3Quantum: Algebra Geometry Information

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Abstract. The basic information about the international QQQ Conference 3Quantum: Algebra Geometry Information (Tallinn, July 2012) \[AstrAlgo eWeb 2012 QQQ II\] is presented and the concise review of the QQQ Proceedings is given by the proceedings volume editors.

1. Introduction
This proceedings volume presents results obtained by the participants of the 6th ECM satellite QQQ Conference 3Quantum: Algebra Geometry Information \[1\] held at the Tallinn University of Technology \[2\], Estonia, 10-13 July 2012. The meeting was organized by joint initiative of two networks: ITGP n/w of ESF and AGMP \[3\] Baltic-Nordic n/w, under the general endorsement and sponsorship of the European Science Foundation (ESF) and it was dedicated to memory of Professor Jean-Louis Loday (1946-2012).

In the QQQ Conference scientific program, the invited thematic session Gravitation and Cosmology was dedicated to centenary of Professor Harald Keres (1912-2010) who was the initiator of Einstein’s gravity studies in Estonia.

The QQQ Conference international Organizing Committee was: E. Paal (chair), A. Fialowski, J. Fuchs, M. Schlichenmaier, V. Shumeiko, A. Stolin and P. Zusmanovich.

The QQQ Conference was an official satellite meeting of:

- 6th European Congress of Mathematics ECM6
  Associated Mini-symposia 25 Years of Quantum Groups: From Definition to Classifications was organized by A. Stolin with invited speakers: Y. I. Manin, M. Semenev-Tian-Shansky, S. Woronowicz and E. Zelmanov
- 17th International Congress of Mathematical Physics ICMP12
- 13th Marcel Grossmann Meeting on General Relativity MG13

2. Description of QQQ Conference
The main purpose of the QQQ Conference was to stimulate and promote interactions between three major research areas: (quantum) algebra, (quantum) geometry and gravitation, and (quantum) information theory. Thus, the basic concept

\[geometry \equiv gravitation\] and the cubic acronym \[QQQ\]

were fixed, that determined the face of the QQQ Conference & Proceedings.
In total, 96 scientists from 27 countries attended the *QQQ* Conference. The scientific program of the *QQQ* Conference contained 79 registered talks, including 20 plenary talks, and 59 talks in the following 8 invited thematic sessions:

- **Deformation Theory and Quantization** with 7 speakers
- **Geometry and Topology in Quantum Gravity** with 9 speakers
- **Gravitation and Cosmology** with 12 speakers
dedicated to centenary of Professor Harald Keres (1912-2010)
- **Hopf Algebra, Quantum Algebra** with 11 speakers
- **Jordan Structures in Mathematical Physics** with 4 speakers
- **Modularity and Non-Perturbative Strings** with 5 speakers
- **Nonassociative Algebra, Hom-Algebra, Applications** with 6 speakers
- **Operads, Noncommutative Algebra and Geometry** with 5 speakers

The reader can find more information, including the *QQQ* Conference Abstracts and other related materials, in [1].

### 3. Review of *QQQ* Proceeding

The research articles, carefully selected for the *QQQ* Proceedings volume cover a wide range of topics representing the main directions of the conference and the topics of the special thematic sessions. All articles submitted for this volume were refereed, and the Editors take this opportunity to thank all referees for their work as well as all contributors for the collaboration. Among contributors to the volume there are 49 researchers from many different countries.

The *QQQ* Proceedings volume includes 30 contributions, including 5 contributions from the *QQQ* Conference plenary speakers – L. Faddeev, P. Kulish, O. A. Laudal, Y. I. Manin and S. Waldmann – that we review here in the alphabetical order by the first author, by paying more attention on the contributions of the *QQQ* Conference plenary speakers, and apology for the monotonic presentation.

D. V. Artamonov and V. A. Golubeva in the article *Knizhnik-Zamolodchikov type equations for the root system B and Capelli central elements* turn attention that the construction of the famous Knizhnik-Zamolodchikov equations uses the central element of the second order in the universal enveloping algebra for some Lie algebra. But in the universal enveloping algebra there exist central elements of higher orders as well. Thus, it seems desirable to use these elements for the construction of Knizhnik-Zamolodchikov type equations. In the present paper they give a construction of such Knizhnik-Zamolodchikov type equations for the root system $B$ associated with Capelli central elements in the enveloping algebra for the orthogonal algebra.

