A NEW EQUILIBRIUM FOR ACCRETION DISKS AROUND BLACK HOLES

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ABSTRACT

Accretion disks around black holes in which the shear stress is proportional to the total pressure, the accretion rate is more than a small fraction of Eddington, and the matter is distributed smoothly are both thermally and viscously unstable in their inner portions. The nonlinear end state of these instabilities is uncertain. Here a new inhomogeneous equilibrium is proposed that is both thermally and viscously stable. In this equilibrium, the majority of the mass is in dense clumps, while a minority reaches temperatures $\sim 10^9$ K. The requirements of dynamical and thermal equilibrium completely determine the parameters of this system, and these are found to be in good agreement with the parameters derived from observations of accreting black holes, both in active galactic nuclei and in stellar binary systems.

Subject headings: accretion, accretion disks — galaxies: active — X-rays: general

1. INTRODUCTION

If the dissipation rate in an accretion disk is proportional to the pressure (Shakura & Sunyaev 1973), the disk is both viscously (Lightman & Eardley 1974) and thermally unstable (Shakura & Sunyaev 1976) wherever the radiation pressure exceeds the gas pressure. Unfortunately, this is exactly what happens in the inner parts of disks around black holes whenever the accretion rate is greater than a small fraction of Eddington. Because the innermost radii are where most of the energy is released, these instabilities may disrupt the most interesting portions of these disks.

The ultimate result of these instabilities (i.e., the actual structure of disks occurring in nature) remains unknown. Shapiro, Lightman, & Eardley (1976) suggested that the endpoint is a two-temperature fluid in which the ions are much hotter than the electrons, with the contrast maintained by the inefficiency of heat transfer by Coulomb collisions. However, this equilibrium is also thermally unstable (Piran 1978). Ichimaru (1977), Rees et al. (1982), and Narayan & Yi (1995) have argued that the two-temperature solution can be stabilized by the inward advection of heat. However, ions are hotter than electrons only if most of the dissipated heat is given to the ions, a much disputed point (e.g., Bisnovatyi-Kogan & Lovelace 1997; Quataert 1997; Blackman 1997; Gruzinov 1997). In addition, there may be particle-wave interactions that greatly accelerate ion-electron heat transfer (Begelman & Chiueh 1988). Another possible stable equilibrium is one in which the dissipation is concentrated in a disk corona (Svensson & Zdziarski 1994). However, there is no known mechanism that can take most of the accretion energy liberated inside the body of the disk and transport it with only minimal losses to its surface.

The thermal instability [whose growth rate is larger than that of the viscous instability by $\sim (\nu h)^2$, for disk thickness $h$ at radius $r$] is driven by a feedback loop in which increased radiation pressure causes increased dissipation, which then, because the surface density, and hence the optical depth, is fixed on the thermal timescale, leads to greater radiation pressure. If increasing pressure caused the effective optical depth to diminish, the instability could be quenched. Clumping of the disk matter would lead to just this result because photons find their way out through the most transparent channels.

In this Letter, a new equilibrium is proposed, motivated by the qualitative argument of the previous paragraph. In this equilibrium, the disk remains geometrically thin, but most of the disk mass is found in very dense clumps, with only a small fraction left behind to form a volume-filling substrate through which the clumps move. The multiple requirements of dynamical and thermal equilibrium strongly constrain the allowed parameters of such an arrangement; as shown in § 3, these compare well with those inferred empirically.

Others have proposed that accretion disks around black holes might contain numerous small clumps, but their efforts have all focused on the internal state of the clumps and how they reradiate the energy they absorb (Guilbert & Rees 1988; Celotti, Fabian, & Rees 1992; Kuncic, Blackman, & Rees 1996; Kuncic, Celotti, & Rees 1997). Here we examine not just the thermal properties of the clumps but the overall equilibrium of the accretion disk.

2. EQUILIBRIUM SOLUTION

The existence of this new equilibrium depends on one key assumption: that whatever process forms the clumps, it leaves them magnetically connected to the volume-filling plasma. This would be the result if, for example, they are created by thermal instabilities or a “photon bubble” instability (Gammie 1998). The specific structure of these connections is unimportant. All that is required is for most of every clump’s mass to be attached to a field line that ultimately makes its way out into the external medium.

