A New Simple Model for High Frequency Quasi Periodic Oscillations in Black Hole Candidates

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ABSTRACT
Observations of X-ray emissions from binary systems have long since been considered important tools to test General Relativity in strong-field regimes. The high frequency quasi-periodic oscillations (HFQPOs) observed in binaries containing a black hole candidate, in particular, have been proposed as a means to measure more directly the black hole properties such as its mass and spin. Numerous models have been suggested to explain the HFQPOs and the rich phenomenology accompanying them. Many of these models rest on a number of assumptions and are at times in conflict with the most recent observations. We here propose a new, simple model in which the HFQPOs result from basic \textit{p}-mode oscillations of a small accretion torus orbiting close to the black hole. We show that within this model the key properties of the HFQPOs can be explained simply, given a single reasonable assumption. We also discuss observational tests that can falsify the model.

Key words: X-ray; binaries – accretion discs – relativity – oscillations

1 INTRODUCTION
One of the strongest motivations for studying X-ray binaries has been the hope that these systems could be used as probes of fundamental physics. Stellar mass black holes represent a laboratory for studying strong field General Relativity, while neutron stars allow us to test equations of state for nuclear matter. The high frequency quasi-periodic oscillations (HFQPOs) seen from these sources have been held forth as one of the most promising diagnostics, with the potential to measure, for example, the masses and radii of neutron stars (e.g. Kluzniak et al., 1990; Miller et al., 1998) or the masses and spins of black holes (e.g. Wagoner et al. 2001; Abramowicz and Kluzniak 2001).

A great deal more work has been done explaining the HFQPO phenomena in neutron star systems than those in black hole systems, and this is motivated in large part by the much larger number of observed systems and the richer data sets of these systems. A correlation between the frequencies of the high frequency QPOs and the break frequency for the broadband noise component of the Fourier power spectrum that fits both neutron stars and black holes (Psaltis, Belloni, van der Klis 1999 - PBK) has been suggested to provide evidence that the same mechanism must be taking place in all these systems. The extension of this correlation to include white dwarf systems (Mauche 2002) bolstered these claims.

If the PBK and Mauche (2002) correlations are indeed universal and the interpretation that a single mechanism is at work in all cases is correct, then the mechanism cannot depend on the presence of a stellar surface, of a strong magnetic field, or of a strong (i.e. relativistic) gravitational field. Instead, a model taking advantage of the properties of a non-uniform rotating fluid would be a strong candidate for explaining the HFQPOs (see e.g. Titarchuk 2003; Osherovich and Titarchuk 1999).

More recently, though, additional phenomenology has emerged which indicates some fundamental differences between the neutron star and black hole systems, and may suggest that different models apply to these sources after all. The kilohertz QPOs are typically seen in multiples. In the neutron star systems, the separation in the HFQPO frequencies is, in general, nearly constant, with the frequency separation shrinking as the frequencies increase (see e.g. van der Klis et al., 1997; Mendez et al., 1998). In the black hole systems, on the other hand, the kilohertz QPO frequencies seem to drift by much smaller amounts (Strohmayer 2001a, 2001b) and to be found in ratios of small integers (i.e. 1:2, 2:3, or 1:2:3; Abramowicz and Kluzniak, 2001; Remillard et al., 2002; Homan et al. 2003b). There are also some claims for harmonic structure in the probable black hole transient XTE J 1550-564 (Homan et al. 2003a), with peaks seen at 110, 140, 210, and 270 Hz, but the identification of a harmonic structure is not as clear here as the frequencies are not all identified simultaneously and, in fact, seem to drift. In addition to this, recent observations of the probable black hole transient XTE J 1550-564 indicate that its HFQPOs do not always fit on the PBK correlation (Remillard et al., 2002). These differences suggest that the PBK correlation may not apply to all of the HFQPOs seen; models requiring a stellar surface or general relativistic effects, or both (e.g. Stella and Vietri 1998) need not be rejected. As we wish to explain the integer ratios of...
the frequencies of HFQPOs from the dynamically confirmed black hole candidates, we will concentrate here on models which do not require a stellar surface.

