Most attractive channel for self-breaking of grand unification group symmetry

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Abstract
Most attractive channel is studied for SU(5) grand unified theory in five dimensions where $5^*$ and $10$ fermions and a $24$ gauge boson propagate in the bulk. If there are bulk fermions with zero mode for the chirality opposite to quark and lepton multiplets, they can contribute to forming $5, 24, 50$ and $75$ scalar bound states. We find that the inclusion of a simple anomaly-free set for zero mode yields scalar bound states whose binding strengths are the largest for $24$ and the second largest for $5$. This corresponds to the breaking pattern of the SU(5) to color and electromagnetism via the standard model gauge group.
1 Introduction

While elementary particles have mass and charge, the origin still remains unknown. When the generation of masses is related to symmetry breaking, the corresponding problem is how symmetry is broken. In the standard model, the Higgs mechanism describes physics of elementary particles very well up to the weak scale. However, it has been found in the own framework of the standard model that this picture is not valid at higher energy scales. In addition, the standard model does not have any explanation for quantization of charge. Hence, we need a deeper understanding for mass and charge, or symmetry breaking and charge beyond the standard model.

If extra dimensions are included in an effective theory, the strength of force would behave unlike four dimensions. The gauge bosons of the standard model propagating in extra dimensions can rapidly become strongly coupled and form scalar bound states of quarks and leptons. The self-breaking of the standard-model gauge symmetry was proposed in Ref. [1]. The authors proposed that the existence of a Higgs doublet is a consequence of the standard-model gauge symmetry and three generation of quarks and leptons provided the gauge bosons and fermions propagate in appropriate extra dimensions compactified at a TeV scale. It has been shown earlier that electroweak symmetry may be broken by fields propagating in extra dimensions in Ref. [2][3]. Also in the Randall-Sundrum warped space [4][5], transition of the strength of force and the type of bound states have been studied [6]-[11].

As for quantization of charge, grand unification has attracted much attention. The charges belong to subgroups of a unification group. Quantum numbers for quarks and leptons are fixed by the group structure. The SU(3), SU(2) and U(1) couplings meet at a single point and the couplings are evaded from blowup of the values by renormalization group evolution. Beyond the unification scale, the energy dependence of the three couplings are shown graphically as one identical line due to contributions of twelve $X$ and $Y$ bosons. Thus, a grand unification with extra dimensions might contain the generation of masses and the quantization of charges automatically.

In this paper, we study most attractive channel for an SU(5) grand unification in five dimensions. Our setup is that components in each SU(5) multiplet obey the same boundary condition with respect to extra dimensions although gauge symmetry breaking by boundary conditions might be an interesting possibility [12] [13]. The energy dependence of couplings in gauge theories with extra dimensions has been found [14] [15] and has been examined in detail [16] [17]. In particular, the energy dependence of the three couplings should be shown as one identical line beyond the unification scale. It has been emphasized in Ref. [18] that this is not the case for gauge symmetry breaking by boundary conditions where the same boundary conditions do not span irreducible representations of the unification group. In order to keep gauge coupling unification above the unification scale, we consider only the self-breaking for the origin of symmetry breaking. The fundamental fields are a gauge boson and fermions. A gauge boson belongs to $24$. Quarks and leptons are included in $5^*$ and $10$ Dirac fermions whose left-handed components have zero mode. If these fields are only objects propagating in the bulk, there are no scalar bound states composed of zero mode. When an anomaly-free set of fields with zero mode for the right-handed component is added, we find scalar bound states whose binding strengths are the largest for $24$ and the second largest for $5$, correspondingly to the breaking pattern $SU(5) \rightarrow SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times U(1)$. 

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The paper is organized as follows. In Section 2, we start with the basics such as the scenario, framework, field content and group structure. With consideration for anomaly-free sets, scalar bound states and the binding strengths are given in Section 3. Here we find various composites. The simple model to form 24 and 5 for appropriate symmetry breaking is proposed. Furthermore, the composites 50 and 75 are discussed for the doublet-triplet splitting. We conclude in Section 4.

