A combined limit on the neutrino mass from neutrinoless double-beta decay searches in multiple isotopes

P Guzowski
School of Physics and Astronomy, University of Manchester, Manchester M13 9PL, United Kingdom
E-mail: pawel.guzowski@manchester.ac.uk

Abstract. We set a combined limit on the effective Majorana neutrino mass \( m_{\beta\beta} \) from experimental searches for neutrinoless double-beta decay of multiple isotopes. The limits on \( m_{\beta\beta} \) range between 130–310 meV, depending on the choice of nuclear matrix element calculation. The limits on \( m_{\beta\beta} \) can also be translated into a limit on the neutrino mass and mixing parameters of a fourth sterile neutrino.

1. Introduction
The observation of neutrinoless double-beta decay of an isotope would show that the neutrino is a Majorana fermion, and assuming no other new physics, would allow the measurement of the absolute mass of the neutrino. The rate \( \Gamma \) of this process is given by

\[
\Gamma = G^{0\nu} |M^{0\nu}|^2 \frac{m_{\beta\beta}^2}{m_e^2},
\]

where \( m_e \) is the electron mass, \( M^{0\nu} \) a nuclear matrix element (NME) factor and \( G^{0\nu} \) is a phase space factor. \( m_{\beta\beta} \) is the effective Majorana mass:

\[
m_{\beta\beta} = \sum_{j=1}^{N_\nu} m_j |U_{ej}|^2 e^{i\alpha_j},
\]

where \( j \) labels the \( N_\nu \) neutrino mass eigenstates (\( N_\nu = 3 \) in the Standard Model), \( m_j \) are their masses, \( U_{ej} \) are the electron-row elements of the neutrino mass-flavour mixing matrix, and \( \alpha_j \) the Majorana phases (with \( e^{i\alpha_j} = 1 \) by convention).

There have been several recent experimental limits set on the half-lives of this process in multiple isotopes. Using Eq. 1 together with calculations of the phase space factors and NMEs for each isotope, a combined limit can be set on \( m_{\beta\beta} \).

There has been no attempt prior to this work (published in Ref. [1]) to produce a combined limit for the effective Majorana mass that is derived from individual experimental data across multiple isotopes.
2. Experimental inputs
The six experimental inputs combined in this work are recent results of CUORICINO [2], CUORE-0 [3], EXO [4], GERDA [5], KamLAND-Zen [6], and NEMO-3 [7]. The isotopes studied and half-life limits of these experiments is summarised in Table 1.

Table 1. The six experiment results which are combined in this work. The isotope examined buy the experiment, the exposure, the experiments’ published limits, and our observed and expected limits given the energy spectrum histograms that the experiments published.

| Experiment   | Isotope | Exposure | Published limit | Observed limit | Expected limit |
|--------------|---------|----------|-----------------|----------------|----------------|
| CUORICINO    | $^{130}$Te | 19.75 kg y | $2.8 \times 10^{24}$ y | $2.8 \times 10^{24}$ y | $2.9 \times 10^{24}$ y |
| CUORE-0      | $^{130}$Te | 9.8 kg y | $2.7 \times 10^{24}$ y | $3.0 \times 10^{24}$ y | $3.0 \times 10^{24}$ y |
| EXO          | $^{136}$Xe | 100 kg y | $1.1 \times 10^{25}$ y | $1.3 \times 10^{25}$ y | $2.1 \times 10^{25}$ y |
| KamLAND-Zen  | $^{136}$Xe | 89.5 kg y | $1.9 \times 10^{25}$ y | $1.7 \times 10^{25}$ y | $1.1 \times 10^{25}$ y |
| GERDA        | $^{76}$Ge | 21.6 kg y | $2.1 \times 10^{25}$ y | $2.0 \times 10^{25}$ y | $2.1 \times 10^{25}$ y |
| NEMO-3       | $^{100}$Mo | 34.7 kg y | $1.1 \times 10^{24}$ y | $1.1 \times 10^{24}$ y | $0.9 \times 10^{24}$ y |

As a cross-check of our procedure, we attempted to replicate the experiments’ results. We digitised the experimental energy distributions, shown in Figure 1. We used the $CL_s$ limit-setting algorithm [8] with the log-likelihood ratio as the test statistic. The observed and expected 90% CL limits that we calculated are shown in Table 1. In general there is good agreement.

Figure 1. The input energy spectrum histograms for (a) CUORICINO (fig. 9 of [2]); (b) CUORE-0 (fig. 3 of [3]); (c) EXO (fig. 4(a) of [4]); (d) KamLAND-Zen (fig. 1(a) of [6]); (e) GERDA (fig. 1 of [5]); and (f) NEMO-3 (fig. 2 of [7]).
between the published values and our limits. There is a maximum difference of 15% in the case of KamLAND-Zen, but this is not unexpected given that we are using different limit-setting methods, and also do not have access to the unbinned experimental data.

3. Combination
The combination of the experimental inputs was performed by summing the log-likelihood ratios of the six experimental histograms. The relative normalisation of the signal for each experiment, for a fixed value of the universal physics parameter $m_{\beta\beta}$, was calculated using Eq. 1, using published calculations of $G^0_{\nu}$ [9] and NMEs $M^0_{\nu}$ [10, 11, 12, 13, 14, 15] for the four isotopes. A different combined limit is set for each NME model, as each one predicts different signal normalisation for a fixed effective neutrino mass in each isotope.

