Inherent Synchronization in Electric Power Systems with High Levels of Inverter-based Generation

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Abstract

The synchronized operation of power generators is the foundation of electric power system stability and key to the prevention of undesired power outages and blackouts. Here, we derive the condition that guarantees synchronization in electric power systems with high levels of inverter-based generation when subjected to small perturbations, and perform a parametric sensitivity to understand synchronization with varied types of generators. Contrary to the popular belief that achieving a stable synchronized state is tied chiefly to system inertia, our results instead highlight the critical role of generator damping in achieving this pivotal state. Additionally, we report the feasibility of operating interconnected electric grids with a 100% power contribution from renewable generation technologies with assured system synchronization. The findings of this paper can set the basis for the development of advanced control architectures and grid optimization methods and has the potential to further pave the path towards the decarbonization of the electric power sector.

Index Terms

Power system dynamic stability, Frequency response, Renewable energy, Synchronization

I. Introduction

DECARBONISATION of power systems is an ongoing global effort and is rapidly accelerating, incentivized by the continuous cost decline of energy production with variable renewable technologies such as solar photovoltaic and wind power plants that are now cost competitive with conventional generation in most locations [1]. Accordingly, it is anticipated that renewable power generation technologies will continue to dominate the new installed power generation capacity over the next two decades, spurring a transition to 100% renewable-based power systems with an entirely altered landscape for their planning, management, stability and control [2], [3].

This paper studies inherent synchronization in electric power systems with high levels of inverter-based generation, near 100%. This is a dynamic problem whose impetus is to understand whether a
system remains stable following a disturbance. Ensuring synchronization in electric power systems is pivotal because the lack thereof will lead to sustained spontaneous oscillations and a loss of stability after any disturbance [4]. Synchronization in a power system can be interpreted as a stable state when the pace of evolution of the electric angle of all generators across the system is identical; in a power system with \( n \) generators, it can be mathematically described by:

\[
\dot{\delta}_1 = \dot{\delta}_2 = \ldots = \dot{\delta}_n = \omega_{\text{sync}}
\]

(1)

where \( \delta_i \) represents the electric angle of the \( i \)-th generator and \( \omega_{\text{sync}} \) is the synchronized speed. The synchronized operation of generators guarantees that the flow of electric power across the system remains stable and yields a homogeneous frequency at all nodes; i.e., a system frequency \((f \approx (2\pi)^{-1}\omega_{\text{sync}})\). In real-world instances, this system frequency is continuously fluctuating within a small, stable equilibria as a result of numerous slight perturbations but can be approximated as a quasi-steady state system frequency due to the boundedness of the deviations. Instabilities have been observed in power systems’ synchronized frequency on several occasions over the past half century leading to interruptions of power delivery [5]–[10]. Most recently, the European continental power system experienced a loss of synchronization on Jan 8, 2021 [10], as shown in Fig. 1.

The synchronous power grid of Continental Europe was split into two separated grid regions on January 8th, 2021

Fig. 1: Frequency traces of the European synchronized continental power system recorded by Phasor Measurement Units across this system on January 8, 2021. In this plot, two distinct frequencies are visible after a loss of synchronization when the system was split into two sub-systems with an interruption in power delivery for a duration of 1 hour and 3 minutes.

The problem inherent synchronization aims to assess system dynamics and aims to assess the ability of the system to maintain its synchronized operation given its natural properties sans auxiliary automatic controllers such as the automatic voltage regulator (AVR) or excitation systems of synchronous machines and cascade power regulators in power electronics. Among the existing body of literature, we identify several approximations of the power system as nonlinear oscillators including Kuramoto oscillators [11]–[16], Lienard oscillators [17]–[19], and Van der Pol oscillators [20]. For oscillator approximations, several critical factors are simplified and neglected so

\[ \dot{x} = \frac{4}{\pi^2} x \]

1\(^t\)the dot notation indicates the time derivative; \( \dot{\delta} = \frac{d}{dt} \delta \)
that the power system equation of motion (commonly known as the swing equation) can resemble the oscillator of interest. We also recognize more comprehensive studies in which closed-form solutions have been developed \cite{21}, \cite{22}. The former study \cite{21} offers provisional conditions and the latter study \cite{22} assumes homogeneity of coupling damping factors.

