Fuzzy programming model and its application of the optimization design for smart home system

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\textbf{ABSTRACT}

Owing to the extensive applications of the smart home system in the family level, the optimization design for the smart home system has received considerable research attention. In this paper, based on the practical commercial operation, the fuzzy programming model and its application are studied for the optimization design problem of the smart home system. A fuzzy multi-objective evaluation model is first proposed for the optimization design scheme. Then, the optimization design scheme and the optimization production programming are investigated based on the fuzzy set theory. Finally, the fuzzy programming model of the optimization design of the smart home system and its application are discussed.

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\section{1. Introduction}

The concept of the smart home system was first born in the United States in the 1970s, and then spread to Japan and some developed countries in Europe where such a concept is developing very fast. The smart home system is a result of combination of the modern electronic technology, the automation technology, and the communication technology. The purpose of the smart home system is to monitor, control, and manage all information-related communication equipment, appliances, and home security devices in the house centrally or remotely by connecting them to an intelligent system, and maintain harmony between the equipment and the environment in the meantime (Chen & Lin, 2019; Curumsing et al., 2019).

The smart home system can automatically control and manage the home appliances and monitor the safety of the environment in the house, thus providing a safe, comfortable, and convenient environment for the living and working of the residents. After the first smart building was constructed in the Connecticut of the United States in 1984, the smart home system has become a fast-growing area since developed countries and regions, such as United States, Canada, Australia, Europe, and Southeast Asia, have put forward many programs concerning smart home system Bharathi & Chatterjee, 2015).

Recently, the research studies that concern smart home system are quite extensive in the world, such as Interactive Workspace (Johanson, Fox, & Winograd, 2002) of Stanford, Intelligent Room (Coen et al., 2000) of Artificial Intelligence Laboratory of MIT, Aware Home (Kientz et al., 2008) of Georgia Tech, EasyLiving (Brumitt, Meyers, Krumm, Kern, & Shafer, 2000) of Microsoft Research, iLand (Streitz, Tandler, Mueller-Tomfelde, & Konomi, 2001) of GMD and House_n (http://architecture.mit.edu/house_n/) of MIT. As for the family smart space, one special case of the smart space, many research institutes in both academia and industry have carried out corresponding research projects. The Aware Home program (Tang, 1985a) of Georgia Tech has investigated the tracking of human eye and face and identification of voice and image by using the distributed cognitive system. The AIRE program (Larson, 2000) of the MIT CSAIL has proposed an agent-based software platform design method based on the Intelligent Room prototype system (Tamura, Togawa, Ogawa, & Yoda, 1998). The Intelligent Home program (Lesser et al., 1999) of the Massachusetts university has solved the problem of resource sharing and resource coordination by using the TAEMS task model framework. The Interactive Workspace program (Mokhtari, Feki, Abdulrazak, & Grandjean, 2004) of the Stanford university aims to achieve the ‘integration of interactive devices’. In this program, a large number of different devices communicate with each other via wireless or wired networks, and the firewall technology is used to protect the safety of the information of the smart home. The Easy Living program (Brumitt et al., 2000) of the Microsoft is devoted...
to the development of smart environment. This program focuses on the experience of the users in the smart environment with lots of interactive devices, and the main research interests include machine vision, automatic and semi-automatic calibration of multiple sensors and communications that are independent of the devices.

The fuzzy set theory has been developed to be a major scientific topic which includes fuzzy sets, logic, algorithm, and programming, and considerable research results have been available for the application of the fuzzy set theory in the multi-objective evaluation problem (Hu & Wei, 2014; Zhou, Pedrycz, Kuang, & Zhang, 2016). For example, a multi-objective data envelopment analysis model has been developed in (Zhou et al., 2016) to evaluate the most appropriate sustainable suppliers by using fuzzy sets of type-2. The suppliers selection problem under multi-product purchases has been discussed in (Hu & Wei, 2014) by proposing a fuzzy multi-objective integer programming model. To solve the aforementioned problem, by considering the actual commercial operation of the smart home system, the optimization design problem for the smart home system is discussed in this paper. By using the fuzzy set theory, the fuzzy multi-objective evaluation model of the optimization design scheme of the smart home system is investigated. Moreover, the fuzzy programming model with variable coefficient of the optimization production programming of the smart home system and its applications are discussed. The main contributions of this paper can be highlighted as follows: 1) the optimization design problem is studied for the smart home system based on the practical commercial operation; 2) a fuzzy multi-objective evaluation model is proposed for the optimization design scheme and a fuzzy programming model with variable coefficient is constructed for the optimization production programming of the smart home system; and 3) a practical application is provided to demonstrate the effectiveness of the proposed models.

