Development of Unrestricted Fuzzy Linear Fractional Programming Problems Applied in Real Case

Sapan Kumar Das\textsuperscript{a}, S. A. Edalatpanah\textsuperscript{b} and T. Mandal\textsuperscript{a}

\textsuperscript{a}Department of Mathematics, National Institute of Technology Jamshedpur, Jamshedpur, India;
\textsuperscript{b}Department of Applied Mathematics, Lahijan Branch, Islamic Azad University, Lahijan, Iran

ABSTRACT

**Purpose:** We formulate a linear fractional programming (LFP) problem in which costs of the objective functions and constraints all are taken to be triangular fuzzy numbers.

**Methodology:** The fuzzy LFP problem is transformed into an equivalent crisp line fractional programming (CLFP) problem by using the centroid ranking function. This proposed method is based on crisp LFP and has a simple structure.

**Findings:** To show the efficiency of our proposed method a real life problem has been illustrated. The discussion of the practical problem will help decision makers to realise the usefulness of the CLFP problem.

**Value:** Using centroid ranking function, we overcome the all limitations of our day to day real life problem. Finally, a result analysis is also established for applicability of our method.

1. Introduction

The decision-making problems can be derived as the minimised of several fractional terms and this is the well-known fractional programming (FP) problem. If these fractional terms are in linear terms, appearing in objective function subject to a linear constraint, then these types of problem are called linear FP (LFP) problems. LFP problems have attracted the interest of many researchers due to its application in decision-making such as production planning, marketing and media selection, university planning and student admissions, financial and corporate planning, health care and hospital planning, etc. see [1–5] and references therein. In the literature, many researchers have been recommended to solve LFP problems. Isbell and Marlow [6] first identified an example of LFP problem and solved it by a sequence of linear programming problems. Charnes and Cooper [5] considered the variable transformation method to solve LFP problems. Bitran and Novaes [7] considered the updated objective functions method to solve LFP problems by solving a sequence of linear programmes. Martos [8], Swarup [9], Pandy and Punnen [10], Das and Mandal [11] solved the LFP problem by various types of solution procedures based on the simplex method; see also [12–15]. These methods are interesting; however, in daily life circumstances, due to ambiguous information supplied by decision-makers, the parameters are often illusory.
and it is very hard challenge for decision-maker to make a decision. In such a case, it is more appropriate to interpret the ambiguous coefficients and the vague aspiration parameters by means of the fuzzy set (FS) theory. The concept of fuzzy set and fuzzy numbers was first introduced [16] and applied efficiently for linear optimisation; see [17–32] and references therein. Furthermore, several researchers have investigated linear fractional programming problems in the fuzzy framework. For example, Sakawa and Yano [33] proposed a method to solve a multi-objective LFP (MOLFP) problem under a fuzzy environment. Dutta et al. [34] established the sensitivity analysis in the fuzzy LFP (FLFP) problems. Some authors solved the FLFP problems by the fuzzy goal programming approach [35–39]. De and Deb [40] considered a fuzzy linear fractional programming problem using the sign distance ranking method where all the terms are triangular fuzzy numbers. Youness et al. [41] imported a design to find a bi-level multi-objective fractional integer programming problem that consists of fuzzy numbers in the right-hand side of the constraints. Pop and Minasian [42] proposed a method for solving fully falsified linear fractional programming problems where all the parameters and variables are triangular fuzzy numbers. In [43,44], they considered the same problem of [42] for solving fully fuzzy linear fractional programming problems. Veeramani and Sumathi [45] proposed a solution procedure for solving fuzzy linear fractional programming problem by using the fuzzy mathematical programming approach. Very recently, a number of papers have exhibited their interest to solve the FLFP problems [46–51]. Recently, several researchers had focused for solving linear programming by using multi-objective and transformation technique under fuzzy circumstances, see refer [52–56]. The main works and features of this paper are that we consider a new type of fuzzy arithmetic for triangular numbers in which the coefficients of the objective function and the constraints were represented by triangular fuzzy numbers with inequality constraints utilised in daily life problems. The proposed technique is very easy and involves mathematical calculation. The rest of our work is organised as follows: In Section 2, we review some concept and arithmetic between two triangular fuzzy numbers. In Section 3, formulation of FFLFP problems and use of ranking function are discussed. The new method for solving FFLFP problems is affirmed in Section 4. In Section 5, the numerical examples and the obtained results are given for illustrating the new method. In Section 6, advantages of the proposed method are discussed. Finally, the conclusion is given in Section 7.

