The Interference Channel Revisited: Eliminating Interference with a Two Antenna Receiver

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Abstract

It is shown that a receiver equipped with two antennas may null an arbitrary large number of spatial directions to any desired accuracy, while maintaining the interference-free signal-to-noise ratio, by judiciously adjusting the distance between the antenna elements. The main theoretical result builds on ergodic theory. The practicality of the scheme in moderate signal-to-noise systems is demonstrated for a scenario where each transmitter is equipped with a single antenna and each receiver has two receive chains where the desired spacing between antenna elements is achieved by selecting the appropriate antennas from a large linear antenna array. We then extend the proposed scheme to show that interference can be eliminated also in specular multipath channels as well as multiple-input multiple-output interference channels. To demonstrate the performance of the scheme, we show significant gains at low and moderate signal-to-noise ratios (0-20 dB) with 4 and 6 users scenarios. The robustness of the proposed technique to small direction estimation errors is also explored.

I. INTRODUCTION

The information-theoretic model of an interference channel is an abstraction that is motivated by the physical channel model of transmitter-receiver pairs that communicate over a shared wireless medium. While abstraction often leads to insights that may then be translated to more complicated real-life models, it is now recognized that the interference channel is an example that generalization also carries with it the risk of over-abstraction, i.e., losing some key features of the true problem. Therefore, it is worthwhile to re-examine the problem formulation from time to time as has been demonstrated, e.g., in the case of magnetic recording channels; see e.g., [1] for an overview of the evolution of the physical models and its impact on the relevant information-theoretic and coding techniques. Another example is the evolution that led to the V.90 voice-band modem [2], [3].

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Indeed, works on interference alignment \cite{4, 5, 6} reveal that a two-user model is non-representative and further that the linear Gaussian model allows for elegant schemes that do not carry over to the general interference channel model. Nonetheless, interference alignment techniques have faced serious difficulties in translating the theoretical asymptotic gains to the operating conditions of communication systems.

In the present paper, we argue that taking a further step in bringing back into the model some simple considerations stemming from the physical propagation medium yields new insights on how near interference-free transmission can be approached by an effective signal processing method, thereby resolving some of the drawbacks of interference alignment techniques. Specifically, the proposed method substitutes simultaneous alignment of interference by simultaneous near nulling. This difference is crucial as it eliminates the power penalty involved in interference alignment techniques and thus the associated gains are not restricted to the very high signal-to-noise ratio (SNR) regime.

To convey the essence of the advocated approach, consider the standard interference nulling performed in multi-antenna wireless communication. It is well known that given an adaptive array with \( N_r \) receive antennas, one can null out \( N_r - 1 \) (single-antenna) interferers and enjoy a full degree-of-freedom (DoF) for one (single-antenna) desired source. This leads to low utilization of the receive antennas, since only \( \frac{1}{N_r} \) of the degrees of freedom convey useful information. The main advantages of receive beamforming are its ease of implementation and its robustness, since channel state information (CSI) feedback is not required.

Nevertheless, as mentioned, works on achievable rates for the interference channel \cite{4, 5, 6} demonstrate that half of the degrees of freedom can be achieved, independent of the number of interferers, even when employing a single antenna at each node. While appealing from a theoretical point of view, interference alignment techniques face some major challenges in real-life applications; see, e.g., \cite{7}. Beyond knowledge of full CSI of the complete interference network at all transmitters being required, the results are highly asymptotic. The SNR at which a tangible improvement over naive schemes is achieved is extremely high or requires very specific system configurations.

Most of the work on the interference channel concentrated on a simplified channel model which assumes that the wireless channel is represented by an arbitrary matrix with random elements. However, as is recognized for many years in the communications theory literature, wireless channels are better represented as a combination of a small number of reflections with complex random coefficients caused by the small scale fading at the reflectors. Examples of such models include the well known Saleh and Valenzuela model \cite{8}, that is prevalent in recent applications of wireless communications (see e.g., \cite{9} and the references therein), as well as ray-based MIMO models \cite{10, 11, 12, 13, 14}. These models are characterized by a finite (typically small) set of reflection clusters with well defined direction-of-departure (DoD) and direction-of-arrival angles (DoA) together with fading coefficients.

We consider the particular class of line-of-sight (LOS) interference channels as well as specular multi-path channels that are the basis for physical modeling of recent of wireless communication systems. We
then extend the results to the MIMO interference channel.

Our main result is as follows: given a large $\lambda$-spaced array with $N_r$ receive antennas and the possibility of selecting two antennas, one can approximately null out any number of sources in the plane, affording (with probability one) a full DoF to a single desired source. Accordingly, for a $K$-user interference channel with single-antenna transmitters, we achieve $K$ degrees of freedom, i.e., a utilization factor of one half, similar to the best achievable DoF of interference alignment schemes. In contrast to the latter, the scheme only requires receive-side CSI. Moreover, it achieves substantial gain at practical values of SNR. The scheme can be implemented using a simple array and selection mechanism as depicted in Figure 1.

