How Does the Earth’s Rotation Affect Predictions of Gravitational Wave Strong Lensing Rates?

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Abstract

The next generation of ground-based gravitational wave (GW) detectors, e.g., the Einstein Telescope, is expected to observe a significant number of strongly lensed GW events as predicted in many previous papers. However, all these works ignored the impact of the Earth’s rotation on this prediction. Multiple lensed images arrive at the Earth at different time, thus the ground-based detector has different responses to the lensed images due to different orientations of the detector relative to the GW source direction. Therefore the amplitudes of the GW signal from different images are modulated appropriately, in addition to the lensing magnification. In order to assess this effect, we performed Monte Carlo simulations to calculate the event rate of lensed GW signals. Our conclusion is that the Earth’s rotation has a non-negligible impact on the event rate of lensed GW images. The updated event rates decrease by factors of ~40%, ~20%, and ~10%, for NS–NS, BH–NS, and BH–BH systems, respectively.

Key words: gravitational lensing: strong – gravitational waves

1. Introduction

After the first detection of gravitational wave (GW) event GW150914, which was produced by merging binary black holes (BBH; Abbott et al. 2016a), and other subsequent detections (Abbott et al. 2016b, 2017a, 2017b, 2017c, 2017d), GW astronomy came into being and a brand new window into the universe was opened. The event GW170817 registered by the LIGO-Virgo GW detector network (Abbott et al. 2017a) accompanied by the detection of an electromagnetic counterpart was a breakthrough that commenced a new era in multimessenger astronomy creating unique opportunities to deepen our understanding of the universe. One may expect that the LIGO-Virgo network will keep providing new detections and the next generation of ground-based detectors such as the Einstein Telescope (ET) operating with increased sensitivities will eventually yield $10^3$–$10^7$ inspiral events per year reaching the redshift $z = 17$ (Abernathy et al. 2011). Reaching so deep, a significant number of such signals could be lensed by intervening galaxies.

Gravitational lensing of GWs has been extensively studied in many works since the pioneering paper (Wang et al. 1996). In particular, the effect of lensing on the parameter extraction of GW signals was discussed by Cao et al. (2014). It was also suggested that cosmological parameters can be significantly constrained using time delay measurements of strongly lensed GW events (Sereno et al. 2010; Liao et al. 2017). Moreover, strongly lensed GW signals can be used to test fundamental physics. For example, the speed of gravity can be tested with strongly lensed GW events accompanied by electromagnetic counterparts (Collett & Bacon 2017; Fan et al. 2017). Admittedly, constraints on the speed of GWs (expressed as bounds on the graviton mass or equivalently on its Compton wavelength) obtained by the LIGO Collaboration Abbott et al. (2016c) with unlensed events are already very stringent. However, they strongly rely on the PPN waveform templates fitted to the data. The idea here is that if the Compton wavelength of the graviton is finite, lower frequencies would propagate slower than higher frequencies, leading to the dispersion that would modify phasing of the coalescing signal. Similarly, the bound obtained from the coalescing NS–NS system (Abbott et al. 2017e) was a conservative one assuming that GW and EM signals were emitted simultaneously and the observed delay was attributed solely to the difference of propagation speeds. Lensed GW-EM signals are free from such pre-assumptions. Therefore, the accurate prediction of the lensed GW event rate becomes an important issue.

Lensed event rates for the ET detector were studied in Piórkowska et al. (2013), Biesiada et al. (2014), and Ding et al. (2015). The intrinsic merger rates of the whole class of double compact objects—DCOs thereafter—i.e., (NS–NS,BH–NS, BH–BH systems) located at different redshifts were taken from the StarTrack population synthesis evolutionary code (Dominik et al. 2013). Optical depth for lensing was calculated using the singular isothermal sphere (SIS) lens model and Schechter-like velocity dispersion distribution in the population of lenses according to Choi et al. (2007). The general conclusion was that the ET would register about 50–100 strongly lensed inspiral events per year. These would be dominated by BH–BH events contributing 91%–95% to the total rate, depending on details of evolutionary scenarios considered. Recently, Li et al. (2018) extended these predictions to more realistic lens properties allowing for the ellipticity of the lens (and thus the quadruple lensed images), the lens environment (modeled as an external shear), and magnification bias. Meanwhile, the prediction of the lensing rate for Advanced LIGO was revisited by Ng et al. (2018), where it had been noticed that the detection is correlated with source position and detector’s orientation. Comparing to ground-based detectors, the lensing rates for the space-borne detector LISA were also discussed in Sereno et al. (2010, 2011).

