An explanation of Belle states $Z_b(10610)$ and $Z_b(10650)$

D V Bugg

Queen Mary, University of London, London E14NS, UK

Abstract

Belle report data on $\Upsilon(5S) \rightarrow \Upsilon(1S, 2S, 3S)\pi^+\pi^-$ and $\chi_b(1P, 2P)\pi^+\pi^-$; they observe peaks in $\Upsilon\pi^\pm$ and $\chi_b\pi^\pm$ consistent with $J^P = 1^+$. They interpret the peaks as molecular states $Z_b(10608)$ and $Z_b(10653)$. Their masses are just above $\bar{B}B^*$ and $\bar{B}^*B^*$ thresholds at 10604.6 and 10650.2 MeV. An explanation in terms of cusps at these thresholds is presented here. The product of the rising phase space for $\bar{B}B^*$ and $\bar{B}^*B^*$ with the cusps creates peaks a few MeV higher in $\bar{B}B^*$ and $\bar{B}^*B^*$, and these peaks can de-excite to $\pi^+\pi^-\Upsilon(1S, 2S, 3S)$ and $\pi^+\pi^-\chi_b(1P, 2P)$.

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Belle [1] report peaks in $\pi^\pm\Upsilon(1S, 2S, 3S)$ and $\pi^\pm\chi_b(1P, 2P)$ with masses and widths:

\begin{align}
M[Z_b(10610)] &= 10608.4 \pm 2.0 \text{ MeV} \quad \Gamma = 15.6 \pm 2.5 \text{ MeV} \quad (1) \\
M[Z_b(10650)] &= 10653.2 \pm 1.5 \text{ MeV} \quad \Gamma = 14.4 \pm 3.2\text{MeV}. \quad (2)
\end{align}

Angular distributions are consistent with $J^P = 1^+$ in both cases.

Thresholds for $B^*\bar{B}$ and $B^*\bar{B}^*$ are at 10604.6 MeV and 10650.2 MeV and have the same mass separation as the two $Z_b$ peaks within experimental errors. This suggests strongly that the $Z_b$ peaks are associated with those thresholds. There is an explanation how this can arise via cusps at the $B^*\bar{B}$ and $B^*\bar{B}^*$ thresholds.

Consider $B^*\bar{B}^*$ elastic scattering first. The imaginary part of the T-matrix for $B^*\bar{B}^* \rightarrow B^*\bar{B}^*$ S-wave scattering is

\[ \text{Im} T \propto g^2 \rho(s), \quad (3) \]

where $\rho$ is the phase space for $B^*\bar{B}^*$ and $g$ is the coupling constant to this channel. Analyticity requires a real part to the amplitude [2]

\[ \text{Re} T \propto \frac{1}{\pi} P \int_{s_{thr}}^{M^2[\Upsilon(5S)]} d\sqrt{s'} g^2(\sqrt{s'}) \rho(\sqrt{s'}) \frac{\sqrt{s'}}{s' - s}, \quad (4) \]

where $s_{thr}$ is mass squared at threshold; $P$ denotes the principal value integral. For $B^*\bar{B}^*$, the S-wave phase space is

\[ \rho(\sqrt{s'}) = \frac{2k}{\sqrt{s'}} FF(\sqrt{s'}), \quad (5) \]

where $k$ is the momentum of each $B^*$ in the $B^*\bar{B}^*$ rest frame and $FF(s')$ is a form factor. There are further form factors for $\Upsilon(5S) \rightarrow B^*\bar{B}^*$, but these change little over the cusps and will be neglected. It is important to be aware that this cusp effect is not an ‘optional extra’. It is a basic requirement at the opening of any new channel, but has been ignored in the majority of calculations of ‘molecules’. It is also known as the ‘coupled channel effect’.

1email: d.bugg@rl.ac.uk
Let us now turn to the process $\Upsilon(5S) \to \pi^+ \pi^- \Upsilon(nS)$ shown in Fig. 1. It goes via rearrangement of the four quarks in $B^* \bar{B}^*$. This process has the same analytic structure as $B^* \bar{B}^* \to B^* \bar{B}^*$, since that amplitude obviously factorises. Let us consider first $B^* \bar{B}^* \to \Upsilon(nS) \pi$ via $\pi$ exchange. Heavier exchanges will be discussed later. The momentum transfer in the production is quite large and varies little over the cusp, though the phase space for the intermediate $B^*$ is included. For free $B^*$, pion decay is forbidden by the small mass difference between $B^*$ and $B$. However, for the triangle graph shown in Fig. 1, intermediate $B^*$ are off-shell, allowing $\pi$ exchange in $B^* \bar{B}$ scattering at the right-hand vertex. Suppose the form factor is a Gaussian

$$FF = \exp \left( -k^2 R^2 / 3 \right)$$

with $R = 1.41 \text{ fm}$, corresponding to the mass of the pion, though the exponential may be an approximation.

