Heat transfer enhancement induced by electrically generated convection in a plane layer of dielectric liquid

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Abstract. The electro-thermo-convective motion in a plane horizontal dielectric liquid layer subjected to simultaneous action of electric field and thermal gradient is numerically investigated. We consider the case of a strong unipolar charge injection $C=10$ from above or below. Therefore in this context, we only take into account the Coulomb force, disregarding the dielectric one. The effect of the electric field on the heat transfer is analyzed through the characterization of the time history of the Nusselt number as well as its evolution according to the characteristic dimensionless electric parameter $T$. It is demonstrated that the electric effects dominate the buoyancy ones resulting in an electrically induced convection which significantly enhance the heat transfer.

1. Introduction

The combined effects of an electric field and a thermal gradient simultaneously applied to a horizontal dielectric liquid layer (Electro-Thermo-Hydro-Dynamic: ETHD) leads to complex physical interactions in the flow and have received much attention 0-0. One of the reasons for this interest is that the development of electro-thermo-convective instabilities in liquids could be a promising way to increase the heat transfer by means of electrical forces. Such combined effects are therefore worthy of examination not only from a fundamental point of view but also because of potential technological advantages that may result from them [1-3]. The injection of charges between two electrodes provides an efficient way to enhance heat transfer as continuous work can be done by with the release of electrical potential energy.

Since the solutions of the governing ETHD equations are not readily amenable to an exact mathematical analysis, this problem has been essentially tackled by stability analysis that we can find in a series of papers by [7-11].

It has been well established also from an experimental point of view that electro-convection could significantly increase the heat transfer in a liquid layer subjected to a thermal gradient [2],[12]. Considering the complexity of the mathematical models, few ETHD flows have been numerically solved in the past. In [3], we developed a direct numerical simulation based on finite volume method to solve the entire set of ETHD problems.

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In this paper we present a numerical study of electro-thermo-convection in a 2D cavity to demonstrate the enhancement of heat transfer in an horizontal dielectric liquid layer subjected to the development of both Rayleigh-Bénard and electro-convection instabilities. In the next section the mathematical model of a dielectric layer subjected to a thermal gradient and an electric field applied between two electrodes is given. After that the numerical method to solve the entire set of coupled equations is presented. In section 4 the heat transfer enhancement is characterized. Finally a conclusion is drawn in section 5.

2. Formulation of the problem

2.1 Conservation equations

Let us consider a dielectric liquid layer of depth $H$ enclosed between two parallel electrodes of length $L$ (figure 1). The layer is subjected to a potential difference $\Delta V = V_0 - V_1$ and a thermal gradient $\Delta \theta = \theta_1 - \theta_0$ (figure 1). The $z$-axis is taken perpendicular to the electrodes and the gravity is considered acting in the negative $z$-direction. The liquid is heated from below in order to induce Rayleigh-Bénard instabilities and charge injection can occur at Electrode 0 or Electrode 1.

The general set of equations for an incompressible fluid expressing the conservation of mass and momentum including buoyancy and electrical effects, energy balance equation under the Boussinesq assumption, the Gauss theorem, charge density conservation, the definition of the electric field towards electric potential $V$, and the temperature variations of density, permittivity and ionic mobility takes the following form

$$\nabla \cdot (\vec{u}) = 0$$  \hspace{1cm} (1)

$$\rho_0 \left( \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} \right) = -\nabla p + \nabla \tau + \rho \vec{g} + \vec{f}_{el}$$  \hspace{1cm} (2)

$$\rho_0 C_p \left( \frac{\partial \theta}{\partial t} + \vec{u} \cdot \nabla \theta \right) = \nabla \cdot (\lambda \nabla \theta) + \vec{E} \cdot \vec{j}$$  \hspace{1cm} (3)

$$\nabla \cdot (\varepsilon \vec{E}) = q$$  \hspace{1cm} (4)

$$\frac{\partial q}{\partial t} + \nabla \cdot \vec{j} = 0$$  \hspace{1cm} (5)

$$\vec{E} = -\nabla V$$  \hspace{1cm} (6)

$$\rho(\theta) = \rho_0 \left[ 1 - \beta (\theta - \theta_0) \right]$$  \hspace{1cm} (7)

$$\varepsilon(\theta) = \varepsilon_0 \left[ 1 - \varepsilon_1 (\theta - \theta_0) \right]$$  \hspace{1cm} (8)

$$K(\theta) = K_0 \left[ 1 - k_1 (\theta - \theta_0) \right]$$  \hspace{1cm} (9)
where \( u \) is the velocity, \( \rho_0 \) is the mass density, \( p \) is the pressure, \( g \) the gravity, \( f_{el} \) stands for the electric forces induced by the electric field \( \vec{E} \), \( C_p \) the thermal capacity, \( \theta \) the temperature, \( \theta_0 \) a reference temperature, \( \lambda \) the conductivity, \( \varepsilon_0 \) the permittivity of the fluid, \( q \) the volumic electric charge density in the liquid. \( K \) is the ion drift velocity and \( V \) the electric potential. \( \beta \) is the volumetric thermal expansion coefficient, \( e_1, k_1 \) are respectively the relative change of permittivity and mobility with temperature.

