On the measurement of cosmological parameters

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ABSTRACT

We have catalogued and analysed cosmological parameter determinations and their error bars published between the years 1990 and 2010. Our study focuses on the popularity of measurements, their precision and their accuracy. The accuracy of past measurements is gauged by comparison with the most recent WMAP results of Komatsu et al. (2011). The 637 measurements in our study are of 12 different parameters and we place the techniques used to carry them out into 12 different categories. We find that the popularity of parameter measurements (published measurements per year) in all 12 cases except for the dark energy equation of state parameter $w_0$ peaked between 1995 and 2004. Of the individual techniques, only Baryon Oscillation measurements were still rising in popularity at the end of the studied time period. The quoted precision (fractional error) of most measurements has been declining relatively slowly, with several parameters, such as the amplitude of mass fluctuations $\sigma_8$ and the Hubble constant $H_0$ remaining close to the 10$\%$ precision level for a 10-15 year period. The accuracy of recent parameter measurements is generally what would be expected given the quoted error bars, although before the year 2000, the accuracy was significantly worse, consistent with an average underestimate of the error bars by a factor of $\sim 2$. When used as complement to traditional forecasting techniques, our results suggest that future measurements of parameters such as $f_{\mathrm{NL}}$ and $w_0$ will have been informed by the gradual improvement in understanding and treatment of systematic errors and are likely to be accurate. However, care must be taken to avoid the effects of confirmation bias, which may be affecting recent measurements of dark energy parameters. For example, of the 28 measurements of $\Omega_\Lambda$ in our sample published since 2003, only 2 are more than 1 $\sigma$ from the WMAP results. Wider use of blind analyses in cosmology could help to avoid this.

Key words: Cosmology: observations

1 INTRODUCTION

Modern cosmological parameters have been measured since Hubble’s (1929) discovery of the expansion of the Universe. The number of model parameters increased during the late 1980s with the introduction of what is often referred to as the “Standard cosmological model” (e.g., Dodelson 2005). The idea of “Precision cosmology” emerged more recently, and by the present time, many of the parameters in this model are well known (see e.g., Komatsu et al. 2011, hereafter WMAP7). This presents us with an interesting opportunity: by comparing the past measurements of parameters and their error bars with the currently known values, we can evaluate how well the measurements were carried out in the past, how realistic the quoted uncertainties were, and which methods gave the most statistically reliable results. We can also study how both their precision and accuracy has varied with time. Such research will help us in our quest to make critical evaluations of what will be possible in the future, and by working with past data serves as a complement to more conventional future extrapolations of technology and techniques (e.g., the report of the Dark Energy Task Force, hereafter DETF, Albrecht et al., 2006). In the present paper we make a first attempt at such a study, by compiling published parameter values taken from the NASA Astrophysics Data System over the years 1990-2010.

Previous studies of cosmological parameter determinations have tended to focus on the Hubble Constant, $H_0$, for which there is a longer than 80 year baseline for analysis. Several papers have used the comprehensive database compiled by John Huchra\cite{https://www.cfa.harvard.edu/~dfabricant/huchra/} to generate their dataset, such as the
study of the non-Gaussian error distribution in those measurements (Chen et al. 2003). Gott et al. (2001) used median statistics in a metanalysis of these $H_0$ measurements to find the most probable value (and also analysed early measurements of $\Omega_L$). This median statistics approach has also been used to combine individual estimates of $\Omega_m$, the present mean mass density in non-relativistic matter by Chen and Ratra (2003). In the present paper we do not seek to combine the measurements from various works into best determinations of parameters. Instead we start from the assumption that the parameters we look at have been well measured (and their correct values are close to the WMAP7 values) and see what this implies about past measurements.

We therefore will be starting with the assumption that $\Lambda CDM$ is the correct cosmological model. This should be borne in mind when interpreting our results. Even if the true cosmology turns out to be something else, we expect that the effective values of the $\Lambda CDM$ parameters are not likely to be very different (given the good fits to current data), so that our approach will have some value even then. Parameters which at the moment are unknown, or very poorly constrained, such as the non-Gaussianity parameter $f_{NL}$ (e.g., Slosar et al. 2008), or the time derivatives of the dark energy equation of state parameter $w$ (e.g., Chevallier & Polarski 2001) can obviously not be studied at present with our approach. Instead we hope that the general lessons from the past about the reliability of error bars, methods and achievable precision and accuracy can usefully to inform future efforts to measure those parameters.

The DETF report explains how four different techniques are being used and will be used in the future to constrain dark energy parameters. These techniques, gravitational lensing, baryon oscillations, galaxy cluster surveys and supernova surveys all have a history and have been involved in a large number of previous measurements of different parameters. It is interesting to see how they have performed in the past, and evaluate them based on this data. By looking over the published record, we can also show how measurement precision has changed, in terms of the quoted fractional error bars, and see how this compares with predicted future trends. One can ask whether for example the earlier error bars were unrealistically small, so that the quoted precision of measurements has not changed much. This should have consequences for the accuracy of measurements, which we will define and measure. In general, our motivation for this study can be summarized by the idea that once cosmological parameter measurements are published, for the most part they are ignored when future work arrives. The dataset left behind can instead become a valuable resource to inform future work.

