Parametric image amplification in optical cavities

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Abstract

We show the possibility of noiseless amplification of an optical image in cavities containing a parametric oscillator. We consider a confocal ring cavity with plane mirrors and compare with the case of planar cavity. In the latter case the system operates with severe spatial limitations, while in the confocal case, there is the possibility of preserving the signal-to-noise ratio while amplifying uniformly the entire image.

1 Introduction

It is well known that phase-insensitive amplifiers introduce at least 3 dB extra noise in the output, whereas phase-sensitive amplifiers may preserve the input signal-to-noise ratio, and in this sense are “noiseless” \(^1\). A spectral analysis of some cavity-based noiseless amplifiers has been given in Ref.\(^2\), but these investigations are carried out exclusively in the time domain, while spatial aspects are neglected by introducing the plane wave approximation, i.e. by considering only one spatial mode.

However, the spatial domain is relevant for the subject of noiseless amplification. Indeed, many areas of physics would benefit from having a possibility of noiseless amplification of faint optical images. Here, based on this motivation, we analyse the parametric image amplification in two different cavity geometries \(^3\) \(^4\).

2 The Optical Scheme

A possible realization of a parametric image amplifier is shown in Fig.1. A faint image which is to be amplified is located in the object plane \(O\). This image is projected by a lenses system in the input plane of a ring-cavity degenerate optical parametric amplifier. The amplified image from the output plane is then projected in the image plane \(I\) by another lenses system. Each lens has a focal length \(f\). The presence

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of the dashed part in the scheme depends on the geometry one wants to consider, as we shall see. Instead, the presence of a pupil $P$ of finite area $S_p$, is necessary for evaluation of the noise properties. Let us now introduce the two-component transverse wave vector $\vec{q} \equiv (k_x, k_y)$ and position vector $\vec{\rho} \equiv (x, y)$. Moreover, let $a(\vec{\rho}, t)$, $a(\vec{\rho}, t)$, and $e(\vec{\rho}, t)$, $e(\vec{\rho}, t)$ the photon annihilation and creation operators of the field in the object plane and image plane respectively. We shall assume that the field in the object plane is in a coherent state with real amplitude $s(\vec{\rho})$ modulated (and even) in space. The observable, in the image plane, is the surface photocurrent density $i(\vec{\rho}, t) = \eta \langle e(\vec{\rho}, t) e(\vec{\rho}, t) \rangle$, with $\eta$ the photodetection efficiency. However, the quantity of interest for us is the number of photodetections $N_I(\vec{\rho}, t)$ registered by the pixel, of area $S_d$, centered at the point $\vec{\rho}$ in the image plane in the time window $[t - T_d/2, t + T_d/2]$, i.e.

$$N_I(\vec{\rho}, t) = \int_{S_d} d\vec{\rho}' \int_{T_d} dt' i(\vec{\rho}', t').$$

We shall consider the mean number $\langle N_I(\vec{\rho}, t) \rangle$ of registered electrons as the amplified signal of our scheme. Its variance characterizes the noise properties of the image

$$\langle \Delta N^2_I(\vec{\rho}, t) \rangle = \int_{S_d} d\vec{\rho}' \int_{T_d} dt' \int_{S_d} d\vec{\rho}'' \int_{T_d} dt'' \langle \delta i(\vec{\rho}', t'), \delta i(\vec{\rho}'', t'') \rangle.\ (2)$$

The power signal-to-noise ratio (SNR) of the amplified image is given by

$$R_I = \langle N_I(\vec{\rho}, t) \rangle^2 / \langle \Delta N^2_I(\vec{\rho}, t) \rangle. \ (3)$$

Equivalent quantities, $\langle N_O(\vec{\rho}, t) \rangle$, $\langle \Delta N^2_O(\vec{\rho}, t) \rangle$ and $R_O$, can be built up in the object plane. Finally, by definition, the noise figure is

$$F = R_O / R_I, \ (4)$$

and we refer to the situation of $F \to 1$ as the noiseless amplification. In order to investigate this possibility we have to express the image field in terms of object field.

