Quantum chaos and critical behavior on a chip

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The Dicke model describes $N$ qubits (or two-level atoms) homogeneously coupled to a bosonic mode. Here we examine an open-system realization of the Dicke model, which contains critical and chaotic behaviour. In particular, we extend this model to include an additional open transport qubit (TQ) (coupled to the bosonic mode) for passive and active measurements. We illustrate how the scaling (in the number of qubits $N$) of the superradiant phase transition can be observed in both current and current-noise measurements through the transport qubit. Using a master equation, we also investigate how the phase transition is affected by the back-action from the transport qubit and losses in the cavity. In addition, we show that the non-integrable quantum chaotic character of the Dicke model is retained in an open-system environment. We propose how all of these effects could be seen in a circuit QED system formed from an array of superconducting qubits, or an atom chip, coupled to a quantized resonant cavity (e.g., a microwave transmission line).

I. INTRODUCTION

Understanding and categorizing complex modes of behavior, such as quantum phase transitions¹,² and quantum chaos³, is an important part of quantum many-body theory. Recently, concepts and formalisms from quantum information theory have been used to understand and classify several aspects of criticality⁴,⁵,⁶,⁷. However, the realization of strong coupling regimes, coherent dynamics, and careful readout necessary to observe these phenomena in laboratory conditions is challenging.

Our goal here is to show how a particular quantum phase transition, the Dicke superradiant transition⁸,⁹,10,11,12,13,14,15,16,20,21, behaves when coupled to the environment and measured using transport techniques, as is the case in realistic experimental conditions. The Dicke model describes $N$ two-level “atoms” or qubits coupled to a common single-mode cavity. We focus on this model because of the recent advances in on-chip `circuit-QED'¹⁰,¹¹,¹²,¹³,¹⁴,¹⁵,¹⁶,¹⁷, where the strong coupling regime is accessible, and which allow for coupling to a range of artificial atoms and measurement apparatuses. In particular, we propose a dispersive measurement scheme to observe this transition by coupling either a superconducting qubit array, or an atom chip, to a cavity which is simultaneously (dispersively) coupled to a non-equilibrium measurement device (a so-called “transport” qubit), realisable with a superconducting single electron transistor, or double quantum dot. The geometry of the proposed device is shown in Fig. 1 and described in detail in its caption.

We begin by outlining the salient features of the phase transition in the Dicke model, and existing work in this area (section II), and discussing the closed (section III) and open (section IV) descriptions of the model. We then investigate how coupling to a transport qubit allows read-out of the phase transition properties, and give analytical and numerical results for the current and current-noise in the zero back-action limit (section V). We then discuss our main result: that the current can be used as an observable order parameter to detect the phase transition (section VI). This complements a recent surge of interest in identifying signatures of complex behavior in mesoscopic transport measurements¹⁷,¹⁸,¹⁹. We then consider back-action and decoherence (cavity-loss) effects using a master equation approach (section VII).

We show that both transport qubit back-action and cavity loss appear to only have a weak affect on the current measurement near the critical point. In addition, we show that the Liouvillian describing the open-system dynamics has an eigenvalue spectrum similar to that of the Wigner-Dyson distribution of random matrix theory, as in the closed system case (section VIII). Finally, we briefly discuss practical schemes to realize this model in an experiment¹⁰,¹¹,¹²,¹³,¹⁴,¹⁵,¹⁶,¹⁷, (section IX).

We point out that the properties we investigate here require the precise control of the couplings between qubits and the resonator, and access to a very strong coupling regime, both of which are difficult to achieve. However it was recently shown²¹ that the generalized Dicke model, a variation of the Dicke model where the couplings between the $N$ qubits and the cavity are inhomogeneous, still has all the critical properties of the standard Dicke model. This indicates the universality of the Dicke phase transition, as well as making an experimental realization more feasible. Furthermore, the critical point in the transition relies on the relative values of the coupling strengths and the level-splittings of the qubits. Thus, while the qubit-boson coupling strengths are not tunable in a real experiment, the level splittings of the qubits can typically be controlled by external parameters (e.g., in the case of superconducting flux qubits via an external magnetic flux). Furthermore, a realization using Raman transitions in atoms in an optical cavity has been proposed as a method to reach a controllable strong coupling regime²².
II. Dicke Superradiant Phase Transition.