Our plan for the paper is as follows. In Section 2, we detail the source for the cosmological parameter estimates and how the data was collated. We explain the different categorizations of measurements and methods and standardization that was carried out. In Section 3 we outline the steps involved in our analysis of the data, and present results including historical trends in some individual parameters and the precision and accuracy of measurements. In Section 4 we summarize our findings and discuss our results.

2 DATA

We have made use of the NASA Astrophysics Data System (ADS) to generate our dataset by carrying out an automated search of publication abstracts for the years (1990-2010). We limited the search to published papers which include cosmological parameter values and their error bars in the paper abstract itself. It is of course possible to carry out a more extensive analysis by searching the main text of each paper, and we estimate from a random sampling that approximately 40% of parameter estimates are missed by our abstract-only technique. We make the assumption that this does not bias our sample. The total number of parameter measurements in the 20 year period shown is 637.

2.1 Parameters

The search we use in the ADS abstract query form is a search for the following terms: “$\sigma_8$”, “$H_0$”, “Omega”, “$\Omega_m$”, “$\Omega_b$”, “baryon”. We also restrict our search to the following journals: MNRAS, Astrophysical Journal, ApJ Letters, ApJ supplement, and Physical Review Letters. This parameter search query appears restrictive, but enables results for 12 different parameters to be found, including associated parameters. These 12 are:

(i) $\Omega_M$, the ratio of the present matter density to the critical density,
(ii) $\Omega_L$, the cosmological constant as a fraction of the critical density,
(iii) $H_0$, the Hubble constant,
(iv) $\sigma_8$, the amplitude of mass fluctuations,
(v) $\Omega_b$, the baryon density as a fraction of the critical density,
(vi) $n$, the primordial spectral index
(vii) $\beta$, equivalent to $\Omega_m^{0.6}/b$ where $b$ is the galaxy bias,
(viii) $m_\nu$, the neutrino mass,
(ix) $\Gamma$, equivalent to $\Omega_m H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$,
(x) $\Omega_m^{0.6}\sigma_8$, a combination that arises in peculiar velocity and lensing measurements,
(xi) $\Omega_k$ the curvature,
(xii) $w_0$, the equation of state parameter for Dark Energy.

The measurements are generally quoted with 1 $\sigma$ errors on the parameters but 7% have 2 $\sigma$ errors. In this case, in order to have a uniform sample, we halve the 2 $\sigma$ error bars. We have tested the effect of ignoring excluding these 7% of measurements on our results (Section 3.3) and find that our conclusions are insensitive to this. Some of the measurements are also quoted with separate systematic and statistical error bars (6% of the sample). In this case we sum the statistical and systematic errors to make a total error bar. We also test the effect of adding them in quadrature, or ignoring the systematic part altogether (see Section 3.4).

Given that our approach is to assume that the WMAP7 results are correct within their quoted errors and that the $\Lambda CDM$ model describes the observations well, we use the $\Lambda CDM$ model values to convert combinations of published parameters into those listed above. For example, when measurements of $\Omega_m h^2$ are given we convert these into a value



http://adsabs.harvard.edu/
Table 1. Fiducial values for the cosmological parameters used in this paper. These values are used when computing the accuracy of past measurements. All parameters are taken from the last column of Table 1 in the WMAP7 paper, which are mean of the posterior distribution of combined WMAP+BAO+$H_0$ measurements (we have also tried the maximum likelihood parameters, with no difference in our results), except parameters (ii),(viii),(xi) and (xii) for which we have assumed that an exactly flat $Λ$CDM model holds with $m_ν = 0 \pm 0.1$ eV. The quoted error bars are derived from the WMAP7 error bars, with the exception of parameter (vii) for which an error bar of 0.1 is used to approximately take into account differences in galaxy bias between different samples.

We explore the effect of adding these error bars in quadrature to the error bars of past measurements in Section 3.4.

| Parameter | Central value | 1 $σ$ error bar |
|-----------|---------------|------------------|
| (i) $Ω_M$ | 0.274         | 0.013            |
| (ii) $Ω_A$ | 1.0±0.274      | 0.013            |
| (iii) $H_0$ | 70.2 $\text{km s}^{-1}\text{Mpc}^{-1}$ | 1.4 $\text{km s}^{-1}\text{Mpc}^{-1}$ |
| (iv) $σ_8$ | 0.816         | 0.024            |
| (v) $Ω_b$ | 0.0458        | 0.0016           |
| (vi) $n$ | 0.968         | 0.012            |
| (vii) $β$ | 0.460         | 0.1              |
| (viii) $m_ν$ | 0.0 eV         | 0.1 eV           |
| (ix) $Γ$ | 0.193         | 0.006            |
| (x) $Ω_0^{\text{CDM}}σ_8$ | 0.376       | 0.015            |
| (xi) $Ω_b$ | 0.0           | 0.0              |
| (xii) $w_0$ | -1.0          | 0.0              |

for $Ω_b$ using the WMAP7 value of $h = 0.702$. For reference we give our fiducial values of each parameter in Table 1. As stated in the caption, most of these are taken from Table 1 in WMAP7, but others are assumptions based on $Λ$CDM (e.g. $w_0 = -1$ exactly).