### Table 2.
The combined limits (observed and expected) for each NME model. For the QRPA model, four different calculations are used, using (A) Argonne or (B) CD-Bonn nucleon-nucleon potentials with new or old parametrisations (further details in Ref. [13]). The improvement of the combined limit over the best individual experiment for that NME calculation (labelled as (G) GERDA, (E) EXO or (K) KamLAND-Zen) is also given.

| NME         | $m_{\beta\beta}^{obs}$ (meV) | $m_{\beta\beta}^{exp}$ (meV) | Improvement Limit | Sensitivity |
|-------------|-------------------------------|-------------------------------|-------------------|-------------|
| GCM [10]    | 130                           | 120                           | 14% (K)           | 10% (E)     |
| IBM-2 [11]  | 170                           | 170                           | 16% (K)           | 12% (E)     |
| NSM [12]    | 310                           | 290                           | 14% (K)           | 10% (E)     |
| QRPA: [13]  |                               |                               |                   |             |
| A-new       | 200                           | 200                           | 23% (G)           | 25% (E)     |
| A-old       | 180                           | 180                           | 26% (G)           | 25% (E)     |
| B-new       | 180                           | 180                           | 28% (K)           | 24% (E)     |
| B-old       | 170                           | 160                           | 28% (K)           | 23% (E)     |
| pnQRPA [14] | 170                           | 170                           | 19% (K)           | 16% (E)     |
| (R)QRPA [15]| 250                           | 240                           | 25% (G)           | 25% (E)     |

Table 2 shows the combined 90% CL limits on $m_{\beta\beta}$ for each of the six NME models examined. The limits range from 130–310 meV. The results show that there can be up to a 28% improvement in using the combined limit over the best individual experiment. With the mass limit generally varying as the 4th-root of the exposure, these improvements can represent a factor of 1.5–2.5 increase of exposure of the best individual experiment.

Figure 2 shows these combined limits on a plot of the value of $m_{\beta\beta}$ predicted at various values of the mass of the lightest neutrino, using neutrino oscillation fits for the mass splittings and mixing parameters [16] in Eq. 2.

4. Limits on sterile neutrinos
We also translate the limits on $m_{\beta\beta}$ into limits on the existence of a sterile Majorana neutrino state “$\nu_4$”. An upper limit on $m_{\beta\beta}$ can be converted into an upper limit on $m_4 |U_{e4}|^2$ using
Figure 2. The limits on \(m_{\beta\beta}\) in the context of predicted values of \(m_{\beta\beta}\) (filled area) for different values of the mass of lightest neutrino, and the Planck limit on that mass [17]. Light bands represent the best fit values of neutrino oscillation fits, and darker bands include 3\(\sigma\) uncertainties on those fits.

Eq. 2, and measured neutrino mass splittings and mixing matrix parameters. The Majorana phases and mass of the lightest neutrino ("\(m_0\)") are unknown, and we produce these limits for various combinations of these unknown parameters. Figure 3 shows the upper limit contours in

\[
\begin{align*}
\text{Normal Hierarchy} & : \pi = \gamma = 0; \beta = \alpha_{NSM}; \gamma' = 0 \\
\text{Inverted Hierarchy} & : \pi = \mathring{\gamma} = 0; \beta = \alpha_{GCM}; \gamma' = 0 \\
\end{align*}
\]

\[
\begin{align*}
\text{Normal Hierarchy} & : m_0 = 50 \text{ meV} \\
\text{Inverted Hierarchy} & : m_0 = 150 \text{ meV} \\
\end{align*}
\]

\[\Delta m^2_{41} = 1.78 \text{ eV}^2 \quad \sin^2 2\theta_{41} = 0.09\]

Figure 3. 90% CL limits on the sterile Majorana parameters \(m_4, |U_{e4}|^2\) derived from the combined limits on effective Majorana neutrino mass. The Majorana phases \((\alpha_2, \alpha_3, \alpha_4)\) are labelled as \((\alpha, \beta, \gamma)\) in the legend. The star represents the best fit value from global fits of sterile neutrino oscillation experiments [18]. The Normal Hierarchy has \(m_0 = m_1 < m_2 < m_3\), while the Inverted Hierarchy has \(m_0 = m_3 < m_1 < m_2\).

the \(m_4, |U_{e4}|^2\) plane, with regions to the top-right excluded at 90% CL, for two extreme choices of NME model and Majorana phases, and for various values of \(m_0\). For a fixed \(m_0\), other choices
of Majorana phases or NME models will produce upper limit contours between the two extreme cases.

As an example, if \( m_4 |U_{e4}|^2 > 500 \text{ meV} \) (and also assuming all 4 neutrino species are Majorana fermions, no other NME model produces more conservative limits than the NSM model, and the mass of the lightest neutrino is less than 150 meV), neutrinoless double-beta decay should have been observed.

The global fit of sterile neutrino appearance evidence [18] is also presented on the plots, showing that neutrinoless double-beta decay searches can be sensitive to this candidate for some choices of NME model, Majorana phases, and \( m_0 \), assuming all 4 neutrino species are Majorana fermions.

5. Summary
We have produced the first combined limits on \( m_{\beta\beta} \) from experimental data across multiple isotopes. The combined limits depend on the choice of NME model, having a range between 130 meV and 310 meV. These limits represent an up to ~ 30% improvement of a single-experiment limit, corresponding to a factor ~ 2 increase of exposure.

We can also translate these limits into limits on the mass and mixing parameters of a fourth sterile Majorana neutrino state, showing that for some values of these masses and parameters, the existence of this state should have produced a positive signal of neutrinoless double-beta decays.

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