Solving the Job Shop Scheduling Problem with Ant Colony Optimization

Alysson Ribeiro da Silva
Computer Science Graduate Program
Federal University of Minas Gerais
Belo Horizonte, Brazil

Abstract—The Job Shop Schedule Problem (JSSP) refers to the ability of an agent to allocate tasks that should be executed in a specified time in a machine from a cluster. The task allocation can be achieved from several methods, however, this report it is explored the ability of the Ant Colony Optimization to generate feasible solutions for several JSSP instances. This proposal models the JSSP as a complete graph since disjunct models can prevent the ACO from exploring all the search space. Several instances of the JSSP were used to evaluate the proposal. Results suggest that the algorithm can reach optimum solutions for easy and harder instances with a selection of parameters.

Index Terms—Ant Colony Optimization, Natural Computing, Schedule, Planning

1 INTRODUCTION

Ant Colony Optimization is an optimization method from Evolutionary Computing that was inspired by ant colony behavior [1][2]. It allows for optimization processes through traces of pheromones left by simulated ants through the best-found path towards a valid solution in a graph. For example, several ants explore the search space, in a directed graph, and when reaching a terminal vertex, the solution is extracted from the found path. At the end of the process, the simulated ants deposit pheromones that are utilized by other ants to reach the same solution or to explore the space near it. In this report, the ACO was used to solve the Job Shop Schedule Problem (JSSP).

The JSSP can be typically modeled as a disjunctive graph, however, due to its nature in this report, the problem was modeled as a complete graph. The distance between each graph edge was calculated as a delta Makespan, where the global Makespan is generated with an efficient algorithm.

All the routines were implemented in C++ due to performance issues. The experiments were conducted in two steps. The first step comprises the parameter evaluation, whereas in the second the algorithm’s best solutions were evaluated. Four instances of the JSSP from [3] were used for evaluation, the ft06, la01, la29, and la49. The parameter evaluation was conducted with ft06 and la29. Differently, the algorithms’ performance was evaluated in all instances. The proposal shows convergence for all instances towards the global optimum. At the end of this report is shown the evaluation of solutions obtained from the ACO for several other JSSP instances.

2 JOB SHOP SCHEDULING PROBLEM

The JSSP can be defined as an optimization problem, where the objective is to minimize the total Makespan. Let $J = \{j_1, \ldots, j_n\}$ be the set of all services, $M = \{m_1, \ldots, m_m\}$ the set of machines, $O_{ij} = \{O_{i1}, \ldots, O_{ik}\}$ the set of operations $J_i$ that need to be executed in the machine $M_k$ for $\tau_{ik}$, and $\tau_{ik} = \{\tau_{i1}, \ldots, \tau_{ik}\}$ time period, the objective is to:

$$
\text{Minimize } C_{\text{max}} = \max(t_{ik} + \tau_{ik}), \forall j \in J \text{ and } M \in M
$$

$$
t_{ik} - \tau_{ih} \geq 0 \text{ if } O_{ih} \text{ precedes } O_{ik}
$$

$$
t_{pk} - t_{ik} + K(1 - \gamma_{ipk}) \geq \tau_{ik}
$$

$$
t_{ik} - t_{pk} + K(\gamma_{ipk}) \geq \tau_{pk}
$$

$$
\gamma_{ipk} = 1 \text{ if } O_{ik} \text{ precedes } O_{pk}
$$

$$
\gamma_{ipk} = 0 \text{ otherwise}
$$

(1)

where $i, p \in J$ and $K > \sum_{i}(\sum_{k} \tau_{ik} - \min(\tau_{ik}))$.

3 METHOD

This proposal uses the ACO to find the best possible Makespan. The problem was modeled as a complete graph similar to a disjunctive graph approach [2]. Next, the ACO is presented alongside the JSSP model.

3.1 Elitist ACO

For the optimization of the Makespan, it is proposed to use an elitist ACO, where the transition between vertices is done through the Equation 2:

$$
p_{ij} = \frac{(\tau_{ij})^\alpha(n_{ij})^\beta}{\sum_l(\tau_{il})^\alpha(n_{il})^\beta}
$$

(2)

where, $p_{ij}$ is the transition probability from $i$ to $j$, $\tau_{ij}$ is the pheromone amount for arc $i \rightarrow j$, $\alpha$ is the exponent of pheromone amount, $\beta$ is the exponent of distance, $\gamma$ is the weighting factor, $n$ is the number of possible arcs, $\tau_{ij}$ is the path length, $\tau_{il}$ is the path length from $l$ to $i$ for all $l$, $\eta_{il}$ is the path length from $l$ to $i$ for all $l$. This proposal models the JSSP as a disjunctive graph approach [2]. Next, the ACO is presented alongside the JSSP model.
between $i$ and $j$, and $\sum_j (\tau_{ij})^\alpha (\eta_{ij})^\beta$ is the sum of the pheromones times the distance for all available transitions from vertex $i$.

