\(\bar{D}N\) interaction in a color-confining chiral quark model

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We investigate the low-energy elastic \(DN\) interaction using a quark model that confines color and realizes dynamical chiral symmetry breaking. The model is defined by a microscopic Hamiltonian inspired in the QCD Hamiltonian in Coulomb gauge. Constituent quark masses are obtained by solving a gap equation and baryon and meson bound-state wave functions are obtained using a variational method. We derive a low energy meson-nucleon potential from a quark-interchange mechanism whose ingredients are the quark-quark and quark-antiquark interactions and baryon and meson wave functions, all derived from the same microscopic Hamiltonian. The model is supplemented with (\(\sigma, \rho, \omega, a_0\)) single-meson exchanges to describe the long-range part of the interaction. Cross-sections and phase shifts are obtained by iterating the quark-interchange plus meson-exchange potentials in a Lippmann-Schwinger equation. Once model parameters in meson exchange potential are fixed to describe the low-energy experimental phase shifts of the \(K^+N\) and \(K^0N\) reactions, predictions for \(\bar{D}^0N\) and \(\bar{D}^-N\) reactions are obtained without introducing new parameters.

I. INTRODUCTION

The interaction of heavy-flavored hadrons with ordinary hadrons like nucleons and light mesons is focus of great contemporary interest in different contexts. One focus of interest is in experiments of relativistic heavy ion collisions. In heavy ion experiments, charm and bottom quarks are produced in the initial stages of the collision by hard scattering processes. Since they are much heavier than the light partons making up the bulk of the matter produced in the collision, it is likely that they will not equilibrate with the surrounding matter and therefore they might be ideal probes of properties of the expanding medium – for a recent review on the subject and an extensive list of references see Ref. [1]. However, the heavy quarks will eventually hadronize and information on the medium will be carried out of the system by heavy-flavored hadrons. In their way out of the system, the heavy-flavored hadrons will interact with the more abundant light-flavored hadrons and a good understanding of the interaction is crucial for a reliable interpretation of experimental data. Another focus of interest is an exciting physics program that will be carried out with the 12 GeV upgrade of the CEBAF accelerator at the Jefferson Lab in the USA and the construction of the FAIR facility in Germany. At the Jefferson Lab charmed hadrons will be produced by scattering electrons off nuclei and at FAIR they will be produced by the annihilation of antiprotons on nuclei. One particularly exciting perspective is the possibility of creating new exotic nuclear bound states by nuclei capturing charm - a very successful model for the \(K^+N\) reaction, Ref. [13] extended the \(\bar{D}\) mesons with nucleons in free space. There is a complete lack of experimental information on this reaction, all of what is presently known about the interaction in free space has been apprehended from model calculations based on hadronic Lagrangians motivated by flavor \(SU(4)\) symmetry [10–12], models using hadron and quark degrees of freedom [13] and heavy-quark symmetry [14, 15], and the nonrelativistic quark model [16]. Although the first lattice QCD studies of the interaction of charmed hadrons like \(J/\Psi\) and \(\eta_c\) with nucleons are starting to appear in the literature [17–19], the \(\bar{D}N\) interaction does not seem to have been considered by the lattice community. In view of this situation, the use of models seems to be the only alternative for making the urgently needed predictions for e.g. low-energy cross sections of reactions involving charmed hadrons. However, as advocated in the works of Refs. [13, 20–22], minimally reliable predictions of unknown cross-sections need to be founded on models constrained as much as possible by symmetry arguments, analogies with other similar processes, and the use of different degrees of freedom. In the specific case of the \(DN\) reaction, Ref. [13] extended a very successful model for the \(KN\) reaction [23], in which the long-range part of the interaction is described within a meson-exchange framework [24, 25] and the short-distance part is described by a quark-interchange mechanism from one-gluon-exchange (OGE) of a nonrelativistic quark model (NRQM) [26–31]. The model of
Ref. [23] describes the available low-energy experimental data for the $K\bar{N}$ reaction and was used to set limits on the width of the hypothetical $\Phi^+(1540)$ pentaquark state [32]. For the $D\bar{N}$ reaction, the model predicts cross-sections that are on average of the same order of magnitude but larger by a factor roughly equal to 2 than those of the analogous $K^+N$ and $K^0\bar{N}$ reactions for center-of-mass kinetic energies up to 150 MeV. Two interesting findings of the study of Ref. [13] are noteworthy: (1) quark interchange contributes about the same amount as meson exchanges to the $DN$ s-wave phase shifts, and (2) among the meson exchanges processes, scalar ($\sigma$) and vector ($\omega, \rho$) are the most important contributors – single $\Lambda_c$, $\Sigma_c$ baryon-exchange diagrams and higher-order box diagrams involving $D^*N$, $D\Delta$, and $D^*\Delta$ intermediate states contribute very little. Recall that single-pion exchange is absent in this reaction. The same model was also used to examine the possibility to extract information on the $DN$ and $D\bar{N}$ interactions in an antiproton annihilation on the deuteron [20].

The fact that quark-interchange plays a prominent role in the $DN$ reaction is very significant to the quest of experimental signals of chiral symmetry restoration via changes in the interactions of $D$ mesons in matter. As said, such changes would be driven at the microscopic level by modifications of the properties of the light $u$ and $d$ constituent quarks. However, within a NRQM there is no direct way to link $D\chi$SB in medium and the effective hadron-hadron interactions, since constituent quark masses and microscopic interactions at the quark level are specified independently in the model [33]. Also, any temperature or density dependence on constituent quark masses has to be postulated in an ad hoc manner [34, 35] to account for effects of $D\chi$SB restoration. As a step to remedy this limitation of the NRQM, in the present paper we use a model that realizes $D\chi$SB in a way that the same microscopic interaction that drives $D\chi$SB also confines the quarks and antiquarks into color singlet hadronic states, and in addition is the source of the hadron-hadron interaction. The model is defined by a microscopic Hamiltonian inspired in the QCD Hamiltonian in Coulomb gauge, in that an infrared divergent interaction models the full non-Abelian color Coulomb kernel in QCD and leads to hadron bound states that are color singlets – the Hamiltonian also contains an infrared finite interaction to model transverse-gluon interactions and leads to, among other effects, hyperfine splittings of hadrons masses [36–38]. We implement an approximation scheme that allows to calculate with little computational effort variational hadron wave functions and effective hadron-hadron interactions that can be iterated in a Lippmann-Schwinger equation to calculate phase shifts and cross-sections. An early calculation of the $KN$ interaction within such a model, but using a confining interaction only was performed in Ref. [39].

