Diffractive production of jets at high-energy in the QCD shock-wave approach

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Abstract. In view of future phenomenological applications, we study the impact factor for the photon to quark, antiquark and gluon transition within Balitsky’s shock-wave formalism. The aim of the present program is to extend existing results beyond approximations discussed in the literature. We present our results of the real contribution, and report on recent progress in calculating the virtual contributions for the photon to quark, antiquark transition.

1. Introduction

One of the major achievements of HERA was the experimental evidence \cite{1,2} shown in Fig. 1 that among the whole set of $\gamma^* p \rightarrow X$ deep inelastic scattering events, almost 10\% are diffractive (DDIS), of the form $\gamma^* p \rightarrow X Y$ with a rapidity gap between the proton remnants $Y$ and the hadrons $X$ coming from the fragmentation region of the initial virtual photon.

Diffraction can be theoretically described according to several approaches. The first one involves a resolved Pomeron contribution (with a parton distribution function inside the Pomeron), see Fig. 2 (left), while the second one relies on a direct Pomeron contribution involving the coupling of a Pomeron with the diffractive state, see Fig. 2 (right).

The diffractive states can be modelled in perturbation theory by a $q\bar{q}$ pair (for moderate $M^2$, where $M$ is the invariant mass of the diffractively produced state $X$) or by higher Fock states as a $q\bar{q}g$ state for larger values of $M^2$. Based on such a model, with a two-gluon exchange picture for the Pomeron, a good description of HERA data for diffraction \cite{3,4} could be achieved \cite{5}. One of the important features of this approach is that the $q\bar{q}$ component with a longitudinally polarized photon plays a crucial role in the region of small diffractive mass $M$, although it is a twist-4 contribution. In the direct components considered there, the $q\bar{q}g$ diffractive state has been studied in two particular limits. The first one, valid for very large $Q^2$, corresponds to a collinear approximation in which the transverse momentum of the gluon is assumed to be much smaller than the transverse momentum of the emitter \cite{6}. The second one \cite{7,8}, valid for very...
Figure 1. Excess of events exhibiting a rapidity gap with the proton remnants with respect to Monte Carlo DIS predictions, providing an evidence for diffractive events, at ZEUS (1993) (left panel) [1] and H1 (1994) (right panel) [2].

Figure 2. Resolved (left panel) and direct Pomeron (right panel) contributions to two jets production.

large $M^2$, is based on the assumption of a strong ordering of longitudinal momenta, encountered in BFKL equation [9]. Both these approaches were combined in order to describe HERA data for DDIS [10].

Based on these very successful developments led at HERA in order to understand the QCD dynamics with diffractive events, it would be appropriate to look for similar hard diffractive events at LHC. The idea there is to adapt the concept of photoproduction of diffractive jets, which was performed at HERA [11, 12], now with a flux of quasi-real photons in ultraperipheral collisions (UPC) [13], relying on the notion of equivalent photon approximation. In both cases, the hard scale is provided by the invariant mass of the tagged jets.

We here report on our computation [14] of the $\gamma^* \rightarrow q\bar{q}g$ impact factor at tree level with an arbitrary number of $t$-channel gluons described within the Wilson line formalism, also called QCD shockwave approach [15]. As an aside, we rederive the $\gamma^* \rightarrow q\bar{q}$ impact factor. In particular, the $\gamma^* \rightarrow q\bar{q}g$ transition is computed without any soft or collinear approximation for the emitted gluon, in contrast with the above mentioned calculations. These results provide
necessary generalization of building blocks for inclusive DDIS as well as for two- and three-jet diffractive production. Since the results we derived can account for an arbitrary number of $t$-channel gluons, this could allow to include higher twist effects which are suspected to be rather important in DDIS for $Q^2 \lesssim 5$ GeV$^2$ [16].