A model applied primarily to the HFQPOs from black hole candidates is the “discospaceic” model which asserts that $g$ modes should become trapped in the potential well of a Keplerian disc in a Kerr potential (e.g. Nowak et al., 1997). The size of the region where the modes are trapped depends on both the mass and the spin of the accreting black hole. Additional frequencies of oscillation should be expected from $p$ (pressure) modes and $c$ (correction) modes. The predictions of the model are well summarised by Kato (2001). Given pairs of high frequency QPOs and a proper identification of the frequencies with the particular modes, one can measure both the black hole mass and spin to relatively high accuracy (Wagoner et al., 2001 and references therein). The discoveries of three systems where the HFQPOs show a harmonic structure with relatively strong peaks seen in integer ratios 1:2, 2:3, or 1:2:3 seem to cast some doubt upon this model. However, the discospaceic model remains viable for the intermediate frequency QPOs in GRS 1915+105, seen at 67 Hz (Morgan et al., 1997) and 40 Hz (Strohmayer et al., 2001b). We emphasize that such harmonic structure has been seen only from systems thought to contain a black hole as the compact accretor.

Being the first to point out that the frequencies of QPOs in some black hole and neutron star sources were in a ratio of small integers, Abramowicz and Kluzniak (2001, 2002, 2003a) have proposed the “resonance” model, in which a harmonic relationship in the HFQPO frequencies can be produced as a result of orbital resonances. In particular, the model suggests that an initial perturbation is amplified at a radius where the radial epicyclic frequency for point-like masses is in resonance with the latitudinal epicyclic frequency, with the two frequencies being in (small) integer ratios (In a Schwarzschild spacetime the latitudinal epicyclic frequency and the orbital one coincide.). These annuli tend to be close to the black hole event horizon for the observed frequencies and, hence, given a mass estimate for the black hole and an identification of the ratio of the frequencies in resonance, a black hole spin could be measured.

It should be noted that given the observed frequencies of 162 and 324 Hz in GRS 1915+105 it is not possible to produce the 40 and 67 Hz QPOs with a discospaceic model and the higher frequency QPOs with a resonance model while retaining the same values for the black hole mass and spin (Maccarone 2002). Another potential problem for these two last models is the observational evidence for a frequency “jitter” in the HFQPOs of XTE J1550-564, i.e. for small variations of about 10% in frequencies for about 15% of the time. This difficulty could be particularly severe for the resonance model, whose relevant frequencies define the resonance to a narrow region in radial coordinates [This situation is worsened for the 1:3 resonance for which the radial variation of the frequencies is even more rapid (Remillard et al., 2002)]. More recently, however, Abramowicz et al., (2003b) have considered a perturbative approach to the standard resonance model and were able to show that the resonance can take place also near the radial positions at which the epicyclic frequencies are in an exact 2:3 ratio. While this result offers at least in part a possible explanation for the frequency jitter, the radial extension over which the resonance takes place (which is $\sim 0.1 GM/c^2$ at most, Karas 2003) may still be too small to produce the observed modulation in the emissivity.

We here propose an explanation for the high frequency QPOs in black hole candidates that has connections with both the discospaceic and the resonance model. More precisely, as in the discospaceic model, we are here focussed on global oscillation modes of a fluid orbiting in the vicinity of the black hole and, as the resonance model, we consider fundamental the observational evidence of a 2:3 ratio in the HFQPO frequencies. Our model, however, has distinct novel features and, most importantly, is based on the existence of a non-Keplerian (geometrically thick) disc orbiting in the vicinity of the rotating black hole. Using this single assumption, we show that the HFQPO phenomenology finds a simple explanation.