In the article *On quantum deformations of (anti-)de Sitter algebras in (2+1) dimensions*, A. Ballesteros, F. J. Herranz and F. Musso revisit deformations of (anti-)de Sitter $(A)dS$ algebras in (2+1) dimensions, and review several features of these quantum structures. In particular, the classification problem of (2+1) $(A)dS$ Lie bialgebras is presented and the associated noncommutative quantum $(A)dS$ spaces are also analysed. Moreover, the flat limit (or vanishing cosmological constant) of all these structures leading to (2+1) quantum Poincaré algebras and groups is simultaneously given by considering the cosmological constant as an explicit Lie algebra parameter in the $(A)dS$ algebras. By making use of this classification, a three-parameter generalization of the $\kappa$-deformation for the (2+1) $(A)dS$ algebras and quantum spacetimes is given. Finally, the same problem is studied in (3+1) dimensions, where a two-parameter generalization of the $\kappa$-$(A)dS$ deformation that preserves the space isotropy is found.

L. Borsten in the article *3-qubit entanglement: A Jordan algebraic perspective* turns attention that it is by now well known that three qubits can be totally entangled in two physically distinct
ways. The author reviews work classifying the physically distinct forms of qubit entanglement using the elegant framework of Jordan algebras, the Freudenthal-Kantor triple systems (F-KTS) and groups of type $E_7$. In particular, it is shown that the four Freudenthal-Kantor ranks correspond precisely to the four 3-qubit entanglement classes: (1) Totally separable $A-B-C$, (2) Biseparable $A-BC$, $B-CA$, $C-AB$, (3) Totally entangled $W$, (4) Totally entangled GHZ. The rank 4 GHZ class is regarded as maximally entangled in the sense that it has non-vanishing quartic norm, the defining invariant of the Freudenthal-Kantor triple system. While this framework is specific to three qubits, the author shows here how the essential features may be naturally generalised to an arbitrary number of qubits. In the article Stringlike structures in the real and complex Kerr-Schild geometry, A. Burinskii studies the four-dimensional Kerr-Schild (KS) geometry that displays remarkable relationships with quantum world and theory of superstrings. In particular, the Kerr-Newman (KN) solution has gyromagnetic ratio $g = 2$, as that of the Dirac electron and represents a consistent background for gravitational and electromagnetic field of the electron. As a consequence of very big spin/mass ratio, black hole horizons disappear, exposing the naked Kerr singular ring. The author considers four-decade history of development of this structure which took finally the form of a point-string-membrane-bubble complex which is reminiscent of the enhancon model of string/M-theory. A complex string obtained in the complex structure of the Kerr geometry gives an extra dimension to the world-sheet of the real Kerr string, forming a membrane by analogue with the string/M-theory unification. By analysis of the orientifold parity of the complex Kerr string, the author obtains that the principal null congruence of the Kerr geometry determined by the Kerr theorem is described by a quartic equation in projective twistor space $CP^3$, and therefore, it creates the known Calabi-Yau twofold (K3 surface) in twistor space of the 4d KS geometry. The author connects it with $N=2$ superstring which has (complex) critical dimension two and may be embedded into complex KS geometry.

In the article 3J-Symbol for the Modular Double $SL_q(2, \mathbb{R})$ Revisited the authors S. E. Derkachev and L. D. Faddeev first recall that the modular double of quantum group $SL_q(2, \mathbb{R})$ with $|q| = 1$ has a series of selfadjoint irreducible representations $\pi_s$ parameterized by $s \in \mathbb{R}_+$. Ponsot and Teschner in [2001 Comm. Math. Phys. vol 224 p 613] considered a decomposition of the tensor product $\pi_{s_1} \otimes \pi_{s_2}$ into irreducibles. In the present article, authors give the detailed derivation and some new proofs.

