Model-independent Estimations for the Cosmic Curvature from the Latest Strong Gravitational Lensing Systems

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Abstract

Model-independent measurements for the cosmic spatial curvature, which is related to the nature of cosmic spacetime geometry, play an important role in cosmology. On the basis of the distance sum rule in the Friedmann–Lemaître–Robertson–Walker metric, (distance ratio) measurements of strong gravitational lensing (SGL) systems, together with distances from SNe Ia observations, have been proposed to directly estimate the spatial curvature without any assumptions for the theories of gravity and contents of the universe. However, previous studies indicated that a spatially closed universe was strongly preferred. In this paper, we re-estimate the cosmic curvature with the latest SGL data, which includes 163 well-measured systems. In addition, possible factors, e.g., a combination of SGL data from different surveys and stellar masses of the lens galaxy, which might affect estimations for the spatial curvature, are considered in our analysis. We find that, except for the case where only SGL systems from the Sloan Lens ACS Survey are considered, a spatially flat universe is consistently favored at very high confidence levels by the latest observations. It has been suggested that an increasing number of well-measured strong lensing events might significantly reduce the bias of estimation for the cosmic curvature.

Unified Astronomy Thesaurus concepts: Cosmological parameters (339); Observational cosmology (1146); Galaxy distances (590); Gravitational lensing (670)

1. Introduction

If the universe satisfies the cosmological principle, i.e., the space of the universe is homogeneous and isotropic at large scales, we can use the Friedmann–Lemaître–Robertson–Walker (FLRW) metric to describe the spacetime geometry of the universe. In the standard ΛCDM model, a spatially flat universe is favored at very high confidence levels by several popular observations, including the latest Planck-2015 results of cosmic microwave background (CMB) observations (Ade et al. 2016). However, almost all of these constraints on the cosmic curvature were indirectly obtained by assuming some specific dark energy models. It should be noted that, on one hand, spatial curvature is related to the nature of spacetime geometry. On the other hand, it also relates to the evolution of the universe and the various states of the universe in many cosmological models. For example, there is a strong degeneracy between the equation of state of dark energy and the spatial curvature in some standard dark energy scenarios (Clarkson et al. 2007; Gong & Wang 2007). Therefore, on the basis of the fundamental cosmological principle assumption, model-independent measurements for the spatial curvature have always been an important task in cosmology. That is, model-independent measurements for the spatial curvature are helpful for breaking this degeneracy and thus play an important role in exploring the nature of dark energy. In addition, small changes of the spatial curvature have a huge impact on the early inflation models (Eisenstein et al. 2005; Tegmark et al. 2006; Wright 2007; Zhao et al. 2007).

Recently, Clarkson et al. (2008) proposed tetomg the radial homogeneity of the universe in a model-independent way by directly measuring the spatial curvature at different redshifts with observations of the expansion rate and distance. Later, this test was widely implemented with updated observations (Shafieloo & Clarkson 2010; Mörtssell & Jönsson 2011; Li et al. 2014; Sapone et al. 2014; Cai et al. 2016). Results consistently suggested that the cosmological principle is valid. Moreover, in this way, one can simultaneously achieve model-independent estimations for the spatial curvature. However, derivatives of distance with respect to redshift usually lead to large uncertainty. Therefore, dodging the derivative of distance with respect to redshift has been recently proposed to obtain constraints on the curvature with greater precision from observations of expansion rate and distance (Li et al. 2016; Yu & Wang 2016; Wei & Wu 2017). Meanwhile, a similar test has also been presented to check the validity of the FLRW metric using parallax distances and angular diameter distances (Räsänen 2014). In parallel, the distance sum rule (DSR), which characterizes the relation of distances along null geodesics in the FLRW background, has been proposed to be a practical measurement of the curvature of the universe by studying the cross-correlation between foreground mass and gravitational shear of background galaxies (Bernstein 2006). More recently, DSR has been put forward as a consistency test (Räsänen et al. 2015). That is, on one hand, the FLRW spacetime will be ruled out if DSR is disfavored by observations; on the other hand, if observations are in agreement with DSR, this test achieves a model-independent estimation for the spatial curvature of the universe. In Räsänen et al. (2015), using strong gravitational lensing (SGL) data selected from the Sloan Lens ACS Survey (SLACS; Bolton et al. 2008) and the Union2.1 compilation of SN Ia (Suzuki et al. 2012), they found that the spatial curvature parameter is constrained to be $\Omega_\text{K} = -0.55^{+0.18}_{-0.12}$ at the 95% confidence level, which slightly favors a spatially closed universe. The following studies have been carried out with the latest SGL (Cao et al. 2015) and SNe Ia (Betoule et al. 2014) observations available at the time: Xia et al. (2017), Li et al. (2018a). In addition, Qi et al. (2018) have used...
radio quasars as distance indicators (Gurvits et al. 1999; Cao et al. 2017a) together with SGL observations to extend the analysis to higher redshift. They obtained that, compared with constraints on the curvature obtained in Räsänen et al. (2015), a spatially flat universe is not preferred at higher confidence levels. In this paper, following the DSR method, we re-estimate the spatial curvature with the latest 163 well-measured SGL systems (Cao et al. 2015; Shu et al. 2017; Chen et al. 2019).

