Temporal and spatial superbunching effects from a pair of modulated distinguishable classical light

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Abstract. From the Feynman path integration theory of view, the Hanbury Brown-Twiss effect would not be observed for one definite two-photon propagation path, as well as the superbunching effect. Here, temporal and spatial superbunching effects are measured from a pair of modulated distinguishable classical light. These interesting phenomena are realized by passing two orthogonal polarized laser beams through two rotating ground glass plates in sequence. To understand the underlying physical process, the intensity fluctuation correlation theory is developed to describe the superbunching effect in the temporal and spatial domain, which agrees with experimental results well. Such experimental results are conducive to the study of superbunching effect which plays an important role in improving the performance in related applications, such as the contrast of ghost imaging.

1. Introduction

Since Hanbury Brown and Twiss (HBT) first observed the two-photon bunching effect in 1956 [1, 2], they found that photons emitted by thermal light source are more inclined to arrive in pairs rather than randomly. The degree of second-order correlation of thermal light in the HBT interferometer was measured as 2, which greatly promoted the development of quantum optics [3–7]. Then the discovery of the superbunching effect that the degree of second-order correlation is greater than 2 has also attracted the attention of researchers [8–19]. It is generally accepted that the HBT effect is all well explained by both classical and quantum interpretations which can be unified. Classical intensity fluctuation correlation theory was usually employed to interpret this phenomenon [20], which emphasized whether the two detectors could detect specks with the same intensity fluctuation in the HBT interferometer. It can also be interpreted
by quantum theory, in which two-photon bunching is interpreted by superposition of different alternatives to trigger a two-photon coincidence count \cite{3,5,21}. In 2017, Bin Bai et al. reported an experiment that bunching effect was observed without two-photon interference since there is only one alternative to trigger a joint detection event \cite{22}. It seems to indicate the discrepancy between classical and quantum theory. Ultimately, he used Glauber’s quantum optical correlation theory to explain this interesting phenomenon. Later, Yu Zhou et al. proposed quantum two-photon interference theory to explain superbunching pseudothermal light based on multi-path interference \cite{17}. However, there is no classical intensity fluctuation correlation theory analyzing the such superbunching effect with distinguishable interference paths in the temporal and spatial domain.

In this paper, we proposed and demonstrated a method to observe temporal and spatial superbunching effect from a pair of modulated distinguishable classical light. The normalized temporal and spatial second-order correlation function were measured as $g^{(2)}(0) = 3.83 \pm 0.02$ and $g^{(2)}(0) = 3.42 \pm 0.02$, respectively. At the same time, the theoretical results derived from classical intensity fluctuation correlation theory are in good agreement with the experimental results. Based on above study, it further demonstrates the unity and complementarity between quantum theory and classical theory, which is not only replenish the theoretical study of superbunching effect, but also helpful to understand the physical essence of superbunching effect.

The remaining parts of the paper are organized as follows. In Sec. 2 classical intensity fluctuation correlation theory is employed to interpret the superbunching effect, then the theoretical derivation results is verified by the designed experimental scheme. The discussions and conclusions are in Sec. 3 and 4 respectively.

2. Methods

2.1. Classical intensity fluctuation correlation theory

Instead of the quantum two-photon interference theory, the classical intensity fluctuation correlation theory is used to explain the superbunching effect. This is mainly due to the correlation between the electromagnetic waves propagating to two detectors located at different space-time coordinates $(r_1, t_1)$ and $(r_2, t_2)$. Therefore, the function of the second-order correlation of optical field can be expressed as

$$G^{(2)}(r_1, t_1; r_2, t_2) = \langle E_1(r_1, t_1)E_2(r_2, t_2)E_1^*(r_1, t_1)E_2^*(r_2, t_2)\rangle,$$

where $\langle \cdots \rangle$ is the ensemble average by taking all the possible realizations into account, $E_i(r_i, t_i)$ is the electric field intensity on the surface of detector $D_i$, where $E_i^*(r_i, t_i)$ is the complex conjugate of $E_i(r_i, t_i)$, $i = 1, 2$.