2. The multi-objective evaluation model

2.1. Construction of the model

The model is constructed based on the following procedures: (1) decide the evaluation standards for the multiple design schemes and use these standards to constitute a set of vectors; (2) use the points in the n-dimensional space to respectively represent the design schemes and pick out the point which represents the optimal design scheme/worst design scheme; (3) calculate the distances between the design schemes and the optimal design scheme/worst design scheme based on the Euclidean Distance; and (4) sort the design schemes according to the distances.

Before proceeding further, the following notations are first introduced:

\[ X = (x_1, x_2, \cdots, x_n) \] represents the design scheme vector which needs to be evaluated;

\[ f = (f_1, f_2, \cdots, f_m) \] represents the evaluation standard vector of the design schemes;

\[ f(X) = (f_1(X), f_2(X), \cdots, f_m(X)) \] represents the goal set of the evaluation of the design scheme.

Supposing that the information, such as preference of the customer, are known exactly, and can be represented by the goal weight \( \omega \) which can be determined by using the AHP method or the expert prediction method and satisfies

\[ \omega = (\omega_1, \omega_2, \cdots, \omega_n), \quad \omega_i > 0 \text{ and } \sum_{i=1}^{m} \omega_i^2 = 1. \]

Then, in a general case, the evaluation model can be constructed as

Model 1 : \[ \min z = (f(X)). \]

Some evaluation standards, such as punctual delivery rate and quality of service, of the design schemes are better when they are larger, while some evaluation standards, such as price and return rate, of the design schemes are better when they are smaller. Based on this fact, by improving the Model 1, we can construct the Model 2 which is non-unity as

Model 2 : \[
\begin{cases}
\min f^-(X) = (f_1(X), f_2(X), \cdots, f_k(X)) \\
\max f^+(X) = (f_{k+1}(X), f_{k+2}(X), \cdots, f_m(X)).
\end{cases}
\]

where \( f^-(X) = (f_1(X), f_2(X), \cdots, f_k(X)) \) represents \( k \) standards that are better when they are smaller and \( f^+(X) = (f_{k+1}(X), f_{k+2}(X), \cdots, f_m(X)) \) represents \( m - k \) standards that are better when they are larger.

In the general multi-objective decision problem, the weighted method is always used to transform the multi-objective problem to a single objective problem. In the following, by using the relative inferior subordination degree and the quadratic normal form, Model 2 is transformed to a single objective problem.

In order to transform the decision matrix to relative superior subordination degree matrix and eliminate the influence from different physical dimensions to the decision result, a set of relative inferior subordination degrees
where \( v_i(x_j) \) represents the degree of quality of the \( i \)th standard of the \( j \)th design scheme and

\[
\bar{f}_i = \max \{ f_i(x_1), f_i(x_2), \ldots, f_i(x_n), \ i = 1, 2, \ldots, k, \}
\]

\[
f_\circ = \min \{ f_i(x_1), f_i(x_2), \ldots, f_i(x_n), \ i = k + 1, k + 2, \ldots, m, \}
\]

Moreover, based on the most inferior principle, choose the worst point \( f^* \) as

\[
f^* = (f_1^*, f_2^*, \ldots, f_k^*) = (\bar{f}_1, \bar{f}_2, \ldots, \bar{f}_k, \bar{f}_{k+1}, \bar{f}_{k+2}, \ldots, \bar{f}_m).
\]

Then, the relative subordination degree matrix can be shown as

\[
A = (a_{ij})_{m \times n} = (x_i(x_j))_{m \times n} = (A_1, A_2, \ldots, A_n).
\]

Introduce a weighted vector \( s(x_j) \) such that

\[
s(x_j) = (\omega_1(1 - a_{1j}), \omega_2(1 - a_{2j}), \ldots, \omega_m(1 - a_{mj})),
\]

then the quadratic normal of \( s(x_j) \) can be written as

\[
s(x_j)^2 = \sum_{i=1}^{m} (\omega_i^2 - (1 - a_{ij})^2),
\]

By denoting \( d(x_j) = \sqrt{s(x_j)^2} \), it can be seen that \( d(x_j) \) represents the Euclidean distance between the \( j \)th design scheme and the worst design scheme under the relative subordination degree. A larger \( d(x_j) \) means that the integrated index of the design scheme is better. Then, an evaluation model can be constructed as

\[
\text{Model 3 : } \max d(x_j) = \sqrt{s(x_j)^2}.
\]

According to the existence theorem, the optimal solution of the Model 3 is the efficient solution of Model 2. Therefore, the multi-objective decision problem is transformed to a single objective problem.

