Pseudoscalar (Charged) Higgs Boson Production with 
\( Z^0 \) ( \( W^\pm \) )-Boson at Muon Colliders in the Models 
with Several Higgs Doublets and Singlets

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Abstract

Pseudoscalar and charged Higgs bosons production in the processes \( \mu^+ \mu^- \rightarrow P_0 Z^0 \), \( \mu^+ \mu^- \rightarrow H^\pm W^\mp \) within the models with several Higgs doublets and singlets is considered.
1. Introduction

As known at muon colliders Higgs bosons single production in resonance is possible:

\[ \mu^+\mu^- \rightarrow H^0(P^0) \]  

with large cross section. However, mass of Higgs bosons is not fixed in the theory and, thus, we don’t know which energies are necessary for Higgs bosons production in resonance.

In this article we continue consideration of pseudoscalar and charged Higgs bosons production in association with gauge bosons in models with Higgs sector with several Higgs doublets and singlets (an examples of such models see e.g. in the context of Minimal Supersymmetric Standard Model (MSSM) and also in the context of general two Higgs doublet model and also for supersymmetric models containing two doublets and one singlet of Higgs bosons).

Previously in has been considered Higgs bosons production in association with photon at muon colliders:

\[ \mu^+\mu^- \rightarrow H^0(P^0)\gamma \]  

by model independent way and also has been obtained standard Higgs bosons case and enhancement in supersymmetric Higgs bosons case in comparison with standard Higgs bosons production in reaction (2) The process (2) (for standard and supersymmetric Higgs bosons production in association with photons in $e^+e^-$-collisions see [34], [35], [36] and references therein, scalar Higgs bosons production in association with Z-bosons (ZZH) couplings has been considered in [3], [4] (see also references therein).
standard Higgs bosons has been calculated also in [8] [9] however authors of this papers do not consider enhancement in theories with extended Higgs sector and besides authors of [8] do not consider loop effects which as was shown in [4] and [5] are very important. It must be noted also that our analytical calculations are essentially differ from calculations of [8].

In [12] has been considered supersymmetric charged and pseudoscalar Higgs bosons production at muon colliders in association with $W^\pm$, $Z^0$-bosons:

\begin{align}
\mu^+\mu^- & \rightarrow H_3^0Z^0, \\
\mu^+\mu^- & \rightarrow H^\pm W^\mp
\end{align}

This processes are similar to the processes calculated in theories with $R$-parity violation [18]-[24]:

\begin{align}
l_i^+l_j^- & \rightarrow \tilde{\nu}_{kL}Z^0, \\
l_i^+l_j^- & \rightarrow \tilde{l}_{kL}^\pm W^\pm.
\end{align}

also considered in [12]

In this article we consider by model independent way pseudoscalar (charged) Higgs bosons production at lepton antilepton colliders in association with $Z^0(W^\pm)$ bosons in model which contain many pseudoscalars and many scalars (such large number of scalars and pseudoscalars appear e.g. in models which

\textsuperscript{2}Previously the processes $e^+e^- \rightarrow \tilde{\nu}_{kL}Z^0, \tilde{l}_{kL}^\pm W^\pm$ have been discussed in [23]. However, our results [12] are essentially differ from results of the [23]. For instance, formulas for cross section of the process $e^+e^- \rightarrow \tilde{\nu}_{kL}Z^0$ in [23] do not contain $a_{L,R}$ in contrast to our result. Feynmann diagrams corresponding to the process $e^+e^- \rightarrow \tilde{\nu}_{kL}\gamma$ has been also discussed in [23] however its cross section has not been calculated. Case of different flavors of colliding leptons in this paper also has not been considered.
Higgs sector consist of several doublets + several singlets. Part of the
scalars may have masses so that resonances in $S$-channel diagram is possi-
bile (in contrast to MSSM). I.e. condition $m_j > m_P + m_Z$ for some masses of
scalars $m_j$ are possible.

From the Fig.2 of the paper we see that after taking into account
radiative corrections (for radiative corrections see references in [3]) situation
$m_P + m_Z < m_H$ within the MSSM is possible.

