertainty of SCADA and epistemic uncertainty of expert. In the wind farm operation and maintenance community to the best of our knowledge, there have been limited applications of Bayesian methods [16] where a crack model of a wind turbine was
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analyzed. Finally the contribution of this article is introduction of Subjective Logic to Bayesian preventive maintenance in the context of preventive maintenance to address two stake-holders expert wind farm manager and manufacturer with several experts presenting opinions to address both aleatory uncertainty of SCADA and epistemic uncertainty of expert.

The rest of the paper is organized as follows. Section 2 will review the basic theory of reliability. Section 3 will summarize Bayesian update strategy. Section 4 will summarize key findings of subjective opinion setting in Bayesian update strategy. Section 5 will assess the approach and present key numerical findings.

2. Reliability

Failure (pause start  pause end/service start  service end): A failure event starts when the turbine state changes to Alarm (Pause Start) followed by a downtime period until human intervention is detected (Pause End / Service Start) and ending when the WTG starts operating again.

The reliability function based on Weibull failure model with Weibull density defines time to first failure as

\[ f(t|\alpha, \beta) = i(t) \exp(-t^\alpha) = t^{\beta - 1} \beta \alpha \exp(-t^\beta \alpha). \] (1)

Based on the Weibull distribution, the likelihood can be defined as interval for size for replacement time \(t_B\). When \(n\) system failures are observed at \(0 < t_1 < \cdots < t_n < t_B\) then the likelihood can be defined as

\[ L(t^n|\alpha, \beta) = \prod_{i=1}^{n} t_i^{\beta - 1} \beta \alpha \exp(-t_i^\beta \alpha) \] (2)

where the case for no failures will be reduced to

\[ L(t^0|\alpha, \beta) = \exp(-t_B^\beta \alpha) \] (3)

for \(\alpha\). Then an appropriate prior for distribution of \(\alpha\) is selected based on the gamma distribution based on previous studies

\[ \text{gamma}(\alpha) = \frac{\tau^n}{\Gamma(\eta)} \alpha^{n-1} e^{-\tau \alpha}, \alpha > 0 \] (4)

where \(\eta, \tau > 0\) are specified parameters.

For the prior distribution of \(\beta\), a discrete distribution by using a discretization of the Beta probability density functions (Beta PDFs) on \((\beta_L, \beta_U)\) is chosen based on previous studies [12] [17]. The beta density is given by
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\[ \text{beta}(\beta) = \frac{\Gamma(\mu + \nu)(\beta - \beta_L)^{\mu-1}(\beta_U - \beta)^{\nu-1}}{\Gamma(\mu)\Gamma(\nu)(\beta_U - \beta_L)^{\mu+\nu-1}} \]

\[ 0 \leq \beta_L \leq \beta \leq \beta_U \]

(5)

where \( \beta_L, \beta_U, \mu, \nu > 0 \) are specified parameters. The distribution of \( \beta \) is as

\[ P_l = P(\beta = \beta_l) = \int_{\beta_l-\delta/2}^{\beta_l+\delta/2} \text{gamma}(\beta) d\beta \]

(6)

where \( \beta_l = \beta_L + \delta(2l - 1)/2 \) and \( \delta = (\beta_U - \beta_L)/k \) for \( l = 1, \ldots, k \). The quantities \( \alpha \) and \( \beta \) are assumed as independent. The joint prior distribution is given as the product of Equations 4 and Equation 5. The analysis involves expressing our uncertainty about the unknown coefficients \( \alpha \) and \( \beta \) via prior distributions.

3. Posterior distribution with Bayes Theorem

Using Bayes Theorem [12] [13] [14] and focusing only first case block replacement protocol with minimal repair [12] of case 5 of [13] [14] while other cases can be addressed in the same manner, the joint posterior distribution of \( \alpha \) and \( \beta \) is obtained from failures that were observed at times \( 0 < t_1 < \cdots < t_l < t_\beta \), the posterior joint distribution of \( \alpha \) and \( \beta \) employing Bayes Theorem soyer. This presents the following distribution update of \( \alpha \) and \( \beta \) based on Equation 2,

\[
\begin{align*}
    f(\alpha, \beta_l|t^{(n)}) &= \frac{\prod_{j=1}^{n} t_j^{\beta_j-1} \beta_l^{n-1} \exp\{-\alpha \Delta_l\}}{\sum_{i=1}^{k} \{\prod_{j=1}^{n} t_j^{\beta_j-1} \beta_l^{n-1} \Gamma(\mathbb{H}) P_l / (\Delta_l)^\mathbb{H}\}} P_l \\
    &= f(\alpha, |\beta_l, t^{(n)}) P_l
\end{align*}
\]

