A rapid stability charts analysis method for rock slopes based on Generalized Hoek-Brown criterion

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ABSTRACT
Stability charts for slopes remain useful in engineering practice because of their convenience and practicality. This paper presents a new stability chart analysis method based on the Generalized Hoek-Brown (GHB) criterion to rapidly calculate the factor of safety (FOS) of rock slopes. First, critical failure state charts (CFSCs) are produced to represent the relationships among the slope height, unit weight of the rock mass and all the parameters involved in the GHB criterion when the slopes lead to a critical failure state (i.e. FOS = 1.0). Second, based on the CFSCs, the formulas of the FOS derived from m, and dimensionless parameter H/H are presented for slopes with different slope angles. Thus, the FOS for any slope with a GHB rock mass can be directly determined by taking the GHB parameters (i.e. γ, m, GSI, and D), the unit weight (γ) and the slope’s geometrical parameters (i.e. height H and slope angle β) as inputs. Finally, 25 cases selected from the published literature are used to illustrate the applicability and validity of the proposed method. The results for these cases show good agreement with those obtained with the limit equilibrium method (LEM).

1. Introduction
It is generally accepted that rock masses are inhomogeneous and exhibit discontinuous failure characteristics. Thus, predicting the stability of a rock mass slope is a classical issue and remains a great challenge in geotechnical engineering (Hammah et al. 2005; Erik 2012). With the development of computer technology, many numerical simulation methods (i.e. the finite element method, finite difference method, discrete element method, etc.), along with the limit equilibrium method (LEM) and strength reduction method (SRM), have been used for slope stability analysis. However, regardless of the progress of computer technology development, the stability chart analysis method (SCAM), a traditional method that was first proposed by Taylor in 1937, remains useful in engineering practice because of its convenience and practicality. At present, the most widely used stability charts are established on the basis of the linear Mohr-Coulomb failure criterion (MC), which takes the shear strength parameters cohesion c and friction angle φ as the main inputs to estimate the factor of safety (FOS) of the soil slope (Baker, 2003; Michalowski, 2002, 2010; Steward et al., 2011; Sun et al., 2017). However, rock masses always have strongly nonlinear failure characteristics, which cannot always be accurately represented by the linear MC criterion. Recently, the Generalized Hoek-Brown (GHB) criterion has developed into one of the most broadly adopted failure criteria in rock slope engineering (Chen Y. et al. 2018). Producing stability charts combined with the GHB criterion is a promising approach for rock slope stability analysis.

The latest version of the GHB criterion was presented in 2002, and its equation in principal stress space is expressed as (Hoek et al., 2002):

\[
\sigma_1 = \sigma_3 + \sigma_c \left( \frac{m_b \sigma_1}{\sigma_c} + s \right)^a
\]

(1)

where \( \sigma_c \) is the uniaxial compressive strength of intact rock; \( m_b \), \( s \), and \( a \) are empirical parameters reflecting the characteristics of different fractured rock masses, given by:

\[
\begin{align*}
    m_b &= \exp\left(\frac{1}{10}\left(GSI-100\right)\right) m_l \\
    s &= \exp\left(\frac{1}{10}\left(GSI-10\right)\right) \\
    a &= 0.5 + \frac{1}{10} \left[\exp\left(-\frac{10}{30}\right) - \exp\left(-\frac{10}{3}\right)\right]
\end{align*}
\]

(2)

where GSI is the geological strength index, which can be estimated according to the structural features of the discontinuities in the rock mass, varying from 5 (for a highly fractured and damaged rock mass) to 100...
(for intact rock) (Sonmez & Ulusay, 1999); D reflects the degree of damage done to the rock mass by blasting or stress relaxation due to excavation, ranging from 0.0 (for undisturbed in situ rock masses) to 1.0 (for disturbed rock masses) (Hoek et al., 2002); and \( m \) is the hardness degree of the intact rock. Therefore, \( \sigma_c \), GSI, D and \( m \), which can be determined directly from laboratory test data and field observations, are the final input parameters of the GHB criterion for representing the failure of a rock mass. There is no doubt that rock mass weight \( (\gamma) \), slope height \( (H) \) and slope angle \( (\beta) \) can significantly affect the slope stability. Thus, the rock slope stability is controlled by these seven parameters: \( \gamma, H, \beta, \sigma_c, \text{GSI, D and } m \). The equation of the MC criterion in principal stress space is expressed as (Yuan et al., 2018):

