Supernovae and Cosmology

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Abstract

These lecture notes intend to form a short pedagogical introduction to the use of typical type Ia-Supernovae (hereafter SNIa) as standard candles to determine the energy density of the universe. Problems of principle for taking SNIa as cosmological probes are pointed out, and new attempts at solving them are indicated including the empirical width-luminosity relation (WLR) and its possible explanations.

Finally, the observations of SNIa at high redshift carried out by two major teams are briefly reviewed and their interpretation as evidence for an accelerating universe is also rapidly discussed.

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1 Introduction

A supernova is a very powerful event: a star which suddenly brightens to about $10^9$-$10^{10}$ $L_\odot$. There are two different types of supernovae (see Table 1): SNII and SNIa, such as:

- SNII are core-collapse induced explosions of short-lived massive stars ($M_\star > 8 M_\odot$) which produce more $O$ and $Mg$ relative to $Fe$.
- SNIa are thermonuclear explosions of accreting white-dwarfs in close binaries which produce mostly $Fe$ and little $O$; the exact companions stars of

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white-dwarfs are generally not identified but must be relatively long-lived stars.

There are also particular supernovae: \textit{SNIb} and \textit{SNIa} with most properties of \textit{SNII} except their spectra which have no \textit{H} line (such as SNIa) but they believed to be core-collapse supernovae (such as SNII).

| Type          | SNIa                     | SNII                     |
|---------------|--------------------------|--------------------------|
| Hydrogen      | no                       | yes                      |
| Optical spectrum | Metal lines  
deep 6180 Å | P Cyg lines  
Balmer series |
| Absolute luminosity at max. light | ~ 4×10^9 L⊙  
small dispersion  
standard candles ? | ~ 10^9 L⊙  
large dispersion |
| Optical light curve | homogeneous | heterogeneous |
| UV spectrum   | very weak                | strong                   |
| Radio emission | no detection            | strong  
slow decay               |
| Location      | all galaxies             | spirals and irregulars   |
| Stellar population | old                     | young                    |
| Progenitors   | WD in binary systems     | massive stars            |

For some time astronomers had focused on all types of supernovae to measure Hubble’s constant. Now, with the use of Hubble Space Telescope (HST) observations of Cepheids in the host galaxies of these supernovae, there is a consensus that \(H_0\) is in the range of 60-70 km s\(^{-1}\) Mpc\(^{-1}\) -see Branch (1998) for a review.

But very recently, two observational groups working independently to use SNIa as standard candles to measure distance versus redshift presented evidence that the Hubble expansion has been accelerating over time. In effect, these two independent groups (Perlmutter et al. 1998, Schmidt et al. 1998) had to develop not only a method of discovering SNIa at high redshift by careful observations -with large format CCDs, large apertures- and scanning of plates but they also had to determine if these SNIa could be used as standard candles. This recent and important progress has been possible because an empirical relation (Phillips 1993) between the duration of the peak phase of a SNIa’s light curve and its luminosity (\textit{broader is brighter}) has been taken into account by both teams.

This is precisely the work of these two groups that we are trying to present, in
an educational manner, in these lessons. In this scope, in section 2, we summarize our current understanding of SNIa: their possible progenitors; some models of their explosions; observed optical spectra of SNIa, \(\gamma\)-rays from SNIa, observed optical light curves and finally the empirical width- luminosity relation (WLR) which is also called the Phillips Relation. In section 3, we give a cosmological background by recalling what is the \((\Omega_{\Lambda}, \Omega_M)\)-plane and the luminosity distance. In section 4, we present the observations of SNIa at high redshift by the two major teams: the \textit{Supernovae Cosmology Project} (SCP) and the \textit{High z supernova search team} (HZT). In section 5, we discuss some aspects of their results and finally give a conclusion.

2 On type Ia supernovae

There are spectroscopic and photometric indications that SNIa result from the thermonuclear explosions of accreting carbon/oxygen white dwarfs (hereafter WD). However, the progenitor systems of SNIa -their nature, their evolution- the hydrodynamical models for SNIa- the mass of the WD at ignition, the physics of the nuclear burnings- are still uncertain. Moreover, because they are among the brightest optical explosive events in the universe, and because their light curves and their spectral evolution are \textit{relatively uniform}, SNIa have been tentatively used as standard candles. However variations of light curves and spectra among SNIa have recently been extensively studied; in particular, the relation between the duration of the peak phase of their light curves and their luminosities -\textit{broader and brighter}- Phillips (1993).

This section briefly summarizes our knowledge of these SNIa properties and also their controversial issues. However, for more insight into the underlying physics of SNIa and their progenitors, see the various excellent reviews of Woosley-Weaver (1986), Nomoto et al. (1997), Livio (2000) and most of the papers of the following proceedings \textit{Supernovae} (Petscheck, 1990), \textit{Thermonuclear Supernovae} (Ruiz-Lapente et al. 1997); \textit{Type Ia Supernovae and Cosmology} (Niemeyer Truran 2000), \textit{Cosmic Explosions} (Holt & Zhang 2000).

2.1 On the progenitors of SNIa

There are two classes of models proposed as progenitors of SNIa:

\textit{i)} The Chandrasekhar mass model in which a mass accreting \(C+O\) WD grows in mass up to the critical mass: \(M_{Ia} \sim 1.37 - 1.38\, M_\odot\) -which is near the Chandrasekhar mass and explodes as a SNIa.

\textit{ii)} The sub-Chandrasekhar mass model, in which an accreted layer of helium,
atop a $C + O$ WD ignites off-center for a WD mass well below the Chandrasekhar mass.
The early time spectra of the majority of SNIa are in excellent agreement with the synthetic spectra of the Chandrasekhar mass models. The spectra of the sub-Chandrasekhar mass models are too blue to be comparable with the observations. But let us only give some crude features about the progenitor evolution.

