Utility Maximization for an Investor with Asymmetric Attitude to Gains and Losses over the Mean–Variance Efficient Frontier

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Abstract. This paper examines the problem of choosing the optimal portfolio for an investor with asymmetric attitude to gains and losses described in the cumulative prospect theory (CPT) of A. Tversky and D. Kahneman. We consider the problem of finding the portfolio that maximizes CPT-utility over the set of all feasible portfolios along the mean–variance efficient frontier under some conditions on the stochastic behavior both of the portfolio price and the discount factor. It should be noted that since the value function of the CPT-utility is S-shaped and not-differentiable in the origin, numerical approach to the optimization of CPT-utility is not trivial. We introduce a model with stochastic behavior of both the portfolio price and a discount factor. Following routing procedure for Black-Scholes environment we find the closed-form solution for CPT-utility in a simple particular case. Nevertheless, it turned out that in the general case the closed-form solution can not be obtained. Therefore, we use computational methods for studying the properties of CPT-utility. The paper shows that this approach to the optimal portfolio choice for CPT-investor might be promising.

1. Introduction
One of the classic problems of the portfolio investment theory is to find an optimal portfolio. Portfolio is defined as a set of assets with weights, the sum of which is equal to 1 (the budget constraint).

In this paper we consider the problem of finding the optimal portfolio for an investor with asymmetric attitudes to gains and losses described in the cumulative prospect theory of A. Tversky and D. Kahneman. Cumulative prospect theory (CPT) was proposed in [1] and is the further development of prospect theory studied in [2]. The difference between this version and the original version of prospect theory is that cumulative probabilities are transformed, rather than the probabilities itself. Modern economic literature considers the cumulative prospect theory as one of the best models explaining the behavior of the players, the investors in the experiment and in decision-making under risk.

The paper [3] shows that the prospect theory can resolve a number of decision making paradoxes, but the author notes that it is not a ready-made model for economic applications. Nevertheless, recent years show increasing interest in the problems lying in the intersection of prospect theory and portfolio optimization theory. It should be noted, that due to the
computational difficulties connected to the complexity of the numerical evaluation of the CPT-utility, there are not so much works devoted to the portfolio optimization problem under the framework of both prospect theory [4], [5] and cumulative prospect theory [6], [7], [8], [9], [10]. While the papers contain some numerical results, only simple cases (2-3 artificially created assets) of the portfolio selection problem are considered. Besides, most of the papers are based on the assumption that testing data are normally distributed. However, it is well known, that many asset allocation problems involve non-normally distributed returns since commodities typically have fat tails and are skewed.

The paper [5] tries to select the portfolio with the highest prospect theory utility amongst the other portfolios in the mean variance efficient frontier. Developing this idea, the work [6] shows that an analytical solution of the problem is mostly equivalent to maximising the CPT-utility function along the mean-variance efficient frontier. Recent developments in the cumulative prospect theory may be found in papers [11, 12, 13, 14, 15, 16, 17, 18]. The paper [19] incorporates CPT-based criteria into a stochastic optimization framework and illustrates the usefulness of some proposed algorithms for optimizing CPT-based criteria in a traffic signal control application.

First, we briefly present the main ideas of this theory, and then proceed to the problem of finding the function for the assessment of the prospects under some assumptions on the stochastic behavior of the discount factor and the portfolio price. Preliminary closed-form results on the CPT-utility maximization under some assumptions can be also found in [20, 21].

2. CPT-investor

We will consider the development of the prospect theory, Cumulative Prospect Theory, published in 1992 [1]. Prospect theory (PT) has three essential distinctions from Expected Utility Theory:

- investor makes investment decisions based on deviation of his/her final wealth from a reference point and not according to his/her final wealth, i.e. PT-investor concerned with deviation of his/her final wealth from a reference level, whereas Expected Utility maximizing investor takes into account only the final value of his/her wealth.
- utility function is $S$-shaped with turning point in the origin, i.e. investor reacts asymmetrical towards gains and losses; moreover, he/she dislikes losses with a factor of $\lambda > 1$ as compared to his/hers liking of gains.
- investor evaluates gains and losses based on transformation of real probability distribution and not according to the real probability distribution per ce, such that investor’s probability assessments are transformed in the way that small probability (high probability) are over-(under-) valued.

