A comment on anti–brane singularities in warped throats

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Abstract

We compute the imaginary self–dual (ISD) and imaginary anti–self–dual (IASD) fluxes for the Klebanov–Strassler background perturbed by a stack of $p$ anti–D3 branes. We show that, at linear order in $p$, they both have a singularity in the near–brane region. While one can argue that the IASD flux may disappear at full non–linear level, no such argument exists for the ISD mode. An analogy with anti–D6 backreaction suggests that such singularity may survive once full backreaction is taken into account and may be a universal feature of anti–brane solutions.
1 Introduction

Recently, much attention has been devoted to the study of a non–supersymmetric background (first described by Kachru, Pearson and Verlinde [1]) obtained by adding $p$ anti–D3 branes to the Klebanov–Strassler solution [2]. This configuration is expected to be metastable and to describe a metastable vacuum of the dual $\mathcal{N} = 1$ supersymmetric gauge theory.

To confirm this it is important to consider the effects of the backreaction of the anti–branes and some preliminary steps were taken in [3, 4, 5]. The full first–order backreacted solution possibly describing these anti–branes was obtained in [6, 7, 8], and the parameters of this anti–brane solution were schematically described in [9] and computed in [8]. Analogous investigations have been performed in similar models in M–theory [10, 11, 12] and type IIA string theory [13].

One of the key results of the analysis of [6, 7, 8] is that the first–order backreacted anti–D3 brane solution must have a certain singularity in the infrared. While there are no arguments that this singularity might be physical, it has been suggested that it may be an artifact of perturbation theory, and may go away once full backreaction is taken into account [9, 5]. Indeed, as we will review below, the imaginary anti–self–dual (IASD) flux couples to the inverse warp factor, which is singular in perturbation theory but is expected to be regular at the non–linear level.

In this paper we will show that besides a singular IASD flux, the perturbative solution of [6, 7, 8] also contains a singular imaginary self–dual (ISD) flux for which there is no argument that it will go away in the fully non–linear solution.

This is supported by the construction of the fully backreacted solution for anti–D6 branes in a flux background [14, 15, 16], where a singularity in the $H$-flux is unavoidable. This solution can be thought as a toy model for ours: by T-dualizing it three times along the D6 worldvolume one obtains a solution for anti–D3 branes in $\mathbb{R}^3 \times T^3$, which is an increasingly better approximation of the KS near–tip region as the $S^3$ radius grows or as the number of anti–D3 branes becomes ever more smaller than the number of fractional branes.

Hence, our result suggests that at least the singular ISD flux, already visible in the linearized solution, will persist in the fully backreacted regime. It would be interesting to verify this by doing a fully backreacted calculation. The problem would then be whether this ISD singularity can be resolved, and it has been hinted [9] that this might happen via brane polarization in the near–brane region [19], much like in Polchinski–Strassler [18]. At first glance this seems unlikely, at least for smeared branes: the divergent ISD flux we find does not seem to have the appropriate leg needed for brane polarization.
2 Anti–D3 branes on the deformed conifold

We will consider the setup in which a stack of \( p \) anti–D3 branes has been introduced in the warped deformed conifold. The dynamics of these anti–branes has been studied in the probe approximation in [1]. Due to the warping and the five–form flux the anti–branes are attracted to the tip of the geometry (namely, a three–sphere \( S^3 \) of radius \( \epsilon \), related to the confinement scale of the dual gauge theory) and there they polarize (by the Myers effect [19]) into an \( NS5 \)–brane wrapping a two–sphere \( S^2 \subset S^3 \). This state will decay to a supersymmetric state, but for sufficiently small \( p/M \) (where \( M \) denotes the units of RR 3-form flux over the \( S^3 \)) the probe analysis shows that this process is non–perturbative and the leading channel is via bubble nucleation, thus providing a realization of a metastable supersymmetry–breaking state. From the gauge theory point of view, this setup would describe a dynamical supersymmetry breaking scenario in the Klebanov–Strassler theory. A description of such a state from the gauge theory side is lacking due to the strong coupling regime, thus motivating the search for a gravity dual.

We now want to describe the effects of the backreaction of the stack of anti–D3 branes on the ambient geometry. In order to make the calculation tractable, one can make two simplifying assumptions: i) smearing the sources on the \( S^3 \) of the deformed conifold and ii) working in perturbation theory around the supersymmetric background, at first–order in the parameter \( p/M \), which we suppose small in order to avoid perturbative decay to the supersymmetric vacuum.

In this approximation it is possible to find an analytic solution in terms of integrals for the metric and flux modes that perturb the Klebanov–Strassler background. This solution was obtained in [7], building on the results of [6]. The anti–D3 solution was then obtained in [8] by imposing appropriate boundary conditions on the space of linearized deformations around the warped deformed conifold. Let us briefly review the main conclusions of this analysis.