The paper is organized as follows. In the next Section we present the microscopic quark-antiquark Hamiltonian of the model. We discuss $D\chi$SB in the context of two models for the infrared divergent potentials that mimic full non-Abelian color Coulomb kernel in QCD, and obtain numerical solutions of the constituent quark mass function for different current quark masses. In Section III we discuss a calculation scheme for deriving effective low-energy hadron-hadron interactions within the context of the Hamiltonian of the model. A low-momentum expansion of the quark mass function is used to obtain variational meson and baryon wave functions and explicit, analytical expressions for the effective meson-baryon potential. In Section IV we present numerical results for phase shifts and cross-sections for the $K^+N$ and $K^0\bar{N}$ and $D^0N$ and $D^-\bar{N}$ reactions at low energies. Initially we use the short-ranged quark-interchange potential derived within the model and add potentials from one-meson exchanges to fit experimental $s$–wave phase shifts of the $K^+N$ and $K^0\bar{N}$ reactions. Next, without introducing new parameters, we present the predictions of the model for the $D^0N$ and $D^-\bar{N}$ reactions. Our conclusions and perspectives are presented in Section V. The paper includes one Appendix, that presents the meson Lagrangians and respective one-meson exchange potentials.

II. MICROSCOPIC HAMILTONIAN AND THE CONSTITUENT QUARK MASS FUNCTION

The Hamiltonian of the model is given as

$$H = H_0 + H_{int},$$

where $H_0$ and $H_{int}$ are given in terms of a quark field operator $\Psi(x)$ as

$$H_0 = \int d\mathbf{x} \bar{\Psi}^T(x)(-i\mathbf{\alpha} \cdot \nabla + \beta m)\Psi(x),$$

and

$$H_{int} = \frac{-1}{2} \int d\mathbf{x} \int d\mathbf{y} \rho^a(\mathbf{x}) V_C(|\mathbf{x} - \mathbf{y}|) \rho^a(\mathbf{y})$$

$$+ \frac{1}{2} \int d\mathbf{x} \int d\mathbf{y} J^a_i(\mathbf{x}) J^b_j(\mathbf{y}) D^{ij}(|\mathbf{x} - \mathbf{y}|).$$

In the above $m$ is the current-quark mass matrix of the light $l = (u, d)$, strange $s$, and charm $c$ quarks:

$$m = \begin{pmatrix}
    m_u & 0 & 0 & 0 \\
    0 & m_d & 0 & 0 \\
    0 & 0 & m_s & 0 \\
    0 & 0 & 0 & m_c
  \end{pmatrix},$$

and $\rho^a(\mathbf{x})$ is color charge density

$$\rho^a(\mathbf{x}) = \bar{\Psi}^T(\mathbf{x}) T^a \Psi(\mathbf{x}),$$

$J^a_i(\mathbf{x})$ is the color current density

$$J^a_i(\mathbf{x}) = \bar{\Psi}^T(\mathbf{x}) T^a \alpha_i \Psi(\mathbf{x}),$$
with \( T^a = \lambda^a / 2 \), where \( \lambda^a \) are the \( SU(3) \) Gell-Mann matrices. \( V_C \) and \( D^{ij} \) are the effective Coulomb and transverse-gluon interactions; the transversity of \( D^{ij} \) implies

\[
D^{ij}(|x - y|) = \left( \delta^{ij} - \frac{\nabla^i \nabla^j}{v^2} \right) D_T(|x - y|). \tag{7}
\]

The problem of \( \Delta \chi \Sigma B \) with such an Hamiltonian has been discussed in the literature since long time in Bardeen-Cooper-Schriefer (BCS) mean-field type of approaches, via Bogoliubov-Valatin transformations or Dyson-Schwinger equations in the rainbow approximation [40–54]. For our purposes in the present paper, it is more convenient to follow the logic of the Bogoliubov-Valatin transformations. In this approach, \( \Delta \chi \Sigma B \) is characterized by a momentum-dependent constituent-quark mass function \( M(k) \), so that the quark field operator of a given color and flavor can be expanded as

\[
\Psi(x) = \sum_s \int \frac{dk}{(2\pi)^3} [u_s(k)q_s(k) + \bar{v}_s(k)\bar{q}_s(-k)]e^{ikx}, \tag{8}
\]

where \( u_s(k) \) and \( v_s(k) \) are Dirac spinors:

\[
u_s(k) = \sqrt{\frac{E(k) + M(k)}{2E(k)}} \left( \begin{array}{c} 1 \\ \sigma \cdot k \\ E(k) + M(k) \end{array} \right) \chi_s, \tag{9}
\]

\[
u_s(k) = \sqrt{\frac{E(k) + M(k)}{2E(k)}} \left( \begin{array}{c} \sigma \cdot k \\ E(k) + M(k) \\ 1 \end{array} \right) \chi_s, \tag{10}
\]

with \( E(k) = [k^2 + M^2(k)]^{1/2} \), \( \chi_s \) is a Pauli spinor, \( \chi_s^c = -i \sigma^2 \chi_s^c \), and \( q_s(k) \), \( \bar{q}_s(k) \) are creation and annihilation operators of constituent quarks; \( q_s(k) \) and \( \bar{q}_s(k) \) annihilate the vacuum state \(|\Omega\rangle\):

\[
q_s(k)|\Omega\rangle = 0, \quad \bar{q}_s(k)|\Omega\rangle = 0. \tag{11}
\]

For \( m = 0 \), the Hamiltonian is chirally symmetric, but \(|\Omega\rangle\) is not symmetric, \( \langle \Omega | \Psi \Psi | \Omega \rangle \neq 0 \).

Substituting in Eqs. (2) and (3) the expansion of \( \Psi \), Eq. (8), and rewriting \( H \) in Wick-contracted form, one obtains an expression for \( H \) that can be written as a sum of three parts:

\[
H = \mathcal{E} + H_2 + H_4, \tag{12}
\]

where \( \mathcal{E} \) is the c-number vacuum energy, and \( H_2 \) and \( H_4 \) are normal-ordered operators, respectively quadratic and quartic in the creation and annihilation operators. The mass function \( M(k) \) is determined by demanding that \( H_2 \) be diagonal in the quark operators. This leads to the gap equation for the constituent quark mass function \( M_f(k) \) of flavor \( f \):

\[
M_f(k) = m_f + \frac{2}{3} \int \frac{dq}{(2\pi)^3} \left[ F_f^{(1)}(k,q) V_C(|k - q|) + 2 G_f^{(1)}(k,q) D_T(|k - q|) \right], \tag{13}
\]

where \( m_f \) is the current quark mass and

\[
F_f^{(1)}(k,q) = \frac{M_f(q)}{E_f(q)} - \frac{M_f(k)}{E_f(k)} \frac{k \cdot q}{k \cdot \hat{q}}, \tag{14}
\]

\[
G_f^{(1)}(k,q) = \frac{M_f(q)}{E_f(q)} + \frac{M_f(k)}{E_f(k)} \frac{(k \cdot q - k^2)(k \cdot q - q^2)}{k q |k - q|^2}, \tag{15}
\]

and

\[
V_C(k) \text{ and } D_T(|x|) \text{ are the Fourier transforms of } V_C(|x|) \text{ and } D_T(|x|). \tag{16}
\]

The quadratic Hamiltonian \( H_2 \) is given by

\[
H_2 = \sum_{s,f} \int dk \varepsilon_f(k) \left[ q_s(k)q_{sf}(k) + \bar{q}_{sf}(k)\bar{q}_s(k) \right], \tag{17}
\]

where \( \varepsilon_f(k) \) is the constituent-quark single-particle energy of flavor \( f \), given by

\[
\varepsilon_f(k) = \frac{k^2 + m_f M_f(k)}{E_f(k)} + \frac{2}{3} \int \frac{dq}{(2\pi)^3} \left[ F_f^{(1)}(k,q) V_C(|k - q|) + 2 G_f^{(1)}(k,q) D_T(|k - q|) \right], \tag{18}
\]

and

\[
G_f^{(1)}(k,q) = \frac{M_f(k)}{E_f(k)} \frac{M_f(q)}{E_f(q)} + \frac{(k \cdot q - k^2)(k \cdot q - q^2)}{E_f(k)E_f(q)|k - q|^2}. \tag{19}
\]

The four-fermion term \( H_4 \) is simply the normal-ordered form of \( H_{int} \):

\[
H_4 = : H_{int} : . \tag{20}
\]

The color confining feature of the Hamiltonian will be discussed in the next Section.

