Modeling of the impact of GDP and human population on CO₂ emission by using Cobb-Douglas model and particle swarm optimization

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Abstract. High economic growth and population can cause a negative impact on the environment, one of which is the increase of CO₂ emissions in the air. In the paper, modelling the impact of GDP and human population on CO₂ emissions in Indonesia. In this paper the modelling is done by using Cobb-Douglas model, and for parameter estimation done with particle swarm optimization (PSO). The data used in this modeling is GDP growth and human population over the period 1967-2014. Based on the results of the analysis it can be shown that the impact of GDP and human population on CO₂ emissions, significantly follow the Cobb-Douglas model. The parameter estimation using PSO yields elasticity estimators for GDP and the human population are 0.195517125 and 1.558049947, respectively. Since the sum of elasticity estimators is greater than one, this shows the characteristic proportion of the increase in GDP and human population, will increase CO₂ emissions by a larger proportion.

1. Introduction
Economic development is an attempt to increase the economic growth of a country, which is usually measured by the growth rate of Gross Domestic Product (GDP). However, if the GDP growth is not properly scrutinized, there will be an imbalance between large GDP growth and declining environmental quality. GDP growth is not only a successful measure of development of a country [1, 2]. However, a good environment and natural preservation are also factors of development success. GDP growth also has a negative and significant influence on environmental quality through CO₂ emissions. If GDP increases every 1 percent, the environmental quality will decrease by 9.11 percent. These results show that GDP growth will lead to high levels of environmental quality decline as CO₂ emissions increase [3].

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Similarly, an increase in human populations leads to an increase in housing demand, resulting in increased timber demand and many illegal logging. Increasing human populations also led to an increase in the number of food needs, so that forest trees were opened to agriculture to meet food needs [4, 5]. Whereas forests are an efficient C absorbent (sequester) ecosystem and C sinks are important. Forest destruction contributes to the increase in carbon dioxide (CO₂) emissions. An increase in population will also increase the amount of fuel energy consumption for transportation and industry. The remaining combustion of fuel energy may cause air pollution such as in the form of CO₂ emissions [6, 7].

Thus GDP growth and increased human populations have an impact on increasing CO₂ emissions in the air [8]. How big the growth of GDP and the increase in human population each contributes to the increase of CO₂ emissions, this is a problem that needs to be done a study. Other studies that have been done by previous researchers include: Aye and Edoja [2], examining the effect of economic growth on CO₂ emissions using panel data analysis. Analyzed panel data consists of 31 developing countries. Based on the analysis results show that economic growth has a negative effect on CO₂ emissions on low growth conditions, but has a positive effect on high growth conditions with higher marginal effects on high growth conditions. It was also found that increased energy consumption and population growth had a positive effect on CO₂ emissions. Zhu and Peng [8], examined the impact of population size and carbon emissions levels in China from 1978 to 2008, using the ridge regression method. The results show that an increase in the human population and the level of energy consumption, is a major impact factor for the increase in carbon emissions that are highly correlated. Similar studies have also been conducted by Mehrizi et al. [4] and Omri [9].

Furthermore, Senthil et al. [10], in his research proposed a decrease in CO₂ emissions in gas power plants, using the swarm partition optimization algorithm approach. The results showed that particle swarm optimization algorithm is more efficient in terms of the number of evolutions to reach the global optimum point. It also shows that particle swarm optimization algorithm provides practical and valid solutions for real-time applications. Other studies that have done the same are Mohammadi and Nasiraghdam [11]. Therefore, based on the exposure of previous researchers in this paper, research on the modeling of the impact of GDP and human population on CO₂ emission by using Cobb-Douglas model and particle swarm optimization. As research object is GDP growth data and human population in Indonesia in period 1967-2014. The goal is to obtain a significant model estimate, and can be used to predict the development of CO₂ emissions with minimum errors.

2. Methodology

In the analytical methodology here the discussion includes: Cobb-Douglas production function, model parameter estimation using particle swarm optimization (PSO), model significance test, and forecasting (prediction).