2.1.1 Chandrasekhar mass models

For the evolution of accreting WD toward the Chandrasekhar mass model, two scenarii have been proposed: a) a double degenerate (DD) scenario, i.e. the merging of the $C+O$ WD with a combined mass surpassing the Chandrasekhar mass $M_{ch}$; b) a single degenerate (SD) scenario, i.e. the accretion of hydrogen rich matter via mass transfer from a binary companion. The issue of DD versus SD scenario is still debated. Moreover, some theoretical modeling has indicated that the merging of WD lead to the accretion-induced collapse rather than a SNIa explosion.

2.1.2 Sub-Chandrasekhar mass models

In the sub-Chandrasekhar mass model for SNIa, a WD explodes as a SNIa only when the rate of the mass accretion rate $\dot{M}$ is in a certain narrow range. Moreover, for these models, if $\dot{M} > \dot{M}_{\text{crit}}$, a critical rate, the accreted matter extends to form a common envelope. However, this difficulty has been recently overcome by a WD wind model. For the present binary systems which grow the WD mass to $M_{Ia}$ there are two possible systems:

a) a mass-accreting WD and a lobe-filling main sequences star (WD+MS system).

a) a mass-accreting WD and a lobe-filling less massive red giant star (WD+RG system).

Let us conclude this subsection by saying that the evolution of the progenitor is undoubtedly the most uncertain part of a fully predictive model of a SNIa explosion.

2.2 On explosion models

As already noted above, the first important question is about ignition, which can be formulated as: when and how does the burning start? We have also
seen that there are eventually two candidates:

*i)* the ignition of $^{12}\text{C} + ^{12}\text{C}$, in the core of a WD composed of $\text{C} + \text{O}$, is due to compressive heating from accretion; this is the case of Chandrasekhar models, for which: $M_{\text{WD}} \sim M_{\text{ch}} \sim 1.4\,M_\odot$.

*ii)* the ignition of He in a shell around a $\text{C} + \text{O}$ core is followed by the explosive burning of the $\text{C} + \text{O}$ core and the helium shell; this is the case of sub-Chandrasekhar mass models for which $0.7\,M_\odot < M_{\text{WD}} < 1.4\,M_\odot$.

The second important question is *once ignited, does the flame propagate supersonically by detonation (new fuel heated by shock compression)?* or *subsonically by deflagration (new fuel heated by conduction)?*.

The physics involved in these processes is very complex and relative to the physics of thermonuclear combustion. In particular, the physics of turbulent combustion and the possible spontaneous transition to detonation are probably the most important and least tractable effects. Multidimensional simulations are presently conducted to understand these processes.

Because this work is a pedagogical introduction to the use of SNIa as cosmological probes, let us only consider Chandrasekhar mass models as *standard SNIa models*: they provide a point of convergent evolution for various progenitor systems and therefore could offer a natural explanation of the assumed uniformity of display. Here, we only mention two Chandrasekhar mass models (groups of Nomoto and Woosley) that can account for the basic features of the so-called *standard SNIa*, i.e.:

i) an explosion energy of about $10^{51}\,\text{ergs}$;

ii) the synthesis of a large amount of $^{56}\text{Ni}$ ($\sim 0.6\,M_\odot$);

iii) the production of substantial amounts of intermediate-mass elements at expansion velocities of about $10^4\,\text{km s}^{-1}$ near the maximum brightness of SNIa explosions. All these features are very important to explain observed optical displays of SNIa: spectra and light curves.

### 2.2.1 The carbon deflagration model W7

This model (Nomoto et al. 1984) has been seen for a long time as the standard explosion model of SNIa. In this model, a subsonic deflagration wave propagates at an average speed of $1/5$ of the sound speed from the center to the expanding outer layers. The explosion synthesis of $\sim 0.58\,M_\odot$ of $^{56}\text{Ni}$ in the inner region is ejected with a velocity of $1-2 \times 10^4\,\text{km s}^{-1}$.

### 2.2.2 Delayed detonation models

Since 1984, there has been great progress in applying concepts from terrestrial combustion physics to the SNIa problem. In particular, it has been shown that the outcome of carbon deflagration depends critically on its flame velocity
which is still uncertain. If the flame velocity is much smaller than in W7, the
WD might undergo a pulsating detonation; the deflagration might also induce
a detonation, for instance model DD4 of Woosley & Weaver (1994) or model
WDD2 of Nomoto et al. (1996).
Thus the full details of the combustion are quite uncertain; in particular:
progenitor evolution, ignition densities, effective propagation, speed of the
burning front, type of the burning front (deflagration, detonation, both at
different stages and their transitions).
However, constraints on these still uncertain parameters in models such as
W7, DD4, WDD2 can be provided by comparisons of synthetical spectra and
light curves with observations, by comparisons of predictive nucleosynthesis
with solar isotopic ratio.

2.3 On the observations of SNIa

Here, we only review the observations of SNIa which are relevant for our
cosmological problem which can be formulated by the following question: Can
one consider SNIa as a well-defined class of supernovae in order to use them as
standard candles? Therefore, we briefly present some of the observed optical
spectra and optical light curves of the so-called SNIa. Moreover, we also give
some considerations on \( \gamma \)-rays from SNIa.

