1 Introduction

The KTEV experiment at Fermilab was designed primarily to measure the direct CP violating parameter $\Re(\epsilon'/\epsilon)$ with much better precision than previous measurements at Fermilab and CERN in the early 1990's. The measured quantity is the double ratio of decay rates,

$$R \equiv \frac{\Gamma(K_L \to \pi^+\pi^-)/\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^+\pi^-)/\Gamma(K_S \to \pi^0\pi^0)}$$

where $R \approx 1 + 6\Re(\epsilon'/\epsilon)$. The CP-violating $K_L \to 2\pi$ decays can be explained by the parameter $\epsilon \sim 2 \times 10^{-3}$, which describes the tiny CP-violating asymmetry in the $K^0 \leftrightarrow \bar{K}^0$ mixing. If the double ratio $R$ differs from unity, i.e., $\Re(\epsilon'/\epsilon) \neq 0$, this would be a clear indication of direct CP-violation in the decay amplitude. In short, $R \neq 1$ implies that

$$A(K^0 \to \pi^+\pi^-) \neq \bar{A}(K^0 \to \pi^+\pi^-)$$
$$A(K^0 \to \pi^0\pi^0) \neq \bar{A}(K^0 \to \pi^0\pi^0)$$

which would demonstrate that a particle and its anti-particle partner can decay differently. Note that CPT requires $\sum |A_i|^2 = \sum |\bar{A}_i|^2$.

The experimental challenge is to measure the double ratio $R$ based on millions of decays, and then to control the systematic uncertainties to much better than a percent. The key is to collect $K_L$ and $K_S$ decays at the same time so that common charged or neutral mode systematic uncertainties will cancel in the $K_L/K_S$ single ratios.
Figure 1. KTEV Detector
Figure 2. Plan view of KTEV Detector.
The $\pi^+\pi^-$ and $\pi^0\pi^0$ mass resolutions are both 1.5 $MeV/c^2$. The decay distributions for $K_L$ and $K_S$ decays is shown in Figure 3. The difference in these two distributions results in different acceptances that are corrected using a Monte Carlo [MC] simulation. The quality of the simulation is shown by comparing the data and MC decay vertex distributions, and is shown in Figures 4 and 5 for neutral and charged decays in the $K_L$ beam. There are 2.5 million $K_L \rightarrow \pi^0\pi^0$ decays, which agree very well with the 20 million MC events. To further check our MC, 39 million $K_L \rightarrow 3\pi^0$ decays are compared with an equal size MC sample, and the agreement is excellent. In charged mode, 10 million $K_L \rightarrow \pi^+\pi^-$ decays agree very well with the MC sample of 60 million. As a charged mode cross-check, a sample of 170 million $K_L \rightarrow \pi^+\pi^-$ decays are compared with an MC sample of 40 million; there is no $z$-slope in data/MC vs. decay vertex, but there are non-statistical fluctuations suggesting subtle problems that might be related to the reconstruction with a missing neutrino.

Figure 3. Decay vertex distributions for $K_L$ and $K_S$ decays.

Figure 4. Data/MC comparison of decay vertex for $K_L \rightarrow \pi^0\pi^0$ (left) and for $K_L \rightarrow 3\pi^0$ (right). The lower plots show the data/MC ratio vs. decay vertex.

3 Regenerator Beam Results

The $K_{LS} \rightarrow \pi^+\pi^-$ decay distribution in the regenerator beam is shown in Figure 6. The kaon momentum range 40-50 GeV illustrates the high precision with which we can measure parameters associated with the interference. These data (40-160 GeV) are used to make precision measurements of $\tau_S$, $\Delta m$, $\Delta \Phi$ and $\Phi_{+-}$. The distribution of decays in the regenerator beam is given by

$$\frac{dN}{dt} = |\rho|^2 e^{-\Gamma_S t} + |\eta|^2 e^{-\Gamma_L t} + 2|\rho||\eta|e^{(\Gamma_S + \Gamma_L)t/2} \cos(\Delta mt + \phi_p - \Phi_\eta)$$

where $\Phi_\eta = \Phi_{+-}, \Phi_{00}$ for charged, neutral decays. For all fits described in the subsections below, the regeneration phase $\phi_p$ is determined from analyticity.

