Multicriteria optimization of the dynamics of integrated logistics of the forest industry

S M Bazarov, Y I Belenky, V A Aleksandrov, V A Sokolova*, A N Soloviev
Institute of Technological Machines and Forest Transportation, St. Petersburg State Forest Technical University, 5 Institutskiy Lane, St. Petersburg 194021, Russian Federation

*Corresponding email: sokolova_vika@inbox.ru

Abstract. Under complex dynamic market conditions, the sustainable development of a coherent integrated spatio-temporal structure of production-transport-consumption is determined by many objective functions, the optimization of which necessitates solving the problem of making a multi-criteria compromise. This problem is solved using a set of the main criteria for technical and economic efficiency: a minimum of cost, a minimum of functional time and space, and a minimum of energy costs. The formulated criteria allow, from a multifaceted perspective, to best structure the material, energy and financial flows in the integrated logistics of the timber industry complex. The article explores the three-step structure of the movement of logistics flows from the forest to the consumer. Mathematical programming determines a discrete set of optimization criteria, on the basis of which a Pareto compromise set is built. On this set, the decision maker formulates a compromise.

1. Introduction

In the forest industrial complex (FC), the production, transport and marketing of timber and wood products form a single technological spatial-temporal structure, from the forest area to the final consumer. The requirement of the sustainable development of the FC makes it necessary to formulate a multitude of objective functions that reflect the multifaceted dynamics of market relations. From the point of view of generalized concepts of logistics, the FC is a connected integrated system of production and movement of material, energy, finances and information, each of which is assigned a target function and criteria for its optimization [1, 2]. From the standpoint of the group theory and system analysis, objective functions make a single multidimensional functional space-time, in which the problems of optimizing complexes as a whole are defined and solved [3, 4].

Let us formulate the main optimization criteria that must be determined for the sustainable development of the forest industry in the market: minimum cost, minimum time (speed), minimum energy cost, and minimum transport and technological distance. In [5], a solution is presented for the first two optimization criteria and only for the transport problem. Market dynamics require solving optimization problems using a broader set of criteria for the entire industrial-transport-consumer complex of the FC.

The aim of the study is mathematical programming [6-8] to determine the discrete optimal values of the formulated criteria in the functional space-time of integrated logistic flows of the FC, and to build a Pareto compromise set on their basis [8, 9]. Using Pareto sets, the decision maker (DM) formulates the best compromised option.
2. Methods and Materials
The integrated industrial-transport-consumer system of the FC can be represented as a continuous three-stage space-time group structure (figure 1) [10]:

- the first group of steps consists of timber harvesting enterprises (denoted by $A_i$, $i = 1, 2, 3, ..., n$);
- the second group of steps unites timber treating enterprises (denoted by $B_j$, $j = 1, 2, ..., m$);
- the third group of steps includes timber processing enterprises (denoted by $C_k$, $k = 1, 2, 3, ..., l$);
- the fourth group unites consumers of timber products (denoted by $D_0$, $0 = 1, 2, 3, .., e$).

Material flows between the represented groups of producers - consumers are provided by appropriate vehicles.

In the presented scheme, the next group of enterprises in relation to the previous one can be considered as a consumer which processes incoming wood into the material necessary for the subsequent group.

The analytical method for solving the problem consists of applying a systematic approach to solving the production-transport-consumption optimization problem by mathematical programming for each optimization criterion, and constructing a compromise Pareto set for DM.

![Figure 1. Three-stage production-transport-consumption scheme in the FC.](image)

3. Results and Discussion
Production operations in the FC can be divided into technological and relocation (transport), each of which is characterized by productivity, energy capacity and financial support, on the basis of which the functional space-time of the connectedness of integrated flows is determined.

The generalized formula for determining the productivity of technological operations of wood processing can be represented as:

$$P = \frac{V}{t},$$

where $V$ is the volume of wood, and $t$ is the operation time.

A related performance parameter is the functional time per unit volume of production.
The generalized formula for determining the performance of a relocation (transport) operation can be written as

$$ PT = \frac{V}{t}, $$

(3)

where, $V$ is the volume, and $t$ is the total time of the transport operation.

Here, the functional time of moving a volume unit by a transport operation is

$$ q = \frac{1}{PT}, $$

(4)

We introduce a representation of the technological speed of a transport operation

$$ U = \frac{L}{t}, $$

(5)

where, $L$ is the transport distance.