![Fig. 1. Setting antenna spacing via selection.](image)

To gain insight into the proposed approach, consider a four-user line-of-sight interference channel where we focus on the receiver of user 1. The direction of the transmitters were $[175^\circ, 59^\circ, 151^\circ, 133^\circ]$ with respect to user 1 array. Selecting antennas having a separation of $5\lambda$, yields the beam-pattern depicted in Figure 2. The desired user’s gain is close to 2 which is the interference-free gain, while the gains corresponding to the signals of all other users are almost completely suppressed. Theorem 1 proves that such a beam pattern is almost always achievable provided that the array is sufficiently large.

### A. Related Work

Handling interference efficiently is a major challenge in multi-user wireless communication. Recently, it has become clear that this challenge can sometimes be overcome via interference alignment [15], [16]. For instance, consider the $K$-user Gaussian interference channel, where $K$ transmitter-receiver pairs wish to communicate simultaneously. Through clever encoding strategies, the received signals can be aligned so that each receiver only observes its desired signal along with a single effective interferer. As a result, each user can achieve asymptotically roughly half the rate that would be available if there were no interference whatsoever; i.e., $K/2$ DoF are available. However, many schemes, such as the Cadambe-Jafar framework
Fig. 2. Optimal beam pattern of user 1. Four-user interference channel. $d_{\text{max}} = 25\lambda$. Directions: $[175^\circ, 59^\circ, 151^\circ, 133^\circ]$. Optimal $d = 5\lambda$. The powers of all users are $P = 1$.

[16] and ergodic interference alignment [17], require a large number of independent channel realizations to achieve near-perfect alignment and suffer from a significant SNR penalty due to channel inversion. In certain settings, this level of channel diversity may not be attainable; ideally, we would like to achieve alignment over a single channel realization.

The capacity region of the static Gaussian $K$-user interference channel [18] is unknown in general, although significant progress has been made recently, in part due to the discovery of interference alignment and the shift from exact capacity results to capacity approximations [19], [20]. Motahari et al. showed that $K/2$ DoF are achievable for almost all channel realizations [21] but thus far this result has not been
translated into real gains outside of the very high SNR regime.

More recent work has concentrated on extending the basic zero-forcing approach suggested by Cadambe and Jafar [4] by obtaining more favorable channel conditioning through various methods; see, e.g., [22] and references therein. These are practical only for particular system configurations. In general, at present time, the gains of interference alignment have yet to be realized.

Apart from the obvious connection to works on the interference channel, the idea of altering the physical propagation channel bears some similarity to “media-based modulation”, “spatial modulation” and “index modulation” schemes; see [23], [24], [25], [26] for an overview of these inter-related concepts. In all of these works, the physical medium is *modulated* based on the information-bearing signal. In contrast to these works, the present work only requires sub-sampling of the spatial channel at the receiver and can cope with an arbitrary number of interfering signals.

II. THE INTERFERENCE CHANNEL - REVISITED

We extend the standard interference channel model to a model where for each user the number of transmit/receive chains is not necessarily equal to the number of transmit/receive antennas, respectively. To that end, consider a $K$-user interference channel where each transmitter has $t$ transmit chains and $N_t$ antennas and each receiver has $r$ receive chains and $N_r$ antennas; see Figure 1 depicting a link between one transmitter and one receiver. This configuration, where all transmitter/receiver pairs have the same parameters, is denoted as the symmetric $t/N_t/N_t/r$ interference channel. We note that this model is of significance for modern systems which utilize massive antenna arrays where the number of Tx/Rx chains differs from the number of antennas. We begin with the traditional case where $t = N_t$ and $r = N_r$.

Consider an interference channel with $K$ transmitters and $K$ corresponding receivers. We assume for simplicity that all transmitters are equipped with the same number of antennas $N_t$ and all receivers are equipped with $N_r$ antennas. Denoting by $H_{ij}$, the channel matrix from transmitter $j$ to receiver $i$, the received signal is given by

$$y_i = \sum_{j=1}^{K} H_{ij}x_j + z_i, \quad i = 1 \ldots K,$$

where $z_i$ is i.i.d. (between users and over time) circularly-symmetric complex Gaussian noise.

Several variants of this problem have been addressed. For instance, the case of $N_t = N_r = 1$ and real time-varying (which can be thought of as a diagonal matrix) coefficients has been studied in [4] where it was shown that for almost all channel coefficients, interference alignment attains half a DoF per user. A similar result was shown for scalar but time-invariant channels in [5] through alignment on the signal scale using lattice codes.

Both of these approaches are very asymptotic in nature and require high resolution transmit-side CSI as well as very high SNR conditions to start to play a beneficial role. Extensions to more general MIMO channels have subsequently revealed, e.g., [27], [28] that the DoF alignment gains are much more modest
under more realistic assumptions. As a partial remedy, antenna switching has been proposed as a means for improving the channel coefficients to facilitate alignment [29].

We note that the $t/N_t/N_r/r$-interference channel is equivalent to requiring that for each user $i$, the transmitter and receiver must employ linear front-end selection matrices $S_T \in \{0,1\}^{N_t \times t}$, $S_R \in \{0,1\}^{N_r \times r}$, each having exactly $t$ and $r$ non-zero elements which are not in the same row or column, respectively. Applying selection matrices $S_{T,i}, S_{R,i}$ at both ends of the link of each user, (1) becomes

$$y_i = \sum_{j=1}^{K} S_{R,i}^H H_{ij} S_{T,j} x_j + z_j,$$

$$= \sum_{j=1}^{K} H_{ij}^S x_j + z_j. \quad (2)$$

From a practical perspective, implementing the selection mechanism yields a substantial reduction in hardware complexity.

