Physical properties of double-mode RR Lyrae stars based on pulsation and evolution models

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Abstract. We determine the fundamental stellar parameters of altogether 20 double-mode RR Lyrae (RRd) stars from the Galactic Field and the Large Magellanic Cloud, following our approach employed on the field RRd star BS Comae [1]. The stars were selected to cover wide ranges of periods and period ratios, implying diverse stellar parameters. From the possible observed quantities we use only the periods and determine stellar parameter combinations that satisfy both current helium-burning evolutionary models and grids of linear non-adiabatic purely radiative pulsational models. Thus, the periods of an object determine a sequence of solutions for its mass, luminosity, effective temperature and metallicity, parametrized by the time elapsed from the zero age horizontal branch. The derived sets of solutions yield various important theoretical relations between the physical parameters of the stars which are, in the case of some parameter combinations, nearly independent of the age. We get very tight simple linear relations between log $P_0$ and log $R$, log $\rho$, log $g$ and the Wesenheit index $W(B-V)$. This latter period-luminosity-color relation is in good agreement with the one derived on empirical basis [2] and calibrated by Baade-Wesselink results [3].

Keywords: RRd stars, fundamental parameters

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INTRODUCTION

Double-mode RR Lyrae (RRd) stars pulsate simultaneously and stably in the first two radial modes with a period ratio close to 0.75. Their two precisely observable periods make them a particularly important subtype of RR Lyrae stars by supplying an additional constraint on their structure. Eventually, the double-mode nature offers the possibility of determining the fundamental parameters of these stars from radial pulsation models. The solid theory of linear radial pulsation has long been used for this purpose by fitting models simultaneously to the periods of both pulsation modes. By measuring or giving an estimate on the chemical composition and effective temperature of the star we can derive its mass and luminosity. The consistency of these values can be confirmed by independently obtained distances [see, e.g., 4].

Nonlinear hydrodynamical modeling of double-mode pulsation has also became possible in the past decade [5, 6]. Sustained double-mode pulsation can be reproduced in a consistent range of physical parameters, a star-by-star modeling, however, is still not available.

For a fully consistent RRd model it is important to take horizontal branch (HB) stellar evolution also into account. In a novel study of this endeavor, to involve evolutionary constraints in explaining the observed period distribution of RRd stars of the Large Magellanic Cloud (LMC) by means of physical parameter ranges, [7] calculated pulsation model sequences along theoretical tracks of HB evolution to compare the theoretical period distribution with the observed one. In the present study we follow a different approach by taking the observed periods as an input to our calculations and search for a set of physical parameters that satisfy the combined parameter space of pulsational and evolutionary models. Thus obtaining strongly constrained values for the fundamental parameters of a representative sample of RRd stars, we can probe the full parameter ranges they populate, as well as use them to deduce theoretical relations among their physical parameters.

METHOD AND DATA

Our goal is to determine the fundamental parameters of an RRd star solely from its periods by restricting the parameter space of pulsation models by the constraints of evolutionary models. The basic assumption behind this approach is that there is a good level of consistency between the two kinds of models. We obtain the fundamental physical parameters of a star by the same procedure as described in [1]. and hereby we only briefly summarize the essentials of the method.

On the one hand we calculate a grid of linear pulsation models fitting the observed pair of periods. These models give stellar mass and luminosity for specified effective temperature and metallicity values. On the other hand, isochrones of Helium-burning models also determine mass and luminosity as a function of effective temperature, metal content, and age. To obtain the set of parameters that satisfy both models we have to calculate the intersection of these two restricted parameter-subspaces
FIGURE 1. An example for the combined pulsational and evolutionary solution sequences calculated for an RRd star in the LMC. Age dependence of the various physical parameters are shown up to \( \log t = 7.86 \) (\( \approx 72.5 \) Myr).

FIGURE 2. Petersen diagram showing RRd stars in the LMC, taken from the MACHO catalog [4, dots], together with all known Galactic Field RRd stars taken from the literature [10, 11, 12, 13, 14, 15, 16, 17, crosses]. Stars analyzed in this study are denoted by circles.

(formally vector-vector functions that we sample on a grid). By fixing the periods to the observed ones, evolutionary models have one variable in surplus, the age, i.e. the time elapsed from the zero age horizontal branch (ZAHB), which will be a free parameter of the solution.

