Type-II Fuzzy Multi-Product, Multi-Level, Multi-Period Location–Allocation, Production–Distribution Problem in Supply Chains: Modelling and Optimisation Approach

Sarah J.-Sharahi\textsuperscript{a}, Kaveh Khalili-Damghani\textsuperscript{a}, Amir-Reza Abtahi\textsuperscript{b} and Alireza Rashidi-Komijan\textsuperscript{c}

\textsuperscript{a}Department of Industrial Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran; \textsuperscript{b}Department of Information Technology Management, Faculty of Management, Kharazmi University, Tehran, Iran; \textsuperscript{c}Department of Industrial Engineering, Firoozkooh Branch, Islamic Azad University, Firoozkooh, Iran

ABSTRACT
In this study, the application of type-II fuzzy sets is addressed to design a multi-product, multi-level, multi-period supply chain networks. The proposed model provides integrated approach to make optimal decisions such as location–allocation, production, procurement and distribution subject to operational and tactical constraints. In the context of fuzzy linear programming, this study involves type-II fuzzy numbers for the right-hand side of constraints regarding three sources of uncertainty: demand, manufacturing and supply. According to fuzzy components considered, a type-II fuzzy mixed-integer linear programming is converted into an equivalent auxiliary crisp model using linear fuzzy type-reducer models. The final models are linear and the global optimum solutions can be achieved using commercial OR softwares. The contributions of this study are three folds: (1) introducing a new integrated supply chain network design problem; (2) considering a solution procedure based on type-II fuzzy sets and (3) presenting a linear fuzzy type-II reducer. Finally, the proposed model and solution approach are illustrated through a numerical example to demonstrate the significance.

ARTICLE HISTORY
Received 8 May 2016
Revised 5 January 2017
Accepted 12 January 2018

KEYWORDS
Type-II fuzzy integer linear programming; fuzzy type-II reducer; location–allocation problem; production–distribution problem; supply chain planning

Highlights
- A model is proposed to formulate the integrated supply chain network design problem.
- Uncertainties in real supply chains are modelled using type-II fuzzy sets.
- A new solution procedure based on type-II fuzzy sets is developed.
- A linear fuzzy type-II reducer is proposed.
- Numerical example is proposed to illustrate the efficacy of proposed approach.

1. Introduction
Supply Chain (SC) is a network of organisations, suppliers and distributors that produces value in the form of products and services and bring them to ultimate customer or market

CONTACT Kaveh Khalili-Damghani kaveh.khalili@gmail.com

© 2018 The Author(s). Published by Taylor & Francis Group on behalf of the Fuzzy Information and Engineering Branch of the Operations Research Society of China & Operations Research Society of Guangdong Province. This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0/), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.
Supply Chain Management (SCM) is a set of management activities for planning, implementing and controlling the supply chain operations. SCM needs to cope with the high uncertainty and imprecision related to real-world character [3]. Peidro et al. [4] have analysed and classified the sources of uncertainty in SC into three groups as demand, manufacturing and supply. Uncertainty is presented as variability and inexact forecasting of supplier operations and demand, and also poorly reliable production and manufacturing process [5].

In SC planning, uncertainty is a main factor that can influence the SCM processes. The uncertainty in SC has already been modelled through several paradigms such as robust optimisation [6, 7], stochastic programming [8, 9], fuzzy mathematical programming [5, 10] and hybrid models [11]. There also exists hybrid models where stochastic fuzzy programming or robust fuzzy programming models were combined [12]. In supply chains, a series of decisions such as which plants to establish, which distribution centre supplies which customer, production level of plants, capacity of plants and distribution strategies are needed to address accurately while the information contains a great amount of uncertainties.

Type-II fuzzy sets and systems generalise type-I fuzzy sets and systems so that more uncertainty can be handled. From the very beginning of fuzzy sets, criticism was made about the fact that the membership function of a type-I fuzzy set has no uncertainty associated with it, something that seems to contradict the word fuzzy, since that word has the connotation of lots of uncertainty. So, what does one do when there is uncertainty about the value of the membership function? The answer to this question was provided by Zadeh [13] when he proposed more sophisticated kinds of fuzzy sets, the first of which he called a type-II fuzzy set. A type-II fuzzy set lets us incorporate uncertainty about the membership function into fuzzy set theory, and is a way to address the above criticism of type-II fuzzy sets head-on. And, if there is no uncertainty, then a type-II fuzzy set reduces to a type-I fuzzy set, which is analogous to probability reducing to determinism when unpredictability vanishes.

Very limited applications of type-II fuzzy sets were reported in recent years. Due to our best knowledge, there is no study which addresses the type-II fuzzy numbers on integrated location–allocation, production and distribution activities in supply chain. So, the main contributions of this paper can be summarised as follows:

- Introducing a strategic and tactical SC problem through integrating dynamic location–allocation, production and distribution planning into a multi-echelon, multi-product, multi-period network.
- Modelling the aforementioned problem using mixed-integer mathematical modelling.
- Considering different sources of uncertainty in demand, manufacturing and supply processes affecting the SC.
- Considering the interval type-II fuzzy (IT2F) number for uncertain parameters and solving the SC problem based on fuzzy type-II programming.

The rest of the paper is organised as follows. The literature of past works on SC under fuzziness are presented and classified in Section 1. Section 2 introduces a fuzzy mixed-integer linear programming (FMILP) model for the integrated multi-product, multi-level, multi-period location–allocation, production–distribution problem in supply chains. In Section 3, a solution methodology is developed to transform the proposed FMILP into an auxiliary crisp mixed-integer linear programming model. A numerical example and results
are presented in Section 4 to illustrate the mechanism of proposed model and the applicability of solution procedure. Finally, the conclusions and future research directions are presented in Section 5.

2. Literature Review of Past Works

The literature of uncertain supply chain models and problems as well as the fuzzy solution procedures is reviewed in this section.

2.1. Uncertain Integrated Supply Chain Problems

The aim of location–allocation problem is to decide the facilities to be opened and the assignment of customers to the opened facilities. In literature, there is a vast effort for combining allocation decisions with other decisions such as production, inventory, procurement and capacity decisions [12]. Several authors have studied these integrated SC problems from a deterministic point of view. Tsiakis and Papageorgiou [14] developed a mixed-integer linear programming for global supply chain networks to decide about production, allocation, production capacity, purchase and network configuration with financial aspects of exchange rates, duties and production costs. Ahmadi-Javid and Hoseinpour [15] presented a non-linear mixed-integer programming model to determine location, allocation, price and order-size decisions of SC with two scenarios of uncapacitated and capacitated distribution centres. Jang et al. [16] studied integrated inventory-allocation and shipping problem in SC. Tsao and Lu [17] developed an integrated facility location and inventory-allocation problem in SC that considering two types of transportation cost discounts simultaneously: quantity discounts for inbound transportation cost and distance discounts for outbound transportation cost. Jonrinaldi and Zhang [18] proposed a model for coordinating integrated production and inventory cycles in manufacturing supply chain involving reverse logistics for multiple items with finite multiple periods. Shahabi et al. [19] studied mathematical models to coordinate facility location, allocation of supplier to warehouses and retailers and inventory control for a four-echelon supply chain network. The goal in this study is to minimise the facility location, transportation and the inventory costs.

Latha Shankara et al. [20] proposed a single product, four-echelon supply chain architecture consisting of suppliers, production plants, distribution centres and customer zones to determine the number and location of plants in the system, the flow of raw materials from suppliers to plants, and the quantity of products to be shipped from plants to distribution centres and from distribution centres to customer zones, so as to minimise the facility location and shipment costs. A multi-objective hybrid particle swarm optimisation algorithm was proposed as the solution method for the proposed model. In a study by Bandyopadhyay and Bhattacharya [21] a tri-objective problem for a two-echelon serial supply chain with objectives of minimisation of the total cost of the supply chain, minimisation of the variance of order quantity and minimisation of the total inventory was developed. To solve the model, a proposed modified non-dominated sorting genetic algorithm was applied.

Mousavi et al. [22] developed an inventory–location–allocation problem in two-echelon SC in which the all unit discount strategy was applied for purchasing costs. Diabat et al. [23] considered a closed-loop location-inventory problem with forward and reverse supply chain consisting of a single echelon. The problem is to choose which distribution centres
and remanufacturing centres are to be opened and to associate the retailers with them. Subulan et al. [24] designed a closed-loop, multi-objective, multi-echelon, multi-product and multi-period logistics network model using mixed-integer linear programming with recovery options such as remanufacturing, recycling, and energy recovery and interactive fuzzy goal programming approach was utilised to solve the proposed model.