**IR behavior.** The boundary conditions that we should impose in the near–brane region are those consistent with smeared anti–branes at the tip: singular warp factor and five–form flux, coming from the anti–D3 source with equal mass and charge, and regularity in all other modes. This requirement fixes half of the sixteen integration constants of the general linearized deformation around the conifold in terms of a physical quantity: the number \( p \) of anti–D3 branes at the tip. In particular, this fixes the value of the mode (indicated by \( X_1 \) in [8]) that gives rise to the force felt by a probe D3 brane in the backreacted geometry

\[
F_{D3} = \frac{82^{2/3}\pi p}{h_0^2} j' (\tau) ,
\]

where \( j(\tau) \) is the Green’s function at the linear order, defined in (A.13). This agrees with the computation à la KKLMMT [20] and is a nice check that the boundary conditions
are the correct ones to describe anti–branes. Once all those requirements are fulfilled, one finds that remnant nonzero perturbations to three–form fluxes near $\tau = 0$ cause the energy density of such fluxes to diverge \cite{5, 6}. We stress that this is purely an infrared phenomenon, in the sense that the presence of the singularity is insensitive to the UV boundary conditions. The scope of this note is to dissect this singularity.

3 Three-form flux singularities

The Klebanov–Strassler background is parametrized by eight scalars $\phi^a(\tau)$ which take values in a scalar manifold $\mathcal{M}$ and which satisfy a first–order system of ODE’s \cite{2}. The space of linearized deformations around this solution, which preserve its $SU(2) \times SU(2) \times \mathbb{Z}_2$ symmetry, is described by a system of eight second–order Euler–Lagrange equations for the perturbation modes $\delta \phi^a(\tau)$. It is useful to use a Hamiltonian approach and recast this system in terms of sixteen first–order equations for the modes $\delta \phi^a(\tau)$ and their conjugates variables $\xi^a(\tau)$. As a consequence of perturbing around a first–order system, the equations of motion for the $\xi^a(\tau)$ modes decouple \cite{21}. These modes provide a useful description of the general solution since they parametrize supersymmetry breaking\footnote{While often stated in the literature, this is not quite true. The modes $\xi^a$ parametrize the breaking of the first–order description for the zeroth–order solution, which for the Klebanov–Strassler case is not equivalent to the conditions imposed by supersymmetry. In particular there exist solutions to the first–order system with a non–vanishing $(0,3)$–flux which break the supersymmetry \cite{23, 24}. As a result, if all the $\xi^a$ are zero, the perturbation is not necessary supersymmetric. The converse is however true, if any $\xi^a$ is different from zero, the perturbation breaks supersymmetry. In our case the only supersymmetry–breaking solution with $\xi^a = 0$ is divergent in the UV.}. We will organize the metric and flux modes as follows

$$\{\Phi_{\pm}, G_{\pm}, \phi, g_{mn}\}, \quad (3.1)$$

and we will concentrate on the modes $\Phi_{\pm}, G_{\pm}$, defined as

$$G_\pm = \ast_6 G_3 \pm i G_3, \quad \Phi_{\pm} = e^{4A} \pm \alpha, \quad (3.2)$$

where $G_\pm$ are the ISD and IASD parts of the three–form flux, $\alpha$ is the RR 4–form and $e^{-4A}$ is the warp factor (we refer to appendix A for the notations). The dynamics of these modes is described by the equation of motion \cite{22}

$$\left(d + i \frac{d\tau}{\text{Im} \, \tau} \wedge \text{Re} \right)(\Phi_- G_+ + \Phi_+ G_-) = 0. \quad (3.3)$$

In our Ansatz we have $\tau = e^{-\phi}$ since $C_0 = (\mathbb{P})$\footnote{The axion/dilaton $\tau$ in equation (3.3) should not be confused with the radial direction of the conifold.}
3.1 Linearized perturbations

