T_c suppression in co-doped striped cuprates

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We propose a model that explains the reduction of T_c due to the pinning of stripes by planar impurity co-doping in cuprates. A geometrical argument about the planar fraction of carriers affected by stripe pinning leads to a linear T_c suppression as a function of impurity concentration z. The critical value z_c for the vanishing of superconductivity is shown to scale like T_c^2 in the under-doped regime and becomes universal in the optimally- and over-doped regimes. Our theory agrees very well with the experimental data in single- and bi-layer cuprates co-doped with Zn, Li, Co, etc...

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One of the most striking properties of high-temperature superconductors (HTSC) is the sensitivity of the critical temperature, T_c, to planar impurities which are introduced with the substitution of the Cu atoms in the CuO_2 planes. T_c is suppressed with a few percent of doping almost independently of the magnetic nature of the impurity. In particular it has been shown experimentally that the HTSC undergo a superconductor to insulator transition due to Zn doping. The fast depression of T_c with Zn has been studied extensively by NMR, µSR, infrared techniques and a heated debate has been developed on the interpretation of the experimental data. It has also been established that this suppression is more robust in the under-doped compounds. The reduction of T_c has been assigned to formation of magnetic defects, electron scattering by disorder in the presence of a d-wave order parameter, unitary scattering, local variation of the superconducting gap, and weak localization. However, although the existing theories describe well the behavior of these systems at low impurity concentration, they systematically deviate from the experimental data at higher doping concentration.

In this work we introduce a scenario for the destruction of superconductivity which is based on the stripe picture. We propose that Zn pins stripes and slows down their dynamics in a region of size R in its surrounding. This effect resembles the “swiss cheese”-like model with Zn sites creating voids in the superconducting mesh and destroying superfluid density. In our model, however, the superfluid density remains unaltered upon Zn co-doping, whereas the stripe inertia increases locally. Since the stripe mass density ρ is inversely proportional to the hopping energy t, a local decrease of t by pinning will enhance the stripe effective mass, ρ. Magnetic studies of the effect of Zn doping in cuprates indicate that co-doping indeed leads to charge carrier localization. T_c(x, z) is a function of the carrier density x and z and it is determined by the ratio between the 3D superfluid density n_s and the ab-plane charge carriers effective mass m^{ab}_s. Within the stripe picture we must have T_c ∝ n_s/ρ. Hence, the pinning of stripes in the vicinity of the impurity implies a reduction of T_c by the enhancement of ρ. As a result of our model we obtain T_c(x, z)/T_c(x, 0) = 1 − z/z_c(x), where z_c(x) is the critical concentration required to completely suppress superconductivity. In under-doped compounds, z_c(x) ∝ x^2 and scales with T_c^2(x, 0) in contrast with the case of ordinary superconductors. In optimally- and over-doped materials z_c(x) is independent of x and exhibits universal behavior. Our results give a very good description of the experimental data in La_{2−x}Sr_xCuO_4 (LSCO), YBa_2Cu_3O_7−δ (YBCO), and Bi_2Sr_2Ca_1−yY_yCu_2O_{8+δ} (BSCO) co-doped with different impurities, such as Zn, Li, Co, etc.

FIG. 1. Effect of Zn on the stripe grid. Dark dashed lines mark the regions of stripes with finite superfluid density. The circle shows the region where hopping is suppressed and the superfluid effective mass is increased.

Striped phases of holes have been observed experimentally in the superconductor La_{1.6−x}Nd_{0.4}Sr_xCuO_4 and in La_{2−x}Sr_xNiO_{4+y}, which is an antiferromagnetic insulator. Moreover, stripe formation provides a
simple explanation for the observed magnetic incommensurability in LSCO \[20\] and YBCO \[21\]. The magnetic incommensurability appears in neutron scattering by the splitting of the commensurate peak at \(Q = (\pi/a, \pi/a)\) \((a \approx 3.8\text{Å} \text{ is the lattice spacing})\) by a quantity \(\delta\), which is inversely proportional to the average inter-stripe distance \(\ell\) \[22\]. In under-doped LSCO, the charge stripes behave as an incompressible quantum fluid and \(\delta\) is proportional to \(x\) \[22\]. In this regime \(T_c\) scales with \(\delta\). \[23\]

\[
T_c(x) = \frac{569}{\ell(x)} \propto \delta
\]

where \(\ell\) is given in Å. On the other hand, in the optimally and over-doped compounds the incommensurability \(\delta\) saturates to a constant value and \(\ell\) becomes nearly independent of \(x\) \[22\]. To date, there is strong evidence for stripe formation in LSCO compounds \[22,23\]. Recent ion-channeling \[24\], NMR \[23\], μSR \[25\] and neutron scattering data \[26\] also indicate that stripes might be present in YBCO, whereas a lack of experimental data still persists for BSCO. In what follows we will concentrate on the physics of HTSC co-doped with planar impurities, like \(\text{La}_{1.82-x}\text{Sr}_x\text{Cu}_{1-y}\text{Zn}_y\text{O}_4\). We will verify that our results can describe equally well YBCO and BSCO co-doped with Zn, Li, and Co.

