NOISE-INDUCED SELF-OSCILLATION (FLUTTER) SUPPRESSION IN THE KELDYSH MODEL

An equation for the evolution of the energy of a dynamical system (Keldysh model with one degree of freedom), which contains a source of white noise, is constructed. It is shown that self-oscillations (flutter) are suppressed if the intensity of white noise exceeds a critical value.

Keywords: Keldysh model, limit cycle, noise-induced flutter.

1. INTRODUCTION

More than seventy years ago, nonlinear differential equations were constructed (the Keldysh model with one and two degrees of freedom [1]), the self-oscillating solutions of which made it possible to explain the nature of flutter. The Keldysh model [1] continues to attract the attention of researchers [2]. The question of the influence of noise on the Keldysh model remained open all these years. A similar problem of the effect of noise on the van der Pol oscillator was solved for the first time in [3], to which attention was drawn in [4]. It was shown in [3] that if the noise intensity exceeds a certain critical value, then the self-oscillations disappear. A similar result was obtained in [5] using the stochastic equation for the evolution of the energy of a dynamical system with one degree of freedom [6]. In this paper, the stochastic equation for the evolution of the energy of a dynamical system with one degree of freedom [6] is applied to the Keldysh model with one degree of freedom, which includes an additive source of white noise.

2. SOLUTION OF A NONLINEAR DIFFERENTIAL EQUATION IN THE KELDYSH MODEL

The nonlinear differential equation (see, for example, the Keldysh model with one degree of freedom [2]) has the form

\[ J\ddot{x} + kx = -\mu \dot{x} - (\Phi + \kappa \dot{x}^2) \text{sign} \dot{x}, \]

where constants \( J = k = \kappa = 1, \mu < 0, \Phi > 0 \) are defined in [2].

If the energy of an unperturbed dynamical system has the form

\[ E = \frac{J\dot{x}^2}{2} + \frac{kx^2}{2}, \]

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then the trajectory turning points have the form
\[ x_{1,2} = \pm \sqrt{\frac{2E}{k}}. \tag{3} \]

The stochastic equation for the evolution of the energy of a dynamical system with one degree of freedom [6] has the form
\[ \frac{dE}{dt} = \frac{2}{T} \int x \left( \bar{f} + \delta f \right) dx, \tag{4} \]
where \( T = 2\pi \sqrt{\frac{J}{k}} \) — oscillation period of the unperturbed system;
\[ \bar{f} = -\mu \dot{x} - (\Phi + \kappa x^2) \text{sign} \dot{x}; \quad \delta f = \sqrt{2D\xi(t)}; \quad D \) — the intensity of the random source noise; \( \xi = \xi(t) \) — random variable with unit variance.

We find the speed of the dynamic variable using (2)
\[ \dot{x} = \pm \sqrt{\frac{2E - kx^2}{J}}. \tag{5} \]

Using (5), we rewrite equation (4) in the form
\[ \frac{dE}{dt} = \mp(a_0E^2 + a_1E + a_2E^2) + a_3E^2 \delta f, \tag{6} \]
where \( a_0 = \frac{2\sqrt{2}}{\pi} \cdot \frac{\Phi}{\sqrt{J}}, \quad a_1 = \frac{\mu}{J} < 0, \quad a_2 = \frac{8\sqrt{2}}{3\pi} \cdot \frac{\kappa}{J^2}, \quad a_3 = \frac{2\sqrt{2}}{\pi} \cdot \frac{1}{\sqrt{J}}. \]
The minus sign on the right side of equation (6) corresponds to the plus sign in formula (5). Equation (6), in which there is no fluctuating source,
\[ \frac{dE}{dt} = \mp(a_0E^2 + a_1E + a_2E^2) \tag{7} \]
has three stationary points \( E = 0 \) and
\[ \sqrt{E_{1,2}} = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_0a_2}}{2a_2}, \tag{8} \]
where the discriminant \( a_1^2 - 4a_0a_2 = \frac{(\mu^2 - \delta^2_K)}{J^2}; \quad \delta^2_K = \frac{128\Phi\kappa}{3\pi^2}. \)

Let us show that for \( \delta^2_K < \mu^2 \) the dynamic system (Keldysh model) relaxes either to an external \( \sqrt{E_1} \), either to the internal \( \sqrt{E_2} \) limit cycle depending on the minus or plus sign in equation (7).

The solution to equation (7), on the right-hand side of which there is a minus sign, has the form
\[
\sqrt{E} = \sqrt{E_1} - \sqrt{E_2} A \cdot \exp(-\nu \tau), \tag{9}
\]

where \( A = \frac{\sqrt{E_0} - \sqrt{E_1}}{\sqrt{E_0} - \sqrt{E_2}} \); \( E_0 = E(t_0 = 0) \) – the initial value of the energy of a dynamical system (Keldysh model with one degree of freedom) at the moment of time \( t_0 = 0 \); the relaxation time to a stationary state has the form

\[
\tau = \frac{2J}{\sqrt{\mu^2 - \delta_k^2}}. \tag{10}
\]

It is seen that at \( t \gg \tau \) dynamic system (Keldysh model) relaxes to external \( \sqrt{E_1} \) limit cycle. The solution for equation (7), on the right-hand side of which there is a plus sign, coincides with formula (9) with the replacement \( t \rightarrow -t \). It is seen that in this case the dynamical system (the Keldysh model) relaxes to the internal \( \sqrt{E_2} \) limit cycle.

3. Calculation results

The stationary solution of equation (6), on the right-hand side of which there is a minus sign, is constructed using (see, for example, [7])

\[
g(E) = \frac{1}{N \sqrt{E}} \exp \left[ - \left( \frac{1}{2a_0 E^2 + a_1 E + \frac{2}{3}a_2 E^2} \right) \frac{3}{D a_3^2} \right], \tag{11}
\]

where \( N \) – normalization factor.

The graph of the function (11) of the probability density of detecting a dynamic system (Keldysh model) depending on the energy of the dynamic system is constructed for two values of the intensity of a random source \( D = 0.2 \) in Fig. 1 and \( D = 0.4 \) in Fig. 2. Constants \( J = k = \kappa = 1; \mu = -1.2987; \Phi = 0.2 \) are defined in [2].

Using formula (8), we can calculate \( E_1 \approx 0.845 \) and \( E_2 \approx 0.027 \). The radii of the outer and inner limit cycle in Fig. 1a in [2] are close to the values \( \sqrt{2E_1} \approx 1.303 \) and \( \sqrt{2E_2} \approx 0.23 \), respectively. It can be seen that the graph of function (11) in Fig. 1 reaches its maximum value at the energy of the dynamic system \( E = 0 \) and with an energy that is slightly less than \( E_1 \approx 0.845 \). A similar result was obtained in [5] when studying the effect of noise intensity on the position of the maximum of the probability density of detecting a dynamical system, which is described by the van der Pol equation.
Fig. 1. The graph of the probability density of detecting a dynamic system (Keldysh model) depending on the energy of the dynamic system is plotted for the value of the intensity of a random source $D = 0.2$.

Fig. 2. The graph of the probability density of detecting a dynamic system (Keldysh model) depending on the energy of the dynamic system is plotted for the value of the intensity of a random source $D = 0.4$. 
4. CONCLUSION

It is seen that at \( D \geq D_{\text{crit.}} \approx 0.4 \) self-oscillations are suppressed, since the maximum of function (11) disappeared, which describes the probability density of detecting a dynamical system in the vicinity of the limit cycle. The dynamical system passes into a quasi-stationary state of rest, since self-oscillations reappear with a decrease in the intensity of a random source.

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