2. An introduction to the shockwave formalism

As stated before, we use Balitsky’s shockwave formalism. Its application shows that this method is very powerful in determining evolution equations and impact factors at next-to-leading order for inclusive processes [17], at semi-inclusive level for $p_t$-broadening in $pA$ collisions [18] or in the evaluation of the triple Pomeron vertex beyond the planar limit [19], when compared with usual methods based on summation of contributions of individual Feynman diagrams computed in momentum space. It is an effective way of estimating the effect of multigluon exchange. Its formulation in coordinate space makes it natural in view of describing saturation [20].

We use the following notation. We introduce the light cone vectors $n_1$ and $n_2$

$$n_1 = (1, 0, 0, 1), \quad n_2 = \frac{1}{2} (1, 0, 0, -1), \quad n_1^+ = n_2^- = n_1 \cdot n_2 = 1$$

Then, one introduces Wilson lines as

$$U_i = U_{\bar{z}_i} = U (\bar{z}_i, \eta) = P \exp \left[ ig \int_{-\infty}^{+\infty} b_{\eta}^- (z_i^+, \bar{z}_i) \, dz_i^+ \right].$$

The operator $b_{\eta}^-$ is the external shock-wave field built from slow gluons whose momenta are limited by the longitudinal cut-off defined by the rapidity $\eta$

$$b_{\eta}^- = \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot \bar{z}} b^-(p) \theta \left( e^{\eta} - \frac{|p^+|}{P^+} \right),$$

where $P^+$ is the typical large $+$ momentum of the problem, to be identified with $p_\gamma^+$ later on. We will denote the longitudinal cut-off $\sigma = e^{\eta} P^+ = \alpha P^+$. We use the light cone gauge $A \cdot n_2 = 0$, with $A$ being the sum of the external field $b$ and the quantum field $A$

$$A^\mu = A^\mu + b^\mu, \quad b^\mu (z) = b^-(z^+, \bar{z}) n_2^\mu = \delta (z^+) B (\bar{z}) n_2^\mu,$$

where $B(\bar{z})$ is a profile function.

Indeed, let us consider an external gluon field $b^\mu$ in its rest frame and boost it along the $+$ direction. One obtains:

$$b^+ (x^+, x^-, \bar{x}) \rightarrow \frac{1}{\lambda} b^+ \left( \lambda x^+, \frac{1}{\lambda} x^-, \bar{x} \right),$$

$$b^- (x^+, x^-, \bar{x}) \rightarrow \lambda b^- \left( \lambda x^+, \frac{1}{\lambda} x^-, \bar{x} \right),$$

$$b^i (x^+, x^-, \bar{x}) \rightarrow b^i \left( \lambda x^+, \frac{1}{\lambda} x^-, \bar{x} \right).$$

Assuming that the field vanishes at infinity, one immediately gets that only its minus component survives the boost in the limit $\lambda \rightarrow \infty$, and that it does not depend on $x^-$ and contains $\delta (x^+)$, thus justifying the form of $b^\mu$ in Eq. (4).

The natural and extensively used operator appearing in studies of diffractive processes within the shock wave approach is the dipole operator $U_{12} = \frac{1}{N_c} \text{tr} \left( U_1 U_2^\dagger \right) - 1$ constructed from the Wilson line [2].
Figure 3. Diagram contributing to the impact factor for two jet production

3. Impact factor for $\gamma \rightarrow q\bar{q}$ transition
In the leading order the diagram contributing to the impact factor for $\gamma \rightarrow q\bar{q}$ transition is shown in Fig. 3 in which $z$'s denote the coordinates of interaction points with the photon and the shock wave. After projection on the color singlet state and subtraction of the contribution without interaction with the shock wave, the contribution of this diagram can be written in the momentum space as (factorizing out a global QED factor $-ieq$)

$$M^\alpha = N_c \int d\vec{z}_1 d\vec{z}_2 F(p,q,z_0,\vec{z}_1,\vec{z}_2)^\alpha U_{12}.$$  (5)