This paper is organized as follows. First, we briefly introduce the DSR and describe the observational data in Section 2. Next, in Section 3, we show corresponding constraint results. Finally, conclusions and discussions are presented in Section 4.

2. Methodology

Following the assumption that the cosmological principle is satisfied, the FLRW metric can be used to describe the spacetime geometry of the universe (in units where $c = 1$):

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1 - kr^2} + r^2d\Omega^2\right),$$

where $k$ is a constant related to the spatial curvature that takes the value $-1, 0, 1$. Let $d_l(z_l, z_s)$ be the angular diameter distance of a source at redshift $z_s$ (emission time $t_e$) as seen at redshift $z_l$. Then we can define a dimensionless comoving angular diameter distance

$$d(z_l, z_s) = \frac{1}{\sqrt{|\Omega_k|}} f\left(\frac{z}{z_l}E(z)\right),$$

where

$$f(x) = \begin{cases} \sin(x), & \Omega_k < 0, \\ x, & \Omega_k = 0, \\ \sinh(x), & \Omega_k > 0, \end{cases}$$

and $\Omega_k \equiv -k/(H_0^2 a_0^2)$. Here $H_0$ is the present value of the Hubble parameter and $a_0 = a(0)$ is the scale factor at the present time.

In order to get model-independent estimates of the spatial curvature via Equation (4), we can in principle, use different kinds of distance indicators, such as standard candles, rulers, and sirens for providing distances $d_l$ and $d_s$ on the right side of Equation (4). Here, we use SNe Ia and intermediate-luminosity quasars (ILQSO) observations to obtain $d_l$ and $d_s$.

2.1. Distances: $d_l$ and $d_s$

In order to get model-independent estimates of the spatial curvature via Equation (4), we can in principle, use different kinds of distance indicators, such as standard candles, rulers, and sirens for providing distances $d_l$ and $d_s$ on the right side of Equation (4). Here, we use SNe Ia and intermediate-luminosity quasars (ILQSO) observations to obtain $d_l$ and $d_s$.

(i) SNe Ia data: in previous works (Xia et al. 2017; Li et al. 2018a), the sample from a joint light-curve analysis (JLA) of SNe Ia observations obtained by the SDSS-II and SNLS collaborations (Betoule et al. 2014) was considered. The data set includes several low-redshift samples ($z < 0.1$), all three seasons from SDSS-II ($0.05 < z < 0.4$), and three years from SNLS ($0.2 < z < 1$), and it totals 740 spectroscopically confirmed SNe Ia with high-quality light curves. For these well-measured events, the SALT2 model is used to reconstruct light-curve parameters ($x_1$, stretching of the light curve, $c$—the color of SNe Ia at maximum brightness, and $m_{B,0}$—the observed peak magnitude in the rest-frame $B$ band). In this case, the distance modulus $\mu = m_B - M_B + \eta \times x_1 - \nu \times c$ includes two nuisance parameters characterizing the stretch-luminosity and color-luminosity relationships. In other words, they represent the well-known broader-brighter and bluer-brighter relationships, respectively. The value of $M_B$ is another nuisance parameter that denotes the absolute magnitude of a fiducial SNe. It was found that the absolute magnitude of a fiducial SNe Ia depends on the properties of host galaxies, e.g., the host stellar mass ($M_{stellar}$). In the JLA SNe Ia, this dependence is approximately corrected with a simple step function when the mechanism has not been fully understood (Conley et al. 2011; Sullivan et al. 2011).

Recently, Scolnic et al. (2018) released a new data set called Pantheon SNe Ia, which consists of 1048 SNe Ia in the redshift range $0.01 < z < 2.3$. Like the previously mentioned JLA SNe Ia, the nuisance parameters $\eta$ and $\nu$ are usually regarded as free parameters and are constrained together with cosmological parameters (Li et al. 2011, 2012). However, this approach might result in cosmological model dependence, therefore the distance calibrated in a specific cosmological model should not be directly used for other cosmological implications. To dodge this problem, Kessler & Scolnic (2017) proposed a new method called BEAMS with Bias Corrections (BBC) to calibrate the SNe. This method is based on the approach proposed by Marriner et al. (2011) but includes extensive simulations for correcting the SALT2 light curve fitter. Moreover, the simulation also depends on an input cosmology, but the changes in the input cosmology within typical statistical uncertainties are generally negligible. With the BBC method, Scolnic et al. (2018) reported the corrected apparent magnitude $m_{B,corr} = m_{B,corr}^\mu + \eta \times x_1 - \nu \times c + \Delta_B$ for all the SNe Ia. Therefore, we just need to subtract $M_B$ from $m_{B,corr}$ to calculate the distance moduli, relating to the luminosity distance $D_L$ via