III. LINE-OF-SIGHT INTERFERENCE CHANNELS

In this section we describe a novel approach to the interference channel. The classical signal processing literature deals primarily with Nyquist-resolution beamformers, where at least some antennas are separated by at most $\lambda / 2$. In this case, the array has a single main lobe in the desired direction, and the resolution of the array is determined by the farthest elements. This is because that when all distances between antennas are larger than $\lambda / 2$, an ambiguous beam pattern occurs. An example of this phenomenon is depicted in Figure 2. Interestingly, an ambiguous beam pattern, can prove extremely advantageous when dealing with interference, since such patterns have multiple nulls. We will show that by judiciously designing the beam pattern, we can point multiple nulls at the multiple interferers simultaneously. In fact, with a highly under-sampled array, any number of interferers in almost any set of directions can be suppressed. This follows from an ergodic theory argument.

In what follows we assume that $N_t = t = 1$ and $N_r \gg r = 2$. The spacing is assumed to be $\lambda$ and the receiver selects two out of $N_r$ antennas to be switched into the receive chains. We assume that the receiver has full directional CSI.

To develop the general framework for receiver antenna array design, we first introduce the line-of-sight interference channel model. The use of high-frequency communication in general and mm-wave and THz frequency communication has prompted recent interest in LOS communication channels [30], [31], [32]. Moreover, such channels form the basis for the more elaborate channel models described in Sections V and VI. Specifically, we make the following assumptions:

A1 We assume a single transmit chain and transmit antenna per user. Hence $S_{T,i}$ is trivial for all $i$.

A2 We assume two receive chains per user, i.e. the matrices $H_{ij}^S$ in (2) are reduced to $2 \times 1$ LOS vectors $h_{ij} = h(\theta_{ij}; d)$.

A3 The vectors $h(\theta_{ij}; d)$ consist of array manifold vectors which depend on the selection matrix, setting the spacing $d$ (in units of $\lambda$) between the chosen antennas.
A4 Let \( h(\theta; d) \) be the array response towards direction \( \theta \). Thus, the array response is given by
\[
 h(\theta; d) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1, e^{j 2\pi d \cos \theta} \end{bmatrix}^T. \tag{3}
\]

A5 We assume that each receiver has perfect CSI w.r.t. all channel gains corresponding to impinging signals.

A6 Transmitters on the other hand need not have access to any CSI beyond the rate at which they should communicate with their respective receiver.

A7 For simplicity, we assume a linear array and planar geometry where all sources are far field point sources.

A8 Without loss of generality, we use the array manifold as the channel, since the signal attenuation can be absorbed in the power of the signal \( x_j \).

A9 We assume that the locations of all transmitters and receivers are independently uniformly distributed in angle with respect to the origin.

A10 We assume that the power of all transmitters is bounded by \( P \).

Note that by A9, the incidence angle of each received signal is uniformly distributed as well. Under this setting, it suffices to focus on the receiver of a single user \( i \) as the operations at all receivers will be similar.

It follows that (1) becomes:
\[
y_i = \sum_{j=1}^{K} h(\theta_{ij}; d_i) x_j + z_j, \quad i = 1 \ldots K, \tag{4}
\]

Using a received beamforming vector \( w_i = \frac{1}{\sqrt{2}} [1, e^{j \phi_i}] \), the received signal of the \( i \)'th user becomes:
\[
y_i = \sum_{j=1}^{K} \kappa(\theta_{ij}; d_i) x_j + z_i, \tag{5}
\]

where
\[
\kappa(\theta_{ij}; d_i) = w_i^T h(\theta_{ij}; d_i). \tag{6}
\]

Therefore,
\[
g(\theta_{ij}; d_i) = \kappa(\theta_{ij}; d_i) = \frac{1}{2} \left| 1 + e^{j(2\pi d_i \cos \theta_{ij} + \phi_i)} \right|^2. \tag{7}
\]

Straightforward algebraic simplification yields:
\[
g(\theta_{ij}; d_i) = 1 + \cos(2\pi d_i \cos(\theta_{ij}) + \phi_i). \tag{8}
\]

In the next section we show that by properly selecting \( d_i, \phi_i \), we can obtain the following:
\[
g(\theta_{ij}; d_i) \approx \delta_{ij}, \quad j = 1, \ldots, K, \tag{9}
\]

where \( \delta_{ij} \) is Kronecker’s delta function.
IV. Eliminating Interference via Ergodic Nulling

We now show that by judiciously adjusting the distance between the receive antennas, we can (with probability 1) suppress all interferers to any desired level. Furthermore, we show that this can be achieved by a separation which is an integer multiple of the wavelength and hence, implementation via antenna selection applied to a linear array is possible.