\[
\sigma_1 = \frac{1 + \sin \varphi}{1 - \sin \varphi} \sigma_3 + \frac{2c \cos \varphi}{1 - \sin \varphi}
\]

where \( c \) and \( \varphi \) are the cohesion and friction angle, respectively. Once \( c \) and \( \varphi \) are determined, the relationship between the maximum and minimum principal stress in the MC criterion is linear, while their relationship in the GHB criterion is nonlinear, as shown in Eq. 1. Clearly, compared with the stability charts based on the MC criterion, there are more input parameters involved in calculating the FOS of the rock slopes, which creates a great challenge to establish stability charts on the basis of the GHB criterion.

Many scholars have devoted effort to building stability charts for the calculation of the FOS directly from the GHB criterion. Li et al. (2008), (2009), (2011) applied the ultimate analysis method to provide rock slope stability charts that take the excavation disturbance effect and seismic effect into account. A stability number \( N \) is introduced into Li et al., charts (Li et al., 2009, 2008, 2011), which is defined as \( N = \sigma_c / \gamma H F \) (\( F \) is the factor of safety). The stability charts feature \( N \) as the ordinate and \( m \) as the abscissa. The use of these charts is easy. The first step is to calculate the dimensionless parameters \( \sigma_c / \gamma H \), and then the value of \( N \) is determined according to the \( m \), and GSI of the rock mass of the slope by referring to stability charts. Finally, the factor of safety \( F \) is calculated as \( \sigma_c / \gamma H N \). Taheri and Tani (2010) proposed a slope stability rating (SSR) system to assess the stability of rock slopes. The value of the SSR originates from the modification of the initial GSI of the slope rocks, which considers five additional parameters that affect the stability of fractured rock slopes (i.e. \( \sigma_c, m, D \), saturation of slope and horizontal earthquake acceleration). Finally, a number of design charts used for representing the relationship between SSR, H and \( \beta \) are established for different FOS values (only FOS = 1.0, 1.2, 1.3, and 1.5 are listed in Abbas’s study). Shen et al. (2013) presented new stability charts for analysing the stability of rock slopes satisfying the GHB criterion. The core of Shen’s charts is designed for a slope with a specified slope angle \( (\beta = 45^\circ) \). On this basis, a slope angle weighting factor \( f_\beta \) is introduced to illustrate the influence of the slope angle \( \beta \). In addition, a disturbance weighting factor \( f_d \) is introduced to reflect the effect of various disturbances in engineering on the stability of the rock slope. Thus, the FOS for any assigned rock slope can be calculated by combining \( f_\beta \) and \( f_d \) with stability charts based on \( \beta = 45^\circ \). Jiang et al. (2016) used the LEM to produce fundamental stability charts for calculating the FOS of rock slopes with a slope angle \( \beta = 30^\circ \) in static and pseudostatic states. In addition, scaling factors of the horizontal seismic acceleration coefficient \( (f_s) \) and slope angle \( (f_\beta) \) were introduced to illustrate the influence of the horizontal seismic load and slope angle on the stability of rock slopes, respectively. Xu et al. (2018) established stability charts for 3-dimensional rock slopes based on the GHB criterion. Similar to Li et al., charts, Xu J. et al., charts are also shown according to \( N \) as the ordinate and \( m \) as the abscissa. To investigate the 3-dimensional effect of the slope stability, a dimensionless parameter \( B/H \) (\( B \) is the width of the slope) is introduced in Xu J. et al., charts. In addition, different hydraulic conditions acting on the rock slopes are taken into account.

A review of the above literature shows that there are some striking similarities among the present stability charts for rock slopes, i.e. charts are an essential database tool for slope stability analysis. In addition, almost all previously published stability charts include correction coefficients to reflect the influence of different slope angles, disturbances, underground water and earthquakes on the slope stability. The fundamental stability charts considering only one slope angle and soil gravity were established first, and then the correction coefficients were introduced to the fundamental stability charts. In this study, we present new rapid stability charts for rapidly calculating the FOS of rock slopes based on the GHB criterion. All the parameters associated with the slope stability, such as \( \gamma, H, \beta, \sigma_c, \text{GSI, D and } m \), are regarded as input parameters, and the FOS can be directly obtained from the explicit expression. The proposed method in this paper provides a convenient tool for analysing the stability of rock slopes.