$$\mu = 5\log\frac{D_L}{Mpc} + 25 \quad \text{(please refer to Scolnic et al. 2018 for detailed information on the Pantheon SNe Ia)}:$$

$$\mu^{SN} = m_{B,corr}^\mu - M_B.$$
Pantheon SNe Ia data have extended redshifts to 2.3 and subtract some nuisance parameters, so we choose this as a distance indicator. With the distance–duality relation, which holds in any spacetime (Etherington 1933; Ellis 2009), the dimensionless comoving angular diameter distance \( d = H_0 D_d/(1 + z) \) can be obtained by normalizing \( H_0 \). Here, the result from CMB measurements (Ade et al. 2016), \( H_0 = 67.74 \pm 0.46 \), is used.\(^1\)

(ii) ILQSO data: the possibility of using compact radio sources to study cosmological parameters became very attractive (Zhu & Fujimoto 2002; Chen & Ratra 2003). In order to extend our analysis to higher redshifts, we use the sample of 120 ILQSO as distance indicators in the redshift range \( 0.46 < z < 2.76 \) (Cao et al. 2017a). Our procedure follows the phenomenological model, which quantifies the luminosity \( L \) and redshift \( z \) dependence of the linear sizes of quasars as (Gurvits 1994; Gurvits et al. 1999; Cao et al. 2017a; Qi et al. 2018)

\[
l_m = l^* (1 + z)^n,
\]

where \( l \) is the linear size scaling factor, and \( \gamma \) and \( n \) quantify the dependence of the linear size on source luminosity and redshift, respectively. Following Cao et al. (2017a), for a sample of 120 ILQSO, the linear size \( l_m \) is independent of both redshift luminosity \( L \) and redshift \( z \) (\( |\gamma| \approx 10^{-3}, |n| \approx 10^{-2} \)), and the linear size \( l_m \) was 11.03 pc at 2.29 GHz. We will use the above value of the linear size \( l_m \) to calculate the angular diameter distances to the ILQSO sample

\[
D_h(z) = \frac{l_m}{\theta(z)},
\]

where \( \theta(z) \) is the angular size at redshift \( z \) with error \( \sigma_\theta = (\sigma_{\theta_{\text{ILQSO}}}^2 + \sigma_{\theta_{\text{SIE}}}^2)^{1/2}, \) and \( \sigma_{\theta_{\text{ILQSO}}} \) and \( \sigma_{\theta_{\text{SIE}}} \) are observational uncertainty and 10% systematic uncertainty, respectively. Then one can use angular diameter distances to the quasars to obtain the dimensionless distances \( d = H_0 (1 + z) D_h(z) \).

In our analysis, as proposed in Räsänen et al. (2015), we model-independently determine the function of dimensionless angular diameter distance with respect to redshift (i.e., \( d(z) \)) by fitting a polynomial to SNe Ia data and ILQSO data. Here, we use a simple third-order polynomial function with initial conditions, \( d(0) = 0 \) and \( d'(0) = 1 \), to fit the cosmology-free distances of SNe Ia. This polynomial is expressed as

\[
d(z) = z + a_1 z^2 + a_2 z^3,
\]

where \( a_i \) are two free parameters that need to be constrained simultaneously with light-curving fitting parameters. It has been suggested that, with current data, it does not make a significant difference which function is used, as long as it is more flexible than a second-order polynomial (Räsänen et al. 2015). We show redshift distributions of both Pantheon SNe Ia samples, and ILQSO samples in Figure 1.

\[\text{Figure 1. Redshift distributions for observational data sets including sources and lenses from all 163 SGL samples, Pantheon SN Ia, and ILQSO.}\]

2.2. The Distance Ratio: \( d_h/d_s \)

Since the discovery of the first gravitational lens Q0957+561 (Walsh et al. 1979), and with the increasing number of SGL systems detected, strong lensing has become an important astrophysical and cosmological probe (Futamase & Yoshida 2001; Biesiada 2006; Grillo et al. 2008), e.g., tests of general relativity (Cao et al. 2017b), tests of dark energy (Biesiada et al. 2010, 2011; Cao et al. 2012a; Cao & Zhu 2014; Chen et al. 2015; Mario et al. 2019), and constraints on cosmological models (Cao & Zhu 2012; Cao et al. 2012b). If general relativity holds on the scale of the lensing system, distance ratio \( d_h/d_s \) can be measured from observations for angular separation between lensed images of the same source and the structure of the lens. We will consider two types of lens models that have been extensively used in strong lensing studies.

