COLLISIONS OF SMALL NUCLEI IN THE THERMAL MODEL*

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An analysis is presented of the expectations of the thermal model for particle production in collisions of small nuclei. The maxima observed in particle ratios of strange particles to pions as a function of beam energy in heavy-ion collisions are reduced when considering smaller nuclei. Of particular interest is that the $\Lambda/\pi^+$ ratio shows the strongest maximum which survives even in collisions of small nuclei.

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1. Introduction

A large effort is presently under way to study not only heavy- but also light-ion collisions. This is being motivated by the results obtained in heavy-ion collisions such as Pb–Pb and Au–Au, for the $K^+/\pi^+$, and also other particle ratios. It has been conjectured that these indicate a phase change in nuclear matter [1].

A consistent description of particle production in heavy-ion collisions, up to the LHC energies, has emerged during the past two decades using a thermal-statistical model (referred to simply as a thermal model in the remainder of this paper). It is based on the creation and subsequent decay of hadronic resonances produced in chemical equilibrium at the unique temperature and baryon chemical potential. According to this picture, the bulk of hadronic resonances made up of the light flavors $u$, $d$ and $s$ valence quarks are produced in chemical equilibrium.

Indeed, some particle ratios exhibit very interesting features when studied as a function of the beam energy which deserve attention: (i) a maximum in the $K^+/\pi^+$ ratio, (ii) a maximum in the $\Lambda/\pi$ ratio, (iii) no maximum in the $K^-/\pi^-$ ratio. The maxima occur at a center-of-mass energy of around 10 GeV [2–4]. It is interesting to note that the occurrence of these maxima happens in an energy regime where a maximum baryon density occurs [5] and a transition from baryon-dominated freeze out to a meson-dominated one takes place [4]. An alternative interpretation is that these maxima reflect a phase change [1] to deconfined state of matter.

The maxima mark a distinction between heavy-ion collisions and $p$–$p$ collisions as they are not observed in the latter. This shows a clear difference between the two systems which is worthy of further investigation.

It is the purpose of the present contribution to report on an analysis [6] studying the transition from a small system like a $p$–$p$ collision to a large system like a Pb–Pb or Au–Au collision and to follow explicitly the genesis of the maxima in certain particle ratios.

2. The model

A relativistic heavy-ion collision will go through several stages. At one of the later stages, the system will be dominated by hadronic resonances. The identifying feature of the thermal model is that all the resonances as listed by the Particle Data Group [7] are assumed to be in thermal and chemical equilibrium. This assumption drastically reduces the number of free parameters and thus this stage is determined by just a few thermodynamic variables namely, the chemical freeze-out temperature $T$, the various chemical potentials $\mu$ determined by the conserved quantum numbers and
by the volume $V$ of the system. The latter plays no role when considering ratios of yields. It has been shown that this description is also the correct one [8–11] for a scaling expansion as first discussed by Bjorken [12].

In general, if the number of particles carrying quantum numbers related to a conservation law is small, then the grand-canonical description no longer holds. In such a case, conservation of quantum numbers has to be implemented exactly in the canonical ensemble [13,14]. In the case considered here, there are two volume parameters: the overall volume of the system $V$, which determines the particle yields at fixed density and the strangeness correlation (cluster) volume $V_c$, which reflects the canonical suppression factor and reduces the densities of strange particles. Assuming spherical geometry, the volume $V_c$ is parameterized by the radius $R_C$ which serves as a free parameter and defines the range of local strangeness equilibrium.

### 3. Origin of the maxima

According to the thermal model, the baryon chemical potential decreases continuously with increasing beam energy. At the same time, the temperature increases rather quickly until it reaches a plateau. Following the rapid rise of the temperature at low beam energies, the $\Lambda/\pi^+$ and $K^+/\pi^+$ also increase rapidly. This halts when the temperature reaches its limiting value. However, simultaneously, the baryon chemical potential keeps on decreasing. Consequently, the $\Lambda/\pi$ and $K^+/\pi^+$ ratios follow this decrease due to the strangeness conservation as $K^+$ is produced in associated production together with a $\Lambda$. The two effects combined lead to maxima in both cases. For very high energies, the baryo-chemical potential no longer plays a role ($\mu_B \approx 0$) and the temperature is constant hence, these ratios hardly vary [4].

