Abstract

In this article, an energy-efficient gait planning algorithm that utilizes both 3D body motion and an allowable zero moment point region (AZR) is presented for biped robots based on a five-mass inverted pendulum model. The product of the load torque and angular velocity of all joint motors is used as an energy index function (EIF) to evaluate the energy consumption during walking. The algorithm takes the coefficients of the finite-order Fourier series to represent the motion space of the robot body centroid, and the motion space is gridded by discretizing these coefficients. Based on the geometric structure of the leg joints, an inverse kinematics method for calculating grid intersection points is designed. Of the points that satisfy the AZR constraints, the point with the lowest EIF value in each network line is selected as the seed. In the neighborhood of the seed, the point with the minimum EIF value in the motion space is successively approximated by the gradient descent method, and the corresponding joint angle sequence is stored in the database. Given a distance to be traveled, our algorithm plans a complete walking trajectory, including two starting steps, multiple cyclic steps, and two stopping steps, while minimizing the energy consumption. According to the preset AZR, the joint angle sequences of the robot are read from the database, and these sequences are adjusted for each step according to the zero-moment-point feedback during walking. To determine the effectiveness of the proposed algorithm, both dynamic simulation and walking experiment in the real environment were carried out. The experimental results show that compared with algorithms based on the fixed body height or vertical body motion, our gait algorithm has a significant energy-saving effect.

Keywords

Biped robot, five-mass model, gait planning, zero moment point, spatial gridding, gradient approximation

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Introduction

Bipedal robots have human-like structures and appearances, which can adapt to the human environment, and are ideal robots for replacing human work. In the past 50 years, research institutions and scholars at home and abroad have carried out much research on biped robots and have achieved remarkable progress.1,2 For example, humanoid robot HRP-4 can drive vehicles in the middle of the road,3 and ATLAS can traverse obstacles and climb stairs.4

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Bipedal walking is the most important and challenging problem in research on biped robot motion. According to different research ideas, the proposed biped walking methods can be divided into three approaches. One is to utilize high-speed optical motion capture systems to obtain data on human motion according to external characteristics of human walking and apply these features to the generation of robot motion modes. The other approach is to use a central pattern generator to simulate the neural network control of human walking to generate rhythmic signals, thereby solving the problem of robot gait generation. The third approach is to simplify the biped robot into a linear inverted pendulum model consisting of a point mass and a massless telescopic leg or a cart-table model consisting of a table without mass and a cart driving on the table with all the mass concentrated in it.

At present, the structural complexity of existing biped robots is much less than that of humans, which indicate that gait methods based on human characteristics and central pattern generators have limitations. Researchers prefer to adopt the method that involves simplifying the biped robot model. The accuracy of the simplified model determines the accuracy of the control effect. Based on the single-mass inverted pendulum model and considering the weight of the legs, Shimmyo et al. proposed biped walking pattern generation using preview control based on a three-mass model. Luo and Chen presented a three-mass angular momentum model and a five-mass momentum model using model predictive control to obtain better motion control accuracy. However, as model complexity increases, model nonlinearity increases and gait control becomes more difficult. Considering that the leg mass of the motor-driven biped robot is mainly distributed according to the position of the motor, the five-mass model, which includes five mass points representing the body, legs, and feet, is regarded as the best tradeoff.

To simplify the complexity of the algorithm used in gait control for biped robots, constraint conditions for the robot body are introduced, which cause unnatural walking motions, consume large amounts of energy, and limit the operation time of battery-powered robots. Hong et al. allowed vertical body motion and alleviated the problem of bending knee joints. Shin and Kim further removed constraints on the motion trajectory, which improved walking speed and reduced actuator energy consumption. If the robot body is allowed to move with more degrees of freedom, a better control effect can be expected. In this article, the bipedal body center of mass is allowed to move in three dimensions.

