Spin–isospin excitation of \(^3\)He with three-proton final state

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Spin–isospin excitation of the \(^3\)He nucleus by a proton-induced charge exchange reaction, \(^3\)He\((p, n)ppp\), at forward neutron scattering angle is studied in a plane wave impulse approximation (PWIA). In PWIA, cross sections of the reaction are written in terms of proton–neutron scattering amplitudes and response functions of the transition from \(^3\)He to the three-proton state by spin–isospin transition operators. The response functions are calculated with realistic nucleon–nucleon potential models using a Faddeev three-body method. Calculated cross sections agree with available experimental data in substance. Possible effects arising from the uncertainty of proton–neutron amplitudes and three-nucleon interactions in the three-proton system are examined.

Subject Index D00, D05, D22

1. Introduction

Three-nucleon (3N) systems—\(^3\)H, \(^3\)He, nucleon–deuteron elastic and breakup reactions, etc.—have been playing important roles in the quest for the details of interactions among nucleons. These systems are essentially total isospin \(T = \frac{1}{2}\) states. (Although the breaking of charge symmetry in nuclear interaction and the Coulomb interaction allow a mixture of \(T = \frac{3}{2}\) components, the percentage is quite small (Ref. [1]).) On the other hand, knowledge of interactions among three nucleons, especially of three-nucleon interactions, in \(T = \frac{3}{2}\) states is expected for studies on heavier nuclei, neutron-rich nuclei, neutron star matter, etc. Since there is no bound state of three-neutron (3n) and three-proton (3p) systems, which are typical \(T = \frac{3}{2}\) states (Ref. [2]), observables related to these systems may be obtained from nuclear reactions that produce them as final continuum states. The reaction mechanism of such a reaction needs to be as simple as possible to reduce ambiguity in extracting information on the nuclear interaction.

In the present paper, I will study a charge exchange reaction, namely the \(^3\)He\((p, n)ppp\) reaction at incident proton energies of several hundreds of MeV and a reaction angle of \(\theta_n = 0^\circ\). Although this is a four-body reaction for which it is still difficult to perform rigorous calculations at high energies, the cross section of the reaction in a plane wave impulse approximation (PWIA) is written in terms of \(n(p, n)p (pn)\) scattering amplitudes and response functions of the 3N system. The former can be taken from nucleon–nucleon (NN) databases (Refs. [3,4]). The latter corresponds to a transition from the initial \(^3\)He bound state to final 3p continuum states, in which one needs to solve a three-body problem.
The present author has developed a method to solve the quantum mechanical three-body problem applying the Faddeev method (Ref. [5]). This method is based on solving the Faddeev equation as integral equations in coordinate space, which even includes long-range Coulomb force effects (Refs. [6,7]), and has been successfully applied for proton–deuteron systems (Ref. [8]) and three-alpha-particle systems (Ref. [9]). In this paper, this method will be applied for calculating the response functions of the $3p$ final states.

In Ref. [10], the cross section $I(0^\circ)$ for the $^3\text{He}(p, n)ppp$ reaction at the incident proton energy $T_p = 200$ MeV was measured. In Ref. [11], the polarization transfer coefficient in the transverse direction, $D_{NN}(0^\circ)$, and that in the longitudinal direction, $D_{LL}(0^\circ)$, as well as $I(0^\circ)$ were measured at $T_p = 346$ MeV. One of the measured polarization transfer coefficients, $D_{NN}(0^\circ)$, is consistent with the corresponding $pn$ values. However, the other one, $D_{LL}(0^\circ)$, deviates from the $pn$ values. The authors of Ref. [11] show that this discrepancy may be attributed to a $3p$ resonance with spin parity $\frac{1}{2}^-$. The existence of resonant states in multi-neutron or multi-proton states has been a long-standing problem in nuclear physics. A recent compilation of the mass number $A = 3$ systems (Ref. [2]) reports negatively for the existence of $A = 3$ resonance. In Ref. [12], the possibility of existence of a four-neutron (tetraneutron, $4n$) resonant state was reported. In Ref. [13], it was shown that the existence of the $4n$ resonant state demands an attractive $T = \frac{3}{2}$ three-nucleon potential (3NP) that is tremendously strong. The effects of such a 3NP on the $3p$ system will be studied.

