Simulation and measurement of electromagnetic undulator prototype for production of THz radiation

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Abstract. This research focused on magnetic field simulation and measurement of an electromagnetic undulator prototype for generation of terahertz (THz) radiation from electron beam with energy of about 10 MeV at the PBP-CMU Electron Linac Laboratory of the Plasma and Beam Physics Research Facility. This undulator prototype has 6.5 periods with a period length of 64 mm. Its magnetic field was measured by using a Hall probe with a measuring system, which its position movement can be precisely controlled with automatic computer interface. The magnetic field of end-poles was compensated by adjusting the end-pole currents. It was found that at the main-pole current of 0.76 A, the optimal end-pole currents at the entrance and exit of the undulator were 0.27 A and 0.19 A, respectively. At these currents the undulator has the transverse good field region of 19 mm. For magnetic field simulation, the computer program RADIA was used to model the undulator prototype for the ideal design. This was done for comparing the magnetic fields from the simulation and the measurement. The magnetic field of the two cases were not the same because the constructed prototype has different gaps for different longitudinal positions. However, they could give the radiation in the THz regime, which were 97.6 μm for the simulation and 96.7 ± 0.1 μm for the measurement.

1. Introduction

Terahertz (THz) radiation is the electromagnetic wave, which covers the frequency between 100 GHz to 10 THz [1]. This radiation regime corresponds well to rotational and vibrational modes of many molecules. Furthermore, the radiation has different absorption efficiencies in materials with different densities, thus it can be utilized in non-destructive material detection via THz imaging technique.

One of interesting techniques to generate the THz radiation from accelerator-based light source is the THz undulator radiation. There are two types of the undulator magnet; permanent magnet and electromagnetic magnet. At the PBP-CMU Electron Linac Laboratory, the electromagnetic undulator was chosen due to its low-cost for construction and its magnetic field is conveniently adjusted by changing the excitation current of its coils. This undulator magnet will be installed downstream the electron beam accelerator system for producing THz radiation from electron beam with energy of about 10 MeV. This research focused on magnetic field simulation and measurement of an electromagnetic undulator prototype, which was designed by K. Thaijai-un [2]. The undulator prototype has 6.5 periods with 13 pairs of poles and 532 turns of coils per pole.
This research is separated in to two parts; measurement and simulation. In the measurement part, firstly, we measured the undulator gap size along the undulator axis \((z\text{-axis})\). Then, we measured the horizontal magnetic field \((B_x)\) at the central of the undulator gap \((x = 0, y = 0)\) along the undulator axis by giving the main-pole current of 0.76 A. Then, we varied the end-pole currents for the magnetic field compensation. Lastly, we measured \(B_x\) along the \(y\)-axis to find the good field region. In all magnetic field measurements, we used a Group 3 Hall effect probe [3], which was mounted on the translation stage and was controlled by the computer program. The Hall probe can move along the undulator axis with high precision. Figure 1 presents the top view of the undulator prototype and the magnetic field measurement set up. In simulation part, we used the software RADIA [4] to simulate and model the undulator prototype for ideal case. Then, we compared the results of the magnetic field distribution, the electron deflecting angle and the electron vertical displacement for the cases of measurement and simulation.

![Figure 1. Top view (a) and magnetic field measurement set up (b) of the undulator prototype.](image)

To calculate the electron deflecting angle and vertical displacement, we started with the horizontal magnetic field of the undulator, which is a function of sinusoidal field that can be written as \(B_x(z) = B_o \sin(k_o z)\), where \(k_o = 2\pi / \lambda_o\), which \(\lambda_o\) is the undulator period length. When an electron moves in the undulator magnetic field, we get the equation of motion as

\[
y'' + \frac{eB_o}{\gamma m_o \beta c} \sin(k_o z) = 0.
\]

The deflecting angle of the electron can be calculated by integrating equation (1) and we get

\[
y' = -\frac{e}{\gamma m_o \beta c} \int_{-\infty}^{\infty} B_o \sin(k_o z) dz = -\frac{e}{\gamma m_o \beta c} \int_{-\infty}^{\infty} B_o(z) dz.
\]

The vertical electron displacement can then be calculated by integrating equation (2) as

\[
y(z) = -\frac{e}{\gamma m_o \beta c} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} B_o \sin(k_o z) dz dz = -\frac{e}{\gamma m_o \beta c} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} B_o(z) dz dz.
\]

The radiation wavelength \(\lambda_{ph}\) of the fundamental mode can be calculated from

\[
\lambda_{ph} = \frac{\lambda_o}{2\gamma^2} \left(1 + \frac{K^2}{2}\right),
\]

where \(\gamma\) is the Lorentz factor and \(K\) is the undulator strength, which corresponds to

\[
K = \frac{eB_o}{m_o c k_o} = 0.0934 B_o[T] \lambda_o[mm].
\]
2. Magnetic field measurements

2.1. Measurement of undulator gap
The undulator prototype has thirteen pairs of poles that correspond to six and a half undulator periods. To check the quality of the undulator fabrication, we measured the dimensions of all components, especially, the pole gaps which affect significantly to the magnetic field intensity. The results of the undulator gap measurement were 10.4 ± 0.1 mm for the pole’s numbers 1, 3, 4, 5 and 7, 10.5 ± 0.1 mm for the pole’s numbers 2, 6, 8, 10, 12, and 13, and 10.6 ± 0.1 mm for the pole’s number 9 and 10. These data show that the constructed prototype has different gaps for different longitudinal positions. The different gap sizes generate different magnetic fields as shown in figure 2.

