Nonlinear dynamics of soft fermion excitations in hot QCD plasma II: Soft-quark–hard-particle scattering and energy losses

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Abstract

In general line with our first work [1] within the framework of semiclassical approximation a general theory for the scattering processes of soft (anti)quark excitations off hard thermal particles in hot QCD-medium is thoroughly considered. The dynamical equations describing evolution for the usual classical color charge $Q^a(t)$ and Grassmann color charges $\theta^i(t), \theta^{\dagger i}(t)$ of hard particle taking into account the soft fermion degree of freedom of the system are suggested. On the basis of these equations and the Blaizot-Iancu equations iterative procedure of calculation of effective currents and sources generating the scattering processes under consideration is defined and their form up to third order in powers of free soft quark field, soft gluon one, and initial values of the color charges of hard particle is explicitly calculated. With use of the generalized Tsytovich principle a connection between matrix elements of the scattering processes and the effective currents and sources is established. In the context of the effective theory suggested for soft and hard fermion excitations new mechanisms of energy losses of high-energy parton propagating through QCD-medium are considered.

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1 Introduction

In the second part of our work we proceed with our analysis of dynamics of fermion excitations in hot QCD-medium at the soft momentum scale, started in [1] (to be referred to as “Paper I” throughout this text) in the framework of the hard thermal loop effective theory. Here we focus our research on study of the scattering processes of soft quark plasma excitations off hard thermal (or external) particle within real time formalism based on Boltzmann type kinetic equations for soft quark, antiquark and gluon modes. This problem has been already discussed earlier in our work [2] in somewhat other technique of calculation than presented here. In this work we consider this problem in great detail and by systematic way using the technique of construction of effective currents and sources generating the scattering processes we are interested in.

Our approach is based on the complete system of dynamical equations derived by Blaizot and Iancu [3] complemented by the Wong equation [4] describing a change of the classical color charge $Q = (Q^a, a = 1, \ldots, N_c^2 - 1$ of hard color-charged particle and by the dynamical equations for the Grassmann color charges $\theta = (\theta^i)$ and $\theta^i = (\theta^i)$, $i = 1, \ldots, N_c$ for the first time proposed in the papers [5]. The use of the last equations from above-mentioned ones is the main new ingredient of the effective theory for nonlinear interaction of hard and soft modes in a hot quark-gluon plasma developed in the present work. Introduction in consideration of the Grassmann color charges of hard particle on the equal footing with usual color charge enables us to enter a so-called color (Grassmann) source of a spin-1/2 hard particle along with classical color current. We add this Grassmann source to the right-hand side of the Dirac equation for soft-quark field just as we add the usual color current of hard particle to the right-hand side of the Yang-Mills equation [6]. This allows us to obtain closed self-consistent description of nonlinear interaction dynamics of soft and hard excitations both Fermi and Bose statistics (within the framework of semiclassical approximation).

Unfortunately, Wong’s equation and equations for the Grassmann charges in that form as they have been derived in original works [4, 5] are insufficient for obtaining complete and gauge-invariant expressions for matrix elements of the scattering processes under consideration. The reason of this is that these equations have been obtained on the assumption that there exists only (regular and/or stochastic) gluon field $A^a_\mu(x)$ in the system. In this work, we suggest a minimal extension of these equations to the case of the presence of soft (stochastic) quark field $\psi^a_\mu(x)$ in system. These generalized equations generate new gauge-covariant additional color currents and sources of the hard test particle, which we add to the right-hand side of the corresponding field equations. In this case only we are able to calculate complete and gauge-covariant expressions for effective currents and sources generating the scattering processes of soft quark excitations
We apply the current approach to study of the propagating a high energy parton (gluon or quark) through the hot QCD-medium and energy losses associated with this motion. We show that the account of an existence in the medium of soft excitations obeying Fermi statistics results in appearance of new channels for energy losses and write out complete explicit expressions determining the energy losses to the first orders in interaction with soft stochastic fields of the plasma. As a special case we have obtained the expression for so-called polarization losses caused by large distance ‘inelastic’ collisions under which a type of initial hard parton changes. This expression supplements well-known expression for the polarization losses caused by large distance ‘elastic’ scattering \[7\].

The paper II is organized as follows. In Section 2, preliminary comments with regard to derivation of a system of the Boltzmann equations describing a change of the number densities for soft quark and soft gluon excitations due to their scattering off hard thermal partons of medium are given. In Section 3, the self-consistent system of the nonlinear integral equations for the gauge potential $A^a_\mu(k)$ and quark wave function $\psi_i^\alpha(q)$ taking into account presence in the system of color current and color Grassmann source of hard test particle is written out. On the basis of perturbative solutions of these equations the notions of the effective currents and sources are introduced. In Section 4, examples of calculation of the simplest effective currents and sources are given. It is shown that some of the expressions obtained are either incomplete or gauge-noncovariant. In Section 5, the more general expressions of dynamical equations describing a change of the usual color charge $Q^a(t)$ and the Grassmann color charges $\theta^i(t)$, $\theta^{i\dagger}(t)$ of the hard test particle on interaction of the last one with both soft gluon and soft quark fluctuation fields of system are suggested. On the basis of these equations it is determined all necessary additional color currents and sources allowing to derive complete and gauge covariant expressions for the effective currents and sources at least up to third order in powers of free soft gluon field $A^{(0)a}_\mu(k)$, free soft quark field $\psi^{(0)i}_\alpha(q)$, and initial value of the color charges $Q^a_0$, $\theta^i_0$, and $\theta^{i\dagger}_0$. In Sections 6 and 7, and Appendices B, D, and E, the explicit examples of such calculations are given. In Section 8, due to the Tsytovich correspondence principle the effective currents and sources derived in previous sections are used for calculation of the matrix elements of the scattering processes under consideration. Section 9 is concerned with analysis of a structure of simplest scattering probability of plasmino by hard particle. Here a problem of generalization of results obtained for the case when hard test parton is in a partial polarization state is also considered. In Section 10 we deal with definition of semiclassical expressions for energy losses of energetic color particle propagating through hot quark-gluon plasma taking into account the scattering processes off soft-plasma excitations carrying a half-integer spin. In particular an expression for the polarization losses induced by large distance collisions with a change of statistics of
energetic particle is given.

In Sections 11 and 12, thoroughly the so-called 'non-diagonal' contributions to the energy losses are analyzed and conditions for cancellation of off mass-shell singularities of complete expressions for the energy losses are written out. In Conclusion some features of dynamics of soft and hard excitations of Fermi and Bose statistics considered in this work are briefly discussed. Finally, in Appendix A we give an explicit form for the action varying which one obtains the equation of motion for color charge of hard test parton and also the soft gluon and quark field equations. In Appendix C origin of singularity generated by classical soft-quark and soft-gluon one-loop corrections (Section 7) to the effective currents and sources is analyzed and in Appendix F all formulae of the paper [2] necessary for analysis of a structure of the simplest scattering probability are given.

2 Preliminaries

As was mentioned in Introduction in this paper we consider a change of the number densities of colorless soft-quark \( n_{q}^{\pm ij} = \delta^{ij}n_{q}^{\pm} \), soft-antiquark \( \bar{n}_{q}^{\pm ij} = \delta^{ij}\bar{n}_{q}^{\pm} \), and soft-gluon \( N_{k}^{t,lab} = \delta^{ab}N_{k}^{t,lab} \) excitations as a result of their scattering off hard thermal quarks, antiquarks and hard transverse gluons. Hereafter, we denote momenta of the soft-quark fields by \( q, q', q_1, \ldots \), momenta of the soft-gauge fields by \( k, k', k_1, \ldots \), and momenta of the hard thermal particles by \( p, p', \ldots \).

We expect that a time-space evolution of scalar functions \( n_{q}^{(f)} \), \( f = \pm \) and \( N_{k}^{(b)} \), \( b = t, l \) will be described by a self-consistent system of kinetic equations

\[
\frac{\partial n_{q}^{(f)}}{\partial t} + v_{q}^{(f)} \cdot \frac{\partial n_{q}^{(f)}}{\partial x} = -n_{q}^{(f)} \Gamma_{d}^{(f)}[n_{q}^{\pm}, N_{k}^{t,lab}, f_{p}^{G,Q}] + (1 - n_{q}^{(f)}) \Gamma_{i}^{(f)}[n_{q}^{\pm}, N_{k}^{t,lab}, f_{p}^{G,Q}] \tag{2.1}
\]

\[
\equiv -C^{(f)}[n_{q}^{\pm}, N_{k}^{t,lab}, f_{p}^{G,Q}],
\]

\[
\frac{\partial N_{k}^{(b)}}{\partial t} + v_{k}^{(b)} \cdot \frac{\partial N_{k}^{(b)}}{\partial x} = -N_{k}^{(b)} \Gamma_{d}^{(b)}[n_{q}^{\pm}, N_{k}^{t,lab}, f_{p}^{G,Q}] + (1 + N_{k}^{(b)}) \Gamma_{i}^{(b)}[n_{q}^{\pm}, N_{k}^{t,lab}, f_{p}^{G,Q}] \tag{2.2}
\]

\[
\equiv -C^{(b)}[n_{q}^{\pm}, N_{k}^{t,lab}, f_{p}^{G,Q}],
\]

where \( v_{q}^{(f)} = \partial \omega_{q}^{(f)}/\partial q \), \( v_{k}^{(b)} = \partial \omega_{k}^{(b)}/\partial k \) are the group velocities of soft fermionic and bosonic excitations and \( f_{p}^{G} \equiv f^{G}(p, x) \), \( f_{p}^{Q} \equiv f^{Q}(p, x) \) are distribution functions of hard thermal gluons and quarks, which in general case obey own kinetic equations. Here for the sake of brevity we drop a dependence on soft and hard antiquark occupation numbers \( \bar{n}_{q}^{\pm} \) and \( f_{p}^{G} \) on the right-hand side of Eqs. (2.1) and (2.2). The equation for \( \bar{n}_{q}^{(f)} \) is obtained from (2.1) by replacement \( n_{q}^{(f)} \rightarrow (1 - \bar{n}_{q}^{(f)}) \).
We present, as it usually is, decay and regenerating rates in the form of the functional expansion in powers of the soft-(anti)quark and soft-gluon number densities

\[
\Gamma_d^{(f,b)}[n_q^\pm, N_{k}^{t,l}, f_p^{G,Q}] = \sum_{n=1}^\infty \Gamma_d^{(f,b)(2n+1)}[n_q^\pm, N_{k}^{t,l}, f_p^{G,Q}],
\]

(2.3)

\[
\Gamma_i^{(f,b)}[n_q^\pm, N_{k}^{t,l}, f_p^{G,Q}] = \sum_{n=1}^\infty \Gamma_i^{(f,b)(2n+1)}[n_q^\pm, N_{k}^{t,l}, f_p^{G,Q}],
\]

(2.4)

where \(\Gamma_d^{(f,b)(2n+1)}[n_q^\pm, N_{k}^{t,l}, f_p^{G,Q}]\) collect the contributions of the total \(n\)th power in \(n_q^\pm, \bar{n}_q^\pm, \) and \(N_{k}^{t,l}\). As well as in a case of the nonlinear interaction of soft fermion and soft boson excitations among themselves (Paper I), the general structure of the expressions for arbitrary \(n\) is rather cumbersome and therefore here we write out only in an explicit form the decay and regenerating rates to lowest order in the nonlinear interaction \((n = 0, 1)\).

The fermion decay rate is written in the form

\[
\Gamma_d^{(f)}[n_q^\pm, N_{k}^{t,l}, f_p^{G,Q}] = \int \frac{dP}{(2\pi)^3} \left[ f_p^G \left(1 - f_p^{Q}\right) + f_p^{Q} \left(1 + f_p^{G}\right) \right]
\]

(2.5)

\[
\times \left\{ \sum_{b = t, l} \int dT_{q \rightarrow g}(f; b) \left[ w(T; b) (p; q; k) \left(1 + N_k^{(b)}\right) \right] 
\]

\[
+ \sum_{b_1, b_2 = t, l} \left[ \int dT_{q \rightarrow g}(f; b_1 b_2) \left[ w(T; b_1 b_2) (p; q; k_1, k_2) \left(1 + N_{k_1}^{(b_1)}\right) \left(1 + N_{k_2}^{(b_2)}\right) \right] 
\]

\[
+ \int dT_{q \rightarrow g}(f; b_1 b_2) \left[ w(T; b_1 b_2) (p; q; k_1, k_2) N_{k_1}^{(b_1)} \left(1 + N_{k_2}^{(b_2)}\right) \right] 
\]

\[
+ \sum_{f_1, f_2 = -1} \left[ \int dT_{q \rightarrow q}(f; f_1 f_2) \left[ w(T; f_1 f_2) (p; q; q_1, q_2) \left(1 - \bar{n}_{q_1}^{(f_1)}\right) \left(1 - n_{q_2}^{(f_2)}\right) \right] 
\]

\[
+ \int dT_{q \rightarrow q}(f; f_1 f_2) \left[ w(T; f_1 f_2) (p; q; q_1, q_2) \left(1 - n_{q_1}^{(f_1)}\right) \left(1 - n_{q_2}^{(f_2)}\right) \right] \right\} + \ldots 
\]

\[
+ \sum_{\zeta = Q, G} \int \frac{dP}{(2\pi)^3} \left[ f_{p, \zeta}^G \left(1 + f_{p, \zeta}^{Q}\right) \right] \left\{ \sum_{f_1 = \pm} \int dT_{q \rightarrow q}(f; f_1) \left[ w(T; f_1) (p; q; q_1) \left(1 - n_{q_1}^{(f_1)}\right) \right] 
\]

\[
+ \sum_{b = t, l} \sum_{f_1 = \pm} \left[ \int dT_{q \rightarrow g}(f; b_1 f_1) \left[ w(T; b_1 f_1) (p; q; k, q_1) \left(1 + N_{k}^{(b)}\right) \left(1 - n_{q_1}^{(f_1)}\right) \right] 
\]

\[
+ \int dT_{q \rightarrow q}(f; b_1 f_1) \left[ w(T; b_1 f_1) (p; q; k; q_1) N_{k}^{(b)} \left(1 - n_{q_1}^{(f_1)}\right) \right] 
\]

\[
+ \int dT_{q \rightarrow g}(f; f_1 b) \left[ w(T; f_1 b) (p; q; q_1; k) \left(1 + N_{k}^{(b)}\right) \left(1 - \bar{n}_{q_1}^{(f_1)}\right) \right] \right\} + \ldots .
\]
The fermion regenerating rate is

$$\Gamma_1^{(f)}[n_{q^+}, N_{k^+}^{(t,l)}, f_p^{G,Q}] = \int \frac{dp}{(2\pi)^3} \left[ f_p^G \left( 1 + f_{p'}^G \right) + f_{p'}^G \left( 1 - f_p^G \right) \right]$$

(2.5)

$$\left\{ \sum_{b=t,l} \int dT_{q \rightarrow g} w^{(f;b)}(p|q;k) N_{k}^{(b)} + \sum_{b_1,b_2=t,l} \left[ \int dT_{q \rightarrow g} w^{(f_1,b_1;b_2)}(p|q,k_1,k_2) N_{k_1}^{(b_1)} N_{k_2}^{(b_2)} \right. \right. \right.$$  

$$+ \int dT_{q \rightarrow g} w^{(f_2)}(p,k_1,k_2) \left[ 1 + N_{k_1}^{(b_1)} \right] N_{k_2}^{(b_2)} \right\}$$

$$+ \sum_{f_1,f_2 = \pm} \left[ \int dT_{q \rightarrow g} w^{(f_1,f_2)}(p,q_1,q_2) \left( 1 - n_{q_1}^{(f_1)} \right) n_{q_2}^{(f_2)} \right]$$

$$+ \sum_{b=t,l} \sum_{f_1 = \pm} \left[ \int dT_{q \rightarrow g} w^{(f_1)}(p,q_1) N_{k}^{(b)} n_{q_1}^{(f_1)} \right]$$

$$+ \int dT_{q \rightarrow g} w^{(f_1)}(p,q,k) \left( 1 + N_{k}^{(b)} \right) n_{q_1}^{(f_1)}$$

$$+ \int dT_{q \rightarrow g} w^{(f_1)}(p,q,k) \left( 1 - n_{q_1}^{(f_1)} \right) \right\} + \ldots .$$

Here, for brevity we have used a somewhat symbolical denotation. We have taken out factors \( f_p^G (1 - f_p^G) + f_{p'}^G (1 + f_{p'}^G) \) etc. as the general multipliers. However it is necessary to mean that they differ for each term in braces by values of momenta \( p' \) and \( p'' \). Thus, for example, for terms linear in \( N_{k}^{(b)} \) and \( n_{q_1}^{(f_1)} \) it is necessary to mean \( p' \equiv p + q - k \), \( p'' \equiv p + q - q_1 \). Furthermore for the terms quadratic in \( N_{k}^{(b)} \) and \( n_{q_1}^{(f_1)} \) it is necessary to mean \( p' \equiv p + q - k_1 - k_2 \) (or \( p' \equiv p + q - q_1 - q_2 \)), \( p'' \equiv p + q - k - q_1 \) and so on. The function \( w^{(f_1)}(p|q;k) \) defines a probability of two processes: (1) the process of absorption of soft-quark excitation with frequency \( \omega_q^{(f)} \) (and momentum \( q \)) by hard thermal gluon with consequent conversion of the latter into hard thermal quark and radiation of soft-gluon excitation with frequency \( \omega_k^{(b)} \) (and momentum \( k \)) and (2) the process of annihilation of soft-quark excitation with hard thermal antiquark into soft-gluon excitation and hard transverse gluon. In other words, this probability defines the scattering processes of ‘inelastic’ type

\[ q + G \rightarrow g + Q, \quad q + \bar{Q} \rightarrow g + G, \]

\footnote{In the subsequent discussion hard thermal particles of plasma undergoing the scattering processes off soft excitations, will be called also test particles.}
where \( q, g \) are plasma collective excitations and \( G, Q \) and \( \bar{Q} \) are excitations with typical momenta of temperature order \( T \) and above. Furthermore, the function \( w^{(\bar{q})}_{q \rightarrow q}(p| q; q_1) \) defines a probability for an ‘elastic’ scattering of soft-quark excitation off hard thermal quark, antiquark, and gluon:

\[
q + Q \rightarrow q + Q, \quad q + \bar{Q} \rightarrow q + \bar{Q}, \quad q + G \rightarrow q + G.
\]

The functions \( w^{(f; b_1 b_2)}_{q \rightarrow q}(p| q; k_1, k_2) \), \( w^{(f; f_1 f_2)}_{q \rightarrow q}(p| q; q_1, q_2) \), \( w^{(\bar{q}) f_1 f_2}_{q \rightarrow q}(p| q; k_1, q_1) \), and so on define probabilities of more complicated scattering processes connected with an interaction of three soft plasma waves with a hard test particle. Thus the first two of them are the scattering probabilities for the processes of type

\[
q + G \rightarrow g + g + Q, \quad q + \bar{Q} \rightarrow g + g + G,
\]

\[
q + G \rightarrow \bar{q}_1 + q_2 + Q, \quad q + \bar{Q} \rightarrow \bar{q}_1 + q_2 + G
\]
correspondingly, and the third function is the scattering probability for the processes of type

\[
q + G \rightarrow q_1 + g + G, \quad q + Q \rightarrow q_1 + g + Q, \quad q + \bar{Q} \rightarrow q_1 + g + \bar{Q}.
\]

Let us specially emphasize that in the first case the type of the hard particle changes and in the second one it doesn’t. Such a division of the probabilities will take place to all orders of nonlinear interaction of soft and hard modes. This is reflected in particular in our notation of generalized rates (2.4) and (2.5).

The phase-space measures for these processes are

\[
\int d\mathcal{T}^{(f; b)}_{q \rightarrow q} = \int \frac{d\mathbf{k}}{(2\pi)^3} 2\pi \delta \left( E_p + \omega^{(f)}_q - E_{p + q - \mathbf{k}} - \omega^{(b)}_k \right), \quad (2.6)
\]

\[
\int d\mathcal{T}^{(f; f_1)}_{q \rightarrow q} = \int \frac{d\mathbf{q}_1}{(2\pi)^3} 2\pi \delta \left( E_p + \omega^{(f)}_q - E_{p + q - \mathbf{q}_1} - \omega^{(f_1)}_{\mathbf{q}_1} \right),
\]

\[
\int d\mathcal{T}^{(f; b_1 b_2)}_{q \rightarrow q} = \int \frac{d\mathbf{k}_1}{(2\pi)^3} \frac{d\mathbf{k}_2}{(2\pi)^3} 2\pi \delta \left( E_p + \omega^{(f)}_q - E_{p + q - \mathbf{k}_1 - \mathbf{k}_2} - \omega^{(b_1)}_{\mathbf{k}_1} - \omega^{(b_2)}_{\mathbf{k}_2} \right),
\]

\[
\int d\mathcal{T}^{(f; f_1 f_2)}_{q \rightarrow q} = \int \frac{d\mathbf{q}_1}{(2\pi)^3} \frac{d\mathbf{q}_2}{(2\pi)^3} 2\pi \delta \left( E_p + \omega^{(f)}_q - E_{p + q - \mathbf{q}_1 - \mathbf{q}_2} - \omega^{(f_1)}_{\mathbf{q}_1} - \omega^{(f_2)}_{\mathbf{q}_2} \right),
\]

\[
\int d\mathcal{T}^{(f; b_1)}_{q \rightarrow q} = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{d\mathbf{q}_1}{(2\pi)^3} 2\pi \delta \left( E_p + \omega^{(f)}_q - E_{p + q - \mathbf{k} - \mathbf{q}_1} - \omega^{(b)}_k - \omega^{(f_1)}_{\mathbf{q}_1} \right),
\]

etc. Here \( E_p \equiv |p| \) for massless hard gluons and (anti)quarks. The \( \delta \)-functions in Eqs. (2.6) represent the energy conservation for the scattering processes under consideration.
We particularly note that for generalized decay and regenerated rates (2.4) and (2.5) it was assumed that the scattering probabilities satisfy symmetry relations over permutation of incoming and outgoing soft quark and gluon momenta. For the simplest probabilities, e.g., we have

\[ w_{q \rightarrow g}^{(f:b)}(p | q; k) = w_{q \rightarrow g}^{(b:f)}(p | k; q), \quad w_{q \rightarrow q}^{(f):f_1}(p | q; q_1) = w_{q \rightarrow q}^{(f):f}(p | q; q_1). \]  

(2.7)

These relations are a consequence of more general relations for exact scattering probabilities depending on initial and final values of momenta of hard particles, namely,

\[ w_{q \rightarrow g}^{(f:b)}(p, p' | q; k) = w_{g \rightarrow q}^{(b:f)}(p', p | k; q), \quad w_{q \rightarrow q}^{(f):f_1}(p, p'' | q; q_1) = w_{q \rightarrow q}^{(f):f}(p'', p | q; q_1). \]

They present detailed balancing principle in scattering processes and in this sense they are exact. The scattering probabilities in Eqs. (2.4), (2.5) are obtained by integrating initial probabilities over \( p' \) or \( p'' \) with regard to momentum conservation law:

\[ w_{q \rightarrow g}^{(f:b)}(p | q; k)2\pi \delta(E_p + \omega_q^{(f)} - E_{p+q-k} - \omega_k^{(b)}) \]

\[ = \int w_{q \rightarrow g}^{(f:b)}(p, p' | q; k)2\pi \delta(E_p + \omega_q^{(f)} - E_{p'} - \omega_k^{(b)})2\pi \delta(p + q - p' - k) \frac{dp'}{(2\pi)^3}, \]

\[ w_{q \rightarrow q}^{(f):f_1}(p | q; q_1)2\pi \delta(E_p + \omega_q^{(f)} - E_{p+q-q_1} - \omega_{q_1}^{(f_1)}) \]

\[ = \int w_{q \rightarrow q}^{(f):f_1}(p, p'' | q; q_1)2\pi \delta(E_p + \omega_q^{(f)} - E_{p''} - \omega_{q_1}^{(f_1)})2\pi \delta(p + q - p'' - q_1) \frac{dp''}{(2\pi)^3}. \]

The scattering probabilities obtained satisfy relations (2.7) in the limit of interest to us, i.e., when we neglect by (quantum) recoil of hard particles. In the general case expressions (2.7) are replaced by more complicated ones (see, e.g., Ref. [6], Section 10).

For the boson sector of plasma excitations the generalized rates \( \Gamma_{d}^{(b)} \) and \( \Gamma_{i}^{(b)} \) to lowest order in the nonlinear interactions of soft and hard modes have a similar structure. For this reason, their explicit form is not given here.

Taking into account that in semiclassical approximation we have \( |p| \gg |k|, |q|, |q_1|, \ldots, \) the energy conservation laws can be represented in the form of the following resonance conditions:

\[ \omega_q^{(f)} - \omega_k^{(b)} - v \cdot (q - k) = 0, \quad \omega_q^{(f)} - \omega_{q_1}^{(f_1)} - v \cdot (q - q_1) = 0, \]

(2.8)

\[ \omega_q^{(f)} - \omega_{k_1}^{(b_1)} - \omega_{k_2}^{(b_2)} - v \cdot (q - k_1 - k_2) = 0, \]

\[ \omega_q^{(f)} - \omega_{q_1}^{(f_1)} - \omega_{q_2}^{(f_2)} - v \cdot (q - q_1 - q_2) = 0, \]

\[ \omega_q^{(f)} - \omega_k^{(b)} - \omega_{q_1}^{(f_1)} - v \cdot (q - k - q_1) = 0, \]

and so on. Here, \( v = p/|p| \) is a velocity of the hard test particle.
In subsequent discussion for the sake of simplicity of the problem we suppose that a characteristic time for nonlinear relaxation of the soft oscillations is much less than a time of relaxation of the distribution of hard partons $f_p^G$, $f_p^Q$ and $f_p^Q$. In other words under the conditions when the intensity of soft plasma excitations are sufficiently small and they cannot essentially change such 'crude' equilibrium parameters of plasma as particle density, temperature, and thermal energy, we can neglect by a space-time change of the distribution functions of hard partons assuming that these functions are specified and describe the global (baryonfree) equilibrium state of hot non-Abelian plasma

$$f^G_p = 2 \frac{1}{e^{E_p/T} - 1}, \quad f^Q_p = 2 \frac{1}{e^{E_p/T} + 1}.$$ 

Here the coefficient 2 takes into account that hard gluon and hard (anti)quark have two helicity states.

In Section 8 for determining an explicit form of the required probabilities we need in some specific approximation of collision terms. We use the fact that the occupation numbers $N^{(b)}_p$ are more large than one, i.e., $1 + N^{(b)}_p \simeq N^{(b)}_p$. Furthermore, we present the integration measure as

$$\int \frac{d\mathbf{p}}{(2\pi)^3} = \int \frac{|\mathbf{p}|^2 d|\mathbf{p}|}{2\pi^2} \int d\Omega_\mathbf{v},$$

where the solid integral is over the directions of unit vector $\mathbf{v}$. Setting $f^G_p \simeq f^G_p$, $f^Q_p \simeq f^Q_p$ and also $1 \pm f^G_p f^Q_p \simeq 1 \pm f^G_p f^Q_p \simeq 1$ by virtue of $f^G_p f^Q_p \ll 1$, in the limit of vanishing fermion intensity $n^{(f)}_q \to 0$ we have the following expression for collision term $C^{(f)}[n^\pm_q, N^{\pm 1}_q, f^G_p, f^G_p]$

$$C^{(f)}[n^\pm_{q_i}, N^{\pm 1}_{q_k}, f^G_p] \simeq -\int \frac{|\mathbf{p}|^2 d|\mathbf{p}|}{2\pi^2} \int f_p^Q f_p^G \int d\Omega_\mathbf{v} \left\{ \sum_{b = t, l} \int dT_{q \to \bar{q}} w_{q \to \bar{q}}^{(f; b)}(p | q; k) N^{(b)}_k \right. 

+ \sum_{b_1, b_2 = t, l} \int dT_{q \to g, \bar{g}} w_{q \to g, \bar{g}}^{(f; b_1 b_2)}(p | q; k_1, k_2) N^{(b_1)}_{k_1} N^{(b_2)}_{k_2} 

+ \sum_{f_1, f_2} \int dT_{q \to g, q_1, q_2} w_{q \to g, q_1, q_2}^{(f; f_1 f_2)}(p | q; q_1, q_2) \bar{n}^{(f_1)}_{q_1} n^{(f_2)}_{q_2} + \ldots \right\} 

- \sum_{z = Q, Q, G} \int \frac{|\mathbf{p}|^2 d|\mathbf{p}|}{2\pi^2} f_p^{(z)} \int d\Omega_\mathbf{v} \left\{ \sum_{f_1 = \pm} \int d\Omega_\mathbf{v} \int dT_{q \to q} w_{q \to q}^{(f; f_1)}(p | q; q_1) n^{(f_1)}_{q_1} 

+ \sum_{b = t, l} \sum_{f_1 = \pm} \int dT_{q \to g, q_1} w_{q \to g, q_1}^{(f; b f_1)}(p | q; k, q_1) N^{(b)}_{k} n^{(f_1)}_{q_1} + \ldots \right\}.$$

The kinetic equation (2.1) with collision term in the form of (2.9) defines a change of the soft-quark number density $n^{(f)}_q$ caused by so-called spontaneous scattering processes of soft-quark excitations off hard thermal partons.
3 Soft-field equations

We consider SU\((N_c)\) gauge theory with \(n_f\) flavors of massless quarks. The color indices for the adjoint representation \(a, b, \ldots\) run from 1 to \(N_c^2 - 1\) while ones for the fundamental representation \(i, j, \ldots\) run from 1 to \(N_c\). The Greek indices \(\alpha, \beta, \ldots\) for the spinor representation run from 1 to 4.

In this section we discuss the equations of motion for soft-gluon and soft-quark plasma excitations, which will play a main role in our subsequent discussion. We have already written out these equations in Paper I (Eqs. (I.3.1), (I.3.2)). Here they should be correspondingly extended to take into account the presence of a color current caused by hard test particle passing through the hot QCD plasma.

One expects the world lines of the hard modes to obey classical trajectories in the manner of Wong [4] since their coupling to the soft modes is weak at a very high temperature. Considering this circumstance, we add the color current of color point charge

\[
  j_Q^a(x) = gv^\mu Q^a(t)\delta(3)(x - vt)
\]

to the right-hand side of the Yang-Mills field equation (I.3.1). Here, \(Q^a = Q^a(t)\) is a color classical charge satisfying the Wong equation

\[
  \frac{dQ^a(t)}{dt} + igv^\mu A^b_\mu(t, vt)(T^b)^{ac}Q^c(t) = 0, \quad Q^a_0 = Q^a(t)|_{t=0},
\]

where \(v^\mu = (1, v, \ldots)\), \(T^a)^{bc} \equiv -if^{abc}\) and \(t\) is a coordinate time. The gauge potential \(A^b_\mu(t, x)\) in Eq. (3.2) is determined on a straight-line trajectory of a hard parton, i.e., at \(x = vt\). The explicit form of a solution of Eq. (3.2) in the momentum representation is given in Ref. [6]. Thus we lead to the following integral equation for gauge potential \(A^a_\mu(k)\) instead of Eq. (I.3.1)

\[
  ^*D^{-1}\mu\nu(k)A^{\alpha\nu}(k) = -j^{A(2)a}_\mu(A, A)(k) - j^{A(3)a}_\mu(A, A, A)(k) - j^{(0,2)a}_\mu(\bar{\psi}, \psi)(k)
\]

\[
  - j^{(1,2)a}_\mu(A, \bar{\psi}, \psi)(k) - j^{(0)a}_\mu(Q_0)(k) - j^{(0)a}_\mu(A, A)(k)
\]

\[
  \equiv -j^{a}_\mu[A, \bar{\psi}, \psi, Q_0](k).
\]

On the right-hand side in the expansion of the induced currents \(j^A, j^\psi\) and current of hard color-charged parton we keep the terms up to the third order in interacting fields and initial value of color charge \(Q_0^a\). The explicit form of induced currents \(j^{A(2)a}_\mu, j^{(0,2)a}_\mu\) and \(j^{(1,2)a}_\mu\) in the hard thermal loop (HTL) approximation is defined by Eq. (I.3.3). The expansion terms of hard parton current have the following structure:

\[
  j^{(0)a}_\mu(k) = \frac{g}{(2\pi)^3} v^\mu Q^a \delta(v \cdot k),
\]
\( j^{(1)\mu}_Q(A)(k) = \frac{g^2}{(2\pi)^3} v^\mu \int \frac{1}{(v \cdot k_1)} (v \cdot A^{a_1}(k_1)) \delta(v \cdot (k - k_1)) dk_1 (T^{a_1})^{ab} Q^b, \)

\( j^{(2)\mu}_Q(A, A)(k) = \frac{g^3}{(2\pi)^3} v^\mu \int \frac{1}{(v \cdot (k_1 + k_2)) (v \cdot k_2)} (v \cdot A^{a_1}(k_1)) (v \cdot A^{a_2}(k_2)) \delta(v \cdot (k - k_1 - k_2)) dk_1 dk_2 (T^{a_1} T^{a_2})^{ab} Q^b. \)

Now we consider the Dirac field equations (I.3.2). To take into account an existence of current of a test color particle in system it is necessary to put a certain additional terms into the right-hand side of soft-quark field equations. For determination of an explicit form of these terms we introduce a Grassmann color charge

\( \theta^i = \theta^i(t) \) (and conjugate color charge \( \theta^i = \theta^i(t) \)) of hard particle satisfying the following fundamental equation proposed in [5]

\[
\frac{d\theta^i}{dt} + igv^\mu A^a_{\mu}(t, vt)(t^a)^{ij} \theta^j(t) = 0, \quad \theta^i_0 = \theta^i(t) |_{t=0} \tag{3.5}
\]

and correspondingly

\[
\frac{d\theta^{i^j}}{dt} - igv^\mu A^a_{\mu}(t, vt)\theta^{i^j}(t)(t^a)^{ji} = 0, \quad \theta^{i^j}_0 = \theta^{i^j}(t) |_{t=0}. \tag{3.6}
\]

The Grassmann color charge \( \theta^i \) is associated with the usual color charge \( Q^a \) by relation

\[
Q^a(t) = \theta^{i^j}(t)(t^a)^{ij} \theta^j(t). \tag{3.7}
\]

By analogy with classical color current (3.1) we write the following expression for a Grassmann color ‘current’ (further called a Grassmann color source) of hard test particle

\[
\eta^i_{\theta^a}(x) = g\theta^i(t)\chi_\alpha \delta^{(3)}(x - vt) \tag{3.8}
\]

and correspondingly its conjugation

\[
\bar{\eta}^i_{\theta^a}(x) = g\theta^{i^j}(t)\bar{\chi}_\alpha \delta^{(3)}(x - vt). \tag{3.9}
\]

Here, \( \chi_\alpha \) is a spinor independent of time \( t \). The physical sense of this spinor will be discussed in the following sections.

