Passive positioning method and error analysis of UAV based on TDOA

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Abstract. At present, the phenomenon of “black fly” and “flooding” of drones frequently occurs, which brings various security risks to the country, society and individuals. Aiming at the passive positioning problem of UAV, this paper proposes a method based on TDOA for direct closure to solve the target position. The four-station time difference method is analyzed. The position coordinates of UAV are expressed as a function of the radial distance between the UAV and the main station. Then solve the quadratic equation of the radial distance between the UAV and the main station. Finally, obtain the position coordinates of the UAV by radial distance. According to the distribution of the solution of the quadratic equation, the unique solution zone, the fuzzy solution zone and the non-solution zone are analyzed, and the fuzzy solution is solved. The method of defuzzification is proposed in the area. The positioning error is studied, and three factors affecting the positioning error are discussed: the distance between base stations, the height of the main station and the measurement accuracy of distance difference. The research conclusion has important guiding significance for the station design.

1. Introduction

In recent years, with the continuous development and maturity of technologies in the fields of control, information and communication, the development of the Unmanned Aerial Vehicle (UAV) industry has been promoted. However, at the same time, the rapid development and wide application of UAV brings various hidden dangers, such as public safety, personal safety, personal privacy, etc. [1,2]. The number of the user of UAV is increasing. But some of them are still unconscious, and the supervision of relevant departments is not in place, which leads to the frequent occurrence of "black fly" [3] and "discarding". Even the use of UAV for criminal activities has occurred...
frequently. Therefore, for some important places and airspace, it is particularly important to use technical means to monitor and drive off drones.

The detection of the drone is mainly to obtain the location information of the drone, and provide information and intelligence support for the subsequent anti-UAV combat operations. Currently, methods commonly used for positioning include: Time Difference of Arrival (TDOA), Time of Arrival (TOA), Angle of Arrival (AOA), and Received Signal Strength Indication (RSSI), etc. [4] The TDOA positioning system, also known as the hyperbolic positioning system [5,6], was developed from the "Roland" positioning system [7,8], which locates by measuring the time difference of the radiation source signals arriving at each base station. The TDOA positioning system has the advantages of long distance and accurate positioning. Compared with the TOA positioning system, the TDOA positioning system has lower requirements for time synchronization. In general, two-dimensional positioning requires three base stations, and three-dimensional positioning requires at least four base stations. In this paper, the four-station TDOA positioning system is used to locate the UAV and its positioning error is analyzed.

2. TDOA positioning principle

As shown in Figure 1, four-station positioning system based on TDOA consists of one main station and three auxiliary stations.

Assuming the target position is \((x, y, z)\), each base station position is \((x_i, y_i, z_i)\) \((i = 0, 1, 2, 3)\), when \(i = 0\), it is the primary station, when \(i = 1, 2, 3\), it is a secondary station. The relationship between the arrival time difference parameter and the position of the UAV is

\[
\sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} - \sqrt{(x_0 - x)^2 + (y_0 - y)^2 + (z_0 - z)^2} = \Delta r_i, i = 1, 2, 3
\]

(1)

Move the item to the above formula, sort it, simplify it, and get the formula as follows

\[
(x_i - x_0)x + (y_i - y_0)y + (z_i - z_0)z = \frac{1}{2}\left(\|x_i\|^2 - \|x_0\|^2 - \Delta r_i^2\right) - r_0\Delta r_i, i = 1, 2, 3
\]

(2)

Among them,

\[
\|x_i\|^2 = x_i^2 + y_i^2 + z_i^2
\]

\[
r_0 = \sqrt{(x_0 - x)^2 + (y_0 - y)^2 + (z_0 - z)^2}
\]

\[
\Delta r_i = c\Delta t_0 = r_i - r_0
\]
Hypothesis

\[ k_i = \frac{1}{2} \left( \|v_i\|^2 - \|x_0\|^2 - c^2 t_{i0}^2 \right), i = 1, 2, 3, \]

Then equation (2) can be expressed as a form of nonlinear equations, as follows

\[ Ax = f \quad (3) \]

Among them,

\[
A = \begin{bmatrix}
(x_1 - x_0) & (y_1 - y_0) & (z_1 - z_0) \\
(x_2 - x_0) & (y_2 - y_0) & (z_2 - z_0) \\
(x_3 - x_0) & (y_3 - y_0) & (z_3 - z_0)
\end{bmatrix},
\]

\[
x = \begin{bmatrix}
x \\
y \\
z
\end{bmatrix},
\]

\[
f = \begin{bmatrix}
k_1 - r_0 \Delta r_1 \\
k_2 - r_0 \Delta r_2 \\
k_3 - r_0 \Delta r_3
\end{bmatrix}.
\]

