Abstract

Quantum weak measurement attracts much interests recently [Rev. Mod. Phys. 86, 307 (2014)], as it could amplify some weak signals and provide a technique to observe the nonclassical phenomenons. Here, we apply this technique to study the interaction between the free atoms and the vacuum in a cavity. Due to the gradient field in the vacuum cavity, the external orbital motions and the internal electronic states of the atoms can be weakly coupled via the atom-field electric-dipole interaction. We show an interesting phenomenon that, within the properly post-selected internal states, the weak atom-vacuum interaction could generate a large change to the external motions of atoms.
I. INTRODUCTION

The notion of quantum weak measurement was introduced by Aharonov, Albert, and Vaidman (AAV) in 1988 [1]. They considered a quantum measurement process that the coupling between the pointer and system (needed to be measured) is very weak. For example, the weakly coupled external motion (the pointer) and internal spin state (the measured system) of the free electrons. They measure the pointer in certain sub-ensemble of the system, i.e., with the post-selected spin state $|S_f\rangle$. They found an interesting result that, the average value of the pointer has a shift proportional to the value

$$A_w = \frac{\langle S_i|\hat{A}|S_f\rangle}{\langle S_i|S_f\rangle}. \quad (1)$$

Where, $|S_i\rangle$ and $\hat{A}$ are the initial state and the observable of the measured system. This value is obtained under the condition of the weakly pointer-system coupling, so that it is called usually weak value. Comparing to the usual measurement $\langle S_i|\hat{A}|S_i\rangle$, the weak value provides an improved approach to detect $\hat{A}$ by the induced post-selection $|S_f\rangle$, i.e., more controllable parameters are induced [2].

Recently, the weak value attracts much interests as it could amplify some weak signals [3–7] and improve the signal-to-noise ratio [8, 9]. It is further used to study the foundational questions and the new effects, e.g., the Hardy’s paradox [10], the Leggett-Garg inequality [11], the Heisenberg’s uncertainty relation [12], the superluminal tunnelling [13], the direct measurement of quantum wave function [14], the average trajectories of single photons [15], and the spin Hall effect of light [16], etc. On the physical systems, the previous studies of weak measurement used the light both as the pointer and the measured system, see, e.g., [14–17]. There are several interesting work implementing weak measurement by the condensed-matter system, e.g., the quantum dot [18], superconducting phase qubit [19], and the semiconducting Aharonov-Bohm interferometer [20]. A recent Ref. [21] studied the weak measurement of cold atoms system based on the dynamics of spontaneous emission.

On the other hand, the cavity quantum electrodynamics predicted many non-classical phenomena in a vacuum cavity, i.e., nothing in the cavity. For example, the vacuum Rabi splitting [22–24], the vacuum induced atom-atom couplings [25, 26], and the Casimir effects [27–31]. The so-called vacuum Rabi splitting means that a vacuum cavity can split the transition frequency between the two internal levels of the atom under the condition of atom-cavity large detuning. Under the similar condition, the internal states (encoded as the qubits) of two atoms can be di-
rectly coupled by the vacuum cavity. This coupling is due to the virtual excitation of photon, and used often to implement quantum information processing between the qubits, see, e.g., [32–34]. Remarkable, the Casimir effect means that, there is a weak attractive force between two mirrors due to the changes of vacuum energy caused by the mirrors.

Here, we apply the weak measurement technique to study the free atoms interacting with the vacuum cavity. We show that, due to the gradient field in the vacuum cavity, the internal electronic states and external orbital motions of free atoms could be weakly coupled with a Hamiltonian similar to the standard one of weak measurement theory [1]. After this coupling, we perform a single qubit operation on the two internal states of atoms and post-select an internal state. Consequently, we obtain a weak value, its real and imaginary parts decide the shifts of the average momentum and position of the atoms’ external motions, respectively. These shifts could be amplified (in the certain degree) by the weak values as it shown by AAV [1]. Our paper is organized as follows. In the Sec. II, we present the vacuum induced weak coupling between the internal and external motions of free atoms. In the Sec. III, we get the desirable weak values by the single qubit operation and post-selection. In Sec. IV, we give a conclusion of this work.

