Magnetic Field Concentrator Based on the Superconducting Films with Nanosize Cuts

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Abstract. Optimal active strip nanostructuring of a magnetic field concentrator based on superconducting films allows to further increase the concentration ratio of the device. The magnetically sensitive element was placed between two concentrator rings lying in the same plane without crossing. Calculated concentration coefficients \( F \) and \( F_0 \) of a planar concentrator with an active strip with nanosize cuts and without them. Different position of the cuts in the active strip of the concentrator were investigated, as well as different values of the magnetically sensitive element width \( w_0 \) and the London penetration depth \( \lambda \). In the calculations it was assumed that the width of the cut \( w_p \) coincides with the distance \( w_a \) between the ends of the near concentrator and magnetically sensitive element. The active strip width \( w_s \) and width of the superconducting branch were multiples of \( w_a \). It turned out that as \( w_0 \) decreases, \( F_0 \) increases and \( F \) decreases but the total concentration coefficient \( F^* = F_0 F \) increases. \( F^* \) value for a concentrator based on the niobium film (\( \lambda \sim 50\,nm \)) is higher than for the concentrator based on films Y-123 or Bi-2223 (\( \lambda \geq 250\,nm \)). The considered concentrator with nanosize cuts will increase the efficiency of combined magnetic field sensors, SQUIDs, and other sensors with a resolution of \( \leq 1 \) pT.

1. Introduction

Ultra-sensitive magnetic sensors are in great demand in many applications, for example: aerospace navigation, exploration of the Earth resources, non-destructive testing of materials, structures and nanosize objects, biomedical diagnostics. In the latter case, ultra-weak magnetic fields with values \( B_0 \leq 10^{-9}\,T \) arising in the physiology of biological elements (neurons, cells, tissues, organs, etc.) need to be registered non-invasively. In some cases, the magnetic fields of magnetic particles that are pre-entered into the body for diagnosis or to control the targeted delivery of medical drugs are fixed.
At present, ultra-weak magnetic fields are measured by various types of manometers [1-4]: SQUIDs (Superconducting Quantum Interference Device), quantum magnetometers, fluxgate transducers (FGT), etc. Of these, the most sensitive are SQUID devices based on the effect of tunneling superconducting electrons through a weak coupling (Josephson effect, Josephson contact or Josephson junction). For SQUIDs, the threshold sensitivity, that is, the minimum recorded magnetic field, is at level of $\sim 10^{-15}$ T.

To improve the basic parameters (in particular, reducing of $\delta B_0$) of magnetic field sensors (MFS), it is necessary to use magnetic field concentrators (MFC), also called magnetic flux transformers (MFT). For this purpose, the property of superconductors is often used to preserve the magnetic flux in a closed loop without loss. In MFS, superconducting film ring often act as MFC or MFC, and various magnetoresistive structures can serve as magnetically sensitive element (MSE).

In most magnetic field sensors, high resolution, i.e. low threshold sensitivity $\delta B_0 \leq 10^{-9}$ T, is achieved through the use of superconducting film concentrators. They reduce $\delta B_0$ but due to their large geometrical dimensions, the overall size of the MFS increases [5]. For example, commercial SQUID’s are in the form of a prism or ring a few millimeters thick and with a base area $\sim 10 \times 10^{-3}$ mm² [6-8].

In [9], it was shown that optimal nanostructuring of the active strip of MFT leads to an additional increase in its concentration factor $F$, that is, the multiplication factor of the magnetic field. This further reduces $\delta B_0$, and thereby increases the effectiveness of MFS. Many sensors are sensitive to magnetic flux (for example, FGT or SQUID) and for them MFC play the role of MFT [4]. For planar MFS with superconducting film MFC, in the previous work the $F$ values were calculated, however, the possibility of varying the MSE sizes and inductance of the receiving rings of the concentrator receiving rings were not taken into account [10].

This paper presents calculations of the magnetic field concentration factor in planar sensor, when the active strip of the concentrator is in both nanostructured and non-nanostructured states. While in $F$ the inductances of the receiving rings of the concentrator are taken into account.

