THE COUPLINGS OF THE PION TO TWO GAUGE FIELDS AND TO LEPTONS
IN A DYNAMICALLY BROKEN GAUGE THEORY

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Abstract: we show how a spontaneously broken gauge theory of fermions endowed with a composite scalar multiplet becomes naturally anomaly-free, and yet describes the correct couplings of the pion to two gauge fields and to leptons: the first coupling is the same as computed from the chiral anomaly, and the second identical with that obtained from the ‘Partially Conserved Axial Current’ hypothesis. For the sake of simplicity, we only study here the abelian case.

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1 Introduction

Spontaneously broken gauge theories, embodied by the Standard Model of electroweak interactions [1], face two major issues:

* anomalies [2]: gauge anomalies are an obstacle to renormalizability and are required to cancel between quarks and leptons [3]; chiral anomalies yield however the correct decay of the neutral pion into two photons [4].

* the lack of prediction concerning their scalar sector.

Considering the latter as composite went along, up to now, with the introduction of another scale of interaction [5], but 'technicolour' theories run into serious problems [6]. This has led to the blockage that we are facing nowadays, worsened by the growing feeling that supersymmetry may have nothing to do with nature [7].

We present here an abelian dynamically broken gauge theory which provides the correct coupling of the (neutral) pion to leptons and, though being anomaly free, also yields its usual coupling to two gauge fields. The non-abelian case is included in the more general work [8]. We do not worry either about renormalizability and only mention the arguments in favour of it, which are developed in [8, 9, 10].

This work is completed by [11] where we show how the Standard Model of leptons can be reconciled with an anomaly-free purely vectorial theory, thus cutting the link between the hadronic and leptonic sectors [3].

2 The hadronic sector

We consider a $U(1)_L$ spontaneously broken gauge theory; the generator of the gauge group $G$ is

$$T_L = \frac{1 - \gamma_5}{2} T;$$

(1)

it acts on the gauge field $\sigma_\mu$ and on the $N$ fermions. $\Psi$ is the $N$-vector: $N$ is the number of 'flavours'. we embed $G$ into the chiral group $U(N)_L \times U(N)_R$ and consider $T$ as a $N \times N$ matrix.

When $T^2 = 1$

$$\Phi = (H, \varphi) = \frac{v}{\mu^3} (\nabla \Psi, -i \nabla \gamma_5 T \Psi)$$

(2)

is a 2-dimensional representation of the gauge group (this condition is generalized in the Standard Model to the existence of an associative algebra [8]). Both $H$ and $\varphi$ are real. $H$ will be called the Higgs boson by analogy. We take for example in the following $T = 1$, and, from the action of $G$ on the fermions, we then deduce

$$\begin{align*}
T_L \varphi &= iH, \\
T_L H &= -i \varphi.
\end{align*}$$

(3)

The gauge symmetry is spontaneously broken by $\langle H \rangle = v$, equivalent to $\langle \Psi \psi \rangle = \mu^3$. The gauge and chiral symmetry breaking are thus two aspects of the same phenomenon. We write as usual

$$H = v + h.$$  

(4)
The Lagrangian is chosen as
\[ L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\Psi} \gamma^\mu (\partial_\mu - ig \sigma_\mu T_L) \Psi + \frac{1}{2} (D_\mu H D^\mu H + D_\mu \varphi D^\mu \varphi) - V(H^2 + \varphi^2) - \frac{\partial_\mu \xi}{v} \bar{\Psi} \gamma^\mu T_L \Psi, \] (5)
where the real field
\[ \xi = -\varphi (1 - \frac{h}{v}) + \cdots \] (6)
is defined, together with
\[ \tilde{H} = v + \eta \] (7)
by
\[ \tilde{H} = e^{-i \frac{\xi}{v} T_L} (H + i \varphi) \] (8)
When
\[ \Psi \to e^{-i \vartheta T_L} \Psi, \] (9)
it transforms, like a $U(1)$ Wess-Zumino [12] field, by
\[ \xi \to \xi - \vartheta v \] (10)
while $\tilde{H}$ stays invariant. One has $\tilde{H}^2 = H^2 + \varphi^2$. $L$ in equation (5) differs from the ‘standard’ Lagrangian by the additional coupling
\[ - \frac{\partial_\mu \xi}{v} \bar{\Psi} \gamma^\mu T_L \Psi; \] (11)
its presence is however the natural consequence of taking $L$ as a function of the ‘generic’ gauge field $\sigma_\mu - (1/g) \partial_\mu \xi/v$ instead of $\sigma_\mu$ alone; indeed, $L$ can also be written (see for example [13])
\[ L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\Psi} \gamma^\mu \left( \partial_\mu - ig \left( \sigma_\mu - \frac{1}{g} \frac{1}{v} \partial_\mu \xi \right) T_L \right) \Psi \]
\[ + \frac{1}{2} (\partial_\mu \tilde{H})^2 + \frac{1}{2} g^2 \left( \sigma_\mu - \frac{1}{g} \frac{1}{v} \partial_\mu \xi \right)^2 \tilde{H}^2 - V(\tilde{H}^2). \] (12)
This has been advocated in [14] to lead to the recovery of gauge invariance for anomalous gauge theories, and thus to be the right procedure of quantization. For more comments in this precise case of a composite Wess-Zumino field, we also refer the reader to [8, 9, 10].

