INFLATION, STRUCTURE FORMATION AND DARK MATTER

ANDREW R. LIDDLE AND PEDRO T. P. VIANA

Astronomy Centre, University of Sussex
Brighton BN1 9QH Great Britain

Abstract

The formation of structure in the Universe offers some of the most powerful evidence in favour of the existence of dark matter in the Universe. We summarize recent work by ourselves and our collaborators, using linear and quasi-linear theory to probe the allowed parameter space of structure formation models with perturbations based on the inflationary cosmology. Observations used include large and intermediate angle microwave background anisotropies, galaxy clustering, the abundance of galaxy clusters and object abundances at high redshift. The cosmologies studied include critical density models with cold dark matter and with mixed dark matter, cold dark matter models with a cosmological constant and open cold dark matter models. Where possible, we have updated results from our journal papers.

1 Introduction

The demise of the standard cold dark matter (CDM) model, and the wide range of minor variants which have surfaced in its wake, have forced us to come to terms with the fact that structure formation models depend on a wide range of parameters. Cosmological parameters, such as the Hubble parameter, the density parameter and a possible cosmological constant, all have a significant effect on the evolution of structure. So too does the assumed matter content; almost all models contain cold dark matter, but we must also worry about the appropriate value of the baryon density, the possibility that some of the dark matter may be hot, and even the present energy density in the form of massless species. To add to that, there is uncertainty concerning the form of the initial perturbations from which structure grows; for example, even the simplest versions of the inflationary universe paradigm typically generate more complicated spectra than the scale-invariant Harrison–Zel’dovich spectrum, giving yet further freedom, whilst topological defect models such as cosmic strings lead to a yet more complex situation.

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Such a wide parameter space is not amenable to detailed investigation via numerical simulations, which provide the most rigorous test of cosmological models. In order to discover the parameters best suited to matching the observational data, and as importantly to determine which possibilities are definitely excluded, it is necessary to use more economical techniques. In that vein, we and our collaborators have recently been investigating the parameter space through the use of linear and quasi-linear perturbation theory, which allows a semi-analytical comparison of theory against a range of observations. We have concentrated on constraints which can be applied directly to the matter and radiation power spectra, rather than becoming involved in the delicate issues of biased galaxy formation introduced when the galaxy correlation function is studied in detail. This has enabled us to carry out an investigation of wide areas of parameter space, which we have published in a series of papers. These cover the case of a critical density universe [1] (both the cold dark matter and mixed dark matter cases), a flat universe with a cosmological constant [2] and an open universe [3]. Finally, we returned to the case of critical density cold dark matter models [4] to investigate whether their perilous position could be alleviated by allowing a substantial increase to the baryon density. Throughout this work, we have placed particular focus on taking the inflationary paradigm seriously, and consequently considering a range of possible initial power spectra.

In this brief article there is no room to give the full details of these papers. Instead, we aim to give an overview of some of the possibilities. Where possible we have taken the opportunity to update constraints from our published work, especially with regard to intermediate scale microwave background anisotropies. The present situation is described in great detail in Viana’s thesis [5]. An extensive list of references can be found in the papers cited here.

2 Theoretical Input

2.1 Inflationary perturbations

During an inflationary epoch, quantum fluctuations swept up in the rapid expansion lead to perturbations in the present universe. A review of this mechanism, and of the modelling of an inflationary epoch, is given in Ref. 6. Inflation produces not only a spectrum of density perturbations, but also one of gravitational waves. These latter are capable of influencing large-angle microwave background anisotropies, as witnessed by the COBE satellite, and must be taken into account in a full comparison with observations.

In absolute generality, inflationary models can lead to a wide range of different spectra of density perturbations. However, almost all models fall within the slow-roll paradigm, within which physical conditions only vary slowly as inflation proceeds. In that situation, one finds that the density perturbations can be accurately described by a power-law spectrum \( n \), sometimes known as a tilted spectrum. We shall work within that regime.

Slow-roll inflation can therefore be characterized by three parameters. The overall amplitude of density perturbations has always been considered a free parameter in large-scale structure studies. Along with that comes the spectral index of the density perturbations \( n \), and a parameter \( R \) which measures the extent to which gravitational waves contribute to

\[ n \]

\[ R \]

Often the tilt can be given the following interpretation. If inflation is to end, it must move away from the ‘scale invariance’ of exponential expansion. Although perturbations on observable scales were generated some time before inflation came to an end, typically one does see the beginnings of the approach to the end of inflation, with the tilt arising in the incipient breaking of the expansion away from exponential.
Since gravitational waves only affect the large-angle microwave background observations, only a single parameter need be introduced to quantify their effect.

