Dualities in $D = 5$, $N = 2$ Supergravity, Black Hole Entropy, and AdS Central Charges

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Abstract: The issue of microstate counting for general black holes in $D = 5$, $N = 2$ supergravity coupled to vector multiplets is discussed from various viewpoints. The statistical entropy is computed for the near-extremal case by using the central charge appearing in the asymptotic symmetry algebra of $\text{AdS}_2$. Furthermore, we show that the considered supergravity theory enjoys a duality invariance which connects electrically charged black holes and magnetically charged black strings. The near-horizon geometry of the latter turns out to be $\text{AdS}_3 \times S^2$, which allows a microscopic calculation of their entropy using the Brown-Henneaux central charges in Cardy’s formula. In both approaches we find perfect agreement between statistical and thermodynamical entropy.

1 Introduction

The study of black hole solutions in $N = 2$ five-dimensional supergravity coupled to vector and hypermultiplets plays an important role in the understanding of the non-perturbative structure of string and M-theory [1, 2]. In this setting the interplay between classical and quantum results is exemplified at its best.

In this paper we consider general charged black holes of the $D = 5$, $N = 2$ theories, not necessarily those obtained from compactification of eleven-dimensional supergravity on a Calabi-Yau threefold. The analysis is simplified by the rich geometric structure of the $N = 2$ theories. Black hole solutions are given in terms of a rescaled cubic homogeneous prepotential which defines very special geometry [3]. In the extremal BPS case, half of the vacuum supersymmetries are preserved, while at the horizon supersymmetry is fully restored [4].

Here we focus on the asymptotic symmetries of the near-horizon geometry of the general near-extremal solution: the aim is the computation of the entropy from a counting of microstates to be compared to the macroscopic, thermodynamical entropy.

We will see that the calculation of the microscopic entropy of small excitations above extremality is equivalent to a microstate counting for certain black holes in two-dimensional
anti-de Sitter space. This can then be done by using the central charge of the $AdS_2$ asymptotic symmetry algebra in Cardy’s formula.

The main result however presented here is an explicit duality transformation, which realizes an invariance of the $N = 2$ supergravity action \[5\]. This duality turns the $AdS_2 \times S^3$ near-horizon geometry of the extremal black hole solution into $AdS_3 \times S^2$. The key point underlying the duality is the fact that the three-sphere can be written as a Hopf fibration over the base $S^2$. For $AdS_3$ the counting of microstates is performed using the Brown-Henneaux central charges \[6\] in Cardy’s formula, and it is shown that this reproduces correctly the Bekenstein-Hawking entropy.

In the case where the $D = 5$, $N = 2$ supergravity action is obtained by Calabi-Yau (CY) compactification of M-theory, the considered duality transformation, which maps electrically charged black holes onto magnetically charged black strings, corresponds to the duality between M2 branes wrapping CY two-cycles and M5 branes wrapping CY four-cycles. According to \[7\], M-theory compactified on $AdS_3 \times S^2 \times M$, where $M$ denotes some Calabi-Yau threefold, is dual to a $(0,4)$ superconformal field theory living on an M5 brane wrapping some holomorphic CY four-cycle. This fact has been used in \[8\] to compute the entropy of five-dimensional BPS black holes\[9\]. We stress that our method for microstate counting applies to any near-extremal black hole in $N = 2$, $D = 5$ supergravity, independent of whether it is obtained by CY compactification or not.

In section \[2\] the black hole solutions of $N = 2$, $D = 5$ supergravity coupled to vector multiplets are briefly reviewed. We thereby focus on the STU model as a simple example, which nonetheless retains all the interesting features of the general solutions. In section \[3\] we compute the statistical entropy of small excitations near extremality, using the $AdS_2$ central charge \[10\], and find perfect agreement with the Bekenstein-Hawking entropy. In section \[4\] we construct the duality transformation for the supergravity action, and in \[5\] we finally perform the state counting, using the fact that the near-horizon geometry of the dual solution includes an $AdS_3$ factor. In this way, we obtain a microscopic entropy which agrees precisely with the corresponding thermodynamical result.

