**ABSTRACT** This article explores the problems of robust information fusion for wireless sensor networks with state delay, parameter uncertainty, and communication constraints. Based on a data-driven transmission strategy, the robust fusion estimator proposed in this article can greatly reduce the possibility of network congestion, while ensuring the accuracy of the estimation fusion. The uncertainty of random parameters in the model is not limited to special forms, which means that this estimator is applicable to a wide range of situations. The pseudo cross covariance matrix used for estimation fusion is derived by local robust state estimation algorithm, which is based on state augmentation and expectation minimization of estimation errors. Furthermore, we prove that the error variance of this fusion estimator is uniformly bounded, and the corresponding condition is given. According to this condition, the selection of key matrices and vectors in the transmission strategy is determined. Finally, some numerical simulations are utilized to verify the performance of this algorithm.

**INDEX TERMS** Robust fusion estimation, wireless sensor networks, state delay, parameter uncertainty, data transmission strategy.

**I. INTRODUCTION**

With the development of industrial production and control technology, state estimation technology and wireless sensor networks (WSNs) technology complement each other, and are widely used in various industries [1], [2]. Compared with a single sensor, a network of multiple sensors can obtain higher quality information, which means that the estimation performance of fusion estimator is more accurate. In wireless sensor networks, the energy consumption of sensors during data transmission accounts for a considerable part of the total energy consumption. Frequent information exchange and many unnecessary transmissions will directly cause energy waste and transmission congestion [3]. Therefore, how to overcome the above problems as much as possible while ensuring the filtration performance is particularly important. Data technology aims to make full use of large amounts of data to obtain useful information, which is of great significance for saving transmission energy. Therefore it is widely used in control system design [4], [5], and the staged results of this technology are summarized in [6]. Common fusion estimation structures are divided into three types: centralized, distributed, and hybrid [7]. The centralized fusion method is easy to obtain the global optimal state estimation. But its frequent data transmission runs counter to the desire to save energy and reduce the possibility of network congestion. Distributed fusion estimation with communication between sensor nodes is more flexible, and it has always been a hot
spot in fusion research. Under this structure, there have been many achievements for estimation fusion and track fusion (see [11], [12]). For the former, the common state weighting methods include: covariance intersection (CI) (see [8], [9]), sequential covariance intersection (SCI) [10], vector weighting, scalar weighting and matrix weighting (see [13], [14]). In [15], a relatively simple distributed fusion estimation method is advanced, which uses the weighted average of adjacent node data to update each node’s own data. A strategy based on a send-on-delta method is proposed in [16], which expresses that only when the sensor data change exceeds a fixed value, the data will be transmitted. In [17], the authors put forward a similar transmission strategy correlated to the transmission probability of each node, which adjusts the relationship between communication cost and estimated performance. In [18], by minimizing the weighting functions of transmission energy consumption and times, the author proposes a global event-triggered communication strategy to achieve the purpose of saving energy.

From the perspective of system theory, the past state of actual system will undoubtedly affect the current state [19]. In recent years, for the filtering problem of time-delay systems, the existing research results are very rich [20]. In addition to common methods such as state augmentation, linear matrix inequalities (LMI) [21] and partial differential equations (PDE) [22] are often used to solve such problems. In view of the existence of communication delay, packet loss and variable delay in sensor networks, many theoretical results on fusion estimation of time-delay systems have emerged, see [23], [24], and [25]. For the systems with random transmission delay, a distributed fusion estimator based on event trigger is proposed in [8]. In [26], an online selective linear fusion method is used for fusion estimation in the presence of data delay or loss in the system. But most of the existing achievements depend on an assumption: the system model can be measurable or the model errors characteristics are known. In actual production, this assumption has great limitations.

In practical application, model errors are inevitable and have a huge impact on the filtering effect. There have been numerous research results on the robustness of linear and nonlinear systems (see [27], [28]). For linear systems, Riccati difference equation [29], innovation analysis approach, and LMI [30] were used to analyze the system with uncertain noise in the state matrix and the observation matrix. Based on existing relevant conclusions, many filtering methods for time-delay uncertain systems have also emerged. But most of them are aimed at systems in specific situations, such as the variance of the model errors can be determined [31], or the parameter uncertainty caused by correlated additive or multiplicative noises [32]. In [33], using the meaning of linear minimum variance, scholars derive a robust fusion Kalman filter for multisensor systems with random time delay and packet loss, but its model uncertainty only exists in the state matrix. In [33], the author proposes a filter based on regularized least squares (RLS) and estimation errors sensitivities, which overcomes the limitations in [31] and [32], and has been innovatively applied to fusion estimation in [34]. In [36], a filter based on expectation minimization of estimation errors is proposed, which provides a new idea for solving the problem of random uncertainty of system model errors.

To solve the fusion estimation problem of discrete linear wireless sensor networks with state delay and uncertain model errors while taking the data transmission frequency into account, a robust fusion algorithm was proposed. In this article, the robust estimator based on state augmentation and the expectation minimization of estimation error is used in the multisensor networks with state delay. We derive the analytical expression of this robust fusion estimator based on a data-driven strategy. It is worth noting that when the system model errors affect the system parameters in arbitrary way, the fusion estimator is still applicable. Therefore, it has a wide range of applications. Some conditions are proposed to guarantee that the estimation error covariance matrix of the proposed estimator is bounded, and the relevant proof is given. It is reflected by some numerical simulations that the estimator can maintain the outstanding performance. The following description is organized into five parts. In Section 2, we introduce the problems to be solved and the data-driven transmission strategy. In Section 3, the robust state estimation algorithm for local sensors is introduced, and then in conjunction with the transmission strategy, the covariance matrix for information fusion is also obtained in this section. In Section 4, the steady-state characteristics and conditions of this estimator are given. Some numerical simulation examples reflect the performance of the estimator are shown in Section 5. Finally, we summarize the full paper in Section 6.

