Abstract. After a brief update on the prospects for dark matter in the constrained version of the MSSM (CMSSM) and its differences with models based on minimal supergravity (mSUGRA), I will consider the effects of unifying the supersymmetry-breaking parameters at a scale above $M_{GUT}$. One of the consequences of superGUT unification, is the ability to take vanishing scalar masses at the unification scale with a neutralino LSP dark matter candidate. This allows one to resurrect no-scale supergravity as a viable phenomenological model.

1. Introduction

While often used synonymously, the constrained minimal supersymmetric standard model (CMSSM) [1, 2, 3, 4, 5, 6] differs in two important ways from supersymmetric models based on minimal supergravity (mSUGRA) [7, 8, 9, 10]. The latter class of theories is in fact a subset of the former and can be thought of as a very constrained version of the theory [11]. The often studied CMSSM, is a 4+ parameter theory. Starting with the superpotential,

$$W = \left[ y_e H_1 L^e + y_d H_1 Q^d + y_u H_2 Q^u \right] + \mu H_1 H_2, \quad (1)$$

we can obtain the soft supersymmetry-breaking part of the Lagrangian

$$\mathcal{L}_{\text{soft}} = -\frac{1}{2} M_\alpha \lambda^\alpha \phi^\alpha - m^2_{ij} \phi^i \phi^j - A_e y_e H_1 L^e - A_d y_d H_1 Q^d - A_u y_u H_2 Q^u - B \mu H_1 H_2. \quad (2)$$

In the CMSSM, the four parameters are: the gaugino mass, $M_\alpha = m_1/2$, unified at some high energy input scale, $M_{in}$, usually assumed to be the grand unified (GUT) scale, $M_{GUT}$; the scalar masses $m^2_{ij} = \delta_{ij} m^2_0$; and the trilinear terms, $A_f = A_0$, are all unified at the same input scale, $M_{in}$; and the ratio of the two Higgs expectation values, $\tan \beta$. In the CMSSM, $|\mu|$ and $B$ are determined by the electroweak symmetry-breaking (EWSB) conditions by fixing $M_Z$ and $\tan \beta$. In mSUGRA models, there is an additional constraint, namely

$$B_0 = A_0 - m_0. \quad (3)$$

In this case, it is no longer possible to satisfy the EWSB conditions (i.e., the minimization of the Higgs potential) and choose $\tan \beta$ independently. As a consequence, we have a 3+ parameter theory which is now specified by $m_1/2$, $m_0$, and $A_0$. The “+” in the number of parameters refers to the sign of $\mu$. While the magnitude of $\mu$ is fixed by the EWSB conditions, its sign is left undetermined. In true minimal supergravity models, there is also a relation between $m_0$ and the
the gravitino mass, namely $m_0 = m_3/2$. Thus, the gravitino mass is no longer independent and as we will see, often in mSUGRA models, the gravitino is found to be the lightest supersymmetric particle (LSP). The gravitino mass is generally not counted as one of the parameters in the CMSSM, as it can be set to a sufficiently large value so as to render it irrelevant.

No-scale supergravity models [12] are still more restrictive, as the boundary conditions on the scalar masses and bi- and trilinear supersymmetry-breaking terms become

$$m_0 = A_0 = B_0 = 0.$$  \hspace{1cm} (4)

Thus, we are left with a 1+ parameter theory specified by $m_{1/2}$. Generally, the condition $m_0 = 0$ is largely incompatible with phenomenology except for a restricted set of values of tan $\beta$, but might be more viable if the supersymmetry-breaking scale were pushed above the GUT scale [13].

Below, I will try to highlight some of the differences between the CMSSM and mSUGRA models and explore the phenomenological consequences of pushing the scale at which the supersymmetry-breaking parameters are unified above the GUT scale [14, 15]. After a brief review on the status of the CMSSM, and its differences with mSUGRA, I will demonstrate the difficulties with no-scale supergravity models. Then, I will describe the effects of raising the supersymmetry-breaking scale above the GUT scale and apply this to no-scale models.