In Sect. 2, I will summarize the formalisms to calculate the response functions and then observables in the $^3\text{He}(p, n)ppp$ ($\theta_n = 0^\circ$) reaction. In Sect. 3, I will show some results of calculations and compare them with available experimental data. A summary will be given in Sect. 4. In the appendix, some kinematical values related to the reaction will be summarized.

2. Theoretical background

In this section, I will consider the charge exchange reaction $^3\text{He}(\vec{p}, \vec{n})ppp$ ( $\theta_n = 0^\circ$) by PWIA, in which the $n(\vec{p}, \vec{n})p$ ( $\theta_n = 0^\circ$) scattering amplitude and response functions for the transition from $^3\text{He}$ to $3p$ continuum states are the basic elements. (See Refs. [14,15], e.g., for the general formalism of PWIA.) The kinematics of the reaction is characterized by the incident proton energy in the laboratory system $T_p$, and the energy transfer in the laboratory system $\omega_{Lab}$ defined by Eq. (A.2a) in the appendix. The direction of the incident proton and thus of the outgoing neutron is taken to be the $z$-axis.

First, I introduce the $3p$ Hamiltonian in the center of mass (c.m.) system,

$$\hat{H}_{3p} = \hat{H}_0 + \hat{V},$$

(1)

where $\hat{H}_0$ is the kinetic energy operator of the three-body system, and $\hat{V}$ is an interaction potential, which consists of two-nucleon potentials (2NPs) and 3NPs.

Let $|\Psi_{m_1m_2m_3}(\vec{q}, \vec{p})\rangle$ be an eigenstate of the Hamiltonian $\hat{H}_{3p}$ associated with an asymptotic $3p$ state, in which the relative momentum between two protons is $\vec{q}$, the momentum of the third proton with respect to the c.m. of the proton pair is $\vec{p}$, and the spin projection of proton $i$ is $m_i$. The superscript $(\pm)$ expresses the outgoing (+) or incoming (−) boundary condition.

The eigenvalue problem is written as

$$\hat{H}_{3p} |\Psi_{m_1m_2m_3}^{(\pm)}(\vec{q}, \vec{p})\rangle = E(\vec{q}, \vec{p}) |\Psi_{m_1m_2m_3}^{(\pm)}(\vec{q}, \vec{p})\rangle,$$

(2)
with

$$E(q,p) = \frac{q^2}{m_p} + \frac{3p^2}{4m_p},$$

(3)

where $m_p$ is the mass of the proton.

A response function corresponding to the transition from the $^3\text{He}$ state with spin projection $M$, $|\Psi_M\rangle$, to $^3p$ continuum states with energy $E$ by an operator $\hat{O}$ is given by

$$R(E) = \frac{1}{2} \sum_{M = \pm \frac{1}{2}} \sum_{m_1, m_2, m_3} \int dq dp \left| T(q,p,m_1,m_2,m_3,M) \right|^2 \delta (E - E(q,p)),$$

(4)

where $E$ is related to kinematical values of the reaction as Eq. (A.9), and the transition amplitude is defined by

$$T(q,p,m_1,m_2,m_3,M) = \langle \Psi_{m_1m_2m_3}^{-}(q,p) | \hat{O} | \Psi_M \rangle.$$

(5)

Using the completeness of the $^3p$ states, we have

$$R(E) = \frac{1}{2} \sum_{M = \pm \frac{1}{2}} \langle \Psi_M | \hat{O}^\dagger \delta (E - \hat{H}_{3p}) \hat{O} | \Psi_M \rangle$$

$$= -\frac{1}{2\pi} \sum_{M = \pm \frac{1}{2}} \text{Im} \langle \Psi_M | \hat{O}^\dagger \frac{1}{E + i\epsilon - \hat{H}_{3p}} \hat{O} | \Psi_M \rangle.$$

(6)

Here, I introduce a wave function $|\Xi_M\rangle$ describing the disintegration process (Ref. [16]),

$$|\Xi_M\rangle = \frac{1}{E + i\epsilon - \hat{H}_{3p}} \hat{O} | \Psi_M \rangle,$$

(7)

from which the transition amplitude is calculated as follows:

$$T(q,p,m_1,m_2,m_3,M) = \langle \Phi_{m_1m_2m_3}^{3p} (q,p) | \hat{O} | \Psi_M \rangle + \langle \Phi_{m_1m_2m_3}^{3p} (q,p) | \hat{V} | \Xi_M \rangle,$$

(8)

where $|\Phi_{m_1m_2m_3}^{3p} (q,p)\rangle$ is the initial state corresponding to $|\Psi_{m_1m_2m_3}^{(+)} (q,p)\rangle$.