2.1 Cobb-Douglas production function

Referring to Omri [9] and Obradovic & Lojanica [6], the Cobb-Douglas production function is an equation involving two or more variables, a dependent variable and several independent variables. The production function of Cobb-Douglas model with multiplicative error term is given as equation:

\[ G_t = \phi X_{1t}^{\theta_1} X_{2t}^{\theta_2} e^{\varepsilon_t} \]  

(1)

where in this study, \( G_t \) is output as CO₂ emissions, \( X_1 \) is input as GDP at current prices per capita, \( X_2 \) is the input of the human population, \( \phi, \theta_1 \), and \( \theta_2 \) are the Cobb-Douglas production function parameters, and \( \varepsilon_t \) is exponential of residual.

Referring to Felipe and Adams [12], the elasticity of production \( \rho \) is the percentage change in output, divided by the percentage of input changes. Production elasticity is the ratio of the relative change of output produced, to the relative changes in the number of inputs that affect. The output elasticity of GDP, \( \rho_1 \) measured by using the equation:
\[ \rho_1 = \frac{\% \Delta G_t}{\% \Delta X_{1t}} = \theta_1 \]

The output elasticity of GDP can also be measured using coefficient parameters \( \theta_1 \) of the Cobb-Douglas production function. While the output elasticity of the human population, \( \rho_2 \) measured by using the equation:

\[ \rho_2 = \frac{\% \Delta G_t}{\% \Delta X_{2t}} = \theta_2 \]

The output elasticity of the human population can also be measured using coefficient parameters \( \theta_2 \) of the Cobb-Douglas production function.

Referring to Felipe and Adams [12], the sum of the elasticity of production \( \theta_1 + \theta_2 \) explains the size of a venture scale or called return to scale. There are three characteristics of return to scale conditions as follows:

- When \( \theta_1 + \theta_2 = 1 \), then the function shows the scale with constant return (constant return to scale), meaning the increase of input will be followed by the increase of output proportionally.
- When \( \theta_1 + \theta_2 < 1 \), then the function shows the scale with decreasing return (decreasing return to scale), meaning the percentage increase output smaller than the percentage of input addition.
- When \( \theta_1 + \theta_2 > 1 \), then the function shows the scale with the increase (increasing return to scale), meaning the percentage of output addition is greater than the percentage of input addition.

Equation (1), if the left and right segments are taken natural logarithm, the following linear equations are obtained:

\[ \ln G_t = \ln \phi + \theta_1 \ln X_{1t} + \theta_2 \ln X_{2t} + \varepsilon_t \]

If you can \( Y_t = \ln G_t \), \( \theta_0 = \ln \phi \), \( M_{1t} = \ln X_{1t} \), and \( M_{2t} = \ln X_{2t} \), then the last equation can be expressed as:

\[ Y_t = \theta_0 + \theta_1 M_{1t} + \theta_2 M_{2t} + \varepsilon_t \]

(2)

Thus, the estimator obtained from the regression equation (2) is:

\[ \hat{Y}_t = \theta_0 + \theta_1 M_{1t} + \theta_2 M_{2t} \]

(3)

Equation (2) is linear in the parameters of \( \theta_0 \), \( \theta_1 \), and \( \theta_2 \) as well as residuals \( \varepsilon_t \). Thus it is shaped as a linear regression model. Constants \( \theta_0 \) is intercept, \( \theta_1 \) and \( \theta_2 \) is a parameter of elasticity of production.

For the purposes of parameter estimation, from equation (2) an optimization equation can be formed as follows:

\[ \Sigma \varepsilon_t^2 = \Sigma (Y_t - \hat{Y}_t)^2 = \Sigma (Y_t - \theta_0 - \theta_1 M_{1t} - \theta_2 M_{2t})^2 \]

(4)

That is, estimation is aimed at determining the values of \( \theta_0 \), \( \theta_1 \), dan \( \theta_2 \) which can minimize the number of squares of the residuals \( \Sigma \varepsilon_t^2 \). The process of minimizing the amount of residual squares, in this study was carried out using the Particle Swarm Optimization (PSO) algorithm.

2.2 Particle swarm optimization (PSO)

According to Pao-La-Or [13], the particle swarm optimization (PSO) algorithm is a population-based optimization technique developed by Eberhart and Kennedy in 1995, inspired by the social behavior of birds or fish. PSO used to solve optimization problems. PSO performs a search using the population (swarm) of the individual (particle) to be updated from the iteration. The PSO has several parameters such as position, velocity, and inertial coefficients. PSO has more or even superior performance search ratios for many optimization problems with faster and more stable convergence rates.