Specifically, we demonstrate that for almost all angles of arrival, one can approach the interference-free rate of any desired user. This is proved using the uniform distribution property of sequences modulo 1. Moreover, we show that without loss of generality, the beamforming vector \( \mathbf{w}_i \) can be chosen as\( \mathbf{w}_i = \frac{1}{\sqrt{2}} \left[ 1, 1 \right]^T \).

The proof utilizes an integer antenna spacing (in terms of wavelength \( \lambda \)). Hence, it can be translated to an implementation using antenna selection applied to an array of \( N_r \gg 2 \) antennas with \( \lambda \)-spacing.

**Theorem 1 (Main Theorem).** Assume that the directions \( \theta_{i,1}, \ldots, \theta_{i,K} \) are such that \( \cos(\theta_{i,1}), \ldots, \cos(\theta_{i,K}) \) are independent over \( \mathbb{Q} \). Then, for every \( \delta > 0 \), one can find a spacing \( d \in \mathbb{N} \) such that applying receive beamforming with the vector \( \mathbf{w}_i = \frac{1}{\sqrt{2}} \left[ 1, 1 \right]^T \) yields:

\[
g(\theta_{i,k};d) < \delta, \quad k \neq i
\]

\[
g(\theta_{i,id}) > 1 - \delta.
\]

Before we prove the main theorem, note that it provides a full DoF per transmitter. This is the case since the gain in the desired direction can be made arbitrarily close to 1 while the total interference is suppressed to any desired level.

**Proof.** To prove the main theorem, recall the following definition by Weyl (see \[33]\):

**Definition 1.** A \( K \)-dimensional sequence of real vectors \( \mathbf{x}_m : m \in \mathbb{N} \) is uniformly distributed modulo 1 if for every box

\[
B = \prod_{k=1}^{K} [a_k, b_k], \quad B \subseteq [0,1)^K
\]

\[
\lim_{M \to \infty} \frac{|\{1 \leq m \leq M : (\mathbf{x}_m \mod 1) \in B\}|}{M} = \prod_{k=1}^{K} (b_k - a_k) \tag{11}
\]

Weyl \[34]\ proved that whenever \( \mathbf{x} = [x_1, \ldots, x_K]^T \) is a vector of irrational real numbers that are linearly independent over \( \mathbb{Q} \), the sequence \( \{m \mathbf{x} \mod 1 : m \in \mathbb{N} \} \) is uniformly distributed modulo 1. In the present context, assume that \( \cos(\theta_{i,1}), \ldots, \cos(\theta_{i,K}) \) are linearly independent over \( \mathbb{Q} \). Note that this holds with probability one. By Weyl’s theorem the sequence \( m [\cos(\theta_{i,1}), \ldots, \cos(\theta_{i,K})], m \in \mathbb{N} \), is uniformly distributed modulo 1. Define a box

\[
B = \prod_{k=1}^{K} B_k \tag{12}
\]
where

\[ B_k = \begin{cases} [0, \epsilon'] & k = i \\ \left[ \frac{1-\epsilon', 1+\epsilon'}{2}, 1 \right] & k \neq i \end{cases} \]

where \( \epsilon' = \frac{\epsilon}{2\pi} \). Therefore, we can find a \( d \) such that

\begin{align*}
2\pi d \cos(\theta_{i,i}) &< \epsilon \mod 2\pi \\
\pi - \frac{\epsilon}{2} &< 2\pi d \cos(\theta_{i,j}) < \pi + \frac{\epsilon}{2} \mod 2\pi
\end{align*}

By continuity of \( g(\theta_{ij};d) \), as given in (13), for a given \( \delta \), we can find an \( \epsilon \) such that (10) is satisfied.

V. MULTIPATH CHANNELS AND RELATED INTERFERENCE CHANNEL MODELS

We now show that the proposed approach generalizes to the case of multipath [35], [36] with a finite number of reflections.

We assume a physical channel model such that the carrier frequency is much larger than the signal bandwidth, which is typical in cellular and indoor wireless communications. We first show that in the case of a \( 1/1/N_r/2 \) interference channel with inter-symbol interference (ISI), we can approach the single-user interference-free rate.

Adhering to discrete time and allowing a different path loss for each reflection, the channel, as given in (2), now generalizes to

\[ y_i(t) = \sum_{k=1}^{K} \sum_{\ell=1}^{L_{i,j}} \gamma_{i,k,\ell} h(\theta_{i,k,\ell};d_i) x_k(t-\tau_{i,k,\ell}) + z_i(t), \]  

for \( i = 1, \ldots, K \), where \( L_{i,j} \) is the number of reflections of the \( j \)'th signal received by user \( i \), and \( \gamma_{i,k,\ell} \) is the complex path loss of the signal arriving from direction \( \theta_{i,k,\ell} \) and \( h \) is defined in (3). Following standard models, we can assume that \( \gamma_{i,k,\ell} \) is either Rician or Rayleigh, or even deterministic.

