Ground state energies and decay widths of particle unstable nuclei are calculated within the Hartree-Fock approximation by performing a complex scaling of the many-body Hamiltonian. Through this transformation, the wave functions of the resonant states become square integrable. The method is implemented with Skyrme effective interactions. Several Skyrme parametrizations are tested on four unstable nuclei: $^{10}\text{He}$, $^{12}\text{O}$, $^{26}\text{O}$ and $^{28}\text{O}$.
The drip lines of the nuclear chart have been mapped experimentally up to Z=19 on the proton rich side and to Z=7 on the neutron side \[^4\]. The availability of radioactive ion beams will soon allow an extension of our knowledge on these limits of stability. It will give access to the study of nuclei beyond the drip lines which are unstable by delayed particle emission but have life times long enough to determine some of their spectroscopic properties. One-proton radioactivity has been discovered long ago \[^6\]. Very recently, two-proton emission from an unbound ground state has been observed in the three-body decay of \(^{12}\)O \[^8\]. The mass and excited states of the long searched heavy helium isotope \(^{10}\)He (which has the largest observed neutron to proton ratio) has been measured \[^9\] using a beam where both the projectile and the target are radioactive nuclei.

The description of nuclei close to the drip lines has to deal with many specific difficulties. In mean field methods, these difficulties are related to the fact that the Fermi level is close to the continuum. As a result, the last occupied orbitals have long tails not easily described by conventional expansions on an oscillator basis. Pairing correlations also create an interaction between bound and continuum single particle levels which cannot be treated within the BCS approximation. These processes affect the shell structure of these isotopes and modify the magic numbers close to the drip lines. A proper way to solve these problems within the mean-field approximation is the use of the Hartree-Fock-Bogoliubov approximation with a solution of the HFB equations on a mesh \[^5\]. This method has recently been implemented for spherical nuclei \[^2\] and in a slightly different way for nuclei with quadrupole triaxial deformations \[^3\].

A further complication arises for nuclei beyond the drip lines for which the last particle orbitals are unbound and have a width. This is the problem that we address in this paper.

Radioactive decay has been the subject of numerous theoretical studies since the beginning of nuclear physics. The most successful approaches have been based on the time independent formulation of nuclear collisions and rely on an analysis of the collision matrix. In the R matrix formulation \[^4\], the configuration space is divided into an interaction region and an asymptotic one corresponding to the daughter nuclei. Analysis of the R matrix and the use of its relation to the collision matrix permits to define the energies and widths of resonances narrow enough not to interfere in a too complicated way. This method has been mainly used to analyze experimental data or within phenomenological models. Recently, a parameter free description of \(^\alpha\) radioactivity based on a unification of the shell and the cluster models has been applied to the decay of \(^{212}\)Po \[^6\]. For light nuclei, the microscopic R matrix method enables also to determine the decay width of unstable states in a purely microscopic approach \[^7\].

Decay widths can be roughly explained by considering only the penetrability through the nuclear, Coulomb and centrifugal barriers, as Gamow did in the beginning of quantum mechanics. This was recently done \[^6\] in a study of the diproton decay of \(^{48}\)Ni with the additional assumption that the diproton moves in the self-consistently determined field of \(^{46}\)Fe.

In this letter we present a method suitable for the calculation of the total decay width of an unstable nucleus. It is based on a general technique to find resonance states: the complex scaling (CS) method. This method can be combined with any model that defines the spatial coordinates of the particles. It has already been applied in nuclear physics \[^8\] to different variants of the cluster model. The combination of the CS and the Hartree-Fock (HF) methods that we use here has already been performed in atomic and molecular physics (for a recent reference, see for instance Ref. \[^9\]).

The starting point of the CS method is a transformation of the Hamiltonian \(\hat{H}\). First one defines the unbounded non-unitary scaling operator \(\hat{U}(\theta)\):

\[
\hat{U}(\theta)\Psi(\vec{r}) = \exp(3i\theta/2)\Psi(\hat{r}\exp(i\theta)),
\]

where \(\theta\) is real. The transformed complex scaled Hamiltonian is of the form:

\[
\hat{\tilde{H}}_\theta = \hat{U}(\theta)\hat{H}\hat{U}(\theta)^{-1},
\]

where \(\theta < \pi/4\) is the scaling parameter. If we deal with a many body system each single-particle coordinate has to be transformed according to this prescription.