2.2 Measurement Methods

For each published measurement, we also choose a category based on the type of data and method used to extract the cosmological parameter. There are obviously many different possible choices of categorization possible and with different coarseness. We choose the following 12 categories in order to have a reasonable number of measurements in each (the mean is 53):

(i) Cosmic Microwave Background (CMB), specifically measurement of primary anisotropies,
(ii) Large-Scale Structure (LSS), which includes clustering of galaxies, galaxy clusters (BAO measurements and redshift distortions are considered separately), the Ly$α$ forest, quasar absorption lines, and quasars.
(iii) Peculiar velocities, which includes measurements of galaxy peculiar velocities inferred from distance measurements and redshifts, and the cosmic dipole.
(iv) Supernovae, which includes techniques that use supernova distance measurements.
(v) Lensing, which includes constraints from the number of strong gravitational lenses, weak lensing shear, and gravitational lens time delay.
(vi) Big Bang Nucleosynthesis (BBN),
(vii) Clusters of galaxies including their abundance and their masses. Includes Sunyaev-Zeldovich measurements,
(viii) Baryonic Acoustic Oscillation measurements from large-scale structure of galaxies and clusters,
(ix) The Integrated Sachs Wolfe effect (ISW),
(x) $z$ distortions, redshift distortions of clustering
(xi) Other, includes Tully Fisher distance estimates, galaxy ages and/or colours, globular cluster distances, internal structure of galaxies, cepheid distances, surface brightness fluctuations, reverberation mapping, radio source size, and Gamma Ray Burst distances.
(xii) Combined, includes measurements that result from a combination of techniques or past measurements, without the addition of new measurements.

In Figure 1 we show a scatter plot of method vs parameter for our dataset. We can see that the most popular parameter/method combination is $Ω_M$ measured using galaxy clusters, but that in general there is a fairly wide selection of method and parameter, with just over half (76 out of 144) of the combinations covered by at least one published abstract.

3 ANALYSIS

Our analysis is in two parts, the first being a study of general trends in the number of parameter measurements and popularity of different methods by year, as well as a looking at the measurement value vs year for a subset of parameters (Sections 3.1 and 3.2). In the second part (Sections 3.3 and 3.4), we compute the precision and accuracy of the measurements and see how these have varied with time.
3.1 Number of studies by year

In Figure 2 we show how the number of parameter measurements per year has varied with time. The results are shown averaged in bins of 3 years.

It is immediately noticeable that nearly all of the parameters have a peak in the number of measurements around the years 2000-2003, and then a decline in the post-WMAP1 (Spergel et al. 2003) era. Exceptions to this are measurements of \( w_0 \), which are still increasing in number, and constraints on \( \sigma_8 \). Of course this historical trend is largely guaranteed by our selection of the parameter set we have chosen, which in large part are considered to have been well measured already. Other parameters such as \( f_{NL} \), \( w_a \), or the modified gravity parameter \( E_G \) (see e.g., Reyes et al. 2011) would still be increasing on such a plot. Figure 2 can also be viewed as a measure of the extent to which parameters are considered to be well measured. For individual parameters such as \( \sigma_8 \), there are still many measurements published even at the current time, but the decline is still there.

Another way to present the data is shown in Figure 3, where the popularity of different methods with time can be examined. Here it can be seen that “combined” methods are the exception to the general post WMAP1 decline. In overall number, galaxy clusters have proven the most popular cosmological probe, with a sharp start in the early 1990s. Supernovae and Large-Scale structure measurements have remained fairly constant since 2000, and the popularity of gravitational lensing per year has not been much different from that of galaxy clusters, except lagging behind by about 8 years. BAO measurements are the only technique still on the rise, reflecting the current and future large-scale structure surveys targeted at BAO (e.g., Eisenstein et al. 2011, Blake et al. 2011, Schlegel et al. 2011).

3.2 History of individual parameter measurements

It is instructive to study the distribution of data points and their error bars as a function of time, and we do this for a subset of parameters in Figures 4 through 9. In each case we show the WMAP7 best fit value for the parameter as a horizontal line. This type of plot is most familiar from the studies of Huchra for the Hubble constant, where the initial values reported by Hubble were over 5 times the currently accepted values.

