Hyper-Dilaton Weyl Multiplet of 4D, $\mathcal{N} = 2$ Conformal Supergravity

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Abstract

We define a new dilaton Weyl multiplet of $\mathcal{N} = 2$ conformal supergravity in four dimensions. This is constructed by reinterpreting the equations of motion of an on-shell hypermultiplet as constraints that render some of the fields of the standard Weyl multiplet composite. The independent bosonic components include four scalar fields and a triplet of gauge two-forms. The resulting, so-called, hyper-dilaton Weyl multiplet defines a $24 + 24$ off-shell representation of the local $\mathcal{N} = 2$ superconformal algebra. By coupling the hyper-dilaton Weyl multiplet to an off-shell vector multiplet compensator, we obtain one of the two minimal $32+32$ off-shell multiplets of $\mathcal{N} = 2$ Poincaré supergravity constructed by Müller in 1986. On-shell, this contains the minimal $\mathcal{N} = 2$ Poincaré supergravity multiplet together with a hypermultiplet where one of its physical scalars plays the role of a dilaton, while its three other scalars are dualised to a triplet of real gauge two-forms. Interestingly, a $BF$-coupling induces a scalar potential for the dilaton without a standard gauging.
1 Introduction

Conformal supergravity has played an important role in several research avenues in the last five decades — we refer the reader to a few books for reviews and a more detailed list of references [1–4]. The main aim of our paper is to revise some of the ingredients of the superconformal tensor calculus for matter-coupled Poincaré supergravity focusing on the four-dimensional (4D), $\mathcal{N} = 2$ case, and more generally on theories with eight real supercharges. For instance, we will show how to define a new off-shell $24 + 24$ Weyl multiplet of $\mathcal{N} = 2$ conformal supergravity.

After the seminal papers on 4D, $\mathcal{N} = 1$ supergravity [5–8], the superconformal tensor calculus for the 4D, $\mathcal{N} = 2$ case was first constructed in the 80s in [9–13] and also extended to the cases of 6D, $\mathcal{N} = (1, 0)$ in [14]; 5D, $\mathcal{N} = 1$ in [15–20]; and more recently to three space-time dimensions in [21–22]. Similar to superspace approaches (see [1] for introductory reviews and, e.g., [23–35] and references therein, for the 4D, $\mathcal{N} = 2$ case) a main advantage of the superconformal tensor calculus is to provide an off-shell
description of general supergravity-matter couplings. This allows one to formulate general supergravity-matter couplings where supersymmetry is engineered in a completely model independent way. The approach has been very successful in helping to decipher many of the intricate geometrical structures associated to (two-derivatives) sigma-models in supergravity-matter systems with eight real supercharges, see e.g., \[4,36-40\]. The off-shell nature of the formalism has been a central ingredient in its employment to the study of supersymmetric localisation and supersymmetric quantum field theories on curved spacetimes — see \[41\] for a recent extensive review. Moreover, off-shell supersymmetry has also been a crucial ingredient when using the superconformal tensor calculus to construct higher-derivative supergravity invariants \[22,42-62\]. These play an important role, e.g., in the study of black-hole entropy in next to leading order AdS/CFT — see the recent works \[63-65\] and references therein.

Within the superconformal tensor calculus, general supergravity-matter couplings are engineered by a few ingredients. First of all, one needs a conformal supergravity multiplet — named the Weyl multiplet — which forms an off-shell representation of the local superconformal algebra and contains the vielbein as one of its independent fields. This multiplet defines the geometry (soft algebra) associated with the gauging of the superconformal space-time symmetry. Next, one identifies off-shell matter multiplets with local superconformal transformation rules in a Weyl multiplet background. These two ingredients provide the kinematic data of a specific supergravity-matter system. Finally, one engineers locally superconformal invariant action principles constructed out of these multiplets to obtained well-defined supergravity theories.

Assuming the matter multiplets contain enough “compensating” degrees of freedom, one can suitably gauge fix part of the superconformal group, specifically dilatations, special conformal transformations, $S$-supersymmetry, and $R$-symmetry, to obtain supergravity models where only the super-Poincaré symmetry survives and is gauged. For instance, pure 4D, $N = 2$ Poincaré supergravity can arise by the coupling of the standard Weyl multiplet \[9,13\] to two compensating multiplets. There is significant freedom in doing so. Typically, one uses a vector multiplet and a hypermultiplet (e.g., a linear, non-linear, or hypermultiplet with or without a central charge) as compensators — see \[4\] for a recent review. Note that, in this endeavour, for forty years the first step has predominantly been the same (standard Weyl multiplet), while most of the freedom that has been used
concerned the matter (compensators) side of this story. However, it is natural to ask whether it is possible to use alternative Weyl multiplets and if it is useful to do so. These are the types of questions that led to our paper.

Answers to these questions are already known. For instance, if one considers the superconformal tensor calculus for 6D, \( \mathcal{N} = (1, 0) \) supergravity, it has been known since 1986 [14] that there can be more than one Weyl multiplet. In 6D, the existence of a variant dilaton Weyl multiplet engineered as a standard Weyl multiplet coupled to an on-shell tensor multiplet has been a key ingredient to obtain 6D, \( \mathcal{N} = (1, 0) \) Poincaré supergravity by using superconformal techniques. Similar ideas were employed to construct a variant dilaton Weyl multiplet for 5D, \( \mathcal{N} = 1 \) conformal supergravity as the standard Weyl multiplet coupled to an on-shell vector multiplet [18]. Among interesting applications of these variant Weyl multiplets, it is worth mentioning that both in 5D and 6D the use of the dilaton Weyl multiplet allowed the construction of the component actions for the supersymmetric extensions of all curvature squared combinations — Riemann-squared, Ricci-squared, and scalar-curvature-squared — in Poincaré supergravities, see [42, 46, 47, 50, 53, 58, 59].

For the 4D, \( \mathcal{N} = 2 \) case the existence of a variant representation of the Weyl multiplet of conformal supergravity was argued in [68] and was explicitly constructed only recently in [69]. For reasons that will soon be clear, we will refer to this multiplet as the vector-dilaton Weyl multiplet. Its construction closely mimics the 5D case [18]. More specifically, in [69], the system described by a 4D, \( \mathcal{N} = 2 \) on-shell vector multiplet in a standard Weyl multiplet background was interpreted as a new 24 + 24 multiplet of conformal supergravity. Using the equations of motion for the vector multiplet, the existing covariant matter fields of the standard Weyl multiplet, i.e., the real antisymmetric tensor, \( T_{ab}^{ij} \), the real scalar field, \( D \), and the spinor field, \( \chi^i \), together with the \( U(1)_R \) symmetry connection, were traded for fields of the on-shell vector multiplet [69]. The complex scalar field, \( X \), of the vector multiplet then becomes an independent physical field whose real part plays the role of a dilaton in a Poincaré supergravity constructed in this framework. This Poincaré supergravity was constructed by coupling the vector-dilaton Weyl multiplet to a 8 + 8 linear multiplet compensator [62, 69]. Upon gauge fixing dilatation, special conformal transformations, \( S \)-supersymmetry, and \( U(1)_R \times SU(2)_R \) \( R \)-symmetry (up to a residual \( U(1)_R \)), the resulting 32 + 32 Poincaré supergravity multiplet comprises the following set of fields

\[
\{ e_m^a, \psi_{m\alpha}^i, \bar{\psi}_{m\dot{\alpha}}^i, a_m, a_m', \chi^i, \bar{\chi}^i, C, t_{mn}, e_m, e_m', \bar{e}_{mn}, M, b_{a_{ij}} \}. \tag{1.1}
\]

Here, \( \{ e_m^a, \psi_{m\alpha}^i, \bar{\psi}_{m\dot{\alpha}}^i, a_m \} \) are the fields of the \( \mathcal{N} = 2 \) on-shell supergravity multiplet,
respectively: the vielbein, the gravitino and its conjugate, and the real graviphoton gauge
vector field. The fields \( \{ v_m, t'_{mn}, M, b_{a}^{ij} \} \), that are respectively a real vector, a real
antisymmetric gauge two-form, a real scalar, and a triplet of real vectors, are all auxiliary
fields. The remaining fields, \( \{ a'_{m}, \lambda'_{\alpha}, \bar{\lambda}'_{\dot{\alpha}}, C, t_{mn} \} \), are physical and describe an on-shell
vector multiplet where the imaginary part of the complex scalar field \( X \) has been traded
for a dual antisymmetric real gauge two-form, \( t_{mn} \).

The previous 32 + 32 off-shell Poincaré supergravity turned out to coincide with the
one engineered in 1986 by Müller in [70] — we will refer to this as the vector-Müller
supergravity. It is useful to compare Müller’s supergravity to the well-known 40 + 40
off-shell supergravity of [9][71][73]. One initial feature is that, from the point of view of
off-shell supersymmetry, Müller’s multiplet is irreducible while the 40+40 multiplet is not.
In fact, the vector-Müller multiplet arises from a 24+24 conformal supergravity multiplet
coupled to a single 8 + 8 off-shell compensator, while the 40 + 40 multiplet requires two
8 + 8 compensating multiplets. The on-shell theories are however different. The 40 + 40
off-shell supergravity leads to a dynamical system containing only the irreducible on-shell
\( \mathcal{N} = 2 \) supergravity while Müller’s 32 + 32 off-shell supergravity comes with an extra
physical on-shell “dilaton” vector multiplet.

Interestingly, in 1986 Müller constructed another minimal 32 + 32 off-shell \( \mathcal{N} = 2 \)
Poincaré supergravity [74]. We will refer to this as the hyper-dilaton Poincaré supergra-
vity. The multiplet of [74] comprises the following fields

\[
\{ e^{a}_{m}, \tilde{\psi}_{m\alpha}, \bar{\psi}_{m\dot{\alpha}}, a_{m}, C, t_{mn}^{ij}, \rho^{i}_{\alpha}, \bar{\rho}_{\dot{\alpha}}^{i}, X^{ij}, W_{ab}, b_{a} \}.
\tag{1.2}
\]

Besides the fields of minimal \( \mathcal{N} = 2 \) on-shell supergravity, \( \{ e^{a}_{m}, \tilde{\psi}_{m\alpha}, \bar{\psi}_{m\dot{\alpha}}, a_{m} \} \), the fields
\( \{ X^{ij}, W_{ab}, b_{a} \} \) are respectively a real SU(2) triplet of Lorentz scalars, a real antisymmet-
ric tensor, and a real vector. These three are auxiliary fields [74]. The remaining fields,
\( \{ C, t_{mn}^{ij}, \rho^{i}_{\alpha}, \bar{\rho}_{\dot{\alpha}}^{i} \} \), are physical and describe an on-shell hypermultiplet where three of the
four hypermultiplet’s scalar fields have been traded for an SU(2) triplet of dual antisym-
metric real gauge two-forms, \( t_{mn}^{ij} = t_{mn}^{ji} = -t_{nm}^{ij} \). Precisely as for the vector-Müller
supergravity, even though off-shell the hyper-dilaton Poincaré multiplet is irreducible
with 32 + 32 degrees of freedom, the on-shell theory contains an extra 4 + 4 dilaton mul-
tiplet which, in this case, is described by a variant version of a hypermultiplet where \( C \)
plays the role of a dilaton field. Considering the recent superconformal description of the
vector-Müller supergravity [62][69], it is natural to ask if and how one can engineer the
hyper-dilaton Poincaré supergravity by using the superconformal tensor calculus. The
main aim of our paper is to show how this can be done by using a 24 + 24 variant rep-
representation of the $\mathcal{N} = 2$ conformal supergravity multiplet that we will refer to as the hyper-dilaton Weyl multiplet.

The definition of the hyper-dilaton Weyl multiplet is fairly simple. In fact, it closely mimics the description of vector-dilaton Weyl multiplets with the crucial difference that one starts with an on-shell hypermultiplet in a standard Weyl multiplet background, rather than with an on-shell vector multiplet. The constraints that arise by requiring the algebra of local superconformal transformations to close on the fields of the hypermultiplet can then be interpreted as algebraic equations for some of the fields of the standard Weyl multiplet. More precisely, in the hyper-dilaton Weyl multiplet, the standard Weyl multiplet’s matter fields $(\Sigma^{\alpha i}, \bar{\Sigma}^{\dot{\alpha} i})$ and $D$, together with the SU(2)$_R$ symmetry connection $\phi^{ij}_m$ become composite fields. On the other hand, the four bosonic $q^{i\alpha}$ and four fermionic $(\rho^i_{\alpha}, \bar{\rho}^i_{\dot{\alpha}})$ fields of the hypermultiplet, together with an emerging triplet of real gauge two-forms $b_{mn} ij = b_{mn} j i = -b_{nm} ij$, are independent and not subject to any equations of motion. By then coupling the $24 + 24$ hyper-dilaton Weyl multiplet to a single $8 + 8$ off-shell vector multiplet compensator, upon gauge fixing dilatation, special conformal transformations, $S$-supersymmetry, and the whole $U(1)_R \times SU(2)_R$ symmetry, one readily obtains the $32 + 32$ hyper-dilaton Poincaré supergravity with the field content described in (1.2).

To the best of our knowledge, despite their simplicity, the reinterpretation that we advocate in this paper for the on-shell hypermultiplet as the hyper-dilaton Weyl multiplet and their connection to Müller’s supergravity in a superconformal framework has never explicitly appeared before in the literature. The advantage of our novel superconformal formulation compared to the original work of Müller is the potentially straightforward extension to more general matter couplings. As a simple example, in our paper we show this by extending Müller’s Poincaré supergravity action by including a new invariant that leads to a non-trivial potential for the dilaton field. Intriguingly, such a scalar potential is generated without a standard gauging which, in an $\mathcal{N} = 2$ standard Weyl multiplet setting, is associated to integrating out an independent auxiliary field given by the SU(2)$_R$ gauge connection.

This paper is organised as follows. In section 2 we first review the definition of the standard Weyl multiplet of off-shell $\mathcal{N} = 2$ conformal supergravity and, by using our notation, present results we need for the rest of the paper. We then describe the structure of an on-shell hypermultiplet in a standard Weyl multiplet background and explain how such a system can be reinterpreted as a variant hyper-dilaton Weyl multiplet of off-shell $\mathcal{N} = 2$ conformal supergravity. Section 3 is devoted to first prove how the off-shell
Poincaré supergravity theory constructed by Müller in [74] can be engineered as the hyperdilaton Weyl multiplet coupled to an off-shell vector multiplet conformal compensator. We then extend the results of [74] by adding a new $BF$-coupling which induces a scalar potential for the dilaton without a standard $R$-symmetry gauging. Section 4 includes a final discussion and an outline of some future directions based on the results of our paper.

2 The hyper-dilaton Weyl multiplet

The aim of this section is to construct the $24 + 24$ hyper-dilaton Weyl multiplet of off-shell $\mathcal{N} = 2$ conformal supergravity. Subsection 2.1 reviews well-known results about the standard Weyl multiplet and serves to introduce the notation that we employ. Subsection 2.2 describes the on-shell hypermultiplet in a standard Weyl multiplet background and the resulting interpretation of this system as an independent multiplet of conformal supergravity.

