Strong Dynamics at the Muon Collider: Working Group Report

Pushpalatha C. Bhat† and Estia Eichten†

Fermi National Accelerator Laboratory, Batavia, IL 60510

Working group members: Gustavo Burdman, Bogdan Dobrescu, Daniele Dominici, Takanobu Handa, Christopher Hill, Stephane Keller, Kenneth Lane, Paul Mackenzie, Kaori Maeshima, Stephen Parke, Juan Valls, Rocío Vilar and John Womersley

Abstract. New strong dynamics at the energy scale \( \approx 1 \) TeV is an attractive and elegant theoretical ansatz for the origin of electroweak symmetry breaking. We review here, the theoretical models for strong dynamics, particularly, technicolor theories and their low energy signatures. We emphasize that the fantastic beam energy resolution \((\sigma_E/E \sim 10^{-4})\) expected at the first muon collider \((\sqrt{s} = 100-500\) GeV\) allows the possibility of resolving some extraordinarily narrow technihadron resonances and, Higgs-like techniscalar produced in the s-channel. Investigating indirect probes for strong dynamics such as search for muon compositeness, we find that the muon colliders provide unparallel reaches. A big muon collider \((\sqrt{s} = 3-4\) TeV\) would be a remarkable facility to study heavy technicolor particles such as the topcolor \(Z'\), to probe the dynamics underlying fermion masses and mixings and to fully explore the strongly interacting electroweak sector.

INTRODUCTION

The success of the Standard Model of Particle Physics has been spectacular, thus far! But, new physics beyond the Standard Model (SM) seems inevitable since some critical issues remain unresolved. The cause of electroweak symmetry breaking

\footnote{Summary talk presented by P.C. Bhat at “Workshop on Physics at the First Muon Collider and at the Front End of a Muon Collider”, November 6-9, 1997, Fermilab, Batavia, IL 60510}

\footnote{Co-convenors}
Technicolor is not experimentally established, nor is the origin of fermion masses and mixings known. There are two enticing theoretical approaches to understand EWSB: introducing supersymmetry or invoking new strong interactions. Our working group explored the latter scenario, that of a new strong dynamics, and, its search and study at the First Muon Collider.

In this report, we review and summarize the activities of the working group. We first explore the existing theories for new strong dynamics. These models provide an intuitively attractive (though presently disfavored) approach to origin of EW symmetry breaking. A large fraction of this report is devoted to topics related to technicolor, topcolor and their variants. We give an overview of the technicolor models, their low energy signatures, the potential of the First Muon Collider (FMC with $\sqrt{s} = 100-500$ GeV) for direct searches and detailed measurements of these signatures. Some comments on how these compare with what would be attainable at the Tevatron, LHC and the possible NLC machines are included. We also explore the indirect probes for strong dynamics such as tests for compositeness and comment on constraints from rare B and K decays. We briefly state the long-range opportunities to discover and study new strong dynamics with a big muon collider (BMC with $\sqrt{s} = 3–4$ TeV).

At the working group meetings, technicolor theories [1] and relevant issues for the FMC were described in detail by Ken Lane [2]. The specific details for topcolor theories [3] and other recent new ideas in strong dynamics were expounded by Chris Hill [4]. Cross sections for production of some low mass technihadrons at a $\mu^+\mu^-$ collider and their signatures were discussed by John Womersley [5]. A model of technicolor with scalars and the prospects for discovering non-standard Higgs-like scalars in the s-channel at the FMC were presented by Bogdan Dobrescu [6]. A study of vector resonances in the framework of BESS (breaking electroweak symmetry strongly) model [7] was presented by Daniele Dominici [8]. A search for technicolor particles using the Tevatron Run I data and the resulting 95% confidence level (C.L.) upper limits on the production cross-section and exclusion of certain mass regions for these particles were reported by the CDF collaboration [9]. Studies of various indirect tests of strong dynamics such as compositeness tests (Eichten and Keller [10]), strong WW scattering (Gunion [11]) and constraints on strong dynamics from rare B and K decays (Burdman [12]), were also presented. During the workshop, prompted by comments from Hill and Mackenzie, stressing the importance of narrow neutral technipion production at a muon collider, Eichten and Lane calculated the cross sections for resonance production of these particles at the FMC. Subsequently, Dominici et al., have also studied the production of such particles (called pseudo-Nambu-Goldstone bosons – PNGBs) in the framework of the BESS model [8].

**TECHNICOLOR AND VARIANTS**

Technicolor (TC) is a new strong interaction of fermions and gauge bosons at
FIGURE 1. Generation of quark and lepton masses via ETC interactions.

the scale $\Lambda_{TC} \sim 1$ TeV, which causes dynamical breaking of electroweak symmetry [1]. No elementary scalar bosons (such as the Higgs) are required. Technicolor model, in its simplest form, is a scaled-up version of QCD with massless technifermions that strongly interact at a scale $\Lambda_{TC} \sim 1$ TeV and acquire a dynamical mass $O(\Lambda_{TC})$. The chiral symmetry is spontaneously broken through technifermion condensation, producing three massless Goldstone bosons. These Goldstone bosons (technipions) have Higgs-like coupling to fermions and correspond to the longitudinal components $W_L^\pm$ and $Z_L^0$ of the weak gauge bosons. If left- and right-handed technifermions are assigned to weak SU(2) doublets and singlets, respectively, then $M_W = M_Z \cos \theta_W = \frac{1}{2} g F_\pi$ where $F_\pi=246$ GeV is the technipion decay constant, analogous to $f_\pi = 93$ MeV for the pion. In non-minimal technicolor model, with a large number of technifermion doublets, additional Goldstone bosons arise from technifermion chiral symmetry breaking. The technicolored and the SM fermions however remain massless. They can acquire masses if they couple to technifermions via additional gauge interactions as shown in Fig 1. In this **Extended Technicolor (ETC)** model [13], the quark and lepton masses ($m_f$) proportional to the dynamical mass of the technifermions (condensate $< T \bar{T} >$) are generated:

