Characteristics of problems for developing higher-order thinking skills in mathematics

Makmuri¹, T A Aziz¹, * and S A A Kharis¹

¹Universitas Negeri Jakarta, Jakarta Timur, DKI Jakarta

* E-mail: tian_aziz@unj.ac.id

Abstract. A large number of studies in the literature have illuminated the issue of higher-order thinking skills with different perspectives. The purpose of the study was to establish a synthesis of the literature on the theoretical basis of higher-order thinking skills in mathematics learning. This study reviews and analyses all resources, including articles published in academic journals and books. Based on a content analysis of selected resources, this article formulates a definition of higher-order thinking skills and proposes ten substantial characteristics of problems for developing students’ higher-order thinking skills in mathematics learning. The present study makes several significant improvements to the field of mathematics education by providing a framework for test construction and an agenda for future research.

1. Introduction

Generally, thinking is considered as an activity that is carried out and occurs when a person faces a situation or problem that must be solved. Thinking could influence individual learning ability and acceleration. Accordingly, thinking ability tends to be related to the learning process. Thinking activities are commonly categorized into lower-order thinking and higher-order thinking [1]. Currently, the curriculum's demand in the focus of learning mathematics is to develop students' higher-order thinking skills. Higher-order thinking skills are essential elements in the teaching and learning process. It is one of the essential abilities which students need to develop, especially in mathematics learning. Therefore, in the last decade, there is an educational transformation from rote learning to learning for developing higher-order thinking skills [2].

The importance of thinking at a higher level for students results from the demands of the 21st-century curriculum that are increasingly requiring students to solve a non-standard problem that cannot be tackled merely by memorizing the pre-learned steps of the process. Students should develop creative and logical thinking [3] to deal with the puzzlement in constructing knowledge and to accomplish their assignments [4]. Higher-order thinking contributes positively to students’ learning achievement, such as a high level of grade point average or GPA [5].

The past decade has seen the rapid development of studies related to high order thinking. Considerable studies in the literature have examined the definition of higher-order thinking skills. To explicate the definition of higher-order thinking skills, most educators refer to available learning taxonomies, such as a revision of Bloom’s taxonomy [6], Marzano’s taxonomy [7], and SOLO taxonomy [8].

According to a revision of Bloom’s taxonomy [9], higher-order thinking skills involve students’ thinking activities in analyzing, evaluating, and creating. The taxonomy, which consists of six thinking levels, implies that lower-order thinking skills are required to perform higher-order thinking skills. In
literature, most educators and researchers employ this classification to explain higher-order thinking skills ([10]–[12]). Meanwhile, Marzano [7] described higher-order thinking as a comparison, classification, inductive and deductive reasoning, error analysis, support construction, abstraction, perspectives analysis, decision making, investigation, problem-solving, experimental inquiry, and invention. Several researchers took into account the Marzano’s taxonomy to define higher-order thinking in their studies (i.e., [13], [14]). Besides, SOLO taxonomy [8] claims that the last two stages of their five stages are categorized into higher-order thinking, namely relational (i.e., outline, analyze, classify, contrast, and categorize) and extended abstract (i.e., create, synthesize, hypothesize, and predict). The taxonomy was initially designed to evaluate students’ responses and their enterprise during assessments. Therefore, the quality of learning could be determined by observing students’ performance. Several researchers used the SOLO taxonomy to define higher-order thinking in their studies (i.e. [15]–[17]).

Several researchers also defined higher-order thinking skills based on their purposes and contexts. For instance, Lo and Feng [18] referred higher-order thinking skills to creative thinking, critical thinking, problem-solving ability, and reflective thinking in the context of teaching gifted students. In addition, Richland and Begolli [19] considered analogical reasoning as part of higher-order thinking skills. Meanwhile, Resnick [20] expressed that higher-order thinking skill characteristics include non-algorithmic, complex in nature, multiple solutions, variations in decision making and interpretation, applications of multiple criteria, and effortful. Besides, Bruer [21] highlighted higher-order thinking skills as integrating a broad domain knowledge and meta-cognition to deal with unfamiliar problems. What is more, by considering Ausubel’s learning theory, Ivie tried to point out three associated criteria of higher-order thinking, namely, the application of an abstract system for thinking, management of information into a unified structure, and the employment of logic and evaluation [22].

