Upper security bounds for coherent-one-way quantum key distribution

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Introduction.—Quantum key distribution (QKD) allows two distant parties (Alice and Bob) to distribute a secret key despite the computational power of an eavesdropper (Eve). Due to channel loss, however, the secret key rate of point-to-point QKD is fundamentally limited; it scales at most linearly with the system’s transmittance \( \eta \) for long distances \([3, 4]\). To mitigate this limitation, Alice and Bob could use intermediate nodes together with, say, twin-field QKD \([5, 8]\), satellite to ground links \([9, 10]\), or, in the long term, quantum repeaters \([11, 13]\).

Besides channel loss, device imperfections also severely limit the performance of QKD. One main imperfection are multi-photon pulses (MPPs) emitted by laser sources generating weak coherent pulses (WCPs). Indeed, multi-photon signals provide Eve with full information about the part of the key generated with them by means of the photon-number-splitting (PNS) attack \([14, 15]\). As a result, the achievable secret key rate of the standard BB84 protocol \([16]\) with WCPs is of order \( O(\eta^2) \) \([17]\).

To enhance the performance of point-to-point QKD, three main approaches have been proposed. The first one is decoy-state QKD \([18, 20]\), where Alice uses phase-randomized WCPs of different intensities. The second solution uses strong reference pulses together with WCPs \([21, 23]\). Both options deliver a key rate of order \( O(\eta) \), thus matching the best possible scaling. The third approach is distributed-phase-reference (DPR) QKD, where Bob performs joint measurements on the incoming signals. There are two main types of DPR QKD protocols: differential-phase-shift (DPS) QKD \([24, 26]\), and coherent-one-way (COW) QKD \([27, 30]\). In the former, Alice’s bit values are encoded into the phase difference between two adjacent coherent pulses of equal intensity. In the latter, Alice’s encoding is done by combining vacuum and coherent pulses. Both options share the practical advantage of a simple experimental implementation, while being probably robust against the PNS attack. Indeed, long-distance demonstrations of the DPS (COW) protocol, beyond 200 km (300 km), have been reported recently \([26, 30]\). The COW protocol is even used in commercial setups \([31]\). Despite these promising results, however, the actual performance of DPR QKD has not been fully established yet. For instance, in \([32, 33]\) it is shown that DPS QKD can provide a key rate of order \( O(\eta^{3/2}) \) in the high loss regime, given that the error rate is sufficiently small. Also, a scaling almost linearly with \( \eta \) is possible by employing a receiver able to check coherence between randomly chosen signals \([34, 35]\).

In this Letter, we fill in this gap by providing upper security bounds for the COW scheme, suggesting a key rate at most of order \( O(\eta^2) \), thus matching the scaling of the lower security bounds \([37]\). That is, in contrast to what has been claimed, this protocol does not seem to be appropriate for long-distance QKD transmissions. Our analysis uses a special type of intercept-resend attacks, the so-called sequential attacks \([39, 41]\), which are particularly suited to attack DPR QKD. Intercept-resend attacks transform the quantum channel into an entanglement breaking channel and, thus, no secret key can be generated \([42]\). In doing so, we explicitly show, for instance, that all long-distance implementations of the COW scheme performed so far are insecure.

Coherent-one-way QKD.—Let us start by introducing briefly the COW protocol \([27, 30]\). The basic setup is illustrated in Fig. 1. Alice uses a laser source, together with an intensity modulator, to generate a sequence of coherent states \(|0\rangle|\alpha\rangle, |\alpha\rangle|0\rangle\) and \(|\alpha\rangle|\alpha\rangle\) that she sends to Bob, with \(|0\rangle\) representing the vacuum state. These signals correspond, respectively, to a bit value 0, a bit value 1, and a decoy signal. They are generated by Alice with \( a \) priori probabilities \( P_0 = P_1 = (1 - f)/2 \) and \( P_d = f \), respectively, for a given \( f \). At Bob’s side, a beamsplitter of transmittance \( t_B \) distributes the incoming signals into

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two lines: the data line and the monitoring line. The former measures the arrival time of each signal with the detector $D_d$. This allows Bob to perfectly discriminate between the bit states $|0\rangle\langle1|$ and $|1\rangle\langle0|$; it is used to generate the raw key. If he observes a “click” in $D_d$ in the first (second) time slot, he assigns to it a bit value 0 (1). If he observes a “click” in both time slots of a sequence, this is assigned a random bit value. Once the communication phase of the protocol finishes, Bob announces over an authenticated classical channel which signals produced a “click” in $D_d$ without disclosing the particular time slot where the “click” occurred. For each of these signals, Alice informs Bob whether it belongs to a key generation round (i.e., it corresponds to $|0\rangle\langle\alpha|$ or $|\alpha\rangle\langle0|$), in which case Bob keeps his result as a sifted key bit, or it belongs to a parameter estimation round (i.e., it corresponds to $|\alpha\rangle\langle\alpha|$). On the other hand, the monitoring line measures the coherence between adjacent non-empty pulses to check for eavesdropping. This is done using a Mach-Zehnder interferometer that measures the coherence between adjacent pulses. In the figure: IM, intensity modulator; $\Delta t$, time delay between adjacent pulses; $D_d$, $D_{M1}$, and $D_{M2}$, single-photon detectors.

