Cosmology, Particle Physics, and Superfluid $^3$He

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Abstract

Many direct parallels connect superfluid $^3$He with the field theories describing the physical vacuum, gauge fields and elementary fermions. Superfluid $^3$He exhibits a variety of topological defects which can be detected with single-defect sensitivity. Modern scenarios of defect-mediated baryogenesis can be simulated by the interaction of the $^3$He vortices and domain walls with fermionic quasiparticles. Formation of defects in a symmetry-breaking phase transition in the early Universe, which could be responsible for large-scale structure formation and for microwave-background anisotropy, also may be modelled in the laboratory. This is supported by the recent observation of vortex formation in neutron-irradiated $^3$He-B where the "primordial fireball" is formed in an exothermic nuclear reaction.

1 INTRODUCTION

There are many similarities between superfluid $^3$He and particle physics stemming from the fact that both systems are described by the quantum field
theory. The superfluid $^3$He is a unique system among the other condensed matter, because it has the maximum symmetry breaking, which can be compared only with the vacuum in the elementary particle physics, and due to the rich set of the fermionic and bosonic excitations interacting with gauge-like fields of numerous collective modes \[1, 2\]. Most of the common phenomena are yet to be confirmed in the particle physics while they are quite routinely observed or can be investigated in $^3$He. These include the variety of topological defects with their potential astrophysical and cosmological consequences.

Till now in particle physics only the perturbative features of the modern quantum field theories have been put to the test and it would be worthwhile to have a better understanding of the non-perturbative aspects. Such an understanding may be crucial, for example, for testing the hypothesis that the baryon number of the Universe was produced (baryogenesis) during the electroweak or other phase transition in the early Universe. To gain some intuition, we need such condensed matter system as superfluid $^3$He, which has the closest similarity with the physical vacuum. This makes the superfluid $^3$He a working laboratory for modelling different processes which can occur in the physical vacuum and in Universe.

2 VORTICES and COSMIC STRINGS

2.1 Topological defects

The complicated topology of the $^3$He vacuum results in a variety of defects in $^3$He, such as monopoles, hedgehogs, boojums, solitons, domain walls, textures, vortices, which have many direct analogies with topological defects in quantum vacuum. In particular, due to the similar symmetry breaking in $^3$He-A and in the electroweak vacuum \[3\], some of four vortices observed in $^3$He-A \[3\] share the common properties with the $Z$– and $W$– strings of the electroweak model \[4, 5\]. There are also combinations of topological defects of different dimensionality found in Helsinki rotating cryostat: In $^3$He-A it is the vortex sheet – the soliton plane filled with the vortex lines \[6\], which topology is similar to that of Bloch lines within the Bloch wall in ferromagnets. In $^3$He-B it is the soliton terminating on the line defect \[7\], this combined object is topologically similar to the cosmic wall terminating on cosmic string \[8\].
2.2 Phase transitions in vortices and strings.

Several phase transitions related to the quantized vortices have been observed in Helsinki. (i) Spontaneous breaking of continuous symmetry in the vortex core was experimentally verified in $^3$He-B: a new Goldstone mode, related to the twist of the deformed vortex core, has been excited in NMR experiments\cite{9}. In cosmic strings the analogous breaking of $U(1)$ symmetry in the core results in the superconducting current along the twisted core\cite{10}. The instability towards the symmetry breaking in the core can be triggered by fermions living in the core, see \cite{11} for cosmic strings and \cite{12} for vortices.

(ii) The textural phase transitions with the change of the topological invariants have been identified in rotating $^3$He-A\cite{3}. In some of them the transformation of the vortex can occur by motion of point defects along the vortex line\cite{13}. The relevant point defect can be the monopole (analog of Dirac magnetic monopole) or the hedgehog (analog of ’tHooft-Polyakov monopole). In the future this scenario of the phase transition will be tested in detail.

