THE DISTANCE DUALITY RELATION FROM STRONG GRAVITATIONAL LENSING

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ABSTRACT

Under very general assumptions of the metric theory of spacetime, photons traveling along null geodesics and photon number conservation, two observable concepts of cosmic distance, i.e., the angular diameter and the luminosity distances are related to each other by the so-called distance duality relation (DDR) $D^d = D^L (1 + z)^2$. Observational validation of this relation is quite important because any evidence of its violation could be a signal of new physics. In this paper we introduce a new method to test the DDR based on strong gravitational lensing systems and type Ia supernovae (SNe Ia) under a flat universe. The method itself is worth attention because unlike previously proposed techniques, it does not depend on all other prior assumptions concerning the details of cosmological model. We tested it using a new compilation of strong lensing (SL) systems and JLA compilation of SNe Ia and found no evidence of DDR violation. For completeness, we also combined it with previous cluster data and showed its power on constraining the DDR. It could become a promising new probe in the future in light of forthcoming massive SL surveys and because of expected advances in galaxy cluster modeling.

Key words: distance scale – gravitational lensing: strong – methods: data analysis – supernovae: general

1. INTRODUCTION

There exists a very fundamental relation connecting the observed luminosity distance $D^L$ and angular diameter distance $D^d$ at the same redshift $z$, namely, $D^d(z)(1 + z)^2/D^L(z) = 1$. This is the so-called “distance duality relation” (Etherington 1933, 2007) (DDR thereafter). This reciprocity relation always holds true under the following three general conditions.

1. The spacetime is described by a metric theory of gravity,
2. The Photons travel along null geodesics, and
3. The number of photons is conserved.

Conditions 1 and 2 are very basic, related to the foundations of the theory of gravity (Adler 1971; Bassett & Kunz 2004). Violation of condition 3 can be achieved much easier by cosmic opacity (Liao et al. 2015a). The number of photons arriving from standard candles, such as type Ia supernovae (SNe Ia), may be altered simply by the absorption and scattering on dust on their way to the observer. More exotic scenarios invoked in the literature comprise the conversion of photons (in the presence of extragalactic magnetic fields) to very light axions (Sikivie 1983; Raffelt 1999), gravitons (Chen 1995), Kaluza-Klein modes associated with extra-dimensions (Deffayet & Úzan 2000), or a chameleon field (Khoury & Weltman 2004; Burrrage 2008). It is therefore quite important and necessary to test the DDR because any violation of it could be a clue of new physics.

Starting with the first paper on this subject (Bassett & Kunz 2004), many efforts have been devoted to validate the DDR with observational data (e.g., Úzan et al. 2004; Bernardis et al. 2006; Holanda et al. 2010, 2012; Khedekar & Chakraborti 2011; Li et al. 2011; Nair et al. 2011; Meng et al. 2012; Ellis et al. 2013). In these works, the most common and mature strategy was to compare luminosity distances from SNe Ia (based on Union 2 and Union 2.1 SN Ia compilations Amanullah et al. 2010; Suzuki et al. 2012) and angular diameter distances from galaxy clusters at almost the same redshifts. The advantage of such a strategy is that the number of well-measured SNe Ia increased rapidly during recent years. However, the sample size of $D^d$ from galaxy clusters is still limited to several dozen. What is even more important, the angular diameter distances estimated from X-ray and SZ-studies of clusters are sensitive to the assumptions about the hot gas density profile (simple $\beta$ or double $-\beta$ profile) and the underlying geometry of the cluster (spherical or elliptical). All these unknowns contribute to systematic uncertainties which are hard to quantify. Some researchers took the opposite approach, namely assuming that the DDR was valid, they tried to draw conclusions regarding galaxy clusters (Holanda et al. 2011; Cao et al. 2016). Other papers (Nair et al. 2012) used angular diameter distance data from Baryon Acoustic Oscillations. However, there is one more disadvantage of the DDR studies performed so far. Namely, most of them used distance data (both angular diameter and luminosity distances) which were obtained under the assumption of a certain particular cosmological model (usually flat $\Lambda$CDM). Of course the DDR as a fundamental relation based only on conditions 1–3 does not depend on a cosmological model. However, the distances inferred under the assumption of a specific cosmology may be biased, hence obscuring the issue of how stringent the resulting constraints on possible violation of the DDR might be. The violations of the DDR found in previous studies might originate from these (Holanda et al. 2010; Li et al. 2011).

