Numerical investigation on sloshing of a viscous fluid in a rigid tank at resonant frequency

W F Wu¹, C W Zhen², K B Zhu¹, J W Zhang¹ and J S Lu¹

¹ School of Port and Transportation Engineering, Zhejiang Ocean University, Zhoushan, Zhejiang, China.
² School of Naval Architecture and Mechanical-electrical Engineering, Zhejiang Ocean University, Zhoushan, Zhejiang, China.
E-mail: 18368091723@163.com

Abstract. A three-dimensional numerical calculation model is established and a numerical method is proposed based on the theory of viscous flow and compared with experiment to investigate the sloshing of viscous fluids in a tank at resonant frequency. The results of numerical simulation and experiment are well agreed. The characteristics analysis on sloshing of viscous fluid at resonant frequency is developed based on the numerical model. The results show that the free surface at left tank wall reaches the top of the tank in the third cycle and reaches the top of the tank in each cycle thereafter. It also concludes that the dynamic pressure below and away from the free surface is the shortest, and the dynamic pressure near the free surface is the next, and the dynamic pressure at the free surface is the largest by comparing the dynamic pressure at different monitoring points.

1. Introduction
Liquid sloshing is an extremely complex form of fluid motion that is highly non-linear and random. In previous studies, the study mainly focused on tank sloshing at non-resonant frequencies [1-2] or considering the low viscosity of the liquid in the tank [3-5]. The relationship between the sloshing pressure experienced by the tank wall and the wave elevation of the liquid surface over time is analyzed. The main purpose of this paper is to obtain the sloshing characteristics of the larger viscous fluid by analyzing the variation of the wave elevation of the free surface with the fluctuation of the liquid level at the left wall and the dynamic pressure at different points.

2. Mathematical model

2.1. Physics model
The three-dimensional rectangular rigid tank calculation model is created by Fluent shown in Figure 1. Model size \( L=100\,\text{cm}, \, B=40\,\text{cm}, \, H=60\,\text{cm} \), liquid carrier rate is 50%.
2.2. Numerical model

2.2.1. Controlling equation. It is assumed that the fluid is an incompressible viscous fluid and is kept at a constant temperature during the sloshing. The tank is a rigid tank.

Continuity equation:

\[ \frac{\partial u_i}{\partial x_i} = 0 \]  

Momentum equation:

\[ \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\mu}{\rho} \frac{\partial^2 u_i}{\partial x_j^2} + b_i \]  

Where \( u_i \) is the velocity vector, \( m/s \). \( P \) is the pressure, \( Pa \). \( \rho \) is the fluid density, \( kg/m^3 \). \( \mu \) is the dynamic viscosity, \( Pa\cdot s \). \( b_i \) is the volume force, \( N \).

Since only the two phases of gas phase and liquid phase are involved in this paper, the VOF method can solve the gas-liquid two-phase interface tracking by solving the continuous equation of the volume fraction of one phase. The fluid volume equation is:

\[ \frac{\partial}{\partial t} (\alpha_q \rho_q) + \frac{\partial}{\partial x_i} (\alpha_q \rho_i \mu_i) = 0 \]  

\( \alpha_q \) represents the volume fraction of the q phase fluid in the cell, and the calculation of the main phase volume fraction is based on the following constraints:

\[ \sum_{q=1}^{n} \alpha_q = 1 \]  

2.2.2. Determination of natural frequency of liquid. In the case of roll excitation, the natural frequency of the fluid in the tank is \( f_n \), and the natural frequency of the sloshing liquid in the rectangular tank can be estimated by the expression of the natural frequency given by Faltinsen [7]:

\[ \omega_n = g \frac{(2n+1)^\pi}{L} \tanh \left( \frac{(2n+1)^\pi}{L} h \right) \]  

\[ \omega_n = 2\pi f_n \]  

In the formula: \( L \) represents the tank length, \( m \); \( h \) represents the liquid level height, \( m \); \( g \) represents the gravitational acceleration, \( m/s^2 \); \( \omega_n \) represents the angular frequency velocity, \( rad/s \); \( f_n \) represents the sloshing frequency, \( Hz \); \( n \) represents the modal number. In this paper, we mainly consider the natural frequency of the tank liquid in the low-order mode when \( n=0 \), that is the resonant frequency.

