We give a simple, semianalytic theory of the development of the hadronic and neutrino/muon components of a cascade induced by a primary which produces hadrons at the initial interaction. The main purpose of the theory is to allow the user to obtain quick, but reasonably reliable estimates of the longitudinal properties of such cascades developing in a medium, such as a stellar interior, the atmosphere and/or water. As an application, we discuss the possibility of discovering physics beyond the Standard Model by means of neutrino telescopes. Some of those events may have spectacular signatures in neutrino telescopes.

1 Introduction

“Back of the envelope” estimates play an important role in physics. First, if one wants to decide the feasibility of an experiment, or the observability of a phenomenon a theorist just discovered, typically, one does not want to spend hours of CPU time on a substantial computer in order to learn the answer. (It is “no” in 90% of the cases, anyway…) Second, even after an elaborate computation has been performed, one usually wants to obtain an intuitive understanding of the result; in part, in order to learn how to proceed, in part, perhaps, in order to discover some bug in the program the presence of which is not obvious otherwise.

Cascade theory, necessary in order to conduct feasibility studies for neutrino telescopes or to understand the generation of neutrinos inside the point sources in the sky, among other things, is notoriously time consuming and unintuitive if it is to be sufficiently accurate. It is desirable therefore to design some simple “back of the envelope” cascade theory for the purpose outlined above. In the age before large scale computation became feasible, cascade theory was full of — often unreliable — approximations and clumsy numerical computations (cf. the review

---

1Invited talk given at the third NESTOR workshop, Pylos 1993; to be published in the Proceedings
2E–MAIL: SKD@JHUP4.PHA.JHU.EDU
This was rendered largely obsolete by the arrival of high speed computers. Nevertheless, one can learn some tricks from the “classics” (for good reviews, see e.g. Rossi’s book, [4] or Nishimura’s review article just cited) and invent a few new ones in order to reduce the amount of numerical computation necessary for obtaining quick estimates. The purpose of the work reported here is a first step in this direction; a similar approach to the problem was taken recently by Lipari [3].

2 Approximations

We outline the approximations used in developing the theory. Most of them can be improved upon at some cost in computing time. However, even in its present form, the theory is suitable for, say, astrophysical calculations, where uncertainties in the input data (e.g. stellar structure, etc.) are substantially bigger than the errors introduced by the approximations.

- Throughout this work, we use a one dimensional cascade theory in the diffusion approximation. Transverse development can be added on later in the diffusion approximation with relative ease. However, the present version is adequate for purposes of a first orientation on ultra high energy (UHE) processes.

- We assume the validity of Feynman scaling in the one particle inclusive cross sections, viz.

\[ E \frac{d\sigma}{dE} = \sigma F \left( \frac{E}{E_{in}} \right), \]  

(1)

where \( E \) and \( E_{in} \) stand for the observed and initial particle energies, respectively and \( \sigma \) stands for the total inelastic cross section.

In practice, a two parameter fit to \( F(z) \) viz.

\[ F(z) = A (1 - z)^n \Theta (1 - z) \]  

(2)

gives a fair approximation with \( n \approx 3 \), see, e.g. [4]. The distribution needs an infrared cutoff, in order to get a finite total multiplicity. We choose a finite cutoff in \( z \) in order to maintain Feynman scaling and thus simplify the solution. In order to determine the value of the infrared cutoff, somewhat arbitrarily, we chose a median shower energy, \( E_L \approx 5 \text{ PeV} \), corresponding to a CMS energy, \( \sqrt{s} \approx 3 \text{ TeV} \). At this energy, the total average multiplicity is \( \langle N \rangle \approx 30 \), cf.[5]. One knows that the infrared cutoff is of the order of \( \Lambda_{QCD}/\sqrt{s} \) which, with these numbers gives \( z_0 \approx 10^{-4} \); in turn, that leads to \( A \approx 4.1 \). (Most of our results are not very sensitive to the precise value of \( z_0 \)).
It is known that Feynman scaling is violated due to QCD loop effects; in particular, the average multiplicity of a produced hadron,

$$\langle N \rangle = \int \frac{dz}{z} F(z)$$

increases with $E_{\text{lab}}$. However, all such violations of Feynman scaling have a logarithmic dependence on the energy; within the accuracy of the present calculations it is legitimate to neglect them. For the sake of consistency, one should then either take all logarithmic dependences into account or none of them.