In this article, we use many standard symbolic representations: the notation $E[a]$ stands for the mathematical expectation of matrix $a$, $\mathbb{R}^n$ represents $n$-dimensional Euclidean space, $N(\cdot; \lambda, \Sigma)$ denotes a Gaussian distribution probability density function with mean $\lambda$ and variance $\Sigma$. $\|\|_W$ stands for the Euclidean norm of weighted coefficient $W$, $\text{diag} \{ a, \ldots, b \}$ denotes the block-diagonal matrix with matrices $\{a, \ldots, b\}$, $\text{tr} \{\ast\}$ and $\text{col} \{\ast\}$ stand for the trace of a matrix and the stacking vector or matrix respectively.

II. PROBLEM FORMULATION AND PRELIMINARY KNOWLEDGE

Consider the following linear discrete-time uncertain stochastic system with $d$ steps state delay composed of $K$ sensors:

$$
\begin{gathered}
x_{i+1} = A_{1i}(\varepsilon_i) x_i + A_{2i}(\varepsilon_i) x_{i-d} + B_i(\varepsilon_i) w_i, \quad d > 0, \\
y_i' = C_i^l(\varepsilon_i) x_i + v_i', \\
l = 1, 2, \ldots, K,
\end{gathered}
$$

(1)

where $i$ is discrete time, $x_i \in \mathbb{R}^n$ is the state vector, $w_i$ is the process noise, $y_i' \in \mathbb{R}^m$ and $v_i' \in \mathbb{R}^{n'}$ are the measurement output and measurement error of $l$-th sensor node, respectively. $d$ is a positive integer, and it indicates the system lag time. $A_{1i}(\varepsilon_i)$, $A_{2i}(\varepsilon_i)$, $B_i(\varepsilon_i)$, and $C_i^l(\varepsilon_i)$ are matrices with corresponding dimensions of model parameter errors $\varepsilon_i$.
Their dimensions are \( n \times n, n \times n, n \times n, \) and \( m \times n. \) For System (1), we make two assumptions as follows:

**Assumption 1:** \( w_i \) and \( v_i \) are Gaussian white noises with zero mean and variance matrices \( Q_i \) and \( R_i', \) respectively. The initial condition \( x_0 \) is a random vector independent of noises and errors, which meet the following relationship:

\[
E \left( (x_0 - E(x_0)) (x_0 - E(x_0))^T \right) = \Pi_0,
\]

where \( \Pi_0 \) is a known positive definite matrix.

**Assumption 2:** The model parameter errors \( \varepsilon_i \) changes with time \( i, \) consists of \( I \) real-valued scalar-bounded uncertainties \( \varepsilon_{ij}, j = 1, \ldots, I. \) It can be seen from System (1) that the model errors in this article are not limited to a particular form, which ensures that our model has a wider range of application.

Then the problem to be solved in this article is to calculate the fusion estimate \( \hat{x}_{ij} \) based on the observation data \( \{y_i^j, l = 0\} \) in the case of limited transmission.

In order to achieve the purpose of reducing energy waste, we adopted the data-driven transmission strategy in [17]. The main idea of this strategy is as follows:

Consider a sensor observation channel:

\[
Y = H \xi + \nu.
\]

In which, \( Y \in \mathbb{R}^n \) is the output, \( H \) is the observation matrix of the corresponding dimension, \( \xi \in \mathbb{R}^n \) indicates state, \( \nu \in \mathbb{R}^n \) is the measurement noise. To reflect whether the observation data is transmitted, we define a binary variable \( \mu. \) If the sensor measurement data is transmitted to the fusion center, \( \mu \) is assigned a value of 1. Otherwise, it is assigned a value of 0. The data transmission rate of every sensor is defined as \( \alpha' \). By setting the conditions for whether the observations are transmitted, the transmission rate \( \alpha' \) is satisfied. Then the data-driven transmission strategy can be described as follows:

\[
\mu = \begin{cases} 
0, & Y - \tilde{y} \in \Xi, \\
1, & \text{others.}
\end{cases}
\]

Among them, the vectors \( \tilde{y} \in \mathbb{R}^m \) and \( \Xi \in \mathbb{R}^m \) are measurable sets. In which, the set \( \Xi \) meets that \( \int_{\Xi} \hat{y} \, d\hat{y} = 0. \) Formula (3) shows that when the measurement output \( Y \) is far from the set value \( \tilde{y}, \) \( Y \) is transmitted to the fusion center by Equation (2). On the other hand, it is easy to understand that the measurement noise will change when the data is not transmitted. According to [17], when \( \mu = 0, \) a virtual transmission is defined:

\[
z = \tilde{y} = H \xi + \nu - g,
\]

where \( g \in \Xi \) is uncorrelated to \( \xi \) and \( \nu, \) and it follows uniform distribution. Suppose \( f_\xi(x) = N(x; \bar{x}, \Sigma_x), f_\nu(v) = N(v; \bar{v}, \Sigma_v), \) then we can get \( f_y(y) = N(y; \tilde{y}, \Sigma_y) \) and \( \Sigma_y = \Sigma_v + H \Sigma_x H^T \) from Formula (2). So the transmission conditions are equivalent as follows:

\[
\|Y - H\bar{x}\|^2_{\Sigma_y^{-1}} \geq \delta
\]

\[
\delta = y_m^{-1}(1 - \alpha)
\]

Based on mathematical statistics knowledge, random variables \( \|Y - H\bar{x}\|^2_{\Sigma_y^{-1}} \) obey \( \chi^2 \) distribution with \( m \) degrees of freedom. In which \( y_m \) is a chi-square distribution function with degrees of freedom \( m. \)