The pheromones decay is computed with Equation $3$, where $d$ represents the decaying factor for the pheromone quantity $\tau_{ij}$.

$$\tau_{ij} = (1 - d)\tau_{ij} \tag{3}$$

Pheromones update is achieved through Equation $4$, where $L_k$ is the total Makespan generated by the Ant $k$ if $k$ passed through the arc $i \rightarrow j$.

$$\tau_{ij} = \tau_{ij} + \sum_k \frac{Q}{L_k} \tag{4}$$

### 3.2 JSSP Model

The graph model used to represent tasks and resources is inspired by disjunctive graphs. A disjunctive graph is defined as $G = (V, C \cup D)$, where $V$ is the set of all tasks, $C$ is the edges that connect tasks from the same service, and $D$ represents edges that connect tasks from the same machine. The problem with this representation regards the fact that it does not allow an Ant to generate all possible feasible solutions. Consequently, in this research the search graph is defined as $G' = (V, C \cup D \cup F)$, where $F$ contains the complement of $G$. To avoid invalid paths, each Ant should check for two restrictions in this model. The first prevents from selecting tasks with parents yet running and the second prevents from choosing tasks already chosen.

### 3.3 Distance Between Tasks

A distance between tasks is computed as $1/d$ to minimize the total Makespan, where $d$ is the time necessary to complete a task. However, this computation can lead to distances that do not represent the real impact of the chosen task in reality. Consequently, it is proposed that a distance $\eta_{ij}$ between $i$ and $j$ to be computed as $\Delta m = s - c$, where $m$ is the current Makespan from a partial path to $i$, and $s$ is the future possible Makespan if $j$ is chosen. To extract a solution from the generated path, a Gantt diagram is built by following the best-found policy in $G'$.

### 4 Experimental Configuration

Two-stage experimentation was conducted on an AMD CPU using Ubuntu 20.04. In the first stage, the impact of the parameters of the ACO was evaluated. Differently, the second stage evaluates the solutions obtained by the algorithm when solving the JSSP. The ACO was deployed in C++ with OpenMP due to performance concerns.

Four instances of the JSSP were primarily used for the experiments and they are described in Table 1. For the parameter evaluation, only the ft06 and la01 were used since they portray traits of all four instances. Finally, the quality of the generated solution was evaluated for the best-observed parameters for all instances of Table 1. At the end of the document, it is provided the evaluation of the algorithm for other instances.

All results convey the statistics from 30 execution with 1000 iterations each. Each execution was done in parallel with OpenMP. The number of iterations was fixed at 1000 to avoid large memory consumption when running in parallel. The initial and minimum pheromone value for all evaluations was set to 1 to avoid normalization issues in case $\eta_{ij} < 1$.

### 5 Parameter Analysis

All parameters were evaluated according to the configurations in Table 2. Each cell in the table that is depicted with the orange color represents the main parameters. The columns Elit., Init., and Inc. represent flags that can be active or not the elitism, random initialization, and pheromones deposit policy, respectively. In general, the flag can assume three values that define how the initial vertex is chosen for each Ant, such as:

- 0) Random.
- 1) Only random at the first iteration and fixed at the others.
- 2) Only one Ant for service. If the operation mode is set to 2, the Ants quantity is ignored and only one Ant is created for each service.

Differently, the flag defines which policy will be used to deposit pheromones. If set to 0, then the Equation 4 is used. Otherwise, Equation 5 is used,

$$\tau_{ij} = \tau_{ij} + \left( \frac{Q}{L_k} \right)^P |P| - i \tag{5}$$

where $P$ represents the path generated by the Ant $K$, $P$ is the total path length, and $i$ represents a vertex from $P$. Consequently, it is possible to ensure that vertices closest to the object will receive more pheromones.

The results for all the configuration parameter selection from Table 2 is shown in Fig. 1 for the instance ft06. The $Y$ axis represents the average Makespace for 40 executions and the $X$ axis shows each parameter configuration. According to the obtained results, the worst performance was 58.7 with a deviation of 1.28. This behavior could indicate that in this particular instance the pheromones can help Ants far from the optimum. Differently, the configurations Beta, Evap. $C$, and Inc. were able to obtain near optimum Makespans. In particular, the configuration Evap. $C$, which regards modifications in the pheromone’s evaporation rate, obtained the optimum Makespan with 0 deviation. The observed behavior could indicate that a smoother pheromones evaporation helps Ants to retain more memory regarding the environment exploration and consequently to explore new areas surrounding a solution. Other configurations show an average Makespan between 56 and 58 with subtle changes in deviation.