$$m_f(M_{ETC}) \approx \frac{g_{ETC}}{M_{ETC}} < T \bar{T} >_{ETC}$$

where $g_{ETC}$ is the coupling strength of the fermions to the ETC boson and $M_{ETC}$ is the mass of the ETC boson. The ETC symmetry which pertains to a larger gauge group into which technicolor, color and flavor symmetries are embedded, is broken at a scale $\Lambda_{ETC} = O(100$ TeV).

To avoid large flavor-changing neutral currents and to obtain quark masses of a few GeV, the strong technicolor coupling $\alpha_{TC}$ must run very slowly or “walk”, all the way up to the ETC scale of several hundred TeV [14]. **Walking Technicolor** needs a large number of technifermions for $\alpha_{TC}$ to “walk”.

Another major turning point in the development of technicolor theories came with the discovery of the top quark and the measurement of its large mass ($m_t$)
The direct measurements from the CDF and DØ collaborations yield a value of $m_t = 175.6 \pm 5.5 \text{ GeV}/c^2$ (current world average) [18,19]. To generate this large $m_t$, the ETC models would have to violate experimental constraints on the $\rho$ parameter ($\rho = \frac{M_W}{M_W \cos \theta_W}$) or the $Z \to b \bar{b}$ decay rate. To resolve this problem, a new strong Topcolor interaction was introduced by Hill [4] and Topcolor-assisted Technicolor (TC2) was born [15]. The top quark is very heavy compared to all the other quarks and leptons. This fact suggests that it might be strongly coupled to the mechanism of mass generation and to the dynamics of EWSB itself. It is conceivable that the top quark has unique dynamics. The simplest TC2 model has the following group structure:

$$G_{TC} \times SU(2)_{EW} \times SU(3)_3 \times SU(3)_{1,2} \times U(1)_3 \times U(1)_{1,2} \rightarrow G_{TC} \times SU(2)_{EW} \times SU(3)_C \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{EM}$$

where $G_{TC}$ and $SU(2)_{EW}$ are the technicolor and electroweak gauge groups; $SU(3)_3$ and $U(1)_3$ are topcolor gauge groups coupled to the third generation fermions (with stronger couplings) while $SU(3)_{1,2}$ and $U(1)_{1,2}$ couple to first and second generations only. Technicolor causes most of the EWSB, while topcolor contributes only feebly with $f_t \approx 60 \text{ GeV}$. The light quark and lepton masses are generated via ETC dynamics which contributes only a GeV to the third generation masses. The strong topcolor dynamics (top quark pair condensate) generates $m_t \approx 175 \text{ GeV}$. The $U(1)_3$ provides the difference that causes only top quarks to condense. Thus, the top quark mass may be perceived as being generated by a combination of a dynamical condensate component, $(1 - \epsilon)m_t$ (from topcolor dynamics) and a small fundamental component, $\epsilon m_t$ (from e.g., technicolor) with $\epsilon << 1$. A number of additional particles called “top-pions” $\pi_t$ and “top-rho” $\rho_t$, are expected. The small ETC component of the top quark implies that the masses of the top-pions depend on $\epsilon$ and $\Lambda$. The top-pion mass induced from the fermion loop can be estimated as,

$$M_{\pi_t}^2 = \frac{N \epsilon m_t^2 M_B^2}{8 \pi^2 f_\pi^2} = \frac{\epsilon M_B^2}{\log(M_B/m_t)}$$

where the Pagels-Stoker formula is used for $f_\pi^2$. For $\epsilon = (0.03, 0.1)$, $M_B \approx (1.5, 1.0) \text{ TeV}$, and $m_t = 180 \text{ GeV}$, this predicts $M_{\pi_t} = (180, 240) \text{ GeV}$. The bare values of $\epsilon$ generated at $\Lambda_{ETC}$ is subject to large radiative enhancements (~10) by topcolor and $U(1)_3$. Thus, we expect that even a bare value of $\epsilon \sim 0.005$ can produce sizeable $M_{\pi_t}$ (>$m_t$). The breaking of $U(1)_3 \times U(1)_{1,2} \rightarrow U(1)_Y$ in the vicinity of 2 TeV leads to eight color-octet vector bosons $V_8$ or $B$ (colorons or top-gluons) and an additional $Z$ boson, $Z'$. The mass of the $Z'$ is expected to be in the range of 1-3 TeV.
Top See-saw Model

The topcolor models have met with some problems in their implications for limits on custodial symmetry violation and other phenomenological constraints. The proximity of the measured top quark mass, \( m_t \), to the electroweak scale, however, suggests that EWSB may have its origin in dynamics associated with the top quark. An explicit realization of this idea is the top condensation mechanism \([20]\), in which the top-antitop quark pair acquires a vacuum expectation value, much like the chiral condensate of QCD or the electron condensate of superconductivity (BCS theory). The EWSB occurs via the condensation of the top quark in the presence of an extra vector-like, weak-isosinglet quark. The mass scale of the condensate is \( \sim 0.6 \) TeV corresponding to the electroweak scale \( v \approx 246 \) GeV. The vector-like isosinglet then naturally exhibits a see-saw mechanism, yielding the physical top quark mass, which is then adjusted to the experimental value. The choice of \( \sim \)TeV scale for the topcolor dynamics determines the mass of the weak-isoscalar see-saw partner. The model also implies the existence of PNGBs. The lower bound on the mass of a PNGB that couples to the top quark is less than \( m_t \).

