Search for the Birefringence of Gravitational Waves with the Third Observing Run of Advanced LIGO-Virgo

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Abstract

Gravitational waves would attain birefringence during their propagation from distant sources to the Earth, when the charge, parity, and time reversal (CPT) symmetry is broken. If it was sizeable enough, such birefringence could be measured by the Advanced LIGO, Virgo, and KAGRA detector network. In this work, we place constraints on the birefringence of gravitational waves with the third observing run of this network, i.e., two catalogs GWTC-2 and GWTC-3. For the dispersion relation $\omega^2 = k^2 \pm 2\zeta k^3$, our analysis shows the up-to-date strictest limit on the CPT-violating parameter, i.e., $\zeta = 4.07^{+3.59}_{-5.70} \times 10^{-17}$ m, at 68% confidence level. This limit is stricter by $\sim$5 times when compared to the existing one ($\sim 2 \times 10^{-16}$ m) and stands for the first $\sim$10 GeV-scale test of the CPT symmetry in gravitational waves. The results of the Bayes factor strongly disfavor the birefringence scenario of gravitational waves.

Unified Astronomy Thesaurus concepts: Gravitational wave astronomy (675); Gravitational waves (678); General relativity (641)

1. Introduction

The charge, parity, and time reversal (CPT) transformation is a fundamental symmetry in modern physics (Peskin & Schroeder 1995). The results from numerous experiments in laboratories and astronomy have been shown to be consistent with the predictions of the CPT symmetry to high precision (Kostelecký & Russell 2011; Will 2014), since it was first proposed in the 1950s (Schwinger 1951). Although no definitive signals of CPT violation have been uncovered, there are quantities of motivations, e.g., candidate theories of quantum gravity on Planck scale (Kostelecký & Samuel 1989; Kostelecký & Potting 1991; Amelino-Camelia et al. 1998; Amelino-Camelia 2013; Mielczarek & Trześniewski 2018) to perform careful investigations on possible mechanisms and manifestations of CPT symmetry breaking. However, almost all tests of the CPT symmetry have been implemented in the electromagnetic or/and neutrino sectors, rather than the pure-gravitational sector (see reviews Kostelecký & Russell 2011; Will 2014 and references therein).

The discovery of gravitational waves (GWs) by the Advanced Laser Interferometer Gravitational-Wave Observatory (LIGO; Abbott et al. 2016) in 2015 September opened a new observational window to testing the CPT symmetry, in particular, in the gravitational sector (Kostelecký & Mewes 2016; Wang & Zhao 2020; Shao 2020; Wang et al. 2021a, 2021b, 2021c; Niu et al. 2022). When the CPT symmetry is violated, GWs would attain birefringence during their propagation from distant sources to our detectors (Kostelecký & Mewes 2016). The birefringence could slightly widen or split the peak of the gravitational waveform (Yamada & Tanaka 2020). The first constraint on the dimension-5 CPT-violating operators is reported to be smaller than $2 \times 10^{-14}$ m (Kostelecký & Mewes 2016), by considering the width of peak at the maximal amplitude of the first event, i.e., GW150914. The birefringence could also lead to a rotation between the plus and cross modes of GWs (Mewes 2019; Wang 2020). In the spirit of effective field theory, the leading-order CPT-violating contribution to the gravitational waveform was evaluated quantitatively (Mewes 2019; Wang 2020). Based on this, we performed the first full Bayesian test of the CPT symmetry in Wang & Zhao (2020) by analyzing the events observed during the Advanced LIGO-Virgo’s first (O1) and second (O2) observing runs, i.e., GWTC-1 Abbott et al. (2019). No evidence of the CPT violation yielded a constraint on the CPT-violating parameter, i.e., $1.4^{+2.2}_{-3.1} \times 10^{-16}$ m, which is two orders of magnitude stricter than before. Similar results were reported soon in Wang et al. (2021b), Shao (2020). Recently, Wang et al. (2021a, 2021c) reported a new upper limit $4.5 \times 10^{-15}$ m by using the third open GW catalog (Nitz et al. 2021), which includes the events during the first half of the third observing run (O3a). Model-dependent studies on the CPT symmetry have also been broadly investigated.5 However, in either case, there has not been an analysis based on all three observing runs of Advanced LIGO-Virgo.