S. Groote and R. Saar in the article Group theory aspects of chaotic strings first note that the chaotic strings are a special type of non-hyperbolic coupled map lattices, exhibiting a rich structure of complex dynamical phenomena with a surprising correspondence to physical contents and are generated by the Chebyshev maps $T_2(\phi)$ and $T_3(\phi)$. In this paper the authors connect the Chebyshev maps via the Galois theory to the cyclic groups $Z_2$ and $Z_3$ and give some ideas how this fundamental connection might lead to the emergence of the familiar Lie group structure of the particle physics and, in particular, even to the emergence of space-time. The authors present here the most important results associated with these papers. In the article Lorentz invariance and gauge equivariance, S. Groote, R. Saar and H. Liivat turn attention that when trying to place Lorentz and gauge transformations on the same foundation, it turns out that the first one generates invariance, the second one equivariance, at least for the abelian case. The authors note that this similarity is not an hypothesis but is supported by and is a consequence of the path integral formalism in quantum field theory. S. Groote, H. Veermäe and C. Beck in the article Renormalization aspects of chaotic strings state that chaotic strings are a class of non-hyperbolic coupled map lattices, exhibiting a rich structure of complex dynamical phenomena with a surprising correspondence to physical contents. In this article the authors...
introduce different types and models for chaotic strings, where 2B-strings with finite length are considered in detail and demonstrate possibilities to extract renormalized quantities, which are expected to describe essential properties of the string. In the article *Aspects of multimetric gravity*, M. Hohmann presents a class of gravity theories containing \( N \geq 2 \) metric tensors and a corresponding number of standard model copies. In the Newtonian limit gravity is attractive within each standard model copy, but different standard model copies mutually repel each other. The author discusses several aspects of these multimetric gravity theories, including cosmology, structure formation, the post-Newtonian limit and gravitational waves. The most interesting feature the author finds is an accelerating expansion of the universe that naturally becomes small at late times. P. Jastrzębski and A. Tralle in the article *A note on Clifford-Klein forms* consider the difficult problem of finding the Clifford-Klein forms in a class of homogeneous spaces determined by inclusions of real Lie algebras of a special type which they call strongly regular. This class of inclusions is described in terms of their Satake diagrams. For example, the complexifications of such inclusions contain the class of subalgebras generated by automorphisms of finite order. Authors show that the condition of strong regularity implies the restriction on the real rank of subalgebras. This in part explains why the known examples of Clifford-Klein forms are rare. The authors make detailed calculations of some known examples from the point of view of the Satake diagrams. In the article *Parametrizations in scalar-tensor theories of gravity and the limit of general relativity*, L. Järv, P. Kuusk, M. Saal and O. Vilson consider a general scalar-tensor theory of gravity and review briefly different forms it can be presented. The authors investigate the conditions under which its field equations and the parametrized post-Newtonian parameters coincide with those of general relativity and demonstrate that these so-called limits of general relativity are independent of the parametrization of the scalar field, although the transformation between scalar fields may be singular at the corresponding value of the scalar field. In particular, the limit of general relativity can equivalently be determined and investigated in the commonly used Jordan and Einstein frames.

In the article

**Deformed Richardson-Gaudin Model**

the authors P. Kulish, A. Stolin and L. H. Johannesson study the Richardson-Gaudin model that describes strong pairing correlations of fermions confined to a finite chain. The integrability of the Hamiltonian allows the algebraic construction of its eigenstates. The authors show that the quantum group theory provides a possibility to deform the Hamiltonian preserving integrability. More precisely, the authors use the so-called Jordanian \( r \)-matrix to deform the Hamiltonian of the Richardson-Gaudin model. In order to preserve its integrability, one needs to insert a special nilpotent term into the auxiliary \( L \)-operator which generates integrals of motion of the system. Moreover, the quantum inverse scattering method enables to construct the exact eigenstates of the deformed Hamiltonian. These states have a highly complex entanglement structure which certainly requires further investigation.