Based on (3) and (5), we determine the volumetric load per unit of the transport path

$$ v_l = \frac{PT}{U}, \text{[m}^3/\text{m}], $$

(6)

and its associated functional transport and technological distance of movement of a unit volume

$$ l = \frac{U}{PT}, $$

(7)

The functional time of the operation per unit of energy is

$$ \tau = \frac{1}{N}, \text{[hour/kWh]}, $$

(8)

where, $N$ is the power of a mechanism.

The energy intensity of the production operation is defined by the formula

$$ E = \frac{N}{P}, \text{[kWh/m}^3], $$

(9)

for technological operations, and the formula

$$ e = \frac{N}{PT}, $$

(10)

for transport operations.

The functional time of the cost of production operations (transport) per financial unit is

$$ T = \frac{1}{C_1}, \text{[hour/Rub]}, $$

(11)

where, $C_1$ is the cost of the operation per unit time.

The functional cost of the produced (moved) unit volume of wood in the technological operation is
and accordingly for transport

\[ C = \frac{C_i}{P_i} \text{, [Rub / m}^3\text{]}, \]

The presented formulas show the versatility of the functional time of production-displacement of a unit of the subject of labor (volume of wood) in integrated logistics of the FC. Based on these formulas, a multidimensional functional space-time is built, in which, from the standpoint of the theory of system analysis and groups, it becomes possible to study multi-criteria optimization of the dynamics of the entire technical and economic process, from the forest area to wood processing, transportation and consumption. In this multidimensional functional space-time, the problem of finding the best compromise solution is solved for the difficult conditions of the existence of the FC in the market.

Each forest industry enterprise, as a rule, produces a sufficiently large number of products; its movement in the space-time of forest industry becomes multi-parametric. In order to somewhat simplify this complex movement, we assume that selected groups of enterprises produce an appropriate volume of a product of general type. Consideration of the dynamics of the movement of many types of products by enterprises will lead to an algebraic complication of the mathematical model, but not to its fundamental representation.

From the standpoint of the system analysis, we will sequentially perform a study of the dynamics of the integrated FC production-transport-consumption in a functional time-space built on the formulated optimization criteria.

The minimum cost of production and product movement in the production-transport-consumer structure will be determined sequentially in the functional time-space from step to step in the representations of the system analysis and mathematical programming. We introduce the functional time of the connectedness of production-transport operations by complicating the classical static transport problem. In the first stage, enterprises \( A_1, A_2, ..., A_m \) with the corresponding productivity of the equipment \( P_1, P_2, ..., P_m \) produce the corresponding volumes of wood products \( a_1, a_2, ..., a_m \), this volumes should be delivered to consumers – producers \( B_1, B_2, ..., B_n \) by vehicles with productivity \( PT_1, PT_2, ..., PT_n \), with the corresponding demand in amount \( b_1, b_2, ..., b_n \), the total volume of production equals total demand

\[ \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j, \]

the cost of transporting a unit volume of products from \( A_i \) to \( B_j \) is determined by a formula that takes into account the coherence of production-transport operations in a single functional time,

\[ c_{ij} = \frac{C_i \cdot c_y}{C_i + c_y}, \]

Here the problem is formulated as follows: to minimize the cost of production – first-stage transport by finding an extreme linear functional that satisfies the condition

\[ Z_{min} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \rightarrow \min \]
where, \(x_{ij}\) is the number of units of a product (matrix), moved from \(A_i\) to \(B_j\) subject to the restrictions

\[
\begin{align*}
\sum_{j=1}^{n} x_{ij} &= b_j, \quad j = 1, 2, \ldots, n, \\
\sum_{i=1}^{m} x_{ij} &= a_i, \quad i = 1, 2, \ldots, m,
\end{align*}
\]

and is solved as a generalized transportation problem [6].

Formulas (14), (15) and (16) describe the functional production and economic dynamic connectivity between enterprises of the first and second groups.

We will construct a similar picture for the second connectivity level of the second and third groups: enterprises \(B_1, B_2, \ldots, B_n\) with the corresponding productivity of equipment \(P_1, P_2, \ldots, P_n\) produce the following volumes of products \(b^*_1, b^*_2, \ldots, b^*_n\); these volumes should be delivered to manufacturing consumers \(C_1, C_2, \ldots, C_r\) by vehicles with capacities \(PT_1, PT_2, \ldots, PT_r\) with the corresponding demand in quantities \(c_1, c_2, \ldots, c_r\). The total volume of production is equal to the total demand

\[
\sum_{j=1}^{n} b^*_j = \sum_{k=1}^{r} c_k, \quad (17)
\]

the cost of transporting a unit volume of products from \(B_j\) to \(C_k\) is determined by a formula that takes into account the coherence of production-transport operations in a single functional time,