2. Preliminaries

In this section, the basic definitions, involving fuzzy sets, fuzzy numbers and operations on fuzzy numbers, are outlined. For detailed information on the fuzzy set theory, we refer the interested reader to [20,57].

**Definition 2.1:** Let $X$ denote a universal set. Then a fuzzy subset $\tilde{A}$ of $X$ is defined by its membership function $\mu_{\tilde{A}} : X \to [0, 1]$, which assigns a real number $\mu_{\tilde{A}}(x)$ in the interval $[0, 1]$, to each element $x \in X$, where the values of $\mu_{\tilde{A}}(x)$ at $x$ show the grade of membership of $x$ in $\tilde{A}$. A fuzzy subset $\tilde{A}$ can be characterised as a set of ordered pairs of element $x$ and grade $\mu_{\tilde{A}}(x)$ and is often written $\tilde{A} = (x, \mu_{\tilde{A}}(x)) : x \in X$ is called a fuzzy set.
Definition 2.2: A fuzzy number $\tilde{A} = (b, c, a)$ is said to be a triangular fuzzy number if its membership function is given by

$$
\mu_{\tilde{A}}(x) = \begin{cases} 
(x - b) & b \leq x \leq c, \\
(c - b) & c \leq x \leq a, \\
0 & \text{else.}
\end{cases}
$$

Definition 2.3: A triangular fuzzy number $(b, c, a)$ is said to be non-negative fuzzy number if $b \geq 0$.

Definition 2.4: Two triangular fuzzy numbers $\tilde{A} = (b, c, a)$ and $\tilde{B} = (e, f, d)$ are said to be equal if $b = e, c = f, a = d$.

Definition 2.5: A convenient method for comparing of the fuzzy numbers is by the use of ranking function. So, we define a ranking function $\mathcal{R}: \mathbb{F}(\tilde{A}) \to \mathbb{R}$ which maps for each fuzzy number in to real line. The centroid of centroid ranking of triangular fuzzy numbers is

$$
\mathcal{R}(\tilde{A}) = \frac{2b + 14c + 2a}{6},
$$

here $w = 1$.

Definition 2.6: Let $\tilde{A} = (b, c, a), \tilde{B} = (e, f, d)$ be two triangular fuzzy numbers, then:

(i) $\tilde{A} + \tilde{B} = (b, c, a) + (e, f, d) = (b + e, c + f, a + d)$,
(ii) $\tilde{A} - \tilde{B} = (b, c, a) - (e, f, d) = (b - d, c - f, a - e)$,
(iii) If $\tilde{A} = (b, c, a)$ be any triangular fuzzy number and $\tilde{B} = (e, f, d)$ be a non-negative triangular fuzzy number, then

$$
\tilde{A} \otimes \tilde{B} = \tilde{AB} = \begin{cases} 
(be, cf, ad) & \text{if } b \geq 0, \\
(bd, cf, ad) & \text{if } b < 0, a \geq 0, \\
(bd, cf, cd) & \text{if } c < 0,
\end{cases}
$$

Definition 2.7: Let $\tilde{A} = (b, c, a), \tilde{B} = (e, f, d)$ be two triangular fuzzy numbers. We say that $\tilde{A}$ is relatively less than $\tilde{B}$, if:

(i) $c < f$ or
(ii) $c = f$ and $(a - b) > (d - e)$ or
(iii) $c = f, (a - b) = (d - e)$ and $(a + b) < (d + e)$.

