Pokemon: Protected Logic Qubit Derived from the 0-π Qubit

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We propose a new protected logic qubit called pokemon, which is derived from the 0-π qubit by harnessing one capacitively shunted inductor and two capacitively shunted Josephson junctions embedded in a superconducting loop. Similar to the 0-π qubit, the two basis states of the proposed qubit are separated by a high barrier, but their wave functions are highly localized along both axis directions of the two-dimensional parameter space, instead of the highly localized wave functions along only one axis direction in the 0-π qubit. This makes the pokemon qubit more protected. For instance, the relaxation of the pokemon qubit is exponentially reduced by two equally important factors, while the relaxation of the 0-π qubit is exponentially reduced by only one factor. Moreover, we show that the inductor in the pokemon can be replaced by a nonlinear inductor using, e.g., a pair or two pairs of Josephson junctions. This offers an experimentally promising way to implement next-generation superconducting qubits with even higher quantum coherence.

Introduction.—Quantum computers can outperform their classical counterparts in simulating many-body quantum systems [1, 2] and implementing important algorithms [3], owing to their exponentially large capacity in storing and processing information. These quantum advantages can be demonstrated when qubits in quantum computers achieve sufficiently high quantum coherence. In recent years, superconducting qubits [4–11] based on Josephson-junction circuits have indeed been considerably improved in their quantum coherence, showing quantum advantage using tens of superconducting qubits [12, 13].

This quantum-coherence enhancement of superconducting qubits was achieved by shunting a large capacitance to the small Josephson junction in the circuit to reduce the sensitivity of the qubits to the charge noise [14, 15]. This was proposed in the capacitively shunted flux qubit [16] and later implemented experimentally [17, 18]. Also, it was proposed for the capacitively shunted Cooper-pair box, called the transmon [19]. To be tunable and easily coupled to the neighboring qubits, the transmon was latter modified as the Xmon [20] by both replacing the small Josephson junction with a superconducting quantum interference device (SQUID) and connecting a cross-shaped electrode to the SQUID. In addition, the gatemon [21], which is also a transmon-like device, and the capacitively shunted fluxonium [22] were implemented. Generally speaking, these high-coherence superconducting qubits are encoded only on one degree of freedom related to the small Josephson junction (or SQUID) in the circuit, whereas other circuit elements are to adjust the anharmonicity of the qubit [15, 23].

However, a practical quantum computer should be fault-tolerant, requiring a significantly increased system scale [24]. This needs qubits with even higher quantum coherence. A protected logic qubit called the 0-π qubit was proposed [25] and implemented very recently [26], which harnesses two Josephson junctions and two inductors embedded in a superconducting loop and shunted with two intersecting capacitors [see Fig. 1(a)]. This superconducting qubit is encoded on the two degrees of freedom of the Josephson junctions, with two isolated wave functions in the two-dimensional (2D) parameter space acting as the basis states of the qubit. While these two states are separated by a high barrier, they are highly localized only along one axis direction in the 2D parameter space. Also, the circuitry of the 0-π qubit becomes complex when implementing each of the two inductors with a large number of Josephson junctions [26].

In this work, we propose a simplified structure to implement a new protected logic qubit, called pokemon, which is derived from the 0-π qubit by using one capacitively shunted inductor and two capacitively shunted Josephson junctions embedded in a superconducting loop. The two basis states of the proposed qubit are also separated by a high barrier. Moreover, they are highly localized along both axis directions of the 2D parameter space, instead of the highly localized basis states along only one axis direction in the 0-π qubit. These make the pokemon more protected. For instance, the relaxation of the proposed qubit is exponentially reduced by two equally important factors, while the relaxation of the 0-π qubit is exponentially reduced by only one factor. Also, we show that the inductor in the pokemon can be replaced by a nonlinear inductor using, e.g., a pair or two pairs of Josephson junctions, instead of using a large number of Josephson junctions to achieve a linear inductor. This provides an experimentally promising way to implement next-generation superconducting qubits with even higher coherence to carry out more demanding tasks in quantum computing.

Pokemon derived from the 0-π qubit.—In contrast to the 0-π qubit in Fig. 1(a), the protected logic qubit called pokemon is composed of two (identical) capacitively shunted Josephson junctions and a capacitively shunted inductor embedded in a superconducting loop [see Fig. 1(b)]. Also, we harness a large shunting capacitance for each Josephson junction to reduce
with the commutation relations \[ [\hat{T}, \hat{C}] = \hbar \pi, \] two identical conductors \( L \), and two intersecting capacitors of the same capacitance \( C \).