(7)

and the posterior distribution of \( \beta \) can be defined by rearrangement of terms

\[
\begin{align*}
    P_l^* &= f(\alpha, \beta_l|t^{(n)})/f(\alpha, |\beta_l, t^{(n)}) \\
    &= \frac{\{\prod_{j=1}^{n} t_j^{\beta_j-1} \beta_l^{n-1}\}}{\sum_{i=1}^{k} \{\prod_{j=1}^{n} t_j^{\beta_j-1} \beta_l^{n-1} / (\Delta_i)^\mathbb{H}\}} P_l
\end{align*}
\]

(8)

which includes the Bayesian updates from data and posterior update of \( \beta \) is dependent on both \( \alpha \) and \( \beta \) distributions [12] and where the posterior case the distribution parameters will be replaced by the updated parameters of the distribution \( \mathbb{H}^* = \mathbb{H} + n \) and \( \Delta^* = \Delta + t_B^{\beta_l} \). The optimal strategy for the block replacement model with minimal repair is obtained as the value of block replacement time \( t_B \) which minimizes the expected cost

\[ E[C(t_B)] = E_{\alpha, \beta}[E_T[C(t_B)|\alpha, \beta]] = \sum_{l=1}^{k} \frac{c_P + c_F(\frac{\alpha}{t_B}) t_B^{\beta_l}}{t_B} P_l \]

(9)

The minimum is easily obtained with numerical techniques [12].
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4. Subjective opinion representations with assigned probabilities

It is possible to compile information from experts of wind energy convergence systems and implement into Bayesian cycles with Subjective opinion which is being introduced here since both Subjective Logic and Bayesian cycles employs Beta PDFs.

The subjective opinion model extends the traditional belief function model of belief theory in the sense that opinions take base rates into account, whereas belief functions ignore base rates. The subjective opinion model extends the traditional belief function model of belief theory by making it possible to define a bijective mapping between subjective opinions and Beta and Dirichlet PDFs to which this article contributes by implementing Beta PDFs in the Bayesian cycles that employs Beta PDFs to introduce subjective opinions to Bayesian cycles.

In the approach presented here, the expert can intervene in Bayesian cycle updates if the expert opinion is invited. If there is any disbelief, belief or uncertainty involved in these findings, experts can implement opinions in Beta PDFs. In order to achieve this, the $\mu$ and $\nu$ parameters of Beta PDFs are remapped to opinion parameters. From the last cycle of Bayesian data update, the $P_l^*$ of Beta PDFs is available. The Beta defines aging of the components and since the article is addressing aging components, the Beta is $\beta > 1$. After the normalization $\beta^* = (\beta - \beta_L)/\beta_U - \beta_L$ via the definitions of Beta PDFs implements the interval $0 \leq \beta^* \leq 1$ where the Beta PDFs is defined. The $\mu, \nu$ parameters can be derived as,

$$
\mu = E[\beta^*] \left( \frac{E[\beta^*](1 - E[\beta^*])}{\sigma^2} - 1 \right)
$$

$$
\nu = (1 - E[\beta^*]) \left( \frac{E[\beta^*](1 - E[\beta^*])}{\sigma^2} - 1 \right)
$$

hence the beta distribution parameter $\mu$ and $\nu$ can be defined where $0 \leq \beta^* \leq 1$ and $\mu, \nu > 1$ \ref{10}. By employing the last cycle $\mu, \nu$ parameters, the aim is to map the $\mu, \nu$ parameters to subjective opinion parameters so that it can be controlled with the expert opinion. The subjective opinions that are being implemented here are equivalent to Dirichlet and Beta probability density functions (Beta PDFs) that links the update of Bayesian cycles that employs Beta PDFs. Through this equivalence, subjective logic is readily available for Bayesian cycles as a calculus for reasoning with probability density functions. The respective parameters for the quantification of expert subjective opinion that is employed in this relations are defined as

- $b_\beta$: belief mass in support of $\beta$ being TRUE (i.e. $B = \beta$),
- $d_\beta$: disbelief mass in support of $\beta$ being FALSE (i.e. $B = \overline{\beta}$),
- $u_\beta$: uncertainty mass representing the vacuity of evidence,
- $a_\beta$: base rate, i.e. prior probability of $\beta$ without any evidence.

