Radiative corrections to theta term in the left-right supersymmetric models

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Abstract

We calculate the radiative correction to the theta term in the generic left-right supersymmetric model due to the Kobayashi-Maskawa source of CP-violation. We found that the value of $\theta$ is very sensitive to the relations between vacuum expectation values of bidoublet scalars $\langle \Phi_1 \rangle = \text{diag}(\kappa_1, \kappa'_1)$ and $\langle \Phi_2 \rangle = \text{diag}(\kappa'_2, \kappa_2)$. The minimal value of $\theta$ in the model is found to be of order $10^{-9}$ for $\kappa_1/\kappa_2 \sim 1$, $\kappa'_1 = \kappa'_2 = 0$ in agreement with the experimental constraint without an axion mechanism or fine tuning. In other regions of the parameter space, the radiatively induced $\theta$ gives unacceptably large contributions to the electric dipole moment of the neutron.

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1 Introduction

The strong CP problem is one of the most intriguing issues of modern particle physics. The additional term in the QCD Lagrangian

\[ \mathcal{L} = \theta \frac{g_3^2}{16\pi^2} G^a_{\mu \nu} \tilde{G}^a_{\mu \nu} \]  

violates P and CP symmetries \cite{1}. In the electroweak theory, the diagonalization of the quark mass matrices \( M_u \) and \( M_d \) involves chiral rotations and brings the additional contribution to the theta term:

\[ \tilde{\theta} = \theta + \arg(\det M_u M_d) \]  

The current experimental limits on the electric dipole moment (EDM) of the neutron put a severe constraint on the \( \tilde{\theta} \) parameter. The chiral algebra calculation of the neutron EDM induced by the theta term \cite{2} gives the following prediction:

\[ d_n \simeq 3.6 \times 10^{-16} \tilde{\theta} \, e \cdot cm. \]  

Together with the current neutron EDM constraints it implies the limit \( \tilde{\theta} < 10^{-10} \). Bearing in mind other alternative ways to calculate EDM and the big diversity of the results (See, for example the review \cite{3}) we shall assume here the following milder limit for \( \tilde{\theta} \):

\[ \tilde{\theta} < 10^{-9}. \]  

The extreme smallness of \( \tilde{\theta} \) could be explained theoretically in different manners. The most popular solution for strong CP problem is to allow the dynamical relaxation of \( \tilde{\theta} \) through the axion mechanism \cite{4}. Since no axion, visible or invisible, is found so far, one has to consider other alternative ways to obtain naturally small \( \tilde{\theta} \) \cite{5, 6}.

2 Radiative corrections to \( \tilde{\theta} \)

In recent works Kuchimanchi \cite{7} and Mohapatra and Rasin \cite{8, 9} proposed a solution for the strong CP problem in the framework of the supersymmetric models conserving parity. The theta parameter in the Lagrangian is simply set to zero above some scale \( M_{WR} \) where parity and CP are the exact symmetries of the theory. After spontaneous symmetry breaking, at the scale where \( W_R \) becomes massive, the \( \tilde{\theta} \) parameter picks up no contribution from \( \arg(\det M_u M_d) \). This is because the minimum of the superpotential corresponds to the real vacuum expectations values of scalar fields which leads to hermitean mass matrices \cite{1, 7, 8, 9}. It does not mean, however, that the strong CP problem is solved; the theta term can be generated through radiative corrections if there is a CP-violating source in the theory.

It is clear that to ensure these radiative corrections \( \tilde{\theta}_{rad} \) to satisfy the limit (4) and thus to solve the CP-problem completely, one has to eliminate all extra sources of CP-violation beyond the Kobayashi-Maskawa (KM) phase. The latter provides a minimal content of
CP-violation. If the contribution to $\bar{\theta}$ from KM phase happens to be large, this means that one cannot obtain the viable solution to the strong CP problem without fine tuning. This question was studied in the framework of pure SM [10, 11], where radiative corrections to $\bar{\theta}$ arise first in the order $\alpha_s G_F^2 m_c^2 m_s^2$ times the CP-odd KM invariant [11], and in the MSSM with the KM mechanism of CP-violation [12] where the result also is found to be much smaller than $10^{-9}$.

Here we address the same question to the generic left-right supersymmetric model and calculate the theta term induced by the radiative corrections through the KM type of CP-violation. The simple estimate of the upper limit for $\bar{\theta}$, $\bar{\theta} < \frac{\alpha_s}{64 \pi} \times \log(\frac{M_W^R}{M_W^L}) \sim 10^{-8}$, presented in the work [9] is not satisfactory because it can predict the electric dipole moment of the neutron one order of magnitude above the present experimental limit. This estimate does not take into account the dependence of the quark masses which should be associated with the KM-type of CP-violation.