For any given inflation model, \( n \) and \( R \) are easy to compute. The range of possible values is wide; indeed, wider than present observations permit. Consequently, some inflation models have already been excluded (extended inflation being the most notable example), a promising development we can expect to see much more of in the future. Given this, it is satisfactory to treat \( n \) and \( R \) as completely free parameters in seeking viable large-scale structure models. Needless to say, the extra freedom which they represent has a serious influence on what can be said concerning the other cosmological parameters and the nature of the dark matter.

2.2 Cosmological parameters

In our studies, we permitted variation of five cosmological parameters, in addition to the inflationary parameters. They are the Hubble parameter \( h \), the density parameter \( \Omega \), the cosmological constant \( \Lambda \), the amount of hot dark matter \( \Omega_\nu \) and the amount of baryonic matter \( \Omega_B \). We didn’t attempt to consider all possible combinations however; we only considered a hot component in universes with critical density (since other models can comfortably fit the data without a hot component, and to have too many different components with similar densities seems contrived), and a cosmological constant was only introduced with the appropriate value to make the universe spatially flat. In all cases, cold dark matter was used to make up the required total density.

While this wide range of parameters gives our analysis a large degree of generality, we might as well mention that it is far from exhausting the range of possibilities one might contemplate trying. Other approaches, all of which would have important consequences, would be to change the number of massless particle species, introduce exotic dark matter (degenerate species, non-thermal production, warm dark matter, decaying or annihilating dark matter), allow exotic inflation giving non-power-law initial perturbations or to utilize topological defects (cosmic strings, textures, etc.) as the source of initial perturbations. There is no indication from observations that any of these are required, but neither can they all be excluded.

3 Observations

The cited papers give extensive description of the observations we use.

As stated above, the gravitational waves only affect \( COBE \). The density perturbations, by contrast, are responsible for the complete range of structure seen in the Universe. Present observations of microwave anisotropies are useful primarily through \( COBE \) on large angular scales and via a host of experiments sampling the acoustic (sometimes called Doppler) peak at about a degree. Observations constraining the matter power spectrum are bulk motions, the shape of the galaxy correlation function and the object abundances, the most useful being galaxy clusters today and that of quasars, damped Lyman alpha systems and galaxies at high redshift.

To give a feel for some of the data, Fig. 1 shows the observed matter power spectrum, under the assumption of a critical density Universe. It is shown for illustration only. We plot \( \sigma(R) \), the variance of the matter distribution smoothed on a scale \( R \). It is greatly encouraging to see that the data form a smooth curve across orders of magnitude in both linear scale and in the size of the perturbations. The normalization of the galaxy correlation...
Figure 1: The observational data, with $1\sigma$ uncertainties and 95 per cent confidence lower limits. We represent the COBE data schematically at $4000h^{-1}$ Mpc; they are indicated by a filled square whose size represents the uncertainty. The galaxy correlation function data are shown by circles; an uncertainty in overall normalization has not been illustrated. The bulk flow constraint is represented by a star, and the cluster abundance constraint by a cross. The lower limits are damped Lyman alpha systems (left) and quasars (right).

function depends on the bias parameter; what is crucial for us is the shape of this function, which is expected to be almost independent of the bias on the large scales used here.

Fig. 2 shows some sample models plotted against this data. Here the variance has been normalized by the standard CDM model. Although this model clearly doesn’t fit the data, it gets us within a factor two or so across the relevant scales, allowing us to bring the model into agreement by variation of the underlying assumptions. Shown are models with a tilted initial spectrum, a very low Hubble parameter or a hot component added. Each of these options, corresponding to a single-parameter variation from standard CDM, is a reasonable eyeball fit to the data. While it is encouraging that the observations are so consistent with one another, we see that there is not a huge amount of information contained within the matter power spectrum, in comparison to the number of parameters we have available to construct models.

In the future, it seems likely that microwave anisotropy measurements alone may give us all the information we need on cosmological parameters. At present, the observational situation is beginning to become very interesting but one needs to consider both the matter and radiation power spectra to draw the strongest conclusions. Fig. 3 shows the present observational status. The leftmost four points represent the COBE four-year data, and the rest are from the many experiments probing the acoustic peak — the squares are Saskatoon and the remaining circles, from left to right, are SP94, MAX, Python, MSAM, CAT (2 points), White Dish, OVRO and ACTA (see Ref. 4 for details).
Figure 2: The data normalized to the prediction of the standard CDM model. The solid line is standard CDM; the others modify one parameter from this fiducial model, as indicated in the key. All models are precisely COBE normalized; the COBE point at 4000$ h^{-1}$ Mpc is illustrative. One can shift the entire galaxy correlation function data set vertically, corresponding to changing the bias.

