Phenomenology of the Minimal Supergravity $SU(5)$ Model

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Abstract

The minimal grand unified supergravity model is discussed. Requiring radiative breaking of the electroweak gauge symmetry, the unification of $b$ and $\tau$ Yukawa couplings, a sufficiently stable nucleon, and not too large a relic density of neutralinos produced in the Big Bang constrains the parameter space significantly. In particular, the soft breaking parameter $m_{1/2}$ has to be less than about 130 GeV, and the top quark Yukawa coupling has to be near its (quasi) fixed point. The former condition implies $m_{\tilde{g}} \leq 400$ GeV and hence very large production rates for gluino pairs at the LHC, while the latter constraint implies that the lighter stop and sbottom eigenstates are significantly lighter than the other squarks, leading to characteristic signatures for gluino pair events.

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1) Introduction

Supersymmetry (SUSY) now seems to be the most popular extension of the Standard Model (SM). There are several reasons for this. First of all, SUSY solves the (technical) hierarchy problem (also known as finetuning or naturalness problem), i.e. stabilizes the weak scale against radiative corrections that otherwise tend to pull it up towards the GUT or Planck scale. This is also true for SUSY’s main competitor, technicolor (TC), although through a completely different mechanism. However, it seems increasingly difficult to find realisations of the TC idea that are not ruled out, or at least strongly disfavoured, by LEP measurements, in particular of the so-called $S$ parameter and of the $Z \rightarrow b\bar{b}$ partial width. In contrast, sparticles decouple quickly, i.e. do not affect predictions for LEP observables noticeably if sparticle masses exceed 100 GeV or so, so that LEP measurements can only rule out SUSY if they also exclude the SM (with a light Higgs boson) at the same time. Moreover, if sparticles are light, agreement between LEP measurements and predictions is often (slightly) improved compared to the non-supersymmetric SM.

Another strong motivation for SUSY is that the minimal supersymmetric standard model (MSSM) allows for a predictive grand unification of all gauge interactions, in contrast to the SM. Actually GUTs were in better agreement with data with than without SUSY already before LEP was turned on, but the higher precision achieved by LEP experiments has made this argument much more convincing.

All these arguments are independent of the way SUSY is broken, provided only that the breaking is “soft” and sparticle masses do not (greatly) exceed 1 TeV; if either of these conditions were violated, the naturalness problem would re-appear. Unfortunately no completely convincing dynamical mechanism of SUSY breaking has yet been suggested. For example, the one thing we know for sure about SUSY breaking in superstring theories is that it does not happen at any order in perturbation theory, i.e. SUSY breaking is an intrinsically nonperturbative problem and thus not easily treatable. At present we are therefore forced to use a phenomenological approach to SUSY breaking.

In many analyses of SUSY signals at colliders or elsewhere, it has been assumed that the sparticle spectrum has a high degree of degeneracy at the weak energy scale, $O(100)$ GeV, where present and near-future experiments operate. Specifically, it has often been assumed that all squarks (with the possible exception of stops) are exactly degenerate in mass with each other as well as with all sleptons. On the other hand, in these same analyses the parameters of the Higgs sector have usually been chosen “by hand”, independently of the sparticle spectrum. Both these assumptions are, in my view at least, rather unnatural. First of all, we know experimentally that the running gauge couplings meet (unify) at $M_X \approx 2 \cdot 10^{16}$ GeV. In other words, starting from a seemingly complicated situation (described by three “independent” gauge couplings) at low energies we are led to a
much simpler scenario with only a single gauge coupling at very high energies. On the other hand, if we similarly run the soft breaking parameters from the weak to the GUT scale, starting from a degenerate spectrum at the weak scale leads to a complicated, highly non-degenerate spectrum at the GUT scale. This violates what can be called the “unification dogma”, which stipulates that nature should become simpler, i.e. more symmetric, at higher energies. Secondly, the main beauty of SUSY is that it naturally includes (elementary) scalar particles, which seem to be required for the breaking of the electroweak symmetry in accordance with LEP data. Some of this beauty is lost if we treat matter (sfermion) and Higgs scalars differently.

It thus seems much more natural to me to assume a highly degenerate (s)particle spectrum at the GUT (or even Planck) scale, and to extend this degeneracy to include the Higgs bosons. This is called the minimal supergravity (mSUGRA) scenario \[12\], the idea being that local supersymmetry or supergravity is spontaneously broken in a “hidden sector”, and that this is communicated to the visible (gauge/Higgs/matter) sector only through flavour-blind gravitational-strength interactions. One very attractive feature of this scenario is that it (almost) automatically leads to the correct symmetry breaking pattern; that is, even though all scalars get the same nonsupersymmetric (positive) mass term at scale \(M_X\), at the weak scale the Higgs fields (and only the Higgs fields) acquire nonvanishing vacuum expectation values (vevs), provided only that the top quark is not much lighter than the \(W\) boson, which we now know to be true \[13\]. This “miracle” occurs since radiative corrections involving Yukawa interactions reduce the squared masses of the Higgs bosons, eventually driving a combination of these masses to negative values \[12, 14\], at which point the electroweak gauge symmetry is broken. Notice that this mechanism of radiative gauge symmetry breaking “explains” both the existence of the gauge hierarchy and the large mass of the top quark, in the sense that it only works if \(\log (M_X/M_Z) \gg 1\) and the top Yukawa \(h_t \sim O(1)\).

Of course, this approach to SUSY breaking is most naturally combined with grand unification of all gauge interactions. Here I only consider the simplest GUT group, \(SU(5)\). Moreover, only the minimal necessary number of fields will be assumed to exist at the GUT scale as well as at lower energies. As discussed in more detail in sec. 2, this leads to a fairly predictive (constrained) scenario \[15\], although some freedom in the choice of parameters, and of the resulting phenomenology, still exists. It should be emphasized that it is not at all trivial that this simplest of all SUSY GUTs is still experimentally viable.\[‡\]

The remainder of this contribution is organized as follows. Sec. 2 contains a description of the model as well as a discussion of the constraints that have been imposed. In sec. 3 the resulting (s)particle spectrum will be discussed in some detail; this section contains the only material that cannot be found in our publication \[16\]. In sec. 4 signals for sparticle production at the LHC are treated for a few characteristic spectra; the emphasis will be on methods to distinguish these spectra from each other as well as from the kind of spectrum that has been studied previously. Finally, sec. 5 is devoted to summary and conclusions.

\[‡\]For example, the simplest TC model has been ruled out many times.
2) The model

As explained in the Introduction, I will assume a simple form for the sparticle and Higgs spectrum at very high energy scales. In fact, one ultimately hopes to describe all of SUSY breaking by a single parameter, the equivalent of the electroweak symmetry breaking scale \((\sqrt{2}G_F)^{-1/2} = 246 \text{ GeV}\). At present we are still far from this ambitious goal, so it is prudent to allow at least a few free parameters to describe the spectrum. Here I will follow the standard assumptions \(^{[12]}\) and introduce four independent SUSY breaking parameters: \(m_0^2\), which contributes to the squared masses of all scalar bosons; a common gaugino mass \(m_{1/2}\); and common nonsupersymmetric trilinear and bilinear scalar interaction strengths \(A\) and \(B\), respectively. \(^{[1]}\) In addition one has to introduce a supersymmetric Higgs(ino) mass \(\mu\) in order to avoid the existence of a (visible) axion and to give masses to both up– and down–type quarks. Altogether the masses of the two \(SU(2)\) doublets of Higgs bosons needed in any realistic SUSY model are then

\[
m^2_H(M_X) = m^2_\bar{H}(M_X) = m_0^2 + \mu^2(M_X).
\] (1)

One can show quite easily that spontaneous \(SU(2) \times U(1)_Y\) breaking is not possible as long as \(m^2_H = m^2_\bar{H}\). Fortunately this degeneracy is lifted by radiative corrections. The reason is that \(\bar{H}\) only has Yukawa couplings to up–type quarks, while \(H\) only couples to down–type quarks and leptons. Of course, the \(t\) quark belongs to the former category, and has by far the largest Yukawa coupling of all SM fermions (unless \(\tan \beta \gg 1\); see below). This implies that radiative corrections from Yukawa interactions will be much larger for \(m^2_\bar{H}\) than for \(m^2_H\). The crucial point is that these corrections reduce the (running) squared mass when going to lower energy scales, eventually leading to nonzero vevs for \(H\) and \(\bar{H}\). As already emphasized in the Introduction, this mechanism will only work if \(\log M_X/M_Z \gg 1\) and the top Yukawa coupling \(h_t\) is large.\(^{†}\)

The running of the soft SUSY breaking parameters as well the of the gauge and Yukawa couplings is described by a set of renormalization group equations (RGE) \(^{[17]}\). As a consequence of the assumption of minimal particle content the model contains no intermediate scales between \(M_X\) and the weak scale. The RGE therefore allow to uniquely determine the values of parameters at the weak scale from the input parameters at \(M_X\). Of course, at the weak scale certain equalities have to be satisfied. First of all, we know that

\[
\frac{g^2 + g'^2}{2} \left( \langle H^0 \rangle^2 + \langle \bar{H}^0 \rangle^2 \right) = M_Z^2.
\] (2)

It is often convenient to introduce the weak scale parameter \(\tan \beta \equiv \langle \bar{H}^0 \rangle / \langle H^0 \rangle\). With the

\(^{*}\)From supergravity or superstring theories one might expect such a spectrum to emerge from an effective theory at scales below the Planck scale \((M_P = 2.4 \cdot 10^{18} \text{ GeV})\) or perhaps the string compactification scale \((M_c \simeq 5 \cdot 10^{17} \text{ GeV})\). Here I will assume that this ansatz is still valid at the GUT scale \(M_X \simeq 2 \cdot 10^{16} \text{ GeV}\); in the scenario presented here the running of parameters from \(M_P\) or \(M_c\) down to \(M_X\) is not expected to play a major role.

