Dilepton photoproduction on a deuteron target

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We investigate the sensitivity of the cross section for lepton pair production off a deuteron target, $\gamma d \rightarrow l^+ l^- d$, to the deuteron charge radius. We show that for small momentum transfers the Bethe-Heitler process dominates, and that it is sensitive to the charge radius such that a cross section ratio measurement of about 0.1% relative accuracy could give a deuteron charge radius more accurate that the current electron scattering value and sufficiently accurate to distinguish between the electronic and muonic atomic values.

Over the past decade, the extractions of the proton charge radius from the Lamb shift measurements in muonic hydrogen [1, 2] resulted in a significant discrepancy in comparison with measurements with electrons [3–5], amounting to a 5.6 $\sigma$ difference according to a recent re-evaluation [6]. The resolution of this “proton radius puzzle” has triggered a lot of activity, see e.g. Refs. [7–9] for recent reviews. Corresponding measurements on the deuteron have not only confirmed the puzzle [10], but have also revealed a 3.5 $\sigma$ difference between the spectroscopic measurements in muonic versus ordinary deuteron. The deuteron charge radius as extracted from elastic electron scattering [11] has at present a too large error bar to distinguish between both spectroscopic values. In this letter, we investigate the sensitivity of the complementary lepton pair production process off a deuteron target, $\gamma d \rightarrow l^+ l^- d$, to the deuteron charge radius.

We consider $\gamma d \rightarrow l^+ l^- d$ in the limit of very small spacelike momentum transfer, defined as $\Delta \equiv p' - p$, with four-momenta as indicated in Fig. 1. Furthermore, we will use in the following the Mandelstam invariant $s = (k + p)^2 = M^2_d + 2M_d E_p$, with $M_d$ the deuteron mass and $E_p$ the photon lab energy, the Mandelstam invariant $t = \Delta^2$, as well as the squared invariant mass of the lepton pair, defined as $M^2_{ll} \equiv (l_+ + l_-)^2$. In the limit of small $-t$, the Bethe-Heitler (BH) mechanism, shown in Fig. 1 dominates the cross section of the $\gamma d \rightarrow l^+ l^- d$ reaction, as we shall show.

The deuteron electromagnetic structure entering the hadronic vertex in the BH process of Fig. 1 is described by three elastic electromagnetic form factors (FFs), corresponding to the Coulomb monopole ($G_C$), magnetic dipole ($G_M$), and Coulomb quadrupole ($G_Q$) FFs, respectively. The definitions and normalizations of $G_C$, $G_M$, and $G_Q$ follow from [12],

$$
\langle p', \lambda' | J^\mu (0) | p, \lambda \rangle = \varepsilon_{\alpha}(p, \lambda) \varepsilon_{\beta}^*(p', \lambda') \left\{ -g^{\alpha \beta} 2p^\mu G_1(t) + \left( g^{\alpha \mu} \Delta^\beta - g^{\beta \mu} \Delta^\alpha \right) G_M(t) + \Delta^\alpha \Delta^\beta \frac{p^\mu}{M_d^2} G_3(t) \right\},
$$

where $P = (p + p')/2$, and $\varepsilon_{\alpha}$ and $\varepsilon_{\beta}^*$ are deuteron polarization vectors. The charge and quadrupole FFs accordingly,

$$
G_C = G_1 + \frac{2}{3} \tau_d G_Q, \quad G_M = G_1 - G_M + (1 + \tau_d) G_3,
$$

with normalizations $G_C(0) = 1$, $G_M(0) = \mu_d$ (magnetic moment in units $e/(2M_d)$), $G_Q(0) = Q_d$ (quadrupole moment in units $e/M^2_d$), and where $\tau_d \equiv -t/(4M^2_d)$. For numerical evaluation, we will use the parameterization of the deuteron FFs, obtained from scattering and tensor polarization data, and given as fit II by Abbott et al. [13]; see also [14] for details. In addition, we have the deuteron FF parameterization based on scattering data, but including a treatment of two and more photon exchange corrections, by Sick and Trautmann [11].