The description of includes three important parts: a value function over outcomes, $v(\cdot)$; a weighting function over cumulative probabilities, $w(\cdot)$; CPT-utility as unconditional expectation of the value function $v$ under probability distortion $w$.

**Definition 1.** The value function derives utility from gains and losses and is defined as follows [1]:

$$v(x) = \begin{cases} x^\alpha, & \text{if } x \geq 0, \\ -\lambda(-x)^\beta, & \text{if } x < 0. \end{cases}$$  \hspace{1cm} (1)$$

Note that the value function is convex over losses if $0 \leq \beta \leq 1$ and it is strictly convex if $0 < \beta < 1$. Moreover, the value function reflects loss aversion when $\lambda > 1$. It follow from the fact that individual investors are more sensitive to losses than to gains. D. Kahneman and A. Tversky estimated [2] the parameters of the value function $\alpha = \beta = 0.88$, $\lambda = 2.25$ based on experiments with gamblers.
Definition 2. Let $F_\xi(x)$ be cumulative distribution function (cdf) of a random variable $\xi$. The probability weighting function $w : [0, 1] \to [0, 1]$ is defined by
\[
w(F_\xi(x)) = \frac{(F_\xi(x))^\delta}{((F_\xi(x))^\delta + (1 - F_\xi(x))^\delta)^{1/\delta}}, \quad \delta \leq 1
\] (2)

Definition 3. The CPT-utility of a gamble $G$ with stochastic return $\xi$ is defined as [3]
\[
U_{CPT}(G) = \int_{-\infty}^{0} v(x)dw(F_\xi(x)) - \int_{0}^{\infty} v(x)dw(1 - F_\xi(x)),
\] (3)
where $F_\xi(x)$ is cumulative distribution function of $\xi$.

If we apply integration by part, then CPT-utility of $G$ defined in (3) can be rewritten as
\[
U_{CPT}(G) = \int_{0}^{\infty} w(1 - F_\xi(x))dv(x) - \int_{-\infty}^{0} w(F_\xi(x))dv(x).
\] (4)

3. Analysis of CPT-utility in $(\sigma, \mu)$-space

We will use the paradigm of cumulative prospect theory introduced in [1], and consider the problem of finding the optimal portfolio by CPT-investor on the set of all feasible portfolios. We suppose that the investor has an initial wealth level $S_0$. The cumulative prospect theory argues that the investor evaluates the expected deviation of final value of wealth from the reference value $X$ at the time moment $T$ and makes an investment decision on the basis of this assessment. In other words, the investor evaluates not the utility of final wealth $S_T$ but the gain or the loss, i.e. the difference $S_T - X$.

Examining expected utility maximization many researches use the following approach: the expected utility is maximized along the MV efficient frontier and the portfolio obtained this way is used in the comparison with mean-variance method. We will employ this technique in the CPT-optimization problems. It means that we examine the following problem:
\[
U_{CPT}(G) \to \max_{G \in \Omega}
\]
where $\Omega$ denotes the set of portfolios along the MV efficient frontier.

Let $S = S_t$ be the price of the portfolio at the time moment $t$. Let $X$ be the reference point (investor compares the portfolio price $S_T$ with $X$ at the moment $t = T$ and if $S_T > X$, they consider $S_T - X$ a gain; in the case $S_T < X$, they think it to be a loss $X - S_T$). We follow discount factor approach [22], i.e. at each time we construct a discount factor that prices the stock and risk-free asset, and use this discount factor to price the difference $S_T - X$.

The fair value of a gain (a loss) can be found by discounting the value of the gain $S_T - X$ (or the loss $X - S_T$) using the discount factor $\Lambda$.

We will assume that the price $S$ of portfolio follows the stochastic differential equation
\[
\frac{dS}{S} = \mu dt + \sigma dz,
\] (5)
where $\mu$ is a drift, and $\sigma$ is a standard deviation. We will assume that there is also a money market security that pays the real interest rate $rdt$.