Taking into account (3.8) and (3.9), we lead to the nonlinear integral equations for soft-quark interacting fields \( \psi^i \) and \( \bar{\psi}^i \) instead of (I.3.2)

\[
\hat{S}^{-1}_{\alpha\beta} \psi^i_{\alpha}(q) = -\eta^{(1)i}_\alpha(A, \psi)(q) - \eta^{(2)i}_\alpha(A, A, \psi)(q) - \eta^{(0)i}_\alpha(A)(q) - \eta^{(1)i}_\alpha(A)(q) - \eta^{(2)i}_\alpha(A, A)(q) \equiv -\eta^{i}_\alpha[A, \psi, \theta_0](q) - \eta^{(0)i}_\alpha(q), \tag{3.10}
\]
\[
\bar{\psi}_\beta (-q)^* S_{\beta \alpha}^{-1} (-q) = \bar{\eta}_{\alpha}^{(1,1)i} (A^*, \bar{\psi})(-q) + \bar{\eta}^{(2,1)i}_{\alpha} (A^*, A^*, \bar{\psi})(-q) + \bar{\eta}_{\theta \alpha}^{(0)i} (-q) + \bar{\eta}_{\theta \alpha}^{(0)i} (A^*)(-q) + \bar{\eta}_{\theta \alpha}^{(2)i} (A^*, A^*) (-q) \equiv \bar{\eta}_{\alpha}^{i} [A^*, \bar{\psi}, \theta_{0}^1] (-q) + \bar{\eta}_{\theta \alpha}^{(0)i} (-q),
\]

where on the right-hand side in the expansion of induced sources \( \eta \), \( \bar{\eta} \) and the Grassmann test particle sources \( \eta_0 \), \( \bar{\eta}_0 \), we keep the terms up to the third order in interacting fields and initial values of the Grassmann color charges \( \eta \), \( \bar{\eta} \). Equations (1.3.4) and (1.3.5) define an explicit form of induced sources \( \eta_{\alpha}^{(1,1)i} \), \( \eta_{\alpha}^{(2,1)i} \), \ldots in the HTL approximation. By virtue of identical structure of evolution equations (3.2) and (3.5) by trivial replacements it is easy to define the expansion terms from (3.4) for the Grassmann source \( \eta_{\theta \alpha}^{i} \) in the momentum representation

\[
\bar{\eta}_{\theta \alpha}^{(0)i} (q) = \frac{g}{(2\pi)^{3}} \theta_{0}^1 \chi_{\alpha} \delta (v \cdot q), \tag{3.11}
\]

\[
\eta_{\theta \alpha}^{(1)i} (A)(q) = \frac{g^{2}}{(2\pi)^{3}} \chi_{\alpha} \int \frac{1}{(v \cdot q_1)} (v \cdot A^{a_1} (q_1)) \delta (v \cdot (q - q_1)) \, dq_1 \, \left(t^{a_1}\right)^{ij} \theta_{0}^j, \tag{3.12}
\]

\[
\eta_{\theta \alpha}^{(2)i} (A, A)(q) = \frac{g^{3}}{(2\pi)^{3}} \chi_{\alpha} \int \frac{1}{(v \cdot (q_1 + q_2)) (v \cdot q_2)} (v \cdot A^{a_1} (q_1)) (v \cdot A^{a_2} (q_2)) \times \delta (v \cdot (q - q_1 - q_2)) \, dq_1 \, dq_2 \, \left(t^{a_1}\right)^{ij} \theta_{0}^j.
\]

The expressions for conjugate terms \( \bar{\eta}_{\theta \alpha}^{(s)i} \), \( s = 0, 1, 2 \) can be obtained from (3.11) by rule:

\[
\bar{\eta}_{\theta \alpha}^{(s)i} (-q) = \eta_{\theta \beta}^{(s)i} (q) \bar{\eta}_{\beta \alpha}^{(0)i}.
\]

Now we rewrite equations (3.3) and (3.10) in the following form:

\[
A_{\mu}^{\alpha} (k) = A_{\mu}^{(0)a} (k) - * D_{\mu \nu} (k) j_{Q}^{(0)av} (k) - * D_{\mu \nu} (k) j^{av} \left[A, \bar{\psi}, \psi, Q_0\right] (k),
\]

\[
\psi_{\alpha}^{i} (q) = \bar{\psi}_{\alpha}^{(0)i} (q) - * S_{\alpha \beta} (q) \eta_{\theta \beta}^{(0)i} (q) - * \bar{S}_{\alpha \beta} (q) \eta_{\beta \alpha}^{(0)i} (q), \tag{3.12}
\]

\[
\bar{\psi}_{\alpha}^{(s)i} (-q) = \bar{\psi}_{\alpha}^{(0)i} (-q) + \bar{\eta}_{\theta \alpha}^{(0)i} (-q) * S_{\beta \alpha} (-q) + \bar{\eta}_{\beta \alpha}^{(0)i} (-q) * S_{\beta \alpha} (-q),
\]

where \( A_{\mu}^{(0)a} (k) \), \( \psi_{\alpha}^{(0)i} (q) \) and \( \bar{\psi}_{\alpha}^{(0)i} (q) \) are free-field solutions. The second terms on the right-hand sides of these equations represent fields induced by a test ‘bare’ particle moving in the medium. In the weak-field limit the system of the nonlinear integral equations can be perturbatively solved by the approximation scheme method. Formally, a solution of system (3.12) can be presented in the following form:

\[
A_{\mu}^{a} (k) = A_{\mu}^{(0)a} (k) - * D_{\mu \nu} (k) j_{Q}^{(0)av} (k) - * D_{\mu \nu} (k) j^{av} \left[A^{(0)}, \bar{\psi}^{(0)}, \psi^{(0)}, Q_0, \theta_{0}^1, \theta_{0}^1\right] (k),
\]

\[
\psi_{\alpha}^{i} (q) = \psi_{\alpha}^{(0)i} (q) - * S_{\alpha \beta} (q) \eta_{\theta \beta}^{(0)i} (q) - * \bar{S}_{\alpha \beta} (q) \eta_{\beta \alpha}^{(0)i} (q), \tag{3.13}
\]

\[
\bar{\psi}_{\alpha}^{i} (-q) = \bar{\psi}_{\alpha}^{(0)i} (-q) + \bar{\eta}_{\theta \beta}^{(0)i} (-q) * S_{\beta \alpha} (-q) + \bar{\eta}_{\beta \alpha}^{(0)i} (-q) * S_{\beta \alpha} (-q).
\]
Here the functions \( \tilde{j}^\mu_a, \tilde{\eta}_i^\beta \) and \( \tilde{\bar{\eta}}_i^\beta \) represent some new (effective) currents and sources being functionals of free fields and initial values of color charges. Our main purpose is calculation of an explicit form of these effective currents and sources.

Unfortunately, as was earlier shown \[1\] a direct employment of the approximation scheme method for determination of desired effective currents and sources is very complicated already on the second step of iteration and as a consequence it is ineffective. Here we use a simpler approach to calculation of effective currents and sources suggested in our previous works \[8, 6, 9, 1\]. It is based on the fact that by virtue of the structure of the right-hand sides of Eqs. (3.12) and (3.13) the following system of equations should be carried out identically

\[
j^\mu_a[A, \bar{\psi}, \psi, Q_0](k) = \tilde{j}^\mu_a[A^{(0)}, \bar{\psi}^{(0)}, \psi^{(0)}, Q_0, \theta_0^i, \theta_0^\dagger](k), \tag{3.14}
\]

\[
\eta_i^\beta[A, \psi, \theta_0](q) = \tilde{\eta}_i^\beta[A^{(0)}, \psi^{(0)}, Q_0, \theta_0](q), \tag{3.15}
\]

\[
\bar{\eta}_i^\beta[A^*, \bar{\psi}, \theta_0^\dagger ](-q) = \tilde{\bar{\eta}}_i^\beta[A^{(0)*}, \bar{\psi}^{(0)}, Q_0, \theta_0^\dagger ](-q), \tag{3.16}
\]

under the condition that on the left-hand sides of Eqs. (3.14) – (3.16) the interacting fields \( A^a_\mu, \psi_i^\alpha, \) and \( \bar{\psi}_i^\alpha \) are defined by expressions (3.13). For deriving an explicit form of the effective currents and sources we functionally differentiate the right- and left-hand sides of equations (3.14) – (3.16) with respect to free fields \( A_\mu^{(0)}, \psi_i^{(0)}, \bar{\psi}_i^{(0)} \) and initial color charges \( Q_0^a, \theta_0, \theta_0^\dagger \) considering Eqs. (I.3.3), (3.4), (I.3.4), (I.3.5), (3.11) and so on for differentiation on the left-hand sides, and set \( A_\mu^{(0)} = \psi_i^{(0)} = \bar{\psi}_i^{(0)} = Q_0^a = \theta_0^i = \theta_0^\dagger = 0 \) at the end of calculations. In Sections 4, 6, and 7 we will give some examples.

### 4 Effective currents and sources of lower order

The first simplest effective source arises from second derivative of the source \( \eta_i^\alpha[A, \psi, \theta_0](q) \) with respect to free soft-quark field \( \psi^{(0)} \) and initial usual color charge \( Q_0 \). Taking into account Eq. (I.3.4) and the derivative

\[
\frac{\delta A_\mu^a(k)}{\delta Q_0^a} \bigg|_0 = -\frac{g}{(2\pi)^3} \delta^{ab} \delta_a^\mu (v \cdot k), \tag{4.1}
\]

we find

\[
\frac{\delta^2 \eta_i^\beta[A, \psi, \theta_0](q)}{\delta \psi_i^\alpha(q) \delta Q_0^a} \bigg|_0 = \frac{\delta^2 \tilde{\eta}_i^\beta[A^{(0)}, \psi^{(0)}, Q_0, \theta_0](q)}{\delta \psi_i^\alpha(q) \delta Q_0^a} \bigg|_0 \tag{4.2}
\]

\[
= -\frac{g^2}{(2\pi)^3} \Gamma_i^{(Q)}(q-q_1, q_1, -q) \delta^{ab} \delta^\mu_a \delta_a^\mu (v \cdot (q-q_1)).
\]
Here and henceforth the symbol “$|_0$” means that derivatives are taken for values $A^{(0)} = \psi^{(0)} = Q_0 = \theta_0 = 0$. Thus in linear approximation in free quark field $\psi^{(0)}$ and color charge $Q_0$, we have

$$\tilde{\eta}^{(1)}_\alpha(\psi^{(0)}, Q_0)(q) = \frac{g^2}{(2\pi)^3} (t^a)_{ii} Q_0^a \int K^{(Q)}_{\alpha\alpha_1}(\chi, \bar{\chi}| q, -q_1) \psi^{(0)}_{\alpha_1}(q_1) \delta(v \cdot (q - q_1)) dq_1,$$  
(4.3)

where the coefficient function in the integrand is

$$K^{(Q)}_{\alpha\alpha_1}(\chi, \bar{\chi}| q, -q_1) \equiv -\Gamma^{(Q)\mu}_{\alpha\alpha_1}(q - q_1; q_1, -q) \ast D^{\mu
u}(q - q_1)v^{\nu}.$$  
(4.4)

The effective source (4.3) within the framework of semiclassical approximation generates the simplest elastic scattering process of soft-quark excitation off the hard test particle, i.e., the scattering process occurring without change of statistics of soft and hard excitations (Fig. 1).

![Figure 1](image)

Figure 1: The process of elastic scattering of soft fermion excitation off the hard test particles. The blob stands for the HTL resummation and the double line denotes hard test particles ($G$ is a hard gluon, $Q$ ($\bar{Q}$) is a hard quark (antiquark)).

Furthermore, we calculate the second derivative of the source $\eta^{(1)}_\alpha[A, \psi, \theta_0](q)$ with respect to $A^{(0)}$ and $\theta_0$. Taking into account Eq. (3.11), (I.3.4) and derivative

$$\frac{\delta \psi^{(0)}_\alpha(q)}{\delta \theta^j_0} \bigg|_0 = -\frac{g}{(2\pi)^3} S_{\alpha\beta}(q) \chi_{\beta} \delta^{ij} \delta(v \cdot q)$$  
(4.5)

after simple calculations, we get

$$\frac{\delta^2 \tilde{\eta}^{(1)}_\alpha[A, \psi, \theta_0](q)}{\delta A^{(0)\alpha}_\mu(k) \delta \theta^j_0} \bigg|_0 = \frac{\delta^2 \tilde{\eta}^{(1)}_\alpha[A^{(0)}, \psi^{(0)}, Q_0, \theta_0](q)}{\delta A^{(0)\alpha}_\mu(k) \delta \theta^j_0} \bigg|_0$$

$$= \frac{g^2}{(2\pi)^3} (t^a)^{ij} K^{(Q)\alpha\mu}(v, \chi| k, -q) \delta(v \cdot (k - q)).$$
where the coefficient function $K^{(Q)\mu}_\alpha$ is defined by the expression

$$K^{(Q)\mu}_\alpha(v, \chi| k, -q) \equiv \frac{v^\mu \chi_\alpha}{v \cdot q} - \ast \Gamma_{\alpha\beta}^{(Q)\mu}(k; q - k, -q) \ast S_{\beta\beta'}(q - k) \chi_{\beta'}.$$  \hspace{1cm} (4.6)

Thus we have found one more effective source additional to (4.3) linear in free gauge field $A^{(0)}$ and the Grassmann color charge $\theta_0$

$$\tilde{\eta}^{(1)i}_{\alpha}(A^{(0)}, \theta_0)(q) = \frac{g^2}{(2\pi)^3} (t^a)^{ij} \theta_0 \int K^{(Q)\mu}_\alpha(v, \chi| k, -q) A^{(0)a}_\mu(k) \delta(v \cdot (k - q)) dk. \hspace{1cm} (4.7)$$

This effective source generates now somewhat more complicated scattering process of soft-quark excitation off hard thermal particles bringing into change of statistics of hard and soft modes. The diagrammatic interpretation of two terms in expression (4.6) is presented in Fig. 2, where in first line a ‘direct’ channel of scattering is depicted and in the second one ‘annihilation’ channel is drawn. \footnote{In the semiclassical approximation, the first term in coefficient function (4.6) also contains processes, where the soft gluon is emitted prior to soft-quark absorption. Hereafter for the sake of brevity the diagrams of these scattering processes will be sequentially omitted.}

For convenience of the further references we write

out also an explicit form of effective source Dirac conjugate to (4.7)

$$\tilde{\eta}^{(1)i}_{\alpha}(A^{(0)}, \theta_0^\dagger)(-q) = \frac{g^2}{(2\pi)^3} (t^a)^{ji} \theta_0 \int \bar{K}^{(Q)\mu}_\alpha(v, \bar{\chi}| k, -q) \ast A^{(0)a}_\mu(k) \delta(v \cdot (k - q)) dk. \hspace{1cm} (4.8)$$

Figure 2: The lowest order scattering process of soft fermion excitations off the hard test particles with a change of statistics of hard and soft excitations.
Here the coefficient function $\tilde{K}^{(Q)\mu}_\alpha(k, -q)$ equals

$$\tilde{K}^{(Q)\mu}_\alpha(v, \bar{v}\check{\alpha} k, -q) = \frac{v^\mu \bar{v}^\check{\alpha}}{v \cdot q} + \bar{\chi}_\beta S_{\beta'\beta}(-q + k) \Gamma^{(Q)\mu}_\beta(-k; -q + k, q). \quad (4.9)$$

It should be mentioned that in calculating derivatives with respect to the Grassmann charges $\theta_0^i$ and $\theta_0^{i\dagger}$ we must consider variables $Q_0^0$, $\theta_0^i$ and $\theta_0^{i\dagger}$ as completely independent, in spite of the fact that there exists relation (3.7). Otherwise in calculating effective currents $\tilde{j}^{(Q)\mu}(k)$ we obtain the coefficient functions containing terms of different order in the coupling constant.

Now we consider derivatives of the current $j^{a\nu} [A, \bar{\psi}, \psi, Q_0](k)$. To lowest order in the coupling here there exist two nontrivial derivatives linear in free soft-quark fields $\bar{\psi}^{(0)}$, $\psi^{(0)}$ and the Grassmann charges $\theta_0^i$, $\theta_0^{i\dagger}$. The first of them has a form

$$\frac{\delta^2 j^{a\nu} [A, \bar{\psi}, \psi, Q_0](k)}{\delta \bar{\psi}^{(0)i}(q) \delta \theta_0^i} \bigg|_0 = \frac{\delta^2 j^{a\nu} [A^{(0)}, \bar{\psi}^{(0)}, \psi^{(0)}, Q_0, \theta_0^i, \theta_0^{i\dagger}](k)}{\delta \bar{\psi}^{(0)i}(q) \delta \theta_0^i} \bigg|_0 \quad (4.10)$$

$$= \frac{g^2}{(2\pi)^3} (\Gamma^{(G)\mu}_\alpha(k; q, -k - q) S_{\beta'\beta}(k + q) \chi_{\beta'} \delta(v \cdot (k + q))).$$

In deriving the expression we have used Eqs. (1.3.3) and (4.5). Another variation has a similar structure

$$\frac{\delta^2 j^{a\nu} [A, \bar{\psi}, \psi, Q_0](k)}{\delta \psi^{(0)j}(q) \delta \theta_0^{j\dagger}} \bigg|_0 = \frac{\delta^2 j^{a\nu} [A^{(0)}, \bar{\psi}^{(0)}, \psi^{(0)}, Q_0, \theta_0^i, \theta_0^{i\dagger}](k)}{\delta \psi^{(0)j}(q) \delta \theta_0^{j\dagger}} \bigg|_0 \quad (4.11)$$

$$= \frac{g^2}{(2\pi)^3} (\Gamma^{(G)\mu}_\alpha(k; -k + q, -q) \delta(v \cdot (k - q))).$$

Let us recall that we consider the quark wave functions $\psi^{(0)}_\alpha$ and $\bar{\psi}^{(0)}_\check{\alpha}$ as Grassmann variables similarly to color charges $\theta_0^i$ and $\theta_0^{i\dagger}$, i.e., they obey anticommutation relations. Besides we require in addition fulfilment of rules

$$\{ \bar{\psi}^{(0)}_\check{\alpha}, \theta_0^i \} = \{ \psi^{(0)}_\alpha, \theta_0^{i\dagger} \} = \{ \bar{\psi}^{(0)}_\check{\alpha}, \theta_0^{i\dagger} \} = \{ \psi^{(0)}_\alpha, \theta_0^i \} = 0,$$

where $\{ , \}$ denotes anticommutator\footnote{In conjunction operation of product of two (and more) anticommuting functions it is necessary to follow the rule

$$(ab) = b^\dagger a^\dagger,$$

i.e., $a^\dagger$ and $b^\dagger$ are rearranged without change of a sign \cite{10}.}. Consideration of these relations is important for obtaining correct signs ahead of the terms in the coefficient functions of the effective currents and sources.
It is easy to show that the coefficient functions in the effective sources (4.3) and (4.7) are gauge invariant if they are determined on mass-shell of soft plasma modes. In the first case this implies independence of the function (4.4) from a choice of gauge for soft-gluon propagator. Let us consider in more detail the second case. We convolve function (4.6) with longitudinal projector \( \bar{u}_\mu(k) = k^2 u_\mu(k \cdot u) \) in a covariant gauge. Making use the effective Ward identity

\[
\ast \Gamma^{(Q)}_{\mu}(k-q-k,-q) k_\mu = \ast S^{-1}(q-k) - \ast S^{-1}(q),
\]

we find

\[
\bar{u}_\mu(k) K^{(Q)}_{\alpha}(v, \chi | k, -q) = k^2 \left\{ \frac{\chi_\alpha}{v \cdot q} - \ast \Gamma^{(Q)\mu}_{\alpha\beta}(k; q-k,-q) S^{-1}_{\beta\beta'}(q-k) \chi_{\beta'} \right\}
\]

\[
- k^0 \left\{ \chi_\alpha - (\ast S^{-1}_{\alpha\beta}(q-k) - \ast S^{-1}_{\alpha\beta}(q)) S^{-1}_{\beta\beta'}(q-k) \chi_{\beta'} \right\}.
\]

On mass-shell of soft fermion excitations the equation \( \ast S^{-1}(q) = 0 \) is fulfilled and therefore, the second term on the right-hand side of Eq. (4.13) vanishes. Furthermore, we consider a convolution of coefficient function (4.6) with longitudinal projector in the temporal gauge \( \tilde{u}_\mu(k) = k^2 (u_\mu(k \cdot u) - k_\mu(k \cdot u)) / (k \cdot u) \). The reasonings similar to previous ones result the convolution \( \tilde{u}_\mu(k) K^{(Q)}_{\alpha}(v, \chi | k, -q) \) on-shell in the expression, which exactly equals the first term on the right-hand side of Eq. (4.13). By this means we have shown that at least in the class of temporal and covariant gauges coefficient function (4.6) is gauge invariant.

If we repeat the same reasonings for coefficient function generated by derivatives (4.10) (or (4.11)) using the Ward identity for HTL-resummed vertex \( \ast \Gamma^{(G)}_{\mu}(k; q,-k-q) \), then we see that the function is not gauge invariant. The analysis of this fact suggests that for restoration of gauge symmetry of effective currents it is necessary to add an additional current to the right-hand side of the Yang-Mills equation (3.3). This current is directly connected with an existence of spin-half hard partons with the Grassmann color charge. The next section is concerned with consideration of this problem in more details.

5 Additional color currents and sources

Let us define an additional current, which should be added to the right-hand side of the Yang-Mills equation in order to restore a gauge invariance of the coefficient function determined by derivative (4.11) (and also (4.11)). At first we consider this problem in the coordinate representation. It is clear that this current is to be real, gauge-covariant, vanishing in the absence of the hard test particle with the Grassmann charge (i.e., for \( \theta^\dagger_0 = \theta_0 = 0 \)). Besides, the current as far as possible should not generate additional terms,
which not needed for restoration of gauge invariance of coefficient functions in the effective
currents. Let us write out usual color current (3.1) taking into account the relation (3.7)
\[ j^a_{Q\mu} [A](x) = g v_\mu \theta^{(t)}(t^a)^{ij}(t) \bar{\psi}_j(t) \delta^{(3)}(x - vt) \]
and transform this current as follows. We perform the replacements
\[ \theta^{(t)}(t) \rightarrow \theta^{(t)}(t) \left( \equiv \theta_0^{(t)} U^{(t)}(t_0, t) \right), \]
\[ \theta^j(t) \rightarrow ig \int_{t_0}^t U^{ji}(t, \tau) \left( \bar{\chi}_\alpha \psi^j_\alpha(\tau, v \tau) \right) d\tau, \]
where \( U(t, \tau) = T \exp \left\{ -ig \int_{\tau}^t (v \cdot A^a(\tau', v \tau')) t^a d\tau' \right\} \) is the evolution operator in the fund-
damental representation. Furthermore, in the same way we replace in the initial current
\[ \theta^{(t)}(t) \rightarrow -ig \int_{t_0}^t \left( \bar{\psi}_{ij}(\tau, v \tau) \chi_\alpha \right) U^{ij}(\tau, t) d\tau, \]
\[ \theta^j(t) \rightarrow \theta^j(t) \left( \equiv U^{jj}(t, t_0) \theta_{0}^{(t)} \right) \]
and combine together two in such a manner the expressions obtained. As a result we
obtain new color current in the following form:
\[ j^a_{\theta\mu} [A, \bar{\psi}, \psi](x) = ig^2 v_\mu \int_{t_0}^t \left( \bar{\psi}^j_\alpha(\tau, v \tau) \chi_\alpha \right) U^{ji}(\tau, t) d\tau \left( t^a \right)^{ij} \theta^j(t) \delta^{(3)}(x - vt) \] (5.1)
\[ -ig^2 v_\mu \theta^{(t)}(t) \left( t^a \right)^{ij} \int_{t_0}^t U^{ji}(t, \tau) \left( \bar{\chi}_\alpha \psi^j_\alpha(\tau, v \tau) \right) d\tau \delta^{(3)}(x - vt). \]
In order to distinguish this current\(^4\) from (3.1) we use notation \( j_\theta \) instead of \( j_Q \). The current (5.1) satisfies all properties listed above. Unlike color current (3.1) and color
sources (3.8), (3.9) expression (5.1) explicitly (linearly) depends on interacting soft-quark
fields \( \bar{\psi} \) and \( \psi \). Now we turn to the momentum representation
\[ j^a_{\theta\mu} [A, \bar{\psi}, \psi](k) = \int e^{ik \cdot x} j^a_{\theta\mu}[A, \bar{\psi}, \psi](x) \frac{dt}{2\pi} \frac{d\mathbf{x}}{(2\pi)^3} \] (5.2)
\(^4\)Note that by using differentiation rule of the link operator \( U(t, \tau) \), we can represent also current (5.1)
in more compact form
\[ j^a_{\theta\mu} [A, \bar{\psi}, \psi](x) = g \delta^{(3)}(x - vt) \]
\[ \times \frac{\delta}{\delta A^{\alpha\mu}(t, v t)} \left\{ \theta_0^{(t)} \int_{t_0}^t U^{ij}(t_0, \tau) \left( \bar{\chi}_\alpha \psi^j_\alpha(\tau, v \tau) \right) d\tau - \int_{t_0}^t \left( \bar{\psi}^i_\alpha(\tau, v \tau) \chi_\alpha \right) U^{ij}(\tau, t_0) d\tau \theta_0^j \right\}. \]

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and define the first term in expansion of $j^{(1)a}_{0\mu}$ in powers of interacting fields $A, \psi, \bar{\psi}$, and initial values of the Grassmann color charges $\theta^0_0, \theta^1_0$. For this purpose in (5.1) we set

$$\theta^i(t) \to \theta^i_0, \quad U^{ij}(\tau, t) \to \delta^{ij}, \quad \left(\bar{\psi}^i_\alpha(\tau, v\tau)\chi_\alpha \right) = \int e^{-i(v-q)\tau} \left(\bar{\psi}^i_\alpha(-q)\chi_\alpha \right) dq$$

and so on. Discarding all terms containing initial time $t_0$, we obtain after simple calculations

$$j^{(1)a}_{0\mu}(\bar{\psi}, \psi)(k) = \frac{g^2}{(2\pi)^3} v_\mu \int \frac{1}{(v \cdot q)} \left(\bar{\psi}^i_\alpha(-q)\chi_\alpha \right) (t^a)^{ij} \theta^j_0 \delta(v \cdot (k + q)) dq \quad (5.3)$$

where the coefficient function $K^{(G)\mu}_\alpha$ is defined by

$$K^{(G)\mu}_\alpha(v, \chi|k, q) \equiv \frac{v_\mu \chi_\alpha}{v \cdot q} - \ast \Gamma^{(G)\mu}_{\alpha\beta}(k; q, -k - q) \ast S^{(G)\beta\gamma}(k + q) \chi_\gamma,$$  \hspace{1cm} (5.4)

and correspondingly instead of (4.11), we have

$$\frac{\delta^2 \left< j^{a\mu}[A, \bar{\psi}, \psi, Q_0](k) + j^{a\mu}[A, \bar{\bar{\psi}}, \bar{\psi}](k) \right>}{\delta \psi^{(0)i}_\alpha(q) \delta \theta^j_0 \mid_0} = \frac{\delta^2 \bar{j}^{a\mu}[A^{(0)}, \bar{\psi}(0), \bar{\psi}(0), Q_0, \theta_0, \theta_0^\dagger](k) \mid_0}{\delta \bar{\psi}^{(0)i}_\alpha(q) \delta \theta^j_0 \mid_0},$$

$$= - \frac{g^2}{(2\pi)^3} (t^a)^{ij} \bar{K}^{(G)\mu}_\alpha(v, \bar{\chi} | k, -q) \delta(v \cdot (k - q)),$$  \hspace{1cm} (5.5)

where

$$\bar{K}^{(G)\mu}_\alpha(v, \bar{\chi} | k, -q) \equiv \frac{v_\mu \bar{\chi}_\alpha}{v \cdot q} + \bar{\chi}_\beta^\gamma \ast S^{(G)\beta\gamma}(k - q) \ast \Gamma_\beta^{(G)\mu}(k; -k + q, -q).$$

Now we can write out a total expression for the effective current linear in free soft-quark fields $\bar{\psi}^{(0)}, \bar{\psi}(0)$ and the Grassmann charges $\theta^0_0, \theta^1_0$:

$$\bar{j}^{(1)a}_{\bar{\theta}\mu}(\bar{\psi}^{(0)}, \bar{\psi}(0))(k) = \bar{j}^{(1)a}\mu_{\bar{\psi}^{(0)}}(-k) \theta^i_0 + \theta^1_0 \bar{j}^{a\mu}_{\bar{\psi}^{(0)}}(\psi^{(0)})(k),$$  \hspace{1cm} (5.6)
where

\[
\tilde{j}^{ai}_\mu(\psi^{(0)}) (k) \equiv g^2 \frac{1}{(2\pi)^3} (t^a)^{ij} \int \bar{K}^{\alpha,\mu}_\chi (v, \bar{\chi} | k, -q) \psi^{(0)j\alpha}_\alpha (q) \delta(v \cdot (k - q)) dq, \tag{5.7}
\]

\[
\tilde{j}^{\dagger ai}_\mu(\psi^{(0)}) (-k) \equiv g^2 \frac{1}{(2\pi)^3} \int \bar{\psi}^{(0)j\alpha}_\alpha (q) (t^a)^{ji} K^{\alpha,\mu}_\chi (v, \chi | k, q) \delta(v \cdot (k + q)) dq.
\]

This effective current generates the scattering processes that are inverse to the scattering processes depicted in Fig. 2. By a direct calculation it is easy to verify that this current satisfies the condition of reality: \( \tilde{j}^* (k) = \tilde{j} (-k) \), and coefficient functions (5.4) and (5.5) in contracting with a longitudinal projector are gauge invariant in the sense as it was discussed in the previous section.

Further, we consider a derivation of current (5.1) proceed from the corresponding generalization of evolution equations (3.5) and (3.6). As was already mentioned in Introduction these equations was obtained in Ref. [5] under the assumption that in medium there is only mean (and/or stochastic) gauge field \( A^a_\mu (x) \) in which the classical color-charged particles move. Here the following question arises: How are the equations modified if in addition there exist stochastic soft-quark fields \( \bar{\psi}^{i\alpha}_\alpha (x), \psi^{i\alpha}_\alpha (x) \) in the medium? We assume that a minimal extension of equations (3.5) and (3.6) to the soft-fermion degree of freedom of system with retention of gauge symmetry has the following form:

\[
\frac{d\bar{\vartheta}^i(t)}{dt} + ig v^\mu A^a_\mu (t, v t)(t^a)^{ij} \bar{\vartheta}^j(t) + ig \langle \bar{\chi}^\alpha \psi^{j\alpha}_\alpha (t, v t) \rangle = 0, \quad \vartheta^i_0 = \vartheta^j(t) \bigg|_{t=t_0}, \tag{5.8}
\]

\[
\frac{d\vartheta^{i\dagger} (t)}{dt} - ig v^\mu A^a_\mu (t, v t) \vartheta^{j\dagger} (t) (t^a)^{ji} - ig \langle \bar{\psi}^{j\alpha}_\alpha (t, v t) \chi^\alpha \rangle = 0, \quad \vartheta^{i\dagger}_0 = \vartheta^{j\dagger} (t) \bigg|_{t=t_0}.
\]

The general solution of these equations is

\[
\vartheta^i(t) = U^{ij}(t, t_0) \theta^j_0 - ig \int_{t_0}^t U^{ij}(t, \tau) \langle \bar{\chi}^\alpha \psi^{j\alpha}_\alpha (\tau, v \tau) \rangle d\tau, \tag{5.9}
\]

\[
\vartheta^{i\dagger} (t) = \theta^{j\dagger}_0 U^{ji} (t_0, t) + ig \int_{t_0}^t \langle \bar{\psi}^{j\alpha}_\alpha (\tau, v \tau) \chi^\alpha \rangle U^{ji} (\tau, t) d\tau.
\]

Here we have introduced new symbols \( \vartheta^i \) and \( \vartheta^{i\dagger} \) for the Grassmann charges having kept the old symbols \( \theta^i \) and \( \theta^{i\dagger} \) for solutions of homogeneous equations (3.5), (3.6).

If we now substitute these solutions into (3.7) and take into account the identity

\[
U (\tau, t) t^a U (t, \tau) = \tilde{U}^{ab} (t, \tau) t^b, \tag{5.10}
\]

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where $\tilde{U}(t, \tau) = T \exp\{-i g \int_{t_0}^{t}(v \cdot A^a(\tau', v \tau')) T^a d\tau'\}$ is the evolution operator in the adjoint representation, then we obtain the following expression\footnote{The function (5.12) is formally a solution of the equation}

$$Q^a(t) = \tilde{U}^{ab}(t, t_0) Q^b_0 + i g \left\{ \frac{t}{t_0} \left( \bar{\psi}_\alpha^j(\tau, v \tau) \chi_\alpha \right) U^{ij} \left( t, t \right) d\tau \right\}_{j_1} \left[ U^{i\eta}(t, t_0) \partial_0 \eta \right]_a$$

(5.12)

$$= - \left[ \partial_0^{i\eta} U^{i\eta}(t_0, t) \right] \left( t_0 \right)^{ij} \int \left[ U^{j\eta}(t, \tau) \left( \bar{\chi}_\alpha \psi_\alpha^j(\tau, v \tau) \right) d\tau \right]$$

$$+ g^2 \int_{t_0}^{t} \int \left( \bar{\psi}_\alpha^j(\tau, v \tau) \chi_\alpha \right) U^{ij} \left( t, t' \right) \left( t_0 \right)^{ij} \left( \bar{\chi}_\alpha \psi_\alpha^j(t, v \tau') \right) d\tau d\tau',$$

where we also have introduced a new symbol $Q^a$ for usual color charge, having kept the old symbol $Q^a$ for the solution of the Wong equation (3.2). If we now insert (5.12) into expression for current (3.1), then the first term on the right-hand side of Eq. (5.12) generates the usual ‘classical’ color current of hard parton. The expression in braces in Eq. (5.12) determines current (5.11), which we have introduced above for recovering gauge invariance of scattering amplitude. The physical meaning of the last term in (5.12) is less clear, as well as the physical meaning of the last terms on the right-hand sides of solutions (5.9) in inserting the last into Grassmann color sources (3.8) and (3.9). Unfortunately, these terms generate additional contributions redefining the effective currents and sources. For example, in color source (3.8) there exists a term linear in interacting soft-quark field $\psi$

$$\eta_{\alpha} = \eta_{\alpha}^{(0)} + \frac{g^2}{(2\pi)^3} \frac{\chi_\alpha}{\left( v \cdot q \right)} \int \left( \bar{\chi}_\beta \psi_\beta(q') \right) \delta \left( v \cdot (q - q') \right) dq' + \ldots .$$

This in particular leads to the fact that it is necessary to redefine the left-hand side of the Dirac field equation (3.10) to the following form

$$^a S_{\alpha\beta}^{-1}(q) \psi_\beta(q) \rightarrow \int dq' \left\{^a S_{\alpha\beta}^{-1}(q') \delta(q - q') + \frac{g^2}{(2\pi)^3} \frac{\chi_\alpha \bar{\chi}_\beta}{\left( v \cdot q \right)} \delta \left( v \cdot (q - q') \right) \right\} \psi_\beta(q').$$

There are additional contributions to all HTL-induced sources: $\eta^{(1,1)}(A, \psi), \eta^{(2,1)}(A, A, \psi)$ and so on. The last term on the right-hand side of Eq. (5.12) generates new contributions to the HTL-induced currents $j^{\psi(0,2)}(\tilde{\psi}, \psi)$, $j^{\psi(1,2)}(A, \tilde{\psi}, \psi)$, $\ldots$. By virtue of this fact we can say nothing about physical sense of these new contributions, during all work we
simply ignore them. However the account of the functions generating these contributions is very important. With the help of these functions we can construct new gauge-covariant currents and sources, which generate new terms in scattering amplitudes. These terms have already quite concrete physical sense. This will be the subject of discussion just below. Here we would like to make one more additional remark. As it is well known evolution equation (3.2) admits an integral of motion

\[ Q^a(t) Q^a(t) = Q^a_0 Q^a_0 = \text{const}, \]

and equations (3.5), (3.6) admit, correspondingly

\[ \theta^{\dagger}i(t) \theta^{i}(t) = \theta^{\dagger}i_0 \theta^{i}_0 = \text{const}. \]

The new evolution equations (5.8) and (5.11) allows no such integrals of motion. In other words, in the presence of soft-quark field fluctuations in the medium, ‘the length’ of color vector of classical particle is not conserved any more.

Now we introduce the following notations:

\[ \Omega^{i}(t) \equiv -ig \int_{t_0}^{t} U^{ij}(t, \tau) \left( \bar{\chi}^{a}_\alpha \psi^{j}_\alpha (\tau, v_\tau) \right) d\tau, \quad \Omega^{ji}(t) = ig \int_{t_0}^{t} \left( \bar{\psi}^{j}_\alpha (\tau, v_\tau) \chi^{a}_\alpha \right) U^{ji}(\tau, t) d\tau, \quad (5.13) \]

\[ \Xi^{a}(t) \equiv g^2 \int_{t_0}^{t} \int_{t_0}^{t} U^{i_1 i}(t, t') U^{j_1 j}(t', t') U^{i_2 i}(t', \tau) U^{j_2 j}(\tau, \tau') \left( \bar{\chi}^{a}_\alpha \psi^{j_1}_\alpha (\tau', v_{\tau'}) \right) \left( \bar{\chi}^{a'}_\beta \psi^{j_2}_\beta (\tau, v_{\tau}) \right) d\tau d\tau' \]

\[ = i \frac{g}{\delta(v_{\mu} A^{a_\mu}(t, v_\tau))} \left( \frac{\delta \Omega^{i_2}(t) \Omega^{j_2}(t)}{\delta(v_{\mu} A^{a_\mu}(t, v_\tau))} \right). \]

In the last equality we have used the differentiation rule of the evolution operator

\[ \frac{\delta U^{ij}(t, \tau)}{\delta(v_{\mu} A^{a_\mu}(t', v_\tau'))} = -ig U^{ii'}(t, t')(t'^{a_\mu})^{ij'} U^{jj'}(t', \tau). \]

Let us define a new additional source setting by the definition

\[ \eta^a_{Qa}(x) \equiv i \alpha \chi^{a}_\alpha Q^a(t) \left( \frac{\delta \Omega^{i}(t)}{\delta(v_{\mu} A^{a_\mu}(t, v_\tau))} \right) \delta^{(3)}(x - v_\tau) \]

\[ = -i \alpha g^2 \chi^{a}_\alpha Q^a(t) (t^{a})^{ij} \int_{t_0}^{t} U^{i_1 i}(t, \tau) \left( \bar{\chi}^{a'}_\beta \psi^{j_1}_\beta (\tau, v_\tau) \right) d\tau \delta^{(3)}(x - v_\tau) \]

\[ \equiv \alpha g \chi^{a}_\alpha Q^a(t) (t^{a})^{ij} \Omega^{i}(t) \delta^{(3)}(x - v_\tau). \]

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Here $\alpha$ is a constant, which should be defined from some physical reasons. Some of them will be discussed later. The multiplier ‘$i$’ is introduced for convenience subsequently.

Under the gauge transformation non-Abelian charge is transformed by the rule

$$Q^a t^a \to S Q^a t^a S^{-1},$$

therefore, as it is not difficult to see from explicit expression (5.14), this source is transformed properly: $\eta Q_a \to S \eta Q_a$. We turn to the momentum representation. From (5.14) it follows the next nontrivial derivative of the second order with respect to $\psi(0)$ and $Q_0$

$$\frac{\delta^2 \eta^{(0)}_{Q_0} [A, \psi, \theta_0](q)}{\delta \psi_1^{(0)}(q_1) \delta Q_0^a} \bigg|_0 = \alpha \frac{g^2}{(2\pi)^3} \left( t^a \right)^{ij} \frac{\chi \overline{\chi}}{(v \cdot q_1)} \delta(v \cdot (q - q_1)).$$

If we now add this expression to (4.2), then we obtain total expression for the effective source $\eta^{(1)}_{Q_0}$ linear in free soft-quark field $\psi(0)$ and color charge $Q_0$, instead of (4.3), where now as the coefficient function $K^{(Q)}_{\alpha_1}(\chi, \overline{\chi} | q, -q_1)$ it is necessary to understand the following expression

$$K^{(Q)}_{\alpha_1}(\chi, \overline{\chi} | q, -q_1) = \alpha \frac{\chi \overline{\chi}}{v \cdot q_1} * \Gamma^{(Q)\mu}_{\alpha_1}(q - q_1; q_1, -q) * D_{\mu\nu}(q - q_1)v^\nu. \quad (5.15)$$

The diagrammatic interpretation of new ‘eikonal’ term on the right-hand side of Eq. (5.15) is presented in Fig. 3. These graphs should be added to Fig. 1. Note that the fundamental difference between coefficient function (5.15) and coefficient functions (4.6) and (5.4) consists in the fact that two terms in (5.15) are not associated to one another by the

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Figure 3: The eikonal contributions to the elastic scattering process of soft fermion excitation off the hard test particle (Fig. 1).
requirement of gauge invariance of the scattering amplitude that, for example, unambiguously would fix the constant $\alpha$.

For convenience of the further references we write out an explicit form of the effective source $\tilde{\eta}$ Dirac conjugate to (13)

$$\tilde{\eta}_0^{(1)}(\bar{\psi}(0), Q_0)(-q) = \frac{g^2}{(2\pi)^3} (t^a)_{i_1i_2}Q_0^a \int K_{a_1a_1}(x, \bar{x}|q, -q_1) \bar{\psi}_{a_1}^{(0)i_1}(-q_1) \delta(v \cdot (q - q_1)) dq_1,$$

(5.16)

where

$$K_{a_1a_1}(x, \bar{x}|q, -q_1) \equiv \alpha^* \frac{\chi_{a_1} \bar{\chi}_a}{v \cdot q_1} - *\Gamma_{a_1a_1}(q + q_1; -q_1, q)^* \mathcal{D}_{\mu a}(q - q_1) v^\mu.$$

(5.17)

From functionals (5.13) one can form two some more additional sources:

$$\eta_{\Xi, \alpha}(x) = \beta g \chi_\alpha (t^a)^{ij} \theta^j(t) \Xi^a(t) \delta^{(3)}(x - vt)$$

(5.18)

and

$$\eta_{\Omega, \alpha}(x) = \beta_1 g \chi_\alpha (t^a)^{ij} \theta^j(t) \Xi^a(t) \delta^{(3)}(x - vt),$$

(5.19)

where $\beta, \beta_1$ are some constants. The sources (5.18), (5.19) generate contributions to the scattering processes of higher order in the coupling (see the next sections). For example, the first nontrivial derivative of source (5.18) in the momentum representation

$$\frac{\delta^3 \eta_{\Xi, \alpha}[A, \bar{\psi}, \psi, Q_0](q)}{\delta \psi^{(0)ij}_{\alpha_1} q_2 \delta \bar{\psi}^{(0)ij}_a q_2 \delta \theta^0} \bigg|_{0} = \beta \frac{g^3}{(2\pi)^3} (t^a)^{ij} (t^a)^{ij_2} \chi_{\alpha} \chi_{\alpha_1} \bar{\chi}_{\alpha_2} \frac{1}{(v \cdot q_1)(v \cdot q_2)} \delta(v \cdot (q + q_1 - q_2))$$

(5.20)

defines eikonal contribution to nonlinear interaction process of three soft-quark excitations with the hard parton (Eqs. (6.6), (6.7) and Fig. 6). The source (5.19) generates a similar contribution with replacements in Eq. (5.20): $\beta \rightarrow \beta_1$ and $(t^a)^{ij} (t^a)^{ij_2} \rightarrow (t^a)^{ij} (t^a)^{ij_2}$. By virtue of a structure of the functionals $\Omega^i(t)$, $\Omega^j(t)$ and $\Xi^a(t)$, sources (5.18), (5.19) are transformed by covariant way under the gauge transformation.

Finally, we can define another color current supplementing (5.1) setting by definition

$$j^c_{\Xi, \mu}(A, \bar{\psi}, \psi, Q_0)(x) = i\sigma v_\mu Q_\mu(t) \frac{(t^a)^{ij} \chi_{\alpha_1 \alpha_2}}{\delta(v_\nu \mathcal{A}^\nu(t, vt))} \delta^{(3)}(x - vt)$$

(5.21)

$$= \sigma v_\mu g^3 \int_{t_0}^t \int (\bar{\psi}^\alpha(t, \tau) \chi_{\alpha_1}) U^{ij}(t, \tau) \{t^a, t^b\}^{ij} U_j^{j'}(t, \tau') \left(\bar{\chi}_{\alpha_1} \psi_{\alpha_2}^{j'}(\tau', vt)\right) d\tau d\tau'$$

$$\times Q_\mu(t) \delta^{(3)}(x - vt),$$

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where \( \sigma \) is a new constant. The additional color current (5.21) similar to additional sources (5.13), (5.19) generates contributions only to higher orders nonlinear interaction of soft and hard modes. The first nontrivial variation of current (5.21) (in the momentum representation) defines eikonal contribution to nonlinear interaction process of two soft-quark and one soft-gluon excitations with the hard parton (Eqs. (6.1), (6.2) and Fig. 4).

\[
\frac{\delta^3 j_{\mu}^{\alpha \beta} [A, \bar{\psi}, \psi, Q_0](k)}{\delta \psi^{(0)}_{\alpha}(q_1) \delta \bar{\psi}^{(0)}_{\beta}(-q) \delta Q_0^{b}} \bigg|_0 = \sigma \frac{g^3}{(2\pi)^3} v_\mu \{t^a, t^b\}_{ij} \frac{\chi_{\alpha} \bar{\chi}_{\beta}}{(v \cdot q)(v \cdot q_1)} \delta(v \cdot (k + q - q_1)). \quad (5.22)
\]

The currents and sources written out in this section and Section 3 enables us to calculate complete expressions for scattering amplitudes of soft QGP modes off hard thermal particles, at least up to the third order in free soft-field amplitudes and initial values of color charges of hard partons. In two latter sections this will be proved by explicit calculation of effective currents and sources generating processes of nonlinear interaction of three plasma waves with the test particle and soft-loop corrections to the scattering processes considered in Section 4. Unfortunately, we cannot prove whether it will be necessary to introduce more complicated in structure additional currents and sources in calculating the total scattering amplitudes of higher order in interaction. The direct computations here become very cumbersome and therefore for the proof of closure (or non-closure) of the theory it is necessary to use methods and approaches that are not related to an expansion in a series in powers of soft-field amplitudes and initial values of color charges.

In Appendix A we give an explicit form of the action \( S \) varying which we get the equation for color charge evolution of the hard test parton and the Yang-Mills and Dirac equations for soft gluon and quark excitations. The action suggested generates all additional sources introduced in present section.

### 6 Third-order effective currents and sources

In this section examples of calculation of effective currents and sources next-to-leading order in the coupling \( g \) will be given. As a first step we consider the third order functional derivative of relation (3.14) with respect to \( \psi^{(0)}, \bar{\psi}^{(0)} \) and \( Q_0 \). By current \( j_{\mu}^{\alpha} \) on the left-hand side it is necessary to realize a sum of the initial current and additional currents (5.4) and (5.21), i.e.,

\[
\int_{\mu}^{\alpha} [A, \bar{\psi}, \psi, Q_0, \bar{\theta}_0, \theta_0](k) = j_{\mu}^{\alpha} [A, \bar{\psi}, \psi, Q_0](k) + j_{\mu}^{\alpha} [A, \bar{\psi}, \psi, \theta_0](k) + j_{\mu}^{\alpha} [A, \bar{\psi}, \psi, Q_0](k).
\]
Here, we have

\[
\frac{\delta^2 j^a_\mu[A, \bar{\psi}, \psi, Q_0, \theta^0_0, \theta^0_0](k)}{\delta \psi^{(ij)}(q_2) \delta \bar{\psi}^{(0)}(0)(-q_1) \delta Q_0^b} = \frac{\delta^3 j^a_\mu[A^{(0)}, \bar{\psi}^{(0)}, \psi^{(0)}, Q_0, \theta^0_0, \theta^0_0](k)}{\delta \psi^{(ij)}(q_2) \delta \bar{\psi}^{(0)}(0)(-q_1) \delta Q_0^b} \bigg|_0
\]

\[
= \int \left\{ \frac{\delta^3 j^{\Psi (1,2)_\mu}(k)}{\delta A^\nu \mu_i'(k_1') \delta A^\nu \mu_i'(k_2')} \frac{\delta^2 A^\nu \mu_i'(k_1')}{\delta \psi^{(ij)}(q_2) \delta \bar{\psi}^{(0)}(0)(-q_1)} dk_1' \frac{dk_1'}{dk_2'} \frac{dk_1'}{dq_1' dq_2'} \right\}
\]

where on the right-hand side we keep the terms different from zero only. Taking into account Eqs. (1.3.3), (4.11), (3.3), (1.2), (5.15) and (5.22) from the last expression we find the effective current in the following form

\[
\tilde{J}^{(2)a}_{\mu}(\bar{\psi}^{(0)}, \psi^{(0)}, Q_0)(k) = \frac{g^3}{(2\pi)^3} \int K_{\mu, \alpha \beta}^{(G)ab, ij}(v, \chi, \bar{\chi})(k; q_1, -q_2) \bar{\psi}^{(0)i}_\alpha(-q_1) \psi^{(0)j}_\beta(q_2)
\]

\[
\times \delta(v \cdot (k + q_1 - q_2)) dq_1 dq_2 Q_0^b,
\]

where the coefficient function \( K_{\mu, \alpha \beta}^{(G)ab, ij} \) is

\[
K_{\mu, \alpha \beta}^{(G)ab, ij}(v, \chi, \bar{\chi})(k; q_1, -q_2) \equiv -\delta \bar{\Gamma}_{\mu \nu, \alpha \beta}^{(G)ab, ij}(k, -k - q_1 + q_2; q_1, -q_2) \ast \mathcal{D}^{0 \nu'}(k + q_1 - q_2) \bar{\psi}^{i}_\alpha(0)(-q_1) \psi^{j}_\beta(0)(q_2)
\]

\[
+ [t^a, t^b]_{ij} k_{\mu \nu}(v, w)(k; q_1, -q_2) \ast \mathcal{D}^{0 \nu'}(-q_1 + q_2) \ast \Gamma^{(G)}_{\mu \nu, \alpha \beta}(q_1 + q_2; q_1, -q_2) \ast \mathcal{D}^{0 \nu'}(-q_1 + q_2) \ast \mathcal{D}^{0 \nu'}(k + q_1 - q_2)
\]

\[
+ (t^a t^b)_{ij} k_{\mu \nu}(v, w)(k; q_1, -q_1) \ast S^{(G)}_{\gamma \gamma'}(k + q_1) K^{(Q)}_{\mu \nu, \alpha \beta}(v \cdot k_1 + q_1, -q_2)
\]

\[
+ [t^a, t^b]_{ij} k_{\mu \nu}(v, w)(k; q_1, -q_1) \ast S^{(G)}_{\gamma \gamma'}(k + q_1) K^{(Q)}_{\mu \nu, \alpha \beta}(v \cdot k_1 + q_1, -q_2)
\]

\[
+ \sigma v_{\mu} \{t^a, t^b\}_{ij} \frac{\chi_\alpha \chi_\beta}{(v \cdot q_1)(v \cdot q_2)}.
\]

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Here the function

\[ K_{\mu\nu}(\mathbf{v}, \mathbf{v} | k, q_1 - q_2) \equiv -\frac{v_\mu v_\nu}{v \cdot (q_1 - q_2)} + \Gamma_{\mu\nu\lambda}(k, q_1 - q_2, -k - q_1 + q_2) \, \mathcal{D}^{\lambda\nu}(k + q_1 - q_2) v_\lambda \]

was introduced in Ref. [6]. It defines (on mass-shell of soft modes) the amplitude for elastic scattering of soft-gluon excitations off hard particle. The coefficient functions \( K^{(Q)}_{\gamma/\beta}(\chi, \bar{\chi} | k + q_1, -q_2) \) and \( K^{(Q)}_{\alpha\gamma}(\chi, \bar{\chi} | k - q_2, q_1) \) in the third and fourth lines are defined by equations (5.15) and (5.17) correspondingly. The last term on the right-hand side of Eq. (6.2) is produced by additional current (5.21), and two the last but one terms contain the eikonal contributions induced by additional source (5.14) and its conjugation. The effective current (6.1) generates two processes of interaction of three soft plasma excitations with hard test particle: (i) the scattering process of soft-gluon excitation off hard particle with consequent soft-quark pair creation and (ii) the scattering process of soft-gluon and soft-quark excitations with consequent soft-quark radiation. Diagrammatic interpretation of different terms in coefficient function (6.2) is shown in Fig. 4. Here the scattering process of soft-gluon and soft-quark excitations only with consequent soft-quark radiation is presented.

To define an effective current generating the scattering process with participation of two soft-gluon excitations and one soft-quark excitation, it should be considered a functional derivative of the relation (5.11) with respect to \( \theta_0^\dagger, A(0), \bar{\psi}(0) \) and \( \theta_0, A^*(0), \bar{\psi}(0) \) with the over-all current \( j^a(\mathbf{A}, \bar{\psi}, \psi, Q_0, \theta_0^\dagger, \theta_0)(k) \) on the left-hand side. Omitting the details of calculations, we result at once in the final expression (cp. (5.6))

\[ \bar{\psi}^{(2)a}(A(0), \bar{\psi}(0), \psi(0), \theta_0^\dagger, \theta)(k) = \bar{\psi}^{(1)}(A(0), \psi(0))(-k) \theta_0^\dagger + \theta_0 \bar{\psi}^{(1)}(A(0), \psi(0))(k), \] (6.3)

where

\[ \bar{\psi}^{(1)}(A(0), \psi(0))(k) \equiv \frac{g^3}{(2\pi)^3} \int K^{(G)_{a1\alpha}}_{\mu_1\alpha}(\mathbf{v}, \mathbf{v}, \bar{\chi} | k; -k_1, -q) A^{(0)a1\mu_1}(k_1) \psi^{(0)}(q) \]

\[ \times \delta(v \cdot (k - k_1 - q)) \, dqdk_1, \]

\[ \bar{\psi}^{(1)}(A(0), \psi(0))(-k) \equiv \frac{g^3}{(2\pi)^3} \int K^{(G)_{a1\alpha}}_{\mu_1\alpha}(\mathbf{v}, \mathbf{v}, \bar{\chi} | k; k_1, q) A^{*(0)a1\mu_1}(k_1) \bar{\psi}^{(0)}(q) \]

\[ \times \delta(v \cdot (k + k_1 + q)) \, dqdk_1. \]

Here the coefficient function \( K^{(G)_{a1\alpha}}_{\mu_1\alpha}(\mathbf{v}, \mathbf{v}, \bar{\chi} | k; -k_1, -q) \) is defined by the expression

\[ K^{(G)_{a1\alpha}}_{\mu_1\alpha}(\mathbf{v}, \mathbf{v}, \bar{\chi} | k; -k_1, -q) \equiv -\bar{\chi}_{\beta}^* S_{\beta\gamma}(k - k_1 - q) \delta \Gamma^{(G)_{a1\gamma}}_{\mu_1\beta\alpha}(k, -k_1; -k + k_1 + q, -q) \]

\[ - [t^a, t^{a1}]^{ij} \delta \Gamma^{(G)_{\mu_1\beta\alpha}}_{\mu_1\beta\alpha}(k, -k + k_1, -k_1) \mathcal{D}^{\nu\nu'}(k - k_1) K^{(G)_{\nu\nu'}}_{\nu\nu'}(\mathbf{v}, \bar{\chi} | k - k_1, -q) \] (6.5)

\[ - (t^a t^{a1})^{ij} \delta \Gamma^{(G)_{\mu_1\beta\alpha}}_{\mu_1\beta\alpha}(\mathbf{v}, \bar{\chi} | k, -q - k_1) S_{\beta\gamma}(q + k_1) \Gamma^{(G)_{\mu_1\beta\alpha}}_{\mu_1\beta\alpha}(k_1; q, -q - k_1) \]
Figure 4: The processes of absorption of soft-gluon and soft-quark excitations by hard parton, accompanied by soft-quark radiation.
\[-(t^{a_1}t^{a_2})^{ij} \tilde{K}_{\mu \nu}^{\alpha}(v, \tilde{\chi} | k_1, k - q) S_{\beta \gamma}(k - q) \Gamma_{\alpha \beta}(k; q - k, -q) \]
\[+ (t^{a_1}t^{a_2})^{ij} \frac{\nu_{\mu}u_{\nu} \tilde{\chi}_{\alpha}}{(v \cdot q)(v \cdot k)} - (t^{a_1}t^{a_2})^{ij} \frac{\nu_{\mu}u_{\nu} \tilde{\chi}_{\alpha}}{(v \cdot q)(v \cdot k_1)}.\]

The second, fourth and fifth terms contain eikonal contributions generated by additional current (5.1). Diagrammatic interpretation of different terms in the coefficient function (6.5) are presented in Fig. These diagrams should be supplemented with graphs describing interaction of soft modes with hard test antiquark $\bar{Q}$. The example of such diagrams is shown on the first line in parentheses.

Now we consider a third order functional derivatives of relation (3.15). By the source on the left-hand side it is necessary to mean a sum of initial source (the right-hand side of Eq. (3.10)) and additional sources (5.14), (5.18), and (5.19):

\[\eta_{\alpha}^{(3)}[A, \bar{\psi}, \psi, Q, \theta_0](q) \equiv \eta_{\alpha}^{(3)}[A, \psi, Q, 0, \theta_0](q) + \eta_{\bar{\alpha}}^{(3)}[A, \bar{\psi}, \psi, 0, \theta_0](q) + \eta_0^{(3)}[A, \bar{\psi}, \psi, 0, \theta_0](q).\]

Differentiation of (3.15) with respect to $\bar{\psi}^{(0)}$, $\psi^{(0)}$ and $\theta_0^j$ yields

\[\frac{\delta^3 \eta_{\alpha}^{(3)}[A, \bar{\psi}, \psi, Q, \theta_0](q)}{\delta \psi^{(0)}_{\alpha_2}(q_2) \delta \psi^{(0)}_{\alpha_1}(q_1) \delta \theta_0^j} = \frac{\delta^3 \eta_{\alpha}^{(3)}[A^{(0)}, \bar{\psi}^{(0)}, \psi^{(0)}, Q, 0, \theta_0](q)}{\delta \psi^{(0)}_{\alpha_2}(q_2) \delta \psi^{(0)}_{\alpha_1}(q_1) \delta \theta_0^j} \bigg|_0 \]

\[\left\{ \frac{\delta^2 \eta_{\alpha}^{(3)}(q)}{\delta A^{(0)}_{\alpha_2}(k_1) \delta \psi^{(0)}_{\alpha_1}(q_1)} \frac{\delta \psi^{(0)}_{\alpha_1}(q_1)}{\delta \theta_0^j} \frac{\delta^2 A^{(0)}_{\alpha_1}(k_1)}{\delta \psi^{(0)}_{\alpha_1}(q_1)} \frac{\delta \psi^{(0)}_{\alpha_1}(q_1)}{\delta \theta_0^j} \right\} \frac{dk_1^i dq_1^i}{dq_1^i} \]

\[+ \frac{\delta^2 \eta_{\alpha}^{(3)}(q)}{\delta A^{(0)}_{\alpha_2}(k_1) \delta \psi^{(0)}_{\alpha_1}(q_1)} \frac{\delta^2 A^{(0)}_{\alpha_1}(k_1)}{\delta \psi^{(0)}_{\alpha_1}(q_1)} \frac{\delta \psi^{(0)}_{\alpha_1}(q_1)}{\delta \theta_0^j} \frac{dk_1^i dq_1^i}{dq_1^i} \]

\[+ \frac{\delta^2 \eta_{0}^{(3)}(q)}{\delta \theta_0^j} \frac{\delta^2 A^{(0)}_{\alpha_1}(k_1)}{\delta \psi^{(0)}_{\alpha_1}(q_1)} \frac{\delta \psi^{(0)}_{\alpha_1}(q_1)}{\delta \theta_0^j} \frac{dq_1^i dq_2^i}{dq_1^i dq_2^i} \bigg|_0.\]

We kept again only the terms different from zero on the right-hand side. Taking into account Eqs. (1.3.4), (1.3.3), (5.4), (5.11) and (5.20), we easily derive an explicit form of effective source generating the process of nonlinear interaction of three soft-quark excitations with the hard test particle

\[\tilde{\eta}_{\alpha}^{(3)}(\bar{\psi}^{(0)}, \psi^{(0)}, \theta_0)(q) = \frac{g^3}{(2\pi)^3} \int K_{\alpha \beta \gamma}(X; X; \tilde{\chi} | q, q_1; -q_2) \tilde{\psi}_{\alpha_1}(q_1) \tilde{\psi}_{\alpha_2}(q_2) \tilde{\psi}_{\alpha_3}(q_3) \times \delta(v \cdot (q + q_1 - q_2)) \, dq_1 dq_2 \theta_0^j.\]
Figure 5: The scattering process of soft-gluon excitation by hard parton followed by soft-gluon and soft-quark radiation.
where the coefficient function $K^{(Q)ij}_{\alpha\alpha_1\alpha_2}$ is

$$K^{(Q)ij}_{\alpha\alpha_1\alpha_2}(\chi, \chi, \bar{\chi}| q, q_1; -q_2) \equiv (t^a)^{ij}(t^a)^{i_1j_1} \Gamma^{(Q)\mu}_{\alpha\alpha_2}(q - q_2; q_2, -q) * D_{\mu\nu}(q - q_2) K^{(G)\mu}_{\alpha_1}(v, \chi| q - q_2, q_1)$$

$$- (t^a)^{ij}(t^a)^{i_1j_1} K^{(Q)\mu}_{\alpha}(q, \chi| q - q_1 + q_2, -q) * D_{\mu\nu}(q_1 + q_2) \Gamma^{(G)\mu}_{\alpha\alpha_2}(q - q_1 + q_2; q_1, -q_2)$$

$$+ \beta (t^a)^{ij}(t^a)^{i_1j_1} \chi_{\alpha} \chi_{\alpha_1} \bar{\chi}_{\alpha_2} \frac{(v \cdot q_1)(v \cdot q_2)}{(v \cdot q_1)} + \beta_1 (t^a)^{ij}(t^a)^{i_1j_1} \chi_{\alpha} \chi_{\alpha_1} \bar{\chi}_{\alpha_2} \frac{(v \cdot q_1)(v \cdot q_2)}{(v \cdot q_1)}.$$

Diagrammatic interpretation of different terms in coefficient function (6.7) is presented in Fig. 4.

Furthermore, we can define just one more third-order effective source $\tilde{n}_\alpha^{(2)i}$ taking variation of relation (3.15) with respect to $A^{(0)}$, $\psi^{(0)}$ and $Q_0$. Omitting calculation details, we give at once final:

$$\tilde{n}_\alpha^{(2)i}(A^{(0)}, \psi^{(0)}, Q_0)(q) = \frac{g^3}{(2\pi)^4} \int K^{(Q)\mu}_{\mu, \alpha \alpha_1}(v, \bar{\chi}, \chi| q, -q_1, -k) A^{(0)\mu}(k) \psi^{(0)i}_{\alpha_1}(q_1)$$

$$\times \delta(v \cdot (q - q_1 - k)) dq_1 dk Q_0,$$

where the coefficient function $K^{(Q)\mu}_{\mu, \alpha \alpha_1}$ is defined by the following expression:

$$K^{(Q)\mu}_{\mu, \alpha \alpha_1}(v, \bar{\chi}, \chi| q, -q_1, -k) \equiv - \Gamma^{(Q)\mu}_{\nu, \alpha \alpha_1}(q - q_1 - k; q_1, -q) * D_{\nu\nu}(q - q_1 - k)$$

$$+ (t^b)^{i_1j_1} \Gamma^{(Q)\mu}_{\alpha \alpha_2}(q - q_1; q_1, -q) * D_{\nu\nu}(q_1 - q_1 - k) K^{(G)\mu}_{\alpha_1}(v, \chi| q - q_1, -k)$$

$$+ (t^b)^{i_1j_1} K^{(Q)\mu}_{\alpha}(q, \chi| q_1 - k) * S_{\beta\beta}(k + q_1) \Gamma^{(G)\mu}_{\beta\beta}(k; q_1, -q_1 - k)$$

$$+ (t^b)^{i_1j_1} \Gamma^{(Q)\mu}_{\mu, \alpha \alpha_1}(k; q_1 - k) * S_{\beta\beta}(k - k) K^{(Q)\mu}_{\beta\alpha_1}(q, \chi| q - q_1, -k)$$

$$- \alpha (t^b)^{i_1j_1} \frac{v_{\mu} \chi_{\alpha} \bar{\chi}_{\alpha_1}}{(v \cdot q_1)(v \cdot k)} + \alpha (t^b)^{i_1j_1} \frac{v_{\mu} \chi_{\alpha} \bar{\chi}_{\alpha_1}}{(v \cdot q_1)(v \cdot k)}.$$

The effective source (6.8) generates the scattering processes, which are reverse to the scattering processes depicted in Fig. 4.

