Dynamic System Stability when Machining with Cutter

D V Vasilkov¹, A V Nikitin², V S Cherdakova¹

¹ Baltic State Technical University “Voennmeh” named after D.F. Ustinov, 1, 1-ya Krasnoarmeiskaya, 190005, St. Petersburg, Russia
² Peter the Great St. Petersburg Polytechnic University, 29, Polytechnic Street, 195251, St. Petersburg, Russia

E-mail: vasilkovdv@mail.ru

Abstract. The stability analysis of the machine dynamic system when machining with a cutter performing such operations as turning and milling is considered. A general approach is proposed aiming to determine the boundaries of the stability region on the basis of simplified models. Parameterization of the model is carried out with the reference to specific machining conditions.

1. Introduction
The choice of modes and machining conditions ensuring smooth, vibration free cutting is very important during the edge cutting machining of critical parts of thermal control systems of spacecraft, aerospace, mining and power engineering. The machining with a high level of vibration leads to premature wear of the equipment and a reduction in the overhaul cycle, as well as to an intensive wear of the cutting tool. When machining by cutting, for example, turning and milling, there is a problem to ensure the accuracy with specified dimensions and shape of a product. It is connected with the need to select the cutting mode range which ensures the stability of the technological system [1-5].

2. Materials and methods
The kinematics of shape-generating movements during the turning is shown in Fig. 1. The cutting force \( P_z \) acts in the direction opposite to the cutting speed \( V \). The mass \( m_z \), stiffness \( c_z \) and damping \( b_z \) coefficients are given to the cutter in this direction. The force \( P_y \) acts in \( y \) direction. The mass \( m_y \), stiffness \( c_y \) and damping \( b_y \) coefficients are given to the cutter in this direction. The cross-section of the cut is made by the thickness \( a \) and width \( b \), which are respectively equal: \( b = \frac{t}{\sin \phi} \); \( a = S \sin \phi ; t \) is the cut depth; \( S \) is the feed; \( \phi \) is the main angle (Fig. 1). The components of the cutting force will be written in the form of static characteristics

\[
P_z = k_z a ; \quad P_y = k_y a
\]

where \( k_z \), \( k_y \) are the reduction coefficients, \( k_z = p b ; \quad k_y = \mu p b \cos \phi \) is the coefficient of friction between the front surface of a tool and the chips; \( p \) is the specific cutting force, \( p = (1,3 \ldots 1,4) \sigma_u \xi \); \( \sigma_u \) is the ultimate strength of processed material; \( \xi \) is the shrinkage ratio of chips; \( f \) is the coefficient of friction between the front surface of a tool and the chips.
As a model of minimum dimension during the turning a single-mass two-loop bending dynamic model with lumped parameters (Fig. 2) was taken, which is quite applicable for calculating the stability of machines [6, 7].

With regard to the dynamic issue from the point of view of stability of the technological system the normal contour ($OY$ axis) and the tangent contour ($OZ$ axis) are unequal:

- The normal contour determines the accuracy of shape generation, as well as it forms a vibrating trace on the surface and makes contribution to the formation of stability region boundary;
- The tangent contour improves the surface roughness, takes away part of the energy generated during the cutting process and, accordingly, expands the stability region.

The exception in the $OZ$ tangent contour model narrows the stability region to about 10% [6]. Admissibility of transition to a single-loop dynamic model is confirmed by numerous dynamic tests of machines [6-9]. However, such simplification of the dynamic system requires periodic identification of parameters during the performance of dynamic tests.

There is a delay in the cutting force change with respect to the thickness variation of the cut in the dynamics of the cutting process [6, 7]. These changes are made with respect to the position of dynamic equilibrium and formalized by the dynamic characteristic of cutting, taking into account (1) in the form of an aperiodic link reduced to the contour $OY$

$$T_p P_y + P_y + k^*_y y = 0,$$

where $T_p$ is the delay time constant, $T_p = \frac{l_p}{V_s}$; $l_p$ is the chip formation path equal to the length of a chip contact line with the front surface of a tool; $k^*_y$ is the transmission factor of a loop, $k^*_y = k_y \sin \varphi$.

Taking into account the expression (3), one can obtain the mathematical expression of a single-loop dynamic model of the technological system

$$m_y \ddot{y} + b_y \dot{y} + c_y y = P_y; \quad T_p \ddot{P}_y + P_y = -k^*_y y.$$

Let us consider the conditions of force interaction during the milling. To do so, let us consider the kinematics of the shaping motions in the contour milling (Fig. 3). Here, the angle $\alpha$ is a position angle of the milling cutter measured between the normal line to the shape-generating point $A$ of a workpiece and the rotation axis of a milling cutter. The instantaneous values of both thickness $a_i$ and width $b_i$ of the cut for the $i^{th}$ tooth of a milling cutter are determined by the following expressions [8]

$$a_i = s \sin \psi_i; \quad b_i = r \tan \beta \sin \psi_i 1 + \frac{f}{\cos \alpha},$$

where $s$ is the working feed to the milling cutter tooth; $r$ is the milling cutter radius; $\psi_i$ is the instantaneous angle of the turn of a milling cutter.

Such components of the cutting force acting on the $i^{th}$ tooth of the milling cutter as the tangent $P_{ti}$, the radial $P_{ri}$, and the axial $P_{oci}$ components are determined by the expressions

$$P_{ti} = p a_i b_i; \quad P_{ri} = f P_{ti}; \quad P_{oci} = P_{ti} \tan \beta,$$

where $\beta$ is the inclination angle of the milling cutter tooth screw line.