When a single-mode continuous-wave laser light passes through two rotating ground glass (RG) plates continuously, the scattered pseudothermal light, which fluctuates in time and space, will arrive at the surfaces of the two detectors. The theoretical model of superbunching effect is shown in figure[1] There will be eight different electromagnetic
Temporal and spatial superbunching effects

Figure 1. The theoretical model diagram of a joint detection event triggered by different electromagnetic waves. RG$_1$ and RG$_2$ are two rotating ground glass plates. D$_1$ and D$_2$ are two single-photon detectors in the HBT interferometer. CC is the two-photon coincidence count detection system.

waves triggering the two detectors, $E_{111}$, $E_{112}$, $E_{121}$, $E_{122}$, $E_{211}$, $E_{212}$, $E_{221}$, $E_{222}$. Taking $E_{121}$ as an example, it represents the electromagnetic wave scattered by the laser passing through position 1 of the RG$_1$, then propagates to the position 2 of the RG$_2$, finally arrives at the surface of D$_1$.

For the meaning of $E_{121}$, it is actually a compound electric field, which is obtained by multiplying a initial electromagnetic waves with the propagation function under different conditions. According to Green function for a point light source in classical optics $^{23}$, the electromagnetic wave which arrived at the surface of detector without the spatial part can be expressed as

$$E_{ijk} = E_i \cdot T_{ji} \cdot T_{kij|i} = e^{-i\omega_0(t_i - t_0)} \int_{\omega_0 - \frac{1}{2} \Delta \omega_1}^{\omega_0 + \frac{1}{2} \Delta \omega_1} e^{-i\omega_j(t''_j - t'_i)} d\omega_j \int_{\omega_0 - \frac{1}{2} \Delta \omega_2}^{\omega_0 + \frac{1}{2} \Delta \omega_2} e^{-i\omega_k(t_k - t''_j)} d\omega_k, (2)$$

where $E_i$ is the electromagnetic wave which propagated by laser passing through position $i$ of the RG$_1$, $T_{ji}$ represents the propagation function of pseudothermal light when $E_i$ passes through the position $j$ of the RG$_2$, and $T_{kij|i}$ is the propagation function of electromagnetic wave when it arrives at the surface of D$_k$ under the condition of $E_i \cdot T_{ji}$.

$\omega_0$ is the center frequency of the laser, $\omega_{0i}$ is the frequency which the laser scattered by the position $i$ of the RG$_1$, and $\omega_j$ is the frequency of the light which scattered by the position $j$ of the RG$_2$. $\Delta \omega_1$ and $\Delta \omega_2$ represent the spectral widths of pseudothermal light scattered by the RG$_1$ and RG$_2$, respectively. $t'_i$, $t''_j$ and $t_k$ are the time when the electromagnetic wave arrives at position $i$ of the RG$_1$, position $j$ of the RG$_2$ and the detector $k$ surface respectively, $i, j, k = 1, 2$.

Then there are four types of coincidence current detected by joint detection system as shown in the figure $^1$ $E_{111}E_{222}$, $E_{112}E_{221}$, $E_{121}E_{212}$, $E_{122}E_{211}$. Therefore, the total current measured by the detector can be expressed as

$$E_1(r_1, t_1)E_2(r_2, t_2) = \frac{1}{4} E_{111}E_{222} + \frac{1}{4} E_{112}E_{221} + \frac{1}{4} E_{121}E_{212} + \frac{1}{4} E_{122}E_{211}. \quad (3)$$

Substituting equation (3) into equation (1), the second-order correlation function can be written as

$$G^{(2)}(r_1, t_1; r_2, t_2) = \frac{1}{16} \langle (E_{111}E_{222} + E_{112}E_{221} + E_{121}E_{212} + E_{122}E_{211}) \rangle$$
\[
\langle E_{111}E_{222}E_{111}^{*}E_{222}^{*} \rangle = \left( E_{11}^{*}E_{222}^{*} + E_{112}^{*}E_{221}^{*} + E_{121}^{*}E_{212}^{*} + E_{122}^{*}E_{211}^{*} \right).
\]

There will be 16 terms after expanding equation \([4]\). It is obvious that there are 12 cross-correlation terms besides four auto-correlation terms. The whole second-order correlation function can be categorized into four groups. We calculate one term for each group, and the other terms are calculated in the same way. Substituting equation \([2]\) into equation \([4]\), one can get a concrete expression.