In this paper we propose a new method of testing the DDR, which is completely free from prior assumptions concerning the details of cosmological model except the geometry, i.e., a flat FRW cosmology, while other state-of-the-art methods furthermore depend on information like Hubble constant or dark energy ($\Lambda$CDM or $\omega$CDM). The idea is to consider jointly the SNe Ia and strong gravitational lensing systems. Cosmological model independence is achieved at the prize of keeping some nuisance parameters associated with the SN Ia light curve calibration (Yang et al. 2013) and strong lens model assumption. Let us also stress that by its construction, our test becomes independent of the SN absolute magnitude.
This paper is organized as follows. In Section 2 we introduce the observations of strong lensing (SL) and SNe Ia. Then we describe the methodology of testing DDR and present the results in Section 3. Finally, we discuss and summarize the results in Section 4.

2. OBSERVABLES AND DATA

Strong gravitational lensing has become an important and powerful tool to study both the background cosmology and the structure of galaxies acting as lenses; more specifically, the combined measurements of stellar central velocity dispersion $\sigma_0$ and image separations (the Einstein radius) $\theta_E$ supplemented with an assumption about lens mass density profile can provide us the angular diameter distance ratio $R^A(z_l, z_s) = D_A^h/D_A^s$, where $D_A^h$ and $D_A^s$ are the angular diameter distances from the lens to the source and from the observer to the source, respectively. For example, within the Singular Isothermal Sphere (SIS) model of the lens, distance ratio $R^A$ is related to observable quantities in the following way (Biesiada et al. 2010):

$$R^A(z_l, z_s) = \frac{c^2\theta_E}{4\pi\sigma^2_{\text{SIS}}},$$  \hspace{1cm} (1)

where $c$ is the speed of light (later on in Equation (3) we introduce another quantity denoted $c$, but the distinction from the speed of light should be clear from the context), $\theta_E$ is the Einstein radius, and $\sigma_{\text{SIS}}$ denotes the velocity dispersion due to lens mass distribution. Let us stress that $\sigma_{\text{SIS}}$ need not to be exactly equal to the observed stellar velocity dispersion $\sigma_0$ (White & Davis 1996). To account for this, we introduce a phenomenological free parameter $f_\epsilon$ (Kochanek 1992; Ofek et al. 2003; Cao et al. 2012) defined by the relation $\sigma_{\text{SIS}} = f_\epsilon \sigma_0$. It is worth repeating that $f_\epsilon$ accounts not only for systematic errors caused by taking the observed stellar velocity dispersion $\sigma_0$ as $\sigma_{\text{SIS}}$, but also for deviation of the real mass density profile from the SIS and the effects of secondary lenses (nearby galaxies) and line of sight contamination (Ofek et al. 2003).

The fractional uncertainty of $R^A$ is given by

$$\delta R^A = \frac{\Delta R^A}{R^A} = \sqrt{(\Delta\sigma_{\text{SIS}})^2 + \Delta(\theta_E)^2}. \hspace{1cm} (2)$$

Following the strategy adopted by the Sloan Lens ACS Survey (SLACS) team, we take the fractional uncertainty of the Einstein radius at the level of 5% for all lensing systems. The uncertainties of $\sigma_0$ measurements are taken from the data. This approach has already been used to constrain cosmological parameters and the evolution of elliptical galaxies (Biesiada et al. 2010; Ruff et al. 2011; Cao et al. 2012). We use a new compilation of 118 galactic SL systems from SLACS, the BOSS Emission-line Lens Survey, the Lenses Structure and Dynamics Survey, and the Strong Legacy Survey presented in Cao et al. (2015).

The possibility of getting $R^A$ from SL systems is the starting point of our idea. To constrain the DDR violation, $R^A$ from strong lenses will be compared with analogous (although not exactly the same) luminosity distance ratios $D_L$ from the SNe Ia.

As a source of SN Ia data, we use the JLA compilation (Betoule et al. 2014) obtained by the SDSS-II and SNLS collaborations. It contains several low-redshift samples ($z < 0.1$), all three seasons from the SDSS-II (0.05 < z < 0.4), and three years from SNLS (0.2 < z < 1).