The projections of the cutting force acting on the $i^{th}$ tooth of a milling cutter in the cutting area in the coordinate system $O_1X_1Y_1Z_1$ and taking into account (5) is determined by the expressions
**Figure 1.** Kinematics of shape-generating movements during the turning

**Figure 2.** Model of a two-loop dynamic system during the turning

**Figure 3.** Kinematics of shape-generating movements during the milling

**Figure 4.** Two-loop dynamic system model during the milling
It is possible to determine the cutting force depending on the number of simultaneously cutting teeth using the expressions (6). The cutting force obtained in this way is a periodic function, which makes the solution of the stability problem more complicated and requires an appropriate linearization. An analysis of the change nature of cutting forces during the milling has shown [6, 8] that when two or more teeth are used simultaneously, the milling irregularity does not exceed 10% with respect to the average value. If to neglect this value, then the calculation of cutting forces during the milling is simplified. This is achieved by the introduction of average integral slice thickness \( a_m \), which is determined by the following expression [6]

\[
a_m = s_z (1 - \cos \psi_B) \cos \alpha \frac{\cos \alpha}{\psi_B},
\]

where \( \psi_a \) is the cutting angle (Fig. 3).

\( \psi_m \) and \( b_m \) are the integral values determined from the relations and corresponding to the value \( a_m \)

\[
\psi_m = \arcsin \left( \frac{a_m}{s_z \cos \alpha} \right); \quad b_m = r \frac{t}{\cos \alpha} \tan \beta \left( \sin \psi_m - 1 \right) + \frac{t}{\cos \alpha}.
\]

The cutting force components \( P_r \) and \( P_{oc} \) can be determined with the help of formulas (5) by substituting the mean integral values from (8).

\[
P_r = f' P a_m b_m; \quad P_{oc} = p' a_m b_m \tan \beta,
\]

The technological system during the contour milling process is shown in Fig. 4. The same surface shape-generating rules can be applied as to the model during the turning (Fig. 2), i.e. it can be mathematically represented by a single-loop dynamic model (3) in the direction of \( OY \) axis. In this case the loop transmission coefficient will take a new value

\[
k_y = p' b_m \sin \alpha \left( \sin \alpha - f \cos \alpha \right).
\]

Thus, owing to the achieved simplifications it became possible to generate an overall model of a technological machining system aimed at studying stability and evaluating the ultimate capabilities of the machine. The stability of a dynamical system is determined on the basis of the algebraic Hurwitz criterion [10] defined by the following inequality

\[
(T_{p_e} c_y + b_y)(T_{p_b} s_y + m_y) > T_{p_b} m_y (c_y + k_y^*)
\]

The stability region boundary is constructed by means of a stepwise verification of the criterion (14) on the basis of D-decomposition method or the Loeb algorithm [8].

3. Discussion of research results

To provide grounds for the developed models, some series of experiments were carried out using measuring and computing complex for dynamic testing. A full-scale simulation was performed during the turning. The following parameters of the technological system were selected: 1A616 machine with a high degree of wear, the material of a sample being processed is Steel 38XMA; the cutting tool parameters are transverse dimensions 25x30, the main angle in plan is 60°, the cutting part material is the hard alloy T5K10; the nominal values of processing parameters are as follows, cutting depth \( t = 2 \) mm, working feed \( S = 0.1 \) mm/rev., cutting speed \( V_c = 50 \) m/min, sample diameter 50 mm. Variable in the course of the experiment parameters are as follows \( V = 10 ... 100 \) m/min, \( S = 0.05 ... 0.35 \) mm/rev., \( t = 0.5 ... 5 \) mm.

In the course of experiment, the following pairwise varied parameters were considered: cutting speed - depth of cutting; cutting speed - working feed.
Let us consider the implementation of one of the above modes. The calculated boundaries of the stability region are shown in Fig. 5

![Figure 5. Calculated stability limit and experimental points: 1 - n=530 rpm; 2 - n= 600 rpm](image)

Let us consider the modes 1 and 2 (Fig. 5). They correspond to two neighboring values of the rotational speed of a sample, i.e. 530 and 600 rpm. The first mode corresponds to a smooth cutting in the stability region as evidenced by the vibration displacement diagram (Fig. 6a). During the transition to the second mode there is a loss of stability. The vibration displacement diagram (Fig. 7, a) shows that the oscillation amplitude increased 3-4 times.

![Figure 6. Diagram of vibration displacements, mode 1: a - vibration displacements; b - amplitude spectrum](image)
The boundaries between stable and unstable modes were visually observed along the treated surface relief in the shape of chips. The calculated and experimentally determined boundaries of the stability region pass fairly close (rotational speed and working feed are set discretely) with deviations not exceeding 17%. Due to the absence of continuous adjustment of the machine operating conditions, it was not possible to refine the experimentally established boundary of the stability region.

4. Conclusion
The study carried out to find the stability of the machine dynamic system during the machining with cutter (turning and milling) disclosed the applicability of general approach when determining the boundary of the stability region on the basis of simplified models. The solutions obtained make it possible to ensure the quality requirements for the manufacturing of precision parts taking into account the current state of metal cutting equipment.

The calculated and experimental results prove practical applicability of the presented model solutions.

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