Note: It is clear from Definition 2.7 that $\tilde{A} \preceq \tilde{B}$ if $c = f, (a - b) = (d - e)$ and $(a + b) = (d + e)$.
3. Linear Fractional Programming (LFP) Problem

In this section, the general form of LFP problem is discussed. Furthermore, Charnes and Cooper’s linear transformation is summarised.

\[
\begin{align*}
\text{Max } Z(x) &= \sum c_j x_j + p \sum d_j x_j + q = F(x) \quad G(x) \\
\text{Subject to } x &\in S = \{x \in \mathbb{R}^n. Ax \leq b, x \geq 0\}
\end{align*}
\]

where \(j = 1,2,\ldots,n, A \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m, c, d \in \mathbb{R}^n, p, q \in \mathbb{R}\). For some values of \(x\), \(G(x)\) may be equal to zero. To avoid such cases, one requires that either \(\{x \geq 0, Ax \leq b, \Rightarrow G(x) > 0\}\) or \(\{x \geq 0, Ax \leq b, \Rightarrow G(x) < 0\}\). For convenience, assume that LFP problem satisfies the condition that

\[
\{x \geq 0, Ax \leq b, \Rightarrow G(x) > 0\}
\]

(2)

**Theorem 3.1 ([3]):** Assume that no point \((z,0)\) with \(z \geq 0\) is feasible for the following linear programming problem.

\[
\begin{align*}
\text{Max } c^t z + pt \\
\text{Subject to } d^t z + qt &= 1, \\
A z - bt &= 0, \\
t &> 0, z \geq 0, z \in \mathbb{R}^n, t \in \mathbb{R}.
\end{align*}
\]

Then, with the condition of relation (2), the LFP problem (1) is equivalent to the linear programming problem model (3).

Now, consider the two related problems

\[
\begin{align*}
\text{Max } tF(z/t) \\
\text{Subject to } A(z/t) - b &\leq 0, \\
tG(z/t) &= 1, \\
t &> 0, z \geq 0.
\end{align*}
\]

(4)

and,

\[
\begin{align*}
\text{Max } tF(z/t) \\
\text{Subject to } A(z/t) - b &\leq 0, \\
tG(z/t) &\leq 1, \\
t &> 0, z \geq 0.
\end{align*}
\]

(5)

where model (4) is obtained from model (1) by the transformation \(t = 1/G(x), z = tx\) and model (5) differs from model (4) by replacing the equality constraints \(tG(z/t) = 1\) by an inequality constraint \(tG(z/t) \leq 1\).

**Theorem 3.2 ([3]):** If model (1) is a standard concave-convex programming problem which reaches a maximum at a point \(x^*\), then the corresponding transformed problem model (5)
attains the same maximum value at a point \((t^*, z^*)\) where \(z^*/t^* = x^*.\) Moreover, model (5) has a concave objective function and a convex feasible set.

Suppose that:

\[
\begin{align*}
\text{Max} & \quad Z(x) = \frac{F(x)}{G(x)} \\
\text{Subject to} & \quad x \in S = \{x \in \mathbb{R}^n \mid Ax \leq b, x \geq 0\},
\end{align*}
\]

where \(F(x)\) is concave and negative for each \(x \in S\) and \(G(x)\) is concave and positive on \(S\), then

\[
\begin{align*}
\text{Max} & \quad tF(z/t) \\
\text{Subject to} & \quad A(z/t) - b \leq 0, \\
& \quad tG(z/t) \leq 1, \\
& \quad t > 0, z \geq 0.
\end{align*}
\]

4. Fuzzy Linear Fractional Programming Problem and Its Solution

Consider the following fuzzy linear fractional programming (FLFP) problem. We are going to approach \(m\) fuzzy equality constraints and \(n\) fuzzy variables where all the terms are triangular fuzzy numbers.

\[
\begin{align*}
\text{Maximize} & \quad Z = \frac{\tilde{c}^t x + \tilde{\alpha}}{\tilde{d}^t x + \tilde{\beta}} \\
\text{Subject to} & \quad \tilde{A} \otimes x \leq \tilde{b}, \\
& \quad x \geq 0,
\end{align*}
\]

where \(\tilde{c}^t = [\tilde{c}_j]\) is \(1\) by \(n\) matrix; \(\tilde{d}^t = [\tilde{d}_j]\) is \(1\) by \(n\) matrix; \(x = [x_j]\) is \(n\) by \(1\) matrix; \(\tilde{A} = [\tilde{a}_{ij}]\) is \(m\) by \(n\) matrix; \(\tilde{b} = [\tilde{b}_j]\) is a \(m\) by \(1\) matrix; \(\tilde{\alpha} = [\tilde{\alpha}_j]\) and \(\tilde{\beta} = [\tilde{\beta}_j]\) are the scalars. Here all the parameters \(\tilde{c}_j, \tilde{d}_j, \tilde{a}_{ij}\) are a set of fuzzy numbers.

