Bottom up approach to Quantum Gravity

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ABSTRACT: A general introduction is given to what can be predicated about quantum gravity once the lessons from the standard model of particle physics are taken into account. In particular, the effective lagrangian point of view is briefly commented upon.
Contents

1. Introduction .................................................. 2
2. Symmetries and observables ............................. 3
3. The Effective Lagrangian Approach to Quantum Gravity 5
1. Introduction

Can we assert some proposition about quantum gravity with some confidence?

The mass scale associated to this problem just by sheer dimensional analysis is Planck’s mass, which is given in terms of Newton’s constant, $G$ by

$$m_p \equiv \sqrt{\frac{\hbar c}{8\pi G}} \sim G^{-1/2} \sim 10^{19} \text{GeV}$$  \hspace{1cm} (1.1)

(we shall always work in units so that $\hbar = c = 1$). If we remember that $1 \text{GeV} (= 10^3 \text{MeV})$ is the rough scale of hadronic physics (the mass and inverse Compton wavelength of a proton, for example), this means that quantum gravity effects will only be apparent when we are able to explore concentrated energy roughly $10^{19}$ times bigger (or an scale distance correspondingly smaller; these two statements are supposed to be equivalent owing to Heisenberg’s principle). To set the scale, the Large Hadron Collider works roughly at the $\text{TeV} (= 10^3 \text{GeV})$ scale, so there is a long way to go before reaching expected quantum gravity effects in accelerators.

If we want to get experimental information, we have to turn our attention towards Cosmology, or perhaps look for some clever precision experiment in the laboratory. Lacking any experimental clue, the only thing we can do is to think and try to look for logical (in)consistencies.

It has been repeatedly argued by many particle physicists that the practical utility of the answer to this question will not presumably be great. How would we know for sure beforehand? There has always been a recurrent dream, exposed vehemently by Salam [18] that the inclusion of the gravitational interaction would cure many of the diseases and divergences of quantum field theory, through the inclusion in the propagator of terms of the type

$$e^{-\frac{1}{m_p^2 r^2}}$$

So that for example, the sum of tree graphs that leads to the Schwarzschild solution as worked out by Duff [8]

$$\frac{1}{r} + \frac{2M}{m_p^2 r^2} + \frac{4M^2}{m_p^4 r^3} + \ldots$$

would get modified to

$$\frac{1}{r} e^{-\frac{1}{m_p^2 r}} + \frac{2M}{m_p^2 r^2} e^{-\frac{2}{m_p^2 r}} + \ldots \sim \frac{1}{r e^{-\frac{1}{m_p^2 r}} - 2 \frac{M}{m_p^4}}$$

shifting the location of the horizon and eliminating the singularity at $r = 0$. Nobody has been able to substantiate this dream so far.

Any ant rate quantum gravity is nevertheless a topic which has fascinated whole generations of physicists, just because it is so difficult. There seems to be a strong
tension between the beautiful, geometrical world of General Relativity and the no
less marvelous, less geometrical, somewhat mysterious, but very well tested exper-
imentally, world of Quantum Mechanics.

As with all matters of principle we can hope to better understand both quantum
mechanics and gravitation if we are able to clarify the issue.

The most conservative approach is of course to start from what is already known
with great precision about the standard model of elementary particles associated
to the names of Glashow, Weinberg and Salam. This can be called the bottom-up
approach to the problem. In this way of thinking Wilson taught us that there is
a working low energy effective theory, and some quantum effects in gravity can be
reliably computed for energies much smaller than Planck mass. There are two caveats
to this. First of all, we do not understand why the observed cosmological constant
is so small: the natural value from the low energy effective lagrangian point of view
ought to be much bigger. The second point is that one has to rethink again the lore
of effective theories in the presence of horizons. We shall comment on both issues in
due time.

There is not a universal consensus even on the most promising avenues of research
from the opposite top-down viewpoint. Many people think that strings [17] are
the best buy (I sort of agree with this); but it is true that after more than two
decades of intense effort nothing substantial has come out of them. Others [16] try
to quantize directly the Einstein-Hilbert lagrangian, something that is at variance
with our experience in effective field theories. But it is also true that as we have
already remarked, the smallish value of the observed cosmological constant also cries
out of the standard effective theories lore.

