Generation of continuous variable squeezing and entanglement of trapped ions in time-varying potentials

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Abstract

We investigate the generation of squeezing and entanglement for the motional degrees of freedom of ions in linear traps, confined by time-varying and oscillating potentials, comprised of an DC and an AC component. We show that high degrees of squeezing and entanglement can be obtained by controlling either the DC or the AC trapping component (or both), and by exploiting transient dynamics in regions where the ions’ motion is unstable, without any added optical control. Furthermore, we investigate the time-scales over which the potentials should be switched in order for the manipulations to be most effective.

1 Introduction and scope of the paper

The manipulation of continuous variable quantum information is by now a well established area of research [1], that has led to a number of remarkable experimental and technological advances [2]. So far, the physical system of choice in the arena of continuous variables has certainly been the electromagnetic field, mostly because of the ease with which it can be coherently manipulated and distributed in space, making it exceptionally suitable for quantum communication tasks. Yet, notwithstanding the clear benefits and successes of quantum optical degrees of freedom, there are reasons why one would be interested in exploring alternatives.

Firstly, while the entanglement between two continuous variable degrees of freedom could be, in principle, unbounded, the degrees of entanglement achievable in practice for quantum optical systems are severely limited by the photons’ reluctance to interact with each other. Entanglement (and squeezing) are obtained in continuous variable systems through interactions mediated by parametric crystals: to the best of our knowledge, assuming the highest reported degrees of squeezing (corresponding to a noise reduced to 0.1 vacuum units) [3] and perfect mixing operations, one can
achieve a logarithmic negativity $E_N \simeq 3\, \text{ebits}$ (see also \cite{5} for a “measured”, i.e. inferred from state reconstruction, value of 1.6 ebits).

Secondly, while extremely good at traveling, electromagnetic fields don’t make such good static degrees of freedom: even though they can be trapped in cavities, one is often confronted with a challenging and impractical trade-off between keeping the cavity open to external fields in order to access the quantum information and isolating the cavity to reduce losses and thus decoherence. This problem could be partially solved by mapping the quantum state of light into polarised atomic clouds memories \cite{6,7}. Though such a technology has been pioneered and successfully tested, its performance is still far from ideal, whence it may be desirable to resort to other static degrees of freedom allowing for the direct manipulation of continuous variable quantum information.

An extremely promising candidate to this aim, which might potentially address both the aforementioned issues, is represented by the motional degrees of freedom of trapped ions. The control of positions and momenta of trapped atoms and ions has been successfully implemented in several past experiments, both for its own sake \cite{8,9,10,11,12,13} and to address internal degrees of freedom (e.g. in realising prototypes of the Cirac-Zoller ionic quantum computer \cite{14,15,16,17}). In the case of ions, Coulomb interactions between the ions could be exploited to generate entanglement between motional degrees of freedom, while the long achievable trapping times would account for the need of good static degrees of freedom.

Indeed, the quantum properties of mechanical degrees of freedom have gained a large amount of interest recently, either in terms of squeezing or in terms of entanglement properties. Most of such investigations have focused around nannomechanical and micromechanical oscillators \cite{16,17,18} but, recently the entanglement of motional degrees of ions was created and measured as well \cite{19}.

Here we shall focus on the transverse (“radial”) motions, which can be individually addressed, and where phonons can be locally defined, thanks to the tightness of the transverse confinement \cite{20,21}. In a previous contribution from the authors \cite{22}, it was shown that comprehensive manipulations of such radial degrees of freedom could be realised for two and more ions in a linear Paul trap by controlling the radial trapping frequencies. More specifically, it was shown that the capability to control each individual trapping potential in the array would allow for the implementation of any linear operation on the motions, including squeezing. Moreover, creation of high degrees of bipartite and multipartite entanglement was shown to be possible with only global control of the trap potential. The transmission of quantum information through the chain of ions, in both qubit and genuine continuous variable form, was also studied and shown to be achievable. Besides, it was indicated how multipartite entanglement of three ions could be put to use to violate Bell-like inequalities and demonstrate quantum non-locality.

