On the viability of gravitational Bose–Einstein condensates as alternatives to supermassive black holes

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ABSTRACT

It is argued that astrophysical Bose–Einstein condensates (BECs) most likely form through a quasi-static contraction of ultradense cores of neutron stars. Such an evolutionary track would ensure that there is sufficient time left for the nuclear matter to stably liberate the excess of thermal energy, enable the core’s matter to intercommunicate and undergo a phase transition to form stellar BECs. This slow contraction would enable their gravitational redshifts to be sufficiently high not to shine, but not too large to still escape collapse into black holes. The collapse can be made avoidable if the core’s fluid is perfectly incompressible and non-contractible, not even on the scale of $10^{-24}$ of a nanometre. It is shown that a slowly contracting core would maintain causal communication with the crust and it is preferable over the direct dynamical collapse of massive stars. In the latter case, the post-collapsed matter would be too energetic to enter a superfluidity phase and would turn the cores turbulent. Moreover, it is shown that super massive Bose–Einstein condensate (SMBEC) cores most likely would suffer from an extensive deceleration of their rotational frequency as well as of vortex dissipation induced by the magnetic fields threading the crust, hence accelerating the transition of cores into the BEC phase. Given that rotating superfluids between two concentric spheres have been verified to be dynamically unstable to non-axisymmetric perturbations, we conclude that gravitational forces generally govern astrophysical BECs, if these exist in nature, and would have a rather stabilizing effect, such as maintaining the perfect incompressibility of the nuclear matter.

Key words: black hole physics – stars: neutron – dark energy.

1 INTRODUCTION

Before 228 years ago and 97 after Newton’s gravitation (1687), John Michell (1784) was the first to anticipate the existence of a critical radius, $R_{\text{H}}$, below which even the light cannot escape the gravitational pull of the central point-mass object. Based on Newtonian gravity, he equated the potential energy $E_p$ of the object of mass $M$ to the kinetic energy $E_K$ and obtained $R_{\text{crit}} = \frac{2GM}{c^2}$, where $c$ and $G$ are the speed of light and the gravitational constant, respectively, although the speed of light was still uncertain and the possible existence of such light-capturing objects in nature was merely a fictitious proposal.

In 1915 Einstein presented his profound theory of gravitation, therein postulating the energy as a field in 4D space–time and postulating as a source for curvature. Mathematically, Einstein field equations read

$$G_{\mu\nu} = \kappa T_{\mu\nu},$$

where $G_{\mu\nu}$ and $T_{\mu\nu}$ are the rank 2 Einstein’s geometrical and energy-momentum tensors, respectively. The coefficient $\kappa = \frac{8\pi G}{c^2} \approx 1.863 \times 10^{-27}$.

Obviously, unless there is an unusually strong accumulation of energy in a relatively small volume, the left- and right-hand sides of equation (1) can be conveniently decoupled, therefore justifying the Newtonian theory in the weak field limit in flat space–time.

Shortly after the general relativity (GR) was published (in 1915), Karl Schwarzschild presented a solution for the field equations that corresponds to a spherically symmetric object of mass $M$ embedded in vacuum. This solution was the first theoretical proof that black holes (BHs) are inevitable products of GR, where the space–time becomes indefinitely warped.

Several years later, Subrahmanyan Chandrasekhar (in 1931) proposed a framework for the BH creation. Accordingly, sufficiently massive stars could in principle collapse under their own self-gravity, as neither the thermal nor the degenerate pressures could counterbalance the enormous gravitational attraction.

From the point of view of Schwarzschild’s solution, a massive star could contract in a quasi-stationary manner to finally undergo a dynamical collapse into indefinitely small size at the centre. However,
a sufficiently distant observer would be able to observe the collapsing matter up to the horizon, but not beyond.

In the early times, this scenario was rejected both by theorists and observers alike and in particular by Einstein and Eddington. Only at the beginning of the 1960s the BH proposal was revived, when BHs appeared to be the only reasonable objects to explain the origin of the vast energy output observed to liberate from quasars.

Since then the BH proposal has been repeatedly adopted and their mass regime has been continuously expanded to finally span the whole mass spectrum, ranging from the microgram scale up to billion solar masses.

Although BHs are unobservables by definition, the dynamical behaviour of matter and stars in their vicinity, in principle, should disclose the properties of these objects. In particular, the distinguished depth of their gravitational well enables BHs to convert a significant portion of the potential energy of matter or objects approaching the BH into other forms of energy, such as thermal or magnetic energy. In fact, there are additional observational techniques that are used for predicting the depth of the gravitational well of BHs, e.g. the iron Kα emission lines associated with rotating plasmas in accretion discs, the Lorentz factor of ejected plasmas from their vicinity and possibly through gravitational wave detectors to be employed in the near future (see Mueller 2007 for additional detection methods).

