The Ubiquitous Throat

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Abstract

We attempt to quantify the widely-held belief that large hierarchies induced by strongly-warped geometries are common in the string theory landscape. To this end, we focus on the arguably best-understood subset of vacua – type IIB Calabi-Yau orientifolds with non-perturbative Kähler stabilization and a SUSY-breaking uplift (the KKLT setup). Within this framework, vacua with a realistically small cosmological constant are expected to come from Calabi-Yaus with a large number of 3-cycles. For appropriate choices of flux numbers, many of these 3-cycles can, in general, shrink to produce near-conifold geometries. Thus, a simple statistical analysis in the spirit of Denef and Douglas allows us to estimate the expected number and length of Klebanov-Strassler throats in the given set of vacua. We find that throats capable of explaining the electroweak hierarchy are expected to be present in a large fraction of the landscape vacua while shorter throats are essentially unavoidable in a statistical sense.
1 Introduction

Since the seminal work of Giddings, Kachru and Polchinski [1] following on from the foundational papers of Refs. [2] and [3], it has become common knowledge that strongly warped regions or throats are a natural feature of type IIB flux compactifications (see [4] for a recent review). Moreover, thanks to the KKLT construction [5], the very same class of models has become the nucleus of the large and growing collection of metastable de-Sitter vacua of string theory (known with a varying degree of rigour) which are generally referred to as the ‘string theory landscape’ [6]. Following the line of thought developed by Douglas and collaborators [7–9], it is then natural to attempt to link the presence of throats quantitatively to the assumption that we live in one of the numerous type IIB orientifold models with 3-form flux. It is the aim of the present paper to understand to which extent throat and multi-throat geometries can be considered a prediction of the type IIB landscape proposal.

To be specific, we will focus on the oldest and arguably simplest situation [5] in which, given a model where all complex structure moduli are stabilized by 3-form flux, the single Kähler modulus is stabilized non-perturbatively by gaugino condensation or D3-brane instantons. We have every reason to expect that our conclusions, which will mainly be related to the distribution of 3-form flux quanta on the various 3-cycles, remain valid if Kähler moduli are stabilized by the interplay of perturbative and nonperturbative physics [10, 11] or even in an entirely perturbative fashion [12, 13]. Similarly, we do not expect our conclusions to be affected by the modifications and extensions of the stabilization mechanism required in situations with more than one Kähler modulus (see e.g. [11]).

Given that all geometric moduli are stabilized in a supersymmetric AdS vacuum as described above, we assume, following KKLT [5], that a small supersymmetry breaking effect, such as the presence of anti-D3-branes in one of the warped regions, uplifts this vacuum to a de-Sitter vacuum with realistic cosmological constant. We choose to focus on this (by now classic) scenario since the metastability of such uplifted vacua is essentially guaranteed in the limit of a parametrically small AdS cosmological constant before the uplift. We will comment on this in more detail below. However, we emphasize again that our decision to be so restrictive in our choice of models is motivated solely be the desire to keep the non-essential parts of our analysis short and simple. We expect that the distribution of throats emerging from our analysis will be similar in a much wider class of flux vacua.

Given the above considerations, we focus on the distribution of throats in type IIB Calabi-Yau orientifolds with a large number of 3-cycles and under the restriction that the total flux superpotential $W_0$ at the SUSY minimum is parametrically small. It is natural to expect that, as a result of the random choice of a large number of independent flux quanta for the various 3-cycles, some of these 3-cycles will automatically carry only a small number of flux. If this occurs for a 3-cycle that can shrink to produce a conifold singularity [14] (which may be the generic situation) and if the flux carried by the dual cycle is not small, a Klebanov-Strassler throat [15, 16] with an exponentially
large hierarchy develops. This is the naive expectation and at the same time our main result: The more detailed analysis described in bulk of the paper confirms that one has to expect large hierarchies of scales \[3, 17, 18\] and multiple throats \[19–21\] in generic orientifold models of the landscape.