Next, we describe briefly a slightly simplified version of the sequential attack that we consider, which already captures its main features. A detailed description of the attack and the parameters that Eve can tune can be found in the Supplemental Material. In particular, Eve first measures each signal emitted by Alice with a measurement strategy which resembles that introduced in Ref. [45]. That is, her measurement lies between the so-called minimum error discrimination (MED) strategy [50, 51], and the USD strategy [43, 44]. Each measurement provides Eve four possible outcomes, i.e., either it identifies Alice’s state or it provides an inconclusive result. The probability of this latter event, which we shall call $q_{inc}$, depends on the overall system loss, and is selected a priori by Eve to reproduce the expected gain at Bob’s data line (i.e., the probability that Bob observes a detection event per signal sent by Alice). For any given value of $q_{inc}$, Eve’s measurement minimizes the error probability to distinguish Alice’s states conditioned signal ($i.e., |0\rangle\langle\alpha|\langle\alpha|0\rangle$), and the other sequences $s \in S$ are defined similarly. Finally, Alice and Bob estimate the QBER and correct their sifted key bit strings. Also, they apply a privacy amplification step (which depends on the estimated visibilities $V_s$) to distill a secret key.

Upper security bounds.—So far, upper bounds on the secret key rate of the COW protocol have been established by considering mainly collective attacks [30, 38]. That is, they assume that Eve attacks each of Alice’s signals independently from each other, and she uses the same individual strategy to interact with each of them. This approach provides bounds that scale linearly with $n$ [30, 38].

Below, we derive tighter upper bounds on the performance of the COW scheme by using a special type of intercept-resend attacks, the so-called sequential attacks [39, 41]. Importantly, in a sequential attack Eve decides jointly the signals that she sends to Bob. Precisely, such signals can now depend on all the measurement results obtained by Eve after measuring all the signals emitted by Alice. This property makes these attacks particularly suited to attack DPR QKD [39, 41]. Furthermore, Eve can take advantage of two special properties of Alice’s signals in the COW protocol. First, they are linearly independent. This means that she could use an unambiguous state discrimination (USD) strategy [43, 45] to distinguish the signals $|0\rangle\langle\alpha|, |\alpha\rangle\langle0|$ and $|\alpha\rangle\langle\alpha|$ without introducing any error [45]. And, second, Alice’s signals contain the vacuum state, which breaks the coherence between adjacent pulses. That is, in contrast to DPS QKD, here Eve could send Bob blocks of signals, separated by vacuum states, without reducing the visibility. Indeed, she only needs to maintain the coherence between those consecutive non-empty pulses sent by Alice that resends to Bob. As we will show below, thanks to these properties together, it turns out that for any value of $\alpha$, there is always a loss regime in which the signals sent by Eve result in QBER$=0$ and $V_s = 1$ for all $s \in S$. Moreover, we remark that intercept-resend attacks do not allow the distribution of a secure key [42].
on outputting a conclusive result. This implies that, in the limit where \( q_{\text{inc}} = 0 \) (\( q_{\text{inc}} \geq q_{\text{usd}} \)), her measurement matches the MED (USD) strategy, with \( q_{\text{usd}} < 1 \) being the failure probability of the optimal USD measurement able to distinguish Alice’s states. See the Supplemental Material for a detailed description of Eve’s measurement.