where the subjective binomial opinion of the expert as belief mass in support of $\beta$ being TRUE (i.e. $B = \beta$) and disbelief mass in support of $\beta$ being FALSE (i.e. $B = \overline{\beta}$) is
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defined based on findings [6] [15]. A binary domain of opinions consists of only two values. The variable is typically fixed to one of the two values. Let $p$ denote the continuous probability function $p : B \rightarrow [0,1]$ where $p(\beta) + p(\overline{\beta}) = 1$. Herein $p(\beta) = \beta$ so essentially same as $p(\beta)$ so for compactness of notation we define $\beta \equiv p(\beta)$ and $\overline{\beta} \equiv p(\overline{\beta})$. The parameter $\beta$ controls the rate at which the component ages so any change in $\beta$ impacts preventive maintenance time. More disbelief makes the maintenance time more conservative. A binomial opinion about the truth/presence of value $\beta$ is given as the ordered quadruplet via the above defined parameters [15]

$$\omega_{\beta} = (b_{\beta}, d_{\beta}, u_{\beta}, a_{\beta})$$

where the additivity requirement is required

$$b_{\beta} + d_{\beta} + u_{\beta} = 1$$

For the case of $u_{\beta} \neq 0$ the $\mu$ and $\nu$ coefficients are remapped to $\omega_{\beta} = (b_{\beta}, d_{\beta}, u_{\beta}, a_{\beta})$ parameters via the below transformations,

$$\mu = \frac{b_{\beta}W}{u_{\beta}} + a_{\beta}W \quad ; \quad \nu = \frac{d_{\beta}W}{u_{\beta}} + (1 - a_{\beta})W$$

$$1 = b_{\beta} + d_{\beta} + u_{\beta} \quad ; \quad E(\beta) = \frac{\mu}{\mu + \nu}$$

which are derived in the findings of [6] [15]. From Equations 11, it is possible to solve for $\omega_{\beta} = (b_{\beta}, d_{\beta}, u_{\beta}, a_{\beta})$ or for $\mu$ and $\nu$. The parameter $\mu$ represents evidence/observations of $B = \beta$, and the parameter $\nu$ represents evidence/observations of $B = \overline{\beta}$. This information about Subjective Opinion of expert can be implemented on a barycentric coordinate system as illustrated in Figure 1.

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Figure 1. Subjective opinion on a barycentric coordinate system with mapping to measure of opinion optimal $t_B$. 
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If the opinion of two experts A and B were taken into consideration, the namely $\omega^A_\beta$ and $\omega^B_\beta$ the following cumulative fusion rules $\omega^{(A\cup B)}_\beta$ which will be defined by the fusion operator due to [15] such that

\begin{itemize}
  \item $b^{(A\cup B)}_\beta = \frac{b^A_\beta + b^B_\beta}{u^A_\beta + u^B_\beta - u^A_\beta u^B_\beta}$
  \item $d^{(A\cup B)}_\beta = \frac{d^A_\beta + d^B_\beta}{u^A_\beta + u^B_\beta - u^A_\beta u^B_\beta}$
  \item $u^{(A\cup B)}_\beta = \frac{u^A_\beta + u^B_\beta - u^A_\beta u^B_\beta}{u^A_\beta + u^B_\beta - u^A_\beta u^B_\beta}$
  \item $a^{(A\cup B)}_\beta = \frac{a^A_\beta + a^B_\beta - (a^A_\beta + a^B_\beta) u^A_\beta u^B_\beta}{u^A_\beta + u^B_\beta - 2u^A_\beta u^B_\beta}$ if $u^A_\beta \neq 1$ and $u^B_\beta \neq 1$
  \item $d^{(A\cup B)}_\beta = \frac{a^A_\beta + a^B_\beta}{2}$ if $u^A_\beta = u^B_\beta = 1$
\end{itemize}

In Figure 1, a measure for the expert to be able to judge where the expert can readily place the point, is required. This is to replace $\beta$ parameter that will need more significant statistical expertise from the expert. To avoid this, the block replacement $t_B$ optimal time is used as a measure for each opinion while the expert updates the choice, the cost function solution of Equation 12 provides the optimal $t_B$ via the updated Beta PDFs defined via the remapping since the cost function Equation 12 is dependent on discrete integration of Beta PDFs as in

$$E[C(t_B)] = \frac{c_P + c_F(n)}{t_B} + \sum_{l=1}^{k} \int_{\beta_l - \delta_l/2}^{\beta_l + \delta_l/2} \frac{\Gamma(\mu + \nu) (\beta - \beta_L)^{\mu-1} (\beta_U - \beta)^{\nu-1}}{\beta^{\mu+\nu-1}}$$

\[0 \leq \beta_L \leq \beta \leq \beta_U\] (12)

via the $\mu$ and $\nu$ parameters that are redefined to reflect the subjective opinions impact on the optimal $t_B^{opt}$. Once the expert opinion is injected, further Bayesian cycles can continue.

