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The effect of adaptive behavior on risk propagation in industrial symbiosis networks

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A B S T R A C T

The complex symbiotic relationship in the industrial symbiosis network (ISN) may cause new risks for firms. In view of this problem, previous studies mainly regard the ISN as a static system, without considering the adaptive behavior of firms. This paper establishes a risk propagation model of the ISN based on the change of firm state, proposes four kinds of reconnection strategies to model the adaptive behavior, and uses numerical simulation to investigate the effect of adaptive behavior on risk propagation. The results demonstrate that all the reconnection strategies play an inhibitory role in the risk propagation. Therein, the effectiveness of PP strategy is the best, followed by RR strategy, and DP (SP) strategy. In any case, the effect of reconnection strategies on risk propagation will improve with the increase of the disconnection probability and network resilience. Additionally, the more decentralized weight distribution will weaken the inhibition of adaptive behavior on risk propagation.

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1. Introduction

Econophysics is a new interdisciplinary subject, which aims to use the concepts and methods in physics to study economic problems [1,2]. During the last fifteen years, many physical theories have been widely used in the field of economic management. For instance, Suzuki et al. [3] analyzed the implied volatility smile and the skewness premium in the option pricing based on an agent-based model. Rejichi and Aloui [4] applied the Hurst exponent to verify the long-term memory of all Middle East and North African stock returns. Fang et al. [5] found that the success of mergers and divestitures mainly depended on the ability and the development status of entities by building an evolutionary network model. To some extent, physical methods are conducive to obtain some new insights into economic problems. In this paper, a propagation dynamics model on complex networks is established to investigate the effect of adaptive behavior on risk propagation in the industrial symbiosis network (ISN). Since the extensive economic growth mode neglects the basic ecological criteria [6,7], this study not only can improve the robustness of ISN, but further mitigate serious environmental pollution and resource shortage.

Industrial symbiosis, as a strategy to promote sustainable development, enables traditionally separate firms to cooperate with each other in resource sharing [8]. Through the effective substance circulation, it can avoid the secondary pollution caused by traditional waste treatment (e.g., incineration and landfill) to the environment, and realize the echelon utilization of waste [9]. When many relatively independent firms form a complex system by symbiosis, an ISN emerges [10]. Although the ISN is helpful to improve environmental and social benefits, due to complicated symbiosis relationships, firms may encounter new risk problems, such as resource mismatch [11,12]. Suppose that the two firms are in symbiotic cooperation, considering that industrial waste is not generated according to a specific demand [13,14], under such resource allocation, if one exits the network, the other may occur risk owing to the imbalance of supply and demand. This phenomenon that the risk diffuses among firms can be regarded as the risk propagation of the ISN.

At present, the research on the risk of the ISN is still infancy, and there are few related literatures. Based on network analysis, Chopra and Khanna [15] recommended that diversity, redundancy, and versatility should be considered in the construction of an industrial symbiosis system. In order to avert the cascading failure, Kuznetsova et al. [16] adopted the input-output inoper-
ability model to evaluate the risk caused by destructive events, providing guidance for formulating recovery measures against destructive events. Based on the vulnerability framework, Wang et al. [17] constructed a modified cascaded failure model for directed weighted networks, and stressed that the interaction between economic fluctuation and network structure was associated with the vulnerability of the ISN. Wu et al. [18] quantified the environmental risk of the iron and steel ISN, the result demonstrated that the coking plant was at a high environmental risk level and the ironworks-thermal power plant link had the highest risk propagation coefficient. Finally, Xu et al. [19] depicted the failure propagation characteristics of the water network in the ISN, and found out corresponding weak links by using the ant colony algorithm.

The existing studies have shown plentiful results, but there is room for further discussion. Most literature only focuses on static networks, i.e., the network structure will not change in the process of risk propagation. In real networks, the structure evolves with the change of nodes’ state [20]. For example, during the outbreak of COVID-19, in social networks, susceptible people will keep the social distance away from infected individuals. Similarly, in the ISN, the network can adjust the structure by itself. For any firm, if it currently faces a large risk, from the perspective of risk aversion, its symbiotic partners may terminate relevant cooperation (i.e., disconnection) to reduce the loss of profits. In this case, due to the adjustment of the network structure, the original risk propagation path and propagation process will alter accordingly. That is to say, the disconnected firms will be no longer affected by risk. This kind of network with a synergy of network structure and propagation dynamics is called an adaptive network. Since Gross et al. [21] proposed such a concept for the first time, it has been applied in many fields, such as neural systems [22,23], epidemic model [24,25], and social games [26,27], etc. Thus, integrating the adaptive behavior into the research framework is reasonable and necessary, and it is of great significance to investigate the effect of adaptive behavior on risk propagation in the ISN, for maintaining its stable development, optimizing resource allocation, and promoting green economic development.