Mention: Let \(\tilde{x}\) a fuzzy optimal solution of FFLFP problem. If there exists a fuzzy number \(\tilde{y}\), it satisfies the following conditions:

(i) \(\tilde{y}\) is a non-negative fuzzy number,

(ii) \(\tilde{A} \otimes \tilde{y} \leq \tilde{b}\),

(iii) \(\Re(\tilde{c}^t \otimes \tilde{x}) = \Re(\tilde{c}^t \otimes \tilde{y})\),

(iv) \(\Re(\tilde{d}^t \otimes \tilde{x}) = \Re(\tilde{d}^t \otimes \tilde{y})\),

then, \(\tilde{y}\) is also an exact optimal solution of the problem (8) and is called a substitute optimal solution.

Consider the model (8), let all the parameters \(\tilde{x}, \tilde{c}, \tilde{\alpha}, \tilde{d}, \tilde{\beta}, \tilde{b}\) and \(\tilde{z}\) are represented by triangular fuzzy numbers (\(p, q, r\), \((\alpha_1, \alpha_2, \alpha_3)\), \((u, v, w)\), \((\beta_1, \beta_2, \beta_3)\), \((b_1, b_2, b_3)\) and \((z_1, z_2, z_3)\),
respectively. Then we can rewrite the mentioned FLFP as follows:

\[
\text{Max } (z_1, z_2, z_3) = (p, q, r)^t \otimes x + (\alpha_1, \alpha_2, \alpha_3) \\
(u, v, w)^t \otimes x + (\beta_1, \beta_2, \beta_3)
\]

Subject to

\[
(b, c, a) \otimes x \leq (b_1, b_2, b_3) \\
x \geq 0
\]

Hence, from the above demonstrations, the steps of our method can be written as follows:

**Step 1:** Write the FFLFP problem as follows:

\[
\text{Max } \tilde{z} = \tilde{c}_j x_j + \tilde{a} \\
\text{Subject to } \tilde{a}_{ij} x_j \leq \tilde{b}_i \\
x_j \geq 0.
\]

**Step 2:** If all the terms represent the triangular fuzzy numbers, then write the FFLFP problem as follows:

\[
\text{Max } \tilde{z} = (p_j, q_j, r_j)^t \otimes x_j + (\alpha_1, \alpha_2, \alpha_3) \\
(u_j, v_j, w_j)^t \otimes x_j + (\beta_1, \beta_2, \beta_3)
\]

Subject to

\[
(b_{ij}, c_{ij}, a_{ij}) \otimes x_j \leq (b_i, g_i, h_i) \\
(x_j, y_j, z_j) \geq 0.
\]

**Step 3:** By utilising Definition 2.5, the new centroid ranking function of the problem should be transformed into a crisp LFP problem. The model can be written as:

**Step 4:** The above problems are crisp linear fractional programming problems, which can be changed into crisp LP problems by using a Charnes-Cooper transformation model, as discussed in Section 3.

**Step 5:** Solved the crisp LP problem by utilising any technique.

**Step 6:** Write the solution of FLFP problems in the form of \( x \) and obtain the optimal solution as \( \tilde{z} \).

**Step 7:** Finally, by using Definition 2.7, compare the results.

### 5. Application of Our Proposed Method

In this section, we take some real-life problems and proved the ability of our proposed method:

**Example 5.1:** In TATA Hospital Jamshedpur, India has two nutritional experiments (Vitamin A and Calcium) with two products Milk (glass) and Salad (500 mg) with profit around 6 dollars and around 2 dollars per unit, respectively. However, the cost for each unit of the above product is around 1 dollar. Consider that a fixed cost of around 2 dollars is added to the cost function. Determine the maximum profit of these two products.

