Statefinder diagnostic of logarithmic entropy corrected holographic dark energy with Granda-Oliveros IR cut-off

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Abstract In this work, we have studied the logarithmic entropy corrected holographic dark energy (LECHDE) model with Granda-Oliveros (G-O) IR cut-off. The evolution of dark energy (DE) density $\Omega_D$, the deceleration parameter, $q$, and equation of state parameter (EoS), $\omega_{\Lambda}$, are calculated. We show that the phantom divide may be crossed by choosing proper model parameters, even in absence of any interaction between dark energy and dark matter. By studying the statefinder diagnostic and $\omega_{\Lambda} - \omega_{\Lambda}'$ analysis, the pair parameters $\{r, s\}$ and $(\omega_{\Lambda} - \omega_{\Lambda}')$ is calculated for flat GO-LECHDE universe. At present time, the pair $\{r, s\}$ can mimic the $\Lambda$CDM scenario for a value of $\alpha/\beta \simeq 0.87$, which is lower than the corresponding one for observational data $(\alpha/\beta = 1.76)$ and for Ricci scale $(\alpha/\beta = 2)$. We find that at present, by taking the various values of $(\alpha/\beta)$, the different points in $r - s$ and $(\omega_{\Lambda} - \omega_{\Lambda}')$ plans are given. Moreover, in the limiting case for a flat dark dominated universe at infinity ($t \to \infty$), we calculate $\{r, s\}$ at G-O scale. For Ricci scale $(\alpha = 2, \beta = 1)$ we obtain $\{r = 0, s = 2/3\}$.

1 Introduction

It is widely accepted among cosmologists and astrophysicists that our universe is experiencing an accelerated expansion. The evidences of this accelerated expansion are given by numerous and complementary cosmological observations, like the SNIa (Perlmutter et al., 1999; Astier et al., 2006), the CMB anisotropy, observed mainly by WMAP (Wilkinson Microwave Anisotropy Probe) (Bennett et al., 2003; Spergel et al., 2003), the Large Scale Structure (LSS) (Tegmark et al., 2004; Abazajian et al., 2004, 2005) and X-ray (Allen et al., 2004) experiments.

In the framework of standard Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmology, a missing energy component with negative pressure (known as Dark Energy (DE)) is the source of this expansion. Careful analysis of cosmological observations, in particular of WMAP data (Bennett et al., 2003; Spergel et al., 2003; Peiris et al., 2003) indicates that almost 70 percent of the total energy of the universe is occupied by DE, whereas DM occupies almost the rest (the baryonic matter represents only a few percent of the total energy density). The contribution of the radiation is practically negligible.

The nature of DE is still unknown and many candidates have been proposed in order to describe it (see Copeland et al., 2006; Padmanabhan, 2003; Peebles, & Ratra, 2003 and references therein for good reviews).

The time-independent cosmological constant $\Lambda$ with equation of state (EoS) parameter $\omega = -1$ is the earliest and simplest DE candidate. However, cosmologists know that $\Lambda$ suffers from two main difficulties: the fine-tuning and the cosmic coincidence problems (Copeland et al., 2006). The former asks why the vacuum energy density is so small (about $10^{-123}$ times smaller than what we observe) (Weinberg, 1988) and...
the latter says why vacuum energy and DM are nearly equal today (which represents an incredible coincidence if no internal connections between them are present). Alternative candidates for DE problem are the dynamical DE scenarios with no longer constant but time-varying $\omega$. It has been shown by observational data analysis of SNe-Ia that the time-varying DE models give a better fit compared with a cosmological constant. A good review about the problem of DE, including a survey of some theoretical models, can be found in (Li et al., 2011).

