CONCENTRATIONS OF DARK HALOS FROM THEIR ASSEMBLY HISTORIES

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ABSTRACT

We study the relation between the density profiles of dark matter halos and their mass assembly histories using a statistical sample of halos in a high-resolution N-body simulation of the ΛCDM cosmology. For each halo at \( z = 0 \), we identify its merger history tree and determine concentration parameters \( c_{\text{vir}} \) for all progenitors, thus providing a structural merger tree for each halo. We fit the mass accretion histories by a universal function with one parameter, the formation epoch \( a_c \), defined when the log mass accretion rate \( d \log M / d \log a \) falls below a critical value \( S \). We find that late-forming galaxies tend to be less concentrated, such that \( c_{\text{vir}} \) observed at any epoch \( a_c \) is strongly correlated with \( a_c \) via \( c_{\text{vir}} = c_1 a_c / a_c \). Scatter about this relation is mostly due to measurement errors in \( c_{\text{vir}} \) and \( a_c \), implying that the actual spread in \( c_{\text{vir}} \) for halos of a given mass can be mostly attributed to scatter in \( a_c \). We demonstrate that this relation can also be used to predict the mass and redshift dependence of \( c_{\text{vir}} \) and the scatter about the median \( c_{\text{vir}}(M, z) \) using accretion histories derived from the extended Press-Schechter (EPS) formalism, after adjusting for a constant offset between the formation times as predicted by EPS and as measured in the simulations; this new ingredient can thus be easily incorporated into semianalytic models of galaxy formation. The correlation found between halo concentration and mass accretion rate suggests a physical interpretation: for high mass infall rates, the central density is related to the background density; when the mass infall rate slows, the central density stays approximately constant, and the halo concentration just grows as \( R_{\text{vir}} \). Because of the direct connection between halo concentration and velocity rotation curves and because of probable connections between halo mass assembly history and star formation history, the tight correlation between these properties provides an essential new ingredient for galaxy formation modeling.

Subject headings: cosmology: theory — dark matter — galaxies: evolution — galaxies: formation — galaxies: structure

On-line material: color figures

1. INTRODUCTION

The theory of cold dark matter (CDM; see, e.g., Peebles 1982; Blumenthal et al. 1984; Davis et al. 1985) provides a remarkably successful framework for understanding galaxy assembly and structure formation in the universe. Within this picture, dark matter collapses first into small halos, and these halos merge to form progressively larger halos over time. As this process continues, baryons that initially trace the dark matter cool and condense to form galaxies in halo centers. New supplies of gas and galaxy mergers are closely linked to the mass accretion histories of the halos they inhabit. A detailed understanding of how this mass accretion occurs and how individual halo properties depend on their merger histories is of fundamental importance for predicting galaxy properties within the CDM theory and, similarly, for using observed galaxy properties (e.g., rotation curves) to test the paradigm.

The basic theory for the buildup of structure in the universe, and for the evolution of the properties of gravitationally bound structures, is fairly well developed; it has been extensively simulated at increasingly high resolution, and analytic formulations have been developed to describe this behavior. The Press-Schechter formalism (Press & Schechter 1974; Bond et al. 1991) has provided a useful framework for understanding the mass function of dark halos and, with subsequent improvements, has been shown to work well in comparison to N-body simulations (Gross et al. 1998; Sheth & Tormen 1999, 2002; Sheth, Mo, & Tormen 2001; Jenkins et al. 2001). The theory has been extended using an excursion set formalism to an extended Press-Schechter (EPS) theory, which describes the statistics of the buildup of individual halos through time (Bond et al. 1991; Lacey & Cole 1993, hereafter LC93). This EPS theory has been implemented to construct random realizations of merger trees, each specifying a full assembly history for a halo (see, e.g., LC93; Kauffmann, White, & Guiderdoni 1993; Somerville & Kolatt 1999, hereafter SK99). Detailed comparisons of EPS with simulations have highlighted the general success of the theory and quantified the level of accuracy of its predictions (see, e.g., Somerville et al. 2000, hereafter SLKD; Gardner 2001; Cohn, Bagla, & White 2001).

Similar advances have been made in understanding halo density structure (Elstathiou et al. 1988; Frenk et al. 1988; Dubinski & Carlberg 1991). Navarro, Frenk, & White (1995, 1996, 1997; hereafter collectively NFW) have proposed that halo profiles can be universally fitted by a two-parameter functional form:

\[
\rho_{\text{NFW}}(r) = \frac{\rho_s}{(r/R_s)(1 + r/R_s)^2}
\]

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where $R_s$ is a characteristic “inner” radius and $\rho_s$ a corresponding inner density. One of the inner parameters can be replaced by a “virial” parameter, either the virial radius ($R_{\text{vir}}$), mass ($M_{\text{vir}}$), or velocity ($V_{\text{vir}}$), defined such that the mean density inside the virial radius is $\Delta_{\text{vir}}$ times the mean universal density $\rho_u$ at that redshift:

$$M_{\text{vir}} = \frac{4\pi}{3} \Delta_{\text{vir}} \rho_u R_{\text{vir}}^3.$$  (2)

The critical overdensity at virialization $\Delta_{\text{vir}}$ is motivated by the spherical collapse model; it has a value $\approx 180$ for the Einstein–de Sitter cosmology and $\approx 340$ for the $\Lambda$CDM cosmology assumed here. A useful alternative parameter for describing the shape of the profile is the concentration parameter $c_{\text{vir}}$, defined as $c_{\text{vir}} = R_s / R_{\text{vir}}$. NFW found that this functional form provides a good fit to halos over 2 decades in radius for a large range of halo masses and for several different cosmological scenarios. They tested it for the Einstein–de Sitter model with a standard CDM power spectrum of initial fluctuations (SCDM), a flat cosmological model with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and a corresponding CDM power spectrum (ACDM), and several models with power-law power spectra (confirmed by Craig 1997 and Kravtsov, Klypin, & Khokhlov 1997).

Later studies (e.g., Moore et al. 1999; Ghigna et al. 2000; Klypin et al. 2001) have indicated that the inner logarithmic slopes of at least some halo density profiles in CDM cosmologies are closer to $-1.5$ than to $-1.0$. However, Klypin et al. (2001) showed that even for halos that are better fitted by $-1.5$ slopes, an NFW fit is perfectly adequate outside the inner $\approx 1/3$ of the virial radius (this corresponds to $\approx 3$ kpc for a Milky Way–type halo). An important implication (as pointed out by Bullock et al. 2001, hereafter B01) is that even if halos are not perfectly described by the NFW form (see also Avila-Reese et al. 1999 and Jing 2000), fitted parameters derived using this profile provide a useful general characterization of the density structure, relating, for example, the central density of a halo to that of the background via two fitted parameters (e.g., $c_{\text{vir}}$ and $R_{\text{vir}}$).

The two-parameter NFW characterization of halo profiles has provided several useful insights into the nature of halo collapse. Among the most interesting results, first noticed by NFW, was that, for a given cosmology, the central density $\rho_s$ varies inversely with halo mass, or, equivalently, $c_{\text{vir}}$ tends to increase with decreasing $M_{\text{vir}}$. A natural reason for this fact is that low-mass halos typically collapse earlier, when the universe was denser, and the central density somehow reflects this higher background density. In a toy model to explain this correlation, NFW (1997) assumed that $\rho_s$ is a constant multiple $k$ of the universal density at collapse redshift $z_c$. They defined the collapse redshift as the time when half of the halo’s mass was first in progenitors more massive than $f$ times the halo’s mass. The general trend of the relation between the two profile parameters at $z = 0$ is reproduced well using EPS to predict $z_c$ and a proper choice of values for the constants $k$ and $f$ (favored parameters for $\Lambda$CDM were $f = 0.01$ and $k = 3.4 \times 10^3$).

Subsequent analyses revealed additional complexities and trends in halo structure. First, it has been realized that while the $M_{\text{vir}}$-$c_{\text{vir}}$ trend holds on average, halos of fixed mass show significant scatter in their $c_{\text{vir}}$ values (Bullock 1999; Jing 2000; B01). In the context of the proposed correlation between concentration and collapse time, the scatter in $c_{\text{vir}}$ is a natural reflection of the expected scatter in collapse time for halos of a given mass.

