Constraining the gravitational-wave spectrum from cosmological first-order phase transitions using data from LIGO-Virgo first three observing runs

Yang Jiang$^{a,b}$ and Qing-Guo Huang$^{a,b,c,d,e,*}$

$^a$CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China
$^b$School of Physical Sciences, University of Chinese Academy of Sciences, No. 19A Yuquan Road, Beijing 100049, China
$^c$School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, China
$^d$Center for Gravitation and Cosmology, College of Physical Science and Technology, Yangzhou University, Yangzhou 225009, China
$^e$Shanghai Frontier Science Research Center for Gravitational Wave Detection, School of Aeronautics and Astronautics, Shanghai Jiao Tong University, Shanghai 200240, China

E-mail: jiangyang@itp.ac.cn, huangqg@itp.ac.cn

Received March 10, 2023
Revised May 9, 2023
Accepted June 9, 2023
Published June 23, 2023

Abstract. We search for a first-order phase transition (PT) gravitational wave (GW) signal from Advanced LIGO and Advanced Virgo’s first three observing runs. Due to the large theoretical uncertainties, four shapes of GW energy spectral from bubble and sound wave collisions widely adopted in literature are investigated, separately. Our results indicate that there is no evidence for the existence of such GW signals, and therefore we give the upper limits on the amplitude of GW energy spectrum $\Omega_{pt}(f_*)$ in the peak frequency range of $f_* \in [5, 500]$ Hz for these four theoretical models, separately. We find that $\Omega_{pt}(f_* \simeq 40 \text{ Hz}) < 1.3 \times 10^{-8}$ at 95% credible level, and roughly $H_* / \beta \lesssim 0.1$ and $\alpha \lesssim 1$ at 68% credible level in the peak frequency range of $20 \lesssim f_* \lesssim 100$ Hz corresponding to the most sensitive frequency band of Advanced LIGO and Advanced Virgo’s first three observing runs, where $H_*$ is the Hubble parameter when PT happens, $\beta$ is the bubble nucleation rate and $\alpha$ is the normalized latent heat.

Keywords: cosmological phase transitions, gravitational waves / sources

ArXiv ePrint: 2203.11781

*Corresponding author.
Contents

1 Introduction 1

2 SGWB from the first-order PT 2

3 Method of data analysis 3

4 Results 5

5 Discussion and conclusions 5

1 Introduction

The evolution of the Universe can be carried out in a smooth or abrupt way, and these sudden changes are what we called phase transitions (PTs) [1–3]. The early stage of the Universe might contain a variety of PTs since many extensions of Standard Model, such as grand unification model [4], suppersymmetric model [5] and so on, predict the occurrence of PTs. See some other relevant models in [6–14]. At QCD scale, PT could happen due to the confinement between quarks and gluons [15, 16]. At a higher energy level, the destruction of electroweak gauge symmetry may lead to electroweak PT [17]. Generally, every break of new symmetry introduced by theory might lead to the occurrence of PT. Therefore, the observation of PT becomes a way to explore new physics.

Among all kinds of PTs, the first-order PT is of great significance to gravitational wave (GW) astronomy, because they are usually strong enough to be interesting sources of GW radiation [18, 19] in the Universe. When the temperature of the Universe drops to a certain value, bubbles containing true vacuum will nucleate in the meta-stable vacuum. Subsequently, these bubbles expand, merge, fill the Universe in the end and GWs are produced during this process [20–22]. Since the nucleation and collision occur in a random manner, the radiated GWs form a stochastic gravitational wave background (SGWB). Because the gravitational interaction is very weak, SGWB decouples from the primordial plasma rapidly. Detecting these GW signals becomes an important way to explore the early Universe which is difficult to be observed by any other methods due to a longer decoupling time [23].