An important advance in the study of black hole theory and string theory is the suggestion of the so called holographic principle: according to it, the number of degrees of freedom of a physical system should be finite, it should scale with its bounding area rather than with its volume (’t Hooft, 1993) and it should be constrained by an infrared cut-off (Cohen et al., 1999). The Holographic DE (HDE), based on the holographic principle proposed by (Fischler & Susskind, 1998), is one of the most interesting DE candidates and it has been widely studied in literature (Enqvist et al., 2005; Shen et al., 2003; Zhang & Wu, 2005; Zhang, 2006; Shevkhi, 2010; Huang & Li, 2004; Hsu, 2004; Guberina et al., 2002; 2006; Gong, 2004; Elizalde et al., 2005; Jamil & Farooq, 2010a, 2010b, 2011; Karami et al., 2011; Setare & Jamil, 2010a, 2010b; Shevkhi et al., 2012; Jamil & Farooq, 2010b; Pasqua et al., 2012; Setare, 2006; 2007a, b; Setare & Vagenas, 2009; Setare & Jamil, 2010b, 2011; Khodam-Mohammadi & Malekjani, 2009).

Applying the holographic principle to cosmology, the upper bound of the entropy contained in the universe can be obtained (Fischler & Susskind, 1998). Following this line, Li (2004) suggested the following constraint on the energy density:

$$\rho_A \leq 3c^2M_p^2L^{-2},$$

where $c$ is a numerical constant, $L$ indicates the IR cut-off radius, $M_p = (8\pi G)^{-1/2} \approx 10^{18}$ GeV is the reduced Planck mass ($G$ is the gravitational constant) and the equality sign holds only when the holographic bound is saturated. Obviously, in the derivation of HDE, the black hole entropy (denoted with $S_{BH}$) plays an important role. As it is well known, $S_{BH} = A/(4G)$, where $A \approx L^2$ is the area of the horizon. However, this entropy-area relation can be modified as (Banerjee & Majhi, 2008a, b; Banerjee & Modak, 2009):

$$S_{BH} = A/4G + \alpha \log \left( A/4G \right) + \beta,$$

where $\alpha$ and $\beta$ are dimensionless constants. These corrections can appear in the black hole entropy in Loop Quantum Gravity (LQG). They can also be due to quantum fluctuation, thermal equilibrium fluctuation or mass and charge fluctuations. The quantum corrections provided to the entropy-area relationship leads to curvature correction in the Einstein-Hilbert action and viceversa (Cai et al., 2005; Nojiri & Odintsov, 2001; Zhu & Ren, 2009). Using the corrected entropy-area relation given in Eq. (2), the energy density $\rho_A$ of the logarithmic entropy-corrected HDE (LECHDE) can be written as (Wei, 2009):

$$\rho_A = 3\alpha M_p^2L^{-2} + \gamma_1L^{-4} \log \left( M_p^2L^2 \right) + \gamma_2L^{-4},$$

where $\gamma_1$ and $\gamma_2$ are two dimensionless constants. In the limiting case of $\gamma_1 = \gamma_2 = 0$, Eq. (3) yields the well-known HDE density.

The second and the third terms in Eq. (3) are due to entropy corrections: since they can be comparable to the first term only when $L$ is very small, the corrections produce make sense only at the early evolutionary stage of the universe. When the universe becomes large, they reduce to the ordinary HDE.

It is worthwhile to mention that the IR cut-off $L$ plays an important role in HDE model. By assuming particle horizon as IR cut-off, the accelerated expansion can not be achieved (Hsu, 2008), while for Hubble scale, event horizon, apparent horizon and Ricci scale, this fact may be achieved (Shevkhi, 2010; Duran & Pavon, 2011; Nojiri & Odintsov, 2006; Pavon & Zimdahl, 2003; Zimdahl & Pavon, 2007).

Recently, Granda and Oliveros (G-O), proposed a new IR cut-off for HDE model, namely ‘new holographic DE’, which includes a term proportional to $H$, and one proportional to $H^2$ (Granda & Oliveros, 2009, 2008). Despite of the HDE based on the event horizon, this model depends on local quantities, avoiding in this way the causality problem.

