Tetra-quark mesons with exotic quantum numbers

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Tetra-quark mesons with exotic quantum numbers, their production rates and decay properties are studied, because they are useful to establish existence of tetra-quark mesons.

Tetra-quark mesons can be classified into the following four groups in accordance with difference of symmetry property of their flavor wavefunctions (wfs.) \( \Box \) (and \( ] \)),

\[
\{qq\bar{q}\bar{q}\} = \{qq\bar{q}\bar{q}\} \oplus \{\bar{q}q\bar{q}\bar{q}\} \oplus \{qq\bar{q}\bar{q}\} \oplus \{\bar{q}q\bar{q}\bar{q}\}\]

(1)

with \( q = u, d, s \) (and \( c \)), where parentheses and square brackets denote symmetry and anti-symmetry, respectively, of flavor wfs. under exchange of flavors between them. Each term on the right-hand-side (r.h.s.) of Eq. (1) is again classified into two groups with \[ \Box \] \( 3_c \times 3_c \) and \( 6_c \times 6_c \) of the color \( SU_c(3) \), because these two can lead to color-singlet tetra-quark states. The force between two quarks \( qq \) is attractive (or repulsive) \( ] \) when they are of \( 3_c \) (or \( 6_c \)) of \( SU_c(3) \), so that the \( 3_c \times 3_c \) state is taken as the (dominant part of) lower lying one, unless these states largely mix with each other \( ] \). Spin \( (J) \) and parity \( (P) \) of (dominant components of) \{qq\bar{q}\bar{q}\} \text{ and } \{\bar{q}q\bar{q}\bar{q}\} \text{ mesons are } J^P = 0^+, 1^+, 2^+ \text{, respectively, while } \{qq\bar{q}\bar{q}\} \text{ and } \{\bar{q}q\bar{q}\bar{q}\} \text{ mesons are } J^P = 0^+, 1^+, 2^+, 3^+, 4^+. \text{ However, we ignore this } (qq\bar{q}\bar{q}) \text{ because no signal of scalar } (K\pi)_{I=3/2} \text{ state which can come from the } (qq\bar{q}\bar{q}) \text{ state } \Box \text{ has been observed in the region } \lesssim 1.8 \text{ GeV } \Box, \text{ although there is an argument that an iso-tensor state might have been observed around 1.6 GeV in the } \gamma\gamma \rightarrow \rho\rho \Box \text{.}

In inclusive \( e^+e^- \) annihilation \( [7, 8] \), the charm-strange scalar \( D_{s0}^+(2317) \) was discovered in the \( D_{s}^+\pi^0 \) mass distribution, while no signal has been observed in the \( D_{s}^{+}\gamma \) channel, and, therefore, the above results lead to a severe constraint \( [8] \),

\[
\frac{\Gamma(D_{s0}^+(2317) \rightarrow D_{s}^{+}\gamma)}{\Gamma(D_{s0}^+(2317) \rightarrow D_{s}^{+}\pi^0)} \bigg|_{\text{CLEO}} < 0.059.
\]

(2)

This implies \( ] \) that \( D_{s0}^+(2317) \) is an iso-triplet scalar state. The simplest way \( ] \) to realize \( D_{s0}^+(2317) \) as an iso-triplet scalar state is to assign it to \( [en][\bar{s}n]_{I=1} \), \( (n = u, d) \) in Eq. (1) which is denoted as \( \tilde{F}_s^+ \). In this case, the observed narrow width of \( D_{s0}^+(2317) \) is understood by a small overlap of color and spin wfs. \( ] \) (1). The above assignment is consistent \( ] \) with the observations in \( B \) decays, i.e., signals of charm-strange scalar mesons have been observed in the \( D_s^+\gamma \) channel as well as in the \( D_s^+\pi^0 \), and branching fractions of \( B \) decays producing them have been measured as \( ] \) \( Br(B \rightarrow \bar{D}\bar{D}_{s0}^+(2317)[D_s^+\pi^0]) = (8.5^{+2.1}_{-1.9}\pm2.6) \times 10^{-4} \) and \( Br(B \rightarrow \bar{D}\bar{D}_{s0}^+(2317)[D_s^+\gamma]) = (2.5^{+2.0}_{-1.8}(< 7.5)) \times 10^{-4} \), where these signals observed in the \( D_s^+\gamma \) and \( D_s^+\pi^0 \) are denoted as \( \tilde{D}_{s0}^+(2317)[D_s^+\gamma] \) and \( \tilde{D}_{s0}^+(2317)[D_s^+\pi^0] \), respectively. Therefore, it is natural to identify \( \tilde{F}_s^+ \) and its iso-singlet partner \( \tilde{F}_0^+ \sim [en][\bar{s}n]_{I=0} \) with \( \tilde{D}_{s0}^+(2317)[D_s^+\pi^0] \) and \( \tilde{D}_{s0}^+(2317)[D_s^+\gamma] \), respectively, because \( \tilde{F}_s^+ \) decays dominantly into \( D_s^+\pi^0 \) while \( \tilde{F}_0^+ \) into \( D_s^+\gamma \). For more details, see Refs. \( 2, 9, 11 \) and \( ] \).