All the works done so far have ignored the impact of the Earth’s rotation. The orientation between the source and the
detector was considered as fixed. However, because different lensed signals travel along different paths and probe different depths of the potential of the lens (Shapiro effect), they actually arrive at the Earth at different times. The typical time delay of multiple signals varies from days to hundreds of days. After a time delay, the rotation of the Earth changes the orientation of the ground-based detector with respect to the direction of the GW source. As we will see, this affects the strength of the observed signal, in addition to the relative magnification due to lensing. If not taken into account, it would bias the predictions.

In this paper, we are filling this gap by accounting for the Earth’s rotation in estimating the rates of lensed GW signals. We base our forecasts on the Monte Carlo simulation. In Section 2, we describe our methodology. In Section 3, we present and discuss the results. Finally, the conclusions are summarized in Section 4.

2. Methodology

Our predictions are focused on the ET, which will consist of three nested detectors placed underground at depths of 100–200 m, arranged in a triangular pattern. The initial design assumed that each detector would be built from a single interferometer, where the high power needed to achieve good high-frequency performance compromises the low-frequency performance. The next step in the ET design is the so-called “xylophone configuration,” where each detector is split into two interferometers, one specialized for detecting low-frequency GWs and the other one for the high-frequency part.

We briefly review the detection rate for unlensed events in Section 2.1 and lensing statistics in Section 2.2. Then, we describe the details of our Monte Carlo simulation and update the event rate of lensed GW sources that could be detected by the ET in Section 2.3.

2.1. Detection Rate of Unlensed Events

Since both the rudiments and the details of GW detection theory have been introduced many times (Finn & Chernoff 1993; Taylor & Gair 2012; Piórkowska et al. 2013; Biesiada et al. 2014), we only recap the main points for clarity. The matched filtering is a standard technique applied in GW data analysis to efficiently search for GW signals with known characteristics (templates) hidden in noisy data. Consequently, the strength of the signal is measured by the signal-to-noise ratio (S/N). The optimal matched-filtering S/N of an inspiralling DCO system at the redshift \( z_r \) registered by a single detector is

\[
\rho = 8\Theta \frac{r_0}{d_L(z_r)} \left( \frac{M_6}{1.2 M_6} \right)^{5/6} \sqrt{\zeta(f_{\max})},
\]

where \( d_L \) is the luminosity distance to the inspiralling DCO and \( r_0 \) is the detector’s characteristic distance parameter. For two configurations considered here, i.e., the ET initial design and the advanced “xylophone” design, \( r_0 = 1527 \text{ Mpc} \) and \( r_0 = 1918 \text{ Mpc} \), respectively. \( M_6 \) is the observed (redshifted) chirp mass (i.e., \( M_6 = M_0 (1 + z) \)). We have assumed that \( M_0 = 1.2 M_6 \) for NS–NS, 3.2 \( M_6 \) for BH–NS, and 6.7 \( M_6 \) for BH–BH systems respectively. According to Dominik et al. (2012), these values represent average chirp mass for each category of DCO simulated by population synthesis. They were also used in Dominik et al. (2013). \( \zeta(f_{\max}) \) is the dimensionless function reflecting the overlap between the GW signal and the detector’s effective bandwidth. \( \Theta \) is the orientation factor determined by four angles, according to:

\[
\Theta = 2F_z^2(1 + \cos^2 \psi)^2 + 4F_z \cos \psi \sin \psi, \quad (2)
\]

where

\[
F_z = \frac{1}{2}(1 + \cos^2 \psi)\cos 2\psi - \cos \psi \sin 2\psi
\]

and

\[
F_z = \frac{1}{2}(1 + \cos^2 \psi)\cos 2\psi + \cos \psi \sin 2\psi
\]

are the dimensionless expansion rate of the universe at redshift \( z \) and \( \frac{\sin \psi}{\cos \psi} \) are independent and one can assume that \( (\cos \theta, \phi, \psi, \cos t) \) are distributed uniformly over the range \([-1, 1]\). Due to rotation of the Earth, \( (\theta, \phi) \) of a given source would change appropriately, while \( (\psi, t) \) would be the same. Note that different values of \( (\theta, \phi) \) could modify the value of \( \Theta \) up to an order of magnitude, thus the effect of rotation of the Earth is nontrivial. For this reason, \( \rho \) of lensed images is not constant but is a function of time \( \rho(t) \). A single inspiral GW signal crosses the detector at one well defined moment \( t_0 \), \( S/N \) of such an event can be considered as a single value \( \rho(t_0) \). GW signal is detectable if its \( S/N \) exceeds the threshold, i.e., \( \rho > \rho_0 = 8 \). Then, the yearly detection rate of DCO sources can be expressed as:

\[
\dot{N}(>\rho_0) = \int_0^{z_r} \frac{d\dot{N}(>\rho_0)}{dz_r} dz_r
\]

where \( \frac{d\dot{N}(>\rho_0)}{dz_r} \) is the yearly merging rate of DCO sources in the redshift interval \([z_r, z_r + dz_r]\). In one such redshift interval, the rate of DCO events is

\[
d\dot{N} = 4\pi \left( \frac{c}{H_0} \right)^3 \frac{n_0(z_r) \tilde{r}^2(z_r)}{1 + z_r E(z_r)} dz_r,
\]

where \( n_0(z_r) \) denotes intrinsic inspiral rate at redshift \( z_r \), \( \tilde{r}(z_r) \) is the dimensionless comoving distance to the source, and \( E(z_r) \) is the dimensionless expansion rate of the universe at redshift \( z_r \). Concerning the cosmological model, we assume a flat \( \Lambda \)CDM model with \( \Omega_m = 0.3 \) and \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \) in order to comply with Dominik et al. (2013). Biesiada et al. (2014), and Ding et al. (2015). More importantly, we use the intrinsic merger rates \( \tilde{n_0}(z_r) \) predicted by the population synthesis model (using StarTrack code) in Dominik et al. (2013). We have taken data from the website http://www.syntheticuniverse.org, more specifically the so-called “rest frame rates” in the cosmicological scenario. In this code binary systems were evolved from ZAMS until the compact binary formation (after supernova explosions) making a number of assumptions about star formation rate, galaxy mass distribution, stellar populations, their metallicities, and galaxy metallicity evolution with redshift. In order to investigate the uncertainties of the chemical evolution of the universe, Dominik et al. (2013) employed two distinct scenarios for metallicity evolution with redshift, called “low-end” and “high-end” scenarios. These assumptions result in different DCO merger rate predictions and consequently to distinct lensed GW event rates. Because the compact object formation depends critically on the physics of the common
envelope phase of evolution and on the SN explosion mechanism and both of them are to some degree uncertain, Dominik et al. (2013) considered four scenarios: the standard one and three of its modifications—Optimistic Common Envelope, delayed SN explosion, and high BH kicks scenario. Some of the underlying assumptions could be constrained in light of existing GW detections (Chruslinska et al. 2018). However, the aim of this work is to evaluate the effect of the Earth’s rotation, hence we focus only on the standard scenario to facilitate the comparison with previous works.

2.2. Lensing Statistics

Similarly as in Biesiada et al. (2014) and Ding et al. (2015), we assume the SIS model of the lens. Of course, adopting more sophisticated lens models (such as SIE or power-law profile) could make the prediction more realistic and would enable us to study lensing systems with quadruple images. In the recent paper Li et al. (2018), in which the authors used the SIE model, the results turned out to be consistent with those by the SIS model (Biesiada et al. 2014; Ding et al. 2015); thus, we conclude that the SIS model is sufficient for the purpose of our study, which is the prediction of the event rates. From the physical point of view, mass distribution of lensing galaxies is the most relevant parameter for gravitational lensing. In the SIS model assumed here, the same information is conveyed by the stellar velocity dispersion, which is much easier to assess.

Einstein radius \( \theta_E \) gives the characteristic angular scale of lensing phenomenon. In the SIS model, it can be expressed as \( \theta_E = 4\pi \frac{z_l}{c} \frac{d_A(z_s)}{d(z_s, z_l)} \), where \( \sigma \) denotes velocity dispersion of lensing galaxy, \( d_A(z_s) \) and \( d_A(z_l, z_s) \) are angular diameter distances to the source and between the source and the lens, respectively. Introducing the angle \( \beta \), which is the angle between directions to the source location and to the center of the lens, one concludes that if \( \beta < \theta_E \), two strong lensed images (brighter one \( I_+ \) and fainter one \( I_- \)) will appear at locations \( \theta_{\pm} = \theta_E \pm \beta \). Introducing dimensionless quantities: \( x = \frac{x_l}{\theta_E} \) and \( y = \frac{y_l}{\theta_E} \), the strong lensing condition becomes \( y > 1 \) and positions of the images are \( x_{\pm} = 1 \pm y \) with magnifications \( \mu_{\pm} = \frac{1}{y} \pm 1 \). Gravitationally lensed GW signals, corresponding to these two images would have S/Ns:

\[
\rho_{\pm} = \rho_{\text{intr.}, \pm} \sqrt{\mu_{\pm}} = \rho_{\text{intr.}, \pm} \sqrt{\frac{1}{y} \pm 1}.
\]