More work is needed to extend the scheme to generate masses and mixing for all quarks and leptons, and to construct attractive schemes for topcolor breaking.

Technicolor Production and Signatures at the FMC

In the minimal technicolor model, with just one technifermion doublet, the only prominent signals at the hadron and lepton colliders would be the enhancements in longitudinally-polarized weak vector boson production. These are due to the s-channel production of color-singlet technirho resonances near \( 1.5-2.0 \) TeV and the subsequent decay into vector boson pairs (\( \rho_T^0 \to W_L^+W_L^- \) and \( \rho_T^\pm \to W_T^\pm Z^0 \)). Observing these enhancements would be extremely difficult, since the \( \mathcal{O}(\alpha^2) \) production cross sections are small at such high technirho masses and efficiency for reconstructing vector boson pairs low.

The non-minimal technicolor models, however, predict a rich spectrum of light, color-singlet technihadrons—the isotriplet vectors \( \rho_T^0, \rho_T^\pm \) and their isoscalar partner \( \omega_T \), and pseudoscalars \( \pi_T^0, \pi_T^\pm \) and \( \pi_T^{0\prime} \)—accessible at the Tevatron, LHC and the FMC. (A search at the Tevatron by CDF has been discussed later). Since techni-isospin is likely to be a good approximate symmetry, \( \rho_T \) and \( \omega_T \) are approximately degenerate and so are the technipions. The masses are expected to be: \( m_{\pi_T} \approx 100 \) GeV and \( m_{\rho_T} \approx m_{\omega_T} \approx 200 \) GeV. The technipions with Higgs-like ETC couplings to quarks and leptons decay to the heaviest fermion pairs allowed. The isosinglet component of neutral technipions, \( \pi_T^{0\prime} \), may decay into a pair of gluons if the constituent technifermions are colored. Thus the predominant decay signatures of the light technipions would be:
\[ \pi^0_\tau \rightarrow b\bar{b}, c\bar{c}, \tau^+\tau^- \]
\[ \pi^+_\tau \rightarrow gg, b\bar{b}, c\bar{c}, \tau^+\tau^- \]
\[ \pi^-_\tau \rightarrow c\bar{b}, c\bar{s}, \tau^+\nu. \]

(4)

The signatures for technirhos and techniomegas are as follows:

\[ \rho^+_\tau \rightarrow W^+Z, W^0, Z^\pm \pi^0, \pi^+\pi^- \]
\[ \rho^0_\tau \rightarrow W^+W^-, W^\pm, \pi^0, q\bar{q}, \ell^+\ell^-, \nu\bar{\nu} \]
\[ \omega_\tau \rightarrow \gamma\pi^0_\tau, Z\pi^0_\tau, q\bar{q}, \ell^+\ell^-, \nu\bar{\nu}. \]

(5)

If the large ratio of \( \frac{<\bar{T}T>_{ETC}}{<\bar{T}T>_{TC}} \) significantly enhances technipion masses relative to technivector masses, then \( \rho_\tau \rightarrow \pi\pi_\tau \) decay channels may be closed.

If technicolor exists and technihadrons have masses low enough to be produced at the FMC, they will most probably be first discovered at the Tevatron or at the LHC. An interesting aspect of the technihadrons is that several of them, particularly the neutral ones are very narrow. Therefore, a \( \mu^+\mu^- \) collider which is expected to have very fine energy resolution is ideally suited for their studies.

Figure 2 shows the production cross section for \( \rho_\tau \) at a muon collider as a function of \( \sqrt{s} \), for \( M_{\rho_\tau}=210 \) GeV and \( M_{\pi^0_\tau}=110 \) GeV. The peak cross section is \( \sim 1 \) nb which translates to \( 10^6 \) events/year with \( \int Ldt=10^{32} \text{ cm}^{-2} \text{ s}^{-1} \). Figure 3 shows the cross section for \( \omega_\tau \) production (\( M_{\omega_\tau}=210 \) GeV). The peak cross section here is even larger, \( \sim 10 \) nb, that would provide an yield of \( 10^7 \) events/year for the same luminosity. Also, note that the peak is extremely narrow, with a width \( < 1 \) GeV. The production cross sections decrease, if \( \rho_\tau \), \( \omega_\tau \) are heavier. For \( M_{\rho_\tau}=400 \) GeV and \( M_{\pi^0_\tau}=150 \) GeV, the event rate is still \( 10^4 \) events/year.

The neutral technipions, like the SM Higgs boson, are expected to couple to \( \mu^+\mu^- \) with a strength proportional to \( m_\mu \). In the ansatz of the non-minimal technicolor model with \( N_D \) technifermion doublets, the coupling is enhanced by a factor of \( \sqrt{N_D} \). Therefore, the FMC can serve as a neutral technipion factory with phenomenal rates for production in the s-channel, far exceeding those at any other collider.

Once a neutral technipion has been found in \( \rho_\tau \) or \( \omega_\tau \) decays at a hadron collider, it should be relatively easy to locate the precise position of the resonance at the FMC and, take data at the resonance. The cross sections for \( f\bar{f} \) and \( gg \) final states are isotropic. The \( \pi^0_\tau \) production cross sections and the \( Z^0 \) backgrounds are shown in Fig. 4 for \( M_{\pi^0_\tau} = 110 \) GeV (for description of other parameters see ref. [2]). The peak signal rates approach \( 1 \) nb. The \( b\bar{b} \) dijet rates are much larger than the \( Z^0 \rightarrow b\bar{b} \) backgrounds, while the \( gg \) rate is comparable to \( Z^0 \rightarrow q\bar{q} \). Details of these and other calculations in this section, including the effects of the finite beam energy resolution, will appear in Ref. [21].

