We study the acceleration of very small dust grains including polycyclic aromatic hydrocarbons arising from electrostatic interactions of dust grains that have charge fluctuating randomly in time. Random charge fluctuations of very small grains due to discrete charging events (i.e., sticking collisions with electrons and ions in plasma, and emission of photoelectrons by UV photons) are simulated using the Monte Carlo (MC) method. The motion of dust grains in randomly fluctuating electric fields induced by surrounding charged grains is studied using MC simulations. We identify the acceleration induced by random charge fluctuations as a dominant acceleration mechanism for very small grains in the diffuse interstellar medium (ISM). We find that this acceleration mechanism is efficient for environments with a low degree of ionization (i.e., large Debye length), where charge fluctuations are slow but have a large amplitude. The implications of the present acceleration mechanism for grain coagulation and shattering in the diffuse ISM and dark clouds are also discussed.

Key words: acceleration of particles – dust, extinction – ISM: kinematics and dynamics

Online-only material: color figures

1. INTRODUCTION

Dust is an important constituent of the interstellar medium (ISM), molecular clouds, and accretion disks (see Whittet 2003; Draine 2011). It is involved in many key astrophysical processes, including the heating and cooling of the ISM (see Draine 2003; Tielens 2005) and the diagnostics of magnetic fields through grain alignment (see Lazarian 2007 for a review). In particular, thermal dust emission interferes with attempts to measure the temperature anisotropy and polarization of the cosmic microwave background (CMB) radiation (see Lazarian & Finkbeiner 2003; Fraisse et al. 2009; Dunkley et al. 2009).

Very small dust grains (of size $a \lesssim 100 \, \text{Å}$) with a notable fraction of polycyclic aromatic hydrocarbons (PAHs, hereafter very small grains and PAHs are used interchangeably) radiate electric dipole emission that contaminates the CMB radiation (Draine & Lazarian 1998; Hoang et al. 2010; Hoang et al. 2011). Moreover, PAHs in protoplanetary disks (PPDs) may affect the magnetorotational instability through their influence on the ionization degree (Bai 2011; Perez-Becker & Chiang 2011).

Most properties of dust, including light extinction, electron photoemission, and chemical activity, not only depend on grain chemical composition, but also on grain size distribution. The latter is determined by grain coagulation and shattering, which depend on grain relative velocities.

Traditionally, it is believed that the motion of dust grains in the ISM arises from radiative force, ambipolar diffusion, and hydrodrag (see Draine 2011). The grain motion induced by these processes, which is sub-Alfvénic, has in general been assumed in the models of dust coagulation (Ossenkopf 1993; Dullemond & Dominik 2008).

Astrophysical environments are practically all magnetized and turbulent, and turbulence is expected to be an important factor in accelerating dust grains. Studies of grain acceleration for magnetized turbulent environments were initiated by Lazarian & Yan (2002), who dealt with the acceleration by incompressible Alfvénic turbulence. Comprehensive studies of the acceleration in realistically compressible environments were performed in Yan & Lazarian (2003, hereafter YL03) and Yan et al. (2004, hereafter YLD04) using quasi-linear theory (Schlickeiser & Miller 1998). Those studies identified gyroresonant interactions of grains with fast MHD modes as a new powerful mechanism of grain acceleration.

The resonance acceleration of charged grains due to MHD turbulence tends to decrease with decreasing grain size. Such a decrease arises from the fact that the Larmor radius of charged grains becomes smaller as the grain mass decreases. Therefore, grains are only able to interact with turbulent fluctuations of small scales and low power. In addition, compressible fluctuations (i.e., fast modes), which are identified in YL03 as the most efficient source of acceleration, are suppressed at small scales due to plasma viscous damping, while the Alfvénic mode becomes inefficient for acceleration at these scales due to the anisotropy of turbulent cascades (see YL03 for more discussion). The resonance acceleration for grains of size $\lesssim 10^{-5} \, \text{cm}$ was shown to be rather inefficient in most phases of the ISM, except the warm ionized medium (WIM; see YLD04). These conclusions were also confirmed by Hoang et al. (2012), who studied the gyroresonance acceleration and the transit time damping acceleration (TTD) using nonlinear theory.

Recently, a new mechanism of astrophysical grain acceleration induced by random charge fluctuations was sketched in Ivlev et al. (2010), where rough estimates of grain velocities were provided for the ISM conditions. The study employed the Fokker–Planck (FP) equation, while assuming that charge fluctuations are fast (i.e., the relaxation timescale for charge fluctuations is much shorter than the gaseous damping time) and have small amplitude (i.e., $\delta Q \ll \bar{Q}_0$).

The rate of grain charging and also of charge fluctuations decreases with decreasing grain size. In the conditions of the ISM, it is usually very fast for large grains and becomes rather slow for very small grains. Moreover, for the latter case, the charge fluctuations also correspond to large amplitude, i.e., $\delta Q \gg \bar{Q}_0$, in contrast to the charge fluctuations of large grains (see Draine & Lazarian 1998). As a result, the key assumptions
of fast rate and small amplitude for charge fluctuations in the FP equation approach become inappropriate for PAHs. Then, if the FP equation is employed for estimating grain velocities, unrealistically high velocities were obtained as in Ivlev et al. (2010).

The primary goal of our present study is to find exact grain velocities induced by random charge fluctuations using Monte Carlo (MC) simulations. The apparent advantage of MC simulations is that they can handle the general case of charge fluctuations, i.e., slow and fast fluctuations. Moreover, this numerical method allows us to investigate the effect of charge fluctuations for tiny grains of size \( a \lesssim 10 \text{ Å} \), which are an important source of microwave emission.

The structure of the paper is as follows. We first discuss the major processes responsible for grain charging and charge fluctuations in Section 2. In Section 3, we describe an algorithm to simulate charge fluctuations using the MC method. In Section 4, we present a numerical simulation method to investigate grain acceleration, by combining dynamical equations with MC simulations. Velocities of very small grains in the ISM obtained from numerical simulations are presented in Section 5. Discussions and summary are presented in Sections 6 and 7, respectively.

2. GRAIN CHARGING AND CHARGE FLUCTUATIONS

The charging processes for a dust grain in the ISM consist of its sticking collisions with charged particles in plasma (Draine & Sutin 1987, hereafter DS87) and photoelectric emission induced by UV photons (Weingartner & Draine 2001). In the former case, the grain acquires charge by capturing electrons and ions from the plasma, while, in the latter case, the grain loses charge by emitting photoelectrons. After a sufficiently long time, these processes result in the statistical equilibrium of ionization, and the grain charge can be described by a mean value \( \langle Z \rangle \). The instantaneous grain charge fluctuates randomly around its equilibrium value \( \langle Z \rangle \).

To characterize the fluctuations of grain charge, following DS87, we define the charge distribution function \( f_Z(Z) \), which indicates the probability of finding the grain with a charge \( Ze \). Detailed calculations for \( f_Z \) are presented in DS87.

![Figure 1. Mean grain charge, \( \langle |Z| \rangle \) (upper), and charge dispersion in units of the mean charge, \( \sigma_Z/\langle |Z| \rangle \) (lower), of graphite grains as a function of grain size \( a \) for various ISM phases. Strong charge fluctuations are observed for grains smaller than \( a_{\text{eq}} \sim 10^{-5} \text{ cm} \) for the CNM and WIM, and \( a_{\text{eq}} \sim 10^{-7} \text{ cm} \) for the WNM.](image)

Table 1

| Parameters\(^a\) | CNM | WNM | WIM |
|-----------------|-----|-----|-----|
| \( n_{\text{H}} \) (cm\(^{-3}\)) | 30  | 0.4 | 0.1 |
| \( T_{\text{gas}} \) (K)         | 100 | 6000| 8000|
| \( \chi \)                      | 1   | 1   | 1   |
| \( \chi_{\text{HI}} \)          | 0.0012| 0.1| 0.99|
| \( \chi_{\text{M}} \)           | 0.0003| 0.0003| 0.0001 |

Note. \(^a\) Here, \( n_{\text{H}} \) is the gas density, \( T_{\text{gas}} \) is the gas temperature, \( \chi_{\text{HI}} \) is the hydrogen ionization fraction, \( \chi_{\text{M}} \) is the metal ionization fraction, and \( \chi \) is the ratio of radiation energy density to the interstellar radiation energy density (ISRF).

and Weingartner & Draine (2001), here we describe them briefly.

Basically, in a steady-state approximation, the distribution function \( f_Z \) of grain charge is constrained by the statistical equilibrium

\[
f_Z(Z)[I_{\text{pe}}(Z) + J_{\text{ion}}(Z)] = f_Z(Z+1)J_{\text{ion}}(Z+1),
\]

which means that the number of positively charged particles the grain absorbs per second to change the charge state from \( Z \) to \( Z+1 \) must be equal to the number of electrons that the grain absorbs per second to cascade from \( Z+1 \) to \( Z \). Above, \( J_{\text{pe}} \) and \( J_{\text{ion}} \) are the rate of sticking collisions of the grain with electrons and ions, and \( J_{\text{pe}} \) is the rate of emission of photoelectrons induced by UV photons (see Appendix A). To find \( f_Z \), we solve Equation (1) using an iterative method as in DS87.