Historically, the Dicke Hamiltonian (DH) describes the dipole interaction between $N$ "atoms" and $n_b$ bosonic field modes. Typically, the atoms are considered to be at fixed sites within a cavity of volume $V$. The atoms are assumed to be well separated, and thus non-interacting. Hereafter we refer to the atoms as ‘qubits’, and the additional measurement qubit as the ‘transport qubit’. To observe critical phenomena we consider the single-mode case with $n_b = 1$. We do not make the rotating wave approximation, allowing the model to describe both weak and strong coupling regimes (and we omit the $A^2$ term).

Previous work on this model has shown an exact analytical solution in the limit $N \rightarrow \infty$. Furthermore, the transition was characterized as a breaking of parity symmetry at a particular value of the coupling between qubits and cavity (denoted by $\lambda$, with the critical value being $\lambda_c$). Both the qubits and the cavity bosonic degrees of freedom become ‘macroscopically occupied’ (i.e., of $O(N)$, the number of qubits) in the regime above the critical point $\lambda > \lambda_c$. For finite arrays of qubits, $N$, the system is known to exhibit power-law scaling, quantum chaos, and critical entanglement.

Several proposals for an experimental realisation of this system have already been made. For example, Dimer et al. proposed a cavity QED realisation, and discussed in detail the effect of the cavity decay on the phase transition. In another work, Chen et al. proposed using superconducting charge qubits coupled to an optical cavity, so that the critical properties can be observed in the optical mode using homodyne detection. In addition they proposed observing the phase transition as a function of level splitting, as discussed earlier.

III. Dicke Hamiltonian.

The single-mode Dicke Hamiltonian is defined as

$$H_D = \omega_0 \sum_{i=1}^{N} s_i^{(i)} + \omega a^\dagger a + \sum_{i=1}^{N} \frac{\lambda}{\sqrt{N}} (a^\dagger + a) (s_i^{(i)} + s_i^{(\dagger)}) = \omega_0 J_z + \omega a^\dagger a + \frac{\lambda}{\sqrt{N}} (a^\dagger + a) (J_+ + J_-),$$

where $J_z = \sum_{i=1}^{N} s_i$, $J_\pm = \sum_{i=1}^{N} s_\pm$ are collective angular momentum operators for a pseudo-spin of length $j = N/2$. These operators obey the usual angular momentum commutation relations, $[J_z, J_\pm] = \pm J_\pm$ and $[J_+, J_-] = 2J_z$. The frequency $\omega_0$ describes the qubit level splitting, $\omega$ is the oscillator field frequency, and $\lambda$ the qubit-field coupling strength. Because of their mutual interaction with the oscillator field the qubits are not independent. The $\lambda/\sqrt{N}$ scaling is important to realise the thermodynamic limit. It essentially bosonises the low-energy part of the state space of the collective angular momentum. Physically, this scaling implies that the density of qubits is constant, so that the cavity volume becomes larger as $N$ is increased, consequently reducing the electric field density and thus the effective interaction with each individual qubit.

First, we will show analytical results for an entirely passive measurement of the system, using an off-resonance ancillary qubit with current transport. Secondly, we will treat the back-action of the ancillary qubit as a fully quantum interaction, with Markovian transport properties, and including decay terms for the cavity. We will see how this alters the final current measurements, as well as how it changes the properties of the phase transition.

IV. Master Equation.

To take into account the back-action of the transport qubit on the Dicke Hamiltonian, we can model the whole system using a master equation,

$$\frac{d}{dt}\rho(t) = L[\rho(t)] = -i[H, \rho(t)] + L_0[\rho(t)]$$

$$H = H_D + H_{\text{TQ}} + H_{\text{int}}, \quad L_0 = L_{\text{TQ}} + L_{\text{C}}$$
where

$$H_{TQ} = \epsilon \sigma_z + \Delta \sigma_x,$$

(3)

is the Hamiltonian of the transport qubit,

$$H_{\text{int}} = g \sigma_z a^\dagger a,$$

(4)

where $H_{TQ} = \epsilon \sigma_z + \Delta \sigma_x$, is the Hamiltonian of the transport qubit (TQ). Here $\epsilon$ is the level splitting, and $\Delta$ the coherent tunneling within the TQ. $H_{\text{int}} = g \sigma_z a^\dagger a$, is the off-resonance dispersive interaction between Dicke system and TQ. $L_{TQ}$ contains the transport properties of the TQ, and $L_C$ contains cavity damping properties (e.g., photons leaking from the cavity).

