Combining Black-Litterman model with clustering on portfolio construction

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Abstract. This research aims to propose a new strategy for an investor when using Black Litterman Model (BLM) in order to gain higher performance on their portfolio. The proposed strategy is starting to cluster stocks then combine it with allocation weight following the Black-Litterman rules. We describe two scenarios combination cluster and BLM in the processing of portfolio construction then we investigate both performances measured with the Sharpe index. In this research, we limited for expressing views which only use absolute views. We find that a result from combining the cluster technique will help investors to determine which assets that will be given certain views on their portfolio.

1. Introduction
Asset diversification is an effective way in modern portfolio theory that can be used in order to reduce the risk of obtaining improved portfolio performance. Along with the investment objective of obtaining a high profit then it is very likely to have the possibility of high risk as well. Therefore, the desire of an investor to gain high profits with minimum risk seems to be impossible. Many factors may influence in building a portfolio such as selection assets in investment, decision making to invest, and optimal allocation.

The best portfolio is a well-diversified portfolio since the portfolio is predicted to have a minimum risk when it is diversified well. We can diversify our portfolio by choosing stocks that come from different sectors. Although these stocks come from a different field, we should ensure that those resulted in the same expectation of profit. There are several ways or methods to select assets for constructing a portfolio and one of them is cluster analysis. Cluster analysis method attempt to classify assets with the same character in one group. Some clustering studies related to construction portfolios are Kheyrkhah [1] and Haqiqi [2].

Portfolio diversification can also be conducted by selecting assets with the highest Sharpe ratio from several different clusters. In general, there are many ways to cluster process, such as Haqiqi [2] applied ant colony approach in portfolio construction while the results of Kheyrkhah [1] showed that the most compact cluster for stock classification data is Fuzzy C-means Cluster as compared to another method with the highest Sharpe Index.

In practice, an investor or manager has a certain feeling when facing the movement of the asset price in the market. It is not easy to construct a portfolio based on the classic model, Mean-Variance model because, in fact, the influence of information or news circulating against market prices can be
quite significant and random. In this allocation problem, BLM is a well-known method for portfolio optimization which starting from CAPM and collaborate with feeling or investor’s views [3]. This article will discuss how to combine the procedure of selecting asset through the clustering process into modeling the portfolio using Black-Litterman formula and the implementation in the Indonesian market. The expectation of this new procedure is gaining a higher performance index of a portfolio than using classical Mean-Variance.

2. Literature Review

This section summarizes the formula of a portfolio return, clustering approach in the portfolio, investment management process and BLM.

2.1. Return

The formula of expected return will be given as:

\[ R_i = \frac{P_i - P_{i(t-1)}}{P_{i(t-1)}} \]

Where:
- \( R_i \) = Total Return
- \( P_i \) = Asset Price \( i \) at the \( t \) without dividend
- \( P_{i(t-1)} \) = Asset Price \( i \) at the \( t-1 \) without dividend

2.2. Cluster approach in portfolio

The implementation of clustering as a way of stock selection in a portfolio has been done by several other studies such as [1] [2], and [4-7]. Clustering techniques used as an alternative to select assets that can be performed by various methods such as hierarchy, non-hierarchy or using meta-heuristic. In this study, we still work from the previous research using ant colony as an approach to classify stocks, in addition, it has been shown empirically the same results using both the average linkage and k-means clustering on the same data.

2.3. Investment Management Process

There are several different phases of the investment management process [8], where strategic asset allocation is the first and essential phase, mostly the investment time is about 5 years. Another stage of tactical asset allocation is the different phases in which market timing is the best method, therefore a performance evaluation method is needed to determine the market's rising or falling as expected. The phase of asset/stock determination is when the investment manager ensures optimal asset selection in its portfolio so that modeling for different portfolio optimization and asset evaluations needs to be implemented.

2.4. Black-Litterman

Black-Litterman’s formulation is one of the model of portfolio formation which is still developing. It is well known that the expertise investment or manager in reading the emerging market situation and its influence on financial markets can affect the process of portfolio compilation. Hence the formation of portfolios through the BLM introduced by Robert Litterman and Fischer Black in the 1990s became an alternative method that could be used [3].