B01 also found that halo concentrations (at fixed mass) are systematically lower at higher redshifts, $c_{\text{vir}} \propto 1/(1 + z)$, implying a much stronger trend than that predicted by the NFW toy model. Since by definition the concentration at redshift $z$ roughly relates the halo central density to the universal background density at that time via $c_{\text{vir}}(z) \propto \langle \rho_c(z) / \bar{\rho}(z) \rangle^{1/3}$; we have $c_{\text{vir}}(z) \propto \rho_c(z)^{1/3} / (1 + z)$. The observed relation implies that a halo’s central density (or collapse time) is set only by the halo’s mass, independent of the redshift when the halo is observed. One reason for the shortcoming of the NFW toy model in reproducing the proper redshift dependence is that the collapse time as defined by this model also depends on the redshift of observation. In order to properly match the time evolution, B01 proposed a modified toy model in which the collapse time, denoted now by $a_c$ (instead of $a_t$ to avoid later confusion) is set by the mass only and is defined as the epoch at which the typical collapsing mass $M_{\text{vir}}(a_c)$ equals a fixed fraction $F$ of the halo mass at epoch $a$, $M_{\text{vir}}(a_c) \equiv FM_{\text{vir}}$. The typical collapsing mass is defined by $\sigma[M_{\text{vir}}]/M_{\text{vir}} = 1.686 / D(a)$, where $\sigma(M)$ is the rms density fluctuation on the comoving scale encompassing a mass $M$, extrapolated using linear theory to the present, $a = 1$, and $D(a)$ is the linear growth rate. The implied concentration is given by $c_{\text{vir}}(a_c) = K a / a_c$, which, with appropriate values of $F$ and $K$, reproduces quite well the dependence of concentration on both mass and redshift as measured in simulations (see B01 for details).

Simplified models of the type discussed so far provide a qualitative understanding and useful parameterization of the complex processes seen in simulations (for further insights and an exploration of nonhierarchical power spectra, see Eke, Navarro, & Steinmetz 2001). However, our understanding remains somewhat schematic, and several important questions remain open. First, how do individual halo density profiles build up over time? How do the mass accretion histories affect the final halo concentrations, and how can physically realistic mass accretion histories be connected to the simplified definition of formation time advocated by B01? Can EPS be used to predict halo concentrations? What physical process is responsible for the scatter in $c_{\text{vir}}$ at fixed mass? This work builds on the work of B01, using the same simulations analyzed there together with a new “structural merger tree” described below. The goal is to see if we can directly correlate the assembly history of halos with their dark matter halo density profiles, test the model proposed by B01, and develop a physical model for the range of halo concentrations seen in simulations, including scatter and dependence on mass and redshift.

These questions are interesting from a theoretical perspective, but they also have direct observational implications. The shape of the halo density profile directly affects the rotation curve of the galaxy that forms within it; the $1 \times$ scatter in concentration observed by B01 corresponds to, for a $1 \times 10^{11} \ h^{-1} M_\odot$ halo, a scatter in
values of $\approx 85$–105 km s$^{-1}$. Thus, scatter in halo density profiles is directly related to scatter in the Tully-Fisher relation and also has implications for the relative contributions of halo and disk to velocity rotation curves. In addition, density profiles may affect other aspects of the galaxy formation process, such as the efficiency of gas cooling or the star formation rate. If halo concentrations are related to their mass assembly histories, it may indicate that halo concentration is correlated with galaxy type. This would have implications for both the zero point and scatter in the Tully-Fisher relation and could also have implications for the rotation curves of low surface brightness galaxies. Possible correlations between halo density profiles and galaxy observables (with the exception of mass) have been neglected in previous modeling efforts, but it is clear that such correlations are likely to be quite important.

We begin in § 2 by summarizing the relevant details of the $N$-body simulation, halo finder, and density profile fitting. In § 3 the ‘‘structural merger tree’’ developed for this project is described. We continue in § 4 by detailing how mass accretion histories are constructed and then describe a new method for defining a characteristic formation epoch for each halo. We then show how this formation epoch can be related to the halo concentration and how this can explain the dependence of concentration on mass and redshift as well as explaining the origin and magnitude of the scatter in these relations. In § 8 we show how extended Press-Schechter theory can be used to predict concentrations for individual halos using our model for relating halo concentration to characteristic formation epoch. This model can reproduce the scatter, mass, and redshift trends observed in $N$-body simulations. In § 9 we discuss the scatter in $c_{\text{vir}}(M_{\text{vir}})$ and how it depends on the merging history of halos. We discuss the implications of our results and conclude in § 10.

2. SIMULATED HALOS

In the work that follows, we consider only one cosmology whose evolution has been simulated with the Adaptive Refinement Tree (ART) code (Kravtsov et al. 1997), a flat $\Lambda$CDM model with $\Omega_m = 0.3$, $h = 0.7$, and $\sigma_8 = 1.0$. The simulation is the same as that used in B01. It followed the trajectories of $256^3$ cold dark matter particles within a cubic, periodic box of comoving size $60$ h$^{-1}$ Mpc from redshift $z = 40$ to the present. A $512^3$ uniform grid is used, with up to six refinement levels in the regions of highest density, implying a dynamic range of 32,768. The formal resolution of the simulation is thus $f_{\text{esc}} = 1.8$ h$^{-1}$ kpc, and the mass per particle is $m_p = 1.1 \times 10^9$ h$^{-1}$ M$_\odot$. For the purpose of constructing accurate merger trees, here we analyze the simulation data from 36 output times thinly spaced between $z = 7$ and 0. It should be noted that the methods described here for finding and fitting halos and constructing merger trees are completely generalizable to other simulations or cosmologies.

The halo-finding algorithm used here is based on the bound density maxima technique (Klypin & Holtzman 1997) described in Bullock (1999) and B01, but we have modified and optimized it for the purpose of building a structural merger tree. The essential elements are presented briefly here, and details are described in Appendix A. For each density maximum, we step out in radial shells until the mean overdensity falls below $\Delta_{\text{vir}}(z)$, or the radial profile shows a significant upturn. We denote this radius as $R_8$ and the mass determined by counting particles within this radius as $M_8$. We attempt to identify halos containing as few as $N_{\text{min}} = 20$ particles; our halo catalog thus includes $\sim 14,000$ halos above a mass threshold of $2.2 \times 10^{10}$ h$^{-1}$ M$_\odot$. By comparing our simulation with a smaller, higher resolution realization of the same cosmology, at this mass we estimate our completeness to be 70%, and we are $\sim 100\%$ complete at $6.6 \times 10^{10}$ h$^{-1}$ M$_\odot$. NFW profiles are fitted to halos with more than $N_{\text{min}} = 200$ particles, corresponding to halos more massive than $2.2 \times 10^{11}$ h$^{-1}$ M$_\odot$. A profile fitted to a halo of only a few hundred particles carries large errors, but as long as the fit is done properly to include these errors, reliable concentrations estimates can be obtained (B01). Halos in the mass range $(2-5) \times 10^{11}$ h$^{-1}$ M$_\odot$ have typical fitted mass errors of about 10% and $c_{\text{vir}}$ errors of 15%–20%. However, a few percent of the time the errors are significantly larger than that. We therefore make a rigorous attempt to estimate the errors and take them into account in every step of the process. Poor fits are marked by large errors that are incorporated in the analysis and the results we present. The outcome at any output time is a statistical halo catalog that includes all the bound virialized systems in the simulation above the mass threshold of $2.2 \times 10^{10}$ h$^{-1}$ M$_\odot$. At $z = 0$ there are 14,219 such halos. The output for each halo includes the list of its bound particles, the location and velocity of its center, and the NFW profile parameters (e.g., $c_{\text{vir}}$ and $M_{\text{vir}}$) and corresponding errors ($\sigma_c$ and $\sigma_{M_{\text{vir}}}$) when applicable. (We also include information about the halo angular momentum properties; this is not used in the present work, but see also Wechsler 2001, Vitvitska et al. 2001, and R. H. Wechsler et al. 2002, in preparation.) The mass function of this revised halo catalog and a detailed comparison with the halo catalog used in the work of B01 has been presented in Wechsler (2001).

