Circuit Theory Based on New Concepts and Its Application to Quantum Theory

10. Extended Telegrapher’s Equations Obtained by Using Riccati Differential Equation

Nobuo Nagai (Hokkaido University) and Takashi Yahagi (Signal Processing Technology Laboratory)

E-mail: nagai@es.hokudai.ac.jp, yahagi@risp.jp

Abstract  Physical phenomena showing both particle and wave natures at the same time exhibit resonance. In addition, resonance is a physical phenomenon occurring in the steady state. Transient and steady-state responses can be discriminated when transmission lines are used. We demonstrate that (1) extended telegrapher’s equations can be obtained using the Riccati differential equation, (2) lossless transmission lines called Pasteur, Tellegen, and bi-isotropic (BI) media can be characterized using the extended telegrapher’s equations, and (3) the resonance conditions can be determined for each of these media.

Keywords: extended telegrapher’s equation, Riccati differential equation, Pasteur medium, Tellegen medium, bi-isotropic medium, voltage and current, lossless circuit, resonance, steady-state, active and reactive powers

1. Introduction

Quanta such as photons and electrons are considered to have both particle and wave natures at the same time. In this lecture series, we consider that physical phenomena showing the nature of both particles and waves exhibit resonance. In the previous sessions, we demonstrated that resonance can be determined by using Maxwell’s equations and the Schrödinger equation as well as by using an LC ladder circuit.

Quanta such as electrons and protons are considered to have charges and spins. The scalar and vector potentials of Maxwell’s equations are considered to be closely related to charges and spins and are regarded as important physical quantities in conventional quantum mechanics. In addition, the Dirac equation is used as an extended Schrödinger equation that satisfies the theory of relativity.

As mentioned above, we consider, in this lecture series, that physical phenomena showing both particle and wave natures at the same time exhibit resonance. Because waves with the resonance frequency can reach all parts in a circuit, signals applied to the waves can be transmitted throughout the circuit. We aim to explain quantum theory using a theoretical system based on a transmission theory that treats important physical phenomena, e.g., those in which causality is satisfied in the steady state.

In Ref. [1], Pasteur and Tellegen media were obtained on the basis of extended Maxwell’s equations, with the aim of expanding the application of microwave circuits. Circuit elements obtained using these media are lossless. The method used to extend the equations was similar to that used to extend the Schrödinger equation to the Dirac equation in Ref. [2], [3] and made use of the Riccati differential equation [4].

In this session, we will obtain extended telegrapher’s equations using the Riccati differential equation, considering the above method of extension in the framework of circuit theory. The properties of the circuit elements obtained from the extended telegrapher’s equations will be examined from the viewpoint of circuit theory [5], [6]. The circuit-theory properties of the lossless telegrapher’s equations can be determined by obtaining a unit element expressed by a cascade matrix. We will use this method to obtain the cascade matrices of circuit elements expressed by the extended telegrapher’s equations and examine the circuit-theory properties of the circuit elements using the cascade matrices.

2. Extension of Partial Differential Equation Using Riccati Differential Equation

To examine the circuit-theory properties of spin-1/2 electrons, we should first discuss a circuit element obtained
Schrödinger equation is given by [2], [3]. However, here, we will give a broader discussion, that is, we will examine the properties of a circuit element, obtained using extending Maxwell’s equations.

In Ref. [1], Pasteur and Tellegen media were obtained on the basis of extended Maxwell’s equations to expand the application of microwave circuits. Circuit elements obtained using these media are lossless. When the extension method is applied to the lossless telegrapher’s equation, we obtain obtained using extending Maxwell’s equations.

When the extended Maxwell’s equations are given by Eq. (10.1), the Dirac equation obtained by extending the Schrödinger equation is given by [2], [3]

$$\frac{d}{dx} \begin{bmatrix} V(x) \\ I(x) \end{bmatrix} = \begin{bmatrix} 0 & L \\ C & 0 \end{bmatrix} \begin{bmatrix} V(x) \\ I(x) \end{bmatrix} + j \omega \begin{bmatrix} \xi \\ -\xi \end{bmatrix} \begin{bmatrix} V(x) \\ I(x) \end{bmatrix}$$

Here, \(\xi\) and \(\zeta\) are complex numbers, which were assumed to be complex conjugates in Ref. [1].

