Mathematical model of changing the combined cutting tool durability

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Abstract. The article provides information on the development and study of a mathematical model for changing the durability of a new progressive combined tool. Currently, one effective way to increase productivity in automated manufacturing is to concentrate operations or transitions. With a high degree of operations concentration, the complexity of the kinematics of the equipment increases, its reliability decreases, therefore, downtime and the production cost are rising. The transitions concentration leads to the tool design complexity increasing, while the equipment can have simple kinematics of movements. The compatibility of operations or transitions depends on the required machining accuracy and surface roughness. The accepted procedure for performing operations and transitions determines necessary tool type. The most efficient concentration of operations or transitions can be achieved by using combined tools. Combined tools can be used in the treatment of both stepped and smooth cylindrical holes, which make up 35% of all surfaces used in mechanical engineering. Therefore, this study is devoted to solving the problem of increasing productivity and reducing the cost of threaded holes processing through the use of a new combined tool called the drill-tap.

1. Introduction

The cutting process result is determined by many parameters, which can have hidden nature sometimes. Non-uniformity of the treated material physical and mechanical properties, its anisotropy, deformation and heating during cutting, various physical and chemical effects (sticking, oxidizing films, etc.) appear and disappear during cutting. Visually inconspicuous factors significantly affect the cutting tool durability. Instability of tool material, heat treatment and tool sharpening, its uncontrolled parameters (microgeometry of cutting edge, radius of its rounding), change of stiffness zone in connection with machine operation modes have more effect on tool stability and, therefore, on productivity [1].

The purpose of the work is to study the tap durability in the combined tool, which will first of all make it possible to justify the parameters of both tools in the combination drill-tap, since the durability of the combined tool depends on the durability of the tap. In contrast to the tap, the drill has an unlimited regrinding number, so its durability is not a limiting factor.

2. Mathematical model of combined tool durability change

The study is carried out according to the "optimal planning method" developed when studying the optimization of chemical technology processes [2-4].

Figure 1 shows a sketch of a combined tool called "drill-tap". To experimentally determine its durability a mathematical model that describes this organized system behavior when a number of factors change is created.
The YSSG-E tool material has the following chemical composition: C – 0.93%, Cr – 4.2%, Mo – 6.4%, W – 6.4%, V – 1.8%, Co – 5% [5].

The tap durability was studied on a vertical CNC drilling machine model 2R135F2. The processed material was 8 mm thick rolled steel 08kp (USA – 1008, EU – DC01), steel 20 (USA – 1020, EU – 1.1151) and steel 45 (USA – 1044, EU – 1.1191).

The change of tool durability depending on the cutting speed, feed per tooth and the processed material hardness is determined by a mathematical model [6]:

\[ T^m = C \cdot v \cdot S_z^p \cdot HB^q, \]  

(1)

where \( T \) is the tool durability, min;
\( v \) is cutting speed, m/min;
\( S_z \) is feed per tooth, mm/tooth;
\( HB \) is the processed material hardness;
\( m, p, q \) are coefficients.

In equation (1) we assume that \( T \) is a dependent variable and \( v, S_z, HB \) are independent variables.

When taking the logarithm of equation (1) we obtain the first degree polynomial:

\[ \ln T = \frac{\ln C}{m} + \frac{\ln v}{m} + \frac{p}{m} \ln S_z + \frac{q}{m} \ln HB \]  

(2)

or, given the errors of the experiment

\[ y = b_0 + b_1 \cdot x_1 + b_2 \cdot x_2 + b_3 \cdot x_3 + \varepsilon, \]  

(3)

where \( b_i \) are coefficients;
\( x_i \) are variable factors.

We plan the experiment on the basis of a multi-factor analysis, that is, we simultaneously vary all the variables: the cutting speed is the variable \( x_1 \), the feed to the tooth is the variable \( x_2 \), and the hardness of the processed material is the variable \( x_3 \). It is need to determine the constant \( b_0 \) and coefficients \( b_1, b_2, b_3 \). Each of these variables varies at two levels \( (2^k) \), where \( k \) is the number of variables.

