Interval Type-2 fuzzy logic controller design for the speed control of DC motors

Hossein Hassani and Jafar Zarei*

Department of Electrical and Electronics Engineering, Shiraz University of Technology, Shiraz, Iran

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In this paper, an optimal interval Type-2 Fuzzy controller is designed for the speed control of DC motors. In this way, first, the importance and position of Type-2 fuzzy systems are mentioned. In addition, some properties of Type-2 operators are investigated as well as the properties of membership degree of Type-2 fuzzy sets. A comparison between different parts of Type-1 and Type-2 fuzzy systems, such as fuzzifier, fuzzy inference engine, rule-base and defuzzifier is given. Finally, an Interval type-2 Fuzzy logic controller is implemented for the speed control of DC motor for the cases of series and shunt. The motor is considered under both the load disturbances and disturbance free conditions. The obtained results for different conditions are compared in tables and figures. The results show that in the disturbance free case, both controllers have acceptable performance, however, when the system is affected by disturbance interval Type-2 controller has better performance.

Keywords: interval type-2 fuzzy logic systems; interval type-2 fuzzy sets; interval type-2 fuzzy membership functions; theoretical operations on type-2 fuzzy sets

1. Introduction

Series and shunt connected DC motors are widely used in control applications. These motors have relatively high torque for their weight, especially when compared to a similar size permanent magnet motors. Permanent magnet motors are linear while shunt and series motors are nonlinear. The non-linearity of the series and shunt connected DC motors complicates their use in applications that require automatic speed control. However, classical control approaches are not able to handle these issues. Therefore, the major challenge in the control problem of DC motors is overcoming to the nonlinear behavior.

Fuzzy sets (Type-1 fuzzy sets) were first introduced by Zadeh in 1965 (Zadeh, 1965). Type-1 fuzzy sets (T1FSs) are exploited to design type-1 fuzzy logic controllers (T1FLCs) (Mamdani and Assilian, 1975). The successful applications of T1FLCs are reported in many researches. For example, in control and modeling (Wang, 1999; Yager and Filev, 1994), predictions of time series (Kasabov and Song, 2002; Liao, Tang, and Liu, 2004; Versaci and Morabito, 2003) and other applications (Azar, 2010, 2012; Wang and Mendel, 1992).

In Yousef and Khalil (1995), two T1FLCs are used for the speed and current control of series DC motors. Control of uncertain highly nonlinear biological processes based on type-1 Takagi-Sugeno fuzzy models is investigated in Bououiden, Chadli, and Karimi (2015). An adaptive fuzzy control scheme is considered to estimate the concentration in substrate at the outlet bioreactor. Fuzzy controller for electric power steering system is introduced in Saifia, Chadli, Karimi, and Labiod (2014). In order to overcome the friction and disturbances of the road, which are the main sources of nonlinearity in the electric power steering systems, Takagi-Sugeno fuzzy model is used to represent the non-linearity of the system, and stabilization conditions are established based on linear matrix inequality. In Zhao, Pawlus, Karimi, and Robbersmyr (2014), an adaptive neural-fuzzy inference systems are used in data-based modeling of vehicle crash. A robust observer for unknown input Takagi-Sugeno models is designed in Chadli and Karimi (2013).

Despite the apparent advantages of T1FSs, it has been shown that it is not able to handle the effect of uncertainties completely (Hagras, 2004, 2007; Mendel, 2001). This is because a T1FS is certain in the sense that its membership grades are crisp values. Type-2 fuzzy sets (T2FSs) were also introduced by Zadeh as an extension of T1FSs (Zadeh, 1975). T1FSs have certain membership functions, while T2FSs have membership functions that are fuzzy themselves. In the other hand, the membership grade of type-1 membership functions are crisp numbers, whereas the membership degree of type-2 membership functions can be any subset in the interval $[0, 1]$ that are called primary membership function (PMF). In addition, according
to any PM, there is a value that is called secondary membership function (SMF) that defines the probability of PMFs. Since this improvement increases the computational burden, interval type-2 fuzzy logic controllers (IT2FLCs) in which SMFs are zero or one, are developed (Liang and Mendel, 2000).