As mentioned before, deterministic SC is unrealistic in real terms of SC problems and coefficients and parameters are often imprecise because of incomplete and unavailable information. There is increasing attention to development fuzzy or stochastic supply chain model and offer possible solutions that leading to more realistic models. SC planning under fuzzy uncertainty have been reviewed in several researches [4, 5, 12]. Peidro et al. [5] modelled integrated procurement, production and distribution planning in SC with different sources of uncertainty in demand, process and supply. All uncertain parameters were assumed to be type-I fuzzy numbers. The proposed model was solved with adaptation approach of Jimenez et al. [25]. Bilgen [26] developed an integrated production and distribution plans in multi-echelon SC for determining the allocation of production among production lines and transporting products with fuzzy available resources capacity and fuzzy costs. Paksoy et al. [27] proposed fuzzy multi-objective linear programming for minimising the end customer’s dissatisfaction and transportation costs. The production capacity and amount of demands were type-I fuzzy numbers. Figueroa [28] proposed multi-period production planning with IT2F demand. Diaz-Madronero et al. [10] proposed fuzzy multi-objective integer linear programming model for transportation and procurement planning in three-level, multi-product and multi-period automobile supply chain. They assumed the maximum capacity of the available truck and minimum percentage of demand in stock as type-I fuzzy numbers. They used an interactive solution methodology to solve the problem.

Mousavi and Niaki [29] studied a capacitated location allocation problem with fuzzy demands and stochastic locations of the customers that follow the normal probability distribution in which the distances between the locations and the customers are taken Euclidean and squared Euclidean. The fuzzy expected cost programming, the fuzzy $\beta$-cost minimisation model and the credibility maximisation model are three types of fuzzy programming that are developed to model. Azadeh et al. [30] proposed a multi-objective, multi-period, multi-echelon fuzzy linear programming model to optimise natural gas supply chain based on two economic and environmental objectives and fuzzy parameters, including demand, capacity and cost. A combination of the possibilistic programming approach based on the defuzzification method and interactive fuzzy approach was used to deal with uncertainty. Ramezani et al. [31] designed a multi-product, multi-period, closed-loop supply chain network with three objective functions: maximisation of profit, minimisation of delivery time, and maximisation of quality and three fuzzy components of constraints, coefficients, and goal. To cope with fuzziness, a fuzzy optimisation approach was adopted to convert the proposed fuzzy multi-objective mixed-integer linear programme into an equivalent auxiliary crisp model.

Gholamian et al. [32] developed multi-site, multi-period and multi-product aggregate production planning in supply chain to minimise the total cost of the SC, improve customer satisfaction, minimise the fluctuations in the rate of changes of workforce, and maximise the total value of purchasing under fuzzy parameters. Alizadeh Afrouzy et al. [33] designed a multi-echelon, multi-period, multi-product and aggregate procurement and production
planning model which considered three objectives of maximise the profit of the supply chain due to new product development, customer satisfaction, and maximising the production of the developed and new products. The proposed model considered uncertainties of the environment such as customer demands and supplier capacities which is modelled by fuzzy stochastic programming.

Following the stochastic viewpoint, Lin and Wu [34] proposed product pricing and integrated supply chain operations plan under conditions of price-dependent stochastic demand under two approaches of maximising a manufacturer’s expected profit and cost minimisation. Marufuzzaman et al. [35] developed two-stage stochastic programming model to design and extend the classical two-stage stochastic location–transportation model. Their model was designed to optimise costs and emissions in the supply chain and capture the impact of biomass supply and technology uncertainty on tradeoffs that exist between location and transportation decisions; and the tradeoffs between costs and emissions in the supply chain.

About decision-making in inventory management, Panda et al. [36] developed economic production lot size for single-period multi-product model in which the demand rate is stochastic under stochastic and/or fuzzy budget and shortage constraints. The stochastic constraints have been represented by chance constraints and fuzzy constraints in the form of possibility/necessity constraints which are transformed to equivalent deterministic ones. Wang et al. [37] proposed single-item and multi-item single-period inventory models for short life-cycle products when demands are assumed to be uncertain random variables. For deriving the optimality condition for optimal order quantity the uncertain random models are transferred to equivalent deterministic forms by considering expected profit and providing more information of chance distributions. Integrated inventory model in supply chain is developed in some researches. Bag and Chakraborty [38] addressed optimal order quantity of the retailers, production rate of the producer and the production rate of the suppliers to minimise the supply chain cost in which there are imprecise chance constraints on the transportation costs for producer, retailer and also a fuzzy space constraint for producer is considered. Jana et al. [39] developed ordering policy for deteriorating products with allowable shortage and permissible delay in payment under inflation and time value of money in fuzzy rough environment which the proposed model is converted to deterministic one using expected value method techniques.

2.2. Fuzzy Linear Programming as Solution Procedure

Fuzzy linear programming (FLP) is an optimisation technique applied in real-world problems with imprecise data. FLP has an extensive literature. In this research, two main categories of FLP are reviewed: (1) FLP with fuzzy numbers as coefficient in objective function, right-hand sides (RHSs), and technological coefficient, (2) FLP with fuzzy parameters and fuzzy decision variables.

In the first category, numerous researches have proposed different techniques to solve the FLP problems. Zimmermann [40] formulated and solved the FLP problem for the first time. Herrera and Verdegay [41] proposed three models for three types of FLPs. They solved the problems with fuzzy RHS or fuzzy coefficient of constraint using auxiliary parametric linear programme. They also used two fuzzy ranking methods to cope with FLPs with fuzzy
objective functions. Xinwang [42] developed FLP with fuzzy technological coefficients and the RHSs of the constraints and solved FLPs based on the new ranking method of fuzzy numbers for satisfaction degree of the constraints. Zhang et al. [43] presented FLPs with fuzzy coefficients of objective function and showed how to convert such FLPs into equivalent deterministic multi-objective optimisation problem. Jimenez et al. [25] presented full FLPs in which all parameters were fuzzy numbers. They also presented a resolution method with interactive participation of decision maker during solving procedure. Wu [44] described the FLP which involve fuzzy numbers for the coefficients of variables in the objective function and developed the optimality conditions for FLPs through proposing two solving methodologies that were similar to non-dominated solution in multi-objective programming. The problems with fuzzy numbers for the coefficients of the objective function, the technological coefficients in the constraints and the RHS of the constraints were developed by Mahdavi-Amiri and Nasseri [45], Hatami-Marbin and Tavana [46] and Saati et al. [47].

The method for finding the optimal solution of FLPs with type-II fuzzy numbers presented by Figueroa, [48] and Figueroa and Hernandez [49]. They proposed a method to solve FLPs with interval type-II RHSs.

In the second category, FLP with fuzzy numbers for the decision variables, the coefficients in the objective function and the RHS of the constraints was developed by Ganesan and Veeramani [50] that led to the solution without converting FLPs to crisp linear programming problems.

Mahdavi-Amiri and Nasseri [51] studied the FLP problems with fuzzy numbers for the decision variables and the right-hand-side of the constraints and developed solution method based on auxiliary problems that applied a linear ranking function to order trapezoidal fuzzy variables. Then, Ebrahimnejad et al. [52] proposed a new primal-dual algorithm for solving FLP problems with fuzzy variables by using the duality method investigated by Mahdavi-Amiri and Nasseri [51]. Kumar et al. [53] proposed a method to improve the method proposed by Hosseinizadeh Lotfi et al. [54] to find the optimal solution of the FLPs with fuzzy parameters and fuzzy variables that called Fully FLP.

Based on our knowledge, the integrated SC model in a fuzzy environment with different sources of uncertainty especially with type-II fuzzy sets has not been well studied. Therefore, this paper attempts: (1) to introduce an integrated location–allocation, production, procurement and distribution planning problem in supply chains, (2) to model the proposed problem in presence of uncertainty in demand, manufacturing and supply processes through mathematical programming and (3) to develop a solution methodology to solve the proposed model optimally considering interval type-II fuzzy sets.

Most of the supply chain researches assumed the fuzzy parameters to be of type-I fuzzy sets such as Peidro et al. [5], Bilgen [26], Paksoy et al. [27] and Diaz-Madronero et al. [10]. In decision-making problems like location–allocation, production–distribution, the parameters cannot be always exactly known. For example, demand or flow of products in supply chain management depends upon several variables such as price, labour charges and repair time. Each of these variables fluctuates and also membership degree of each point cannot be exactly determined. So it is not easy to predict the deterministic or type-I fuzzy sets. Assuming fuzzy type-II parameters, i.e. fuzziness in membership function, for such variables is a proper idea, although fuzzy type-II parameters impose very high computational complexity to the problem. In this paper, the fuzzy type-II mathematical programming is reduced into fuzzy type-I mathematical programming
while the main properties of original programming is retained. Moreover, the procedure is generalised for the cases in which the resultant model may be a non-linear mathematical programming. So, formally talking, the main advantages of proposed procedure of this study over the previous methods in the literature can be summarised as follows:

- Considering complicated uncertainties in membership value of a fuzzy variable in terms of fuzzy type-II parameters.
- Reducing the fuzzy type-II mathematical programming into a fuzzy type-I mathematical programming while the main properties of original programming is retained.
- Proposing a procedure for linearisation of the resultant models in the sense of generalisation of the cases in which the resultant model may be a non-linear mathematical programming.

