Generation Method of Air Defense Missile Weapon Interception Scheme

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Abstract: The research on the generation method of the anti-aircraft missile interception scheme before the war can lay the foundation for the rational allocation of the next air defense missile. In this paper, the clustering algorithm based on fuzzy C-means is used to cluster anti-ship missiles. Based on this, the number of anti-aircraft missiles launched during the first interception is determined. Then, by analyzing the depth of the launching area and the interception process, the number of interception of each layer of air defense missiles in the air defense interception operation is calculated. Finally, the anti-aircraft interception scheme that can achieve the expected damage probability of anti-ship missiles can be solved. The case is verified by the case that the calculation efficiency is relatively high and has certain application value.

1. Preface
At present, the three-layer air defense interception with air defense missiles as the main defensive weapon is the main method of surface warship anti-aircraft anti-missile operations. At the same time, the use of universal vertical launchers on ship platforms has changed the way missiles are deployed, and surface ships can flexibly configure ship-borne missiles based on operational requirements. However, there is no specific study on the configuration methods of various types of ship-borne missile weapons before the war. Therefore, this paper aims to determine the generation method of the pre-war air defense missile interception scheme by appropriately simplifying the main links of air defense and anti-missile warfare, thus laying a foundation for the rational allocation of the next air defense missile [1, 2].

2. Problem analysis and solution process

2.1. Solution process of formation air defense missile interception scheme
(1) Firstly, cluster analysis of anti-ship missiles. In air defense operations, different types of anti-ship missiles have different penetration capabilities and damage capabilities, and different interception schemes must be adopted. In the multi-layer air defense interception operation, the first interception method is particularly important. Based on the penetration capability index and the damage capability index as the reference variables, the anti-ship missiles are clustered and the cluster center points are determined. Based on this, the number of anti-aircraft missile launches during the first interception is determined[3,4]

(2) Calculation of the number of interceptions. The boundary of the three-layer interception area is divided based on the maximum tracking distance of the radar and the maximum range of the far,
medium and short-range anti-aircraft missiles. The calculation of the depth of the launch area is appropriately simplified and the maximum number of intercepts at each layer is calculated and calculated.

(3) The air defense missile interception plan is determined. Based on the maximum interception times of the long-, medium-, and near-three-layer air defense interception operations and the interception probability of air defense missiles, various air defense missile interception schemes for the expected interception probability of incoming missiles are calculated[5].

2.2. Anti-ship missile clustering analysis

On the one hand, when launching anti-aircraft missiles to intercept anti-ship missiles, the first interception effect determines the success or failure of interception operations to a certain extent. The greater the threat of incoming missiles, the greater the number of ship-to-air missiles launched during the first interception. On the other hand, the different target characteristics of different incoming missiles increase the difficulty of determining the air defense missile interception scheme before the war. Therefore, cluster analysis method is used to comprehensively analyze different incoming targets according to the characteristic indicators, determine the cluster center, and then determine the number of anti-aircraft missile launches during the first interception according to the cluster center feature values.

2.2.1. Cluster Analysis Based on Fuzzy C-Means. Fuzzy clustering is an important component in the field of pattern recognition. Fuzzy C-means clustering is currently the more common method. It clusters the data by calculating the geometric closeness, so that the target achieves aggregation based on the main features. The general idea of its calculation is: The number of samples is divided into c categories, denoted as , and the cluster center of each category is expressed as , and we use the degree of membership of the sample pair to satisfy the condition. Express the objective function as:

\[ J_b(v) = \sum_{i=1}^{n} \sum_{k=1}^{c} (\mu_{ik})^b \]  

Clustering Analysis Based on Simulated Annealing Ant Colony Algorithm

The simulated annealing ant colony algorithm generally adopts the calculation process of ant colony algorithm, but updates the population by simulated annealing algorithm to avoid falling into local optimum[6,7]. The flow chart of fuzzy C-means clustering algorithm based on simulated annealing ant colony algorithm is shown in Fig. 1.