$$L_{TQ}[\rho(t)] = -\frac{\Gamma_L}{2} \left[ s_L s_\dagger L \rho(t) - 2 s_\dagger L s_L \rho(t) + \rho(t) s_L s_\dagger L \right]$$

$$- \frac{\Gamma_R}{2} \left[ s_R s_\dagger R \rho(t) - 2 s_\dagger R s_R \rho(t) + \rho(t) s_R s_\dagger R \right]$$

(5)

$$L_C = -\frac{\gamma_b}{2} \left[ a^\dagger a \rho - 2 a \rho a^\dagger + \rho a^\dagger a \right]$$

(6)

where

$$s_L = |0\rangle \langle L|, \quad s_\dagger L = |L\rangle \langle 0|,$$

$$s_R = |0\rangle \langle R|, \quad s_\dagger R = |R\rangle \langle 0|,$$

(7)

(8)

$\Gamma_L$ and $\Gamma_R$ are the left/right tunneling rates for the TQ, and $\gamma_b$ is the decay rate of photons out of the cavity (throughout, we set $\hbar = 1$). Here $\rho(t)$ is the density matrix describing the state of the qubit-array, cavity, and transport qubit system.

$$S(0) = 2e I \left[ 1 - 8 \epsilon T_c^2 (2 + \epsilon g a^\dagger a)^2 \left( (\epsilon - \epsilon_-)^2 \right) - 2 \right] \left[ 4T_c^2 (2 \Gamma_L + \Gamma_R) + \Gamma_L \Gamma_R + 4(\epsilon + g a^\dagger a)^2 \Gamma_L^2 \right].$$

(11)

V. Passive Measurement.

If we assume no back-action from the transport qubit onto the Dicke model, the problem is very simple. However, the form of the interaction between the transport qubit and the effective cavity is still important. As mentioned, off-resonance, $\epsilon \ll \omega$, we assume the interaction is dispersive, $H_{\text{int}} = g \sigma_z a^\dagger a$. For an entirely non-destructive passive measurement (with no feedback), the state of the ancillary transport qubit is then just shifted by the occupation of the transmission line (i.e. considering the mean-field of Eq. [3])

$$H_{TQ} \approx (\epsilon + g(a^\dagger a)) \sigma_z.$$

(9)

We are able to calculate the analytical values of $\langle a^\dagger a \rangle$ in the limit $N \to \infty$. The transport properties are easily calculated using a counting-statistics approach, which has been well summarised elsewhere. Thus, the current and zero-frequency current-noise measured through the ancillary qubit is simply given by,

$$I = e \frac{T_c^2 \Gamma_R}{(2 + \Gamma_L/\Gamma_R) + \Gamma_R^2/(4 + (\epsilon + g(a^\dagger a))^2)}.$$

(10)

$$\frac{I}{e} = \frac{T_c^2 \Gamma_R}{(2 + \Gamma_L/\Gamma_R) + \Gamma_R^2/(4 + (\epsilon + g(a^\dagger a))^2)}.$$

In the limit $N \to \infty$ the Dicke Hamiltonian has two distinct solutions, corresponding to the two phases of the transition. In the superradiant phase both cavity and qubit array have a macroscopic mean field displacement.

In the lower, 'normal phase', we define the occupation of the cavity $\langle a^\dagger a \rangle$ by an effective temperature $T$ and frequency $\Omega$,

$$\langle a^\dagger a \rangle = \left( \frac{m \Omega}{4\omega} + \frac{\omega}{4m \Omega} \right) \coth \left( \frac{\Omega}{2T} \right) - \frac{1}{2}.$$  

(12)

Where $\Omega$ and $T$ depend on the eigenenergies of $H$:

$$[\epsilon^{(1)}_\pm]^2 = \frac{1}{2} \left( \omega^2 + \omega_0^2 \pm \sqrt{(\omega_0^2 - \omega^2)^2 + 16 \lambda^2 \omega \omega_0} \right),$$