At a more technical level, we will replace the above heuristic argument about ‘accidentally’ small 3-cycles by the quantitatively well-known fact that vacua accumulate near conifold points \[8, 22–24\]. If many such conifold points are present in the moduli space of a given Calabi-Yau, the probability of being far away from any of them becomes extremely small. In this sense, the presence of throats becomes a prediction of the given branch of the string theory landscape.

## 2 The relevant set of vacua

Following \[8\], we consider the orientifold limit of an F-theory compactification based on a four-fold with Euler number \(\chi_4\). The 3-form flux on this orientifold can be quantified by a flux vector \(N \in \mathbb{Z}^{2K}\). Its dimension is given by \(2K = 4(h^{2,1} + 1)\), where \(h^{2,1}\) is the number of complex structure moduli.\(^1\) Allowing for a contribution \(N_{D3}\) to the total D3-brane charge from freely moving D3 branes \((N_{D3} > 0)\) or anti-D3-branes \((N_{D3} = -N_{\bar{D}3} < 0)\), the tadpole cancellation condition reads

\[
\frac{\chi_4}{24} = \frac{1}{2} N^T \Sigma N + N_{D3}, \quad \text{where} \quad \Sigma \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.
\]

Assuming that the orientifold planes of the model preserve the same supersymmetry as D3 (rather than anti-D3) branes and focusing on supersymmetric vacua,\(^2\) one requires \(N_{D3} > 0\). The flux vector is then subject to the constraint

\[
\frac{1}{2} N^T \Sigma N \equiv L \leq L_* \equiv \frac{\chi_4}{24}.
\]

The number of SUSY vacua available in this situation was estimated in \[8\] to be

\[
\mathcal{N}_{suy}(L \leq L_*) \sim \frac{L^K}{K!}.
\]

We are, however, interested specifically in realistic vacua (i.e. vacua with small positive cosmological constant) originating from non-perturbative Kähler stabilization combined with an anti-D3-brane uplift based on a small positive \(N_{\bar{D}3}\). To ensure stability, no freely moving D3 branes should be present in this construction. Furthermore, to guarantee perturbative control and a sufficiently long lifetime of the metastable anti-D3-brane

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\(^1\)The index ‘−’ is used since, as an alternative to the F theory construction, one may think of orientifolding a smooth Calabi-Yau 3-fold to obtain a given model. The relevant cycles are those which are odd under the orientifold projection. Note that our \(K\) is \(K/2\) in the notation of \[8\].

\(^2\)To be more precise, these are SUSY-breaking no scale vacua which turn into supersymmetric AdS vacua once non-perturbative Kähler stabilization is taken into account.
configuration at the bottom of the throat, we require $N_{D3} \leq N_{D3, \text{max}}$. An upper bound on $N_{D3, \text{max}}$ is provided by the classically-allowed decay process studied in [25] which limits the range of metastability to $N_{D3} < 0.08 M$ where $M$ is the RR-flux quantum. As in our counting of flux vacua we scan over flux quanta up to $L_*$ we take the parametric dependence $N_{D3, \text{max}} \ll L_*$.

The number of such ‘uplifted’ vacua can be estimated by an appropriate modification of Eq. (3):

$$N_{\text{uplift}} = N_{\text{susy}}(L_* < L \leq L_* + N_{D3, \text{max}}) \sim N_{D3, \text{max}} \frac{L_*^{K-1}}{(K-1)!} \sim \frac{L_*^K}{K!},$$

(4)

where we have Taylor expanded in $N_{D3, \text{max}}$ and dropped irrelevant non-exponential factors in the last expression to simplify the final formula. Thus, $N_{\text{uplift}}$ has the same parametric behaviour as $N_{\text{susy}}$. Clearly, the cosmological constants of these uplifted vacua can have both signs and vary widely in their value. The source for this variation is the flux superpotential

$$W = \int G_3 \wedge \Omega,$$

(5)

