Study of influence of boundary conditions on deformation and stresses in a cooled piston of a diesel engine – part A - mathematical models

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Abstract. The paper presents influence of boundary conditions on mechanical and thermal stresses of an oil cooled piston in diesel engine Scania DC09 074A and its deformation due to high combustion pressure and variable temperature. Due to different fixing of the pin in the piston hub the deformation of the piston shape and von Misses stresses differ at the same boundary conditions. This part of paper describes mathematical model of the piston load especially of heat transfer to wall taking into account three elements: convection, radiation and conductivity of the piston material. Calculations of the piston loads were carried out by using 0D mathematical model at assumption of the Vibe combustion model and the Woschni heat transfer model for homogenous mixture. For precise determination of the gas temperature in the combustion chamber the CFD technique recently is used and for that case the simulation of compression, combustion and expansion processes was carried out by using the program Fluent. The numerical calculations were done for two kinds of fuel: diesel oil and CNG due to possibility of changing the oil fuelling system into gaseous fuelling system. In order to proper work of CNG engine the compression ratio should be decreased. This was done by reducing the volume of the combustion chamber. Most calculations have done by researchers at assumption that the piston hubs are fixed, but it is not right assumption. In summary the paper gives indications how to set boundary conditions for “floating” piston pin. The paper presents shortly mathematical model of heat balance in the piston and FEM solution of stresses and deformation which are based on strains. This study is an introduction to calculations showed in the part B.

1. Introduction

Measurement of heat transfer in internal combustion engines encounters great difficulties due to unsteady heat flow from the variable temperature of the hot gas in the cylinder to the cooling medium, in most cases water. The biggest temperature of the cylinder charge takes place at the end of the combustion process at initial period of expansion stroke. On the other hand the lowest temperature of the charge occurs during intake stroke. Sometimes temperature of the cooling medium is higher than temperature of the cylinder charge. From technical point of view there is problem with measurement of variable temperature in the cylinder and therefore also assessment of the heat flows. Increasing of total efficiency of IC engines requires a decreasing of thermal losses to the cooling system, decreasing of exhaust losses (lower energy flowing into outflow system) and decreasing of friction losses (increasing of mechanical efficiency) [12]. For this reason determination of the mechanical and thermal stresses of the piston and its deformation depends on assumed initial and boundary conditions, it means determination of values of pressure, temperature, heat transfer, properties of materials and
others. Because the thermal and mechanical loads of the piston change every angle of the crankshaft rotation, it is difficult to accept the proper boundary conditions. The maximum pressure and maximum temperature in the cylinder lasts only a few degrees of crank rotation. The author analysed two cases of the piston load: at maximum pressure and at mean pressure (mean indicated pressure). However in the paper only the first case is considered. Many authors made their calculations only for the piston itself [4, 13, 10]. The piston pin is mounted in the piston hubs and mainly the pin is sliding in the hubs (‘floating pin”). There is a contact of the pin with the upper surface of the hub’s hole. Due to the contact with the hubs the piston is bent by the force in the small-end of the connecting rod. Also the piston hubs are deformed. The traditional calculation method of the piston stresses and deformation is not correct. Taking into account the contact of the pin with the piston hubs it gives the real work of these two parts. In many calculation works shown often in the internet it is assumed that the upper surfaces of the pin holes are fixed, which influences on smaller deformation of the piston in this region. Most of the engines have a sliding pin in the hubs and it causes the piston to propagate along the hole axis due to mechanical forces acting on the hubs. The determination of the piston’s deformation and stresses requires the proper definition of pressure, temperature, radiation, thermal conductivity of the piston material, heat transfer resulting from convection in different surfaces of the piston. The hardest is to determine the convection coefficients in piston crevices and on the bottom surfaces of the piston. However some engineers try to measure temperature of the piston by sophisticated technique [14]. In the work heat transfer coefficient was determined based on the Woschni hypothesis [5, 6]. The correct values of stresses, strains and deformations can be obtained only by taking into account the contact of the pin and piston’s hubs. The considerations were concerned to Scania DC09 074A engine with piston made from aluminium alloy. In diesel engine the burning of the fuel begins in the central combustion chamber and the surfaces of this part are under high temperature. After several degree of crankshaft rotation the flame propagates also radially into cylinder walls. Today the calculations are carried out by sophisticated computer programs using FEM technique. However, in order to determine temperature distribution in the cylinder space the CFD technique is used.

