Gluon effects may rule out the existence of color superconducting strange stars

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Compact astrophysical objects are a window for the study of strongly interacting nuclear matter given the conditions in their interiors, which are not reproduced in a laboratory environment. Much has been debated about their composition with possibilities ranging from a simple mixture of mostly protons and neutrons to deconfined quark matter. Recent observations on the mass of two pulsars, PSR J1614-2230 and PSR J0348+0432, have posed a great restriction on their composition, since the equation of state must be hard enough to support masses of about at least two solar masses. The onset of quarks tends to soften the equation of state, but it can get substantially stiffer since in the high-dense medium a repulsive vector interaction channel is opened. Nevertheless, in this letter we show that once gluon effects are considered, the equation of state of quark matter in the color-flavor-locked phase of superconductivity becomes significantly smoother constraining the maximum stellar mass that can be reached to values much smaller than the observed ones. This may indicate that stars made entirely of color superconducting matter are not favored to describe compact stars.

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Introduction. Recent very precise mass measurements for two compact objects, PSR J1614-2230 and PSR J0348+0432 with $M = 1.97 \pm 0.04M_{\odot}$ \[1\] and $M = 2.01 \pm 0.04M_{\odot}$ \[2\], respectively, where $M_{\odot}$ is the mass of the Sun, have provided an important and reliable parameter for constraining the interior composition of the class of objects known as neutron stars. These high mass values imply that the equation of state (EoS) of the corresponding stellar medium should be rather stiff at high densities.

On the other hand, in the highly dense cores of compact objects, the neutron-rich matter can give rise to more degrees of freedom, like hyperons, and perhaps even transitioning to a quark matter phase (see \[3\] for review). Given that cold strange quark matter has been argued to be absolutely stable \[4\], a phase transition could occur in the core of a compact star that would favor quark matter in all its interior giving rise to a strange star. Nevertheless, already from the first indications of $2M_{\odot}$ stars \[3\]-\[7\], there has been claims \[6\], \[7\] that quark matter had to be ruled out as a possible core phase, since it would produce too soft an EoS, incapable to stabilize a $2M_{\odot}$ star against gravitational collapse. However, once the vector interaction channel, always present in a dense medium of quarks \[8\], was taken into account, the EoS became stiffened and quark matter was back as a possible core phase of massive stars (see \[9\], for example). One may wonder if this last result could be challenged by other effects present in the complicated physics of super dense quark matter.

To study quark matter in compact objects one has to rely on effective models like Nambu-Jona-Lasinio (NJL) theories with parameters matched to nuclear data or lattice calculations. The one-gluon exchange interaction of QCD contains an attractive interaction in the diquark channel that can be incorporated as a four-fermion interaction term in NJL models aimed to explore color superconductivity \[10\], \[11\]. At asymptotically high densities these models, as well as weakly coupled QCD predict that the most favored phase is the three-flavor color-flavor-locked (CFL) phase. In these scenarios, gluons degrees of freedom are usually disregarded as negligible at zero temperature. On the other hand, in a pure CFL phase, the breaking of the Lorentz symmetry and the existence of the diquark condensate yield Debye $(m_D)$ and Meissner $(m_M)$ masses for the gluons, both of which depend on the baryonic chemical potential as \[12\].

$$m_D^2 = \frac{21 - 8 \ln 2}{18} m_G^2, \quad m_M^2 = \frac{21 - 8 \ln 2}{54} m_G^2$$ \tag{1}

where $m_G^2 = g^2 \mu^2 N_f / 6\pi^2$, with $N_f$ being the number of flavors of massless quarks, $\mu$ the baryonic chemical potential, and $g$, the effective gauge coupling.

However, gluons might give rise to qualitatively important effects. Notice that while the energy eigenvalue $\epsilon = 0$ is always allowed in the partition function of non-relativistic bosons in a big box with periodic boundary conditions, so at zero temperature these particles can always occupy the zero energy ground state with zero momentum, and consequently zero pressure, things can be different for relativistic bosons. In the relativistic case, if the bosons have nonzero rest energy, they will produce a positive contribution to the system’s energy density and a negative term in the pressure. Since gluons in the color superconductor acquire rest energy due to to their density-dependent Debye and Meissner masses \[13\], these energies will affect the EoS of the CFL matter at $T = 0$. 

[1] E. J. Ferrer and V. de la Incera
[2] L. Paulucci
[3] M.~J.0348+0432 with $m$ and $g$ and Meisner masses (1), these energies will affect the EoS of the CF L matter at the pressure. Since gluons in the color superconductor acquire rest energy due to their density-dependent Debye and Meissner masses (1), these energies will affect the EoS of the CFL matter at $T = 0$.
The main goal of this paper is to investigate the effect of gluons in the mass-radius relation of quark stars with color superconductivity. We will show that in the CFL phase the effect of gluons causes the EoS to become too soft to support the observed $2M_{\odot}$ masses for this class of compact objects.

