Genetic Resampling Particle Filter with Randomly Delayed Measurements and Its Application to Line-of-Sight Rate Estimation

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To deal with randomly delayed measurements and glint noise, a novel recursive filter referred as a randomly delayed genetic resampling particle filter (RD-GRPF) is proposed in this paper. By making use of Bernoulli random variables, the measurement model is modified to describe the random delay. Then, the weight update equation is reformulated based on this model. To avoid the particle degeneration and sample impoverishment that always arise in the application of a particle filter, the genetic resampling method is utilized to resample the particles. Then, the RD-GRPF is obtained. Therefore, the filter proposed not only copes with randomly delayed measurements, it also keeps the advantage of the standard particle filter, which ensures good performance in the case of non-Gaussian noise. In addition, the RD-GRPF proposed is applied to line-of-sight (LOS) rate estimation, the model for which is also presented in this paper. A simulation was conducted and the results demonstrate the superiority of the RD-GRPF.

Key Words: Particle Filter, LOS Rate Estimation, Glint Noise, Genetic Resampling

Nomenclature

RD: Randomly Delayed
GRPF: Genetic Resampling Particle Filter
LOS: Line-Of-Sight
UKF: Unscented Kalman Filter
CKF: Cubature Kalman Filter
EKF: Extended Kalman Filter
RPF: Regularized Particle Filter
GBR: Ground-Based Radar

Subscripts

i: the ith particle
k: time k
0: initial
d: delay step
γ: elevation
η: azimuth
α: body azimuth
β: body elevation

1. Introduction

Due to the low cost, compact structure, high reliability and unlimited line-of-sight (LOS) rate tracking capability, strapdown imaging seekers have been widely used in many missiles, such as the Javelin missile and the JDAM missile.1) Compared with traditional seekers, strap-down imaging seekers are rigidly mounted to the missile body, but can only measure body LOS (BLOS) angles and not BLOS angle rate. The LOS angles and rate, which are indispensable to the proportional guidance law, couple with the body attitude motion, which finally results in the strong nonlinearity of the system. In the process of tracking, the changes in the target aspect toward the seeker may cause irregular electromagnetic wave reflections and give rise to glint noise.2,3) Moreover, the measurements received by the data processing center are randomly delayed due to network congestion.4) Therefore, it’s a challenge to distil the LOS angles and rate from the BLOS angle measurements. Therefore, there are four problems when applying a strap-down imaging seeker: (1) The seeker can’t measure the LOS angles and LOS angle rates, (2) the system is nonlinear, (3) the measurement noise of the seeker is glint noise instead of Gaussian noise, and (4) the measurements are randomly delayed.

In order to deal with the strong nonlinearity, several filtering algorithms have been utilized to design the LOS rate estimator. The best known filtering algorithm for LOS rate estimation is the extended Kalman filter (EKF), which is based on first-order Taylor-series linearization.5) But the performance of the EKF is unsatisfactory due to the truncations. To reduce the error caused by the truncations, an unscented Kalman filter (UKF) was proposed based on the unscented transformation method and applied to the LOS rate estimation in Sun et al.6) In addition, the cubature rule-based cubature Kalman filtering (CKF), which is more numerically stable than UKF, has also been applied to the LOS rate estimation.7) The research results indicate that the UKF and CKF perform well in the case of Gaussian noise.

Unfortunately, the measurements of the strap-down imaging seeker are always corrupted by glint noise, which is a typical non-Gaussian noise. In this situation, the performance of the above filters decreases seriously. To estimate the LOS rate in such a situation, the particle filter was proposed and utilized to design the LOS rate estimator.8) When implement-
ing a particle filter, particle degeneration and sample impoverishment always arise, which decrease accuracy dramatically and even leads to divergence.  