Let us define a characteristic relaxation time of charge fluctuations, \( \tau_Z \), which is equal to the time required for the grain charge to relax from \( Ze \) to the equilibrium state (Draine & Lazarian 1998):

\[
\tau_Z = \frac{(\langle Z \rangle - \langle Z \rangle^2)}{\sum Z f_Z J_{\text{tot}}(Z)} = \frac{\langle Z \rangle^2}{\sum Z f_Z J_{\text{tot}}(Z)}.
\]

where \( J_{\text{tot}}(Z) = J_{\text{e}} + J_{\text{ion}} + J_{\text{pe}} \) is the total charging rate at the charge state \( Z \). Here, we averaged \( Z \) over all possible charge states to find \( \tau_Z \).

As mentioned earlier, in this paper, we are interested in very small grains for which charge fluctuations are important. Moreover, it is well known that very small grains mostly consist of graphite material (Mathis et al. 1977; Draine & Li 2007).

Therefore, in the following, we only consider graphite grains.

Figure 1 shows the mean grain charge (upper plot) and the charge dispersion in units of the mean charge, \( \sigma_Z/\langle |Z| \rangle \) (lower plot), as a function of the grain size for the cold neutral medium (CNM), warm neutral medium (WNM), and WIM. Physical parameters for these phases, including the gas density \( n_{\text{H}} \), gas temperature \( T_{\text{gas}} \), hydrogen ionization fraction \( \chi_{\text{HI}} \), and metal ionization fraction \( \chi_{\text{M}} \), are given in Table 1.

Here, \( \chi = u_{\text{rad}}/u_{\text{ISRF}} \), where \( u_{\text{rad}} \) is the energy density of the radiation field under interest and \( u_{\text{ISRF}} \) is the energy density of the interstellar radiation field (ISRF) from Mathis et al. (1983). The gray area indicates the region, where charge fluctuations are strong with large amplitude, i.e., \( \tilde{\sigma}_Z \geq 1 \). Let \( a_{\text{eq}} \) be the grain size corresponding to \( \tilde{\sigma}_Z = 1 \). Figure 1 shows that charge fluctuations are important for \( a < a_{\text{eq}} \). Figure 1 shows that charge fluctuations are important for \( a < a_{\text{eq}} \), where \( a_{\text{eq}} \sim 10^{-5} \text{ cm} \) for the CNM and WIM, and \( a_{\text{eq}} \sim 10^{-7} \text{ cm} \) for the WNM.

We calculate the relaxation time of charge fluctuations \( \tau_Z \) and the gas damping time \( \tau_{\text{drag}} \) (Equation (B2)) for graphite grains in various phases of the ISM. Here, \( \tau_{\text{drag}} \) comprises the damping
due to dust–neutral and dust–ion interactions (see Appendix B for details). The results that were obtained are shown in Figure 2. In the WNM and WIM, the charge fluctuations are fast (i.e., \( \tau_z \ll \tau_{\text{drag}} \)) for the entire range of grain sizes. In the CNM, the charge fluctuations are slow, i.e., \( \tau_z \) comparable to \( \tau_{\text{drag}} \) for \( a \ll 5 \times 10^{-8} \) cm. The slow charge fluctuations arise from the fact that the degree of ionization is rather low in the CNM, which makes the charging much less frequent than the gas–grain collisions.\(^5\)

3. MONTE CARLO SIMULATIONS OF CHARGE FLUCTUATIONS

For very small grains (e.g., PAHs) in the ISM that are of interest, the charging is infrequent, so that charge fluctuations may not be adequately characterized by the steady-state distribution \( f_z \). In addition, what the present paper is interested in is the time-dependent grain charge, instead of the steady distribution. Thus, in the following, we first describe an algorithm to simulate the random charge fluctuations using the MC method and find the time-dependent charge. Then, we compare the resulting charge distribution function with the steady-state charge distribution obtained using the statistical equilibrium approximation.

3.1. Monte Carlo Simulations

MC simulations of random charge fluctuations for dust grains in plasma have been studied in Cui & Goree (1994), but they only considered the charge fluctuations arising from collisions with electrons and ions in the plasma. Here, we consider a general case of charge fluctuations arising from both the sticking collisions and the photoemission induced by UV photons.

The underlying idea of our MC simulations is that we identify the moment of a charging event that occurs randomly,\(^6\) and assume that the grain charge changes instantly at this moment. We keep track of grain charge and the time interval between two successive charging events. The algorithm for modeling random charge fluctuations is presented in Figure 3. At the initial step \( i = 0 \) and time \( t = 0 \), the grain is assumed to be neutral with charge \( Z_e = 0 \). The charging rates \( J_e, J_{\text{ion}}, \) and \( J_{\text{pe}} \) are used to calculate the mean time between two successive charging events \( \Delta t_i \) and the waiting time to the next charging event \( \tau_i \). The simulations are iterated until the elapsed time \( t_{i+1} \) exceeds the drag time \( \tau_{\text{drag}} \) (see the text).

\(^{5}\) The definition of slow and fast fluctuations is arbitrary. Ivlev et al. (2010) defined by comparing \( \tau_z \) with the characteristic timescale of interactions \( \tau_{\text{int}} = \lambda_{\text{int}}/\nu_{\text{int}} \).

\(^{6}\) A charging event can be the absorption of an electron (ion) or emission of a photoelectron.
When the waiting time to the next charging event $\Delta t$ is known, we have to determine which particle among $e$, ion, and pe the grain will capture or emit at the next charging event. We note that the probability of capturing or emitting $\omega$ particle is $P_\omega = f_\omega$, with $\omega = e$, ion, and pe. The probability that the next charging event will correspond to the absorption/emission of the $x$ particle among $e$, ion, and pe is given by $P_{x}/P_{\text{tot}}$, where $P_{\text{tot}} = P_e + P_{\text{ion}} + P_{\text{pe}}$. The next charging event corresponds to the absorption/emission of the $x$ particle when $P_x$ is the maximum.

We begin by generating a random number $R_1$ in the range $[0, 1]$ from a uniform distribution. If $R_1 < P_{\text{max}}/P_{\text{tot}}$, then we know that a charging event corresponding to $P_{\text{max}}$ just occurred. Now, if $P_{\text{max}} = P_e$, then one electron is captured in this charging event, and the grain charge $Z$ is advanced by $-1$. If $P_{\text{max}} \neq P_e$, then the grain either captures an ion or emits a photoelectron in this charging event. For this case, $Z$ is advanced by +1.

If $R_1 > P_{\text{max}}/P_{\text{tot}}$, then we know that the next charging event with $P_{\text{max}}$ has not occurred, and the charging event has occurred with one of two remaining particles. To determine what particle the grain actually absorbs/emits, we continue by generating a random number $R_2$, and comparing it with $P_e$, $P_{\text{ion}}$, and $P_{\text{pe}}$. Then, the grain charge $Z$ is updated, and the time is advanced as $t_{i+1} = t_i + \Delta t$. We tabulate the charging time $t_{i+1}$ and grain charge $Z(t)$. This process is iterated until the elapsed time $t_{i+1}$ is equal to a few gaseous damping times $\tau_{\text{drag}}$.

Figure 4 shows the time-dependent grain charge obtained from MC simulations for a 10, 20, and 100 Å grain in the WIM. The discrete nature of charging can be easily seen for a very small grain $a = 10$ Å in which $Z$ varies slowly.

### 3.2. Charge Distribution Function

To characterize the fluctuations of grain charge from MC simulations, we find the steady-state charge distribution function $f_Z$ using the tabulated data $Z(t)$. Figure 5 compares $f_Z$ obtained from MC simulations with $f_Z$ obtained using the statistical equilibrium approximation (see Equation (1)) for different grain sizes in the ISM. For $a = 100$ Å grains, the results from two different approaches are similar, while there exists some difference in $f_Z$ for smaller grains (e.g., $a = 10$ and 20 Å). Such a difference stems from the fact that charge fluctuations occur at a much lower rate for very small grains, so that the statistical equilibrium approximation becomes invalid.

Figure 6 illustrates the mechanism of acceleration induced by charge fluctuations. Basically, during the interaction time, the grain charge changes due to the absorption of UV photons, which results in the random variation of grain electric field. As a result, the grain velocity changes randomly. After one encounter, grain velocities change from $(v_1, v_2)$ to $(v_1', v_2')$. The average over many interactions gives rise to a net increase of grain kinetic energy.