Here, the environmental coefficients, such as profit (due to market situations), cost (due to market conditions), vitamin A and calcium (due to the presence of the suppliers), are
imprecise numbers with triangular possibility distributions over the planning horizon due
to incomplete information. For example, the profit of the product A is \((4, 6, 8)\) dollars. Sim-
ilarly, the other parameters and variables are assumed to be triangular fuzzy numbers. Hence, the above problem can be formulated as the following FFLFP problem (Table 1).

**Solution:** In this case, let \(x_1\) and \(x_2\) be the amount of units of Vitamin A and Calcium to be produced, respectively. Then the above problem can be formulated as

\[
\begin{align*}
\text{Max} & \quad \bar{z} = \frac{6x_1 + 2x_2}{x_1 + x_2 + \frac{2}{3}} \\
\text{Subject to} & \quad x_1 + x_2 \leq 7 \\
& \quad 2x_1 + 3x_2 \leq 17 \\
& \quad x_1, x_2 \geq 0.
\end{align*}
\]

Now we consider the coefficients \(\bar{7} = (3, 7, 11), \bar{1}\bar{7} = (7, 17, 27), \bar{6} = (4, 6, 8), \bar{2} = (1, 2, 3), \bar{3} = (2, 3, 4)\) and \(\bar{1} = (0, 1, 2)\). The problem can be written as follows:

\[
\begin{align*}
\text{Max} & \quad z = \frac{(4, 6, 8) \otimes x_1 + (1, 2, 3) \otimes x_2}{(0, 1, 2) \otimes x_1 + (0, 1, 2) \otimes x_2 + (1, 2, 3)} \\
\text{Subject to} & \quad (0, 1, 2) \otimes x_1 + (0, 1, 2) \otimes x_2 \leq (3, 7, 11), \\
& \quad (1, 2, 3) \otimes x_1 + (2, 3, 4) \otimes x_2 \leq (7, 17, 27), \\
& \quad x_1, x_2 \geq 0.
\end{align*}
\]

As the Step-3, the fuzzy linear fractional programming problems will be

\[
\begin{align*}
\text{Max} & \quad z = \frac{21 \otimes x_1 + 7 \otimes x_2}{3.5 \otimes x_1 + 3.5 \otimes x_2 + 24.5} \\
\text{Subject to} & \quad 3.5 \otimes x_1 + 3.5 \otimes x_2 \leq 24.5, \\
& \quad 7 \otimes x_1 + 10.5 \otimes x_2 \leq 59.5, \\
& \quad x_1, x_2 \geq 0.
\end{align*}
\]

We transform the crisp LFP problem into a crisp LP problem by using our Step-4:

\[
\begin{align*}
\text{Max} & \quad Z = 21 \otimes y_1 + 7y_2 \\
\text{Subject to} & \quad 3.5 \otimes y_1 + 3.5 \otimes y_2 + 7 \otimes t = 1, \\
& \quad 3.5 \otimes y_1 + 3.5 \otimes y_2 - 24.5 \otimes t = 0, \\
& \quad 7 \otimes y_1 + 10.5 \otimes y_2 - 59.5 \otimes t = 0 \\
& \quad y_1, y_2, t \geq 0.
\end{align*}
\]

Then, we obtain the optimal solution as

\[x_1 = 4.097, \text{ and } x_2 = 3.064\]

And the optimal value of the problem as \(Z = 3.33\). In Veeramani and Sumathi methods [51] the optimal values of the problems are

| Nutrient   | Milk (glass) | Salad (500 mg) | Min nutrient required |
|------------|--------------|----------------|-----------------------|
| Vitamin A  | 1            | 1              | 7                     |
| Calcium    | 2            | 3              | 17                    |
In Figure 1, we compare the membership function for the proposed method and the existing methods [51,58]. Graph (Figure 1) shows that the modified technique yields better values of most of the membership functions and individual objective functions in comparison to the existing methods [51,58]. It is clear that both the approaches are closer, but the modified methodology is efficient and requires less computation than earlier technique in terms of considering the solution preferences by the decision-maker at each level. In Figure 1, Z is an objective function and $Z_{\pi}$ is a membership function.