Let us emphasize that the \( \rho_{\text{intr.}, \pm} \) and \( \rho_{\text{intr.}, \pm} \) are now different due to the rotation of the Earth.

In previous works, only the \( I_- \) image was considered necessary for the detection in order to establish the lensed nature of the signal. This was because its S/N \( \rho_- = \rho_{\text{intr.}, \pm} \sqrt{\mu_-} = \rho_{\text{intr.}, \pm} \sqrt{\frac{1}{y} - 1} \) was mistakenly assumed as being always lower than the S/N of \( I_+ \), i.e., \( \rho_+ = \rho_{\text{intr.}, \pm} \sqrt{\frac{1}{y} + 1} \), which lead to a misconception that detecting the \( I_- \) ensures the ability to detect \( I_+ \). However, due to the rotation of the Earth during the time delay between images it need not be the case. Therefore, we should treat the \( I_- \) and \( I_+ \) images separately and we propose that the lensed GW event registered by the ET requires both \( \rho_- \) and \( \rho_+ \) exceeding the threshold \( \rho_0 = 8 \). This means that

\[
\sqrt{\frac{1}{y} \pm 1} \rho_{\text{intr.}, \pm} > \rho_0 \text{ is required, which leads to the condition:}
\]

\[
\gamma_\pm \leq \gamma_{\pm,\text{max}} = \left[\left(\frac{8}{\rho_{\text{intr.}, \pm}^2} \pm 1\right)^{-1}\right].
\]

Thus, the elementary cross section for lensing reads:

\[
\Sigma_{\text{cr}, \pm}(\sigma, z_l, z_s, \rho_\pm) = \pi \theta_E^2 \gamma_{\pm,\text{max}}^2
\]

\[
= 16\pi^3 \left(\frac{\sigma}{c}\right)^4 \left(\frac{r}{r_0}\right)^2 \gamma_{\pm,\text{max}}^2.
\]

Finally, total optical depth \( \tau \) describes probability that GW source at redshift \( z_l \) would be lensed and detected by the ET. Since the images \( I_+ \) and \( I_- \) are considered separately, we split \( \tau \) into two terms, i.e., \( \tau_\pm \) corresponding to \( I_\pm \), respectively:

\[
\tau_\pm(z_l, \rho_\pm) = \frac{1}{4\pi} \int_0^{z_s} dz_l \int_0^\infty d\sigma \frac{4\pi}{c} \frac{c}{H_0} \int_0^{\tau_{\pm}} d\tau E(z_l) S_{\tau, \pm}(\sigma, z_l, z_s, \rho_\pm) \frac{dn}{d\sigma},
\]

where \( z_l \) is the redshift of the lens (we assume that lenses are distributed homogeneously in redshift). Moreover, we model the velocity dispersion distribution in the population of lensing galaxies as a modified Schechter function

\[
\frac{dn}{d\sigma} = n_\sigma \left(\frac{\sigma}{\sigma_\sigma}\right)^\alpha \exp\left(-\frac{\sigma}{\sigma_\sigma}\right)^\beta \frac{1}{\Gamma(\gamma)},
\]

with the parameters \( n_\sigma, \sigma_\sigma, \alpha, \) and \( \beta \) taken after Choi et al. (2007). This is the same assumption as that made in Ding et al. (2015)—we reproduced it for comparison of results. Admittedly, there exist more recent data for the velocity distributions of galaxies like, e.g., Bernardi et al. (2010). However, these fits were made for galaxies of all types, unlike Choi et al. (2007), who used early-type galaxies only.

2.3. Monte Carlo Method

Our goal is to update the event rates of lensed GW signals, explicitly considering variation of the detector’s orientation due to Earth’s rotation, which will modulate the S/Ns of \( I_+ \) and \( I_- \). This can hardly be done analytically, hence we need to perform the appropriate Monte Carlo simulation. We describe the details of the simulation in this section.