As usual, the potentially large CP-violating effects emerge through the one loop induced by quark-squark-gluino interaction. Following the works [7, 8, 9] we take all new CP-violating phases specific for supersymmetric models to be equal to zero as the result of the parity conservation at $\Lambda_{GUT}$ scale:

$$ A = A^*; \quad B = B^*; \quad m_{\lambda_i} = m_{\lambda_i}^*; \quad \mu_{ij} = \mu_{ij}^*. $$

The left-right symmetry imposed on the interaction of the quarks with Higgs bidoublets $\Phi_1$ and $\Phi_2$ requires the hermiticity of the Yukawa matrices:

$$ \mathcal{L}_Y = Y_1 \bar{Q}_L \Phi_1 Q_R + Y_2 \bar{Q}_L \Phi_2 Q_R + H.c. \\
Y_1 = Y_1^\dagger; \quad Y_2 = Y_2^\dagger $$

We do not specify here the particular content of the Higgs sector giving masses to $M_{W_R}$ in order to obtain the maximum of generality. The reality of the vacuum expectation values (VEV’s) for Higgs bidoublets $\Phi_1$ and $\Phi_2$,

$$ \langle \Phi_1 \rangle = \left( \begin{array} {cc} \kappa_1 & 0 \\ 0 & \kappa'_1 \end{array} \right); \quad \langle \Phi_2 \rangle = \left( \begin{array} {cc} \kappa'_2 & 0 \\ 0 & \kappa_2 \end{array} \right), $$

corresponds to the minimum of the superpotential [7, 8, 9]. It ensures the hermiticity of the mass matrices $M_u$ and $M_d$ and provides the same KM matrices for left- and right-handed charged currents. To get the simplest relations between mass matrices and Yukawa couplings and to avoid the problems with flavour changing neutral currents, we assume for the moment that $\kappa'_1 = \kappa_2 = 0$. Then $M_u$ and $M_d$ read as follows:

$$ M_u = \kappa_1 Y_1 \equiv \kappa_u \lambda_u; \quad M_d = \kappa_2 Y_2^\dagger \equiv \kappa_d \lambda_d, $$

where $\kappa_u$ and $\kappa_d$, $\lambda_u$ and $\lambda_d$ are introduced from matter of convenience. As in the MSSM there is one additional free parameter, $\tan \beta = \kappa_u/\kappa_d$.

Let us now turn to the squark mass sector. The mass matrix for the down type squarks has the following general form:

$$ (\tilde{D}_L^\dagger \tilde{D}_R^\dagger) \left( \begin{array} {cc} m_{\tilde{d}_L}^2 + c_u \lambda_u^2 + c_d \lambda_d^2 & A_d \\ A_d^\dagger & m_{\tilde{d}_R}^2 + c'_u \lambda_u^2 + c'_d \lambda_d^2 \end{array} \right) (\tilde{D}_L \tilde{D}_R^\dagger), $$

$\tilde{D}_L \tilde{D}_R^\dagger$.
where \( A_d = (A - \mu \tan \beta)(M_d + a_d \lambda^2 d M_d + a_u \lambda^2 u M_d + a' u M_d \lambda^2 u) \).

The coefficients \( c_u, c'_u, c_d, c'_d, a_d, a_s, a'_u \) appear either at the tree level or in the one-loop renormalization from \( \Lambda_{\text{GUT}} \). The obvious requirement of the L-R symmetry is:

\[
m_L = m_R, \quad c_d = c'_d, \quad c_u = c'_u, \quad a_u = a'_u. \tag{10}
\]

As a result the mass matrix (10) differs from that of the MSSM where \( c'_u = 0 \) and \( a'_s = 0 \). The values of all these coefficients depend on many additional parameters and we simply assume here the following estimate: \( c_u \sim c'_u \sim m_{\text{susy}}^2 (16\pi^2)^{-1} \ln(\Lambda_{\text{GUT}}^2 / M_{\text{susy}}^2) \sim \mathcal{O}(m_{\text{susy}}^2) \).