2.1 The standard Weyl multiplet

The standard Weyl multiplet of 4D, $\mathcal{N} = 2$ conformal supergravity is associated with the local off-shell gauging in space-time of the superconformal group $SU(2,2|2)$ [9], see also [10–13] and [3,4] for reviews. The multiplet comprises $24 + 24$ physical components described by a set of independent gauge fields: the vielbein $e^n_a$ and a dilatation connection $b_m$; the gravitino $(\psi^i_m, \bar{\psi}^i_m)$, associated with the gauging of $Q$-supersymmetry; a $U(1)_R$ gauge field $A_m$; and $SU(2)_R$ gauge fields $\phi^{ij}_m$. The fields associated to the remaining generators of $SU(2,2|2)$, specifically the Lorentz connections $\omega^{cd}_m$, $S$-supersymmetry connection $(\phi^i_m, \bar{\phi}^i_m)$ and the special conformal connection $f_{ma}$, are composite fields. The connections define the locally superconformal covariant derivatives

$$\nabla_a = e^m_a \nabla_m = e^m_a \left( \partial_m - \frac{1}{2} \psi^i_m Q^i - \frac{1}{2} \bar{\psi}^i_m \bar{Q}^i - \frac{1}{2} \omega^{cd}_m M_{cd} - i A_m Y - \phi^{kl}_m J_{kl} \right) ,$$

where $\nabla_a = e^m_a \nabla_m$ is the locally superconformal derivative.
Together with the independent gauge connections, the standard Weyl multiplet includes a set of covariant matter fields that are necessary to close the local superconformal algebra off-shell: an anti-symmetric real tensor $W_{ab} = W_{ab}^+ + W_{ab}^-$, which decomposes into its imaginary-(anti-)self-dual components $W_{ab}^\pm$; a real scalar field $D$; and fermions ($\Sigma^{\alpha i}$, $\bar{\Sigma}^{\dot{\alpha}i}$). The covariant derivatives satisfy the algebra
\[
[\nabla_a, \nabla_b] = -R(P)_{abc}^{\quad d} \nabla_c - R(Q)_{ab}^{\quad \alpha} Q^{\alpha} - R(\bar{Q})_{ab}^{\quad \dot{\alpha}} \bar{Q}^{\dot{\alpha}} - \frac{1}{2} R(M)_{abc}^{\quad cd} M_{cd} - R(\mathbb{D})_{abc} \mathbb{D} \\
i R(Y)_{ab} Y - R(J)_{ab}^{\quad kl} J_{kl} - R(S)_{ab}^{\quad i} S_i^{\alpha} - R(\bar{S})_{ab}^{\quad \dot{i}} \bar{S}^{\dot{i}} - R(K)_{abc} K^c . \tag{2.2}
\]

It is also useful to list the non-trivial conjugation properties
\[
(\psi_m^{\alpha})^* = \bar{\psi}_m^{\dot{\alpha}}, \quad (\phi_m^{i})^* = \bar{\phi}^{i \dot{\alpha}}, \quad (\phi_{m\alpha}^{ij})^* = \bar{\phi}^{ij \dot{\alpha}}, \quad (\Sigma^{\alpha i})^* = \bar{\Sigma}^{\dot{\alpha}i}, \quad (R(Q)_{ab}^{\alpha})^* = R(\bar{Q})_{ab}^{\dot{\alpha}}, \quad (R(S)_{ab}^{i})^* = R(\bar{S})_{ab}^{\dot{i}}, \quad (R(J)_{ab}^{kl})^* = R(J)_{abkl} . \tag{2.3a-2.3b}
\]

while all the other fields and curvatures are real.

A set of conventional constraints express the superconformal curvatures in terms of connections and covariant matter fields and render the connections $\omega_m^{cd}$, $(\phi_m^{i}, \bar{\phi}^{i \dot{\alpha}})$, and $f_{ma}$ composite. There is large freedom in the choice of conventional constraints, and, in fact, different papers often make different choices. In our paper we adopt the conventional constraints used in \cite{31} adapted to our conventions. They are given by
\[
R(P)_{abc} = 0 , \tag{2.4a} \\
R(Q)_{abj} \sigma^b = -\frac{3}{4} \Sigma_j \sigma_a , \quad R(\bar{Q})_{ab}^{\dot{j}} \bar{\sigma}^b = \frac{3}{4} \bar{\Sigma}^{\dot{j}} \bar{\sigma}_a , \quad R(M)_{abc}^{\quad cd} = R(\mathbb{D})_{abc} + 3 \eta_{ab} D - \eta^{cd} W^+_{ac} W^+_{bd} . \tag{2.4b-2.4c}
\]

Later in this section we will present the expressions of the superconformal curvatures that we need while we refer the reader to \cite{31,32} for more detail and for the relation with the results of \cite{9}.

In presenting the multiplet we restrict to all local superconformal transformations except local translations (covariant general coordinate transformations). Such transformations are identified by $\delta$ and defined by the following operator
\[
\delta = \xi_i Q^i + \bar{\xi}_i \bar{Q}^i + \lambda^{ab} M_{ab} + \lambda^{ij} J_{ij} + \lambda_{\mathbb{D}} \mathbb{D} + i \lambda_Y Y + \lambda_a K^a + \eta_{i}^{\alpha} S_i^{\alpha} + \eta_{i}^{\dot{\alpha}} \bar{S}^{\dot{i}} . \tag{2.5}
\]

The local superconformal transformation of the fundamental fields of the standard Weyl multiplet are then given by
\[
\delta e_m^a = i \xi_i \sigma^a \bar{\psi}_m^{i} + i \bar{\xi}^{\dot{\alpha}} \bar{\sigma}^a \psi_{mi} - \lambda_{\mathbb{D}} e_m^a + \lambda_a b e_m^b , \tag{2.6a}
\]
\[
\delta \psi_m^i = \left( 2\partial_m \xi^a_i + \omega^a_{m} (\xi_i \sigma_{a \lambda})^\alpha + 2\phi^a_m \xi^a_j + 2iA_m \xi^a_i + b_m \xi^a_i \right) - \frac{i}{2} (\xi_{i \sigma_m} \sigma_{cd})^\alpha W_{cd}^+ \\
- \frac{1}{2} \lambda^{ab} (\bar{\psi}_m \sigma_{ab})^\alpha - \lambda^j \psi^a_j - i\lambda Y \psi^a_i - \frac{1}{2} \lambda^a \psi^a_i + 2i(\bar{\eta} \sigma_m)^\alpha ,
\]
\[
\delta \bar{\psi}^i_m = \left( 2\partial_m \bar{\xi}_i + \omega^a_{m} (\bar{\xi}_i \bar{\sigma}_{ab})_\lambda^\alpha - 2\phi^a_m \bar{\xi}^i + 2iA_m \bar{\xi}_i + b_m \bar{\xi}_i \right) + \frac{i}{2} (\xi_{i \sigma_m} \sigma_{cd})_\lambda^\alpha W_{cd}^- \\
- \frac{1}{2} \lambda^{ab} (\bar{\psi}_m \sigma_{ab})_\lambda^\alpha + \lambda^j \bar{\psi}^a_j + i\lambda Y \bar{\psi}^a_i - \frac{1}{2} \lambda^a \bar{\psi}^a_i + 2i(\bar{\eta} \sigma_m)_\lambda^\alpha ,
\]
\[
\delta \phi^i_m = \left( \partial_m \alpha_i^\lambda - 2\phi_m (i^k \lambda^j)^i \right) + \frac{3i}{2} \xi(i \sigma_m \Sigma^j) + \frac{3i}{2} \xi(i \bar{\sigma}_m \Sigma^j) - \phi_m (i \xi^j) + \bar{\phi}_m (i \bar{\xi}^j) \\
+ 2\psi_m (i \bar{\eta}^j) - 2\bar{\psi}_m (i \eta^j) ,
\]
\[
\delta A_m = \partial_m \lambda^\lambda Y - \frac{3}{8} \xi_i \sigma_m \Sigma^i - \frac{3}{8} \bar{\xi}_i \bar{\sigma}_m \Sigma^i + \frac{1}{2} \xi_i \phi^i - \frac{1}{2} \xi^i \bar{\sigma}_m + \frac{1}{2} \bar{\xi}_i \eta^\lambda + \frac{1}{2} \bar{\bar{\eta}}_i \bar{\sigma}_m ,
\]
\[
\delta b_m = \partial_m \lambda^\lambda \bar{\sigma} - \frac{3i}{4} \xi_i \sigma_m \Sigma^i + \frac{3i}{4} \bar{\xi}_i \bar{\sigma}_m + \xi \phi^i + \bar{\xi}_i \bar{\sigma}_m + \psi_m \eta^i - \bar{\psi}_m \bar{\sigma}_m - 2\lambda m ,
\]
\[
\delta W_{ab} = -4\xi_k R(Q)_{abk}^c - 3\xi_k R(Q)_{abk}^c - 2\lambda [a \bar{W}^c_{bc} + \lambda \bar{D}^c_{ab} - 2i\lambda Y \Sigma^c_{ab} + 2i\lambda Y \bar{W}^c_{ab} ] ,
\]
\[
\delta D = -i\xi^a \sigma^a \nabla_a \Sigma_{\lambda \delta} - i\bar{\xi}_a \sigma^a \nabla_a \Sigma_{\lambda \delta} + 2\lambda D ,
\]
\[
\delta \Sigma^i = \xi^a D + \frac{4i}{3} (\xi^a \sigma^{ab})^\alpha R(Y)_{abj} + \frac{2}{3} (\xi^a \sigma^{ab})^\alpha R(J)_{abj} - i\frac{3}{2} (\xi^a \sigma^{ab})^\alpha \nabla_a W^+_{cd} \\
- \frac{1}{2} \lambda^{ab} (\Sigma^a \sigma_{ab})^\alpha + \lambda^i \sigma^{ij} + \frac{3}{2} \lambda^a \Sigma^a + i\lambda Y \Sigma^i + \frac{2}{3} (\eta^a \sigma^{ab})_\lambda^\alpha W^+_{cd} ,
\]
\[
\delta \Sigma_{\lambda i} = -\xi_{\lambda i} D + \frac{4i}{3} (\xi_{\lambda i} \sigma^{ab})_\lambda^\alpha R(Y)_{ab} + \frac{2}{3} (\xi_{\lambda i} \sigma^{ab})_\lambda^\alpha R(J)_{abj} - i\frac{3}{3} (\xi_{\lambda i} \sigma^{ab})_{\lambda \delta} \nabla_a W^-_{cd} \\
- \frac{1}{2} \lambda^{ab} (\Sigma_{\lambda i} \sigma_{ab})_\lambda - \lambda^i \Sigma_{\lambda j} + \frac{3}{2} \lambda^a \Sigma_{\lambda a} + i\lambda Y \Sigma_{\lambda i} + \frac{2}{3} (\bar{\eta}_{\lambda i} \sigma_{cd})_{\lambda \delta} W^-_{cd} ,
\]
where
\[
\nabla_a W_{bc} = D_a W_{bc} + 2\psi_{ak} R(Q)_{bek} + 2\bar{\psi}_{\lambda k} R(Q)_{bek} ,
\]
\[
\nabla_a \Sigma_{\lambda i} = D_a \Sigma_{\lambda i} - \frac{1}{2} \psi^a_{\lambda i} D - \frac{2i}{3} (\psi^a_{\lambda i} \sigma^{ab})^\alpha R(Y)_{cd} - \frac{1}{3} (\psi^a_{\lambda i} \sigma^{ab})^\alpha R(J)_{cdj} \\
+ \frac{i}{6} (\bar{\psi}_{\lambda i} \sigma^{ab})^\alpha \nabla_a W^+_{cd} + (\phi^a_{\lambda i} \sigma^{ab})^\alpha W^+_{cd} ,
\]
\[
\nabla_a \Sigma_{\lambda i} = D_a \Sigma_{\lambda i} + \frac{1}{2} \bar{\psi}_{\lambda i} D - \frac{2i}{3} (\bar{\psi}_{\lambda i} \sigma^{ab})_{\lambda} R(Y)_{cd} - \frac{1}{3} (\bar{\psi}_{\lambda i} \sigma^{ab})_{\lambda} R(J)_{cdj} \\
+ \frac{i}{6} (\psi_{\lambda i} \sigma^{ab}) \nabla_a W^-_{cd} + (\bar{\phi}_{\lambda i} \sigma^{ab}) \nabla_a W^-_{cd} ,
\]
and the derivatives $D_a$ are
\[
D_a = e_{m}^a D_m = e_{a}^m \left( \partial_m - \frac{1}{2} \omega^m_{cd} M_{cd} - \phi_m i j_{ij} - i A_m Y - b_m D \right) .
\]
The composite Lorentz and $S$-supersymmetry connections are respectively

\[
\omega_{abc} = \omega(e)_{abc} - 2\eta_{a[bc]} - \frac{1}{2}(\overline{\psi}_{aj} \sigma_{\bar{b}c} \overline{\psi}_{\bar{a}}^j + \overline{\psi}_{[bj} \sigma_{c]} \overline{\psi}_{\bar{a}}^j + \overline{\psi}_{[aj} \sigma_{\bar{a}c]} \overline{\psi}_{\bar{a}}^j),
\]

(2.9)

and

\[
\begin{align*}
\phi^j_{m \beta} &= \frac{1}{4} \left( \sigma^{bc}_{m} \sigma_{\bar{a}c} - \frac{1}{3} \sigma^{bc}_{m} \sigma_{\bar{a}c} \right) \overline{\Psi}_{bc} \delta_{\beta}^j + \frac{1}{3} W_{mb}^{-} \overline{\psi}_{j \beta}^b - \frac{1}{3} W_{mb}^{-} (\sigma^{bc}_{m} \overline{\psi}_{j \beta}^c) \beta + \frac{i}{4} (\sigma_{m} \Sigma^{j})_{\beta}, \\
\Phi^j_{m \beta} &= \frac{1}{4} \left( \sigma^{bc}_{m} \sigma_{\bar{a}c} - \frac{1}{3} \sigma^{bc}_{m} \sigma_{\bar{a}c} \right) \overline{\Phi}_{bc} \delta_{\beta}^j - \frac{1}{3} W_{mb}^{+} \overline{\psi}_{j \beta}^b + \frac{1}{3} W_{mb}^{+} (\sigma^{bc}_{m} \overline{\psi}_{j \beta}^c) \delta - \frac{i}{4} (\sigma_{m} \Sigma^{j})_{\delta}.
\end{align*}
\]

(2.10a

(2.10b)

The field $\omega(e)_{abc}$ in (2.9) is the usual torsion-free Lorentz connection given in terms of the anholonomy tensor $C_{mn}^a(e)$ as

\[
\omega(e)_{abc} = -C_{a[bc]}(e) + \frac{1}{2} C_{bca}(e), \quad C_{mn}^a(e) := 2\partial_{[m}e_{n]}^a, \quad C_{ab}^c(e) := e_a^m e_b^n C_{mn}^c(e),
\]

(2.11)

while the fields $(\Psi_{ab}^\gamma, \overline{\Psi}_{ab}^k)$ are the gravitini field strengths

\[
\begin{align*}
\Psi_{ab}^\gamma &= 2e_m^a e_n^b D_{[m} \overline{\psi}_{n]}^\gamma, \quad \overline{\Psi}_{ab}^k = 2e_m^a e_n^b D_{[m} \overline{\psi}_{n]}^k.
\end{align*}
\]

(2.12)

Note that $(R(Q)_{ab}^k, R(\overline{Q})_{ab}^k)$ are the $Q$-supersymmetry curvatures and satisfy

\[
\begin{align*}
R(Q)_{ab}^k &= \frac{1}{2} \Psi_{ab}^\gamma - i(\overline{\phi}_{[a} \overline{\sigma}_{b]} \gamma) + \frac{1}{4} (\overline{\psi}_{[a} \overline{\sigma}_{b]} \sigma^{cd}) \gamma W_{cd}^+, \\
R(\overline{Q})_{ab}^k &= -\frac{1}{2} \overline{\Psi}_{ab}^\gamma - i(\phi_{[a} \sigma_{b]} \gamma) - \frac{1}{4} (\psi_{[a} \sigma_{b]} \sigma^{cd}) \gamma W_{cd}^-.
\end{align*}
\]

(2.13a

(2.13b)

while $R(Y)_{ab}$ and $R(J)_{ab}^k l$ are

\[
\begin{align*}
R(Y)_{ab} &= 2e_m^a e_n^b \partial_{[m} A_{n]} - \frac{i}{2} \overline{\psi}_{[aj} \phi_{b]}^j + \frac{i}{2} \overline{\psi}_{[aj} \overline{\phi}_{b]}^j + \frac{3}{8} \overline{\psi}_{[aj} \sigma_{b]} \Sigma^j + \frac{3}{8} \overline{\psi}_{[aj} \overline{\sigma}_{b]} \Sigma^j, \\
R(J)_{ab}^k l &= 2e_m^a e_n^b \partial_{[m} \phi_{n]}^l - 2\phi_{[a} (k \overline{\phi}_{b]}^l) + 2\psi_{[a} (k \overline{\phi}_{b]}^l) - 2\overline{\psi}_{[a} (k \phi_{b]}^l) - \frac{3}{2} \overline{\psi}_{[a} (k \overline{\sigma}_{b]} \Sigma^l) - \frac{3}{2} \psi_{[a} (k \sigma_{b]} \Sigma^l).
\end{align*}
\]

(2.14a

(2.14b)

We are not going to present the expression for the composite special conformal connection $f_{ma}$ and other superconformal curvatures. We will however use $e_m^a f_m^a$ which is given by

\[
\begin{align*}
f_a^a &= -\frac{1}{12} R + D - \frac{1}{24} \varepsilon_{mnpq} (\overline{\psi}_m \sigma_n D_p \overline{\psi}_{pq}) + \frac{1}{24} \varepsilon_{mnpq} (\overline{\psi}_{mn} \sigma_n D_p \overline{\psi}_{pq}) \\
&\quad - \frac{i}{8} \overline{\psi}_{aq} \sigma_a \Sigma^j + \frac{i}{8} \overline{\psi}_{aq} \overline{\sigma}_a \Sigma_j - \frac{1}{12} W^{ab+} (\overline{\psi}_{a}^j \psi_{bj}) + \frac{1}{12} W^{ab-} (\overline{\psi}_{aj} \psi_{b}^j),
\end{align*}
\]

(2.15)

where $R = e_m^a e_n^b R_{mn}^{ab}$ is the scalar curvature constructed from the Lorentz curvature

\[
R_{mn}^{cd} = 2\partial_{[m} \omega_{n]}^{cd} - 2\omega_{[m}^c e \omega_{n]}^d.
\]

(2.16)
Remember also that the spin connection $\omega_{m}^{cd}$ is a composite field of the vielbein, the gravitini, and the dilatation connection, eq. (2.9).

