Field-dependent quantum gauge transformation

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Abstract – In this paper we generalize the quantum gauge transformation of the Maxwell theory obtained through gaugeon formalism. The generalization is made by making the bosonic transformation parameter field dependent. The Jacobian of vacuum functional under field-dependent quantum gauge transformation is calculated explicitly. We show that the quantum gauge transformation with a particular choice of field-dependent parameter connects the gaugeon actions of the Maxwell theory in two different gauges. We establish the result by connecting two well-known gauges, namely, Lorentz gauge and axial gauge.

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Introduction. – Gauge theories are the most successful theories explaining the dynamics of elementary particles and play a very crucial role in the unification of fundamental interactions. Being first in all the gauge theories, the Maxwell theory of electromagnetism has become one of the main pillars of modern theoretical physics playing the key role in the formulation and the development of Einstein’s special theory of relativity. Quantum electrodynamics (QED) is an extension of the Maxwell theory describing the electromagnetic interaction of the nature.

In the quantization of QED, people generally consider the gauge symmetry at the classical level but not at the quantum level because the quantum theory can be defined properly only after fixing the gauge. However, being gauge-fixed the theory does not remain (local) gauge invariant. The quantum gauge transformation for QED was first studied by Yokoyama utilizing a different formalism which is commonly known as gaugeon formalism [1]. Yokoyama gaugeon formalism provides a wider framework to quantize the general gauge theories [1–5]. The main idea behind the gaugeon formalism is to study the quantum gauge freedom by extending the configuration space with the introduction of some set of extra fields, so-called gaugeon fields, in the effective theory. Since then, the gaugeon fields have not appeared in the physical processes. Therefore, they are not the physical fields. In fact, the gaugeon fields yield negative normed states in the theory, which results in a negative probability [1]. Hence, one needs to remove the unphysical modes present in the theory associated with gaugeon fields. Firstly, the Gupta-Bleuler-type subsidiary condition was implemented to remove the unphysical gaugeon modes. But this Gupta-Bleuler-type subsidiary condition had found certain limitations [1]. Further, the properties of BRST symmetry were utilized to get rid of such limitations of the physical subsidiary conditions by replacing them into a single Kugo-Ojima-type condition [6–9]. Along with BRST symmetry the extended gaugeon action possesses the quantum gauge transformation under which the action remains form invariant. Incidentally, Hayakawa and Yokoyama have founded that within the renormalization procedure the gauge parameters get shifted from their original values [10]. However, within the framework of gaugeon formalism a shift in gauge parameter occurs naturally which gets identified with the renormalized parameter [1]. The gaugeon formalism has already been utilized by many physicists in the context of various gauge theories [11–17]. Recently, the generalization of BRST symmetry has been analysed in the context of gaugeon formalism [18].

Although the generalization of the BRST transformation by making the transformation parameter finite and field dependent has been studied extensively [19–30], the generalization of the quantum gauge transformation in the similar fashion has not yet been investigated. Therefore, we take this opportunity to generalize the quantum gauge transformation of the Maxwell theory within gaugeon formalism. A remarkable difference in these symmetry transformations is that the transformation parameters of the BRST transformation and the quantum gauge transformation follow different statistics. For instance,
the parameter of the BRST transformation is fermionic in nature; however, the parameter of the quantum gauge transformation is bosonic in nature. Therefore, the novelty of the present work is to generalize the quantum gauge transformation by making the bosonic parameter field dependent.

In the present work, we first emphasize the effective Maxwell theory analysing the quantum gauge symmetry through Yokoyama gaugeon formalism. For this purpose, we extend the configuration space by introducing the gaugeon fields and corresponding ghost fields. Then, we investigate the quantum gauge symmetry for the extended Maxwell action, which we call the gaugeon-Maxwell action, incorporating some extra quantum fields. Furthermore, the quantum gauge symmetry is generalized by making the transformation parameter field dependent.

The resulting field-dependent quantum gauge transformation leads to the non-trivial field-dependent Jacobian within the functional integral. We compute the Jacobian of the field-dependent quantum gauge transformation explicitly. Remarkably, for a particular choice of the bosonic field-dependent parameter the Jacobian changes the gaugeon-Maxwell action from Lorentz gauge to axial gauge. Although we establish the results with the help of a specific example, these latter hold for any arbitrary gauge.

This paper is organized in the following manner. In the next section, we discuss the quantum gauge transformation for the Maxwell theory within the framework of Yokoyama gaugeon formalism. Furthermore, in the third section, we generalize the quantum gauge transformation by making the transformation parameter field dependent. The novelty of such field-dependent quantum gauge transformation is described in the fourth section. We summarize the present work in the last section.

