Free surface flow over two-dimensional dunes under different flow regimes

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Abstract. To explore the influence of different flow regimes on fluid dynamics over dunes after the construction of the Three Gorges Dam, a model is applied by utilizing a large eddy simulation, immersed boundary method and level set method to address the turbulence, dune morphology and free water surface, respectively. Seven simulations with incrementally increased Froude numbers are simulated based on previous experimental work. The vertical profiles of the non-dimensional double-averaged streamwise velocity agree well with the experimental data. The profiles display an inflection caused by the effects of the dunes within the form-induced sublayer, where the individual profiles are reduced by wake flow. As the Froude number increases, the flow regime transitions from subcritical flow to supercritical flow, and the mean streamwise velocity and the shear velocity increase. Additionally, the reattachment location of the recirculation zone downstream of the crest moves upwards, while the location of the highest water elevation moves downwards, approaching the crest of the dune. The fluctuation in the free water surface increases as the Froude number increases.

1. Introduction

After the start of the operation of the Three Gorges Reservoir (TGR), many scientific problems have arisen that are worth studying. Due to the water and sediment regulation function of the TGR, it has a certain impact on the flood control, shipping, irrigation and ecological environment in the midstream and downstream reaches of the Yangtze River[1-4]. Hence, one-, two- and three-dimensional mathematical models have been established to investigate sediment transport, environmental pollution and ecological change in this area[5-11].

Because flow regimes can be diversified downstream of the TGR by discharge control, the corresponding Froude number can be less than, equal to or larger than 1. Thus, the dynamic flow is more complex after the operation of the TGR. How the flow influences the near-bed shear velocity and then affects the sediment transport and the subaquatic morphology of dunes is quite important to investigate. Because there are few corresponding studies, this is the main topic of our study.

In this study, in order to investigate the influence of different flow regimes on fluid dynamics over dunes, a combination model of large eddy simulation, immersed boundary method and level set method is used to address the turbulent flow, dune morphology and free water surface fluctuation, respectively. Based on a previous experiment, seven simulations are performed with an incrementally increased flow Froude number. The flow regimes transition from subcritical flow to supercritical flow. Then, the results of the model are validated and analysed in detail.
2. Mathematical model

In this study, a large eddy simulation is used to capture the three-dimensional turbulent characteristics of open channel flow. An immersed boundary method with direct forcing is applied to capture the interaction between fluid and dunes. A level set method is introduced to simulate the dynamic free water surface.

2.1. Large eddy simulation for turbulence

In this model, a large eddy simulation in-house code (Hydro3D), which has been validated by many complex studies relevant to different kinds of turbulent flow conditions, is adopted to simulate turbulent open channel flow conditions[12-16]. It is suitable to use the finite differences method on a staggered Cartesian grid to solve the filtered Navier-Stokes equations for unsteady, incompressible and viscous flow:

\[
\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial (2\nu S_{ij})}{\partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} + f_{imb}
\]

(2)

where \( u_i, u_j, x_i \) and \( x_j \) are the flow velocity and spatial variables towards the \( i \) and \( j \) directions, respectively, and 1, 2, and 3 represent the \( x, y, \) and \( z \) directions. Additionally, \( p, \rho \) and \( \nu \) represent the pressure, fluid density and fluid kinematic viscosity coefficient, respectively, and \( f_{imb} \) denotes the direct force per unit volume exerted on the flow from the dunes. In addition, \( S_{ij} \) is the strain-rate tensor, equal to \( (\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})/2 \), and \( \tau_{ij} \) is the subgrid stress (SGS) tensor, defined as \( \tau_{ij} = -2\nu S_{ij} \).

In addition, the Wall-Adapted Local Eddy viscosity (WALE) model is utilized to compute the SGS stress[17]. Second-order and fourth-order central differences schemes (CDSs) are used to calculate convective and diffusive velocity terms in the code, and time advancement is achieved by a second-order explicit Runge-Kutta scheme.

2.2. Immersed boundary method for fluid over dunes

Another key method is the immersed boundary method, which is applied here to calculate the interaction between fluid and dunes. In the code, the dunes are composed of a multitude of solid bodies that are imposed by means of adequately formulated source term \( f_{imb} \) added to the Navier–Stokes equations. This fictitious forcing term is provided as

\[
 f_{imb} = \begin{cases} 
 C - D + \nabla P + \frac{\Delta V}{\Delta t} (U_{i,t} - U_i^*) & \text{inside the solid bodies} \\
 0 & \text{elsewhere}
\end{cases}
\]

(3)

where \( C \) is the discrete convection term, \( D \) is the discrete diffusion term, \( P \) is the resolved pressure divided by the density, \( V \) is the volume of the water, \( t \) is the time, \( U_i^* \) is the resolved velocity in the \( i \) direction at the previous time step and \( U_{i,t} \) is the target velocity (here, \( U_{i,t} = 0 \)). Hence, when the fluid approaches the dunes, its velocity will decrease. The velocity is completely suppressed on the surfaces of the dunes, resulting in zero velocity there.