The main purpose of the work was to demonstrate the influence of boundary conditions on mechanical and thermal stresses and also on deformation of outer shape of the piston. This is very important for engine designers and for those who modify the pistons for other applications. The main task is to show the significant impact of the piston pin contact with the piston hub.

2. Piston heat transfer

The thermodynamic model of heat exchange in piston internal combustion engines takes into account variable temperature of the cylinder charge and constant temperature of the cooling medium. There are many studies of heat transfer in diesel engines even working in ceramic coatings as adiabatic engines [7, 8]. In this paper the water is considered as cooling medium, because most of the modern diesel engines are currently equipped with liquid cooling system. In order to increase total efficiency of the engines the difference between higher and lower heat sources should reach higher value. Usually higher temperature $T_{\text{max}}$ of the combustion process influences on higher total efficiency.

$$\eta_0 = \frac{T_{\text{max}} - T_{\text{min}}}{T_{\text{max}}} = 1 - \frac{T_{\text{min}}}{T_{\text{max}}}$$

Heat transfer in the piston engines is carried out through three phenomena: convection, radiation and conductivity. Usually calculation is carried out for assessment of heat transfer in IC engines. Temperature of the charge in the cylinder changes during piston motion from value near 3000 K to 350 K in SI engine and from 2000 K to 350 K in diesel engines, thus the heat stream to the cooling medium changes rapidly and fluctuation of heat transfer can be defined in a function of crank rotation. Heat transfer as a result of radiation is only due the temperature difference between the gas and walls and emissivity of the surfaces and the cylinder gas.
In order to decrease the heat flow stream from the cylinder wall to water it is considered application of the special coating with the material of lower conductivity, usually a ceramic coating. The surface heat flow rate \( \dot{q} \) through the wall with thickness \( l \) and wall conductivity \( \lambda \) can be expressed as follows [2]:

\[
\dot{q} = -\lambda \frac{dT}{dt} \quad \text{[W/m}^2\text{]} \quad (2)
\]

For constant wall conductivity \( \lambda \) this expression can be written as a function of the temperature difference \( \Delta T \), thickness \( g \) of the wall or the heat transfer resistance \( R_\lambda \):

\[
\dot{q} = -\lambda \frac{\Delta T}{g} = -\frac{\Delta T}{R_\lambda} \quad (3)
\]

Convection coefficient \( \alpha_1 \) inside the cylinder (heat flow from the gas to the inner cylinder wall) can be calculated using Woschni experimental formulas for intake, compression, expansion and exhaust processes given in different sources [1, 3]. On the other hand inflow of the heat to the liquid medium is a linear function of convection coefficient \( \alpha_2 \) and can be obtained from Nusselt number, which is a function of Reynolds and Prandtl numbers. However, the simple expression as a function of water velocity given by Heywood [1] was used. Convection coefficient for heat outflow from the bottom surface of the piston head is calculated using Grashoff and Prandtl numbers for laminar flow from the crankcase side. Temperature of the gas inside the crankcase is almost constant (about 350 K) and gas flow near the piston walls is laminar.

Radiation heat between cylinder gas containing also soot and walls is determined from the following formula given by Horlock and Winterbone [3]:

\[
\dot{q}_r = \frac{F_b}{F_1} \sigma_0 \varepsilon_z \left[ T_g^4 - T_1^4 \right] \quad (4)
\]

where \( \sigma_0 = 5.67 \) is Boltzman constant, \( \varepsilon_z \) is emissivity of central body, \( F_b \) and \( F_1 \) are central and surroundings surfaces, respectively.

Temperature of the piston material is variable due to change of gas temperature in the cylinder. The heat to the cylinder body is delivered by radiation and convection and temperature on the piston crown cannot be assumed as gas temperature. Due to the strong gas motion in diesel engine in compression process a high values of Reynolds and Nusselt numbers are expected. The heat transfer coefficient from the gas to the piston crown can be calculated on the Woschni theory based on fluid flow similarity and is defined as follows:

\[
h_w = 130 \cdot D^{-0.2} \cdot \rho^{0.8} \cdot T^{-0.53} \left( C_1 \cdot c_m + C_2 \frac{V_s \cdot T_1}{p_1 \cdot V_1} (p_1 - p_0) \right)^{0.8} \quad (5)
\]

where during the process of exchange gas the constant \( C_1 \) and \( C_2 \) are determined by the formulas:

\[
C_1 = 6.18 + 0.417 \frac{c_m}{c_r} \quad \text{and} \quad C_2 = 0 \quad (6)
\]

and in compression, combustion and expansion processes take the following form:

\[
C_1 = 2.28 + 0.308 \frac{c_r}{c_m} \quad \text{and} \quad C_2 = 0.00324 \quad (7)
\]

but for compression process \( C_2 = 0 \).