It seems that literature does not deliver a common agreement related to the definition of higher-order thinking skills as it tends to be unclear and inconsistent. Resnick [20] argues that defining higher-order thinking skills is not an easy task. Nevertheless, the aforementioned researchers have a common argument that higher-order thinking requires sophisticated cognitive enterprise, such as generating multiple solutions, synthesizing knowledge, solving problems, reasoning, and others. Besides, most of the previous studies took into account different aspects to describe higher-order thinking skills. Explaining higher-order thinking skills should consider the aspect of skill or ability and other aspects, such as situations or environment and outcome.

To the authors’ best knowledge, few publications can be found in the literature that describe various aspects required for increasing students’ higher-order thinking skills. Therefore, this study aims to develop an overarching framework to portray the characteristics of problems for promoting higher-order thinking skills in mathematics learning. The research question of the present study is: "What are substantial characteristics of problems for developing higher-order thinking skills in mathematics learning?" The current study contributes to our knowledge by addressing theoretical and practical issues. The study could have a theoretical contribution to the literature of higher-order thinking skills as it tries to propose a general conception of higher-order thinking skills by considering numerous studies. Practically, the result of the study could be used by researchers or mathematics educations for developing mathematical tasks to measure or enhance students’ higher-order thinking skills.

2. Method

Employing a multi-search tool, such as google scholar and ERIC, we applied search terms: higher-order thinking, taxonomy, and high-level thinking. Relevant sources were compelled for further analysis, and a few sources were attained by snowballing via listed reference. There were primary and secondary sources used in this study since there was considerable literature related to higher-order thinking skills.

A content analysis of the collected sources was carried out, and it resulted in classifying sources into several categories. Each collected source was analyzed based on how the term of higher-order thinking was defined and explicated. We considered various research methods, such as literature reviews,
descriptive surveys, experimental design, design research approach, etc. The present study involved three researchers who have long-term experiences in the field of mathematics education.

Regular online meetings were conducted involving three researchers within two months. Before the meeting, each researcher thoroughly reviewed and analyzed relevant materials. Each researcher then presented and shared the analysis results during the meeting and received several questions and feedback from other researchers. Discussion and reflection were conducted together, focusing on describing characteristics of problems for developing higher-order thinking skills. At the end of the meeting, each author presented their categories, and similar categories were determined. After the similar categories were confirmed, we arrived at a mutually agreed decision that there are ten substantial characteristics of problems for promoting students’ higher-order thinking skills in mathematics learning.

3. Results and Discussion

Based on the analysis results, we formulated ten substantial characteristics of problems required for developing students’ mathematics higher-order thinking skills. Each of the characteristics will be described and supported by relevant literature.

3.1. Non-routine problems.

This type of problem was described briefly as a problem that has not been presented before, and it tends to be more difficult and complicated than the routine one. Routine problems are problems that students could deal with effortlessly, while non-routine problems are problems that students are challenging to cope with and require certain skills in solving them. This definition is consistent with what has been illustrated by Lewis and Smith [1], that non-routine refers to a problem whose procedures or strategies used to reach the intended solution are unfamiliar for students. The solution to non-routine problems is not simple as the problems contain curiosity and confusion. Therefore, it necessitates a high-level of thinking, including creativity, reasoning, and critical thinking, to deal with non-routine problems. An example of a non-routine problem is a real-world problem. The teaching and learning process in mathematics should integrate this type of problem to develop students’ higher-order thinking skills.

3.2. Problems that activate students’ previous and current knowledge and students are required to analyze or synthesize the knowledge and relate them to new situations (contexts).

This characteristic ties well with prior studies in which higher-order thinking skills occur when students develop an ability to receive new information, build relations between new and prior knowledge, and apply this combination of knowledge to solve a complicated problem [23] [24]. The authors found that the SOLO taxonomy is aligned with the results of the present study. In the SOLO taxonomy, we also found that the last two levels, to wit: relational and extended abstract, could be associated with this type of problem. Having sufficient prior knowledge assists students in applying higher-order thinking to deal with unfamiliar problems. According to King, Goodson, and Rohani [25], higher-order thinking skills are related to prior knowledge. Besides, taking previous, new knowledge and the context at hand into consideration and making linkage among them are critical in higher-order thinking as it could form rules that contribute to successful problem solving [26].