We quantize the theory by the functional integral formalism. The scalars and the fermions are not independent degrees of freedom; so, to integrate on both, we include in the generating functional constraints
\[ \prod_x \delta \left( H - \frac{1}{\mu v} \bar{\Psi} \Psi \right)(x), \]
\[ \prod_x \delta \left( \varphi + i \frac{1}{\mu v} \bar{\Psi} \gamma_5 T_L \Psi \right)(x), \] (13)
that we exponentiate into the effective Lagrangian
\[ L_c = \lim_{\beta \to 0} -\frac{A^2}{2\beta} \left( H^2 + \varphi^2 - 2 \frac{v}{\mu^3} (H \bar{\Psi} \Psi - i \varphi \bar{\Psi} \gamma_5 T_L \Psi) + \frac{v^2}{\mu^6} \left( (\bar{\Psi} \Psi)^2 - (\bar{\Psi} \gamma_5 T_L \Psi)^2 \right) \right). \] (14)
Λ is an arbitrary mass scale. We thus define the theory by
\[ Z = \int D\Psi D\overline{\Psi} D\phi D\sigma \mu e^{i \int d^4x (\mathcal{L}(x) + \mathcal{L}_c(x))}, \]  
(15)
eventually adding a gauge fixing term. (Remark: the integration over \( \tilde{H} \) and \( \xi \) may be preferred to that over \( H \) and \( \phi \), since \( D\xi \) can be interpreted as the measure over the gauge group \([14]\); they only differ by a \( \xi \)-independent Jacobian \([9]\)).

\( \mathcal{L}_c \) introduces
- an infinite bare fermion mass, appearing when \( \langle H \rangle = v \):
\[ m_0 = -\frac{\Lambda^2 v^2}{\beta \mu^3}; \]  
(16)
- infinite 4-fermions couplings
\[ \zeta_0 = -\frac{\zeta_5^0}{2\mu^3} \]  
(17)
At the classical level, the infinite fermion mass in \( \mathcal{L}_c \) is cancelled by the 4-fermions term \( \propto (\overline{\Psi}\Psi)^2 \) when \( \langle \overline{\Psi}\Psi \rangle = \mu^3 \); however, staying in the ‘Nambu-Jona-Lasinio approximation’ \([15]\), equivalent to keeping only diagrams leading in an expansion in powers of \( 1/N \), the fermion mass and the effective 4-fermions coupling satisfy the two coupled equations
\[ \zeta(q^2) = \frac{\zeta_0}{1 - \zeta_0 A(q^2, m)}, \]
\[ m = m_0 - 2\zeta(0)\mu^3, \]  
(18)
graphically depicted in fig. 1 and fig. 2. \( A(q^2, m) \) is the one-loop fermionic bubble. The above cancellation represents only the first two terms of the series depicted in fig. 2.

\[ \zeta(0) \propto -A(0, m)^{-1}, \]  
and \( A \) involves a term proportional to \( m^2 \) (see

\[ \mu^3 \] being finite, \( m = m_0 \) is a solution of the equations above as soon as \( \zeta(0) \) goes to 0. This is the case here since \( \zeta(0) \propto -A(0, m)^{-1} \), and \( A \) involves a term proportional to \( m^2 \) (see
for example \[16\]). (The presence of eventual other solutions is beyond our reach because
it requires knowing exactly \(A(q^2, m)\)). This also makes the effective 4-fermions coupling
\(\zeta(q^2)\) (and similarly \(\zeta^5(q^2)\)) go to 0 like \(\beta^2\).

The fact that the fermions have an infinite mass, in addition to making them unobservable as asymptotic states (see \[9, 8\]) makes the theory anomaly-free. The Pauli-Villars
regularization of the triangular diagram, which gives the (covariant) anomaly, writes, \(M\) being the mass of the regulator (see fig. 3)

\[ k^\mu \left( T_{\mu\nu\rho}(m) - T_{\mu\nu\rho}(M) \right) = mT_{\nu\rho}(m) - MT_{\nu\rho}(M). \] (19)

We have

\[ \lim_{M \to \infty} MT_{\nu\rho}(M) = -A(g, \sigma_\mu), \] (20)

where \(A(g, \sigma_\mu)\) is the anomaly; so, when \(m \to \infty\), the Ward Identity \[19\] now shows that
the anomaly gets cancelled.