The contribution of this paper is threefold. First, we advance the findings of \cite{22} by introducing heterogeneous coupling damping factors and deriving the necessary and sufficient condition for inherent synchronization in power systems. Second, we conduct a parametric sensitivity analysis for this condition and develop a mechanism for enhancement of synchronization robustness by adjusting the key contributing generator parameters. Third, leveraging this formalism, we demonstrate the feasibility of operating electric power systems with 100% renewable energy based technologies with enhanced synchronization capabilities. We wish to emphasize that what distinguishes our work is that the consideration of heterogeneous coupling damping factors allows us to study the wide range of different configurations and combinations of conventional and emerging renewable generation technologies, enabling us to directly address the challenge of power system decarbonization.

II. Dynamic Model of Power systems

A. System model

Electric power systems can be modeled as a complex network with power generators, substations, and load centers constituting its nodes and the electric transmission lines that interconnect them the links. In this study, we focus on the impacts of generator parameters on the system frequency synchronization. Thus, we model system generators as dynamic elements and transmission lines interconnecting them and the load centers as algebraic elements \cite{23}. The power generation technologies considered here can be primarily divided into two main categories: (1) synchronous generator-based power plants and (2) power electronics-based power plants. The first technology is the synchronous generator and the overwhelming majority of existing generation facilities are equipped with this type of converter including hydro, nuclear, natural gas, and coal fleet \cite{24}, \cite{25}. The second and third technologies are based on power electronic inverters, which interface variable renewable energy sources and energy storage units with the grid. They can be categorized as (i) grid-following inverters (referred to as GFL henceforth) and (ii) grid-forming inverters (referred to as GFM henceforth). The GFL are the most common currently used class of inverters and the GFM are an emerging and promising technology.

The analysis of inherent synchronization considers only the dynamics associated with frequency response sans auxiliary automatic controllers \cite{22}, \cite{26}. For the synchronous generator and GFM, the equations of motion are thus very similar and describe the evolution of the electric angle, $\delta$, as given by \cite{24}, \cite{26}–\cite{30}:

\[
\ddot{\delta} = M^{-1}(P^* - P_e - D\dot{\delta})
\]  

where $P^*$ and $P_e$ are the power input (which is mechanical power for the synchronous generator and direct current electrical power for GFM) and power export (alternating current electrical power), respectively. The $M$ and $D$ are mechanical and virtual inertia and damping coefficients for synchronous
generator and the GFM, respectively. The chief contrast between the two technologies is that in a synchronous generator the inertia and damping coefficients are dependent on each other and are often expressed by the $M^{-1}D$ ratio of a generator \cite{24}, \cite{26} whereas in the GFM the inertia and damping coefficients are independent of each other. This is because power electronic inverters have kHz switches driven by digital controllers which allows the device to exhibit a dynamic response with desired damping and inertia components, granted available positive headroom energy reserve \cite{31}, \cite{32}, which differs from synchronous generators where these are physical instead of digital parameters. The GFM model represents two of the most common GFM inverters; for $M \approx 0$, it represents multi-loop droop class of GFM \cite{28} and otherwise $M >> 0$ it represents virtual synchronous machine (VSM), a class of GFM that uses digital control to emulate the dynamics of a synchronous generator \cite{29}, \cite{30}. The GFL, on the other hand, simply follows the grid’s frequency measured at its point of interconnection to the grid, using the estimation provided by a phase locked loop (PLL). Therefore, the governing equation are given by:

$$0 = P^* - P_e$$ \hspace{1cm} (3)

where $P^*$ and $P_e$ are the power setpoint and power export. Effectively, the GFL can be modeled as a negative constant load. We assume that transmission line parameters constant because of the timescale of interest \cite{22}, \cite{24}, \cite{26} and develop the admittance matrix to consider the topology of the system. Next, we apply Kron reduction to acquire a lower dimensional electrical-equivalent system of a system when keeping only the nodes with a dynamic element directly interconnected \cite{33} by eliminating all algebraic elements including substations, loads, and GFL inverter-backed generators. This technique is extensively used in power system dynamic analysis \cite{33}–\cite{39}. Kron reduction produces an approximate admittance between boundary nodes – nodes with a dynamic element in this study – by approximating the admittance of an interconnecting link between nodes $i$ and $k$ when node $p$ being eliminated as \cite{36}:

$$Y_{ik} = Y_{0_{ik}} - \frac{Y_{0_{ip}}Y_{0_{pk}}}{Y_{0_{pp}}}, \hspace{1cm} i \neq k, \ i, k = 1, \ldots, n$$ \hspace{1cm} (4)

The resultant system is an equivalent power system with dynamic generation interconnected to its nodes. The eliminated nodes and loads are integrated through adjusted admittance values between boundary nodes.