- The total inelastic $\pi N$ and NN cross sections differ by about 30% or so at high energies. Most of the secondaries are pions, with pions of either charge being produced in roughly equal numbers. There are few mesons produced containing $s$, $c$, $b$ (and $t$?) quarks. The qualitative reason is that those quarks have to be pulled out of a quark sea containing a small fraction of heavy quarks. Likewise, baryon pair production is suppressed due to the fact that a pair of three quarks has to be created coherently. As a consequence, the total high energy baryon contents of a high energy interaction is small: most of the baryons (with the exception of a leading baryon in a NN interaction) come from target fragmentation and they are of low energy in the LAB system. These qualitative arguments are generally borne out by the experimental data, wherever they are available, see e.g. [5]. In view of the above, it is justified to use only one hadron distribution function as a first approximation. Instead of a coupled system of integro–differential equations, we now have a single equation describing the evolution of the cascade. Due to the linearity of the equations, whatever method of solution is used for the single equation, it can be immediately generalized to the more accurate, coupled system by a straightforward use of matrix methods. However, the increase in computing time is not negligible, since one has to invert matrices repeatedly.

- Neutral pions drop out from the hadronic part of the cascade development due to their short lifetime at all energies of interest. Therefore, it is justified to multiply the hadron distribution by a factor of $(2/3)$ in the cascade evolution: due to Pomeranchuk’s theorem, the $\pi^\pm N$ cross sections being very nearly equal. Conversely, in the development of the electromagnetic component, the source of photons is fairly represented by $1/3^{\text{rd}}$ of the hadron distribution. (The electromagnetic component of a cascade in this approximation will be described elsewhere.)

- Keeping the needs of neutrino telescopes in mind, it is fair to say that a neutrino–rich shower is necessarily a hadron–rich one: the main source of UHE neutrinos is the process $\pi^\pm \to \mu + \nu_\mu$, where we have not distinguished
between $\nu_\mu$ and $\nu_\mu$. (The decay of muons is another source of neutrinos; however, if one concentrates on the the UHE part of the neutrino spectrum, $\mu$-decay neutrinos represent only a small correction, see Ref. [6].) Consequently, if the initial interaction produces hadrons in substantial numbers, either because the primary is a hadron or because of some “new physics”, such as the one conjectured in Ref. [7] or ’t Hooft’s B+L violating process, cf. A. Ringwald’s contribution to these Proceedings, there will be a substantial number of neutrinos present. Likewise, if one wants to determine, say, the UHE neutrino spectrum emerging from an AGN or a binary system of stars, one can concentrate on the accelerated hadrons ($p$, n, He, etc.) interacting with the target material, which often consists of hadrons.

In turn, this circumstance drastically simplifies the description of the evolution of the cascade. Due to the fact that

$$\sigma(\gamma, \text{hadron}) \approx K \sigma(\text{hadron}, \text{hadron}),$$

($\alpha$ being the fine structure constant and $K$ a number of O(1)), the feedback of the electromagnetic component into the hadronic one represents a correction of about 1% to the hadronic development. Neutrino–hadron interactions have a much smaller cross section, typically scaled down by an additional factor of $\left[m_h/m_{\text{gauge}}\right]^2$, where the masses involved are a typical hadron mass (say, 1 GeV) and a gauge boson (W, Z) mass, of the order of 100 GeV.

Consequently, the evolution of the hadronic component is, in essence, an autonomous one. For the purposes of determining the neutrino (and electron–photon) spectrum, the hadronic component acts only as a source evolving autonomously.

3 The Development of the Cascade

We denote the differential distribution of hadrons by $H(E, x)$, where $x$ stands for the depth measured in units of the hadronic interaction mean free path. In this manner, in the UHE region, one can adjust the units as new data become available. With current data extrapolated to CMS energies of the order of 50 to 100 TeV and averaging over $\pi N$ and NN cross sections, one gets $\lambda \approx 30g/cm^2$. Using the approximations described in the previous Section, we get the diffusion equation:

$$\frac{\partial H}{\partial x} = -\left(1 + \frac{2}{3}D\right)H + \frac{2}{3}\int_0^\infty \frac{dE'}{E} H(E') F\left(\frac{E}{E'}\right) \Theta(E' - E).$$

Here the quantity $D$ stands for the loss of charged hadrons due to decay; its expression is:

$$D(E, x) = \frac{\lambda m}{E_\tau \rho(x)},$$

4
where $\rho(x)$ stands for the density of the medium expressed as a function of $x$. To a good approximation, $m$ can be replaced by the pion mass and $\tau$ by the charged pion lifetime, i.e. $\tau \approx 7.8$ meters.