We now want to compute the ISD and IASD flux once the backreaction of the anti–D3 branes is taken into account. As explained in the previous section, we linearize the problem by expanding in the parameter $\gamma = p/M$:

$$G_\pm = G_\pm^0 + G_\pm^1(\gamma) + \mathcal{O}(\gamma^2),$$

$$\Phi_\pm = \Phi_\pm^0 + \Phi_\pm^1(\gamma) + \mathcal{O}(\gamma^2).$$

For the Klebanov–Strassler background we have

$$\Phi_+^0 = \Phi_+^0 = 0,$$

while

$$\Phi_+^0 = \frac{2}{h(\tau)},$$

$$G_+^0 = (f_0 - k_0) (g_1 \wedge g_3 \wedge g_5 + g_2 \wedge g_4 \wedge g_5 + ig_1 \wedge g_3 \wedge g_6 + ig_2 \wedge g_4 \wedge g_6) + 2i(2P - F_0) g_3 \wedge g_4 \wedge g_5 + 2iF_0 g_1 \wedge g_2 \wedge g_5$$

$$+ 2e^{2\gamma_0}(2P - F_0) g_1 \wedge g_2 \wedge g_6 + 2e^{-2\gamma_0}F_0 g_3 \wedge g_4 \wedge g_6,$$

where the function $h(\tau)$ is the KS warp factor defined in (A.12), while the other KS functions are given in (A.11). At the linear order in $\gamma$, by using the expansions of [8] (to which we refer for details of the solution) we find the flux modes:

$$G_-^1 = 2e^{-4A_0} \left[ (ig_1 \wedge g_2 \wedge g_5 - e^{-2\gamma_0}g_3 \wedge g_4 \wedge g_6) \left( \tilde{\xi}_5 - \tilde{\xi}_6 \right) \right.$$

$$- \left( e^{2\gamma_0}g_1 \wedge g_2 \wedge g_6 - ig_3 \wedge g_4 \wedge g_5 \right) \left( \tilde{\xi}_5 + \tilde{\xi}_6 \right)$$

$$- \left( g_1 \wedge g_3 \wedge g_5 + g_2 \wedge g_4 \wedge g_5 - ig_1 \wedge g_3 \wedge g_6 - ig_2 \wedge g_4 \wedge g_6 \right) \tilde{\xi}_7 \right],$$

$$G_+^1 = e^{-4A_0} \left[ 2ig_3 \wedge g_4 \wedge g_5 \left( \tilde{\xi}_5 + \tilde{\xi}_6 - e^{4A_0}\tilde{\phi}_7 \right) + 2ig_1 \wedge g_2 \wedge g_5 \left( \tilde{\xi}_5 - \tilde{\xi}_6 + e^{4A_0}\tilde{\phi}_7 \right) \right.$$

$$+ 2e^{2\gamma_0}g_1 \wedge g_2 \wedge g_6 \left( \tilde{\xi}_5 + \tilde{\xi}_6 + e^{4A_0}(4P\tilde{\phi}_2 - 2F_0\tilde{\phi}_2 - \tilde{\phi}_7) \right)$$

$$+ 2e^{-2\gamma_0}g_3 \wedge g_4 \wedge g_6 \left( \tilde{\xi}_5 - \tilde{\xi}_6 - e^{4A_0}(2F_0\tilde{\phi}_2 - \tilde{\phi}_7) \right)$$

$$- \left( g_1 \wedge g_3 \wedge g_5 + g_2 \wedge g_4 \wedge g_5 + ig_1 \wedge g_3 \wedge g_6 + ig_2 \wedge g_4 \wedge g_6 \right) \cdot$$

$$\left( 2\tilde{\xi}_7 + e^{4A_0}(\tilde{\phi}_6 - \tilde{\phi}_5 + (f_0 - k_0)\tilde{\phi}_8) \right).$$
Here the modes $\tilde{\xi}^a$ and $\tilde{\phi}^a$ are respectively linear combinations of the conjugate–momenta $\xi^a$ and the perturbations modes $\delta \phi^a$ (we refer to appendix A for their definition). By using the definition (3.2), we find the expressions for the $\Phi_{\pm}$ modes at the linearized level in terms of the modes $\tilde{\xi}^a$, $\tilde{\phi}^a$

\[
\frac{d\Phi_{\pm}}{d\tau} = \pm \frac{2}{3} e^{-2x_0(\tau)}\tilde{\xi}_1,
\]

\[
\frac{d\Phi_{\pm}}{d\tau} = \pm \frac{2}{3} e^{-2x_0}\tilde{\xi}_1 + \frac{4\tilde{\phi}_4 h'(\tau) - 4h(\tau)\tilde{\phi}'_4}{h(\tau)^2}.
\] (3.12)

The first equation can be integrated by using the equation of motion for $\tilde{\xi}_5$ (B.4) and gives

\[
\Phi_{\mp} = -\frac{2}{P} \tilde{\xi}_5 + \text{const} = \frac{32}{3} X_1 j(\tau) + \text{const},
\] (3.13)

where $P = M/4$ and $j(\tau)$ is the Green’s function defined in (A.13) and $X_1$ is proportional to the number of anti–branes $p$

\[
X_1 = \frac{3\pi}{4h_0^2} p,
\] (3.14)

where $h_0 = 18.2373 P^2$. The equation for $\Phi_{\mp}$ can be easily integrated to get

\[
\Phi_{\mp} = -(\frac{32}{3} X_1 j(\tau) + \frac{4\tilde{\phi}_4}{h(\tau)}) + \text{const}.
\] (3.15)