2 Basics of the model

We consider a gauge theory with two branes on fixed points of the orbifold $S^1/Z_2$ in five dimensions. Bulk fields can have strong coupling compared with the corresponding four-dimensional theory so that the ordinary four-dimensional massless gluon and quarks does not give rise to gauge symmetry breaking and that the five-dimensional effect may change potentials for composites. If non-vanishing vacuum expectation values are generated, symmetry is broken. The pattern of symmetry breaking depends on the attractive force of composites and the masses of the constituents. The analysis can be applied for both of flat spacetime and warped spacetime.

In the present model, there are no fundamental scalar fields. The gauge boson $A_M$ has the four-dimensional vector component $A_\mu$ and the four-dimensional scalar component $A_5$. The boundary conditions at the location of the branes are given by Neumann for all $A_\mu$ and Dirichlet for all $A_5$. Quarks and leptons are included as zero mode for $5^*$ and $10$ Dirac fermions, $\bar{\psi}_{5^*}$ and $\psi_{10}$. For the left-handed component, the Neumann boundary condition is imposed. Since singlet neutrinos do not produce attractive force, they can be omitted in the present analysis. For these boundary conditions, fields which belong to irreducible representations of SU(5) obey the same boundary conditions so that at higher energy scales above the scale of symmetry breaking by any condensation the theory behave as SU(5). For simplicity, the number of the generation is chosen as one.

The source of symmetry breaking is scalar bound states represented by the form $\bar{\psi}_I\psi_J$ where $I, J$ are denoted as the species of fermions. The attractive force is dominantly originated from exchange of a gauge boson between fermions and is determined by the gauge coupling and the group structure. As for the group structure, we take the notation $tr(t^a_I t^b_J) = C(r)\delta^{ab}$ where $t^a_I$ is denoted as the representation matrices in the irreducible representation $r$ and $C(r)$ is a constant for each representation $r$. The relation $d(r)C_2(r) = d(\text{Adj})C(r)$ is fulfilled where $C_2(r)$ is the quadratic Casimir operator for each representation and $d$ is the dimension of each representation. The values $C$, $C_2$ and $d$ are summarized for several irreducible representations of SU(N) in Table 1. For $N = 5$, the diagonal generator $t^8 = \text{diag}(2, 2, 2, -3, -3)/(2\sqrt{15})$ plays the same role as the U(1) hypercharge up to a overall factor. We call its charge $Q_Y$. For SU(3)×SU(2)×U(1), $\psi_{5^*}$ and $\psi_{10}$ are decomposed into $(3^*, 1)_{Q_Y=-\sqrt{15}/15} \oplus (1, 2)_{Q_Y=\sqrt{15}/15}$ and $(3, 2)_{Q_Y=-\sqrt{15}/30} \oplus (3^*, 1)_{Q_Y=2\sqrt{15}/15} \oplus (1, 1)_{Q_Y=-\sqrt{15}/5}$, respectively.

For the quark and lepton multiplets, possible combinations of the bound states are $\bar{\psi}_{5^*}\psi_{5^*}$, $\bar{\psi}_{10}\psi_{10}$ and $\bar{\psi}_{5^*}\psi_{10}$. They are written in terms of four-dimensional fields dependent on the five-dimensional coordinates as chirality mixing operators $\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$. Because $\psi_R$ consists of Kaluza-Klein massive mode without zero mode, it is difficult to have potentials for bound states to yield nonzero vacuum expectation values. Hence, we need to add new fields with zero mode for the right-handed component. In order that $\psi_{5^*}$ and...
ψ_{10} do not form brane mass terms together with the new fields, candidates are except for 5* and 10. A singlet is also excluded because it does not contribute to attractive force.

For forming the fundamental 5 or the anti-fundamental 5* scalar bound states together with quark and lepton multiplets, possible bulk fermions are 5, 10* and 24. For forming an adjoint 24 scalar bound state together with quark and lepton multiplets, possible bulk fermions are 15, 40* and 45. Added new fermions have zero mode only for the right-handed component. In general they are chiral in four dimensions. Therefore new fields need to be added in such a way that anomaly is canceled. An anomaly coefficient $A(r)$ is defined by

$$\text{tr} \left[ t^a_r \{ t^b_r, t^c_r \} \right] = \frac{1}{2} A(r)d^{abc}$$

(2.1)

where $\{ t^a_N, t^b_N \} = \frac{1}{N} S^{ab} + d^{abc} t^c_N$ for SU(N). For $N = 5$, the generators associated with $Q_Y$ yield $d^{888} = -1/\sqrt{15}$. The anomaly coefficients for the above irreducible representations are given by

$$A(5) = -\frac{1}{2}, \quad A(10*) = \frac{1}{2}, \quad A(15) = \frac{9}{2}, \quad A(24) = 0, \quad A(40*) = -8, \quad A(45) = 3.$$  

(2.2)

Thus the minimal anomaly-free set to obtain 5 or 5* scalar bound states and a 24 scalar bound state is (10*, 15, 40*, 45) Dirac fermions. In the next section, we will examine the binding strengths for bound states in the most attractive channel approximation.