2.2. The application example

The key step in applying the evaluation model of the design scheme is choosing the evaluation standard of the design scheme. By considering the results from Dickson and Weber and consulting the expert in relevant field, in this model, we choose the preset time (T), procurement cost (C), quality (Q), after-sale service (S), and supply ability (F) as the critical factors of the evaluation. Since the procurement cost is hard to calculate, the procurement price can be used to substitute the procurement cost. However, it is better to use the procurement cost if the company has a perfect cost accounting system. The supply ability can be calculated according to the average annual production capacity. The quality and after-sale service are the indexes which are difficult to quantize, but they can be obtained by employing the data correlation method. The evaluation index values for the different design schemes are given in Table 1.

First, denote the design scheme as \( X = (x_1, x_2, x_3) \) and set the index weights as \( \omega = [0.5128, 0.2615, 0.1289, 0.0634, 0.0333] \). By introducing \( f_1(X) \) to represent the index value of the preset time (T), \( f_2(X) \) to represent the index value of the procurement cost (C), \( f_3(X) \) to represent the index value of the quality (Q), \( f_4(X) \) to represent the index value of the after-sale service (S), and \( f_5(X) \) to represent the index value of the supply ability (F), the evaluation model can be constructed as

\[
\begin{cases}
\min f^- (X) = (f_1(X), f_2(X)) \\
\max f^+ (X) = (f_3(X), f_4(X), f_5(X)).
\end{cases}
\]

Based on the most inferior principle, choose the worst point \( f^* \) as

\[
f^* = (\bar{f}_1, \bar{f}_2, \bar{f}_3, \bar{f}_4, \bar{f}_5) = (6, 150, 80, 80, 800).
\]

Then, the relative inferior subordination degree matrix can be shown as

\[
A = \begin{bmatrix}
0.5 & 0.67 & 1 & 0.89 & 0.67 \\
0.67 & 0.8 & 0.89 & 0.8 & 1 \\
1 & 1 & 0.8 & 1 & 0.73
\end{bmatrix}.
\]

Finally, the weighted Euclidean distance can be obtained which is shown in Table 2.
It can be seen from Table 2 that the optimal design scheme is the design scheme 1, and the design scheme 2 is better than the design scheme 3.

3. Fuzzy linear programming model with variable coefficient

3.1. Linear programming model with variable coefficient

The linear programming problem with variable coefficient is a conditional extremum problem shown as follows (Tang, 1985b):

\[
\text{max } F^{(k)} = \sum_{j=1}^{n} c_j^{(k)} x_j^{(k)} ,
\]

\[
\text{st. } \sum_{j=1}^{n} a_{ij}^{(k)} x_j^{(k)} = b_i^{(k)} , \quad i = 1, 2, \ldots, m ,
\]

\[
x_j^{(k)} \geq 0 , \quad j = 1, 2, \ldots, n ,
\]

where \( k \) is the discrete flow time, \( c_j^{(k)} \) and \( a_{ij}^{(k)} \) are time-varying coefficients, \( b_i^{(k)} \) is a constant (or time-varying) value, and \( x_j^{(k)} \) is an unknown variable. Equation (1) is the objective function and Equations (2) and (3) are the constraint conditions. The solution \( \{x_1^{(k)}, x_2^{(k)}, \ldots, x_n^{(k)}\} \) of the Equation (2) is also called the permissible solution if it satisfies the nonnegative constraint (3). The solution that satisfies Equation (1) is called the optimal solution.