The remainder of this paper is organized as follows. Section 2 proposes the generation algorithm of ISNs, and constructs a WFW model for risk propagation. Section 3 designs four reconnection strategies to model adaptive behavior. Section 4 performs the relevant numerical simulation. Finally, the conclusions are summarized in Section 5.

2. Risk propagation in the ISN

This section is divided into two subsections. Section 2.1 provides the general generation algorithm of ISNs, while Section 2.2 illustrates the process of risk propagation.

2.1. Construction of ISNs

The ISN is essentially a win-win cooperative network. Firms in the network take on different functions [28], which can obtain economic, environmental, or social benefits through the transmit-receive of waste with each other [29]. In this paper, the ISN will be modeled as a graph $G(V, E)$, where $V = \{1, 2, \ldots, N\}$ and $E = \{\{i, j\}, i, j \in V\}$ represent the set of nodes (i.e., firms in the ISN) and the set of edges (i.e., symbiotic relationship between firms), respectively.

According to the evolution of several existing industrial ecosystems, the ISN shows obvious power-law distribution [30]. Specifically, in the initial phase, networks are characterized by a small number of firms participating in fewer synergies. As the network develops, one or more core groups may form. In the mature phase, a lot of firms are involved in numerous synergies with some anchors in the system, e.g., Kalundborg in Denmark. At the same time, considering that the extent of symbiosis among firms is different, the ISN can be treated as a weighted network. Thus, the generation algorithm of ISN is based on BBV model [31], which is presented as follows:

(1) Initialization: Starting from a small number ($m_0$) of vertices connected by links with assigned weight $w_0$.

(2) Preferential attachment: A new node will be implanted at each time, which connects to existing $m$ ($m < m_0$) nodes with the similar weight $w_0$. The probability $p_i$ that the new node will be connected to node $i$ is defined as

$$p_i = \frac{s_i}{\sum_{j \in V} s_j}$$

where $s_i$ indicates strength of node $i$, whose value is equal to the sum of weights between node $i$ and its symbiotic partners, i.e., $\sum_{j \in \Omega_i} w_{ij}$.

(3) Weight evolution: After the new node is connected to node $i$, the strength of node $i$ will increase $w_0$. Meanwhile, due to the influence of the new node, the strength also has an additional increment $\delta$, which will be proportionally distributed among previous symbiotic partners. Thus, the change of strength is shown as below

$$s_i \rightarrow s_i + w_0 + \delta$$

and the weights of node $i$’s edges will also alter correspondingly, i.e.,

$$w_{ij} \rightarrow w_{ij} + \Delta w_{ij}$$

$$\Delta w_{ij} = \delta \frac{w_{ij}}{s_i}$$

2.2. Risk propagation

The structure and resource flow of ISNs are complicated, and individuals in the network usually are not separate. The mutual interaction between firms brings not only the collective competitive advantage but also the corresponding risks. Once a firm fails, its symbiotic partners may also be triggered by the interrupt of the supply-demand balance. Obviously, risk propagation is a dynamic process. The firm occurring risk can recover to the working state following a certain probability. As time goes on, such a firm may be in the failure state again. Therefore, based on the framework of epidemic dynamics, we propose the WFW model.

Combining the reality of ISNs, the following assumptions are posited:

(1) Firms in the ISN have two states, i.e., working state ($W$) and failure state ($F$). The state of firm $i$ at time $t$ can be expressed as $\pi_i(t)$, when firm $i$ is in the working state, the $\pi_i(t)$ is equal to 0, otherwise it equals 1.

(2) The firm only accepts risk when it is in the working state. Contrarily, when the firm is in the failure state, it will propagate risk.

If there is risk propagation in the ISN, then, firms in the working state will occur risk with probability $\alpha$, and that in the failure state will recover with probability $\beta$. The state transition is illustrated in Fig. 1.
Considering the heterogeneity of firms, it may not be appropriate to utilize the same probability to depict the state change of each firm. Next, we will give the definition of probability $\alpha_i$ and $\beta_i$ in detail.

