High Degree Modes & Instrumental Effects

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Abstract.

Full-disk observations taken with the Michelson Doppler Imager (MDI) on board the Solar and Heliospheric Observatory (SOHO) spacecraft, or the upgraded Global Oscillations Network Group (GONG) instruments, have enough spatial resolution to resolve modes up to \( \ell \leq 1500 \). The inclusion of such high-degree modes (i.e., \( \ell \leq 1000 \)) improves dramatically inferences near the surface. Unfortunately, observational and instrumental effects cause the characterization of high degree modes to be quite complicated.

Indeed, the characteristics of the solar acoustic spectrum are such that, for a given order, mode lifetimes get shorter and spatial leaks get closer in frequency as the degree of a mode increases. A direct consequence of this property is that individual modes are resolved only at low and intermediate degrees. At high degrees the individual modes blend into ridges and the power distribution of the ridge defines the ridge central frequency, masking the underlying mode frequency. An accurate model of the amplitude of the peaks that contribute to the ridge power distribution is needed to recover the underlying mode frequency from fitting the ridge.

We present a detailed discussion of the modeling of the ridge power distribution, and the contribution of the various observational and instrumental effects on the spatial leakage, in the context of the MDI instrument. We have constructed a physically motivated model (rather than an \textit{ad hoc} correction scheme) that results in a methodology that can produce unbiased estimates of high-degree modes. This requires that the instrumental characteristics are well understood, a task that has turned out to pose a major challenge.

We also present our latest results, where most of the known instrumental and observational effects that affect specifically high-degree modes were removed. These new results allow us to focus our attention on changes with solar activity.

Finally, we present variations of mode frequencies resulting from solar activity over most of solar cycle 23. We present the correlation of medium and high degree modes with different solar indices. Our results confirm that the frequency shift scaled by the relative mode inertia is a function of frequency alone and follows a simple power law.

1. Introduction

High-degree modes are trapped near the solar surface, which make them exceptional diagnostic tools to probe that region of the sun. For example, a mode of degree \( \ell = 500 \) and frequency around 3 mHz sample the solar interior down to 0.99\( R_\odot \). In this shallow region below the solar surface, effects of the equation of state are felt most strongly, and dynamical effects of convection and processes that excite and damp the solar oscillations are predominantly concentrated. It has been shown in [1] that the inclusion of high-degree modes (i.e., \( \ell \) up to 1000) has the potential to improve dramatically the inference of the sound speed in the outermost 2 to 3\% of the solar radius. Furthermore, inversions of artificial mode frequency differences resulting from models
computed with two different equations of state (i.e., MHD and OPAL) recovered the intrinsic difference in $\Gamma_1$, the adiabatic exponent, throughout the second helium ionization zone and well into the first helium and hydrogen ionization zones, with error bars far smaller than the differences resulting from using two different equations of state. These results were obtained using the relatively large observational uncertainties associated to high degree, but with the implicit assumption that there were no systematic errors.

Unfortunately, since spherical harmonics are not orthogonal over a hemisphere—resulting in spatial leakage in the spherical harmonic decomposition of the observed solar Doppler images—and high degree modes are short lived while spatial leaks get closer together in frequency at higher degree, we are unable to resolve individual modes above $\ell = 300$ for the f-mode and above somewhat lower degrees for the p-modes (i.e., above $\ell = 200$ and 150 for $p_1$ and $p_7$, respectively). In power spectra of high degree spherical harmonic coefficient time series the peaks from the target mode and its spatial leaks overlap and soon blend into a power ridge. The amplitudes of these leaks have been shown to be asymmetric [2] and thus the ridge central frequency does not correspond to the target mode frequency. To recover the underlying mode frequency, an accurate model of the amplitude of the peaks that contribute to the ridge power distribution (i.e., the leakage matrix) is needed.