### 3 Composites and binding strengths

In this section we derive possible bound states and their quantum numbers and binding strengths by adding bulk fields with zero mode for the right-handed component, $\chi$. Here $\bar{\psi} \chi$ includes $\bar{\psi}_L \chi_R^{(0)}$ composed of only zero mode and strong attractive force can lead to potentials with phase transition. First we examine the minimal anomaly-free set. Taking
into account the pattern of symmetry breaking more, we study a simple anomaly-free set. In addition, the 50 and 75 bound states associated with the doublet-triplet splitting are discussed.

**The minimal anomaly-free set**

The quark and lepton multiplets are $\psi_5^*$ and $\psi_{10}$. The minimal set of addition fermions is given by $\chi_{10^*}$, $\chi_{15}$, $\chi_{40^*}$ and $\chi_{45}$. It is necessary to examine the binding strengths for eight possible scalar bound states

$$\bar{\psi}_5^* \chi_{10^*}, \bar{\psi}_5^* \chi_{15}, \bar{\psi}_5^* \chi_{40^*}, \bar{\psi}_5^* \chi_{45}, \bar{\psi}_{10} \chi_{10^*}, \bar{\psi}_{10} \chi_{15}, \bar{\psi}_{10} \chi_{40^*}, \bar{\psi}_{10} \chi_{45}. \quad (3.1)$$

They are decomposed further in terms of irreducible representations into twenty four bound states

$$\begin{align*}
5 \otimes 10^* &= 5^* \oplus 45, \\
5 \otimes 15 &= 35 \oplus 40, \\
5 \otimes 40^* &= 10^* \oplus 15^* \oplus 175_A^*, \\
5 \otimes 45 &= 24 \oplus 75 \oplus 126, \\
10^* \otimes 10^* &= 5 \oplus 45^* \oplus 50^*, \\
10^* \otimes 15 &= 24 \oplus 126, \\
10^* \otimes 40^* &= 24 \oplus 75 \oplus 126^* \oplus 175_B^*, \\
10^* \otimes 45 &= 10 \oplus 15 \oplus 40^* \oplus 175 \oplus 210.
\end{align*} \quad (3.2-3.9)$$

The binding strength for $\bar{\psi} \chi$ is given by

$$\frac{1}{2} g^2 \left[ C_2(\bar{\psi}) + C_2(\chi) - C_2(\bar{\psi} \chi) \right]. \quad (3.10)$$

The coupling constant $g$ is common for types of fields in the present model. For SU(5), the values of $C_2$ for the irreducible representations appearing here are summarized in Table 2. Using these values, we find the binding strengths for the twenty four bound states as shown in Table 3.

| $r$ | 5  | 10 | 15 | 24 | 35 | 40 | 45 | 50 | 75 | 126 | 175_A | 175_B | 210 |
|-----|----|----|----|----|----|----|----|----|----|-----|-------|-------|-----|
| $C_2$ | $\frac{12}{5}$ | $\frac{18}{5}$ | $\frac{21}{5}$ | 5 | $\frac{45}{5}$ | $\frac{42}{5}$ | $\frac{42}{5}$ | 8 | 10 | $\frac{45}{5}$ | 12 | $\frac{42}{5}$ | $\frac{42}{5}$ |

For the minimal field content, it is found that the binding strengths for $10$ and $10^*$ are larger than that of the adjoint 24. Usually SU(5) symmetry is broken by 24 or 75. The results given in Table 3 means SU(5) symmetry would not be broken desirably unless there is some mechanism such that the binding for $10$ and $10^*$ decreases effectively or the binding for 24 increases. In the following we consider a non-minimal but simple anomaly-free set where this problem does not arise.
The simple anomaly-free set

As given in the previous section, a 24 scalar bound state can be made of 15, 40* and 45 Dirac fermions with quark and lepton multiplets. From Table 3, it is read that a composite 24 has the largest binding strength only for a 15 χ field and the other composites are largest for different quantum numbers: the composite 10* for a 40* χ field and the composite 10 for a 45 χ field. Therefore the fermion with zero mode for the right-handed component to form a 24 scalar bound state is uniquely fixed as a 15 χ field.

