Identifying and engineering Majorana bound states [1–4] for topological quantum computing [5–7] remains a challenge. Among various candidates, the semiconductor-superconductor nanowires [8, 9] have received considerable attention [10–25] due to their high tunability [26]. The Majorana bound states always come in a pair and are localized at the two ends of the wire. They are supposed to have zero energy, but in reality, they are partially successful, e.g., assumed temperatures have been experimentally observed to oscillate as a function of magnetic field, showing a signature of overlapped Majorana bound states. However, the oscillation amplitude either dies away after an overshoot or decays, sharply opposite to the theoretically predicted enhanced oscillations for Majorana bound states. We reveal that a steplike distribution of spin-orbit coupling in realistic devices can induce the decaying Majorana oscillations, resulting from the coupling-induced energy repulsion between the quasi-particle spectra on the two sides of the step. This steplike spin-orbit coupling can also lead to decaying oscillations in the spectrum of the Andreev bound states. For Coulomb-blockade peaks mediated by the Majorana bound states, the peak spacings have been predicted to correlate with peak heights by a $\pi/2$ phase shift, which was not experimentally observed and may be explained by the steplike spin-orbit coupling. Our work will inspire more works to re-examine effects of the non-uniform spin-orbit coupling, which is generally present in experimental devices.

In this Letter, we reveal that the oscillation patterns in the experiments [30–34], including both the decay in amplitude and increase in period, can be well captured [Figs. 1(b)-(d) experiment, (e)-(g) theory] by a simple, but realistic assumption: spin-orbit coupling strength along the nanowire has a steplike distribution [see the green curve in Fig. 1(a)]. The steplike spin-orbit coupling is reasonable because the gates apply a non-uniform electrostatic potential and spin-orbit coupling depends on the electrostatic fields perpendicular to the nanowire [40–47]. Moreover, the presence of the superconductor can greatly modify the electrostatic field in the nanowire due to screening effect and work-function mismatch between the superconductor and semiconductor [25]. Thus the spin-orbit coupling is well expected to be non-uniform from the nanowire covered with superconductor to the part (tunnel barrier region) without the superconductor. Additionally, we find that these decaying oscillations caused by the steplike spin-orbit coupling also exist in the energy spectrum of Andreev bound states [Figs. 3(d) and (e)]. To distinguish Majorana from Andreev bound states, a recent theory [48] predicted a $\pi/2$ phase shift between the spacings and heights of the Coulomb-blockade peaks mediated by the Majorana bound states in nanowire islands [Fig. 4(b)], but was not observed in experiments [34], which may be explained by considering the steplike spin-orbit coupling [Figs. 4(c) and (d)]. These results highlight the non-uniform spin-orbit coupling generally existing in experiments but ignored in most simulations.
FIG. 1. (a) Schematic of the semiconductor-superconductor nanowire island [30–34], its two ends may host a pair of Majorana bound states (MBSSs). [(b)–(d)] Adapted from Ref. [30]. The Majorana bound states can hybridize as a function of the magnetic field $B$. However, opposite to the experiments, Majorana theory predicts that $E_0$, the first zero-energy crossing at $V_{\text{Z}}$, develops after the first zero-energy crossing at $V_{\text{Z}}$ [Fig. 2(b)]. Between $V_{\text{Z}}$ and $V_{\text{Z}}$, the spectrum of the entire nanowire depends on the competition between $E_{\text{L/R}}$ and $V_{\text{ee/eh}}$. Figure 2(b) shows that $E_{\text{L}}$ increases with increasing amplitude as a function of $V_{\text{Z}}$, consistent with the known result for uniform spin-orbit coupling [28]. Also, the Majorana wavefunctions are known to move towards the nanowire ends under [see top of (a)], so they suppress the enhanced oscillations in (b) into decaying oscillations with increasing periods [blue solid line] in Fig. 1(f). (e) $V_{\text{ee/eh}}$ increase with increasing $V_{\text{Z}}$ since the wavefunctions move towards the nanowire ends [see top of (a)], so they suppress the enhanced oscillations in (b) into decaying oscillations with increasing periods [blue solid line] in (d). [(f) and (g)] Majorana wavefunctions $\psi_{A/B}$ at $V_{\text{Z}}$ marked by the scatters in Fig. 1(f).

