A General Retraining Framework for Scalable Adversarial Classification

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Abstract

Traditional classification algorithms assume that training and test data come from similar distributions. This assumption is violated in adversarial settings, where malicious actors modify instances to evade detection. A number of custom methods have been developed for both adversarial evasion attacks and robust learning. We propose the first systematic and general-purpose retraining framework which can: a) boost robustness of an arbitrary learning algorithm, in the face of b) a broader class of adversarial models than any prior methods. We show that, under natural conditions, the retraining framework minimizes an upper bound on optimal adversarial risk, and show how to extend this result to account for approximations of evasion attacks. Extensive experimental evaluation demonstrates that our retraining methods are nearly indistinguishable from state-of-the-art algorithms for optimizing adversarial risk, but are more general and far more scalable. The experiments also confirm that without retraining, our adversarial framework dramatically reduces the effectiveness of learning. In contrast, retraining significantly boosts robustness to evasion attacks without significantly compromising overall accuracy.

1 Introduction

Machine learning has been used ubiquitously for a wide variety of security tasks, such as intrusion detection, malware detection, spam filtering, and web search [1, 2, 3, 4, 5]. Traditional machine learning systems, however, do not account for adversarial manipulation. For example, in spam detection, spammers commonly change spam email text to evade filtering. As a consequence, there have been a series of efforts to both model adversarial manipulation of learning, such as evasion and data poisoning attacks [6, 7, 8], as well as detecting such attacks [2, 9] or enhancing robustness of learning algorithms to these [10, 11, 12, 13, 14]. One of the most general of these, due to Li and Vorobeychik [10], admits evasion attacks modeled through a broad class of optimization problems, giving rise to a Stackelberg game, in which the learner minimizes an adversarial risk function which accounts for optimal attacks on the learner. The main limitation of this approach, however, is scalability: it can solve instances with only 10-30 features. Indeed, most approaches to date also offer solutions that build on specific learning models or algorithms. For example, specific evasion attacks have been developed for linear or convex-inducing classifiers [6, 7, 8], as well as neural networks for continuous feature spaces [15]. Similarly, robust algorithms have typically involved non-trivial modifications of underlying learning algorithms, and either assume a specific attack model or modify a specific algorithm. The more general algorithms that admit a wide array of attack models, on the other hand, have poor scalability.
We propose a very general retraining framework, RAD, which can boost evasion robustness of arbitrary learning algorithms using arbitrary evasion attack models. We show that RAD minimizes an upper bound on optimal adversarial risk. This is significant: whereas adversarial risk minimization is a hard bi-level optimization problem and poor scalability properties (indeed, no methods exist to solve it for general attack models), RAD itself is extremely scalable in practice, as our experiments show. We develop RAD for a more specific, but very broad class of adversarial models, offering a theoretical connection to adversarial risk minimization even when the adversarial model is only approximate. In the process, we offer a simple and very general class of local search algorithms for approximating evasion attacks, which are experimentally quite effective. Perhaps the most appealing aspect of the proposed approach is that it requires no modification of learning algorithms: rather, it can wrap any learning algorithm “out-of-the-box”. Our work connects to, and systematizes, several previous approaches which used training with adversarial examples to either evaluate robustness of learning algorithms, or enhance learning robustness. For example, Goodfellow et al. [16] and Kantchelian et al. [17] make use of adversarial examples. In the former case, however, these were essentially randomly chosen. The latter offered an iterative retraining approach more in the spirit of RAD, but did not systematically develop or analyze it. Teo et al. [12] do not make an explicit connection to retraining, but suggest equivalence between their general invariance-based approach of RAD, but did not systematically develop or analyze it. Teo et al. [12] do not make an explicit connection to retraining, but suggest equivalence between their general invariance-based approach using column generation and retraining. However, the two are not equivalent, and Teo et al. did not study their relationship formally.

In summary, we make the following contributions:

1. RAD, a novel systematic framework for adversarial retraining,
2. analysis of the relationship between RAD and empirical adversarial risk minimization,
3. extension of the analysis to account for approximate adversarial evasion, within a specific broad class of adversarial models,
4. extensive experimental evaluation of RAD and the adversarial evasion model.

We illustrate the applicability and efficiency of our method on both spam filtering and handwritten digit recognition tasks, where evasion attacks are extremely salient [18, 19].