(i) Singular isothermal ellipsoid (SIE) model: for the SIE lens profile, we have

\[
d_h/d_s = \frac{\theta_E}{4\pi \alpha^2 \beta^2},
\]

where \( \theta_E \) is the Einstein radius. In our analysis, we assign an error of 5% on \( \theta_E \) (Cao et al. 2015). \( \sigma \) is the velocity dispersion of the lens and \( f \) is a phenomenological coefficient that characterizes uncertainty due to the difference between the velocity dispersion of the observed stars and the underlying dark matter, and other systematic effects. In general, observations suggest the range \( 0.8 < f^2 < 1.2 \) (Ofek et al. 2003). We take \( f \) as a free parameter, which is assumed as a flat prior, on the same weight as \( \Omega_c \) (Li et al. 2018a).

(ii) Extended power-law (EPL) model: in addition to the SIE lens profile, we also consider a more complicated profile for the mass distribution of the lens, named the EPL model. In this case, the distance ratio is written as (Chen et al. 2019)

\[
d_h/d_s = \frac{\theta_E}{4\pi \sigma_j^2 f^2(\alpha, \beta, \delta)}\left(\frac{\theta_E}{\theta_h}\right)^{2-\alpha},
\]

where

\[
f(\alpha, \beta, \delta) = \frac{(\xi - 2\beta)(3 - \xi)\lambda(\alpha)\lambda(\delta)}{2\sqrt{\pi}(3 - \delta)(\lambda(\xi) - \beta\lambda(\xi + 2))},
\]

\(^1\) We also checked the dependence of our analysis on \( H_0 \) using the result from the latest local direct measurements (Riess et al. 2019), \( H_0 = 74.03 \pm 1.42 \), to normalize it, and found that this change leads to very tiny influence on estimations for cosmic curvature.

\(^2\) We also checked the dependence of our analysis on the form of \( d(z) \) using the logarithmic parameterization (Risaliti & Lusso 2018) to fit the SNe Ia data and ILQSO data, and found that there is a tiny difference between results when the polynomial expression and the logarithmic parameterization are considered separately.
with $\xi = \alpha + \delta - 2$ and $\lambda(\xi) = \Gamma(\frac{\xi+1}{2})/\Gamma(\frac{\xi}{2})$. Moreover, we use two different kinds of velocity dispersion with different $\theta$. From the spectroscopic data, one can measure the velocity dispersion $\sigma_{ap}$ inside the circular aperture with the angular radius $\theta_{ap}$. For the sake of comparison and in consideration of the effect of the aperture size on the measurements of velocity dispersions, all velocity dispersions $\sigma_{ap}$ measured within apertures of arbitrary sizes are normalized to a typical physical aperture, $\sigma_0 = \sigma_{ap} \theta_{ap}/(2\theta_{ap})$, with the radius $\theta_{ap}/2$, where $\theta_{eff}$ is the half-light radius of the lens galaxy. In this work, we adopt the best-fitting values of $\eta$ as $-0.066 \pm 0.035$ from Cappellari et al. (2006). Then, the total uncertainty of $\sigma_0$ consists of the following three ingredients:

$$\Delta \sigma_0^{\text{tot}} = (\Delta \sigma_0^{\text{stat}})^2 + (\Delta \sigma_0^{\text{AC}})^2 + (\Delta \sigma_0^{\text{sys}})^2,$$

where $\Delta \sigma_0^{\text{stat}}$ is the statistical error from the measurement error of $\sigma_{ap}$. The error due to the aperture correction, $\Delta \sigma_0^{\text{AC}}$, is propagated from the uncertainty of $\eta$. In addition to the measurement errors, the uncertainty of $\sigma_{ap}$ should include a systematic error of 3%, which mainly comes from the uncertainty in the projected gravitational mass $M_{\text{proj}}$ (Jiang & Kochanek 2007). In addition, $\alpha$ and $\delta$ are free parameters that are from the total mass–density profile $\rho(r) \sim r^{-\alpha}$ and the luminosity-density profile $\nu(r) \sim r^{-\delta}$, respectively. The parameter $\beta(r) = 1 - \sigma_0^2 / \sigma_0^2$ denotes the anisotropy of the stellar velocity dispersion, and is also called the stellar orbital anisotropy, where $\sigma_0$ and $\sigma_r$ are the tangential and radial velocity dispersions, respectively. In the literature, $\beta$ is usually assumed to be independent of $r$ (Schwab et al. 2010; Cao et al. 2016; Xia et al. 2017; Qi et al. 2018; Chen et al. 2019). Following these previous works, we also treat $\beta$ as a nuisance parameter and marginalize over it using a Gaussian prior with $\beta = 0.18 \pm 0.13$, based on the well-studied sample of nearby elliptical galaxies. In addition, we assume a flat prior for each remaining parameter of the EPL model ($\alpha$, $\delta$).