2.3 Half-quantum vortices

The vortices with the fractional winding number $N = 1/2$ \cite{14} are the counterpart of Alice strings, which appear in some models of particle physics (see eg \cite{15}). The particle travelling around some type of the Alice string changes its electric charge to the opposite \cite{16}. In $^3$He-A, the analogous effect is the reversal of the spin of the quasiparticle upon circling the 1/2 vortex. This behavior results also in the peculiar Aharonov-Bohm effect, which has been discussed for 1/2 vortices in $^3$He-A \cite{17, 13} and has been modified for the cosmic Alice strings in \cite{18, 19}. The 1/2-vortex can also occur in the $d$-wave superconductors \cite{20} and recently it has been identified in high-temperature superconductor \cite{21}.

3 PROBING OF VACUUM

3.1 Vacuum instability

The instability of the physical vacuum can occur in strong electric or gravity fields. The former can be realized in collision of two heavy nuclei\cite{22} (see
also Refs. [23, 24]), while the latter can happen in the vicinity of the black hole. The analog of the former instability is realized in the moving $^3$He-B, if its velocity reaches the Landau value: The creation of quasiparticles in supercritical regime, detected in vibrating wire experiments [25], is similar to the creation of the electron-positron pairs by strong field.

### 3.2 Gravity and $^3$He

In many cases the motion of the elementary excitations in condensed matter corresponds to the motion of the particle in the effective gravity field represented by the metric tensor [26, 27, 1, 28]. In superfluid $^3$He the gravity field is simulated by the order parameter texture. This condensed matter analog of Sakharov’s effective gravity [29] allows to model several phenomena related to strong or quantum gravity: (i) In the effective gravity the cosmological constant is many orders of magnitude larger than the experimental upper limit [30]. Superfluid $^3$He possibly gives some hint how to resolve this paradox [27, 1]. (ii) The problem of the black hole entropy and quantum radiation [31] can be modelled by moving texture (soliton or interface between $^3$He-A and $^3$He-B). When the velocity of the texture exceeds the critical value (which is rather low and is easily achieved in the $^3$He experiment) the metric resembles that of the black hole: the event horizon appears [32]. This is the realization of the idea of Unruh who introduced the event horizon in condensed matter [26]. This should help to understand such problems in quantum gravity as possibility of the nonunitarity of quantum state and the behavior of the vacuum in strong gravity. The black hole analogy was also discussed for closed string [33] and vortex [34].

### 3.3 Homogeneously precessing vacuum

Nonlinear spin-coherent dynamics of the $^3$He-B vacuum discovered in [35] is the subject of intensive investigations. Several dynamical vacua, precessing in the applied magnetic field, have been found and identified in NMR experiments (see the latest Refs. [36, 37]). The analogy with the nonperturbative dynamics of the physical vacuum is to be found.
3.4 Other vacuum effects

Vacuum polarization and zero charge phenomena have their counterpart in $^3$He \cite{1}. The vacuum pressure (Casimir effect) and vacuum friction (quantum radiation from the mirror moving in the physical vacuum, see Refs. in \cite{39}) are reproduced by the moving interface between $^3$He-A and $^3$He-B \cite{38}.

4 VORTEX MOTION vs BARYOGENESIS

The popular hypothesis of the baryogenesis is that the baryon number of the Universe was produced during the electroweak phase transition in the early hot Universe. According to modern scenarios \cite{8, 40} the baryogenesis was mediated by topological defects. Similar situation occurs in superfluid $^3$He and superconductors, where the dynamics of vortices and domain walls leads to the production of the fermionic charges. In this Section we discuss the ”momentogenesis” – production of the fermionic linear momentum during the vortex motion. This leads to the additional nondissipative force on the vortex which competes with conventional Magnus force.

4.1 Three forces on moving vortex

The dynamics of the vortex is governed by interaction of 3 object: (1) Superfluid vacuum moving with the velocity $v_s$. (2) The system of excitations (matter) which form the normal component of the liquid. It moves with the velocity $v_n$ (heat bath or normal velocity). (3) The vortex moving with the velocity $v_L$. The vortex also carries the excitations localized in the vortex core \cite{41}.