Let

\[ B_i = \prod_{k=1}^{K} \prod_{\ell=1}^{L_{i,j}} B_{i,k,\ell} \]  

where for all \( 1 \leq \ell \leq L_{i,k} \):

\[ B_{i,k,\ell} = \begin{cases} [0, \epsilon'] & k = i, \ell = 1, \ldots, L_{i,i} \\ \left[ \frac{1-\epsilon', 1+\epsilon'}{2}, 1 \right] & k \neq i, \ell = 1, \ldots, L_{i,k} \end{cases} \]

and \( \epsilon' = \frac{\epsilon}{2\pi} \). As in the previous section, by invoking Weyl’s theorem, there exists a \( d \) such that:

\[ 2\pi d \cos(\theta_{i,i,\ell}) < \epsilon \mod 2\pi \]  

for all \( \ell = 1, \ldots, L_{i,i} \) and

\[ \pi - \frac{\epsilon}{2} < 2\pi d \cos(\theta_{i,j,\ell}) < \pi + \frac{\epsilon}{2} \mod 2\pi \]  

for all \( \ell = 1, \ldots, L_{i,i} \) and \( j \neq i \). By continuity of \( g(\theta;\ell) \), for a given \( \delta \), there exists an \( \epsilon \) such that:

\[ g(\theta_{i,i,\ell};d_i) > 1 - \delta, \quad \ell = 1, \ldots, L_{i,i} \]  

\[ g(\theta_{i,j,\ell};d_i) < \delta, \quad \ell = 1, \ldots, L_{i,j}, j \neq i. \]
We conclude that one can suppress all specular multipath components of the interference signals to any desired level. Hence, the resulting received signal is given by:

$$y_i(t) = \sum_{\ell=1}^{L_{i,j}} \kappa(\theta_{i,j,\ell};d_i) \gamma_{i,j,\ell} x_i(t - \tau_{i,j,\ell}) + z_i(t)$$

(21)

where

$$z_i(t) = z_i(t) + z'_i(t)$$

is composed of the receiver noise as well as the residual interference at receiver $i$,

$$z'_i(t) = \sum_{\ell=1}^{L_{i,j}} \sum_{k \neq i} \kappa(\theta_{i,j,k,\ell};d_i) \gamma_{i,j,k,\ell} x_k(t - \tau_{i,j,k,\ell}).$$

Note that the power of the residual interference satisfies:

$$E \| z'_i(t) \|^2 < \delta \sum_{k \neq i} E |x_k(t)|^2 \sum_{\ell=1}^{L_{i,j}} |\gamma_{i,j,k,\ell}|^2$$

(22)

By selecting $\delta$ sufficiently small, $E |z'_i(t)|^2$ can be made arbitrarily small. Moreover, for all desired signal paths $g(\theta_{i,j,k,\ell};d_i) = |\kappa(\theta_{i,j,k,\ell};d_i)|^2$ are (simultaneously) arbitrarily close to 1 by a proper choice of $\delta$. It follows that (21) amounts to a standard ISI channel, with coefficients arbitrarily close to the interference-free ISI channel.

VI. ERGODIC NULLING FOR THE MIMO INTERFERENCE CHANNEL

We now turn to analyze the MIMO interference channel where for simplicity we assume that the number of transmit and receive antenna elements as well as RF chains are the same for all transmitter and receiver pairs, i.e., of dimensions $N_t, N_r$ and $t, r$.

Following the vast literature of physical spatial point-to-point MIMO channel models, we note that the $N_r \times N_t$ MIMO channel between the transmit antennas of user $j$ and the receive antennas of user $i$ can be described as

$$H_{i,j} = \sum_{\ell=1}^{L_{i,j}} \gamma_{i,j,\ell} a_{R,i}^{(\theta_{i,j,\ell})} a_{T,j}^{(\psi_{i,j,\ell})}$$

(23)

where, $\psi_{i,j,\ell}, \theta_{i,j,\ell}$ are the DoD between transmit array $j$ and reflector $\ell$ and the DoA between receive array $i$ and reflection $\ell$. Without loss of generality we also assume the $\gamma_{i,j,\ell}$ are monotonically decreasing in $\ell$. As is common in the MIMO literature, we assume that the scattering is sufficiently rich. In the present context, this requires that $L_{i,j} \geq t$ for all $i$ so that (almost surely) for all $i$, we have $\text{rank}(H_{i,j}) \geq t$. The following theorem holds:

**Theorem 2.** Let $t, N_t$ be given and assume that $r = t + 1, L_{i,j} \geq t$ and $N_r = t$. Further, assume that each receiver has directional CSI. Then, for any $\delta > 0$, there is a sufficiently large $N_r$ and a selection matrix $S_{R,i}$, such for user $i$ any rate satisfying

$$R_i \leq \log \left| I + \frac{P}{\sigma^2} G_i A_{T,i} A_{T,i}^H G_i^H \right| - \delta$$

(24)
is achievable in the $t/N_t/N_r/(t+1)$ interference channel, where
\[ G_i = \text{diag}\{\gamma_{i,1}, \ldots, \gamma_{i,t}\} \]
and
\[ A_T = [a_{T,i}(\psi_{i,1}), \ldots, a_{T,i}(\psi_{i,t})] \]
Furthermore, if the transmitter has CSI, then the rate
\[ \max_{Q} \log \left| I + G_i A_T Q A_T^H A_i^H \right| - \delta \]
is achievable where $Q$ is a positive semi-definite matrix satisfying $\text{tr}(Q) = P$.