The investigation of cosmological quantities such as the EoS parameter $\omega_A$, deceleration parameter $q$ and statefinder diagnosis have attracted a great deal of attention in new cosmology. Since the various DE models give $H > 0$ and $q < 0$ at the present time, the Hubble and deceleration parameters can not discriminate various DE models. A higher order of time derivative of scale factor is then required. Sahni
et al. [Sahni & Shtanov, 2003] and Alam et al. [Alam et al., 2003], using the third time derivative of scale factor $a(t)$, introduced the statefinder pair \{r,s\} in order to remove the degeneracy of $H$ and $q$ at the present time. The statefinder pair is given by:

$$r = \frac{\dddot{a}}{aH^3},$$
$$s = \frac{r - 1}{3(q - 1/2)}.$$

Many authors have been studied the properties of various DE models from the viewpoint of statefinder diagnostic [Khodam-Mohammadi, & Malekjani, 2011b; Malekjani, & Khodam-Mohammadi, 2010; Malekjani et al. 2011a,b; Malekjani, & Khodam-Mohammadi, 2012, 2013].

This paper is organized as follows. In Section 2, we describe the physical contest we are working in and we derive the EoS parameter $\omega_L$, the deceleration parameter $q$ and $\Omega_L$ for GO-LECHDE model. In Section 3, the statefinder diagnosis and $\omega - \omega t$ analysis of this model are investigated. We finished our work with some concluding remarks.

2 cosmological properties

The energy density of GO-LECHDE in Planck mass unit (i.e. $M_P = 1$) is given by

$$\rho_L = \frac{3}{L_{GO}^2} \left[ 1 + \frac{1}{3} L_{GO}^2 (2\gamma_1 \log L_{GO} + \gamma_2) \right] = \frac{3}{L_{GO}^2} \Gamma,$$

where we defined $\Gamma = 1 + \frac{1}{3} L_{GO}^2 (2\gamma_1 \log L_{GO} + \gamma_2)$ for simplicity. The Granda-Oliveros IR cutoff given by [Granda, & Oliveros, 2009; Khodam-Mohammadi, 2011];

$$L_{GO} = \left( \alpha H^2 + \beta \dot{H} \right)^{-1/2},$$

where $\alpha$ and $\beta$ are two constant.

The line element of FLRW universe is given by:

$$ds^2 = dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 \right),$$

where $t$ is the cosmic time, $a(t)$ is a dimensionless scale factor (which is function of the cosmic time $t$), $r$ is referred to the radial component, $k$ is the curvature parameter which can assume the values $-1$, $0$ and $+1$ which yield, respectively, a closed, a flat or an open FLRW universe and $(\theta, \varphi)$ are the angular coordinates.

The Friedmann equation for non-flat universe dominated by DE and DM has the form:

$$H^2 + \frac{k}{a^2} = \frac{1}{3} (\rho_L + \rho_m),$$

where $\rho_L$ and $\rho_m$ are, respectively, the energy densities of DE and DM.

We also define the fractional energy densities for DM, curvature and DE, respectively, as:

$$\Omega_m = \frac{\rho_m}{\rho_c},$$
$$\Omega_k = \frac{\rho_k}{\rho_c},$$
$$\Omega_L = \frac{\rho_L}{\rho_c}.$$

where $\rho_c = 3H^2$ represents the critical energy density. Recent observations reveal that $\Omega_k \approx 0.02$ [Spergel et al., 2007], which support a closed universe with a small positive curvature.

Using the Friedmann equation given in Eq. [9], Eqs. [10], [11] and [12] yield:

$$1 + \Omega_k = \Omega_m + \Omega_L.$$

In order to preserve the Bianchi identity or the local energy-momentum conservation law, i.e. $\nabla_{\mu} T^{\mu\nu} = 0$, the total energy density $\rho_{tot} = \rho_L + \rho_m$ must satisfy the following relation:

$$\dot{\rho}_{tot} + 3H (1 + \omega_{tot}) \rho_{tot} = 0,$$

where $\omega_{tot} = \rho_{tot}/\rho_{tot}$ represents the total EoS parameter. In an non-interacting scenario of DE-DM, the energy densities of DE and DM $\rho_L$ and $\rho_m$ are preserved separately and the equations of conservation assume the following form:

$$\dot{\rho}_L + 3H \rho_L (1 + \omega_L) = 0,$$
$$\dot{\rho}_m + 3H \rho_m = 0.$$