Let us now estimate the CP-violating mass term for quarks induced by the squark-gluino loop. The characteristic loop momenta are of order \( \Lambda_{\text{susy}} \). To sufficient accuracy we can take also \( \Lambda_{\text{GUT}} \) = \( \Lambda_{\text{susy}} \). For our purposes, however, it is sufficient to use the trivial integration, we arrive to the following form of the CP-violating quark masses:

\[
(A - \mu \tan \beta) \sum_{n,m} \frac{c^2_n c^2_m (V^+ \lambda^2 u (V M_d V^+ + a_u \lambda^2 u V M_d V^+ + a'_u V M_d V^+ \lambda^2 u) \lambda^2 u V)}{(p^2 - m^2)^{n+1}(p^2 - m^2)^{m+1}}, \tag{11}
\]

where \( V \) is the usual KM matrix and the subscript \( ii \) denotes the projection on the initial flavour \( i \). We have dropped also all \( c_d \)-proportional terms as they are further suppressed by the D-quark Yukawa couplings. It is clear that if the conditions of the left-right symmetry (10) are held, the expression (11) is purely CP-conserving. In other words, in the mass eigenstate basis, the mixing matrices in the quark-squark-gluino couplings are identical for \( (10) \) are held, the expression (11) is purely CP-conserving. In other words, in the mass eigenstate basis, the mixing matrices in the quark-squark-gluino couplings are identical for left- and right-handed particles and the CP-violating phase drops out at the one-loop level. However the further running of the mass parameters from the scale of parity violation down to the electroweak scale necessarily implies the departure from the exact relations (10). As a result of that, the CP-violation can be developed, and the lowest-order term where it arises is \( \lambda^2 u \lambda^2 u \). The explicit extraction of the CP-violating part from Eq. (11) for the external \( d \)-flavour leads to the following expression:

\[
\frac{2 a_u c_u (c_u - c'_u) + 2 c^2_u (a_u - a'_u)}{(p^2 - m^2)^4} + \frac{2 a_u c^2_u (m^2_R - m^2_L)}{(p^2 - m^2)^5} + \frac{2 c^2_u (c_u - c'_u)}{(p^2 - m^2)^6} + \frac{c^2_u (m^2_R - m^2_L)}{(p^2 - m^2)^6} \tag{12}
\]

The differences between the coefficients \( c_u \) and \( c'_u \), \( m_L \), and \( m_R \) cannot be calculated without the knowledge of all masses below \( M_{\text{W_R}} \). For our purposes, however, it is sufficient to use the reliable estimate for mass difference \( m^2_L - m^2_R \sim m_{\text{susy}}^2 6 g_2^2 (16\pi^2)^{-1} \ln(M_{\text{W_R}}^2 / M_{\text{W_L}}^2) \) and similar relations for other coefficients. Combining together all these factors and performing the trivial integration, we arrive to the following form of the CP-violating quark masses:

\[
\mathcal{L}_5 \sim \text{Im}(V^* V_{\text{th}} V_{\text{cb}} V_{\text{cd}}) \frac{\alpha_s}{4\pi} 3 \alpha_w \ln \frac{M_{\text{W_R}}^2}{M_{\text{W_L}}^2} \frac{\lambda^2 u^2}{\lambda^2 u'} \frac{m_{\text{G}_2}}{m_{\text{susy}}} (A - \mu \tan \beta) F(m_{\text{G}_2} / m_{\text{susy}}) \times \left[ (m_b - m_s) \bar{d} \gamma_5 d + (m_d - m_b) \bar{s} \gamma_5 s + (m_s - m_d) \bar{b} \gamma_5 b \right] \tag{13}
\]

The exact form of the function \( F \) is not important to us and we can take it \( F \sim \mathcal{O}(1) \). All three CP-odd masses are suppressed by the square of the charm quark Yukawa coupling as it should be. To sufficient accuracy we can take also \( \lambda^2 u \sim 1 \) because no \( \lambda u \)-expansion can be made.
The analogous calculation of the radiatively induced CP-violating gluino mass term yields the following result:

\[ m_{\tilde{G}} - m_{\tilde{G}^*} \sim i \text{Im}(V_{td}^* V_{tb} V_{cb}^* V_{cd}) \frac{\alpha_s 3\alpha_w}{4\pi} \frac{M_{W_R}^2}{2\pi} m_{\tilde{G}} \left( A - \mu \tan \beta \right) \frac{\lambda_e^2 \lambda_s}{m_{\text{susy}}^2} \]  

\[ (14) \]