In Figure 4 we show how Hubble constant determinations have changed over the last 20 years, with the beginning of this time period overlapping with the end of the \(~ 20 \) year timeframe during which measurements of \( H_0 \) were largely divided into two groups, one group closer to \( 50 \) km s\(^{-1} \) Mpc\(^{-1} \) (e.g., Sandage & Tammann 1975), and one closer to \( 100 \) km s\(^{-1} \) Mpc\(^{-1} \) (e.g., Devaucouleurs et al. 1979). These two camps can be seen prior to 1995 in Figure 4 where it is also obvious that their error bars are largely not compatible, or indeed compatible with the eventual currently favoured value of \( H_0 = 70 \) km s\(^{-1} \) Mpc\(^{-1} \). The HST Key Project (hereafter KP) to measure the extragalactic distance scale published its first results in 1994 (Freedman et al. 1994), and final results in 2001 (Freedman et al. 2001). The main contribution of the project was to extend the Cepheid-based rung of the distance ladder to cosmological distances. Freedman et al. 2001 combined this with other datasets (Type Ia and type II SN, the galaxy Tully-Fisher relation, surface brightness fluctuations and galaxy fundamental plane) to yield different measurements which were all consistent with \( H_0 = 72 \pm 8 \) km s\(^{-1} \) Mpc\(^{-1} \), meeting the goal of a \(~ 10\% \) measurement of \( H_0 \).

This post-1994 period of activity related to the KP is immediately apparent in Figure 4. It can also be seen that different methods have produced results which were some-
what divergent at first but which eventually became consistent with the final result by the end of the 1994-2001 KP period. An example of this is the determination from type IA supernovae, where it can be seen that the green points representing these track steadily upwards from 1993 onwards. A large cluster of gravitational lensing time delay measurements also exhibits a similar trend, and indeed some other measures such as galaxy cluster Sunyaev-Zeldovich measurements are somewhat lower than $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$. This set of lower results largely disappears by 2003, which is when the next sudden tightening of determinations occurs, coincident with the WMAP1 data release. The WMAP1 best fit value of $H_0$ was $72 \pm 5$ km s$^{-1}$ Mpc$^{-1}$, and after that date essentially all measurements are consistent with it. Of course the WMAP1 result was strikingly similar to the KP result even though it involved radically different physics. The evidence of Figure 4 is that the combination of the two sets of measurements was enough to convince most researchers that the measurement goal had been reached. In the future, a measurement of $H_0$ to even higher accuracy will be needed.
to make truly accurate constraints on dark energy parameters (see e.g. the DETF report).

There are a few obviously discordant points, for example, Leith et al. (2008) find $H_0 = 61.7^{+1.7}_{-1.1}$ km s$^{-1}$ Mpc$^{-1}$ from a combined analysis of several datasets. Their analysis is not in the context of the LCDM model, but in one in which cosmological averaging can be used to understand the acceleration of the Universe (Wiltshire 2007).

In Figure 6, we show the history of measurements of $\Omega_m$, the most frequently measured parameter in our dataset. In this case we can see that before 1999 approximately 1/3 of the measurements were consistent with high values of $\Omega_m \sim 0.5 - 1.0$, and that the most popular technique in this early period involved the use of galaxy peculiar velocities. The error bars were large, although there are several points which are not consistent with the eventual WMAP7 value of $\Omega_m = 0.274 \pm 0.013$. After 1999, although peculiar velocities continued to be popular, the measurements were no longer sampling the high $\Omega_m$ end of parameter space. As with the $H_0$ results a second significant tightening of published values around the final range took place in the years 2004-2005, shortly after the WMAP1 results.

The amplitude of mass fluctuations, $\sigma_8$ is examined in Figure 8. In this case we can see that the abundance of galaxy clusters is easily the most popular method used to measure this parameter, and the effort started in earnest around 1995. The cluster measurements of $\sigma_8$ are roughly evenly spread around the WMAP7 value of $\sigma_8 = 0.816 \pm 0.024$ until after the WMAP1 release, when low values (below $\sigma_8 \sim 0.8$) ceased to be published. As with the other parameters, the evidence of post WMAP1 tightening is there. Lensing determinations of $\sigma_8$ seemed to favour high values, $\sigma_8 \sim 1$ until after WMAP1.

Turning to the baryon density parameter $\Omega_b$ in Figure 8 we can see that the measurements are mainly concentrated in an 8 year period between 1996 and 2004. Over this time span two features can be clearly seen, the first being the steady rise in $\Omega_b$ measured using BBN, and other being the start of CMB measurements around 2000. Because a high Deuterium to Hydrogen ratio (easier to see) implies a low value of $\Omega_b$ this may account for the difficulty encountered in early BBN measurements. Both CMB and BBN were consistent, however well before the WMAP tightening which occurred around 2003-2004, as with the other parameters.

In Figure 8 we plot the measurements of $\Omega_\Lambda$. In this case, many of the early points are upper limits which were just consistent with the eventually measured value. The first Type 1A supernova results showing acceleration appeared at the end of this era of upper limits. The probably WMAP-related tightening of results around 2003 is especially pronounced in this plot, where one can see the published error bars sizes immediately dropping. It is interesting to note that after 2002, almost all measurements of $\Omega_\Lambda$ are consistent with the fiducial value from Table 1. Of the most recent 28 measurements shown in Figure 8 (these are those that contribute to the last 2 points in the $\Omega_\Lambda$ accuracy plot, Figure 12), only 2 are more than 1 $\sigma$ from the “correct” value. The sum of $\chi^2$ values when we compare to the $\Omega_\Lambda$ from Table 1 per data point is 22.7 for these 28 measurements, which does not sound very small. However, this includes the measurement of Cabre et al., (2006), which is 4.0 $\sigma$ from the Table 1, value. Without this outlier, the $\chi^2$ per data point is only 0.26. This could be a signature of overestimation of the error bar size, or perhaps of “confirmation bias”. We will return to this in Section 4.