The derivative with respect to the cosmic time $t$ of $L_{GO}$ is given by:

$$L_{GO} = -H^3 L_{GO}^3 \left( \frac{\dot{H}}{H^2} + \beta \dot{\dot{H}} \right).$$

Using Eq. [17], the derivative with respect to the cosmic time $t$ of the energy density $\rho_L$ given in Eq. [2] can be written as:

$$\dot{\rho}_L = 6H^3 \left( \frac{\dot{H}}{H^2} + \beta \dot{\dot{H}} \right) \times$$
$$\left\{ 1 + \frac{1}{3} L_{GO}^{-2} (2\gamma_1 (4 \log L - 1) + 2\gamma_2) \right\}.$$
Differentiating the Friedmann equation given in Eq. (9) with respect to the cosmic time \( t \) and using Eqs. (12), (13), (16) and (18), we can write the term \( \frac{\dot{H}}{H^2} + \frac{\ddot{H}}{2H^3} \) as:

\[
\alpha \frac{\dot{H}}{H^2} + \beta \frac{\ddot{H}}{2H^3} = \frac{1 + \frac{\ddot{H}}{2H^3} + \left( \frac{2}{3} - 1 \right) \Omega_L}{\left( 1 + \frac{1}{3}L_{GO}^2 \left[ \gamma_1 \left( 4 \log L_{GO} - 1 \right) + 2 \gamma_2 \right] \right)}
\]

where \( u = \rho_m/\rho_\Lambda = \Omega_m/\Omega_\Lambda = (1 + \Omega_k)/\Omega_\Lambda - 1 \) is the ratio of energy densities of DM and DE. Using the expression of \( L_{GO} \) given in Eq. (8) and the energy density of DE given in Eq. (3), we obtain that the \( \dot{\Omega}_m \) can be written as:

\[
\dot{\Omega}_m = \frac{6H^3\Omega_m}{\beta} \left( \frac{1}{\Gamma} - \frac{\alpha - \beta}{\Omega_\Lambda} + \frac{\beta (u - 2)}{2} \right)
\]

Differentiating the expression of \( \Omega_\Lambda \) given in Eq. (12) with respect to the cosmic time \( t \) and using the relation \( \Omega_\Lambda = H^2/\Omega_L' \), we obtain the evolution of the energy density parameter as follow:

\[
\Omega_L' = \frac{2\Omega_L}{\beta} \left( \frac{1}{\Gamma} - \frac{\alpha - \beta}{\Omega_\Lambda} + \frac{\beta u}{2} \right)
\]

The dot and the prime denote, respectively, the derivative with respect to the cosmic time \( t \) and the derivative with respect to \( x = \ln a \).

Finally, using Eqs. (12), (13) and (21), the EoS parameter \( \omega_\Lambda \) and the deceleration parameter (defined as \( q = -1 - \frac{\ddot{H}}{H^2} \)) as functions of \( \Omega_\Lambda \) and \( \Gamma \) are given, respectively, by:

\[
\omega_\Lambda = -\frac{2}{3\Omega_L} \left[ 1 - \frac{\alpha}{\beta} + \frac{\Omega_\Lambda}{\beta \Gamma} \right] - \frac{1 + u}{3},
\]

\[
q = \left( \frac{\alpha}{\beta} - 1 - \frac{\Omega_\Lambda}{\beta \Gamma} \right).
\]

We can easily observe that the EoS parameter \( \omega_\Lambda \) and the deceleration parameter \( q \) given, respectively, in Eqs. (23) and (24) are related each other by the following relation:

\[
\omega_\Lambda = \frac{2}{3\Omega_L} q - \frac{1 + u}{3}.
\]

Moreover, using Eqs. (12) and (24), we can derive that:

\[
L_{GO}^{-2} H_{GO}^{-2} = \frac{\Omega_\Lambda}{\Gamma} = \alpha - \beta - \beta q = \alpha - \beta (1 + q).
\]

From Eqs. (15) and (16), the evolution of \( u \) is governed by:

\[
u' = 3u\omega_\Lambda.
\]