We stress that the transformations (2.6) form an algebra that closes off-shell on a local extension of SU(2,2). We will not need the explicit form of the algebra here, though it can be straightforwardly derived from results of [9] and [31,32]. To conclude this subsection, for convenience, we include Table 1 which summarises the non-trivial chiral and dilatation weights of the fields and local gauge parameters of the standard Weyl multiplet.

| $e_{m}^{a}$ | $\psi_{mi}$, $\xi_{i}$ | $\bar{\psi}_{mi}$, $\bar{\xi}_{i}$ | $\phi_{m}^{i}$, $\eta^{i}$ | $\bar{\phi}_{m}$, $\bar{\eta}_{i}$ | $W_{ab}^{+}$ | $W_{ab}^{-}$ | $\Sigma^{i}$ | $\bar{\Sigma}_{i}$ | $D$ |
|----------|-----------------|----------------|----------------|----------------|-----------------|----------------|-----------------|----------------|----------------|
| $D$      | $-1$            | $-1/2$        | $1/2$          | $1/2$          | $1$             | $1$            | $3/2$           | $3/2$          | $2$             |
| $Y$      | $0$             | $-1$          | $1$            | $-1$           | $0$             | $-2$          | $2$             | $-1$           | $1$             | $0$             |

Table 1: Summary of the non-trivial dilatation and chiral weights in the standard Weyl multiplet.

### 2.2 On-shell hypermultiplet and hyper-dilaton Weyl multiplet

A single on-shell hypermultiplet comprises $4 + 4$ degrees of freedom described by a Lorentz scalar field $q^{\hat{i}}$ and spinor fields $(\rho_{\hat{i}}^{\alpha}, \bar{\rho}_{\hat{i}}^{\dot{\alpha}})$ — see [10,13,75,76] together with [3,4,38] and references therein for superconformal approaches to systems of on-shell hypermultiplets. The index $\hat{i} = 1,2$ is an SU(2) flavour index and the fields satisfy the following reality conditions

$$(q^{\hat{i}})^{*} = q_{\bar{\hat{i}}} , \quad (\rho_{\hat{i}}^{\alpha})^{*} = \bar{\rho}_{\bar{\hat{i}}}^{\dot{\alpha}} ,$$

together with the following dilatation and chiral weight identities

$$D q^{\hat{i}} = q^{\hat{i}} , \quad D \rho_{\hat{i}}^{\alpha} = \frac{3}{2} \rho_{\hat{i}}^{\alpha} , \quad D \bar{\rho}_{\bar{\hat{i}}}^{\dot{\alpha}} = \frac{3}{2} \bar{\rho}_{\bar{\hat{i}}}^{\dot{\alpha}} ,$$

$$Y q^{\hat{i}} = 0 , \quad Y \rho_{\hat{i}}^{\alpha} = \rho_{\hat{i}}^{\alpha} , \quad Y \bar{\rho}_{\bar{\hat{i}}}^{\dot{\alpha}} = -\bar{\rho}_{\bar{\hat{i}}}^{\dot{\alpha}} .$$

The multiplet, which has the field $q^{\hat{i}}$ as its superconformal primary, is characterised by the following local superconformal transformations [3,4,10,13,38]

$$\delta q^{\hat{i}} = \frac{1}{2} \xi^{i} \rho_{\hat{i}}^{\alpha} - \frac{1}{2} \bar{\xi}^{i} \bar{\rho}_{\bar{\hat{i}}}^{\dot{\alpha}} + \lambda_{k}^{i} q^{k\hat{i}} + \lambda_{D} q^{\hat{i}} ,$$

$$\delta \rho_{\hat{i}}^{\alpha} = -4i (\sigma^{a} \bar{\xi}_{k})_{\alpha} \nabla_{a} q^{k\hat{i}} + \frac{1}{2} \lambda_{ab} (\sigma^{ab} \rho_{\hat{i}}^{\alpha})_{\alpha} + i \lambda_{Y} \rho_{\hat{i}}^{\alpha} + \frac{3}{2} \lambda_{D} \rho_{\hat{i}}^{\alpha} + 8 \eta_{\alpha}^{k} q^{k\hat{i}} .$$
\[
\delta \bar{\rho}^\alpha_i = 4i(\bar{\sigma}^a \xi^k) \nabla_a q_{k\bar{i}} + \frac{1}{2} \lambda_{ab}(\bar{\sigma}^{ab})^\alpha_{\bar{i}} - i \lambda_Y \bar{\rho}^\alpha_{\bar{i}} + \frac{3}{2} \lambda_D \bar{\rho}^\alpha_{\bar{i}} - 8i\bar{\eta}^\alpha_{\bar{i}} q^k_{\bar{i}}, \quad (2.19c)
\]

where

\[
\nabla_a q^i = D_a q^i - \frac{1}{4} \bar{\psi}_a^i \rho^i + \frac{1}{4} \bar{\psi}_a^i \bar{\rho}^i . \quad (2.20)
\]

In contrast with the standard Weyl multiplet described in the previous subsection, the algebra of the local transformations (2.19) closes only when equations of motion for the fields are imposed, see for example [4, 38] for a detailed analysis. In our notations, the covariant equations of motion of \( q^i \) and \( (\rho^\alpha_a, \bar{\rho}^\alpha_{\bar{i}}) \) are:

\[
(\nabla_a \rho^i - \bar{\sigma}^a)_{\bar{i}} = \frac{i}{2}(\bar{\rho}^i \bar{\sigma}^{cd})\alpha W_{ca}^{-} + 6i\Sigma_{a\bar{k}} q^k_{\bar{i}}, \quad (2.21a)
\]

\[
(\nabla_a \bar{\rho}_{\bar{i}} - \bar{\sigma}^a)_{\bar{i}} = -\frac{i}{2}(\rho^i \sigma^{cd})\alpha W_{ca}^{+} + 6i\Sigma_{a\bar{k}} q_{\bar{k}i}, \quad (2.21b)
\]

\[
\Box q^i = -\frac{3}{2} D q^i , \quad \Box := \nabla^a \nabla_a . \quad (2.21c)
\]

The expressions for \( \nabla_a \rho^i, \nabla_a \bar{\rho}_{\bar{i}}, \) and \( \Box q^i \) in terms of the derivatives \( D_a \) are given by

\[
\nabla_a \rho^i = D_a \rho^i + 2i(\sigma^b \bar{\psi}_{ab})_{\alpha} \left(D_b q^i - \frac{1}{4} \bar{\psi}_b^k \rho^i + \frac{1}{4} \bar{\psi}_b^k \bar{\rho}^i + 4\phi_{a\bar{k}} q^k_{\bar{i}}\right), \quad (2.22a)
\]

\[
\nabla_a \bar{\rho}_{\bar{i}} = D_a \bar{\rho}_{\bar{i}} - 2i(\bar{\sigma}^b \psi^k)_{\alpha} \left(D_b q_{\bar{i}} - \frac{1}{4} \psi_{bk} \rho^i + \frac{1}{4} \psi_{bk} \bar{\rho}^i - 4\bar{\phi}_{a\bar{k}} q^k_{\bar{i}}\right), \quad (2.22b)
\]

\[
\Box q^i = D^a D_a q^i - 2f_{a\bar{a}} q^i - \frac{1}{4} \bar{\rho}^i D_a \rho^i + \frac{1}{4} \bar{\rho}^i D_a \bar{\rho}^i - \frac{1}{2} \bar{\psi}_a^i D_a \rho^i + \frac{1}{2} \bar{\psi}_a^i D_a \bar{\rho}^i - \frac{1}{4} \phi_a^i \sigma^a_{\bar{i}} + \frac{1}{4} \bar{\phi}_a^i \sigma^a_{\bar{i}} + i(\bar{\psi}_a^i \sigma^b \bar{\psi}_{\bar{b}a})D_b q^i + \frac{3i}{4}(\bar{\psi}_a^i \sigma^a \Sigma_i)q^i + \frac{3i}{4}(\psi_a^i \sigma^a \Sigma_i)q^i
\]

\[
- \frac{1}{16}(\bar{\psi}_a^i \sigma^a \bar{\sigma}^{cd} \rho^i)W_{ca}^{-} - \frac{1}{16}(\psi_a^i \sigma^a \bar{\sigma}^{cd} \bar{\rho}^i)W_{ca}^{+} - (\psi_a^i \phi^a_{\bar{k}})q_{\bar{k}i} + (\bar{\psi}_a^i \phi^a k)q_{\bar{k}i} - \frac{1}{4}(\bar{\psi}_a^i \sigma^b \bar{\psi}_{\bar{b}a})(\psi_{bk} \rho^i) + \frac{1}{4}(\psi_a^i \sigma^c \psi_{\bar{a}c})(\psi_{ck} \bar{\rho}^i) . \quad (2.22c)
\]

It is important to stress that equations (2.21) are typically read as equations of motion for the hypermultiplet fields, see e.g., [3, 4, 10, 13, 38]. They certainly are dynamical equations for \( q^i \) and \( (\rho^\alpha_a, \bar{\rho}^\alpha_{\bar{i}}) \) in a flat background (with no central charges as in our case) where all conformal supergravity fields are set to zero [75, 76]. For this reason, the multiplet is typically referred to as the on-shell hypermultiplet. However, such an interpretation is not necessary in a curved background described by the standard Weyl multiplet. In fact, the equations (2.21) can be interpreted as algebraic equations for the standard Weyl multiplet that determine the fields \( (\Sigma^\alpha_i, \bar{\Sigma}_{\bar{\alpha}i}) \) and \( D \) in terms of \( q^i \) and
(ρthew, ρtw) together with the other independent fields of the standard Weyl multiplet. If we
assume that qii is an invertible matrix, which is equivalent to imposing

\[ q^2 := q^{ii}q_{ii} = \varepsilon_{ij}\varepsilon_{ik}q^{ij}q^{jk} = 2 \det q^{ii} \neq 0 \]

then the following relations hold

\[
\Sigma^{ai} = 2q^{-2}q^{ii}[ -\frac{i}{2}(D_{a}\bar{\rho}^{a})^{\alpha} + (\bar{\psi}_{a}^{i}j^{b}\bar{\sigma}^{a})^{\alpha} \left( D_{b}q_{ij} - \frac{1}{4}\bar{\psi}_{bj}\rho^{a} + \frac{1}{4}\bar{\psi}_{bj}\bar{\rho}^{a} \right) \\
+ \frac{2}{3}(\bar{\psi}_{abj}\sigma^{ab})^{\alpha}q_{ij} + \frac{1}{4}(\rho_{2}\sigma^{cd})^{\alpha}W_{cd}^{+} + \frac{i}{6}(\bar{\psi}_{aj}\bar{\sigma}^{a}\sigma^{cd})^{\alpha}q_{ij}W_{cd}^{+} ] , \tag{2.24a}
\]

\[
\Sigma_{ia} = 2q^{-2}q_{ii}[ -\frac{i}{2}(D_{a}\rho^{a})_{\dot{\alpha}} - (\bar{\psi}_{a}^{i}\dot{j}^{b}\sigma^{a})_{\dot{\alpha}} \left( D_{b}q_{ij} - \frac{1}{4}\psi_{bij}\rho^{a} + \frac{1}{4}\bar{\psi}_{bij}\bar{\rho}^{a} \right) \\
- \frac{2}{3}(\bar{\psi}_{abj}\sigma^{ab})_{\dot{\alpha}}q_{ij} - \frac{1}{4}(\rho_{2}\bar{\sigma}^{cd})_{\dot{\alpha}}W_{cd}^{-} + \frac{i}{6}(\psi_{aj}\bar{\sigma}^{a}\dot{\sigma}^{cd})_{\dot{\alpha}}q_{ij}W_{cd}^{-} ] , \tag{2.24b}
\]

\[
D = q^{-2}q_{ii} \left[ D^{a}D_{a}q^{ii} + \frac{1}{6}Rq^{ii} - \frac{i}{8}(\bar{\psi}_{a}^{i}\bar{\sigma}^{a}\sigma^{cd}j^{i})W_{cd}^{+} - \frac{1}{2}\phi_{a}^{i}\sigma^{a}j^{i} - \frac{1}{2}\rho^{a}D_{a}\psi_{ai} \\
- \bar{\psi}_{ai}D_{a}\bar{j}^{i} + 2(\bar{\psi}_{a}^{i}\phi^{aj})q_{ij}^{i} + \frac{3i}{2}(\bar{\psi}_{a}^{i}\sigma^{a}\Sigma_{ij})q_{ij}^{i} + \frac{i}{2}(\psi_{aj}\sigma^{a}\Sigma^{ij})q_{ij}^{i} \\
+ \frac{i}{6}\varepsilon^{mnpq}(\bar{\psi}_{m}\bar{\sigma}_{n}D_{p}\psi_{q})q_{ij}^{i} + \frac{1}{3}W_{ab}^{+}(\bar{\psi}_{a}^{i}\bar{\psi}_{bj})q_{ij}^{i} + i(\psi_{a}(i)j^{b}\bar{\psi}_{j}^{a})D_{b}q_{ij}^{i} \\
- \frac{i}{2}(\bar{\psi}_{a}(i)j^{b}\bar{\psi}_{j}^{a})(\psi_{b}\rho_{2}) \right] + c.c. . \tag{2.24c}
\]

In the expression for D, eq. \((2.24c)\), remember that (Σi, Σi) and (ϕi, ϕi), together with
the spin connection ωm, are composite fields. Note that so far we have only used one of
the four equations that are equivalent to \((2.21c)\) to solve for D in eq. \((2.24c)\). It is simple
to show that the remaining independent three equations are equivalent to the following

\[
\nabla^{a}(q^{ij}q_{ij}) = 0 . \tag{2.25}
\]

As we are going to explain in detail below, this equation is solved by turning the SU(2)R
connection ϕm into a composite field.