**B. Model linearization**

For the assessment of the system frequency synchronization when subjected to small perturbations (representing frequent events such as load fluctuations and generation dispatch changes), we linearize the model and appoint one generator as the reference generator \cite{24}, \cite{26} and all other angles are analyzed relative to the angle of the reference generator, $\delta_{i,n}$.

For linearization of our nonlinear model, we approximate the system behavior around equilibrium points. In power systems, system equilibria are determined by solving the system power-flow equations. First, let us consider operations at a stable equilibrium where the generator power setpoints
and power exports are equal, \( P^* = P_e \), for both the GFM and Synchronous generators. Following a disturbance, there will be a mismatch between these known as the acceleration power, defined by \( P = P^* - P_e \) as pertaining to changes in generators’ power output. Inspired by the analysis of power networks with only synchronous generators considered in [26], linearization of the governing equations for the GFM and Synchronous machines interconnected at the \( i \)-th node (2) yields:

\[
\Delta \dot{\delta}_i = \Delta \omega_i
\]

\[
\Delta \dot{\omega}_i = \frac{1}{M_i} \Delta P_i - \frac{D_i}{M_i} \Delta \omega_i
\]

where \( \Delta P_i \) is the change in the generator power injection. For any node \( i \), the active power injection at the node with voltage of \( V_i = |V_i| \angle \delta_i \) can be expressed by the power-flow equation as [26]:

\[
P_i = V_i^2 \cdot G_{ii} + \sum_{j=1;j\neq i}^{n} V_i \cdot V_j \cdot [B_{ij} \cdot sin(\delta_i - \delta_j) + G_{ij} \cdot cos(\delta_i - \delta_j)]
\]

where \( Y_{ij} = G_{ij} + j \cdot B_{ij} \) here are the effective admittance values obtained from Kron reduction; \( G \) is the conductance and \( B \) is the susceptance. The linearized form of (6) can be expressed in a matrix form as:

\[
\Delta P_n = H_n \Delta \delta_n
\]

where \( H \) is the system’s Laplacian (effective admittance), a \((n \times n)\) matrix whose elements can be computed by:

\[
H_{ij} = -V_i \cdot V_j \cdot [B_{ij} \cdot cos(\delta_i - \delta_j) - G_{ij} \cdot sin(\delta_i - \delta_j)]
\]

\[
H_{ii} = \sum_{j=1;j\neq i}^{n} V_i \cdot V_j \cdot [B_{ij} \cdot cos(\delta_i - \delta_j) - G_{ij} \cdot sin(\delta_i - \delta_j)]
\]

Replacing (7) into (5) and appointing the \( n \)-th bus as the reference generator (or slack generator) produces:

\[
\begin{bmatrix}
\Delta \delta_{n-1} \\
\Delta \omega_{n-1}
\end{bmatrix} =
\begin{bmatrix}
0 & 1_{-1} \\
- M^{-1}H_{n-1} & - M^{-1}D_n
\end{bmatrix}
\begin{bmatrix}
\Delta \delta_{n-1} \\
\Delta \omega_{n-1}
\end{bmatrix}
\]

In (9), \([0]\) represents a \((n \times n)\) matrix of zeros and \([1_{-1}]\) represents a matrix of identity with an extended column whose arrays are all equal to (-1). \(H_{n-1}\) is an \(n \times (n-1)\) matrix and the angles presented in \( \Delta \delta_{i,n} \) are the relative angles between node \( i \) and the reference angle at the reference generator \( n \). This is a set of \((2n - 1)\) equations representing \((n - 1)\) electric angles of all generators relative to the reference generator and \( n \) actual electric speeds that would include the reference generator’s electric speed.

**III. Condition of Stability**

Here we solve the necessary condition for stable frequency synchronization in an electric power network with heterogeneous coupling damping factors. The linear model previously developed and
described in (9) can alternatively be described as:

\[
\begin{bmatrix} \Delta \delta_i \\ \Delta \dot{\omega}_i \\ \Delta \dot{\omega}_n \end{bmatrix} = \begin{bmatrix} 0 & I & -1 \\ h_i & d_i & 0 \\ h_n & 0 & d_n \end{bmatrix} \begin{bmatrix} \Delta \delta_i \\ \Delta \omega_i \\ \Delta \omega_n \end{bmatrix}
\]  

(10)

where \( \Delta \) is the linear operator, \( \delta_{n-1} \) is the vector of \((n - 1)\) relative electric angles and \( \omega_{n-1} \) and \( \omega_n \) are absolute electric speeds. \([0]\) and \([I]\) are matrices of zero and identity and \([-1]\) is a matrix of \(1 \times (n - 1)\) whose arrays are (-1). \(h_n = -M_n^{-1}H_{i,n}, \forall i = 1, 2, \ldots, (n - 1)\) is a \(1 \times (n - 1)\) matrix, \(d_n = -M_n^{-1}D_n\) is a single array, and \(h_i = -M_i^{-1}H_{i,j}\) and \(d_i = M_i^{-1}D_{i,j}, \forall i, j = 1, 2, \ldots, (n - 1)\).