\[
c_{jk} = \left[ \frac{C_j c_{jk}}{C_j + c_{jk}} \right], \quad (18)
\]

the task is formulated as follows: to minimize the cost of production-transportation of the second stage

\[
Z_{c2} = \sum_{k=1}^{r} \sum_{j=1}^{n} c_{jk} x_{jk} \rightarrow \min, \quad (19)
\]

where \(x_{jk}\) is the number of units transported from \(B_j\) to \(C_k\) under the restrictions

\[
\begin{align*}
\sum_{k=1}^{r} x_{jk} &= b^*_j, \quad j = 1, 2, \ldots, n, \\
\sum_{j=1}^{n} x_{jk} &= c_k, \quad k = 1, 2, \ldots, r.
\end{align*}
\]

Formulas (18), (19) and (20) describe the functional production and economic dynamic connectivity between enterprises of the second and third groups.

A similar picture will be presented for the third stage relationship between the third and fourth groups: enterprises \(C_1, C_2, \ldots, C_r\) with the corresponding productivity of equipment \(P_1, P_2, \ldots, P_r\) produce the following volumes of products \(c^*_1, c^*_2, \ldots, c^*_r\), which will be delivered to consumers \(D_1, D_2, \ldots, D_w\) by vehicles with productivity \(PT_1, PT_2, \ldots, PT_r\) given the corresponding demand in amount \(d_1, d_2, \ldots, d_w\), the total volume of production is equal to the total demand

\[
\sum_{k=1}^{r} c^*_k = \sum_{o=1}^{w} d_o, \quad (20)
\]

the cost of transporting a unit volume of products from \(C_k\) to \(D_o\) is determined by a formula that takes into account the coherence of production-transport operations in a single functional time,
the task is formulated as follows: to minimize the cost of production-transportation of the third stage

\[ Z_{c3} = \sum_{k=1}^{r} \sum_{o=1}^{w} c_{ko} x_{ko} \rightarrow \min, \tag{22} \]

where \( x_{ko} \) is the number of units transported from \( C_k \) to \( D_o \) under the restrictions

\[
\begin{align*}
\sum_{o=1}^{w} x_{ko} &= c_k, \quad k = 1, 2, \ldots, r, \\
\sum_{k=1}^{r} x_{ko} &= d_o, \quad o = 1, 2, \ldots, w.
\end{align*}
\]

Formulas (20), (21) and (22) describe the functional production and economic dynamic connectivity of enterprises of the third group with consumers.

For the presented three-stage complex production-transport-consumption, the task of minimizing the cost is reduced to determining a linear functional satisfying the condition

\[ Z_{\nu} = \sum_{i=1}^{n} \sum_{j=1}^{r} \sum_{k=1}^{w} \sum_{o=1}^{w} \left[ \frac{C_{ijk} + c_{iko}}{P_{ijk} + PT_{iko}} \right] \rightarrow \min, \tag{23} \]

In the previously formulated list of optimization criteria, the minimum functional time of the connectivity of the system manufacturers-transport-consumers is the second criterion. Based on the constructed analytical algorithm for solving the cost minimization problem, we immediately write down the conditions for minimizing the functional time of the connectivity of the three-stage producer-transport-consumer system in steps.

For the first stage, minimizing the production-transportation functional time is a condition

\[ Z_{q1} = \sum_{i=1}^{n} \sum_{j=1}^{r} q_{ji} x_{ji} \rightarrow \min, \tag{24} \]

where,

\[ q_{ji} = \frac{1}{\frac{1}{P_{ji}} + \frac{1}{PT_{ji}}}, \]

for the second stage

\[ Z_{q2} = \sum_{k=1}^{r} \sum_{o=1}^{w} q_{ko} x_{ko} \rightarrow \min, \tag{25} \]
where,

\[ q_{jk} = \frac{1}{P_j} + \frac{1}{PT_{jk}}. \]

and for the third stage

\[ Z_{q3} = \sum_{k=1}^{r} \sum_{a=1}^{w} q_{ko} x_{ko} \rightarrow \min, \tag{26} \]

where,

\[ q_{ko} = \frac{1}{P_k} + \frac{1}{PT_{ko}}. \]

For the presented three-stage complex production-transport-consumption, the solution to the problem of minimizing the functional connection time is reduced to the condition

\[ Z_q = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{r} \sum_{a=1}^{w} \left( \frac{1}{P_{ijk}} + \frac{1}{PT_{ikko}} \right) \rightarrow \min. \tag{27} \]

In the previously formulated list of optimization criteria the minimum energy in the system manufacturers-transport-consumers is the third criterion. Based on the constructed analytical algorithm for solving the problem of minimizing cost and functional time, we immediately write down the conditions for minimizing energy in a three-stage producer-transport-consumer system in steps.