To solve the gap equation on needs to specify the interactions \( V_C \) and \( D_T \). For the confining Coulomb term \( V_C \), we use two analytical forms, that we name Model 1 and Model 2, to assess the sensitivity of results with respect to \( V_C \). Model 1 is a parametrization of the lattice Coulomb gauge of Ref. [55]:

\[
V_C(k) = \frac{8\pi \sigma_{\text{Coul}}}{k^4} + \frac{4\pi C}{k^2}, \tag{21}
\]

with \( \sigma_{\text{Coul}} = (552 \text{ MeV})^2 \) and \( C = 6 \). Model 2 for \( V_C \) was used in recent studies of glueballs [56] and heavy hybrid quarkonia [57]; it is written as

\[
V_C(k) = V_l(k) + V_s(k), \tag{22}
\]
where

\[ V_t(k) = \frac{8\pi\sigma}{k^4}, \quad V_s(k) = \frac{4\pi\alpha(k)}{k^2}, \quad (23) \]

with

\[ \alpha(k) = \frac{4\pi Z}{\beta^{3/2} \ln^{3/2} \left( c + k^2 / \Lambda_{QCD}^2 \right)} . \quad (24) \]

The parameters here are \( \Lambda_{QCD} = 250 \text{ MeV}, \ Z = 5.94, \ c = 40.68, \) and \( \beta = 121/12. \) For the transverse-gluon interaction \( D_T \) we use

\[ D_T(k) = -\frac{4\pi\alpha_T}{(k^2 + m^2) \ln^{1/2} (\tau + k^2 / m_g^2)}. \quad (25) \]

This choice is guided by previous studies of spin-hyperfine splittings of meson masses \([38]\) using an Hamiltonian as in the present work. Moreover, the Yukawa term multiplying the log term is used to conform lattice results that the gluon propagator in Coulomb gauge is finite in the infrared \([55, 58]\). Further ahead we will discuss the impact of this choice of infrared behavior on our numerical results. Parameters here are \( m_g = 550 \text{ MeV}, \ m = m_g/2, \ \tau = 1.05, \) and \( \alpha_T = 0.5. \) We use the same \( D_T \) for both models. We note that we could have used a log running similar to the one in \( V_c, \ Eq. (24), \) but results would not change in any significant way.

We have solved the gap equation in Eq. (13) by iteration. The angular integrals can be performed analytically, but special care must be taken with the strongly-peaked confining term \( 1/[k - q]^4 \) at \( q \approx k \) in the numerical integration over \( q. \) There is no actual divergence here: the terms \( M(q)/E(q) \) and \( M(k)/E(q) (q/k) \cdot \hat{q} \) in Eq. (14) cancel exactly when \( q = k, \) but this cancellation can be problematic in the numerical integration. This problem can be handled in different manners, like using a momentum mesh containing node and half-node points with e.g. \( k \) at nodes and \( q \) at half-nodes \([44]\), or introducing a mass parameter \( \mu_{IR} \) such \( k^4 \to (k^2 + \mu_{IR}^2)^2 \) in Eq. (21) and varying \( \mu_{IR} \) until results become independent of \( \mu_{IR} \) \([59]\). In the present paper we use a different method \([60]\); we add a convenient zero to the gap equation:

\[-\int \frac{dq}{(2\pi)^3} \frac{1}{|k - q|^4} \frac{M(k)}{E(k)} \frac{1}{k} \cdot (k - q), \quad (26)\]

and rewrite \( F^{(1)}(k, q) \) in Eq. (13) as

\[ F^{(1)}(k, q) \to F^{(1)}(k, q) - \frac{M(k)}{E(q)} \frac{1}{k} \cdot (k - q) \]

\[ = \frac{M(q)}{E(q)} - \frac{M(k)}{E(q)} \]

\[ - \left( \frac{q}{k} \frac{\hat{k} \cdot \hat{q}}{} - 1 \right) \frac{M(k)}{E(q)} - \frac{M(k)}{E(k)}, \quad (27) \]

so that angle-independent and angle-dependent terms vanish independently when \( q = k. \) This feature makes the numerical cancellation of the divergences very stable.

\[ \begin{align*}
\text{FIG. 1. Constituent quark mass } M(k) \text{ as a function of the momentum for different values of current-quark masses.}
\end{align*} \]

In Fig. 1 we present solutions \( M(k) \) of the gap equation for different values of current-quark masses. The dramatic effect of \( D_\chi \) mass-generation is seen clearly in the figure: for the fictitious limit of a zero current-quark mass (solid line), the mass function \( M(k) \) acquires a sizable value in the infrared, \( M(k = 0) \approx 250 \text{ MeV}. \) In the ultraviolet, the mass function runs logarithmically with \( k, \) in a manner dictated by the running of the microscopic interactions. Since the model interactions used here have a different ultraviolet running from the one dictated by perturbative QCD, the logarithm running of the quark-mass function must be different; the interactions here fall off faster than those in QCD. One consequence of this is that the momentum integral over the trace of the quark propagator, which gives the quark condensate, does not run and is ultraviolet finite in the present case; we obtain \( \langle -\langle \bar{\Psi} \Psi \rangle \rangle^{1/3} = 280 \text{ MeV.} \) For nonzero current-quark masses, Fig. 1 shows that the effect of mass generation diminishes as the value of \( m_f \) increases – here \( m_u = m_d = 10 \text{ MeV}, m_c = 150 \text{ MeV} \) in both models, and \( m_s = 950 \text{ MeV in Model 1 and } m_s = 600 \text{ MeV in Model 2}. \) On the logarithm scale, one
sees that the mass function varies substantially in the ultraviolet, from $k \simeq 3\Lambda_{QCD} = 750$ MeV onwards. It is worth noticing that although the momentum dependence of the mass function charm quark is much less dramatic than for light quarks, there is still a significant dressing effect, $M_c(0)/m_c \sim 1.5 - 3$, similarly to what is found with covariant Dyson-Schwinger equations [61–63].