\(^{†}\)Strictly speaking, any nonzero \(h_t\) will give a finite region of input parameter space where radiative gauge symmetry breaking can be achieved. However, this region would have been small, i.e. some fine–tuning would have been needed, if \(h_t\) were much smaller than the gauge couplings. Radiative symmetry breaking is therefore more natural for heavy top.
help of this parameter, eq.(2) can be solved for $\mu^2$ at the weak scale $Q_0$:

$$
\mu^2(Q_0) = \frac{m_1^2(Q_0) - m_2^2(Q_0)}{\tan^2 \beta - 1} - \frac{1}{2} M_Z^2,
$$

(3)

where $m_1^2, m_2^2$ are the nonsupersymmetric contributions to the squared Higgs masses, i.e. $m_H^2 = m_1^2 + \mu^2$, $m_{\tilde{H}}^2 = m_2^2 + \mu^2$. For heavy top, $m_2^2(Q_0)$ is negative, giving a positive (and usually quite large) contribution to $\mu^2$. A similar equation determines $B \cdot \mu$ at scale $Q_0$ in terms of $m_1^2, m_2^2$ and $\tan \beta$.

Having fixed $M_Z$, we are left with the parameters $m_0, m_{1/2}$ and $A$ (at the GUT scale) as well as $\tan \beta$ (at the weak scale). In addition the mass $m_t$ of the top quark is an important parameter, since the top Yukawa coupling plays a vital role in radiative gauge symmetry breaking. At this point the assumption of a minimal $SU(5)$ GUT helps to further reduce the number of free parameters. The reason is that minimal $SU(5)$ implies the equality of $b$ and $\tau$ Yukawa couplings at scale $M_X$. As has been shown by several groups [20], this can only be brought into agreement with the experimentally measured ratio $m_b/m_\tau$ if either $h_t$ is close to its upper bound or if $h_b \simeq h_t$, which implies $\tan \beta \simeq m_t/m_b$. The second choice not only necessitates some finetuning in this minimal scenario [21], it also makes it more difficult to satisfy proton decay constraints (see below). I therefore only consider the first solution here. Within the precision of a 1–loop calculation it can simply be implemented by taking

$$
h_t(M_X) = 2.
$$

(4)

This ensures that at low energies $h_t$ is very close to its infrared quasi fixed point [20], which implies

$$
m_t(m_t) \simeq 190 \text{ GeV} \cdot \sin \beta,
$$

(5)

where $m_t(m_t)$ is the running (MS) top mass at scale $m_t$; the on–shell (physical) top mass is about 5% larger. Eq.(3) reduces the number of free parameters from five to four in this scenario. Assuming $m_t(\text{pole}) \leq 185$ GeV, as indicated by LEP data [22] if the Higgs is light, then implies

$$
\tan \beta \leq 2.5.
$$

(6)

In addition to the relations discussed so far a number of conditions has to be satisfied. These can all be expressed in terms of inequalities, i.e. they define allowed ranges of parameter values rather than determining them uniquely. One important constraint emerges from the requirement that nucleons should be sufficiently long–lived. In minimal SUSY $SU(5)$ the main contribution to nucleon decay comes from the exchange of the fermionic superpartners of the $SU(5)$ partners of the ew. Higgs bosons, i.e. from higgsino triplet exchange [23]. The reason is that the corresponding diagrams are only suppressed by one power of a mass $O(M_X)$, as compared to two such powers for $SU(5)$ gauge boson exchange.

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1 Eq.(3) is valid if the tree–level Higgs potential is used with running parameters to include radiative corrections $\propto \log(M_X/Q_0)$. However, it is always possible to chose the scale $Q_0$ where the RG running is terminated such that corrections to eq.(3) are small even when the full 1–loop Higgs potential is used [18, 19], which is almost independent of $Q_0$. The same choice of $Q_0$ also minimizes corrections to the mass of the pseudoscalar Higgs boson.

2 This conclusion remains valid if $h_b/h_\tau$ is allowed to vary by $\sim 10\%$ from unity due to threshold effects or “Planck slop”.

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4
However, while higgsino exchange suffices to violate both baryon and lepton number, it leads to two sfermions, rather than two fermions, in the final state. These sfermions have to be transformed into a lepton and an anti–quark by gaugino (mostly chargino) exchange, i.e. the decay only occurs at 1–loop level. The matrix element therefore contains a so–called dressing loop function, which scales like $m_{1/2}/m_0^2$ for $m_0 \geq m_{1/2}$. Altogether one thus has \[ \mathcal{M}(p \to K\nu) \propto \frac{\tan \beta m_{1/2}}{m_{H_3}}, \] (7) where $m_{H_3}$ is the mass of the Higgsino triplet, and the factor $\tan \beta$ appears since Yukawa couplings to $d$ and $s$ quarks grow $\propto \tan \beta$. The experimental lower bound on the proton lifetime $\tau_p$ gives an upper bound on the matrix element (7); assuming conservatively that $m_{H_3}$ could be as much as ten times larger than the scale $M_X$ where the gauge couplings meet, and taking into account that we are only interested in rather small values of $\tan \beta$, eq.(8), a conservative interpretation of the constraint imposed by the bound on $\tau_p$ is \[ m_0 \geq \min(300 \text{ GeV}, 3m_{1/2}). \] (8)

Another important constraint can be derived from the requirement that relic LSPs produced in the Big Bang do not overclose the universe, in which case it would never have reached its present age of at least $10^{10}$ years. In minimal SUGRA the lightest supersymmetric particle (LSP) is always the lightest of the four neutralino states. Moreover, the large value of $h_t$, eq.(4), and small $\tan \beta$ (3) imply that $|\mu(Q_0)|$ is quite large, see eq.(3). The LSP is therefore always a gaugino (mostly bino) with small higgsino component. The LSP relic density is essentially inversely proportional to its annihilation cross section, summed over all accessible channels. Gaugino–like LSPs mostly annihilate into $f\bar{f}$ final states \[24\], where $f$ stands for any SM fermion with mass below that of the LSP. This final state is accessible via $\tilde{f}$ exchange in the $t$ or $u$ channel, as well as via the exchange of the $Z$ boson or one of the neutral Higgs bosons in the $s$ channel. However, the constraint (8) implies that the $\tilde{f}$ exchange contribution is strongly suppressed, due to the large sfermion masses. (Recall that all sfermions get a contribution $+m_0^2$ to their squared masses at scale $M_X$.) Moreover, since both $m_0$ and $|\mu|$ are large, most Higgs bosons are very heavy. Finally, the LSP–LSP–$Z$ coupling needs two factors of the small higgsino component of the LSP, while the LSP–LSP–Higgs couplings only need one such factor. Therefore the only potentially large contribution to LSP annihilation comes from the exchange of the light neutral Higgs boson $h^0$. Fortunately it has recently been shown \[23\] that this contribution suffices to reduce the LSP relic density to acceptable values for a substantial range of LSP masses, provided it is below $m_h/2$. The upper bound (9) on $\tan \beta$ implies that $m_h \leq 110 \text{ GeV}$ even after radiative corrections \[26, 19\] are included, while Higgs searches at LEP imply [27] $m_h \geq 63 \text{ GeV}$. (The couplings of $h^0$ are very similar to that of the SM Higgs boson in the given scenario.) Since the mass of a bino–like neutralino is about $0.4 m_{1/2}$, this implies \[ 60 \text{ GeV} \leq m_{1/2} \leq 130 \text{ GeV}. \] (9)

\*Strictly speaking the Higgs sector of minimal $SU(5)$ is not realistic, since it leads to wrong predictions for the masses of SM fermions of the first two generations. Adding new Higgs fields and/or non–renormalizable interactions to solve this problem can in general also change predictions for $\tau_p$. However, refs.\[23\] already use the physical quark masses as input. Furthermore, the constraint \[8\] is really rather lenient; it can therefore be expected to hold even in slightly extended models.
It should be emphasized that such a strong upper bound on $m_{1/2}$ only holds in this specific scenario. If we give up on the unification of $b$ and $\tau$ Yukawa couplings, we can allow smaller $h_t$ and/or larger $\tan\beta$, leading to smaller values of $|\mu(Q_0)|$, and hence a larger higgsino component of the LSP and stronger annihilation into final states containing Higgs and/or gauge bosons. If we allow for a sufficiently non-minimal GUT Higgs sector the bound on the proton lifetime can be satisfied even if the constraint (8) is violated, allowing for efficient LSP annihilation through $\tilde{f}$ exchange. Finally, if we allow new, $R-$parity violating interactions, relic LSPs could have decayed a long time ago, and no constraint on LSP annihilation could be given.

