Moving Charges, Detectors and Mirrors in a Quantum Field with Backreaction

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Abstract

This is a progress report on our current work on moving charges[1, 2], detectors[3], and moving mirrors[4, 5] in a quantum field treated in a fully relativistic way via the Feynman-Vernon influence functional method[6], which preserves maximal quantum coherence of the system with self-consistent back-reaction from the field.

1 Introduction

The interplay of moving charges, detectors or boundaries (mirrors) with a quantum field (scalar and electromagnetic considered) can be analyzed at different levels of precision and sophistication. At the lowest level of approximation described in textbooks, one computes the charge motion as determined by a fixed background field. One can then examine the modifications in the field by the moving charge, which take the forms of emitted classical radiation, acceleration field, polarization cloud and changing field correlations. A detailed account of how these familiar classical behavior arise from a field of quantum origin is already a non-trivial task. For a self-consistent treatment one must allow for full backreaction of the quantum field on the charge. At the classical level, a well-known backreaction effect is radiation reaction. At the quantum level additional backreaction effects arise from quantum fluctuations in the form of quantum dissipation. (Claims that classical radiation reaction can be derived from quantum fluctuations are misleading.[7]) The importance of a coherent self-consistent treatment of the mutual interaction between a particle and a
quantum field arises in many situations, becoming particularly important under relativistic and strong field conditions. (For QED see, e.g., [8]).

A deep result of Unruh [9] in 1976 states that a uniformly accelerated detector (a physical object with some internal degree of freedom which can respond to external influences, such as a two-level atom) registers thermal radiance at a (Unruh) temperature $T_U$ proportional to its proper acceleration. This can be interpreted as a kinematical effect, i.e., amplification of quantum noise (vacuum fluctuations of the quantum field) by the moving detector. Its exact thermality arises from an exponential red-shifting brought about by the uniform acceleration [10]. The cases of nonthermal radiation in non-uniformly accelerated detectors have also been studied [3, 11].

It was known even earlier that imposition of boundary conditions on a quantum field such as introducing two conducting plates (mirrors) or dielectric surfaces also leads to discernible physical effects. A celebrated example is the attractive Casimir force between two parallel conducting plates which is discussed extensively in this conference series. The effects related to a moving mirror [14] is sometimes referred to as the dynamical Casimir effect, which has close similarity with cosmological particle creation. A moving mirror imparts a changing boundary condition on the quantum field which has detectable effects nearby and afar.

Investigations of these three cases: moving charges, moving detectors, and moving mirrors carry both theoretical and practical values. In addition to the issues of radiation reaction and vacuum fluctuations mentioned above, these problems probe directly into the physical nature of the vacuum, such as vacuum viscosity [11] in relation to the backreaction of cosmological particle creation or vacuum friction [12] in the dynamical Casimir effect. Moving detectors and moving mirrors were studied in the 70’s as analogs to the Hawking effect from black holes [13, 14, 9]. Readers interested in coherent back-action of a cavity quantum field on a moving atom are referred to a companion paper [15] dealing with the Casimir-Polder effect [16] based on moving atoms through a quantum field in the presence of a mirror[17] and an analysis of a recent proposal for the detection of the Unruh effect in a QED cavity [18].

2 Self-Consistent Trajectories of Moving Charges in a Quantum Field

The problem of self-consistent radiation reaction between particle and field is, of course, an old one. The famous Abraham-Lorentz-Dirac (ALD) equation obtained for classical theory is ultimately inconsistent at the point particle level, giving either runaway or pre-accelerating (acausal) solutions [19]. Markovian quantum treatments for nonrelativistic particles have given essentially the same results [20]: a charge with structure, or a UV regulated field, can yield consistent results, but pathologies return in the point particle limit. Below, we discuss the connections between causal and pathology-free particle motion, self-consistent
non-Markovian quantum backreaction, and the stochastic regime for backreaction.

