New Possibilities for Testing Local Realism in High Energy Physics

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Abstract

The three photons from the dominant ortho-positronium decay and two vector mesons from the $\eta_c$ exclusive decays are found to be in tripartite and high-dimensional entangled states, respectively. These two classes of entangled states possess the Hardy type nonlocality and allow a priori for quantum mechanics vs local realism test via Bell inequalities. The experimental realizations are shown to be feasible, and a concrete scheme to fulfill the test in experiment via two-vector-meson entangled state is proposed.

\textbf{Key words:} High Energy Process, Entanglement, Quantum Nonlocality

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Bell’s theorem \cite{1} states that no local hidden variable theory (LHVT) can reproduce all the possible results of quantum mechanics (QM). This can be testified via either inequalities, the Bell inequalities (BIs), satisfied by the expectation values of LHVT while violated by QM predictions, or logic contradictions remaining in LHVT vs QM. The original BI and its generalizations mainly concern about two-qubit systems and have been verified with various physical systems, see, e.g., \cite{2,3,4}. However, the analogous tests by virtue of high-dimensional and multipartite entangled states, especially in high energy experiment are very limited \cite{3,5}.

In late 1980s, Greenberger, Horne, and Zeilinger (GHZ) showed that certain three(or more)-partite entangled states may exhibit nonclassical effects more prominently than by bipartite state \cite{6}. Soon afterwards, Hardy \cite{7} invented a cute method to demonstrate the essential contradiction between QM and LHVT without employing the BI, in which the non-maximally entangled state works. A three-photon GHZ state was obtained in \cite{8} by manipulating the parametric downconversion (PDC) photons and the experimental result deviates the local realistic expectation by more than eight standard deviations \cite{9}. For the Hardy’s type of violation, it was proved that the argument may take the form of Clauser and Horne \cite{10} (CH) inequality and then reduce the required detector efficiencies for loophole-free tests of Bell inequalities \cite{11,12}. Experimental measurement of the Hardy fraction was obtained in \cite{13}. It is noteworthy that the existing entanglement tests are performed mainly in the regime of visible light. The non-maximally entangled kaon pairs were once suggested to realize the Hardy’s type of criterion in experiment \cite{14,15,16}, however there has been no such experiment carried out yet to the best of our knowledge.

In this Letter we propose to investigate the quantum non-local entanglement in two well-established high energy processes, in which the multipartite or/and high-dimensional entanglement is naturally realized, i.e., the three photons in the dominant ortho-positronium
decay and the two vector mesons in the $\eta_c$ exclusive decay. In [17], Acin, Latorre and Pascual already realized that the ortho-positronium decay can provide an entangled state of three space-separated photons. However, in this work we find that there is a Hardy type contradiction between QM and LHVT, and the CH type inequality would be violated by the quantum correlation of this state.

In the middle of the last century, the polarization correlation of the two photons in para-positronium decay was used to determine its parity [18, 19, 20]. In quantum mechanics these two correlated photons are in the state of [21]

\[ |\Psi\rangle = \frac{1}{\sqrt{2}}\{|x\rangle|y\rangle - |y\rangle|x\rangle\}, \] (1)

where $|x\rangle$ and $|y\rangle$ stand for the photon polarization states in coordinate space. The ortho-positronium, the spin triplet state of electron and positron with $J^{PC} = 1^{-+}$, decays overwhelmingly to three photons. In this case, the photon polarization information can be extracted from the decay amplitude, which is proportional to [22]

\[ V \propto \sum_{\text{cyc}} [\epsilon_1 (\epsilon_2 \cdot \epsilon_3 - \delta_2 \cdot \delta_3) + \delta_1 (\epsilon_3 \cdot \delta_2 + \epsilon_2 \cdot \delta_3)], \] (2)

where $\epsilon_i$ denote the photon polarization vectors, $k_i$ represent their momenta, and $\delta_i = k_i \times \epsilon_i$. In the form of circular polarization, $\epsilon_i^\pm = \epsilon_i \pm i\delta_i$ and the amplitude reads

\[ V \propto \sum_{\text{cyc}} [\epsilon_1^+ (\epsilon_2^- \cdot \epsilon_3^+) + \epsilon_1^- (\epsilon_2^+ \cdot \epsilon_3^+)]. \] (3)

Obviously, $\epsilon_i^\pm$ represent the left- and right-handed polarizations of photons, respectively.
Thus the three photon polarization state should be in the following form

\[
|3\gamma\rangle = \frac{1}{\sqrt{6}} \{ |RRL\rangle + |RLR\rangle + |LRR\rangle \\
+ |LLR\rangle + |LRL\rangle + |RLL\rangle \} .
\] (4)

Here, \( L \) and \( R \) denote the left and right circularly polarized photons, respectively.