II. THE VACUUM INDUCED COUPLING BETWEEN THE INTERNAL AND EXTERNAL MOTIONS OF FREE ATOMS

Following the original work of AAV [1], we consider the weak measurement experiment as showing in Fig. 1. The atoms are injected into the cavity through a pinhole located about the point of (0, 0, 0). Such a hole generates a Gaussian wave packet

\[ \phi(x) = (2\pi\Delta^2)^{-1/4}e^{-\frac{x^2}{4\Delta^2}} \]  

in the \( x \) direction, with \( \Delta \) being the root-mean-square (rms) width of the wave packet. It is easily to confirm the following results of the initial state. The average position \( \langle x \rangle = \int_{-\infty}^{+\infty} \phi^* x \phi dx = 0 \) and its uncertainly \( \Delta = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \Delta \). The average momentum \( \langle p \rangle = \int_{-\infty}^{+\infty} \phi^* (-i\hbar \partial/\partial x) \phi dx = 0 \) and the momentum uncertainly \( \Delta_p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \hbar/(2\Delta) \). Physically, the uncertainly \( \Delta \) (or \( \Delta_p \)) means the main distribution range of particles’ positions (or momentums). Out of this range, the probability to find the particles is negligible. Below, we study the vacuum field (in the cavity 1) induced change on the initial wave packet \( \phi(x) \) within a very short duration (i.e., the free diffraction of atom is negligible).
FIG. 1: A sketch for weak measurement process. The two-levels atoms are prepared in the certain internal state \( |S_0\rangle \) (i.e., the pre-selection), and pass through a pinhole with an average velocity along the \( z \) direction. This pinhole generates a Gaussian wave packet \( \phi(x) \) of atoms’ external motion in the \( x \) direction. The vacuum field (with a \( x \)-directional gradient) in cavity 1 generates a weak coupling between the atoms’ internal states and the external \( x \)-directional motions. The cavity 2, with the classical light, resonantly excites the atoms and generates the desirable single-qubit operation \( \hat{U}_s \). The applied voltage \( \pm V \) ionize the atoms in excited state (similar to the procedure in the experiments of Haroche group [36–38]) and leave the ground state to be detected. In the post-selected ensemble of ground state, the atoms have a shift (along \( x \) direction) of the average position on the deposition plate, due to the vacuum induced coupling in cavity 1. This shift can be described by the so-called weak value, which depends on the pre-selection \( |S_0\rangle \) and the single-qubit operation \( \hat{U}_s \).

In the cavity 1, we suppose the field taking the form [35]

\[
\vec{E} = \vec{\tau}E_0 \sin(kx + kx_0)(\hat{a}^{\dagger} + \hat{a})
\]  

which excites the coming atoms. Here, \( \vec{\tau}, E_0 \) and \( k \) are respectively the polarization-vector, amplitude, and wave-number of the standing wave. \( \hat{a}^{\dagger} \) and \( \hat{a} \) are respectively the creation and annihilation operators of the cavity ground mode with the frequency \( \omega_c \). Consequently, the electric dipole interaction between the atom and the ground mode can be written as

\[
\hat{H}_{\text{int}} = \hbar \Omega_0 \sin(kx + kx_0)(\hat{a}^{\dagger} + \hat{a})\hat{\sigma}_x
\]  

with the so-called Rabi frequency \( \Omega_0 = E_0 \mu / \hbar \) [35]. Where, \( \hbar \) is the Planck constant divided by \( 2\pi \), \( \hat{\sigma}_x = |e\rangle \langle g| + |g\rangle \langle e| \) is the transition operator of the two-level atom with the ground state \( |g\rangle \) and exciting state \( |e\rangle \), and \( \mu \) is the transition matrix element of the two-level atom.
We consider \( kx \ll 1 \), then the Hamiltonian (4) can be approximately written as

\[
\hat{H}_{\text{int}} = \hbar \Omega (x + x_c) (\hat{a}^\dagger + \hat{a}) \hat{\sigma}_x
\]  

(5)

with the constants: \( \Omega = k \cos(kx_0)\Omega_0 \) and \( x_c = \tan(kx_0)/k \). Here, we have used the well-known trigonometric function \( \sin(kx + kx_0) = \cos(kx_0) \sin(kx) + \cos(kx) \sin(kx_0) \) and neglected the high order of \((kx)^2\). Note that, \( kx \ll 1 \) means that the range of atomic motion in \( x \) direction is much smaller than the wave length of the ground mode. The range of \( x \) depends on the initial uncertainty \( \Delta \), and the wave packet spread (i.e., the diffraction). As mentioned earlier, the diffraction of the atom is negligible as the duration of the cavity-atom interaction is very short, i.e., \( t \ll m \Delta^2/\hbar \). Thus, the value of \( x \) is on the order of its initial uncertainly \( \Delta \) (e.g., 10 \( \mu \)m), which can be much smaller than the wave length of cavity ground mode (e.g., 1 cm [38]).