Initialization parameters including population size, size, population MaxGen, MaxGen, Cross probability P_c, mutation probability P_m, Annealing initial temperature T_0, termination temperature T_end, and temperature cooling factor k.

The selected genetic algorithm is encoded as follows: using binary encoding. Each chromosome consists of c cluster centers, thereby m chromosomes constitute the sample variables to be optimized. Each variable is binary coded with k bits. Fitness function Target function Target function Target function Target function Target function Target function Target function Target function Target function.

\[ F_{intV} = \text{ranking}(J_b) \]  

2.2.2. Inbound missile clustering analysis process.

(1) MATLAB solving process of simulated annealing ant colony algorithm
Generally speaking, the faster the anti-ship missile's penetration speed, the stronger the anti-interference ability, the lower the penetration height, and the stronger the penetration capability. At the same time, the stronger the target detection and selection ability, the greater the lethality of the warhead, and the greater the damage it poses to our ship platform [8,9]. The penetration capability index and the damage capability index of different anti-ship missiles are characterized by the value of interval [0,1]. These two characteristic indicators reflect the threat degree of the incoming missile and are used to determine the missile launch during the first interception. Quantity.

Because of the difficulty in obtaining samples, MATLAB randomly generated the penetration capability index and the damage capability index of 400 anti-ship missiles. Since the sample is too large, it will not be expanded here.

Set the maximum number of iterations to 20. The termination tolerance of the objective function is $e^{-6}$, and the number of categories is 4; Simulated annealing algorithm parameters: $k=0.8, T_0=100, T_{end}=1$; Genetic algorithm parameters: size pop=10, MAXGEN=10, $P_c=0.7, P_m=0.01$.

According to the above method, the cluster analysis is performed. The results are as follows: the termination temperature is 0.922, the objective function $J_b=3.4566$, and finally the coordinates of the four cluster centers are $(0.17, 0.41), (0.72, 0.18), (0.55, 0.55)$ and $(0.57, 0.81)$, clustering diagram is as Fig 2 shows:

![Flow chart of clustering algorithm](image-url)
(2) Analysis and application of target clustering results

A comprehensive analysis of the penetration capability index and the damage index of the obtained cluster center point is performed to determine the number of air defense missile launches for the first interception of the incoming missile belonging to each cluster center. As shown in Table 1:

| Cluster center | Number of first-shot missiles |
|----------------|------------------------------|
| (0.17,0.41)    | ≥1                           |
| (0.72,0.18)    | ≥2                           |
| (0.55,0.55)    | ≥2                           |
| (0.57,0.81)    | ≥3                           |

2.3. Formation process of formation air defense missile interception plan

2.3.1. Target intercept count calculation.

(1) The kill zone of an air defense missile against an air target is an area composed of points of an air defense missile that can produce an expected damage effect after the target encounters, and the area is usually an irregular three-dimensional area. The kill zone only describes the encounter between the anti-aircraft missile and the incoming target. To make the decision on the timing of the missile launch, it is necessary to calculate the launch zone. The launching area is the area that can be used to make the missile and the target of the air-defense missile launched by the target in the killing zone. The near and far boundaries of the kill zone are usually determined by the missile's comprehensive tactical indicators and ship platform indicators. Its calculation formula is [10]:

\[
D_{by} = \sqrt{D_{sy}^2 + (V_m \cdot t_{dy})^2 + 2V_m \cdot t_{dy} \sqrt{D_{sy}^2 - H^2 - p^2}}
\]

\[
D_{bn} = \sqrt{D_{sj}^2 + (V_m \cdot t_{dj})^2 + 2V_m \cdot t_{dj} \sqrt{D_{sj}^2 - H^2 - p^2}}
\]

