PAIR PRODUCTION IN LOW-LUMINOSITY GALACTIC NUCLEI

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ABSTRACT

Electron–positron pairs may be produced near accreting black holes by a variety of physical processes, and the resulting pair plasma may be accelerated and collimated into a relativistic jet. Here, we use a self-consistent dynamical and radiative model to investigate pair production by $\gamma\gamma$ collisions in weakly radiative accretion flows around a black hole of mass $M$ and accretion rate $\dot{M}$. Our flow model is drawn from general relativistic magnetohydrodynamic simulations, and our radiation field is computed by a Monte Carlo transport scheme assuming the electron distribution function is thermal. We argue that the pair production rate scales as $\dot{M}^{-2}M^{\eta}$. We confirm this numerically and calibrate the scaling relation. This relation is self-consistent in a wedge in $M$, $\dot{M}$ parameter space. If $M$ is too low the implied pair density over the poles of the black hole is below the Goldreich–Julian density and $\gamma\gamma$ pair production is relatively unimportant; if $M$ is too high the models are radiatively efficient. We also argue that for a power-law spectrum the pair production rate should scale with the observables $L_X \equiv X$-ray luminosity and $M$ as $\dot{M}^{2}M^{-4}$. We confirm this numerically and argue that this relation likely holds even for radiatively efficient flows. The pair production rates are sensitive to black hole spin and to the ion–electron temperature ratio which are fixed in this exploratory calculation. We finish with a brief discussion of the implications for Sgr A* and M87.

Key words: accretion, accretion disks – black hole physics – Galaxy: center – magnetohydrodynamics (MHD) – radiative transfer

Online-only material: color figures

1. INTRODUCTION

Models of zero-obliquity black hole accretion—in which the accretion flow angular momentum is parallel to the black hole spin—typically exhibit a low-density “funnel” over the poles of the black hole. The funnel is empty because the funnel plasma is free to fall into the hole or be ejected to large radius. Magnetic fields do not prevent this: in magnetohydrodynamic (MHD) simulations of radiatively inefficient accretion flows (RIAFs) the funnel magnetic field typically runs in a smooth spiral from the event horizon to large radius (De Villiers et al. 2003; McKinney & Gammie 2004; Komissarov 2005; Hawley & Kritikos 2006; Beckwith et al. 2008). Because the field lines do not leave the funnel there is no way for the disk plasma to resupply the funnel plasma.

What process, then, populates the funnel with plasma? And what controls the temperature (or distribution function) of the funnel plasma? These questions bear directly on two interesting problems in black hole jet theory: are jets made of pairs or an electron–ion plasma? And which is more luminous: the base of the jet or the accretion flow? The purpose of this paper is to investigate these questions in the specific context of hot, underluminous accretion flows where nearly ab initio models are computationally feasible.

There are several pair-creation processes that might populate the funnel with plasma. Plasma close to the event horizon in an RIAF is relativistically hot, and thus can form electron–positron pairs $e^\pm$ through particle–particle ($ee$, $ep$), particle–photon ($e\gamma$, $p\gamma$), or photon–photon collisions ($\gamma\gamma$). The cross section near the $e^\pm$ energy threshold is largest for $\gamma\gamma$ interactions, which have a cross section $\sim\sigma_T \equiv$ the Thomson cross section. In the funnel, the photon density vastly exceeds the particle density, so $\gamma\gamma$ collisions dominate $e^\pm$ production (Stepney & Guilbert 1983; Phinney 1983; Phinney 1995; Krolik 1999). Pair production by these processes is discussed in, e.g., Kusunose & Mineshige (1996) and Esin (1999) in the context of advection-dominated accretion flows (ADAFs; Narayan & Yi 1994). These works, however, focus on the energetic role of pairs in ADAF disks rather than the population and dynamics of pairs in the funnel.

Other processes are important when the density is below the Goldreich & Julian (1969) charge density. Then the plasma can have $\mathbf{E} \cdot \mathbf{B} \neq 0$, and the electric field can directly accelerate particles to high Lorentz factors. The energetic particles Compton upscatter background photons that collide with other background photons and produce a shower of pairs in a pair-photon cascade (Blandford & Znajek 1977; Phinney 1983; Beskin et al. 1992; Hirota & Okamoto 1998, and recently Vincent & Lebohec 2010 and Beskin 2011).

In this paper, we model production of an $e^\pm$ plasma by photon–photon collisions in the funnel above a hot, underluminous accretion disk. At low accretion rates $\dot{M}$ (\lesssim $M_{\text{crit}} \sim 10^{-6}L_{\text{Edd}}/(0.1c^2)$, where $L_{\text{Edd}} \equiv$ Eddington luminosity), the disk cools on a timescale longer than the accretion timescale; it is a RIAF. In this regime, the radiative and dynamical losses are decoupled and it is practical to treat both on a nearly ab initio basis. Throughout the range of $\dot{M}$ we consider that the funnel pair plasma is tenuous enough that annihilation is negligible, so pair production will be balanced by advective losses such as accretion into the black hole or loss in a wind.

We draw our RIAF model from two- and three-dimensional general relativistic magnetohydrodynamics simulations (GRMHD, using the HARM code; Gammie et al. 2003; Noble et al. 2009) of an accreting, magnetized torus with zero cooling. The radiation field is calculated as a post-processing step using a Monte Carlo method (gammonty; Dolence et al. 2009), and pair production rates are estimated from snapshots of the radiation field using a procedure described in detail below.
This paper is organized as follows. In Section 2, we describe the basic model for accretion flow dynamics and radiative transfer. In Section 3, we write down the pair production model and present a test problem for our Monte Carlo scheme. Scaling formulae are presented in Section 4. In Section 5, we show results for a range of black hole masses and accretion rates. We briefly discuss implications for Sgr A* and M87 in Section 6 and summarize in Section 7.

2. ACCRETION FLOW MODEL

We use a numerical model for the accretion flow and for the radiation field; together these nearly ab initio models form a numerical laboratory for investigating physical processes near a black hole in a self-consistent way.

2.1. Dynamical Model

We use a relativistic MHD model for the accreting plasma (see, e.g., Gammie et al. 2003). The initial condition is an equilibrium torus (Fishbone & Moncrief 1976) in orbit around a Kerr black hole with $a_\ast = 0.94$, where $a_\ast GM/c^2$ is the hole angular momentum. The torus is seeded with poloidal, concentric loops of weak magnetic field that are parallel to density contours. Small perturbations are added to the internal energy and this seeds the magnetorotational instability, which leads to the development of MHD turbulence in the disk and accretion onto the central black hole. The model extends from slightly inside the event horizon to $r = 40GM/c^2$.

We solve the evolution equations until a quasi-equilibrium accretion flow is established, meaning that the mean structure of the flow is not evolving on the dynamical timescale. Our (untested) hypothesis is that at $r < 15GM/c^2$ the model accurately represents the inner portions of a relaxed accretion flow extending over many decades in radius.

A few of the physical assumptions in the GRMHD model are worth stating explicitly. The equation of state is

$$p = (\gamma_{ad} - 1)u, \quad (1)$$

where $\gamma_{ad} = 13/9$ (appropriate for ion temperature $T_i < m_p c^2/k = 1.1 \times 10^{11}$ K and electron temperature $T_e > m_e c^2/k = 5.9 \times 10^9$ K), $p \equiv$ pressure, and $u \equiv$ internal energy density. Particle number is also conserved:

$$(\rho_0 u^\mu),_\mu = 0, \quad (2)$$

where $\rho_0 \equiv$ rest-mass density and $u^\mu \equiv$ four-velocity, in the dynamical evolution. That is, pair production is not included in the dynamical model. The model is therefore consistent only if pair creation is weak enough not to alter the flow dynamics or energetics.

We evolve the GRMHD equations using the harm code (Gammie et al. 2003; Noble et al. 2009). harm is a conservative scheme that evolves the total energy rather than internal energy of the flow. The MHD equation integration is performed on a uniform grid in modified Kerr–Schild coordinates (Gammie et al. 2003). The coordinates are logarithmic in Kerr–Schild radius $\rho$ and nonuniform in Kerr–Schild colatitude $\theta$ (Boyer–Lindquist and Kerr–Schild $r$ and $\theta$ are identical), with zones concentrated toward the midplane of the accretion disk. The inner and outer radial boundaries use outflow boundary conditions. The axisymmetric models use a 256 $\times$ 256 grid, and the single three-dimensional run uses a 192 $\times$ 192 $\times$ 128 grid. For details of the numerical method, the initial setup and the flow evolution in two dimensions, see Gammie et al. (2003) and McKinney & Gammie (2004). A snapshot of the density, temperature, and magnetic field strength from one of our runs is shown in Figure 1.