2. The CMSSM

For given values of tan $\beta$, $A_0$, and $sgn(\mu)$, the regions of the CMSSM parameter space that yield an acceptable relic density and satisfy other phenomenological constraints may be displayed in the $(m_{1/2}, m_0)$ plane. In Fig. 1a, the dark (blue) shaded region corresponds to that portion of the CMSSM plane with tan $\beta = 10$, $A_0 = 0$, and $\mu > 0$ such that the computed relic density yields the WMAP value [16] of

$$\Omega h^2 = 0.111 \pm 0.006.$$  \hspace{1cm} (5)

The bulk region at relatively low values of $m_{1/2}$ and $m_0$, tapers off as $m_{1/2}$ is increased. At higher values of $m_0$, annihilation cross sections are too small to maintain an acceptable relic density and $\Omega h^2$ is too large. At large $m_{1/2}$, co-annihilation processes between the LSP and the next lightest sparticle (in this case the $\tilde{\tau}$) enhance the annihilation cross section and reduce the relic density. This occurs when the LSP and NLSP are nearly degenerate in mass. The dark (red) shaded region has $m_{\tilde{\tau}} < m_{\chi}$ and is excluded. The effect of coannihilations is to create an allowed band about 25-50 GeV wide in $m_0$ for $m_{1/2} \lesssim 950$ GeV, or $m_{\chi} \lesssim 400$ GeV, which tracks above the $m_{\chi} = m_{\chi}$ contour [17]. Also shown in the figure are some phenomenological constraints from the lack of detection of charginos [18], or Higgses [19] as well as constraints from $b \rightarrow s\gamma$ [20] and $g_\mu - 2$ [21]. The locations of these constraints are described in the caption.

At larger $m_{1/2}, m_0$ and tan $\beta$, the relic neutralino density may be reduced by rapid annihilation through direct-channel $H, A$ Higgs bosons, as seen in Fig. 1(b) [1, 3]. Finally, the relic density can again be brought down into the WMAP range at large $m_0$ in the ‘focus-point’ region close the boundary where EWSB ceases to be possible and the lightest neutralino $\chi$ acquires a significant higgsino component [22]. The start of the focus point region is seen in the upper left of Fig. 1b.

A global likelihood analysis enables one to pin down the available parameter space in the CMSSM. One can avoid the dependence on priors by performing a pure $\chi^2$-based fit as done in [6] which used a Markov-Chain Monte Carlo (MCMC) technique to explore efficiently the likelihood function in the parameter space of the CMSSM.

The best fit point is shown in Fig. 2, which also displays contours of the $\Delta \chi^2$ function in the CMSSM (solid curves outline the 68 and 95% CL regions). The parameters of the best-fit CMSSM point are $m_{1/2} = 310$ GeV, $m_0 = 60$ GeV, $A_0 = 130$ GeV, and tan $\beta = 11$, yielding
the overall $\chi^2/N_{\text{dof}} = 20.6/19$ (36% probability) and nominally $M_h = 114.2$ GeV [6]. The best-fit point is in the coannihilation region of the $(m_0, m_{1/2})$ plane. The C.L. contours extend to somewhat large values of $m_0$. However, the qualitative features of the $\Delta \chi^2$ contours indicate a preference for small $m_0$ and $m_{1/2}$.

**Figure 1.** The $(m_{1/2}, m_0)$ planes for (a) $\tan \beta = 10$ and $\mu > 0$, assuming $A_0 = 0, m_t = 173.1$ GeV and $m_0(m_0)^{\text{MS}} = 4.25$ GeV. The near-vertical (red) dot-dashed lines are the contours $m_h = 114$ GeV, and the near-vertical (black) dashed line is the contour $m_{\chi^\pm} = 104$ GeV. The medium (dark green) shaded region is excluded by $b \to s\gamma$, and the dark (blue) shaded area is the cosmologically preferred region. In the dark (brick red) shaded region, the LSP is the charged $\tilde{\tau}_1$. The region allowed by the E821 measurement of $g_\mu - 2$ at the 2-$\sigma$ level, is shaded (pink) and bounded by solid black lines, with dashed lines indicating the 1-$\sigma$ ranges. In (b), $\tan \beta = 55$.

**Figure 2.** The $\Delta \chi^2$ function in the $(m_0, m_{1/2})$ plane for the CMSSM. We see that the coannihilation region at low $m_0$ and $m_{1/2}$ is favored.
The frequentist analysis described above can also be used to predict the neutralino-nucleon elastic scattering cross section [6]. The value of $\sigma_p^{SI}$ shown in Fig. 3a is calculated assuming a $\pi$-N scattering $\sigma$ term $\Sigma_{\pi N} = 64$ MeV. We see in Fig. 3 that values of the $\chi^0$-proton cross section $\sigma_p^{SI} \sim 10^{-8}$ pb are expected in the CMSSM, and that much larger values seem quite unlikely. The 2D $\chi^2$ function in the $(m_{\chi}, \sigma_p)$ plane is shown in Fig. 3b.

![Figure 3](image_url)

**Figure 3.** The likelihood function for the spin-independent $\tilde{\chi}_1^0$-proton scattering cross section $\sigma_p^{SI}$ in the CMSSM (left panel). The correlation between the spin-independent DM scattering cross section $\sigma_p^{SI}$ and $m_{\tilde{\chi}_1^0}$ in the CMSSM (right panel).