Finally, we give an explicit expression for the effective source $\tilde{n}_\alpha^{(2)i}$ arising in calculating the third order derivative of relation (3.15) with respect to $A^{(0)}(k_1)$, $A^{(0)}(k_2)$ and $\theta_0$. The calculations similar to previous ones result in the following expression for $\tilde{n}_\alpha^{(2)i}$:

$$\tilde{n}_\alpha^{(2)i}(A^{(0)}, A^{(0)}, \theta_0)(q) = \frac{1}{2} \frac{g^3}{(2\pi)^4} \int K^{(Q)\mu_{12}, \alpha_2}_{\mu, \alpha_1}(v, \chi| q, -k_1, -k_2) A^{(0)\alpha_1\mu_1}(k_1) A^{(0)\alpha_2\mu_2}(k_2)$$

$$\times \delta(v \cdot (q - k_1 - k_2)) dk_1 dk_2 \theta_0^i.$$

The explicit form of the coefficient function $K^{(Q)\alpha_{12}}_{\mu_1 \mu_2, \alpha}$ and also diagrammatic interpretation of different terms are given in Appendix B. The effective source (6.10) closes a set of the effective currents and sources determining nonlinear interaction processes of three soft elementary excitations with hard test particle in the linear approximation in initial values of the usual and Grassmann color charges.
Figure 6: The processes of interaction of three soft-quark elementary excitations with hard thermal parton.
7 Soft-loop corrections

The effective currents and sources written out in previous sections define the scattering processes of soft modes by hard thermal particles in tree approximation. However, as we have shown in the case of purely soft-gluon excitations [6], there exists an infinite number of effective currents, in which coefficient functions define so-called ‘classical’ soft-gluon one-loop corrections to the tree scattering amplitudes. Although these effective currents are suppressed by powers of the coupling constant in comparison with tree-level effective currents, their accounting in some cases is rather important. Thus, for example, in our work [9] it was shown that soft one-loop corrections to soft-gluon bremsstrahlung generate ‘off-diagonal’ contributions to the radiation energy loss of fast parton connected with the coherent double gluon exchanges. These contributions within the frameworks of a light-cone path integral approach were first considered by Zakharov [11] to ensure unitarity. The presence of soft-quark excitations in the medium results in a new feature: appearing ‘classical’ soft-quark loops. As well as in the case of purely soft-gluon excitations [6] effective currents and sources containing soft-quark loops arise in calculating derivatives of higher order with respect to color charges $Q^0$, $\theta^0$ and $\bar{\theta}^0$ of the basic relations (3.14) – (3.16). Below we will give some examples of concrete calculations and diagrammatic interpretation of the results.

The first nontrivial example of this type arises in calculating the derivative of relation (3.14) with respect to the Grassmann charges $\theta^0$ and $\bar{\theta}^0$ (with the over-all current on the left-hand side)

$$\frac{\delta^2 j^a_\mu[A, \bar{\psi}, \psi, Q_0, \theta^0, \bar{\theta}^0](k)}{\delta \theta^0_i \delta \bar{\theta}^0_j} \bigg|_0 = \frac{\delta^2 j^a_\mu[A^{(0)}, \bar{\psi}^{(0)}, \psi^{(0)}, Q_0, \theta^0, \bar{\theta}^0, \theta^0](k)}{\delta \theta^0_i \delta \bar{\theta}^0_j} \bigg|_0 = \left( \frac{\delta^2 j^{(1)a}_\mu(\bar{\psi}, \psi)(k)}{\delta \theta^0_i \delta \bar{\theta}^0_j} + \frac{\delta^2 j^{(2)a}_\mu(\bar{\psi}, \psi)(k)}{\delta \theta^0_i \delta \bar{\theta}^0_j} \right) \bigg|_0,$$

where in the last line we keep the terms different from zero only. Taking into account Eqs. (5.3), (4.5) and (I.3.3), it is not difficult to show that the right-hand side of this equation can be resulted in the following form (for the sake of simplicity here we suppress of the summation over spinor indices):

$$\frac{g^3}{(2\pi)^3} (t^a)^{ij} \left\{ \frac{v_\mu}{(v \cdot q')} \left[ \left( \bar{\chi}^* S(q') \chi \right) - \left( \bar{\chi}^* S(-q') \chi \right) \right] + \left( \bar{\chi}^* S(k-q') \right) \right\} \delta(v \cdot q') dq' \delta(v \cdot k).$$

From this point on we will designate virtual momenta by letter with prime. The diagrammatic interpretation of different terms is presented in Fig. [7]. The region of integration
in loops is restricted by the Cherenkov cone $v \cdot q = 0$. Moreover, by virtue of condition $v \cdot k = 0$ (the last $\delta$-function in Eq. (7.1)) the derivative calculated is not equal to zero only for off mass-shell plasma excitations. We note especially that it is impossible to obtain Eq. (7.1) literally from the right-hand side of Eq. (5.6) by simple replacements

$$\bar{\psi}_i^{(0)}(-q) \rightarrow \frac{g}{(2\pi)^3} \bar{\chi}_\beta S_{\beta\alpha}(-q) \delta(v \cdot q),$$

since in this case we obtain superfluous term with vertex $\bar{\psi}_i^{(0)}(q) \bar{\chi}_\beta S_{\beta\alpha}(-q) \delta(v \cdot q)$. Finally, the integrand in (7.1) contains a singularity generated by the eikonal term. Brief analysis of this singularity is given in Appendix C.

We only point to the fact that an origin of this singularity is the assumption of linearity of a hard parton trajectory used by us during all this work. Note that an expression similar to (7.1) exists in purely gluonic case [6]. Here, instead of the HTL-resummed quark propagators and vertex $\bar{\psi}_i^{(0)}(q) \bar{\chi}_\beta S_{\beta\alpha}(-q) \delta(v \cdot q)$ we have correspondingly the gluon propagators and triple gluon vertex $\Gamma_{3g}$. However, this expression within the framework of HTL-approximation vanishes by virtue of color factor: $f^{abc}Q^b_0Q_0^c = 0$.

Another nontrivial example of this kind arises in calculating the derivative of relation (3.15) with respect to the usual $Q_0$ and Grassmann $\theta_0$ charges with the total source on the left-hand side

$$\frac{\delta^2 \eta_{i\alpha}^A[A, \bar{\psi}, \psi, Q_0, \theta_0](q)}{\delta Q_0^b \delta \theta_0^j} \bigg|_0 = \frac{\delta^2 \eta_{i\alpha}^A[A^{(0)}, \bar{\psi}^{(0)}, \psi^{(0)}, Q_0, \theta_0](q)}{\delta Q_0^b \delta \theta_0^j} \bigg|_0 = \left( \frac{\delta^2 \eta_{i\alpha}^{(1)}}{\delta Q_0^b \delta \theta_0^j}(\psi)(q) \right) + \left( \frac{\delta^2 \eta_{i\alpha}^{(1)}}{\delta Q_0^b \delta \theta_0^j}(A)(q) \right) + \left( \frac{\delta^2 \eta_{i\alpha}^{(1,1)i}}{\delta Q_0^b \delta \theta_0^j}(A, \psi)(q) \right) \bigg|_0.$$
Taking into account Eqs. (5.14), (3.11), (4.1), and (4.5) we lead the right-hand side of the last equation to the following form:

\[
- \frac{g^3}{(2\pi)^3} \left\{ \int \frac{\chi_\alpha}{(v \cdot q')} \left( \tilde{\chi}_\beta * S_{\beta\beta'}(q') \chi_{\beta'} \right) \delta(v \cdot q') d q' + \int \frac{\chi_\alpha}{(v \cdot k')} \left( v^\mu * D_{\mu\nu}(k') v^{\nu'} \right) \delta(v \cdot k') d k' \right. \\
- \int \left[ \Gamma^{(Q)}_{\alpha\beta}(q-q'; q', -q) * S_{\beta\beta'}(q') \chi_{\beta'} \right] * D_{\mu\nu}(q-q') v^{\nu'} \delta(v \cdot q') d q' \left. \right\} \delta(v \cdot q).
\]

Diagrammatic interpretation of the different terms in braces is presented in figure 8. The expression (7.3) also cannot be obtained by simply replacements (7.2) and from a sum of effective sources (4.3) and (4.7). For such a replacement a superfluous contribution with the HTL-induced vertex \(\Gamma^{(Q)}_{\alpha\beta}(q-q'; q', -q)\) arises.

Physically, more thoughtful examples arise in calculating higher order derivatives of relations (3.14) – (3.16) that contain at least one soft free field. We consider at first third
order derivative of relation \((3.14)\) with respect to the usual and Grassmann charges \(Q_0, \theta_0^i\) and soft-quark field \(\psi^{(0)}\). Somewhat cumbersome calculations result in

\[
\frac{\delta^3j^\alpha_\mu[A, \bar{\psi}, \psi, Q_0, \theta_0^i, \theta_0^j](k)}{\delta Q_0^i \delta \theta_0^j \delta \psi^{(0)i}_\alpha(q)} = \frac{\delta^3j^\alpha_\mu[A^{(0)}, \bar{\psi}^{(0)}, \psi^{(0)}, Q_0, \theta_0^i, \theta_0^j](k)}{\delta Q_0^i \delta \theta_0^j \delta \psi^{(0)i}_\alpha(q)} \tag{7.4}
\]

\[
= \frac{g^4}{(2\pi)^6} \left\{ \int \chi^\beta \Gamma^{(G)ab,ij}(k, -k' - q) \delta K_{\mu,\beta\alpha}(v, q) \delta(\nu' \cdot k') dk' - [t^a, t^b]^{ij} \int A_{\mu}(v, q) k, -q + k') \delta K_{\nu'\alpha}(v, \chi) - k' + q, -q) \delta(\nu' \cdot k') dk' + (t^a t^b)^{ij} \int A_{\mu}(v, q) k, -q)' \delta K_{\nu'\alpha}(v, \chi) k + q, -q) \delta(\nu' \cdot k') dq' + \frac{v_{\mu} \chi_{\alpha}}{(v \cdot q)(v \cdot k)} \int \delta(\nu' \cdot k') dk' \right\}
\]

Diagrammatic interpretation of the different terms on the right-hand side of this expression is presented in Fig. 9. The dots here are referred to graphs describing interaction with hard gluon in initial state. The example of a graph of that kind is presented in the first line in parentheses.

Just one more interesting example arises in differentiating equality \((3.14)\) with respect to the Grassmann charges \(\theta_0^i, \theta_0^j\) and free soft-gluon field \(A^{(0)}\). The explicit form of derivative \(\delta^3j^\alpha_\mu[A, \bar{\psi}, \psi, Q_0, \theta_0^i, \theta_0^j](k)/\delta \theta_0^j \delta \theta_0^i \delta A^{(0)i\mu_1}(k_1)\) is given in Appendix D. The effective current defined by this derivative generates the soft one-loop corrections to the nonlinear Landau damping process studied early in \[15\]. The diagrams with soft-quark loop (Fig. 14) in Appendix D should be added to those of Fig. 4 in Ref. [6], which include soft-gluon loop.

Now we consider differentiation of relation \((3.15)\). The calculation of third order derivative with respect to \(\theta_0^i, \theta_0^j\) and \(\psi^{(0)}\) leads to the following expression

\[
\frac{\delta^3\eta^{\alpha}_\alpha[A, \bar{\psi}, \psi, Q_0, \theta_0^i, \theta_0^j](q)}{\delta \theta_0^j \delta \theta_0^i \delta \psi^{(0)i\alpha}_\alpha(q_1)} = \frac{\delta^3\eta^{\alpha}_\alpha[A^{(0)}, \bar{\psi}^{(0)}, \psi^{(0)}, Q_0, \theta_0^i, \theta_0^j](q)}{\delta \theta_0^j \delta \theta_0^i \delta \psi^{(0)i\alpha}_\alpha(q_1)} \tag{7.5}
\]

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Figure 9: The one-loop corrections to the scattering process generated by the effective current (5.6).
\[
\frac{g^4}{(2\pi)^6} \left\{ (t^a)^{ij} (t^a)^{1i1j} \int K^{(Q)\nu\nu}(v,\chi|q-k',-q) \hat{D}_{\nu\nu'}(q-k') \hat{K}^{(G)\nu\nu'}(v,\bar{\chi}|q-k',-q_1) \delta(v\cdot k') \, dk' \\
+ (t^a)^{i1i}(t^a)^{jjj} \Gamma^{(Q)\nu\alpha}(q-q_1;q_1,-q) \hat{D}_{\nu\nu'}(q-q_1) \\
\times \int \left( \frac{v^{\nu'}}{(v\cdot q')} \left( \left[ \bar{\chi}^* S(q') \chi \right] - \left[ \bar{\chi}^* S(-q') \chi \right] \right) \\
+ \left[ \bar{\chi}^* S(-q') \Gamma^{(G)\nu\nu'}(q-q_1;q_1,-q+q_1-q') \right] \delta(v\cdot q') \, dq' \right) \delta(v\cdot (q-q_1)).
\]

Diagrammatic interpretation of the different terms on the right-hand side of this expression is presented in Fig. 10. These graphs must be supplemented with graphs describing interaction with the hard test antiquark \( \bar{Q} \). However, soft-loop corrections (7.5) do not exhaust all corrections to the elastic scattering process of soft-quark excitation off hard parton drawn on Figs. 1 and 3. The remaining corrections are determined by variation of relation (3.15) with respect to usual color charges \( Q_a^0, Q_b^0 \) and free soft-quark field \( \psi(0) \): \( \delta^3 \eta_a[A, \bar{\psi}, \psi, Q_0, \theta_0](q)/\delta Q^a_0 \delta Q^b_0 \delta \psi^{(0)i_1}(q_1)|_0 \). The explicit form and diagrammatic interpretation of the variation are given in Appendix E.

In conclusion of this section we note run ahead that in our next paper [12] we point to another independent way of deriving the effective currents and sources of (7.1), (7.3) type (and also the other more complicated expressions obtained in this section) based on the expressions for the effective currents and sources generating soft-quark bremsstrahlung in the tree approximation. As known for production of bremsstrahlung it is necessary that at least two hard color-charged particles have been involved in the interaction process. The essence of this approach reduces to a simple identifying in a final expression for the effective currents and sources (generating bremsstrahlung) of these two hard particles. Under this identification soft-quark and soft-gluon propagators describing the interaction process of two particles among themselves are effectively closed into loops (with soft virtual momentum) attached to straight line of hard parton. From the geometric point of view it can be presented as the imposition of the straight line of the first hard parton on the straight line of the second hard parton. Note that there exist two ways for deriving soft-quark loop resulting in different directions of circuit of the fermion loop. Fig. 11 gives graphic interpretation of the procedure described above.

It should be particularly emphasized that these loop corrections are not quantum ones. The notion of loops in our consideration has somewhat conventional character. By this we simply mean the effect of self-interaction of color classical partons induced by the surrounding medium in which they move and with which they interact. This process of interaction with medium is more evidently seen from Fig. 11 where on the right-hand side hard test parton 1 radiates virtual oscillation absorbed by hard thermal particle 2 and on the left-hand side the same virtual oscillation is radiated and absorbed by the same hard
Figure 10: One-loop corrections to the scattering process generated by the effective source \( \text{Eq. 10.3} \) with coefficient function \( \text{Eq. 5.15} \) (Figs. 1 and 3).
Figure 11: Graphic illustration of examples for obtaining the effective currents and sources of the present work including soft one-loop corrections from the effective currents and sources for the processes of soft gluon and soft quark bremsstrahlung [12].
test parton. The coefficient functions (7.1), (7.3), (7.4) so on and the effective currents (sources) connected with them can be interpreted as “dressing” initial bare current (source) of the test color particle generated by interaction of this current (sources) with hot bath.

8 Scattering probabilities

Making use of the explicit form for the effective currents and sources obtained in previous sections, one can define scattering probabilities of soft quark and soft gluon excitations off hard test particle. For this purpose according to the Tsytovich correspondence principle [6, 9] it is necessary to substitute the effective current \( \tilde{j}_a^\mu[A^{(0)}, \bar{\psi}^{(0)}, \psi^{(0)}, Q_0, \theta_0, \theta_0^\dagger](k) \) and effective source \( \tilde{\eta}_i^a[A^{(0)}, \bar{\psi}^{(0)}, \psi^{(0)}, Q_0, \theta_0](q) \) into expressions (I.8.1) and (I.8.2) correspondingly and define the emitted radiant power of soft plasma excitations. However, it needs to be preliminary carried out a little generalization of expressions (I.8.1), (I.8.2) taking into account a specific of the problem under consideration.

At first we perform an averaging of the right-hand sides of Eqs. (I.8.1), (I.8.2) over initial value of the usual color charge \( Q_0 \) by adding an integration over the colors with measure

\[
dQ_0 = \prod_{a=1}^{d_A} dQ_0^a \delta(Q_0^a Q_0^b - C^{(C)}_2), \quad d_A = N_c^2 - 1, \quad \zeta = G, Q, \bar{Q}
\]

with the second Casimir \( C^{(G)}_2 = C_A (= N_c) \) for hard gluons and correspondingly \( C^{(Q, \bar{Q})}_2 = C_F (= (N_c^2 - 1)/2N_c) \) for hard (anti)quarks normalized such that \( \int dQ_0 = 1 \), and thus

\[
\int dQ_0 Q_0^a Q_0^b = \frac{C_A}{d_A} \delta^{ab}. \tag{8.1}
\]

Besides, an averaging over distributions of hard particles in thermal equilibrium should be added. This statistic factor in a certain way depends on power of Grassmann color charges in expansions of the effective current and source in powers of soft free fields and initial value of color charges. If the term in the expansion contains ‘not compensated’ Grassmann charge \( \theta_0 \) (or \( \theta_0^\dagger \)), then it is necessary to introduce an averaging over hard particle distributions in the following form:

\[
\int \frac{dp}{(2\pi)^3} \left[ f_p^Q + f_p^G \right] \quad \text{(or} \quad \int \frac{dp}{(2\pi)^3} \left[ f_p^Q + f_p^G \right] \right).
\]

\(^6\)As it will be shown below there is no necessity to enter an averaging over initial values of the Grassmann charges \( \theta_0^\dagger \) and \( \theta_0^\dagger \). These charges will always appear in final expressions in the form of combination \( \theta_0^\dagger \theta_0 = C_\theta = \text{const.} \)
Otherwise, when every charge \( \theta_0 \) is compensated by \( \theta_0^1 \), it is necessary to use an average in the form

\[
\sum_{\zeta=Q,G} \int \frac{d\mathbf{p}}{(2\pi)^3} f_p(\zeta).
\]

In the former case the effective currents and sources generate the scattering processes, which change the type of the hard test parton and correspondingly in the latter case the type of the hard test parton is not varying. This feature was discussed in Section 2 after Eq. (2.5). In the remainder of the paper we restrict our consideration only to linear terms in expansion of the effective current and source, i.e., we set

\[
\tilde{\gamma}^\mu(v, \chi; Q_0, \theta_0| k) \simeq \tilde{\gamma}^\mu(\chi; Q_0, \theta_0| k) + \left[ \tilde{\gamma}^{\psi_{\alpha\beta}}(v, \bar{\chi} | k) - k \theta^0_0 + \theta^{ij}_0 \tilde{\gamma}^{\psi_{\alpha\beta}}(v, \bar{\chi} | k) \right],
\]

(8.2)

\[
\tilde{\eta}^i_\alpha(v, \chi; Q_0, \theta_0| q) \simeq \tilde{\eta}^i_\alpha(v, \chi; Q_0, \theta_0| q) + \tilde{\eta}^{ij}_\alpha(v, \chi; Q_0, \theta_0| q).
\]

In the notations \( \tilde{\gamma}^\mu(v, \chi, Q_0, \theta_0| k), \tilde{\eta}^i_\alpha(v, \chi, Q_0, \theta_0| q), \ldots \) we take into account that for the globally equilibrium system the effective currents and sources depend on hard momentum \( \mathbf{p} \) through velocity \( v = \mathbf{p}/|\mathbf{p}| \) and also spin state of hard parton described by \( \chi \). Dependence on soft fields is implicitly implied.

Taking into account above-mentioned we use the following expressions for emitted powers \( \mathcal{I}_B \) and \( \mathcal{I}_F \) instead of (I.8.1), (I.8.2):

\[
\mathcal{I}_B = \pi \lim_{\tau \to \infty} \frac{(2\pi)^4}{\tau} \left( \int \left| \mathbf{p} \right|^2 \frac{d|\mathbf{p}|}{2\pi^2} \left[ f_p^Q + f_p^G \right] \right) \left( \theta^0_0 \theta^{j}_0 \right) \int dQ_0 \int d\Omega_v \int dk^0 \text{sign}(k^0) \]

\[
\times \left\{ Q^{\mu\nu}(k) \left[ \tilde{\gamma}^{\psi_{\alpha\beta}}(v, \bar{\chi} | k) \tilde{\gamma}^{\psi_{\alpha'\beta'}}(v, \bar{\chi} | k) + \tilde{\gamma}^{\psi_{\alpha\beta}}(v, \bar{\chi} | k) \tilde{\gamma}^{\psi_{\alpha'\beta'}}(v, \bar{\chi} | k) \right] \delta(\text{Re}^* \Delta^{-1}(k)) \right. \]

\[
+ P^{\mu\nu}(k) \left[ \tilde{\gamma}^{\psi_{\alpha\beta}}(v, \bar{\chi} | k) \tilde{\gamma}^{\psi_{\alpha'\beta'}}(v, \bar{\chi} | k) + \tilde{\gamma}^{\psi_{\alpha\beta}}(v, \bar{\chi} | k) \tilde{\gamma}^{\psi_{\alpha'\beta'}}(v, \bar{\chi} | k) \right] \delta(\text{Re}^* \Delta^{-1}(k)) \}
\]

(8.3)

\[
\mathcal{I}_F = - \pi \lim_{\tau \to \infty} \frac{(2\pi)^4}{\tau} \left( \int \left| \mathbf{p} \right|^2 \frac{d|\mathbf{p}|}{2\pi^2} \left[ f_p^Q + f_p^G \right] \right) \left( \theta^0_0 \theta^{j}_0 \right) \int dQ_0 \frac{d\Omega_v}{4\pi} \int dk^0 \text{sign}(k^0) \]

\[
\times \left\{ Q^{\mu\nu}(k) \right. \left\{ \tilde{\gamma}^{\psi_{\alpha\beta}}(v, \chi | k) \tilde{\gamma}^{\psi_{\alpha'\beta'}}(v, \chi | k) \right\} \delta(\text{Re}^* \Delta^{-1}(k)) \right. \]

\[
+ P^{\mu\nu}(k) \left. \left\{ \tilde{\gamma}^{\psi_{\alpha\beta}}(v, \chi | k) \tilde{\gamma}^{\psi_{\alpha'\beta'}}(v, \chi | k) \right\} \delta(\text{Re}^* \Delta^{-1}(k)) \right. \}
\]

(8.4)

\[\text{and correspondingly}\]

\[
\mathcal{I}_F = - \pi \lim_{\tau \to \infty} \frac{(2\pi)^4}{\tau} \left( \int \left| \mathbf{p} \right|^2 \frac{d|\mathbf{p}|}{2\pi^2} \left[ f_p^Q + f_p^G \right] \right) \left( \theta^0_0 \theta^{j}_0 \right) \int dQ_0 \frac{d\Omega_v}{4\pi}
\]

(Strictly speaking the effective currents and sources are also implicitly dependent on energy \( E \) of hard test particle through spinor \( \chi \) (see Eq. (C.2)). It is this dependence that is meant in notation of the scattering probabilities as functions of hard momentum \( \mathbf{p} \) in Eqs. (2.4) and (2.5).)

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\[ \times \int dq \, q^0 \text{sign}(q^0) \left\{ (h_+(\bar{q}))_{\alpha'\alpha} \langle \tilde{n}^{ij}_\alpha (v, \bar{\chi}) - q )\tilde{n}^{ij'}_\alpha (v, \chi | q) \rangle \delta(\text{Re}^*\Delta^{-1}_+(q)) \right. \\
+ \left. (h_- (\bar{q}))_{\alpha'\alpha} \langle \tilde{n}^{ij}_\alpha (v, \bar{\chi}) - q )\tilde{n}^{ij'}_\alpha (v, \chi | q) \rangle \delta(\text{Re}^*\Delta^{-1}_+(q)) \right\} \\
- \pi \lim_{\tau \to \infty} \frac{(2\pi)^4}{\tau} \left( \sum_{\zeta=Q, Q, G} \int \frac{|p|^2 \, dq |p|}{2\pi^2} \int dQ_{\bar{Q}_0} Q_{\bar{Q}_0} \int d\Omega_{\bar{Q}} \right) \int dq \, q^0 \text{sign}(q^0) \\
\times \int dq \, q^0 \text{sign}(q^0) \left\{ (h_+(\bar{q}))_{\alpha'\alpha} \langle \tilde{n}^{ib}_\alpha (v, \bar{\chi}) - q )\tilde{n}^{ib'}_\alpha (v, \chi | q) \rangle \delta(\text{Re}^*\Delta^{-1}_+(q)) \right. \\
+ \left. (h_- (\bar{q}))_{\alpha'\alpha} \langle \tilde{n}^{ib}_\alpha (v, \bar{\chi}) - q )\tilde{n}^{ib'}_\alpha (v, \chi | q) \rangle \delta(\text{Re}^*\Delta^{-1}_+(q)) \right\}. \\
\]

In Eq. (8.3) we keep only a contribution from the current \( \tilde{j}_\mu^a \) including soft-quark fields and the Grassmann color charges. The contribution from the current \( \tilde{j}_\mu^A \) was considered in Ref. [6]. Besides, unlike (I.8.1) and (I.8.2) we remove an integration over volume of the system, since in the definition of phase-space measures (2.6) the momentum conservation laws are explicitly considered. According to the corresponding principle for determining the scattering probabilities \( w_{q \rightarrow g}^{(f; b)}(p | q; k) \), \( w_{q \rightarrow q}^{(C; f; f)}(p | q; q_1) \) etc. the expression obtained from (8.4) should be compared with expression determining a change of energy of soft fermionic plasma excitations generated by the spontaneous processes of soft-quark and soft-gluon emission only

\[
\left( \frac{d\mathcal{E}}{dt} \right)_{\text{sp}} = \sum_{f=\pm} \frac{d}{dt} \left\{ \int \frac{dq}{(2\pi)^3} \omega_{\bar{q}q}^{(f)} n_{\bar{q}q}^{(f)} + \int \frac{dq}{(2\pi)^3} \omega_{qq}^{(f)} \bar{n}_{qq}^{(f)} \right\} 
\]

\[
= \sum_{f=\pm} \left( \sum_{b=t, l} \int \frac{|p|^2 \, dq |p|}{2\pi^2} \left[ f_p^Q + f_p^G \right] \int \frac{d\Omega_{\bar{Q}}}{4\pi} \int \frac{dq}{(2\pi)^3} \int dT_{q \rightarrow \bar{g}}^{(f; b)} \omega_{\bar{q}q}^{(f)} \right) \\
\times \left\{ w_{q \rightarrow \bar{g}}^{(f; b)}(p | q; k) + w_{q \rightarrow \bar{g}}^{(f; b)}(p | q; k) \right\} \eta_{q}^{(f)} \]

\[
+ \sum_{f=\pm} \left( \sum_{b=t, l} \left( \sum_{\zeta=Q, Q, G} \int \frac{|p|^2 \, dq |p|}{2\pi^2} \left( f_p^{(C)} \right) \int \frac{d\Omega_{\bar{Q}}}{4\pi} \int \frac{dq}{(2\pi)^3} \int dT_{q \rightarrow \bar{g}}^{(f; f_1)} \omega_{\bar{q}q}^{(f)} \right) \\
\times \left\{ w_{q \rightarrow \bar{g}}^{(C; f_1; f)}(p | q; q_1) n_{q_1}^{(f_1)} + w_{q \rightarrow \bar{g}}^{(C; f; f_1)}(p | q; q_1) \left( 1 - \bar{n}_{q_1}^{(f_1)} \right) \right\} \right\}. \\
\]

In deriving the right-hand side we have taken into account an equality of integration measures for the processes with participation of soft-quark and soft-antiquark modes

\[
\int dT_{q \rightarrow \bar{g}}^{(f; b)} = \int dT_{q \rightarrow \bar{g}}^{(f; f_1)}; \quad \int dT_{q \rightarrow \bar{g}}^{(f; f_1)} = \int dT_{q \rightarrow \bar{g}}^{(f; f_1)} \\
\]

and also the fact that in conditions of global equilibrium and zero quark chemical potential an equality \( f_p^Q = f_p^Q \) takes place. What is more the kinetic equation (2.1) with collision
term in the limit of a small intensity \( n_0(q) \to 0 \) (Eq. (2.9)) was used. The dots designate contributions of higher order scattering processes.

With all required formulas at hand now one can define the simplest scattering probabilities \( w_q^{(f;g)} \), \( w_q^{(C;f)} \) etc. To be specific, we consider the scattering processes with participation of normal soft-quark and transverse soft-gluon modes, i.e., we set \( f = f_1 = + \) and \( b = t \). As the first step we single out on the right-hand side of (8.4) the contribution of normal mode of soft fermion excitations. For this purpose we set

\[
\delta(\text{Re} \Delta_\pm^{-1}(q)) = \sum_{\lambda = \pm} u_\alpha(q, \lambda) \bar{u}_\alpha' (q, \lambda), \quad (h_-)(q) = \sum_{\lambda = \pm} v_\alpha(q, \lambda) \bar{v}_\alpha' (q, \lambda).
\]

Substituting (8.6), (8.7) into (8.4) and integrating with respect to \( dq_0 \), we define contribution from the normal soft-quark modes to the emitted power

\[
\mathcal{I}_F = -\pi \lim_{\tau \to \infty} \frac{(2\pi)^4}{\tau} \left( \int \frac{|p|\,d|p|}{2\pi^2} \left[ f_p^G + f_p^C \right] \right) \int dq_0 \frac{d\Omega_q}{4\pi} \frac{d\Omega_v}{4\pi} \times \sum_{\lambda = \pm} \int dq \, \omega_\pm^+ Z_\pm(q) \left\{ \left( \langle \tilde{u}_\alpha(q, \lambda) - q \rangle u_\alpha(q, \lambda) \right) \left( \bar{u}_\alpha'(q, \lambda) \tilde{u}_\alpha''(q, \lambda) \right) \right\}_{\omega_\pm^+}
\]

and expand the spinor projectors \( h_\pm(q) \) in terms of simultaneous eigenspinors of chirality and helicity

\[
(h_+(q))_{\alpha\alpha'} = \sum_{\lambda = \pm} u_\alpha(q, \lambda) \bar{u}_\alpha'(q, \lambda), \quad (h_-(q))_{\alpha\alpha'} = \sum_{\lambda = \pm} v_\alpha(q, \lambda) \bar{v}_\alpha'(q, \lambda).
\]

The first contribution on the right-hand side of the last equation enables us to define the scattering probabilities \( w_q^{(+;t)}(p; q; k) \) and \( w_q^{(+;t)}(p; q; k) \). Let us perform the following replacements

\[
\tilde{u}_\alpha'(q, \lambda) \to \tilde{u}_\alpha^{(1)'}(A^{(0)})(q), \quad \tilde{u}_\alpha'(q, \lambda) - q \to \tilde{u}_\alpha^{(1)}(A^{(0)})(-q),
\]
where the effective sources \( \tilde{\eta}^{(1)}_{\alpha ij} \), \( \tilde{\eta}^{(1)}_{\alpha ij} \) are defined by Eqs. (4.7), (4.6) and (4.8), (4.9).