5.1. Result Analysis

This section provides a comparative study of the proposed method with the existing method for fuzzy linear fractional programming problems.
• In our proposed model, our results are better than the existing results. In our model, we introduced a new centroid ranking function method for solving the fuzzy LFP problem and transformed into a crisp LFP problem.
• In fig-1, we have compared our proposed technique with other existing technique, we have found that the objective value of our proposed method is more than that of the existing method [51,58].
• Our model is very simple and efficient compared to the existing method [51,58].
• Our model is applied in a real-life problem and also in a large-scale problem.

6. Conclusion
In the past few years, growing interests are shown in fuzzy linear fractional programming and currently there are several methods for solving FLFP. However, to the best of our knowledge, a few efficient optimal solutions were found in fuzzy linear fractional programming (FLFP). In this paper, we proposed a new efficient method for solving FLFP problems, in order to obtain the fuzzy optimal solution. Furthermore, the limitations of other existing methods have been pointed out. To show the efficiency of the proposed method, some numerical examples are illustrated. We concluded that our proposed model is very easy to handle, efficient and shows better outcomes than other models.

Acknowledgements
The authors would like to thank the anonymous reviewers for their helpful comments and suggestions.

Disclosure statement
No potential conflict of interest was reported by the author(s).

Notes on contributors
Sapan Kumar Das is currently with the Department of Revenue, Ministry of Finance, Government of India. He received his Ph.D. degree in Operation Research in 2017 from National Institute of Technology Jamshedpur, India. He received his BSc in Mathematics in 2010 and his M.Sc. in Applied Mathematics in 2012, both from the Fakir Mohan Autonomous College. His research interests include fuzzy optimisation, Neutrosophic Optimisation applied in real-life problem, Linear Fractional Programming, Transportation Problem. He has published more than 30 international journals including SCI and SCOPUS journal. He serves as Editorial Member of several international journals.

Seyyed Ahmad Edalatpanah is Assistant Professor of Ayandegan Institute of Higher Education of Iran. He received his Ph.D. in Applied Mathematics from the University of Guilan, Rasht, Iran. He is also an academic member of Guilan University and Islamic Azad University of Iran. His fields of interest are numerical modelling, soft computing and optimisation. He has published over 100 journal and conference proceedings papers in the above research areas. He serves on the editorial boards of several international journals. He is Editor-in-chief of the International Journal of Research in Industrial Engineering at www.riejournal.com.

Tarni Mandal is Professor of National Institute of Technology Jamshedpur, India. He received his Ph.D. in Applied Mathematics from Ranchi University, India. His field of interest are statistics, operation research, Fuzzy Optimisation and Linear Programming. He has published more than 50 international journals in the above research areas.
References