### 3. mSUGRA Models

As noted above, mSUGRA models are a very constrained version of the MSSM, as $\tan \beta$ is no longer a free parameter. In minimal $N=1$ supergravity, the Kähler potential can be written as

$$G = K(\phi^i, \phi^*_i, z, z^*) + \ln(|W|^2)$$

with

$$K = \kappa^2(\phi^i \phi^*_i + zz^*)$$

where $W = f(z) + g(\phi)$ is the superpotential, assumed to be separable in hidden sector fields, $z$, and standard model fields, $\phi$. $\kappa^{-1} = M_P/\sqrt{8\pi}$ and the Planck mass is $M_P = 1.2 \times 10^{19}$ GeV.

The scalar potential can be derived once the superpotential is specified. The simplest choice for a single hidden sector field is a superpotential [7] $f(z) = m(z + b)$. Using this in Eq. (6) and dropping terms inversely proportional to the Planck mass, we can write [10]

$$V = \frac{\partial g}{\partial \phi}^2 + m_{3/2}^2(\phi \frac{\partial g}{\partial \phi} - \sqrt{3}g + h.c.) + m_{3/2}^2 \phi \phi^*,$$

where the vacuum expectation value $\langle \kappa z \rangle = \sqrt{3} - 1$ (with $\kappa b = 2 - \sqrt{3}$, chosen to cancel the vacuum energy density at the minimum) has been inserted and the superpotential has been rescaled by a factor $e^{-\kappa b}$.

The first term in Eq. (8) is the ordinary $F$-term part of the scalar potential of global supersymmetry. The next term, proportional to $m_{3/2}$ represents a universal trilinear $A$-term. This can be seen by noting that $\sum \phi \partial g/\partial \phi = 3g$, so that in this model of supersymmetry breaking, $A_0 = (3 - \sqrt{3})m_{3/2}$. Note that if the superpotential contains bilinear terms, we would find $B_0 = (2 - \sqrt{3})m_{3/2}$. The last term represents a universal scalar mass of the type advocated...
in the CMSSM, with $m_0^2 = m_Z^2/2$. The generation of such soft terms is a rather generic property of low energy supergravity models [23] and the relation $B_0 = A_0 - m_0$ is derived from the minimal form of the Kähler potential and does not depend on the specific form of $f(z)$.

The analogue of the CMSSM ($m_1 = 2$, $m_0$) plane for mSUGRA models is shown in Fig. 4 for two fixed values of $A_0/m_0$ [11]. In the left panel, the Polonyi model choice of $A_0/m_0 = 3 - \sqrt{3}$ is made. As one can see, each point on the plane corresponds to a specific value of $\tan \beta$ as specified by the dot-dashed contours. Also seen in this panel is a solid (brown) diagonal line. Below this line, the gravitino is the LSP, since $m_\chi \approx 0.43 m_{1/2} < m_0 = m_3/2$. Below this line, there is a diagonal dotted (red) line, which separates the region for which the next lightest supersymmetric particle is the neutralino (above) or the stau (below). The area below this line is not shaded since in principle, this area could be allowed as the stau is not stable. However, constraints from big bang nucleosynthesis excluded much of gravitino LSP region in mSUGRA models [24].

![cmssm_2.png](attachment:cmssm_2.png)

**Figure 4.** As in Fig. 1, the $(m_{1/2}, m_0)$ planes for (a) $A_0/m_0 = 3 - \sqrt{3}, B_0 = A_0 - m_0$, and $\mu > 0$. The dot-dashed contours show curves of constant $\tan \beta$. Below the dark (brown) line the gravitino is the LSP. In (b), $A_0/m_0 = 2$.

In the right panel of Fig. 4, $A_0/m_0 = 2$. In this case, the boundary of $m_{\tau} = m_\chi$ is above the boundary for $m_{\chi} = m_{3/2}$. As a consequence, there is now a viable co-annihilation strip with neutralino dark matter. Below that, there is an excluded region (shaded dark red) which has a stau LSP. Below the shaded region, the gravitino is again the LSP subject to BBN constraints [24]. Funnel and focus point regions are generally absent in mSUGRA models.