With this assumption, we begin defining this equilibrium by applying the requirement of angular momentum conservation. Here and throughout this Letter, we deal exclusively with vertically integrated (or vertically averaged) quantities. The dominant angular momentum transport mechanism is likely to be clump-clump collisions. Magnetic torques may also play a role but are likely to be smaller. This is because the $r - \phi$ component of the magnetic stress cannot exceed $B^2/8\pi$, yet the magnetic energy density is at most comparable to the gas pressure, which, as we estimate below, is probably well below the kinetic energy density of the clumps.

When (as can be expected here) the collision frequency between clumps is comparable to or smaller than the orbital frequency, the effective viscosity due to collisions is reduced (Goldreich & Tremaine 1978). Taking the unit of stress to be the momentum flux density of clumps $P$, we describe the viscous transfer of angular momentum by clump collisions in
terms of an effective “α” parameter (as in Shakura & Sunyaev 1973): $\alpha_\text{c} = \min (C, C^{-1})$, where $C$ is the covering fraction of clumps along a vertical line of sight. In the same units, any other $r - \phi$ stresses (e.g., magnetic) add an amount $\alpha_\text{m}$. The angular momentum conservation equation may then be written as a constraint on the total (i.e., hot phase plus cold) Thomson optical depth measured from the midplane to the surface:

$$\tau_T = \left( \frac{9}{5} \right) \left( \frac{\dot{m}}{m_\text{c}} \right) \frac{\dot{m} R_T}{(\alpha_c + \alpha_m) M^2 \Theta x^{3/2}} = 64 \frac{R_T}{(\alpha_c + \alpha_m) M^2} \left( \frac{\dot{m}}{0.1} \right) \left( \frac{\Theta}{0.1} \right)^{-1} \left( \frac{x}{10} \right)^{-3/2}.$$  (1)

Here $\dot{m}/m_\text{c}$ is the ratio of the mean mass per particle to the electron mass, $\dot{m}$ is the accretion rate in Eddington units (for unit efficiency), $\Theta$ is the hot phase temperature in units of $m_\text{c} c^2/\hbar$, $M$ is the velocity dispersion of the clumps in units of the hot phase sound speed, $x = r c^2/GM$, and $R_T$ is the relativistic correction factor to the integrated stress (Novikov & Thorne 1973). Note that at $x = 10$, $R_T = 0.1-0.5$, depending on the black hole spin. We expect, therefore, that if (as estimated below) the other fiducial factors are all an order of unity, in sub-Eddington accretion, $\tau_T$ is never more than several tens and could be rather less.

The origin of the clumps’ random motions lies in their magnetic connections. Let us consider a flux tube that passes through two clumps initially at the same azimuthal angle but at slightly different radii. As the inner one moves ahead of the outer one, the field energy grows as the tube lengthens. The associated force transfers angular momentum outward, and the clumps’ radial separation grows. If the field is initially weak (in the sense that the magnetic force between a pair of clumps is smaller than the central mass’s gravity), this is an unstable process, very similar to the continuum fluid magnetorotational instability whose importance to accretion disk evolution was pointed out by Balbus & Hawley (1991). Numerical integration of the equations of motion demonstrates that, if left uninterrupted, the end result is to give clumps energies of random motion comparable to the initial difference in gravitational potential energy between linked clumps. At the end of this section, we will estimate the typical separation of clumps to show that the expected random speeds are then on the order of the hot phase sound speed. The field line–stretching adds energy to the magnetic field, which may saturate at a level that is roughly in equipartition with the gas pressure.

Next let us consider energy conservation in the hot phase. Several heating mechanisms act, all having roughly constant rate per unit volume. Field lines running through a clump and the adjacent hot gas are pulled along by the clump and exert a drag force on the external plasma. The energy of the random motions so induced is eventually dissipated into heat. The magnetic field can be dissipated by forced reconnection (when clumps attempt to pull field lines across one another) and other mechanisms. Because the dissipation draws energy from the random motions of the clumps relative to the local mean velocity, it heats the hot, low-density phase but exerts no torque.

In disk-stress units, the hot phase heating rate is $\dot{\alpha}_\text{H} P_\Omega$, where $\Omega$ is the orbital frequency. We expect that $\dot{\alpha}_\text{H} \sim C M (\tau_T/r_T) R$ where $r_T$ is the (half) Compton optical depth of the hot phase and $R$ is the ratio between the effective drag cross section of a clump and its geometrical cross section. $R$ can be rather more than unity, but it cannot be less than ~1.