2.2. Radiative Model

Our radiative model is identical to that applied by Mościbrodzka et al. (2009) to Sgr A*, although here we restrict attention to a thermal plasma with $T_e = T_i$ (except in one case noted below). Synchrotron emission and absorption are included, as is Compton scattering.

Bremsstrahlung is not important in the inner parts of the accretion flow. For a thermal plasma with $\Theta_\gamma \equiv kT_e/(m_e c^2) > 1$ the ratio synchrotron/bremsstrahlung cooling $\sim \Theta_\gamma^2/(a\beta)$, where $a \equiv$ fine structure constant and $\beta \equiv 8\pi p/B^2$. At the radii of interest here, $\Theta_\gamma \sim 10^2$ and $\beta \sim 10$, so in an energetic sense synchrotron dominates the direct production of photons.

Synchrotron emission occurs at a characteristic frequency $v_s \sim (eB/(2\pi m_e c))\Theta_\gamma^2$ which is $\ll m_e c^2/h$ for any astrophysically reasonable combination of $M$ and $M^3$. Potentially, pair-producing photons must therefore be produced by Compton scattering.

We compute the radiation field using the general relativistic Monte Carlo radiative transfer code grmontefy (Dolence et al. 2009). The radiation field is represented by photon packets (photon rays or “superphotons”). Each superphoton is characterized by a weight $w \equiv$ number of physical photons/superphoton, and a wave four-vector $k^\mu$. Superphotons are produced by sampling the emissivity. The wavevector is transported according to the geodesic equation. Along a geodesic $w$ is decremented to account for synchrotron absorption. Compton scattering is incorporated by sampling scattering events. When a superphoton scatters it is divided into a scattered piece with new wavevector $k^\mu$ and new weight $w$, and an unscattered piece along the original wavevector with weight $w-w'$. The distribution of scattered $k^\mu$ is consistent with the full Klein–Nishina differential cross section.

We use a “fast light” approximation in treating the radiative transfer. The data from a single time slice $t_n$ (e.g., $\rho_0(t_n, x^1, x^2, x^3)$) are used to calculate the emergent radiation field as if the data, and therefore photon field, were time independent. We have checked the fast light model against a time-dependent radiative transfer model (J. C. Dolence et al. 2011, in preparation) and verified that this approximation does not introduce significant errors.

2.3. Model Scaling

The properties of the accretion flow model are independent of the absolute value of the density (provided the magnetic field

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5 For the synchrotron emissivity, we use the approximate expression of Leung et al. (2010)\footnote{Leung et al. (2010)}

$$f_s = \frac{\sqrt{2\pi e^2 n_0 v_s}}{3\zeta_2 G_0^2} \left(\frac{1}{2} + \frac{1}{2} \right) \exp(-X^{1/3}), \quad (3)$$

where $X = v/v_s, v_s = 2/9(eB/(2\pi m_e c))\Theta_\gamma^2 \sin \theta$ is the synchrotron frequency, $\theta$ is an angle between the magnetic field vector and emitted photon, and $K_2$ is a modified Bessel function of the second kind. The fractional error for this approximate formula is smaller than 1% for $\Theta_\gamma > 1$ (where most of the emission occurs) and increases to 10% and more at low frequencies for $\Theta_\gamma \leq 1$ (where there is very little emission). The synchrotron emissivity function peaks at $v \approx 8v_s$.}
strength is scaled appropriately), but the radiative model is not. To scale the model we specify the length unit

\[ L \equiv GM/c^2, \quad (4) \]

time unit

\[ T \equiv GM/c^3, \quad (5) \]

and mass unit \( \mathcal{M} \), which is proportional to the mass accretion rate. \( M \) does not set a mass scale because it appears only in the combination \( GM \). Since \( \mathcal{M} \ll M \) the accretion flow does not affect the gravitational field.

Given \( M \) and \( \mathcal{M} \) the radiative transfer calculation is well posed. Typically \( M \) can be estimated directly from observations, while \( \mathcal{M} \) is varied until the model submillimeter flux matches the observed flux.

2.4. Model Limitations

An important limitation of our model is that it treats accreting plasma as a nonradiating ideal fluid. This implies that electrons and ions have an isotropic, thermal distribution function. The potentially important effects of pressure anisotropy and conduction (e.g., Sharma et al. 2006; Johnson & Quataert 2007) are therefore neglected, as are the radiative effects of a nonthermal component in the electron distribution function.

Cooling is also neglected. This is a good approximation in low accretion rate systems like Sgr A*, but a poor approximation in higher accretion rate systems like M87. If one were to turn on cooling but hold the synchrotron flux fixed, the density and magnetic field strength (i.e., the mass unit \( \mathcal{M} \)) would increase.

3. PAIR PRODUCTION

3.1. Basic Equations

For a population of photons with distribution function \( dN_\gamma/d^3xk \) (here \( d^3k \equiv dk_1dk_2dk_3 \) and 1, 2, and 3 are the spatial coordinates), the invariant pair production rate per unit volume is

\[
\dot{n}_\pm \equiv \frac{1}{\sqrt{-g}} \frac{dN_\pm}{d^3xdt} = \frac{1}{2} \int \frac{d^3k}{\sqrt{-gk_1^1}} \frac{d^3k'}{\sqrt{-gk_2^2}} \frac{dN_\gamma}{d^3xk} \frac{dN_\gamma}{d^3xk'} \epsilon^2_{\mathrm{CM}} \sigma_{\gamma\gamma} c, \quad (6)
\]

where \( g \) is the determinant of \( g_{\mu\nu} \), and the factor of \( 1/2 \) prevents double counting. Here, \( \sigma_{\gamma\gamma} \) is the cross section for
\[
\gamma + \gamma \rightarrow e^+ + e^- \quad (\text{Breit} & \text{ Wheeler 1934}):
\]
\[
\frac{\sigma_{\gamma\gamma}}{\sigma_T} = \frac{3}{8\epsilon_{[\text{CM}]}} \left[ \left( 2\epsilon_{[\text{CM}]}^4 + 2\epsilon_{[\text{CM}]}^2 - 1 \right) \cosh^{-1} \epsilon_{[\text{CM}]} - \epsilon_{[\text{CM}]^3} \epsilon_{[\text{CM}]} - 1 \right],
\]
\[
\epsilon_{[\text{CM}]} = -u_{[\text{CM}]}k^\mu = -u_{[\text{CM}]}k^\mu = \left( \frac{-k_jk^\mu}{2} \right)^{1/2}
\]
is the energy of either photon in the center-of-momentum ([CM]) frame of the two photons, and \(u_{[\text{CM}]}\) is the four-velocity of the [CM] frame.

Equation (6) is coordinate invariant since \(\sqrt{-g}d^3x\,dt\) is invariant, the distribution function is invariant (because \(d^3x\) is invariant), \(\epsilon_{[\text{CM}]}\) is a scalar, the cross section is invariant, and \(d^3k/\sqrt{-g}k\) is invariant. It reduces to the correct rate (cf. Equation (12.7) of Landau & Lifshitz 1971) in Minkowski space, and is therefore the correct general expression for the pair production rate. Because \(\eta_\gamma\) itself is invariant it also describes the pair-creation rate in the fluid frame.

We will need the rate of four-momentum transfer from the radiation field to the plasma via pair creation:
\[
G^\mu \equiv \frac{1}{\sqrt{-g}} \frac{dP_\pm^\mu}{d^3xd\tau} = \frac{1}{2} \int \frac{d^3k}{\sqrt{-g}k} \frac{d^3k'}{\sqrt{-g}k'} \frac{dN_\gamma}{d^3x'} \frac{dN_\gamma}{d^3x} \times (k^\mu + k'^\mu) \epsilon_{[\text{CM}]}^2 \sigma_{\gamma\gamma} c.
\]
Here, \(A\) is a constant that makes the equation dimensionally correct.

3.2. Monte Carlo Estimate of Pair Creation Rate

We estimate the integrals (6) and (9) using a Monte Carlo scheme. Given a sample of photons on a time slice \(\tau\) within a small three-volume \(\Delta^3x\), a naive estimate is
\[
\frac{1}{\sqrt{-g}} \frac{dN_\pm}{d^3xd\tau} \approx \frac{1}{2} \sum_{i,j} \left( \frac{w_i}{\Delta^3x} \right) \times \left( \frac{w_j}{\Delta^3x} \right) \frac{1}{\sqrt{-g}k^i} \frac{1}{\sqrt{-g}k^j} \epsilon_{[\text{CM}]}^2 \sigma_{\gamma\gamma} c.
\]
where \(i\) and \(j\) label superphotons.

If there are \(N_\gamma\) superphotons in \(\Delta^3x\) then there are \(O(N_\gamma^2)\) pairs of superphotons and the computational cost of Equation (10) is \(O(N_\gamma^2)\). One might hope that the error would scale as \(1/\sqrt{N_{\gamma}^2}\) because there are \(O(N_{\gamma}^2)\) pairs, but this is wrong. There are only \(N_\gamma\) independent samples and so the error scales as \(1/\sqrt{N_{\gamma}}\).