The ABC theorem \[^10\] gives the properties of the spectrum of the transformed Hamiltonian. A bound state eigenvalue of \(\hat{H}\) remains also an eigenvalue of \(\hat{\tilde{H}}_\theta\). A resonance pole \(E = E - i\Gamma/2\) of the Green-operator of \(\hat{H}\) becomes an eigenvalue of \(\hat{\tilde{H}}_\theta\) provided \(\theta > \arg(E)/2\). There are no other eigenvalues of \(\hat{\tilde{H}}_\theta\). The important point is that now the wave functions of resonant states are square integrable. The continuous part of the spectrum of \(\hat{\tilde{H}}_\theta\) differs drastically from that of \(\hat{H}\). It is rotated down into the complex energy plane by the angle \(2\theta\).

Complex scaling can be combined with the HF theory when resonances are to be included in the formalism. In such a situation two cases can be distinguished. The first one corresponds to a nucleus with a particle stable ground state. The CS permits then to determine the resonant states corresponding to the self-consistent mean-field. An RPA theory using such resonant states was recently developed and successfully applied \[^11\] starting from a phenomenological mean-field.

The second case concerns nuclei with particle unstable ground states which therefore are resonant states. In these cases, the CS of the many-body Hamiltonian has to be carried out at the very first place. This defines \(\hat{\tilde{H}}_\theta\). Choosing a Slater-determinant as trial wave function and applying a bi-variational principle to get the
“best” single-particle orbits, one derives new mean field equations. The use of a bi-variational principle instead of the usual Rayleigh-Ritz variational principle is due to the non-self-adjoint nature of \( \hat{H}_R \). It was proved that the new mean-field equation is identical with the complex scaling of the original Fock-operator \( \hat{h} \). The total binding energy \( E_{CSHF} = E - i \frac{\hbar^2}{2m_2} \) resulting from the application of the complex scaled Hartree-Fock (CSHF) procedure is complex. Its interpretation in the many-body case is the same as for a two-body resonance. The half-life time of the ground state is \( \hbar \ln 2/\Gamma \) and the real part of \( E_{CSHF} \) is interpreted as the total binding energy which has to be compared with the experimental mass.

We have constructed a spherical CSHF code for Skyrme like effective interactions. Let’s consider an even-even doubly closed shell nucleus with time-reversal invariance and assume spherical symmetry. In this case the HF integro-differential equation becomes an ordinary second order differential equation

\[
\frac{\hbar^2}{2m(r)} \left( -R''(r) - \frac{2}{r} R'(r) + \frac{l(l+1)}{r^2} R(r) \right) - \frac{\hbar^2}{2m'f^2} R''(r) + V(r)R(r) = \epsilon R(r),
\]

where \( R(r) \) is the radial part of the single-particle orbit. The effective mass \( m(r) \) and the mean field potential \( V(r) \) are complicated functionals of the filled single particle orbits. The actual form of these functionals are given in \( \hat{U} \). The left hand side of Eq. 3 is actually the action of the Fock-operator \( \hat{h} \) onto the function \( R(r) \). In order to find the resonances one has to derive the complex scaled Fock-operator \( \hat{h}_\theta = \hat{U}(\theta)\hat{h}\hat{U}(\theta)^{-1} \), which is a rather straightforward task. The action of \( \hat{h}_\theta \) on an arbitrary function \( f(r) \) is

\[
\hat{h}_\theta f(r) = \frac{\hbar^2}{2m(r\eta)} \eta^{-2} \left( -f''(r) + \frac{2}{r} f'(r) + \frac{l(l+1)}{r^2} f(r) \right) - \eta^{-1} \frac{\hbar^2}{2m'(r\eta)} f'(r) + V(r\eta)f(r)
\]

where \( \eta = \exp(i\theta) \). The eigenvalue problem of \( \hat{h}_\theta \) is solved by direct numerical integration and the usual iteration technique is used except that the initial orbits were calculated using a complex scaled radial Schrödinger equation. Further details of the present calculation and a discussion on the choice of the basis set used in the CSHF approach will be published elsewhere.