Most BHs in binaries are considered to have masses of the order of 10 M⊙. The massive BHs are generally found at the centre of galaxies with masses ranging from several million up to several billion solar masses. In most cases they are observed to be radiatively active, implying therefore that they are accreting from or ejecting matter into their surrounding.

Nevertheless, these techniques hint at the existence of objects that differ from normal or compact stars, such as old stars, white dwarfs and neutron stars; however, they definitely are unable to determine their nature.

1.1 Difficulties with the classical BH proposal

Consider an object of mass M and spin 0 embedded in vacuum (see Fig. 1), how does the space–time around this object look like?

From the point of view of GR, the equations to be solved are

\[ G_{\mu
u} = 0. \]

The solution to this problem is a metric, which was forwarded by Einstein by Schwarzschild just two months after the former published his theory of GR. The metric reads as

\[ ds^2 = c^2(1 - r_s/r) \, dt^2 - \frac{dr^2}{1 - r_s/r} - d\Omega^2, \quad (2) \]

Figure 1. The gravitational field of a non-rotating object of mass M embedded in vacuum is equivalent to a curved space–time having the event horizon as a boundary. The hyperspace on the right is a projection of the 4D curved space–time.

where ds, c, t and rs correspond to the proper distance between two events in this space–time, the speed of light, the time measured by a fixed observer at infinity and the Schwarzschild radius rs = 2GM/c², respectively. The term \( d\Omega^2 \) = \( r^2(d\theta + \sin^2\theta \, d\phi^2) \) corresponds to a differential area on the surface of a sphere of radius r.

As shown below, the Schwarzschild’s proposal raises various fundamental questions in GR rather than providing a solution to a problem.

(i) The Schwarzschild solution contains two singularities: \( r = 0 \) and \( r_s \). While the former is intrinsic and unremovable, the latter can be removed by appropriate coordinate transformation, e.g. using the Eddington–Finkelstein coordinates (Hobson, Efstathiou & Lasenby 2006). The contraction of the Riemann curvature tensor at the event horizon, \( R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \), yields a well-defined finite number, which implies that the curvature at this radius is well behaved and that a comoving observer will feel nothing special as she/he crosses the event horizon.

In fact, it was shown by Penrose (1965) and Hawking (1967) that if Einstein’s theory of general relativity is correct and the stress-energy tensor satisfies certain positive-definite inequalities, then space–time singularities are inevitable. This may imply that all quantum information generally associated with the matter building the BH will be completely destroyed.

Moreover, as the light cones tend to close up near the event horizon, \( r = r_s \), our connection to observers approaching the event horizon will be continuously weaker and will be completely lost inside \( r = r_s \), which implies that we will never know if this observer ever crossed the horizon.

This raises a serious question about the progenitor of BHs. If these were massive stars that run out of energy generation via nuclear fusion at their centres followed by a dynamical self-collapse under their own self-gravity, then from our point of view as distant observers they must be still collapsing and would cross the event horizon with the speed of light after infinite time. Equivalently, the progenitors of BHs are massive stars that collapsed into rings of matter that surround the event horizon and whose particles are approaching the horizon, but will never reach or cross it.

(ii) A photon of energy \( E_\lambda = h\nu_\lambda \) measured in our frame will be infinitely blueshifted at the event horizon. Assuming global energy conservation, this indefinite gain of energy must be extracted from the field and therefore would cause a non-negligible perturbation to the space–time. As a consequence, each photon that approaches the horizon will cause a measurable change of the space–time curvature around the BH, which mathematically means that the solution may not be stable against external perturbations.

(iii) The entropy problem of BHs has been investigated by Bekenstein (1973), who found that the entropy of a BH scales as the number of Planck spheres accommodatable within the 2D projection area of a BH, or equivalently

\[ S \approx k_B \left( \frac{r_s}{\ell_p} \right)^2 \approx k_B \times 10^{77} \left( \frac{M}{M_\odot} \right)^2, \quad (3) \]

where \( k_B, \ell_p \) and \( M_\odot \) denote the Boltzmann constant, the Planck length and the solar mass, respectively.

Assuming a true association of the thermodynamical variables with the BH thermodynamics, the entropy of the progenitor appears to increase by at least 20 orders of magnitude during a transition into a BH phase. This may imply that a large amount of information is hidden behind the horizon and that a deep holographical connection between the horizon as a surface and the BH as bulk is at work.
(Padmanabhan 2006). For a comprehensive discussion of the status of the horizon from the quantum mechanics point of view, see Mottola (2010).

(iv) The information paradox repeatedly discussed by theoreticians has not been satisfactorily solved yet.

Noting that the phenomena of particle–wave duality in quantum theory would enable particles to cross the event horizon as waves, the information about the pre-collapsed state of matter must be still found in the non-vanishing wave tail. Such extraction of information is possible due to the unitarity requirement of the Hamiltonian operator describing the quantum state of the infalling particles. However, the only retrievable information from BHs is the Hawking quanta, which is of blackbody (BB) type and therefore featureless (see Fig. 2).

