Study of vertical milling machine layout in terms of rigidity

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Abstract. The paper explores the signs of static elastic deformation when operating milling machines. The research is concerned with the issue of simulation of a machine-device-tool-workpiece system for preliminary evaluation of elastic deformation on treatment in order to eliminate rejects. Linear and angular deformations were defined. Total rigidity of a milling machine depends not only on its inherent stiffness and contact rigidity of its components, but also on geometrical characteristics of manufacturing system layout. These characteristics could be evaluated through axes of rigidity. Errors in surface machining depend on both tool deflection and workpiece distortion. That is why it is necessary to analyze workpiece and tool load-deformation curve separately. Especially that their rigidity is determined by different layouts of bearing system. Main deformation values are defined and vectors of axes of rigidity are created. Using the results of the analysis it is possible to select a cutting pattern or change manufacturing system layout so that the resultant of cutting forces comes closer to highest rigidity axis, which reduces elastic deformations and improves working accuracy.

1. Introduction

1.1. The subject in today’s machine-building industry
Improvement of accuracy of machined parts requires close consideration of all factors that could affect working accuracy [1-4]. In the context of real-life machining, rigidity of the entire manufacturing system should be considered [5-7]. Total rigidity of a milling machine depends not only on its inherent stiffness [8, 9] and contact rigidity of its components [10, 11], but also on geometrical characteristics of manufacturing system layout. These characteristics could be evaluated through axes of rigidity. As shown in [12], at any particular point in a body there are three orthogonally-related axes where linear deformations display extreme values. These axes are called principal axes of strain and linear deformations along principal axes are called main deformations.

1.2. Problem statement
Within the topic, the axes of highest and lowest rigidity, along which the elastic deformations have minimal and maximal values respectively, hold a lot of significance. The importance of these axes is based on the following notions [13]:

1. If the manufacturing system layout is organized in such a way that the resultant of cutting forces is equal or close to the axis of highest rigidity, then deformations would be the lowest and elastic strain-related working errors would be minimized.
2. Axes of rigidity of main deformations have such properties that if the resultant of forces is applied to them lengthwise it produces only linear deformations along these axes with no angular deformations. As a result, the elastic strain is minimized.

3. Alignment of resultant of cutting forces with axes of rigidity increases dynamic stability of the manufacturing system.

2. Determination of stiffness axes

In the studies of rigidity of milling machines, the researchers use a method involving either a full-scale or a simulation experiment. All that requires a lot of time and, as in the first case, involves material costs. This paper offers a theoretical approach to finding position of elastic axes. As shown in [12], when any load is applied to a body, its linear and angular deformations form a deformation tensor:

\[
\begin{bmatrix}
\varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\
\varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\
\varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33}
\end{bmatrix}
\]

(1)

where \( \varepsilon_{ij} \) – linear and angular deformations.

Values of main deformations \( \varepsilon_i \) could be found from cubic equation [3]:

\[
\varepsilon^3 - I_1 \varepsilon^2 + I_2 \varepsilon + I_3 = 0
\]

(2)

where \( I_i \) are deformation tensor invariants (1).

Deformation components of equation (1) could be found through a single load experiment or a finite element analysis of the manufacturing system [14]. Manufacturing system layout corresponded to an actual case of cavity milling. Linear and angular deformations were identified through an estimation.

Errors in surface machining depend on both tool deflection and workpiece distortion. That is why it is necessary to analyze workpiece and tool load-deformation curve separately. Especially that their rigidity is determined by different layouts of bearing system.

2.1. Determination the axes of rigidity for a curve of a tool

Solution to the equation (2) has provided the values of main deformations.

\[
\varepsilon = \begin{bmatrix}
-0.003 \\
-0.001 \\
0.003
\end{bmatrix}
\]

(3)

Components \( \varepsilon_1 \) and \( \varepsilon_3 \) conform with the highest main deformations, their direction corresponds to the lowest rigidity axis; component \( \varepsilon_2 \) conforms with the lowest main deformation, it corresponds to the axis of highest rigidity.