### 4. NUMERICAL SIMULATIONS OF GRAIN ACCELERATION

#### 4.1. Basic Equations

Let us consider an ensemble of $N_p$ grains in a plasma with the atomic density $n_H$, ionization fraction $x_H$, and temperature $T_{\text{gas}}$. The velocity of a grain $i$ of mass $m_i$ changes due to the gas drag force as well as the Coulomb force induced by charged grains...
in its vicinity. The conventional equation of motion reads
\[
dv_i = -\frac{\mathbf{v}_i}{\tau_{\text{drag}}} + R_i + \frac{\mathbf{F}_i}{m_i},
\]
(6)
where \(\tau_{\text{drag}}\) is the damping time due to gas drag (see Equation (B2)), \(R_i\) is a random force due to Brownian motions, which is described by
\[
\{R_i^2\} = G_i,
\]
(7)
with \(G_i = 2\tau_{\text{drag}}^{-1}k_B T_{\text{gas}}\) as the translational excitation coefficient due to dust–gas collisions. Here, \(\mathbf{F}_i\) is the Coulomb force acting on \(i\) grain induced by all surrounding grains \(j \neq i\). Assuming the Yukawa potential with the screened length \(\lambda = \lambda_D\) (see Equation (C4)), \(\mathbf{F}_i\) is given by
\[
\mathbf{F}_i(r, t) = \sum_{j \neq i} \mathbf{Q}_j(t)\mathbf{Q}_i(t) \exp(-r_{ij}/\lambda)/r_{ij},
\]
(8)
where \(Q_i(t)\) and \(Q_j(t)\) are the instantaneous charge of \(i\) and \(j\) grains at the time \(t\), and \(r_{ij}\) is the separation between them.

The position of the \(i\) grain at the time \(t\) is determined by the equation
\[
v_i = \frac{dr_i}{dt},
\]
(9)
Solving Equation (6) with the force term given by Equation (8) is rather challenging, because it involves the stochastic term \(R_i\) and the time-dependent force \(\mathbf{F}_i\), which depends on the instantaneous charge \(Q_i(t)\).

4.2. Numerical Integration

To solve Equation (6) combined with Equation (8) for grain velocities, we first need to know the instantaneous charge \(Q_i(t)\) at the time \(t\). The detailed treatment of \(Q_i(t)\) using data from MC simulations will be addressed in the next section. Here, we present a general method to solve Equation (6) for an arbitrary time-dependent force \(\mathbf{F}(r, t)\).

Let \(h_n\) be the time interval at the step \(n\) in which the force term is nearly constant. The velocity of a grain at the step \(n + 1\) is given by
\[
\mathbf{r}_{n+1} = \mathbf{r}_n + \mathbf{v}_n h_n,
\]
(10)
\[
\mathbf{v}_{n+1} = \mathbf{v}_n + \left[ -\frac{\mathbf{v}_n}{\tau_{\text{drag}}} + R_i + \frac{\mathbf{F}_i + \mathbf{F}_f}{2m_i} \right] h_n,
\]
(11)
\[
\mathbf{r}_{n+1} = \mathbf{r}_n + \frac{1}{2} (\mathbf{v}_n + \mathbf{v}_{n+1}) h_n,
\]
(12)
where \(\mathbf{F}_i = \mathbf{F}_i(r_n, t_n)\) and \(\mathbf{F}_f = \mathbf{F}_f(r_{n+1}, t_n)\).

In simulations, we adopt dimensionless units
\[
t' = \frac{t}{\tau_{\text{drag}}, \quad \mathbf{v}' = \frac{\mathbf{v}}{v_T},
\]
(13)
where \(v_T = (2k_B T_{\text{gas}}/m_i)^{1/2}\) is the thermal velocity of a grain with mass \(m_i\) defined such that its total kinetic energy is equal to \(k_B T_{\text{gas}}\). The grain position is normalized over the screened length of plasma \(\lambda\):
\[
r' = \frac{r}{\lambda},
\]
(14)
Using the dimensionless units, Equations (10)–(12) can be rewritten as
\[
\mathbf{r}'_{n+1} = \mathbf{r}'_n + \mathbf{v}'_n h',
\]
(15)
\[
\mathbf{v}'_{n+1} = \mathbf{v}'_n + \left[ -\mathbf{v}'_n + R_i' + \frac{F_i + F_f}{2m_i} \right] h',
\]
(16)
\[
\mathbf{r}'_{n+1} = \mathbf{r}'_n + \frac{1}{2} (\mathbf{v}'_n + \mathbf{v}'_{n+1}) h',
\]
(17)
When the velocities of grains are known, we can estimate the grain rms velocity for a given size as follows:
\[
\langle v^2 \rangle = \frac{1}{N_p(a)} \sum_{i=1}^{N_p(a)} \frac{v_i^2}{2},
\]
(18)
where \(N_p(a)\) is the number of grains of size \(a\) in our simulations.

To obtain grain velocities from the above equations, one must find \(h_n\) in which the force term \(\mathbf{F}\), depending on \(Q(t)\), is constant at every time step. Therefore, first of all, one needs to treat the time dependence of \(Q(t)\) properly.

4.3. Algorithm

Below, we present an algorithm to numerically solve the equations of motion for grains with randomly fluctuating charge for both cases of slow and fast charge fluctuations.

4.3.1. Tiny Grains \(a \leq 10\,\text{Å}\): Slow Fluctuations

Tiny grains with \(a \leq 10\,\text{Å}\) have slow charge fluctuations (i.e., infrequent charging; see Figure 2). Thus, the time interval between two successive charging events is long, and they spend most of the time in the charge states \(Z = 0, \pm 1, \pm 2\).

Since the grain charge \(Q\) is a function of time, the time step \(h_n\) in Equations (15)–(17) must be chosen such that the charge of grains in the simulation box remains constant during this time step. The easy way is to choose the time step coincident with the charging time of all grains.

We consider \(N_p\) grains in the simulation box. For each grain, we generate \(N_Z\) random charging events, and tabulate the moment of charging \(t_Z\) and the time interval between two charging events \(\Delta t_Z\) using MC simulations (here, \(t_Z \equiv t_i\) in Section 3). As a result, we have a total of \(N_{\text{chrg}} = N_p \times N_Z\) charging events in the entire simulation box. When the charging moment \(t_Z\) is known, we can sort it into ascending order, and obtain \(t_{\text{chrg}}^n = n - 1\) for the sequence of \(N_{\text{chrg}}\) charging events in the simulations. The time step is chosen as the time interval between two successive charging events in the sequence, i.e., \(h_n = t_{\text{chrg}}^n - t_{\text{chrg}}^{n-1}\). When sorting \(t_{\text{chrg}}\), we also mark the grain that experiences the charging event. Therefore, at each time step, only this marked grain has a charge that changes instantly by \(\pm 1\), while the charge of the other grains remains the same.

When the integration time \(t_n\) is equal to the charging time \(t_{Z,i}\), then the charge of \(i\) grain changes.

If the time step \(h_n\) results in a large increment of \(v_{n+1}\), then we need to adjust \(h_n\) so that the increment of grain velocity...
is not larger than an \( \epsilon \) fraction of \( v_\text{ch} \). We choose \( \epsilon = 0.1 \) for our simulations. When the time interval between two successive charging events is very large, we need to limit \( \Delta t = 10^{-5} \) to keep the numerical integration stable.

### 4.3.2. Grains of Size \( a > 10 \, \text{Å} \): Fast Fluctuations

For grains larger than 10 Å, we fully enter into a regime of fast charge fluctuations (see Figure 2). The grain charging occurs frequently, so that the time interval between two successive charging events \( t_{\text{ch}} \) becomes rather small (about \( 10^{-9} \tau_{\text{drag}} \)). As a result, we need to integrate Equation (6) over a huge number of time steps to achieve the terminal velocity after \( t_{\text{drag}} \), which is obviously impractical. To overcome this obstacle, we choose a constant time step \( \Delta t \) such that it is much larger than the charging time. Therefore, we need to account for the effect of charge fluctuations during one time step. In the zero approximation, the grain charge can be approximated to its average value \( \langle Z_e \rangle \), and the term of Coulomb force containing \( Q_i(t)Q_j(t) \) in Equation (8) is replaced by \( Z_iZ_j e^2 \). However, this may not be a good approximation because of the large charge dispersion, \( \sigma_Z \sim \langle |Z| \rangle \), thus we have to use the diffusion approximation as in Ivlev et al. (2010).

Let us consider the interaction between a pair of grains with rapidly fluctuating charge. For this case, the increment of kinetic energy due to charge fluctuations during the time step \( \Delta t \gg \tau_Z = 1/v_\text{ch} \) can be inferred from Equation (6) in Ivlev et al. (2010):

\[
\frac{\delta \epsilon_{\text{kin}}}{\Delta t} = \frac{1}{2m_d} \left( \frac{d\phi_0}{dr} \right)^2 \sigma_Z^2 \tau_Z (1 - e^{-\Delta t/\tau_Z}),
\]

where the interaction between two grains of equal size is considered (see also Appendix C).

