Impact of Heat Transfer and Viscous Dissipation on Unsteady MHD oscillatory Dusty Fluid Flow Through a Vertical Porous Channel

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Abstract. This work discusses the effect of heat transfer and viscous dissipation on unsteady MHD (Magneto Hydro Dynamic or Hydromagnetic) oscillatory dusty fluid flow through a vertical porous channel. The governing equations for both fluid and particle phase are solved separately to get the exact solution for velocity and temperature profiles. The effect of various dimensionless parameters are displayed graphically and discussed qualitatively.

1. Introduction
For a viscous fluid flow, the fluid takes energy from the motion of the fluid and convert it into internal energy. This phenomenon is called viscous dissipation.

The existence of viscous dissipation stimulates the rate of heat transfer in many industrial problems. Megahed et al. \cite{1} analysed the influence of Joule and viscous dissipation on MHD oscillatory dusty fluid. Cookey et al. \cite{11} studied the impact of viscous dissipation and thermal radiation on unsteady hydromagnetic flow with suction based on time.

Later, Chen \cite{2} discussed the heat and mass transfer effects with viscous dissipation in MHD oscillatory vertical surface. Abo et al. \cite{3} made a research on the influence of viscous dissipation and Joule heating effects in the presence of Hall current. Mansour et al. \cite{4} studied the chemical reaction effects and viscous dissipation on hydromagnetic flows through porous medium. Prasad et al. \cite{5} investigated the influence of thermal radiation and mass transfer on an unsteady MHD semi-infinite vertical porous plate. Sekhar et al. \cite{6} worked out the impact of chemical reaction and viscous dissipation on hydromagnetic oscillatory flow.

Uwanta et al. \cite{9} investigated the problem of chemical reaction effects and viscous dissipation on semi-infinite porous plate. Vyas et al. \cite{12} discussed the impact of thermal radiation and viscous dissipation on MHD transient flow along a vertical porous channel. Vyas et al. \cite{13} also studied the influence of thermal radiative viscous dissipation through porous medium.

Neild \cite{14} made a resolution of a paradox with viscous dissipation and non-linear drag. Salawu \cite{10} discussed the effects of thermal radiation and viscous dissipation in an non-Darcy medium.

Saffman\cite{7} discussed the stable nature of the laminar dusty fluid flow, in which the dust particles are distributed evenly. Cogley \cite{8} assessed the differential approximation for radiative transfer in a non-grey gas near equilibrium.
To be more specific, Kiran Kumari et al. [15] discussed the effects of Viscous Dissipation and Mass Transfer on hydromagnetic oscillatory fluid flow through a vertical porous channel. 

Upto the knowledge of the researcher, the topic Impact of Heat Transfer and Viscous Dissipation on Unsteady MHD oscillatory Dusty Fluid Flow Through a Vertical Porous Channel is not yet carried out by any researcher so far. This paper deals with this topic.

2. Mathematical Formulation
Consider the unsteady flow of a viscous, incompressible, hydromagnetic, oscillatory, optically thin dusty fluid moving along two vertical porous parallel channels.

Figure 1 explains the flow geometry in Cartesian coordinates \((x, y)\) where \(x\) component is taken along the centre of the channel and \(y\) component is the distance calculated normal to it.

![Figure 1. Geometry of Flow](image)

The following presumptions are made here.

(i) A magnetic field of uniform strength is put perpendicular to the plane of the plates
(ii) Boussinesq approximation is considered while framing the governing equations.
(iii) The impact of Joule’s dissipation is ignored in the energy equation.
(iv) The dust particles are considered to be spherical in shape and are spread uniformly throughout the fluid.
(v) Chemical reactions and heat radiations are not present in the dust particles.
(vi) The magnetic Reynolds number is meagre, so that Hall effect is ignored.
(vii) \(u\) and \(u_d\) denote the velocity components of the fluid and dust particles in the \(x\)-direction.
(viii) The walls of the channel are maintained at temperatures \(T_0\) and \(T_w\).
(ix) Viscosity \(\nu\) and Darcy’s resistance terms \(Da\) are taken into account.
(x) In addition, the heat transfer from the dust particle to the fluid is taken for solving the problem.
GOVERNING EQUATIONS

Fluid Phase

Momentum equation

\[ \frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho} \frac{\partial P^*}{\partial x^*} + \nu \frac{\partial^2 u^*}{\partial y^*^2} - \frac{\nu}{K} u^* + \frac{\sigma_c B_0^2 u^*}{\rho} + g \beta_T (T^* - T_0^*) + \frac{K_0 N_0 (u_d^* - u^*)}{\rho} \]  