Many modified particle filters have been proposed to solve these issues, and they can be classified into two categories. The filters contained in the first category are modified by choosing the optimal importance density function,\(^1\)\(^1\)-\(^1\)\(^2\) such as the extended Kalman particle filter, unscented particle filter and cubature particle filter. Although these filters could effectively alleviate the degeneracy phenomenon, they also increase the computational burden. The filters contained in the second category are proposed by modifying the resampling method,\(^1\)\(^2\)-\(^1\)\(^3\) such as the regularized particle filter, unscented particle filter and cubature particle filter. These filters, especially the GRPF, can ensure validity and effectively improve diversity. However, the above particle filters were derived based on the assumption that the measurement is available in real-time. But in practice, the measurements are always randomly delayed. Therefore, many studies have been conducted, and the most common ones were proposed based on backward prediction and forward prediction.\(^1\)\(^4\)-\(^1\)\(^6\) These filters perform well coping with delayed measurements, but the latency time is indispensable. Inspired by the development of Gaussian filters with delayed measurements,\(^1\)\(^7\)-\(^1\)\(^9\) the paper reformulates the weight update equation based on the modified measurement model and then the RD-GRPF is obtained. Compared with the EKF, UKF and CKF, the computational burden of the RD-GRPF proposed is bigger, but it has two advantages: (1) The filter proposed is robust to glint noise and has better performance than EKF, UKF and CKF in this case; (2) The filter proposed is capable of coping with randomly delayed measurements and performs better than GRPF when the measurements are randomly delayed. The framework of this paper is given in Fig. 1.

The rest of the paper is organized as follows. Section 2 formulates the problem investigated in this paper. Section 3 derives the randomly delayed genetic resampling particle filter. In Section 4, the RD-GRPF is applied to line-of-sight rate estimation. A simulation is conducted in Section 5 and the conclusion is given in Section 6.

### 2. Problem Formulation

Consider the following discrete-time nonlinear stochastic system:

\[
x_k = f_{k-1}(x_{k-1}) + w_{k-1}
\]

\[
y_k = h(x_k) + v_k
\]

where, \(k \in \{1, 2, \cdots\}\) is the discrete-time, \(x_k \in \mathbb{R}^n\) is the \(n\)-dimensional state vector at time \(k\), \(y_k \in \mathbb{R}^m\) is the \(m\)-dimensional ideal measurement vector, \(w_{k-1} \in \mathbb{R}^n\) and \(v_k \in \mathbb{R}^m\) are uncorrelated noises, and \(f\) and \(h\) denote the known nonlinear state function and measurement function, respectively.

In the practice of engineering, the measurements are always delayed due to network congestion. To describe the multiple-step randomly delayed measurements, the measurement equation is modified as:

\[
z_k = (1 - \beta_k^1)y_k + \beta_k^1(1 - \beta_k^2)y_{k-1} + \cdots + \left(\prod_{i=1}^{j-1} \beta_i^k\right)(1 - \beta_k^s)y_{k-s} + \sum_{d=0}^{s-1} \alpha_k^d y_{k-d}
\]

where, \(\beta_k^i \in [0, 1]\) are the Bernoulli random variables with \(\beta_k^0 = 1\) and \(\beta_k^s = 0\), \(s\) is the maximum number of delay, and \(\alpha_k^d\) is defined by

\[
\alpha_k^d = \left(\prod_{j=0}^{d-1} \beta_j^k\right)(1 - \beta_k^{d+1})
\]

According to the property of Bernoulli random variables, the following equation is easily obtained.

![Fig. 1. The framework of this paper.](image-url)
3) Until

\( p(\beta_i^t = 1) = E[\beta_i^t] = p_t \)

\( p(\alpha_i^t = 0) = 1 - E[\beta_i^t] = 1 - p_t \)

where, \( p_t \) is the probability of \( \beta_i^t \) taking value one.

Then, the probability that the measurement is \( d \)-time step delayed is

\[
p_k^d = E[\alpha^d_i] = E\left[ \left( \prod_{j=0}^{d} \beta_j^t \right) E\left[ (1 - \beta_j^{t+1}) \right] \right] = \left( \prod_{j=0}^{d} p_j \right) (1 - p_{d+1})
\]

To solve the problem formulated in Eqs. (1)–(6), a novel particle filter is derived in the following section.

3. Randomly Delayed Genetic Resampling Particle Filter

3.1. Genetic resampling

Genetic resampling, which is one of the optimization methods, is proposed based on the mechanics of the natural evolutionary process. It consists of three parts: selection, crossover and mutation. The purpose of the selection is to retain particles with big fitness value, while that of the crossover and mutation is to generate new particles. The main steps are given as follows:

1) Initialization

Set selection probability \( P_s \), crossover probability \( P_c \) and mutation probability \( P_m \), respectively.

