Constraining The Magnetic Field in Gamma Ray Burst Blast Waves

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We point out that already existing literature on relativistic collisionless MHD shocks show that the parameter $\sigma \equiv$ upstream proper magnetic energy density/upstream rest mass energy density, plays an important role in determining the structure and accelerating properties of such shocks. By adopting the value of $\sigma \approx 0.002$ which corresponds to the relativistic shock associated with the Crab nebula, and by using appropriate relativistic shock jump conditions, we obtain here a generous upper-limit on the value of (proper) magnetic field, $B_{sh} \approx 1.5 \times 10^{-3} \eta n_1^{1/2} G$, for gamma ray burst (GRB) blast wave. Here, $\eta \equiv E/Mc^2$, where $E$ is the energy and $M$ is the mass of the baryons entrained in the original fireball (FB), and $n_1$ is the proper number density of the ambient medium. Further, we point out that, in realistic cases, the actual value $B_{sh}$ could be as low as $\sim 5 \times 10^{-6} \eta n_1^{1/2} G$. realistic cases.

Subject headings: gamma rays: bursts- hydrodynamics-relativity - shock waves
1. Introduction

Understanding the phenomenon of GRBs is one of the important problems of recent astrophysics. Fortunately, following the discovery of a cosmological redshift in the May 08, 1997 event, it is certain now that some or all of them could be of cosmological origin Metzger et al. (1997). Whether cosmological or galactic, GRB phenomenon is broadly understood in terms of a standard model developed by Cavallo & Rees (1978), Goodman (1986), Paczynski (1986), Eichler et al. (1989), Shemi & Piran (1990). Nonetheless, as far as the origin of the complex nonthermal observed GRB spectra are concerned, an important development took place with the work of Rees & Meszaros (1992) and Meszaros & Rees (1993) suggesting that the cosmic fireballs (FB) with an optimal amount of baryonic pollution, $\eta \sim 10^2 - 10^3$, could explain such spectra, where $\eta \equiv E/Mc^2$, where $E$ is the energy and $M$ is the mass of the baryons entrained in the original fireball (FB). Meszaros & Rees suggested that as the baryon polluted FB deposits half of its original momentum onto the ambient medium, presumably, the bare interstellar medium (ISM), at $r = r_d$, the so-called deceleration radius, the blast wave becomes very strong and radiates part of its energy. For further appreciation of this paper it would be appropriate to crudely visualize the geometry associated with the blast wave in terms of a 1-D simple diagram (Mitra 1998, henceforth M98). Here region (1) is the ambient ISM, the lab frame, $S_1$ is the forward shock moving ahead of the contact discontinuity $S$, the location of the original FB boundary. Region (2) represents the (forward) shocked fluid and it is this region which is the site for the particle acceleration and gamma ray production in this standard model. The region (4) is the unperurbed FB and (3) is the part of the FB compressed by the reverse shock front $S_2$. It was shown in M98 that, in the context of this standard model, the reverse shock plays an insignificant role in the overall energy balance and may be neglected for dynamical purposes.
The gamma rays are likely to be produced either by a synchrotron process or a self-synchrotron-Compton process occurring near the region (2) and the most crucial factor for the success of such processes is the value of the comoving magnetic field $B_2' = B_{sh}$. Here prime denotes respective comoving quantities, i.e. respective *proper values*, whereas ‘*’ denotes quantities measured in the rest frame of the forward shock $S_1$. The question of probable generation of a magnetic field in a relativistic (or nonrelativistic) shock is a poorly understood topic, and, practically, most of the authors have therefore been compelled to use an equipartition argument to estimate the same (Meszaros & Rees 1993, Cheng & Wei 1996, Vietri 1995, Waxman 1995):

$$\frac{B_{sh}^2}{8\pi} \sim \eta^2 n_1 mc^2$$  \hspace{1cm} (1)$$

But by recalling the basic definition of

$$r_d \approx \left(\frac{3E}{2\pi c^2 \eta^2 mn_1}\right)^{1/3} \approx 7 \times 10^{15} E_{51}^{1/3} \eta_3^{-2/3} n_1^{-2/3} \text{ cm}$$  \hspace{1cm} (2)$$

where $\eta_3 = 10^{-3}\eta$ and $E_{51} = E/10^{51}\text{ erg s}^{-1}$, it can be easily verified that the energy density shown on the R.H.S. of eq.(1) directly corresponds to the region (4), i.e, the unperturbed FB, having a proper density (M98)

