Charge density oscillations in a quasi-two-dimensional electron gas for integer filling factors

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Abstract

The possibility of charge density oscillations in a finite-thickness two-dimensional system is investigated for strong magnetic fields and integer filling factors. Using an effective action formalism, it is shown that an oscillatory charge density (OCD) is generated in a self-consistent way and is favored energetically over homogeneous distributions. It is smooth on the scale of the sample thickness and of the magnetic length. The modulus of its wave vector is shown to be experimentally accessible. The Hall voltage and the current density are shown to oscillate with the same wave vector when a weak current is applied. The stability of the charge oscillations against impurity potentials is discussed.

73.40Hm - Quantum Hall effect
71.45.Lr - Charge-density-wave systems
11.10.Wx - Finite temperature field theory
Two-dimensional electron gas systems (2DEG) subject to a magnetic field normal to the plane of confinement exhibit a rich phase structure. This includes the manifestation of the integer and fractional quantum Hall effect \[1\] at low temperature and the expected Wigner crystallization at low densities \[2\].

In this letter, we study a system of interacting nonrelativistic charged fermions in 3+1 dimensions embedded in a uniform neutralizing background of opposite charge. This system is subjected to a uniform perpendicular magnetic field and the particles interact with each other through the repulsive Coulomb interaction. The goal of the study is to analyze the low energy effective theory describing this system and to explicitly show the possibility of charge density oscillations.

From the study of strictly two-dimensional systems in the absence of magnetic fields we know that the ground state is a fluid of constant density \[3\]. Furthermore, it has been realized that a magnetic field driven charge density redistribution in the bulk is possible as long as no Landau level is partially occupied \[4,5\]. However, in these studies, the self-consistent equations used to describe the charge density do not allow for oscillatory type solutions. Here, we show that these are permissible and favored energetically over the uniform distribution for real systems with finite-thickness when only the lowest subband is occupied.

The approach we use relies on the effective action formalism combined with a derivative expansion technique \[6,7\]. Here, the effective action is simply defined as the sum of all connected one-particle irreducible vacuum diagrams in the presence of any background field \[8\]. This method has been extremely useful, among other things, in showing the origin of the Chern-Simons term in 2+1 dimensions \[9\] and in analyzing the phase diagram of planar superconductivity \[10\]. Here, after removing the four-Fermi Coulomb term in favour of a Yukawa-type coupling to an auxiliary field $\phi$, the effective action is obtained by integrating out the fermions. The background fields in the expansion take account of the magnetic field which gives rise to Landau levels as underlying single particle states and also of a self-consistent electrostatic potential. The result of the latter is to shift the center of the Landau
wavefunctions and thereby make the electron density position dependent. In this way it is possible to maintain an integer filling factor throughout the sample despite charge redistribution. Most importantly, we show explicitly that an oscillatory charge density (OCD) can be generated in a self-consistent way as long as each Landau level is fully occupied. It is smooth on the scale of the magnetic length \( \ell_B \) and of the thickness \( d \). The modulus \( q \) of its wave vector, in the plane of the 2DEG, is given by

\[ q \approx q_w - \frac{1}{\nu} \frac{\epsilon_0 \hbar \omega_c}{e^2} \]  

(1)

Here \( q\ell_B \ll 1, qd_w \ll 1, \omega_c = |e|B/m^* \) is the cyclotron frequency, \( m^* \) the effective mass, and \( \epsilon_0 \) the static dielectric constant which is assumed spatially homogeneous. The Hall potential and the current density both oscillate with the same wave vector. A possible consequence of this oscillatory charge density is the destruction of incompressibility in line with experimental and theoretical studies of finite-thickness effects in a different context \[11,12\].

The action \( S \), at finite temperature \( T \), describing the interacting finite-thickness 2DEG is given by

\[ S = \int_0^\beta \delta \int d^3x \psi^\dagger (\partial_\tau + \mu - H_0) \psi - \frac{e^2}{2\epsilon_0} \int_0^\beta \delta \int \frac{d^3x d^3y}{|x - y|} \left[ \psi^\dagger \psi(x, \tau) - \bar{\rho} \right] \left[ \psi^\dagger \psi(y, \tau) - \bar{\rho} \right] ; \]

(2)

the first term is the kinetic energy with a parabolic dispersion relation for the fermions; \( H_0 = (-i\hbar \nabla + eA)^2/2m^* + U(z) \), the vector potential is \( A = (-By, 0, 0) \), \( \beta = 1/T \) \( (k_B = 1) \) and \( \mu \) is the chemical potential. The potential \( U(z) \) describes the confinement of the 2DEG in the \( z \) direction. It can be that of a square, triangular or parabolic well. Finally, the last term describes the Coulomb interaction between the charged fermions.