3. Problem Descriptions and Modelling

In this section, the problem of multi-echelon, multi-product, multi-period location–allocation production and distribution supply chain network is proposed. Then, the associated mathematical programming is developed. The following assumptions are considered in the proposed model:

- Number of possible plants are known and determined in advance.
- Number of possible distribution centres are known and determined in advance.
- Each product can be produced at several plants.
- Distribution centres can be supplied from more than one plant and can supply more than one customer.
- Customers have uncertain demands of multiple products during planning periods.
- Production and transportation costs are assumed to be known and deterministic.
- Production and transportation capacity are known and uncertain.

The following dynamic decisions are to be determined:

- Establish or shut down distribution centres
- Product plan, production rate and utilisation per plants
- Distribution plan between plants, distribution centres and customers
- Procurement plan
- Assignment of distribution centres to plants and customer to distribution centres

The objective is the minimisation of the total cost of the strategic, tactical and operational costs.

The graphical representation of the three-stage, multi-product, multi-period supply chain network is shown in Figure 1. The objective is to find which distribution centres are to be opened, which customers are served from opened distributors, and which procurement and production strategy is to be planned so that the total cost is minimised.
3.1. Notations

Indices, parameters and decision variables are presented in Table 1.

Three types of uncertainty are assumed in the proposed model. Table 2 presents the sources and types of uncertainties.

3.2. Model formulation

A new multi-echelon, multi-product, multi-period location–allocation production and distribution supply chain network model with type-II fuzzy sets with due attention to the model proposed by Tsiakis and Papageorgiou [14] is developed.

\[
\begin{align*}
\text{Min } Z & = \sum_{k=1}^{K} \sum_{t=1}^{T} EC_{kt}^D \cdot Y_{kt}^D + \sum_{k=1}^{K} \sum_{t=1}^{T} SC_{kt}^D (1 - Y_{kt}^D) + \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{t=1}^{T} PC_{ijt} \cdot P_{ijt} \\
& + \sum_{i=1}^{I} \sum_{k=1}^{K} \sum_{t=1}^{T} BC_{it} \cdot P_{Qijkt} + \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{t=1}^{T} TC_{ijkt}^P \cdot TQ_{ijkt}^P \\
& + \sum_{i=1}^{I} \sum_{k=1}^{K} \sum_{d=1}^{D} \sum_{t=1}^{T} TC_{ikdt}^D \cdot TQ_{ikdt}^D \\
\end{align*}
\]

(1)

\[
X_{jkt}^P \leq Y_{kt}^D \quad \forall \ j,k,t
\]

(2)

\[
\sum_{j=1}^{J} X_{jkt}^P \geq Y_{kt}^D \quad \forall \ k,t
\]

(3)

\[
X_{kdt}^D \leq Y_{kt}^D \quad \forall \ k,d,t
\]

(4)

\[
\sum_{d=1}^{D} X_{kdt}^D \geq 1 \quad \forall \ k,t
\]

(5)
Table 1. Indices, parameters and decision variables.

Indices

| Index | Description |
|-------|-------------|
| I     | Set of products $i = \{1, 2, \ldots, I\}$ |
| J     | Set of plants $j = \{1, 2, \ldots, J\}$ |
| K     | Set of possible distribution centre $k = \{1, 2, \ldots, K\}$ |
| D     | Set of customers $d = \{1, 2, \ldots, D\}$ |
| T     | Set of planning periods $t = \{1, 2, \ldots, T\}$ |

Parameters

- $ECD_{kt}$: cost of establishing a distribution centre at location $k$ in time $t$
- $SCD_{kt}$: cost of closing a distribution centre at location $k$ in time $t$
- $PC_{ijt}$: unit production cost for product $i$ at plant $j$ in time $t$
- $BC_{it}$: unit procurement cost for product $i$ from third parties in time $t$
- $TCP_{ijkt}$: unit transport cost for product $i$ from plant $j$ to distribution centre $k$ in time $t$
- $TCD_{ikdt}$: demand of customer $d$ for product $i$ in time $t$
- $\tilde{D}_{ddt}$: Hours of operating of plant $j$
- $M_j$: Hours of maintenance of plant $j$
- $\tilde{C}_{ppj}$: maximum production capacity of plant $j$ for product $i$
- $\tilde{C}_{ppj}$: maximum rate of flow of products transferred from plant $j$ to distribution centre $k$
- $\tilde{C}_{ppk}$: maximum rate of flow of products transferred from distribution centre $k$ to customer $d$
- $r_j$: daily production rate of product $i$ in plant $j$
- $\beta$: change-over coefficient in hours
- $\varepsilon$: utilisation parameter in hours

Decision Variable

- $PQ_{ikt}$: out-sourced product $i$ delivered to distribution centre $k$ in time $t$
- $P_{ijt}$: production rate of product $i$ in plant $j$ in time $t$
- $TQP_{ijkt}$: rate of flow of product $i$ transferred from plant $j$ to distribution centre $k$ in time $t$
- $TQD_{ikdt}$: rate of flow of product $i$ transferred from distribution centre $k$ to customer $d$ in time $t$
- $U_{jt}$: hours allocated for production of product $i$ in plant $j$ in time $t$
- $\alpha_t$: maximum allowed difference in utilisation of plants in time $t$
- $\gamma^*_j$: 1 if production plant $j$ is assigned to distribution centre $k$ in time $t$, 0 otherwise
- $\gamma^*_{kd}$: 1 if distribution centre $k$ is assigned to customer $d$ in time $t$, 0 otherwise
- $\omega_{jt}$: 1 if production plant $j$ is to produce product $i$ in time $t$, 0 otherwise

Table 2. Sources of uncertainty.

| Source of uncertainty | Description | Parameter |
|-----------------------|-------------|-----------|
| Demand                | Demand of product $i$ at customer $d$ in period $t$ | $\tilde{D}_{ddt}$ |
| Process (manufacturing) | Production capacity of plant $j$ for product $i$ | $\tilde{C}_{ppj}$ |
| Supply                | Rate of flow from plant $j$ to distribution centre $k$ | $\tilde{C}_{ppk}$ |
|                       | Rate of flow from distribution centre $k$ to customer $d$ | $\tilde{C}_{ppk}$ |

\[
\sum_{i=1}^{I} TQP_{ijkt} \leq \tilde{C}_{ppj} \cdot X^P_{jkt} \quad \forall j, k, t
\]  

(6)

\[
\sum_{i=1}^{I} TQD_{ikdt} \leq \tilde{C}_{ppk} \cdot X^D_{kdt} \quad \forall k, d, t
\]  

(7)

\[
P_{ijt} = \sum_{k=1}^{K} TQP_{ijkt} \quad \forall i, j, t
\]  

(8)
\[
\sum_{j=1}^{J} TQ_{ijkt}^P + PQ_{ikt} = \sum_{d=1}^{D} TQ_{ikdt}^D \quad \forall \, i, k, t
\] (9)

\[
\sum_{k=1}^{K} TQ_{ikdt}^D \geq \tilde{D}e_{idt} \quad \forall \, i, d, t
\] (10)

\[
P_{ijt} \leq c^P \cdot p_{ijkl} \cdot w_{ijt} \quad \forall \, i, j, t
\] (11)

\[
\sum_{i=1}^{I} T_{ijt}^P \leq (H_j - M_j) - \beta \sum_{i=1}^{I} w_{ijt} \quad \forall \, j, t
\] (12)

\[
P_{ijt} \leq r_{ij} \cdot T_{ijt}^P \quad \forall \, i, j, t
\] (13)

\[
U_{jt} = \sum_{i=1}^{I} T_{ijt}^P \quad \forall \, j, t
\] (14)

\[
\alpha_t = U_{jt} - U_{jt}^\prime \quad \forall \, j, t \neq \hat{j}
\] (15)

\[
\alpha_t \leq \varepsilon \quad \forall \, t
\] (16)

\[
PQ_{ikt} \geq 0 \quad \forall \, i, k, t
\] (17)

\[
P_{ijt} \geq 0, \quad T_{ijt}^P \geq 0, \quad w_{ijt} \in \{0, 1\} \quad \forall \, i, j, t
\] (18)

\[
TQ_{ijkt}^P \geq 0 \quad \forall \, i, j, k, t
\] (19)

\[
TQ_{ikdt}^D \geq 0 \quad \forall \, i, k, d, t
\] (20)

\[
U_{jt} \geq 0 \quad \forall \, j, t
\] (21)

\[
\alpha_t \geq 0 \quad \forall \, t
\] (22)

\[
Y_{kt}^D = Bin \quad \forall \, k, t
\] (23)

\[
X_{jkt}^P = Bin \quad \forall \, j, k, t
\] (24)

\[
X_{kdt}^D = Bin \quad \forall \, k, d, t
\] (25)

Equation (1) minimises the total cost of supply chain. The objective function consists of costs of the distribution centre infrastructure, the costs of production and procurement, and costs of transportation. Set of constraints (2), which is written for each plant, distribution center and planning period, assures that a plant can services to a distribution center if an only if the distribution center had been established. Set of constraints (3), which is written for each distribution center and planning period, assures
that one of the production plants is assigned to each distribution center in each time period.

Set of constraints (4), which is written for each distribution center, customer and planning period, assures that a distribution center can services to a customer if and only if the distribution center had been established. Set of constraints (5), which is written for each distribution center and planning period, assures that a distribution center must be assigned at least to a customer in each time period.

