Effects of the \(\gamma\)-rays Scattered Backward by Metals on the Nuclear Energy Level Width

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Abstract

By placing a \(^{133}\text{Cs}\) \(\gamma\)-ray source embedded in a solid at the center of a platinum (gold) cylinder, we try to change the width of the 81-keV level. Our results show a narrowed energy level and, equivalently, a prolonged lifetime. With a 0.5-mm-thick, 5-cm-long, 2-mm-diameter platinum cylinder, we obtain a width narrower by 6.1\% at 4.2 K.

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I. INTRODUCTION

Recently, the $\gamma$-ray backward scattering cross sections were measured and compared with the theoretical predictions [1, 2]. The results show that some of the $\gamma$-rays can be scattered completely backward in a coherent and elastic way. This fact implies that a $\gamma$-ray emitted from a radioactive nucleus can return to the source nucleus without any energy loss when a suitable reflector is applied.

The nuclear energy level width has been known as a solidly determined and unchangeable quantity. Although there have been several attempts to change it by altering the chemical state [3, 4, 5] and by applying high pressure [6] or low temperature [7], only negligible changes, i.e., less than 0.6%, were observed. Thus, attempts at substantial modifications of the nuclear lifetime have failed.

Nevertheless, it is a very attractive problem to modify the lifetime, equivalently the energy level width, because the nuclear waste problem must be solved. If the lifetime becomes shorter, waste cleaning processes may be accelerated. While, prolongation of the lifetime implies suppression of radioactivity. Furthermore, narrowing the width (equivalently lengthening the lifetime) would be useful for longer storage of radioactive material for some purposes. A more important point is that precision measurement of the $\gamma$-ray spectra may be improved. Namely, the accuracy of the Mössbauer experiment might be improved if the width of its absorption spectrum could made narrower.

On the other hand, it is also known that spatial structure of the vacuum field can change the atomic and nuclear energy levels and widths [8, 9]. That is to say, if space is limited by two perfect conducting plates on the surface of which all wave functions vanish, the vacuum field becomes discrete and, therefore, induces some modification of physical quantities. However, observable effects could only be obtained for plates with separations on the order of micrometers [8, 9, 10, 11, 12, 13, 14].

A Mössbauer experiment carried out to observe nuclear energy level shifts [11, 12, 13] discovered that the width became narrower [15]; then, the data were carefully reanalyzed [16]. Usually, broadening occurs easily due to various noises but narrowing is very difficult. Therefore, it is very interesting to investigate the mechanism of such a phenomenon which must be different from the effects of the chemical environment. We should stress here that we are not talking about a reduction of broadening due to various noises but a reduction of
the natural line width itself.

A novel idea has been proposed \[16\] to explain the phenomenon found in the Mössbaur experiment. If the photon emitted from the source could partially return and be reabsorbed by the original source, the duration for the source nucleus to stay in the excited state would effectively increase; and consequently, the nuclear half-life could be prolonged, equivalently, the width of this state could become narrower. In free space, return of the emitted photon to the source is impossible. However, the photon may be forced to return to the source by operating reflectors, say metallic plates or a cylinder. A process in which even a part of the photon returns and is reabsorbed by the source nucleus after backscattering on the metal surface would cause suppression of photon emission. When such a process is repeated many times, the lifetime is finally prolonged. Of course, all these processes such as emission, backscattering and absorption should occur elastically, i.e., without any energy loss. Therefore, the source nucleus should be implanted in a solid. One step of photon reabsorption would cause only a tiny modification of the lifetime, but iterative processes would proceed step by step until the probability for finding the nucleus in the excited state would become half \[17, 18, 19, 20, 21\].