For the first stage, minimization of production-transportation energy is a condition

\[ Z_{q1} = \sum_{i=1}^{m} \sum_{j=1}^{n} e_{ij} x_{ij} \rightarrow \min, \tag{28} \]

where,

\[ e_{ij} = E_j + e_{ij}, \]

for the second stage

\[ Z_{q2} = \sum_{k=1}^{r} \sum_{a=1}^{w} e_{jk} x_{jk} \rightarrow \min, \tag{29} \]

where,

\[ e_{jk} = E_j + e_{jk}, \]

and for the third stage

\[ Z_{q3} = \sum_{k=1}^{r} \sum_{a=1}^{w} e_{ko} x_{ko} \rightarrow \min, \tag{30} \]

where,

\[ e_{ko} = E_k + e_{ko}. \]
For the presented three-stage complex production-transport-consumption, the solution to the problem of minimizing the functional connection time is reduced to the condition

\[ Z_N = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{r} \sum_{a=1}^{u} \left( E_{ijk} + e_{a} \right) \rightarrow \min . \]  

We write down the conditions of the minimum transport and technological path, respectively, for the first stage as

\[ Z_{L1} = \sum_{i=1}^{m} \sum_{j=1}^{n} l_{ij} x_{ij} \rightarrow \min , \]  

for the second stage as

\[ Z_{L2} = \sum_{j=1}^{n} \sum_{k=1}^{r} l_{jk} x_{jk} \rightarrow \min , \]  

and for the third stage as

\[ Z_{L3} = \sum_{k=1}^{r} \sum_{a=1}^{u} l_{ko} x_{ko} \rightarrow \min . \]

For an integrated three-stage complex of the FC, the condition for finding the minimum transport and technological path in the production-transport-consumer system takes the form

\[ Z_{L} = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{r} \sum_{a=1}^{u} l_{ijko} \rightarrow \min \]  

**4. Conclusion**

The conjugated connectivity of the formulated optimization criteria with each other should be noted:

- The minimum cost is associated with the total functional time, energy and transport path

\[ Z_c = Z_c \left( Q_c, E_c, L_c \right) , \]  

- The minimum functional time is associated with the total cost, energy and transport path

\[ Z_q = Z_q \left( C_q, E_q, L_q \right) , \]  

- Minimum energy correspond to the total cost, functional time and transport path

\[ Z_N = Z_N \left( C_N, Q_N, L_N \right) , \]  

- The minimum transport and technological path is associated with the total cost, functional time and energy

\[ Z_L = Z_L \left( C_L, Q_L, E_L \right) . \]  

Discrete representations (36)–(39) allow us to build connection compositions, the analytical generalization of which determines the Pareto set, on which a DM makes the most informative compromise decision. The analytical approach (36) – (39) can also be applied to construct the Pareto set in separate steps in the producer-transport-consumer system.

**References**

[1] Protsenko O D and Protsenko I O 2010 *Logistics and supply chain management - a look into the future* (Moscow: Delo) pp 5–12
[2] Salminen E O, Borozna A A and Tyurin N A 2001 *Timber Logistics* (St. Petersburg: SPb SFTA) pp 10–30

[3] Kargapolov M I and Merzlyakov Y I 1982 *Fundamentals of group theory* (Moscow: Nauka) pp 15–22

[4] Bazarov S M, Belenky Y I and Soloviev A N 2018 *The basics of a systems analysis of production processes* (St Petersburg: SPb SFTU) pp 8–15

[5] Bazarov S M, Belenky Y I, Sokolova V A and Soloviev A N 2019 Two-criteria technical and economic optimization of the forest transportation problem. *Proc. Sc.-Tech. Conf. on Forests of Russia: Policy, Industry, Science and Education* (St Petersburg: SPb SFTU) vol 316 pp 01064.

[6] Korobov P N 2006 *Mathematical programming and modeling of economic processes* (St. Petersburg: DNK) pp 27–30

[7] Korobov P N and Gurov S V 2010 *Mathematical modeling in the theory of forestry complex management* (St. Petersburg: SPb SFTA) pp 3–9

[8] Andreev V N and Gerasimov Y O 1999 *Making the best decisions. Theory and application in the forest complex* (Joensuu: University of Joensuu) pp 137–152

[9] Gurov S V 2008 *Theory of systems analysis and decision making* (St. Petersburg: SPb SFTA) pp 67–73

[10] Lien-Tsen F and Chu-Sen W 1967 *Discrete maximum principle* (Moscow: Mir) pp 13–18