According to Pitono et al. [14], mathematically \( A \subset R \) as search space and \( f: A \rightarrow Y \subset R \) is the purpose function. As described earlier, the PSO is a population-based algorithm, that is, by exploiting
the population of solutions to investigate the search space simultaneously. In PSO, the population is called a swarm and the individual is called a particle. Swarm is defined as set:

\[ S = \{\theta_0, \theta_1, \theta_2\} \]  

(5)

of the three particles (potential solution), defined as follows:

\[ C_i = (\theta_{i1}, \theta_{i2}, \ldots, \theta_{in})^T \in A, i = 0,1,2 \text{ and } n \text{ the number of data}. \]  

(6)

The particles are moving in their search space, this movement is done by adjusting their position by using velocity, which is defined as follows:

\[ V_i = (V_{i1}, V_{i2}, \ldots, V_{in})^T \in A, i = 0,1,2 \text{ and } n \text{ the number of data}. \]  

(7)

The speed is adjusted gradually so that the expected particles can visit each region \( A \). If \( t \) shows the number of iterations, then the current position of the particles to-\( i \) and their respective velocities are denoted as \( C_i(t) \) and \( V_i(t) \). Speed is updated based on information obtained from previous algorithm steps. It is implemented in a memory, where each particle can store the best position ever visited during the search, so that \( S \) contains the current particle position. Memory maintained by PSO is \[ P = (P_{0\text{best}}, P_{1\text{best}}, P_{2\text{best}}) \]  

which contains best positions.

\[ P_{i\text{best}} = (P_{i1}, P_{i2}, \ldots, P_{in})^T \in A, i = 0,1,2 \text{ and } n \text{ the number of data}. \]  

(9)

and have been visited by each particle. So this position is defined as follows:

\[ P_{i\text{best}}(t) = \arg \min f_i(t) \]  

(10)

Where \( \arg \min f_i(t) \) is a particle with \( f_i(t) \) that is minimum.

PSO is based on a social behavior simulation model, so an information exchange mechanism must exist to allow particles to communicate with each other. The algorithm will approach the global minimum with the best position ever visited by all the particles. Suppose given \( g \) as the best position with the lowest inner function value \( P \) on \( t \) the given iteration, \[ g_{\text{best}}(t) = \arg \min f(P_{\text{best}}(t)) \]  

(11)

where \( \arg \min f(P_{\text{best}}(t)) \) is \( P_{\text{best}} \) with \( f(P_{\text{best}}(t)) \) that is minimum.

Each particle updates its speed and its position with the following equation \[ V_{in} = wV_{in} + c_1r_1(P_{i\text{best}} - C_{in}) + c_2r_2(g_{\text{best}} - C_{in}) \]  

(12)

\[ C_{in} = C_{in} + V_{in} \]  

(13)

where \( w \) is the inertial coefficient at intervals \([0,1]\), \( c_1, c_2 \) is the acceleration coefficient in the form of positive constants, and \( r_1, r_2 \) is a random number at interval \([0,1]\). Here is the basic PSO algorithm:

1. Initialize the initial position of the particle \( C_i \) and initial speed \( V_i \), with \( i = 0,1,2 \) is a swarm size.
2. Evaluate the value of the objective function for each particle \( f(C_i) \).
3. Define \( P_{i\text{best}} \) beginning, and \( g_{\text{best}} \) beginning.

\[ g_{\text{best}} = \begin{cases} P_{i\text{best}} & f(P_{i\text{best}}) < f(g_{\text{best}}) \\ P_{i\text{best}} & f(P_{i\text{best}}) \geq f(g_{\text{best}}) \end{cases} \]  

(14)

4. Update speed with equations (12).
5. Update the position of new particles with equations (13).
6. Reevaluate \( f(C_i) \). If \( f(C_i) \leq f(P_{i\text{best}}) \), then \( P_{i\text{best}} = C_i \). After getting \( P_{i\text{best}} \) new, then get it \( g_{\text{best}} \) new.
7. If the stopping condition is met then the algorithm stops. If it has not been met then go back to step 4.
2.3 Test the significance of model parameters

The significance test of model parameters is done by using simultaneous test (test-F) and partial test (test-\(t\)). In this research, test of model parameter significance is done by using Microsoft Excel 2013 help.