2 Learning and Evasion Attacks

Let $X \subseteq R^n$ be the feature space, with $n$ the number of features. For a feature vector $x_i \in X$, we let $x_{i,j}$ denote the $j$th feature. Suppose that the training set $(x_i, y_i)$ is comprised of feature vectors $x_i \in X$ generated according to some unknown distribution $x_i \sim F$, with $y_i \in \{-1, +1\}$ the corresponding binary labels, where $-1$ means the instance $x_i$ is benign, while $+1$ indicates a malicious instance. The learner aims to learn a classifier with parameters $\beta$, $g_\beta : X \rightarrow \{-1, +1\}$, to label instances as malicious or benign, using a training data set of labeled instance $D = \{(x_1, y_1),..., (x_m, y_m)\}$. Let $I_{bad}$ be the subset of datapoints $i$ with $y_i = +1$. We assume that $g_\beta(x) = \text{sgn}(f_\beta(x))$ for some real-valued function $f_\beta(x)$.

Traditionally, machine learning algorithms commonly minimize regularized empirical risk:

$$\min_\beta L(\beta) \equiv \sum_i l(g_\beta(x_i), y_i) + \alpha \|\beta\|_p^p,$$

where $l(\hat{y}, y)$ is the loss associated with predicting $\hat{y}$ when true classification is $y$. An important issue in adversarial settings is that instances classified as malicious (in our convention, corresponding to $g_\beta(x) = +1$) are associated with malicious agents who subsequently modify such instances in order to evade the classifier (and be classified as benign). Suppose that adversarial evasion behavior is captured by an oracle, $O(\beta, x)$, which returns, for a given parameter vector $\beta$ and original feature vector (in the training data) $x$, an alternative feature vector $x'$. Since the adversary modifies malicious instances according to this oracle, the resulting effective risk for the defender is no longer captured by Equation 1, but must account for adversarial response. Consequently, the defender would seek to minimize the following adversarial risk (on training data):

$$\min_\beta L_A(\beta; O) = \sum_{r: y_r = -1} l(g_\beta(x_r), -1) + \sum_{r: y_r = +1} l(g_\beta(O(\beta, x_r), +1) + \alpha \|\beta\|_p^p.$$

(2)
The adversarial risk function in Equation 2 is extremely general: we make, at the moment, no assumptions on the nature of the attacker oracle, $O$. This oracle may capture evasion attack models based on minimizing evasion cost [6, 10, 15], or based on actual attacker evasion behavior obtained from experimental data [20].

3 Adversarial Learning through Retraining

A number of approaches have been proposed for making learning algorithms more robust to adversarial evasion attacks [21, 10, 11, 12, 14]. However, these approaches typically suffer from three limitations: 1) they usually assume specific attack models, 2) they require substantial modifications of learning algorithms, and 3) they commonly suffer from significant scalability limitations. For example, a recent, general, adversarial learning algorithm proposed by Li and Vorobeychik [10] makes use of constraint generation, but does not scale beyond 10-30 features.

Recently, retraining with adversarial data has been proposed as a means to increase robustness of learning [16, 17, 12]. However, to date such approaches have not been systematic.

We present a new algorithm, $RAD$, for retraining with adversarial data (Algorithm 1) which systematizes some of the prior insights, and enables us to provide a formal connection between retraining with adversarial data, and adversarial risk minimization in the sense of Equation 2. The $RAD$ algorithm is quite general. At the high level, it starts with the original training data $X$ and iterates between computing a classifier and adding adversarial instances to the training data that evade the previously computed classifier, if they are not already a part of the data.

A baseline termination condition for $RAD$ is that no new adversarial instances can be added (either because instances generated by $O$ have already been previously added, or because the adversary’s can no longer benefit from evasion, as discussed more formally in Section 4.1). If the range of $O$ is finite (e.g., if the feature space is finite), $RAD$ with this termination condition would always terminate. In practice, our experiments demonstrate that when termination conditions are satisfied, the number of $RAD$ iterations is quite small (between 5 and 20). Moreover, while $RAD$ effective increases the importance of malicious instances in training, this does not appear to significantly harm classification performance in a non-adversarial setting. In general, we can also control the number of rounds directly, or use an additional termination condition, such as that the parameter vector $\beta$ changes little between successive iterations. However, we assume henceforth that there is no fixed iteration limit or convergence check.

To analyze what happens if the algorithm terminates, define the regularized empirical risk in the last iteration of $RAD$ as:

$$L^R_N(\beta, O) = \sum_{i \in D \cup N} l(g_\beta(x_i), y_i) + \alpha ||\beta||^p_p,$$

1Indeed, neither Teo et al. [12] nor Kantchelian et al. [17] focus on retraining as a main contribution, but observe its effectiveness.
where a set \( N = \bigcup_i N_i \) of data points has been added by the algorithm (we omit its dependence on \( \mathcal{O} \) to simplify notation). We now characterize the relationship between \( \mathcal{L}_N^R(\beta, \mathcal{O}) \) and \( \mathcal{L}_A(\mathcal{O}) = \min_{\beta} \mathcal{L}_A(\beta, \mathcal{O}) \).