II. DESCRIPTION OF THE PROCESS

A. Decay Equation and the Half-life

Let \[|\psi_0(t)|^2\] be the probability for finding the system in the state \(\psi_0(t)\) at the time \(t\), whether a single-particle or many-particle system. This state is assumed to be unstable and, therefore, decays, i.e., by \(\gamma\) emission in our case. Then, its decay equation reads

\[
\frac{d}{dt}|\psi_0(t)|^2 = -\lambda|\psi_0(t)|^2,
\]

(1)

where \(\lambda\) is the decay constant. Thus, the state can generally be expressed as

\[
\psi_0(t) = A \exp \left[ -\frac{i}{\hbar} (E_0 - \frac{i\Gamma}{2})t \right].
\]

(2)

The state has a complex energy eigenvalue because it is unstable, and \(\Gamma\) is the width of the state. The state \(\psi_0(t)\) given in Eq. (2) satisfies, of course, Eq. (1), and \(\lambda = \Gamma/\hbar\).

The validity of the expression in Eq. (2) for \(\psi_0(t)\) can immediately be verified by taking
a Fourier transform of $\psi_0(t)$, i.e.,

$$\phi(E) = \frac{A}{\sqrt{2\pi}} \frac{i\hbar}{[(E-E_0) + i\frac{\Gamma}{2}]}; \quad (3)$$

then,

$$|\phi(E)|^2 = \frac{A^2}{2\pi} \frac{\hbar^2}{[(E-E_0)^2 + (\frac{\Gamma}{2})^2]}. \quad (4)$$

This last equation represents a Lorentzian spectrum, and $\Gamma$ is definitely the width of the state $\psi_0(t)$.

Since the decay rate cannot be measured experimentally with a single-particle system, the measurement is always carried out by means of a particle assembly. However, one can obtain an identical answer by repeating the measurements again and again with a single particle under the same conditions. This is due to the fundamental concept of quantum mechanics. Therefore, we investigate the process with an assembly of nuclei instead of a single nucleus, which is fundamentally the same as phenomena occurring with a single nucleus.

If emitted photons return once to the source after being scattered coherently by a metal surface and are reabsorbed, the decay equation with the decay constant $\lambda$ is given as

$$dN = -\lambda N \, dt + \Sigma \lambda N \, dt \equiv -\lambda^{(1)} N \, dt, \quad (5)$$

where $\lambda^{(1)} = (1 - \Sigma)\lambda$. Even if $N$ is replaced by $|\psi_0(t)|^2$, this equation holds as it is. $\Sigma$ denotes the probability associated with photon reabsorption. Therefore, the second term stands for the effect of $\gamma$ reabsorption. Equation (5) is valid for the time interval $t_0 \leq t < 2t_0$ ($t_0 = 2R/c$, $c=$speed of light, and $R$ is the distance between the source and the metallic surface where the photon is scattered) during which the photon returns only once. Thereby, the decay constant $\lambda$ is effectively modified as $\lambda^{(1)}$.

For the $m$th return of the photon, i.e., for the time interval $mt_0 \leq t < (m + 1)t_0$, the decay equation is generally expressed as

$$dN_{m+1} = -\lambda^{(m)} N_{m+1} \, dt. \quad (6)$$

Integration of this equation over that time interval yields

$$N_{m+1} = N_m \exp[-\lambda^{(m)} (t_0 - \epsilon)], \quad (7)$$

where the limit $\epsilon \to 0$ should be taken in the final stage and

$$\lambda^{(m)} = (1 - \Sigma)^m \lambda. \quad (8)$$
For $0 \leq t < t_0$, the photon has no time to return to the source, i.e., $m = 0$; therefore, $N_1 = N_0 \exp[-\lambda(t_0 - \epsilon)]$, where $\lambda = \lambda^{(0)}$. By iteration, we find

$$N_{m+1} = N_0 \prod_{s=0}^{m} \exp \left(-\lambda^{(s)} t_0 \right) = N_0 \exp \left[-\lambda \frac{1 - (1 - \Sigma)^{m+1}}{\Sigma} t_0 \right]$$