3.1 $\tau_S$ and $\Delta m$

The acceptance-corrected data was fit to the function in Eq. 3. $\tau_S$ and $\Delta m$ are floated and CPT is assumed so that the phase of $\eta$ is given by the superweak phase, $\Phi_\eta = \Phi_{SW} = \tan^{-1}(2\Delta m/\Delta \Gamma)$. The combined
charged+neutral results are
\[
\tau_S = [89.67 \pm 0.04_{\text{stat}} \pm 0.04_{\text{syst}}] \text{ psec} \\
\Delta m = [52.62 \pm 0.08_{\text{stat}} \pm 0.13_{\text{syst}}] \times 10^8 \text{ fs}^{-1}
\]
and are shown in Figures 3-8. Our new \(\tau_S\) value is consistent with previous measurements, and it is 2.5\(\sigma\) above the PDG2000 value. Similarly our new \(\Delta m\) value is consistent with previous measurements, and it is 2.1\(\sigma\) below the PDG2000 value.

3.2 CPT Tests: \(\Delta \Phi\) and \(\Phi_{+-}\)
CPT-symmetry demands that
\[
\Delta \Phi = \Phi_{+-} - \Phi_{00} \simeq 0. 
\]
with the caveat that final state interactions can lead to \(\Delta \Phi \sim 0.05^\circ\). In this fit, the uncertainty in \(\phi_\rho\) cancels between the charged and neutral phases. The result from combining 96+97 data is
\[
\Delta \Phi = [0.41 \pm 0.22_{\text{stat}} \pm 0.53_{\text{syst}}] \circ
\]
and is compared with previous results in Figure 3. Our value is consistent with CPT-symmetry. The systematic uncertainty of 0.53\(^\circ\) is more than twice the statistical uncertainty, and is due mainly to the neutral energy reconstruction. Our \(\Delta \Phi\) result is related to \(\Im(\epsilon'/\epsilon)\) by
\[
\Im(\epsilon'/\epsilon) = -\Delta \Phi/3 \\
= [-24 \pm 13_{\text{stat}} \pm 31_{\text{syst}}] \times 10^{-4}
\]
Note that the uncertainty on \(\Im(\epsilon'/\epsilon)\) is \(\times 12\) larger than the uncertainty on \(\Re(\epsilon'/\epsilon)\) (next section).

CPT symmetry also demands that \(\Phi_{+-} = \Phi_{SW}\). From PDG2000, \(\Phi_{+-} - \Phi_{SW} = [-0.2 \pm 0.5]^\circ\). However, note that previous experiments had fixed \(\tau_S\) and/or \(\Delta m\) to PDG values in their fits; our KTEV data show that these kaon parameters differ by more than 2\(\sigma\) from PDG values (Figures 3-8). Since our measurements of \(\tau_S\) and \(\Delta m\) assume CPT-symmetry, we cannot use our values to make a CPT test; on the other hand, the PDG values may not be appropriate either. We therefore float both \(\Delta m\) and \(\tau_S\) in our fit at the expense of increasing both the statistical and systematic errors. Our preliminary result floating both \(\tau_S\) and \(\Delta m\) is
\[
\Phi_{+-} - \Phi_{SW} = [+0.6 \pm 0.6_{\text{stat}} \pm 1.1_{\text{syst}}]^\circ
\]
An external measurement of \(\tau_S\) that does NOT assume CPT would help to reduce the KTEV errors as follows,
- \(\sigma_{\text{stat}}(\Phi_{+-} - \Phi_{SW}) \rightarrow 0.3^\circ\)
- \(\sigma_{\text{syst}}(\Phi_{+-} - \Phi_{SW}) \rightarrow 0.7^\circ \oplus [7.3^\circ \times \sigma_{\tau_S}(\text{psec})]\)

If an external CPT-independent \(\tau_S\) measurement has the same error as KTEV (0.06 psec), then our error on \(\Phi_{+-} - \Phi_{SW}\) would be reduced from 1.3\(^\circ\) (Eq. 3) down to 0.9\(^\circ\). There are good prospects for such an external measurement from the CERN-NA48 collaboration.

We have made another CPT test based on a suggestion to look for diurnal variations
in $\Phi_+$. The fit-values of $\Phi_{+-}$ vs. sidereal time are shown in Figure 10, which shows that $\Phi_{+-}$ is constant over “kaon beam direction” to within 0.37° at 90% confidence. A similar fit to $\tau_S$ limits diurnal variations to be less than 0.0015 × $\tau_S$.

Figure 6. Decay distribution in regenerator beam.

Figure 7. History of $\tau_S$ measurements.