Clearly, the term $D$ is quite small in the UHE region and at a first approach, it may be neglected.

The expression of $\rho(x)$ depends on the model of the medium used. For instance, assuming a radial incidence of the primary, we have for some typical models:

1. Exponential atmosphere:

$$\rho(l) = \rho_0 \exp \left( -\frac{l}{h_0} \right),$$

$$\rho(x) = \frac{x\lambda}{h_0}.$$  

2. Atmosphere with a power law for the density:

$$\rho = \rho_1 \left( \frac{l}{h_1} \right)^{-\kappa}, \quad (\kappa > 1),$$

$$\rho(x) = \rho_1 \left( \frac{x\lambda}{\rho_1 h_1} \right)^{\frac{1}{\kappa - 1}}.$$  

3. For a (nearly) incompressible medium, like water, one may even contemplate replacing $\rho$ by a constant, say, the value of the density halfway between the entrance of the primary and the depth of the detector.

For a non–radial incidence, $l$ is to be replaced by the slant depth: the changes in the formulae are obvious and will not be exhibited here.

### 4 Solutions

#### 4.1 General techniques

Neglecting the decay term in eq. (4), — an approximation valid at the highest energies — one can solve the equation by means of a Mellin transformation in a standard fashion. The introduction of a new function, $h(E, x)$ by means of the substitution

$$H(E, x) = \exp(-x) h(E, x)$$

---

3 Throughout this work we use natural units, i.e. $\hbar = c = 1$. 
reduces the equation to the form:

\[
\frac{\partial h(E, x)}{\partial x} = \int_0^\infty \frac{dE'}{E} F\left(\frac{E}{E'}\right) \Theta (E' - E) h(E', x).
\]  

(11)

This equation is to be solved with an initial condition suitable for the physical problem at hand: we outline two types of such problems.

1. **Problems related to the development of a shower in an astrophysical environment or a hadronic shower in the atmosphere.**

In that case, the primary spectrum can be well approximated by a power spectrum for a substantial energy range. However, a power spectrum over all energies is unphysical, since it has an infinite energy contents. We also remark in passing that a pure power spectrum has no Mellin transform either; thus, the popular factorized solution, (see e.g. \[2\], \[8\] or \[3\]) cannot be obtained by using Mellin transforms, just by substituting the *ad hoc* Ansatz, \( h = E^{-\alpha} f(x) \) into eq. (11). One has to introduce at least an ultraviolet cutoff, reflecting the fact that no physical system is capable of producing particles of arbitrarily high energy. Often, an infrared cutoff is also needed; however, in the present case, this is not necessary: we are concentrating on the UHE part of the cascade. Thus, energy–momentum conservation alone provides an effective infrared cutoff.

In the following examples we use a spectrum of the form:

\[
H(E, 0) = N \left(\frac{E}{E_M}\right)^{-\alpha} \Theta (E_M - E)
\]  

(12)

2. **Problems connected with exploring some “new physics”.** In all models of physics beyond the Standard Model, one has to take into account the fact that the Standard Model is highly accurate up to LEP and TEVATRON energies. As a consequence, it is commonly assumed (for instance in \[7\]) that the onset of the “new physics” is sudden and that it can be characterized by a “pseudothreshold” in \( p_T \) and/or energy. Translated to the language of cascade development, this means that all but the first interaction is described, in essence, by the Standard Model. Consequently, the effect of the “new physics” can be simulated by an initial condition imposed at the depth of the first interaction,

\[
x_0 = \frac{\lambda_{\text{new}}}{\lambda},
\]  

(13)

where \( \lambda_{\text{new}} \) is the mfp. corresponding to the new physics introduced. (In practice, one may impose the initial condition at depth zero, but \( x \) has to be replaced by \( x - x_0 \).)

\[4\] It is the consequence of this infinite “energy reservoir” that a pure power spectrum passes through an absorber without getting distorted.
There is an important consequence of this picture, often ignored in the literature. Some authors have claimed that processes like 't Hooft’s instanton induced B+L violating process and other processes, presently considered exotic ones, can be recognized by searching for the occurrence of high multiplicity muon bundles in neutrino detectors, roughly resembling a heavy nucleus hitting the target (the atmosphere or water). While this appears to be true for, say, \( \nu \) and Pb induced showers started at the same depth, in practice, \( x_0 \gg 1 \) in any model available: thus, care is needed in counting the multiplicity of muon bundles in a neutrino telescope in order to find the onset of “new physics” . . .