![Figure 2](image)

Figure 2. Deviation of magnetic field for different undulator gaps along the undulator magnet length.

2.2. Magnetic field measurement and end-pole compensation
For the magnetic field measurement, a power supply with a voltage of 50 V was used to provide the electric current to the undulator coils. Due to high resistance of the coils, the maximum current that we can achieve with this power supply is only 0.76 A. Therefore, we set the current of the main pole to be at 0.76 A and the current of the two end-poles was varied for magnetic field compensation along the undulator length. The measurement results of magnetic field along the undulator axis as well as the deflecting angle and the vertical displacement of 10-MeV electron for different end-coil currents are shown in figure 3. It can be seen that the electron is deflected with different angles along the undulator length. This leads to non-parallel vertical displacements along the magnet. Moreover, the electron does not exit the magnet at the vertical centre of the magnet. This can cause the electron to emit radiation with low intensity because the radiation does not have constructive interference.

To obtain more parallel electron trajectories, we varied the exit-pole currents with three values, which are 0.09, 0.19 and 0.27 A by keeping the entrance-pole current at 0.27 A. The results of this optimization are presented in figure 4. Among these three exit-pole currents, most parallel electron trajectory is the blue line, which is corresponding to the exit-pole current of 0.19 A. Therefore, the optimal main-pole current, entrance-pole current and exit-pole current for the measurement are 0.76, 0.27 and 0.19 A, respectively.
Figure 3. (a) The horizontal magnetic field, (b) the deflecting angle and (c) the vertical trajectory of 10-MeV electron along the undulator with a main-pole current of 0.76 A for different end-pole currents.

Figure 4. (a) The horizontal magnetic field, (b) the deflecting angle and (c) the vertical trajectory of 10-MeV electron along the undulator with a main-pole current of 0.76 A, an entrance-pole current of 0.27 A and exit-pole currents of 0.09, 0.19 and 0.27 A.
2.3. Transverse magnetic field measurement
We measured the transverse magnetic field along the y-axis at the center of the eighth pole \((x = 0, z = 338.25 \text{ mm})\) along the y-axis. During this measurement, the currents of main-pole, entrance-pole and exit-pole current were 0.76, 0.27 and 0.19 A, respectively. The measurement result in figure 5 shows that the percentage error of the magnetic field is less than 0.1 % for the good field region of 19 mm.

![Figure 5](image)

*Figure 5.* Horizontal magnetic field and magnetic field error along the vertical axis.

3. Simulations of undulator magnet
The RADIA software [4] was used to construct the 3D model (shown in figure 6 (a)) of the undulator prototype with ideal dimensions; ignoring the imperfections from the construction. Comparisons of the measured and the simulated horizontal magnetic field, the deflecting angle and the vertical displacement of the 10-MeV electron are shown in figure 6 (b) to figure 6 (d). These two cases are the optimal cases that we obtained from the measurement and simulation. However, the maximum magnetic field from the measurement does not fit with the simulation case because the undulator gaps are different as mentioned in section 2.1. This also affects to the deflecting angle and the vertical displacement of electron trajectory in the magnetic field.

![Figure 6](image)

*Figure 6.* (a) The 3D RADIA model of the undulator prototype, (b) The horizontal magnetic field, (c) the deflecting angle, and (d) the vertical trajectory of 10-MeV electron along the undulator magnet for the cases of simulation and measurement.
Finally, we determined the undulator period length, the maximum magnetic field, the maximum undulator parameter, and the expected radiation wavelength for both two cases and the results are shown in Table 1.

| Parameter                      | Measurement value | Simulation value |
|--------------------------------|-------------------|------------------|
| Undulator period length $\lambda_u$ | 64.02 ± 0.26 mm   | 64.00 mm         |
| Maximum magnetic field $B_0$      | 0.094 ± 0.001 T   | 0.097 T          |
| Maximum undulator parameter $K$   | 0.56 ± 0.09       | 0.58             |
| Expected radiation wavelength $\lambda_{ph}$ | 96.7 ± 0.1 μm   | 97.6 μm          |

4. Conclusion

The measurement and simulation of the undulator prototype were done. In the measurement, the optimal main-pole, entrance-pole and exit-pole currents were 0.76, 0.27 and 0.19 A, respectively. The good field region of the transverse magnetic field was 19 mm. From the measurement results, we found that the gaps between each pair of poles are not the same. This causes the difference of the maximum magnetic fields along the undulator axis. It is also the main reason of the magnetic field difference between the results from measurement and simulation. The maximum magnetic field obtained from measurement and simulation are 0.094 ± 0.001 T and 0.097 T, respectively. The expected radiation wavelengths of both cases are 96.7 ± 0.1 μm and 97.6 μm, which still lie in the THz regime.

Acknowledgments

This research work was partially supported by Chiang Mai University and the Thailand Center of Excellence in Physics (ThEP-61-EQP-CMU1). Furthermore, P. Kitisri would like to acknowledge the scholarship support by the Development and Promotion of Science and Technology Talents projects (DPST). This work would not be completed without the technical support from M.W. Rhodes, N. Kangrang and P. Wichaisirimongkol.

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