In Section 2 we introduce the basic properties of our model and discuss how, using this single assumption, we can explain the most important aspects of the HFQPO phenomenology. Finally, in Section 3, we explain how the model can be used to deduce the black hole properties and the ways in which it can be refined or refuted.

## 2 THE MODEL

In contrast to a Keplerian accretion disc, which is in principle infinitely extended, a non-Keplerian disc can easily be constructed to have a finite size, the extent being determined uniquely by the distribution of the specific angular momentum and by the pressure gradients. Because pressure gradients play such an important role, non-Keplerian discs tend to be geometrically thick and look like tori rather than thin discs. Depending then on the pressure gradients and angular momentum distribution, the orbiting fluid is confined to a finite-size region which can behave as a cavity in which global oscillation modes could be trapped. This is a fundamental difference from Keplerian discs, which have no definite outer boundary and in which outward propagating waves cannot grow (Kato 2001).

When a disc (either Keplerian or non) initially in equilibrium is perturbed in some way, restoring forces appear to compensate for the perturbation. A first restoring force is the centrifugal force, which is responsible for inertial oscillations of the orbital motion of the disc and hence for epicyclic oscillations (this is the restoring force at work in the resonance model). A second restoring force is the gravitational field in the direction vertical to the orbital plane and which will produce a harmonic oscillation across the equatorial plane if a portion of the disc is perturbed in the vertical direction (these oscillations are extensively studied in galactic dynamics). A third restoring force is provided by pressure gradients and the oscillations produced in this way are closely related to the sound waves propagating in a compressible fluid. In a geometrically thick disc, the vertical and horizontal oscillations are in general coupled and more than a single restoring force can intervene for the same mode. A detailed discussion of how to classify the different modes of oscillation through its main restoring force can be found in Kato (2001), but it is here sufficient to remind that $c$ modes are essentially controlled by the vertical gravitational field, that $g$ modes are mainly regulated by centrifugal and pressure-gradient forces, and that all of the restoring forces discussed above play a role in the case of $p$ modes. It is also useful to underline that because we are here interested in modes with a prevalent horizontal motion and frequencies above the epicyclic one, we are essentially selecting “inertial-acoustic” modes having centrifugal and pressure gradients as only restoring forces. Hereafter we will refer to these simply as $p$ modes.

Following a recent investigation of the nonlinear dynamics of perturbed relativistic tori orbiting around a Schwarzschild black hole (Zanotti et al., 2003), we have analysed the global oscillation properties of such systems and, more specifically, we have performed a perturbative analysis of the axisymmetric modes of oscillation of relativistic tori in a Schwarzschild spacetime and in
the Cowling approximation (Rezzolla et al., 2003a). The eigenvalue problem that needs to be solved to investigate consistently the oscillation properties of fluid tori is simplified considerably if the vertical structure is accounted for by an integration in the vertical direction. Doing so removes one spatial dimension from the problem, which can then be solved integrating simple ordinary differential equations. While an approximation, this simpler model reproduces with a precision of less than a few percent, the numerical results obtained with fully nonlinear, 2D numerical calculations (see Rezzolla et al., 2003a for a comparison).

One of the important results of the perturbative investigation was that the eigenfunctions and eigenfrequencies found were those corresponding to the $p$ modes of the torus. Furthermore, the behaviour of the fundamental frequencies was observed to converge to the epicyclic frequency $\Omega_e$ at the position of maximum density in the torus $r_{\text{max}}$ as the torus’s size was progressively reduced to zero (see Fig. 4 of Rezzolla et al., 2003a). Finally and most importantly, the eigenfrequencies computed both for the fundamental mode of oscillation as well as for the first few overtones, were found to be in a harmonic sequence 2:3:4:... to within 5–10%, the exact value depending on the specific model for the torus (in general larger tori have progressively smaller ratios). Note that such a relation among the eigenfrequencies is not a standard property of $p$-mode oscillations. In stars, for instance, this happens only for purely radial oscillations. Despite showing this important property, the fundamental frequencies were smaller than the observed lower HFQPO frequencies when the estimated masses of the black hole candidates were used.