(1)

Energy equation

\[ \frac{\partial T^*}{\partial t^*} = \frac{k}{\rho C_p} \frac{\partial^2 T^*}{\partial y^*^2} + \frac{1}{\rho C_p} \frac{\partial q^*}{\partial y^*} + \frac{\mu}{\rho C_p} \left( \frac{\partial u^*}{\partial y^*} \right)^2 + \frac{\rho_d C_d (T_d^* - T^*)}{\rho C_p \gamma_T} \]  

(2)

Particle Phase

Momentum equation

\[ m_d \frac{\partial u_d^*}{\partial t^*} = K_0 (u^* - u_d^*) \]  

(3)

Energy equation

\[ \frac{\partial T_{d^*}}{\partial t^*} = -\frac{1}{\gamma_T} (T_{d^*} - T^*) \]  

(4)

Boundary Conditions

\[ u^* = 0, u_d^* = 0, T^* = T_0^*, T_d^* = T_0^*, \text{ at } y^* = 0 \]  

(5)

\[ u^* = 0, u_d^* = 0, T^* = T_w^*, T_d^* = T_w^*, \text{ at } y^* = h \]  

(6)

According to Cogley [8],

\[ \frac{\partial q^*}{\partial y^*} = 4 \alpha^2 (T^* - T_0^*) \]  

(7)

The governing equations are changed into non-dimensional form by using the dimensionless parameters.

Dimensionless parameters

\[ x = \frac{x^*}{h}, \quad y = \frac{y^*}{h}, \quad t = \frac{t^* U}{h}, \quad \omega = \frac{\omega^* h}{U}, \quad u = \frac{u^*}{U}, \quad u_d = \frac{u_d^*}{U}, \quad Re = \frac{U h}{\nu}, \quad \theta = \frac{T^* - T_0^*}{T_w^* - T_0^*}, \]

\[ \theta_d = \frac{T_d^* - T_0^*}{T_w^* - T_0^*}, \quad \phi = \frac{C_w - C_0^*}{C_w - C_0^*}, \quad Da = \frac{K_s}{h^2}, \quad s^2 = \frac{1}{Da}, \quad Sc = \frac{U h}{\nu}, \quad M^2 = \frac{\sigma_c B_0^2 h^2}{\rho \nu}, \]

\[ P = \frac{P^* h}{\nu \nu U}, \quad Pr = \frac{\gamma \rho C_p}{k}, \quad N^2 = \frac{4 \alpha^2 h^2}{k}, \quad K_c = \frac{K_c^* h}{U}, \quad Gr = \frac{g \beta_T (T_w^* - T_0^*) h^2}{\nu U}, \]

\[ Gc = \frac{g \beta_T (C_w^* - C_0^* h^2}{\nu U}, \quad R = \frac{K_0 N_0 h^2}{K_0 h^2}, \quad G = \frac{m_d \nu}{K_0 h^2}, \quad L_0 = \frac{h}{U \gamma_T}, \quad \gamma_T = \frac{3 Pr \gamma_p C_d}{2 C_p}, \]

\[ \gamma_p = \frac{2 \rho_s D^2}{9 \mu}, \quad \rho_s = \frac{3 \rho_d}{4 \pi D^3 N_0}, \quad Ec = \frac{U^2}{C_p (T_w^* - T_0^*)} \]  

(8)

where \( \rho \) density of the fluid, \( \rho_d \) density of dust particles, \( \gamma_T \) temperature relaxation time parameter, \( C_d \) specific heat capacity of dust particles, \( C_p \) specific heat capacity of fluid at constant pressure, \( m_d \) average mass of dust particles, \( K_0 = 6\pi \rho_U D \) Stoke’s constant, \( \gamma_p \) velocity relaxation time parameter, \( \rho_s \) material density of dust particles, \( Re \) Reynolds number, \( Gr \) Grashof number for heat transfer, \( Ec \) Eckert number respectively.
3. Solution of the problem
After applying the dimensionless parameters, equations (1 - 6) become

**Fluid Phase**

\[
\begin{align*}
\text{Re} & \left( \frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} - (s^2 + M^2 + R) u + R u_d + Gr \theta \right) \\
\text{Re} & \left( Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + N^2 \theta + Ec \left( \frac{\partial u}{\partial y} \right)^2 + \frac{2}{3} R (\theta_d - \theta) \right)
\end{align*}
\]