From the above equation, we can obtain the value of grain velocity increment

\[
|\delta v| = \sqrt{2\delta \epsilon_{\text{f}}/m_d},
\]

and then \( \delta v_x = |\delta v|(v_x/v) \) and \( \delta v_y = |\delta v|(v_y/v) \) using the fact that the direction of the velocity increment is through these grains. The total increment of velocity due to both the mean value and the fluctuation part of the grain charge is given by

\[
\Delta v = \frac{Q_0^2 \mathbf{r}}{r^2} \frac{\dot{r}}{r} \Delta t + \delta v,
\]

where the first term is due to the elastic interaction between two grains with their mean charge, and the second term is induced by the charge deviation from their mean value. Then, we sum over all possible pairs of grains to obtain the total increment of grain velocity per step.

### 4.4. Boundary and Initial Conditions

We adopt periodic boundary conditions for our simulations. The periodic boundary ensures that when a dust grain leaves the box at one boundary, another grain is added to the box at its opposite boundary. The periodic boundary is useful for simulating large-scale systems consisting of a huge number of particles.

We start simulations with \( N_p \) particles randomly distributed in a square box of \( L^2 \). Their initial velocities are drawn from the Gaussian distribution with a kinetic temperature \( T_0 \).

### 4.5. Code Benchmarking: Brownian Motion

We test the code by simulating the Brownian motion of dust grains in a plasma of temperature \( T_{\text{gas}} \). We consider an ensemble of \( N_p \) neutral grains of equal size in the conditions of the CNM and WIM. Simulations are started with an initial kinetic energy \( E_0 < k_B T_{\text{gas}} \). We found that the grain kinetic energy is driven to thermal equilibrium with the plasma after about a gaseous damping time \( \tau_{\text{drag}} \).

### 5. GRAIN ACCELERATION DUE TO CHARGE FLUCTUATIONS IN THE ISM

#### 5.1. Model Parameters

Below, we are going to find the exact velocities of very small grains induced by random charge fluctuations in various phases of the ISM. Since very small grains are found to be graphite (see Mathis et al. 1977; Draine & Li 2007), below, we study the acceleration for graphite grains only. The standard physical parameters of these phases are listed in Table 1. We consider the translational damping due to grain–neutral and grain–ion collisions only (see Appendix B), and disregard subdominant damping processes, such as dipole–dipole and grain–dipole interactions. We also neglect the sticking grain–grain collisions due to the fact that the mean distance between grains in the ISM is substantially larger than the grain size (see Figure 1 in Ivlev et al. 2010).

#### 5.2. Effect of Grain Size Distribution

##### 5.2.1. Interactions of Grains of the Same Size

First, let us assume for the sake of simplicity that all dust grains have the same equivalent size \( a_{eq} \) with density \( n_d(a_{eq}) \). Here, the density \( n_d(a_{eq}) \) is constrained by the condition that the mass density of \( a_{eq} \) grains is equal to the total mass density of dust integrated over the entire size distribution from \( a_{\text{min}} \) to \( a_{\text{max}} \):

\[
M_d \equiv \frac{4\pi}{3} a_{eq}^3 n_d(a_{eq}) = \int_{a_{\text{min}}}^{a_{\text{max}}} \frac{4\pi}{3} a^3 \frac{dn}{da} \, da,
\]

where \( a_{\text{min}} = 3.56 \, \text{Å} \) and \( a_{\text{max}} = 2.5 \times 10^{-5} \, \text{cm} \) are assumed. Assuming the grain size distribution from Mathis et al. (1977, hereafter MRN): \( dn/da = n_{H} A_{\text{MRN}} a^{-3.5} \) with \( A_{\text{MRN}} = 10^{-25.16} \, \text{cm}^{-2.5} \) for diffuse clouds, we obtain

\[
n_d(a_{eq}) \approx 10^{-6} n_{H} \left( \frac{a_{eq}}{10^{-7} \, \text{cm}} \right)^{-3}.
\]

The above equation can give rise to an overestimate of the dust density of the smallest grains because most of the dust mass in the MRN distribution is concentrated in large grains. However, it is useful for us to have a rough estimate of grain velocities due to charge fluctuations. The detailed calculations of grain velocities using the grain size distribution are presented in the next subsection.

We perform MC simulations for different equivalent sizes \( a_{eq} \) from \( a_{\text{min}} \) to \( a_{\text{max}} \). For each value of \( a_{eq} \), we consider \( N_p = 10^3 \) grains and the density is inferred from Equation (23).

##### 5.2.2. Interactions between Grains of Different Size

Now, let us consider the electrostatic interactions between grains of different sizes. We divide the grain size distribution
from $a_{\text{min}}$ to $a_{\text{max}}$ into $N_{\text{bin}}$ with a constant log bin size, $\Delta \ln a = \ln(a_{\text{max}}/a_{\text{min}})/(N_{\text{bin}})$. The grain size in the $i$ bin is given by

$$\ln a_i = \ln a_{\text{min}} + (i - 1) \Delta \ln a,$$

(24)

and the fraction of grains in the $i$ bin, $n_d(a_i)$, to the total dust density $n_d$ reads

$$\frac{n_d(a_i)}{n_d} \equiv \frac{(dn/da)\Delta a_i}{n_d} = \frac{f(a_i)a_i}{\sum_{a=a_{\text{min}}}^{a_{\text{max}}} f(a)a},$$

(25)

where $dn/da = f(a)$ is the size distribution function. For graphite grains, we adopt the grain size distribution from Draine & Li (2007), which exhibits an enhancement of the dust mass of the smallest grains due to PAHs compared to MRN.

Assuming the total number of particles in the simulation box is $N_p$, the box size $L$ is constrained by

$$\sum_{i=1}^{N_{\text{bin}}} n_d(a_i)L^2 = n_dL^2 = N_p.$$

(26)

We run simulations for $N_p = 10^4$ grains of various sizes $a$, and find the rms velocity of grains with the same size. For each grain size $a$, the dominant contribution to the acceleration arises from its interaction with other grains of size $a' > a$. Therefore, the Coulomb force in Equation (8) corresponds to the summation over all grains larger than the grain of mass $m_i$ under interest.

5.3. Grain Velocities

We first present grain velocities as a function of size obtained from MC simulations for the simplest case in which all grains have the same equivalent size. For each grain size, we follow the time evolution of grain velocity and calculate the rms velocity $v = \langle v^2 \rangle^{1/2}$ at each time step. We identify the time interval in which $v$ varies slowly and then saturates. The terminal velocity is obtained by averaging $v$ over the time interval ranging from $\tau_{\text{drag}}$ to the total integration time $T$. Here, we adopt the total integration time $T = 1.5\tau_{\text{drag}}$.

Figure 7 (upper) presents the temporal evolution of the rms velocity of grains normalized over the thermal velocity in the CNM and WIM for three grain sizes. As expected, the rms grain velocity increases exponentially as a function of time from the initial value, then slows down gradually and saturates after about a gas damping time. To see the dependence of the terminal velocity with grain size $a$, in Figure 7 (lower), we show results for three phases of the ISM. We truncate at the size $a = 10^{-6}$ cm because the charge fluctuations are negligible for large grains. We show the rapid increase of velocity with decreasing grain size. We can see that the acceleration by charge fluctuations is more efficient in the CNM than the WIM, which can accelerate grains up to about four times its thermal velocity (see red lines). The reasons for that are as follows. First, the CNM has stronger charge fluctuations, which correspond to a larger charge dispersion $\sigma Z / \langle Z \rangle$ (see Figure 1), resulting in stronger fluctuating electric fields. Second, since the CNM has a lower degree of ionization, which corresponds to a larger Debye length, the shielding effect of plasma on electrostatic interactions between charged grains is much less than in the WIM.

To study the effect of grain size distribution on grain acceleration, we perform simulations for $N_{\text{bin}} = 32$ size bins from $a = a_{\text{min}}$ to $a_{\text{max}}$ and take into account the electrostatic interactions of grains of different sizes. The velocities obtained for three different grain sizes in the CNM as a function of time are shown in Figure 8 (upper) and grain rms velocities as functions of grain size are shown in Figure 8 (lower).

From the lower panel in Figures 7 and 8, one can see that grain velocities obtained when the size distribution is taken into account decrease compared to the velocities obtained for grains of equivalent size (Figure 7, lower). Such a decrease arises from the fact that when the grain size distribution is accounted for, the density of the smallest dust grains (PAHs) decreases substantially because most of grain mass is localized at a large size, so that the mean distance between two grains increases, resulting in a significant decrease of the Coulomb force.

5.4. Comparison with the Results from the FP Equation

The FP equation was used by Ivlev et al. (2010) to study the grain acceleration by charge fluctuations (see also Appendix C). The key assumptions in their FP equation are as follows: (1) grain charge fluctuations are fast and (2) the charge fluctuations is more efficient in the CNM than the WIM. Second, since the CNM has a stronger fluctuating electric fields. Third, since the CNM has a lower degree of ionization, which corresponds to a larger Debye length, the shielding effect of plasma on electrostatic interactions between charged grains is much less than in the WIM.