Another candidate \( ] \) of tetra-quark meson is \( X(3872) \) with \( ] \) \( J^P = 1^+ \), as seen below. It was discovered in the \( \pi^+\pi^- J/\psi \) mass distribution by the Belle \( ] \) \( 19 \), and confirmed \( ] \) \( 20 \) by the CDF, D0 and Babar. (Hereafter, we describe \( J/\psi \) as \( \psi \) to save space.) Its charge conjugation parity (\( C \)) can be even, because it decays into the \( \gamma\psi \) \( 21, 22 \). However, it decays into two states with opposite \( G \)-parities \( ] \),

\[
\frac{Br(X(3872) \rightarrow \pi^+\pi^-\pi^0\psi)}{Br(X(3872) \rightarrow \pi^+\pi^-\psi)} = 1.0 \pm 0.4 \pm 0.3.
\]

(3)

This is puzzling because the well-known strong interactions conserve \( G \)-parity. In addition, it has been noted \( ] \) \( 21, 22 \) that the decay \( X(3872) \rightarrow \pi^+\pi^-\psi \) proceeds through \( \rho^0\psi \). Because a search for its charged partners \( X(3872) \) has given a negative result \( ] \), however, it would be an iso-singlet state, and isospin conservation does not work in the decay. Besides, it has been pointed out \( ] \) \( 21, 22 \) that the \( X(3872) \rightarrow \pi^+\pi^-\pi^0\psi \) decay proceeds through the sub-threshold \( X(3872) \rightarrow \langle\psi\rangle \). If isospin is conserved in this decay, \( X(3872) \) would be an iso-singlet state. This is consistent with the above negative result on \( X(3872) \).

Although various approaches \( ] \) \( 26, 27 \) have been proposed to solve the above puzzle, they are unnatural because the well-known \( \omega\rho^0 \) mixing \( ] \) as the origin of the isospin non-conservation in nuclear forces, which is compatible \( ] \)
with the measured rate for the \( \omega \rightarrow \pi^+\pi^- \) decay \[29\], has not been considered. Assuming that \( X(3872) \) is a tetra-quark system like \( \{\bar{c}u|\bar{n}d\} + \{\bar{c}n|\bar{d}u\} \) \[16\] (or a \( D^0\bar{D}^{*0} \) molecule) with even C-parity and that the isospin non-conservation under consideration is caused by the \( \omega R \) mixing \[17\], we have reproduced \[14, 17\] the measured ratios,