Finally, we comment on the impact of the momentum dependence of $D_T(k)$ in the infrared on the mass function. In the complete absence of $D_T$, using only $V_{c\gamma}$ with the parameters above, $M(0)$ would be of the order of 100 MeV in the chiral limit. On the other hand, using a $D_T(k)$ that vanishes at $k = 0$, like in the form of a Gribov formula [64], would give $M(0) \sim 200$ MeV in the chiral limit. This is in line with the recent finding of Ref. [65], which studies $D_{XSB}$ in a framework based on a quark wave functional determined by the variational principle using an ansatz which goes beyond the BCS-type of wave functionals.

### III. BARYON-MESON INTERACTION

In this Section we set up a calculation scheme for deriving effective low-energy hadron-hadron interactions with the Hamiltonian discussed above. We seek a scheme that makes contact with traditional quark-model calculations and can be systematically improved with some computational effort. Since the early 80’s, the great majority of calculations of hadron-hadron scattering observables in the quark-model are based on methods to handle cluster dynamics adapted from nuclear or atomic physics – for reviews on methodology and fairly complete lists of references see Refs. [66–68]. Such methods require a microscopic Hamiltonian and hadron bound state wave functions given in terms of quark degrees of freedom. Within the context of the model discussed in the previous Section, the starting point is the microscopic Hamiltonian of Eq. (1), with quark field operators given in terms of the constituent quark mass function $M(k)$ obtained from the numerical solution of the gap equation in Eq. (13).

The sector of the Hamiltonian relevant for the elastic meson-baryon interaction can be written in a compact notation as

$$H_{q\bar{q}} = \varepsilon(\mu) \hat{q}_\mu \hat{q}_\mu + \varepsilon(\nu) \hat{\bar{q}}_\nu \hat{\bar{q}}_\nu + \frac{1}{2} V_{q\bar{q}}(\mu\nu; \sigma\rho) \hat{q}_\mu \hat{\bar{q}}_\nu \hat{q}_\sigma \hat{\bar{q}}_\rho + \frac{1}{2} V_{\bar{q}\bar{q}}(\mu\nu; \sigma\rho) \hat{\bar{q}}_\mu \hat{\bar{q}}_\nu \hat{\bar{q}}_\sigma \hat{\bar{q}}_\rho,$$  

(28)

where the indices $\mu, \nu, \rho$, and $\sigma$ denote collectively the quantum numbers (orbital, color, spin, flavor) of quarks and antiquarks. The first two terms in Eq. (28) are the quark and antiquark single-particle self-energies coming from Eq. (16), and $V_{q\bar{q}}$, $V_{\bar{q}\bar{q}}$, and $V_{\bar{q}\bar{q}}$ are respectively the quark-quark, quark-antiquark and antiquark-antiquark interactions from $H_q$ in Eq. (20). The one-baryon state in the BCS approximation can be written in the schematic notation as [47, 51, 52]

$$|a\rangle = B_a^\dagger |\Omega\rangle = \frac{1}{\sqrt{3}} \psi_{a1\mu1\nu1\rho1} \hat{q}_\mu \hat{q}_\nu \hat{q}_\rho |\Omega\rangle,$$  

(29)

with $\psi_{a1\mu1\nu1\rho1}$ being the Fock-space amplitude, with $a$ denoting the (orbital, spin, flavor) quantum numbers of the baryon. Likewise, the one-meson states is written as

$$|a\rangle = M_a^\dagger |\Omega\rangle = \phi_{a\mu\nu\sigma\rho\lambda} \hat{q}_\mu \hat{q}_\nu \hat{q}_\sigma \hat{q}_\rho \hat{q}_\lambda |\Omega\rangle,$$  

(30)

where $\phi_{a\mu\nu\sigma\rho\lambda}$ is the corresponding Fock-space amplitude, with $a$ representing the quantum numbers of the meson.

The Fock-space amplitudes $\psi$ and $\phi$ can be obtained by solving a Salpeter-type of equation [47, 51, 52].

Given a microscopic Hamiltonian $H_{q\bar{q}}$ and hadronic states $|a\rangle$ as in Eqs. (28)-(30), an effective low-energy meson-baryon $V_{MB}(ab, cd)$ potential for the process $\text{Meson}(a) + \text{Baryon}(b) \rightarrow \text{Meson}(c) + \text{Baryon}(d)$ can be written as [67]

$$V_{MB}(ab, cd) = -3 \phi_{a\mu\nu\sigma\rho\lambda} \psi_{b\mu\nu\sigma\rho\lambda} V_{q\bar{q}}(\mu\nu; \sigma\rho) \phi_{\bar{a}\mu\nu\sigma\rho\lambda} \psi_b$$

$$- 3 \phi_{a\mu\nu\sigma\rho\lambda} \psi_{b\mu\nu\sigma\rho\lambda} V_{\bar{q}\bar{q}}(\mu\nu; \sigma\rho) \phi_{\bar{a}\mu\nu\sigma\rho\lambda} \psi_b$$

$$- 6 \phi_{a\mu\nu\sigma\rho\lambda} \psi_{b\mu\nu\sigma\rho\lambda} V_{q\bar{q}}(\mu\nu; \sigma\rho) \phi_{\bar{a}\mu\nu\sigma\rho\lambda} \psi_b$$

$$- 6 \phi_{a\mu\nu\sigma\rho\lambda} \psi_{b\mu\nu\sigma\rho\lambda} V_{\bar{q}\bar{q}}(\mu\nu; \sigma\rho) \phi_{\bar{a}\mu\nu\sigma\rho\lambda} \psi_b,$$  

(31)

where $\phi_{a\nu\sigma\rho\lambda}, \psi_{b\nu\sigma\rho\lambda}, \ldots$ are the meson and baryon Fock-space amplitudes of Eqs. (29) and (30), and $V_{q\bar{q}}, V_{\bar{q}\bar{q}}$, and $V_{q\bar{q}}$ are respectively the quark-quark, quark-antiquark and antiquark-antiquark interactions in $H_{q\bar{q}}$ of Eq. (28). The expression in Eq. (31) is completely general, in that it is valid for any low-energy meson-baryon process for which the baryon and meson Fock-space states and $H_{q\bar{q}}$ are as given above. It can be iterated in a Lippmann-Schwinger equation to obtain scattering phase shifts and cross sections – for details, see Ref. [67]. There is, however, one difficulty: $V_{MB}$ involves multidimensional integrals over internal quark and antiquark moments of baryon and meson wave functions and products of Dirac spinors $u(k)$ and $\bar{v}(k)$ that depend on the quark mass function $M(k)$. Although this is not a major difficulty, an approximation can be made noting that the bound-state amplitudes $\psi$ and $\phi$ are expected to fall off very fast in momentum space for momenta larger than the inverse size of the hadron, and therefore only low-momentum quark and antiquark processes contribute in the multidimensional integrals. In view of this, and the fact that $M(k)$ changes considerably only at large momenta, a natural approximation scheme to simplify the calculations without sacrificing the low-energy content of the effective interaction, is to retain the first few terms in the low momentum expansion for the mass function $M(k)$:

$$M(k) = M + M'(0) k + \frac{1}{2} M''(0) k^2 + \cdots,$$  

(32)

where $M'(k) = dM(k)/dk$ and $M''(k) = d^2M(k)/dk^2$. In particular, retaining terms up to $O(k^2/M^2)$ in the expansion, it is not difficult to show that the Dirac spinors...
$u_s(k)$ and $v_s(k)$ in Eqs. (9) and (10) become

$$u_s(k) = \left(1 - \frac{k^2}{2M^2}\right) \chi_s,$$

$$v_s(k) = \left(\frac{2M}{k + \sigma}\right) \chi_s.$$  

(33)
(34)

Using these spinors in Eq. (3), the expressions one obtains for $V_{qq}, V_{qg},$ and $V_{qq}$ are very similar to those of the Fermi-Breit expansion of the OGE interaction [69].