In this paper, probability $\alpha_i$ is mainly affected by the symbiotic cooperative extent and symbiotic dependence, which measure the breadth and depth of symbiosis respectively. For instance, firm $i$ and $j$ have symbiotic cooperation in many aspects, but these symbioses will not involve the core business of firm $i$. In this case, the symbiotic cooperative extent between them is high, but the dependence of firm $i$ on $j$ is low. On the contrary, if there is only one kind of symbiosis between firm $i$ and $k$, but such cooperation is quite important to firm $i$. Then, although their cooperation amount is not much, firm $i$ has a strong dependence on firm $k$. We adopt the edge’s weight to characterize symbiotic cooperative extent, and introduce the parameter $\mu_{ij} \in (0, 1)$ to represent the symbiotic dependence of firm $i$ on firm $j$. It is noteworthy that $\mu_{ij}$ and $\mu_{ji}$ are usually unequal. Finally, the probability $\alpha_i$ that firm $i$ occur risk is given by

$$\alpha_i = \frac{\mu_{ij} W_{ij}}{s_i}$$  \hspace{1cm} (5)

As for probability $\beta_i$, first, it is related to the network resilience since this feature reflects the ability of the ISN to keep operating after disruptions, which has been verified by several studies [32, 33]. The more the resilience, the higher the recovery probability of firms. Second, the importance of the firm also has an impact on the probability $\beta_i$. If a firm with the dominant role is in the failure state, it needs more resources and time to recover, so as to lead a relatively low recovery probability. Here, we use degree ($k_i$) and strength ($s_i$) to reflect the importance of each firm. Therefore, the probability $\beta_i$ can be defined as follows

$$\beta_i = \frac{\varepsilon}{\theta_1 k_i + \theta_2 s_i}$$  \hspace{1cm} (6)

where $\varepsilon$ indicates the network resilience. Meanwhile, $\theta_1$ and $\theta_2$ are tunable parameters, which control the proportion of degree and strength, respectively.

3. Structural adaptive behavior

After building the process of risk propagation in the ISN, we need to explain the adaptive behavior of network structure. When the risk begins to spread in the ISN, the firm will terminate the cooperation with the symbiotic partner that is in the failure state for the purpose of reducing the loss and avoiding the risk. At the same time, in order to maintain the balance of resource flow, the firm will choose to establish a new symbiotic relationship with other working firms. This process is the adaptive behavior of network structure.

Specifically, if a firm fails, its symbiotic partners will disconnect their symbioses with probability $\lambda$. Then, the working firms search for new connections based on their own preferences. Here, we propose four kinds of reconnection strategies. Let $f_1$ indicate the firm which needs to construct a new symbiotic relationship, all reconnection strategies are shown as below:

1. Reconnect randomly (RR). In this case, $f_1$ will randomly connect to the firm in working state as a new symbiotic partner.

2. Degree preference (DP). This strategy means that $f_1$ preferentially connects to the firm with the maximum degree in the ISN.

3. Strength preference (SP). This strategy means that $f_1$ preferentially connects to the firm with the maximum strength in the ISN.

4. Path preference (PP). In this strategy, $f_1$ preferentially connects to the closest firm. In this paper, we use the principle of similarity weight, that is, weight is inversely proportional to distance, the greater the weight, the closer the distance. Therefore, the reciprocal of the weight will be applied to express the distance between two firms.

It should be noted that if some candidate firms happen to have the same degree, strength, or distance, $f_1$ will randomly choose one of them.

Obviously, in the process of reconnection, the change of network structure is accompanied by the transfer of weights. Thus, we suppose that the weight of the disconnected edge will be given to the newly generated edge. In parallel, considering that reconnection can change the degree of some firms, the original weights of relevant firms will also alter correspondingly.

When a edge of firm $i$ is broken, the change of the weight between firm $i$ and its remaining neighbors obeys

$$W_{ij} \rightarrow W_{ij} - \Delta W_{ij}$$  \hspace{1cm} (7)

where the term $\Delta W_{ij}$ is the same as Eq. (4). When a new edge is connected to firm $i$, the change of the weight between firm $i$ and its remaining neighbors is similar to Eq. (3).

In summary, the structural adaptive behavior is shown in Fig. 2. Therein, the black and white circles represent the firms with and without risks, respectively. Since firm $i$ is in the failure state, in order to keep the working state, firm $j$ terminates the symbiotic relationship with $i$, and establishes a new connection with firm $l$. For other neighbors that are still connected to firm $i$, all of them will occur risk with a certain probability, e.g., firm $k$. In this process, reconnection can lead to the variations of edges’ weights. Superscript “−” and “+” stand for the decrease and increase of weights, respectively.