IT2FLCs are used widely because of their reasonable computations. A type-2 fuzzy controller is designed for liquid-level control in Wu and Tan (2004). Using genetic algorithm, an optimal type-2 fuzzy controller is implemented for the velocity regulation of a DC motor in Maldonado and Castillo (2012). In Hsiao, Li, Lee, Chao, and Tsai (2008), an interval type-2 fuzzy sliding mode controller is proposed for linear and nonlinear systems. This controller is the combination of IT2FLC and sliding mode controller. In order to reduce the effect of uncertainty associated with the available information, a T2FLC is designed to control a buck DC–DC converter (Lin, Hsu, and Lee, 2005).

In this paper, A denotes a T1FS, and \( \mu_A(x) \) shows the membership degree of \( x \) in the T1FS \( A \); \( \bar{A} \) denotes a T2FS and \( \mu_{\bar{A}}(x) \) denotes the membership degree of \( x \) in the T2FS \( \bar{A} \); i.e. \( \mu_{\bar{A}}(x) = \int_{\sigma} f_\sigma(u) \, du \), where \( u \in J \subseteq [0, 1] \); \( \cap \) is used to show the meet operator; and, \( \cup \) denotes join operation.

The remainder of this paper is organized as follows. In Section 2, type-1 and type-2 fuzzy sets are introduced. Two examples of T2FSs are given in Section 2.1. The theoretical operations of T2FSs are presented in Section 2.2. The structure of IT2FLSs is introduced in Section 3. The simulation results are given in Section 4.

2. Type-1 and Type-2 fuzzy sets

This section introduces T1FSs and T2FSs. An example of T1 fuzzy set \( A \) is demonstrated in Figure 1, while only integer numbers are considered in the \( x \) domain. This fuzzy set can be represented as \( \{0.1, 1, 1.67, 0.33, 0.67, 1\} \), where for example \( 0.67/6 \) means that the number 6 in the domain of \( A \), has the membership degree of 0.67.

Membership functions are in different shapes such as Trapezoidal, Triangular, Gaussian and etc. The parameters of membership functions can be designed by experts or tuned using optimization methods (Horikawa, Furuhashi, and Uchikawa, 1992; Jammeh, Fleury, Wagner, Hagras, and Ghanbari, 2009; Wang and Mendel, 1992).

It is shown that T1FSs have some limitations to model and reduce the effects of uncertainties (Hagras, 2004, 2007; Wu and Wan Tan, 2006). Therefore, T2FS was developed as a powerful alternative for addressing these issues (Zadeh, 1974). The main disadvantage of the T2FLSs is computational burden. This is because of the type-reducer computations. Using IT2FLSs computations reduces to reasonable volume that make the implementation of IT2FLC easy.

IT2FSs as a special case of T2FSs, are currently the most widely used for their reduced computational burden. An example of IT2FS is illustrated in Figure 1. According to this figure, in T1FSs the membership degree of each element \( x \) in the domain of the T1FS is a crisp number, while the membership degree of each element of \( x \) in the domain of IT2FS is an interval. For example, the membership degree of 0.5 is the interval \([0.9, 1]\).

As it is illustrated in Figure 1, IT2FSs are bounded from up and down with two T1FMs that are called upper membership function (UMF) and lower membership function (LMF), respectively. The area between UMF and LMF is called footprint of uncertainty.

2.1. Two examples of T2 fuzzy sets

Example 1 Consider a Gaussian membership function with the mean \( m \) and standard deviation \( \sigma \) that can take values in \([\sigma_1, \sigma_2]\), i.e.

\[
\mu(x) = e^{-\frac{1}{2}(x-m)^2} ; \quad \sigma \in [\sigma_1, \sigma_2].
\]

(1)

Corresponding to each \( \sigma \), we will get a different membership curve. Therefore, membership degree of each element \( x \) (except \( x = m \)) according to each values of \( \sigma \) can change.

![An example of T1FS.](image)

![An example of IT2FS.](image)

Figure 1. T1 and IT2 fuzzy sets. (a) An example of T1FS. (b) An example of IT2FS.
functions. Suppose that they are characterized by their membership
sets are defined using

\[ \mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x) = e^{-\frac{1}{2}(x-m)^2}; \quad m \in [m_1, m_2]. \]

Consider a case that mean changes in the interval [0.45, 0.55] and standard deviation is 0.12 as shown in
Figure 2(a).