2. Research methods

Figure 1 shows a sketch of the MFC.

![Sketch of the MFC](image)

Shown here: $a$ – MFC in the form of superconducting rings between which MSE is enclosed; $b$ – superconducting rings active strips and MSE on an enlarged scale; $c$ – the cuts are located at the same distance along the width of the active strip; $d$ – the cuts are located at the same distance by half the width of the active strip, which is far from the MSE; $e$ – the cuts are located at the same distance by half the width of the active strip, which is close to MSE.

The width of the active strip of the concentrator $w_0$ is less than the width of the remaining sections by an order of magnitude or more. This leads to a multiple increase in the shielding current density in the active strip, therefore, to an increase in the concentration of the external magnetic field near the active strip and on the MST.

Concentration coefficients were calculated for cases when there are no cuts in the active strip $F_0$, that is, MFC is non-nanostructured (Fig. 1, $b$), and when there are no cuts in the active strip $F$, that is, MFC is nanostructured (Fig. 1, $c$, $d$, $e$). The location of the slits in the active strip, the width of the MSE and the value of the London penetration depth $\lambda$ varied. Only the projection of the magnetic field perpendicular to the substrate surface was taken into account.

In the external magnetic field $B_0$, the magnetic flux that shields the ring 1 (Fig. 1), is defined as $\phi = A \cdot B_0$, where $A = \pi D^2/4$ is ring area, $D$ is ring diameter.

Shielding current $I_S$ has a magnitude $I_S = \phi/(L+M)$, where $L$ is ring inductance, $M$ is the sum of mutual inductances between parts of MFC and MSE.

It is known that the value of $L$ is an order of magnitude or more greater than the total inductance of $M$. Then for $I_S$ we write:
where \( w_L \) – ring width, \( \mu_0 = 4\pi \cdot 10^{-7} \text{ H/m} \) – magnetic constant. The inductance of the MFC ring \( L \) is much greater than the inductance of the active strip \( L_S \). In the case where the latter consists of several branches, each with an inductance \( L_i \), their total inductance slightly increases relative to \( L_S \). The formula (2) is given for the "thin ring" (\( D, w_L \gg d \), \( d \) is the ring thickness), but in order of logarithmic accuracy it almost coincides with \( L \) for the "thick ring" (\( D, w_L \geq d \)).

In the calculations, known formulas were used, in which the Meissner-Oxenfeld effect and the dependence of the magnetic field \( B \) on the superconducting current \( I \) in the branch of the active strip are reflected [7,8]:

\[
B_i = \frac{\mu_0 \cdot I_i}{8\pi \cdot \lambda \cdot h} \left[ \int_{-2h-l}^{0} \frac{e^{-\frac{x}{\lambda}}}{(y_0-y)^2+(x_0-x)^2} \, dx \, dy + \int_{0}^{l} \frac{e^{-\frac{l-x}{\lambda}}}{(y_0-y)^2+(x_0-x)^2} \, dx \, dy \right]
\]

\[
F_0 = \frac{\langle B_i \rangle}{B_0};
\]

\[
F = \frac{\langle B \rangle}{\langle B_i \rangle} \cdot \frac{1}{K_L};
\]

\[
K_L = \frac{\sum_{i=1}^{n} L_i^{-1}}{L} - \frac{w_i}{\sum_{i=1}^{n} w_i},
\]

where \( w_s \) – width of an active strip; \( w_i \) – width of \( i \)-th superconducting branch; \( n \) – number of superconducting branches; (\( n=1 \) – solid (non-nanostructured) active strip, \( n \geq 2 \) – nanostructured active strip), \( l \) and \( h \) – half-width and half-sickness of the \( i \)-th branch, respectively;

\[
I_S = \sum_{i=1}^{n} I_i; \quad \frac{I_i}{4\lambda h} \leq J_c,
\]
where $J_c$ – critical current density of the MFC superconducting film; $\lambda$ – London’s penetration depth of a magnetic field into superconductor; $K_L$ – growth factor total inductance of the active strip; $\langle B \rangle$, $\langle B_c \rangle$ – averaged over the width of the MSE magnetic fields generated by the current in the active strip with and without cuts, respectively. The values $B_i$, $B$ and $B_a$ were determined according to (1) ÷ (3), averaged over the width of the active strip and their average values $\langle B \rangle$ and $\langle B_c \rangle$ were used in (4) and (5).

