Discrete Element Modelling of Rock Creep in Deep Tunnels using Rate Process Theory

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Abstract. Rock creep behaviour is a key aspect of many engineering projects, such as deep tunnels in which squeezing problems could occur. Many theories have been published in the literature to reproduce rock creep behaviour; however, most of them are not able to capture the last phase of creep (i.e., tertiary creep, or the accelerating strains that occur prior to failure). In this work, the Distinct-Element Method (DEM) approach is employed, in conjunction with Rate Process Theory (RPT), to simulate the effect of rock creep in deep tunnels. To do that, the DEM models are constructed using particles, whose interactions are simulated with a hybrid mixture of the Flat Joint Contact Model (FJCM) and the Linear Model (LM) contact models; the RPT is implemented into DEM models using a Visual C++ function. Results show that the DEM plus RPT combination can suitably reproduce the tunnel convergences due to rock creep.

1. Introduction
The time-dependent (creep) behaviour of rocks is a key aspect on the stability and performance of deep tunnels; in particular, in deep tunnels excavated in soft rocks [1][2]. Creep is a progressive deformation with time that occurs in rocks (or other materials) subjected to a constant stress level [3]. Figure 1 presents an idealized creep behaviour under a stress increase that is kept constant afterwards; it consists of an instantaneous elastic strain due to the applied stress (1→2); then, if such stress is kept constant with time (and it is high enough to lead the full process to develop), the following 3-stages can occur: (i) Primary creep, where strains increase with a decreasing strain rate (2→3), (ii) Secondary creep, with a quasi-constant strain rate (3→4), and (iii) Tertiary creep, with a non-linear strain acceleration until rock failure is observed (4→5).

Several theories have been developed in the literature to analyze creep behaviour observed in rock laboratory tests [5], and to predict creep tunnel convergences [2][6]. For instance, the Burger model has been used to simulate the creep behaviour of intact siltstone samples [7]; however, such approach cannot reproduce the rock failure due to tertiary creep. As an alternative, recent works have demonstrated that the Rate Process Theory (RPT) proposed by Eyring [8] could be successfully employed to simulate all stages of creep in soils [3] or in rocks [4]. The aim of this paper is to examine the applicability of RPT, and its implementation through the Distinct-Element Method (DEM), to simulate rock creep convergences in deep tunnels.
2. Rate Process Theory (RPT)

The RPT was formulated by Eyring [8] to describe the motion of particles (at molecular level) as a function of the energy barriers that control their equilibrium positions; therefore, to obtain a relative displacement between particles, an activation energy is needed. On the basis of RPT, previous works [3][4] have simulated creep behavior using the following constitutive equation:

\[ \dot{s} = \lambda \frac{2kT}{h} e^{-\frac{\Delta F}{RT}} \sinh \left( \frac{1}{2kT n_1 f^n} \right) \]  

(1)

where \( \dot{s} \) is the sliding velocity at each contact between particles, \( \lambda \) is the flow unit, \( k \) is Boltzmann’s constant (1.381 \times 10^{-23} \text{ J K}^{-1}), \( T \) is the absolute temperature (K), \( h \) is Planck’s constant (6.626 \times 10^{-34} \text{ J s}), \( \Delta F \) is the activation energy (kJ mol\(^{-1}\)), \( R \) is the universal gas constant (8.314 \times 10^{-3} \text{ kJ K}^{-1} \text{ mol}^{-1}), n_1 \) is the number of bonds per unit of normal contact force (bonds/N), and \( f^\tau / f^n \) is the tangential-to-normal force at particles contacts, which could be expressed as the friction coefficient at such contacts (\( \mu = f^\tau / f^n \)). The Eq. (1) can be rewritten as [4]:

\[ \dot{s} = \alpha \sinh(\beta \mu) \]  

(2)

where:

\[ \alpha = \lambda \frac{2kT}{h} e^{-\frac{\Delta F}{RT}} \]  

(3)

\[ \beta = \frac{1}{2kT n_1} \frac{1}{f^n} \]  

(4)

As explained later, Eq. (2) is implemented in the numerical model as a Visual C\(^++\) function that modify the friction coefficients between particles.