The final parameter for which we examine the individual measurements is the dark energy equation of state parameter $w_\Lambda$, which we show in Figure 9. In this case there are no measurements or limits before the SN measurements

![Figure 5](image_url)
of the acceleration of the Universe in the late 1990s (Perlmutter et al. 1999, Riess et al. 1998). At around the time of WMAP1 the first measurements rather than limits on $w_0$ started to be published, and since then SN have continued to be the most popular probe of this parameter. A trend more apparent in this more recently measured parameter is the large number of points from “combined” measurements.

Although one could argue that $w_0$ is not at present as well known as some of the other parameters, we have plotted the fiducial value on this graph as $w_0 = 1$ exactly. All measurements since 2004 (bar 2) are consistent with this value at the 1 $\sigma$ level.

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**Figure 6.** Individual published values of the amplitude of mass fluctuations, $\sigma_8$, as a function of year. We show one sigma error bars on the points, and the point colour (shown in the legend) denotes the technique used to make the measurement (see section 2.2 for more details).

**Figure 7.** Individual published values of the baryon density as a fraction of the critical density, $\Omega_b$, as a function of year. We show one sigma error bars on the points, and the point colour (shown in the legend) denotes the technique used to make the measurement (see section 2.2 for more details). 1 (2) sigma upper and lower limits are shown using single (double) arrows.
3.3 Precision

One of the common themes which has emerged in the past few years is that we are now in the era of “precision cosmology”. It is instructive to study what the data reveals about how we reached this point and what precision is currently achievable for the different parameters and using the different techniques. We quantify the precision of measurements to be the size of the 1σ error bar as a percentage of the fiducial (WMAP7) value for each parameter. We have also tried using the error bar size as a percentage of the quoted central value of each measurement, finding no significant difference in our results (except for the case of $\Omega_\Lambda$, for which the latter is not a useful way of examining earlier data). In the case of the neutrino mass, $m_\nu$ for which only upper limits are available, we have taken the precision to be the limit in $m_\nu$ divided by the value of $m_\nu$ required for the closure den-
ity (i.e., $\Omega_m = 1$) in neutrinos, which is $m_\nu = 93h^2$eV. For $\Omega_k$ we divide the measurement error by 1.0 as the fiducial value of $\Omega_k = 0$. Our definition of precision in this case is therefore more directly related to the precision on the measured distance to the surface of last scattering (see e.g., Hu & Dodelson 2002).

In Figure [10] we show the average precision for each parameter as a function of year. We have binned the measurements into bins of width 4 years, and when computing the average precision compute an unweighted mean from the measurements in each bin. We show Poisson error bars on the mean precision. Apart from $m_\nu$, we have not included any upper or lower limits on parameters in this plot, only published measurements of values with error bars.

It is apparent from the general appearance of Figure [10] that the precision of most measurements has not increased very steeply. The log scale of the $y$-axis is partly responsible for this impression, but even so, of the 12 parameters shown, 6 have a mean precision in the latest bin which is compatible (within $1\sigma$) of that in the earliest bin. It is possible that this situation has arisen because of greater understanding of the role of possible systematic errors as time has gone on. The value of $\sigma_8$ is now known to better than 10% for an average measurement, for example, after a long period in which the precision did not improve. Currently the most precisely known parameters are the curvature $\Omega_k$ and the primordial spectral index $n_s$, which are both known to about 1%. A large group of parameters are currently known to about 10% precision, from $\Omega_M$ (17%), through $\Omega_b$, $\Omega_\Lambda$, $\sigma_8$ and $H_0$ (7%).

In Figure [11] we show the precision of measurements as a function of the technique used. As many of the techniques are used to measure several different parameters, it is worth bearing in mind that decreases in precision with time could be related to the switch to a less well measured parameter. We can see that this may indeed be happening in some cases, or else that again systematic errors are being confronted more as time goes in. We can differentiate between these possibilities by considering the averaged accuracy of measurements, which we do in the next Section. For now, we can see that lensing, redshift distortion and peculiar velocity measurements have exhibited no improvement in quoted precision with time. The CMB on the other has improved by about an order of magnitude over the 20 year period, and cluster measurements by about a factor of 3. Supernova measurements are also more precise now than they were in the late 1990s by a factor of 2.

### 3.4 Accuracy

Our assumption that the “correct” values of the different cosmological parameter values are available allows us to compute a potentially powerful statistic, the accuracy of measurements. We define this to be the absolute value of the difference between a measured value of a parameter and our fiducial value for that parameter (as listed in Table [1]), divided by the quoted $1\sigma$ error bar for that measurement. The accuracy can therefore be written as $N_\sigma$, the average number of standard deviations measurements are from the correct value. We note that for a Normal distribution of errors, the average value of $N_\sigma = 1$. Values smaller than 1 indicate that the error bars have been overestimated, and for values larger than 1 the error bars have been underestimated. Alternatively, values smaller than 1 may also indicate evidence for “confirmation bias”, in which values closer to the expected ones are favoured (not necessarily consciously). We have chosen to use $N_\sigma$ as our statistic rather than the $\chi^2$ as it is more robust to outliers (not being dependent on the square of the difference between a measurement and the true value). Qualitatively similar conclusions would result if we did use the $\chi^2$ of measurements with respect to the “known” values as a measure of accuracy, however.