2.3.1 On optical spectra of SNIa

Optical spectra of SNIa are generally homogeneous but one can also observe
some important variations. One recalls that SNIa have not a thick \( H \)-rich
envelope so that the elements which are synthesized during their explosions
must be observed in their spectra. Therefore, à-priori, a comparison between
synthetic spectra and observations can be a powerful diagnostic of dynamics
and nucleosynthesis of SNIa models. Spectra have been calculated for various
models and can be used for comparisons with observations. For instance:

\( i \) Nugent et al. (1997) present agreement between observed spectra of SN
1992A (23 days after the explosion), SN 1994A (20 days after the explosion)
and synthetic spectra of the W7 model for the same epochs after the explo-
sions.
Moreover, the various spectral features can be identified as those of \( Fe \), \( Ca \),
\( S \), \( Si \), \( Mg \) and \( O \). As already noted above, synthetic spectra for the sub-
Chandrasekhar mass models are less satisfactory.

\( ii \) For heterogeneity Nomoto et al. (1997) present observed optical spectra
of SNIa (SN 1994D, SN 1990N) about one week before maximum brightness
which show different features from those of SN1991T observed at the same
epoch. Let us note also that SN1991T is a spectroscopically peculiar SNIa.

2.3.2 On \(\gamma\)-rays from SNIa

SNIa synthesise also radioactive nuclei such as \(^{56}Ni\), \(^{57}Ni\), \(^{44}Ti\). The species \(^{56}Ni\) is certainly the most abundant radioactivity produced in the explosion of a SNIa. As we will see below, the amount of \(^{56}Ni\) (\(\sim 0.6 M_\odot\)) is very important for the physics of the optical light curve of the SNIa. But in fact, one must consider the following decay chain:

\[
^{56}Ni \xrightarrow{9 \text{ days}} ^{56}Co \xrightarrow{112 \text{ days}} ^{56}Fe + e^+ + \nu_e
\]

And, for a certain time after the explosion the hard electromagnetic spectrum is dominated by \(\gamma\)-rays from the decays of \(^{56}Co\). During each decay of \(^{56}Co\) a number of \(\gamma\)-rays lines are produced. The average energy is about 3.59 MeV including 1.02 MeV from \(e^+ - e^-\) annihilations. The most prominent lines are the 847 keV line and the 1238 keV line. After \(^{56}Co\) decays, other unstable nuclei become also important both for the energy budget and for producing observable hard emissions. One must calculate, for every SNIa model, the \(\gamma\)-ray light curve, the date, the flux at the maximum of the light curve for the 847 keV line and 1238 keV line -see for instance many \(\gamma\)-rays papers in Ruiz-Lapente et al. 1997).

From the observational point of view:

- i) SN 1986G (in NGC 5128) was observed by SMM (Solar Maximum Mission). An upper limit of: \(F_{847} \leq 2.2 \times 10^{-4} \gamma \text{ cm}^{-2} \text{ s}^{-1}\) was reported leading to an upper limit of

\[
M_{56}(3\sigma) < 0.4 \left( \frac{D}{3 \text{ Mpc}} \right)^2 M_\odot.
\]

Later, SN 1986 G was estimated as an under luminous SNIa (Phillips 1993).

- ii) SN1991T (in NGC 4527) was observed by CGRO (Compton Gamma Ray Observatory) on day 66 after the explosion. A detection by the instrument COMPTEL/CGRO of the 847 keV line has been reported with a flux of \(F_{847} = 5.3 \pm 2.1 \times 10^{-5} \gamma \text{ cm}^{-2} \text{ s}^{-1}\). During the same date, no detection, by the instrument OSSE/CGRO, of the 847 keV line has been reported. A 3\(\sigma\)-upper limit \(F \leq 4.5 \times 10^{-5} \gamma \text{ cm}^{-2} \text{ s}^{-1}\) has been given only (see the \(\gamma\)-ray papers in Ruiz-Lapente et al. 1997 and references therein). Let us also recall that SN 1991 T was quoted as a peculiar SNIa, from the point of view of optical spectroscopy (see above 2.3.1).

- iii) An interesting test for the nature of SNIa explosion models has been found and published just after Cargese 2000, Pinto et al. (2000) present results of X and \(\gamma\)-ray transport calculations of \(M_{ch}\) and sub-\(M_{ch}\) explo-
sion models for SNIa and they have shown that X-ray and \(\gamma\)-ray spectral evolution of both models are very different. In particular, they show that:

- The \(\gamma\)-ray light curves in sub-\(M_{ch}\) models are much brighter and peak much earlier than in \(M_{ch}\) SNIa. The \(^{56}\text{Ni}\) \(\gamma\)-ray line emission (at 847 keV) from a bright sub-\(M_{ch}\) explosion at 15 Mpc would be just at the limit for detection by the ESA satellite INTEGRAL (International Gamma Ray Astrophysics Laboratory) which will be launched in 2001.
- The presence of surface \(^{56}\text{Ni}\) in sub-\(M_{ch}\) SNIa would make them very bright emitters of iron peak K-shell emission visible for several hundred days after explosion. K-shell emission from a bright sub-\(M_{ch}\) located near Virgo would be just above the limit for detection by the XMM observatory.

Anyway, let us only notice that a first true detection of the 847 keV line of a true typical SNIa by INTEGRAL may help to better understand the SNIa models and the optical light curves of SNIa (see below 2.3.3).

2.3.3 On optical light curves of SNIa, The Phillips Relation

For a long time, the optical light curve shapes of SNIa were generally supposed to be homogeneous i.e. all SNIa were identical explosions with identical light curve shapes and peak luminosities (Woosley & Weaver 1986, see also our table 1). This was also supported by observations: for instance SN 1980N and SN 1981D in the galaxy NGC 1316 exhibited almost identical brightness and identical shapes of their light curves. It was this uniformity in the light curves of SNIa which had led to their primary use as standard candles in cosmology. Then, recent work on large samples of SNIa with high quality data has focused attention on examples of differences within the SNIa class: in particular there are observed deviations in luminosity at maximum light, in colors, in light curve widths etc... However, despite all these differences, the discovery of some regularities in the light curve data has also emerged, in particular, the Phillips relation: the brightest supernovae have the broadest light curve peaks.