3. Numerical Simulation

3.1. Particle Flow Code (PFC)

PFC [10] is a commercial code implementing the DEM approach employed in this work. In PFC, the media is discretized by a set of rigid and finite-sized particles that can translate and rotate independently to each other; the motion of the particles follows Newton’s second law and, together with a force-displacement law, they are both used to control the interactions between particles [10].
Following Gutiérrez-Ch et al. [4], the creep behavior in this work is simulated through: (i) a hybrid model composed of particles interacting with the Linear Model (LM) [10] and the Flat-Joint Contact Model (FJCM) [11], and with a LM-to-FJCM contact ratio of 17.5%; (ii) the implementation of RPT into such models. To do that, the micromechanical parameters of the LM and of FJCM should be calibrated first. Usually, such calibration is conducted by comparison of DEM simulation results against laboratory tests—e.g., uniaxial compressive strength (UCS) test, direct tensile test, etc.—[10][12][13]. This calibration was conducted by the authors in a previous work where the creep behavior in intact slate samples was simulated and, for brevity, is not presented again here (see [4] for details). The calibration results for the LM and for the FJCM are listed in Table 1. Note that the cohesion, the tensile strength, and the control gap reported in Table 1 is modified from [4] to obtain a UCS of 4.5 MPa; thus, the influence of discontinuities that weaken the rock mass—instead of using the intact UCS, as calibrated in [4]—is considered.

Table 1. Particle and micro-mechanical properties fitted from the 2D DEM UCS tests (data from [4]).

| Particle micromechanical properties | FJCM micromechanical properties |
|-------------------------------------|---------------------------------|
| Effective modulus, \( E^* \) (GPa) | 34.15                           |
| Normal-to-shear stiffness ratio, \( k^* = k_n/k_s \) | 1.80                            |
| Friction angle, \( \phi \) (°) | 30                              |
| Ball density, \( \rho \) (kg/m\(^3\)) | 2737                            |
| Minimum radius, \( R_{min} \) (mm) | 0.60                            |
| \( R_{max}/R_{min} \) | 1.50                            |

| LM micromechanical properties |
|--------------------------------|
| Effective modulus, \( E_{LM}^* \) (GPa) | 34.15 |
| Normal-to-shear stiffness ratio, \( k_{LM}^* \) | 1.80 |

| Hybrid model |
|--------------|
| Control gap, \( g_c \) (mm) \( \approx 1.15 \times 10^{-2} \) |

\( ^* \) Modified to obtain a uniaxial compressive strength of 4.5 MPa.

3.2. Implementation of the RPT

Next, the RPT is implemented into the contacts laws (i.e., LM and FJCM) of the DEM models, as follows (see Figure 2): (i) the sliding velocity (\( \dot{s} \)) is computed, (ii) the friction coefficient (\( \mu \)) is calculated using Eq. (2) with the parameters listed in Table 2, and (iii) \( \mu \) is modified depending on the contact model. For additional details see [3][4].

Table 2. Constants employed for RPT implementation (data from [4]).

| Variable | Value |
|----------|-------|
| Absolute temperature, \( T \) (K) | 293 |
| Flow unit, \( \lambda \) (m) | \( 3 \times 10^{-10} \) |
| Activation energy, \( \Delta F \) (kJ/mol) | 100 |
| Number of bonds per unit of normal contact force, \( n_t \) (bonds/N) | \( 1 \times 10^9 \) |
3.3. Set-up of the Numerical Model

At this stage, the creep convergence of a deep tunnel with a 5m radius is examined. To do that, a 2D DEM model of a circular tunnel is employed (see Figure 3). To take advantage of the symmetry of the problem and to reduce the computational cost, note that: (i) a quarter of section (rather than the whole tunnel section) is considered, and (ii) external stresses to simulate the in-situ horizontal ($\sigma_h$) and vertical ($\sigma_v$) stresses acting on the tunnel, are employed.