The first group is all the autocorrelation group. The calculated results are all constant. Take \(E_{111}E_{222}E_{111}^{*}E_{222}^{*}\) as an example
\[
\langle E_{111}E_{222}E_{111}^{*}E_{222}^{*} \rangle = \left( E_{11}^{*}E_{222}^{*} \right) \left( E_{22}^{*}E_{111}^{*} \right) \left( E_{11}^{*}E_{222}^{*} \right) \left( E_{22}^{*}E_{111}^{*} \right)
\]
\[
= (\Delta \omega_1)^2 (\Delta \omega_2)^2,
\]
where \(\sin c(x) = \sin(x)/x\), the other terms \(E_{112}E_{221}E_{111}^{*}E_{222}^{*}, E_{121}E_{212}E_{112}^{*}E_{211}^{*}, E_{122}E_{211}E_{122}^{*}E_{211}^{*}\) in the same group have the same result as the one of equation \([5]\).

The second term that needs to be calculated is \(E_{111}E_{222}E_{112}^{*}E_{221}^{*}\). With the same method above, it can express as
\[
\langle E_{111}E_{222}E_{112}^{*}E_{221}^{*} \rangle = \left( E_{11}^{*}E_{222}^{*} \right) \left( E_{22}^{*}E_{112}^{*} \right) \left( E_{11}^{*}E_{222}^{*} \right) \left( E_{22}^{*}E_{112}^{*} \right)
\]
\[
= (\Delta \omega_1)^2 (\Delta \omega_2)^2 \text{sinc}^2 \left[ \frac{1}{2} \Delta \omega_2 (t_1 - t_2) \right],
\]
where \(\sin c(x) = \sin(x)/x\), the other terms \(E_{112}E_{221}E_{111}^{*}E_{222}^{*}, E_{121}E_{212}E_{112}^{*}E_{211}^{*}, E_{122}E_{211}E_{122}^{*}E_{211}^{*}\) in the same group have the same result as the one of equation \([6]\). All of the terms in the second group add up to \(4(\Delta \omega_1)^2(\Delta \omega_2)^2 \text{sinc}^2 \left[ \frac{1}{2} \Delta \omega_2 (t_1 - t_2) \right]\).

The third term that needs to be calculated is \(E_{111}E_{222}E_{112}^{*}E_{221}^{*}\), it can be obtained by simplification as
\[
\langle E_{111}E_{222}E_{112}^{*}E_{221}^{*} \rangle = \left( E_{11}^{*}E_{222}^{*} \right) \left( E_{22}^{*}E_{112}^{*} \right) \left( E_{11}^{*}E_{222}^{*} \right) \left( E_{22}^{*}E_{112}^{*} \right)
\]
\[
= (\Delta \omega_1)^2 (\Delta \omega_2)^2 \text{sinc}^2 \left[ \frac{1}{2} \Delta \omega_1 (t_1 - t_2) \right] .
\]