In particular, see the upper plot of figure 2 in section 4. In effect, Phillips (1993) quantified the photometric differences among a set of nine well-observed SNIa using the following parameter \(\Delta m_{15}(B)\) which measures the total drop, in \(B\)-magnitudes, from maximum to \(t = 15\) days after \(B\) maximum. This Phillips relation is also called the width luminosity relation (WLR). Finally, it is this empirical brightness decline relation (WLR) which allows the use of SNIa as calibrated candles in cosmology - see section 4-

Because SNIa are certainly more complex that can be described as a single-parameter supernova family, several groups of theorists - experts in supernovae - have done a lot of work on SNIa in the early times and on their possible evolution until now - see most of the papers written in the proceedings edited by Niemeyer & Truran (2000) and those edited by Holt & Zhang (2000). But let us briefly recall how we can explain the present SNIa light curves and the
Phillips relation or WLR. See, in particular, Woosley & Weaver (in Petscheck, 1990), Arnett (1996), Nomoto et al. (1997) and more recently Pinto & Eastman (2000a, 2000b, 2000c).

In theoretical models of light curves, the explosion energy goes into the kinetic energy of expansion $E$. The light curves are also powered by the radioactive decay chain:

$$^{56}\text{Ni} \rightarrow^{56}\text{Co} \rightarrow^{56}\text{Fe}.$$ 

The theoretical peaks of the light curve is at about 15-20 days after the explosion. Their decline is essentially due to the increasing transparency of the ejecta to $\gamma$-rays and to the decreasing input of radioactivity. The light curve shape depends mainly on the diffusion time:

$$\tau_D \sim \left(\frac{\kappa M}{v_{\text{esc}} c}\right)^{1/2}$$

where $\kappa$ is the opacity, $v_{\text{esc}} \sim (E/M)^{1/2}$. Or, in other words, the theoretical light curve of a SNIa is essentially determined by three effects:

- i) The deposition of energy from radioactive decays.
- ii) The adiabatic conversion of internal energy to kinetic energy of expansion.
- iii) The escape of internal energy as the observed light curve.

A priori, the light curve models of SNIa depends on, at least, four parameters: 1) the total mass $M$; 2) the explosion energy $E$; 3) the $^{56}\text{Ni}$ mass or $M_{56}$; 4) the opacity $\kappa$.

Therefore, the question can be the following one: how these four (or five) parameters collapse to one for leading to the WLR? Finally, one must only mention again the work of Pinto & Eastman (2000c) which shows that:

- 1) The WLR is a consequence of the radiation transport in SNIa.
- 2) The main parameter is the mass of radioactive $^{56}\text{Ni}$ produced in the explosion and the rate of change of $\gamma$-ray escape.
- 3) The small differences in initial conditions which might arise from evolutionary effects between $z \sim 0$ and $z \sim 1$ are unlikely to affect the SN cosmology results.

To conclude this section 2, one can say that although there are uncertainties in the nature and evolution of SNIa progenitors, in the ignition time and nature of the burning front, the empirical Phillips relation may be due to the radiation transport in SNIa and to atomic physics and not to hydrodynamics of the SNIa explosion (if the results of Pinto & Eastman are confirmed).

Anyway, in section 3, we will recall the cosmological background needed to use any standard candle for studying the geometry of the Universe - and in section 4, we will show how cosmologists use the Phillips relation to calibrate SNIa and use them as calibrated candles.
3 Cosmological Background

In order to understand all the cosmological implications of observations of SNIa at high redshift, one must recall some theoretical background.

3.1 The \((\Omega_m,\Omega_\Lambda)-plane\)

The cosmological term -i.e. the famous \(\Lambda\) term- was introduced by Einstein when he applied General Relativity to cosmology:

\[
G_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}
\]  

(1)

where \(g_{\mu\nu}\) is the metric, \(G_{\mu\nu}\) is the Einstein tensor, \(\Lambda\) is the cosmological constant and \(T_{\mu\nu}\) is the energy- momentum tensor such as \(\nabla_\nu T^{\mu\nu} = 0\).

For a homogeneous and isotropic space-time, \(g_{\mu\nu}\) is the Robertson- Walker metric:

\[
ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - R^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]
\]  

(2)

where \(R(t)\) is the cosmic scale factor, \(k\) is the curvature constant, \(T_{\mu\nu}\) has the perfect fluid form:

\[
T_{\mu\nu} = pg_{\mu\nu} + (\rho + p)u_\mu u_\nu,
\]

with \(\rho, p, u_\mu\) being respectively the energy density, the pressure and the 4-velocity:

\[
u_\mu = \frac{dx^\mu}{ds}.
\]

Therefore, the Einstein field equations (1) simplify in:

- The Friedman-Lemaitre equation for the expansion rate \(H\), called the Hubble parameter:

\[
H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G \rho}{3} + \frac{\Lambda}{3} - \frac{k}{R^2}
\]  

(3)

- An equation for the acceleration:

\[
\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}
\]  

(4)

where \(\dot{R} = dR/dt\) and \(k\) is the curvature constant which equals respectively to -1,0,+1 for a universe which is respectively open, flat and closed. Equation (3) says us that three competing terms drive the expansion: a term of energy
(matter and radiation), a term of dark energy or cosmological constant, and a term of geometry or curvature term.