in the limit $\epsilon \to 0$. For $m \to 0$, we have $N_1 = N_0 \exp(-\lambda t_0)$. Similarly, for $\Sigma \to 0$, it becomes $N_{m+1} = N_0 \exp[-\lambda(m+1)t_0] = N_0 \exp(-\lambda t)$, where $(m+1)t_0 = t$. This result is usual; i.e., nothing changes because of $\Sigma = 0$. Equation (9) indicates that the decay constant is changed step by step at every stage in the reabsorption of returning photons. Let us find the value of $m$ by setting $N_{m+1} = \frac{1}{2} N_0$ because the number of radioactive nuclei becomes half of the initial amount at the $(m+1)$th step. Namely,

$$\ln 2 = \lambda \frac{1 - (1 - \Sigma)^{m+1}}{\Sigma} t_0,$$

which gives

$$m + 1 = \frac{\ln[1 - (\tau_{1/2}/t_0) \Sigma]}{\ln(1 - \Sigma)} = \frac{\ln[1 - (c \tau_{1/2}/2R) \Sigma]}{\ln(1 - \Sigma)},$$

where we used $(\ln 2)/\lambda = \tau_{1/2}$ and $t_0 = 2R/c$. By introducing the effective decay constant $\tilde{\lambda}$ in Eq. (9) for $\Sigma \neq 0$, it can be rewritten as

$$N_{m+1} = N_0 \exp(-\tilde{\lambda} t)$$

with

$$\tilde{\lambda} = \lambda \frac{1 - (1 - \Sigma)^{m+1}}{(m + 1) \Sigma}.$$

Rewriting Eq. (11) as

$$(1 - \Sigma)^{m+1} = 1 - \left(\frac{c \tau_{1/2}}{2R} \right) \Sigma$$

and substituting this result with Eq. (11) into Eq. (13), we obtain the expression of $\tilde{\lambda}$ as

$$\tilde{\lambda} = \lambda \left(\frac{c \tau_{1/2}}{2R} \right) \frac{\ln[1 - (c \tau_{1/2}/2R) \Sigma]}{\ln(1 - \Sigma)}.$$
Because $0 < \frac{\tilde{\lambda}}{\lambda} < 1$, the decay is delayed; thus, the level width appears narrower. Accordingly, the modified lifetime is now found to be

$$\tilde{\tau}_{1/2} = \left( \frac{2R}{c} \right) \frac{ln[1 - (c \tau_{1/2}/2R)\Sigma]}{ln(1 - \Sigma)}. \quad (16)$$

As we have seen above, the photon reabsorption process is repeated $m$ times before the half-life of the state is formed. Indeed, that process directly participates to build the half-life. It is not simple radiation trapping, but in the course of photon reabsorption, the nuclear lifetime has been gradually build up step by step.

**B. The Backscattering Cross Section**

Let us now investigate the $\gamma$–backscattering by the metallic cylinder. The elastic scattering of $\gamma$–rays from the metal surface is coherent and mostly caused by atomic electrons. For an incident photon energy much larger than the atomic binding energy, the scattering can be described in a good approximation by the seagull term of the corresponding Feynman diagrams, i.e., by

$$\frac{d\sigma}{d\Omega} = \sum_{\alpha, \alpha'} \left( \frac{e^2}{m_e c^2} \right)^2 |\epsilon^{(\alpha)} \cdot \epsilon^{(\alpha')}|^2 |F(E_\gamma, \theta)|^2, \quad (17)$$

where $\epsilon^{(\alpha)}$ ($\alpha = 1, 2$) is the photon polarization vector, $m_e$ and $e$ are the electron mass and charge, respectively, $c$ is the velocity of light, and $F(E_\gamma, \theta)$ is the form factor given by the $\gamma$–ray energy $E_\gamma$ and scattering angle $\theta$. With the Cartesian components of $\epsilon^{(\alpha)} = (1, 0, 0)$ for $\alpha = 1, 2$ and

$$\epsilon^{(\alpha')} = \begin{cases} (\sin\phi, -\cos\phi, 0) & (\alpha' = 1), \\ (\cos\theta \cos\phi, \cos\theta \sin\phi, -\sin\theta) & (\alpha' = 2), \end{cases} \quad (18)$$

we find

$$\sum_{\alpha, \alpha'} |\epsilon^{(\alpha)} \cdot \epsilon^{(\alpha')}|^2 = \sin^2\phi \cos^2\theta \cos^2\phi. \quad (19)$$