2. Critical failure state charts based on the GHB criterion

The general definition of FOS is as follows (Wei et al., 2020):

\[
FOS = \frac{\tau_l dl}{\tau_m dl}
\]

where \( l \) denotes the potential sliding surface; \( \tau_m \) is the driving shear stress, which greatly depends on the
gravitational load of the overlying rock mass and the slope angle; and \( \tau_r \) is the resistant shear stress, which depends on the normal stress at the potential sliding surface and the mechanical parameters of the rock mass (Wei et al., 2020). Thus, the slope height (H), slope angle (\( \beta \)), rock mass unit weight (y), and essential parameters involved in the GHB criterion (\( \sigma_c \), GSI, \( m_i \), D) work together to determine the stability of rock slopes. The FOS can therefore be expressed as the function of these seven parameters:

\[
FOS = f(y, H, \beta, \sigma_c, \text{GSI}, m_i, D)
\]  

(5)

Supposing FOS is equal to 1.0, Eq. 5 means that all the parameters associated with the slope stability should satisfy this equation when a slope reaches the critical failure state. If the relationships among these parameters are shown as curves in a user-defined coordinate system, we can call the plots critical failure state charts (CFSCs).

It is undoubtedly highly challenging to solve Eq. 5 with FOS = 1.0. Therefore, CFSCs cannot be produced by theoretical analysis. Thus, we are required to build numerical solutions of CFSCs by means of a numerical simulation method. The work discussed in this study requires hundreds of runs on a microcomputer to obtain various combinations of \( \sigma_c \), GSI, \( m_i \), and D to exactly match slopes with different geometries upon reaching the critical failure state. All such runs were performed using the LEM software SLIDE 6.0 based on the Morgenstern–Price method. The procedures of the numerical simulation are as follows:

1. A series of slope models with different \( \beta \) and H values is established. \( \beta \) varies from 10° to 80° with an interval of 10°. Three cases of H are tested: 20 m, 50 m, and 100 m. Figure 1 shows the profile of the slope model.
2. For each slope model, \( y \) remains constant at 25.0 kN/m², and \( \sigma_c \), GSI, \( m_i \), and D are adjusted to lead the slope to the critical failure state.
3. The value of \( \sigma_{\text{cmax}}/yH \) is calculated, and the pairs of \( (m_i, \sigma_{\text{cmax}}/yH) \) are drawn in the established coordinate system, in which \( \sigma_{\text{cmax}}/yH \) is taken as the ordinate and \( m_i \) is taken as the abscissa. Note that \( \sigma_{\text{cmax}} \) means the uniaxial compressive strength of the fractured rock mass, which is expressed as:

\[
\sigma_{\text{cmax}} = \sigma_c \cdot s^a
\]

(6)

where \( s \) and \( a \) are empirical parameters reflecting the characteristics of different fractured rock masses, and their formulas are shown as Eq. 2.

1. According to the multiple nonlinear fitting method, the functional relationships between \( m_i \) and \( \sigma_{\text{cmax}}/yH \) under the conditions of different slope angles are obtained.

Based on the proposed procedures (1) to (4), the CFSCs for different slope angles are shown in Figure 2. All CFSCs exhibit the same characteristics, i.e. \( \sigma_{\text{cmax}}/yH \) first sharply drops and subsequently slowly decreases with the increase in \( m_i \). When \( m_i \) has a relatively small value (\( m_i \leq 13.0 \)), \( \sigma_{\text{cmax}}/yH \) has an appreciable influence on the slope stability. When the value of \( m_i \) is greater than 13.0, \( \sigma_{\text{cmax}}/yH \) has only a slight influence on the slope stability. Through Levenberg-Marquardt’s optimisation algorithm contained in business software for nonlinear fitting (First Optimisation, 1stOpt), Eq. 7 can be used for fitting the relationship between \( \sigma_{\text{cmax}}/yH \) and \( m_i \):

\[
\frac{\sigma_{\text{cmax}}}{yH} = p_1 \cdot m_i^{p_2 \cdot p_3 \cdot \ln(m_i)} + p_4 \cdot m_i
\]