3. CONSTRUCTING A MERGER TREE

As a base for the structural merger tree, we use the distinct halo catalogs described above at each of 36 output times of the simulations, from $z = 7$ to the present. The output times are well spaced in redshift; the cosmological scale factors associated with the saved output times are shown in Figure 1. Between each set of output times, we track the trajectories of all of the particles. Given a halo and an output time, we tag all of its particles and track them back to the previous output time. We then make a list of all halos at that earlier output time containing tagged particles, recording the number of tagged particles contained in each one. A similar list of halos and tagged particle numbers is obtained for the neighboring future output time. In addition, we record the number of particles that are not in any halo in the previous output time and the number of particles that do not end up in a halo in the subsequent output time.

We have two criteria for halo 1 at one output time to be labeled a “progenitor” of halo 2 at the subsequent output
time. In our language, halo 2 will then be labeled an “offspring” of halo 1. First, more than half the particles in halo 1 must end up in halo 2. In addition, since, on occasion, a halo’s particles do not end up in any halo in the subsequent output time, we also demand that more than 70% of the particles in halo 1 that end up in any halo must end up in halo 2. Thus, a halo is allowed to have only one offspring, but there is no limit on the number of progenitors a halo may have. In much of the work that follows, we will focus on a “most massive progenitor” halo in the previous output time. In general, this is the progenitor halo that contributes the largest number of particles; however, if the halo’s progenitor mass is not at least half of its mass, we additionally require that the progenitor’s most bound particle be included in the halo. Even if this is the case, we also check that the halo’s most bound particle in the present output time was a member of the progenitor; if it was not, the halo is only designated as the most massive progenitor if it is more massive than the minimum mass of the halo catalog and it is at least one-third the mass of the offspring halo. These criteria were designed to maximize the redshift extent of as many mass accretion trajectories as possible, without counting trajectories that may have been affected by the completeness of the catalog or severe errors in fitting.

We have used the criteria outlined above to construct the merger tree of every halo at every output time. Examples of such a merger tree are shown in Figure 2, which shows the mass accumulation history of a small cluster halo of mass \( M_{\text{vir}} = 2.8 \times 10^{14} \, h^{-1} M_{\odot} \) at \( z = 0 \) and a galaxy mass halo \( (M_{\text{vir}} = 2.9 \times 10^{12} \, h^{-1} M_{\odot} \) at \( z = 0 \). Each halo in the tree is represented by two circles, one with radius proportional to the halo’s virial radius and one with radius proportional to the halo’s best-fit NFW \( R_s \) parameter.

4. MASS GROWTH CURVES

For the purposes of understanding the evolution of halo concentrations, it is useful to characterize the history of mass assembly in each halo by tracking the evolution of its most massive progenitor. The mass assigned to the most massive progenitor at each output time is typically the best-fit virial mass \( M_{\text{vir}} \). However, for cases in which the fit errors on \( M_{\text{vir}} \) were large, we used an iterative procedure, described in Appendix B, to determine a corrected mass; this mass is based on either the fit mass, the measured mass within \( R_{\text{vir}} \), or an interpolated mass between the previous and subsequent time steps. In addition, our halo mass trajectories do not always extend as far back as the first analyzed output time of the simulation. This usually happens because the most massive progenitor at some redshift falls below our completeness limit and cannot be identified, although there are also rare cases in which our criteria for progenitor are simply not met (see Wechsler 2001). In order to have complete trajectories for a reasonable number of halos and in order to have reliable fits for most halos, we limit our analysis to halos more massive than \( 10^{12} \, h^{-1} M_{\odot} \) at \( z = 0 \). In this mass range, which includes \( \sim 900 \) halos, \( \sim 95\% \) of the halo trajectories extend back to \( z = 2 \), and \( \sim 90\% \) extend back to \( z = 3 \).

Figure 3 shows the history of mass growth for the major progenitors of several different halos, spanning a range of masses and concentration parameters. Notice that more massive halos tend to show substantial mass accumulation up to late times, but the growth curves for less massive halos tend to flatten out earlier. This tendency is illustrated in Figure 4, which shows the average mass accretion histories for halos binned by final mass. Again, the high-mass halos form later than low-mass halos, as expected in CDM models.

Since mass accretion is a continuous process, the loose term “formation time” is ambiguous, and it requires an agreed, measurable, quantitative definition. The trends of formation time with mass and redshift may depend on this definition. One common choice is to set the formation time equal to the time when the mass in progenitor halos (or in the most massive progenitor) is equal to some fraction of the halo’s final mass \( M_0 \) (see, e.g., NFW 1997; LC93). Definitions of this type have a common feature: the formation time is a relative measure that depends, for a given halo trajectory, on the redshift \( z_0 \) at which the halo is observed. As \( z_0 \) is increased, the formation redshift \( z_f \) also increases. However, it increases more slowly since mass accretion proceeds more rapidly at high redshift in CDM models. Thus, both the formation redshift \( z_f \) and the ratio of \( (1 + z_f)/(1 + z_0) \) change with time in such a model. As mentioned in § 1, this feature of the formation time definition is the reason that the redshift dependence proposed for the NFW model fails to match accurately the dependence seen in simulations. Another shortcoming of this kind of definition is that it is limited to using the value of the halo trajectory at one time, which may introduce noise and miss relevant information. It would be useful to find a quantity that more fully characterizes the whole assembly history of the halo and preferably one that does not depend on the redshift of observation \( z_0 \) (at a possible cost that such a definition may be allowed to have formation times in the future).

By examining a range of full mass assembly histories for our sample of halos, we have found a useful parameterized
form that captures many essential aspects of halo growth over time. Remarkably, we find that both average mass accretion histories and mass accretion histories for individual halos, as observed at $z = 0$, can be characterized by a simple function:

$$M(a) = M_0 e^{-a z}, \quad a = (1 + z)^{-1}.$$  \hspace{1cm} (3)

Although individual halo trajectories may deviate from this form significantly in places (e.g., at the time of a major merger), this one-parameter model (in addition to the halos' final mass $M_0$) provides a remarkably good characterization of the range of halo mass accretion trajectories. Fits to this equation are shown in Figure 3 for several representative individual halos. Van den Bosch (2002) has independently shown that a similar, two-parameter, functional form can be used to represent halo mass accretion histories for a variety of cosmologies and over a large mass range.
The single free parameter in the model $a_c$ can be related to a characteristic epoch for formation $a_c$, defined as the expansion scale factor $a$ when the logarithmic slope of the accretion rate $d \log M / d \log a$ falls below some specified value $S$. The functional form defined in equation (3) implies $a_c = \alpha/S$.

The same formation epoch can be defined equivalently for any "observing" epoch $z_o$ of that halo by replacing $a$ in equation (3) with $a/a_o$. In this case, the characteristic formation time is related to $\alpha$ via

$$a_c = \frac{a_o \alpha}{S}.$$  

Thus, at any such observing redshift, with scale factor $a_o = 1/(1 + z_o)$ and mass $M_o = M(z_o)$, the mass growth is fitted by

$$M(a) = M_o \exp \left[ -a_c S \left( \frac{a_o}{a} - 1 \right) \right].$$  

This implies that for any halo whose mass accretion trajectory resembles equation (3), the characteristic formation time is the same regardless of the redshift $z_o$ at which the halo is observed. In what follows we have chosen $S = 2$.

Since the value of $S$ is not used for the fit but only serves to define $a_c$, this choice is arbitrary.$^8$

The range of formation scale factors defined in this manner exhibits a lognormal distribution whose mean value increases with increasing mass—low-mass halos typically accrete their mass early, while high-mass halos are typically still accreting mass rapidly in the present epoch. However, for a given mass, there is a large scatter in formation epoch. The scatter in the mass accretion trajectories for a given mass can be seen in Figure 4; we resume a detailed discussion of the mass dependence of $a_c$ and the scatter about this relation in § 7.

