Modified engineering method for ultimate load prediction of composite hat-stiffened panels

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Abstract. In this paper, the ultimate load of a typical hat type composite stiffened panel under axial compression is simulated by finite element method. Based on the ultimate load value and failure mode obtained by numerical simulation, the thickness of the lower flange of the stiffener in the engineering method is modified by using the stiffness equivalent method. The modified engineering method is in good agreement with the finite element results, which has a certain reference significance for the rapid calculation and analysis of stiffened plate in the early design stage.

1. Introduction

In recent years, composite stiffened structures have been widely used in modern aircraft design due to their low specific gravity, high specific strength and specific modulus. One of the main failure modes of stiffened panels is buckling instability. Generally, composite stiffened panels used in aircraft still have strong post buckling capacity after local buckling of skin occurs.

At present, a series of exploration and research on the stability of composite stiffened panels have been carried out, mainly focusing on the comparative study of finite element numerical analysis method and engineering theoretical calculation combined with axial compressive test. Commonly used methods for estimating the ultimate strength of composite stiffened panels include the segmented method [1] and the effective width method [2-3]. Bisagni [4] equated the stiffeners to torsion bars, and studied the linearized local skin buckling load and nonlinear post buckling behavior of composite stiffened panels based on Ritz method. Mo et al. [5] proposed an engineering calculation method for predicting the buckling load of stiffened curved panels.

In the past, the local buckling of stiffeners was considered as the failure criterion of stiffened panels, while the supporting effect of skin on the lower flange of stiffener was not considered, which led to the conservative results of engineering method. Based on the finite element analysis of the ultimate load of a typical composite hat-stiffened panel, this paper summarizes the calculation formula of the effective support range of the skin to the stiffeners, and modifies the thickness of the lower flange. The results show that the modified engineering method is in good agreement with the finite element results. The application of the modified engineering method to calculate the ultimate load of stiffened panel has certain reference significance in the early stage of structural design.
2. Theory and formula

2.1. Local buckling analysis

The local buckling of stiffened panels consists of two different modes: the local buckling of skin between stiffeners and the local buckling of the strips forming the stiffeners. Different formulas can be derived according to different boundary conditions [6]. The buckling load of web, hat top and skin can be calculated as the axial buckling load of rectangular laminated panel simply supported on four edges, which can be shown as equation (1).

\[ N_s = \frac{\pi^2 D_{22}}{b^2} \left[ \frac{D_{11}}{D_{22}} \left( \frac{b}{a} \right)^2 m^2 + 2 \left( \frac{D_{12} + 2D_{66}}{D_{22}} \right) + \left( \frac{a}{b} \right)^2 \frac{1}{m^2} \right] \]  

In the above equation, \( a \) and \( b \) are the length and width of the laminate respectively, and \( D_{11}, D_{22} \) and \( D_{66} \) are the equivalent bending stiffness. \( N_s \) is the axial compression buckling load per unit length, and \( m \) is the number of buckling half waves along the the length of the panel. When the value of \( N_s \) is the minimum, then it can be defined as the buckling load of the panel. When calculating the buckling load of skin, the effective width of skin should be selected as \( b_{\text{eff}} = b_s + b_f \), where \( b_s \) is the spacing of stiffeners and \( b_f \) is the width of lower flange shown in Figure 1.

\[ b_{\text{eff}} = b_s + b_f \]

2.2. Ultimate load calculation

It is assumed that the local buckling of skin between stiffeners occurs firstly, and the overall failure mode of stiffened panel is the local buckling of stiffeners. In order to calculate the local buckling load of stiffeners, the stiffened panel are divided into individual plate elements, then the buckling load of plate elements is calculated by using the buckling equations of laminated panels. The local buckling loads of skin, hat strip and two web strips of the hat-stiffener all can be calculated by equation (1), while for the lower flange, the local buckling load can be calculated by equation (2). Figure 2 is the flow chart of ultimate strength of stiffened panels, and the parabola in Figure 2 is shown in equation (3), where \( \sigma_{co} \) is the average failure stress of stiffened panel, \( \sigma_{cc} \) is minimum buckling stress of
plate element, $\sigma_{scr}$ is the local buckling stress of skin between stiffeners, and $\sigma_r$ is the global buckling stress calculated by Euler formula[6].

$$\bar{\sigma}_{co} = 1 - \left( \frac{\sigma_{scr}}{\sigma_c} \right) \frac{\sigma_{scr}}{\sigma_r}$$

Equation (4) is equation for calculating the effective width.

$$b_{ef} = \frac{1}{2} b_s \left( \sigma_{scr} / \sigma_c \right)^{3/2}$$

2.3. Modification of lower flange equivalent thickness

The original engineering method does not consider the strengthening effect of skin on the lower flange, which leads to conservative results when the stiffeners are weak. In present study, the thickness of the lower flange is modified according to the stiffness equivalent correction method given in equation (5), where $ET_{sx}$ is the in-plane stiffness coefficient of skin, $ET_{fx}$ is the in-plane stiffness coefficient of lower flange, and the calculation of them is shown in reference [8]. The parameter $b_{ess}$ is the effective support width of the skin to the stiffeners, which is related to the in-plane stiffness ratio, bending stiffness ratio and the remaining bonding area of the skin.

$$t_{adj} = t_f + t_s \left( ET_{sx} b_{ess} / ET_{fx} b_f \right)$$

**Figure 3** (a) shows the distribution of the adhesive stiffness degradation zone and the out of plane displacement of a typical composite hat-stiffened panel under axial compression. **Figure 3** (b) shows the zone where the CSDMG=1, which means the skin and the stiffeners is debonded. The number of stiffness degradation regions is exactly equal to the number of buckling waves if the two half stiffness degradation regions near the loading end are regarded as a whole one.