Figure 7. Upper panel: evolution of rms grain velocity normalized to its thermal velocity as a function of time from MC simulations for three grain sizes in the CNM. Graphite grains are considered. Charge fluctuations accelerate very small grains in the CNM to suprathermal velocities after about the gas drag timescale $\tau_{\text{drag}}$. Lower panel: terminal velocities of graphite grains as a function of grain size for the CNM, WNM, and WIM due to charge fluctuations are shown by solid, dotted, and dashed lines, respectively. Thin lines show their thermal velocities in the corresponding ISM phases.

(1) a = 0.0 cm to a = 10$^{-6}$ cm and take into account the electrostatic interactions of grains of different sizes. The velocities obtained for three different grain sizes in the CNM as a function of time are shown in Figure 8 (upper) and grain rms velocities as functions of grain size are shown in Figure 8 (lower).

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dispersion is small compared to the equilibrium charge. These assumptions are, in general, valid for large grains, but they fail for very small grains (see Figure 1). To see the consequence of such unrealistic assumptions on predicting PAH velocities, we conservatively apply the FP equation for very small grains and compute grain velocities using Equation (D13), in which the actual values of $\sigma_Z$ and $\langle Z \rangle$ from Figure 1 are adopted.

Figure 8 (lower) compares grain velocities obtained from MC simulations with those predicted by the FP equation for the ISM. We can see that the FP equation predicts much larger grain velocities than the actual values from MC simulations. The difference in grain velocities obtained from the two methods decreases when $a$ increases and becomes negligible for sufficiently large grains.

The reason that the FP equation predicts unrealistically high velocities for PAHs is the following. When adopting the FP equation to estimate grain velocities, we are implicitly assuming an unrealistic situation in which PAHs have fast fluctuating charges but with a large amplitude. It corresponds to the situation in which each PAH is subject to unusually strong and fast fluctuating electric fields. As a result, PAHs gain a substantial amount of energy after a short interval of time and are accelerated to extremely high velocities. However, for very small grains (e.g., PAHs), we showed in Figures 1 and 2 that the grain charge indeed has a large amplitude, but it fluctuates rather slowly, which makes the acceleration much less efficient than predicted by the FP equation.

For highly ionized media, the fast charge fluctuations can be applied, but we found that the results are still much lower than predicted by the FP equation. Such a difference may arise mainly from the fact that the plasma screening effect is disregarded in the estimate of Ivlev et al. (2010), while highly ionized media have a short Debye length, which reduces the electrostatic dust–dust interactions.

6. DISCUSSION

6.1. Grain Acceleration by Various Mechanisms

Grain acceleration by hydrodrag (Draine 1985) was considered the dominant mechanism driving grain motions in theISM as well as in PPDs. Recently, thanks to significant progress in our understanding of MHD turbulence, Yan & Lazarian (2003), YLD04, and Yan (2009) identified gyroresonant interactions of charged grains with fast modes in MHD turbulence as the dominant mechanisms for accelerating large grains ($a > 10^{-5}$ cm) to super-Alfvénic velocities. For tiny grains (e.g., PAHs and nanoparticles), the gyroresonance acceleration is inefficient for most phases of the ISM, because PAHs and nanoparticles with a smaller gyroradius only resonantly interact with turbulent eddies on small scales with weak magnitude.

Hoang et al. (2012) revisited the treatment of gyroresonance acceleration in MHD turbulence by accounting for the fluctuations of the grain guiding center with respect to the mean magnetic field. They improved the estimates of the previous authors and proposed a new way of acceleration through the TTD of fast MHD modes, which is efficient for super-Alfvénic grains (see solid lines in Figure 9).

6.2. Acceleration Induced by Random Charge Fluctuations

In the present paper, we have numerically investigated a new grain acceleration mechanism induced by random charge fluctuations for very small grains (e.g., PAHs). The combination of MC simulations of charge fluctuations with direct simulations...
of electrostatic interactions between grains with fluctuating charges allows us to follow the evolution of grain velocities in time, and estimate their terminal rms velocities. We showed that the acceleration induced by random charge fluctuations is indeed an important mechanism to accelerate PAHs to suprathermal motion.

In particular, we found that grain velocities obtained from MC simulations are between two and three orders of magnitude lower than the predictions by the FP equation by Ivlev et al. (2010). The overestimate of the FP equation arises from the fact that charge fluctuations of very small grains in the ISM are slow, i.e., in contrast to the assumption of fast charge fluctuations adopted for the FP equation, which makes electric fields fluctuate rather slowly. Moreover, the FP equation disregarded the screening effect of plasma for simplicity. However, for the ISM, this is a poor approximation because the mean separation between two grains is comparable to the Debye length.

Figure 9 compares grain velocities induced by charge fluctuations (shaded area) with those due to the resonance interactions in MHD turbulence. As shown, the acceleration by MHD turbulence is efficient for large grains, while the acceleration by charge fluctuations is more important for tiny grains.

6.3. Effect of Grain Acceleration on the Evolution of the Interstellar Dust

Grain coagulation is a process in which small dust grains collide with one another to form aggregates of submicron size (Spitzer 1978). This process can occur in dense clouds (e.g., Ossenkopf 1993; Ormel et al. 2009), as well as in PPDs (see van Boekel et al. 2003). Grain coagulation is not only important for dust evolution, but is also considered to be a major mechanism for the formation of planetesimals (Weidenschilling 1980 and Weidenschilling 1995).

Grain coagulation and shattering result from grain–grain collisions, which depend on the grain relative velocity, denoted by $v_{dd}$. Following Chokshi et al. (1993), the threshold velocity for grain shattering is a function of the grain size:

$$v_{\text{shat}} = 2.7 \left( \frac{a}{10^{-7} \text{ cm}} \right)^{-5/6} \text{ km s}^{-1}. \quad (27)$$

If $v_{dd} < v_{\text{shat}}$, then grains collide and stick together. When $v_{dd} > v_{\text{shat}}$, the collisions between grains of high velocity produce shock waves inside grains, and shatter them into smaller fragments. For $v_{dd} \rightarrow 20$ km s$^{-1}$, the evaporation of dust grains occurs and grains are destroyed.

Figure 9 shows that the resonant acceleration by MHD turbulence is able to force large grains into moving rapidly, with velocities greater than $v_{\text{shat}}$ (see the orange line). As a result, the shattering of large grains is efficient to replenish very small grains into the ISM and plays an important role in constraining the upper limit of grain size distribution (see Hirashita & Yan 2009). In particular, the shattering in the WIM seems to be very strong and occurs for much smaller grains—grains as small as $10^{-7}$ cm.

Theoretical modeling and observational studies (see, e.g., Draine & Li 2007) indicate that about 5% of dust mass is contained in tiny grains of the size $\leq 10$ Å, which is believed to be PAHs. Thus, if PAHs are actually moving suprathermal due to random charge fluctuations as we predict, then they apparently enhance the grain coagulation and modify the grain size distribution. Future models of dust evolution should take into account the acceleration of very small grains.

7. SUMMARY

The present paper applies MC simulations to study the efficiency of a new acceleration mechanism for very small dust grains (e.g., PAHs) induced by random charge fluctuations. Our principal results can be summarized as follows.

1. Random charge fluctuations of small grains due to both sticking collisions with electrons and ions in the plasma, and the emission of photoelectrons by UV photons are simulated using the MC method. The charge distribution obtained by MC simulations is similar to the steady charge distribution for large grains, but they are different for very small grains.

2. Grain acceleration arising from electrostatic interactions between dust grains with randomly fluctuating charge is investigated using MC simulations. This numerical method allows us to investigate both cases of slow and fast charge fluctuations. We found that the acceleration induced by charge fluctuations is efficient for very small grains, which can increase grain velocities from their thermal velocities. This mechanism is more efficient for the CNM, with the lower ionization fraction (i.e., larger Debye length) and larger charge dispersion, than the WNM and WIM.

3. The increase of relative velocities of PAHs and nanoparticles due to random charge fluctuations that we reported has certain effects on grain coagulation and shattering, and should be accounted for in the models of grain evolution in the ISM as well as in dense clouds.

A.L. and T.H. acknowledge the support of the NSF-funded Center for Magnetic Self-Organization (CMSO) and the NASA Grant NNX11AD32G. We thank Bruce Draine for providing us with the data of grain charge distribution and for fruitful discussions and comments. We thank Hideko Nomura for providing us with the physical parameters of protoplanetary disks, and Huirong Yan and Satoshi Okuzumi for valuable comments. We thank the anonymous referee for helpful comments and suggestions that significantly improved the paper. A.L. acknowledges the Humboldt Award at the Universities of Bochum and Cologne and the Visiting Fellowship at the International Institute of Physics (Brazil).

APPENDIX A

GRAIN CHARGING

In plasma, grains can acquire charge due to sticking collisions with electrons and ions, and the emission of photoelectrons due to UV photons (see DS87). Below, we briefly describe these charging processes.