- **Test with statistic-F**

  This test is conducted with the aim to know that all parameter estimators simultaneously significantly affect the dependent variable. In this test the hypothesis used in the test of statistic-\(F\) as follows: \(H_0: \theta_0 = \theta_1 = \theta_2 = 0\) (independent variable does not significantly affect the dependent variable), against alternative hypothesis is \(H_1: \exists \theta_0 \neq \theta_1 \neq \theta_2 \neq 0\) (there is at least one independent variable that significantly affects the dependent variable). The value of count statistics \(F_{Stat}\) determined using equation [16, 17]:

\[
F_{Stat} = \frac{MS_{Reg}}{MS_{Error}} = \frac{\sum_{i=1}^{n-1}(\hat{Y}_t-y_t)^2}{\sum_{i=1}^{n-1}(y_t-\hat{y}_t)^2}
\]  

(15)

Next, the calculated statistical value \(F_{Stat}\) compared with statistical critical values \(F(a, df)\) table with a significant level \(\alpha\) and \(df = (k - 1, n - k)\). Where \(df\) degree of freedom, with \(n\) number of samples, and \(k\) the number of parameters. The testing criterion is reject the hypothesis \(H_0\), and accept the hypothesis \(H_1\), if \(F_{Stat} > F(a, df)\). This means that simultaneously significant independent variables influence the dependent variable. Conversely, accept the hypothesis \(H_0\), and reject the hypothesis \(H_1\), if \(F_{Stat} \leq F(a, df)\). Meaning simultaneously significant independent variable has no effect on the dependent variable.

- **Test with statistic-\(t\)**

  This test was conducted in order to find out whether each parameter estimator significantly contributed to affect the dependent variable. In this test the hypothesis used is \(H_0: \theta_i = 0, (i = 0, 1, 2)\), the independent variable does not significantly influence the dependent variable, against the alternative hypothesis \(H_1: \theta_i \neq 0, (i = 0, 1, 2)\), independent variables significantly influence the dependent variable. Value statistic \(t_{Stat}\) calculated using equation [16, 17]:

\[
t_{Stat} = \frac{\theta_i}{SE(\theta_0)}
\]  

(16)

Next, the calculated statistical value \(t_{Stat}\) compared with statistical critical values \(t_{(\alpha/2, df)}\) with a significant level \(\alpha\) and \(df = (n - k)\). Where \(df\) degree of freedom, \(n\) number of samples, and \(k\) the number of parameters. Criteria testing reject hypothesis \(H_0\), and accept the hypothesis \(H_1\) be accepted, if \(|t_{Stat}| > t_{(\alpha/2, df)}\). This means that each independent variable significantly affects the dependent variable. Instead accept the hypothesis \(H_0\), and reject the hypothesis \(H_1\), if \(|t_{Stat}| \leq t_{(\alpha/2, df)}\). This means that each independent variable does not significantly affect the dependent variable.

- **Test assumption of residual normality**

  This test is performed in order to know that the residual spreads a normal distribution with a mean of zero and a certain variance. In this test the hypothesis is used \(H_0\): the residual data follows the normal distribution, by fighting the alternate hypothesis \(H_1\): the residual data does not follow the normal distribution. Kolmogorov-Smirnov statistical calculation \(D_{Stat}\) can be determined using equation [16]:

\[
D_{Stat} = \max|S(z) - P(z)|
\]  

(17)
Next, the value of the statistical count $D_{Stat}$ compared with statistical critical values $D_{(\alpha,n)}$, with $\alpha$ level of significance, and $n$ the number of samples. The criteria for testing is to reject the hypothesis $H_0$, and accept the hypothesis $H_1$, if $D_{Stat} > D_{(\alpha,n)}$. This means that residual data comes from abnormal distribution. Instead, accept the hypothesis $H_0$, and reject the hypothesis $H_1$, if $D_{Stat} > D_{(\alpha,n)}$. This means that the residual data comes from the normal distribution. In addition to testing the assumption of normality, on the residual data also estimated the amount of indigo mean and variance.