The values of cutting speeds were taken as 3, 5, 8 m/min, which corresponds to the tool rotation speeds of 120, 200, 320 \( \text{min}^{-1} \) respectively. The feed per tooth is 0.18, 0.25, 0.38 mm/tooth. For steel 08kp hardness \( HB=126 \), for steel 20 \( HB=159 \), for steel 45 \( HB=193 \) [7, 8].

The plan of twelve experiments provided for two series of six experiments in each: six main and six additional. The selection of processing modes is carried out in such a way as to simplify coding, taking into account the capabilities of the equipment and the maximum processing modes, as shown in table 1.

These levels are encoded using conversion levels so that the upper level answers +1 and the lower level answers -1.
\[
x_1 = \frac{2(\ln v - \ln 8)}{\ln 8 - \ln 3} + 1
\]
\[
x_2 = \frac{2(\ln S - \ln 0.38)}{\ln 0.38 - \ln 0.18} + 1
\]
\[
x_3 = \frac{2(\ln HB - \ln 193)}{\ln 193 - \ln 126} + 1
\]

(4)

**Table 1.** Processing modes selection.

| Level     | Processing modes under study | Code mark |
|-----------|------------------------------|-----------|
|           | \(v\), m/min | \(S_z\), mm/tooth | \(HB\) | \(x_1\) | \(x_2\) | \(x_3\) |
| Top       | 8               | 0.38       | 193    | 1       | 1       | 1       |
| Average   | 5               | 0.25       | 159    | 0       | 0       | 0       |
| Lower     | 3               | 0.18       | 126    | -1      | -1      | -1      |

Here, in the experimental plan, for example, the value \(\frac{\ln 8 - \ln 3}{2}\) is taken as the unit of cutting speed. Thus, the cutting speed is converted by dividing it by the accepted unit.

Similarly, the feed to the tooth and the hardness of the processed material are converted. To simplify calculations the composite plan is constructed according to the so-called "Italian cube", which built on three orthogonal coordinates with the beginning in the center of the cube (figure 2).

The vertices of the cube indicate the numbers of experiments, cutting modes and hardness of the workpiece material according to table 2.

Experimental points are marked with circles according to the first four experiments; additionally, two experiments are placed in the center of the cube to test the adequacy hypothesis (experiments 9 and 10). If the hypothesis of adequacy does not pass, we perform six more experiments: the cube vertices are being completed (indicated by circles) and two more experiments in the center.

**Figure 2.** "Italian cube".
The cutting modes, material hardness, code values, and tool durability are shown in table 2.

**Table 2.** The experiment plan.

| No. | No. series | The cutting modes | Code mark | T, min | y = ln T |
|-----|-----------|-------------------|-----------|--------|---------|
|     |           | v, m/min          | S₂, mm/tooth | HB | x₁ | x₂ | x₃ |
| 1   | 2         | 3                 | 0.18      | 126 | -1 | -1 | -1 | 90 | 4.49 |
| 2   | 1         | 8                 | 0.18      | 126 | 1  | -1 | -  | 60 | 4.09 |
| 3   | 1         | 3                 | 0.38      | 126 | -1 | 1  | -1 | 70 | 4.24 |
| 4   | 2         | 8                 | 0.38      | 126 | 1  | 1  | -1 | 45 | 3.81 |
| 5   | 1         | 3                 | 0.18      | 193 | -1 | -1 | 1  | 35 | 3.55 |
| 6   | 2         | 8                 | 0.18      | 193 | 1  | 1  | 1  | 25 | 3.21 |
| 7   | 2         | 5                 | 0.25      | 159 | 0  | 0  | 0  | 40 | 3.68 |
| 8   | 2         | 5                 | 0.25      | 159 | 0  | 0  | 0  | 45 | 3.8  |
| 9   | 1         | 5                 | 0.25      | 159 | 0  | 0  | 0  | 40 | 3.68 |
| 10  | 1         | 5                 | 0.25      | 159 | 0  | 0  | 0  | 45 | 3.8  |
| 11  | 2         | 5                 | 0.25      | 159 | 0  | 0  | 0  | 40 | 3.68 |
| 12  | 2         | 5                 | 0.25      | 159 | 0  | 0  | 0  | 45 | 3.8  |