If the distance \( r_0 \) between the primary station and the target is regarded as a known quantity, the nonlinear equation of the above equation can be degenerated into a linear non-homogeneous equation, so the appropriate base station position is selected such that \( \text{rank}(A) = 3 \), so

\[ x = A^{-1} f \quad (4) \]

If hypothesis

\[ A^{-1} = \begin{bmatrix} a_{ij} \end{bmatrix}_{3 \times 3}, \]

Then there is

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix} \begin{bmatrix}
k_1 - r_0 \Delta r_1 \\
k_2 - r_0 \Delta r_2 \\
k_3 - r_0 \Delta r_3
\end{bmatrix} = \begin{bmatrix}
m_1 - n_1 r_0 \\
m_2 - n_2 r_0 \\
m_3 - n_3 r_0
\end{bmatrix} \quad (5)
\]

Among them,

\[
m_i = a_{i1} k_1 + a_{i2} k_2 + a_{i3} k_3,
\]

\[
n_i = a_{i1} \Delta r_1 + a_{i2} \Delta r_2 + a_{i3} \Delta r_3.
\]

Substitute equation (5) into the expression of \( r_0 \) to get the formula, after simplification and integration, the formula can be obtained

\[
(\begin{array}{c}
n_1^2 + n_2^2 + n_3^2 - 1 \end{array}) r_0^2 + 2 \left[ n_1 (x_0 - m_1) + n_2 (y_0 - m_2) + n_3 (z_0 - m_3) \right] r_0 + (x_0 - m_1)^2 + (y_0 - m_2)^2 + (z_0 - m_3)^2 = 0.
\]

Hypothesis

\[
\begin{cases}
a = n_1^2 + n_2^2 + n_3^2 - 1 \\
b = 2 \left[ n_1 (x_0 - m_1) + n_2 (y_0 - m_2) + n_3 (z_0 - m_3) \right], \\
c = (x_0 - m_1)^2 + (y_0 - m_2)^2 + (z_0 - m_3)^2.
\end{cases}
\]

Then get the unary quadratic equation about \( r_0 \), as follows

\[ ar_0^2 + br_0 + c = 0 \quad (6) \]

Solving the quadratic equation can get \( r_0 \), and then bring the value of \( r_0 \) into (5) to get the position \((x, y, z)\) of the target.
3. Distribution of solutions

3.1. Analysis of the situation of the solution

Solving the quadratic equation \( ar_0^2 + br_0 + c = 0 \) about \( r_0 \). According to the characteristics of the quadratic equation, there are three cases [9, 10].

(1) When \( \Delta = b^2 - 4ac > 0 \), the equation has two different solutions, and there may be a problem of positioning ambiguity. If the two solutions are positive, the positioning is fuzzy, and other means are needed to eliminate the positioning blur; if the two solutions are positive and negative, the negative solution is discarded, and the positive solution is brought into (5) to calculate the target position; if the two solutions are negative, the positioning has no solution.

(2) When \( \Delta = b^2 - 4ac = 0 \), the equation has two identical solutions, namely the unique solution, and there is no problem of positioning ambiguity. If the solution is a positive number, it can be located without blurring; if the solution is negative, the positioning has no solution.

(3) When \( \Delta = b^2 - 4ac < 0 \), the equation has no solution and the positioning has no solution.

Simulate the distribution of the solution, and set the base station coordinates to: main station \((0, 0, z_0)\), auxiliary station \((0, R, 0)\), \((-\sqrt{3}/2R, -R/2, 0)\), \((\sqrt{3}/2R, -R/2, 0)\). The array is a circular array, the station spacing is \( R = 500m \), and the height of the main station is \( z_0 = 1m \). Assume that the UAV has a flying height of 200m.

The simulation area is set to \( x = (-5000m, 5000m) \), \( y = (-5000m, 5000m) \) and \( x = (-50000m, 50000m) \), \( y = (-50000m, 50000m) \), the fuzzy solution is marked as black, the unique solution is marked as green, and the unsolved mark is red. The result is shown in Figure 2.

![Figure 2. Distribution of the solution.](image)

This paper only considers the range of 5000m according to the actual situation, as shown in Figure 2(a). All the points in the figure are black points, indicating that all are fuzzy solutions, and the fuzzy area occupies the entire area. Therefore, the positioning of the UAV needs to solve the fuzzy problem.

3.2. Analysis of Fuzzy solution

The fuzzy solution means there are two different position coordinates, so for the above fuzzy solution region, the two coordinates solved are plotted, as shown in Figure 3.

![Figure 3. Distribution of two coordinates.](image)
As shown in Fig. 3, two solutions of the fuzzy solution case, one of which is consistent with the preset UAV coordinates. It is the real solution coordinate. The other is the fuzzy solution coordinate. It is below the ground. Since the drone is flying in the air, the coordinates under the ground are not in line with reality. The final solution can be considered to have a unique solution.