With this, the solution of eq. (13) is given by

\[
H(E, x) = e^{-x} \frac{1}{2\pi i} \int_C ds E^{-s} \tilde{H}(s, 0) \exp \left[ \tilde{F}(s-1) x \right],
\]  

(14)

where \( \tilde{H} \) and \( \tilde{F} \) stand for the Mellin transforms of the initial condition and of \( F \), respectively. The contour \( C \) runs from \(-i\infty \) to \(+i\infty \) in the complex \( s \) plane, in the strip where both \( \tilde{F} \) and \( \tilde{H} \) exist.

Equation (14) can be a starting point of a numerical evaluation of the hadronic distribution. The initial conditions mentioned above (and several variations on them) as well as the expression of \( F \) are simple enough so that their Mellin transforms can be computed analytically; yet, they are sufficiently accurate so as to give a fair representation of the initial spectra and of the inclusive cross sections. In general, however the integral in eq. (14) cannot be evaluated in a closed form. Asymptotic methods used in the distant past (saddle point, etc.) are, in general, not sufficiently accurate.

Another way, very convenient from the point of view of a numerical treatment, is to solve the diffusion equation by successive approximations. The convenient starting point for this is eq. (11). Upon integrating both sides with respect to \( x \) and putting the equation in the form of a recursion relation, we obtain:

\[
h^{(n+1)}(E, x) = h(E, 0) + \int_0^x \int_E^\infty dx' \frac{dE'}{E'} F \left( \frac{E}{E'} \right) h^{(n)}(E', x') .
\]

(15)

Here \( h^{(n)} \) stands for the \( n^{\text{th}} \) iteration of the solution, with

\[
h^0 = h(E, 0).
\]

One notices that the integration over the variable \( x' \) can be performed in a closed form at every step of the iterative procedure. Moreover, with the simple representation of the inclusive distribution suggested in this section and with the simple initial spectra described above, the integral over the energy can also be evaluated in a closed form, although the result is somewhat clumsy. In this way, the iterative solution reduces to the evaluation of a series. (This, however, may not be true for more complicated representations of the initial inclusive distribution and/or of the initial spectrum.)

A few remarks are in order here.
1. One knows that the iterative process just described is a convergent one: eq. (15) is a Volterra equation which (due to the compactness of its kernel) can always be solved by successive approximations. (For purposes of proving this fact, one should use $1/E'$ as an independent variable in eq. (14)). As a consequence, if one uses the simple representations of the inclusive cross section and initial spectra as described above, the resulting series is a convergent one (in practice, it converges quite rapidly).

2. The method of successive approximations described above is, in essence, equivalent to the method of successive collisions invented by Bhabha and Heitler in 1937, see e.g. [2].

3. It is often advantageous to combine the numerical evaluation of the cascade development with symbolic manipulation programs, such as MATHEMATICA(C). In our experience, often a substantial amount of computing time may be saved in this manner.

### 4.2 A Very Simple Solution

If one is interested in the crudest type of estimates only, one does not worry too much about the precise shape of the inclusive distribution. In particular, one may just assume that there is no leading particle present at all and the energy is shared equally among all the secondaries, i.e.

$$F(z) = \delta \left( z - \frac{1}{\langle N \rangle} \right).$$ (16)

The validity of this approximation has been discussed elsewhere, see [6]. We concluded in that reference that the relative error committed by using eq. (16) as opposed to a more realistic form of the inclusive distribution is about a factor of 2. Hence, even this extremely crude approximation is adequate for the purposes of qualitative estimates. Using eq. (16), one easily arrives at an explicit solution. Assuming a primary power spectrum cut off at $E = E_M$, one gets:

$$H(E, x) = E^{-\alpha} e^{-x} \sum_{k=0}^{k_{max}} \frac{(x\langle N \rangle^{2-\alpha})^k}{k!}. \quad (17)$$

The maximal value of $k$ is given by:

$$k_{max} = \left[ \frac{E_M}{E \ln \langle N \rangle} \right].$$

Here $[\cdots]$ stands for the integer part of a number.