Denoting $Z_{12} = \sqrt{x_q x_{\bar{q}}}$, we get for a longitudinally polarized photon

$$F(p,q,k,\vec{z}_1,\vec{z}_2)^\alpha = \theta(p_+^q) \theta(p_+^{\bar{q}}) \frac{\delta(k^+ - p^+_{\bar{q}} - p^+_q)}{(2\pi)^2} e^{-iq_{\bar{q}}z_1 - im_{\bar{q}}z_2}(-2i)\delta_{\lambda_q,-\lambda_{\bar{q}}} x_q x_{\bar{q}} Q K_0 (Q Z_{12}),$$  (6)

whereas for a transversally polarized photon

$$F(p,q,k,\vec{z}_1,\vec{z}_2)^j = \theta(p_+^q) \theta(p_+^{\bar{q}}) \frac{\delta(k^+ - p^+_{\bar{q}} - p^+_q)}{(2\pi)^2} e^{-iq_{\bar{q}}z_1 - im_{\bar{q}}z_2} \xi_{\bar{T}} \delta_{\lambda_q,-\lambda_{\bar{q}}} (x_q - x_{\bar{q}} + s\lambda_q) \frac{z_{12} \cdot \xi_{\bar{T}}}{Z_{12}} Q Z_{12} K_1 (Q Z_{12}).$$  (7)

4. Impact factor for $\gamma \rightarrow q\bar{q}g$ transition
In the case of the $q\bar{q}g$ Fock final state the contributing diagrams are shown in Fig. 4. After projection on the color singlet state and subtraction of the contribution without interaction with the shock wave, the result can be put in the form

$$M^\alpha = N_c^2 \int d\vec{z}_1 d\vec{z}_2 d\vec{z}_3 F_1 (p,q,p,q,p,g,z_0,\vec{z}_1,\vec{z}_2,\vec{z}_3)^\alpha \frac{1}{2} (U_{32} + U_{13} - U_{12} - U_{32} U_{13})$$

$$+ N_c \int d\vec{z}_1 d\vec{z}_2 F_2 (p,q,p,q,p,g,z_0,\vec{z}_1,\vec{z}_2)^\alpha N_c^2 - \frac{1}{2N_c} U_{12}.$$  (8)

The first and the second line of this equation correspond to contributions to the impact factor, respectively, of the diagrams 1 and 2 of Fig. 4 and of the diagrams 3 and 4 of it. For a
longitudinally polarized photon, the functions $F_i$ read

$$F_1 (p_q, p_g, p_g, k, \bar{z}_1, \bar{z}_2, \bar{z}_3)^\alpha \varepsilon_{L\alpha} = 2 Q_g \frac{\delta(k^+ - p^+_g - p^+_q - p^+_s) \theta(p^+_q - \sigma)}{\pi \sqrt{2p^+_q}} e^{-i\bar{p}_q \cdot \bar{z}_1 - i\bar{p}_q \cdot \bar{z}_2 - i\bar{p}_q \cdot \bar{z}_3}$$

$$\times \delta_{\lambda_q, -\lambda_q} \left\{ \left( x_q + x_g \delta_{-s_g \lambda_q} \right) x_{\bar{g}} x_{\bar{q}} x_{\bar{q}} - \bar{x}_q \frac{\bar{z}_{q12} \cdot \bar{z}_{q}}{\bar{z}_{q12}^2} - \left( x_q + x_g \delta_{-s_g \lambda_q} \right) x_{\bar{q}} \right\} K_0(Q Z_{123}), \quad (q \leftrightarrow \bar{q}), \quad (q \leftrightarrow \bar{q}), \quad (q \leftrightarrow \bar{q}) \tag{9}$$

while for a transversally polarized photon, we have

$$F_1 (p_q, p_q, p_g, k, \bar{z}_1, \bar{z}_2, \bar{z}_3)^\alpha \varepsilon_{T\alpha} = -2i Q g \delta(k^+ - p^+_g - p^+_q - p^+_s) \theta(p^+_q - \sigma) \frac{e^{-i\bar{p}_q \cdot \bar{z}_1 - i\bar{p}_q \cdot \bar{z}_2 - i\bar{p}_q \cdot \bar{z}_3}}{\pi Z_{123} \sqrt{2p^+_q}} e^{-i\bar{p}_q \cdot \bar{z}_1 - i\bar{p}_q \cdot \bar{z}_2 - i\bar{p}_q \cdot \bar{z}_3}$$