For simplicity, we assume that the Zn atoms are located half distance between superconducting stripes and suppress stripe fluctuations within a circle of radius \(R\) around their position, as shown in Fig. 1. This distance \(R\) is assumed to scale with the average inter-stripe distance \(\ell\). Thus, we parameterize \(R = \gamma \ell/2\) with the doping independent phenomenological parameter \(\gamma > 1\). As it is shown in Fig. 1, the stripe length which is pinned per Zn atom is \(4 \sqrt{R^2 - \ell^2}/4 = 2\ell \sqrt{\gamma^2 - 1}\). If we assume that all the \(N_{Zn}\) Zn atoms take part in pinning the stripes, the total pinned length is \(N_{Zn} 2\ell \sqrt{\gamma^2 - 1}\). The transverse kinetic energy density of the stripe is \(t/a\), where \(t\) is the single hole kinetic energy. Thus, the suppressed energy density in the plane is \(\delta \tau = (t/a) 2\ell \sqrt{\gamma^2 - 1} N_{Zn}/L^2\) where \(L\) is the sample size. Defining the impurity fraction \(z = N_{Zn}(a/L)^2\) and the planar kinetic energy density \(\tau = t/(a\ell)\), we find that the suppressed energy density reads

\[
\delta \tau = \frac{2\ell \sqrt{\gamma^2 - 1}}{a^2} \tau(x,0). \tag{2}
\]

The energy density of the co-doped system is given by \(\tau(x, z) = \tau(x,0) - \delta \tau\), which leads to

\[
\frac{\tau(x, z)}{\tau(x,0)} = 1 - \frac{z}{z_c(x)}; \tag{3}
\]

where

\[
z_c(x) = \frac{1}{2\sqrt{\gamma^2 - 1}} \left(\frac{a}{\ell(x)}\right)^2. \tag{4}
\]

Superconducting long range order, and therefore \(T_c\), is obtained when the stripe array attains phase coherence. This coherence can be achieved by the Josephson coupling between stripes which may occur via exchange of Cooper pairs \[28\] or \(d_{x^2-y^2}\) bosons \[29\]. In this case the finite transition temperature is in the 2D XY universality class (stabilized by the inter-planar coupling). Thus, \(T_c\) is given by

\[
T_c = \frac{\pi\hbar^2 L_c}{2k_B m_{ab}} n_s
\]

where \(L_c\) is the inter-planar distance. Notice that \(n_s \sim 1/(2a\ell(x) L_c)\) (Ref. \[18\]) and \(m_{ab}\) is proportional to the stripe linear mass density \(\rho = \hbar^2/(ta^3)\) \[12\]. Therefore we conclude that

\[
T_c(x, z) \propto \frac{t(z)}{\ell(x)} \propto \tau(x, z). \tag{6}
\]

Combining (1) with (3) we find:

\[
\frac{T_c(x, z)}{T_c(x,0)} = \frac{\tau(x, z)}{\tau(x,0)} = 1 - \frac{z}{z_c(x)}.
\]

![FIG. 2. \(T_c(x, z)/T_c(x,0)\) versus the Zn doping concentration \(z\) for the lanthanate-compound. Dashed and dotted lines correspond to Eq. (3) with \(z_c\) given by Eqs. (8) and (9), respectively. Red circles denote \(x = 0.10\) \[30\], circles \(x = 0.15\) \[1\], lozenges \(x = 0.18\) \[22\], triangles \(x = 0.20\) \[21\], squares \(x = 0.20\) \[30\].](image)

It is very illuminating to compare the expressions for \(z_c(x)\), that is, the critical Zn doping for which \(T_c\) vanishes, in the various regimes. For under-doped lanthanates \((x < 1/8)\), neutron scattering data show that \(T_c(x,0) \propto \delta\) \[22\]. Comparing (3) with (1), one finds an unforeseen result, which is

\[
z_c(x) = \left(\frac{a(\AA)}{2}\right)^2 \frac{T_c^2(x,0)}{805} \sqrt{\gamma^2 - 1} \tag{8}
\]

in the under-doped regime. Moreover, in the optimally and over-doped compounds the average stripe separation
saturates to a value $\ell_s$, and therefore we expect $z_c$ to be an universal constant, independent of $x$,

$$z_c = \frac{1}{2\sqrt{\gamma^2 - 1}} \left(\frac{a}{\ell_s}\right)^2. \quad (9)$$

In Fig. 3 we plot the experimental data for LSCO co-doped with Zn [22]. The dashed and dotted lines are the theoretical result (8) with $z_c$ given by Eqs. (8) and (9), respectively. The agreement between theory and experiment is excellent. One can easily observe that indeed the data for different $x$ collapse into a single line for optimally and over-doped compounds. In addition, in Fig. 3 we show the experimental data for $z_c \times T_c$, as well as the theoretical prediction (8). In both figures, we used $\gamma \sim 1.42$ so that $R \approx 0.71f$. Hence, for the optimally- and over-doped materials, for which the mean stripe distance $\ell_s = 4a = 15.2\AA$, we estimate $R \approx 10\AA$. The value of $R$ can be independently obtained in neutron scattering experiments in Zn doped samples. At this point in time, however, we are not aware of any data in this regard for LSCO.