The cross sections for technipion production via the decay of technirho and techniomega s-channel resonances are calculated using vector meson \( (\gamma, Z^0) \) dominance [22–25]. For \( M_{\rho_\tau} = M_{\omega_\tau} = 210 \) GeV, \( M_{\pi^0_\tau} = 110 \) GeV, and other parameters as above, the total peak cross sections are [21]:
FIGURE 2. Cross section (pb) for technirho production at a muon collider as a function of $\sqrt{s}$ (GeV), for $M_{\rho_T}=210$ GeV and $M_{\pi_T}=100$ GeV. The solid curve is the total cross section, the dashed curve is for $\rho_T \rightarrow W\pi_T$ and the dotted curve is for $\rho_T \rightarrow W^+W^-$. 

\[
\sum_{AB} \sigma(\mu^+\mu^- \rightarrow \rho^0_T \rightarrow \pi_A\pi_B) = 1.1 \text{ nb} \\
\sigma(\mu^+\mu^- \rightarrow \omega_T \rightarrow \gamma\pi^0_T) = 8.9 \text{ nb}. 
\] (6)

The technirho decay rate is 20% $W^+W^-$ and 80% $W^\pm\pi_T^\mp$.

Further, there might be a small nonzero isospin splitting between $\rho_T^0$ and $\omega_T$. This would appear as a dramatic interference in the $\mu^+\mu^- \rightarrow f\bar{f}$ cross section, provided the FMC energy resolution is good enough in the $\rho_T-\omega_T$ region. The cross section is most accurately calculated [2] by using the full $\gamma-Z^0-\rho_T-\omega_T$ propagator matrix ($\Delta$).

Figure 5 shows the theoretical $\rho^0_T-\omega_T$ interference effect in $\mu^+\mu^- \rightarrow e^+e^-$ for input masses $M_{\rho_T} = 210$ GeV and $M_{\omega_T} = 212.5$ GeV. The propagator shifts the nominal positions of the resonance peaks by $O(\alpha/\alpha_{\rho_T})$. The theoretical peak cross sections are 5.0 pb at 210.7 GeV and 320 pb at 214.0 GeV. This demonstrates the importance of precision resolution in the 200 GeV FMC.

The detectors [28] at the muon collider should be capable of identifying and measuring electrons, muons, taus, jets and, of tagging $b$-jets with high efficiency. It would be useful if $c$-jets could be distinguished from $b$-jets.
FIGURE 4. Theoretical (un smeared) cross sections for $\mu^+\mu^- \rightarrow \pi^0_{T'} \rightarrow b\bar{b}$ (dashed), $gg$ (dot-dashed) and total (solid) for $M_{\pi_T} = 110$ GeV and other parameters defined in the text. The solid horizontal lines are the backgrounds from $\gamma, Z^0 \rightarrow b\bar{b}$ (lower) and $Z^0 \rightarrow q\bar{q}$ (upper). Note the energy scale.

Topcolor Signatures

Topcolor-assisted technicolor introduces additional particles called top-pions ($\pi_t$), top-gluons ($B$ or $V_8$) and topcolor $Z'$, as discussed in the previous section. Top-pions can be as light as $\sim 150$ GeV, in which case they would emerge as a detectable branching fraction of top quark decay. However, not to violate constraints on $Z \rightarrow b\bar{b}$ rate, $M_{\pi_t} \geq 300$ GeV may be required. Top-gluons are expected to have mass in the range of 0.5-2 TeV and topcolor $Z'$ in the range of 1-3 TeV. The decays are expected to be:

\begin{align*}
\pi_t & \rightarrow t\bar{b}, \quad \text{or} \quad t \rightarrow \pi_t b \\
B & \rightarrow b\bar{b}, t\bar{t} \\
Z' & \rightarrow t\bar{t}.
\end{align*}

Top-pions may be produced copiously at the FMC in the $s$-channel as previously discussed in the case of technipions. The LHC experiments should be sensitive over the entire range of the expected masses for both top-gluons and topcolor $Z'$. If topcolor $Z'$ is not found at LHC, it can be discovered at the big muon collider ($\sqrt{s} = 3-4$ TeV). There are a number of other effects of topcolor that can be observed at the FMC [4]. For example, new effects in $Z$ physics involving the third generation such as $Z \rightarrow b\bar{b}$, might be observed. The generational structure of topcolor may induce GIM violation in low energy processes such as $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and lepton family number violation such as $\mu\bar{\mu} \rightarrow \tau\bar{\mu}$. There may be induced
FIGURE 5. Theoretical (unsmeared) cross sections for $\mu^+\mu^- \rightarrow \rho_0^T, \omega_T \rightarrow e^+e^-$ for input masses $M_{\rho_T} = 210$ GeV and $M_{\omega_T} = 212.5$ GeV and other parameters as defined in the text.

FCNC interactions giving rise to anomalous $\mu\bar{\mu} \rightarrow b\bar{s}$. The FMC and the front-end of the FMC provide great opportunities to study such effects that are enhanced due to topcolor w.r.t. SM.