In all calculations it was assumed that the width of the cut $w_p$ coincides with the width of the gap $w_a$ between the nearest edges of the MFC and MSE, and the width of the active stripe and its branches are multiples of $w_a$. In addition, it was assumed that the lengths of the active strips and the slits in them are much larger than the geometric dimensions of the MSE.

3. Results

The research was carried out of the dependence of the values of $F_0$ and $F$ on the MSE width $w_0$. Table 1 shows the result of calculations. Three $w_0$ values were considered: 0.2 $\mu$m, 1 $\mu$m and 5 $\mu$m. The following numerical values were taken for calculations: $\lambda = 50$ nm, $250$ nm; $J_c = 10^{-10}$ A/m$^2$; $h = 10$ nm; $w_i = 30$ $\mu$m; $r_l = D/2 = 1$ mm – ring radius; $w_i = 0.8$ mm – ring depth; $w_a = w_p = 20$ nm – the distance between MFC and MSE, the width of the cuts, respectively. The cuts were applied evenly across the width of the active strip (see Fig. 1, c).

Table 1. The dependence of the concentration coefficient on the width of the magnetically sensitive element

| $w_0$, $\mu$m | $F_0$ | $F$ | $F^*$ |
|---------------|-------|------|-------|
| $\lambda = 50$ nm | $\lambda = 250$ nm | $\lambda = 50$ nm | $\lambda = 250$ nm | $\lambda = 50$ nm | $\lambda = 250$ nm |
| 5.0 | 210 | 160 | 1.265 | 1.350 | 270 | 210 |
| 1.0 | 680 | 440 | 1.095 | 1.145 | 740 | 500 |
| 0.2 | 1770 | 950 | 1.035 | 1.070 | 1830 | 1020 |

From Table 1 it can be seen that with decreasing MSE width, $F_0$ increases and $F$ decreases, but the overall concentration ratio $F^* = F_0 \cdot F$ increases. The threshold sensitivity depends on the concentration ratio as $\delta B_0 \sim 1/F^*$ [10], therefore using low-temperature superconducting materials (LTSC, for example, heteroepitaxial niobium layers, $\lambda = 50$ nm) as MFC films is most effective compared to MFC from high-temperature superconducting materials (HTSC, for example, systems Y-123, and Bi-2223, $\lambda \geq 250$ nm). Especially low $\delta B_0$ value, that is, high efficiency of MFC is realized when the MSE has a narrow width (0.2 mm) and $F^* \sim 1830$.

The dependences $F$ ($w_0$) at $\lambda = 50$ nm and various locations of the cuts are shown in Fig. 2. It can be seen that the most optimal is the split, when the cuts are closer to the MSE (see Fig. 1, e). In this case, $F$ is approximately 1.5 times larger than in the case when the cuts are located in the farthest part of the active strip from the MSE.
Fig. 2. Dependence $F(w_0)$ at various locations of the cuts in the active strip: ◯ – near to MSE (Fig. 1, e); □ – evenly (Fig. 1, c); ◈ – far from MSE (Fig. 1, d).

On the basis of the calculations done, it was concluded that the most optimal is the splitting of the active strip when the cuts are closer to the MSE, and the width of the element is $w_0 = 0.2 \mu m$.

The following calculations were carried out in two stages. At the first stage, sequential fragmentation of the active strip into a branch and cuts was carried out. Indent from the edge of the active strip for applying the first cut was determined by the step value. The step is also the value by which the cut was shifted along the width of the active strip when selecting the optimal location. This stage allowed us to find the initial indent, that is, a step from the edge or from the previous strip, when the highest value of $F$ is reached, that is, $F_n$. At the second stage, at a fixed step value, the dependence $F(n)$ was calculated for various numbers of cuts $n$ and their widths $w_p$.