When the extended Maxwell’s equations are given by Eq. (10.1), the Dirac equation obtained by extending the Schrödinger equation is given by [2], [3]

$$\frac{d}{dx} \begin{bmatrix} V(x) \\ I(x) \end{bmatrix} = \begin{bmatrix} 0 & L \\ C & 0 \end{bmatrix} \begin{bmatrix} V(x) \\ I(x) \end{bmatrix} + j \omega \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} V(x) \\ I(x) \end{bmatrix}$$

The simultaneous differential equations given by Eqs. (10.1) and (10.2) are considered to be related to the Riccati differential equation described in Ref. [4]. In Ref. [5], the following argument that the Riccati differential equation is related to circuit theory was given.

In Reid’s work (Ref. [4] in this paper), the relation between the Riccati differential equation and the telegrapher’s equations was stated. Terms corresponding to the voltage and current waves were defined for the Riccati differential equation. Descriptions related to the telegrapher’s equations extracted from Reid’s work are given below.

The scalar Riccati differential equation \(R[w](x)\) is given by

$$R[w](x) = \frac{d}{dx} v(x) + [a(x) + d(x)]w(x) + b(x)v^2(x) - c(x) = 0$$

This is related to the following first-order homogeneous linear differential equations.

$$L_1[u, v](x) = -\frac{d}{dx} v(x) + c(x)u(x) - d(x)v(x) = 0$$

$$L_2[u, v](x) = \frac{d}{dx} u(x) - a(x)u(x) - b(x)v(x) = 0$$

For the scalar Riccati differential equation, the following theorems hold.

[Theorem 10.1]

Equation (10.3) has a solution \(v(x)\) only when a solution \([u(x), v(x)]\) of Eqs. (10.4a) and (10.4b) exists, assuming that \(u(x) \neq 0\) and \(w(x) = v(x)/u(x)\).

[Theorem 10.2]

If \(w_0(x)\) is a solution of Eq. (10.3), the following functions are defined.

$$g(x, x_0 | w_0) = \exp \{-\int_{x_0}^{x} [d(y) + w_0(y)b(y)]dy\}$$

$$h(x, x_0 | w_0) = \exp \{-\int_{x_0}^{x} [a(y) + h(y)w_0(y)]dy\}$$

$$f(x, x_0 | w_0) = \int_{x_0}^{x} g(y, x_0 | w_0)b(y)h(y, x_0 | w_0)dy$$

In this case, we define \(\xi\) as

$$\xi = w(x_0) - w_0(x_0)$$

\(w(x)\) is a solution of Eq. (10.3) only when

$$1 + \xi f(x, x_0 | w_0) \neq 0$$

and is given by

$$w(x) = w_0(x) + g(x, x_0 | w_0) h(x, x_0 | w_0) \frac{\xi}{1 + \xi f(x, x_0 | w_0)}$$

A demonstration of the above theorems can be found in Reid’s work and is omitted here.

According to Theorem 10.1 in Ref. [4], the Riccati differential equation is expressed by \(w(x) = v(x)/u(x)\), where \(u(x) \neq 0\). Therefore, when \(v(x)\) and \(u(x)\) are assumed to be the voltage and current, respectively, the solution \(w(x)\) corresponds to the impedance, implying a circuit-theory phenomenon. Thus, the Riccati differential equation is similar to electromagnetic wave equations such as the telegrapher’s equations. In physics, the motion and rotation (spin) of particles and electromagnetic waves have been focused on, but impedance and impedance matching, which are related to the connection of transmission lines of particles and electromagnetic waves at interfaces, have not been considered. When the extended Maxwell’s equations are regarded as the Riccati differential equation, the connection of transmission lines of particles and waves at interfaces may be discussed on the basis of circuit theory, for example, by considering impedance matching.