**Technicolor with Scalars**

Technicolor models that include scalars are an interesting class of models for dynamical EWSB. In the current model [6], in addition to SM fermions, one doublet of technifermions, $P$ and $N$, and three scalars, $\phi$, $\chi$ and $\Phi$ are considered. The gauge group considered is $SU(4)_{TC} \times SU(3)_C \times SU(2)_{EW} \times U(1)_Y$. Only the third generation couples to the technicolor fields, and, as in QCD, the $SU(4)_{TC}$ technicolor interactions trigger the formation of technifermion condensates $<P\bar{P}> \simeq <N\bar{N}> \simeq 2\sqrt{3}\pi f^3$, which breaks the electroweak symmetry at a scale $f$. This also results in the generation of masses for $t, b$ and $\tau$. The masses of the first and second generations are generated by coupling to a scalar $\Phi$ that behaves like a Higgs doublet under gauge transformations. The scalar acquires a small vacuum expectation value (VEV) by coupling to the new strong interactions sector and would have Yukawa couplings to the first and second generations which are larger than in the SM.

If this model is the correct description of physics up to a TeV scale, then the components of the $\Phi$ scalar should be accessible at a $\mu^+\mu^-$ collider with $\sqrt{s}$ below the first technihadron resonance. Since the Yukawa couplings are proportional to the fermion mass, the s-channel production is very large at a muon collider. The
scalar $\Phi$ decomposes into an isosinglet $\sigma$ and an isotriplet $\pi^3$, $a=1,2,3$. The neutral real scalar $\sigma$ and the charged scalars $\pi^\pm = (\pi^1 \mp i\pi^2)/\sqrt{2}$ are almost degenerate, with a mass $M_\Phi$. For $M_\Phi < \sqrt{s} < 2M_\Phi$, only the $\sigma$ and $\pi^3$, can be produced.

The total decay widths of the $\sigma$ and $\pi^3$ scalars are equal. The VEV of $\Phi$ is taken to be in the range,

$$1\, GeV \leq f' \leq 10\, GeV,$$

where the lower bound is chosen to avoid Yukawa coupling constants larger than 1.0, and upper bound is chosen to satisfy condition $f' << f$. The width for decay into pairs of gauge bosons, $\Gamma(W^+W^- + ZZ)$, is at most a few percent of the width for $\sigma, \pi^3 \rightarrow c\bar{c}$, and is neglected here. The widths of the $\sigma$ and $\pi^3$ scalars are dominated only by the $c\bar{c}$ final state:

$$\Gamma \approx \frac{3m^2 M_\Phi}{8\pi f'^2} \approx 13.2\, GeV \left( \frac{3\, GeV}{f'} \right)^2 \left( \frac{M_\Phi}{500\, GeV} \right)^2.$$

(9)

Given the enhanced couplings to the second generation, the $s$-channel production of the neutral scalars at a $\mu^+\mu^-$ collider is large. The natural spread in the muon collider beam energy, $\sigma\sqrt{s}$, is rather small, and can be ignored in computing the effective $s$-channel resonance cross section:

$$\bar{\sigma}(\mu^+\mu^- \rightarrow \sigma, \pi^3 \rightarrow c\bar{c}) \approx \frac{4\pi \Gamma^2}{(s-M_\Phi^2)^2 + M_\Phi^2 \Gamma^2} B(\sigma, \pi^3 \rightarrow \mu^+\mu^-) B(\sigma, \pi^3 \rightarrow X).$$

(10)

For the final state is $X \equiv c\bar{c}$, this cross section becomes,

$$\bar{\sigma}(\mu^+\mu^- \rightarrow \sigma, \pi^3 \rightarrow c\bar{c}) \approx \frac{4\pi \Gamma^2}{(s-M_\Phi^2)^2 + M_\Phi^2 \Gamma^2} \left( \frac{m^2_\mu}{3m^2_c} \right).$$

(11)

The main background comes from $\mu^+\mu^- \rightarrow \gamma^*, Z^* \rightarrow c\bar{c}$, and amounts to

$$\sigma(\mu^+\mu^- \rightarrow c\bar{c}) \approx 0.7 pb \left( \frac{500\, GeV}{s} \right)^2.$$

(12)

The discovery potential of a $\mu^+\mu^-$ collider operating at a maximum center of mass energy of 500 GeV has been studied. Two scan points, at 300 and 500 GeV, are sufficient to find the neutral scalars with masses roughly between 200 and 600 GeV.

Once the resonance is found, the beam energy can be adjusted to the peak (even if this requires a significant reduction in the luminosity) and then the production cross section becomes very large:

$$\bar{\sigma}(\mu^+\mu^- \rightarrow \sigma, \pi^3 \rightarrow c\bar{c}) \approx \frac{8\pi}{M_\Phi^2} \left( \frac{m^2_\mu}{3m^2_c} \right) \approx 80 pb \left( \frac{500\, GeV}{M_\Phi} \right)^2.$$

(13)

With a luminosity of $2 \times 10^{34} \text{cm}^{-2}\text{s}^{-1}$, and a $c$-tagging efficiency of 30%, the observed rate should be $10^7$ ($\sim 2 \times 10^5$) events per year.
BESS Model Study of SEWS

The BESS model is an effective Lagrangian parametrization approach to the symmetry breaking mechanism. The symmetry group of the theory is $G' = SU(2)_L \times SU(2)_R \times SU(2)_V$, where $SU(2)_V$ is the hidden symmetry through which new vector particles are introduced. The spontaneous breakdown of the symmetry group $G' \rightarrow SU(2)$ gives rise to six Goldstone bosons. Three of these are absorbed by new vector particles while the other three give mass to the SM gauge bosons when gauging of the subgroup $SU(2)_L \times SU(2)_V \subset G$ is performed.