As shown in [23], under stochastic local operations and classical communication (SLOCC) pure three qubits entangled states have six inequivalent classes, and among them two are genuinely entangled, named as GHZ and W states. By virtue of the method proposed in [24], one can easily confirm that the state (4) is SLOCC equivalent to GHZ state. On the other hand, from the point of entanglement measure, 3-tangle \( \tau \) defined in [25], the three-qubit entangled state (4) has \( \tau(|3\gamma\rangle) = 1/3 \). Since \( \tau \) is none zero only in the case of GHZ or its equivalent states [23], we can also conclude that (4) is in SLOCC equivalence to the GHZ state. Therefore, we know that the three photons from the ortho-positronium decay constitute the simplest multipartite entangled system, the three-qubit state.

To observe the non-locality of state (4), one needs also to measure the linear polarization of the state, e.g., the horizontal and vertical polarizations \( H \) and \( V \). The transformation between circular and linear polarizations reads

\[
\begin{pmatrix}
R \\
L
\end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & i \\
1 & -i
\end{pmatrix}
\begin{pmatrix}
H \\
V
\end{pmatrix}.
\] (5)

Under the linear polarization bases, the three-photon polarization wave function can be reexpressed as

\[
|3\gamma\rangle = \frac{1}{\sqrt{12}} \{ 3|HHH\rangle + |HVV\rangle + |VHV\rangle + |V VH\rangle \} .
\] (6)
To show the incompatibility of quantum mechanics with local realism (LR), we need an additional form of the state \( |3\gamma\rangle \). Here, we express one photon in linear polarization, but left the other two in circular bases. In this case the wave function is

\[
|3\gamma\rangle = \frac{1}{\sqrt{12}} \left( (|RR\rangle + |LL\rangle + 2|RL\rangle + 2|LR\rangle)|H\rangle \right)
- i(|RR\rangle - |LL\rangle)|V\rangle .
\] (7)

From Eqs. (4), (6) and (7) one can easily get the following probabilities:

\[
P_{|3\gamma\rangle}(i = V, j = V) = \frac{1}{4} , \quad P_{|3\gamma\rangle}(C_j = C_k | i = V) = 1 ,
\]

\[
P_{|3\gamma\rangle}(C_i = C_k | j = V) = 1 , \quad P_{|3\gamma\rangle}(C_i = C_j = C_k) = 0 .
\] (8)

Here, \( P_{|3\gamma\rangle}(i = V, j = V) \) denotes the probability of two of the photons being vertically polarized; \( P_{|3\gamma\rangle}(C_j = C_k | i = V) \) denotes the conditional probability of photons \( j \) and \( k \) possessing the same circular polarization while the photon \( i \) being vertically polarized; and \( P_{|3\gamma\rangle}(C_i = C_j = C_k) \) represents the probability of all three photons having the same circular polarization.

The probabilities in (8) can be embedded into CH type inequality [26], that is

\[
P_{|3\gamma\rangle}(i = V, j = V) - P_{|3\gamma\rangle}(i = V, C_j \neq C_k)
- P_{|3\gamma\rangle}(C_i \neq C_k, j = V) - P_{|3\gamma\rangle}(C_i = C_j = C_k) \leq 0 .
\] (9)

This inequality is a constraint imposed by the LHVT. While inputting the quantum me-
chanics results on $P_{\langle 3\gamma \rangle}$, i.e., Eq. (8), into (9), one readily finds that

$$\frac{1}{4} - 0 - 0 - 0 \leq 0 ,$$

which is obviously a contradiction. Note that inequality (9) is valid only in the ideal case of perfect detection efficiencies, by which only a restricted class of Local Realistic Theories can be tested.