Stage I. The dependence of the concentration ratio of the magnetic field on the location and the number of cuts in the active strip was investigated. Three step values were chosen: 20 nm, 200 nm, and 1000 nm. The selection of the most optimal partitioning was carried out as follows: a cut was fixed on the active strip at a step distance from the edge (closer to MSE), the corresponding concentration coefficient was calculated. Then this cut was displaced in the direction from the edge of the active strip by the step value. This way the cut went through the entire active strip. The cut position was fixed when the concentration ratio $F$ reached the maximum value $F_m$. From this cut in the direction from MSE at a distance of a step, a second cut was made, which also shifted along the width of the active strip, and its position was fixed when reaching $F_m$. On the same way, the third, fourth, etc. cuts were applied and the $F_m$ were calculated.

The width of the cuts for all three cases was the same $w_p = 20 \text{ nm}$.

The application of cuts in the amount of 20 pieces was investigated. Fig. 3 and Fig. 4 show the $F_n(n)$ curves, that is, the dependence of the concentration coefficient on the number of cuts in the active strip, provided that the cuts have an optimal location so that they are listed in the course of the calculations. Fig. 3 shows the case when the active strip is based on LTSC ($\lambda = 50 \text{ nm}$), and in Fig. 4 – based on HTSC ($\lambda = 250 \text{ nm}$).

Analysis of the obtained curves allows us to conclude that with the increase in the number of cuts, it is possible to increase the value of the concentration coefficient $F$. In this regard, we observe that the increase in the step value does not allow to detect the maximum values of the concentration ratio: the smaller the step, the higher the $F_m$. The maximum values were reached at a step 20 nm at that is, for the NTSC superconductor (see Fig.3): the concentration coefficient almost doubled from $F_m = 1.83$ for one cut to $F_m = 3.57$ for 20 cuts. The optimal splitting of the active strip of one of the rings of a magnetic field concentrator with such parameters is as follows (beginning from MSE, numbers indicate the dimensions of the branches in nm, dotted lines denote cuts of 20 nm): 140 – 180 – 220 – 240 – 260 – 280 – 300 – 320 – 340 – 360 – 380 – 380 – 400 – 400 – 420 – 420 – 440 – 440 – 440 – 23000. By the nature of the curves in Fig.3 and Fig.4 it possible to judge that a further increase in the number of cuts...
(from 20 pieces) will not lead to further increase in $F_m$, since the increase in the curve becomes less noticeable. From Fig.3 and Fig.4 it can be seen that at different values of the step 20 nm and 200 nm, the concentration coefficients are slightly different from each other ($\leq 7\%$). For the case LTSC and under the same conditions for the case HTSC, the curves almost coincide with an accuracy of $\sim 1\%$.

A noticeable difference in the magnitudes of the $F_m$ is observed at values of steps 20 nm and 1000 nm for both types of superconductors. This behavior of the $F_m(n)$ curves appears to be caused by the influence of the step size on the uniform current distribution in the active strip. Indeed, when the step size (20 nm, 200 nm) is comparable to or less than the parameter $\lambda$, the current in the active strip from the edges is redistributed to its middle, thus the average magnetic field created by the superconducting branches on the MSE increases. $F_m$ grows also with correlating. This effect is less expressive at the step size 1000 nm, since it is much larger than the $\lambda$, and the current redistribution from the edges to the middle is insignificant.

Stage II. Based on previous calculations, step 20 nm was chosen for the following calculations. This time the width of cuts $w_p$ took three different values: 20 nm, 100 nm and 500 nm. In this case, the selection of the location of the cuts was made similar to stage I. In Fig. 5 and Fig. 6 shows the dependences $F_m(n)$ for a low-temperature superconductor and a high-temperature superconductor, respectively. As shown in Fig. 5 and Fig. 6, with an increase in the number of cuts an increase in the magnetic field concentration coefficient also occurs. After reaching a certain value of the cut number, this effect is no longer observed and the concentration coefficient value decreases. For example, in Fig. 5 (2) a peak is observed when the number of cuts is 13, and in Fig. 5 (3) when the number of cuts is 4. Thus, reducing the size of the cut leads to an increase in the number of cuts required to obtain the maximum values of the concentration coefficient, but these values are much higher. For example, for the case of 19 cuts in Fig. 6 (1), concentration coefficient $F_m = 2.40$, and for the case of 5 cuts in Fig. 6 (3), $F_m = 1.54$ (and this is the maximum value), that is, the difference is almost 1.5 times.

