Electroweak chiral Lagrangian with a light Higgs

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Abstract

In this talk we discuss the structure of electroweak low-energy effective theories where the Higgs is non-linearly realized, typically in scenarios where the Higgs is a pseudo Nambu-Goldstone boson (pNGB) of some beyond Standard Model (BSM) symmetry. The organization of the perturbative counting and the relevance of the various next-to-leading order contributions is studied. We discuss some new results on the structure of the one-loop ultraviolet divergences and the contribution from tree-level heavy resonance exchanges to the low-energy effective theory, which are applied to a couple of explicit examples in order to show how, in the non-linear effective theory –the electroweak chiral Lagrangian with a light Higgs (ECLh)–, one-loop corrections can be as important as the contribution from higher dimension operators.

1 Chiral power counting in non-linear effective theories

There are two usual approaches to low-energies electroweak (EW) effective field theories (EFT) according to how the Higgs $h$ and the EW Goldstones $\omega^a$ are introduced:

1. Linear EFT: The Higgs $h$ and EW Goldstone $\omega^a$ fields conform a complex doublet $\Phi$ of the EW symmetry.

2. Non-linear EFT: The Higgs field $h$ is introduced as a singlet and the EW Goldstones are non-linearly realized through the unitary matrix $U(\omega^a)$.

The non-linear approach is indeed more general: it includes also the linear-Higgs EFT, as one can always write down the doublet $\Phi$ in its polar form in terms of the modulus $(v + h)/\sqrt{2}$ and a unitary matrix $U(\omega^a)$. The non-linearity of the model is indeed a quality that is related to the separation from the linear scenario rather than whether one chooses to express $h$ and $\omega^a$ in a non-linear way. Many BSM frameworks show this non-linear structure, e.g., models where $h$ is a pNGB and, hence, it transforms non-linearly under the spontaneously broken generators of the BSM symmetry.

The non-linear EFT Lagrangian is sorted out according to the “chiral” dimension of its operators, not through their canonical dimension which one uses in linear EFT’s:

$$\mathcal{L}_{ECLh} = \mathcal{L}_p^2 + \mathcal{L}_p^4 + \ldots$$

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where derivatives and masses of the particles count as $\mathcal{O}(p)$, $h/v$ count as $\mathcal{O}(p^0)$ — and the same occurs with other boson fields, and fermion fields scale like $\psi/v \sim \mathcal{O}(p^{1/2})$. $v = (\sqrt{2} G_F)^{-1/2} = 246$ GeV is the Higgs vacuum expectation value. A simple dimensional analysis shows that the amplitudes have an expansion of the form (e.g. for a $2 \rightarrow 2$ process)

$$
\mathcal{M} \sim \frac{p^2}{v^2} + \left( c_L^{(\text{LO (tree)})} - \frac{\Gamma_{k,n} p}{16 \pi^2} \frac{\ln p}{\mu} + \ldots \right) \frac{p^4}{v^4} + \mathcal{O}(p^6),
$$

(2)

The three singled-out contributions have different origins:

- **The LO amplitude** is given by the tree-level diagrams provided by the vertices from the LO Lagrangian $\mathcal{L}_{\mu^2}$.

- **The NLO amplitude** has two types of contributions:
  1. **One-loop diagrams** with vertices only from the LO Lagrangian $\mathcal{L}_{\mu^2}$. These contributions are typically suppressed with respect to (wrt) the LO in the form $p^2/\Lambda_{\text{non-in}}^2$, with this scale $\Lambda_{\text{non-in}} \sim 4\pi v$ directly related to the non-linearity of the BSM scenario: these corrections vanish when the Higgs can be linearly realized through a complex doublet $\Phi^0$.
  2. **Tree-level diagrams** with one vertex of higher dimension, from $\mathcal{L}_{\mu^4}$. In the underlying BSM theory these constants can get contributions from tree-level exchanges of heavy states of mass $M_R$ not included in the EFT. Although the renormalized EFT couplings also get corrections from resonance loop diagrams, the suppression of these NLO corrections can be estimated as $c_k^2 p^2/v^2 \sim p^2/M_R^2$.

In the case when the scale $\Lambda_{\text{non-in}}$ that governs the non-linearity is much higher than the masses $M_R$ of the intermediate heavy states NLO loops are highly suppressed wrt NLO tree-level corrections and the linear-Higgs EFT approach is more appropriate. However, in the case when both scales $\Lambda_{\text{non-in}}$ and $M_R$ are similar, one must account for both the tree-level and one-loop NLO corrections to have reliable determinations of the observables at that precision and the linear-Higgs EFT is then inappropriate.