2. Methodology & Strategy

The methodology we have adopted to produce unbiased high degree mode parameters is to fit the power ridge, using an adequate estimate of the limit spectrum, and then apply a ridge-to-mode correction. The limit spectrum is estimated by subdividing the time series in shorter intervals and then averaging the respective power spectra. This procedure reduces the effective spectral resolution and the realization noise, while the length of the intervals is chosen so as to produce a resolution adequate to fit the ridges. One non-negligible benefit of this approach is that it blends modes at lower degrees. This allows us to check our methodology at in the range in degree that overlap with resolved mode fitting. We fit an asymmetric profile to the ridge power density, namely:

$$P_{n,\ell}(\nu) = \tilde{A}_{n,\ell} \frac{1 + \alpha_{n,\ell}(\tilde{x}_{n,\ell}(\nu) - \tilde{\alpha}_{n,\ell}/2)}{1 + \tilde{x}_{n,\ell}(\nu)^2} + \tilde{B}_{n,\ell}$$

where

$$\tilde{x}_{n,\ell}(\nu) = \frac{\nu - \tilde{\nu}_{n,\ell}}{\tilde{\gamma}_{n,\ell}}$$

and $\tilde{A}_n, \tilde{\nu}_{n,\ell}, \tilde{\gamma}_n$, and $\tilde{\alpha}$ are the ridge amplitude, central frequency, full width at half max and asymmetry respectively, and $\tilde{B}_{n,\ell}$ the background term.

Our strategy is then to model the ridge power distribution by computing synthetic power spectra by overlapping individual modes as they leak into each spherical harmonic coefficient time series, and this for $100 \leq \ell \leq 1000$. We use the complete leakage matrix (i.e., horizontal and vertical components), in which most instrumental effects are modeled and we also account for the distortion of the eigenfunction by differential rotation [3]. The number of included leaks is selected to match the fitting residual level of 5\%, namely we include leaks whose contribution is above this threshold:

$$|\delta m| = 6, |\delta \ell| = \begin{cases} 12 & \text{for } \ell < 600 \\ \ldots & \\ 20 & \text{for } \ell = 1000 \end{cases}$$

We thus compute a sophisticated model of the underlying modes that contribute to the ridge power distribution, as explained in greater detail in [4] and [5]. We fit that model to produce
ridge-to-mode corrections for each \((n, \ell, m)\) mode:

\[
\Delta_\nu = \nu_{n,\ell,m}^{(\text{model})} - \nu_{n,\ell,m}^{(\text{ridge})} \tag{4}
\]

\[
\Delta_\gamma = \gamma_{n,\ell,m}^{(\text{model})} - \gamma_{n,\ell,m}^{(\text{ridge})} \tag{5}
\]

\[
\Delta_\alpha = \alpha_{n,\ell,m}^{(\text{model})} - \alpha_{n,\ell,m}^{(\text{ridge})} \tag{6}
\]

e tc.

We then validate this model, as described below, and produce an upper limit on any residual systematics (\textit{i.e.}, residual bias). We can thus estimate the uncertainty of the corrections themselves.

3. Data Set

We have to date some 11 years of MDI observations, starting in 1996. Since SOHO is at the L1 points, telemetry restrictions have allowed us to collect full disk images for only a two to three month epoch each year. We have analyzed 10 such epochs, known as \textit{Dynamics} runs, listed in table 1. We also list the actual length of each epoch, the relative solar activity index, and the average amount of instrumental defocus. During the first three years of operation of MDI, the image was kept on purpose out of focus by nearly two focus steps. Thereafter, the image was kept close to focus, \textit{i.e.} within one focus step\(^1\).

| Starting Date | Length [days] | Average Defocus\(^1\) | Solar Index [% of max] |
|---------------|---------------|------------------------|------------------------|
| 1996-05-23    | 63            | 2.07                   | 2                       |
| 1997-04-13    | 93            | 2.53                   | 5                       |
| 1998-01-09    | 92            | 1.75                   | 20                      |
| 1999-03-13    | 77            | 0.10                   | 40                      |
| 2000-05-27    | 45            | 0.45                   | 69                      |
| 2001-02-28    | 90            | 0.57                   | 61                      |
| 2002-02-23    | 72            | 0.77                   | 80                      |
| 2003-10-18    | 38            | 0.92                   | 51                      |
| 2004-07-04    | 65            | 0.20                   | 36                      |
| 2005-06-25    | 67            | 0.34                   | 30                      |

\(^1\) in focus steps, 1 step \(\equiv 0.529\) pixel

4. Instrumental Effects & Model Validation

The following instrumental effects had to be taken into account:

(i) The plate scale: the actual image radius is changing with time as a result of the change of focus. A small short term variation correlated with the annual temperature change of the front window due to the satellite orbit around the Sun is causing an unexpected change of focus, see for further detail [6] or [4].