The frequency of GWs depends on the moment of PT, and then radiated GWs would be redshifted due to the expansion of the Universe. As a result, the frequency band of SGWB today is related to the temperature $T^*$ at which PT occurs. Pulsar Timing Arrays (PTAs) [24–27] are used to search for the SGWB at frequencies of several nHz, corresponding to $T^*$ at the order of MeV. Even though a stochastic process has been detected by PTA data sets [28–31], a SGWB detection consistent with general relativity cannot be claimed now because there is no statistically significant evidence of quadrupolar spatial correlations [32–34]. By assuming this correlation is caused by GWs, the constraints on the first-order PT parameters from PTA data sets are presented in [35, 36]. The astrophysical foreground like compact binary coalescences (CBCs) also contributes to SGWB and PTA detection can hardly distinguish them now. After adopting the integrated bound set by cosmic microwave background [37], big bang nucleosynthesis [38] and astrometric observation [39, 40], this degeneracy can be alleviated [41]. Future space-based interferometers projects like LISA [42] aim to search for GW
signals between $10^{-4} \sim 10^{-1}$ Hz, which means that they are promising programmes probing PT at electroweak scale [43, 44]. On the other hand, the terrestrial GW observatories like Advanced LIGO [45] and Advanced Virgo [46] are designed to be sensitive to the frequency of $10 \sim 10^3$ Hz, roughly corresponding to the PT temperature in the range of $10^5 \sim 10^{10}$ GeV. So far, LIGO/Virgo/KAGRA Collaboration has accumulated three generations (O1-O3) of GWs observing data and there is no evidence for the SGWB signal [47, 48]. The related constraints on the GW energy spectrum generated by the first order PT have been investigated in [49] based on the broken power law and two phenomenological PT models.

In this letter, we present a comprehensive analysis of the first-order PT models utilizing data from Advanced LIGO and Advanced Virgo’s first three observing runs by extending and improving the analysis in [49] in several aspects. First of all, we take into account four different GW energy spectral shapes from bubble and sound wave collisions widely adopted in literature due to the large theoretical uncertainties, and provide the constraints on the amplitude of the GW energy spectrum in the peak frequency range of $f_* \in [5, 500]$ Hz for these four theoretical models, separately. Secondly, although CBCs background is supposed to contribute SGWB as well, it is not strong enough to produce a detectable correlation according to the current sensitivities of both Advanced LIGO and Advanced Virgo [47]. Therefore, different from [49], we do not include the contribution of CBCs in our analysis. Thirdly, the parameters related to the amplitude of the GW spectrum are chosen to be uniform distributions in our analysis to result more conservative upper limits than the log-uniform priors adopted in [49]. Finally, we do not keep the bubble nucleation rate $\beta$ and the PT temperature $T_*$ fixed for deriving the upper limits of the GW spectrum because they should be free from the viewpoint of data analysis.

2 SGWB from the first-order PT

Isotropic SGWB can be described by the fractional energy density spectrum in the frequency domain:

$$\Omega(f) = \frac{1}{\rho_c} \frac{d\rho_{gw}}{d\ln f}, \hspace{1cm} (2.1)$$

where $\rho_c = 3H_0^2 c^2/(8\pi G)$ denotes the critical density of the current universe. It has been known that there are three main sources of GW produced by a first-order PT: bubble collisions, collisions of sound waves and magnetohydrodynamic turbulence [44, 50]. In this letter, the contribution from turbulence is not considered due to the lack of understanding about its energy spectrum [43, 51–53]. Besides, magnetohydrodynamic turbulence is usually subdominant compared with sound waves. The amplitude of the GW energy spectrum from a first-order PT is characterized by the bubble nucleation rate $\beta$ and the ratio of energy released to total radiation energy density $\alpha$. Quantitatively, the following energy density spectrum can be used to fit the SGWB from both bubble collisions and sound waves [35, 43, 54–56]:

$$h^2 \Omega_{pt}(f) = R g_*^{\frac{2}{3}} \Delta(v_w) \left( \frac{\kappa \alpha}{1 + \alpha} \right)^{\frac{1}{2}} \left( \frac{H_*}{\beta} \right)^{\frac{1}{2}} S \left( \frac{f}{f_*} \right), \hspace{1cm} (2.2)$$

where $h$ is the dimensionless Hubble constant, the factor $R \approx 7.69 \times 10^{-5}$, $\Delta(v_w)$ is a function of bubble walls velocity $v_w$, $g_*$ is the number of relativistic degrees of freedom which is fixed to be 100 in our analysis. $H_*$ is the Hubble parameter when phase transition happens and $\kappa$ is the fraction of vacuum energy converted. $S(x)$ represents the shape of the spectrum and
it comes to its maximum when $x = 1$. Here, $f_*$ is the peak frequency of SGWB at present:

$$f_* \simeq 1.13 \times 10^{-7} \left( \frac{\tilde{f}_*}{\beta} \right) \left( \frac{\beta}{H_*} \right) \left( \frac{T_*}{10 \text{ GeV}} \right) \left( \frac{g_*}{10} \right)^{1/6} \text{Hz},$$