Eqs.(4), (6), (8) and (9) describe the basic allowed parameter space of the model, except for the $A$ parameter. The bounds on this parameter are intimately linked to to the details of the (s)particle spectrum, which is the topic of the next section.

3) The spectrum

In this section I discuss the sparticle and Higgs spectrum of the model defined in the previous section, with emphasis on features that are relevant for “new physics” searches at colliders; see also refs.[28, 29] for recent discussions of the spectrum in mSUGRA models with top Yukawa coupling close to its fixed point.

A general feature of all mSUGRA models is that the Higgs(ino) mass parameter $\mu$ is not an independent variable, but can be computed from the SUSY breaking parameters, $m_t$ and $\tan\beta$ (as well as $M_Z$, of course), as described by eq.(3). In general, the r.h.s. of that equation is a complicated function of all input parameters, which has to be computed by solving the relevant RGE [17] numerically. However, the following analytical expression are often sufficient for practical purposes:

$$m^2(Q_0) \simeq m^2 + 0.5m^2_{1/2}; \quad (10a)$$

$$m^2_2(Q_0) \simeq m^2_1(Q_0) - \frac{X_2}{\sin^2\beta}, \quad \text{where}$$

$$X_2 = \left(\frac{m_t}{150 \text{ GeV}}\right)^2 \left\{ 0.9m^2_0 + 2.1m^2_{1/2} \right.$$  
$$+ \left[ 1 - \left(\frac{m_t}{190 \text{ GeV} \sin\beta}\right)^3 \right] \left(0.24A^2 + A \cdot m_{1/2}\right) \right\} \quad (10c)$$

Eqs.(10) reproduce the exact numerical results to 10% or better if $Q_0$ is around 350 GeV (which corresponds [13] to squark masses around 600 GeV), and if $h_t \ll 1$, which is always true here due to the upper bound (3) on $\tan\beta$. Note that $m_t$ in eqs.(10) is the running top mass $m_t(m_t)$. The expression in square brackets in eq.(10) will therefore always be small if $h_t$ is close to its fixed point, see eq.(3). The resulting very weak dependence of $X_2$ on $A$ has also been observed in ref.[28].

*Eqs.(10) are valid also away from the IR fixed point of $h_t$. They differ from the analytical expressions given in ref.[28] partly because in that paper $Q_0 = M_Z$ has been assumed, which is not appropriate [13, 14] for the heavy scalar spectrum implied by the condition (8). (All numerical results in ref.[28] use the 1-loop...
Note that eqs. (10) imply that $m_2^2(Q_0) < 0$ if $m_t/\sin\beta > 158$ GeV; this is certainly true in the given scenario. Specifically, if $h_t$ is close to the fixed point one has

$$
\mu^2(Q_0) > m_0^2 \frac{1 + 0.44\tan^2\beta}{\tan^2\beta - 1} > 0.72m_0^2,
$$

(11)

where I have used (3) in the second inequality; the lower bound (8) on $m_0$ then implies that the LSP is indeed always a very pure gaugino, as stated in the previous section. It also means that the two heavier neutralinos and the heavier chargino, whose masses are all very close to $|\mu(Q_0)|$, will be very difficult to detect: Their production cross section at the LHC is much too small to yield a viable signal, while they are too heavy to be produced at all at a linear $e^+e^-$ collider with $\sqrt{s} \simeq 500$ GeV, which is likely to be the next high–energy $e^+e^-$ collider.

Another useful identity is [19]

$$
m_P^2 = m_{\tilde{\nu}}^2 + \mu^2(Q_0) \sin^2\beta,
$$

(12)

where

$$
m_{\tilde{\nu}}^2 \simeq m_0^2 + 0.5m_{1/2}^2 + 0.5M_Z^2 \cos 2\beta
$$

(13)

is the squared sneutrino mass and $m_P$ is the mass of the pseudoscalar Higgs boson, which is almost degenerate with the charged and heavy neutral scalar Higgs bosons if $M_Z^2 \gg M_2^2$ [31]. Note that eq. (12) is exact, unlike eqs. (10), up to corrections of order $h_b^2$. In our fixed point scenario eq. (12) and the bounds (8) and (11) imply that $m_P$ can easily exceed 1 TeV even for quite modest values of $m_0$; such heavy SUSY Higgses are very difficult to detect (or even produce) experimentally. On the other hand, this also implies that the couplings of the light neutral scalar Higgs boson $h^0$ to SM fermions and gauge bosons are practically identical to those of the SM Higgs [30], so that $h^0$ will be produced copiously at the LHC, at future $e^+e^-$ linacs, and perhaps even at the second stage of LEP.†

Since we have $m_0^2 \gg m_{1/2}^2$ in our model, all sleptons have very similar masses; eqs. (13) and (8) then imply that they are most likely too heavy to be detectable at the LHC, while they cannot be produced at all at a 500 GeV $e^+e^-$ collider.

Clearly the best chance for a decisive test of the model is given by the upper bound (11) on $m_{1/2}$. Since $|\mu(Q_0)|$ is so large the masses of the lighter chargino and the next–to–lightest neutralino lie within a few GeV of the low–energy $SU(2)$ gaugino mass $M_2 \simeq 0.82m_{1/2}$. In particular, the lighter chargino always lies below 105 GeV [15], offering a good chance for its discovery at LEP, especially if the machine energy can be boosted beyond the currently foreseen value of 176 GeV.

The situation is slightly more complicated for the gluino. The bound (11) implies a rather low upper bound for the running (\overline{MS} or \overline{DR}) gluino mass, $m_{\tilde{g}}(m_{\tilde{g}}) \simeq 2.5m_{1/2} \leq 330$ GeV, corrected potential, and hence do not depend significantly on the choice of $Q_0$.) Since most parameters run much more quickly just above $Q_0$ than just below $M_X$ changing $Q_0$ by a factor 4 is not completely irrelevant. Notice also that ref. [28] uses the opposite sign convention for $A$.

†While by construction the invisible decay of $h^0$ into two LSPs is always allowed in this model, the corresponding branching ratio is very small, being proportional to the square of the small higgsino component of the LSP.
GeV. However, it has recently been pointed out [32] that the on–shell gluino mass can be substantially larger than this, especially if squarks are heavy:

\[
m_{\tilde{g}}(\text{pole}) \simeq m_{\tilde{g}} \left[ 1 + \frac{\alpha_s(m_{\tilde{g}})}{\pi} \left( 3 + \frac{1}{4} \sum_q \log \frac{m_{\tilde{g}}}{m_{\tilde{g}}} \right) \right],
\]

(14)

where I have only included the leading logarithmic correction from squark–quark loops. Requiring somewhat arbitrarily

\[
m_0 \leq 1 \text{ TeV}
\]

(15)
in order to avoid excessive finetuning in eq.(3), we see that the “threshold correction” (14) can change the gluino mass by about 30% if \( m_{1/2} \) is close to its lower bound and \( m_0 \) is at its upper bound. This can change both the cross section and the signal for gluino pair production quite significantly. Including the correction (14), the bound (8) implies \( m_{\tilde{g}}(\text{pole}) \leq 400 \) GeV if (13) is imposed.

The lower bound (8) on \( m_0 \) also implies that the squarks of the first two generations are significantly heavier than the gluinos; they will thus dominantly decay into \( \tilde{g} + q \), although the left–handed (\( SU(2) \) doublet) squarks also have \( \mathcal{O}(10\%) \) branching ratios into an elw. gaugino and a quark. The masses of the first two generations of squarks lie between about 330 GeV and a little above 1 TeV in this model, where the upper bound merely reflects the somewhat arbitrary constraint (13).

So far the spectrum resembles a special case of the type of models whose signatures were discussed in the existing literature [1, 11], with heavy squarks and sleptons, large |\( \mu \)| and \( m_P \), but rather light gluino and elw. gauginos. This is not surprising, since in these earlier studies \( \mu \) and \( m_P \) were considered to be free parameters; they could thus be chosen to be large, whereas the present model requires them to be large. However, as already remarked in the Introduction, in these papers all squarks were also assumed to have the same mass at the weak scale. It is here that the present model makes specific predictions which cannot be mimicked by these earlier treatments, in spite of the larger number of free parameters.

The reason is that the same kind of radiative corrections that reduce the Higgs mass parameter \( m_2 \) to negative values also reduce the masses of the stop and left–handed sbottom squarks, as compared to the masses of first generation squarks. Specifically, one finds

\[
m_{\tilde{b}_L}^2 = m_{\tilde{t}_L}^2 \simeq m_q^2 - \frac{X_2}{3 \sin^2 \beta};
\]

(16a)

\[
m_{\tilde{t}_R}^2 \simeq m_q^2 - \frac{2X_2}{3 \sin^2 \beta},
\]

(16b)

where

\[
m_q^2 \simeq m_0^2 + 6m_{1/2}^2
\]

(17)
is a typical first generation squark mass (at scale \( Q = m_{\tilde{g}} \)), and \( X_2 \) has been given in eq.(10c).