$$\times \delta_{\lambda_q, -\lambda_q} \left\{ \left( x_q + x_g \delta_{-s_g \lambda_q} \right) x_{\bar{q}} x_{\bar{q}} x_{\bar{q}} - \bar{x}_q \frac{\bar{z}_{q12} \cdot \bar{z}_{q}}{\bar{z}_{q12}^2} - \left( x_q + x_g \delta_{-s_g \lambda_q} \right) x_{\bar{q}} \right\} - (q \leftrightarrow \bar{q}), \quad (q \leftrightarrow \bar{q}), \quad (q \leftrightarrow \bar{q}) \tag{10}$$

We denote $F_2 (p_q, p_q, p_g, z_0, \bar{z}_1, \bar{z}_2)^\alpha = \bar{F}_2 (p_q, p_q, p_g, z_0, \bar{z}_1, \bar{z}_2)^\alpha + \int d\bar{z}_3 F_1 (p_q, p_q, p_g, z_0, \bar{z}_1, \bar{z}_2, \bar{z}_3)^\alpha$. 

Figure 4. Diagrams contributing to the impact factor for three jet production.
5. **2- and 3-gluon approximation**

Let us notice that the dipole operator $U_{ij}$ involves terms at least of order $g^2$. Hence for only two or three exchanged gluons one can neglect the quadrupole term in the amplitude $M^\alpha$ which results in the simpler expression

$$M^\alpha \equiv \frac{1}{2} \int d\vec{z}_1 d\vec{z}_2 U_{12} \left[ (N_c^2 - 1) \tilde{F}_2 (\vec{z}_1, \vec{z}_2)^\alpha + \int d\vec{z}_3 \{ N_c^2 F_1 (\vec{z}_1, \vec{z}_3, \vec{z}_2)^\alpha + N_c^2 F_1 (\vec{z}_3, \vec{z}_2, \vec{z}_1)^\alpha - F_1 (\vec{z}_1, \vec{z}_2, \vec{z}_3)^\alpha \} \right].$$  \hspace{1cm} (12)

For $\vec{p}_q = \vec{p}_{\bar{q}} = \vec{p}_q = \vec{0}$, those integrals can be performed analytically. Otherwise they can be expressed as a simple convergent integral over $[0, 1]$ that can be performed numerically for any future phenomenological study.

6. **Towards the next-to-leading-order corrections**

The calculation of virtual corrections to the $\gamma^* \rightarrow q\bar{q}$ involves two kinds of contributions. The diagrams contributing to virtual corrections in which the radiated gluon doesn’t cross the shock wave are shown in Fig. 5, whereas the diagrams in which the radiated gluon interacts with the shock wave are illustrated in the Fig. 6.

This calculation is much more complicated than the computation of the real contribution to the $\gamma^* \rightarrow q\bar{q}g$ impact factor discussed above. Although it only involves one-loop diagrams, the complications arise due to the presence of many different scales. Indeed, our aim is to obtain results in the general kinematics where the virtuality of incoming photon, the $t$–channel momentum transfer and the invariant mass $M^2$ of the diffractive two-jet state are arbitrary. Additionally, this impact factor is a function of the virtuality of $t$–channel exchanged gluons.
This work is in progress. We here present the matrix element corresponding to the first three diagrams of Fig. 5. We work in dimensional regularization for the transverse momentum space, i.e. $d = D - 2 = 2 + 2\epsilon$, and introduce the regularization scale $\mu$, and the related dimensionless scale $\tilde{\mu}^2 = \mu^2/Q^2$. Denoting $p_{ij} \equiv p_i - p_j$, we introduce $p_\perp = p_{q1\perp}$, $\vec{p}^2 = -p_\perp^2$ and $w = \vec{p}^2/Q^2$. For simplicity, we write $x = x_q$. We get for the case of a longitudinally polarized photon