We used the canonical HB evolution models of [8] with solar-scaled heavy element mixture. These models have a fine mass and time spacing, suitable for the detailed investigation of the instability strip. We obtained isochrones by interpolating the tracks to a uniform mass and time base. We generated linear, non-adiabatic (LNA), fully radiative pulsation models. The pulsation model grid was calculated for the same chemical compositions as those of the evolutionary tracks, and with a fine temperature step of 50 K. The deep and dense sampling of the stellar envelopes resulted in an accuracy of \( \sim 10^{-5} \) in the period fit. See [1] for further details on the model construction.

As a result of the method, for a pair of observed periods we obtain a 1-D solution sequence giving the effective temperature, metallicity, stellar mass, and luminosity as a function of the time parameter. Thus, in most cases we stop scanning in age between \( \log t = 7.85 \) and 7.9.

Figure 2 shows the Petersen diagram of all known Galactic Field RRd stars and those found in the LMC by the MACHO project. The ranges of the \( P_1/P_0 \) vs. \( P_0 \) distribution populated by these stars encompass the vast majority of all known double-mode RR Lyrae stars in any stellar system. We selected a sub-sample of 20 stars (3 from the Field: AQ Leo, BS Com, and CU Com, and 17 more from the LMC) to survey full ranges of physical parameters, as implied by the periods and period ratios. Sequences for fundamental physical parameters were obtained for each of the stars in the way discussed above. We present the analysis of the derived sets of solutions in the following section.

RESULTS

Based on the Petersen diagram, the physical parameters obtained for the 20 objects investigated in this work are representative for all known RRd stars (and their luminosities, presumably, for a dominant fraction of single-mode RR Lyrae stars). Therefore, we use them to measure the fundamental parameter ranges populated by these objects and probe important theoretical relations.
defined by the solutions of our method.

Figure 3 shows the distribution of the solutions on the Herzschprung-Russell diagram (HRD) together with various evolutionary tracks for reference. The expected general trend, that stars with lower metallicity are cooler and brighter, is clearly exhibited. The gradual topological change of the sequences originates from the smoothly changing sizes and shapes of the blueward loops in the constituting tracks, mainly as a function of metallicity. As it is also shown in Fig. 1, the temperature dependence of a solution increases considerably for higher values of age (i.e., after reaching the high temperature extrema – the ‘blue noses’ – of the constituting tracks). Therefore we would need further specific information on each star (i.e., on their spectral energy distributions) to put more significant constraints on their temperatures. However, according to the combined models, RRd stars populate an overall temperature range of at least 300 K, as it is clear from Fig. 3.

Figure 4 shows the solutions on the $B - V$ vs. $V - I$ two-color diagram. Stars are concentrated into a very narrow overall range of color index pairs. Here we note that by a very precisely measured pair of color indices one would be able to further constrain the time parameter, thus reducing the ranges of uncertainty in all other parameters, as it was successfully done in the case of BS Com [see 1].

In Fig. 5 we examine the luminosity and mass as functions of the metallicity. Values corresponding to the ZAHB states show a tight and, apparently, somewhat non-linear correlation with [Fe/H] in both cases. Allowing later stages of HB evolution complicates the topology, but the models set minimal luminosity and maximal mass values at any specified metal content. Accordingly, for specified mass the models give a minimal luminosity, and, vice-verse, for a fixed luminosity, they put a constraint on the maximum value of the mass. Since we selected our stars to cover nearly the full RRd period ranges of the LMC (and by this selection, that of almost all known RRd stars as well), the solutions inherently yield the full theoretical [Fe/H], mass, and $M_V$ ranges spanned by the constituting objects. Although the individual sequences suffer from an uncertainty of roughly $0.1 - 0.3$ dex in metallicity (depending on the allowed range of the time parameter), the upper limit for the overall [Fe/H] range turns out to be $\sim 0.9$ dex with extrema of approximately $-1.3$ and $-2.2$. We note that, as implied by the Petersen diagram (i.e., the short period end is more populated in the case of the LMC), the majority of the stars are in the upper part of this metallicity range. By direct spectrophotometric measurements of 6 RRd stars in the LMC, [18] found a wider metallicity range of $(-2.28, -1.09)$. Besides the individual errors of the few measurements and the sensitivity of the $\Delta S$ method to different calibrations, this moderate difference may result from the possible non-solar heavy element mixture of these stars (we recall that throughout this paper solar-scaled mixtures of heavy elements are used).

The stellar masses corresponding to our metallicity domain span in a large $0.17 M_\odot$ range from $0.64 M_\odot$ to $0.81 M_\odot$. As for absolute magnitudes, the luminosity interval permitted by the solutions on (see Fig. 5) can be translated into an $M_V$ range of $(0.46, 0.70)$ which, as we will see later, is in a good agreement with Baade-Wesselink results.