For unpolarized beams, the differential cross-section appears in the form

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \left[ \frac{d\sigma}{d\Omega} (\phi = 0) + \frac{d\sigma}{d\Omega} (\phi = \frac{\pi}{2}) \right] \quad (20)$$

$$= \left( \frac{e^2}{m_e c^2} \right)^2 |F(E_\gamma, \theta)|^2 \frac{1}{2} (1 + \cos^2\theta).$$
Since photon coherent scattering by protons in the nucleus may take place simultaneously, one has to take it into account. This scattering can be described in analogy with the atomic case, provided the electron mass is replaced by the proton mass. However, its contribution is actually negligible compared to that of the scattering by atomic electrons because the proton mass is much larger than the electron mass.

Now, only photons scattered entirely backward can successfully return to the source to be reabsorbed. Assuming the nucleus to be a point particle because the nucleus is much smaller than the photon wavelength, one may express the backward scattering cross section $\sigma_\pi$ as

$$\sigma_\pi = \int \frac{d\sigma}{d\Omega} \delta(\cos\theta - \cos\pi) \delta(\phi - \phi_0) d\Omega = \left(\frac{e^2}{m_ec^2}\right)^2 |F(E_\gamma, \pi)|^2. \tag{21}$$

Since the photon is scattered by the atoms in a metallic cylinder of thickness $d$, we must count the number of atoms per $cm^2$. This number can be given by $nd$, where $n$ is the number of atoms per $cm^3$, and can be obtained from the density divided by the atomic mass: $n = \rho/M = \rho N_A/A$ with Avogadro’s number $N_A$. When the $\gamma$--ray comes from a direction at an angle of $\psi$ from the normal direction of the cylinder surface, $d$ must be replaced by $d_1 = d/cos\psi$. For this case, the number of atoms per $cm^2$ should be $n_1 = nd_1 = nd/cos\psi$.

Since the scatterers are bounded in the solid, the effect of lattice vibration should be taken into account. It can be done by introducing the Debye-Waller factor [21]. Therefore, backward scattering cross section $\sigma_\pi$ should be multiplied by this factor.

C. Expression of $\Sigma$

Generally, radioactive nuclei emit $\gamma$--rays isotropically, so the total number of photons emitted during the time $dt$ is given by

$$\int \rho_u dS_u = \lambda N dt, \tag{22}$$

where $\lambda$ is the decay constant, $N$ is the number of radioactive nuclei at a certain time, $dS_u$ is an element of area on a sphere of arbitrary radius $u$, and $\rho_u$ is the surface density of photons passing through this area, when the initial number of photons is $\lambda N dt$. $\rho_u$ is given by

$$\rho_u = \frac{\lambda N dt}{4\pi u^2}. \tag{23}$$

This relation holds for the sphere of any arbitrary radius, i.e. $\rho_R = \lambda N dt/(4\pi R^2)$.
Let us consider a element of area $dS_z$ at a point on the cylinder surface that is located at the distance $u$ from the center. Since $z = R \tan \psi$, we find

$$dS_z = dz \, R \, d\theta = \frac{R^2}{\cos^2 \psi} \, d\psi \, d\theta$$

$$= \left( \frac{R}{u} \right)^2 \frac{1}{\sin \psi \cos^2 \psi} \left( u^2 \sin \psi \, d\psi \, d\theta \right)$$

$$= \frac{1}{\sin \psi} \, dS_u , \quad (24)$$

where the relations $R = u \cos \psi$ and $dS_u = u^2 \sin \psi \, d\psi \, d\theta$ are used. Then, we obtain

$$\rho_u \, dS_u = \left( \frac{\lambda N dt}{4\pi u^2} \right) \sin \psi \, dS_z = \left( \frac{\lambda N dt}{4\pi R^2} \right) \left( \frac{R}{u} \right)^2 \sin \psi \, dS_z$$