**Particle Phase**

\[
\begin{align*}
\text{Re} & \left( \frac{\partial u_d}{\partial t} = u - u_d \right) \\
\frac{\partial \theta_d}{\partial t} &= -L_0 (\theta_d - \theta)
\end{align*}
\]

**Boundary Conditions**

\[
\begin{align*}
u & = 0, u_d = 0, \theta = 0, \theta_d = 0, \text{ at } y = 0 \\
u & = 0, u_d = 0, \theta = 1, \theta_d = 1, \text{ at } y = 1
\end{align*}
\]

Equations (9 - 14) are coupled, non-linear partial differential equations and these cannot be
solved in closed form. However, these equations can be reduced to a set of ordinary
differential equations, which can be solved analytically.

Since the flow is purely oscillatory and Eckert number (\( Ec << 1 \)), assume

\[
\begin{align*}
-\frac{\partial P}{\partial x} &= A_0 + A_1 Ec e^{i \omega t} \quad \text{; } u(y, t) = u_0(y) + u_1(y) Ec e^{i \omega t} \quad \text{; } \theta(y, t) = \theta_0(y) + \theta_1(y) Ec e^{i \omega t} \\
\begin{align*}
\theta_d & = \theta_0 \\
u_d & = 0 \\
\theta_d & = \left( \frac{L_0}{L_0 + i \omega} \right) \theta_1 \\
u_d & = \left( \frac{1}{1 + i \omega Re \ G} \right) u_1 \\
\theta_0'' + N^2 \theta_0 & = -Ec \ Pr u_0^2 \\
u_0'' - m_0^2 u_0 & = -A_0 - Gr \ \theta_0 \\
\theta_1'' + m_1^2 \theta_1 & = -2 Ec \ Pr u_0 u_1' \\
u_1'' - m_1^2 u_1 & = -A_1 - Gr \ \theta_1
\end{align*}
\]

**Boundary Conditions**

\[
\begin{align*}
u_0 = 0, u_1 = 0, u_d = 0, u_d = 0, \theta_0 = 0, \theta_1 = 0, \theta_d = 0, \theta_d = 0 \text{ at } y = 0 \\
u_0 = 0, u_1 = 0, u_d = 0, u_d = 0, \theta_0 = 1, \theta_1 = 0, \theta_d = 1, \theta_d = 1 \text{ at } y = 1
\end{align*}
\]
where \( m_1^2 = N^2 + \frac{2}{3} \frac{R L_0}{(L_0 + i \omega)} - \frac{2}{3} \frac{R - i \omega Re Pr}{1 + i \omega Re G} \) 
\( m_2^2 = s^2 + M^2 \) 
\( m_3^2 = s^2 + M^2 + R + i \omega Re - \frac{R}{1 + i \omega Re G} \)

Solve equations (16 - 23) using equations (24 - 25).

**TEMPERATURE**

**Fluid Phase**

\[ \theta(y, t) = \theta_0(y) + \theta_1(y) Ec e^{i \omega t} \]  (26)

**Particle Phase**

\[ \theta_d(y, t) = \theta_{d0}(y) + \theta_{d1}(y) Ec e^{i \omega t} \]  (27)

**VELOCITY**

**Fluid Phase**

\[ u(y, t) = u_0(y) + u_1(y) Ec e^{i \omega t} \]  (28)

**Particle Phase**

\[ u_d(y, t) = u_{d0}(y) + u_{d1}(y) Ec e^{i \omega t} \]  (29)

where \[ u_0(y) = C_1 e^{m_2 y} + C_2 e^{-m_2 y} + t_1 + t_2 \theta_{d0} \]
\[ \theta_0(y) = C_3 \cos(Ny) + C_4 \sin(Ny) - C_2^2 t_3 t_4 e^{2 m_2 y} - C_2^3 t_3 t_4 e^{-2 m_2 y} + t_3 t_5 t_6 \]
\[ u_1(y) = C_5 e^{m_3 y} - C_6 e^{-m_3 y} + t_7 + t_8 t_9 \theta_{d1} \]
\[ \theta_1(y) = C_7 \cos(m_1 y) + C_8 \sin(m_1 y) - t_{10} e^{(m_2+m_3) y} - t_{11} e^{(m_2-m_3) y} + t_{12} e^{-(m_2+m_3) y} + t_{13} e^{-(m_2-m_3) y} \]  (33)