Directions of axes of main deformations could be calculated through directional cosines \( l_i, m_i, n_i \) with simultaneous solution of the set of equations [15]:

\[
\begin{align*}
(\varepsilon_i - \varepsilon_{11}) l_i - \varepsilon_{12} m_i - \varepsilon_{13} n_i &= 0 \\
-\varepsilon_{12} l_i + (\varepsilon_i - \varepsilon_{22}) m_i - \varepsilon_{23} n_i &= 0 \\
-\varepsilon_{13} l_i + \varepsilon_{23} m_i - (\varepsilon_i - \varepsilon_{33}) n_i &= 0 \\
l_i^2 + m_i^2 + n_i^2 &= 1
\end{align*}
\]

(4)
Based on the estimated values vectors of axes of rigidity are created ‘figure 1’. Axis, along which the deformations are highest, is the axis of lowest rigidity \( j_{\text{min}} \); axis, along which the deformations are lowest, is the axis of highest rigidity \( j_{\text{max}} \).

![Figure 1. Axes of rigidity of a tool branch.](image)

2.2. Determination the axes of rigidity for curve of a workpiece

Solution to the equation (2) has provided the values of main deformations.

\[
\varepsilon = \begin{bmatrix}
-0.001 \\
-0.001 \\
0.002
\end{bmatrix}
\]  

(5)

Components \( \varepsilon_3 \) conform with the highest main deformations, their direction corresponds to the lowest rigidity axis; component \( \varepsilon_1 \) and \( \varepsilon_2 \) conforms with the lowest main deformation, it corresponds to the axis of highest rigidity.

Based on the estimated values vectors of axes of rigidity are created ‘figure 2’.

Using the stiffness axes defined above, especially the characteristics of their arrangement expressed by the guide cosines, two tasks can be solved:

1. Performing the layout of the technological system so as to bring the axis of stiffness to the resultant cutting forces, thereby reducing the deformation in the technological system [16].

2. Modification the parameters of the cutting modes so that the resultant cutting force is close to the axes of rigidity of the unchanged technological system [17-19].

Next, we will consider the solution of the second task.
3. Determination of optimal cutting conditions
In this paper, we are interested in the effect of deformations on the error of the treated surface. From this point of view, the most important projection of deformation on the normal to the treated surface:

\[ y_l = \begin{bmatrix} \frac{P(S_z,t,B) \cos(\eta_1(S_z,t,B))}{f_1} \\ \frac{P(S_z,t,B) \cos(\eta_2(S_z,t,B))}{f_2} \\ \frac{P(S_z,t,B) \cos(\eta_3(S_z,t,B))}{f_3} \end{bmatrix} \cdot \begin{bmatrix} \cos(\theta n_1) \\ \cos(\theta n_2) \\ \cos(\theta n_3) \end{bmatrix} \]  

(6)

where

\[ \cos(\eta_1(S_z,t,B)) = ca(S_z,t,B) \cos(\alpha_1) + cb(S_z,t,B) \cos(\beta_1) + cg(S_z,t,B) \cos(\gamma_1) \]  

(7)

\[ \cos(\eta_2(S_z,t,B)) = ca(S_z,t,B) \cos(\alpha_2) + cb(S_z,t,B) \cos(\beta_2) + cg(S_z,t,B) \cos(\gamma_2) \]  

(8)

\[ \cos(\eta_3(S_z,t,B)) = ca(S_z,t,B) \cos(\alpha_3) + cb(S_z,t,B) \cos(\beta_3) + cg(S_z,t,B) \cos(\gamma_3) \]  

(9)

where \( S_z \) – the feed per tooth; \( t \) – cutting depth; \( B \) – width of cut; \( P \) – cutting force; \( \cos(\eta_1), \cos(\eta_2), \cos(\eta_3) \) – direction cosines for the projection of the cutting force on the axle rigidity; \( \theta n_i \) – the angle between the axes of rigidity and the surface normal the angle of rotation of the point of
application of resultant vector; \( \cos(\alpha_1) \), \( \cos(\beta_1) \), \( \cos(\gamma_1) \) – direction cosines for the projection of the components of the cutting forces on the axis of rigidity.

