Mathematical models of image formation for the eye with diffractive intraocular lens

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Abstract. The article is a review of analysis of mathematical models of different types of diffractive intraocular lenses. Bi-, three- and multifocal lenses were the subjects of comparison. There are two approximations of work of intraocular lens: geometric optical approximation and the scalar theory of diffraction. The advantages and disadvantages of bi-, three- and multifocal lenses are shown from the position of energy effectively, function of transportation of modulation and point spread function.

1. Introduction

While choosing the method of eyesight correction it is important to understand, which methods are good for each patient and which difficulties can take place. The statistic researches of eyes can’t give the opportunity of comparison of each desired parameters of the different types of intraocular lenses (IOLs) so the creation of mathematical models of IOLs is actual. Diffractive IOLs have a huge expansion, because there is an opportunity of making several focuses, which give us possibility of making an image on the middle (0.5-1 m) distances with the small aberrations. The investigations of diffractive IOLs take place in domestic [6,7,18,19,32,33,36,43,49,43-56,60-75,79,80] and foreign [1,2,4,13,14,20,31,39,40,44,45,59] organizations from the past century to today. The opportunity of the correction of cornea’s aberrations also takes place. The most convenient method of modeling of the behavior of an eye with an IOL is to use «Zemax». But the diffractive elements can be made only as binary and it is no possibility to evaluate the diffraction efficiency. In the program «Code V», only the gratings can be made. The program «VirtualLab» can give the possibility of modeling the stepwise DOE and counting the distribution of energy in different orders of diffraction.

2. Choosing of a model of eye

In article [3] the comparison of the models of Gullstrand, Shviegerling, Emsley, Helmgoltz-Lourens, Liou and Brennan was done. The model of Liou and Brennan (fig. 1 and table 1) is the most realistic because the natural decentration of the pupil was viewed. That’s why the graph of the PSF is asymmetric (fig. 2). In article [82] were compared models of Navarro, Arizona and Liou-Brennan and the model of Liou-Brennan was accepted as the most realistic, too.

That model is the most widespread at the IOL’s counting. In article [15] the program «Zemax» and the model of Liou-Brennan were used. The results are shown with the help of the PSF. The crystalline is shown as an complex lens. The model of an eye in article [16] was shown with the diffractive effects and there is an opportunity of adding an IOL. The AHEM is a commercially available software tool from Breault Research Organization.

In article [58] the fullest geometrooptical model with the laminated structure is shown. In article [48] was demonstrated an artificial gradient refractive lens, which can be used as artificial crystalline.
3. The comparison of mathematical models of lenses

In article [34] the results of an analysis of chromatism of IOLs are shown. The opportunities of correction of spherochromatism of IOLs are discussed. The types of IOLs are not shown because it is not important. The chromatism depends only on the material of a lens and existence of the diffractive structure.

In article [35] was shown that the influence of the ghost images is not important. In articles [51-52] the another approach was applied and the multi-order IOLs were made with the aim of decreasing the chromatism.

Also in article [81] the physical model was shown, there was considered the question of the deep of focus of IOL.

Table 1. The parameters of the components of an eye for the Liou-Brennan model.

| Surface | Radius, mm | Thickness, mm | Aspherisity | Refractive coefficient (for 555 nm) |
|---------|------------|---------------|-------------|-----------------------------------|
| 1       | 7.77       | 0.50          | -0.18       | 1.376                             |
| 2       | 6.40       | 3.16          | -0.60       | 1.336                             |
| 3       | 12.40      | 1.59          | -0.94       | Grad A                             |
| 4       | ∞          | 2.43          | -           | Grad P                             |
| 5       | -8.10      | 16.27         | 0.96        | 1.336                             |

Grad A = 1.368 + 0.049057z − 0.015427z² − 0.001978r²

Grad P = 1.407 − 0.006605z² − 0.001978r²

\n\n\[ n(\lambda) = n(0.555 \text{ мкм}) + 0.0512 - 0.1455\lambda + 0.0961\lambda^2 \]
In article [51] trifocal lenses with the sine profile are shown. The influence of background intensity on the image quality is shown as MTF.