In total, it contains 740 data points. As was pointed out by Yang et al. (2013), data reported in the form of distance moduli actually depend on the details of cosmological model assumed. For example, the Union 2 (Amanullah et al. 2010) and Union 2.1 (Suzuki et al. 2012) compilations calibrated the SNe Ia with wCDM cosmology, while the JLA used flat $\Lambda$CDM model. Therefore, it is inappropriate to use these distance moduli data directly as they depend on the cosmological model assumed. In our work, we instead use the original measurements of the observed B band magnitude $m_B$, the stretch factor $x$, and color parameter $c$ in the distance modulus

$$\mu_B(z; \alpha, \beta, M_0) = 5 \log D_L(z) + 25$$

$$= m_B(z) - M_B + \alpha x(z) - \beta c(z), \hspace{1cm} (3)$$

where $D_L(z)$ is in Mpc. Note that $m_B$, $x$, $c$ are directly extracted from the light curve and are independent of cosmological model, and $\alpha$, $\beta$ are nuisance parameters related to the well-known broader-brighter and bluer-brighter relationships. The absolute B band magnitude $M_B$ is another calibrating parameter, but as we will see it will cancel in the distance ratios. Hence, the uncertainty of luminosity distance is

$$\Delta D_L = (\ln 10/5) D_L(\Delta m_B)^2 + \alpha^2(\Delta x)^2 + \beta^2(\Delta c)^2. \hspace{1cm} (4)$$

Redshift distributions of lenses, sources and supernovae used in our analysis are shown in Figure 1. As can be seen, lenses and sources overlap sufficiently with the supernovae.

3. METHOD AND RESULTS

One possible way to test the DDR is to use the following parametrization:

$$\frac{D_A^L(1 + z)^2}{D_L^S} = \eta(z) \approx 1 + \eta_0 z. \hspace{1cm} (5)$$

Standard DDR implies $\eta(z) \equiv 1$. All deviations from it, which might occur at different redshifts, are encoded in the function

![Figure 1. Redshift distributions of lenses, sources, and SNe Ia from our 118 lensing systems and JLA sample.](image)
Since all redshifts of the objects we use satisfy \( z \leq 1 \), we just take the first-order term of the Taylor expansion of \( \eta(z) \).

The most straightforward way to test the DDR would be to compare the luminosity distance and angular diameter distance at the same redshift via Equation (5). However, we can also test it using the distance ratios, because the SL system provides us angular diameter distance \( (D_A) \) ratio \( R^A \). Next, by virtue of the DDR, we should express it through the luminosity distance ratio \( R^L \). However, the numerator \( D_A^l \) is the angular diameter distance from lens to source and one can not find an observable corresponding to a similar luminosity distance from the supernova data. Therefore, taking advantage of the fact that in flat cosmological model comoving distance \( r(z) = (1 + z)D_A(z) \) between lens and source is simply \( r_s = r_l - r_l \), one can rewrite the \( R^A \) in a simple way which contains the ratio of \( D_A^l \) and \( D_s^l \):

\[
R^A = 1 - \frac{(1 + z_s)D_A^l}{(1 + z_l)D_A^s}.
\]  

Note that in a non-flat universe, \( R^A \) as a function of \( D_A^l \) and \( D_A^s \) can be quite complex (Räsänen et al. 2015) and it is extremely difficult to make the calculation with the covariance matrix of JLA data. For robustness and simplicity, we only consider the flat case in this work as a first step. Then, using the Equation (5) one can write

\[
R^A(z_l, z_s) = 1 - \frac{D_A^l(1 + z_l)(1 + \eta_l z_l)}{D_A^s(1 + z_l)(1 + \eta_l z_l)} = 1 - R^L(z_l, z_s)p(\eta_l; z_l, z_s)
\]

where

\[
p(\eta_l; z_l, z_s) = \frac{(1 + z_s)(1 + \eta_l z_l)}{(1 + z_l)(1 + \eta_l z_l)}.
\]

This is the main formula used to infer the value of \( \eta_l \) (capturing any possible deviations from the DDR). It involves observables \( R^A(z_l, z_s) \) and \( R^L(z_l, z_s) \). Note that \( R^A = D_A^l/D_A^s \) and \( R^L = D_A^l/D_A^s \) here are not exact counterparts, but they can be derived from the observed quantities. Let us also point out that by virtue of Equation (3), \( \log R^L(z_l, z_s) = 0.2[(\eta_l z_l - m_B(z_s)) + \alpha (x(z_l) - x(z_s)) - \beta (c(z_l) - c(z_s))] \), the absolute magnitude of a fiducial SN Ia cancels out. The redshifts here have the meaning of redshifts of the supernovae located at \( z_l \) and \( z_s \), respectively. Therefore, for each lensing system we have to find two SNe Ia located at the lens and the source redshifts. To achieve this, many approaches have been proposed. We adopt the simplest and efficient method of Holanda et al. (2010) and Li et al. (2011), which applies the following criterion: if the redshift difference between the lens or the source and its matched SN Ia is smaller than 0.005, it can be ignored. To avoid correlations among individual DDR tests, which would occur if we could select the same SN Ia pair for different lensing systems, we adopt the procedure that if certain SN Ia pair is matched to some lensing system it cannot be used again. This procedure resulted in selecting a sample smaller than original 118 lenses. The number of filtered lensing systems is 60 and the redshift differences are summarized in Figure 2.

