The model identification of buildings horizontal displacements with the use of a free geodetic network

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Abstract. The geodetic monitoring of engineering structures, their displacements, and deformations, carried out permanently or periodically, allows obtaining information on the technical condition of facilities. The achieved information enables determining the necessary changes in using objects and minimizing future errors in the similar object's design. The measurement results are subject to geometric interpretation based on the determined displacement parameters of the object's shape and the approximation of the vector displacement field. Due to the influence of random factors characterized by a change in time and varying intensity, the deformation measurements performed during the operation of the facilities are of great importance for the safety of structures and engineering structures. In actual tasks of determining the object's deformation and building a geometric model of displacements, the dominant method is the differential method, the advantage of eliminating systematic errors in measurement results while maintaining the geometric structure of the measurement and control network. The displacement's geometric model, built based on measurements and calculations, can build a dynamic model of a building object, additionally considering such causes of deformation as, for example, own and usable weight, wind pressure, changes in ambient temperature, or ground vibrations. The article proposes approaches using the free alignment of linear and angular observations made in a geodetic network to determine horizontal displacements of an engineering object. This method may be necessary to study displacements of various parts of the object, thus analyzing its deformation. Free alignment allows for an optimal fit of the equalized network into the approximate network by imposing additional conditions (compared to the classic least squares method) on the vector of estimates of increments to approximate coordinates and the value of the covariance matrix. As an example of applying the proposed approach, the actual data received from the geodetic monitoring of the building structure was used. The structure was a road viaduct located along Wojska Polskiego Street in Bydgoszcz. The object of measurements and analyses was represented by finite sets of fixed points, subject to periodic observations over two years. The authors tested the effectiveness of the proposed algorithm and compared the obtained results with the values of horizontal displacements, which were calculated based on the classic study of geodetic monitoring results using the least-squares method. The accuracy analysis of the obtained values of the geodetic network horizontal displacements using free alignment and the least-squares method was also performed. The results indicate the possibility of using the presented approach.
to identify the geometric model of horizontal displacements without losing the accuracy of their determination.

1. Introduction

Geodetic buildings monitoring includes the measurement of vertical and horizontal displacements using appropriate measurement methods and the interpretation of the results. Interpretation of the results is preceded by constructing a geometric model of displacements allowing for the identification of the reference base [1, 2]. Conducting geodetic monitoring can be continuous or periodic while considering the need to observe the dynamics of changes taking place in the facility. Systems for monitoring and diagnostics of the condition of building objects may consider loads of individual structural elements with an analysis of the influence of external factors: prevailing winds, environmental pollution and thermal and chemical interactions [3-5]. Correctly carried out measurement and determination of displacements allow for identifying the object's technical condition and predicting its behaviour in a long-term perspective. That, in turn, allows planning the optimal use of the facility and carrying out activities aimed at maintaining the facility in a non-deteriorated condition [6-9].

A wide range of measurement methods and techniques can be used to measure displacements and deformations. Still, the measurement should always be preceded by an analysis of the measurement conditions in the facility, the development of an optimal measurement and control network architecture and an indication of a possible calculation method [10, 11]. All analyses should consider the technological, numerical, economic, environmental protection and safety of people working in the facility and using the facility [12, 13]. When optimizing the geodetic network architecture for monitoring changes taking place in building objects, it should consider the possibility of making measurements and the computational correctness of determining displacements and the possibility of capturing changes that occur. That elements depend on the average measurement error resulting from the adopted technological assumptions [14, 15]. The measurement results are subject to geometric interpretation, which is the determination of the displacement parameters of the object's body and the approximation of the vector field of displacements [16, 17]. Due to the influence of random factors characterized by a change in time and varying intensity, the deformation measurements performed during the operation of the facilities are of great importance for the safety of structures and engineering structures. The results of alignment of periodic measurements of measurement and control networks, assumed to determine the displacements of building objects, are compared with the results obtained for the initial measurement [18, 19]. One of the methods that can be used in the alignment of measurement and control networks is the free alignment method, which may be necessary to study displacements of various parts of the object, thus analyzing its deformation. Free alignment allows for an optimal fit of the aligned network into the approximate network by imposing additional conditions (compared to the classic least squares method) on the vector $\mathbf{d}_X$ of the estimates of increments to approximate coordinates and the value of the covariance matrix $\mathbf{C}_X$ [20, 21].