The excellent agreement between theory and experiment is however not restricted to the Zn doped lanthanates. In Fig. 4 we display the experimental data for YBCO (red points) co-doped with Li (squares) and Zn (circles) [32], as well as for BSCO co-doped with Co (black points) [22]. The universal behavior at high doping is very clear (Fig. 4a) as well as the different slopes characteristic of the under-doped compounds (Fig. 4b). Observe that YBCO and BSCO exhibit exactly the same features in terms of Zn doping, with the same value of $z_c$ in the over-doped regime. We assign this behavior to the fact that both systems are bi-layers while LSCO is a single-layer system. Moreover, it has been shown recently that the relationship of $T_c$ and $\delta$ for YBCO is similar to the one in LSCO [24]. Using the value of the incommensurability $\delta = 0.11$ measured by neutron scattering in YBCO$_{6.6}$ [27], which has $T_c = 60K$, we can determine the stripe separation in this under-doped compound, $\ell(x) \approx 4.55a$. Then, by fitting the data from Ref. [33] for the same compound (red circles in Fig. 4b, $z_c = 0.04$) to Eqs. (7) and (8), we determine the only free parameter in our model, $\gamma = 1.17$, corresponding to $R \sim 10\AA$. Now, we use this value of $\gamma$ to analyze the overdoped regime. The universal line in Fig. 4a defines the critical doping in over-doped bi-layer cuprates, $z_c \sim 0.13$. By replacing $z_c$ and $\gamma$ in Eq. (8), we can estimate the saturation value of the average stripe distance. We then find $\ell_s \sim 3a$, corresponding to $R = \gamma \ell_s/2 \sim 6\AA$. These results agree with magnetic measurements in over-doped YBCO with Zn impurities: NMR studies suggest the existence of two different relaxation times ascribed to $^{63}$Cu sites away from and near Zn impurities [9] and inelastic neutron scattering data [35] indicate that Zn induces a magnetic perturbation on a range of $R \sim 7\AA$, very close to our estimates. Moreover, we predict that the saturation of the incommensurability in YBCO should start at optimal doping, where $\delta_s \sim 0.2$ and $\ell_s \sim 3a$, in contrast with the case of LSCO where the saturation starts at $x \sim 0.12$ and $\ell_s \sim 4a$. This result agrees with recent neutron scattering data in YBCO [33] and establishes a very important relationship between two different kinds of experiments, namely, magnetization and neutron scattering. By determining the doping concentration $x$ beyond which the $T_c(x, z)/T_c(x, 0)$ versus $z$ line becomes universal, one is automatically finding the value of $x$ beyond which a saturation should be reached in the $\delta$ versus $x$ plot. It is also a straightforward conclusion of the data in Fig. 4 that BSCO has a behavior very similar to YBCO although there is no direct evidence for incommensurability or stripes in BSCO. It is interesting to notice that $T_c(x, z)/T_c(x, 0)$ when plotted versus the in-plane 2D residual resistance, $\rho_0^{2D}$, exhibits universal behavior in the under-doped but not in the over-doped regime [2]. However, as a function of $z$, as we have shown here, the universal behavior is found in the optimally- and over-doped regimes. This fact is due to the linear dependence of $\rho_0^{2D}$ with $z$ [3].

We finally would like to point out that the prediction that $z_c \propto T_c^2(x, z = 0)$ is highly unusual. In BCS-like models the Abrikosov-Gor’kov [36] approach predicts that $T_c$ is suppressed when the scattering rate $1/\tau_s \propto z$ becomes of the order of the superconducting gap $\propto T_c(x, 0)$ and therefore one would expect $z_c \propto T_c(x, z = 0)$ in disagreement with the data.

In conclusion, we presented a “geometrical” model for the suppression of $T_c(x, z)$ by planar impurities within the stripe model. It is based on the assumption that Zn impurities work as local pinning centers and suppress the transverse kinetic energy density of the stripes in their immediate vicinity. Our model leads to a linear $T_c$ suppression with doping, with a slope inversely proportional...
to the critical impurity concentration \(z_c\), as given by Eq. (3). Moreover, we find that \(z_c\) exhibits a non-trivial behavior in under-doped compounds, \(z_c(x) \propto T_c^2(x)\), and a universal one, \(z_c = \text{const. at optimally- and over-doped compositions.}\) The obtained results agree very well with the experimental findings and allow to establish a connection between neutron scattering and magnetization measurements.

Fig. 4. NMR and \(\mu\)SR data for yttrium-and bismuth-based compounds. Red points correspond to \(\text{YBa}_2\text{Cu}_3\text{O}_{7-x}\) co-doped with Li (squares) and Zn (circles), whereas open symbols correspond to \(\text{Bi}_2\text{Sr}_{2-x}\text{Y}_x\text{Cu}_3\text{O}_{6+y}\) \(x = 0.2\). (a) optimally- and over-doped compounds; (b) under-doped materials.

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