(b) The circuit for the protected logic qubit called potassium, consisting of a superconducting loop with two capacitively shunted Josephson junctions and an inductor \( L \) shunted by a capacitance \( C \). Each junction has a coupling energy \( E_J \) and capacitance \( C \), and is also shunted with a large capacitance \( C_s \). Without the (green) capacitance \( C_s \), the circuit is reduced to a bifluxon qubit [27], but each junction is shunted by a large capacitance. In both (a) and (b), as well as in Fig. 3 below, each (red) arrow indicates the assigned phase-drop direction for the junction or inductor. The reduced magnetic flux threading the loop is \( \phi = \Phi_{\text{ext}}/\Phi_0 \).

the noise effect on the qubit.

In this potassium, the phase drops across the two Josephson junctions and the inductor are constrained by the fluxoid quantization condition: \( \phi_1 - \phi_2 - \phi_3 + 2\pi f = 0 \), with \( f = \Phi_{\text{ext}}/\Phi_0 \), where \( \Phi_{\text{ext}} \) is the applied magnetic flux threading the loop and \( \Phi_0 = h/2e \) is the flux quantum. The voltages across the two Josephson junctions and the inductor are related to the corresponding phase drops by \( V_1 = (\Phi_0/2\pi)\phi_1 \). The electric energy of the potassium is \( E = \frac{1}{2}(C_1 + C_2)(V_1^2 + V_2^2) + \frac{1}{2}C_1 V_1^2 + \frac{1}{2}(C_1 + C_2) V_2^2 \). Using the relationship between the voltage and phase drop, as well as the canonical coordinates \( \theta \equiv \frac{1}{2}(\phi_1 + \phi_2) \) and \( \varphi \equiv \frac{1}{2}(\phi_1 - \phi_2) \), we can express the electric energy as \( E = (\Phi_0/2\pi)^2((C_1 + C_2)\theta^2 + (C_1 + C_2 + 2C_L)\varphi^2) \).

The Lagrangian of the potassium is \( L = T - U \), where the potential is \( U = E_J(1 - \cos \phi_1) + E_J(1 - \cos \phi_2) + (\Phi_0/2\pi)^2(\varphi^2)_{1} - (\Phi_0/2\pi)^2(\varphi^2)_{2} \). The canonical momenta are \( P_\theta = \frac{\partial L}{\partial \dot{\theta}} = 2(\Phi_0/2\pi)^2(C_1 + C_2)\theta \), and \( P_\varphi = \frac{\partial L}{\partial \dot{\varphi}} = 2(\Phi_0/2\pi)^2(C_1 + C_2 + 2C_L)\varphi \). Then, the Hamiltonian of the potassium, \( H_\text{pot} = P_\theta^2 + P_\varphi^2/L \), can be expressed as

\[
H_\text{pot} = \frac{P_\theta^2}{4(\Phi_0/2\pi)^2(C_1 + C_2)} + \frac{P_\varphi^2}{4(\Phi_0/2\pi)^2(C_1 + C_2 + 2C_L)} + U, \tag{1}
\]

with the commutation relations \( [\theta, P_\varphi] = i\hbar \), and \( [\varphi, P_\varphi] = i\hbar \).

When introducing number operators for Cooper pairs, \( \hat{N}_\theta = -i\frac{P_\theta}{\hbar} \) and \( \hat{N}_\varphi = -i\frac{P_\varphi}{\hbar} \). Hamiltonian (1) is converted to

\[
H_\text{pot} = H_J + E_L\varphi^2, \tag{2}
\]

with the inductive energy \( E_L = 2(\Phi_0/2\pi)^2/L \), and

\[
H_\text{pot} = 4E_\varphi\hat{N}_\varphi^2 + 2E_J - 2E_J \cos \theta \cos (\varphi - \pi f). \tag{3}
\]

Here \( E_\varphi = e^2[4(C_1 + C_2)]^{-1} \), \( E_J = e^2[4(C_1 + C_2)]^{-1} \), and \( E_\text{osc} = 4E_\varphi\hat{N}_\varphi^2 + 2E_J \), are single-electron charging energies relevant to the two junctions.