4. Simulation

In this section, we perform some numerical simulations. Stipulating that at time $t$, the proportion of firms in the failure state is expressed as $f(t) = \sum_i \pi_i(t)/N$. According to the generation algorithm of ISN, the relevant parameters are set as follows: $N = 500$, $m_0 = 3$, $w_0 = 1$, and $m = 1$. In the initial stage, we randomly
choose 10% of all firms as the risk source, which indicates that these firms are in the failure state at the beginning. Then, we analyze the effects of adaptive behavior on risk propagation in the ISN from three aspects: reconnection strategy, disconnection probability, and network resilience.

4.1. The effects of different reconnection strategies

In order to probe the effects of four adaptive behaviors on risk propagation in the ISN, we let $\lambda = 0.04$, $\theta_1 = 0.3$, $\theta_2 = 0.3$, $\varepsilon = 0.6$, and $\delta = 0.5, 1, 1.5$ in turn. At the same time, the process of risk propagation in the static network (SN) is also given, so as to better compare the effect of each reconnection strategy. The simulation results are shown in Fig. 3. Concretely, Fig. 3(a), (b), (c) illustrate the effects of four adaptive behaviors on risk propagation when $\delta$ is equal to 0.5, 1, and 1.5, respectively.

As shown in Fig. 3, compared with the static network, in the symbiosis network with the adaptive behavior, the scope of risk propagation will decrease significantly. Essentially, the risk propa-
gation of the ISN is the transfer of failure state. In the initial stage, since the dominant firm usually has a lot of symbiotic partners, it will be more vulnerable to the risk, and then enter a failure state. After that, such kernel firms can further affect others, which enhances the propagation ability of the risk, and lead to a rapid increase in the proportion of failed firms. However, as time goes on, due to the adaptive behavior, the transmission route of risk is disconnected according to a probability, the network structure will change gradually, and firms can also slowly begin to recover. Thus, the scope of risk propagation will show a stable trend or a downward-stable trend, which depends on the strength of the adopted reconnection strategy.

In Fig. 3, it is not difficult to find that PP strategy has the most effective inhibitory impact on risk propagation in the ISN, followed by RR, DP, and SP. Under the PP strategy, the firm in the working state will preferentially connect with the relatively near nodes. On the one hand, such a strategy can promote the formation of small-sized communities in the network, make the symbiotic cooperation in the local area closer, and impel the enhancement of community structure. On the other hand, the principle of proximity reduces the dependence of firms on kernel nodes, in other words, it also relieves the pressure of kernel nodes. Therefore, this kind of diversified network can restrain the risk propagation in the ISN. Consequently, there will be a decline in the proportion of the firm that is in the failure state. In the case of DP, firms will preferentially connect to the node with a high degree. As can be seen from Fig. 3, although DP dwindles the scope of risk propagation, its effect is the weakest. The reason is mainly that when each firm considers the value of the degree as the connection indicator, the ISN will show a core-periphery form, which shortens the average path length of the network. Once the high-degree node occurs risk, then, more firms in the working state will be affected. Under this strategy, after $f(t)$ goes through the rising period, the failure and recovery of firms may keep a dynamic equilibrium. Thus, the trajectory of the corresponding curves does not display a downward trend. With regard to SP, considering that there is a positive relationship between degree and strength, the effects of SP on risk propagation are similar to that of DP, which can be easily verified by the simulation results. As for RR strategy, since the firm will choose the target node randomly in this situation, the firm with the highest degree (or strength), the firm with the nearest path, and other normal firms could all be the target of a connection. This results in the performance of this strategy lie between RR and DP (SP).

Based on the above analysis, generally, PP strategy can always minimize the scope of risk propagation in the ISN, in contrast, DP and SP have a relatively weak inhibitory effect. This suggests that blindly attaching to high-degree or high-strength firms is not the optimal choice. To some extent, the principle of proximity does not need to obtain information about all the firms in the network, which reduces the search cost of the target firm. Thus, it is a more effective measure to prevent risk propagation in the ISN.

Additionally, with the increase of weight dispersion (the value of $\delta$), the impact of all strategies on risk propagation in the ISN will
gradually be impaired. The mechanism of \( \delta \) can be explained in two aspects: the occurrence and recovery of risk. Under the same network structure, if the value of \( \delta \) is greater, it can be proved easily that the probability of firms occurring risk is higher accordingly (see Appendix A). On the other hand, according to Eq. (6), in this case, the recovery probability of firms will also decrease. Consequently, the increase of \( \delta \) has a positive effect on risk propagation.