Because the other parameters in the estimate for $\alpha_\text{H}$ are all regulated to be ~1 (see below), and generally $\tau_T < 30$, although $\alpha_\text{H}$ could be smaller than $\alpha_\text{c}$, it cannot be too small.

The hot phase is cooled by inverse Compton scattering. Pietrini & Krollik (1995) show that when $\tau_T \sim 1$, the thermal balance of a plasma cooled by inverse Compton scattering may be described approximately by an expression, which, in this context, becomes

$$\frac{\Theta}{0.1} \tau_T = (C + \alpha_c/\alpha_\text{H})^{-1/4}.$$  (2)

The clumps are heated both by dissipative collisions and by absorbing X-rays radiated by the hot phase. If they reradiate thermally and $C < 1$, their outer surface temperature is

$$T_e = 6.2 \times 10^6 \left( \frac{\alpha_c/C + \alpha_\text{H}}{\alpha_c + \alpha_\text{H}} \right)^{1/4} \left( \frac{\dot{m}}{0.1} \right)^{1/4} \times \left( \frac{x}{10} \right)^{-3/4} m^{-1/4} R_e^{1/4} K,$$  (3)

where $m = M/M_\odot$ and $R_e$ is the relativistic correction factor for the dissipation rate per unit area (Novikov & Thorne 1973). If, as is likely, $\alpha_c/C > \alpha_\text{H}$, so that most of the dissipation associated with $\alpha_c$ occurs deep inside the clumps, the temperature at their centers is larger than $T_c$ by a factor of $\sim (1 + \chi)^{1/4}$, where $\tau_c$ is the Rosseland mean optical depth through a clump.

In contrast to, for example, Kuncic et al. (1997), who advocate magnetic confinement of the clumps, we suppose that the clumps are magnetically linked to the hot phase. Consequently, motion along field lines is unimpeded by magnetic forces, and clump survival requires pressure balance along field lines. Although it is true, as they argue, that maintenance of a smooth internal pressure distribution matched to the external pressure requires clumps small enough for sound waves to cross during a dynamical time, the picture proposed here does not depend on the clumps maintaining a smooth internal pressure distribution. All that is really necessary is that the pressure inside a clump not vary by large factors. If $M < 1$, the variations in the external pressure at a clump’s edge are only of order unity, so there is no reason to impose such a strict upper bound on the clump size.

The gas pressure in the hot phase is generally somewhat greater than the radiation pressure. Approximate pressure balance with the clumps then implies a gas density in the clumps:

$$n_{\text{cl}} \approx 3 \times 10^2 \tau_c \left( \frac{\alpha_c/C + \alpha_\text{H}}{\alpha_c + \alpha_\text{H}} \right)^{-1/4} \left( \frac{\dot{m}}{0.1} \right)^{-1/4} \times \left( \frac{\Theta}{0.1} \right)^{1/2} \left( \frac{x}{10} \right)^{-3/4} m^{-3/4} R_e^{1/2} R^{-1/4} \text{ cm}^{-3},$$  (4)

where $R_e$ is the relativistic correction factor for the vertical gravity (Abramowicz, Lanza, & Percival 1997). At such high densities, the approximation of thermal radiation (eq. [3]) should be reasonably valid, even when (as in active galactic nuclei [AGNs]) $m \sim 10^3$. Thermal equilibrium in the presence of heat conduction (which, in this context, must flow along field lines) constrains both the size of the clumps and the external pressure. As shown
by McKee & Begelman (1990), when conduction is in the classical regime, the relative motion between the clumps and the hotter gas around it is subsonic, and the geometry is plane-parallel, there is a unique internal pressure at which there is neither condensation nor evaporation. The plane-parallel flow occurs in one of two ways: if the clumps are larger than the length scale on which the temperature varies or if the magnetic field lines are parallel. With the assumption that the path length integrated along field lines is not grossly different from the straight-line path length, the characteristic length for the temperature gradient at which radiative cooling becomes comparable to thermal conduction is the field length $\lambda_F \equiv n_e^{-1} (\kappa T/e)^{1/2}$, where $n_e$ is the electron density in the conducting gas, $\kappa$ is its conductivity, and $\Lambda$ is its cooling function.

