Probability learning trajectory: Students' emerging relational understanding of probability through ratio

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Abstract. Several studies on learning mathematics indicate that students have difficulty in understanding the concept of probability. Students are more likely to be introduced by the use of the formula without involving the concept itself. This underlies the researcher to design a probability learning trajectory in ratio which always starts from concrete to abstract level. The probability learning trajectory explained of student thinking and learning over time about to with concerning set of tasks and activities developed. The probability learning trajectory helped by context problem. This study aims to look at relational understanding ability eighth-grade students in solving the problem of a probability. The method used is design research which describes student learning activities in solving problems using ratio. The data collection technique is done by video recorder, documentation and test question. The instruments are a video that is to see student activities during the study, photos to see the results of student work and work the answer to see student answers to the questions given. The results of the research carried out obtained that students experienced an increase in relational understanding and the gave a good response to in solving the probability questions.

1. Introduction
Probability is mathematical learning that is useful in human life activity. We often use the probability in daily life because it is related to predict something. Many of the things we do and the events that occur around us have a consistent result and fully predictable. Even though everyone understands about chance, there are still many people who find the difficult to determine the chance of an event precisely even in a context and a fairly simple calculation [1]. Statistics and probability material, some of which are a difficult subject for students [2]. Students found many difficult situations are unpredictable while solving probability problems with the intuitive nature of the concept of probability [3]. Many empirical studies since the 1970s have shown conclusively that this is far from reality [4, 5, 6, 7]. According to Watson [8] all of the adolescents, the probability problem always to be a serious challenge and only a few secondary schools’ students can solve the problem of probabilistic properly and adequately react in situations of uncertainty. Furthermore, the various misconceptions about the probability seriously undermines the formal probabilistic reasoning in high school [9]. Several studies from different theoretical perspectives seem to indicate that students tend to have a perception that hinders their learning about the concept of probability.

Students have a fundamental difficulty with the basic ideas of probability, probability ideas seem contrary to the experience of the students on how they view the world [10]. Students are not able to understand the concept of probability so well. Students only know the concept in accordance with what is being taught and not been able to build their own concept. Understanding as an actor appeared in
expressions such as: "Oh, I understand now!". Understanding is as a way of knowing (how to know) "I understand it this way" [11]. One of understanding that must be considered in the study is a relational understanding because when students develop their relational understanding, they can get away to move easily and skillfully [12]. Relational understanding capability can be enhanced by innovative and creative learning. Noyes said many children are trained to study mathematical calculations instead be educated to think mathematically [13]. How to approach mathematics learning is strongly influenced by the opinions of the teachers and the students in learning mathematics. Some things such as innovative mathematical problem solving often requires inquiry and discovery, the result may trigger interest to learn about mathematics. The basic idea is to make the students beyond the obedient student, who only answers teacher’s questions [14]. Teachers should pay more attention to how to make math meaningful for all students, it takes to make it fit with the development, working with the question, allowing for in-depth and offer exciting activities.

Simon [15] offers a learning trajectory as a way to explain important aspects of pedagogical thinking involved in teaching mathematics for understanding. Learning trajectory has the potential to support the understanding of the students to learn better, allowing more effective teaching strategies and curriculum design guide for the better [16]. Learning trajectory very useful in mathematics because this trajectory helps students find knowledge used by teachers as a guide during the learning process and it is critical for students to bridge non-formal knowledge towards formal knowledge. In particular, it illustrates how mathematics educators such as teachers, researchers and curriculum developers oriented by the constructivist perspective and especially math learning goals for students, thinking about the design and use of mathematical tasks to promote conceptual learning of mathematics [17]. The learning process is made in this study will refer to the steps of learning with a realistic mathematics education approach.

A problem called "realistic", if the problem can be imagined or real in the minds of students. The problem realistically is not only used as a foundation for building a mathematical concept but also known as a source for learning. In addition, students must be given the chance to find their own math strategies [18]. The underlying philosophy of realistic mathematics education is the students should develop an understanding of mathematics to work from the context that makes sense for students [19]. It becomes something important in the learning trajectory of this research. Learning trajectory is useful for students to find the right concept with its own language and students better able to understand probability well. The Activities in the learning trajectory made a very detailed and a guide for teachers to help students find the concept of probability in order not to be mistaken in understanding the meaning. This research will answer this research question is how we can introduce probability as a ratio and to help students for building relational understanding.

2. Method
This study uses a design research method which is an appropriate way to answer the research questions and achieve the research objectives that start from preliminary designs, teaching experiments, and retrospective analysis [20,21]. There are two important aspects related to design research as the Hypothetical Learning Trajectory (HLT) and Local Instruction Theory (LIT). Both will be learning activities as learning paths that may be taken by students in their learning activities. Teachers can select appropriate learning activities as a basis to stimulate students to think and act when constructing a relational understanding. The stages of design research as guidelines in this study were shown in Figure 1.
The first stage became the most important thing in this study was the probability learning trajectory as seen in Table 1. There were the three components in learning trajectory: learning objectives, learning activities and hypothetical about the learning process (predictions about how the students' thinking, and understanding will evolve in the context of activities learning). When creating your learning trajectory should also be considered how to support the development of students' reasoning tasks and a set of mathematical learning objectives. The choices made by the teacher when respond students’ contribution was important in shaping the development of student’s understanding [23]. Therefore, the learning trajectory was a hypothesis that included consideration of mental activities that engaged students in-depth, because the students’ participation in learning activities that were expected. Data collecting use to support adjusting the learning trajectory plan.

**Figure 1.** Phase Design Research [22]
The second phase was the teaching experiment. The teaching experiment led to the revision and improvement of the learning trajectory that has been compiled. The problems in the learning trajectory useful context for students to discover concepts from probability through comparison. The issue would be used consistently from the first meeting until the final meeting to facilitate the students to understand the learning.

The third stage was the retrospective analysis. The specific objective of this phase was to evaluate the success of the learning activities that have been carried out, observing the learning progress of the students and inform the progress of learning activities. This stage consisted was a retrospective analysis of data analysis, reflection, interpretation of the findings, and the formulation of recommendations for the next research. We used a retrospective analysis technique, row after row of teaching trial transcript (examined in coordination with video recording) [24] to pay attention to changes that occur in students during the learning process and pay attention to the development of relational understanding. All

| Meeting | Learning Context | Learning Activities |
|---------|------------------|---------------------|
| 1       | Indicator: Understanding the sample space. Context: List price of pulses (Telkomsel, Indosat, XL and AXIS). The sample space introduced through the set of all pulses price list. sample point introduced by members of the list price of pulses. | - Students worked on the activity sheet context problems associated with pulse price list. - Students discussed with their friends. - Students presented the results of their discussion. - Students’ quiz. - Students and teacher concluded learning together. |
| 2       | Indicator: Finding a theoretical probability concepts and determine the theoretical probability. Context: List price of pulses (Telkomsel, Indosat, XL and AXIS). The theoretical probability introduced through assisted ratio with the student’s knowledge of the sample space in pulses price list. | |
| 3       | Indicator: Finding empirical probability concepts and determine the empirical probability. Context: Proceeds from sale of pulses based on the list prices of pulses (Telkomsel, Indosat, XL and AXIS). The empirical probability also introduced through the ratio of pulses sales. | |
| 4       | Indicator: Finding complement probability and define concepts complement probability. Context: List price of pulses (Telkomsel, Indosat, XL and AXIS). Complement probability introduced by comparison, still through the list price of pulses aided by an understanding of the theoretical probability. | |
sessions were transcribed and upgraded to a document includes not only verbal speech but also all the images of student work, a description of the relevant action cues, and other nonverbal actions [25]. The results of a retrospective analysis form were the basis for adjusting the trajectory of learning and to answer research questions. In this research, audiovisual and written data were used to get various data. The research is being held in the eighth class of Junior High School, Sumatera Barat, Indonesia.

3. Result and Discussion
The result is used to answer two research questions of this study. First, how is the role of a probability learning trajectory in supporting students’ ability to understanding the probability concept? Second, how is a student’s learning trajectory in learning of probability using the list price of pulses?

3.1. Teaching Experiment Results
This research uses four meetings of teaching experiment: first meeting (to explain about sample space), second meeting (to explain about theoretical probability), third meeting (to explain about empirical probability) and fourth meeting (to explain about complement probability).

3.1.1. The First Meeting. The first meeting was to discuss the concept of sample space by using the context of the problem that price list of pulses. Students understood the concept of space as a set of samples from all price list and sample point/ event as a member of the set are each priced pulses as shown in Figure 2.

![Figure 2. Price List of Pulses](image)

Next, the students solve the problem with to complete statement from help to students to find the concept about sample space as shown in Figure 3.

![Figure 3. Student's Work](image)

Students find a concept about sample space through assistance completing the statement, it is instructed by the sentence that the price of all pulses is said to be the sample space and the price of each pulse as the sample point or event. Look at a few differences of the students' answers about sample space's concept and sample point as shown in Figure 4.
The teacher estimates that students will answer the sample space is a set of all the events that might occur from an experiment while members of the sample space are called sample point. The notes in Figure 4 (a) show that students deduce the sample space is the set of sample point and the sample point is part of the sample space while in Figure 4 (b) students deduce the concept of sample space as a set (not formal concept) such as the sample room is a collection of pulse prices and sample points is of a small part selected from the list of pulse prices. From the answers given by the students, it appears that students have been able to build the basic concepts of the sample space which will later be used in the concept of probability well.

3.1.2. The Second Meeting. In the second meeting discussed the concept of theoretical probability. Almost all of the students were able to understanding the concept of theoretical probability through comparison assisted with the price list context of the pulse sales, it looked from the discussion process undertaken and the results of the student activity sheet as shown in Figure 5.

Students used the ratio as an early knowledge to know the opportunities seen in Figure 5. Based on the answers to the problems that solved by the students showed that almost all of the students were able to solve the problems given the well as shown in Figure 6.
The teacher estimates that students will answer the theoretical probability is ratio about sample space for a occur with the all of sample space. From the answers, it appears that students are able to construct concepts about theoretical probability well as shown in Figure 6. Students construct the concept of probability as a ratio seen from Figure 6 (b), which is the comparison between sample points and sample space, while for Figure 6 (a) still not too clear in defining theoretical probability but already understanding theoretical probability as a probability. Almost all students answer as shown in Figure 6 (b), this indicates that students have been able to build their own concepts assisted with material that has been studied previously. All students are able to make formulas for theoretical probability as shown in Figure 7.

**Figure 6. Results of Answers Students**

From the figure, we could see that the students were able to determine probability through ratio and can be seen in Figure 8 the conclusion of the students' discussion with their friends.

**Figure 7. Results of Answers Students**

3.1.3. *The Third Meeting.* The third meeting discussed the empirical probability concept. Students understood the concept of empirical probability through assisted ratio with the context of the sale of pulses as shown in Figure 8.

**Figure 8. Students Use Ratio**
The teacher estimates that students will answer the empirical probability is the ratio of frequency for an occurrence with all of the experiments conducted. Almost all students answered like Figure 9(b), students define empirical probability as a theoretical probability. Only one student was able to find the concept of empirical probability correctly can be shown in Figure 9(a). Based on the results of the interviews, it had been conducted with students, almost all of the students stated that they were confused to distinguish between theoretical probability with the empirical. From the discussion process also showed when the teacher shared the student activity sheet, some students said that this student activity sheet they had completed the previous meeting. Students to make formulas for empirical probability as shown in Figure 10.

![Figure 9. Results of Answers Students](image1)

![Figure 10. Results of Answers Students](image2)

The teacher gives instructions with \( n(A) \) as the many events and \( n(S) \) as the many experiments so that all students answer that the formula for the theoretical probability as shown in Figure 7. So, some of the students can formulate an empirical probability correctly as shown in Figure 10(a), while some others answer as shown in Figure 10(b). Some students who answered like Figure 10(b) were asked why the formula became like that, students answered that the empirical probability with the theoretical was different. So, the formula cannot be the same so the formula is reversed like that. It can be concluded that students experience a misunderstanding because of the symbol used to express the empirical probability formula.

3.1.4. The Fourth Meeting. The fourth meeting discussed the concept of complement probability. Students understood the concept of complement probability is through ratio as shown in Figure 11.
The first stage was to use the ratio shown in Figure 1 and to find the concept of the complement probability assisted with theoretical probability value as shown in Figure 12.

The teacher estimates that students will answer the complementary probability is the ratio between a non-event and all sample space. Almost all of the students were able to understand the complement probability well as shown in Figure 12(a) but some students who still had not concluded the concept of complement probability in mathematics language as shown in Figure 12(b).

Students can make formulas for complement probability with the help of theoretical probability as shown in Figure 13. In general, the students understood the concept of complement probability as an event that was not chosen by a card of the existing problem.

3.2. Roles from List Price of Pulses in Supporting Relational Understanding Students for Probability Concepts

How could that list motivate students in mathematics learning in the class were shown from the students’ engagement during the learning process, the student’s readiness to solve the given problems and the students’ ability to involve parts of the story when they solve the problems. This was as shown from students’ enthusiasm when the teacher told the context of the price list of pulses in life. Students’ motivation in mathematics learning influences the students’ readiness and engagement during the learning process. In the next activities, points of the context also influence students’ motivation in solving the problems, so that the students consider the solution for the given problems as the solutions for the contextual problem. Students are also able to relate some knowledge about learning related to solving problems. During the discussion process, solving problems and the results of other audio-visual data the show of students’ relational understanding can be well built through ratio with the use list of price pulses.
3.3. Discussion

The probability learning trajectory in eight's class had been conducted in this research, the development of the relational understanding of students about the material good chances. Something unexpected happened in this study, students with lower-class capabilities were able to compete with students with high abilities. Based on the interviews had been conducted with students who had lower ability, they understood the concept of probability that was given as a proportion and the students also felt the learning at that time made them more excited than usual. The other thing that was unexpected too, in arguing during the process about empirical probability, the students' lower ability could do better than the students' upper ability. In general, all of the students in the class including upper and lower students were able to answer all questions relating to the material opportunities very quickly and well, it was visible when the teacher gave the task in each of the ends of learning without using the quiz.

The hypothesis had been made by researchers in this study was proved to be realized well so that the goal of this research was to develop the probability of learning trajectory in learning the use of the list of price pulses to be building a relational understanding, it can be done well. Although, there were still some learning trajectories that had to be repaired so that students were able to understand the probability concept with good later.

4. Conclusion

Based on the learning process, quiz results, and also interviews with several students, including the subject of the research concluded that one of the learning paths that still had to be improved was the empirical probability. Students in the interview process almost all of them said that the concept of empirical probability was very difficult and made them confused to distinguish between the empirical and theoretical probability. Possibly it caused by the creation of the problem the context was rudimentary because the students did not understand the concept of empirical probability. In general, this probability learning trajectory can help students in building relational understanding well.

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