2.1 Relativistic worldlines and quantum fields

Consider a spinless particle of charge $e$ and mass $m_0$ moving through a massless scalar field $\phi$ with action

$$S_\phi + S_{\text{int}} + S_x = \frac{1}{2} \int dy (\partial_\mu \phi)^2 + \int dy j(y) \phi(y) + \int d\tau u(\tau) [m_0 + V(x)], \quad (1)$$

where the charge current $j(y, x) = e \int d\tau u(\tau) \delta(y - x(\tau))$ is a functional of the particle’s parametrized spacetime trajectory $x^\mu(\tau)$, $V(x)$ is an external field, and $u(\tau) = (\dot{x}^\mu \dot{x}_\mu)^{1/2}$ is the reparametrization-invariant factor giving the equations of motion for a relativistic particle. The matrix elements of the quantum evolution operator for the particle-field system may be obtained by summing over all field and particle (i.e. worldline) histories with amplitudes determined by the above action.

For particle-motion backreaction problems, the worldline path integral representation can be more efficient than the usual quantum field theoretic techniques because it directly employs only the particle’s trajectory rather than the infinite set of field degrees of freedom. This approach, dating back to Feynman and Schwinger,\cite{21} has been applied to a range of problems from action at a distance QED,\cite{22} particle production and high-order background field QED and QCD calculations,\cite{23} to studies of finite size effect,\cite{24} reparametrization invariance in quantum cosmology,\cite{25} and string theory.\cite{26} We use the worldline path integral representation together with the Feynman-Vernon influence functional formalism for describing particle motion with self-consistent backreaction. We employ an open-systems approach by coarse-graining the field to obtain the influence functional, from which we find equations of motion for the particle trajectory.

2.2 Coarse-grained effective action for backreaction on worldlines

For technical convenience we make the simplifying assumption that at an initial time $t_i$, the states of the particle and field factorize: $\hat{\rho}(t_i) = \hat{\rho}_{x,i} \otimes \hat{\rho}_{\phi,i}$. We also introduce a UV momentum cutoff $\Lambda$ for the field.\footnote{While more general initial conditions are possible, no fully satisfactory method has yet been developed to describe completely physical initial correlated states for the kind of particle-field systems considered here.\cite{27}}

The full particle-field system evolves unitarily according to

$$\hat{\rho}(t_f) = \hat{U}(t) (\hat{\rho}_{x,i} \otimes \hat{\rho}_{\phi,i}) \hat{U}(t)^\dagger.$$
The reduced density matrix of the particle at time $t_f$ is found by integrating out the field degrees of freedom:

$$\hat{\rho}_x(t_f) = T\rho_x(t_f).$$

It can be expressed as

$$\rho_x(x_f, x'_f; t_f) = \int \rho_x(x, x'_i; t_i) J_r(x_f, x'_f; x_i, x'_i) dx dx'_i,$$

where $J_r$ is the density matrix evolution operator. It has a path integral representation of the form

$$J_r(x_f, x'_f; x_i, x'_i) = \int dx dx'_i M[x, x' ] e^{i S_{CGEA}[x,x']},$$

where the path integrals involve summing over worldlines histories, with the appropriate boundary conditions and the class of allowed paths enforced by the functional measure $M[x, x']$. The coarse-grained effective action $S_{CGEA}$ for the particle degrees of freedom is

$$S_{CGEA} = S[x] - S[x'] + S_{IF}[x, x'],$$

the influence action $S_{IF}$ is defined by

$$S_{IF}[x, x'] = -i\hbar \ln F[x, x'],$$

and $F[x, x']$ is the Feynman-Vernon influence functional. Since determining the influence action $S_{IF}$ only requires integrating-out the field path integrals, we will not need the explicit form of $M$ here. For Gaussian field states, $S_{IF}$ is given exactly by

$$S_{IF} = -\int \int dy dy' \left[ j^-(y) G_R(y, y') j^+(y') + j^-(y) G_H(y', y') j^-(y') \right],$$

where $G_{R,H}$ are the retarded and Hadamard Green’s functions of the scalar field, respectively, and $j^\pm (y) = j(y, x] \pm j(y, x]$. We consider first the semiclassical limit, described by a pathology-free time-dependent modification of the ALD equation, and then discuss the ALD-Langevin equations describing the quantum field-induced fluctuations in the motion of the particle around its average (semiclassical) trajectory.