The plane-parallel condition is met if the clumps are at least as large as $\lambda_F$. The unique equilibrium pressure is defined by the condition that radiative heating and cooling exactly balance when integrated across the regions of intermediate temperature. When there is no net evaporation or condensation, the gas in the interface is static (modulo any relative bulk motion). Its pressure is therefore nearly constant. Since the rate of Compton cooling per unit volume is proportional to the electron pressure so long as the electrons are subrelativistic, it is independent of temperature within the conductive interface. Consequently, if the heating rate is fixed per unit volume, the difference between the heating rate and the Compton cooling rate is also constant throughout the interface. This fact means that the relevant cooling rate for determining $\lambda_F$ is the (small) part due to bremsstrahlung.

Relative to the disk scale height, $\lambda_F$ is

$$\lambda_F/h = 0.26 \tau_h^{-3} \left( \frac{\Theta}{0.1} \right)^{3/2} = 0.26 \tau_h^{-3/2} (C + \alpha_c/\alpha_H)^{-3/8},$$

where the second expression has used equation (2) to fix the temperature in terms of the optical depth. The numerical values in equation (5) are computed by assuming the Spitzer conductivity, $\kappa = 5.6 \times 10^{-12}$ ergs cm$^{-1}$ K$^{-1}$. Equation (5) shows that in order to achieve evaporative balance, the clumps must be an interesting fraction of a scale height across (at least in the plane perpendicular to the local magnetic field direction). This equation also places an implicit lower bound on $\tau_h$. The reason is that if $\tau_h$ were so small that $\lambda_F/h > 1$, clouds large enough to have plane-parallel conductive interfaces would also be so large that the surrounding pressure could not everywhere be equal to the equilibrium pressure.

An upper bound on $\tau_h$ also follows. The differential equation of thermal balance that allows for dissipative heating in the hot phase, Compton cooling, bremsstrahlung, and heat conduction may be solved subject to the boundary condition that the temperature gradient goes to zero at a large distance from a clump. The condition of evaporative balance then reduces to

$$H_{\text{net}} = \int_0^1 dt \, t^{-5/2} |(4/7) H_{\text{net}} (1 - t^{7/2}) - (1 - t^2)|^{-1/2},$$

where $H_{\text{net}}$ is the ratio of the net heating rate (i.e., after subtracting the Compton cooling rate) to the bremsstrahlung cooling rate, evaluated far from any clump. The solution to equation (6) is $H_{\text{net}} \approx 1.765$. Rewriting $H_{\text{net}}$ in terms of $\tau_h$ then gives

$$0.15 \left( \frac{m}{0.1} \right)^{-1} \left( \frac{x}{10} \right)^{3/2} R_1^{1/2} R_2^{-1} \tau_h^{1/2} \left( C + \alpha_c/\alpha_H \right)^{1/2} \times \left( \frac{\Theta}{0.1} \right) \tau_h - (1 + \alpha_c/\alpha_H)^{-1} = 0.$$  \hspace{1cm} (7)

When Compton cooling dominates, $\Theta$ adjusts to permit an equilibrium for any value of $\tau_h$. However, if $\tau_h$ is too large, $\Theta$ falls to the point that bremsstrahlung may contribute significantly. The critical optical depth is

$$\tau_{h,\max} \approx 2.5 (1 + \alpha_c/\alpha_H)^{-1/2} \left( \frac{m}{0.1} \right)^{1/2} \left( \frac{x}{10} \right)^{-3/4} R_1^{1/2} R_2^{-1}.$$  \hspace{1cm} (8)

When $\tau_h$ becomes this large, an evaporative balance fixes $\tau_h = \tau_{h,\max}$. Thus, bounded by the requirement that $\lambda_F < h$ and by equation (8), $\tau_h$ must always be $\sim 1$.

The covering factor may be found from the geometric relation $C \approx (T_{\text{cl}}/T_h) \left( \tau_h/\tau_c \right) \left( \theta^2 h \right)$ if the typical clump temperature is $\sim T_h$ and the typical clump size is $\sim \lambda_F$. Evaluating this expression and solving for $C$ (assuming $C < 1$), we find

$$C \approx 0.30 \frac{(1 + \alpha_H/\bar{C})^{9/11}}{(1 + \alpha_c/C)^{2/11}} \times \frac{M^{-16/11} (\alpha_H/0.1)^{-7/11} m^{-2/11} \left( \frac{m}{0.1} \right)^{10/11}}{\tau_h^{26/11} \left( \frac{x}{10} \right)^{-18/11} R_1^{2/11} R_2^{-11/11} \tau_h^{1/4} \alpha_c/\alpha_H},$$

where we have assumed that $\alpha_c/\alpha_H < \alpha_c < 1$. In this equation, we scaled the central mass to $10^7 M_\odot$ because $C$ rises toward unity as the central mass declines toward the solar scale. When $m \sim 1$, $C < 1$ is no longer a good approximation, although $C$ does not exceed unity because of its strong dependence on $\tau_h$, whose allowed range shifts slowly downward with increasing $C$ (eqs. [5] and [8]).