In the article

**The Structure of \( Ph^* \), Generalized de Rham, and Entropy**

the author O. A. Laudal continues the study of the non-commutative phase space functor, \( Ph(A) \), defined for any associative algebra \( A \), and its derived differential co-simplicial algebra, \( Ph^*(A) \). The main focus of this research is on its relationship to the classical de Rham complex, to the dynamics of the finite dimensional \( Ph^\infty(A) \)-modules, and to the notion of Entropy. These subjects are treated within the set-up of author’s recently published monograph [Geometry of Time-Spaces 2011 (World Scientific)] and in the forthcoming monograph [Cosmos and its Furniture].
A. Leibak and P. Puusemp in the article *On determinability of some classes of medial quasigroups by their endomorphisms* study the endomorphisms of idempotent medial quasigroups and determinability of some classes of medial quasigroups by their endomorphisms. The authors introduce the endomorphism algebra of idempotent medial quasigroup and prove that if the endomorphism algebras of quasigroups are isomorphic, then the corresponding quasigroups are isomorphic as well. In the article *A short survey of Lie antialgebras*, S. Leidwanger and S. Morier-Genoud follow the presentation at the QQQ conference. Authors give a brief survey of the existing theory of Lie antialgebras and suggest open questions. R. K. Loide and P. Suurvarik in the article *Supersymmetry: superfield equations of motion* consider supersymmetry and superfields in connection with the Poincaré superalgebras. The original formalism of the projection operators for deriving the wave equations for ordinary fields and superfields is developed. The superfield equations of motion in the case of massive and massless fields are presented together with an application in linear supergravity. In the article *Drinfeld twisting elements on Hom-bialgebras*, A. Makhlouf and B. Torrecillas have the aim to introduce the concept of a twisting element based on a Hom-bialgebra and to use it to provide twists or deformations of Hom-associative algebras. Also, the authors review the module theory in Hom-setting and, in particular, show that a twisting element based on a bialgebra gives rise to a twisting element based on a Hom-bialgebra.

The QQQ Conference key speaker Y. I. Manin presents in the article

**COMPLEXITY vs ENERGY:**

Theory of Computation and Theoretical Physics

a thorough survey based upon author’s plenary lecture [4] at the QQQ Conference as well as his recent research results in this field. It is dedicated to a deep analogy between the notions of complexity in theoretical computer science and energy in physics. The author emphasizes that this analogy is not metaphorical and describes three precise mathematical contexts, suggested recently, in which mathematics related to (un)computability is inspired by and to a degree reproduces formalisms of statistical physics and quantum field theory.

H. Melikyan and P. Zusmanovich in the article *Melikyan algebra is a deformation of a Poisson algebra* prove, using computer, that the restricted Melikyan algebra of dimension 125 is a deformation of a Poisson algebra. In the article *Categorical generalization of spinfoam models*, A. Miković and M. Vojinović discuss problems of the nonrenormalizability of the general relativity and consider the main research directions which have the aim to resolve this problem. The attention of authors focuses on the approach of loop quantum gravity, specifically on the spinfoam models. These models have some issues concerning the semiclassical limit and coupling of matter fields. The recent developments in category theory provide the necessary formalism to introduce a new action for general relativity and perform covariant quantization. Z. Oziewicz in the article *Centre-of-mass for the finite speed of light* first recalls that already in 1632 Galilei was aware of relativity of velocity and that this implies relativity of spaces-of-locations and during centuries the latter was ignored. Author notes that Professor Harald Keres (1912-2010) considered the space-of-locations as a congruence of world-lines and there is no universal absolute three-dimensional space-of-locations. Author shows that the centre-of-mass of many-body interacting (bound) system for the case of finite light-speed is a well defined concept within the group-free approach using algebra epimorphisms as splits and considers the Keres space-of-locations as the Grassmann factor-algebra of differential forms. In the article *Note on homological modeling of the electric circuits*, E. Paal and M. Umbleja explain on a simple example how the homological analysis may be used for modeling of the electric circuits. The homological
branch, mesh and nodal analyses are presented and the geometrical interpretations are given. E. Paal and J. Virkepu in the article *Operadic quantization as a tool for discrete geometry* use the operadic Lax representations of the harmonic oscillator to construct the quantum counterparts of 3d real Lie algebras in the Bianchi classification. The Jacobi operators of these quantum algebras are studied. The authors show how the energy conservation is related to the Jacobi identity and how the quantization leads to an anomaly – the quantum violation of the Jacobi relations. By using the non-vanishing quantum Jacobi operators, the derivative quantum algebra for a triple of 3d real Lie algebras is constructed. It is proposed that the derivative algebra is the 3d real Heisenberg algebra, from which it follows that in this model only the discrete values of the spatial coordinates are physically allowed. In the article *On quantum groups and Lie bialgebras related to sl(n)*, A. Stolin and I. Pop study for given an arbitrary field $F$ of characteristic 0 the Lie bialgebra structures on $sl(n, F)$, based on the description of the corresponding classical double. It turns out that for any Lie bialgebra structure $\delta$, the classical double $D(sl(n, F), \delta)$ is isomorphic to $sl(n, F) \otimes A$, where $A$ is either $F[\varepsilon]$, with $\varepsilon^2 = 0$, or $F \oplus F$ or a quadratic field extension of $F$. In the first case, the classification leads to quasi-Frobenius Lie subalgebras of $sl(n, F)$. In the second and third cases, a Belavin-Drinfeld cohomology can be introduced which enables one to classify the Lie bialgebras on $sl(n, F)$, up to gauge equivalence. The Belavin-Drinfeld untwisted and twisted cohomology sets associated to an $r$-matrix are computed. E. Randla in the article *PPN parameters for multiscalar-tensor gravity without a potential* proposes a generic multiscalar-tensor gravity action functional in the Jordan conformal frame. After redefining the scalar fields, the parameterised post-Newtonian parameters are calculated, neglecting the potential term. The field equations are shown to be nearly identical with those of scalar-tensor gravity up to the first post-Newtonian approximation, differing only in the definitions of constant terms. In the article *Examples of cosmological rips in the scalar-tensor cosmology*, the authors M. Saal and A. Frantskijvitšius examine whether a cosmological rip scenarios are possible in the scalar-tensor gravity by parametrizing the equation of state of dark energy in an appropriate way. The parameters of the model are constrained by the observational data of type Ia supernovas collected to Union 2.1 compilation.