$$= \left( \frac{\lambda N dt}{4\pi R^2} \right) \cos^2 \psi \sin \psi \, dS_z \equiv \rho_z(\psi) \, dS_z , \quad (25)$$

where

$$\rho_z(\psi) = \left( \frac{\cos^2 \psi \sin \psi}{4\pi R^2} \right) \lambda N dt \equiv \hat{\rho}_z(\psi) \lambda N dt \quad (26)$$

is the surface density of photons on an element of area $dS_z$ of the cylinder when an initial number of photons, $\lambda N dt$, comes. $\hat{\rho}_z$ is the surface density of photons on $dS_z$ when a single incident photon comes in. Of course, the total number of photons emitted during $dt$ can be obtained by integrating over a cylinder surface of infinite length

$$\int \rho_z(\psi) \, dS_z = 2 \int_0^{\pi/2} \left( \frac{\lambda N dt}{4\pi R^2} \cos^2 \psi \sin \psi \right) \frac{2\pi R^2}{\cos^2 \psi} \, d\psi = \lambda N dt . \quad (27)$$

In addition, a photon that needs to travel a distance $u$ contributes $(R/u)$ times as much as one travelling $R$.

Thus, $\Sigma$ is expressed by an integral over the cylinder surface; i.e.,

$$\Sigma = \zeta \int \left( \frac{R}{u} \, n_1 f \, \sigma_\pi \right) \hat{\rho}_z \, dS_z$$

$$= \zeta \int \left[ \cos \psi (\frac{nd}{\cos \psi}) f \, \sigma_\pi \right] \left( \frac{\cos^2 \psi \sin \psi}{4\pi R^2} \right) \frac{R^2}{\cos^2 \psi} \, d\psi \, d\theta$$

$$= 2\zeta \frac{2\pi}{4\pi} \int_0^{\psi_L} \left( nd f \, \sigma_\pi \right) \sin \psi \, d\psi$$

$$= \zeta \left( nd \, f \, \sigma_\pi \right) \left[ 1 - \left( \frac{L_0}{2R} \right)^2 \right]^{-1/2} ,$$

where $f$ is the Debye-Waller factor [21] and $\zeta$ is the photon absorption probability of the source. $R$ and $L_0$ are the radius and the length of the cylinder, respectively; therefore, $\psi_L = \arctg(L_0/2R)$. 

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D. Photon Absorption Probability

The photon absorption probability may be given by the ratio of the $\gamma$-absorption cross section to the total cross section:

$$\zeta = \frac{\sigma_N^\gamma f_1}{\sigma_{tot}},$$

(29)

where

$$\sigma_{tot} = f_1 (\sigma_N^\gamma + \sigma_{coh}^N) + \sigma_{pe}^N + \sigma_{incoh}^N + f_1 \sigma_{coh}^A + \sigma_{pe}^A + \sigma_{incoh}^A.$$  

(30)

The superscripts $N$ and $A$ denote nuclear and atomic processes, respectively. $f_1$ is again the Debye-Waller factor for the source nuclei. $\sigma_N^\gamma$ is the $\gamma$-absorption cross section given as

$$\sigma_N^\gamma = 2\pi \bar{\lambda}_\gamma^2 \frac{2J_f + 1}{2J_i + 1} \frac{1}{1 + \alpha},$$

(31)

where $\bar{\lambda}_\gamma = \hbar c/E_\gamma$ and $\alpha$ is the internal conversion coefficient. $J_i$ and $J_f$ are the spins of the initial and the final nuclear states, respectively. The nuclear photoelectric absorption cross section $\sigma_{pe}^N$ can be obtained from the relation $\sigma_{pe}^N = \sigma_N^\gamma \alpha$. The cross sections of coherent and incoherent scattering of gamma rays from a nucleus are calculated by the formulae given in Ref. 23. They are negligible small compared with $\sigma_{pe}^N$ and $\sigma_N^\gamma$. The cross sections of coherent and incoherent scattering and of photoelectric absorption of photons by atoms are also given in Ref. 23.