Once Eve has measured all the signals emitted by Alice, she prepares new signals that she sends to Bob. For a given value of the gain at Bob’s side, her goal is to minimize (maximize) the QBER (visibilities \( V_s \)). For this, she proceeds as follows. Whenever her measurement result is inconclusive, or the number of consecutive conclusive measurement results is below a certain threshold value, say \( M_{\text{min}} \), Eve sends Bob vacuum signals to avoid errors. Note that in the conservative untrusted device scenario, where Eve can modify the parameters of Bob’s detectors (particularly their detection efficiency and dark count rate), vacuum signals do not produce a “click” at Bob’s side. On the other hand, if the number of consecutive conclusive measurement results is greater than or equal to \( M_{\text{min}} \), she sends Bob a sequence of signals that may contain non-empty pulses via a lossless channel. These signals correspond in principle to the results obtained with her measurements but could be slightly adjusted. Precisely, depending on the expected gain, Eve may optimize the intensity \( |\beta|^2 \) of the non-empty pulses she sends Bob to enhance the probability that he detects them. Moreover, Eve might slightly process each sequence of signals before she sends it to Bob to increase the resulting visibilities, as we explain below. This is because even if a sequence of signals prepared by Eve perfectly matches that emitted by Alice (except for the intensity), only those sequences whose first (last) signal is \( |\beta \rangle \langle 0| \) (\( |0 \rangle \langle |\beta| \rangle \)) can guarantee perfect visibility results at Bob’s side. Note that, as already explained, Alice and Bob only check coherence between adjacent non-empty pulses, and the presence of a vacuum state in \( |\beta \rangle \langle 0| \) (\( |0 \rangle \langle |\beta| \rangle \)) breaks the coherence between this signal and the preceding (following) one. This means, in particular, that to improve the visibilities Eve should favor the transmission of those blocks of signals which start (end) with the signal \( |\beta \rangle \langle 0| \) (\( |0 \rangle \langle |\beta| \rangle \)). For this, with probability \( q^p \) Eve selects the largest subsequence of signals (within the original sequence) whose first (last) signal is \( |\beta \rangle \langle 0| \) (\( |0 \rangle \langle |\beta| \rangle \)) and sends this sequence to Bob (after adding the necessary vacuum signals), while, with probability \( 1 - q^p \), she directly sends Bob the original sequence of signals without making any adjustment. In doing so, Eve can use the parameter \( q^p \) to tune the gain and visibilities observed by Bob. Precisely, by increasing \( q^p \) Eve can decrease (increase) Bob’s gain (visibilities \( V_s \)).

We find, therefore, that for those values of the gain which can be reproduced by Eve by selecting \( q_{\text{inc}} \geq q_{\text{usd}} \) and \( q^p = 1 \), her attack achieves QBER=0 and \( V_s = 1 \) for all \( s \in S \). We call this scenario the perfect USD regime. For higher values of the gain, Eve can optimize the parameters of her attack to minimize (maximize) the QBER (visibilities \( V_s \)) observed by Bob. We refer the reader to the Supplemental Material for analytical expressions of the resulting gain, QBER and visibilities \( V_s \) as a function of Eve’s parameters.

**Evaluation.**—We first apply the sequential attack introduced above to the long-distance experimental implementations of the COW protocol reported in [29, 30]. After-

![FIG. 2. QBER and visibilities versus gain at Bob’s side in the sequential attack considered. The stars represent experimental data from (Subfigure (a)) and (Subfigure (b)). In Subfigure (a), \( V_{\text{av}} = V_{d1} \), \( V_{\text{av}} \approx V_{dd} \), and the uncertainty bars shown in one graphic match the information provided in [29]. In Subfigure (b), \( V_{\text{av}} \) refers to the average visibility considered in [30]. Also, the two lines correspond to those experiments using the weakest and the strongest intensity \( |\alpha|^2 \). For other experiments, the results lie exactly between these two lines, and are omitted for simplicity.](image-url)
ward, we evaluate a simple upper bound on its secret key rate, \( K \), which suggests a scaling of order \( O(\eta^2) \).

In Fig. 2 we show the QBER and visibilities which are achievable by Eve, as a function of the gain at Bob’s side, for the different experiments in [29, 30]. For this, we optimize numerically the analytical expressions that describe Eve’s attack over all parameters controlled by Eve. Precisely, in Fig. 2(a) (Fig. 2(b)) we maximize the minimum value of all the visibilities \( V_s \) (the average visibility \( V_{\text{ave}} \)). We note that the experiments in [30] only consider the average visibility \( V_{\text{ave}} \), which is defined by using a weighted combination of the conditional detection probabilities \( p_{\text{click}}(D_M|s) \) [52]. Due to the symmetry of Alice’s signal states as well as Eve’s attack, it turns out that \( V_{bb} = V_{dd} \). Moreover, as a result of the numerical optimization, we find that the results in Fig. 2(a) satisfy \( V_d \approx V_{dd} \) (i.e., these visibilities cannot be distinguished with the resolution of the figure), though this is not true in general. As expected, when the gain at Bob’s side decreases, the achievable QBER by Eve also decreases and the visibilities \( V_s \) increase, till they reach the perfect USD regime where QBER = 0 and \( V_s = 1 \) for all \( s \in S \). Importantly, this latter regime is reached very rapidly by Eve (i.e., for relatively large values of the gain) unless Alice selects the amplitude \( \alpha \) of her signals very small (which in turn significantly reduces the secret key rate). Indeed, by decreasing \( \alpha \), Alice’s signals become less orthogonal to each other, which means that Eve must increase \( q_{\text{inc}} \) to be able to distinguish them unambiguously. In turn, this reduces the maximum value of the gain at which the perfect USD regime is possible. Each particular experimental implementation reported in [29, 30] is indicated in Fig. 2 with a star symbol both in the QBER and visibilities’ graphics. These implementations range from 100 km to 250 km (104 km to 307 km) in [29] ([30]).

