Spectroscopy using the Anisotropic Clover Action

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The calculation of the light-hadron spectrum in the quenched approximation to QCD using an anisotropic clover fermion action is presented. The tuning of the parameters of the action is discussed, using the pion and $\rho$ dispersion relation. The adoption of an anisotropic lattice provides clear advantages in the determination of the baryonic resonances, and in particular that of the so-called Roper resonance, the lightest radial excitation of the nucleon.

The calculation of the properties of excited states of hadrons composed of light constituents is complicated by two factors. Firstly, lattice interpolating operators can be constructed to transform under the irreducible representations of the cubic group of the lattice, but then the corresponding excited states belong to many irreducible representations of the full continuum rotation group. Secondly, the signal-to-noise ratio for such correlators degrades rapidly at increasing temporal separations. The first problem can be addressed by measuring a matrix of correlators, and identifying particular states common to the lattice irreducible representations in the approach to the continuum limit. The second problem can be addressed through the use of an anisotropic lattice, having a much smaller lattice spacing in the temporal direction allowing the behaviour of the correlators at small separations to be examined over many more time slices. In this poster, we will investigate the second technique.

We use an anisotropic version of the standard (unimproved) Wilson action which can be constructed from the standard isotropic action by a simple rescaling of the fields. In addition to the usual coupling $\beta$, there is a tunable parameter $\xi_0$, the bare anisotropy, which we tune to give the required renormalised anisotropy $\xi$ where $\xi = a_s/a_t$, the ratio of lattice spacings in the spatial and temporal directions \cite{2}. We use the anisotropic version of the usual clover-fermion action\cite{2,3} with Dirac operator:

$$\mathcal{M} = m_0 - \nu_0 W_0 - \frac{\nu}{\xi_0} \sum_k W_k - \frac{1}{2} \left[ \omega_0 \sum_k \sigma_{0k} F_{0k} + \frac{\omega}{\xi_0} \sum_{k<l} \sigma_{kl} F_{kl} \right],$$

(1)

where

$$W_\mu = \frac{1}{2} \left[ (1 - \gamma_\mu) U_\mu(x) \delta_{x+\hat{\mu},y} + (1 + \gamma_\mu) U_\mu^T(x - \hat{\mu}) \delta_{x-\hat{\mu},y} \right].$$

(2)

This form of the action preserves the projection property of the Wilson action for $r = 1$. There are two tunable parameters for the unimproved Wilson fermion action: $\kappa$, and either $\nu$ or $\nu_0$. We will set $\nu_0 \equiv 1$, and then tune $\nu$. The clover term introduces a further two parameters, $\omega$ and $\omega_0$.

In the case of free fermions, where the clover term vanishes, imposition of the continuum dispersion relation requires

$$\nu^2 = e^{a_t m} \frac{\sinh a_t m}{a_t m} - r \xi \sinh a_t m.$$  

(3)

To determine the coefficient of the clover term, we now impose a constant background field, and the dispersion relations then require

$$\omega_0 = \frac{1}{\xi} \left( \frac{e^{a_t m}}{a_t m} - \frac{\nu}{\sinh a_t m} \right)$$

(4)

$$\omega = \frac{1}{\xi} \left( \frac{e^{a_t m}}{a_t m} - \frac{\nu^2}{\sinh a_t m} \right).$$

(5)
Table 1
Fermion parameters for $\nu = 0.810$.

| $\kappa$ | Configurations | $m_\pi a_t$ | $m_\rho a_t$ |
|----------|----------------|-------------|-------------|
| 0.2650   | 67             | 0.137(2)    | 0.186(2)    |
| 0.2660   | 159            | 0.108(1)    | 0.167(1)    |
| 0.2665   | 167            | 0.091(1)    | 0.158(2)    |

To proceed beyond the classical level, we take the bare anisotropy $\xi_0$ from ref. [1] to give the required renormalised anisotropy $\xi$. We then determine the fermion parameters ($\kappa, \nu$) to give the required pion mass, and so that the pion and $\rho$ have the correct dispersion relations. There remains the question of what to use for the clover coefficients, and here we impose the classical constraint of eqn. (3) to obtain a value of $m$, whence we obtain $\omega_0$ from eqn. (4). There are two solutions to eqn. (3); for the case of $\nu \rightarrow 1$, the correct solution for light-quark physics corresponds to $m = 0$, and this is the solution we use at our non-zero light-quark masses. The remaining clover coefficient $\omega$ is given by eqn. (5) and both the $\omega$ and $\omega_0$ are then tadpole improved using the mean values of the spatial and temporal links.

The calculation is performed on $24^3 \times 64$ lattices at $\beta = 6.1$ with a renormalised anisotropy $\xi = 3$; the spatial lattice spacing $a_s(r_0)^{-1} = 2.04$ GeV giving a spatial lattice size $L \sim 2.3$ fm [1].

We choose a range of quark masses corresponding to pion masses between around 500 MeV and 900 MeV, and a range of values of $\nu$ around 0.80. The determination of the $\kappa$ renormalisation $\nu$ proved straightforward at heavy quark masses, but more problematical at light quark mass. For our final analysis of the spectrum, we choose $\nu = 0.810$ and the remaining parameters are listed in table 1. The extent to which the dispersion relations are satisfied for both the pion and the $\rho$ is shown in figure 1. While the dispersion relation is clearly well satisfied for the $\rho$ meson, this is less true for the pion where the tuning of the dispersion relation is increasingly delicate at lighter quark masses; a possible explanation might be that at the lightest mass $a_t m_\pi / a_t \sim |\vec{p}|$.

To illustrate the efficacy of anisotropic lattices in the calculation of the excited hadron spectrum, we will consider the extraction of the masses of nucleon resonances, and in particular of the nucleon, of its parity partner, and of the first radial positive-parity excitation; the latter is of particular interest as the observed light Roper resonance $N(1440)$ is hard to incorporate within standard quark models. Propagators are computed from both local and smeared sources, and fits performed to the smeared-source/local-sink correlators for the usual nucleon operator $N_1$, and for the “bad” nucleon operator $N_2$ which couples upper and lower quark components and is conventionally assumed to overlap the first excited nucleon state. The effective masses for these correlators are shown in figure 2. A clear plateau is exhibited
for each of these channels, and this persists to the lightest pseudoscalar mass studied.

A reliable calculation of the masses of the lowest-lying nucleon resonances of both parities can be made, something not readily possible with the isotropic clover action, and the measured spectrum is shown in figure 3. The calculation reveals that, for the quark masses probed in this study, the ordering of the masses of the states is \( N_{1/2}^+ > N_{1/2}^- > N_{1/2}^+ \) as expected in a simple quark model; furthermore we find reasonable consistency between the masses of the lightest negative-parity state determined using the two interpolating operators \( N_1 \) and \( N_2 \). The large mass of the first positive-parity excitation was originally suggested using the matrix-correlator method [5], but there is evidence that it may not persist to light-quark masses [6].

In this work we have demonstrated the efficacy of using an anistropic lattice for the calculation of the masses of nucleon resonances. Whilst the tuning of the action in the heavy quark sector is straightforward, the tuning is more problematic at light quark masses. A more extensive analysis will include the use of the full planoply of techniques aimed at delineating the spectrum, notably the measurement of a matrix of correlators and the use of Bayesian statistics both to extract higher radial excitations and to effectively use data close to the temporal source. Further work will include the determination of the light-quark hybrid meson spectrum.

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