### 3 Plane Cavity

We first consider the case of planar cavity [3]. Hence, the dashed part in Fig.1 has to be neglected. The behavior of the slowly varying part of the field operator $b(\xi, t)$ inside the cavity is described by the following Langevin equation

$$\partial_t b(\xi, t) - i c \nabla^2_\perp / 2k = -\gamma \left[ (1 + i \Delta) b(\xi, t) - A_p b(\xi, t) \right] + \sqrt{2\gamma} b_{\text{in}}(\xi, t), \ (5)$$

where $k = 2\pi/\lambda$ is the wave number, $\gamma$ is the cavity decay rate, and $\Delta$ is the cavity detuning. $A_p$ is the constant of parametric interaction proportional to the amplitude of the pump field taken as a classical quantity. The symbol $\nabla^2_\perp$ denotes the transverse
laplacian. The above equation can be solved by means of spatio-temporal Fourier transformations; furthermore, with the aid of transformations at lenses

$$b_{in}(\xi, t) = \int \frac{d\tilde{\rho}}{Nf} a(\tilde{\rho}, \Omega) e^{-i\frac{2\pi}{\lambda f}\tilde{\rho} \cdot \xi}, \quad e(\tilde{\rho}, t) = \int \frac{d\xi}{Nf} \mathcal{P}(\xi) b_{out}(\xi, t)e^{-i\frac{2\pi}{\lambda f}\tilde{\rho} \cdot \xi}$$

we arrive at

$$e(\tilde{\rho}, \Omega) = \frac{1}{Nf} \int d\rho' \varphi(\rho - \rho') \left[ U(\rho', \Omega)a(\rho', \Omega) + V(\rho', \Omega)a^\dagger(\rho', -\Omega) \right]$$

where \( \varphi \) is the impulse response function, i.e. the Fourier transform of the pupil function \( \mathcal{P} \) (for an infinitely large pupil \( \varphi(\rho) = \lambda f \delta(\rho) \)). The coefficients \( U, V \) are

$$U(\rho, \Omega) = \frac{[1 - i\delta(\rho, \Omega)][1 - i\delta(\rho, -\Omega)] + A_p^2}{[1 + i\delta(\rho, \Omega)][1 - i\delta(\rho, -\Omega)] - A_p^2},$$

$$V(\rho, \Omega) = \frac{2A_p}{[1 + i\delta(\rho, \Omega)][1 - i\delta(\rho, -\Omega)] - A_p^2}.\quad (9)$$

Here, \( \delta(\rho, \Omega) = \Delta - \Omega + (\rho/\rho_0)^2 \) is the local mismatch function, with \( \rho_0 = f \sqrt{\lambda \gamma/\pi c} \) a characteristic transverse length.

Now, the quantities of interest can be calculated by using Eq.(3), and some simplifying assumptions, i.e.: \( \lambda f / S_p^{1/2} \) much smaller than the typical scale of change of \( U, V \) and \( s \), so we can take the latter functions out of integral when they enter as a product with \( \psi \); the size of each pixel is the smallest of all the spatial scales, so that we can substitute integration over the pixel area by multiplication by \( S_d \); observation time \( T_d \) long compared with the inverse cavity bandwidth \( \gamma^{-1} \). All that leads to

$$\langle N_f(\rho, t) \rangle = \eta S_d T_d s^2(\rho) G(\rho) + \text{noise},$$

with \( G(\rho) \) the gain factor

$$G(\rho) = \left\{ [(1 + A_p)^2 - \delta(\rho, 0)]^2 + 4\delta^2(\rho, 0) \right\}/\left\{ [1 + \delta^2(\rho, 0) - A_p^2] \right\}.$$  \( (10) \)

For the variance we get

$$\langle \Delta N^2_f(\rho, t) \rangle = \eta S_d T_d s^2(\rho) G(\rho) \left\{ 1 - \eta \left[ \cos^2 \theta(\rho) e^{2R(\rho)} + \sin^2 \theta(\rho) e^{-2R(\rho)} \right] \right\},$$

where we have introduced the squeezing parameter \( \exp[\pm R(\rho)] = |U(\rho, 0)|/|V(\rho, 0)| \), and the orientation angle \( \theta(\rho) = \arg[U(\rho, 0) + V(\rho, 0)] - \arg[U(\rho, 0)] - \arg[V(\rho, 0)] \). The condition to neglect the (unspecified) noise terms in Eqs.\((10), (12)\) reads

$$s^2(\rho)(\lambda^2 f^2 / S_p)(2\pi/\gamma) \gg 1.$$  \( (13) \)