The markings in the formulas determine following values: \( c_r \) - peripheral gas velocity, \( c_m \) - mean piston velocity, \( p_0 \) - pressure in working cycle without combustion process, \( V_s \) - stroke cylinder
volume, $D$ - cylinder diameter, $l$ – index determining the initial of compression or combustion process, $V$ – volume, $p$ – pressure.

3. Thermal and mechanical loads

The scheme of the piston load by pressure and temperature acting on the crown is shown in Fig 1a. The heat transfer from the piston sliding surfaces $Q_{cool}$ to the cylinder walls and heat $Q_{cc}$ from the bottom surfaces to the air in the crankcase decreases the heat capacity in the piston material. Temperature $T_{cyl}$ of the gas under the piston is not equal. On the bottom surface of the small-end of the connecting rod acts the pressure $p_{cc}$ which deforms the piston pin due to its bending (Fig. 1b).

![Figure 1. Scheme of pressure and temperature load (a) and deformation of piston pin and piston hubs (b)](image)

Knowing all heat transfer resistances from the cylinder gas to the external cooling medium the surface heat flow rate $\dot{q}$ can be determined from (1). The variable temperature of the cylinder gas is obtained from simulation process of engine work with taking into account unsteady gas flow in inlet and outlet systems. One of the recent research works concerning the thermal loads of piston was carried by Sroka [11], where optimization of the piston was done.

4. Heat balance in piston

The heat inflow to the piston surfaces from the gas and heat outflow from the sliding faces and piston rings to the cylinder walls and also from the internal piston surfaces to the crankcase causes an increment of heat capacity of the piston body $Q_p$. The heat balance can be written as follows:

$$Q_p = \sum_{i=1}^{k} A_{pi} h_{pi} (T_g - T_{pi}) - \sum_{j=1}^{m} A_{pj} h_{pj} (T_{pj} - T_{oj}) - A_d h_d (T_i - T_{oil}) = \sum_{i=1}^{n} m_i c_i \Delta T_i$$  

(8)

where $A$ denotes elementary area of the surface, $h_{pj}$ is convection coefficient describing a heat flow from an elementary area of the sliding or internal walls to the gas in the crevices or the air in the crankcase, $T_g$ is temperature in the cylinder, $T_{pj}$ determines temperature of elementary area of the piston walls, $T_{oj}$ denotes a gas temperature in the crevices and internal piston walls. The heat balance equation includes an internal cooling heat of the piston by the oil with temperature $T_{oil}$ in the duct.
with area $A_d$ around the ring part. The increment of temperature $\Delta T_l$ in the element $l$ can be calculated from the above equation at known values of heat capacity $c_l$ for the piston material and its density, which determines the mass $m_l$ of elementary volume.

The Scania piston is cooled by the oil which is injected from the nozzle in the crankcase to the hole located in the bottom internal surface, when the piston is at BDC. At assumption of the nozzle diameter $d_n$, pressure in the lubricating system $p_l$ and diameter of the oil duct $d_p$, the velocity of the oil in the piston duct is determined from the Bernoulli equation and oil mass flow rate with density $\rho_{oil}$ from the following equation:

$$u_p = \beta \left( \frac{d_n}{d_p} \right)^2 \sqrt{\frac{2(p_l - p_0)}{\rho_{oil}}} \quad (9)$$

The convection coefficient $h_d$ in the cooling oil duct is determined from the Reynolds, Prandtl and Nusselt numbers at assumption of a hypothesis of flow similarity.

### 5. Mathematical model of stresses and deformation

Usually the material used for piston is aluminium alloy containing silicon and this material is treated as linear material. Also piston pin which contacts with the piston is made from alloy steel and has linear properties concerning deformation. For such materials the stress is related to the strain:

$$\{\sigma\} = [D]\{\varepsilon\} \quad (10)$$

where: $\{\sigma\}$ - stress vector $= [\sigma_x, \sigma_y, \sigma_z, \sigma_{xy}, \sigma_{xz}, \sigma_{yz}]^T$,

$[D]$ - elasticity or elastic stiffness matrix or stress-strain matrix,

$\{\varepsilon\}$ - elastic strain vector,

$\{\varepsilon\}_{th}$ - thermal strain vector.