Maxwell theory in Gaugeon formalism. – In this section, we discuss the Yokoyama gaugeon formalism analysing the quantum gauge freedom for the Maxwell theory in covariant and non-covariant gauges. In order to achieve the goal, we begin with the effective action for the Maxwell theory in Lorentz (covariant) gauge defined by

$$S^L_M = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - A_\mu \partial^\mu B + \frac{\lambda}{2} B^2 - i \varphi^\mu c_\star \partial_\mu c \right],$$

where $F_{\mu\nu}$ is the usual antisymmetric field-strength tensor for the gauge field $A_\mu$. Here $B, c$ and $c_\star$ are the multiplier (auxiliary) field, the Faddeev-Popov ghost field and the anti-ghost field, respectively.

However, in accordance with above, the effective Maxwell action described in the axial (non-covariant) gauge is defined by

$$S^A_M = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - A_\mu \eta^\mu B + \frac{\lambda}{2} B^2 - i \varphi^\mu c_\star \partial_\mu c \right],$$

where $\eta_\mu$ is an arbitrary constant four-vector. The effective actions $S^L_M$ and $S^A_M$ are invariant under the following nilpotent BRST transformations (i.e. $\delta^2 = 0$):

$$\delta A_\mu = -\partial_\mu c \eta, \quad \delta c = 0,$$

$$\delta B = 0,$$

where $\eta$ is an infinitesimal, anticommuting but global parameter.

Now, the gaugeon effective action corresponding to the Maxwell theory (1) is obtained by introducing the gaugeon field $Y$ and its associated field $Y_\star$ (both subjected to the Bose-Einstein statistics) as follows [31]:

$$S^L_Y = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - A_\mu \partial^\mu B + \partial_\mu Y_\star \partial^\mu Y + \frac{\varepsilon}{2} (Y_\star + \alpha B)^2 - i \varphi^\mu c_\star \partial_\mu c \right],$$

where $\alpha$ denotes the group vector valued gauge-fixing parameter and $\varepsilon$ refers the $(\pm)$ sign factor. The quantum gauge transformation, which leaves the quantum action (4) form invariant, is demonstrated as

$$A_\mu \rightarrow \hat{A}_\mu = A_\mu + \alpha \partial_\mu Y \tau,$$

$$B \rightarrow \hat{B} = B,$$

$$Y_\star \rightarrow \hat{Y}_\star = Y_\star - \alpha B \tau,$$

$$Y \rightarrow \hat{Y} = Y,$$

$$c \rightarrow \hat{c} = c,$$

$$c_\star \rightarrow \hat{c}_\star = c_\star,$$

where $\tau$ is a bosonic transformation parameter. The form invariance of quantum action (4), under the above quantum gauge transformation, reflects a natural shift in parameter $\alpha$ as follows:

$$\alpha \rightarrow \hat{\alpha} = \alpha + \alpha \tau.$$
where $\Box = \partial_\mu \partial^\mu$. Furthermore, such kind of limitation is improved by introducing the Faddeev-Popov ghosts $K$ and $K_\tau$ corresponding to the gaugeon fields $Y$ and $Y_\tau$ in the Yokoyama effective action (5) as follows:

$$
S_{YB}^L = \int d^4x \left[ -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - A_\mu \partial^\mu B + \partial_\mu Y_\tau \partial^\mu Y + \frac{\epsilon}{2} (Y_\tau + \alpha B)^2 - i\partial^\mu c_\tau \partial_\mu c - i\partial^\mu K\tau \partial_\mu K \right].
$$

Now, the effective action, $S_{YB}^L$, admits the following nilpotent BRST symmetry transformations:

$$
\begin{align*}
\delta_\alpha A_\mu &= -\partial_\mu \eta, & \delta_\alpha c &= 0, \\
\delta_\alpha Y &= -B \eta, & \delta_\alpha K &= 0, \\
\delta_\alpha c_\tau &= -i\partial_\tau Y_\tau, & \delta_\alpha Y_\tau &= 0.
\end{align*}
$$