### 2.3 Semiclassical level: ALD equation and non-Markovian cure of its pathologies

If the initial particle state is well-localized in phase-space, then

$$\left( \frac{\delta S_{CGEA}}{\delta x^-} \right)_{x^\pm = 0} = 0$$

gives the following semiclassical equations of motion for the average trajectory $\bar{V}$:

$$m(\tau) \dddot{x}_\mu(\tau) - \partial_\mu V(z) = f_\mu^{RR}(\tau) = e^2 g(\tau) \left( \dot{x}_\mu \ddot{x}^2 + \dddot{x}_\mu \right) + O(\Lambda^{-1}),$$
where the $\mathcal{O} (\Lambda^{-1})$ represents higher-derivative terms which are suppressed at low energies. The worldline parameter $\tau$ is chosen, for simplicity, so that $\tau (t_i) = 0$. Equation (7) has the usual ALD form except for the coefficients $m (\tau)$ and $g (\tau)$ whose time-dependence is a consequence of dynamical redressing of the particle by the field on a time-scale determined by the cutoff $\Lambda$. This redressing is a consequence of our having assumed an initially factorized particle-field state. For late times defined by $\tau \gg m_0 r_0 / \Lambda$, the particle is dressed, the dynamics are effectively those of a physical particle state with $g (\infty) = 1$ and renormalized mass $m (\infty) = m (0) - \kappa e^2 \Lambda / 8 \pi$; $\kappa$ is a constant of order one that depends on the details of how the UV field is regulated. For the solutions to be runaway free at late times the bare mass must satisfy $m (0) = m_0 > \kappa e^2 \Lambda / 8 \pi$ (e.g. see the analysis of causality by Ford, Lewis, and O’Connell [30]).

The short-time regime is of interest because this is where acausality of the traditional ALD equation arises. Taking into account the dynamical redressing of the particle, we find $g (0) = 0$, and thus $f_{\mu R}^R (0) = 0$. The suppressed $\mathcal{O} (\Lambda^{-1})$ terms are also time-dependent, but they too have the generic property of vanishing at $\tau = 0$. Consequently, Eq. (7) has a causal solution that is uniquely determined by initial position, $x (0)$, and velocity, $\dot{x} (0)$, data. The time scale for radiation reaction forces to grow from zero to their asymptotic ALD value is given by $m_0 r_0 / \Lambda$, which is bounded from below by the time it takes for light to cross the classical radius of the particle.

### 2.4 Stochastic Level: Decoherence and the ALD-Langevin equation

When decoherence of the quantum histories is sufficiently strong, the worldline path integrals may be approximated by replacing the coarse-grained effective action with a stochastic effective action $S_{\chi}$. Then $(\delta S_{\chi} / \delta z) |_{z = 0} = 0$ gives ALD-Langevin (ALDL) equations of motion, where $z (\tau)$ is the fluctuation (deviation coordinate) of the particle around the average worldline $x (\tau)$.

To find a self-consistent ALDL equation we first solve for the quantum-average worldline $x (\tau)$, including the effects of radiation reaction, by using Eq. (7). The ALDL equations are then

$$\eta (\tau) = m \ddot{z}_{\mu} (\tau) + z_{\nu} \frac{\partial V (x)}{\partial x_{\mu} \partial x_{\nu}} - \frac{e^2}{8 \pi} (S_{\mu \nu} \dot{z}^\nu + R_{\mu \nu} \dot{z}^\nu),$$

where $\eta (\tau)$ is the stochastic noise. At late-times, $R_{\mu \nu} = (g_{\mu \nu} - \dot{x}_{\mu} \dot{x}_{\nu})$ and $S_{\mu \nu} = (\dddot{x}^2 g_{\mu \nu} - \dddot{x}_{\mu} \dddot{x}_{\nu})$. At $\tau = 0$, $R$ and $S$ vanish and the stochastic equations are causal and unique. The noise is given by

$$\eta_{\mu} (\tau) = e (\ddot{x}_{\mu} (\tau) + \dot{x}_{[\mu} \dot{x}_{\nu]} \chi (x (\tau))),$$

where $\chi (y)$ is a stochastic field evaluated along the average worldline, whose noise correlator is

$$\langle \{ \chi (y), \chi (y') \} \rangle = \hbar G^H (y, y').$$
Note that the stochastic field correlations are nonlocal since the Hadamard function is non-vanishing for spacelike separated points. Not surprisingly, quantum fluctuation induced noise is colored (nonlocal).

