The Problem of Monopoles in the Standard and Family Replicated Models

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Abstract

The aim of the present talk is to show that monopoles cannot play any role in the Standard Model (SM), and in its usual extensions, up to the Planck scale: $M_{Pl} = 1.22 \cdot 10^{19}$ GeV, because they have a huge charge and are completely confined or screened. The possibility of the extension of the SM with a Family Replicated Gauge Group (FRGG) symmetry of the type $(SMG)^N = [SU(3)_c]^N \times [SU(2)_L]^N \times [U(1)_Y]^N$ is briefly discussed. It was shown that the Abelian monopoles (existing also in non-Abelian theories) in FRGG model have $N^*$ times smaller magnetic charge than in the SM, where $N^* = N(N+1)/2$. These monopoles can appear at the high energies in the FRGG-model and give additional contributions to the beta–functions of the renormalisation group equations for the running constants $\alpha_i(\mu)$, where $i=1,2,3$ correspond to the U(1), SU(2) and SU(3) gauge groups of the SM.

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1. The Problem of Monopoles in the Standard Model

The gauge symmetry group in the SM is:

$$SMG = SU(3)_{c(color)} \times SU(2)_{L(left)} \times U(1)_{Y(hypercharge)},$$

which describes the present elementary particle physics up to the scale $\approx 100$ GeV.

The aim of the present talk is to show that monopoles cannot be seen in the Standard Model and in its usual extensions, known in the literature, up to the Planck scale [1,2]:

$$M_{Pl} = 1.22 \cdot 10^{19} \text{ GeV},$$

because they have a huge magnetic charge and are completely confined or screened.

Supersymmetry does not help to see monopoles.

Let us consider the "electric" and "magnetic" running constants:

$$\alpha = \frac{g^2}{4\pi} \quad \text{and} \quad \tilde{\alpha} = \frac{\tilde{g}^2}{4\pi},$$

where $g$ is the coupling constant, and $\tilde{g}$ is the dual coupling constant.

In QED:

$$g = e \quad \text{electric charge},$$

$$\tilde{g} = m \quad \text{magnetic charge}.$$  

The Renormalization Group Equation (RGE) for monopoles is:

$$\frac{d(\log \tilde{\alpha}(t))}{dt} = \beta(\tilde{\alpha}).$$

Here $t$ is the evolution variable:

$$t = \log\left(\frac{\mu^2}{\mu_R^2}\right),$$

where $\mu$ is the energy scale and $\mu_R$ is the renormalisation point.

The scalar monopole beta–function is taken from the dual scalar electrodynamics by Coleman and Weinberg [3]:

$$\beta(\tilde{\alpha}) = \frac{\tilde{\alpha}}{12\pi} + \left(\frac{\tilde{\alpha}}{4\pi}\right)^2 + ... = \frac{\tilde{\alpha}}{12\pi} \left(1 + 3\frac{\tilde{\alpha}}{4\pi} + ...ight).$$

The last equation shows that the theory of monopoles cannot be considered perturbatively at least for

$$\tilde{\alpha} > \frac{4\pi}{3} \approx 4.$$
And this limit is smaller for non–Abelian monopoles.

Let us consider now the evolution of the SM running fine structure constants $\alpha_i(t)$, where $i=1,2,3$ correspond to U(1), SU(2) and SU(3) gauge groups of the SM.

The usual definition of the SM coupling constants is given in the Modified minimal subtraction scheme ($\overline{MS}$):

$$\alpha_1 = \frac{5}{3} \alpha_Y, \quad \alpha_Y = \frac{\alpha}{\cos^2 \theta_{\overline{MS}}},$$

$$\alpha_2 = \frac{\alpha}{\sin^2 \theta_{\overline{MS}}},$$

$$\alpha_3 \equiv \alpha_s = \frac{g_s^2}{4\pi},$$

where $\alpha$ and $\alpha_s$ are the electromagnetic and SU(3) fine structure constants respectively, $Y$ is the weak hypercharge, and $\theta_{\overline{MS}}$ is the Weinberg weak angle in $\overline{MS}$ scheme.

Using RGEs with experimentally established parameters, it is possible to extrapolate the experimental values of three inverse running constants $\alpha_Y^{-1}(\mu)$ and $\alpha_i^{-1}(\mu)$ (for $i=2,3$) from the Electroweak scale to the Planck scale (see Fig.1).