(2.3)

where $\tilde{f}_*$ is the peak frequency when PT happens. For sound waves, a suppression factor $\Upsilon$ should be multiplied in eq. (2.2) due to the finite lifetime [57, 58]:

$$\Upsilon = 1 - \left( 1 + 2\tau_{sw}H_* \right)^{-1/2}.$$  

(2.4)

The time scale $\tau_{sw}$ is usually taken to be the timescale for the onset of turbulence [44]: $\tau_{sw} \simeq R_*/U_f$, where $R_* = (8\pi)^{1/3} \beta^{-1} \text{Max}(v_w, c_s)$ [58, 59] and $U_f^2 \simeq 3\rho_{sw} \alpha/[4(1 + \alpha)]$ [44]. More details about the parameters in eq. (2.2) and eq. (2.3) are listed in Table 1.

In fact, there are still large theoretical uncertainties in predicting the PT GW energy spectrum, especially for the signal from bubble collision. Analytically, GW energy spectrum can be calculated assuming that energy is concentrated on the infinitesimal bubble wall and it vanishes once the bubbles collide with others. This method is the so-called envelope approximation [21, 55]. Numerically, 3D lattice simulations [60, 61] can be used to break through these assumptions. However, large solved volume needed to accommodate multiple bubbles and very dense lattices to fit thin bubble walls usually lead to substantial costs. Hence, semi-analytic [62] methods are served as alternative ways. The difference of these models is reflected by the asymptotic behavior of $S(x)$ which is characterized by $(a, b, c)$ appeared in Table 1. Table 2 illustrates the details of different GW energy spectra and the typical shapes of these spectra are shown in figure 1. In the high frequency band, the numerical simulation results show faster attenuation than envelope approximation, while the opposite is true at low frequency band. Besides, notice that the spectral shape of sound wave contribution is parameterized by the same broken power law with $a = 3, b = 4, c = 7/2$.

### 3 Method of data analysis

The measurement of SGWB depends on the correlations [63, 64] between multiple detectors and Bayesian approach is used to calculate the possibility of models [65]. Correlated magnetic noise budget shows that such signal is negligible [47, 66] since the intensity of this correlation is much lower than the sensitivity of the current detectors. Therefore, a Gaussian distributed

| $\Delta(v_w)$ | $0.48v_{w}^{1+5/3}$ | $0.513v_{w}$ |
| $\kappa$ | $f(\alpha, v_w)$ |
| $p$ | $2$ | $2$ |
| $q$ | $2$ | $1$ |
| $S(x)$ | $x^3 \left( \frac{7}{4+3\alpha^2} \right)^{7/2}$ |
| $\tilde{f}_*/\beta$ | $0.35$ | $0.536$ |

Table 1. Parameters for the GW energy spectrum in eq. (2.2) and eq. (2.3).
Table 2. Energy spectra of bubble collisions for envelope approximation, semi-analytic approach and lattice simulations. In this letter, we take $(a, b, c) = (1, 2, 2, 2)$ and $(a, b, c) = (0.7, 2.3, 1)$ for semi-analytic and numerical methods respectively.

|           | Envelope | Semi-analytic | Numerical |
|-----------|----------|---------------|-----------|
| $a$       | 3        | $1 \sim 2.3$  | $1.6 \sim 0.7$ |
| $b$       | 1        | $2.2 \sim 2.4$ | $1.4 \sim 2.3$ |
| $c$       | 1.5      | $2 \sim 4.2$  | 1         |
| $\hat{f}_*/\beta$ | $\frac{0.35}{1+0.07v_w+0.69v_w^2}$ | 0.1 | 0.2 |

Figure 1. SGWB energy spectra for bubble collision and sound wave. The peak frequency $f_*$ is chosen to be 25 Hz and $h^2\Omega_{pt}(f_*) = 5 \times 10^{-8}$.

The energy spectrum is characterized by parameters $\theta$ to be estimated. $\hat{C}_{IJ}$ denotes the spectrum of correlation and $\sigma_{IJ}$ is related to the noise intensity of detector pair $IJ$. Summing the index $IJ$ means multiplying the likelihoods from all detector pairs since the correlations between different baselines can be neglected [64]. Here $\lambda$ denotes the calibration uncertainties [67] of the detector baselines and can be marginalized by the method given in [68].