We also computed $\log g$, $\log R$, and $\log \rho$ values for all solution sequences. These parameters are highly degenerate, i.e., very insensitive to the time parameter. They also show very tight, linear correlations with the fundamental period. Figure 6 shows these simple theoretical relations provided by our results. It is remarkable how
tiny individual parameter ranges are allowed by the solutions. Thus, for the sake of simplicity, linear formulae given on the same figure were obtained by fitting to the zero age solutions only.

One of the tightest empirical relations for RR Lyrae stars is the period-luminosity-color (PLC) relation, or in other words the period – reddening-free Wesenheit-index relation. We can directly compare it with the one derived from the synthetic $M_V$ and color index values of RRd stars calculated above. Figure 7 shows the theoretical PLC relation for $B$ and $V$ colors stretched by our RRd model results. In this representation of the solutions we notice a vast insensitivity again to evolution effects. For comparison we show the empirical relation derived from a large number of RRab stars from Galactic globular clusters (GCs) by [2], together with the 3 $\sigma$ range of the calibration sample. The zero-point of this relation was set by the Baade-Wesselink (BW) results for Galactic Field RRab stars (also shown in Fig. 7) as given by [3]. The zero points of the two relations show a fine agreement. Although the RRd sample clearly defines a significantly steeper slope, it does not exceed the 3 $\sigma$ ranges of the GC sample. We note that the situation is very similar in the case of the $V$ and $I$ colors.

The reason for the slope difference is not yet clear and may be caused by a combination of different effects. Since the PLC relation is governed by two parameters (i.e., metallicity and effective temperature) out of four of the pulsation equation, it has an intrinsic scatter within which the subset of RRd stars could follow their own, tighter relation, determined by the specific stellar parameter subspace they populate.

We can also speculate on the following. In the derivation of the empirical PLC relations by [2], no direct calibration (i.e., using independently measured absolute magnitudes) was made. Instead, the relative dereddened distance moduli of the constituting globular clusters were obtained as output parameters of the least-squares minimization procedure, together with the regression coefficients. Thus, a uniform slope was obtained for all stars, nicely fitting the subsamples of the individual clusters, and the zero-point differences were interpreted as the result of the relative distances. Later [19] found a somewhat steeper $W(V,I) - \log P_0$ relation for LMC RRab stars than that of [2], but the scatter was much higher. The HB stars of a GC have generally little if any metallicity dispersion. Therefore, if an additional slight, intrinsic metallicity dependence of the Wesenheit index does rather manifest in zero point than in slope differences between subsamples of stars with constant metallicity, its effect could merge with that of the relative distances and remained hidden in the above case. Furthermore, if this is so, then our RRd sample will yield a steeper relation because it scans a considerable, nearly 1 dex range in [Fe/H], which, in this case, is more correlated with $\log P_0$. However, we would need very precise independent relative distance determinations for the calibrating clusters to give a more secure assessment of this problem.

**CONCLUSIONS**

We presented a method and its applications for stellar parameter determination of double-mode RR Lyrae stars. The method combines theoretical model results of linear stellar pulsation and canonical HB stellar evolution. Based on the periods of an object, the joint constraints of the models allow sequences of fundamental stellar parameters with age as a free parameter. Additional observables like color index or metallicity can be used to put further constraints on the solutions. By using a representative sample of objects to probe the observed period ranges of the known RRd’s with our method, we obtained the theoretical ranges of physical parameters populated.
FIGURE 6. Surface gravitational acceleration, stellar radius, and stellar density as a function of the period of the fundamental mode. Notation is the same as in Fig. 3. Lines show linear fits to the zero age solutions, yielding the formulae given in each panel.

FIGURE 7. PLC relation for the RRd sample of this study are shown with the same notation as in Fig. 3. Crosses denote BW results of [3]. Solid line shows the linear fit to the RRd data. PLC relation of [2] with zero-point fixed by the BW sample is delineated by a dashed line, with a shaded area illustrating the 3σ ranges of its calibrating sample.

by these objects. We also extracted important theoretical relations from the derived sets of stellar parameters among which we found nearly age-independent ones. We derived a very tight linear PLC relation with a magnitude zero-point being in a fine agreement with that of Baade-Wesselink results. We found a slope difference, which might be explained by a hitherto hidden metallicity dependence of the relation.

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