As a next step in the construction of the hyper-dilaton Weyl multiplet, we note that,
accompanied to an on-shell hypermultiplet there is always a triplet of composite linear
multiplets [10] 1[12] 76. An N = 2 off-shell linear multiplet [11] 1[77] 83 comprises the
following covariant fields: an SU(2)R triplet of Lorentz scalar fields Gij subject to the
reality condition \((G^{ij})^* = G_{ij}\); spinor fields \((\chi_{ai}, \tilde{\chi}^a_i)\); a complex scalar field \((F, \tilde{F})\;\); and a covariant real closed anti-symmetric three-form \(H_{abc}\), which is equivalent to a conserved dual vector \(\tilde{H}^a := \frac{1}{6}e^{abcd}H_{bcd}\). Their local superconformal transformations in a standard Weyl multiplet background are given by

\[
\delta G_{ij} = 2\xi(i\chi_j) + 2\tilde{\xi}(i\tilde{\chi}_j) - 2\lambda(i\chi_{ik}) - 2\tilde{\lambda}(i\tilde{\chi}_{ik}) + 2\lambda_b G_{ij}, \tag{2.26a}
\]

\[
\delta \chi_{ai} = -\xi_a F - 4i\tilde{H}_a(\sigma^a \tilde{\chi}_i) + i(\sigma^a \tilde{\chi}_i) + 4\eta_a G_{ij},
\]

\[
\delta \tilde{\chi}^a_i = -\tilde{\xi}^a F + 4i\tilde{H}_a(\tilde{\sigma}^a \chi^i) + i(\tilde{\sigma}^a \chi^i) + 4\tilde{\eta}^a G^{ij},
\]

\[
\delta F = -2i\tilde{\xi}^a A^a G_{ij} + 6(\tilde{\chi}^i \chi^j) + 4\tilde{\eta}_i \chi^j + 4\lambda F, \tag{2.26d}
\]

\[
\delta \tilde{F} = -2i\xi_a A^a G_{ij} + 6(\chi^i \tilde{\chi}^j) + 4\eta_i \tilde{\chi}^j + 4\lambda \tilde{F}, \tag{2.26e}
\]

\[
\delta \tilde{H}_a = \frac{1}{2} \xi_a(\tilde{\sigma}^a \tilde{\chi}_i) - \frac{1}{16} (\xi_a(\tilde{\sigma}^a \tilde{\chi}_i) - \frac{3i}{8} (\xi_a \tilde{\chi}_i) G^{ij},
\]

\[
\delta \tilde{H}_a = \frac{1}{2} \tilde{\xi}_a(\sigma_a \chi^i) - \frac{1}{16} (\tilde{\xi}_a(\sigma_a \chi^i) - \frac{3i}{8} (\tilde{\xi}_a \chi^i) G_{ij},
\]

\[
\delta H_{abc} = \frac{1}{2} \xi_a(\psi_b \chi^i) + \frac{1}{2} \tilde{\xi}_a(\tilde{\psi}_b \tilde{\chi}_i) - \frac{3i}{4} \tilde{\psi}_b \tilde{\chi}_i + \frac{3i}{4} \psi_b \chi^i, \tag{2.26f}
\]

where

\[
\nabla_a G_{ij} = \mathcal{D}_a G_{ij} - \psi_a(i\chi_j) - \tilde{\psi}_a(i\tilde{\chi}_j), \tag{2.27a}
\]

\[
\nabla_a \chi_{ai} = \mathcal{D}_a \chi_{ai} + \frac{1}{2} \psi_{ai} F + 2i(\sigma^b \psi_{ai}) \chi_b G_{ij} - \frac{1}{2} (\sigma^b \psi_{ai}) \chi_b G_{ij} - 2\phi^a_{ai} G_{ij}, \tag{2.27b}
\]

\[
\nabla_a \tilde{\chi}^a_i = \mathcal{D}_a \tilde{\chi}^a_i + \frac{1}{2} \tilde{\psi}_a \tilde{F} - 2i(\tilde{\sigma}^b \tilde{\psi}_a) \tilde{\chi}_b G_{ij} - \frac{1}{2} (\tilde{\sigma}^b \tilde{\psi}_a) \tilde{\chi}_b G_{ij} - 2\tilde{\phi}^a \tilde{G}^{ij}, \tag{2.27c}
\]

The covariant conservation equation for \(\tilde{H}_a\) is

\[
\nabla^a \tilde{H}_a = 3\chi_i + \frac{3}{8} \chi_i. \tag{2.28}
\]

The constraint implies the existence of a gauge two-form potential, \(b_{mn} = -b_{nm}\), and its exterior derivative \(h_{mnp} := 3\partial_{[m} b_{np]}\). The solution of \(2.28\)

\[
\tilde{H}_a = \frac{1}{6} e^{bcd} \left( h_{bcd} - \frac{3i}{4} \psi_b \sigma_{cd} \xi^i - \frac{3i}{4} \tilde{\psi}_b \tilde{\sigma}_{cd} \tilde{\xi}_{i} - \frac{3}{4} (\psi_{[m} \sigma_{n]} \xi^i - \psi_{[m} \sigma_{n]} \tilde{\xi}^i) G_{ij} \right), \tag{2.29}
\]

where \(h_{abc} = e_a^m e_b^n e_c^p h_{mnp}\). The locally superconformal transformations of \(b_{mn}\) are

\[
\delta b_{mn} = \frac{1}{2} \xi_r b_{mn} \chi^i + \frac{i}{2} \tilde{\xi}^r \sigma_{mn} \tilde{\chi}_i + \frac{1}{2} \left( \psi_{[m}^i \sigma_{n]} \xi^j - \psi_{[m}^i \sigma_{n]} \tilde{\xi}^j \right) G_{ij} + 2\partial_{[m} b_{n]}, \tag{2.30}
\]
where we have also included the vector gauge transformation \( \delta b_{mn} = 2\partial_{[m}l_{n]} \) that leaves \( h_{mnp} \) and \( \tilde{H}^a \) invariant. For convenience, we have summarised the dilatation and chiral weights of the fields of the linear multiplet in Table 2.

|       | \( G_{ij} \) | \( \chi^{\alpha i} \) | \( \bar{\chi}^{\dot{\alpha} i} \) | \( F \) | \( \bar{F} \) | \( \tilde{H}^a \) | \( b_{mn} \) |
|-------|-------------|----------------|----------------|------|------|------|------|
| \( \mathbb{D} \) | 2           | 5/2            | 5/2            | 3    | 3    | 3    | 0    |
| \( Y \)  | 0           | 1              | -1             | 2    | -2   | 0    | 0    |

Table 2: Summary of the dilatation and chiral weights in the off-shell linear multiplet.

Now that we have reviewed the structure of a locally superconformal linear multiplet, a straightforward analysis shows that, assuming \( q^i \) and \( (\rho^i, \bar{\rho}^i) \) describe an on-shell hypermultiplet in a standard Weyl multiplet background with transformation rules (2.19), the following composite fields define a triplet of linear multiplets [13]

\[
G_{ij}^{\underline{ij}} = q_i q_j^{\underline{ij}} = q_i (q_j^{\underline{ij}}), \quad (G_{ij}^{\underline{ij}})^* = G_{\underline{ij}}^{ij},
\]

\[
\chi^{\alpha \underline{i}} = \frac{1}{2} q^{(i} \rho^{j)}, \quad \bar{\chi}^{\dot{\alpha} \underline{i}} = -\frac{1}{2} q^{(i} \bar{\rho}^{\dot{j})}, \quad (\chi^{\alpha \underline{i}})^* = \bar{\chi}^{\dot{\alpha} \underline{i}},
\]

\[
F^{\underline{ij}} = \frac{1}{8} \rho^{(i} \rho^{j)}, \quad \bar{F}^{\underline{ij}} = \frac{1}{8} \bar{\rho}^{(i} \bar{\rho}^{j)}, \quad (F^{\underline{ij}})^* = \bar{F}^{\underline{ij}},
\]

\[
\tilde{H}^{a \underline{ij}} = -\frac{1}{4} q^{(i} \nabla^a q^{j)} + \frac{i}{32} \rho^{(i} \bar{\sigma}^a \bar{\rho}^{j)}, \quad (\tilde{H}^{a \underline{ij}})^* = \tilde{H}^{a \underline{ij}}.
\]

These fields all transform according to (2.26) and each of the previous fields is symmetric in \( \underline{i} \) and \( \underline{j} \). Within the previous composite fields, the field \( \tilde{H}^{a \underline{ij}} \) is particularly interesting. In fact, equation (2.31d) together with (2.29) represent the solution to the constraint (2.25) and can be used to express the SU(2) connection \( \phi_m^{ij} \) as a composite field. By introducing the derivative

\[
D_a = e_a^m \left( \partial_m - \frac{1}{2} \omega_m^{cd} M_{cd} - i A_m Y - b_m \mathbb{D} \right) = D_a + e_a^m \phi_m^{ij} J_{ij},
\]

and by using (2.20), eq. (2.31d) can be rearranged for the SU(2) gauge connection as follows

\[
\phi_a^{ij} = 4q^{-4} q^{(i} q_j^{j)} \left[ q^{k} D_a q_k - \frac{1}{4} q^{k} (\psi_{ak} \rho^{j}) + \frac{1}{4} q^{k} (\bar{\psi}_{ak} \bar{\rho}^{j}) - \frac{i}{8} \rho^{(i} \bar{\sigma}^a \bar{\rho}^{j)} + 4\tilde{H}_a^{ij} \right].
\]

This concludes the definition of the hyper-dilaton Weyl multiplet. The final result of our analysis is that we have identified a new representation of the off-shell local 4D,
\[ N = 2 \) superconformal algebra in terms of the following independent fields: \( e_m^a, b_m, A_m, W_{ab}, q^{\check m}, b_{mn}\check k, (\psi_{mi}, \bar{\psi}_m^i), \) and \((\rho_i, \bar{\rho}_i)\). The multiplet has precisely the same number of off-shell degrees of freedom as the standard Weyl multiplet, \( 24 + 24 \). Table 3 summarises the counting of degrees of freedom, underlining the symmetries acting on the fields. Note

| \( e_m^a \) | \( \omega_{ab}^m \) | \( b_m \) | \( f_{ma} \) | \( \phi_{mij} \) | \( A_m \) | \( \psi_{mi} \) | \( \phi_{m}^i \) | \( W_{ab} \) | \( q^{\check m} \) | \( b_{mn}\check k \) |
|---|---|---|---|---|---|---|---|---|---|---|
| 16B | 0 | 4B | 0 | 0 | 4B | 32F | 0 | 6B | 8F | 4B | 18B |
| \( P_a \) | \( M_{ab} \) | \( \mathbb{D} \) | \( K_a \) | \( J^{ij} \) | \( Y \) | \( Q \) | \( S \) | \( \lambda_{m\check k}-\text{sym} \) | | |
| -4B | -6B | -1B | -4B | -3B | -1B | -8F | -8F | -9B | | |

Table 3: Degrees of freedom and symmetries of the hyper-dilaton Weyl multiplet. Row one gives all the fields in the multiplet. Row two gives the number of independent components of these fields – composite connections are counted with zero degrees of freedom. Row three gives the gauge symmetries. Note that the parameter \( \lambda_{m\check k} \) describes the vector symmetry associated with the gauge two-forms \( b_{mn}\check k \) with field strength three-forms \( h_{mnp}\check l \) and \( \tilde{H}^{\check ml} \). Row four gives the number of gauge degrees of freedom to be subtracted when counting the total degrees of freedom. Row five gives the resulting number of degrees of freedom.

that with the ingredients provided so far, it is a straightforward exercise to obtain the locally superconformal transformations of the fundamental fields of the hyper-dilaton Weyl multiplet written only in terms of fundamental fields. These are given by \((2.6a)\)–\((2.6c)\), \((2.6e)\)–\((2.6g)\), \((2.30)\), and \((2.19a)\)–\((2.19c)\) after using the appropriate identities for all the composite fields \( \omega_{m}^{cd} \), \( f_{ma} \), \( \phi_{mij} \), \( (\phi_{mi}, \bar{\phi}_m^i) \), \( (\Sigma^{ai}, \Sigma_{ai}) \), and \( D \) respectively given by eqs. \((2.9)\), \((2.33)\), \((2.10)\), and \((2.24)\).

It is important to underline that the local gauge transformations of the hyper-dilaton Weyl multiplet form an algebra that closes off-shell on a local extension of \( SU(2, 2|2) \). In fact, by construction the resulting algebra is identical to the one of the standard Weyl multiplet transformations \((2.6)\) (see \([9]\) and \([31, 32]\) for detail on the local algebra), with the only important subtlety being that the structure functions will have more composite fields.

### 3 Gauge fixing and Müller’s Poincaré supergravity

As explained in the introduction, one of the motivations of our analysis was to show that the \( 32 + 32 \) off-shell multiplet of 4D, \( N = 2 \) Poincaré supergravity constructed by
Müller in [74] could be derived by superconformal techniques starting from the hyper-
dilaton Weyl multiplet. In this section we explain how this goes. We first focus on the
structure of the multiplet and then explain how to construct the Poincaré supergravity
action derived in [74]. At the end of this section we also extend the results of [74] by
adding a new $BF$-coupling which induces a scalar potential for the dilaton without a
standard $R$-symmetry gauging.

3.1 Hyper-Dilaton Poincaré supergravity multiplet

To recover a multiplet of Poincaré supergravity, compensating multiplets must be
coupled to the off-shell conformal supergravity multiplet to fix some of the local super-
conformal symmetries — see [3, 4] for reviews. Below we will describe how to recover
the multiplet described in [74] which we denote as the hyper-dilaton Poincaré multiplet.
The construction is straightforward. We simply need to couple the hyper-dilaton Weyl
multiplet to a single off-shell vector multiplet compensator and then appropriately gauge
fix to eliminate all symmetries except local supersymmetry, Lorentz, and the vector gauge
symmetry of the gauge two-forms $b_{mn}^{ij}$.

It is straightforward to define an off-shell 4D, $N = 2$ Abelian vector multiplet in
a hyper-dilaton Weyl multiplet background. As a first step consider an Abelian vector
multiplet [75, 84] in a standard Weyl multiplet background [9, 13, 36, 37]. This is described
by a complex scalar field $\phi$ and its conjugate $\bar{\phi} = (\phi)^*$, gaugini $(\lambda^i, \bar{\lambda}^{\dot{i}})$ such that $(\lambda^i)^* = \bar{\lambda}^{\dot{i}}$, a triplet of auxiliary fields $X^{ij} = X^{ji}$ satisfying the reality condition $(X^{ij})^* = X_{ij}$, and a real Abelian gauge connection $v_m$ or, equivalently, its covariant real field strength $F_{ab}$ given by

$$F_{ab} = e_a^m e_b^n f_{mn} - \frac{i}{2} \psi_{[ab} \sigma_{|ij]} \bar{\lambda}^k + \frac{i}{2} \bar{\psi}_{[a} \bar{\sigma}_{|kj]} \lambda^k - \frac{1}{2} (\psi_{ak} \psi_{b}) \phi + \frac{1}{2} (\bar{\psi}_{a}^{\dot{k}} \bar{\psi}_{b}) \bar{\phi} , \quad (3.1)$$

where $f_{mn} = 2 \partial_{[m} v_{n]}$. By construction $F_{ab}$ satisfies the Bianchi identity

$$\nabla_{[a} F_{bc]} = -\frac{i}{2} R(Q)_{[abj} \sigma_{|c]} \bar{\lambda}^{j} + \frac{i}{2} R(\bar{Q})_{[abj} \bar{\sigma}_{|c]} \lambda_{j} , \quad (3.2)$$

that is solved by (3.1). The non-trivial dilatation and chiral weights of the vector multiplet
fields are summarised in Table 4.