$$n_4' = \frac{E(\gamma_F/\eta)}{4\pi r^3 c^2 m} \approx 5 \times 10^7 E_{51} r_{15}^{-3}(\gamma_F/\eta) \text{ cm}^{-3},$$  \hspace{1cm} (3)$$

i.e., actually,

$$\frac{B_4^2}{8\pi} \approx \gamma_F^2 n_1 mc^2$$  \hspace{1cm} (4)$$

Here $\gamma_F$ is the bulk Lorentz factor (LF) of $S_1$ in the lab frame (1), $m$ is the mass of a proton, and $n_1$ is the particle number density of the ambient medium in units of $1\text{ cm}^{-3}$. Note that for the region (1) the comoving frame coincides with the lab frame and at $r = r_d$, we will have $\gamma_F \approx \eta/2$ in the Meszaros & Rees scenario. On the other hand, we need to apply the equipartition argument in the downstream of the shock, i.e., in the region, which
is expected to be turbulent, and which is, in any case, the site for the particle acceleration. And, it will be seen in the next section that, equipartition argument yields approximately the same value of $B'_s$ in the shocked fluid, and this is important, because, energy density, in itself, is not a Lorentz invariant quantity.

Nevertheless, the question we want to pose here is how justified is this assumption of equipartition in the context of GRBs and whether by adopting this brute assumption we are running into conflict with some well established feature of relativistic collisionless shocks. As was stressed in Mitra (1996), equipartition, as a general physical concept may be found to be valid in steady-state situations like the ISM where the plasma interacts with the particles and currents over astronomically significant time scales. As to dynamic situations, there are hints that many young supernova remnants are endowed with freshly generated magnetic fields which are considerably higher than the bare ISM values $\sim 3 \times 10^{-6}$G. Nonetheless, even in such cases, the age of the supernova could be thousands of years and the enhanced magnetic field is usually much smaller than what is obtained by naive equipartition arguments. In fact, it was clearly anticipated by Meszaros, Rees, & Papathanassiou (1994) that the equipartition argument can at best serve as a broad guide to determine the actual value of $B_{sh}$, and accordingly, they introduced a completely free parameter, $\lambda \leq 1$, tagged onto the naively obtained value of $B_{sh}^{eq}$:

$$B_{sh} \sim 4 \times 10^2 n_1^{1/2} \eta_3 \lambda^{1/2} \ G$$

Further, it could be possible to apply the basic equipartition idea at $t = 0$ to the initial FB, and then evaluate the value of the instantaneous $B_{FB}$ or $B_{sh}$ by using the flux-freezing condition. And again, in this case, if we symbolize our ignorance through the free parameter, $\xi \leq 1$, it follows that (Meszaros, Rees & Papathanassiou 1994)

$$B_{sh} \sim 0.4 \xi^{1/2} F^{-1/6}_{51} n_1^{2/3} \eta_3^{2} \ G$$

Thus, in this paper, we would attempt to invoke a known feature of relativistic collisionless
MHD shocks to obtain physically significant upper limits on $B_{sh}$.

2. Shock Dynamics

Following the work on relativistic strong shock jump conditions by Taub (1949) and Blandford & McKee (1976) it was discussed in M98 that the ratio of the comoving particle densities in the region (1) and (2) is

$$\frac{n'_2}{n'_1} = \frac{\Gamma_2 \gamma_{12} + 1}{\Gamma_2 - 1}$$

(7)

where $\Gamma_2 \approx 4/3$ is the effective ratio of the specific heat of (2) and $\gamma_{12}$ is the bulk LF of (2) with respect to (1). Therefore, we have

$$n'_2 \approx (4\gamma_{12} + 3)n'_1 \approx 4\gamma_{12}n_1 \approx 2\sqrt{2}\gamma_F n_1$$

(8)

where we have used the fact that the maximum value of the LF of the (forward) shocked fluid with respect to the FB is only $\sqrt{2}$ (M98):

$$\frac{\gamma_F}{\sqrt{2}} \leq \gamma_{12} \leq \gamma_F$$

(9)