Set of constraints (6), which is written for each plant, distribution center and time period, assures that flow of material from production plant to distribution center can take place only if the connection exists. Set of constraints (7), which is written for each distribution center, customer and time period, assures that flow of material from distribution center to customer can take place only if the corresponding connection exists.

Set of constraints (8) assures that the production rate of product type $i$ in plant $j$ in time period $t$ is equal to all sent products type $i$ to all distribution centres in time period $t$.

Set of constraints (9) assures that all products type $i$ which has received by distribution center $k$ from all plants in time period $t$ plus the products type $i$ which has received by distribution center $k$ from out-sourcing supplier in time period $t$ is equal to all products type $i$ which has received by all customers $d$ from distribution center $k$ in time period $t$.

Set of constraints (10), which is written for all products, all customers and all time periods, assures that total flow of each product received by each customer $d$ in every planning period $t$ from all distribution centers must at least satisfies the demand of customer $d$.

Set of constraints (11), which is written for all products, all plants and all time periods, assures that if production of product $i$ in plant $j$ is planned, the plant cannot produce product $i$ more than its production capacity.

Set of constraints (12), which is written for each plant and each time period, assures that the number of available working days in plant $j$ during planning period $t$ is restricted by available operating days minus the maintenance days considering change over (set up) days.

Set of constraints (13), which is written for all products, all plants and all time periods, assures that the production rate of product $i$ in plant $j$ in planning period $t$ is less than or equal to hours allocated for the production of product $i$ in plant $j$ in time $t$ multiply by daily production rate of product $i$ in plant $j$.

Set of constraints (14), which is written for all products, all plants and all time periods, assures that the utilisation of plant $j$ in planning period $t$ is equal to hours allocated for the production of all products in plant $j$ in time $t$.

Set of constraints (15)–(16) ensure that utilisation of two arbitrary plants in time period $t$ is constant and less than a predetermined value. Set of constraints (17)–(25) define the corresponding decision variables of the model.

3.3. Model validation

In order to validate the process of mathematical modelling, several extreme state instances are considered. The solution of these instances can be expected easily as the parameters are meaningfully determined in a biased form. Then, the output of the proposed mathematical model (1)–(25) is compared with the expected results for these instances. For example in one of these instances, the production rate and procurement costs were
Table 3. Benchmark instance for validation of proposed model.

|                    | Plant 1 | Plant 2 | Plant 3 | Distribution 1 | Distribution 2 | Distribution 3 |
|--------------------|---------|---------|---------|----------------|----------------|----------------|
| Product 1          | 0.0     | 0.0     | 0.0     | 9666.549       | 22662.45       | 20783.00       |
| Product 2          | 0.0     | 0.0     | 0.0     | 4446.451       | 3337.549       | 1343.000       |

set equal to zero in all plants, and the production costs are set to a very high value. Table 3 shows a partial report of the model (1)–(25) on this benchmark instance with three plants, three distribution centers and two demand zone for one month. As, we expected the model suggest the out-sourcing. The results are an evidence to validate the proposed model. Several aspects of model (1)–(25) were tested using several extreme state instances.

4. Proposed Fuzzy Type-II Solution methodology

In this context, the approach by Figueroa [48] and Figueroa and Hernandez [49] is adopted to solve FLP problems with interval type-II RHS.

Figueroa [48] proposed a Type-II fuzzy mathematical programming method. The approach reduced a fuzzy Type-II mathematical programming into a Type-I mathematical programming. The main idea of the method by Figueroa [48] was formed on the basis of linearity of mathematical programming. In many real mathematical programming models, such as supply chain network design, distribution and production, the resultant model is not linear. In this paper, the method by Figueroa [48] has been generalised for the nonlinear class of mathematical problems. A linearisation approach is proposed and adopted to overcome the shortage of the method by Figueroa [48].

Let us now consider the model (26) as a linear programming problem, where \( \tilde{b} = (\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_n) \) is interval type-II fuzzy (IT2F) RHS defined by its lower primary membership functions \( \mu_{\tilde{b}}(x) \) with parameters \( \tilde{b}^V \) and \( \tilde{b}^A \) and upper primary membership function \( \mu_{\tilde{b}}(x) \) with parameters \( \overline{\tilde{b}}^V \) and \( \overline{\tilde{b}}^A \). The graphical representation of interval fuzzy type-II RHS is shown in the Figure 2.

\[
\text{Min } Z = C^T x
\]

\[
x \in (A, \tilde{b}) = \{x \in \mathbb{R}^m | a_i x \geq \tilde{b}_i, i = 1, \ldots, m, x \geq 0 \}
\]

Applying the approach proposed by Figueroa [48], the method is summarised as follow:

1. Calculate \( Z^{\min} \) using \( b^A - \Delta \) as RHS; where, \( \Delta \) is auxiliary variable weighted by \( C^A \). The optimum value of \( \Delta^* \) and \( Z^{\min} \) are obtained by solving the LP problem (27).

\[
\text{Min } Z = C^T x + C^A \cdot \Delta
\]

subject to:

\[
A \cdot x \geq \tilde{b}^A - \Delta
\]

\[
\Delta \leq \tilde{b}^A - \overline{\tilde{b}}^A
\]

\[
\Delta \geq 0, \quad x \geq 0
\]
(2) Calculate $Z^{\text{max}}$ using $b^\nabla - \nabla$ as RHS; where $\nabla$ is auxiliary variable weighted by $C^\nabla$. The optimum value of $\nabla^*$ and $Z^{\text{max}}$ are obtained by solving the LP problem (28).

\[
\begin{align*}
\text{Min } Z &= C^I \cdot x + C^\nabla \cdot \nabla \\
\text{subject to : } \\
A \cdot x &\geq b^\nabla - \nabla \\
\nabla &\leq b^\nabla - \nabla^*
\end{align*}
\tag{28}
\]

$C^\Delta$ and $C^\nabla$ are assumed as incremental costs which are used to increase the consumption of resources in models (27) and (28), respectively. Therefore, $\Delta$ and $\nabla$ operate as fuzzy type-II reducers. Using the values of $\Delta$, and $\nabla$, for each uncertain RHS, a best fuzzy set embedded on the footprint of uncertainty (FOU) is obtained such that $b^{\text{max}} = b^\Delta - \Delta^*$ and $b^{\text{min}} = b^\nabla - \nabla^*$. The type-I fuzzy value $\tilde{b}$ is determined using parameters $b^{\text{max}}$, and $b^{\text{min}}$ as linear membership function. Application of such fuzzy type-II reducers conducts the rest of algorithm based on Zimmermann [40] soft constraints method.

(3) Define a Fuzzy Set $\tilde{Z}$ with bounds $Z^{\text{min}}$ and $Z^{\text{max}}$ and linear membership function as (29).

\[
\mu_Z = \begin{cases} 
1 & Cx \leq Z^{\text{min}} \\
\frac{Z^{\text{max}} - Cx}{Z^{\text{max}} - Z^{\text{min}}} & Z^{\text{min}} \leq Cx \leq Z^{\text{max}} \\
0 & Cx \geq Z^{\text{max}}
\end{cases}
\tag{29}
\]

(4) Consider an auxiliary variable $\lambda$ to represent the $\lambda - \text{cut}$ overall satisfaction degree between $\tilde{Z}$ and $\tilde{b}$. Calculate the optimum value of $\lambda$ by solving the LP problem (30).

\[
\begin{align*}
\text{Max } \lambda \\
\text{subject to : } \\
Cx + \lambda(Z^{\text{max}} - Z^{\text{min}}) &= Z^{\text{max}} \\
Ax - \lambda(b^{\text{max}} - b^{\text{min}}) &\geq b^{\text{min}} \\
\lambda &\geq 0, \quad x \geq 0
\end{align*}
\tag{30}
\]
4.1. Application of Proposed Fuzzy Type-II Approach

The described approach is applied on model (1)–(25). Considering linear interval type-II fuzzy numbers for the RHS of model (1)–(25), three auxiliary crisp mixed-integer programming models are yielded as follows. The values of $Z^{\min}$ and $\Delta^*$ are calculated using model (31)–(43).