Numerical solution of Eq. (7) is obtained by a method based on the Faddeev three-body theory (Ref. [5]), whose formal and technical details are essentially the same as those used for the proton–deuteron scattering (Refs. [7,8]) and three-alpha-particle (Ref. [9]) problems.

The $n(p, n)p$ ($\theta_n = 0^\circ$) amplitude consists of three independent terms:

$$f_{pn} = \mathcal{V}_c + \mathcal{V}_L (\sigma_p \cdot \hat{z}) (\sigma_n \cdot \hat{z}) + \mathcal{V}_T (\sigma_p \times \hat{z}) \cdot (\sigma_n \times \hat{z}),$$

(9)

where $\sigma_p$ ($\sigma_n$) is the Pauli spin matrix of the incident proton (the outgoing neutron), and $\mathcal{V}_c$, $\mathcal{V}_L$, and $\mathcal{V}_T$, are the spin-scalar, spin-longitudinal, and spin-transverse components of the amplitude, respectively.
The $pn$ observables, differential cross section and polarization transfer coefficients, are given as follows:

\begin{align}
\sigma^{pp}(0^\circ) &= |\mathcal{V}_c|^2 + |\mathcal{V}_L|^2 + 2 |\mathcal{V}_T|^2, \\
D_{LL}^{pp}(0^\circ) &= \frac{|\mathcal{V}_c|^2 + |\mathcal{V}_L|^2 - 2 |\mathcal{V}_T|^2}{|\mathcal{V}_c|^2 + |\mathcal{V}_L|^2 + 2 |\mathcal{V}_T|^2}, \\
D_{NN}^{pp}(0^\circ) &= \frac{|\mathcal{V}_c|^2 - |\mathcal{V}_L|^2}{|\mathcal{V}_c|^2 + |\mathcal{V}_L|^2 + 2 |\mathcal{V}_T|^2}.
\end{align}

In the process considered, there are three operators corresponding to each term of Eq. (9): the isovector spin-scalar operator $\hat{O}_c$, the isovector spin-longitudinal operator $\hat{O}_L$, and the isovector spin-transverse operator $\hat{O}_T$, which are defined by

\begin{align}
\hat{O}_c &= \sum_{i=1}^{3} e^{i Q_{c.m.}\hat{z}} r_i t_i^{(+)} , \\
\hat{O}_L &= \sum_{i=1}^{3} e^{i Q_{c.m.}\hat{z}} (\hat{z} \cdot \sigma_i) t_i^{(+)} , \\
\hat{O}_T &= \sum_{i=1}^{3} e^{i Q_{c.m.}\hat{z}} (\hat{z} \times \sigma_i) t_i^{(+)} ,
\end{align}

where $Q_{c.m.}$ is the momentum transfer, Eq. (A.8b) in the appendix, $t_i^{(+)}$ is an isospin operator that transforms the neutron $i$ in $^3$He to proton $i$ in the final $3p$ state, and $r_i (\sigma_i)$ is the coordinate vector in the $3N$ c.m. system (the Pauli spin matrix) of particle $i$. The corresponding response functions will be denoted as $R_c(E)$, $R_L(E)$, and $R_T(E)$, respectively.

The unpolarized differential cross section $I(0^\circ)$ and the polarization transfer coefficients, $D_{LL}(0^\circ)$ and $D_{NN}(0^\circ)$, for the $^3\text{He}(p,n)ppp (\theta_n = 0^\circ)$ reaction are expressed as

\begin{align}
I(0^\circ) &= N_K \left( |\mathcal{V}_c|^2 R_c + |\mathcal{V}_L|^2 R_L + 2 |\mathcal{V}_T|^2 R_T \right), \\
D_{LL}(0^\circ) &= \frac{|\mathcal{V}_c|^2 R_c + |\mathcal{V}_L|^2 R_L - 2 |\mathcal{V}_T|^2 R_T}{|\mathcal{V}_c|^2 R_c + |\mathcal{V}_L|^2 R_L + 2 |\mathcal{V}_T|^2 R_T}, \\
D_{NN}(0^\circ) &= \frac{|\mathcal{V}_c|^2 R_c - |\mathcal{V}_L|^2 R_L}{|\mathcal{V}_c|^2 R_c + |\mathcal{V}_L|^2 R_L + 2 |\mathcal{V}_T|^2 R_T},
\end{align}

where the kinematical factor $N_K$ is given in Eq. (A.10) in the appendix.

