Analysing the thermal state of voltage transformer based on resistive voltage divider

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Abstract. We performed a simulation of the thermal state of a resistive voltage divider based on an equation of heat conductivity with internal sources of heat, solving this equation by using two numerical procedures. We also conducted experimental research regarding transformer thermal state on a laboratory stand. We obtained numerical results of the above heat conductivity equation, taking into account the supply of heat energy from internal sources, and compared the results of our calculations with our experimental data. Transformer thermal state simulation and numerical solution procedures enable us to formulate and resolve the problems of choosing optimal transformer design and operating modes, ensuring maximum measuring accuracy when limiting the thermal state of resistive elements.

1. Introduction

As part of the creation of electronic measuring instruments for Smart Grids it is vital to develop a high-voltage measuring transformer based on a resistive voltage divider. The use of solid state insulation material creates additional heat resistance, hindering heat removal from resistive elements. Warming of resistive elements may lead to changes in their active resistance and therefore reduced accuracy in voltage measurement using a resistive divider. Increased resistance leads to reduced self-heating of resistive elements and thus smaller errors. However, increasing active resistive element resistance reduces total current passing through the voltage divider, so that with significant resistive element resistance measuring accuracy becomes subject to displacement volume current in insulator and environment dielectrics, and also leakage current along the insulator’s surface. Most significantly, the impact of volume (displacement) current increases the phase angle.

Thus, to achieve a compromise solution it will be necessary to increase current passing through the resistive divider, resulting in lower active resistive element resistance, which reduces the current’s capacitive component. To reduce the impact of resistive element self-heating on accuracy, resistive elements with a minimal temperature resistance coefficient (TRC) should be selected.

It should be noted that this approach leads to greater heat release in the resistive elements, with possible overheating and damage. To limit maximum temperatures as required by guidelines for heat testing [1] and use of resistive elements, we selected a maximum positive temperature value for resistive element surfaces of 100º C.

The purpose of this study is to analyse the thermal state of resistive voltage divider transformers, to choose the optimal position of resistive elements and their characteristics, the dielectric material for the support insulator and filler having the appropriate thermo-physical and dielectric properties.

To achieve this goal the following tasks must be fulfilled in the given order:
• conduct experimental studies of non-steady temperature fields inside a transformer;
• develop mathematical models of non-steady heat conductivity in a cylinder with internal sources of heat energy due to heat release from resistive elements;
• identify mathematical models with the help of obtained experimental data;
• estimate transformer thermal state within the framework of the developed model, and compare the obtained data with the results of experimental studies;
• find the optimal position of resistive elements in the developed model.

2. Experimental examining the thermal field and mathematical simulation of resistive voltage divider transformers

For experimental research purposes an experimental prototype resistive divider transformer (see Fig.1, a) was designed and made. The experimental studies were carried out with real time temperature measurements, using a measuring system comprising a National Instruments Compact RIO 9074 controller with NI 9223 enter-release modules, thermo-resistive temperature sensors and a source of direct current for those sensors. The temperature sensors for temperature field measurement were placed in the insulator body. The experimental model was heated by applying voltage to the resistive divider until a steady-state mode was achieved.

The results of the experimental studies are shown in figure 2 as dependence of maximum resistor surface temperature on time, and in figure 3 as a steady-state temperature distribution along the transformer’s radius.

For mathematical simulation of transformer thermal state we used the thermal conductivity equation recorded in the cylindrical coordinates system. The directions of its axes are shown in figure 1, c.

\[
\frac{\partial T}{\partial t} = a \left[ \frac{1}{r \partial r} \left( r \frac{\partial T}{\partial r} \right) \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \varphi^2} + \frac{\partial^2 T}{\partial z^2} \right] + \frac{q_v}{c \cdot \rho},
\]

(1)

where \( T \) – temperature, \( r, \varphi, z \) - coordinates (axis \( z \) coincides with cylinder axis), \( t \) – process time, \( a \) – temperature conductivity coefficient, \( q_v \) – heat source related to heat release in resistive elements, \( c \) – heat capacity, \( \rho \) – density. The parabolic equation (1) is resolved by specifying boundary single-valuedness conditions. Transformer operation is planned on an open distribution device, therefore third genus boundary conditions are accepted on the outer surface, as

\[
q_i = \alpha (T_w - T_a),
\]

(2)

where \( \alpha \) –coefficient of heat release from wall to air, indexes “w” and “a” refer to wall and air temperatures, respectively, \( q_v \) – heat flow via cylinder outer surface.

Equation (1) is solved using two methods: modified finite volume method and finite element method.