2.5 Radiation reaction and quantum fluctuations

At the semiclassical level described by the ALD equation there is radiation reaction independent of quantum fluctuations [i.e. there is no $\hbar$ in Eq. (7)]. At the stochastic level described by the ALD-Langiven (ALDL) equation, there are additional quantum backreaction effects manifesting as quantum dissipation which is balanced by quantum fluctuations. Historically there are incorrect claims that (classical) radiation reaction is balanced by quantum fluctuations. This mistake comes from the failure to distinguish between these two levels of backreaction effects.\[7\]

As an explicit example illustrating the difference between semiclassical and stochastic radiation reaction, consider an external field that uniformly accelerates a charged particle through the scalar field vacuum. It is a well-known consequence of the semiclassical ALD equation that the (average) radiation reaction force vanishes, even though the particle does radiate. However, the ALDL equation, describing the quantum field-induced fluctuations in the particle motion around it’s average worldline, shows that the particle responds to the scalar vacuum as if it were a thermal state at the Unruh temperature.\[9\] These quantum fluctuations are balanced by the (non-vanishing) dissipative backreaction term in the ALDL equations, despite the fact that the semiclassical radiation reaction force is zero.

Analysis of the nonrelativistic limit for QED has shown that Bremsstrahlung is the primary source of decoherence for charged particles. An ALD-Langevin equation for nonrelativistic QED in the late time limit has also been derived by a number of authors.\[30\] Currently PRJ and BLH are in the process of extending these results and methods to a relativistic particle in the quantum electromagnetic field.\[2\] This formulation should provide a way for describing relativistic particle backreaction, including quantum-induced stochastic fluctuations, under conditions of sufficient decoherence.

3 Moving mirror and detector in a quantum field with backreaction

We now turn our attention to moving mirrors in a quantum field. As background material refer to\[14, 3, 4\]. Consider in $d + 1$ dimensional flat spacetime a point-like detector moving through and (linearly) coupled to a massive scalar field that is constrained to vanish at the $n$-dimensional surface of a mirror ($n < d$). To allow for backreaction from the field, the detector and mirror are assumed to move along arbitrary, unprescribed trajectories determined by the dynamics under the influence of the quantum field in a self-consistent manner. In the open system approach the influence functional, which captures the effects of the
coarse-grained quantum field, generates correlation functions of the constrained field at various times along the detector worldline and the variation of the related stochastic effective action leads to the stochastic equations of motion for both objects and their responses.

Denote by \( x_1^\alpha(\tau_1) \) and \( x_2^\beta(\tau_2, \tilde{\sigma}_2) \) the detector and mirror paths with affine parameters \( \tau_1, \tau_2 \), respectively, with the shape of the mirror parametrized by \( \tilde{\sigma}_2 \), by \( Q_1 \) the internal degree of freedom of the detector assumed to be a harmonic oscillator with natural frequency \( \Omega \), and by \( j_1[Q_1(\tau_1)] \) the detector portion of the interaction with the constrained field \( \phi_c(x) \). The total action describing the motions of the detector and the mirror, the excitation of the detector (by an energy \( \hbar \Omega \)), and the quantum field constrained by the moving mirror, is given by\(^2\)

\[
S_{\text{tot}} = S_{\phi_c[\phi_c]} + S_Q[Q_1] + S_{x_1}[x_1] + S_{x_2}[x_2] + S_{\text{int}}[j_1[Q_1], x_1, x_2, \phi_c] \\
= \frac{1}{2} \int d^{d+1}y \left\{ \partial_\mu \phi_c(y) \partial^\mu \phi_c(y) - m^2 \phi_c^2(y) \right\} \\
+ \frac{1}{2} \int d\tau_1 \left\{ \dot{Q}_1^2(\tau_1) N_1^{-1}(\tau_1) - \Omega^2 Q_1^2(\tau_1) N_1(\tau_1) \right\} \\
- \frac{m_1}{2} \int d\tau_1 \left\{ \ddot{x}_1^\mu(\tau_1) \dot{x}_1^\mu(\tau_1) N_1^{-1}(\tau_1) + N_1(\tau_1) \right\} \\
+S_{x_2}[x_2] + e \int d\tau_1 j_1[Q_1(\tau_1)] \phi_c(x_1^\alpha(\tau_1)) .
\]