The article is organized as follows to present the possibility of using free alignment to determine horizontal displacements of a building object:

- based on the cyclical monitoring of the building structure, the possibility of using free alignment to determine horizontal displacements was investigated. The approach has been implemented in the MATLAB environment;
- the obtained results were analyzed, and quantitative analysis was carried out to test the accuracy of the obtained displacements to the horizontal displacements of the measurement and control network points. The displacements were obtained using classical computational techniques used in the geodetic analysis of deformation and changes in buildings.
The rest of the article is organized as follows: Section 2 describes how the countervailing problem is solved for the applied free-form approach, Section 3 presents the results and discussions. Section 4 contains the essential findings and conclusions and indicates different research lines.

2. Geodetic networks free alignment

In the classical alignment of geodetic networks using the least-squares method with minimal restrictions on the degrees of freedom, the correction vector is subject to the condition [20-23]:

\[ V^T P V = \min \]  

where: \( V \) – matrix of corrections to be observed, \( P \) - weight matrix. Using free alignment, additional conditions are imposed on the Euclidean norm of the vector of approximate coordinate increments \( \hat{d}_X \) and on the \( C_X \), covariance matrix, these conditions take the form:

\[ \hat{d}_X P \hat{d}_X = \min \]  
\[ \text{trace}(C_X) = \min \]  

In the process of free alignment, there weren't assume minimal restrictions on the degrees of freedom. Therefore, it was determined that the increments to the coordinates of all geodetic network points subject to alignment. Such a course of action results in the external network defect occurrence, which is equal to the number of degrees of freedom and calculated as:

\[ de = \text{cols}(A) - \text{rank}(A), \]  

where: \( A \) is a matrix of coefficients occurring with unknowns. Using the formula (4), it can determine the external defect, which is \( de = 1 \) for vertical networks and \( de = 3 \) for horizontal networks. The external defect of geodetic networks is in practice eliminated by establishing the network to points with known coordinates or by defining a local coordinate system (e.g. by assuming the constant coordinates of one of the points in vertical networks). The principle of free equalization assumes a non-zero external defect. If, with this assumption, the system of observational equations is written as:

\[ A\hat{d}_X = L + V, \]  

where: \( L \) - intercept matrix. In the group of possible solutions of the system (5), there is one particular solution that meets both the postulate of the least-squares method (1) and the condition (2). It should be noted, however, that with a non-zero external defect of the network, the matrix of coefficients \( A \) of the system of correction equations \( V = A\hat{d}_X + L \) is a columnar incomplete-order matrix. That is, the matrix of coefficients of a system of standard equations \( A^T P \hat{d}_X + A^T PL = 0 \) it is a singular matrix, and its classical inverse does not exist. The solution can be obtained using the Moore-Penrose pseudo-inverse, i.e. the generalized inverse. For the generalized reciprocal of \( A^+ \) the vector of estimated increments to approximate coordinates, satisfying the solution of the contradictory system of equations \( A\hat{d}_X + L = 0 \), will take the form:

\[ \hat{d}_X = -A^+ L. \]  