3. Results and discussion

In this section, calculations of the observables for the $^3\text{He}(p,n)ppp (\theta_n = 0^\circ)$ reaction at $T_p = 346$ MeV and 200 MeV will be presented and compared with available experimental data.

The calculations are performed as follows: the three-body equation, Eq. (7), is solved for each of the transition operators, Eqs. (11a)–(11c), from which the transition amplitude, Eq. (5), is calculated.
Table 1. Empirical and calculated values for the NN scattering length parameters of $^1S_0$ pp, nn, and pn states. For the $pp$ system, the scattering length after subtracting the effect of the Coulomb force is used. Experimental values are taken from Ref. [21].

|       | $a_{np}^0$ (fm) | $a_{nn}$ (fm) | $a_{pn}$ (fm) |
|-------|----------------|---------------|---------------|
| Empirical | $-17.3 \pm 0.4$ | $-18.9 \pm 0.4$ | $-23.740 \pm 0.020$ |
| AV18  | $-16.6$        | $-18.3$       | $-23.7$       |
| AV8’  | $-19.3$        | $-19.3$       | $-19.3$       |
| AV14  | $-23.7$        | $-23.7$       | $-23.7$       |
| AV14(CD) | $-17.7$     | $-18.9$       | $-23.7$       |
| dTRS  | $-18.0$        | $-18.0$       | $-18.0$       |
| dTRS(CD) | $-16.6$     | $-18.0$       | $-24.1$       |

by Eq. (8). Then the response functions are calculated from Eq. (4). Using the response functions together with the $pn$ amplitudes in Eq. (9), the observables are calculated by Eqs. (10a)–(10c).

In solving three-body equations, 3N partial wave states for which 2NPs and 3NPs are active are restricted to those with total NN angular momenta $J \leq 6$ for bound state calculations, and $J \leq 4$ for continuum state calculations. For continuum state calculations, 3N states with total angular momenta $J_0 = \frac{1}{2}$ and $\frac{3}{2}$ are taken into account. The error in these truncating procedures is estimated to be at most 2% from comparisons of results with $J \leq 4$ NN states and those with $J \leq 3$ ones, and contributions from $J_0 = \frac{5}{2}$ states, which demonstrates that it is good enough for the purposes of the present work.

As realistic models of 2NP, the Argonne $V_{18}$ model (AV18; Ref. [17]) and its $V_8$ version (AV8’; Ref. [18]), the Argonne $V_{14}$ model (AV14; Ref. [19]), and a super-soft core model (dTRS; Ref. [20]) are used. The NN scattering length parameters of these models for $^1S_0$ states $pp$, $nn$, and $pn$ are compared with empirical values (Ref. [21]) in Table 1. As this table shows, the AV8’, AV14, and dTRS models are charge independent. In this work, charge-dependent versions of AV14 and dTRS in Table 1 are introduced by adding potentials that break the charge independence as done in Ref. [22]. Such potentials for AV14 and dTRS are denoted by AV14(CD) and dTRS(CD), respectively.

The $^3$He wave function is calculated using each 2NP model with the Brazil model of the two-pion exchange three-nucleon potential given in Ref. [23], whose cutoff mass parameter of the $\pi NN$ vertex $\Lambda_\pi$ is tuned to reproduce the empirical binding energy (Ref. [8]). The values of $\Lambda_\pi$ in MeV are 660, 610, 670, and 650 for AV18, AV8’, AV14(CD), and dTRS(CD), respectively. It is noted that the $^3$He wave function of the CD version of AV14 (dTRS) is used for calculations of the original version of AV14 (dTRS).

The $n(\vec{p},\vec{n})p (\theta_n = 0^\circ)$ scattering amplitudes in Eq. (9), $\mathcal{V}_c$, $\mathcal{V}_L$, and $\mathcal{V}_T$, are calculated by Eqs. (10a)–(10c) with the $pn$ observables, $\sigma_{pn}(0^\circ)$, $D_{LL}^{pn}(0^\circ)$, and $D_{NN}^{pn}(0^\circ)$, taken from the SP07 solution (Refs. [3, 24]), which are shown in Table 2.