which provides a negative contribution $\sim |W_0|^2$ in each vacuum, to be (under- or over-) compensated by the uplift $\sim N_{D3}$. Given that both $\text{Re}W_0$ and $\text{Im}W_0$ depend linearly on the flux vector, one expects a uniform distribution of vacua in the central region of the complex $W_0$ plane. This, in turn, implies a uniform distribution of $|W_0|^2$ on the positive real axis. If we ignore any moderate volume suppression and non-exponential factors depending on $L_*$ and $K$, the maximal size of $W_0$ is string scale (i.e. $\mathcal{O}(1)$ in our units). Thus, the probability that the negative contribution $\sim |W_0|^2$ compensates a fixed positive $V_{\text{uplift}}$ with enough precision to come close to the observed cosmological constant $\Lambda$ is approximately equal to $\epsilon \sim \Lambda \sim 10^{-120}$. ($V_{\text{uplift}}$ should be small enough to allow perturbative control but large enough to avoid any peculiarity that the $W_0$ distribution might have very close to the origin.) We conclude that one needs geometries with $N_{\text{uplift}} \sim 10^{120}$ to have $\mathcal{O}(1)$ probability for a (cosmologically) realistic vacuum to exist.

We are interested in an estimate for the lowest $K$ that is consistent within the present framework. Thus, we choose $L_* = \chi/24$ as large as possible (within the presently known set of Calabi-Yau 4-folds) and estimate $K$ on the basis of

$$\frac{L_*^K}{K!} \sim \frac{1}{\epsilon}.$$

(6)

Ignoring non-exponential factors in Stirling’s formula and assuming that $\log(e L_*/K) \simeq \mathcal{O}(1)$, one finds $K_0 \sim \log(1/\epsilon)$. A better estimate of $K$ follows from replacing $K!$ with $(K_0/\epsilon)^K$ on the lhs of Eq. (6) leading to

$$K \sim \frac{\log(1/\epsilon)}{\log(e L_*/ \log(1/\epsilon))}.$$

(7)

For $L_* \sim 10^4$ (see, e.g., [26]), we find $K \sim 60$, corresponding to $h_{2.1}^2 \sim 30$. 

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It is important to keep in mind that this is just a lower bound and that, most probably, the number of cycles of a ‘typical’ flux compactification with realistic cosmological constant is significantly larger. For example, one might say that with typical Calabi-Yaus having \( h^{2,1} \sim 100\ldots200 \) (see e.g. [27–29])\(^3\) we can very naively expect that \( h^{2,1}_{\rm min} \sim 50\ldots100 \) is typical. Moreover, the scaling of the number of vacua (as implied by Eq. (4) for \( K < L_s \)) suggests that CY’s with the largest possible value of \( K \) are exponentially preferred in that they allow a far greater number of flux vacua. For example, \( \mathcal{N}_{\text{susy}}(K = 200)/\mathcal{N}_{\text{susy}}(K = 60) \sim 10^{270} \) for fixed \( L_s \sim 10^4 \).

Given our ignorance of the model describing our vacuum as well as of the mechanism choosing physical compactification manifolds, we keep \( h^{2,1} \) and \( \chi_4 \) (or equivalently \( K \) and \( L_s \)) as unknown parameters with the order of magnitude given above.

The complex structure moduli spaces of such complicated orientifold models have not been analyzed in detail. It is clear that they will contain various regions where certain 3-cycles blow up or shrink to zero size. We will henceforth ignore the former ‘large complex structure’ regions although they might, in fact, be interesting and important to study. Instead, in this paper we focus on the singularities arising when one or more of the 3-cycles shrink. We want to argue that, in many cases, these singularities are ‘nodes’ or ‘ordinary double points’, which are particularly common singularities of complex varieties. Nodal 3-folds arise naturally in algebraic topology, one of the prominent examples being the various singular limits of the quintic hypersurface in 4d complex projective space. In this specific case, it is known that the ‘generic’ singular space has a single node [30]. From the perspective of the Calabi-Yau 3-fold defined in this way, such a point corresponds to a conifold singularity [14] (which develops as one of the 3-cycles shrinks). Furthermore, a large set of smooth Calabi-Yaus is linked by conifold transitions into a ‘Web’ (including, in particular, the quintic) [31]. In each case, the singular intermediate situation is approached from one side of the transition by the shrinking of a number of 3-cycles with \( S^3 \)-topology (the conifold limit mentioned above) [32]. From this we conclude that the conifold limit is a common (possibly the generic) way in which a 3-cycle of a Calabi-Yau shrinks. More specifically, we assume in the following that an \( \mathcal{O}(1) \) fraction of the possible limits of shrinking 3-cycles of the models under consideration correspond to conifold points. It is an interesting question (which goes beyond the scope of this work) to understand for how many of the known Calabi-Yau orientifolds this assumption holds.

