Chirped Biphotons and Their Compression in Optical Fibres

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We show that broadband biphoton wavepackets produced via Spontaneous Parametric Down-Conversion (SPDC) in crystals with linearly aperiodic poling can be easily compressed in time using the effect of group-velocity dispersion in optical fibres. This result could foster important developments in quantum metrology and lithography.

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One of the central problems in quantum optics is generation of nonclassical light with given spectral and spatiotemporal properties. In particular, for the needs of quantum metrology and quantum lithography it is important to obtain two-photon wavepackets with small correlation times. Such wavepackets should naturally manifest a broad frequency spectrum. Several ideas have been put forward in this direction, all based on two-photon states produced via Spontaneous Parametric Down-Conversion (SPDC). Among them, one can mention prisms or diffraction gratings introducing a frequency chirp [4], SPDC in aperiodically poled crystals [2, 3, 4], and SPDC in crystals with temperature gradients [2]. However, a broad spectrum of two-photon light does not necessarily imply small correlation times, although the inverse is true [2, 3, 8]. This is similar to the fact that a broadband pulse does not have to be short in time, although the spectrum of a short pulse is always broad. The spectrum of a short pulse is always broad, the way to eliminate the chirp was not specified.

In this work we show that, under certain conditions, biphoton wavepackets can be made nearly Fourier transform-limited and hence compressed by injecting one of the photons of a pair in a standard optical fibre and exploiting the effect of group-velocity dispersion (GVD). No specially engineered fibres (for instance, with negative GVD) are necessary. This suggests an easy way of achieving extremely short correlation times for two-photon light.

Consider generation of two-photon light via spontaneous parametric down-conversion (SPDC) from a cw pump in an aperiodically poled crystal. From the viewpoint of applications and for simplifying the calculation, it is convenient to assume that signal and idler photons are distinguishable, due to either frequency nondegenerate or type-II phase matching. Below, we consider phase matching to be type-II, collinear, and frequency degenerate, with idler (extraordinary) and signal (ordinary) radiations centered at frequency \( \omega_0 \). The two-photon state can be written as

\[
|\psi\rangle = \int d\Omega F(\Omega)|\omega_0 - \Omega_i|\omega_0 + \Omega_s, \tag{1}
\]

where \( |\omega_i(s)\rangle \) denotes the idler (signal) photon state with frequency \( \omega \). The two-photon spectral amplitude (TPSA) \( F(\Omega) \) determines all spectral and temporal properties of two-photon light. In particular, its squared module gives the frequency spectra of signal and idler radiation, \( I_{i,s}(\omega) \propto |F(\omega - \omega_0)|^2 \). Its Fourier transform can be called time two-photon amplitude (TTPA) \( \hat{F}(\tau) \),

\[
F(\tau) = \int d\Omega e^{i\tau\Omega} F(\Omega), \tag{2}
\]

its squared module giving the second-order Glauber’s correlation function [2, 5]:

\[
G^{(2)}(\tau) = |F(\tau)|^2. \tag{3}
\]

Here, \( L \) is the crystal length and \( k_i, k_s, k_p \) are wavevectors of the idler, signal, and pump waves, respectively. Let the spatial dependence of the quadratic nonlinearity be \( \chi(z) = \chi_0 e^{ikK(z)(z + L/2)} \), where the inverse grating vector \( K \) has a linear dependence on the coordinate, \( K(z) = K_0 - \alpha(z + L/2) \). The group-velocity matching: \( k_i(\omega_0) + k_s(\omega_0) - k_p + K_0 = 0 \) at the center of the crystal. It is convenient to expand the wavevectors around the exact quasiphasematching frequency:

\[
k_i = k_i(\omega_0) - k_i(\Omega) + \frac{1}{2}k_i''(\Omega)^2,
\]

\[
k_s = k_s(\omega_0) + k_s(\Omega) + \frac{1}{2}k_s''(\Omega)^2. \tag{4}
\]

Here, \( k_i'' \) and \( k_s'' \) are the first and second derivatives of the dispersion law evaluated at \( \omega_0 \), related to the group
velocity and group velocity dispersion (GVD), respectively.

Denoting $D \equiv k''_s - k'_i$ and $\kappa \equiv \frac{1}{2}(k''_i + k''_o)$, we obtain the TPSA in the form

$$F(\Omega) \propto e^{-iD\Omega \frac{\pi}{2} - i\kappa\Omega^2 \frac{\pi}{2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\xi \chi_0 e^{i(D\Omega + \kappa\Omega^2)\xi - i\alpha\xi^2},$$

(5)

where $\xi = z + L/2$.

Suppose that the spectrum is not too broad compared to the difference of group velocities of the signal and idler radiation, so that the condition

$$\left| \frac{\kappa\Omega}{D} \right| \ll 1$$

(6)

holds true. Then the wavevector mismatch can be written up to linear terms in frequency detuning $\Omega$, and the TPSA becomes

$$F(\Omega) \propto e^{-iD\Omega \frac{\pi}{2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\xi \chi_0 e^{iD\Omega\xi - i\alpha\xi^2},$$

(7)

which yields, similarly to Ref. [3],

$$F(\Omega) \propto \exp\{-iD\Omega\frac{L}{2} + iD^2\Omega^2 \frac{\pi}{4\alpha}\} \times \left\{ \text{erf} \left[ \sqrt{\frac{i}{\alpha} \frac{L\alpha - D\Omega}{2}} \right] + \text{erf} \left[ \sqrt{\frac{i}{\alpha} \frac{L\alpha + D\Omega}{2}} \right] \right\},$$

(8)

In the case of large aperiodicity $\alpha$, the spectral amplitude has a rectangular shape. Indeed, let us introduce the ‘rectangle function’ $\Pi(x, a, b) \equiv 1$ for $a \leq x \leq b$ and $\Pi(x, a, b) \equiv 0$ otherwise. Then, rewriting the integral in (7) in terms of the rectangle function and applying the convolution theorem, we get

$$F(\Omega) \propto e^{-iD\Omega \frac{\pi}{2} + i\frac{D^2\Omega^2}{4\alpha}} \int_{-\infty}^{\infty} dx \text{sinc}\{Dx \frac{L}{2}\} e^{iD^2\Omega^2 \frac{\pi}{4\alpha}},$$

(9)

where $\text{sinc}(x) \equiv \sin(x)/x$. The first exponential term in the integral, $\exp\{i\frac{D^2\Omega^2}{4\alpha}\}$, can be omitted if the typical scale of its variation is much larger than the sinc-function width, $\pi/DL$. This is the case if the aperiodicity is large enough,

$$|\alpha| \gg \frac{\pi^2}{4L^2}.$$  

(10)

Note that condition (10) is well satisfied in Refs. [2] and [4]. Then, the integral in Eq. (9) becomes

$$F(\Omega) \propto e^{-iD\Omega \frac{\pi}{2} + i\frac{D^2\Omega^2}{4\alpha}} \Pi(\Omega, -\frac{\alpha L}{D}, \frac{\alpha L}{D}).$$

(11)

We see that the spectrum of SPDC in a crystal with linear $K(z)$ dependence and large $\alpha$ is a rectangular function of width $\Delta\Omega = \frac{2\alpha L}{D}$. The condition (10) means physically that the aperiodicity should induce a substantial spectrum broadening. Now we can explicitly write the condition (10) for the GVD of the nonlinear crystal to be negligible:

$$\left| \frac{\kappa\Omega}{D} \right| \ll 1.$$  

(12)

For given $\alpha$ and $L$, this condition is realized if the separation of signal (ordinary) and idler extraordinary group velocities is large enough.

Increasing the aperiodicity $\alpha$, one can make the spectrum as broad as desired. At the same time, this does not make the TTPA (2) short in time because, due to the nonlinear frequency-dependent phase factor in (8), the TPSA is not Fourier transform-limited. Here we would like to stress that because the squared module of TTPA is the second-order Glauber’s correlation function, its width gives the correlation time of the biphoton, i.e., the biphoton entanglement time [6, 7]. It is this time that is important for two-photon effects such as two-photon absorption, two-photon ionization, or up-conversion, and which can be measured for two-photon light using these techniques [12]. At the same time, coherence time of biphoton light is defined as the width of the first-order Glauber’s correlation function, which is the Fourier transform of the spectrum $|F(\Omega)|^2$. This is why coherence time does not depend on the phase factor in (8) and it is given by the inverse width of the spectrum [4].

The Fourier transform of TPSA (7) is easily obtained by introducing the rectangle function under the integral, extending the integration to infinite limits and using the convolution theorem. As a result, we get

$$F(\tau) \propto e^{-i\alpha(\frac{\pi}{2} - \frac{\tau}{\tau^2})} \Pi(\tau, 0, DL),$$

(13)

with the amplitude being the same as in the case of a bulk or periodically poled crystal of length $L$. This means that the TTPA of an aperiodically poled crystal is as broad as in the absence of the aperiodicity $\alpha$.

Consider now propagation of the extraordinary photon of the biphoton field through an optical fibre of length $l$ with the inverse group velocity given by $k''_f \equiv \frac{d^2}{dx^2} |_{\omega = \omega_0}$ and the GVD given by $\kappa_f \equiv \frac{1}{2} \frac{d^4}{dx^4} |_{\omega = \omega_0}$. Propagation through such a fibre leads to a phase factor $\exp\{ik''_f (\Omega + \kappa_f \Omega^2)l\}$ in the two-photon spectral amplitude [6, 7]. The first term in the phase is linear in frequency and hence only shifts the two-photon wavepacket in time. The second term, being quadratic in frequency, can compensate for the TPSA chirp. This will happen under the condition

$$\kappa_f l + \frac{D^2}{4\alpha} = 0.$$  

(14)

For a fibre with positive GVD, this condition can be satisfied for negative $\alpha$, i.e., for the case where the poling
period reduces along the pump propagation through the crystal.

In the case of large aperiodicity $\alpha$ satisfying $|\alpha| \leq \alpha_0$, the resulting TTPA can be calculated analytically as the Fourier-transform of expression (11) with the quadratic phase term removed. Clearly, it has the form of a sinc-function with the width being almost $\alpha L^2$ times narrower than a periodically poled sample of the same length.

Figure 1 shows the spectrum of SPDC radiation calculated for the case of aperiodically poled KTP crystal with $L = 0.8$ cm, $K_0 = 2441.8$ cm$^{-1}$ and $\alpha = 1200$ cm$^{-2}$, which corresponds (using the first-order quasi-phasematching) to the poling period varying from 18.47 to 42.40 $\mu$m. The pump at 458 nm is $y$-polarized, as well as the idler radiation, and all three wavevectors, as well as the inverse grating vector, are directed along $x$. The dispersion dependencies are given by Sellmeier equations from Ref. [13], without any additional assumptions. One can see that the spectrum of the biphoton field is quite broad (from 800 to 1200 nm) and has nearly rectangular shape. This is due to the fact that condition (10) is fulfilled very well. In fact, the aperiodicity leads to the spectral broadening of more than two orders of magnitude. For comparison, the same figure shows the spectra of signal and idler radiation for a crystal with the same length but constant poling period $K = K_0$, corresponding to $\alpha = 0$. Condition (12) is reasonably satisfied, since $|\alpha L^2| / 2K_0 \approx 0.16$.

Figure 2 shows the second-order correlation function calculated as the squared module of the Fourier transform of expression (8). For comparison, second-order correlation function of a crystal with constant poling period is plotted in the same graph. Clearly, both distributions have the same width, which means that the biphoton with the broadened spectrum has the same correlation time as the narrowband biphoton, i.e., it is not Fourier transform-limited. Evolution of the correlation time in an optical fibre is demonstrated in Fig.3 for the cases of $\alpha > 0$ and $\alpha < 0$, when only the idler photon is transmitted through the fibre. We see that at $\alpha > 0$, propagation through the fibre only broadens the biphoton wavepacket, while in the case $\alpha < 0$, which is achieved by simply exchanging the input and output faces of the crystal, the biphoton is compressed. For the calculation, we used the GVD of bulk fused silica [14], because the waveguide contribution into GVD far from zero dispersion point is negligibly small [13]. The value of $\kappa_f$ we used in the calculation was $1.359 \times 10^{-28}$ s$^2$/cm. The largest compression of the biphoton wavepacket is achieved at the fibre length $l = 16.927$ cm. The second-order correlation function in this case has a typical shape of squared sinc-function with the FWHM equal to 12 fs. At other lengths of the fibre, the two-photon wavepacket is broader. As the length of the fibre increases, the shape of $G^{(2)}(\tau)$ becomes similar to the shape of the spectrum, an effect that was studied in detail in Refs. [4,14]. Fig.4 shows the shapes of $G^{(2)}(\tau)$ after the biphoton propagation through fibres of different length; the corresponding points are shown in Fig.3.

The model we have been using so far is based on the linear dependence of the wavevector mismatch on the frequency detuning from exact phasematching. This is valid for type-II or frequency non-degenerate type-I SPDC [10], under the condition (12). Similarly, the dispersion law of the fibre in our consideration was described by a quadratic dependence, i.e., third-order GVD of the fibre.
and the fibre will be presented in the nearest future [17].

... was neglected. In order to see the effect of higher-order GVD terms, we have performed exact numerical calculation for the same case as considered above. The results (Fig.5) show that the effect of compression is slightly reduced but there still remains a significant narrowing of the TTPA, useful for applications. Even without any optimization, the correlation time is reduced by more than an order of magnitude. An exhaustive study of this effect and a search for optimized parameters of both the crystal and the fibre will be presented in the nearest future [17].

An interesting feature of Fig.5 is that at the output of the crystal, TTPA widths for the cases $\alpha > 0$ and $\alpha < 0$ are different. This can be explained as follows. Although at the center of the crystal signal and idler photons are generated with the same frequencies, biphotons generated at the back face are non-degenerate. Due to GVD, there is a delay accumulated between the photons of a pair in the course of its propagation through the crystal. At $\alpha < 0$, this delay is compensated by the one appearing due to birefringence and at $\alpha > 0$, both delays add up.

In conclusion, a biphoton whose spectrum is broadened due to a linear aperiodicity of the crystal poling can be compressed in time using normal GVD of an optical fibre. To describe the compression, it is sufficient to take into account first-order terms in the crystal dispersion dependence and second-order terms in the fibre dispersion dependence. Exact calculation shows that higher-order terms reduce the compression but the effect is still present. It is worth mentioning an interesting result: the two-photon correlation time as well as its evolution due to the propagation of the biphoton through an optical fibre strongly depend on the position of the crystal.

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