Polarization in lamp-post model of black-hole accretion discs

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Abstract. We revisit the lamp-post geometry of the black-hole accretion disc with a primary illuminating source on the rotational axis. The primary X-ray power-law radiation is Compton reflected from the disc towards the observer. The gravitational field of a rotating black hole influences the photon properties on its way from the primary source to the disc and from the primary source and accretion disc to a distant observer. We study the polarization properties of the radiation how they would be observed in this scenario. The degree and the angle of polarization are examined as functions of the black hole spin, observers inclination angle and the position of the primary source.

1. The lamp-post geometry
The lamp-post geometry was originally proposed by [8] with the aim to provide a simple common scheme for the origin of the X-ray power-law continuum and the relativistic spectral features seen in accreting black hole sources. According to this arrangement, the primary ‘lamp’ moves...
along the black hole axis, typically at heights of only up to $10 - 20$ gravitational radii above the disc plane, see Fig. 1. The primary emission that is assumed to be unpolarized illuminates the accretion disc where it is reprocessed and part of it is emitted towards the observer. In this scenario the path of the light that travels from the lamp-post through some point in the disc up to the observer is set for each position on the disc unambiguously, thus defining the scattering geometry at every point in the disc plane. Here, we consider only direct photons, i.e., we assume optically thick disc and we neglect the photons that might bend near the photon orbit in such a way that they would circle around the black hole and only then either reach the observer or strike the surface of the disc.

We compute the flux and polarization as the observer at infinity would measure in this scenario, including both the reprocessed and direct emission. We use four energy bands (2–6 keV, 6–10 keV, 10–20 keV and 20–50 keV), three different inclination angles (30, 60 and 80 degrees) and various values of the height, $h$, of the primary source. Two different values of the angular momentum of the black hole have been explored: a static, $a=0$, and an extremely rotating black hole, $a=1$, where, as customary, $a$ indicates the angular momentum per unit mass in units of the gravitational radius. We use these units for the remainder of this paper. The accretion disc in our computations extends from the marginally stable orbit ($r_{\text{in}} = r_{\text{ms}}$) up to the radius $r_{\text{out}} = 1000GM/c^2$.

2. Local polarization of the reflected emission

We assume that the primary source of unpolarized emission radiates isotropically with the specific intensity being the usual power-law dependence on the energy, $I_{\text{lamp}}(E) = N_{\text{lamp}} E^{-\alpha}$. In our computations we use the value of the power-law index $\alpha = 1$ and the normalization factor $N_{\text{lamp}}$ to be unity in the local static frame. In the equatorial plane of the system we assume a neutral cold geometrically thin and optically thick Keplerian accretion disc. Intensity of the reflected radiation in the local frame co-moving with the accretion disc was computed by the

![Figure 2](image.png)

**Figure 2.** The properties of the local reprocessed radiation depend on the position on the disc. This is due to their dependence on the local scattering geometry. *Left:* The local polarization degree can have any value between 0 and 1. We show the dependence on the relative azimuthal angle $\Delta \Phi$ for three pairs of incident and emission cosines. *Right:* The local polarization angle obtains any value between $-90^\circ$ and $90^\circ$. The dependence on the relative azimuthal angle $\Delta \Phi$ for three pairs of incident and emission cosines is shown. A polarization angle of $0^\circ$ represents the direction perpendicular to the disc and it is rotated counter-clockwise for positive values when looking towards the approaching photon.
Monte Carlo multi-scattering code NOAR [6]. The single scattering approximation [2] is used for the local polarization of the reflected continuum component, see [5] for the details. The polarization properties of the reflected radiation depend highly on the scattering geometry, i.e., the incident and the emission angles, $\delta_i$ and $\delta_e$, as well as the relative azimuthal angle, $\Delta\phi$, of the incident and emitted light rays. This is illustrated in Fig. 2. We stress the dependence on the incident and relative azimuthal angles for the lamp-post geometry. This feature is additional to the pure directionality dependence on the emission angle in the case of an extended corona when the disc is usually assumed to be illuminated isotropically.

3. Overall polarization far from source

To get the resultant polarization far from the source, one has to integrate the emission over the disc surface and thus one integrates the local polarization properties. From the previous section it is obvious that the result of such integration is not easily estimated because of the complicated dependence on the local geometry of scattering. Moreover, relativistic effects enhance radiation from some parts of the disc. They also rotate the polarization angle and thus the overall observed polarization properties will be given by an interplay of the local polarization properties on the disc and relativistic change acting on photons on their way to the observer. The dependence of the polarization degree and angle at infinity on the lamp-post height and observer inclination is shown in Fig. 3 and 4.

![Graphs showing polarization degree vs. Height for different inclinations and energies](image)

**Figure 3.** The dependence of the polarization degree on the height of the primary source (left) and inclination (right) as the observer at infinity would measure. The radiation from the primary source is taken into account as well. The value of the observer inclination, $\theta_o$, the height, $h$, and the energy ranges are depicted inside the graphs. The solid lines show the results for the non-spinning Schwarzschild black hole, $a = 0$, while the dotted lines are for highly spinning Kerr black hole, $a = 1$. The polarization degree shown would be zero for unpolarized light and unity for totally polarized light.
The polarization degree is highest when the radiation from the inner accretion disc has a large contribution to the overall flux. This is opposite to the de-polarization one gets in the inner disc for the Comptonized thermal radiation, see [4, 9]. The cause of the thermal de-polarization is the relativistic rotation of the polarization angle because the local polarization is constant in that case. In the reflection model the local polarization angle changes due to its dependence on the local scattering geometry. When integrating the contributions from the disc, the local polarization angle adds up with the relativistic rotation in such a way that the overall polarization is much less de-polarized.