**Proposition 3.1.** \( \mathcal{L}_A(\mathcal{O}) \leq \mathcal{L}_N^R(\beta, \mathcal{O}) \) for all \( \beta, \mathcal{O} \).

**Proof.** Let \( \bar{\beta} \in \arg \min_{\beta} \mathcal{L}_N^R(\beta, \mathcal{O}) \). Consequently, for any \( \beta \),

\[
\mathcal{L}_N^R(\beta, \mathcal{O}) \geq \mathcal{L}_N^R(\bar{\beta}, \mathcal{O}) \\
= \sum_{i:y_i = -1} l(g_{\bar{\beta}}(x_i), -1) + \sum_{i:y_i = +1,j \in N_i \cup x_i} l(g_{\bar{\beta}}(x_i), +1) + \alpha ||\bar{\beta}||_p
\]

\[
\geq \sum_{i:y_i = -1} l(g_{\bar{\beta}}(x_i), -1) + \sum_{i:y_i = +1} l(g_{\bar{\beta}}(\mathcal{O}(\bar{\beta}, x_i)), +1) + \alpha ||\bar{\beta}||_p
\]

\[
\geq \min_{\beta} \mathcal{L}_A(\beta; \mathcal{O}) = \mathcal{L}_A^*(\mathcal{O}),
\]

where the second inequality follows because in the last iteration of the algorithm, \( new = \emptyset \) (since it must terminate after this iteration), which means that \( \mathcal{O}(\bar{\beta}, x_i) \in N_i \) for all \( i \in I_{bad} \).

In words, retraining, systematized in the RAD algorithm, effectively minimizes an upper bound on optimal adversarial risk.

This offers a conceptual explanation for the previously observed effectiveness of such algorithms in boosting robustness to adversarial evasion. Formally, however, the result above is limited for several reasons. First, for many adversarial models in prior literature, adversarial evasion is NP-Hard. While some effective approaches exist to compute optimal evasion for specific learning algorithms [17], this is not true in general. Although approximation algorithms for these models exist, using them as oracles in RAD is problematic, since actual attackers may compute better solutions, and Proposition 3.1 no longer applies. Second, we assume that \( \mathcal{O} \) returns a unique result, but when evasion is modeled as optimization, optima need not be unique. Third, there do not exist effective general-purpose adversarial evasion algorithms the use of which in RAD would allow reasonable theoretical guarantees. Below, we investigate an important and very general class of adversarial evasion models and associated algorithms which allow us to obtain practically meaningful guarantees for RAD.

**Clustering Malicious Instances:** A significant enhancement in speed of the approach can be obtained by clustering malicious instances: this would reduce both the number of iterations, as well as the number of data points added per iteration. Experiments (in the supplement) show that this is indeed quite effective.

**Stochastic gradient descent:** RAD works particularly well with online methods, such as stochastic gradient descent. Indeed, in this case we need only to make gradient descent steps for newly added malicious instances, which can be added one at a time until convergence.

## 4 Modeling Attackers

### 4.1 Evasion Attack as Optimization

In prior literature, evasion attacks have almost universally been modeled as optimization problems in which attackers balance the objective of evading the classifier (by changing the label from \(+1\) to \(-1\)) and the cost of such evasion [6, 10]. Our approach is in the same spirit, but is formally somewhat distinct. In particular, we assume that the adversary has the following two competing objectives: 1) appear as benign as possible to the classifier, and 2) minimize modification cost. It is also natural to assume that the attacker obtains no value from a modification to the original feature vector if the result is still classified as malicious. To formalize, consider an attacker who in the original training data uses a feature vector \( x_i \) (\( i \in I_{bad} \)). The adversary \( i \) is solving the following optimization problem:

\[
\min_{x \in X} \min \{0, f(x) \} + c(x, x_i).
\]

\[\text{Note that the bound relies on the fact that we are only adding adversarial instances, and terminate once no more instances can be added. In particular, natural variations, such as removing or re-weighing added adversarial instances to retain original malicious-benign balance lose this guarantee.}\]
We assume that $c(x, x_i) \geq 0$, $c(x, x_i) = 0$ iff $x = x_i$, and $c$ is strictly increasing in $\|x - x_i\|_2$ and strictly convex in $x$. Because Problem 4 is non-convex, we instead minimize an upper bound:

$$
\min_x Q(x) \equiv f(x) + c(x, x_i).
$$

In addition, if $f(x_i) < 0$, we return $x_i$ before solving Problem 5. If Problem 5 returns an optimal solution $x^*$ with $f(x^*) \geq 0$, we return $x_i$; otherwise, return $x^*$. Problem 5 has two advantages. First, if $f(x)$ is convex and $x$ real-valued, this is a (strictly) convex optimization problem, has a unique solution, and we can solve it in polynomial time. An important special case is when $f(x) = w^T x$. The second one we formalize in the following lemma.

**Lemma 4.1.** Suppose $x^*$ is the optimal solution to Problem 4, $x_i$ is suboptimal, and $f(x^*) < 0$. Let $\bar{x}$ be the optimal solution to Problem 5. Then $f(\bar{x}) + c(\bar{x}, x_i) = f(x^*) + c(x^*, x_i)$, and $f(\bar{x}) < 0$.

The following corollary then follows by uniqueness of optimal solutions for strictly convex objective functions over a real vector space.

**Corollary 4.1.** If $f(x)$ is convex and $x$ continuous, $x^*$ is the optimal solution to Problem 4, $\bar{x}$ is the optimal solution to Problem 5, and $f(x^*) < 0$, then $\bar{x} = x^*$.

A direct consequence of this corollary is that when we use Problem 5 to approximate Problem 4 and this approximation is convex, we always return either the optimal evasion, or $x_i$ if no cost-effective evasion is possible. An oracle $O$ constructed on this basis will therefore return a unique solution, and supports the theoretical characterization of $RAD$ above.

The results above are encouraging, but many learning problems do not feature a convex $f(x)$, or a continuous feature space. Next, we consider several general algorithms for adversarial evasion.

### 4.2 Coordinate Greedy

We propose a very general local search framework, *CoordinateGreedy (CG)* (Algorithm 2 for approximating optimal attacker evasion. The high-level idea is to iteratively choose a feature, and greedily update this feature to incrementally improve the attacker’s utility (as defined by Problem 5). In general, this algorithm will only converge to a locally optimal solution. We therefore propose a version with random restarts: run $CG$ from $L$ random starting points in feature space. As long as a global optimum has a basin of attraction with positive Lebesgue measure, or the feature space is finite, this process will asymptotically converge to a globally optimal solution as we increase the number of random restarts. Thus, as we increase the number of random restarts, we expect to increase the frequency that we actually return the global optimum. Let $p_L$ denote the probability that the oracle based on coordinate greedy with $L$ random restarts returns a suboptimal solution to Problem 5. The next result generalizes the bound on $RAD$ to allow for this, restricting however that the risk function which we bound from above uses the 0/1 loss. Let $L_{A,01}(O)$ correspond to the total adversarial risk in Equation 2, where the loss function $l(g_{\beta}(x), y)$ is the 0/1 loss. Suppose that $O_L$ uses coordinate greedy with $L$ random restarts.

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3Here we exhibit a particular general attack model, but many alternatives are possible, such as using constrained optimization. We found experimentally that the results are not particularly sensitive to the choice of the attack model.
Proposition 4.2. Let $B = |I_{bad}|$. $\mathcal{L}_{A,\text{01}}^* (\mathcal{O}) \leq \mathcal{L}_B^* (\beta, \mathcal{O}_L) + \delta(p)$ with probability at least $1 - p$, where $\delta(p) = B \left( p_L + \sqrt{\frac{\log^2 p^{-8BpL \log p - \log p}}{2B}} \right)$, and $\mathcal{L}_B^* (\beta, \mathcal{O}_L)$ uses any loss function $l(g_\beta(x), y)$ which is an upper bound on the 0/1 loss.

Experiments suggest that $p_L \to 0$ quite rapidly for an array of learning algorithms, and for either discrete or continuous features, as we increase the number of restarts $L$ (see the supplement for details). Consequently, in practice retraining with coordinate greedy nearly minimizes an upper bound on minimal adversarial risk for a 0/1 loss with few restarts of the approximate attacker oracle.