As a result the equation for the balance of forces acting on the moving vortex contains 3 nondissipative forces and the friction one \cite{12}:

$$F_M + F_1 + F_{sf} + D(v_n - v_L) = 0$$

Here $D$ is the parameter of the friction force, which arises when the vortex moves with respect to the heat-bath. The nondissipative forces contain (i) the conventional Magnus force on the vortex moving with respect to the superfluid vacuum

$$F_M = \rho N \vec{K} \times (v_L - v_s).$$

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Here $N$ is the vortex winding number, $\kappa$ is the circulation quantum, $\rho$ the density of the liquid.

(ii) Iordanskii force is the result of the Aharonov-Bohm effect, experienced by excitations in bulk liquid, when they interact with the vortex:

$$ F_1 = N\kappa \times \tilde{\rho}_n(T)(v_s - v_n) \ . \quad (4.3) $$

Here $\tilde{\rho}_n(T)$ is the (tensor) density of the normal component. This effect is analogous to the gravitational Aharonov-Bohm topological effect discussed for the spinning cosmic string, i.e., for the strings with an angular momentum.

(iii) The so-called spectral-flow force is responsible for the "momentogenesis" which we discuss below

$$ F_{sf} = NC(T)\kappa \times (v_n - v_L) \ . \quad (4.4) $$

The parameter $C(T)$ is determined by the anomalous dynamics of fermions in the core.

### 4.2 Baryogenesis and axial anomaly.

There is a close connection between the spectral flow force and the nonconservation of the baryon number $N_B$ in the presence of the $SU(2)$ and $U(1)$ field strengths $F_{\mu\nu}$ and $F_{\mu\nu}$

$$ \partial_t N_B = \frac{N_F}{32\pi^2} \int d^3r \left( -F_{\mu\nu}F^{\mu\nu} + F_{\mu\nu}\tilde{F}^{\mu\nu} \right) \quad (4.5) $$

Here $N_F$ is the number of families. The nonconservation of $N_B$ is the result of the phenomenon of the axial anomaly predicted by Adler and Bell and Jackiw. The fields $F_{\mu\nu}$ and $F_{\mu\nu}$ can be generated in the core of the cosmic strings evolving in the expanding Universe.

### 4.3 Momentogenesis in $^3$He-A.

Similar nonconservation of the fermionic charge occurs in $^3$He-A. The $^3$He-A quasiparticles in the vicinity of the point gap nodes are chiral: like neutrino they are either left-handed or right-handed. For such gapless chiral fermions the Adler-Bell-Jackiw anomaly of the kind in Eq. (4.5) takes place.
The "chiral charge" of quasiparticles \( N_{ch} \) is not conserved in the presence of the electric and magnetic fields \( E = \partial_t A, \ B = \vec{\nabla} \times A \):

\[
\partial_t N_{ch} = \frac{1}{2\pi^2} \int d^3 r \ \partial_t A \cdot (\vec{\nabla} \times A) . \tag{4.6}
\]

In \(^3\text{He}-\text{A}\) the vector potential \( A \) acting on the quasiparticles is simulated by the hat texture, \( A = p_F \hat{l} \), where \( \hat{l} \) is a unit vector in the direction of the gap node and \( p_F \) the Fermi momentum.

For us it is important that the left quasiparticle carries the linear momentum \( p_F \hat{l} \), and the equal momentum is carried by the left quasihole. As a result the counterpart of the axial anomaly in condensed matter leads to the net product of the fermionic linear momentum \( P \) in the time-dependent texture:

\[
\partial_t P = \frac{1}{2\pi^2} \int d^3 r \ p_F \hat{l} \ (\partial_t A \cdot (\vec{\nabla} \times A)) . \tag{4.7}
\]

Since the total linear momentum is conserved in condensed matter, the Eq.(4.7) means that the momentum is transferred from the superfluid vacuum to the matter (the normal component) in the presence of the time-dependent texture.