2.4 Coefficient of determination

The value of determination and correlation coefficient is used to measure how strong the correlation between all independent variables with non-free variable. Value of coefficient of determination $R^2$ can be calculated using equation [13]:

$$R^2 = \frac{SS_{Reg}}{SS_{Corr}} = \frac{\sum_{t=1}^{n}(Y_t - \bar{Y})^2}{\sum_{t=1}^{n}(Y_t - \bar{Y}_t)^2}$$  \hspace{1cm} (18)

If the value of $R^2 = 1$, means the percentage of independent variable contributions $M_1t$ and $M_2t$ to the dependent variable $Y_t$ of 100%. If the value $R^2 = 0$, means the independent variable $M_1t$ and $M_2t$ does not contribute to the dependent variable $Y_t$.

2.5 Mean square error (MSE) of prediction

To measure the model’s equity can be used in prediction, it is measured based on the error generated from the prediction. According to Sukono et al. [16], to measure errors is usually done using Mean Square Error (MSE). MSE is the mean of the squared value of the prediction error. To determine the value of MSE can be done using the following equation:

$$MSE = \frac{\sum_{t=1}^{n}e_t^2}{n} = \frac{\sum_{t=1}^{n}(Y_t - \hat{Y}_t)^2}{n}$$ \hspace{1cm} (19)

A model is said to be appropriate, if the error resulting from the prediction is very small close to zero.

3. Results and Analysis

In this section we discussed the result and analysis which included: data analyzed, descriptive data statistics, model parameter estimation using PSO, model significance test, deterministic coefficient value, and predictive error measurement.

3.1 Data analyzed

The data analyzed in this study is secondary data, ie GDP data at current prices, human population, and CO$_2$ emissions data from 1967-2014. The data is obtained from the official website of the World Bank. Descriptive statistics to provide an overview of the quantitative data used in this study. Presentation of descriptive statistical data of this research using Microsoft excel assistance 2013. The result of descriptive statistical calculation can be seen in Table 1

| $Y_t$ | $M_{1t}$ | $M_{2t}$ |
|-------|----------|----------|
| Mean  | 1.02753743 | 974.481835 | 181239285.4 |
| Median| 0.89894000 | 584.263600 | 1.8300000 |
| Maximum| 2.55975023 | 3687.954000 | 2.5500000 |
| Minimum| 0.23191548 | 53.5161517 | 105907403 |
| Std.Dev| 0.57736700 | 1011.1570000 | 446693252 |
Where $Y_t$, CO$_2$ emissions, $M_{1t}$ GDP at current prices, and $M_{2t}$ the number of human populations. As for graphs of CO$_2$ emissions data, GDP at current prices, and the number of human populations can be seen in Figure 1.

![GDP at current prices](a)  ![Number of human populations](b)  ![CO$_2$ emissions](c)

**Figure 1.** GDP at current prices, number of human populations, and CO$_2$ emissions

3.2 Estimation of model parameters

Data of GDP at current prices, the number of human population and CO$_2$ emissions obtained from the World Bank official website is converted into natural logarithms by reference equation (2). The data that has been transformed into a natural logarithmic form is modeled into the Cobb-Douglas model by referring to equation (1). Estimation of model parameters was performed using particle swarm optimization (PSO) with reference to section 2.2. Estimation of model parameters is done with the help of Matlab R2015a software, with the following steps:

- Click start → choose Matlab R2015a.
- On the File menu, select New Script.
- Then write the objective function along with the CO$_2$ emissions data as $Y_t$, GDP at current prices $M_{1t}$ and the human populations $M_{2t}$ on the sheet script. Then save script by name `obj1_function.m`. The objective function is refer to equation (4).
- Open Command Windows to call script which has been made.
- To get the parameter values from the data entered into the function, run it `obj1_function.m` with a click run then type `Y =particleswarm(@obj1_function,3)` on Command Windows then press enter, then the parameter value will appear.