### 4. No-Scale Models

No-scale supergravity models [12] are characterized by a Kähler potential defined by

$$K = -3 \ln \left[ \kappa(z + z^* - \frac{1}{3} \kappa^2 \phi_i^\dagger \phi_i \right].$$

(9)

The scalar potential takes the simple form

$$V = e^{G - \frac{1}{4} K} \left| \frac{\partial g}{\partial \phi_i} \right|^2.$$

(10)
Thus one immediately finds that $m_0 = A_0 = B_0 = 0$, and the only source of supersymmetry breaking is transmitted through the gaugino masses (if the gauge kinetic function is a non-trivial function of $z$). In Fig. 5, an $(m_{1/2}, m_0)$ plane with $A_0/m_0 = B_0/m_0 = 0$ is shown. Like the mSUGRA model, specification of $B_0$ fixes $\tan \beta$ at each point on the plane. However, unlike mSUGRA models, the gravitino mass is rather arbitrary and can be set independently from other supersymmetry-breaking scales. No-scale models run along the $m_0 = 0$ axis of this plane. As one can see, along $m_0 = 0$, there is no way to achieve a sufficiently high relic density to match the WMAP determination and for $m_{1/2} \lesssim 150$ GeV, there is a problem with the branching ratio for $B_s \rightarrow X_s \gamma$, while at larger $m_{1/2}$, the LSP is either the stau or the gravitino. Furthermore, the LEP limit on the Higgs mass requires $m_{1/2} > \sim 340$ GeV. Thus it would appear that no-scale models are not phenomenologically viable.

\[ A_0/m_0 = 0; \mu > 0 \]
\[ B_0/m_0 = 0 \]

**Figure 5.** As in Fig. 1, the $(m_{1/2}, m_0)$ planes for (a) $A_0/m_0 = B_0/m_0 = 0$, and $\mu > 0$. The dot-dashed contours show curves of constant $\tan \beta$.

5. SuperGUT Models

In all of the models discussed above, it was assumed that the renormalization scale at which scalar and gaugino masses are unified, $M_{\text{in}}$, is the same scale at which gauge coupling unification occurs, $M_{\text{GUT}}$. This need not be the case. Indeed, since supersymmetry breaking may occur either below or above $M_{\text{GUT}}$, it is quite possible that $M_{\text{in}} \neq M_{\text{GUT}}$. For example, as $M_{\text{in}}$ is decreased below $M_{\text{GUT}}$, the differences between the renormalized sparticle masses diminish and the regions of the $(m_{1/2}, m_0)$ planes that yield the appropriate density of cold dark matter move away from the boundaries [25]. Eventually, for small $M_{\text{in}}$, the coannihilation and focus-point regions of the conventional GUT-scale CMSSM merge and for very small $M_{\text{in}}$ they disappear entirely.

On the other hand, increasing $M_{\text{in}}$ increases the renormalization of the sparticle masses which tends to increase the splittings between the physical sparticle masses [26]. Furthermore, this in turn has the effect of increasing the relic density in much of the $(m_{1/2}, m_0)$ plane. As a consequence, the coannihilation strip is squeezed to lower values of $m_{1/2}$ [27, 14], particularly
for $\tan \beta \sim 10$, and even disappears as $M_{\text{in}}$ increases. At the same time, the focus-point strip often moves out to ever larger values of $m_0$. The allowed region of parameter space that survives longest is the rapid-annihilation funnel at large $m_{1/2}$ and $\tan \beta$. In the CMSSM with $M_{\text{in}} = M_{\text{GUT}}$, the funnel region also requires large $m_0$ and would make a contribution to $g_\mu - 2$ that is too small to explain the experimental discrepancy with Standard Model calculations based on low-energy $e^+e^-$ data. However, for large $M_{\text{in}}$, the funnel region extends to low $m_0$ (including $m_0 = 0$) and in some cases will be compatible with the $g_\mu - 2$ measurements.

For superGUT models with $M_{\text{in}} > M_{\text{GUT}}$, one must specify in addition to Eq. (1), the GUT superpotential which for minimal SU(5) takes the form,

$$W_5 = \mu_5 Tr \hat{\Sigma}^2 + \frac{1}{6} \lambda' Tr \hat{\Sigma}^3 + \mu_H \hat{H}_{1a} \hat{H}_{2a}^c + \lambda \hat{H}_{1a} \hat{\Sigma}_{23} \hat{H}_{2a}^c + (\hat{h}_5)_{ij} \hat{\psi}_i^{\alpha} \hat{\psi}_j^{\beta} \hat{\phi}_a \hat{\phi}_a \hat{H}_{1a}. \quad (11)$$

where the $d^c$ and $L$ superfields of the MSSM reside in the $\Sigma$ representation, $\hat{\phi}_a$, while the $Q$, $u^c$ and $e^c$ superfields are in the $10$ representation, $\hat{\psi}_i$. $\Sigma(24)$ is the SU(5) adjoint Higgs multiplet and the two Higgs doublets of the MSSM, $H_1$ and $H_2$ are extended to five-dimensional SU(5) representations $\hat{H}_1(5)$ and $\hat{H}_2(5)$ respectively. There are now two new couplings: $\lambda$ and $\lambda'$. $\lambda$ affects directly the running of the soft Higgs masses, adjoint and Yukawa couplings, while $\lambda'$ affects only the adjoint. Accordingly there are also new soft masses and $\mu$ terms associated with SU(5).