To perform the experimental test on above inequality becomes realistic in recent years. Intense pencil beams of ortho-positronium $(30-1.7 \times 10^4 \text{s}^{-1})$ can be produced based on a low energy positron storage ring equipped with an electron cooling system [27]. This may accumulate about $10^{12}$ three-photon events in ortho-positronium decay a year. Unlike in the case of Bell inequality where a group of measurements with relative angles in the measurement bases should be performed, here only two kinds of polarization measurements are necessary. However, we should point out that the measurement of photon polarization in positronium decay is a bit challenging since the photon energy lies in the regime of hard X-ray. The entangled photons from ortho-positronium decay are of hundreds of keV which is about 5 orders greater than ordinary photons in visible region. In experiment, the X-ray polarimeter has made the measurements on linear and circular polarization components feasible in a wide range of energy. With ordinary inorganic crystal GaAs, people had already realized the measurement on polarizations of photon with energy span of 3-20 keV [28]. Recently, a novel kind of X-ray polarimeter based on hexagonal crystal, which can separate different polarization components simultaneously, is available [29]. Although its working energy is a bit lower than that of photon energy from positronium decay, further development on X-ray polarization analyzer is quite realistic in near future.

As mentioned in the beginning, we find that a $3 \times 3$ entangled state in spin can be
produced in the process of \( \eta_c \) exclusive decay into two light vector mesons, i.e., \( \eta_c \rightarrow \rho \rho, \phi \phi \), etc., as shown in Figure 1. In the following part we briefly prove this, show its nature, and propose a scheme on how to carry out the test on LR by using of the two-vector-meson entangled state. Details will be presented elsewhere.

According to the laws of angular momentum and parity conservation in strong interaction we know that those two vector mesons are in the state of total spin 1. From the Clebsch-Gordan decomposition, its zero component in z-direction is

\[
\psi_{|1,0\rangle} = \frac{1}{\sqrt{2}}(|1\rangle - 1\rangle - |1\rangle - 1\rangle)_z ,
\]

where the z-axis is chosen to be the moving direction of one vector meson in another meson’s rest frame, as shown in Figure 1. Expressing the entangled state in bases perpendicular to z-axis, one has

\[
\psi_{|1,0\rangle} = \frac{1}{\sqrt{2}}(|0\rangle_{\alpha \perp} |0\rangle_{\alpha} - |0\rangle_{\alpha} |0\rangle_{\alpha \perp}) .
\]

Here, the subscript \( \alpha \) indicates an arbitrary axis perpendicular to z, and \( \alpha \perp \) means another base which is perpendicular to both z and \( \alpha \) in terms of right-handed system. The plane fixed by axes \( \alpha \) and \( \alpha \perp \) is in fact the x-y plane, and \( \alpha \) is the relative angle between x- and \( \alpha \)-axes(or y- and \( \alpha \perp \)-axes). Then one can immediately get four different probabilities within the x-y plane based on the QM, which are measurable quantities in experiment. They are

\[
P(J_\beta = 0, J_\alpha = 0) = \frac{1}{2} \sin^2 (\alpha - \beta) ,
\]

\[
P(J_\beta \neq 0, J_\gamma = 0) = \frac{1}{2} \cos^2 (\beta - \gamma) ,
\]

\[
P(J_x = 0, J_\alpha \neq 0) = \frac{1}{2} \cos^2 \alpha ,
\]

\[
P(J_x = 0, J_\gamma = 0) = \frac{1}{2} \sin^2 \gamma ,
\]

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with

\[
P(J_\beta = 0, J_\alpha = 0) = |\beta \langle 0 |_\alpha \langle 0 | \Psi \rangle_{1,0}|^2,
\]

\[
P(J_\beta \neq 0, J_\gamma = 0) =
\]

\[
P(J_\beta = 1, J_\gamma = 0) + P(J_\beta = -1, J_\gamma = 0),
\]

\[
\hat{J}_\alpha = \hat{J}_x \cos \alpha + \hat{J}_y \sin \alpha.
\]

Here, \(P(r_1, r_2)\) means the probability under the conditions of \(r_1\) and \(r_2\), which are satisfied by the two partites of 1 and 2 of the entangled state, respectively. \(|0\rangle_\alpha\) stands for the eigenvector of angular momentum operator \(\hat{J}_\alpha\) with eigenvalue \(J_\alpha = 0\) in the \(x-y\) plane.