For the 0-\( \pi \) qubit in Fig. 1(a), the phase drops across the two Josephson junctions and the two inductors are constrained by the fluxoid quantization condition: \( \phi_1 - \phi_2 - \phi_3 + 2\pi f = 0 \). The Hamiltonian can be written as [28] \( H_\text{pot} = H_\text{pot} + H_\text{osc} \), where \( H_\text{osc} \) has the same form as in Eq. (2), but \( E_\varphi = E_\varphi\hat{N}_\varphi^2 + 2E_J \cos \theta \cos (\varphi - \pi f) \). Hamiltonian (1) is written as \( H_\text{osc} = H_\text{osc} + H_\text{osc} \), where \( H_\text{osc} = 4E_\varphi\hat{N}_\varphi^2 + 2E_J \cos \theta \cos (\varphi - \pi f) \). Since \( H_\text{osc} \) is fully decoupled from \( H_\text{osc} \), the Hamiltonian of the 0-\( \pi \) qubit can be reduced to \( H_\text{osc} \).

For both the 0-\( \pi \) qubit in Fig. 1(a) and the potassium in Fig. 1(b), when the external magnetic flux is absent (i.e., \( f = 0 \)), the potential without the inductive energy, i.e., \( U_0 = E_J - 2E_J \cos \theta \cos (\varphi - \pi f) \), has degenerate minima at points \( (\theta, \varphi) = (0,0) \) and \( (\pi, \pi) \) in the 2D parameter space [see Fig. 2(a)]. Nevertheless, the inductive potential \( E_L\varphi^2 \)
removes this degeneracy [see the contour plot in Figs. 2(b) and 2(c)]. When \( E_J \gg E_L \), as in the 0-\( \pi \) qubit [25], we can use the lowest-energy states localized around (0,0) and \((\pi,\pi)\) to encode thepokemon qubit. Therefore, both the Pokémon and the 0-\( \pi \) qubit are actually logic qubits encoded on two degrees of freedom \( \theta \) and \( \varphi \). However, there are distinct differences between them. First, the pokemon has a simpler circuitry, providing an advantage in sample fabrication. Second, the pokemon has two small single-electron charging energies \( E_{\text{c}0} = \tilde{E}_{\text{c}0} \) and \( E_{\text{c}q} = \tilde{E}_{\text{c}q} \), because \( C_x \gg C_y \). This corresponds to a heavy particle with anisotropic masses \( M_0 = 2(\Phi_0/2\pi)^2(C_x + C_y) \) and \( M_\varphi = 2(\Phi_0/2\pi)^2(C_x + C_\varphi + 2C_L) \), moving in quantum wells separated by a barrier of height \( E_J \gg E_{\text{c}0},E_{\text{c}q} \). Owing to these large masses in both \( \theta \) and \( \varphi \) directions as well as the higher inter-well barrier, the ground and first excited states \( |0\rangle \) and \(|1\rangle \) of the pokemon are highly localized in the vicinity of \((0,0)\) and \((\pi,\pi)\), respectively, along both \( \theta \) and \( \varphi \) directions [Fig. 2(c)]; and the transition frequency of the pokemon qubit is mainly determined by the inductive-potential difference between these two points. As for the 0-\( \pi \) qubit, the effective mass along the \( \theta \) direction is as large as \( M_\theta \) of the pokemon, but the effective mass along the \( \varphi \) direction, \( M_\varphi = 2(\Phi_0/2\pi)^2C_\varphi \), is much smaller. The ground and first excited states of the 0-\( \pi \) qubit are hence only highly localized at \((0,0)\) and \((\pi,\pi)\) along the \( \theta \) direction [Fig. 2(b)]. This indicates that the pokemon can become more protected.

In addition, when the capacitance \( C_T \) is absent in Fig. 1(b), the pokemon is reduced to the circuit configuration of the biluxon [27], but without the gate, and each Josephson junction there is now shunted by a large capacitance \( C_T \). This capacitively shunted biluxon has the same Hamiltonian as in Eq. (2), but with \( E_{\text{c}0} = E_{\text{c}q} = \tilde{E}_{\text{c}0}, \text{i.e.,} \) the effective masses become isotropic.