The forth term that needs to be calculated is \(E_{111}E_{222}E_{112}^{*}E_{221}^{*}\), which is
\[
\langle E_{111}E_{222}E_{112}^{*}E_{221}^{*} \rangle = \left( E_{11}^{*}E_{222}^{*} \right) \left( E_{22}^{*}E_{112}^{*} \right) \left( E_{11}^{*}E_{222}^{*} \right) \left( E_{22}^{*}E_{112}^{*} \right)
\]
\[
= (\Delta \omega_1)^2 (\Delta \omega_2)^2 \text{sinc}^2 \left[ \frac{1}{2} \Delta \omega_1 (t_1 - t_2) \right] .
\]
And the rest of the terms, $E_{112}E_{221}E_{121}^*E_{212}^*$, $E_{121}E_{212}E_{112}^*E_{221}^*$, $E_{122}E_{211}E_{111}^*E_{222}^*$ in the same group have the same result as the one of equation [8]. We’re going to add up to $4(\Delta \omega_1)^2(\Delta \omega_2)^2 \text{sinc}^2 \left[ \frac{1}{2} \Delta \omega_1 (t_1'' - t_2'') \right] \text{sinc}^2 \left[ \frac{1}{2} \Delta \omega_2 (t_1 - t_2) \right]$. 

Finally, when all the above terms are added together, the second-order temporal correlation function with two RGs in the scheme as shown in figure [1] is 

$$G^{(2)}(t_1 - t_2; t_1'' - t_2'') = \frac{1}{4} (\Delta \omega_1 \Delta \omega_2)^2 (1 + \text{sinc}^2 \left[ \frac{1}{2} \Delta \omega_1 (t_1 - t_2) \right] + \text{sinc}^2 \left[ \frac{1}{2} \Delta \omega_2 (t_1 - t_2) \right]
+ \text{sinc}^2 \left[ \frac{1}{2} \Delta \omega_1 (t_1'' - t_2'') \right] \text{sinc}^2 \left[ \frac{1}{2} \Delta \omega_2 (t_1 - t_2) \right]), \quad (9)$$

where $\tau_1 = t_1'' - t_2'', \tau_2 = t_1 - t_2$, they represent the coherent time that the laser passing through the first and second RG, respectively. Taking these equations into equation (9), the normalized second-order temporal correlation function can be expressed as 

$$g^{(2)}(\tau_1; \tau_2) = \left[ 1 + \text{sinc}^2 \left( \frac{1}{2} \Delta \omega_1 \tau_1 \right) \right] \cdot \left[ 1 + \text{sinc}^2 \left( \frac{1}{2} \Delta \omega_2 \tau_2 \right) \right]. \quad (10)$$

When the value of $\tau_1$ and $\tau_2$ approaches infinity, $g^{(2)}(\tau_1; \tau_2)$ equals 1, which means detection events are independent each other. When $\tau_1$ and $\tau_2$ all equals zero, $g^{(2)}(\tau_1; \tau_2)$ equals 4, which means superbunching effect can be observed. Also the result of theoretical derivation is in agreement with the conclusion derived from the quantum two-photon interference theory [17].

### 2.2. Experimental verification

The experimental setup for observing two-photon superbunching from a pair of modulated distinguishable classical light is shown in figure [2]. The initial polarization of two employed He-Ne lasers are horizontal. A half-wave plate (HWP) behind laser 1 is employed to change the horizontal polarization to vertical polarization. GP$_1$ and GP$_2$ are two Glan prisms, which are set to purify the polarization of these two lasers. L$_1$ and L$_2$ are convergent lenses with focal lengths of 50 mm. The distance between RG$_1$ and P$_1$ is 300 mm. The transverse coherence length of pseudothermal light generated by RG$_1$ is 1.9 mm in the plane of P$_1$. The diameter of the P$_1$ is 1 mm, which is less than the coherence length in order to pass through P$_1$ within one same coherence area. L$_2$ is also employed to focus the light onto RG$_2$. The distance between L$_2$ and RG$_2$ needs to be larger than the focus length of L$_2$ mainly because pseudothermal light scattered by RG$_1$ is not parallel. D$_1$ and D$_2$ are two single-photon detectors (PerkinElmer, SPCM-AQRH-14-FC). C.C is a two-photon coincidence count detection system (Becker & Hickl GmbH, DPC-230). The experiment mainly includes the following three steps.