Why Majorana oscillations decay.—Before showing the numerical simulations of the decaying Majorana oscillations in Fig. 1, we first use Fig. 2 to give the mechanism underneath. Suppose that a wire of 2 µm is divided at 0.55 µm into two uncoupled parts, with smaller (L) and larger (R) spin-orbit coupling, respectively [Fig. 2(a)]. Their energy spectra are quite different due to different length and spin-orbit coupling strength [49, 50]: on the left [Fig. 2(b)], the enhanced oscillations emerge simultaneously after the first zero-energy crossing at $V_{\text{Z}}^L$; on the right [Fig. 2(c)], two near-zero-energy bound states develop after $V_{\text{Z}}^R$ ($> V_{\text{Z}}^L$).

Turning on the coupling between the two parts, the lowest-energy spectrum can be modeled by

$$H_{\text{eff}} = \sum_{i=L,R} E_i c_i^\dagger c_i + (V_{\text{ee}} c_L^\dagger c_R + V_{\text{eh}} c_L^\dagger c_R + H.c.),$$

where $E_{\text{L/R}}$ stand for the lowest-energy spectra in Figs. 2 (b) and (c), and $V_{\text{ee}}$ and $V_{\text{eh}}$ are the particle-particle and particle-hole couplings (details in Sec. SI of [51]). between the lowest-energy states of the two parts [see top of Fig. 2(a)]. Between $V_{\text{Z}}^L$ and $V_{\text{Z}}^R$, the spectrum of the entire nanowire depends on the competition between $E_{\text{L/R}}$ and $V_{\text{ee/eh}}$. Figure 2(b) shows that $E_{\text{L}}$ oscillates with increasing amplitude as a function of $V_{\text{Z}}$, consistent with the known result for uniform spin-orbit coupling [28]. Also, the Majorana wavefunctions are known to move towards the nanowire ends under [see top of (a)], so they suppress the enhanced oscillations in (b) into decaying oscillations with increasing periods [blue solid line] in (d). [(f) and (g)] Majorana wavefunctions $\psi_{A/B}$ at $V_{\text{Z}}$ marked by the scatters in Fig. 1(f).
nanowire will show the decaying oscillations [blue solid in Fig. 2(d)], consistent with the exact spectrum in Fig. 1(f).

In contrast, there will be enhanced oscillations if the repulsion by $V_{ee/eh}$ is not strong enough. Therefore, the competition between $E_{L/R}$ and $V_{ee/eh}$ can account for the decaying or enhanced oscillations (Sec. SII of [51]).

Model.— To verify our physical picture, we perform simulations by using the steplike spin-orbit coupling. We model the nanowire island by the Hamiltonian $H = \int_0^L dx \psi^\dagger(x) \mathcal{H} \psi(x)$, where

$$H = \frac{\mathbf{p}^2}{2m^*} - \mu(x) - \sigma_y \left( \alpha(x) p_x \right) / 2\hbar \tau_z + V_Z \sigma_x + \Delta \tau_z,$$

where $L$, $m^*$, $p_x = -i\hbar \partial_x$, $\Delta$, and $V_Z$ are the wire length, effective electron mass, momentum operator, effective pairing, and Zeeman energy induced by $B$, respectively. $\mu(x)$ and $\alpha(x)$ denote the position-dependent chemical potential and spin-orbit coupling, respectively.

