Chemical reaction and radiation effects on MHD flow past an exponentially stretching sheet with heat sink

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Abstract. In this study, the problem of MHD boundary layer flow past an exponentially stretching sheet with chemical reaction and radiation effects with heat sink is studied. The governing system of PDEs is transformed into a system of ODEs. Then, the system is solved numerically by using Runge-Kutta-Fehlberg fourth fifth order (RKF45) method available in MAPLE 15 software. The numerical results obtained are presented graphically for the velocity, temperature and concentration. The effects of various parameters are studied and analyzed. The numerical values for local Nusselt number, skin friction coefficient and local Sherwood number are tabulated and discussed. The study shows that various parameters give significant effect on the profiles of the fluid flow. It is observed that the reaction rate parameter affected the concentration profiles significantly and the concentration thickness of boundary layer decreases when reaction rate parameter increases. The analysis found is validated by comparing with the results previous work done and it is found to be in good agreement.

1. Introduction

In 1904, the concept of boundary layers proposed by Ludwig Prandtl has revolutionised the understanding of fluid mechanics [1]. Prandtl’s boundary layer concept is used in his theory of thin airfoil which made the practical calculation of airfoil properties possible [2].

Magnetic fields influence many natural and forced flows, such as in the process of heating, pumping and stirring. Magnetohydrodynamic (MHD) is formally being related to the relation between fluid flow and magnetic fields, and the fluid must be an electrical conductor and has non-magnetic characteristics such as liquid metals, hot ionised gases (plasma) and strong electrolytes [3]. Ishak [4], Seini and Makinde [5] and Jhankal and Manoj [6] analysed the heat transfer on the flow of MHD boundary layer over a stretching plate and found out that when magnetic field parameter increases, there is decrease in velocity of fluid flow. This is due to the presence of Lorentz force, a force that is resistive in nature which causes deceleration in the velocity of fluid [7]. Devi et al. [8] studied the effect of the heat and mass transfer on the flow of MHD boundary layer of a thick, radiating and incompressible fluid pass an exponentially stretching sheet, and found out that magnetic parameter thickens the fluid.

In many industrial processes, chemical reaction is one of the important applications such as hot rolling, chemical coating of flat plates, and extrusion of polymer [9]. The existence of pure water or air naturally is not possible as there might be distant mass in the air or water [10]. Therefore, the presence of those mixtures could cause chemical reaction within the substances. Sinha [11] studied the unsteady MHD free convective flow and effects of chemical reaction past a permeable plate under sloping temperature and found out that the chemical reaction parameter increment causes reaction rate
to increase. The radiation effects are considered by Chaudhary et al. [7] and Ishak [4] for the MHD flow past an exponentially stretching sheet. The heat transfer rate hikes with the increase in Prandtl number, but drops with both radiation and magnetic parameters. The temperature increases with radiation parameter. Research that combines both the effects of radiation and chemical reaction on MHD flow is important in many areas especially in manufacturing industries. Seini and Makinde [5] studied the flow of MHD boundary layer on exponentially stretching sheet under the effects of chemical reaction and radiation, and the concentration of the boundary layer increases with increasing reaction rate parameter. The results reveal that the mass and heat transfer rate depend strongly on the heat generation and reaction rate parameters.

The governing equations for the problems of heat and mass transfer on MHD boundary layer flow over a stretching sheet with heat sink under the effects of chemical reaction and radiation are discussed in this paper. The equations that govern the boundary layer in this study are continuity, momentum, energy and concentration equations. By using similarity transformation, the system of PDEs that govern the boundary layer problems is transformed into a system of ODEs. Then the system is solved using numerical method; (RKF45) method available in MAPLE 15 software.

2. Mathematical Formulation
This study discusses the effects of radiation, magnetic field, chemical reaction and the characteristics of heat transfer on the MHD boundary layer flow past an exponentially stretching sheet. The fluid is assumed to be incompressible, thick and radiating fluid over an exponentially stretching sheet along x-axis. A variable magnetic field \( B(x) \) is applied normal to the sheet where \( B_0 \) is constant. A variable chemical reaction rate is assumed to be \( k_i(x) \) and variable heat sink parameter \( Q(x) \) where \( k_0 \) and \( Q_0 \) are constants. The flow is restricted to \( y > 0 \). Due to small magnetic Reynolds numbers, the induced magnetic field can be neglected. Hall effect is negligible, since the presented magnetic field is not strong enough. Joule heating, which can transport the extra thermal energy from the environment to the heated item is also negligible. Other than that, the effects of Soret and Dufour are abandoned as the concentration level of foreign mass is assumed to be low.