There are several studies that focus on how the portfolio is compiled using the BL method and some of them show more favorable results than other portfolio methods [9-12], given the importance of the investor feeling in declaring a view as an input in the model. Based on these problems, in this research, we propose an alternative way to arrange the portfolio by utilizing stock grouping procedure which then combined with stock weighting based on the BL method.
The basic idea to start using the BL formula is Mean-Variance Markowitz and the starting point is Capital Asset Pricing Model (CAPM) with some additional components that must be prepared such as investor statement about the future return on assets. In this model, we are introduced with other components are some additional parameters such as tau and delta aversion risk. The concept of portfolio formation using CAPM is the assumption of a linear relationship that occurs between risk and market return which used a classical regression technique that requires the existence of normally distributed data.

The formula of $\pi$ (CAPM) is

$$\pi = r_f + \beta \left( \bar{r}_m - r_f \right)$$  \hspace{1cm} (1)

and Black-Litterman return can be derived from the following equation:

$$\mu_{BL} = E(r_{BL}) = \pi + \tau \Sigma P'(\Omega + \tau \Sigma P')^{-1}q - \pi$$  \hspace{1cm} (2)

where $E(r_{BL})$ is to be interpreted as the expected return of the Black Litterman model, $\pi$ is a vector $k \times 1$ for return equilibrium which is related with free risk rate and market return, $\tau$ is a value to represent the confidence of views (range from 0 to 1), $\Sigma$ is a covariance matrix of return, $\Omega$ is a diagonal matrix of covariance of views, $P$ is a pick matrix $k \times n$ for return views and $q$ is a vector $k \times 1$ of return views. The return formula of BL has been explained through several works of literature, one of those is the reference [13]. The problem of this Black-Litterman optimization can be written as follows:

$$\text{Max } E(R_p) = W'\mu_{BL}$$  \hspace{1cm} (3)

$$\text{Var}(R_p) = W'\Sigma W$$  \hspace{1cm} (4)

3. Result and Discussion

This research was conducted in order to observe whether the addition of clustering strategy as a new way before forming a portfolio using Black-Litterman method showed an increase in the results of portfolio performance as measured by Sharpe Index. The data used in this study is still using the data from the previous research [4-5]. The previous research which focused on the Indonesian stock market data has selected seven shares with different sectors namely WIKA, UNVR, UNTR, SMGR, SSMS, TLKM and SMRA [4]. Both studies have used clustering techniques to assist in the selection of asset in the portfolio, wherein [4] proposed ant colony as an approach to classify shares and [5] discussed the uses of the average linkage and k-means clustering in portfolio construction using Mean-Variance Markowitz. The results obtained from both are 6 assets without SSMS as a first cluster and SMSS in another one.

Table 1. Cluster result using Mean-Variance Markowitz

|       | Cluster 1       | Cluster 2       |
|-------|-----------------|-----------------|
| Mean  | 0.0023          | -0.0026         |

Furthermore, Mean-Variance (MV) method has been used for the weighting of the portfolio in the previous research and we summarize the results obtained show that the first portfolio involved cluster without SSMS and the second one is arranged to involve SSMS without clustering. Table 2 following is the brief result of allocation:

Table 2. The brief result of an allocation

|       | WIKA | UNVR | UNTR | SMGR | TLKM | SMRA | SSMS | Return | Risk | Sharpe |
|-------|------|------|------|------|------|------|------|--------|------|--------|
| P1    | 22%  | 19%  | 1%   | 20%  | 38%  | 0    | 0    | 0.18%  | 2.3% | 7.7%   |
| P2    | 20.1%| 13.8%| 0.1% | 12.9%| 33.2%| 19.3%| 0.3% | 0.08%  | 2.1% | 3.9%   |

In this research, the idea of clustering grouping continued with optimal allocation using the BLM. The BLM is represented by the posterior return formula in equation (2) which is the result of a
combination of CAPM equilibrium return and an investor's feeling. Therefore, the calculation here involves two returns namely the formula CAPM return \( p_i \) and views return \( Q \).

There are two scenarios that we propose in this research in terms of building a portfolio with a combination of cluster and Black-Litterman process:

Scenario 1: Selected cluster results continued using BLM as a new portfolio.
Scenario 2: The cluster results will be used as a stock group given the statement of the view in BLM. In implementing both scenarios, we then continue to state the views as a prediction of the portfolio. We consider time series method especially the simple one which is moving average (MA) to assist in making return views statement. As a preliminary study, we use MA in combining a new strategy of clustering for the implementation of BLM even variety of previous studies has used more complex time series methods such as RBFNN [11] and Fuzzy [14].