Quite different from the previous theories which assume a constant spin-orbit coupling [35–37], we model that spin-orbit coupling has a profile [see also the green curve in Fig. 1(a)].

$$\alpha(x) = \frac{A}{2} \left[ \tanh \left( \frac{\lambda_L}{\lambda_L} \right) + \tanh \left( \frac{\lambda_R}{\lambda_R} \right) \right] + \alpha_0,$$

where $A$, $\alpha_0$, $x_{L/R}$, and $\lambda_{L/R}$ are the parameters that describe the profile. $\mathcal{H}$ is written in terms of the Nambu spinor $\{u_i(x), v_i(x), \uparrow(x), \downarrow(x)\}$. The Pauli matrices $\sigma$ and $\tau$ act on the spin and particle-hole spaces, respectively. The anticommutator in $\mathcal{H}$ ensures the Hermiticity [41, 46, 47]. In realistic experiments, the parameters intertwine when changing the gate voltages [22, 24, 25, 53–56], and the superconductor can induce renormalization effects [57, 58]. Nevertheless, to focus on the effect of the steplike spin-orbit coupling, all the parameters in $H$ are assumed to be independently adjustable. By diagonalizing $H$ on a lattice, the energy spectrum and wavefunctions are obtained. The lowest energy is the bound state energy $E_0$, the hybridization energy mentioned above.

Decays of Majorana oscillations.— Figures 1(e)-(g) show our numerical results. We use the parameters $m^* = 0.026 m_e$, $\Delta = 0.25$ meV, $\alpha_0 = 0.04$ eVÅ, $A = 0.4$ eVÅ, and lattice constant $a = 10$ nm. To focus on the effect of the steplike spin-orbit coupling, first we consider only one step of spin-orbit coupling, so that $\alpha(x) = \alpha_0 + A \Theta(x - x_L)$ [Fig. 2(a)], i.e., let $x_R = L$ and $\lambda_L = \lambda_R = a$ in Eq. (2), and the chemical potential $\mu = 0$. Our simulations agree with the experiments, not only for the decaying amplitude, but also including the lowest-energy crossing [Figs. 1(b) and (c), (e) and (f)], anti-crossing [Figs. 1(d) and (g)], and increasing oscillation period at small Zeeman energies [Figs. 1(c) and (f)]. We note that our results are generic and do not depend on the detailed parameters, e.g., the step shape (smoothness), effective pairing $\Delta$, chemical potential $\mu$, and spin-orbit coupling strength (Sec. SIII of [51]). Further increasing the Zeeman energy, we find that the oscillations turn from decay to increase for those magnetic fields at which the the superconductivity is suppressed in the experiments, thus less likely to be observed.

Decays of Andreev oscillations.— Are these decaying oscillations unique for Majorana bound states? Our answer is no. It has been suggested that the same device can also host the Andreev bound states [59–63]. Whether the decaying oscillations are from Andreev or Majorana bound states can be checked from the spatial profiles of the lowest-energy Majorana wavefunctions at the Zeeman energies indicated in Fig. 1(f). The Majorana wavefunctions can be constructed by projecting the lowest-energy wavefunctions to the Majorana basis [62, 64], i.e., $\psi_A = (1/\sqrt{2})(\psi_{E_0} + \psi_{-E_0})$ and $\psi_B = (1/\sqrt{2})(\psi_{E_0} - \psi_{-E_0})$. $\psi_A$ and $\psi_B$ are localized at the opposite wire ends for the Majorana bound states, while they are strongly overlapping or separated by a distance comparable with the penetration length for the Andreev bound states [62]. For $V_Z$ far smaller than $V_Z^0$ [Fig. 2(f)], the two wavefunctions are squeezed in the region with small spin-orbit coupling ($0 < x < x_L$), implying that they are two Andreev bound states. For $V_Z$ larger than $V_Z^0$ [Fig. 2(g)], the two wavefunctions are well localized at the opposite ends, forming a pair of near-zero-energy Majorana bound states with a slight overlap.