In order to make \( S \) bilinear in terms of the fermion fields, we add to it a term of the form

\[ \delta S = -\frac{\epsilon_0}{8\pi} \int d\tau \int d^3x [\nabla \phi(x, \tau)]^2 \]  

(3)
and perform the shift \( \phi(\mathbf{x}, \tau) \rightarrow \phi(\mathbf{x}, \tau) + (ie/\epsilon_0) \int (d^3y/|\mathbf{x} - \mathbf{y}|) \psi^\dagger(y, \tau) \psi(y, \tau) \), which cancels the Coulomb interaction term in (2). The partition function is obtained by integrating over the fermion field \( \psi \) as well as over the scalar field \( \phi \). Clearly, this procedure has no effect since the integration over \( \phi \) gives a physically irrelevant constant; however, it allows us to cancel the quartic term and thereby make \( S \) bilinear in the fermion fields. The linearized action reads

\[
S = \int_0^\beta d\tau \int d^3x \left\{ \psi^\dagger (\partial_\tau + \mu - H_0 - ie\phi(\mathbf{x}, \tau)) \psi - \frac{\epsilon_0}{8\pi} [\nabla \phi(\mathbf{x}, \tau)]^2 + ie\bar{\rho}\phi(\mathbf{x}, \tau) \right\}
\]

(4)

It is important to emphasize that the scalar field \( \phi \) in Eq. (4) couples to the charge density \( \psi^\dagger\psi \); its fluctuations in the presence of the uniform magnetic field will describe the magnetoplasmon collective excitations.

The advantage of the action (4) is that its bilinear nature in the fermion fields allows one to eliminate them exactly. The result of this operation is the normalized effective action \( S_{\text{eff}} \) given by

\[
S_{\text{eff}} = \int d\tau d^3x \left\{ \frac{\epsilon_0}{8\pi} [\nabla \phi(\mathbf{x}, \tau)]^2 - ie\bar{\rho}\phi(\mathbf{x}, \tau) \right\}
- \text{Tr} \ln(\partial_\tau + \mu - H_0 - ie\phi)(\partial_\tau + \mu - H_0 - V)^{-1},
\]

(5)

where \( \text{Tr}(\cdots) \) denotes the functional trace operation. The potential \( V(\mathbf{x}) \) in Eq. (5) will be specified later.

In this formalism, a uniform charge distribution corresponds to taking \( \phi(\mathbf{x}, \tau) \) purely fluctuating. Here, we assume a nontrivial vacuum structure corresponding to some average electric potential \( \phi_0(\mathbf{x}, \tau) = \phi_0(\mathbf{x}) \). This can be obtained by minimizing the effective action \( \delta S_{\text{eff}}/\delta \phi(\mathbf{x}, \tau) = 0 \); the result after the substitution \( e\phi_0(\mathbf{x}) = -iV(\mathbf{x}) \), is the usual Poisson equation

\[
\nabla^2 V(\mathbf{x}) = \frac{4\pi e^2}{\epsilon_0} \left[ \bar{\rho} - \langle \mathbf{x}, \tau | \frac{1}{\partial_\tau + \mu - H_0 - V} | \mathbf{x}, \tau \rangle \right] = -\frac{4\pi e^2}{\epsilon_0} \delta \rho(\mathbf{x}).
\]

(6)

Furthermore, the fluctuations around this configuration are described by
\[ S_{\text{eff}} = -\frac{\beta \epsilon_0}{8\pi e^2} \int d^3x [\nabla V(x)]^2 + \frac{\epsilon_0}{8\pi} \int d\tau d^3x [\nabla \phi(x, \tau)]^2 \]
\[ -\text{Tr} \ln \left[ 1 - \frac{ie}{\partial_\tau + \mu - H_0 - V(x)} \phi(x, \tau) \right]. \tag{7} \]