There is, however, one important difference here: while for the OGE one has $V_C(k) = D_T(k) \approx 1/k^2$, in the present case $V_C(k)$ and $D_T(k)$ are different and represent very different physics; $V_C$ is a confining interaction and $D_T$ is a (static) transverse gluon interaction.

The evaluation of multidimensional integrals can be further simplified using the variational method of Refs. [47, 51] with Gaussian ansätze for the bound-state amplitudes $\psi$ and $\phi$, instead of solving numerically Salpeter-type of equations. Specifically:

$$\psi_P(k_1, k_2, k_3) = \delta(P - k_1 - k_2 - k_3)$$

$$\times \left(\frac{3}{\pi \alpha^2}\right)^{3/4} e^{-\sum_{i=1}^{3} (k_i - P/3)/\alpha^2}.$$  

(35)

$$\phi_P(k_1, k_2) = \delta(P - k_1 - k_2)$$

$$\times \left(\frac{3}{\pi \beta^2}\right)^{3/4} e^{-(M_1 k_1 - M_2 k_2)^2/8\beta^2},$$  

(36)

where $\alpha$ and $\beta$ are variational parameters, $P$ is the center-of-mass momentum of the hadrons and

$$M_1 = \frac{2M_q}{(q_1 + q_2)}; \quad M_2 = \frac{2M_q}{(q_1 + q_2)},$$  

(37)

with $M_q$ and $M_{\bar{q}}$ being the zero-momentum constituent-quark masses. The variational parameters are determined by minimizing the hadron masses:

$$M_a = \frac{\langle a | (H_2 + H_4) | a' \rangle}{\langle a | a' \rangle},$$  

(38)

where we left out the constant $\mathcal{E}$, defined in Eq. (12), which cancels in the hadron mass differences – see Table I.

In this work we consider the elastic scattering of the pseudoscalar mesons ($K^+, K^0$) and ($D^0, D^-$) off nucleons, with both the mesons and nucleons in their ground states. Using the Gaussian forms for the amplitudes $\psi$ and $\phi$, Eqs. (35) and (36), one obtains for the nucleon mass, $m_N$, and for the pseudoscalar meson mass, $m_P$, the following expressions:

$$M_N = 3 \left(\frac{3}{\pi \alpha^2}\right)^{3/2} \int dk \left[ \frac{k^2}{E_l(k)} + m_t \frac{M_l(k)}{E_l(k)} \right] e^{-3k^2/\alpha^2}$$

$$+ \left(\frac{3}{2\pi \alpha^2}\right)^{3/2} \int \frac{dk dq}{(2\pi)^2} \left[ \frac{2}{3} \left| F^{(2)}_{l, h}(k, q) + F^{(2)}_{h, l}(k, q) \right| \right]$$

$$+ \left(\frac{3}{2\pi \alpha^2}\right)^{3/2} \int \frac{dk dq}{(2\pi)^2} \left[ 4G^{(2)}_{l, h}(k, q) - \frac{(k - q)^2}{3M_l^2} e^{-(k - q)^2/2\alpha^2} \right] D_T(|k - q|) e^{-k^2/\alpha^2},$$  

(39)

$$M_P = \left(\frac{1}{\pi \beta^2}\right)^{3/2} \int dk \left[ \frac{k^2}{E_l(k)} + m_t \frac{M_l(k)}{E_l(k)} + \frac{k^2}{E_h(k)} + m_h \frac{M_l(h)}{E_h(k)} \right] e^{-k^2/\beta^2}$$

$$+ \left(\frac{1}{\pi \beta^2}\right)^{3/2} \int \frac{dk dq}{(2\pi)^2} \left[ \frac{2}{3} \left| F^{(2)}_{l, h}(k, q) + F^{(2)}_{h, l}(k, q) \right| + C_P e^{-(k - q)^2/2\beta^2} \right] D_T(|k - q|) e^{-k^2/\beta^2}$$

$$+ \frac{4}{3} \left(\frac{1}{\pi \beta^2}\right)^{3/2} \int \frac{dk dq}{(2\pi)^2} \left[ G^{(2)}_{l, h}(k, q) + G^{(2)}_{h, l}(k, q) + \frac{1}{2} \frac{(k - q)^2}{M_l M_h} e^{-(k - q)^2/2\beta^2} \right] D_T(|k - q|) e^{-k^2/\beta^2},$$  

(40)

where $F^{(2)}_{l, h}(k, q)$ and $G^{(2)}_{l, h}(k, q)$ are given in Eqs. (18) and (19) and the indices $l$ and $h$ refer to light and heavy flavors, $l = (u, d)$ and $h = (s, c)$. The matrix elements of the color matrices, $C_N = \langle [N]T^aT^b[N]\rangle$ and $C_P = \langle [P]T^a(-T^b)^T[P]\rangle$, in the terms proportional to the Coulomb potential $V_C$ – that come from $H_4$ – are written in a way to emphasize the color-confinement feature of the model [47]: for $k = q$, these terms are divergent, unless the color matrix elements are such that the corresponding expressions vanish. The terms $F^{(2)}_{l, h}(k, q)$ come from $H_2$ – which is diagonal in color. For baryon and meson color singlet states, the contributions from $H_2$
and \( H_4 \) cancel exactly for \( k = q \), since:
\[
C_N = \frac{\epsilon^{c_1 c_2 c_3 c'_1 c'_2}(T^a)^{c_1 c'_1}(T^a)^{c_2 c'_2}}{3!} = \frac{2}{3} \tag{41}
\]
\[C_P = \frac{\delta^{c_1 c_2} \delta^{c'_1 c'_2}}{3} (T^a)^{c_1 c'_1} (-T^a)^{c'_2 c_2} = \frac{4}{3}. \tag{42}\]

and
\[
\lim_{k \to q} F^{(2)}_{V_{1,0}}(k, q) = 1. \tag{43}\]

Such a cancellation of infrared divergences also plays an important role in context of a conjectured [70] new high-density phase of matter composed of confined but chirally symmetric hadrons [71, 72].