In Fig. 1, the calculated differential cross section $I(0^\circ)$ and polarization transfer coefficients, $D_{NN}(0^\circ)$ and $D_{LL}(0^\circ)$, for the $^3$He($p,n$)pp reaction at $T_p = 346$ MeV as functions of $\omega_{Lab}$ are compared with the experimental data of Ref. [11]. In Fig. 2, the calculated values of $I(0^\circ)$ for $T_p = 200$ MeV are compared with the experimental data (Ref. [10]). In both figures, the calculations of all 2NP models in Table 1 fall within narrow bands, which demonstrates the small $pp$–2NP dependency of the observables.

The calculations of $I(0^\circ)$ at $T_p = 346$ MeV reproduce the data well except for some deviations around $\omega_{Lab} = 20$ MeV. Those at $T_p = 200$ MeV reproduce the line shape of the data with a reduction of about 30%.
Table 2. Observables and scattering amplitudes in \( n(\vec{p}, \vec{n})p \) reaction at forward angle \( \theta_n = 0^\circ \) taken from the SP07 solution (Refs. [3,24]). Those used for the calculations of the \( ^3\text{He}(p,n)ppp \) reaction at \( T_p = 200 \text{ MeV} \) and 346 MeV are shown.

| \( T_p = 200 \text{ MeV} \) | \( T_p = 346 \text{ MeV} \) |
|----------------------------|----------------------------|
| \( \sigma^{pn}(0^\circ) \) [mb/sr] | 12.47 | 11.32 |
| \( D^p_{nn}(0^\circ) \) | -0.1831 | -0.3942 |
| \( D^p_{NN}(0^\circ) \) | -0.3269 | -0.2396 |
| \( |V_c|^2 \) [mb/sr] | 0.5085 | 0.3583 |
| \( |V_L|^2 \) [mb/sr] | 4.5849 | 3.0705 |
| \( |V_T|^2 \) [mb/sr] | 3.6883 | 3.9456 |

Fig. 1. Differential cross section \( I(0^\circ) \) (a) and polarization transfer coefficients, \( D_{NN}(0^\circ) \) (b) and \( D_{LL}(0^\circ) \) (c), for the \( ^3\text{He}(p,n)ppp \) reaction at \( T_p = 346 \text{ MeV} \). Calculations with all 2NP models in Table 1 are shown by bands (light magenta). The experimental data (black points and histogram) are taken from Ref. [11]. The dashed horizontal lines (green) in (b) and (c) are the corresponding \( pn \) values in Table 2.

Fig. 2. Differential cross section \( I(0^\circ) \) for the \( ^3\text{He}(p,n)ppp \) reaction at \( T_p = 200 \text{ MeV} \). Calculations with all 2NP models in Table 1 are shown by the band (light magenta). The experimental data are taken from Ref. [10].
The calculated and experimental values of $D_{NN}(0^\circ)$ and calculated $D_{LL}(0^\circ)$ are almost consistent with the $pn$ values, which are expressed by the dashed horizontal lines in Figs. 1(b) and (c). On the other hand, the experimental values of $D_{LL}(0^\circ)$ deviate from the $pn$ values with an energy transfer dependence, from which the authors of Ref. [11] predicted the existence of a $3p$ resonance in the $1\frac{1}{2}^-$ state centered at $\omega_r = 16 \pm 1$ MeV with a width of $\Gamma_1 = 11 \pm 3$ MeV.

Using the three observables measured in Ref. [11] along with the $pn$ amplitudes in Table 2, the response functions, $R_c$, $R_L$, and $R_T$, are calculated by Eqs. (12a)–(12c). The response functions thus obtained are compared with those calculated with AV18 in Fig. 3. The figure shows that the extracted $R_L$ and $R_T$ have similar shape and magnitude to the calculations, but the extracted $R_c$ is a few times larger than the calculation. The resonance-like behavior in $D_{LL}(0^\circ)$ as a function of $\omega_{Lab}$ is reflected in $R_c$, but not in $R_L$ and $R_T$. However, the calculations are not able to reproduce this tendency.