Remarks: i) if one introduces the equation of state: \( p = \omega \rho \), let us recall that \( \omega_R = 1/3 \) for the radiation, \( \omega_M = 0 \) for the cold matter, \( \omega_\Lambda = -1 \) for the cosmological constant. ii) Let us notice that one must, a priori, not only consider a static, uniform vacuum density (or cosmological constant) but also a dynamical form of evolving, inhomogeneous dark energy (or quintessence); for this last case: \( -1 < \omega_Q < 0 \).

It is convenient to assign symbols to the various fractional contributions of the energy density at the present epoch such as:

\[
\Omega_M = \frac{8\pi G \rho_o}{3H_o}, \quad \Omega_\Lambda = \frac{\Lambda}{3H_o} \text{ and } \Omega_k = \frac{k}{R_o^2 H_o^2}
\]  

(5)

where the index \( o \) refers to the present epoch. Then equation (3) can be written:

\[
\Omega_m + \Omega_\Lambda + \Omega_k = 1
\]  

(6)

and the astronomer’s cosmological constant problem can be formulated by the following simple observational question:

Is a non-zero \( \Omega_\Lambda \) required to achieve consistency in equation (6)?

In comparison with \( H_o \) and \( \Omega_M \), the attempts to measure \( \Omega_\Lambda \) are infrequent and modest in scope. Moreover signatures of \( \Omega_\Lambda \) are more subtle than signatures of \( H_o \) and \( \Omega_M \), at least from an observational point of view. Anyway, studying possible variations in equation (6) having dominant versus negligible \( \Omega_\Lambda \)-term is quite challenging!

Let us consider now the expansion dynamics in non-zero cosmological constant models. If:

\[
a \equiv \frac{1}{1+z} = \frac{R}{R_o}
\]  

(7)

is the expansion factor relative to the present epoch and if \( \tau = H_o t \) is a dimensionless time variable (i.e. time in units of the measured Hubble time \( 1/H_o \)), the Friedmann equation (3) can be rewritten:

\[
\left( \frac{da}{d\tau} \right)^2 = 1 + \Omega_M \left( \frac{1}{a} - 1 \right) + \Omega_\Lambda (a^2 - 1)
\]  

(8)

\( \Omega_M \) and \( \Omega_\Lambda \) here, are constant that parametrize the past (or future) evolution in terms of quantities of the present epoch.
Equivalently, one can also parametrize the evolution with $\Omega_M$ and the deceleration parameter $q_o$:

$$q_o = -\left(\frac{R\ddot{R}}{R^2}\right)_o$$  \hspace{1cm} (9)

which can also be written as:

$$q_o = \frac{\Omega_M}{2} - \Omega_\Lambda.$$  \hspace{1cm} (10)

Finally, the expansion dynamics can be given by the system:

$$\left(\frac{\dot{a}}{a}\right)^2 = \Omega_M\left(\frac{1}{a}\right)^3 + (1 - \Omega_M - \Omega_\Lambda)\left(\frac{1}{a}\right)^2 + \Omega_\Lambda$$  \hspace{1cm} (11)

$$\frac{\ddot{a}}{a} = \Omega_\Lambda - \frac{\Omega_M}{2}\left(\frac{1}{a}\right)^3$$  \hspace{1cm} (12)

For different values of $(\Omega_M, \Omega_\Lambda)$ one gets different expansion histories. Here, we only give the figure of the $(\Omega_M, \Omega_\Lambda)$-plane which displays various possible regimes - see figure 1 of the $(\Omega_M, \Omega_\Lambda)$-plane. Of course, the results of high $z$-supernova observations will be first represented by the Hubble diagram, but their analysis will be given in this $(\Omega_M, \Omega_\Lambda)$-plane where it can be confronted to other major recent cosmological observational results.

Of course, the most direct and theory independent way to measure $\Lambda$ would be to actually determine the value of the scale factor $a$ as a function of time. But, it is very difficult! However with sufficiently precise information about the dependence of a distance-measure on $z$, one will try to disentangle the effects of matter, cosmological constant and spatial curvature.

3.2 The luminosity distance $D_L$

The Hubble diagram is a graphic representation of the luminosity distance, i.e. magnitude, of some class of objects -called standard candles- as a function of their redshift. Before presenting and discussing the recent results concerning type Ia-supernovae taken as standard candles, we recall some basic facts.

3.2.1 On the luminosity distance $D_L$

In cosmology, several different distance measurements are in use. They are all related by simple $z$-factors (see for instance Weinberg 1972, Peebles 1993,
Hoggs 1999). The one which is relevant for our study is the luminosity-distance:

$$D_L = \left( \frac{\mathcal{L}}{4\pi F} \right)^{1/2}$$

(13)

where $\mathcal{L}$ is the intrinsic luminosity of the source and $F$, the observed flux. For Friedmann-Lemaître models, one can show that:

$$D_L(z) = \frac{c}{H_o}d_L(z; \Omega_M; \Omega_{\Lambda}) = D_H d_L$$

(14)

where $D_H \equiv c/H_o$ is the Hubble distance and $d_L$ is a known dimensionless function of $z$, which depends parametrically on $\Omega_M$ and $\Omega_{\Lambda}$ defined by equations (5). In effect, the luminosity-distance $D_L$ is related to the transverse-comoving distance $D_m$ through:

$$D_L = (1 + z)D_m$$

(15)
with:

- \( D_m = D_H \Omega^{-1/2} \sinh[\Omega_{k}^{1/2} D_c / D_H], \) if \( \Omega_k > 0 \)  \hspace{1cm} (16)