## 2 Black Holes in $N = 2$, $D = 5$ Supergravity

$N = 2$, $D = 5$ supergravity coupled to an arbitrary number $n$ of Maxwell supermultiplets was first considered in \[1\]. In this theory, the scalar manifold can be regarded as a hypersurface in an $(n + 1)$-dimensional Riemannian space $\mathcal{R}$ with coordinates $X^I$. The equation of the hypersurface is $\mathcal{V} = 1$ where $\mathcal{V}$, the prepotential, is a homogeneous cubic polynomial in the coordinates of $\mathcal{R}$, $\mathcal{V}(X) = \frac{1}{6} C_{IJK} X^I X^J X^K$. One can then parametrize the hypersurface in terms of the $n$ scalar fields $\phi^i$ appearing in the vector multiplets, $X^I = X^I(\phi^i)$.

The bosonic part of the Lagrangian is given by

$$
e^{-1} \mathcal{L} = \frac{1}{2} R - \frac{1}{4} G_{IJ} F_{\mu \nu}^I F^{\mu \nu J} - \frac{1}{2} G_{ij} \partial_\mu \phi^i \partial_\mu \phi^j + \frac{e^{-1}}{48} \epsilon_{\mu \nu \rho \sigma \lambda} C_{IJK} F_{\mu \nu}^I F_{\rho \sigma}^J A^K.$$

The vector and scalar metric are completely encoded in the function $\mathcal{V}(X)$,

$$G_{IJ} = -\frac{1}{2} \partial_I \partial_J \ln \mathcal{V}(X)|_{\mathcal{V} = 1}, \quad \mathcal{G}_{ij} = G_{IJ} \partial_i X^I \partial_J X^J |_{\mathcal{V} = 1},$$

\[3\]The work in \[8\] includes as a special case also the results obtained in \[9\].
where \( \partial_i \) and \( \partial_I \) refer, respectively, to partial derivatives with respect to the scalar fields \( \phi^i \) and \( X^I \). Note that for Calabi-Yau compactifications of M-theory, \( C_{IJK} \) denote the topological intersection numbers, \( \mathcal{V}(X) \) represents the intersection form, and \( X^I = \frac{1}{6} C_{IJK} X^J X^K \) correspond, respectively, to the size of the two- and four-cycles of the Calabi-Yau threefold. In what follows, we will concentrate on the STU model \([1, 12]\), i.e. \( X^0 \equiv S, X^1 \equiv T, X^2 \equiv U, \mathcal{V}(X) = STU \). This model can be obtained by compactification of heterotic string theory on \( K_3 \times S^1 \) \([13]\).

The field equations following from the action (1) admit the non-extremal static black hole solutions \([14]\)

\[
 ds^2 = -e^{-4V} f dt^2 + e^{2V} (f^{-1} dr^2 + r^2 d\Omega_3^2), \\
 F^I_{rt} = -H^{-2}_I \partial_r \tilde{H}_I, \quad X^I = H^{-1}_I e^{2V}, \\
\]

where \( d\Omega_3^2 \) denotes the standard metric on the unit \( S^3 \). The \( H_I \) and \( \tilde{H}_I \) are harmonic functions,

\[
 H_I = 1 + \frac{Q_I}{r^2}, \quad \tilde{H}_I = 1 + \frac{\tilde{Q}_I}{r^2}, \\
\]

where the \( \tilde{Q}_I \) denote the physical electric charges. \( V \) and \( f \) read

\[
 e^{2V} = (H_0 H_1 H_2)^{1/3}, \quad f = 1 - \frac{\mu}{r^2}, \\
\]

with the nonextremality parameter \( \mu \). The physical charges are related to the \( Q_I \) by the equations

\[
 Q_I = \frac{\mu}{2} \sinh \beta_I \tanh \frac{\beta_I}{2}, \quad \tilde{Q}_I = \frac{\mu}{2} \sinh \beta_I. \\
\]

The extremal (BPS) limit is reached when \( \beta_I \to \infty, \mu \to 0 \), with \( \mu \sinh \beta_I \) kept fixed.

For the ADM mass \( M_{\text{ADM}} \), the Hawking temperature \( T_H \), and the Bekenstein-Hawking entropy \( S_{\text{BH}} \), one obtains

\[
 M_{\text{ADM}} = \frac{\pi}{4G_5} \left( \sum_I Q_I + \frac{3}{2} \mu \right), \quad T_H = \frac{\mu}{\pi \prod_I (\mu + Q_I)^{1/2}}, \\
 S_{\text{BH}} = \frac{A_{\text{hor}}}{4G_5} = \frac{\pi^2}{2G_5} \prod_I (\mu + Q_I)^{1/2}. \\
\]

In the extremal case, the near-horizon geometry becomes \( AdS_2 \times S^3 \).