Assuming the existence of the Dirac relation for renormalised charges $g$ and $\tilde{g}$ [4]:

$$g\tilde{g} = 2\pi n, \quad n \in Z,$$

we have for minimal charges $n=1$ and the following expression:

$$\alpha(t)\tilde{\alpha}(t) = \frac{1}{4}. \quad (12)$$

Using this relation, it is easy to estimate (in the simple SM) the Planck scale value of $\tilde{\alpha}(\mu_{Pl})$ (minimal for $U(1)_Y$ gauge group):

$$\tilde{\alpha}(\mu_{Pl}) = \frac{5}{3} \alpha_Y^{-1}(\mu_{Pl})/4 \approx 55.5/4 \approx 14. \quad (13)$$

This value is really very big compared with our previous estimate (7) and, of course, with the critical coupling $\alpha_{crit} \approx 1$, corresponding to the confinement—deconfinement phase transition in the lattice U(1) gauge theory.

Clearly we cannot make a perturbation approximation with such a strong coupling $\tilde{\alpha}$.

It is hard for such monopoles not to be confined.

There is an interesting way out of this problem if one wants to have the existence of monopoles, namely to extend the SM gauge group so cleverly that certain selected linear combinations of charges get bigger electric couplings than the corresponding SM couplings. That could make
the monopoles which, for these certain linear combinations of charges, couple more weakly and thus have a better chance of being allowed "to exist".

An example of such an extension of the SM that can impose the possibility of allowing the existence of free monopoles is just Family Replicated Gauge Group Model (FRGGM).

2. Family Replicated Gauge Group as an extension of the Standard Model

The extension of the Standard Model with the Family Replicated Gauge Group:

$$G = (SMG)^{N_{fam}} = [SU(3)_c]^{N_{fam}} \times [SU(2)_L]^{N_{fam}} \times [U(1)_Y]^{N_{fam}}$$

was first suggested in the paper [5] and developed in the book [6] (see also the review [7]).

Here $N_{fam}$ designates the number of quark and lepton families.

If $N_{fam} = 3$ (as our theory predicts [6] and experiment confirms), then the fundamental gauge
The group \( G \) is:
\[
G = (SMG)^3 = SMG_{1st\ fam.} \times SMG_{2nd\ fam.} \times SMG_{3rd\ fam.},
\]
(15)

or
\[
G = (SMG)^3 = [SU(3)]^3 \times [SU(2)]^3 \times [U(1)]^3.
\]
(16)

A new generalization of our FRGG–model was suggested in papers [8-10].

The group :
\[
G_{\text{ext}} = (SMG \times U(1))_B^{-L} \equiv [SU(3)]^3 \times [SU(2)]^3 \times [U(1)]^3 \times [U(1)]_B^{-L}
\]
(17)

is the fundamental gauge group, which takes right-handed neutrinos and the see–saw mechanism into account. This extended model can describe all modern neutrino experiments, giving a reasonable fit to all the quark-lepton masses and mixing angles in the SM.

The group \( G_{\text{ext}} \) contains:
\[
3 \times 8 = 24 \quad \text{gluons},
\]
\[
3 \times 3 = 9 \quad \text{W-bosons},
\]

and
\[
3 \times 1 + 3 \times 1 = 6 \quad \text{Abelian gauge bosons}.
\]

The gauge group
\[
G_{\text{ext}} = (SMG \times U(1))_B^{-L} \equiv [SU(3)]^3 \times [SU(2)]^3 \times [U(1)]^3 \times [U(1)]_B^{-L}
\]
undergoes spontaneous breakdown (at some orders of magnitude below the Planck scale) to the Standard Model Group SMG which is the diagonal subgroup of the group \( G_{\text{ext}} \).

As was shown in the paper [8], 6 different Higgs fields:
\[
\omega, \rho, W, T, \phi_{WS}, \phi_{B-L}
\]
break our FRGG–model to the SM.

The field \( \phi_{WS} \) corresponds to the Weinberg–Salam Electroweak theory. Its vacuum expectation value (VEV) is fixed by the Fermi constant:
\[
< \phi_{WS} > \approx 246 \quad \text{GeV},
\]
so that we have only 5 free parameters – five remaining VEVs – to fit the experiment in the framework of the SM.

These five adjustable parameters were used with the aim of finding the best fit to experimental data for all fermion masses and mixing angles in the SM, and also to explain the neutrino oscillation experiments.