Due to this possibility as was shown in this article resonant production (1)
$\mu^+\mu^- \rightarrow H_1^0$ with subsequent decay $H_1^0 \rightarrow Z^0 + H_3^0$ is possible ($H_1^0$ is heaviest
scalar Higgs boson in the MSSM, $H_3^0$ is pseudoscalar Higgs boson in the MSSM).
This situation take place at small $\tan\beta \sim 1$ and not heavy pseudoscalar
($m_P < 50–100$ GeV depend on scalar quarks masses and $\tan\beta$). For example
at $\tan\beta \approx 1$, stop mass 1 TeV possible $m_{H_3} = 100$ GeV $m_{H_1} = 200$ GeV at
stop mass 0.5 TeV $m_{H_3} < 50$ GeV $m_{H_1} = 140$ GeV). I.e. studied situation
is possible even in MSSM.

Situation which considered in this paper is similar to the consideration
of the analogous process $l_i^+l_j^- \rightarrow \tilde{l}_{kL}^\pm W^\pm$ in which this phenomena (reso-
nance enhancement via intermediate sneutrino exchange even far from reso-
nance, scalar neutrino is scalar + pseudoscalar with same masses) also take
place [12]. There are many parallels between scalar neutrinos (scalars lepto-
s) production in theories with R-parity violation and neutral (charged) Higgs
bosons.

\footnote{for sneutrino resonant production see [13]–[17]}
Also related phenomena has been found in [26] where in the framework of Two Higgs -doublet Model has been considered enhancement of the process (4) ( in comparison with previous consideration of the process (4) in the framework of MSSM in [12] ) which appear due to resonance of intermediate scalar and pseudoscalar Higgs bosons on S -channel diagramm.

2. Results for $\mu^+\mu^- \rightarrow P_i^0 Z^0$

In our model independent calculations we suppose that the couplings of the scalar and pseudoscalar Higgs bosons couplings to leptons are following:

$$\mathcal{L} = i h_j \bar{l}l + h_P \bar{l} \gamma_5 l P^0$$ (7)

Interaction $Z^0 H_j^0 P^0$ is following:

$$h_{jZ}(k_1 \times Z)$$ (8)

Our results for differential cross section of the process $\mu^+\mu^- \rightarrow P_i^0 Z^0$, are following:

$$\frac{d\sigma(\mu^+\mu^- \rightarrow P_i^0 Z^0)}{dt} = \frac{\alpha}{8\sin^2 \theta_W \cos^2 \theta_W s^2} (B_1 + B_2 + B_3),$$ (9)

where:

$$B_1 = \frac{s}{4m_Z^2} |b|^2 - m_{P,s}^2 \sum_j \frac{h_j h_{jZ}}{s - m_j^2 + i\Gamma_j m_j}^2,$$ (10)

$$B_2 = \frac{sm_P^2}{2}(\frac{1}{t} + \frac{1}{u})h_P \sum_j Re(\frac{h_j h_{jZ}}{s - m_j^2 + i\Gamma_j m_j}),$$ (11)

$$B_3 = h_P^2 [(a_L^2 + a_R^2)(\frac{1}{t^2} + \frac{1}{u^2})(tu - m_P^2 m_Z^2) + \frac{4a_L a_R (t - m_P^2)(u - m_P^2)}{tu}],$$ (12)
\[ b = h_P + (s - m_P^2 - m_Z^2) \sum_j \frac{h_j h_{jZ}}{s - m_j^2 + i \Gamma_j m_j} \]  

(13)

Requirement of unitarity lead to the condition:

\[ 0 = h_P + \sum_j h_j h_{jZ} \]  

(14)

\[ t_- < t < t_+ , \]  

(15)

where

\[ t_{\pm} = \frac{m_P^2 + m_Z^2 - s \pm \sqrt{(m_P^2 + m_Z^2 - s)^2 - 4m_P^2 m_Z^2}}{2}. \]  

(16)