This study also assumed that \( T = T_w = T_w + T_0 e^{x/L}, \quad C = C_w + C_0 e^{x/L}, \quad T_w > T_x \) and \( C_w > C_x \) where the temperature of the sheet is \( T_w \) and its concentration is \( C_w \), and the corresponding ambient concentration and temperature are \( C_x \) and \( T_x \). \( T_0 \) is the reference temperature \( C_0 \) is the reference concentration. These are the assumptions under consideration together with the boundary layer approximations. The system of equations, following the work done by Devi et al. [8] and Seini and Makinde [5] which models the flow is given by:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + \nu \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho} u \tag{2}
\]

\[
u \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{Q(x)}{\rho c_p} (T - T_w) \tag{3}
\]

\[
u \frac{\partial C}{\partial x} + \nu \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_i(x) (C - C_w) \tag{4}
\]

Where \( u \) and \( v \) are the velocity components along the \( x \) and \( y \) axis respectively, \( \nu \) is the kinematic viscosity, \( \rho \) is the fluid density, \( T \) is the temperature of the fluid, \( \kappa \) is the thermal conductivity of the
fluid, \( c_p \) is the specific heat, \( C \) is the boundary layer fluid concentration, \( q_r \) is the radiative heat flux, and \( D \) is the mass diffusivity coefficient.

The boundary conditions associated for the profiles of temperature, velocity and concentration are

\[
\begin{align*}
u &= U_w = U_0 e^{\frac{x}{L}}, v = 0, T = T_w = T_\infty + T_0 e^{\frac{x}{L}}, C = C_w = C_\infty + C_0 e^{\frac{x}{L}} \text{ at } y = 0 \\
u &\to 0, T \to T_\infty, C \to C_\infty, \text{ as } y \to \infty
\end{align*}
\]

Where, \( U_w \) is the uniform velocity of the sheet and \( L \) is the reference length.

2.1. Similarity Transformation

A set of ordinary differential equations are produced by transforming the modified governing equations of boundary layer with the boundary conditions that make up the boundary layer using similarity transformation variables. The equations are simplified by writing them in the following non-dimensional variables introduced by Mukhopadhyay [12]. By employing Rosseland approximation,

\[
q_r = -\frac{4\sigma^*}{3K} \frac{\partial T^4}{\partial y}
\]

where \( K \) is the mean absorption coefficient. Meanwhile, \( \sigma^* \) is the Stefan-Boltzmann constant. \( T^4 \) is expanded into the Taylor series about \( T_\infty \) to linearise the Rosseland approximation into the form

\[
T^4 = 4T_\infty^3 T - 3T_\infty^4
\]

With reference to equation (6) and equation (7), the equation (3) is reduced to

\[
\begin{align*}
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= -\frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \left[ -\frac{16\sigma^*}{3K'} \frac{\partial^2 T}{\partial y^2} + \frac{Q^* (x)}{\rho c_p} (T - T_\infty) \right]
\end{align*}
\]

By introducing the following similarity transforms

\[
\eta = \left( \frac{U_0}{2vL} \right)^{\frac{1}{2}} e^{\frac{x}{2L}} y, \quad u = U_0 e^{\frac{x}{2L}} f' (\eta), \quad v = -\left( \frac{vU_0}{2L} \right)^{\frac{1}{2}} e^{\frac{x}{2L}} (f (\eta) + \eta f' (\eta)),
\]

\[
T = T_\infty + T_0 e^{\frac{x}{2L}} \theta (\eta), \quad C = C_\infty + C_0 e^{\frac{x}{2L}} \phi (\eta), \quad B(x) = B_0 e^{\frac{x}{2L}}, \quad k_1 (x) = k_0 e^{\frac{x}{2L}}, \quad Q^* (x) = Q_0 e^{\frac{x}{2L}}
\]

where the stream function \( \psi \) is defined as \( u = \partial \psi / \partial y \) and \( v = -\partial \psi / \partial x \) which satisfies equation (1). Then, by using equation (9), equations (2) – (4) are transformed into

\[
f'' - 2f' \theta' = -Mf' = 0
\]

\[
\left( 1 + \frac{4}{3} R \right) \theta'' + Pr f \theta' - Pr f' \theta + Pr Q \theta = 0
\]

\[
\phi'' + Sc f \phi' - Sc f' \phi - Sc \beta \phi = 0
\]

The boundary conditions in equation (5) is reduced to

\[
\begin{align*}
f (0) &= 0, \quad f' (0) = 1, \quad \theta (0) = 1, \quad \phi (0) = 1 \\
f' (\eta) &= 0, \quad \theta (\eta) = 0, \quad \phi (\eta) = 0 \text{ as } \eta \to \infty
\end{align*}
\]