![Fig. 1. Values of the $K^+/\pi^+$ (left panel) and the $\Lambda/\pi^+$ (right panel) ratios in the $T$–$\mu_B$ plane. Lines of constant values are indicated. The dash-dotted line is the freeze-out curve obtained in [3], while the dashed line uses the parameterization given in [18]. Note that the maxima do not occur in the same position.](image-url)
To show this in more detail, we present as an example in Fig. 1 lines where the $K^+ / \pi^+$ and the $\Lambda / \pi^+$ ratios remain constant in the $T-\mu_B$ plane. It should be noted that the maxima of these ratios do not occur in the same position, which remains to be confirmed experimentally. It is also worth noting that the maxima are not on but slightly above the freeze-out curve.

4. Particle ratios for small systems

To consider the case of the collisions of smaller nuclei we have to take into account the strangeness suppression according to the canonical model, i.e. the concept of strangeness correlation in clusters of a sub-volume $V_c \leq V$ [15–17].

A particle with strangeness quantum number $s$ can appear anywhere in the volume $V$ but it has to be accompanied by another particle carrying strangeness $-s$ to conserve strangeness in the correlation volume $V_c$. Assuming spherical geometry, the volume $V_c$ is parameterized by the radius $R_c$ which is a free parameter that defines the range of local strangeness equilibrium.

In the following, we show the trends of various particle ratios as a function of $\sqrt{s_{NN}}$. The dependence of $T$ and $\mu_B$ on the beam energy is taken from heavy-ion collisions [3]. For $p-p$ collisions, slightly different parameters are more suited [19]. Therefore, the calculations shown give the general trend. We have ignored the variations of other parameters with system size.

We focus on the system-size dependence of the thermal parameters with particular emphasis on the change in the strangeness correlation radius $R_c$. The parameters $R = 10$ fm (which is the value for central Pb–Pb collisions) and $\gamma_S = 1$ are kept fixed. The freeze-out values of $T$ and $\mu_B$ will vary with the system size [17], however this has not been taken into account in the present work which aims to give a qualitative description of the effect.

The smaller system size is described by decreasing the value of the correlation radius $R_c$. This ensures that strangeness conservation is exact in $R_c$, and that strangeness production is suppressed with decreasing $R_c$.

In Fig. 2, we show the energy and system size dependence of two particle ratios calculated along the chemical freeze-out line. In Fig. 2, a maximum is seen in the $K^+ / \pi^+$ ratio which gradually disappears when the correlation radius decreases. A different effect is seen in $\Lambda / \pi^\pm$ ratio. Here, the gradual decrease of the maximum is also seen but, contrarily to the $K^+ / \pi^+$ ratio, it remains quite prominent even for a small correlation radius. Also, the maximum shifts, for smaller systems, towards higher $\sqrt{s_{NN}}$. For $pp$ collisions which correspond to an $R_C$ of about 1.5 fm [17], they will hardly be observed. It should also be noted that in the thermal model, the maxima happen at different beam energies.
Fig. 2. Values of the $K^+ / \pi^+$ (left panel) and the $\Lambda / \pi^+$ (right panel) ratios as a function of invariant beam energy for various strangeness correlation radii $R_c$, calculated using the thermal model [20]. The correlation radius is varied from 3.0 (top curve) to 2.5, 2.0, 1.5 and finally 1.0 fm (bottom curve). Note that the $\Lambda / \pi^+$ ratio is the ratio where the maximum stays most pronounced as the system size is reduced.

It must be emphasized that the results presented here are of a qualitative nature. In particular, there could be changes due to variations with the system size of the temperature and the baryon chemical potential. In addition, the strangeness equilibration volume $V_c$ could be energy-dependent and system-size-dependent.

5. Conclusions

The thermal model qualitatively describes the presence of maxima in the $K^+ / \pi^+$ and the $\Lambda / \pi^\pm$ ratios at a beam energy of $\sqrt{s_{NN}} \approx 10$ GeV. In this paper, we have described what could possibly happen with different strange particles and pion yields in collisions of smaller systems due to constraints imposed by exact strangeness conservation. In particular, the $\Lambda / \pi^+$ ratio still shows a clear maximum even for small systems. The pattern of these maxima is also quite special as they are not always at the same beam energy.

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