The initial spatial decomposition did not use the actual instantaneous plate scale. All of the 10 epochs of full disk data listed in table 1 have since been (re-)decomposed using our best estimate of the image plate scale.

\(^1\) The focus of the MDI instrument is adjusted by inserting (or not) blocks of glass of varying thickness into the light path.
(ii) Image distortion: it has been shown, [4], that the MDI image distortion results from two effects:
(a) radial distortion by the instrument optics, and
(b) image ellipticity caused by $\approx 4.6^\circ$ tilt of the CCD.
This distortion is now taken into account by the spatial decomposition.

(iii) Image orientation: effective $P$ angle. While the SOHO spacecraft attitude is adjusted so as to keep the solar rotation axis aligned with the CCD detector column direction, a small misalignment was measured, about $0.2^\circ$. Again the spatial decomposition was modified to include this.

(iv) Carrington elements: it has been shown in [7], and later confirmed in [8], that the angles used to specify the orientation of the solar rotation axis are off by $\sim 0.1^\circ$. This introduces a time dependent correction in the calculation of the rotation axis projection in the plane of the sky, and the roll angle, $B_0$. As for the effective $P$ angle, the spatial decomposition was modified to include this.

(v) Instrument point spread function (PSF): the MDI instrument PSF remains to be understood. By comparing results from analyzing large defocus to near focus observations we realized that the magnitude of the effect of a change in PSF is larger than what was anticipated in [4].

For at least one epoch, and whenever possible, we have corrected one effect at a time, and compared the model prediction to the actual offset to validate quantitatively our modeling. This is a key step in our strategy, and only by validating quantitatively our model can we be confident that the magnitude of any residual bias is below a known threshold.

5. Status & Results of Validation
5.1. Plate Scale
All the Dynamics data have been spatially decomposed or re-decomposed using the instantaneous image radius, and thus the correct plate scale. The largest effect was for the 1999 data, since the initial spatial decomposition had a 0.27% plate scale error. Our model of a plate scale error was proven to be quantitatively correct, as illustrated in figure 1, except at very high frequencies (i.e., above 4.5 mHz), see [9] for additional detail.

5.2. Image Distortion
A ray trace model of the MDI optics predicts a radial distortion, namely:

$$\frac{\Delta r}{r} = b(r^2 - r_m^2)$$

where $r$ is the radius in the image plane, $r_m$ the mean solar disk radius and $b = 7 \times 10^{-9}$ pixels$^{-2}$, corresponding to a distortion that causes the solar disk to be larger by $\sim 0.8$ pixels (or 17 $\mu$m). The solar disk appears elliptical in the MDI instrument image plane (the semi-major axis is some 0.6 pixels larger than the semi-minor axis). This is believed to be caused by a tilt of the CCD with respect to the plane normal to the optical axis. See [4], [5] and [10] for further detail. Figure 2 validates our model of the distortion. An image distortion can be thought of as a non-uniform image scale, and thus our model should work (or not) in both cases.

5.3. Other Effects
The error in the image orientation ($0.2^\circ$) and the Carrington elements ($0.1^\circ$) proved to result in minute effects. Our initial model, see [4], had over-estimated the effective $P$ effect. We now realize that the modeling of a $P$ angle error in the leakage matrix is a tricky one, and thus not easily carried out accurately.
Figure 1. Frequency offset resulting from a plate scale error (0.27% in 1997), computed by re-decomposing the images using the correct radius (black & blue points for the f- and p-modes resp.). The red and orange points are the same offset (f- and p-modes resp.) when computed by our modeling approach. The errors bars correspond to the uncertainties resulting from ridge fitting. See [9] for additional detail.

5.4. Conclusions on Model Validation

All the instrumental effects, except the PSF and the distortion by differential rotation, can be, and have been, included in the spherical harmonic decomposition. With the exception of the effective $P$, our model has been shown to be valid and can predict reliably any residuals systematics.