As the momentum transfer $t$ is the argument appearing in the form factor (FF) in the BH process, a measurement of the cross section in the small $-t$ kinematic regime, where the BH process dominates, will allow accessing the deuteron charge FF $G_C$ at small spacelike momentum transfer. The deuteron charge radius $R_d$ is determined from $G_C$ through

$$
G_C(t) = 1 + \frac{1}{6} R^2_G t + \mathcal{O}(t^2).
$$

We quote several current values for the deuteron charge radius $R_d$, all in femtometers.

$$
R_d = \begin{cases} 
0.12771(22) & \mu-H \text{ Lamb shift & isotope shift [10]}, \\
2.12562(78) & \mu-d \text{ Lamb shift [10]}, \\
2.1413(25) & \text{CODATA 2014 [5]}, \\
2.1415(45) & \text{atomic deuterium spectroscopy [15]}, \\
2.130(10) & e-d \text{ elastic scattering [11]}, \\
2.088 & Abbott et al. fit [13], 
\end{cases}
$$

FIG. 1. Mechanisms for $\gamma d \rightarrow l^+ l^- d$. The momenta of the external particles are $k$ for the photon, $p(p')$ for initial (final) deuterons, and $l_-, l_+$ for the lepton pair. The upper diagrams show the Bethe-Heitler mechanism; the lower diagram shows the Compton mechanism.

FIG. 1. Mechanisms for $\gamma d \rightarrow l^+ l^- d$. The momenta of the external particles are $k$ for the photon, $p(p')$ for initial (final) deuterons, and $l_-, l_+$ for the lepton pair. The upper diagrams show the Bethe-Heitler mechanism; the lower diagram shows the Compton mechanism.
Of the two purely or mainly atomic values, atomic deuteron spectroscopy uses only fits to energy splittings measured in deuterium, while CODATA uses the proton radius obtained electronically and the isotope shift (the very accurate measurement of \( R_d^2 - R_p^2 \) [16], using ordinary hydrogen). The last listed radius measurement also uses the isotope shift, this time combined, supposing the absence of new physics, with the proton radius measured from the muonic hydrogen Lamb shift. Notable is the definite incompatibility between the deuteron radius measured using ordinary and muonic atoms.

As a preliminary observation, the effect of the radius modifications on the calculated \( e^+d \) elastic scattering cross section is shown in Fig. 2, where cross sections are shown relative to the Abbott et al. results. Results using the Sick-Trautmann \( G_C \) form factor as,

\[
G_C(t) = \frac{G_C,\text{Sick-Trautmann}(t)}{1 - \frac{1}{6} \left( R_d^2 - R_d,\text{Sick-Trautmann}^2 \right) t}.
\]

This will allow studying the dependence on \( R_d \) while keeping the same curvature terms as used in the Sick-Trautmann parameterization of the \( G_C \) FF.

The elastic cross sections are obtained from the no structure parameterization are labeled scattering, and results obtained for other values of \( R_d \) are obtained by modifying the Sick-Trautmann \( G_C \) form factor as,

\[
\frac{d\sigma}{d\Omega} = \left. \frac{d\sigma}{d\Omega} \right|_\text{NS} \left[ A(t) + B(t) \tan^2(\theta_e/2) \right],
\]

with \( \theta_e \) the electron lab scattering angle, and where

\[
A(t) = G_C^2(t) + \frac{2}{3} \tau_d G_M^2(t) + \frac{8}{9} \tau_d^2 G_O^2(t),
\]

\[
B(t) = \frac{4}{3} \tau_d (1 + \tau_d) G_M^2(t).
\]

Experimental data for Simon et al. [17] and Platchkov et al. [18] are also shown. The two photon corrections as given by McKinley and Feshbach [19] have been applied to the data.

Turning to lepton production, the differential cross section for the BH process is strongly peaked for leptons emitted in the incoming photon direction, and as we aim to maximize the BH contribution in this work in order to access \( G_C \), we will study the \( \gamma d \rightarrow l^- l^+ d \) process when (only) detecting the recoiling deuteron’s momentum and angle, thus effectively integrating over the large lepton peak regions. The lab momentum of the deuteron is in one-to-one relation with the momentum transfer \( t \) :

\[
|\vec{p}^{\prime}\rangle^{lab} = 2M_d \sqrt{\tau_d} (1 + \tau_d).\]

Furthermore, for a fixed value of \( \tau_d \), the recoiling deuteron lab angle \( \Theta_d^{lab} \) is expressed in terms of invariants as:

\[
\cos \Theta_d^{lab} = \frac{M_\mu^2 + 2(s + M_\mu^2) \tau_d}{2(s - M_\mu^2) \sqrt{\tau_d} (1 + \tau_d)}.
\]