5. CONCENTRATION AND ASSEMBLY HISTORY

We find that the concentration of a halo is tightly correlated with the characteristic formation epoch as defined in the previous section. Figure 5 shows the average evolution of the concentration parameter for halos in different mass...
ranges, corresponding to the average mass trajectories shown in Figure 4. From this figure it is clear that halo concentrations have a stronger trend with less scatter when binned on $a_c$ (right) than when binned on mass (left).\footnote{Note that the figure only shows this directly for $z = 0$, although it is true at any redshift $z_c$ when $a_c$ is measured at $z_c$. However, the scatter about the average trajectory increases with $z$ since a halo cannot uniquely predict its future.} We therefore investigate how $c_{\text{vir}}$ is related to $a_c$ directly.

Figure 6 shows the relation between concentration and $a_c$ for halos at $z = 0$. The concentration of a halo is strongly correlated with its characteristic formation time, and a good fit is obtained with the inverse relation:

$$c_{\text{vir}} = \frac{c_1}{a_c}, \quad (6)$$

where $c_1 = 4.1$ is the typical concentration of halos forming today. The scatter about this relation is already not too large for all the halos, but we note that most outliers fall in one of the following three special categories:

1. The halo has a truncated trajectory that does not extend far back to the past, and thus $a_c$ is not well determined.

2. The halo has a significant discontinuity in its $c_{\text{vir}}$ trajectory at the final output time only, so that this value is not representative of the whole trajectory (this can occur if there is a merger or other disruption occurring at the final output time).

3. The assembly history includes a merger that is substantially larger than the average accretion rate at that time, and thus equation (3) does not provide a good description of the actual history.

To deal with special case 1, halos with trajectories that do not extend as far back as $z = 1$ are excluded from further analysis (fewer than 5% of cases; these halos are indicated by plus signs in Fig. 6). For case 2, outlined by squares in Figure 6, we find that a much better agreement with the median relation is obtained when the last discrepant value of $c_{\text{vir}}$ is replaced by the value of $c_{\text{vir}}$ in the preceding output time. We do not attempt to cure the problem associated with case 3, except for keeping in mind that the outliers remaining in the $c_{\text{vir}}$-$a_c$ relation are often due to a failure of equation (3) to adequately model the history of that halo. With these modifications, the scatter in $c_{\text{vir}}$ for a given $a_c$ is $\Delta(\log c_{\text{vir}}) \approx 0.09$, without removing additional scatter due to large NFW fit errors (see also Fig. 9), and $\Delta(\log c_{\text{vir}}) \approx 0.05-0.06$ when errors in $c_{\text{vir}}$ are corrected for.

We have also examined the dependence of $c_{\text{vir}}$ on the merger history of halos, which is correlated with but distinct from the mass accretion history of the most massive progenitor discussed above. Since halos that did not undergo a recent major merger are more likely to have accreted most of their mass early, they typically have earlier formation times and higher concentration values (see Fig. 17). However, we find that the parameter $a_c$ we have defined based on the mass assembly history is more useful; for halos with a given $a_c$-value, the occurrence of a recent merger is not an important factor affecting the concentration. This can be seen in Figure 6, which demonstrates that halos that have

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**Figure 4.**—Average mass accretion histories, normalized at $a = 1$. Left: binned in three bins by final halo mass: $M_0 = (1-4) \times 10^{12} \, h^{-1} \, M_\odot$ (solid line), $M_0 > 3 \times 10^{12} \, h^{-1} \, M_\odot$ (dashed line). The three curves connect the averages of $M(a)/M_0$ at each output time. The pair of dotted lines shows the 68% spread about the middle case (the spread is comparable for the other bins). We see that massive halos tend to form later than lower mass halos, whose mass accretion rate peaks at an earlier time. Right: binned in three bins by formation epoch $a_c$. The dot-dashed lines correspond to early formers (typically low-mass halos) and the dashed lines to late formers (typically higher mass halos). The averages and spread are displayed in analogy to the left-hand panel.
experienced recent major mergers (circles; defined here as a merger ratio of greater than $1/3$) form later on average but follow the same trend in $c_{\text{vir}}$ versus $a_c$ (albeit with more scatter; see §9) as halos with no recent major mergers (triangles).

6. DEPENDENCE ON REDSHIFT OF MEASUREMENT

We now investigate how the correlation found in the previous section between $c_{\text{vir}}$ and $a_c$ as measured at $z = 0$ behaves as a function of the redshift $z_o$. We found that $c_{\text{vir}}$ is inversely proportional to $a_c$, or equivalently to the fit parameter $C_{11}$ from equation (3). If the determining factor in a halo’s concentration is the shape of its mass growth curve, then we expect that a similar dependence would hold for all redshifts. Equation (4) would then imply that the concentration should be inversely proportional to $a_c/a_o$ for any $a_o$ when the halo is observed. In Figure 7 the measured concentration values are plotted versus this scaled formation epoch for halos at $z_o = 0, 1,$ and $2$. Halos appear to follow the same trend regardless of mass or redshift.

In order to obtain the most reliable estimate of the proportionality constant $c_1$ in the linear regression of $c_{\text{vir}}$ and $a_c/a_o$, we use halos in the mass range $(1-5) \times 10^{12} h^{-1} M_\odot$ and consider the errors in both $a_c$ and $c_{\text{vir}}$ while excluding outlying points that are more than $2 \sigma$ away from the best-fit line. Considering halos at $z_o = 0, 1,$ and $2$ together yields the same result as obtained for $z_o$ halos alone, properly scaled by $a_o$:

$$c_{\text{vir}} = \frac{c_1 a_o}{a_c},$$  \hspace{1cm} (7)\

where $c_1 = 8.2/S$ (here we have used $S = 2 \rightarrow c_1 = 4.1$). As before, the parameter $c_1$ is the typical concentration of halos whose formation time is at the time of measurement, $a_c = a_o$. For the $\Lambda$CDM cosmology considered here and $a_o = 1$, this is the typical concentration of halos of
Figure 7 shows that this formula provides a good description of the observed correlation between concentration and formation time for halos at all masses and redshifts.

7. PROPERTIES OF THE FORMATION TIME

As discussed above, B01 showed that the average concentration value measured for halos of a fixed mass scales as $c_{\text{vir}} \propto a_0$. This trend was understood using a simple model in which the central densities of halos are set by the density of the universe at a characteristic collapse time $a_c$, implying $c_{\text{vir}} \propto a_0/a_c$. Crucial to the success of this model was that the definition of collapse time $a_c$ was independent of $a$ for fixed-mass halos. In the previous sections we showed that a similar result is obtained using a different definition of the characteristic formation time: $c_{\text{vir}} \propto a_0/a_c$, where $a_c$ is derived based on the actual history of mass accretion in individual halos rather than on a simple universal scaling argument as in B01. In order to obtain a consistent trend with redshift, we expect that $a_c$ (like $a_c$) will be independent of the epoch $z_o$ when the measurement is performed for a given halo mass. This assumption is tested in Figure 8, which shows the dependence of $a_c$ on mass for halos identified at three distinct redshifts in the simulation. Within the errors, it shows roughly the same mass trend regardless of redshift; this is a key feature that our $a_c$ parameter has in common with the collapse epoch $a_c$ defined by B01. In fact, these parameters can be directly associated (for an appropriately chosen value of $S$). For example, we find that $a_c$ follows the same mass trend as our derived $a_c$ (with $S = 2$) if it is defined as in B01 [$M_\odot (a_c) = FM$] with $F = 0.015$.

The value of $a_c$ for a given mass halo also shows significant scatter, as can be seen in Figure 8. The amount of scatter ($\sim 0.13$ in the log) can almost completely account for the scatter seen in the $c_{\text{vir}}$ versus $M$ relation (discussed in detail in §9). Figure 9 shows probability distributions of $a_c$ and $c_{\text{vir}}$ for a given mass range. The intrinsic scatter in $c_{\text{vir}}$ for a given $a_c$ is relatively small compared to the scatter in $c_{\text{vir}}$ for a given mass, and we find that the measured distribution of $c_{\text{vir}}$ can be practically reproduced if equation (7) is used to transform each measured $a_c$-value into a concentration.
Thus, the scatter in concentration can be understood as deriving almost exclusively from the range of accretion histories for a given halo mass.