From the generalization of Hardy’s argument to spin-\(s (s = \frac{1}{2}, 1, \frac{3}{2}, \cdots)\) systems \([30]\), we obtain the following LHVT constraint

\[
P(J_x = 0, J_\gamma = 0) \leq P(J_x = 0, J_\alpha \neq 0)
\]

\[
+ P(J_\beta \neq 0, J_\gamma = 0)
\]

\[
+ P(J_\beta = 0, J_\alpha = 0).
\]

Substituting the quantum mechanics results \((13) - (16)\) into \((17)\), one immediately obtains

\[
\frac{1}{2} \sin^2 \gamma \leq \frac{1}{2} [\cos^2 \alpha + \cos^2 (\beta - \gamma) + \sin^2 (\alpha - \beta)].
\]

This inequality is maximally violated by QM predictions while \(\alpha = 3\pi/8, \beta = \pi/4, \gamma = 5\pi/8\), which gives a contradictive result of \(\frac{2 + \sqrt{2}}{8} \leq \frac{6 - 3\sqrt{2}}{8}\).

In principle, in experiment one can determine the polarizations of \(\rho s\) or \(\phi s\) through measuring the distributions of their two-body exclusive decay products, the \(\pi s\) or \(K s\), and then find the difference between LR and QM predictions. Here, we find that there exists
another practical method to find out the difference in the experiment, that is to perform the measurement on the quantities in CH inequality event by event. Following we explain it in detail.

Suppose the probabilities of a count being triggered by the decays of $V_1$, $V_2$ polarizing along $\vec{n}_{1,2}$ being $p(\vec{n}_1, \lambda)$ and $q(\vec{n}_2, \lambda)$ respectively, then we can readily get a CH inequality \[10, 32\]

$$P(\vec{n}_1, \vec{n}_2) - P(\vec{n}_1, \vec{n}'_2) + P(\vec{n}'_1, \vec{n}_2) - P(\vec{n}'_1) - P(\vec{n}_2) \leq 0. \quad (19)$$

Here, $P(\vec{n}_1, \vec{n}_2) = \int p(\vec{n}_1, \lambda)q(\vec{n}_2, \lambda)\rho(\lambda)\,d\lambda$, and hence $P(\vec{n}_1) = \int p(\vec{n}_1, \lambda)\rho(\lambda)\,d\lambda$ with $\lambda$ being a set of hidden variables.

In the quantum field theory, the differential decay width of $\eta_c \to V_1(p, \vec{\epsilon}')V_2(q, \vec{\epsilon}'') \to P(p_1)P(p_2)P(q_1)P(q_2)$, as shown in Figure 2, takes the following form

$$\frac{d\Gamma_{\eta_c \to V_1V_2 \to \cdots}}{d\varphi} \propto |\langle \vec{n}_1|\langle \vec{n}_2|\Psi \rangle|^2 \quad (20)$$

with

$$|\langle \vec{n}_1|\langle \vec{n}_2|\Psi \rangle|^2 \equiv P(\vec{n}_1, \vec{n}_2), \quad (21)$$

where unit vectors $\vec{n}_1$ and $\vec{n}_2$ are normalized projections of momenta $\vec{p}_1$ and $\vec{q}_1$ respectively of the final pseudoscalar mesons in $x$-$y$ plane, and

$$|\Psi \rangle = \frac{1}{\sqrt{2}}(|\epsilon_x \rangle|\epsilon'_y \rangle - |\epsilon_y \rangle|\epsilon'_x \rangle) \quad (22)$$

with $\epsilon_{x,y}$ and $\epsilon'_{x,y}$ being the transverse components of the polarization vectors $\vec{\epsilon}$ and $\vec{\epsilon}'$. The
wave function of (22) explicitly shows that the two vector mesons are entangled, the same as (12). Note that our concerned process is similar to the $B \to V_1V_2$ [31], and hence the differential decay width (20) can be analogously obtained. The details of how to derive (20) and others will be presented elsewhere.

Given that the four final pseudoscalars move with momenta $p_1$, $p_2$, $q_1$, and $q_2$, the azimuthal angle $\varphi$ between two decay planes of the entangled vector meson pair then equals to the angle between $\vec{n}_1$ and $\vec{n}_2$, as shown in Figure 2. The magnitudes of $P(\vec{n}_1, \vec{n}_2)$ in the CH inequality are therefore experimentally measurable, which is obviously the probability density, up to an overall normalization factor, from the definition of $P(\vec{n}_1, \vec{n}_2)$. That is

$$P(\vec{n}_1, \vec{n}_2) = \kappa \frac{N(\varphi + \Delta \varphi) - N(\varphi)}{N \Delta \varphi}.$$  

(23)