Similar to the emitted BB radiation from the Sun’s photosphere, these photons transmit highly diffused information about their paths inside the Sun due to their random-walk motions in optically thick media. Thus, the Hawking radiation carries information about the physical conditions at or outside the interface, where they make abrupt transition from the opaque space–time domain inside the event horizon into the optically thin and roughly flat space–time outside it (see Preskill 1992 for further details).

Also, it is not clear whether energy states of matter and the behaviour of atoms in the vicinity of the event horizon would obey a Maxwell–Boltzmann-like distribution and whether the Planck function would continue to properly describe the photon spectrum.

(v) The Schwarzschild solution (equation 2) unequivocally shows that GR is capable of singling out two events in the space–time and declare them to be singular in a deterministic manner. However, this approach is in complete contradiction to the fundamental principles of quantum field theory and in particular to the Heisenberg uncertainty principle (Fig. 2).

(vi) While the progenitors of stellar BH are well understood, the evolution of supermassive black holes (SMBHs) is still a controversial issue. It is believed that the Population III stars in the early universe must have been sufficiently massive, as the corresponding Jeans mass was of the order of $10^5 M_\odot$ (Bromm, Coppi & Larson 1999).

Such metal-free massive stars must have formed at cosmological redshifts at $8 \leq z \leq 20$ and should have collapsed relatively fast to form the first massive BHs. Since then their mass must have grown exponentially through repeated mergers and accretion of matter to acquire typical quasar masses of the order of $10^9 M_\odot$. However, the cosmological simulations were able to show certain condensations of clouds but not the final mass that exclusively form the massive star. In fact, such modelling is associated with various numerical difficulties that severely limit the reliability of such large-scale simulations.

Furthermore, astrophysical and cosmological observations reveal that there are two distinct ranges of BH masses: the stellar mass range of $5 \leq M_{\rm BH}/M_\odot < 100$ and the very massive range of $10000 \leq M_{\rm BH}/M_\odot$. It is however not clear why BHs with masses in between are missing, in particular as these range of masses could be conveniently covered through the collapse of Population III-type stars.

Figure 2. The modern picture of a quantum horizon governed by quantum fluctuations and quantum tunnelling, giving rise to the emission of quanta. The semiclassical approach of Hawking predicted that a BH of mass $M$ emits BB radiation at temperature $T_{\rm BH} = 1/(8\pi GM)$.

2 THE LONG WAY TOWARDS BH ALTERNATIVES

During the last decade, several alternatives to BHs have been proposed. A considerable attention was given to dark energy stars (henceforth DESs) and gravitational vacuum stars (henceforth gravastars) that have been proposed by Chapline et al. (2001) and Mazur & Motolla (2004).

Inspired from the $\Lambda$ cold dark matter cosmology (ACDM; Peacock 2011), in which the vacuum energy is considered to be responsible for the accelerating expansion of the universe, one may replace the inner-horizon region of a BH by a vacuum-like core or a de Sitter space–time.

In this case, the effect of gravity is repulsive rather than impulsive. As in the case of dark energy in cosmology, the energy density in the core is set to be constant whereas the pressure is equal to the negative energy, i.e. $P = -\rho$.

Such a negative pressure is a typical phenomenon on the scale of quantum fluctuations as the Casimir effect shows. Furthermore, the equation of state (EoS) $P = -\rho$ is an inevitable consequence of Einstein’s field equations describing an isotropic and homogeneous universe. Using the Robertson–Walker metric, the Friedmann equations yield the following evolutionary equation for the scaling factor $a$:

$$\ddot{a}/a = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3},$$

where $\Lambda$ denotes the cosmological constant and $\ddot{a}$ corresponds to the second time derivative of $a$ (see Peacock 2011 for further details).

As revealed by cosmological observations, including Wilkinson Microwave Anisotropy Probe (WMAP) and high-redshift Type Ia supernovae (Komatsu et al. 2011), $\ddot{a} > 0$, which implies that the right-hand side of equation (4) must be positive. However, if vacuum energy density is due to zero-point energy of quantized fields, then $\Lambda$ must be a Lorentz invariant and therefore can be traced back in the history to make it negligibly small compared to the other terms in the equation. Consequently, the term $\rho + 3p$ must be negative, hence $P < -\rho/3$, or generally $P = -\omega\rho$, where $\omega < -1/3$. In fact, recent WMAP cosmological observations reveal a very narrow range for $\omega$, $-1.24 \leq \omega \leq -0.86$ (Komatsu et al. 2011), implying therefore that $P = -\rho$ as the EoS for the vacuum core can be safely used.

In the case of a gravastar (see e.g. Mazur & Motolla 2004; Chirenti & Rezzolla 2007), the object consists of the following three main domains (see Fig. 3).