Vector \( \hat{d}_X \) meets the following conditions:

\[ (A\hat{d}_X + L)^T P(A\hat{d}_X + L) = \min \]  
\[ \hat{d}_X^T M \hat{d}_X = \min, \]  

where: \( P \) is the weight matrix of the measurement results vector, while the matrix of weights known before aligning the coordinates of the points. With these assumptions, solution (6) is a solution of the correction equation system for which conditions (1) to (3) are met.
After determining the vector $\mathbf{d}_X$ fulfilling condition (1), another equalization task should be formulated. It is an additional optimization problem compared to the classical equalization methods. The optimization problem [23] relates to the conditional method known in the equation theory and takes the form:

$$
\mathbf{B}\mathbf{d}_X + \Delta = 0 \tag{9}
$$

$$
\min\{\mathbf{V}^T\mathbf{M}\mathbf{V}\} = \mathbf{d}_X^T\mathbf{M}\mathbf{d}_X, \tag{10}
$$

where: $\mathbf{B}$ is a matrix of coefficients with unknowns, and the matrix $\Delta$ contains intercepts. The optimization problem (10) is a primary problem with constraints that allow solutions from the set $\Psi = \{\mathbf{d}_X : \mathbf{B}\mathbf{d}_X + \Delta = 0\}$. When solving the discussed problem, the primary problem is transformed into a dual problem without limits, which can be written as:

$$
\min\{\mathbf{V}^T\mathbf{M}\mathbf{V} - 2\mathbf{\kappa}^T(\mathbf{B}\mathbf{d}_X + \Delta)\} = \mathbf{d}_X^T\mathbf{M}\mathbf{d}_X. \tag{11}
$$

The solution to the dual problem (11) is the estimator:

$$
\hat{\mathbf{d}}_X = \mathbf{M}^{-1}\mathbf{B}^T\mathbf{\kappa}, \tag{12}
$$

where $\mathbf{\kappa}$ is a correlate vector designated as:

$$
\mathbf{\kappa} = -\left(\mathbf{B}\mathbf{M}^{-1}\mathbf{B}^T\right)^{-1}\Delta. \tag{13}
$$

Ultimately, the estimator of increments to approximate coordinates is given by the following formula:

$$
\hat{\mathbf{d}}_X = -\mathbf{M}^{-1}\mathbf{B}^T(\mathbf{B}\mathbf{M}^{-1}\mathbf{B}^T)^{-1}\mathbf{A}^T\mathbf{P}\mathbf{L}, \tag{14}
$$

while the covariance matrix can be written in general form:

$$
\mathbf{C}_X = m_0^2\mathbf{M}^{-1}, \tag{15}
$$

where: $m_0^2$ is an estimate of the coefficient of variance defined as $m_0^2 = \frac{1}{\text{trace}(\mathbf{P})}\mathbf{V}^T\mathbf{P}\mathbf{V}$.

3. Results and discussions

As an example of applying the proposed approach, the actual data obtained from the results of the geodetic monitoring of the building structure was used. The structure was a road viaduct located along Wojska Polskiego Street in Bydgoszcz. The object of measurements and analyses was represented by finite sets of fixed points, subject to periodic observations over two years [24]. Measurements of the geodetic control network points displacement were used for the tests; the method of stabilizing the measurement control network points is presented in Figures 1a and 1b. The network measurements were performed with the Rec-Elta 15 electronic total station by Zeiss with accessories. The device is characterized by the mean error of the distance measurement $\pm (3 \text{mm} + 2 \text{ppm})$ and the mean error of the angle measurement $-\pm 10''$ $(\pm 3^\circ)$. Angular measurements of the network were made from each network position using the directional method, while each side lengths were performed in both directions. The geodetic monitoring carried out at the facility included the performance of four series (epochs) of periodic measurements in the form of angular and linear observations. The baseline measurement was marked as Epoch 0, and the individual periodic measurements as Epoch 1, Epoch 2, and Epoch 3.
The discretization of the object in the form of the selection of controlled points (Figure 2a) and the observations made (Figure 2b) allowed for the determination of horizontal displacements of the controlled points, which in turn was the basis for direct geodetic monitoring on the analyzed object. Calculations of the displacements of the controlled points were made using the free alignment as presented in Section 2 and defining the geometric model of displacements based on the alignment by the classical method of the least squares with the conditions for the reference system.