This Table 3 reports the view return from MA with two periods as input for each stock and return BL for each strategy.

**Table 3. The view return from MA(2)**

|        | WIKA | UNVR | UNTR | SMGR | TLKM | SMRA | SSMS |
|--------|------|------|------|------|------|------|------|
| Prediction | 2.14% | 4.14% | 2.54% | 1.2% | -1.73% | 2.09% | -1.53% |

We prefer to put the absolute views due to the positive prediction for WIKA, UNVR, UNTR, SMGR, and SMRA. Then, we illustrate the matrix \( P \), \( Q \) and the summary of each result as follows based on table prediction:

\[
Q = \begin{bmatrix}
  q_{SMGR} \\
  q_{SMRA} \\
  q_{UNVR} \\
  q_{WIKA} \\
  q_{UNTR}
\end{bmatrix} = \begin{bmatrix}
  0.0121 \\
  0.0209 \\
  0.0414 \\
  0.0215 \\
  0.0254
\end{bmatrix}, \text{ while } P = \begin{bmatrix}
  1 & 0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 \\
  0 & 0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

We use \( \tau \) 0.5, the risk-free rate for CAPM is 0.055 and matrix \( P, Q \) in equation 1 and 2. The return of BLM is reported in Table 4 while the weight of CAPM and Black Litterman are displayed in Figure 1 and 2.

**Table 4. The BL return for each scenario**

|        | WIKA | UNVR | UNTR | SMGR | TLKM | SMRA | SSMS |
|--------|------|------|------|------|------|------|------|
| RBL-1  | 2.61% | 1.62% | 2.08% | 1.50% | 1.39% | 1.26% | -    |
| RBL-2  | 2.59% | 1.76% | 1.83% | 1.62% | 1.37% | 1.55% | 3.88% |

The bar chart in Figure 1 reports how the difference between the change of weight of CAPM result and Black Litterman results for both scenarios so we can see the correction of each allocation on the first scenario compared to the second one.
The change of the allocation on the portfolio gives the correction on each stock when it was built from an equilibrium condition. In the second scenario, the change of weight each stock from CAPM to BL is not relatively significant but without ignoring SSMS, portfolio 2 considered on all stocks from both clusters in order to concern on the correlation between clusters.

The Sharpe Index of both scenarios in Table 5 put the portfolio 2 as the best portfolio due to the higher index not only for the CAPM result but also for the BL result. In this case, the Sharpe Index of
BL-1 was higher than CAPM-1 and CAPM-2, even though BL-2 is the highest one. From this result, we can use the alternative of the BLM for increasing the performance of a portfolio without concerning the correlation of internal cluster but consider the other assets in another cluster. It means we involve all assets from each cluster. Sharpe Index of portfolio 1 rose from 0.3 to 0.75 and its performance portfolio 2 almost doubled from 0.59 to 1.03 through BLM.

The clustering technique works in the procedure of building the views in the BLM, especially to help in processing or selection stock which is better to give the views. Many factors probably affect in BL perform due to building the views, we focus on absolute views in this research rather than relative views in terms of the limitation on the computation program.

| Table 5. Result of Sharpe Index from the clustering technique |
|---------------------------------------------------------------|
|                | CAPM-1     | BL-1       | CAPM-2     | BL-2       |
| Return         | 0.0072929  | 0.017822   | 0.012631   | 0.0219     |
| Risk           | 0.023659   | 0.023669   | 0.021304   | 0.021305   |
| Sharpe Index   | 0.30825    | 0.75297    | 0.59288    | 1.0299     |

4. Conclusion
The Sharpe Index for portfolio BL-2 is 1.029 showed the performance of that portfolio is relatively higher than the others and it could be considered as the best choice portfolio when involving the cluster techniques in making views on Black-Litterman strategy. This effort will improve the performance of a portfolio based on Sharpe Index significantly. In addition, when the portfolio BL-1, ignored the other cluster, we found that the performance of the Sharpe index will rise from 0.3 to 0.75. It means, in this case, a portfolio which is constructed from an investor through clustering procedure in the BLM will outperform the result of Sharpe index of classical Mean-Variance.

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