The lapse function along the detector worldline is \( N_1(\tau_1) \) and is closely related to the induced metric along the worldline \( h_{1}(\tau_1) = \dot{x}_1^\mu \dot{x}_1^\mu \) with an over-dot denoting differentiation with respect to \( \tau_1 \). The factor \( j_1[Q_1(\tau_1)] \) \( d\tau_1 \) is a reparametrization-invariant coupling that describes various interactions with the constrained field, e.g. the monopole coupling \( j_1 = \sqrt{m} \ Q_1 \) and the minimal coupling \( j_1 = Q_1 \). The mirror action \( S_{x_2}[x_2] \) is difficult to write down for the general case of an \( n \)-dimensional object moving relativistically in which case the mirror cannot be regarded as infinitely rigid. In any case, the explicit form of \( S_{x_2} \) is irrelevant at the formal level of this discussion. In what follows, a \( \phi \) appearing without a subscript \( c \) denotes a field configuration that does not respect the constraint imposed by the mirror.

Note that there is no explicit interaction Lagrangian appearing in \( S_{\text{tot}} \) describing the direct coupling of the mirror to the field. Rather, it is through the Dirichlet boundary conditions in the constrained field configurations in \( (11) \). The interaction term \( e \int d\tau_1 j_1[Q_1(\tau_1)] \phi_c(x_1^\alpha(\tau_1)) \) describes the mutual effects of the mirror motion on the field as \( \phi_c \) couples to the detector through \( j_1 \) and the detector’s motion. While this is linear in \( \phi_c \) it is highly nonlinear as a function of the detector worldline \( x_1 \). The manifestation of the mirror worldvolume \( x_2 \) appears here in that \( j_1 \) couples to the constrained field which is evaluated at the position of the detector \( \tilde{x}_1 \). The constrained field transmits the information and

\(^2\)Natural units are used so that \( \hbar = c = 1 \) and the flat spacetime metric is \( \eta_{\mu\nu} = \text{diag}(1, -1, \ldots, -1) \).
mediates the influence on the motion of the mirror and the detector together. These effects must be determined self-consistently which is why the influence functional formalism is so valuable in studying problems like this one involving backreaction dynamics.

In the open system approach the influence functional $\mathcal{F}_c$ describes the coarse-grained effect of the constrained field $\mathcal{E} = (\phi_c)$, regarded as the ‘environment’, on the self-consistent evolution of the detector internal degree of freedom and the paths of the detector and mirror, taken together as the ‘system’ $\mathcal{S} = (Q_1, x_1, x_2)$. In the path integral representation it is given by

$$
\mathcal{F}_c[j_1, j'_1; x_1^0, x_1^0; x_2, x_2^0] = \text{Tr}_{\phi_c} \left\{ \hat{U}_c(t_f, t_i; x_1, x_2) \hat{U}_c^\dagger(t_f, t_i; x'_1, x'_2) \right\}
$$

$$
= \int_{\Sigma_f} D\phi_c^i(y) \int_{\Sigma_i} D\phi_c^j(y) \int_{\Sigma'_i} D\phi_c^i(y) \rho_{\phi_c,i}(\phi_c^i, \phi_c^j)
$$

$$
\times \int_{\phi_c^i(y)}^\phi_c \mathcal{D}\phi(y) \delta(\phi(x_2(t_2, \bar{\sigma}_2))) e^{\frac{i}{\hbar} \int_{t_i}^{t_f} \mu \phi(y) \partial^\mu \phi(y) - m^2 \phi^2(y)}
$$

$$
\times e^{\frac{i}{\hbar} \int_{t_1}^{t_f} d\tau_1 j_1(Q_1(\tau_1)) \phi(x_1^0(\tau_1))}
$$

$$
\times e^{\frac{i}{\hbar} \int_{t_{i_1}}^{t_f} d\tau_{i_1} j_{i_1}(Q_{i_1}(\tau_{i_1})) \phi(x_{i_1}^0(\tau_{i_1}))}
$$

$$
\times e^{-\frac{i}{\hbar} \int_{t_f}^{t_{i_1}} d\tau_{i_1} j_{i_1}(Q_{i_1}(\tau_{i_1})) \phi(x_{i_1}^0(\tau_{i_1}))}
$$

(12)

where the trace is over constrained field configurations at the final time $t_f$. We assume that at the initial time $t_i$ the combined $\mathcal{S} + \mathcal{E}$ is in a factorized state. Notice that the initial and final time functional integrals are over constrained field configurations while the transition amplitudes $U$ are over unconstrained fields.