The Fig.1 actually represents a lab-frame vision where the shock front $S_1$ is moving. However, shock dynamics is often better studied in the rest frame of $S_1$ (*) where, the region (1) (upstream) would be seen to gushing in with a large bulk LF, $\gamma_1 = \gamma_s$. On the other hand, the region (2) (downstream) would be seen to run away with small LF $\gamma_2 \lesssim \sqrt{2}$ (M98). The shock jump conditions also show that the proper relativistic internal energy density of the shocked fluid is

$$e'_2 \approx \gamma_{12}n'_2c^2 \approx mn_1\eta^2c^2$$

(10)

which shows that an equipartition argument in region (2) approximately leads to a value of $B_{sh}$ as is given by eq. (1):

$$B'_2 = B_{sh} \sim 15n_1^{1/2} n_1^{1/2} \text{ G}$$

(11)
If the plasma is supposed to have infinite conductivity, all comoving electric fields must vanish. Then, for a parallel shock where the embedded magnetic field is parallel to the flow direction (x-axis) one obtains the following simple shock jump condition (de Hoffman & Teller 1950, henceforth HT50):

\[ B'_{2x} = B'_{1x}; \quad B'_y = B'_z = 0 \]  

(12)

Further, since under Lorentz transformation, the parallel component of magnetic field remains unchanged, for a parallel shock, we find

\[ B'_{1x} = B_{1x*} = B'_{2x} = B_{2x*} = B_{sh} \]  

(13)

### 2.1. Perpendicular shock

For a perpendicular shock, the magnetic field changes under Lorentz transformation and accordingly shock jump conditions are different from their parallel counterpart. If the magnetic field lies in the y-direction \( B_x = B_z = 0 \), Lorentz transformation leads to (HT50)

\[ B'_{1y} = \gamma_2 B'_{1y}; \quad B'_{2y} = \gamma_2 B'_{2y}, \]  

(14)

where \( \gamma_2 \approx \sqrt{2} \) is the LF of the downstream fluid in the shock frame (M98), and all other components of magnetic field in any other frame becomes zero. Further for all transverse cases, the frozen-in magnetic field line densities become proportional to the respective comoving particle densities (HT50):

\[ \frac{B'_{2y}}{B'_{1y}} = \frac{n'_2}{n'_1} \]  

(15)

Then eq. (14) and (15) together imply

\[ \frac{B'_{2y}}{B'_{1y}} = \frac{\gamma_2}{\gamma_2} \frac{n'_2}{n'_1} \approx 4 \]  

(16)

Thus, for the transverse case too, the magnetic field jump condition looks similar to the corresponding non-relativistic case if quantities are measured in the shock frame. However,
actually, there is a strong enhancement of the magnetic field in the downstream and this becomes clear if we write the above equation in terms of respective proper fields by using eq. (8):

\[ B'_{2} \approx 4\gamma_{12}B'_{1} \approx 4\gamma_{*}B'_{1} \approx 2\sqrt{2}\gamma_{F}B'_{1} \approx \sqrt{2}\eta B'_{1} \quad (17) \]

3. Relativistic MHD shocks

Although, the micro-physics of relativistic shocks, in particular, collisionless MHD shocks is poorly understood, at least in comparison to their non-relativistic counterparts, there have been considerable amount of observational, numerical as well as analytical studies of relativistic collisionless shocks. Although, such studies have largely been focussed for the perpendicular or transverse shocks, presumably, to harness the shock drift mechanism inherent in the \( \vec{v} \times \vec{B} \) term, we will see that some fundamental aspect of such studies may be extendable to the parallel cases too. To be precise, it is found that the following parameter

\[ \sigma \equiv \frac{v_{1*}(B_{1v}'/4\pi)}{v_{1*}(2mn_{1*}\gamma_{1*}c^{2})}, \quad (18) \]

where \( v_{1*} \) is the speed of the upstream fluid in the shock frame, is very important for studying the structure and accelerating properties of the shock (Kennel & Coroniti 1984a, 1984b, Alsop & Arons 1988, Hoshino et al. 1992). Physically \( \sigma \) is the ratio of the upstream Poynting energy flux and the particle energy density as measured in the shock frame. Stated this way, it may appear that \( \sigma \) may not have any relevance for the parallel case. However, by noting that \( B_{1v}' = \gamma_{*}B_{1v}' = \gamma_{*}B_{1}' \) and that the shock frame upstream particle density \( n_{1*} = \gamma_{*}n_{1} \), one can reformulate eq. (18) as

\[ \sigma = \frac{B_{1}'^{2}/8\pi}{mn_{1}c^{2}} \quad (19) \]
One can clearly see that the above definition of $\sigma$ is Lorentz covariant and of a general nature:

$$\sigma = \frac{\text{upstream proper magnetic energy density}}{\text{upstream rest mass energy density}}$$  \hfill (20)

