GRAPHS WITH 4-RAINBOW INDEX 3 AND n − 1

XUELIANG LI\(^1\), INGO SCHIERMEYER\(^2\)
KANG YANG\(^1\) AND YAN ZHAO\(^1\)

\(^1\) Center for Combinatorics and LPMC-TJKLC
Nankai University
Tianjin 300071, China

\(^2\) Institut für Diskrete Mathematik und Algebra
Technische Universität Bergakademie Freiberg
09596 Freiberg, Germany

e-mail: lxl@nankai.edu.cn
Ingo.Schiermeyer@tu-freiberg.de
yangkang@mail.nankai.edu.cn
zhaoyan2010@mail.nankai.edu.cn

Abstract

Let G be a nontrivial connected graph with an edge-coloring \( c : E(G) \to \{1, 2, \ldots, q\} \), \( q \in \mathbb{N} \), where adjacent edges may be colored the same. A tree \( T \) in \( G \) is called a rainbow tree if no two edges of \( T \) receive the same color. For a vertex set \( S \subseteq V(G) \), a tree that connects \( S \) in \( G \) is called an \( S \)-tree. The minimum number of colors that are needed in an edge-coloring of \( G \) such that there is a rainbow \( S \)-tree for every set \( S \) of \( k \) vertices of \( V(G) \) is called the \( k \)-rainbow index of \( G \), denoted by \( rx_k(G) \). Notice that a lower bound and an upper bound of the \( k \)-rainbow index of a graph with order \( n \) is \( k − 1 \) and \( n − 1 \), respectively. Chartrand \textit{et al.} got that the \( k \)-rainbow index of a tree with order \( n \) is \( n − 1 \) and \( n \) and the \( k \)-rainbow index of a unicyclic graph with order \( n \) is \( n − 1 \) or \( n − 2 \). Li and Sun raised the open problem of characterizing the graphs of order \( n \) with \( rx_k(G) = n − 1 \) for \( k \geq 3 \). In early papers we characterized the graphs of order \( n \) with 3-rainbow index 2 and \( n − 1 \). In this paper, we focus on \( k = 4 \), and characterize the graphs of order \( n \) with 4-rainbow index 3 and \( n − 1 \), respectively.

\textbf{Keywords:} rainbow \( S \)-tree, \( k \)-rainbow index.

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