Due to the sharp cutoff of the spectrum, $H(E, x)$ is a function which has discontinuities. On could get rid of those discontinuities by smoothing out in some...
way the $\Theta$ function occurring in the spectrum: in that case, however, one would be dealing with an infinite series. Due to the crudeness of this approximation, we shall not discuss this question any further. One notices that the approximate solution written down here is a simple generalization of “Heitler’s caricature” of an electromagnetic cascade, cf. [9].

### 4.3 Muons and Muon Neutrinos

In the approximation used in the present work (UHE neutrinos and muons only, no decaying muons, cf. the previous Section), there are practically no $\nu_e$ present: the branching ratio of $\pi^\pm \to e + \nu$ is of the order of $10^{-4}$, cf. [5]. Further, the number of neutrinos and charged muons is equal and determined by the autonomously evolving hadronic component:

$$N (\nu + \bar{\nu}) = N (\mu^+ + \mu^-) \equiv N.$$

We get in an obvious manner:

$$N (E, x) = \frac{2}{3} \int_0^x dx' D (2E, x') H (2E, x'). \quad (18)$$

Thus, once the hadronic component is computed by using one of the solutions described in the previous subsections, the number of UHE muons and muon neutrinos is obtained by a quadrature. (It is known of course that, either in the case of a pure power spectrum, or going to lower energies in the present calculation, electron neutrinos begin to appear. At energies substantially lower than the ultraviolet cutoff, the ratio of muon and electron neutrinos is about 2.)

Given the simplifications made in the course of the calculations, one should check the accuracy and internal consistency of the results.

First, an internal consistency check. We stated earlier that the iterative solution to the cascade equation should be convergent on general grounds. In practice, however, the speed of the convergence is an important factor from the computational point of view. For this reason, we computed the hadronic flux and the atmospheric neutrino flux generated by it at a depth of 1000 g/cm$^2$ and with a primary spectrum $\propto E^{-\alpha}$ cut off at $E_M = 10^{11}$GeV. For the sake of definiteness, we chose $\alpha = 2.7$.

The iterative solution can be written in the form:

$$H (x, E) \propto E^{-\alpha} e^{-x} \sum_{n=0}^{\infty} (ax)^n f_n (E/E_M), \quad (19)$$

where the constant $a$ is determined in terms of fractional moments of the fragmentation function. One easily verifies that as $E_M \to \infty$, all $f_n \to 1$ and the solution goes over into the one obtained with the factorization assumption.

In order to exhibit the speed of the convergence, we chose $f_0 = 1$. In Fig. [9] we exhibited the first few coefficients $f_n (E/E_M)$. 

9
Figure 1: The first six coefficients of the iterative solution. The consecutive coefficients are plotted on a varying gray scale, $f_1$ being the darkest and widest line.

One sees that the iterative solution is converging quite well: the area under the consecutive coefficients $f_n(E/E_M)$ decreases rapidly.

Next, we compute the hadron distribution and from it, the neutrino flux in the manner described above. For the sake of simplicity, we used a standard expression of the flux, ref. [8] all the way to the cutoff. In Fig 2 we plot the integral spectrum of neutrinos, multiplied by $E^{2.7}$.

If the factorized solution were valid everywhere, this quantity would be approximately constant in the energy range we are considering.

We see that this is reasonably well satisfied up to $E/E_M \approx 10^{-3}$. In this region, our result agrees to within 30% or so, with Lipari’s [3], and also by MC calculations. Given the simplicity of the model developed here, the agreement is satisfactory.

At still higher energies, the effect of the ultraviolet cutoff can be clearly seen. In
principle, one could infer the existence of such a cutoff from atmospheric neutrino observations. The main obstacle to such an observation is, of course, the scarceness of events.

5 Looking for “New Physics”

In this Section, we illustrate the method developed in the previous Sections by asking a simple question:

Assuming that there exists some “new physics” beyond the Standard Model, do we have any hope of detecting it in a neutrino telescope?

The answer, of course, depends on the cross section and energy of the onset of the new processes. We have in mind, in particular, either the phenomenon conjectured in ref.[7] or the multiple production of gauge Bosons (with or without B+L violation) as discussed by A. Ringwald in these Proceedings.