The master station height $z_0$ and the station spacing $R$ are changed, and different simulation conditions are set. The results are as shown in Figure 4.

![Figure 4](image-url)

**Figure 4.** Distribution of two coordinates of different $R$ and $z_0$.

Taking Fig. 4(c) as an example, the difference between the real solution coordinate and the fuzzy solution coordinate of the partial region is large, and the target position can be determined by the method of geographic location defuzzification. In the other regions, the difference between the real solution coordinate and the fuzzy solution coordinate is reduced. So the defuzzification method proposed in this paper is not applicable. But other methods can be used to solve the blur. Analyze and compare the graph in Figure 4. When the height of the main station is constant, the larger the station spacing is, the larger the area that can be deblurred is. When the station spacing is constant, the smaller the height of the main station is, the larger the area that can be deblurred is.

### 4. Analysis of positioning error

The base station is set to a circular array with coordinates: main station $(0,0,z_0)$, auxiliary stations $(0,R,0)$, $(-\sqrt{3}R/2,-R/2,0)$, $(\sqrt{3}R/2,-R)/2,0)$. Where, $R$ represents the station spacing, $z_0$ represents the height of the main station. And the flying height of the UAV is 200m.
4.1. Influence of station spacing

Assume that the height of the main station is \( z_0 = 1 \text{m} \), the measurement accuracy of the distance difference is \( \sigma_d = 10 \text{m} \), change the station spacing \( R \), and the detection range is \( D = 3000 \text{m} \), that is, the simulation area is set to \( x = (-3000 \text{m}, 3000 \text{m}) \), \( y = (-3000 \text{m}, 3000 \text{m}) \). The z-axis represents the positioning error. The simulation result is shown in Figure 5.

Figure 5. Effect of station spacing on positioning error.

As shown in Figure 5, the following conclusions can be obtained.

(1) In the X, Y, and Z directions, the image shows a phenomenon of depression in the middle, indicating that the closer the UAV is to the base station, the smaller the positioning error is; the farther away from the base station, the larger the positioning error is.

(2) The positioning errors in the X, Y, and Z directions are compared respectively. When the height of the master station and the measurement accuracy of the distance difference is constant, the positioning error of the UAV becomes smaller as the station spacing \( R \) increases.

Change the detection range, and the results of the positioning error are shown in Table 1.

As shown in Table 1, the following conclusions can be obtained.

(1) The positioning error in the X direction when the station spacing \( R \) is 3000 m is taken as an example. When the detection range \( D \) is 1000 m, 3000 m, 5000 m, the positioning errors are 10.24 m, 52.76 m, and 123.55 m, respectively. The farther away from the base station, the larger the positioning error for the UAV is.

(2) The positioning error in the X direction when the detection range \( D \) is 3000 m is taken as an example. When the station spacing \( R \) is 1000 m, 3000 m, and 5000 m, the positioning errors are 326.31 m,
52.76m and 12.86m, respectively. It indicates that the larger the station spacing is, the smaller the positioning error for the UAV is.

| Table 1 | Effect of station spacing on positioning error. |
|----------|-----------------------------------------------|
|          | D=1000m                                      |
|          | X:47.51m                                     |
|          | Y:39.32m                                     |
| R=1000m  | Z:96.60m                                     |
|          | X:10.24m                                     |
| R=3000m  | Y:9.92m                                      |
|          | Z:79.30m                                     |
| R=5000m  | Y:9.75m                                      |
|          | Z:72.57m                                     |

4.2. Influence of the height of the main station

Assume that the station spacing is R=1000m, the measurement accuracy of the distance difference is $\sigma_d=10$m, change the height of the main station $z_0$, and the detection range is D=3000m, that is, the simulation area is set to $x=(-3000m, 3000m)$, $y=(-3000m, 3000m)$. The $z$-axis represents the positioning error. The simulation result is shown in Figure 6.

Figure 6. Effect of the height of the main station on the positioning error.
As shown in Figure 6, the following conclusions can be obtained.

(1) In the X, Y, and Z directions, the image shows a phenomenon of depression in the middle, indicating that the closer the UAV is to the base station, the smaller the positioning error is; the farther away from the base station, the larger the positioning error is.

(2) The positioning errors in the X, Y, and Z directions are compared respectively. When the station spacing and the measurement accuracy of distance difference is constant, as the height of the main station increases, the positioning error of the UAV becomes larger and larger.