Fewer constraint conditions result in an increase in parameter space in gait control. It would be optimal if a solution could be found by artificial intelligence. Dau et al. applied a genetic algorithm to optimize the seven key parameters defining the hip and foot trajectories. Elhosseinia et al. designed a whale optimization algorithm with a random parameter “a” and weight parameter “C” to find the optimal setting for the hip parameters. Wu and Li used fuzzy logic to control the dynamic gait pattern generator of a humanoid robot, resulting in better responses to external force disturbances. Wright and Jordanov summarized the commonly used intelligent approaches to robot locomotion control. Generally, the range and accuracy of parameters need to be included in the optimization algorithm. This article presents a hierarchical optimization algorithm, that is, the grid computing method, to find the best region in a large range of parameter value spaces. Then, the gradient approximation method is used to find the optimal gait parameters for the robot with high accuracy.

The rest of this article is organized as follows. In the second section, we describe the simplified biped robot model and formulate the problem of gait planning. The third section focuses on the gait planning algorithm from two aspects: optimal minimization of energy consumption and real-time motion planning. According to the algorithm in this article and various control methods proposed in the related literature, the fourth section presents the simulation and experimental results for the performance analysis of the proposed algorithm. In the fifth section, the algorithm is summarized, and possible follow-up work is presented.

Problem formulation

System structure

The humanoid robot used in this article has two arms and two legs, thereby imitating the walking motion of the human body. Each leg has five degrees of freedom (DoFs), including two DoFs at the hip, one DoF at the knee, and two DoFs at the ankle, and the joint vector can be expressed as $q = [q_1, q_2, \ldots, q_{10}]^T$. The origin in the global coordinate system is located at the midpoint of the two legs when the robot is upright. The structure of the experimental robot is shown in Figure 1. Four force-sensing resistor (FSR)
sensors are installed under the sole of each foot to measure the distribution of force on the feet. The parameters for the body and right leg are summarized in Table 1, and the parameters for the left leg are the same as those for the right leg.

In general, some constraints are needed to simplify the scope of an analysis of robot walking. The proposed walking gait assumes the following conditions:

(a) The upper body remains upright at all times. The pitch rotation of the human trunk is small, within \(3^\circ \)–\(25^\circ\) and energy consumption increases as the trunk is leaned forward.\(^26\) Therefore, most relevant research studies\(^3,5,10-23\) have shown that this assumption is acceptable.

(b) Both feet are always parallel to the ground. Most common humanoid robots have no toes and cannot utilize the toes to improve driving the lifting and planting of the feet.\(^27,28\)

(c) One step of duration \(T\) contains a double support phase (DSP) \(T_{DSP}\) and a single support phase (SSP) \(T_{SSP}\) and defines the duty ratio of the DSP as \(\sigma = \frac{T_{DSP}}{T}\). In the process of human walking, \(\sigma\) is approximately 15–25\%\(^25\); \(\sigma = 25\%\) was selected for the algorithm proposed in this study.

### Zero moment point equations

Among various dynamic stability standards for biped robots, the most widely used technique is the zero moment point (ZMP).\(^29\) The ZMP is the point on the ground where the horizontal component of the total moment generated by gravity and inertia forces is zero. The five mass points of the biped robot are the body and right leg. The five mass points of the horizontal component of the total moment generated by point (ZMP).\(^29\) The ZMP is the point on the ground where robots, the most widely used technique is the zero moment point equations

\[
\mathbf{n} \times \mathbf{M}_{ZMP} = \mathbf{n} \times (\mathbf{M}_s - \mathbf{r}_{ZMP} \times \mathbf{F}) = 0 \quad (1)
\]

where

\[
\mathbf{F} = \sum_{i \in P_c} \mathbf{m}_i (\mathbf{g} - \mathbf{\dot{r}}_i)
\]

\[
\mathbf{M}_s = \sum_{i \in P_c} \left( \mathbf{r}_i \times \mathbf{m}_i (\mathbf{g} - \mathbf{\dot{r}}_i) - \mathbf{I}_i \times \mathbf{\ddot{\theta}}_i \right) \quad (3)
\]