When comparing large defocus observations (1996 – 1998) to near focus ones we see that the effect of a change in the instrument PSF turns out to be larger than expected. However, adequately modeling the effect of the unknown MDI instrumental PSF remains a challenge.

One other approach for validating our model is to compare corrected mode frequencies resulting from ridge fitting to resolved mode fitting at intermediate degrees, where both methods overlap. Figure 3 compares the ridge-to-mode frequency corrections resulting from our model to the differences between ridge frequency and mode frequency measured for resolved, intermediate-$\ell$ modes. The comparison is not as good as we would like it to be, but we have recently realized that our low degree ridge fitting is likely to be affected by the fact that our spectral resolution is still too high to fully blend the resolved modes into ridges. This is likely to be the source of the remaining discrepancies.

Finally, let us add that the ridge fitting precision is in the 0.1 – 0.6 $\mu$Hz range, while the ridge-to-mode correction is in the 0.8 – 4.0 $\mu$Hz range. A 10% precision in the correction will assure a residual systematic smaller but commensurable with the fitting precision. A 2.5 – 3% precision in the correction will assure a residual systematic one fifth smaller than the fitting precision.
Figure 2. Frequency offset resulting from image distortion (using 2000 observations), computed by re-decomposing the images and accounting for the distortion (black points). The red points are the same offset when computed by our modeling approach. The errors bars correspond to the uncertainties resulting from ridge fitting. See [9] for additional detail.

Figure 3. Comparison of the ridge-to-mode frequency corrections resulting from our model to the differences between ridge frequency and mode frequency measured for resolved, intermediate-$\ell$ modes. See [5] for additional detail and discussion.
6. Changes with Epoch & Solar Activity

To look for changes with epoch and solar activity levels, we have ignored observations taken with a large defocus (1996 – 1998) and assume that the remaining unaccounted for effect of the instrumental PSF is constant with time. Figure 4 shows changes in solar activity indices over the 10 years analyzed, while figure 5 shows correlations between the frequency change over time (i.e., difference with respect to 2005) and the corresponding activity index (UV radiance), for 4 high degree modes with similar frequencies (i.e., around 3 mHz).

Figure 6 shows how the frequency change, scaled for 100% change of the activity index from the correlation slope ($\delta \nu_{n,\ell}^e$), varies with frequency. It shows that the frequency change scales as a power law in frequency:

$$\delta \nu_{n,\ell}^e = C_\gamma \frac{(\nu_{n,\ell})^\gamma}{Q_{n,\ell}}$$

Figure 4. Solar activity indices as a function of time, and their averages corresponding to the Dynamics runs analyzed.
Figure 5. Correlations between the frequency change over time (i.e., difference with respect to 2005, for 1999 to 2004) and the corresponding activity index (UV radiance), for 4 high degree modes ($\ell = 200, 400, 600, 800$) with similar frequency (i.e., around 3 mHz).

where

$$Q_{n,\ell} = \frac{I_{n,\ell}}{I_{n,0}(\nu_{n,\ell})}$$

(9) and $I_{n,\ell}$ are the mode mass inertia. See additional detail and discussion in [11].

7. Conclusions
While ridge fitting in itself is simple, correcting the ridge parameters to modal values is a complex task, that requires a very detailed and precise understanding of the instrument. We can today say with some confidence that all these effects, except the PSF, are well understood. Our modeling strategy has been validated quantitatively for most of these effects.

The effect of the PSF turned out to be larger than expected, and remains hard to track down. We have little, if any, guidance to get a handle on MDI’s PSF, since the instrument detector package was re-assembled shortly prior to launch and no test data could be taken after that re-assembly.

Fortunately, the MDI’s environment onboard SOHO is stable, and once the instrument was kept near focus we can safely assume that the effects of the instrument PSF are constant with time. This allows us to measure changes with solar activity levels. We have shown here only a few of these changes; a complete description can be found in [5].
Figure 6. Frequency change, $\delta \nu_{n,\ell}^e$ (see text), as a function of frequency.
Finally, we do expect to quantify the effect of the instrument PSF in the near future, and produce finally unbiased tables of frequencies.

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