and \[ t_1 = \frac{A_0}{m_2^2}; t_2 = \frac{Gr}{m_2^2}; t_3 = Ec Pr m_2^2; t_4 = \frac{1}{4 m_2^2 + N^2}; t_5 = \frac{1}{N^2}; t_6 = 2 \frac{C_1 C_2}{m_2}; \]
\[ t_7 = \frac{A_1}{m_3^2}; t_8 = \frac{Gr}{m_3^2}; t_9 = \frac{L_0 + i \omega}{L_0}; t_{10} = \frac{2 Ec Pr C_1 C_5 m_2 m_3 e^{(m_2+m_3) y}}{(m_2 + m_3)^2 + m_4^2}; \]
\[ t_{11} = \frac{2 Ec Pr C_1 C_6 m_2 m_3 e^{(m_2-m_3) y}}{(m_2 - m_3)^2 + m_4^2}; t_{12} = \frac{2 Ec Pr C_2 C_5 m_2 m_3 e^{-(m_2-m_3) y}}{(m_2 - m_3)^2 + m_4^2}; \]
\[ t_{13} = \frac{2 Ec Pr C_2 C_6 m_2 m_3 e^{-(m_2+m_3) y}}{(m_2 + m_3)^2 + m_4^2}; \]
\[ C_1 = \frac{-t_1 e^{-m_2} + t_1 + t_2}{e^{-m_2} - e^{m_2}}; C_2 = \frac{-t_1 - t_2 + t_1 e^{m_2}}{e^{-m_2} - e^{m_2}}; C_3 = C_2^2 t_3 t_4 + C_2^3 t_3 t_4 - t_3 t_5 t_6; \]
\[ C_4 = \frac{1}{\sin(N)} [1 - C_3 \cos(N) + C_4^2 t_3 t_4 e^{2 m_2} + C_4^2 t_3 t_4 e^{-2 m_2} - t_3 t_5 t_6]; \]
\[ C_5 = \frac{-t_7 e^{m_3} + t_7}{e^{m_3} - e^{m_3}}; C_6 = \frac{-t_7 + t_7 e^{m_3}}{e^{-m_3} - e^{m_3}}; C_7 = t_{10} + t_{11} - t_{12} - t_{13}; \]
\[ C_8 = \frac{1}{\sin(m_1)} [t_{10} e^{(m_2+m_3)} - t_{11} e^{(m_2-m_3)} + t_{12} e^{-(m_2+m_3)} + t_{13} e^{-(m_2-m_3)} - C_7 \cos(m_1)] \]
SKIN FRICITION  
Fluid Phase

\[ \tau = \mu \frac{\partial u}{\partial y} \text{ at } y = 0, 1. \quad (34) \]

\[ \tau = \mu \frac{\partial}{\partial y} \left( u_0 + E c u_1 e^{i \omega t} \right) \text{ at } y = 0, 1. \quad (35) \]

Particle Phase

\[ \tau_d = \mu \frac{\partial u_d}{\partial y} \text{ at } y = 0, 1. \quad (36) \]

\[ \tau_d = \mu \frac{\partial}{\partial y} \left( u_{d0} + E c u_{d1} e^{i \omega t} \right) \text{ at } y = 0, 1. \quad (37) \]

NUSSELT NUMBER (Rate of Heat Transfer)  
Fluid Phase

\[ N u = - \left[ \frac{\partial \theta}{\partial y} \right] \text{ at } y = 0, 1. \quad (38) \]

\[ N u = - \left[ \frac{\partial}{\partial y} \left( \theta_0 + E c \theta_1 e^{i \omega t} \right) \right] \text{ at } y = 0, 1. \quad (39) \]

Particle Phase

\[ N u = - \left[ \frac{\partial \theta_d}{\partial y} \right] \text{ at } y = 0, 1. \quad (40) \]

\[ N u = - \left[ \frac{\partial}{\partial y} \left( \theta_{d0} + E c \theta_{d1} e^{i \omega t} \right) \right] \text{ at } y = 0, 1. \quad (41) \]

4. Graphical Results and Discussions
The above mathematical expressions are resolved analytically and the computed results are graphically plotted using MATLAB 8.3. For all numerical computations, take Prandtl number as \( Pr = 0.71 \), that corresponds to air at \( 20 \degree C \).

Figures 2 - 11 illustrate the impact of various significant parameters on velocity of fluid and dust particles. The parabolic nature of velocity profiles is noted in all the graphs. Generally, the velocity profiles of fluid and dust particles are maximum at the middle of the channel and minimum near the walls of the channel. (i.e.) All the velocity profiles accelerate steadily from the left end of the plate \( (y = 0) \), attain maximum at its centre and converges near the other end of the channel \( (y = 1) \), satisfying the prescribed boundary conditions.