Here, \(\mathbf{n} = [0,0,1]^T\) is the ground-surface unit normal vector, \(\mathbf{g} = [0,0,g]^T\) is the gravitational acceleration vector, \(g = 9.80 \text{ m/s}^2\), and \(\mathbf{I}_i\) and \(\mathbf{\ddot{\theta}}_i\) are the rotational inertia and angular acceleration at position \(i\), respectively. Kajita et al. proved that the influence of \(\mathbf{\ddot{\theta}}\) is small and can be ignored.\(^10\) By substituting equations (2) and (3) into equation (1), the simplified ZMP equations are

\[
\begin{align*}
x_{ZMP} &= \frac{\sum_{i \in P_c} m_i (\dot{z}_i x_i + g x_i - z_i \ddot{x}_i)}{\sum_{i \in P_c} m_i (\dot{z}_i + g)} \quad (4) \\
y_{ZMP} &= \frac{\sum_{i \in P_c} m_i (\dot{z}_i y_i + g y_i - z_i \ddot{y}_i)}{\sum_{i \in P_c} m_i (\dot{z}_i + g)}
\end{align*}
\]

### Allowable zero moment point region

Modeling errors inevitably exist when modeling biped robots. Hong et al. set the ZMP trajectory near the centerline of the support foot to achieve the greatest stability during walking,\(^11\) but this is not an energy-efficient method. In the region of the support foot, some edge

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**Table 1. Basic parameters of the experimental robot.**

| Symbol | Description       | Value  |
|--------|-------------------|--------|
| \(m_b\) | Upper body mass   | 1.2 kg |
| \(m_{rl}\) | Right leg mass    | 0.2 kg |
| \(m_{rf}\) | Right foot mass   | 0.2 kg |
| \(l_h\) | Hip width         | 0.08 m |
| \(l_1\) | Link length from 1 to 2 | 0.04 m |
| \(l_2\) | Link length from 2 to 3 | 0.085 m |
| \(l_3\) | Link length from 3 to 4 | 0.075 m |
| \(l_4\) | Link length from 4 to foot | 0.04 m |
| \(l_f\) | Foot length       | 0.12 m |
| \(l_w\) | Foot width        | 0.06 m |

---

**Figure 2. Five-mass biped robot model:** (a) Sagittal plane and (b) frontal plane.
regions are drawn out to compensate for the modeling errors, and the ZMP trajectory is located in the allowable ZMP region (AZR) of the middle subregion of the support foot. This is a better method for balancing modeling errors and walking efficiency. For a biped robot with foot length \( l_{F} \) and foot width \( l_{FW} \), the step length is \( s \), and the \( y \)-axis distance between the two feet is \( w \). The AZR when the left foot (LF) is supporting and the right foot (RF) is swinging is shown in Figure 3. The AZR during the first DSP is a hexagon with \((r_1, r_2, \ldots, r_6)\), that during the SSP is a square with \((r_3, r_4, r_5, r_6)\), and that during the second DSP is a hexagon with \((r_5, r_6, \ldots, r_{10})\). In our gait planning algorithm, \( \eta_{AZR}^{u} \) and \( \eta_{AZR}^{v} \) are used to represent the unit values of the AZR subregions on the \( x \) and \( y \) axes, respectively. \( \eta_{AZR}^{v} \) takes a constant value, so the default is \( \eta = \eta_{AZR}^{u} \).

**Energy consumption index function**

The energy consumption \( E \) of a biped robot can be divided into two parts: the energy consumption \( E_m \) for robot motion and the energy consumption \( E_a \) for nonmotion. \( E_m \) is the main component of \( E \), which is the integral of the instantaneous power consumption vector \( p_m(t) \) for all joint motors over time \( t \). \( E_a \) is used for sensors, controllers, and so on. Their power \( P_a \) is relatively stable during operation and can be expressed as a linear function of time. \( E \) can be described by

\[
E = E_m + E_a = \int_{0}^{t} p_m(\xi)d\xi + P_at
\]  

In general, \( E_m \) can be significantly reduced through gait trajectory optimization, \(^{30} \) which is suitable for evaluating the performance of the gait algorithm. For biped robots with many motors, it is difficult to accurately measure \( p_m(t) \). \( p_m(t) \) is equal to the sum of the output power consumption \( p_2(t) \) and subsidiary electrical loss \( p_{ed}(t) \); \( p_2(t) \), its main component, can be calculated by multiplying the motor output torque \( \tau_2(t) \) and angular velocity \( \dot{q}(t) \). \(^{31} \) When the robot is moving, \( \tau_2(t) \) is used to overcome the load moment \( \tau_d(t) \) formed by all CoMs on the motor rotor, where \( \tau_d(t) = [\tau_1(t), \tau_2(t), \ldots, \tau_{10}(t)]^T \) and \( \tau_f(t) \) can be written as