Using the Noether’s theorem we calculate the conserved charge corresponding to the BRST symmetry (10) as follows:

$$
Q = \int d^3x \left[ -F^{\mu \nu} \partial_\nu c - \partial_\tau Y_\tau \right],
$$

which helps in replacing the two Yokoyama subsidiary conditions (namely, of Kugo-Ojima type and Gupta-Bleuler type (7)) by a single Kugo-Ojima-type condition (for details see, e.g., [7]). The effective action (9) also admits the following quantum gauge transformations ($\delta_\alpha$):

$$
\begin{align*}
\delta_\alpha A_\mu &= \alpha \partial_\mu \tau, & \delta_\alpha c &= 0, \\
\delta_\alpha Y &= -\alpha \partial_\tau Y_\tau, & \delta_\alpha K &= 0, \\
\delta_\alpha c_\tau &= -i\partial_\tau Y_\tau, & \delta_\alpha \tau &= 0.
\end{align*}
$$

Field-dependent quantum gauge transformation. In this section, we investigate the methodology of the field-dependent quantum gauge transformation characterized by the field-dependent bosonic parameter. To achieve this goal, we first define the general nilpotent quantum gauge transformation for the generic field $\phi_\alpha(x)$, written compactly, as

$$
\delta_\tau \phi_\alpha(x) = \phi_\alpha(x) - \phi_\alpha(x) = R_\alpha(\phi) \tau,
$$

where $R_\alpha(\phi)$ is the generic variation of field $\phi_\alpha(x)$ under the quantum gauge transformation satisfying $\delta_\tau R_\alpha(x) = 0$ and $\tau$ is the parameter of transformation satisfying Bose-Einstein statistics.

Now, we propose the field-dependent quantum gauge transformation defined by

$$
\delta_\tau \phi_\alpha(x) = \phi_\alpha(x) - \phi_\alpha(x) = R_\alpha(\phi) \tau[\phi],
$$

where the parameter of transformation $\tau[\phi]$ depends on the fields explicitly. Now, it is obvious that such field-dependent quantum gauge transformations do not remain nilpotent any more. In spite of being non-nilpotent, the field-dependent quantum gauge transformation (17) leaves the quantum action $S_{YB}^L$ given in (9) form invariant. However, the functional measure defined in (13) is not invariant under such field-dependent quantum gauge transformation. If we apply the field-dependent quantum gauge transformation defined in (17) on the generating functional (13), the generating functional gets transformed as follows:

$$
\begin{align*}
\delta_\tau Z^L[0] &= \int D\phi (\text{Det} J[\phi]) e^{iS_{YB}^L[\phi]}, \\
&= \int D\phi e^{iS_{YB}^L[\phi] - i\text{Tr} \ln J[\phi]}. \quad \text{(18)}
\end{align*}
$$

Furthermore, we calculate the Jacobian matrix of field-dependent quantum gauge transformation (17) as

$$
J_{\alpha}^{\beta}[\phi] = \frac{\delta \phi_{\beta}}{\delta \phi_{\alpha}} = \delta_{\alpha}^{\beta} + \delta R_{\alpha}(\phi) \frac{\delta R_{\alpha}(\phi)}{\delta \phi_{\beta}} \tau[\phi] + R_{\alpha}(x) \frac{\delta \tau[\phi]}{\delta \phi_{\beta}},
$$

$$
= \delta_{\alpha}^{\beta} + R_{\alpha}[\phi] \tau[\phi] + R_{\alpha}(\phi) \tau^{\beta}[\phi]. \quad \text{(19)}
$$

Utilizing the nilpotency property of quantum gauge transformation (i.e. $\delta_\tau R_\alpha(\phi) = 0$) and relation (19), we compute

$$
\text{Tr} \ln J[\phi] = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \text{Tr} \left( R_{\alpha}^{\beta} \tau + R_{\alpha} \tau^{\beta} \right)^n,
$$

$$
= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (\delta_\tau \tau^{\alpha}[\phi])^n, \quad \text{Tr} \ln J[\phi] = \ln(1 + \delta_\tau \tau[\phi]), \quad \text{(20)}
$$

where $\tau[\phi]$ is considered up to linear order which reflects the infinitesimal nature of the parameter even though it
depends on the fields explicitly. Consequently, expression (18) reads
\[ \delta_\eta Z^L[0] = \int D\phi \, e^{i\int S^L_{\phi}[\phi] - i\ln(1 + \delta_\eta \tau[\phi])}, \] (21)
which is nothing but the expression of the generating functional for the gaugeon-\(\phi\)Maxwell theory having an additional term, \(-i\ln(1 + \delta_\eta \tau[\phi])\), in the effective action due to the Jacobian. Therefore, we conclude that under the field-dependent quantum gauge transformation, the generating functional changes from one effective action to another.