According to the numerical setting in 2.1, the liquid carrier rate is calculated to be 50%, and the natural frequency of the liquid in the tank is 0.758 Hz.

3. Numerical methods and validation
3.1. Numerical methods

The tank material is aluminum, which is rigid by default in the Fluent. The surrounding walls of the tank are all set as non-slip walls; the fluid in the tank is crude oil, and the viscosity is set to 0.229 Pa·s due to the wide viscosity of the crude oil (0.03-34.29 Pa·s), combined with the viscosity of the crude oil in the oil tanker operation [9-10], the density is 950 kg/m³. The gas pressure inside the tank is set to 1 standard atmosphere, and the effect of evaporation of the crude oil on the gas pressure is ignored.

The sloshing is a transient unsteady process, and the accuracy of the algorithm is required to be high, therefore the PISO algorithm in the non-coupling implicit algorithm based on pressure solution is used for solving the problem, and the convection terms are discretized with the second-order upwinding difference, and the VOF is used, which was used to track the free surface changes. The RNG-k-ε turbulence model to calculate for taking into account the liquid viscosity.

The movement of the tank is achieved by loading the UDF [8]. The tank is sinusoidally rolled and the sloshing center is set to the center of the bottom of the tank. When the sloshing starts, the tank moves to the left. The specific parameters are shown in Table 1.

| External excitation | Excitation frequency (Hz) | Sloshing cycle (s) | External incentive cycle (s) | Excitation amplitude (m) | Carrier rate (%) | Monitoring point of pressure |
|---------------------|---------------------------|-------------------|----------------------------|------------------------|-----------------|----------------------------|
| Roll                | 0.758                     | 1.32              | 1.32                       | 0.0872                 | 50%             | A, B, C, D                  |

3.2. Validation of numerical methods

This paper selects the model test in [6] to verify the correctness of the numerical methods. The size of the model is 92×46×62 cm, and the monitoring point selects the point on the vertical line on the right side of the model, 17 cm from the bottom surface. The liquid in the tank is tap water. The sloshing period is 4s and the amplitude is 4°. The sloshing center is the body of the tank. The VOF model and RNG-K-ε turbulence model are used, combined with dynamic grid technology to calculate and solve the problem, besides, the pressure of the monitoring point on the side wall surface of the tank is selected as a characteristic parameter. The calculation results with the experimental results of the literature [6] are compared as shown in Figure 2. It can be seen that the numerical method results in this paper are in good agreement with the experimental results, indicating the accuracy of the numerical method.

![Figure 2. Validation of numerical methods for tank sloshing.](image)

4. Results and discussion

4.1. Analysis of free surface wave elevation at the left tank wall

According to the above model, the tank is sloshed with the conditions in Table 1. In the case of the resonant frequency sloshing, the time history of the wave elevation at the left tank wall is shown in Figure 3.
Figure 3. Time history of wave elevation at left tank wall.

It can be seen from the figure that the wave elevation increases gradually from the first cycle; the increase in wave elevation is greatest in the second cycle; the maximum wave is as high as 30 cm in the third cycle, which indicates that in the third cycle, the liquid in the tank Peaking occurs. From the fourth cycle, the wave elevation always maintains its maximum value and changes periodically. The reason for analyzing the above phenomenon is that in the first cycle, as the tank starts to shake, the liquid in the tank starts to move from rest. The sloshing range of the tank is smaller, and the elevation of the liquid level climbs is smaller. In the second cycle, the liquid in the tank already has a certain speed. Under the action of kinetic energy and inertial force, the wave elevation increases rapidly. In the third cycle, the liquid internal energy continuously accumulates, and the peaking phenomenon occurs. Because the external excitation does not change, therefore from the third cycle, the wave elevation changes periodically.