Next, we use the PyMC Python module\(^4\) that implements Bayesian statistical models and fitting algorithms, including Markov chain Monte Carlo to calculate full posterior PDF:

\[
P(\eta_l, \alpha, \beta, f_s | z_l, z_s, \theta_E, \sigma_0, m_B, x, c) = \mathcal{L}(z_l, z_s, \theta_E, \sigma_0, m_B, x, c | \eta_l, \alpha, \beta, f_s)P(\eta_l, \alpha, \beta, f_s).
\]

The likelihood is derived from the \( \chi^2 \) function \( \mathcal{L} \sim \exp(-\chi^2_{\text{likelihood}}/2) \), where \( \chi^2_{\text{likelihood}} \)

\[
O_l = R^A_l - 1 + pR^L_l.
\]

Prior probability for the parameters \( P(\eta_l, \alpha, \beta, f_s) \) is the product of respective priors assumed to be uniform distributions:

\[
P(\eta_l) = U[-2.0, 2.0], P(\alpha) = U[-0.6, 0.6], P(\beta) = U[-1.0, 10.0], P(f_s) = U[0.85, 1.15].
\]

Note that for JLA data (Betoule et al. 2014), the errors include both statistic and systematic ones. The systematic error can be embodied in the full covariance matrix of all data points, which can make the uncertainties larger. In this work, we only consider a small part of JLA data and the corresponding systematic errors. The different feature here is the constructed observational quantities are distance ratios, therefore we have to consider the systematic errors for pairs.

Our results are summarized in Table 1 and shown in Figure 3. Constraint contours were plotted using the publicly available package “corner.py.”\(^5\) One can see that \( \eta_l = 0 \) is located near the center of 1σ contour, the best-fitted central value being \( \eta_l = -0.005^{+0.351}_{-0.213} \), indicating that the DDR is in a very well agreement with the observations and there is no signs of its violation in light of SN Ia and SL data.\(^6\)

To see how stringent the results of joint analysis would be, we have also considered galaxy clusters with angular diameter

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\(^4\) http://github.com/pymc-devs/pymc

\(^5\) http://github.com/dflow/corner.py

\(^6\) We have specified the cosmological parameter \( \Omega_m = 0 \). However, like the analysis in Liao et al. (2015a), the independent determination of the flatness of our universe at the percent level implies that corrections coming from deviations from flatness should be at least one order of magnitude smaller than the errors considered here.
distances determined from X-ray data combined with Sunyaev–Zeldovich effect measured in these clusters. Let us stress that principle cluster distances are also obtained in a model independent way. However, systematic uncertainties are different. For SL systems, they come form the ansatz of SIS augmented with \( f_e \) factor, whereas for clusters they come from the model assumptions concerning gas distribution profile. We have considered two separate data sets. First, there were 38 clusters of Bonamente et al. (2006) where spherical symmetry was assumed and the cluster plasma was assumed to be in hydrostatic equilibrium with dark matter following Navarro–Frenk–White profile. The second sample comprised 25 clusters analyzes by De Filippis et al. (2005) within elliptical model. The analysis details are similar to Yang et al. (2013). In particular, we used the Equation (5) to derive distance modulus to the cluster \( \mu_{\text{cluster}}(z; \eta_0) = 5 \log[\eta(z) D_{\text{cluster}}^A(z)(1 + z)^2] + 25 \) (\( \eta(z) \) here is the inverse in Yang et al. 2013) and then we considered the chi-square function of the form

\[
\chi^2_{\text{cluster}} = O^T C^{-1} O, \quad \text{where}
\]

\[
O_{\alpha} = \mu_{\text{cluster}}(z; \alpha, \beta, M_B) - \mu_{\text{cluster}}(z; \eta_0).
\]

The covariance matrix includes the corresponding systematic errors of the SNe we use (38 or 25 points). Here, the supernova absolute magnitude \( M_B \) should be included as an extra nuisance parameter. The rest of analysis is analogous to (Yang et al. 2013) but with the newest JLA sample (Betoule et al. 2014) instead of Union 2 (Amanullah et al. 2010). The results are presented in Figures 4 and 5. One can see that joint analysis gave considerably more stringent constraints.