The observables in this work are largely determined by the $pn$ amplitudes, which are related to the $pn$ observables by Eqs. (10a)–(10c). Since there are few experimental data for corresponding $pn$ observables (Ref. [25]), the uncertainty in the $pn$ amplitudes used in this work is not small. Thus, I have evaluated the $pn$ amplitudes inversely from the calculated response functions, $R_c$, $R_L$, and $R_T$, and the experimental data of $I(0^\circ)$, $D_{NN}(0^\circ)$, and $D_{LL}(0^\circ)$ by Eqs. (12a)–(12c).

The $pn$ amplitudes obtained in this way depend on $\omega_{Lab}$. Taking the average, one obtains: $|V_c|^2 = 1.43 \pm 0.57$ mb/sr, $|V_L|^2 = 2.90 \pm 0.38$ mb/sr, and $|V_T|^2 = 3.60 \pm 0.75$ mb/sr, from which the $pn$ observables are calculated as: $\sigma^{pn}(0^\circ) = 11.5 \pm 1.7$ mb/sr, $D_{LL}^{pn}(0^\circ) = -0.24 \pm 0.11$, and $D_{TT}^{pn}(0^\circ) = -0.13 \pm 0.05$. The errors are only statistical ones with respect to the averaging procedure, and the effects of the experimental error are not taken into account.

Figure 4 shows the calculated observables with the above fitted $pn$ amplitudes (only the central values are used), which gives a better agreement with the data, although the rapid dependence of $D_{LL}(0^\circ)$ is not reproduced.

Next, I will study effects of 3NP on the observables. Recently, the possibility of a resonant $4n$ state at low energy was indicated experimentally in Ref. [12]. In Ref. [13], the effects of $T = \frac{3}{2}$ 3NP on the $4n$ system as well as the $3n$ system are studied. The functional form of 3NP used in Ref. [13] is as follows:

$$V^{3NP}(T) = \sum_{n=1}^{2} W_n(T) e^{-\frac{q^2_g+q^2_h+r^2_{gh}}{b^2_n}+P_{ijk}(T),}$$

(13)
where \( T = \frac{1}{2} \) or \( \frac{3}{2} \), \( r_{ij} \) is the distance between the \( i \)th and \( j \)th nucleons, and \( \mathcal{P}_{ijk}(T) \) is a projection operator on the 3N isospin \( T \) state.

The range parameters used in Refs. [13,26] are \( b_1 = 4.0 \) fm and \( b_2 = 0.75 \) fm. The strength parameters of the shorter-range term \( W_2(T) \) for both \( T = \frac{1}{2} \) and \( T = \frac{3}{2} \) are fixed to be 35.0 MeV in Ref. [13], and also in this work. The required value of the strength parameter for the longer-range term \( W_1(\frac{3}{2}) \) for the \( J^\pi = 0^+ 4\pi \) state to bind as the lower bound of the experimental value (Ref. [12]) is \(-36.14 \) MeV (Ref. [13]). This value contrasts with \( W_1(\frac{1}{2}) = -2.04 \) MeV, which is determined to reproduce the binding energies of \(^3\)H, \(^3\)He, and \(^4\)He in combination with the Argonne \( V_8' \) (AV8') NN potential (Ref. [18]).

In the following I will use AV18, which is more repulsive than AV8' in the 3N\((T = \frac{1}{2})\) bound state. As a consequence of this, a more attractive value, \( W_1(\frac{1}{2}) = -2.55 \) MeV, is used to reproduce the \(^3\)He binding energy. However, this difference may not be essential in the present case.

In Fig. 5, calculated values with \( V^{3NP}(\frac{3}{2}) \) taking \( W_1(\frac{3}{2}) = -36.0 \) MeV are compared with the AV18 calculations. The introduction of \( V^{3NP}(\frac{3}{2}) \) shifts the peak of the cross section to higher in the magnitude and lower in the position, which makes the agreement with the experimental data worse than the AV18 calculation. On the other hand, the effects of the 3NP on \( D_{NN}(0^\circ) \) and \( D_{LL}(0^\circ) \) are quite small. These rather small effects of the 3NP on the \( 3p \) system in spite of the large value of \( W_1(\frac{3}{2}) \) are, however, consistent with the analysis of 3\( n \) systems in Ref. [13], and should be due to a large separation among the three protons by the Pauli principle.