### 3 Statistical Entropy from \( AdS_2 \) Central Charge

We would now like to use the near-horizon geometry \( AdS_2 \times S^3 \) to count the microstates which give rise to the black hole entropy \([8]\). As we are mainly interested in the \( AdS_2 \) factor, we perform a Kaluza-Klein reduction of the \( D = 5, N = 2 \) supergravity action \([4]\) to two dimensions. As we only consider nonrotating black holes carrying electric charge, we can consistently truncate the Chern-Simons term in \([3]\). The reduction ansatz for the metric is

\[
 ds^2 = \Phi^{-\frac{1}{2}} ds_2^2 + l_P^2 \Phi^{\frac{1}{2}} d\Omega_3^2, \\
\]

where \( \Phi \) denotes the dilaton and \( l_P \) is the Planck length in five dimensions. In two dimensions, the field strenghts \( F^I \) are proportional to the volume form and hence they
can be integrated out. In this way, one arrives at the two-dimensional action

$$I = \frac{\pi}{8} \int d^2 x \sqrt{-g} \left[ \Phi R + \frac{6}{l_P^2 \Phi^{1/3}} - \Phi G_{ij} \partial_i \phi^j \partial^a \phi^j - \frac{G^{IJ} \tilde{Q}_I \tilde{Q}_J}{l_P^5 \Phi^{5/3}} \right].$$

(10)

Let us now expand the nonextremal black hole solution (3) near extremality. To this end, we introduce an expansion parameter $\epsilon (\epsilon \to 0)$, and set

$$t = \frac{\tilde{t}}{\epsilon}, \quad r = \sqrt{\frac{2l_P^2 \epsilon x}{(\tilde{Q}_0 \tilde{Q}_1 \tilde{Q}_2)^{1/6}}} + \frac{\mu}{2}, \quad \mu = \mu_0 \epsilon,$$

$$\Phi = \frac{(\tilde{Q}_0 \tilde{Q}_1 \tilde{Q}_2)^{1/2}}{l_P^3} + \frac{4}{\pi} \eta \quad (\eta = O(\epsilon)), \quad \phi^j = \tilde{Q}_i^{-1} (\tilde{Q}_0 \tilde{Q}_1 \tilde{Q}_2)^{1/3} + \epsilon \tilde{\phi}^j.$$

(11)

One thus arrives at

$$ds^2 = -(\lambda^2 x^2 - a^2)dt^2 + (\lambda^2 x^2 - a^2)^{-1} dx^2$$

for the two-dimensional metric, with $\lambda$ and $a$ given by

$$\lambda = \frac{2l_P}{(\tilde{Q}_0 \tilde{Q}_1 \tilde{Q}_2)^{1/3}}, \quad a^2 = \frac{\mu_0^2}{4l_P^2 (\tilde{Q}_0 \tilde{Q}_1 \tilde{Q}_2)^{1/3}}.$$ 

(13)

The action at lowest order in the expansion parameter $\epsilon$ reads

$$I = \frac{1}{2} \int d^2 x \sqrt{-g} \eta [R + 2\lambda^2],$$

(14)

so the leading order is governed by the Jackiw-Teitelboim model (12), together with the linear dilaton

$$\eta = \eta_0 x, \quad \eta_0 = \frac{\Omega_0}{16 \pi l_P^2} (\tilde{Q}_0 \tilde{Q}_1 \tilde{Q}_2)^{2/3} \sum_I \tilde{Q}_I^{-1},$$

(15)

represents a black hole solution of this model (10), with mass, thermodynamical entropy and temperature given by

$$M_{(2)} = \frac{1}{2} \eta_0 a^2 \lambda, \quad S_{(2)} = 2\pi \eta_{hor} = 2\pi \eta_0 a, \quad T_{(2)} = \frac{a \lambda}{2\pi}.$$ 

(16)

This black hole spacetime has constant curvature, i.e. it is locally $AdS_2$. Now it is known that the asymptotic symmetries of two-dimensional anti-de Sitter space form a Virasoro algebra (10), similar to the case of $AdS_3$, where one has two copies of Virasoro algebras as asymptotic symmetries (3). This algebra was shown to have a central charge $c = 12 \eta_0$ (15).