At Ricci scale, i.e. when \( \alpha = 2 \) and \( \beta = 1 \), Eqs. (23) and (24) reduce, respectively, to:

\[
\omega_\Lambda = -\frac{2}{3\Omega_\Lambda} \left[ \frac{\Omega_\Lambda}{\Gamma} - 1 \right] - \frac{1 + u}{3},
\]

\[
q = 1 - \frac{\Omega_\Lambda}{\Gamma},
\]

and the evolution of the energy density parameter given in Eq. (22) reduces to:

\[
\Omega_L' = \left( 2 - \frac{2}{3\Omega_\Lambda} \left( \frac{\Omega_\Lambda}{\Gamma} - 1 \right) \right) + u\Omega_\Lambda = -\Omega_\Lambda (1 + 3\omega_\Lambda).
\]

By choosing the proper model parameters, it can be easily shown that the equation of state parameter \( \omega_\Lambda \) given in Eqs. (23) and (24) may cross the phantom divide. Moreover, from Eqs. (23) and (24), we can see that the transition between deceleration to acceleration phase can be happened for various model parameters. In a flat dark dominated universe, i.e. when \( \gamma_1 = \gamma_2 = 0 \) or at infinity \((t \to \infty)\), \( \Omega_\Lambda = 1 \), \( \Omega_k = 0 \) and \( u = 0 \), we find that the Hubble parameter \( H \) reduces to:

\[
H = \beta \left( \frac{1}{t} \right).
\]

Moreover, the EoS parameter \( \omega_\Lambda \) and the deceleration parameter \( q \) given in Eqs. (23) and (24) reduce, respectively, to:

\[
\omega_{\Lambda}^\infty = -\frac{2}{3} \left( \frac{1 - \alpha}{\beta} \right) - 1,
\]

\[
q_{\Lambda}^\infty = \frac{\alpha - 1}{\beta} - 1.
\]

Also in this case the phantom wall can be achieved for \( \alpha < 1 \), \( \beta > 0 \). In Ricci scale in this limit, Eqs. (32) and (33) reduce to

\[
\omega_{\Lambda}^\infty = -\frac{1}{3}, \quad q_{\Lambda}^\infty = 0,
\]

which corresponds to an expanding universe without any acceleration.

3 Statefinder diagnostic

We now want to derive the statefinder parameters \( \{ r, s \} \) for GO-LECHDE model in the flat universe.
The Friedmann equation given in Eq. (9) yields, after some calculations:

\[ \frac{\dot{H}}{H^2} = -\frac{3}{2} (1 + \omega_A \Omega_L). \]  

(35)

Taking the time derivation of Eq. (33) and using Eq. (22), we obtain:

\[ \frac{\ddot{H}}{H^3} = \frac{9}{2} \left[ 1 + \omega_A^2 \Omega_L (1 + \Omega) + \frac{7}{3} \omega_A \Omega_L - \frac{1}{3} \omega_A' \Omega_L \right]. \]  

(36)

Using the definition of \( H \) (i.e. \( H = \dot{a}/a \)), the statefinder parameter \( r \) given in Eq. (4) can be written as:

\[ r = 1 + 2 \frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H}. \]  

(37)

Substituting Eqs (20), (24) and (36) in Eqs. (37) and (38), we can write:

\[ \begin{align*}
    r & = 1 + 6 \omega_A \Omega_L + \frac{9}{2} \omega_A^2 \Omega_L (1 + \Omega) - \frac{3}{2} \omega_A' \Omega_L, \quad \text{(38)} \\
    s & = \beta \Gamma \Omega_L \left[ \frac{4 \omega_A' + 3 \omega_A^2 (1 + \Omega) - \omega_A'}{\Gamma (2 \alpha - 3 \beta) - 2 \Omega_L} \right]. \quad \text{(39)}
\end{align*} \]

At early time, when \( \omega_A \to 0 \), the pair relations (38) show that statefinder parameters tends to \( \{ r = 1, s = 0 \} \), which coincides with the location of the \( \Lambda \)CDM fixed point in \( r - s \) plane.