(i) Region I: $0 \leq r \leq R_c$, where $R_c$ is the core radius. This region is governed by the de Sitter space–time governed by the EoS $P = -\rho$. 
(ii) Region III: $r > R_\star$, where $R_\star$ is the radius of the object. In this domain $T_{\mu\nu}^{(\text{mater})}$ and $\Lambda$ are set to vanish completely, hence the EoS reads $P = \rho = 0$.

(iii) Region II: $R_* < r < R$, corresponds to a spherical shell filled with normal matter that obeys the EoS $P = \rho$.

The aim is then to search for a global solution to the field equations

$$G_{\mu\nu} = \kappa G_{\mu\nu} + g_{\mu\nu}\Lambda$$

that fulfils the boundary conditions across the different interfaces separating the three regions. The solution procedure relies on proposing the following metric solution:

$$\text{ds}^2 = f(r) \, \text{d}t^2 - \frac{\text{dr}^2}{h(r)} - r^2 \, \text{d}\Omega^2,$$  

where $f(r)$ and $h(r)$ are metric coefficients to be found. Unlike Newtonian physics in Euclidian geometry, the distribution of energy in each region may have a significant impact on the curvature of space–time. Hence, a GR-consistent matching procedure, such as the Israel junction condition (Israel 1966), is required in order to construct a global solution that satisfies the conditions in the three different domains.

One possible solution runs as follows.

(i) In the core region, $f(r) = C \, h(r) = C \left(1 - H_0^2 r^2\right)$, which is similar to the metric coefficients in the case of a de Sitter space–time in cosmology. The coefficient $H_0$ is an integration constant that is analogous to the cosmological Hubble constant, though it has a completely different value.

(ii) In the outermost region, the solution is identical to that of Schwarzschild, i.e.

$$f(r) = h(r) = 1 - \frac{r_\star}{r},$$

(iii) In the intermediate region and in the limit $R_* \rightarrow R_\star$, the following analytical solution was obtained:

$$h(r) = 1 - \frac{2Gm(r)}{rc^2} \approx \epsilon,$$

where $m(r)$ is a continuous mass function, so that $\text{dm}(r) = 4\pi r^2 \text{d}r$. $\epsilon$ is an integration constant of the order of the Planck mass $M_\text{P}$ divided by the mass of the object. Most importantly, the integration constant $\epsilon$ is strictly positive, which implies that $r_\star$ is less than $r_\star$, therefore prohibiting the formation of an event horizon.

Following Mazur & Motolla (2004), the thickness of the shell was estimated to be of the order of $\ell \sim \sqrt{\epsilon} r_\star \approx 10^{-14}$ cm, where $\ell_\text{P}$ is the Planck length. Thus, for a 1-M$_\odot$ gravastar, $\ell$ would hardly exceed the radius of a proton.

Thus, the shell is actually an extremely thin membrane rather than a normal matter-contained shell. Moreover, the model requires a conversion mechanism of unknown nature to operate efficiently at the base of the membrane, whose function is to enforce normal matter to undergo a phase transition into a de Sitter vacuum state.

## 2.1 The reliability of the gravitational Bose–Einstein condensates as BH alternatives

Similar to cosmology, vacuum-dominated cores must expand. The effect of the membrane is then to limit/decelerate the expansion rate, so as to maintain these objects in hydrostatic equilibrium. Ghezzi (2011) showed that an anisotropic pressure is required for ensuring dynamical stability of dark energy objects (DEOs), though a physical origin was not provided.

On the other hand, all normal astrophysical objects known to date, including the relativistic ones, are found to have compactness parameters that are strictly less than 1/2 (see Table 1).

Martin & Visser (2003, see also references therein) found that under a variety of stellar-structure conditions, the compactness parameter has an upper limit: $C < 4/9$. Therefore, a 1-M$_\odot$ DEO with radius $R_\star = r_\star + \ell \approx (1 + \sqrt{\epsilon}) r_\star$ would have the compactness parameter $C = 0.5 - 10^{-14}$, which implies that DEOs, if isolated, are almost indistinguishable from their BH counterparts.

However, the extraordinary compactness of these objects, while having solid surfaces, raises several fundamental questions about their reliability and viability in nature as elaborated in the following.

(i) While the gravastar model is based on a fluid approach, the connection to quantum effects in vacuum has been performed through the integration constant $\epsilon$ solely, which is rather an ad hoc approach. It is unclear, what are those microscopic quantum effects that would lead to different integration constants for different masses in a universe of well-defined universal constants. A reasonable analysis should ensure a scaling out of the dependence of $\epsilon$ on the mass in order to yield a universal constant that applies for a reasonable range of the mass function characterizing astrophysical BHs.