\textbf{Proof.} Let the transmitter use an i.i.d. isotropic Gaussian codebook of dimension $t$ and with power $P/t$ per dimension. Let $e_i$ denote the $(t+1)$-dimensional standard unit vectors. The receiver uses a selection matrix $S_{R,i}$ followed by a beamforming matrix $W_i = [w_1, \ldots, w_t]$ where
\[ w_{i,\ell} = \frac{1}{\sqrt{2}} \left( e_0 + e_{n_i,\ell} \right). \]
Here, $(S_{R,i})_{\ell, n_i,\ell} = 1$ if and only if the antenna $n_i,\ell$ is selected such that the beamformer $w_{i,\ell}$ receive only direction $\theta_{i,\ell}$ (and approximately nulling all other directions, both from the desired user as well as from all others users). Recall that by Theorem [1] this is possible.

Thus, user $i$ obtains an equivalent MIMO channel
\[ y_i = \tilde{H}_i x_i + \tilde{z}_i \tag{25} \]
where
\[ \tilde{H}_i = W_i S_{R,i} A_{R,i} G_i A_{T,i} \]
and
\[ \tilde{z}_i = z_i + z_i' \]
is composed of the receiver noise as well as the residual interference at receiver $i$. Note that the power of the residual interference can be made as small as desired.

By construction
\[ W_i S_{R,i} A_{R,i} = I + D_i \]
and $\|D_i\|_{\infty} < \delta$. Hence, $\tilde{H}_i$ can be made arbitrarily close to the channel $\tilde{H}_i' = G_i A_{T,i}$.

Thus, an achievable rate for this channel is given by
\[ R(\tilde{H}_i') = \log \left| I + \frac{P}{t^2 \sigma^2} G_i A_{T,i} A_{T,i}^H G_i^H \right|. \]
The case of full CSIT follows by standard MIMO techniques. \hfill \square

This should be compared against the isotropic transmission interference-free benchmark of
\[ \tilde{R}(\tilde{H}_i') = \log \left| I + \frac{(t+1)P}{t^2 \sigma^2} G_i A_{T,i} A_{T,i}^H G_i^H \right|. \]
For large $t$, the two rates nearly coincide.

Example: Three-user $2/2/N_r/3$ MIMO interference channel. Assuming a specular multipath model with at least two reflections for every desired user (at the respective receiver), we note that we can achieve a total of 2 DoFs per user. This should be compared to the $3/2$ DoFs per user achieved (in the generic) MIMO interference channel [4, 27, 28].

VII. Optimizing the beamformer

We now discuss the practical implementation of the proposed method. While Theorems 1 and 2 guarantee that interference can be suppressed to any desired level, they do not exploit the full optimization parameter space on the one hand. On the other hand, in practice $N_r$ is fixed a-priori. Ultimately, our goal is to maximize the signal-to-interference-plus-noise ratio by properly choosing $d$ and $\phi$. To simplify notation, we focus on a single receiver since the design of all receivers is equivalent.

Explicitly, the desired solution for user $i$ is given by

$$
(d_i, \phi_i) = \arg \max_{d,\phi} \frac{P_i g(\theta_i; d)}{\sum_{k \neq i} P_k g(\theta_i; d) + \sigma^2}
$$

(26)

where $P_r, P_k$ are the receive power of the desired and interfering signals, and $\theta_{i,i}, \theta_{i,k}$ are the directions of the desired and interfering signals, respectively. While this equation is highly non-linear, given the received signal and interference CSI, we need to optimize the receive array by enumerating over $d, \phi$. This is a two dimensional search with a moderate complexity. Note that in the MIMO case, when $r > 2$ we can further optimize the receive beamforming matrix, given the selection of the antennas using an MMSE criterion and allowing a full (combinatorial) search over the subsets of $r = t + 1$ receive antennas.

VIII. Simulations

To test the proposed ergodic interference nulling scheme we performed several simulations. In the first set of simulations, we tested the performance as a function of SNR and the sensitivity to directional errors. For SNR values ranging from $-5$dB to 20dB, we generated 100 LOS interference channels, with four users. We repeated the experiment for three values $d_{\text{max}} = 50, 100, 500\lambda$ which is a reasonable number for practical massive MIMO scenarios. We tested the capacity of user 1 with all transmitters randomly located at directions chosen between 0 and 180 degrees. All interferers were assumed to be received with the same power. As a benchmark for comparison we took non-naive time-division multiple access (TDMA), with two users transmitting per time slot assuming MMSE nulling of the undesired signal. We calculated the average achievable rate over all the channel realizations, optimized over $d, \phi$ using full search with $1^\circ$ resolution in $\phi$ and a $\lambda/2$ uniform linear array. To test for robustness, we also tested the performance of a receiver suffering from i.i.d receiver directional errors with $\sigma_\theta = 0.1^\circ, 0.05^\circ, 0.01^\circ$ for $d_{\text{max}} = 50, 100, 500\lambda$ respectively. The results are depicted in Figures 3-5. The interference-free rate is clearly attained up to an SNR of roughly 10dB for $d_{\text{max}} = 100\lambda$, and even at 20dB when $d_{\text{max}} = 500\lambda$. The slowing of the growth of the attained rates of proposed nulling scheme is due to the limited size of the array. We can see that the scheme shows reasonable sensitivity to small errors in direction.
Finally, we have tested a 6-user interference channel scenario, where $d_{\text{max}} = 200\lambda$. As expected, the gain over TDMA is smaller. This indicates that combining TDMA with ergodic nulling can be beneficial over pure ergodic nulling, in particular, at high SNR.