In the first step, turn on laser 1 and turn off laser 2, let the vertically polarized light pass through RG$_1$ and RG$_2$ successively. The traditional second-order temporal correlation function of the ordinary pseudothermal superbunching effect is measured to verify the reliability of the experimental test system. RG$_1$ and RG$_2$ are rotating at 100 Hz and 50 Hz, respectively. As a result of the vertical polarization, the modulated light passes through the PBS$_2$ and only arrives at D$_1$. The experimental results are shown...
Temporal and spatial superbunching effects

Figure 2. Experimental setup for measuring superbunching effect from a pair of modulated distinguishable classical light. Laser 1 and Laser 2 are single-mode continuous-wave He-Ne lasers. PBS$_1$ and PBS$_2$ are two polarized beam splitters. RG$_1$ and RG$_2$ are two rotating ground glass plates that can adjust the rotational speed. P$_1$ and P$_2$ are the pinholes. The measuring system is a HBT-like intensity interferometer.

In figure 3(a). The coincidence count is almost constant for 50s of collection time. It has no superbunching effect obviously because there is almost no light reaching D$_2$. Keeping experimental devices unchanged, when we rotate additional HWP to make it $45^\circ$ polarized light respect to the horizontal direction which is placed between the PBS$_1$ and the L$_1$ (not shown in figure 2), the pseudothermal light will be split into two beams with the same intensity after passing through the PBS$_2$. Finally, the HBT intensity interferometer will be triggered by two pseudothermal lights. The results are shown in figure 3(b). The degree of second-order correlation $g^{(2)}(0) = 3.91 \pm 0.02$ was observed, which means the superbunching effect of pseudothermal light was achieved. Also the measured full width at half maximum (FWHM) reaches about $1.7 \mu s \pm 0.02$ and the visibility of the peak is about 59.2%.

In the second step, turn on the laser 1 and laser 2 simultaneously. Vertically polarized light from laser 1 and horizontally polarized light from laser 2 are combined into one beam at the PBS$_1$, and then focus on RG$_1$ by the lens L$_1$. Note that two beams focus on different areas of RG$_1$ at the moment. That means RG$_1$ will produce two completely different sets of speckles. When two sets of speckles with mutually perpendicular polarization directions pass through PBS$_2$, the vertically polarized light from the laser 1 will reach the detector D$_1$, and the horizontally polarized light from laser 2 will reach the detector D$_2$. The measurement result is shown in figure 3(c), $g^{(2)}(\tau)$ is flat and no superbunching effect occurs.

In the third step, except for the condition that the combination of the two light beams from laser 1 and laser 2 through PBS$_1$ focus on the same area of RG$_1$, the rest of experimental steps are the same as the second steps. Hence the speckle patterns scattered by RG$_1$ and RG$_2$ are almost the same, and they have the same temporal and spatial fluctuations. The measurement result is shown in figure 3(d). The circles is the measured result, the red line is theoretical fittings by employing equation (9). The FWHM is $1.71 \mu s \pm 0.02$ and the visibility of the peak is about 58.5%. The degree of second-order correlation is $g^{(2)}(0) = 3.83 \pm 0.02$ as depicted in the figure 3(d). It is
Temporal and spatial superbunching effects

obviously that $g^{(2)}(0)$ is greater than 2, which means the temporal superbunching effect of pseudothermal light was observed successfully.

Based on the the third step, we proceed to study the superbunching effect in the spatial domain. With $D_1$ fixed at $x_1 = 0$, $D_2$ is then moved transversely in steps of 2 mm through $x_2 = \pm 12 mm$. The collection time for every steps is 50s. The spatial normalized second-order correlation function $g^{(2)}(x_1 - x_2)$ is calculated and the second-order correlation pattern as shown in figure 4. The FWHM of this peak is $4.8 \pm 0.01 mm$, while the visibility of the peak is about 22.4%. The degree of second-order correlation of pseudothermal light was measured as $g^{(2)}(0) = 3.42 \pm 0.02$, which means superbunching.
effect in the spatial domain was also observed.