Since for the small values of \( \tan \beta \) of interest here \( \tilde{b}_L - \tilde{t}_R \) mixing is almost negligible, \( \tilde{b}_L \) is to good approximation the mass eigenstate \( \tilde{b}_1 \), with mass given by eq.(16a). The ratio \( m_{\tilde{b}_1}/m_{\tilde{u}_L} \) (including small mixing effects and \( D–\)terms) is shown in fig. 1 as a function of
\( A_0 \equiv A/m_0 \), for various values of \( m_0 \) and \( m_{1/2} \) and \( \mu < 0 \). We observe that the dependence on \( A \) is indeed quite weak here, as claimed earlier. The allowed range for \( A_0 \) is determined by various constraints: The squared mass of the lightest stop eigenstate (see below) must be larger than \( +(45 \text{ GeV})^2 \); the scalar potential at the weak scale should not have minima \(^{[34]} \) in the directions \( \langle \tilde{\tau}_R \rangle = \langle \tilde{\tau}_L \rangle = \langle H^0 \rangle \) or \( \langle \tilde{t}_R \rangle = \langle \tilde{t}_L \rangle = \langle H^0 \rangle \) that are deeper than the desired minimum where only \( \langle H^0 \rangle \) and \( \langle \tilde{H}^0 \rangle \) are nonzero; and the potential must be bounded from below at the GUT scale, which requires \( m_{\tilde{b}_1}^2 + \mu^2 (M_X) \geq 2 |B(M_X) \mu (M_X)| \).

The ratio \( m_{\tilde{b}_1}/m_{\tilde{u}_L} \) does depend somewhat on the ratio \( m_{1/2}/m_0 \), however, being maximal where \( m_{1/2}/m_0 \) is minimal (and vice versa). We see that in our fixed–point scenario \( m_{\tilde{b}_1} \) is reduced by typically 20 to 30\% compared to first generation squark masses. This can be quite significant, since partial widths for three–body decays of gluinos or elw. gauginos that involve squark exchange scale approximately like the inverse fourth power of the squark mass. A reduction of the squark mass by 20\% (30\%) therefore leads to an increase of the corresponding partial width or branching ratio by a factor of 2.1 (2.9). This leads to \( b \)–rich final states, as will be discussed in more detail in sec. 4.

Eq.\(^{(16b)} \) implies that the mass of the \( \tilde{t}_R \) current state is reduced even more than \( m_{\tilde{b}_L} \).

Moreover, \( \tilde{t}_L - \tilde{t}_R \) mixing is not negligible; it reduces the mass \( m_{\tilde{t}_i} \), of the lighter \( \tilde{t} \) mass eigenstate even further. In the convention of ref.\(^{[33]} \) the \( \tilde{t} \) mass matrix is given by \(^{[33]} \):

\[
M_{\tilde{t}}^2 = \begin{pmatrix}
  m_{\tilde{t}_L}^2 + m_{\tilde{t}}^2 + 0.35 M_Z^2 \cos 2\beta & -m_{\tilde{t}} (A_t + \mu \cot \beta) \\
- m_{\tilde{t}} (A_t + \mu \cot \beta) & m_{\tilde{t}_R}^2 + m_{\tilde{t}}^2 + 0.16 M_Z^2 \cos 2\beta
\end{pmatrix}.
\]

At scale \( M_X \), \( A_t = A \), but it is subject to radiative corrections due to both gauge and Yukawa interactions. For small \( \tan \beta \), one has approximately:

\[
A_t(Q_0) \simeq A \left[ 1 - \left( \frac{m_t}{190 \text{ GeV} \sin \beta} \right)^2 \right] + m_{1/2} \left[ 3.5 - 1.9 \left( \frac{m_t}{190 \text{ GeV} \sin \beta} \right)^2 \right],
\]

where I have again assumed \( Q_0 \simeq 350 \text{ GeV} \). Right at the fixed point of \( h_t \), where eq.\(^{[3]} \) becomes exact, the weak–scale value of \( A_t \) is again independent of \( A \) \(^{[28]} \). However, even for values of \( h_t(M_X) \) as large as 2, \( m_{\tilde{t}_i} \) can still show substantial \( A \)–dependence. The reason is that in the expression for \( m_{\tilde{t}_i} \) strong cancellations occur between the contributions from diagonal and off–diagonal entries of the stop mass matrix; relatively small changes of these elements, of the size shown in fig. 1, can therefore have sizable effects on \( m_{\tilde{t}_i} \). This is especially true for \( \mu > 0 \), since there all effects go in the same direction: Increasing \( A \) increases \( X_2 \), which simultaneously reduces \( m_{\tilde{t}_L}^2 \) and \( m_{\tilde{t}_R}^2 \) in the diagonal entries, eq.\(^{(10)} \), and increases the off–diagonal elements by increasing \( \mu \), see eqs.\(^{(8)} \) and \(^{(11)} \); note that \( A_t \) and \( \mu \) have the same sign in this case.

\( m_{\tilde{t}_i} \) also depends quite strongly on \( m_t \) in the fixed–point scenario. The reason is that a larger \( m_t \) implies larger \( \tan \beta \), see eq.\(^{(2)} \), and hence smaller \( |\mu| \), eq.\(^{(3)} \); in addition, the off–diagonal term in the stop mass matrix \(^{(18)} \) is reduced, due to the explicit \( \cot \beta \) factor. Therefore larger \( m_t \) also imply larger \( m_{\tilde{t}_i} \) here. (This is not generally true if \( h_t \) is significantly below its fixed point \(^{[33]} \).)

\(^{4}\)Note that eq.\(^{(3)} \) only determines the absolute value of \( \mu \). In this paper I assume \( m_{1/2} > 0 \), \( \tan \beta > 0 \) without loss of generality; however, \( A, B \) and \( \mu \) must then in general all be allowed to have either sign. The sign conventions for \( \mu \) and \( m_{1/2} \) used here are the same as in refs.\(^{[32]} \), \(^{[33]} \).
In all of parameter space allowed in our model one finds that \( m_{\tilde{t}_1} \) is close to or even well below the gluino mass. This is partly due to the correction \( \frac{m_{\tilde{g}}}{m_{\tilde{t}_1}} \) (fig. 2a). Not surprisingly, this statement remains true \( m_{\tilde{g}} > m_{\tilde{t}_1} + m_{\tilde{t}_1} \) and also whether the decay \( \tilde{t}_1 \to b + \tilde{W}_1 \) is allowed, where \( \tilde{W}_1 \) is the lighter chargino; if (and only if) this decay is forbidden, \( \tilde{t}_1 \to c + \tilde{Z}_1 \) via loop diagrams \( \frac{m_{\tilde{g}}}{m_{\tilde{t}_1}} \).

The ordering of \( m_{\tilde{g}}, m_{\tilde{t}_1}, m_t \) and \( m_{\tilde{W}_1} \) depends quite strongly on \( A_t, m_{1/2} \) and \( m_t \), as well as on the sign of \( \mu \). This is demonstrated by figs. 2 a–d, which show regions in the \( (A_0, m_{1/2}) \) plane for \( m_0 = 500 \) GeV; results for other values of \( m_0 \) allowed by \( \delta \) are quite similar. Figs. 2a,b are for \( m_{\tilde{t}}(m_t) = 160 \) GeV, corresponding to \( m_{\tilde{t}}(pole) = 168 \) GeV, while c,d are for \( m_{\tilde{t}}(m_t) = 175 \) GeV or \( m_{\tilde{t}}(pole) = 183 \) GeV; \( \mu \) has been chosen positive in a,c and negative in b,d. In all these figures the regions outside the dotted lines are excluded by the constraints described in the discussion of fig. 1. Along the solid line one has \( m_{\tilde{g}} = m_{\tilde{t}_1} + m_t \), i.e. on one side of this line \( \tilde{g} \to \tilde{t}_1 + t \) decays are allowed; since all other squarks are considerably heavier, this is the only possible two–body decay mode of the gluino and will hence have a branching ratio of nearly 100% if allowed at all. Similarly, the long dashed lines separate regions where \( \tilde{t}_1 \to \tilde{W}_1 + b \) is open, in which case it has a branching ratio very close to 100%, from those where \( \tilde{t}_1 \to \tilde{Z}_1 + c \). The decay \( \tilde{t}_1 \to t + \tilde{Z}_1 \) is almost never allowed here (it opens up if both \( m_{1/2} \) and \( m_0 \) are near their upper bounds); instead, the short dashed lines are contours of constant \( m_{\tilde{t}_1} = m_t \). In all cases \( m_{\tilde{t}_1} \) increases with increasing \( m_{1/2} \), and it generally also increases with decreasing \( |A_0| \), although the maximum (for given \( m_{1/2} \)) is not exactly at \( A_0 = 0 \).