We obtain an estimate with accuracy that is the same order as Equation (10) at \(O(N_{\gamma})\) cost by selecting an unbiased sample of \(N_{\gamma}\) pairs of superphotons and evaluating
\[
\frac{1}{\sqrt{-g}} \frac{dN_\pm}{d^3xd\tau} \approx \frac{1}{2} \frac{N_{\gamma}}{2} \sum_{i,j} \left( \frac{w_i}{\Delta^3x} \right) \times \left( \frac{w_j}{\Delta^3x} \right) \frac{1}{\sqrt{-g}k^i} \frac{1}{\sqrt{-g}k^j} \epsilon_{[\text{CM}]}^2 \sigma_{\gamma\gamma} c.
\]
An identical procedure is used to evaluate \(G^\mu\).

3.3. Test Problem

Does our Monte Carlo procedure accurately estimate the pair production rate? As a check, we evaluate pair production rates near two point sources of \(E = 4m_\gamma c^2\) photons. The calculation is done in Minkowski space and Cartesian coordinates (so \(\sqrt{-g} = 1\), and the optical depth to pair creation is assumed small. At each point, we compare the analytic and numerical result.

The expected pair production rate is given by Equation (6). The energy of two colliding photons in their center-of-momentum frame is a function of the cosine \(\mu\) of the angle between the rays from the two sources; \(\epsilon_{[\text{CM}]} = (1/2)(1 - \mu)k^\mu\). The photon momentum space distribution is a \(\delta\) function. The number density of photons \(dN_\gamma/d^3x = N_\gamma/(4\pi r^2 c)\) at distance \(r\) from the source, where each source produces photons at the rate \(N_\gamma\).

Figure 2 shows a two-dimensional map of the numerically evaluated pair production rate in the plane of the two sources. Figure 3 shows the analytic and numerical pair production rates along the black contour shown in Figure 2 (upper panel), and their difference (lower panel). The error in the numerical rate is \(\propto N_\gamma^{-1/2}\), as demonstrated in Figure 4, where \(N_\gamma\) is the number of photon packets emitted by each source. Evidently, the Monte Carlo method produces an unbiased, convergent estimate of the pair production rate.

4. RIAF SCALING LAWS

In this section, we derive scaling relations for the pair production rate in two cases: (1) \(M\) and \(M\) are known and the flow is radiatively inefficient; (2) the spectrum \(\nu L_\nu\) and \(M\) are known. In case (1) we can numerically evaluate the pair production rate self-consistently and check it against the scaling relation. In case (2) we can do the same, but we also obtain a method for estimating pair production rates from observations that may also apply to flows that are radiatively efficient.
First consider the pair production rate density at a single point in the flow where the plasma-frame photon spectrum is a power law with high-energy cutoff at \( \epsilon = E/(m_e c^2) = \epsilon_{\text{max}} \gg 1 \):

\[
\frac{dn}{dE} = \frac{n_0}{m_e c^2} \epsilon^{-\epsilon/\epsilon_{\text{max}}}.
\]

We evaluated the pair production rate density numerically for this energy distribution. A fit to the result over \(-3 < \alpha < 2\) and \(10 < \epsilon_{\text{max}} < 160\) gives

\[
\frac{\dot{n}_\|}{n_0^2 \sigma_T c} \approx \frac{1}{16} e^{2\alpha/3} \left( \frac{4}{3} + \epsilon_{\text{max}}/\epsilon_{\text{max}} \right)^4 \ln \left( \frac{\epsilon_{\text{max}}}{2} \right) \quad (13)
\]

(Zdziarski (1985) gives a similar expression in the \( \epsilon_{\text{max}} \gg 1 \) limit). At worst the fit is \( \approx 2 \) too small for \( \alpha \approx 0 \) and \( \epsilon_{\text{max}} = 160 \). For \( \alpha < -2 \), which is typical of our models, the relative error is smaller than 60%.

For \( \alpha > 0 \) (\( d \ln v_{\text{L}} / d \ln \nu > 2 \)), pair production is dominated by photons with \( \epsilon \sim \epsilon_{\text{max}} \), and \( \dot{n}_\| \) is therefore sensitive to \( \epsilon_{\text{max}} \). For \( \alpha < 0 \), pair production is dominated by pairs with \( \epsilon \sim 1 \) in the center-of-momentum frame. In this case, there is an equal contribution from each logarithmic interval in energy in the plasma frame, and the pair production rate density is therefore weakly (logarithmically) dependent on \( \epsilon_{\text{max}} \).

Our models have \( \alpha \lesssim -2 \) so \( \dot{n}_\| \) is insensitive to \( \epsilon_{\text{max}} \). Therefore, the effective number density of pair-producing photons is \( n_0 \sim L_{\text{S12}} / (4\pi L^2 m_e c^2) \), where \( L_{\text{S12}} \equiv \nu L_{\nu}(512 \text{ keV}) \) and \( \dot{n}_\| \sim n_0^2 \sigma_T c \). Then

\[
\dot{n}_\| \approx \left( \frac{L_{\text{S12}}}{m_e c^2 L^2 c} \right)^2 \sigma_T c \times f \left( \frac{r}{L}, \mu \right),
\]

where \( f \) is a dimensionless function of the Kerr–Schild radius \( r \) and colatitude \( \theta = \cos^{-1} \mu \).

What do we expect for the spatial distribution of pair production \( f \)? The pair-producing photons are made by upscattering synchrotron photons in a ring of hot gas near the innermost stable circular orbit (ISCO). Away from this ring the density of photons will fall off as \( \sim 1/r^2 \). The pair production rate also depends on the angle \( \psi \) between photon trajectories in the coordinate frame through the geometrical factor \( \epsilon_{\text{CM}}^2 k_x^2 \alpha \approx 1 - \cos \psi \).

At large \( r, \psi \approx 0 \) so \( 1 - \cos \psi \propto 1/r^2 \). Then \( \dot{n}_\| \sim r^{-6} \). Compton upscattered photons are also beamed into the plane of the disk by the relativistic orbital motion. If the intensity of photons to be scattered is nearly independent of \( \theta \) then the pair production rate density should fall off away from the midplane as the density of upscattering electrons \( \sim \exp(-\mu^2/(2\sigma^2)) \) where \( \sigma \approx 0.3 \). Gathering these estimates together we expect

\[
f \approx \exp(-\mu^2/(2\sigma^2)) / \sigma^6 \quad (14)
\]

where \( \sigma \approx \sigma_\| \approx \sigma_\perp \).

4.1. Scalings with Model Parameters

Now suppose we know the mass \( M = m_8 M_8 \) (\( M_8 \equiv 10^8 M_\odot \)) and the accretion rate \( \dot{M} = \dot{m} M_{\text{edd}} \), where \( M_{\text{edd}} \equiv L_{\text{edd}} / (\epsilon_{\text{ref}} c^2) \) and \( \epsilon_{\text{ref}} = 0.1 \) is a reference accretion efficiency. We assume that photons are produced in a low-frequency synchrotron peak and then scattered to \( \sim 512 \text{ keV} \) by \( n_{\text{sc}} \) Compton scatterings, where \( n_{\text{sc}} \) is 1 or 2.

For a plasma that is optically thin to synchrotron absorption at peak, the total number of synchrotron photons at the peak frequency produced per unit time is \( \dot{N}_{\text{peak}} \approx 4\pi v_{\text{peak}} j_{\text{peak}} L^3 / (h\nu_{\text{peak}}) \), where \( j_{\text{peak}} \) is the synchrotron emissivity. The number density of synchrotron photons is then \( n_{\text{peak}} \approx \dot{N}_{\text{peak}} / (4\pi L^2 c) \).

A fraction \( \nu_{\text{sc}} \) of the peak photons are upscattered to 512 keV, where \( \nu = \sigma_T n_{\perp} L \) is the Thomson depth of the plasma, so \( n_{\text{sc}} = n_{\text{peak}} / (27 m_e c^2) \). The mean number of Compton scatterings is then \( n_{\text{sc}} = \log(m_e c^2 / h\nu_{\text{peak}}) / \log A \), where \( A \approx 16\theta_\| L \) is the photon
energy enhancement in a single scattering by a relativistic electron, so

\[ n_{sc} \simeq a_1 + a_2 \log \frac{m_8}{\dot{m}}. \tag{15} \]

We determine \( a_1 \) and \( a_2 \) numerically but for a reference model with \( \dot{m} = 10^{-8} \) and \( m_8 = 4.5 \times 10^{-2} \) (Sgr A*), the average value of \( n_{sc} \approx 1-2 \).