(13)

where $\epsilon_{\pm}$ is only real for $\lambda \leq \lambda_c$, giving the range of this solution. The dependence of $T$ and $\Omega$ on the eigenvalues is via the relations,

$$\cosh \beta \Omega = \left[ 1 + \frac{2 \epsilon - \epsilon_+}{(\epsilon_- - \epsilon_+)^2 \epsilon^2 s^2} \right],$$

(14)

$$m \Omega = \left[ \left( 1 + \frac{2 \epsilon - \epsilon_+}{(\epsilon_- - \epsilon_+)^2 \epsilon^2 s^2} \right) - 1 \right]^{1/2},$$

(15)

$$c \equiv \frac{\cos \gamma^{(1)}}{2}, \quad s \equiv \frac{\sin \gamma^{(1)}},$$

(16)

$$\tan (2 \gamma^{(1)}), \quad \frac{\lambda}{\omega \omega_0} \Omega = \frac{4 \lambda \sqrt{\omega \omega_0}}{(\omega_0^2 - \omega^2)}.$$ 

(17)

where $\beta = 1/k_B T$. These define two equations linking the three parameters of the cavity/qubit system $\omega$, $\omega_0$, $\lambda$, and the three effective parameters of a thermal oscillator $\beta$, $\Omega$, $m$. By setting one energy scale of the original
system such that $\omega = 1$, and that of the thermal oscillator such that $m = 1$, we can uniquely define the correspondence between the two systems. We use the relations,

$$
cosh(\beta \Omega') = 1 + 2\epsilon_-\epsilon_+/D, \quad (19)$$
$$
D = \left[sc(\epsilon_- - \epsilon_+)\right]^2, \quad (20)$$
$$
2\Omega'/\sinh(\beta \Omega') = D/(\epsilon_-s^2 + \epsilon_+c^2), \quad (21)$$
$$
\Omega \sinh(\beta \Omega') = \frac{2\epsilon_-\epsilon_+ (1 + \epsilon_-\epsilon_+ + D)}{\epsilon_-s^2 + \epsilon_+c^2}, \quad (22)$$
$$
\coth(\beta \Omega/2) = \frac{[cosh(\beta \Omega) + 1]/\sinh(\beta \omega)}{\phantom{2}}, \quad (23)$$

to obtain,

$$
\langle a^\dagger a \rangle = \frac{(\epsilon_-s^2 + \epsilon_+c^2)^2}{4} \left( \frac{m}{\omega} + \frac{\omega}{m\epsilon_-\epsilon_+} \right). \quad (24)$$

Thus, in this passive measurement regime, in the large $N$ limit, the occupation of the bosonic mode (which is an order parameter of the phase transition) diverges as $\epsilon_- \to 0$ when $\lambda \to \lambda_c$. In the next section we discuss the effect of this on the current-measurement.

VI. POWER-LAW SCALING IN TRANSPORT PROPERTIES.

A. Results

We plot the current and current-noise in Figs. 2 and 4. We immediately see that, at the critical point $\lambda_c$, the large occupation of the cavity mode (which is proportional to the number of qubits $N$) acts to blockade the current flow (by “pushing apart” the internal energy levels of the transport qubit). Similarly, the zero-frequency noise becomes strictly Poissonian at the critical point. This is a consequence of the slow current and charge-dominated dynamics. Thus, both the current and current-noise are operating as signatures, or order parameters, of the phase transition, because of their direct dependence on $\langle a^\dagger a \rangle$.

As mentioned earlier, in previous work the phase transition was studied as a function of multi-qubit-oscillator coupling $\lambda$. However, the transition can also be observed for a given constant $\lambda$, by tuning the energy level of the qubits $\omega_0$. This is a more realistic approach with superconducting qubits as a possible realisation. Qualitatively, the properties of the transition are the same. For instance, for $\lambda = 0.1\omega$ the transition occurs when $\omega_{0,c} \to 0.04\omega$. The sub-radiant phase occurs for $\omega_0 > \omega_{0,c}$, while the super-radiant phase appears when $\omega_0 < \omega_{0,c}$, both of which are experimentally-accessible regimes. However, because the interaction is off-resonance, the convergence to the correct scaling behaviour requires much larger $N$.