The distribution of flux vacua in the vicinity of such conifold points has been analyzed at least for certain simple examples. It has been found that vacua accumulate near these points. This can be understood intuitively by recalling that the distance \(|z|\) from a conifold point is given by [1]

\[
|z| \sim \exp(-2\pi P/g_s M).
\]

Here \( z \) is the complex structure modulus corresponding to the shrinking 3-cycle while \( M \) and \( P \) are the numbers of flux quanta on the conifold cycle and on its dual. It is then

\(^3\)Extreme cases of \( h^{2,1} \sim 500 \) are known, see, eg. [http://hep.itp.tuwien.ac.at/~kreuzer/CY/]
clear that a smooth distribution of flux quanta can lead to a strong enhancement of the number of vacua with exponentially small $z$.

More specifically, it was shown in [8] that, in a given model with one conifold point at $z = 0$ and a fixed tadpole constraint $L_*$, the fraction of vacua with conifold cycle smaller than $|z|$ decays as

$$\mathcal{N}(z) \sim \frac{1}{\log(1/|z|)}$$

for $z \to 0$. Clearly, this implies an enhancement of the number of vacua very close to the conifold point relative to naive expectations that one might have on the basis of the canonical measure on $\mathbb{C}$.

### 3 Stability issues

Before turning to our main interest, the distribution of throats, we would like to address the stability of the above set of vacua after uplifting.

Naively, one might expect the following situation: We focus on the complex structure moduli $z_i$ and the dilaton modulus $\tau$. A generic flux induced modulus mass is $\mathcal{O}(1)$ in string units (if we ignore any volume suppression $\sim \mathcal{O}$(few)). Making the vacuum value $W_0$ of the superpotential $W$ parametrically small by tuning fluxes, we obtain a parametrically small cosmological constant $\Lambda_{\text{AdS}} \sim -|W_0|^2$. Consider now the scalar potential

$$V = e^K (K_{ab} D_a W D_b W - 3|W|^2)$$

near the supersymmetric point, where $D_a W = 0$ and $W = W_0$ (the index $a$ labels the moduli $\phi_a = (\tau, z_i)$). The scalar mass matrix near this point gets an $\mathcal{O}(1)$ contribution from the first term, which is positive definite since the inverse Kähler metric has this feature. It also gets (potentially negative) contributions

$$\sim -e^K K_{ab} |W_0|^2 \quad \text{and} \quad \sim -e^K (D_a D_b W) \bar{W}_0$$

from the second term (where we again used the fact that $D_a W$ vanishes in the vacuum). Thus, in the generic case, all masses should be positive and $\mathcal{O}(1)$ if $W_0$ is parametrically small. (This can also be argued by appealing to the known stability of supersymmetric vacua in combination with the Breitenlohner-Freedman bound [33]: If all mass squares are $\mathcal{O}(1)$ and $\Lambda_{\text{AdS}}$ is small, all mass squares must be positive.)

However, in the conifold limit of the one-modulus case analyzed in [8], this naive expectation was found to be violated and tachyonic directions (implying the danger of physical instabilities after uplifting) were found to be generically present. At the same time, it was argued that this problem will not persist in models with more than one complex structure modulus. We agree with this expectation and we would like to supply an explicit argument in its favour:
Ignoring the $O(1)$ prefactor $e^K$ and using the parametric smallness of $W_0$, the second-order expression for the supergravity scalar potential, Eq. \((10)\), takes the form

\[ V \sim \delta \phi_a W_{ab} K^{bc} \overline{W}_{cd} \delta \phi_d \]  