A superGUT version of the CMSSM based on SU(5) is now a 7+ parameter theory [14] specified by $m_{1/2}, m_0, A_0, \tan \beta$, and $\text{sgn}(\mu)$ as in the CMSSM, as well as $M_{\text{in}}, \lambda$, and $\lambda'$. At $M_{\text{in}} > M_{\text{GUT}}$, the universality conditions become,

$$m_{\Sigma,1} = m_{10,1} = m_{\Sigma} = m_{h1} = m_{h2} = m_{\Sigma} \equiv m_0,$n$$

$$A_{\Sigma} = A_{10} = A_{\lambda} = A_{\lambda'} \equiv A_0,$n$$

$$M_5 \equiv m_{1/2}. \quad (12)$$

These parameters are evolved down to $M_{\text{GUT}}$ and matched to their Standard Model CMSSM counterparts.

Some examples of $(m_{1/2}, m_0)$ planes in a superGUT model are shown in Fig. 6 for $\tan \beta = 10$ and two values of $M_{\text{in}}$ and the specific choices $\lambda = 1, \lambda' = 0.1$ [14]. These results are quite insensitive to the value of $\lambda'$ and can be compared with the CMSSM plane shown in Fig.1a. For $M_{\text{in}} = 2.5 \times 10^{16}$ GeV, we see two dramatic effects from the modest increase in $M_{\text{in}}$. One is the rapid disappearance of the stau LSP region, which has retreated to $m_{\tilde{\tau}} < 0$. In the particular example shown, the coannihilation strip extends to $m_{1/2} \sim 450$ GeV, and there is a healthy portion compatible with the $g_\mu - 2$ constraint. Note that the chargino, $m_h$, $g_\mu - 2$ and $b \to s\gamma$ constraints are relatively stable in the $(m_{1/2}, m_0)$ plane with respect to changes in $M_{\text{in}}$. The other noticeable feature in panel (a) of Fig. 6 is the retreat of the EWSB constraint to smaller $m_{1/2}$ and larger $m_0$. This effect is quite sensitive to the value of $\lambda$, whereas the fate of the coannihilation region is relatively insensitive to its value.

For the choice $M_{\text{in}} = 10^{17}$ GeV, shown in panel (b) of Fig. 6, these effects are more pronounced: both the coannihilation and the focus-point strips have disappeared entirely. There is a small piece of the $(m_{1/2}, m_0)$ plane where the relic density falls within the WMAP range, but this is incompatible with $m_h$ and gives too large a value of $g_\mu - 2$.

For $\tan \beta = 55$, we see in Fig. 7 that as $M_{\text{in}}$ increases, the renormalization effects cause the stau LSP region to retreat as in the $\tan \beta = 10$ case, though more slowly, and it does not disappear entirely, even for $M_{\text{in}} = 2.4 \times 10^{18}$ GeV. Likewise, whilst the coannihilation strip shrinks with increasing $M_{\text{in}}$, it does not disappear, and much of it remains consistent with $m_h$.
**Figure 6.** As in Fig. 1, the \((m_{1/2}, m_0)\) planes for \(A_0 = 0\), \(\tan \beta = 10\), \(\lambda = 1\) and \(\lambda' = 0.1\) for different choices of \(M_{\text{in}}\): (a) \(2.5 \times 10^{16}\) GeV and (b) \(10^{17}\) GeV.

\(b \to s\gamma\) and \(g_\mu - 2\). The rapid-annihilation funnel also persists as \(M_{\text{in}}\) increases, staying in a similar range of \(m_{1/2}\), but shifting gradually to lower values of \(m_0\). In particular, we note that for the case \(M_{\text{in}} = 2.4 \times 10^{18}\) GeV, the no-scale possibility \(m_0 = 0\) [12, 13] is allowed, on one or both flanks of the rapid-annihilation funnel. Finally, we note that the EWSB boundary disappears entirely for the displayed choices of \(M_{\text{in}} > M_{\text{GUT}}\), as does the focus-point WMAP strip.

**Figure 7.** As in Fig. 6, for \(A_0 = 0\), \(\tan \beta = 55\), \(\lambda = 1\) and \(\lambda' = 0.1\) for different choices of \(M_{\text{in}}\): (a) \(10^{17}\) GeV and (b) \(2.4 \times 10^{18}\) GeV.
6. No-Scale Models Resurrected

As discussed above, no-scale models with GUT scale universality conditions are not phenomenologically viable. However, we have also seen when the universality scale is increased above the GUT scale, no-scale requirements such as \( m_0 = 0 \) may yield satisfactory models \([13, 28, 15]\).