The validity of Eq. (29) was already examined for the CsCl compound [21]. All necessary cross sections were obtained from the XCOM Photon Cross Section Database [24] and $\alpha = 1.72$ [25]. The result was

$$\zeta_{CsCl} = 8.3 \times 10^{-3}$$

(32)

at $T = 4.2 K$, i.e., the probability of $\gamma$-absorption by $Cs$ in the CsCl compound is about 0.8%. This value can be compared to the relative depths of the absorption spectra observed in Mössbauer experiments, which are all about $0.7 \sim 4\%$ [16, 26, 27, 28, 29, 30, 31, 32]. The agreement is good, so the value of $\zeta$ calculated using Eq.(29) is reliable.

III. NUMERICAL CALCULATION

Let us examine our theory. As above, the conditions to maximize the effect is to select a material which has a large Debye temperature and which induces large backward scattering. Furthermore, the energy of emitted gamma-ray, $E_{\gamma}$, should be less than 100 keV. Otherwise,
the Debye-Waller factor becomes very small, and the effect is greatly reduced. If \( E_\gamma \) is less than 10 keV, various noises associated with detectors become large, and clean data may not be obtained.

A. Gamma-ray Source

Considering the above conditions, we try to examine the first excited state, the \( ^{5/2}+ \) state of \(^{133}\text{Cs}\), which is 81-keV level with a lifetime of 6.27 ns. To eliminate the recoil effect, this nucleus should be implanted in a solid. When a compound \(^{133}\text{BaTiO}_3\) is taken, \(^{133}\text{Ba}\) decays into \(^{133}\text{Cs}\) through the electron conversion process because \(^{133}\text{Ba}\) is radioactive, so \(^{133}\text{Cs}\) remains in the compound. Of course, \(^{133}\text{Cs}\) is in the first excited \(^{5/2}+ \) state and emits a 81-keV gamma ray when it drops into the ground \(^{7/2}+ \) state.

The compound \(^{133}\text{BaTiO}_3\) has a perovskite structure with a rather high Debye temperature, \( \theta_D = 431.8 \text{ K} \) \[31\]. Although, the Debye temperature of the perovskite resulting from the decay of \(^{133}\text{Ba}\) into \(^{133}\text{Cs}\) is not actually known, it may be assumed to be the same as that of \(^{133}\text{BaTiO}_3\) because both of them have the perovskite structure and \(^{133}\text{Ba}\) simply converts to \(^{133}\text{Cs}\) through the EC process. Therefore, the Debye temperature \( \theta_D = 431.8 \text{ K} \) is taken for \( \text{Cs}_2\text{TiO}_3 \). Then, the Debye-Waller factors can be found as \( f_1 = 0.3434, 0.3407, \) and 0.2750 at temperatures \( T = 4.2 \text{ K}, 15 \text{ K}, \) and 77 K, respectively.

All the cross sections necessary to estimate the gamma absorption probability are obtained using Eq. (31) and the relations \( \sigma_{pe}^N = \alpha \sigma_{\gamma}^N \) and \( \sigma_{incoh}^N = (A/Z)^2 \sigma_{coh}^N \), where \( A \) and \( Z \) denote the nuclear mass number and the atomic number, respectively. \( \sigma_{coh}^N \) has been estimated with Eq. (20), where the electron mass was replaced by the the proton mass and with the fact that the normalized nuclear form factor is almost unity in the energy region considered here.

The cross sections for the atomic processes should be calculated with the \( \text{Cs}_2\text{TiO}_3 \) compound. They can be obtained by using the XCOM Photon Cross Section Database with the modified relativistic form factor \[24\]. The results are listed in Table 1. Thus, the absorption probabilities are \( \zeta = 0.166, 0.165, \) and 0.138 at \( T = 4.2 \text{ K}, 15 \text{ K}, \) and 77 K, respectively.
B. Materials of the cylinder

Platinum is suitable for a large backward scattering of $\gamma$-rays because the atomic number of platinum is $Z = 78$ and its Debye temperature is $\theta_D = 240 \, K$. Therefore, the Debye-Waller factors for $E_\gamma = 81 \, keV$ are $f = 0.271$, 0.263, and 0.125 at $T = 4.2 \, K$, 15 $K$, and 77 $K$, respectively.

