Exotic circuit elements from zero-modes in hybrid superconductor–quantum-Hall systems

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The fractional quantum Hall effect and superconductivity, remarkable phenomena in their own right, can harbour even more exotic physics at their interface. In particular, coupling quantum Hall edges with a superconductor can create emergent excitations known as non-Abelian anyons that trap widely coveted Majorana fermion zero-modes and generalizations thereof. We uncover non-local transport signatures of these zero-modes that not only provide striking experimental signatures of the anyons, but moreover allow one to construct novel circuit elements, including superconducting current and voltage mirrors, fractional charge transistors and flux-based capacitors. Underlying this unusual transport is a phenomenon that we term ‘perfect Andreev conversion’—whereby quasiparticles propagating chirally at the edge reverse their electric charge as a result of hybridization with the zero-modes. Our findings suggest numerous experimental directions in the study of quantum-Hall–superconductor systems hybrids and highlight a fundamentally new application of non-Abelian anyons.

Non-Abelian anyons provide a fascinating illustration of Anderson’s ‘more is different’ paradigm. These quasiparticles, which emerge from interacting collections of ordinary bosons and fermions, produce a ground-state degeneracy that scales exponentially with the number of anyons present in the host system. Moreover, braiding the anyons around one another non-commutatively rotates the system’s quantum state within this degenerate manifold. These remarkable properties have led to great interest in non-Abelian anyons for use in fault-tolerant quantum information processing devices1,7. Our goal here is to propose an entirely new application of non-Abelian anyons (as well as another route to their detection), namely the construction of unusual circuit elements such as transistors for fractional charge, current/voltage mirrors and flux-based capacitors.

As a primer, let us first consider a one-dimensional (1D) topological superconductor6, obtained when an odd-channel wire acquires a bulk pairing gap. Suppose that a 1D superconductor breaks up into alternating topological and trivial domains as in Fig. 1a (for example, by varying the number of channels spatially along the wire). Here the endpoints of the topological regions realize ‘Ising’ non-Abelian anyons, which bind Majorana zero-modes that encode a two-fold ground-state degeneracy per topological segment. Physically, the degeneracy reflects the fact that each topological domain can switch its fermion parity without affecting the energy density—contrary to conventional superconductors. Numerous sources of 1D topological superconductivity have been proposed, which involve coupling a bulk superconductor to systems such as two-dimensional (2D) topological insulator edges6, spin–orbit-coupled nanowires3,5, magnetic-atom chains2, or counterpropagating sets of integer quantum Hall edge modes2,10 (for reviews see refs 11–14).

Among these platforms, the integer quantum Hall architecture most naturally generalizes to fractionalized set-ups that harbour richer phenomena stemming from the interplay between superconductivity and strong correlations. Consider, for instance, a ‘wire’ synthesized from counterpropagating fractional quantum Hall edge states separated by a narrow trench (see Fig. 1b for an example at filling ν = 2/3). This ‘wire’ can acquire a gap either through electron backscattering across the trench or via Cooper pairing. Because the edge states support fractionally charged excitations, the ends of pairing-gapped regions correspond to exotic non-Abelian anyons binding generalizations of Majorana zero-modes8,10,15. These parafermionic zero-modes16 encode a larger ground-state degeneracy than the usual Majorana case, since here each superconducting-gapped region can acquire fractional charge without changing its energy density. Similar effects may arise in other fractionalized set-ups17,18, including quantum Hall bilayers in which interlayer tunnelling plays the role of Cooper pairing19,20.

In this paper we predict novel non-local transport signatures of Majorana and parafermionic zero-modes in quantum-Hall–superconductor systems hybrids that, in turn, provide a foundation for the unusual circuit elements mentioned above. The experiments we propose relate closely to the ‘zero-bias anomaly’ arising when a single-channel normal lead probes a Majorana zero-mode in a 1D topological superconductor24–26. In such a set-up—sketched in Fig. 2a—the Majorana mode is predicted to mediate perfect Andreev reflection as temperature T and bias voltage V approach zero. That is, in this asymptotic limit an incoming electron from the normal lead reflects off the topological superconductor as a hole with unit probability, yielding a quantized zero-bias conductance of 2e2/h. Importantly, this is twice the conductance of an ideal single-channel wire, with the factor of two arising because of the added contribution of the reflected hole.

