The influence of network structure on quality of satellite corrections for precise point positioning in GNSS

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Abstract. Network solution and user solution for precise point positioning in global navigation satellite systems are considered in the study. GPS ionosphere-free observation model with decoupled clocks is used for applying the procedure of ambiguity resolution of phase measurements. Four different variants of network solution based on European stations from a network of International GNSS service are considered. The other two variants of network solution are based on stations from a network of Russian satellite based augmentation system. Six investigated variants of network solution are varied in number of stations, total area, receiver types, configuration and measurements quality. Network products computed in different variants of network solution are then used for the user solution with applying the procedure of ambiguity resolution of phase measurements. The influence of network structure on accuracy and convergence time of precise point positioning with ambiguity resolution of phase measurements is analyzed. The properties of network structure are listed and described. Obtained precise point positioning results are averaged and discussed.

1. Introduction

Recently precise point positioning methods (PPP) [1] have been actively being developed in global navigation satellite systems (GNSS). The key features of PPP methods are using of precise ephemerides, compensation of multiple geophysical effects and applying of ambiguous phase measurements. Nowadays traditional ionosphere-free observation model with phase ambiguities estimated as real values is well developed (Float PPP methods, [2]). Main disadvantage of these methods is long convergence time. More complex methods which save integer nature of phase ambiguities and support the procedure of their ambiguity resolution are in progress currently (Integer PPP or PPP-AR methods [3]). Convergence time for PPP-AR methods is much shorter. The main problem of these methods is rank deficiency of corresponding system of linearized equations. There is a rank issue because of equipment biases in mathematical models of measurement and necessity to estimate phase ambiguities as integers. There are a few published PPP-AR approaches [4-6]. Author uses approach suggested in Natural Resources Canada which is based on Decoupled clock model [6].

Two types of solution can be distinguished for PPP in GNSS: network solution and user solution. Hereinafter “user solution” is determination of the user receiver coordinates in the PPP-AR methods with available precise satellite corrections. “Network solution” stands for determination of mentioned...
satellite corrections by common processing of code and carrier phase measurements formed by a network of ground stations with known coordinates. Precise satellites corrections (network products) are transmitted to user by means of communication lines. Approaches [4-6] differ in that part of network products which consists of satellite clocks and satellite equipment biases. Precise satellite coordinates are necessary for any approach. It is shown in [7] that in user solution network products corresponding to different approaches [4-6] can be strictly converted from one to another. In practice, however, there are some errors of such conversion because of specific aspects of network solutions for each approach.

In this study different variants of GPS network solution based on European stations from a network of International GNSS service (IGS) and stations from a network of Russian satellite based augmentation system (SBAS) are described. Investigated variants of network solution vary in number of stations, total area, receiver types, configuration and measurements quality. PPP user solution quality (accuracy and convergence time) with use of different network products is analyzed.

2. Methods and materials

2.1. Decoupled clock GPS observation model

Following ionosphere-free GPS observation model with decoupled clocks is used in the work [6]:

\[
\begin{align*}
\begin{bmatrix}
P_j^i \\
L_j^i \\
A_j^i
\end{bmatrix} &= \begin{bmatrix}
R_j^i + m^i \Delta D_{ew} + d t_{p3} - d t_{l3} + \varepsilon_{p3}^j \\
R_j^i + m^i \Delta D_{ew} + d t_{l3} - d t_{l3} - \lambda_3(7 N_j^i + 60 N_j^4) + \varepsilon_{l3}^j \\
b_{44} - b_{44} - \lambda_4 N_j^4 + \varepsilon_{44}^j
\end{bmatrix},
\end{align*}
\]