3. Method of Solving Riccati Differential Equation

The simultaneous differential equations given by Eqs. (10.1) and (10.2) are related to the Riccati differential equation. We assume a uniform transmission line by using Eq. (10.3). The coefficient is assumed to be a constant rather than a function of \(x\). With this assumption, using complex numbers \(a\) and \(d\), first-order linear differential equations that satisfy the Riccati differential equation can be obtained as the extended telegrapher’s equations given by

$$-\frac{d}{dx} V(x) = d \cdot V(x) + j \omega L \cdot I(x)$$

$$-\frac{d}{dx} I(x) = j \omega C \cdot V(x) - a \cdot I(x)$$

These equations are a system of first-order linear differential equations and can be solved as an eigenvalue problem of a matrix. Using vectors and a matrix, Eqs. (10.8a) and (10.8b) can be rewritten as

$$\frac{d}{dx} \begin{bmatrix} V(x) \\ I(x) \end{bmatrix} + \begin{bmatrix} d & j \omega L \\ j \omega C & -a \end{bmatrix} \begin{bmatrix} V(x) \\ I(x) \end{bmatrix} = 0$$
This is a simultaneous linear differential equation. The eigenvalues and eigenvectors of the matrix in Eq. (10.9) are determined below. Namely, the determinant of the matrix is given by
\[
\det \begin{pmatrix} d - \gamma & j\omega L \\ j\omega C & -a - \gamma \end{pmatrix} = 0 \tag{10.10}
\]
This is expressed as
\[
\gamma^2 - (d - a)\gamma - (ad - \omega^2 LC) = 0 \tag{10.11}
\]
From Eq. (10.11), the eigenvalues are given by
\[
\gamma = \frac{(d - a) \pm \sqrt{(d - a)^2 + 4(ad - \omega^2 LC)}}{2} \tag{10.12}
\]
What do the two eigenvalues obtained above represent? This question can be answered by considering the solution of the telegrapher’s equations. To solve the telegrapher’s equations, the propagation constant should be determined. However, it cannot be easily determined from the above simultaneous equation. To obtain the propagation constant from the equation, the eigenvalues should be determined. Namely, the eigenvalues are the propagation constant.

Now that we have determined the eigenvalues of the matrix in Eq. (10.9), we next determine the eigenvectors corresponding to the eigenvalues. Then, what do the eigenvectors represent? Although they are rarely considered in physics, an important physical quantity that determines whether or not energy can be transmitted can be obtained from the eigenvectors, that is, the characteristic impedance of the transmission line. When the real part of the impedance is positive, the complex number corresponding to the voltage is the eigenvector for the current component of the eigenvectors should be 1 for the forward and backward waves, respectively, and satisfy Eq. (10.15) by referring to Ref. [1] and discuss the extended telegrapher’s equations in detail.

4. Pasteur Media

Unit elements can be devised using the lossless telegrapher’s equations. In this section, we examine whether the circuit elements obtained using the extended telegrapher’s equations have the same properties as those of unit elements and, if they have different properties, whether the properties are related to spins. As the first step, the circuit-theory properties of Pasteur media, described in Ref. [1], are clarified. Next, the connection of two identical Pasteur media is attempted.

4.1 Circuit-theory properties of Pasteur media

A transmission line in which \(a\) and \(d\) are real numbers and satisfy Eq. (10.15) was introduced in Ref. [1]. This line is referred to as a Pasteur medium. The equations for Pasteur media are given below.

The extended telegrapher’s equations for Pasteur media are given by
\[
\frac{d}{dx} \begin{pmatrix} V(x) \\ I(x) \end{pmatrix} + \begin{pmatrix} -\omega \kappa_s & j\omega L \\ j\omega C & \omega \kappa_s \end{pmatrix} \begin{pmatrix} V(x) \\ I(x) \end{pmatrix} = 0 \tag{10.16}
\]
Here, the signs of the coefficients are opposite those in Ref. [1].