## 2 One-loop NLO corrections: ultraviolet divergences

The background field method in path integral allows one to study the loop corrections to the effective action. Expanding the Lagrangian $\mathcal{L}_{\mu^2}$ in powers of the fluctuation $\vec{\eta}^T = (\Delta^a, \epsilon)$ (with $\Delta^a$ and $\epsilon$ providing the Goldstone and Higgs fluctuations, respectively) around the solutions of the equations of motion (EoM) one obtains:

$$
\mathcal{L}_{\mu^2} = \mathcal{L}_{\mu^2}^{\mathcal{O}(\eta^0)} + \mathcal{L}_{\mu^2}^{\mathcal{O}(\eta^1)} + \mathcal{L}_{\mu^2}^{\mathcal{O}(\eta^2)} + \mathcal{O}(\eta^3),
$$

(3)

where the $\mathcal{O}(\eta^0)$ term yields the tree-level diagrams with LO vertices, the requirement that the linear term vanishes provides the EoM at LO and $\mathcal{L}_{\mu^2}^{\mathcal{O}(\eta^2)}$ gives the one-loop NLO amplitude. The remaining terms provide the amplitudes at two loops and higher, i.e., at next-to-next-to-leading order (NNLO) and higher in the chiral expansion. $\mathcal{L}_{\mu^2}^{\mathcal{O}(\eta^2)}$ can be rearranged as the quadratic form

$$
\mathcal{L}_{\mu^2}^{\mathcal{O}(\eta^2)} = -\frac{1}{2} \vec{\eta}^T (d_\mu d^\mu + \Lambda) \vec{\eta},
$$

(4)

in terms of the corresponding operators $\Lambda$ and $d_\mu$ operators determined by the structure of $\mathcal{L}_{\mu^2}$. The integration of this term over the fluctuations $\vec{\eta}$ yields

$$
S^{1\ell} = \frac{1}{2} \Gamma \log (d_\mu d^\mu + \Lambda) = -\frac{\mu^{d-4}}{16\pi^2(d-4)} \int d^4x \{ \frac{1}{12} [d_\mu, d_\nu] [d^\mu, d^\nu] + \frac{1}{2} \Lambda^2 \} + \text{finite}
$$

[5] In this case the $\mathcal{O}(p^4)$ couplings are of the order of $c_k^2 \sim (16\pi^2)^{-1}$. 10.

[6] It is instructive to observe that from the chiral counting point of view $\Lambda \sim \mathcal{O}(p^2)$ and $d_\mu \sim \mathcal{O}(p)$.
where \( \langle ... \rangle \) stands for the matrix trace, and \( \mu \) is the renormalization scale and \( d \) is the space-time dimension in dimensional regularization. The 1st line has been reexpressed in the 2nd line in terms of the basis of EFT operators \( \mathcal{O}_k \). The ultraviolet (UV) divergences are cancelled out by means of appropriate \( \mathcal{O}(p^4) \) counter-terms of the form

\[
\mathcal{L}_p^k = \sum_k c_k \mathcal{O}_k, \quad \text{with the renormalizations } c_k = c_k^{\text{r}}(\mu) + \frac{\mu^{d-4} \Gamma_k}{16\pi^2(d-4)}.
\]  

The couplings of the EFT Lagrangian that get renormalized due to Higgs and EW Goldstone loops are listed in Ref. 9.

Of course, though important, this is not the end of the story in non-linear EFT’s: one must compute the full one-loop amplitude for every particular process under study, with the full structure of logs, polylogs and rational 12 pieces, not just the running 9.

3 Tree-level NLO corrections: predictions from composite resonance exchanges

The exchange of heavy resonances leads at low energies to NLO and higher order EFT operators suppressed by powers of \( p/M_R \) 8,13. At NLO one only needs the resonance Lagrangian compatible with the SM symmetries that are linear in the resonance fields \( R \) and have at most two derivatives (or analogous light scales \( p \)) 8,13. For instance, in the case of a parity preserving strongly coupled model, the resonance Lagrangian with triplet vector (\( V \)) and axial-vector (\( A \)) multiplets that is relevant for \( \mathcal{L}_p^4 \) has the form 8:

\[
\mathcal{L}_V + \mathcal{L}_A = \sum_{R=V,A} \langle R_{\mu\nu} \chi_{R}^{\mu\nu} \rangle,
\]

with \( \chi_V^{\mu\nu} = \frac{F_V}{2\sqrt{2}} f_+^{\mu\nu} + \frac{i G_V}{\sqrt{2}}[u^{\mu},u^{\nu}] + ... \) and \( \chi_A^{\mu\nu} = \frac{F_A}{2\sqrt{2}} f_+^{\mu\nu} + ... \) The spin–1 resonances \( R_{\mu\nu} \) are described in the antisymmetric tensor formalism 13, and one has the tensors \( f_+^{\mu\nu} = u^i W_{\mu\nu} u \pm u^i B_{\mu\nu} u \) given by the field strengths of the \( W_\mu \) and \( B_\mu \) fields and \( u_{\mu} = i u D_{\mu} u^{\dagger} u \), with \( u^2 = U(\omega^a) \) 14. \( \chi_{R}^{\mu\nu} \) contains only particles in the low-energy EFT. Integrating out the spin–1 resonances at low energies, one obtains the tree-level contribution to the \( \mathcal{O}(p^4) \) ECLh 8