Finally, we conclude that our theory with the FRGG–symmetry is very successful in describing experiment.
3. Monopoles in the Family Replicated Gauge Group Model.

In theories with the FRGG symmetry the charge of monopoles is essentially diminished. Then monopoles can appear near the Planck scale and change the evolution of the running constants $\alpha_i(t)$.

Family replicated gauge groups of type:

\[ [SU(N)]^{N_{fam}} \]

lead to the lowering of the magnetic charge of the monopole belonging to one family:

\[ \tilde{\alpha}_{one\ family} = \frac{\tilde{\alpha}}{N_{fam}}. \] (18)

For $N_{fam} = 3$, for $[SU(2)]^3$ and $[SU(3)]^3$, we have:

\[ \tilde{\alpha}^{(2,3)}_{one\ family} = \frac{\tilde{\alpha}^{(2,3)}}{3}. \] (19)

For the family replicated group $[U(1)]^{N_{fam}}$ we obtain:

\[ \tilde{\alpha}_{one\ family} = \frac{\tilde{\alpha}}{N^*}, \] (20)

where

\[ N^* = \frac{1}{2} N_{fam} (N_{fam} + 1). \]

For $N_{fam} = 3$ and $[U(1)]^3$, we have:

\[ \tilde{\alpha}^{(1)}_{one\ family} = \frac{\tilde{\alpha}^{(1)}}{6}, \] (21)

Six times smaller!

This result was obtained previously in the paper [11].

According to the FRGGM, at some point $\mu = \mu_G < \mu_{Pl}$ (or really in a couple of steps) the fundamental group $G = G_{ext}$ undergoes spontaneous breakdown to its diagonal subgroup:

\[ G \rightarrow G_{diag.subgr.} \equiv \{ g, g, g | g \in SMG \}; \] (22)

which is identified with the usual (low–energy) group SMG.

The aim of this investigation is to show that we have the influence of monopoles with masses:

\[ M_{mon} > 10^{14} \text{ GeV}, \] (23)
if the $G$–group undergoes the breakdown to its diagonal subgroup (that is, SMG) at
\[ \mu_G \sim 10^{14} \quad \text{or} \quad 10^{15} \text{ GeV}, \]  
(24)
that is, before the intersection of $\alpha_2^{-1}(\mu)$ with $\alpha_3^{-1}(\mu)$ at $\mu \approx 10^{16}$ GeV.

In this case, in the region
\[ \mu_G < \mu < \mu_{Pl} \]
there are three $SMG \times U(1)_{B-L}$ groups for the three FRGG families, and we have a lot of new fermions, mass protected or not mass protected, belonging to usual families or to mirror ones, because \textbf{in the FRGGM the additional 5 Higgs bosons, with their large VEVs, are responsible for the mass protection of a lot of new fermions appearing in the region $\mu > \mu_G$.}

In this region we denote the total number of fermions $N_F$, which is different to $N_{fam}$.

Also the role of monopoles can be important in the vicinity of the Planck scale: they can give contributions to the corresponding beta-functions and change the evolution of $\alpha^{-1}(\mu)$.

Here it is necessary to comment: in the FRGG model, near the Planck scale, monopole charges, together with electric ones, are sufficiently small, and their $\beta$-functions can be considered perturbatively:
\[ \beta(\alpha) = \beta_2(\alpha/4\pi) + \beta_4(\alpha/4\pi)^2 + \ldots \quad (25) \]
\[ \beta(\tilde{\alpha}) = \beta_2(\tilde{\alpha}/4\pi) + \beta_4(\tilde{\alpha}/4\pi)^2 + \ldots \quad (26) \]

As was shown in the paper [4], there exists a region when both running constants $\alpha$ and $\tilde{\alpha}$ are perturbative. Approximately this region is given by the following inequalities:
\[ 0.2 \lesssim (\alpha, \tilde{\alpha}) \lesssim 1, \quad (27) \]

In this region the two-loop contribution to beta-function is not larger than 30% of the one-loop contribution, and the perturbation theory can be realized in this case.