By the definition of eigenvalues, it can be written

\[
Jw = \lambda w
\]

(11)

with \( J \) being the system’s Jacobian along its solution trajectory. For this system, it can be presented in compact form as:

\[
\begin{bmatrix} 0 & I & -1 \\ h_i & d_i & 0 \\ h_n & 0 & d_n \end{bmatrix} \begin{bmatrix} w' \\ w'' \\ w''' \end{bmatrix} = \lambda \begin{bmatrix} w' \\ w'' \\ w''' \end{bmatrix}
\]  

(12)

Expansion of this expression provides:

\[
I \cdot w'' - 1w''' = \lambda \cdot w'
\]

\[
h_i \cdot w' + d_i \cdot w'' = \lambda \cdot w''
\]

\[
h_n \cdot w' + d_n \cdot w''' = \lambda \cdot w'''
\]  

(13)

where \( \cdot \) is the inner product operator and \(1\) is a matrix of all ones of \((n - 1) \times 1\).

From the third equation, it can be derived:

\[
w''' = (\lambda - d_n)^{-1} h_n \cdot w'
\]  

(14)

From the first equation:

\[
1w''' = I \cdot w'' - \lambda \cdot w'
\]  

(15)

Replacing (14) into (15) produces

\[
w'' = \left( (\lambda - d_n)^{-1} [1 \otimes h_n] + \lambda I \right) \cdot w'
\]  

(16)

where \( \otimes \) is the outer product operator.

Let us rearrange the second equation:

\[
h_i \cdot w' = (\lambda I - d_i) \cdot w''
\]  

(17)
Replacing (16) in this expression produces:

\[ 0 = \left( (\lambda I - d_i) \cdot \left( (\lambda - d_n) \cdot (1 \otimes h_n) + \lambda I \right) - h_i \right) \cdot w' \]  

(18)

Since \( w' \) cannot be zero, then the first expression should be equal to zero. Accordingly, we define a characteristic matrix for the system described in (10) as:

\[ p(\lambda) = \alpha \lambda^3 + \beta \lambda^2 + \gamma \lambda + \xi = 0 \]

\[ \alpha = I \]
\[ \beta = -d_i - d_n I \]
\[ \gamma = d_i d_n - h_i + [1 \otimes h_n] \]
\[ \xi = h_i d_n - d_i \cdot [1 \otimes h_n] \]  

(19)

The necessary and sufficient condition for existence of eigenvalues is that this characteristic matrix is singular and thus \( \det(p(\lambda)) = 0 \). Satisfying this condition yields a total of \( (2n - 1) \) eigenvalues whose eigenvectors are linearly independent for a system with \( n \) generators, and a stable equilibrium exists if and only if all roots of the polynomial \( p(\lambda) \) possess non-positive real parts (non-positive Lyapunov exponents). One may recognize this formalism as the stability criteria in the sense of Lyapunov’s First Method [40]. In the next section, we present the solution for Eq. (19) in a form convenient for parametric analysis.

IV. Parametric Analysis of Synchronization

The formal expression of the solution for Eq. (19) can be written in the form of

\[ p(\lambda) = \prod_{i=1}^{(n-1)} \left( \lambda^2 + 2 \zeta \omega_n \lambda + \omega_n^2 \right) \cdot (\lambda + k_d) = 0 \]  

(20)