(ii) Similar to this is the proposed formation of gravitational Bose–Einstein condensates (GBECs) and superfluidity of their cores. Although globally stable condensates have not been verified experimentally yet, these are expected to form when the constituents are cooled down to a temperature near absolute zero. The particles then congregate into a single macroscopic quantum state. Following Chapline (2004), when ordinary particles enter the quantum critical shell they morph into heavy bosonic particles.
In fact, the long time-scale of the post-glitch recovery of the Crab and Vela pulsars is strong observational evidence that superfluidity might be a natural phase governing the flow dynamics in the cores of relativistic neutron stars. Although the core’s temperature in an NS is of the order of several million degrees, this is still too to three orders of magnitude lower than the dominant electron Fermi or the effective Coulomb temperatures characterizing the core’s matter. These conditions are equivalent to a terrestrial superfluid with $T \approx 10^{-3}$ K. However, superfluidity in cores of pulsars is a favoured phase due to the presence of neutron–proton two-liquid nature with $n_u/n_p \approx 30$, which gives rise to $n$–$n$ and $p$–$p$ pairing. Given the high rotational speed of pulsars, the superfluid ought to consist of a discrete array of quantized vortex lines elongated parallel to their rotation axis. The Crab pulsar, for example, is expected to contain $N \approx 5.3 \times 10^{18}$ vortex lines. Due to the superconductivity of the core’s matter, those vortex lines that are coupled to the magnetic field must migrate randomly outwards, leading to the loss of radiation reaction torques and subsequently decelerating the core’s rotational speed (Alpar & Sauls 1988).

When extending this scenario to super massive Base–Einstein condensates (SMBECs), several difficulties may emerge that could potentially prohibit their formation.

Consider, for example, the time evolution of the pressure, $P(t)$, at the centre of a neutron star. While the dominant contribution due to the non-thermal degenerate pressure, these objects still have to spend several million years in order to liberate the vast thermal energy trapped in the core during the dynamical collapse of the progenitor. A newly born neutron star is expected to have a central temperature of $T_C \approx 10^{11}$–$10^{12}$ K, but which decreases quickly thereafter to reach several million degrees through extensive neutrino emission. Without extensive neutrino emission, the stored thermal energy in the core would alter the force balance and may lead to completely different evolutionary tracks. In particular, the superfluidity would diminish.

As the progenitors of a stellar mass Bose–Einstein condensate (BEC) must be much more massive than that of an NS, the total thermal energy trapped in the core is of the order of $U \approx V^2 \cdot M_{\text{BECC}}$, where $V_s$ is the sound speed, assuming isothermal core. As this thermal energy would neither disappear in a singularity, as in the case of BHs, nor being expelled to the surrounding regions, $U$ must eventually be comparable to the rest energy of the core. Thus, $V_s$ is relativistic and therefore is much larger than the critical speed, beyond which terrestrial superfluidity would normally be destroyed, unless astrophysical BECs are indefinitely stable against perturbations propagating at ultrarelativistic speeds.

Furthermore, as the crust is made of normal matter, it would serve as a source for electromagnetic radiation. This loss of energy would cause the core to shrink and subsequently collapse into a BH.

We note that in the absence of self-gravity, it was experimentally verified that external magnetic fields tend to shrink the condensates, enhance the self-interactions and reconnection of vortices and subsequently lead to their collapse or explosion as bosonova (Donley et al. 2001).

When the BEC phase is reached via a quasi-stationary contraction of a normal matter core, the heat capacity of the matter in the crust must have been continuously decreasing to reach a small, but still a positive critical value, below which the BEC would cease to thermally interact with the surrounding media. Only a negligibly small fraction of $\xi = \sqrt{1 - 2\zeta}$ of the total internal energy would find its way outwards, while the rest is being trapped in the core or diffuses backwards from the crust into the core.

To conclude, the enormous thermal energy trapped in the core during the collapse of the progenitor in combination with magnetic fields and large conversion efficiency of kinetic into thermal energy via shocks at the surface would eventually act to suppress the superfluidity and superconductivity of the core and lead to a reverse phase transition.

A further uncertainty of SMBECs is the causality problem. Consider for example the SMBH powering the high-redshifted quasar PKS 1020–103, whose mass is estimated to be $M \approx 2.62 \times 10^9 M_\odot$ and accretion rate of several solar masses per year. The light-crossing time of this object is $t_{\text{LC}} \approx 2 \times c \approx 7$ h, which is the shortest possible time-scale required for a global readjustment of the core to external perturbations.

Let us consider the case in which the core consists of a single bosonic condensate with a specific macroscopic quantum state. Unlike white dwarfs and neutron stars, in which the Pauli exclusion principle prevents their collapse, in the case of boson-like condensates it is the Heisenberg uncertainty principle that opposes self-collapse. The zero-energy state would allow the density of bosonic objects to be much larger than in fermionic objects. The critical mass of bosonic objects most likely obeys the correlation $M_{\text{CB}} \sim M_{\odot}^2/m_B$ and that of fermionic objects obeys $M_{\text{CF}} \sim M_{\odot}^2/m_F$. Here, $m_B$ and $m_F$ denote the mass of an individual bosonic and fermionic particle, respectively. Provided that $m_B$ is much less than the proton mass, boson objects in principle could be much more massive than their fermionic counterparts.