Assuming \( M > 4\pi \rho c^2 \), the magnetic pressure is comparable to the gas pressure and both are \( \sim \rho c^2 \); the plasma density, magnetic field strength, and plasma temperature (close to the virial temperature) scale as

\[ n_e \simeq \frac{1}{\epsilon_{ref}} \left( \frac{c^2}{G M_8 \sigma_T} \right) \left( \frac{\dot{m}}{m_8} \right), \tag{16} \]

\[ B^2 \simeq \frac{1}{8\pi} \left( \frac{\rho \dot{c}^4}{G M_8 \sigma_T} \right) \left( \frac{\dot{m}}{m_8} \right), \tag{17} \]

and

\[ \Theta_e \simeq \frac{1}{30} \frac{m_p}{m_e}. \tag{18} \]

The mean emission \( \Theta_e \) corresponds to the mean value near the ISCO, and therefore increases with \( \alpha_c \). Combining,

\[ \dot{n}_\pm \simeq A \left( \frac{1}{r_0^3 \sigma_T} \right) \epsilon_{ref}^{-2(n_e+3)} a^2 \left( \frac{m_p}{m_e} \right) m_8^{3.2n_e} f \left( \frac{r}{L_c}, \mu \right), \tag{19} \]

where \( A \) is a dimensionless constant to be determined numerically, \( r_0 \equiv \) classical electron radius, and \( \epsilon_{ref} \equiv \) the fine structure constant. From now on unless stated otherwise we will set \( \epsilon_{ref} = 0.1 \) and the mean number of scatterings \( n_{sc} \approx 1.5 \) (numerical results, below, show \( 1.4 < n_{sc} < 1.6 \) for relevant \( M, \dot{M} \)). Then,

\[ \dot{n}_\pm \simeq 9 \times 10^{39} A m_8^{-1} m_6^{6} f \left( \frac{L_c}{\sigma_T}, \mu \right). \tag{20} \]

To estimate the jet kinetic luminosity, we need the pair production rate:

\[ \dot{N}_\pm = \int_{r>r_{	ext{hor}}} \sqrt{-g} d^3 x \dot{n}_\pm, \tag{21} \]

where \( r_{	ext{hor}} \) is the horizon radius. Then,

\[ \dot{N}_\pm \simeq \dot{n}_\pm L_c^3 \simeq 10^{78} A m_8^{3.2n_e} m_8^2 s^{-1}. \tag{22} \]

Only pairs made inside the funnel and at \( r > r_{sa} \equiv \) stagnation radius can escape to large radius (they are “free pairs”); those made at smaller radius or inside the accretion flow are advected into the hole. The free pair fraction is therefore a small multiple of Equation (22).

Evidently, the pair production rate density is sensitive to the mass accretion rate, \( \dot{n}_\pm \sim \dot{m}^6 \). This steep dependence shuts off pair production at low accretion rates, making it difficult for low \( \dot{m} \) systems like Sgr A* to populate their funnel with pairs. Below we will show that the implied funnel pair density for Sgr A* falls below the Goldreich–Julian charge density (see Section 5.4).

4.2. Scalings with Observables

Assume that from observations we know \( M \), the X-ray luminosity \( L_X \equiv l_X L_\odot \) (assuming isotropic emission), and the spectral index \( \alpha = d \log (\nu L_\nu)/d \log \nu \).5 Self-consistent models then permit us to calibrate the relation between these quantities and the pair production rate density. Since this relation depends only on the distribution of pair-producing photons within the source, it seems likely that it can be applied to sources with \( M > M_{\text{crit}} \), in which cooling is important.

If the spectrum is a power law from the keV to MeV energies,

\[ L_{512}(L_X) \approx L_X e^{4.92a}, \tag{23} \]

\[ \dot{n}_\pm \simeq B \left( \frac{c^3 \sigma_T L^2_{512}}{m_8^2 G^4 M_8^8} \right) \frac{L_c^3}{\sigma_T^2} \approx 9 \times 10^{39} A m_8^{-1} m_6^{6} f \left( \frac{r}{L_c}, \mu \right), \tag{24} \]

where \( B \) is a constant to be determined numerically and \( (c^3 \sigma_T L^2_{512}/m_8^2 G^4 M_8^8) \approx 10^{-8} \) cm\(^{-3}\) s\(^{-1}\). This assumes that the observed spectrum and the plasma-frame spectrum near the black hole are identical. We checked the plasma-frame spectrum and found it to be a slightly blueshifted version of the observed spectrum; the blueshifting does not change the scaling relation.

The pair production rate is

\[ \dot{N}_\pm \simeq \dot{n}_\pm L_c^3 \simeq B \left( \frac{\sigma_T L^2_{512}}{m_8^2 G^4 c^2} \right) \frac{L_c^3}{\sigma_T^2} e^{9.26a} m_8^{-1}, \tag{25} \]

where \( (\sigma_T L^2_{512}/m_8^2 G^4 c^2) = 10^{31} \) s\(^{-1}\). The dependence on black hole mass changes between Equations (24) and (25) because \( L_c \propto m_8 \).

5. PAIR PRODUCTION IN RIAF—NUMERICAL RESULTS

We now evaluate the pair production rate numerically, check whether it matches the expected scaling laws, and evaluate \( f(r, \mu) \). To do this, we have run simulations with a range of \( M \) and \( \dot{M} \), assuming that the models have equal ion and electron temperatures, \( T_e = T_i \). A list of model parameters is given in Table 1.

5.1. Pair Creation Rate

5.1.1. Dependence on Model Parameters: \( \dot{m}, m \)

The \( \dot{n}_\pm \) in models A through H (see Table 1) is well fit by

\[ \dot{n}_\pm(r, \mu) = 3 \times 10^{10} \dot{m}^{3.2n_e} m_8^{-1} \times \left( \frac{r}{L_c} \right)^{-6} e^{-\mu^2/(2\mu^2)} \text{ cm}^{-3} \text{ s}^{-1}, \tag{26} \]

or \( A \simeq 3 \) in Equation (20). The constant in Equation (26) is derived from models with \( T_i/T_e = 1 \). The constant is sensitive to \( T_i/T_e \); for \( T_i/T_e = 3 \) it is \( 10^{-4} \) times smaller. As expected, \( \dot{n}_\pm \sim r^{-6} \) at large \( r \); surprisingly, however, this is also good fit at all \( r \).

The pair production scale height \( \sigma_\pm \approx 0.3 \) independent of \( \dot{m}, m \). This is nearly identical to \( \sigma_c \), the plasma scale height. Notice that \( \sigma_\pm \) also controls \( \dot{f}_\text{jet} \) the fraction produced inside the funnel. The funnel wall is at

\[ \mu^2 \approx \mu_f^2 \approx \frac{r + 0.4GM/c^2}{r + 4GM/c^2}, \tag{27} \]

and \( \dot{f}_\text{jet} \approx 10\% \). Figure 5 shows a two-dimensional contour map of \( \dot{n}_\pm \) corresponding to model C and Equation (26), and a...
μ is a steeply declining function of averaged fractional difference between time-averaged MHD contour marking the approximate funnel boundary. The grid averaged fractional difference between model and data in this case is <40%. Black contours mark the black hole horizon and the funnel wall. The pair production rate is well fit by

\[ \dot{N}_\pm = 4 \times 10^{30} \dot{m}^{3.2} m_8^2 \, \text{s}^{-1} \]  

Figure 5. Spatial distribution of \( \dot{N}_\pm \) in RIAF model C (points) and the contours of corresponding fitting function given by Equation (26) (lines). The fractional difference between model and data in this case is <40%. Black contours mark the black hole horizon and the funnel wall.

(A color version of this figure is available in the online journal.)

Figure 6. Pair production rate dependence on the model parameters. Comparison of the total pair production rate \( \dot{N}_\pm \) to the fitting formula for models with various mass accretion rates \( \dot{m} \) (A–E, blue filled symbols), and black hole masses \( m_8 \) (F–H, red open symbols). The \( \dot{N}_\pm \) (analytical) is given by Equation (28).

(A color version of this figure is available in the online journal.)

where a fit gives

\[ n_{sc} = 1 + 0.03 \ln(m_8/\dot{m}) \].  

(29)

Figure 6 compares the time-averaged numerical \( \dot{N}_\pm \) to Equation (28).