In all these coherent manipulations, the only kind of experimental control supposed was the possibility of tuning and changing the electric trapping potentials: no optical control through laser pulses was required. However these theoretical findings, promising as they are, were all derived assuming two major idealisations:

- the changes of the trapping potential, which are the main way to manipulate the quantum states, were assumed to be instantaneous;
- the potentials were assumed to be static in time, which is only approximately true in a Paul trap, if the static component of the trapping field is large with respect to the amplitude of the oscillating component.
In this note, we will relax these two assumptions and study how the dynamics of the radial modes is affected if finite switching times and oscillating trapping potentials are taken into account.

The paper is organised as follows. In section 2 we describe the time-dependent Hamiltonian governing the evolution of the system and define the experimental parameters under control. In section 3 we address the generation of squeezing in the position and momentum of a single trapped ion, while in section 4 we will present results on the generation of entanglement of two trapped ions, considering the effect of finite switching times and of oscillating potentials. Notice that continuous variable squeezing and entanglement are closely related as, essentially, entanglement manifests itself in the squeezing of combined quadratures, like in the Einstein Podolski Rosen seminal example. Finally, some concluding words and future perspectives are given in section 5.

2 The trapping potential

We shall consider radial modes (along a transverse direction with respect to the trap’s axis) of one or two ions of mass \( m \) and charge \( ze \) in a linear Paul trap. Let \( \hat{X} \) and \( \hat{P} \) be the position and momentum operators of a single ion associated to the considered radial degree of freedom, then the Hamiltonian governing the dynamics of \( \hat{X} \) and \( \hat{P} \) in the quadrupole trapping field is

\[
\hat{H}(a, q) = \frac{\hat{P}^2}{2m} + \frac{m\Omega^2}{8} (a + 2q \cos(\Omega t)) \hat{X}^2,
\]

where \( \Omega \) is the frequency of the oscillating trapping potential, while \( a \) and \( q \) are dimensionless parameters which determine, respectively, the amplitude of the DC trapping field and of the AC oscillating trapping field. The parameters and factors are chosen so that the resulting \( a \) and \( q \) are the same as in the classical Mathieu equation, whose solutions discriminate between stable (generally trapped) and unstable motions of the ions in the trap 23.

In the case of two ions, the Hamiltonian contains also an interaction term due to the Coulomb repulsion between the ions: we shall approximate this term to the second order in the displacement, thus obtaining a quadratic term coupling the oscillations of the two ions. Notice that this “harmonic” approximation is very accurate in our case, where transverse to longitudinal potential ratios will be larger than 0.1. Under such conditions, the ratio between radial displacements and distance between neighbouring ions is at most about 0.02. Hence, fourth and higher order terms in the displacements are at least \((0.02)^2 \approx 4 \times 10^{-4}\) times smaller than the considered second order terms and can be safely neglected. However, we should note that this would not be the case anymore for very large amounts of radial squeezing: in this case the anharmonic corrections would have to be taken into account. In the present study we shall restrict to cases where the squeezing, while large, is still small enough for the harmonic approximation to hold. The Hamiltonian \( \hat{H}_2 \) for two ions read

\[
\hat{H}_2(a, q) = \frac{\hat{P}_1^2 + \hat{P}_2^2}{2m} + \frac{m\Omega^2}{2} \left[ \frac{1}{4} (a + 2q \cos(\Omega t)) - \frac{\xi \Omega^2}{\Omega^2} \right] \left( \hat{X}_1^2 + \hat{X}_2^2 \right) + 2\xi \Omega^2 \hat{X}_1 \hat{X}_2.
\]
The factor $\xi$ comes from the Coulomb interaction at lowest order and is actually equal to $z^2 e^2/(4\pi \varepsilon_0 m \Omega_L^2 d^3) \approx 0.5$, where $d$ is the distance between the ions and $\varepsilon_0$ is the vacuum electric permittivity [24]. Notice that $\xi$ itself does not depend on the mass of the ions nor on the longitudinal trapping frequency $\Omega_L$, although the Hamiltonian terms in which it enters can be tuned by adjusting $\Omega_L$.