**Gauged-CFL model with vector interaction.** Consider the Lagrangian density of the gauged-CFL model with vector interaction,

$$
\mathcal{L} = -\bar{\psi}_i^a (\gamma^\mu p_\mu + \mu \gamma^0) \psi^a_i + \frac{G_D}{4} \sum_{\eta} (\bar{\psi}_\eta \gamma^T (\psi^T P_\eta \psi) - G_V (\bar{\psi}_i^a \gamma_0 \psi^a_i)^2 + \mathcal{L}_G. \tag{2}
$$

Here, $\psi^a_i$ are quark fields with flavors $i = u, d, s$ and colors $a = r, g, b$, $G_D$ is the diquark coupling constant, $P_\eta = C\gamma_5 \epsilon^{abc} \epsilon_{ij} \eta$, $G_V$ is the vector interaction coupling constant, and the effective gluon Lagrangian density is

$$
\mathcal{L}_G = -\frac{1}{4} G_{\mu \nu}^A G_A^{\mu \nu} + \frac{1}{2} G_{\mu \nu}^A \Pi_{AB} G^B_{\mu \nu}, \tag{3}
$$

where $G_{\mu \nu}^A$ is the field strength tensors of the gluon fields; and $\Pi_{AB}^{\mu \nu}$ is the polarization operator in the hard-loop approximation of the CFL phase [12]. Since we will be interested in the system behavior at zero temperature, we only need to consider the leading contribution of the polarization operator in the infrared limit ($p_0 = 0, |\vec{p}| \to 0$), which depends on the baryon chemical potential,

$$
\Pi_{\mu \nu}^{AB}(x, y) = [m_D^2 \delta_{\mu 0} \delta_{\nu 0} + m_M^2 \delta_{\mu \nu}] \delta(x - y) \delta^{AB}. \tag{4}
$$

The thermodynamic potential at $T = 0$ obtained from (2) in the mean-field approximation with condensates $\Delta_q = \langle \bar{\psi}^T P_\eta \psi \rangle$ and $\rho = \langle \bar{\psi}_i^a \gamma_0 \psi^a_i \rangle$ is given by

$$
\Omega = \Omega_q + \Omega_G + \frac{3 \Delta^2}{G_D} - G_V \rho^2 - \Omega_{Vac} \tag{5}
$$

with

$$
\Omega_G = \frac{1}{4\pi^2} \int_0^\infty dp p^2 e^{-p^2/\Lambda^2} (8 \sqrt{p^2 + \bar{m}_D^2} + 24 \sqrt{p^2 + \bar{m}_M^2}), \tag{6}
$$

$$
\Omega_q = -\frac{1}{4\pi^2} \int_0^\infty dp p^2 e^{-p^2/\Lambda^2} (16|\epsilon| + 16|\bar{\epsilon}|) - \frac{1}{4\pi^2} \int_0^\infty dp p^2 e^{-p^2/\Lambda^2} (2|\epsilon'| + 2|\bar{\epsilon}'|) \tag{7}
$$

where

$$
\epsilon = \pm \sqrt{(p - \mu)^2 + \Delta_{CFL}^2}, \quad \bar{\epsilon} = \pm \sqrt{(p + \mu)^2 + \Delta_{CFL}^2},
$$

$$
\epsilon' = \pm \sqrt{(p - \mu)^2 + 4\Delta_{CFL}^2}, \quad \bar{\epsilon}' = \pm \sqrt{(p + \mu)^2 + 4\Delta_{CFL}^2} \tag{8}
$$

are the quasiparticle dispersion relations with the chemical potential modified by the vector-field condensate as follows: $\mu = \mu - 2G_V \rho$. The vacuum contributions is given by $\Omega_{Vac} = \Omega(\mu = 0, \Delta = 0)$. In [12] the Debye $(\bar{m}_D)$ and Meissner $(\bar{m}_M)$ masses are formally given by Eq. [11], but with the chemical potential shifted as $\mu \to \tilde{\mu}$. The contribution of gluons to the thermodynamic potential of a gauged-NJL model was also investigated in a recent paper [13], but this work did not consider color superconductivity and besides the study was done in the presence of a magnetic field.

The solution for $\Delta$ and $\rho$ in terms of the chemical potential can be obtained by solving the equations

$$
\frac{\partial \Omega}{\partial \Delta} = 0, \quad \rho = -\frac{\partial \Omega_q}{\partial \mu}. \tag{9}
$$

In doing that, we use the following set of model parameters $G_D = 4.32 \times 10^{-6}$ GeV$^{-2}$, $g^2 = \Lambda^2 G_D$, and $\Lambda = 1000$ MeV. In general, $G_D$ is expected to be of the same order as the quark-antiquark coupling $G_S$. We will explore the regime of intermediate coupling strength with $G_D \approx 0.6 G_S$ and $G_V = 0.5 G_S$.