\[
\tau_f(t) = l_i^T(t)C_i(t)m, \ i = 1, 2, \ldots, 10
\]  

where \( m = [m_b, m_{lb}, m_{ld}, m_{rb}, m_{rd}]^T \) is the CoM vector, \( l_i(t) \) is the effective distance vector between \( m \) and the \( i \)-th rotor, and \( C_i(t) \) is the \( 5 \times 5 \) diagonal matrix describing the proportion of \( m \) acting on the rotor.

It is assumed that the sampling period \( t_s \) of the control system is small enough between the two sampling points with \( \tau_i(n) = \tau_i(n \cdot t_s) \approx \tau_i((n-1) \cdot t_s) \), and \( \ddot{q}_i(n \cdot t_s) \approx \ddot{q}_i((n-1) \cdot t_s) \), where \( i = 1, 2, \ldots, 10 \). In general, the joint motors of the robot carry out cyclical motion with the period \( N \), and even if gravity and the direction of motion are the same, most robots are not able to regenerate energy. Therefore, we define the energy consumption index function \( E \) for the gait algorithm as follows

\[
E = \frac{1}{t_s} \sum_{n=1}^{N} \sum_{i=1}^{10} \tau_i(n) |\dot{q}_i(n) - \dot{q}_i(n-1)|
\]

**Problem definition**

Based on the previous analysis, our energy-efficient gait planning problem can be formulated as follows:

(a) Given the step length and AZR of a biped robot, find the optimal gait parameters, that is, the parameters that minimize \( E \), while allowing 3D body motion and satisfying the constraints in “System structure” section.

(b) For a goal distance \( d \) of bipedal walking, control the value of \( \eta \) for the AZR in real time, thereby minimizing the total energy consumption of the complete gait trajectory, including two starting steps, multiple cyclic steps, and two stopping steps.

**Gait planning algorithm**

**Overview**

(a) In the gait planning optimization (GPO) algorithm, given the step length set \( S \) and the AZR set \( H \) for bipedal walking, 18 parameters are represented by the body trajectory \( S \) and the range \( U_e \) of the parameters is gridded. After the inverse kinematics calculation, the parameter subset \( R_s \) satisfying the stability and physical constraints is obtained, and the seed set \( P_s \) meeting the \( \eta \in H \) requirement is selected from \( R_s \). Then, in the neighborhood of \( p \in P_s \), the joint angle sequence \( g^0 = \{q_i(n) \mid n = 1, 2, \ldots, N\} \) with minimum \( E \) is obtained by iterative calculation according to the gradient descent method and stored in an offline database. The GPO algorithm takes a long time and
is suitable for offline operation. It completes the calculation of $S$ and $H$ and copies the offline database to the online database, which is called in real time in the gait synthesis (GSYN) algorithm.

(b) Given the required walking distance $d$ and the value of $\eta$ for the AZR, the GSYN algorithm plans the step sequence $S^*$ to achieve the minimum $E$. Each step length $s_i \in S^*$ and initial value $\eta_0$ of the AZR are removed in turn, and the online database is queried to obtain the motor angle sequence $g_i$ for gait control. By inputting $g_i$ into the joint controllers, the robot can walk. During walking, according to the foot pressure set $F_i$, the real-time ZMP trajectory is calculated. In the AZR control, the deviation between $r_{ZMP}$ and $r_{AZR}$ is calculated from $r_{AZR}$, which is required for stability, and the PI correction method is used to correct $\eta_0$, optimizing the tradeoff between low energy consumption and robustness of robot motion.

**Gait planning optimization algorithm**

The GPO algorithm is used to generate a low-energy joint angle sequence $g^*_{ij}$ for gait control; this is the core of our proposed algorithm. According to the accuracy requirements of bipedal walking, the range in step length, from minimum to maximum, is discretized to set $S_i$ and $H$ selects several levels from the interval $[0, 1]$ to adapt to different robustness requirements. The GPO algorithm selects the elements $s$ and $\eta$ from $S$ and $H$, respectively, to calculate the distribution gradient and obtain $g^*_{ij}$ for the minimum $E$.