The parameters of the BESS model are the masses of the new bosons $M_V$, their self-coupling $g''$, and a parameter $b$ that characterizes the coupling strengths of $V$ to the fermions. Taking $b \rightarrow 0$ and $g'' \rightarrow \infty$, the new bosons decouple and the SM is recovered. Bounds on the parameter space obtained by an analysis of $d\sigma(\ell^+\ell^- \rightarrow W_{L,T}^+W_{L,T}^-)/d\cos \theta$ ($\theta$ being the scattering angle of the $W$ in the center of mass), are shown in Fig. 6. The solid lines show the case relevant to an $e^+e^-$ machine with $\sqrt{s}=500$ GeV and $\int L dt = 20 fb^{-1}$, the dashed lines correspond to a $\mu^+\mu^-$ machine with same $\sqrt{s}$ and luminosity. The $\mu^+\mu^-$ collider provides some improvement in the bounds. The result for LHC with $pp \rightarrow W^\pm V^\mp \rightarrow W^\pm Z$ is shown by dot-dash curves for comparison.

Partial wave unitarity bounds from WW scattering deduced in the $(M_V, g/g'')$ and $(\Gamma_V, M_V)$ planes (see Fig. 7) imply that one or more of the heavy vector resonances should be discovered at the LHC, NLC or a $\sqrt{s} \sim 500$ GeV muon collider or, for certain, at a 3-4 TeV muon collider, unless $g''$ is very large and $b$ is very small so that they are largely decoupled.

Since the workshop, the production of the lightest neutral PNGB ($P^0$) in the $s$-channel, and the potential for discovering it at the FMC have also been studied using an extension of the model with $SU(8) \times SU(8)$ symmetry [8].

Search for Technicolor at CDF

The CDF collaboration reported on their search for technipion and technirho signals in the $W + 2$ jets + $b$-tag channel in the Run I data. The signatures sought are for the processes:

$$q\bar{q} \rightarrow W^{\pm*} \rightarrow \rho^\pm_T \rightarrow W^\pm \pi^0_T$$

and

$$q\bar{q} \rightarrow Z^*, \gamma^* \rightarrow \rho^0_T \rightarrow W^\pm \pi^\mp_T$$

with $W^\pm \rightarrow \ell\nu$ ($\ell = e$ or $\mu$) and $\pi^0_T \rightarrow b\bar{b}$, $\pi^\pm_T \rightarrow b\bar{c}$, $c\bar{b}$ ($\approx 95\%$) and $\pi^\mp_T \rightarrow c\bar{s}$, $s\bar{c}$ ($\approx 5\%$).

The candidate event selection requires an isolated electron (muon) with $E_T(p_T) > 20$ GeV within $|\eta| < 1.0$, $E_T > 20$ GeV and two or more jets with $E_T > 15$ GeV. At
least one of the jets is required to be a $b$-jet, tagged by the silicon vertex detector (SVX). The $Z$ boson candidates are rejected by requiring $|M_{ee} - M_Z| > 15$ GeV/$c^2$. A total of 42 events are selected while the expected number of background from $W^{b\bar{b}}$, $W^{c\bar{c}}$, $W^c$, top production, mis-tags, $Z$+heavy flavor amount to $31.6 \pm 4.3$ events.

The technicolor signal is modeled using PYTHIA MC and GEANT-based detector simulation. Signal MC events are generated at a number of $(\pi_T, \rho_T)$ mass values. The combinations with more than 5 pb cross section are used. The technicolor model parameters used are the ones from ref \cite{24}. Further cuts on kinematic variables $\Delta \phi(jj)$ (the azimuthal angle between two jets) and $p_T(jj)$ ($p_T$ of the dijet system) \cite{5} are employed to enhance the expected signal to background ratio in the selected sample. Finally, $M(jj)$ and $M(Wjj)$ are required to be within $\pm 3\sigma$ of the expected mean values for the signal. No significant excess is seen in the data. The 95% C.L. upper limits on the production cross section then exclude certain region of the $(M(\pi_T), M(\rho_T))$ plane as shown in Fig. 8.

**PROBING MUON COMPOSITNESS AT THE FMC**

The generational pattern of quarks and leptons hints possibly at a substructure (with an associated strong interaction at energy scale $\Lambda$) that might manifest at high energies. The existence of such substructure, however, is expected to result in four-fermion “contact” interactions which differ from those arising from the SM, at energies well below $\Lambda$. The signals can be sought in a number of ways—exclusive
FIGURE 8. The 95% C.L. exclusion region in (\(M(\pi_T), M(\rho_T)\)) plane. Some production cross section contours are also shown.
jet production, Drell-Yan production, Bhabha scattering etc. CELLO at the $e^+e^-$ collider PETRA with a $\sqrt{s}=35$ GeV and $\int L dt=86$ pb$^{-1}$ was able to set a limit on the electron compositeness scale $\sim$2-4 TeV using Bhabha scattering. These limits are similar to the ones from the Tevatron ($p\bar{p}$, $\sqrt{s} = 1.8$ TeV) [26]. Clearly, the lepton colliders seem to hold great potential for probing lepton compositeness. Probing the muon compositeness using Bhabha scattering measurements and the reach attainable as a function of $\sqrt{s}$ at the muon colliders has been investigated by Eichten and Keller [10].