We show that quantum-Hall–superconductor systems hybrids yield an interesting variation on this transport anomaly, in particular when the native edge states serve as leads that probe zero-modes generated in such set-ups. The basic transport architecture is shown in Fig. 2b and contains two new features compared to the

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1D topological superconductor problem. First, in the fractional quantum Hall case the (parafermionic) zero-mode at the outer trench edge mediates perfect conversion of incoming quasiparticles carrying fractional charge $e^\alpha$ into $-e^\gamma$ quasiholes as $T, V \to 0$, thereby transmitting charge $2e^\alpha$ into the superconductor. Such events are possible since the pairing-gapped trench can resonantly absorb fractional charge owing to the ground-state degeneracy. (Ref. 23 briefly explored an analogous transport phenomenon in a bilayer set-up.) Second—and more importantly—if the (ungapped) quantum Hall edge used as a lead supports purely chiral charge excitations, the outgoing quasihole continues in the same direction around the edge, as Fig. 2b illustrates. We refer to this process as Andreev conversion to distinguish it from standard Andreev reflection, in which the hole retraces the incoming particle’s path. (One can view Andreev conversion as a chiral analogue of crossed Andreev reflection\(^\text{[15,35]}\)). The superconductor and quantum Hall edges thus form a trijunction in which the current and voltage in each leg exhibits a strong dynamical constraint. When multiple superconducting trenches are immersed in the same quantum Hall fluid, this dynamical constraint underlies various non-local transport anomalies. In what follows we theoretically establish the perfect Andreev conversion noted above under rather general circumstances and then discuss several novel circuit elements that follow.

**Perfect Andreev conversion**

One can access the transport phenomena we describe using many different quantum Hall phases. The conceptually simplest corresponds to a $v = 1$ integer quantum Hall state, although coupling to superconductivity in this case seems non-trivial owing to spin polarization. Alternatively, the $v = 1$ edge mode can arise from the magnetically gapped surface of a 3D topological insulator\(^\text{[36,37]}\). In this realization one can use ordinary $s$-wave superconductors; moreover, orbital magnetic fields are not required. Quantum Hall phases with multiple edge channels also suffice. We will simply require that the edge supports a single, chiral charge mode that at low energies decouples from all other neutral modes. This decoupling occurs, for example, for hierarchical states at $v = n/(2np + 1)$ ($n$ and $p$ are integers) due to disorder, as shown by Kane and Fisher\(^\text{[38]}\). The spin-unpolarized $v = 2/3$ state is particularly advantageous, in part because here too one can induce pairing via an $s$-wave superconductor. Another virtue is that the instabilities leading to the zero-modes occur at weak coupling for a range of realistic parameters. We expound on this important technical point in the Supplementary Information, which explores the $v = 2/3$ case in greater detail.

Suppose that we etch a trench into a quantum Hall system and then fill the void with a superconductor, as shown in Fig. 2b. Provided the trench width does not exceed the superconductor’s coherence length, the proximity effect can then gap out the adjacent edge modes through Cooper pairing. As noted earlier, one thereby creates zero-modes at the ends of the trench that encode a ground-state degeneracy for the intervening pairing-gapped region. We will now explore transport phenomena resulting when the gapless chiral charge mode impinging on the trench hybridizes with one of these zero-modes.

The key physical mechanism is that at low energies the gapped trench imposes certain boundary conditions that relate incoming and outgoing quasiparticles from the adjacent gapless edge. Consider first the limit in which the gapless edge state completely decouples from the superconducting trench. Concretely, one could envisage adding near the boundary a small tunnelling-gapped region, as in Fig. 1b, to block coupling to the zero-mode. In this case incoming quasiparticles continue along the edge uninterrupted by the trench, and the action governing the charge mode of interest is simply that of an unperturbed edge\(^\text{[39]}\):

$$S_{\text{charge}} = -\frac{1}{4\pi\tau} \int dx \, dt \, \partial_t \phi (\partial_t + v_\alpha \partial_x) \phi$$

with $\phi$ a field that determines the total edge charge density through $\rho = e\partial_t \phi / (2\pi)$. Commutation relations implicit in the action imply that $e^{i\phi}$ is an operator that adds charge $e^\alpha = ve$. Throughout we assume that this charge mode decouples from all neutral modes—should any exist. Equation (1) describes a fixed point for the edge at which particles undergo ‘perfect normal transmission’ on hitting the trench.