Based on the estimation by using PSO obtained estimator parameter value of model respectively is $\theta_0 = -30.96678719$, $\theta_1 = 0.195517125$, and $\theta_2 = 1.558049947$. Thus, by reference to equation (2), the multiple linear regression model of the estimate can be expressed as follows:

$$Y_t = -30.96678719 + 0.195517125M_{2t} + 1.558049947M_{2t} + \epsilon_t$$  \hspace{1cm} (20)

Furthermore, to ensure that the model obtained from the estimation using PSO is significant, the next step is to test the significance of the model.

3.3 Testing the significance of model estimators

Testing the significance of model estimators performed in this section includes: simultaneous parameter estimator significance test, individual parameter estimator significance test, and assumption of residual normality test.

- Test of simultaneous significance

On the significant test of these simultaneous parameters, the hypothesis is $H_0: \theta_0 = \theta_1 = \theta_2 = 0$, independent variables together do not significantly affect the dependent variable, with the alternative hypothesis is $H_1: \exists \theta_0 \neq \theta_1 \neq \theta_2 
eq 0$, there is at least one independent variable that has a significant effect on the dependent variable.
Statistic value of $F_{Stat}$ is calculated by using equation (15), and the statistical value is obtained $F_{Stat} = 1114.495605$. Meanwhile, using the level of significance $\alpha = 0.05$, and with degrees of freedom $df = (k - 1; n - k) = (2, 45)$, from the table of $F$ distribution obtained critical value $F_{0.05;2,45} = 3.23$. Thus, it is clearly shown that $F_{Stat} > F_{0.05;2,45}$, thus the hypothesis $H_0$ rejected, and therefore the hypothesis $H_1$ be accepted. This shows that at a significant 5% or 95% confidence level, GDP at current prices and the number of human populations together, or at least one parameter estimator has an effect on CO$_2$ emissions.

- Test of individual significance
In testing the significance of these individual parameters, each parameter estimator is tested one by one. It starts with testing for parameters $\theta_0$, with the hypothesis used is $H_0 : \theta_0 = 0$, constants $\theta_0$ has no partial effect on $Y_t$, with an alternative hypothesis is $H_1 : \theta_0 \neq 0$, constants $\theta_0$ has a partial effect on $Y_t$.

Value of statistic $t_{Stat}$ determined by using equation (16), and obtained statistical value $t_{Stat} = 0.092009976$. Meanwhile, using a significant level $\alpha = 0.05$, and degrees of freedom $df = n - k = 45$, from the table $t$ distribution obtained statistical critical value $t_{0.025;45} = 2.014$. This shows that $|t_{Stat}| < |t_{0.025;45}|$, thus the hypothesis $H_0$ rejected and therefore hypothesis $H_1$ be accepted. Means that with a significance level of 5% or a 95% confidence level a constant parameter estimator, individually has an effect on CO$_2$ emissions.

Testing also needs to be done on estimator coefficient parameters of $\theta_1$ and $\theta_2$. Using the same way, the test results show that the parameter estimator coefficients $\theta_1$ and $\theta_2$, significantly each also has an effect on CO$_2$ emissions.

- Test assumption of residual normality
Testing assumption of residual normality in this section is done by using statistical test Kolmogorov-Smirnov. In testing the assumption of this residual normality, the hypothesis is $H_0$: the residual data follows the normal distribution, with the alternate hypothesis being $H_1$: the residual data does not follow the normal distribution.

Determination of statistical value of Kolmogorov-Smirnov $D_{Stat}$ can be done by using equation (17), and obtained statistical value $D_{Stat} = 0.07004871$. Meanwhile, using a significant level $\alpha = 0.05$ and the number of data $n = 48$, from the Kolmogorov-Smirnov table obtained the critical value of statistics $D_{0.05;48} = 0.198$. So it clearly shows that $D_{Stat} > D_{0.05;48}$, thus the hypothesis $H_0$ rejected, and therefore hypothesis $H_1$ be accepted. This shows that the residual data is normally distributed. From the process of testing the residual normality also obtained the mean value $\mu_\varepsilon = 0.005327 \equiv 0$, and with variance $\sigma_\varepsilon^2 = 0.015567$. Thus, it can be shown that $\varepsilon_t \sim N(0, 0.015567)$.