After performing integration within the limits (13), (14) we obtain for the total cross sections the following result:

\[ \sigma(\mu^+ \mu^- \rightarrow P_i^0 Z^0) = \frac{\alpha}{4 \sin^2 \theta_W \cos^2 \theta_W s^2} (A_1 \log \frac{t_+}{t_-} + A_2 (t_+ - t_-)), \]  

(17)

where:

\begin{align*}
A_1 &= h_P^2 (a_L^2 + a_R^2) (m_P^2 + m_Z^2 - s) + 2a_L a_R \frac{m_P^2 (s - m_Z^2)}{m_P^2 + m_Z^2 - s} + \frac{sm_P^2}{2} \sum_j h_P h_j h_{jZ} \frac{(s - m_j^2)}{(s - m_j^2)^2 + \Gamma_j^2 m_j^2}, \\
A_2 &= \frac{s}{8m_Z^2} |b|^2 - \frac{sm_P^2}{2} \sum_j \frac{h_j h_{jZ}}{(s - m_j^2)^2 + i \Gamma_j m_j} |^2 + 2h_P^2 (a_L a_R - (a_L^2 + a_R^2)),
\end{align*}  

(18)

In the vicinity of resonance of the one of the scalar on the s-channel diagram \((s \approx m_j^2)\) we obtain Breit Wigner cross section (as in [3]) :

\[ \sigma(\mu^+ \mu^- \rightarrow P_i^0 Z^0) \approx \frac{4 \pi \Gamma(H_j^0 \rightarrow \mu^+ \mu^-) \Gamma(H_j^0 \rightarrow Z^0 P^0)}{(s - m_j^2)^2 + \Gamma_j^2 m_j^2} \]  

(20)

as it must be.
In the limit \( s, m_j^2 \gg m_P^2 \) \((m_j \text{-is arbitrary})\) the previous formulas are reduced and we have:

\[
\sigma(\mu^+\mu^- \to P_i^0 Z^0) = \frac{\alpha}{4 \sin^2 \theta_W \cos^2 \theta_W s} (2 h_P^2 (a_L^2 + a_R^2) s \log(s/m_p m_Z) + A_2),
\]

(21)

In the MSSM at large \( \tan\beta \) it is necessary to put \( h_P = \frac{g m_t \tan\beta}{\sqrt{2} m_W} \) and besides mass of scalar is equal to the mass of pseudoscalar and we obtain formula (25) of [5] in the limit \( s \gg m_j^2, m_P^2 \).

We will present numerical results for specific models in our next article (for example for two Higgs doublet model). However it is obviously that even off resonance the cross section may be essentially enhanced. The situation is very similar to the [5] (see Fig.4) where due to resonance of scalar neutrino exchange the process (2,3) essentially enhanced even far from resonance. Analogous enhancement in the process (4) has been obtained in [26] in the framework of two HDM model (see Fig.4 in this paper).

3. Results for \( \mu^+\mu^- \to H^\pm W^\mp \)

In our model independent calculations we suppose that the couplings of the charged Higgs bosons to leptons are following:

\[
\mathcal{L} = h\bar{l}l P \nu H^- + h.c. + \sum_j h_j \bar{l}l + \sum_j h_{Pj} \bar{l}\gamma_5 l
\]

(22)

Interactions \( W^+H_j^0 H^- \) and \( W^+P_j^0 H^- \) are following:

\[
H_{jW} W^+H_j^0 H^-, P_{jW} W^+H_j^0 H^-\]

(23)
Amplitude of the process (3) may be written as:

\[
M = \frac{g}{\sqrt{2}} \bar{u}(k_1) \left[ (h_{k4W}^2 + (k_4W)a_+(j)H_{jW}h_j) P_L + \left( (k_4W) \sum_j a_-(j)H_{jW}h_j \right) P_R \right] u(k_2).
\]

(24)

where:

\[
a_\pm(j) = \frac{1}{s - m_j^2 + i\Gamma_j m_j} \pm \frac{1}{s - M_j^2 + i\Gamma'_j M_j}
\]

(25)

where \(\Gamma_j, \Gamma'_j\) are widths of scalars and pseudoscalars, \(m_j, M_j\) are masses of scalars and pseudoscalars.