We simulate the near-zero-energy Andreev bound states by employing a smoothly varying chemical potential $\mu(x)$ [59–63], as shown in Fig. 3(a). For uni-
form spin-orbit coupling (i.e., \( x_L = 0 \)), two near-zero-energy bound states persists for a wide range of Zeeman energy before the topological phase transition point \( V_Z^* = \sqrt{\max[\mu(x)]^2 + \Delta^2} \) [about 0.91 meV in Fig. 3(b)] at which the superconducting gap nearly closes and reopens. These bound states are partially separated Andreev bound states [62] since the constituent Majorana eigenstates are separated by a distance comparable with the penetration length [Fig. 3(c)]. After including a steplike distribution of spin-orbit coupling, Figs. 3(d) and (e) show that there are also decaying oscillations for the Andreev bound states at \( V_Z < V_Z^* \). The oscillations turn to increase at \( V_Z > V_Z^* \) for Majorana bound states. The Andreev or Majorana nature is determined by the spatial profiles of the projections of the lowest-energy wavefunctions onto the Majorana basis and these decaying oscillations are also due to the competition between \( E_{L/R} \) and \( \langle E_{ee} \rangle \), similar to Fig. 2(d) (Sec. SIV of [51]).

\[ [51]. \]

**Phase shift between peak spacing and height oscillations.** In the floating nanowire island [Fig. 1(a)] [65–69], adding an electron costs a finite charging energy due to its small capacitance, leading to the Coulomb blockade peaks in the two-terminal conductance measurement [Fig. 4(a)]. Because of the hybridization energy \( E_0 \), charging a pair of unoccupied Majorana bound states to occupied \((e \rightarrow o)\) differs in energy from the process \( o \rightarrow e \) in the next charging event. In this way, \( E_0 \) can be extracted from the difference between two consecutive Coulomb blockade peak spacings in gate voltage (Sec. SV of [51]). The blue lines in Figs. 4(c) and (d) show the calculated Majorana oscillations of \( E_0 \). Different from those in Figs. 1(e)-(g), here we consider two steps of spin-orbit coupling and the steps are smoothed by using finite \( \lambda_{L/R} \), as depicted in Fig. 1(a). In addition, Figs. 4(b)-(d) also present the calculated Coulomb blockade peak height ratio \( \Lambda = G_{e \rightarrow o}/(G_{e \rightarrow o} + G_{o \rightarrow e}) \) as a function of the Zeeman energy \( V_Z \) (orange lines). The corresponding conductance peak heights \( G_{e \rightarrow o} \) and \( G_{o \rightarrow e} \) are shown in Sec. SV of [51]. The zero-temperature peak heights are assumed independent of \( V_Z \); and are formulated as \( G_{e \rightarrow o} = (e^2/h)(\Gamma_L\Gamma_R|u_L|^2|u_R|^2)/(|\Gamma_L|u_L|^2 + \Gamma_R|u_R|^2) \) [48], where \( \Gamma_{L/R} \) is the tunneling rate between the left (right) end of the nanowire and its nearest metallic lead, and \( |u_{L/R}(\sigma)|^2 = \sum_{\sigma = \uparrow, \downarrow} |u_{L/R}(\sigma)|^2 \) with \( u_{L/R}(\sigma) \) the lowest-energy wavefunction component at the leftmost (rightmost) lattice site of the wire. \( G_{o \rightarrow e} \) is obtained by replacing all \( u_{L/R}(\sigma) \) in \( G_{e \rightarrow o} \) with \( u_{L/R}(\sigma) \), which means that \( G_{e \rightarrow o} \) and \( G_{o \rightarrow e} \) are related to the electron-like and hole-like components of the lowest-energy state, respectively. It has been predicted [48] that the oscillations of \( \Lambda \) are correlated to those of \( E_0 \) by a \( \pi/2 \) phase shift for the Majorana bound states. Specifically, \( E_0 \) is zero at the extremals of \( \Lambda \), and \( \Lambda = 1/2 \) at the extremals of \( E_0 \) [Fig. 4(b)]. While for the Andreev bound states, there is no such correlated \( \pi/2 \) phase shift [48]. When considering the steplike spin-orbit coupling in our model, the correlations for our decaying Majorana oscillations show clear deviations from the exact \( \pi/2 \) phase shift [Figs. 4(c) and (d)]. This implies that the steplike spin-orbit coupling may be one of the reasons why the correlation between \( E_0 \) and \( \Lambda \) is ambiguous in a recent experiment [34], since not only the Andreev bound states, but also the Majorana states, can give uncorrelated oscillation patterns between \( E_0 \) and \( \Lambda \) when spin-orbit coupling is non-uniform.

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