Continuous Feature Space: For continuous feature space, we assume that both $f(x)$ and $c(x, \cdot)$ are differentiable in $x$, and propose using the coordinate descent algorithm, which is a special case of coordinate greedy, where the GreedyImprove step is: $x^{k+1} \leftarrow x^k - \tau_k e_{i_k} \frac{\partial Q(x^k)}{\partial x_{i_k}}$, where $\tau_k$ is the step size and $e_{i_k}$ the direction of $i_k$th coordinate. Henceforth, let the original adversarial instance $x_i$ be given; we then simplify cost function to be only a function of $x$, denoted $c(x)$. If the function $f(x)$ is convex and differentiable, our coordinate descent based algorithm 2 can always find the global optima which is the attacker best response $x^*$ [22], and Proposition 3.1 applies, by Corollary 4.1. If $f(x)$ is not convex, then coordinate descent will only converge to a local optimum.

Discrete Feature Space: In the case of discrete feature space, GreedyImprove step of CG can simply enumerate all options for feature $j$, and choose the one most improving the objective.

5 Experimental Results

The results above suggest that the proposed systematic retraining algorithm is likely to be effective at increasing resilience to adversarial evasion. We now offer an experimental evaluation of this (additional results are provided in the supplement). We present the results for the exponential cost model, where $c(x, x_i) = \exp \left( \lambda \sum_j (x_j - x_{ij})^2 + 1 \right)^{1/2}$. Additionally, we simulated attacks using Problem 5 formulation. Results for other cost functions and attack models are similar, as shown in the supplement. Moreover, the supplement demonstrates that the approach is robust to cost function misspecification.

Comparison to Optimal: The first comparison we draw is to a recent algorithm, SMA, which minimizes $l_1$-regularized adversarial risk function (2) using the hinge loss function. Specifically, SMA formulates the problem as a large mixed-integer linear program which it solves using constraint generation [10]. The main limitation of SMA is scalability. Because retraining methods use out-of-the-box learning tools and does not involve non-convex bi-level optimization, it is considerably more scalable.

We compared SMA and RAD using Enron data [18]. As Figure 1(a) demonstrates, retraining solutions of RAD are nearly as good as SMA, particularly for a non-trivial adversarial cost sensitive $\lambda$. In contrast, a baseline implementation of SVM is significantly more fragile to evasion attacks. However, the runtime comparison for these algorithms in Figure 1(b) shows that RAD is much more scalable than SMA.
Effectiveness of Retraining: In this section we use the Enron dataset [18] and MNIST [19] dataset to evaluate the robustness of three common algorithms in their standard implementation, and in RAD: logistic regression, SVM (using a linear kernel), and a neural network (NN) with 3 hidden layers. In Enron data, features correspond to relative word frequencies. 2000 features were used for the Enron and 784 for MNIST datasets. Throughout, we use precision, recall, and accuracy as metrics. We present the results for a continuous feature space here. Results for binary features are similar and provided in the supplement.

Figure 2(a) shows the performance of logistic regression, with and without retraining, on Enron and MNIST. The increased robustness of RAD is immediately evident: performance of RAD is essentially independent of $\lambda$ on all three measures, and substantially exceeds baseline algorithm performance for small $\lambda$. Interestingly, we observe that the baseline algorithms are significantly more fragile to evasion attacks on Enron data compared to MNIST: benign and malicious classes seem far easier to separate on the latter than the former. This qualitative comparison between the Enron and MNIST datasets is consistent for other classification methods as well (SVM, NN). These results also illustrate that the neural-network classifiers, in their baseline implementation, are significantly more robust to evasion attacks than the (generalized) linear classifiers (logistic regression and SVM): even with a relatively small attack cost attacks become ineffective relatively quickly, and the differences between the performance on Enron and MNIST data are far smaller. Throughout, however, RAD significantly improves robustness to evasion, maintaining extremely high accuracy, precision, and recall essentially independently of $\lambda$, dataset, and algorithm used.

In order to explore whether RAD would sacrifice accuracy when no adversary is present, Figure 3 shows the performance of the baseline algorithms and RAD on a test dataset sans evasions. Surpris-
ingly, *RAD* is never significantly worse, and in some cases better than non-adversarial baselines: adding malicious instances appears to increase overall generalization ability. This is also consistent with the observation by Kantchelian et al. [17].

**Oracles based on Human Evasion Behavior:** To evaluate the considerable generality of *RAD*, we now use a non-optimization-based threat model, making use instead of observed human evasion behavior in human subject experiments. The data for this evaluation was obtained from the human subject experiment by Ke et al. [20] in which subjects were tasked with the goal of evading an SVM-based spam filter, manipulating 10 spam/phishing email instances in the process. In these experiments, Ke et al. used machine learning to develop a model of human subject evasion behavior. We now adopt this model as the evasion oracle, \( \mathcal{O} \), injected in our *RAD* retraining framework, executing the synthetic model for 0-10 iterations to obtain evasion examples.