Requirement of unitarity lead to the conditions:

\[
0 = -h + \sum_j \frac{1}{2} h_j H_{jW}
\]

(26)

\[
0 = h + \sum_j \frac{1}{2} h_{jP} P_{jW}
\]

(27)

Formulas presented in [26] for cross section of the process (4) jointly with formulas (26),(27) of this paper may be used in principle for any Higgs sector although in this paper has not been mentioned this possibility. In particularly in theories with more than two doublets and one or more singlets may exist several resonances of the scalars and several resonances of pseudoscalar which may very essentially to enhance the corosse section of the process (4).

4. Loop Diagrams

Besides tree diagrams also exist loop diagrams (Fig.2) which also contribute to the processes

\[
l^+l^- \rightarrow P^0 Z^0, l^+l^- \rightarrow H^\pm W^\pm.
\]

Blocks \(H^+ W^-, Z^0 Z^0 P^0, H^+ W^- \gamma H^0 Z^0 \gamma, P^0 Z^0 \gamma, H^- \mu^+ \nu, H_i^0 \mu^+ \mu^-\) on diagrams on the Fig.2 has been considered previously in [28]-[36], [3] (see also
references therein). For loop vertexes $H^0 Z^0 \gamma$, $P^0 Z^0 \gamma$ and full process process $e^+ e^- \rightarrow H^0 (P^0) + \text{photon}$ see \cite{36} and references therein. In papers \cite{32} and \cite{33} has been considered process $e^+ e^- \rightarrow P^0 Z^0$ due to block $Z^0 Z^0 P^0$, all another diagrams of Fig.2 in this papers has not been considered.

In \cite{12}, (and in \cite{27} in the form presented in this article on the Fig.2) has been considered also box diagrams and diagrams containing $H^- \mu^+ \nu$, $H^0_i \mu^+ \mu^-$-vertexes (including supersymmetric particles contribution: charginos, neutralinos, scalar neutrinos, charged scalar leptons...) with some estimates and conclusions where this diagrams may be important. Obviously this loop diagrams are dominant in $e^+ e^-$ annihilation where tree diagrams are nonimportant due to small mass of electron.

It is interesting to compare our tree results and estimates of loop contribution to the processes $\mu^+ \mu^- \rightarrow P^0 Z^0$, $\mu^+ \mu^- \rightarrow H^\pm W^\mp$ of the Fig.2 o with exact calcultions of the processes $e^+ e^- \rightarrow P^0 Z^0$, $e^+ e^- \rightarrow H^\pm W^\mp$ also based on diagrams of the Fig.2 which has been performed in the later papers (\cite{37}, \cite{38}, \cite{39}, \cite{40} and in paper \cite{38} within MSSM with SUSY particles in loops).

For example from paper \cite{40} we see that cross section of charged Higgs bosons production+$W$-bosons via loops in $e^+ e^-$-annihilation is less than 0.01 fb at $\sqrt{s} = 500 GeV$ and $\tan \beta > 10$ and decrease with $\tan \beta$ growth. At $\sqrt{s} = 1000 GeV$ and $\tan \beta > 10$ the author of this paper obtain the cross section less than 0.003 fb.

From paper \cite{39} we see that cross section of the process $e^+ e^- \rightarrow H^\pm W^\mp$ is smaller than 0.003 fb in the MSSM case at $\tan \beta > 10$. From Fig.5 of this paper we see that cross section is smaller than 0.1 fb at $\tan \beta > 10$ in 2HDM.
case and essentially decrease with growth $\tan \beta$.

From paper [37] we see that number of pseudoscalars $+ Z^0$-bosons produced via loop of Fig.2 (without SUSY contribution) at $\sqrt{s} = 500$ GeV and $10 < \tan \beta$ the cross section $< 0.001$ fb. At $6 < \tan \beta$ the cross section $< 0.01$ fb has been obtained.