The obtained results of the free alignment were compared with the results obtained from the network measurement using the classic engineering and geodetic approach made with the least-squares method. In comparison, the RMSE root of the mean square error was used [25, 26], and it was calculated for individual calculation variants in individual measurement epochs. The comparison considers the differences in the obtained aligned coordinates (Table 1) and the value of displacements for individual measurement epochs (Table 2). A graphical representation of the obtained results, in the form of the resultant horizontal displacements, for the last measurement epoch is presented in Figure 3.

**Figure 1.** Research object (a) and the method of the geodetic network points stabilization (b)

**Figure 2.** Measurement and control network points (a) and a diagram of the observations (b)

| Table 1. RMSE error values of corrections to coordinates for individual measurement epochs |
|---------------------------------------------|---------------------------------------------|
| Free alignment RMSE [mm]                  | Classical alignment using the least-squares method RMSE [mm] |
| Epoch 0                                  | 3.6                                       | 3.2 |
| Epoch 1                                  | 3.2                                       | 3.4 |
| Epoch 2                                  | 3.4                                       | 3.3 |
| Epoch 3                                  | 1.9                                       | 1.8 |
Table 2. RMSE error values of displacements of network points for individual measurement epochs

| Epoch 0-Epoch 1 | Free alignment RMSE [mm] | Classical alignment using the least-squares method RMSE [mm] |
|-----------------|-------------------------|----------------------------------------------------------|
| Epoch 0-Epoch 1 | 1.8                     | 1.5                                                      |
| Epoch 0-Epoch 2 | 3.0                     | 3.2                                                      |
| Epoch 0-Epoch 3 | 2.0                     | 1.8                                                      |

Figure 3. Resultant values of horizontal displacements in the last measurement period for free alignment (a) and least-squares alignment (b)

When analyzing the results presented in Table 1 and Table 2, it can be seen that the obtained coordinate values and horizontal displacements in individual measurement periods differ within the range not exceeding the average error of the measurements performed. The largest difference in the coordinate values obtained from the free alignment and the least-squares alignment was obtained for Epoch 0, and it was 0.4 mm. On the other hand, the greatest difference in the values of horizontal displacements was observed for the Epoch 0-Epoch 1 period and amounted to 0.3 mm. The obtained results indicate the possibility of applying the presented approach while avoiding the need to adopt fixed points in the adjustment process.

4. Conclusions

A free-form network with a non-zero external defect can be used to determine the displacements of monitored buildings, using a free reference system for description. In the measurement of displacements and deformations, such a network is a full-fledged geodetic system that allows determining internal changes in the shape of the tested object. The proposed solution allows for the determination of the aligned values of the coordinates of the measurement and control network points, thus determining the values of horizontal displacements experienced by the object.
The obtained results have several clear implications, indicating the direction of development of measurement and calculation methods used in the building structures monitoring. The most important implications of the presented research are:

- using free alignment, was avoid subjective assumptions related to the adoption of fixed points in the alignment process while maintaining a geometric model of displacements similar to that which can be obtained from the least-squares alignment with the conditions for the reference system,
- based on the numerical experiments carried out, and it can be seen that the proposed approach allows for obtaining results similar to the classic methods of equalization carried out using the least-squares method and modelling the phenomena and processes taking place on buildings subject to geodetic monitoring,
- the proposed method can be used both for the alignment of observations obtained using classical measurement methods (e.g. tachymetric measurements) and methods based on the GNSS system,
- it is worth noting that free alignment can be used not only to align measurement and control geodetic networks but also to search for outlier adjustment points.

The proposed approach to the alignment of observations made during geodetic monitoring refers to advanced measurement methods. It is a significant engineering issue in assessing the safe operation of buildings and constitutes an original contribution to the current research. Due to the satisfactory results in previous studies and the present one, in our future work, the authors plan to use the presented approach to equalize observations made in more extensive measurement networks established for large engineering facilities (e.g. areas under the influence of mining, earth dams and construction reinforced concrete or areas with landfills). The authors also plan to apply the approach to equalize observations obtained from other measurement methods (e.g. vectors obtained using GNSS measurements) and identify those with gross errors (outliers).

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