Stimulated by this mismatch, we have extended the calculations to the case of a torus orbiting around a Kerr black hole. The details on these calculations will be presented in a separate paper (Rezzolla, and Yoshida, 2003b) but the mathematical setup and the numerical methods used follow closely those presented in Rezzolla et al. (2003a). Here, we will concentrate on summarizing the results and discussing how they can be used to construct a new model to explain the HFQPOs in black hole candidates.

Figure 1 shows a typical example of the solution of the eigenvalue problem for a black hole with mass $M = 10 M_\odot$ and its comparison with the observations of XTE J1550-564 (other sources could equally have been used). In particular, we have plotted the value of the different eigenfrequencies found versus the radial extension $L$ of the torus, expressed in units of the gravitational radius $r_g \equiv GM/c^2$. The sequences have been calculated for a distribution of specific angular momentum following a power-law (see Rezzolla et al., 2003a for details), keeping constant $r_{\text{max}} = 3.489$, and for a black hole with dimensionless spin parameter $a \equiv J/M^2 = 0.94$ to maximize the value for $L$. Indicated with a solid line are the fundamental frequencies $f$, while the first overtones $o_1$ are shown with a dashed line; each point on the two lines represents the numerical solution of the eigenvalue problem. The asterisks represent the frequencies of the HFQPOs detected in XTE J1550-564 at 148 and 276 Hz, respectively. Note that the two plotted eigenfrequencies are close to a 2:3 ratio over the full range of $L$ considered (this is shown in the inset) and that while they depend also on other parameters in the problem (e.g. the position of $r_{\text{max}}$, the angular momentum distribution, the polytropic index), these dependences are very weak so that the frequencies depend effectively on $M$, $a$ and $L$. As a result, if $M$ and $a$ are known, a diagram as the one shown in Fig. 1 could be used to determine the dimension of the oscillating region $L$ rather accurately.

On the basis of these results, we suggest that the HFQPOs observed in black hole candidate systems can be interpreted in terms of $p$-mode oscillations of a small-size torus orbiting in the vicinity of a black hole. Such a configuration could be produced whenever an intervening process (e.g. large viscosity, turbulence, hydrodynamical and magnetohydrodynamical instabilities, etc.) modifies the Keplerian character of the flow near the black hole.

Two remarks are worth making at this point. The first one is about the existence of a non-Keplerian fluid motion: this is required only very close to the black hole (the inner edge of the torus can be in principle be located at the marginally bound orbit) and beyond this region the fluid motion can be Keplerian. Stated differently, the HFQPOs observed could be produced by the inner parts of a standard, nearly-Keplerian, geometrically thin accretion disc, where a variety of physical phenomena can introduce pressure-gradients. The second remark is about the stability of these tori to non-axisymmetric oscillations. It is well-known that a stationary (i.e. non-accreting) perfect fluid torus flowing in circular orbits around a black hole is subject to a dynamical instability triggered by non-axisymmetric perturbations (Papaloizou and Pringle, 1984). It is less well-known, however, that the instability can be suppressed if the flow is non-stationary. Stated differently, a fluid torus around black could be stable to non-axisymmetric perturbations if mass-accretion takes place (Blaes 1987; Blaes and Hawley 1988; Hawley 1991; De Villiers and Hawley 2002). Because the tori discussed here are assumed to be the terminal part of standard accretion discs, we expect them to be stable to non-axisymmetric perturbations as long as magnetic fields are unimportant.

In what follows we discuss how the HFQPO phenomenology finds simple explanations within this model.