Next, we obtain an explicit expression for the effective meson-nucleon interaction \( V_{MB} \), given generically in Eq. (31). This effective interaction is generated by a quark-interchange mechanism. The use of Gaussian wave functions is very helpful for getting a \( V_{MB} \) in closed form [67]: it can be written as a sum of four contributions (see Eqs. (9) and (10) of Ref. [13]), each contribution corresponding to a quark-interchange diagram shown in Fig. 2:
\[
V_{MB} (p, p') = \frac{1}{2} \sum_{i=1}^{4} \omega_i \left[ V_i(p, p') + V_i(p', p) \right], \tag{44}\]

where \( p \) and \( p' \) are the initial and final center-of-mass momenta, and the \( V_i(p, p') \) are given by
\[
V_i(p, p') = \left[ \frac{3g}{(3 + 2g) \pi \alpha^2} \right] e^{-a_i p^2 - b_i p'^2 + c_i \cdot p p'}
\times \int \frac{dq}{(2\pi)^3} v(q) e^{-d_i q^2 + e_i \cdot q}, \tag{45}\]

where the \( a_i, b_i, c_i, d_i, \) and \( e_i \) involve the hadron sizes \( \alpha \) and \( \beta \) and the constituent quark masses \( M_u, M_s, \) and \( M_c, \) and \( q = (\alpha/\beta)^2. \) Since we are using the same Fock-space amplitudes \( \psi \) and \( \phi \) as those used in Ref. [13], the expressions for \( a_i, b_i, \cdots \) are the same as there. It is, however, important to note that the essential difference here as compared to Ref. [13] is that the constituent quark masses \( M_u, M_s, \) and \( M_c, \) the width parameters \( \alpha \) and \( \beta, \) and the meson-baryon interaction are all derived from the same microscopic quark-gluon Hamiltonian. Another very important difference here is \( v(q); \) while in Ref. [13] it comes from the OGE, here \( v(q) = V_C(q) \) for the spin-independent interaction and \( v(q) = 2q^2/(3M_1M') D_T(q), \) for the spin-spin interaction, with \( M' = M_1 \) for diagrams (a) and (c), and \( M' = M_0 \) for diagrams (b) and (d). The \( \omega_i \) are coefficients that come from the sum over quark color-spin-flavor indices and combinatorial factors, whose values for the \( \bar{D}N \) interaction are given in Table 3 of Ref. [13].

The corresponding coefficients for the \( KN \) reaction can be extracted from that table by identifying \( D^0 \) with \( K^+ \) and \( D^− \) with \( K^0. \) Note that \( V_{MB} (p, p') \) is symmetric under interchange of \( p \) and \( p' \) and therefore possess post-prior symmetry [67, 73], a feature that is not always satisfied when using composite wave functions that are not exact eigenstates of the microscopic Hamiltonian [74]. As mentioned earlier, the quark-interchange mechanism leads to an effective meson-baryon potential that is of very short range. It depends on the overlap of the hadron wave-functions and contributes mostly to \( s \)-waves. As shown in Ref. [23], in order to describe experimental data for the \( K^+N \) and \( K^0N \) reactions, light-meson exchange processes are required to account for medium- and long-ranged components of the force. Quark-interchange generated by OGE accounts only to roughly 50 % to the experimental \( s \)-wave phase shifts and vector (\( \omega \) and \( \rho \)) and scalar exchanges are crucial for the correct description of this wave as well as higher partial waves [23]. Moreover, in order to describe the correct isospin dependence of the data, it is essential to include the exchange of the scalar-isovector \( a_0 \) meson [23]. With these facts in mind, we follow Ref. [13] and include meson-exchanges in the \( \bar{D}N \) system. As in that reference, we parameterize correlated \( \pi \pi \) exchange in terms of a single \( \sigma \)-meson exchange — this is not a bad approximation for the \( I = 0 \) channel, but for \( I = 1 \) channel it underestimates the total strength by 50% [13]. In Appendix A we present the effective Lagrangians densities and corresponding one-meson-exchange amplitudes.

IV. NUMERICAL RESULTS FOR PHASE SHIFTS AND CROSS SECTIONS

As mentioned previously, not much is known experimentally about the \( \bar{D}N \) interaction at low energies. In view of this and in order to have a comparison standard, we also consider within the same model, without changing any parameters besides the current quark masses, the \( K^+N \) and \( K^0N \) elastic processes, for which there are experimental data. It is important to reiterate that the effective short-range meson-baryon potential derived from quark-interchange driven by the microscopic inte-
interactions of Model 1 or Model 2 are very constrained, in the sense that they are determined by the microscopic interactions $V_C$ and $D_T$ via a chain of intermediate results for hadron properties driven by $D_{SB}$. That is, the microscopic interactions $V_C$ and $D_T$ determine the constituent quark masses $M_l = (M_u, M_d)$ and $M_h = (M_s, M_c)$, and these quark masses together with $V_C$ and $D_T$ determine the hadron wave functions. The effective meson-baryon interaction is determined by all these ingredients simultaneously, since it depends explicitly on the hadron wave functions, the quark masses and $V_C$ and $D_T$. Such an interdependent chain of results determining the effective meson-baryon interaction is absent in the NRQM with OGE, where quarks masses are independent of the microscopic quark-antiquark Hamiltonian.

Having determined the constituent quarks masses $M_l = (M_u, M_d)$ and $M_h = (M_s, M_c)$, the next step is the determination of the variational parameters $\alpha$ and $\beta$ of the $\psi$ and $\phi$ wave functions. Minimization of $M_N$ and $M_P$ with respect to $\alpha$ and $\beta$ leads to the results shown in Table I; also shown are the parameters used with the OGE model in Refs. [13, 23].

As expected, because the charm quark is heavier than the strange quark, the charmed $D$ mesons are smaller objects than the kaons - recall that the meson root mean square radii are inversely proportional to $\beta$. One also sees that hadron sizes of Models 1 and 2 are smaller than those used in the OGE model. Although the sizes of the wave functions have an influence on the degree of overlap of the colliding hadrons, the microscopic interactions $V_C$ and $D_T$ also play a role in the effective meson-baryon potentials. As we shall discuss shortly ahead, there is an interesting interplay between these two effects. For a recent discussion on the influence of hadron sizes on the quark-interchange mechanism, see Ref [75]. Regarding the hadron masses, one sees a discrepancy of 25\% (Model 1) and 50\% (Model 2) on the kaon-nucleon mass difference; this does not come as a surprise in view of the pseudo-Goldstone boson nature of the kaon, in that a BCS variational form for the kaon wave function is a poor substitute for the full Salpeter amplitude [51, 52].

On the other hand, the $DN$ mass difference is within 10\% of the experimental value in both Model 1 and Model 2, from the quark-interchange mechanism and one-meson exchanges, using the method discussed in Section 2.4 of Ref [76]. The importance of going beyond quark-Born diagrams by iterating the quark-interchange potentials in a scattering equation has been stressed previously [13, 23, 68]. For the specific case of the OGE quark-interchange in $DN$ scattering, unitarization of the scattering amplitude by iteration in a Lippmann-Schwinger equation leads to a decrease of the cross-section at low energies by a factor of three as compared to the nonunitarized quark-Born amplitude [13].

![Figure 3](image-url)

**FIG. 3.** $KN$ s-wave phase shifts for isospin $I = 0$ (upper panel) and $I = 1$ (lower panel) channels from the quark-interchange meson-baryon potential driven by the microscopic interactions of Model 1 (solid lines) and Model 2 (dashed lines). The curve with diamond symbols are the results from the phase shift analysis from the George Washington Data Analysis Center (DAC), Ref [77].