A no-scale superGUT model could be constructed from the same superpotential (11) with the additional requirements that \( m_0 = A_0 = B_0 = 0 \) at \( M_{in} \). The latter condition translates to \( B_\Sigma = B_H = B_0 = 0 \) at \( M_{in} \). And it would appear therefore that we now have a 4+ parameter theory specified by \( m_1, M_{in}, \lambda, \lambda' \), and \( sgn(\mu) \). However, the model really only depends on the ratio of the two Higgs couplings, \( \lambda/\lambda' \) and we are in fact left with a 3+ parameter theory.

As in the mSUGRA models discussed above, \( \tan \beta \) is determined from the EWSB conditions though the boundary condition is now \( B_0 = 0 \). Since this condition is applied at \( M_{in} \), the MSSM (subGUT) \( B \) parameter must be matched to the GUT \( B \) parameters, \( B_\Sigma \) and \( B_H \) \([29]\),

\[
B = B_H - \frac{6\lambda}{\mu \lambda'} [(2B_\Sigma - A'_\lambda)(2B_\Sigma - A'_\lambda) + m_\Sigma^2],
\]

(13)

In Fig. 8, some examples of \( (m_{1/2}, M_{in}) \) planes are shown for fixed \( \lambda' = 2 \) with \( \lambda = 0, -0.14, -0.15, \) and -0.16 \([15]\). We see that in the first panel of Fig. 8 the WMAP-compatible region takes the form of a thin L-shaped strip in the \( (m_{1/2}, M_{in}) \) plane with a rounded corner: points above and to the right of the L have values of \( \Omega h^2 \) that are too large. The near-horizontal part of the line is located at \( M_{in} \sim 5 \times 10^{16} \) GeV and extends from \( m_{1/2} \sim 200 \) GeV to \( \sim 1000 \) GeV, larger values being excluded by the requirement that the LSP not be charged, and we find that \( \tan \beta \in (16, 30) \). All the base strip is compatible with the LEP chargino constraint, and with \( b \to s\gamma \). However, only the portion with \( m_{1/2} \gtrsim 300 \) GeV is compatible with the LEP lower limit on \( m_h \). The near-vertical part of the no-scale strip is always incompatible with the LEP Higgs constraint and (mostly) \( b \to s\gamma \).

Panel (b) of Fig. 8 for \( \lambda = -0.14 \) has a rather different appearance, but is in fact a natural continuation of the trends seen in the previous panel. In particular, the near-vertical part of the WMAP-compatible strip has moved to larger \( m_{1/2} \sim 400 \) GeV, and is compatible with both \( m_h \) and \( b \to s\gamma \), and the near-horizontal part of the strip has risen to \( M_{in} \sim 3 \times 10^{17} \) GeV. More dramatically, the WMAP-compatible strip now becomes a loop, with a right side connecting
Figure 8. The \((m_{1/2}, M_{\text{in}})\) planes for the no-scale supergravity model with \(\lambda' = 2\) and (a) \(\lambda = 0\), (b) \(\lambda = -0.14\), (c) \(\lambda = -0.15\) and (d) \(\lambda = -0.16\). Shading and line types are as in Fig. 1 with the exception that in the orange region we find no consistent solutions to the RGEs.

the previous two strips at relatively large \(m_{1/2}\) and \(M_{\text{in}}\) and \(\tan \beta \gtrsim 50\) (though the loop closes only when \(M_{\text{in}} > M_P\)). We emphasize that all of the loop has a neutralino LSP, and that the stau-LSP contour surrounds this loop: the region within the loop has too much dark matter.

In panel (c) of Fig. 8 for \(\lambda = -0.15\) we see just a ‘blob’ with \(m_{1/2} \in (500, 650)\) GeV and \(M_{\text{in}} \in (5 \times 10^{17}, 3 \times 10^{18})\) GeV on the edge of the region preferred by \(g_\mu - 2\). This remaining ‘blob’ disappears for larger \(-\lambda\) as shown in panel (d) for \(\lambda = -0.16\). Here, the area within the ‘window’ is phenomenologically allowed, though the relic density lies below the WMAP range.