5.1.2. Dependence on \( l_x, \alpha, m \)

The self-consistent radiative model enables us to calculate the emergent spectrum, from which we can measure a 2–10 keV contour marking the approximate funnel boundary. The grid averaged fractional difference between time-averaged MHD models A through H and Equation (26) is < 60%. Since \( \mu_\pm \) is a steeply declining function of \( \mu^2 \), almost all free pairs are made near the funnel walls.

| ID | \( a_c \) | \( m_8 \) | \( \langle n \rangle_t \) | \( L_{\text{Bol}}/L_{\text{Edd}} \) | Radiative Efficiency | \( L_{\pm}/(L_{\text{Edd}} T_f) \) | Note |
|----|----------|----------|----------------|-----------------|-------------------|-------------------|------|
| A  | 0.94     | 4.5 \times 10^{-2} | 2 \times 10^{-9} | 10^{-11} | 7 \times 10^{-4} | 10^{-17} | 2D |
| B  | 0.94     | 4.5 \times 10^{-2} | 6 \times 10^{-9} | 10^{-10} | 2 \times 10^{-3} | 10^{-15} | 2D |
| C  | 0.94     | 4.5 \times 10^{-2} | 1 \times 10^{-8} | 4 \times 10^{-10} | 4 \times 10^{-3} | 10^{-13} | 2D |
| D  | 0.94     | 4.5 \times 10^{-2} | 5 \times 10^{-8} | 2 \times 10^{-8} | 0.02 | 10^{-10} | 2D |
| E  | 0.94     | 4.5 \times 10^{-2} | 1 \times 10^{-7} | 6 \times 10^{-8} | 0.04 | 10^{-8} | 2D |
| F  | 0.94     | 4.5 \times 10^{-3} | 1 \times 10^{-8} | 5 \times 10^{-10} | 5 \times 10^{-3} | 10^{-14} | 2D |
| G  | 0.94     | 4.5 \times 10^{-1} | 1 \times 10^{-8} | 4 \times 10^{-10} | 4 \times 10^{-3} | 10^{-12} | 2D |
| H  | 0.94     | 4.5     | 1 \times 10^{-8} | 3 \times 10^{-8} | 3 \times 10^{-3} | 10^{-11} | 2D |
| I  | 0.94     | 4.5 \times 10^{-2} | 2.7 \times 10^{-8} | 5 \times 10^{-10} | 2 \times 10^{-3} | 10^{-11} | 3D-quiescent, \( T_i/T_e = 3 \) |
| J  | 0.94     | 4.5 \times 10^{-2} | 5.3 \times 10^{-8} | 1 \times 10^{-9} | 3 \times 10^{-3} | 10^{-9} | 3D-weak flare, \( T_i/T_e = 3 \) |
| K  | 0.94     | 30      | 1.5 \times 10^{-6} | 3 \times 10^{-4} | 16.5 | 0.1 | 2D—w/o cooling |
| L  | 0.94     | 30      | 1 \times 10^{-6} | 3 \times 10^{-6} | 0.3 | 4 \times 10^{-5} | 2D—w/o cooling |

Notes. From left to right columns are: model ID, dimensionless spin of the black hole, the black hole mass in units of \( M_8 \), the rest-mass accretion rate through the black hole horizon in units of Eddington mass accretion rate (\( \dot{m}_{Edd} = \frac{2.22m_8 M_⊙}{\text{yr}} \)), averaged over later times of the simulation (\( \Delta t = 1500–2000 T_f \)), the Eddington ratio \( \dot{L}_{\text{Bol}}/L_{\text{Edd}} \), the model radiative efficiency \( \eta = L_{\text{Bol}}/m^2 L_{\text{Bol}} \) (\( L_{\text{Bol}} \) is the RIAF luminosity integrated over emitting angles and frequencies), ratio of kinetic to electromagnetic luminosity, and comments on models. Models I and J correspond to Sgr A* while K and L model M87. Run L accounts for cooling terms in the dynamical solution so the pair production rate is reduced.
luminosity $l_X$ and a spectral slope $\alpha$. The pair production rate density can be measured in the same models. The numerical results are well fit by

$$\dot{n}_\pm(r, \mu) = 10^{-8} l_X^2 e^{0.26a} m_s^{-4} \times \frac{L}{E}^{-0} e^{-\mu^2/2\sigma^2} \text{ cm}^{-3} \text{ s}^{-1},$$

or $B \simeq 1$ in Equation (24). The fractional error of the fit is $<50\%$ for time-averaged models A–L.

The pair-creation rate is well fit by

$$\dot{N}_\pm = 5 \times 10^{30} l_X^2 e^{0.26a} m_s^{-1} \text{ s}^{-1}. \quad (31)$$

Figure 7 compares $\dot{N}_\pm$ to the semianalytic formula given by Equation (31) for different snapshots of the simulations with different mass accretion rates (models A–C) and black hole masses (models F–H). The semianalytic and numerical results agree well, and the scaling constants are close to those estimated in Section 4.

Although Equations (30) and (31) are strictly valid only for radiatively inefficient flows with $\dot{m} < \dot{m}_{\text{crit}}$, they depend mainly on the geometry of the radiation field and not on the radiative efficiency of the flow. We speculate that they provide a good estimate of the pair production rate even in more efficient systems, if $\sigma_\pm$ is set to the scale height of the Comptonizing corona and the spectrum extends to sufficiently high energy.

### 5.2. Pair Power and Electromagnetic Luminosity of a Funnel

We define the funnel pair-creation “luminosity”

$$L_\pm \equiv f_{\text{jet}} \dot{N}_\pm 2m_e c^2 \Gamma_{\text{jet}}, \quad (32)$$

where $\Gamma_{\text{jet}}$ is the jet bulk Lorentz factor at large $r$ (assuming cold flow). Then,

$$L_\pm \simeq 6 \times 10^{74} f_{\text{jet}} \dot{m}^{3+2\alpha} m_s^2 \Gamma_{\text{jet}} \text{ erg s}^{-1} \quad (33)$$

and

$$L_\pm \simeq 10^{55} f_{\text{jet}} l_X^2 e^{0.26a} m_s^{-1} \Gamma_{\text{jet}} \text{ erg s}^{-1}. \quad (34)$$

It is interesting to compare this with the Blandford–Znajek (BZ), or electromagnetic, luminosity of the funnel

$$L_{\text{BZ}} = 2\pi \int d\theta \sqrt{-g} T'_r, \quad (35)$$

where $T'_r = b^2 u^r u_t - b^\mu b_\mu$ is the electromagnetic part of the stress-energy tensor and is computed directly from the simulation data. The BZ luminosity is well fit by

$$L_{\text{BZ}} \approx 8 \times 10^{55}(1 - \sqrt{1 - a_r^2})^2 \dot{m} m_s \text{ erg s}^{-1}. \quad (36)$$

The scaling with $a_r$ is taken from Equation (61) of McKinney & Gammie (2004), which is a fit to numerical data.

For mass comparable to that of Sgr A* and $a_r = 0.94$, $L_{\text{BZ}} > L_\pm/\Gamma_{\text{jet}}$ for $\dot{m} < \dot{m}_{\text{crit}} \approx 10^{-6}$ (for $\dot{m}_{\text{crit}}$, see Section 6.1). At low accretion rates, the BZ luminosity completely dominates the pair luminosity, because the pair luminosity is such a steep function of accretion rate. Because $L_\pm \sim m_s^2$ while $L_{\text{BZ}} \sim m_s^3$, the $\dot{m}$ at which $L_{\text{BZ}} \sim L_\pm$ is higher for lower mass black holes. The $L_\pm/L_{\text{BZ}}$ ratio is shown in the Table 1. For reasonable $\Gamma_{\text{jet}}$, the funnel luminosity is therefore electromagnetically dominated for radiatively inefficient flows.

#### 5.3. Energy-Momentum Deposition

Some of the pairs created in the funnel will escape to large radius and some will fall into the black hole. In an MHD model, the escaping fraction and asymptotic Lorentz factor will depend on the run of pair-creation rate with radius, the magnetic field structure, the energy density of the pair plasma, and the pair-creation four-force $G^\mu$.

Based on numerical calculations, the spatial distribution of $G^\mu$ is well fit, with $x \equiv r/L$, by

$$G_{\text{code}}^0(r, \mu) = G^0 \frac{L^2 T^2}{M} \approx \frac{300}{x} \frac{\dot{n}_\pm(x, \mu) m_e c}{M/(L^2 T^2)}, \quad (37)$$

$$G_{\text{code}}^1(x, \mu) = G^1 \frac{L^2 T^2}{M} \approx \frac{20(x - x_a)}{x^2} \frac{\dot{n}_\pm(x, \mu) m_e c}{M/(L^2 T^2)}, \quad (38)$$

$$G_{\text{code}}^2(x, \mu) = G^2 \frac{L^2 T^2}{M} \approx \frac{\mu}{x^2} \frac{\dot{n}_\pm(x, \mu) m_e c}{M/(L^2 T^2)}, \quad (39)$$

$$G_{\text{code}}^3(x, \mu) = G^3 \frac{L^2 T^2}{M} \approx \frac{150}{x^2} \frac{\dot{n}_\pm(x, \mu) m_e c}{M/(L^2 T^2)}, \quad (40)$$