Values $\{\varepsilon\}_{th}$ are strains which cause the stresses. For the 3-D case calculation, the thermal strain vector is:

$$\{\varepsilon\}_{th} = \Delta T \left[ \alpha_x^{se}, \alpha_y^{se}, \alpha_z^{se}, 0, 0 \right]^T \quad (11)$$

where $\alpha_x^{se}$ is a secant coefficient of thermal expansion in the x direction and $\Delta T = T - T_{ref}$ and $T$ is a current temperature at the point in question and $T_{ref}$ = reference (strain-free) temperature.

The flexibility or compliance matrix, $[D]^{-1}$ is:

$$[D]^{-1} = \begin{bmatrix}
1/E_x & -v_{xy}/E_x & -v_{xz}/E_x & 0 & 0 & 0 \\
-v_{yx}/E_y & 1/E_y & -v_{yz}/E_y & 0 & 0 & 0 \\
-v_{zx}/E_z & -v_{zy}/E_z & 1/E_z & 0 & 0 & 0 \\
0 & 0 & 0 & 1/G_{xy} & 0 & 0 \\
0 & 0 & 0 & 0 & 1/G_{yz} & 0 \\
0 & 0 & 0 & 0 & 0 & 1/G_{xz}
\end{bmatrix} \quad (12)$$

where: $E_x$ - Young's modulus in the x direction,

$v_{xy}$ - major Poisson's ratio,
\( \nu_{xy} \) - minor Poisson's ratio,

\( G_{xy} \) - shear modulus in the xy plane

For isotropic material after expansion of the above equations the strain for example in direction x has a following form:

\[
\varepsilon_x = \alpha_x \Delta T + \frac{\sigma_x}{E_x} - \frac{\nu_{xy} \sigma_y}{E_x} - \frac{\nu_{xz} \sigma_z}{E_x} \quad \text{and} \quad \varepsilon_{xy} = \frac{\sigma_{xy}}{G_{xy}}
\]  

(13)

The given above mathematical model of thermal and mechanical loads enables to calculate stresses, deformation, strains and safety coefficients for a given materials and determined boundary conditions.

6. Combustion model and tasks of research work

The simulation research was carried out on CI four stroke engine SCANIA DC09 074A, which geometrical parameters are presented in Table 1.

| Geometrical parameters           | Data   |
|----------------------------------|--------|
| Cylinder diameter                | 130 mm |
| Stroke                           | 140 mm |
| Compression ratio                | 18     |
| Length of connecting rod         | 255.5 mm|
| Number of valves                 | 2+2    |
| Number of cylinders              | 5      |
| Opening and closing of inlet valve| 8° CA BTDC/ 42° CA ABDC |
| Opening and closing of exhaust valve| 59° CA BBDC / 8° CA ATDC |

In order to perform the simulation test of influence of temperature and pressure on the stresses and deformation of the piston it should be calculated the engine work parameters. The calculation computer program was made by author, which takes into account all thermodynamic processes taking place during one engine cycle. The combustion processes was modelled by using Vibe function and gas flow in engine ducts was treated as an unsteady flow with taking into account the waveform of pressure. Heat release depends also on the initial temperature of the charge in the cylinder. Validation of the assumed combustion model for all calculation models is required in the future. In this work the Vibe [16] coefficients \( a \) and \( m \) were taken from the experimental work done before on the conventional engine Suzuki by Soczowka [9] with using of the following exponential Vibe formula:

\[
x_b = 1 - \exp \left[ a \left( \frac{\phi - \phi_b}{\Delta \phi_b} \right)^m \right]
\]  

(15)

where:

- \( x_b \) - fuel burnt ratio,
- \( \phi \) - current angle of crank rotation,
- \( \phi_b \) - crank angle of beginning of fuel burning,
- \( \Delta \phi_b \) - duration of crank angle rotation of full combustion process.

The following values of coefficients were taken from calibration of the standard engine: \( a = -6.908 \) and \( m = 3 \).