In this study, we will consider situations where the parameters $a$ and $q$ can be controlled and changed over time by a hypothetical experimentalist. However, we will not assume such changes to occur instantaneously. The state of the system will thus evolve under the time-varying Hamiltonians $\tilde{H}(a(t), q(t))$ and $\tilde{H}_2(a(t), q(t))$.

As initial states, we will assume the ground state of the Hamiltonian $\hat{H}_{pw}$ with trapping frequency $\omega_{pw} = \Omega^2 (a(0) + q(0)^2)/2$:

$$\hat{H}_{pw} = \frac{\hat{p}^2}{2m} + \frac{m}{2} \omega_{pw}^2 \hat{X}^2.$$ 

(3)

This is the initial effective trapping frequency in the so-called “potential well model” [7], when the positions of the ions in the trap can be separated into a comparatively small and fast micro-motion, and a comparatively large and slow dominant term. Note that this initial state is Gaussian (i.e., it has Gaussian Wigner and characteristic functions and is hence completely characterised by the first and second statistical moments of positions and momenta), while the subsequent dynamics is linear and thus preserves the Gaussian character of the state. Therefore the dynamics can, under such conditions, be integrated numerically with straightforward techniques. To this aim, we employed the Runge-Kutta (RK4) method and cross checked it against the piece-wise exponentiation of the Hamiltonian matrix: the two methods yielded essentially coincident results for small enough time-steps (actually, at variance with RK4, the piece-wise exponentiation carries a small second order error, which would however be hardly noticeable in the results we will present).

As mentioned above a Gaussian state $\rho$ is completely determined by its first and second moments: first moments will not be of any concern here, as they can be unitarily adjusted and do not affect the quantities we set to study. The second moments can be conveniently grouped together in the “covariance matrix” (CM) $\sigma$, with entries $\sigma_{jk} = \text{Tr} [\hat{R}_j \hat{R}_k \rho]/2 - \text{Tr} \hat{R}_j \rho \text{Tr} \hat{R}_k \rho$, in terms of the vector of canonical operators: $\hat{R} = (\hat{X}, \hat{P})$ for one ion and $\hat{R} = (\hat{X}_1, \hat{X}_2, \hat{P}_1, \hat{P}_2)$ for two ions [11, 25, 26].

**3 Generation of squeezing**

In order to study the generation of squeezing by time-varying potentials we will consider a single ion, starting from the ground state of the Hamiltonian (3), and evolving in time under the Hamiltonian (3) for properly chosen $a(t)$ and $q(t)$. However, we will consider the rescaled quadratures $\hat{x} = \sqrt{m \omega_{pw}} \hat{X}$ and $\hat{p} = \hat{P}/\sqrt{m \omega_{pw}}$, so that this ground state $\rho_0$, which will constitute our reference for the vacuum, reduces to a Gaussian state with covariance matrix equal to the identity (in our units). Once the dynamics is solved and the CM $\sigma_t$ at subsequent time is obtained, it will thus suffice to evaluate the smallest eigenvalue $\lambda$ of $\sigma_t$ as a signature of squeezing: the smallest $\lambda$ (compared to 1) the largest the squeezing.

Fig. 1 shows two cases with small $a$’s and $q$’s, and with $|a(0)| \ll |q(0)|$, where the potential well model is very accurate. For the red (dashed) curve, $q(t) = 0.1$ at all times while $a$ starts from $a(0) = -0.001$ and then switches, linearly over a time...
Figure 1: Squeezing (smallest eigenvalue of the CM) for an initial state $\sigma_0$ (ground state of $\hat{H}_{pw}$) evolving under the Hamiltonian $\hat{H}$ for varying $a$'s and $q$'s. Red (dashed) curve: $a(t) = -0.001 + (-0.1 + 0.001)\Omega t/4$ for $0 \leq t \leq 4/\Omega$, $a(t) = -0.1 + (-0.001 + 0.1)(\Omega t/4 - 1)$ for $4/\Omega \leq t \leq 8/\Omega$, $a(t) = -0.001$ for $t \geq 8/\Omega$, and $q(t) = 0.1\ \forall t$. Blue (continuous) curve: $a(t) = 0.0001 + (0.01 + 0.0001)\Omega t/4$ for $0 \leq t \leq 4/\Omega$, $a(t) = 0.01 + (0.0001 - 0.01)(\Omega t/4 - 1)$ for $4/\Omega \leq t \leq 8/\Omega$, $a(t) = 0.0001$ for $t \geq 8/\Omega$, and $q(t) = 0.01\ \forall t$.