(i) The harmonic relations between the HFQPO frequencies in black hole candidates are naturally explained within this model. When the torus is sufficiently small, in fact, it can be thought of as a cavity in which the $p$ modes effectively behave as trapped sound waves. If the sound speed in the cavity were constant, the frequen-
cies of these standing waves would be in an exact integer ratio. In reality the sound speed is not constant but the eigenfrequencies found are in a sequence very close to 2:3:4 in the parameter range we are interested in.

(ii) Being global modes of oscillation, the same harmonicity is present at all radii within the torus. This removes the difficulty encountered in the resonance model and provides also a larger extent in radial coordinates where the emissivity can be modulated (cf. Fig. 1).

(iii) By construction, the frequencies scale like $1/M$. This is in agreement with the observations made of XTE J1550-564 and GRO J1655-40 as long as the spins are similar (Remillard et al., 2002). On the other hand, a rather narrow range of black hole spins has been suggested as a possible explanation for the narrow range of radio-to-X-ray flux ratios in the Galactic X-ray binary systems (Fender 2001).

(iv) The frequency jitter can be naturally interpreted in terms of variations of the size of the oscillating cavity $L$. Indeed, the frequencies may drift arbitrarily over a large range with the harmonic structure preserved (cf. Fig. 1).

(v) The observed variations in the relative strength of the peaks can be explained as a variation in the perturbations the torus is experiencing. (This has been reproduced with numerical simulations; Rezzolla et al., 2003a). Furthermore, while the low frequency overtones are energetically favoured and the corresponding eigenfunctions possess less nodes, any number of harmonics could in principle be observed.

(vi) The evidence that an overtone can be stronger than the fundamental in the harder X-ray bands (Strohmayer 2001a), can be explained simply given that the overtone is an oscillation preferentially of the innermost (and hottest) part of the accretion flow (see Rezzolla et al., 2003a for the eigenfunctions).

3 BLACK HOLE PROPERTIES AND MODEL TESTS

Because the $p$ modes discussed here represent the basic oscillation modes of the torus, some of their features will not be very sensitive to general relativistic corrections. The harmonic relation between the fundamental frequency and the first overtones is one of such features and this has been encountered not only in Schwarzschild and Kerr spacetimes, but also when we have considered a simple Newtonian gravitational potential (Rezzolla et al., 2003a). While these qualitative features remain unchanged in relativistic regimes, quantitative difference emerge and since these depend on the mass and spin of the black hole, they can be used to measure the black hole properties. This weak dependence on relativistic effects is an important way in which our model for HFQPOs differs from the ones proposed so far.

As discussed above, the solution of the eigenvalue problem in a Schwarzschild spacetime has shown that the fundamental $p$-mode frequency tends to the radial epicyclic frequency at $r_{\text{max}}$ in the limit of a vanishing torus size. For a Kerr black hole, this frequency is a function of its mass and spin through the relation $\kappa_r^2(a, M) = (GM/r)^2(1 + a/\sqrt{2} - 2 - 6/r + 8a/\sqrt{2} - 3a^2/r^2)$, where $\bar{r} \equiv r/r_g$ (Kato 2001). Exploiting this property and the solution of the eigenvalue problem for a large number of different models for the tori, (i.e. with different pressure and density profiles, distributions of specific angular momentum, polytropic indices, etc.) we suggest that a first estimate of the black hole spin and of the size of the oscillating torus can be obtained once the lower frequency in the HFQPOs is measured accurately. In this case, and as shown schematically in Fig. 2, given a black hole candidate with measured mass $M_*$, a lower limit on the black hole spin $\bar{a}$ can be deduced as the value of $a$ at which the maximum epicyclic frequency is equal to the lower observed HFQPO frequency, i.e. $\bar{a} \leq a \leq 1$, with max $[\kappa_r(\bar{a}, M_*)] = $ lower HFQPO frequency. Because for small tori $L \lesssim r_{\text{max}}$ (cf. Rezzolla et al., 2003a), once $\bar{a}$ has been determined, the radial position $\bar{r}$ of the maximum of the epicyclic frequency provides an upper limit on the size of the torus, i.e. $0 \leq L \lesssim \bar{r}$, where $d\kappa_r(a, M_*)/dr = 0 = \bar{r}$. These two estimates should be considered first approximations only and should be used to further define the properties of the oscillating region; in the case of XTE J1550-564, for instance, the values estimated in this way are $\bar{r} \simeq 3.49 r_g$, and $a \simeq 0.89$ (cf. Table 1), while the solution of the full eigenvalue problem built around these estimates yields the more accurate limits: $\bar{r} \simeq 1.75 r_g$, and $a \simeq 0.94$ (cf. Fig. 1). The values of $\bar{a}$ and $\bar{r}$ for the black hole candidates with HFQPO observations and for which an estimate of the mass is available, are presented in Table 1, which also shows a surprisingly small scatter. Given these estimated sizes and the location of the inner and outer edges of the tori, it is easy to calculate that the gravitational potential energy change across the torus is $\gtrsim 6\%$ of the energy at the outer edge; this relative energy loss could account for the modulated emission observed in QPOs.