Example 2 As another case, consider a Gaussian membership function with standard deviation of \( \sigma \) and uncertain mean \( m \), that can take values in \([0.1, 0.15]\) and mean is 0.5. This fuzzy set is shown in Figure 2(a).

2.2. Set theoretic operations on type-2 fuzzy sets

Set theoretic operations are frequently used to implement an IT2FLS. Theoretical operations on type-2 fuzzy sets are defined using Extension Principle (Dubois, 1980; Zadeh, 1974). Consider two T2FSs \( \tilde{A} \) and \( \tilde{B} \) in the universe \( X \). Suppose that they are characterized by their membership functions \( \mu_{\tilde{A}}(x) \) and \( \mu_{\tilde{B}}(x) \), respectively. These fuzzy sets can be described as

\[ \mu_{\tilde{A}}(x) = \sum_i f_i(u_i) / u_i; \quad u_i \in J \subseteq [0, 1], \]

\[ \mu_{\tilde{B}}(x) = \sum_i g_i(w_i) / w_i; \quad w_i \in J \subseteq [0, 1]. \]

Using Extension Principle, the membership degree of the union, intersection and negation of the T2FSs \( \tilde{A} \) and \( \tilde{B} \) can be written as (Mizumoto and Tanaka, 1976)

\[ \tilde{A} \cup \tilde{B} \iff \mu_{\tilde{A} \cup \tilde{B}}(x) = \mu_{\tilde{A}}(x) \bigcup \mu_{\tilde{B}}(x) \]

\[ = \sum_{i,j} (f_i(u_i) \ast g_j(w_j)) / (u_i \lor w_j), \]

\[ \tilde{A} \cap \tilde{B} \iff \mu_{\tilde{A} \cap \tilde{B}}(x) = \mu_{\tilde{A}}(x) \bigcap \mu_{\tilde{B}}(x) \]

\[ = \sum_{i,j} (f_i(u_i) \ast g_j(w_j)) / (u_i \land w_j), \]

\[ \tilde{A}^c \iff \mu_{\tilde{A}^c}(x) = 1 - \mu_{\tilde{A}}(x), \]

where \( \ast \) represents maximum t-conorm and \( \cdot \) is used to show the t-norm operator. Using Equations (5)–(7), the meet, join and negation operators can be obtained under minimum and product t-norm.

2.3. Join and Meet operations under min t-norm

Theorem 1 (Karnik and Mendel, 1998) Suppose that we have two convex, normal, type-1 real fuzzy sets \( F \) and \( G \) characterized by membership functions \( f \) and \( g \), respectively. Let \( v_0 \in R \) and \( v_1 \in R \) is such that \( v_0 \leq v_1 \) and \( f(v_0) = g(v_1) = 1 \). Then, the membership functions of join and meet of \( F \) and \( G \), using maximum t-conorm and minimum t-norm, can be expressed as

\[ \mu_{F \cup G}(\theta) = f(\theta) \lor g(\theta); \quad \theta < v_0, \]

\[ = g(\theta); \quad v_0 \leq \theta \leq v_1, \]

\[ = f(\theta) \lor g(\theta); \quad \theta > v_1 \]

and

\[ \mu_{F \cap G}(\theta) = f(\theta) \land g(\theta); \quad \theta < v_0, \]

\[ = f(\theta); \quad v_0 \leq \theta \leq v_1, \]

\[ = f(\theta) \land g(\theta); \quad \theta > v_1. \]

Figure 3 shows Theorem 1 for Gaussian membership functions.
2.4. Join under product t-norm