2) Selection

Draw a random sample \( u_i \) from the uniform distribution over \((0, 1)\). If \( u_i \) satisfies

\[
\sum_{j=1}^{m-1} \omega_{k,j} < u_i \leq \sum_{j=1}^{m} \omega_{k,j}
\]

the particle \( j \) is chosen to be the resampling particle, where \( \omega_{k,j} \) is the weight of particle \( j \) at time \( k \). Repeat Step 2) until \( N_s = P_s \times N \) particles are obtained, where \( N \) is the particle number.

3) Crossover

Randomly select two particles \( x_{k,i} \) and \( x_{k,j} \). Then, two new particles \( \tilde{x}_{k,i} \) and \( \tilde{x}_{k,j} \) are generated using Eq. (7):

\[
\tilde{x}_{k,i} = \alpha x_{k,i} + (1 - \alpha) x_{k,j} \\
\tilde{x}_{k,j} = \alpha x_{k,j} + (1 - \alpha) x_{k,i}
\]

where, \( \alpha \) is a random number between 0 and 1. Repeat Step 3) until \( N_c = P_c \times N \) numbers are obtained.

4) Mutation

Randomly select one particle \( x_{k,i} \). Then, the new particle \( \tilde{x}_{k,i} \) is generated by Eq. (8):

\[
\text{if } \alpha < 0.5, \quad \tilde{x}_{k,i} = (1 + \beta) x_{k,i} \\
\text{else, } \tilde{x}_{k,i} = (1 - \beta) x_{k,i}
\]

where, \( \alpha \) is a random number between 0 and 1, and \( \beta \) is a random positive real number. Repeat Step 4) until \( N_m = P_m \times N \) numbers are obtained.

5) Combine

Combine the selection, crossover and mutation particle sets as the new particle set.

3.2. Randomly delayed genetic resampling particle filter

Since the actual measurement \( z_k \) is the ideal mixture, measurements \( \{y_{k-d}^{t}\}_{d=0}^{t} \) and the state vector \( \{x_{k-d}^{t}\}_{d=0}^{t} \) are joined together, and the augmented state \( X_{k-1} \) is given as

\[
X_{k-1} = [x_{k-1}, x_{k-2}, \cdots, x_{k-2}, x_{k-1}]^T
\]

Then the state equation could be modified as

\[
X_k = F_{k-1}(X_{k-1}) = \\
\begin{bmatrix}
X_{k-1} \\
F_{k-1}(X_{k-1}) + w_{k-1}
\end{bmatrix}
\]

It can be seen that most elements in Eq. (10) are available in Eq. (9) except the last element \( x_k \), which is computed based on the state equation.

According to the Bayesian theory, the posterior conditional probability density function of \( X_k \) is given by:

\[
p(X_k | Z_k) = \frac{p(z_k | X_k)p(X_k | Z_{k-1})}{p(z_k | Z_{k-1})}
\]

where,

\[
p(X_k | Z_{k-1}) = \int p(X_k | X_{k-1})p(X_{k-1} | Z_{k-1})dX_{k-1}
\]

and

\[
Z_k = [z_j]_{j=1}^{k}
\]

is the measurements sequence.

By resorting the Monte Carlo techniques and importance sampling method, the posterior probability density function \( p(X_k | Z_k) \) could be approximated based on the \( N \) samples.

\[
p(X_k | Z_k) \approx \frac{1}{N} \sum_{i=1}^{N} \omega_{k,i} \delta(x_k - X_k^i)
\]

where, \( \delta(\cdot) \) is the Dirac delta function, \( 0 \leq \omega_{k,i} \leq 1 \) is the weight of \( i \) particles, and \( q \) is a proposal distribution function.

Based on the standard particle filtering algorithm, the priori PDF \( p(X_k | X_{k-1}, i) \) could be chosen as the proposal dis-
tribution function, hence
\[
\hat{\alpha}_{k,i} = \hat{\alpha}_{k-1,i} p(z_k | X_{k,i})
\]  
(17)

By defining
\[
\alpha_k = \left[ \alpha_k^0, \alpha_k^1, \cdots, \alpha_k^d \right]^T,
\]
Eq. (3) can be rewritten as:
\[
z_k = \left[ y_k, y_{k-1}, \cdots, y_{k-i} \right] \cdot \alpha_k
\]  
(18)

According to the definition of \(\alpha_k^d\) given in Eq. (4), there is one and only one element of \(\alpha_k\) taking the value 1, and the others taking the value 0. Therefore,
\[
p\left(\alpha_k = [0, \cdots, \alpha_k^d, \cdots 0]^T, \alpha_k^d = 1 \right) = p(\alpha_k^d = 1) = p_k^d
\]  
(19)