A.1. Collisional Charging

Let us briefly describe the charging due to the collisions of grains with charged particles in the plasma. Consider a beam of charged particles $p$ with velocity $v$ colliding with a spherical grain of charge $Q = Ze$ and size $a$. The number of particles sticking to the grain per second is defined as the collisional charging rate:

$$J_{p,v}(Z) = s_p n_p v \sigma_{\text{coll}} = s_p n_p v \pi a^2. \quad (A1)$$
where \( s_p \) is the sticking coefficient, \( n_p \) is the number density of particles (\( p = \) ion or electron), and the critical impact factor \( b_{cri} \) is defined such that the closest distance that the particle can approach the grain \( r_{min} \leq a \) (i.e., requirement for the direct collisions).

Assuming that grain velocities follow the Maxwell distribution, the collisional charging rate can be rewritten as

\[
J_p(Z) = \int s_p n_p v \pi b_{cri}^2 f_{Max}(v) 4\pi v^2 dv \\
= \int s_p n_p \pi b_{cri}^2 (k_B T_{gas}/2\pi)^{-3/2} \times \exp\left(\frac{-m v^2}{2k_B T_{gas}}\right) 4\pi v^2 dv. \tag{A2}
\]

DS87 have studied in detail the collisional charging, and derived the final charging rates \( J_{ion} \) and \( J_e \) for collisions of grains with ions and electrons, respectively. Thus, in the present paper, we adopt their expressions for calculations of the collisional charging rates.

### A.2. Photoemission

The number of electrons emitted by the grain due to UV photons per second is given by

\[
J_{pe}(Z) = \pi a^2 \int Y_e Q_{abs} \frac{u_v c}{h v} dv, \tag{A3}
\]

where \( n_v = u_v/(h v) \) is the density of a photon with energy \( E = h v > 13.6 \text{ eV} \), \( Y_e \) is the photoemission yield, and \( Q_{abs} \) is the absorption efficiency (see detail in Weingartner & Draine 2001). For the ISM, we calculate the photoemission rate \( J_{pe} \) using the interstellar radiation from Mathis et al. (1983). We use the tabulated data \( Q_{abs} \) for graphite grains from Draine & Li (2007).

In the ISM, the photoemission is dominant, but in dense regions with a high gas density (e.g., PPDs), the collisional charging dominates in the mid-plane region where UV photons from the newborn stars are shielded. Close to young stars or further from the mid-plane, the photoemission dominates due to the abundance of UV photons.

### APPENDIX B

#### GAS DRAG

The interactions of dust grains with the ambient gas represent a primary mechanism of dissipating the streaming motion of grains. The damping rate of translational motion arising from the interaction of a grain with neutral gas is essentially the inverse time for collisions with the mass of the gas equal to that of the grain (Purcell 1969),

\[
\gamma_n = \tau_{dn}^{-1} = \frac{1}{\tau} \frac{n_n}{\pi a \rho_d} \left( m_n k_B T_n \right)^{1/2} \\
= 2.4 \times 10^{-12} \left( \frac{10^{-6} \text{ cm}}{a} \right) \left( \frac{n_n}{\text{cm}^3} \right) \left( \frac{m_n}{m_H} \right) \left( \frac{T_n}{100 \text{ K}} \right)^{1/2}, \tag{B1}
\]

where \( m_n, n_n, \) and \( T_n \) are the mass, volume density, and temperature of neutral atoms, \( \rho_d \) is the mass density of dust grains, and \( a \) is the grain size.

When the ionization level is sufficiently high, the interaction of charged grains with ions in plasma becomes important (Draine & Salpeter 1979). The ion–grain cross section due to the long-range Coulomb force is larger than the atom–grain cross section. As a result, the rate of translational motion damping is modified (Draine & Salpeter 1979).

For subsonic motions, the total damping rate due to neutral atoms and ions is given by

\[
\tau_{drag}^{-1} = \gamma_n + \gamma_{ion} = \alpha \gamma_n, \tag{B2}
\]

where the renormalizing factor reads

\[
\alpha = 1 + \frac{n_H}{2n_n} \sum_i x_i \left( \frac{e^2}{a \k_B T_i} \right)^2 \left( \frac{m_i}{m_n} \right)^{1/2} \sum Z^2 f(Z) \times \ln \frac{3}{2\sqrt{\pi} |Z| e^3 (x n_H)^{1/2}}. \tag{B3}
\]

Here, \( x_i \) is the abundance of an ion \( i \) (relative to hydrogen) with mass \( m_i \) and temperature \( T_i \), \( x = \sum_i x_i \), and \( f(Z) \) is the grain charge distribution function. When the grain velocity \( v_d \) relative to the gas becomes supersonic, the dust interactions with the plasma is diminished, and the damping rate in this case is renormalized due to the gas-dynamic correction (Baines et al. 1965; Purcell 1969; Draine & Salpeter 1979)

\[
\alpha = \left( 1 + 9 \pi \frac{v_d^2}{128 C_s^2} \right)^{1/2}, \tag{B4}
\]

where \( C_s = \sqrt{k_B T_n/m_n} \) is the sound speed.

### APPENDIX C

THEORETICAL TREATMENT FOR ACCELERATION DUE TO CHARGE FLUCTUATIONS

#### C.1. Charge Evolution for Large Grains

In Sections 2 and 3, we discussed the charging and charge fluctuations for very small grains and present MC simulations to model charge fluctuations. For the sake of completeness, here we consider the case of large grains for which charge fluctuations can be modeled using Langevin equations.

Assuming that some small deviation from the mean grain charge is introduced, then the charging rate changes accordingly to bring the charge to an equilibrium state. If \( \tau_{ch} \) is the relaxation time for which charge fluctuations return to the equilibrium state, then the charging frequency becomes

\[
\nu_{ch} = \frac{1}{\tau_{ch}} = \sum_k \frac{dJ_k}{dQ} \delta_{Q_0}, \tag{C1}
\]

where \( J_k \) with \( k = \) ion, \( e, \) and \( pe, \) is the charging rate from the \( k \) process.

For the collisional charging, using the charging rates \( J_{ion} \) and \( J_e \) for (C1), one can obtain \( \nu_{ch} \) and the charge dispersion \( \sigma_Q \) for a grain of size \( a \)

\[
\nu_{ch} = \frac{d(J_{ion} - J_e)}{dQ} = \frac{1 + \frac{2\pi}{\lambda_D} a}{2\pi} \omega_i, \tag{C2}
\]

\footnote{This drag consists of both close collisions and Coulomb distant interactions of dust with ions and electrons. Since the momentum of electrons is much smaller than that of ions, drag due to collisions with electrons are disregarded.}
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\[ \sigma_Q^2 = \frac{1 + \frac{z}{x(2 + z)}}{z} |eQ_0|, \]  
(C3)

where \( z = Q_0 e/(ak_B T_i) \) and

\[ \lambda_D = \frac{v_{Ti}}{\omega_Q} = \left( \frac{k_B T_i}{m_i} \right)^{1/2} \left( \frac{m_i}{4\pi e^2 n_i} \right)^{1/2} = \left( \frac{k_B T_i}{4\pi e^2 n_i} \right)^{1/2} \]
\[ \approx 2 \times 10^2 \left( \frac{T_i}{100 K} \right)^{1/2} \left( \frac{0.1 \text{ cm}^{-3}}{n_i} \right)^{1/2}, \]  
(C4)

is the Debye length due to ions with the plasma frequencies:

\[ \omega_{pi}^2 = \frac{4\pi e^2 n_i}{m_i}, \quad \omega_{pd}^2 = \frac{4\pi Q_d^2 n_d}{m_d}. \]  
(C5)

C.2. Continuous Approximation

The charge fluctuations of large grains can be described by a Gaussian random process. The time evolution of charge can be modeled by a stochastic differential equation:

\[ \frac{dQ}{dt} = -v_{ch}Q + \Gamma_Q, \]  
(C6)

where \( \Gamma_Q \) is given by an autocorrelation function:

\[ \langle \Gamma_Q(t)\Gamma_Q(t + \tau) \rangle = 2v_{ch} \sigma_Q^2 \delta(\tau). \]  
(C7)

Above, \( \sigma_Q \) is the standard deviation of charge from equilibrium or charge dispersion (see a review by Morfill & Ivlev 2009). The autocorrelation function of charge is a decaying law

\[ \langle Q(t)Q(t + \tau) \rangle = Q_0^2 \exp(-v_{ch} \tau). \]  
(C8)