interval $4/\Omega$, to $a(4/\Omega) = -0.01$ to finally decrease back to the initial value $-0.001$. The system evolves then under the original Hamiltonian from $t = 8/\Omega$ on. The static DC field is always repulsive in this case, but the ion’s stability is guaranteed by the AC component. Besides, even though the initial and final conditions are stable, the ion briefly goes through a region of instability through this motion. As can be seen, if such passages can be carried out quickly enough not to lose the ion in the unstable region (that is, on the time-scale of $\Omega^{-1}$), then the average squeezing resulting from the change of potential can be remarkably high, even up to 0.1 vacuum units like here. The blue (continuous) curve shows instead the case $q(t) = 0.01$ at all times and $a$ increasing from 0.0001 to 0.01 for a time $0 < t < 4/\Omega$, and then decreasing back to 0.0001 for $4/\Omega < t < 8/\Omega$. In this case as well a relatively small, but rapid, change in the DC field yields a considerable degree of squeezing. However these values of squeezing, around $\lambda = 0.25$, are well below the previous instance, when the system underwent unstable regimes (whereas in this case the ion’s motion is stable for all $a$’s).

The left plot of Fig. 2 illustrates the effect of a change in the AC component with a steady DC trapping component. In the case portrayed $a(t) = 1$ at all times, while $q$ starts from $q(0) = 0$ and increases linearly up to 0.5 over a time interval $10/\Omega$ (after a time $4/\Omega$ where the system is kept at the initial potential and does not evolve). The parameter $q$ then goes back to 0 over the same time interval. Finally, the system evolves for another interval lasting $10/\Omega$ under the initial static trapping Hamiltonian. Even in this case, a degree of squeezing of the 10% ($\lambda = 1$) can be achieved, by changing only the AC potential. Finally, in the right plot, we report a case where both $a$ and $q$ vary (see the caption for details). Let us just point out that, as apparent from the plot, a change in both $a$ and $q$ does not in general grant significant advantages over the, arguably more practical, changes in only one of the parameters.
Figure 2: Squeezing (smallest eigenvalue of the CM) for an initial state $\sigma_0$ (ground state of $\hat{H}_{pw}$) evolving under the Hamiltonian (1) for varying $a$’s and $q$’s. Left plot: $q(t) = 0$ for $0 \leq t \leq 4/\Omega$, $q(t) = 0.5(\Omega t/10 - 2/5)$ for $4/\Omega \leq t \leq 14/\Omega$, $q(t) = 0.5 - (0.5)(\Omega t/10 - 7/5)$ for $14/\Omega \leq t \leq 24/\Omega$, $q(t) = 0$ for $t \geq 24/\Omega$, and $a(t) = 1$ $\forall t$. Right plot: $q(t) = 0$ for $0 \leq t \leq 10/\Omega$, $q(t) = 0.5(\Omega t/10 - 1)$ for $10/\Omega \leq t \leq 20/\Omega$, $q(t) = 0.5$ for $20/\Omega \leq t \leq 30/\Omega$, $q(t) = 0.5 - (0.5)(\Omega t/10 - 3)$ for $30/\Omega \leq t \leq 40/\Omega$, $q(t) = 0$ for $t \geq 40/\Omega$, and $a(t) = 1 - 0.9\Omega t/10$ for $0 \leq t \leq 10/\Omega$, $a(t) = 0.1$ for $10/\Omega \leq t \leq 20/\Omega$, $a(t) = 0.1 + 0.9(\Omega t/10 - 2)$ for $20/\Omega \leq t \leq 30/\Omega$, $a(t) = 1$ for $t \geq 30/\Omega$.