near the vacuum. Here $\delta \phi_a$ is the deviation of $\tau$ (for $a = 1$) or any of the complex structure moduli (for $a > 1$) from its vacuum value. This is, of course, just the familiar rigid-SUSY expression. It is obvious from Eq. \((12)\) that a parametrically small eigenvalue of $W_{ab}$ leads to a light scalar field which, taking into account the full supergravity expression (including non-vanishing $W_0$) and the uplift, entails the risk of a tachyonic direction. Indeed, such a small eigenvalue arises in the one-complex-structure-modulus case near the conifold point: The explicit form of the superpotential

\[ W = A(z) + \tau B(z), \]  

implies $W_{11} = 0$ and the singular behaviour of the integral over the dual conifold cycle,

\[ \int_B \Omega = \frac{z}{2\pi i} \log(z) + \text{holomorphic}, \]  

implies $W_{22} \sim 1/z$. The 2×2 matrix $W_{ab}$ then develops a parametrically small eigenvalue by the usual see-saw mechanism, which makes the tachyonic direction found in \([8]\) possible.

The situation changes drastically in the case of two or more complex structure moduli. Let $W_{ab}$ be an $n \times n$ matrix and let $a = n$ correspond to the conifold modulus $z$ of Eq. \((14)\). While it is still true that $W_{11} = 0$ and $W_{nn}$ is parametrically large, this no longer implies the existence of a small eigenvalue. This can be seen by considering the characteristic equation

\[ \det(W_{ab} - \lambda \delta_{ab}) = 0. \]  

Clearly, the largeness of $W_{nn}$ implies the existence of a large eigenvalue $\lambda \approx W_{nn}$. Any further eigenvalue, however, has to solve the equation

\[ \det \left( W_{ab} - \lambda \delta_{ab} \big|_{\{a,b=1...n-1\}} \right) = 0, \]  

approximately. The solutions are simply the eigenvalues of an $(n - 1) \times (n - 1)$ matrix with vanishing upper-left element, which is otherwise generic. For $n > 1$, neither of these eigenvalues is generically small. Thus, we have no reason to expect that the problem of a tachyonic direction observed in the case of a single complex structure modulus will persist.

Independently of the above, it may also be useful to observe that the special feature $W_{11} = 0$ of the KKLT construction is not generic and can easily be avoided, e.g., by including gaugino condensation on stacks of D3 branes at singularities.\(^4\)

Our main conclusion for the following is that the difficulties observed in the one-modulus case do not represent an argument against the existence of many uplifted near-conifold vacua of fluxed multi-modulus Calabi-Yaus.

\(^4\)Related discussions of the stability of the KKLT construction can be found, e.g., in [34, 35].
4 The distribution of throats

We now turn to our main point, which is the interplay between the expected large number of 3-cycles (and hence of potential conifold singularities) and the enhancement of the number of vacua in the vicinity of each of those singularities.

Consider the complex structure moduli space of an orientifold model with $\sim K$ 3-cycles, as discussed in the previous section. We assume, motivated by the example of the quintic and the 'Web of Calabi-Yaus', that an $O(1)$ fraction of these cycles produce, when they shrink, conifold singularities. Furthermore, we excise all large complex structure regions, ending up with a compact moduli space of complex dimension $K/2$. The various conifold points are described by $O(K)$ subspaces of complex co-dimension one which, in general, intersect each other.

Let us first focus on one of these conifold points (more properly: on one of the subspaces along which a certain conical singularity persists) and parameterize the moduli space such that the coordinate $z_i$ characterizes the shrinking cycle. Making use of the distribution of vacua near a conifold singularity implied by Eq. (9), we expect that a randomly chosen flux vacuum will have probability

$$ p_i(|z_i|) \simeq \frac{1}{c_i \log(1/|z_i|)} $$

(17)

to be less than $|z_i|$ away from the conifold point under consideration.