The total Hamiltonian of atom reads

\[
\hat{H}_x = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \frac{\hbar \omega_a}{2} \hat{\sigma}_z + \hbar \omega_c (\hat{a}^\dagger \hat{a} + \frac{1}{2}) + \hbar \Omega (x + x_c) (\hat{a}^\dagger + \hat{a}) \hat{\sigma}_x
\]  

(6)

Where, the first term describes the atoms’ external motion. The second term describes the atomic two internal levels (defined by the Pauli operator \( \hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g| \) with the transition frequency \( \omega_a \)). The third term is the free Hamiltonian of the cavity. The last term describes the coupling between the considered three degrees of freedom, e.g., a position-dependent Jaynes-Cummings interaction. In the momentum space, the Hamiltonian (6) reads

\[
\hat{H}_p = \frac{p^2}{2m} + \frac{\hbar \omega_a}{2} \hat{\sigma}_z + \hbar \omega_c (\hat{a}^\dagger \hat{a} + \frac{1}{2}) + \hbar \Omega (x + x_c) (\hat{a}^\dagger + \hat{a}) \hat{\sigma}_x
\]  

(7)

with the well-known operator \( \hat{x} = i\hbar \partial/\partial p \). This Hamiltonian can be further written as

\[
\hat{H}_p = \frac{\hbar \omega_a}{2} \hat{\sigma}_z + \hbar \omega_c (\hat{a}^\dagger \hat{a} + \frac{1}{2}) + \hbar \Omega (\hat{x} + x_c + \frac{pt}{m}) (\hat{a}^\dagger + \hat{a}) \hat{\sigma}_x
\]  

(8)

in the rotating frame defined by \( \hat{U}_1 = \exp[-ip^2t/(2m\hbar)] \). With such a transform, the free term \( p^2/(2m) \) is eliminated. Considering the atom rapidly crosses the cavity (i.e., the effective interaction duration \( t \) is sufficiently short), there is an impulse atom-field interaction corresponding to the von Neumann Hamiltonian of quantum measurement. Thus, \( pt/m \rightarrow 0 \) \[1, 2\], and the Hamiltonian (8) can be reduced to

\[
\hat{H}_p = \frac{\hbar \omega_a}{2} \hat{\sigma}_z + \hbar \omega_c (\hat{a}^\dagger \hat{a} + \frac{1}{2}) + \hbar \Omega (\hat{x} + x_c) (\hat{a}^\dagger + \hat{a}) \hat{\sigma}_x
\]  

(9)
In the rotating frame defined by \( \hat{U}_2 = \exp[-i\omega_c t(\hat{a}^\dagger \hat{a} + 1/2) - it\omega_c \hat{\sigma}_z/2] \), the Hamiltonian (9) reduces to
\[
\hat{H}_p = \hbar \Omega (\hat{x} + x_c) \left[ \hat{a}^\dagger \hat{\sigma}_- e^{-i\delta t} + \hat{a} \hat{\sigma}_+ e^{i\delta t} \right]
\] (10)
with the detuning \( \delta = \omega_a - \omega_c \). Here, the usual rotating wave approximation is preformed, i.e., neglected the terms relating to the sum-frequency \( \omega_a + \omega_c \).