\[
\mathcal{L}_p^{\text{from } V,A} = - \sum_{R=V,A} \frac{1}{M_R^4} \left( \langle \chi_{R}^{\mu\nu} \chi_{R}^{\nu\mu} \rangle - \frac{1}{2} \langle \chi_{R}^{\mu\nu} \rangle^2 \right) - \frac{i F_V G_V}{4 M_V^2} \langle f_+^{\mu\nu} [u_\mu, u_\nu] \rangle + ... \tag{8}
\]

obtaining a prediction for the \( \mathcal{O}(p^4) \) constants in terms of the \( V \) and \( A \) masses and couplings 8

\[
a_1|_{V,A} = - \frac{F_V^2}{4 M_V^2} + \frac{F_A^2}{4 M_A^2} \quad \text{UV compl.} \equiv - \frac{v^2}{2} \left( \frac{1}{M_V^2} + \frac{1}{M_A^2} \right),
\]

\[
a_2 - a_3|_{V,A} = - \frac{F_V G_V}{2 M_V^2} \quad \text{UV compl.} \equiv - \frac{v^2}{2 M_V^2}, \tag{9}
\]

where we have used some UV-completion hypotheses in the last equalities of each line: in the case of \( a_1 \) we assume the strongly coupled theory is asymptotically free and one has the two Weinberg sum-rules \( F_V^2 = F_A^2 + v^2 \) and \( F_V^2 M_V^2 = F_A^2 M_A^2 \) 15,16, in the case of \( a_2 - a_3 \) the electromagnetic form-factor into two composite EW Goldstone bosons in the theory with resonances is \( F_{\gamma \omega}(s) = 1 + (F_V G_V/v^2)(s/(M_V^2 - s)) \) and demanding that it vanishes at infinite momentum transfer \( s \to \infty \) yields the constraint \( F_V G_V = v^2 \) used above 8.

4 The importance of being earnest and keeping the full NLO: two examples

The importance of the different NLO pieces becomes obvious if one consider the previous asymptotically-free strongly coupled benchmark scenario. At one-loop in the resonance theory the oblique parameters lead to the constraint \( M_A^2 = M_V^2 / \kappa_W \) and the 95% confidence
Taking the lower allowed limits for $M_V$ and $\kappa_W$ (which produce the maximal possible effects) one gets the predictions

- **Oblique S–parameter (NLO tree dominance):** at NLO one has the structure\(^\text{7}\)

\[
S = -16\pi \left[ a_1^\prime(M_V)_{\text{Tree}} + \frac{\kappa_W^2 - 1}{192\pi^2} \left( \frac{\ln M_Z^2}{m_H^2} + \frac{5}{6} \right) \right] ,
\]

with $a_1^\prime(\mu) = -v^2/4(M_U^2 + M_A^2) + \left\{ [8/3 + \ln(\mu^2/M_Z^2)] - \kappa_W^2 [8/3 + \ln(\mu^2/M_A^2)] \right\} / (192\pi^2)\).

- **$\gamma\gamma \rightarrow W^+W^-$ scattering (NLO loop dominance):** in the energy region $m_{W,Z,H}^2 \ll s \ll \Lambda_{ECLh}$ this amplitude is given at NLO by the scalar function\(^\text{7}\)

\[
A_{\gamma\gamma \rightarrow \gamma\gamma}^{\text{NLO}} = \frac{1}{v^2} \left[ 2\kappa_W c_γ^\prime \left( a_1^\prime - a_2^\prime + a_3^\prime \right)_{\text{tree}} + \frac{\kappa_W^2 - 1}{8\pi^2} \left( a_1^\prime - a_2^\prime + a_3^\prime \right)_{\text{loop}} \right] ,
\]

with the $a_1^\prime - a_2^\prime + a_3^\prime$ estimate from the previous Section. The value of the $h \rightarrow \gamma\gamma$ $O(p^4)$ coupling $c_γ^\prime$ is in principle undetermined in our analysis\(^\text{8}\). If the impact of $c_γ^\prime$ is as important as that from $a_1^\prime - a_2^\prime + a_3^\prime$ one realizes that the one-loop correction is more important that the tree-level NLO amplitude and should not be dropped.

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\(^d\) The $h \rightarrow WW$ coupling is normalized such that $\kappa_{WW}^{SM} = 1$. 