It is very interesting that the above-mentioned region coincides with the region of critical couplings for the phase transition "confinement-deconfinement" obtained in the lattice compact QED:
\[ \alpha_{\text{crit}}^{\text{lat}} \approx 0.20 \pm 0.015, \quad (28) \]
\[ \tilde{\alpha}_{\text{crit}}^{\text{lat}} \approx 1.25 \pm 0.10, \quad (29) \]

obtained in Refs.[12-14], what confirms the idea of Ref.[11] that at the Planck scale we have the Multiple Critical Point.
4. The Evolution of Running Fine Structure Constants

Finally, we obtain the following RGEs:

\[
\frac{d(\alpha_i^{-1}(\mu))}{dt} = \frac{b_i}{4\pi} + \frac{N_M^{(i)}}{\alpha_i} \beta^{(m)}(\tilde{\alpha}_{U(1)}),
\]

where \(b_i\) are given by the following values:

\[
b_i = (b_1, b_2, b_3) = (-\frac{4N_F}{3} - \frac{1}{10}N_S, \frac{22}{3}N_V - \frac{4N_F}{3} - \frac{1}{6}N_S, 11N_V - \frac{4N_F}{3}).
\]

The integers \(N_F, N_S, N_V, N_M\) are respectively the total numbers of fermions, Higgs bosons, vector gauge fields and scalar monopoles in the FRGGM considered in our theory.

In our FRGG model we have:

\[N_V = 3,\]

because we have 3 times more gauge fields \((N_{fam} = 3)\), in comparison with the SM, and one Higgs scalar monopole in each family.

We have obtained the evolutions of \(\alpha_i^{-1}(\mu)\) near the Planck scale by numerical calculations for:

\[\mu_G = 10^{14} \text{ GeV},\]

\[M_{mon} > 10^{14} \text{ GeV},\]

\[N_F = 18,\]

\[N_S = 6,\]

\[N^{(1)}_M = 6,\]

\[N^{(2,3)}_M = 3.\]

Fig.2 shows the existence of the unification point.

In this connection, it is very attractive to include gravity. The quantity:

\[\alpha_g = \left(\frac{\mu}{\mu_{Pl}}\right)^2\]

plays the role of the running "gravitational fine structure constant" and the evolution of its inverse is presented in Fig.2 together with the evolutions of \(\alpha_i^{-1}(\mu)\).
In Fig. 2 we see that in the region \( \mu > \mu_G \) a lot of new fermions and a number of monopoles near the Planck scale change the one-loop approximation behaviour of \( \alpha_i^{-1}(\mu) \) which we had in the SM.

In the vicinity of the Planck scale these evolutions begin to decrease, as the Planck scale \( \mu = M_{Pl} \) is approached, implying the suppression of asymptotic freedom in the non-Abelian theories.

Fig. 2 gives the following Planck scale values of \( \alpha_i \):

\[
\alpha_1^{-1}(\mu_{Pl}) \approx 13, \\
\alpha_2^{-1}(\mu_{Pl}) \approx 19, \\
\alpha_3^{-1}(\mu_{Pl}) \approx 24.
\]

Fig. 2 demonstrates the unification of all gauge interactions, including gravity (the intersection of \( \alpha_y^{-1} \) with \( \alpha_i^{-1} \)), at

\[
\alpha_{GUT}^{-1} \approx 27 \quad \text{and} \quad x_{GUT} \approx 18.4.
\]
It is easy to calculate that for one family we have:

\[ \tilde{\alpha}_{\text{GUT,one fam.}} = \frac{\tilde{\alpha}_{\text{GUT,one fam.}}}{4} = \frac{\tilde{\alpha}_{\text{GUT}}}{4 \cdot 6} \approx \frac{27}{24} \approx 1.125, \]  

(37)

and

\[ \alpha_{\text{GUT,one fam.}} \approx 0.22, \]  

(38)

what means that at the GUT scale electric and monopole charges are not large and can be considered perturbatively.

Here we can expect the existence of

\[ [SU(5)]^3 \] or \[ [SO(10)]^3 \]

SUSY or not SUSY unification.

If SUSY, then we have superparticles of masses:

\[ M \approx 10^{18.4} \text{ GeV}. \]  

(39)

The scale \( \mu_{\text{GUT}} = M \), given by Eq.(39), can be considered as a SUSY breaking scale.

Considering the predictions of such a theory for the low-energy physics and cosmology, maybe in future we shall be able to answer the question:

"Does the unification of [SU(5)]\(^3\) or [SO(10)]\(^3\), SUSY or not SUSY, really exist near the Planck scale?"

Recently P. Ramond and Fu-Sin Ling [15] have considered [SO(10)]\(^3\) group of symmetry and have shown that it explains the observed hierarchies of fermion masses and mixings.

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