By setting

\[
c^{(k)} = (c_1^{(k)}, c_2^{(k)}, \ldots, c_n^{(k)})^T ,
\]

\[
A^{(k)} = \begin{bmatrix}
a_{11}^{(k)} & \cdots & a_{1n}^{(k)} \\
\vdots & \ddots & \vdots \\
a_{m1}^{(k)} & \cdots & a_{mn}^{(k)}
\end{bmatrix} ,
\]

\[
b^{(k)} = (b_1^{(k)}, b_2^{(k)}, \ldots, b_m^{(k)})^T ,
\]

\[
x^{(k)} = (x_1^{(k)}, x_2^{(k)}, \ldots, x_n^{(k)})^T ,
\]

\[
A_i^{(k)} = (x_1^{(k)}, x_2^{(k)}, \ldots, x_n^{(k)})^T , \quad i = 1, 2, \ldots, m,
\]

the linear programming problem with variable coefficient can be rewritten as

\[
\text{max } F^{(k)} = (c^{(k)})^T x^{(k)} ,
\]

\[
\text{st. } (A_i^{(k)})^T x^{(k)} = b_i^{(k)} , \quad i = 1, 2, \ldots, m ,
\]

\[
x^{(k)} \geq 0
\]

or

\[
\text{st. } (A^{(k)})^T x^{(k)} = b^{(k)} ,
\]

\[
x^{(k)} \geq 0.
\]

Moreover, if the preset objective function is \( \min (c^{(k)})^T x^{(k)} \), then it can be transformed to \( \max (-c^{(k)})^T x^{(k)} \).

If there exist the following inequations in the preset constraint condition:

\[
(A_i^{(k)})^T x^{(k)} \leq b_i^{(k)}
\]

or

\[
(A_i^{(k)})^T x^{(k)} \geq b_i^{(k)}
\]

then the inequations are equivalent to

\[
(A_i^{(k)})^T x^{(k)} + x_{n+i}^{(k)} = b_i^{(k)} , \quad x_{n+i}^{(k)} \geq 0
\]

or

\[
(A_i^{(k)})^T x^{(k)} - x_{n+i}^{(k)} = b_i^{(k)} , \quad x_{n+i}^{(k)} \geq 0 ,
\]

where \( x_{n+i}^{(k)} \) is called a slack variable.

The main procedures of employing the linear programming method with variable coefficient are shown as follows (Tang, 1985b, 1990):

1. Summarize the discussed practical problem to a corresponding linear programming problem with variable coefficient and transform it to a standard form.
2. Build the corresponding prediction model based on the data that relate to the time-varying coefficients, then obtain the predicted value by employing the adaptive prediction method.
3. Obtain the optimal solution of each programming time by using the appropriate method.

The linear programming method with variable coefficient combines the prediction problem and the linear programming problem and it can be used to solve a series of dynamic programming and optimization decision problems.

3.2. Fuzzy linear programming problem with variable coefficient

Suppose that there exist the following constraint conditions in the above linear programming problem with variable coefficient:

\[
A^{(k)} x^{(k)} = b^{(k)} \text{ or } A^{(k)} x^{(k)} \leq b^{(k)} \text{ or } A^{(k)} x^{(k)} \geq b^{(k)}
\]

and

\[
x^{(k)} \geq 0 ,
\]

then the problem that maximizes the objective function \( F^{(k)} = (c^{(k)})^T x^{(k)} \) becomes fuzzy linear programming problem with variable coefficient (Tang, 1985a).
The membership function of the fuzzy constraint is defined as follows: when a component $b^{(k)}_i$ increases to $b^{(k)}_i + d^{(k)}_i$, the degree of membership that can be increased is $\mu_i(d^{(k)}_i)$ where $d^{(k)}_i \geq 0$ and $\mu_i$ are strictly monotonic decreasing functions.

The fuzzy goal $\varphi_G^{(k)}(x^{(k)})$ can choose a function that is normalized to $[0, 1]$

$$\varphi_G^{(k)}(x^{(k)}) = (c^{(k)})^T x^{(k)}/u^{(k)}$$  \hspace{1cm} (14)

where

$$u^{(k)} = \max_{x^{(k)} \in R^{(k)}} (c^{(k)})^T x^{(k)},$$  \hspace{1cm} (15)

$$R^{(k)} = \{x^{(k)} | A^{(k)} x^{(k)} = b^{(k)}, x^{(k)} \geq 0\},$$  \hspace{1cm} (16)

and $sup \mu_i$ is the closure of the set $\{d^{(k)}_i | \mu_i(d^{(k)}_i) \neq 0\}$.