It is encouraging that our definition of formation time is robust both in terms of measuring the same value at different epochs along the growth curve of a given halo and in terms of its average value for halos of a given mass when measured at different redshifts. We explore this further in the next section. Although a given halo mass accretion history that is a perfect fit to equation (3) will have a constant value of \( \alpha_{c} \), regardless of when it is observed, we find that the early part of a halo’s trajectory is not a good indicator of the latter part. This is shown in Figure 10, which demonstrates that the formation time measured using the first half of a halo’s history is not correlated with the formation time measured using the second half.

8. REPRODUCING THE RESULTS WITH EPS

We have demonstrated that the tight correlation found in the simulations between \( \alpha_{c} \) and that \( c_{\text{vir}} \) can account for the mass and redshift trends and the scatter in \( c_{\text{vir}}(M) \). However, it would be useful to have a way to model the concentrations without such a computationally expensive simulation for incorporation into analytic or semianalytic models. Of course, the model of B01 did this to some extent for average halo properties as a function of mass and redshift, but this neglects the tight correlation found here between individual halo density profiles and their mass accretion histories. In this section we compare the mass accretion trajectories from our simulation with trajectories generated using the EPS formalism and test whether the correlation we found, \( c_{\text{vir}} \propto \alpha_{c}^{-1} \), can be used to predict concentrations for individual halos semianalytically.

Comparisons of specific aspects of mass accretion histories as derived semianalytically and as extracted from simulations have been performed (SLKD; Gardner 2001), but here we focus on the quantity relevant for our modeling of \( c_{\text{vir}} \), namely, the range of values of \( \alpha_{c}(M, z) \), which has not been investigated previously. There are essentially four relevant questions in this analysis:

1. Do EPS trees produce the same range of \( \alpha_{c} \)-values for a given mass as halo trees extracted from simulations?
2. Do EPS trees produce the same trend of \( \alpha_{c} \) with \( M \), which could then explain the \( c_{\text{vir}} \) trend with \( M \)?
3. Is \( \alpha_{c}(M) \) constant with redshift for EPS trees, which could then explain the \( c_{\text{vir}}(z) \) trend?
4. Does the scatter in \( \alpha_{c} \) for a given mass in EPS trees account for the scatter in the measured \( c_{\text{vir}}(M) \) relation?

8.1. \( \alpha_{c} \) versus \( M_{\text{vir}} \) and the EPS Offset

In order to answer these questions, we first generate a random ensemble of mass accretion histories. This is done by coupling the extended Press-Schechter formalism, which predicts the probability of accreting a given mass in a given time, with a method for generating Monte Carlo merger trees. Here we use the scheme introduced by SK99, with an additional modification proposed by Bullock, Kravtsov, & Weinberg (2000). For completeness, we outline the fundamental aspects of the method in Appendix C.

We consider the list of all halos found in the simulation with masses larger than the minimum-fit mass \( 2 \times 10^{11} \ h^{-1} M_{\odot} \ (\sim 3000 \ \text{halos}) \). We then generate 10 Monte Carlo realizations of mass accretion trajectories for each of these halos, based solely on its \( z = 0 \) mass, keeping track of the growth of the most massive progenitor as a function of time. We start at \( z = 0 \) and trace histories back to \( z = 7.2 \). Using this most massive progenitor trajectory, \( \alpha \) is calculated by fitting the trajectory to equation (3). The value of \( \alpha_{c} \) is then defined by equation (4), with \( S = 2 \).

Figure 11 shows the distribution of \( \alpha_{c} \)-values found for different given mass ranges for both the simulated halo trajectories and the EPS trajectories. We see that the distribution of \( \alpha_{c} \)-values from EPS trees is systematically offset toward later formation times for all mass ranges and also appears to be slightly broader. A similar discrepancy was found by SLKD, although they did not investigate the same
quantity—they found the average mass of the largest progenitor to be larger in the EPS trees than in the simulated trees at low redshift and smaller at high redshift. This implies later formation times for the simulated halos, as seen here. This discrepancy can be understood in terms of comparing the conditional progenitor mass functions; SLKD showed that EPS overpredicts it for low masses and underpredicts it for high masses, with the mass scale of the crossover decreasing with increasing redshift. This discrepancy in formation epochs seems to directly reflect the more well-known finding that PS overpredicts the number density of halos below \( M^* \) and underpredicts the number density above \( M^* \) (see, e.g., Gross et al. 1998; Sheth & Tormen 1999). Although only one cosmological model (ΛCDM) has been investigated here, the disagreement of the halo mass function with Press-Schechter is generic, and we expect that the formation time discrepancy would be seen to some extent in any CDM cosmogony.

Although we find a discrepancy in the median \( a_c \)-values as a function of mass, we find that they are offset by a constant multiplicative factor. Figure 12 shows the median and 68% scatter for \( a_c(M) \) from the EPS trajectories and the simulated trajectories for \( z = 0 \) halos. The constant offset is in the sense that the characteristic formation epochs derived from the EPS trajectories are roughly 25% larger than those measured from the simulation trees. The scatter is quite comparable, although it is slightly larger for the EPS-derived values.

If we assume that the \( a_c \)-values found using EPS trajectories are correct except for this constant 25% offset, due to the known discrepancies in PS theory, then these values can be used in combination with equation (7) to estimate \( c_{\text{vir}} \) for
Figure 14 shows the $a_c$ must be uniquely set by the mass, independent of redshift. When halos are combined with the proper mass weighting, they thus conspire to keep the same relation regardless of redshift. Figure 15 demonstrates this in detail. In the upper panel, the $a_c(M)$ trend is shown at $z = 0$ for halos in two distinct mass ranges. In the lower panel, these same halos are shown at $z = 3$: the $a_c$-values for each halo remain essentially unchanged, but they are now plotted against their $z = 3$ masses. Halos in any given mass range at $z = 3$ consist of a combination of low-$a_c$ halos that will have low $z = 0$ masses and high-$a_c$ halos that will have high $z = 0$ masses. These add in a manner that maintains the shape and normalization of $a_c(M)$. The fact that $a_c(M)$ is constant with redshift is an indication that the model proposed by B01 is a reasonable one—i.e., that (on average) the formation time of a halo is set only by its mass and can be related to the time that mass was a fixed fraction of $M_*$.

In summary, the formation times of halos in EPS (using an improved version of the SK99 method to generate merger trees) are somewhat later than those found in the ART simulation. However, if we measure $a_c$-values for these semianalytic trajectories (using eq. [5]), multiply these values by a constant factor (0.8), and translate these formation epochs to concentrations using the relation $c_{\text{vir}} = c_{\text{ao}}a_c/a_o$, we are able, to a good approximation, to match the scatter, mass, and redshift dependence of the $c_{\text{vir}}$-values found for simulated halos. This method can be used in semianalytic models to estimate halo concentrations that are both based on their individual mass growth histories and have the correct distribution at every redshift.

The only disadvantage of the above method is that it requires generating full mass accretion histories for a sample of halos. Since equation (5) is a one-parameter model, one might be tempted to directly calculate the more prevalent formation redshift $z_f$, when the most massive progenitor mass was half of $M_0$ and whose probability distribution can be calculated directly from EPS without generating merger trees, and then translate this value into $z_c$ using equation (5) in order to derive a concentration. However, while this method predicts the mean values relatively well, in fact, there is substantially more scatter in values of $z_f$ than in values of $z_c$. This is due to the fact that individual trajectories are noisy, and at any one point in the trajectory, there is likely to be significantly more scatter than in the shape of the halo as a whole.

9. SCATTER IN THE $c_{\text{vir}}(M_{\text{vir}})$ RELATION

9.1. Evaluating a Corrected Scatter

The scatter in the concentration parameter has been estimated by B01 and Jing (2000), and it may have a number of important observational implications. Jing (2000) found a scatter of $0.08-0.1$ in $\log_{10} c_{\text{vir}}$, while B01 derived a somewhat larger scatter of $\Delta \log_{10} c_{\text{vir}} = 0.14$. A revised scatter
estimate from our improved halo catalogs, which also relies on the mass trajectories, is presented here. We also discuss how the scatter in $c_{\text{vir}}$ is affected by the merging history of halos.