Figure 4(a) shows the recall results for the dataset of 10 malicious emails (the classifiers are trained on Enron data, but evaluated on these 10 emails, including evasion attacks). Figure 4(b) shows the classifier performance for the Enron dataset by applying the synthetic adversarial model as the oracle. We can make two high-level observations. First, notice that human adversaries appear significantly less powerful in evading the classifier than the automated optimization-based attacks we previously considered. This is a testament to both the effectiveness of our general-purpose adversarial evaluation approach, and the likelihood that such automated attacks likely significantly overestimate adversarial evasion risk in many settings. Nevertheless, we can observe that the synthetic model used in *RAD* leads to a significantly more robust classifier. Moreover, as our evaluation used actual evasions, while the synthetic model was used only in training the classifier as a part of *RAD*, this experiment suggests that the synthetic model can be relatively effective in modeling behavior of human adversaries. Figure 4(b) performs a more systematic study using the synthetic model of adversarial behavior on the Enron dataset. The findings are consistent with those only considering the 10 spam instances: retraining significantly boosts robustness to evasion, with classifier effectiveness essentially independent of the number of queries made by the oracle.

### 6 Conclusion

We proposed a general-purpose systematic retraining algorithm against evasion attacks of classifiers for arbitrary oracle-based evasion models. We first demonstrated that this algorithm effectively minimizes an upper bound on optimal adversarial risk, which is typically extremely difficult to compute (indeed, no approach exists for minimizing adversarial loss for an arbitrary evasion oracle). Experimentally, we showed that the performance of our retraining approach is nearly indistinguishable from optimal, whereas scalability is dramatically improved: indeed, with *RAD*, we are able to easily scale the approach to thousands of features, whereas a state-of-the-art adversarial risk optimization method can only scale to 15-30 features. We generalize our results to show that a probabilistic upper bound on minimal adversarial loss can be obtained even when the oracle is computed approximately by leveraging random restarts, and an empirical evaluation which confirms that the resulting bound relaxation is tight in practice.

We also offer a general-purpose framework for optimization-based oracles using variations of coordinate greedy algorithm on both discrete and continuous feature spaces. Our experiments demonstrate
that our adversarial oracle approach is extremely effective in corrupting the baseline learning algorithms. On the other hand, extensive experiments also show that the use of our retraining methods significantly boosts robustness of algorithms to evasion. Indeed, retrained algorithms become nearly insensitive to adversarial evasion attacks, at the same time maintaining extremely good learning performance on data overall. Perhaps the most significant strength of the proposed approach is that it can make use of arbitrary learning algorithms essentially “out-of-the-box”, and effectively and quickly boost their robustness, in contrast to most prior adversarial learning methods which were algorithm-specific.

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Proof of Lemma 4.1

If \( f(x^*) < 0 \), then \( \min \{ 0, f(x^*) \} + c(x^*, x_i) = f(x^*) + c(x^*, x_i) \). By optimality of \( \bar{x} \), \( f(\bar{x}) + c(\bar{x}, x_i) \leq f(x^*) + c(x^*, x_i) \). Since \( x_i \) is suboptimal in Problem (4) and \( c \) strictly positive in all other cases, \( f(x^*) + c(x^*, x_i) < \min \{ 0, f(x^*) \} + c(x^*, x_i) = 0 \). By optimality of \( x^* \), \( f(x^*) + c(x^*, x_i) \leq \min \{ 0, f(\bar{x}) \} + c(\bar{x}, x_i) \), which implies that \( f(\bar{x}) + c(\bar{x}, x_i) = f(x^*) + c(x^*, x_i) \). Consequently, \( f(\bar{x}) + c(\bar{x}, x_i) < 0 \), and, therefore, \( f(\bar{x}) < 0 \).

Proof of Proposition 4.2

Let \( \bar{\beta} \in \arg \min_\beta \mathcal{L}^R_N(\beta, \mathcal{O}_L) \). Consequently, for any \( \beta \),

\[
\mathcal{L}^*_{A,01}(\mathcal{O}_L) = \min_\beta \mathcal{L}_{A,01}(\beta; \mathcal{O}_L) \leq \sum_{i:y_i=-1} l_{01}(g_{\bar{\beta}}(x_i), -1) + \sum_{i:y_i=+1} l_{01}(g_{\bar{\beta}}(\mathcal{O}(\bar{\beta}, x_i)), +1) + \alpha ||\bar{\beta}||_p^p,
\]