**Gridding method.** Our proposed gridding method is a process of discretizing multidimensional parameters by appropriate intervals, constructing parameter space grids, and then calculating the value of the function for each gridded line intersection point. If the walking step length is $s$, the robot body $b$ performs a 3D cyclic motion satisfying the constraint (a) in “System structure” section, and the position $r_{b}(n) = [x_b(n) \ y_b(n) \ z_b(n)]^T$ can be expressed by a Fourier series of finite terms as follows

$$
x_b(n) = \frac{s}{N}n + \sum_{k=1}^{3} a_k \sin(k\omega_0n) + \sum_{k=4}^{6} a_k \cos((k-3)\omega_0n) + \sum_{k=7}^{11} a_k \sin((k-6)\omega_0n) + \sum_{k=12}^{15} a_k \sin((k-12)\omega_0n) + \sum_{k=16}^{18} a_k \cos((k-15)\omega_0n)
$$

where $N$ is the gait period, $\omega_0 = \pi/N$, $n = 1, 2, \cdots, 2N$, and $r_b(n)$ are represented by the parameters $\{a_k : k = 1, 2, \cdots, 18\}$. If $a_k \in [d_k, u_k]$ is discretized according to the interval $\mu_k$ and its range is the set $A_k = \{d_k, d_k + \mu_k, d_k + 2\mu_k, \cdots, u_k\}$, then the intersection set $U_i$ formed by the gridding method is the Cartesian product of $A_{18}$, that is, $U_i = A_1 \times A_2 \times \cdots \times A_{18}$.

**Inverse kinematics calculation.** To utilize the parallel acceleration of the GPU, the inverse kinematics algorithm is designed for the geometric structure of the robot leg. Without loss of generality, suppose that the robot starts walking during the DSP with the left leg in front and the right leg in the rear; then, the right leg swings forward. The starting position of the RF leg is $r_{rf}(0) = [0 \ 0 \ 0]^T$, and the motion satisfies constraint (c) in “System structure” section; then, the motion trajectory of $r_{rf}(n) = [x_{rf}(n) \ y_{rf}(n) \ z_{rf}(n)]^T$ is

$$
x_{rf}(n) = \begin{cases} 0, & 1 \leq n \leq N_1 \\ s \left(1 - \cos \left(\frac{(n-N_1)\pi}{K+1}\right)\right), & N_1 < n \leq N_2 \\ 2s, & N_2 < n \leq 2N \\ \frac{w_i}{2}, & 1 \leq n \leq 2N \\ \end{cases}
$$

$$
y_{rf}(n) = \begin{cases} 0, & 1 \leq n \leq N_1 \\ h_k \sin \left(\frac{(n-N_1)\pi}{K+1}\right), & N_1 < n \leq N_2 \\ 0, & N_2 < n \leq 2N \end{cases}
$$

$$
z_{rf}(n) = \begin{cases} \frac{s}{N}n + \sum_{k=1}^{3} a_k \sin(k\omega_0n), & 1 \leq n \leq N_1 \\ \sum_{k=4}^{6} a_k \cos((k-3)\omega_0n), & N_1 < n \leq N_2 \\ \sum_{k=7}^{11} a_k \sin((k-6)\omega_0n), & N_1 < n \leq N_2 \\ \sum_{k=12}^{15} a_k \sin((k-12)\omega_0n), & N_1 < n \leq N_2 \\ \sum_{k=16}^{18} a_k \cos((k-15)\omega_0n), & N_1 < n \leq N_2 \end{cases}
$$

where $w_i$ is the step width, $h_k$ is the step height, $N$ is the gait cycle, $N_1 = N/8$, and $N_2 = 7N/8$. Cyclic bipedal walking has the property of symmetry, and the LF trajectory is $r_{lf}(n) = [x_{lf}(n+N) + x_{rf}(n+N) \ y_{rf}(n+N) \ z_{rf}(n+N)]^T$.

Given $r_{b}(n)$, according to constraint (a) in “System structure” section, the right hip joint trajectory is $r_{rh}(n) = r_{b}(n) - [0 \ 0.5h_k \ l_b]^T$. Figure 4 shows the geometric relations between the joint angles of the robot leg.