where $M, T, L,$ and $\dot{n}_\pm$ are given in cgs units. The four-force vector components are given in a Kerr–Schild coordinate basis and in code units; we divide $\check{G}^\mu$ in cgs units by the unit of the four-force density $M/L^2 T^2$. All components of the four-force depend more steeply on radius than $\dot{n}_\pm$. The radial component of the four-force is positive at large radius, zero at $x_a \approx x_{\text{ISCO}}(1 + \mu^2/2)$, and negative at small radius. The sign of $G^2$ changes at the equatorial plane, as should.
Pairs are created in the funnel with an initial distribution function, which is immediately isotropized with respect to rotation around the magnetic field. Later evolution of the distribution function depends on ill-understood relaxation processes; the pairs may not relax. Whether or not relaxation occurs the initial mean energy of the particles is of interest. So, what is the pairs may not relax. Whether or not relaxation occurs the rate, in the plasma frame, is

\[ \dot{n}_\pm \left( \frac{\gamma_e}{\gamma_{F}} \right) \]

where \( \gamma_e \) is the Lorentz factor of the new leptons in the fluid frame is

\[ n \equiv -\dot{n}_\mu J_\mu = a_s B' c^2 \left( 1 + 2 x \right)^{1/2} \cos \theta \frac{\pi e c}{X^3}, \]

where the field rotation frequency in the funnel \( \Omega = (a_s / B') c^2 / (2GM) \) in the BZ model at \( a_s \lesssim 1 \). For our standard Sgr A* model with \( B \sim 30 \text{ G}, a_s \sim 0.94, M \sim 4.5 \times 10^8 M_\odot \), so \( n_{GJ} \lesssim 10^{-3} \text{ cm}^{-3} \).

A better estimate uses the BZ model for a monopole magnetosphere. We use Kerr–Schild coordinates \( (t, r, \theta, \phi) \) and define the Goldreich–Julian density as the charge density measured in the frame of the normal observer, who to lowest (zeroth) order in \( a_s \) has four-velocity \( n_\pm = \left( -1 / 2 r / r \right)^{-1/2} \), \( 0, 0, 0 \). Using the current density \( J_\mu \) derived from the BZ monopole solution as given in McKinney & Gammie (2004) we find, to lowest order in \( a_s \),

\[ n_{GJ} = \frac{\Omega B}{4 \pi e c} = \frac{a_s B c^2}{3 \pi G M e c}, \]

where \( \Omega = (a_s / B') c^2 / (2GM) \) in the BZ model at \( a_s \lesssim 1 \). For our standard Sgr A* model with \( B \sim 30 \text{ G}, a_s \sim 0.94, M \sim 4.5 \times 10^8 M_\odot \), so \( n_{GJ} \lesssim 10^{-3} \text{ cm}^{-3} \).

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\[ n_{GJ} = \frac{\Omega B}{4 \pi e c} = \frac{a_s B c^2}{3 \pi G M e c}, \]

where \( \Omega = (a_s / B') c^2 / (2GM) \) in the BZ model at \( a_s \lesssim 1 \). For our standard Sgr A* model with \( B \sim 30 \text{ G}, a_s \sim 0.94, M \sim 4.5 \times 10^8 M_\odot \), so \( n_{GJ} \lesssim 10^{-3} \text{ cm}^{-3} \).

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\[ n_{GJ} = \frac{\Omega B}{4 \pi e c} = \frac{a_s B c^2}{3 \pi G M e c}, \]

where \( \Omega = (a_s / B') c^2 / (2GM) \) in the BZ model at \( a_s \lesssim 1 \). For our standard Sgr A* model with \( B \sim 30 \text{ G}, a_s \sim 0.94, M \sim 4.5 \times 10^8 M_\odot \), so \( n_{GJ} \lesssim 10^{-3} \text{ cm}^{-3} \).
6. DISCUSSION

6.1. Self-consistency of the Models

Our models are self-consistent when they are radiatively inefficient and \( n_{\pm} \) is greater than the Goldreich–Julian density. Figure 9 shows the self-consistent \( \dot{m}, M \) as a shaded region. The region is bounded at low accretion rates by the solid line, where \( n_{\pm} = n_{\text{GJ}} \) (for \( a = 0.94 \)). The region is bounded at high accretion rates by \( \dot{m} = \dot{m}_{\text{crit}} \) (vertical dashed line), where the model becomes radiatively efficient (here defined as \( L_{\text{bol}}/M c^2 = 0.1 \)). Numerically, \( \dot{m}_{\text{crit}} \approx 10^{-6} \) at \( T_i/T_e = 1 \). The models are fully self-consistent in the resulting wedge in parameter space. They are never applicable to stellar mass black holes, which cannot produce enough pairs to exceed the Goldreich–Julian density even at \( \dot{m}_{\text{crit}} \).

6.2. Sgr A*

The bright radio source associated with the \( M = 4.5 \times 10^6 M_\odot \) black hole in the Galactic center, Sgr A*, is a weak X-ray source (“quiescent” emission \( L_X \lesssim 10^{33} \text{ erg s}^{-1} \)) and is strongly sub-Eddington \( (L/L_{\text{Edd}} \approx 10^{-9}) \). Mościbrodzka et al. (2009) have presented models of Sgr A* that suggest the most probable spin of the black hole \( a_* = 0.94 \), temperature ratio \( T_i/T_e = 3 \), and \( \dot{m} = 2 \times 10^{-8} \). The \( n_{\pm} \) rate for these parameters is lower than the scaling laws of Section 5 because \( T_i/T_e > 1 \). For the purposes of this subsection only, we consider three-dimensional GRMHD models that have \( n_{\pm} \) very similar to the two-dimensional models. All models assume the accretion flow lies in the equatorial plane of the black hole.

During the quiescent state the X-ray luminosity is \( L_X < 1 \). Near the horizon in the funnel, in model L, \( n_{\pm} \approx 10^{-9} \text{ s}^{-1} \). The light-crossing time is \( T \approx 20 \text{ s} \), so a typical pair density near the horizon in the funnel is \( n_{\pm} \approx 10^{-8} \text{ cm}^{-3} \). This is five orders of magnitude below \( n_{\text{GJ}} \approx 10^{-3} \text{ cm}^{-3} \). In quiescence, the funnel must therefore (for the assumed spin) be populated by a process other than the \( \gamma\gamma \) pair production process considered here, for example, a pair cascade.

Sgr A* exhibits intraday variability at all observed wavelengths (radio, submillimeter, NIR, and X-rays). In particular, in 2–10 keV luminosity may increase up to 160 times (the brightest flare detected has luminosity of \( L_X = 5.4 \times 10^{34} \) (Porquet et al. 2008) from the “quiescent” level and last a few ks. During a bright flare, the X-ray slope can change, and the implied pair density may reach or exceed \( n_{\text{GJ}} \). Even during a flare the funnel kinetic luminosity is far below the BZ luminosity (see Table 1) for any reasonable \( T_{\text{jet}} \). We conclude that close to the black hole any jet in Sgr A* is electromagnetically dominated. For an interesting discussion of pair production rates near Sgr A* in the past, see Totani (2006).

6.3. M87

The core of M87 hosts a sub-Eddington black hole with \( M = 3 \times 10^9 M_\odot \) (Marconi et al. 1997; but see Gebhardt & Thomas 2009) at distance \( D \approx 16 \text{ Mpc} \). M87 has a prominent radio jet resolved from \( 100 G M/c^2 = 1 \text{ kpc} \). Reynolds et al. (1996b) have argued that models in which the jet is made of a pair plasma are favored over those in which the jet is composed of an ion–electron plasma. It is difficult to apply our model to M87 because the spectral energy distribution (SED) of M87 from \( r < 100 G M/c^2 \) includes contributions from both the accretion flow and the jet.

We assume \( a_* = 0.94 \), set the inclination \( i = 30 \text{ deg} \) (Heinz & Begelman 1997), assume that the accretion disk lies in the equatorial plane of the black hole, that the electron distribution function is thermal, and that \( T_i/T_e = 3 \). The SED is normalized via \( f_\nu (\nu = 230 \text{ GHz}) = 1770 \text{ mJy} \) at 16 Mpc (Tan et al. 2008). We find that the resulting zero cooling, two-dimensional model with \( \dot{m} = 1.5 \times 10^{-6} \) (model K) is radiatively efficient (see Table 1) and therefore not self-consistent.

To find a more self-consistent model, we have run a GRMHD simulation with synchrotron and bremsstrahlung cooling, but not Compton cooling, included (model L, with \( \dot{m} = 10^{-6} \)). The cooling rates for synchrotron and free–free emission are presented in Appendices A and B, respectively, and the cooling algorithm is described in Appendix C. The efficiency is reduced to \( \approx 30\% \). Figure 10 shows the SED for model L; the dotted line is the bremsstrahlung contribution, which is negligible. The model has \( L_X = 10^{31} \text{ erg s}^{-1}, L_\nu = 7 \times 10^{36} \text{ erg s}^{-1}, \) and \( L_{\text{BZ}} \approx 10^{41} \text{ erg s}^{-1} \).