Next we incorporate weak coupling to the zero-mode nearest to the edge, which allows charge $e^\alpha$ to pass between the gapless edge and the pairing-gapped region. Such processes perturb the above fixed-point action with a term

$$\delta S_{\text{zero-mode}} = \int_{\partial L} \left[ e^{i\phi} e^{-i\phi_{\text{tun}}(x = \ell)} + \text{H.c.} \right]$$

where $x_0$ is the position where the trench and gapless charge mode intersect, $\Gamma$ denotes the bare coupling strength, and $e^{i\phi_{\text{tun}}}$ is an operator that cycles the charge on the pairing-gapped region of the trench by $e^{i\phi}$ (mod $2e$). Under renormalization $\Gamma$ flows according to $\partial_t \Gamma = (1 - \Delta) \Gamma$, where $\ell$ is a logarithmic rescaling factor and $\Delta = v/2$ is the scaling dimension of the tunnelling operator in equation (2). For $v < 2$—to which we specialize hereafter—hybridization with the zero-mode thus constitutes a relevant perturbation that destabilizes the perfect normal transmission fixed point.

The system then flows to a different fixed point at which coupling to the zero-mode imposes non-trivial boundary conditions on the gapless charge mode at $x = x_0$. This boundary condition can be expressed as

$$\phi_{\text{out}} = 2\phi - \phi_{\text{in}}$$

with $\phi_{\text{out/in}} = \phi(x = x_0 \pm \ell^a)$ denoting gapless charge-sector fields evaluated just above and below the trench. Equation (3) causes incoming charge-$e^\alpha$ quasiparticles ($e^{i\phi}$) added relative to the superconductor’s potential to continue as outgoing $-e^\alpha$ quasiholes ($e^{-i\phi}$), with the pairing-gapped trench absorbing the deficit charge $2e^\alpha (e^{i\phi})$. This is precisely the Andreev conversion process described earlier. There is, however, a competing effect, whereby electrons
The potential on the upper quantum Hall edge, where the outgoing Andreev-converted carriers flow, follows from current conservation. More precisely, since the same current \( I = GV \) passing through the superconductor traverses the quantum Hall fluid, the current must additionally satisfy \( I = \sigma_H V_{\text{H}} \), with \( V_{\text{H}} \) the Hall voltage. Consistency requires that the Hall voltage, like the conductance, is also doubled compared to the case where both source and drain are normal electrodes. That is, the potential on the upper edge is \(-V\), opposite that of the lower edge. This implies that the Andreev-converted carriers do not equilibrate with the superconductor; rather equilibration occurs at the normal electrode in Fig. 2b.

Consider next the more general situation where the potential for the superconductor is \( V_{\text{SC}} \) and that of the incoming/outgoing quantum Hall edges is \( V_{\text{in/out}} \). Owing to the doubled conductance the current flowing out of the superconductor is \( I = 2 \sigma_H (V_{\text{in}} - V_{\text{SC}}) \), whereas the (same) current crossing the Hall fluid obeys \( I = \sigma_H (V_{\text{in}} - V_{\text{out}}) \). It follows that

\[
V_{\text{out}} = 2V_{\text{SC}} - V_{\text{in}} \tag{5}
\]

which we will frequently employ in the forthcoming discussion. These results apply when temperature and the superconductor/quantum Hall voltage differences are small compared to the zero-mode hybridization energy \( T^* \) (recall equation (2)) and sufficiently small that the irrelevant electron tunnelling terms (equation (4)) remain inoperative; this is the regime where perfect Andreev conversion holds.