Combining $r_{b} = [r_{rh}, r_{hi}]$ and $r_{f} = [r_{rf}, r_{lf}]$ that need to be calculated, the process for obtaining the inverse kinematics solution presented in this article is shown in Algorithm 1.

Generally, $U_i$ has a large number of elements. Considering the locomotion characteristics of $r_{b}(n)$, unreasonable elements in $U_i$ can be excluded, thus reducing the number of inverse kinematics calculations needed. Common characteristics of $r_{b}(n)$ include the following

(a) $x_b(n)$ increases predictably during bipedal walking.
(b) $y_b(n)$ is convex during RF swing $N$ and concave during LF swing $N$.  

For $r_s$ then, the corresponding ZMP trajectory can be obtained.

(a) Physical constraint: for $z_b$ distance are less than the leg length, that is, $\max |z_b| < L_s$, which satisfies the following:

\[
(z_b) = \left\{ \begin{array}{ll}
0 & \text{for } 0 < z_b < L_s \\
L_s & \text{for } z_b < 0 \\
-L_s & \text{for } z_b > L_s 
\end{array} \right.
\]

(b) ZMP condition: the ZMP trajectory corresponding to $a_i$ satisfies the stability requirement.

The wider AZR consumes less energy but yields less walking stability. In gait control of the biped robot, multiple $\eta$ values are taken to constitute the set $H$. The dynamic adjustment of $\eta$ during walking can achieve a better compromise between energy consumption and stability. The element $a_0$ in $R_s$ that meets the requirement of $\eta$ is used to calculate $E(a_0)$ according to equation (7). If the range of $E(a_0)$ contains a finite number of convex sets, the seed set $P_s = \{p_1, p_2, \ldots, p_N\}$, where $p_i \in P_s$, satisfies the following:

(a) There is a linear relationship between the number of seeds $\Lambda$ and the number of parameters. That is, $\Lambda \leq n_1 + n_2 + \cdots + n_{18}$, where $n_k$ is the number of $a_k$ values in $\forall a_0$ and $k = 1, 2, \ldots, 18$.

(b) The energy consumption of the same column grid is the lowest. Namely, $p_j = a_0$ if and only if $E(a_0) = \min(E(A_1))$, where $A_k$ is the set containing the $j$th value of $a_k$ in $\forall a_0$, where $j = 1, 2, \ldots, n_k$.

Gradient optimization. Obviously, the seed $p_j \in P_s$ is a gridded intersection in $U_s$. If $E(p_j)$ of $p_j = [a_1, a_2, \ldots, a_{18}]^T$ achieves convexity in adjacent grids, then the search space $L_s = B_1 \times B_2 \times \cdots B_{18}$ is formed with $p_j$ as the center and $s_j = [b_1, b_2, \ldots, b_{18}]^T$ as the search interval, where $B_k = [a_{k-1} - \beta_k, a_k + \beta_k]$ and $k = 1, 2, \ldots, 18$.

For $\xi \in L_s$, the inverse kinematics and $E$ are calculated. If $\xi$ satisfies $E(\xi) = \min(E(\xi) \in L_s)$ and its ZMP meets the requirements of $\eta$, then taking the gradient $\nabla E(\xi) = \xi - p_j = [\beta_1, \beta_2, \ldots, \beta_{18}]$, a new search space $L_s = B_1 \times B_2 \times \cdots B_{18}$ is obtained, where $B_k = B_k + \beta_k$. The specific calculation process is shown in Algorithm 2.