We note that $G_{\mp}$ and $\Phi_{\mp}$ are parametrized by the modes $\tilde{\xi}^a$ only, and thus vanish if the perturbation is supersymmetric. One can check (see appendix B) that the equation of motion (3.3) is equivalent to the equations for the modes $\tilde{\xi}_{5,6,7}$. Those expressions are valid for all $\tau$ and can be evaluated by numerically integrating the explicit solution for the $\tilde{\xi}^a$ and $\tilde{\phi}^a$ modes found in [8].

### 3.2 Infrared behavior

We now discuss the behavior of the three–form flux in the near–brane region, namely at small $\tau$, and we will show that both the ISD and IASD modes are singular. The presence of a singularity in the IASD flux mode was first noticed in [5, 6]. An explanation of this behavior was given in [5, 9], where the singularity was interpreted as coming from the coupling of anti–D3 branes to the mode $\Phi_{-}$, which is singular in the linearized solution, as we will show in (3.27). We remark that the $G_{+}$ mode also presents a singularity at linearized level and discuss the possible implications of this behavior.

The infrared expansions for the modes $\tilde{\xi}^a$ and $\tilde{\phi}^a$, as well as the anti–D3 boundary conditions can be read from [8]. For the IASD flux $G_{-}$ we only need the expansions
for the scalars conjugate to the flux perturbation modes $\tilde{\xi}_{5,6,7}$ which are given in (3.16). From them and (3.9) we get

$$G_1^- = \frac{1}{\tau} \left( \frac{32}{3} \right)^{1/3} \left( \frac{2}{3} \right) P h_0 X_1 \right) (g_3 \wedge g_4 \wedge g_6 + 3i g_3 \wedge g_4 \wedge g_5 + O(\tau^0) , \quad (3.16)$$

where $X_1$ is defined in (3.14). For the ISD flux $G_+^\pm$ we also need the expansions for the modes $\tilde{\phi}^a$ which can be found in section 6.4 of [8]. The final result is

$$G_1^+ = \frac{1}{\tau} \left( \frac{32}{3} \right)^{1/3} \left( \frac{2}{3} \right) P h_0 X_1 \right) (g_3 \wedge g_4 \wedge g_6 + 3i g_3 \wedge g_4 \wedge g_5 + O(\tau^0) . \quad (3.17)$$

We note that $G_+^\pm$ shows the same kind of singularity as the $G_-$ mode. However we remark that, as we can see from (3.10), two contributions enter in (3.17): one is from the $\tilde{\xi}^a$ modes, the other is from the $\tilde{\phi}^a$ terms and both give rise to the singularity. We are now going to rederive these results in a way that will makes clear their interpretation.

Let us introduce a set of functions $\lambda(\tau)_A$ that parametrize the breaking of the ISD condition (A.9)

$$H_3 = - \sum_A \lambda(\tau)_A e^\phi \star F_3^A , \quad (3.18)$$

where the index $A$ runs over the components of the three–forms. A straightforward calculation shows that the ISD and IASD fluxes are given by

$$G_\pm = \sum_A \left[ (1 \pm \lambda(\tau)_A) \star F_3^A + i \left( \pm 1 + \lambda(\tau)_A \right) F_3^A \right] . \quad (3.19)$$

The functions $\lambda(\tau)_A$ can be obtained from the Ansatz (A.3), (A.4). By expanding at first–order in $\gamma = p/M$ around the Klebanov–Strassler solution (for which the fluxes are imaginary–self–dual), one finds the following non–vanishing components:

$$\lambda(\tau)^{345} = e^{-2y-\phi} f' \frac{2}{2P - F} = 1 + 2 e^{-4A_0} \frac{2}{2P - F_0} (\xi_5 + \xi_6) + O(\gamma^2) , \quad (3.20)$$

$$\lambda(\tau)^{125} = \frac{e^{2y-\phi} k'}{F} = 1 + 2 e^{-4A_0} \frac{2}{F_0} (\xi_5 - \xi_6) + O(\gamma^2) , \quad (3.21)$$

$$\lambda(\tau)^{136} = \lambda(\tau)^{246} = \frac{e^{-\phi}(f - k)}{2F'} = 1 + \frac{4 e^{-4A_0}}{f_0 - k_0} \xi_7 + O(\gamma^2) . \quad (3.22)$$

