Scaling behavior of domain walls at the $T = 0$ ferromagnet to spin-glass transition

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Introduction
Techniques
Results
Summary
Model

- $N = L \times L$ Ising spins $\sigma_i = \pm 1$ on square lattice
- Periodic boundary conditions in one direction
- Edwards-Anderson Hamiltonian: $\mathcal{H}(\sigma) = -\sum_{\langle ij \rangle} J_{ij} \sigma_i \sigma_j$

Interaction strength:

- $J_{ij} > 0$
- $J_{ij} < 0$

Quenched disorder

Frustration:

- Here: “Gaussian-like” distributed bonds

$$P(J) = (1 - \rho) e^{-J^2/2}/\sqrt{2\pi} + \rho \delta(J - 1)$$

- $\rho < \rho_c$: Spin-glass (SG)
- $\rho > \rho_c$: Ferromagnet (FM)
Domain Walls (DWs)

- Exact ground states (GSs) using sophisticated matching algorithms (up to \( L = 512 \)).
- DWs defined relative to 2 spin configurations \( \sigma^{(1)}/(2) \)
  - \( \sigma^{(1)}: \)
  - \( \sigma^{(2)}: \)
- Separates regions of agreeing/disagreeing spin config.

[A.K. Hartmann and H. Rieger, *Optimization Algorithms in Physics*]

**DW energy:**

\[
\delta E = 2 \sum_{\langle ij \rangle \in \mathcal{D}} J_{ij} \sigma_i^{(1)} \sigma_j^{(1)}
\]

\( \mathcal{D} \equiv \) bonds satisfied by only 1 config.
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DW energy:

$$\delta E = 2 \sum_{\langle ij \rangle \in D} J_{ij} \sigma_i^{(1)} \sigma_j^{(1)}$$

$D \equiv$ bonds satisfied by only 1 config.
Construct weighted graph $G = (V, E, \omega)$

- $V(G)$: elementary plaquettes (EP)
- $E(G)$: connect EP with common side
- $\omega$: energy contribution to DW

Consider GS $\sigma$ for periodic BCs:

(i) Bond satisfied for $\sigma$, e.g.
\[ \uparrow \quad \uparrow : \omega \geq 0 \]

(ii) Bond not satisfied for $\sigma$, e.g.
\[ \uparrow \quad \uparrow : \omega \leq 0 \]
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no loops with negative weight:

$$
\omega(C) = \sum_{\langle ij \rangle \in C} J_{ij} \sigma_i \sigma_j \geq 0
$$

DW: minimum-weight (top, bottom) path
Minimum-Weight Paths

- $G$: undirected graph, allowing for negative edge weights
- Here: standard minimum-weight path algorithms, e.g. Bellman-Ford, Floyd-Warshall, *don’t work*
- Minimum-weight path problem on dual requires matching techniques
  - i) Dual graph $\rightarrow$ auxiliary graph
  - ii) Find *minimum-weighted perfect matching* (MWPM)
  - iii) Interpret MWPM as min.-weight path

[R.K. Ahuja, T.L. Magnanti and J.B. Orlin, *Network flows*]
Excitation energy of DWs:
\[ \langle |\delta E| \rangle \sim L^\theta, \quad \theta = -0.287(4) \]
[AKH and A.P. Young, PRB 2001]

Scaling behavior of DWs:
\[ \langle \ell \rangle \sim L^{d_f}, \quad d_f = 1.274(2) \]
\[ \langle r \rangle \sim L^{d_r}, \quad d_r = 1.008(11) \]
[OM and AKH, PRB 2007]

DWs can be described by Schramm-Loewner evolutions (SLEs)
[Amoruso et. al., PRL 2006], possibility to relate exponents via
\[ d_f = 1 + 3/[4(3 + \theta)] \]

Universality: SLE scaling relation also valid for \( \rho > 0 \)?
Magnetization: \( m_L = \sum_i \sigma_i / L^2 \)

Binder ratio: \( b_L = (3 - \frac{\langle m^4 \rangle}{\langle m^2 \rangle^2})/2 \)

finite size scaling:
\[ b_L \sim f[(\rho - \rho_c)L^{1/\nu}] \]
\[ \rho_c = 0.660(1) \]
\[ \nu = 1.49(7) \]
\[ S = 1.3 \]

\( S \) = “quality” of the scaling assumption
Scaling behavior of DWs

Scaling analysis up to $L = 512$

| $\rho$ | $d_f$    | $d_r$    | $\theta_2$ |
|-------|---------|---------|-----------|
| 0.00  | 1.274(2)| 1.008(11)| -0.287(4) |
| 0.60  | 1.275(1)| 1.003(3)| -0.28(2)  |
| 0.64  | 1.275(2)| 1.012(4)| -0.28(4)  |
| 0.66  | 1.222(1)| 1.002(2)| 0.16(1)   |
| 0.68  | 1.05(2) | 0.74(3) | 0.35(3)   |
| 0.72  | 1.022(1)| 0.698(6)| 0.27(2)   |

where

$$\sigma(\delta E) = \sqrt{\langle \delta E^2 \rangle - \langle \delta E \rangle^2} \sim L^{\theta_2}$$

Spin glass phase up to $\rho$ close to $\rho_c$: Scaling behavior of DW energy and DW length consistent with scaling relation

$$d_f = 1 + 3/[4(3 + \theta)]$$

derived from SLE processes.
Summary

- Groundstate study on 2D Ising spin glasses with short ranged interactions
- DWs obtained via minimum-weight path approach
- Scaling behavior of DWs near SG-FM transition at $T = 0$
- $\rho < \rho_c$: SLE scaling relation consistent with exponents found from numerical simulations