Gait synthesis algorithm

For a given walking distance $d$, there is a sequence $S' = \{s_1, s_2, \ldots, s_c\}$ with minimum $E$. To make the walking process smooth, both $s_1$ and $s_2$ are planned as starting steps, both $s_{c-1}$ and $s_c$ are stopping steps, and $s_m$ is a cyclic step, where $m = 3, 4, \ldots, c - 2$. We let $s_1 = s_m$, $s_2 = s_{c-1}$, $\forall s_m$ cover equal distances and $d_b = 2s_1 + 2s_2$ be controlled in $1 \sim 2s_m$, that is, $s_m < d_b \leq 2s_m$. When $s_m$ is known, the calculation of $s_1$ and $s_2$ is shown in equation (10).
Algorithm 2. Gradient optimization algorithm

\[
\begin{align*}
g^0_i & \leftarrow \min\{E(p_1), E(p_2), \ldots, E(p_i)\}. \\
g^1_i & \leftarrow g^0_i + (c - 4)E(s_i^*) + E(s_i^0) \\
E(s_i^0) & = E(s_i^0) + E(s_i^0) + E(s_i^0) \quad (11)
\end{align*}
\]

where \( y_{ZMP}(n) = \sum_{j=1}^{c_n} y_j(n)f_j(n) \bigg/ \sum_{j=1}^{c_n} f_j(n), \)
and \( f_j(n) \in F_c(n) \) correspond to the sensor position and pressure in the \( Y \) direction, respectively. For the SSP, \( c_n = 4, \)
and for the DSP, \( c_n = 6. \)
When \( y_{AZR}(n) \geq 0, c_n = 1; \)
and \( y_{AZR}(n) < 0, c_n = -1. \)

The incremental PI regulator with transfer function \( D(s) = k_P \left(1 + \frac{1}{sC_20}\right) \)
is used to calculate \( \eta_{i+1}, \) that is
\[
\eta_{i+1} = \eta_i + \Delta\eta_{i+1} \tag{13}
\]
where \( \Delta\eta_{i+1} = k_P(e_i - e_{i-1}) + k_D \frac{\Delta\eta_{i+1}}{\Delta t}. \) Because \( (e_i - e_{i-1}) \)
easily causes high frequency interference, the first-order inertia link \( G(s) = \frac{1}{sC_{21}} \) is introduced to smooth its output, so that
\[
\Delta\eta_{i+1} = K_0\Delta\eta_i + K_1e_i + K_2e_{i-1} \tag{14}
\]
where \( K_0 = \frac{T_s}{T_s + T_c}, K_1 = k_P\left(\frac{T_s}{T_s + T_c} + \frac{T_c}{T_c}\right), \)
and \( K_2 = -k_P\frac{T_s}{T_s + T_c}. \)

Experimental evaluation

According to the structure of the biped robot described in “System structure” section, a 3D simulation model is constructed in Webots to evaluate our proposed algorithm. In the GPO algorithm, we take \( S = \{s_i \; | ; s_i = 1, 2, \cdots, 13\} \) and \( H = \{|h_i; h_i = 0, 1, 0.2, \cdots, 1\} \) and calculate the gait trajectory \( g^p \) from their combination. Taking \( s = 10 \) as an example, \( r_s(n) \) is described by shown in equation (8), where \( N = 16, \mu_k = 0.2 \) cm, and \( k = 1, 2, \ldots, 18; r_s(n) \) is shown in equation (9), where \( w_s = 8 \) cm and \( h_6 = 1 \) cm. Using the features extracted from \( r_s(n), \) the number of effective grid elements is 1 763 449 134, and the inverse kinematics calculation is carried out with Algorithm 1. According to the physical constraints and ZMP conditions in “Seed extraction” section, 17 024 963 elements of \( R \) are obtained, and the seeds corresponding to different \( \eta \) are selected.

When \( \eta = 0.8, \) the number of seeds selected is \( \Lambda = 102, \) as presented in Table 2, where \( k \) is the label of \( a_k, \) and \( n_k \) is the actual number of seeds obtained by removing the repeatedly selected seeds. \( P_k \subset P_{10}^{0.8} \) and contains \( n_k \) seeds, and \( E_{\min}(P_k) \) is the minimum \( E \) of seeds in \( P_k. \) To obtain a gait with lower energy consumption, gradient optimization is performed twice with Algorithm 2 with \( \delta_k = 0.05 \) cm and \( \delta_k = 0.01 \) cm. \( E_{\min}(P_k) \) and \( D_1 \% \) and \( E_{\min}(P_k) \) and \( D_2 \% \) are the minimum values of \( E \) and the percent reduction in \( E \) in the two calculations, respectively. The \( E_{\min} \) curve of \( P_1 \sim P_{20} \) in the gradient optimization calculation is shown in Figure 6. In the first gradient optimization calculation, \( P_1 \sim P_{10}^{0.8} \), \( E_{\min}(P_{10}^{0.8}) = 182.3 \) mJ, and the results are marked with “a.” In the second calculation, \( E_{\min}(P_{10}^{0.8}) = 178.4 \) mJ, and the results are marked with “b.”