One of the simplest anomaly-free fermion content with 15 is 9 χ5 and 1 χ15. The composite are given by $\bar{\psi}_{5*}\chi_{15}$, $\bar{\psi}_{5*}\chi_{15}$, $\bar{\psi}_{10}\chi_{5}$, and $\bar{\psi}_{10}\chi_{15}$ as well as 8 copies of χ5. They are decomposed into eight irreducible representations,

$$5 \otimes 5 = 10 \oplus 15, \quad (3.11)$$

$$10* \otimes 5 = 5* \oplus 45, \quad (3.12)$$

with Eqs. (3.3) and (3.7). For this simple set, the binding strengths are given in Table 4.

| Composite | Constituents | Binding strength |
|-----------|--------------|------------------|
| 24        | $\psi_{10}\chi_{15}$ | 21/10 |
| 5*        | $\psi_{10}\chi_{5}$ | 9/5       |
| 40        | $\psi_{5*}\chi_{15}$ | 7/10      |
| 10        | $\psi_{5*}\chi_{5}$ | 3/5       |
| 45        | $\psi_{10}\chi_{5}$ | -1/5      |
| 126       | $\psi_{10}\chi_{15}$ | -2/5      |
| 15        | $\psi_{5*}\chi_{5}$ | -2/5      |
| 35        | $\psi_{5*}\chi_{15}$ | -4/5      |

Table 4: Binding strengths for $\bar{\psi}\chi$ for the simple set in unit of $g^2$.
breaking of the SU(5) to SU(3) × SU(2) × U(1). After the SU(5) is broken, nine 5 and 5* composite scalar fields contribute for the breaking of SU(2) × U(1) to U(1).

Now we compare the binding strengths given in Table 4 with the results of the standard model gauge group. When quarks Q, U, D, leptons L, E, N and SU(3) × SU(2) × U(1) gauge bosons propagate in five dimensions, the binding strengths for the composites with zero mode are given in Table 5. Here Q, L are SU(2) doublets and U, D, E, N are SU(2) singlets.

| Composite | Constituents | SU(3) × SU(2) × U(1) | Binding strength | Binding for SU(3) × SU(2) × U(1) |
|-----------|-------------|----------------------|-----------------|----------------------------------|
| H$_1$     | QU          | (1, 2, $\frac{1}{2}$) | $\frac{2}{3}g_2^2 + \frac{1}{3}g_1^2$ | $(7/5) g^2$                      |
| H$_2$     | QD          | (1, 2, $-\frac{1}{2}$) | $\frac{4}{3}g_2^2 - \frac{2}{3}g_1^2$ | $(13/10) g^2$                    |
| S$_{10}$  | EQ          | (3, 2, $\frac{1}{6}$) | $-\frac{1}{3}g_1^2$ | $(-1/10) g^2$                    |
| S$_{11}$  | LU          | (3, 2, $\frac{5}{6}$) | 0 | 0                                |
| S$_{12}$  | LD          | (3, 2, $\frac{5}{6}$) | $\frac{1}{6}g_1^2$ | $(1/10) g^2$                     |
| S$_{13}$  | LE          | (1, 2, $-\frac{1}{2}$) | $\frac{2}{3}g_1^2$ | $(3/10) g^2$                     |
| S$_{14}$  | N or $\bar{N}$ is included | | 0 | 0                                |

From Tables 4 and 5, the binding strengths for 24 and 5* are larger than that of the Higgs doublet $H_1$. Because the self-breaking of the standard model gauge group has been claimed [1] [2] [3], the symmetry breaking associated with the larger binding strengths can occur. In other words, the symmetry breaking of SU(5) to color and electromagnetism can be expected.

**On the composites 50, 50* and 75**

A solution to the doublet-triplet splitting has been to break the SU(5) symmetry by the real representation 75 instead of 24 and add 50 and 50* [20] [21]. Now we consider the composites 50, 50* and 75.