### 3.4 Value of determination coefficient
In this section is done calculation of coefficient of determination, which can be used to measure the correlation strength between the free variable with the dependent variable. Value of coefficient of determination $R^2$ using equation (18), and the result is $R^2 = 0.980211005$. This means that the contribution of the GDP at current prices and the number of the human population of the variation (fluctuation) of CO$_2$ emissions by 98%, while the remaining 2% are caused by other factors beyond the GDP at current prices and population of Indonesia.

### 3.5 Prediction and error measurement
The predictive error measurement in this section is very important to measure the accuracy of the model estimator when used to make predictions. The model obtained from the estimation process made using PSO is given in (20). Referring to equation (3), equation (20) means having the model estimator as follows:

$$\hat{Y}_t = -30.96678719 + 0.195517125M_{4t}$$
While it is known that 
\[ Y_t = \ln G_t, \quad \theta_0 = \ln \phi, \quad M_{1t} = \ln X_{1t}, \quad \text{and} \quad M_{2t} = \ln X_{2t}, \]
so this last equation can be expressed as follows:
\[
\hat{G}_t = e^{-30.96678719}X_{1t}^{0.195517125}X_{2t}^{1.55804994} \quad (21)
\]
On equation (21), the sum of elasticity \( \theta_1 + \theta_2 = 1.753567065 \), this shows that \( \theta_1 + \theta_2 > 1 \). So equation (21) is a Cobb-Douglas production function that has a characteristic increasing return to scale, meaning that there is an increase of input unit values \( X_{1t} \) and \( X_{2t} \), there will be an increase of greater than the proportion of the input unit \( X_{1t} \) and \( X_{2t} \). Furthermore, equation (21) is used to predict the data in sample and out sample.

Error measurements are performed to determine the level of model accuracy. Determination of error value here is done by using data in sample. Error value is calculated by using MSE as in equation (19), so obtained value of MSE = 0.007428467. So it can be interpreted that the accuracy of GDP modeling on the basis of current prices and the number of human population on CO2 emissions, using the PSO algorithm is 99.2571533\%. The graph of actual data and predicted results as shown in Figure 2, as follows:

![Figure 2. Actual and predicted data in sample](image)

Next, suppose the estimated values \( X_{1t} \) and \( X_{2t} \) as given in Table 2. Predicted out sample CO2 emission values in 2015, 2016, 2017, and 2018, by including the value of each variable that increased from the previous years. Prediction is done using equation (21) result can be seen in Table 2, column \( \hat{G}_t \).

| Years (t) | \( \hat{G}_t \) | \( X_{1t} \) | \( X_{2t} \) |
|-----------|-----------------|-------------|-------------|
| 2015      | 2.219717049     | 3336.106686 | 258162113   |
| 2016      | 2.289576952     | 3570.294888 | 261115456   |
| 2017      | 2.335585477     | 3609.644389 | 264107626   |
| 2018      | 2.380178086     | 3620.832557 | 267229312   |

From Table 2, it appears that there is an increase of input unit in the form of GDP based on prevailing prices and human population in Indonesia, causing an increase in larger CO2 emissions.

4. Conclusion

In this paper we have estimated the impact model of GDP growth and human population on CO2 emissions, using Cobb-Douglas model whose parameters are estimated using particle swarm optimization (PSO) algorithm. Based on the results of the analysis, it can be concluded that the impact of GDP growth and human population on CO2 emissions is significantly a Cobb-Douglas production function with an elasticity estimator \( \theta_1 = 0.195517125 \), and \( \theta_2 = 1.558049947 \). Because \( \theta_1 + \theta_2 > 1 \), the obtained production function estimator has the characteristic of increasing return to scale, that the
percentage of output addition multiplied from the percentage of input addition. Furthermore, the prediction based on the data in sample, the error rate measured using MSE obtained value of 0.007428467. This means that the estimator accuracy model of GDP impact on current prices and the population of CO2 emissions, using the PSO algorithm is 99.2571533%. This suggests that the Cobb-Douglas production function estimator obtained is well suited to predict increases in CO2 emissions, as a result of GDP growth and human populations. Considering such a high rate of CO2 emissions, such modeling is expected to be taken into consideration by the authorities in the policy making of CO2 emission control in Indonesia.

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