Suppose that the discount factor $\Lambda$ follows the stochastic equation
\[
\frac{d\Lambda}{\Lambda} = -rdt - \frac{(\mu - r)}{\sigma}dz - \sigma_w dw, \quad E(dwdz) = 0.
\] (6)
It is easy to check that it is valid discount factor, since $E(d\Lambda/\Lambda) = -r dt$ and $E(dS/S) - r dt = -E(d\Lambda/\Lambda dS/S)$.

It is well-known [22] that

$$
\ln S_T = \ln S_0 + \left( \mu - \frac{\sigma^2}{2} \right) T + \sigma \sqrt{T} \varepsilon,
$$

(7)

$$
\ln \Lambda_T = \ln \Lambda_0 - \left( r + \frac{1}{2} \left( \frac{\mu - r}{\sigma} \right)^2 \right) T - \frac{\mu - r}{\sigma} \sqrt{T} \varepsilon,
$$

(8)

where $\varepsilon \sim N(0,1)$, and $S_0$ is the price of the portfolio at $t = 0$.

Let $S_T$ and $\Lambda_T$ be the solutions of equations (5) and (6) defined by (7) and (8) respectively. Let $\Omega = \{ (\mu, \sigma) \}$ denote the set of all feasible portfolios along the MV efficient frontier. The set $\Omega = \{ (\mu, \sigma) \}$ is the Markovitz frontier (a set or a line on the $(\mu, \sigma)$-plane). Then investor's decision is to chose a portfolio from the set $\Omega$. Every such portfolio is characterized by $(\mu, \sigma)$ and generates a prospect $S_T^{(\mu, \sigma)}$. In the following we will omit $(\mu, \sigma)$ subscripts and use $S_T$ for simplicity.

Denote

$$
D(S_T, \Lambda_T) := \frac{\Lambda_T}{\Lambda_0} (S_T - X)
$$

the $\Lambda$-discounted gain (or loss) $S_T - X$. The value function defined by (1) at the point $D(S_T, \Lambda_T)$ is equal to

$$
u (D(S_T, \Lambda_T)) = \begin{cases} (D(S_T, \Lambda_T))^\alpha, & S_T \geq X, \\ -\lambda (-D(S_T, \Lambda_T))^\beta, & S_T < X. \end{cases}
$$

Then CPT-utility of the prospect $S_T = S_T^{(\mu, \sigma)}$ (and respectively decision $(\mu, \sigma)$) can be written as

$$
U(\mu, \sigma) = - \int_{S_T=X}^{+\infty} v (D(S_T, \Lambda_T)) \, dw(1 - \Phi(\varepsilon)) + \int_{S_T=-\infty}^X v (D(S_T, \Lambda_T)) \, dw(\Phi(\varepsilon)),
$$

(9)

where $\Phi(\varepsilon)$ is the normal cumulative distribution function defined by

$$
\Phi(\varepsilon) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\varepsilon e^{-\frac{1}{2}t^2} \, dt
$$

(10)

is the normal cumulative distribution function, and $w(\cdot)$ is defined in (2).

Portfolio optimization problem for CPT-investor can be stated as follows:

$$
U(\mu, \sigma) \rightarrow \max_{(\mu, \sigma) \in \Omega}
$$

(11)

We will assume that $X = S_0 e^{r T}$ is the reference point as it is natural from investor’s point of view, since $S_0 e^{r T}$ is the amount of wealth the investor would have received on the date $t = T$ after investing $W_0$ with the continuously compounding rate $r$.

Denote

$$
I_1(\mu, \sigma) := - \int_{S_T=X}^{+\infty} v (D(S_T, \Lambda_T)) \, dw(1 - \Phi(\varepsilon)),
$$

$$
I_2(\mu, \sigma) := \int_{S_T=-\infty}^X v (D(S_T, \Lambda_T)) \, dw(\Phi(\varepsilon)).
$$

For the analysis of portfolio selection, it will be useful to introduce the concept of an indifference curve in $(\sigma, \mu)$-space.