The methodology to simulate the creep behaviour of the 2D DEM tunnel model developed herein can be summarized as follows:

i. Initial particle assembly: the container that represents the geometry of the 2D DEM tunnel model is filled with an assembly of particles that are randomly placed. To minimize the computational cost, the rock has been discretized into three zones, with the smallest particles located in the area adjacent the tunnel (see Figure 3).

ii. Application of the linear and flat-joint contact model: the LM and the FJCM (with the micromechanical properties listed in Table 1) are installed. According to Gutiérrez-Ch et al. [4], a LM-to-FJCM contact ratio of about 17.5% is used to simulate rock creep behaviour in this case. To achieve that contact distribution, a control gap of $g_c = -7.75\times10^{-1}$ mm is employed as shown in Figure 4. Additional details are available in [4].

iii. Initialization of in-situ stress: then, a gravity load and the pressure loads are applied on the model to induce the in-situ stress. The pressure loads are computed depending on the tunnel depth (i.e., on the overburden stress acting on the model), and a lateral earth pressure coefficient of $K_o = 1$ (i.e., $\sigma_x = \sigma_y$), is used. Such stresses are applied on the top and right sides the model, while their left and bottom sides are fixed (see Figure 3).

iv. Removal of the tunnel wall: subsequently, a FISH function [10] is used to register the reaction forces acting on each particle-wall (of the tunnel) contact. After that, the tunnel wall is removed, and a force equivalent to 0.4% of the value recorded by the FISH function is applied on each particle (i.e., on blue particles of Figure 3); thus, the sudden fall of particles as a consequence of removing the tunnel wall, is avoided.

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**Figure 2.** Flowchart of RPT implementation at each ball-ball contact.
v. Creep simulation: Finally, the DEM tunnel model is allowed to creep while the external stresses are kept constant for a specified number of steps. At this stage, the RPT is implemented (see Section 3.2) for each cycle, tunnel convergences are measured by tracking the displacement of the “gauge particle” located in the rock body (red particle, see Figure 3).

Figure 3. 2D DEM tunnel model.

(a) 
(b) 
(c) 

Figure 4. Application of the FJCM and LM contact models: (a) idealization of the gap ($g$) between particles, (b) close-up view of the contact network, (c) final contact network of tunnel model.

4. Results
To evaluate the creep tunnels convergences, three numerical simulations were conducted on 2D DEM models developed with the methodology described in Section 3.3. To simulate different tunnel depths, in-situ stresses replicating $h$ =1000 m, 2000 m and 4000 m depths, with an average unit weight of 27 kN/m$^3$, were used.
Figure 5 shows the evolution of the tunnel strain ($\varepsilon$) computed from the “gauge particle” (see Figure 3) during DEM simulations for all models. The tunnel strain was computed as:

$$\varepsilon(\%) = \left( \frac{d_i - d_f}{d_i} \right) \times 100$$  (5)

where $d_i$ and $d_f$ are the initial and final distances, respectively, from the center of the tunnel to the center of the “gauge ball”. Note that Eq. (5) takes into account the convergence of the whole tunnel section; for instance, the tunnel strain 1-1’ in Figure 5 represents the creep deformation of the whole tunnel section, where 1’ is the location symmetrical to “gauge ball” 1.

As it can be seen in Figure 5, an initial instantaneous strain occurs in each model after the tunnel wall is removed. Next, the strain rate decreases with increasing number of cycles (i.e., primary creep). Then, the models with $h = 1000$ m and 2000 m show a quasi-constant strain rate (i.e., secondary creep), while the strain rate for the model with $h = 4000$ m increases rapidly and failure occurs (i.e., tertiary creep). These numerical results clearly show that the DEM and RPT approach can reproduce tunnel convergences where all phases of creep behaviour are observed.