3.3. Problems that require students to generate and test hypotheses.

In mathematical learning, generating and testing hypotheses are part of higher-order thinking skills. It is supported by Lewis and Smith’s [1] study, which reveals that generating and testing hypotheses are part of constructing an argument in critical thinking. Underbakke, Borg, and Peterson [27] add that generating and testing hypotheses are crucial aspects of the process of problem-solving. Providing opportunities for students to generate hypotheses and prove the truth of the proposed hypotheses also might help them learn meaningfully. Therefore, students are required to propose plausible hypotheses, establish strategies to evaluate these hypotheses, and revise their initial hypotheses when inappropriate.
3.4. Problems that measure creativity, namely, inventions, formulations, or generalizations.
There is considerable literature relating creativity to higher-order thinking [28]. Creative thinking is best described as creating novel ideas of intradisciplinary or interdisciplinary knowledge, utilizing or deliberately breaking with existing symbolic rules and procedures [29]. Students sometimes encounter difficulty in problems that demand them to create novel ideas [4]. Creativity consists of three substantial factors, namely, synthesis, articulation, and imagination [30]. Creative thinking might occur when students develop different strategies to solve mathematical problems or come up with different plausible solutions. Considering the definition and factors mentioned earlier, problems that encourage students to activate their creativity could enhance their higher-order thinking skills.

3.5. Problems that combine and use knowledge, understanding, application of synthesis, or analysis in evaluating or making judgments.
It is in line with the explanation of higher-order thinking based on Bloom’s taxonomy. According to the taxonomy, students’ ability to analyze, synthesize, and evaluate is associated with higher-order thinking. Besides, problems that require students to decide the validity of the statement given are categorized as problems of higher-order thinking skills. Lower-level thinking should not be neglected as it is a prerequisite, and along with content and context, it would influence their cognitive development and help students enhance their higher-order thinking skills [25]. Also, in making a valid judgment, students are required to be critical thinkers. This type of problem might ask students to evaluate the process or solution of problems or mathematical concepts given whether it makes sense or not.

3.6. Problems that measure the ability to think reflective and metacognitive.
Metacognition is commonly expressed as thinking about thinking [31]. Metacognitive strategies are considered as complex thinking. According to Marzano’s taxonomy, it is beyond the cognitive system [7]. It is not a single concept, yet it consists of two broad dimensions, namely, knowledge of cognition and regulation of cognition. Also, metacognition is a fundamental aspect of developing critical thinking, which is part of higher-order thinking [32]. Besides, Barak and Shakhman [33] add that problems that encourage students to carry out reflection could foster thinking ability. Students’ metacognitive and reflective thinking skills could be amplified by presenting them with an environment that enables them to regulate their thinking process during problem-solving. Students could regulate their thinking by presenting questions, such as comprehension, connection, strategic, and reflection questions [34].

3.7. Problems that require students to be persistent, flexible, and open.
This characteristic is based on King, Rohani, and Goodson's [35] explanation that students are required to be assisted in developing their persistence, flexibility, and open-mindedness. Even though there is an evident relationship between higher-order thinking skills and difficulty, most higher-order thinking skill problems tend to have a high level of difficulty. Therefore, to get to grips with this difficulty, students should pay more attention, have high determination, open their minds, and have flexibility. Besides, the development of students’ higher-order thinking should be viewed in many aspects; apart from the cognitive aspect, it is necessary to pay attention to affective aspects. However, few studies describe the role of the affective aspect of higher-order thinking skills. The results of the present study also suggest that it is necessary to investigate the affective aspects required for students to have in dealing with difficult mathematical problems.

3.8. Divergent problems that allow students to provide various answers according to the thinking process and point of view used.
According to Kwon, Park, and Park [36], students’ divergent thinking could be cultivated through open-ended problems. Divergent problems are commonly interpreted as open-ended problems. Most mathematics teachers tend to consider closed-ended questions in their classroom evaluations. It might lead to impeding them in investigating different ideas. Therefore, teachers should help students develop their thinking by proposing questions or problems that might be solved by various strategies and
generate multiple solutions. Open-ended problems enable students to improve their critical and creative thinking. That is why an open-ended test could be employed to investigate students’ higher-order thinking [5], [36].

3.9. Problems that require students to find answers using multiple representations.
Mathematics is a subject that has a rich representation. There are internal and external representations. The external representation consists of visual forms, such as tables, graphics, symbols, etc. In dealing with mathematical problems, generally speaking, students use various strategies, one of which is drawing strategy. Using this strategy, students consider various visual representations, choose the appropriate ones, and construct them properly. Also, it requires students to move from one type of external representation to another, which is called flexibility [37]. In this case, students are led to activate their thinking activities, such as analysis, evaluation, and creation. The relation of multi-representational ability and higher-order thinking skills were portrayed explicitly by several studies (i.e. [38] and [23]).