The last term in \( S_{\text{eff}} \) can be evaluated by a derivative expansion technique \[7\]. In a diagrammatic language, it corresponds to a sum of an infinite number of loops with external legs representing the field \( \phi \). The quadratic term in this expansion, defines an induced propagator for the field \( \phi \), and the higher order terms represent self-interaction. Here we will restrict ourselves only to the quadratic term giving

\[ -\frac{\epsilon^2 e^2}{2\beta} \sum_\omega \int d^3x d^3y \phi(y, -\omega) \Pi(x, y; \omega) \phi(x, \omega), \tag{8} \]

where \( \Pi(x, y; \omega) \) is the polarization tensor, represented by the diagram in Fig. (1) and \( \omega = 2\pi n/\beta \) the Matsubara frequency. The ground state energy in the presence of the nontrivial electrostatic potential is obtained by integrating out the fluctuations \( \phi \). In terms of the potential \( V(x) \) and the polarization tensor \( \Pi(x, y; \omega) \), the result is

\[ E_G = -\frac{\epsilon_0}{8\pi e^2} \int d^3x [\nabla V(x)]^2 + \lim_{\beta \to \infty} \frac{1}{2\beta} \text{Tr} \ln \left[ 1 + \frac{4\pi e^2}{\epsilon_0} \frac{1}{\nabla^2} \Pi(x, y; \omega) \right]. \tag{9} \]

The evaluation of \( \Pi(x, y; \omega) \) requires the knowledge of the single-particle states of the Schrödinger operator \( H_0 + V(x) \). Before we proceed into that we exploit the fact that when a small electric field is applied in the \( x \)-direction, a Hall voltage develops in the \( y \)-direction. As a consequence, the electrostatic potential that enters the Schrödinger equation depends only on the \( y \) coordinate. Then the eigenfunctions of the Schrödinger operator take the form \( \exp(ik_x x) \psi_\ell(y - y_0) X_0(z)/\sqrt{L_z} \) with \( y_0 = \ell_B^2 k_x \), \( X_0(z) \) the lowest subband wavefunction in the \( z \)-direction and \( \psi_\ell(y) \) satisfying

\[ \left( -\frac{\hbar^2}{2m^*} \frac{d^2}{dy^2} + \frac{1}{2} m^* \omega_c^2 y^2 + V(y_0 + y) - \varepsilon_{\ell, y_0} \right) \psi_\ell(y) = 0; \tag{10} \]

here, \( \ell \) is the Landau-level index, and \( \varepsilon_{\ell, y_0} \) the eigenvalue measured relative to the lowest subband energy. Provided that \( V(y) \) is a slowly varying function on the scale of \( \ell_B \), i. e., \( V''(y_0) \ll m \omega_c^2 \), we can expand \( V(y + y_0) \) in a Taylor series and replace it by \( V(y_0) + y V'(y_0) \). Then \( \psi_\ell(y - y_0) = \Phi_\ell(y - y_0') \) and
where \( y_0' = y_0 - V'(y_0)/m^*\omega_c^2 \), and \( \Phi_\ell(y) \) is a harmonic oscillator function. The last two terms of Eq. (11) can also be obtained by considering \( V(y) \) as a perturbation and using non-degenerate perturbation theory.

As in Ref. [4] we assume that no Landau level is partially occupied and that the initial charge distribution is homogeneous in the 2D plane when \( V(y) \equiv 0 \). Then the charge density \( \delta\rho(y,z) \) entering Poisson’s equation is given by

\[
\delta\rho(y,z) = \frac{e}{L_x} \sum_{\ell,k_x} \left[ \Phi_\ell^2(y - y_0') - \Phi_\ell^2(y - y_0) \right] X_0^2(z) = \frac{e\nu}{2\pi\hbar\omega_c} V''(y)X_0^2(z),
\]