### 4.4 Momentogenesis by continuous vortex.

The continuous vortex moving in 3He-A generates both the "electric" and "magnetic" fields and thus represents the right texture, which leads to the momentogenesis. Integration of the anomalous momentum transfer in Eq.(4.7) over the cross-section of the soft core of the moving vortex gives the lost of the linear momentum [7]. This corresponds to the force acting on the vortex in the form of Eq.(4.4):

\[
\mathbf{F}_{sf} = N C_0 \vec{\kappa} \times (\mathbf{v}_n - \mathbf{v}_L) . \tag{4.8}
\]

The parameter \( C(T) \) appears to be temperature independent, \( C_0 = m_3 p_F^3 / 3\pi^2 \), and is close to the mass density \( \rho \). The continuous vortex provides an example of the extreme spectral-flow force, which effectively cancels the Magnus force in Eq.(4.2).
4.5 Spectral-flow force on singular vortex.

For singular vortices in $^3$He-B and superconductors the spectral flow is suppressed due to discrete character of the fermionic spectrum in the vortex core. The anomalous exchange between the core fermions and the heat bath now depends on the kinetics, determined by the level spacing $\omega_0$ and the life-time $\tau$ of the core fermions:

$$C(T) \approx C_0 \left[ 1 - \frac{\omega_0^2 \tau^2}{1 + \omega_0^2 \tau^2} \tanh \frac{\Delta(T)}{2T} \right]. \quad (4.9)$$

The extreme limit, $C(T) = C_0$, takes place when the interlevel distance can be neglected compared to the life time: $\omega_0 \tau \ll 1$, i.e. when the fermionic spectrum can be considered as continuous and the spectral flow is not suppressed. In $^3$He-A and in dirty enough superconductors this occurs at practically arbitrary temperature, while for clean superconductors and for $^3$He-B this regime appears only at high temperature (for $^3$He-B at $T > 0.5 T_c$, see below).

The dynamics of the core fermions also determines the friction parameter $D$:

$$D \approx \kappa C_0 \tan \frac{\Delta(T)}{2T} \frac{\omega_0 \tau}{1 + \omega_0^2 \tau^2}. \quad (4.10)$$

4.6 Experiment on vortex dynamics

The forces on the vortex have been recently measured in superfluid $^3$He-B in a broad temperature range. The results are expressed in terms of

$$d_\parallel = \frac{D(T)}{\kappa \rho_s(T)} \quad d_\perp = \frac{C(T) - \rho_n(T)}{\rho_s(T)} \quad (4.11)$$

The experimental bell shape of $d_\parallel(T)$ follows the temperature dependence of $\omega_0 \tau/(1 + \omega_0^2 \tau^2)$ in the Eq.(4.10) with $\omega_0 \tau \gg 1$ at $T \ll T_c$ and $\omega_0 \tau \ll 1$ close to $T_c$. The temperature dependence of the observed $d_\perp(T)$ reproduces the Eq.(4.9): At low $T$ the observed negative sign of $d_\perp(T)$ reflects the suppression of the spectral flow according to Eq.(4.9): $C(T) \ll \rho_n(T) \ll \rho$. The observed upturn of $d_\perp(T)$ demonstrates an increase of $C(T)$ with increasing $T$. The positive value of $d_\perp(T)$ at $T > 0.5 T_c$ shows that at these $T$ the spectral flow already dominates: $C(T) > \rho_n(T)$. And finally at higher
$T$, where $\omega_0 \tau \ll 1$, the experimental $d_\perp (T)$ approaches 1, which shows that $C(T) \rightarrow \rho$, ie the spectral flow reaches its extreme limit.

5 BIG BANG SIMULATIONS

5.1 Introduction

The string scenario of the baryogenesis implies that the strings appear at some stage of the expanding Universe. These strings could be also responsible for large-scale structure formation and for microwave-background anisotropy of the Universe[8]. In a popular theory the topological defects can be formed, if during the cooling of the Universe a symmetry-breaking phase transition occurs at some critical temperature $T_c$ [56, 57]. According to Zurek modification[58] of the Kibble mechanism[57], the topological defects appear due to the nonequilibrium dynamics of the phase ordering. The nonequilibrium processes take place because the cool-down through $T_c$ occurs with finite rate, as a result the order parameter fluctuations above $T_c$ are quenched below $T_c$.