**Definition 4.** The indifference curve in $(\sigma, \mu)$-space, relative to a given utility level $V$, is the locus of points $(\sigma, \mu)$ along which expected utility is constant, i.e., equal to $V$. 


3.1. The case \( \alpha = \beta \) and \( \delta = 1 \)

Let \( \epsilon^* \) be such that \( e^{(\mu - \sigma^2/2)T + \sigma \sqrt{T} \epsilon^*} = e^{rT} \), i.e. \( \epsilon^* = \frac{-(\mu - r - \sigma^2/2)}{\sigma \sqrt{T}} \), or

\[
\epsilon^* = \left( -\frac{\mu - r}{\sigma} + \frac{\sigma}{2} \right) \sqrt{T}.
\]  

(12)

Note that if \( V \neq 0 \) then \( D(V) \geq 0 \) if the random \( \epsilon \geq \epsilon^* \), and \( D(V) < 0 \) otherwise.

It is impossible to find a closed-form solution of the problem (11). In this subsection we will assume \( \alpha = \beta, \delta = 1 \) and derive a closed-form solution in the case \( \alpha = \beta = 1 \).

**Lemma 1.** Let \( \alpha = \beta \) and \( \delta = 1 \). Then

\[
U(\mu, \sigma) = \frac{S_0^\alpha}{\sqrt{2\pi}} e^{\frac{1}{2}(1-\alpha)(1-\mu^2/\sigma^2)}(\mu-r)T \int_0^{+\infty} \left( e^{-\frac{1}{2}(x+\frac{1}{2}\sigma \sqrt{T})^2+\frac{\mu-r}{\sigma} \sqrt{T}(1-\alpha)x (e^{\sigma \sqrt{T}x} - 1)^{1/\alpha} \right.
\]

\[
- \lambda e^{-\frac{1}{2}(x+\frac{1}{2}\sigma \sqrt{T})^2-\frac{\mu-r}{\sigma} \sqrt{T}(1-\alpha)x (1-e^{-\sigma \sqrt{T}x})} dx.
\]  

(13)

**Proof.** Let \( \epsilon^* \) be defined by (12). Changing variables by \( \epsilon = x + \epsilon^* \) we get

\[
I_1(\mu, \sigma) = \int_0^{+\infty} e^{(\mu - \sigma^2/2)T + \sigma \sqrt{T} x - \mu \sqrt{T} x} dx = \frac{S_0^\alpha}{\sqrt{2\pi}} e^{\frac{1}{2}(1-\alpha)(1-\mu^2/\sigma^2)}(\mu-r)T \int_0^{+\infty} \left( e^{-\frac{1}{2}(x+\frac{1}{2}\sigma \sqrt{T})^2+\frac{\mu-r}{\sigma} \sqrt{T}(1-\alpha)x (e^{\sigma \sqrt{T}x} - 1)^{1/\alpha} \right.
\]

\[
- \lambda e^{-\frac{1}{2}(x+\frac{1}{2}\sigma \sqrt{T})^2-\frac{\mu-r}{\sigma} \sqrt{T}(1-\alpha)x (1-e^{-\sigma \sqrt{T}x})} dx.
\]  

(14)

Similarly we can get

\[
I_2(\mu, \sigma) = \frac{S_0^\alpha}{\sqrt{2\pi}} e^{\frac{1}{2}(1-\alpha)(1-\mu^2/\sigma^2)}(\mu-r)T \int_0^{+\infty} \left( e^{-\frac{1}{2}(x+\frac{1}{2}\sigma \sqrt{T})^2+\frac{\mu-r}{\sigma} \sqrt{T}(1-\alpha)x (e^{\sigma \sqrt{T}x} - 1)^{1/\alpha} \right.
\]

\[
- \lambda e^{-\frac{1}{2}(x+\frac{1}{2}\sigma \sqrt{T})^2-\frac{\mu-r}{\sigma} \sqrt{T}(1-\alpha)x (1-e^{-\sigma \sqrt{T}x})} dx.
\]  

(15)

Then Lemma follows from (14) and (15).
Theorem 2. Let $\alpha = \beta = 1$ and $\delta = 1$. Then

$$U(\mu, \sigma) = S_0(\lambda - 1) \left( \Phi \left( -\frac{1}{2} \sigma \sqrt{T} \right) - \Phi \left( \frac{1}{2} \sigma \sqrt{T} \right) \right),$$

where $\Phi$ is the cumulative density function of normal distribution.