- \( D_m = D_c = D_H \int_0^z \frac{dz'}{E(z')} \) if \( \Omega_k = 0 \) \hspace{1cm} (17)

with \( E(z) = \Omega_m (1 + z)^3 + \Omega_k (1 + z)^2 + \Omega_{\Lambda}^{1/2} \)

- \( D_m = D_H \Omega^{-1/2} \sin[\Omega_{k}^{1/2} D_c / D_H], \) if \( \Omega_k < 0 \).  \hspace{1cm} (18)

Therefore, from equations (6) and (14-18) we check that, for different possible values of \( \Omega_k \):

\[
D_L = D_L(z, \Omega_M, \Omega_{\Lambda}, H_0) \quad \text{and} \quad d_L = d_L(z, \Omega_M, \Omega_{\Lambda}) \hspace{1cm} (19)
\]

### 3.3 On the magnitude redshift relation

The apparent magnitude \( m \) of an object is related to its absolute magnitude \( M \) through the distance modulus by:

\[
m - M = 5 \log_{10} \frac{D_L}{\text{Mpc}} + 25. \hspace{1cm} (20)
\]

Therefore, from equation (14-18) and (20) we obtain a relation between the apparent magnitude \( m \) and \( z \) with \( \Omega_{\Lambda} \) and \( \Omega_M \) as parameters:

\[
m = 5 \log_{10} d_L(z, \Omega_M, \Omega_{\Lambda}) + \mathcal{M} \hspace{1cm} (21)
\]

Let us only note here that:

- i) \( \mathcal{M} \equiv M - 5 \log H_0 - 25 \) is a non-important fit parameter.
- ii) It is the comparison of the theoretical expectation (21) with data which can lead to constraints on the parameters \( \Omega_M \) and \( \Omega_{\Lambda} \).

In other words and as we will see, the likelihoods for cosmological parameters \( \Omega_M \) and \( \Omega_{\Lambda} \) will be determined by minimizing the \( \chi^2 \) statistic between the measured and predicted distances/magnitudes of SNIa taken as standard candles.
4 Observations of SNIa at high redshift

There are two major teams investigating high-z SNIa:

- i) The *Supernova Cosmology Project* (SCP) which is led by S. Perlmutter (see S. Perlmutter et al. (1997, 1998, 1999 and references therein).
- ii) The *High z-Supernovae Search Team* (HZT) which is led by B. Schmidt (see B. Schmidt et al. (1998 and all references therein).

Both groups have published almost identical results.

In Section (2), we briefly presented SNIa and emphasized that there are large uncertainties in the theoretical models of these explosive events as well as on the nature of their progenitors. We have also seen that although they cannot be considered as perfect *standard candles*, convincing evidence has been found for a correlation between light curve shape and luminosity of nearby SNIa -brighter implies broader- which has been quantified by Phillips (1993).

In this section, we briefly present the cosmological use of SNIa at high redshift with: the observations, the results, a discussion of these results and their cosmological implications.

4.1 The observations

Both groups -the SCP and the HZT- developed a strategy that guarantee the discovery of many SNIa on a certain date. Here, we briefly review the strategy described by the SCP for its early campaigns.

4.1.1 On the strategy

Just after a new moon, they observed some 50-100 high galactic latitude fields -each containing about 1000 high-z galaxies- in two nights, at the Cerro-Tololo 4m telescope in Chile with a Tyson and Bernstein’s wide-field-camera. They returned three weeks later to observe the same fields. About two dozen SNIa were discovered in the image of the ten thousands of galaxies which were still brightening since the risetime of a SNIa is longer than 3 weeks. They observed the supernovae with spectroscopy at maximum light at the Keck-Telescope and with photometry during the following two months at the Cerro-Tololo International Observatory (CITO), Isaac Newton Telescope (INT) and -for the highest redshift SNIa- with the Hubble Space Telescope (HST).
4.1.2 On spectra

With the Keck (10m-telescope): 

i) they confirmed the nature of a large number of candidate supernovae; 

ii) they searched for peculiarities in the spectra that might indicate evolution of SNIa with redshift. In effect, as a supernova brightens and fades, its spectrum changes showing on each day which elements, in the expanding atmosphere, are passing through the photosphere. In principle, their spectra must give a constraint on high-z SNIa: they must show all of the same features on the same day after the explosion as nearby SNIa; or else, one has evidence that SNIa have evolved between the epoch $z$ and now.

4.1.3 On light curves

As already noted, in Section 2, nearby SNIa show a relationship between their peak luminosity and the timescale of their light curve: brighter implies broader. This correlation, often called Phillips relation or luminosity width relation (LWR) has been refined and described through several versions of a one-parameter brightness decline relation: i) $\Delta m(15)$ (Hamuy et al. 1996); ii) multicolor light curve shape (MLCS) (Riess et al. 1996); iii) Stretch parameter (Perlmutter et al. 1998). Moreover, there exists a simple linear relation between the absolute magnitude and the stretch parameter. Fig. (2) shows how the stretch correction aligns both the light curve width and the peak magnitude for the nearby Hamuy supernovae.

4.1.4 On the analysis

One can summarize the three-steps analysis of the 42 high-z SNIa presented in Perlmutter et al. (1998). For each supernova:

- i) The final image of the host galaxy alone is substracted from the many images of the given SNIa spanning its light curve.
- ii) Perlmutter et al. (1998) computed a peak-magnitude in the $B$-band corrected for galaxy extinction and the stretch parameter that stretches the time axis of a template SNIa (see 4.1.3) to match the observed light curve.
- iii) Then, all of the SN magnitudes -corrected for the stretch-luminosity relation- are plotted in the Hubble diagram as a function of their host galaxy redshift.
B Band

as measured

Calan/Tololo SNe Ia

light-curve timescale
“stretch-factor” corrected

$M_B = 5 \log (h/65)$

Kim, et al. (1997)

Fig. 2.: The upper plot shows nearby SNIa light curves which exhibit LWR; the lower plot shows the nearby SNIa light curves, measured in the upper plot, after the stretch parameter correction.