Recall that the legs 1 and 2 are on the shrinking $S^2$, while legs 3, 4 and 5 are on the $S^3$. While these expressions are valid for the whole conifold, we need their near–tip behavior. The infrared expansions (A.16) yields

$$\lambda(\tau)^{345} \sim \frac{1}{\tau} \left( 16 \left( \frac{2}{3} \right)^{1/3} P h_0 X_1 \right) , \quad \lambda(\tau)^{125} \sim -\frac{1}{\tau} \left( 16 \left( \frac{2}{3} \right)^{1/3} P h_0 X_1 \right) , \quad (3.23)$$
while \( \lambda(\tau)^{136} = \lambda(\tau)^{246} = \mathcal{O}(\tau) \). We thus see that only two components of \( \lambda(\tau)_A \) are relevant for the infrared physics. We can now compute \( G^1_\pm \) by expanding the expression (3.19) at first–order in \( \gamma \). We find

\[
G^1_+ = \sum_A \left[ 2 (\ast F^A_3)^1 + \lambda(\tau)^1_A (\ast F^A_3)^0 + i \lambda(\tau)^1_A (F^A_3)^0 \right], \tag{3.24}
\]

\[
G^1_- = \sum_A \left[ - \lambda(\tau)^1_A (\ast F^A_3)^0 + i \lambda(\tau)^1_A (F^A_3)^0 \right], \tag{3.25}
\]

where we indicated by the superscript 0,1 the order of the expansion in \( \gamma \). We can now analyse the near–tip behavior of the ISD and IASD fluxes, namely find the leading terms in an expansion near \( \tau = 0 \). We are interested in the origin of the singular behavior of such modes.

For the imaginary part, we see from the infrared expansions of the KS fields [A.14] that the only component that contributes to the singularity is \( F^{345}_3 \). Since \( 2P - F_0 \sim 2P - \tau^2/6 \), from \( \lambda(\tau)^{345} \) in (3.23) we recover the imaginary part of the \( G_\pm \) fluxes (3.16), (3.17). For the real part, we find that the only relevant component is \( F^{125}_3 \). We have \((\ast F^{125}_3)^0 = e^{-2y_0} F_0 g_3 \wedge g_4 \wedge g_6 \sim \frac{2}{3} P g_3 \wedge g_4 \wedge g_6 \), while \((\ast F^{125}_3)^1 = e^{-2y_0} (\phi_0^0 - 2F_0 \tilde{\phi}_2) g_3 \wedge g_4 \wedge g_6 \).

Since

\[
e^{-2y_0} (\phi_0^0 - 2F_0 \tilde{\phi}_2) = \frac{1}{\tau} \left( \frac{32}{3} \left( \frac{2}{3} \right)^{1/3} P h_0 X_1 \right) + \mathcal{O}(\tau^0), \tag{3.26}
\]

we see that the two terms in the real part of \( G_+ (3.24) \) give the same singularity as in the real part of \( G_- \), in agreement with (3.16), (3.17). Before discussing the interpretation of these results, let us show the expansions of the modes \( \Phi_\pm \) at the first order in \( \gamma \)

\[
\Phi_-^1 = -\frac{1}{\tau} \left( 16 \left( \frac{2}{3} \right)^{1/3} X_1 \right) + \frac{32 j_0 X_1}{3} - \frac{32}{15} \left( \frac{2}{3} \right)^{1/3} X_1 \tau + \mathcal{O}(\tau^2), \tag{3.27}
\]

\[
\Phi_+^1 = -\frac{32 j_0 X_1}{3} - \frac{4Y_{1IR}}{h_0^2 P^4} + \mathcal{O}(\tau^2). \tag{3.28}
\]

The singularity in \( \Phi_-^1 \) is expected since the Green’s function \( j(\tau) \) at the linearized level diverges at the tip. The regular behavior of \( \Phi_+^1 \) is one of the infrared boundary conditions that was imposed in \([8]\) and ensures that no regular D3 branes are present at the tip.

Let us summarize our findings. The ISD and IASD three–form fluxes in the linearized anti–D3 solution have a singularity of order \( \tau^{-1} \) in the infrared. Another mode in the solution has the same \( \tau^{-1} \) singularity, namely the mode \( \Phi_- \) which is coupled to the anti–D3 branes.

We can see that the singularity in \( G_-^1 \) compensates the singularity in \( \Phi_- \) in the equation of motion (3.3) \([9, 5]\). Indeed, at \( \tau \sim 0 \) we find

\[
\Phi_0^1 G_-^1 + \Phi_+^1 G_0^1 = \mathcal{O}(\tau). \tag{3.29}
\]
Based on this observation, it was argued in [9, 5] that at the non-linear level, since $\Phi_-$ will be finite at the tip, the $G_-$ singularity will disappear in the full backreacted solution. We remark however that in the linearized solution also the $G_+$ mode (3.17) have a singular behavior near $\tau = 0$, as shown in figure 1.