APPENDIX D

STOCHASTIC ACCELERATION DUE TO CHARGE FLUCTUATIONS

D.1. Stochastic Acceleration in Binary Collisions

We consider dilute dust clouds, and hence only focus on the binary interactions between grains. Binary collisions can be conveniently studied in terms of the center-of-mass and relative coordinates. Below, we consider grains of the same mass, \( m_{d1} = m_{d2} = m_d \), although all results can be generalized in a straightforward manner for the arbitrary mass ratio (Ivlev et al. 2004): e.g., for \( m_{d1}/m_{d2} \ll 1 \), one should substitute the mass of a light particle instead of the reduced mass (1/2\( m_d \)). For a pair of particles with momenta \( p_1 \) and \( p_2 \), the center-of-mass and relative momenta are \( \mathbf{p}_c = (1/2)(p_1 + p_2) \) and \( \mathbf{p}_r = p_1 - p_2 \), respectively. The kinetic energy of the pair can be expressed via \( p_c \) and \( p_r \) as follows:

\[ p_c^2/2m_d + p_r^2/2m_d = p_c^2/m_d + p_r^2/4m_d \equiv \varepsilon_c + \varepsilon_r. \]

The center-of-mass momentum, and hence the energy \( \varepsilon_c \), are conserved during the collision (here, we neglect momentum exchange due to collisions with electrons and ions), whereas the relative momentum is changed,

\[ p_c' = p_c; \quad p_r' = p_r + q. \]

For constant charges, \( \varepsilon_c \) is obviously conserved during the collision, but when charges vary, \( \varepsilon_r \) varies as well. Thus, in the presence of charge fluctuations, the exchange of the relative momentum can be divided into the elastic and inelastic parts,

\[ q = q_0 + \delta q. \]

The elastic part keeps the magnitude of the relative momentum constant, \( |p_c + q_0| = |p_c| \), so that the energy variation after the collision is \( \delta \varepsilon_c = (p_c + q_0) \cdot \delta q/2m_d + (\delta q)^2/4m_d \). The inelastic momentum exchange \( \delta q \) is generally a function of \( p_c \) and \( p_r \), and is determined by the stochastic properties of charge fluctuations.

In order to calculate the energy variation, we suppose that charge fluctuations are uncorrelated for different particles, so that for a pair of particles with charges \( Q_1(t) = Q_0 + \delta Q_1(t) \) and \( Q_2(t) = Q_0 + \delta Q_2(t) \), we have \( \langle \delta Q_1(t)\delta Q_2(t) \rangle = 0 \). Assuming that \( |\delta Q/Q_0| \ll 1 \), the inelastic momentum exchange due to the collision can be calculated as \( \delta q \approx \int \delta Q_1(t) \delta Q_2(t) \nabla \varphi_0 dt \), where \( \varphi_0(r) \) is the potential of the equilibrium charge \( Q_0 \) and the integration is performed along the equilibrium collision trajectory \( r(t) \). The resulting energy variation is obtained by averaging over the fluctuations, i.e.,

\[ \delta \varepsilon_r \equiv \langle (\delta q)^2 \rangle = \frac{1}{2m_d} \left( \int \delta Q \nabla \varphi_0 dt \right)^2 \int \delta Q \nabla \varphi_0' dt'. \]  
(D1)

To calculate \( \delta \varepsilon_r \), we also assume \( |\delta Q/Q_0| \ll 1 \) and therefore can use the following charge autocorrelation function (Matsoukas & Russell 1997):

\[ \langle \delta Q(t + \tau)\delta Q(t) \rangle = \sigma_Q^2 e^{-\nu_{ch} |\tau|}. \]  
(D2)

Here, \( \sigma_Q^2 \) is the charge dispersion and \( v_{ch} \) is the “charging frequency” (which plays the role of the scale parameter in charge fluctuations spectrum). The assumption about small charge fluctuations significantly simplifies the derivation of Equation (D1), and also allows us to treat the charge as a continuous variable and hence employ Equation (D2).

There are several mechanisms that can result in the fluctuations (Yan et al. 2004; Khrapak et al. 1999). In this paper, for the sake of convenience, we focus on the most elementary charging process, which is due to the discreteness of the elementary charge. Usually, this is a Gaussian process with \( \sigma_Q^2 = ((1 + z)/z(2 + z))eQ_0 |\delta q_0| \) and \( v_{ch} = 1/(\sqrt{2\pi})(1 + z)\omega_Q^2/\nu_{ch} \) (Matsoukas & Russell 1997), where \( \omega_Q = \sqrt{4\pi e^2 n_i/m_i} \) is the ion plasma frequency, \( v_{Ti} = \sqrt{k_B T_i/m_i} \) is the ion thermal velocity, and \( z = e(Q_0/ak_B T_i) \) is the dimensionless charge of a dust particle with radius \( a \) (for different gases, \( z = 1 \sim 3 \); Tsytovich 1997; Fortov et al. 2005). Note that other charging processes can also play an important role (in particular, the photoelectric emission) and require separate careful consideration.

One can identify two limiting collisional regimes associated with the charge fluctuations. In the limit of rapid fluctuations, the charges vary many times in each act of binary collisions between dust particles. This regime corresponds to \( v_{ch,\text{fast}} \gg 1 \), where \( v_{ch,\text{fast}} \) is the characteristic timescale of the dust–dust interaction during the collision. By employing Equation (D2), we readily obtain from Equation (D1) the following limiting expression for the energy variation:

\[ \delta \varepsilon_r \approx \frac{\sigma_Q^2}{m_d \nu_{ch}} \int \frac{d\varphi_0}{dr} \left| \frac{d\varphi_0}{dr} \right|^2 dt + O(v_{ch}^{-2}). \]  
(D3)

Equation (D3) shows that in this limit each collision between dust particles results in the increase of their kinetic energy.

D.2. Evolution of Grain Kinetic Temperature

In order to understand the effect of the energy variation in binary dust–dust collisions on the mean kinetic energy of the
whole dust ensemble, one should employ the kinetic approach. The kinetics are described in terms of the velocity distribution function \( f_d(p, t) \). There are two principal contributions to the dust kinetics—one is due to the mutual dust collisions and another to dust interactions with the ambient gas.

Three known interaction processes between dust–gas are dust–neutral, dust–ion collisions, and plasma drag, which arises from the interaction of passing ions with grain dipole moment. We assume that these dust–gas interactions lead to the local thermal equilibrium of gas and dust, characterized by a temperature \( T_g \), at different rates \( \gamma_{\alpha} \), \( \gamma_{\beta} \), and \( \gamma_{\gamma} \) (see Appendix B).

The resulting kinetic equation has the following form:

\[
\frac{df_d}{dt} = St_{dd} f_d + St_{dg} f_d, \tag{D4}
\]

where \( St_{dd} \) and \( St_{dg} \) denote the collision operators (integrals) describing the dust–dust and dust–gas interactions, respectively. Although the analysis of Equation (D4) can be performed for arbitrary \( f_d(p) \), for the sake of convenience we assume that dust particles have a Maxwell distribution \( f_M(p) \). Then, the equation for evolution of mean kinetic energy (kinetic temperature) \( T_d \), at different rates \( \gamma_{\alpha} \), becomes:

\[
T_d = \frac{1}{3} \int \frac{p^2}{m_d} (St_{dd} f_M + St_{dg} f_d) dp. \tag{D5}
\]

Ivlev et al. (2010) calculated integral (D5) and derived the equation for evolution of mean kinetic energy (kinetic temperature) \( T_d \), at different rates \( \gamma_{\alpha} \),

\[
\frac{d}{dt} = \frac{2}{Q_{v, ch}} \frac{\alpha C}{\gamma_{\alpha}} T_d - 2 \gamma_{\alpha}(T_d - T_g), \tag{D6}
\]

where \( \alpha C = \frac{\sigma_T^2 \omega_{pd}}{Q_{v, ch}} \).

Using typical parameters, we obtain the ratio of source-to-sink from Equation (D6):

\[
\frac{\alpha C}{\gamma_{\alpha}} = 2 \times 10^{-3} \left( \frac{\sigma_T}{0.1} \right)^2 \left( \frac{Q_d}{100 e} \right) \left( \frac{n_d}{10^{12} \text{ cm}^{-3}} \right) \times \left( \frac{1}{v_{ch}} \right) \left( \frac{1}{\gamma_{\alpha}} \right) \left( \frac{10^{-12} \text{ g}}{m_d} \right). \tag{D8}
\]

In order to have an increase of dust kinetic energy, the source term must exceed the sink term, i.e., \( \alpha C/\gamma_{\alpha} > 1 \).

In the low-temperature regime, the dust–dust collision is similar to the hard-sphere collision. Thus, the variation of \( T_d \) in time is governed by

\[
T_d = \frac{\sigma_T^2}{m_d Q_{v, ch}^2 \lambda} T_d^2 - 2 \gamma_{\alpha}(T_d - T_g), \tag{D9}
\]

where \( l = 1/(n_0\sigma_0) = 1/(n_0\pi a_{HS}^2) \) with \( a_{HS} = 2\lambda \ln(Q_0^2/\lambda \epsilon), \)

Without charge fluctuations, when the collisions between dust particles conserve the energy, the equilibrium temperature of dust grain is determined by interactions with the ambient gas, so that \( T_d = T_g \). Random charge fluctuations provide an additional energy source, and if the coefficient of the first (source) term in the right-hand side of Equation (D6) exceeds the damping rate \( 2\gamma_{\alpha}^{-1} \), then the dust temperature grows exponentially with time.