The differential cross section for the dominating BH process to the \( \gamma d \rightarrow l^- l^+ d \) reaction, differential in \( t, M_\mu^2 \), and the lepton solid angle \( \Omega^{l^- l^+}\text{cm} \) in the c.m. frame of the dilepton pair is given by

\[
\frac{d\sigma}{dt \, dM_\mu^2 \, d\Omega^{l^- l^+}\text{cm}} = \frac{\alpha^3 \beta}{16\pi (s - M_\mu^2)^2 t^2} L_{\mu\nu} H^{\mu\nu},
\]

with \( \alpha \equiv e^2/4\pi \approx 1/137 \), and \( \beta \equiv \sqrt{1 - 4M_\mu^2/M_\text{light}^2} \) the lepton velocity in the \( l^- l^+ \) c.m. frame, with \( m \) the lepton mass. Furthermore in Eq. (9), \( L_{\mu\nu} \) is the unpolarized lepton tensor, averaged over the initial photon polarizations, given by:

\[
L_{\mu\nu} = -\frac{1}{2} \left\{ \left( I^- + m \right) \left[ \gamma^\mu f^- - \bar{\gamma}^\nu \bar{\gamma}^\mu f^+ + \bar{\gamma}^\nu \bar{\gamma}^\mu \bar{f}^+ + m^2 \right] \right\} \times \left( I^- - m \right) \left[ \gamma^\nu f^- - \bar{\gamma}^\mu \bar{\gamma}^\nu \bar{f}^+ + m^2 \right],
\]

and \( H^{\mu\nu} \) is the unpolarized hadronic tensor defined by:

\[
H^{\mu\nu} = \frac{1}{3} \sum_{\lambda = 0,\pm 1} \sum_{\lambda' = 0,\pm 1} \langle p', \lambda' \mid J^{\mu}(0) \mid p, \lambda \rangle \times \langle p', \lambda' \mid J^{\nu}(0) \mid p, \lambda \rangle^*.\]

Using Eq. (1), the hadronic tensor for the BH process can be expressed as:

\[
H^{\mu\nu} = \left( -g^{\mu\nu} + \frac{\Delta^\mu \Delta^\nu}{\Delta^2} \right) \left[ \frac{8}{3} M_{\mu}^2 \tau_d (1 + \tau_d) G_M^2 \right] + 4 p^\mu p^\nu \left[ G_C^2 + \frac{2}{3} \tau_d G_M^2 + \frac{8}{9} \tau_d^2 G_O^2 \right],
\]

and the FFs are functions of the momentum transfer \( t \).
When detecting only the deuteron momentum and angle, the cross section integrated over the deuteron angles is

$$
\frac{d\sigma^{\text{BH}}}{dt \, dM_{ll}^2} = \frac{4\alpha^2 \beta}{(s - M_d^2)^2 (M_d^2 - |t|)^4} \times \left\{ C_E \left[ C_C^2 + \frac{8}{3} \tau_2 G_Q^2 \right] + C_M \frac{2}{3} \tau_4 G_M^2 \right\},
$$

where

$$
C_{E,M} = C_{E,M}^{(1)} + C_{E,M}^{(2)} \frac{1}{\beta} \ln \left( \frac{1 + \beta}{1 - \beta} \right).
$$

The coefficients $C_{E,M}^{(1)}$ and $C_{E,M}^{(2)}$ are expressed through invariants as

$$
C_E^{(1)} = t \left( s - M_d^2 \right) (s - M_d^2 - M_H^2 + t) \times \left[ M_H^4 + 6M_H^2 t + t^2 + 4m_d^2 M_H^2 \right]
\times \left[ (M_H^2 - t)^2 \left[ t^2 M_d^2 + M_H^2 (M_H^2 + t)^2 + 4m_d^2 M_d^2 M_H^2 \right] \right],
$$

$$
C_E^{(2)} = -t \left( s - M_d^2 \right) (s - M_d^2 - M_H^2 + t) \times \left[ M_H^4 + t^2 + 4m_d^2 \left( M_H^2 + 2t - 2m_d^2 \right) \right]
\times \left[ (M_H^2 - t)^2 \right]
\times \left[ -M_H^2 (M_H^2 + t^2) + 2m_d^2 \left( t^2 - 2M_H^2 M_d^2 + 4m_d^2 M_d^2 \right) \right],
$$

$$
C_M^{(1)} = C_E^{(1)} - 2M_H^2 (1 + \tau_d) (M_H^2 - t)^2 \left[ M_H^4 + t^2 + 4m_d^2 M_H^2 \right],
$$

$$
C_M^{(2)} = C_E^{(2)} + 2M_H^2 (1 + \tau_d) (M_H^2 - t)^2 \times \left[ M_H^4 + t^2 + 4m_d^2 \left( M_H^2 - t - 2m_d^2 \right) \right].
$$

The absolute Bethe-Heitler cross sections are shown in Fig. 3. The abscissa is the dilepton mass-squared $M_{ll}^2$, with both electrons and muons represented, and showing three different values of $-t$. The plot is similar to the one for protons [20], but the cross sections are smaller because of the faster falloff with $|t|$ of the deuteron FFs.