In fig. 2a, \( \tilde{g} \to \tilde{t}_1 + t \) decays are allowed everywhere except within the small triangular region delineated by the solid line. There are also substantial regions where \( \tilde{t}_1 \to \tilde{W}_1 + b \) is not possible, and \( m_{\tilde{t}_1} < m\tilde{t}_1 \) almost everywhere except for the small region enclosed by the short dashed curve. Changing the sign of \( \mu \) (fig. 2b) or increasing \( m_{\tilde{t}}(m_t) \) to 175 GeV (fig. 2c) changes the situation quite drastically, however. The region where \( \tilde{g} \to \tilde{t}_1 + t \) is allowed is now confined to the narrow strips between the solid and dotted lines, and \( m_{\tilde{t}_1} < m_{\tilde{W}_1} + m_b \) only in the even narrower strips between the long dashed and dotted lines. In contrast, the region where \( m_{\tilde{t}_1} > m_t \) (above the short dashed lines) is now quite large. Finally, if \( m_t \) is close to its upper bound and \( \mu < 0 \), fig. 2d, the gluino can never decay into \( \tilde{t}_1 + t \), and \( \tilde{t}_1 \) always decays into \( \tilde{W}_1 + b \); moreover, now \( m_{\tilde{t}_1} > m\tilde{t}_1 \) over most of the allowed part of the \( (A_0, m_{1/2}) \) plane.

Since \( \tilde{t}_1 \) and the gluino are by far the lightest strongly interacting sparticles in our model, SUSY signals at the LHC (or other hadron colliders) will obviously depend quite sensitively on which of the regions depicted in figs. 2 is picked. This is the subject of the next section, where these signals are discussed in more detail.

\footnote{The difference between running and on–shell masses is much smaller \( |\mathcal{R}| \) for squarks than for gluinos, since squarks are color triplets (rather than octets) and no sum over flavours occurs, unlike in eq.(14).}
4) Signals at the LHC

In this section I discuss the specific signatures produced by the kind of spectrum described in the previous section. The signatures at $e^+e^-$ colliders are rather straightforward: A light chargino (below 105 GeV), a rather light neutral scalar Higgs boson (below 110 GeV), and a light $\tilde{t}_1$ (below $\sim 300$ GeV), all of which can be detected in a straightforward way.\footnote{The only possible problem could occur if $\tilde{t}_1$ is very close in mass to the LSP, which is in principle possible.} Due to numerous backgrounds the situation at hadron colliders is much more complicated. Moreover, at these machines the largest signals come from the production of strongly interacting sparticles. Since these are usually heavier than many other sparticles they tend to decay via lengthy cascades\footnote{The model also predicts the existence of a light $\tilde{Z}_2$, with mass very close to $m_{\tilde{W}_1}$, but its production cross section at $e^+e^-$ colliders is small, since it is a rather pure gaugino, so that $Z$ exchange is suppressed by small couplings, while selectron exchange is suppressed by the large selectron mass.}. While this makes signals more difficult to analyze, it also offers the opportunity to determine various branching ratios, which greatly helps to distinguish between different SUSY scenarios.

| parameter      | A1  | A2  | B1  | B2  | B3  | B4  |
|----------------|-----|-----|-----|-----|-----|-----|
| $m_0$          | 500 | 500 | 300 | 1000| 400 | 1000|
| $m_{1/2}$      | 120 | 130 | 60  | 70  | 130 | 130 |
| $A_0/m_0$      | 0.6 | 3.75| 0.1 | 0.0 | 0.0 | 0.0 |
| $\tan\beta$   | 1.49| 2.2 | 1.94| 1.32| 2.22| 2.11|
| $\mu$          | 697.1| 580 | -313.3| -1571.7| 430 | 964.7|
| $m_t$          | 160 | 175 | 170 | 155 | 175 | 175 |
| $m_{\tilde{g}}$| 346 | 371 | 185 | 231 | 364 | 400 |
| $m_{\tilde{q}}$| 568 | 578 | 328 | 1011| 496 | 1039|
| $m_{\tilde{t}_1}$| 131 | 83.6| 198 | 121 | 225 | 288 |
| $m_{\tilde{t}_2}$| 521 | 500 | 304 | 781 | 470 | 799 |
| $m_{\tilde{b}_L}$| 437 | 426 | 250 | 732 | 396 | 765 |
| $m_{\tilde{\ell}}$| 508 | 501 | 305 | 1001| 412 | 1005|
| $m_{\tilde{Z}_1}$| 43.6| 48.0| 27.1| 28.8| 46.1| 50.7 |
| $m_{\tilde{Z}_2}$| 85.1| 93.5| 63.5| 59.8| 89.3| 99.9 |
| $m_{\tilde{Z}_3}$| 698 | 582 | 316 | 1572| 432 | 966 |
| $m_{\tilde{Z}_4}$| 710 | 595 | 333 | 1577| 450 | 973 |
| $m_{\tilde{W}_1}$| 85.2| 99.7| 61.6| 86.0| 91.3| 101 |
| $m_{\tilde{W}_2}$| 707 | 593 | 331 | 1576| 448 | 972 |
| $m_{H^0}$      | 1038| 841 | 492 | 2339| 652 | 1543|
| $m_{H^\pm}$    | 1038| 845 | 492 | 2343| 653 | 1544|
| $m_{P}$        | 1038| 845 | 492 | 2343| 653 | 1544|

Here I summarize the results of ref.\cite{ref16}, where a full Monte Carlo study of signals and backgrounds was performed, using the latest version of ISAJET which contains the ISASUSY

[^1]: Table 1: Parameters and masses for six SUGRA cases A1, A2 and B1–B4.

[^2]: Table 1: Parameters and masses for six SUGRA cases A1, A2 and B1–B4.
program package to compute sparticle masses and decay branching ratios. The program includes initial and final state showering, fragmentation, and crude detector modelling, where we took present designs of LHC detectors as guidelines; see ref.\textsuperscript{38} for more details on this program package.

A simulation of this type consumes a substantial amount of CPU time\textsuperscript{1}. We therefore limited ourselves to the study of six spectra that can occur within SUGRA SU(5), see table 1. We saw already in the previous section that over a substantial region of parameter space the gluino can decay into $t + \tilde{t}_1$. The two ‘A’ spectra were picked from that region, with A1 being an example where $\tilde{t}_1 \rightarrow \tilde{W}_1 + b$ while in case A2 the light stop decays into charm plus LSP. In the remaining four ‘B’ cases the gluino has no two–body decay modes; here we picked points where $m_{\tilde{g}}$ and $m_{1/2}$ are minimal or maximal: $(m_0, m_{1/2}) = (0.3 \text{ TeV, } 60 \text{ GeV})$ (B1); $(1 \text{ TeV, } 70 \text{ GeV})$ (B2); $(0.4 \text{ TeV, } 130 \text{ GeV})$ (B3); and $(1 \text{ TeV, } 130 \text{ GeV})$ (B4).

These six spectra show all the features discussed earlier: Light $\tilde{Z}_1$, $\tilde{Z}_2$, $\tilde{W}_1$, $h^0$ and $\tilde{g}$; quite heavy squarks and sleptons, except for $\tilde{t}_1$, with $\tilde{t}_1$ significantly below first generation squarks; heavy $\tilde{Z}_3$, $\tilde{Z}_4$ and $\tilde{W}_2$; and very heavy Higgs bosons $P$, $H^0$ and $H^\pm$.

In order to see how mSUGRA signals differ from the signals of “conventional SUSY” models of the type considered in ref.\textsuperscript{11}, we also studied four cases (labelled BTW1 through BTW4) where all squarks and sleptons are assumed to be degenerate. In all ‘BTW’ cases we took $m_{\tilde{g}} = 300 \text{ GeV}$, and picked $(m_{\tilde{q}}, \mu) = (320 \text{ GeV, } –150 \text{ GeV})$ (BTW1); $(600 \text{ GeV, } –150 \text{ GeV})$ (BTW2); $(320 \text{ GeV, } –500 \text{ GeV})$ (BTW3); and $(600 \text{ GeV, } –500 \text{ GeV})$ (BTW4)\textsuperscript{2}.

In table 2 SUSY event fractions as well as important sparticle decay branching ratios are listed. In the ‘A’ cases with very light $\tilde{t}_1$, more than half of all SUSY events are $\tilde{t}_1\tilde{t}_1^*$ pairs, while in the four ‘B’ cases, gluino pairs constitute the most copiously produced supersymmetric final state. Pairs of electroweak gauginos are produced only in a few percent or even few permille of all SUSY events; their detection therefore necessitates a dedicated search\textsuperscript{39}, as opposed to the “generic SUSY search” presented here.