Figures 12 - 21 show the impact of various physical parameters on temperature of fluid and dust particles. It is also observed that the nature of temperature profiles is parabolic. Moreover, the temperature profiles of fluid and dust particles reaches its maximum in the centre of the channel \( (y = 0.5) \).

The skin friction coefficient and the rate of heat transfer for fluid and dust particles by varying various parameters are tabulated in Tables 1 - 4.
From Figure 2, it is noted that rise in the value of Reynolds number ($Re$) increases the velocity of fluid and dust particles. $Re$ has the same influence on velocity of fluid and dust particles.

From Figure 3, it is clear that while augmenting Grashof number for heat transfer ($Gr$), the velocity of fluid and dust particles’ gradually increase due to buoyancy force. $Gr$ has minor impact on velocity of fluid and dust particles. For the same flow geometry, this result coincides with Kiran Kumari et al. [15] for the MHD oscillatory fluid flow without dust parameters.

From Figure 4, it is seen that the velocity of fluid and dust particles’ decelerate for a gradual rise in the value of Hartmann number ($M$). It is true that the transverse magnetic field produces a resistive force called Lorentz force, which retards the fluid motion. $M$ has significant effect on velocity of fluid and dust particles. For the same flow geometry, this result coincides with Kiran Kumari et al. [15] for the MHD oscillatory fluid flow without dust parameters.
From Figure 5, it is apparent that the velocity of fluid and dust particles accelerate gradually, while enhancing the value of radiation parameter ($N$). $N$ shows a major variation on velocity of fluid and dust particles.

From Figure 6, it is seen that a gradual increase in particle mass parameter ($G$) shows that the velocity of fluid and dust particles increases steadily. $G$ has significant effect on velocity of fluid and dust particles.

From Figure 7, it is apparent that while augmenting the value of particle concentration parameter ($R$), the velocity of fluid and dust particles increase. $R$ has a minor impact on velocity of fluid and dust particles.
Figure 8. Impact of $L_0$ on (a) $u$, (b) $u_d$, with $Pr = 0.71$; $Re = 2$; $Gr = 2$; $M = 3$; $N = 3$; $G = 3$; $R = 3$; $s = 2$; $A = 0.1$; $w = 1$; $t = 0.5$; $Ec = 0.5$.

From Figure 8, it is noteworthy to mention that an increase in temperature relaxation time parameter ($L_0$) rises the velocity of fluid and dust particles. $L_0$ has the same influence on velocity of fluid and dust particles.

Figure 9. Impact of $s$ on (a) $u$, (b) $u_d$, with $Pr = 0.71$; $Re = 2$; $Gr = 2$; $M = 3$; $N = 3$; $G = 3$; $R = 3$; $L_0 = 1$; $A = 0.1$; $w = 1$; $t = 0.5$; $Ec = 0.5$.

From Figure 9, it is clear that a rise in the value of porous medium shape factor ($s$) reduces the velocity of fluid and dust particles steadily. $s$ shows a minor variation on velocity of fluid and dust particles.

Figure 10. Impact of $\omega$ on (a) $u$, (b) $u_d$, with $Pr = 0.71$; $Re = 2$; $Gr = 2$; $M = 3$; $N = 3$; $G = 3$; $R = 3$; $L_0 = 1$; $s = 2$; $A = 0.1$; $t = 0.5$; $Ec = 0.5$.

From Figure 10, it is clear that augmenting the value of frequency of oscillation parameter ($\omega$), the velocity of fluid and dust particles rise steadily. $\omega$ shows a slight change on velocity of fluid and dust particles.
Figure 11. Impact of $t$ on (a) $u$, (b) $u_d$, with $Pr = 0.71; Re = 2; Gr = 2; M = 3; N = 3; G = 3; R = 3; L_0 = 1; s = 2; A = 0.1; w = 1; Ec = 0.5$.

From Figure 11, it is apparent for an unsteady flow, as time ($t$) increases, the velocity of fluid and dust particles increase. Time ‘$t$’ has the same impact on velocity of fluid and dust particles.

Figure 12. Impact of $Re$ on (a) Temperature of Fluid ($\theta$), (b) Temperature of Dust Particles ($\theta_d$), with $Pr = 0.71; Gr = 2; M = 3; N = 3; G = 3; R = 3; L_0 = 1; s = 2; A = 0.1; w = 1; t = 0.5; Ec = 0.5$.