The first part is the generators’ internal modes involving the electric angle and speed of the generators, which presents a pair of complex conjugate eigenvalues. The second part is the system’s coupling mode and presents as a real eigenvalue whose value is a function of the generators damping coefficient. Recalling Eq. (10), let us suppose a special condition where the damping factors, \( d_i = -\frac{D_i M_i}{M} \) are homogeneous (i.e., \( d_s = d_1 = d_2 = \cdots = d_n \)). With this assumption, we can approximate the internal modes, given the dimension associated with the coupling mode can be reduced. The stability criteria for this special condition is developed in [22] and [26] and we borrow their formalism here to determine the roots of the characteristic equation (19) for this special condition. The assumption of homogeneity leads to a reduced system Jacobian to \( 2(n - 1) \) by representing electric speeds as relative electric speeds with respect to the electric speed of the reference generator. To this end, let us assume the coupling damping factor is uniform for all generators across the system and equivalent to \( d_s \). For any
\(i\)-th generator, with the \(n\)-th generator being the reference generator, it can be written \([26]\):

\[
\Delta \dot{\omega}_{i,n} = -\left(\frac{\Delta P_i}{M_i} - \frac{\Delta P_n}{M_n}\right) - d_s (\Delta \omega_i - \Delta \omega_n)
= -\left(\frac{\Delta P_i}{M_i} - \frac{\Delta P_n}{M_n}\right) - d_s \Delta \omega_{i,n}
\]  (21)

where \(\omega_{i,n} = \omega_i - \omega_n\) is the electric speed deviation from the reference machine. Substituting (21) into (5) produces

\[
\begin{bmatrix}
\Delta \delta_{n-1} \\
\Delta \dot{\omega}_{n-1}
\end{bmatrix}
= \begin{bmatrix} 0 & I \\ h & d \end{bmatrix}
\begin{bmatrix}
\Delta \delta_{n-1} \\
\Delta \omega_{n-1}
\end{bmatrix}
\]  (22)

where \(\Delta \delta_{n-1}\) and \(\Delta \omega_{n-1}\) are vectors of \((n - 1)\) relative electric angles and electric speeds and \([0]\) and \([I]\) are \((n - 1) \times (n - 1)\) matrices of zero and identity. \(d\) is a diagonal matrix of damping coefficients with all of its diagonal arrays equivalent to \(d_s\) and \(h\) is a \((n - 1) \times (n - 1)\) matrix of relative network coefficients.

To assess the stability of this system, one can calculate the eigenvalues of (22), denoted by \(\lambda\) with corresponding eigenvectors of \(w\). The definition of eigenvalues and eigenvectors provides \([22], [26]\):

\[
\begin{bmatrix} 0 & I \\ h & d \end{bmatrix}
\begin{bmatrix} w' \\ w'' \end{bmatrix}
= \lambda
\begin{bmatrix} w' \\ w'' \end{bmatrix}
\]  (23)

where \(w = [w'w''^T]^T\). From (22), it can be said that \(w'' = \lambda w'\) and \(h \cdot w' + dw'' = \lambda w''\) or \(h \cdot w' = (\lambda - d_s)w''\). Replacing \(w'' = \lambda w'\) into \(h \cdot w' = (\lambda - d_s)w''\) produces \(h \cdot w' = (\lambda - d_s)\lambda w'\) which implies \(\lambda_h = \lambda^2 - d_s \lambda\) is the eigenvalue of the submatrix \(h\). It can be rearranged as

\[
\lambda^2 - d_s \lambda + \lambda_h = 0
\]  (24)

Eq. (22) is equivalent of Eq. (10) under the assumption of \(d_i\) homogeneity yielding at a zero coupling mode \(k_{d_i} = 0\) in (22). The eigenvalues of (22) can be found by solving (24) for \(\lambda\) as

\[
\lambda = 0.5d_s \pm 0.5\sqrt{d_s^2 + 4\lambda_h}
\]  (25)

where \(\lambda_h\) is the eigenvalue of the submatrix \(h\). Eq. (25) yields two internal modes for each generator, complex conjugates in a stable system, which pertain to the relative electric angle and relative speed of the generator, both with respect to those of the reference generator. The location of these eigenvalues determines the state of the system and as they approach the \(y\)-axis, the imaginary axis in the complex plane, the system’s transient response becomes increasingly oscillatory and the settling equilibrium becomes farther away from the pre-disturbance equilibrium. Eq. (25) suggests that the real part of these modes is proportional to the generator damping coefficient and thus, a reduction in the generator damping coefficient, \(D\), directly moves the eigenvalues to the right closer to the \(y\)-axis, whilst its increase directly moves them to the left farther away from the \(y\)-axis. Both the generator damping factor \(d\) and the system interconnection Laplacian \(h\) have an inverse relationship with its inertia coefficient \(M\), given \(d_s = -\frac{D}{M}\) and \(h_{ik} = -\frac{H_{ik}}{M}\). Therefore, for a stable system, a reduction in generator inertia increases the values of both the real and the imaginary terms and results in
Fig. 2: Loci of migration of eigenvalues in the complex plane and time-domain response as generator parameters are varied.