A complete description of all parameters characterizing these experiments can be found in the Supplemental Material. Importantly, Fig. 2 suggests that all experimental implementations in [29, 30] are insecure, as their experimental QBER value (visibilities) is (are) above (below) the achievable values by Eve’s sequential attack.

Similarly, if we compare the performance of the sequential attack above with the upper bounds derived in [38], which only consider collective two-pulse attacks, it can be shown that significantly overestimates the gain region where the COW protocol could be secure at all. See the Supplemental Material for further details.

Finally, we use the sequential attack studied to obtain an explicit upper security bound on the secret key rate \( K \) of the COW protocol. Our starting point is a trivial upper bound on \( K \), which is the probability that Alice sends Bob a bit signal state and he observes a “click” in his data line. That is, it holds that

\[
K \leq (1 - f) \left[ 1 - e^{-\eta t_B |\alpha|^2} \right] < \eta t_B |\alpha|^2 < \eta |\alpha|^2. \tag{2}
\]

In the second inequality we use the fact that \( f > 0 \) and \( 1 - \exp(-\eta t_B |\alpha|^2) \leq \eta t_B |\alpha|^2 \), while in the third inequality we exploit that \( t_B < 1 \). The next step is to determine the maximum value of \( |\alpha|^2 \), which we shall call \( |\alpha_{\text{max}}|^2 \).

Precisely, for a given value of the gain at Bob’s side and for given threshold values of the QBER and visibilities, say \( Q^\text{th} \) and \( V^\text{th} \), we use the analytical expressions characterizing Eve’s attack to search numerically for the maximum intensity \( |\alpha_{\text{max}}|^2 \) such that the sequential attack above is unsuccessful. That is, we require that whenever \( |\alpha|^2 \leq |\alpha_{\text{max}}|^2 \) the QBER and visibilities produced by Eve’s attack do not simultaneously satisfy QBER \( \leq Q^\text{th} \) and \( V_s \geq V^\text{th} \) for all \( s \in S \). Conversely, this implies that if \( |\alpha|^2 > |\alpha_{\text{max}}|^2 \) then Eve’s attack is successful. Given \( |\alpha_{\text{max}}|^2 \), and using Eq. (2), we have that

\[
K < \eta |\alpha_{\text{max}}|^2 \equiv R. \tag{3}
\]

This upper bound is illustrated in Fig. 3 for the cases \( Q^\text{th} = 0 \) and \( V^\text{th} = 1 \), and \( Q^\text{th} = 0.05 \) and \( V^\text{th} = 0.95 \). The corresponding values of \( |\alpha_{\text{max}}|^2 \) can be found in the Supplemental Material; they decrease linearly with the system’s transmittance \( \eta \). We find, therefore, that in both cases \( R \) scales quadratically with \( \eta \), as explicitly shown in Fig. 3.

**Conclusion.**—We have derived simple upper security bounds for coherent-one-way (COW) quantum key distribution (QKD). They exploit sequential attacks and the fact that Alice’s signals are linearly independent and, moreover, they contain vacuum states, which naturally break the coherence between adjacent pulses. By using a simple eavesdropping strategy, we have explicitly shown that all long-distance implementations of the COW protocol reported so far appear to be insecure. Most importantly, our results suggest that the key rate of this protocol scales at most quadratically with the system’s transmittance, given that Alice and Bob only monitor the QBER and visibilities. This renders this scheme inappropriate for long-distance quantum communications.

**Note added.**—We shared the finished manuscript with ID Quantique before its submission for publication. The...
company stated that their commercial QKD products based on the COW protocol run on relatively short distances and use a sufficiently weak mean number of photons, such that they can detect sequential attacks. However, the value of mean number of photons employed would not allow ID Quantique products to detect sequential attacks if the distances were much larger than their present limit.

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The average visibility is defined as $V_{\text{ave}} = \frac{p_{\text{click}}(D_{M1}) - p_{\text{click}}(D_{M2})}{p_{\text{click}}(D_{M1}) + p_{\text{click}}(D_{M2})}$, where the probabilities $p_{\text{click}}(D_{Mi}) = \sum_{s \in S} P_s p_{\text{click}}(D_{Mi}|s)$ with $i = 1, 2$. Here, $P_s$ denotes the probability that Alice emits the sequence $s$. 

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