B01 devised a method for correcting the scatter estimate for errors in the fits and actually plotted this corrected scatter in Figure 4. Our new halo catalogs have significantly fewer halos with large fit errors and thus a smaller uncorrected scatter, but encouragingly, when we use the method of B01 for correcting this for fit errors and Poisson errors, we get almost identical results (shown in Fig. 16). This method involves doing 500 Monte Carlo realizations of each halo, in which their $c_{\text{vir}}$-value is chosen from a one-sided Gaussian deviate with a standard deviation error on the measured $c_{\text{vir}}$-value. This value is then added or subtracted to the measured value depending on whether the measured value was below or above the median value for that mass. Poisson errors due to finite statistics are then subtracted in quadrature to obtain an estimate for the intrinsic scatter—which we also find to be $\Delta(\log c_{\text{vir}}) \approx 0.14$.

As mentioned previously, there may be halos that for a short period of time are not well fitted by an NFW density profile; in many cases this occurs when a halo is undergoing merging or disruption. In most cases, this results in an artificially low value of $c_{\text{vir}}$, which usually persists only for one time step. In order to remove the effects of badly estimated concentration parameters, which persist only for a short time, we use the merging history of a halo to identify those cases in which the concentration has jumped significantly since the previous output time. The $c_{\text{vir}}(M)$ relation excluding these halos (which make up $\approx 8\%$ of the total) is plotted in Figure 16 compared to the whole sample; the scatter for this sample is reduced from $\approx 38\%$ to $\approx 31\%$. Note that the exclusion of these halos does not change the median value significantly. We regard the scatter estimate from excluding these halos as a lower limit. With this correction, our analysis is closer to the scatter estimate presented by Jing (2000). It is possible that some of the remaining discrepancy could be due to an underestimate of fit errors; the total scatter for the very massive halos in our analysis is slightly smaller.

![Fig. 11.—Comparison of $a_c$-values in the simulation with those derived from EPS trees for various mass ranges in the different panels. The filled histogram represents simulated halos, and the shaded histogram represents 10 realizations of EPS trajectories for the same set of masses. Abnormal halos (whose trajectories end prematurely) in the simulations are counted in the leftmost bin; the number of these cases is small and negligible above $10^{12} h^{-1} M_\odot$.](image-url)
However, it should also be pointed out that Jing (2000) only considered relaxed halos, and our analysis contains halos with a full range of properties, including those that have been recently disrupted—and these add significantly to the scatter.

9.2. Scatter and Halo Merger History

As discussed in §5, halos with recent major mergers display the same trend between concentration and \(a_c\) (eq. [3]) but with somewhat more scatter. We show in Figure 17 that these halos also follow the same basic trend with mass. This figure compares this trend and scatter in \(c_{\text{vir}}(M)\) for all halos with that seen in halos that have not had a major merger since \(z = 2\). The scatter is reduced from about 31% (38%) for all halos to \(\sim 26\%\) (28%) for those halos without recent major mergers, where the first listed estimate is when halos with large \(c_{\text{vir}}\) jumps are excluded, and the number in parenthesis includes all halos. (The implied correction for halos without recent mergers is not large since most of the halos with jumps are in the process of merging.) The sample of halos that have had a major merger since \(z = 2\) do not have reduced scatter compared to the whole sample. Since halos with recent major mergers have later formation times on average, when they are excluded from the sample, the remaining halos have higher \(c_{\text{vir}}\)-values by a factor of about \(\sim 10\%\) compared to the complete sample; this can also be seen in Figure 17.
The amount of scatter in the concentration parameter for a given mass is of particular interest because of its possible implications for scatter in the Tully-Fisher relation (see, e.g., B01; van den Bosch 2000), and there is particular interest in the amount of scatter in the concentrations of spiral galaxies. In many scenarios for galaxy formation, major mergers destroy disks, and thus halos with recent major mergers are unlikely to host spiral galaxies. As described above, not counting these halos reduces the scatter of the whole sample and thus may reduce consequential scatter in Tully-Fisher to a level that can be matched with observations. Whether these remaining halos host spiral galaxies may be influenced by how much mass they have accreted since their last disruption; this could bring the concentrations of halos hosting spiral galaxies lower than the median shown here. It should also be noted that in this analysis we have only considered distinct halos. To get a full estimate of scatter and normalization for galactic halos, we would have to include subhalos in the analysis, which B01 have shown have somewhat larger scatter in \( c_{\text{vir}} \). We defer a full analysis of these issues to a later work.

10. DISCUSSION AND CONCLUSIONS

Making use of a large sample of dark halos simulated in a cosmological volume, we have studied the relation between mass accretion history and the density concentration of halos. Remarkably, halo mass growth curves (normalized to the final halo mass) can be accurately described by a one-parameter function in which mass accretion occurs rapidly at early times and slows at late times. The characteristic

![Fig. 15.—Analysis of the robustness of \( a_c(M) \) to redshift in EPS trajectories. Top: \( a_c(M) \) at \( z = 0 \), broken into two mass ranges. Bottom: \( a_c(M) \) at \( z = 3 \) for the same mass ranges determined at \( z = 0 \) (dark thick line: high mass; light thin line: low mass). The ratio \( \frac{M(z=3)}{M(z=0)} \) is related to \( a_c \) such that for a given \( z = 0 \) mass, halos with high \( a_c \) will have lower \( z = 3 \) masses. The dashed line shows the sum of the mass bins, which follows a similar trend with mass as the \( z = 0 \) halos. The apparent upturn in the leftmost bin of the dashed curve is due to the exclusion of halos less massive than \( 4 \times 10^{11} M_\odot \) at \( z = 0 \), which contribute lower \( a_c \)-values.

![Fig. 16.—Scatter in the \( c_{\text{vir}}-M_{\text{vir}} \) relation for halos at \( z = 0 \). The thick lines represent all the halos. The thin lines represent all halos whose concentration has not jumped by more than a factor of 2 in either direction since the previous output time (at \( z = 0.01 \)). In each case, the solid lines represent the median value of \( c_{\text{vir}} \), plotted with Poisson error bars, and the dashed lines represent the scatter corrected for errors in the individual profile fits and for Poisson scatter in the bins. In the mass range in which the results are most reliable (\( \sim 1 \times 10^{12} h^{-1} M_\odot \)), the value of the corrected scatter in these two samples is \( \Delta (\log c_{\text{vir}}) \approx 0.14 \) and 0.12, respectively.

"formation" time \( \alpha_c \), defined as the time when the log mass infall rate drops below a fixed value, fully defines the trajectory. We find that the value of \( \alpha_c \) for each halo trajectory is independent of the epoch at which the halo is observed, \( \alpha_c \). In addition, the average value of \( \alpha_c \) for halos of fixed mass is independent of redshift.

The NFW halo concentration parameter \( c_{\text{vir}} \), which in combination with the halo mass uniquely sets the shape of the density profile, is tightly related to \( c_{\text{vir}} \) via \( c_{\text{vir}} = c_1 \alpha_c / \alpha_c \). The central density of a halo at fixed radius grows rapidly when the mass accretion rate is high and approaches a constant value as the mass accretion slows. This result is consistent with a picture in which the mass accretion rate determines how fast accreted mass moves into the center of a halo: for high mass accretion rates, accreted material makes it far into the center of the halo, but as it slows, new material builds up on the outside. Thus, central densities of halos asymptote to a value that is proportional to the density of the universe at the time when the mass accretion rate slows. In late-forming halos this process is delayed, and thus the final central density is lower. To demonstrate the model, in Figure 18 we plot the evolution of several variables for an early- and late-forming halo: the mass, concentration, log slope of the mass accretion rate, scale radius \( R_s \), and the density within a fixed radius. Each parameter is calculated analytically; this is done using equation (1), which specifies the density profile, equation (5), which specifies the mass accretion history, and equation (7), which specifies the relationship between them.
We showed that scatter in \( c_{\text{vir}} \) for a given mass can be explained almost exclusively by scatter in \( a_{\text{d}} \) for haloes of that mass. Thus, this model, based on the \( c_{\text{vir}}-a_{\text{d}} \) correlation, captures successfully the main properties of the concentration parameter, including its mass dependence, redshift trend, and the scatter about these relations.

