INTRODUCTION TO THE SPECIAL ISSUE
“NONLINEAR WAVE PHENOMENA IN CONTINUUM PHYSICS: SOME RECENT FINDINGS”

The papers presented herein were, with one exception\textsuperscript{1}, authored/co-authored by invitees to the special session

Nonlinear wave phenomena in continuum physics: Some recent findings
of The 10th IMACS International Conference on Nonlinear Evolution Equations and Wave Phenomena: Computation and Theory (or, more briefly, WAVES 2017), which was held 29 March – 01 April 2017 at the University of Georgia, Athens, GA, USA.

The diversity of topics investigated reflects the wide range of areas encompassed by this important field of classical physics. In most of these works, both analytical and numerical methods are used to examine propagation problems described by nonlinear model systems. The methodologies employed include those of perturbation, optimization, and singular surface theories, to cite just a few; the phenomena investigated range from shocks and solitary waves to fractals and pattern formation. Collectively, these contributions capture a cross-section of recent progress in the modeling and study of wave phenomena in media for which the continuum assumption holds.

We hope that this special issue not only showcases some of the latest research in the field, but that it also brings about new exchanges and collaborations among authors. Below, we briefly summarize each of the 13 (peer-reviewed) articles contained herein.

In the contribution by Boyd, a diverse collection of rather unconventional perturbation techniques applicable to strongly nonlinear problems is surveyed. The models considered include variants of the Kortweg–deVries and Benjamin–Ono water wave equations, as well as the Blasius and renormalized quantum anharmonic oscillator ODEs. In addition to assessing the techniques presented, the author also provides a number of engrossing historical vignettes relating to perturbation theory.

In her paper, Carillo examines a model of a rigid heat conductor with memory. In particular, she modifies the existing Fabrizio–Gentili–Reynolds (FGR) model to allow for the case of a singular kernel, i.e., the case in which the heat flux relaxation function exhibits a singularity at the initial time. To deal with the relaxed regularity requirements, the author must also modify a number of the definitions associated with the FGR model.

\textsuperscript{1}Since they were unable to accept our invitation to submit to the last special issue of this Journal which we oversaw as Guest editors [“Mathematics of nonlinear acoustics: New approaches in analysis and modeling,” Evolution Equations & Control Theory, 5(3) (2016)], and given the topic of their work, we thought it only fitting to also invite Pellicer and Sola-Morales to contribute to the present special issue.
In their contribution, Carillo, Lo Schiavo, and Schiebold investigate links between a wide range of nonlinear evolution equations based on Bäcklund transformations. The authors’ focus is on using Bäcklund transformations as a tool to connect different nonlinear evolution equations in both the Abelian (commutative) and non-Abelian (non-commutative) settings. The physical models considered include Burgers' equation and various variants/generalizations of the Kortweg–deVries equation.

The propagation of thermal waves in a nonlinear, un-insulated, rigid conductor composed of a dielectric material is investigated by Christov. The resulting temperature transport equation, which is based on both the Maxwell–Cattaneo and Stefan–Boltzmann laws, is derived and shown to be $C$-integrable, specifically, reducible to the linear telegrapher’s equation. In addition to deriving a number of exact solutions, the author is also able to both determine the time-evolution of a thermal shock and place a restriction on the thermal relaxation time in such conductors.

Gaididei, Marschler, Sørensen, Christiansen, Rasmussen, and Starke, in their paper, consider pattern formation, flows of asymmetrically interacting particles, and pedestrian dynamics. In particular, these authors investigate the influence of asymmetry in the coupling between repulsive particles, an example of which is the social force model for pedestrian dynamics in a long corridor. After taking the continuum limit, these authors employ perturbation and numerical methods to analyze their model system. They conclude their work by pointing out a number of possible follow-on studies.