B. Scaling with the number $(N)$ of qubits

To observe power-law scaling with $N$, we must look at the derivative of both the current and current-noise with respect to the Dicke multi-qubit-oscillator coupling $\lambda$. The minimum value of these derivatives will act as a signature of “precursor behaviour”, and from them we can extract the power-law dependence. In Fig. 3(b) we show the derivative of the current, and in Fig. 3(a) we see that the position of the minimum of the current derivative scales as a power law in $N$ via

$$
(\lambda_m - \lambda_c) \propto N^{-0.68 \pm 0.05}. \quad (25)$$

This matches a previous result for the scaling of the entanglement entropy. Similarly the value of the current at this minimum point scales logarithmically as

$$
\frac{d(I/c)}{d\lambda_m} \propto (0.81 \pm 0.05) \log_2 N, \quad (26)$$

as shown in Fig. 3(b). The value of the current derivative obeys similar scaling laws.

Vidal et al. studied the scaling, in $N$, at the critical point $\lambda_c$ of several properties of the Dicke model. They predicted a scaling exponent for $1/\langle a^\dagger a \rangle^2$ of $\alpha = 2/3$. These exponents are different from those we observe here, as they describe behaviour of quantities measured exactly at $\lambda_c$. To extract the same exponents from our numerics would require very large values of $N$. However a recent numerical study by Chen et al. describes a scheme where such exponents can be calculated efficiently for large $N$, and confirmed the correct exponents for some of these quantities.

VII. BACK-ACTION AND CA VITY LOSS.

To take into account both the back-action of the transport qubit, and the loss of photons from the cavity due to coupling to the environment, we must solve the entire master equation numerically. This is a non-trivial task, even with state-of-the-art numerics, and requires careful use of sparse-matrix techniques to increase efficiency.

Dimer et al. investigated the thermodynamic limit of the Dicke model including losses from the bosonic cavity. They found that the critical point was shifted from its normal position as a function of the cavity loss $\gamma_b$. In Figs. 2(c) and 2(d) we do the same for the finite-$N$ case, comparing the three possible regimes: zero back-action and no cavity loss, zero back-action with cavity loss, and a full treatment of cavity loss and back-action.

In figure 2(c) we see that around the critical point the occupancy of the bosonic cavity is almost exactly the same for both master equation treatments, but differs slightly from the ground state Dicke case. Furthermore the strong coupling limit for the full master equation treatment saturates because of the bosonic Hilbert space.
FIG. 2: (Color online) (a) The current $I/e$ versus multi-qubit-oscillator coupling $\lambda$ through the transport qubit for $T_c = 0.1, \Gamma_L = \Gamma_R = 0.1, \epsilon = 0, \omega = \omega_b = 1, g = 0.1$ for $N = 4, 8, 16, 20, 24, \infty$. (b) The derivative of the current through the transport qubit for the same parameter set, versus $\lambda$. Figures (c) and (d) show one particular data curve ($N = 4, \gamma_b = 0.1$) for the bosonic occupancy $\langle a^\dagger a \rangle$ and the current $I/e$ for the three different approximations; zero back-action (ground state of the pure Dicke model), master equation with cavity damping, and master equation with cavity damping and transport qubit feedback.

FIG. 3: (Color online) (a) shows the scaling with $N$ and scaling exponent of the position $(\lambda_m - \lambda_c)$ of the minimum of the current derivative: $(\lambda_m - \lambda_c) \propto N^{-0.68 \pm 0.05}$. Figure (b) shows the scaling of the value of the current at this minimum point to be $(\frac{d(I/e)}{d\lambda})_m \propto (0.81 \pm 0.05) \log_2 N$. The parameters used here are $T_c = 0.1, \Gamma_L = \Gamma_R = 0.1, \epsilon = 0, \omega = \omega_b = 1, g = 0.1$ with data taken at $N = 4, 8, 16, 20, 24, 40, 60$. However the coupling to the qubit, and the loss of energy from the cavity, has less obvious effects on the properties of the phase transition itself. In particular, the parity,

$$\Pi = \exp[i\pi(a^\dagger a + J_z + j)]$$

is no longer conserved, and the steady state will contain components of both the ground state and excited states of $H_D$. Because of this, and the restrictions on the number of spins we can efficiently model, it is not possible to extract exponents from this data. However, we expect the large-$N$ limit to still exhibit features of the phase transition, as predicted by Dimer et al.23