When computing the accuracy, one must decide how the uncertainty on the true values of the parameters affects the results. Two choices which approximately bracket the range of potential effects are to either add the $1\sigma$ error bars on the values in Table 1 in quadrature to the error bars on each published measurement, or else to assume no additional uncertainty beyond the quoted error bar for each published measurement. We have tried both, finding almost imperceptible quantitative differences which do not affect any of our conclusions. This can be understood from the fact that the error bars in Table 1 are much smaller than those on past measurements. In making our plots, we have chosen the second option, on the grounds that some of the uncertainty from the other measurements has already been incorporated into the WMAP7 results we use in Table 1.

In Figure [12] we show the accuracy for the different parameters as a function of year, with the binning by year carried out as for the precision (Figure [10]), an unweighted average of the measurements in that bin. Poisson error bars have been computed as before. In general, one can see that the measurements in the different panels are not extremely offset from the $N_\sigma = 1$ line, indicating that the accuracy of parameter determinations has not been wildly off. That said, however, the $N_\sigma = 1$ line is a good fit by eye in only 1 panel, that for the shape parameter $\Gamma = \Omega_h$.

Turning to individual parameters in Figure [12] we can see that the accuracy of measurements of $H_0$ and $\sigma_8$, has improved over the last 20 years, so that the most recent measurements appear to have realistic error bars. The error bars on $\Omega_\Lambda$ appear to be overestimated, as do the most recent error bars on $\Omega_M$. From this it would appear that significant work successfully understanding the overall levels of measurement uncertainty has been carried out for $H_0$ and $\sigma_8$, but that this has not happened for some of the other parameters. We return to this topic in Section 4.2.

If the varying accuracy is more tightly related to the choice of technique than parameter, then we can expect the plot of accuracy for different techniques (Figure [13]) to be more instructive. Here we can see that there are indeed some techniques which have a better track record than others. For example the use of peculiar velocities to measure parameters has resulted in an underestimate of error bars by a factor of roughly 2 on average (there is some improvement in the most recent two points, which are consistent at $1\sigma$ with $N_\sigma = 1$ ). The CMB and redshift distortions have on the other hand proven accurate sources of measurements for the whole period. Galaxy clusters were sources of measurements with underestimated errors in the 1990s, but in the last 12 years have tracked $N_\sigma = 1$ very well.

The most recent 2 points for SN and 3 for BAO appear to have overestimated error bars, significantly so in the case of SN (by a factor of 3). SN measurements are those which
most often quote systematic error bars (which we have added directly to the statistical errors). We have tried two other ways of dealing with the systematic errors, either adding them in quadrature, or ignoring them altogether. We find that with the latter most conservative treatment, the SN results yield $N_\sigma = 0.5 \pm 0.07$ and $0.77 \pm 0.12$ for the most recent two points. This is an improvement, indicating that the SN systematic error bars may well be too conservative. It is still an underestimate, but now of similar magnitude to the differences seen between the accuracy=1 line and some data points on the “other”, “combined” and “LSS” panels. If we allow for the possibility that the Poisson error bars on our data points in Figure 13 are underestimates, and that there may be correlations between measurements in different years then this may go some way to reconciling the measurements and their hoped for accuracy. We return to this point in our discussion below (Section 4.2).

One question which is not easy to answer from the multipanel Figures 12 and 13 is how the overall accuracy of measurements is changing by year. Are cosmological measurements improving as both theoretical knowledge and expertise in dealing with experimental uncertainties improve? We can see that this does appear to be the case by considering Figure 14 which plots accuracy by year for results published in the two main journals, MNRAS and ApJ (including ApJL and ApJS). These account for 35% and 55% of all results in our compilation, respectively. The results before the year ~2003 are significantly inaccurate, but steadily improve with time until after this date they become consist-
Figure 11. The quoted precision of measurements as a function of year for the different measurement techniques. The precision is defined to be the size of the 1σ error bar as a percentage of the fiducial parameter value in Table 1. Each panel therefore includes measurements made of many different parameters. Error bars are Poisson errors computed from the number of measurements in each bin.

Figure 15. The accuracy of measurements published in a paper as a function of the number of citations to it. The accuracy is defined to be the difference between the quoted measurement and the fiducial parameter value in Table 1 in units of the quoted measurement 1σ error bar.
Figure 12. The accuracy of measurements as a function of year for the different parameters. The accuracy is defined to be the difference between the quoted measurement and the fiducial parameter value in Table 1 in units of the quoted measurement 1σ error bar. Error bars are Poisson errors computed from the number of measurements in each bin. The dashed line is the expectation for Gaussian statistics, \( N_\sigma = 1 \).