To compare the performance of different algorithms, the fixed hip height (FHH) is taken as the FHH algorithm, the
vertical variation in the hip height (VHH) according to the cosine waveform is taken as the VHH algorithm, and the other conditions are the same as the algorithm in this article. After optimization, $E_{\text{min}} = 222.8$ mJ for FHH algorithm and 198.7 mJ for VHH algorithm, which means that the energy consumption is reduced by 19.93% and 10.22%, respectively. In a single gait cycle, the $x_{\text{ZMP}}(n)$ trajectories of the three algorithms and the corresponding $E$ of each sampling point are identified in Figure 7, where $\sigma = 0.25$, $N = 16$, and $t_e = 0.1$ s. It can be seen from the
that compared with that of both FHH algorithm and VHH algorithm, the $E$ distribution of our algorithm is more balanced, and the total energy consumption value is lower.

According to the GPO algorithm, \(s_m \in S\) and \(\forall \eta = H\), the calculated $E_{\min}^s$ is shown in Figure 8. Obviously, as the value of $\eta$ decreases, the required $E_{\min}^s$ gradually increases to achieve the same $s$.

In the GSYN algorithm, if \(d = 100\) cm and \(\forall s_m \in S\), according to equations (10) and (11), $J_h^\eta(S)$ can be calculated, as shown in Figure 9, where $\eta = 0.2, 0.3, \cdots, 1$, and $s_m = 10$ cm. $J_h^\eta(S)$ is the minimum in the same column data. In this case, $c = 8, s_1 = 3$ cm and $s_2 = 7$ cm, that is, $S^* = \{3, 7, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 7, 3\}$.

If we let $\eta = 0.8$ and set the starting and stopping positions of walking to be two feet standing side by side, the swing length of the RF is $L_{rf} = \{3, 17, 20, 20, 17, 3\}$, and the swing length of the LF is $L_{lf} = \{10, 20, 20, 20, 10\}$. The two 3-cm elements in $L_{rf}$ correspond to the starting step and stopping step, respectively, where $E_{\eta}^0(3) = 65.2$ and $E_{\eta}^0(3) = 66.8$ mJ. There are two 17-cm elements in $L_{rf}$ and two 10-cm elements in $L_{lf}$, which correspond to the case when the movement distance of the swing foot is unequal before and after it becomes the support foot. From equation (11), $J_h^0(10) = 1.8967$ J, where $E_{\eta}^0(10) = 515.9$ mJ. The gait array for each step $g_i = [q_1, q_2, \cdots, q_{16}]$, which is obtained from the database, is used to control the
The algorithm has universal applicability for different biped robots that improve the energy efficiency of robot walking experiment is shown in Figure 11.

In the experiment, we set \( d = 100 \text{ cm}, \ \dot{h} = 0.5, \ T_S = 1.6 \text{ s}, \ k_P = 0.875, \ T_F = 3.2 \text{ s}, \ T_L = 1.6 \text{ s}, \) and we set \( y_{AZR}(n) \) along the \( \eta = 0.8 \) sideline during the SSP. The initial conditions for walking are \( e_0 = e_{-4} = 0 \) and \( \eta_0 = \dot{h} = 0.5. \) The \( \eta \) value of the AZR controller is stable at 0.8 after the third step. The calculation data are presented in Table 3, where the data in parentheses in the column titled \( s_i \) correspond to the movement distance of the swinging leg in front of and behind the support foot, \( E_i^{[0]} \) is the energy consumption calculated according to the actual \( \eta_i \), and \( E_i^{[0.5]} \) is the energy consumption value when \( \dot{h} = 0.5. \) It is calculated that \( J_s^{[0]}(10) = 1.929 \) and \( J_s^{[0.5]}(10) = 2.195J; \) that is, the AZR controller adjusts \( \eta_i \) to reduce the energy consumption by 12.13%.

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