5.2 Advantages of 3He for simulations

The physics of the nonequilibrium phase transition in expanding Universe is the same as in condensed matter, and thus may be modelled in the laboratory where the cooling rate can be controlled [58]. Laboratory experiments intended to test different theories of cosmological defect formation have recently been conducted in liquid crystals [59] and superfluid $^4$He [60]. The superfluid phases of liquid $^3$He open a new page in these Big-Bang simulations. $^3$He has several advantages over other systems: (i) It is more closely resembles the complicated physical vacuum. (ii) It has a variety of bosonic and fermionic elementary particles which are very important for the physics of the evolution of the topological defects. The fermionic quasiparticles can be detected with sensitive vibrating wire detector[61]. (iii) The superfluid $^3$He exhibits several phase transitions[62], this allows us to investigate the defects formation after the 1-st and the 2-nd order transitions. (iv) We can investigate formation of different types of the topological defects, some of them can be detected with NMR methods with single-defect sensitivity [63].
(v) Of particular relevance to the quench experiment is the fact that liquid $^3$He can be locally efficiently heated with thermal neutrons \cite{64, 61, 65, 66}. Thermal neutrons produce the ‘primordial fireball’ due to the nuclear reaction $\nu + ^3\text{He} = p + ^3\text{H} + 0.76\text{MeV}$. In Helsinki experiments \cite{65} it was found that the subsequent rapid cooling of the locally overheated liquid through the superfluid transition $T_c$ results in the formation of vortex rings. If the $^3\text{He-B}$ container rotates, sufficiently large vortex rings grow under the influence of the Magnus force, escape from the heated bubble, and are detected individually with nuclear magnetic resonance.

### 5.3 Primordial fireball by nuclear reaction

In Helsinki experiment \cite{65} the neutrons, produced with a paraffin moderated Am-Be source of 10 mCi activity, are incident upon the $^3\text{He}$ sample container. At the minimum distance of 22 cm between source and sample, $\nu \approx 20$ neutrons/min are absorbed by the $^3\text{He}$ liquid. For thermal neutrons the mean free path is 0.1 mm in liquid $^3\text{He}$ and thus all absorption reactions occur close to the walls of the container. Since the incident thermal neutron has low momentum, the 573 keV proton and 191 keV triton fly apart in opposite directions, producing 70 and 10 $\mu$m long ionization tracks, respectively. The subsequent charge recombination yields a ‘primordial fireball’ – a heated region of the normal liquid phase. This process was discussed in details in connection to the $^3\text{He-A} \rightarrow ^3\text{He-B}$ transition mediated by neutrons \cite{64}: the fireball may lead to the formation of the critical bubble of the $^3\text{He-B}$ phase in the supercooled $^3\text{He-A}$. As in \cite{64} we assume for simplicity that the fireball has a spherically symmetric shape.

This bubble of normal fluid cools by the diffusion of quasi-particle excitations out into the surrounding superfluid with a diffusion constant $D \approx v_F l$ where $v_F$ is their Fermi velocity and $l$ the mean free path. The difference from the surrounding bulk temperature $T_0$ as a function of the radial distance $r$ from the centre of the bubble is given by $T(r, t) - T_0 \approx \left(\frac{E_0}{(4\pi D t)^{3/2}C_v}\right) \exp\left(-\frac{r^2}{4Dt}\right)$ where $E_0$ is the energy deposited by the neutron as heat and $C_v$ the specific heat. The maximum value of bubble radius $R_b$ with fluid in the normal phase, $T(r) > T_c$, is

$$R_b \sim \left(\frac{E_0}{C_v T_c}\right)^{1/3}(1 - T_0/T_c)^{-1/3},$$  \hspace{1cm} (5.1)

which is typically of order 10 $\mu$m. The bubble cools and shrinks away rapidly.
with the characteristic time $\tau_Q \sim R_b^2/D \sim 10^{-6}$ s.