(12)

where \( \nu \) is the filling factor. The last equality follows from a Taylor expansion and the assumption \( \delta y_0 = 2\pi\ell_B^2/L_x \ll \ell_B \); it can also be obtained if \( \Phi_\ell(y - y_0') \) is evaluated by perturbation theory. In either case the entire Hilbert space is used to obtain Eq. (12) as the perturbation \( V(y) \approx V(y_0) + yV'(y_0) \) connects a Landau level with the higher and lower levels, e.g., \( \Phi_0(y - y_0') \) is not orthogonal to \( \Phi_1(y - y_0) \). This is in sharp contrast with usual treatments, e.g., those of Refs. [13][14], which are restricted to the subspace of the lowest Landau level wavefunctions. Eq. (11) and the last term in Eq. (12) cannot be obtained if we restrict ourselves to this subspace. Notice that Eq. (12) can be written as

\[
\delta\rho(y,z) = \delta\rho(y)X_0^2(z) \quad \text{with} \quad \delta\rho(y) = e\nu V''(y)/2\pi\hbar\omega_c.
\]

Alternatively, this redistribution of the electron density will create a potential \( V(y,z) \) given by

\[
V(y,z) = -\frac{e}{\epsilon_0} \int dy'dz' \delta\rho(y',z') \ln \left[ (y - y')^2 + (z - z')^2 \right],
\]

(13)

where use has been made of the charge-neutrality condition

\[
\int_{-L_y/2}^{L_y/2} dy \delta\rho(y) = \frac{e\nu}{2\pi\hbar\omega_c} \left[ V'(L_y/2) - V'(-L_y/2) \right] = 0.
\]

(14)

Notice that Eq. (13) merely expresses the potential produced at a point \( (y,z) \) by a strip parallel to the \( x \) axis, at point \( (y',z') \), with charge density \( \delta\rho(y',z') \).
We assume, in line with most experiments, that only the lowest subband in the z-direction is occupied. The electrons can then be thought of as extended rod-like charges in the z-direction which are allowed to move only in the xy plane. For these reasons, the potential that enters the Schrödinger equation is approximated by the average \( V(y) \approx \langle X_0(z)|V(y,z)|X_0(z) \rangle \). Notice that when the thickness of the 2DEG is zero, the integral over \( z \) is easily carried out and yields Eq. (12) of Ref. [4].

Differentiating Eq. (13) twice with respect to \( y \) and using Eq. (12), gives the following integral equation

\[
\delta \rho(y) = -\frac{e^2}{\epsilon_0 \pi \hbar c} \int X_0^2(z)dz \int X_0^2(z')dz' \int_{-L_y/2}^{L_y/2} dy' \delta \rho(y') \frac{(z' - z)^2 - (y' - y)^2}{[(z' - z)^2 + (y' - y)^2]^2}.
\]

(15)

An oscillatory charge density \( \delta \rho(y) = \delta \rho(q) \sin(qy) \), with \( q > 0 \) is a solution. When substituted on both sides of Eq. (15), it gives the dispersion relation that \( q \) has to satisfy

\[
\eta = \int dz X_0^4(z) - \frac{q}{2} \int X_0^2(z)dz \int X_0^2(z')dz' e^{-q|z'-z|},
\]

(16)

where \( \eta = \epsilon_0 \hbar \omega_c/2e^2 \nu \). An oscillatory charge density of the form \( \delta \rho(y) = \delta \rho(q) \cos(qy) \) is another solution to this integral equation; however, it is not physically permissible since it violates the charge neutrality condition (14) which allows only odd solutions.

We evaluate this dispersion for a square well \( X_0(z) = \sqrt{2/d} \cos(\pi z/d), -d/2 \leq z \leq d/2 \). With \( s = qd \), we obtain

\[
\eta d = \frac{2\pi^2}{s^2 + 4\pi^2} + \frac{16\pi^4}{s(s^2 + 4\pi^2)^2}(1 - e^{-s}).
\]

(17)

When \( qd \ll 1 \), this gives Eq. (1) with \( q_w = 3/d \). For a triangular well we use \( X_0(z) = (b_0^3/\sqrt{2})z \exp(-b_0 z/2) \) with \( 0 \leq z, z' \leq \infty \). For the average thickness \( d_0 = 3/b_0 \) we obtain, for \( qd_0 \ll 1 \), again Eq. (1) with \( q_w = 9/8d_0 \). Eq. (1) is also valid for a parabolic well of frequency \( \omega_z \), for which \( X_0(z) = (1/\pi \ell_z^2)^{1/4} \exp(-z^2/2\ell_z^2) \), with \( q_w = \sqrt{2\pi/\ell_z} \).

