Spectral properties of quarks above $T_c$ in quenched lattice QCD

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We analyze the quark spectral function above the critical temperature for deconfinement ($T_c$) in quenched lattice QCD using Wilson fermions in Landau gauge. We show that the temporal quark correlator is well reproduced by a two-pole approximation for the spectral function and analyze the bare quark mass dependence of both poles as well as their residues. In the chiral limit we find that the quark spectral function has two collective modes which correspond to the normal and plasmino excitations. At large values of the bare quark mass the spectral function is dominated by a single pole.

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To explore the properties of hot and dense matter formed by quarks and gluons above the critical temperature for deconfinement ($T_c$) is an intriguing problem that has been addressed in many studies. Recent experimental results on the properties of the matter created in heavy ion collisions at the Relativistic Heavy-Ion Collider (RHIC) suggest, that its time evolution above $T_c$ is well described by ideal hydrodynamics down to the freeze-out temperature in the vicinity of $T_c$. In order to understand better the structure of matter in this non-perturbative region, it is desirable to identify the basic degrees of freedom of the system and their quasi-particle properties.

At asymptotically high temperatures almost free quarks and gluons are most certainly the basic degrees of freedom that control the properties of the Quark-Gluon plasma (QGP). In this regime properties of the QGP can be analyzed using perturbative techniques. At lower temperatures the application of hard-thermal loop (HTL) resummation still allowed to define gauge independent propagators for quarks and gluons that can be used to study properties of the QGP perturbatively. From these perturbative analyses it is known that the collective excitations of gluons and quarks develop a mass gap (thermal mass) that is proportional to $gT$, while the temperature dependence of the spectral function for fermions with scalar mass $m$ has been considered in QED. In this model, the spectral function at zero temperature has two poles at energies $\omega = \pm m$, while in the high temperature limit, $T/m \to \infty$, it approaches the HTL result, which has four poles. The one-loop calculation performed in clearly showed that the two limiting forms of the spectral function are connected continuously; in the spectral function a peak corresponding to the plasmino gradually appears and becomes larger with increasing temperature, in addition to the normal quasi-particle peak.

In this Letter, we analyze dynamical properties of quarks above $T_c$ in quenched lattice QCD. These calculations have been performed in Landau gauge. So far, there have been only a few studies that address this problem in lattice calculations.

In order to understand the origin of the plasmino mode in the quark propagator in the high temperature limit, it is instructive to consider the quark propagator at intermediate temperature by introducing some energy scale of the order of the temperature. In the high temperature dependence of the spectral function for fermions with scalar mass $m$ has been considered in QED. In this model, the spectral function at zero temperature has two poles at energies $\omega = \pm m$, while in the high temperature limit, $T/m \to \infty$, it approaches the HTL result, which has four poles. The one-loop calculation performed in clearly showed that the two limiting forms of the spectral function are connected continuously; in the spectral function a peak corresponding to the plasmino gradually appears and becomes larger with increasing temperature, in addition to the normal quasi-particle peak.

In this Letter, we analyze the quark propagator at two values of the temperature, $T = 1.5T_c$ and $3T_c$, as a function of the bare quark mass. To simplify the present analysis, all our calculations have been performed for zero momentum. The dynamical properties of quarks at zero momentum are encoded in the quark spectral function $\rho(\omega)$ which is related to the Euclidean correlation function

$$S(\tau) = \frac{1}{V} \int d^3x d^3y \langle \bar{\psi}(\tau, x) \psi(0, y) \rangle,$$

through an integral equation

$$S(\tau) = \int_{-\infty}^{\infty} d\omega \frac{e^{(1/2-\tau)\omega/T}}{e^{\omega/2T} + e^{-\omega/2T}} \rho(\omega),$$

where $\rho(\omega)$ is the spectral density of the quark propagator.
The correlation function \( S(\tau) \) has been calculated at two values of the temperature, \( T = 1.5T_c \) and \( 3T_c \), in quenched QCD using non-perturbatively improved clover Wilson fermions \([13, 14]\). To control the dependence of our results on the finite lattice volume, \( N_\tau^3 \times N_z \), and lattice spacing, \( a \), we analyze the quark propagator on lattices of three different sizes. The gauge field ensembles used for this analysis have been generated and used previously by the Bielefeld group to study screening masses and spectral functions \([15]\). The different simulation parameters are summarized in Table I \([15]\). For each lattice size, 51 configurations have been analyzed. On the \( 64^3 \times 16 \) lattices and at our smallest temperature, \( T = 1.5T_c \), we observed for the largest values of the hopping parameter, i.e. closest to \( \kappa_c \), an anomalous behavior of the quark propagator on a few gauge field configurations. The appearance of such exceptional configurations in calculations with light quarks in quenched QCD is a well-known problem in calculations with Wilson fermions \([16]\). We identified 7 such configurations, which we excluded from our analysis. The properties of the quark propagator on these configurations will be discussed in more detail elsewhere \([17]\). Quark propagators have been calculated after fixing each gauge field configuration to Landau gauge. For this we used a conventional minimization algorithm with a stopping criterion, \( (1/3)\text{tr}|\partial_\mu A^\mu|^2 < 10^{-11} \). In the Wilson fermion formulation the bare mass, \( m_0 \), is related to the hopping parameter \( \kappa \), through the standard relation