### 4. DISCUSSIONS AND CONCLUSIONS

Validation of DDR using extragalactic observations is an important issue in modern cosmology. Any strong enough evidence of its violation could be a signal of new physics either in the theory of gravity or in the particle physics sector. Currently, the most common method to test DDR is to compare luminosity distances from standard candles (SNe Ia) and angular diameter distances from standard rulers (galaxy clusters). Technically, such an approach demands that some specific cosmological model be assumed together with the values of its parameters like the Hubble constant, cosmic equation of state, or matter density parameter \( \Omega_m \). This results in additional sources of uncertainties and bias.

In this paper, we proposed a new method to test the DDR which is free from such shortcomings (though it still assumes a flat universe). The idea is to use SL systems with known redshifts to the lenses and to the sources, the Einstein rings, and

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**Table 1**

The Best-fit Values of Parameters \( \eta_0, \alpha, \beta, f_e, M_B \) with 1\( \sigma \) and 2\( \sigma \) Uncertainties for Three Cases: Strong Lensing (SL), Strong Lensing + Clusters with Spherical Model (S) (Bonamente et al. 2006), Strong Lensing + Clusters with Elliptical Model (E) De Filippis et al. (2005), Respectively

| SL | SL+38 Clusters(S) | SL+25 Clusters(E) |
|----|------------------|------------------|
| \( \eta_0 \) | \( -0.055^{+0.051}_{-0.047} (1\sigma) \) | \( -0.051^{+0.152}_{-0.103} (1\sigma) \) | \( -0.003^{+0.234}_{-0.160} (1\sigma) \) |
| \( \alpha \) | \( 0.129^{+0.092}_{-0.045} (1\sigma) \) | \( 0.169^{+0.059}_{-0.026} (1\sigma) \) | \( 0.065^{+0.080}_{-0.035} (1\sigma) \) |
| \( \beta \) | \( 6.700^{+2.017}_{-0.425} (1\sigma) \) | \( 6.173^{+0.864}_{-0.764} (1\sigma) \) | \( 5.599^{+1.195}_{-0.895} (1\sigma) \) |
| \( f_e \) | \( 1.023^{+0.029}_{-0.018} (1\sigma) \) | \( 1.019^{+0.026}_{-0.014} (1\sigma) \) | \( 1.024^{+0.026}_{-0.014} (1\sigma) \) |
| \( M_B \) | \( ... \) | \( -19.055^{+0.111}_{-0.139} (1\sigma) \) | \( -19.113^{+0.156}_{-0.193} (1\sigma) \) |
central velocity dispersions. These observables allow us to infer the angular diameter distance ratios which—by virtue of the DDR—could be linked to luminosity distance ratios of matched pairs of SNe Ia, and hence used to test the violations of the DDR. Using a recent compilation of 118 SL systems and matched pairs of SNe Ia from the JLA compilation, we were able to select 60 matched (SL–SN) pairs to perform the test. Our result confirmed the validity of the DDR. The method we proposed is an interesting new technique, complementary to previous methods used by the others. By combining it with previous methods that may be improved in the future, one can effectively eliminate systematic errors from single test and break the constraint degeneracy, thus giving a more robust effect. To show this, we combined it with galaxy cluster data from spherical model and elliptical model, respectively. Despite the cluster modeling problem, the results showed the potential power of our method. If the cluster modeling problem is solved in the future, the combination will give a cosmological model independent result at better precision than previous cosmological model dependent approaches (Holanda et al. 2010; Li et al. 2011) can get.

The forthcoming new generation of sky surveys like the EUCLID mission, Pan-STARRS, LSST, and JDEM are estimated to discover from thousands to tens of thousands of SL systems. Even with a few percent fraction of them having spectroscopic follow-ups, this would enlarge the SL sample considerably making the constraints on DDR violation much more stringent. Furthermore, with a sufficient number of well-measured time delays (Liao et al. 2015b), it is worth noting that our method can be extended using time-delay distance consisting of a combination of three angular diameter distances.

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