Fig. 2(b) shows $\text{Im} T$ as the dashed curve, normalised to 1 at its peak. Then $\text{Re} T$, shown as the dotted curve with the same normalisation, has a sharp cusp exactly at the $B^* \bar{B}^*$ threshold. $\text{Im} T$ peaks at a mass of 10656 MeV. $\text{Re} T$ goes negative just above this mass. For higher masses,
there is repulsion between $B^*$ and $\bar{B}^*$; this will keep them apart and suppress de-excitation to $\pi \Upsilon(nS)$, where $n = 1 - 3$. As a result, the intensity $|T|^2$ for $\Upsilon(5S) \rightarrow \pi^+\pi^-\Upsilon(nS)$ is proportional to the full curve. It rises at the threshold 10650.2 MeV to the scattering length given by the cusp and switches off at 10652.7 MeV. Folding in the 5.2 MeV experimental resolution quoted by Belle gives a somewhat rounded line-shape with a full-width at half-height of 10.0 MeV, rather less than the Belle value of 14.4 $\pm$ 3.2 MeV.

Consider next $B^*\bar{B}$ scattering via a diagram like Fig. 1 except for a $\bar{B}^-$ running from left- to right-hand vertices. This requires a P-wave interaction and

$$\rho(B^*\bar{B}) = k^3 \exp(-k^2 R^2)/(1 + k^2 R^2),$$

(7)

assuming a centrifugal barrier with the same radius as the exponential; this is an approximation where the centrifugal barrier is replaced by an equivalent square barrier. It is necessary to preserve the P-wave form for the amplitude at threshold and the dispersion integral becomes

$$\text{Re} T \propto \frac{k}{\pi} P \int_{s_{\text{thr}}}^{M^2[\Upsilon(5S)]} \frac{ds' g^2(s')\rho(s')/k'}{s' - s}. \quad (8)$$

Fig. 2(a) shows the corresponding line-shape for $\text{Re} T/k$ as the dotted curve; as on Fig. 2(b), it peaks at threshold. When it is multiplied by the factor $k$ in Eq. (8), the line-shapes for $\text{Re} T$ and $|T|^2$ are shown by the full curves and $\text{Im} T$ is shown by the dashed curve. In this case, the threshold is at 10604.6 MeV and the $B^*\bar{B}$ interaction becomes repulsive at 10613.9 MeV. The peak is centred at 10609.2 MeV and its full width becomes 9.5 MeV when folded with experimental resolution, compared with the experimental value 15.6 $\pm$ 2.5 MeV.

There are two key points in favour of the cusp mechanism. The first is that it predicts almost the same widths for $Z_b(10610)$ and $Z_b(10650)$, in agreement with data. In the molecular picture, these widths have no obvious relation. Secondly, the measured branching ratios of $\Upsilon(1S, 2S, 3S)\pi\pi$ sum to $\sim 2\%$ of the total width of $\Upsilon(10860)$, hence accounting for only a 2 MeV width for each peak [3]. There is a factor 7 discrepancy with experimental values.

Let us now consider heavier exchanges, for example due to $\sigma$. A full calculation is plagued by unknowns. Firstly, it is unclear whether the $\sigma$ will couple via the $\sigma$ pole (as in BES II data for $J/\Psi \rightarrow \omega\pi\pi$ [4]) or as in elastic scattering, where the Adler zero appears in the numerator of the amplitude. Next, it is necessary to fold the radius of interaction (as a function of $\sigma$ mass) with the folded radius of two $B^*$ (or $\bar{B}B^*$). A calculation gives an RMS radius of $\sim 0.65$ fm after this folding procedure. The cut-off at high $Z_b$ mass varies with the folded mass and is rounded off both below and above the sharp cut-off as shown below in Fig. 3. However, the key point is that, classically, the cross section for each de-excitation process, e.g. $\bar{B}^*B^* \rightarrow \Upsilon(nS)\pi$, is proportional to the mean $< R^2 >$. As a result $\pi$ exchange plays a strong role.