A migration farther away from the origin to the left hand side, whilst its increase will bring them closer to the origin. A concurrent reduction in generator inertia and damping coefficients increases the imaginary term whilst the real part, \( d_s = -\frac{D_i}{M_i} \), remains unaffected because its numerator and denominator change at an identical rate. On the other hand, reduction of the generator inertia coefficient and improvement of its damping coefficient will move the eigenvalues rapidly to the left hand side of the plane given that both the real and imaginary parts will increase.

Now that the internal modes are established, we assume that the damping factors are heterogeneous and, therefore, \( k_d \neq 0 \), as the main distinguishing feature of our formalism from the existing body of
literature [22], [26]. This assumption brings the model more in line with the nature of power grids, where damping factors are heterogeneous because the power grids operate on generation portfolios which encompass diverse sources and technologies for electricity generation with different damping capability. Under the heterogeneity assumption, there will be an additional real mode (the coupling mode described in (20)) and it is a function of generators damping $D_i$ coefficient and the inverse of their inertia coefficient $M_i^{-1}$, as $d_i = \frac{D_i}{M_i}$, and therefore directly migrates as a function of changes in these parameters.

To verify this hypothesis, we analyzed a 3-generator benchmark [24] and results for the sensitivity of eigenvalues of a non-reference generator to parametric changes, Gen 2, and the system’s coupling mode are shown in Fig. 2. The results from Gen 3 were similar and thus not shown. We considered six scenarios in addition to the base case and in each scenario, one or more parameters were changed by overall order of 2. All plots shown here include two internal modes manifested as complex eigenvalues that are a conjugate pair and one coupling mode manifested as a real eigenvalue. In time-domain results, the higher the nadir frequency is, the more robust the frequency dynamic response. These results from individual parametric analysis are conclusive that the reduction of inertia and increase of damping moves the imaginary part of internal modes farther away from the y-axis, resulting in improved dynamic response, whilst the increase of inertia and reduction of damping result in the opposite impact.

For concurrent parametric analysis; scenario 5 represents substitution of synchronous generators with GFL inverters and scenario 6 represents replacement with GFM inverters. These results exhibit the superior capability of the multi-loop droop GFM over GFL in improving system dynamics. The GFM-VSM replacement is not presented here because it uses parameters equivalent to those of a synchronous generator in order to emulate its identical behavior and, thus, does not pose any parametric changes.

Next, we quantify the system’s frequency response when subjected to a small perturbation. The system’s time-domain response is presented in Fig. 3 and the dynamic frequency responses are quantified and summarized in Table I. In this table, the swiftness of system response is measured by the rise time, $t_r$, and the peak time, $t_p$; the smaller they are, the swifter the response. The severity of frequency excursion is measured by the nadir value, $p$; the larger it is, the more robust the response.

| Case                  | Damping  | Inertia  | $t_r$  | $t_p$  | $p$      |
|----------------------|----------|----------|--------|--------|----------|
| All Synchronous Generator | $D_i$   | $M_i$    | 16.389 | 28.958 | 59.533   |
| 90% GFL              | $0.1 D_i$ | $0.1 M_i$| 16.316 | 29.052 | 59.533   |
| All GFL              | 0        | 0        |        |         | Infeasible|
| All GFM-VSM         | $D_i$    | $M_i$    | 16.389 | 28.958 | 59.533   |
| All GFM-Droop       | $2 D_i$  | $0.01 M_i$| 0.072  | 0.144  | 59.997   |

There are three main points to note in the results presented in Fig. 3 and Table I. First, the frequency response for the three scenarios of All Synchronous Generator (SG), All GFM-VSM, and 90% GFL are approximately identical. Second, operating a system with 100% GFL is infeasible because mathematically, the denominator of the Jacobian’s elements cannot be zero (100% GFL...
implies \( M = 0 \) and practically, there will be no source to construct the synchronization frequency for the GFLs to follow. The stability limit for synchronization is \(|C| > 0, C = \{\exists M_i | M_i > 0, \forall i = 1, 2, \ldots, n\} \) and \( D_i > 0, \forall i = 1, 2, \ldots, n\), reaffirming that the minimum of one non-GFL generator interconnected to the system is necessary to constitute the synchronization frequency when the damping coefficients of all generators are negative. Third, and perhaps the most significant point, when operating with 100% multi-loop droop GFM, the effective inertia value can be reduced to near zero, and the GFM’s frequency response can be seen effectively as a first-order response and is less likely to experience severe frequency excursions. This observation is consistent with the observations reported in [41], [42].