To estimate the Compton mechanism, the lower graph in Fig. 1, we estimate the $S$-matrix amplitude

$$
\mathcal{M}_C = -\frac{e^3}{q^2} \bar{\epsilon}_\nu (k, \lambda) \bar{\epsilon}_\lambda (l_-, s-) \gamma_\nu v(l_+, s_+)
\times \int d^4x \bar{\epsilon}^{\nu q} (\ell') \lambda' |T^{\mu} (x)|^2 (0) |p, \lambda \rangle,
$$

where $T^{\mu \nu}$ is the unpolarized timelike real Compton tensor and $\lambda$ is the photon polarization.

For near-real, near-forward kinematics, $M_{ll}^2, |t| \ll s$ the unpolarized TCS amplitude can be approximated by

$$
T_{\text{TCS}}^{\mu \nu}(k,q',P) \approx \left( g^{\mu \nu} - \frac{q'^\mu k^\nu}{q'^2} \right) T_{ld} (v,t,M_d^2),
$$

where $T_{ld}$ denotes the leading scalar amplitude, and $v$ the crossing symmetric variable, defined as

For $M_d^2, |t| \ll s$, we can further approximate

$$
T_{ld} (v,t,q'^2) \approx \frac{v}{4\pi} \sigma (v) \frac{\pi \alpha}{M_d^2} F_{ld} (v,0).
$$

Analyticity and the low-energy theorem value of $f(0)$ allow us to obtain the real part of $f$ from a once-subtracted dispersion relation,

$$
\text{Re} f (v) = -\frac{\alpha}{M_d^2} + \frac{v^2}{2\pi^2} \int v_0^\infty dv' \frac{\sigma (v')}{v'^2 - v^2}.
$$
where $v_0$ is the inelastic threshold, $v_0 = ( (M_{n} + M_{p})^2 - M_d^2 ) / (2M_d) \approx 2.23$ MeV.

The Compton contribution to the $\gamma d \rightarrow l^+l^-d$ differential cross section, integrated over lepton angles, is

$$
\frac{d\sigma^{\text{TCS}}}{dt dM^2_{ll}} = \frac{2M^2_{\gamma} \alpha \beta}{(s - M^2_{d})^2 M^2_{ll}} \left( 1 - \frac{\beta^2}{3} \right) \left| f(v) \right|^2 .
$$

(21)

Figure 4 shows the Compton cross section, compared to the Bethe-Heitler, for particular values of $t$ and $M^2_{ll}$, with $E_{\gamma}$ on the abscissa. We obtained $\sigma_v$ or $F_1(v,0)$ in the quasi-elastic region from the fits of [21], and in the nucleon inelastic region from Bosted-Christy [22] deuteron fits when $\delta_{\text{nucleon}} < (3.1 \text{ GeV}^2)$ and from Capella et al. [23], isospin modified for the neutron, above that. There is no interference between the Compton and Bethe-Heitler contributions when we integrate over the lepton angles. The Compton cross section is more than two orders of magnitude smaller than the Bethe-Heitler cross section, for this energy and momentum transfer range.

The sensitivity of the differential inelastic cross section for different FFs and different deuteron radii is shown in Fig. 5, as a function of $-t$, with a photon beam energy 0.65 GeV, corresponding with a minimum in the Compton contribution. The plot shows the cross section relative to a reference value $t_0 = -0.01 \text{ GeV}^2$, and normalized to results from the Abbott et al. parameterization. The outgoing deuteron angle has been fixed. The fixed angle can allow better experimental calibration than in an elastic scattering experiment, where different momentum transfers require different scattering angles, for a given beam energy. Measurements of such a fixed angle ratio at different $t$ with 0.1% relative accuracy would allow distinguishing the various fits.

In this work we have studied dilepton photoproduction off a deuteron with the aim of extracting the deuteron charge radius. By studying the momentum transfer dependence of the outgoing deuteron at a fixed angle, we have seen that a cross section ratio measurement of about 0.1% accuracy will allow extracting a deuteron charge radius more accurate than the present value from elastic scattering, and can distinguish between the values obtained from ordinary and muonic deuterium, which are currently at variance by around 3.5 $\sigma$.

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