Similar to terrestrial BECs, we may assume that the pre-phase of astrophysical condensates is a superfluid that consists of quantized vortices. In the ideal case, self-gravity would squeeze these vortices into a single one of radius $r_s$, but which still obeys the equality of quantized circulation:

$$\int V \cdot dl = \frac{h}{2m},$$

Here, $V$, $l$, $h$, $m$ and $n$ correspond to the circulation velocity, path around the vortex line, Planck’s constant, the mass involved in the vortex core and the total number of vortex lines, respectively.

We may employ the Stokes theorem and approximate the resulting integral as follows:

$$\nabla \times V ds \approx \int \nabla \times \langle V \rangle ds = 2 \Omega 4\pi r^2,$$

where $\langle V \rangle$ and $\Omega$ denote the average superfluid velocity and the uniform rotational frequency of the SMBEC, respectively. Expressing $r_s$ in terms of mass, we end up with the following prediction:

$$\Omega_{\text{BECC}} = 6 \times 10^{-72} \left( \frac{M_\odot}{M_{\text{BECC}}} \right)^3$$

However, this rotational frequency is too low to explain the origin of power of highly collimated jets observed in microquasars, high-redshifted quasars as well as at the centre of active galaxies. These are considered to originate from the vicinity of fast spinning BHs, where direct extraction of rotational energy from the BH via Blandford–Znajek mechanism or from the boundary layer (BL) via dynamo action is considered to be most efficient (see Blandford & Znajek 1977; Brezinski & Huijers 2011, and references therein). Consequently, unless SMBECs are born massive, their cosmological growth will normally be associated with an increase of their rotational energy, which would enforce the cores to rotate at a much higher speed than the above estimated value. However, a higher $\Omega_{\text{BECC}}$ would enhance the rotational interaction between the core
and the crust, thereby enabling energy transfer from the energetic crust into the condensate, which may therefore undergo a phase transition into a higher energy state.

Let us now consider the case in which the core of the above-mentioned quasar is being hit by a neutron star (Fig. 4). Noting that the mean surface density of the quasar is $\langle \Sigma \rangle_{\text{BEC}} = M/(4\pi r_c^2) \approx 10^{11} \text{ g cm}^{-2}$, the ratio of the surface energy density of the NS to that of the SMBEC at the time of the crash is

$$\frac{E_{NS}}{E_{\text{BEC}}} \approx 10^9. \quad (8)$$

In evaluating the last expression, we have assumed $M_{\text{NS}} = M_{\odot}$ and $R_{\text{NS}} = 10^6 \text{ cm}$. On the other hand, for a gravastar, for example, the total mass of the membrane is roughly equal to the Planck mass. Thus, the mass of the portion of the membrane possibly involved directly in this crash can be estimated to be $\sim M_{\text{PL}}(S_{\text{NS}}/S_{\text{MEMBE}}) \approx 10^{-21} \text{ g}$, where $S_{\odot}$ means the surface of the corresponding object. Thus, the local energy available for resisting the NS–SMBEC crash is of the order of several erg. Also, the local time-scale characterizing the crash time is $t_{\text{LOC}} \sim R_{\text{NS}}/c \approx 10^{-9} t_{\text{GD}}$, where $t_{\text{GD}}$ is the global dynamical time-scale of the SMBEC.

Given that the input of energy through the crash exceeds the locally available one by several orders of magnitude and that the NS would crash into the membrane almost with the speed of light, we conclude that a considerable portion of the membrane and the enclosed portion of the SMBEC would be completely destroyed early enough before the condensate could react dynamically to maintain global stability. In the case of stellar mass BECs, the input of energy associated with such a crash would destroy the whole condensate.

The important question that arises is whether SMBECs may still re-form after crash events? As the mechanisms leading to the formation of astrophysical BECs are neither uncertain nor discussed in the literature, we propose the following two evolutionary scenarios.

(a) Astrophysical BECs are the final phase in the evolution of compact objects that are reached through a quasi-static contraction of their cores.

(b) Astrophysical BECs are formed directly through the dynamical collapse of massive stars, but grow cosmologically through repeated collisions and mergers with other objects.

Viewing gravitation as a mechanism by which bosonic particles condensate to reach the lowest possible microscopic and macroscopic energy states, we conclude that the latter scenario is unlikely, as the energy budget associated with such a collapse event would be too large to form a low-energy condensate. The post-collapsed matter in this case would be too energetic to enter a superfluidity phase and/or subsequently become a macroscopic condensate. The large gravitational redshift of such condensates minimizes the energy loss to the surrounding and turns them into poor radiators, hence into dark objects.

