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A new derivation of singularity theorems with weakened energy hypotheses. (English)

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Summary: The original singularity theorems of Penrose and Hawking were proved for matter obeying the null energy condition or strong energy condition, respectively. Various authors have proved versions of these results under weakened hypotheses, by considering the Riccati inequality obtained from Raychaudhuri’s equation. Here, we give a different derivation that avoids the Raychaudhuri equation but instead makes use of index form methods. We show how our results improve over existing methods and how they can be applied to hypotheses inspired by quantum energy inequalities. In this last case, we make quantitative estimates of the initial conditions required for our singularity theorems to apply.

MSC:

83C75 Space-time singularities, cosmic censorship, etc.
83C40 Gravitational energy and conservation laws; groups of motions
58J20 Index theory and related fixed-point theorems on manifolds
81S07 Uncertainty relations, also entropic

Keywords:
general relativity; singularity theorems; energy conditions; quantum inequalities

Software:

DLMF

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