2. Symmetries and observables

It is generally accepted that General Relativity, a generally covariant theory, is akin
to a gauge theory, in the sense that the diffeomorphism group of the apace-time
manifold, \( \text{Diff}(M) \) plays a role similar to the compact gauge group in the standard
model of particle physics. There are some differences though. To begin with, the
group, \( \text{Diff}(M) \) is too large; is not even a Lie group [13]. Besides, its detailed struc-
ture depends on the manifold, which is a dynamical object not given a priori. Other
distinguished subgroups (such as the area-preserving diffeomorphisms,[2]) are per-
haps also arguable for. Those leave invariant a given measure, such as the Lebesgue
measure, \( d^nx \).

It also seems clear that when there is a boundary of space-time, then the gauge
group is restricted to the subgroup consisting on those diffeomorphisms that act
trivially on the boundary. The subgroup that act not-trivially is related to the set
of conserved charges. In the asymptotically flat case this is precisely the Poincaré
group, \( SO(1,3) \) that gives rise to the ADM mass.
In the asymptotically anti-de Sitter case, this is presumably related to the conformal group $SO(2, 3)$.

It is nevertheless not clear what is the physical meaning of keeping constant the boundary of spacetime (or keeping constant some set of boundary conditions) in a functional integral of some sort.

A related issue is that it is very difficult to define what could be observables in a diffeomorphism invariant theory, other than global ones defined as integrals of scalar composite operators $O(\phi_a(x))$ (where $\phi_a, a = 1 \ldots N$ parametrizes all physical fields) with the pseudo-riemannian measure

$$O \equiv \int \sqrt{|g|} d^4x O(\phi_a(x))$$

Some people claim that there are no local observables whatsoever, but only pseudolocal ones [10]; the fact is that we do not know. Again, the exception to this stems from keeping the boundary conditions fixed; in this case it is possible to define an $S$-matrix in the asymptotically flat case, and a conformal quantum field theory (CFT) in the asymptotically anti-de Sitter case. Unfortunately, the most interesting case from the cosmological point of view, which is when the space-time is asymptotically de Sitter is not well understood.

Incidentally, it is well known that the equivalence problem in four-dimensional geometries is undecidable [3]. In three dimensions Thurston’s geometrization conjecture has recently been put on a firmer basis by Hamilton and Perelman, but it is still not clear whether it can be somehow implemented in a functional integral without some drastic restrictions. Those caveats should be kept in mind when reading the sequel.

A radically different viewpoint has recently been advocated by ’t Hooft [19] by insisting in causality to be well-defined, so that the conformal class of the space-time metric should be determined by the physics, but not necessarily the precise point in a given conformal orbit. If we write the spacetime metric in terms of a unimodular metric and a conformal factor

$$g_{\mu\nu} = \omega^2(x)\hat{g}_{\mu\nu}$$

with

$$det \hat{g}_{\mu\nu} = 1$$

then the unimodular metric is in some sense intrinsic and determines causality, whereas the conformal factor depends on the observer in a way dictated by black hole complementarity.

Finally, there is always the (in a sense, opposite) possibility that space-time (and thus diffeomorphism invariance) is not a fundamental physical entity in such a way that the appropriate variables for studying short distances are non geometrical. Something like that could happen in string theory, but our understanding of it is still in its infancy.
3. The Effective Lagrangian Approach to Quantum Gravity

The main purpose of this talk is to review the least ambitious approach possible to the topic. Actually, one can discuss what is the notion of causality when the spacetime metric is a fluctuating object, or whether there is an adequate notion of time to be used in writing down Schrödinger’s equation for the Universe itself. But if our previous experience with the other interactions is to be of any relevance here, there ought to be a regime, experimentally accessible in the not too distant future, in which gravitons propagating in flat spacetime can be isolated. This is more or less unavoidable, provided gravitational waves are discovered experimentally, and the road towards gravitons should not be too different from the road that lead from the discovery of electromagnetic waves to the identifications of photons as the quanta of the corresponding interaction, a road that led from Hertz to Planck.

Any quantum gravity theory that avoids identifying gravitational radiation as consisting of large numbers of gravitons in a semiclassical state would be at variance with all we believe to know about quantum mechanics.