From paper [38] where has been considered process $e^+ e^- \rightarrow Z^0 P^0$ (loop contribution with SUSY particles as our diagram 2) we see that again the cross section is smaller than 0.1 fb in most favorable cases in 2HDM model. In case of MSSM the cross section is smaller than 0.01 fb in most favorable range of parameters. However it very difficult to consider all ranges of parameters and probably in some area various SUSY contribution will even to cancel to each over (see discussion in [5]). The authors of [38] consider only several set of parameters and probably at another parameters the situation will be essentially differ. Such partial cancellation depend on SUSY parameters take place in the loop process $e^+ e^- \rightarrow photon + P^0(H^0)$.

In accordance with [12] total cross sections of processes (3), (4), may be obtained from cross section of the processes of scalar neutrinos and leptons ($l_i^+ l_j^- \rightarrow \bar{\nu}_{kL} Z^0, l_i^+ l_j^- \rightarrow \bar{\nu}_{kL} W^\pm$) by the following replacements (because scalar and pseudoscalar Higgs bosons interactions with leptons are similar to scalar leptons interaction with leptons):

$$h_{ijk} \rightarrow \frac{g m_{\mu}}{\sqrt{2} m_W} \tan \beta, m_{\bar{\nu}} \rightarrow m_3, m_i \rightarrow m_4 \quad \text{and} \quad \Gamma_{\bar{\nu}} = 0 \quad (28)$$

where $m_{\mu}$ is $\mu$-meson mass. To value $h_{ijk} = 10^{-2}$ of Yukawa couplings of scalar neutrino and lepton to ordinary leptons correspond $\tan \beta = 17.5$ and we obtain from Fig.4,5 of [12] that about $\approx 1200(\frac{\tan \beta}{17.5})^2 - 60(\frac{\tan \beta}{17.5})^2$ events
per year at \( L = 1000 \) fb\(^{-1}\) to 50 fb\(^{-1}\), \( \sqrt{s} = 1000 \) GeV, \( m_4 = 300 - 700 \) GeV.

About 1100 events per year at \( L = 1000 \) fb\(^{-1}\) to 50 fb\(^{-1}\), \( \sqrt{s} = 500 \) GeV, \( m_4 = 300 \) GeV.

For MSSM pseudoscalar Higgs production with \( Z^0\)-bosons in [12] has been obtained \( \approx 600(\frac{\tan \beta}{17.5})^2 - 30(\frac{\tan \beta}{17.5})^2 \) pseudoscalar Higgs bosons per year at \( \sqrt{s} = 500 - 1000 \) GeV and masses \( m_P = 500 - 700 \) GeV. At kinematical peak we have about 1000 events per year at same \( \tan \beta \) and \( m_P = 500 \) GeV. At smaller \( m_P \) the cross section may be also enhanced.

In any cases we see that at \( \tan \beta > 10 \) (and sometimes at smaller \( \tan \beta \)) the tree contribution considered in [12] in the processes (3),(4) is much more significant than loop contributions considered in results of this papers [37],[38],[39],[40].

Thus we confirm our previous result [12] in accordance with at \( \tan \beta \sim 7 \) or higher tree contribution is more essential than loop contribution.

However if resonances at third s-channel diagrams of the processes (3) is exist (i.e. masses of some of the scalars is larger than \( m_P + m_Z \)) the cross section of the tree process (3) may be essential even at small Higgs bosons interactions with leptons.

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Figures captions:

Fig. 1 Tree diagrams corresponding to the processes (2), (3). $H^0$ denoted scalars $P^0_j$-pseudoscalars.

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Fig. 2 Loop diagrams corresponding to the processes (2), (3) within the MSSM. Shaded ring corresponds to the diagrams with loop-induced $\gamma W^\pm H^\pm, Z^0 W^\pm H^\pm, \gamma Z^0 H_i^0, Z^0 Z^0 H_i^0, H^- \mu^+ \nu, H_i^0 \mu^+ \mu^- $-vertexes. Diagrams with loop-induced virtual gauge bosons-Higgs bosons mixing are not shown.
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