where \(P_j^i, L_j^i\) – code and phase ionosphere-free measurements of the j-th GPS satellite (m); \(A_j^i\) – ionosphere-free Melbourne–Wübbena linear combination of code and phase measurements of the j-th satellite (m); \(R_j^i\) – geometrical distance to j-th satellite (m); \(\Delta D_{ew}\) – uncompensated wet troposphere delay (m); \(m^i\) – mapping function for j-th satellite; \(d t_{p3}, d t_{l3}\) – code and phase ionosphere-free receiver clock offsets against GPS time scale including receiver equipment biases (m); \(b_{44}, b_{44}\) – equipment biases for receiver and j-th satellite in Melbourne–Wübbena combination (m); \(\lambda_3, \lambda_4\) – wavelengths for GPS ionosphere-free combinations \(L_j^i\) and \(A_j^i\) (m); \(N_j^i\) and \(N_j^4\) – integer ambiguities for j-th GPS satellite (cycles); \(N_j^4 = N_j^i - N_j^4\) – widelane integer ambiguities (cycles); \(\varepsilon_{p3}, \varepsilon_{l3}, \varepsilon_{44}\) - measurements errors for combinations \(P_j^i, L_j^i, A_j^i\); \(J\) - the total number of available GPS satellites.

In user solution the model (1) contains \(7 + 2J\) estimated parameters. Because of the presence of decoupled clocks in (1) the corresponding system of equations is singular and the rank deficiency equals two. In network solution the model (1) contains \(3J + 4M + 2JM\) estimated parameters (\(M\) - the number of stations in network solution). Because of the presence of decoupled clocks in (1) the corresponding system of equations is singular and the rank deficiency equals \(3 + 2(J + M - 1)\). According to the basic property of singular GNSS linear systems described in [3, 8], part of initial estimated parameters can be estimated as linear combinations. Due to this property inconsistent and underdetermined system of equations (1) can be transformed to inconsistent and overdetermined system of equations with unique least-square solutions.

In Float PPP methods traditional ionosphere-free observation model [1] is applied and the same receiver and satellite clock offsets are used in mathematical models of ionosphere-free code and phase measurements (unlike the model (1) with different clock offsets for \(P_j^i, L_j^i, A_j^i\) combinations). In this case ambiguity of phase measurement absorbs equipment biases and becomes real value estimate, rank
deficiency disappears. As a result, Float PPP methods are simple in practice implementation (both network and user solutions), but they converge to 1 cm positioning accuracy longer PPP-AR methods [4-6].

2.2. Aspects determining accuracy and convergence time of precise point positioning

Let us consider the aspects which influence accuracy and convergence time of precise point positioning using ionosphere-free observation model with decoupled clocks (1) (meaning the aspects connected with quality of network products):

- **Statistical characteristics of network products** (standard deviation values of the satellite corrections $dt_{\text{p}_i}^j, dt_{\text{l}_i}^j, b_{\text{A}_i}^j, j = \overline{1, J}$). During the first 1-2 hours of network solution the satellite corrections $dt_{\text{p}_i}^j, dt_{\text{l}_i}^j, b_{\text{A}_i}^j, j = \overline{1, J}$ still have convergence period. The user needs to use some threshold values for standard deviations of $dt_{\text{p}_i}^j, dt_{\text{l}_i}^j, b_{\text{A}_i}^j, j = \overline{1, J}$, which can indicate that convergence period is over and satellite corrections can be applied to precise point positioning. The issue here is that mentioned satellite corrections may have different convergence period depending on network configuration, quality of measurements and elevation angles of observed satellites.

- **Network configuration of stations** (number of stations, ranges between them and relative positions). Given a small number of stations with relative ranges 500-1000 km (local network solution), the quantity of network products for high elevation satellites can be not enough for PPP with ambiguity resolution of phase measurements. Increasing the number of stations and extending the network may lead to the lack of uniform (homogeneous) station receivers and their ineffective (irregular) relative distributing. Ineffective relative distributing of the stations inside the network also decreases the quality of the network products because from some areas of the network some satellites can be observed just with low elevation angles. Regular distributing of stations for network solution can not always be guaranteed. For instance, density of IGS stations in Russia is extremely low compared to Europe. In Russia to the north of Yekaterinburg (56 degrees north latitude) there are just five IGS stations, and three of them are located closely to the continental edges, i.e. near the edges of basic servicing area in Russia (stations in Tiksi, Saint Petersburg and Magadan).