### III. ROBUST FUSION ESTIMATOR BASED ON DATA-DRIVEN TRANSMISSION STRATEGY

In the presence of state delay and uncertain model errors, a robust state estimator in [20] is used to obtain the state estimation value of the local sensor nodes. This robust state estimation algorithm uses the state augmentation method to simplify the system matrix, and achieves excellent filtering effect based on the principle of minimum errors expectation. Meanwhile, several key matrices in this algorithm can be calculated off line, which makes it show considerable advantages when dealing with random model errors. It is worth noting that this method is similar in form and computational complexity to the kalman filtering. Due to the time delay in the case of limited transmission, the augmented state vector is defined as \( X_i = [x_i^T \ x_{i-1}^T \ \cdots \ x_{i-d}^T]^T, \) then System (1) can be reconstructed into the following equivalent model without state delay:

\[
\begin{align*}
X_{i+1} &= \tilde{A}_i \ X_i + \tilde{B}_i \ \nu_i, \\
y_i^j &= \tilde{C}_i^j \ x_i + v_i^j, \quad i = 1, 2 \ldots K.
\end{align*}
\]

In which,

\[
\tilde{A}_i(\varepsilon_i) = \begin{bmatrix} A_{1i}(\varepsilon_i) & 0_{n \times n} & \cdots & 0_{n \times n} & A_{2i}(\varepsilon_i) \\
I_n & 0_{n \times n} & \cdots & 0_{n \times n} & 0_{n \times n} \\
\vdots & \ddots & \vdots & \vdots & \vdots \\
0_{n \times n} & \cdots & 0_{n \times n} & 0_{n \times n} & 0_{n \times n} \end{bmatrix},
\]

\[
\tilde{B}_i(\varepsilon_i) = \begin{bmatrix} B_{1i}^T(\varepsilon_i) \\
0_{n \times d_n} \end{bmatrix}^T,
\]

\[
\tilde{C}_i^j(\varepsilon_i) = \begin{bmatrix} C_{i}^j(\varepsilon_i) & 0_{m \times d_n} \end{bmatrix}.
\]

Then the dimensions of \( \tilde{A}_i(\varepsilon_i), \tilde{B}_i(\varepsilon_i) \), and \( \tilde{C}_i^j(\varepsilon_i) \) are \( n (d + 1) \times n (d + 1), n (d + 1) \times n, \) and \( m \times n (d + 1), \) respectively. Several key matrices are defined to form recursive equations:

\[
H_{i1}' = E \left[ \begin{bmatrix} A_{1i}^T(\varepsilon_i) \\
\tilde{B}_{i}^T(\varepsilon_i) \end{bmatrix} \right] \left( \tilde{C}_{i+1}^j(\varepsilon_{i+1}) \right)^T \left( R_i^{l+1} \right)^{-1} \times \tilde{C}_{i+1}^j(\varepsilon_{i+1}) \tilde{A}_i \tilde{B}_i,
\]

\[
H_{i2}' = E \left[ \begin{bmatrix} A_{1i}^T(\varepsilon_i) \\
\tilde{B}_{i}^T(\varepsilon_i) \end{bmatrix} \right] \left( \tilde{C}_{i+1}^j(\varepsilon_{i+1}) \right)^T, 
\]

\[
\tilde{G}_i^j = \begin{bmatrix} G_{i1}^j & G_{i2}^j \\
G_{i3}^j & G_{i4}^j \\
G_{i5}^j & G_{i6}^j \\
G_{i7}^j & G_{i8}^j \end{bmatrix},
\]

\[
= H_{i1}' \left[ \begin{bmatrix} 0 \\
A_{1i} \tilde{B}_i \end{bmatrix} \right] \left( \tilde{C}_{i+1}^j(0) \right)^T \left( R_i^{l+1} \right)^{-1} \times \tilde{C}_{i+1}^j(0) \tilde{A}_i \tilde{B}_i \right].
\]
At the same time, the pseudo covariance matrix $\hat{P}_{ii}^l$ and $P_{ii}^l$ are defined here, so the local estimate $\hat{X}_{ii}^l$ can be computed by means of the following recursion:

**A. INITIALIZATION**

$$P_{00}^l = \left( \Delta_0^{-1} + E \left( \left( \bar{C}_0^l (\epsilon_0) \right)^T \left( R_0^l \right)^{-1} \bar{C}_0^l (\epsilon_0) \right) \right)^{-1},$$

$$\hat{X}_{00} = P_{00} E \left( \left( \bar{C}_0^l (\epsilon_0) \right)^T \left( R_0^l \right)^{-1} \right) y_0,$$

where $\Delta_0 = E \left( \left( X_0^l - E(X_0^l) \right) \left( X_0^l - E(X_0^l) \right)^T \right)$.

**B. PARAMETER CORRECTION**

$$\left( \hat{p}_{ii}^l \right)^{-1} = \left( P_{ii}^l \right)^{-1} + G_{i11},$$

$$\left( \tilde{Q}_i^l \right)^{-1} = \left( Q_i^l \right)^{-1} + G_{i22} - G_{i12} \hat{P}_{ii}^l G_{i12}^T,$$

$$\hat{A}_i^l = \left( A_i (0) - \tilde{B}_i \tilde{Q}_i^l \left( G_{i12} \right)^T \left( I_{n(d+1)} \right) - \hat{P}_{ii}^l G_{i11} \right),$$

$$\tilde{B}_i^l = \tilde{B}_i \left( 0 \right) - \tilde{A}_i \left( 0 \right) \tilde{P}_{ii} G_{i12},$$