If the level $\alpha^{(k)}$ is given, then the $\alpha^{(k)}$-level set $C^{(k)}$ can be expressed as

$$C^{(k)} = \{x^{(k)} | A^{(k)} x^{(k)} = b^{(k)}, x^{(k)} \geq 0\},$$  \hspace{1cm} (17)

where

$$\begin{align*}
\bar{b}^{(k)} &= (b^{(k)}_1 + d^{(k)}_1, b^{(k)}_2 + d^{(k)}_2, \ldots, b^{(k)}_m + d^{(k)}_m)^T, \\
\delta_a^{(k)} &= \max\{d^{(k)}_i | d^{(k)}_i \in sup \mu_i | i = 1, 2, \ldots, m\}. \\
N_a(k) &= \{d^{(k)}_i | \mu_i(d^{(k)}_i) \geq \alpha^{(k)}\}, \quad i = 1, 2, \ldots, m
\end{align*}$$

Unlike $\alpha^{(k)}$, the level set $C^{(k)}$ and $\max_{x^{(k)} \in C^{(k)}} \varphi_G^{(k)}(x^{(k)})$ can be obtained by using the general linear programming method with variable coefficient. Then, the procedures of the fuzzy linear programming method with variable coefficient can be shown as follows:

1. Set $l = 1$, and choose arbitrary $\alpha^{(k)}$, $\forall \epsilon^{(k)} > 0$ and $0 \leq \alpha^{(k)}_1 \leq 1$;
2. Calculate

$$\varphi_G^{(k)} = \max_{x^{(k)} \in C^{(k)}} \varphi^{(k)}(x^{(k)}) = \max_{x^{(k)} \in C^{(k)}} \varphi^{(k)}(x^{(k)}) = (c^{(k)})^T x^{(k)}/u^{(k)};$$

3. Calculate $\epsilon^{(k)} = \alpha^{(k)}_1 - \varphi_G^{(k)}$, if $\epsilon^{(k)} \geq \epsilon^{(k)}$, then turn to

4. Set $\alpha^{(k)} = \alpha^{(k)}_1 - \gamma^{(k)}_l \epsilon^{(k)}$ and $l = l + 1$, then turn to

5. Set $\alpha^{(k)} = \alpha^{(k)}_1$ and calculate $\hat{x}^{(k)}$. If $\hat{x}^{(k)}$ satisfies $\varphi_G^{(k)}(\hat{x}^{(k)}) = \max_{x^{(k)} \in C^{(k)}} \varphi_G^{(k)}(x^{(k)})$, then $\hat{x}^{(k)}$ is the optimal solution;
6. Stop.

It can be seen from the above steps that the step 2 and step 5 are general linear programming with variable coefficient.

4. Example of the production optimization programming of smart appliance

4.1. Practical problem

In the design of the production optimization programming of smart appliance for a smart appliance company, due to the existence of some fuzzy conditions, the above fuzzy linear programming method with variable coefficient is used. First, some practical problems and conditions are listed as follows:

1. The upper and lower bounds of the output of the smart appliance in the $k$th ($k = 1, 2, 3$) year are shown in Table 3, where the upper bounds are constants, while the lower bounds are fuzzy numbers.

Moreover, since the production capacity of each product is different, the outputs are transformed to standard outputs, which are shown in the brackets, to unify the dimension.

2. The mutual ratios of each product. By analysing and comparing the ratios of the upper and lower bounds in each month and considering the demand of the market, the mutual ratios of each product are established.

The mutual ratios of each product are shown as follows:

$$A_1 : A_2 = 10.47$$  \hspace{1cm} (18)

$$A_2 : A_3 = 1.375$$  \hspace{1cm} (19)

$$A_3 : A_4 = 0.935$$  \hspace{1cm} (20)

$$A_4 : A_5 = 0.25$$  \hspace{1cm} (21)

$$A_5 : A_1 = 0.455$$  \hspace{1cm} (22)

3. The prediction of the value of production is calculated according to the standard annual output, and the results are shown in Table 4.