From Figure 12, it is evident that the temperature of fluid and dust particles increase with an increase in value of Reynolds number ($Re$). $Re$ has slight effect on the temperature of fluid and dust particles.

Figure 13. Impact of $Gr$ on (a) $\theta$, (b) $\theta_d$, with $Pr = 0.71; Re = 2; M = 3; N = 3; G = 3; R = 3; L_0 = 1; s = 2; A = 0.1; w = 1; t = 0.5; Ec = 0.5$.

From Figure 13, it is visible that the temperature of fluid and dust particles increase with an increase in the value of Grashof number for heat transfer ($Gr$). $Gr$ has major influence on the temperature of fluid and dust particles.
Figure 14. Impact of $M$ on (a) $\theta$, (b) $\theta_d$, with $Pr = 0.71$; $Re = 2$; $Gr = 2$; $N = 3$; $G = 3$; $R = 3$; $L_0 = 1$; $s = 2$; $A = 0.1$; $w = 1$; $t = 0.5$; $Ec = 0.5$.

From Figure 14, it is seen that for a gradual increase in Hartmann number ($M$) or magnetic parameter, the temperature of fluid and dust particles decrease in the left part of the channel and increase in the right side. $M$ has the same influence on the temperature of fluid and dust particles.

Figure 15. Impact of $N$ on (a) $\theta$, (b) $\theta_d$, with $Pr = 0.71$; $Re = 2$; $Gr = 2$; $M = 3$; $G = 3$; $R = 3$; $L_0 = 1$; $s = 2$; $A = 0.1$; $w = 1$; $t = 0.5$; $Ec = 0.5$.

From Figure 15, it is apparent that the temperature of fluid and dust particles increase with an increase in the value of radiation parameter ($N$). $N$ has major impact on the temperature of fluid when compared to dust particles.

Figure 16. Impact of $G$ on (a) $\theta$, (b) $\theta_d$, with $Pr = 0.71$; $Re = 2$; $Gr = 2$; $M = 3$; $N = 3$; $R = 3$; $L_0 = 1$; $s = 2$; $A = 0.1$; $w = 1$; $t = 0.5$; $Ec = 0.5$.

From Figure 16, it is clear that the temperature of fluid and dust particles increase for an increase in the value of particle mass parameter ($G$). $G$ has the same influence on the temperature of fluid and dust particles.
Figure 17. Impact of $R$ on (a) $\theta$, (b) $\theta_d$, with $Pr = 0.71; Re = 2; Gr = 2; M = 3; N = 3; G = 3; L_0 = 1; s = 2; A = 0.1; w = 1; t = 0.5; Ec = 0.5$.

From Figure 17, it is evident that the temperature of fluid and dust particles increase for a rise in the value of particle concentration parameter ($R$). $R$ has significant effect on temperature of fluid compared to that of dust particles.

Figure 18. Impact of $L_0$ on (a) $\theta$, (b) $\theta_d$, with $Pr = 0.71; Re = 2; Gr = 2; M = 3; N = 3; G = 2; R = 2; s = 1; A = 0.1; w = 1; t = 0.5; Ec = 0.5$.

From Figure 18, it is noteworthy to mention that an increase in temperature relaxation time parameter ($L_0$) rises the temperature of fluid and dust particles gradually. $L_0$ has the same influence on temperature of fluid and dust particles.

Figure 19. Impact of $s$ on (a) $\theta$, (b) $\theta_d$, with $Pr = 0.71; Re = 2; Gr = 2; M = 3; N = 3; G = 3; R = 3; L_0 = 1; A = 0.1; w = 1; t = 0.5; Ec = 0.5$.

From Figure 19, it is notable that an increase in the value of porous medium shape factor ($s$) decreases the temperature of fluid, whereas it shows an opposite effect for the dust particles. $s$ has significant effect on temperature of fluid and dust particles.
Figure 20. Impact of $\omega$ on (a) $\theta$, (b) $\theta_d$, with $Pr = 0.71$; $Re = 2$; $Gr = 2$; $M = 3$; $N = 3$; $G = 3$; $R = 3$; $L_0 = 1$; $s = 1$; $A = 0.1$; $t = 0.5$; $Ec = 0.5$.

From Figure 20, it is noteworthy to mention that while augmenting the value of frequency of oscillation parameter ($\omega$), the temperature of fluid and dust particles increase gradually. $\omega$ shows a major variation on temperature of fluid compared to dust particles.