The methodology described above is implemented for the sample of 163 galactic scale SGL systems from the SLACS (Bolton et al. 2008; Auger et al. 2009, 2010; Shu et al. 2015; Shu et al. 2017), Baryon Oscillation Spectroscopic Survey (BOSS) emission-line lens survey (BELLS; Brownstein et al. 2012; Shu et al. 2016, 2016), Lens Structure and Dynamics (LSD; Koopmans & Treu 2002, 2003; Treu & Koopmans 2002, 2004) and Strong Lensing Legacy Survey (SL2S; Ruff et al. 2011; Sonnenfeld et al. 2013a, 2013b, 2015) assembled by Chen et al. (2019). The SLACS consists of spectroscopic lens surveys in which candidates are selected from Sloan Digital Sky Survey (SDSS) data. Then candidates were followed up with Hubble Space Telescope (HST) photometry from the Advanced Camera for Survey (ACS). The SDSS spectroscopy provides precise measurements, e.g., the stellar velocity dispersion and the redshifts of the lens and source, while high-resolution HST imaging yields a detailed view of the lensed background source and the surface brightness profile of the lensing galaxy. BELLS consists of spectroscopic lens surveys in which candidates are selected from the BOSS, which has been initiated by upgrading SDSS-I optical spectrographs (Eisenstein et al. 2011). Those candidates from SLACS and BELLS, which have multiple images or Einstein rings, have been classified as confirmed lenses. The LSD survey was a predecessor of SLACS that involved combining velocity dispersion data from ground-based instruments (e.g., Keck) and photometric data from space-based instruments (HST), and using different methods, comparing SLACS and BELLS to select lens systems. Therefore, in order to comply with SLACS and BELLS, there are just five reliable lenses from LSD (Cao et al. 2015; Chen et al. 2019). Finally, the SL2S is a project dedicated to finding galaxy-scale lenses in the Canada–France–Hawaii–Telescope Legacy Survey. The targets are massive red galaxies that can be followed up with HST and spectroscopy.

In Figure 1, we also show the redshift distribution of lens and source galaxies of SGL systems. A model-independent method of Gaussian processes (Seikel et al. 2012) was applied to reconstruct the dimensionless comoving distance from the ILQSO data in a straightforward way, without any parametric assumption regarding the cosmological model (Qi et al. 2018). However, the fidelity of the reconstructed function from most currently available observational data with Gaussian processes should be further discussed and tested (Zhou & Li 2019). Therefore, we combine Pantheon SNe Ia and ILQSO as distance indicators to extend redshift to 2.8. In this case, the maximum source redshift should be cut off at $z = 2.8$. This full sample contains 152 SGL systems and is named “Sample-152.” Moreover, the stellar mass of the lens galaxy also might result in possible bias, which has implications for SGL observations (Cao et al. 2012a, 2016; Xia et al. 2017; Li et al. 2018a). Therefore, systems with a typical velocity dispersion of the lens galaxy ranging from 200 to 300 km s$^{-1}$ were selected as a subsample for the sake of comparison. This subsample consists of 106 lensing systems and is named “Sample-106.” If we only use Pantheon SNe Ia as distance indicators, the maximum source redshift should be cut off at $z = 2.3$. This subsample contains 137 SGL systems and is named “Sample-137.” To avoid possible systematic bias from combining the SGL data of different surveys, we estimate the cosmic curvature with the latest SGL data, which only come from SLACS and include 97 well-measured systems. This subsample is named “Sample-97.” Because the maximum source redshift of SLACS is 1.3, we just use Pantheon SNe Ia as distance indicators for “Sample-97.”

3. Results

As previously mentioned, the aim of this work is to re-estimate the spatial curvature with 163 systems with strong gravitational lensing data. We infer the value of $\Omega_k$ via Equation (4) by comparing measurements of the latest distance indicators (Pantheon SNe Ia and ILQSO) with SGL observations. In our analysis, we perform a global fitting with emcee (Foreman-Mackey et al. 2012), using the Python module including Markov chain Monte Carlo. By marginalizing the light-curve fitting parameters ($M_0$), and the polynomial coefficients ($\alpha_1$, $\alpha_2$), we obtain the marginalized distributions with $1\sigma$ and $2\sigma$ confidence level contours for the curvature $\Omega_k$ and lens profile parameter(s) $f$ (or $\alpha$, $\beta$, and $\delta$).