First we examine the composite 75. For quark-lepton multiplets $\psi_{5*}$ and $\psi_{10}$, fermions $\chi$ with the representations 10, 40, 45 and 50 can form 75 bound states as

$$5 \otimes 45 = 24 \oplus 75 \oplus 126,$$

$$5 \otimes 50 = 75 \oplus 175_B,$$

$$10^* \otimes 10 = 1 \oplus 24 \oplus 75,$$

$$10^* \otimes 40^* = 24 \oplus 75 \oplus 126^* \oplus 175_B^*.$$ 

Among these constituents, only the 50 $\chi$ field yields a 75 bound state without the adjoint 24. The binding strengths for the constituents $\bar{\psi}_{5^*}\chi_{50}$ are 12/5 for the composite 75 and $-3/5$ for the composite 175$_B$. This point is favorable. However, in addition to the coupling with $\psi_{5^*}$, $\bar{\psi}_{10}\chi_{50}$ needs to be taken into account where $10^* \otimes 50 = 10 \oplus 175 \oplus 315$. The binding strength for the constituents $\bar{\psi}_{10}\chi_{50}$ are 21/5 for the composite 10. This means that the composite 75 with the quark-lepton multiplets for the constituents involves other composites bound with larger strengths.
Next we examine the composites $50$ and $50^*$. When the $5^*$ and $10$ quark-lepton multiplets and their conjugates coupled to fermions with right-handed zero mode form $50$ or $50^*$ bound states, the composites would include $45$ or $45^*$ bound states simultaneously. Since the quadratic Casimir operators have the relation $C_2(50) < C_2(45)$, the composite $45$ is bound stronger than the composite $50$. This gives rise to SU(2) doublet mass terms.

It is because the representation $45$ for SU(5) has a color singlet and weak doublet state for SU(3)×SU(2) and the composites $45$ and $75$ can become an SU(5) singlet with the composite $5$ as seen in Eq. (3.13).

Therefore doublet-triplet splitting by the composites $50$, $50^*$ and $75$ would not occur minimally. The model would need to be modified for application of the idea.

4 Conclusion

If extra dimensions are included in an effective theory, the strength of force would behave unlike four dimensions. We have studied most attractive channel for an SU(5) grand unification in five dimensions. We have assumed that components in each SU(5) multiplet obey the same boundary condition with respect to extra dimensions to keep the gauge coupling unification above the unification scale. In addition to quarks, leptons and gauge bosons, $\psi_{5^*}$, $\psi_{10}$ and $A_M$, anomaly-free sets of fields $\chi$ with zero mode for the right-handed component lead to various scalar bound states.

The minimal anomaly-free set to obtain $5$ and $24$ scalar bound states is $(\psi_{5^*}, \psi_{10}, A_M)$ plus $(\chi_{10^*}, \chi_{15}, \chi_{40^*}, \chi_{45})$. For this field content, the binding strengths for $10$ and $10^*$ have been found to be larger than that of $24$. Then, before SU(5) symmetry is broken to the standard model gauge group by $24$, symmetry breaking can occur by $10$ and $10^*$.

A simple way to overcome this problem is to adopt $(\chi_{5}^{I=1,\cdots,9}, \chi_{15})$ instead of the anomaly-free set $(\chi_{10^*}, \chi_{15}, \chi_{40^*}, \chi_{45})$. Here the $15$ $\chi$ field is uniquely chosen because it is the only representation to form a $24$ scalar bound state whose binding strength is large compared with other composites. The largest strength for the composite $24$ corresponds to the breaking of the SU(5) to the standard model gauge group. After the SU(5) is broken, $5$ in $\bar{\psi}_{10}\chi_5$ contribute for the breaking of SU(2)×U(1) to U(1). In addition, we have found that the binding strengths for $24$ and $5$ are larger than the known results for the composite Higgs doublet for the standard model gauge group.

We have shown that the symmetry breaking of the grand unification group to color and electromagnetism may occur by $24$ and $5$. It would be more interesting to identify other fermion sets to assure the double-triplet splitting. However, we have found that the doublet-triplet splitting by the composites $50$, $50^*$ and $75$ does not seem to occur minimally.

Finally, our analysis can be developed in various directions such as modification of field contents. It needs to be examined in more detail how masses and charges of elementary particles are derived appropriately.
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