Deformation is defined as a function of the angles of the resultant cutting force relative to the axes of stiffness. These angles are also functions of the cutting forces:

\[
ca(S_z,t,B) = \frac{Ph(S_z,t,B)}{P(S_z,t,B)}
\]

\[
cb(S_z,t,B) = \frac{P\vartheta(S_z,t,B)}{P(S_z,t,B)}
\]

\[
cg(S_z,t,B) = \frac{Px(S_z,t,B)}{P(S_z,t,B)}
\]

where \( Ph, Pv, Px \) – projections of the components of the cutting force on the machine axis \( x \), \( y \) and \( z \), respectively.

\[
Ph(S_z,t,B) = \sum Pz \cos(\varepsilon) + \sum Py \sin(\varepsilon)
\]

\[
P\vartheta(S_z,t,B) = \sum Pz \sin(\varepsilon) + \sum Py \cos(\varepsilon)
\]

\[
P(S_z,t,B) = \sqrt{Ph(S_z,t,B)^2 + P\vartheta(S_z,t,B)^2 + Px(S_z,t,B)^2}
\]

where \( Pz \) – tangential cutting force, summed on all working edges of the cutter; \( Py \) – radial cutting force, summed on all working edges of the cutter; \( \varepsilon \) – the angle position of the cutting edge.

On the basis of the proposed model in equations (6) – (15), static elastic deformations are calculated on the example of groove milling on the part [20].

Figure 3. The calculated strain values.

From the analysis of the graphs ‘figure 3’, it can be seen that, in general, with increasing cutting conditions, cutting forces and static elastic deformations increase. However, with some combination of cutting modes (in this case \( t = 2.5 \text{ mm}, B = 0.5 \text{ mm} \)), static elastic deformations are reduced. This decrease is explained by the fact that in this combination of cutting modes, the resultant cutting forces approach the axis of rigidity. ‘Figure 4’ shows that in the above cutting modes, the resulting cutting force is closest to the maximum stiffness axis.
Thus, the relationship between the location of the resultant cutting forces and static elastic deformation is visible. The solution of this mathematical model was found optimal cutting conditions when milling this part element: \( t = 2.5 \text{ mm}, B = 0.5 \text{ mm} \) - for roughing; \( t = 1 \text{ mm}, B = 0.2 \text{ mm} \) - for finishing.

4. Experimental study

The experiment was performed in the form of milling grooves with different combinations of width and depth of cut. The experiment was carried out on a vertical milling machine JMD 3CNC, end mill diameter 4 mm, material of the workpiece is steel 45, material of the mill is steel R6M5.

After the experiment, the deviation from the machined surface and the width of the machined grooves were measured directly on the machine.

From the analysis of the results of the experiment, it can be seen that static elastic deformations affect the change in the size of the groove. The deviations of the groove walls are not the same. Deviations of the wall, when milling which the cutting force is closer to the axis of maximum
stiffness, more responsive to changes in cutting conditions. The deviations of this wall depending on the cutting modes are shown in the ‘figure 5’.

As can be seen, the experimental data are close to the calculated data, the difference averaged 5.5%, which confirms the adequacy of the developed model. Thus, this model can be used to optimize cutting conditions to minimize static elastic deformations.

5. Conclusion
Application of the suggested approach enables to determine the position of axes of rigidity with minimal effort. Using this result, it is possible to select a cutting pattern or change manufacturing system layout so that the resultant of cutting forces comes closer to highest rigidity axis, which reduces elastic deformations and improves working accuracy.

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