(7)

where \( p_1, p_2, p_3 \) and \( p_4 \) are the undetermined coefficients varying with the change of the slope angle, and their fitting results are shown in Table 1. The relevant coefficients of Eq. 7 for different slope angles are all greater than 0.99, which means that Eq. 7 could be regarded as an optimal explicit expression of the CFSCs. If a given slope angle is not contained in Table 1, its corresponding undetermined coefficients are recommended to be calculated by the linear interpolation method. The general formula is given by:

![Figure 1. Schematic diagram for the slope model.](image-url)
where $p_i$; $\beta$ and $p_i$; $\alpha_1$ denote the $i$th undetermined coefficients for slope angles $\beta$, $\alpha_1$ and $\alpha_2$, respectively. The values of directly acquired by looking them up in Table 1.

### Table 1. Coefficients of critical failure state charts for different slope angles.

| Slope angle | $p_1$  | $p_2$  | $p_3$  | $p_4$  | Relevant coefficient |
|-------------|--------|--------|--------|--------|----------------------|
| 10°         | 0.0932 | -1.9765| 0.0774 | 0.00005| 0.9999               |
| 20°         | 0.1676 | -1.1867| -0.2249| 0.00013| 0.9999               |
| 30°         | 0.2188 | -0.5254| -0.427 | 0.00002| 0.9997               |
| 40°         | 0.2626 | -0.0926| -0.5099| 0.00004| 0.9995               |
| 50°         | 0.297  | 0.1019 | 0.4663 | 0.00081| 0.9995               |
| 60°         | 0.3346 | 0.232  | -0.4337| 0.00133| 0.9996               |
| 70°         | 0.4093 | 0.1318 | -0.2869| 0.00269| 0.9999               |
| 80°         | 0.5147 | 0.1293 | -0.1958| 0.00467| 0.9914               |

The values of directly acquired by looking them up in Table 1.

### 3. Calculation method of the FOS for different slope angles

Here, a new parameter $\omega$ is introduced, and it is defined as:

$$\omega = \frac{(\sigma_{\text{cmax}} / \gamma H_c)}{(\sigma_{\text{cmax}} / \gamma H_i)}$$  \hspace{1cm} (9)$$

where $(\sigma_{\text{cmax}} / \gamma H_i)$ represents the dimensionless parameter resulting from the rock slope whose stability needs to be analysed (i.e. the analysis object) and $(\sigma_{\text{cmax}} / \gamma H_c)$ represents the dimensionless parameter resulting from the critical failure state charts. Figure 3

![Figure 3](image3.png)

**Figure 3.** Schematic diagram for calculating the value of $\omega$.
shows the schematic diagram of the calculation of $\omega$. The critical failure state chart in Figure 3 clearly divides the $m_i, \sigma_{cmax}/Y^H$ coordinate system into two parts, that is, if a point is above the curve ($\omega > 1.0$), the slope is stable, if a point is below the curve ($\omega < 1.0$), the slope is unstable. Additionally, if a point is exactly on the critical failure state curve ($\omega = 1.0$), the slope is in the critical state. Thus, $\omega$ has definite physical meaning.

According to Eq. 2 and Eq. 6, $\sigma_{cmax}$ is a function of $\sigma$, GSI and $D$. Thus, the FOS is finally simplified as a function of $\omega$, $\beta$ and $m_i$ as follows:

$$FOS = f(\omega, m_i, \beta)$$

Similarly, it is difficult to establish the theoretical expression of the FOS for any specified rock slope. Thus, we also use the numerical simulation tool based on the LEM to study the relationship among $\omega$, $m_i$ and FOS for different slope angles. The procedures of the numerical simulation are as follows:

(1) A series of slope models with different $\beta$ values are established. $\beta$ varies from 10° to 80° with an interval of 10°. The value of $H$ is set to 50.0 m. For each slope model, $\gamma$ remains constant at 25.0 kN/m³.

(2) According to the slope angle $\beta$, the undetermined coefficients $p_1$, $p_2$, $p_3$, and $p_4$ are acquired, and the corresponding CFSCs are determined.