(i) SIE model: for the simple SIE model, graphical and numerical results from each SGL data set are shown in Figures 2–4 and Table 1. For the full sample (Sample-152), $\Omega_k = -0.223^{+0.511}_{-0.305}$ and $\Omega_k = 0.224^{+0.698}_{-0.428}$ are obtained at the 95% confidence level when $\sigma_{ap}$ and $\sigma_0$ are used, respectively. It is suggested that, compared to results in Räsänen et al. (2015; $\Omega_k = -0.55^{+0.11}_{-0.67}$ at the 95% confidence level) and Xia et al. (2017; $\Omega_k < 0.6$ at the 95% confidence level), the precision of
constraints on $\Omega_k$ has been significantly improved due to an increase of the number of well-measured strong lensing systems. In addition, the latest distance indicators (Pantheon SNe Ia and ILQSO) extend the redshift of $d_l$ or $d_s$ data from 1.3 to 2.8, which allows us to add about 40 strong gravitational lens systems in this redshift range. Note that, in our analysis, we estimate all free parameters in a global fit without taking any priors for both $\Omega_k$ and $f$ into consideration. Moreover, we find that a spatially flat universe is consistently favored by these observations at high confidence levels. This is somewhat different from the results of Räsänen et al. (2015), which found a slight trend toward a spatially closed universe. Interestingly, these model-independent estimations are in good agreement with the conclusions of the latest CMB observations (Ade et al. 2016). In addition, as shown in Figure 2, there is a small discrepancy between the estimations of $f$ ($f = 1.039^{+0.017}_{-0.019}$ and $f = 1.020^{+0.021}_{-0.019}$ at the 95% confidence level) when $\sigma_{ap}$ and $\sigma_0$ are used separately. Results regarding $f$ suggest that the mass distribution profiles of lens galaxies are marginally consistent with the simplest singular isothermal sphere (SIS) model ($f = 1$). These results are similar to the results of Xia et al. (2017), where the SIS profile was almost disfavored at more than the 95% confidence level when the prior for the curvature from the Planck-2015 CMB observations ($\Omega_k > -0.1$) was considered. On one hand, this discrepancy might imply that the SIS profile is too simple to characterize the mass distribution of lens galaxies. On the other hand, the deviations of constraints on $f$ from the SIS model also might indicate that lensing systems dominated by groups/clusters would affect fitting results (Faure et al. 2011). Finally, as shown in Figures 2–4 and Table 1, there are obvious differences in both cosmic spatial curvature and lens model parameters when $\sigma_{ap}$ and $\sigma_0$ are used separately.

(ii) EPL model: for the EPL model, graphical and numerical results from each SGL data set are shown in Figures 5–7 and Table 2. In the framework of the EPL model, comparing to the results of SIE model, a spatially flat universe is more strongly supported by these available observations. In addition, as shown in Figure 5 and Table 2, the estimation of the spatial curvature is insensitive to different velocity dispersion, which means that the EPL model is more suitable for estimating the cosmic curvature. Previous observational constraints on the total mass–density profile were $\alpha \sim 2$ (Koopmans et al. 2009; Schwab et al. 2010; Sonnenfeld et al. 2013b; Oguri et al. 2014; Cao et al. 2016; Xia et al. 2017; Chen et al. 2019). As shown in Figures 6–7 and Table 2, our constraints on it are marginally...
consistent with $\alpha = 2$ at the 95% confidence level when $\sigma_{ap}$ is used. However, the constraints of the total mass–density profile are more consistent with $\alpha = 2$, which means the velocity dispersion $\sigma_0$ is more suitable for constraining the lens model parameters. In addition, the EPL model in which mass traces light ($\alpha = \delta = 2$) is excluded at the >95% confidence level, which might be helpful for understanding the difference in mass–density distributions of dark matter and luminous baryons in early-type galaxies. As shown in Figures 5–7, there are not only different degrees of degeneracy between parameters characterizing the lens mass profile and $\Omega_k$ lens model parameters themselves also degenerate with each other. In this case, additional observational information, such as stellar velocity dispersion of the lens galaxy, can be collected to provide complementary constraints on the slope of the total mass–density profile and thus are helpful for constraining $\alpha$, $\beta$, $\delta$, or even estimating $\Omega_k$. In addition, auxiliary data can be used to improve constraints on $\alpha$, $\beta$, and $\delta$ in the future (Cao et al. 2017b). For example, $\alpha$ can be inferred for individual lenses from high-resolution imaging of arcs (Suyu et al. 2007; Vegetti et al. 2010; Collett & Auger 2014; Wong et al. 2015), while constraints on $\beta$ and $\delta$ can be improved with integral field unit data (Barnabé et al. 2013). Although the result of “Sample-97” can be compatible with a spatially flat universe at the 95% confidence level, it is obvious that the cosmic curvature of “Sample-97” is very different from “Sample” in the SIE model. However, these differences are alleviated in the EPL model, which means that the EPL model is more reasonable for describing gravitational lenses and estimating cosmic curvature. On the whole, it is suggested that the EPL model with velocity dispersion $\sigma_0$ is more suitable for constraining the parameters of both the lens and cosmic curvature.