3. Discussion

The superbunching effect was observed in the first step. From a classical theory of view, when the 45° linearly polarized light passes through RG$_1$, RG$_2$ and PBS$_2$ in sequence, two single photon detectors at symmetric position are triggered by the pseudothermal light which possesses the same intensity fluctuation and photon statistical distribution. Therefore, the classical intensity fluctuations theory [24, 25] could be explained why the superbunching effect can be observed. In the quantum two-photon interference theory [26], when we changed the horizontal light to 45° linearly polarized light by rotating the HWP, the pseudothermal lights pass through PBS$_2$, there are two different and indistinguishable paths to trigger two detectors. It is the reason why we can use quantum two-photon interference theory to explain the superbunching effects, simultaneously the superbunching effects of $g^{(2)}(0) = 3.91 \pm 0.02$ was observed.

On the basis of the first step, turn on the laser 1 and laser 2 simultaneously, the second step and third step are a group of comparative experiments. The difference between them is whether two lights focus on same areas of RG$_1$ or not. From the classical theory of intensity fluctuation correlation, the second step and the third step will get different results. When the lasers focus on the RG$_1$ in different positions, it means the two-photon coincidence count system is triggered by two completely different sets of speckles. Therefore, the superbunching effects can not be observed corresponding to figure 3(c). The third step is the opposite of the second step, the superbunching effects is observed corresponding to figure 3(d) mainly because the lasers focus on the RG$_1$ in same area. The two set of speckles that pass through RG$_1$ and RG$_2$ own identical fluctuations in the same coherent region.

However, the quantum two-photon interference theory will give same prediction results between the second step and the third step. It is well known that two-photon interference theory emphasizes whether the interference path is different and indistinguishable. In the second and third step, the polarization directions of two incident lasers are all orthogonal. After passing through PBS$_2$, the horizontally polarized light enters the detector D$_2$, and the vertically polarized light enters the detector D$_1$. There is only one path to trigger the two photon coincidence count detection system. Under the prediction of quantum two-photon interference theory, there is no superbunching effect in the second and third step. But we observed the superbunching effect of pseudothermal light in the third step, which the degree of second-order temporal correlation was measured as $g^{(2)}(0) = 3.83 \pm 0.02$ as shown in figure 3(d). It is contrary to the prediction of classical theory of intensity fluctuation correlation. The temporal superbunching effect of classical light in the figure 3(d) seems to break the unity of classical and quantum theory. The quantum two-photon interference theory can not explain this strange phenomenon. Here we used the the classical intensity fluctuations theory to explain the superbunching effect without two-photon interference.
On account of that the polarization of two beams are orthogonal and they focus on the same area of RG$_1$, two pseudothermal light beams own the different polarization direction but generate the same fluctuation. It means the distribution of electric field $E(r, t)$ is the same. After transferring through RG$_1$ and RG$_2$ in series, the two-photon coincidence count system is triggered by two sets of same speckles. The specific derivation is shown in the above Sec. 2. Therefore, the superbunching effect can be well explained by the classical intensity fluctuations theory.

In the fourth step, the background is equal to 2 in the spatial superbunching correlation diagram. This is mainly due to the fact that P$_1$, which is set in front of RG$_2$, performs a spatial mode selection on the light passing through RG$_2$ so that the speckle has the same fluctuation in the same spatial mode. However, there is no change in the longitudinal correlation length related to the superbunching effect in the time domain. Therefore, the temporal superbunching effects is different from the spatial superbunching effect, and the measured background of the superbunching effect is 1 and 2, respectively.

It is well known that there exists space-time duality between the equations that describe the paraxial diffraction of light beams [27–29]. Similar to the above methods for calculating the temporal superbunching effect, the classical intensity fluctuation correlation theory can also be used to calculate the spatial superbunching effect. The expression of electromagnetic wave which transmitted to the surface of detector can be expressed as

$$E_{ijk} = E_i \cdot T_{ji} \cdot T_{k|i}$$

$$= e^{-ik_0(r_i - r_0)} \int_{-\frac{d_1}{2}}^{\frac{d_1}{2}} e^{-ik_{0i}(r''_j - r'_i)} dr_0 \int_{-\frac{d_2}{2}}^{\frac{d_2}{2}} e^{-ik_k(r_k - r''_j)} dr_k. \quad (11)$$