Bayesian analyses for the circumstances of bubble collision dominant and sound wave dominant cases will be done separately because it is ambiguous to determine which one plays a leading role. Python software *bilby* [69] is used for parameter estimation and generate 2D posterior distribution corner-plots. For the bubble collision dominant case, all three models are considered, and $\kappa$ and $v_w$ are set to unity based on the assumption that the bubble wall interacts weakly with the plasma and runaway regime has been reached. For the sound wave dominant case, $\kappa$ is related to $\alpha$ and $v_w$ [70]. In our statistical analysis, $\theta = (\alpha, H_*/\beta, f_*)$ are free parameters to be determined in the bubble collision dominant
Parameter Prior
\(\alpha\) Uniform\((10^{-3}, 10)\)
\(H_s/\beta\) Uniform\((10^{-3}, 1)\)
\(v_w\) Uniform\((10^{-2}, 1)\)
\(f_s\) LogUniform\((5, 500)\)

Table 3. Prior distributions of the parameters used for Bayesian analysis, and the distribution of \(v_w\) is set to be \(\delta(1)\) for the bubble collision dominant cases.

cases, and \(\theta = (\alpha, H_s/\beta, f_s, v_w)\) are free parameters in the sound wave dominant case. Generally, a log-uniform prior is preferred for variable spanning several orders of magnitude. However, the lower bound of a log-uniform distribution cannot be taken into zero, and how to choose such a specific value is unclear. Because there is no correlation found in the raw data, the result of posterior depends on the choice of priors greatly. The initial sampling points are mostly concentrated in small values of parameter for log-uniform prior and might lead to an under-estimation of the upper bound of the GW energy spectrum. In this sense, we adopt uniform prior distributions to avoid ambiguities in the choice of priors. Besides, we note that eq. (2.2) may not be applicable for \(\alpha \gtrsim 10\) and \(H_s/\beta \gtrsim 1\) \cite{71, 72}. Details about the priors are listed in table 3.

4 Results

The posterior distributions of parameters are illustrated in figure 2 for all four theoretical models. Roughly speaking, we find \(H_s/\beta \gtrsim 0.1\) and \(\alpha \gtrsim 1\) are excluded at 68% confidence level in the peak frequency range of \(f_s \in [20, 100]\) Hz for all four theoretical models considered in this letter. This frequency range roughly corresponds to the most sensitive frequency band of Advanced LIGO and Advanced Virgo since 99% of the sensitivity measuring the SGWB comes from the range of \(20 \sim 76.6\) Hz during O3 observation run \cite{47}. Outside this frequency band, the lack of sensitivity will weaken the limitation of \(\alpha\) and \(H_s/\beta\) which are related to the amplitude of GW signal. It also leads to the bimodal distribution of \(\log f_s\) on either side. The Bayes factors between SGWB caused by PT and pure noise \(\log B_{\text{PT}}\) are \(-0.64, -0.74, -0.70\) and \(-0.63\) for the bubble collision dominant cases fitted by envelope approximation, semi-analytic, numerical methods, and the sound wave dominant case, respectively. This result indicates that there is no evidence to claim such SGWB signals in the data.

In order to effectively demonstrate the constraints on the GW energy spectrum in the frequency band, we provide the upper limits on the amplitude of the GW energy spectrum \(\Omega_{\text{pt}}(f_s)\) for the peak frequency \(f_s\) in the range of \(f_s \in [5, 500]\) Hz. This result is illustrated in figure 3 and we find that the constraints are different for these four different theoretical models. However, the most stringent constraint appears at around \(f_s \simeq 40\) Hz with the upper limits of \(\Omega_{\text{pt}}(f_s \simeq 40\) Hz) < \(1.3 \times 10^{-8}\) at 95% credible level for all four scenarios.