Now,

\[
\sum_{i:y_i=+1} l_{01}(g_{\bar{\beta}}(\mathcal{O}(\bar{\beta}, x_i)), +1) \leq \sum_{i:y_i=+1} l_{01}(g_{\bar{\beta}}(\mathcal{O}_L(\bar{\beta}, x_i)), +1) + \delta(p)
\]

with probability at least \( 1 - p \), where \( \delta(p) = Bp_L + \sqrt{\log^2 \frac{p}{2} Bp_L \log 2} \), by the Chernoff bound, and Lemma 4.1, which assures that an optimal solution to Problem (5) can only over-estimate mistakes. Moreover,

\[
\sum_{i:y_i=+1} l_{01}(g_{\bar{\beta}}(\mathcal{O}_L(\bar{\beta}, x_i)), +1) \leq \sum_{i:y_i=+1} \sum_{j \in N_i} l(g_{\bar{\beta}}(\mathcal{O}_L(\bar{\beta}, x_i)), +1),
\]

since \( \mathcal{O}_L(\bar{\beta}, x_i) \in N_i \) for all \( i \) by construction, and \( l \) is an upper bound on \( l_{01} \). Putting everything together, we get the desired result.

Convergence of \( p_L \) with Increasing Number of Restarts \( L \)

![Figure 5: The convergence of \( p_L \) based on different number of starting points for (a) Binary, (b) Continuous feature space.](image)

Attacks as Constrained Optimization

A variation on the attack models in the main paper is when the attacker is solving the following constrained optimization problem:

\[
\min_x \min \{ 0, f(x) \} \quad \text{s.t.:} \quad c(x, x_i) \leq B
\]
for some cost budget constraint $B$ and query budget constraint $Q$. While this problem is, again, non-convex, we can instead minimize the convex upper bound, $f(x)$, as before, if we assume that $f(x)$ is convex. In this case, if the feature space is continuous, the problem can be solved optimally using standard convex optimization methods [23]. If the feature space is binary and $f(x)$ is linear or convex-inducing, algorithms proposed by Lowd and Meek [6] and Nelson et al. [8]. Figure 6, 7 and 8 show the performance of RAD based on the optimized adversarial strategies for various learning models, respectively.

Figure 6: Performance of baseline (adv-) and RAD (rob-) as a function of adversarial budget for Enron dataset with binary features testing on adversarial instances using Naive Bayesian. (a) query budget $Q = 30$, (b) query budget $Q = 40$.

Figure 7: Performance of baseline (adv-) and RAD (rob-) as a function of adversarial budget for Enron dataset with binary features testing on adversarial instances using SVM with RBF kernel. (a) query budget $Q = 30$, (b) query budget $Q = 40$.

Figure 8: Performance of baseline (adv-) and RAD (rob-) as a function of adversarial budget for Enron dataset with binary features testing on adversarial instances using 3-layer NN. (a) query budget $Q = 30$, (b) query budget $Q = 40$. 
Experiments with Continuous Feature Space

![Figure 9: Example modification of digit images (MNIST data) as $\lambda$ decreases (left-to-right) for logistic regression, SVM, 1-layer NN, and 3-layer NN (rows 1-4 respectively).](image)

In Figure 9 we visualize the relative vulnerability of the different classifiers, as well as effectiveness of our general-purpose evasion methods based on coordinate greedy. Each row corresponds to a classifier, and moving right within a row represents decreasing $\lambda$ (allowing attacks to make more substantial modifications to the image in an effort to evade correct classification). We can observe that NN classifiers require more substantial changes to the images to evade, ultimately making these entirely unlike the original. In contrast, logistic regression is quite vulnerable: the digit remains largely recognizable even after evasion attacks.

Experiments with Discrete Feature Space

Considering now data sets with binary features, we use the Enron data with a bag-of-words feature representation, for a total of 2000 features. We compare Naive Bayes (NB), logistic regression, SVM, and a 3-layer neural network. Our comparison involves both the baseline, and RAD implementations of these, using the same metrics as above.

![Figure 10: Performance of baseline (adv-) and RAD (rob-) implementations of (a) Naive Bayes, (b) logistic regression, (c) SVM, and (d) 3-layer NN, using binary features testing on adversarial instances.](image)

Figure 10 confirms the effectiveness of RAD: every algorithm is substantially more robust to evasion with retraining, compared to baseline implementation. Most of the algorithms can obtain extremely
high accuracy on this data with the bag-of-words feature representation. However, a 3-layer neural network is now less robust than the other algorithms, unlike in the experiments with continuous features. Indeed, Goodfellow et al. [16] similarly observe the relative fragility of NN to evasion attacks.