Finally, we estimate $a$, the mean separation between clumps, in terms of $C$:

$$a/h \approx 0.4 C^{-7/12} (1 + \alpha_H)^{-1/4} \tau_h^{-5/6} \left( \frac{\alpha_H}{0.1} \right)^{1/4}.$$  \hspace{1cm} (10)

Thus, $a/h$ is regulated to be $\sim 1$. The difference in potential energy between neighboring clumps is then great enough so that, given weak magnetic connections, they are stirred up to $\mathcal{M} \sim 1$.

3. DISCUSSION

So long as the hot phase cools predominantly by inverse Compton scattering, it should be thermally stable, because the Compton cooling rate automatically rises with the energy density of photons available. On the other hand, should $\tau_h$ rise to $\tau_{h,\max}$, the optical depth at which bremsstrahlung becomes significant, the hot gas would become unstable to thermal perturbations with the wavelengths being short compared to a scale height. The result would be the creation of more cool clumps,
so that $\tau_c$ would be brought back into the permitted range. Thus, $\tau_{c,\max}$ is truly an upper bound on the optical depth that may be found in the hot phase. The equilibrium is viscously stable because combining equations (1), (2), and (9) shows that $\dot{m} \propto \tau_c^{-2/7}$.

At our fiducial values, there is only a factor of several between the largest and smallest possible $\tau_c$. However, the dependence of $\tau_{c,\max}$ on $C$ is such that the net scaling with both $\dot{m}$ and $x$ is extremely slow: $\tau_{c,\max} \propto \dot{m}^{0.01} x^{-0.022}$. Consequently, $\tau_c$ is constrained to within factors of a few for any values of $\dot{m}$ and $x$ for which the canonical disk equilibrium indicates radiation pressure dominance. This tight constraint also ensures that $\dot{p}/\dot{P} \propto (\dot{m}/\mu_\ast) \tau_c x^{3/2} (mR_{T_c}) \ll 1$.

Finally, we compare the observables predicted by this equilibrium with those actually seen. Although the estimates made in this Letter are highly approximate, each of them can be refined with more detailed calculation. The most important prediction of this equilibrium is the simultaneous presence of both a thermal and a “coronal” component in the spectrum. More specifically, it predicts, as is seen in real black hole systems (Zdziarski et al. 1996), that $\tau_c \sim 1$ and $0 \sim 0.1$, independent of the central mass. It also predicts that the covering fraction of the clumps depends weakly on $\dot{m}$, although it rises with increasing $\dot{m}$ and falls outward. In the inner rings, $C \sim 0.1$ for AGNs but is closer to $\sim 1$ for stellar black holes. In AGNs, each clump is very optically thick to Compton scattering, so they are individually (and collectively) effective absorbers of soft X-rays and reflectors of hard X-rays. In disks accreting onto stellar black holes, the individual clump optical depth may not be so great, so that the Compton reflection feature may be weakened. $C$ is in general less than unity, so we would not expect (as we do not generally see) strong soft X-ray absorption. The slope of the Comptonized power law depends on $(C + \alpha_{c}/\alpha_{\mu})^{0.04}$ (Pietrini & Krolik 1995). If $\alpha_{\mu}$ is not too much smaller than $\alpha_c$, the slope predicted by this relation is consistent with observations (i.e., $F \propto \nu^{-0.9}$ in AGNs [Mushotzky, Done, & Pounds 1993]; in Galactic black hole systems, the power-law index ranges from $\sim 0.3$ to $\sim 1.5$ [Tanaka 1989; Ballet et al. 1994; Gil’fanov et al. 1994]). The ratio of hard X-ray luminosity to thermal emission (ultraviolet in AGNs, soft X-ray in stellar-mass black hole systems) is given by $\alpha_{\mu}/(\alpha_c + C\alpha_{\mu})$.

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