Robustness against both charge and flux noises.—Below we reveal the robustness of the pokemon against both charge and flux noises, which are usually two major decoherence sources in superconducting circuits. For the 0-\( \pi \) qubit, since the ground and first excited states \(|0\rangle \) and \(|1\rangle \) are less localized around \((0,0)\) and \((\pi,\pi)\) along the \( \varphi \) direction, it is suitable to numerically study its quantum coherence [28]. On the contrary, the ground and first excited states \(|0\rangle \) and \(|1\rangle \) of the pokemon are highly localized around \((0,0)\) and \((\pi,\pi)\) along both \( \theta \) and \( \varphi \) directions. These two basis states of the pokemon can be well approximated by \(|0\rangle = \sqrt{\frac{\alpha_0}{\alpha_\varphi}} e^{-\frac{1}{2} \alpha_\varphi (\theta-\theta_0)^2+\alpha_\varphi (\varphi-\varphi_0)^2} \rangle \), \(|1\rangle = \sqrt{\frac{\alpha_\varphi}{\alpha_0}} e^{-\frac{1}{2} \alpha_0 (\theta-\theta_0)^2+\alpha_0 (\varphi-\varphi_0)^2} \rangle \), with \( \alpha_\varphi^2 = \frac{1}{2} \sqrt{E_J/E_L} \), and \( \alpha_0^2 = \frac{1}{2} \sqrt{E_{\text{c}0}/E_{\text{c}q}} \). Due to the inductive potential, the center of \(|1\rangle \) is shifted to \( \frac{E_J}{E_L+E_{\text{c}q}} \pi \) in the \( \varphi \) direction, and the level difference between \(|1\rangle \) and \(|0\rangle \) is \( \delta E_{10} = \frac{E_{\text{c}q} E_J^2}{E_J+E_{\text{c}q}} \sim E_L \pi^2 \) for \( E_J \gg E_L \).

When both charge and flux fluctuations are considered, the Hamiltonian (2) is changed, at \( f = 0 \), to

\[
H = 4E_{\text{c}0}(\hat{\Delta}_0 - \delta N_0)^2 + 4E_{\text{c}q}(\hat{\Delta}_q - \delta N_q)^2 + 2E_J - 2E_J \cos \theta \cos \varphi - E_J \varphi^2,
\]

where \( \delta N_{\theta(\varphi)} = \delta Q_{\theta(\varphi)}/2e \) is the reduced charge fluctuation and \( \delta f \equiv \delta \Phi_{\text{ext}}/\Phi_0 \) is the reduced flux fluctuation. Up to second-order perturbations, the Hamiltonian (4) can be written as \( H = H_0 + H'_\theta + H'_\varphi + H'_f \), with \( H_0 \) given by Eq. (2) and

\[
H'_\theta = X_{\theta(\varphi)}^{(1)} \delta N_{\theta(\varphi)} + \frac{1}{2} X_{\theta(\varphi)}^{(2)} \delta N_{\theta(\varphi)}^2,
\]

\[
H'_f = X_f^{(1)} \delta f + \frac{1}{2} X_f^{(2)} \delta f^2,
\]

where \( X_{\theta(\varphi)}^{(1)} = \frac{\partial}{\partial \delta N_{\theta(\varphi)}} \langle \delta \hat{\Delta}_\theta(\varphi) \rangle_{\delta N_{\theta(\varphi)}=0} = -8E_{\text{c}0} \delta N_{\theta(\varphi)} \), \( X_{\theta(\varphi)}^{(2)} = \frac{\partial^2}{\partial \delta N_{\theta(\varphi)}^2} \langle \delta \hat{\Delta}_\theta(\varphi) \rangle_{\delta N_{\theta(\varphi)}=0} = -2\pi E_J \cos \theta \sin \varphi \), and \( X_f^{(2)} = \frac{\partial^2}{\partial \delta f^2} \langle \delta \hat{\Delta}_f \rangle_{\delta f=0} = 2\pi E_J \cos \theta \cos \varphi \).

Note that the perturbation arising from the charge noise terminates at the second order owing to the specific form of the Hamiltonian (4). We can derive that \( \langle \delta \hat{\Delta}_f \rangle_{\delta f=0} = 0 \), and \( \langle \delta \hat{\Delta}_f \rangle_{\delta f=0} = 0 \). These imply that the charge fluctuation does not induce dephasing to the pokemon. However, for the flux noise, we have nonzero \( A_f \equiv \frac{1}{2} \langle \langle \delta \hat{\Delta}_f \rangle_{\delta f=0} \rangle - \langle \delta \hat{\Delta}_f \rangle_{\delta f=0} \rangle \);
where each charge-noise power spectrum is defined as

$$S_i(\omega) = \frac{1}{\pi} \int_0^{\infty} dt \langle \delta N_i(t') \delta N_i(t) e^{-i\omega t} \rangle, \quad i = \theta, \phi,$$

and \(\omega_{10} = \epsilon_{10}/\hbar\) is the transition frequency of the system. The total relaxation rate is

$$\Gamma_1 = \Gamma_{1,0} + \Gamma_{1,\phi},$$

which gives the relaxation time

$$T_1 = \frac{1}{\Gamma_1}.$$

For superconducting circuits, low-frequency noise plays a pivotal role in decoherence, which can be modeled as 1/f noise via

$$K_{\text{char}}(\omega) = K_i/|\omega|, \quad i = \theta, \phi, \text{ and } f.$$