Since \(\alpha_k\) is independent of state vectors, it is highly likely that the probability density function can be computed using
\[
p(z_k | X_{k,i}) = p(z_k | x_{k,i}, x_{k-1,i}, \cdots, x_{k-i,i})
\]  
\[= \int p(z_k, \alpha_k | x_{k,i}, x_{k-1,i}, \cdots, x_{k-i,i})d\alpha_k
\]  
\[= \int p(z_k | \alpha_k, x_{k,i}, x_{k-1,i}, \cdots, x_{k-i,i})p(\alpha_k)d\alpha_k
\]  
\[= \sum_{d=0}^{d} p_k^d p_{\alpha_k} (z_k = [0, \cdots, \alpha_k^d, \cdots, 0]^T, \alpha_k^d = 1, x_{k,i}, x_{k-1,i}, \cdots, x_{k-i,i})
\]  
\[= \sum_{d=0}^{d} p_k^d p_{\alpha_k} (z_k = h_{k-d}(x_{k-d,i}))
\]  
(20)

where, \(p_{\alpha_k}(\cdot)\) denotes the probability density function of the measurement noise \(v_{k-d}\).

Substituting Eq. (20) into Eq. (17) yields
\[
\hat{\alpha}_{k,i} = \hat{\alpha}_{k-1,i} \sum_{d=0}^{d} p_k^d p_{\alpha_k} (z_k = h_{k-d}(x_{k-d,i}))
\]  
(21)

Finally, the genetic resampling introduced in Section 3.1 is utilized to avoid sample impoverishment, thereby obtaining the novel randomly delayed genetic resampling particle filter.

4. Application to Line-of-sight Estimation

4.1. State model

The state vector is defined as:
\[
x = \left[ q_y, \dot{q}_y, q_z, \dot{q}_z \right]^T
\]  
(22)

where, \(q_y, \dot{q}_y, q_z\) and \(\dot{q}_z\) are LOS elevation, elevation rate, azimuth and azimuth rate, respectively.

According to the relative motion between missile and target, the relative dynamic equation is shown as follows:
\[
\left\{ \begin{align*}
\dot{r} - r (\dot{q}_y)^2 - r (\dot{q}_z) \cos q_y & = \alpha_y, \\
\dot{\dot{r}} + 2 \dot{q}_y \dot{q}_z \sin q_y \cos q_y + \dot{r} \dot{q}_z - 2 \dot{q}_z \dot{q}_z \sin q_y - 2 \dot{q}_z \dot{q}_z \cos q_z - \dot{q}_z \cos q_y & = \alpha_z
\end{align*} \right.
\]  
(23)

where, \(\alpha_y, \alpha_z\) are components of missile–target relative accelerations along the axis.

Then, the standard four-dimensional state equations can be presented as follows:
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{-2r}{r} (\dot{x}_2 - x_2^2 \sin x_1 \cos \frac{a_s}{r}) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= 2x_2 x_4 \tan x_1 - \frac{2r}{r} x_4 + \frac{a_s}{r} \cos x_1
\end{align*}
\]  
(24)

where, \(r\) is the relative range, and \(x_1, x_2, x_3\) and \(x_4\) are \(q_y, \dot{q}_y, q_z\) and \(\dot{q}_z\), respectively. Since the targets are mainly fixed, \(\alpha_y, \alpha_z\), and \(\alpha_s\) can regarded as 0 in engineering. Then, the state equation can be rewritten as:
\[
\begin{align*}
\dot{q}_y &= x_1 = x_2 \\
\dot{q}_y &= x_2 = \frac{-2r}{r} (\dot{x}_2 - x_2^2 \sin x_1 \cos x_1) \\
\dot{q}_z &= x_3 = x_4 \\
\dot{q}_z &= x_4 = 2x_2 x_4 \tan x_1 - \frac{2r}{r} x_4
\end{align*}
\]  
(25)

4.2. Measurement model

The strap-down imaging seeker can directly measure LOS angles \(q_y\) and \(q_z\), after which the measurement vector is defined as:
\[
y = \left[ q_y, q_z \right]^T
\]  
(26)