The modified relativistic form factor for $Z = 78$ is estimated, using the XCOM program [24], as $F(81 \, keV, \pi) = 3.3442$ at $\theta = \pi$. Accordingly, the cross section of the 81-keV gamma-ray backward scattered by platinum is $\sigma_\pi = 0.888 \times 10^{-24} \, cm^2$. The validity of the XCOM program has been verified by experiments [1, 2]. Since the density of platinum is $\rho = 21.41 \, (g/cm^3)$, the number of atoms per $cm^2$ is $n = 6.58 \times 10^{22} \, cm^{-2}$. A platinum cylinder with a thickness of $d = 0.05 \, cm$, inner diameters of $2R = 0.2$, 0.3, 0.5, and 1.0 $cm$, and a length of $L_0 = 5 \, cm$ is used.

C. Results for the Energy Level Width and Lifetime

With all the information obtained above, the level width and lifetime can be calculated using eqs. (15) and (16). The results are given in Table 2. They are within the measurable range. The value of $m$ in Eq. (15) depends on both the temperature and the cylinder radius $R$ and ranges between 190 and 650.

Gold is also a suitable material as a photon reflector. Its Debye temperature is 165 $K$, which gives the Debye-Waller factors of 0.150, 0.137, and 0.0102 at $T = 4.2 \, K$, 15 $K$, and 77 $K$, respectively, for $E_\gamma = 81 \, keV$. Since the modified relativistic form factor is $F(81 \, keV, \theta = \pi) = 3.4057$, the total cross section at $\theta = \pi$ is $\sigma_\pi = 0.921 \times 10^{-24} \, cm^2$. The density of gold is 18.85 $(g/cm^3)$ and, so $n = 5.76 \times 10^{22} \, cm^{-2}$. The results for this case are shown in parentheses of in Table 2.

Notice that $\tilde{\Gamma} \rightarrow \Gamma$ and $\tilde{\tau}_{1/2} \rightarrow \tau_{1/2}$ in the limit of $R \rightarrow \infty$. Their temperature dependence appears through the Debye-Waller factor of the metallic cylinder. At room temperature, the width and the lifetime do not change.
IV. CONCLUSION

As is seen above, the decay constant $\lambda$ is altered step by step at every stage of $\gamma$ reabsorption. $\lambda$ is actually related to the level width as $\Gamma = \hbar \lambda$. Therefore, the process definitely changes the level width, and equivalently the half-life. Our results imply that the accuracy in Mössbauer measurements can be improved by setting both the source and the absorber, respectively, between two plates.

The spatial structure of vacuum field can also change the atomic and the nuclear lifetimes \cite{14, 34}. However, in that case, plausible effects appear only for much smaller separations between the two plates. Therefore, it is negligible in the present investigation.

Other processes, so-called "radiation trapping", were also investigated \cite{35, 36}, and a prolonged nuclear lifetime was observed \cite{37}. The interpretation was that the time evolution of gamma-ray emission was modulated as a result of the time consumed during photon exchange between two radioactive nuclei. Namely, the photon is delayed in coming out of the system while the two nuclei play with the photon. However, it has no relevance to the lifetime unless the energy level width is modified. The level width and lifetime can be changed only when population of the excited state is increased through the mechanism discussed above. In conclusion, a sharper spectrum can be obtained by using the method proposed here, and the accuracy in measuring the $\gamma$—ray spectrum can be improved. Radiation trapping can occur even without modification of energy levels, but such a simple "trapping" cannot have any effect on the Mössbauer spectrum connected directly to the energy level width. Measurements should be carried out at low temperatures with specific detectors with good timing performance and good energy resolution.
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FIG. 1: Geometry of a cylinder.

FIG. 2: Decay scheme.