3. Research results and discussion

Based on the results of series 1 six experiments, it is convenient to estimate four coefficients in an empirical formula:

\[ y = b_0 + b_1 \cdot x_1 + b_2 \cdot x_2 + b_3 \cdot x_3. \]  

(5)

According to the least squares method, the sum of the squared deviations of the actual values obtained by equation (4) \( y_{x_1,x_2,x_3} \), must be the smallest:

\[ \sum (y - y_{x_1,x_2,x_3})^2 = \text{min}. \]  

(6)

The left part of expression (6) is denoted by the \( f \) as a function of the unknown parameters \( b_0, b_1, b_2, b_3 \). The minimum of this function is found using the equations:

\[ \frac{df}{db_0} = 0; \frac{df}{db_1} = 0; \frac{df}{db_2} = 0; \frac{df}{db_3} = 0. \]  

(7)

After differentiating, we write a system of these equations in the final form:

\[ \sum y = nb_0 + b_1 \sum x_1 + b_2 \sum x_2 + b_3 \sum x_3; \]

\[ \sum yx_1 = b_0 \sum x_1 + b_1 \sum x_1 x_1 + b_2 \sum x_1 x_2 + b_3 \sum x_1 x_3; \]

\[ \sum yx_2 = b_0 \sum x_2 + b_1 \sum x_2 x_1 + b_2 \sum x_2 x_2 + b_3 \sum x_2 x_3; \]

\[ \sum yx_3 = b_0 \sum x_3 + b_1 \sum x_3 x_1 + b_2 \sum x_3 x_2 + b_3 \sum x_3 x_3. \]

(8)

Equations (8) make it possible to determine

\[ b_0 = \frac{\sum y}{n}, \]  

(9)

where \( n \) is the number of experiments, \( n = 6 \),

\[ b_1 = \frac{\sum yx_1}{\sum x_1^2}; \quad b_2 = \frac{\sum yx_2}{\sum x_2^2}; \quad b_3 = \frac{\sum yx_3}{\sum x_3^2}. \]  

(10)

It should be added that the other terms of equations (8) will be zero, since the vectors \( x_1, x_2, x_3 \) are orthogonal and \( \sum x_i = 0 \).
Obviously, it would be easier not to solve equations (6) – (8), but to use the matrices \( (x'x) \) and \( (x'x)^{-1} \):

\[
(x'x) = \begin{bmatrix}
6 & 0 & 0 & 0 \\
0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4
\end{bmatrix}, \quad (x'x)^{-1} = \begin{bmatrix}
\frac{1}{6} & 0 & 0 & 0 \\
0 & \frac{1}{4} & 0 & 0 \\
0 & 0 & \frac{1}{4} & 0 \\
0 & 0 & 0 & \frac{1}{4}
\end{bmatrix},
\]

(11)

We present the dependencies for determining the coefficients \( b_0 \ldots b_3 \), which according to equation (11) will have the form:

\[
b_0 = \frac{1}{6} \cdot (y_2 + y_3 + y_5 + y_8 + y_9 + y_{10}),
\]

\[
b_1 = \frac{1}{4} \cdot (y_2 - y_3 - y_5 + y_8),
\]

\[
b_2 = \frac{1}{4} \cdot (-y_2 + y_3 - y_5 + y_8),
\]

\[
b_3 = \frac{1}{4} \cdot (-y_2 - y_3 + y_5 + y_8).
\]

(12)