In this work, we revisit the Bayesian test of the CPT symmetry with the data in GWTC-2, GWTC-2.1 (Abbott et al. 2021a, 2021b), and GWTC-3 (Abbott et al. 2021c). The former

5 For example, see Kostelecký (2011), Yagi & Yang (2018), Yagi et al. (2012), Crisóstomi et al. (2018), Nishizawa & Kobayashi (2018), Horava (2009), Gao & Hong (2020), Conroy & Kovistó (2019), Alexander & Yunes (2009), Jackiw & Pi (2003), Wu et al. (2022), Gong et al. (2021), Takahashi & Soda (2009), Yoshida & Soda (2018), Wang et al. (2013), Zhu et al. (2013), Wang (2017), Amelino-Camelia (2001), Amelino-Camelia (2002), Kowalski-Glikman (2001), Magueijo & Smolin (2002), Gambini & Pullin (1999), Affaro et al. (2002), Carroll et al. (2001), Douglas & Nekrasov (2001), Kamada et al. (2021).
includes the events observed during O3a, while the latter includes those during the second half of the third observing run (O3b). Besides a significant enlargement of the number of events, there are improvements on the performance of detectors and on the accuracy of template approximates (Abbott et al. 2021a, 2021b, 2021c). All of these possibly enable more accurate extraction of physical parameters from the observed events. Therefore, we expect to perform a stricter test of the CPT symmetry in this paper.

This paper is organized as follows. In Section 2, we review the gravitational waveform under the hypothesis of CPT violation and the method used for data analysis. In Section 3, the results and discussions are shown explicitly. Finally, our conclusions can be found in Section 4. Throughout this paper, the results and discussions are shown explicitly. Finally, our

2. Theory and Data Analysis

The birefringence alters in an opposite way the phases of two circular polarization modes of GWs; though it contributes little to the amplitude of GWs (Zhao et al. 2020). For the leading-order CPT-violating effect, which is characterized by an independent parameter \( \zeta \), the dispersion relation of GWs is given by

\[
\omega^2 = k^2 \pm 2\zeta k^3, \tag{1}
\]

where the symbol \( \pm \) stands for the birefringence, \( \omega \) and \( k \) denote the energy and momentum of GWs, respectively. We take \( + \) and \( - \) for the left- and right-handed polarization modes, respectively. The effects of higher-order CPT violation are neglected in this work, since they are expected to be suppressed by high energy scales (Kostelecky 2004).

The GW strain involving the birefringence is given as

\[
h_{L,R} = h_{GR}^{L,R} e^{\pm i \delta \Psi}, \tag{2}
\]

where \( h_{GR}^{L,R} \) denotes the strain in general relativity (GR). For the left- and right-handed polarization modes, the phase is shifted by a factor, i.e., Mewes (2019), Wang (2020),

\[
\delta \Psi = 4\pi z f'^2 \int_0^z \frac{1 + z'}{H(z')} dz', \tag{3}
\]

where \( z \) is the redshift of the source, \( f = \omega/2\pi \) is the frequency of GWs in the observer frame, and \( H(z') \) is the Hubble parameter at redshift \( z' \). Throughout this work, we use in our evaluation the cosmological parameters measured by Planck satellite 2015 (Ade et al. 2016).

During the process of data analysis, the GWs are commonly decomposed in terms of the plus and cross modes, which are related to the left- and right-handed modes by following \( h_{L,R} = h_+ \pm ih_\times \). Therefore, the waveform involving the birefringence is given by Mewes (2019), Wang (2020):

\[
\begin{pmatrix}
h_+ \\
h_\times
\end{pmatrix} = \begin{pmatrix}
\cos(\delta \Psi) & -\sin(\delta \Psi) \\
\sin(\delta \Psi) & \cos(\delta \Psi)
\end{pmatrix}
\begin{pmatrix}
h_{GR}^+ \\
h_{GR}^\times
\end{pmatrix}, \tag{4}
\]

where \( h_{GR}^+ \) and \( h_{GR}^\times \) stand for the plus and cross modes of the GR waveform generated with the state-of-the-art “IMRPhenomXPHM” method (Pratten et al. 2021). Based on Equations (2) and (3), when the parameter \( \zeta \) vanishes, the birefringence waveform would be recovered to the GR waveform, as expected. In the following, we estimate the allowed value of \( \zeta \) as well as the independent parameters of GR waveform by performing data analysis.

To infer the parameter space, we perform a Bayesian analysis of the transient events in two recently released catalogs GWTC-2 (Abbott et al. 2021a, 2021b) and GWTC-3 (Abbott et al. 2021c). Specifically, we analyze the data of binary black holes (BBHs) by using a modified version of pBilby (Smith et al. 2020) and dyneasty (Skilling 2004; Speagle 2020). As discussed in Wang & Zhao (2020), the compact binary coalescence events involving neutron stars would be discarded in such an analysis, since they might be related to unknown matter effects rather than the pure-gravity effect. Therefore, the current work stands for an analysis of 65 BBH events in total. In addition, to check our inference configuration, we have reproduced the results of GWTC-2 and GWTC-3 without considering the birefringence effect.