Optimal splitting, when the concentration coefficient is maximum, was obtained for the case of a low-temperature superconductor (see Fig. 5) $F_m = 3.57$ and coincides with the optimal splitting, obtained at stage I.

Note that superconducting films MFC or MFT can significantly lower the threshold sensitivity of MFS. In particular, in the combined sandwich-type MFC consisting of a superconducting film concentrator and a spintronics structure as MSE, it was possible to obtain $\delta B_\delta \leq 5\mu T$ resolution at the SQUID resolution level. At the same time, the calculated values $F_m$ are in good agreement with the measured values [11,12].
It should also be noted that, according to (3), (4) in order of magnitude $F_0 \sim D/(w_0 + w_p)$, that is, to increase $F_0$, it is required to increase the diameter $D$ of the receiving rings, which leads to an increase in the geometric dimensions of a solid non-nanostructured MFC. However, nanostructuring of MFC allows, due to $F > 1$, to further increase the overall concentration ratio $F^* = F_0 \cdot F$ and the need to increase $D$ for $F_0$ growth is no longer necessary. For example, for a solid (non-nanostructured) MFC, a calculated value $F_0 = 1770$ is obtained with a diameter $D = 2 \text{ mm}$ and the parameters reflected in the table. However, after nanostructuring the active strip while maintaining the previous parameters and the total concentration ratio $F^* = 1770$, the value $F_0$ can be reduced as $F_0 = F^*/F$ and, accordingly, the former value of $D$ can be reduced $F$-fold. Obviously, such a situation is preferable when too large an $F^*$ value does not make it possible to reduce $\delta B_0$, since it is limited by MSE own noise or MFS design noise. It can be seen that the nanostructuring of the active strip allows to reduce either the resolution or the geometric dimensions of the MFC, which is an significant factor for increasing its efficiency. Obviously, in the latter case, the geometric dimensions of the MFS sharply decrease, since they are practically determined by the size of the MFC (see Fig. 1).

4. Conclusion

From the analysis of the obtained results it follows: nanostructuring of the active strip of the superconducting film concentrator of the magnetic field into parallel branches and cuts, changing the width of the magnetically sensitive element, and using low-temperature superconductive materials allows to achieve a significant increase in the concentration ratio of the magnetic field and, thereby, lowering the threshold sensitivity of the magnetic field sensor or reduce its geometric dimensions. In particular, when the diameter of the rings of the magnetic field concentrator is 2 mm, non-nanostructured (continuous) active strips 30 μm wide and a magnetically sensitive element 0.2 μm wide, the concentration ratio 1770 (LTSC material) is achieved. However, the nanostructuring of the active strips of the concentrator in the form of alternating parallel superconducting branches and slits with a uniform distribution across the width and with nanosize widths (20 nm) the total concentration ratio increases to $F^* = 1830$. Nanostructuring reduces the diameter of the concentrator rings. The positive effect becomes more significant if you use the case of the optimal location of the cuts in the active strip and the corresponding values of $F_m$. For example, when $F_m \approx 3.57$ (see Fig. 5) the $F^*$ value can reach 6300, or it is possible to reduce the diameter of the rings to 0.6 mm.

In modern medicine, relevant biocompatible materials (nanomaterials with ferromagnetic or superparamagnetic particles, carbon nanotubes, etc.), as well as non-invasive diagnosis and control of active implanted devices (artificial heart, various stimulants, measuring blood flow velocity, etc.) are
relevant. The sought-after task are likely to be solved using magnetic field sensors with superconducting film magnetic field concentrators with nanostructured active strips.

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