Figure 21. Impact of $t$ on (a) $\theta$, (b) $\theta_d$, with $Pr = 0.71$; $Re = 2$; $Gr = 2$; $M = 3$; $N = 3$; $G = 3$; $R = 3$; $L_0 = 1$; $s = 1$; $A = 0.1$; $w = 1$; $Ec = 0.5$.

From Figure 21, it is noted that as time ($t$) increases, the temperature of fluid and dust particles enhance steadily, in the case of unsteady flow. Time ‘$t$’ has the same influence on temperature of fluid and dust particles.

5. Tables and Discussions

Table 1. Skin friction coefficient for Fluid Phase ($\tau$) and Dust Particle Phase ($\tau_d$) by varying $t$ with $Pr = 0.71$; $Re = 2$; $Gr = 2$; $M = 3$; $N = 3$; $G = 3$; $R = 3$; $L_0 = 1$; $s = 2$; $A = 0.1$; $w = 1$; $Ec = 0.5$.

| $t$ | Fluid Phase | Dust Particle Phase |
|-----|-------------|---------------------|
|     | $\tau$ at $y = 0$ | $\tau$ at $y = 0.5$ | $\tau$ at $y = 1$ | $\tau$ at $y = 0$ | $\tau$ at $y = 0.5$ | $\tau$ at $y = 1$ |
| 0.1 | 0.2832      | 0.0474              | -0.6470            | 0.3006      | 0.0619              | -0.6136            |
| 0.2 | 0.2830      | 0.0467              | -0.6481            | 0.2990      | 0.0592              | -0.6169            |
| 0.3 | 0.2828      | 0.0464              | -0.6487            | 0.2974      | 0.0592              | -0.6199            |
| 0.4 | 0.2827      | 0.0462              | -0.6487            | 0.2960      | 0.0579              | -0.6226            |
| 0.5 | 0.2817      | 0.0461              | -0.6488            | 0.2948      | 0.0568              | -0.6251            |

It is evident from Table 1 that as time ($t$) increases, the skin friction coefficient of fluid and dust particles decrease at all positions across the channel, maximum at the left end of the channel ($y = 0$) and minimum at the right end ($y = 1$) and decrease along the ends and the middle of the channel.
Table 2. Skin friction coefficient for Fluid Phase ($\tau$) and Dust Particle Phase ($\tau_d$) by $R$ and $G$ with $Pr = 0.71$; $Re = 2$; $Gr = 2$; $M = 3$; $N = 3$; $L_0 = 1$; $s = 1$; $A = 0.1$; $w = 1$; $t = 0.6$; $Ec = 0.5$.

| $R$ | $G$ | Fluid Phase | Dust Particle Phase |
|-----|-----|-------------|---------------------|
|     |     | $\tau$ at $y = 0$ | $\tau$ at $y = 0$ | $\tau$ at $y = 1$ | $\tau_d$ at $y = 0$ | $\tau_d$ at $y = 0.5$ | $\tau_d$ at $y = 1$ |
| 1   | 1   | 0.2846      | 0.0465              | $-0.6470$           | 0.2938               | 0.0558                 | $-0.6272$           |
| 2   | 1   | 0.2876      | 0.0502              | $-0.6398$           | 0.2961               | 0.0585                 | $-0.6226$           |
| 3   | 1   | 0.2899      | 0.0529              | $-0.6347$           | 0.2978               | 0.0605                 | $-0.6193$           |
| 3   | 2   | 0.2922      | 0.0555              | $-0.6315$           | 0.3058               | 0.0654                 | $-0.6066$           |
| 3   | 3   | 0.2930      | 0.0563              | $-0.6305$           | 0.3086               | 0.0669                 | $-0.6019$           |

It is evident from Table 2 that by varying particle concentration parameter ($R$) and particle mass parameter ($G$), the skin friction coefficient of fluid and dust particles decrease at all positions across the channel, maximum at the left end of the channel ($y = 0$) and minimum at the right end ($y = 1$) and increase along the ends and the middle of the channel.