### Table 1

| parameters | Sample-152 | Sample-137 | Sample-106 | Sample-97 |
|------------|------------|------------|------------|------------|
| $\Omega_k$ | $-0.223^{+0.211}_{-0.124}$ | $0.030^{+0.112}_{-0.082}$ | $-0.428^{+0.230}_{-0.141}$ | $-0.752^{+0.701}_{-0.414}$ |
| $f$        | $1.039^{+0.009}_{-0.008}$ | $1.038^{+0.010}_{-0.010}$ | $1.028^{+0.015}_{-0.012}$ | $1.024^{+0.008}_{-0.012}$ |
| $M_B$      | $-19.383^{+0.006}_{-0.006}$ | $-19.429^{+0.007}_{-0.007}$ | $-19.383^{+0.006}_{-0.006}$ | $-19.420^{+0.007}_{-0.007}$ |
| $\omega_1$ | $-0.296^{+0.007}_{-0.008}$ | $-0.215^{+0.013}_{-0.014}$ | $-0.296^{+0.008}_{-0.008}$ | $-0.226^{+0.013}_{-0.013}$ |
| $\omega_2$ | $0.021^{+0.004}_{-0.004}$ | $-0.007^{+0.012}_{-0.011}$ | $0.021^{+0.005}_{-0.005}$ | $0.004^{+0.011}_{-0.011}$ |

| $\sigma_0$ | $0.224^{+0.302}_{-0.184}$ | $0.483^{+0.385}_{-0.239}$ | $-0.105^{+0.111}_{-0.069}$ | $-0.969^{+0.774}_{-0.504}$ |
| $f$        | $1.026^{+0.010}_{-0.010}$ | $1.020^{+0.011}_{-0.010}$ | $1.005^{+0.011}_{-0.010}$ | $0.992^{+0.016}_{-0.010}$ |
| $M_B$      | $-19.383^{+0.006}_{-0.006}$ | $-19.429^{+0.007}_{-0.007}$ | $-19.383^{+0.006}_{-0.006}$ | $-19.426^{+0.007}_{-0.007}$ |
| $\omega_1$ | $-0.295^{+0.008}_{-0.008}$ | $-0.216^{+0.013}_{-0.014}$ | $-0.295^{+0.008}_{-0.008}$ | $-0.226^{+0.013}_{-0.013}$ |
| $\omega_2$ | $0.026^{+0.005}_{-0.005}$ | $-0.006^{+0.011}_{-0.011}$ | $0.020^{+0.005}_{-0.005}$ | $0.004^{+0.011}_{-0.011}$ |

Figure 5. 1D and 2D marginalized distributions with 1σ and 2σ confidence contours for the parameters $\Omega_k$, $\alpha$, $\beta$, and $\delta$ constrained from the “Sample-152” using the EPL model for the lens mass profile. The solid line and dotted lines represent $\sigma_{ap}$ and $\sigma_0$, respectively.

Figure 6. 1D and 2D marginalized distributions with 1σ and 2σ confidence contours for the parameters $\Omega_k$, $\alpha$, $\beta$, and $\delta$ constrained from four sub-samples using the EPL model for the lens mass profile when $\sigma_{ap}$ is used.
Table 2

Constraints on All Parameters from the Pantheon SNe Ia, ILQSO, and SGL Observations Using the EPL Model for the Lens Mass Profile

| parameters | Sample-152 | Sample-137 | Sample-106 | Sample-97 |
|------------|------------|------------|------------|------------|
| $\Omega_k$ | 0.024$^{+0.234}_{-0.150}$ | 0.138$^{+0.396}_{-0.076}$ | $-0.194^{+0.270}_{-0.163}$ | $-0.053^{+0.184}_{-0.049}$ |
| $\alpha$ | 1.919$^{+0.036}_{-0.033}$ | 1.912$^{+0.033}_{-0.039}$ | 1.894$^{+0.045}_{-0.043}$ | 1.931$^{+0.048}_{-0.043}$ |
| $\delta$ | 2.207$^{+0.092}_{-0.095}$ | 2.245$^{+0.075}_{-0.093}$ | 2.167$^{+0.090}_{-0.099}$ | 2.218$^{+0.068}_{-0.099}$ |
| $M_B$ | $-19.382^{+0.006}_{-0.006}$ | $-19.430^{+0.004}_{-0.006}$ | $-19.383^{+0.006}_{-0.006}$ | $-19.426^{+0.007}_{-0.007}$ |
| $a_1$ | $-0.296^{+0.007}_{-0.008}$ | $-0.213^{+0.012}_{-0.014}$ | $-0.295^{+0.008}_{-0.014}$ | $-0.222^{+0.013}_{-0.014}$ |
| $a_2$ | 0.021$^{+0.005}_{-0.004}$ | $-0.010^{+0.013}_{-0.008}$ | 0.020$^{+0.005}_{-0.005}$ | 0.004$^{+0.012}_{-0.012}$ |