The real constant $c_i$ is related to the detailed distribution of vacua away from the conifold point and to the ambiguities which arise in excluding the large complex structure regions. To see this, assume for simplicity that we have excised the region $|z_i| > |z_{i,\text{max}}|$. Clearly, away from the small-$z_i$ region $p_i$ has some more complicated functional form (not explicitly known in general) and it has to satisfy the normalization condition

$$ p_i(|z_i|) \to 1 \quad \text{for} \quad |z_i| \to |z_{i,\text{max}}|. $$

(18)

All that we can infer from Eq. (8) is that $p_i$ is proportional to $1/\log(1/|z_i|)$ at small $z_i$; the normalization is inextricably linked to the behaviour of $p_i$ at $|z_i| \sim |z_{i,\text{max}}| \sim O(1)$. What is worse, $c_i$ is in general a function of the other complex structure moduli, $c_i = c_i(z_1, \ldots, z_{i-1}, z_{i+1}, \ldots, z_{K/2})$ (which we ignored in the above), thereby making a detailed analysis of the full probability distribution highly non-trivial. Thus, all that we can do at the moment is to assume that the various $c_i$ do not vary too rapidly and are not parametrically large or small (for which there is no obvious reason). We will parameterize our ignorance assigning a universal unknown value of the order of one to all these coefficients, $c_i = c$.

Let us now recall that, if $z_i$ is stabilized near zero, a strongly warped region or throat with a hierarchy of mass scales

$$ h_i \sim |z_i|^{-1/3} $$

(19)

between the Klebanov-Strassler region (IR end) and the Calabi-Yau region (UV end) develops [1] (see also [36]). We conclude from the above that the probability for finding
a throat with a hierarchy larger than $h_i$ is

$$p_i(h_i) \simeq \frac{1}{3c_i \log h_i}. \quad (20)$$

Intuitively, this characterizes the probability for being, in the given moduli space, within a slice of a certain thickness that surrounds the hypersurface defined by $z_i = 0$.

Assuming that these probabilities are uncorrelated for the various conifold hypersurfaces (i.e. for the various $z_i$, $i = 1 \ldots K$), we can estimate the probability for finding precisely $n$ throats with hierarchy larger than $h_*$. It is given by the probability for being inside $n$ of the $K$ slices and outside the remaining $K - n$ slices, multiplied by a combinatorial factor for choosing inside which slices to be:

$$p(n, h > h_*|K) \sim \binom{K}{n} p^n (1 - p)^{K-n} \quad \text{with} \quad p \equiv \frac{1}{3c \log h_*}. \quad (21)$$

The fact that this ‘multi-throat probability’ is given simply by a binomial distribution with parameters $K$ and $p$ represents one of our main results (or, given the various assumptions above, our main conjecture). Many interesting and potentially phenomenologically relevant questions can now be addressed.

For example, given a certain hierarchy factor $h_*$, we can inquire about the expected number of throats with a larger hierarchy. It is given by the well-known mean of the binomial distribution,

$$\bar{n}(h > h_*|K) = \sum_{n=0}^{K} n \binom{K}{n} p^n (1 - p)^{K-n} = K p = \frac{K}{3c \log h_*}. \quad (22)$$

The crucial but certainly not unexpected point here is that $\bar{n}$ goes to zero very slowly as $h_*$ grows. The variance of $n$, again a familiar result, is

$$\text{var}(n) = \sum_{n=0}^{K} (n - \bar{n})^2 \binom{K}{n} p^n (1 - p)^{K-n} = K p (1 - p), \quad (23)$$

which is very close to $\bar{n}$ for $p \ll 1$. Thus, the expected number of throats is

$$\bar{n} \pm \sqrt{\bar{n}}, \quad (24)$$

with $\bar{n}$ as given in Eq. (22).

For a given $K$ the probability that at least one throat has hierarchy exceeding some specified $h_*$ is given by

$$P(h > h_*|K) = \left(1 - \frac{1}{3c \log h_*}\right)^K \left[\left(1 + \frac{1}{3c \log h_* - 1}\right)^K - 1\right]. \quad (25)$$

Figure 1 shows this function against $\log h_*$ for $c = 1$ and $K$ taking the values 60 and 200. It is noteworthy that there is a very slow decrease of the probability with throat length,
and that at 50% likelihood there exist throats of hierarchy greater than \( \exp(28) \sim 10^{12} \), and \( \exp(95) \sim 10^{41} \) as \( K \) varies from 60 to 200. Not surprisingly, Eq. (25) coincides with Eq. (22) if \( K/3c \log(h_*) \ll 1 \).