What we expect instead to be confirmed by observations somewhere in the future is that the number of gravitons per unit volume with frequencies between $\omega$ and $\omega + d\omega$ is given by Planck’s formula

$$n(\omega)d\omega = \frac{\omega^2}{\pi^2} \frac{1}{e^{\frac{\omega}{kT}} - 1} d\omega$$

It is natural to keep an open mind for surprises here, because it can be argued that gravitational interaction is not alike any other fundamental interaction in the sense that the whole structure of space-time ought presumably be affected, but it cannot be denied that this is the most conservative approach and as such it should be explored first, up to its very limits, which could hopefully indicate further avenues of research.

From our experience then with the standard model of elementary particles, and assuming we have full knowledge of the fundamental symmetries of our problem, we know that we can parametrize our ignorance on the fundamental ultraviolet (UV) physics by writing down all local operators in the low energy fields $\phi_i(x)$ compatible with the basic symmetries we have assumed.

$$L = \sum_{n=0}^{\infty} \frac{\lambda_n(\Lambda)^n}{\Lambda^n} O^{(n+4)} (\phi_i)$$

Here $\Lambda$ is an ultraviolet (UV) cutoff, which restricts the contributions of large euclidean momenta (or small euclidean distances) and $\lambda_n(\Lambda)$ is an infinite set of dimensionless bare couplings.

Standard Wilsonian arguments imply that irrelevant operators, corresponding to $n > 4$, are less and less important as we are interested in deeper and deeper
infrared (IR) (low energy) variables. The opposite occurs with relevant operators, corresponding to \( n < 4 \), like the masses, that become more and more important as we approach the IR. The intermediate role is played by the marginal operators, corresponding to precisely \( n = 4 \), and whose relevance in the IR is not determined solely by dimensional analysis, but rather by quantum corrections. The range of validity of any finite number of terms in the expansion is roughly

\[
\frac{E}{\Lambda} \ll 1
\]

where \( E \) is a characteristic energy of the process under consideration.

In the case of gravitation, we assume that general covariance (or diffeomorphism invariance) is the basic symmetry characterizing the interaction. We can then write

\[
L_{\text{eff}} = \lambda_0 \Lambda^4 \sqrt{|g|} + \lambda_1 \Lambda^2 R \sqrt{|g|} + \lambda_2 R^2 + \frac{1}{2} g^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi \sqrt{|g|} + \lambda_3 \frac{1}{\Lambda^2} R^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi \sqrt{|g|} + \lambda_4 \frac{1}{\Lambda^4} R^4 \sqrt{|g|} + \lambda_5 \phi^4 \sqrt{|g|} + \nonumber \\
+ \bar{\psi} (e^\mu_\alpha \gamma^\alpha (\partial_\mu - \omega_\mu) \psi - m) \psi + \frac{\lambda_5}{\Lambda^2} \bar{\psi} e^\mu_\alpha \gamma^\alpha R (\partial_\mu - \omega_\mu) \psi + \ldots
\] (3.1)

where \( e^\mu_\alpha \) is the tetrad, such that

\[ e^\mu_\alpha e^\nu_\beta \eta^{\alpha\beta} = g^{\mu\nu} \]

\( \eta^{\alpha\beta} \) being Minkowski’s metric. The quantities \( \omega_\mu \) are the spin connection.

The need to recover General Relativity in the classical IR limit means

\[
\lambda_1 \Lambda^2 = -\frac{c^3}{16\pi G} \equiv -2M_p^2
\]

This in turn, means that if

\[
\lambda_0 \Lambda^4
\]

is to yield the observed value for the cosmological constant (which is of the order of magnitude of Hubble’s constant, \( H_0^4 \)), which is a very tiny figure when expressed in particle physics units, \( H_0 \sim 10^{-33} eV \) then

\[
\lambda_0 \sim 10^{-244}
\]

This is one aspect of the cosmological constant problem; it seems most unnatural that the cosmological constant is observationally so small from the effective lagrangian point of view. I do not have anything new to say on this.