The four-fermion contact interaction is assumed to be described by the effective Lagrangian proposed by Eichten, Lane and Peskin [27]:

$$\mathcal{L} = \frac{g^2}{2\Lambda^2}[\eta_{LL}j_Lj_L + \eta_{RR}j_Rj_R + \eta_{LR}j_Lj_R]$$ (14)

where $j_L$ and $j_R$ are the left-handed and right-handed currents, respectively; $\Lambda$ is the compositeness scale and $\frac{g^2}{4\pi}=1$ is assumed (strong coupling). The quantity $\eta$ is used to set the sign of the coupling i.e., $|\eta| = \pm 1$. Four typical coupling scenarios are considered in the present work: $LL(\eta_{LL} = \pm 1, \eta_{RR} = \eta_{LR} = 0), RR(\eta_{RR} = \pm 1, \eta_{LL} = \eta_{LR} = 0), VV(\eta_{LL} = \eta_{RR} = \eta_{LR} = \pm 1)$ and $AA(\eta_{LL} = \eta_{RR} = -\eta_{LR} = \pm 1)$. The angular distribution of scattered muons (scattering angle $\theta$) are then calculated for each of the models, with and without compositeness hypothesis. The fractional change in the differential cross section due to compositeness,

$$\Delta = \frac{\frac{d\sigma}{dcos\theta}}{\frac{d\sigma}{dcos\theta}}$$

is shown in Figures. 9 and 10 for the four different models and for both signs of $\eta$'s. The plots are made with $\Lambda$ chosen to provide an average correction of 10% due to compositeness.

The 95% C.L. limits on the compositeness scale $\Lambda$ are computed by employing an analytical approach that approximates $\chi^2$ fitting of ideal data to theory. The limits extracted for various $\sqrt{s}$ of the muon collider and for various models, together with the expected limits attainable at LEP are shown in Table 1. Since the detectors at a muon collider may not provide coverage down to small angles due to large

| $\sqrt{s}$ (in GeV) | LEP(91) | LEP(175) | 100 | 200 | 350 | 500 | 4000 |
|---------------------|---------|----------|-----|-----|-----|-----|------|
| $L(fb^{-1})$        | .15     | .1       | .6  | 1.  | 3.  | 7.  | 450. |
| LL                  | 4.0     | 5.8      | 4.8 | 10  | 20  | 29  | 243  |
| RR                  | 3.8     | 5.7      | 4.9 | 10  | 19  | 28  | 228  |
| VV                  | 6.9     | 12.      | 12  | 21  | 36  | 54  | 435  |
| AA                  | 3.8     | 7.2      | 12  | 13  | 21  | 32  | 263  |
FIGURE 9. The variable $\Delta$ versus $\cos \theta$ at $\sqrt{s}=100$ GeV for the four models, LL, RR, VV, and AA, for the two signs of the $\eta$'s, indicated by + and − on the plot.

FIGURE 10. The variable $\Delta$ versus $\cos \theta$ at $\sqrt{s}=500$ GeV for the four models, LL, RR, VV, and AA, for the two signs of the $\eta$'s, indicated by + and − on the plot.

TABLE 2. 95% CL limits (in TeV) for different on the scattering angle $\theta$ cuts ($\sqrt{s} = 500$ GeV, $\mathcal{L} = 7 fb^{-1}$).

| $|\cos \theta|$ | 6 | 8 | 9 | .95 |
|----------------|---|---|---|----|
| LL             | 26| 29| 31| 32 |
| RR             | 24| 28| 30| 30 |
| VV             | 50| 54| 56| 57 |
| AA             | 28| 32| 34| 35 |
CONSTRAINTS FROM RARE B AND K DECAYS

The new strong dynamics scenario for EWSB or for the origin of fermion masses can produce sizeable effects in low energy observables at energies much smaller than the scale of new physics. Such effects in rare $B$ and $K$ decays have been studied by Burdman [12], in the framework of an effective Lagrangian Model. These effects in FCNC processes seem to originate from the insertion of anomalous triple gauge boson coupling vertices and four-fermion operators.

In the four-fermion operator scenario, it has been shown that branching ratios for $B \rightarrow q\ell^+\ell^-$, $b \rightarrow q\nu\bar{\nu}$, $b \rightarrow qq'q'$ can have large deviations (up to a factor of $\sim 2$) from the SM expectations. However, no significant deviation is expected in $b \rightarrow s\gamma$ decay. The effects are very similar in rare $K$ decays such as $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and $K_L \rightarrow \pi^0\nu\bar{\nu}$. The effective Lagrangian approach for non-SM couplings of fermions to gauge bosons has been examined in the topcolor class of theories. The presence of the relatively light top-pions, and other additional bound states, imposes severe constraints on the topcolor models due to their potential loop effects in low energy observables such as $R_b$ and rare $B$ and $K$ decay rates. These depend not only on $f_{\pi_t}$ and $m_{\pi_t}$, but typically also on one or more elements of the quark rotation matrices necessary to diagonalize the quark Yukawa couplings. So, it can be shown for example, for $f_{\pi_t} \approx 120$ GeV, the effect of a 400 GeV $\pi_t$ in $b \rightarrow s\ell^+\ell^-$ is an enhancement of more than 5% with respect to SM expectations. Similar effects are expected to be present in $K^+ \rightarrow \pi^+\nu\bar{\nu}$. Thus, the measurements of $R_b$ and the rare $B$ and $K$ decay modes can constrain strong dynamics models such as the topcolor model.