If we identify \( L_{\text{BZ}} \) as the jet luminosity then the model is inconsistent with existing estimates (see the useful compilation of estimates in Table 3 of Li et al. 2009), which range from \( 3 \times 10^{42} \text{ erg s}^{-1} \) (Young et al. 2002) to \( 10^{44} \text{ erg s}^{-1} \) estimated by Bicknell & Begelman (1996). The discrepancy between \( L_{\text{BZ}} \) in model L and observations is by 1–3 orders of magnitude, but the lowest “observed” value of \( L_{\text{BZ}} \) could be possibly reached in a model which combines \( T_i/T_e \approx 1 \) and radiative cooling.

The jet is optically thin to pair annihilation: \( \tau_{\pm} \approx n_{\pm} L \sigma_T \approx 10^{-10} \). It is also optically thick to pair production for TeV photons, \( \tau_{\gamma\gamma} \sim \sigma_T n_{\pm} L \approx 10^3 (n_{\text{IR}} \approx 10^{13} \text{ cm}^{-3}) \) is the infrared photon density calculated from the Monte Carlo simulations.

The shape of the spatial distribution of pair production in model L is similar to that in models without cooling (although the scaling of the distribution changes). The implied pair density \( n_{\pm} = n_{\pm} T \approx 10 \text{ cm}^{-3} \) (\( T \approx 10^4 \text{ s} \)), which is 107 times larger than \( n_{\text{GJ}} \approx 10^{-6} \text{ cm}^{-3} \) in almost the entire computational domain.
Because of the shortcomings of the model, however, it is useful to use a more nearly model-independent estimate of the total pair production rate based on Equation (31). For $L_X \approx 3 \times 10^{41} (7 \times 10^{40} \text{ from Di Matteo et al. (2003)}$ corrected upward to an isotropic X-ray luminosity because our models beam X-rays into the equatorial plane and $a_X = 0, N_{\pm} \approx 10^{35} s^{-1}$. This implies $L_X = f_{\text{jet}} N_{\pm} m_e c^3 \Gamma_{\text{jet}} = 8 \times 10^{38} \Gamma_{\text{jet}} f_{\text{jet}}$. The implied pair density exceeds $n_{\text{GJ}}$ for model L by $\sim 10^8$. Since $n_{\text{GJ}} \propto B \propto m_{\pm}^{1/2}$ and $n_{\pm} \propto L_{512}^2$, the implied pair density will fall below the Goldreich–Julian density only for $(m/10^{-5})^{1/2} (L_{512}/10^{41.5})^{-2} < 10^8$. Even if $m \sim 10^{-4}$ this would require $L_{512} \sim 10^{38}$, which seems implausibly low given the $\sim 10^{40} 	ext{ erg s}^{-1} \text{ TeV luminosity}$ (Aharonian et al. 2006). Therefore, the main conclusion of this section does not change even if a more self-consistent model is found.

There are significant limitations on the model. We have considered only one value of $a_*$; estimates and preliminary models not described here show that the pair production rate is a steeply increasing function of $a_*$. Further preliminary models and a comparison of the $T_i/T_e = 3$ model for Sgr A* with the scaling relation for $T_i/T_e = 1$ models also show that the pair production rate declines sharply as $T_i/T_e$ increases. But the allowed values of $T_i/T_e$ are strongly constrained by submillimeter VLBI (Fish & Doeleman 2010), because as $T_i/T_e$ increases so does the size of the synchrotron photosphere.

After submission of this article, Levinson & Rieger (2010) released a paper focused on modeling TeV emission and pair production in M87 (and Sgr A*). These authors use an ADAF model, assume that $T_s$ saturates at few$\times 10^5$ K ($\Theta_e \sim 1$), and set $m \approx 10^{-4}$. The model is semi-analytic and does not include general relativistic effects. Bremsstrahlung is the dominant source of photons near the pair-production threshold, and the resulting radiation field is inadequate to raise the pair density above $n_{\text{GJ}}$. Levinson & Rieger (2010) therefore invoke a gap/pair cascade model to produce pairs.

We have investigated the Levinson & Rieger (2010) model by calculating images and an SED for a GRMHD/radiative transfer model with $\Theta_e = 1$ everywhere, $m = 10^{-4}$, and $a_* = 0.94$. The model includes synchrotron, Compton, and bremsstrahlung. We find $f_{\text{synch}} \approx 1$ at $(i = 30 \text{deg})$, and $L_{\text{BB}} = 10^{43} \text{ s}^{-1}$, consistent with observations. Free–free cooling dominates over synchrotron cooling only at $r > 20 GM/c^2$. Levinson & Rieger neglect Compton cooling, but we find that Compton $\gamma \gamma$ cancels that the pair production rate due to $\gamma \gamma$ collisions is small. The model is also constrained by VLBI measurements. An optically thick spherical source of radius $r$ and distance $D$ in the Rayleigh–Jeans regime has flux $f_{\nu} \approx 2\pi \Theta_e m_e c^2 (r/D)^2/\nu^2$. Small $r$ inferred from VLBI therefore requires high $\Theta_e$. At 230 GHz Fish & Doeleman (2010) report structure on scales of a “few Schwarzschild” radii, while we find the Levinson & Rieger model has a photosphere at $\sim 30 GM/c^2$. In comparison, our model L has a photosphere at $\sim 7 GM/c^2$. This argues against the Levinson & Rieger model if the reported structure arises from the accretion flow rather than the jet.

7. SUMMARY

We have studied electron–positron pair production in black hole magnetospheres by $\gamma \gamma$ collisions. Our pair production rate simulations are based on a GRMHD time-dependent model of a magnetized disk around a spinning black hole. The disk is a source of high-energy radiation formed in multiple Compton scatterings of synchrotron photons. The pair production rates are calculated nearly ab initio within $40 GM/c^2$ of the event horizon, using Monte Carlo methods.

The main results of this work are the fitting formulae for the rate and spatial distribution of pair production in terms of $m_8$ and $i$ (Equation (26)) and in terms of $m_8$, $L_X$, and $a_*$ (Equation (30)). These indicate that $\gamma \gamma$ pair production is concentrated close to the event horizon, and is sensitive to model parameters such as $a_*$. The pair production rate is also sensitive to black hole spin $a_*$ and the electron–ion temperature ratio $T_i/T_e$, but probing the dependence on these parameters is beyond the scope of this paper.

We also find that the pair plasma is created with a power-law-like energy distribution. Most of the pairs are created in the equatorial plane of the thick disk because MeV photons created by Compton scattering are beamed into the equatorial plane. The pair plasma has negligible effect on the accretion flow dynamical evolution, consistent with previous results by Esin (1999) and Kusunose & Mineshige (1996), assuming that it escapes on the viscous timescale.

Only a few percent of all pairs are created in the magnetized funnel (black hole magnetosphere), and most of pairs in the funnel are created near its wall. Pair jets will have spectra with a turnover frequency at around $v_i = 10^{-3} n_\pm L \text{ Hz}$ (for example,
for M87 $L = 4 \times 10^{14}$ and $n_{\perp} = 10$, turnover frequency $\nu_T = 10^{12}$ Hz.

We also find that the general relativistic RIAF models are self-consistent up to $m_{\text{crit}} \approx 10^{-6}$, which is consistent with the $m_{\text{crit}} = 5 \times 10^{-6}$ reported by Fragile & Meier (2009). For higher $m$, one must couple the radiative cooling and forces into the dynamical model.

Models with $\dot{m} < m_{\text{crit}}$ have force-free, Thomson thin jets with the BZ luminosity much larger than pair kinetic luminosity. In models with very small $\dot{m}$, the pair plasma density in the funnel is below the Goldreich–Julian density $n_{\text{GJ}}$, suggesting that another process, such as a pair cascades, will operate and populate the funnel.

We have applied versions of our model to Sgr A* and to M87. These models suggest that $n_{\perp} > n_{\text{GJ}}$ in M87 and $n_{\perp} < n_{\text{GJ}}$ in Sgr A*, with the important caveat that there are parameters ($a_e$ and $T_e/T_\text{c}$) that we have not varied, and effects (Compton cooling, and nonthermal electrons) that we have not included.