In this case the contribution under discussion to the emitted power (8.8) will be equal

\[
- \frac{1}{2\pi} g^4 \lim_{\tau \to \infty} \frac{1}{\tau} \left( \int \frac{d|p|^2 d|p|}{2\pi^2} \left[ f_p^Q + f_p^G \right] \right) \left( \theta_0^{ij} (\dot{\theta}^b)^{ijj} \theta_0^b \right) \int dQ_0 \int \frac{d\Omega_v}{4\pi} (8.9)
\]

\[
\times \sum_{\lambda = \pm} \int dQ \omega_\alpha \left( \frac{Z_\pm(Q)}{2} \right) \left\{ \left[ \left( K_\alpha^{(Q)}(v, \bar{\chi}| k, -q) u_\alpha(q, \lambda) \right) \left( K_\alpha^{(Q)\mu'}(v, \chi| k', -q) \bar{u}_\alpha(q, \lambda) \right) \right. \right.
\]

\[
\times \left. \delta(v \cdot (q - k)) \delta(v \cdot (q - k')) \right\} q^\mu = \omega^\mu_q + \left. \left[ \left( K_\alpha^{(Q)}(v, \bar{\chi}| k, -q) u_\alpha(q, \lambda) \right) \left( K_\alpha^{(Q)\mu'}(v, \chi| k', -q) \bar{u}_\alpha(q, \lambda) \right) \delta(v \cdot (q - k)) \delta(v \cdot (q - k')) \right] q^\mu = -\omega^\mu_q \right\}
\]

\[
\times \left\langle A^{(0)b}_\mu(k) A^{(0)b'}_{\mu'}(k') \right\rangle \; dk dk'.
\]

The integral in \( dQ \) is equal to unit by virtue of the normalization. The correlation function of random soft-gluon field in conditions of stationary and homogeneous state of the quark-gluon plasma can be written in the form

\[
\left\langle A^{(0)b}_\mu(k) A^{(0)b'}_{\mu'}(k') \right\rangle \simeq \delta^{bb'} \delta_{\mu\mu'} \delta(k - k'),
\]

where in the spectral density \( I_{\mu\mu'}(k) \) we keep only ‘transverse’ part: \( P_{\mu\mu'}(k) I_k^L \). In the frame, where \( u^\mu = (1, 0, 0, 0) \) the transverse projector \( P_{\mu\mu'}(k) \) reduces to three-dimensional transverse projector \( P^{\mu\mu'}(\hat{k}) \), \( \hat{k} = k/|k| \). For the transverse mode we introduce polarization vectors \( e^i(\hat{k}, \xi), \xi = 1, 2 \) possessing the properties

\[
k \cdot e^i(\hat{k}, \xi) = 0, \quad e^i(\hat{k}, \xi) \cdot e^{i'}(\hat{k}, \xi') = \delta_{\xi\xi'}.
\]

The three-dimensional transverse projector is associated with the polarization vectors by relation

\[
P^{\mu\mu'}(\hat{k}) = (\delta^{\mu\mu'} - \hat{k}^i \hat{k}^i) = \sum_{\xi = 1, 2} e^{\ast i}(\hat{k}, \xi) e^{ij}(\hat{k}, \xi).
\]

In what follows we take the spectral function \( I_k^L \) in the form of the quasiparticle approximation. Turning into the number density of transverse soft gluons \( N_k^L (\equiv -2(2\pi)^3 2\omega_k^L Z_k^{-1}(k) I_k^L) \), we finally obtain a substitution rule for the spectral density \( I_{\mu\mu'}(k) \):

\[
I_{\mu\mu'}(k) \to \frac{1}{(2\pi)^3} \frac{Z_t(k)}{2\omega_k^4} \left[ N_k^L \delta(k^0 - \omega_k) + N_k^L \delta(k^0 + \omega_k) \right]
\]

\[
\times \sum_{\xi = 1, 2} e^{\ast i}(\hat{k}, \xi) e^{ij}(\hat{k}, \xi).
\]

To take into account weak non-homogeneity and slow evolution of the medium in time, it is sufficient to replace (within the accuracy accepted) the equilibrium number density \( N_k^L \) by off-equilibrium one in the Wigner form slowly depending on \( x = (t, \mathbf{x}) \).
For the ‘color’ factor with the Grassmann charges considering \( \delta^{bb'} \) in (8.11), we have

\[
\theta^i_j (t^b t^{b'}) \theta^j_0 = C_F \theta^i_0 \theta^j_0 \equiv C_F C_0.
\]

The exact value (more precisely, the values) of the constant \( C_0 \) will be defined in the next section. Here we only need to know that \( C_0 < 0 \). Substituting (8.10), (8.11) into Eq. (8.9), performing integration with respect to \( dk' \) and \( dk_0 \), and making use of the relation

\[
[ \delta (v \cdot (k - q)) ]^2 = \frac{\tau}{2 \pi} \delta (v \cdot (k - q)),
\]

we finally obtain an expression for the desired emitted power instead of (8.9)

\[
-g^4 C_F C_0 \left( \int \frac{|p|^2 dp}{2\pi^2} \left[ f^i_p + f^i_p' \right] \right) \int \frac{d\Omega_v}{4\pi} \int \frac{d\mathbf{k}'}{(2\pi)^3} \frac{d\mathbf{q}}{(2\pi)^3} \frac{d\mathbf{k}}{(2\pi)^3} \omega^+_k N_k \left( \frac{Z_+(\mathbf{q})}{2} \right) \left( \frac{Z_l(\mathbf{k})}{2\omega^+_k} \right)
\]

\[
\times \sum_{\lambda = \pm} \sum_{\xi = 1, 2} \left\{ \left[ e^{i\xi} (\hat{\mathbf{k}}_v, \xi) \tilde{K}^{(Q)}_{\alpha} (\mathbf{v}, \bar{\mathbf{v}} | k - q) u_\alpha (\mathbf{q}, \lambda) \right] \left[ \tilde{u}_{\alpha'} (\mathbf{q}, \lambda) K^{(Q)'i} (\mathbf{v}, \chi | k - q) e^{i\xi} (\hat{\mathbf{k}}_v, \xi) \right] \right\}
\]

\[
\times 2\pi \delta (\omega^+_k - \omega^+_k - v \cdot (\mathbf{q} + \mathbf{k})).
\]

In deriving this equation we have omitted the contribution containing the delta-function \( \delta (\omega^+_q + \omega^+_k - v \cdot (\mathbf{q} + \mathbf{k})) \). It defines the processes of simultaneous emission (absorption) of soft-quark and soft-gluon excitations by hard particle. For the second term in brackets we have made replacements of variables: \( \mathbf{q} \rightarrow -\mathbf{q}, \mathbf{k} \rightarrow -\mathbf{k} (\omega^+_q \rightarrow \omega^+_q, \omega^+_k \rightarrow \omega^+_k) \). Now we introduce the following matrix elements for soft-quark – hard-particle and soft-antiquark – hard-particle ‘inelastic’ scattering, respectively

\[
\mathcal{M}^{(+i)}(\mathbf{v}, \chi | \mathbf{q}; \mathbf{k}) \equiv g^2 (C_F |C_0|)^{1/2} \left( \frac{Z_+(\mathbf{q})}{2} \right)^{1/2} \left( \frac{Z_l(\mathbf{k})}{2\omega^+_k} \right)^{1/2}
\]

\[
\times \left[ \tilde{u}_\alpha (\mathbf{q}, \lambda) K^{(Q)}_{\alpha} (\mathbf{v}, \chi | k - q) e^{i\xi} (\hat{\mathbf{k}}_v, \xi) \right]_{\text{on-shell}},
\]

\[
\tilde{\mathcal{M}}^{(+i)}(\mathbf{v}, \chi | \mathbf{q}; \mathbf{k}) \equiv g^2 (C_F |C_0|)^{1/2} \left( \frac{Z_+(\mathbf{q})}{2} \right)^{1/2} \left( \frac{Z_l(\mathbf{k})}{2\omega^+_k} \right)^{1/2}
\]

\[
\times \left[ \tilde{u}_{\alpha'} (\mathbf{q}, \lambda) K^{(Q)'i} (\mathbf{v}, \chi | k - q) e^{i\xi} (\hat{\mathbf{k}}_v, \xi) \right]_{\text{on-shell}}.
\]

Confronting expression obtained (8.12) with the corresponding terms in (8.5), one identifies the desired probabilities \( w^{(+i)}_{q \rightarrow g} \) and \( w^{(+i)}_{q \rightarrow g} \),

\[
w^{(+i)}_{q \rightarrow g} (\mathbf{p} | \mathbf{q}; \mathbf{k}) = \sum_{\lambda = \pm} \sum_{\xi = 1, 2} | \mathcal{M}^{(+i)}(\mathbf{v}, \chi | \mathbf{q}; \mathbf{k}) |^2,
\]
\[ u^{(+,t)}_{q\to g}(p| q; k) = \sum_{\lambda=\pm} \sum_{\xi=1,2} \left| \hat{\mathcal{N}}^{(+,t)}_{\chi_{\xi}}(v, \chi| q; k) \right|^2 . \]

The probabilities of the scattering processes with (anti)plasmino and plasmon are obtained from Eqs. (8.13) - (8.15) with the help of corresponding replacements such as

\[
\left( \frac{Z_+(q)}{2} \right)^{1/2} u_{\alpha}(q, \lambda) \to \left( \frac{Z_-(q)}{2} \right)^{1/2} v_{\alpha}(q, \lambda),
\]

\[
\left( \frac{Z_i(k)}{2\omega_k^i} \right)^{1/2} e^i(k, \xi) \to \left( \frac{Z_i(k)}{2\omega_k^i} \right)^{1/2} \left( \frac{\tilde{u}^{\mu}(k)}{\sqrt{\tilde{u}^2(k)}} \right),
\]

e tc. and proper choice of mass-shell conditions on the right-hand side of Eqs. (8.13), (8.14).

Now we turn to deriving the probabilities for the elastic scattering of soft-quark and soft-antiquark modes off the hard test particle. As in the previous case we restrict our consideration only to the normal quark excitations. In the second term on the right-hand side of (8.8) we perform the following replacements:

\[
\tilde{\eta}_{\alpha}^i q, \bar{\chi} \to \tilde{\eta}_{\alpha}^i q, \bar{\chi} \to \tilde{\eta}_{\alpha}^i q, \bar{\chi}, \eta_{\alpha'}^i q, \bar{\chi} \to \eta_{\alpha'}^i q, \bar{\chi},
\]

where the effective sources \( \tilde{\eta}_{\alpha}^i \), \( \tilde{\eta}_{\alpha'}^i \) are defined by Eqs. (5.16), (5.17) and (4.3), (5.15) correspondingly. As a result of this kind replacement the second term on the right-hand side of Eq. (8.8) is written in the following form:

\[
\frac{1}{2}\pi^4 \lim_{\tau \to \infty} \frac{1}{\tau} \left( \sum_{\xi=Q, Q', G} \int \frac{d^4p}{2\pi^2} \frac{d^4\xi}{2\pi^2} \right) \left( t^{\bar{\alpha} \beta} \right)^{ij} \int dQ_0 Q_0^i Q_0^j \int \frac{d\Omega}{4\pi}
\]

\[
\times \sum_{\lambda = \pm} \int d\Omega \omega_{\xi}^\pm \left( \frac{Z_+(q)}{2} \right) \left\{ \left[ \tilde{K}_{\alpha' \alpha}^{(Q)}(\bar{\chi}, \chi| q, -q_1) u_{\alpha}(\bar{\chi}, \lambda) \right] \left[ \tilde{v}_{\beta}^{(Q)}(\bar{\chi}, \chi| q, -q_1') \delta(v \cdot (q - q_1)) \delta(v \cdot (q - q_1')) \right] \right\}_{q^0 = \omega_{\xi}^\pm}
\]

\[
\times \left\{ \tilde{v}_{\alpha'}^{(Q)}(-q_1) \psi_{\beta'}^{(Q)}(q_1') \right\} dq_1 dq_1'.
\]

Under the conditions of stationary and homogeneous state of QGP the correlation function for random soft-quark field can be presented as

\[
\left\langle \tilde{v}_{\alpha'}^{(Q)}(-q_1) \psi_{\beta'}^{(Q)}(q_1') \right\rangle = \delta^{ij} \gamma_{\beta' \alpha'}(q_1') \delta(q_1' - q_1),
\]
where in the spectral density \( \Upsilon_{\beta\alpha'}(q'_1) \) we keep only the normal quark part in the form of the quasiparticle approximation

\[
\Upsilon_{\beta\alpha'}(q'_1) \rightarrow \frac{1}{(2\pi)^3} \left( \frac{Z_+(q'_1)}{2} \right) \sum_{\lambda_1=\pm} \left[ u_{\beta'}(\hat{q}'_1, \lambda_1) \bar{u}_{\alpha'}(\hat{q}'_1, \lambda_1) n_{q'_1}^+ \delta(q'_1^0 - \omega_{q'_1}^+) 
+ v_{\beta'}(\hat{q}'_1, \lambda_1) \bar{v}_{\alpha'}(\hat{q}'_1, \lambda_1) (1 - \bar{n}_{-q'_1}^+) \delta(q'_1^0 + \omega_{q'_1}^+) \right].
\] (8.19)

Within the accepted accuracy we also can replace the equilibrium number densities \( n_{q'_1}^+ \), \( \bar{n}_{-q'_1}^+ \) by off-equilibrium ones slowly depending on \( x \).

Furthermore, making use of the formula for color averaging (8.1), triviality in a color space of the correlation function (Eq. (8.18)), and the equality \( \text{tr}(t^a t^a) = d_A T_F \), where \( T_F \) is index of the fundamental representation, we find that the color factor in (8.17) equals \( T_F C_2^{(c)} \), \( \zeta = Q, \bar{Q}, G \). Taking into account the above-mentioned and performing integration with respect to \( dq'_1 dq_1^0 \), we define final expression for the desired emitted power instead of (8.17)

\[
g^4 T_F \left( \sum_{\zeta=Q,\bar{Q},G} C_2^{(c)} \int \frac{|p|^2 d|p|}{2\pi^2} \int_{|p|}^{f_\zeta(L)} \int_{(2\pi)^3} \int_{(2\pi)^3} \omega_{q}^+ \left( \frac{Z_+(q)}{2} \right) \left( \frac{Z_+(q_1)}{2} \right) \right.
\]

\[
\times \sum_{\lambda = \pm} \sum_{\lambda_1 = \pm} \left\{ n_{q'_1}^+ \left| \bar{u}_\alpha(\hat{q}_1, \lambda) K_{\alpha\alpha_1}^{(Q)}(\chi, \tilde{x} \mid q, -q_1) u_{\alpha_1}(\hat{q}'_1, \lambda_1) \right|^2 \right\}_{on-shell}
\]

\[
+ (1 - \bar{n}_{-q'_1}^+) \left| \bar{v}_\alpha(\hat{q}_1, \lambda) K_{\alpha\alpha_1}^{(Q)}(\chi, \tilde{x} \mid q, -q_1) v_{\alpha_1}(\hat{q}'_1, \lambda_1) \right|^2 \right\}_{on-shell}
\]

\[
\times \left[ 2\pi \delta(\omega_{q}^+ + \omega_{q_1}^+) - \mathbf{v} \cdot (\mathbf{q} + \mathbf{q}_1) \right].
\]

In deriving this expression we have dropped the contribution containing delta-function \( \delta(\omega_{q}^+ + \omega_{q_1}^+) - \mathbf{v} \cdot (\mathbf{q} + \mathbf{q}_1) \) and for the second term in braces we have made the replacement of integration variables: \( q \rightarrow -\mathbf{q} \) and \( q_1 \rightarrow -\mathbf{q}_1 \). We introduce the following matrix elements for soft-quark – hard-particle and soft-antiquark – hard-particle ‘elastic’ scatterings, respectively

\[
\mathcal{M}_{\lambda\lambda_1}^{(c)(+,-)}(\chi, \tilde{x} | \mathbf{q}, \mathbf{q}_1) \equiv g^2 \left( T_F C_2^{(c)} \right)^{1/2} \left( \frac{Z_+(q)}{2} \right)^{1/2} \left( \frac{Z_+(q_1)}{2} \right)^{1/2}
\]

\[
\times \left[ \bar{u}_\alpha(\hat{q}, \lambda) K_{\alpha\alpha_1}^{(Q)}(\chi, \tilde{x} \mid q, -q_1) u_{\alpha_1}(\hat{q}_1, \lambda_1) \right]_{on-shell},
\]

\[
\mathcal{M}_{\lambda\lambda_1}^{(c)(+,-)}(\chi, \tilde{x} | \mathbf{q}, \mathbf{q}_1) \equiv g^2 \left( T_F C_2^{(c)} \right)^{1/2} \left( \frac{Z_+(q)}{2} \right)^{1/2} \left( \frac{Z_+(q_1)}{2} \right)^{1/2}
\]

\[
\times \left[ \bar{v}_\alpha(-\hat{q}, \lambda) K_{\alpha\alpha_1}^{(Q)}(\chi, \tilde{x} \mid q, -q_1) v_{\alpha_1}(-\hat{q}_1, \lambda_1) \right]_{on-shell}.
\]
Comparing the expression obtained (8.20) with the corresponding terms in (8.5), we identify the desired probabilities

\[ w^{(f)}_{q \rightarrow \bar{q}}(p | q, q_1) = \sum_{\lambda = \pm} \sum_{\lambda_1 = \pm} | \mathcal{M}^{(f)}_{\lambda \lambda_1}(\chi, \bar{\chi} | q, q_1) |^2 \]  

These probabilities depend on type of the hard test particle (through the Casimirs \( C^f_2 \)) on which the scattering of soft-quark modes takes place. The probabilities of the scattering processes with participation of plasmino and antiplasmino can be derived from (8.22), (8.21) with the help of the corresponding replacements of the quark wave functions and mass-shell conditions.

We demand that scattering probabilities (8.22) satisfy balance relations (2.7) for the direct and reverse scattering processes. A straightforward calculation shows that these relations take place only under fulfilment of the following conditions:

\[ \alpha^* = \alpha, \quad *\Gamma^{(Q)\mu}(q - q_1; -q, q_1) = *\Gamma^{(Q)\mu}(q - q_1; q_1, -q). \]

The first condition implies that the constant \( \alpha \) in definition of additional source (5.14) is real. The second condition holds only when the linear Landau damping of soft-quark on-shell excitations is absent in the medium.

9 Structure of scattering probability \( w^{(f; b)}_{q \rightarrow g} \). Determination of constant \( C_\theta \)

In this section we consider in more details a structure of the scattering probability \( w^{(f; b)}_{q \rightarrow g}(v | q, k) \) obtained in the previous section and define also an explicit value of the constant \( C_\theta (\equiv \theta_0^\dagger \theta_0) \). Here we make use some results of our early work [2]. For the sake of convenience of further references all required formulae from this work are given in Appendix F.

First of all we recall that in the paper [2] the scattering probability of plasmino off hard parton with transition to plasmon \( w^{(-; l)}_{q \rightarrow \bar{g}}(v | q; k) \) was defined in a different way by a direct calculation of the following expression:

\[ \text{Im} \text{Sp} \left[ *\tilde{\Gamma}^{(Q)\alpha_1 \alpha_2, ij}_{\mu_1 \mu_2}(k_1, k_2; q_1, -q) \delta^{\alpha_1 \alpha_2} \bar{u}^{\mu_1}(k_1) \bar{u}^{\mu_2}(k_2) h_- (\bar{q}) \right]_{k_1 = -k_2 = k, q_1 = q}, \]

where ‘Sp’ denotes the Dirac trace and \( \bar{u}^{\mu}(k) \) is the longitudinal projector in the covariant gauge (Eq. (C.6)). The effective amplitude \( *\tilde{\Gamma}^{(Q)\alpha_1 \alpha_2, ij}_{\mu_1 \mu_2} \) was defined in Paper I (Eq. (I.4.9)).
The amplitude determines the elastic scattering process of soft-quark excitation on soft-gluon excitation without momentum-energy exchange with hard thermal partons. Within the framework of the approach [2] for description of the scattering process under consideration there was no necessity to introduce the Grassmann color charges $\theta^i_0$ and $\theta^i_0$ of hard parton (and consequently, the constant $C_\theta$). Comparing, for example, the kinetic equations for the plasmino number density $n_q$ obtained in these two approaches, one can define unknown constant $C_\theta$. However, preliminary we recast the scattering probability $w_q^{(-l)}$ in the form suggested in Ref. [2]. This makes it possible in particular correctness of unusual at first sight structure of the scattering kernel $Q(q,k)$ given by Eqs. (F.9) – (F.11) to be independently proved.

At the beginning we write out an explicit form of the matrix elements defining the scattering processes of plasmino and antiplasmino off hard parton with the subsequent transition in plasmon. For this purpose we perform replacements (8.16) in Eqs. (8.13), (8.14) and relevant replacements of the mass-shell conditions. Here it is more convenient to use the temporal gauge. This means that in last replacement in Eq. (8.16) instead of the projector $\tilde{u}^\mu(k)$ it is necessary to use the projector $\tilde{u}^\mu(k) \equiv k^2(u^\mu(k \cdot u) - k^\mu)/(k \cdot u)$. In the rest system $u^\mu = (1, 0, 0, 0)$, we get

$$M^{(-l)}(v,\chi \mid q; k) \equiv g^2 (C_F|C_\theta|)^{1/2} \left( \frac{Z_q(q)}{2} \right)^{1/2} \left( \frac{Z_t(k)}{2\omega_k^2} \right)^{1/2} \left( \frac{k^2}{k^2(\omega_k^2)^2} \right)^{1/2}$$

(9.1)

$$\times \left[ \tilde{v}_\alpha(q,\lambda) \left( K^{(Q)_i}_\alpha(v,\chi \mid k, -q) k^i \right) \right]_{o-n-shell},$$

$$\tilde{M}^{(-l)}(v,\chi \mid q; k) \equiv g^2 (C_F|C_\theta|)^{1/2} \left( \frac{Z_q(q)}{2} \right)^{1/2} \left( \frac{Z_t(k)}{2\omega_k^2} \right)^{1/2} \left( \frac{k^2}{k^2(\omega_k^2)^2} \right)^{1/2}$$

(9.2)

$$\times \left[ \tilde{u}_\alpha(-q,\lambda) \left( K^{(Q)_i}_\alpha(v,\chi \mid -k, q)(-k^i) \right) \right]_{o-n-shell}.$$

Let us consider at first matrix element (9.1). Making use of the definition of the coefficient function $K^{(Q)_i}_\alpha$, Eq. (4.6), we write out the function $\tilde{v}_\alpha(q,\lambda)(K^{(Q)_i}_\alpha k^i)$ in an explicit form (for the sake of brevity we suppress spinor indices)

$$\frac{(v \cdot k)}{v \cdot q} (\tilde{v}(q,\lambda) \chi) - \tilde{v}(q,\lambda)[*\Gamma^{(Q)_i}(k; q - k, -q)k^i] *S(q - k) \chi = \frac{(v \cdot k)}{v \cdot q} (\tilde{v}(q,\lambda) \chi)$$

(9.3)

$$+ \tilde{v}(q,\lambda) \left[ h_- (\hat{l}) (T^i_+ k^i) + h_+ (\hat{l}) (T^i_- k^i) - 2 h_- (\hat{l}) (T^i_+ k^i) \right] \left[ h_+ (\hat{l}) t^i \chi + h_+ (\hat{l}) *\Delta (l) + h_- (\hat{l}) *\Delta^* (l) \right] \chi,$$

where $l \equiv q - k$. On the right-hand side of the last equation we have used expansion (F.3) for the vertex $*\Gamma^{(Q)_i}$, the properties (F.2), and the representation of the soft-quark propagator $*S(l)$ in the form of an expansion in terms of the spinor projectors $h_\pm(q)$. Now we multiply together two expressions in square brackets taking into account the property of nilpotency $h_\pm (\hat{l}) h_\pm (\hat{l}) = 0$ and the fact that the wave function $\tilde{v}(q,\lambda)$ satisfies the
equation $\bar{v}(\hat{q}, \lambda)h_{-}(\hat{q}) = 0$. As a result we obtain the second term on the right-hand side of Eq. (9.3) in more simple and symmetric form:

$$\left[ \bar{v}(\hat{q}, \lambda)h_{-}(\hat{1})h_{+}(\hat{1})\chi \right] (T_{+}^{ij}k^{i})\Delta_{+}(l) + \left[ \bar{v}(\hat{q}, \lambda)h_{+}(\hat{1})h_{-}(\hat{1})\chi \right] (T_{-}^{ij}k^{i})\Delta_{-}(l).$$

Let us emphasize that the simplification is a direct consequence of a choice of the expansion (F.3) for convolution $*\Gamma^{(Q)ij}k^{i}$. The first term on the right-hand side of Eq. (9.3) can be also written down in a symmetric form with respect to the matrixes $h_{\pm}(\hat{1})$ if we use an identity

$$1 = h_{-}(\hat{1})h_{+}(\hat{1}) + h_{+}(\hat{1})h_{-}(\hat{1}).$$

(9.4)

Furthermore, we collect similar terms and square of the absolute value of the expression (9.3). Summing over polarization states of soft-quark excitations, we obtain an initial expression for the subsequent analysis

$$\left( \sum_{\lambda = \pm} |\bar{v}(\hat{q}, \lambda)h_{-}(\hat{1})h_{+}(\hat{1})\chi|^{2} \right) |M_{\pm}^{(-, \lambda)}(q, k)|^{2} + \left( \sum_{\lambda = \pm} |\bar{v}(\hat{q}, \lambda)h_{+}(\hat{1})h_{-}(\hat{1})\chi|^{2} \right) |M_{\pm}^{(-, \lambda)}(q, k)|^{2}$$

$$+ 2 \sum_{\lambda = \pm} \text{Re} \left( [\bar{v}(\hat{q}, \lambda)h_{-}(\hat{1})h_{+}(\hat{1})\chi][\bar{v}(\hat{q}, \lambda)h_{+}(\hat{1})h_{-}(\hat{1})\chi]^{*}M_{\pm}^{(-, \lambda)}(q, k)M_{\pm}^{(-, \lambda)}(q, k) \right).$$

(9.5)

where

$$M_{\pm}^{(-, \lambda)}(q, k) \equiv \frac{v \cdot k}{v \cdot q} + *\Delta_{\pm}(l)(T_{\pm}^{ij}(k; l, -q)k^{i}).$$

(9.6)

Let us define an explicit form of coefficients of the scalar amplitudes $M_{\pm}^{(-, \lambda)}(q, k)$. At first we consider the coefficient of $|M_{+}^{(-, \lambda)}(q, k)|^{2}$. Taking into account identity (9.4) and definition of density matrix for a fully unpolarized state of the hard test (anti)quark (Eq. (C.2)), we have the following chain of equalities:

$$\sum_{\lambda = \pm} |\bar{v}(\hat{q}, \lambda)h_{-}(\hat{1})h_{+}(\hat{1})\chi|^{2} = -\bar{\chi}h_{-}(\hat{1})h_{+}(\hat{1})h_{-}(\hat{q})h_{+}(\hat{1})\chi + \bar{\chi}h_{-}(\hat{q})h_{-}(\hat{1})h_{+}(\hat{1})\chi$$

$$= -\frac{1}{2E} \left\{ \text{Sp} \left[ h_{-}(\hat{1})h_{+}(\hat{1})h_{-}(\hat{q})h_{+}(\hat{1})\chi \right] \chi - \text{Sp} \left[ h_{-}(\hat{q})h_{-}(\hat{1})h_{+}(\hat{1})\chi \right] \right\}$$

$$= -\frac{1}{2E} \left\{ \frac{1}{2} (v \cdot (\hat{1} \times (\hat{q} \times \hat{1}))) - \frac{1}{2} \rho_{+}(v; \hat{q}, \hat{1}) \right\},$$

(9.7)

where the function $\rho_{+}(v; \hat{q}, \hat{1})$ is defined by Eq. (F.10). The coefficient of $|M_{+}^{(-, \lambda)}(q, k)|^{2}$ in (9.5) is defined from the coefficient above by formal replacement $\hat{1} \rightarrow -\hat{1}$. This reduces to a simple replacement $\rho_{+} \rightarrow \rho_{-}$ on the rightmost expression in Eq. (9.7). Finally, it is not difficult to see that calculation of the coefficient in the interference term of Eq. (9.5) is reduced to calculation of the first trace on the second line of Eq. (9.7) and therefore it is equal to

$$-\frac{1}{2} (v \cdot (\hat{1} \times (\hat{q} \times \hat{1}))) (\equiv -\frac{1}{2} \sigma(v; \hat{q}, \hat{1})).$$

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Taking into account the above-mentioned and collecting similar terms at the function \( \sigma(\mathbf{v}; \hat{\mathbf{q}}, \hat{l}) \), we define instead of (9.5)

\[
\frac{1}{2E} \left\{ \frac{1}{2} \rho_+(\mathbf{v}; \hat{\mathbf{q}}, \hat{l}) \left| \mathcal{M}^{(+,l)}(\mathbf{q}, \mathbf{k}) \right|^2 + \frac{1}{2} \rho_-(\mathbf{v}; \hat{\mathbf{q}}, \hat{l}) \left| \mathcal{M}^{(-,l)}(\mathbf{q}, \mathbf{k}) \right|^2 \right\}
\]

(9.8)

\[-\frac{1}{2} \left\{ \frac{1}{2} \rho_-(\mathbf{v}; \hat{\mathbf{q}}, \hat{l}) \left| \mathcal{M}^{(+,l)}(\mathbf{q}, \mathbf{k}) - \mathcal{M}^{(-,l)}(\mathbf{q}, \mathbf{k}) \right|^2 \right\}.
\]

A structure of this expression differs from a structure of the integrand of the scattering kernel (F.9) only by the last term with the coefficient \( \sigma(\mathbf{v}; \hat{\mathbf{q}}, \hat{l}) \). However, this term vanishes (see below) when we average over the directions of the velocity \( \mathbf{v} \) of the hard test parton. Thus we have proved by another way a correctness of a structure of the scattering kernel \( Q(\mathbf{q}, \mathbf{k}) \) obtained in [2] by a direct calculation of the imaginary part of the effective amplitude for the plasmino-plasmon elastic scattering.

Now we turn our attention to matrix element (9.2) defining the scattering process with participation of antiplasmino. By using a property of the coefficient function \( K^{(Q)}(\hat{\mathbf{q}}) \):

\[
K^{(Q)}(\mathbf{v}, \chi; -\mathbf{k}, \mathbf{q}) = \gamma^0 \left[ \bar{K}^{(Q)}(\mathbf{v}, \bar{\chi}; -\mathbf{k}, \mathbf{q}) \right]^\dagger
\]

and definition of the (Dirac) conjugate coefficient function \( \bar{K}^{(Q)}(\hat{\mathbf{q}}) \), Eq. (4.9), we find that the modulus squared of expression in braces in matrix element (9.2) can be written in the following form:

\[
\left| \frac{\mathbf{v} \cdot \mathbf{k}}{v \cdot q} \left( \mathbf{v} \cdot (\mathbf{q} - \mathbf{k}) \right) - \bar{\chi} \cdot S(\mathbf{q} - \mathbf{k}) \right|^2.
\]

The advantage of this representation consists in the fact that both the propagator and vertex here have the same signs at momenta as on the left-hand side of Eq. (9.3). This enables us at once to perform calculations fully similar to previous ones and show that finally we lead to the same expression (9.8). Thus the scattering probabilities of plasmino and antiplasmino off the hard test particle are equal among themselves when the hard test particle is in fully unpolarized state, i.e.,

\[
\left. w_{q \to g}^{(-;l)}(\mathbf{p} | \mathbf{q}; \mathbf{k}) \right|_{\text{unpol}} = \left. w_{q \to g}^{(-;l)}(\mathbf{p} | \mathbf{q}; \mathbf{k}) \right|_{\text{unpol}}.
\]

(9.9)

It is evident that this conclusion is valid and for the scattering processes with participation of soft normal quark and soft transverse gluon modes.