5.4 Defect formation by quench

The distance $\xi_v(t)$ between the defects in the process of the phase ordering after the quench is of primary interest. The real defects appear at the moment $t^*$ when they can be distinguished from the background fluctuations of the order parameter. The density of the defects at $t^*$ (the so called initial vortex density) was recently the subject of controversy. In the original Kibble scenario $t^*$ was identified with the moment when the system cools below the Ginzburg temperature $T_G$ above which the thermal fluctuations prevail, i.e. $T(t^*) = T_G$. This implies that the initial distance between the vortices is the coherence length at $T_G$, i.e. $\xi_v(t^*) = \xi(T_G) = \xi_0/\sqrt{1 - (T_G/T_c)}$, and does not depend on the cooling time $\tau_Q$. In superfluid $^3$He the critical fluctuation region is extremely narrow, $1 - (T_G/T_c) \sim (a/\xi_0)^4$, because the zero temperature coherence length $\xi_0 \sim 0.1 \mu m$ is two orders of magnitude larger than the interatomic spacing $a$ [12]. As a result $\xi(T_G) \sim \xi_0(\xi_0/a)^2$ exceeds the bubble size $R_b$, which means that no vortices can be created in this scenario.

Fig. 1. Two length scales characterizing the nonequilibrium phase transition into the ordered phase: coherence length $\xi(t)$ and intervortex distance in the infinite cluster $\xi_v(t)$. At $t > t^*$ one has $\xi_v(t) > \xi(t)$ and the vortices become well defined in the ordered phase.
In the alternative theory put forward by Zurek \cite{58, 67} the initial vortex density is essentially determined by the cooling rate \(1/\tau_Q = \partial_t T/T_c\) (if \(t = 0\) is the moment of phase transition one has \(1 - T(t)/T_c = t/\tau_Q\)). To get the idea of this theory let us consider an oversimplified model, in which the normal state above \(T_c\) is represented as an infinite cluster of vortices with the intervortex distance \(\xi_v\) determined by the coherence length \(\xi(T)\). The equilibrium state below \(T_c\) does not contain an infinite cluster, but in the nonequilibrium phase transition this cluster persists even in the ordered phase. When \(T \rightarrow T_c\) the intervortex distance \(\xi_v(t)\) diverges together with the thermodynamic coherent length, but at some moment \(t_0\) (see Fig.1) the velocity \(\partial_t \xi_v\) of defects approaches the limiting speed of light \(c\). At \(t > t_0\) the distance between the vortices \(\xi_v(t)\) can increase only with the speed \(c\). As a results in nonequilibrium the system has two length scales: \(\xi_v(t)\) characterizes the infinite cluster, while the coherence length \(\xi(t)\) is determined by the other degrees of freedom (including the small closed loops).

After transition into the ordered state (which occurs at \(t = 0\)) the \(\xi(t)\) decreases and \(\xi_v(t) > \xi(t)\) at \(t > t^*\) (see Fig.1). Starting from that moment \(t^*\) the vortices in the infinite cluster become well defined. The initial distance between the vortices in this model, determined from the equation \(\xi_v(t^*) = \xi(t^*)\), is \(\xi_v(t^*) \sim \xi_0(\tau_Q/\tau_0)^{1/3}\), where \(\tau_0 = \xi_0/c\). In \(^3\)He the limiting speed \(c\) relevant for the order parameter defects is the velocity of propagation of the order parameter, which is of order of spin-wave velocity: \(c \sim v_F \sqrt{1 - (T/T_c)}\). Then one has

\[
\xi_v(t^*) \sim \xi_0(\tau_Q/\tau_0)^{1/4},
\]

where \(\tau_0 = \xi_0/v_F\). With \(\tau_Q\) from Sec.(5.3) this gives the initial distance between the defects \(\xi_v(t^*) \sim 1 \mu m\). This is well within bubble radius \(R_b\), ie after cooling through \(T_c\) the bubble should contain many well defined vortex lines.