Proof. Since $\alpha = 1$ we have

$$I_1(\mu, \sigma) = \frac{S_0}{\sqrt{2\pi}} \int_0^\infty e^{-\frac{1}{2}(x+\frac{1}{2} \sigma \sqrt{T})^2} (e^{\sigma \sqrt{T}x} - 1) dx =$$

$$= \frac{S_0}{\sqrt{2\pi}} \left( \int_0^\infty e^{-\frac{1}{2}(x-\frac{1}{2} \sigma \sqrt{T})^2} dx - \int_0^\infty e^{-\frac{1}{2}(x+\frac{1}{2} \sigma \sqrt{T})^2} dx \right). \quad (16)$$

We have

$$\frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-\frac{1}{2}(x-\frac{1}{2} \sigma \sqrt{T})^2} dx = -\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\frac{1}{2} \sigma \sqrt{T}} e^{-\frac{1}{2}y^2} dy = -\Phi \left( -\frac{1}{2} \sigma \sqrt{T} \right) \quad (17)$$

and

$$\frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-\frac{1}{2}(x+\frac{1}{2} \sigma \sqrt{T})^2} dx = -\Phi \left( \frac{1}{2} \sigma \sqrt{T} \right). \quad (18)$$

It follows from (17), (18) and (18) that

$$I_1(\mu, \sigma) = S_0 \left( \Phi \left( -\frac{1}{2} \sigma \sqrt{T} \right) - \Phi \left( \frac{1}{2} \sigma \sqrt{T} \right) \right). \quad (19)$$

Analogously we can get

$$I_2(\mu, \sigma) = \lambda S_0 \left( \Phi \left( \frac{1}{2} \sigma \sqrt{T} \right) - \Phi \left( -\frac{1}{2} \sigma \sqrt{T} \right) \right). \quad (20)$$

Then Lemma follows from (19) and (20).

Corollary 1. Let $\alpha = \beta = 1$ and $\delta = 1$. Then

(i) $U(\mu, \sigma)$ does not depend on $\mu$;
(ii) if $\lambda > 1$ then $U'_\sigma(\mu, \sigma) < 0$ on $(0, \infty)$;
(iii) if $\lambda = 1$ then $U(\mu, \sigma) = 0$;
(iv) maximum of $U(\mu, \sigma)$ is equal to 0 and is achieved at point $\sigma = 0$.

It is impossible to find a closed-form expression for the integral (13). We can use numerical approach to find its value. We have realized the algorithm for the solution of this problem in C++. We have carried out numerical experiments with the use of the following model parameters $S_0 = 100$, $r = 0.01$, $T = 1$, $\delta = 1$. Besides, we took $\alpha = \beta = 0.88$, $\lambda = 2.25$ which was proposed in [1, 2].

Computational results are shown in Fig. 1, 2, 3 and 4.

Fig. 1 plots the dependence of $U(\mu, \sigma)$ from $\sigma$ and $\mu$. It can be seen that the surface is continuous. The Fig. 2 shows dependance the value of $U(\mu, \sigma)$ from $\mu$ for different values of $\sigma$. 

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It can be seen that there is the global maximum of the function $U(\mu, \sigma)$ with the fixed value of $\sigma$. The Fig. 3 presents dependance the value of $U(\mu, \sigma)$ from $\sigma$ for different values of $\mu$. Again we can see that if we fix $\mu$ then there is the global maximum of the function $U(\mu, \sigma)$.

Fig. 4 plots the dependance the optimal values $\mu$, which maximize the function $U(\mu, \sigma)$, from $\sigma$. The plot shows that the behavior of CPT-investor is the same as the behavior of the well-known expected utility investor: she/he demands a higher return with increasing of risk.