Typically, \(K_\theta(\omega) \sim 1.7 \times 10^{-6}\) for charge noise [31] and \(K_\phi \sim 3 \times 10^{-12}\) for flux noise [32]. For \(E_{\text{ech}}(\omega), E_L \ll E_J, A_i\), reduces to a small quantity

$$A_i \sim 2\pi^2 E_L,$$

in comparison with \(E_J\). Also, because \(K_f\) is several orders of magnitude smaller than \(K_{\text{char}}(\omega)\), a long dephasing time \(T_\phi\) is given to satisfy

$$\eta(T_\phi) = 1$$

in Eq. (7), so it gives a small dephasing rate \(\Gamma_\phi\). Then, we can ignore this dephasing rate and write the decoherence rate as

$$\Gamma_2 = \frac{1}{2} \Gamma_1 + \Gamma_\phi \approx \frac{1}{2} \Gamma_1.$$  

Moreover, it follows from Eqs. (8)-(10) that the relaxation rate is proportional to \(e^{-2^2}\), which is exponentially reduced by both

$$\sqrt{\frac{E_J}{E_{\text{cho}}}} \text{ and } \sqrt{\frac{E_J + E_L}{E_{\text{cho}}} \text{.}}$$

Because \(E_{\text{cho}} < E_J\), these two quantities are both large, yielding a small relaxation rate as well. Therefore, thepokemon is a well-protected logic qubit, owing to its small rates in both dephasing and relaxation. In contrast, the 0-\(\pi\) qubit has \(E_{\text{cho}} \ll E_J\) but not \(E_{\text{cho}} \ll E_J\), so its relaxation rate is only exponentially reduced by

$$\sqrt{\frac{E_J + E_L}{E_{\text{cho}}}},$$

without a further exponential reduction by

$$\sqrt{\frac{E_J + E_L}{E_{\text{cho}}}} \approx \sqrt{\frac{E_J}{E_{\text{cho}}}} \text{ in the pokemon.}$$

**Pokemon with the loop inductance replaced by a nonlinear inductance.**—For both the 0-\(\pi\) qubit in Fig. 1(a) and thepokemon in Fig. 1(b), it is required that \(E_J \gg E_L\), needing a large loop inductance \(L\). Here we propose to implement thepokemon by replacing the loop inductance with a nonlinear one.

We first replace the capacitively shunted inductor in Fig. 1(b) with two Josephson junctions, both having the same coupling energy \(E_J\) and capacitance \(C_J\). The total Hamiltonian of thepokemon can be written as

$$H_{\text{tot}} = H_J + 2E_J(1 - \cos \varphi) \cos \chi + H_\chi,$$

where \(H_J\) has the same form as in Eq. (3), but with \(E_{\text{cho}} = E_{\text{ch}}\), and \(E_{\text{cho}} = e^2[4(C_J + C_J + C_J)]^{-1} \). The inductive potential of a nonlinear inductor, which yields an energy-level difference between these two basis states of thepokemon, is

$$\epsilon_{10} = 2E_J(1 - \cos \varphi).$$

When replacing the capacitive shunted inductor with four (i.e., two pairs of Josephson junctions, each junction having coupling energy \(E_J\) and capacitance \(C_J\)), the Hamiltonian of thepokemon can be reduced to [29]

$$H_{\text{tot}}^\prime = H_J + E_J(1 - \cos \varphi).$$

Thus, the flux noise does not induce relaxation but weak dephasing to thepokemon, while the charge noise does not cause dephasing and the induced relaxation can be exponentially reduced by the large capacitors shunted to the two Josephson junctions. Furthermore, we show how to achieve thepokemon by replacing the capacitive shunted inductor with a few (one or two) pairs of Josephson junctions. This offers an experimentally feasible method to implement protected logic qubits with even higher quantum coherence. In the near future, an experimental comparison between the properties of apokemon and...
a transmon would be insightful. It would be the superconducting qubit analog of Godzilla meets King Kong.

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