\[
R^0_{\text{belle}} = 0.14 \pm 0.05 \quad \text{and} \quad R^0_{\text{Babar}} = 0.33 \pm 0.12, \tag{4}
\]
given by the Belle \[21\] and Babar \[30\], respectively, where \( R^0 \) is defined by \( R^0 \equiv \text{Br}(X(3872) \rightarrow \gamma \psi)/\text{Br}(X(3872) \rightarrow \pi^+\pi^-\psi) \). In the above, we have considered the \( X(3872) \rightarrow \gamma \psi \) decay in place of \( X(3872) \rightarrow \pi^+\pi^-0\psi \) in Eq. \[33\], because both of them are controlled by the same subthreshold \( X(3872) \rightarrow \omega \psi \) decay when the vector meson dominance (VMD) \[31\] is applied to the radiative decay, while the kinematics of the former is much simpler than the latter. In contrast, if \( X(3872) \) were assumed to be a charmonium \( X_{cc} \), the \( \psi \) pole contribution, \( X_{cc} \rightarrow \psi \psi \rightarrow \gamma \psi \), would be dominant in the radiative decay because of the OZI rule \[32\], and as the result, \( (R^0_{\text{Belle}}) \geq (R^0_{\text{Babar}}) \sim (R^0_{\text{Belle}}) \) would be obtained \[14, 17\]. Therefore, we see that a tetra-quark interpretation of \( X(3872) \) is favored over the charmonium, although a small mixing of \( X_{cc} \) is not excluded. In addition, it should be noted that production \[33\] of the prompt \( X(3872) \) seems to favor a compact object like a tetra-quark meson over a loosely bound molecule \[34\]. Although an argument against the above conclusion was proposed \[35\], it does not explicitly prove that a loosely bound molecule provides a sufficiently large cross section for \( X(3872) \) production. Full widths of the above interpretations. However, for example, neutral and doubly charged partners, \( \hat{c}\eta \pi \) to a peak in the \( D^* \) mesons listed above are of the same order of magnitude (in the region \( 0 < \rho < 2.0 \) GeV). Therefore, if all the sizes of amplitudes are of the same order of magnitude, all the rates for the decays under consideration, which are expected to be dominantly of S-wave, would be of the same order of magnitude except for the CKM suppressed \( H^+_{\text{acc}} \) production. Full widths of \( B_u, B_d \) and \( B_c \) are approximately proportional to (parent mass)\(^3\), so that the width of \( B_c \) is larger by about a factor 2.4 than those of \( B_u \) and \( B_d \) (but within the same order of magnitude). Therefore, amplitudes (with the same order of magnitude) for decays under consideration lead to branching fractions of the same order of magnitude.

Productions of \( K^+_{\text{acc}} \) and \( K^+_{\text{acc}} \) listed above can be described by the quark-line diagram, Fig. 1(c), which is of the

\[\includegraphics[width=\textwidth]{fig1.png}\]

Fig. 1. Productions of tetra-quark scalar \( \hat{E}^0 \) and axial-vector \( E_{A(\bar{c}d)}^0, E_{A(\bar{c}d)}^{(0,+)} \), \( K^+_{\text{acc}} \) and \( H^+_{\text{acc}} \) with exotic quantum numbers.
same type as Fig. 2(a) and Fig. 3(b) in Ref. \[31\] describing $B_u^+ \to \bar{D}^0 \bar{F}^+_I$ and $B_d^0 \to D^0 \bar{D}^+_I$, respectively. Because $\text{Br}(B_u^+(B_d^0) \to \bar{D}^0(D^-)D^+_s(2317))_{\text{exp}} \sim 10^{-(4-3)}$ as mentioned before, production rates for $K_{A\text{acc}}^+$ would be very crudely estimated as

$$\text{Br}(B_c^+ \to D^+ K_{A\text{acc}}^+) \sim \text{Br}(B_c^- \to \bar{D}^0 K_{A\text{acc}}^+) \sim 10^{-(4-3)}.$$ (5)

Production of $H_{A\text{acc}}^+$ with $I = 0$ is described by the diagram Fig. 1(d). However, it is now the CKM suppressed decay, so that the rate for $H_{A\text{acc}}^+$ production would be more suppressed by a factor $\sim 1/10 - 1/20$ than the above case, although it is described by the same type of diagram. Productions of scalar and axial-vector tetra-quark mesons with $C = -S = 1$ can be described by the diagrams, (a) and (b) in Fig. 1. These diagrams are of the same type as that of Fig. 4(c) in Ref. \[37\] describing $B_d^0 \to K - \bar{F}^+_I$ whose rate also has already been measured as $\text{Br}(B_d^0 \to K - \bar{D}^+_s(2317)) \cdot \text{Br}(D_s^0(2317) \to D^+_s\pi^0) = (5.3_{-1.3}^{+1.5} \pm 0.7 \pm 1.4) \times 10^{-5}$. By taking $\text{Br}(D_s^0(2317) \to D^+_s\pi^0) \sim 1$ as expected in Eq. \[4\], it follows that $\text{Br}(B_d^0 \to K - \bar{D}^+_s(2317)) \sim 10^{-5(5-4)}$. However, this is smaller by about one order of magnitude than $\text{Br}(B_d^0 \to D^- D_s^0(2317))_{\text{exp}}$. This would be because the former includes an $s\bar{s}$ creation while the latter an $u\bar{d}$ creation (but no $s\bar{s}$) as discussed in Ref. \[37\]. In fact, a phenomenological analysis and a recent lattice QCD suggest that $s\bar{s}$ component is much smaller compared with $u\bar{u}$ and $d\bar{d}$ components in the sea quarks of nucleon (0.07 - 0.22 in the former \[42\] and most likely 0.05 in the latter \[43\]). Therefore, we expect that the diagrams (a) and (b) in Fig. 1 would not be suppressed compared with the other ones, because they now involve no $s\bar{s}$ creation, i.e.,