Equation (10.16) is a simultaneous linear differential equation. The eigenvalues and eigenvectors of the matrix in this equation are determined as follows. Namely, the determinant of the matrix is given by
\[
\det \begin{pmatrix} -\omega \kappa_s - \gamma & j\omega L \\ j\omega C & \omega \kappa_s - \gamma \end{pmatrix} = 0 \tag{10.17}
\]
This is rewritten as
\[
\gamma^2 + (-\omega^2 \kappa_s^2 + \omega^2 LC) = 0 \tag{10.18}
\]
The eigenvalues can be obtained as the roots of Eq. (10.18). If the eigenvalue represents the propagation constant and the circuit is lossless, a purely imaginary phase constant is desired. Therefore, we express the eigenvalues as
\[
\gamma = \pm j\omega \sqrt{LC - \kappa_s^2} = \pm j\beta_p \tag{10.19}
\]
\(\beta_p\) in Eq. (10.19) is the phase constant and is different from the phase constant of the lossless telegrapher’s equations by \(\kappa_s^2\), as shown by Eq. (10.19). It can be
considered that this difference corresponds to the change in
the number of spins.

The eigenvector corresponding to the eigenvalue given by
\[ \gamma_1 = j\omega \sqrt{LC - \kappa_a^2} = j\beta_p \] (10.20a)
is expressed by
\[ \begin{pmatrix} \frac{\kappa_a}{C} + \sqrt{\frac{LC - \kappa_a^2}{C}} \\ \frac{1}{C} \end{pmatrix} = \begin{pmatrix} Z_{0f} \\ 1 \end{pmatrix} = \begin{pmatrix} R_p + jX_p \\ 1 \end{pmatrix} \] (10.20b)

The eigenvector corresponding to the eigenvalue given by
\[ \gamma_2 = -j\omega \sqrt{LC - \kappa_a^2} = -j\beta_p \] (10.21a)
is expressed by
\[ \begin{pmatrix} -j\frac{\kappa_a}{C} + \sqrt{\frac{LC - \kappa_a^2}{C}} \\ -1 \end{pmatrix} = \begin{pmatrix} Z_{0b} \\ -1 \end{pmatrix} = \begin{pmatrix} Z_{0f}^* \\ -1 \end{pmatrix} \] (10.21b)

The characteristic impedances of the forward and backward waves, \( Z_{0f} \) and \( Z_{0b} \), respectively, are complex conjugates.

Using the eigenvalues given by Eqs. (10.20a) and (10.21a) and the eigenvectors given by Eqs. (10.20b) and (10.21b), the simultaneous differential equations for Pasteur media given by Eq. (10.16) are expressed by
\[ V(x) = N_f Z_{0f} \exp(-j\beta_p x) + N_b Z_{0b} \exp(j\beta_p x) \] (10.22a)
\[ I(x) = N_f \exp(-j\beta_p x) - N_b \exp(j\beta_p x) \] (10.22b)

Here, \( N_f \) and \( N_b \) are integral constants.

From Eqs. (10.22a) and (10.22b), the cascade matrix of length \( l \) of Pasteur media in circuit theory is given by
\[
\begin{bmatrix}
A_p & B_p \\
C_p & D_p
\end{bmatrix}
= \begin{bmatrix}
1 & \frac{Z_{0f} e^{j\beta_p l} + Z_{0b} e^{-j\beta_p l}}{Z_{0f} + Z_{0b}} & Z_{0f} e^{j\beta_p l} - e^{-j\beta_p l} & \frac{1}{Z_{0f} + Z_{0b}} \left( R_p \cos \beta_p l - X_p \sin \beta_p l \right) & j \sin \beta_p l \left( R_p^2 + X_p^2 \right) \\
\frac{1}{R_p} & \frac{1}{j \sin \beta_p l} & \frac{R_p}{R_p} & R_p & R_p \cos \beta_p l + X_p \sin \beta_p l \\
\end{bmatrix}
\] (10.23)

This cascade matrix is that of a lossless reciprocal circuit element. In addition, the characteristic impedances of the forward and backward waves, \( Z_{0f} \) and \( Z_{0b} \), respectively, are complex conjugates. Namely, the impedance of the circuit viewed from the left end is \( Z_{0f} \), whereas that viewed from the right end is \( Z_{0b} \). Therefore, the circuit is considered to have a matrix equivalent to the cascade matrix expressed using the iterative parameters of an asymmetric LC ladder circuit. That is, changing the rotation (spin) of a unit element causes the input and output impedances to become
conjugate complex numbers, resulting in an asymmetric LC ladder circuit.