Expanding the ADM mass $M_{ADM}$ (4) and Bekenstein-Hawking entropy $S_{BH}$ (8) of the black hole (3) in five dimensions for $\mu \to 0$, one obtains that small excitations above extremality have the energy and entropy

$$\Delta M_{ADM} = \frac{\pi \mu^2}{2l_P^2} \sum_I \tilde{Q}_I^{-1}, \quad \Delta S_{BH} = \frac{\pi^2 \mu}{8l_P^3} (\tilde{Q}_0 \tilde{Q}_1 \tilde{Q}_2)^{1/2} \sum_I \tilde{Q}_I^{-1}.$$ 

(17)

Comparing this with the two-dimensional results (16), one finds $\Delta S_{BH} = S_{(2)}$ and $\Delta M_{ADM} = \epsilon M_{(2)}$. The factor $\epsilon$ appearing in the relation between the two masses stems
from the fact that $M_{ADM}$ was computed with respect to the Killing vector $\partial_t$, whereas $M_{(2)}$ is related to $\partial_{\tilde{t}} = \epsilon \partial_t$. This means that up to these normalizations the five- and two-dimensional energies and entropies match.

Let us now compute the statistical entropy. Inserting the conformal weight $L_0 = M_{(2)}/\lambda$ together with the central charge in Cardy’s formula $S_{\text{stat}} = 2\pi \sqrt{c L_0}/6$ yields a statistical entropy which agrees precisely with the thermodynamical entropy $\Delta S_{BH}$ of the small excitations above extremality.

4 Duality Invariance of the Supergravity Action

In this section we will show that in presence of a Killing vector field $\partial_z$, the supergravity action (1) is invariant under a certain generalization of T-duality. The key observation is then that the three sphere $S^3$ appearing in the black hole geometry can be written as a Hopf fibration, i.e. as an $S^1$ bundle over $\mathbb{C}P^1 \approx S^2$. Performing then a duality transformation along the Hopf fibre untwists the $S^3$, and transforms the electrically charged black hole into a magnetically charged black string, which has $AdS_3 \times S^2$ as near-horizon limit in the extremal case.

To begin with, we reduce the action (1) to four dimensions, using the usual Kaluza-Klein reduction ansatz for the five-dimensional metric,

$$ds^2 = e^{k/\sqrt{3}} dz^2 + e^{-2k/\sqrt{3}} (dz + A_\alpha dx^\alpha)^2,$$

where $k$ denotes the dilaton, and early greek indices $\alpha, \beta, \ldots$ refer to four-dimensional spacetime. One thus arrives at the four-dimensional action

$$I_4 \equiv \frac{L}{16\pi G_5} \int d^4x \sqrt{-g_4} \left[ R_4 - \frac{1}{2} (\nabla k)^2 - \frac{1}{4} e^{-\sqrt{3}k} F^2 - \frac{1}{2} e^{-k/\sqrt{3}} G_{ij} \partial_\alpha \phi^i \partial^\alpha \phi^j \right] ,$$

where $L$ denotes the length of the circle parametrized by $z$, $\mathcal{F}$ is the field strength associated to the Kaluza-Klein vector potential $A$, and

$$\mathcal{F}^2 = F_{\alpha\beta} F^{\alpha\beta}, \quad F^2 = G_{IJ} F^I_{\alpha\beta} F^J{\alpha\beta}.$$

We now dualize both $\mathcal{F}$ and $F^I$, which yields

$$I_4 = \frac{L}{16\pi G_5} \int d^4x \sqrt{-g_4} \left[ R_4 - \frac{1}{2} (\nabla k)^2 - \frac{1}{4} e^{-\sqrt{3}k} (\star \mathcal{F})^2 
 - \frac{1}{2} e^{k/\sqrt{3}} \frac{1}{4} G^{IJ} \star F_{I\alpha\beta} \star F^\alpha_{\beta} - G_{ij} \partial_\alpha \phi^i \partial^\alpha \phi^j \right] ,$$

where we defined

$$\star F_{\alpha\beta} = \frac{1}{2} e^{-\sqrt{3}k} \epsilon_{\alpha\beta\gamma\delta} F^{\gamma\delta}, \quad \star F_{I\alpha\beta} = e^{-k/\sqrt{3}} G_{I\alpha\beta} \epsilon_{\alpha\beta\gamma\delta} F^{\gamma\delta}.$$