4.2. The effects of different disconnection probability

In order to probe the effects of disconnection probability on risk propagation in the ISN, we let \( \lambda \) equal to 0.02, 0.03, 0.04, 0.05, 0.06, and other parameters are the same as the assignment in Section 4.1. Fig. 4, Fig. 5, and Fig. 6 illustrate the results with \( \delta = 0.5, 1, 1.5 \), and subfigures (a), (b), (c), and (d) show the effects of disconnection probability under the strategies of RR, DP, SP, and PP. Due to the similarity of the curves, we take Fig. 4 as an example to elaborate.

Obviously, regardless of reconnection strategy, the higher the disconnection probability, the lower the proportion of firms in the failure state, i.e., the scope of risk propagation has a negative relationship with the disconnection probability. As illustrated in Fig. 4, when \( \lambda \) rises from 0.02 to 0.06, once it exceeds a certain value, \( f(t) \) under each strategy will show a downward process. For RR strategy, such a value is equal to 0.04. Under DP and SP strategies, only when \( \lambda = 0.06 \), the proportion of failed firms will decrease slightly. As regards PP strategy, it has a strong inhibitory effect, even when \( \lambda \) is equal to 0.02. This verifies the effectiveness of PP strategy again. The increase of the disconnection probability means that the risk propagation path is more easily cut off. Firms will be more sensitive to the state changes of symbiotic partners, thereby protecting their own interests as much as possible. From the simulation results, the high disconnection probability is always advantageous. However, according to reality, there may be an optimal solution for the disconnection probability. Because on the one hand, the firm has a certain recovery ability, and on the other hand, before the establishment of a symbiotic relationship, both parties will sign a contract to clarify the rights and obligations. The frequent replacement of symbiotic partners will increase the cost of firms, which is also not conducive to the overall stability of the network. Thus, it is necessary for decision-makers to weigh the gain and loss of benefits, so as to identify the appropriate disconnection probability.

4.3. The effects of different network resilience

In order to probe the effects of network resilience on risk propagation in the ISN, we let \( \varepsilon \) equal to 0.4, 0.5, 0.6, 0.7, 0.8. The remaining parameters are the same as the assignment in Section 4.1. The results when \( \delta = 0.5, 1, 1.5 \) are illustrated in Fig. 7, Fig. 8, and Fig. 9, and subfigures (a), (b), (c), and (d) show the effects of
network resilience under the strategies of RR, DP, SP, and PP. Similarly, we still take Fig. 7 as an instance.

Fig. 7 shows that the effect of each strategy on risk propagation rises with the increase of the network resilience, which is similar to the disconnection probability. In the process of risk propagation, the network resilience can help the failed firms recover to work faster, as presented in Eq. (6). According to Fraccascia et al. [34], the resilience of ISNs can be considered from two aspects, i.e., resource generation and consumption, and the improvement of resilience implies that the increase of the types and quantity of resources in the network. When the network resilience is low, the failed firm caused by the imbalance of supply and demand may not be able to recover through resource scheduling in time. The lack of resources is easy to lead to the situation of network rigidity, which is not favorable to the control of risk propagation. As shown in Fig. 7, although reconnection has an inhibitory effect on risk, the scope of risk propagation is still at a relatively high level. Nevertheless, if the network resilience increases gradually (from 0.4 to 0.8), the proportion of firms in the failure state will only show a small increase in a short period of time, and then decline continuously. Therefore, the network resilience is an important factor to control risk propagation in the ISN. The decision-makers should carefully consider the variety of firms in the process of planning network construction, thereby enhancing the resilience of the ISN.

5. Conclusion

As an innovative way of resource utilization, the ISN can alleviate the enormous pressure on the ecological environment. However, frequent resource exchange and complex resource flow may reduce the stability of the network. In order to deal with this potential risk, the working firm will tend to avoid symbiosis with the failed nodes, which has a significant impact on risk propagation. Considering the interaction between network topology and risk propagation, this study proposes four reconnection strategies, and explores the effect of adaptive behavior on risk propagation in the ISN. The main conclusions are drawn as follows:

First, in all situations, the effectiveness of PP strategy is better than other strategies. This indicates the superiority of the principle of proximity. No matter whether we know the global information or not, it is not a wise way to blindly attach to the dominant firm. Second, the scope of risk propagation will decrease with the increase of the disconnection probability. Since every reconnection strategy has a certain inhibition on risk propagation, for any firm, it may not easily change its reconnection preference in the short term, then, improving the disconnection probability can avoid the risk better. Third, with the increase of network resilience, the effect of adaptive behavior on risk propagation will improve accordingly. Considering that network resilience is strongly associated with each firm, the access principle of organizations should be enhanced, to ensure that the introduced firms have stable management and technology. Finally, on the premise of

Fig. 7. The effects of network resilience on risk propagation when $\delta = 0.5$. 
The unchanged network scale, the more decentralized weight distribution will weaken the inhibition of adaptive behavior on risk propagation.