The log-likelihood function for a strain signal with Gaussian noise is defined as follows (Finn 1992, Thrane & Talbot 2019):

\[
\log \mathcal{L}(s|\xi, h) = \langle s, h(\xi) \rangle - \frac{1}{2}s(\xi)\cdot h(\xi), \tag{5}
\]

where \( s \) denotes the strain signal, and \( h(\xi) \) is the waveform template with independent parameters \( \xi \). For the birefringence waveform in Equation (3), \( \xi \) also includes \( \zeta \), besides the GR-related parameters. In Equation (4), the inner product is defined as

\[
\langle a, b \rangle = 4\pi \int_0^\infty a(f) b^*(f) df, \tag{6}
\]

where \( S_n(f) \) is the noise power spectral density (PSD) of a detector. In this work, the noise PSD for each event, as well as the duration and minimum frequency cutoff configuration, is the same as that in either GWTC-2 or GWTC-3. To reduce the computational burden, the analytic marginalization procedures for the coalescence time and distance are used (Ashton et al. 2019). We assume the noise of multiple detectors to be uncorrelated, implying that the likelihoods of them can be multiplied.

Given a prior probability distribution function (PDF) \( p(\xi) \) and the likelihood function \( \mathcal{L}(h|\xi) \), following Bayes’ rule, we evaluate the posterior PDF of \( \xi \), i.e., Ramos & Arregui (2018),

\[
p(\xi|s, h) = \frac{p(\xi|h)\mathcal{L}(s|\xi, h)}{\mathcal{Z}(s|h)}, \tag{7}
\]

where the Bayesian evidence is defined as follows (Kass & Raftery 1995):

\[
\mathcal{Z}(s|h) = \int \mathcal{L}(s|\xi, h) p(\xi|h) d\xi, \tag{8}
\]

In our parameter inference, the priors for the GR-related parameters are the same as those used in GWTC-2 and GWTC-3. For the prior of \( \zeta \), we introduce a uniform distribution over \([-4, 4]\) in units of \(10^{-14}\) m.\(^6\)

To perform the model comparison, we employ the Bayes factor (BF) which is a ratio between the Bayesian evidence of

\(^6\) For GW191204, \(110529\), we employ a uniform distribution over \([-40, 40]\) instead of \([-4, 4]\), since the latter would produce a posterior that touches the boundaries of the prior.
The value of BF shows which model is more favored by the

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Table 1
Median Value and 90% Confidence Intervals of the Parameter ζ for Each Event in GWTC-2 (Left Columns) and GWTC-3 (Right Columns)