3.10. The problem given requires the ability to relate, interpret, apply, and integrate knowledge in classroom learning to solve problems in a real context.
The last substantial characteristic of problems for developing students’ higher-order thinking skills is the use of real-world problems. Real-world problems are commonly used in mathematical modeling activities. To cope with real-world problems, students are required to (i) comprehend, breaking down, and portraying context, (ii) assume, define, and construct mathematical model, (iii) implement mathematical strategies, (iv) interpret the obtained result, (v) evaluate the result and revise, and (vi) communicate [39]. Henceforth, finding solutions to real-world problems requires multiple thinking activities referred to a high level of thinking. Also, the literature showed that real-world problems or authentic problems, which are an essential part of project-based learning, might assist in promoting students higher-order thinking skills [40].

4. Conclusion
The purpose of the current study was to formulate a definition of higher-order thinking skills and describe several substantial characteristics of problems for increasing students’ higher-order thinking skills in mathematics learning. One of the most significant findings from this study is that we found and propose ten substantial characteristics of higher-order thinking skill problems that could be applied to enhance students’ higher-order thinking skills. Attention was concentrated not only on proposing the ten substantial characteristics but also on defining higher-order thinking skills. When defining higher-order thinking skill problems, it should consider three aspects: environment, skill, and outcome. Therefore, in this study, we propose a description of higher-order thinking skills as an ability to use multi-dimensional skills in situations that make students challenging to come up with or transform concepts and theorems to the relevant situation, resulting in outcomes not obtained from students’ previous knowledge.

The current findings might add to a growing body of literature on higher-order thinking skills. Further research will be required to validate these ten substantial characteristics empirically. Besides, these ten substantial characteristics might be used as a framework for assessing and developing students’ higher-order thinking skills in mathematics learning.

5. Acknowledgment
The authors gratefully acknowledge the generous financial support of the Faculty of Mathematics and Natural Science, Universitas Negeri Jakarta.

References
[1] Lewis A and Smith D, Jun. 1993 Defining higher order thinking Theory Pract. 32, 3 p. 131–137
[2] Zohar A and Alboher Agmon V, Apr. 2018 Raising test scores vs. teaching higher order thinking (HOT): senior science teachers’ views on how several concurrent policies affect classroom
practices Res. Sci. Technol. Educ. 36, 2 p. 243–260.

[3] Herbert S and Pierce R, 2004 Gifted are lifted higher: An exploration of the development of higher order thinking skills of gifted students playing strategy games TalentEd 22, 1 p. 22–30

[4] Heong Y M, Yunos J M, Othman W, Hassan R, Kiong T T and Mohamad M M, Oct. 2012 The Needs Analysis of Learning Higher Order Thinking Skills for Generating Ideas Procedia - Soc. Behav. Sci. 59 p. 197–203

[5] Tanujaya B, Mumu J and Margono G, 2017 The Relationship between Higher Order Thinking Skills and Academic Performance of Student in Mathematics Instruction. Int. Educ. Stud. 10, 11 p. 78–85

[6] Anderson L W and Krathwohl D R, 2001 A revision of Bloom’s taxonomy of educational objectives A Taxon. Learn. Teach. Assessing. Longman, New York.

[7] Marzano R J and Kendall J S, 2006 The new taxonomy of educational objectives Corwin Press

[8] Biggs J and Collis K, 1982 The Structure of Observed Learning Outcomes (SOLO) Taxonomy Teach. Learn. Support.

[9] Anderson L W and Krathwohl D R, 2001 A taxonomy for learning, teaching, and assessing: A revision of Bloom’s taxonomy of educational objectives Theory Pract. Complete e p. xxix 352

[10] Agarwal P K 2019 Retrieval practice &amp; Bloom’s taxonomy: Do students need fact knowledge before higher order learning? J. Educ. Psychol. 111 2 189–209

[11] Karim F A and Marzita P, 2019 The Development of Higher Order Thinking Skills (HOTS) Assesment Instrument for Word Problems Int. J. Acad. Res. Bus. Soc. Sci. 9 6 1079–1083

[12] Jensen J L McDaniel M A Woodard S M and Kummer T A 2014 Teaching to the Test…or Testing to Teach: Exams Requiring Higher Order Thinking Skills Encourage Greater Conceptual Understanding Educ. Psychol. Rev. 26 2 307–329

[13] Dubas J M and Toledo S A, 2016 Taking higher order thinking seriously: Using Marzano’s taxonomy in the economics classroom Int. Rev. Econ. Educ. 21 12–20

[14] Reyes-Lozano C A Meda-Campaña M E and Morales Gamboa R 2015 Flipped classroom as educational technique to teach Math on a competencies-based Approach: Case study Conf. LACLO 5 1

[15] Bhagwat M S 2017 Developing and Implementing Instructional Strategy on the Structure of Observed Learning Outcomes (SOLO) Taxonomy for Mathematics of Class–IX. Maharaja Sayajirao University of Baroda (India)

[16] Wells C 2015 The structure of observed learning outcomes (SOLO) taxonomy model: How effective is it?