The time-evolution operator of the Hamiltonian (10) can be given by the Dyson-series:
\[
\hat{U}(t) = 1 + \left( -\frac{i}{\hbar} \right) \int_0^t \hat{H}_p(t_1) dt_1 + \left( -\frac{i}{\hbar} \right)^2 \int_0^t \hat{H}_p(t_1) \int_0^{t_1} \hat{H}_p(t_2) dt_2 dt_1 + \cdots
\] (11)
Under the conditions of large detuning: \( \Omega \ll \delta \), the time-evolution operator can be approximately written as
\[
\hat{U}(t) \approx e^{-\frac{i}{\hbar} \hat{H}_{\text{eff}} t}
\] (12)
with the effective Hamiltonian \( \hat{H}_{\text{eff}} = (\hbar \Omega^2 / \delta)(\hat{x} + x_c)^2(\hat{a}^\dagger \hat{a} \hat{\sigma}_z + |e\rangle \langle e|) \). Considering the cavity is in the vacuum state \(|0\rangle\), i.e., \( \hat{a}^\dagger \hat{a} |0\rangle = 0 \), the effective Hamiltonian reduces to
\[
\hat{H}_{\text{eff}} = \hbar g_0 (\hat{x} + x_c)^2 |e\rangle \langle e|
\] (13)
with \( g_0 = \Omega^2 / \delta \). Obviously, this Hamiltonian describes a position dependent vacuum Rabi splitting, and the time-dependent state of the system reads
\[
\psi = e^{-i2g_0tx_c |e\rangle \langle e|} e^{-ig_0t^2 |e\rangle} \psi_i
\] (14)
with \( \psi_i = \exp(-ig_0tx_c^2 |e\rangle \langle e|) \psi(0) \) and the initial state \( \psi(0) \).

III. THE WEAK VALUES

As mentioned earlier, the \( x \)-directional motion of the atom is initially in a Gaussian state \( \phi(x) \), which can be written as \( \phi(p) = \phi_0 \exp(-\Delta^2 p^2 / \hbar^2) \) in the momentum space, with the normalized coefficient \( \phi_0 = [2\Delta^2 / (\pi \hbar^2)]^{1/4} \). Obviously, it can be rewritten as \( \phi(\tilde{p}) = \phi'_0 \exp(-\tilde{p}^2) \) with the coefficient \( \phi'_0 = (2\pi)^{-1/4} \Delta \tilde{p}^{-1/2} \) and the dimensionless numbers \( \tilde{p} = p \Delta / \hbar \). Then, we can write the final state (14) as
\[
\psi = e^{ig_c |e\rangle \langle e| \frac{\partial}{\partial \tilde{p}} e^{ig'_c |e\rangle \langle e| \frac{\partial}{\partial \tilde{p}}} \phi(\tilde{p}) |S_i\rangle}
\] (15)
by using the relation \( \hat{x} = i\hbar \partial / \partial p = i\Delta \partial / \partial \tilde{p} \). Here, \( g_c = 2g_0tx_c\Delta \) and \( g_c' = g_0t\Delta^2 \) are the dimensionless coupling parameters, and \( |S_i\rangle = \exp (-ig_0tx_c^2|e\rangle \langle e|) |S_0\rangle \) with \( |S_0\rangle \) being the initial state of the atomic qubit. We consider \( \Delta \ll x_c \), i.e., \( g_c' \ll g_c \), then the state (15) can be approximately written as
\[
\psi = e^{g_c|e\rangle \langle e| \partial / \partial \tilde{p}} |S_i\rangle.
\]
(16)

For simplicity, we re-define \( |S_i\rangle = \alpha |g\rangle + \beta \exp(i\theta) |e\rangle \) being the initial internal state of the atoms (which can be generated by the well-known single qubit operations). Here, \( \theta \) is the phase of the superposition state, and \( \alpha \) and \( \beta \) are the superposition coefficients (real number) satisfying the normalized condition \( \alpha^2 + \beta^2 = 1 \). Immediately, we have the following state evolution
\[
\phi(p)|S_i\rangle \longrightarrow \alpha \phi(p)|g\rangle + \beta e^{i\theta} \phi(p + \hbar g_c/\Delta)|e\rangle.
\]
(17)

Then, we have the expectation value of momentum:
\[
\langle p \rangle \approx \beta^2 \frac{\hbar g_c}{\Delta} = 2\beta^2 g_c \Delta p
\]
(18)

The Eq. (18) means that, the vacuum in cavity 1 generates a shift \( \langle p \rangle - 0 = \langle p \rangle \) to the average momentum of atoms. Because \( \beta^2 \leq 1 \), the shift \( \langle p \rangle \rightarrow 0 \) for the very weak coupling \( g_c \rightarrow 0 \).