We are now in position to discuss numerical results for scattering observables. We solve numerically the Lippmann-Schwinger equation for the potentials derived

| TABLE I. Variational size parameters of the hadron amplitudes and hadron mass differences. All values are in MeV. |
|---------------------------------------------------------------|
| Experiment | $\alpha$ | $\beta_K$ | $\beta_D$ | $\Delta M_{NK}$ | $\Delta M_{DN}$ | $\Delta M_{DK}$ |
|------------|---------|---------|---------|----------------|----------------|----------------|
| Model 1    | 568     | 425     | 508     | 350            | 990            | 1345           |
| Model 2    | 484     | 364     | 423     | 205            | 1010           | 1220           |
| OGE Refs.  | 400     | 350     | 383.5   |                |                |                |

In Fig. 3 we present results for the s-wave phase shifts for the isospin $I = 0$ and $I = 1$ of the $K^+N$ and $K^0N$ reactions. Here we show the results derived from the baryon-meson potential obtained from the quark-
interchange mechanism driven by the microscopic interactions of Model 1 and Model 2. Also shown in the figure are results from phase shift analysis from the George Washington Data Analysis Center (DAC) [77]. As in the case of the model with OGE [23], both Model 1 and Model 2 reproduce the experimental fact that the $s$–wave phase shifts for $I = 0$ are much smaller than for $I = 1$. This is due to the combined effects that for $I = 0$, the confining interaction $V_C$ does not contribute at all and $D_T$ contributes only via diagrams c and d of Fig. 2.

Also seen in Fig. 3 is the fact that both Model 1 and Model 2 provide large repulsion – the confining interaction $V_C$ of Model 2 provides a little more repulsion than the $V_C$ from Model 1. As in the case of OGE, meson-exchanges can be added to obtain a fair description of the data, as we discuss next.

In Fig. 4 we present results for the phase shifts when $(\sigma, \omega, \rho$, and $a_0)$ one-meson exchanges are added to the quark-interchange potential. The input parameters for the meson-exchange potential are cutoff masses in form factors and coupling constants – we take the values used in Refs. [13, 23], with the exception of the $\sigma$ and $a_0$ couplings, the product $g_{\sigma MM}g_{\sigma BB}$ is increased by four and the product $g_{a_0 MM}g_{a_0 BB}$ is increased by three for a reasonable description of the phases - this is because the quark-interchange potential gives strong repulsion. Once the $K^+N$ and $K^0N$ phase shifts are fitted, we use the same values of the cutoff masses and couplings to make predictions for the $\bar{D}N$ system, which we discuss next.

In Fig. 5 we present the $s$–wave phase shifts for isospin $I = 0$ and $I = 1$ states of the $\bar{D}^0N$ and $\bar{D}^-N$ reactions. Results are obtained with a meson-baryon potential from quark-interchange driven by the interactions of Model 1 and Model 2. Like in the similar $KN$ system, the phases for the $I = 1$ channel are much bigger than those for the $I = 0$ channel. Also, one sees that Model 2 gives a stronger repulsion.

Adding $(\sigma, \omega, \rho$, and $a_0)$ one-meson exchanges to the quark-interchange potential leads to the results shown
in Fig. 6. Parameters of the meson-exchange potentials are the same used for the KN. The predictions for the $s$–wave phase shifts for $\bar{D}N$ system are qualitatively similar to the results for the $KN$ system, but are roughly a factor of two larger than the latter ones.

The quark-interchange meson-baryon potential leads to much smaller phase shifts for the higher partial waves. Like in the $KN$ system, meson exchanges play a much more important role in these waves. Although not shown here, many important features of the meson-exchanges discussed in Ref. [13], as the interference pattern between $\rho$ and $\omega$ contributions – destructive for $I = 0$ and constructive for $I = 1$ – are seen in the present model as well. We do not show these detailed results here, as they concern the meson exchange part of the full interaction, but present in Fig. 7 the final predictions for $\bar{D}N$ cross sections.

The results from both confining models are qualitatively similar, although the results from Model 1 for the $I = 1$ state are on average larger by a factor roughly equal to two. It is important to note that in the present paper we are approximating the correlated two-pion exchange contribution by single $\sigma$-meson exchange. As seen in Ref. [13], this seems to be a reasonable approximation for the $I = 0$ channel, but underestimates the $I = 1$ cross section by a factor roughly equal to two. We expect a similar feature in the present case. As a final remark, we note that the results for the cross sections are of the same order of magnitude as those from OGE quark-interchange, but the shapes of the curves for the $I = 1$ cross sections from Model 1 and Model 2 are different at very low energies from those from OGE. This last feature can possibly be attributed to the drastically different momentum dependences of $V_C(k)$ and $V_T(k)$ from the $1/k^2$ dependence of the OGE.
V. CONCLUSIONS AND PERSPECTIVES

There is great contemporary interest in studying the interaction of charmed hadrons with ordinary hadrons and nuclei. Of particular interest is the study of $D$ mesons in medium, mainly in connection with the possibility of creating new exotic nuclear bound states by nuclei capturing charmonia states like $J/\Psi$ and $\eta_c$ [2–5], or $D$ and $D^*$ mesons [6–8]. One major difficulty here is the complete lack of experimental information on the free-space interactions that would be of great help to guide model building – this is not the case for the similar problems involving strange hadrons. In a situation of lack of experimental information, one way to proceed in model building is to use symmetry constraints, analogies with other similar processes, and the use of different degrees of freedom [13, 20–22]. With such a motivation, we have implemented a calculation scheme for deriving effective low-energy hadron-hadron interactions based on a phenomenological Hamiltonian inspired in the QCD Hamiltonian in Coulomb gauge. The model Hamiltonian, defined in Eqs. (1)-(7), confines color and realizes Dyson-Schwinger equation studies in QCD. The scheme makes contact with traditional constituent quark models [33], but it goes beyond such models: the constituent quark masses are derived from the very same interactions that lead to hadron bound states and hadron-hadron interactions, while in the traditional quark models, quark masses and quark-quark interactions are specified independently. Moreover, the model confines color, in that only color-singlet hadron states are of finite energy. These features of the model are essential for studies seeking signals of in-medium modifications of hadron properties.