SUMMARY

We have reviewed various theories that currently offer to explain the breaking of electroweak symmetry dynamically. In particular, technicolor and related theories have been examined in detail. Direct searches for signals of new strong dynamics as well as indirect tests for existence of strong dynamics, at the FMC, have been studied. Long-range opportunities at the high energy muon colliders (BMC with $\sqrt{s}=3-4$ TeV) are examined. The experimental prospects for strong dynamics at the FMC can be summarized as follows:

- If low energy technicolor signatures (extended walking technicolor, topcolor-assisted technicolor) exist, they would be found at the Tevatron or at the LHC. In this case, the first muon collider will be a remarkable facility to
make detailed studies and precision measurements. The narrow neutral technihadrons—π_T, ρ_T and ω_T—would appear as spectacular resonances at the FMC (√s=100–200 GeV and energy resolution σ_E ≤ 10^{-4}). One can operate on the resonance and study all the decays and branching fractions of the technirhos, techniomegas and technipions. We emphasize that the all-hadronic modes would be very difficult to study at the hadron colliders.

- The good beam resolution achievable at the FMC with √s ∼ 200 GeV would enable studies of ρ_T − ω_T interference effects in detail, using fermion-antifermion final states.

- A variety of other models such as technicolor with scalars and top see-saw model predict s-channel resonance production of new particles. Muon collider has a big advantage over other colliders to discover and study these particles.

- Compositeness tests using Bhabha scattering give reaches of several tens of TeV at the FMC. At the big muon collider, the reaches for 95% confidence level limits on compositeness scale are unparalelled, far exceeding the reaches possible at any other collider. The reaches would be of the order of 200-300 TeV, at which scale we would be probing the structure of the dynamics that gives rise to fermion masses and mixings.

- Studies of rare B and K decays using the front-end of the FMC can provide tight constraints on strong dynamics and help distinguish between universal (EWSB sector) vs. non-universal (flavor dynamics) scenarios.

These are extremely strong physics motivations to build the first muon collider. A full exploration of the strong electroweak sector can be accomplished at the big muon collider with √s=3-4 TeV.

REFERENCES

1. S. Weinberg, Phys. Rev. D 19, 1277 (1979); L. Susskind, Phys. Rev. D 20, 2619 (1979).
2. K. Lane, these proceedings.
3. C. T. Hill, Phys. Lett. B 345, 483 (1995).
4. C. Hill, these proceedings.
5. J. Womersley, these proceedings.
6. B. Dobrescu, these proceedings.
7. R. Casalbuoni, S. De Curtis, D. Dominici, and R. Gatto, Phys. Lett. B 155, 95 (1985); Nucl. Phys. B 282, 235 (1987).
8. D. Dominici et al., these proceedings.
9. T. Handa, K. Maeshima, J. Valls, R. Vilar (for the CDF collaboration), these proceedings.
10. E. Eichten and S. Keller, these proceedings.
11. J. Gunion, these proceedings.
12. G. Burdman, these proceedings.
13. S. Dimopoulos and L. Susskind, *Nucl. Phys.* B **155**, 237 (1979); E. Eichten and K. Lane, *Phys. Lett.* B **90**, 125 (1980).
14. B. Holdom, *Phys. Rev.* D **24**, 1441 (1981); *Phys. Lett.* B **150**, 301 (1985); T. Appelquist, D. Karabali and L. C. R. Wijewardhana, *Phys. Rev. Lett.* **57**, 957 (1986); T. Appelquist and L. C. R. Wijewardhana, *Phys. Rev.* D **36**, 568 (1987); K. Yamawaki, M. Bando and K. Matumoto, *Phys. Rev. Lett.* **56**, 1335 (1986); T. Akiba and T. Yanagida, *Phys. Lett.* B **169**, 432 (1986).
15. K. Lane and E. Eichten, *Phys. Lett.* B **352**, 382 (1995); K. Lane, *Phys. Rev.* D **54**, 2204 (1996).
16. F. Abe, *et al.*, The CDF Collaboration, *Phys. Rev. Lett.* **74**, 2626 (1995).
17. S. Abachi, *et al.*, The DØ Collaboration, *Phys. Rev. Lett.* **74**, 2632 (1995).
18. F. Abe, *et al.*, The CDF Collaboration, *Phys. Rev. Lett.* **??**, ?? (1997).
19. S. Abachi, *et al.*, The DØ Collaboration, *Phys. Rev. Lett.* **79**, 1197 (1997).
20. Y. Nambu, in *New Theories in Physics*, Proceedings of the XI International Symposium on Elementary Particle Physics, Kazimierz, Poland, 1988, edited by Z. Ad- juk, S. Pokorski and A. Trautmann (World Scientific, Singapore, 1989); Enrico Fermi Institute Report EFI 89-08 (unpublished); V. A. Miransky, M. Tanabashi and K. Yamawaki, *Phys. Lett.* B **221**, 171 (1989); *Mod. Phys. Lett.* A4, 1043 (1989); W. A. Bardeen, C. T. Hill and M. Lindner, *Phys. Rev.* D **41**, 1647 (1990).
21. E. Eichten, K. Lane and J. Womersley, “Narrow Technihadron Production at the First Muon Collider”, in preparation.
22. E. Eichten, I. Hinchliffe, K. Lane and C. Quigg, *Rev. Mod. Phys.* **56**, 579 (1984); *Phys. Rev.* D **34**, 1547 (1986).
23. K. Lane and E. Eichten, *Phys. Lett.* B **222**, 274 (1989); K. Lane and M. V. Ramana, *Phys. Rev.* D **44**, 2678 (1991).
24. E. Eichten and K. Lane, *Phys. Lett.* B **388**, 803 (1996).
25. E. Eichten, K. Lane and J. Womersley, *Phys. Lett.* B **405**, 305 (1997).
26. F. Abe, *et al.*, The CDF Collaboration, *Phys. Rev. Lett.* **79**, 2198 (1997).
27. E. Eichten, K. Lane and M. Peskin, *Phys. Rev. Lett.* **50**, 811 (1983).
28. P. Lebrun and R. Roser, these proceedings.