| Event               | ζ (10^{-14} m) | BF | Event               | ζ (10^{-14} m) | BF |
|---------------------|----------------|----|---------------------|----------------|----|
| GW190408_181802     | -0.00 ± 0.25  | 17.47 | GW191103_012549     | -0.00 ± 0.25  | 35.11 |
| GW190412            | 0.07 ± 0.25   | 16.99 | GW191105_143521     | 0.00 ± 0.25   | 38.59 |
| GW190413_052954     | 0.09 ± 0.35   | 8.74  | GW191109_010717     | -0.16 ± 0.15  | 0.69  |
| GW190415_134308     | -0.01 ± 0.70  | 4.67  | GW191113_077532     | -0.61 ± 0.99  | 1.44  |
| GW190421_213856     | -0.08 ± 0.78  | 5.35  | GW191126_115259     | 0.00 ± 0.25   | 42.29 |
| GW190424_180648     | 0.03 ± 0.20   | 6.23  | GW191127_050227     | 0.03 ± 0.77   | 4.93  |
| GW190503_185404     | -0.55 ± 0.64  | 3.50  | GW191129_134029     | 0.01 ± 0.04   | 18.56 |
| GW190512_180714     | 0.01 ± 0.15   | 24.49 | GW191204_110529     | 0.05 ± 0.95   | 3.52  |
| GW190513_205428     | -0.06 ± 0.24  | 13.85 | GW191204_171526     | 0.00 ± 0.01   | 320.34 |
| GW190514_065416     | 0.03 ± 0.66   | 6.05  | GW191215_223052     | 0.02 ± 0.09   | 21.67 |
| GW190517_055101     | 0.14 ± 0.24   | 15.77 | GW191216_213338     | 0.00 ± 0.02   | 348.53 |
| GW190519_153544     | 0.02 ± 0.62   | 8.87  | GW191222_033557     | 0.02 ± 0.82   | 4.34  |
| GW190521            | 0.22 ± 1.05   | 0.60  | GW191230_180458     | -0.04 ± 0.20  | 7.90  |
| GW190521_074359     | 0.03 ± 0.59   | 4.27  | GW200012_155838     | 0.00 ± 0.15   | 33.65 |
| GW190527_092055     | 0.02 ± 0.46   | 14.43 | GW200128_022011     | -0.00 ± 0.26  | 12.79 |
| GW190602_175927     | 0.01 ± 0.79   | 4.73  | GW200129_065458     | -0.05 ± 0.05  | 28.21 |
| GW190602_203421     | 0.09 ± 0.66   | 5.89  | GW200202_154313     | -0.01 ± 0.13  | 142.85 |
| GW190630_185205     | -0.01 ± 0.34  | 17.57 | GW200208_130117     | -0.07 ± 0.16  | 25.14 |
| GW190701_203306     | 0.22 ± 0.59   | 8.36  | GW200208_222617     | 0.33 ± 0.60   | 1.02  |
| GW190706_222641     | 0.14 ± 0.69   | 3.62  | GW200209_085452     | 0.00 ± 2.34   | 27.21 |
| GW190708_233457     | -0.01 ± 0.10  | 42.56 | GW200210_092255     | -0.05 ± 0.57  | 11.79 |
| GW190719_215514     | -0.01 ± 1.44  | 9.93  | GW200212_226084     | 0.19 ± 0.52   | 5.51  |
| GW190727_060333     | -0.01 ± 0.26  | 13.91 | GW200219_094415     | 0.09 ± 0.60   | 5.41  |
| GW190731_140936     | 0.00 ± 0.40   | 13.17 | GW200220_061928     | 0.15 ± 1.78   | 2.75  |
| GW190803_022701     | 0.03 ± 0.41   | 14.45 | GW200220_124850     | -0.03 ± 0.46  | 10.73 |
| GW190828_063405     | -0.00 ± 0.31  | 14.21 | GW200224_122234     | 0.01 ± 0.22   | 28.68 |
| GW190828_065509     | -0.28 ± 0.16  | 1.34  | GW200225_066041     | 0.01 ± 0.09   | 11.67 |
| GW190909_114149     | -0.04 ± 1.30  | 2.10  | GW200302_015811     | 0.00 ± 0.14   | 41.93 |
| GW190910_112807     | 0.00 ± 0.36   | 13.85 | GW200306_093714     | 0.01 ± 0.95   | 11.15 |
| GW190924_021846     | 0.72 ± 1.50   | 1.14  | GW200308_173609     | -0.09 ± 0.36  | 1.11  |
| GW190929_012149     | 0.03 ± 1.60   | 1.93  | GW200311_115853     | 0.00 ± 0.08   | 34.94 |
| GW190930_133541     | -0.00 ± 0.14  | 28.02 | GW200316_215756     | 0.25 ± 1.27   | 7.65  |
|                    |                |      | GW200322_091133     | -0.08 ± 0.28  | 0.90  |

Note. Bayes factor is shown to compare GR and the birefringence scenario.

GR and birefringence models, i.e., Ramos & Arregui (2018),

\[
BF = \frac{Z(s|h^{GR})}{Z(s|h)},
\]

where \(h^{GR}\) and \(h\) denote the waveforms in GR and the birefringence scenario, respectively. For multiple events, the total BF is obtained by multiplying the BFs of them together. The value of BF shows which model is more favored by the data (Reyes 2019; Trotta 2008).

3. Results and Discussion

The results of this work are shown in Table 1 and Figure 1. We show in Table 1 the median value and 90% confidence interval of ζ for each event in GWTC-2 (left columns) and GWTC-3 (right columns). We also show the results of the Bayes factor to compare GR and the birefringence scenario. In Figure 1, we depict the posterior PDF of ζ from a joint analysis of all events.

Based on Table 1, we find that each event in GWTC-2 and GWTC-3 is well compatible with null birefringence, i.e., \(ζ = 0\). There is not significant evidence for the CPT violation and birefringence in GWs. However, we obtain the up-to-date best constraints on ζ. Among all events, GW191204_171526 and GW191216_213338 in GWTC-3 reveal the strictest bounds on
\( \zeta \), i.e., \( \zeta = 0.8^{+6.4}_{-6.8} \times 10^{-17} \text{m} \) and \( \zeta = 2.3^{+6.6}_{-7.6} \times 10^{-17} \text{m} \) at 68% confidence level, respectively. These bounds are stricter by \( \sim 3 \) orders of magnitude than the first upper bound \( 2 \times 10^{-14} \text{m} \) (Kostelecký & Mewes 2016), which was obtained from the first event GW150914. They are also stricter by \( \sim 1 \) order of magnitude than the upper bound \( |\zeta| < \text{few} \times 10^{-16} \text{m} \) (Wang & Zhao 2020; Shao 2020), which was obtained from GWTC-1. Moreover, they are stricter than the upper bound \( \sim 4.5 \times 10^{-16} \text{m} \) (Wang et al. 2021a, 2021c), which was obtained via a joint analysis of GWTC-1 and GWTC-2. Interestingly, we find that the aforementioned two events alone can lead to better bounds on \( \zeta \) than those events in GWTC-1 and GWTC-2.