Due to the additional suppressions by the D-quark masses, this imaginary gluino mass gives just a negligible contribution to the theta-term. The main contribution to \( \bar{\theta} \) comes from \( \bar{d} \gamma_5 d \)-operator:

\[ \bar{\theta} \sim \text{Im}(V_{td}^* V_{tb} V_{cb}^* V_{cd}) \frac{\alpha_s 3\alpha_w}{4\pi} \frac{M_{W_R}^2}{2\pi} m_{\tilde{G}} \left( A - \mu \tan \beta \right) \frac{\lambda_e^2 m_b}{m_{\text{susy}}^2} \]  

\[ (15) \]

The interesting feature of this formula is a sort of “chiral” enhancement \( m_b/m_d \) which is natural in the framework of the left-right model and simply impossible in MSSM where the chirality flip is always proportional to the mass of the external fermion. (This formula is valid only for the situation when the coefficient in front of \( \bar{d} \gamma_5 d \) is much smaller than \( m_d \)). Substituting the numbers into (15), we get the following estimate for the theta term developed in the generic left-right supersymmetric model with the KM-source of CP-violation:

\[ |\theta| \sim 10^{-9} \begin{cases} \tan \beta & \text{for } \tan \beta \gg 1 \\ \mathcal{O}(1) & \text{for } \tan \beta \sim 1 \\ \tan^{-2} \beta & \text{for } \tan \beta \ll 1 \end{cases} \]  

\[ (16) \]

When obtaining (16) out of Eq. (15), we took \( \text{Im}(V_{td}^* V_{tb} V_{cb}^* V_{cd}) \simeq 2.5 \cdot 10^{-5}; \ m_{\tilde{G}} \sim |A| \sim |\mu| \sim m_{\text{susy}} \) and \( \ln(M_{W_R}^2/M_{W_L}^2) \simeq 7 \).

## 3 Discussion

The common wisdom that the KM mechanism always gives the negligibly small contribution to the CP-violating flavour-conserving observables apparently is not true in the case of the left-right supersymmetric model. We have shown that the radiative corrections to the \( \theta \)-parameter in the generic left-right supersymmetric model are large, just about the edge of the current experimental constraint. The only contribution to theta term comes from the radiative corrections to the \( d \)-quark mass. The main difference of our answer (16) in comparison with the simple estimate quoted in [9] is in the additional multiplier \( \lambda_e^2 m_b/m_d \) which is of the order \( 5 \cdot 10^{-2} \) for \( \kappa_1 \sim \kappa_2 \) and \( \kappa'_1 = \kappa'_2 = 0 \). In this domain of the parameter space, the radiatively induced \( \bar{\theta} \) is hard but not impossible to reconcile with the current experimental limit. One way for that would be to make the ratio \( m_{\tilde{G}} (A - \mu \tan \beta)/m_{\text{susy}}^2 \) reasonably small, of order \( 10^{-1} \).

It turns out that the value of \( \bar{\theta} \) is very sensitive to the relations between different VEV’s of the model. Thus, the Eq. (14) suggests that both small and large \( \kappa_1/\kappa_2 \) are almost excluded. In the more general formulation of the model \( \kappa'_1, \kappa'_2 \) also differ from zero. In that case we observe another contribution to \( \bar{\theta} \) which is suppressed only by the first power of the charm quark Yukawa coupling. This contribution comes from the cubic term \( a_u (\lambda_u^2 M_d + M_d \lambda_u^2) \simeq a_u (\kappa'_1/\kappa'_2) M_u M_d M_u \) in the mixing of the left- and right-handed
squarks. The overall factor $\lambda_2^2$ in the estimate (13) is then substituted for $\lambda,\kappa'_1/\kappa_1$. To keep this contribution in agreement with the experimental limit, one has to assume that $\kappa'_1/\kappa_1 < 10^{-2}$. This constraint is held even in the limit of very large $m_{\text{susy}}$ and $M_{W_R}$ where many other phenomenological constraints (such as the flavour changing neutral currents) are trivially satisfied. In the limit $M_{W_R} \rightarrow \infty$ the squark mass matrix keeps the nonvanishing remnants of the left-right symmetry resembling the case of the supersymmetric $SO(10)$ models [13] where the radiative corrections to $\theta$ are also known to be large (the last Ref. in [13]).

If the strong CP-problem is cured by the axion, CP-violating mass term (13) has no effect on the physical observables. The EDM of the neutron, in this case, originates from operators of dimension bigger than 4, such as EDM of quarks, color EDM of quarks, etc. We give a crude estimate for the EDM of the neutron using the size of coefficient in front of $\bar{d}\gamma_5 d$ in (13) multiplied by $e/m_{\text{susy}}^2$ which is of the order $10^{-29} e \cdot cm$ for $m_{\text{susy}}$ taken close to electroweak scale.

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