Many of the branching ratios shown in table 2 can be understood directly from the sparticle masses listed in table 1, keeping in mind that two–body final states will always overwhelm three–body final states if both are accessible at tree level. A few features are worth pointing out, however. For example, in cases B1, B2 the $\tilde{b}_1\tilde{b}_L$ coupling is “accidentally” suppressed. The branching ratio for $\tilde{g} \rightarrow \tilde{Z}_1\tilde{b}_1\tilde{b}_L$ is therefore dominated by $\tilde{b}_R$ exchange, whose mass is not reduced compared to first generation squark masses; therefore $\tilde{g} \rightarrow \tilde{Z}_1\tilde{b}_1\tilde{b}_L$ is not enhanced significantly over $\tilde{g} \rightarrow \tilde{Z}_1\tilde{d}_1\tilde{d}_1$ in these cases. Nevertheless $\tilde{Z}_2 \rightarrow \tilde{Z}_1\tilde{b}_1\tilde{b}_1$ is enhanced considerably over $\tilde{Z}_2\tilde{d}_1\tilde{d}_1$. The reason is that virtual $h^0$ exchange diagrams, which contribute much more strongly to $\tilde{b}\tilde{b}$ final states, are not negligible here. This can be understood from the observation that $h^0$ exchange is only suppressed \textsuperscript{30} by one power of the small higgsino component of $\tilde{Z}_2$ or $\tilde{Z}_1$, while $Z$ exchange needs \textsuperscript{1} two such powers, and $\tilde{f}$ exchange is suppressed by the large sfermion masses.

$h^0$ exchange contributes negligibly to $\tilde{Z}_2 \rightarrow \tilde{Z}_1 e^+ e^-$ decays, so it reduces the corresponding branching ratio.\textsuperscript{4} The leptonic branching ratio of $\tilde{Z}_2$ can nevertheless exceed considerably

\textsuperscript{1}A typical gluino pair event has $O(1000)$ hadrons and photons in the final state.

\textsuperscript{2}Recall that $\mu$ is assumed to be a free parameter in this “conventional” treatment. The calculated values of $\mu$ in the six mSUGRA cases are listed in table 1.

\textsuperscript{3}Note that, at least in the limit $m_0 \ll m_{\tilde{Z}_2}$, $h^0$ exchange does not interfere with sfermion or $Z$ exchange; it therefore always contributes positively.
that of the $Z$ boson, even if (most) squarks and sleptons have almost the same mass, as is the case here; this is largely due to interference between $Z$ and sfermion exchange diagrams, which can however also result in very small leptonic branching ratios for $\tilde{Z}_2$, see cases B3 and B4. In contrast, the branching ratios of the light chargino very closely track that of the $W$ bosons, since the $\tilde{\chi}^\pm_1$ couplings get contributions from both the higgsino and $SU(2)$ gaugino components of $\tilde{W}_1$ and $\tilde{Z}_1$, and are therefore much less suppressed than the $\tilde{Z}_2\tilde{Z}_1$ coupling. The only exception occurs in case A2, where the two–body decay $\tilde{W}_1 \rightarrow \tilde{t}_1 + b$ is allowed and hence completely dominates all $\tilde{W}_1$ decays.

Table 2: (a) Fractions of SUSY particle pairs produced in $pp$ collisions at the LHC; and (b) branching fractions of selected decay modes, for six SUGRA cases A1, A2 and B1–B4, where $\tilde{g}$ stands for all squarks except stops, and $\tilde{\chi}\tilde{\chi}$ stands for all possible chargino and neutralino pairs.

| SUSY particles\Case | A1   | A2   | B1   | B2   | B3   | B4   |
|---------------------|------|------|------|------|------|------|
| (a) Sparticle Pairs Produced |
| $\tilde{g}\tilde{g}$ | 0.30 | 0.093 | 0.72 | 0.74 | 0.44 | 0.73 |
| $\tilde{t}_1\tilde{t}_1^*$ | 0.51 | 0.84  | 0.011| 0.21 | 0.10 | 0.083|
| $\tilde{g}\tilde{q}$ | 0.13 | 0.050 | 0.23 | 0.013| 0.33 | 0.081|
| $\tilde{g}\tilde{g}$ | 0.018| 0.007 | 0.029| 3×10^{-4}| 0.067| 0.005|
| $\tilde{W}_1^\pm\tilde{Z}_2$ | 0.018| 0.006 | 0.004| 0.019| 0.027| 0.066|
| $\tilde{\chi}\tilde{\chi}$ | 0.026| 0.009 | 0.007| 0.027| 0.042| 0.088|
| (b) Important Decay Modes |
| $\tilde{g} \rightarrow \tilde{t}_1\tilde{t}_1, \tilde{t}_1\tilde{t}_1$ | 1.0  | 1.0  | -    | -    | -    | -    |
| $\tilde{g} \rightarrow \tilde{W}_1^-tb, \tilde{W}_1^+bt$ | 1.6×10^{-4}| 1.4×10^{-4} | - | 0.091 | 0.12 | 0.25 |
| $\tilde{g} \rightarrow \tilde{W}_1^-ud, \tilde{W}_1^+d\bar{u}$ | 4.6×10^{-4}| 2.9×10^{-4} | 0.21 | 0.10 | 0.19 | 0.16 |
| $\tilde{g} \rightarrow \tilde{Z}_1dd$ | 3.1×10^{-5}| 1.9×10^{-5} | 0.012| 0.005| 0.014| 0.01 |
| $\tilde{g} \rightarrow \tilde{Z}_1bb$ | 6.8×10^{-5}| 5.4×10^{-5} | 0.012| 0.006| 0.035| 0.018|
| $\tilde{g} \rightarrow \tilde{Z}_1bb$ | 4.3×10^{-4}| 3.9×10^{-4} | 0.21 | 0.10 | 0.18 | 0.15 |
| $\tilde{t}_1 \rightarrow \tilde{W}_1^+b$ | 1.0 | -    | 0.95 | 1.0  | 0.93 | 0.29 |
| $\tilde{t}_1 \rightarrow \tilde{Z}_1t$ | -    | -    | 0.05 | -    | 0.07 | 0.64 |
| $\tilde{t}_1 \rightarrow \tilde{Z}_2t$ | -    | -    | -    | -    | -    | 0.07 |
| $\tilde{t}_1 \rightarrow \tilde{Z}_1c$ | -    | 1.0  | -    | -    | -    | -    |
| $\tilde{Z}_2 \rightarrow \tilde{Z}_1dd$ | 0.11 | 0.18 | 0.024| 0.028| 0.21 | 0.17 |
| $\tilde{Z}_2 \rightarrow \tilde{Z}_1bb$ | 0.37 | 0.39 | 0.057| 0.22 | 0.38 | 0.42 |
| $\tilde{Z}_2 \rightarrow \tilde{Z}_1e^+e^-$ | 0.047| 0.018 | 0.14 | 0.12 | 0.007| 0.012|
| $\tilde{W}_1^+ \rightarrow \tilde{Z}_1e^+\nu_e$ | 0.11 | 6.9×10^{-5} | 0.11 | 0.11 | 0.11 | 0.11 |

It is by now quite well known that, in addition to the “classical” missing $E_T$+jets signature, sparticle production at hadron colliders can also give rise to final states containing hard leptons \[11, 40\]. In the present study a hard lepton is defined as an electron or muon with $p_T > 20$ GeV, pseudorapidity $|\eta| < 2.5$, and visible (hadronic or electromagnetic) activity in
a cone $\Delta R = 0.3$ around the lepton less than $5 \text{ GeV}$. Hadronic clusters with $E_T > 50 \text{ GeV}$ in a cone $\Delta R = 0.7$ are labelled as jets. We can then define various mutually exclusive event classes:

- “Missing $E_T$” events have no hard leptons, $n_j \geq 4$ jets, missing $E_T > 150 \text{ GeV}$, transverse sphericity $S_T > 0.2$, and total scalar (calorimetric) $E_T > 700 \text{ GeV}$. This last cut is not strictly necessary, but greatly enhances signal/background.

- “$n$ lepton” events have exactly $n \ (\geq 1)$ hard leptons, and missing $E_T > 100 \text{ GeV}$. If $n = 1$, we in addition required the scalar $E_T$ to exceed $700 \text{ GeV}$, and demanded that the transverse mass computed from the missing $p_T$ and the leptonic $p_T$ does not fall in the interval between $60$ and $100 \text{ GeV}$, where the background from real $W$ decay has a Jacobian peak. For $n = 2$ we distinguish opposite sign (OS) and same sign (SS) events, and required total $E_T > 700 \text{ GeV}$ in the OS sample in order to suppress $t\bar{t}$ backgrounds.

Cross sections after cuts for these final states are listed in table 3, for the six mSUGRA cases, 4 ‘BTW’ cases and leading sources of background, i.e. $t\bar{t}$, $W$+jets and $Z$+jets events. (We checked that backgrounds from $W^+W^-$, $c\bar{c}$ and $b\bar{b}$ production are always very small.) The table extends out to $n = 4$ leptons, but the results for $n = 4$ already suffer from substantial statistical errors (we generated 50,000 events for each SUSY spectrum, and several hundred thousand background events). We observe that the missing $E_T$, SS and (except in case A2) $3\ell$ signals are all well above background; the $1\ell$ and OS signals are also always larger than, or at least similar to, backgrounds.