Temperature and other non-idealities (for example, possible resistive channels between the edge state and the superconducting lead) will generically decrease the voltage drop across the contact relative to equation (5). One can extract finite-temperature effects from the scaling dimension of the electron tunnelling operator in equation (4), yielding a correction to the outgoing voltage of \( \Delta V \sim T^*/V_{\text{in}}^2 \). For the most experimentally promising fractional cases at \( \nu = 2/3 \) or \( \nu = 2/5 \), \( \Delta V \) decays fairly rapidly, respectively vanishing as \( T^* \) and \( T^8 \). A more detailed analysis of the contact, including, for example, the effects of disorder, would tell us the bounds within which we can expect this scaling limit to hold, and is left for future work. Here we simply note that we can model all such non-idealities with a single quality factor \( \eta \) defined such that

\[
V_{\text{out}} = V_{\text{in}} + \eta (V_{\text{SC}} - V_{\text{in}}) \tag{6}
\]

where \( \eta \) ranges between 0 (no contact) and 2 (ideal superconducting contact). We stress that, for a normal contact, \( \eta \) could never exceed 1 under reasonable physical conditions. The possibility of \( \eta > 1 \) arising from a stable Andreev conversion fixed point is a key result that we will now use to explore several exotic circuit elements made possible in this regime. To emphasize the unusual behaviour of these elements, we will assume the ideal limit \( \eta = 2 \), corresponding to equation (5), unless specified otherwise. Any real-world implementation should, however, expect \( \eta < 2 \).

**Transistor for fractional charge**

Figure 3 illustrates a simple example of non-local effects resulting when multiple equipotential superconducting trenches appear in the quantum Hall system. Unlike in Josephson junctions, their relative phases are inconsequential, as we assume that any notion of phase coherence is lost along the edges connecting the leads. Charge conservation nevertheless strongly restricts current flow in these set-ups. In Fig. 3a—where the number of superconductors is even—current can only flow from one superconductor to another if it flows in all four simultaneously, with relative orientations specified in the figure. Sending current from the left to bottom...
supercconductors, for instance, necessarily yields the same current flow from right to top, a behaviour reminiscent of the non-local transport mediated by excitons in bilayer quantum Hall systems\textsuperscript{40}. The restricted current arises because quasiparticles undergo perfect Andreev conversion at each trench and a given superconductor receives the same charge under subsequent ‘round trips’ along the edge. By contrast, with an odd number of equipotential superconductors, as in Fig. 3b, (direct) current can simply not flow. This becomes evident on tracing the path of a single quasiparticle around the edge. On the first round trip, the quasiparticle deposits charge on each superconductor owing to Andreev conversion, but on the next pass removes these same charges because the number of superconductors is odd. Figure 3 illustrates the relevant quasiparticle processes in both cases. One can use this even/odd effect to create a transistor for fractional charge by using gates to controllably isolate one of the superconductors from the rest of the system.

**Voltage/current mirror**

Suppose we have a device with four superconducting trenches (as in Fig. 3a) but now allow their potentials to vary. The constraints imposed by equation (5), together with the doubled conductance relating voltage to current, leave the system with only three remaining degrees of freedom. The first corresponds to current flow in the pattern shown by Fig. 3a. A second degree of freedom appears in Fig. 4a, where we ground the upper and lower superconductors while raising the voltage of the left superconductor (the control). The voltage of the remaining superconductor (the output) necessarily goes down, mirroring the change. As in Fig. 3a, the current carried away by the output is opposite that flowing into the control; excess current flows to ground via the top and bottom superconductors. We are left, then, with a device that reverses both the voltage and the current flow from input to output. One can access the final degree of freedom by changing the relative voltage of the two grounded superconductors. The response to such a change may be interpreted as a superposition of Fig. 4a with the same set-up rotated by 90°.

**Voltage/current swap**

In a system with three ideal equipotential superconducting trenches, we have already shown that no current can flow (recall Fig. 3b). However, suppose that—as in Fig. 4b—we ground only the top superconductor and set the current and voltage on the bottom (control) superconductor. The third superconductor functions as the output. In this case the constraints of equation (5) lead to the unusual result that the two independent information channels for a superconductor, current and voltage, are swapped at the output relative to the input. For instance, if the current flowing into the control terminal is $I_{in}$ then the voltage at the output is $I_{out}/(2\sigma_1)$;
likewise, if the control terminal voltage is $V$, the output current is $I_{\text{out}} = 2e\Phi V$. Excess current again flows to ground.