A similar situation was found for the full backreaction of anti–D6 branes in [16]. While this latter setup differs in many aspects from the Klebanov–Strassler background, it displays the same kind of singular behavior of our linearized solution, as we will now explain. One can perform three T–dualities along the worldvolume of the anti–D6 branes and finds that this setup will describe anti–D3 branes on $R^3 \times T^3$. If one regards the three–torus as a large radius limit of the finite $S^3$ at the tip of the Klebanov–Strassler throat, we expect that the anti–D6 solution will describe the behavior of the three–form flux $F_3$ with legs on the three–sphere. From the result of [16], we then expect that for this flux the full backreacted solution will be described by the relation

$$H = -\lambda(\tau) e^{\Phi} \ast F_3,$$

with a divergent $\lambda(\tau)$ in the near–brane region (with $\lambda(\tau) \to +\infty$). We can now compare this expectation to our result for the linearized anti–D3 solution (3.23). We see that the three–form flux with legs on the $S^3$ (i.e. the $g_3 \wedge g_4 \wedge g_5$ component) is precisely described by a relation of the form (3.30), with $\lambda(\tau) = \lambda(\tau)^{345}$. As we established in (3.24), (3.25), this analogy would point towards a divergency in the imaginary part of both the ISD and IASD fluxes at the full non–linear order. However, the leg structure of the three–form flux in our linearized anti–D3 solution is more complicated, and there is another component of the flux, $F_3^{125}$, which contributes to the singularity in $\lambda(\tau)$, making the anti–D6 analogy alone not fully conclusive.
Recently, a discussion on the interpretation of the behavior described by (3.30) appeared in [17], where it was argued that it describes an $H$–flux accumulation which will eventually lead to a critical value for which the barrier against brane/flux annihilation is destroyed. It would be interesting to confirm by a full non–linear near–brane analysis whether this picture is valid for anti–D3 branes. While in principle the IASD flux singularity may disappear in the full backreaction by the argument presented in [9,5], we still find a singular ISD flux which may survive at the non–linear order.

A possible way out, as schematically depicted in [3,9], is to argue that in the very near–tip region the solution might be altered by the polarization process in which the anti–branes form a fuzzy five–brane wrapping an $S^2 \subset S^3$. In particular, one may hope that the three–form flux singularity will be cured much as in the Polchinski–Strassler solution [18]. If the geometry is smoothed–out in this way, then the effects of the backreaction will alter only quantitatively the KPV model, by making the bound on $p/M$ for the existence of a metastable state more strong. However, at least for smeared anti–D3 branes, the singular ISD flux (3.17) does not have the correct legs needed for brane polarization, as also mentioned in [6].

4 Conclusion

We have computed the ISD and IASD fluxes for the linearized backreaction of $p$ anti–D3 branes on the Klebanov–Strassler geometry. Both these modes have an infrared singularity. While it has been suggested [9,5] that the IASD mode may be regular in the full non–linear solution, this argument does not apply to the ISD flux. In fact, by analogy with the anti–D6 backreaction [14,15,16], one can even argue that some components of the IASD flux could be singular at the non–linear level as well. It would be interesting to verify this by computing the full backreaction near the anti–D3 branes. If confirmed, one should address the question whether the interpretation of this singularity is compatible or not with the existence of a metastable state.

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A Notations

A.1 Klebanov–Strassler Ansatz

The line element in the Klebanov–Strassler Ansatz is

\[ ds_{10}^2 = e^{2A} ds_{1,3}^2 + e^{-6p-x} g_6^2 + e^{x-y} (g_1^2 + g_2^2) + e^{x+y} (g_3^2 + g_4^2) + e^{-6p-x} g_5^2, \]  

(A.1)

while the fluxes are parametrized as follows

\[ H_3 = \frac{1}{2} (k - f) (g_1 \wedge g_3 \wedge g_5 + g_2 \wedge g_4 \wedge g_5) + f' g_1 \wedge g_2 \wedge g_6 + k' g_3 \wedge g_4 \wedge g_6, \]  

(A.2)

\[ F_3 = F g_1 \wedge g_2 \wedge g_3 + (2P - F) g_4 \wedge g_5 + F' (g_1 \wedge g_3 \wedge g_6 + g_2 \wedge g_4 \wedge g_6), \]  

(A.3)

\[ F_5 = \mathcal{F}_5 + \ast \mathcal{F}_5, \quad \mathcal{F}_5 = \left( F k + f (2P - F) \right) g_1 \wedge g_2 \wedge g_3 \wedge g_4 \wedge g_5, \quad C_0 = 0, \]  