The formation times derived from semianalytic realizations of EPS mass accretion histories were compared with those measured in the N-body simulation and found to be systematically larger by \( 25\% \). This offset reflects known inaccuracies in the Press-Schechter approximation. By adjusting the formation times for this offset, we have successfully used the \( c_{\text{vir}}-a_{\text{d}} \) correlation to reproduce the mass and redshift dependence of \( c_{\text{vir}} \) and the scatter about these relations using mass accretion histories derived from EPS. This technique is likely to be very useful for inclusion in semianalytic models of galaxy formation, which so far have at best drawn \( c_{\text{vir}} \) randomly from an assumed global probability distribution, with no dependence on the halo's history.

We have presented an estimate for the scatter in \( c_{\text{vir}} \), with preliminary hints for how this scatter may depend on galaxy type. For haloes of a given mass that have not had a major merger, we estimate that the scatter is as low as \( \Delta(\log c_{\text{vir}}) \approx 0.1 \). This scatter is roughly consistent with the scatter observed in the Tully-Fisher relation. However, we have not included subhaloes in this estimate, so our results only apply to isolated field galaxies; including subhaloes may increase the scatter. It is also interesting that such haloes (those without recent major mergers, which could conceivably host disk galaxies) are also somewhat more concentrated, which is perhaps contrary to naive expectations.

It is worth pointing out that this is not the only attempt to connect the full mass accretion history of haloes to their density structure. For example, Avila-Reese, Firmani, & Hernández (1998), building on the work of Zaroubi & Hoffman (1993), used an analytic approach based on shell-by-shell mass accretion to connect halo structure with mass accretion (see also Ryden & Gunn 1987). Although their results differ in detail from what we have found using N-body simulations, the general trends associated with early and late formation seem to agree. Our results may serve as a useful benchmark against which specific analytic and semianalytic models of halo structure formation can be tested.

The correlation we have found between individual halo assembly histories and their density profiles is likely to have important consequences for a large range of observable galaxy properties. For example, halo density profiles directly affect galaxy rotation curves and are likely to play an important role in determining galaxy shapes, gas infall, and star formation rates. We thus expect that the inclusion of such a correlation in the context of semianalytic models that track galaxy formation and evolution with simple recipes will affect a number of predictions of these models and thus will provide a significantly more realistic theoretical framework for understanding galaxy populations, the origin of galaxy type, and the variation in galaxy properties.

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APPENDIX A

FINDING AND FITTING HALOS

The details of our procedure for finding and fitting halos are as follows:

1. We construct density field values by a cloud-in-cell (CIC) process (Hockney & Eastwood 1981) on the largest grid of the simulation $\Delta L$ and rank the particles according to their local density as determined on this grid. We then search for the possible halo centers using two sets of smoothing spheres: one with a small radius $r_{sp1}$ in order to locate the centers of tight, small clumps and the other with a larger radius $r_{sp2}$ in order to locate the centers of halos with diffuse cores. The larger radius $r_{sp2}$ is set equal to $R^\text{min}_\text{vir}$, the virial radius of a halo of mass $M^\text{min}_\text{vir}$. The smaller radius is set to $r_{sp1} = 5R^\text{min}_\text{vir}/N^\text{min}_p$, a rough approximation to the radius within which our smallest halo would be expected to contain ~5 particles.

For each set of spheres, we take from the ranked list the particle with the highest local density and place a sphere about its location. A second sphere is placed about the next particle in the list not contained in the first sphere. The process is continued until all of the particles are contained within at least one sphere. Because we are only interested in centers of halos more massive than $M^\text{min}_\text{vir}$, we discard each sphere with fewer than a set number of particles. The minimum number of particles required for a kept sphere is determined separately for each radius.

For the $r_{sp1}$ spheres, we use the following conservative halo density profile:

$$
\rho(r) = \begin{cases} 
C/r_{sp1}^2, & r < r_{sp1}, \\
C/r_{sp1}^2, & r > r_{sp1}
\end{cases}
$$

(A1)

where $C$ is determined by fixing the minimum halo mass to be $M^\text{min}_\text{vir}$ in order to estimate the minimum number of particles within $r_{sp1}$:

$$
N_{sp1} = \frac{N^\text{min}_p}{1 + 6[(R^\text{min}_\text{vir}/r_{sp1})^{1/2} - 1]}.
$$

(A2)

For the $r_{sp2}$ output of the 60 $h^{-1}$ Mpc simulation we analyze, $N_{sp1} = 3$ (rounding to the next lowest integer). Spheres of size $r_{sp1}$ with fewer than $N_{sp1}$ particles are discarded. Similarly, all of the $r_{sp2}$ spheres containing fewer than $N_{sp2} = N^\text{min}_p$ particles are discarded.

The final list of candidate halo centers is made up of all of the (small) $r_{sp1}$ spheres together with each of the $r_{sp2}$ spheres that do not contain an $r_{sp1}$ sphere.

2. For each sphere of radius $r_{sp} = r_{sp1}$ or $r_{sp2}$, whichever applies, we use the particle distribution within the sphere to find the center of mass and iterate until convergence. We repeat the procedure using a smaller radius, $r = r_1$, where $r_1 = r_{sp}/2^{1/2}$. We continue this method until $r_1 = r_L$, where $r_L$ is defined by the criterion $r_L > 2f_{res} > r_{L-1}$, or until reduction leads to a sphere with fewer than $N_{sp1}$ particles.

3. We unify the spheres whose centers are within $r_L$ of each other. The unification is performed by making a density weighted guess for a common center of mass and then iterating to find a center of mass for the unified object by counting particles. The size of sphere used to determine the center of mass is the smallest radius that will allow the new sphere to entirely contain both candidate halo spheres.

4. For each candidate halo center, we step out in radial shells of $3f_{res}$, counting enclosed particles, in order to find the outer radius of the halo: $R_h = \min(R_{\text{vir}}, R_t)$. The radius $R_{\text{vir}}$ is the virial radius, and $R_t$ is a “truncation” radius, defined as the radius ($< R_{\text{vir}}$) in which a rise in (spherical) density is detected ($d \log \rho / d \log r > 0$). This is our method for estimating when a different halo starts to overlap with the current halo and is important for halos in crowded regions. We estimate the significance of a measured upturn using the Poisson noise associated with the number of particles in the radial bins considered. Only if the signal-to-noise ratio of the upturn is larger than $\sigma_{R_h}$ do we define a truncation radius. The value of $\sigma_{R_h}$ is a free parameter. We use $\sigma_{R_h} = 5.11$.

5. Among the halo candidates for which we have found an $R_{\text{vir}}$, we discard those with $N_{\text{vir}} < N^\text{min}_p$, where $N_{\text{vir}}$ is the number of particles within $R_{\text{vir}}$. Among the halo candidates for which we have found a rise in spherical density, we discard those that

\[\text{The choice was motivated by several tests using mock catalogs of halos in clusters designed to determine how varying } \sigma_{R_h} \text{ affects our ability to fit the density profiles of subhalos. Although our results were not strongly dependent on this choice, we did obtain the best fits using } \sigma_{R_h} = 5.\]
contain fewer than \( N_{R_t}^{\text{min}} \) particles, where \( N_{R_t}^{\text{min}} = N_{p}^{\text{min}} \) if \( R_t > R_{\text{vir}}^{\text{min}} \); otherwise

\[
N_{R_t}^{\text{min}} = N_{p}^{\text{min}} \left( \frac{R_t}{R_{\text{vir}}^{\text{min}}} \right) .
\]

The above constraint follows from an extrapolation of the minimum-mass halo using an isothermal profile \( \rho(r) \propto 1/r^2 \).