\[
m_0 = \frac{1}{2\kappa} \left( \frac{1}{\kappa} - 1 \right),
\]

where \( \kappa_c \) denotes the critical hopping parameter corresponding to the chiral limit, or vanishing quark mass.

To evaluate Eq. \( \text{11} \) numerically, we solve the linear equation \( K\psi_{\text{result}} = \psi_{\text{source}} \) for a given source \( \psi_{\text{source}} \), with \( K \) being the fermion matrix. For this procedure, we use the wall source, \( \psi_{\text{wall}} = (1/V) \sum_x \psi(0, x) \), which we found to be very efficient in reducing the statistical error in the propagator calculation. To reduce the statistical error further, we define the correlation function \( S^\text{corr.}(\tau) \) for each configuration by

\[
S^\text{corr.}(\tau) = \frac{1}{12} \text{tr}[S(\tau)\gamma^0\lambda_+ + S(\beta - \tau)\gamma^0\lambda_-],
\]

with the trace taken over Dirac and color indices.

In Fig. 4 we show the numerical results for \( S^\text{corr.} \) for several values of \( \kappa \) calculated on a lattice of size \( 64^3 \times 16 \) at \( T = 3T_c \). One sees that the shape of \( S^\text{corr.} \) approaches that of a single exponential function for smaller \( \kappa \), while it becomes symmetric as \( \kappa \) approaches \( \kappa_c \). In the vicinity of the wall source, \( i.e. \) at small and large \( \tau \), we see deviations from this generic picture which can be attributed to distortion effects arising from the presence of the source. We thus exclude points with \( \tau < \tau_{\text{min}} \) and \( N_z - \tau < \tau_{\text{min}} \) from our fits to the ansatz given in Eq. (4). The resulting correlation functions obtained from correlated fits with \( \tau_{\text{min}} = 3 \) are shown in Fig. 4. One sees that \( S^\text{corr.} \) is well reproduced by our fitting ansatz \([22]\); the \( \chi^2/\text{dof} \) of our fits is between 2 and 3 at 0.1335 \( \lesssim \kappa \lesssim 0.134 \), while it gradually increases as \( \kappa \) becomes smaller than \( \kappa = 0.1335 \). A similar behavior is also observed for our other lattice sizes \([17]\).

In Fig. 4 we show the dependence of \( E_{1,2} \) and \( Z_2/(Z_1 + Z_2) \) on the bare quark mass \( m_0 \) for \( T = 1.5T_c \) and \( 3T_c \). The results have been obtained from two-pole fits on lattices of size \( 64^3 \times 16 \). Errorbars have been estimated

| \( T/T_c \) | \( N_\tau \) | \( N_z \) | \( \beta \) | \( \epsilon_{SW} \) | \( \kappa_c \) | \( a [\text{fm}] \) |
|-------|-------|-------|-------|-------|-------|-------|
| 3     | 16    | 64    | 48    | 7.457 | 1.3389 | 0.13390 0.015 |
| 12    | 48    | 7.192 | 1.3550 | 0.13437 | 0.021 |
| 1.5   | 16    | 64    | 48    | 6.872 | 1.4125 | 0.13495 0.031 |
| 12    | 48    | 6.640 | 1.4579 | 0.13536 | 0.041 |

**Table I: Simulation parameters \([13]\).**

with the quark field \( \psi \), the spatial volume \( V \), and the imaginary time \( \tau \) which is restricted to the interval \( 0 < \tau < 1/T \). The Dirac structure of \( \rho(\omega) \) is decomposed as

\[
\rho(\omega) = \rho_0(\omega)\gamma^0 + \rho_\pm(\omega) = \rho_+(\omega)\Lambda_+ \gamma^0 + \rho_-(\omega)\Lambda_- \gamma^0,
\]

where the residues \( \rho_\pm(\omega) \) correspond to the normal and plasmino modes, \( \rho_+(\omega) \) and \( \rho_-(\omega) \) are related through

\[
\rho_+(\omega) = \rho_0(\omega) + \rho_-(\omega) \quad \text{and} \quad \rho_- = \rho_0(\omega) - \rho_-(\omega)
\]