### Table 1

| Inst. | Jobs x Machines | Optimum |
|-------|----------------|---------|
| ft06  | 6x6            | 35      |
| la01  | 10x3           | 666     |
| la29  | 20x10          | 1157    |
| la40  | 15x13          | 1222    |

| TABLE 1 | Instances for the JSSP problem. |
TABLE 2
Configurations to check parameters influence in the ACO’s performance.

| Config. | Ite. | Ants. | Elig. | Alpha | Beta | Evap. | Q  | Init | Exec. | Inc. |
|---------|------|-------|-------|-------|------|-------|----|------|-------|------|
| elitism | 1000 | 100   | 1     | 1     | 1    | 0.1   | 1  | 0    | 30    | 0    |
| alpha   | 1000 | 100   | 0     | 2     | 1    | 0.1   | 1  | 0    | 30    | 0    |
| beta    | 1000 | 100   | 0     | 1     | 1    | 1.0   | 1  | 1    | 30    | 0    |
| evap. A | 1000 | 100   | 0     | 1     | 1    | 0.01  | 1  | 1    | 30    | 0    |
| evap. B | 1000 | 100   | 0     | 1     | 1    | 0.1   | 1  | 1    | 30    | 0    |
| evap. C | 1000 | 100   | 0     | 1     | 1    | 1     | 1  | 1    | 30    | 0    |
| init. A | 1000 | 100   | 0     | 1     | 1    | 0.1   | 1  | 1    | 30    | 0    |
| init. B | 1000 | 100   | 0     | 1     | 1    | 0.1   | 1  | 1    | 30    | 0    |
| init. C | 1000 | 100   | 0     | 1     | 1    | 0.1   | 1  | 2    | 30    | 0    |
| ite. A  | 100  | 100   | 0     | 1     | 1    | 0.1   | 1  | 0    | 30    | 0    |
| ite. B  | 500  | 100   | 0     | 1     | 1    | 0.1   | 1  | 0    | 30    | 0    |
| ite. C  | 1000 | 100   | 0     | 1     | 1    | 0.1   | 1  | 2    | 30    | 0    |

6 PARAMETER SELECTION
The selected parameters for evaluation are described in Table 3, where the selection was made based on convergence, deviation, and best obtained Makespan.

The total iterations were set to 1000 to avoid memory consumption and statistics were collected during 30 executions. Evaporation was set to 0.1 for slow convergence and stability. Next, elitism was used to ensure the best solution for newly developed generations. Differently, the Beta parameter was set to 2 for a greater influence of the delta Makespan in the generated paths and to avoid pheromone degradation. Finally, Q was set to 1 for normalization purposes in the final solution.

7 RESULTS
All the results of the algorithm’s performance are shown in Table 4. It was observed that the algorithm was able to reach a global optimum in both, fl06 and la01 instances, instances. In fl06, the average Makespan was 56.57±0.49. Differently, la01 portrayed an average Makespace of 673.07±2.08. Furthermore, the algorithm obtained sub-optimum average Makespans of 1429.67±7.17 for instance la29 and 1361.67±6.96 for instances la40, respectively. The observed behavior could have happened due to several factors, for example, the found path, pheromones reinforce parameter, Ants path selection policy, evaporation rate, and other parameters. Also, the fitness landscape could also have influenced the performance of the algorithms, since the aforementioned instances are harder.

8 CONCLUSION
In this technical report, the capability of the ACO (Ant Colony Optimization) was evaluated to solve the JSSP (Job Shop Schedule Problem) problem. Several instances were evaluated and the algorithm was able to reach the optimum value for simple instances. Furthermore, it reached near-optimum ones in harder instances. Nevertheless, evaluation of several other instances of the JSSP presented in Appendix A, shows that it can solve the problem with low standard deviation in several scenarios. The parameter selection plays an important role in the algorithm’s performance since for different parameters the algorithm was not able to achieve optimum results.
TABLE 3
Selected parameters for evaluation.

| Inst. | Re. | Ants. | Elit. | Alpha | Beta | Evap. | Q | Init | Exec. | Inc. |
|-------|-----|-------|-------|-------|------|-------|---|------|-------|------|
| ft06  | 1000| -     | 1     | 1     | 2    | 0.01  | 1 | 2    | 30    | 1    |

TABLE 4
Obtained Makespan for all evaluation instances.