5 Discussion and conclusions

We search for the GW signals generated by first-order PT in various theoretical models widely adopted in literature using data from Advanced LIGO and Advanced Virgo’s first three observing runs. To demonstrate the influence of theoretical uncertainties, three models
Figure 2. Posterior distributions of parameters for bubble collision and sound wave dominant cases. Here 68% and 95% exclusion contours are shown. Horizontal solid green lines denote the priors used in analysis. The vertical dotted lines and digits denote 16% and 84% quantiles. \( v_w \) is 1 for bubble contribution analyses and \((a,b,c)\) is set to be \((3,1,1.5)\) for envelope approximation. For semi-analytic and numerical simulation, we take \((1,2,2,2)\) and \((0.7,2,3,2)\) respectively. See eq. (2.2), eq. (2.3) and table 1 for more details about the expressions under consideration in each case.

of bubble collision and one sound wave model are investigated separately. The result of Bayesian analysis does not show preference between different models and implies that it is impossible to tell which model is more preferred. In all, we find that, roughly speaking, \( H_s/\beta \lesssim 0.1 \) and \( \alpha \lesssim 1 \) at 68% credible level in the peak frequency range of \( 20 \lesssim f_\star \lesssim 100 \) Hz corresponding to the most sensitive frequency band of Advanced LIGO and Advanced Virgo’s first three observing runs.
Figure 3. Upper limit of $\Omega_{pt}(f_*)$ at 95% credible level for certain values of peak frequency $f_*$. 

In fact, a joint scenario including both bubble collision and sound wave is more realistic. The energy spectra from these two processes are distinguishable due to the different shapes. However, up to now, the SGWB has not been detected by Advanced LIGO and Advanced Virgo and then we can not distinguish one from the other. Therefore, the different theoretical models are treated separately and the upper limit of $\Omega_{pt}(f_*)$ for the joint analysis is expected to be a “weighted average” of our results, but our constraints on the model parameters (i.e. $H_*/\beta$ and $\alpha$) should still be reliable.

Since there is no correlated signal examined from the observing periods, we put the upper limits on the amplitude of the GW energy spectrum in the peak frequency range of $f_* \in [5, 500]$ Hz. Because the GW spectra of the four theoretical models considered in this letter are roughly the same around the peaks, the constraints on the amplitudes of the GW spectra are roughly the same for them if the peak frequency of the GW energy spectra stays within the most sensitive frequency band of Advanced LIGO and Advanced Virgo, such as $f_* \in [20, 80]$ Hz. But, once the peak frequency stays outside the most sensitive frequency band, the shape of GW spectra plays an important role on the data analysis and the constraints should be different for different theoretical models. The results in figure 3 are consistent with what we expect.

Acknowledgments

We would like to thank Li Li, Jiang-Hao Yu and Yue Zhao for their useful conversations. We acknowledge the use of HPC Cluster of ITP-CAS and HPC Cluster of Tianhe II in National Supercomputing Center in Guangzhou. This work is supported by the National Key Research and Development Program of China Grant No.2020YFC2201502, grants from NSFC (grant No. 11975019, 11991052, 12047503), Key Research Program of Frontier Sciences, CAS, Grant NO. ZDBS-LY-7009, CAS Project for Young Scientists in Basic Research YSBR-006, the Key Research Program of the Chinese Academy of Sciences (Grant NO. XDPB15).
References

[1] A.D. Linde, Phase transitions in gauge theories and cosmology, *Rept. Prog. Phys.* **42** (1979) 389 [arXiv:1903.04977] [SPIRE].

[2] T.W.B. Kibble, Some implications of a cosmological phase transition, *Phys. Rept.* **67** (1980) 183.

[3] A. Mazumdar and G. White, Review of cosmic phase transitions: their significance and experimental signatures, *Rept. Prog. Phys.* **82** (2019) 076901 [arXiv:1811.01948] [SPIRE].

[4] D. Croon et al., GUT physics in the era of the LHC, *Front. in Phys.* **7** (2019) 76 [arXiv:1903.04977] [SPIRE].

[5] S.J. Huber, T. Konstandin, G. Nardini and I. Rues, Detectable gravitational waves from very strong phase transitions in the general NMSSM, *JCAP* **03** (2016) 036 [arXiv:1512.06357] [SPIRE].

[6] E. Megías, G. Nardini and M. Quirós, Cosmological phase transitions in warped space: gravitational waves and collider signatures, *JHEP* **09** (2018) 095 [arXiv:1806.04577] [SPIRE].

[7] L. Fromme, S.J. Huber and M. Seniuch, Baryogenesis in the two-Higgs doublet model, *JHEP* **11** (2016) 003 [arXiv:1606.07812] [SPIRE].

[8] P. Huang, A.J. Long and L.-T. Wang, Probing the electroweak phase transition with Higgs factories and gravitational waves, *Phys. Rev. D* **94** (2016) 075008 [arXiv:1608.06619] [SPIRE].