(3) $\sigma$, GSI, $m_i$ and $D$ are adjusted, and the values of $\omega$ are calculated according to the different combinations of $\sigma$, GSI, $m_i$, and $D$ by means of Eq. 9.

(4) $\sigma$, GSI, $m_i$ and $D$ are taken as input parameters and applied to the slope models. The FOS under the different conditions of input parameters and slope models is calculated by the LEM.

(5) Taking FOS as the ordinate and $\omega$ or $m_i$ as the abscissa, the pairs of ($m_i$, FOS) are drawn in the $m_i$-FOS coordinate system and the pairs of ($\omega$, FOS) are drawn in the $\omega$-FOS coordinate system. Finally, the relationship between $\omega$ and FOS for different slope angles is established.

Based on the proposed procedures (1) to (5), the variation in the FOS with increasing $m_i$ for different slope angles is shown in Figure 4, and the relationship between the FOS and $\omega$ for different slope angles is shown in Figure 5. It can be found that all the curves shown in Figure 4 exhibit the same characteristics, that is, the FOS first decreases sharply and then tends to be constant with increasing $m_i$. Specifically, when $m_i$ is probably less than or equal to 9.0, it has a significant influence on the FOS. Once $m_i$ is greater than 9.0, its effect on the FOS can be ignored. In addition, as shown in Figure 5, the FOS monotonically increases with increasing $\omega$ for different slope angles. Through the optimisation algorithm, Eq. 11 is used to fit the relationship between the FOS and $\omega$. The fitting results are shown in Table 2.

$$FOS = q_1 + q_2 \cdot \omega^{q_3}$$

where $q_1$, $q_2$, and $q_3$ are the undetermined coefficients varying with the change in the slope angle $\beta$ and $m_i$. For other $m_i$ and $\beta$ values not included in Table 2, $q_1$, $q_2$, and $q_3$ can be estimated by the interpolation method. Next, two situations are discussed in calculating the values $q_1$, $q_2$ and $q_3$ in detail.

If $m_i \leq 9$, as shown in Figure 6, suppose that $q_1$, $q_2$, and $q_3$ at points ①, ②, ③ and ④ are all known in the $\beta - m_i$ coordinate system; thus, $q_1$, $q_2$ and $q_3$ at point ⑤ located in the rectangle formed by points ①, ②, ③ and ④ can be calculated (Murti & Valliappan, 1986):

$$q_1 = \sum_{i=1}^{4} N_i \cdot q_1, \quad q_2 = \sum_{i=1}^{4} N_i \cdot q_2, \quad q_3 = \sum_{i=1}^{4} N_i \cdot q_3$$

(12)

where is the shape function and expressed as (Murti & Valliappan, 1986):

$$N_i = \frac{(\beta - \beta_1) \cdot (m_i - m_{i2})}{(\beta_1 - \beta_2) \cdot (m_1 - m_{i2})} \cdot \frac{N_1}{\beta_1 - \beta_2}$$

(13)

If $m_i > 9$, the general formula for calculating the value of $q_1$, $q_2$ and $q_3$ is given by:

$$q_{i, \beta} = \frac{q_{i, \beta1} - q_{i, \beta2}}{\beta_2 - \beta_1} (\beta - \beta_1) + q_{i, \beta1} \qquad (i = 1, 2, 3)$$

(14)

$$|\beta_1 - \beta| \leq 10°, \quad |\beta_2 - \beta| \leq 10°, \quad \beta_2 - \beta_1 = 10°$$

where $q_{i, \beta}$, $q_{i, \beta1}$, and $q_{i, \beta2}$ denote the $i$th undetermined coefficients for slope angles $\beta$, $\beta_1$, and $\beta_2$, respectively. $\beta_1$ and $\beta_2$ are the slope angles shown in Table 2. The values of $q_{i, \beta}$, $q_{i, \beta1}$, and $q_{i, \beta2}$ are directly acquired by looking them up in Table 2 for the case of $m_i > 9$.