The delta functionals constrain the field to vanish at the surface of the mirror and are introduced by considering the functional measure $D\phi_c(y) = D\phi(y) \delta(\phi(x_c^0(t_2, \bar{\sigma}_2)))$ which does not affect the limits of the integration. A simple way to realize the delta functional is to introduce an auxiliary field $Q_2(t_2, \bar{\sigma}_2)$ on the surface of the mirror

$$
\delta(\phi(x_c^0(t_2, \bar{\sigma}_2))) = \int_{-\infty}^{\infty} DQ_2 e^{\frac{i}{\hbar} \int_{t_f}^{t_{i_1}} d\tau_2 \sqrt{-h_{21}(\tau_2, \bar{\sigma}_2)} Q_2(\tau_2, \bar{\sigma}_2) \phi(x_c^0(\tau_2, \bar{\sigma}_2))}
$$

(13)

The boundary condition along $x_2$ is therefore implemented as a Lagrange multiplier on $\phi$ and then summing over all possible multipliers $Q_2$.

For simplicity we assume the initial state of the field is in a Gaussian form (e.g. vacuum, thermal). A quick look at (12) using (13) shows that the exponentials are quadratic in the field. Since the influence functional $\mathcal{F}_c$ involves only integrals over the field, it can be evaluated in principle. From this point onward, there are two possible routes to take: One can use $\mathcal{F}_c$ to obtain correlation functions of the constrained field at various times along the detector.
worldline. Or, by setting to zero the variation of the stochastic effective action, which is related to the coarse-grained effective action containing $F_c$, i.e.,

$$S_{CGEA} = S_{x_1}[x_1] - S_{x_1}[x'_1] + S_{x_2}[x_2] - S_{x_2}[x'_2] + S_{Q_1}[Q_1] - S_{Q_1}[Q'_1] - i \ln F_c,$$

with respect to the appropriate system variable, one obtains a set of coupled, nonlinear, and nonlocal Langevin equations. This offers a formal solution to the backreaction problem, but for arbitrary detector and mirror motions an explicit solution is too difficult to obtain. It is important, however, to realize that even though the constrained field cannot be calculated for arbitrary mirror and detector motions, the influence functional can be used to construct expectation values of the constrained field without the knowledge of the form of $\phi_c(y)$.

In many physical situations the mirror mass is sufficiently large that the recoil experienced by impinging radiation and radiation reaction is negligible on its center of mass motion. A further simplification assumes that the detector mass is much larger than $\hbar \Omega$ so that the recoil effects due to absorption and emission of quanta do not significantly influence the detector’s motion. In this approximation, the detector and mirror trajectories have no action and may be prescribed from the outset. A point-like mirror moving in one spatial dimension is the case assumed in most previous work\textsuperscript{[14, 4]}. One can use this simplest case to study statistical mechanical features such as the derivation of generalized fluctuation-dissipation relations and vacuum friction. Including backreaction effects on the detector and mirror’s (generically relativistic) motions will allow us to follow the causal dynamics and the consistent evolution of correlations and the effect on their trajectories. The nonrelativistic limit of these results are useful for quantum/atom optics applications. One area is in quantum computer designs where maximal quantum coherence of the system in the face of environmental influences requires a coherent backreaction treatment. The other area of application is in quantum noise reduction schemes in interferometer gravitational wave detectors such as advanced LIGO\textsuperscript{[31]}.

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\textsuperscript{3}For this particular example the mirror action describes the free dynamics of a massive point particle, $S_{x_2}[x_2] = -\frac{\mu}{2} \int d\tau_2 \left\{ \dot{x}_2^\mu \dot{x}_2^\nu N_2^{-1}(\tau_2) + N_2(\tau_2) \right\}$ with $N_2(\tau_2)$ the mirror lapse function and differentiation with respect to $\tau_2$ denoted by an overdot.
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