5.5 Evolution of the string network

Below \(T_c\) the vortex spaghetti is nonequilibrium and thus finally decays. At high temperature used in \cite{65} the decay occurs within the former fireball. At low \(T\) of \cite{66} the friction is small and vortices have time to propagate into the bulk liquid before they collapse. The formed vortices can be effectively stabilized in the rotating container which produces the superfluid velocity \(v_s = \Omega R\).
near the wall (here $R$ is the container radius and $\Omega$ the rotation velocity). If the radius of the vortex loop exceeds the value $r_\circ(v_s) = (\kappa/4\pi v_s) \ln r_\circ/\xi$, where $\kappa = \pi \hbar/m_3$ is the circulation quantum, the loop expands under the action of the Magnus force. An expanding vortex ring eventually results in a rectilinear vortex line which is pulled to the centre of the container, where it is detected by NMR. This allows us to measure the number of loops in the cluster with $\xi_v > r_\circ(v_s)$.

For the velocities $v_s$ used in the experiments, the radius $\xi_v(t^\star)$ of the initial loops is smaller than the critical radius $r_\circ(v_s)$, that is why the formed vortex network remains in the bubble and the vortex cluster decays according to the general properties of the strings dynamics. If in this process the $\xi_v(t)$ reaches $r_\circ(v_s)$, the vortices escape and are detected. Let us discuss this in more detail.

During the decay of the cluster the network structure remains scale-invariant, while $\xi_v(t)$ gradually increases [8]. The number of loops $n(l)$ per unit length and unit volume with line lengths $l > \xi_v$ is given [8] by $n(l) = C \xi_v^{-3/2} l^{-5/2}$ where $C \sim 0.3-0.4$. The network is also characterized by the average straight-line dimension $D$ of a loop: $D = \beta(l \xi_v)^{1/2}$, which minimal value is $D_{\min} = \alpha \xi_v(t)$ (from numerical simulations one finds that $\beta$ and $\alpha$ are close to unity). In terms of $D$ the loop size distribution, $n(D) dD \approx 2C D/dD/D^4$, does not depend on $\xi_v$: The evolution of the network leads to an increasing lower cutoff of the distribution $D_{\min} = \xi_v(t)$ while the upper cutoff is the diameter of the bubble $D_{\max} = 2R_b$. When the average radius of curvature, determined by $\xi_v$, exceeds $r_\circ(v_s)$ the vortices start to expand. Thus the number of detected vortices per one Big-Banh event, $N(v_s)$, is the number of loops with $r_\circ(v_s) < D < 2R_b$ in the bubble volume $V_b$:

$$N(v_s) = V_b \int_{r_\circ}^{2R_b} dD \ n(D) = \frac{\pi C}{9} \left[ \left( \frac{2R_b}{r_\circ(v_s)} \right)^3 - 1 \right]$$  \hspace{1cm} (5.3)

The Eq.(5.3) shows that there should be the critical velocity $v_{cn}$, below which no vortices can be detected: the vortices can be extracted only if $r_\circ(v_s) < 2R_b$, which gives

$$v_{cn} = (\kappa/8\pi R_b) \ln(R_b/\xi) .$$  \hspace{1cm} (5.4)

According to Eq.(5.1) $v_{cn}$ has the temperature dependence $v_{cn} \propto (1-T/ T_c)^{1/3}$. 

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In terms of $v_{cn}$ one obtains the universal curve

$$\mathcal{N}(v_s/v_{cn}) = \frac{\pi C}{9} \left[ \left( \frac{v_s}{v_{cn}} \right)^3 - 1 \right]$$

(5.5)

5.6 NMR measurement of vortex formation

In the inset of Fig. 2 two NMR absorption records are shown, starting from the moment when the neutron source is placed in position. The absorption events, which lead to formation of rectilinear vortices, are visible as distinct steps. The step height gives the number of vortex lines per event. The number of vortex lines created per unit time $\dot{N} = \nu N$ (main frame of Fig. 2) reproduces the theoretical $v_s^3$ dependence in Eq.(5.5) and is of the correct order of magnitude. The Fig. 3 demonstrates the universality feature of $\dot{N}$: The measuring conditions depend on temperature, pressure, and magnetic field, but all this dependence is contained in the single parameter, the critical velocity $v_{cn}$. The temperature dependence and the order of magnitude of $v_{cn}$ are in agreement with Eq.(5.4). This $v_{cn}$ is much smaller than the critical velocity $v_c$ at which a vortex is created at the container wall in the absence of neutrons[63].