Figure 8. History of $\Delta m$ measurements.
**Figure 9.** History of $\Delta \Phi$ measurements.

**Figure 10.** $\Phi_{+-}$ vs. sidereal time.
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Table 1. Summary of KTEV $\Re(\epsilon'/\epsilon)$ results. The 1997 and “PRL” samples are statistically independent. The uncertainties reflect statistical, systematic and MC-statistics.

| data sample | $\Re(\epsilon'/\epsilon) \times 10^{-4}$ |
|-------------|--------------------------------------|
| 1997        | $19.8 \pm 1.7_{\text{stat}} \pm 2.3_{\text{syst}} \pm 0.6_{\text{MC}}$ |
| “PRL” (update) | $23.2 \pm 3.0_{\text{stat}} \pm 3.2_{\text{syst}} \pm 0.7_{\text{MC}}$ |
| 96+97       | $20.7 \pm 1.5_{\text{stat}} \pm 2.4_{\text{syst}} \pm 0.5_{\text{MC}}$ |

4 $\Re(\epsilon'/\epsilon)$ Results

Our new and updated results on $\Re(\epsilon'/\epsilon)$ are shown in Table 1, and a comparison with other results is shown in Figure 11. Using many improvements developed since the original result, the $\Re(\epsilon'/\epsilon)$ update of the published sample has changed by $-4.8 \times 10^{-4}$, and the changes are illustrated in Figures 12 and 13. The changes are due to analysis improvements, better measurements of $\tau_S$ and $\Delta m$, and to MC statistical fluctuations. Except for a mistake in the regenerator-scatter background (top entry in Fig. 13), the changes are consistent with the systematic errors assigned in the published result. All of the changes to $\Re(\epsilon'/\epsilon)$ are uncorrelated. The updated result has a slightly larger systematic error, $3.2 \times 10^{-4}$ compared with published value of $2.7 \times 10^{-4}$; this is because the data/MC decay vertex comparisons, which determine the acceptance error, are slightly worse using the improved techniques, even though the larger independent [1997] data set shows much better agreement in the same distributions.

4.1 $\Re(\epsilon'/\epsilon)$ Systematic Errors

The dominant systematic errors are shown in Table 2. The neutral energy uncertainty is determined mainly from $\pi^0\pi^0$ pairs produced from hadronic interactions in the vacuum window located 159 meters from the pri-
mary target. The vacuum window location is known to within 1 mm based on charged track vertex analyses; the $\pi^0\pi^0$ vertex is off by $2.5 \pm 0.4$ cm, leading to most of the error in the first row of Table 2.

The neutral background from regenerator scatters is based on measuring the $p_t^2$ distribution from $K_{L,S} \rightarrow \pi^+\pi^-$ decays in the regenerator beam, and then using this measurement to simulate $K_{L,S} \rightarrow \pi^0\pi^0$ decays that scatter in the regenerator. The $\sim 1\%$ scatter background in neutral mode must be known to about 5% of itself to get the $\Re(\epsilon'/\epsilon)$ systematic down to $1 \times 10^{-4}$. The background uncertainty is due to the charged mode reconstruction and acceptance ($0.8 \times 10^{-4}$), to the possibility that the beam-hole veto used only in neutral could alter the $p_t^2$ shape ($0.3 \times 10^{-4}$) and to slight imperfections in the parametrization of the $p_t^2$ using scattered $K_{L,S} \rightarrow \pi^+\pi^-$ decays ($0.4 \times 10^{-4}$).

The charged acceptance uncertainty in row 3 of Table 2 is due mainly to a large data/MC z-slope in the first 20% of the 1997 data set. The $\Re(\epsilon'/\epsilon)$ charged acceptance error is $2.2 \times 10^{-4}$ in the first 20% of the data and $0.5 \times 10^{-4}$ for the remaining 80%.

During data collection, a Level 3 software filter was used to remove most of the $K_{L} \rightarrow \pi e\nu$ and $K_{L} \rightarrow \pi \mu\nu$ events. One percent of the charged triggers were saved without any online filter to check for a bias in $K_{L,S} \rightarrow \pi^+\pi^-$ decays. We find a small bias with $2.2\sigma$ significance, which leads to a systematic error of $0.5 \times 10^{-4}$ on $\Re(\epsilon'/\epsilon)$. More detailed studies on the higher statistics sample of un-filtered $K_{L} \rightarrow \pi e\nu$ decays shows no Level 3 bias in the decay vertex distribution, nor any difference in loss between the two beams.