(A.4)

We assume that the functions \( \phi^a = (x, y, p, A, f, k, F, \phi) \) only depend on the radial variable \( \tau \). For the forms \( g_i \) we use the same conventions as in [2], which we reproduce here for the reader’s convenience

\[ g_1 = \frac{1}{\sqrt{2}} \left( -\sin \theta_1 d\phi_1 - \cos \psi \sin \theta_2 d\phi_2 + \sin \psi d\theta_2 \right), \]  

(A.5)

\[ g_2 = \frac{1}{\sqrt{2}} \left( d\theta_1 - \sin \psi \sin \theta_2 d\phi_2 - \cos \psi d\theta_2 \right), \]  

\[ g_3 = \frac{1}{\sqrt{2}} \left( -\sin \theta_1 d\phi_1 + \cos \psi \sin \theta_2 d\phi_2 - \sin \psi d\theta_2 \right), \]  

\[ g_4 = \frac{1}{\sqrt{2}} \left( d\theta_1 + \sin \psi \sin \theta_2 d\phi_2 + \cos \psi d\theta_2 \right), \]  

\[ g_5 = d\psi + \cos \theta_2 d\phi_2 + \cos \theta_1 d\phi_1, \]  

\[ g_6 = d\tau. \]

The ISD and IASD fluxes \( G_\pm \) and the modes \( \Phi_\pm \) are defined as

\[ G_\pm = \ast_6 G_3 \pm i G_3, \quad G_3 = F_3 + ie^{-\phi} H_3 \]  

(A.6)

\[ \Phi_\pm = e^{4A} \pm \alpha, \]  

(A.7)

where

\[ \alpha = -\int \left[ F(\tau') k(\tau') + f(\tau') (2P - F(\tau')) \right] e^{4A(\tau') - 2x(\tau')} d\tau' \]  

(A.8)

is the RR 4–form \( C_4 = \alpha dx_0 \wedge \cdots \wedge dx^3 \) and \( e^{-4A} \) is the warp factor. In the convention of [3], the ISD condition is

\[ e^\phi \ast F_3 + H_3 = 0, \]  

(A.9)

which from the definition (A.6) is equivalent to \( G_- = 0. \)

\[ ^3 \text{Our convention for } A \text{ is related to the one used in [3] as follows: } 2A_{\text{here}} = 2A_{\text{there}} + 2p - x. \]
A.2 Linearized anti–D3 solution and infrared expansions

The fields $\phi^a$ that enter in the KS Ansatz are expanded at first–order in $\gamma = p/M$ around their respective background values $\phi^a_0$

$$\phi^a = \phi^a_0 + \phi^a_1(\gamma) + \mathcal{O}(\gamma^2).$$

The Klebanov–Strassler solution $\phi^a_0$ is given by

$$e^{x_0} = \frac{1}{4} h(\tau)^{1/2} \left( \frac{1}{2} \sinh(2 \tau) - \tau \right)^{1/3},$$

$$e^{y_0} = \tanh(\tau/2),$$

$$e^{6p_0} = 24 \left( \frac{1}{2} \sinh(2 \tau) - \tau \right)^{1/3} \frac{h(\tau)}{\sinh^2 \tau},$$

$$e^{6A_0} = \frac{1}{3} \cdot 2^9 h(\tau) \left( \frac{1}{2} \sinh(2 \tau) - \tau \right)^{2/3} \sinh^2 \tau,$$

$$f_0 = -P \frac{(\tau \coth \tau - 1)}{\sinh \tau} \left( \cosh \tau - 1 \right),$$

$$k_0 = -P \frac{(\tau \coth \tau - 1)}{\sinh \tau} \left( \cosh \tau + 1 \right),$$

$$F_0 = P \frac{(\sinh \tau - \tau)}{\sinh \tau}, \quad \phi_0 = 0.$$

We define the following integrals, which correspond to the Klebanov–Strassler warp factor $h(\tau)$ and the linearized Green’s function $j(\tau)$

$$h(\tau) = 32 P^2 \int_{\tau}^{\infty} \frac{u \coth u - 1}{\sinh^2 u} (\cosh u \sinh u - u)^{1/3} du,$$

$$j(\tau) = -\int_{\tau}^{\infty} \frac{du}{(\cosh u \sinh u - u)^{2/3}}.$$  

We provide here the infrared expansions of the KS fields that are relevant for computing the near–tip behavior of the three–form fluxes

$$h(\tau) = h_0 - \frac{16}{3} \left( \frac{2}{3} \right)^{1/3} P^2 \tau^2 + \mathcal{O}(\tau^4), \quad e^{y_0} = \frac{\tau}{2} - \frac{\tau^3}{24} + \mathcal{O}(\tau^5),$$