In summary, the core of calculating the FOS is to obtain the value of $\omega$, and the prerequisite for obtaining the value of $\omega$ is establishing the CFSC of the slope according to its slope angle. According to the procedures to establish the CFSCs, it can be concluded that the proposed method in this study is only suitable for slopes with straight surfaces, and this method only considers the effect of rock gravity on slope stability. Other factors, such as groundwater, rainfall and earthquakes, are not considered in the slope stability charts. Thus, the procedures for calculating the FOS based on CSFC are finally shown as follows:

(1) Obtain all the parameters associated with the rock slope stability, i.e. $\gamma$, $H$, $\beta$, $\sigma$, GSI, $D$ and $m_i$.
(2) According to the slope angle $\beta$, determine the coefficients of CSFC for the rock slope whose stability needs to be analysed, i.e. $p_1$, $p_2$, $p_3$ and $p_4$.

(3) Calculate the dimensionless parameter of the analysis object $(\sigma_{\text{cmass}}/\gamma H)_t$.

(4) Substitute $m_i$ into the equation of the critical failure state charts to obtain $(\sigma_{\text{cmass}}/\gamma H)_c$.

(5) Calculate the value of $\omega$.

(6) According to the slope angle $\beta$ and $m_i$, determine the coefficients required to calculate the FOS, that is, $q_1$, $q_2$ and $q_3$.

(7) Calculate the FOS.

According to the above discussion, this study provides an effective method to calculate the FOS of the rock slopes. In the previous paper of the first author recently published in Computers and Geotechnics (Wei et al., 2020), an approach for calculating the FOS based on Generalized Hoek-Brown criterion has also been proposed. Next, we try to state the differences between the previous published paper and this study. In the published paper, the first and most significant step is to establish the CFSC of the slope, based on the CFSC, it is supposed that the strength of the rock mass weakens along the most possible attenuation path to make the slope reach the critical state, and then...
a reduction strategy with precise physical meaning is provided to find an optimal set of parameters that trigger rock slope failure. That is, the previous paper has established a new calculating method for FOS, and it is an essential numerical simulation method and needs to build geomechanical slope model and execute numerical iterative operation. However, the proposed method in this study belongs to the stability chart analysis method, its significant innovation is that the CFSC is also established based on numerical simulation method to build the stability charts. According to the stability charts, the FOS could be directly acquired by virtue of Eq. (7)–Eq. (14). Thus, although the CFSC is the common core and precondition in the previous published paper and this study, the former focuses on the establishment of a new numerical analysis method to calculate the safety factor, and the latter focuses on the establishment of a practical stability charts for the rapid calculation of safety factors based on the existing numerical analysis method.

4. Illustrative examples
To illustrate how to calculate the FOS by the proposed method in this study and discuss its accuracy, the following 25 cases with a wide range of rock mechanical properties and slope geometries have been
selected from previously published literature to be the objects of study. Specifically, Case 1 to Case 8 originate from Taheri and Tani (2010), Case 9 to Case 11 originate from Shen et al. (2013), Case 12 to Case 14 originate from Jiang et al. (2016), and Case 15 to Case 25 originate from Sun C.W. et al. (2016). The required input parameters for calculating the FOS are shown in Table 3.

Case 1 is taken as an example to show the calculation procedures using the proposed method in this study:

1. $\beta=55^\circ$ means that the linear interpolation method is used to obtain $p_1$, $p_2$, $p_3$, and $p_4$ as follows:

$$
p_1 = 0.5 \times (0.2970 + 0.3346) = 0.3158,
\quad p_2 = 0.5 \times (0.1019 + 0.2320) = 0.1669
$$

$$
p_3 = 0.5 \times (-0.4663 - 0.4337) = -0.4500,
\quad p_4 = 0.5 \times (0.00081 + 0.00133) = 0.0011
$$

(1) The values of GSI and D are substituted into Eq. 2 to calculate $s$ and $\alpha$, that is, $s = 0.000222$, $\alpha = 0.507050$. Thus, $\alpha_{\text{max}} = 153 \times 0.000222 \times 0.507050 = 2.1485$ MPa, and $(\alpha_{\text{max}}/H)^2 = 2.1485 \times 100/27 = 0.4325$.

(2) The values of $m$, $p_1$, $p_2$, $p_3$ and $p_4$ are all substituted into Eq. 7; thus, $(\sigma_{\text{max}}/H)^2 = 0.3158 \times 9 \times 0.1669 - 0.4892 \times 0.0011 \times 9 \times 0.0011 = 0.0615.$

(3) $\omega = 0.4325/0.0615 = 7.0280$.