In the article

**Fréchet-Algebraic Deformation Quantizations**

the author S. Waldmann presents some recent results on the convergence properties of formal star products. Based on a general construction of a Fréchet topology for an algebra with countable vector space basis, the author discusses several examples from deformation quantization: the Wick star product on the flat phase space gives a first example of a Fréchet algebraic framework for the canonical commutation relations. More interesting, the star product on the Poincaré disk can be treated along the same lines, leading to a non-trivial example of a convergent star product on a curved Kähler manifold.

G. P. Wene in the article *Permutations of cubical arrays* notes that the structure constants of an algebra determine a cube called the cubical array associated with the algebra. The permuted indices of the cubical array associated with a finite semifield generate new division algebras. Author does not require that the algebra be finite and asks "Is it possible to choose a basis for the algebra such any permutation of the indices of the structure constants leaves the algebra unchanged?" What are the associated algebras? Author shows that the property "weakly quadratic" is invariant under all permutations of the indices of the corresponding cubical array and presents two algebras for which the cubical array is invariant under all permutations of the indices. In the article *Geometrodynamics and Lorentz symmetry*, D. K. Wise studies dynamics of the gauge theory and general relativity using fields of local observers, thus maintaining local Lorentz symmetry despite a space/time splitting of fields. Author starts
with Yang-Mills theory, where the observer fields are defined as normalized future-timelike vector fields, then defines observers without a fixed geometry and finds that these play two related roles in general relativity: splitting fields into spatial and temporal parts, and breaking the gauge symmetry, effectively reducing the spacetime $SO(n,1)$ connection to an observer-dependent spatial $SO(n)$ connection. In both gauge theory and gravity, the observer field reduces the action to canonical form, without using gauge fixing. In the 4d gravity case, the result is a manifestly the Lorentz covariant counterpart of the Ashtekar-Barbero formulation. A. Zuevsky in the article *Lie-algebraic symmetries of generalized Davey-Stewartson equations* identifies the full Lie-algebraic structure of the generalized Davey-Stewartson system of equations with symmetries of a specific continual Lie algebras. The author shows that these are related to two copies of the Poisson bracket continual Lie algebra.

4. Conclusions

In conclusion, we can state that all main topics and sessions of the *QQQ* Conference are well represented in the *QQQ* Proceedings and we hope that this international interdisciplinary volume will be a source of further inspiration for the international research community working in contemporary physics and mathematics.

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