**(C. STATE ESTIMATE UPDATE)**

$$P_{i+1|i}^l = \hat{A}_i^l \left( 0 \right) \tilde{P}_{ii}^l \tilde{Q}_i^l \left( 0 \right) + \hat{B}_i^l \hat{Q}_i^l \left( \hat{B}_i^l \right)^T,$$

$$\tilde{R}_{i+1} = \hat{R}_{i+1} + \tilde{C}_{i+1} \left( 0 \right) P_{i+1|i}^l \left( \tilde{C}_{i+1} \left( 0 \right) \right)^T,$$

$$P_{i+1|i}^l = P_{i+1|i}^l - P_{i+1|i}^l \left( \tilde{C}_{i+1} \left( 0 \right) \right)^T \tilde{R}_{i+1}^{-1} \tilde{C}_{i+1} \left( 0 \right) P_{i+1|i}^l,$$

$$\hat{X}_{i+1|i}^l = \hat{A}_i^l \hat{X}_{i|i}^l + P_{i+1|i}^l \tilde{X}_{i+1|i}^l - \hat{P}_{ii}^l \hat{Q}_i^l \left( \hat{B}_i \right)^T \left( R_i^l \right)^{-1} \tilde{C}_{i+1} \left( 0 \right) A_i^l \hat{X}_{i|i}^l,$$

where $\delta^l$ represents a positive real number corresponding to $l$, and it can be obtained from Equation (6). $\hat{X}_{i|i}^l$ is a vector with the same dimensions as $X_{i|i}^l$, $\Sigma_i^l$ is a key positive definite matrix, the local transmission rate $\alpha^l$ for every node determines their value. It is not difficult to see that the information transmitted from each local node to the fusion center is divided into two types. According to the transmission strategy, when $\mu_1^l = 1$, the state estimation value $\hat{X}_{i+1|i+1}^l$ of the $l - th$ node at time $i$ will be transmitted, then the transmission value $\tilde{z}_i^l$ received at the fusion center can be expressed as Formula (17).

$$\tilde{z}_i^l = \hat{X}_{i|i}^l = X_{i|i} + \left( \hat{X}_{i|i}^l - X_{i|i} \right) - g_i^l.$$  

**Same as Formula (4), where $g_i^l$ is a variable independent of virtual measurement noise, and is uniformly distributed within the ellipsoid mentioned in (16). Then in this case, $\sigma_i^l$ is equal to $\left( \hat{X}_{i|i}^l - X_{i|i} \right) - g_i^l$. So the value transmitted to the fusion center and the corresponding measurement noise $\sigma_i^l$ can be summarized as**

$$\sigma_i^l = \left\{ \tilde{X}_{i|i} - X_{i|i}, \mu_i^l = 1, \right\}, \left\{ \tilde{X}_{i|i} - X_{i|i}, \mu_i^l = 0. \right\}$$

Therefore, the transmission value $z_i$ for information fusion and the corresponding measurement noise $\sigma_i$ are defined as follows:

$$z_i = col \left\{ \left( \mu_i^l \hat{X}_{i|i} + \left( 1 - \mu_i^l \right) \hat{X}_{i|i} \right) I_{1|1}^K \right\},$$

$$\sigma_i = col \left\{ \left( \hat{X}_{i|i} - X_{i|i} \right) I_{1|1}^K - \hat{X}_{i|i} \left( 1 - \mu_i^l \right) g_i^l I_{1|1}^K \right\}.$$  

Define $I_0 = col \{ I_{1|1}^K \}$, there is the following relationship:

$$z_i = I_0 X_i + \sigma_i.$$  

According to the best linear unbiased estimator (BLUE) method in [37], we can derive the state estimation value $\hat{X}_{i|i}$ and estimation error covariance matrix $\hat{P}_i$ of the fusion center as follows:

$$\hat{X}_{i|i} = \left( \hat{P}_i^l P_i^{-1} I_0 \right)^{-1} I_0^T P_i^{-1} z_i,$$

$$\hat{P}_i = E \left\{ \left( X_i - \hat{X}_{i|i} \right) \left( X_i - \hat{X}_{i|i} \right)^T \right\} = \left( \hat{P}_i^l P_i^{-1} I_0 \right)^{-1}.$$  

**FIG. 4:**
where $P_i$ is the covariance matrix of $\sigma_i$. From the expression of $\sigma_i$ in (20) and the distribution characteristics of $g_i$, the expression of $P_i$ can be obtained as follows:

$$P_i = E \left\{ \begin{bmatrix} \text{col} \left[ (\hat{X}_{ij} - X_i) |_{i=1}^K \right] \\ \times \text{col} \left[ (\hat{X}_{ij} - X_i) |_{i=1}^K \right]^T \right\} + \text{diag} \left\{ (1 - \mu_i) \frac{\delta}{n + 2} (\Sigma_i)^{-1} \right\}_{i=1}^K. \tag{23}$$

Define $\hat{P}_i$ as the pseudo covariance matrix of $\text{col} \left[ (\hat{X}_{ij} - X_i) |_{i=1}^K \right]$. Then $\hat{P}_i$ can be obtained by the local pseudo covariance matrix $P_{i+1|i}^{j,k}$ and the pseudo cross covariance matrix $P_{i+1|i+1}^{j,k}$ of the robust fusion estimator, such as follows:

$$\hat{P}_i = \begin{bmatrix} P_{i+1|i}^{1,1} & \cdots & P_{i+1|i}^{1,K} \\ \vdots & \ddots & \vdots \\ P_{i+1|i}^{K,1} & \cdots & P_{i+1|i}^{K,K} \end{bmatrix}. \tag{24}$$

Bring (24) into (23), the expression of $P_i$ can be get as follows:

$$P_i = \hat{P}_i + \text{diag} \left\{ (1 - \mu_i) \frac{\delta}{n + 2} (\Sigma_i)^{-1} \right\}_{i=1}^K. \tag{25}$$

Next we consider the following multisensor fusion estimation system:

$$\begin{bmatrix} X_{i+1} = \hat{A}_i^l X_i + \hat{B}_i^l w_i, \\ y_i' = \hat{C}_i^l X_i + v_i', \quad l = 1, 2 \ldots K. \end{bmatrix} \tag{26}$$

Then the following relationship can be easily obtained:

$$X_{i+1} - \hat{X}_{i+1|i+1}^l = \left( I_{n(d+1)} + P_{i+1|i}^l (\hat{C}_i^l + \hat{B}_i^l w_i) \right)^T \left( R_i^l + C_i^l \right)^{-1} - \left( (P_{i+1|i}^l)^{-1} + (C_i^l)^T \left( R_i^l + C_i^l \right)^{-1} C_i^l \right)^{-1} \times \left( \hat{C}_i^l \right)^T \left( R_i^l \right)^{-1} v_{i+1}. \tag{27}$$