Using Eq. (23), the evolution of EoS parameter \( \omega_A \) can be written as:

\[ \begin{align*}
    \omega_A' & = \frac{2 \Omega_L}{3 \beta \Omega_L^2} \left[ \frac{3}{2} \beta - \alpha \right] \\
    & + \frac{4}{3 \beta} \left[ \frac{L_{GO}'}{L_{GO}} \right] \left( 1 + \frac{2 \gamma}{3 L_{GO}^2} - \Gamma \right), \quad \text{(40)}
\end{align*} \]

where from Eqs. (12) and (17), the term \( \left( \frac{L_{GO}'}{L_{GO}} \right) \) can be calculated as:

\[ \begin{align*}
    \frac{L_{GO}'}{L_{GO}} & = -\frac{\Gamma}{\Omega_L} \left( \frac{\dot{H}}{H^2} + \beta \frac{\ddot{H}}{H^3} \right) \\
    & = \frac{3 \Gamma}{2} \left[ \frac{1 + \omega_A}{1 + \frac{1}{3} L_{GO}^{-2} (4 \log L_{GO} - 1) + 2 \gamma} \right]. \quad \text{(41)}
\end{align*} \]

At present epoch of the Universe (\( \Omega_L \approx 0.72, u \approx 0.4 \)), the EoS parameter \( \omega_A \) given in Eq. (25) reduces to:

\[ \omega_A \approx 0.93 q - 0.47. \]  

(42)

Then, the universe exists in accelerating phase (i.e \( q < 0 \) if \( \omega_A < -0.47 \) and the phantom divide \( \omega_A = -1 \), may be crossed provided that \( q \lesssim -0.5 \). This condition implies \( \frac{H}{\Omega} \gtrsim -0.58 \) and, from Eq. (20), we derive:

\[ L_{GO}^2 - 2 \Omega_0 H_0^2 \approx - \alpha - 0.42 \beta, \]  

(43)

\[ \Omega_0 \beta \approx \frac{\alpha}{\beta} - 0.42. \]  

(44)

By inserting the above quantities in Eqs. (22) and (40), we have \( \omega_A \approx -1.86 (\alpha/\beta - 3/2) \), which gives:

\[ \begin{align*}
    r_0 & \approx 2 \left( \frac{\alpha}{\beta} \right) - 0.75, \quad \text{(45)} \\
    s_0 & \approx -0.62 \left( \frac{\alpha}{\beta} \right) + 0.54. \quad \text{(46)}
\end{align*} \]

Recently, Wang and Xu (Wang, & Xu, 2010) have constrained the new HDE model in non-flat universe using observational data. The best fit values of \( (\alpha/\beta) \) with their confidence level they found are \( \alpha = 0.8824_{-0.1163}^{+0.2180} (1 \sigma)_{-0.1375}^{+0.2213} (2 \sigma) \) and \( \beta = 0.5016_{-0.0871}^{+0.0955} (1 \sigma)_{-0.1102}^{+0.1247} (2 \sigma) \). Using these values, the pair parameters \( \{ r, s \} \), at present epoch, become \( \{ r = 2.77, s = -0.55 \} \), which are far from \( \Lambda \)CDM model values (i.e., \( \{ r = 1, s = 0 \} \)). Moreover, it shows that \( s < 0 \), which corresponds to a phantom-like DE. However, in order to mimic these parameters to \( \Lambda \)CDM scenario at present epoch, the ratio of \( \alpha/\beta \) must be approximately 0.87, which is lower than the value obtained with observational data.

At Ricci scale (i.e., when \( \alpha/\beta = 2 \)), at present time, pair parameters assume the values \( \{ r = 3.25, s = -0.70 \} \). It is worthwhile to mention that by increasing the value of \( \alpha/\beta \) from 0.87, the distance from \( \Lambda \)CDM fixed point in \( r - s \) diagram become longer.