There is a large amount of theoretical uncertainty in the nature of the phenomena in which any physics beyond the Standard Model would manifest itself. However, one can determine with relative ease the reactions in which it is virtually
hopeless to look for manifestations of some post–Standard–Model physics, unless
the effects are unexpectedly dramatic. Any such argument is based, in essence,
on the unitarity of the S–matrix. Probability is conserved, hence, *one should not
look for the new physics in reactions with a large number of open channels*. Unless
a firm theoretical prediction is available — as it was in the case of the discovery
of the W and Z bosons in a hadronic machine — it is virtually impossible to find
the proverbial needle in a haystack...

This criterion immediately tells us that hadronic reactions are, in all proba-

dility, unsuitable in a search for new physics: there are just too many channels open,
producing mostly mundane physics: typically, relatively soft pions. Likewise,
electron pair production in γ – nucleus collisions dominates in a medium with an
average Z of the order of 3 or larger (water, earth, the atmosphere). Hence, again,
“mundane physics” suppresses any appearance of new physics. (This question
was discussed in some detail in ref. [10]. In a recent work, Morris and Ringwald,
ref. [11] reached a similar conclusion by means of a rather sophisticated Monte
Carlo simulation.)

What remains therefore, is the realm of neutrino induced reactions: any pro-
cess predicted by the Standard Model has a cross section of the order of a few
nanobarns at LAB. energies around an EeV or so. By contrast, the conjectured
new processes may reach cross sections of the order of 10^{-2}mb, cf. ref.[7], [11].
Therefore, they should be observable in neutrino induced reactions better than
anywhere else.

Neglecting all effects of nuclear structure, there is a very simple relationship
between a mfp (λ) and the cross section of a reaction (cf. [6] and references
discussed there):

\[ \lambda \, [g/cm^2] = \frac{1670}{\sigma \, [mb]} \]  \hspace{1cm} (20)

(At high energies, this relationship should hold to an accuracy better than about
20% for medium heavy nuclei.)

Thus, a \( \nu \)–induced reaction of a cross section, \( \sigma \approx 5 \times 10^{-3}mb \) and incidence
at a zenith angle, \( \theta = 0 \), would, in the mean, produce its first interaction very
close to a detector like NESTOR. (At incidences lower than vertical, the range
of cross sections one can explore below the one at vertical incidence, depends on
the geology around the detector and it needs a more detailed investigation. In
general, however, one can explore several orders of magnitude in the initial cross
section by scanning at lower zenith angles. For purposes of illustration therefore,
we assume vertical incidence and a cross section of the order of magnitude just
quoted.)

Due to the theoretical uncertainties surrounding the “new physics”, we made
two simplifying assumptions.

1. The onset of the “new physics” is sudden: in practice, we approximate it by
a \( \Theta \) function in the energy, as in the articles in ref. [7]. As a consequence,
after the initial interaction, the resulting shower tends to evolve according
to the Standard Model. (At extremely high energies where this is not the case, the primary fluxes are expected to be very low.)

2. The rapidity distribution of the produced particles in the initial interaction follows an “equipartition law”, viz.

\[ F(z) = \delta \left( z - \frac{1}{N_1} \right), \]  

(21)

where \( N_1 \) stands for the average hadronic multiplicity in the first interaction: this quantity parametrizes what we call “new physics” in this model.

Rather than compute the spectrum of the hadronic component for a given primary neutrino spectrum, we investigated the profile of single events, with a primary energy \( E = 10^{16} \) GeV, corresponding to about \( \sqrt{s} \approx 5 \) TeV in the neutrino–nucleon CMS.

We computed the development of the hadronic cascade with two initial multiplicities \( (N_1 = 5 \) and \( N_1 = 20) \).

Both cascades are supposed to have a primary incident at zenith angle, \( \theta = 0 \) and an initial cross section around the value just quoted. The results are displayed in Fig. 3 and Fig. 4 respectively.

In both Figures, we plotted the integral hadron spectrum as a function of \( x = t/\lambda \) for energies \( E > 10 \) GeV and \( E > 100 \) GeV, respectively. (Obviously, the higher curves correspond to the lower threshold energies in both Figures.) In the Figures, \( \lambda \) is the usual hadronic mfp, given the assumption that after the first interaction, the cascade develops according to the Standard Model.