3.2. The general case

As it was mentioned earlier, it is impossible to find a closed-form solution of the problem (11). On the other hand, we might use computational approach to solve the problem. It follows from...
(4) that CPT-utility of decision \((\mu, \sigma)\) is

\[
U(\mu, \sigma) := \int_{\varepsilon^*}^{+\infty} w(1 - \Phi(\varepsilon)) \, d\varepsilon \left( \alpha \left( -\left( r + \frac{1}{2} \left( \frac{\mu - r}{\sigma} \right)^2 \right) T - \frac{\mu - r}{\sigma} \sqrt{T \varepsilon} \right) \right)^\alpha \left( S_0 e^{\left( \mu - \frac{\mu^2}{2} \right) T + \sigma \sqrt{T \varepsilon} - X} \right)^\alpha + \\
+ \lambda \int_{-\infty}^{\varepsilon^*} w(\Phi(\varepsilon)) \, d\varepsilon \left( \beta \left( -\left( r + \frac{1}{2} \left( \frac{\mu - r}{\sigma} \right)^2 \right) T - \frac{\mu - r}{\sigma} \sqrt{T \varepsilon} \right) \right)^\beta \left( X - S_0 e^{\left( \mu - \frac{\mu^2}{2} \right) T + \sigma \sqrt{T \varepsilon} - X} \right)^\beta.
\]

Therefore, the problem of optimal portfolio choice for the investor with asymmetrical attitudes towards gains and losses described in cumulative prospect theory, can be brought to the problem of maximization of the value \(U(\mu, \sigma)\) on the set of all feasible pairs \((\mu, \sigma)\). We have realized the algorithm for the solution of this optimization problem in C++. We have carried out numerical experiments with the use of the following model parameters \(S_0 = 100, r = 0.02, T = 1, X = (1 + r)S_0 = 102, \delta = 0.65\). Besides, we took \(\alpha = \beta = 0.88, \lambda = 2.25\) based on [2, 1].

To examine the behavior of CPT-investor, in our research we consider four different efficient frontier \(\Omega\) in \((\mu, \sigma)\)-space:

- all portfolio lying on the segment between points \((0,0.01)\) and \((0.028,0.16)\) (Fig. 5, red line);
- all portfolio lying on the segment between points \((0,0.01)\) and \((0.016,0.2)\) (Fig. 6, red line);
- all portfolio lying on the segment between points \((0,0.01)\) and \((0.0449, 0.1595)\) (Fig. 7, red line);
- all portfolio lying on the segment between points \((0,0.01)\) and \((0.2, 0.05)\) (Fig. 8, red line).

\[\text{Figure 5. The efficient frontier (red line) and the indifference curve (blue line), } U_{\max} = 0.2249.\]

\[\text{Figure 6. The efficient frontier (red line) and the indifference curve (blue line), } U_{\max} = 0.1127.\]

Using gradient optimization approach we can find the maximum value of \(U(\mu, \sigma)\) on the set \(\Omega\) of all feasible pairs \((\mu, \sigma)\) lying on the corresponding efficient frontier:

\[U_{\max} := \max_{(\mu, \sigma) \in \Omega} U(\mu, \sigma).\]

Then we draw the indifference curve \(U(\mu, \sigma) = U_{\max}\) in the same \((\mu, \sigma)\)-space. Results are in Fig. 5, 6, 7 and 8 (blue lines). The maximum values of \(U(\mu, \sigma)\) are achieved in tangents points.
Fig. 7. The efficient frontier (red line) and the indifference curve (blue line), $U_{\text{max}} = 0.1797$.

Fig. 8. The efficient frontier (red line) and the indifference curve (blue line), $U_{\text{max}} = 0.0$.