Here, $\kappa$ is a calculable normalization constant independent of specific theory for our aim, $N$ is the total event number, and $N(\varphi)$ is the event number within azimuthal angle $\varphi$, the angle between $\vec{n}_1$ and $\vec{n}_2$. In above expression, apart from the constant $\kappa$ the right-hand side is experimentally measurable, i.e., the differential decay width of $\eta_c$ to four pseudoscalar mesons divided by its total width via intermediate vector meson $\phi$s, and is hence also theoretically calculable in quantum theory. Inputting the theoretical and experimental results into (19), one may in principle find the incompatibility of quantum theory with LR. However, in practice, to perform the test of incompatibility the experiment efficiency should be taken into account. The general inequality efficiency and background levels was once discussed by Eberhard [11, 12], and for the wave function (22) the violation of inequality (19) yields the threshold efficiency $\eta > 82.8\%$ [33].

To carry out the test of Bell type inequality, the decay angles $(\vec{n}_1, \vec{n}_2)$ should generally be chosen actively by experimenters, but this cannot be realized for mesons due to the passive
character of their decays [34]. Thus, here only a restricted class of LR can be tested, like in [35]. A genuine Bell test also requires the decay events of two vector mesons $V_1$ and $V_2$ to be space-like separated. For the strongly decayed vector mesons ($\phi, \rho$, etc.), we cannot guarantee for each particular event of $\eta_c \rightarrow VV \rightarrow (PP)(PP)$ that the decays of two vector mesons are space-like separated. The non-space-like decay events may induce the locality loophole in experiment. Fortunately the ratio of the space-like separated events can be determined by the magnitude of $\beta$ [36]. And, for $\eta_c \rightarrow \phi\phi$, it is easy to find that $\beta_{\phi} = 0.729$ [32], which is greater than 0.59, the lower bound required to test the LR [35]. Note that the $\beta$ has a negative effect on the threshold counter efficiency $\eta$, and which then requires a higher detection efficiency in experiment.

In summary, in this work we show that three photons in the decay of ortho-positronium, or other similar onium, form a tripartite entangled state, which is SLOCC equivalent to the GHZ state. Since in experiment the ortho-positronium decays to three-photon process can be well measured and huge data samples exist, new technology development on the X-ray polarimeter may enable a direct measurement on the hard photon polarization and hence find the difference between quantum theory and LR. Due to the detection efficiency and the unavoidable noise and losses, this cannot be considered as a loophole-free experiment. Nevertheless, the present method appears to be of broad interest because unlike the general production of GHZ state in optical cases the non-locality of the three photons in positronium decay arises in a dynamical process without postselection [37]. The investigation of three partite systems will show different features not only to that of bipartite qubit systems, but bipartite qutrit and in general bipartite qudit systems as well.

We find that two vector mesons in $\eta_c$ exclusive decay automatically form a three-dimensional non-maximally entangled state. A concrete scheme to test the LR in experiment via the two-vector-meson entangled state is proposed. Since experimental measurements on exclusive two
body decay processes

\[ \rho \rightarrow \pi\pi, \; \phi \rightarrow K^+K^-(K_L^0K_S^0) \tag{24} \]

are well-established, and give large branching fractions of \( \rho \rightarrow \pi\pi \sim 100\% \) and \( \phi \rightarrow K^+K^-(K_L^0K_S^0) \sim 49.2 \pm 0.6\% \) \((34.0 \pm 0.5\%) \) \cite{38}, we believe that in current running experiments the CH type inequality can be readily measured. It should be mentioned that the non-space-like decay events may induce the so-called locality loophole in experiment, which hinders the proposed tests to refute the LR definitely. Nevertheless, the experimental realization of the proposals in this work may extend the test on local realism into high energy regime with multipartite and high dimension, which will give us a more explicit conclusion in comparison with that from the bipartite qubit results.

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**Figure Captions**

**Figure 1:** The schematic Feynman diagram of processes $\eta_c \rightarrow VV$. Here $V$ stands for vector meson $\phi$ or $\rho$.

**Figure 2:** $\eta_c \rightarrow V_1(p, e^*)V_2(q, e'^*) \rightarrow [P(p_1)P(p_2)][P(q_1)P(q_2)]$ decay kinematics in the rest frame of $V_1$. Here, $V_1, V_2$ stand for vector meson pair of $\phi$s or $\rho$s, $P(p_i)$ and $P(q_i)$ for the pseudoscalar mesons in $V_1$ and $V_2$ decays, the $K$s or $\pi$s.
Figure 1:

Figure 2: