Modeling pulsar time noise with long term power law decay modulated by short term oscillations of the magnetic fields of neutron stars

Shuang-Nan Zhang\textsuperscript{1,2} & Yi Xie\textsuperscript{1}
\textsuperscript{1}National Astronomical Observatories, Chinese Academy Of Sciences, Beijing 100012, China.
\textsuperscript{2}Key Laboratory of Particle Astrophysics, Institute of High Energy Physics, Chinese Academy of Sciences Beijing 100049, China.

Received Day Month Year
Revised Day Month Year
Communicated by Managing Editor

We model the evolution of the magnetic fields of neutron stars as consisting of a long term power-law decay modulated by short term small amplitude oscillations. Our model predictions on the timing noise $\dot{\nu}$ of neutron stars agree well with the observed statistical properties and correlations of normal radio pulsars. Fitting the model predictions to the observed data, we found that their initial parameter implies their initial surface magnetic dipole magnetic field strength $B_0 \sim 5 \times 10^{14}$ G when $t_0 = 0.4$ yr and that the oscillations have amplitude $K \sim 10^{-8}$ to $10^{-5}$ and period $T$ on the order of years. For individual pulsars our model can effectively reduce their timing residuals, thus offering the potential of more sensitive detections of gravitational waves with pulsar timing arrays. Finally our model can also re-produce their observed correlation and oscillations of $\dot{\nu}$, as well as the “slow glitch” phenomenon.

\textit{Keywords:} Neutron Star; Magnetar; Magnetic Field Decay; Timing Noise.

1. Introduction

Many studies on the possible magnetic field decay of neutron stars have been done previously, based on the observed statistics of their periods ($P$) and period derivatives ($\dot{P}$). Some of these studies relied on population synthesis that includes complicated observational selection effects, unknown initial parameters of neutron stars, and not completely understood models of radio emission of pulsars\textsuperscript{1,2,3,4,5,6,7,8}; consequently no firm conclusion can be drawn on if and how the magnetic fields of neutron stars decays (see Refs.\textsuperscript{9} for reviews). Alternatively some other studies used the spin-down or characteristics ages, $\tau_c = (P - P_0)/2\dot{P}$ ($P_0$ is the initial spin period of the given neutron star), or $\tau_c = P/2\dot{P}$ if $P \gg P_0$, as indicators of the true ages of neutron stars and found evidence for their dipole magnetic field decay\textsuperscript{11,12,13,14}. However, as we have shown recently, their spin-down ages are normally significantly larger than the ages of the supernova remnants physically associated with them, which in principle should be the unbiased age indicators of...
Pulsars are generally very stable natural clocks with observed steady pulses. However, significant timing irregularities, i.e., unpredicted times of arrivals of pulses, exist for all pulsars studied so far (see Ref. [19] for an extensive review of many previous studies on timing irregularities of pulsars). The timing irregularities of the first type are “glitches”, i.e., sudden increases in spin rate followed by a period of relaxation; it has been found that the timing irregularities of young pulsars with $\tau_c < 10^5$ years are dominated by the recovery from previous glitch events [19]. In many cases, the neutron star recovers to the spin rate prior to the glitch event and thus the glitch event can be removed from the data satisfactorily without causing significant residuals over model predictions. However, in some cases, glitches can cause permanent changes to both $P$ and $\dot{P}$ of the neutron star, which cannot be removed from the data satisfactorily. These changes can be modeled as a permanent increase of the surface dipole magnetic field of the neutron star; consequently some of these neutron stars may grow their surface magnetic field strength gradually this way and eventually have surface magnetic field strength comparable to that of magnetars over a lifetime of $10^{5-6}$ years [20]. These are the first studies that linked the timing irregularities of pulsars with the long-term evolution of magnetic fields of neutron stars.

The timing irregularities of the second type have time scales of years to decades and thus are normally referred to as timing noise. Hobbs et al. carried out so far the most extensive study of the long-term timing irregularities of 366 pulsars [19]. Besides ruling out some timing noise models in terms of observational imperfections, random walks, and planetary companions, some of their main conclusions are: (1) Timing noise is widespread in pulsars and is inversely correlated with $\tau_c$; (2) Significant periodicities are seen in the timing noise of a few pulsars, but quasi-periodic features are widely observed; (3) The structures seen in the timing noise vary with data span, i.e., more quasi-period features are seen for longer data span and the magnitude of $|\ddot{\nu}|$ for shorter data span is much larger than that caused by magnetic braking of the neutron star; and (4) The numbers of negative and positive $\ddot{\nu}$ are almost equal in the sample.

Proper understanding of these important observations not only can lead to better understanding of the internal structures of neutron stars, but can also be important for using some millisecond pulsars for gravitational wave detections. In particular it is of high interests if some of the long-term timing noise can be modeled and removed reliably from data, thus improving the sensitivity of gravitational wave detections substantially. It is extremely desirable if a physical model of neutron stars can catch and even predict the main features and long-term variations of the timing noise of pulsars, since such a model will ensure that no signal of gravitational
Neutron star magnetic field evolution produces timing noise

waves is removed when modeling the observed timing noise. Some of the observed periodicities have been suggested as due to free precessions of neutron stars. Some strong quasi-periodicities have been identified recently as due to abrupt changes of the magnetospheric regulation of neutron stars, perhaps due to varied particle emissions. It was suggested that such varied magnetospheric particle emissions is also responsible for the observed long term timing noise. Alternatively it has been suggested that the propagation of Tkachenko waves in a neutron star may modulate its rotational moment of inertia, and thus produce the observed periodic or quasi-periodic timing noise. Because none of the above processes may permanently change the properties of neutron stars, we can collectively represent all the above modulations as some sort of oscillations of the observed surface magnetic field strengths of neutron stars, as shown below.

In this work we first model the evolution of the dipole magnetic field of a neutron star as a long term power-law decay modulated by one or several components of oscillations. We then calculate the spin-down history of a neutron star within the magnetic braking model with the prescribed dipole magnetic field. The timing noise is then calculated and compared with data. We show that all main observed features of neutron star’s timing noise can be reproduced satisfactorily. In the end we compare our model with other recently developed models of neutron star’s timing noise.

2. Evolutionary model of the dipole magnetic field of neutron stars

Attributing the observed $T_{\text{SNR}}/\tau_c \ll 1$ to the long term decay of the magnetic field of a neutron star, we found that the magnetic field decay should follow a power-law form, $B \propto t^{-\alpha} (\alpha \approx 0.5)$, in good agreement with the field decay process dominated by the ambipolar diffusion mechanism with constant core temperatures. We further found that the core temperatures of magnetars, normal radio pulsars and millisecond pulsars are approximately $10^8$ K, $10^7$ K, and $10^5$ K, respectively. The neutron star free precession, inferred from the observed periodic timing noise, is phenomenologically equivalent to periodic oscillation of its apparent surface magnetic field $B = \sqrt{B_s^2 \sin^2 \chi}$, where $\chi$ is the angle between the magnetic axis and spin axis of the neutron and varies slightly as the neutron star precesses periodically. Similarly the observed strong quasi-periodicities of timing noise can also be phenomenologically modeled as modulations of the observed surface magnetic fields of these neutron stars, which are obtained from the observed $P$ and $\dot{P}$ of neutron stars. We therefore model the evolution of the dipole magnetic field of a neutron star as a long term power-law decay modulated by one or several components of oscillations,

$$B = B_0 \left( \frac{t}{t_0} \right)^{-\alpha} \left( 1 + \sum K_i \sin(\phi_i + 2\pi \frac{t}{T_i}) \right),$$

where $t$ is the neutron star age, $t_0$ and $B_0$ are the starting time and initial surface dipole magnetic field strength of the neutron star, $K_i$, $\phi_i$ and $T_i$ are the amplitude,
phase and period of the oscillating magnetic field of the \( i \)-th component, respectively.

Assuming pure magnetic dipole radiation as the braking mechanism for a pulsar’s spin down, we have,

\[
I \dot{\Omega} = - \frac{(BR^3)^2}{6c^3} \Omega^4,
\]

and

\[
\dot{\nu} = -AB^2 \nu^3,
\]

where \( A = \frac{(2\pi R^3)^2}{6c^3} \) is assumed to be a constant, \( B \) is its dipole magnetic field at its magnetic pole, \( R \) is its radius, \( I \) is its moment of inertia. We then have,

\[
\ddot{\nu} = -3AB^2 \nu^2 \dot{\nu} - 2AB \dot{B} \nu^3.
\]

Simple calculations show that the value of the first term in the right part of the above equation is much less than the observed \( \ddot{\nu} \) as reported in Ref. 19. Therefore the observed timing noise characterized by \( \ddot{\nu} \) may be dominated by the second term containing \( \dot{B} \). From Eq. (4), we have,

\[
\ddot{B} = B(t)(-\frac{\alpha}{t} + \sum \frac{2\pi K_i}{T_i} \cos(\phi_i + 2\pi \frac{t}{T_i})).
\]

Assuming \( K_i \ll 1 \) and ignoring the first term in Eq. (4), we have,

\[
\dot{\nu} \simeq -2\dot{\nu}(\frac{\alpha}{t} - \sum \frac{2\pi K_i}{T_i} \cos(\phi_i + 2\pi \frac{t}{T_i})),
\]

where the first and second terms in the right part of the above equation are contributed by the long term magnetic field decay and short term magnetic field oscillations, respectively. The age of the pulsar (not to be confused with its characteristic or spin-down age \( \tau_c \)) is given,

\[
t \simeq t_0 \left( \frac{B_0}{B(t)} \right)^{\frac{1}{2}}.
\]

For a young pulsar with \( t \lesssim 10^6 \) yr, it is very likely that the first term in Eq. (6) dominates and thus it is generally anticipated that \( \dot{\nu} > 0 \). For example, for \( t \sim 5 \times 10^3 \) yr, \( \alpha \sim 0.5 \), \( P \sim 1 \) s and \( \dot{\nu} \sim 10^{-13} \) Hz s\(^{-1} \), the first term in Eq. (5) gives \( \dot{\nu} \sim 10^{-24} \) Hz s\(^{-1} \) s\(^{-1} \), consistent with observations. On the other hand, the second term in Eq. (6) dominates for an old pulsar, and thus both positive and negative \( \dot{\nu} \) should be observed with almost equal possibilities. This is in general agreement with observations\(^{19} \).

3. Statistical properties of the spin evolution of pulsars

In estimating \( \ddot{\nu} \) from observations, \( \nu, \dot{\nu}, \) and \( \ddot{\nu} \) are obtained by fitting the phases of all pulses observed to the third order of its Taylor expansion over a period of time,

\[
\Phi(t) = (\Phi_0 + \nu t + \frac{1}{2} \dot{\nu} t^2 + \frac{1}{6} \ddot{\nu} t^3).
\]
We can therefore estimate $\dot{v}$ for a pulsar from
\begin{equation}
\dot{v} \sim -2\nu(\frac{\alpha}{t} \pm f),
\end{equation}
where $f = 2\pi \max(K_i/T_i)$. As shown later, $K_i \ll 1$ and $T_i$ is on the order of years, therefore $f \ll 1/t$ for young pulsars. Substitute Eq. (7) (with $\alpha = 0.5$) into Eq. (9), our model of power-law magnetic field decay in neutron stars predicts,
\begin{equation}
\dot{v} \sim g_0 \frac{P}{\tau_c^2} \pm \frac{f}{P\tau_c},
\end{equation}
where $g_0 = (3.2 \times 10^{19})^2/(4t_0B_0^2)$ is the initial parameter of pulsar’s magnetic field. In Fig. 1 (left) we show the observed correlation of $\dot{v} \sim P/\tau_c^2$ for young pulsars with $\dot{v} > 0$ and $\tau_c < 2 \times 10^6$ yr. Fitting this correlation with that predicted by Eq. (10) with $f = 0$, we get $g_0 = 83.74$, which is considered to be a constant for all pulsars in the rest of this paper, unless specified otherwise. Assuming $B_0 = 5 \times 10^{14}$ G, we have $t_0 = 0.39$ yr, suggesting that these pulsars may be fast rotating magnetars when $t = t_0$. In Fig. 1 (right) we show that the observed and model calculated $\dot{v}$ are very consistent with each other for these young pulsars, with $f = 0$ in Eq. (10); this is clear evidence for the power-law decay of neutron stars’ magnetic fields, at least for young pulsars.

In Fig. 2 we show the observed correlation of $\dot{v} \sim \tau_c$ for all pulsars with $\tau_c < 10^9$ yr (i.e. recycled millisecond pulsars are not considered); the left and right panels displays all pulsars with $\dot{v} > 0$ and $\dot{v} < 0$, respectively. Eq. (11) means that $\dot{v}$ is determined by both $P$ and $\tau_c$, given $g_0$ and $f$. Since the observed $P$ of these pulsars is narrowly distributed around a median value of 0.6 s, therefore the observed main correlation should be with $\tau_c$, rather than $P$, consistent with data\cite{19}. In order to make straightforward comparison between data and that predicted by Eq. (10), we
March 13, 2012 0:17 WSPC/INSTRUCTION FILE  ms

Fig. 2. Correlation of $\dot{\nu} \sim \tau_c$ for all pulsars with $\tau_c < 10^9$ yr, i.e. recycled millisecond pulsars are not considered. Left: pulsars with $\dot{\nu} > 0$. Right: pulsars with $\dot{\nu} < 0$. The solid and dashed lines are the model predicted correlation of $\dot{\nu} \sim \tau_c$ by taking the ‘+’ and ‘−’ signs in Eq. (10), respectively; different values of $f$ are labeled in the figure.

simply take $P = 0.6$ s in Eq. (10) to calculate the model predicted correlation of $\dot{\nu} \sim \tau_c$ by choosing several values of $f$; the calculations with the ‘+’ and ‘−’ signs in Eq. (10) are shown as solid and dashed lines, respectively. It should be noted that the calculations with the ‘−’ sign in Eq. (10) can also produce positive $\dot{\nu}$, and thus dashed lines appear in both panels. For $f = 10^{-13}$ and $T = 3$ yr, we have $K \approx 10^{-6}$.

In Fig. 3 we show the comparison between the observed and model predicted $\dot{\nu}$. It is clear that with just two parameters ($g_0$ is fixed and $f$ ranges between $10^{-15}$ and $10^{-11}$), the general properties of the long term spin evolution of all pulsars can be described successfully, except for the recycled millisecond pulsars which we will address separately. This can be considered as a strong support to our model of the long term (power-law decay) and short term (small amplitude oscillation) magnetic field evolution of neutron stars.

Fig. 3. Comparison between the observed and model predicted $\dot{\nu}$ with $f = 3 \times 10^{13}$ or $10^{12}$ for $\dot{\nu} > 0$ or $< 0$, respectively; here the observed value of $P$ of each pulsar is used in Eq. (10).
4. Timing noise of individual pulsars

Eq. (10) with fixed $g_0$ and $f$ cannot describe accurately the timing noise of individual pulsars for at least the following reasons: (i) $g_0$ and $f$ may be different for different pulsars; (ii) multiple oscillating components may exist with different phases, periods and amplitudes; (iii) the observed $\dot{\nu}$ for each pulsar is determined by fitting the observed pulse phases with Eq. (5) over a certain observation time span $T_0$; (iv) the observed $\dot{\nu}$ correlates with $T_0$, which cannot be predicted by Eq. (10); (v) in using Eq. (5) to fit data, all higher order terms are absorbed in the four terms in Eq. (5); and (vi) substantial residuals are often found in using Eq. (5) to fit data. None of the last three issues can be addressed with Eq. (10). We therefore should calculate numerically the history of a pulsar’s pulse phase $\Phi(t)$ within our model and compare it with observations.

The pulse phase change is given by

$$d\Phi = (\nu + \dot{\nu}t)dt,$$

where $\nu$ and $\dot{\nu}$ can be calculated numerically by combining Eqs. (1) and (3). The entire history of $\Phi(t)$ for a pulsar can thus be calculated by integrating Eq. (11) numerically, given the values of these parameters $t_0$, $B_0$, $\alpha$, $\phi_i$, $K_i$ and $T_i$. The calculated $\Phi(t)$ can then be fitted with Eq. (8) to determine $\ddot{\nu}$ and residuals.

4.1. Periodic and quasi-periodic residuals of timing noise

For many pulsars the residuals in fitting data with Eq. (5) show period or quasi-periodic behaviors. From the FFT of the residuals of B1540−0619, we can determine the phase and period of the main peak in its power spectrum, which are taken as $\phi$ and $T$ in Eq. (4) for a single oscillation component. The only remaining parameter is $K$. With different values of $K$, we repeat the above procedure to calculate $\Phi(t)$, and fit it with Eq. (5) to determine residuals, which are then compared with data until the minimum $\chi^2$ of the match is found. Fig. 4 (left) shows the comparison between the observed and model predicted residuals with $K = 7.41 \times 10^{-10}$; the rms of the residuals is reduced from 11.7 ms to 7 ms by introducing just one parameter $K$, after subtracting the best fit residuals from the observed ones.

When several quasi-periodic components are found in the residuals, we take an iterative approach to find the phase and period of each component: (1) find $\phi$, $T$ and the amplitude of the strongest component in the FFT; (2) remove this periodic component from the residuals; and (3) repeat the above until no significant component is found in the FFT of the residual after all identified components are subtracted. We then still use the $\chi^2$ optimization method to find $K_i$ of each component. In Fig. 4 (right) we show an example for B1826−17, the FFT of the residuals of which has two prominent components; the rms of the residuals is reduced from 33 ms to 14 ms in this case.
Fig. 4. Residuals after fitting Eq. (8) to the observed and calculated pulse phases for B1540−06 (left panel) and B1826−17 (right panel), respectively. The dashed and solid lines in the top panels are from the observed and calculated phases, respectively. The bottom panels are their differences.

4.2. Oscillating $\ddot{\nu}$ and slow glitches

We then try to model the timing noise of B0329+54, which shows $\ddot{\nu}$ correlates with $T_s$ and even switches between positive and negative values; the parameters we choose are given in the caption of Fig. 5, the left panel of which shows a similar correlation of $\ddot{\nu} \sim T_s$ in Fig. 12 of Ref. 19. This shows that a single pulsar can produce $\ddot{\nu}$ over almost the entire range of $\ddot{\nu}$ for all pulsars, depending on the chosen observation time span $T_s$; this may be the main source of the observed wide scatter in Fig. 2. In Fig. 5 (right), the calculated correlation of $\ddot{\nu} \sim T_s$ is shown with $K = 10^{-7}$, exhibiting clear oscillations of $\ddot{\nu}$ between positive and negative values. This explains naturally the almost equal numbers of positive and negative values of $\ddot{\nu}$ reported19.

Fig. 5. Left: Calculated correlation of $\ddot{\nu} \sim T_s$ for B0329+54, with one oscillation component of $T = 15.5$ yr and $K = 10^{-8}$. Right: The same as the left panel, except that $K = 10^{-7}$.

We then increase $K$ to $K = 2.5 \times 10^{-5}$ and show in Fig. 6 the calculated correlations of $\dot{\nu} \sim T_s$ and $\ddot{\nu} \sim T_s$. We find that sometimes $\dot{\nu}$ switches to positive
values, quite similar to the recently found slow glitches\textsuperscript{30,31}, the observed change of sign of $\dot{\nu}$ is also consistent with our model prediction. However the calculated $\dot{\nu}$ becomes negative most of the times. This means that the phenomenon of slow glitches may just be a manifestation of unusually large amplitude oscillations of the apparent magnetic field of a neutron star, consistent with the observed unusually large rms of residuals for the two pulsars observed with slow glitches, i.e., 1284.3 ms and 613.7 ms for PSR J1825$-$0935 (B1822$-$09) and PSR J1835$-$1106, respectively.

![Fig. 6. Calculated correlations of $\dot{\nu} \sim T_s$ (left panel) and $\ddot{\nu} \sim T_s$ (right panel) with the following parameters: $T = 15.5$ yr and $K = 2.5 \times 10^{-5}$.](image)

5. Conclusions and discussions

Our previous work has found evidence that the magnetic fields of neutron stars have long term decays in a power-law form, agreeing with the prediction of the ambipolar diffusion mechanism. Through examining the observed (apparent) short term variations of the magnetic fields of neutron stars, we propose that the (apparent) magnetic fields of neutron stars have oscillations. We thus model the evolution of a neutron star’s magnetic field as consisting of a long term power-law decay modulated by short term small amplitude oscillations. Our model predicted $\dot{\nu}$, a parameter widely used to characterize the timing noise of pulsars, agrees well with the observed statistical properties and correlations of all pulsars (except those recycled millisecond pulsars). This can be considered as a strong support to our model of magnetic field evolution of neutron stars.

Fitting the model predictions to the observed data, we found that their initial parameter implies their initial surface dipole magnetic field strength $B_0 \sim 5 \times 10^{14}$ G when $t_0 = 0.4$ yr and that the oscillations have amplitude $K \sim 10^{-8}$ to $10^{-5}$ and period $T$ on the order of years. We then calculated the timing properties of individual pulsars with our model. We modeled the observed timing residuals of two pulsars and demonstrated that our model can effectively reduce their timing residuals, thus offering the potential of more sensitive detections of gravitational
waves with pulsars. Depending on the combination $K$ and $T$, we can re-produce the observed correlation and oscillations of $\dot{\nu}$, as well as the “slow glitch” phenomenon.

We did not study the timing noise properties of the recycled millisecond pulsars with our model, because their initial properties may be substantially different from the normal radio pulsars we studied in this work.

Acknowledgments

We thank interesting discussions with Renxin Xu on timing noise. SNZ acknowledges partial funding support by the National Natural Science Foundation of China under project no. 11133002, 10821061, 10725313, and by 973 Program of China under grant 2009CB824800.

References

1. Holt, S. S. & R. Ramaty, Natur 228 (1970) 351
2. Bhattacharya, D., et al., A&A 254 (1992) 198
3. Han, J. L., A&A 318 (1997) 485
4. Regimbau, T. & J. A. de Freitas Pacheco, A&A 374 (2001) 182
5. Gonthier, P. L., et al., ApJ 565 (2002) 482
6. Guseinov, O. H., A. Ankay, & S. O. Tagieva, IJMPD 13 (2004) 1805
7. Aguilera, D. N., J. A. Pons, & J. A. Miralles, ApJ 673 (2008) L167
8. Popov, S. B., et al., MNRAS 401 (2010) 2675
9. Harding, A. K. & D. Lai, RPPh 69 (2006) 2631
10. Ridley, J. P. & D. R. Lorimer, MNRAS 404 (2010) 1081
11. Lorimer, D. R., in High-Energy Emission from Pulsars and their Systems, Astrophysics and Space Science Proceedings (2011) 21
12. Pacini, F., Natur 224 (1969) 160
13. Ostriker, J. P. & J. E. Gunn, ApJ 157 (1969) 1395
14. Gunn, J. E. & J. P. Ostriker, ApJ 160 (1970) 979
15. Lyne, A. G., R. T. Ritchings, & F. G. Smith, MNRAS 171 (1975) 579
16. Geppert, U., D. Page, & T. Zannias, A&A 345 (1999) 847
17. Ruderman, M., in The Electromagnetic Spectrum of Neutron Stars, Proceedings of the 6th NATO ASI series (2005) 47
18. Zhang, S.-N. & Y. Xie, in the Proceedings of the 9th Pacific Rim Conference on Stellar Astrophysics (2011) arXiv:1110.3154
19. Hobbs, G., A. G. Lyne, & M. Kramer, MNRAS 402 (2010) 1027
20. Lin, J. R. & S. N. Zhang, ApJ 615 (2004) L133
21. Stairs, I. H., A. G. Lyne, & S. L. Shemar, Natur 406 (2000) 484
22. Lyne, A., G. Hobbs, M. Kramer, I. Stairs, & B. Stappers, Sci 329 (2010) 408
23. Liu, X.-W., X.-S. Na, R.-X. Xu, & G.-J. Qiao, ChPhL 28 (2011) 019701
24. Tkachenko, V. K. Sov. Phys. JETP 23 (1966) 1049.
25. Ruderman, M. Nature 225 (1970) 619.
26. Haskell, B., PhRvD 83 (2011) 043006
27. Goldreich, P. & A. Reisenegger, ApJ 395 (1992) 250
28. Heyl, J. S. & S. R. Kulkarni, ApJ 506 (1998) L61
29. Xie, Y. & S.-N. Zhang, in the Proceedings of the 9th Pacific Rim Conference on Stellar Astrophysics (2011) arXiv:1110.3869
30. Zou, W. Z., et al., MNRAS 354 (2004) 811
31. Shabanova, T. V., *Ap&SS* **308** (2007) 591