We will apply the CSHF method to calculate the widths of the ground state of the nuclei \( ^{10}He \), \( ^{12}O \), \( ^{26}O \) and \( ^{28}O \). The parameters of effective nucleon-nucleon interactions are fitted to reproduce properties of selected stable nuclei as well as nuclear and neutron matters. These interactions are often more reliable than phenomenological approaches in the determination of nuclear properties far from stability. However, the isospin dependence of the interaction is probably rather poorly known. In this work, we not only determine the energies of nuclei close to the drip lines but we also calculate the width of resonant states, a property which has not at all been included in the adjustment of the forces. We will therefore test farther a large number of interactions which have been rather successful in the description of a large variety of nuclear properties. The parametrization SIII has been the more widely used, in particular to describe deformation properties of long series of isotopes. The interaction SLy4 \( \hat{U} \) has been recently adjusted with a special care on the properties of nuclear and neutron matters and is expected to be particularly reliable far from stability. The force Skm1 \( \hat{U} \) is a slightly modified version of Skm*. It should have the same properties close to stability line and be well suited to describe extremely deformed nuclear states. However, the very bad isospin dependence of Skm* has been corrected by a reduction of its symmetry energy from 30.0MeV to 25.8MeV. Although this value seems to be too small, Skm1 leads to very good prediction on masses of nuclei far from stability in the HF approach. Finally, we have also used the parametrization SkP \( \hat{U} \) which has been constructed to be used at the same time in the mean field and in the pairing channels. This interaction is the only one to have an effective mass equal to 1, and therefore produces a larger density of levels in the vicinity of the Fermi energy. This difference could be important for nuclei unstable by particle emission.

In table I the absolute value of the binding energies of even-even He and O isotopes and of \( ^{10}C \) are given. In the case of a partially filled shell, the uniform filling approximation has been used. For most calculations, the Fermi level of the nuclei were found to be bound and the standard HF procedure has been applied. In the case of the Skm1 parametrization, the Fermi energy is in the continuum for \( ^{10}He \), \( ^{12}O \), \( ^{26}O \) and \( ^{28}O \) and for SIII in the case of \( ^{10}He \). The CSHF method was used in these circumstances and the absolute value of the real part of the total binding energy is displayed in Table I. In order to test the accuracy of the the CSHF method, we have carried out CSHF calculations in a few bound isotopes. Because of the finite accuracy of the numerical integration of the differential equation (4) the total energy has a small imaginary part. In each cases, its absolute value was less than \( 10^{-4}MeV \). This gives an estimation of the overall error and indicates that we could not attempt to describe a system with a decay width smaller than \( 10^{-4}MeV \).

On the average, the parametrization Skm1 leads to the best agreement with the experimental energies, with the noticeable exception of \( ^{10}He \). The fact that Skm1 leads to a very fast decrease of the binding energies for the most neutron rich isotopes may probably be related to its behaviour in pure neutron matter. Compared to realistic calculations, this parametrization leads to an over-
estimation of the energy of neutron matter at densities lower than 0.75 times the saturation density and to a collapse for larger densities. The underestimation of the energy of $^{10}\text{He}$ is thus probably related to the presence of regions of low densities due to the large neutron excess. Note that the interaction $\text{Skm}^*$ from which $\text{Skm}1$ is derived describes much more satisfactorily neutron matter. Differences between $\text{Skm}1$, $\text{SIII}$ and $\text{Sly4}$ are large only for the most neutron rich isotopes while systematically larger differences are obtained with $\text{SkP}$. Note that when pairing correlations are included, $\text{SIII}$, $\text{Sly4}$ and $\text{SkP}$ predict $^{26}\text{O}$ to be bound by at least 2.0 MeV and $^{28}\text{O}$ to be either slightly unbound (SIII) or slightly bound.