Combining the posterior PDFs of all events together, we can obtain a joint constraint on \( \zeta \), which is stricter than the bounds from the individual events mentioned above. By employing Monte Python (Audren et al. 2013), we obtain the combined bound to be

\[
\zeta = 4.07^{+5.91}_{-5.79} \times 10^{-17} \text{m}
\]

at 68% confidence level. It stands for an upper bound of \( |\zeta| < \text{few} \times 10^{-17} \text{m} \), which roughly corresponds to an energy scale of \( \sim 10 \text{GeV} \). It becomes \( 4.1^{+12.4}_{-12.2} \times 10^{-17} \text{m} \) and \( 4.1^{+9.2}_{-20.1} \times 10^{-17} \text{m} \) at 95% and 99.7% confidence levels, respectively. Or equivalently, it is \( \zeta = 4.07^{+10.2}_{-5.91} \times 10^{-17} \text{m} \) at 90% confidence level. Based on this joint analysis, we show a violin depiction for the posterior PDF of \( \zeta \) in Figure 1. We represent the median value with a white point while the 68% confidence interval is represented by a black solid line. The confidence level, or equivalently, it is \( \zeta = 4.07^{+10.2}_{-5.91} \times 10^{-17} \text{m} \) at 90% confidence level, which is obviously stricter than those of individual events. We found that the above limits are tighter than all the existing ones in the literature. On the other hand, we found 3 events to indicate weak evidence for the birefringence model, while the other 62 events indicate strong or moderate or weak evidence for GR.

With the combination of all the events analyzed in this work, the total BF indicates very strong evidence for GR rather than the birefringence scenario. To be specific, we find 3 events, i.e., GW191204_171526, GW191216_213338, and GW200202_154313, to have the largest BFs, i.e., 320, 349, and 143, respectively. These events indicate strong evidence for GR, since they have BF \( \in [45, 740] \). It is interesting to find that the strictest bounds on \( \zeta \) also arise from the first two events. However, we do not find any special ingredients, e.g., the mass ratio or the higher harmonic modes, to account for why these two events have such large BFs when compared to others. Other events do not show strong evidence for either scenario. However, there are 59 events having BFs greater than one. Among them, 41 events with \( \text{BF} \in [5.5, 45] \) show moderate evidence for GR, while 18 events with \( \text{BF} \in [1, 5.5] \) indicate weak evidence. Exceptions include 3 events, i.e., GW190521, GW191109_010717, and GW200322_091133. We show their BFs to be smaller than one. Due to \( \text{BF} \in [0.18, 1.0] \), they just indicate weak evidence for the birefringence model.

### 4. Conclusion

In this work, we constrained the CPT violation and birefringence in GWs by performing a Bayesian analysis of GWTC-2 and GWTC-3. We reported no significant evidence for the deviations from GR. The limits on the CPT-violating parameter \( \zeta \) were shown to be few \( \times 10^{-17} \text{m} \), corresponding to \( \sim 10 \text{GeV} \) energy scale. These are the up-to-date strictest limits from individual events. Meanwhile, we obtained the combined constraint on \( \zeta \) by performing the joint analysis of GWTC-2 and GWTC-3. The result was shown to be \( \zeta = 4.07^{+10.2}_{-5.91} \times 10^{-17} \text{m} \) at 68% confidence level, or equivalently, \( \zeta = 4.07^{+9.2}_{-10.4} \times 10^{-17} \text{m} \) at 90% confidence level, which is obviously stricter than those of individual events. We found that the above limits are tighter than all the existing ones in the literature. On the other hand, we found 3 events to indicate weak evidence for the birefringence model, while the other 62 events indicate strong or moderate or weak evidence for GR. The total BF was shown to indicate very strong evidence for GR rather than the birefringence scenario. In this work, we have ignored possible unknown effects of CPT violation on the generation of GWs in the sources. However, these effects may be interesting and would be studied in the future. In addition, our method is suitable to analyze the data from upcoming Advanced LIGO observing runs (Abbott et al. 2018).

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