This has two consequences. Firstly, the highest polarization degrees are achieved for low lamp-post heights when the inner region of the disc is more illuminated than its more distant parts. Secondly, the radiation from the extreme Kerr black-hole accretion disc is more polarized than that in the Schwarzschild case where there is a hole in the disc below marginally stable orbit. Note, however, that there are some exceptions to the latter behaviour, see the results for some intervals of the lamp-post heights and for the observer inclination $30^\circ$ and $80^\circ$.

From left panels in Fig. 3 and 4 one can see that the polarization degree and angle for different spins are almost the same for heights around $3GM/c^2$ and above. This is due to the fact that at these heights the innermost accretion disc already makes quite a small contribution to the overall reflected radiation.

The polarization degree has a maximum for low observer inclinations, e.g. in the highest studied energy band, $20 - 50$ keV, for the lamp-post height $h = 3GM/c^2$ it peaks in the extreme Kerr case at inclination $25^\circ$ where it reaches almost $15\%$ polarization (see the right panel in Fig. 3).

The polarization degree for zero inclination angle has to be zero because of the symmetry. As soon as the symmetry is broken by non-zero inclination the total polarization increases,
determined by the region below critical radius\(^1\) which for low inclinations is far from the center. As soon as the emission above this radius starts to contribute significantly the overall polarization starts to decrease with the inclination angle (the larger the inclination the lower the critical radius). This turn-around is at lower inclinations for larger heights because higher lamp-post illuminates better farther radii (i.e. radii above the critical one). There is another turn-around at the inclination when the critical radius moves close to the center. The contribution to the polarization from below and above critical radius cancel each other and the dependence of the polarization degree on the observer inclination reaches its minimum. For even higher inclinations the polarization again increases, it is determined mainly by the emission from far above the critical radius with lower and lower contamination from the regions around and below this radius. For very high inclinations the reflection is small and thus the overall polarization decreases again.

The polarization angle at infinity is quite sensitive to the details described in the points mentioned above and its dependence on height and inclination is rather complex. As discussed above for the polarization degree, the polarization angle for low inclinations is also determined mainly by the region below the critical radius while for high inclinations by the region above it. The change in the behaviour is very well visible by the rapid change of polarization angle with the inclination in Fig. 4. The transition happens at the inclination when the polarization above critical radius starts to dominate for higher inclinations. The transition depends on the lamp-post height and energy band and it can be either gradual or quite abrupt. Notice that there is no transition (or a very mild one) in the bottom panel in Fig. 4 for the extreme Kerr black hole and for a very low height of the primary source. This is due to the fact that the inner region below the critical point still has large impact on the polarization at infinity even for very high inclinations. In this case also the dip in the dependence of the polarization degree on inclination is not so deep, see the dotted graphs of bottom right panel in Fig. 3.

4. Predictions for the AGN case — MCG–6-30-15
In our simulations we model the polarization observations of the Seyfert 1 galaxy MCG–6-30-15. We assume that the lowest flux state of MCG–6-30-15 observed in [7] corresponds to the

![Figure 5](image-url)

**Figure 5.** The correlation of the degree (left) and of the angle of polarization (right) with the flux for MCG–6-30-15. Each point corresponds to a different height of the source, specified within the figure (in units of \(GM/c^2\)). The dashed and the dotted lines are the expected dependence for spin 1 and 0, respectively.

\(^1\) for the definition of the critical point and its role in the relativistic rotation of the polarization angle see [3]
primary source height \( h = 3 GM/c^2 \). This choice, though arbitrary, is supported by conclusions of other authors, who suggest that the illuminating source is quite close to the central black hole. We assume that the observation inclination is 30° and that the black hole in MCG–6-30-15 is maximally rotating.

In the lamp-post geometry the polarization usually increases with the energy (see Fig. 3). Therefore we use the Gas Pixel Detector, see [1] for details, onboard a medium sized satellite mission sensitive in the interval 8–30 keV, as an example of the observation simulations. In Fig. 5 we show the correlations of the degree and angle of polarization with the flux. Both the flux and the polarization are integrated in the energy range 8–30 keV to avoid the iron line which is expected to be unpolarized. Each point represents an observation of 500 ks and corresponds to a different state of the source, that is a different height as indicated in the figure. Although the angle of polarization is almost constant with the flux, a certain evolution of the degree of polarization should be detectable. Moreover, we report for comparison the evolution of the polarization expected in the case of a Schwarzschild black hole to show that such evolution is somehow dependent on the spin of the central black hole.

5. Conclusions

In this paper we have discussed the observational properties of the polarization in the lamp-post geometry (the light bending model) and we used our theoretical computations to model possible future observations by next-generation X-ray satellite missions equipped with an X-ray polarimeter on board.

The polarization degree in a lamp-post geometry is higher at the highest studied energy band, 20–50 keV. This is a natural result coming from the fact that the primary source has a power-law spectrum with a negative index. Thus the best results for polarimetry of reflection spectra should be achieved in the Compton hump energy region. We also get higher polarization degree for large values of spin of the black hole, small height of the primary source and low inclination of the observer.

The behaviour of the polarization degree and polarization angle in the light bending model is quite complex. Because the result depends on the interplay of several parameters, the polarization properties may be degenerate with respect to different parameter values (e.g. for larger heights we cannot distinguish a difference between a rotating and a non-rotating black hole). Nevertheless, when combined with spectral and timing observations, polarimetry is yet another important channel that can help us to uncover the physical parameters of the black-hole accretion disc systems, such as the black hole spin, the system inclination and the height of the illuminating source.

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