The transformation rules of the vector multiplet fields in a standard Weyl multiplet
background are

$$\delta \phi = \xi \lambda^i + \lambda_{\Sigma} \phi - 2 i \lambda_{Y} \phi , \quad (3.3a)$$
Table 4: Summary of the dilatation and chiral weights in the off-shell Abelian vector multiplet.

|   | φ | δφ | λ² | X² | Fαβ | vαβ |
|---|---|----|----|----|-----|-----|
| D | 1 | 1 | 3/2 | 3/2 | 2   | 2   |
| Y | -2 | 2 | -1 | 1   | 0   | 0   |

\[
\delta\phi = \bar{\xi}^i \bar{\lambda}_i + \lambda_D \bar{\phi} + 2i\lambda_Y \bar{\phi}, \tag{3.3b}
\]

\[
\delta\lambda^i = 2(\sigma^{ab}\xi^i)F_{ab} + (\sigma^{ab}\xi^i)_{a}W^+_{ab}\bar{\phi} - \frac{1}{2}\xi_{a j}X^i_{j} + 2i(\sigma^a\bar{\xi}^i)\nabla_a \phi
\]

\[
+ \frac{1}{2}\lambda^{ab}(\sigma_{ab}\lambda^i) + \bar{\lambda}^i_j\lambda^j + \frac{3}{2}\lambda_D \bar{\lambda}^i + i\lambda_Y \lambda^i + 4\eta^i, \tag{3.3c}
\]

\[
\delta\bar{\lambda}^i = -2(\bar{\sigma}^{ab}\xi^i)\bar{\phi} - (\bar{\sigma}^{ab}\xi^i)\bar{\phi}W^-_{ab}\phi - \frac{1}{2}\xi_{a j}\bar{X}^i_{j} + 2i(\bar{\sigma}^a\xi_i)\bar{\nabla} a \bar{\phi}
\]

\[
+ \frac{1}{2}\lambda^{ab}(\bar{\sigma}_{ab}\bar{\lambda}^i) - \lambda^j \bar{\lambda}^i_j + \frac{3}{2}\lambda_D \bar{\lambda}^i + i\lambda_Y \bar{\lambda}^i + 4\eta^i, \tag{3.3d}
\]

\[
\delta X^i_{j} = -4i\xi^i(\sigma^a \nabla_a \bar{X}^j) - 4i\xi^i(\bar{\sigma}^a \nabla_a \lambda^j) + 2\lambda^i(k X^j) + 2\lambda_D X^i_{j}, \tag{3.3e}
\]

\[
\delta F_{ab} = \left[-i\xi_{k}\sigma_{[a} \nabla_{b]} \bar{\lambda}^k + 2(\xi_{k} R(Q)_{ab}^k) \bar{\phi} - \frac{1}{2}(\xi_{k} \lambda^k)W^-_{ab} + 2\eta^k \sigma_{ab} \lambda^k + \text{c.c.} \right]
\]

\[
+ 2\lambda_D F_{ab} - 2\lambda_{[a} \bar{c} F_{bc]}, \tag{3.3f}
\]

\[
\delta v_{m} = (\xi_{k} \psi_{m}^k) \bar{\phi} - (\bar{\xi}^k \bar{\psi}_{mk}) \phi + \partial m \lambda_V, \tag{3.3g}
\]

where

\[
\nabla_a \phi = D_a \phi - \frac{1}{2}
\]

\[
\nabla_a \bar{\phi} = D_a \bar{\phi} - \frac{1}{2}
\]

\[
\nabla_a \lambda_a = D_a \lambda_a - (\sigma^{cd} \psi_{cd} a) \frac{1}{2}(F_{cd}^+ + \frac{1}{2}W_{cd}^+ \bar{\phi}) + \frac{1}{4}\psi_{a a j} X^i
\]

\[
- i(\sigma^b \bar{\psi}_{a}^i) \nabla_b \phi - 2\phi^i a \phi, \tag{3.4a}
\]

\[
\nabla_a \bar{\lambda}^i_a = D_a \bar{\lambda}^i_a + (\sigma^{cd} \bar{\psi}_{cd} a) \frac{1}{2}(F_{cd}^- + \frac{1}{2}W_{cd}^- \bar{\phi}) + \frac{1}{4}\psi_{a a j} \bar{X}^i
\]

\[
- i(\bar{\sigma}^b \psi_{a}^i) \nabla_b \phi - 2\phi^i a \bar{\phi}, \tag{3.4b}
\]

and we have also included in (3.3g) the gauge field transformation parametrised by the local real parameter \(\lambda_V\). The transformations of the vector multiplet in a hyper-dilaton Weyl multiplet background are precisely the same with the only subtlety that one has to interpret several standard Weyl multiplet fields as composite of \(q^i\), \(p^i_{\alpha}\), \(p^i_{\bar{\alpha}}\), and \(b_{mn}^i\).

The compensating vector multiplet contains 8 + 8 off-shell degrees of freedom. Once added to the hyper-dilaton Weyl multiplet we obtain the right number of off-shell degrees
of freedom, $32 + 32$, of the hyper-dilaton Poincaré multiplet [74] but in a manifestly superconformal setting. We can then obtain the structure of the Poincaré multiplet, including its local transformation rules, after gauge fixing.

The first set of gauge fixing conditions are

$$\phi = 1 , \quad \bar{\phi} = 1 ,$$

$$b_m = 0 .$$

(3.5a)

(3.5b)

The condition (3.5a) fixes dilatation and $U(1)_R$ symmetries, while (3.5b) fixes special conformal $K^a$ symmetry. Next we impose

$$\lambda^i_\alpha = 0 , \quad \bar{\lambda}^{\dot{\alpha}}_i = 0 ,$$

(3.5c)

which gauge fixes $S$-supersymmetry. A characterising feature of the hyper-dilaton Weyl multiplet is that it contains an $SU(2)_R$ compensator, the $q^i_\lambda$ fields. As a last gauge fixing condition we then impose

$$q^i_\lambda = -\varepsilon^i_\lambda e^{-U} \iff q^i_\lambda = \delta^i_\lambda e^{-U} \iff q^i_\lambda = -\delta^i_\lambda e^{-U} \iff q^i_\lambda = \varepsilon^i_\lambda e^{-U} ,$$

(3.5d)

which breaks $SU(2)_R$. After imposing the previous gauge fixing conditions, the remaining fundamental fields in the multiplet match those of the hyper-dilaton Poincaré supergravity multiplet [74] as summarised in table 5. The fundamental fields are the vielbein $e^a_m$,

| $e^a_m$ | $\omega^c_m$ | $A_m$ | $(\psi^\alpha_m, \bar{\psi}^{\dot{\alpha}}_m)$ | $W_{ab}$ | $(\rho^{\alpha}_a, \bar{\rho}^{\dot{\alpha}}_a)$ | $U$ | $b^i_{mn}$ | $X^{ij}$ | $v_m$ |
|-------|-------------|------|--------------------------------|--------|-----------------|------|------|------|------|
| 16B   | 0           | 4B   | 32F                           | 6B     | 8F              | 1B   | 18B  | 3B   | 4B   |
| $P_a$ | $M_{ab}$    |      | $Q$                           |        | $(\lambda^i_\lambda)$ |      | $(\lambda^i_V)$ |
| $-4B$ | $-6B$       |      | $-8F$                         |        | $-9B$           |      | $-1B$ |

Result: $32 + 32$ degrees of freedom

Table 5: Hyper-Dilaton Poincaré multiplet. Row 1 gives all fields in the multiplet. Row two gives the number of independent components of these fields. Row three gives the surviving gauge symmetries. Row four gives the number of gauge degrees of freedom to be subtracted when counting the total degrees of freedom. Row five gives the resulting degrees of freedom. The parameter $\lambda^i_\lambda$ describes the vector symmetry associated with the triplet of gauge two-form $b^i_{mn}$. The gauge parameter $\lambda_V$ describes the scalar symmetry of $v_m$.

the gravitini $(\psi^\alpha_m, \bar{\psi}^{\dot{\alpha}}_m)$, a real vector field $A_m$, a real antisymmetric tensor $W_{ab}$, a real scalar field that plays the role of a dilaton $U$, a real triplet of scalar fields $X^{ij}$, a triplet of gauge two forms $b^i_{mn}$, a gauge field $v_m$ that plays the role of the graviphoton, and spinor
fields \((\rho_\alpha, \bar{\rho}_\dot{\alpha})\). The residual gauge transformations of the multiplet are described by covariant general coordinate transformations \((\xi^a)\), local Lorentz transformations \((\lambda_{ab})\), local supersymmetry \((\xi^a, \bar{\xi}_\dot{a})\), and Abelian scalar \((\lambda_V)\) and vector \((\lambda_m \tilde{\lambda})\) gauge transformations. Note that we have kept the distinction of \(\text{SU}(2)_R\) and \(\text{SU}(2)\) flavour indices. However, thanks to the second gauge condition in \(3.5d\), after gauge fixing the two indices can be identified.

The transformation rules of the resulting Poincaré supergravity multiplet \([74]\) are those that preserve the previous gauge conditions \(3.5\). To preserve the gauge condition \(3.5a\) we need to impose \(\lambda_D \equiv 0\) and \(\lambda_Y \equiv 0\). Since \(Q\)-supersymmetry do not preserve the gauge, it is necessary to accompany these transformations with appropriate \(S\)-supersymmetry, special conformal, and \(\text{SU}(2)_R\) compensating transformations. To preserve the gauge condition \(3.5c\), by examining the transformations \(3.3c\) and \(3.3d\), it is straightforward to show that any \(Q\)-supersymmetry transformation has to be accompanied by a compensating \(S\)-supersymmetry transformation with parameter

\[
\eta_a^i(\xi) = -\frac{1}{2}(\sigma^{cd} \xi^i)_{\alpha} \left(F^+_{cd} \xi^j + \frac{1}{2}W^+_{cd} \right) + \frac{1}{8} \xi_{\alpha j} X^{ji} + (\sigma^a \bar{\xi})_{\alpha} A_a , \tag{3.6a}
\]

\[
\bar{\eta}_\dot{a}^i(\xi) = \frac{1}{2}(\bar{\sigma}^{cd} \bar{\xi}_i)_{\dot{a}} \left(F^-_{cd} \bar{\xi}^j + \frac{1}{2}W^-_{cd} \right) + \frac{1}{8} \bar{\xi}_{\dot{a} j} X_{ji} - (\bar{\sigma}^a \xi)_{\dot{a}} A_a . \tag{3.6b}
\]

A similar analysis shows that to preserve the gauge condition \(b_m = 0\) one needs to enforce nontrivial compensating special conformal \(K\)-transformations with a parameter \(\lambda^a(\xi)\). However, since all the other supergravity fields are conformal (not necessarily superconformal) primaries, not transforming under special conformal boosts, in practice we will never have to worry about inserting the compensating \(\lambda^a(\xi)\) parameter (whose expression is quite involved) in any Poincaré supergravity transformations. The last gauge fixing condition which is not preserved is \(3.5d\). It is straightforward to check that we can consistently have \(\delta q^{ii} = 0\) by implementing in \(2.19a\) a compensating \(\text{SU}(2)_R\) transformation with the following parameter

\[
\lambda^{i}(\xi) = -e^U \left[\xi^{i}(\rho^j - \bar{\xi}^{i}(\bar{\rho}^\dot{j})\right] , \tag{3.7}
\]

where \(\rho^i = \delta^i_\alpha \rho^\alpha\) and \(\bar{\rho}_i = \delta^i_\dot{a} \bar{\rho}_\dot{a}\).

At this stage, one has all the ingredients to obtain the transformation rules of any matter multiplet in the gauge fixed, Poincaré supergravity frame, by appropriately implementing the previous compensating gauge parameters in the superconformal transformation rules. The resulting local transformation rules of the hyper-dilaton Poincaré multiplet form an algebra that closes off-shell on a local extension of the \(\mathcal{N} = 2\) super-Poincaré algebra with no residual \(R\)-symmetry. The structure of the algebra coincides,
up to notation, with results in [74]. A detailed presentation of the hyper-dilaton Poincaré multiplet and its coupling to matter in the superconformal framework of this section will be given elsewhere as it is not necessary for the rest of our paper. It is worth underlining that, as explained by Müller in [74], the resulting 32 + 32 multiplet describes an irreducible off-shell representation. This differs from the case of the standard 40 + 40 multiplet of off-shell $N = 2$ Poincaré supergravity [9,71–73] which, for example, can arise by coupling the standard Weyl multiplet to two compensators given by an off-shell vector and a hypermultiplet (the simplest of which is probably an off-shell linear multiplet).

3.2 Hyper-Dilaton Poincaré supergravity action

With the hyper-dilaton Poincaré’s multiplet recovered using a superconformal approach, we can now describe how to obtain the Poincaré supergravity action first constructed in [74]. Once more, the construction is straightforward. In fact, the action derives from the kinetic action of the vector multiplet compensator in a hyper-dilaton Weyl multiplet background after imposing the gauge fixing conditions (3.5). We will describe this construction by focusing only on the bosonic fields.

As a starting point we consider the bosonic sector of the chiral density formula for a system of vector multiplets possessing scalar fields $\phi^I$ with prepotential $F(\phi^I)$ in a standard Weyl multiplet background [13, 32, 36, 37]. This supersymmetric invariant has the following bosonic Lagrangian

$$e^{-1} L_{\text{bosonic}} = \mathcal{F}_I \Box \bar{\phi}^I + 3 \mathcal{F}_I \phi^I D + \frac{1}{32} \mathcal{F}_{IJ} X^{Ij} X^j_I - \mathcal{F}_{IJ} f^{+Iab} f_{+J}^{ab} - \frac{1}{2} \mathcal{F} W^{-ab} W_{-ab}$$

$$- \mathcal{F}_I W^{-ab} f_{ab}^I - \mathcal{F}_{IJ} W^{+ab} f_{+ab}^J - \frac{1}{4} \mathcal{F}_I \bar{\phi}^J \bar{\phi}^J W^{+ab} W_{-ab} + \text{c.c.}, \quad (3.8)$$

where $\mathcal{F}_I = \frac{\partial F(\phi)}{\partial \phi^I}$ and $\mathcal{F}_{IJ} = \frac{\partial^2 F(\phi)}{\partial \phi^I \partial \phi^J}$. We refer the reader to equation (3.30) of [32] for a derivation of the previous Lagrangian. In our case, we have only one vector multiplet and the only possible function $F$ we can choose which leads to a locally superconformal invariant is given by

$$F(\phi) = -\frac{1}{4} \phi^2. \quad (3.9)$$

Here the overall factor is chosen for later convenience. Once we insert (3.9) into (3.8) and take into consideration that we are working with a hyper-dilaton Weyl multiplet rather than a standard Weyl multiplet (meaning that (2.24c) has to be used), we obtain the following

$$e^{-1} L_{\text{bosonic}} = -\frac{1}{4} |\phi|^2 R - \frac{|\phi|^2}{q^2} q_a^i D^a D_a q^i_{-} - \frac{1}{2} \phi D^a D_a \bar{\phi} - \frac{1}{64} X^{ij} X_{ij} + \frac{1}{2} f^{+ab} f_{+ab}$$
\[ + \frac{1}{2} \phi W^{-ab} f^- + \frac{1}{2} \phi W^{+ab} f^+ + \frac{1}{8} \phi^2 W^{-ab} W^- + \frac{1}{8} \phi^2 W^{+ab} W^+ + \text{c.c.} \]  
(3.10)

Note that in the previous Lagrangian there is a dependence upon the triplet of gauge two-forms \( b_{mn}^{ij} \) which is still hidden in the SU(2)_R connection inside the \( D_a \) derivatives, see eq. (2.33).

The final step to obtain the bosonic sector of the Poincaré supergravity of [74] is to impose the gauge fixing conditions (3.5). Upon implementing these conditions, the resulting Poincaré supergravity Lagrangian turns out to be

\[ e^{-1} \mathcal{L}_{\text{bosonic}} = -\frac{1}{2} R + \frac{1}{2} f^{ab} f_{ab} + W^{ab} f_{ab} + \frac{1}{4} W^{ab} W_{ab} - 2(\partial_m U) \partial^n U \]
\[ + 16 e^{4U} \tilde{h}^a_{kl} \tilde{h}^a_{kl} - \frac{1}{32} X^{ij} X_{ij} + 4 A^a A_a . \]  
(3.11)

Note that here \( b_{mn}^{kl} = \delta^k_i \delta^l_j b_{mn}^{ij} \) and \( \tilde{h}^a_{kl} = \delta^k_i \delta^l_j \tilde{h}^a_{ij} \) since we have stopped distinguishing between underlined and non-underlined SU(2) indices after gauge fixing.

The structure of our and Müller's Lagrangian in [74] coincide up to change of notation. It is a straightforward exercise to derive the fermionic extension of the previous Lagrangian. This result will be presented elsewhere together with a discussion of more general supergravity-matter couplings based on the hyper-dilaton Weyl multiplet and the associated hyper-dilaton Poincaré supergravity.