Fig. 2. Formation of vortices during neutron irradiation. Inset: Change in NMR absorption as a function of running time $t$ at high and low rotation velocity. Each step corresponds to one neutron absorption event. The height of the step gives the number of vortex lines created and is denoted by the adjacent number.
Main frame: The rate \( \dot{N} \) at which vortex lines are created during neutron irradiation, plotted as a function of the normalized superflow velocity \( v_s/v_{cn} \). Below the critical threshold at \( v_{cn} \) the rate vanishes while above \( v_{cn} \) the rate follows the fitted cubic dependence \( \dot{N} = \nu \tilde{N}(v_s/v_{cn}) \), where \( \nu \approx 20 \) neutrons/min is the neutron flux and \( \tilde{N} \) is given by Eq.(5.5) with the fitting parameter \( C \sim 0.2 \).

Fig. 3. The rate \( \dot{N} \) as a function of \( v_s^3 \) demonstrates the scale invariance of the vortex formation in different conditions. In agreement with Eq. (5.5) the dependence on temperature, pressure, and magnetic field is contained in the critical velocity \( v_{cn} \). The latter is obtained as the intercept of the fitted lines with the horizontal axis. The common intercept with the vertical axis gives \( \nu \pi C/9 \).

5.7 Formation of other defects

There is an experimental evidence of the other type of the topological defects, formed in the mini-Big-Bang events. This is the spin-mass vortex (with the mass current and spin current circulating together around the vortex line), which is also the termination line of the soliton\(^7\). Due to the mass flow circulation the spin-mass vortex is influenced by the Magnus force in the same manner as conventional (mass) vortex, and can be also extracted under rotation. The events interpreted as formation of spin-mass vortices occur rather rare, as one can expect for the combined defects.

Another object which should form in the rapid quench is the interface between \(^3\)He-B and \(^3\)He-A, however without the bias field the A-B interfaces
decay within the Big-Bang volume. Situation changes in the supercooled 3He-A: the energy difference between 3He-A and 3He-B gives the required bias for the growth of the interfaces, which finally results in A→B transition. This provides an explanation of the observed collapse of the supercooled 3He-A by neutron and γ irradiation, which is alternative to the ”baked Alaska” scenario.

In the future the formation of half-quantum vortices (Alice strings) and point defects (monopoles) in the Big-Bang events will be studied.

5.8 Discussion

The experiment demonstrates that vortices are created in a rapid quench to the superfluid state. The measured number of vortices created as a function of superfluid velocity $v_s$ is consistent with the Vachaspati-Vilenkin scaling law discussed for cosmic strings. This is in favour of the Zurek modification of the Kibble mechanism of defect formation. On the other hand it is difficult to find some other mechanism of the defect nucleation by neutron irradiation, which could be responsible for all the found phenomena, including that observed at low $T$. It is believed now that the $v_s$ itself is not the reason of the vortex formation: $v_s$ mainly allows the formed vortices to escape, to expand and finally to reach a stable state in the rotating container so that they can be detected one by one. With increasing $v_s$, smaller loops, which represent an earlier stage in the evolution of the network, are extracted. One can expect that when the smallest possible size of loops is reached, which is limited by the initial inter-defect distance $\xi_v(t^*)$, the cubic scaling law will be violated. At the moment, the smallest size of the loops extracted at the highest rotation velocity is above though close to $\xi_v(t^*)$.

6 CONCLUSION

There is a close connection between superfluid 3He and high energy physics. Many nonperturbative processes in particle physics and cosmology can be modelled in 3He. On the other hand some effects observed in 3He still wait for their analogs in physical vacuum. Also the 3He is a very pure system and can serve for the particle physics experiments, eg as the dark matter detector.
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