Let us consider now a question of determining the constant \( C_\theta \). For this purpose we rewrite kinetic equation (2.11) for \( f = - \) in the following form:

\[
\frac{\partial n_-}{\partial t} + \mathbf{v}_q \cdot \frac{\partial n_-}{\partial \mathbf{x}} = \Gamma_i(\mathbf{n}_q^+, \mathbf{N}_q, \mathbf{f}_P^GQ) - \mathbf{n}_q \left\{ \Gamma_i(\mathbf{n}_q^+, \mathbf{N}_q, \mathbf{f}_P^GQ) + \Gamma_d(\mathbf{n}_q^+, \mathbf{N}_q, \mathbf{f}_P^GQ) \right\}.
\]
The first and second terms on the right-hand side define the spontaneous and induced scattering processes correspondingly. Here we are interested in the second term. This term is precisely one, which necessary for comparing with the right-hand side of the kinetic equation (F.7). Within the framework of approximations listed at the end of Section 2 this term equals

$$-2n_q \left\{ \int \frac{|p|}{4\pi^2} \left[ f_p^Q + f_p^G \right] \int \frac{d\Omega}{4\pi} \int dT_{q\rightarrow g}^{(-\ell)} w_{q\rightarrow g}^{(-\ell)}(v|q;k) N_k + \ldots \right\},$$

(9.10)

where the dots denote the contributions of higher order in the coupling and the contribution containing soft gluon transverse mode. We have somewhat redefined the scattering probability, having explicitly separated dependence on $E = |p|$, setting by definition

$$w_{q\rightarrow g}^{(-\ell)}(p|q;k) = \frac{1}{2E} w_{q\rightarrow g}^{(-\ell)}(v|q;k).$$

For the global equilibrium plasma statistical factor is equal to

$$\int \frac{|p|}{4\pi^2} \left[ f_p^Q + f_p^G \right] = T^2/8.$$

Furthermore, the scattering probability $w_{q\rightarrow g}^{(-\ell)}$ up to kinematic factors equals expression obtained (9.8). From the definitions of the scalar amplitudes $M_{\pm}^{(-\ell)}(q,k)$ it follows that the difference $M_+^{(-\ell)}(q,k) - M_-^{(-\ell)}(q,k)$ is independent of the velocity $v$ of hard parton. Therefore substituting (9.8) into (9.10) and considering definition of the measure of integration $\int dT_{q\rightarrow g}^{(-\ell)}$ (Eq. (2.6) with approximation (2.8)) we find that the last term in (9.8) gives contribution proportional to the integral

$$\int \frac{d\Omega}{4\pi} (v \cdot (\hat{q} \times \hat{k}) - v \cdot (q - k)) \sim (1 \cdot (\hat{1} \times (\hat{q} \times \hat{1}))) = 0.$$

Considering the above-mentioned and comparing the term written out explicitly in (9.10) with the term on the right-hand side of the kinetic equation (F.7), we find that

$$C_\theta = -C_F.$$

From the other hand we have kinetic equation (F.8) determining a change of the plasmon number density $N_k^{l}$. It contains on the right-hand side the same scattering kernel $Q(q,k)$ as the first equation (F.7) with different common color multiplier. For determining a connection of this equation with kinetic equation (2.2), we write the last one for $b = l$ rearranging the terms on the right-hand side:

$$\frac{\partial N_k^{l}}{\partial t} + v_k \cdot \frac{\partial N_k^{l}}{\partial x} = \Gamma_1^{(l)}[n^+,N_k^{l},f_p^Q] + N_k^{l} \left[ \Gamma_1^{(l)}[n^+,N_k^{l},f_p^Q] - \Gamma_d^{(l)}[n^+,N_k^{l},f_p^G] \right].$$

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Here we are also interested only in the last term since it is necessary for comparing with the right-hand side of kinetic equation (F.8). Within the framework of the approximation used for Eq. (9.10) this term is equal to

$$2N_k \left\{ \int \frac{|p| d|p|}{4\pi^2} \left[ f_p^Q + f_p^G \right] \int d\Omega v \int d\Omega v' \int d\Omega v'' \int d\Omega v''' \right\},$$

(9.11)

where

$$\int d\Omega v' = \int \frac{d\Omega}{(2\pi)^2} 2\pi \delta(\omega_k - \omega_q' - v \cdot (k - q)).$$

As the scattering probability $w_{l;q'}(v|k; q)$ here we mean the same expression (9.8). Carrying out reasonings completely similar previous ones and comparing the term written out in (9.11) with the term on the right-hand side of Eq. (F.8), we find in this case

$$C_\theta = -n_f T_F.$$ 

Thus we have a simple rule: the constant $C_\theta$ entering into definition of various probabilities in soft-quark decay and regenerating rates (2.4), (2.5) should be considered equal to $(-C_F)$, and in the probabilities of the soft-gluon decay and regenerating rates in kinetic equation (2.2) it should be set equal to $(-n_f T_F)$. The last case is rather obvious since the factor $n_f$ takes into account the number of possible with respect to flavour channels of interaction of the soft-gluon excitations with soft-quark ones.

One can somewhat extend the previous results if we consider the scattering of soft-quark excitations off hard particle taking place in a partial polarization state. In this case instead of polarization matrix (C.2) we should use more general expression [13]

$$\hat{\rho} = \hat{\rho}(v, \tilde{\zeta}) = \frac{1}{2} (v \cdot \gamma) \left[ 1 + \gamma^5 (\pm \tilde{\zeta}_\parallel + \tilde{\zeta}_\perp \cdot \tilde{\gamma}_\perp) \right],$$

(9.12)

where $\gamma^5 = -i\gamma^0 \gamma^1 \gamma^2 \gamma^3$. The sign (+) concerns to quark and the sign (−) belongs to antiquark. The vector $\tilde{\zeta}$ is a double average value of a spin vector in the frame of rest of the test particle. $\zeta_\parallel$ is the vector component parallel to (for $\zeta_\parallel > 0$) or antiparallel (for $\zeta_\parallel < 0$) to the momentum of particle; $\tilde{\zeta}_\perp = \vec{\tilde{\zeta}} - v (v \cdot \vec{\zeta})$. Now we return to the problem of calculation of the coefficients in Eq. (9.5). The simple calculations show that basic expression (9.8) remains invariable here, additional interference term only appears

$$\mp \zeta_\parallel \frac{(v \cdot n)}{|q||l|} \text{Im}\left\{ \mathcal{M}_+^{(-,l)}(q, k), \mathcal{M}_-^{(-,l)}(q, k) \right\}.$$ 

(9.13)

The expression obtained (9.13) suggests that if the hard test particle is in a partial polarization state, then the scattering probability of plasmino off hard gluon with transformation to plasmon and hard quark (the process $qG \rightarrow gQ$) in general case is not equal to the
scattering probability of annihilation of plasmino with hard antiquark into plasmon and hard gluon (the process \( q \bar{Q} \rightarrow g G \)). Nevertheless, it is easy to see taking into account a definition of the functions \( M^{(\pm)}_{\pm}(v) \) in integrating expression (9.13) over

\[
\int \frac{dQ_\nu}{4\pi} \delta (\omega^-_q - \omega^+_k - v \cdot (q - k)),
\]

this additional contribution vanishes. For this reason in the semiclassical approximation in definition of generalized decay and regenerating rates (2.4), (2.5) we did not make distinction between the scattering processes such as mentioned above and thus we have collected the statistical factors \( f_G p (1 - f_Q p) \) and \( f_{\bar{Q} p} (1 + f_G p) \) together.

If we consider the scattering process with participation of antiplasmino (\( \bar{q} \)), then we result in the fact that the scattering probability \( w^{(-)})_{\bar{q} \rightarrow g} \) will be proportional not to a sum of two expressions (9.8) and (9.13) but to their difference. Therefore instead of equality (9.9) we have more nontrivial statements

\[
w^{(-)}_{\bar{q} \rightarrow g} (v | q; k) = w^{(-)}_{q \rightarrow g} (v | q; k),
\]

\[
w^{(-)}_{\bar{q} \rightarrow g} (v | q; k) = w^{(-)}_{\bar{q} \rightarrow g} (v | q; k).
\]

10 Energy losses of energetic parton

In this section we give general formulae defining energy losses of high-energy parton (quark or gluon) traversing the hot QCD medium induced by scattering off soft-quark excitations. These formulae supplement expression for energy loss generated by the effective current \( \tilde{j}^{Ad} (A^{(D)})(k) \) (Eq. (7.5) in Ref. [6]) and thus enables us to obtain complete (within a framework of semiclassical approximation) expressions for the energy losses of the energetic parton.

As a basic formula for parton energy losses per unit length generated by the effective current \( \tilde{j}^{\psi a}_\mu (v, \chi; Q_0, \theta_0 | k) \) and effective source \( \tilde{n}^{\psi a}_\mu (v, \chi; Q_0, \theta_0 | q) \) we accept the following expression

\[
- \frac{dE}{dx} = \left( - \frac{dE}{dx} \right)_B + \left( - \frac{dE}{dx} \right)_F,
\]

where

\[
\left( - \frac{dE}{dx} \right)_B \equiv - \frac{1}{|v|} \lim_{\tau \to \infty} \frac{(2\pi)^4}{\tau} \int dQ_0 \int k^0 dk^0 dk \left\{ \text{Im} (\hat{\Delta} (k)) \sum_{\xi = 1, 2} \left| \left\langle \tilde{j}^{\psi a}_\mu (v, \chi; Q_0, \theta_0 | k) \cdot e(\hat{k}, \xi) \right| e(\hat{\xi} k, \xi) \right\rangle^2 \right\} + \frac{k^2}{k^0} \text{Im} (\hat{\Delta} (k)) \left\{ \left| \tilde{j}^{\psi a}_\mu (v, \chi; Q_0, \theta_0 | k) \cdot \hat{k} \right|^2 \right\}.
\]
and
\[
\left(-\frac{dE}{dx}\right)_F \equiv \frac{1}{|v|} \lim_{\tau \to \infty} \frac{(2\pi)^4}{\tau} \sum_{\lambda=\pm} \int dQ_0 \int q^0 dq^0 dq \times \left\{ \text{Im}(\Delta_+(q)) \left( |\bar{u}(\hat{q}, \lambda)\bar{\eta}(v, \chi; Q_0, \theta_0| q)|^2 \right) + \text{Im}(\Delta_-(q)) \left( |\bar{v}(\hat{q}, \lambda)\bar{\eta}(v, \chi; Q_0, \theta_0| q)|^2 \right) \right\}.
\]

The right-hand side of Eq. (10.1) has been written in the temporal gauge.

First of all we write the expression for the energy loss associated with the initial Grassmann color source \( \eta_{\theta \dot{\alpha}}(q) = g/(2\pi)^3 \theta^0_\dot{\alpha} \chi_\alpha \delta(v \cdot q) \). Substituting this source into Eq. (10.2) and taking into account
\[
\sum_{\lambda=\pm} |\bar{u}(\hat{q}, \lambda)\chi|^2 = \frac{1}{2E} \text{Sp}[h_+(\hat{q})\varrho(v)] = \frac{1}{2E} (1 - v \cdot \hat{q}),
\]
\[
\sum_{\lambda=\pm} |\bar{v}(\hat{q}, \lambda)\chi|^2 = \frac{1}{2E} \text{Sp}[h_-(\hat{q})\varrho(v)] = \frac{1}{2E} (1 + v \cdot \hat{q}),
\]
we obtain the energy loss to a zeroth-order in the soft fields
\[
\left(-\frac{dE^{(0)}}{dx}\right)_F = \frac{1}{2E} \frac{1}{|v|} \left( \frac{C_{\theta \dot{\alpha}}^2}{2\pi^2} \right) \int q^0 dq^0 dq \text{Im}\left[ \text{Sp}(\varrho(v) S(q)) \right] \delta(v \cdot q) \tag{10.3}
\]
\[
= \frac{1}{2E} \frac{1}{|v|} \left( \frac{C_{\theta \dot{\alpha}}^2}{2\pi^2} \right) \int q^0 dq^0 dq \left\{ (1 - v \cdot \hat{q}) \text{Im}(\Delta_+(q)) + (1 + v \cdot \hat{q}) \text{Im}(\Delta_-(q)) \right\} \delta(v \cdot q),
\]
where \( \alpha_s = g^2/4\pi \). The equation (10.3) defines so-called the polarization losses of energetic parton related to large distance collisions. It supplements the known expression for the polarization losses [7] induced by the ‘elastic’ scattering off hard thermal particles through the exchange of soft virtual gluon. The graphic interpretation of ‘inelastic’ polarization losses (10.3) is presented in Fig. 12. The energy losses (10.3) decrease with the parton energy \( E \) as \( 1/E \). Such suppression results in the fact that polarization losses (10.3) are negligible in comparison with usual ones [7] for asymptotically large parton energy. Nevertheless we can hope that contribution (10.3) is important for intermediate values of \( E \).

The polarization losses give a leading contribution in the coupling constant, provided that the QGP is in thermal equilibrium. However, we can expect that for rather high level of intensity of plasma excitations\(^8\) contributions to energy losses associated with the following terms in expansions of the effective current \( \tilde{j}_\mu^a \) and source \( \tilde{\eta}_a^\mu \) become comparable with the polarization losses. Therefore these contributions to an overall balance of energy losses should be taken into account. Below we write out general expressions for the energy losses to the next-to-leading order.

---

\(^8\)The level of intensity of plasma excitations is defined by values of the soft-quark and soft-gluon occupation numbers.
Figure 12: The energy losses induced by the long-distance collision processes wherein a change of a type of the energetic parton takes place. The dotted lines denote thermal partons absorbing virtual soft-quark excitation.

In present and forthcoming sections we restrict our consideration to the analysis of expression (10.2). The analysis of expression (10.1) will be given in Section 12. Let us take the effective sources \( \tilde{\eta}^{ib}_a \) and \( \tilde{\eta}^{ij}_a \) on the right-hand side of the last equation in (8.2) up to the second order approximation in powers of the soft free fields \( A^{(0)}, \bar{\psi}^{(0)} \) and \( \psi^{(0)} \):

\[
\begin{align*}
\tilde{\eta}^{ib}_a (\nu, \chi \mid q) & \simeq \tilde{\eta}^{(1)ib}_a (\psi^{(0)})(q) + \tilde{\eta}^{(2)ib}_a (A^{(0)}, \psi^{(0)})(q), \\
\tilde{\eta}^{ij}_a (\nu, \chi \mid q) & \simeq \delta^{ij} \eta^{(0)}_{\theta a} (q) + \tilde{\eta}^{(1)ij}_a (A^{(0)})(q) + \left[ \tilde{\eta}^{(2)ij}_a (\bar{\psi}^{(0)}, \psi^{(0)})(q) + \tilde{\eta}^{(2)ij}_a (A^{(0)}, A^{(0)})(q) \right], \\
\eta^{(0)}_{\theta a} (q) & \equiv \frac{g}{(2\pi)^3} \chi_a \delta (v \cdot q).
\end{align*}
\]

Here, in the first line the effective sources \( \tilde{\eta}^{(1)ib}_a (\psi^{(0)}), \tilde{\eta}^{(2)ib}_a (A^{(0)}, \psi^{(0)}) \) are defined by Eqs. (4.3), (5.15) and (6.3), (6.9). The effective source \( \tilde{\eta}^{(1)ij}_a (A^{(0)}) \) in the second line is given by Eqs. (4.7), (4.6) and higher order effective sources \( \tilde{\eta}^{(2)ij}_a (\bar{\psi}^{(0)}, \psi^{(0)}), \tilde{\eta}^{(2)ij}_a (A^{(0)}, A^{(0)}) \) are defined by Eqs. (6.6), (6.7) and (6.10), (B.1), respectively. Substituting (10.4) into (10.2), we find second in importance contribution after (10.3) to expression for the energy losses of the energetic parton. We write down this expression as a sum of two different in structure (and physical meaning) parts:

\[
\left( -\frac{dE^{(1)}}{dx} \right)_F = \left( -\frac{dE^{(1)}}{dx} \right)_{\text{diag}} + \left( -\frac{dE^{(1)}}{dx} \right)_{\text{nondiag}},
\]

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where ‘diagonal’ part equals

\[
\left( \frac{dE^{(1)}}{dx} \right)_{\text{diag}} = - \frac{(2\pi)^3}{|v|} C_2^{(\varsigma)} T_F \left( \frac{\alpha_s}{2\pi^2} \right)^2 \sum_{\lambda, \lambda_1 = \pm} \int q^0 dq_1 dq_1 \tag{10.5}
\]

\[
\times \left\{ \text{Im}(\Delta_+(q)) \left[ \tilde{T}_+(q_1, x) | \tilde{u}_\alpha(\hat{q}, \lambda) K^{(Q)}_{\alpha a} (\chi, \bar{x} | q, -q_1) u_{a_1} (\hat{q}_1, \lambda_1) \right]^2 \\
+ \tilde{T}_-(q_1, x) | \tilde{u}_\alpha(\hat{q}, \lambda) K^{(Q)}_{\alpha a} (\chi, \bar{x} | q, -q_1) v_{a_1} (\hat{q}_1, \lambda_1) \right]^2 \\
+ \left( \Delta_+(q) \rightarrow \Delta_-(q), \quad \tilde{u}(\hat{q}, \lambda) \rightarrow \tilde{v}(\hat{q}, \lambda) \right) \delta(v \cdot (q - q_1))
\]

and in turn ‘nondiagonal’ part equals

\[
\left( \frac{dE^{(1)}}{dx} \right)_{\text{nondiag}} = - \frac{2 (2\pi)^3}{|v|} \left( \frac{\alpha_s}{2\pi^2} \right)^2 \sum_{\lambda, \lambda_1 = \pm} \int q^0 dq_1 dq_1 \left[ \text{Im}(\Delta_+(q)) \right] \tag{10.6}
\]

\[
\times \left\{ \tilde{T}_+(q_1, x) \text{Re} \left[ (\bar{\chi} u(\hat{q}, \lambda)) \left( \tilde{u}_\alpha(\hat{q}, \lambda) u_{a_1} (\hat{q}_1, \lambda_1) K^{(Q)}_{\alpha a} (\chi, \bar{x} | q, q_1; -q_1) u_{a_2} (\hat{q}_1, \lambda_1) \right) \right] \\
+ \tilde{T}_-(q_1, x) \text{Re} \left[ (\bar{\chi} u(\hat{q}, \lambda)) \left( \tilde{u}_\alpha(\hat{q}, \lambda) v_{a_1} (\hat{q}_1, \lambda_1) K^{(Q)}_{\alpha a} (\chi, \bar{x} | q, q_1; -q_1) v_{a_2} (\hat{q}_1, \lambda_1) \right) \right] \\
+ \left( \Delta_+(q) \rightarrow \Delta_-(q), \quad u(\hat{q}, \lambda) \rightarrow v(\hat{q}, \lambda), \quad \tilde{u}(\hat{q}, \lambda) \rightarrow \tilde{v}(\hat{q}, \lambda) \right) \delta(v \cdot q)
\]

\[
+ \frac{(2\pi)^3}{|v|} \left( \frac{\alpha_s}{2\pi^2} \right)^2 \sum_{\lambda, \lambda_1 = \pm} \int q^0 dq_1 dq_1 \left[ \text{Im}(\Delta_+(q)) \right] \\
\times \left\{ I^l(k, x) \sum_{\xi = 1, 2} \text{Re} \left[ (\bar{\chi} u(\hat{q}, \lambda)) \left( \tilde{u}_\alpha(\hat{q}, \lambda) K^{(Q)}_{\alpha a} (\chi, \bar{x} | q; -k, e^{i\hat{q}(\xi)} e^\nu (\hat{k}, \xi)) \right) \right] \\
+ I^l(k, x) \left( \frac{k^2}{k_0^2} \right) \text{Re} \left[ (\bar{\chi} u(\hat{q}, \lambda)) \left( \tilde{u}_\alpha(\hat{q}, \lambda) K^{(Q)}_{\alpha a} (\chi, \bar{x} | q; -k, k \hat{k} k') \right) \right] \\
+ \left( \Delta_+(q) \rightarrow \Delta_-(q), \quad u(\hat{q}, \lambda) \rightarrow v(\hat{q}, \lambda), \quad \tilde{u}(\hat{q}, \lambda) \rightarrow \tilde{v}(\hat{q}, \lambda) \right) \delta(v \cdot q).
\]

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In deriving (10.5), (10.6) we have used the following decompositions of the spectral densities on the right-hand side of Eqs. (8.10) and (8.19)

\[ \Upsilon(q) = h_+(q) \tilde{\Upsilon}_+(q, x) + h_-(q) \tilde{\Upsilon}_-(q, x), \]
\[ I_{\mu\nu}(k) = P_{\mu\nu}(k) I^I(k, x) + \tilde{Q}_{\mu\nu}(k) I^I(k, x), \]

where longitudinal projector \( \tilde{Q}_{\mu\nu}(k) \) in the last equation is determined in the temporal gauge. The dependence on \( x \) takes into account weak non-homogeneity and slow evolution of the medium in time.

The diagonal part (10.5) is not vanishing both for the scattering of the high-energy parton off on-shell and off-shell soft-quark and soft-gluon excitations. In the first case by using the quasiparticle approximation for the spectral densities \( \tilde{\Upsilon}(q_1, x) \), \( I^I(k, x) \) and \( I^I(k, x) \) the integrand in (10.5) can be expressed in terms of the scattering probabilities \( w^{(0)}_{q \rightarrow q, f}(\mathbf{v} | q, q_1), w^{(0)}_{q \rightarrow g}(\mathbf{v} | q, k) \), etc. determined in Section 8. The nondiagonal part (10.6) is different from zero only for the scattering processes by off-shell soft plasma excitations. In the following section we discuss this contribution in full measure.

11 ‘Nondiagonal’ contribution to energy losses

At the beginning we consider the latter contribution in Eq. (10.6) proportional to the gluon spectral densities \( I^I \) and \( I^I \). For this purpose we note first of all that taking into account color decomposition (1.5.19) for the HTL-induced vertex \( \delta \Gamma^{(Q)a_1 a_2}_{\mu_1 \mu_2} \), we can present the coefficient function (B.1) in the following form

\[ K^{(Q)a_1 a_2, ij}_{\mu_1 \mu_2, \alpha}(\mathbf{v}, \mathbf{v}, \chi | q; -k_1, -k_2) = \frac{1}{2} \{ t^{a_1}, t^{a_2} \}^{ij} K^{(S)}_{\mu_1 \mu_2, \alpha}(\mathbf{v}, \mathbf{v}, \chi | q; -k_1, -k_2) \]
\[ + \frac{1}{2} \{ t^{a_1}, t^{a_2} \}^{ij} K^{(A)}_{\mu_1 \mu_2, \alpha}(\mathbf{v}, \mathbf{v}, \chi | q; -k_1, -k_2), \]

where the functions \( K^{(S,A)}_{\mu_1 \mu_2, \alpha} \) possess the properties

\[ K^{(S,A)}_{\mu_1 \mu_2, \alpha}(\mathbf{v}, \mathbf{v}, \chi | q; -k_1, -k_2) = \pm K^{(S,A)}_{\mu_2 \mu_1, \alpha}(\mathbf{v}, \mathbf{v}, \chi | q; -k_2, -k_1). \]

The explicit form of these functions can be easily recovered by (B.1). The coefficient function in the integrand in the last term of (10.6) can be written, in view of (11.1), as

\[ K^{(Q)a_1 a_2, ij}_{\mu_1 \mu_2, \alpha}(\mathbf{v}, \mathbf{v}, \chi | q; -k, -k) = \delta^{ij} C_F K^{(S)}_{\mu_1 \mu_2, \alpha}(\mathbf{v}, \mathbf{v}, \chi | q; -k_1, -k_2) \big|_{k_1 = -k_2 = k}, \]

where

\[ K^{(S)}_{\mu_1 \mu_2, \alpha}(\mathbf{v}, \mathbf{v}, \chi | q; -k_1, -k_2) \big|_{k_1 = -k_2 = k} = - \mathcal{T}^{(S)}_{\mu_1 \alpha \beta}(k; -k; q, -q) \chi \beta', \]

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The function $\mathcal{T}^{(S)}_{\alpha\nu}$ in (11.3) is defined by Eq. (1.5.23). We recall that this function and also $\mathcal{T}^{(A)}_{\alpha\nu}$ enter into the matrix elements of the elastic scattering process of soft-quark excitation off soft-gluon excitation and the process of quark-antiquark annihilation into two soft-gluon excitations with different parity of state of final two–soft-gluon system. The symbols $\mathcal{S}$ and $\mathcal{A}$ belong to states of two gluons being in even and odd states correspondingly (in the c.m.s. of these gluons). The decomposition (11.1) suggests that the scattering process of soft-quark excitation off hard parton with the subsequent radiation of two soft-gluon excitations proceed through two physical independent channels determined by a parity of final two–(soft) gluon system.

Furthermore, the coefficient function $K^{(Q)\,ijij'}_{\alpha\beta}$ in the former contribution on the right-hand side of (10.6) according to definition (6.7) has the following structure:

$$K^{(Q)\,ijij'}_{\alpha\beta}(\chi, \bar{\chi}, q; q_1; -q_1) = \delta^{jj'} C_F \left\{ -M^{(S)}_{\alpha\alpha_1}(-q, -q; q_1, q_1) *S_{\beta\beta'}(q) \chi^\nu \right\} \tag{11.4}$$

where

$$M^{(S)}_{\alpha\alpha_1 \beta \beta'}(-q, -q; q_1, q_1) \equiv *\Gamma^{(Q)\mu}_{\alpha\beta}(q - q_1; q_1, -q) *\mathcal{D}_{\mu\nu}(q - q_1) *\Gamma^{(G)\nu}_{\alpha_1 \beta}(q - q_1; q_1, -q).$$

Here we note also that the function $M^{(S)}_{\alpha\alpha_1 \beta \beta'}$ enters into the matrix element of the elastic scattering process of soft-quark excitation off soft-(anti)quark one (Section 6 in Paper I). The symbol $\mathcal{S}$ means that in nondiagonal contribution (10.6) only ‘symmetric’ part of the function $M^{(S)}_{\alpha\alpha_1 \beta \beta'}$ connected with even parity of final state of soft-quark quasiparticle system survives. For subsequent purposes we present the soft-quark propagator $*S_{\beta\beta'}(q)$ in the first terms on the right-hand side of Eqs. (11.3) and (11.4) in an identical form

$$*S_{\beta\beta'}(q) = \sum_{\lambda' = \pm} \left\{ [u_{\beta}(\bar{q}, \lambda') u_{\beta'}(\bar{q}, \lambda')] *\Delta_{\lambda'}(q) + [v_{\beta}(\bar{q}, \lambda') v_{\beta'}(\bar{q}, \lambda')] *\Delta_{-\lambda'}(q) \right\}. \tag{11.5}$$

Now we substitute Eqs. (11.2) – (11.4) into (10.6) and take into account the equation written just above. We add polarization losses (10.3) to the expression obtained. After some regrouping of the terms we obtain the following final expression:

$$\left( -\frac{dE^{(0)}}{dx} \right)_F + \left( -\frac{dE^{(1)}}{dx} \right)_{\text{nondiag}} = \Lambda_1 + \Lambda_2.$$
Here the function $\Lambda_1$ is

$$
\Lambda_1 = \frac{1}{2E} \frac{1}{|v|} C_\theta \left( \frac{\alpha_s}{2\pi^2} \right) \int q^0 dq \left\{ \text{Im} \left[ \Delta_+ (q) \right] \left[ \sum_{\lambda, \lambda' = \pm} \left( [\bar{u}(\hat{q}, \lambda') \varrho(v) u(\hat{q}, \lambda)] \right) \right] \delta^{\lambda \lambda'} \right. \\
+ \text{Re} \left\{ [\bar{u}(\hat{q}, \lambda') \varrho(v) u(\hat{q}, \lambda)] \Sigma_{++} (q; \lambda, \lambda')^* \Delta_+ (q) \right\} \\
+ \text{Re} \left\{ [\bar{v}(\hat{q}, \lambda') \varrho(v) v(\hat{q}, \lambda)] \Sigma_{+-} (q; \lambda, \lambda')^* \Delta_+ (q) \right\} \left\} \right. \\
+ \text{Im} \left\{ \Delta_- (q) \right\} \left[ \sum_{\lambda, \lambda' = \pm} \left( [\bar{v}(\hat{q}, \lambda') \varrho(v) v(\hat{q}, \lambda)] \right) \delta^{\lambda \lambda'} + \text{Re} \left\{ [\bar{v}(\hat{q}, \lambda') \varrho(v) v(\hat{q}, \lambda)] \Sigma_{+(1)} (q; \lambda, \lambda')^* \Delta_+ (q) \right\} \\
+ \text{Re} \left\{ [\bar{u}(\hat{q}, \lambda') \varrho(v) v(\hat{q}, \lambda)] \Sigma_{+(1)} (q; \lambda, \lambda')^* \Delta_+ (q) \right\} \right\} \delta (v \cdot q),
$$

where in turn we have

$$
\Sigma_{++}^{(1)} (q; \lambda, \lambda') \equiv 2g^2 C_F \sum_{\lambda_1 = \pm} \int dq_1 \left\{ \tilde{T}_+ (q_1, x) \\
\times \left[ \bar{u}_\alpha (\hat{q}, \lambda) \bar{u}_1 (\hat{q}, \lambda_1) M_{\alpha \alpha_1 \beta \beta_1} (-q, q_1) u_{\alpha_2} (\hat{q}, \lambda_1) u_\beta (\hat{q}, \lambda') \right] \\
+ \left( \tilde{T}_+ (q_1, x) \rightarrow \tilde{T}_- (q_1, x), \bar{u}_\alpha (\hat{q}, \lambda) \rightarrow \bar{v}_\alpha (\hat{q}, \lambda_1) \right. \\
\left. \left. \sum_{\lambda_1 = \pm} \int dq_1 \left\{ \tilde{T}_+ (q_1, x) \right. \right. \\
\times \left[ \bar{v}_\alpha (\hat{q}, \lambda) T^{(S)}_{\alpha \beta} (k, -k; q, q_1) u_\beta (\hat{q}, \lambda') e^{i \hat{q}} (\hat{k}, \xi) e^{i \hat{q}} (\hat{k}, \xi) \right] \\
\left. + \left( \tilde{T}_+ (q_1, x) \rightarrow \tilde{T}_+ (q_1, x), e^{i \hat{q}} (\hat{k}, \xi) \rightarrow \sqrt{\frac{k_\perp^2}{k_\perp^2 \hat{k}}}, \right) \right\}.
$$

The function $\Sigma_{++}^{(1)}$ is obtained from $\Sigma_{++}^{(1)}$ by the replacement $u_\beta (\hat{q}, \lambda') \rightarrow v_\beta (\hat{q}, \lambda')$, and the functions $\Sigma_{+-}^{(1)}$, $\Sigma_{-+}^{(1)}$ are accordingly obtained by the replacements

$$
\bar{u}_\alpha (\hat{q}, \lambda) \rightarrow \bar{v}_\alpha (\hat{q}, \lambda), \quad u_\beta (\hat{q}, \lambda') \rightarrow v_\beta (\hat{q}, \lambda')
$$

and $\bar{v}_\alpha (\hat{q}, \lambda) \rightarrow \bar{u}_\alpha (\hat{q}, \lambda)$.

The function $\Lambda_2$ is conveniently represented in the form of a sum of two parts:

$$
\Lambda_2 = \Lambda_2 [\tilde{T}_\pm] + \Lambda_2 [T^{(1)}],
$$

where

$$
\Lambda_2 [\tilde{T}_\pm] \equiv -2 \frac{(2\pi)^3}{|v|} C_\theta C_F \left( \frac{\alpha_s}{2\pi^2} \right)^2 \sum_{\lambda, \lambda_1 = \pm} \int q^0 dq \ dq_1 \left\{ \text{Im} \left[ \Delta_+ (q) \right] \right. \\
\times \left. \left( \tilde{T}_\pm (q_1, x) \right) \text{Re} \left( \beta_1 \left| \Delta_+ (q) \right| \right) \right\},
$$

and

$$
\Lambda_2 [T^{(1)}] \equiv -2 \frac{(2\pi)^3}{|v|} C_\theta C_F \left( \frac{\alpha_s}{2\pi^2} \right)^2 \sum_{\lambda, \lambda_1 = \pm} \int q^0 dq \ dq_1 \left\{ \text{Im} \left[ \Delta_+ (q) \right] \right. \\
\times \left. \left( T^{(1)} (q_1, x) \right) \text{Re} \left( \beta_1 \left| \Delta_+ (q) \right| \right) \right\}.
$$
\[ + \frac{(\bar{u}(\hat{q}, \lambda))(\bar{v}(\hat{q}_1, \lambda_1))}{(v \cdot q_1)} \left[ \bar{u}(\hat{q}, \lambda) * \Gamma^{(Q)\mu}(q - q_1; q_1, -q)u(\hat{q}_1, \lambda_1) * D_{\mu\nu}(q - q_1)v^{\nu} \right] \]

\[ + \bar{\Upsilon} - (q_1, x) \Re \left( \beta_1 \frac{|\bar{\chi}(\hat{q}, \lambda)|^2}{(v \cdot q_1)^2} \right) \]

\[ + \frac{(\bar{\chi}(\hat{q}, \lambda))(\bar{v}(\hat{q}_1, \lambda_1))\chi}{(v \cdot q_1)} \left[ \bar{u}(\hat{q}, \lambda) * \Gamma^{(Q)\mu}(q - q_1; q_1, -q)v(\hat{q}_1, \lambda_1) * D_{\mu\nu}(q - q_1)v^{\nu} \right] \]

\[ + \left( * \Delta_+(q) \to * \Delta_-(q), u(\hat{q}, \lambda) \to v(\hat{q}, \lambda) \right) \delta(v \cdot q) \]

and

\[ \Lambda_2[I_{\mu\nu}] \equiv \frac{(2\pi)^3}{|v|} C_F \left( \frac{\alpha_s}{2\pi^2} \right)^2 \sum_{\lambda = \pm} \int q^0 dq dk \left\{ \Im(*\Delta_+(q)) \right\} \]

\[ \times \left[ I^l(k, x) \sum_{\xi = 1, 2} - \frac{|(e(\hat{k}, \xi) \cdot v)|^2 |\bar{\chi}(\hat{q}, \lambda)|^2}{(v \cdot k)^2} \right] \]

\[ + \frac{1}{(v \cdot k)} \Re \left\{ (\bar{\chi}(\hat{q}, \lambda))(e(\hat{k}, \xi) \cdot v)[\bar{u}(\hat{q}, \lambda) * \Gamma^{(Q)\mu}(k; q - k, -q) * S(q - k)\chi] e^{*s}(\hat{k}, \xi) \right\} \]

\[ - \frac{1}{(v \cdot k)} \Re \left\{ (\bar{\chi}(\hat{q}, \lambda))(e^s(\hat{k}, \xi) \cdot v)[\bar{u}(\hat{q}, \lambda) * \Gamma^{(Q)\mu}(-k; q + k, -q) * S(q + k)\chi] e^s(\hat{k}, \xi) \right\} \]

\[ + \left( I^l(k, x) \to \Gamma^l(k, x), e^s(\hat{k}, \xi) \to \sqrt{\frac{k^2}{k_0^2} k^4} \right) \]

\[ + \left( * \Delta_+(q) \to * \Delta_-(q), u(\hat{q}, \lambda) \to v(\hat{q}, \lambda), \bar{u}(\hat{q}, \lambda) \to \bar{v}(\hat{q}, \lambda) \right) \delta(v \cdot q). \]

First we discuss the function \( \Lambda_1 \). As it will be shown just below the functions \( \Sigma_{++}(q; \lambda, \lambda'), \Sigma_{+-}(q; \lambda, \lambda'), \ldots \) etc. in the integrand of Eq. (11.8) represent nothing but the linear in \( \bar{\Upsilon}_\pm \) and \( I^{l,l} \) corrections to various components of the soft-quark self-energy \( \delta \Sigma(q) \) in the HTL approximation. These corrections take into account a change of dispersion properties of hot QCD plasma induced by the processes of nonlinear interaction of soft-quark and soft-gluon excitations among themselves (see Paper I). For low excited state of the plasma corresponding to level of thermal fluctuations, the corrections \( \Sigma_{++}, \Sigma_{+-}, \ldots \) are suppressed by more power of \( g \) in comparison with \( \delta \Sigma(q) \). However, we can expect that in the limiting case of strong soft fields as far as possible these corrections (and also the corrections of higher powers in \( \bar{\Upsilon}_\pm \) and \( I^{l,l} \)) is the same order in \( g \) as the soft-quark HTL-induced self-energy and therefore consideration of an influence of the nonlinear self-interaction of soft excitations on the energy losses of the energetic parton becomes necessary.