The neutral apertures consist of a collar-anti veto [CA] surrounding the CsI beam-holes, a mask-anti veto just upstream of the regenerator, and the CsI size. The aperture-systematic is dominated by the 100 $\mu m$ uncertainty in the size of the CA. Note that the

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
\textbf{comment} & $\Delta \Re(\epsilon'/\epsilon)/\sigma_{syst}$ relative to PRL 83, 22 (1999) \\
\hline
$2\sigma^2$ background & -1.7 $\times 10^{-4}$ \\
reg screening & -0.3 $\times 10^{-4}$ \\
reg attenuation & -0.3 $\times 10^{-4}$ \\
$\pi^+\pi^-$ reg edge & -0.2 $\times 10^{-4}$ \\
collimator scatter & -0.2 $\times 10^{-4}$ \\
$2\sigma^2$ analysis & 0.1 $\times 10^{-4}$ \\
Mask Anti geometry & 0.26 $\times 10^{-4}$ \\
Absorber scatter & -0.6 $\times 10^{-4}$ \\
$2\sigma^2$ reg edge & -0.2 $\times 10^{-4}$ \\
\hline
\end{tabular}
\end{table}

Figure 13. Detailed list of $\Re(\epsilon'/\epsilon)$ improvements to the published result. The plot at left shows the number of systematic $\sigma$ for each change. The numbers in () at the right show the corresponding change to $\Re(\epsilon'/\epsilon)$.
Table 2. Dominant \( R(\epsilon'/\epsilon) \) systematic uncertainties \((\times 10^{-4})\)

| Source                                      | Value |
|---------------------------------------------|-------|
| Neutral Energy reconstruction               | 1.5   |
| Neutral background from reg-scatters (B/S ~ 1%) | 1.0   |
| Charged Acceptance (data/MC z-slope)        | 0.9   |
| Charged Level 3 online filter (2.2\( \sigma \) effect) | 0.5   |
| Neutral apertures                           | 0.5   |

CA size was determined in-situ using electrons from \( K_L \to \pi e\nu \) decays.

4.2 \( R(\epsilon'/\epsilon) \) From Re-weighting Method

As an additional cross-check, \( R(\epsilon'/\epsilon) \) was measured using a re-weighting technique that is similar to the NA48 method, except that no MC correction is needed due to the identical phase space of the two kaon beams in the KTEV experiment. The method is illustrated by the decay distributions shown before and after re-weighting in Figure 14. After re-weighting the acceptances are the same in the two beams. A comparison of the nominal and re-weighting result using 1997 data sample (PRL sample is not included) is shown in Figure 15. The difference between the two analyses is

\[
\Delta R(\epsilon'/\epsilon) = [1.5 \pm 2.1_{\text{stat}} \pm 3_{\text{syst}}] \times 10^{-4} \quad (8)
\]

where the systematic uncertainty is dominated by cut-variations, particularly on the minimum cluster energy in the \( K_{L,S} \to \pi^0\pi^0 \) analysis.

5 \( K_L \to \pi e\nu \) Charge Asymmetry

The \( K_L \to \pi e\nu \) Charge Asymmetry is defined to be

\[
\delta_{Ke3} = \frac{N(\pi^- e^+\nu) - N(\pi^+ e^-\nu)}{N(\pi^- e^+\nu) + N(\pi^+ e^-\nu)} \quad (9)
\]
\[ = 2\text{Re}(\epsilon - \Delta - Y - X_+) \]

where

\[ \epsilon = CPT \text{ in mixing.} \]
\[ \Delta = CPT \text{ in mixing (}\Delta S = \Delta Q\text{).} \]
\[ Y = CPT \text{ in decay amplitude (}\Delta S = \Delta Q\text{).} \]
\[ X_- = CPT \text{ in decay amplitude (}\Delta S \neq \Delta Q\text{).} \]

The previous best measurement was made at CERN in 1974, and was based on 34 million events. KTEV has a new measurement based on 298 million decays in the \( K_L \) beam (regenerator decays are not used). Each \( N \)-factor in Eq. 9 is replaced with the four-fold geometric mean of the two beams (left,right) and the two magnet polarities in order to cancel acceptances. There is no MC acceptance correction, but there are corrections for particle/antiparticle differences. These differences are measured in-situ using complementary samples such as \( K_L \to \pi^+\pi^-\pi^0 \) to study the \( \pi^+ \) and \( \pi^- \) detection efficiencies.