4.2 Connection of Pasteur media

Here, we attempt to connect Pasteur media. The cascade matrix of Pasteur media given by Eq. (10.23) is iterative. When two identical Pasteur media are iteratively connected, the cascade matrix of the iteratively connected media is given by
\[
\begin{bmatrix}
A_p & B_p \\
C_p & D_p
\end{bmatrix}
= \begin{bmatrix}
\frac{R_p}{R_p} & \frac{1}{R_p} & \frac{R_p}{R_p} & \frac{1}{R_p} \\
\frac{1}{R_p} & \frac{R_p}{R_p} & \frac{1}{R_p} & \frac{R_p}{R_p}
\end{bmatrix}
\] (10.24)

The phase of this matrix is double that of the iterative matrix given by Eq. (10.23) and the rotation (spin) remains unchanged.

In contrast, if the two Pasteur media are image-connected, as explained in Session 9, the phase of the image-connected circuit is different from that of the matrix given by Eq. (10.23). That is, the image connection of Pasteur media results in the change in the phase, i.e., the number of spins. Therefore, image connection is inappropriate for circuits for which the number of spins must be maintained, such as circuits related to spin 1/2 used in quantum theory.

5. Cascade Matrix of Tellegen Media

The Dirac equation given by Eq. (10.2) satisfies Eq. (10.15), whereas Tellegen media do not satisfy Eq. (10.15). Therefore, the Dirac equation should first be discussed from the viewpoint of extending the telegrapher’s equations. However, the Dirac equation is not described in Ref. [1] and is not considered to express a lossless circuit. Hence, we first discuss Tellegen media.

Because Tellegen media do not satisfy Eq. (10.15), we obtain
\[ a - \alpha \neq 0 \] (10.25)

The propagation constant given as the eigenvalue in Eq. (10.12) must also satisfy this condition. If \( d \) and \( a \) have real parts, the propagation constant also has a real part, which is the attenuation or amplification constant. Hence, such \( d \) and \( a \) are undesirable in the extension of lossless transmission lines. Therefore, \( d \) and \( a \) are assumed to have no real parts.

A transmission line generalized by considering the above conditions was shown in Ref. [1] and is referred to as a Tellegen medium. The equation for Tellegen media is given by
By replacing the unit element in Fig. 2.1 with the unit element was described in Session 2 and shown in Fig. 2.1, the internal resistance of the voltage source is \( R_G \). When \( Z_m \) given by Eq. (10.32) is equal to \( R_G \), impedance matching is achieved in the steady state and there is no reflection, indicating resonance. The resonance conditions are determined as follows. \( Z_m \) given by Eq. (10.32) becomes pure resistance given by a real number when \( \exp(j\beta_1 l) \) and \( \exp(-j\beta_2 l) \) are 1 or \(-1\) and the phase difference is \( 2\pi \) or \( \pi \), or when the arguments of the denominator and the numerator are equal. In these cases, the resistance is determined as follows.

(i) When \( \beta_1 l = 2\pi n \), \( \beta_2 l = 2\pi(n-1) \), and the phase difference is \( 2\pi \),

\[
R_G = R_0 \tag{10.33a}
\]

(ii) When the phase difference is \( \pi \),

\[
R_G = R_0^2 \tag{10.33a}
\]

(iii) When the arguments of the denominator and the numerator are equal,

\[
R_0 = R_G = R_0 \tag{10.33c}
\]

Because \( \beta_1 \) and \( \beta_2 \) are different, the phase difference should be predetermined to achieve resonance for the Tellegen media shown here. Tellegen media must have a length of \( n \) waves to achieve resonance, where \( n \) is considered to be quite large.

Next, we examine the resonance of an image-connected circuit element. Because Tellegen media are not reciprocal circuits, image parameters are first discussed.