We write out an effective inverse quark propagator \( \hat{S}^{-1}(q) \) that takes into account additional contributions considering the nonlinear effects of self-interaction of soft plasma.
excitations

\[
\hat{S}^{-1}(q) = \hat{S}^{-1}(q) - \Sigma^{(1)}[\tilde{\Sigma}^-, \bar{I}^{t, i}](q) - \Sigma^{(2)}[\tilde{\Sigma}^-, \bar{I}^{t, i}](q) - \ldots ,
\]

where \(\Sigma^{(1)}, \Sigma^{(2)}, \ldots\) are linear, quadratic, and so on corrections in powers of the spectral densities \(\tilde{\Sigma}^-, \bar{I}^{t, i}\). For low excited state when \(\hat{S}^{-1}(q) \gg \Sigma^{(1)} \gg \Sigma^{(2)} \gg \ldots\) from this equation to the first order in \(\tilde{\Sigma}^-, \bar{I}^{t, i}\), we obtain

\[
\hat{S}(q) \approx \hat{S}(q) + \hat{S}(q)\Sigma^{(1)}[\tilde{\Sigma}^-, \bar{I}^{t, i}](q) \hat{S}(q) + \ldots .
\]

Taking into account (11.5) the last equation can be identically rewritten in the following form:

\[
\hat{S}_{\beta\beta'}(q) \approx \sum_{\lambda, \lambda'=\pm} \left[ \delta^{\lambda\lambda'}(u_{\beta}(\tilde{\chi}, \lambda)\tilde{\upsilon}_{\beta'}(\tilde{\chi}, \lambda')) \Delta_{+}(q) \right.
+ \left. \Delta_{+}(q)(u_{\beta}(\tilde{\chi}, \lambda)\tilde{\upsilon}_{\beta'}(\tilde{\chi}, \lambda')) [\tilde{u}(\tilde{\chi}, \lambda)\Sigma^{(1)}(q)u(\tilde{\chi}, \lambda')] \Delta_{+}(q) \right.
+ \left. \Delta_{+}(q)(u_{\beta}(\tilde{\chi}, \lambda)\tilde{\upsilon}_{\beta'}(\tilde{\chi}, \lambda')) [\tilde{u}(\tilde{\chi}, \lambda)\Sigma^{(1)}(q)v(\tilde{\chi}, \lambda')] \Delta_{-}(q) \right.
+ \left. \Delta_{+}(q) \equiv \Delta_{-}(q), u(\tilde{\chi}, \lambda) \equiv v(\tilde{\chi}, \lambda), u(\tilde{\chi}, \lambda') \equiv v(\tilde{\chi}, \lambda'), \ldots \right].
\]

Hereinafter, we designate for brevity \(\Sigma^{(1)}(q) \equiv \Sigma^{(1)}[\tilde{\Sigma}^-, \bar{I}^{t, i}](q)\). We convolve this expression with \(\lambda_{\beta}\chi_{\beta'}\) and take the imaginary part

\[
\text{Im}(\chi \hat{S}(q)\chi) = \frac{1}{2E} \text{Im} \left[ \text{Sp} \left( \varrho(v) \hat{S}(q) \right) \right]
= \frac{1}{2E} \text{Im} \left( \Delta_{+}(q) \right) \sum_{\lambda, \lambda'=\pm} \left\{ \delta^{\lambda\lambda'}[\tilde{u}(\tilde{\chi}, \lambda')\varrho(v)u(\tilde{\chi}, \lambda)] \right.
+ \text{Re} \left( [\tilde{u}(\tilde{\chi}, \lambda')\varrho(v)u(\tilde{\chi}, \lambda)][\tilde{u}(\tilde{\chi}, \lambda)\Sigma^{(1)}(q)u(\tilde{\chi}, \lambda')] \Delta_{+}(q) \right)
+ \text{Re} \left( [\tilde{v}(\tilde{\chi}, \lambda')\varrho(v)u(\tilde{\chi}, \lambda)][\tilde{u}(\tilde{\chi}, \lambda)\Sigma^{(1)}(q)v(\tilde{\chi}, \lambda')] \Delta_{-}(q) \right)
+ \left. \Delta_{+}(q) \equiv \Delta_{-}(q), u(\tilde{\chi}, \lambda) \equiv v(\tilde{\chi}, \lambda), u(\tilde{\chi}, \lambda') \equiv v(\tilde{\chi}, \lambda'), \ldots \right].
\]

Comparing (11.9) with (11.6), we see that the contributions proportional to \(\text{Im}(\Delta_{+}(q))\) in equation (11.9) exactly reproduces the integrand in (11.6) if we identify

\[
\Sigma^{(1)}_{+\pm}(q; \lambda, \lambda') \equiv [\tilde{u}(\tilde{\chi}, \lambda)\Sigma^{(1)}(q)u(\tilde{\chi}, \lambda')], \quad \Sigma^{(1)}_{+\pm}(q; \lambda, \lambda') \equiv [\tilde{u}(\tilde{\chi}, \lambda)\Sigma^{(1)}(q)v(\tilde{\chi}, \lambda')],
\]

eq etc. Thus if in equation (11.9) there is no corrections proportional to \(\text{Re}(\Delta_{\pm}(q))\), then the function \(\Lambda_1\) could be exclusively interpreted as the polarization losses taking into account
to the first approximation a change of dispersion properties of the QCD medium induced by nonlinear interaction of soft-quark and soft-gluon excitations among themselves. It could be effectively presented as a replacement of the HTL-resummed quark propagator \( *S(q) \) in expression (10.3) by the effective one

\[
*S(q) \rightarrow \tilde{S}(q).
\]

Unfortunately, the existence of the terms proportional to Re \( *\Delta(q) \) in (11.9) doesn’t allow us an opportunity to reduce everything to such a simple replacement. The physical meaning of this circumstance is not clear.

Let us discuss the contribution \( \Lambda_2 \) defined by a sum of Eqs. (11.7) and (11.8). Importance and necessity of accounting this contribution will be completely brought to light from its comparison with ‘diagonal’ contribution (10.5). Taking into account the explicit form of the coefficient functions \( K^{(Q)}_{\alpha\alpha}(\chi, \bar{\chi} | q, -q_1) \) and \( K^{(Q)\mu}_{\alpha}(\nu, \bar{\chi} | k, -q) \), we write out Eq. (10.5) once more opening the modulus squared \( |\bar{u}K^{(Q)}_\mu u|^2, |\bar{u}K^{(Q)\mu\nu}e^\epsilon|^2 \) etc. By analogy with Eqs. (11.7), (11.8) the ‘diagonal’ contribution can be also presented as a sum of two parts:

\[
\left(-\frac{dE^{(1)}}{dx}\right)_{\text{diag}} = \Phi_1[\tilde{\Gamma}_\pm] + \Phi_2[I^{l,l}],
\]

where

\[
\Phi_1[\tilde{\Gamma}_\pm] = -\frac{(2\pi)^3}{|\nu|} C_2^{(Q)} T_F \left( \frac{\alpha_s}{2\pi^2} \right)^2 \sum_{\lambda, \lambda_1 = \pm} \int q^0 dq dq_1 \left\{ \text{Im}(\tilde{\Delta}_\pm(q)) \right\} (11.10)
\]

\[
- \alpha^2 \frac{\bar{u}(\bar{\chi}, q_1, \lambda)\bar{u}(\bar{\chi}, q_1, \lambda_1)}{(v \cdot q_1)^2} \left[ \bar{u}(\bar{\chi}, q_1, \lambda) \right] (11.11)
\]

\[
+ \left( \tilde{\Gamma}_+(q_1, k, \lambda) \rightarrow \tilde{\Gamma}_-(q_1, k, \lambda), \bar{u}(\bar{\chi}, q_1, \lambda) \rightarrow v(\bar{\chi}, q_1, \lambda), \tilde{\Gamma}_+(q_1, k, \lambda) \rightarrow \bar{v}(\bar{\chi}, q_1, \lambda) \right) \delta(v \cdot (q - q_1))
\]

and

\[
\Phi_2[I^{l,l}] \equiv -\frac{(2\pi)^3}{|\nu|} C_g C_F \left( \frac{\alpha_s}{2\pi^2} \right)^2 \sum_{\lambda, \lambda_1 = \pm} \int q^0 dq dk \left\{ \text{Im}(\tilde{\Delta}_\pm(q)) \right\} (11.11)
\]

\[
\times \left[ I^{l}(k, x) \sum_{\xi=1,2} \left( \frac{|\nu(k, \xi \cdot v)|^2}{(v \cdot k)^2} \bar{u}(\bar{\chi}, q_1, \lambda) \right)^2 \right]
\]

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\[-2 \frac{1}{(v \cdot k)} \text{Re} \left\{ (\tilde{\chi} u(\hat{q}, \lambda))(e(\hat{k}, \xi) \cdot \nu) \left[ \tilde{u}(\hat{q}, \lambda) \ast \Gamma^{(Q)i}(k; q - k, -q) \ast S(q - k) \chi \right] e^{\ast}(\hat{k}, \xi) \right\} \]
\[+ \left| \left[ \tilde{u}(\hat{q}, \lambda) \ast \Gamma^{(Q)i}(k; q - k, -q) \ast S(q - k) \chi \right] e^{\ast}(\hat{k}, \xi) \right|^2 \]
\[+ \left( I^l(k, x) \rightarrow I^l(k, x), e^{\ast}(\hat{k}, \xi) \rightarrow \sqrt{\frac{k^2}{k_0^2}} \hat{k}^l \right) \]
\[+ \left( \ast \Delta^+_l(q) \rightarrow \ast \Delta^+_l(q), u(\hat{q}, \lambda) \rightarrow v(\hat{q}, \lambda), \tilde{u}(\hat{q}, \lambda) \rightarrow \tilde{v}(\hat{q}, \lambda) \right) \left\{ \delta(v \cdot (k - q)) \right\} \]

The integrands of Eqs. (11.10), (11.11) contain the factors
\[
\frac{1}{(v \cdot q_1)^2}, \quad \frac{1}{(v \cdot q_1)}, \quad \frac{1}{(v \cdot k)^2}, \quad \frac{1}{(v \cdot k)}. \tag{11.12}
\]

If we define the functions $\Phi_1[\tilde{\Upsilon}_\pm]$ and $\Phi_2[I^{l,l}]$ on mass-shell of soft plasma excitations, i.e., we set

\[
\text{Im} \ast \Delta^+_l(q) \simeq \mp \pi Z_\pm(q) \delta(q^0 - \omega^\pm_q) \pm \pi Z_\mp(q) \delta(q^0 + \omega^\mp_q),
\]

\[
\text{Im} \ast \Delta^+_l(k) \simeq -\pi \text{sign}(k^0) \left( \frac{Z_{l,l}(k)}{2\omega^l_k} \right) \left[ \delta(k^0 - \omega^l_k) + \delta(k^0 + \omega^l_k) \right],
\]

and choose the spectral densities in a form of the quasiparticle approximation, Eqs. (8.11) and (8.19), then factors (11.12) will not be singular. This takes place due to the fact that the linear Landau damping process is absent in the QGP. However, for off mass-shell excitations of the medium when a frequency and momentum of plasma excitations approach to the “Cherenkov cone”

\[
(v \cdot q_1) \rightarrow 0, \quad (v \cdot k) \rightarrow 0,
\]

these factors become singular that results in divergence of the integrals on the right-hand sides of Eqs. (11.10), (11.11).

There exist precisely the same singularities in the integrands of the ‘nondiagonal’ contributions (11.7) and (11.8). In order that the expression for energy loss had a finite value it is necessary that these singularities should exactly compensated with those in Eqs. (11.10), (11.11). From a comparison between (11.11) and (11.8) we see that singularities $1/(v \cdot k)^2$, $1/(v \cdot k)$ are exactly compensated in the limit $(v \cdot k) \rightarrow 0$. We will require that a similar reduction should take place for expressions (11.10) and (11.7). This requirement results in the following conditions of cancellation of the singularities

\[
- \alpha^2 C_2^{(C)} T_F = 2 C_\theta C_F \text{Re} \beta_1, \tag{11.13}
\]

\[
\alpha C_2^{(C)} T_F = C_\theta C_F.
\]

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Hence in particular it immediately follows that
\[ \text{Re} \beta_1 = -\frac{1}{2} \alpha, \quad \alpha \neq 0. \]
The last relation suggests that introduction of additional sources and currents (5.14), (5.19), and so on is necessary ingredient of the theory for its self-consistency. Let us analyze the second relation in (11.13). It is natural to require that the constant \( \alpha \), which enters as a multiplier into the first term of the coefficient function (5.15) would independent of a type of the energetic parton just as the second term. In view of this circumstance it is easy to see that there exist the only reasonable choice of pairs of the constants \( C_2^{(c)} \) and \( C_\theta \) for which the second relation in (11.13) will be fulfilled identically: for the values \( \zeta = Q, \bar{Q} \) it is necessary to take \( C_\theta = -C_F \) and for \( \zeta = G \) it should be set \( C_\theta = -n_f T_F, \) i.e.,
\[ \alpha C_F T_F = -C_F^2, \]
\[ \alpha C_A T_F = -n_f T_F C_F. \]
The requirement for independence of the constant \( \alpha \) of a type of the energetic parton results in the relation
\[ n_f T_F = C_A (\equiv N_c), \]  
then
\[ \alpha = -C_F / T_F. \]
The relation (11.14) is fulfilled for \( n_f = 6, T_F = 1/2 \) and \( N_c = 3. \) Unfortunately, in spite of the fact that relation (11.14) seems quite reasonable from the physical point of view, it is true for extremely high temperatures when one can neglect by mass of the heaviest \( t \) quark. Only under these conditions the complete sum of all (‘diagonal’ and ‘nondiagonal’) contributions to energy losses of the energetic parton will have a finite value for scattering on off-shell soft plasma excitations.

12 ‘Nondiagonal’ contribution to energy losses (continuation)

Now we proceed to discussion of expression (10.1). Let us take the effective currents \( \tilde{j}_\mu^{\psi ab} \) and \( \tilde{j}_\mu^{\psi aj} \) on the right-hand side of the first equation in (8.2) in approximation of the second order in powers of the soft free fields
\[ \tilde{j}_\mu^{\psi ab}(v, \chi | k) \simeq \delta^{ab} \tilde{j}_Q^{(0)}(k) + \tilde{j}_\mu^{(2)ab}(\psi^{(0)}, \psi^{(0)})(k), \quad \tilde{j}_Q^{(0)}(k) \equiv \frac{g}{(2\pi)^3} v_\mu \delta(v \cdot k), \]
(12.1)
\[ \tilde{j}_\mu^{\psi aj}(v, \bar{\chi} | k) \simeq \tilde{j}_\mu^{(1)aj}(\psi^{(0)})(k) + \tilde{j}_\mu^{(2)aj}(A^{(0)}, \psi^{(0)})(k), \]
where the current $\tilde{j}_{\mu}^{(2)ab}(\psi^{(0)}, \tilde{\psi}^{(0)})$ is defined by Eqs. (6.1), (6.2) and the currents $\tilde{j}_{\mu}^{(1)ij}(\psi^{(0)})$, $\tilde{j}_{\mu}^{(2)ij}(A^{(0)}, \psi^{(0)})$ are defined by Eqs. (5.7), (5.8) and (6.1), (6.3), respectively. By the following step we consider the coefficient function $K_{\mu, \alpha \beta}^{(G)ab, ij}$ that enters into the definition of the effective current $\tilde{j}_{\mu}^{(2)ab}(\psi^{(0)}, \psi^{(0)})$. By analogy with (11.1) we present it in the following form of a color decomposition:

$$K_{\mu, \alpha \beta}^{(G)ab, ij}(v, \chi, \bar{\chi}|k; q_1, -q_2) = \frac{1}{2} \{ l^a, l^b \}^{ij} K_{\mu, \alpha \beta}^{(S)}(v, \chi, \bar{\chi}|k; q_1, -q_2)$$

$$+ \frac{1}{2} \{ l^a, l^b \}^{ij} K_{\mu, \alpha \beta}^{(A)}(v, \chi, \bar{\chi}|k; q_1, -q_2).$$

Here 'symmetric' part equals

$$K_{\mu, \alpha \beta}^{(S)}(v, \chi, \bar{\chi}|k; q_1, -q_2) = -M_{\mu, \alpha \beta}^{(S)}(-k; k + q_1 - q_2; -q_1, q_2) * D^{\nu'}(k + q_1 - q_2)$$

$$+ 2\sigma \frac{v_\mu \chi_\alpha \bar{\chi}_{\beta}}{(v \cdot q_1)} + \alpha \frac{1}{(v \cdot q_1)} \chi_\alpha \left[ \bar{\chi}_\gamma S_{\gamma \gamma'}(k - q_2) \Gamma_{\mu, \gamma \gamma'}(k; -q_2, -k + q_2) \right]$$

$$- \alpha \frac{1}{(v \cdot q_2)} \bar{\chi}_\beta \left[ \Gamma_{\mu, \gamma \gamma'}(k; q_1, -k - q_1) S_{\gamma \gamma'}(k + q_1) \chi_\gamma \right].$$

The explicit form of the ‘antisymmetric’ part $K_{\mu, \alpha \beta}^{(A)}$ is not needed for our subsequent consideration and therefore we do not give it here. The function $M_{\mu, \alpha \beta}^{(S)}$ (and also $M_{\mu, \alpha \beta}^{(A)}$) was introduced in Paper I (the expressions following after Eq. (I.7.15)). We recall that these functions enter into the matrix elements of two processes: the elastic scattering process of soft-gluon excitation off soft-quark excitation and the process of pair production by fusion of two soft-gluon excitations.

Further, we substitute effective currents (12.1) into (10.1) taking into account Eqs. (12.2), (12.3). We present again the expression obtained as a sum of two parts different in structure:

$$\left( -\frac{dE^{(1)}}{dx} \right)_{B} = \left( -\frac{dE^{(1)}}{dx} \right)_{\text{diag}} + \left( -\frac{dE^{(1)}}{dx} \right)_{\text{nondiag}}.$$

The ‘diagonal’ part is defined by the following expression:

$$\left( -\frac{dE^{(1)}}{dx} \right)_{\text{diag}} = \left( \frac{2\pi}{|v|} \right)^3 C_\theta C_F \left( \frac{\alpha_s}{2\pi^2} \right)^2 \sum_{\lambda=\pm} \int k^0 dk dq \left\{ \text{Im}(\Delta^I(k)) \right\}$$

$$\times \sum_{\xi=1,2} \left[ \left| \tilde{K}^{(G)\xi}(v, \chi|k, -q) u(\bar{q}, \lambda) e^i(\bar{k}, \xi) \right|^2 \tilde{\Upsilon}_+(q, x) + \tilde{K}^{(G)\xi}(v, \chi|k, -q) v(\bar{q}, \lambda) e^i(\bar{k}, \xi) \right|^2 \tilde{\Upsilon}_-(q, x) \right] \delta(v \cdot (k - q))$$

$$+ \left[ \left| \tilde{u}(\bar{q}, \lambda) K^{(G)\xi}(v, \chi|k, q) e^i(\bar{k}, \xi) \right|^2 \tilde{\Upsilon}_+(q, x) + \tilde{K}^{(G)\xi}(v, \chi|k, q) u(\bar{q}, \lambda) \right|^2 \tilde{\Upsilon}_-(q, x)$$

$$= 67$$
Here the function $\tilde{\Lambda}$ where in turn

$$+ |\bar{v}(q, \lambda)K^{(\xi)}(v, \chi| k, q)e^{i(\hat{k}, \xi)|^2 \tilde{Y}_-(q, x)| \delta(v \cdot (k + q))\}

+ \left\{ \tilde{\Delta}^i(k) \rightarrow \tilde{\Delta}^i(k), \quad e^i(\hat{k}, \xi) \rightarrow \sqrt{\frac{k^2}{k_0}} \hat{k}^i \right\}. $$

We add the expression for the polarization losses connected with the initial color current of hard parton $\tilde{J}^{(0)\mu}(k) \equiv Q^{(0)\mu}_a(k)$

$$\left( -\frac{dE^{(0)}}{dx} \right)_B = -\frac{1}{v} C_2^{(\xi)} \left( \frac{\alpha_s}{2\pi^2} \right) \int k^0 dk \Im \left( \tilde{J}^{(0)\mu}_\nu(k) * D_{\mu\nu}(k) \tilde{J}^{(0)\nu}_\mu(k) \right) \quad (12.5)$$

$$\equiv -\frac{1}{v} C_2^{(\xi)} \left( \frac{\alpha_s}{2\pi^2} \right) \int k^0 dk \left\{ \Im(\tilde{\Delta}^i(k)) \sum_{\xi, \xi' = 1, 2} (e^*(\hat{k}, \xi) \cdot v)(e(\hat{k}, \xi') \cdot v) \delta^{\xi'^i} \right.$$

$$+ \left( \frac{k^2}{k_0} \right) \Im(\tilde{\Delta}^i(k))(v \cdot \hat{k})^2 \} \delta(v \cdot k) \right\}$$

to the ‘nondiagonal’ contribution $(-dE^{(1)}/dx)_{\text{nondiag}}$. On the rightmost side of (12.5) we have taken into account that in the temporal gauge the following replacement holds:

$$*D_{\mu\nu}(k) \rightarrow \tilde{\Delta}^i(k) \sum_{\xi = 1, 2} e^i(\hat{k}, \xi)e^i(\hat{k}, \xi) + \tilde{\Delta}^i(k) \left( \frac{k^2}{k_0} \right) \hat{k}^i \hat{k}^i. \quad (12.6)$$

As in previous section we present expression for a sum of (12.5) and ‘nondiagonal’ contribution in the form of a sum of two terms:

$$\left( -\frac{dE^{(0)}}{dx} \right)_B + \left( -\frac{dE^{(1)}}{dx} \right)_{\text{nondiag}} = \tilde{\Lambda}_1 + \tilde{\Lambda}_2. $$

Here the function $\tilde{\Lambda}_1$ is

$$\tilde{\Lambda}_1 \equiv -\frac{1}{v} C_2^{(\xi)} \left( \frac{\alpha_s}{2\pi^2} \right) \int k^0 dk \left\{ \Im(\tilde{\Delta}^i(k)) \left( \sum_{\xi, \xi' = 1, 2} \left[ (e^*(\hat{k}, \xi') \cdot v)(e(\hat{k}, \xi') \cdot v) \right] \right. \right.$$  

$$\left. + \Re \left[ (e^*(\hat{k}, \xi) \cdot v)(e(\hat{k}, \xi') \cdot v)\Pi^{(1)}_{\mu\nu}(k; \xi, \xi') * \tilde{\Delta}^i(k) \right] \right)$$  

$$\left. + \left( \frac{k^2}{k_0} \right) \sum_{\xi = 1, 2} \Re \left[ (e^*(\hat{k}, \xi) \cdot v)(\hat{k} \cdot v)\Pi^{(1)}_{\mu\nu}(k; \xi) * \tilde{\Delta}^i(k) \right] \right\} \delta(v \cdot k), \quad (12.7)$$

where in turn

$$\Pi^{(1)}_{\mu\nu}(k; \xi, \xi') \equiv -2g^2T_F \sum_{\lambda = \pm} \int dq \tilde{Y}_+(q, x)| e^{*i(\hat{k}, \xi)}u(\hat{q}, \lambda)M^{(S)\mu\nu}(-k, -q, q)u(\hat{q}, \lambda)e^{i(\hat{k}, \xi')} \right\}.$$
\[
\Pi^{(1)}_{tt}(k; \xi, \xi') \equiv e^{\ast i(\hat{k}, \xi)}\Pi^{(1)i\ast}(\hat{\Upsilon}_{\pm})(k)e^{\ast i(\hat{k}, \xi')},
\]
\[
\Pi^{(1)}_{tt}(k; \xi) \equiv \sqrt{\frac{k^2}{k_0^2}}(e^{\ast i(\hat{k}, \xi)}\Pi^{(1)i\ast}(\hat{\Upsilon}_{\pm})(k)\hat{k}^{\ast}),
\]
eq

and so on. The functions \(\Pi^{(1)}_{tt}, \Pi^{(1)}_{tt}\) represent linear in the soft-quark spectral densities \(\hat{\Upsilon}_{\pm}\) corrections to various components of the soft-gluon self-energy \(\delta \Pi_{\mu\nu}(k)\) in the HTL approximation. These corrections take into account a change of dispersion properties of medium induced by the processes of nonlinear interaction of soft-gluon and soft-quark excitations among themselves. It is possible to make sure in this if we define an effective gluon propagator \(\ast\mathcal{D}_{\mu\nu}(k)\) that takes into account additional contributions considering nonlinear effects of the self-interaction of soft excitations

\[
\ast\mathcal{D}_{\mu\nu}^{-1}(k) \equiv \mathcal{D}_{\mu\nu}^{-1}(k) - \{\Pi^{(1)}_{\mu\nu}[\tilde{\Upsilon}_{\pm}](k) + \Pi^{(1)}_{\mu\nu}[I^{t,t}](k)\} - \Pi^{(2)}_{\mu\nu}[\tilde{\Upsilon}_{\pm}, I^{t,t}](k) - \ldots .
\]

For low excited state when \(\ast\mathcal{D}_{\mu\nu}^{-1}(k) \gg \Pi^{(1)}_{\mu\nu} \gg \Pi^{(2)}_{\mu\nu} \gg \ldots\) from equation above to the first order in \(\hat{\Upsilon}_{\pm}\) and \(I^{t,t}\), we obtain

\[
\ast\mathcal{D}_{\mu\nu}(k) \simeq \ast\mathcal{D}_{\mu\nu}(k) + \ast\mathcal{D}_{\mu\nu'}(k)\{\Pi^{(1)i\ast}[\hat{\Upsilon}_{\pm}](k) + \Pi^{(1)i\ast}[I^{t,t}](k)\} \ast\mathcal{D}_{\nu'\nu}(k) + \ldots .
\]

The term with \(\Pi^{(1)i\ast}[I^{t,t}](k)\) was considered in the paper [6]. It connected with self-interaction of soft-gluon excitations. Convolving this expression with \(v_{\mu'}v_{\nu'}\), considering (12.6) and taking the imaginary part, we obtain an expression similar in the form to Eq. (11.9). The terms proportional to \(\text{Im}(\ast\Delta^{t,t}(k))\) exactly reproduce the integrand in (12.7) if we identify

\[
\Pi^{(1)}_{tt}(k; \xi, \xi') = e^{\ast i(\hat{k}, \xi)}\Pi^{(1)i\ast}(\hat{\Upsilon}_{\pm})(k)e^{\ast i(\hat{k}, \xi')},
\]

\[
\Pi^{(1)}_{tt}(k; \xi) = \sqrt{\frac{k^2}{k_0^2}}(e^{\ast i(\hat{k}, \xi)}\Pi^{(1)i\ast}(\hat{\Upsilon}_{\pm})(k)\hat{k}^{\ast}),
\]

etc. If the contributions proportional to \(\text{Re}(\ast\Delta^{t,t}(k))\) are absent, then function (12.7) can be exceptionally interpreted as the polarization losses taking into account to the first approximation a change of the dispersion properties of the QCD medium induced by self-interaction of soft excitations. This could be effectively presented as a replacement of the HTL-resummed soft-gluon propagator \(\ast\mathcal{D}_{\mu\nu}(k)\) in the first line of Eq. (12.5) by the effective one

\[
\ast\mathcal{D}_{\mu\nu}(k) \rightarrow \ast\mathcal{D}_{\mu\nu}(k).
\]

Unfortunately, the existence of the terms proportional to \(\text{Re}(\ast\Delta^{t,t}(k))\) gives no way of reducing everything to such a simple replacement.
Furthermore, the function $\tilde{A}_2$ is

$$\tilde{A}_2 \equiv -2\frac{(2\pi)^3}{|v|} C_2^{(s)} T_F \left( \frac{\alpha_s}{2\pi^2} \right) \sum_{\lambda = \pm} \int k^0 dk dq \left\{ \text{Im}(\ast \Delta^i(k)) \right\}$$

$$\times \tilde{Y}_+(q, x) \sum_{\xi = 1, 2} \left( 2\text{Re} \sigma \frac{|(e(\hat{k}, \xi) \cdot v)|^2 |\tilde{u}(\hat{q}, \lambda)\chi|^2}{(v \cdot q)^2} \right)$$

$$+ \alpha \frac{1}{(v \cdot q)} \text{Re} \left[ (e^*(\hat{k}, \xi) \cdot v) (\tilde{u}(\hat{q}, \lambda)\chi) \left( \tilde{\chi} S(k - q) * \Gamma^{(G)i}(k; -q, -k + q) u(\hat{q}, \lambda) e^i(\hat{k}, \xi) \right) \right]$$

$$- \alpha \frac{1}{(v \cdot q)} \text{Re} \left[ (e(\hat{k}, \xi) \cdot v) (\tilde{\chi} u(\hat{q}, \lambda)) \left( \tilde{u}(\hat{q}, \lambda) * \Gamma^{(G)i}(k; q, -k - q) * S(k + q) \chi e^i(\hat{k}, \xi) \right) \right]$$

$$\times \left( \tilde{Y}_+(q, x) \rightarrow \tilde{Y}_-(q, x), u(\hat{q}, \lambda) \rightarrow v(\hat{q}, \lambda), \tilde{u}(\hat{q}, \lambda) \rightarrow \tilde{v}(\hat{q}, \lambda) \right)$$

$$\text{Re} \sigma = \frac{1}{2} \alpha.$$
13 Conclusion

In this paper within the framework of the semiclassical approximation we have presented successive scheme of construction of the effective theory for the processes of interaction of soft and hard quark-gluon plasma excitations for both the Fermi and Bose statistics. In the third part [12], we completed an analysis of dynamics of soft fermion excitations taking into account also radiative processes. Unfortunately, in view of great amount of this work we did not give the concrete analysis of the expressions obtained for energy losses, as it has been made in our early papers [6], [9]. This is expected to be made in separate publication.

As has been shown during all this work, introduction into consideration of the Grassmann color charges for hard particle and color sources with them associated turns out to be rather powerful method in the analysis of dynamics of the soft (anti)quark modes. This allows obtaining almost completely self-consistent and self-sufficient calculation scheme of the effective color currents and sources and the matrix elements for the scattering processes we are interested in. Note that the effective currents and sources derived within the framework of this calculation scheme possess striking symmetry with respect to free soft-gluon and soft-quark fields, the usual and Grassmann charges. This circumstance suggests that there should be (super?)transformations touching by nontrivial way both boson and fermion degrees of freedom of hard and soft excitations of system under consideration transforming effective currents and sources into each other. In view of observable high symmetry with respect to fermion and boson degrees of freedom it is possible to raise the question about a possibility of supersymmetric generalization of the approach suggested in this work. Supersymmetric formulation of the effective theory would allow to look from the more general point of view on the dynamics of interaction processes in QGP and possibly to predict existence of qualitatively new phenomena. Partly, a tool necessary for such generalization was considered in literature. So, for example, the problem of supersymmetrization of classical point particle with spin and isospin has been investigated in detail both within the framework of the local supersymmetric formulation and in the super-space one (see, e.g., Ref. [18] and references therein). Unfortunately, at present there is no a supersymmetric generalization of the most important ingredient of the effective theory, namely, the concept of the hard thermal loops. This in itself can be a subject for separate research.