\[
\min Z^{\min} = \text{objective function} + C_{jk}^{PD} \cdot \Delta_{jk}^{PD} + C_{kd}^{DD} \cdot \Delta_{kd}^{DD} + C_{idt} \cdot \Delta_{idt} + C_{ij}^{p} \cdot \Delta_{ij}^{p} \quad (31)
\]

subject to:

\[
- \sum_{i=1}^{I} TQ_{ijkt}^{p} \geq \left(-\frac{\text{cap}_{jk}^{PD}}{\Delta_{jk}^{PD}} - \Delta_{jk}^{PD}\right) \cdot X_{jkt}^{p} \quad \forall j, k, t \quad (32)
\]

\[
- \sum_{i=1}^{I} TQ_{ikdt}^{D} \geq \left(-\frac{\text{cap}_{kd}^{DD}}{\Delta_{kd}^{DD}} - \Delta_{kd}^{DD}\right) \cdot X_{kdt}^{D} \quad \forall k, d, t \quad (33)
\]

\[
\sum_{k=1}^{K} TQ_{ikdt}^{D} \geq \text{De} \cdot \Delta_{idt} \quad \forall i, d, t \quad (34)
\]

\[
P_{ijt} \geq \left(-\frac{\text{cap}_{ij}^{p}}{\Delta_{ij}^{p}} - \Delta_{ij}^{p}\right) \cdot w_{ijt} \quad \forall i, j, t \quad (35)
\]

\[
\Delta_{jk}^{PD} \leq \frac{\text{cap}_{jk}^{PD}}{\Delta_{jk}^{PD}} - \frac{\text{cap}_{jk}^{PD}}{\Delta_{jk}^{PD}} \quad \forall j, k \quad (36)
\]

\[
\Delta_{kd}^{DD} \leq \frac{\text{cap}_{kd}^{DD}}{\Delta_{kd}^{DD}} - \frac{\text{cap}_{kd}^{DD}}{\Delta_{kd}^{DD}} \quad \forall k, d \quad (37)
\]

\[
\Delta_{idt} \leq \text{De} \cdot \Delta_{idt} \quad \forall i, d, t \quad (38)
\]

\[
\Delta_{ij}^{p} \leq \frac{\text{cap}_{ij}^{p}}{\Delta_{ij}^{p}} - \frac{\text{cap}_{ij}^{p}}{\Delta_{ij}^{p}} \quad \forall i, j \quad (39)
\]

\[
\Delta_{jk}^{PD} \geq 0 \quad \forall j, k \quad (40)
\]

\[
\Delta_{kd}^{DD} \geq 0 \quad \forall k, d \quad (41)
\]

\[
\Delta_{idt} \geq 0 \quad \forall i, d, t \quad (42)
\]

\[
\Delta_{ij}^{p} \geq 0 \quad \forall i, j \quad (43)
\]

The values of $Z^{\max}$ and $\nabla^*$ are calculated using models (44)–(56).

\[
\min Z^{\max} = \text{objective function} + C_{jk}^{PD} \cdot \nabla_{jk}^{PD} + C_{kd}^{DD} \cdot \nabla_{kd}^{DD} + C_{idt} \cdot \nabla_{idt} + C_{ij}^{p} \cdot \nabla_{ij}^{p} \quad (44)
\]

subject to:

\[
- \sum_{i=1}^{I} TQ_{ijkt}^{p} \geq \left(-\frac{\text{cap}_{jk}^{PD\nabla}}{\Delta_{jk}^{PD\nabla}} - \nabla_{jk}^{PD\nabla}\right) \cdot X_{jkt}^{p} \quad \forall j, k, t \quad (45)
\]
It is notable that the crisp constraints (2)–(5), (8)–(9), (12)–(16) and (17)–(25) should also be included in both models (31)–(43), and (44)–(56). Therefore, a best fuzzy set embedded on footprint of uncertainty is calculated based on Equations (57)–(60).

\[
\begin{align*}
\text{cap}^{\text{PDmin}}_{jk} &= \text{cap}^{\text{PD}}_{jk} + \Delta^{\text{PD}}_{jk}, \\
\text{cap}^{\text{PDmax}}_{jk} &= \text{cap}^{\text{PD}}_{jk} + \nabla^{\text{PD}}_{jk} & \forall j, k \tag{57}
\end{align*}
\]

\[
\begin{align*}
\text{cap}^{\text{DDmin}}_{kd} &= \text{cap}^{\text{DD}}_{kd} + \Delta^{\text{DD}}_{kd}, \\
\text{cap}^{\text{DDmax}}_{kd} &= \text{cap}^{\text{DD}}_{kd} + \nabla^{\text{DD}}_{kd} & \forall k, d \tag{58}
\end{align*}
\]

\[
\begin{align*}
\text{De}^{\text{max}}_{idt} &= \text{De}^{1}_{idt} - \Delta^{*}_{idt}, \\
\text{De}^{\text{min}}_{idt} &= \text{De}^{1}_{idt} - \nabla^{*}_{idt} & \forall i, d, t \tag{59}
\end{align*}
\]

\[
\begin{align*}
\text{cap}^{\text{Pmin}}_{ij} &= \text{cap}^{\Delta}_{ij} + \Delta^{*}_{ij}, \\
\text{cap}^{\text{Pmax}}_{ij} &= \text{cap}^{\Delta}_{ij} + \nabla^{*}_{ij} & \forall i, j \tag{60}
\end{align*}
\]

Then optimum value of $\lambda$ is calculated using the models (61)–(66).

\[
\begin{align*}
\text{Max } \lambda \\
\text{subject to :} \\
\text{objective function } + \lambda(Z^{\text{max}} - Z^{\text{min}}) = Z^{\text{max}} & \tag{61}
\end{align*}
\]

\[
\begin{align*}
- \sum_{i=1}^{I} TQ^P_{ikt} - \lambda(\text{cap}^{\text{PDmax}}_{jk} - \text{cap}^{\text{PDmin}}_{jk}) \geq -\text{cap}^{\text{PDmax}}_{jk} \cdot X^P_{ikt} & \forall j, k, t \tag{62}
\end{align*}
\]
\[- \sum_{i=1}^{I} \sum_{k=1}^{K} T_{ikdt}^{Q} - \lambda (cap_{kd}^{\text{PDmax}} - cap_{kd}^{\text{PDmin}}) \geq - \text{cap}_{kd}^{\text{PDmax}} \cdot x_{kd}^{Q} \quad \forall \ k, d, t (63)\]

\[\sum_{k=1}^{K} T_{ikdt}^{Q} - \lambda (De_{idt}^{\text{max}} - De_{idt}^{\text{min}}) \geq De_{idt}^{\text{min}} \quad \forall \ i, d, t (64)\]

\[- P_{ijt} - \lambda (cap_{ij}^{Pmax} - cap_{ij}^{Pmin}) \geq - \text{cap}_{ij}^{Pmax} \cdot w_{ijt} \quad \forall \ i, j, t (65)\]

\[\lambda \geq 0 (66)\]

Again, the crisp constraints (2)-(5), (8)-(9), (12)-(16) and (17)-(25) should also be included in models (61)-(66).

### 4.2. Linearisation approach

One of the principles of the method proposed by Figueroa [48] is that the resultant auxiliary programming models are linear. In this section, a linearisation approach is proposed to overcome the shortage of the non-linear programming. The models (31)-(43), and the models (44)-(56) which are used to calculate $Z_{min}$, and $Z_{max}$, respectively, are non-linear mixed-integer programming models. The proposed approach in this section transforms them to linear programming (LP) problems. The global optimum solutions can be found easily for LPs.

Both models (31)-(43), (44)-(56) have non-linear terms in which the products of two binary and continuous variables are incorporated. This type of non-linearity can be removed as follows. Let $x_1$ be a binary variable, and $x_2$ be a continuous variable, and $0 < x_2 < u$. A continuous variable, $y$, is introduced to replace the product $y = x_1 x_2$ and the constraints (67)-(70) are to be added to the associated optimisation model.

\[y \leq u x_1 \quad (67)\]

\[y \leq x_2 \quad (68)\]

\[y \geq x_2 - u (1 - x_1) \quad (69)\]

\[y \geq 0 \quad (70)\]

The equations (32)-(33) and (35) are non-linear constraints, and are replaced by Equations (71)-(85). The new continuous variables $F_{jkt}^{P}$, $F_{kdt}^{D}$, $F_{ijt}$ are defined for optimisation of $Z_{min}$.

\[\sum_{i=1}^{I} T_{ijkt}^{P} \leq \text{cap}_{jk}^{PD\Delta} \cdot x_{jkt}^{P} + F_{jkt}^{P} \quad \forall \ j, k, t (71)\]

\[F_{jkt}^{P} \leq (\text{cap}_{jk}^{PD\Delta} - \text{cap}_{jk}^{PD\Delta}) \cdot x_{jkt}^{P} \quad \forall \ j, k, t (72)\]

\[F_{jkt}^{P} \leq \Delta_{jk}^{PD} \quad \forall \ j, k, t (73)\]
\[ F_{jkt}^p \geq \Delta^{PD}_{jk} - (\text{cap}^{PD\Delta}_{jk} - \text{cap}^{PD\Delta}_{jk}) \cdot (1 - X_{jkt}) \quad \forall j, k, t \quad (74) \]

\[ F_{jkt}^p \geq 0 \quad \forall j, k, t \quad (75) \]

\[ \sum_{i=1}^{I} TQ_{ikdt}^D \leq \text{cap}^{DD\Delta}_{kd} \cdot X_{kdt}^D + F_{kdt}^D \quad \forall k, d, t \quad (76) \]

\[ F_{kdt}^D \leq (\text{cap}^{DD\Delta}_{kd} - \text{cap}^{DD\Delta}_{kd}) \cdot X_{kdt}^D \quad \forall k, d, t \quad (77) \]

\[ F_{kdt}^D \leq \Delta_{kd}^D \quad \forall k, d, t \quad (78) \]

\[ F_{kdt}^D \geq \Delta_{kd}^D - (\text{cap}^{DD\Delta}_{kd} - \text{cap}^{DD\Delta}_{kd}) \cdot (1 - X_{kdt}) \quad \forall k, d, t \quad (79) \]

\[ F_{kdt}^D \geq 0 \quad \forall k, d, t \quad (80) \]

\[ P_{ijt} \leq \text{cap}^{p\Delta}_{ij} \cdot W_{ijt} + F_{ijt} \quad \forall i, j, t \quad (81) \]

\[ F_{ijt} \leq (\text{cap}^{p\Delta}_{ij} - \text{cap}^{p\Delta}_{ij}) \cdot W_{ijt} \quad \forall i, j, t \quad (82) \]

\[ F_{ijt} \leq \Delta_{ij}^p \quad \forall i, j, t \quad (83) \]

\[ F_{ijt} \geq \Delta_{ij}^p - (\text{cap}^{p\Delta}_{ij} - \text{cap}^{p\Delta}_{ij}) \cdot (1 - W_{ijt}) \quad \forall i, j, t \quad (84) \]

\[ F_{ijt} \geq 0 \quad \forall i, j, t \quad (85) \]