\[
e_i = -\frac{1}{\varepsilon_0} \left( \frac{\partial \varepsilon}{\partial \theta} \right), \quad k_i = -\frac{1}{K_0} \left( \frac{\partial K}{\partial \theta} \right)
\]

In the conservative equation (5) for the current density, there exists three distinct mechanisms of charge transfer: \( j = q \vec{u} + q\vec{E} - \nabla \cdot q \varepsilon \), where \( D \) denotes the charge-diffusion coefficient. Since the contribution of diffusion to the electric current is much smaller than the other two terms, it is commonly to ignore it [13]. The electric forces are expressed as follows: \( \vec{f}_{el} = q\vec{E} - \frac{E^2}{2} \nabla \varepsilon + \nabla \left( \frac{1}{2} E^2 \left( \frac{\partial \varepsilon}{\partial \rho} \right) \right) \) where the first term is the Coulomb force, the second the dielectrophoretic force and the last one the electrostriction force. The electrostriction force can be combined with the static pressure and term and we define a modified pressure: \( \tilde{p} = p - \nabla \left( \frac{1}{2} E^2 \left( \frac{\partial \varepsilon}{\partial \rho_m} \right) \right) \). The last term in the energy equation (3) represents the Joule heating, which can also be ignored due to the small amplitude of the current density [14].

2.2 Non dimensional set of equations

The above equations are put in dimensionless form by using the following references:

\[
[x, y] = H, \quad [t] = H^2 / \nu, \quad [\vec{u}] = V_H, \quad [\vec{p}] = \rho_0 (v / H)^2, \quad [\theta] = \theta_0 - \theta, \quad [V] = V_0 - V_1, \quad [\vec{E}] = (V_0 - V_1) / H, \quad [q] = q_0, \quad [K] = K_0, \quad [\varepsilon] = \varepsilon_0
\]

We obtained we obtain a complete and fully coupled set of dimensionless equations.

\[
\nabla \vec{u} = 0 \quad (10)
\]

\[
\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} = -\nabla \tilde{p} + \Delta \vec{u} + \frac{T^2}{M^2} \left[ C_q \vec{E} + \frac{E^2}{2} \nabla \varepsilon \right] + \frac{Ra}{Pr} \frac{\theta}{\varepsilon_y} \quad (11)
\]

\[
\frac{\partial \theta}{\partial t} + \vec{u} \cdot \nabla \theta = \frac{1}{Pr} \Delta \theta \quad (12)
\]

\[
\frac{\partial q}{\partial t} + \nabla \cdot \left( q (\vec{u} + RK \vec{E}) \right) = 0 \quad (13)
\]

\[
\nabla \cdot (\varepsilon_0 \nabla V) = -C_q \quad (14)
\]

\[
\vec{E} = -\nabla V \quad (15)
\]

\[
\varepsilon = 1 - LRa \theta \quad (16)
\]

\[
K = 1 + NRa \theta \quad (17)
\]

This leads to the following set of dimensionless parameters:

\[
Ra = \frac{g \beta \Delta \theta H^3}{\nu \kappa} \quad \text{and} \quad Pr = \frac{\nu}{\kappa} \quad \text{the Rayleigh and Prandtl number respectively.}
\]

\[
T = \frac{\varepsilon_0 \Delta V}{\rho_0 \nu K_0} \quad \text{which is the ratio of Coulomb and viscous forces, analogous to the Rayleigh number in thermal convection.}
\]
2. Dimensionless measure of the injection level.

\[ C = \frac{q V}{\varepsilon_0 \Delta V} \]  

is a dimensionless measure of the injection level.

\[ M = \left( \frac{\varepsilon_0}{K_0 \rho_0} \right)^{1/2} \]  

is the ratio between the hydrodynamic and ionic mobility.

\[ N = \frac{\epsilon_0 \nu \kappa}{\beta g H^3} \]  

and \( L = \frac{k_1 \nu \kappa}{\beta g H^3} \) are dimensionless temperature derivatives of respectively dielectric constant and mobility.

We shall also use for convenience \( R = TM^{-2} \).

2.3. Strong injection case

In this study we only consider the case of a strong charge injection \( C=10 \). In this context Rodriguez et al. [8] have shown that the variation of ionic mobility and permittivity with temperature is negligible. Therefore the liquid properties will be taken as independent of temperature so that \( N = L = 0 \) and also the dielectric force could be disregarded.

2.3. Initial and boundary conditions

We start from rest and all the quantities are set to zero. At \( t=0 \) the liquid is heated from below and the electric potential difference between the two electrodes is applied and electric charges are injected in the bulk. The boundary conditions in the case of heating and injection from lower electrode are depicted in figure 2.