Comparing (22) with (19), we observe that the gravitational and gauge field parts of the four-dimensional action, as well as the dilaton kinetic energy, are invariant under the $\mathbb{Z}_4$ transformation

$$k \rightarrow -k, \quad F_{\alpha\beta} \rightarrow \star F_{\alpha\beta}, \quad F^I_{\alpha\beta} \rightarrow \star F_{I\alpha\beta}, \quad G_{IJ} \rightarrow \frac{1}{4} G^{IJ}.$$
The $\mathbb{Z}_4$ is actually a subgroup of the usual symplectic $Sp(2m + 2, \mathbb{R})$ duality group of $D = 4, N = 2$ supergravity (coupled to $m$ vector multiplets). Note that the transformation $G_{IJ} \to G_{IJ}/4$ means that

$$X^I \to 3X_I = \frac{1}{2} C_{IJK} X^J X^K, \quad X_I \to \frac{1}{3} X^I,$$

(24)

so essentially the special coordinates go over into their duals. As (24) does not change the kinetic term of the scalar fields, (23), (24) represent in fact a duality invariance of the four-dimensional action (19). In the special case of the $STU = 1$ model, (24) implies that the moduli $\phi^i$ go over into their inverse, $\phi^i \to 1/\phi^i$. We now wish to apply the duality (23), (24) to the black hole solution (3). To this end, we consider the $S^3$ as an $S^1$ bundle over $S^2$, and write for its metric

$$d\Omega_3^2 = \frac{1}{4} \left[ d\vartheta^2 + \sin^2 \vartheta d\varphi^2 + (d\zeta + \cos \vartheta d\varphi)^2 \right],$$

(25)

where $\zeta$ ($0 \leq \zeta \leq 4\pi$) parametrizes the $S^1$ fibre. Introducing the coordinate $z = \lambda \zeta$, where $\lambda$ denotes an arbitrary length scale, one can write the 5d metric in the KK form (18), where

$$ds^2_{5} = r e^V \left[ -e^{-4V} f dt^2 + e^{2V} f^{-1} dr^2 + e^{2V} r^2 \left( d\vartheta^2 + \sin^2 \vartheta d\varphi^2 \right) \right],$$

(26)

$$e^{-k/\sqrt{3}} = \frac{r e^V}{2\lambda}, \quad \mathcal{A} = \lambda \cos \vartheta d\varphi.$$

We now dualize in 4d according to (23), and then relift the solution to five dimensions. This yields the configuration

$$ds^2 = e^{-2V} \left\{ \frac{\mu}{4\lambda^2} dt^2 + 2dzdt + \frac{4\lambda^2}{r^2} d\zeta^2 \right\} + \frac{r^2}{4\lambda^2} e^{4V} \left[ f^{-1} dr^2 + \frac{r^2}{4} d\Omega_2^2 \right],$$

$$F^{I}_{\vartheta\varphi} = \frac{\tilde{Q}_I}{4\lambda} \sin \vartheta, \quad X^I = H_I e^{-2V}.$$

(27)

One effect of the duality transformation is thus the untwisting of the Hopf fibration. One can further simplify (27) by an $SL(2, \mathbb{R})$ transformation

$$\left( \begin{array}{c} t' \\ z' \end{array} \right) = \left( \begin{array}{cc} 0 & -2\lambda \sqrt{\mu} \\ \frac{\sqrt{\mu}}{2\lambda} & 2\lambda \sqrt{\mu} \end{array} \right) \left( \begin{array}{c} t \\ z \end{array} \right).$$

(28)

Introducing also the new radial coordinate $\rho = r^2/(4\lambda)$, we then get for the metric

$$ds^2 = e^{-2V} \left( -ft'dt'^2 + dz'^2 \right) + e^{4V} \left( f^{-1}d\rho^2 + \rho^2 d\Omega_2^2 \right).$$

(29)

(29), together with the gauge and scalar fields given in (27), represents a nonextremal generalization of the supersymmetric magnetic black string found in [4]. The duality (23) thus maps electrically charged black holes onto magnetically charged black strings.