[1] Charnes A, Cooper WW, Rhodes E. Measuring the efficiency of decision making units. Eur J Oper Res. 1978;2(6):429–444.
[2] Craven B. Fractional programming. Berlin: Heldermann Verlag; 1988.
[3] Schaible S. Fractional programming. In: Frenk BG, editor. Handbook of global optimization. Boston: Springer; 1995. p. 495–608.
[4] Bajalinov EB. Linear-fractional programming theory, methods, applications and software. Vol. 84. Berlin: Springer Science & Business Media; 2013.
[5] Charnes A, Cooper WW. Programming with linear fractional functionals. Nav Res Logist Q. 1962;9(3–4):181–186.
[6] Isbell J, Marlow W. Attrition games. Nav Res Logist Q. 1956;3(1–2):71–94.
[7] Bitran G, Novaes A. Linear programming with a fractional objective function. Oper Res. 1973;21(1):22–29.
[8] Martos B, Whinston V. Hyperbolic programming. Nav Res Logist Q. 1964;11(2):135–155.
[9] Swarup K. Letter to the editor – linear fractional functions programming. Oper Res. 1965;13(6):1029–1036.
[10] Pandey P, Punnen AP. A simplex algorithm for piecewise-linear fractional programming problems. Eur J Oper Res. 2007;178(2):343–358.
[11] Das SK, Mandal T. A single stage single constraints linear fractional programming problem: an approach. Oper Res Appl: Int J. 2015;2:1–5.
[12] Chadha S. Fractional programming with absolute-value functions. Eur J Oper Res. 2002;141(1):233–238.
[13] Odior AO. An approach for solving linear fractional programming problems. Int J Eng Technol. 2012;1(4):298–304.
[14] Tantawy S. Using feasible directions to solve linear fractional programming problems. Aust J Basic Appl Sci. 2007;1(2):109–114.
[15] Tantawy S. A new procedure for solving linear fractional programming problems. Math Comput Model. 2008;48(5):969–973.
[16] Zadeh LA. Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets Syst. 1978;1(1):3–28.
[17] Amin SH, Razmi J, Zhang G. Supplier selection and order allocation based on fuzzy SWOT analysis and fuzzy linear programming. Expert Syst Appl. 2011;38(1):334–342.
[18] Baykasoğlu A, Subulun K. An analysis of fully fuzzy linear programming with fuzzy decision variables through logistics network design problem. Knowl Based Syst. 2015;90:165–184.
[19] Bector C, Chandra S, Vijay V. Duality in linear programming with fuzzy parameters and matrix games with fuzzy pay-offs. Fuzzy Sets Syst. 2004;146(2):253–269.
[20] Das SK, Mandal T, Edalatpanah SA. A mathematical model for solving fully fuzzy linear programming problem with trapezoidal fuzzy numbers. Appl Intell. 2017;46:509–519.
[21] Delgado M, Verdegay JL, Vila M. A general model for fuzzy linear programming. Fuzzy Sets Syst. 1989;29(1):21–29.
[22] Dempe S, Ruziyeva A. On the calculation of a membership function for the solution of a fuzzy linear optimization problem. Fuzzy Sets Syst. 2012;188(1):58–67.
[23] Ebrahimnejad A, Tavana M. A novel method for solving linear programming problems with symmetric trapezoidal fuzzy numbers. Appl Math Model. 2014;38(17):4388–4395.
[24] Edalatpanah S, Shahabi S. A new two-phase method for the fuzzy primal simplex algorithm. Int Rev Pure Appl Math. 2012;8(2):157–164.
[25] Hosseinzadeh A, Edalatpanah S. A new approach for solving fully fuzzy linear programming by using the Lexicography method. Adv Fuzzy Syst. 2016;1:1–6.
[26] Inuiiguchi M, Ramik J. Possibilistic linear programming: a brief review of fuzzy mathematical programming and a comparison with stochastic programming in portfolio selection problem. Fuzzy Sets Syst. 2000;111(1):3–28.
[27] Maleki H, Tata M, Mashinchi M. Linear programming with fuzzy variables. Fuzzy Sets Syst. 2000;109(1):21–33.
[28] Saberi Najafi H, Edalatpanah S. Nash equilibrium solution of fuzzy matrix game solution of fuzzy bimatrix game. Int J Fuzzy Syst Rough Syst. 2012;5(2):93–97.

[29] Saberi Najafi H, Edalatpanah S. A note on ‘a new method for solving fully fuzzy linear programming problems’. Appl Math Model. 2013;37(14):7865–7867.

[30] Saberi Najafi H, Edalatpanah SA, Dutta H. A nonlinear model for fully fuzzy linear programming with fully unrestricted variables and parameters. Alexandria Eng J. 2016;55:2589–2595.

[31] Tanaka H, Okuda T, Asai K. On fuzzy-mathematical programming. J Cybern. 1973;3:37–46.

[32] Zimmermann H-J. Fuzzy programming and linear programming with several objective functions. Fuzzy Sets Syst. 1978;1(1):45–55.

[33] Sakawa M, Yano H. An interactive fuzzy satisficing method for multiobjective linear fractional programming problems. Fuzzy Sets Syst. 1988;28(2):129–144.

[34] Dutta D, Rao JR, Tiwari RN. Sensitivity analysis in fuzzy linear fractional programming problem. Fuzzy Sets Syst. 1992;48(2):211–216.

[35] Baky IA. Solving multi-level multi-objective linear programming problems through fuzzy goal programming approach. Appl Math Model. 2010;34(9):2377–2387.