In the following analysis, we concentrate on a determination of \( \rho_+(\omega) \) instead of \( \rho_0(\omega) \), because excitation properties of quarks are more apparent in these channels \([13]\). In analogy to Eq. \( \text{3} \) we introduce the decomposition of the correlation function \( S(\tau) \) as

\[
S(\tau) = S_+(\tau)\Lambda_+ \gamma^0 + S_-(\tau)\Lambda_- \gamma^0,
\]

where \( S_+ \) and \( S_- \) are related through \( S_+(\tau) = S_-(\beta - \tau) \).

For free quarks with scalar mass \( m_0 \) the spectral functions, \( \rho_\pm(\omega) = \delta(\omega \pm m_0) \), have quark and anti-quark poles at \( \omega = \pm m_0 \), respectively. In the high temperature limit, additional poles, corresponding to the plasmino, appear at negative energy for \( \rho_+(\omega) \) and positive energy for \( \rho_- \) \([9, 11, 12]\). While the positivity of \( \rho_\pm(\omega) \) is ensured by definition, these spectral functions are neither even nor odd functions. In the chiral limit, however, \( \rho_- \) becomes even functions.

To extract the spectral function \( \rho_+(\omega) \) from \( S(\tau) \) using Eq. \( \text{2} \), we assume that \( \rho_+(\omega) \) can be described by a two-pole ansatz,

\[
\rho_+(\omega) = Z_1\delta(\omega - E_1) + Z_2\delta(\omega + E_2),
\]

where the residues \( Z_{1,2} \) and energies \( E_{1,2} \) have to be determined from a fit to \( S_+(\tau) \). The poles at \( \omega = E_1, -E_2 \) correspond to the normal and plasmino modes, respectively \([9]\).

The correlation function \( S(\tau) \) has been calculated at two values of the temperature, \( T = 1.5T_c \) and \( 3T_c \), in quenched QCD using non-perturbatively improved clover Wilson fermions \([15, 16]\). To control the dependence of our results on the finite lattice volume, \( N_\tau^3 \times N_z \), and lattice spacing, \( a \), we analyze the quark propagator on lattices of three different sizes. The gauge field ensembles used for this analysis have been generated and used previously by the Bielefeld group to study screening masses and spectral functions \([15]\). The different simulation parameters are summarized in Table II \([15]\).
The hopping parameters satisfying $Z_{\nu}$ for the lattice of size $64^3 \times 16$ with various values of $\kappa$, and the fitting result with the ansatz Eq. (1).

![FIG. 1: The lattice correlation function $S_{\nu}^{\text{latt.}}(\tau)$ at $T = 3T_c$ for the lattice of size $64^3 \times 16$ with various values of $\kappa$, and the fitting result with the ansatz Eq. (1).](image1)

$E_1$ and $E_2$ are equal within statistical errors at $\kappa = \kappa'$. The spectral function $\rho_+ (\omega)$ thus becomes an even function at this point; the quark propagator becomes chirally symmetric despite the presence of a thermal mass, $m_T \equiv E_1 = E_2$. From Fig. 2 one also finds that the ratio $m_T / T$ is insensitive to $T$ in the temperature range analyzed in this work, while it is slightly larger for lower $T$.

As $m_0$ becomes larger, $Z_2 / (Z_1 + Z_2)$ decreases and $\rho_+ (\omega)$ is eventually dominated by a single-pole. One sees $E_1$ has a minimum at $m_0 > 0$, while $E_2$ is an increasing function of $m_0$. In the one-loop approximation, the peak in $\rho_+ (\omega)$ corresponding to $E_1$ ($E_2$) is monotonically increasing (decreasing) function of $m_0 / T$ [8, 17]. The quark mass dependence of poles found here thus is qualitatively different from the perturbative result. We find, however, that slope of $E_2$ as function of $m_0 / T$ decreases with increasing $T$. This may suggest that the perturbative behavior could eventually be recovered at much larger temperatures.

In order to check the dependence of our results on the lattice spacing and finite volume, we analyzed the quark propagator at $T = 3T_c$ for three different lattice sizes. Results for $E_1$ and $E_2$ are shown in Fig. 3. Comparing the results obtained on lattices with different lattice cut-off, $a$, but same physical volume, i.e. $64^3 \times 16$ and $48^3 \times 12$, one sees that any possible cut-off dependence is statistically not significant in our analysis. On the other hand we find a clear dependence of the quark energy levels on the spatial volume; when comparing lattices with aspect ratio $N_x / N_z = 3$ and 4 we find that the energy levels, $E_{1,2}$, drop significantly. A similar behavior is observed also at $T = 1.5T_c$.