Figure 16. The distribution of measurement errors in units of the quoted standard deviation. For each measurement we divide the difference between the quoted value and the fiducial value in Table 1 by the quoted 1σ error bar. The results are shown as a histogram. We also show the expected curve for a Gaussian distribution of errors (smooth line).

is just due to the overall trend in improving measurements, and a correlation between year of publication and citations \((r = 0.348, p = 0.148)\). This latter is presumably due to the larger number of researchers working in cosmology. Both of these trends combine to produce the trend of citations with precision.

A final issue which we address when looking at the accuracy is the shape of the error distribution. When stating \( N_\sigma = 1 \) is appropriate for an accurate set of measurements we have made the assumption that all quoted errors have a Gaussian distribution. This is an assumption often made (although not by all), and is something which we can examine using our data, by comparing the number of standard deviations that measurements are away from our fiducial values with the curve for a Gaussian distribution. This will tell us for example if there is a long non-Gaussian tail to the error distribution. We show the histogram of \( N_\sigma \) values in Figure 16 along with the Gaussian curve. The data is fairly similar to the Gaussian curve for the low end of the \( N_\sigma \) range where the majority of the data resides, showing that in general error bars are only slightly underestimated (we have seen this already in Figure 14, for example). There is however a long tail extending to high \( N_\sigma \) values, with some measurements being 8 or even 10σ away from their fiducial values. Of course with a Gaussian distribution the chance of such events occurring would be minuscule. We can quantify this further by computing the fraction of measurements which are greater than 2σ away from the correct value. We
4 SUMMARY AND DISCUSSION

4.1 Summary

We have compiled cosmological parameter measurements published between 1990 and 2010 and the techniques used to measure them. Using this data we have carried out an analysis of historical trends in popularity, precision and accuracy. The accuracy of past measurements has been estimated by assuming that WMAP7 parameter values of Komatsu et al. (2011) (combined with ΛCDM standard values for e.g. \( w_0 \)) are the correct ones. Our findings can be summarised as follows:

1. The number of published measurements for different parameters peaks between 1995 and 2004 for all cases, except for \( w_0 \) for which the number was still rising in 2010.
2. Of all techniques used to measure the parameters, only baryon oscillation and “combined” measurements were still rising in terms of publications per year by 2010.
3. The quoted precision of measurements has been declining relatively slowly for most parameters, with several (e.g. \( \sigma_8 \), \( H_0 \)) remaining flat for 10-15 years.
4. The accuracy of recent parameter measurements is generally what should be expected based on the quoted error bars i.e. the error bars overall are neither underestimated nor overestimated (an accuracy, \( N_\sigma = 1.0 \), within the Poisson uncertainty on the measurement). Before 2000, the accuracy \( N_\sigma \) as closer to 2, indicating underestimation of the error bars by a factor of 2. Overall, there is a small non-Gaussian tail to the error distributions (we find that 20% of measurements are more that 2\( \sigma \) away from the true values.
5. The accuracy of most methods has become consistent with \( N_\sigma = 1.0 \), with the historically most inaccurate parameter measurement technique being the use of galaxy peculiar velocities. Measurements of \( \Omega_M \) and particularly \( \Omega_\Lambda \) made since 2000 tend to have accuracy \( N_\sigma \) significantly less than 1.0, indicating “confirmation bias” and/or an overestimation of error bar sizes.

4.2 Discussion

Over the 20 year period covered in this study, it is apparent that many of the parameters in what is now the concordance CDM cosmological model went from the status of no information or only limits to being known at the 10% level or better. It is also apparent from Figure 2 that there was a “golden age” of parameter measurements between \( \sim 1995 \) and \( \sim 2005 \) during which the number of published measurements peaked sharply and then declined. This seems to indicate that for many purposes (such as the use of a background cosmology in galaxy formation models), the precision to which the ΛCDM parameters were known by the time of the first WMAP results is sufficient, and many of the rea-
Inflationary parameters such as the non-Gaussianity $f_{NL}$, or from most of the parameters we have studied in this paper. The concordance CDM model and fall into a different category such parameters involve searching for deviations from the expectations between the error bars so that our estimates of the accuracies of measurements (Figs 12 and 13) we have used much of the post WMAP1 tightening of constraints seen in Figures 4-9. When computing the error bars on the mean of quantitiative study. These include the modified gravity parameter, $E_G$ (Reyes et al. 2010) and the time derivative of the equation of state parameter, $w_a$. Measurements of such parameters involve searching for deviations from the concordance CDM model and fall into a different category from most of the parameters we have studied in this paper. Inflationary parameters such as the non-Gaussianity $f_{NL}$, or tensor to scalar ratio $r$ will pinned down with higher precision in the future, and these should also represent a growth area. The motivation for most future measurements being largely framed in terms of a quest for fundamental physics, it would be logical to assume that they will continue until the cause for the Universe’s acceleration are better understood. Likewise, parameters describing the dark matter particle should be added to this category.