According to the above equation, the pseudo cross covariance matrix $P_{i+1|i+1}^{j,k}$ of local estimation errors can be calculated by (28):

$$P_{i+1|i+1}^{j,k} = \left( I_{n(d+1)} - P_{i+1|i}^l (\hat{C}_i^l + \hat{B}_i^l w_i)^T \right) \left( (\hat{C}_i^l + \hat{B}_i^l w_i)^T \left( R_i^l + C_i^l \right)^{-1} \hat{C}_i^l \right)^{1} \times \left( \hat{A}_i^l P_{i+1|i}^{j,k} \hat{A}_i^l \right)^T + \hat{B}_i^l Q_i (\hat{B}_i^l)^T \times \left( I_{n(d+1)} - P_{i+1|i}^l (\hat{C}_i^l + \hat{B}_i^l w_i)^T \right) \left( (\hat{C}_i^l + \hat{B}_i^l w_i)^T \left( R_i^l + C_i^l \right)^{-1} \hat{C}_i^l \right)^{1} \times \left( \hat{C}_i^l + \hat{B}_i^l w_i \right)^T \left( R_i^l + C_i^l \right)^{-1} C_i^l \right). \tag{28}$$

It should be noted that the robust state estimator based on state augmentation and expectation minimization of estimation errors is used for local sensors, and the state estimation results obtained by this estimator at each local node are closer to the true states than the Kalman filter based on nominal parameters. Therefore, when the number of sensors in the wireless networks increases, the proposed robust fusion estimator can maintain relatively nice performance. In addition, due to the application of the transmission strategy (16), this fusion estimator can also greatly reduce the possibility of information congestion and the energy waste caused by a large amount of information transmission.

### IV. PERFORMANCE ANALYSIS OF THE ROBUST FUSION ESTIMATOR

In this section, we explore the convergence of the fusion estimator, and the key matrices and vectors used in the data transmission strategy are also given in this section. In the assumptions and proofs of this section, the modeling errors $e_{i,j}$ are within set $\mathcal{E} = \{ e | e_{i,j} \leq 1, j = 1, 2, \ldots, I \}$. In order to simplify the proof process, we first define the following matrices:

$$M_i^T = \left( G_{i}^{11} + G_{i}^{12} - (G_{i}^{11})^T (G_{i}^{11})^{-1} G_{i}^{12} \right)^{-1},$$

$$N_i^T = \begin{bmatrix} B_i (0) \\ 0_{n \times d} \end{bmatrix},$$

$$O_i = \begin{bmatrix} \left( R_i^l \right)^{-1} C_i^l (0) & 0_{n \times d} \\ (G_i^l)^T \end{bmatrix},$$

$$T_{ii}^l = \begin{bmatrix} A_{ii} (0) & 0_{n \times (d+1)} \\ I_0 & 0_{d \times n} \end{bmatrix},$$

$$T_{i2}^l = \begin{bmatrix} L_i + (M_i^l)^{-1} (G_i^l)^T (G_i^l)^{-1} G_i^l (M_i^l)^{-1} \end{bmatrix}.$$
Theorem 1: Suppose that the above assumptions hold, and each local sensor sends the estimated value to fusion center according to the data-driven transmission strategy (16) mentioned above. If the weight matrix $\Sigma_i$ of each local sensor meets the following condition:

$$\Sigma_i \succeq \sigma^l I,$$  

(30)

in which $\sigma^l$ are some positive real numbers, then the estimation error $X_i - \hat{X}_{ij}$ is uniformly bounded in mean square error for any possible choice of the sequences $\{\hat{X}_{ij}, i \in \mathbb{Z}_+\}$, that is,

$$\lim_{i \to \infty} \sup \mathbb{E} \left\{ \left\| X_i - \hat{X}_{ij} \right\|^2 \right\} < +\infty.$$  

(31)

Proof: It can be seen from Equation (20) that if the estimated values of the local sensors are all sent to the fusion center, we define the transmission value to be transmitted to the fusion center in this case as $\bar{z}_i$, which is,

$$\bar{z}_i = \text{col} \left\{ \hat{X}_i^l \right\}_{i=1}^K.$$  

(32)

Similar to Equation (22), we let

$$\tilde{X}_{ij} = \left( I_0^T P_i^{-1} I_0 \right)^{-1} I_0^T P_i^{-1} \bar{z}_i,$$  

(33)

then (34) can be obtained:

$$\hat{X}_{ij} = \left( I_0^T P_i^{-1} I_0 \right)^{-1} I_0^T P_i^{-1} (\bar{z}_i + z_i - \bar{z}_i) = \tilde{X}_{ij} + \left( I_0^T P_i^{-1} I_0 \right)^{-1} I_0^T P_i^{-1} (z_i - \bar{z}_i).$$  

(34)

We can deduce that

$$\mathbb{E} \left\{ \left\| X_i - \hat{X}_{ij} \right\|^2 \right\} \leq 2 \mathbb{E} \left\{ \left\| X_i - \tilde{X}_{ij} \right\|^2 \right\} + 2 \left\{ I_0^T P_i^{-1} I_0 \right\}^{-1} \left\{ I_0^T P_i^{-1} I_0 \right\}^{-1} \mathbb{E} \left\{ \left\| z_i - \bar{z}_i \right\|^2 \right\}.  

(35)

According to Formula (33), since the data source of $\tilde{X}_{ij}$ does not include the local virtual transmission values, from the knowledge of the rank of the matrix, the first term on the right side of (35) satisfies:

$$\mathbb{E} \left\{ \left\| X_i - \tilde{X}_{ij} \right\|^2 \right\} \leq \text{tr} \left( I_0^T P_i^{-1} I_0 \right)^{-1}.  