The size of the missing $E_T$ cross section is mostly determined by the gluino mass; direct $\tilde{t}_1\tilde{t}_1^*$ production contributes little to the signal after cuts even in case A2, mostly because we demand at least 4 jets here. However, due to the potentially sizable contribution from $\tilde{g}\tilde{q}$ production shown in table 2, the masses of first generation squarks also play a role. Moreover, even this relatively robust signal can change by a factor of more than 2 depending on gluino branching ratios: Even though case A2 has a slightly heavier gluino than case A1, and hence an almost 50% smaller gluino pair cross section, it produces a two times stronger missing $E_T$ signal than case A1 does. The reason is that $\tilde{t}_1 \to \tilde{Z}_1 + c$ decays give much harder LSPs than $\tilde{t}_1 \to \tilde{W}_1 + b \to \tilde{Z}_1 + q\bar{q} + b$ decays do. Similarly, the missing $E_T$ signal in case B2 is almost as strong as in case B1, inspite of the more than three times smaller sum of $\tilde{g}\tilde{g}$ and $\tilde{g}\tilde{q}$ cross sections. This is mostly due to a large (22%) branching ratio for $\tilde{g} \to \tilde{Z}_1 + g$ loop decays. ($\tilde{g} \to \tilde{Z}_2 + g$ decays also have a branching ratio of about 22%, and about 20% of all $\tilde{Z}_2$ decay invisibly into $\tilde{Z}_1\nu\bar{\nu}$ in this case.) These $\tilde{Z}_1$ produced in $\tilde{g}$ two–body decays are very energetic, much more so than the LSPs produced at the end of a cascade. The branching ratios for these loop decays of the gluino are enhanced in case B2 because ordinary squarks

\footnote{A warning: All cross sections in table 3 are not only subject to unknown NLO QCD corrections, but also depend sensitively on the resolution and coverage of the calorimeters, since both signal and background are backed up against the missing $E_T$ cut. Notice that the predicted missing $E_T$ also depends on, for example, the amount of initial state radiation that is being generated. One will therefore have to understand both the detector and “ordinary” QCD events in some detail before quantitative studies of absolute rates can be undertaken with some confidence. However, the ratios of various cross sections, including signal/background, will hopefully be rather robust against future refinements.}
are very heavy, while $\tilde{t}_1$ (which contributes to loops) is quite light.

**Table 3:** Cross sections in $pb$ for various event topologies after cuts described in the text, for $pp$ collisions at $\sqrt{s} = 14$ TeV. The various SUGRA cases are listed in the first column. The OS/SS ratio is computed with the OS dilepton sample before the scalar $E_T$ cut.

| case     | $E_T^*$ | 1 $\ell$ | OS | SS | OS/SS | 3 $\ell$ | 4 $\ell$ | 3 $\ell$ | 4 $\ell$ |
|----------|---------|----------|----|----|-------|----------|----------|----------|----------|
| A1       | 24.6    | 36.2     | 5.4| 3.7| 2.0   | 1.2      | 0.017    |          |          |
| A2       | 48.0    | 31.4     | 1.5| 2.1| 1.2   | < 0.02   | < 0.02   |          |          |
| B1       | 79.1    | 76.8     | 11.9| 3.4| 6.9   | 1.7      | 0.17     |          |          |
| B2       | 67.7    | 34.5     | 1.5| 2.4| 2.7   | 8.0      | 0.1      |          |          |
| B3       | 51.8    | 21.2     | 0.6| 3.3| 0.09  | < 0.01   |          |          |          |
| B4       | 20.1    | 10.1     | 0.4| 3.5| 0.1   | < 0.004  |          |          |          |
| BTW1     | 105     | 39.8     | 2.8| 1.4| 3.1   | 0.21     | 0.03     |          |          |
| BTW2     | 57.3    | 22.5     | 2.2| 0.85| 3.6  | 0.14     | < 0.02   |          |          |
| BTW3     | 96      | 58.5     | 10.9| 2.9| 6.7   | 1.5      | 0.06     |          |          |
| BTW4     | 52.3    | 23.5     | 2.3| 0.9| 3.7   | 0.2      | < 0.02   |          |          |
| $t\bar{t}$ (160) | 2.9 | 8.0 | 0.1 | 0.01 | 640 | < 0.004 | < 0.004 |          |          |
| $W + jet$ | 0.6 | 3.8 | 0.29 | – | – | – | – |          |          |
| $Z + jet$ | 0.6 | 0.2 | 0.02 | – | – | – | – |          |          |
| total BG | 4.3 | 12.02 | 1.411 | 0.01 |          | < 0.004 | < 0.004 |          |          |

The size of the $1\ell$ signal is generally roughly comparable to that of the missing $E_T$ signal, but backgrounds are about three times larger here. Real top quarks are very efficient in producing hard leptons (and missing $E_T$, needed to pass the cut $E_T > 100$ GeV); in case A1 the $1\ell$ signal is therefore actually larger than the missing $E_T$ signal. Notice also that the effect of increasing $m_0$ to 1 TeV is now about the same for cases B1/B2 and B3/B4, reducing the signal by approximately a factor of two. Many “$1\ell$” events in cases B1, B2 actually come from $\tilde{Z}_2 \rightarrow \tilde{Z}_1 l^+ l^-$ decays where one of the leptons failed to pass the cuts. The large leptonic branching ratio of $\tilde{Z}_2$ in these cases also leads to quite large OS $2\ell$ signals, large OS/SS ratios, and sizable $3\ell$ signals. Case A1 also has substantial OS and $3\ell$ signals, but the OS/SS ratio is much smaller here, since $\tilde{g}\tilde{g}$ events produce $tt$ and $t\bar{t}$ final states with equal abundance as $t\bar{t}$ final states. Case A2 has very few $n \geq 3$ lepton events, since basically all gluinos decay into $t + \bar{c} + \tilde{Z}_1$ (or the charge conjugate thereof) here, giving at most one hard lepton per gluino. Cases B3 and B4 give smaller leptonic signals, due to the larger gluino mass; they differ from each other by a factor of two in the missing $E_T$ and (marginal) $1\ell$ signals, but become more similar to each other as more leptons are required, mostly due to the sizable $\tilde{g} \rightarrow \tilde{W}_1 t b$ branching ratio in case B4.

Finally, the “conventional” BTW cases can produce even larger missing $E_T$ signals than our mSUGRA cases, since we allowed squarks to lie just above gluinos here, leading to large $\tilde{g}\tilde{g}$ production rates. Case BTW3 also has a large leptonic branching ratio of $\tilde{Z}_2$ (12.5% per generation), and hence large cross sections for the $n$ lepton final states; for the other BTW cases the leptonic branching ratio of $\tilde{Z}_2$ is close to that of the $Z$ boson.
This discussion shows that the counting rates in the six or seven independent signal channels listed in table 3 already go a long way towards distinguishing the various mSUGRA cases from each other as well as from the ‘BTW’ cases. Some ambiguities remain, however; for example, cases B1 and BTW3 are very similar at this level (except perhaps in the 4l signal, but statistics is poor here). The remaining ambiguity can be resolved by looking at the events within one signal class in more detail. An example is shown in table 4, where event fractions for the missing \(E_T\) sample are shown, together with the average total scalar \(E_T\) and average missing \(E_T\) per event.

### Table 4: Cross sections and event fractions for missing energy plus jets events for various SUSY cases at the LHC.

| case   | \(\sigma \text{(pb)}\) | \(n_j = 4 - 5\) | \(6 - 7\) | \(\geq 8\) | \(n_b \geq 1\) | \(n_b \geq 2\) | \(\langle \Sigma E_T \rangle\) | \(\langle \bar{E}_T \rangle\) |
|--------|-------------------------|-----------------|-----------|-----------|----------------|----------------|-----------------|-----------------|
| A1     | 24.6                    | 0.54            | 0.35      | 0.10      | 0.65           | 0.28           | 1160            | 212             |
| A2     | 48.0                    | 0.55            | 0.35      | 0.10      | 0.47           | 0.09           | 1112            | 221             |
| B1     | 79.1                    | 0.73            | 0.25      | 0.02      | 0.21           | 0.04           | 964             | 195             |
| B2     | 67.7                    | 0.77            | 0.20      | 0.03      | 0.23           | 0.05           | 999             | 211             |
| B3     | 51.8                    | 0.57            | 0.35      | 0.08      | 0.36           | 0.12           | 1118            | 215             |
| B4     | 20.1                    | 0.54            | 0.36      | 0.10      | 0.44           | 0.15           | 1204            | 217             |
| BTW1   | 105                     | 0.69            | 0.26      | 0.04      | 0.17           | 0.04           | 1006            | 214             |
| BTW2   | 57.3                    | 0.61            | 0.33      | 0.05      | 0.18           | 0.04           | 1091            | 211             |
| BTW3   | 96                      | 0.69            | 0.27      | 0.04      | 0.15           | 0.03           | 1011            | 217             |
| BTW4   | 52.3                    | 0.60            | 0.33      | 0.06      | 0.18           | 0.04           | 1109            | 208             |
| tt(160)| 2.9                     | 0.81            | 0.17      | 0.01      | 0.56           | 0.12           | 895             | 201             |
| Z + jets | 0.63                    | 0.89            | 0.11      | 0.00      | 0.11           | 0.02           | 905             | 281             |