\footnote{It is curious to note that the relation of (11.14) type (without the multiplier $T_F$) was mentioned for the first time in the paper [17] in the context of analysis of various symmetry relations among quark and gluon decay probabilities.}
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Appendix A

The dynamics of the classical color particle in the external gauge and quark fields (when we neglect by the change in trajectory and spin state of particle) can be described by the following action

$$S = \int_{t_0}^{t} \mathcal{L}(t) \, dt, \quad \mathcal{L}(t) = \mathcal{L}_I(t) + \mathcal{L}_{II}(t),$$

where

$$\mathcal{L}_I(t) = i \bar{\psi} \gamma^\mu A^\mu \dot{\psi} - g \bar{v} A^\mu \dot{\psi} \gamma^\mu \bar{\psi} - g \left\{ \bar{\psi} \gamma^\mu (\bar{\chi} \alpha \psi^\mu) + (\bar{\psi}^\mu \chi^\alpha) \right\} \tag{A.1}$$

and

$$\mathcal{L}_{II}(t) = -g \left[ \bar{\psi} \gamma^\mu (t^a)^{ij} \dot{\psi} \right] \left\{ \dot{\alpha} (\bar{\psi}^k \chi^\alpha)(t^a)^{ks} \Omega^s + \dot{\alpha}^* \Omega^{+s}(t^a)^{sk} (\bar{\chi} \psi^k) \right\} \tag{A.2}$$

$$- g \left[ \beta (t^a)^{ij} \dot{\Omega} \right] \left[ (\bar{\psi}^k \chi^\alpha)(t^a)^{ks} \theta^s \right] + \beta^* \left[ \Omega^{+s}(t^a)^{js} \dot{\theta} \right] [\Omega^{+s}(t^a)^{sk} (\bar{\chi} \psi^k)].$$

In Eq. (A.2) $\dot{\alpha}$ and $\beta$ are (complex) parameters.10 The action $S$ is real and it is invariant under gauge transformation

$$A^a \rightarrow S A^a S^{-1} - (i/g) S \partial_\mu S^{-1}, \quad \psi \rightarrow S \psi, \quad \theta \rightarrow S \theta. \tag{A.3}$$

The soft stochastic gauge and quark fields in the Lagrangian $\mathcal{L}$ are determined on the linear trajectory

$$A^a_\mu \equiv A^a_\mu(t, v t), \quad \psi^i_\alpha \equiv \psi^i_\alpha(t, v t), \quad \bar{\psi}^i_\alpha \equiv \bar{\psi}^i_\alpha(t, v t)$$

and the function $\Omega^i = \Omega^i(t)$ is defined by equation (5.13). Here $t$ is the coordinate time.

The action with Lagrangian (A.1) results in equations of motion (5.8) and Lagrangian (A.2) defines additional interaction terms in these equations. Lagrangian (A.2) (at least the first term) describes current-current interaction in a system, where the first current represents color that of hard component of system, and the second current represents color that of soft component. In view of (A.2) the following terms should be added to the left-hand side of the first equation in (5.8)

$$ig \left( t^a \right)^{ij} \dot{\psi} \left\{ \dot{\alpha} (\bar{\psi}^k \chi^\alpha)(t^a)^{ks} \Omega^s + \dot{\alpha}^* \Omega^{+s}(t^a)^{sk} (\bar{\chi} \psi^k) \right\} \tag{A.4}$$

$$+ ig \left\{ \beta (t^a)^{ij} \dot{\Omega} \right[ (\bar{\psi}^k \chi^\alpha)(t^a)^{ks} \theta^s \right] + \beta^* (t^a)^{ij} (\bar{\chi} \psi^k) \right\] [\Omega^{+s}(t^a)^{sk} (\bar{\chi} \psi^k)].$$

The equation of motion (5.8) with additional terms (A.4) has the following general solution

$$\dot{\psi}(t) = U^{ij}(t, \tau) \theta^i_\alpha - ig \int_{t_0}^{t} U^{ij}(t, \tau) \left( \bar{\bar{\chi}} \psi^i_\alpha(\tau, v \tau) \right) d\tau, \tag{A.5}$$

10We introduce the hat above to distinguish the parameters from those in the additional sources in Section 5.
where we have introduced extended evolution operator

\[ U(t, \tau) = T \exp \left\{ -ig \int_{\tau}^{t} v^\mu A_\mu^a(\tau', v\tau') T^a d\tau' \right\} \tag{A.6} \]

\[ -ig \int_{\tau}^{t} \left[ \bar{\chi}_\alpha(\tau', v\tau') \chi_\alpha(t^a \psi_\alpha^k(\tau', v\tau')) + \bar{\psi}_\alpha(\tau', v\tau') A^a_\mu(\tau', v\tau') \right] T^a d\tau' \]

\[ -ig \int_{\tau}^{t} \left[ \bar{\chi}_\alpha(\tau', v\tau') \chi_\alpha(t^a \psi_\alpha^k(\tau', v\tau')) + \bar{\psi}_\alpha(\tau', v\tau') A^a_\mu(\tau', v\tau') \right] T^a d\tau' \]

Here \( \otimes \) is a sign of the direct production. The evolution operator (A.6) takes into account the effect of rotation of color charge in color space induced by both soft gauge and soft quark stochastic fields. The influence of soft quark field on the rotation should be already taken into account in the scattering processes of a third order in the coupling constant. Under gauge transformation (A.3) this evolution operator transforms by covariant fashion

\[ U(t, \tau) \rightarrow S(t) U(t, \tau) S^{-1}(\tau). \]

Instead of identity (5.10) now we have

\[ U(\tau, t) = \tilde{U}^{ab}(t, \tau) T^a d\tau' \]

where the extended evolution operator in the adjoint representation \( \tilde{U}(t, \tau) \) is

\[ \tilde{U}(t, \tau) = T \exp \left\{ -ig \int_{\tau}^{t} v^\mu A_\mu^a(\tau', v\tau') T^a d\tau' \right\} \]

\[ -ig \int_{\tau}^{t} \left[ \bar{\psi}_\alpha(\tau', v\tau') \chi_\alpha(t^a \psi_\alpha^k(\tau', v\tau')) + \bar{\psi}_\alpha(\tau', v\tau') A^a_\mu(\tau', v\tau') \right] T^a d\tau' \]

\[ -ig \int_{\tau}^{t} \left[ \bar{\psi}_\alpha(\tau', v\tau') \chi_\alpha(t^a \psi_\alpha^k(\tau', v\tau')) + \bar{\psi}_\alpha(\tau', v\tau') A^a_\mu(\tau', v\tau') \right] T^a d\tau' \]

The relation (A.7) is easily proved with the Fierz identity for the matrices \( t^a \)

\[ (t^a)^i_j(t^a)^k_l = -\frac{1}{N_c} (t^a)^i_s(t^a)^k_j + \frac{C_F}{N_c} \delta^i_s \delta^k_j. \]

We add the action for the soft gluon and quark fields to the action determining dynamics of hard test particle

\[ -\frac{1}{4} \int d^4x F_{\mu\nu}^a(x) F^{a\mu\nu}(x) + i \int d^4x \bar{\psi}(x) \gamma_\mu D^\mu(x) \psi(x) + \ldots, \]

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where \( D^\mu(x) \equiv \partial/\partial x_\mu + igA^{\mu}(x)t^a \). Hereafter the dots mean contributions caused by HTL effects. The equation of motion for soft quark field \( \psi \) now has the following form
\[
\i (\gamma_\mu D^\mu(x)\psi(x))_\alpha = g\chi_\alpha \partial^t(t) \delta(x-vt) + g\hat{\alpha} \chi_\alpha (t^a)^{ij} \Omega^j(t)[\partial^{1k}(t)(t^a)^{ks}\theta^s(t)] \delta(x-vt)
\]
\[
+ g\hat{\beta} \chi_\alpha (t^a)^{ij} \theta^j(t) [\partial^{1k}(t)(t^a)^{ks}\Omega^s(t)] \delta(x-vt) + \ldots.
\]

Let us substitute solution (5.9) (here we neglect temporarily by contribution of \( \psi \)-fields to the evolution operator \( U(t, t_0) \)) into the second and third terms on the right-hand side of the last equation. If we require fulfillment of condition
\[
\hat{\alpha} + \hat{\beta} = 0, \quad (A.8)
\]
then these terms result in the form
\[
g \left\{ \hat{\alpha} \chi_\alpha (t^a)^{ij} \Omega^j(t)Q^i(t) + \hat{\alpha} \chi_\alpha (t^a)^{ij} \Omega^j(t)[\Omega^{1k}(t)(t^a)^{ks}\theta^s(t)]
\right. 
\]
\[
+ \hat{\beta} \chi_\alpha (t^a)^{ij} \theta^j(t) [\theta^{1k}(t)(t^a)^{ks}\Omega^s(t)] + \hat{\beta} \chi_\alpha (t^a)^{ij} \theta^j(t) \Xi^a(t) \delta(x-vt)
\right. 
\]
\[
\left. + \right. 
\]
\[
\left. \right. \}
\]
(A.9)

The first, second and last terms here reproduce entered by hands additional sources (5.14), (5.19) and (5.18) correspondingly. The next to last term represents new additional rather remarkable source. Its existence suggests necessity of introduction of one more color structure for description of color charge in the semiclassical approximation, namely,
\[
\Theta^{ij} = [(t^a)^{ii'} \theta^i(t)] [\theta^{jj'}(t)(t^a)^{jj'}], \quad \Theta^{ij} = \Theta^{ji}.
\]

When such a combination in final expressions occurs, it is necessary to identify it as independent function, by analogy to the structure \( Q^a(t) \equiv \theta^{1i}(t^a)^{ij} \theta^j(t) \). In particular, to lowest order in the coupling constant the third term in (A.9) defines the eikonal contribution to the effective source
\[
\frac{\delta^2 n^i_\alpha(q)}{\delta \Theta^0_\beta \delta \psi^{(0)k}_\beta(q_1)} \bigg|_0 = \hat{\beta} \frac{g^2}{(2\pi)^3} \frac{\chi_\alpha \bar{\chi}_\beta}{v \cdot q_1} \delta^{ij} \delta^{sk} \delta(v \cdot (q - q_1)).
\]

The equation of motion for the soft gluon field \( A^a_\mu(x) \) have former form
\[
[D^\mu(x), F_{\mu\nu}(x)] = g\nu_\nu [\partial^{1i}(t)(t^a)^{ij} \theta^j(t)] t^a \delta(x-vt) + \ldots, \quad (A.10)
\]
where as the Grassmann charge \( \bar{\theta}^i(t) \) it is necessary to mean solution (A.5). The contribution of additional interaction terms (A.2) here ‘is hid’ in extended evolution operator (A.6). Some new current structures do not appear. Unfortunately, generalized action suggested in this Appendix not enables one to generate additional current (5.21) containing anticommutator \( \{t^a, t^b\} \). The current on the right-hand side of (A.10) enables us to determine the same expression (5.21), but with commutator \( [t^a, t^b] \) only.

\(^{11}\)The condition (A.8) enables one to cancel out the terms not containing color charges at all.
Appendix B

Here we give complete expression for the coefficient function $K_{\mu_1 \mu_2, \alpha}^{(Q)a_1 a_2, ij}(v, \chi | q; -k_1, -k_2)$ defining the scattering process of soft-quark excitation off hard parton followed by emission of two soft-gluon excitations

$K_{\mu_1 \mu_2, \alpha}^{(Q)a_1 a_2, ij}(v, \chi | q; -k_1, -k_2) \equiv -\delta \Gamma_{\mu_1 \mu_2, \alpha \beta}^{(Q)a_1 a_2, ij}(k_1, k_2; q - k_1 - k_2, -q) \ast S_{\beta \beta'}(q - k_1 - k_2)\chi_{\beta'}$

$+ [t^a_1, t^a_2]^{ij} K_{\mu_1 \alpha}^{(Q)}(v, \chi | k_1 + k_2, -q) \ast \mathcal{D}^\mu \nu(k_1 + k_2) \ast \Gamma_{\mu \mu_1 \mu_2}(k_1 + k_2, -k_1, -k_2)$

$- (t^a_1 t^a_2)^{ij} \ast \Gamma_{\mu_1, \alpha \beta}(k_1; q - k_1, -q) \ast S_{\beta \beta'}(q - k_1) \ast K_{\mu_2, \beta'}^{(Q)}(v, \chi | k_2, -q + k_1)$

$- (t^a_2 t^a_1)^{ij} \ast \Gamma_{\mu_2, \alpha \beta}(k_2; q - k_2, -q) \ast S_{\beta \beta'}(q - k_2) \ast K_{\mu_1, \beta'}^{(Q)}(v, \chi | k_1, -q + k_2)$

$+ (t^a_1 t^a_2)^{ij} \frac{1}{(v \cdot q)(v \cdot k_2)} v_{\mu_1} v_{\mu_2} \chi_{\alpha} + (t^a_2 t^a_1)^{ij} \frac{1}{(v \cdot q)(v \cdot k_1)} v_{\mu_1} v_{\mu_2} \chi_{\alpha}$.

The partial coefficient functions $K_{\mu_1 \alpha}^{(Q)}(v, \chi | k_1 + k_2, -q)$ and so on are defined by expression (4.10). The graphic interpretation of various terms on the right-hand side of Eq. (B.1) is presented in Fig. 13. The dots means graphs of the scattering processes with participation of hard thermal antiquark.

Appendix C

In this Appendix we consider in more detail a structure of the terms in Eq. (7.1) containing the singularity $\delta(v \cdot q')/(v \cdot q')$. In the fist stage of our analysis it is more convenient to use the HTL-resummed quark propagator in the representation suggested by Weldon [14]

$S(q) = \frac{1}{(1 + a) q + b\hat{q}} = \frac{(1 + a) q + b\hat{q}}{D},$  \hspace{1cm} (C.1)

where $\hat{q} \equiv q^\mu \gamma_\mu$, $u$ is a global four-velocity of plasma, $a, b$ are Lorentz-invariant functions, and $D = (1 + a)^2 q^2 + 2(1 + a) b(q \cdot u) + b^2$. Let us define a polarization matrix of hard parton (quark or antiquark) $\varrho = (\varrho_{\alpha \beta})$ such that in pure state it is reduced to a product

$\varrho_{\alpha \beta} = \chi_{\alpha} \bar{\chi}_{\beta}$.

We consider a case of fully unpolarized state. By virtue of the fact that this matrix can depend only on the energy $E$ and the velocity $v$ of hard test particle, we can write out at once its explicit form

$\varrho = \varrho(E, v) = \frac{1}{2E} \varrho(v),$  \hspace{1cm} (C.2)
Figure 13: The scattering processes of soft-quark elementary excitation off the hard test particle followed by emission of two soft-gluon excitations.
where

\[ g(v) = \frac{1}{2} (v \cdot \gamma), \quad v = (1, v). \]

The multiplier \(1/2E\) is chosen for reasons of dimension. In this case only both terms in amplitude (5.15) have the same dimension. Thus in view of the structure of soft-quark propagator (C.1) and polarization matrix (C.2) the following replacement is correct:

\[ \bar{\chi}_a^\ast S_{\alpha\alpha'}(q')\chi_{\alpha'} \Rightarrow \frac{1}{2E} \text{Sp} \left[ g(v) \ast S(q') \right] = \frac{1}{2E} \left[ \frac{2(1 + a)}{D} (v \cdot q') + \frac{2b}{D} (v \cdot u) \right]. \]

Furthermore, by using the property \( \ast S(-q') = -\gamma^0 \ast S^\dagger(q') \gamma^0 \) the first two terms in integrand of (7.1) can be resulted in the following form:

\[ \frac{2}{E} \int dq' \text{Re} \left[ \frac{(1 + a)}{D} \right] \delta(v \cdot q') + \frac{2}{E} \int dq' \frac{\delta(v \cdot q')}{(v \cdot q')} \text{Re} \left[ \frac{b}{D} \right]. \tag{C.3} \]

The expression obtained in particular shows that a contribution of the term proportional to \( g \) in soft-quark propagator (C.1) (containing zero-temperature part) has no a singularity. Now we are coming from the functions \( a \) and \( b \) to the scalar propagators \( \ast \Delta_{\pm}(q')\) according to formulae

\[ \frac{1 + a}{D} = \frac{1}{2|q'|} \left\{ \ast \Delta_{+}(q') - \ast \Delta_{-}(q') \right\}, \quad \frac{b}{D} = \frac{1}{2} \left( 1 - \frac{q'^0}{|q'|} \right) \ast \Delta_{+}(q') + \frac{1}{2} \left( 1 + \frac{q'^0}{|q'|} \right) \ast \Delta_{-}(q'). \]

Taking into account the property of the quark scalar propagators \( \ast \Delta_{-}(q') = -\left( \ast \Delta_{+}(-q') \right)^\ast \), we result (C.3) in final form

\[ \frac{2}{E} \int dq' \delta(v \cdot q') \frac{1}{|q'|} \text{Re} \left[ \ast \Delta_{+}(q') \right] + \frac{2}{E} \int dq' \frac{\delta(v \cdot q')}{(v \cdot q')} \left( 1 - \frac{q'^0}{|q'|} \right) \text{Re} \left[ \ast \Delta_{+}(q') \right]. \]

The close analysis of initial presuppositions shows that the assumption of straightness of a hard parton trajectory is origin of the singularity \( \delta(v \cdot q')/(v \cdot q') \). So the function \( \delta(v \cdot q') \) arises under the Fourier transformation of the function \( \delta(x - vt) \) entering into the initial source of hard particle \( \eta_{0\alpha}(x) \) and the factor \( 1/(v \cdot q') \) appears in calculation of the integral

\[ \int_{t_0}^t \left( \bar{\chi}_\alpha \psi^i_\alpha(\tau, v\tau) \right) d\tau = \int dq' \int_{t_0}^t \left[ e^{-i(v \cdot q')\tau} \left( \bar{\chi}_\alpha \psi^i_\alpha(q') \right) \right] d\tau \tag{C.4} \]

\[ = \int \frac{i dq'}{(v \cdot q')} \left[ e^{-i(v \cdot q')t} - e^{-i(v \cdot q')t_0} \right] \left( \bar{\chi}_\alpha \psi^i_\alpha(q') \right). \]

Here the soft-quark field \( \psi^i_\alpha(x) \) is defined on the linear trajectory \( x = vt \). Thus for regularization of the singularity it is necessary to take into account a change of hard parton
trajectory in ‘collisions’ of hard parton with soft fluctuation field of system (although this is beyond the frameworks of the HTL approximation accepted in this work). If we present a weakly perturbative trajectory in the form \( x(t) = vt + \Delta x(t) \), \( |\Delta x(t)| \ll |v| t \), then instead of (C.4), e.g., we have

\[
\int dq^\prime \left( \bar{\chi} \gamma^\prime \chi \right) \int_{t_0}^{t} d\tau e^{-i(q^\prime \cdot \Delta x(\tau)}.
\]

The integral in \( dt \) here should be exactly calculated, since expansion in a series with respect to \( \Delta x(\tau) \) generates immediately the singularity. An explicit form of the function \( \Delta x(t) \) was obtained in Appendix of Ref. [15] in the linear approximation in the soft-gluon field \( A^{(0)} \). This function is suppressed by the factor \( 1/E \), where \( E \) is energy of hard parton with respect to a change of its color charge \( \Delta Q^a(t) \) obtained in the same approximation.

In the remainder of this Appendix we consider a singular contribution connected with the soft-gluon loop (the second term in Eq. (7.3)). We use an explicit expression for gluon propagator (in covariant gauge)

\[
*D_{\mu \nu}(k) = -P_{\mu \nu}(k) *\Delta(k) - Q_{\mu \nu}(k) *\Delta(k) + \xi D_{\mu \nu}(k) \Delta^0(k),
\]

where Lorentz matrices are defined by

\[
P_{\mu \nu}(k) = g_{\mu \nu} - D_{\mu \nu}(k) - Q_{\mu \nu}(k), \quad Q_{\mu \nu}(k) = \frac{\bar{u}_\mu(k) \bar{u}_\nu(k)}{\bar{u}(k)}, \quad D_{\mu \nu}(k) = \frac{k_\mu k_\nu}{k^2},
\]

and \( \xi \) is a gauge fixing parameter. With the help of (C.5), (C.6) the second term in Eq. (7.3) can be presented in the following form:

\[
\int dk^\prime \delta(v \cdot k^\prime) \left[ *\Delta(k^\prime) - *\Delta(k^\prime) \right] \frac{1}{k^{\prime 2}} \left\{ 2k^{\prime 0} - \frac{k^{\prime 2}}{(v \cdot k^\prime)} \right\} - v^2 \int dk^\prime \delta(v \cdot k^\prime) \frac{1}{(v \cdot k^\prime)} *\Delta(k^\prime).
\]

This expression also contains both finite and singular contributions. The last term vanishes in the case of massless hard particle \( (v^2 = 0) \). In opposite case for heavy particle with the mass \( M \) this term is proportional to \( M^2 / E^2 \). The term with the gauge parameter exactly equals zero. The factor \( \delta(v \cdot k^\prime)/(v \cdot k^\prime) \) arises from expression similar to (C.4), where instead of the function \( \left( \chi_\alpha \psi^\dagger_\alpha(\tau, v \tau) \right) \) it is necessary to mean the function \( \left( v^\mu A^\mu_\alpha(\tau, v \tau) \right) \), i.e., the soft-gluon field defined on the parton linear trajectory.

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12In the general case along with a change of the trajectory it is necessary to take into account a change of a polarization state of hard particle defined by the spinor \( \chi \). The equation describing a change of \( \chi = \chi(t) \) in semiclassical approximation may be in principle obtained from the following reasonings. Let us define the spin 4-vector \( S^\mu \) (or the spin tensor \( S^{\mu \nu} \)) setting \( S^\mu(t) \equiv \bar{\chi}(t) \gamma^\mu \chi(t) \) \( (S^{\mu \nu}(t) \equiv \bar{\chi}(t) \gamma^{\mu \nu} \chi(t)) \). By using known the semiclassical equation for spin motion in external field, one can attempt to restore the equation of motion for the spinor \( \chi(t) \). This is in a certain sense similar to restoring equation of motion for the Grassmann charge \( \theta(t) \) entering into the definition of usual charge \( Q^a(t) = \theta^a(t) \theta^a(t) \) when we know the equation of motion for the charge \( Q^a(t) \) (Eq. (3.2)).
Appendix D

The third order derivative of relation (3.14) with respect to the Grassmann charges $\theta^i_0$, $\theta^0_0$ and free soft-gluon field $A^{(0)}$ has the following form:

$$\frac{\delta^3 J^a_\mu[A, \bar{\psi}, \psi, Q_0, \theta^0_0, \theta^i_0](k)}{\delta \theta^i_0 \delta \theta^0_0 \delta A^{(0)a_1\mu_1}(k_1)} \bigg|_0 = \frac{\delta^3 J^a_\mu[A^{(0)}, \bar{\psi}(0), \psi(0), Q_0, \theta^i_0, \theta^0_0](k)}{\delta \theta^i_0 \delta \theta^0_0 \delta A^{(0)a_1\mu_1}(k_1)} \bigg|_0$$

$$= \frac{g^4}{(2\pi)^6} \left\{ \left[ (\bar{\chi}^* S(-q') \delta \Gamma^{(G)ij}_\mu(k, -k_1; q', -k + k_1 - q') S(k - k_1 + q') \chi) \right] - [t^a, t^{a_1}]^{ij} \Gamma_{\mu\nu\mu_1}(k, -k + k_1, -k_1) \delta^{\mu\nu} (k - k_1) \left( \frac{v_{q'}}{(v \cdot q')} \left[ (\bar{\chi}^* S(q') \chi) - (\bar{\chi}^* S(-q') \chi) \right] + \left[ (\bar{\chi}^* S(k - k_1 - q') \delta \Gamma^{(G)}(k - k_1; -k + k_1 + q', -q') S(q') \chi) \right] \right) \\
+ [t^a, t^{a_1}]^{ij} \left[ K^{(G)}(v, \bar{\chi}) | k, -k - q' \rangle \delta S(k + q') K^{(Q)}(v, \chi) | k_1, -k - q' \rangle \right] \\
- [t^{a_1}, t^{a}]^{ij} \left[ K^{(Q)}(v, \bar{\chi}) | -k_1, k - q' \rangle \delta S(k - q') K^{(G)}(v, \chi) | k, -k + q' \rangle \right] \right\} \delta(v \cdot q') \delta(q') \delta(v \cdot (k - k_1)) \right\}

The graphic interpretation of various terms in this expression is presented in Fig. 14.

Appendix E

In this Appendix we give an explicit form of third order derivative of relation (3.15) with respect to the color charges $Q^a_0, Q^b_0$ and free soft-quark field $\psi^{(0)}$:

$$\frac{\delta^3 J^a_\mu[A, \bar{\psi}, \psi, Q^a_0, \theta^i_0](q)}{\delta Q^a_0 \delta Q^b_0 \delta \psi^{(0)i_1}(q_1)} \bigg|_0 = \frac{\delta^3 J^a_\mu[A^{(0)}, \bar{\psi}(0), \psi(0), Q_0, \theta^i_0](q)}{\delta Q^a_0 \delta Q^b_0 \delta \psi^{(0)i_1}(q_1)} \bigg|_0$$

$$= -\frac{g^4}{(2\pi)^6} \left\{ t^a, t^{b_i} \right\} \left\{ \int \alpha \chi_\alpha \bar{\chi}_{\alpha_1} \frac{1}{(v \cdot q)(v \cdot q_1)} \left\{ v_\mu^* \delta^{\mu\nu}(k') v_{\nu'} \right\} + \frac{1}{2} \delta \Gamma^{(Q,S)}_{\mu\nu,\alpha_1 \alpha_1}(k', q - q_1 - k' \cdot q_1, q, q_1) \delta S_{\beta \nu'}(q - q') \left\{ v_\mu^* \delta^{\mu\nu}(q - q_1 - k') v_{\nu'} \right\} \right\} \delta(v \cdot k') \delta(v \cdot q') \delta(v \cdot (q - q_1))

The diagrammatic interpretation of different terms on the right-hand side is presented in Fig. 15. The vertex function $\delta \Gamma^{(Q,S)}_{\mu\nu,\alpha_1}$ is the ‘symmetric’ part of the HTL-induced vertex.
Figure 14: The soft-quark loop corrections to the elastic scattering of soft gluon elementary excitation off the hard test parton drawn on Figs. 1 and 2 in Ref. [6].

between quark pair and two gluons $\delta \Gamma^{(Q)ab,ii}_\mu$. It is defined by Eq. (I.5.19). To the diagram in parentheses in Fig. 15 there corresponds a term

$$\Gamma^{(Q)}_\mu(q - q_1, -q) \epsilon^{\mu\nu\lambda}(q - q_1, -k', -q + q_1 + k') \epsilon^{\lambda\nu}(q - q_1) \delta \left( v \cdot (q - q_1) \right).$$

By virtue of a property of three-gluon HTL-vertex: $\Gamma^{(Q)}_\mu(k, k_1, k_2) = -\Gamma^{(G)}_\mu(k, k_2, k_1)$, the integrand is odd function under the replacement of integration variable $k' \rightarrow q - q_1 - k'$ and therefore in integrating it is equal to zero.

Appendix F

In Section 9, considering a structure of the scattering probability $w^{(f:b)}_{q-g}$ we have faced with contractions of the HTL-resummed vertex between quark pair and gluon $\Gamma^{(Q)}_\mu(k; l, -q)$

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In high-temperature QCD, under conditions when we can neglect by mass of medium constituents, there is no (linear) Landau damping of on-shell soft excitations Fermi and Bose statistics. Consequence of this fact is equality of the HTL-resummed two-quark–one-gluon vertices $\Gamma^{(Q)}_\mu = \Gamma^{(G)}_\mu \equiv \Gamma$, as it is supposed in this Appendix.
Figure 15: The soft-quark loop corrections to the elastic scattering processes of soft quark elementary excitation off the hard test particle (Figs. 1 and 3). These diagrams are additional to ones depicted in Fig. 10.
(≡ *Γ^i) with soft momentum \( k^i \) of plasmon mode and the polarization vectors \( e^i(\hat{k}, \xi) \) of transverse soft-gluon mode. In this Appendix we give various forms of representations of the vertex function *Γ^i, which have been actively used in Section 9.

From analysis of the explicit expression for the vertex *Γ^i derived by Frenkel and Taylor (Eq. (3.38) in Ref. [10]), it is easy to see that this function can be presented in the form of the expansion

\[
*Γ^i = \gamma^{0} \delta Γ_0^i + (l \cdot \vec{γ}) *Γ^i_{||} + ((n \times l) \cdot \vec{γ}) *Γ^i_{\perp} + (n \cdot \vec{γ}) *Γ^i_{1\perp}.
\]

where \( l \equiv q - k, n \equiv q \times k \) and the ‘scalar’ coefficient functions are defined as

\[
\delta Γ_0^i = \omega_0^2 \int \frac{dΩ_\nu}{4\pi} \frac{v^i}{(v \cdot l + i\epsilon)(v \cdot q)}, \quad \epsilon \to +0,
\]

\[
*Γ^i_{||} = \frac{l^i}{l^2} + \delta Γ^i_{||} \equiv \frac{l^i}{l^2} - \frac{ω_0^2}{l^2} \int \frac{dΩ_\nu}{4π} \frac{v^i (v \cdot l)}{(v \cdot l + i\epsilon)(v \cdot q)},
\]

\[
*Γ^i_{\perp} = \frac{(n \times l)^i}{n^2 l^2} + \delta Γ^i_{\perp} \equiv \frac{(n \times l)^i}{n^2 l^2} - \frac{ω_0^2}{n^2 l^2} \int \frac{dΩ_\nu}{4π} \frac{v^i (v \cdot (n \times l))}{(v \cdot l + i\epsilon)(v \cdot q)}.
\]

\[
*Γ^i_{1\perp} = \frac{n^i}{n^2} + \delta Γ^i_{1\perp} \equiv \frac{n^i}{n^2} - \frac{ω_0^2}{n^2} \int \frac{dΩ_\nu}{4π} \frac{v^i (v \cdot n)}{(v \cdot l + i\epsilon)(v \cdot q)}.
\]

The matrix basis in expansion (F.1) is convenient by virtue of its ‘orthogonality’ in computing traces. The ‘transverse’ vertex functions possess obvious properties:

\[
I^i_{\perp} = n^i_{\perp} = 0,
\]

\[
k^i_{\perp} = q^i_{\perp} = 0.
\]

(F.2)

However, in concrete applications it is considerably more convenient to use another representation of the decomposition (F.1) [2]

\[
*Γ^i = -h_{-}(\hat{l}) *Γ^i_{+} + h_{+}(\hat{l}) *Γ^i_{-} + 2h_{-}(\hat{q}) l^2 |q| *Γ^i_{\perp} + (n \cdot \vec{γ}) *Γ^i_{1\perp},
\]

where the ‘scalar’ vertex functions *Γ^i_{\perp} are connected with the previous ones by relations

\[
*Γ^i_{\perp} \equiv -\delta Γ_0^i \mp l |l| *Γ^i_{||} + \frac{n^2}{|q|} \frac{1}{1 \mp q \cdot l} *Γ^i_{1\perp}.
\]

(F.4)

The expansion (F.3) with the matrix \( h_{-}(\hat{q}) \) in the last but one term is specially adapted to studying of plasmino branch of fermion excitations. In the case of a branch describing normal-particle excitations, instead of (F.3), (F.4) it is necessary to use the following decomposition:

\[
*Γ^i = -h_{-}(\hat{l}) *Γ^i_{+} - h_{+}(\hat{l}) *Γ^i_{-} - 2h_{+}(\hat{q}) l^2 |q| *Γ^i_{\perp} + (n \cdot \vec{γ}) *Γ^i_{1\perp},
\]

(F.5)
where the ‘scalar’ vertex functions $\Gamma_\pm$ are defined as

$$
\Gamma_\pm \equiv -\delta \Gamma_0^t \mp |l| \Gamma_\parallel - \frac{n^2}{|q|} \frac{1}{1 \pm \hat{q} \cdot \hat{l}} \Gamma_\perp.
$$

(F.6)

In the paper [2] the system of kinetic equations describing a change of the plasmino and plasmon number densities generated by induced scattering of plasmino (plasmon) off the hard test particle with transition in plasmon (plasmino) has been also obtained:

$$
\frac{\partial n_-^q}{\partial t} + v_q \cdot \frac{\partial n_-^q}{\partial x} = -g^2 C_F n_-^q \int dk \mathcal{Q}(q, k) N_k^l,
$$

(F.7)

$$
\frac{\partial N_k^l}{\partial t} + v_k \cdot \frac{\partial N_k^l}{\partial x} = g^2 n_f T_F N_k^l \int dq \mathcal{Q}(q, k) n_q^-.\n$$

(F.8)

Here in the latter equation the factor $n_f$ accounts all kinematically accessible quark flavors. The scattering kernel $\mathcal{Q}(q, k)$ has the following structure:

$$
\mathcal{Q}(q, k) = \omega_0^2 \left( \frac{Z_l(k)}{2 \omega_k^l} \right) \frac{k^2}{(\omega_k^l)^2 k^2} \int d\Omega_v \frac{1}{4\pi} \left\{ \rho_+(v; \hat{q}, \hat{l}) w_v^+(q, k) + \rho_-(v; \hat{q}, \hat{l}) w_v^-(q, k) \right\}
$$

$$
\times 2\pi \delta (\omega_q^- - \omega_k^l - v \cdot (q - k)),
$$

(F.9)

where the coefficient functions $\rho_\pm$ are

$$
\rho_\pm(v; \hat{q}, \hat{l}) = 1 + v \cdot \hat{q} \mp (\hat{q} \cdot \hat{l} + v \cdot \hat{l}),
$$

(F.10)

the ‘scalar’ scattering probabilities $w_v^\pm$ are defined by expressions

$$
w_v^\pm(q, k) = \left| \frac{v \cdot k}{v \cdot q} + *\Delta_\pm(l) \Gamma_\pm(k; l - q) \right|^2_{\text{on-shell}},
$$

(F.11)

and $\omega_0^2 = g^2 C_F T^2/8$ is the plasma frequency of the quark sector of plasma excitations.
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