The kinetic coefficient \( A_{dd} \) is linearly proportional to the maximum scattering angle, which we set equal to unity. Hence, we implicitly supposed that there are sufficiently small impact parameters \( \rho \) that ensure scattering at large angles, \( \chi \gg 1 \). However, since the lower bound of impact parameters is limited by the particle radius, \( \rho \gg a \), this assumption is only valid if the kinetic temperature is below a certain critical value \( T_d^{s} \).

From the relation \( \chi \sim Q_0^2/\rho T_d \), we readily deduce,

\[
T_d^{s} \sim \frac{Q_0^2}{a} \equiv \frac{Q_0}{e} \varepsilon T_d, \tag{D10}
\]

so that the actual value of the maximum scattering angle is equal to \( T_d^{s} / T_d \). Therefore, the exponential temperature growth described by Equation (D6) proceeds until \( T_d \) reaches the critical value \( T_d^{s} \). At larger temperatures, the source term in Equation (D6) remains constant (which is obtained by replacing \( T_d \) with \( T_d^{s} \)). The temperature growth switches to linear and is eventually saturated due to the translational damping, which determines the ultimate temperature of dust.

Note that when treating the dust–dust collision, we assume that the dust velocity does not change as a result of gas collisions, which corresponds to the orbital-motion-limited approximation.

### D.3. Analytical Estimate for Saturated Kinetic Temperature

When \( \alpha C > \gamma_{\alpha} \), Equation (D6) indicates that the temperature \( T_d \) increases exponentially with time. Thus, we call this regime instability. When \( T_d \) becomes larger than the critical temperature \( T_d^{s} \)-limit of Coulomb interaction, then the source term \( \alpha C \) is constant. As \( T_d \) continues to increase, the sink term increases, while the source term remains constant. Therefore, the temperature is saturated. From Equation (D6), we obtain

\[
T_d^{\infty} = \frac{\sigma_T^2 \omega_{pd}^2}{Q_{v, ch}^2 2a} \tau_{dn} T_d^{s}. \tag{D10}
\]

Plugging \( T_d^{s} \) and parameters \( v_{ch}, \sigma_T, \) and \( \tau_{dn} \) into Equation (D10) it yields

\[
T_d^{\infty} = 4\pi n_i \frac{\lambda^4 n_d^2 \alpha m_d}{a} \frac{\epsilon}{0.3(2 + \zeta)^2}, \tag{D11}
\]

where \( \zeta = Q_{0e}(aK_b T_d) \) has been used. Using the usual expression for thermal velocity

\[
v_d^{\infty} \approx \left( \frac{3k_b T_d}{m_d} \right)^{1/2}, \tag{D12}
\]

and assuming the MRN grain size distribution, we obtain

\[
v_d^{\infty} \approx 6 \times 10^7 \left( \frac{T_d}{100 K} \right)^{3/2} \left( \frac{a}{10^{-6} \text{ cm}} \right)^{-3.25} \text{ cm s}^{-1}, \tag{D13}
\]

where \( n_i \) is in cm\(^{-3}\) and the renormalizing factor \( \alpha \) for the damping rate in the subsonic and supersonic regimes is given by Equation (B3) and (B4), respectively. Note that in the latter
case, \( \alpha \) is the function of dust velocity and hence Equation (D13) should be resolved for \( v_{\infty}^* \).

When the source term is smaller than the sink term, \( \alpha C < \gamma d \), the dust temperature increases with time until the equilibrium with gas is established, and no instability exists. From Equation (D6), we obtain

\[
T_{\text{d sat}} = \frac{1}{1 - \alpha C / \gamma d_n} T_n. \tag{D14}
\]

As \( \sigma_Q \) increases, \( T_{\text{d sat}} \) increases with \( \sigma_Q \).

REFERENCES

Bai, X.-N. 2011, ApJ, 739, 50  
Baines, M. J., Williams, I. P., & Asebiomo, A. S. 1965, MNRAS, 130, 63  
Chokshi, A., Tielens, A. G. G. M., & Hollenbach, D. 1993, ApJ, 407, 806  
Cui, C., & Goree, J. 1994, IEEE Trans. Plasma Sci., 22, 151  
Draine, B. T. 1985, in Protostars and Planets II (A86-12626 03-90) (Tucson, AZ: Univ. Arizona Press), 621  
Draine, B. T. 2003, ARA&A, 41, 241  
Draine, B. T. 2011, Physics of the Interstellar and Intergalactic Medium (Princeton, NJ: Princeton Univ. Press)  
Draine, B. T., & Lazarian, A. 1998, ApJ, 508, 157  
Draine, B. T., & Li, A. 2007, ApJ, 657, 810  
Draine, B. T., & Salpeter, E. E. 1979, ApJ, 231, 77  
Draine, B. T., & Sutin, B. 1987, ApJ, 320, 803  
Dullemond, C. P., & Dominik, C. 2008, A&A, 487, 205  
Dunkley, J., Ambard, A., Baccigalupi, C., et al. 2009, in AIP Conf. Proc. 1141, CMB Polarization Workshop: Theory and Foregrounds: CMBPol Mission Concept Study (Melville, NY: AIP), 222  
Fortov, V. E., Ivlev, A. V., Khrapak, S. A., Khrapak, A. G., & Morfill, G. E. 2005, Phys. Rep., 421, 1  
Frasie, A. A., Brown, J.-A. C., Dobler, G., et al. 2009, in AIP Conf. Proc. 1141, CMB Polarization Workshop: Theory and Foregrounds: CMBPol Mission Concept Study (Melville, NY: AIP), 265  
Goldreich, P., & Ward, W. R. 1973, ApJ, 183, 1051  
Hirashita, H., & Yan, H. 2009, MNRAS, 394, 1061  
Hoang, T., Draine, B. T., & Lazarian, A. 2010, ApJ, 715, 1462  
Hoang, T., Lazarian, A., & Draine, B. T. 2011, ApJ, 741, 87  
Hoang, T., Lazarian, A., & Schlickeiser, R. 2012, ApJ, 747, 54  
Ivlev, A. V., Lazarian, A., Tsytovich, V. N., et al. 2010, ApJ, 723, 612  
Ivlev, A. V., Zhdanov, S. K., Klumov, B. A., et al. 2004, Phys. Rev. E, 70, 66401  
Khrapak, S. A., Nefedov, A. P., Petrov, O. F., & Vaulina, O. S. 1999, Phys. Rev. E (Stat. Phys.), 59, 6017  
Lazarian, A. 2007, J. Quant. Spectrosc. Radiat. Transfer, 106, 225  
Lazarian, A., & Finkbeiner, D. 2003, New Astron. Rev., 47, 1107  
Lazarian, A., & Yan, H. 2002, ApJ, 566, L105  
Mathis, J. S., Mezger, P. G., & Panagia, N. 1983, A&A, 128, 212  
Mathis, J. S., Rumpl, W., & Nordsieck, K. H. 1977, ApJ, 217, 425  
Matsoukas, T., & Russell, M. 1997, Phys. Rev. E (Stat. Phys.), 55, 991  
Morfill, G. E., & Ivlev, A. V. 2009, Rev. Mod. Phys., 81, 1353  
Orrm, C. W., Paszun, D., Dominik, C., & Tielens, A. G. G. M. 2009, A&A, 502, 845  
Ossenkopf, V. 1993, A&A, 280, 617  
Perez-Becker, D., & Chiang, E. 2011, ApJ, 727, 2  
Purcell, E. M. 1969, Physica, 41, 100  
Satronov, V. S., & Zvyagina, E. V. 1969, Icarus, 10, 109  
Schlickeiser, R., & Miller, J. A. 1998, ApJ, 492, 352  
Spitzer, L. 1978, Physical Processes in the Interstellar Medium (New York: Wiley)  
Tielens, A. G. G. M. 2005, The Physics and Chemistry of the Interstellar Medium (Cambridge: Cambridge University Press)  
Tsytovich, V. 1997, Sov. Phys.—Uspekhi, 40, 53  
von Boekel, R., Waters, L. B. F. M., Dominik, C., et al. 2003, A&A, 400, L21  
Weidenschilling, S. J. 1980, Icarus, 44, 172  
Weidenschilling, S. J. 1995, Icarus, 116, 433  
Weingartner, J. C., & Draine, B. T. 2001, ApJS, 134, 263  
Whittet, D. C. B. 2003, Dust in the Galactic Environment (2nd ed.; IOP Publishing: Bristol)  
Yan, H. 2009, MNRAS, 397, 1093  
Yan, H., & Lazarian, A. 2003, ApJ, 592, L33  
Yan, H., Lazarian, A., & Draine, B. T. 2004, ApJ, 616, 895