Our results here establish the mechanism of synchronization in power systems with high levels of inverter-based generation and demonstrates that it can be enhanced by adjusting the digital parameters of GFM inverters including damping and inertia coefficients. Over the past few years, the displacement of synchronous generators has raised concerns over the potential ramifications on system stability, which has been widely characterized as a discussion about the impacts of reduced inertia, or commonly known as 'Low-Inertia Power Grids' [3], [27], [43], [44]. Our results establish that generator damping capability is equally as significant in achieving a stable small-signal state as its inertia. They also indicate the reduced inertia alone does not necessarily deteriorate dynamic performance when responding to small perturbations. In addition, our results demonstrate that emerging multi-loop droop GFM inverter technologies with the capability to provide damping support independent of the inertial contribution [28], offer great promise in enhancing grid stability.

V. Application in Emerging Power Systems

The synchronization condition and mechanism suggest that the stability of these systems is a function of: (1) the generator parameters and (2) the interconnected system conditions. Accordingly, we tested 6 different complex power system benchmarks (common IEEE test cases as explained
in the next subsection), carrying out computer simulations for 1,000 random parameter and system condition combinations for each benchmark, and measured the system nadir frequencies when systems are subjected to stochastic small perturbations. Nadir frequency is the largest value of frequency excursion and is an important value in power grid management and control because if the frequency drops below a standard threshold – 48.5Hz in Europe \cite{45} and 59.5Hz in North America \cite{46} – it can trigger protective equipment that results in an interruption of power supply to some parts of the grid as a preventative measure to avoid cascading failures and blackout. Therefore, the frequency nadir plays a critical role in the operation of power systems and, hence, we consider it as a critical metric in our analysis.

In our simulations, the randomness of generator inertia and damping coefficients represents technological variations to include both conventional and renewable generation. Additionally, the randomness of loading condition resembles the loading variations which determines node voltages and flow of power across the power lines in a power system and, therefore, represents the varying system dynamic states.

![Diagram](attachment:image.png)

(a) 9-bus, 3-gen system  
(b) 30-bus, 6-gen system  
(c) 39-bus, 10-gen system  
(d) 57-bus, 7-gen system  
(e) 118-bus, 54-gen system  
(f) 145-bus, 50-gen system.

Fig. 4: Nadir frequency as a function of generators inertia and damping

A. Dynamic data

We used the network data from the standard IEEE benchmarks including IEEE 9-bus \cite{24}, IEEE 30-bus \cite{47}, IEEE 39-bus \cite{48}, IEEE 57-bus \cite{49}, IEEE 118-bus \cite{50}, and IEEE 145-bus \cite{51} systems. The steady state models for these systems are freely available as part of the MATPOWER tool \cite{52} but the dynamic data for some of these benchmarks does not exist. For this study, we generated synthetic dynamic data that represent real-world power system dynamics and characteristics relying on the data available from those publicly available benchmarks \cite{24, 25, 51, 53}. Creation of synthetic power
grids is a common practice because real-world power system data is not always easily accessible. Our synthetic dynamic data are generated algorithmically such that inertia and damping values are allocated with the upper bounds of 8 for inertia and 0.05 for damping in base cases representing the conventional power system with all generation stations equipped with synchronous generator. The range we chose is consistent with those empirically observed in the data available. The total power (MVA) capacity of generators are also randomly allocated between 100% and 250% of their total power dispatch determined by the power-flow equations to account for reserve margin. Then we validated them by perturbing all generators, one generator at a time, and monitored the dynamic response of all generators that are interconnected to the system to ensure system stability.

B. Results and Discussions

The results for 1,000 random operating conditions on 6 different power system benchmarks are presented in Fig. [4]. In these plots, the observed nadir frequencies are color-coded, with the blue dots being the high nadir values indicating the best system dynamic response, as they are operationally stable, and the red dots being the lowest nadir values reflecting the worst system dynamic response, where protective action was likely to have occurred. The other colors in between should be interpreted accordingly as described by figure legend. The x- and y-axes represent the system aggregated damping and aggregated inertia, respectively. We consider the aggregated inertia as

$$H_{agg} = \frac{\sum_{i=1}^{n} H_i S_i}{\sum_{i=1}^{n} S_i}$$

where $H_i$ and $S_i$ are the inertia constant and the capacity rating of the $i$-th generator in a system with $n$ generators in total. Similarly, we consider the aggregated damping as

$$D_{agg} = \frac{\sum_{i=1}^{n} D_i S_i}{\sum_{i=1}^{n} S_i}$$

where $D_i$ and $S_i$ are the damping constant and the capacity rating of the $i$-th generator in a system with $n$ generators in total.