The model Hamiltonian requires as input a Coulomb-like term $V_C$ and a transverse-gluon interaction $D_T$. We used two analytical forms for $V_C$: Model 1 is a parametrization of the lattice simulation of QCD in Coulomb gauge of Ref. [55] and Model 2 was used in recent studies of glueballs [56] and heavy hybrid quarkonia [57]. For $D_T$, we were guided by previous studies of spin-hyperfine splittings of meson masses [38]. Initially we obtained the constituent quark mass function $M_f(k)$ for different flavors $f$. Next, we derived an effective low-energy meson-baryon interaction from a quark-interchange mechanism, whose input is the microscopic Hamiltonian and hadron bound state wave functions given in terms of quark degrees of freedom [67, 68]. We derived explicit analytical expressions for the effective meson-baryon interaction by using a low-momentum expansion of the constituent quark mass function and variational hadron wave functions [47, 51]. Initially we used the short-ranged quark-interchange potential derived within the model and added one-meson-exchange potentials to fit experimental $s$-wave phase shifts of the $K^+N$ and $K^0N$ reactions. Next, without changing any parameters besides the strange to charm current quark masses, $m_s \to m_c$, we presented the predictions of the model for the $D^0N$ and $D^-N$ reactions. The results for the cross sections obtained with the present model are of the same order of magnitude as those from OGE quark-interchange, of the order of 5 mb for the $I = 1$ state and 10 mb for the $I = 0$ state, on average. However, the shapes of the curves for the $I = 1$ cross sections from Model 1 and Model 2 are different at very low energies from those from OGE – a feature we attribute to the drastically different momentum dependences of $V_C(k)$ and $V_T(k)$ from the $1/k^2$ dependence of the OGE.

The model can be improved in several directions. First of all, without sacrificing the ability of obtaining analytical expressions for the effective meson-baryon interaction, one can expand the the amplitudes $\psi$ and $\phi$ in a basis of several Gaussians and diagonalize the resulting Hamiltonian matrix together with a variational determination of the size parameters of the Gaussians. Another improvement, that certainly will be necessary for studying in-medium chiral symmetry restoration, is to use the full mass function $M_f(k)$ for the light $u$ and $d$ flavors, instead of the low-momentum expansion of Eq. (32). This is because at finite baryon density and/or temperature the mass function $M_f(k)$ for $f = (u, d)$ will loses strength in the infrared [45, 80–83] and the expansion in powers of $k^2/M^2$ evidently loses validity. However, this will have a slight impact on the numerics, since in the case multidimensional integrals for the determination of the hadron sizes and the effective meson-baryon interaction need to be performed numerically. The low-momentum expansion must also be abandoned in the calculation of hadron wave functions when the quark mass function $M_f(k)$ in the infrared is much smaller than the average momentum of the quark in the hadron. Such a situation can happen when $D_T(k)$ is suppressed in the infrared, as in the study of Ref. [65].

Finally, another subject that needs careful scrutiny is the use of SU(4) flavor symmetry, Eq. (A10), to fix the coupling constants in the effective meson Lagrangians. A recent estimation [84], using a framework in which quark propagators and hadron amplitudes are constrained by Dyson-Schwinger equation studies in QCD, finds that while SU(3)-flavor symmetry is accurate to 20%, SU(4) relations underestimate the $DD\rho$ coupling by a factor of 5. On the other hand, a study employing a $3^1P_0$ pair-creation model with nonrelativistic quark model hadron wave functions finds smaller SU(4) breakings [85].

Appendix A: Meson-exchange contributions

The meson-exchange contributions are obtained from the following effective Lagrangian densities [78, 79]:

\[
\mathcal{L}_{NNS}(x) = g_{NNN} \bar{\psi}(N)(x) \tau \cdot \phi(S)(x) \psi(N)(x), \quad (A1)
\]

\[
\mathcal{L}_{NNV}(x) = g_{NNV} \left[ \bar{\psi}(N)(x) \gamma^\mu \psi(N)(x) \tau \cdot \phi(V)(x) \right. \\
+ \left. \left( \frac{\kappa_N}{2M_N} \right) \bar{\psi}(N)(x) \sigma_{\mu\nu} \psi(N)(x) \tau \cdot \partial_\nu \phi(V)(x) \right], \quad (A2)
\]
\[ \mathcal{L}_{PPS}(x) = g_{PPS} \varphi^{(P)}(x) \phi^{(S)}(x) \varphi^{(P)}(x), \]  
\[ \mathcal{L}_{PPV}(x) = ig_{PPV} \left[ \varphi^{(P)}(x) \left( \partial_{\mu} \varphi^{(P)}(x) \right) \tau - \left( \partial_{\nu} \varphi^{(P)}(x) \right) \tau \varphi^{(P)}(x) \right] \phi_{\mu}^{(V)}(x). \]

In these, \( \psi^{(N)} \) denotes the nucleon doublet, \( \phi^{(P)}(x) \) the charmed and strange meson doublet, \( \phi_{\mu}^{(V)}(x) \) the isoscalar triplet of \( \rho \) mesons, and \( \tau \) are the Pauli matrices. The Lagrangians for the \( \sigma \) and \( \omega \) mesons are obtained taking \( \tau \to 1 \) in the expressions above and in addition \( \kappa_{V} = 0 \) for the case of \( \omega \).

The tree-level potentials derived from the above Lagrangians densities lead to the following expressions for the vector-meson exchanges \((\nu = \rho, \omega)\):

\[ V^{\nu}(\mathbf{p}, \mathbf{p}') = -\frac{g_{NNe} g_{PPe}}{2(2\pi)^3} \frac{\sqrt{4m_p m_{\nu}}}{\omega(\mathbf{p}')}(\mathbf{p}' + \mathbf{p})_{\mu} \Delta_{\nu}^{\mu}(q) \times \left[ A_{\nu}(ps, ps') + \left( \frac{\kappa_{e}}{2m_N} \right) B_{\nu}(ps, ps') \right], \]

and for the scalar-meson exchanges \((S = \sigma, \omega)\):

\[ V^{S}(\mathbf{p}', \mathbf{p}) = -\frac{g_{NNS} g_{PPS}}{2(2\pi)^3} \frac{\sqrt{4m_p m_{\omega}}}{\omega(\mathbf{p}')}\Delta_{S}(q)\bar{u}(\mathbf{p}', s')u(\mathbf{p}, s), \]

where \( m_N \) is the nucleon mass and \( \omega(q) = (q^2 + m^2)^{1/2} \) with \( m \) the meson masses, \( \Delta_{\nu}^{\mu}(q) \) and \( \Delta_{S}(q) \) are the vector-meson and scalar-meson propagators, \( u(\mathbf{p}, s) \) are the Dirac spinors of nucleons (same expression as in Eq. (9), with \( M(k) \) replaced by the nucleon mass \( m_N \)), and the quantities \( A_{\mu} \) and \( B_{\mu} \) are given by

\[ A_{\mu}(ps, ps') = \bar{u}(\mathbf{p}', s') \gamma_{\mu} u(\mathbf{p}, s), \]
\[ B_{\mu}(ps, ps') = \bar{u}(\mathbf{p}', s') i\gamma_{\mu} q_{\nu} u(\mathbf{p}, s). \]

To avoid divergences in the Lippmann-Schwinger equation, the meson-exchange potentials are regularized phenomenologically by monopole form factors at each vertex:

\[ F_{i}(q^2) = \left( \frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 + q^2} \right) \]

where \( q = p' - p \), \( m_i \) is the mass of the exchanged meson and \( \Lambda_i \) is a cutoff mass. The coupling constants are fixed by SU(4) symmetry as in Ref. [13]:

\[ g_{DD\omega} = g_{DD\omega} = g_{KK\rho} = g_{KK\omega} = \frac{g_{\pi\rho}}{2}. \]