[17] Lucander H Bondemark L Brown G and Knutsson K, 2010 The structure of observed learning outcome (SOLO) taxonomy: a model to promote dental students’ learning Eur. J. Dent. Educ. 14 3 p. 145–150

[18] Lo C O and Feng L-C 2020 Teaching higher order thinking skills to gifted students: A meta-analysis Gift. Educ. Int. 36, 2 p. 196–217

[19] Richland L E and Begolli K N 2016 Analogy and Higher Order Thinking: Learning Mathematics as an Example Policy Insights from Behav. Brain Sci.

[20] Resnick L B 1987 Education and learning to think (Washington, DC: National Academy Press)

[21] Bruer J T, 1993 Schools for thought 234 (Cambridge, MA: MIT Press)

[22] Ivie S D 1998 Ausubel’s learning theory: An approach to teaching higher order thinking skills High Sch. J. 82 1 35–42

[23] Misrom N B, Muhammad A, Abdullah A, Osman S, Hamzah M and Fauzan A 2020 Enhancing students’ higher-order thinking skills (HOTS) through an inductive reasoning strategy using geogebra Int. J. Emerg. Technol. Learn. 15 3 156–179

[24] Yee M H, Yunos J M, Othman W, Hassan R, Tee T K and Mohamad M M 2015 Disparity of Learning Styles and Higher Order Thinking Skills among Technical Students Procedia - Soc. Behav. Sci. 204 143–152
[25] King F J, Goodson L and Rohani F 1998 Higher order thinking skills: Definition, teaching strategies, assessment Florida: www.cala.fsu.edu.
[26] Gagné R M, Briggs L J and Wager W W 1988 Instructional design (New York: Rinehart Winst. Inc.)
[27] Underbakke M, Borg J M and Peterson D 1993 Researching and developing the knowledge base for teaching higher order thinking Theory Pract. 32 3 138–146
[28] Alkhathib O J 2019 A Framework for Implementing Higher-Order Thinking Skills (Problem-Solving, Critical Thinking, Creative Thinking, and Decision-Making) in Engineering & Humanities in 2019 Advances in Science and Engineering Technology International Conferences (ASET) 1–8
[29] Czikszentmihalyi M 1996 Creativity: Flow and the psychology of discovery and invention (New York)
[30] Sternberg R J 2003 Creative thinking in the classroom Scand. J. Educ. Res. 47 3 325–338
[31] Pableontiou-louca E 2003 The concept and instruction of metacognition Teach. Dev. 7 1 9–30
[32] Magno C 2010 The role of metacognitive skills in developing critical thinking Metacognition Learn. 5 2 137–156
[33] Barak M and Shakhman L 2008 Fostering higher-order thinking in science class: teachers’ reflections Teach. Teach. theory Pract. 14 3 191–208
[34] Mevarech Z and Fridkin S 2006 The effects of IMPROVE on mathematical knowledge, mathematical reasoning and meta-cognition Metacognition Learn. 1 1 85–97
[35] King F J, Rohani F and Goodson L 1997 Statewide assessment of listening and verbal communication skills, information literacy skills, and problem-solving skills (Tallahassee: Florida State University)
[36] Kwon O N, Park J H and Park J S 2006 Cultivating divergent thinking in mathematics through an open-ended approach Asia Pacific Educ. Rev. 7 1 51–61
[37] Aziz T A and Kurniasih M D 2019 External Representation Flexibility of Domain and Range of Function Journal on Mathematics Education 10 1 143–156
[38] Tajudin N M and Chinnappan M 2016 The Link between Higher Order Thinking Skills, Representation and Concepts in Enhancing TIMSS Tasks Int. J. Instr. 9 2 199–214
[39] Stillman G 2011 Applying metacognitive knowledge and strategies in applications and modelling tasks at secondary school, in Trends in teaching and learning of mathematical modelling, (Springer), p. 165–180
[40] Billah A, Khasanah U and Widoretno S, 2019 Empowering higher-order thinking through project-based learning: A conceptual framework in AIP Conference Proceedings 2194 1 20011