Fig. 8 shows that the CPT-investor avoids risk: the optimal point is risk-free investment (0,0.1). On the other hand, it follows from figures 5, 6 and 7 that the CPT-investor is ready to invest in a risky asset only in case of extremely high return. Note that risky assets with parameters (0.028,0.16), (0.016,0.2) and (0.0449, 0.1595) presented in the three plots can not exist in finance practice. We can assume that, in order to fit the CPT-model to real world behavior of financial investor, we need to revisit the value of parameters $\alpha$, $\beta$, $\delta$, $\gamma$.

4. Conclusion
The paper deals with the problem of optimal portfolio choice for CPT-investor described in [10]. The problem can be reduced to the problem of maximization of investors CPT-utility over the all feasible portfolios. We studied the problem of finding the portfolio that maximizes CPT-utility on the set of all feasible portfolios. The problem came into the light in the paper [5] and became popular in the literature later on. It should be noted that since the value function is S-shaped and not-differentiable in the origin, numerical approach to the optimization of CPT-utility is not trivial. Due this fact most of the papers considered only 2 or 3 assets. The paper [23] uses heuristic algorithms to solve this problem with a much larger number of assets. In this paper we apply a different approach. We have introduced a model with stochastic behavior of both portfolio price $S$ and discount factor $\Lambda$. Following routing procedure for Black-Scholes environment we could find the closed-form of CPT-utility in the simple case $\alpha = \beta = 1$ and $\lambda = 1$. Nevertheless, it turned out that in the general case the closed-form solution can not be obtained. Therefore, we relied on computational methods for studying the properties of CPT-utility $U(\mu, \sigma)$. The paper shows that this approach to the optimal portfolio choice for CPT-investor might be promising.

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References
[1] Tversky A and Kahneman D 1992 Journal of Risk and Uncertainty 5 297–323
[2] Kahneman D and Tversky A 1979 Econometrica 62 1291–1326
[3] Barberis N C 2013 Journal of Economic Perspectives 27 173–196
[4] Gomes F J 2005 Journal of Business 78 675–706
[5] Levy H and Levy M 2004 *The Review of Financial Studies* **17** 1015–1041
[6] Pirvu T A and Schulze K 2012 *Mathematics and Financial Economics* **6** 337–362
[7] Zakamouline V and Koekbakker S 2009 *European Financial Managements* **15** 934–970
[8] He X D and Zhou X Y 2011 *Management Science* **57** 315–331
[9] Bernard C and Ghossoub M 2010 *Mathematics and Financial Economics* **2** 277–306
[10] Barberis N C and Huang M 2008 *American Economic Review* **98** 2066–2100
[11] Hens T and Mayer J 2017 *Journal of Computational Finance* **21** 47–73
[12] Lewandowski M 2017 *Central European Journal of Economic Modelling and Econometrics* **275**–321
[13] Zervoudi E K 2017 *Journal of Behavioral Finance* **0** 1–15
[14] Kaucic M and Daris R 2016 *Managing Global Transitions* **14** 359–384
[15] Yang C, Liu B, Zhao L and Xu X 2017 *International Journal of Transportation Science and Technology* **6** 143–158
[16] Häckel B, Pfosser S and Tränkler T 2017 *Energy Policy* **111** 414 – 426 ISSN 0301-4215
[17] Zou B and Zagt R 2017 *Mathematics and Financial Economics* **11** 393–421
[18] Kwak M and Pirvu T A 2018 *SIAM Journal on Financial Mathematics* **9** 54–89
[19] Jie C, A P L, Fu M C, Marcus S and Szepesvari C 2018 *IEEE Transactions on Automatic Control* 1–1
[20] Sidorov S P, Homchenko A A and Mironov S V 2016 *Stochastic models for portfolio choice under prospect theory and cumulative prospect theory* (WORLD SCIENTIFIC) pp 77–90
[21] Sidorov S, Khomchenko A and Mironov S 2017 *Optimal Portfolio Selection for an Investor with Asymmetric Attitude to Gains and Losses* (Cham: Springer International Publishing) pp 157–169
[22] Cochrane J H 2005 *Asset Pricing* (Princeton and Oxford: Princeton University Press)
[23] Grishina N, Lucas C A and Date P 2017 *Quantitative Finance* **17** 353–367