Two comments are in order here. First, once the vector condensate is taken into account $\bar{\mu}$ becomes an effective chemical potential determining the Fermi-Dirac statistics. Second, the second equation in [9] is not a minimum
equation (on the contrary its solution is a maximum [8]), but it defines, as usual in statistics, the particle number density, which coincides with the vector condensate.

**EoS and M-R relation for the Gauged-CFL model.** The EoS for this high-dense phase is obtained from

\[ P = -\Omega, \quad \epsilon = \Omega + \tilde{\mu}\rho \]  \hspace{1cm} (10)

where, as usual, \( P \) is the pressure and \( \epsilon \) the energy density.

Before we investigate the EoS in this medium, it is important to consider the parameters’ range that satisfies the constraint for absolute stability of gauged-CFL matter. This is done by comparing the matter energy density per baryonic number at zero pressure of this medium with that of the iron nucleus (roughly 930 MeV). Depending on whether this energy for quark matter is higher or smaller than this value, a compact star could in principle be converted as a whole to a deconfined quark phase. The energy per baryon number as a function of \( G_V \) at \( P = 0 \) is represented in Fig. 1. There, we can see that for low values of \( G_V \), gauged-CFL matter cannot be absolutely stable. Thus, \( G_V/G_D > 0.27 \) is needed to guarantee absolute stability. This new fact is due to the gluon contribution to the energy density.

On the other hand, it is also important to consider, as an additional constraint, the condition that the sound speed in this medium (\( v_c^2 = dP/d\epsilon \)) should be lower than the speed of light. In Fig. 2 the sound speed is plotted as a function of the baryonic densities for gauged-CFL matter. We can see that the effect of the vector interaction is to increase the sound speed, which can reach values larger than \( c \) at sufficiently high \( G_V \) and density. Taking into account the results plotted in Fig. 2 we should have \( G_V/G_D \leq 0.75 \) for gauged-CFL matter in the energy range under consideration.

In Fig 3 we represent \( P vs  \epsilon \) for \( G_V = (0.3 - 0.75)G_D \), which satisfy the constraints imposed by the results of Figs. 1 and 2. From Fig 3 we can note that the inclusion of gluon degrees of freedom substantially reduces the pressure for a given energy, making the EoS much softer.

Finally, the mass-radius sequence for strange stars is shown in Fig. 4. We observe that when including the gluon contribution, the maximum mass reaches a much too low value, clearly showing that stars composed entirely of CFL matter with gluons within the formalism presented here should be disregarded, as they cannot explain the mass measurements of several compact objects and in particular those of PSR J1614-2230 and PSR J0348+0432. The maximum mass would be reduced even further [14] if quark masses or a bag constant (\( \Omega_{vac} \rightarrow \Omega_{vac} + B \)), for example, were included.

When considering hybrid stars, since gluons cause a substantial decrease in the pressure for a given energy density, they shift the chemical potential for the transition point (when \( P_{CFL} = P_{nuclear} \)) from nuclear matter to the gauged-CFL to very high values (\( \gtrsim 1700 \text{ MeV} \)). For \( G_V/G_D < 0.27 \) (within the conditions for meta-stability given in Fig. 1) the conditions for deconfinement are most likely never achieved in the interior of neutron stars.

**Conclusions.** We have found that gluons significantly soften the EoS of CFL matter. Considering the range of parameters in agreement with the absolute stability of the CFL phase, the maximum stellar mass predicted by this matter is much lower than the reported \( \sim 2M_\odot \). This will exclude pure color superconducting strange stars as possible candidates of compact stars. Unless some new interactions, as the diquark-diquark repulsion studied in Ref. [15], could bring enough outward pressure, the favored composition of neutron stars seems not to be pure superconducting quark matter. Finally, when considering hybrid stars, gluons will push the density for the transition from nuclear to CFL matter to such high values as to prevent the onset of quarks in the interior of neutron stars.
FIG. 2: (Color online) Sound speed as a function of the density for gauged-CFL matter considering different values of $G_V/G_D$. The starting points for each curve correspond to their respective zero pressure value. The horizontal dashed line shows the limiting value of $v_S = c$.

FIG. 3: (Color online) Equations of state for CFL and gauged-CFL matter for different values of $G_V/G_D$.

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FIG. 4: (Color online) Mass-radius relationship of strange stars obtained from the gauged-CFL matter EoS for $G_V/G_D = 0.3$ (dashed line) and 0.5 (full line).
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