This procedure will be reversed here, deducing the RMS radius from the widths of $Z_b(10610)$ and $Z_b(10650)$ and asking whether this is reasonable. Even then, there is a minor problem that Belle have assumed these two peaks are resonant and included interferences between them and have also included interference with a non-resonant background. The two most prominent signals in their data are for $Z_b(10610)$ and $Z_b(10650) \rightarrow \Upsilon(2S, 3S)\pi\pi$, see their Fig. 5. Reading data directly from these graphs, widths come out 17% higher than their values. This is a small discrepancy, but systematic and probably arises from their inclusion of interferences between the two peaks. Using my widths, $\sqrt{< R^2 >}$ comes out to be 0.95 fm. In view of the factor $R^2$ favouring $\pi$ exchange, this appears reasonable within $\pm 0.1$ fm, i.e. $\sim 10\%$. 


Fig. 3 shows resulting line-shapes for this radius of interaction. The sharp cut-offs of Fig. 2 are slightly rounded off, but are still distinctive. With further data of higher statistics (and perhaps improved mass resolution), this line-shape is open to experimental test. In work concurrent with the present paper, Danilkin, Orlovsky and Simonov discuss the general formalism of heavy quarkonia and show results for $Z_b(10610)$ and $Z_b(10650)$; they use a smaller radius of interaction and therefore find wider cusps which are inconsistent with the data.

It is possible that the $Z_b$ states are actually resonant. Cleven, Guo, Hanhart and Meißner assume the molecular state is resonant and close to threshold; they adjust coupling parameters to reproduce the data on $h_b(2P)$ final states. They include the cusp effect and arrive at quite different line-shapes to those presented here. Their essential feature is a sharp cusp right at threshold.

Figs. 4(a) and (b) show Argand diagrams for both $Z_b$ derived from Fig. 3; masses are marked in GeV. In Fig. 4(b) there is no pole, because of the $s$-dependence around the curve. Above a mass where the amplitude goes negative, the variation with $s$ slows down rapidly. However, there is a possibility that meson exchanges will dynamically generate a resonant pole. The evaluation of the cusp made here is exactly equivalent to evaluating the first order loop diagram.
for meson exchanges. Solving the Bethe-Salpeter equation would include loop diagrams of all orders and could dynamically generate a resonance. The more favourable situation is for the loop driven by $\bar{B}B^*$, because of the P-wave factor at threshold. For Fig. 4(a), if the data are fitted with a Breit-Wigner resonance with $s$-dependent width, there is a pole at $10.623 \pm 0.005$ MeV. It has a full width at half-height of 44 MeV and is centred at 10632 MeV. Note that the loop is really a property of the $\bar{B}B^*$ channel. If there is an isospin $I = 0 \chi_{b1}(P)$ state near this threshold, it is likely to be attracted towards the $\bar{B}^*$ threshold. There is also the possibility that the $Z_B(10610)$ becomes an exotic resonance, but that needs to be proved.

There is one detail concerning decays to $\pi^+\pi^-\Upsilon(nS)$. For this case, both $\pi$ are produced in a relative S-wave and are fitted with the $\sigma$ amplitude. To form the final states $\pi^+\pi^-\chi_b(1P,2P)$ observed by Belle, one pion must be in a P-wave. This is consistent with the observation of decays to $\sigma_{L=1}$, where $L$ is the orbital angular momentum of the $\sigma$ with respect to $\chi_b$ states. A further detail is that pion production in the step $B^*\bar{B}$ or $B^*\bar{B} \rightarrow \pi\chi_b$ is likely to flip the spin of the $\chi_b$ final state. This can account for a phase difference with respect to $\pi^+\pi^-\Upsilon(nS)$ of $\sim 180^\circ$, as recorded by Belle. Because of nodes in radial wave functions, significant differences are to be expected between Dalitz plots for $\pi^+\pi^-\Upsilon(nS)$ with different $n$ values. There has been earlier discussion of a possible hybrid in the mass range 10.4–10.8 GeV on the basis of variations of Dalitz plot for $\Upsilon\pi\pi$ with different $n$ values [7].

In conclusion, threshold cusps at $B^*\bar{B}$ and $B^*\bar{B}^*$ thresholds are capable of explaining the peaks observed by Belle. The predicted line-shapes are distinctive, and with higher statistics can be used to test the present hypothesis. The width predicted for these cusps requires some exchange of heavier mesons than pions between $B^*$ and both $\bar{B}$ and $\bar{B}^*$. The cusp mechanism has the virtue of predicting closely similar widths for both $Z_b$ peaks. The experimental width of the peaks is a factor 7 larger than that for $Z_b \rightarrow \Upsilon(1S,2S,3S)\pi\pi$, but is reproduced by the cusp mechanism with a reasonable radius of interaction.

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