**Superconducting flux capacitor**

Consider next Fig. 4b in the limit where the right superconductor is also held to ground. In this case the input current $I_n$ must clearly vanish. Nevertheless, the voltage $V$ on the bottom control superconductor induces a current $2e\Phi V$ flowing from the top to the right superconducting contacts, even though the latter remain at the same potential. Note that this is consistent with Fig. 3b, since in the limit $V = 0$ all currents vanish. This effect persists for larger odd numbers of superconductors as well, in which case current of magnitude $2n e\Phi V$ flows in all grounded terminals, with a sign alternating from lead to lead.

Finally, let us examine the circuit in Fig. 5, where we use analogous physics to create a ‘flux capacitor’ in which a voltage stores magnetic flux. Here a pair of superconducting trenches connect into a loop on the right side, while a normal electrode on the left is held at potential $V$ relative to the superconductors by a battery. Once again using equation (5) to determine the voltages along the circuit, we find that current $I = 2e\Phi V$ flows around the superconducting loop, generating flux $\Phi = C_s V$. The ‘flux capacitance’ $C_s$ is determined by the loop’s inductance. We emphasize that, unlike an ordinary inductor, no current flows through the power source or resistor at steady state in the ideal case for $n = 2$ (for $n < 2$ the capacitor will ‘leak’). Rather, the applied voltage merely sets the level of circulating current in the superconducting loop. See equation (6).

**Discussion**

It is worth reiterating that the non-local transport anomalies in the devices proposed above originate from zero-modes bound to non-Abelian defects induced by the superconductors. Characterization of these circuit elements is therefore a natural first step in the pursuit of topological quantum computation with such systems. To this end, even the simplest circuit element shown in Fig. 2b demonstrates a striking qualitative effect that reveals the non-Abelian anyons. If a voltage is applied between the normal and superconducting leads of that system, a higher voltage drop can be measured transversely across the Hall bar, up to twice the applied voltage in the ideal case. This is very different from the case of two normal contacts, for which the voltage measured transversely can only be lower than the applied voltage. Technically, at finite voltage and/or temperature the presence of the superconductor can lead to enhanced conductance even without a zero-mode via normal, non-fractional Andreev processes (although the fixed-point value $G = 2e^2/h$ would require exquisite fine-tuning). At least at $V \neq 1$, noise measurements in the circuit of Fig. 2b can be used to distinguish these scenarios. With a zero-mode, nearly every quasiparticle moving along the edge past the superconductor is converted to an oppositely charged quasihole. Occasionally, a quasiparticle will make it past the superconductor unaffected, causing shot noise in the current/voltage relation dominated by carriers with charge $2e$ (for $V < 1$). This neatly distinguishes zero-mode-induced Andreev conversion from ordinary Andreev processes that do not involve any fractionally charged edge quasiparticles.

Another tantalizing application of the circuit elements introduced here is the construction of low-power logical circuits. These circuits would have an advantage in the control of low-temperature quantum information devices, as they would coexist within the same low-temperature environment. Depending on the implementation, it may also be possible to produce such logical circuits ‘on-chip’ with quantum information implementations already based on 2D-electron gas and/or superconducting elements. These applications extend to alternative set-ups as well. The topological-insulator-based interferometers proposed in refs 36,37 yield a related mechanism for perfect Andreev conversion—but only in the Majorana case—and thus should also form the backbone of non-trivial circuit elements. Quantum Hall bilayers (without superconductivity) provide another promising venue. Novel d.c. transformers were proposed in this context a decade ago and, given that a bilayer variant of Andreev conversion is already theoretically established, other interesting elements may also be possible.

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Author contributions

D.J.C. contributed the calculations and devised the circuit elements. The manuscript was written by D.J.C. and J.A., while additional reality checks were provided by J.A. and K.S.

Additional information

Supplementary information is available in the online version of the paper. Reprints and permissions information is available online at [www.nature.com/reprints](http://www.nature.com/reprints). Correspondence and requests for materials should be addressed to D.J.C.

Competing financial interests

The authors declare no competing financial interests.