Alternatively, in the process of optimisation of \(Z^{\text{max}}\), Equations (45)–(46) and (48) are non-linear. They are replaced by Equations (86)–(100). The new continuous variables \(O_{jkt}^p\), \(O_{kdt}^D\), \(O_{ijt}^p\) are also defined.

\[ \sum_{i=1}^{I} TQ_{ijkt}^p \leq \text{cap}^{PD\Delta^p}_{jk} \cdot X_{jkt}^p + O_{jkt}^p \quad \forall j, k, t \quad (86) \]

\[ O_{jkt}^p \leq (\text{cap}^{PD\Delta^p}_{jk} - \text{cap}^{PD\Delta^p}_{jk}) \cdot X_{jkt}^p \quad \forall j, k, t \quad (87) \]

\[ O_{jkt}^p \leq \Delta_{jk}^{PD^p} \quad \forall j, k, t \quad (88) \]

\[ O_{jkt}^p \geq \Delta_{jk}^{PD^p} - (\text{cap}^{PD\Delta^p}_{jk} - \text{cap}^{PD\Delta^p}_{jk}) \cdot (1 - X_{jkt}^p) \quad \forall j, k, t \quad (89) \]

\[ O_{jkt}^p \geq 0 \quad \forall j, k, t \quad (90) \]

\[ \sum_{i=1}^{I} TQ_{ikdt}^D \leq \text{cap}^{DD\Delta^D}_{kd} \cdot X_{kdt}^D + O_{kdt}^D \quad \forall k, d, t \quad (91) \]
\[ O_{kdt}^D \leq (\text{cap}_{k}^{DD} - \text{cap}_{k}^{DV}) \cdot \chi_{kdt}^D \quad \forall \ k, d, t \] (92)

\[ O_{kdt}^D \leq \nabla_{k}^{DD} \quad \forall \ k, d, t \] (93)

\[ O_{kdt}^D \geq \nabla_{k}^{DD} - (\text{cap}_{k}^{DD} - \text{cap}_{k}^{DV}) \cdot (1 - \chi_{kdt}^D) \quad \forall \ k, d, t \] (94)

\[ O_{kdt}^D \geq 0 \quad \forall \ k, d, t \] (95)

\[ P_{ijt} \leq \text{cap}_{ij}^{PV} \cdot w_{ijt} + O_{ijt} \quad \forall \ i, j, t \] (96)

\[ O_{ijt} \leq (\text{cap}_{ij}^{PV} - \text{cap}_{ij}^{PV}) \cdot W_{ijt} \quad \forall \ i, j, t \] (97)

\[ O_{ijt} \leq \nabla_{ij}^P \quad \forall \ i, j, t \] (98)

\[ O_{ijt} \geq \nabla_{ij}^P - (\text{cap}_{ij}^{PV} - \text{cap}_{ij}^{PV}) \cdot (1 - W_{ijt}) \quad \forall \ i, j, t \] (99)

\[ O_{ijt} \geq 0 \quad \forall \ i, j, t \] (100)

5. Numerical Example

To demonstrate the applicability and usability of the proposed model, a numerical example with six manufacturing plants, six possible distribution centers and eight customer zones are considered. Each plant can produce six types of products and planning horizon is 2 months. The model described above is coded in LINGO to solve the MILP problem.

5.1. Instance Descriptions

The working hour is equal to 20 per plant for a day. The maintenance lasts for 2 hours in a working day except for plants 5 and 6 in which the maintenance time is equal to 1 hour. The change-over time is 1 hour. The utilisation parameter is assumed to be 50 hours per month. All other parameters of numerical example are given in Tables 4–11. These parameters are repeated for all planning periods.

Capacity of connection between plants and distribution centers (\(\text{cap}_{PD}\)) and capacity between distribution centers and customers (\(\text{cap}_{DD}\)) in kilogram are IT2F set according to Table 4.

The detail information of fuzzy type-II production capacities for all plants are presented in Table 5.

Table 6 presents the rate of production for each product in each plant. Table 7 presents the production cost for each product in each plant.

| Capacity | \(\text{cap}^A\) | \(\text{cap}^V\) | \(\text{cap}^A\) | \(\text{cap}^V\) |
|----------|-----------------|-----------------|-----------------|-----------------|
| \(\text{cap}_{PD}\) | 11000 | 25000 | 12000 | 26000 |
| \(\text{cap}_{DD}\) | 26000 | 28000 | 27000 | 30000 |
Table 8 presents the transportation cost for each product from plant to distribution center. Fixed establishing, shut down, and transportation costs are presented in Table 9. The procurement costs and the customer demands are presented in Tables 10 and 11, respectively.

| Table 5. Fuzzy type-II production capacity (kg/month). |
|-----------------------------------------------|
| **Product** | **Plant PL1** | **PL2** | **PL3** | **PL4** | **PL5** | **PL6** |
|-----------------|--------------|--------|--------|--------|--------|--------|
| **cap**         |              |        |        |        |        |        |
| P1              | 73400        | 12700  | 35100  | 34400  | 11000  | 10000  |
| P2              | 17616        | 3048   | 8424   | 8256   | 2640   | 2400   |
| P3              | 88080        | 15240  | 42120  | 41280  | 13200  | 12000  |
| P4              | 36700        | 6350   | 17550  | 17200  | 5500   | 5000   |
| P5              | 58720        | 10160  | 28080  | 27520  | 8800   | 8000   |
| P6              | 31562        | 5461   | 15093  | 14792  | 4730   | 4300   |
| **cap**         |              |        |        |        |        |        |
| P1              | 69618        | 12049  | 33303  | 32633  | 10376  | 9539   |
| P2              | 16708        | 2892   | 7993   | 7832   | 2490   | 2289   |
| P3              | 83541        | 14459  | 39963  | 39160  | 12451  | 11447  |
| P4              | 34809        | 6025   | 16651  | 16317  | 5188   | 4769   |
| P5              | 55694        | 9639   | 26642  | 26107  | 8301   | 7631   |
| P6              | 29260        | 5064   | 13997  | 13716  | 4361   | 4009   |
| **cap**         |              |        |        |        |        |        |
| P1              | 31511        | 5454   | 15074  | 14771  | 4696   | 4318   |
| P2              | 7563         | 1309   | 3618   | 3545   | 1127   | 1036   |
| P3              | 37813        | 6545   | 18089  | 17725  | 5636   | 5181   |
| P4              | 15756        | 2727   | 7537   | 7385   | 2348   | 2159   |
| P5              | 25209        | 4363   | 12059  | 11817  | 3757   | 3454   |
| P6              | 13550        | 2345   | 6482   | 6351   | 2019   | 1857   |
| **cap**         |              |        |        |        |        |        |
| P1              | 29965        | 5186   | 14334  | 14046  | 4466   | 4106   |
| P2              | 7192         | 1245   | 3440   | 3371   | 1072   | 985    |
| P3              | 35958        | 6223   | 17201  | 16855  | 5359   | 4927   |
| P4              | 14982        | 2593   | 7167   | 7023   | 2233   | 2053   |
| P5              | 23972        | 4149   | 11467  | 11237  | 3573   | 3285   |
| P6              | 12885        | 2230   | 6164   | 6040   | 1920   | 1765   |

| Table 6. Rate of production (kg/hour). |
|---------------------------------------|
| **Plant** | **PL1** | **PL2** | **PL3** | **PL4** | **PL5** | **PL6** |
|-----------|---------|---------|---------|---------|---------|---------|
| P1        | 3660    | 630     | 1770    | 1710    | 540     | 510     |
| P2        | 870     | 150     | 420     | 420     | 120     | 120     |
| P3        | 4410    | 750     | 2100    | 2070    | 660     | 600     |
| P4        | 1830    | 330     | 870     | 870     | 270     | 240     |
| P5        | 2940    | 510     | 1410    | 1380    | 450     | 390     |
| P6        | 1590    | 270     | 750     | 750     | 240     | 210     |

| Table 7. Production cost (money unit/kg). |
|-----------------------------------------|
| **Product** | **Plant PL1** | **PL2** | **PL3** | **PL4** | **PL5** | **PL6** |
|-------------|---------------|--------|--------|--------|--------|--------|
| P1          | 3.18          | 3.03   | 3.67   | 3.54   | 2.77   | 2.76   |
| P2          | 1.73          | 1.67   | 1.71   | 2.37   | 1.62   | 2.47   |
| P3          | 2.68          | 2.69   | 3.6    | 2.85   | 2.21   | 2.20   |
| P4          | 0.98          | 1.21   | 1.04   | 1.24   | 0.93   | 0.92   |
| P5          | 0.43          | 0.49   | 0.55   | 0.41   | 0.28   | 0.27   |
| P6          | 0.65          | 0.65   | 0.72   | 0.90   | 0.57   | 0.56   |
Table 8. Transportation cost from plant to distribution center (money unit/kg).