THEOREM 2 (Karnik and Mendel, 1998) Consider two convex, normal, type-1 real fuzzy sets $F$ and $G$ characterized by membership functions $f$ and $g$, respectively. Let $v_0 \in \mathbb{R}$ and $v_1 \in \mathbb{R}$ be such that $v_0 \leq v_1$ and $f(v_0) = g(v_1) = 1$. Then, the membership functions of the join of $F$ and $G$, using maximum t-conorm and product t-norm, can be expressed as

$$
\mu_{F \cup G}(\theta) = \begin{cases} 
   f(\theta)g(\theta); & \theta < v_0, \\
   g(\theta); & v_0 \leq \theta \leq v_1, \\
   f(\theta) \lor g(\theta); & \theta > v_1.
\end{cases}
$$

(10)

where for $i = 1, 2, \ldots, n$

$$
\tilde{\sigma} = \sqrt{\sigma_1^2 \prod_{i, i \neq 1} m_i^2 + \cdots + \sigma_n^2 \prod_{i, i \neq n} m_i^2}
$$

(12)

Using Equation (12), an approximation of meet under product t-norm is obtained. After defining meet and join, negation operator is introduced as follows.

THEOREM 3 (Karnik and Mendel, 1998) If a T1FS $F$ has a membership function $f(\nu) (\nu \in \mathbb{R})$, then $\neg F$ has the membership function $f(1 - \nu) (\nu \in \mathbb{R})$.

The negation operator for a typical MF is illustrated in Figure 4.

2.5. Meet under product t-norm

There is no closed-form formula for meet operator under product t-norm. However, an approximation approach is suggested in Karnik and Mendel (1998). This method considers $n$ Gaussian membership functions with the mean $m_1, m_2, \ldots, m_n$ and standard deviations $\sigma_1, \sigma_2, \ldots, \sigma_n$, then,

$$
\mu_{F_1 \cap F_2 \cap \cdots \cap F_n}(\theta) \approx e^{-\frac{1}{2}(\theta - m_1 \cdots m_n)/\tilde{\sigma}},
$$

(11)

where for $i = 1, 2, \ldots, n$

$$
\tilde{\sigma} = \sqrt{\sigma_1^2 \prod_{i, i \neq 1} m_i^2 + \cdots + \sigma_n^2 \prod_{i, i \neq n} m_i^2}
$$

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3. Interval type-2 fuzzy logic systems

General structure of IT2FLSs is illustrated in Figure 5. As it is clear, the structure is almost similar to the structure of T1FLSs. The main difference is that at least one of the FSs in the rule base is an IT2FS. Therefore, the outputs of the inference engine are IT2FSs and a type reducer is needed in order to convert them into a T1FS. Then, the T1FSs is defuzzified into a crisp number as the output of the IT2FLS. This process is shown in Figure 5.
A comparison between T1FLSs and IT2FLSs is given in the following. Fuzzifier is a mapping from crisp input to a fuzzy set. Singleton or non-singleton fuzzification can be considered. In the singleton case, in the input fuzzy set, only a single point has a non-zero membership degree and is equal to 1. Non-singleton fuzzifier considers a fuzzy set (T1FS or T2FS) corresponding to each input point. The only difference is that in T2FLSs a type-2 non-singleton fuzzification can take place, i.e., a T2FS is considered for each input point.

In T1FLSs, the rules are in the general form of \( \text{if } \cdot \cdot \cdot \text{ then } \cdot \cdot \cdot \).

For example, the \( f \)th rule can be expressed as

\[
R_f : \text{IF } x_1 \text{ is } F_{1f} \text{ and } \cdot \cdot \cdot \text{ and } x_p \text{ is } F_{pf}, \text{ THEN } y \text{ is } G_f,
\]

where \( x \)'s are inputs, \( F \)'s are antecedent sets, \( y \) is output and \( G \)'s are consequent sets.

The major difference between T1FLSs and IT2FLSs rule base refers to the nature of membership functions, and it does not have any effect on the general form of rules. Therefore, the structure of the rules will remain the same as T1FLSs, and the only difference is that in IT2FLSs membership functions are interval type-2.

Fuzzy inference engines are as a mapping from T1FSs into T1FSs. Antecedent sets in the rules connect to each other using \( t \)-norm (according to the intersection of sets). The input membership degree will be combined by the output membership degree according to the sup-star composition. These steps are also done in IT2FLSs. In addition, in T2FLSs, the computations of union and intersection of IT2FSs are needed as well as the compositions of T2 relations.