![Fig.3: Triangular diagrams involved in the anomalous Ward Identity](image)

The divergence of the fermionic current also receives a contribution from \(L_c\) eq. \[14\]:

\[ \partial_\mu J^\mu_{\psi c} = i \frac{\Lambda_\psi^2}{\beta_\mu^3} (H\overline{\Psi}\gamma_5 T\Psi - i \varphi \overline{\Psi}\Psi) ; \] (21)

it can however be consistently taken as vanishing when the constraints hold.

The conservation of the current appearing in the derivative coupling \[11\] and the vanishing
of the effective 4-fermions coupling constants are arguments in favour of the renormaliz-
ability of the theory; they are developed in \[8, 9, 10\].

3 Leptonic coupling of the pseudoscalar meson

The S-matrix element linking a pseudoscalar meson to two leptons is usually computed by
using an effective current \(\times\) current Fermi interaction, saturating by the vacuum, and
using the so-called ‘Partially Conserved Axial Current’ (PCAC) approximation (see for
example \[17\]). We show here that this PCAC contribution gets exactly cancelled and that
the only contribution to \(\varphi\) into leptons comes from the leptonic equivalent of the coupling
\(11\). Finally we show that a rescaling of the fields allows to identify it with the leptonic
coupling of a usual (neutral) pion.
We introduce the leptonic multiplet $\Psi_\ell$. There are $N_\ell$ leptonic flavours. We take here for granted the $V - A$ structure of the weak leptonic currents. Its origin is studied in [11].

The leptonic Lagrangian is written

$$L_\ell = i \overline{\Psi}_\ell \gamma^\mu \left( \partial_\mu - ig \frac{\sigma_\mu}{2} \right) T_L \Psi_\ell,$$

(22)

where $T_L$ is now an $N_\ell \times N_\ell$ unit matrix. We call $L_\mu$ and $H_\mu$ respectively the leptonic and hadronic currents

$$H_\mu = \overline{\Psi}_\mu T_L \Psi,$$

$$L_\mu = \overline{\Psi}_\ell \gamma^\mu T_L \Psi_\ell.$$  
(23)

The equation for $\varphi$ is

$$D^2 \varphi = -\frac{1}{v} \partial_\mu (L^{\mu} + H^{\mu}) \left( 1 - \frac{h}{v} \right) + \cdots,$$

(24)

giving, neglecting corrections of order $h/v$,

$$\langle 0 | \partial_\mu (L^{\mu} + H^{\mu})(0) | \varphi(k) \rangle_{in} = -v \langle 0 | \partial^2 \varphi + g v \partial_\mu \sigma^\mu + 2g \sigma_\mu \partial^{\mu} H - g^2 \varphi \sigma_\mu^2 (0) | \varphi(k) \rangle_{in}. $$

(25)

The $\partial^2 \varphi$ term yields the customary ‘PCAC’ equation

$$\langle 0 | \partial_\mu (L^{\mu} + H^{\mu})(0) | \varphi(k) \rangle_{in} = v k^2;$$

it is however cancelled by the contribution of $g v \partial_\mu \sigma^\mu$, due to the coupling $g v \sigma_\mu \partial^{\mu} \varphi$ occurring in $1/2D_\mu \varphi D^{\mu} \varphi$, when, in the low energy regime, one takes the propagator of $\sigma_\mu$ as $ig_{\mu\nu}/M_\sigma^2$ ($M_\sigma$ is the mass of the gauge field). This low energy approximation is precisely that used in the usual ‘PCAC’ computation of the S-matrix element for the leptonic decay of a pseudoscalar meson, symbolically depicted in fig. 4; the ‘bubble’ stands for the propagator of the current $L_\mu + H_\mu$, linked by PCAC to that of the fermionic bound state, and the dot with the effective Fermi interaction.

\[Fig. 4: The PCAC contribution to the decay of $\phi$ into leptons\]

We just found that it is now exactly cancelled by the diagram of fig. 5.

\[Fig. 5: Diagram cancelling the PCAC contribution to the decay of $\phi$ into leptons\]
Would the only derivative coupling of $\varphi$ be to the hadronic current, the same cancellation as above would hold, with the ‘bubble’ of fig. 4 now standing for the propagation of $H_\mu$ only. We conclude that, all other contributions cancelling, the disintegrations of $\varphi$ into leptons are mediated by the direct coupling $(\xi/v) \partial_\mu L^\mu$ in (22).