Our new result is

\[ \delta_{K_{e3}} = [3.32 \pm 0.06_{\text{stat}} \pm 0.05_{\text{syst}}] \times 10^{-3} \]

and is shown in Figure 16 with previous measurements. The world average uncertainty is reduced by a factor of 2 with the addition of the KTEV measurement.

CPT limits can be obtained from the relation,

\[ \Re\left( \frac{2}{3} \eta_{\pi^+\pi^-} + \frac{1}{3} \eta_{\pi^0\pi^0} \right) - \delta_L = \Re(Y + X_+ + a) \] (10)

where \( \delta_L \) is the average of \( \delta_{K_{e3}} \) and \( \delta_{K_{\mu3}} \) asymmetries, \( Y \) and \( X_- \) are defined above and \( \Re(a) \) parametrizes \( CPT \) in \( K(K^0) \to 2\pi \) decays. The result is that the sum of \( CPT \) parameters is

\[ \Re(Y + X_+ + a) = (-3 \pm 35) \times 10^{-6} \] (11)

which is less than \( 55 \times 10^{-6} \) with 90% confidence.

| \( \delta_{K_{e3}} \) | \( \delta_{K_{\mu3}} \) |
|----------------|----------------|
| CNTR 69        | 2.46 ± 0.59    |
| CNTR 70        | 3.46 ± 0.33    |
| ASPK 72 (\( K_{\mu3} \)) | 2.78 ± 0.51 |
| CNTR 70        | 3.18 ± 0.38    |
| ASPK 74 (\( K_{\mu3} \)) | 3.13 ± 0.29 |
| ASPK 74        | 3.41 ± 0.18    |
| KTEV 01 (prel) | 3.32 ± 0.06 ± 0.05 |
| New World Ave. | 3.31 ± 0.06    |
| PDG 2000       | 3.27 ± 0.12    |

Figure 16. History of \( \delta_{K_{e3}} \) and \( \delta_{K_{\mu3}} \) results.
6 Rare Kaon Decays

Table 3 lists more than a dozen rare decay results that have been improved by the KTEV collaboration. Typical improvements are based on order-of-magnitude increases in statistics or sensitivity, and includes three first-observations. In the $\mathcal{CP}$ column we have 1800 $K_L \to \pi^+\pi^-e^+e^-$ events which shows a 13\% $\mathcal{CP}$ asymmetry in the angle between the $\pi^+\pi^-$ and $e^+e^-$ planes. The prospects for observing direct $\mathcal{CP}$ in $K_L \to \pi^0\ell\bar{\ell}$ modes ($\pi^0\nu\bar{\nu}$, $\pi^0e^+e^-$ and $\pi^0\mu^+\mu^-$) requires measuring BRs down to $\sim 10^{-11}$ (Fig. 17). In the case of $K_L \to \pi^0\nu\bar{\nu}$, KTEV has improved the upper limit by $\sim 10^2$, but is still a factor of $10^4$ shy of the standard model. KTEV has improved the $K_L \to \pi^0\ell^+\ell^-$ modes by a factor of 10, and remains a factor of $10^2$ above the standard model predictions.

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Table 3. List of rare kaon decay modes by physics topic for which KTEV has made improved BR measurements. The numbers in parentheses indicate the increase in statistics or sensitivity. “(1st obs)” indicates first observation.

| CP        | electromag. structure | hadronic structure | lepton flavor violation |
|-----------|------------------------|-------------------|------------------------|
| $\pi^+\pi^-e^+e^-$ | $\mu^+\mu^-e^+e^-$ | $\pi^0\gamma\gamma$ | $\pi^0\mu\mu$ |
| $\pi^0e^+e^-$     | $e^+e^-e^+e^-$ | $\pi^0e^+e^-$     | $\pi^0e^+e^-$ |
| $\pi^0\mu^+\mu^-$ | $\mu^+\mu^-\gamma\gamma$ | $\pi^0e^-\gamma$     | $\pi^0e^-\gamma$ |
| $\pi^0\nu\bar{\nu}$ | $e^+e^-\gamma\gamma$ | $\pi^0e^+e^-$ | $\pi^0e^+e^-$ |
| $\pi^+\pi^-\gamma$ | $\mu^+\mu^-\gamma\gamma$ | $\pi^0\mu\mu$ | $\pi^0\mu\mu$ |

Figure 17. History of $K_L \rightarrow \pi^0\ell\bar{\ell}$ results.