The parameters that are considered are radiation parameter \( R \), magnetic parameter \( M \), Prandtl number \( Pr \), heat generation parameter \( Q \), Schmidt number \( Sc \) and reaction rate parameter \( \beta \). The physical quantities involve are local Nusselt number, skin friction coefficient and local Sherwood number.
3. Numerical Analysis

The system of ordinary differential equations that has been transformed is solved numerically using Runge-Kutta-Fehlberg fourth fifth order (RKF45) method in MAPLE 15 software together with the boundary conditions. The effects of parameters on the temperature \( \theta(\eta) \), velocity \( f'(\eta) \) and concentration \( \phi(\eta) \) profiles are studied and analysed. The numerical values of local Nusselt number \( -\theta'(0) \), skin friction coefficient \( f''(0) \) and local Sherwood number \( -\phi'(0) \) are tabulated and analysed.

4. Results and Discussions

4.1. Validation of Study

Table 1 below shows the Nusselt number \( -\phi'(0) \) values for various values of Prandtl number while the rest of the parameters are set to be zero where \( M = 0, R = 0, Q = 0, Sc = 0 \) and \( \beta = 0 \). The table shows the comparison between the results obtained to those of Devi et al. [8] and it is shown that results agrees to each other.

| Pr | Devi et al. [8] | Present Study |
|----|----------------|---------------|
| 1  | 0.954811       | 0.954955      |
| 2  | 1.471454       | 1.471421      |
| 3  | 1.869609       | 1.869044      |
| 5  | 2.500128       | 2.500109      |

4.2. Results

The effects of magnetic parameter on temperature, velocity and concentration profiles are shown in figure 1, figure 2 and figure 3 accordingly. Figure 1 shows that velocity reduces when \( M \) is increasing. It is showed that magnetic parameter \( M \) reduces the fluid velocity. The existence of the magnetic field which opposes the flow slows down the fluid. This is related to Lorentz force, a resistive force which is formed as an after effect of the application transverse magnetic field in a fluid that conducts electric. When \( M \) increases, skin friction coefficient increases (Table 2). Hence, the shear wall stress increases and reduces the velocity of the fluid. Then, in figure 2 and figure 3, the temperature and concentration of the fluid increase when \( M \) increases. The magnetic field applied heat up the fluid and hence, there is increase in temperature. The can be seen in the decrease of values of local Nusselt number and local Sherwood number in table 2 when \( M \) increases which indicates an decrease in the rate of heat and mass transfer.

Figure 4 shows how Prandtl number affects the temperature of the fluid. An increase in Pr reduces the temperature. Higher Prandtl number indicates that the fluid have lower thermal conductivity and thinner boundary layer structures [13]. Thus, it reduces the thickness of thermal boundary layer therefore heat diffuse out faster from the sheet. Table 2 shows that when Pr increases local Nusselt number increases; indicating that the rate of heat transfers increases. Hence, heat diffuse faster from the sheet and the temperature decreases.

Figure 5 shows that when heat sink parameter increases, it causes the decrease in temperature. As expected, this is the after effect of the decrease in thermal boundary layer thickness when heat is absorbed. When Q increases, local Nusselt number increases. This shows that the rate of heat transfer increases which indicates that the temperature of the fluid drop gradually. This is because the heat is absorbed from the sheet. Figure 6 reveals that when Schmidt number increase, it has caused a decrease in concentration. The lower the Schmidt number, the lower the mass diffusivity. This explains the
decrease of the thickness of boundary layer concentration when $Sc$ increases. The effects of radiation parameter, $R$ on the temperature profile is as in figure 7. When radiation parameter is higher, the temperature decreases as enhancement of the thermal radiation causes the convection moment in the boundary to increase. When $R$ increases, local Nusselt number decreases as shown in Table 2. The heat transfer rate decreases and increases the temperature of the boundary layer. Reaction rate parameter is the speed of reaction for a reactant or a product in a particular reaction. The impact on concentration profile due to change in the reaction rate parameter $\beta$ is shown in figure 8 where the concentration decreases with the increase in parameter $\beta$. This is because the higher the reaction rate, the thicker the concentration boundary layer.