We rescale the fields by

$$
\begin{align*}
\varphi & = a\pi, \\
H & = aH', \\
\Psi & = a\Psi', \\
\Psi_\ell & = a\Psi_\ell', \\
\sigma_\mu & = a a_\mu, \\
g & = e/a.
\end{align*}
$$

(27)

After a global rescaling by $1/a^2$, the Lagrangian rewrites (we do not mention any longer the scalar potential completely ‘screened’ by the exponentiated constraints)

$$
\begin{align*}
\frac{1}{a^2}(L + L_\ell) &= -\frac{i}{4} f_{\mu\nu} f^{\mu\nu} \\
&+ i \overline{\Psi} \gamma^\mu (\partial_\mu - ie a_\mu T_L) \Psi' + i \overline{\Psi_\ell} \gamma^\mu (\partial_\mu - ie a_\mu T_L) \Psi'_\ell \\
&+ \frac{i}{2} \left((\partial_\mu H' - e a_\mu \pi)^2 + (\partial_\mu \pi + e a_\mu H')^2\right) \\
&- \frac{2}{v} \overline{\Psi_\ell} \gamma_\mu T_L \Psi'_\ell \partial^\mu \pi,
\end{align*}
$$

(28)

where

$$
f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu.
$$

(29)

We have

$$
\langle H' \rangle = \frac{v}{a}, \quad \langle \overline{\Psi'} \Psi' \rangle = \frac{\mu^3}{a^2},
$$

(30)

and

$$
e^2 \langle H'' \rangle^2 = g^2 \langle H \rangle^2,
$$

(31)

yielding the same mass $M_\sigma$ for $a_\mu$ and $\sigma_\mu$. We call now $a_\mu$ “vector boson”, $\pi$ “pion”, identify the ‘primed’ leptons with the observed ones and $e$ with the coupling constant of the theory. Then, in $(L + L_\ell)/a^2$, the term

$$
- \frac{a}{v} \overline{\Psi_\ell} \gamma_\mu T_L \Psi'_\ell \partial^\mu \pi
$$

(32)

rebuilds the correct S-matrix element for the decay of the pion into two leptons if we take

$$
a = \frac{f_\pi}{v}.
$$

(33)

4 The coupling of the pseudoscalar meson to two gauge fields

The $\varphi$ into two $\sigma_\mu$’s transitions are triggered by the coupling of $L_c$

$$
\frac{i\varphi}{v} m \overline{\Psi} \gamma_5 T \Psi.
$$

(34)
Indeed, the quantum contribution to $m\bar{\Psi}\gamma_5 T\Psi$ from the triangle precisely yields, as described in (19) above, $-i \times$ the anomaly, such that contributes at the one-loop level

$$\frac{\varphi}{v} A(g, \sigma_\mu).$$  \hspace{1cm} (35)

Now, after the rescaling (27), (35) describes the customary ‘anomalous’ coupling of a neutral pion to two gauge fields: indeed, we have

$$A(g, \sigma_\mu) = A(e, a_\mu);$$ \hspace{1cm} (36)

consequently, in $\mathcal{L}_c/a^2$, (35) becomes

$$\frac{1}{a v \pi} A(e, a_\mu) = \frac{1}{f_\pi} A(e, a_\mu).$$ \hspace{1cm} (37)

Despite the absence of anomaly, it has been rebuilt from the constraints and the infinite fermion mass that they yield. The fact that the ‘photon’ is massive is only a formal issue; it is shown in [8] that we indeed recover the pion decay into two massless photons when they are present.

## 5 Conclusion; perspectives

We emphasized in this work two phenomenological aspects of our model: we showed that predicting the leptonic couplings of the pion is compatible with dynamical symmetry breaking and does not necessitate the introduction of a new scale of interaction nor that of very massive particles; we also showed that it couples to two gauge fields despite the absence of anomaly in the hadronic sector with the same strength as usual. The scaling factor $a = f_\pi/v$ allows the recovery of both in a model where they were a priori not expected. This can be traced into the presence of the coupling (11), dictated by gauge invariance [14], and into the breaking of the symmetry allowing the presence of a field $\xi$ with the dimension of a mass and of a coupling constant $1/v$ with dimension mass$^{-1}$. We recall that the leptonic sector has been reconciled in [11] with a purely vectorial theory and is also itself anomaly-free. The hadronic and leptonic sectors can consequently be disconnected. The issues of gauge invariance and unitarity, linked in particular to the introduction of a derivative coupling between the scalars and the fermionic current are studied in [9, 8]. We show there how the pion can be gauged into the third polarization of the massive gauge field. The question of the renormalizability is also developed and arguments are given in favour of it; a demonstration at all orders, going beyond the ‘Nambu-Jona-Lasinio approximation’ requires a careful study of how the BRS symmetry [18] is implemented in this precise case. This is currently under investigation [10].

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