4. Simulation results

The design procedure of IT2FLSs with details is presented in Liang and Mendel (2000). These steps are ignored to present in this paper, however, the implementation of controllers are based on the results of Castro, Castillo, and Melin (2007) and Liang and Mendel (2000) and the structure of the proposed method has been shown in Figure 6.
In the current work, the developed toolbox for MAT- LAB in Castro et al. (2007) is used to implement the IT2FLC. This toolbox is considered specially for designing IT2FLCs. Like T1 toolbox of MATLAB, this toolbox has a graphical user interface, and it is also possible to design the system using the command line of MATLAB. Using this toolbox, both Mamdani and Sugeno types of fuzzy systems can be designed. This toolbox is constructed from IT2 fuzzy inference system editor, IT2MF editor, IT2 rule editor, surface and rule viewer. An interesting tool in this toolbox is the ability of type-reduced surface viewer that shows the type-reduced set.

The speed control of DC motors is under consideration. For this purpose, series and shunt connected DC motors are selected. The equivalent circuits of these motors are illustrated in Figure 7. The performance of T1FLC is considered to IT2FLC.

The controller inputs are error $e$ and its derivative $\dot{e}$. The only output of controller is the control signal $u$. Inputs and output membership functions of T1FLC and IT2FLC are demonstrated in Figure 8. As it can be seen from this figure, MFs are considered as a composition of trapezoidal

| $\dot{e}/e$ | NB | NM | NS | Z | PS | PM | PB |
|-------------|----|----|----|---|----|----|----|
| NB          | NB | NB | NB | NM | NS | NS | Z  |
| NM          | NB | NM | NM | NM | NS | Z  | PS |
| NS          | NB | NM | NS | NS | Z  | PS | PM |
| Z           | NB | NM | NS | Z  | PS | PM | PB |
| PS          | NM | NS | Z  | PS | PS | PM | PB |
| PM          | NS | Z  | PS | PM | PM | PM | PB |
| PB          | Z  | PS | PS | PM | PB | PB | PB |

Figure 8. Membership functions of inputs and output of controllers. (a) Type-1 controller. (b) Interval type-2 controller.
and triangular shapes. Rule base for both controllers is in the same form as written in Table 1.

As mentioned in Section 3 about the type reducer, one can see in Figure 9 that the IT2MF converted to a T1MF, which is shown by dashed blue line, as the output of the type reducer. In Figure 10, the performance of both proposed controllers is compared in the speed control of series connected DC motors. The step information are collected in Table 2. From this table, IT2FLC has less IntegralAbsoluteError (IAE), rise-time and settling-time than T1FLC.

In addition, to evaluate the performance of the designed controllers, a load disturbance is inserted at $t = 5$ s. According to Figure 10, it is clear that IT2FLC has the better results than T1FLC. As it can be seen from Figure 10, IT2FLC has smaller overshoot and settling-time compared to T1FLC. In the case of shunt connected DC motor, in the load disturbance free case, T1 has smaller overshoot and settling time, but in general, using IAE index, it is clear that IT2C has better results. As shown in Figure 11, in the load disturbance case for shunt DC motor, better results from IT2C compared to T1C are obtained (Table 3). It is true that T1FLC has better performance in the disturbance free case. As the disturbance appears in the system, it is clear that IT2FLC has a better performance is sense of overshoot. In general, the system is always affected by some disturbances and as can be seen from simulation results, in such cases, IT2FLC has a better performance.

### 5. Conclusion

In this paper, T1FSs and IT2FSs were introduced and two examples of T2FSs were illustrated. Basic operations on T2FSs and T1FSs were described and formulated.
The main idea about IT2FLSs was presented. Differences between T1FLSs and IT2FLSs were highlighted. Next, the IT2FL toolbox introduced in order to implement an IT2FLC. Both T1FLC and IT2FLC were implemented to control series and shunt connected DC motors. In order to evaluate the performance of the controllers, the motor was also considered under load disturbance. Results showed that IT2FLC has better performance, specially in the case of appearance of disturbances. As the future work, the general T2FLC can be considered, or the controllers be optimized using evolutionary algorithms such as PSO or Genetic.

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