| Plant | Product | DC 1 | DC 2 | DC 3 | DC 4 | DC 5 | DC 6 |
|-------|---------|------|------|------|------|------|------|
| PL 1  | P1–P6   | 0.001| 0.062| 0.081| 0.053| 0.051| 0.056|
|       | P5      | 0.003| 0.071| 0.091| 0.061| 0.059| 0.051|
| PL 2  | P1–P6   | 0.065| 0.0890|0.127| 0.077| 0.071| 0.077|
|       | P5      | 0.074| 0.093| 0.139| 0.083| 0.081| 0.083|
| PL3   | P1–P6   | 0.083| 0.126| 0.001| 0.141| 0.125| 0.119|
|       | P5      | 0.091| 0.138| 0.003| 0.153| 0.132| 0.122|
| PL 4  | P1–P6   | 0.107| 0.138| 0.182| 0.001| 0.143| 0.144|
|       | P5      | 0.112| 0.152| 0.194| 0.003| 0.162| 0.164|
| PL5   | P1–P6   | 0.183| 0.236| 0.117| 0.038| 0.001| 0.038|
|       | P5      | 0.193| 0.248| 0.121| 0.052| 0.003| 0.052|
| PL 6  | P1–P6   | 0.168| 0.178| 0.236| 0.117| 0.038| 0.001|
|       | P5      | 0.182| 0.191| 0.258| 0.119| 0.052| 0.003|

Table 9. Fixed infrastructure cost and transportation cost.

| Customer | Product | DC 1 | DC 2 | DC 3 | DC 4 | DC 5 | DC 6 |
|----------|---------|------|------|------|------|------|------|
|          | Fixed establishing and shutting down cost (money unit) |      |      |      |      |      |      |
| Establish| 4300    | 2900 | 3100 | 2200 | 1300 | 1500 |
| Shoot Down| 3100    | 1500 | 1900 | 3000 | 2200 | 2000 |
|          | Transportation cost (money unit/kg) |      |      |      |      |      |      |
| C1       | P1–P6   | 0.07 | 0.068| 0.077| 0.076| 0.064| 0.064|
| C2       | P1–P6   | 0.048| 0.048| 0.048| 0.058| 0.047| 0.060|
| C3       | P1–P6   | 0.063| 0.063| 0.077| 0.065| 0.056| 0.056|
| C4       | P1–P6   | 0.037| 0.041| 0.038| 0.041| 0.036| 0.036|
| C5       | P1–P6   | 0.021| 0.022| 0.024| 0.021| 0.019| 0.019|
| C6       | P1–P6   | 0.032| 0.032| 0.033| 0.036| 0.031| 0.031|
| C7       | P1–P6   | 0.06  | 0.058| 0.067| 0.066| 0.054| 0.051|
| C8       | P1–P6   | 0.027| 0.031| 0.028| 0.031| 0.026| 0.026|

Table 10. Procurement cost per product (money unit/kg).

| Product | P 1 | P 2 | P 3 | P 4 | P 5 | P 6 |
|---------|-----|-----|-----|-----|-----|-----|
|         | 3.41| 2.23| 3.18| 1.00| 0.93| 0.80|

Incremental costs $C^{PD}$, $C^{DD}$, C and $C^P$ are set as 0.04, 0.1, 3 and 1.7, respectively.

5.2. Results

Using the parameters presented in Section 5.1, and Equations (31), (34), (36)–(43) and (71)–(85) for step 1 and Equation (44), (47), (49)–(56) and (86)–(100) for step 2, the optimal solutions are respectively $Z_{\text{max}} = 2404051$, and $Z_{\text{min}} = 914544.5$. The optimal satisfaction degree in the last step is $\lambda = 0.537$, so the optimal value of objective function is $Z^* = 1604404.669$. The graphical representation of fuzzy objective function is shown in Figure 3. Table 12 shows a detailed report of fuzzy objective function.

6. Conclusion and future research directions

This research proposed a new uncertain integrated approach to determine the optimal issues related to the design and operation of supply chains. The main decisions made in
Table 11. Fuzzy customer demand (kg).

| Customer Zone | Product | C 1    | C 2    | C 3    | C 4    | C 5    | C 6    | C 7    | C 8    |
|---------------|---------|--------|--------|--------|--------|--------|--------|--------|--------|
|                | P 1     | 32329  | 20783  | 19269  | 15231  | 7610   | 13099  | 13814  | 10315  |
|                | P 2     | 7784   | 1343   | 5400   | 4985   | 3348   | 2318   | 3859   | 2752   |
|                | P 3     | 52796  | 14427  | 22267  | 22006  | 9905   | 6669   | 19827  | 11044  |
|                | P 4     | 19736  | 2255   | 23053  | 3562   | 4250   | 5351   | 1093   | 6925   |
|                | P 5     | 34745  | 4123   | 21547  | 8698   | 7490   | 6509   | 14886  | 7962   |
|                | P 6     | 6498   | 1414   | 6873   | 25924  | 2873   | 2431   | 3801   | 7139   |

Table 12. Fuzzy objective function.

| Fuzzy Parameters | Z | ZΔ = 2417000 | ZΔ = 916362.2 | ZΔ = 2280701 | ZΔ = 866484.1 |
|------------------|---|---------------|---------------|---------------|---------------|
| DeΔ              | capPDΔ | capDDΔ | capΔZ        |               |               |
| DeΔ              | capPDΔ | capDDΔ | capΔZ        |               |               |
| DeΔ              | capPDΔ | capDDΔ | capΔZ        |               |               |
| DeΔ              | capPDΔ | capDDΔ | capΔZ        |               |               |
| DeΔ              | capPDΔ | capDDΔ | capΔZ        |               |               |
| DeΔ              | capPDΔ | capDDΔ | capΔZ        |               |               |

Figure 3. Fuzzy Type-II set of objective function.

the proposed approach concerned location–allocation, production, procurement and distribution of products based on financial aspects and production balancing among plants in multi-product, and multi-period supply chain networks. The fuzzy model considered three source of uncertainty in demand, manufacturing and supply process, concurrently. The
uncertainties in this research were modelled through type-II fuzzy sets in which a deeper insight towards vagueness and ambiguity in fuzzy membership values was supplied. The whole approach was modelled through a fuzzy mixed-integer mathematical programming. A fuzzy type-reducer methodology was proposed in order reduce fuzzy type-II uncertainties to a type-I fuzziness and to solve the yielded fuzzy type-I mathematical model through existing methods.

A numerical example was supplied to address the mechanism of proposed approach and to illustrate its applicability.

Some further research directions are proposed based on findings of this research as follows: (1) RHS of constrain were considered type-II fuzzy numbers in this research, development of a model with fuzzy type-II objective function or fuzzy type-II decision variables can demonstrate new insights in real supply chains, (2) as the location–allocation, production, procurement and distribution problems in supply chains are usually assumed to be non-deterministic poly-nominal hard (NP-Hard) problems, the application of soft computing techniques or meta-heuristics could be suitable in order to handle large-scale and real life problems, (3) the proposed method can be applied and tested in real case study in order to check its applicability in real-world problems and (4) although the proposed approach models some kind of uncertainty which have never been discussed before, some comparison with existing methods in the literature may be interesting and reveals the advantages of the proposed approach clearly.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This research has been partially supported by Department of Industrial Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran.

ORCID

Alireza Rashidi-Komijan http://orcid.org/0000-0001-7705-980X

References

[1] Christopher M. Logistics and supply chain management. 2nd ed. London: Pitman; 1998.
[2] Lambert DM, Stock JR, Ellram LM. Fundamentals of logistics management. Boston: Irwin/Mc Graw-Hill Publishing; 1998.
[3] Melo MT, Nickel S, Saldanha-da-Gama F. Facility location and supply chain management: a review. Eur J Oper Res. 2009;196(2):401–412.
[4] Peidro D, Mula J, Poler R, et al. Quantitative models for supply chain planning under uncertainty: a review. Int J Adv Manuf Tech. 2009;43(3):400–420.
[5] Peidro D, Mula J, Jimenez M, et al. A fuzzy linear programming based approach for tactical supply chain planning in an uncertainty environment. Eur J Oper Res. 2010;205(1):65–80.
[6] Mirzapour Al-e-hashem SMJ, Malekly H, Aryanezhad MB. A multi-objective robust optimization model for multi-product multi-site aggregate production planning in a supply chain under uncertainty. Int J Prod Econ. 2011;134(1):28–42.
[7] Pishvaee MS, Rabbani M, Torabi SA. A robust optimization approach to closed-loop supply chain network design under uncertainty. Appl Math Model. 2011;35(2):637–649.
[8] Guillen G, Mele FD, Bagajewicz MJ, et al. Multiobjective supply chain design under uncertainty. Chem Eng Sci. 2005;60(6):1535–1553.