Therefore, the foregoing definition of $\sigma$ becomes relevant for all cases. Also, note that, in contrast to non-relativistic shocks, for all practical purposes, ultrarelativistic shocks are bound to be tranverse ones because, the shock may be considered to be parallel only if $\theta_{\text{B}_1} < \gamma^{-1}$, where, $\theta_{\text{B}_1}$ is the angle between the upstream (proper) magnetic field and the flow direction as measured in the upstream proper frame (Begelman & Kirk 1990, Hoshino et al. 1992). In the present case, the upstream proper frame is the ISM and there is no reason that the ISM field (which is in any case distorted over certain length scale) should be practically perfectly aligned with the GRB flow direction. Therefore, irrespective of the Lorentz covariant definition of $\sigma$, the idealized discussion of a perpendicular relativistic shock becomes quite relevant in the present case.

Note that, in case one is performing numerical experiments to study the formation of shocks out of relativistic flows, one must consider a wide range of $\sigma$ including those $> 1$ (Langdon, Arons, & Max 1988). But, it should be realized that a value of $\sigma > 1$ necessarily means that the upstream flow has enough internal energy to support current systems whose energy density exceeds the rest mass energy density. In other words, such numerical experiments correspond to an upstream which is relativistically, “hot”. For most of the realistic astrophysical situations, theoretical and observational arguments suggest that the value of $\sigma \ll 1$ (Piddington 1957, Rees & Gunn 1974, Kennel & Coroniti 1984a,b., Hoshino et al. 1992). And, in any case, for the cold upstream region, which is the appropriate for the present case and most astrophysical situations, an absolute upper limit is $\sigma < 1$. If we, somewhat naively, use this upper limit on $\sigma$ for a parallel shock, we obtain

$$\frac{B_1^2}{8\pi} = mn_1c^2$$  \hfill (21)
And this leads to an absolute upper limit for a supposed parallel relativistic shock is

\[ B_{sh}^\parallel = B_2' = B_1' \approx 0.2 n_1^{1/2} \text{ G} \]  \hspace{1cm} (22)

The basic reason that for the parallel case, we are considering the above mentioned upper limit on \( \sigma \) is that, we are not aware of any theoretical or numerical study of relativistic parallel shocks which may tighten this constraint further. Coming back to the fairly well studied case of perpendicular relativistic shocks, we may adopt a generous upper limit on the value of \( \sigma \approx 0.002 \) by simply adopting the value corresponding to that of Crab (Rees & Gunn 1974, Kennel & Coroniti 1984a,b):

\[ B_{1\text{crab}}' \approx 9 \times 10^{-3} n_1^{1/2} \text{ G} \]  \hspace{1cm} (23)

Or,

\[ B_{sh} = B_2' \approx 4 \gamma_{12} B_{1\text{crab}}' \approx 15 \eta_3 n_1^{1/2} \text{ G} \]  \hspace{1cm} (24)

Given this generous upper-limit on \( \sigma \), depending on the actual obliquity of a given shock, the actual value of \( B_{sh} \) will vary between what is shown by eq. (22) and (24) with more likelyhood of assuming the latter value. This condition that we must have \( \sigma \ll 1 \) (atleast for perpendicular shocks) in order to have a strong and accelerating shock is somewhat akin to the condition that for non-relativistic diffusive shock acceleration the Alfven Mach number \( M_A \gg 1 \) (Begelman & Kirk 1990). This condition means that the Alfven speed must be \( << \) than the flow speed to ensure that MHD scattering centers scatter the test particles vigourously and isotropically. This condition also implies that the magnetic energy density is negligible compared to the kinetic energy flux:

\[ \frac{B_2'^2}{8\pi} \ll \frac{1}{2} mn'_2 v_2'^2 \]  \hspace{1cm} (25)

For nonrelativistic perpendicular shocks too, similar conditions are necessary to ensure that the shock is “strong” (Kundt & Krotschek 1982, Leroy et al. 1982, Appl & Camenzind 1988).
4. Discussion