**Figure 5.** Tunnel convergences for 2D DEM tunnel simulations: strain computed with the “gauge particle” vs steps.

5. Conclusions
Rock creep behavior is a key aspect to be considered in deep tunnels excavated in weak rocks. In this work, the creep convergences that occur in such tunnels were investigated using 2D DEM models which include a Rate Process Theory implementation.

Preliminary numerical results suggest that such DEM + RPT methodology developed herein could be employed to simulate the deformations of deep tunnels, due to their ability to consider the creep behavior of the surrounding rock. Also, DEM results demonstrate that the proposed approach can reproduce all phases of creep behavior, including the tertiary creep. Further analysis will be conducted to confirm these findings and the results will be presented elsewhere.

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References

[1] Jimenez R and Recio D 2011 A linear classifier for probabilistic prediction of squeezing conditions in Himalayan tunnels Eng. Geol. 121 101–109. DOI: 10.1016/j.enggeo.2011.05.006.

[2] Huang M, Zhan J W, Xu C S and Jiang S 2020 New creep constitutive model for soft rocks and its application in the prediction of time-dependent deformation in tunnels Int. J. Geomech. 20(7) 04020096. DOI: 10.1061/(ASCE)GM.1943-5622.0001663.

[3] Kuhn M R and Mitchell J K 1992 Modelling of soil creep with the discrete element method. Eng. Comput. 9(2) 277–287. DOI:10.1108/eb023866.

[4] Gutiérrez-Ch J G, Senent S, Estebanez E and Jimenez R 2020 Discrete element modelling of rock creep behaviour using rate process theory Can. Geotech. J. Just-IN. DOI:10.1139/cgj-2020-0124.

[5] Zhang Y, Xu W Y, Shao J F, Zhao H B, and Wang W 2015 Experimental investigation of creep behaviour of clastic rock in Xiangjiaba Hydropower Project. Water Sci. Eng. 8(1) 55–62. DOI:10.1016/j.wse.2015.01.005.

[6] Feng X, Jimenez R, Zeng P, and Senent S 2019 Prediction of time-dependent tunnel convergences using a Bayesian updating approach. Tunn. Undergr. Space Technol. 94 103118. DOI: 10.1016/j.tust.2019.103118.

[7] Hamza O, and Stace R 2018 Creep properties of intact and fractured muddy siltstone. Int. J. Rock Mech. Min. Sci. 106 109–116. DOI: 10.1016/j.ijrmms.2018.03.006.

[8] Eyring H 1936 Viscosity, plasticity and diffusion as examples of absolute reaction rates. J. Chem. Phys. 4(4) 283–291. DOI:10.1063/1.1749836.

[9] Kwok C Y, and Bolton, M D 2010 DEM simulations of thermally activated creep in soils. Géotechnique 60(6) 425–433. DOI:10.1680/geot.2010.60.6.425.

[10] Itasca Consulting Group Inc 2014 PFC Manual, Version 5.0. (Minneapolis, Minnesota).

[11] Potyondy D O 2012 A flat-jointed bonded-particle material for hard rock. In: 46th US Rock Mechanics/Geomechanics Symposium. Chicago, Illinois.

[12] Gutiérrez-Ch J G, Senent S, Melentijevic S, and Jimenez R 2018 Distinct element method simulations of rock-concrete interfaces under different boundary conditions. Eng. Geol. 240(5) 123–139. DOI: 10.1016/j.enggeo.2018.04.017.

[13] Gutiérrez-Ch J G, Senent S, Melentijevic S, and Jimenez R 2021 A DEM-based factor to design rock-socketed piles considering socket roughness. Rock Mech. Rock Eng. DOI: 10.1007/s00603-020-02347-1.