Wherein, \(D_{fy}\) and \(D_{fy}\) To indicate the near and far boundaries of the launch area; \(D_{sj}\) and \(D_{sy}\) To indicate the near and far boundaries of the kill zone; \(V_m\) Indicates the target flight speed; \(t_{dy}\) Indicates the time required for an air defense missile to reach the far side of the kill zone; \(t_{dj}\) Means the time required for the first anti-aircraft missile to reach the vicinity of the kill zone, \(H\) Indicates the target height, \(p\) indicating the target route shortcut. The depth of the launch area refers to the distance between the far and near points of the launch area. Used\(h_f\) to indicate The period of the target through the depth of the launch area is the period of time during which our missile can be effectively intercepted. To highlight the main contradictions, make the following assumptions:
a. Treat our surface ships and their formations as points, and do not distinguish between the defense angles of the attacked ships and other protective ships.

b. The incoming target passes straight through the speed in our combat airspace at a uniform speed, and the route shortcut is zero. Our air defense missiles pass at a constant speed \( V_d \) along the direction of the missile.

c. Determine the boundary of the far-end air defense intercept zone according to the radar tracking distance and the smaller value of the maximum range of the long-range missile, and determine the boundary of the middle intercept zone and the near-layer intercept zone according to the maximum range of the medium-range and short-range air defense missiles.

d. It can always intercept the incoming target at a certain height, and approximate the depth of the launching area to the horizontal depth of the launching area. In order to meet the defensive operational requirements of warship formation, the requirements for radar detection distance \([11,12]\) are:

\[
R_j \geq d_{sy} + V_t (t_{fy} + t_{xs})
\]  \( \text{(5)} \)

In the formula, \( d_{sy} \) indicates the horizontal distance of the far zone of the killing zone, and \( V_t \) indicates the horizontal speed of the target, \( t_{fy} \) indicating the time required for the air defense missile to fly to the far side of the killing zone, \( t_{xs} \) indicating the reaction time of the air defense missile.

The depth of the launch area \( h_f \) is simply calculated as:

\[
h_f = d_{sy} - d_{sj} + V_m \cdot t_{dy} - V_m \cdot t_{dj}
\]  \( \text{(6)} \)

\( d_{sj} \) indicates the horizontal distance of the vicinity of the killing zone. The other letters have the same meaning as above.

2.3.2 Calculation method for interception times of each layer. Regardless of the flight altitude, the three-layer air defense interception area is divided according to the range of the long-range, medium-range and short-range anti-aircraft missiles, as shown in Fig. 3. Determine the near and far boundaries of each air defense interception zone. The interception number of incoming missiles in each interception zone is calculated as follows: Set the first encounter between the incoming missile and our missile \( P_0 \). The distance is \( D_0 \), by the title, \( D_0 = D_{sy} \); The first encounter point is \( p_{i-1} \), the distance is \( D_{i-1} \), and the following conditions are met:

\[
D_i = D_{i-1} - V_m \cdot t_i
\]  \( \text{(7)} \)

Where \( t_i = t_p + t_{xs} + t_{di} \), \( t_p \) indicating the time of assessment, \( t_{xs} \) indicating the response time of the air defense missile, \( t_{di} \) indicating the time required for our air defense missile to fly to the point of encounter, and

\[
t_{di} = \frac{D_i}{V_d}
\]  \( \text{(8)} \)

When \( D_i < D_{sj} \), the layer interception ends.

![Formation defense zone division](image)

**Figure 3.** Formation defense zone division

2.3.3 Interception scheme.
(1) Calculation method of target intercept probability. In the interception of threat targets, the principle of early defense and time reduction should be grasped. When remote interception is performed, the remote interception is preferred, and the earlier the target is successfully intercepted, the greater the probability of success. Set $P_1, P_2, P_3$ indicate the intercept probability of long-range, medium-range, and short-range missiles, respectively. Representing the number of launches of long-range, medium-range, and short-range missiles, $x_1, x_2$ and $x_3$ respectively, the formula for calculating the intercept probability of incoming threat targets is:

$$P_t = 1 - (1 - P_1)^{x_1}(1 - P_2)^{x_2}(1 - P_3)^{x_3}$$

(2) Anti-aircraft missile interception scheme under expected intercept probability. First, the number of missiles launched during the first interception is determined according to the clustering center to which the incoming anti-ship missile belongs. According to general operational principles, additional air defense intercepts usually only launch one air defense missile. Then, according to the farthest detection distance of the radar, the maximum range of the long-range, medium-range and short-range anti-aircraft missiles, the boundary of the three-layer air defense interception area is determined; according to the above attack, the number of interception levels of each layer is determined. Finally, based on the interception times and interception probabilities of each layer of missiles, a simple arrangement is made to determine the anti-aircraft interception scheme that can achieve the expected interception probability of anti-ship missiles.

3. Case Analysis

We formed a fleet for the destroyer, and the enemy was two fighters; the enemy attacked us and we intercepted the missile. The enemy fighters mounted the air-to-ship missiles when they were formed from our formation with a maximum range of 120km and a height of 5km, and their penetration speed was 2 Mach. The maximum tracking distance of our destroyers for anti-ship missiles is 240km. Our missile types include: regional ship-to-air missile No. 1 with a maximum range of 180km, an average flight speed of Mach 3, a single hit probability of 0.5, regional ship-to-air missile No. 2 with a maximum range of 32km, an average flight speed of 2 Mach, and a single hit probability of 0.7. The short-range defense ship-to-air missile No. 3 has a maximum range of 8km, an average flight speed of Mach 1.5, and a single-shot probability of 0.8 for anti-ship missiles. It is known that the altitude of the ship-to-air missile is within the range of our radar tracking and missile flight altitude. Our launch response time is 5s and the damage evaluation time is 5s. Solve the feasible interception scheme when the ship-to-air missile reaches the intercept probability of more than 85%. The solution process is as follows:

(1) Determine the type of threat target to which the ship-to-air missile belongs. The missile belongs to the cluster center (0.17, 0.41). The penetration capability is poor and the damage capability is general. Therefore, the plan to select the missile single-shot is intercepted for the first time.

(2) The farthest killing distance of our missile is 120km, the radar tracking distance is 240km, and the long-range reference formula of the launching area (5) is 150km < 240km, so the radar can guide the missile when the incoming missile distance is 150km. According to the missile range, the three-layer interception area is simply determined, so that the three-layer air defense interception airspace is 32-120km for the far-end interception area, 8-32km for the middle-level interception area, and 1.5-8km for the short-range interception area.

(3) Taking the calculation of the number of intercepts in the far layer as an example: when the target enters the far boundary of the launching area, the first interception is performed, and a long-range air defense missile is launched. At this time, the first encounter point distance is 120km; the second intercepting distance is Calculate the reference formula (8), the second encounter point distance is 68km; the third intercept distance is 30.6km < 32km, so the far layer can only intercept twice. It is also calculated that the middle layer can be intercepted once, and the short range can be intercepted once. Refer to formula (4) and perform a simple arrangement to obtain three interception schemes as shown in Table 2. According to the economic principle, instead of option one, there are
two interception schemes for the ship-to-air missile. The scheme also conforms to the actual fire interception process in the general combat process, and verifies the practicability of the algorithm.

Table 2. Plan of three-layer air defense

| Interception process | Distal interception | Middle interception | Short-range interception |
|----------------------|---------------------|----------------------|--------------------------|
| Interception scheme one | 2                   | 1                    | 0                        |
| Interception scheme II  | 1                   | 1                    | 0                        |
| Interception scheme three | 0                   | 1                    | 1                        |

4. Conclusion
By analyzing the air-to-air interception process of ship-to-air missiles, this paper determines the interception schemes of various ship-to-air missiles, which can provide a selection interval for the decision-making configuration of ship-to-air missiles. The next step is to further refine the analysis process to better match the actual operational situation.

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