[9] A. Hebecker, J. Jaeckel, F. Rompineve and L.T. Witkowski, Gravitational waves from axion monodromy, *JCAP* **11** (2016) 025 [Erratum ibid. **10** (2010) E01] [arXiv:0906.3434] [SPIRE].

[10] R. Jinno and M. Takimoto, Probing a classically conformal B-L model with gravitational waves, *Phys. Rev. D* **95** (2017) 015020 [arXiv:1604.05035] [SPIRE].

[11] E. Witten, Cosmic separation of phases, *Phys. Rev. D* **30** (1984) 272 [SPIRE].

[12] C.J. Hogan, Gravitational radiation from cosmological phase transitions, *Mon. Not. Roy. Astron. Soc.* **218** (1986) 629 [SPIRE].

[13] A. Kosowsky, M.S. Turner and R. Watkins, Gravitational waves from first order cosmological phase transitions, *Phys. Rev. Lett.* **69** (1992) 2026 [SPIRE].
[40] J. Darling, A.E. Truebenbach and J. Paine, Astrometric limits on the stochastic gravitational wave background, Astrophys. J. 861 (2018) 113 [arXiv:1804.06986] [inSPIRE].

[41] C.J. Moore and A. Vecchio, Ultra-low-frequency gravitational waves from cosmological and astrophysical processes, Nature Astron. 5 (2021) 1268 [arXiv:2104.15130] [inSPIRE].

[42] LISA collaboration, Laser Interferometer Space Antenna, arXiv:1702.00786 [inSPIRE].

[43] C. Caprini et al., Science with the space-based interferometer eLISA. II: gravitational waves from cosmological phase transitions, JCAP 04 (2016) 001 [arXiv:1512.06239] [inSPIRE].

[44] D.J. Weir, Gravitational waves from a first order electroweak phase transition: a brief review, Phil. Trans. Roy. Soc. Lond. A 376 (2018) 20170126 [arXiv:1705.01783] [inSPIRE].

[45] LIGO Scientific collaboration, Advanced LIGO, Class. Quant. Grav. 32 (2015) 074001 [arXiv:1411.4547] [inSPIRE].

[46] VIRGO collaboration, Advanced Virgo: a second-generation interferometric gravitational wave detector, Class. Quant. Grav. 32 (2015) 024001 [arXiv:1408.3978] [inSPIRE].

[47] KAGRA et al. collaborations, Upper limits on the isotropic gravitational-wave background from advanced LIGO and advanced Virgo’s third observing run, Phys. Rev. D 104 (2021) 022004 [arXiv:2101.12130] [inSPIRE].

[48] KAGRA et al. collaborations, Search for anisotropic gravitational-wave backgrounds using data from advanced LIGO and advanced Virgo’s first three observing runs, Phys. Rev. D 104 (2021) 022005 [arXiv:2103.08520] [inSPIRE].

[49] A. Romero et al., Implications for first-order cosmological phase transitions from the third LIGO-Virgo observing run, Phys. Rev. Lett. 126 (2021) 151301 [arXiv:2102.01714] [inSPIRE].

[50] M.B. Hindmarsh, M. Lüben, J. Lumma and M. Pauly, Phase transitions in the early universe, SciPost Phys. Lect. Notes 24 (2021) 1 [arXiv:2008.09136] [inSPIRE].

[51] A. Roper Pol et al., Numerical simulations of gravitational waves from early-universe turbulence, Phys. Rev. D 102 (2020) 083512 [arXiv:1903.08585] [inSPIRE].

[52] T. Kahniashvili et al., Gravitational radiation from primordial helical inverse cascade MHD turbulence, Phys. Rev. D 78 (2008) 123006 [Erratum ibid. 79 (2009) 109901] [arXiv:0809.1899] [inSPIRE].

[53] T. Kahniashvili, L. Kisslinger and T. Stevens, Gravitational radiation generated by magnetic fields in cosmological phase transitions, Phys. Rev. D 81 (2010) 023004 [arXiv:0905.0643] [inSPIRE].

[54] P. Binetruy, A. Bohe, C. Caprini and J.-F. Dufaux, Cosmological backgrounds of gravitational waves and eLISA/NGO: phase transitions, cosmic strings and other sources, JCAP 06 (2012) 027 [arXiv:1201.0983] [inSPIRE].