| Inst. | Optimum | Minimum | Maximum | Average | Std. | Sup. Std. | Inf. Std. |
|-------|---------|---------|---------|---------|------|-----------|-----------|
| ft06  | 55      | 55      | 58      | 56,57   | 1,09 | 0,49      | 0,5       |
| La01  | 666     | 666     | 687     | 673,07  | 6,36 | 3,99      | 2,08      |
| La29  | 1157    | 1388    | 1455    | 1429,67 | 17   | 7,17      | 13,69     |
| La40  | 1222    | 1333    | 1381    | 1361,67 | 12,06| 4,66      | 9,14      |

REFERENCES
[1] C. Blum and M. Sampels, “An ant colony optimization algorithm for shop scheduling problems,” *Journal of Mathematical Modelling and Algorithms*, vol. 3, no. 3, pp. 285–308, Sep 2004. [Online]. Available: https://doi.org/10.1023/B:JMMA.0000038614.39977.6f
[2] A. Puris, R. Bello, Y. Trujillo, A. Nowe, and Y. Martinez, “Two-stage aco to solve the job shop scheduling problem,” in *Progress in Pattern Recognition, Image Analysis and Applications*, L. Rueda, D. Mery, and J. Kittler, Eds. Berlin, Heidelberg: Springer Berlin Heidelberg, 2007, pp. 447–456.
[3] C. Turguner and O. K. Sahingoz, “Solving job shop scheduling problem with ant colony optimization,” in 2014 IEEE 15th International Symposium on Computational Intelligence and Informatics (CINTI), 2014, pp. 385–389.
[4] Zhiqiang Zhang, Jing Zhang, and Shujuan Li, “A modified ant colony algorithm for the job shop scheduling problem to minimize makespan,” in 2010 International Conference on Mechanic Automation and Control Engineering, 2010, pp. 3627–3630.
[5] T. Weise, “jsspinstancesandresults: Results, data, and instances of the job shop scheduling problem,” Hefei, Anhui, China, 2019–2020, a GitHub repository with the common benchmark instances for the Job Shop Scheduling Problem as well as results from the literature, both in form of CSV files as well as R program code to access them. [Online]. Available: https://github.com/thomasWeise/jsspInstancesAndResults
TABLE 5
ACO best-found solution for extra instances.

| Inst. | Optimum | Minimum | Maximum | Average | Std. | Std. Sup. | Std Inf. |
|-------|---------|---------|---------|---------|------|-----------|----------|
| abz5  | 1254    | 1272    | 1303    | 1289,4  | 11,77| 0,90      | 6,34     |
| abz9  | 678     | 610     | 634     | 621,4   | 8,24 | 4,00      | 4,30     |
| orb10 | 944     | 1019    | 1074    | 1046,8  | 16,80| 9,67      | 7,30     |
| swv05 | 1424    | 1757    | 1772    | 1765,8  | 5,81 | 2,83      | 2,50     |
| swv19 | 2843    | 3026    | 3052    | 3039,8  | 9,81 | 1,00      | 4,50     |
| swv20 | 2825    | 2956    | 2997    | 2969,6  | 20,07| 10,03     | 13,30    |
| yn1   | 884     | 1022    | 1046    | 1035,6  | 8,91 | 3,30      | 3,50     |
| yn2   | 872     | 1030    | 1070    | 1057,2  | 14,85| 5,04      | 0,00     |
| yn3   | 859     | 1008    | 1038    | 1026,2  | 10,89| 2,87      | 6,00     |
| yn4   | 929     | 1145    | 1170    | 1157,4  | 8,31 | 4,00      | 4,71     |
| dmu01 | 2501    | 3097    | 3193    | 3150,6  | 37,10| 0,90      | 18,52    |
| dmu20 | 3604    | 4771    | 4864    | 4806,4  | 36,64| 14,50     | 5,73     |
| dmu50 | 3496    | 4648    | 4715    | 4689,2  | 23,73| 9,74      | 17,00    |
| dmu80 | 6439    | 8674    | 8632    | 8757,8  | 54,20| 30,39     | 35,30    |
| ta01  | 1231    | 1375    | 1413    | 1392    | 13,70| 8,65      | 2,90     |
| ta10  | 1241    | 1476    | 1510    | 1488,4  | 11,91| 9,50      | 3,68     |
| ta40  | 1651    | 2125    | 2201    | 2135    | 26,14| 24,00     | 10,71    |
| ta20  | 5183    | 6039    | 6105    | 6066,6  | 24,82| 10,50     | 9,42     |

APPENDIX A
OTHER INSTANCES EVALUATION

In this Appendix, it is shown the performance of the ACO for several other instances from [5]. The parameters used during evaluation are present in Table 3 for 10 executions each. The results are shown in Table 5 where the algorithm suggests that it can achieve optimum or near optimum results for several instances.