We now use the weak measurement technique to amplify the shift \( \langle p \rangle \). First, we perform a single-qubit operation \( \hat{U}_s = \exp (-i\eta \hat{\sigma}_x) \) on the state (16). Alternatively, this single-qubit operation could be realized by the classical resonant light (e.g., the cavity 2 in Fig. 1), and thus the parameter \( \eta = \Omega_s t \) with \( \Omega_s \) being the Rabi frequency (which depends on the strength of the applied classical light). Consequently, we have the final state
\[
\psi' = \hat{U}_s \psi = \hat{U}_s e^{g_c|e\rangle \langle e| \partial / \partial \tilde{p}} \phi(\tilde{p}) |S_i\rangle
\]
(19)

\[
= \hat{U}_s \left[ 1 + g_c(|e\rangle \langle e|) \frac{\partial}{\partial \tilde{p}} + \frac{g^2_c}{2} (|e\rangle \langle e|)^2 \frac{\partial^2}{\partial \tilde{p}^2} + \cdots \right] \phi(\tilde{p}) |S_i\rangle.
\]

Second, we post-select the internal state \( |S_f\rangle \), then the external motion of the atoms collapses on the wave function
\[
\psi'_s = \langle S_f| \psi' \rangle = \langle S_f| \hat{U}_s |S_i\rangle \left[ 1 + g_c A_w \frac{\partial}{\partial \tilde{p}} + \frac{g^2_c A_w}{2} \frac{\partial^2}{\partial \tilde{p}^2} + \cdots \right] \phi(\tilde{p})
\]
(20)

with
\[
A_w = \frac{\langle S_f| (\hat{U}_s|e\rangle \langle e|) |S_i\rangle}{\langle S_f| \hat{U}_s |S_i\rangle}.
\]
(21)
Here, we have used the relation $(|e⟩⟨e|)^n = |e⟩⟨e|$ with $n = 1, 2, 3, \ldots$.

Considering the very weak interactions, i.e., $g_c \ll 1$ and $g_c|A_w| \ll 1$, then the wave function (20) can be approximately written as

$$\psi_s = \frac{\psi_s}{⟨S_f|\hat{U}_s|S_i⟩} = \left(1 + g_c A_w \frac{∂}{∂p}\right) \phi(\tilde{p})$$

$$= \phi(p) - \frac{2g_c Δ}{\hbar} A_w p \phi(p)$$

$$= \phi(p) - \frac{2g_c Δ}{\hbar} Re[A_w] p \phi(p) - i \frac{2g_c Δ}{\hbar} Im[A_w] p \phi(p).$$

Here, the high order of $g_c^2$ has been neglected and then $A_w$ corresponds to the so-called weak value, and $Re[A_w]$ and $Im[A_w]$ are respectively the real and imaginary parts of the weak value. With this approximation, the probability for successfully post-selecting $|S_f⟩$ reads

$$P = |⟨S_f|\psi'⟩|^2 \approx |⟨S_f|\hat{U}_s|S_i⟩|^2.$$

Immediately, we have the expectation value of momentum:

$$\langle \hat{p} \rangle_s ≈ \int_{-∞}^{∞} ψ_s^* p ψ_s dp \approx \hbar \frac{g_c}{Δ} Re[A_w] = 2g_c Δ p Re[A_w].$$

This means that, within the post-selected ensemble the shift of average momentum $⟨p⟩_s - 0 = ⟨p⟩_s$ is proportional to the real-part of the weak value. On the other hand, in the position space, the wave function (22) reads

$$\phi_s ≈ \int_{-∞}^{∞} ψ_s^* x |p⟩ dp$$

$$= \frac{1}{√2π\hbar} \int_{-∞}^{∞} \phi(p) e^{ipx/\hbar} dp + \frac{1}{√2π\hbar} \frac{g_c A_w}{Δ} \int_{-∞}^{∞} e^{ipx/\hbar} \frac{∂\phi(p)}{∂p} dp$$

$$= \phi(x) - \frac{1}{√2π\hbar} \frac{g_c A_w}{Δ} \int_{-∞}^{∞} \phi(p) e^{ipx/\hbar} dp$$

$$= \phi(x) - i \frac{1}{√2π\hbar} \frac{g_c A_w x}{Δ} \int_{-∞}^{∞} \phi(p) e^{ipx/\hbar} dp$$

$$= (1 - i \frac{g_c A_w x}{Δ}) \phi(x).$$

Then, we have the expectation value of position

$$⟨x⟩_s = \int_{-∞}^{∞} ϕ_s^* x ϕ_s dx ≈ 2\frac{g_c}{Δ} Im[A_w] \int_{-∞}^{∞} \phi(x) x^2 \phi(x) dx = 2g_c Δ Im[A_w].$$

(25)
This means that: within the post-selected ensemble the shift of average position \( \langle x \rangle_s - 0 = \langle x \rangle_s \) is proportional to the imaginary-part of the weak value.