We see that already there is extremely strong evidence of a peak on degree scales. This is a crucial constraint, because it limits the amount of tilt permitted in the spectrum; one does not have to tilt very far at all (to about $n = 0.7$) for the acoustic peak to vanish entirely. This constraint is particularly strong on critical-density CDM models, which require a strong tilt to give the right matter power spectrum.

4 Constraints

In this section we give a quick tour of the constraints we find. Much greater detail is available in the cited papers.

4.1 Critical density cold dark matter models

Fig. 4 shows the $n$–$h$ plane, assuming a baryon density given by nucleosynthesis. The strong competition of the acoustic peak and the galaxy correlation constraints rules out these models completely, unless one is willing to tolerate a very low value of $h$ indeed. Adding gravitational waves, as in the lower panel, only makes things worse.

There seems to be only one escape clause from this predicament (without introducing extra forms of matter etc.). That is to raise the baryon density above the standard nucleosynthesis range [4]. Baryons have two beneficial effects; the acoustic peak is boosted by their pressure (as seen in Fig. 3), while they contribute extra damping to reduce power in
the matter spectrum. Somewhere around 12 or 15 percent baryons seems necessary to permit a fit to the data for $h$ of 0.5, about the smallest direct measurement might allow. It is interesting that there would independently be a need for such a baryon fraction to explain the observed baryon fraction in clusters, and that the range of baryon densities permitted by nucleosynthesis extends to higher values in recent analyses.

4.2 Mixed dark matter models

Adding a component of hot dark matter is the simplest way to reconcile critical-density universes with a higher Hubble constant. Recent observational developments aiding these models are the lowering of the COBE normalization between the two and four year data sets, and a weakening of the lower limit on the density of gas in damped Lyman alpha systems at high redshifts. Even without tilt, as shown in Figure 5, a sizeable allowed region exists, all of which produces a satisfactory acoustic peak. Introducing tilt allows $h$ up to at least 0.6.

4.3 Cold dark matter models with a cosmological constant

A truly high $h$, greater than say 0.7, can only be reconciled with the ages of objects in a low-density universe. Models with a cosmological constant again permit a fit to the data for a wide range of parameters, as in Fig. 6. However, one would prefer not to lower $\Omega_0$ much below 0.4; in recent years the ‘fiducial’ value of the low density has risen quite a bit. At lower densities, the normalization of the power spectrum is such that galaxies must be significantly anti-biased, which seems improbable (though this conclusion can be
Figure 4: Critical density cold dark matter models. The top panel is without gravitational waves, the lower one includes those as generated by a specific model — power-law inflation. The constraints are: solid, galaxy correlation function shape; dashed, cluster abundance; dotted, acoustic peak height; dot-dashed, damped Lyman alpha system abundance; dot-dot-dot-dashed, galaxy bulk flows. All constraints are plotted at 95% confidence.

Weakened by including tilt [2]). Versions including gravitational waves tend to have less viable parameter space.

Note also that, amongst models which fit the data, the low-density models are younger. It is only when one looks at constant $h$ that decreasing the density increases the age. An age of 12 Gyrs is probably the lowest the community would be willing to live with at this time.
4.4 Open cold dark matter models

The (re)discovery of inflation models which give an open universe has revived interest in a low-density universe without a cosmological constant. Fig. 6 shows the situation without tilt. Again, these models are viable provide one keeps $\Omega_0$ above 0.4 or so, with the introduction of tilt to $n > 1$ lowering this bound.

5 Conclusions

- The use of linear theory is the only way of probing the large parameter space to obtain an overall view of what is good and bad. Preferred regions can then be subjected to more detailed analysis (see e.g. Primack in these proceedings).

- All the types of model we’ve discussed exhibit viable regions:
  
  **CDM:** Much aided by raising $\Omega_B$. Needs $h \lesssim 0.5$.
  
  **CHDM:** In good shape, provided $h \lesssim 0.6$.
  
  **$\Lambda$CDM:** Under pressure from several sources, forcing $\Lambda$ down.
  
  **Open CDM:** Needs $\Omega_0 \gtrsim 0.4$.

- The common ingredient of these inflation-based models is that a reasonable amount of cold dark matter always seems to be required.

- CMB satellites promise to pin down *all* these parameters to great accuracy. As a by-product, we shall learn much about the mechanism for the origin of perturbations.
Figure 6: The top panel is CDM with a cosmological constant, and the lower one open CDM. Both cases are without tilt or gravitational waves. The dotted lines are of constant age.

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