To test the dependence on $d_{\text{max}}$ we chose SNR= 10 dB, and computed the achievable rate as a function of $d_{\text{max}}$. The results are depicted in figure 3. While attaining the interference-free rate requires about $100\lambda$ separation, there is a very significant performance gain, compared to non-naive TDMA, even at $d = 30\lambda$ where the rate is 50% higher.

IX. DISCUSSION

In this paper we proposed a novel technique for dealing with the interference channel. The approach is based on judiciously setting the distance between the two receive antennas to attain a beamforming vector with approximate nulls in the direction of the interferers. The main theorem shows that we can...
achieve half the degrees of freedom afforded by the system. A significant advantage of the proposed approach compared to traditional interference alignment is the fact that we only require receive-side CSI. Thus, the scheme does not require CSI information at the transmitter beyond transmission rate. Since the scheme is applicable when there is a single transmitter per user, this allows us to operate in the regime where transmit ZF is impossible.

In practice, moving the antennas to set the desired separation may be difficult to implement. To overcome this, one possibility is to use the standard approach taken in massive MIMO systems, where two antennas of a large array are switched into two receiver chains; see, e.g., [29].

In a practical implementation, it is preferable to limit the dimensions of the array. To that end, a receiver could divide the interferers into two groups, a small group of strong interferers for which approximate nulling is required, and a residual that is treated as noise. Moreover, from a system perspective, the users
Achievable rate vs. SNR. $d_{\text{max}}=500$, $K=4$

- **Interference free rate**
- **Ergodic nulling**
- **Ergodic nulling ($\sigma^2=0.01$ error)**
- **Non-naive TDMA**

**Fig. 5.** Four-user interference channel where 100 random channel realizations are drawn. SIR=-5 dB. $d_{\text{max}} = 500\lambda$.

could be partitioned into orthogonal groups in which the number of strong interferers is limited.

Since the proposed approach is capable of suppressing any (finite) number of interferers, it is applicable also for non-symmetric interference channels with $(t_i/N_{T,i}/N_{R,i}/r_i), i = 1, \ldots, K$ configurations as long as all $N_{R,i}$ are large enough and for all $i r_i \geq t_i + 1$.

Similarly, the results can be easily extended to configurations of the interference channel with $i \geq r + 1$ as long as $N_t$ is sufficiently large and $N_r \geq r$ provided that the directional CSI is available at the transmitters.

Finally, we note that the advocated approach easily extends to the model of an interference multiple-access channel. Namely, given $r$ receive chains, $r-1$ single-antenna users can be afforded a full DoF while suppressing an arbitrary number of interferers, thus yielding a DoF utilization factor of $1 - 1/r$. Similar approach applies to the downlink.
Achievable rate vs. SNR. $d_{\text{max}}=200$, $K=6$

Fig. 6. Six-user interference channel where 100 random channel realizations are drawn. SIR=-5 dB. $d_{\text{max}} = 200\lambda$.

REFERENCES

[1] K. S. Immink, P. H. Siegel, and J. K. Wolf, “Codes for digital recorders,” IEEE Transactions on Information Theory, vol. 44, no. 6, pp. 2260–2299, 1998.

[2] D.-Y. Kim, P. A. Humblet, M. V. Eyuboglu, L. Brown, G. D. Forney, and S. Mehrabanzad, “V. 92: the last dial-up modem?” IEEE transactions on communications, vol. 52, no. 1, pp. 54–61, 2004.

[3] P. A. Humblet and M. G. Troulis, “The information driveway [using analog telephone lines],” IEEE Communications Magazine, vol. 34, no. 12, pp. 64–68, 1996.

[4] V. R. Cadambe and S. A. Jafar, “Interference alignment and degrees of freedom of the K-user interference channel,” IEEE Transactions on Information Theory, vol. 54, no. 8, pp. 3425–3441, 2008.

[5] A. S. Motahari, S. Oveis-Gharan, M.-A. Maddah-Ali, and A. K. Khandani, “Real interference alignment: Exploiting the potential of single antenna systems,” IEEE Transactions on Information Theory, vol. 60, no. 8, pp. 4799–4810, 2014.

[6] B. Nazer, M. Gastpar, S. A. Jafar, and S. Vishwanath, “Ergodic interference alignment,” IEEE Transactions on Information Theory, vol. 58, no. 10, pp. 6355–6371, 2012.
Fig. 7. Four-user interference channel where 100 random channel realizations are drawn. 100 random channels. SNR=10 dB, SIR=-5 dB.

[7] O. El Ayach, S. W. Peters, and R. W. Heath, “The practical challenges of interference alignment,” IEEE Wireless Communications, vol. 20, no. 1, pp. 35–42, 2013.

[8] A. A. Saleh and R. Valenzuela, “A statistical model for indoor multipath propagation,” IEEE Journal on selected areas in communications, vol. 5, no. 2, pp. 128–137, 1987.