Another interesting quantity is the hierarchy \( h_1 \) of the longest expected throat. A simply estimate of this quantity is provided by solving

\[
\pi(h > h_1|K) \sim 1
\]

for \( h_1 \). The result is

\[
\log h_1 \sim \frac{K}{3c}.
\] (27)

Alternatively, we can ask for which \( h_1 \) the one-throat-probability is maximized,

\[
\frac{d}{dh_1} p(1, h > h_1|K) = 0 .
\] (28)

The result is consistent with Eq. (27). Yet another way to state the same problem is to ask, at fixed \( h_1 \), for the value of \( K \) which gives the maximal value for \( p(1, h > h_1|K) \). Again, the resulting relation of \( h_1 \) and \( K \) is approximately that of Eq. (27).

Furthermore, a very simple but important quantity is the probability of having no throat with a hierarchy larger than \( h_* \),

\[
p(0, h > h_*|K) \sim (1-p)^K \simeq \exp\left(-\frac{K}{3c \log h_*}\right).
\] (29)

As expected, this is a very small number for large \( K \) and not too large hierarchies. We consider this together with the expected number of throats, Eq. (22), the expected
hierarchy of the longest throat, Eq. (27), and the one-throat-probability of Fig. 1 to be the main results of this section.

Finally, using Eq. (25) it might also be possible to gain information on $K$ independent of the fine-tuning of the cosmological constant by conditioning on the existence of an electro-weak throat with IR scale $\sim \text{TeV}$. Using Bayes’ theorem the conditional probability distribution for $K$ given that there exists at least one throat with $h \geq h_{EW}$ is

$$ P(K | n(h_{EW}) \geq 1) = \frac{P(h > h_{EW} | K)P(K)}{\sum_{K'} P(h > h_{EW} | K')P(K')} .$$

If we conservatively assume a flat prior distribution for $K$, $P(K) = 1/K_{\text{max}}$ and take as an illustrative example $K_{\text{max}} = 200$ and $c = 1$, then a numerical evaluation of Eq. (30) leads to an a posteriori mean $\bar{K} \sim 124$.

5 Possible phenomenological implications

To discuss possible phenomenological implications, we have to quantify the expected hierarchies $h$, which depend crucially on the number of cycles $K$. Since, at the fundamental level, we are ignorant about $K$ and, moreover, $K$ appears in the combination $K/3c$ (with an unknown $O(1)$-constant $c$), we take the following pragmatic approach:

We consider two scenarios, one conservative and one more favourable: In the conservative case, we choose $K = 60$ (roughly the minimal value consistent with fine-tuning $\Lambda$) and $c = 3$, such that the relevant combination of these two parameters takes the low value $K/3c \simeq 7$. In the favourable case, we choose $K = 200$ (consistent with typical Calabi-Yau values, maybe somewhat at the high side, but not extreme). Together with $c = 1/3$ this gives the high value $K/3c \simeq 200$.

In the conservative case, Eq. (27) implies that the longest throat typically has a hierarchy $\sim 10^3$. This clearly also means that, specifying a minimal hierarchy $10^3$, we expect about one throat with a hierarchy above that value. We can also infer that we have to expect about 3 throats with hierarchy 10 or larger. Even though these numbers are not very impressive, they clearly imply that dynamically generated scales of $\sim 10^{-3}M_P$ are natural in the present branch of the landscape. The above short throats can play an important role in inflation or simply to ensure a small (and hence perturbatively controlled) anti-D3-brane uplift. Thus, even though no spectacular low-energy effects can be predicted in this conservative setting, moderate throats are indeed ‘ubiquitous’.