To conclude let us analyse the on-shell structure of (3.11). It is clear that \( W_{ab}, X^{ij}, \) and \( A_a \) are auxiliary fields that can be algebraically integrated out by using the equations of motion

\[ W_{ab} = -2 f_{ab} , \quad X^{ij} = 0 , \quad A_a = 0 . \]  
(3.12)

Once the previous equations are used in (3.11), one obtains the on-shell Lagrangian

\[ e^{-1} \mathcal{L}_{\text{bosonic}} = -\frac{1}{2} R - \frac{1}{2} f^{mn} f_{mn} - 2(\partial_m U) \partial^n U + 16 e^{4U} \tilde{h}^m_{kl} \tilde{h}^m_{kl} . \]  
(3.13)

The first two terms describe the standard kinetic terms for minimal on-shell \( \mathcal{N} = 2 \) Poincaré supergravity with a dynamical graviton and graviphoton. The last two terms describe a dilaton and a triplet of dynamical gauge two-forms which are not part of the minimal on-shell \( \mathcal{N} = 2 \) Poincaré supergravity multiplet. In fact, these fields describe the bosonic sector of an on-shell hypermultiplet where three of the scalars have been dualised into real gauge two-forms [74]. The same holds by including the fermionic sector.
3.3 BF-coupling and dilaton potential

To conclude this section we consider an extension of the original result of Müller from [74] and show how to construct by using superconformal techniques a new off-shell supersymmetric invariant that, e.g., leads to a non-trivial scalar potential for the dilaton.

Given a vector multiplet and a linear multiplet, we consider the local supersymmetric extension of a BF-action in a standard Weyl multiplet background [10]. We refer the reader to [32] for a derivation of the locally superconformal invariant, including fermionic terms, in the notation used in our paper. The bosonic part of such an invariant is

\[ e^{-1} \mathcal{L}_{\text{BF}}|_{\text{bosonic}} = F \phi + \bar{F} \bar{\phi} + \frac{1}{4} G_{ij} \bar{X}^{ij} - 2 \varepsilon^{mnpq} b_{mn} f_{pq}, \]

\[ = F \phi + \bar{F} \bar{\phi} + \frac{1}{4} G_{ij} X^{ij} - 8 \bar{h}^m v_m. \]

By construction, the supersymmetric BF-action is also well defined as an invariant in a hyper-dilaton Weyl background. We can readily construct an invariant of this form by considering the off-shell vector multiplet compensator used in this section and an off-shell linear multiplet given by

\[ G_{ij} := \xi_{ij}, \quad \bar{\chi}_{\alpha i} := \xi_{ij} \chi_{\alpha i}, \quad \bar{\bar{\chi}}_{\dot{\alpha} i} := \xi_{ij} \bar{\chi}_{\dot{\alpha} i}, \quad F_{ij} := \xi_{ij} F_{ij}, \quad \bar{F}_{ij} = \xi_{ij} \bar{F}_{ij}, \quad b_{mnij} := \xi_{ij} b_{mnij}, \quad \bar{H}^{a\dot{a}ij} = \xi_{ij} \bar{H}^{a\dot{a}ij}. \]

(3.15a) Here \( G_{ij}, \chi_{\alpha i}, \bar{\chi}_{\dot{\alpha} i}, F_{ij}, \bar{F}_{ij}, b_{mnij}, \) and \( \bar{H}^{a\dot{a}ij} \) are fields of the composite triplet of linear multiplets (2.31) constructed in terms of fundamental fields of the hyper-dilaton Weyl multiplet, while

\[ \xi_{ij} = \xi_{ij}, \quad (\xi_{ij})^* = \xi_{ij}, \]

is a real triplet of (structure group invariant) constants. The bosonic part of the resulting Lagrangian is

\[ e^{-1} \mathcal{L}_\xi|_{\text{bosonic}} = \xi_{ij} \left( \frac{1}{4} q_{\dot{i}_i j} \dot{X}^{ij} - 2 \varepsilon^{mnpq} b_{mn} f_{pq} \right) = \xi_{ij} \left( \frac{1}{4} q_{\dot{i}_i j} \dot{X}^{ij} - 8 \bar{h}^m v_m \right). \]

(3.17)

After imposing the gauge fixing conditions (3.5), and adding the previous term into (3.11), we obtain the following Lagrangian

\[ e^{-1} \mathcal{L}|_{\text{bosonic}} = -\frac{1}{2} R + \frac{1}{2} f_{ab} f_{ab} + W_{ab} f_{ab} + \frac{1}{4} W_{ab} W_{ab} - 2 (\partial_m U) \partial^m \bar{U} + 4 A^a A_a + 16 e^{4U} \bar{h}^{a} k_l \bar{h}_{a}^{kl} - 2 \xi_{ij} \varepsilon^{mnpq} b_{mn} f_{pq} - \frac{1}{32} X^{ij} X_{ij} + \frac{1}{4} \xi_{ij} e^{-2U} X^{ij}, \]

(3.18)

where, after gauge fixing, we have used \( \xi_{ij} = \delta^i_{ij} \xi_{ij} \) and \( b_{mnij} = \delta^i_{ij} b_{mnij} \). As for the undeformed Lagrangian (3.11), \( W_{ab}, X^{ij}, \) and \( A_a \) are auxiliary fields that can be
algebraically integrated out. With the $\xi$-deformation turned on, the equations of motion obtained from (3.18) are

$$W_{ab} = -2f_{ab}, \quad X^{ij} = -4\xi^{ij}e^{-2U}, \quad A_a = 0.$$  \hspace{1cm} (3.19)

Once these equations are used in (3.18), we obtain the on-shell Lagrangian

$$e^{-1}L|_{bosonic} = -\frac{1}{2}R - \frac{1}{2}f^{mn}f_{mn} - 2(\partial_m U)\partial^m U + 16e^{4U}\tilde{h}_m^{kl}\tilde{h}_m^{kl} + \xi^2 e^{-4U} + 2\xi^{ij}\varepsilon^{mnpq}b_{mn}^{ij}f_{pq},$$  \hspace{1cm} (3.20)

where

$$\xi^2 := \frac{1}{2}\xi^{ij}\xi_{ij} \geq 0.$$  \hspace{1cm} (3.21)

The first line coincides with the on-shell hyper-dilaton Poincaré supergravity (3.13) containing the standard minimal on-shell $\mathcal{N} = 2$ Poincaré supergravity coupled to a dilaton and a triplet of dynamical real gauge two-forms. Interestingly, the $\xi$-deformation induces a scalar potential for the dilaton together with a $BF$-coupling between the graviphoton and one of the three gauge two-forms of the hyper-dilaton Poincaré multiplet (the component $b_{\xi mn} = \xi^{ij}b_{mn}^{ij}$ parallel to the $\xi^{ij}$ direction). We now conclude by commenting the results obtained in this subsection.

By considering a flat limit with $e_m^a \to \delta_m^a, \ b_{mn}^{ij} \to 0, \ U \to 0, \text{ and } q^{ij} \to \delta^{ij}$, and by keeping dynamical the vector multiplet with auxiliary field $X^{ij}$, the Lagrangian (3.17) turns into

$$L_{\xi}^{\text{flat}}|_{bosonic} = \frac{1}{4}\xi^{ij}X^{ij}.$$  \hspace{1cm} (3.22)

This is a standard (electric) Fayet–Iliopoulos (FI) term for an $\mathcal{N} = 2$ vector multiplet [75,85]. The invariant (3.17) can be considered as a curved extension of a FI term in a hyper-dilaton Weyl multiplet background. If one were to choose a different gauge fixing to Poincaré supergravity where $q^{ij} \to \delta^{ij}$ (a condition that would lead to the same model but in a string frame), the dependence upon the dilaton would disappear from the term linear in $X^{ij}$. This straightforwardly shows that if one restricts to a sector with constant dilaton, the $\xi$-deformation leads to a negative cosmological constant $\Lambda = -\xi^2 \leq 0$. Hence, the deformed model (3.20) admits an AdS$_4$ vacuum with constant negative curvature proportional to $\xi^2$.

Another interesting aspect to comment about is how the $SU(2)_R$ symmetry plays a sharply different role for the FI terms in off-shell $\mathcal{N} = 2$ supergravity based on the
standard Weyl multiplet compared to the hyper-dilaton Weyl multiplet case and the \( \xi \)-deformed Müller supergravity described above. When working with the standard Weyl multiplet (and even vector-dilaton Weyl multiplets), there is a close interplay between the SU(2)\(_R\) symmetry, the gauging of isometries of scalar field manifolds, and the emergence of non-trivial scalar potentials. Within the superconformal tensor calculus, this was already noticed in early investigations of systems of Abelian vector multiplets [13, 37], and then extended to general hypermultiplet sigma-models [39, 40]. By working with the standard Weyl multiplet, the SU(2)\(_R\) connection is an auxiliary field. In the presence of FI terms, its equations of motion identify the \( R \)-symmetries of the theory with the symmetries gauged by the \( \mathcal{N} = 2 \) vector multiplets. This leads to non-trivial scalar potentials together with charges and masses for the gravitini — see [4, 91] for reviews. The simplest case is the one of the standard Weyl multiplet coupled to a single vector multiplet and a single hypermultiplet compensator. In this case, the scalar potential is a simple negative cosmological constant and an FI term identifies on-shell the \( R \)-symmetry connection with the graviphoton. In contrast, by using the hyper-dilaton Weyl multiplet, the SU(2)\(_R\) connection is a composite field. After gauge fixing, on-shell the \( R \)-symmetry is completely broken, its connection is identified with the field strength of a dynamical gauge two-form and the \( \xi \)-deformation introduces a dynamical BF-coupling. As a result, one has an alternative procedure to obtain non-trivial scalar potentials compared to a setting based on gaugings in an \( \mathcal{N} = 2 \) standard Weyl multiplet. It will be worth exploring this mechanism for more general off-shell matter systems coupled to a hyper-dilaton Weyl multiplet, potentially including more physical hypermultiplets.

4 Conclusion and future directions

In our paper we have defined a new 24 + 24 so-called hyper-dilaton Weyl multiplet of \( \mathcal{N} = 2 \) conformal supergravity in four dimensions. The construction is based on reinterpreting the equations of motion for an on-shell hypermultiplet as constraints that render some of the fields of the standard Weyl multiplet composite. By coupling the hyper-dilaton Weyl multiplet to an off-shell vector multiplet compensator, we have obtained a minimal 32 + 32 off-shell multiplet of \( \mathcal{N} = 2 \) Poincaré supergravity that was constructed

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4See also [86–90] for discussions concerning gauging and scalar potentials by using alternative on-shell supergravity approaches.

5If one considers higher-derivative interactions, it is possible to construct very general scalar potentials without gauged \( R \)-symmetry by using a standard Weyl multiplet and new types of \( \mathcal{N} = 2 \) FI terms, see [92].
by Müller in [74] and then, by using superconformal techniques, we have shown how to reproduce the supergravity action of [74]. This contains the minimal on-shell $\mathcal{N} = 2$ Poincaré supergravity coupled to a hypermultiplet where one of its physical scalars plays the role of a dilaton while its three other scalars are dualised to a triplet of real gauge two-forms. We have then described how a superconformal $BF$-coupling induces a scalar potential for the dilaton without a standard gauging. There are several future directions that our work is opening up. In the following we are going to mention a few.

As mentioned in the introduction, vector-dilaton Weyl multiplets were used in the past to study off-shell supergravity in five and six space-time dimensions, see [14,18] for descriptions in terms of component fields and also [53,55,59,93] for analyses in superspace. We are currently working towards extending our construction for hyper-dilaton Weyl multiplets in other $D \leq 6$ space-time dimensions. Note also that much of the results obtained in our paper were obtained by using the conformal superspace approach to $\mathcal{N} = 2$ conformal supergravity described in [31,32]. We will present superspace analyses together with more detailed derivations of our results, and extensions to $D \leq 6$ dimensions in the near future.

One of the main motivations of our work was to explore alternative, yet simple, off-shell engineering of non-trivial scalar potentials in 4D, $\mathcal{N} = 2$ supergravity. The results in subsection 3.3 are a first step in this direction. While we have only presented in this paper an off-shell Poincaré supergravity based on the hyper-dilaton Weyl multiplet coupled to a single off-shell vector multiplet compensator, a straightforward generalisation, part of a current work in progress, is to look at generic systems of (Abelian) vector multiplets. It is well known that for these systems, non-trivial scalar potentials in 4D, $\mathcal{N} = 2$ supergravity are associated to Fayet-Iliopoulos (FI) terms. These couplings are known to take two forms, either electric or magnetic FI terms. The electric and magnetic nomenclature arise from the role that extensions of electro-magnetic duality of Maxwell theory play in 4D, $\mathcal{N} = 2$ supersymmetry. In the case of global supersymmetry, electric and magnetic FI terms are well understood both on-shell and off-shell, see [94,100]. They play an important role in the description of spontaneous full and partial breaking of supersymmetry. They are also key ingredients in supergravity descriptions of compactified string theories with fluxes and various patterns of supersymmetry breaking, see e.g., [101] and references therein. In supergravity, the off-shell description of 4D, $\mathcal{N} = 2$ magnetic FI terms (and magnetic gaugings) has not been developed in full generality yet, though they are expected to play an important role in engineering scalar potentials in supergravity models possessing vacua with both positive and negative cosmological constant – see
for instance the recent discussion of magnetic 4D, $\mathcal{N} = 1$ FI terms \[102\]. The curved superspace constraints for off-shell magnetic FI terms were introduced in \[103, 104\] and in depth supergravity analyses in components (though not fully off-shell) were presented earlier in \[103, 106\]. By using a hyper-dilaton Weyl multiplet it is straightforward to engineer generic electric and magnetic FI-type terms by means of composite linear multiplets. We have already described how supergravity extensions of electric $\xi$-deformations can be obtained by using the $BF$-coupling (3.14) in terms of the composite linear multiplet (3.15). Off-shell magnetic FI-type deformations in a hyper-dilaton Weyl multiplet background can easily be engineered in terms of the same composite linear multiplet. This would, for example, appear as an imaginary deformation of the $X_{ij}$-auxiliary real field of a vector multiplet. Such deformations would be parametrised by the composite field $G_{ij} = \zeta_{ij} q_i q_j$ with $\zeta_{ij} = \zeta_{ji}$, $\zeta_{ij}^* = \zeta_{ij}$ constants that generalise the magnetic FI terms of global supersymmetry. Given a system of $n + 1$ vector multiplets with scalar fields $\phi^I$ (with $I = 0, 1, \cdots, n$) coupled to the off-shell hyper-dilaton Weyl multiplet, it is then straightforward to introduce $3(n + 1)$ off-shell deformations each associated to either a $\xi_{ij}^I$ electric deformation or a $\zeta_{ij}^I$ magnetic deformation. These induce non-trivial scalar potentials and vacuum structures. We plan to report in the near future on work in progress based on this direction and to extend these analyses also by including more physical hypermultiplets.

Up until now, dilaton Weyl multiplets for 4D $\mathcal{N} = 2$ conformal supergravity have been constructed by coupling the standard Weyl multiplet to either an on-shell vector multiplet \[69\] or an on-shell hypermultiplet (the latter in our current paper). It is quite clear that other variant dilaton Weyl multiplets might exist. A natural possibility is to couple the standard Weyl multiplet to either an on-shell linear (tensor) multiplet or an on-shell vector-tensor multiplet – see, e.g., \[32, 80, 107, 112\] for references on the vector-tensor multiplet including its coupling to conformal supergravity. It would be interesting to make these constructions explicitly and explore the peculiarities of these possible other dilaton Weyl multiplets in the study of off-shell 4D, $\mathcal{N} = 2$ Poincaré supergravity.

Another natural direction for future research is the construction of higher-derivative actions based on the hyper-dilaton Weyl and the hyper-dilaton Poincaré multiplets. Higher-derivative supergravity naturally arise in the low-energy description of string theory but, despite its importance, it is still poorly understood. Vector-dilaton Weyl multiplets have been successfully used to construct several off-shell higher-derivative supergravities in $4 \leq D \leq 6$ dimensions, see \[12, 46, 17, 50, 53, 58, 59, 62\]. It is natural to look at this problem starting from a hyper-dilaton Weyl multiplet coupled to systems of vector multiplets.
with electric and magnetic FI-type terms. We expect to be able to overcome some of the other Weyl multiplet’s restrictions to engineer off-shell gauged supergravity.

Acknowledgements:
We are grateful to I. Antoniadis, D. Butter, J.-P. Derendinger, J. Hutomo, H. Jiang, S. Kuzenko, A. Van Proeyen, and J. Woods for discussions related to this work. This work is supported by the Australian Research Council (ARC) Future Fellowship FT180100353, and by the Capacity Building Package of the University of Queensland. G.G. and S.K. are supported by the postgraduate scholarships at the University of Queensland.