Take a rectangular area selected in **Figure 3**, (a), and the stiffness degradation area is equivalent to an ellipse size, as shown in **Figure 4**. If the red area represents the damaged area, the remaining bonding area is shown in equation (6), where $a_p$ is the effective length of the stiffened plate, $b_f$ is the width of the stiffener, and $x'$ represents the damage propagation position at the failure moment, which is related to the bending stiffness ratio. The ellipse passes through a fixed point $(b_s/2, a_p/4m)$.

$$S = a_p b_f - \int_{b_{ss}/2}^{x'} b \left( 1 - \left( \frac{x}{a} \right) ^2 \right) ^{1/2} \, dx$$

During debonding process, the damage will propagate along the direction from the lower flange’s edge near the hat's outer skin to the inner skin one, and the limit size of propagation zone is approximately $b_f / 2$. When the bending stiffness ratio is less than a certain value, there is no debonding,
and \( x' = \frac{b_{ss}}{2} \). When the bending stiffness ratio is greater than a certain value, \( x' \) is constant as \( \frac{b_f}{2} \). Equation (7) is the effective support width of the skin to the lower flange, where \( \frac{1}{f} \) is a reduction factor, and \( f \) is related to in-plane stiffness ratio, bending stiffness ratio and stiffeners distance.

\[
b_{ss} = \frac{S}{f a_f}
\]  

(7)

**3. Example analysis and result discussion**

**3.1. Model description**

The structural dimensions of a typical stiffened panel model are shown in Figure 5, and the stiffeners are equidistant. The material properties \( E_X = 154 \text{GPa}, E_Y = 8.5 \text{GPa}, E_{XY} = 4.2 \text{GPa}, \mu_{xy} = 0.35 \), the stacking sequence of the skin is \([45,0_2,-45,90]_s\), the stacking sequence of the stiffeners is \([45,-45,90,45]_s\). In the finite element model, cohesive surfaces are used to simulate the debonding between skin and flange. The cohesive surfaces parameters are shown in Table 1, and the boundary conditions is that one end is fixed and the other end is loaded by displacement.

Eight models were obtained by changing the thickness of the model stiffeners from 1.21mm to 3mm. The specific thickness of the stiffeners is shown in Table 2.

### Table 1. Cohesive parameters.

| \( E_n \) (MPa) | \( E_s \) (MPa) | \( E_t \) (MPa) | \( t^0_n \) (MPa) | \( t^0_s \) (MPa) | \( t^0_t \) (MPa) | \( G^C_n \) (N/mm) | \( G^C_s \) (N/mm) | \( G^C_t \) (N/mm) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-------------------|-------------------|-------------------|
| 10000           | 1500            | 1500            | 10              | 15              | 15              | 3.34              | 1.165             | 1.165             |

**Figure 4. Equivalent damage area.**

**Figure 5. Equivalent damage area.**
3.2. Numerical simulation results
Finite element analysis was carried out for eight models to analyze the degradation law of adhesive stiffness. Figure 6 is the distribution of stiffness degradation in the cohesive surfaces of different models at the failure load. When the thickness is less than 1.44mm, CSDMG is less than 1, and the debonding does not occurred. When the thickness is between 1.44mm and 2.61mm, the stiffness degradation area expands with the increase of thickness. When the thickness is greater than 2.61mm, the area of stiffness degradation zone has no obvious change. The ultimate loads of the eight models are shown in Table 2.

![Figure 6. Distribution of stiffness degradation in cohesive surfaces of different stiffeners’ thickness models.](image)

3.3. Modified engineering method results
Supposed the in-plane stiffness ratio of the model to be A and the bending stiffness ratio be D. When the thickness of the stiffeners is 1.44mm, the corresponding in-plane stiffness ratio be $A_1$, the bending stiffness ratio is $D_1$, and $x' = b_{ss}$. When the thickness of the stiffeners is 2.61 mm, the corresponding in-plane stiffness ratio is $A_2$, and the bending stiffness ratio is $D_2$, while $x' = b_{ss} + b_f/2$. Within this range, $x'$ is linearly interpolated according to the bending stiffness ratio, and beyond this range, the fixed value $b_{ss} + b_f/2$ is taken. The calculation method of $f$ has shown in equation (8).

$$ f = \frac{b_{ss}}{98.36} \left[ 1.7 \frac{A-A_2}{A_2-A_1} + \sqrt{\frac{D^2-D_2^2}{D_2^2-D_1^2}} \right] $$

The modified results are shown in Table 2.

![Table 2. Comparison of the results of the three method.](image)
Table 2 shows the comparison of the results of FEM, engineering method and the modified engineering method. The ratio of the engineering method to FEM ranges from 0.42 to 1.14. When the thickness of the stiffeners is 1.21mm, the ratio of the engineering method to the finite element results is only 0.42, indicating that the results of the engineering method are more conservative when the stiffeners are weak. The comparison between the modified method and the finite element results is in the range of 0.83~1.05, which is obviously improved compared with the results before the modification.

4. Conclusion
1) The results obtained by engineering method without modification are different from the finite element results, and the weaker the stiffeners are, the larger the difference is, the engineering method results are 0.42-1.14 times of the finite element results.

2) The stiffness degradation area is simplified as an ellipse size, and the formula for calculating the effective support width of the skin to the reinforcement is summarized.

3) The equivalent stiffness method is used to modify the thickness of the lower flange, and the modified engineering method results are 0.83~1.05 times of that of FEM, which is significantly improved compared with the results before the modification.

Acknowledgments
The authors thank the National Key Research and Development Program (No. 2019YFA0706803), the National Natural Science Foundation of China (No. 11972106) and the Fundamental Research Funds for the Central Universities of China (DUT2019TD37)

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