(36)

Suppose that the assumptions in this section hold and the pseudo covariance matrix $P_{ij}$ of the local estimator converges to a positive semi-definite matrix [20], then according to the definition in Equation (24), the convergence of $P_i$ can be inferred. Next, we discuss the convergence of $P_i$ in (25). By Condition (30), the last part of Formula (25) satisfies the following formula:

$$\text{tr} \left\{ \left( 1 - \mu_i^l \right) \frac{\delta^l}{n+2} \left( \Sigma_i^l \right)^{-1} \right\} = \left( 1 - \mu_i^l \right) \frac{\delta^l}{n+2} \text{tr} \left( \Sigma_i^l \right)^{-1} \leq \left( 1 - \mu_i^l \right) \frac{\delta^l}{(n+2)} \sigma_i^l n \leq e,$$  

(37)

where $e$ is a scalar. In that way the convergence of $P_i$ can be inferred. Then according to (36) and the definition of $I_0$, the uniform boundedness of the first part on the right side of Equation (35) is derived.

According to condition (30), the following relationship holds:

$$\left\| \tilde{X}_i^l - \bar{X}_{ij} \right\|^2 \geq \sigma^l \left\| \tilde{X}_i^l - \bar{X}_{ij} \right\|^2.$$  

(38)

It can be derived from Equation (16) that, when $\mu_i^l = 0,$

$$\left\| \tilde{X}_i^l - \bar{X}_{ij} \right\|^2 \leq \sigma^l.$$  

At this time, Equation (38) can be reduced to

$$\left\| \tilde{X}_i^l - \bar{X}_{ij} \right\|^2 \leq \sigma^l.$$

Therefore, the proof is accomplished.

According to the condition in Theorem 1, $\tilde{X}_{ij}$ and the weight matrices $\Sigma_i$ can be designed as follows,

$$\tilde{X}_{ij} = \hat{A}_i^l \bar{X}_{ij} + \left( P_i^{l-1} \right) \bar{P}_{ij}^{l-1}.$$  

(40)

In which,

$$\bar{P}_{ij}^{l-1} = \hat{A}_i^l \left( \bar{P}_{ij}^{l-1} + \left( 1 - \mu_i^l \right) \frac{\delta^l}{n+2} \right).$$  

(41)

Which is, each sensor uses the one-step prediction value $\hat{A}_i^l \bar{X}_{ij}$ at the previous moment as $\tilde{X}_{ij}$ in the transmission strategy (16).

V. NUMERICAL SIMULATIONS

In this section, the performance of the derived fusion estimator is demonstrated by comparing with other robust state estimation method and the Kalman filter based on actual and nominal parameters. It should be noted that the three filtering methods participating in the comparison are all used for the derived state augmented model, and the local estimates are fused according to the transmission strategy in Equation (16).

Consider the following system with two-step state delay and uncertain parameters:

$$\begin{align*}
    x_{i+1} &= A_{1i}(\varepsilon_i) x_i + A_{2i}(\varepsilon_i) x_{i-2} + B_i(\varepsilon_i) w_i, \\
    y_i^1 &= C_i^1(\varepsilon_i) x_i + v_i^1, \\
    y_i^2 &= C_i^2(\varepsilon_i) x_i + v_i^2,
\end{align*}$$

this system consists of two sensors. In which,

$$A_{1i}(\varepsilon_i) = \begin{bmatrix} 0.9802 & 0.0010 + 3.9802 \varepsilon_i & 0.0000 \end{bmatrix},$$

$$A_{2i}(\varepsilon_i) = \begin{bmatrix} -0.2802 & 0.0060 + 3.9802 \varepsilon_i & 0.0000 \end{bmatrix}.$$
The transmission rates $\alpha_1$ and $\alpha_2$ are chosen to be 0.6, the initial values given are as follows:

$$
\begin{align*}
x_0 &= \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad x_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \\
x_2 &= \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad y_0^1 = y_0^2 = 0.
\end{align*}
$$

In addition, the process noise $w_i$ and measurement error $v_i$ are produced following normal distributions with zero mean. In these simulations, in order to obtain more convincing comparison results, we perform $5 \times 10^2$ simulations for each sampling point, and use the average value calculated from the above simulation results to reflect the performance of estimators. The calculation expression is,

$$
E \left[ \left\| X_i - \hat{X}_{i|j} \right\|^2 \right] \approx \frac{1}{500} \sum_{j=1}^{500} \left\| X_i - \hat{X}_{i|j}^{(j)} \right\|^2,
$$

in which, $j$ is the number of simulation experiments in each group. That is, the estimation error variance is approximated by the average value of the square of the Euclidean distance from the estimated value to its actual state, which also reflects the standard deviation of the estimators. Furthermore, the values of components $x_{i,1}$ and $x_{i,2}$ of the true state $x_i$ are compared with their estimation values.

For different degrees of model uncertainties, two cases are set to compare the estimation performance of the fusion estimators. In case 1, the model errors $\epsilon_i$ are fixed to be $-0.8508$ in each experiment and do not change with the sampling point. The comparison result of case 1 is intuitively reflected in Figure 1. Because it is not affected by external disturbances, the fusion Kalman filter based on actual parameters shows extraordinary estimation performance.
same conditions, the estimation performance of the estimator proposed in this article is better than that of the Kalman filter based on nominal parameters and the filtering method proposed in [33].

In case 2, the generation of model errors $\epsilon_i$ in parameter matrix follows a truncated normal distribution, the mean and variance of this normal distribution are set to 0 and 1, and the upper limit of $\epsilon_i$ is 1. Figure (2) reflects the comparison of the estimation effect of the estimators in case 2. It can be observed from Figure 2 that the performance of the two robust estimators is equivalent. Compared with the fused Kalman filter based on nominal parameters, the proposed fusion estimator still maintains relatively good performance.