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APPENDIX A

SYNCHROTRON COOLING RATES INCLUDING RADIATION SELF-ABSORPTION

The synchrotron cooling rate for a single electron is

$$\eta^T = \frac{2e^4 B^2 (\gamma^2 - 1) \sin^2 \xi}{3m_e^2 c^3}, \quad (A1)$$

where $\xi$ is the pitch angle between electron velocity and magnetic field (e.g., Rybicki & Lightman 1986). To obtain the total cooling rate from the thermal population of electrons, we integrate Equation (A1) against the relativistic Maxwellian distribution:

$$\frac{d n_e}{d \nu d \cos \xi} = n_e \gamma (\gamma^2 - 1)^{1/2} \exp\left( -\frac{\gamma}{\Theta_e} \right). \quad (A2)$$

The resulting integral over $\cos \xi$ and $\gamma$ is

$$\Lambda_S = \frac{4B^2 e^4 n_e \Theta_e K_3(1/\Theta_e)}{3c^3 m_e^2 K_2(1/\Theta_e)} \cdot (A3)$$

For $x \ll 1$, $K_\nu(x) \rightarrow \Gamma(n)/2(2/x)^n$, so for large $\Theta_e$,

$$\Lambda \rightarrow \frac{16B^2 e^4 n_e \Theta_e^2}{3c^3 m_e^2} \cdot (A4)$$

This agrees with expression (14) in Wardziński & Zdziarski (2000). For $x \gg 1$ ($\Theta_e \ll 1$), $K_\nu(x) \rightarrow (\pi/2x)^{1/2}e^{-x}$, and

$$\Lambda_S = \frac{4B^2 e^4 n_e \Theta_e}{3c^3 m_e^2} \cdot (A5)$$

The ratio of these two expressions is $4\Theta_e$; a reasonable approximation is

$$\Theta_e \frac{K_3(1/\Theta_e)}{K_2(1/\Theta_e)} \approx \left( \Theta_e^n + (2\Theta_e)^{2m} \right)^{1/m}, \quad (A6)$$

where $m = 4/3$ gives at most 4% error.

To account for synchrotron self-absorption, $\Lambda_S$ is multiplied by a factor:

$$f \equiv \frac{1}{\Lambda_{\text{Syn}}} \int_0^\infty dv \int_0^\pi \sin \theta d\theta j_v \exp(-\tau(v, \theta)) \approx \frac{1}{\Lambda_{\text{Syn}}} \int_{\nu_{\text{crit}}}^\infty dv \int_0^\pi \sin \theta d\theta j_v, \quad (A7)$$

where $j_v$ is given by Equation (3). $\Lambda_{\text{Syn}}$ is the first integral without optical depth factor, and $\nu_{\text{crit}}$ is the frequency where self-absorption becomes important. The critical frequency is calculated numerically from

$$\kappa_v(\theta = \pi/2)R = 1, \quad (A8)$$

where $\kappa_v = j_v/B_v$, $B_v$ is the Planck function, and $R = 0.1L$. We find that $f$ is well approximated by

$$f = \frac{1}{2} \left( \exp\left( -\frac{X_{\text{crit}}}{82} \right) + \exp\left( -\frac{X_{\text{crit}}}{360} \right) \right), \quad (A9)$$

where $X_{\text{crit}} = \nu_{\text{crit}}/\nu_T$. This fit gives the error for $f$ less than 1% up to $X_{\text{crit}} = 10^2$ and 5% error at $X_{\text{crit}} = 10^3$. 

APPENDIX B
FREE–FREE COOLING

The electron–ion bremsstrahlung cooling rate is (Stepney & Guilbert 1983)

\[ \Lambda_{ei} = n_en_p \sigma_T c \alpha_f \gamma_e \left[ \frac{9\Theta_e}{2\pi} \left( \ln(2\Theta_e \exp(-\gamma_e)) + 0.42 \right) + 1.5 \right] \Theta_e \geq 1; \]

\[ \Lambda_{ei} = 4 \left( \frac{2\Theta_e}{\pi} \right)^{0.5} \left( 1 + 1.78\Theta_e^{1.34} \right) \alpha_f^2 \Theta_e < 1, \]

where \( \gamma_e = 0.5772 \) is the Euler constant and \( \alpha_f \) is the fine structure constant. The electron–electron bremsstrahlung cooling rate is (Svensson 1982)

\[ \Lambda_{ee} = n_e^2 \sigma_T c \alpha_f m_e c^2 \left[ \frac{12}{\pi} \left( \ln(2\Theta_e \exp(-\gamma_e)) + \frac{5}{4} \right) \right] \Theta_e \geq 1; \]

\[ \Lambda_{ee} = \frac{5}{6\pi^{1.5}} (44 - 3\pi^2) \Theta_e^{1.5} (1 + 1.1\Theta_e + \Theta_e^2 - 1.25\Theta_e^{2.5}) \alpha_f^2 \Theta_e < 1. \]

The cooling rates are in units of erg s\(^{-1}\) cm\(^{-3}\), and are consistent within a factor of two with those provided by, e.g., Maxon (1972) or Gould (1980). Self-absorption for free–free emission is negligible. For \( \Theta_e > 1 \), the ratio of synchrotron to bremsstrahlung cooling rate is approximately \( \Theta_e^2 / \beta \alpha_f \). Synchrotron cooling dominates over the free–free emission in all of models considered here.

APPENDIX C
RADIATIVE COOLING IN MHD CODE

Radiative cooling is governed by

\[ \frac{du}{dt} = \frac{du}{dt} \frac{1}{w'} = -\Lambda_{\text{code}}, \]

where \( u \) is the internal energy per unit proper volume, \( t \) is the proper time, and \( w' \) is the time component of the fluid four-velocity.

Numerically, \( u \) is evolved in an operator-split fashion. After each fluid time step \( \Delta t \), \( u \) is evolved using the second order scheme

\[ u_{n+1} = u_n \exp(-\Delta t / \tau_{\text{cool},n+1/2}) \]

\( \tau_{\text{cool}} = u / \Lambda \).

The cooling rates are calculated in cgs units, and then \( \Lambda_{\text{code}} = \Lambda_{\text{cgs}} LT^3 / M \).

APPENDIX D
BREMSSTRAHLUNG EMISSIVITY IN THE RADIATIVE TRANSFER CALCULATIONS

The emissivity for \( e \rightarrow i \) interactions is (Stepney & Guilbert 1983)

\[ j_{ei}^i = \frac{dE}{dtdVd\omega d\Omega} = \frac{1}{4\pi} n_1 c \int_{1+\omega}^{\infty} \omega d\omega \beta n_e(\gamma) d\gamma, \]

where \( \omega = \nu / m_e c^2 \), \( n_e(\gamma) \) is relativistic Maxwellian electron energy distribution and the cross-section for this reaction is in the ultra-relativistic limit (Jauch & Rohrlich 1976). The 1/4\( \pi \) factor gives emissivity per unit solid angle. The integral is computed numerically using Gauss quadratures. The integration of Equation (D1) over photon energies and solid angle gives the total \( e \rightarrow i \) cooling rate, \( \Lambda_{ei} \).

The emissivity for \( e \rightarrow e \) emission is also from Stepney & Guilbert (1983)

\[ j_{ee}^i = \frac{1}{4\pi} n_1^2 \sigma_T c \alpha_f \gamma_e \exp(-\gamma_e) G(x, \Theta_e). \]

where \( x = (\nu / m_e c^2) / \Theta_e \) and \( G(x, \Theta_e) \) is given in Stepney & Guilbert (1983). This formula is accurate to 5% over 0.1 < \( \Theta_e < 2 \).

For \( \Theta_e < 0.1 \), we use a quadrupole approximation (Maxon 1972):

\[ j_{ee}^i = \frac{2}{4\pi \pi} \frac{2}{n_1^2 \sigma_T c \alpha_f B(x) \sqrt{2\Theta_e / \pi}} \exp \left( -\frac{x}{2} \right) K_0\left( \frac{x}{2} \right), \]

where \( B(x) = 0.85 + 1.35\sqrt{x} + 0.38x \) and \( K_0 \) is the modified Bessel function of the second kind.
For $\Theta_e > 2$, we use the ultra-relativistic approximation (Alexanian 1968; Maxon 1972):

$$j_{ee}^\nu = \frac{1}{4\pi} \frac{3}{4} \frac{n_e^2 \sigma_T c \alpha_f}{\rho} \exp(-x) \left\{ \frac{28}{3} + 2x + \frac{x^2}{2} + 2 \left( \frac{8}{3} + 4 \frac{x}{3} + x \right) \left[ \ln \left( \frac{2kT_e}{m_e c^2} \right) - 0.577 \right] - \exp(x) Ei(-x) \left( \frac{8}{3} - 43x + x^2 \right) \right\}. \tag{D4}$$

Formal formulae (D2)–(D4) connect smoothly at $\Theta_e = 0.1$ and 2. Bremsstrahlung for $e^-e^+$ interactions dominates over $e^-i^-$ ones for $\Theta_e > 1$.

Integration of $j_{ee}^\nu$ over frequencies and solid angle gives the total cooling rate, $\Lambda_{ee}$. For details of the radiative transfer scheme, see Dolence et al. (2009); we sample the bremsstrahlung radiation field in the same way as for synchrotron radiation, except that bremsstrahlung is emitted isotropically in the fluid frame. For the range of parameters considered in this work, energy loss by free–free emission is small in comparison to synchrotron and Compton losses.

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