Table 3. Nusselt number ($Nu$) for Fluid Phase and Dust Particle Phase by varying $t$ with $Pr = 0.71$; $Re = 2$; $Gr = 2$; $M = 3$; $N = 3$; $G = 3$; $R = 3$; $L_0 = 1$; $s = 1$; $A = 0.1$; $w = 1$; $Ec = 0.5$.

| $t$ | Fluid Phase | Dust Particle Phase |
|-----|-------------|---------------------|
|     | $Nu$ at $y = 0$ | $Nu$ at $y = 0.5$ | $Nu$ at $y = 1$ | $Nu$ at $y = 0$ | $Nu$ at $y = 0.5$ | $Nu$ at $y = 1$ |
| 0.1 | $-2.4950$   | $-1.2195$          | 1.4142           | $-2.4937$       | 1.2195           | 1.3744           |
| 0.2 | $-2.4949$   | $-1.2194$          | 1.4125           | $-2.4936$       | 1.2194           | 1.3741           |
| 0.3 | $-2.4948$   | $-1.2193$          | 1.4099           | $-2.4935$       | 1.2193           | 1.3613           |
| 0.4 | $-2.4947$   | $-1.2192$          | 1.4065           | $-2.4934$       | 1.2192           | 1.3539           |
| 0.5 | $-2.4946$   | $-1.2191$          | 1.4022           | $-2.4933$       | 1.2191           | 1.3458           |

It is noted from Table 3 that as time ($t$) increases, the rate of heat transfer for fluid and dust particles increase at all positions across the channel, minimum at the left end of the channel ($y = 0$) and maximum at the right end ($y = 1$) and increase along ($y = 0$), decrease along ($y = 1$).

Table 4. Nusselt Number ($Nu$) for Fluid Phase and Dust Particle Phase by varying $L_0$ and $Pr$ with $Re = 2$; $Gr = 2$; $M = 2$; $N = 2$; $G = 2$; $R = 2$; $s = 2$; $A = 0.1$; $w = 1$; $t = 0.5$; $Ec = 0.5$.

| $L_0$ | $Pr$ | Fluid Phase | Dust Particle Phase |
|-------|------|-------------|---------------------|
|       |      | $Nu$ at $y = 0$ | $Nu$ at $y = 0.5$ | $Nu$ at $y = 1$ | $Nu$ at $y = 0$ | $Nu$ at $y = 0.5$ | $Nu$ at $y = 1$ |
| 0.1   | 0.71 | $-2.4947$   | $-1.2188$          | 1.3933           | $-2.4930$       | 1.2188           | 1.3286           |
| 0.2   | 0.71 | $-2.4947$   | $-1.2188$          | 1.3937           | $-2.4931$       | 1.2189           | 1.3358           |
| 0.3   | 0.71 | $-2.4948$   | $-1.2188$          | 1.3942           | $-2.4933$       | 1.2190           | 1.3434           |
| 0.3   | 1    | $-2.5202$   | $-1.2223$          | 1.4633           | $-2.5179$       | 1.2225           | 1.3897           |
| 0.3   | 6.7  | $-3.0190$   | $-1.3012$          | 2.7049           | $-2.9981$       | 1.2899           | 2.2171           |

It is evident from Table 4 that by varying temperature relaxation time parameter ($L_0$) and Prandtl number ($Pr$), the rate of heat transfer for fluid and dust particles increase at all positions across the channel, very low at one end of the channel ($y = 0$) and gradually rise at the other end ($y = 1$) and decrease along ($y = 0$), increase along ($y = 1$).
6. Conclusion

In view of the results discussed above, the following conclusion is obtained:

- Increase in Grashof number ($Gr$), radiation parameter ($N$), particle mass parameter ($G$), particle concentration parameter ($R$), temperature relaxation parameter ($L_0$), frequency of oscillation parameter ($\omega$) and time ($t$) increase the velocity and temperature of fluid and dust particles.
- Increase in porous medium shape factor ($s$) decreases the velocity and temperature of fluid and dust particles.
- Increase in Reynolds number ($Re$) increases the velocity of fluid and dust particles.
- As time ($t$) increases, the skin friction coefficient of fluid and dust particles decrease at all positions across the channel, maximum at the left end of the channel ($y = 0$) and minimum at the right end ($y = 1$).
- By varying particle concentration parameter ($R$) and particle mass parameter ($G$), the skin friction coefficient of fluid and dust particles decrease at all positions across the channel, maximum at the left end of the channel ($y = 0$) and minimum at the right end ($y = 1$).
- As time ($t$) increases, the rate of heat transfer for fluid and dust particles increase at all positions across the channel, minimum at the left end of the channel ($y = 0$) and maximum at the right end ($y = 1$).
- By varying temperature relaxation time parameter ($L_0$) and Prandtl number ($Pr$), the rate of heat transfer for fluid and dust particles increase at all positions across the channel, very low at one end of the channel ($y = 0$) and gradually rise at the other end ($y = 1$).

Limiting Case: In the absence of dust parameters, the results obtained in this chapter coincide with those of Kiran Kumari [15] for MHD oscillatory flow through a vertical porous channel.

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