The relative range of the missile and target can be represented as:
\[
\begin{align*}
x_b &= r \cos q_y \cos q_z \\
y_b &= r \sin q_y \\
z_b &= -r \cos q_y \sin q_z
\end{align*}
\]  
(27)

\[
\begin{align*}
x_f &= r \cos q_y \cos q_z \\
y_f &= r \sin q_y \\
z_f &= -r \cos q_y \sin q_z
\end{align*}
\]  
(28)

Pre-multiplying \(C^0_i\) on both sides of Eq. (28) yields:
\[
\begin{align*}
x_b &= C^0_i \begin{bmatrix} x_f \\ y_f \\ z_f \end{bmatrix} \\
z_b &= C^0_i \begin{bmatrix} r \cos q_y \cos q_z \\ r \sin q_y \\ -r \cos q_y \sin q_z \end{bmatrix}
\end{align*}
\]  
(29)

where, \(C^0_i\) is the rotation matrix from the inertial coordinate to body coordinate.

Combining Eqs. (27) and (29), we obtain:
\[
C_r^2 \begin{bmatrix}
 r \cos \theta \cos \phi \\
 r \sin \theta \\
-r \cos \phi 
\end{bmatrix} = \begin{bmatrix}
 r \cos \theta \cos \phi \\
 r \sin \theta \\
-r \cos \phi 
\end{bmatrix} (30)
\]

Based on Eq. (30), \(q_a\) and \(q_\theta\) can be given as:
\[
\begin{align*}
q_a &= \arcsin(R_{31} \cos \phi \cos \phi + R_{21} \sin \phi \\
&= -R_{23} \cos \phi \sin \phi \\
q_\theta &= \arctan \left( \frac{R_{33} \sin \phi - R_{31} \cos \phi - R_{32} \tan \phi}{R_{11} \cos \phi + R_{12} \tan \phi - R_{13} \sin \phi} \right)
\end{align*}
\]

where, \(R_{ij}(i, j \in \{1, 2, 3\})\) are elements of \(C_r^2\).

Taking into account measurement noise, the measurement model is given as:
\[
\begin{align*}
q_a &= y_1 \\
&= \arcsin(R_{31} \cos \phi \cos \phi + R_{21} \sin \phi \\
&= -R_{23} \cos \phi \sin \phi \\
q_\theta &= y_2 \\
&= \arctan \left( \frac{R_{33} \sin \phi - R_{31} \cos \phi - R_{32} \tan \phi}{R_{11} \cos \phi + R_{12} \tan \phi - R_{13} \sin \phi} \right) + v_\theta
\end{align*}
\]

where, \(v_\theta\) is the measurement noise.

\[
p(z_k | X_{k,i}) = \sum_{d=0}^{\infty} p^d_{k-d} p(z_k | X_{k,d})
\]

\[
= \sum_{d=0}^{\infty} p^d_{k-d} \left( \frac{1 - \varepsilon}{2\pi}\left| R_1 \right|^{1/2} \exp \left( -\frac{\left[ z_k^* - h_{k-d}(x_{k-d,i}) \right]^T R_1^{-1} \left[ z_k^* - h_{k-d}(x_{k-d,i}) \right]}{2} \right) + \frac{\varepsilon}{2\pi}\left| R_2 \right|^{1/2} \exp \left( -\frac{\left[ z_k^* - h_{k-d}(x_{k-d,i}) \right]^T R_2^{-1} \left[ z_k^* - h_{k-d}(x_{k-d,i}) \right]}{2} \right) \right)
\]

where, \(\varepsilon\) is the measurement function, \(z_k^*\) is the measurement data received at time \(k\), \(m\) is the dimensionality of the measurement vector, and \(R_1\) and \(R_2\) are the covariance of \(p_{G_1}\) and \(p_{G_2}\), respectively.

c) Update each particle’s weight based on the likelihood
\[
\omega_{k,i} = \omega_{k-1,i} : p(z_k | X_{k,i})
\]

\[
= \frac{\omega_{k,i}}{\sum_{i=1}^{N} \omega_{k,i}}
\]

(37)

d) State estimation
\[
\hat{X}_k = \sum_{i=1}^{N} \omega_{k,i} X_{k,i}
\]

and then the origin state \(\hat{x}_k\) can be extracted from \(\hat{X}_k\).

e) Resampling
Calculate the effective sample size \(N_{eff}\):

\[
N_{eff} = \frac{N}{1 + \text{var}(\omega_{k,i})} \approx \frac{1}{\sum_{i=1}^{N} (\omega_{k,i})^2}
\]

(39)

Then, compare \(N_{eff}\) with the threshold \(N_{th}\), which is chosen by the practitioner. If \(N_{eff} < N_{th}\), resampling is conducted to generate a set of posterior particles \(X_{k+1,i}^+\) and their corresponding weights \(\omega_{k+1,i}^+ = \frac{1}{N}\). Otherwise, \(X_{k,1} = X_{k,i}^+, \omega_{k,1}^+ = \omega_{k,i}\).

f) Perform the above steps recursively until the end condition is satisfied.