Figure 3: Entanglement (logarithmic negativity in ebits) for an initial state $\sigma_0$ (ground state of $\hat{H}_{pw}$ for two ions) evolving under the Hamiltonian (2) for varying $a$’s, different switching rates and additional oscillating potentials. On the left hand side, $q(t) = 0$ for all the curves. The system starts off from the ground state for $a(0) = 200$ and then switches linearly in time from $a = 200$ to $a = 2$, over different time intervals $\Delta t$. Red (solid) curve: the switching is instantaneous ($\Delta t = 0$). Blue (dashed) curve: $\Delta t = 0.2/\Omega$. Green (dot-dashed) curve: $\Delta t = 1/\Omega$. Magenta (dotted) curve: $\Delta t = 2/\Omega$. Cyan (solid) curve: $\Delta t = 4/\Omega$. On the right hand side, the case of $\Delta t = 0.1$ is portrayed again (blue, dotted), along with the same case but for $q(t) = 0.1$ $\forall t$ (red, dashed) and $q(t) = 0.5$ $\forall t$.

4 Generation of entanglement

We shall now consider two ions in a trap and address the evolution of the EPR-like entanglement between their canonical operators, under the dynamical conditions spec-
ified above. Because the state of the system is Gaussian at all times, we can quantify such an entanglement by evaluating the logarithmic negativity, a widely used entanglement monotone related to the absolute sum of the negative eigenvalues of the partial transposition of a quantum state \[27, 28, 29, 30\]. If \( \tilde{\rho} \) is the partial transposition of the bipartite state \( \rho \) (transposition with respect to only one the two parties’ Hilbert spaces), then the logarithmic negativity \( E_N \) of \( \rho \) is given by \( E_N = \log_2 \| \tilde{\rho} \|_1 \), where \( \| \cdot \|_1 \) stands for the trace norm. The logarithmic negativity is an upper bound to the distillable entanglement and is customarily expressed in ebits. It can be computed for Gaussian states with standard techniques, essentially because the effect of the partial transposition on positions and momenta is promptly described, and because such a transformation maps bosonic Gaussian states into bosonic Gaussian states \[31\].

As in the case of squeezing, we will always start from a ground state of the Hamiltonian with effective radial trapping frequency \( \omega_{pub} \), and where the corrections due to the Coulomb repulsion are also taken into account. In the case of two ions, the Coulomb interactions and modifications to the local trapping frequencies render the study at hand slightly more delicate. This is essentially because instabilities can arise not only from the configuration of DC and AC trapping fields, but also because of the repulsion between the ions. However, the idea behind the generation of entanglement is analogous to that underlying the generation of squeezing: for large enough initial trapping frequencies, the ions will start the evolution in a very weakly entangled state (often separable to most practical effects). If the initial Hamiltonian does not change, the entanglement will clearly not change either (it will at most oscillate around an average value if the amplitude of the AC component is large). However, if the parameters of the potential, and hence the trapping frequencies, change, the system will perceive such a change as a “deformation” of the canonical coordinates, which are rescaled by the frequencies, that is, essentially, as a squeezing transformation. If, like in the case of two ions, an interaction term is also present, the squeezing will gradually be transferred from local coordinates to a non-local combination of the coordinates. such a squeezing in combined quadratures corresponds essentially to entanglement in continuous variable systems.

Let us first discuss the role of switching times in the entanglement generation. The example on the left of Fig. 3 is extremely clear in this respect: only a static trapping field is considered \( q(t) = 0 \), with \( a(t) \) decreasing from 200 to 2 (notice that this correspond to a change of a factor 10 in the trapping potential, since \( a \) corresponds to a squared trapping frequency) in a time interval \( \Delta t \) which varies from curve to curve, from instantaneous to \( \Delta t = 4/\Omega \) (from top to bottom). It is apparent that faster switching rates allow for a superior entanglement generation. In fact, while the Hamiltonian is changing, the ground state of the system “adapts” to the new Hamiltonian if the change is too slow (much in the spirit of the adiabatic theorem). The actual entanglement generation only begins once the trapping potential reaches the new value, and its magnitude will depend on the rapidity of the change. In general, switching rates of the order of \( 10\sqrt{n}\Omega \) allow for a close to ideal creation of entanglement, but substantial entanglement is also there for switching slower by one order of magnitude.