**Figure 1.** Velocity with Magnetic Parameter $M$ variation and $R=1$, $Pr=1$, $Q=-0.5$, $Sc=0.22$, and $\beta=1$.

**Figure 2.** Temperature with Magnetic Parameter $M$ variation and $R=1$, $Pr=1$, $Q=-0.5$, $Sc=0.22$, and $\beta=1$.

**Figure 3.** Concentration with Magnetic Parameter $M$ variation and $R=1$, $Pr=1$, $Q=-0.5$, $Sc=0.22$, and $\beta=1$.

**Figure 4.** Temperature with Prandtl number $Pr$ variation and $R=1$, $M=2$, $Q=-0.2$, $Sc=0.22$, and $\beta=1$.

**Figure 5.** Temperature with Heat Sink parameter $Q$ variation and $R=1$, $Pr=1$, $M=2$, $Sc=0.22$, and $\beta=1$.

**Figure 6.** Concentration with Schmidt number $Sc$ variation and $R=1$, $Pr=1$, $M=2$, $Q=-0.5$, and $\beta=1$. 
Figure 7. Temperature with Radiation parameter $R$ variation and $Pr=1$, $M=2$, $Q=-0.5$, $Sc=0.22$ and $\beta=1$.

Figure 8. Concentration Reaction Rate parameter $\beta$ variation and $R=1$, $Pr=1$, $M=2$, $Q=-0.5$ and $Sc=0.22$.

Table 2 below shows the absolute values of skin friction coefficient $f''(0)$, values of local Nusselt number $-\theta'(0)$ and local Sherwood number $-\phi(0)$ when various values of all parameters involved are considered.

| M  | Pr | Q  | Sc | R  | $\beta$ | $|f''(0)|$ | $-\theta'(0)$ | $-\phi(0)$ |
|----|----|----|----|----|---------|---------|------------|------------|
| 0  | 1  | -0.5| 0.22| 1  | 1       | 1.281933 | 0.753584   | 0.621791   |
| 1  |    |    |    |    |         | 1.629195 | 0.715307   | 0.600183   |
| 2  |    |    |    |    |         | 1.912633 | 0.690714   | 0.586782   |
| 4  |    |    |    |    |         | 2.379381 | 0.659017   | 0.569903   |
| 2  | 1  | -0.2| 0.22| 1  | 1       | 1.912633 | 0.554890   | 0.586786   |
| 2  | 1  | -0.2| 0.22| 1  | 1       | 1.912633 | 0.873488   | 0.586786   |
| 3  | 1  |    |    |    |         | 1.912633 | 1.132214   | 0.586786   |
| 4  | 1  |    |    |    |         | 1.912633 | 1.555244   | 0.586786   |
| 2  | 1  | 0.0 | 0.22| 1  | 1       | 1.912633 | 1.082728   | 0.690762   |
| 2  | 1  | 0.0 | 0.22| 1  | 1       | 1.912633 | 1.241676   | 0.690762   |
| 2  | 1  | 0.0 | 0.22| 1  | 1       | 1.912633 | 1.373379   | 0.690762   |
| 2  | 1  | 0.0 | 0.22| 1  | 1       | 1.912633 | 1.420840   | 0.690762   |
| 2  | 1  | -0.5| 0.24| 1  | 1       | 1.912620 | 0.293147   | 0.616260   |
| 2  | 1  | -0.5| 0.24| 1  | 1       | 1.912620 | 0.293147   | 1.051421   |
| 2  | 1  | -0.5| 0.24| 1  | 1       | 1.912620 | 0.293147   | 1.195670   |
| 2  | 1  | -0.5| 0.24| 1  | 1       | 1.912620 | 0.293147   | 2.333837   |
| 2  | 2  | -0.5| 0.22| 0  | 1       | 1.912633 | 1.144381   | 0.586786   |
| 2  | 1  | -0.5| 0.22| 0  | 1       | 1.912633 | 0.690717   | 0.586786   |
| 2  | 1  | -0.5| 0.22| 0  | 1       | 1.912633 | 0.690717   | 0.766369   |
| 2  | 1  | -0.5| 0.22| 0  | 1       | 1.912633 | 0.690717   | 0.906532   |
| 2  | 1  | -0.5| 0.22| 1  | 1       | 1.912633 | 0.690717   | 1.131636   |
4.3. Conclusion
The study shows that various parameters affect the velocity, temperature and concentration profiles of the boundary layer flow with heat and mass transfer. The numerical results for the local Nusselt number, skin friction coefficient and local Sherwood number have been tabulated. Then, a comparison is made with the work published previously to support the study that has been done. Magnetic parameter has noteworthy effect on velocity profile, and slight impact on concentration and temperature of the fluid. Most of the applications of boundary layer in industries involve chemical reaction in their manufacturing processes. Therefore, to improve many technologies involving chemical such as food processing and polymer production, the study of the processes could help in advancing the manufacturing industries.

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