Following the calculations of Chapline et al. and Mazur et al. in the last decade, the surface of a $10^{10} M_{\odot}$ BEC would exceed the Schwarzschild radius by an immeasurably small fraction only, which is equivalent to approximately $10^{-24} \text{ nm}$. Practically, $r_{\text{BE}}$ and $r_s$ would be observationally indistinguishable. Thus, the formation of astrophysical BECs most likely would require the gravitational redshift of SMBECs to be extremely large to ensure that they do not shine, but not too large, to still enable their cores to communicate with the crust and avoid their self-collapse into BHs.

Thus, in order to avoid their collapse, their cores are not allowed to contract any longer, not even by a fraction of $10^{-24}$ of a nanometre. If such a universal limit indeed exists, then the transition into the BEC phase is expected to evolve in a quasi-stationary manner, so as to ensure that there is sufficient time left for the matter to liberate the remnant of the thermal energy as well as to enable a global coordination for all available particles to simultaneously enter a BEC phase.

(iii) When feeding a gravitational BEC with the mass rate $M_{\text{max}} = c^3/2G$, the horizon would grow with the speed of light. This is equivalent to injecting the core with $1 M_{\odot}$ per $10^{-5} \text{ s}$. This is much longer than the duration of an NS–SMBEC crash event, which is expected to be of the order of $\epsilon (r_s/c)$. Let us assume that the membrane of a BEC is located less than an epsilon outside the event horizon, i.e., at $r_{\text{BE}} = (1 + \epsilon)r_s$, and that this configuration is applicable to DEOs and independent of their mass, then it is easy to verify from the mass–radius relation that

$$\frac{dr}{dt} = \frac{2G}{c^2} \frac{M}{(1 + \epsilon)} > \frac{dr}{dt}_{\text{BH}}. \quad (9)$$

Consequently, in order to ensure that the surface of the newly formed core (from inside to outside) still lies outside the event horizon, its surface must grow with a superluminal speed. However, as $\epsilon$ and therefore $dr/dt$ are small, an instantaneous violation of causality is quantum-mechanically not forbidden. An elementary particle moving with almost the speed of light in the vicinity of the crust would have extremely large momentum, so that $dr/dt \geq \frac{1}{\hbar}$ can safely fulfill the Heisenberg’s uncertainty principle.

Alternatively, in most astrophysical phenomena the accretion rate $M < M_{\text{max}}$. Taking into account that accreted matter is rotating, we may anticipate the matter forms rings and undergoes successive eruption events, which are necessary for lowering the thermal energy and slowly settling on the crust. This enables the crust to have time to readjust to the enhanced accretion properly, so that the settling of the accreted matter and the core’s contraction occur almost in a quasi-stationary manner without violating the causality condition.

(iv) The settling normal matter from the surrounding would create a multicomponent fluid in the crust, establishing herewith the appropriate conditions for the creation of different Fermi surfaces for fermions and bosons and therefore applying a magnetic tension, which would tend to couple the core to the crust dynamically. As a consequence, the core breaks into a multiple number of vortex lines threaded by magnetic fields. Similar to the superfluid and superconducting cores in NSs, the vortex lines in the core of an SMBEC that are coupled to the magnetic field would migrate from inside to outside, thereby enhancing self-interaction and vortex reconnection and diminishing the superfluidity of the core. In this case, we anticipate the surface of an SMBEC to be magnetically active with intense eruption events, though hardly observable. Similar to cores

Figure 4. A frontal crash of a neutron star with a non-rotating supermassive BEC.

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Figure 5. The Lorentz factor versus the compactness parameter $C$ is plotted for three different jet launching parameters $a_0^2 = 1$, 0.9 and 0.75 denoted with the black, blue and red colours, respectively.

in NSs, the core–crust coupling through magnetic fields would cause SMBEC to decelerate its rotational speed. Moreover, we note that rotating superfluids in BEC cores are most likely similar to rotating superfluids between concentric spheres. The latter was verified to be unstable against non-axisymmetric perturbations compared to normal rotating Couette flows (Barenghi & Jones 1987).

(v) Highly collimated relativistic jets have been observed to emanate from the vicinity of accreting BHs and NSs. The Lorentz factors characterizing the propagation speed of these jets were verified to correlate with the compactness parameter of accreting objects (see Hujeirat, Camenzind & Livio 2002; Hujeirat et al. 2003, and references therein). To first-order approximation, we may assume jet velocities to linearly correlate with the escape velocity, i.e. $V_j = \alpha_0 V_{cs}$, where $\alpha_0$ is a constant coefficient of the order of unity. The Lorentz factor can then be expressed as a function of the compactness parameter as $\Gamma = 1/\sqrt{1 - a_0^2 C}$. Fig. 5 shows that $\Gamma$ is highly sensitive to $a_0$, indicating therefore that large $\Gamma$ factors between 10 and 20 may be obtained if the jet velocity is a significant fraction of the escape velocity. Equivalently, the jet plasma must start its outward-oriented motion either from the very vicinity of the object’s surface or from near the horizon event.