4.2 The results and discussion

One must compare the redshift dependence of observed $m$ with the theoretical expectation given above in Section 3:

$$m = 5 \log d_L(z, \Omega_M, \Omega_\Lambda) + \mathcal{M} \quad (22)$$
The effective magnitude versus redshift can be fitted to various cosmologies, in particular:

- *i)* to the flat universes ($\Omega_M + \Omega_\Lambda = 1$)
- *ii)* to the $\Lambda = 0$-universes.

### 4.2.1 On the Hubble diagram

![Hubble diagram](image)

**Fig. 3.** From Supernova Cosmology Project (SCP): Hubble diagram.

In Fig. (3), one sees the Hubble diagram for 42 high-$z$ SNIa from the SCP and 18 SNIa from the Calán-Tololo Supernovae Survey -after correcting both sets for the LWR (see 4.1.4 above)- on a linear redshift scale to display details at high $z$. The solid curves are the theoretical $m_B^{eff}(z)$ for a range of cosmological models with $\Lambda = 0$, ($\Omega_M, \Omega_\Lambda$) = (0, 0) on top; (1,0) in middle; (2,0) on bottom. The dashed curves are for flat cosmological models ($\Omega_M + \Omega_\Lambda = 1$): ($\Omega_M, \Omega_\Lambda$) =
The best fit (Perlmutter et al. 1998) for a flat Universe ($\Omega_M + \Omega_\Lambda = 1$): $\Omega_M \sim 0.28$; $\Omega_\Lambda \sim 0.72$.

The middle panel of Fig. (3) shows the magnitude residuals between data and models from the best fit flat cosmology: $(\Omega_M, \Omega_\Lambda) = (0.28, 0.72)$. The bottom panel shows the standard deviations. Note that, in their last Hubble diagram Kim (2000), Perlmutter (2000) extend it beyond $z = 1$ by estimating the magnitude of SN1999eq (Albinoni). But, more analysis and more SNIa at $z > 1$ are needed.

4.2.2 On the $(\Omega_M, \Omega_\Lambda)$-plane

Two groups results agree: c.f. Riess et al. (1998)

Fig. 4. : From Supernova Cosmology Project (SCP): the $(\Omega_M, \Omega_\Lambda)$-plane.
Fig. (4) shows the best fit confidence regions in the \((\Omega_M, \Omega_\Lambda)\)-plane for the preliminary analysis of the 42 SNIa fit presented in Perlmutter et al. (1998). Note that the spatial curvature of the universe -open, flat or closed- is not determinative of the future of the universe’s expansion, indicated by the near-horizontal solid line.

From these figures, one can say that the data are strongly inconsistent with the \(\Lambda = 0\), flat universe model. If the simplest inflationary theories are correct and the universe is spatially flat, then the supernova data imply that there is a significant positive cosmological constant.

The analysis of Perlmutter et al. (1998) suggested a universe with:

\[ 0.8\Omega_M - 0.6\Omega_\Lambda \sim -0.2 \pm 0.1 \]

If one assumes a flat Universe \((\Omega_M + \Omega_\Lambda = 1)\), the data imply:

\[ \Omega_M^{\text{flat}} = 0.28^{+0.09}_{-0.08} (1\sigma_{\text{stat}})^{+0.05}_{-0.04} \text{ (syst.)} \]

More recent data of both groups (SCP and HST) and other independent likelihood analysis of the SNIa data provide robust constraints on \(\Omega_M\) and \(\Omega_\Lambda\) consistent with those derived by Perlmutter et al. (1998). For a spatially flat universe, the supernova data require a non-zero cosmological constant at a high degree of significance \((\Omega_\Lambda \geq 0.5 \text{ at } 95\% \text{ confidence level})\).

But before interpreting the results of the analysis of the SNIa data, one must try to discuss the believability of these results.

4.3 Discussion of the results

The current SNIa data set already has statistical uncertainties that are only a factor of two larger than the identified systematic uncertainties. Here, let us only list the main identified sources of systematic uncertainty and only mention some additional sources of systematic uncertainty (from Perlmutter 2000):

- \(i)\) Identified sources of systematic error and their estimated contribution to the uncertainty of the above measurements:
  - Malmquist bias: \(\delta M \sim 0.04\)
  - \(\kappa\)-correction and cross filter calibration: \(\delta M < 0.03\)
  - Non-SNIa contamination: \(\delta < 0.05\)
  - Galaxy extinction: \(\delta M < 0.04\)
  - Gravitational lensing by clumped mass: \(\delta M < 0.06\)
  - Extinction by extragalactic dust: \(\delta M \sim 0.03\)
- \(ii)\) Additional sources of systematic uncertainties:
  - Extinction by \textit{gray dust}
Uncorrected evolution: SNIa behaviour may depend on properties of its progenitor star or binary-star system. The distribution of these stellar properties may evolve in a given galaxy and over a set of galaxies.

Let us only notice that the control of statistical and systematic uncertainties is one of the main goals of the Supernovae Acceleration Probe (SNAP), a 2-m satellite telescope proposed by Perlmutter (2000)\(^2\). Such supernova searches, extended to greater \(z\), can make a much precise determination of the luminosity distance as a function of \(z\), \(d_L(z)\). Note also that a very optimistic estimate of the limiting systematic uncertainty of one percent is expected for such supernovae searches. For a detailed discussion of the present state of all the systematic uncertainties we refer the reader to the paper of Riess (2000) and references therein.