This expansion is fine as long as it is considered a low energy expansion. As Donoghue [7] has emphasized, even if it is true that each time that a renormalization is made there is a finite arbitrariness, there are physical predictions stemming from the non-local finite parts.
The problem is when energies are reached that are comparable to Planck’s mass,

\[ E \sim M_p. \]

Then all couplings in the effective Lagrangian become of order unity, and there is no *decoupling limit* in which gravitation can be considered by itself in isolation from all other interactions. This then seems the minimum prize one has to pay for being interested in quantum gravity; all couplings in the derivative expansion become important simultaneously. No significant differences appear when supergravity is considered.

In conclusion, it does not seem likely that much progress can be made by somehow quantizing Einstein-Hilbert’s Lagrangian in isolation. To study quantum gravity means to study all other interactions as well.

On the other hand, are there any reasons to go beyond the standard model (SM)?

Yes there are some, both theoretical, and experimental. From the latter, and most important, side, both the existence of neutrino masses and dark matter do not fit into the SM. And from the former, abelian sectors suffer from Landau poles and are not believed to be UV complete; likewise the self-interactions in the Higgs sector appear to be a trivial theory. Also the experimental values of the particle masses in the SM are not natural from the effective lagrangian point of view.

The particle physics community has looked thoroughly for such extensions since the eighties: extra dimensions (Kaluza-Klein), supersymmetry and supergravity, technicolor, etc. From a given point of view, the natural culmination of this road is string theory

A related issue is the understanding of the so-called *semiclassical gravity*, in which the second member of Einstein’s equations is taken as the expectation value of some quantum energy-momentum operator. It can be proved that this is the dominant \( 1/N \) approximation in case there are \( N \) identical matter fields (confer [11]). In spite of tremendous effort, there is not yet a full understanding of Hawking’s emission of a black hole from the effective theory point of view (confer, for example, [12]). Another topic in which this approach has been extensively studied is Cosmology. Novel effects (or rather old ones on which no emphasis was put until recently) came from lack of momentum conservation and seem to point towards some sort of instability [14]; again the low energy theory is not fully understood; this could perhaps have something to do with the presence of horizons.

Coming back to our theme, and closing the loop, what are the prospects to make progress in quantum gravity?

Insofar as effective lagrangians are a good guide to the physics there are only two doors open: either there is a ultraviolet (UV) attractive fixed point in coupling space, such as in Weinberg’s *asymptotic safety* [20] or else new degrees of freedom, like in string theory [17] exist in the UV. Even if Weinberg’s approach is vindicated, the fact that the fixed point most likely lies at strong coupling combined with our
present inability to perform analytically other than perturbative computations, mean
that lattice simulations should be able to cope with the integration over (a subclass
of) geometries before physical predictions could be made with the techniques at hand
at the present moment.

It is to be remarked that sometimes theories harbor the seeds of their own de-
struction. Strings for example, begin as theories in flat spacetime, but there are
indications that space itself should be a derived, not fundamental concept. It is
hoped that a simpler formulation of string theory exists bypassing the roundabouts
of its historical development. This is far from being the case at present.

Finally, it is perhaps worth pointing out that to the extent that a purely graviti-
tational canonical approach, as the ones based upon the use of Ashtekar [5] variables
makes contact with the classical limit (which is an open problem from this point of
view) the preceding line of argument should still carry on.

It seems unavoidable with our present understanding, that any theory of quantum
gravity should recover, for example, the prediction that there are quantum corrections
to the gravitational potential given by [4]

\[ V(r) = -\frac{Gm_1 m_2}{r} \left( 1 + 3 \frac{G(m_1 + m_2)}{r} + \frac{41}{10\pi} \frac{G\hbar}{r^2} \right) \]

(the second term is also a loop effect, in spite of the conspicuous absence of \(\hbar\).) Similarly, and although this has been the subject of some controversy, it seems now
established that there are gravitational corrections to the running of gauge couplings,
first uncovered by Robinson and Wilczek [15] and given in standard notation by

\[ \beta(g, E) = -\frac{b_0}{(4\pi)^2} g^3 - 3\frac{16\pi G}{(4\pi)^2\hbar c^3} g E^2 \]

Sometimes these effects are dismissed as perturbative, and therefore trivial. This is
not a healthy attitude.

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