It can be showed from Figures 1 and 2 that when the uncertainty of the model parameters changes, the performance of these two robust fusion estimators is significantly better than that of the fusion Kalman filter based on the nominal parameters. But it should be pointed out that, compared with the filtering method in [33], the proposed estimator only needs to calculate some key matrices offline instead of continuously adjusting the parameters, and it has no limit on model errors.

Furthermore, the proposed fusion estimator is still applicable when the model errors affect the system parameters in arbitrary way. Therefore, the robust fusion estimator proposed in this article has a wider range of applications.

**VI. CONCLUSION**

In this article, we solved the robust information fusion estimation problems of wireless sensor networks with state delay and random parameter uncertainty under communication constraints. Based on the data-driven transmission strategy, this fusion estimator can effectively decrease the waste of energy caused by frequent information exchange. A robust state estimator based on state augmentation and expectation minimization of errors is used to obtain the local state estimation. Besides, we analyze the performance of this fusion estimator and give the conditions of error variance bounded. Finally, the prominent performance of the proposed estimator is verified through numerical simulation examples. The model errors in this article are not limited to a special form, so the proposed estimator is suitable for a wide range of environments. In addition, if considering the existence of random
and uncertain state delays and measurement delays, the model can be closer to reality. Therefore, it is very practical to analyze the robust fusion estimation problem with multiple types of delay and packet drops, which is also one of our future research directions.

REFERENCES

[1] Z. Wang, J. Hu, and L. Ma, “Event-based distributed information fusion over sensor networks,” Inf. Fusion, vol. 39, pp. 53–55, Jan. 2018.

[2] H. Xiong, Z. Mai, J. Tang, and F. He, “Robust GPS/INS/DVL navigation and positioning method using adaptive federated strong tracking filter based on weighted least square principle,” IEEE Access, vol. 7, pp. 26168–26178, 2019.

[3] V. Shnayder, M. Hempstead, B. R. Chen, G. W. Allen, and M. Welsh, “Simulating the power consumption of large-scale sensor network applications,” in Proc. 2nd Int. Conf. Embedded Netw. Sensor Syst. (Sensys), Baltimore, MD, USA, Nov. 2004, pp. 188–200.

[4] S. Yin, X. Li, H. Gao, and O. Kaynak, “Data-based techniques focused on modern industry: An overview,” IEEE Trans. Ind. Electron., vol. 62, no. 1, pp. 657–677, Jan. 2015.

[5] J. K. Yook, D. M. Tilbury, and N. R. Soparkar, “Trading computation for bandwidth: Reducing communication in distributed control systems using state estimators,” IEEE Trans. Control Syst. Technol., vol. 10, no. 4, pp. 503–518, Jul. 2002.

[6] Y. Zhang, Y. Yang, S. X. Ding, and L. Li, “Data-driven design and optimization of feedback control systems for industrial applications,” IEEE Trans. Ind. Electron., vol. 61, no. 11, pp. 6409–6417, Nov. 2014.

[7] X. R. Li, Y. Zhu, J. Wang, and C. Han, “Optimal linear estimation fusion. I. Unified fusion rules,” IEEE Trans. Inf. Theory, vol. 49, no. 9, pp. 2192–2208, Sep. 2003.

[8] L. Li, M. Niu, Y. Xia, H. Yang, and L. Yan, “Event-triggered distributed fusion estimation with random transmission delays,” Inf. Sci., vol. 475, pp. 67–81, Feb. 2019.

[9] Z. Deng, P. Zhang, W. Qi, J. Liu, and Y. Gao, “Sequential covariance intersection fusion Kalman filter,” Inf. Sci., vol. 189, pp. 293–309, Apr. 2012.

[10] H. R. Hashemipour, S. Roy, and A. J. Laub, “Decentralized structures for parallel Kalman filtering,” IEEE Trans. Autom. Control, vol. 33, no. 1, pp. 88–94, Jan. 1988.

[11] M. E. Liggins, C.-Y. Chong, I. Kadar, M. G. Alford, V. Vannicola, and M. G. Alford, “Distributed fusion architectures and algorithms for target tracking,” Proc. IEEE, vol. 85, no. 1, pp. 95–107, Jan. 1997.

[12] S. Mori, W. H. Barker, C.-Y. Chong, and K.-C. Chang, “Track association and track fusion with nondeterministic target dynamics,” IEEE Trans. Aerosp. Electron. Syst., vol. 38, no. 2, pp. 659–668, Apr. 2002.

[13] Z.-L. Deng, Y. Gao, L. Mao, Y. Li, and G. Hao, “New approach to information fusion steady-state Kalman filtering,” Automatica, vol. 41, no. 10, pp. 1695–1707, Oct. 2005.

[14] S.-L. Sun, “Multi-sensor optimal information fusion Kalman filters with applications,” Aerosp. Sci. Technol., vol. 8, no. 1, pp. 57–62, Jan. 2004.

[15] L. Xiao, S. Boyd, and S. Lall, “A scheme for robust distributed sensor fusion based on average consensus,” in Proc. 4th Int. Symp. Inf. Process. Sensor Netw. (IPSN), Boise, ID, USA, 2005, pp. 63–70.

[16] Y. S. Suh, V. H. Nguyen, and Y. S. Ro, “Modified Kalman filter for networked monitoring systems employing a send-on-delta method,” Automatica, vol. 43, no. 2, pp. 332–338, Feb. 2007.

[17] G. Battistelli, A. Benavoli, and L. Chisci, “Data-driven communication for state estimation with sensor networks,” Automatica, vol. 48, no. 5, pp. 926–935, May 2012.

[18] J. Weimer, J. Araújo, and K. H. Johansson, “Distributed event-triggered estimation in networked systems*,” IFAC Proc. Volumes, vol. 45, no. 9, pp. 178–185, 2012.