[9] Santos T, Ahmed S, Goetschalckx M, et al. A stochastic programming approach for supply chain network design under uncertainty. Eur J Oper Res. 2005;167(1):96–115.

[10] Diaz-Madronero M, Peidro D, Mula J. A fuzzy optimization approach for procurement transport operational planning in an automobile supply chain. Appl Math Model. 2014;38(23):5705–5725.

[11] Gümüş AT, Güreri AF, Keles S. Supply chain network design using an integrated neuro-fuzzy and MILP approach: a comparative design study. Expert Syst Appl. 2009;36(10):12570–12577.

[12] Kahraman C, Öztaysi B. Supply chain management under fuzziness, recent developments and techniques. Heidelberg New York: Springer; 2014.

[13] Zadeh LA. The concept of a linguistic variable and its application to approximate reasoning—I. Inf Sci. 1975;8:199–249.

[14] Tsiakis P, Papageorgiou LG. Optimal production allocation and distribution supply chain networks. Int J Prod Econ. 2008;111(2):468–483.

[15] Ahmadi-Javid A, Hoseinpour P. Incorporating location, inventory and price decisions into a supply chain distribution network design problem. Comput Oper Res. 2015;56:110–119.

[16] Jang W, Kim D, Park K. Inventory allocation and shipping when demand temporarily exceeds production capacity. Eur J Oper Res. 2013;227(3):464–470.

[17] Tsao YC, Lu JC. A supply chain network design considering transportation cost discounts. Transp Res Part E Logist Trans Rev. 2012;48(2):401–414.

[18] Jonrinaldi J, Zhang DZ. An integrated production and inventory model for a whole manufacturing supply chain involving reverse logistics with finite horizon period. Omega. 2013;41(3):598–620.

[19] Shahabi M, Akbarinasaji S, Unnikrishnan A, et al. Integrated inventory control and facility location decisions in a multi-echelon supply chain network with hubs. Netw Spat Econ. 2013;13(4):497–514.

[20] Latha Shankara B, Basavarajappab S, Chenc JCH, et al. Location and allocation decisions for multi-echelon supply chain network—a multi objective evolutionary approach. Expert Syst Appl. 2013;40(2):551–562.

[21] Bandyopadhyay S, Bhattacharya R. Solving a tri-objective supply chain problem with modified NSGA-II algorithm. J Manuf Sys. 2014;33(1):41–50.

[22] Mousavi SM, Bahreininejad A, Musa SN, et al. A modified particle swarm optimization for solving the integrated location and inventory control problems in a two-echelon supply chain network. J Intell Manuf. 2014;1–16.

[23] Diabat A, Abdallah T, Henschel A. A closed-loop location-inventory problem with spare parts consideration. Comput Oper Res. 2015;54:245–256.

[24] Subulan K, Tasan AS, Baykasoglu A. Designing an environmentally conscious tire closed-loop supply chain network with multiple recovery options using interactive fuzzy goal programming. Appl Math Model. 2015;39(9):2661–2702.

[25] Jimenez M, Arenas M, Bilbao A, et al. Linear programming with fuzzy parameters: an interactive method resolution. Eur J Oper Res. 2007;177(3):1599–1609.

[26] Bilgen B. Application of fuzzy mathematical programming approach to the production allocation and distribution supply chain network problem. Expert Syst Appl. 2010;37(6):4488–4495.

[27] Paksoy T, Pehlivan NY, Ozceylan E. Application of fuzzy optimization to a supply chain network design: a case study of an edible vegetable oils manufacturer. Appl Math Model. 2012;36(6):2762–2776.

[28] Figueroa-García JC, Kalenaticb D, Lopez-Belloa CA. Multi-period mixed production planning with uncertain demands: fuzzy and interval fuzzy sets approach. Fuzzy Sets Syst. 2012;206:21–38.

[29] Mousavi SM, Niaki STA. Capacitated location allocation problem with stochastic location and fuzzy demand: a hybrid algorithm. Appl Math Model. 2013;37(7):5109–5119.

[30] Azadeh A, Raoofi Z, Zarrin M. A multi-objective fuzzy linear programming model for optimization of natural gas supply chain through a greenhouse gas reduction approach. J Nat Gas Sci Eng. 2015;26:702–710.
[31] Ramezani M, Kiamiagi AM, Karimi B, et al. Closed-loop supply chain network design under a fuzzy environment. Knowl Based Syst. 2014;59:108–120.

[32] Gholamian N, Mahdavi I, Tavakkoli-Moghaddam R, et al. Comprehensive fuzzy multi-objective multi-product multi-site aggregate production planning decisions in a supply chain under uncertainty. Appl Soft Comput. 2015;37:585–607.

[33] Alizadeh Afrouzy A, Nasser SH, Mahdavi I, et al. A fuzzy stochastic multi-objective optimization model to configure a supply chain considering new product development. Appl Math Model. 2016; Article in press.

[34] Lin CC, Wu YC. Combined pricing and supply chain operations under price-dependent stochastic demand. Appl Math Model. 2014;38(5–6):1823–1837.

[35] Marufuzzaman M, Eksioglua SD, Huang Y. Two-stage stochastic programming supply chain model for biodiesel production via wastewater treatment. Comput Oper Res. 2014;49:1–17.

[36] Panda D, Kar S, Maity K, et al. A single period inventory model with imperfect production and stochastic demand under chance and imprecise constraints. Eur J Oper Res. 2008;188:121–139.

[37] Wang D, Qin Z, Kar S. A novel single-period inventory problem with uncertain random demand and its application. Appl Math Comput. 2015;269:133–145.

[38] Bag S, Chakraborty D. An integrated inventory model of imperfect quality products with fuzzy chance constraints. J Uncertain Math Sci. 2014;2014:1–19.

[39] Jana D, Das B, Maity K. Fuzzy rough supply chain model under inflation and credit period with stock dependent consumption rate and partial backlogging shortages via genetic algorithm. Int J Comput Sci Math. 2015;6(6):555–580.

[40] Zimmerman HJ. Fuzzy programming and linear programming with several objective functions. Fuzzy Sets Syst. 1978;1(1):45–55.

[41] Herrera F, Verdegay JL. Three models of fuzzy integer linear programming. Eur J Oper Res. 1995;83(3):581–593.

[42] Liu X. Measuring the satisfaction of constraints in fuzzy linear programming. Fuzzy Sets Syst. 2001;122(2):263–275.

[43] Zhang G, Wu YH, Remias M, et al. Formulation of fuzzy linear programming problems as four-objective constrained optimization problems. Appl Math Comput. 2003;139(2–3):383–399.

[44] Wu HC. Optimality conditions for linear programming problems with fuzzy coefficients. Comput Math Appl. 2008;55(12):2807–2822.

[45] Mahdavi-Amiri N, Nasseri SH. Duality in fuzzy number linear programming by use of a certain linear ranking function. Appl Math Comput. 2006;180(1):206–216.

[46] Hatami-Marbini A, Tavana M. An extension of the linear programming method with fuzzy parameters. Int J Math Oper Res. 2011;3(1):44–55.

[47] Saati S, Hatami-Marbini A, Tavana M, et al. A two-fold linear programming model with fuzzy data. Int J Fuzzy Syst Appl. 2012;2(3):1–12.

[48] Figueroa-García JC. Solving fuzzy linear programming problems with interval type-2 RHS. Conference on Systems, Man and Cybernetics; IEEE, 2009. p. 1–6.

[49] Figueroa-García JC, Hernandez G. A note on Solving Fuzzy Linear programming problems with interval type-2 RHS. IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS); 2013.

[50] Ganesan K, Veeramani P. Fuzzy linear programming with trapezoidal fuzzy numbers. Ann Oper Res. 2006;143(1):305–315.

[51] Mahdavi-Amiri N, Nasseri SH. Duality results and a dual simplex method for linear programming problems with trapezoidal fuzzy variables. Fuzzy Sets Syst. 2007;158(17):1961–1978.

[52] Ebrahimnejad A, Nasseri SH, Lotfi FH, et al. A primal-dual method for linear programming problems with fuzzy variables. Eur J Ind Eng. 2010;4(2):189–209.

[53] Kumar A, Kaur J, Singh P. A new method for solving fully fuzzy linear programming problems. Appl Math Model. 2011;35(2):817–823.

[54] Lotfi FH, Allahviranloo T, Alimardani MA, et al. Solving a fully fuzzy linear programming using lexicography method and fuzzy approximate solution. Appl Math Model. 2009;33(7):3151–3156.