6. For halos with more than \( N_{p}^{\text{min}} \) particles, we model the density profile of each halo using the NFW form (eq. [1]) and determine the best-fit \( R_s \) and \( \rho_s \)-values, which determine \( R_{\text{vir}} \) and \( M_{\text{vir}} \). The fitting procedure uses logarithmically spaced radial bins from \( \max\{2 R_{\text{vir}}, \min\{R_{\text{vir}}, R_t\}\} \) out to \( R_t \). If any bins are empty, we decrease the number of bins by one until this is no longer the case. If the number of bins is reduced below three, we discard the halo as a local perturbation.

The fit takes into account the Poisson error in each bin due to the finite number of particles, and we obtain errors on the fit parameters \( (\sigma_{R_s} \text{ and } \sigma_{\rho_s}) \) using the covariance matrix in the fit routine. The errors on the fit parameters can be translated easily into errors for \( R_{\text{vir}} (\sigma_{R_s}) \) and the estimated NFW mass of each halo \( M_{\text{vir}} (\sigma_{M_s}) \). In some cases, the fit does not converge. When this occurs, we mark the halo as a nonfit. This occurrence is rare for distinct halos but is more common for subhalos (see below). This may reflect a tendency for subhalos defined with the current merging criteria to be poorly described by an NFW form, perhaps as a result of frequent interactions or close neighbors.

7. We unify halos with centers that overlap by \( R_{\text{combine}} \). For a given pair of halos with virial radii \( R_{\text{vir}, 1} \), \( R_{\text{vir}, 2} \), we define this combination radius to be \( R_{\text{combine}} = \min(R_{\text{vir}, 1/2}, R_{\text{vir}, 1/2}) \). If either of the halos does not have a fitted \( R_{\text{vir}} \), we use the halo radius \( R_t \) in place of \( R_{\text{vir}} \). Our criterion is met if two (or more) halo centers are within \( R_{\text{combine}} \) of each other while at the same time having velocities that allow them to be bound to the common system. If such a case occurs, then along with the individual halo NFW fits, we fit another NFW profile about the common center of mass of the two combined halos and decide whether the candidate-united halos are bound/unbound to the common NFW fit using the radial escape velocity determined using the common NFW profile (see below). If both halos are bound, we combine the two halos into one and keep the common fit for the characteristic parameters. If at least one is not bound, we do not combine the halos.

8. For each halo, we remove all unbound particles before we obtain the final fits. We loop over all particles within the halo and declare a particle at a distance \( r \) from the center of a halo to be unbound if its velocity relative to the center of mass velocity of the halo obeys \( v > (2\Phi(r))^{1/2} \). For halos that have fits, we use the radial potential for an NFW density profile\(^{12}\):

\[
\Phi_{\text{NFW}}(r) = -4\pi G \rho_s R_s^2 \frac{\log(1 + x)}{x} ;\]

otherwise, we use the radial potential for a singular isothermal sphere with the same mass.

After removal, we construct a new density profile (and find new NFW fit parameters if \( N_p \geq N_{p, \text{fit}}^{\text{min}} \)). The procedure is repeated until the number of unbound particles becomes less than 1% of the bound particles or until the total number of particles within the halo falls below \( N_{p, \text{fit}}^{\text{min}} \).

9. For each halo in the final catalog, we determine its NFW fit if \( N_p \geq N_{p, \text{fit}}^{\text{min}} \) and record its fit parameters and their errors. We also measure and record its spin parameter \( \lambda \) and the maximum of its circular velocity curve \( V_{\text{max}} \).

As described above, the halo catalog we have developed includes an arbitrary number of levels of substructure within halos. The full catalog with substructure should then include all halos directly around galaxies, above the relevant mass resolution, and thus is useful for a number of direct comparisons with observations. However, for many purposes, a halo catalog including only the “distinct” halos, i.e., halos that are not subhalos of any larger halo, is sufficient and introduces significantly less complications. For this reason, we have culled the full catalog into a smaller catalog that does not include subhalos within halos; this is the catalog analyzed here.

Since there are multiple levels of substructure, the details depend slightly on the algorithm chosen. We take the maximum circular velocity to be the most reliable measure of the halo’s size since it is a measured quantity and does not rely on a fit and can be defined equivalently for all halos. All halos whose centers lie within the virial radius of a larger halo are then designated as subhalos. Note that a halo that lies within the virial radius of subhalos is only removed if it itself is classified as a subhalo of a distinct halo.

**APPENDIX B**

**CORRECTING MASSES**

Our procedure of fitting density profiles is intended to give the best estimate possible of a halo’s virial mass; however, it is subject to large errors when there are a small number of particles or especially when the halo is undergoing merging or disruption and is far from being a relaxed, spherical object. These uncertainties are taken into account in the fit errors, but for many purposes, it is essential to have the best estimate possible of the halo’s mass at each output time. Especially for consideration of the evolution of individual halo mass trajectories, it would be useful to eliminate large jumps in the trajectories that are due to the above-mentioned irregularities and not to real changes in a halo’s mass. In order to do this, for each halo, we compare

\(^{12}\) Note that this potential is not necessarily the physical gravitational potential at the halo location. For a subhalo, for example, the host background potential is not included.
three masses: $M_{\text{vir}}$, as measured from the NFW fit, $M_b$, the measured mass within $R_\text{vir}$, and $M_{\text{traj}}$, which designates the mass interpolated between the most massive progenitor in the previous output time and the offspring halo in the subsequent output time (assuming they both exist). In most cases, $M = M_{\text{vir}}$ (if $M_{\text{vir}}$ exists; otherwise $M = M_b$); it is only changed if this mass seems clearly inconsistent with the other mass estimates and does not seem reasonable. For most halos, the error on $M_{\text{vir}}$ is small, and $M_{\text{vir}} \simeq M_b$; in these cases $M$ always equals $M_{\text{vir}}$. However, if one of these is not the case, $M_{\text{vir}}$ is used if it is close to $M_{\text{traj}}$ and otherwise $M_b$ is used if it is close to $M_{\text{traj}}$. If neither seems consistent with the halo’s trajectory, we use $M = \text{median}(M_{\text{vir}}, M_b, M_{\text{traj}})$. The details of the procedure are slightly more complicated, depending on the error on $M_{\text{vir}}$, and we direct the reader to Wechsler (2001) for a complete description.

APPENDIX C

GENERATING MERGER TREES

For completeness, we outline here the fundamental aspects of EPS and our method for generating merger trees. LC93 introduced a method for calculating the probability that a halo of mass $M$ accretes a given mass in a given time. Let $S(M) = \sigma^2(M)$ be the linear density variance on the mass scale $M$ and $w(t) \equiv \delta_c(t)$ be the linear density for collapsing structures at time $t$ (see, e.g., White 1996). Given a halo of mass $M$ at some time $t$, the probability that it accretes a mass $\Delta M$ in a time $\Delta t$ is then

$$P(\Delta S, \Delta w)d\Delta S = \frac{1}{\sqrt{2\pi(\Delta S)^{3/2}}} \exp \left( -\frac{(\Delta w)^2}{2\Delta S} \right) d\Delta S,$$

where $\Delta S = S(M) - S(M + \Delta M)$ and $\Delta w = w(t) - w(t_1 + \Delta t)$. In order to generate halo merging trees, one must implement this formula iteratively, with some algorithm for choosing progenitors. However, no method has been proposed that simultaneously matches the conditional mass function and progenitor distribution of specified by equation (C1) exactly. We use the scheme suggested by SK99, which enforces mass conservation exactly but only reproduces the progenitor distribution of EPS approximately (for alternative techniques, see Kauffmann et al. 1993 and LC93). In order to keep the trees finite, a minimum progenitor mass $M_m$ is defined. Halo mass growth with $\Delta M < M_m$ is treated as dilute accretion. A key ingredient of this technique is that the time step must be chosen such that $\Delta t \lesssim (M_m dS/dM)^{1/2}$ in order to reproduce the expected conditional mass functions of extended Press-Schechter. Stepping back in time, the tree then provides a list of progenitors and their masses. In order to better match the analytic prediction of the progenitor distribution, we apply an additional fix to the method of SK99, suggested by Bullock et al. (2000), which constrains the number of progenitors at any time step to be close to the mean for that mass and redshift.

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