$$f_0 = -\frac{\tau^3}{6} + \mathcal{O}(\tau^5), \quad k_0 = -\frac{2 P \tau}{3} - \frac{P \tau^3}{90} + \mathcal{O}(\tau^5),$$

$$F_0 = \frac{P \tau^2}{6} - \frac{7 P \tau^4}{360} + \mathcal{O}(\tau^6),$$

where $h_0 = h(0) \approx 18.237 P^2$. For the first–order scalars, we define a rotated basis $\tilde{\phi}^a$

$$\tilde{\phi}^a = \left( -\frac{3}{2} x + 3 p - 5 A, y, x + 3 p, -2 A, f, k, F, \phi \right).$$

12
In the Hamiltonian approach, the equations of motion for the modes $\tilde{\phi}^a$ are given in terms of “conjugate–momenta” $\tilde{\xi}^a$. We refer to [8] for the definition of such modes and for the analytic solution for the modes $\tilde{\phi}^a$. Here we provide the infrared expansions of the flux conjugate modes $\tilde{\xi}_5$, $\tilde{\xi}_6$ and $\tilde{\xi}_7$ which are needed in section 3.2. The expansions for the $\tilde{\phi}^a$ modes can be found in section 6.4 of [8].

\[ \tilde{\xi}_5 = \frac{1}{7} \left( 8 \left( \frac{2}{3} \right)^{1/3} PX_1 \right) - \frac{2}{9} (h_0 + 24 j_0) PX_1 + \frac{16}{15} \left( \frac{2}{3} \right)^{1/3} PX_1 \tau + O(\tau^3), \]

\[ \tilde{\xi}_6 = \frac{1}{7} \left( 8 \left( \frac{2}{3} \right)^{1/3} PX_1 \right) - \frac{2}{9} (h_0 + 24 j_0) PX_1 + \frac{4}{5} 2^{1/3} 3^{2/3} PX_1 \tau + O(\tau^2), \]

\[ \tilde{\xi}_7 = -\frac{2}{27} (h_0 - 40 j_0) PX_1 \tau - \frac{4}{5} 2^{1/3} 2^{2/3} PX_1 \tau^2 + O(\tau^3), \]

where $j_0 = 0.836941$.

**B Equation of motion for $G_\pm$**

We want to show that the equation of motion for the fluxes

\[ d (\Phi_- G_+ + \Phi_+ G_-) = 0 \] (B.1)

at the linearized level is equivalent to the equations of motion for the supersymmetry–breaking modes $\tilde{\xi}_5$, $\tilde{\xi}_6$ and $\tilde{\xi}_7$ of [8]. The $\Phi_\pm$ modes that we need are

\[ \Phi_- = -\frac{2}{P} \tilde{\xi}_5, \quad \Phi_+ = 2 e^{4A_0}. \] (B.2)

By using the Klebanov–Strassler flow equations to eliminate derivatives of the zeroth–order scalars, we get the following expressions in terms of the modes $\tilde{\xi}_{5,6,7}$

\[ d(\Phi_- G_+ + \Phi_+ G_-)^1 = 4i (g_1 \wedge g_2 \wedge g_3 \wedge g_4 \wedge g_5 \wedge g_6) \]

\[ \times \left[ P \tilde{\xi}_7 + (-P + F_0) \tilde{\xi}_5 + P \tilde{\xi}_6 \right] + (g_1 \wedge g_3 \wedge g_5 \wedge g_6 + g_2 \wedge g_4 \wedge g_5 \wedge g_6) \]

\[ \times \left[ \frac{2}{P} (k_0 - f_0) \tilde{\xi}_5^\prime + 4 \left( \sinh(2 y_0) \tilde{\xi}_5 + \cosh(2 y_0) \tilde{\xi}_6 + \tilde{\xi}_7^\prime \right) \right]. \]

If we substitute the derivatives of $\tilde{\xi}_a$ with their equations of motion, which are [8]

\[ \tilde{\xi}_5^\prime = -\frac{1}{3} P e^{-2x_0} \tilde{\xi}_1 \] (B.4)

\[ \tilde{\xi}_6^\prime = -\tilde{\xi}_7 - \frac{1}{3} e^{-2x_0} (P - F_0) \tilde{\xi}_1 \] (B.5)

\[ \tilde{\xi}_7^\prime = -\sinh(2 y_0) \tilde{\xi}_5 - \cosh(2 y_0) \tilde{\xi}_6 + \frac{1}{6} e^{-2x_0} (f_0 - k_0) \tilde{\xi}_1. \] (B.6)

we check that all the components of (B.3) vanish.
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