5. Numerical results

For the wind turbines, the active power control system is an essential component that optimizes efficient use of wind resources with given constraints of safety. The pitch control system which is a very important part of major wind turbine controls aim to optimize efficiency while acting to operate with safety in bad weather conditions. As in any dynamic component, the pitch control system is subjective to several frequent failures with large residence time relative to other components. Pitch control system for a site in Sweden in cold climate will be investigated in this study. The chosen turbine is a 2MW turbine with rotor diameter 82m. There are three identical wind turbines on the site. The methodology used here has focused on the control system maintenance particularly on the pitch control devices which showed several times failure for the duration of data availability from 2009-01-01 to 2015-10-28. For the preventive maintenance of active pitch control systems based on the findings of [18] labor is 480 €.
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This is based on 60 €/h-man, always considering 2 men and 4 hours. The lubricants are 100 €. Parts and displacement of parts can vary so an approximate figure of 580 € was implemented [18] for the variable $c_B$. The maintenance usually consists in replacing the engine of the blade. The cost for engine is taken as 2000 €, sensor placement is taken as 50 €. The 60 €/h-man always considering 2 men are taken into account by tripling the labor of preventive maintenance plus the lubricants so $1540 + 2050$ € is implemented for the variable $c_F$ with the assumption that three turbines are maintained in the same trip or with the assumption that of a major failure where three times more labor is used. The unavailable kWh production when the turbine was at major failure at the end of the life cycle was not taken into consideration while this was addressed in previous studies.

5.1. Data preprocessing

Three wind turbines with available SCADA data were investigated for duration of observations of the pitch control system that was maintained several times during the operation period. This data was preprocessed to find start and stop time of each failure with duration of each failure as presented in Table 1 in days. The information from three turbines are preprocessed into three cycles with each cycle starting with start of SCADA recordings or major failure. Cycle ends with major failure or end of life cycle. Later this data is normalized with duration of maximum cycle days. This is 1846 days from the Table 1 for Bayesian cycles. The life cycle of the pitch control device components is not taken from the manufacturer but based on SCADA data.

| Cycle 1 | 40.22 | 40.30 | 62.98 | 117.06 | 323.93 | 325.086 | 1846.18 |
|---------|-------|-------|-------|--------|--------|---------|---------|
| Cycle 2 | 316.30| 667.47| NaN   | NaN    | NaN    | NaN     | NaN     |
| Cycle 3 | 650.91| 652.22| 655.50| NaN    | NaN    | NaN     | NaN     |

5.2. Bayesian update and Subjective opinion results

In order to test the feasibility of the approach, the failure cycles from Table 1 are implemented in the Bayesian approach for updates. The cost function update for each cycle is presented in Figure 2(a) where the time dependent optimal cost function is presented. The cost and time were normalized.

The belief, disbelief and uncertainty opinions for cycle 4 results of Bayesian adaptive approach are presented in Table 3 as binomial opinion in the barycentric triangle coordinates of binomial opinion for two experts. Bayesian results from the fourth cycle case costs are introduced in Table 3 and this is presented in Figure 2. Once the expert A and B decides to introduce disbelief and uncertainty, they mark the circle in visual
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Table 2. Normalized optimal time of preventive maintenance and normalized optimal costs after each Bayesian cycles and after Subjective Opinion of the expert cycle.

| Cycles | Prior (1) | 2 | 3 | 4 | Expert A and B |
|--------|-----------|---|---|---|----------------|
| Optimal time | 0.5000 | 0.4900 | 0.4800 | 0.4000 | 0.3800 |
| Optimal cost | 0.0396 | 0.1391 | 0.1383 | 0.1479 | 0.1572 |

The impact of the expert opinions on the cost functions is presented in Figure 2. The findings based on these simulations for expert A and B fused opinions on costs and optimal time were presented and the opinion of the fused expert opinions are presented for different estimates with the progress of cycles in Table 2.

Table 3. Quantitative values of opinions presented by experts after Bayesian cycle 4.

|           | Belief | Disbelief | Uncertainty | Base-rate | W   |
|-----------|--------|-----------|-------------|-----------|-----|
| Opinion A | 0.0513 | 0.9042    | 0.0444      | 0.5000    | 2.0000 |
| Opinion B | 0.0513 | 0.8842    | 0.0644      | 0.5000    | 2.0000 |
| Opinion A and B | 0.0527 | 0.9203    | 0.0270      | 0.5000    | 2.0000 |

Figure 2. a) Cost function update after each cycle from prior to posterior. b) Subjective opinion on a barycentric coordinate system with the fused expert opinion marked in circle. c) Subjective opinion impacts of the experts A and B on the last cycle.

6. Conclusions

A Bayesian adaptive preventive maintenance strategy that can address the opinions of several experts have been introduced. This approach can successfully address the expert opinions in the context of the quantitative data for wind turbine maintenance while statistical knowledge from the expert is not expected. The expert can mark the
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opinion in visual aid to be used for statistical cycles. However the knowledge of the expert in addition to existing data can be employed for further prediction cycles once more data becomes available with history of the plant. The approach can be extended to several expert opinions. Bayesian preventive update strategy has been implemented for wind turbine pitch control device maintenance data. The approach is not exclusive and can be implemented to any preventive maintenance data for optimal maintenance time prediction with expert opinions.

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