- **Uniformity of receivers at network stations**. Uniformity (homogeneity) of receivers is based on one the same manufacturer, receiver model, list of measurement type and the same firmware version. It’s difficult to fulfill this requirement with using open access IGS stations measurements. For this reason commercial network products providers choose to set up their own network consisting of uniform receivers. In Float PPP network solution it’s admissible to process measurements from stations with different receivers because traditional ionosphere-free observation model is used for Float PPP methods. It’s not the case for PPP-AR methods.

- **Ineffective location of one/several network stations**. In city area a lot of other powerful receivers and transmitters can operate not far from station receiver (communication systems, airport antenna systems, relay stations and others). As a result, multipath and noise errors can be high, and certain sector of visibility can be shadowed at all. Out of city area GNSS-stations are usually disposed at meteorological and geodetic buildings with limited space and probable shadow region. There are not so many stations with perfect location in terms of all the requirements. For instance, there are 2-3 IGS stations of high quality measurements and low multipath errors which are typically used as reference for network solution with traditional ionosphere-free observation model by IGS Analysis Centers. If station is located in not applicable place, it results in increased noise level and probable systematic biases in measurements which contribute an additional error to the network products.

- **User position inside the network and distance to stations**. Remoteness of the user from the network stations means that observed by the user satellites are visible with low elevation angles from the network stations, so precise satellite corrections can be noisy. PPP quality (accuracy, convergence time, reliability) for the user located at the edge of the network is usually worse compared to user at the center of the network. Most of observed satellites for such near-border users are observed from stations...
with low elevation angles. It reduces the quality of precise satellites corrections and user PPP quality for the locations near the edge of network.

- **Quality of measurements from network stations.** The quality of measurements used for the network solution is determined by their availability, continuity, low noise error and absence of systematic biases, multipath errors and cycle slips.

  When the user applies the procedure of ambiguity resolution (fixed solution), in addition to requirements to accuracy and convergence time it appears the requirement to fixed solution reliability. The reliability of a fixed solution is connected with the value of probability of correct ambiguity resolution of phase measurements during the processing. It’s determined by the quality of network products, i.e. by the all aspects described above.

3. Results and discussion

3.1. Comparative analysis of user positioning using precise satellite corrections obtained from different network configurations

Ultimate (best) results of user PPP accuracy and convergence time were estimated in [3] for the case of network solution with permanent satellite constellation (the simplest case). Three-dimensional error of user positioning not exceeding 1-3 cm is achieved in this case after 3-5 minutes of processing of measurements with 30 sec interval. Strict consideration of changing satellite constellation is quite complicated because of bulkiness of S-transformations theory [9, 10] applied for PPP-AR methods. For this reason obtained PPP-AR results for changing satellite constellation in terms of accuracy and convergence time are worse that mentioned above best results in current conditions (network solution is not optimal).

Four variants of network solution with European IGS stations and two variants of network solutions with Russian SBAS system stations were implemented. Network configuration and specific features for these variants are described in further:

1. Network solution with 10 IGS stations (“Network solution 1” in further) where the receivers are from the same manufacturer. Despite of a high density of IGS stations in Europe, it was impossible to choose 10 stations with good ranges and relative positions for any manufacturer. Network configuration for this variant is shown on figure 1 where 10 used stations are marked with red circles.

2. Network solution with 8 IGS stations (“Network solution 2” in further). In this variant of network solution two the most distant stations from the others shown on figure 1 were excluded. On figure 1 two excluded stations are marked as 1 and 2. Remaining 8 stations were used for processing.