2.3. Level set method for dynamic free surface

A new algorithm is introduced in our model to address the free water surface instead of assuming a rigid lid because the simulated flow conditions here include high Froude numbers. This algorithm is based on the level set method developed by Osher and Sethian[18], which is an interface-capturing method for a two-phase (water and air) flow performed on a fixed grid. The level set method employs a level set
signed distance function $\phi$, which has zero value at the phase interface and is negative in air and positive in water. This method is formulated as:

$$
\phi(x,t) = \begin{cases} 
< 0 & \text{if } x \in \Omega_{\text{air}} \\
0 & \text{if } x \in \Gamma \\
> 0 & \text{if } x \in \Omega_{\text{water}} 
\end{cases}
$$

where $\Omega_{\text{air}}$ and $\Omega_{\text{water}}$ represent the fluid domains in the air and in the water, respectively, and $\Gamma$ is the interface. The interface moves instantaneously with the turbulent flow and can be expressed through a pure advection equation:

$$
\frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial x} = 0
$$

To avoid the numerical instabilities caused by discontinuities of density and viscosity at the interface of water and air, a transition zone $|\phi| \leq \varepsilon$ is introduced to smoothly transition between water and air, where $\varepsilon$ represents half the thickness of the interface, which is two grid spacings in our model. A Heaviside function, $H(\phi)$, accomplishes the transition as follows:

$$
\rho(\phi) = \rho_a + (\rho_v - \rho_a) H(\phi) \tag{6}
$$

$$
\mu(\phi) = \mu_a + (\mu_v - \mu_a) H(\phi) \tag{7}
$$

where $H(\phi)$ is equal to 0 when $\phi < -\varepsilon$, equal to 1 when $\phi > \varepsilon$, and can be calculated as

$$
\frac{1}{2} \left[ 1 + \frac{\phi}{\varepsilon} + \frac{1}{\pi} \sin \left( \frac{\pi \phi}{\varepsilon} \right) \right]
$$

when $|\phi| \leq \varepsilon$. To minimize numerical dissipation, a fifth-order WENO scheme is used. More details can be found in Kara et al. [21].

3. Simulation setup and boundary conditions

The simulations were conducted on the Sunway TaihuLight supercomputer in China using 120 processors. A rectangular channel with a size of $1.65 \times 0.80 \times 0.34$ m$^3$ is used based on the experiment described in Dey et al.[22]. The bed morphology is shaped by two-dimensional dunes, the total wavelength and height of which are 0.55 m, 0.04 m. Specifically, the length of the sloping stoss side is 0.425 m, and the length of the steeply sloping lee side is 0.125 m. Based on these parameters, seven cases (D1-D7) with incrementally increased Froude numbers are simulated, and the flow regimes are divided into three categories: subcritical, critical and supercritical flow. The scheme of the domain is shown in figure 1, and fluid parameters are shown in table 1. As the uniform grid size is 0.005 mm in the $x$, $y$ and $z$ directions, the total number of grid points used in the computational domain is $330000 \times 160000 \times 68000$.

To mimic an infinitely large open-channel flow, periodic boundary conditions were applied to both the streamwise and spanwise directions. The no-slip condition was applied at the channel bed, while the free-slip condition was applied to the free surface.

Figure 1. The computational domain with two-dimensional dunes.
In addition, a variable time step was used based on a maximum CFL of 0.30. The ratio of domain length to mean flow velocity was defined as the flow-through (FT) time. As a precursor simulation, the D1 case was executed for 60 FTs to establish fully developed turbulent flow. Then, another 120 FTs were subsequently calculated for the D1 to D7 cases based on previous simulations to collect the flow information.

| Cases | Water depth h (m) | Mean flow velocity u (ms$^{-1}$) | Reynolds number $Re = uh/v$ ($-$) | Froude number $Fr = u(gh)^{0.5}$ ($-$) | Flow regime |
|-------|-------------------|-----------------------------|---------------------------------|---------------------------------|-------------|
| D1    | 0.207             | 0.225                       | 46575                           | 0.16                            | Subcritical flow |
| D2    | 0.207             | 0.6                         | 124200                          | 0.42                            | Critical flow |
| D3    | 0.207             | 1.0                         | 207000                          | 0.70                            | Supercritical flow |
| D4    | 0.207             | 1.5                         | 310500                          | 1.05                            |              |
| D5    | 0.207             | 2.0                         | 414000                          | 1.40                            |              |
| D6    | 0.207             | 2.5                         | 517500                          | 1.75                            |              |
| D7    | 0.207             | 3.0                         | 621000                          | 2.11                            |              |