In article [50] the analysis of longitudinal distribution of intensity, formed by binary lens (zone plate) by the method of scalar theory of diffraction. It is analytically proved that for lenses with low aperture some local focuses near and behind the main, has the intensity, close to the intensity of main focus. But the sizes of local focuses decreases. In zone near the optical element, where the paraxial approximation can’t be used, the transverse size of focal points stops decreasing and the intensity diminishes proportional to the square of the focal length. Numerous and experimental results shows the accordance to analytical formulas. For the determination of quality the MTF, PSF and longitudinal distribution of intensity were eliminated.

In article [5] the comparison between diffractive efficiency, MTF and chromatical aberrations of bifocal lenses with the rectangle, parabolic, parabolic with holes and sine profiles. For the case of binary lens the energy is between 1 and -1 orders, the energy from another lenses passes to -1 and 0 orders. The MTF of lenses differs insignificant, but the MTF of sine lens is the nearest to the diffractive boundary, the MTF of parabolic lens with holes is the worst. MTF of the binary lens for the near distances and wavelength 440 nm and far distances and wavelength 640 nm is the worst, but for the cases of near distances and 640 nm and far distances and 440 nm is the best. The results of sine and parabolic lenses are the opposite. The lens with holes has the average parameters.

In article [17] was demonstrated a trifocal zone plate without chromatism. All of three focuses on the wavelengths 450 nm, 540 nm and 580 nm are in the same positions. In other trifocal lenses the difference between focuses in the same order achieves ~1 mm. But the chromatism appears, when another wavelengths add.

The trifocal lens with rectangle profile is considered in paper [41,76-77]. The longitudinal distribution of intensity and the energy distribution are shown on fig. 4.

![Figure 4](image)

**Figure 4.** The energy distribution between the focuses. a) Counted relief of a zone plate; b) Normalized distribution of intensity; c) The distribution of intensity in horizontal plane. The dotted line in b) and c) shows the position of focuses.
Phase function of a zone plate is:

\[ \Phi_f(\rho) = \Phi[\Phi_f(\rho)] = \Phi\left[ \text{mod}_{2\pi} \left( \frac{k_0 n_m \rho^2}{2 f_d} \right) \right], \quad \rho \in [0, R] \]  \tag{1}

where R is an aperture radius of a zone plate, \( \Phi_f(\rho) = \Phi\left[ \text{mod}_{2\pi} \left( \frac{k_0 n_m \rho^2}{2 f_d} \right) \right] \) is a paraxial phase function with the focal length \( f_d \) in water, \( k \) is a wavenumber for the free space.

The article [42] is interesting because the testing on patients with IOLs «AcrySofReSTOR» is imitated by the method of Monte-Carlo. The MTF is investigated.

In article [57] were considered bi- and trifocal lenses. MTF, image of mira and energy distribution between the orders of diffraction were analyzed with the help of «Zemax» and «VirtualLab» for refractive, bifocal and trifocal lenses. The comparison of MTF, made by geometrooptical and wave method, is shown on the fig.5. These images were received from the physical model.

![Figure 5. A comparison of MTF for the geometrooptical and wave methods.](image)

In article [78] were reviewed these IOLs on the base of Liou-Brennan model: P359UV (Bausch & Lomb) of PMMA with \( F = 21 \) diopters, AcrySoft MA60BM (Alcon) – double-convex lens with \( F = 21 \) diopters and acrylic lens with \( F = 21 \) diopters by AR40N (Allergan) and Tecnis Z9000 (Advanced Medical Optics Inc.) with \( F = 22 \) diopters of silicon. The model with the natural crystalline has the smallest size of the point, the model of silicon has the littlest point in comparison with another models with IOLs. The different types of chromatism were investigated.

4. Conclusion

The mathematical models of an eye with artificial crystallines are reviewed. The comparison of MTF, chromatism, PSF between different types of IOLs is shown. The comparison between geometrooptical model and model, counted by the wave theory, is shown. There is no any article with the mathematical model investigation of transformation of an image by the optical system of an eye with diffractive IOL.

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