The problem is how to programming the output each year to maximize the total value of production under the above conditions.
Table 3. The upper and lower bounds of the output of the smart appliance unit: 10000 pieces.

| k  | 1      | 2      | 3      |
|----|--------|--------|--------|
|    | Upper bound ($) | Lower bound ($) | Upper bound ($) | Lower bound ($) | Upper bound ($) | Lower bound ($) |
|    | 65 (26) | 46 (18.4) | 72 (28.8) | 55 (22) | 90 (36) | 70 (28) |
| Product A₁ | 3 | 1.5 | 5 | 2.3 | 8 | 3 |
| Product A₂ | 10 (2) | 6 (1.2) | 20 (4) | 10 (2) | 30 (6) | 15 (3) |
| Product A₃ | 15 (3) | 7 (1.4) | 20 (4) | 10 (2) | 24 (4.8) | 15 (3) |
| Product A₄ | 300 (15) | 120 (6) | 400 (20) | 160 (8) | 500 (25) | 240 (12) |

Table 4. The value of production with standard annual output unit: 10000 CNY.

| k  | 1      | 2      | 3      |
|----|--------|--------|--------|
| Product A₁ | 155.05 | 174.88 | 213.17 |
| Product A₂ | 880.00 | 880.00 | 1210.00 |
| Product A₃ | 325.84 | 737.50 | 1013.75 |
| Product A₄ | 124.00 | 122.75 | 132.42 |
| Product A₅ | 136.67 | 145.63 | 143.17 |

### 4.2. The mathematics model

According to the fuzzy linear programming method with variable coefficient, the following notations are set for \( k = 1, 2, 3 \) and \( j = 1, 2, 3, 4, 5 \): 
- \( x_j^{(k)} \) represents the output of the product \( A_j \) in the \( k \)th year (the standard output); 
- \( a_j^{(k)}, b_j^{(k)} \) represent the upper and the lower bound of the output of the product \( A_j \) in the \( k \)th year, respectively; 
- \( c_j^{(k)} \) represents the standard production value of the product \( A_j \) in the \( k \)th year.

Then, the above practical problem can be transformed to fuzzy linear programming problem with variable coefficient which is shown as follows:

Find three groups of nonnegative variables \( x_1^{(k)}, x_2^{(k)}, x_3^{(k)}, x_4^{(k)}, x_5^{(k)} \) \((k = 1, 2, 3)\) such that

\[
\begin{align*}
  & x_j^{(k)} \leq a_j^{(k)}, & & j = 1, 2, 3, 4, 5 \\
  & x_j^{(k)} \geq b_j^{(k)}, & & j = 1, 2, 3, 4, 5 \\
  & -x_1^{(k)} + 10.47x_2^{(k)} \geq 0 \\
  & -x_3^{(k)} + 1.375x_3^{(k)} \geq 0 \\
  & -x_4^{(k)} + 0.935x_4^{(k)} \geq 0 \\
  & -x_5^{(k)} + 0.25x_5^{(k)} \geq 0 \\
  & -x_5^{(k)} + 0.455x_1^{(k)} \geq 0
\end{align*}
\]

\((k = 1, 2, 3)\) are satisfied and the objective functions \( F^{(k)} = \sum_{j=1}^{5} c_j^{(k)} x_j^{(k)} \) \((k = 1, 2, 3)\) are maximized.

### 4.3. The calculation result

In the above mathematic model, the membership functions \( \mu_i \) \((i = 1, 2, \ldots, 10)\) of the fuzzy constraint are sorted by their priority. By considering the fuzzy constraint, the membership functions can be defined as follows:

\[
\mu_1^{(k)}(d) = \mu_5^{(k)}(d) = \mu_8^{(k)}(d) = \mu_9^{(k)}(d) = \left\{
\begin{array}{c}
1 - d \quad 0 \leq d \leq 1 \\
0 \quad d \geq 1
\end{array}
\right.,
\]

\(k = 1, 2, 3\)

\[
\mu_2^{(k)}(d) = \mu_4^{(k)}(d) = \left\{
\begin{array}{c}
1 - 2d \quad 0 \leq d \leq 0.5 \\
0 \quad d \geq 0.5
\end{array}
\right.,
\]

\(k = 1, 2, 3\)

\[
\mu_6^{(k)}(d) = \mu_7^{(k)}(d) = \mu_{10}^{(k)}(d) = \left\{
\begin{array}{c}
1 - 0.2d \quad 0 \leq d \leq 5 \\
0 \quad d \geq 5
\end{array}
\right.,
\]

\(k = 1, 2, 3\)

where the variable level value \( d \geq 0 \).

Make the function value of the membership functions (24)–(26) zero by expanding the limit of the retraining, then \( d \) can be 1, 0.5, or 5. Therefore, calculate the fuzzy goal \( \psi_G^{(k)} \) such that

\[
\psi_G^{(k)} = \sum_{j=1}^{5} \beta_j^{(k)} x_j^{(k)}, \quad k = 1, 2, 3
\]

\((27)\)

where the values of the variable coefficients \( \beta_j^{(k)} \) \((j = 1, 2, 3, 4, 5)\) are shown in Table 5. By setting \( \epsilon^{(k)} = 0.01 \), \( \alpha_{j}^{(k)} = 0.9, \gamma_{j}^{(k)} = 0.5 \) and repeating the above algorithm, the optimal solutions \( \hat{x}^{(k)} = (\hat{x}_1^{(k)}, \hat{x}_2^{(k)}, \hat{x}_3^{(k)}, \hat{x}_4^{(k)}, \hat{x}_5^{(k)})^T \) \((k = 1, 2, 3)\) can be obtained. For \( k = 1 \), the function (27) becomes

\[
\hat{x}_1^{(1)} = 0.0222x_1^{(1)} + 0.0126x_2^{(1)} + 0.0467x_3^{(1)} + 0.0178x_4^{(1)} + 0.0196x_5^{(1)}.
\]

(28)
Table 6. The optimal scheduling plan of the production unit: 10000 pieces.

| k   | 1       | 2       | 3       |
|-----|---------|---------|---------|
| A1  | 50.83 (20.33) | 72 (28.8) | 90 (36) |
| A2  | 2.01    | 2.85    | 3.56    |
| A3  | 10 (2)  | 14.15 (2.83) | 17.7 (3.54) |
| A4  | 9.9 (1.98) | 14 (2.80) | 17.55 (3.51) |
| A5  | 170.2 (8.51) | 241.2 (12.06) | 301.6 (15.08) |

Set \( \alpha_1^{(1)} = 0.9, \ e_1^{(1)} = 0.01, \ y_i^{(1)} = 0.5 \) \( (l = 1, 2, 3 \cdots) \), then we have \( x_G^{(1)} = 1.0025 \) and

\[
|e_1^{(1)}| = |0.9 - 1.0025| = |-0.1025| = 0.1025 > 0.01 \\
= e^{(1)}. \quad (29)
\]

Then by setting \( \alpha_2^{(1)} = 0.9 - 0.5 \times (-0.1025) = 0.9 + 0.0513 = 0.9513 \), then we have \( x_G^{(1)2} = 0.9988 \) and

\[
|e_2^{(1)}| = |0.9513 - 0.9988| = |-0.0475| \\
= 0.0475 > 0.01 = e^{(1)}. \quad (30)
\]

Set \( \alpha_3^{(1)} = 0.9513 - 0.5 \times (-0.0475) = 0.9750 \) and apply the above algorithm repeatedly, then we have the final result \( x_G^{(1)3} = 1.0000 \) with \( \alpha_3^{(1)} = 0.9941 \) and

\[
|e_3^{(1)}| = |0.9941 - 1.0000| = 0.0059 < 0.01 = e^{(1)}. \quad (31)
\]

Finally, by choosing the level as \( \tilde{\alpha}^{(1)} = \alpha_3^{(1)} = 0.9941 \), we can obtain the optimal solution

\[
\tilde{x}^{(1)} = (20.33, 20.01, 2, 1.98, 8.51)^T.
\]

Similarly, we can obtain the optimal solution for \( k = 2, 3 \):

\[
\tilde{x}^{(2)} = (28.8, 2.85, 2.83, 2.83, 12.06)^T, \\
\tilde{x}^{(3)} = (36, 3.56, 3.54, 3.51, 15.08)^T.
\]

After sorting synthetically, the optimal production scheduling plan is shown in Table 6 where the numbers in the brackets are the standard output.

5. Conclusion

In this paper, the interesting topic of the development of the smart home industry has been discussed. The optimization design problem has been studied for the smart home system based on the practical commercial operation by using the fuzzy set theory. The fuzzy multi-objective evaluation model of the optimization design scheme has been proposed and the fuzzy programming model with variable coefficient of the optimization production programming of the smart home has been constructed. The corresponding applications and the prospect of the collaborative innovation development have been discussed which show important theoretical significance and practical value. In our further work, the optimization design problem with fuzzy multi-objective evaluation model will be studied for other practical systems.

Disclosure statement

No potential conflict of interest was reported by the authors.

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