The meaning of the symbol in the above equation (11) is similar to equation (2). $d_1$ and $d_2$ represent the length of the pseudothermal light source after passing through RG$_1$ and RG$_2$, respectively. $k_0$ is the wave vector of the laser, $k_{0i}$ is the wave vector of the light scattered by the position $i$ of the RG$_1$, and $k_j$ is the wave vector of the light scattered by the position of the RG$_2$. $r'_i$, $r''_j$ and $r_k$ are position vectors when the electromagnetic wave arrives at position $i$ of the RG$_1$, position $j$ of the RG$_2$ and the surface $k$ of detector respectively, where $i, j, k = 1, 2$. Substituting the equation (11) into equation (4) and the normalized second-order temporal correlation function is

$$g^{(2)}(\Delta x_1; \Delta x_2) = \left[ 1 + \text{sinc}^2 \left( \frac{\pi L_1}{\lambda d_1} \Delta x_1 \right) \right] \cdot \left[ 1 + \text{sinc}^2 \left( \frac{\pi L_2}{\lambda d_2} \Delta x_2 \right) \right], \quad (12)$$

we define $\Delta x_1 = x'_1 - x'_2$ that means the position difference between two orthogonal beams of light propagating over RG$_1$, $\Delta x_2 = x_1 - x_2$ means the transverse position difference between detectors D$_1$ and D$_2$. $\lambda$ is the wavelength of the light source, $L_1$ is the distance from RG$_1$ to detectors and $L_2$ is the distance from RG$_2$ to detectors.

As a result of two orthogonal light beams focus on the same area of RG$_1$ ($\Delta x_1 = 0$), they generate same fluctuation after passing through RG$_1$. So the equation (12) can be
Temporal and spatial superbunching effects

further simplified as

\[ g^{(2)}(\Delta x^2) = 2 \left[ 1 + \text{sinc}^2 \left( \frac{\pi L^2}{\lambda d^2} \Delta x^2 \right) \right], \]  

(13)

when the value of \( \Delta x^2 \) is infinity, \( g^{(2)}(\Delta x^2) \) equals 2, which means the two detectors are far enough apart; \( g^{(2)}(\Delta x^2) \) equals 4 when \( \Delta x^2 = 0 \), which means two detectors are in the same symmetric position.

The background is close to 2 in our experiment about spatial superbunching effect. The reasons are as follows, taking off RG\(_2\) and repeating the third step above, the measured degree of second-order temporal correlation is 1.90 ± 0.01. It is the bunching effect that \( g^{(2)}(0) = 2 \). When we add the RG\(_2\), the modulated light is modulated again. This is the reason why we can observe spatial superbunching effect. At the same time, the vibration of RG\(_1\) and RG\(_2\) would cause the vibration of the optical table in the actual experiment, so the measurement of the degree of second-order correlation is 1.9 or higher because of the vibration of laser. Therefore, the measured background of spatial superbunching effect accords with our theory that equals 2.

4. Conclusion

In summary, we achieved the superbunching effect from a pair of modulated distinguishable classical light in the temporal and spatial domain. We employed the classical theory of intensity fluctuation correlation to deduce the normalized second-order correlation function of temporal and spatial superbunching effect. It shows good agreement with the measurement as \( g^{(2)}(0) = 3.83 \pm 0.02 \). Also the degree of second-order spatial correlation was measured as 3.42 ± 0.02. Although there are completely opposite predictions by using the classical theory of intensity fluctuation correlation and quantum two-photon interference theory to explain the phenomena in the experiment, it does not mean quantum theory contradicts classical theory. It only shows there is no two-photon interference phenomenon in this experiment. Therefore, studying this interesting phenomenon is not only conducive to the future research about superbunching effect that have potential application in improving the visibility of ghost imaging, but also plays an important role in understanding the relationship between classical theory of intensity fluctuation correlation and quantum two-photon interference theory.

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