[9] T. S. Rappaport, S. Sun, R. Mayzus, H. Zhao, Y. Azar, K. Wang, G. N. Wong, J. K. Schulz, M. Samimi, and F. Gutierrez Jr, “Millimeter wave mobile communications for 5G cellular: It will work!” IEEE access, vol. 1, no. 1, pp. 335–349, 2013.

[10] P. Almers, E. Bonek, A. Burr, N. Czink, M. Debbah, V. Degli-Esposti, H. Hofstetter, P. Kyöstı, D. Laurenson, G. Matz et al., “Survey of channel and radio propagation models for wireless MIMO systems,” EURASIP Journal on Wireless Communications and Networking, vol. 2007, no. 1, p. 019070, 2007.

[11] D.-S. Shiu, G. J. Foschini, M. J. Gans, and J. M. Kahn, “Fading correlation and its effect on the capacity of multielement antenna systems,” IEEE Transactions on communications, vol. 48, no. 3, pp. 502–513, 2000.

[12] H. Bölcskei, D. Gesbert, and A. J. Paulraj, “On the capacity of OFDM-based spatial multiplexing systems,” IEEE Transactions on communications, vol. 50, no. 2, pp. 225–234, 2002.

[13] C. Oestges, V. Erceg, and A. J. Paulraj, “A physical scattering model for mimo macrocellular broadband wireless channels,”
[14] H. Xu, D. Chizhik, H. Huang, and R. Valenzuela, “A generalized space-time multiple-input multiple-output (MIMO) channel model,” IEEE Transactions on Wireless Communications, vol. 3, no. 3, pp. 966–975, 2004.

[15] M. A. Maddah-Ali, A. S. Motahari, and A. K. Khandani, “Communication over MIMO X channels: Interference alignment, decomposition, and performance analysis,” IEEE Transactions on Information Theory, vol. 54, no. 8, pp. 3457–3470, August 2008.

[16] V. R. Cadambe and S. A. Jafar, “Interference alignment and the degrees of freedom for the K-user interference channel,” IEEE Transactions on Information Theory, vol. 54, no. 8, pp. 3425–3441, August 2008.

[17] B. Nazer, M. Gastpar, S. A. Jafar, and S. Vishwanath, “Ergodic interference alignment,” IEEE Transactions on Information Theory, vol. 58, no. 10, pp. 6355–6371, October 2012.

[18] A. B. Carleial, “Interference channels,” IEEE Transactions on Information Theory, vol. 24, no. 1, pp. 60–70, January 1978.

[19] R. H. Etkin, D. N. C. Tse, and H. Wang, “Gaussian interference channel capacity to within one bit,” IEEE Transactions on Information Theory, vol. 54, no. 12, pp. 5534–5562, December 2008.

[20] S. Avestimehr, S. Diggavi, and D. Tse, “Wireless network information flow: A deterministic approach,” IEEE Transactions on Information Theory, vol. 57, no. 4, pp. 1872–1905, April 2011.

[21] S. O.-G. A. Motahari and A. Khandani, “Real interference alignment with real numbers,” IEEE Transactions on Information Theory, Submitted August 2009, also available at [arXiv:0908.1208].

[22] N. Zhao, F. R. Yu, M. Jin, Q. Yan, and V. C. Leung, “Interference alignment and its applications: A survey, research issues, and challenges,” IEEE Communications Surveys & Tutorials, vol. 18, no. 3, pp. 1779–1803, 2016.

[23] T. Gou, C. Wang, and S. A. Jafar, “Aiming perfectly in the dark-blind interference alignment through staggered antenna switching,” IEEE Transactions on Signal Processing, vol. 59, no. 6, pp. 2734–2744, 2011.

[24] K. Witrisal, P. Meissner, E. Leitinger, Y. Shen, C. Gustafson, F. Tufvesson, K. Haneda, D. Dardari, A. F. Molisch, A. Conti et al., “High-accuracy localization for assisted living: 5G systems will turn multipath channels from foe to friend,” IEEE Signal Processing Magazine, vol. 33, no. 2, pp. 59–70, 2016.

[25] R. W. Heath, N. Gonzalez-Prelcic, S. Rangan, W. Roh, and A. M. Sayeed, “An overview of signal processing techniques for millimeter wave MIMO systems,” IEEE journal of selected topics in signal processing, vol. 10, no. 3, pp. 436–453, 2016.

[26] F. Boccardi, R. W. Heath, A. Lozano, T. L. Marzetta, and P. Popovski, “Five disruptive technology directions for 5G,” IEEE Communications Magazine, vol. 52, no. 2, pp. 74–80, 2014.

[27] L. Kuipers and H. Niederreiter, Uniform distribution of sequences. Courier Corporation, 2012.

[28] H. Weyl, “Über die Gleichverteilung von Zahlen mod. Eins,” Mathematische Annalen, vol. 77, no. 3, pp. 313–352, 1916.

[29] D. Tse and P. Viswanath, Fundamentals of Wireless Communication. Cambridge: Cambridge University Press, 2005.

[30] R. G. Gallager, Principles of digital communication. Cambridge University Press Cambridge, UK, 2008, vol. 1.