It also fixes the resolution of the scheme. Once it is satisfied, the noise figure becomes

$$F = \left\{ 1 - \eta \left[ \cos^2 \theta(\rho) e^{2R(\rho)} + \sin^2 \theta(\rho) e^{-2R(\rho)} \right] \right\}/\left\{ \eta G(\rho) \right\}.\quad (14)$$

The optimum condition for noiseless amplification, coming from Eqs.\((11)\) and \((14)\), is \( \delta(\rho, 0) = 0 \). Hence, depending on the value of the detuning, the noiseless amplification takes places in a small region around the optical axis or in an annular region. Therefore, for reconstruction of the whole image one has to use scanning.
4 Confocal cavity

Let us now go to the configuration with confocal cavity \[4\]. In this case the dashed part of the optical scheme in Fig.1 has to be considered. In particular the confocality is guaranteed by means of a specific relation between the focal length of the intracavity lens and the cavity round trip length. The intracavity field \(b\) can be expanded on the basis of the Gauss-Laguerre modes \(\{f_{p,l,i}(\vec{r})\}\) as

\[
b(\vec{r},t) = \sum_{p,l,i} f_{p,l,i}(\vec{r}) b_{p,l,i}(t).
\]  

(15)

All the modes with \(l\) even have the same frequency; the same is true for the modes with \(l\) odd; the frequency separation between the two groups of modes is equal to one half the free spectral range. Due to the frequency degeneracies, the field can be split into even and odd part

\[
b(\vec{r},t) = b^+ + (\vec{r},t) + b^- (\vec{r},t).
\]

The modes with \(l\) even quasi-resonant with the signal. Since the detuning is equal for all these modes, it is possible to get the following Langevin equation for the intracavity field

\[
\partial_t b^+ (\vec{r},t) = -\gamma \left[ (1 + i\Delta^+) b^+ (\vec{r},t) - A_p b^+_{\vec{r}} (\vec{r},t) \right] + \sqrt{2\gamma} b^+_\text{in} (\vec{r},t).
\]

(16)

This equation can be solved in the frequency domain; furthermore, with the aid of transformations at lenses

\[
b^+_\text{in} (\vec{r},t) \equiv a(\vec{r},t), \quad e(\vec{r},t) = \int \frac{d\vec{r}'}{\lambda_f} P(\vec{r}') \int \frac{d\vec{r}''}{\lambda_f} b^\text{out}(\vec{r}'',t) e^{i2\pi\lambda_f (\vec{r}' - \vec{r}) \cdot \vec{r}'},
\]

(17)

we can write

\[
e(\vec{r},\Omega) = \frac{1}{\lambda_f} \int d\vec{r}' \varphi(\vec{r}' - \vec{r}) \left[ U(\Omega)a_+ (\vec{r}',\Omega) + V(\Omega)a_+ (\vec{r}',-\Omega) \right],
\]

(18)

where the coefficients \(U(\Omega)\) and \(V(\Omega)\) are the same of Eqs.(8) and (9) with now the mismatch function no longer dependent from the position vector, i.e. \(\delta(\Omega) = \Delta^+ - \Omega\). Repeating the steps of previous Section, we easily obtain the mean number of photoelectrons

\[
\langle N_I(\vec{r},t) \rangle = \eta S_d T_d s^2 (\vec{r}) G + \text{noise},
\]

(19)

where now the gain factor \(G\) takes a simpler form

\[
G = \left[ (1 + A_p)/(1 - A_p) \right]^2,
\]

(20)

considering the situation of perfect resonance, i.e. \(\Delta^+ = 0\). For the variance we have

\[
\langle \Delta N^2_I(\vec{r},t) \rangle = \eta S_d T_d s^2 (\vec{r}) G \{ 1 - \eta + \eta G \} + \text{self interference of the noise}.
\]

(21)

To neglect the noise terms in Eqs.(19) and (21), we again use the condition (13). Then, the noise figure becomes

\[
F = \{ 1 - \eta + \eta G \}/\{\eta G\}.
\]

(22)

As can be seen from Eqs.(20) and (22), the noiseless amplification occurs uniformly over the transverse plane. Hence, this scheme offer the possibility of amplification of the whole image at once. Moreover, a high gain is required to only compensate the effect of non efficient detection.
References

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FIGURE CAPTIONS
Fig.1 A possible realization of the parametric image amplifier.