Anyway, the believability of the high-\(z\) SNIa results turn on the reliability of SNIa as one-parameter standard candles. The one-parameter is the rate decline of the light curve (the brighter implies the broader) -see subsection 2.3.3 above.

For a while, due to the lack of a good theoretical understanding of the Phillips relation (what is the physical parameter ?), the supernova experts were not completely convinced by the high-\(z\) SNIa observations and results. After several meetings and a lot of theoretical papers -see, in particular, Ruiz-Lapente et al. (1997), Niemayer & Truran (2000), Holt & Zhang (2000), and of course the papers of Pinto & Eastman (2000a, 2000b, 2000c) -the results of (SCP) and (HZT) appear to supernova experts almost quite convincing... Let us only recall the conclusion of Wheeler (summary talk of Cosmic Explosions, December 1999: my personal answer to the question of whether the Universe is accelerating is probably yes. My answer to the query of do we know for sure is not yet !.

4.4 Implications of the results. The dark energy

Therefore, if one accepts the high-\(z\) SNIa results, the confidence levels in the \((\Omega_M - \Omega_{\Lambda})\)-plane -which are consistent for both groups (SCP and HZT)- favor a positive cosmological constant, an accelerating universe and strongly rule out \(\Omega_{\Lambda} = 0\) universes. In other words, the current results of high-\(z\) SNIa observations suggest that the expansion of the Universe is accelerating indicating the existence of a cosmological constant or dark energy.

There are a number of reviews on the various aspects of the cosmological constant. The classic discussion of the physics of the cosmological constant is by Weinberg (1989). More recent works are discussed, for instance, by Straumann (1999), Carroll (2000), Weinberg (2000) and references therein.

\(^2\) see http://snap.lbl.gov
In particular, Weinberg (2000) formulates the two cosmological constant problems as:

- i) Why the vacuum energy density \( \rho_v \) (or \( \Omega_\Lambda \)) is not very large?
- ii) Why \( \rho_v \) (or \( \Omega_\Lambda \)) is not only small but also -as high-\( z \) SNIa results seem indicate- of the same order of magnitude as the present mass density of the universe?

The challenge for cosmology and for fundamental physics is to determine the nature of the dark energy. One possibility is that the dark energy consists of vacuum energy or cosmological constant. In this case -as already noted above in the remarks of the subsection 3.1- the equation of state of the universe is

\[
\omega_\Lambda \equiv \frac{p}{\rho} = -1.
\]

An other possibility is *quintessence* a time-evolving, spatially inhomogeneous energy component. For this case, the pressure is negative and the equation of state is a function of \( z \), i.e. \( \omega_Q(z) \) is such that:

\[-1 < \omega_Q(z) < 0.\]

Let us only notice here that:

- i) these models of dark energy are based on nearly-massless scalar fields slowly rolling down a potential; most of them are called *tracker models* of quintessence for which the scalar field energy density can parallel that of matter or radiation, at least for part of its history. As Steinhardt said (in the introductory talk of the Workshop on *String Cosmology*, Vancouver, August 2000): *There are dumb trackers* (Steinhardt et al. 1998), *smart trackers* (Armandariz-Picon et al. 2000a, 2000b; Bean & Magueijo 2000).
- ii) there are other models of dark energy besides those based on nearly-massless scalar fields; for instance, models based on networks of tangled cosmic strings for which the equation of state is: \( \omega_{\text{string}} = -1/3 \) or based on walls for which \( \omega_{\text{wall}} = -2/3 \) (Vilenkin 1984, Spergel & Pen 1996, Battye et al. 1999).

Anyway, a precise measurement of \( \omega \) today and its time variation could provide important information about the dynamical properties and the nature of dark energy. Telescopes may play a larger role than accelerators in determining the nature of the dark energy. In this context, by observing SNIa at \( z \) about 1, SCP and HZT have found strong evidence for dark energy i.e. a smooth energy component with negative pressure and negative equation of state. For the future -and as already noted in the subsection 4.2.3- the spatial mission SNAP is being planned; it is expected that about 1000-2000 SNIa at greater \( z \) (0.1 < \( z \) < 1.7) will be detected.

SNAP has also very ambitious goals: it is expected that SNAP will test the
nature of the dark energy, will better determine the equation of state of the universe and its time variation i.e. $\omega(z)$. But very recent studies (for instance Maor et al. 2000) show that the determination of the equation of state of the Universe by high-$z$ SNIa searches -by using luminosity distance- are strongly limited.

5 Conclusions

After a brief review of type Ia supernovae properties (Section 2) and of cosmological background (section 3), we presented the work done by two separate research teams on observations of high-$z$ SNIa (Section 4) and their interpretation (Section 5) that the expansion of the Universe is accelerating. If verified, this will proved to be a remarkable discovery. However, many questions without definitive answers remain.

What are SNIa and SNIa progenitors?
Are they single or double degenerate stars?
Do they fit Chandrasekhar or sub-Chandrasekhar mass models at explosion?
Is the mass of radioactive $^{56}\text{Ni}$ produced in the explosion really the main parameter underlying the Phillips relation?
If so, how important are SNIa evolutionary corrections?
Can systematic errors be negligible and can they be converted into statistical errors?
Can a satellite, like SNAP, provide large homogeneous samples of very distant SNIa and help determine several key cosmological parameters with an accuracy exceeding that of planned CMB observations?

Meanwhile, cosmologists are inclined to believe the SNIa results, because of the preexisting evidence for dark energy that led to the prediction of accelerated expansion.

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