The right side of Fig. 3 shows the effect of an added AC component on the same evolution, for \( \Delta t = 0.1 \). As evident from the plot, an AC with \( q \) up to 0.5 affects only rather marginally the evolution of the logarithmic negativity. In general, moreover, the effect on the entanglement of additional oscillating potentials is erratic and does not monotonically depend on the AC amplitude.

Similar performances can be obtained by keeping the same static potential and varying the AC component. Fig. 4 shows a non-trivial instance of such dynamics. The
system starts from the ground state for $a = 10$ and $q = 0$. Then, the parameter $q$ controlling the AC amplitude is linearly increased to 100 and turned off again over different time intervals $2\delta t$'s. The middle curve (blue, dotted) refers to $\delta t = 0.2/\Omega$: in this case the switch is rather fast and the entanglement generation substantial. The lower curve (green, dashed) refers to $\delta t = 1/\Omega$: the switching is slower and thus less entanglement is created. The upper curve (red, solid) refers to $\delta t = 1.3/\Omega$: here the entanglement is larger than in the previous case. In fact, the transient dynamics of the system mostly takes place in a region of parameters which is definitely unstable: as we have seen in the case of squeezing generation, spending a sizeable part of the dynamics in such regions can create very high squeezing and hence, in this case, entanglement. This curve shows a sudden boost in entanglement right after the transient interval, which is a signature of ‘impending’ instability: in fact, higher $\delta t$’s would be impractical, because the ion would probably get lost (the numerics start diverging there). It shouldn’t surprise that the entanglement keeps oscillating over large time-scales after the initial Hamiltonian is re-established. This is due to the fact that, once the squeezing is generated through the varying potentials, the Coulomb interaction keeps rotating the state in phase space, making it undergo cycles of entanglement and disentanglement.

5 Conclusions and outlook

Summing up, in the present note we have shown that:

- both entanglement and squeezing of the motional degrees of freedom of trapped ions can be effectively created by controlling the AC and/or the DC component of the trapping potentials;

Figure 4: Entanglement (logarithmic negativity in ebits) for an initial state $\sigma_0$ (ground state of $\hat{H}_{pw}$, for two ions) evolving under the Hamiltonian $\hat{H}$ for varying $q$’s and different switching rates. $a(t) = 10$ for all curves. In all cases, the system starts from the ground state for $a(0) = 10$ (and $q(0) = 0$), and then switches linearly in time from $q = 0$ to $q = 100$, over different time intervals $\delta t$’s. Red (solid) curve: $\delta t = 1.3/\Omega$. Blue (dotted) curve: $\Delta t = 0.2/\Omega$. Green (dashed) curve: $\Delta t = 1/\Omega$. 

5 Conclusions and outlook

Summing up, in the present note we have shown that:

- both entanglement and squeezing of the motional degrees of freedom of trapped ions can be effectively created by controlling the AC and/or the DC component of the trapping potentials;
• regions of trapping instability can be profitable to boost the generation of motional squeezing and entanglement (if the permanence in such regions is short enough not to lose the ions!);

• switching rates of the order of $10\sqrt{\Omega}$, that is of the order of the effective trapping frequency, are ideal to generate such resources (although one order of magnitude less still yields interestingly good values).

Note that, in principle, the squeezing and entangling operations presented here could be iterated to achieve muchly improved performances (see [22] for details), the ultimate limit being essentially the tolerance of the trap’s geometry to large displacements (large squeezing in positions and momenta implies in fact broad oscillations).

Let us also remark that operations on the ions can be realised also by controlling the RF frequency $\Omega$, while leaving the strenghts of the potentials unchanged. This path has not been followed in the present paper.

To conclude, let us point out that, in view of the considerable potential demonstrated in the present and previous investigations and of the widespread interest in generating and distributing optical squeezing and entanglement, one could argue that the ultimate applicability of this sort of manipulations should be aimed at hybrid systems where, after the resources are generated in situ by controlling the potentials, the ions are then coupled to light through cavities and the squeezing or entanglement are swapped to optical modes. Future work will focus on this possibility [32].

Finally, a further line of investigation could focus on the possibility of realising any quantum gate (not restricted to Gaussian operations) between the motions of two ions, by exploiting anharmonicities and the control of the potentials’ parameters.

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