3. Network solution with 8 IGS stations (“Network solution 3” in further). In this variant of network solution two stations with the most number of cycle slips and gaps in measurements (the worst stations in terms of quality of measurements) were excluded from the stations shown on figure 1. On figure 1 two excluded stations are marked as 3 and 4. Remaining 8 stations were used for processing.

4. Network solution with 6 IGS stations (“Network solution 4” in further). In this variant of network solution two stations with the most number of cycle slips and gaps in measurements and two the most distant stations from the others were excluded from the stations shown on figure 1. On figure 1 four excluded stations are marked as 1, 2, 3 and 4. Remaining 6 stations were used for processing.

5. Network solution with 10 stations of Russian SBAS system (“Network solution 5” in further) where the receivers are from the same manufacturer. Network configuration for this variant is shown on Figure 2 where 10 used stations are marked with red circles.

6. Network solution with 7 stations of Russian SBAS system (“Network solution 6” in further). In this variant of network solution three stations which are not uniform with others and which have the most noisy measurements were excluded from the stations shown on figure 2. On figure 2 three excluded stations are marked as 1, 2 and 3. Remaining 7 stations were used for processing.
Currently the quality of computed decoupled satellite corrections is not stable (PPP convergence time can significantly vary for different receivers and time periods). For this reason to illustrate in some way the current quality of PPP based on network products from considered variants of network solution averaging is applied. Figure 3 shows the averaged three-dimensional errors of PPP-AR fixed and float user solutions corresponding to four variants of network solution with IGS stations. Measurements from five IGS stations not included to corresponding network solutions were used as user measurements. The averaging among the described five stations and among five 50-minutes intervals was applied. On Figure 3 the curve “1” corresponds to the user fixed solution for Network solution 1; the curve “2” corresponds to the user float solution for Network solution 2; the curve “3” corresponds to the user float solution for Network solution 1; the curve “4” corresponds to the user float solution for Network solution 3; the
curve “5” corresponds to the user float solution for Network solution 4 (on figure 3 the most specific curves are shown). The following features can be noted:

- The quality of network products computed in Network solution 1 is the worst (fixed user solution is considerably worse than float user solution in terms of accuracy and convergence time);
- The quality of fixed and float user solution is comparable for the network products from Network solution 2 and Network solution 3;
- Among the considered four variants of the network solution the best quality of network products is provided by Network solution 4 because corresponding fixed and float user solutions have the best accuracy and convergence time. In spite of the fact that stations for Network solution 2 look denser on figure 1 compared to stations for Network solution 4, in case of Network solution 2 two the worst stations (in terms of quality of measurements) degraded obtained PPP results.

Figure 3. Float and fixed solution of averaged. Three-dimensional error of user positioning obtained with network products from network solution with IGS stations.

Figure 4 shows averaged Three-dimensional error of user positioning similar to one on Figure 3 but for network solution based on Russian SBAS system. Averaging among the five stations and among the five 50-minutes intervals was applied. On Figure 4 the curve “1” corresponds to the user fixed solution for Network solution 5; the curve “2” corresponds to the user fixed solution for Network solution 6; the curve “3” corresponds to the user float solution for Network solution 5; the curve “4” corresponds to the user float solution for Network solution 6; Following features can be noted:

- After 30 min of processing averaged Three-dimensional error of user positioning for all four solutions shown on figure 4 are almost concurred;
- The shortest convergence time during the first 30 min of processing is provided by float user solutions with network products from Network solution 5 and Network solution 6;
- During the first 30 min of processing fixed user solution with using network products from Network solution 6 outstrips fixed user solution with using network products from Network solution 5;
3.2. Reliability of fixed solution

Positioning quality for PPP-AR methods is determined not only by stable, quick and correct ambiguity resolution but also by the difference between fixed (with use of ambiguity resolution procedure) and float (without ambiguity resolution procedure) user solutions. On other words, it is of great interest what is contribution of ambiguity resolution procedure to accuracy and convergence time of the user solution. Figures 5 and 6 show three-dimensional error of PPP-AR user positioning (fixed and float solutions). It's clear on figure 5 that fixed solution is reliable since 5 min of measurements processing, it's provide significant improving of convergence time compared with float solution. Figure 6 shows another case for comparison: convergence time for fixed and float solutions is comparable. It means that the procedure of ambiguity resolution hasn't brought any contribution in terms of accuracy and convergence time of user solution.