What is maybe more impressive is the small statistical price that one pays for having a moderately long throat. For example, the expectation value for the number of throats with hierarchy above $10^6$ is 0.5. In other words, low scale SUSY in the KKLT setting (see e.g. [37]) is perfectly plausible and does not require any extra fine-tuning. Even more, demanding a hierarchy of $10^{13}$ or higher, one still finds an expectation value for the throat number of approximately 0.23. In other words, generating the electroweak hierarchy is also very plausible since about 1 in 4 vacua have a sufficiently long throat. However, we can clearly not claim that throats of this length are unavoidable.
We now turn to the case $K/3c = 200$, where things look very different indeed. The longest expected throat produces a huge hierarchy $\sim 10^{80}$. Thus, we expect almost conformal field theories with very low IR cutoff (which are presumably only gravitationally coupled to standard model matter) to be abundant. Specifically, not having a throat with hierarchy $10^{20}$ or larger (corresponding to an IR scale of meV) has probability of about 5% (cf. Eq. (29)). In other words, hidden sectors with dynamical scales $\sim$meV or below are a prediction of this setting. Clearly, the above can have very important cosmological implications as far as dark matter or dark radiation are concerned. Just to give one more numerical implication of the formulae of the last section: The expected number of throats with a hierarchy larger than $M_P/M_{EW} \sim 10^{14}$ is $\bar{n} \sim 6$. Thus, several electroweak-scale hidden sectors are a natural occurrence. The phenomenological and cosmological implications of this scenario clearly depend very strongly on whether ‘we’ are in the throat or on the Calabi-Yau, where inflation took place and how strongly throat sectors are coupled to each other and to light fields localized in the UV. Away from the cosmological context there are two outstanding possibilities for signatures of long throats with IR scale at or below the weak scale which have been partially investigated: invisible Higgs decays to hidden sector particles [38], and kinetic mixing of hypercharge with hidden-sector U(1)’s [39]. All we can say at present is that the various scenarios of this type studied in the literature appear to be everything else but exotic.

6 Conclusions

Based on a number of assumptions, we have quantified the expectation that throats are common in the type IIB landscape. The crucial starting point is the large number of 3-cycles which the compact space is expected to have. This can be quantified in two ways: conservatively, by taking the minimal number which allows for the fine-tuning of $\Lambda$, or moreoptimistically, by taking a number which is typical for the more complicated Calabi-Yau manifolds. Given this large number of cycles (all of which generically carry a certain discrete flux number), one has to expect that by pure chance the flux on some of these cycles will be relatively small. Those cycles are stabilized at small size, which generically leads to the development of a throat and a large hierarchy of scales. We have made this last argument more precise on the basis of the known behaviour of the density of flux vacua near conifold points.

Our main technical results are simple formulae for the expectation value of the number of throats with a certain hierarchy and for the probability of having no throat with a hierarchy larger than some given value. The numerical predictions depend on the uncertain total number of 3-cycles mentioned above and on the details of the flux distribution away from the conifold points. Even with conservative assumptions about both of these unknown quantities, short throats (with hierarchies $\sim 10^3$) are generically expected while longer throats (with electroweak hierarchy) are at least not uncommon. Taking optimistic values for the unknown input data, we find that extreme hierarchies $\sim 10^{80}$ are expected and throats with electroweak hierarchy represent a firm statistical prediction.

While these findings confirm the claim of our title that throats are ubiquitous in the
type IIB landscape, the typical length of those throats is quite uncertain at present. In this respect the main open questions are how complex a Calabi-Yau we should be looking for and a quantitative understanding of the ‘bulk’ of the high-dimensional moduli space of such manifolds. Furthermore, a better understanding of the role played by the large complex structure regions (which we have ignored in this analysis) is highly desirable.

Finally, even though many important questions remain unanswered, we consider one conclusion as relatively firm: Throats are common in the presently best understood part of the string-theory landscape and should thus be taken very seriously both in string-theoretic and phenomenological model building. Given the very general setting we have been working with and the small number of assumptions that we had to make, we are optimistic that throats will become one of the most firm and concrete predictions of the type IIB landscape. We would like to view this as strong support for phenomenological research in 5d Randall-Sundrum-like model, put within the more specific limits of their type IIB realization [40]. At the same time one should, however, keep in mind that, if the ‘favourable’ scenario of many very long throats is confirmed and cosmological problems with the various light fields are established, this whole line of thinking may turn into a serious argument against the type IIB landscape.

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