(4) According to $\beta = 55^\circ$ and $m = 9$, the values of $q_1$, $q_2$ and $q_3$ are calculated as follows:

$$
q_1 = 0.5 \times (0.0888 + 0.4271) = 0.2580
\quad q_2 = 0.5 \times (0.9157 + 0.5795) = 0.7476
\quad q_3 = 0.5 \times (0.3489 + 0.4982) = 0.4236
$$

(1) $FOS = 0.2580 + 0.7476 \times 0.4236 = 1.9654$.

Similarly, the FOS of the other 24 cases could be acquired by means of the same procedures mentioned above, and the results are shown in column 9 of Table 3. In addition, we also use the LEM to calculate the FOSs of the 25 examples, and the results are shown in column 10 of Table 3. To quantitatively represent the accuracy of the proposed method, the absolute error ($\Delta$) and relative error ($\delta$) are defined:

$$
\Delta = |F_p - F_A|,
\quad \delta = \left| \frac{F_p - F_A}{F_A} \right| \times 100\%
$$

where $F_p$ is the FOS resulting from the proposed method and $F_A$ is the FOS resulting from the LEM. Table 3 shows that the minimum value of $\delta$ is 0.11%, the maximum value of $\delta$ is 9.91%, and the average value of $\delta$ is 5.87%. Thus, it can be concluded that the proposed method has good performance in terms of calculating the FOS. In addition, the maximum and minimum absolute errors of the FOS resulting from the two methods are 0.2743 and 0.0035, respectively, and the average absolute error is 0.0947. It is indicated that the FOSs resulting from the two methods provide a consistent assessment of the slope stability.

5. Conclusions

By means of the LEM, this paper has proposed new and rapid stability charts for analysing the stability of rock slopes based on the GHB criterion. The first and most significant step is to establish the CFSCs, which represent the relationship between the dimensionless parameter $\sigma_{\text{max}}/H$ and $m$, when the FOS of the rock slope is equal to 1.0. Based on the CFSCs, the fitting function for calculating the FOS of rock slopes is produced. Finally, we can acquire the FOS of any specified rock slope when these input parameters are all known (i.e. $\gamma$, $H$, $\beta$, $\sigma_c$, $\text{GSI}$, $D$ and $m$).

Table 2. Coefficients of the equations of the FOS.

| Slope angle | $m_1$ | $q_1$ | $q_2$ | $q_3$ | Relevant coefficient |
|-------------|-------|-------|-------|-------|----------------------|
| 10°         | 1     | 0.4365| 0.5699| 0.7795| 0.9999               |
| 20°         | 1     | 0.3658| 0.6402| 0.8993| 0.9999               |
| 30°         | 1     | 0.2825| 0.7201| 0.9238| 0.9999               |
| 40°         | 1     | 0.2028| 0.7971| 0.9461| 0.9999               |
| 50°         | 1     | 0.1252| 0.8751| 0.9526| 0.9999               |
| 60°         | 1     | 0.0888| 0.9157| 0.9489| 0.9999               |

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5. Conclusions

By means of the LEM, this paper has proposed new and rapid stability charts for analysing the stability of rock slopes based on the GHB criterion. The first and most significant step is to establish the CFSCs, which represent the relationship between the dimensionless parameter $\sigma_{\text{max}}/H$ and $m$, when the FOS of the rock slope is equal to 1.0. Based on the CFSCs, the fitting function for calculating the FOS of rock slopes is produced. Finally, we can acquire the FOS of any specified rock slope when these input parameters are all known (i.e. $\gamma$, $H$, $\beta$, $\sigma_c$, $\text{GSI}$, $D$ and $m$).
The use of the proposed stability charts to calculate the FOS of a given slope is quite straightforward. We selected 25 cases with varying rock properties and slope geometries to demonstrate the procedures of its application. The analysis results acquired by the proposed method are in accord with those obtained by the LEM, which indicates that the proposed stability charts have great validity and feasibility in rock slope stability analysis.

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Data availability statement
Some or all data, models, or code generated or used during the study are available from the corresponding author by request.

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