The purpose of this work a special attention was paid to heat transfer from the cylinder gas to the water as a cooling medium with constant temperature 357 K flowing around the cylinder with velocity 0.8 m/s. Input data containing many constant parameters were read by the program from the text table. The results of calculation were transferred to the output files giving many thermodynamic parameters as a function of crank angle rotation for given engine speed, constant ignition point and combustion
duration. Convergence calculations were provided by the iterative cycle work until the indicated pressure difference achieved an assumed value. The calculations were made for cylinder sleeve made from cast iron with constant conductivity 45 W/m. The cylinder head and piston were treated as an aluminium alloy with conductivity equal 180 W/m. The heat transfer from the gas to the piston changes rapidly during compression, combustion and expansion processes. The Woschni hypothesis does not describe precisely a heat exchange during opening exhaust valves and closing of inlet valves. This causes a rapid change of the convective coefficient in these periods. The calculated values of the convective coefficients for the considered Scania engine is presented in Fig. 2 at rotational speed 1500 rpm. The maximum of convective coefficient reaches value 16000 W/(m² K) at combustion process with swirl ratio equal 1.0, but during the exchange gas processes reaches very small values about 200 W/(m² K). In calculation lower value of convection coefficient were assumed because in real conditions the heat transfer is not so fast due to certain heat capacity of the walls and piston body. The engine fuelled by CNG indicates higher mean temperature in the combustion process than that fuelled by diesel oil even at the same air excess ratio and the same absolute charging pressure 1.8 bar as is shown in Fig. 3. The CNG engine had reduced compression ratio to 12.8.

![Figure 2. Calculated convection coefficient at heat transfer in Scania engine at 1500 rpm](image1)

![Figure 3. Comparison of charge temperature in Scania engine fuelled with diesel oil and CNG at air excess ratio λ=1.4](image2)
Mechanical loads of the piston are caused mainly by pressure during compression, combustion and expansion processes. They are reduced by inertia forces, which should be considered for precise calculations. The change of the cylinder pressure is shown in Fig. 4 for turbocharged diesel engine. Maximum of pressure reaches value 14 MPa about 8 deg CA ATDC.

![Figure 4](image)

**Figure 4.** Pressure trace in Scania diesel engine

The fuel was injected at 19º CA BTDC to the combustion chamber in engine working with rotational speed 1500 rpm.

7. Summary
The article shows what are the most important elements which should be taken into consideration while calculation of stresses and deformation of pistons in IC engines. Boundary conditions determined by engineers have big influence on final result of calculated stresses and deformations of the piston shape. Particularly it is important for engineers, who make final decision in prediction of the “barrel-oval” shape of a new piston design. The wrong shape of the side piston surface can cause a seizing and fast damage of the engine. It is most important for heavy duty engines working with a generator or with a propeller in a ship. In this part of the paper the following problems were considered:

1. Before every calculation of the piston strength and deformation it should be done simulations of the engine working cycle. In diesel engines the important factor is determination of temperature under the piston during combustion process.
2. Prediction of pressure during working cycle can be done simply by simulation of whole engine work with using 0-D mathematical models of thermodynamic processes taking places in the cylinder. This is very fast way to obtain many working parameters such as pressure, temperature, heat transfer, radiation, heat losses, fuel consumption and also pressure change in inlet or exhaust pipes by taking into account non-steady gas flow.
3. It should be paid attention on boundary conditions and determination of constraints in the assembly piston-pin, because most cases concern the “floating pin”, which deformation influences also on the piston hubs deformation. The constraints of the hubs and piston should be strictly defined. The next part of the paper shows differences in stresses and deformations of the piston.
4. It was presented the model of heat exchange in the piston based on Woschni hypothesis, where also radiation is taken into account. Such model enables calculation of convection coefficients for setting obtained values as boundary conditions in FEM calculation program.
5. The most important factor influencing on the piston deformation is temperature, which changes in the piston material due to heat flux from the piston crown to the ring zone.
6. Temperature inside the piston material is calculated on a heat balance given in formula (8). For that reason different convection coefficients should be declared for every piston surface.
7. For cooled pistons convection coefficient should be defined at knowing velocity of a cooling fluid and geometry of the channel.
8. Every FEM computer program solves system of non-linear equations for calculation of stresses and deformation by determination of strains inside the piston structure. Shortly it was presented a simple model of strength calculations in the paragraph 5.
9. It was presented the change of convection coefficient between the gaseous charge and cylinder walls and piston in the cylinder of diesel engine Scania DC09 074A at full load. Changing the fuelling system to gaseous fuel the compression ratio should be decreased but for the same air excess ratio temperature in the cylinder in the case of CNG fuelling is higher than in the case of fuelling with diesel oil (see Fig. 3).

The given theoretical considerations and given mathematical models of thermodynamic state of the engine and strength material were helpful in calculation of the Scania piston in FEM Ansys program [15].

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