Having obtained this generous upper-limit let us now ponder how justified we are in adopting a value of $\sigma$ appropriate for Crab. Remember that the Crab shock is practically a standing shock and the upstream region is not the bare ISM. On the other hand, the upstream comprises the plasma ejected by the Crab pulsar during its life time of nearly thousand years. It is very much likely that, in the past, the value of $\sigma$ for Crab was much lower, and the present value has been slowly built up over these thousand years. In contrast the case of the GRB blast wave ploughing through the bare ISM is quite different, and, it is highly improbable that, within a lab frame time scale of few seconds or less, the upstream medium ahead of the shock front (as seen by in the lab frame) can raise its magnetic field from a value of $\sim 3 \times 10^{-6} \text{G}$ to $\sim 9 \times 10^{-3} \text{G}$. We feel that, instead, the following scenario is more plausible: The value of $B'_1$ probably remains close to its unperturbed value $\sim 3 \times 10^{-6} \text{G}$; however the shock could be near perpendicular resulting in a value of $B_{\text{sh}} \sim 4\gamma_{12}B'_1 \sim \sqrt{2}\eta B'_1 \sim 410^{-3}\eta_3 \text{G}$. It is also probable, there may not be any stable shock formation at all invalidating the rigid relations between $B'_1$ and $B'_2$ employed so far, and on the other hand there may be instantaneous spikes in the downstream magnetic field (Langdon, Arons, & Max 1988):

$$\frac{B_{2\text{max}}^*}{B_{\text{sh}}^*} \approx \left[1 + \left(\frac{2}{\sigma}\right)\right]^{1/2}$$

and which may erratically raise the shocked field to a $B_{\text{max}} \sim 0.1 - 1 \text{G}$. However, formation of a shock-like discontinuity, either steady or fluctuating requires that the ambient medium should be such that the leading particles of the FB, i.e., the piston driving the shock, either individually or collectively impart significant amount of their momentum on the ambient medium. We endeavoured to examine this critical but usually overlooked problem in M96. Noting the similarity between the present problem and the one involving propagation of high energy cosmic rays in the ISM, and also recalling that phenomenologically and
observationally obtained parameter, the *spatial diffusion coefficient* describes the entire collective interaction of the cosmic rays and the ISM, we found, that, it is implausible that the FB can produce anything akin to a GRB blast wave in the ISM (M96). This is so for the simple reason that the BF-ISM interaction time scale estimated in this way could be as large as $\sim 10^7$ s!

Probably, we may also examine here whether, the electrons or the left over pairs of the FB, rather than the protons, might carry out the job of imparting energy onto the ambient medium (B. Paczynski, private communication). For a given saturation bulk LF of the FB ($\gamma_F \approx \eta/2$), the electrons or positrons will be less energetic by a factor of $\sim 2000$. And, if we are assuming the Bohm limit of the diffusion coefficient, the deflection length as well as the lab frame deflection time will accordingly be smaller by a factor of $\sim 2000$. If the number of leptons in the FB could overwhelm the protons by a factor of $\sim 2000$, there would have been equal amount of energy residing in protons and leptons, and, the foregoing reduction in time scale would have really meant that the FB-ISM interaction time would be reduced by the same factor. Unfortunately, this is not the case, the number of leptons per proton at the saturation stage of the optimally baryon polluted FB is indeed $\gtrsim 1$. This means that the eventual FB-ISM interaction time scale should be what was obtained in M96, i.e.,

$$t_i \sim 3 \times 10^4 \eta_3 \text{ s}$$

unless it is found that the lepton-ion energy transfer time scale is smaller than this above time scale. It may be probable that at best there may be soliton like discontinuities. At any rate, we do not think, we have the final answer of such questions at this moment, and we realize that, we are simply attempting to understand various aspects subject to our (present author’s) available knowledge.

The upshot of this discussion is that the original GRB is likely to be produced either by internal collisions within the FB as has long been suspected (Paczynski & Xu, Rees
& Meszaros 1994, Papathanssiou & Meszaros 1996) or if the original FB is propagating within a medium which has a modest baryonic mass but is dense enough to absorb the FB momentum either by binary collisions or by collisionless MHD process. Finally, reverting back to the original mandate of this paper, in the case of a supposed GRB blast wave propagating in the ISM and even ignoring the disturbing possibility that no shock like discontinuity may be formed on the GRB time scale, we feel that, the maximum value of the magnetic field in the shocked fluid may not exceed $\sim 1G$. We would like to emphasize here the fact that this conclusion does not at all imply that the FB cannot excite a blast wave (initially relativistic and then non-relativistic) on a much larger time scale of days or weeks and which is necessary for explaining the GRB afterglow in various low energy brackets.
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Figure Caption

Figure 1: Sketch of the FB-shock configurations, for details, see text.

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