The most useful discriminators appear to be the fractions of events with at least one or two tagged \(b\) quarks. Here we have assumed a \(b\) tagging efficiency of 40% if the \(b\)-flavoured hadron has \(p_T > 20\) GeV and pseudorapidity \(|\eta| < 2\), and zero efficiency otherwise; we have ignored the possibility of false tags. The spectra where \(\tilde{g} \to \tilde{t}_1 t\) is allowed clearly lead to the largest \(b\) content; the difference between \(\tilde{t}_1 \to \tilde{W}_1 + b\) (A1) and \(\tilde{t}_1 \to \tilde{Z}_1 + c\) (A2) decays is also evident, especially in the fraction of events with at least two tagged \(b\)'s. Cases B3 and B4 still have fairly large \(b\) content, partly due to \(\tilde{g} \to \tilde{W}_1 tb\) decays which can produce up to four \(b\) quarks in a gluino pair event; enhanced \(\tilde{g} \to \tilde{Z}_2 + \bar{b}b\) and \(\tilde{Z}_2 \to \tilde{Z}_1 + \bar{b}b\) branching ratios also play a role, see table 2. Finally, in the light gluino scenarios B1, B2 the \(b\)-fraction is considerably smaller than in the cases where gluinos can decay into top quarks; the enhanced branching ratios into final states containing \(b\bar{b}\) still lead to \(b\)-fractions that exceed those of the BTW cases, however.

The presence of \(t\) quarks in gluino decays also leads to large average jet multiplicities: 10% of all events in cases A1, A2 and B4 have at least 8 reconstructed jets in them (recall that we require each jet of have \(p_T > 50\) GeV). Moreover, the comparison of BTW2,4 with BTW1,3 shows that the presence of first generation squarks significantly, but not infinitely, heavier than the gluino increases the average number of jets and the average scalar \(E_T\) per event; this is due to \(\tilde{g}\tilde{g}\) production, of course. This effect is less pronounced when comparing
B4 to B3, or B2 to B1, since in cases B4 and B2 first generation squarks are so heavy that they contribute little to the total SUSY signal; see table 2. The average missing $E_T$ seems to be a less useful discriminator, clustering around 210 to 220 GeV in almost all cases. The only exception is case B1, which has a very light gluino; in case B2, where the gluino is also light, the missing $E_T$ is enhanced by the large branching ratios for loop–induced $\tilde{g} \to \tilde{Z}_{1,2} + g$ decays discussed earlier.

Similar tables can be shown for the other samples of signal events [16]; for reasons of space I here merely summarize the most important points. The results for jet multiplicities are very similar for all samples, except that a larger number of leptons implies an overall reduction of the number of jets.\[Recall that our definition of leptonic events does not require a minimal number of jets; this also reduces the average jet multiplicity compared to the missing $E_T$ sample, of course.\]

In the mSUGRA cases B1–B4 the $b$ fraction is considerably larger in the OS sample than in the missing $E_T$ or 1l samples. This effect is especially pronounced in cases B3 and B4, where $\tilde{g} \to \tilde{W}_1 t b$ decays are good sources of both hard leptons and $b$ quarks; indeed, in these two cases the $b$ fraction exceeds that in case A2 in the OS dilepton sample. The correlation between the number of $b$’s and the number of leptons in cases B1 and B2 is due to the fact that here most leptons come from $\tilde{Z}_2$ decays, and $\tilde{g} \to \tilde{Z}_2$ decays frequently lead to a $b\bar{b}$ final state; the enhancement of $b\bar{b}$ final states is much weaker in $\tilde{g} \to \tilde{Z}_1$ decays, as shown in table 1, while $\tilde{g} \to \tilde{W}_1$ decays cannot contain $b$ quarks in these cases.

The contribution from $\tilde{Z}_2 \to \tilde{Z}_1 l^+ l^-$ decays to the OS signal can also be tested by looking at the flavour of the produced leptons: $\tilde{Z}_2$ decays always produce $e^+ e^-$ or $\mu^+ \mu^-$ pairs, while OS events that originate from (semi–)leptonic decays of charginos or $t$ quarks are equally likely to contain an $e\mu$ pair. Indeed, one finds a strong preponderance of like–flavour pairs in cases B1 and B2 as well as in all BTW cases, while all combinations of lepton flavours occur with equal frequency in cases A1 and A2 where $\tilde{Z}_2$ is hardly ever produced; in cases B3 and B4 there is a smaller but still significant preference for like–flavour lepton pairs.

The invariant mass distribution of the two charged leptons in like–flavour opposite–charge dilepton events should allow to determine the $\tilde{Z}_2 - \tilde{Z}_1$ mass difference, which in our mSUGRA scenario is approximately given by 0.4$m_{1/2}$. Unfortunately, other (combinations of) sparticle masses are much more difficult to determine at hadron colliders.

5) Summary and Conclusions

In this contribution I have described the phenomenology of the minimal SUGRA $SU(5)$ model. The underlying theme of this model is unification. First of all, the very existence of a GUT sector implies that the Higgs sector of the SM is ill–behaved at the quantum level unless SUSY exists at an energy scale not much above the weak scale. Moreover, we now know that the particle content of the SM by itself does not allow for unification of all gauge interactions; new degrees of freedom are needed, and minimal SUSY just fits the bill.

Secondly, in this model one can also successfully unify the $b$ and $\tau$ Yukawa couplings, provided that the top Yukawa coupling is large, i.e. close to its IR (quasi) fixed point. The
idea of unification can even be extended into the SUSY breaking sector by assuming a single gaugino mass as well as a single scalar mass. This leads to the very elegant mechanism of radiative breaking of the electroweak gauge symmetry, which hints towards explanations for the large mass of the top quark and the large hierarchy between $M_X$ and $M_Z$.

In this scenario constraints from proton decay, and from the relic density of LSPs produced in the Big Bang, imply that gauginos must be quite light, while most sfermions and Higgs bosons are heavy. Light charginos, and the light scalar Higgs boson predicted by this model, are quite easily detectable at $e^+e^-$ colliders like the second stage of LEP if kinematically accessible. If no chargino with mass below $\sim 105$ GeV or no gluino with mass below $\sim 400$ GeV is found mSUGRA $SU(5)$ has to be discarded, but their discovery can hardly be considered proof of this model. However, it also predicts that left–handed $\bar{b}$ squarks and, much more dramatically, the lighter $\tilde{t}$ eigenstate lie well below the other squarks. Indeed, $\tilde{t}_1$ might even be produced in $\tilde{g}$ decays, giving events with several $b$–quarks. Even if this decay is not possible the reduced masses of $\tilde{t}_1$ and $\tilde{b}_L$ give enhanced branching ratios for $\tilde{g} \rightarrow \tilde{W}_1tb$ and $\tilde{g} \rightarrow \tilde{Z}_1b\bar{b}$ three–body decays and hence again an enhanced $b$–quark content of SUSY events at the LHC; in this case the enhancement is usually less, and increases with the number of hard leptons in the event. Notice that the event rates are so large that a “low” luminosity of a few times $10^{32}$ cm$^{-2}$sec$^{-1}$ is quite sufficient, which should allow for $b$–tagging at least in principle. Further clues to the nature of the spectrum can come, e.g., from the average jet multiplicity per event and from the flavour composition of opposite–sign dilepton events.

It should be emphasized that in this model a SUSY signal should be seen at the LHC in at least three, and possibly as many as seven independent channels, where only the number and charge of hard leptons has been used to classify events. This clearly offers great opportunities for the LHC. Nevertheless the LHC by itself will not suffice to find all the new particles predicted by this model: The heavy Higgs bosons, higgsinos and sleptons are in my opinion impossible to detect at the LHC. Finding first generation squarks on top of the “background” of light gluinos will also be difficult, especially if they are close to the TeV scale in mass. More surprisingly, even the direct pair production of light $\tilde{t}_1$ squarks seems difficult to detect. If we are lucky light stops might be detected at the tevatron [11]. However, if this model is correct the completion of the sparticle and Higgs spectrum will most likely have to await the construction of a TeV scale $e^+e^-$ collider. Finally, one would eventually want to see direct evidence for the existence of GUT particles, the best hope probably being proton decay experiments. The model presented here would therefore keep particle physicists busy for some decades to come.

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Figure Captions

Fig. 1 The ratio $m_{b_1}/m_{\tilde{u}_L}$ as a function of the GUT scale parameter $A_0 \equiv A/m_0$.

Fig. 2 Regions in the $(A_0, m_{1/2})$ plane leading to different gluino and $\tilde{t}_1$ decays. Note that $m_{\tilde{t}_1}$ generally increases with increasing $m_{1/2}$ and decreasing $|A_0|$, although for given $m_{1/2}$ the maximum of $m_{\tilde{t}_1}$ is not exactly at $A_0 = 0$. 
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