A Notation and Conventions

Throughout the paper we follow the 4D notation and conventions used in [2] and [32]. We summarize them here and include a number of useful identities.

The Minkowski metric is $\eta^{ab} = \text{diag}(-1, 1, 1, 1)$ and the four-dimensional sigma matrices are

$$(\sigma^a)_{a\dot{a}} = (1, \sigma) , \quad (\tilde{\sigma}^a)^{\dot{a}\alpha} = \varepsilon^{\alpha\dot{\beta}}\varepsilon^{\alpha\beta}(\sigma^a)_{\beta\dot{\beta}} = (1, -\sigma) .$$

They satisfy

$$(\sigma_a)^{\alpha}_{\dot{a}^{\beta}}(\tilde{\sigma}_b)^{\dot{\beta}^{\beta}} = -\eta_{ab}\delta^{\alpha}_{\dot{a}} - 2(\sigma_{ab})_{\alpha^{\beta}} ,$$

$$(\tilde{\sigma}_a)^{\dot{a}^{\beta}}(\sigma_b)_{\beta^{\dot{\beta}}} = -\eta_{ab}\delta^{\dot{\alpha}}_{\dot{a}} - 2(\tilde{\sigma}_{ab})^{\dot{\alpha}}_{\dot{\beta}} ,$$

together with the following useful identities

$$(\sigma^a)_{a\dot{a}}(\sigma_a)_{\beta\dot{\beta}} = -2\varepsilon_{\alpha\dot{\beta}}\varepsilon_{\dot{\alpha}^{\dot{\beta}}} , \quad (\sigma_{ab})_{\alpha^{\beta}}(\sigma^{ab})_{\gamma^{\delta}} = -2\varepsilon^{\gamma(\alpha}^{\beta)\delta} , \quad (\sigma_{ab})_{\alpha^{\beta}}(\tilde{\sigma}^{ab})_{\gamma^{\dot{\delta}}} = 0 ,$$

$$\varepsilon^{abcd}(\sigma^{cd})_{\alpha^{\beta}} = -2i(\sigma_{ab})_{\alpha^{\beta}} , \quad \varepsilon^{abcd}(\tilde{\sigma}^{cd})_{\alpha^{\dot{\beta}}} = 2i(\tilde{\sigma}_{ab})_{\dot{\alpha}^{\dot{\beta}}} ,$$

$$\text{tr}(\sigma_{ab}\sigma_{cd}) = (\sigma_{ab})_{\alpha^{\beta}}(\sigma_{cd})_{\beta^{\alpha}} - \eta_{a[c}\eta_{d]b} - \frac{1}{2}\varepsilon_{abcd} ,$$

$$\text{tr}(\tilde{\sigma}_{ab}\tilde{\sigma}_{cd}) = (\tilde{\sigma}_{ab})^{\dot{\alpha}^{\dot{\beta}}} (\tilde{\sigma}_{cd})_{\dot{\beta}^{\dot{\alpha}}} - \eta_{a[c}\eta_{d]b} + \frac{1}{2}\varepsilon_{abcd} .$$

Here the Levi-Civita tensor $\varepsilon^{abcd}$ obeys

$$\varepsilon^{0123} = -\varepsilon_{0123} = 1 , \quad \varepsilon^{abcd}\varepsilon^{a'c'd'c} = -4!\delta^{a}_{a'}\delta^{b}_{b'}\delta^{c}_{c'}\delta^{d}_{d'} ,$$

where (anti-)symmetrization of $n$ index includes a $1/n!$ normalization. For example, given a bi-vector $A_{ab}$ and a bi-spinor $B_{a\beta}$ tensors, it holds

$$A_{[ab]} = \frac{1}{2!}(A_{ab} - A_{ba}) , \quad B_{(a\beta)} = \frac{1}{2!}(B_{a\beta} + B_{\beta a}) .$$
For four-dimensional spinors we use the two-component notation. Dotted and undotted spinor indices are raised and lowered using the following conventions
\[\phi_\alpha = \varepsilon_{\alpha\beta} \phi^\beta, \quad \phi^\alpha = \varepsilon^{\alpha\beta} \phi_\beta, \quad \bar{\psi}^\dot{\alpha} = \varepsilon^{\dot{\alpha}\dot{\beta}} \bar{\psi}_{\dot{\beta}}, \quad \bar{\psi}_{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}} \bar{\psi}^{\dot{\beta}},\] (A.7)
where the epsilon matrices obey
\[\varepsilon_{\alpha\beta} = -\varepsilon_{\beta\alpha}, \quad \varepsilon_{\alpha\beta} \varepsilon^{\beta\gamma} = \delta^\gamma_\alpha, \quad \varepsilon^{\dot{\alpha}\dot{\beta}} \varepsilon_{\dot{\beta}\dot{\gamma}} = \delta^\gamma_{\dot{\alpha}}, \quad \varepsilon^{12} = -\varepsilon_{12} = 1.\] (A.8)
Similarly, we raise and lower SU(2) indices using the conventions
\[\phi_i = \varepsilon_{ij} \phi^j, \quad \phi^i = \varepsilon^{ij} \phi_j,\] (A.9)
where \(\varepsilon_{ij}\) and \(\varepsilon^{ij}\) satisfy the same relations (A.8) as the spinor ones. Spinor indices are contracted as follow
\[\phi\psi := \phi^\alpha \psi_\alpha, \quad \bar{\phi}\bar{\psi} = \bar{\phi}_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}}.\] (A.10)
A few examples of such contractions used in the paper are
\[\bar{\xi}_{\dot{\alpha}k} \bar{\sigma}^a \nabla_a \Sigma^k = \bar{\xi}_{\dot{\alpha}k} \bar{\sigma}^{\alpha} \nabla_a \Sigma^k, \quad \psi_{a\dot{\alpha} \sigma} \bar{\sigma}^{cd} \bar{\rho}^b = \psi_{a\dot{\alpha} \sigma} \bar{\sigma}^{\dot{\alpha}} \bar{\sigma}^{\dot{\beta}} \bar{\sigma}^{\dot{\gamma}} \bar{\sigma}^{\dot{\delta}}, \quad (\psi_{a\dot{\alpha} \sigma} \bar{\sigma}^{cd})^\alpha = \psi_{a\dot{\alpha} \sigma} \bar{\sigma}^{\dot{\alpha}} \bar{\sigma}^{\dot{\beta}} \bar{\sigma}^{\dot{\gamma}} \bar{\sigma}^{\dot{\delta}}.\] (A.11a)\( A.11b\)\( A.11c\)
A vector \(T_a\) can be rewritten with spinor indices as
\[T_{\alpha\dot{\beta}} = (\sigma^a)_{\alpha\dot{\beta}} T_a, \quad T_a = -\frac{1}{2} (\bar{\sigma}^{\dot{\alpha}}) \bar{\sigma}^{\dot{\beta}} T_{\alpha\dot{\beta}}.\] (A.12)
A real antisymmetric tensor, \(W_{ab} = -W_{ba}\) is converted to spinor indices as follow
\[W_{\alpha\beta} = \frac{1}{2} (\sigma^{ab})_{\alpha\beta} W_{ab}, \quad \bar{W}_{\dot{\alpha}\dot{\beta}} = -\frac{1}{2} (\bar{\sigma}^{ab})_{\dot{\alpha}\dot{\beta}} W_{ab}, \quad W_{ab} = (\sigma_{ab})^{\alpha\beta} W_{\alpha\beta} - (\bar{\sigma}_{ab})_{\dot{\alpha}\dot{\beta}} \bar{W}^{\dot{\alpha}\dot{\beta}}.\] (A.13)
If \(W_{ab}\) is real, then \(W_{\alpha\beta} = W_{\beta\alpha}\) and \(\bar{W}_{\dot{\alpha}\dot{\beta}} = \bar{W}_{\dot{\beta}\dot{\alpha}}\) are complex conjugates of each others.

References

[1] S. J. Gates, M. T. Grisaru, M. Rocek and W. Siegel, “Superspace Or One Thousand and One Lessons in Supersymmetry,” Front. Phys. 58, 1-548 (1983) [arXiv:hep-th/0108200 [hep-th]].

[2] I. Buchbinder and S. M. Kuzenko. Ideas and methods of supersymmetry and supergravity: Or a walk through superspace, IOP, Bristol (1998).

28
[3] D. Z. Freedman and A. Van Proeyen, *Supergravity*, Cambridge University Press (2012).

[4] E. Lauria and A. Van Proeyen, “$\mathcal{N} = 2$ Supergravity in $D = 4, 5, 6$ Dimensions,” Lect. Notes Phys. **966** (2020), pp. [arXiv:2004.11433 [hep-th]].

[5] S. Ferrara, M. Kaku, P. K. Townsend and P. van Nieuwenhuizen, “Gauging the Graded Conformal Group with Unitary Internal Symmetries,” Nucl. Phys. B **129**, 125-134 (1977).

[6] M. Kaku, P. K. Townsend and P. van Nieuwenhuizen, “Properties of Conformal Supergravity,” Phys. Rev. D **17**, 3179 (1978).

[7] M. Kaku and P. K. Townsend, “POINCARE SUPERGRAVITY AS BROKEN SUPERCONFORMAL GRAVITY,” Phys. Lett. B **76**, 54-58 (1978).

[8] M. Kaku, P. K. Townsend and P. van Nieuwenhuizen, “Gauge Theory of the Conformal and Superconformal Group,” Phys. Lett. B **69**, 304-308 (1977).

[9] B. de Wit, J. W. van Holten and A. Van Proeyen, “Transformation Rules of N=2 Supergravity Multiplets,” Nucl. Phys. B **167** (1980), 186.

[10] B. de Wit, J. W. van Holten and A. Van Proeyen, “Central Charges and Conformal Supergravity,” Phys. Lett. B **95**, 51-55 (1980).

[11] B. de Wit, J. W. van Holten and A. Van Proeyen, “Structure of N=2 Supergravity,” Nucl. Phys. B **184**, 77 (1981) [erratum: Nucl. Phys. B **222**, 516 (1983)].

[12] B. de Wit, P. G. Lauwers, R. Philippe, S. Q. Su and A. Van Proeyen, “Gauge and Matter Fields Coupled to N=2 Supergravity,” Phys. Lett. B **134**, 37-43 (1984).

[13] B. de Wit, P. G. Lauwers and A. Van Proeyen, “Lagrangians of N=2 Supergravity - Matter Systems,” Nucl. Phys. B **255**, 569-608 (1985).

[14] E. Bergshoeff, E. Sezgin and A. Van Proeyen, “Superconformal Tensor Calculus and Matter Couplings in Six-dimensions,” Nucl. Phys. B **264**, 653 (1986) [erratum: Nucl. Phys. B **598**, 667 (2001)].

[15] T. Kugo and K. Ohashi, “Supergravity tensor calculus in 5-D from 6-D,” Prog. Theor. Phys. **104**, 835-865 (2000) [arXiv:hep-ph/0006231 [hep-ph]].

[16] T. Fujita and K. Ohashi, “Superconformal tensor calculus in five-dimensions,” Prog. Theor. Phys. **106**, 221-247 (2001) [arXiv:hep-th/0104130 [hep-th]].

[17] T. Kugo and K. Ohashi, “Gauge and nongauge tensor multiplets in 5-D conformal supergravity,” Prog. Theor. Phys. **108**, 1143-1164 (2003) [arXiv:hep-th/0208082 [hep-th]].

[18] E. Bergshoeff, T. de Wit, R. Halbersma, S. Cucu, M. Derix and A. Van Proeyen, “Weyl multiplets of N=2 conformal supergravity in five-dimensions,” JHEP **06**, 051 (2001) [arXiv:hep-th/0104113 [hep-th]].

[19] E. Bergshoeff, S. Cucu, T. De Wit, J. Gheerardyn, R. Halbersma, S. Vandoren and A. Van Proeyen, “Superconformal N=2, D = 5 matter with and without actions,” JHEP **10**, 045 (2002) [arXiv:hep-th/0205230 [hep-th]].
[20] E. Bergshoeff, S. Cucu, T. de Wit, J. Gheerardyn, S. Vandoren and A. Van Proeyen, “N = 2 supergravity in five-dimensions revisited,” Class. Quant. Grav. 21, 3015-3042 (2004) [arXiv:hep-th/0403043 [hep-th]].

[21] D. Butter, S. M. Kuzenko, J. Novak and G. Tartaglino-Mazzucchelli, “Conformal supergravity in three dimensions: New off-shell formulation,” JHEP 09, 072 (2013) [arXiv:1305.3132 [hep-th]].

[22] D. Butter, S. M. Kuzenko, J. Novak and G. Tartaglino-Mazzucchelli, “Conformal supergravity in three dimensions: Off-shell actions,” JHEP 10, 073 (2013) [arXiv:1306.1205 [hep-th]].

[23] P. S. Howe, “A superspace approach to extended conformal supergravity,” Phys. Lett. B 100, 389 (1981).

[24] P. S. Howe, “Supergravity in Superspace,” Nucl. Phys. B 199, 309-364 (1982).

[25] A. Galperin, E. Ivanov, S. Kalitsyn, V. Ogievetsky and E. Sokatchev, “Unconstrained N=2 Matter, Yang-Mills and Supergravity Theories in Harmonic Superspace,” Class. Quant. Grav. 1, 469-498 (1984); [erratum: Class. Quant. Grav. 2, 127 (1985)].

[26] A. S. Galperin, E. A. Ivanov, V. I. Ogievetsky and E. Sokatchev, “N = 2 Supergravity in Superspace: Different Versions and Matter Couplings,” Class. Quant. Grav. 4, 1255 (1987).

[27] A. S. Galperin, N. A. Ky and E. Sokatchev, “N = 2 Supergravity in Superspace: Solution to the Constraints,” Class. Quant. Grav. 4, 1235 (1987).

[28] A. S. Galperin, E. A. Ivanov, V. I. Ogievetsky and E. S. Sokatchev, Harmonic superspace, Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2007.

[29] S. M. Kuzenko, U. Lindstrom, M. Rocek, and G. Tartaglino-Mazzucchelli, “4D N = 2 Supergravity and Projective Superspace,” JHEP 09, 051 (2008) [arXiv:0805.3683 [hep-th]].

[30] S. M. Kuzenko, U. Lindstrom, M. Rocek and G. Tartaglino-Mazzucchelli, “On conformal supergravity and projective superspace,” JHEP 08, 023 (2009) [arXiv:0905.0063 [hep-th]].

[31] D. Butter, “N=2 Conformal Superspace in Four Dimensions,” JHEP 10 (2011), 030 [arXiv:1103.5914 [hep-th]].

[32] D. Butter and J. Novak, “Component reduction in N=2 supergravity: the vector, tensor, and vector-tensor multiplets,” JHEP 05 (2012), 115 [arXiv:1201.5431 [hep-th]].

[33] D. Butter, “New approach to curved projective superspace,” Phys. Rev. D 92, no.8, 085004 (2015) [arXiv:1406.6233 [hep-th]].

[34] D. Butter, “Projective multiplets and hyperkähler cones in conformal supergravity,” JHEP 06, 161 (2015) [arXiv:1410.3604 [hep-th]].

[35] D. Butter, “On conformal supergravity and harmonic superspace,” JHEP 03, 107 (2016) [arXiv:1508.07718 [hep-th]].

[36] B. de Wit and A. Van Proeyen, “Potentials and Symmetries of General Gauged N=2 Supergravity: Yang-Mills Models,” Nucl. Phys. B 245, 89-117 (1984).
E. Cremmer, C. Kounnas, A. Van Proeyen, J. P. Derendinger, S. Ferrara, B. de Wit and L. Girardello, 
“Vector Multiplets Coupled to N=2 Supergravity: SuperHiggs Effect, Flat Potentials and Geometric 
Structure,” Nucl. Phys. B 250, 385-426 (1985).

B. de Wit, B. Kleijn and S. Vandoren, “Superconformal hypermultiplets,” Nucl. Phys. B 568, 475-
502 (2000) [arXiv:hep-th/9909228 [hep-th]].

B. de Wit, M. Rocek and S. Vandoren, “Hypermultiplets, hyperKahler cones and quaternion Kahler 
gameometry,” JHEP 02, 039 (2001) [arXiv:hep-th/0101161 [hep-th]].