The indexes determine the numbers of experiments (table 2). Substituting the experimental values from table 2 into the equations (12), we obtain:

\[
b_0 = \frac{1}{6} \cdot (4.09 + 4.24 + 3.55 + 2.52 + 3.68 + 3.8) = 3.65,
\]

\[
b_1 = \frac{1}{4} \cdot (4.09 - 4.24 - 3.55 + 2.52) = -0.3,
\]

\[
b_2 = \frac{1}{4} \cdot (-4.09 + 4.24 - 3.55 + 2.52) = -0.22,
\]

\[
b_3 = \frac{1}{4} \cdot (-4.09 - 4.24 + 3.55 + 2.52) = -0.57.
\]

(13)

In this case, for a series 1 of experiments, we obtain a formula for determining the durability period:

\[
\ln T = 3.65 - 0.3x_1 - 0.22x_2 - 0.57x_3.
\]

(14)

Formula (14) must be deciphered according to equations (4):

\[
x_1 = \frac{2(\ln v - \ln 8)}{\ln 8 - \ln 3} + 1 \approx \frac{2(\ln v - 2.07)}{2.07 - 1.09} + 1 = 2.04 \ln v - 4.22;
\]

\[
x_2 = \frac{2(\ln S_z - \ln 0.38)}{\ln 0.38 - \ln 0.18} + 1 \approx \frac{2(\ln S_z + 0.96)}{-0.96 - 1.71} + 1 = 2.66 \ln S_z + 3.55;
\]

\[
x_3 = \frac{2(\ln HB - \ln 193)}{\ln 193 - \ln 126} + 1 \approx \frac{2(\ln HB - 5.26)}{5.26 - 4.83} + 1 = 4.65 \ln HB - 23.46.
\]

(15)

Substituting the values \( x_1, x_2, x_3 \) in equations (14), we obtain a refined formula for determining the logarithm of the durability period:

\[
\ln T = 3.65 - 0.3(2.04 \ln v - 4.22) - 0.22(2.66 \ln S_z + 3.55) - 0.57(4.65 \ln HB - 23.46).
\]

So, the final formula for determining the durability of a combined drill-tap for processing steel parts will take the form:

\[
T = \frac{e^{14.98}}{v^{0.615}S_z^{0.58}HB^{2.65}} = \frac{3.06 \times 10^6}{v^{0.615}S_z^{0.58}HB^{2.65}}.
\]

(16)

After statistical analysis of the results of series 1, when there were very large stability intervals for 95% confidence, six more experiments of series 2 studies were performed.
As a result of all 12 experiments, in accordance with the above calculations and table 2, we obtain refined coefficients for determining the durability of the combined tool:

\[
b_0 = \frac{1}{12} \cdot (4.49 + 4.09 + 4.24 + 3.81 + 3.55 + 3.21 + 2.99 + 2.52 + 3.68 + 3.8 + 3.68 + 3.8) = 3.65, \\
b_1 = \frac{1}{8} \cdot (-4.49 + 4.09 - 4.24 + 3.81 - 3.55 + 3.21 - 2.99 + 2.52) = -0.21, \\
b_2 = \frac{1}{8} \cdot (-4.49 - 4.09 + 4.24 + 3.81 - 3.55 - 3.21 + 2.99 + 2.52) = -0.22, \\
b_3 = \frac{1}{8} \cdot (-4.49 - 4.09 - 4.24 - 3.81 + 3.55 + 3.21 + 2.99 + 2.52) = -0.54.
\]

\[\ln T = 3.65 - 0.21(2.04 \ln v - 4.22) - 0.22(2.66 \ln S_z + 3.55) - 0.54(4.65 \ln HB - 23.46) = 14.85 - 0.42 \ln v - 0.58 \ln S_z - 2.51 \ln HB.\]

\[T = \frac{e^{14.85}}{v^{0.42}S_z^{0.58}HB^{2.51}} = \frac{2.2 \cdot 10^6}{v^{0.42}S_z^{0.58}HB^{2.51}}.\]  

Figure 3 shows the dependence of the combined tool durability on the processed material hardness.

![Figure 3. Changing the combined tool durability depending on the processed material hardness.](image)

4. Conclusions
A mathematical model for changing the new combined cutting tool durability has been developed. This model allows to make an informed choice of drill and tap parameters when they are combined in a combined tool.

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