In Ref. [13], the dependence of the strength parameters in the 3NP on the total angular momentum and parity \( J_0^\pi \) is not considered for simplicity. Here, I will examine the \( J_0^\pi \) dependence of the parameter \( W_1(\frac{3}{2}) \). In Table 3, the results for \( D_{LL}(0^\circ) \) dependence of the parameter \( W_1(\frac{1}{2}) \) with \( W_1(\frac{3}{2}) = -36.0 \) MeV for all four states, \( J_0^\pi = \frac{1}{2}^\pm \) and \( \frac{3}{2}^\pm \), or for only one partial wave state, are shown. Even though the effects are not so large compared to the difference between the data and the AV18 calculation, it looks like only \( V^{3NP}(\frac{3}{2}) \) with \( J_0^\pi = \frac{1}{2}^- \) gives an effective difference.
As Fig. 1; calculations with AV18 are shown by solid (black) lines. Calculations with AV18 plus $V^{3NP}(\frac{1}{2}^-)$ taking $W_1(\frac{1}{2}) = -36.0$ MeV are shown by dashed (red) lines.

**Table 3.** Calculated values of the polarization transfer coefficient $D_{LL}(0^\circ)$ for the $^3$He($p,n$)ppp reaction at $T_p = 346$ MeV and at $\omega_{lab} = 15$ MeV including $V^{3NP}(\frac{1}{2}^-)$ with $W_1(\frac{1}{2}) = -36.0$ MeV for all four states, $J^\pi_0 = \frac{1}{2}^\pm$ and $\frac{3}{2}^\pm$, or for only one partial wave state. $\Delta D_{LL}^{av}(0^\circ)$ is the difference from the AV18 calculation.

|                | $D_{LL}^{av}(0^\circ)$ | $\Delta D_{LL}^{av}(0^\circ)$ |
|----------------|-------------------------|-------------------------------|
| AV18           | -0.389                  |                               |
| AV18+$V^{3NP}(T = \frac{3}{2})$ $[J^\pi_0 = \frac{1}{2}^+, \frac{3}{2}^+]$ | -0.382 0.007 | |
| AV18+$V^{3NP}(T = \frac{1}{2})$ $[J^\pi_0 = \frac{1}{2}^-]$ | -0.391 -0.002 | |
| AV18+$V^{3NP}(T = \frac{3}{2})$ $[J^\pi_0 = \frac{3}{2}^-]$ | -0.379 0.010 | |
| AV18+$V^{3NP}(T = \frac{1}{2})$ $[J^\pi_0 = \frac{3}{2}^-]$ | -0.389 0.000 | |
| AV18+$V^{3NP}(T = \frac{3}{2})$ $[J^\pi_0 = \frac{3}{2}^-]$ | -0.388 0.001 | |

Figure 6 shows the observables calculated with $V^{3NP}(\frac{1}{2}^-)$, which is effective only for the $J^\pi_0 = \frac{1}{2}^-$ state taking $W_1(\frac{1}{2})$ from $-36$ MeV to $-90$ MeV. It looks like the 3NP with $W_1(\frac{1}{2}) = -90$ MeV produces a resonance at $\omega_{lab} = 9$ MeV with a narrow width (about 2 MeV), which produces some visible effects on $D_{LL}(0^\circ)$ and $D_{NN}(0^\circ)$. The width of the resonance is smaller than that reported in Ref. [11].

4. Summary

In this paper, I have presented calculations of the cross section and the polarization transfer coefficients, $D_{NN}$ and $D_{LL}$, in the $^3$He($p,n$)ppp ($\theta_n = 0^\circ$) reaction with the spin–isospin response functions obtained for some realistic NN potential models.

The calculations have little NN potential dependence, and show reasonable agreement with available experimental data, except that the energy transfer dependence of $D_{LL}(0^\circ)$ is much smoother than the data.
Introduction of the attractive 3NP for the 3N state suggested from analysis of the 4n state as well as further strength enhanced 3NPs for the $J^{\pi}_0 = \frac{1}{2}^-$ state so as to produce a 3p resonance state are examined. But they cannot resolve the discrepancy.

These results suggest that the curious energy transfer dependence of the experimental $D_{LL}(0^\circ)$, which was the basis of the existence of the 3p resonance (Ref. [11]), is not consistent with conventional models of nuclear interaction, which indicates the need for further experimental studies of the reaction.

Also, a need for good knowledge of observables in $n(\vec{p}, \vec{n})p$ at very forward angles is stressed to reduce ambiguity in the calculation.