$$\text{Br}(B_u^+ \to D^{*-} E_{A(\text{cs})}^0) \sim \text{Br}(B_u^- \to D^{*-} E_{A(\text{cs})}^0) \sim \text{Br}(B_u^+ \to D^- \bar{E}^0)$$
$$\sim \text{Br}(B_d^0 \to D^{*-} E_{A(\text{cs})}^0) \sim \text{Br}(B_d^0 \to D^{*-} E_{A(\text{cs})}^0) \sim \text{Br}(B_d^0 \to D^- \bar{E}^0)$$
$$\sim 10 \times \text{Br}(B_d^0 \to K - \bar{D}^+_s(2317)) \sim 10^{-5(5-3)}.$$ (6)

because all these decays are dominantly of $S$-wave and the center-of-mass momenta ($p$'s) of the daughter particles are of the same order of magnitude as discussed above. Therefore, the rates estimated above would be large enough to observe these exotic mesons, except for $H_{A\text{acc}}^+$.

Table 1. Scalar and axial-vector tetra-quark mesons with exotic quantum numbers. Mass values are estimated by a quark counting.

| Tetra-quark meson | mass (GeV) | 2-body Threshold | 3-body radiative (or weak) | Possible decays |
|-------------------|-----------|-----------------|--------------------------|-----------------|
| $E^0$             | 2.32      | $(D\bar{K})$    | 2.36                     | $(\bar{K}\pi)K$, ($\bar{K}\pi\pi)K$, ... |
| $H_{A\text{acc}}^+$ | 3.87     | $DD^*$          | 3.88                     | $DD\pi$, $DD\gamma$ |
| $K_{A\text{acc}}$ | 3.97      | $DD^*_D$, $D^*D_s^0$ | 3.98                     | $DD_s\pi$, $DD_s\gamma$ |
| $E_{A(\text{cs})}$, $E_{A(\text{cs})}$ | 2.97 | $\bar{K}D^*$, $(\bar{K}^* D)$ | 2.61 $(2.86)$ | $\bar{K}D\pi$, $\bar{K}D\gamma$ |

Although informations of decay properties of these exotic mesons in addition to their production rates would be useful to search for them, rates for two body decays of axial-vector mesons would be very sensitive to their mass values because they are (very) close to thresholds of their two body decays, as seen in Table 1. Therefore, calculations of rates for these two body decays would be inevitably model dependent at the present stage. In addition, no experimental data as the input data is known, so that they are left as our future subjects, and their possible radiative decays in addition to two- and three-body decays are listed, because they would be useful to search for these mesons. In the case of the scalar $E^0$, its estimated mass has been lower than the $D\bar{K}$ threshold. It implies that $E^0$ could decay neither through the ordinary strong interactions nor through the electromagnetic ones, because it has an exotic set of quantum numbers. Thus, it would decay through weak interactions \[2\], for example, $E^0 \to (D\bar{K}) \to (\bar{K}\pi)K$, $(K\pi\pi)K$, $(K\bar{K}u)K$, etc. Regarding the axial-vector mesons, however, their crudely estimated mass values are (very) close to thresholds of possible two-body decays, so that possible three-body and radiative decays are additionally listed. Radiative three-body decays might be important in search for axial-vector mesons with exotic quantum numbers because of decay properties of $D^*$ and $D^*_s$.

In summary, we have studied scalar and axial-vector mesons with exotic quantum numbers. We have estimated their production rates by comparing quark-line diagrams describing their productions with those of $D^*_s(2317)$, and have seen that they can be large enough to observe in $B$ decays. Their possible decay modes are also studied. However, estimate of their rates is left as a subject in future, because no experimental data as the input data has not been known at the present stage.

In the present scheme, $X(3872)$ has been interpreted as a tetra-quark $\{(cn)(\bar{c}\bar{n}) + (cn)\bar{e}\bar{e}\}_{I=0}$ meson \[10\] with even
C-parity. To confirm this, it is also awaited to observe its opposite C-parity partner \(\{[cn](\bar{c}\bar{n}) - (cn)[\bar{c}\bar{n}]\}\) in the \(\psi\pi^0\pi^0\) channel [14].

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