4. Results

4.1. Validation
To validate that the simulation model is useful, the vertical profiles of the non-dimensional double-averaged streamwise velocity $\langle \bar{u} \rangle/\bar{u}$, from the simulation results of the D1 case are compared with the experimental results from Dey et al. (2020) in figure 2. The simulated data agree well with the experimental data, which means that the simulation result is reasonable. The $\langle \bar{u} \rangle/\bar{u}$ profile displays an inflection caused by the effects of the dunes within the form-induced sublayer, where the individual profiles are reduced by wake flow. This also suggests fluid mixing due to flow separation downstream of the crest. Consequently, the $\langle \bar{u} \rangle/\bar{u}$ profile does not follow the logarithmic law above the form-induced sublayer. In addition, the simulated shear velocity $\bar{u}_s$ is calculated as the square of the product of the time-averaged pressure gradient $\frac{d\bar{p}}{dx}$ and hydraulic radius $h$ in Table 2. The results show that the shear velocity increases monotonically as the Froude number increases from the D1 case to the D7 case. This agrees with the fact that the sediment at the bottom of the bed can be washed up more easily under supercritical flow.

4.2. Fluid analysis and comparison
To compare the details of subcritical flow and supercritical flow, the D2 and D6 cases are selected to plot contours of the mean streamwise velocity at the central plane of the calculation domain in figure 3. The free water surface is also displayed in figure 3. It is evident that the mean streamwise velocity of the D2 case is relatively low, below 1.4 ms$^{-1}$, while that of the D6 case is higher, and the mean streamwise velocity over the crest of the dune exceeds 1.4 ms$^{-1}$. On the lee side of the dune, there is a recirculation zone that begins from the crest of the dune and extends to a point downstream of the crest where the flow reattachment occurs. The distance is approximately five times the dune height for the D2 case, while it decreases for the D6 case. The reattachment location is important because it signifies the location where the high-speed fluid of separated vortices inrushes the dune bed. In contrast, the free water surface slightly and regularly fluctuates for the D2 case, such that the highest water elevation occurs directly above the stoss side of the dune, while the fluctuation in the free water surface of the D6...
case is greater, such that some local fragmentation of the water surface can be detected. In addition, the highest water elevation occurs toward the crest of the dune.

![Figure 2](image2.png)

**Figure 2.** Vertical profiles of non-dimensional double-averaged streamwise velocity \( u'/u_* \) in the D1 simulation (solid line) validated by experimental data presented by Dey et al. (2020) (square points).

**Table 2.** Simulated shear velocities of seven simulations, D1-D7.

| Case | Dimensionless time-averaged pressure gradient \( dp/dx \) | Simulated shear velocity \( u_* \) (m/s) |
|------|-----------------------------------------------------|----------------------------------------|
| D1   | 0.00583                                             | 0.03741                                 |
| D2   | 0.02964                                             | 0.07832                                 |
| D3   | 0.08167                                             | 0.13003                                 |
| D4   | 0.17751                                             | 0.19169                                 |
| D5   | 0.32143                                             | 0.25795                                 |
| D6   | 0.49722                                             | 0.32082                                 |
| D7   | 0.71020                                             | 0.38342                                 |

![Figure 3](image3.png)

**Figure 3.** Mean streamwise velocity and free water surface comparison between the D2 and D6 cases. 
(a) D2 case, (b) D6 case.
5. Conclusions
To investigate the effect of different flow regimes on fluid dynamics over dunes after the construction of the Three Gorges Dam, a combination model is applied by using a large eddy simulation, immersed boundary method and level set method to address the turbulent flow, the dune morphology and the free water surface, respectively. Seven simulations (D1-D7) with incrementally increased Froude numbers are simulated based on the previous experimental setup presented by Dey et al. (2020). The results show that the vertical profiles of the non-dimensional double-averaged streamwise velocity agree well with the previous experiments. The profiles display an inflection caused by the effects of the dunes within the form-induced sublayer, where the individual profiles are reduced by wake flow. In addition, as the Froude number increases, the flow regimes transition from subcritical flow to supercritical flow, and the free water surface fluctuation increases. At the same time, the mean streamwise velocity and the shear velocity increase monotonically. The recirculation zone downstream of the crest is significant, and the reattachment location moves upwards while the location of the highest water elevation moves downwards and approaches the crest of the dune as the Froude number increases.

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