This solution for \( \delta \rho(y) \) and Eq. (12) give \( V(y) = -(2\pi \hbar \omega_c/evq^2)\delta \rho(q) \sin(qy) \). For completeness, we also give the Hall current density, averaged along the z-direction. By noting that \( \dot{x} = \omega_c(y - y_0) \), it is given by
\[ J_x(y) = -\frac{e\omega_c}{L_x} \sum_{\ell,k_x} (y - y_0) \Phi_\ell^2(y - y_0) = \frac{e\nu}{2\pi\hbar} V'(y) \]  
(18)

where the same approximations have been made as in the derivation of Eq. (12) i.e. \( V \ll \hbar \omega_c \) and \( [V'(y_0)]^2/m\omega_c^2 \ll \hbar \omega_c \). As was noted in Ref. [4], Eq. (18) merely expresses the fact that the current density is such that the Lorentz force is locally canceled by the electrostatic potential. Furthermore, by integrating over \( y \) we obtain that the Hall current is proportional to the potential gradient, and the constant of proportionality is \( e^2\nu/2\pi\hbar \).

Now that the electrostatic potential profile is known we are able to evaluate the ground state energy from Eq. (9). The first term in this equation is the Hartree energy and is easily evaluated. The result, per particle, is

\[ w \equiv \frac{E_G}{N_e} = -\frac{\pi^2\hbar^2}{\nu^2q^2m^*} \delta n(q) \],

(19)

where \( \delta n(q) = \delta \rho(q)/e \) and \( N_e = \nu L_x L_y/2\pi\ell_B^2 \). The minus sign shows that the OCD state is energetically favorable. This conclusion remains valid even when we include the next term in the exact expression Eq. (9), here we give only the Fock exchange energy contribution. This is obtained by keeping only the first term when we expand the logarithm and pass to the zero temperature limit which amounts to replacing \( (1/\beta) \sum_\omega \) by \( (1/2\pi) \int d\omega \). The result is

\[ \frac{E_{G,exc}^{\nu}}{N_e} = -\frac{e^2}{\epsilon_0 N_e} \int \frac{d^3Q}{(2\pi)^3} \frac{1}{Q^2} \int d\omega \Im \Pi(Q,\omega), \]

(20)

which after some algebra becomes

\[ \frac{E_{G,exc}^{\nu}}{N_e} = -\frac{e^2 L_x}{2\pi \epsilon_0 N_e} \sum_{\ell,\ell'=0}^{\nu-1} \int dk_x \int dk'_x \int \frac{dQ_y dQ_z}{(2\pi)^2} \frac{F(Q_z) G_{\ell,\ell'}(u)}{(k_x - k'_x)^2 + Q_y^2 + Q_z^2}. \]

(21)

Here

\[ u = \frac{Q_y^2\ell_B^2}{2} + \frac{\ell_B^2}{2} \left[ k'_x - k_x + \frac{V'(\ell_B^2 k_x) - V'(\ell_B^2 k'_x)}{\omega_c} \right]^2, \]

\[ G_{\ell,\ell'}(u) = (\ell! / \ell'!) e^{-u} u^{\ell'-\ell} \left[ L_{\ell'}^{\ell'-\ell}(u) \right]^2, \]

\[ F(Q_z) = \left| \langle X_0 | e^{iQ_z X_0} | X_0 \rangle \right|^2, \]

(22)
and $L_{\ell'}^{\ell}$ is an associated Laguerre polynomial for $\ell' \geq \ell$ (for $\ell' \leq \ell$, interchange $\ell$ and $\ell'$ in this formula). This Fock exchange energy is a very complicated functional of the potential $V$ or equivalently of the charge density amplitude $\delta \rho(q)$. It can be verified after a straightforward algebra that the term linear in $\delta \rho(q)$ is zero. The quadratic term is given, for $\nu = 1$ and $d \propto \ell_B$

$$\frac{\mathcal{E}_G^{\text{exc}}}{N_e} = - \left(0.615 - 0.051 \frac{d}{\ell_B}\right) \frac{e^2}{\epsilon_0 \ell_B} \left[\delta n(q)\right]^2$$  