| $\sigma_{\delta}$ |
|-------------------|
| $\Omega_k$ | 0.015$^{+0.381}_{-0.093}$ | 0.100$^{+0.538}_{-0.114}$ | $-0.078^{+0.118}_{-0.220}$ | $-0.048^{+0.154}_{-0.372}$ |
| $\alpha$ | 2.027$^{+0.024}_{-0.027}$ | 2.026$^{+0.023}_{-0.037}$ | 2.043$^{+0.034}_{-0.026}$ | 2.000$^{+0.036}_{-0.028}$ |
| $\delta$ | 2.259$^{+0.122}_{-0.084}$ | 2.253$^{+0.068}_{-0.107}$ | 2.187$^{+0.185}_{-0.193}$ | 2.182$^{+0.118}_{-0.115}$ |
| $M_B$ | $-19.383^{+0.006}_{-0.006}$ | $-19.430^{+0.007}_{-0.007}$ | $-19.383^{+0.007}_{-0.005}$ | $-19.426^{+0.008}_{-0.007}$ |
| $a_1$ | $-0.295^{+0.008}_{-0.008}$ | $-0.213^{+0.011}_{-0.011}$ | $-0.295^{+0.008}_{-0.008}$ | $-0.226^{+0.013}_{-0.013}$ |
| $a_2$ | 0.020$^{+0.005}_{-0.005}$ | $-0.009^{+0.010}_{-0.010}$ | 0.021$^{+0.005}_{-0.005}$ | 0.004$^{+0.011}_{-0.011}$ |

Figure 7. Same as Figure 6, except using $\sigma_0$. It has been suggested that constraints on cosmic curvature in the DSR method are not dependent on the different distance indicators used and similar results are obtained. That is, model-independent estimations for curvature are mainly determined by SGL observations. Moreover, results derived from only the SLACS catalog, which is characterized by a selection function favoring moderately large separation lenses (Arneson et al. 2012), suggest that this subsample might lead to biased estimation of the cosmic curvature. Therefore, more well-measured strong lensing events might be useful for reducing the bias in estimation of curvature. In this sense, analyzing simulated SGL observations from the Large Synoptic Survey Telescope (LSST) can give some predictions for constraints on cosmological parameters (Collett 2015; Cao et al. 2018; Qi et al. 2018; Ma et al. 2019). Qi et al. (2018) found that, combining about 16,000 strong lensing events combined with the distance information provided by 500 compact radio quasars, one can constrain the cosmic curvature with an accuracy of $\Omega_k \sim 10^{-6}$, which is comparable to the precision of Planck-2015 results. The upcoming LSST, which will monitor nearly half of the sky for 10 years by repeatedly scanning the field and is supposed to find $\sim 10^3$ lensed quasars (Oguri & Marshall 2010), will greatly improve this direct geometrical measurement for cosmic curvature.

4. Conclusion

On the basis of the cosmological principle, we can use the FLRW metric to describe the spacetime geometry of the universe. In this paper, using the DSR in the FLRW metric, we obtain direct geometrical estimations of the spatial curvature of the universe. These estimations are independent of the energy-momentum contents of the universe and the validity of the Einstein equation on cosmological scales. We re-estimate the cosmic curvature by combining the latest SGL data, which includes 163 well-measured systems with the latest distance indicators (Pantheon SNe Ia and ILQSO). Along with the spatial curvature $\Omega_k$, parameters characterizing the mass profiles of lens galaxies and polynomial coefficients are simultaneously constrained in a global fitting. Graphic results for these parameters ($\Omega_k$, and $f$ for $\alpha$, $\beta$, $\delta$) are shown in Figures 2–7 and estimations for all parameters are summarized in Tables 1–2. In summary, compared to the results obtained in Räsänen et al. (2015) and Li et al. (2018a), where a spatially closed universe was preferred, the constraints on the spatial curvature from the latest SGL data in our analysis strongly favor a spatially flat universe. It is suggested that more well-measured strong lensing systems, together with good distance indicators, might reduce the bias in these model-independent estimations. However, it should be pointed out that there still is a large gap between the precision of our model-independent estimations and that obtained from the Planck-2015 CMB observations in the standard $\Lambda$CDM model. Therefore, a large number of SGL systems from the program in the near future, e.g., the Euclid satellite and the LSST, are expected to obtain more precise constraints on the spatial curvature of the universe. In addition, other more promising lensing systems
have been proposed, such as, for example, time delay measurements of strongly lensed transients (such as gravitational waves and fast radio bursts) as a precision probe to constrain some fundamental cosmological parameters including the spatial curvature (Fan et al. 2017; Liao et al. 2017; Li et al. 2019). These upcoming improvements on the precision of model-independent estimation of cosmic curvature will be of great significance for breaking the degeneracy between the curvature and dark energy, and thus will be very helpful for studying the nature of dark energy or even understanding the physical mechanisms of cosmic acceleration.

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