In table II we compare the CSHF ground state decay widths to the experimental data. The four forces predict $^{10}\text{He}$ to be unbound with respect to two proton emission. However with $\text{SkP}$ and $\text{Sly4}$, all single particle orbits are bound. The Fermi level lies in the continuum for SIII and $\text{Skm}1$ which makes possible the calculation of the decay width of $^{10}\text{He}$ by the CSHF method. The SIII width is in good agreement with the experimental data. The values obtained with $\text{Skm}1$ overestimate the experimental data by an order of magnitude for $^{10}\text{He}$ while it largely underestimate it for $^{12}\text{O}$. The reasons for these discrepancies are probably different. $^{10}\text{He}$ is largely under-bound by $\text{Skm}1$ and more correctly described by SIII. The too large width obtained with $\text{Skm}1$ is probably related to this under-binding. $^{12}\text{O}$ is not a closed shell nucleus and is described by the filling approximation. A more realistic description of this nucleus would affect the decay width. Let us also recall that correlations beyond the mean field approximation are important in light nuclei.

The forces $\text{Skm}1$ and $\text{SIII}$ predict that both $^{26}\text{O}$ and $^{28}\text{O}$ are unbound with respect to neutron emission. The interaction $\text{Sly4}$ also gives unbound $^{28}\text{O}$. However only the parametrization $\text{Skm}1$ places the Fermi-level in the continuum so that the decay width can be calculated by the CSHF method. Looking to the results on $^{12}\text{O}$ and $^{10}\text{He}$, one can expect that the widths given in Table II are more reliable for $^{28}\text{O}$, which is a closed shell nucleus than for $^{26}\text{O}$ for which the filling approximation has been used.

In conclusion, we have shown in this paper that nuclei with a particle unstable ground state can be described by the complex scaled Hartree-Fock method. This method permits to determine the energies and the widths for particle emission, while the conventional HF method does not even give access to energies. Our results on $^{10}\text{He}$ have revealed a weakness of the interaction $\text{Skm}1$ which was not apparent on stable nuclei. While the feasibility of the method has clearly been demonstrated, the values of widths that we have obtained are not in agreement with the experimental data. A part of the discrepancy is probably due to the fact that we use effective nuclear interactions in a domain for which they have not been constructed. Before to address this problem, we plan to generalize our method by the inclusion of pairing correlations.

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TABLE I. The negative of the binding energies of light nuclei in MeV with different Skyrme forces using the HF or CSHF method.

|        | $^6\text{He}$ | $^8\text{He}$ | $^{10}\text{He}$ | $^{12}\text{C}$ | $^{14}\text{O}$ | $^{16}\text{O}$ | $^{22}\text{O}$ | $^{24}\text{O}$ | $^{26}\text{O}$ | $^{28}\text{O}$ |
|--------|---------------|---------------|------------------|-----------------|-----------------|-----------------|---------------|---------------|---------------|---------------|
| Exp.   | 29.3          | 31.4          | 30.3             | 60.3            | 58.5            | 98.7            | 127.6         | 162.0         | 168.5         | 168.4         |
| Skm1   | 29.1          | 31.0          | 25.0             | 60.9            | 58.4            | 101.4           | 127.7         | 164.0         | 171.9         | 167.9         |
| SIII   | 29.3          | 33.6          | 29.5             | 59.4            | 59.1            | 100.9           | 128.2         | 165.4         | 171.8         | 170.9         |
| Sly4   | 29.5          | 33.9          | 31.4             | 59.5            | 60.9            | 101.5           | 128.5         | 164.4         | 172.4         | 172.3         |
| SkP    | 30.5          | 34.7          | 32.7             | 60.7            | 61.9            | 101.2           | 127.6         | 164.4         | 173.5         | 174.0         |

aSee the Ref. [22].

TABLE II. The decay width of the ground state of $^{10}\text{He}$, $^{12}\text{C}$, $^{26}\text{O}$ and $^{28}\text{O}$ in MeV using the CSHF model. With the other forces of Table I. decay width could not be assigned to these nuclei.

|        | $^{10}\text{He}$ | $^{12}\text{O}$ | $^{26}\text{O}$ | $^{28}\text{O}$ |
|--------|------------------|-----------------|---------------|---------------|
| Exp.   | 0.3a             | 0.578b         | 0.4           | 0.5           |
| Skm1   | 3.8              | < $10^{-3}$     | 0.4           | 0.5           |
| SIII   | 0.4              |

aSee the Ref. [2].
bSee the Ref. [3].