The presence of a strong volume dependence of the...
quark propagator is not unexpected. In fact, the thermal quark mass arises as collective effect of low momentum gluons; gluons at the soft scale $p \lesssim g T$ play a crucial role to give rise to the thermal mass at high temperatures \cite{2 3}. However, on lattices with given aspect ratio $N_x/N_z$ low momentum gluons are cut-off. The lowest non-vanishing gluon momentum is, $p_{\text{min}}/T = 2\pi (N_x/N_z)$, which still is larger than unity on lattices with aspect ratio $N_x/N_z = 4$. The situation may, nonetheless, be somewhat better in the temperature range explored here as the temperature dependent coupling $g(T)$ is larger than unity. An analysis of quark spectral functions on lattices with even larger spatial volume is needed in the future to properly control effects of small momenta. We attempted to estimate the thermal mass in the $V \rightarrow \infty$ limit by extrapolating the results obtained for two different volumes. Defining $m_T \equiv (Z_1 E_1 + Z_3 E_3)/(Z_1 + Z_3)\kappa = \kappa_c$ and assuming the volume dependence of $m_T$ as $m_T(N_x/N_z) = m_T(0) \exp(c N^3/N^3)$, we obtain $m_T(0)/T = 0.771(18)$ for $T = 3T_c$ and $m_T(0)/T = 0.800(15)$ for $T = 1.5T_c$. This suggests that finite volume effects may still be of the order of 15% in our current analysis of $m_T/T$. Despite these problems, our result clearly shows that light quarks near but above $T_c$ have a mass gap that is of collective nature similar to that in the perturbative regime.

In this Letter, we analyzed the quark spectral function at zero momentum for $T = 1.5T_c$ and $3T_c$ as functions of bare quark mass $m_0$ in quenched lattice QCD with Landau gauge fixing. We found that the two-pole approximation for $\rho_+(\omega)$ well reproduces the behavior of the lattice correlation function. It is argued that the chiral symmetry of the quark propagator is restored at the critical value of $\kappa$ and the shape of the spectral function at this point takes a similar form as in the high temperature limit having normal and plasmino modes with thermal mass $m_T$. As $m_0$ is increased, $\rho_+(\omega)$ approaches a single-pole structure as one can naturally deduce intuitively. The non-perturbative nature of thermal gauge fields is reflected in the behavior of poles as functions of $m_0$, which is qualitatively different from the perturbative result \cite{9 18}. We also note that the ratio $m_T/T$ decreases slightly with increasing $T$, which is expected to happen at high temperature where $m/T$ should be proportional to a running coupling $g(T)$. Although results on the quark propagator are gauge dependent, we expect that our results for its poles suffer less from gauge dependence, because the success of the pole approximation for $\rho_+(\omega)$ indicates that the quark propagator has dynamical poles near the real axis, which are gauge independent quantities \cite{9 18}.

In the present study, we analyzed the quark spectral function in the quenched approximation. Although this approximation includes the leading contribution in the high temperature limit \cite{2} and thus is valid at sufficiently high $T$, the validity of this approximation near $T_c$ is non-trivial. For example, screening of gluons due to the polarization of the vacuum with virtual quark antiquark pairs is neglected in this approximation. The coupling to possible mesonic excitations \cite{13 20}, which may cause interesting effects in the spectral properties of the quark \cite{11}, are not incorporated, either. The comparison of the quark propagator between quenched and full lattice simulations would tell us the strength of these effects near $T_c$.

In the future it will also be interesting to use results on the non-perturbative structure of quark propagators as input for phenomenological studies of the QGP phase. For example, thermal properties of the charm quark \cite{17} should be useful for the understanding of properties of charmonia above $T_c$. \cite{19}. The thermal mass of light quarks can also be used to evaluate details of their dynamics \cite{21}.

Although in this letter we limited our analysis to zero momentum, a proper analysis at finite momentum \cite{17} is needed for the understanding of the entire quark spectral function. In particular, the confirmation of the existence of a minimum in the plasmino dispersion relation at finite momentum \cite{4} clearly is a challenging problem. The exploration of the gluon propagator is also an important subject of further studies. To clarify the origin of the quark mass dependence of $E_1$ and $E_2$, which is qualitatively different from the perturbative result, as well as the $T$ dependence of the thermal mass, are open questions for further numerical and analytic studies.

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