Specially, if the internal state \( |S_f \rangle = |g \rangle \) is post-selected, then the weak value reads

\[
A_w = \frac{\langle g | (\hat{U}_s |e \rangle |e \rangle |S_i \rangle}{\langle g | \hat{U}_s |S_i \rangle} = \frac{\beta e^{i\theta} \langle g | \hat{U}_s |e \rangle}{\alpha \langle g | \hat{U}_s |g \rangle + \beta e^{i\theta} \langle g | \hat{U}_s |e \rangle}.
\]

In principle, this post-selection could be realized by the field-ionization \([36–38]\). Since \( |e \rangle \) and \( |g \rangle \) have the different ionization energies, the ionization is state selective. Supposing the atoms only in exciting state \( |e \rangle \) are effectively ionized with the applied moderate electric field, and then the exciting state atoms will be accelerated in \( y \)-direction. Consequently, the ground state atoms will arrive the plate to be finally detected, as it shown in Fig. 1.

As \( \hat{U}_s |g \rangle = \cos(\eta) |g \rangle - i \sin(\eta) |e \rangle \) and \( \hat{U}_s |e \rangle = \cos(\eta t) |e \rangle - i \sin(\eta) |g \rangle \), the Eq. (26) reads

\[
A_w = \frac{-i \beta e^{i\theta} \sin(\eta)}{\alpha \cos(\eta) - i \beta e^{i\theta} \sin(\eta)} = \frac{1}{A e^{i\theta} + 1}
\]

with \( A = \alpha \cos(\eta)/[\beta \sin(\eta)] \) and \( \theta = (\pi/2) - \eta \). Consequently, we have

\[
\text{Re}[A_w] = \frac{1 + A \cos(\theta)}{A^2 + 2A \cos(\theta) + 1}
\]

and

\[
\text{Im}[A_w] = -\frac{A \sin(\theta)}{A^2 + 2A \cos(\theta) + 1}.
\]

These values can be amplified by properly adjusting the parameters \( A \) and \( \theta \). For example, if \( \cos(\theta) = 1 \), then \( \text{Re}[A_w] = 1/(1 + A) \to \infty \) with \( A \to -1 \). If \( A = -\cos(\theta) \), then \( \text{Im}[A_w] = \cot(\theta) \to \infty \) with \( \theta \to 0 \). According to Eqs. (23) and (25), we have

\[
\frac{\langle p \rangle_s}{2\Delta_p} \approx g_w \text{Re}[A_w], \quad \frac{\langle x \rangle_s}{2\Delta} \approx g_w \text{Im}[A_w].
\]

These equations mean that the controllable \( \text{Re}[A_w] \) or \( \text{Re}[A_w] \) could generate a large shift to the atoms’ average momentum or position, respectively.

We would like to emphasize the following facts. (I) There is no free lunch. The probability \( P \approx |\langle S_f | \hat{U}_s |S_i \rangle|^2 \) for successfully post-selecting \( |S_f \rangle \) decreases rapidly with the increasing \( \text{Re}[A_w] \) or \( \text{Im}[A_w] \). Thus, to get an observable effect the experimenter must use sufficiently more atoms. (II) The shifts \( \langle x \rangle_s \) and \( \langle p \rangle_s \) can not be infinitely amplified, as the weak values were obtained under the weak interaction condition of \( g_c |A_w| \ll 1 \). That is, the displacement of average position/momentum should be less than the rms width of the initial Gaussian wave.
packet, i.e., $\langle x \rangle_s/(2\Delta) \ll 1$ and $\langle p \rangle_s/(2\Delta_p) \ll 1$. Within these regimes, the relevant shifts, e.g., $\langle x \rangle_s = 0.1 \mu m$ and $\Delta = 10 \mu m$, should be experimentally detectable with the modern microscope techniques.

IV. CONCLUSION

In this theoretical paper, we show a position-dependent vacuum Rabi splitting: its Hamiltonian is similar to the standard one of weak measurement theory. This Hamiltonian means a vacuum induced coupling between the internal and external motions of the free atoms. After this coupling, we preform a single-qubit rotation on the atomic internal states and consequently post-select an internal state. Then we obtain a weak value, which could be used to amplify the vacuum induced shifts of the average position or momentum of atoms. These amplified shifts are limited in the certainly ranges and needed sufficiently more atoms to be effectively detected in experiments.

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