B. de Wit, M. Rocek and S. Vandoren, “Gauging isometries on hyperKahler cones and quaternion Kahler manifolds,” 
Phys. Lett. B 511, 302-310 (2001) [arXiv:hep-th/0104215 [hep-th]].

V. Pestun, M. Zabzine, F. Benini, T. Dimofte, T. T. Dumitrescu, K. Hosomichi, S. Kim, K. Lee, 
B. Le Floch and M. Marino, et al. “Localization techniques in quantum field theories,” J. Phys. A 50, no.44, 440301 (2017) [arXiv:1608.02952 [hep-th]].

E. Bergshoeff, A. Salam and E. Sezgin, “A supersymmetric $R^2$-action in six dimensions and torsion,” 
Phys. Lett. B 173, 73 (1986).

G. Lopes Cardoso, B. de Wit and T. Mohaupt, “ Corrections to macroscopic supersymmetric black 
hole entropy,” Phys. Lett. B 451, 309-316 (1999) [arXiv:hep-th/9812082 [hep-th]].

T. Mohaupt, “Black hole entropy, special geometry and strings,” Fortsch. Phys. 49, 3-161 (2001) 
[arXiv:hep-th/0007195 [hep-th]].

K. Hanaki, K. Ohashi and Y. Tachikawa, “Supersymmetric Completion of an R**2 term in Five-
dimensional Supergravity,” Prog. Theor. Phys. 117, 533 (2007) [arXiv:hep-th/0611329 [hep-th]].

F. Coomans and A. Van Proeyen, “Off-shell $N = (1, 0), D = 6$ supergravity from superconformal 
methods,” JHEP 1102, 049 (2011) Erratum: [JHEP 1201, 119 (2012)] [arXiv:1101.2403 [hep-th]].

E. Bergshoeff, F. Coomans, E. Sezgin and A. Van Proeyen, “Higher Derivative Extension of 6D 
Chiral Gauged Supergravity,” JHEP 1207, 011 (2012). [arXiv:1203.2975 [hep-th]].

D. Butter, B. de Wit, S. M. Kuzenko and I. Lodato, “New higher-derivative invariants in N=2 
supergravity and the Gauss-Bonnet term,” JHEP 1312 (2013) 062 [arXiv:1307.6546 [hep-th]].

S. M. Kuzenko, J. Novak and G. Tartaglino-Mazzucchelli, “N=6 superconformal gravity in three 
dimensions from superspace,” JHEP 01, 121 (2014) [arXiv:1308.5552 [hep-th]].

M. Ozkan and Y. Pang, “Supersymmetric Completion of Gauss-Bonnet Combination in Five 
Dimensions,” JHEP 1303 (2013) 158 Erratum: [JHEP 1307 (2013) 152] [arXiv:1301.6622 [hep-th]].

M. Ozkan and Y. Pang, “All off-shell $R^2$ invariants in five dimensional $N = 2$ supergravity,” JHEP 
1308 (2013) 042 [arXiv:1306.1540, arXiv:1306.1540 [hep-th]].

M. Ozkan, “Supersymmetric curvature squared invariants in five and six dimensions,” PhD Thesis 
Texas A&M University, 2013.

D. Butter, S. M. Kuzenko, J. Novak and G. Tartaglino-Mazzucchelli, “Conformal supergravity in five 
dimensions: New approach and applications,” JHEP 1502, 111 (2015). [arXiv:1410.8682 [hep-th]].

31
[54] S. M. Kuzenko and J. Novak, “On curvature squared terms in N=2 supergravity,” Phys. Rev. D 92, no.8, 085033 (2015) [arXiv:1507.04922 [hep-th]].

[55] D. Butter, S. M. Kuzenko, J. Novak and S. Theisen, “Invariants for minimal conformal supergravity in six dimensions,” JHEP 1612, 072 (2016) [arXiv:1606.02921 [hep-th]].

[56] D. Butter, F. Ciceri, B. de Wit and B. Sahoo, “Construction of all N=4 conformal supergravities,” Phys. Rev. Lett. 118, no.8, 081602 (2017) [arXiv:1609.00083 [hep-th]].

[57] D. Butter, J. Novak and G. Tartaglino-Mazzucchelli, “The component structure of conformal supergravity invariants in six dimensions,” JHEP 1705, 133 (2017) [arXiv:1701.08163 [hep-th]].

[58] J. Novak, M. Ozkan, Y. Pang and G. Tartaglino-Mazzucchelli, “Gauss-Bonnet supergravity in six dimensions,” Phys. Rev. Lett. 119, no. 11, 111602 (2017). [arXiv:1706.09330 [hep-th]].

[59] D. Butter, J. Novak, M. Ozkan, Y. Pang and G. Tartaglino-Mazzucchelli, “Curvature squared invariants in six-dimensional N = (1, 0) supergravity,” JHEP 04, 013 (2019) [arXiv:1808.00459 [hep-th]].

[60] D. Butter, F. Ciceri and B. Sahoo, “N = 4 conformal supergravity: the complete actions,” JHEP 01, 029 (2020) [arXiv:1910.11874 [hep-th]].

[61] S. Hegde and B. Sahoo, “New higher derivative action for tensor multiplet in N = 2 conformal supergravity in four dimensions,” JHEP 01, 070 (2020) [arXiv:1911.09558 [hep-th]].

[62] M. Mishra and B. Sahoo, “Curvature squared action in four dimensional N = 2 supergravity using the dilaton Weyl multiplet,” JHEP 04, 027 (2021) [arXiv:2012.03760 [hep-th]].

[63] N. Bobev, A. M. Charles, K. Hristov and V. Reys, “The Unreasonable Effectiveness of Higher-Derivative Supergravity in AdS4 Holography,” Phys. Rev. Lett. 125, no.13, 131601 (2020) [arXiv:2006.09390 [hep-th]].

[64] N. Bobev, A. M. Charles, K. Hristov and V. Reys, “Higher-derivative supergravity, AdS4 holography, and black holes,” JHEP 08, 173 (2021) [arXiv:2106.04581 [hep-th]].

[65] N. Bobev, K. Hristov and V. Reys, “AdS3 Holography and Higher-Derivative Supergravity,” arXiv:2112.06901 [hep-th].

[66] D. Butter, “N=1 Conformal Superspace in Four Dimensions,” Annals Phys. 325, 1026-1080 (2010) [arXiv:0906.4399 [hep-th]].

[67] T. Kugo and S. Uehara, “N = 1 Superconformal Tensor Calculus: Multiplets With External Lorentz Indices and Spinor Derivative Operators,” Prog. Theor. Phys. 73, 235 (1985).

[68] W. Siegel, “Curved extended superspace from Yang-Mills theory a la strings,” Phys. Rev. D 53, 3324-3336 (1996) [arXiv:hep-th/9510150 [hep-th]].

[69] D. Butter, S. Hegde, I. Lodato and B. Sahoo, “N = 2 dilaton Weyl multiplet in 4D supergravity,” JHEP 03, 154 (2018) [arXiv:1712.05365 [hep-th]].

[70] M. Müllner, “Minimal N = 2 Supergravity in Superspace,” Nucl. Phys. B 282, 329 (1987).

[71] E. S. Fradkin and M. A. Vasiliev, Lett. Nuovo Cim. 25, 79-90 (1979); “MINIMAL SET OF AUXILIARY FIELDS IN SO(2) EXTENDED SUPERGRAVITY,” Phys. Lett. B 85, 47-51 (1979).
B. de Wit and J. W. van Holten, “Multiplets of Linearized SO(2) Supergravity,” Nucl. Phys. B 155, 530-542 (1979).

P. Breitenlohner and M. F. Sohnius, “Superfields, Auxiliary Fields, and Tensor Calculus for $N = 2$ Extended Supergravity,” Nucl. Phys. B 165, 483-510 (1980); “An Almost Simple Off-shell Version of SU(2) Poincare Supergravity,” Nucl. Phys. B 178, 151-176 (1981).

M. M¨uller, “Minimal N=2 Off-Shell Supergravity,” Phys. Lett. B 172, 353 (1986).

P. Breitenlohner and M. F. Sohnius, “Superfields, Auxiliary Fields, and Tensor Calculus for $N = 2$ Extended Supergravity,” Nucl. Phys. B 165, 483-510 (1980); “An Almost Simple Off-shell Version of SU(2) Poincare Supergravity,” Nucl. Phys. B 178, 151-176 (1981).

M. F. Sohnius, K. S. Stelle and P. C. West, “Representations of extended supersymmetry,” in Superspace and Supergravity, S. W. Hawking and M. Roˇcek (Eds.) Cambridge University Press, Cambridge, 1981, p. 283.

B. de Wit, R. Philippe and A. Van Proeyen, “The Improved Tensor Multiplet in $N = 2$ Supergravity,” Nucl. Phys. B 219, 143-166 (1983).

R. D’Auria, S. Ferrara and P. Fre, “Special and quaternionic isometries: General couplings in N=2 supergravity and the scalar potential,” Nucl. Phys. B 359 (1991) 705.

L. Andrianopoli, M. Bertolini, A. Ceresole, R. D’Auria, S. Ferrara and P. Fre’, “General matter coupled N=2 supergravity,” Nucl. Phys. B 476 (1996) 397 [hep-th/9603004].

L. Andrianopoli, M. Bertolini, A. Ceresole, R. D’Auria, S. Ferrara, P. Fre and T. Magri, “N=2 supergravity and N=2 superYang-Mills theory on general scalar manifolds: Symplectic covariance, gaugings and the momentum map,” J. Geom. Phys. 23 (1997) 111 [hep-th/9605032].

G. Dall’Agata, R. D’Auria, L. Sommovigo and S. Vaula, “D=4, N=2 gauged supergravity in the presence of tensor multiplets,” Nucl. Phys. B 682 (2004) 243 [hep-th/0312210].

M. Trigiante, “Gauged Supergravities,” Phys. Rept. 680 (2017) 1 [arXiv:1609.09745 [hep-th]].

A. Van Proeyen, “Supergravity with Fayet-Iliopoulos terms and R-symmetry,” Fortsch. Phys. 53 (2005) 997 [hep-th/0410053].
1. Antoniadis, J. P. Derendinger, F. Farakos and G. Tartaglino-Mazzucchelli, “New Fayet-Iliopoulos terms in $\mathcal{N} = 2$ supergravity,” JHEP 07, 061 (2019) [arXiv:1905.09125 [hep-th]].

2. S. M. Kuzenko and G. Tartaglino-Mazzucchelli, “Super-Weyl invariance in 5D supergravity,” JHEP 04 (2008), 032 [arXiv:0802.3953 [hep-th]].

3. E. A. Ivanov and B. M. Zupnik, “Modified N=2 supersymmetry and Fayet-Iliopoulos terms,” Phys. Atom. Nucl. 62, 1043 (1999) [Yad. Fiz. 62, 1110 (1999)] [hep-th/9710236].

4. E. Ivanov and B. Zupnik, “Modifying N=2 supersymmetry via partial breaking,” In *Buckow 1997, Theory of elementary particles* 64-69 [hep-th/9801016].

5. M. Roček and A. A. Tseytlin, “Partial breaking of global D = 4 supersymmetry, constrained superfields, and 3-brane actions,” Phys. Rev. D 59, 106001 (1999) [arXiv:hep-th/9811232].

6. I. Antoniadis, J. P. Derendinger and C. Markou, “Nonlinear $\mathcal{N} = 2$ global supersymmetry,” JHEP 1706 (2017) 052 [arXiv:1703.08806 [hep-th]].

7. I. Antoniadis, H. Jiang and O. Lacombe, “$\mathcal{N} = 2$ Supersymmetry Deformations, Electromagnetic Duality and Dirac-Born-Infeld Actions,” arXiv:1904.06339 [hep-th].

8. I. Antoniadis, H. Jiang and O. Lacombe, “$\mathcal{N} = 2$ Supersymmetry Deformations, Electromagnetic Duality and Dirac-Born-Infeld Actions,” arXiv:1004.06339 [hep-th].

9. S. M. Kuzenko and G. Tartaglino-Mazzucchelli, “New nilpotent $\mathcal{N} = 2$ superfields,” Phys. Rev. D 97 (2018) no.2, 026003 [arXiv:1707.07390 [hep-th]].

10. I. Antoniadis, H. Partouche and T. R. Taylor, “Spontaneous breaking of N=2 global supersymmetry,” Phys. Lett. B 372, 83-87 (1996) [arXiv:hep-th/9512006 [hep-th]].

11. J. Louis, P. Smyth and H. Triendl, “Supersymmetric Vacua in N=2 Supergravity,” JHEP 08, 039 (2012) [arXiv:1204.3893 [hep-th]].

12. I. Antoniadis, J. P. Derendinger, H. Jiang and G. Tartaglino-Mazzucchelli, “Magnetic deformation of super-Maxwell theory in supergravity,” JHEP 08, no.08, 079 (2020) [arXiv:2005.11374 [hep-th]].

13. S. M. Kuzenko, “Super-Weyl anomalies in N=2 supergravity and (non)local effective actions,” JHEP 1310 (2013) 151 [arXiv:1307.7586 [hep-th]].

14. S. M. Kuzenko and G. Tartaglino-Mazzucchelli, “Nilpotent chiral superfield in N=2 supergravity and partial rigid supersymmetry breaking,” JHEP 03, 092 (2016) [arXiv:1512.01964 [hep-th]].

15. M. de Vroome and B. de Wit, “Lagrangians with electric and magnetic charges of N=2 supersymmetric gauge theories,” JHEP 08, 064 (2007) [arXiv:0707.2717 [hep-th]].

16. B. de Wit and M. van Zalk, “Electric and magnetic charges in N=2 conformal supergravity theories,” JHEP 10, 050 (2011) [arXiv:1107.3305 [hep-th]].

17. M. Sohnius, K. S. Stelle and P. C. West, “Off-mass-shell formulation of extended supersymmetric gauge theories,” Phys. Lett. B 92, 123 (1980); “Dimensional reduction by Legendre transformation generates off-shell supersymmetric Yang-Mills theories,” Nucl. Phys. B 173, 127 (1980).

18. B. de Wit, V. Kaplunovsky, J. Louis and D. Lüst, “Perturbative couplings of vector multiplets in N=2 heterotic string vacua,” Nucl. Phys. B 451, 53 (1995) [arXiv:hep-th/9504006].
[109] P. Claus, B. de Wit, M. Faux, B. Kleijn, R. Siebelink and P. Termonia, “The vector-tensor supermultiplet with gauged central charge,” Phys. Lett. B 373, 81 (1996) [arXiv:hep-th/9512143]; P. Claus, P. Termonia, B. de Wit and M. Faux, “Chern-Simons couplings and inequivalent vector-tensor multiplets,” Nucl. Phys. B 491, 201 (1997) [arXiv:hep-th/9612203]; P. Claus, B. de Wit, M. Faux, B. Kleijn, R. Siebelink and P. Termonia, “N=2 supergravity Lagrangians with vector-tensor multiplets,” Nucl. Phys. B 512, 148 (1998) [arXiv:hep-th/9710212].

[110] A. Hindawi, B. A. Ovrut and D. Waldram, “Vector-tensor multiplet in N=2 superspace with central charge,” Phys. Lett. B 392, 85 (1997) [arXiv:hep-th/9609016].

[111] N. Dragon, S. M. Kuzenko and U. Theis, “The Vector-tensor multiplet in harmonic superspace,” Eur. Phys. J. C 4, 717 (1998) [arXiv:hep-th/9706169]; N. Dragon and S. M. Kuzenko, “Self-interacting vector-tensor multiplet,” Phys. Lett. B 420, 64 (1998) [arXiv:hep-th/9709088]; N. Dragon, E. Ivanov, S. Kuzenko, E. Sokatchev and U. Theis, “N=2 rigid supersymmetry with gauged central charge,” Nucl. Phys. B 538, 411 (1999) [arXiv:hep-th/9805152].

[112] S. M. Kuzenko and J. Novak, “Vector-tensor supermultiplets in AdS and supergravity,” JHEP 01 (2012), 106 doi:10.1007/JHEP01(2012)106 [arXiv:1110.0971 [hep-th]].