Finally, it is remarked that precise calculations of observables related to 3n or 3p systems with theoretical models of the nuclear interactions are now available, which enables us to compare and then to study nuclear interactions, whether a 3N resonance does exist or not.

Appendix. Kinematics

In this appendix, kinematical values related to the $^3$He($p, n$)ppp ($\theta_n = 0^\circ$) reaction are summarized.

Let $T_p$ ($T_n$) be the incident proton (outgoing neutron) energy in the laboratory system. The masses of proton, neutron, and $^3$He are denoted by $m_p$, $m_n$, and $m_{^3\text{He}}$, respectively.

- Total energy in laboratory system:

$$E_{tot,Lab} = m_p + T_p + m_{^3\text{He}}. \quad (A.1)$$
Energy transfer and momentum transfer in laboratory system:

\[ \omega_{\text{Lab}} = (m_p + T_p) - (m_n + T_n), \]  
\[ Q_{\text{Lab}} = K_p - K_n, \]  

where

\[ K_p = \sqrt{(m_p + T_p)^2 - m_p^2}, \]  
\[ K_n = \sqrt{(m_n + T_n)^2 - m_n^2}. \]

Total energy of all four particles in c.m. frame of the initial \( p-^3\text{He} \) system:

\[ E_{\text{tot,c.m.}} = \sqrt{E_{\text{tot,Lab}}^2 - K_p^2}. \]

Energy and momentum of the proton and energy of \( ^3\text{He} \):

\[ E_{p,c.m.} = \frac{E_{\text{tot,c.m.}}^2 + m_p^2 - m_{^3\text{He}}^2}{2E_{\text{tot,c.m.}}}, \]  
\[ k_i = \sqrt{E_{p,c.m.}^2 - m_p^2}, \]  
\[ E_{^3\text{He,c.m.}} = E_{\text{tot,c.m.}} - E_{p,c.m.} = \frac{E_{\text{tot,c.m.}}^2 - m_p^2 + m_{^3\text{He}}^2}{2E_{\text{tot,c.m.}}}. \]

Energies and momentum of the neutron, and energy of \( ^3p \) system in the c.m. frame of the final \( n-^3p \) system:

\[ E_{n,c.m.} = \frac{E_{\text{tot,c.m.}}^2 + m_n^2 - (E_{^3p,\text{Lab}}^2 - Q_{\text{Lab}}^2)}{2E_{\text{tot,c.m.}}}, \]  
\[ k_f = \sqrt{E_{n,c.m.}^2 - m_n^2}, \]  
\[ E_{^3p,c.m.} = E_{\text{tot,c.m.}} - E_{n,c.m.} = \frac{E_{\text{tot,c.m.}}^2 - m_n^2 + (E_{^3p,\text{Lab}}^2 - Q_{\text{Lab}}^2)}{2E_{\text{tot,c.m.}}}, \]

where

\[ E_{^3p,\text{Lab}} = m_{^3\text{He}} + \omega_{\text{Lab}}. \]

Energy transfer and momentum transfer in the c.m. system:

\[ \omega_{\text{c.m.}} = E_{p,c.m.} - E_{n,c.m.}, \]  
\[ Q_{\text{c.m.}} = k_i - k_f. \]

Energy in the c.m. system of the final \( ^3p \):

\[ E = \sqrt{E_{^3p,c.m.}^2 - k_f^2} - 3m_p. \]
Kinematical factor in Eq. (12a) is given by

\[ N_K = \left( \frac{2\pi}{\hbar} \right)^2 \mu_i \mu f \frac{k_f}{k_i} \times \left( \frac{K_\mu}{k_f} \right) \frac{dE}{d\omega_{\text{Lab}}}, \]  

(A.10)

where

\[ \mu_i = \frac{E_{p,c.m.}E_{\text{He},c.m.}}{E_{p,c.m.} + E_{\text{He},c.m.}}, \]  

(A.11a)

\[ \mu f = \frac{E_{n,c.m.}E_{3p,c.m.}}{E_{n,c.m.} + E_{3p,c.m.}}, \]  

(A.11b)

and

\[ \frac{dE}{d\omega_{\text{Lab}}} = \frac{E_{\text{tot,Lab}} - E_{R,\text{Lab}} K_\mu}{E + 3m_p}. \]  

(A.12)

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