In summary, we have seen that the CMSSM and mSUGRA are in fact different theories (despite their interchangeable use in the literature). In mSUGRA, \(\tan \beta\) is generally small and funnel regions with the correct relic density do not appear. Focus point regions are also absent due to either a large value of \(A_0/m_0\) or small \(\tan \beta\). In addition, the gravitino is often the LSP in mSUGRA models. For the specific case, of no-scale supergravity with GUT scale universality, we have seen that the model is phenomenologically challenged. For \(M_{\text{in}} > M_{\text{GUT}}\), no-scale supergravity models can however be resurrected. An upcoming challenge will be to differentiate between these models once data from the LHC or direct detection experiments become available.

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References
[1] M. Drees and M. M. Nojiri, Phys. Rev. D 47 (1993) 376 [arXiv:hep-ph/9207234]; H. Baer and M. Brhlik, Phys. Rev. D 53 (1996) 597 [arXiv:hep-ph/9608321]; Phys. Rev. D 57 (1998) 567 [arXiv:hep-ph/9706509]; H. Baer, M. Brhlik, M. A. Diaz, J. Ferrandis, P. Mercadante, P. Quintana and X. Tata, Phys. Rev. D 63 (2001) 015007 [arXiv:hep-ph/0005027].
[2] J. R. Ellis, T. Falk, K. A. Olive and M. Schmitt, Phys. Lett. B 388 (1996) 97 [arXiv:hep-ph/9607292]; Phys. Lett. B 413 (1997) 355 [arXiv:hep-ph/9705444]; J. R. Ellis, T. Falk, G. Ganis, K. A. Olive and M. Schmitt, Phys. Rev. D 58 (1998) 095002 [arXiv:hep-ph/9801445]; V. D. Barger and C. Kao, Phys. Rev. D 57 (1998) 3131 [arXiv:hep-ph/9704403]; J. R. Ellis, T. Falk, G. Ganis and K. A. Olive, Phys. Rev. D 62 (2000) 075010 [arXiv:hep-ph/0004109].
[3] J. R. Ellis, T. Falk, G. Ganis, K. A. Olive and M. Srednicki, Phys. Lett. B 510 (2001) 236 [arXiv:hep-ph/0102098].

[4] J. R. Ellis, K. A. Olive, Y. Santoso and V. C. Spanos, Phys. Lett. B 565 (2003) 176 [arXiv:hep-ph/0303043]; H. Baer and C. Balazs, JCAP 0305, 006 (2003) [arXiv:hep-ph/0303114]; A. B. Lahanas and D. V. Nanopoulos, Phys. Lett. B 568, 55 (2003) [arXiv:hep-ph/0303130]; U. Chattopadhyay, A. Corsetti and P. Nath, Phys. Rev. D 68, 035005 (2003) [arXiv:hep-ph/0303201]; C. Munoz, Int. J. Mod. Phys. A 19, 3093 (2004) [arXiv:hep-ph/0309346]; J. Ellis and K. A. Olive, arXiv:1001.3651 [astro-ph.CO].

[5] J. R. Ellis, K. A. Olive, Y. Santoso and V. C. Spanos, Phys. Rev. D 69 (2004) 095004 [arXiv:hep-ph/0310356]; J. Ellis, S. Heinemeyer, K. Olive and G. Weiglein, JHEP 0502 013, hep-ph/0411216; J. R. Ellis, S. Heinemeyer, K. A. Olive and G. Weiglein, JHEP 0605, 005 (2006) [arXiv:hep-ph/0602220].

[6] O. Buchmueller et al., Phys. Lett. B 657 (2007) 87 [arXiv:0707.3447 [hep-ph]]; O. Buchmueller et al., JHEP 0809 (2008) 117 [arXiv:0808.4128 [hep-ph]]; O. Buchmueller et al., Eur. Phys. J. C 64, 391 (2009) [arXiv:0907.5568 [hep-ph]].

[7] J. Polonyi, Generalization Of The Massive Scalar Multiplet Coupling To The Supergravity, Hungary Central Inst Res - KFKI-77-93.

[8] E. Cremmer, B. Julia, J. Scherk, P. van Nieuwenhuizen, S. Ferrara and L. Girardello, Phys. Lett. B 79, 231 (1978); E. Cremmer, B. Julia, J. Scherk, L. Girardello and P. van Nieuwenhuizen, Nucl. Phys. B 147, 105 (1979).

[9] For reviews, see: H. P. Nilles, Phys. Rep. 110 (1984) 1; A. Brignole, L. E. Ibanez and C. Muno, arXiv:hep-ph/9707209, published in Perspectives on supersymmetry, ed. G. L. Kane, pp. 125-148.

[10] R. Barbieri, S. Ferrara and A. S. Savoy, Phys. Lett. B 119, 343 (1982).

[11] J. R. Ellis, K. A. Olive, Y. Santoso and V. C. Spanos, Phys. Lett. B 573 (2003) 162 [arXiv:hep-ph/0305212], and Phys. Rev. D 70 (2004) 055005 [arXiv:hep-ph/0405110].