In the limiting case of \( t \to \infty \) or for ordinary new HDE \( \gamma_1 = \gamma_2 = 0, \; \Gamma = 1 \), in flat dark dominated universe \( (u = 0, \Omega_L = 1) \), we find that:

\[ \begin{align*}
    r & = \frac{1}{\beta^2} (\alpha - \beta - 1) (2 \alpha - \beta - 4), \quad \text{(47)} \\
    s & = \frac{2 (2 \alpha^2 - 3 \beta \alpha + 5 \beta - 6 \alpha + 4)}{3 \beta (2 \alpha - 3 \beta - 2)}. \quad \text{(48)}
\end{align*} \]

At Ricci scale \( (\alpha = 2, \beta = 1) \), Eqs. (47) and (48) reduce, respectively, to:

\[ \begin{align*}
    r & = 0, \quad s = \frac{2}{3}. \quad \text{(49)}
\end{align*} \]

Moreover the \( \omega - \omega' \) analysis is another tool to distinguish between the different models of DE (Wei, & Cai, 2007). In this analysis the standard \( \Lambda \)CDM model corresponds to the fixed point \( \{ \omega_A = -1, \omega_A' = 0 \} \). At present time, for \( \alpha/\beta = 0.87 \) which corresponds
to $\Lambda$CDM fixed point in $r-s$ diagram, $(\omega_\Lambda = -1, \omega'_\Lambda = 1.17)$. For the observational quantities, $(\alpha/\beta = 1.76)$, we find: $(\omega_\Lambda = -1, \omega'_\Lambda = -0.48)$, and for Ricci scale these are $(\omega_\Lambda = -1, \omega'_\Lambda = -0.93)$. Therefore we see that $\omega'_\Lambda$ become smaller for higher value of $\alpha/\beta$ at present.

4 Conclusion

In this paper, we have extended the work made by Granda and Oliveros (Granda, & Oliveros, 2009) to the logarithmic entropy corrected HDE (LECHDE) model. This model has been arisen from the black hole entropy which may lie in the entanglement of quantum field between inside and outside of the horizon. We obtained the evolution of energy density $\Omega'_\Lambda$, the deceleration parameter $q$ and EoS parameter $\omega_\Lambda$ of the new LECHDE model for non-flat universe. We saw that, by choosing the proper model parameters, the equation of state parameter $\omega_\Lambda$ may cross the phantom divide and also the transition between deceleration to acceleration phase could happen.

At last, we studied the GO-LECHDE model from the viewpoint of statefinder diagnostic and $\omega_\Lambda - \omega'_\Lambda$ analysis, which is a crucial tool for discriminating different DE models. Also, the present value of $\{r, s\}$ can be viewed as a discriminator for testing different DE models if it can be extracted from precise observational data in a model-independent way. The studying at present time, when $\omega_\Lambda$ remains around the phantom wall, $\omega_\Lambda \approx -1$ and our universe evolves in acceleration phase, pair values of $\{r, s\}$ was calculated with respect to model parameters $\alpha, \beta$. By using the observational data which was obtained by Wang and Xu (Wang, & Xu, 2010), where $\alpha/\beta = 1.76$, we obtained $\{r = 2.77, s = -0.55\}$. For Ricci scale, which has $\alpha/\beta = 2$, the pair value assume the values $\{r = 3.25, s = -0.7\}$. Also, choosing $\alpha/\beta = 0.87$, we found $\{r = 1, s = 0\}$ which is corresponds to $\Lambda$CDM scenario. We shaw that increasing value of $\alpha/\beta$, conclude the ascending distance from $\Lambda$CDM fixed point. In the limiting case, at infinity, for flat dark dominated universe at Ricci scale, we found $\{r = 0, s = 2/3\}$, which corresponds to an expanding universe without any acceleration ($q = 0$). In $\omega_\Lambda - \omega'_\Lambda$ analysis at present time, we found that the higher value of $\alpha/\beta$ obtains the smaller value of $\omega'_\Lambda$.

In this model the statefinder pairs is determined by parameters $\alpha, \beta, \gamma_1, \gamma_2$. These parameters would be obtained by confronting with cosmic observational data. Giving the wide range of cosmological data available, in the future we expect to further constrain our model parameter and test the viability of our model.
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