$$T_{f_i|_{\epsilon=\alpha=\gamma}} = -ig^2 N_c^2 - 1 \frac{1}{2N_c} \text{tr}(U(p_{1\perp})U^\dagger(-p_{2\perp})) \delta(p_{q1\perp} + p_{q2\perp}) \delta(p_q^+ - p_q^- + p_q^+ \theta(p_q^+) \theta(p_q^-)) \times \Gamma(1 - \epsilon) \frac{1}{(16\pi^3)^{1+\epsilon}} \frac{x(1-x)p_{\perp}^+ \gamma^+ v_{p_{\perp}^+}}{\sqrt{2p_{\perp}^+} \sqrt{2p_{q1}^+} x(1-x)Q^2 + \vec{p}^2} \times \left(2 \ln \left(\frac{1-x}{x^2}\right) - 3\right) \left(\ln \left(\frac{((1-x)x)^2}{1-x}x\tilde{\mu}^2\right) + \frac{1}{\epsilon}\right) + \ln^2 \left(\frac{x}{1-x}\right) - \frac{\pi^2}{3} + 6 \right).$$

Expanding the photon momentum in the Sudakov basis (11) as

$$p_\gamma = p_\perp^+ n_1 - \frac{Q^2}{2p_\perp^+} n_2$$

one can explicitly check the electromagnetic gauge invariance for this group of diagrams since

$$T_{f_i|_{\epsilon=\alpha=\gamma}} = \frac{Q^2}{2p_\perp^+} T_{f_i|_{\epsilon=\alpha=\gamma}}.$$  

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**Figure 6.** Diagrams contributing to virtual corrections in which the radiated gluon interacts with the shock wave.
Similarly, for the case of a transversally polarized photon, one gets

\[
T_{fi|\text{transverse}} = -i g^2 \frac{N_c^2 - 1}{2 N_c} \epsilon_i \left( U(p_{1\perp}) U^\dagger(-p_{2\perp}) \delta(p_{q1\perp} - p_{i\perp} + p_{q2\perp} - p_j^\perp) \delta(p_q^+ - p_i^+ + p_j^+) \theta(p_q^+ \theta(p_j^+) \right)
\times \frac{\Gamma(1 - \epsilon)}{(16\pi^3)^{1+\epsilon}} \epsilon_i \left( \frac{1}{2 \epsilon_p q_i^+ [\gamma_i^+ p_i^+ + (x - 1)p_i^+ p_i^- + p_i^+]} \right) \times \frac{2}{x(1 - x)(Q^2 + p_i^2)}
\times \left[ 2 \ln \left( \frac{(1 - x)x}{\alpha^2} \right) - 3 \right] \left( \ln \left( \frac{w - x^2 + x}{\mu^2} \right) + \frac{(1 - x)x \ln \left( \frac{1 - x}{w - x^2 + x} \right)}{w} + \frac{1}{\epsilon} \right)
\times \ln^2 \left( \frac{x}{1 - x} - \frac{\pi^2}{3} + 6 \right).
\tag{16}
\]

7. Conclusion
The measurement of dijet production in DDIS was recently performed \cite{21}, and a precise comparison of dijet versus triple-jet production, which has not been performed yet at HERA \cite{22}, would be very useful to get a deeper understanding of the QCD mechanism underlying diffraction. Recent investigations of the azimuthal distribution of dijets in diffractive photoproduction performed by ZEUS \cite{23} show sign of a possible need for a 2-gluon exchange underlying diffraction. Recent investigations of the azimuthal distribution of dijets in diffractive HERA \cite{22}, would be very useful to get a deeper understanding of the QCD mechanism comparison of dijet versus triple-jet production, which has not been performed yet at HERA.

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Diffractive open charm production was measured at HERA \cite{25} and studied in the large \( M \) limit based on the direct coupling between a Pomeron and a \( q\bar{q} \) or a \( q\bar{q}g \) state, with massive quarks \cite{8}. Such a program could also be performed at LHC, again based on UPCs and on the extension of the above mentioned impact factors to the case of a massive quark. It is the subject of presently ongoing research. Beyond jets, this could be further extended to \( J/\Psi \) mesons, which are copiously produced at LHC.

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