We assume that the cascade matrix of the circuit element is given by

\[
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix}
\]

The inverse matrix of this cascade matrix is given by

\[
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix}^{-1} = \frac{1}{AD-BC} \begin{pmatrix}
D & -B \\
-C & A
\end{pmatrix}
\]

The image matrix of the cascade matrix is obtained by changing the negative nondiagonal elements of the inverse matrix to positive to give

\[
\frac{1}{AD-BC} \begin{pmatrix}
D & B \\
A & C
\end{pmatrix}
\]

Therefore, by connecting the image circuit given by Eq. (10.36) to the cascade matrix given by Eq. (10.34), we obtain a circuit with the cascade matrix \([\overline{F}]\) given by
The eigenvalue corresponding to the eigenvalue given by
\[ \gamma_{\beta_2} = j\omega \chi_a - j\omega \sqrt{LC - k_a^2} = -j\beta_{2b} \] (10.42a)
is expressed by
\[ Z_{\beta_2} = \begin{pmatrix} \sqrt{LC - k_a^2} + jk_a C \ 
1 \end{pmatrix} = \begin{pmatrix} R_p & jX_p 
1 \end{pmatrix} \] (10.42b)

Using these equations, \( V(x) \) and \( I(x) \) of the BI media expressed by Eq. (10.40) are given by
\[ V(x) = N_i Z_{\beta_2} \exp(-j\beta_{2b} x) + N_z Z_{\beta_2} \exp(j\beta_{2b} x) \] (10.43a)
\[ I(x) = N_i \exp(-j\beta_{2b} x) - N_z \exp(j\beta_{2b} x) \] (10.43b)

As a result, the cascade matrix of length \( l \) of the BI media is given by
\[ \frac{1}{Z_{\beta_2} + Z_{\beta_1}} \left[ \begin{array}{c} Z_{\beta_1}e^{j\beta_{1b}l} + Z_{\beta_2}e^{-j\beta_{2b}l} 
Z_{\beta_1}e^{-j\beta_{1b}l} \end{array} \right] = \begin{pmatrix} e^{j\beta_{1b}l} & e^{-j\beta_{2b}l} \end{pmatrix} \] (10.44)

Using the elements of this cascade matrix, the following equation can be obtained, similarly to Eq. (10.30).
\[ AD - BC = \exp\left[ j(\beta_{1b} - \beta_{2b})l \right] \] (10.45)

Therefore, BI media cannot be reciprocal circuits, similarly to Tellegen media. Note that BI media are lossless, similarly to Pasteur and Tellegen media.

To determine the resonance condition of BI media, we determine the input impedance \( Z_{\beta_1} \) given by Eq. (2.27b) in Session 2, similarly to the case of Tellegen media. \( Z_{\beta_1} \) is given by
\[ Z_{\beta_1} = \frac{R_p (Z_{\beta_1} e^{j\beta_{1b}l} + Z_{\beta_2} e^{-j\beta_{2b}l}) + Z_{\beta_2} e^{j\beta_{1b}l} e^{-j\beta_{2b}l})}{R_p (e^{j\beta_{1b}l} - e^{-j\beta_{2b}l}) + Z_{\beta_2} e^{j\beta_{1b}l} + Z_{\beta_1} e^{-j\beta_{2b}l}} \] (10.46)

When \( \exp(j\beta_{1b}l) \) and \( \exp(-j\beta_{2b}l) \) are 1 and the phase difference is \( 2\pi \), \( Z_{\beta_1} \) is pure resistance expressed by a real number. Therefore, the resonance condition is as follows.

When the phase difference is \( 2\pi \),
\[ R_{LC} = \frac{R_p}{Z_{\beta_1}} \] (10.47)

To satisfy the resonance condition of BI media, the phase difference should have a predetermined value because \( \beta_{1b} \) and \( \beta_{2b} \) are different, as in the case of Tellegen media. BI media must have a length of \( n \) waves to achieve resonance, where \( n \) is considered to be quite large.

Thus, we characterized the Pasteur, Tellegen, and BI media on the basis of extended telegrapher’s equations using the Riccati differential equation and discussed the resonance conditions required when these media are used as circuit elements. The Riccati differential equation is also used to obtain the Dirac equation [Eq. (10.2)]. We will describe the purpose of using the Riccati differential equation to extend the telegrapher’s equations in the following session, then discuss its effectiveness from the viewpoint of circuit theory.
8. Future Challenges Regarding Transmission and Resonance

The law of energy conservation in mechanics, a branch of physics, is based on the principle that the sum of kinetic and potential energies remains conserved. According to Ref. [8], scalar and vector potentials are important in Maxwell’s equations because the magnetic and electric energies are assumed to correspond to kinetic and potential energies, respectively. In contrast, Heaviside [9], who was a telegrapher, reformulated Maxwell’s equations, which is known as the reformation of Maxwell’s theory, thus providing a summary of the physical properties related to energy transmission in Maxwell’s equations. From the viewpoint of circuit theory, Maxwell’s equations in mechanics treat the response to transient phenomena focusing on the behavior of electrons, whereas Heaviside’s theory treats the response to resonance in the steady state in transmission problems.