[19] D. Zhang and L. Yu, “Survey on the stability analysis of linear time-delay systems,” Kongzhi Yu Juece/Control Decis., vol. 23, no. 8, pp. 841–849, 2008.

[20] J. Wang, Y. Mao, Z. Li, J. Gao, and H. Liu, “Robust state estimation for uncertain linear discrete systems with d-step state delay,” to be published.

[21] J. Ren, “LMI-based fault detection filter design for a class of neutral system with time delay in states,” in Proc. 6th World Congr. Intell. Control Automa., Dalian, China, 2006, pp. 5581–5585.

[22] J.-P. Richard, “Time-delay systems: An overview of some recent advances and open problems,” Automatica, vol. 39, no. 10, pp. 1667–1694, Oct. 2003.

[23] R. Rahman, M. Alanyali, and V. Saligrama, “Distributed tracking in multi-hop sensor networks with communication delays,” IEEE Trans. Signal Process., vol. 55, no. 9, pp. 4656–4668, Sep. 2007.

[24] R. Caballero-Águila, I. García-Garrido, and J. Linares-Pérez, “Information fusion algorithms for state estimation in multi-sensor systems with correlated missing measurements,” Appl. Math. Comput., vol. 226, pp. 548–563, Jan. 2014.

[25] Y. Xia, J. Shang, J. Chen, and G.-P. Liu, “Networked data fusion with packet losses and variable delays,” IEEE Trans. Syst., Man, Cybern. B, Cybern., vol. 39, no. 5, pp. 1107–1120, Oct. 2009.

[26] Q. Liu, X. Wang, and N. S. V. Rao, “Fusion of state estimates over long-haul sensor networks with random loss and delay,” IEEE/ACM Trans. Netw., vol. 23, no. 2, pp. 644–656, Apr. 2015.

[27] Z. Li, J. Wang, and H. Liu, “A robust state estimator for T-S fuzzy system,” IEEE Access, vol. 8, pp. 84063–84069, 2020.

[28] J. Zhang, S. Gao, X. Qi, J. Yang, J. Xia, and B. Gao, “Distributed robust cubature information filtering for measurement outliers in wireless sensor networks,” IEEE Access, vol. 8, pp. 20203–20214, 2020.

[29] F. Yang, Z. Wang, and Y. S. Hung, “Robust Kalman filtering for discrete time-varying uncertain systems with multiplicative noises,” IEEE Trans. Autom. Control, vol. 47, no. 7, pp. 1179–1183, Jul. 2002.

[30] F. Wang and V. Balakrishnan, “Robust Kalman filters for linear time-varying systems with stochastic parametric uncertainties,” IEEE Trans. Signal Process., vol. 50, no. 4, pp. 803–813, Apr. 2002.

[31] Z. Wang, D. W. C. Ho, and X. Liu, “Robust filtering under randomly varying sensor delay with variance constraints,” IEEE Trans. Circuits Syst. II, Exp. Briefs, vol. 51, no. 6, pp. 320–326, Jun. 2004.

[32] T. Tian, S. Sun, and N. Li, “Multi-sensor information fusion estimators for stochastic uncertain systems with correlated noises,” Inf. Fusion, vol. 27, pp. 126–137, Jan. 2016.

[33] T. Zhou, “Sensitivity penalization based robust state estimation for uncertain linear systems,” IEEE Trans. Autom. Control, vol. 55, no. 4, pp. 1018–1024, Apr. 2010.

[34] H. Liu and D. Wang, “Robust state estimation for wireless sensor networks with data-driven communication,” Int. J. Robust Nonlinear Control, vol. 27, no. 18, pp. 4622–4632, Apr. 2017.

[35] B. Chen, L. Yu, W.-A. Zhang, and A. Liu, “Robust information fusion estimator for multiple delay-tolerant sensors with different failure rates,” IEEE Trans. Circuits Syst. I, Reg. Papers, vol. 60, no. 2, pp. 401–414, Feb. 2013.

[36] H. Liu and T. Zhou, “Robust state estimation for uncertain linear systems with random parametric uncertainties,” Sci. China Inf. Sci., vol. 60, no. 1, pp. 157–169, Jan. 2017.

[37] A. S. Behbahani, A. M. Eltawil, and H. Jafarkhani, “Linear decentralized estimation of correlated data for power-constrained wireless sensor networks,” IEEE Trans. Signal Process., vol. 60, no. 11, pp. 6003–6016, Nov. 2012.
ZIQIANG LI was born in Hengyang, Hunan, China, in 1996. He received the bachelor’s degree in automation engineering from Qingdao University, in 2018, where he is currently pursuing the master’s degree in control engineering with the College of Automation Engineering. Since 2018, he has been an Assistant Manager with the Multimedia Laboratory, College of Automation Engineering. His research interests include Siemens PLC programming and application, STM32 microcontroller programming development and application, robust state estimation research, and application in the field of linear systems and nonlinear systems. During his bachelor’s degree, from 2014 to 2018, he won the Excellent Award from the School of Electronic Design Competition and the Excellent Award of the Extracurricular Academic and Technological Works Competition of Qingdao University students.

JUNWEI GAO received the B.S. degree in electrical engineering from the Shandong University of Technology, Zibo, China, in 1995, the M.S. degree in control theory from the Lanzhou University of Technology, Lanzhou, China, in 2000, and the Ph.D. degree in traffic information engineering and control from the China Academy of Railway Sciences, Beijing, China, in 2003. He is currently a Full Professor with the School of Automation, Qingdao University, China. His current research interests include intelligent systems and intelligent control.

HUABO LIU (Member, IEEE) received the B.S. and M.S. degrees in automation and detection technology and automation equipment from Chongqing University, Chongqing, China, in 2001 and 2005, respectively, and the Ph.D. degree in control science and engineering from Tsinghua University, Beijing, China, in 2016. He is currently with the School of Automation, Qingdao University, Qingdao, China. His current research interests include large-scale networked systems, hybrid systems, robust state estimation, and their applications to practical engineering problems.