4.2. Analysis of free surface in tank at resonant frequency

As the sloshing going, the free surface in the tank will be in different states under the influence of inertial force and gravity. Figure 4 shows the free surface deformation of the tank sloshing in three cycles. As can be seen from the figure, in the first cycle, when time is at 0.7 seconds, the wave elevation at left tank wall reaches a maximum. In the second cycle, the wave elevation at left tank wall reaches its maximum again at 2 seconds. When time is at 2.6 seconds (the second cycle), the free surface is lifted on the right side and the liquid reaches the top of the tank. Under the effect of gravity, the tank top liquid falls back to the free surface. In the third cycle, the free liquid reaching its peak at left tank wall is at 3.3 seconds.

![Figure 4. The free surface profiles at different instants.](image)

When time is at 0.7 seconds, the tank starts to move from a standstill, the sloshing amplitude is
small, and the liquid level at the left tank wall reaches a maximum value. When time is at 2.6 seconds, the end of second cycle, the tank moves axially around the center of the bottom surface, and the inertial force of the liquid gradually increases with sloshing. At the same time, the liquid climbs upwards. Finally, the liquid completes the crowning at the right tank wall. The second cycle, the wave height increases, however, the time history of wave elevation at left tank wall shows that the kinetic energy and inertia force at this time are not large enough to complete the the crowning at the left tank wall. The third cycle, the liquid at the left tank wall is peaking when time is at 3.3 seconds. Because of the first two sloshing cycles, the inertia force of liquid accumulation in the tank is increasing, and the free surface at the left tank wall reaches the top of tank under the impetus of inertial force. This also coincides with the time history of wave elevation at left tank wall.

4.3. Dynamic pressure analysis of liquid in tanks under resonant frequency

The dynamic pressure is generated by the fluid motion against the tank wall and is proportional to the speed. Figure 5 is the time history of the dynamic pressure at A, B, C, D.

![Figure 5. Time history of the dynamic pressure at A, B, C, D.](image)

It is found that the farther away from the free surface is below the free surface, the dynamic pressure is smaller, and the dynamic pressure at the free surface is the largest, and the dynamic pressure above the free surface is in the middle by comparing the peak dynamic pressure values of A, B, C, and D. The liquid is below the free surface, which is far from the free surface, the sloshing degree is reduced, and the speed of the liquid in the tank is smaller, meanwhile the dynamic pressure is smaller. During the sloshing, the liquid sloshing at the free surface is the most violent, larger liquid impact pressure on the tank wall, and the dynamic pressure is also greater. D is above the free surface, when sloshing frequency is at the resonant frequency, the liquid in the tank climbs upwards. It needs to overcome the gravity effect of the liquid itself, and it will lose part of the kinetic energy, and the speed will decrease. So the dynamic pressure at D is slightly smaller than that at C.

5. Conclusion

In this paper, Fluent is used to perform numerical experiments on the viscous fluid sloshing in a three-dimensional rigid tank at resonant frequency. The conclusions are as follows:

1. In this paper, a numerical method to solve tank sloshing under resonant condition is obtained by
comparing with physical experiments.

(2) When the liquid carrier rate is 50%, at the resonant frequency, the time at which the free surface wave elevation at the left tank wall reaches its maximum value is within the third cycle, and the free surface reaches the top of the tank. The fourth at the beginning of each cycle, the surface wave elevation changes in a regular state.

(3) The size of the dynamic pressure is related to the elevation of the free surface. Below the free surface, the dynamic pressure far from the free surface is small, and the dynamic pressure at the free surface is the largest, and the dynamic pressure near the free surface is slightly smaller than the dynamic pressure at the free surface.

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