**Experiments with Multi-class Classification**

Discussion so far dealt entirely with binary classification. We now observe that extending it to multi-class problems is quite direct. Specifically, while previously the attacker aimed to make an instance classified as $+1$ (malicious) into a benign instance ($-1$), for a general label set $Y$, we can define a malicious set $M \subset Y$ and a target set $T \subset Y$, with $M \cap T = \emptyset$, where every entity represented by a feature vector $x$ with a label $y \in M$ aims to transform $x$ so that its label is changed to $T$. In this setting, let $g(x) = \arg \max_{y \in Y} f(x, y)$. We can then use the following empirical risk function:

$$\sum_{i: y_i \notin M} l(g_\beta(x_i), y_i) + \sum_{i: y_i \in M} l(g_\beta(O(\beta, x_i)), y_i) + \lambda \|\beta\|_p,$$

(7)

where $O$ aims to transform instances $x_i$ so that $g_\beta(O(\beta, x_i)) \in T$. The relaxed version of the adversarial problem can then be generalized to

$$\min_{x, y \in T} -f(x, y) + c(x, x_i).$$

For a finite target set $T$, this problem is equivalent to taking the best solution of a finite collection of problems identical to Problem 5.

To evaluate the effectiveness of RAD, and resilience of baseline algorithms, in multi-class classification settings, we use the MNIST dataset and aim to correctly identify digits based on their images. Our comparison involves SVM and 3-layer neural network (results for NN-1 are similar). We use $M = \{1, 4\}$ as the malicious class (that is, instances corresponding to digits 1 and 4 are malicious), and $T = \{2, 7\}$ is the set of benign labels (what malicious instances wish to be classified as). The results, shown in Figure 11 are largely consistent with our previous observations: both SVM and 3-layer NN perform well when retrained with RAD, with near-perfect accuracy despite adversarial evasion attempts. Moreover, RAD significantly boosts robustness to evasion, particularly when $\lambda$ is small (adversary who is not very sensitive to evasion costs).

Figure 11: Performance of baseline (adv-) and RAD (rob-) implementations of (a) multi-class SVM and (b) multi-class 3-layer NN, using MNIST dataset testing on adversarial instances.

Figure 12: Visualization of modification attacks with decreasing the cost sensitivity parameter $\lambda$ (from left to right), to change 1 to the set $\{2, 7\}$. The rows correspond to SVM and 3-layer NN, respectively.

Figure 12 offers a visual demonstration of the relative effectiveness of attacks on the baseline implementation of SVM and 1- and 3-layer neural networks. Here, we can observe that a significant
change is required to evade the linear SVM, with the digit having to nearly resemble a 2 after modification. In contrast, significantly less noise is added to the neural network in effecting evasion.

**Evaluation of Cost Function Variations and Robustness to Misspecification**

Considering the variations of cost functions, here we evaluate the classification efficiency for various cost functions as well as different cost functions for defender and adversary, respectively. Figure 13 shows the empirical evaluation results based on Enron dataset with binary features. It is shown that if both the defender and adversary apply the L1 (a) or quadratic cost functions (b), it is easy to defend the malicious manipulations. Even the defender mistakenly evaluate the adversarial cost models as shown in Figure 13 (c), RAD framework can still defend the attack strategies efficiently.

![Figure 13](image)

Figure 13: Performance of baseline (adv-) and RAD (rob-) as a function of cost sensitivity $\lambda$ for Enron dataset with binary features testing on adversarial instances. (a) both defender and adversary use the L1 distance cost function, (b) both defender and adversary use the quadratic distance cost function, (c) adversary uses quadratic cost function while defender estimates it based on exponential cost.

![Figure 14](image)

Figure 14: Performance of baseline (adv-) and RAD (rob-) as a function of cost sensitivity $\lambda$ for Enron dataset with binary features testing on adversarial instances. (a) both defender and adversary use the equivalence-based cost function, (b) adversary uses equivalence-based cost function while defender estimates it based on exponential cost.

**Experiments for Clustering Malicious Instances**

To efficiently speed up the proposed algorithm, here we cluster the malicious instances and use the center of each cluster to generate the potential “evasion” instances for the retraining framework. Figure 15 shows that the running time can be reduced by applying the clustering algorithm to the original malicious instances and the classification performance stays pretty stable for different learning models.
Figure 15: Performance of different learning models based on the number of clusters for Enron dataset testing on adversarial instances. (a) Running time, (b) classification accuracy of $RAD$. 