For the finite volume method [2, 3] of solving equation (1) we developed an algorithm and software package using Markov’s chain theory mathematical apparatus. The method of calculating the process of heat conductivity with a three-dimensional approach to the task is based on cell methodology. Heat distribution among cells in a three-dimensional space was chosen as the required function. The obtained multidimensional cell array is compressed into a single-dimensional matrix (vector-line) from the same cells. Heat distribution among the cells is represented by vector \( S = \{S_i\} \), where index “i” corresponds to the cell number. The algorithm for calculating the required distribution value \( S \) includes the following stages. First, for each space cell we determine the numbers of the cells with which it may interact, then prepare balance equations to determine energy flows between the cells. Certain energy flows allow us to determine the shares of heat transferred to neighboring cells for the time step. To describe the evolution of the state of the system we summarize heat energy transferred from cell j to cell i:
\[ S_{k+1}^i = \sum_{j=1}^{n} S_j^k p_{ij}, \]

where \( p_{ij} \) - share of heat transferred from cell \( j \) to cell \( i \); index \( k \) corresponds to the time step number.

The second numerical method of solving heat conductivity equation is the finite elements method, using COMSOL Multiphysics software package [4] in axially symmetric two dimensional and three dimensional versions. The results of our calculations of transformer thermal state in non-steady and steady-state modes are given in figures 2, 3.

Figure 2 shows the results of numerically solving equation (1) using two methods involving time changes in the maximum resistive element surface temperature. The same chart shows the results of the experimental studies described above. Temperature changes along the transformer radius in steady-state mode are given in figure 3.

The adequacy of describing experimental data by the results of simulation is assessed according to Fisher’s criterion \( F \) [5]. The value of criterion \( F \) is calculated by the ratio of mean and residual dispersion. The calculated value of criterion \( F \) for the finite elements method with COMSOL Multiphysics is 0.7; for the modified finite volumes method it is 1.9. Comparison of obtained values with table criterion value (\( F_{\text{cr}} = 3.04 \)) for a significance level of \( q=0.05 \) has shown that both obtained values are less than the table figure, which confirms the adequacy of the proposed ways of analysing thermal states in the given time domain.

Comparison of time spent on dynamic daily calculations has shown the advantage of the modified matrix method [2] over the finite elements method using COMSOL Multiphysics: required time for calculating one model by the above methods was 100 s and 1000 s, respectively.

For design calculations and analysis of equipment operating modes in the electric power industry, also for the purpose of optimization calculations it is convenient to use the modified matrix method [2], with subsequent itemization of results using the more accurate and resource-intensive (data hungry) COMSOL Multiphysics package method.
Figure 2. Calculated (1 – COMSOL Multiphysics, 2 – recommended matrix method [2]) and experimental (points) of dependence of maximum resistive element surface temperature on time.

Figure 3. Dependence of steady state radial temperature distribution in transformer’s central part; 1 – results of COMSOL Multiphysics calculations; 2 – results of modified method calculation [2], points representing results of experimental measurements.

3. Search of the most optimal position of resistive elements

Experimental research regarding the thermal field of resistive voltage divider transformer was conducted on a verified mathematical model (figure 4) in order to determine the position of resistive elements where their self-heating is minimal.

The research has shown that resistive divider moving to inner wall of the insulator allows decreasing resistive elements temperature from 81 ºC to 54 ºC, and spiral position of resistive elements reduces the temperature by 3.4 ºC.

According to Russian State Standard GOST 1983-2001 voltage transformer should withstand voltage 1.2 Un (of design) and 1.9 Un for 8 hours. The research of self-heating rate of resistive elements was conducted on the developed mathematical model of transformer with spiral position of resistive elements when applied voltage to the resistive divider was equal to 1.9 Un. The research has shown
resistive elements of spiral divider warm up to 170 °C at voltage 1.9 \( U_n \) and an ambient temperature of 70 °C (the most severe conditions according to Russian State Standard GOST 15150-69).

![Temperature comparison](image)

Figure 4. The comparison of thermal fields of resistive voltage divider transformer with axial (a), boundary (b) and spiral (c) position of resistive elements.

![Temperature graph](image)

Figure 5. Dependence of maximum resistor surface temperature on time: 1 – \( U=U_n \); 2 – \( U=1.9 \, U_n \).

4. Conclusions
The mathematical models of resistive voltage divider transformer developed and verified according to experimental data allow determining the most optimal position of resistive elements and their characteristics in compliance with temperature criterion. According to the temperature criterion resistor surface temperature should not exceed 100 °C (at an ambient temperature of 70 °C in conformity with Russian State Standard GOST 15150-69) at voltage 1.9 \( U_n \). The given methods of
simulation and calculation may be used in the design of measuring resistive voltage divider transformers.

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