TABLE I: Cross sections in unit of $10^{-19} \text{cm}^2$. Superscripts $N$ and $A$ denote the nuclear and atomic processes, respectively.

| Process Type | $\sigma$ | $\sigma$ | $\sigma$ | $\sigma$ |
|--------------|---------|---------|---------|---------|
| Nuclear      | $\gamma$ | $\text{pe}$ | $\text{coh}$ | $\text{incoh}$ |
|              | $\sigma^N_{\gamma}$ | $\sigma^N_{\text{pe}}$ | $\sigma^N_{\text{coh}}$ | $\sigma^N_{\text{incoh}}$ |
|              | 1.03    | 1.77    | $9.99 \times 10^{-10}$ | $5.84 \times 10^{-9}$ |
| Atomic       | $\text{pe}$ | $\text{coh}$ | $\text{incoh}$ |
|              | $\sigma^A_{\text{pe}}$ | $\sigma^A_{\text{coh}}$ | $\sigma^A_{\text{incoh}}$ |
|              | $5.60 \times 10^{-3}$ | $2.69 \times 10^{-4}$ | $1.98 \times 10^{-4}$ |
TABLE II: Modified widths and lifetimes. Values are obtained with a platinum cylinder, and those in the parentheses are with a gold cylinder. The standard value is $\tau_{1/2} = (6.27 \pm 0.02) \text{ ns}$ and $\Gamma = 7.28 \times 10^{-8}\text{ eV}$. $\Delta \Gamma = \tilde{\Gamma} - \Gamma$ and $\Delta \tau = \tilde{\tau}_{1/2} - \tau_{1/2}$.

| $T(K)$ | $R(cm)$ | $\Sigma(10^{-4})$ | $\tilde{\Gamma}(10^{-8}\text{ eV})$ | $\Delta \Gamma/\Gamma(\%)$ | $\tilde{\tau}_{1/2}(\text{ ns})$ | $\Delta \tau/\tau_{1/2}(\%)$ |
|---------|---------|-------------------|----------------|----------------|----------------|----------------|
| 4.2     | 0.10    | 1.280(0.6457)     | 6.83(7.05)     | -6.14(-3.07)   | 6.68(6.47)     | +6.54(+3.16)   |
|         | 0.15    | 1.228(0.6194)     | 6.99(7.13)     | -3.90(-1.95)   | 6.52(6.39)     | +4.05(+1.99)   |
|         | 0.25    | 1.176(0.5933)     | 7.11(7.19)     | -2.23(-1.12)   | 6.41(6.34)     | +2.27(+1.13)   |
|         | 0.50    | 1.050(0.5297)     | 7.20(7.24)     | -0.986(-0.496) | 6.33(6.30)     | +0.995(+0.499) |
| 15      | 0.10    | 1.233(0.5836)     | 6.85(7.07)     | -5.91(-2.77)   | 6.66(6.45)     | +6.28(+2.85)   |
|         | 0.15    | 1.183(0.5598)     | 7.00(7.15)     | -3.75(-1.76)   | 6.51(6.38)     | +3.90(+1.79)   |
|         | 0.25    | 1.133(0.5362)     | 7.12(7.20)     | -2.14(-1.01)   | 6.41(6.33)     | +2.19(+1.02)   |
|         | 0.50    | 1.011(0.4787)     | 7.21(7.24)     | -0.949(-0.449) | 6.33(6.30)     | +0.958(+0.451) |
| 77      | 0.10    | 0.4704(0.06621)   | 7.11(7.25)     | -2.22(-0.311)  | 6.41(6.29)     | +2.28(+0.312)  |
|         | 0.15    | 0.4513(0.06351)   | 7.17(7.26)     | -1.42(-0.199)  | 6.36(6.28)     | +1.44(+0.199)  |
|         | 0.25    | 0.4322(0.06084)   | 7.22(7.27)     | -0.813(-0.114) | 6.32(6.28)     | +0.820(+0.114) |
|         | 0.50    | 0.3859(0.05431)   | 7.25(7.27)     | -0.361(-0.0508)| 6.29(6.27)     | +0.363(+0.0508)|

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