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5The fact that Hopf bundles can be untwisted by T-dualities was observed in [17]. The idea of untwisting and twisting fibres to relate strings and black holes, and thus to gain new insights into black hole microscopics, was also explored in [18].
5 Microstate Counting from AdS3 Gravity

We now want to use the near-horizon geometry of the dual solution (29) to count the microstates giving rise to the Bekenstein-Hawking entropy. In [4] it was shown that in the extremal case, the geometry becomes $AdS_3 \times S^2$ near the event horizon. The idea is now to use the central charge of $AdS_3$ gravity [6] in Cardy’s formula, in order to compute the statistical entropy, like it was done by Strominger [19] for the BTZ black hole. As only the $AdS_3$ part is relevant, we would like to reduce the supergravity action from five to three dimensions. To this end, we first Hodge-dualize the magnetic two-form field strength in (27). For the solution under consideration, the field strengths $H_I$ dual to the $F_I$ do not depend on the coordinates of the internal $S^2$. Furthermore, in 3d the three-forms $H_I$ are proportional to the volume form and can be integrated out. For the metric, we use the reduction ansatz

$$ds^2 = \Phi^{-1} ds_3^2 + l_p^2 \Phi^2 d\Omega_2^2,$$

where $d\Omega_2^2$ denotes the standard metric on the unit $S^2$. This gives the reduced action

$$I = \frac{1}{4l_p} \int d^3x \sqrt{-g} \Phi^\frac{3}{2} \left[ R + \frac{2}{l_p^2 \Phi^3} - \frac{3}{2} \Phi^2 (\nabla \Phi)^2 - \frac{G_{IJ} P_I P_J}{\Phi^3 l_p^3} - G_{ij} \partial_\alpha \phi^i \partial^\alpha \phi^j \right],$$

where we introduced the magnetic charges $P_I = \tilde{Q}_I / (4\lambda)$. The idea is now to expand the 3d metric $ds_3^2$ near the horizon and near extremality. This can be done by setting

$$t' = t'' (2\lambda)^4 \sqrt{\frac{l_p}{\mu_0 \lambda \tilde{Q}_0 \tilde{Q}_1 \tilde{Q}_2}}, \quad z' = z'' (2\lambda)^2 \sqrt{\frac{l_p}{\mu_0}}, \quad \rho = \epsilon \tilde{r}^2 \mu_0 l_p (2\lambda)^4, \quad \mu = \mu_0 \epsilon,$$

and taking the limit $\epsilon \to 0$. This leads to the metric

$$ds_3^2 = -\tilde{r}^2 - \tilde{z}^2 dt''^2 + \tilde{r}^2 dz''^2 + \frac{l_{eff}^2 d\tilde{r}^2}{\tilde{r}^2 - \tilde{r}_+^2},$$

where we introduced

$$\tilde{r}_+^2 = \frac{4\lambda^3}{l_p}, \quad l_{eff}^2 = \frac{\tilde{Q}_0 \tilde{Q}_1 \tilde{Q}_2}{16l_p \lambda^3}.$$

We recognize (33) as the BTZ black hole, with event horizon at $\tilde{r} = \tilde{r}_+$. $\Lambda_{eff} = -l_{eff}^2$ is the effective cosmological constant. The effective 3d Newton constant can be read off from the action (31), yielding

$$\frac{1}{16\pi G_{eff}} = \frac{1}{4l_p} \Phi_{hor}^{3/2},$$

where the subscript indicates that the dilaton $\Phi$ is to be evaluated at the horizon. The BTZ black hole mass is given by

$$M_{(3)} = \frac{\lambda^3}{2l_p G_{eff} l_{eff}^2}.$$

We can now apply Strominger’s counting of microstates [19] to reproduce the Bekenstein-Hawking entropy. To this end, one first observes that the central charge appearing in the

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6Cf. also [20], where similar computations for black strings in six dimensions with BTZ $\times S^3$ near-horizon geometry were performed.
asymptotic symmetry algebra of \( AdS_3 \) in our case reads \( c = 3l_{\text{eff}}/(2G_{\text{eff}}) \). Furthermore, we have the relations

\[
M_{(3)} = \frac{1}{l_{\text{eff}}}(L_0 + \bar{L}_0), \quad J = L_0 - \bar{L}_0 = 0
\]

for the mass and angular momentum. We then obtain from Cardy’s formula a statistical entropy which coincides precisely with the thermodynamical entropy \( (3) \) of the 5d black hole \( (3) \).

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