**Connecting different gauges of gaugeon-Maxwell action.** In this section, we explicitly mention the remarkable features of the field-dependent quantum gauge transformation which connects the two different gauges of the gaugeon-Maxwell theory for a particular choice of the field-dependent parameter. In this regard, we start by making the quantum gauge transformation defined in (12) field dependent as follows:
\[ \delta_\eta A_\mu = \alpha \partial_\mu Y \tau[\phi], \quad \delta_\eta B = 0, \]
\[ \delta_\eta Y_\mu = -\alpha B \tau[\phi], \quad \delta_\eta Y = 0, \]
\[ \delta_\eta c = K \tau[\phi], \quad \delta_\eta c_\mu = 0, \quad \delta_\eta K = 0, \]
\[ \delta_\eta K_\mu = -c_\mu \tau[\phi], \quad \delta_\eta \alpha = \alpha \tau[\phi], \] (22)
where \(\tau[\phi]\) is the field-dependent transformation parameter. Here the specific choice of the field-dependent transformation parameter is made by
\[ \tau[\phi] = -\int d^4 x \, (A_\mu \partial^\mu B - \partial_\mu Y_\mu \partial^\mu Y + i\partial^\mu c_\mu \partial^\mu K + i\partial^\mu K_\mu \partial_\mu K - A_\mu \eta^\mu B + \eta^\mu Y_\mu \partial^\mu Y - i\eta^\mu c_\mu \partial^\mu K_\mu \partial_\mu K) - 1. \] (23)
Now, expression (20) for the above field-dependent transformation parameter yields
\[ \ln(1 + \delta_\eta \tau[\phi]) = i \int d^4 x \, (A_\mu \partial^\mu B - \partial_\mu Y_\mu \partial^\mu Y + i\partial^\mu c_\mu \partial^\mu K_\mu \partial_\mu K - A_\mu \eta^\mu B + \eta^\mu Y_\mu \partial^\mu Y - i\eta^\mu c_\mu \partial^\mu K_\mu \partial_\mu K). \] (24)
With this identification, expression (21) gets the following form:
\[ \delta_\eta Z^L = \int D\phi \, \exp \left\{ S^L_{\phi}[\phi] \right\} \]
\[ + \int d^4 x \, (A_\mu \partial^\mu B - \partial_\mu Y_\mu \partial^\mu Y + i\partial^\mu c_\mu \partial^\mu K_\mu \partial_\mu K - A_\mu \eta^\mu B + \eta^\mu Y_\mu \partial^\mu Y - i\eta^\mu c_\mu \partial^\mu K_\mu \partial_\mu K) \right\} \]
\[ = \int D\phi \, e^{iS^L_{\phi}[\phi]} = Z^A. \] (25)

**Conclusions.** In this paper, we have discussed the Maxwell theory in gaugeon formalism to analyse the quantum gauge freedom in great detail. In the framework of gaugeon formalism, we have investigated the quantum gauge transformation characterized by an infinitesimal bosonic parameter which leaves the quantum action form invariant. Under the quantum gauge transformation a natural shift in gauge parameter has been observed. Furthermore, we have constructed the gaugeon-Maxwell action in two different gauges (namely, in the Lorentz and the axial gauges) possessing the BRST as well as the quantum gauge symmetries. The infinitesimal bosonic parameter of the quantum gauge transformation has been made field dependent. Furthermore, the Jacobian of the path integral measure under the field-dependent quantum gauge transformation has been computed. Remarkably, we have observed that under the field-dependent transformation with specific bosonic field-dependent parameter the generating functional of the gaugeon-Maxwell theory changes from the Lorentz gauge to the axial gauge. Although there are many choices of the gauge condition, the physical quantities do not depend on any of them. Therefore, the spectrum of the physical theory remains unaltered under such field-dependent quantum gauge transformation.

We have made all the computations with the source-free partition functions. However, it would be possible to make such an analysis for partition functions having an external source. In my opinion, for the partition functions having an external source such analysis will connect the propagators corresponding to the appropriate gauges because the connection of propagators in the Lorentz and the axial gauges under finite field-dependent BRST transformation had already been established [32]. Also, there are no any ambiguities in dealing with the singularities of the propagators corresponding to the Lorentz and the axial gauges. Naturally, a large number of the practical as well as of the formal calculations have been made in Lorentz gauges. The main disadvantage of the Lorentz gauge choice in non-Abelian gauge theory is, however, that it requires a ghost action which complicates the calculations. For this reason, another set of gauges (namely the axial gauges) has often been found favouring calculations. Therefore, it would be interesting to generalize the results in non-Abelian gauge theory, in higher-form gauge theories and in the perturbative gravity theory as well.
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