In Jordan’s article, acoustic propagation in a particle-laden fluid is studied by means of a model founded on the theories by Marble and Thompson, in 1970 and 1972, respectively, and leading to a weakly nonlinear system of two PDEs. To derive results on singular surface phenomena, two alternative approaches are followed, namely a unidirectional approximation and a linearization of the nonlinear model. Moreover, a traveling waves analysis of the original nonlinear model is carried out. This leads to a number of interesting consequences and, among them, the observation that the particle-to-gas density ratio turns out to be a critical parameter in several instances.

In the contribution by Margolin and Plesko, the topic of discrete regularization of the Euler equations is investigated vis-à-vis the propagation of shock waves in gases. The primary concern of these authors is answering the following question: How to add finite dissipation to the discretized Euler equations so as to ensure the stability and convergence of numerical solutions of high Reynolds number flows? They argue that because measurements are always performed with finite instruments, the equations sought should be those that govern the evolution of volume averages of the state variables. This rational leads to the Finite Scale Navier–Stokes (FSNS) equations, which address some, but not all, of the issues raised in this paper.

Mickens and Oyedeji investigate traveling wave solutions to modified versions of the Burgers and Fisher equations in their contribution. The modifications involve nonlinear terms of fraction-power. Exact solutions, one expressed in terms of the Lambert W-function, are derived and analyzed. The authors show that, because of the fractional power terms, their traveling wave solutions can vanish at a finite value of the independent variable, a consequence of which is a solution profile with a derivative that exhibits a jump discontinuity.
In his article, Morro considers waves in nonlinear electroelastic materials. After presenting the balance equations and constitutive relations for a deformable, thermally conducting dielectric, the author takes up the specific case of weak discontinuity waves in such media. He establishes, after deriving the corresponding (new) compatibility conditions, that weak discontinuity waves are possible. The author then shows that if the state of the medium ahead of the wavefront is undisturbed, then two such waves are allowed: one electromagnetic, the other thermal.

Muhr, Nikolić, Wohlmuth, and Wunderlich address the shape optimization problem of finding the geometry of a lens that is used to focus high intensity ultrasound waves. The underlyng (nonlinear) wave propagation model is the classical Westervelt equation and the numerical optimization is based on a discretization by isogeometric finite elements, which enables an exact representation of the geometry. For a tracking type cost function that penalizes deviation from a prescribed pressure distribution, they provide the shape derivative that serves as the basis for a gradient method for shape optimization. Numerical results of optimized shapes in two space dimensions are provided.

The article by Pellicer and Solà-Morales deals with the Moore–Gibson–Thompson equation, which arises both as a linearization of the Jordan–Moore–Gibson–Thompson equation from nonlinear acoustics and as a model in viscoelastic theory. The central result of the paper is construction of a particular scalar product in which the generator of the semigroup is a normal operator. The construction is based on a refined analysis of the spectrum and works in all but a few exceptional cases. The authors prove exponential decay of solutions, which is optimal in case of normalizability.

Rodero, Conejero, and García-Fernández investigate the formation of shock waves in arteries, as they have been observed in clinical studies as a consequence of too abrupt heart beats. Indeed, for a model complementing continuity and momentum equation by a tube law, they succeed in showing the existence of a shock and provide a formula for the shock formation time. These analytical findings are illustrated by means of computational experiments based on a Discontinuous Galerkin Finite Element implementation of the model.

In their article, Zhang, Nishawala, and Ostoj-Starewski study the transient responses of anti-plane shear in Lamb’s problem for a random mass density field with fractal and Hurst effects. These authors, using cellular automata to predict the dynamic responses with random mass fields, perform an evaluation of different combinations of fractal dimension and Hurst coefficient for both Cauchy and Dagum random fields. They go on to determine to what extent fractal and Hurst effects are significant enough to change the dynamic responses of their model system.

Finally, we express our deep gratitude to the authors and reviewers whose great care, and commitments of time and effort, have helped make this special issue of *Evolution Equations & Control Theory* possible.

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