5. Simulation

To verify the effectiveness of the algorithm proposed, the RD-GRPF proposed and the existing UKF, RD-UKF \(^{18}\) and GRPF are all utilized to design the LOS rate estimator. The initial state estimated error is \(0.5^\circ, 0.5^\circ, 0.05/\text{s}, 0.05/\text{s}^2\); initial relative range is 7425 m; and initial relative velocity is 124.6 m/s. The assumed measurement noise parameters of the strap-down imagine seeker are given as: \(\sigma_1 = 0.4^\circ/3, \sigma_2 = 5\sigma_1\), and \(\varepsilon = 0.2\). The seeker data rate
is 10 Hz, and the particle number selected is 10,000. The maximum number of delays is assumed to be 2, and the probability of one-step delay $p^1_k$ and two-step delay $p^2_k$ is given as:

$$p^1_k = \frac{2}{3} p_{\text{delay}}$$
$$p^2_k = \frac{1}{3} p_{\text{delay}}$$

where, the delay probability is $p_{\text{delay}} = 0.5$.

After 50 independent Monte Carlo runs, the LOS angle and rate estimated error are compared in Figs. 2 and 3, respectively. It can be seen that RD-GRPF and UKF have the best and worst performance among four filters, respectively. That’s because two assumptions (i.e., 1. measurements are available in real-time, and 2. measurement noise is Gaussian white noise), which are the basis of UKF, are violated in this simulation. On the other hand, since the RD-GRPF combines the advantages of GRPF in coping with glint noise and RD-UKF in coping with randomly delayed measurements, it exhibited better performance than other three filters in this simulation.

To compare the ability of four filters to cope with randomly delayed measurements and glint noise, two other simulations are conducted. The simulation results are shown as follows.

In the first simulation, the perturbing parameter $\varepsilon$ is fixed as 0.2 and the delay probability is $p_{\text{delay}} = 0.3, 0.4, \ldots, 0.7$. The simulation results are summarized in Fig. 4. It can be seen that the RD-GRPF performs best among the four filters for all values of $p_{\text{delay}}$. Additionally, as $p_{\text{delay}}$ rises, the performance of GRPF and UKF degrades quicker than that of RD-UKF and RD-GRPF. This is because both RD-UKF and RD-GRPF are capable of coping with randomly delayed measurements, but only RD-GRPF copes with glint noise simultaneously.

In the second simulation, the delay probability $p_{\text{delay}}$ is fixed as 0.5, and the perturbing parameter is $\varepsilon = 0, 0.1, \ldots, 0.4$. As can be seen in Fig. 5, the simulation results show that the accuracy of RD-UKF and RD-GRPF are almost the same when $\varepsilon = 0$. However, the tracking error of
RD-UKF obviously increases with the rise of $\varepsilon$, while that of GRPF and RD-GRPF is unaffected. The reason for this is that RD-UKF is derived based on a Gaussian assumption, which is violated with increasing severity as $\varepsilon$ rises. At the same time, the GRPF and RD-GRPF inherit the advantage of a standard particle filter, which approximates any non-Gaussian distribution.

Therefore, the RD-GRPF exhibits greater robustness to both glint noise and randomly delayed measurements, and the simulation results clearly confirm its superiority.

6. Conclusion

In this paper, a novel filtering algorithm, referred to as RD-GRPF, is proposed to cope with randomly delayed measurements and glint noise. Based on Bernoulli random variables, the random delay is modeled by modifying the measurement equation. Genetic resampling is utilized to avoid particle degeneration and sample impoverishment, and then the weight update equation is reformulated based on the modified measurement equation. Hence, the RD-GRPF proposed is capable of coping with randomly delayed measurements and non-Gaussian noise. Moreover, the RD-GRPF is applied to LOS rate estimation, and the simulation results demonstrate the effectiveness of the filtering algorithm.

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