The biggest difference between our study and existing literature is the adaptive behavior. In this paper, we mainly investigate the role of adaptive behavior on risk propagation in the ISN, which indicates that the adaptive behavior of firms is our main research object. Comparatively, Chopra and Khanna [15] focused on the recovery process of symbiotic networks, rather than the dynamic characteristics of risks. Kuznetsova et al. [16] and Wu et al. [18] aimed to assess the impact of risk, at the same time, their results might have been tailored for Kalundborg and iron and steel ISN, respectively, compared with our study, our results have a wider scope of application. Although Wang et al. [17] also constructed several indicators to explore the cascaded failure of ISN, their model cannot reflect the change of network structure. To overcome this limitation, we allow firms to disconnect or connect with new symbiotic relationships in the evolution process according to different preferences.

CRediT authorship contribution statement

Junliang Yang: Conceptualization, Investigation, Methodology, Software, Validation, Visualization, Writing - original draft, Writing - review & editing. Kai Zheng: Software, Supervision, Validation, Visualization, Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

Suppose that the probabilities of firms occurring risk in two networks with different $\delta$ are $\alpha_{1i}$ and $\alpha_{2i}$, and the corresponding $\delta$ is denoted by $\delta_1$ and $\delta_2$. When the new node is connected to node $i$, we have

$$\alpha_{2i} - \alpha_{1i} = \mu_{ij}\left[\frac{w_{ij} + \frac{w_{ij}}{s_i} \delta_2}{s_i + w_0 + \delta_2} - \frac{w_{ij} + \frac{w_{ij}}{s_i} \delta_1}{s_i + w_0 + \delta_1}\right]$$

$$\mu_{ij}\left[\frac{(w_{ij} + \frac{w_{ij}}{s_i} \delta_2)(s_i + w_0 + \delta_1) - (w_{ij} + \frac{w_{ij}}{s_i} \delta_1)(s_i + w_0 + \delta_2)}{(s_i + w_0 + \delta_2)(s_i + w_0 + \delta_1)}\right]$$

(A.1)

Since $\mu_{ij}$ and $(s_i + w_0 + \delta_2)(s_i + w_0 + \delta_1)$ are always positive, we just need to simplify the value of $(w_{ij} + \frac{w_{ij}}{s_i} \delta_2)(s_i + w_0 + \delta_1) - (w_{ij} + \frac{w_{ij}}{s_i} \delta_1)(s_i + w_0 + \delta_2)$ as follows,

![Fig. 8. The effects of network resilience on risk propagation when $\delta = 1$.](image-url)
\[
(W_{ij} + \frac{W_{ij}}{S_i} \delta_2)(S_i + W_0 + \delta_1) - (W_{ij} + \frac{W_{ij}}{S_i} \delta_1)(S_i + W_0 + \delta_2)
\]
\[
= W_{ij}(S_i + W_0) + W_{ij} \delta_1 + W_{ij} \delta_2 + \frac{W_{ij}}{S_i} \delta_2 W_0 + \frac{W_{ij}}{S_i} \delta_2 \delta_1
\]
\[
- (W_{ij}(S_i + W_0) + W_{ij} \delta_2 + W_{ij} \delta_1 + \frac{W_{ij}}{S_i} \delta_1 W_0 + \frac{W_{ij}}{S_i} \delta_1 \delta_2)
\]
\[
= \frac{W_{ij}}{S_i} \delta_2 W_0 - \frac{W_{ij}}{S_i} \delta_1 W_0 = \frac{W_{ij} W_0}{S_i} (\delta_2 - \delta_1)
\]

It is easy to find out that if \( \delta_2 > \delta_1 \), then \( \alpha_{2i} > \alpha_{1i} \). In other words, the more the \( \delta \), the higher the probability of firms occurring risk.

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Fig. 9. The effects of network resilience on risk propagation when \( \delta = 1.5 \).