VIII. SIGNATURES OF QUANTUM CHAOS.

Quantum chaos is a characteristic of non-integrable quantum systems. Emary et al.9 extensively studied the
electron transport terms here, and focus on the effect of cavity loss terms. Here we ignore the back-action and i

FIG. 4: (Color online) (a) The current-noise \( F(0) = S(0)/2eI \) versus multi-qubit-oscillator coupling \( \lambda \) for: \( T_c = 0.1, \Gamma_L = \Gamma_R = 0.01, \epsilon = 0, \omega = \omega_b = 1, g = 0.1 \) and for \( N = 4, 8, 16, 20, 24, \infty \). (b) The derivative \( \frac{dF(0)}{d\lambda} \) versus \( \lambda \). The peak scales as a power law of \( N \), similar to the minimum of the current derivative.

(closed) Dicke model and its chaotic properties. In the finite-\( N \) regime they showed that the eigenvalue spectrum of the Dicke model fitted that of the Wigner-Dyson distribution\(^{22}\) when the qubit-boson coupling was around the critical point \( \lambda \approx \lambda_c \). Thus, the chaotic behaviour is understood to be a ‘precursor’ of the phase transition, driven by the parity conservation at the critical point.

Here we extend their work by identifying similar distributions in the eigenvalues,

\[
\chi_i = i(E_i^L) + \nu_i
\]

of the Liouvillian \( L \) which include imaginary components \( i(E_i^L) \) from \( H_D \), as well as real components \( \nu_i \) from the cavity loss terms. Here we ignore the back-action and electron transport terms here, and focus on the effect of cavity damping on the level statistics.

For the pure-state case (no cavity losses), the von Neumann equation of motion,

\[
\frac{\rho(t)}{dt} = -i[H, \rho(t)]
\]

can be written as a set of \( N_H^2 \) coupled equations of the matrix elements of \( \rho \), where \( N_H \) is the dimension of the Hilbert space for the system described by the Hamiltonian \( H \). If \( H \) has \( N_H \) eigenvalues \( E_k, k = 1, ..., N_H \), and we take matrix elements according to the eigenbasis of \( H \), then we can write these linear equations as a diagonal matrix with \( N_H^2 \) imaginary eigenvalues

\[
E^L_{i=j \times k} = \sum_{k,j=1}^{N_H} (E_k - E_j).
\]

Every possible energy gap (not just nearest neighbor) in the spectrum of \( H \) has an eigenvalue in \( L \).

In Fig. (4) we show the positive branch of the imaginary components of the eigenvalues of \( L \) for \( N = 6, \lambda = \lambda_c \), after removal of the \( N_H \) zeros, i.e. the stationary states, and the probability distributions of these components. Even though it is not possible to unfold this spectrum, and all possible level spacings are present, still we see some characteristics of the ‘picket-fence’ distribution\(^{30}\) of the Rabi Hamiltonian and the universal Wigner-Dyson distribution\(^{22}\). We point out that the eigenvalues of this matrix, which is a particular representation of the superoperator \( L \), determine many of the higher-order transport properties, like the frequency dependant noise. This is also seen in scattering theory\(^{31}\). Further analytical work needs to be done to make a strong connection between measurable transport quantities, random matrix theory, and quantum chaos.
IX. FROM CIRCUIT QED TO THE DICKE MODEL.

Al-saidi and Stroud[32] have studied a realization of the Dicke model using Josephson Junctions coupled to an electromagnetic cavity. Operating in the regime between charge and flux qubits they showed that, given the right parameters, the higher-lying levels of each junction can be neglected. In the same way, it is possible to derive the Dicke Hamiltonian, Eq. (1), from the Hamiltonian describing superconducting qubits interacting with a cavity. The proposal and realization of cavity QED[10,11,12,13,14,15,16] in a circuit was an important development for quantum optics and condensed matter, and thus the observation of strong many-body effects in these systems is a natural extension of previous work.

Alternatively, a large number of qubits, in the form of two-level atoms in an atom-chip, coupled to a transmission line, was recently proposed as a way to realise the large-N Dicke model[20].

X. CONCLUSIONS

In conclusion, we have shown that current and current-noise measurements could be used to test for criticality in an ‘on chip’ experiment. We extracted scaling exponents for the Dicke phase transition from semi-analytical and numerical modelling, and illustrated how quantum chaos, a precursor behaviour to the phase transition, is retained in an open-system environment.

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