Figure 4. Float and fixed solution of averaged. Three-dimensional error of user positioning obtained with network products from network solution with Russian SBAS stations.

Figure 5. User 3D positioning error, example of reliable fixed solution.

Figure 6. User 3D positioning error, example of unreliable fixed solution.
4. Conclusion
With help of six variants of network solution based on the measurements from IGS network and Russian SBAS network stations the influence of network structure on quality of PPP solution with ionosphere-free decoupled clock observation model was analyzed. It has been figured out that the quality of PPP results is implicitly depends on the network configuration and noisy measurement of certain stations.

For the structure of network involved in the satellite decoupled corrections computing the list of aspects which determine the accuracy and convergence time of user positioning was identified. It was shown that for considered IGS and Russian SBAS system stations there were no way to use uniform receivers in network solution. It leads to reducing of decoupled satellite corrections accuracy and user positioning accuracy. Currently positioning accuracy inside the network for near-border users is significantly influenced by unreliability of ambiguity resolution procedure. In practice, it's not always possible to check PPP quality with receivers which were not included to the network solution (because of extremely low density of IGS stations in Russia).

References
[1] Kouba J 2009 Guide to Using International GNSS Service (IGS) Products, Jet Propulsion Lab., Pasadena, available at: http://igscb.jpl.nasa.gov/components/usage.html
[2] Gao Y, Shen X 2002 A new method for carrier phase based precise point positioning. Navigation 49(2) 109 doi: 10.1002/j.2161-4296.2002.tb00260.x
[3] Podkorytov A N 2014 Precise point positioning in global navigation satellite systems due to ambiguity resolution of phase measurements, PhD thesis, Moscow Aviation Institute (National Research University)
[4] Li P, Zhang X, Ren X, Zuo X, Li Y, Pan Y 2016 Generating GPS satellite fractional cycle bias for ambiguity-fixed precise point positioning. GPS Solutions 20(4) 771 doi: 10.1007/s10291-015-0483-z
[5] Laurichesse D, Mercier F, Berthias J-P, Broca P, Cerri L 2009 Integer Ambiguity Resolution on Undifferenced GPS Phase Measurements and Its Application to PPP and Satellite Precise Orbit Determination. Navigation 56(2) 135 doi: 10.1002/j.2161-4296.2009.tb01750.x
[6] Collins P, Lahaye F, Héroux P, Bisnath S Precise Point Positioning with Ambiguity Resolution using the Decoupled Clock Model. Proc. of ION-GNSS-2008 (Savannah, Georgia, 16-19 September 2008) pp 1315-1322
[7] Seepersad G, Bisnath S 2017 An assessment of the interoperability of PPP-AR network products. The Journal of Global Positioning Systems 15(4) doi:10.1186/s41445-017-0009-9
[8] Povalyayev A A, Podkorytov A N 2015 Precise Point Positioning in Global Navigation Satellite Systems with Ambiguity Resolution of Phase Measurements. J. Commun. Technol. Electron. 60(860) doi: 10.1134/S1064226915070141
[9] Teunissen P J G 1985 Zero Order Design: Generalized Inverses, Adjustment, the Datum Problem and S-Transformations. Optimization and Design of Geodetic Networks (Berlin: Springer-Verlag) pp 11-55
[10] De Jonge P J 1998 A processing strategy for the application of the GPS in networks. (Netherlands Geodetic Commission, Delft, Netherlands)