In all six power systems, the improvement of frequency dynamics proportional to increased damping and reduced inertia is evident (the best dynamic responses appear at the bottom right corner of these plots and the worst dynamic response appear in the top left corner of these plots). It can be seen that higher nadir frequencies are witnessed for low-inertia high-damping conditions; whereas the lowest nadir frequencies are recorded for high-inertia low-damping conditions. These results speak to the relative importance of damping in system stability, as opposed to the commonly studied inertial aspect, and further validate our findings on the synchronization mechanism and the synchronization enhancement mechanism established in the previous section.

It should be noted that for all six power systems considered, 100% of cases whose power flow converged to a stable equilibrium point (meaning that load and generation matched while adhering to voltage and thermal line limits), resulted in a stable case. This observation is consistent for the complete range of varying system inertia and damping values examined and this confirms the feasibility of operating electric power systems on 100% renewable based generation, explained as follows. Any power system relies on multiple forms of electricity generation and in a 100% renewable generation scenario, this mix will include solar, wind, and hydro power, to name but a few potential sources. While hydro power uses synchronous generators with fixed inertia and damping coefficients, almost all other forms of renewable sources rely on power electronics, including GFM and GFL,
and the GFM has the capability to independently offer the desired inertia and damping coefficients through its digital fast responding controllers. The variable and uncertain nature of most renewable resources along with their limited availability constrained by meteorological conditions and time of the day necessitates to rapidly switch between generation technologies in order to utilize them when available and to serve demand without interruptions \cite{2, 3, 54}. Therefore, such systems will have to operate on significantly varying levels of inertia \cite{55, 56} and damping. The only constraining limitation is the requirement for a non-GFL generator to be interconnected to the grid at all times to satisfy the necessary stability condition following small perturbations. This condition can easily be satisfied by GFM-backed or hydro power plants. Hence, stable operation of power grid with 100\% renewable based generation is feasible, from a small-signal stability perspective. This work does not examine transient stability implications of increased inverter-based generation. We emphasize that by definition, we refer to a naturally stable case as a system whose response is bounded, and thus the stable cases we observed here are analytically stable in the sense of Lyapunov’s first theorem and not in the sense of practice.

C. Perspective

The findings of this paper may set a basis for the study of stability, dynamics, control, and operation of bulk power systems with 100\% renewable-based generation. In particular, our results prove that the power systems with 100\% renewable generation naturally possess stable synchronization measures for dealing with small-signal perturbations but their dynamic response may violate the necessary industrial control and operation standards. We emphasize that this is not a fundamental limitation and this shortcoming can be addressed and managed by advanced coordinated control systems that may leverage the stability enhancement mechanism we have established in order to modify the system’s behavior to provide a desired system response.

Power systems have been successfully managed and operated by hierarchical networked control for more than half a century \cite{57, 58}. Most of the control apparatus in power systems can have either a negative or positive damping effect and therefore be destabilizing if they are poorly tuned or poorly structured \cite{59}. As a result, the control systems and structure for conventional power systems, which have been designed for a synchronous generator-dominated system to manage its unique dynamics, are likely inadequate for the operation of a 100\% renewable generation-based grid because of the different underlying dynamics to be managed. Therefore, the solution for reliable, safe, and stable operation of future power systems is to reconsider the control systems and automation currently in place, both the structure and algorithms, and perhaps design and implement modern control systems that are designed and tuned in accordance with the dynamic behaviors and characteristics of power systems with high levels of inverter-based generation, especially by taking advantage of the unique functionalities and responsiveness of the GFM inverters.

VI. Conclusion

Power grids worldwide are changing significantly, transitioning from the currently dominant synchronous generator-based power plants to power electronics-based power plants in order to accompl-
moderate clean and sustainable renewable energy resources. In this paper, we derived the necessary condition for stable synchronized operation in electric power systems, considering both conventional and renewable generation technologies, when subjected to small perturbations. We identified a mechanism to enhance grid synchronization through individual generator parameter adjustment. Contrary to common belief, which ties system stability primarily to system inertia, our results definitively demonstrate the critical role of generator damping in achieving a stable synchronized state of operation. The dynamics of emerging power grids heavily depends upon the technology used for the interconnection of the renewable generation, whether GFL or GFM. Our results here suggest that power grids with 100% renewable generation inherently possess the measures for stable synchronized operation in the Lyapunov sense.

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