[36] Dutta D, Rao J, Tiwari R. Effect of tolerance in fuzzy linear fractional programming. Fuzzy Sets Syst. 1993;55(2):133–142.

[37] Li D, Chen S. A fuzzy programming approach to fuzzy linear fractional programming with fuzzy coefficients. J Fuzzy Math. 1996;4:829–834.

[38] Pal B, Basu I. A goal programming method for solving fractional programming problems via dynamic programming. Optimization. 1995;35(2):145–157.

[39] Pal BB, Moitra BN, Maulik U. A goal programming procedure for fuzzy multiobjective linear fractional programming problem. Fuzzy Sets Syst. 2003;139(2):395–405.

[40] De P, Deb M. Solving fuzzy linear fractional programming problem using signed distance ranking. Advance Computing Conference (IACC), 2013 IEEE 3rd International. IEEE; 2013.

[41] Youness E, Emam O, Hafez M. Fuzzy bi-level multi-objective fractional integer programming. Appl Math Inf Sci. 2014;8(6):2857–2863.

[42] Pop B, Stancu-Minasian I. A method of solving fully fuzzified linear fractional programming problems. J Appl Math Comput. 2008;27(1-2):227–242.

[43] Stanojevic B, Stancu-Minasian I. On solving fully fuzzified linear fractional programs. Adv Model Optim. 2009;11(4):503–523.

[44] Stanojevic B, Stancu-Minasian I. Evaluating fuzzy inequalities and solving fully fuzzified linear fractional programs. Yugosl J Oper Res. 2012;22(1):41–50.

[45] Veeramani C, Sumathi M. Fuzzy mathematical programming approach for solving fuzzy linear fractional programming problem. RAIRO Oper Res. 2014;48(1):109–122.

[46] Das SK, Mandal T, Edalatpanah SA. A new approach for solving fully fuzzy linear fractional programming problems using the multi objective linear programming problem. RAIRO Oper Res. 2017;51(1):285–297.

[47] Deb M, De P. Optimal solution of a fully fuzzy linear fractional programming problem by using graded mean integration representation method. Appl Appl Math. 2015;10(1):571–587.

[48] Sapan KD, Mandal T, Edalatpanah S. A note on ‘a new method for solving fully fuzzy linear fractional programming with a triangular fuzzy numbers’. Appl Math Comput Intell. 2015;4(1):361–367.

[49] Singh SK, Yadav SP. Fuzzy programming approach for solving intuitionistic fuzzy linear fractional programming problem. Int J Fuzzy Syst. 2015;8(2):1–7.

[50] Upmanyu M, Saxena RR. On solving multi objective set covering problem with imprecise linear fractional objectives. RAIRO Oper Res. 2015;49(3):495–510.

[51] Veeramani C, Sumathi M. Solving the linear fractional programming problem in a fuzzy environment: numerical approach. Appl Math Model. 2016;40(11–12):6148–6164.

[52] Das SK, Edalatpanah SA. New insight on solving fuzzy linear fractional programming in material aspects. Fuzzy Optim Modell. 2020;1:1–7.

[53] Das SK, Chakraborty A. A new approach to evaluate linear programming problem in pentagonal neutrosophic environment. Complex Intell Syst 2020. doi:10.1007/s40747-020-00181-0.
[54] Veeramani C, Sharanya S, Ebrahimnejad A. Optimization for multi-objective sum of linear and linear fractional programming problem: fuzzy nonlinear programming approach. Math Sci 2020;14(3):219–233.

[55] Perić T, Babić Z, Omerović M. A fuzzy goal programming approach to solving decentralized bi-level multi-objective linear fractional programming problems. Croat Oper Res Rev. CRORR. 2019;10(1):65–74.

[56] Arya R, Singh P. Fuzzy efficient iterative method for multi-objective linear fractional programming problems. Math Comput Simul. 2019;160:39–54.

[57] Dubois D. Fuzzy sets and systems: theory and applications. Vol. 144. New York: Academic press; 1980.

[58] Stanojević B, Stanojevic M. Solving method for linear fractional optimization problem with fuzzy coefficients in the objective function. Int J Comput Commun Control. 2013;8(1): 136–145.