We note that this model requires rather large values of the spin parameter. This is not too surprising given that the massive star progenitors of the black holes in X-ray binaries routinely have more angular momentum than even a maximally rotating black hole of the mass of the remnant (see, for instance, the velocity measurements in Brown, Verschueren 1997). The large required spins also make it unlikely that this model will be successfully applicable to neutron star HFQPOs; we re-emphasize that this does not represent a major problem for this model, since the neutron star systems do not show simple integer ratios of frequencies as part of the phenomenology of their HFQPOs.

It is also worth noting that if this model is confirmed to be
correct, it will give us a reliable means of placing an upper limit on the mass of a black hole while at the same time requiring the existence of a black hole (see Abramowicz et al., 2002). Observing relatively high-frequency QPOs in pairs with small integer ratios would give us good supportive evidence for the existence of low-mass black holes. At the present, in fact, there is no strong evidence for any compact objects with masses between the \( \frac{1}{4} M_\odot \) of a neutron star and the \( \frac{6.3}{4} M_\odot \) of GRO J 1655–40.

A final comment will be reserved to the possibility of finding support to this model from the observational data. There are now extensive observations of these black hole HFQPOs and interesting spectral and flux correlations are beginning to be found (see, for instance, Remillard et al., 2002a, and references therein). These observations could be used to deduce the presence of a small torus in the terminal part of the accretion discs. Furthermore, being based on a single assumption, our model for HFQPOs is simply constructed, but, equally simply, it can be confuted. We now discuss two observational constraints that if not met will cast serious doubts about whether this model is a satisfactory description of the phenomenology observed in HFQPOs. The first constraint can be easily deduced from Fig. 2 and basically requires the lower HFQPO frequency to be always less than the maximum possible epicyclic frequency for a black hole of mass \( M_* \), i.e. lower HFQPO frequency \( \leq \max(k_{\text{ep}}(a = 1, M_*)) \). The second constraint, on the other hand, is that even if the HFQPOs frequencies change as a result of the change in size of the torus, they should nevertheless appear in a harmonic ratio to within 5-10%. Both of these observational requirements are sufficiently straightforward to assess and therefore provide a simple and effective way of falsifying this model.

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Table 1. Lower limits on the black hole spin \( \bar{a} \), and upper limits on the torus size \( \bar{r} \), as estimated from the lower HFQPO frequency of black hole candidates.

| Source          | \( M/M_\odot \) | \( f \) (Hz) | \( \bar{a} \) | \( \bar{r}/r_g \) |
|-----------------|----------------|-------------|----------|----------------|
| GRS 1915+105    | 14.0           | 162         | 0.9748   | 2.739          |
| XTE J1550-564   | 10.0           | 184         | 0.8909   | 3.489          |
| GRO J1655-40    | 6.3            | 300         | 0.9035   | 3.391          |

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