[12] E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, Phys. Lett. B 133, 61 (1983); J. R. Ellis, C. Kounnas and D. V. Nanopoulos, Nucl. Phys. B 247, 373 (1984).

[13] J. R. Ellis, D. V. Nanopoulos and K. A. Olive, Phys. Lett. B 525, 308 (2002) [arXiv:hep-ph/0109288].

[14] J. Ellis, A. Mustaafayev and K. A. Olive, arXiv:1003.3677 [hep-ph].

[15] J. Ellis, A. Mustaafayev and K. A. Olive, arXiv:1004.5399 [hep-ph].

[16] E. Komatsu et al., arXiv:1001.4538 [astro-ph.CO].

[17] J. Ellis, T. Falk, and K.A. Olive, Phys. Lett. B444 (1998) 367 [arXiv:hep-ph/9810360]; J. Ellis, T. Falk, K.A. Olive, and M. Srednicki, Astr. Part. Phys. 13 (2000) 181 [Erratum-ibid. 15 (2001) 413] [arXiv:hep-ph/9905481].

[18] Joint LEP 2 Supersymmetry Working Group, Combined LEP Chargino Results, up to 208 GeV.

[19] R. Barate et al. [ALEPH, DELPHI, L3, OPAL Collaborations: the LEP Working Group for Higgs boson searches], Phys. Lett. B 565, 61 (2003) [arXiv:hep-ex/0306033]; D. Zer-Zion, Prepared for 32nd International Conference on High-Energy Physics (ICHEP 04), Beijing, China, 16-22 Aug 2004; LHWG-NOTE-2004-01, ALEPH-2004-008, DELPHI-2004-042, L3-NOTE-2820, OPAL-TN-744, http://lepwww.lhep.ac.nl/LEPHIGGS/papers/August2004/MSSM/index.html.

[20] S. Chen et al. [CLEO Collaboration], Phys. Rev. Lett. 87 (2001) 251807 [arXiv:hep-ex/0108032]; P. Koppenburg et al. [Belle Collaboration], Phys. Rev. Lett. 93 (2004) 061803 [arXiv:hep-ex/0403004]; B. Aubert et al. [BaBar Collaboration], arXiv:hep-ex/0207076; E. Barberio et al. [Heavy Flavor Averaging Group (HFAG)], arXiv:hep-ex/0603003.

[21] [The Muon g-2 Collaboration], Phys. Rev. Lett. 92 (2004) 161802, hep-ex/0401008; G. Bennett et al. [The Muon g-2 Collaboration], Phys. Rev. D 73 (2006) 072003 [arXiv:hep-ex/0602035].

[22] J. L. Feng, K. T. Matchev and T. Moroi, Phys. Rev. D 61 (2000) 075005 [arXiv:hep-ph/9909334].

[23] H.-P. Nilles, M. Srednicki, and D. Wyler, Phys. Lett. 120B (1983) 345, L.J. Hall, J. Lykken and S. Weinberg, Phys. Rev. D 27 (1983) 2359.

[24] R. H. Cyburt, J. R. Ellis, B. D. Fields, K. A. Olive and V. C. Spanos, JCAP 0611, 014 (2006) [arXiv:astro-ph/0608562].

[25] J. R. Ellis, K. A. Olive and P. Sandick, Phys. Lett. B 642, 389 (2006) [arXiv:hep-ph/0607002], JHEP 0706, 079 (2007) [arXiv:0704.3446 [hep-ph]], and JHEP 0808, 013 (2008) [arXiv:0801.1651 [hep-ph]].

[26] N. Polonsky and A. Pomarol, Phys. Rev. Lett. 73, 2292 (1994) [arXiv:hep-ph/9406224], and Phys. Rev. D 51 (1995) 6532 [arXiv:hep-ph/9410231].

[27] L. Calibbi, Y. Mambrini and S. K. Vempati, JHEP 0709, 081 (2007) [arXiv:0704.3518 [hep-ph]]; L. Calibbi, A. Faccia, A. Masiero and S. K. Vempati, Phys. Rev. D 74, 116002 (2006) [arXiv:hep-ph/0605139]; E. Carquin, J. Ellis, M. E. Gomez, S. Lola and J. Rodriguez-Quintero, JHEP 0905 (2009) 026 [arXiv:0812.4423 [hep-ph]].

[28] M. Schmaltz and W. Skiba, Phys. Rev. D 62, 095005 (2000) [arXiv:hep-ph/0001172].
[29] F. Borzumati and T. Yamashita, arXiv:0903.2793 [hep-ph].