[55] R. Jinno and M. Takimoto, Gravitational waves from bubble collisions: an analytic derivation, Phys. Rev. D 95 (2017) 024009 [arXiv:1605.01403] [inSPIRE].

[56] M. Hindmarsh, S.J. Huber, K. Rummukainen and D.J. Weir, Shape of the acoustic gravitational wave power spectrum from a first order phase transition, Phys. Rev. D 96 (2017) 103520 [Erratum ibid. 101 (2020) 089902] [arXiv:1704.05871] [inSPIRE].

[57] J. Ellis, M. Lewicki and J.M. No, Gravitational waves from first-order cosmological phase transitions: lifetime of the sound wave source, JCAP 07 (2020) 050 [arXiv:2003.07360] [inSPIRE].

[58] H.-K. Guo, K. Sinha, D. Vagie and G. White, Phase transitions in an expanding universe: stochastic gravitational waves in standard and non-standard histories, JCAP 01 (2021) 001 [arXiv:2007.08537] [inSPIRE].
[59] M. Hindmarsh and M. Hijazi, *Gravitational waves from first order cosmological phase transitions in the sound shell model*, JCAP 12 (2019) 062 [arXiv:1909.10040] [InSPIRE].

[60] D. Cutting, E.G. Escartin, M. Hindmarsh and D.J. Weir, *Gravitational waves from vacuum first order phase transitions II: from thin to thick walls*, Phys. Rev. D 103 (2021) 023531 [arXiv:2005.13537] [InSPIRE].

[61] D. Cutting, M. Hindmarsh and D.J. Weir, *Gravitational waves from vacuum first-order phase transitions: from the envelope to the lattice*, Phys. Rev. D 97 (2018) 123513 [arXiv:1802.05712] [InSPIRE].

[62] M. Lewicki and V. Vaskonen, *Gravitational waves from colliding vacuum bubbles in gauge theories*, Eur. Phys. J. C 81 (2021) 437 [Erratum ibid. 81 (2021) 1077] [arXiv:2012.07826] [InSPIRE].

[63] J.D. Romano and N.J. Cornish, *Detection methods for stochastic gravitational-wave backgrounds: a unified treatment*, Living Rev. Rel. 20 (2017) 2 [arXiv:1608.06889] [InSPIRE].

[64] B. Allen and J.D. Romano, *Detecting a stochastic background of gravitational radiation: signal processing strategies and sensitivities*, Phys. Rev. D 59 (1999) 102001 [gr-qc/9710117] [InSPIRE].

[65] V. Mandic, E. Thrane, S. Giampanis and T. Regimbau, *Parameter estimation in searches for the stochastic gravitational-wave background*, Phys. Rev. Lett. 109 (2012) 171102 [arXiv:1209.3847] [InSPIRE].

[66] LIGO SCIENTIFIC and VIRGO collaborations, *Search for the isotropic stochastic background using data from advanced LIGO’s second observing run*, Phys. Rev. D 100 (2019) 061101 [arXiv:1903.02886] [InSPIRE].

[67] L. Sun et al., *Characterization of systematic error in advanced LIGO calibration*, Class. Quant. Grav. 37 (2020) 225008 [arXiv:2005.02531] [InSPIRE].

[68] J.T. Whelan, E.L. Robinson, J.D. Romano and E.H. Thrane, *Treatment of calibration uncertainty in multi-baseline cross-correlation searches for gravitational waves*, J. Phys. Conf. Ser. 484 (2014) 012027 [arXiv:1205.3112] [InSPIRE].

[69] G. Ashton et al., *BILBY: a user-friendly Bayesian inference library for gravitational-wave astronomy*, Astrophys. J. Suppl. 241 (2019) 27 [arXiv:1811.02042] [InSPIRE].

[70] J.R. Espinosa, T. Konstandin, J.M. No and G. Servant, *Energy budget of cosmological first-order phase transitions*, JCAP 06 (2010) 028 [arXiv:1004.4187] [InSPIRE].

[71] J. Ellis, M. Lewicki, J.M. No and V. Vaskonen, *Gravitational wave energy budget in strongly supercooled phase transitions*, JCAP 06 (2019) 024 [arXiv:1903.09642] [InSPIRE].

[72] J. Ellis, M. Lewicki and J.M. No, *On the maximal strength of a first-order electroweak phase transition and its gravitational wave signal*, JCAP 04 (2019) 003 [arXiv:1809.08242] [InSPIRE].