In this lecture series, we consider that physical phenomena showing both particle and wave natures at the same time exhibit resonance, which may be closely related to the development of Heaviside’s transmission problems. In conventional quantum theory, however, resonance has not been considered a significant physical phenomenon.

In Session 8, we showed that voltage and current can be obtained from the Schrödinger equation, a basic equation in quantum mechanics, and that resonance can also be achieved. Therefore, the Schrödinger equation is considered to be an extension of Maxwell’s and telegrapher’s equations and to lead to the development of Heaviside’s transmission problems. Regarding this development, the following physical phenomena should be discussed.

(1) Voltage and current
(2) Lossless transmission line and circuit
(3) Active and reactive powers
(4) Multiple reflection as a transient phenomenon involving repeated reflection and transmission, and the subsequent steady state and resonance

The above four physical phenomena are considered to be closely related to the Riccati differential equation. In the next session, we will focus on transmission theory using the transmission line expressed by the Dirac equation [Eq. (10.2)] and discuss the effectiveness of using the Riccati differential equation to obtain the Dirac equation.

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Nobuo Nagai received his B.S. and D.Eng. degrees from Hokkaido University in 1961 and 1971, respectively. In 1961, he joined Hokkaido University as an Assistant and in 1972 he became an Associate Professor, and from 1980 to 1992 he was a Professor in the Research Institute of Applied Electricity. From 1992 to 2001, he was a Professor in the Research Institute for Electronic Science, Hokkaido University. In 2001, he retired and became an Emeritus Professor. His research interests are circuit theory and digital signal processing. He is interested in the application of above theory to quantum theory. Dr. Nagai is a Life Fellow of the Institute of Electronics, Information and Communication Engineers, Japan, and a Life Member of IEEE and IEICE, and an Honorary Member of RISP.
Takashi Yahagi received his B.E., M.S. and Ph.D. degrees all from the Tokyo Institute of Technology in 1966, 1968 and 1971, respectively. In 1971, he joined Chiba University as a Lecturer and in 1974 he became an Associate Professor, and from 1984 to 2008 he was a Professor at the same university. Since 2008 he has been with the Signal Processing Research Laboratory. In 1997, he founded the Research Institute of Signal Processing, Japan (RISP). Since 1997 he has been President of RISP. From 1997 to 2013 he was Editor-in-Chief of the Journal of Signal Processing (JSP). Since 2013 he has been Honorary Editor-in-Chief of JSP. He was the author of “Theory of Digital Signal Processing (Vols. 1-3)”, (1985, 1985, 1986), Corona Pub.Co., Ltd. (Tokyo, Japan). He was also the editor and author of “Library of Digital Signal Processing (Vols. 1-10)”, (1996, 2001, 1996, 2000, 2005, 2008, 1997, 1999, 1998, 1997), Corona Pub.Co., Ltd. (Tokyo, Japan). He was the editor of “My Research History (Vols. 1 and 2)” (2003, 2003). RISP. The contents of the Library of Digital Signal Processing are as follows: Vol.1: Digital Signal Processing and Basic Theory (1996), Vol.2: Digital Filters and Signal Processing (2001), Vol.3: Digital Signal Processing of Speech and Images (1996), Vol.4: Fast Algorithms and Parallel Signal Processing (2000), Vol.5: Kalman Filter and Adaptive Signal Processing (2005), Vol.6: ARMA Systems and Digital Signal Processing (2008), Vol.7: VLSI and Digital Signal Processing (1997), Vol.8: Communications and Digital Signal Processing (1999), Vol.9: Neural Network and Fuzzy Signal Processing (1998), Vol.10: Multimedia and Digital Signal Processing (1997). Dr. Yahagi is a Life Fellow of the Institute of Electronics, Information and Communication Engineers, Japan.