The interesting feature of these showers is that charged hadrons containing light quarks (u,d,s), for all practical purposes, do not decay. (Indeed, the interaction mfp is about 80 cm, to be compared with decay mfps substantially larger than 10 meters.) Similarly, the radiation length is approximately 36 cm. Hence, both the hadronic and electromagnetic cascades evolve within a few meters in real space. The ‘anomalous’ showers are characterized by a very large energy deposition within the span of a few meters. In particular, a large amount of Cherenkov light is emitted in such events.

Some features are worth noticing.

- The overall profile of the longitudinal evolution of a shower is rather insensitive to the initial multiplicity: hence, theoretical uncertainties are unlikely to have a drastic effect on the discovery potential of a given neutrino telescope.
The events are “muon poor” for the reason stated above. In practice, the only source of muons is the production of particles containing heavy quarks (c, b, t). However, the production of heavy quarks is expected to be suppressed due to the scarcity of the latter in the quark sea. (In the case of multiple W–production, the source of muons is the leptonic decay mode of the gauge bosons, which is of the order of 30%.)

Overall, one concludes that an underwater neutrino telescope may be the detector of choice for the observation of the events discussed here. Due to the large local energy deposition, such anomalous events may be observable even somewhat beyond the usual attenuation length of Cherenkov light (of the order of 50m at NESTOR). This question needs further study.

The potential beauty of a search for new physics in observations of the type just described is that the signature is a very robust one. Events in which a large amount of energy is released near the neutrino telescope are hard to miss, even though one does not understand all the details of such events at present.
Figure 4: The profile of the hadronic component of an ‘anomalous’ shower with $N_1 = 20$.

One event of an apparently large energy release and multiplicity has been reported recently by the KAMIOKANDE collaboration, ref. [12]. It is amusing to speculate about the possibility that the “unusual event” as described by the authors may be the manifestation of a phenomenon just discussed. Should that be the case, one would have the first experimental evidence for the existence of some physics by means of which the ills of the Standard Model could be cured.

6 Discussion

We believe that the formalism presented in this paper is useful in order to explore new phenomena both in an astrophysical context (point sources, AGN...) and in a particle physics one. Neutrino telescopes are potentially sensitive to the observation of $\nu + ‘quark’$ reactions at energies beyond the reach of current accelerator–based research. Hence, despite the erratic nature of point sources and the uncertainties associated with sources like AGN or gamma – bursters, they may have an important role to play in exploring particle physics beyond the $\sqrt{s} \approx 1$
TeV barrier.

7 Acknowledgement

This research has been supported in part by the U.S. Department of Energy under Grant # DE–FG–02–ER40211.

We thank Paolo Lipari, Christine Kourkoumelis, Al Mann, Kenzo Nakamura, Leonidas Resvanis and Atsuto Suzuki for useful conversations during the Nestor Workshop.

We also thank our hosts, Leonidas Resvanis and his colleagues at the University of Athens as well as the Honorable Mayor J. Vrettakos and the City Council of Pylos for hosting this productive and enjoyable meeting.

References

[1] J. Nishimura in Handbuch der Physik, ed. by S. Flügge, Vol. XLVI/2. Springer, Berlin (1967).

[2] B. Rossi, High Energy Particles. Prentice Hall, New York (1952).

[3] P. Lipari, Astropart. Phys. 1, 195 (1993); ibid. 399 (1993).

[4] V.N. Barger and R.J. Phillips, Collider Physics. Benjamin, New York (1987).

[5] Particle Data Group, Review of Particle Properties, Phys. Rev. 45, 51 (1992).

[6] G. Domokos, B. Elliott and S. Kovesi–Domokos, Jour. Phys. G 19, 899 (1993).

[7] G. Domokos and S. Nussinov, Phys. Letters 187 B, 322 (1987); G. Domokos and S. Kovesi–Domokos, Phys. Rev. D38 2833 (1988).

[8] T.K. Gaisser, Cosmic Rays and Particle Physics, Cambridge University Press, Cambridge (1990).

[9] W. Heitler, The Quantum Theory of Radiation (3rd edition); Oxford University Press, Oxford (1954).

[10] G. Domokos, B. Elliott, S. Kovesi–Domokos and S. Mrenna, in Nuc. Phys. B (Proceedings Supplement) 14A (1990).

[11] D. Morris and A. Ringwald, preprint CERN–TH–6822–93 (1993).

[12] KAMIOKANDE Collaboration (K.S. Hirata et al.) Phys. Rev. D45, 3345 (1993).
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9407219v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9407219v1
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9407219v1
This figure "fig1-4.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9407219v1