So far, we have examined the OCD state at zero temperature. It is not hard to extend the same analysis to include the temperature effects. In order to do so, one has to evaluate the free energy when the charge density oscillates. Denoting by $\delta F$ the difference in free energies between the normal, spatially homogeneous, and the OCD, spatially inhomogeneous, states with fixed $N_e$. This difference can be written in terms of the thermodynamic potential as

$$\delta F = \Omega_{\text{OCD}}(\mu) - \Omega_0(\mu_0) + N_e(\mu - \mu_0).$$

Because $N_e = -\partial \Omega_0/\mu_0$, $\delta F = \Omega_{\text{OCD}}(\mu) - \Omega_0(\mu) + 1/2(\Delta \mu)^2 \partial^2 \Omega_0/\partial \mu_0^2 + \cdots$ where $\Delta \mu = \mu - \mu_0$. Using $V(\theta) = V_0 \sin \theta$, and

$$\Omega_{\text{OCD}}(\mu) = -T \sum_{\ell=0}^{\infty} \int_0^{2\pi} d\theta \frac{\mu}{2\pi} \ln \left(1 + e^{\frac{\mu - \epsilon_{\ell}(\theta)}{T}}\right),$$

we obtain for low temperatures, and $|V_0|/T \gg 1$

$$\delta F = w - \frac{T \sqrt{T}}{\sqrt{2\pi |V_0|}} e^{-\hbar \omega_c \gamma / T},$$

where $\gamma = 1/2 - (\Delta \mu + |V_0|)/\hbar \omega_c$. This gives a negative contribution, and hence proves the stability of the OCD at low temperature.

The OCD remains also stable in the presence of weak disorder as is usually the case in a high-mobility 2DEG. This conclusion is reached by computing the impurity averaged Green’s function which satisfies Dyson’s equation. Now the relation defining the charge density in Eq. (12) is modified by multiplying the squared wavefunctions $\Phi_{\ell}(y)$ by the appropriate spectral functions. A straightforward calculation reveals that the only effect of disorder is Landau level broadening and Eq. (12) is unaffected.

We now comment on the use of the approximation $V(y) \approx <X_0(z)|V(y,z)|X_0(z)>$. It has been used in the past to study the collapse of the FQHE state in a quantum well [12]. It
is reasonable as long as the electron density is not too high and the quantum well is not too wide such that the subbands are not close to each other, otherwise the wavefunction $X_0(z)$ will be modified. Finally, in evaluating the ground state energy in the presence of the OCD we limited ourselves to the Hartree-Fock contribution although Eq. (9) goes beyond this approximation. This justified by the fact that the neglected terms come in higher powers of $(e^2/\epsilon_0\ell_B)[\delta n(q)/n]$ and are small for strong magnetic fields under the assumptions used to derive Eq. (12).

An estimate of the parameters that enter Eq. (1) shows that the OCD is experimentally accessible. For instance, for a square well based on GaAS with $d = 140 \text{Å}$, $m^*/m = 0.07$, $\epsilon = 12$ and $B = 15$ Tesla we obtain $\lambda = 2\pi/q = 2\mu m$. A consequence of the OCD is that the Landau levels given by Eq. (11) oscillate with $\gamma_0$ Fig. 1. This facilitates interlevel transitions through the indirect gaps and may destroy the quantum Hall effect.

In summary we have shown that for fully occupied Landau levels and strong magnetic fields, a self-consistent oscillatory charge density (OCD) is possible in a finite-thickness two-dimensional system, when the lowest subband is occupied. The OCD, which is impossible for the ideal two-dimensional system, is smooth on the scale of the magnetic length and of the thickness. It is energetically favorable over the homogeneous distribution at low temperature and remains stable against weak impurity scattering.
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FIGURES

FIG. 1. Polarization diagram (bubble) entering Eq. (8). Solid lines are fermion propagators, while dashed lines are the scalar fields.

FIG. 2. The first lowest Landau levels ($\ell = 0, \ell = 1$) as a function of $y_0/\ell_B$, obtained from Eq. (11) when $V(y) = V_0 \sin(qy)$ and $q\ell_B = 1/3$. The dashed and solid curves are for $V_0 = 0.1\hbar\omega_c$ and $V_0 = 0.45\hbar\omega_c$. The dotted curve shows the Fermi level $\mu$. 