Change the detection range, and the results of the positioning error are shown in Table 2.

| D=1000m | D=3000m | D=5000m |
|---------|---------|---------|
| z₀=1m   |         |         |
| X:47.51m | X:326.31m | X:963.45m |
| Y:39.32m | Y:299.44m | Y:907.80m |
| Z:96.60m | Z:255.07m | Z:467.94m |
| z₀=30m  |         |         |
| X:52.42m | X:371.68m | X:1208.41m |
| Y:42.99m | Y:357.02m | Y:1242.34m |
| Z:99.86m | Z:303.77m | Z:608.04m |
| z₀=50m  |         |         |
| X:54.83m | X:401.60m | X:2490.72m |
| Y:45.30m | Y:463.49m | Y:2511.10m |
| Z:100.25m | Z:333.78m | Z:1073.91m |

As shown in Table 2, the following conclusions can be obtained.

(1) The positioning error in the Y direction when the height of the main station is z₀=1m is taken as an example. When the detection range D is 1000m, 3000m, 5000m, the positioning errors is 39.32m, 299.44m, and 907.80m, respectively. The farther away from the base station, the larger the positioning error for the UAV is.

(2) The positioning error in the Y direction when the detection range is D=3000m is taken as an example. When the height z₀ of the main station is 1m, 30m and 50m, the positioning errors is 299.44m, 357.02m and 463.49m,respectively. The higher the height of the main station is, the larger the positioning error for the UAV is.

4.3. Influence of the measurement accuracy of distance difference

Assume that the station spacing is R=1000m, the height of the main station is z₀=1m, change the measurement accuracy of the distance difference \( \sigma_d \), and the detection range is D=3000m, that is, the simulation area is set to \( x=(-3000m, 3000m), \ y=(-3000m, 3000m) \). And the z-axis represents the positioning error. The simulation result is shown in Figure 7.

As shown in Figure 7, the following conclusions can be obtained.

(1) In the X, Y, and Z directions, the image shows a phenomenon of depression in the middle, indicating that the closer the UAV is to the base station, the smaller the positioning error is; the farther away from the base station, the larger the positioning error is.

(2) The positioning errors in the X, Y, and Z directions are compared respectively. When the station spacing and the height of the main station is constant, as the measurement accuracy of distance difference increases, the positioning error of the UAV becomes larger and larger.
(a) X direction, $\sigma_d = 5m$  
(c) X direction, $\sigma_d = 15m$  
(d) Y direction, $\sigma_d = 5m$  
(e) Y direction, $\sigma_d = 10m$  
(f) Y direction, $\sigma_d = 15m$  
(g) Z direction, $\sigma_d = 5m$  
(h) Z direction, $\sigma_d = 10m$  
(i) Z direction, $\sigma_d = 15m$

Figure 7. Effect of the measurement accuracy of distance difference on positioning error. Change the detection range, and the results of the positioning error are shown in Table 3.

Table 3. Effect of the measurement accuracy of distance difference on positioning error.

| $\sigma_d$ | D=1000m | D=3000m | D=5000m |
|-----------|---------|---------|---------|
| 5         | X:24.62m Y:20.59m Z:69.86m | X:166.66m Y:150.21m Z:167.44m | X:416.83m Y:393.99m Z:284.68m |
| 10        | X:69.86m Y:39.32m Z:96.60m | X:326.31m Y:299.44m Z:255.07m | X:963.45m Y:907.80m Z:467.94m |
| 15        | X:119.74m Y:62.15m Z:119.74m | X:509.90m Y:495.90m Z:343.38m | X:1593.41m Y:1584.02m Z:787.39m |

As shown in Table 3, the following conclusions can be obtained.

(1) The positioning error in the Z direction when the distance difference measurement accuracy $\sigma_d$ =5m is taken as an example. When the detection range D is 1000m, 3000m, 5000m, the positioning errors are 69.86m, 167.44m, and 284.68m, respectively. The farther away from the base station, the larger the positioning error for the UAV is.

(2) The positioning error in the Z direction when the detection range is D=3000m is taken as an example. When the measurement accuracy of distance difference $\sigma_d$ is 1m, 30m, 50m, the positioning
errors is 167.44m, 255.07m, 343.38m, respectively. It shows that the greater the measurement accuracy of the distance difference is, the larger the positioning error for the UAV is.

5. Conclusion

This paper proposes a four-station three-dimensional time difference positioning method for the problems of UAV "Black Flying" and "Flying Flying". The distribution of the solution of the circular array station method is studied. The reasons of the unique solution, the fuzzy solution and the no solution are analyzed, and the solution of the fuzzy problem is given. After analyzing the three factors affecting the positioning error, such as station spacing, the height of main station and the measurement accuracy of distance difference, it is concluded that increasing the station spacing, reducing the height of main station, and reducing the measurement accuracy of distance difference can reduce the positioning error. At the same time, it is also found that the farther away from the base station, the larger the positioning error is when other conditions are constant. The conclusions obtained in this paper have important guiding significance for the design of the station.

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