Possible behaviours for the precision of future parameter measurements can be predicted by looking at the past results (Figure 10). There is a very wide range, but most parameters improve slowly, with a factor of 10 improvement in precision over the 20 years representing the extreme (2 out of 12 parameters). The precision of some parameters has remained relatively flat for the whole period, so this is a possibility for future so far unconstrained parameters. An argument against this slow progress however is the fact that many new surveys (such as of Baryon Oscillations) are targeted primarily at measures of specific parameters, and this aggressive approach (for example including specific precisions to be obtained at a given time in survey proposal documents) could lead to faster progress.

Our investigation of the accuracy of results could potentially lead to some of the most interesting findings. We have seen that in the earlier half of our studied time period there is evidence that the error bars were significantly underestimated, but that this has changed over time.

When discussing the accuracy, we are should be aware that it was not possible in our analysis to take into account several factors which have the potential to affect our conclusions. For example, we do not keep track of the priors that people have assumed in their measurements, and in many of the later cases, this may include the WMAP results as priors. That this is happening is likely to be responsible for much of the post WMAP1 tightening of constraints seen in Figures 4-9. When computing the error bars on the mean accuracies of measurements (Figs 4,9 and 13) we have used Poisson errors based on the number of measurements in each bin. This will tend to underestimate the uncertainty on the accuracy because some of those measurements could be using the same underlying data, or be using similar priors, or a combination of the two. There will therefore be correlations between the error bars so that our estimates of the accuracy will be affected. Equivalently the chi-square of the fiducial result compared to the data points will be incorrectly determined to be low because of the correlations are not included.

Bearing the above points in mind, we return to the panels in Figures 12 and 13 where the accuracy seems to be significantly below $N_σ = 1$. This is most obvious in the second panel ($Ω_L$) of Figure 12. Such as result could be a sign that either the error bars have been significantly overestimated, or else that researchers have been influenced by prior results (‘confirmation bias’), or a combination of the two. If we return to the data points which led to the last two bins of panel two of Figure 12 we find something especially striking. Of those 28 measurements, only 2 are more than $1σ$ from the fiducial results of Table 1. These 28 measurements were carried out by approximately 11 separate groups (as determined by authorship lists) using several different techniques.

This closeness of published results to the “correct” ones is somewhat worrying for future measurements. One can interpret this as coming partly from error bars being overestimated by cautious cosmologists, for example by including possible systematic errors in the error bars which are not actually present to such a large degree, or in a related point authors marginalizing over parameters which are actually better known than was assumed. We note that including or excluding the actually quoted systematic error bars (Section 3.4) has little effect on this result. An additional question is why some parameters have $N_σ < 1$ and others do not (e.g., $σ_8$). The relatively low number statistics of our whole dataset preclude us from making any strong statements about this issue. If it does partly result from confirmation bias, one can also wonder how observers knew which value of $Ω_L$ (for example) would be the “correct” one, given that our fiducial (mostly WMAP7) results from Table 1 were published in 2011. If this bias is present, it is probably related to the mean level for $Ω_L$ resulting from several prior measurements. For example in Figure 8 and others, the value of the parameter seems to be pretty well determined at least by 2003.

If we look at the techniques which are often associated with dark energy measurements, SN and BAO, we can see in Figure 13 that these two have low $N_σ$ for recent measurements. Of the 23 measurements which where included in the last bins of the SN panel of Figure 13 only 2 are more than $1 σ$ from the fiducial result. We note that this fiducial result from Table 1 does include BAO measurements, but not SN estimates of dark energy. In the case of SN, however, only 4 measurements of $Ω_L$ are included in these bins, and only 2 separate groups of researchers, so that for that subset of data, statistical fluctuations may well be responsible for the low $N_σ$ seen. If confirmation bias is present, on the other hand, one could argue about who is confirming who- certainly the first SN results on dark energy predate those from BAO and from most other techniques. These sorts of questions might be addressed by a more detailed look at the published measurements, including details of priors, jointly used datasets and analysis techniques. Then again small number statistics probably would not allow firm conclusions to be drawn. These hints should instead serve as a warning that care and perhaps concrete steps be taken to avoid any confirmation bias in the future.
In conclusion, we have seen that huge progress has been made in the 20 year period covered by our study. Important questions have been resolved (e.g., is the Universe open?, do massive neutrinos contribute substantially to the dark matter density?), a model has been found which agrees with essentially all observational data so far (ΛCDM), and the parameters of that model have been pinned down at the 1 – 10% level. The first WMAP results (e.g. as presented in Spergel et al. (2003)) form a watershed which is easy to pick up in most plots of parameters with time, and serves as a reminder that statistically measurable progress is not always gradual. Perhaps the most interesting aspect of our study, of the accuracy of past results compared to our most recent knowledge has found that understanding of systematic errors and uncertainties in cosmological measurements has demonstrably improved since the early 1990s. On average, results in the last 10 years are consistent with expectations, given their error bars, something which should instill confidence in future measurements. There are some signs that recent measurements of dark energy parameters are closer to the “expected” values for ΛCDM than statistically likely. These may be explainable by correlations between measurements which we have not included. On the other hand this may serve as a sign that as cosmology collaboration sizes increase carrying out more blind analyses (as in particle physics) may be a good idea.

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