We note that the formation and acceleration of jet plasmas around accreting relativistic objects is considered to be the outcome of complicated coupling processes between the accretion disc, the central object and the jet plasma, whereby magnetic fields play a crucial role (Hujeirat 2004; Brezinski & Hujeirat 2011). Jets emanating from around surface-free objects are generally observed to be radio-dominated and their plasmas propagate with larger Lorentz factors compared to those of neutron stars. Therefore, in the case of accreting DEOs the solid surface in combination with magnetic fields threading the normal-matter-made crust would give rise to an equally significant thermal and radio emission with strong variabilities. These could serve as guide to fix the compactness parameter of the object. However, when the effective gravity vanishes at the surface of a DEO, i.e. $g_{\text{eff}} \approx 0$, the maximum possible frequency would be of the order of $v_{QPO} \approx \sqrt{\alpha_0^2 C} c$. If the microquasar GRS 1915+105 were a DEO, for example, then it would display quasi-periodic oscillation around 400 Hz. This is approximately one order of magnitude larger than the value revealed by observations. Moreover, the dominant radio over thermal emission characterizing this object rules out the possibility of a solid surface.

(vi) All astrophysical objects known to date have been observed to rotate sub-Keplerian. This implies that the effective gravity at their surface is strongly negative; hence, the accreted particle falls almost freely inwards. In the case of slowly rotating astrophysical BECs, the accreted plasma shocks the crust and forms a virially hot BL around the BEC. Due to conservation of magnetic flux, frozen-in magnetic fields must increase inwards as $r^{-2}$ and reach equipartition with the thermal energy of the plasma in the BL (Fig. 6). As a consequence, the plasma in the BL would obey a force balance of the type $\nabla \times \mathbf{B} \times \mathbf{B}/4\pi \approx \rho \nabla \Phi$, which implies that

$$\frac{L}{r} \sim C^{-3} \left(\frac{V_A}{c}\right)^2,$$

where $V_A$ denotes the Alfvén speed. Thus, the BL, which consists mainly of virially hot elementary particles, must have a macroscopic thickness rather than a membrane with a subatomic microscopic width. The strong magnetic fields threading the BL, in principle, are capable of communicating the presence of the solid crust to the surrounding media.

The relativistic charged particles gyrating around these strong magnetic field lines must emit synchrotron radiation at a rate of

$$E_{\text{syn}} = \frac{16}{3} e^2 c n_e \left(\frac{e B}{m_e c}\right)^2 \left(\frac{kT}{m_e c^2}\right)^2 \text{erg cm}^{-3} \text{s}^{-1},$$

where $e, k, n_e, B$ and $T$ correspond to the electron charge, Boltzmann constant, electron number density, magnetic intensity and electron temperature, respectively. Integrating $E_{\text{syn}}$ over the whole volume of the BL, we obtain a total synchrotron luminosity of the order of

$$L_{\text{syn}} \sim 10^{45} n^2 \left(\frac{M_9}{T_5}\right)^3 \text{erg s}^{-1},$$

where $T_5$ and $M_9$ denote the pre-shock temperature of the accreted plasma in $10^5$ K and $10^9 M_\odot$ units, respectively.

Consequently, such a large radio luminosity from a geometrically thin BL would be optically thick to synchrotron emission and the BL would then appear as a bright photospheric ring that surrounds the SMBEC. However, such rings have not been observed yet, though they would be comfortably observable with today’s detectors.

3 SUMMARY

Various aspects of BHs and DEOs as BH candidates have been discussed and the drawbacks of both kinds of objects have been addressed. In this paper, it is argued that a direct formation of astrophysical BECs via stellar collapse of massive stars is unlikely, due to the large energy budget associated with such violent events. The post-collapsed flow here would be too energetic to enter a superfluidity phase and/or subsequently form astrophysical condensates.
Taking into account the fact that stellar condensates must have extremely large gravitational redshifts to be non-luminous objects, we expect the large energies trapped during the collapse to turn their cores turbulent and suppress the superfluidity of the matter, unless these objects undergo successive relaxation events such as supernova or gamma-ray bursts (Hujeirat 2003) so as to substantially lower their energy budgets.

As a consequence, astrophysical BECs would most likely form through a quasi-static contraction of the cores of ultracompact objects. Such an evolutionary track would ensure that there is sufficient time left for the nuclear matter to liberate the excess of energy and undergo a phase transition to form BECs.

It should be noted that the rest energy of stellar BECs would be comparable to the total energy of normal NSs. In this case, crash events would certainly disrupt both kinds of objects completely.

Finally, we conclude that unless SMBECs differ fundamentally from their terrestrial counterparts, their cosmological growth through accretion and mergers requires an exotic mechanism that prevents their self-collapse into massive BHs.

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