On lateral walls we consider for the velocity the no-slip boundary conditions.

\[ u = v = 0 \quad V = 0 \quad \theta = 0 \]

Figure 2. Computational domain and boundary conditions.

3. Numerical method

The equations are integrated using a second order in time and space finite volume method [15]. The primitive variables \( \tilde{u} \) and \( \tilde{p} \) are determined using the Augmented Lagrangian method [16] and Uzawa [17] algorithm. The linear systems are solved using the Bi-CGSTAB [18] method with a preconditioning based on a modified and incomplete Gauss factorization MILU [19]. The calculation is fully transient. Special care must be provided while solving (4) because of its hyperbolic nature. The SMART algorithm [20] has been utilized in that way. Some additional details about the implementation of the numerical method can be found in [21].
4. Results and discussion

4.1 Heat-transfer enhancement.

From the linear stability analysis, it appears that when submitting the liquid layer to charge injection, it is possible to induce a convective movement whereas the Rayleigh number is under the threshold value 1708. Then the question of enhancing the heat-transfer through this mean arises naturally. We have computed the mean Nusselt number 

$$\overline{Nu} = \frac{\int_0^L (\partial \theta )}{\int_0^L (\partial y)} \bigg|_{y=0}$$

along the hot wall.

From figure 3 it is interesting to note that the mean Nusselt number is noticeably increased with $T$ as it has been confirmed by [2],[4],[12].

![Figure 3. Mean Nusselt number versus $T$ for $Ra=10000$, $Pr=10$.](image)

Figure 4 (a) is a caption of the evolution of this mean Nusselt number versus Rayleigh number for different values of $T$. Figure 4 (b) represents experimental results performed by Pérez et al [12]. First we observe a similar tendency for both numerical and experimental results. From these curves another important point can be emphasized. Above a given range of Rayleigh numbers around 15000, and for $T>0$ increasing the rayleigh number does not increase the Nusselt number much. This tendency is more visible on the experimental curves (see figure 4 (b)). In other words the Nusselt number becomes almost independent of the Rayleigh number for high enough values of $T$.

![Figure 4. Mean Nusselt number versus Rayleigh for different values of the $T$ parameter, $Pr=10$.](image)
In figure 5, we present the time evolution of the maximum vertical velocity component of the flow at \( Ra = 10000 \) for several values of \( T \) as well as the flow structures in terms of streamlines isocontours.

Notice that the aspect ratio is 2, for pure thermal problem (\( T=0 \)) as expected we obtain a steady solution with two thermo-convective cells. When \( T \) is increased to 100 and 200 the steady value of the maximum velocity is visibly augmented. The increase of the velocity field explains the heat transfer enhancement in the range of \( T \in (0, 250) \). Indeed in natural convection in enclosure the velocities are small and when electric force is generated these velocities are immediately and significantly augmented [22]. When we further increase \( T \) value to 300, the steady two-cell structure starts to oscillate. These oscillations indicate a better fluid mixing, which can also contribute to the heat transfer enhancement effect. A significant change is observed with \( T=350 \) and \( T=400 \) in the flow structure. Indeed we can observe that the flow bifurcates to a four cells structure. For \( T=350 \) the flow is steady while for \( T=400 \) these four cells start to oscillate too as in the case \( T=300 \).

This four cells pattern will indubitably contributes to increase the vortical activity of the flow and hence fluid mixing. This explain too why the heat transfer can be increased consequently.

![Figure 5](image-url)

**Figure 5.** Time history of the maximum of vertical velocity with the associated flow structure depicted by the streamlines for \( Ra=10000, Pr=10 \) and different values of the \( T \) parameter.
In this final numerical experiment, the fluid is still heated from below and we consider electric charge injection from the bottom or the top.

In figure 8 the time evolution of the mean Nusselt number for $Ra=10000$ and for different $T$ values is presented. We can notice that the value of the mean Nusselt number is notably increased in comparison with the reference case of the pure thermal problem for which $T=0$. For $T=100$ and $T=200$ after a brief transient period a steady state is reached and the final value of the Nusselt number is nearly the same whatever injection is from bottom or top. We observe an augmentation of the mean Nusselt number and therefore of the heat transfer of $10.4\%$ for $T=100$, $30.96\%$ for $T=200$ and $96.23\%$ for $T=400$ with injection from the top. This increase of the mean Nusselt number is directly linked to the dynamic mixture induced by an increase of the vortical activity generated by the development of the electroconvective instability.

![Figure 6. Mean Nusselt number time history for $Ra=10000$, $Pr=10$.](image)

5. Conclusion

The effect of the charge injection on the heat transfer in a dielectric liquid layer is investigated by a direct numerical simulation. The numerical results show that the charge injection significantly increases the heat transfer accordingly to several experimental results from the literature. Injection from the bottom electrode of the upper one leads to almost the same enhancement. The Nusselt number becomes almost independent of the Rayleigh number for high enough values of $T$. Our results are in a good qualitative agreement with the available experimental data.

6. References

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