We build up a mock universe by creating a large sample of DCO events to represent the overall DCO events, and randomly generate the values of their key parameters. For a DCO event at redshift \( z_l \), its \( \rho \) at a specific moment can be randomly assigned with Equation (1). This is done by randomly assigning the orientation factor \( \Theta \) based on sampling the four angles \( (\theta, \phi, \psi, \iota) \). Let us note that, alternatively to the Monte Carlo method, the common practice is to define the \( \Theta \) by taking its averaged numerical probability density as (see formula 3.11 in Finn 1996):

\[
P_\Theta(\Theta) = 5\Theta(4 - \Theta)^{3/2}/256, \quad \text{if } 0 < \Theta < 4
\]

\[
P_\Theta(\Theta) = 0, \quad \text{otherwise.}
\]

It is instructive to recall the origin of Equation (8). First, Finn & Chernoff (1993) numerically estimated \( P(\Theta^2) \) using the Monte Carlo method and both of them are distributed uniformly in the range \([-1, 1] \).
Table 1
Predictions of Yearly Lensed GW Event Rates for which Only \( I_+ \) Image or Both \( I_- \) and \( I_+ \) Images are Magnified above the Threshold \( \rho_0 = 8 \)

| Metallicity Evolution | High | High | Low | Low |
|-----------------------|------|------|-----|-----|
| Which Event Rate      | Only \( I_- \) | Only \( I_- \) and \( I_+ \) | Only \( I_- \) and \( I_+ \) |
| NS–NS                 | 0.7  | 0.4  | 0.6 | 0.4 |
| Initial Design        | 1.4  | 1.1  | 1.2 | 0.7 |
| BH–NS                 | 2.2  | 1.8  | 2.9 | 2.3 |
| Xylophone             | 3.5  | 2.9  | 4.3 | 3.6 |
| BH–BH                 | 106.6| 94.3 | 130.3| 115.4|
| Xylophone             | 143.5| 128.0| 177.6| 159.2|
| Total                 | 109.5| 96.5 | 133.8| 118.1|
| Xylophone             | 148.4| 132.0| 183.1| 163.5|

Note. Results are shown for the standard model of DCO formation and two configurations of the ET. The "high" and "low" represent the "high-end" and "low-end" galaxy metallicity evolution.

Carlo simulation—see their Table 1, where the cumulative probability distribution of \( \Theta^2 \) is reported. Later, Finn (1996) noticed that "to an excellent approximation" \( P_\Theta(\Theta) \) can be described by the Equation (8). Since then this equation has been used abundantly due to its simple analytical form. In Figure 1 we compare our random sampling of \( \Theta \) based on \( 10^7 \) sample points (the same number as that used by Finn & Chernoff 1993) to this numerical probability density. One can see, that the numerical probability distribution Equation (8) is not an excellent approximation.

Identifying lensed DCO system by the ET requires that both lensed images \( I_- \) and \( I_+ \) are detected. We break this identification into two steps. We start with answering how many lensed DCO events have \( I_- \) available to the ET and not considering \( I_+ \) for a moment, and the corresponding optical depth is \( \tau_c(z_{\nu}, \rho_{\text{int}}) \) (see Equation (7)). Knowing this probability, the total number of \( I_- \) detected can be sampled by Monte Carlo simulation by accumulating the events that meet the requirement though all the space, i.e.,

\[
\int_{0}^{\infty} \int_{0}^{\infty} d\rho \tau(z_{\nu}, \rho) \frac{\partial N}{\partial \rho} dz d\rho
\]

Let us remark that quantitatively this number should be equivalent to numerical values reported in previous papers (Biesiada et al. 2014; Ding et al. 2015).

Once the set of \( I_- \) detected events is sampled, we can randomly assign the position \( y \) for each of them.\(^5\) The S/N for corresponding \( I_+ \) images can thus be calculated as \( \rho_+ = \rho_{\text{int}} \sqrt{\Delta t + 1} \). As mentioned above, \( \rho_{\text{int}} \) is different from \( \rho_{\text{int}} \) due to time delay. To take this difference into account, we keep the \( (\psi, \iota) \) values unchanged and regenerate the values of \( (\theta, \phi) \) to derive the \( \rho_{\text{int}} \). This regenerating depends on the location on the ET on Earth and on the time delay between \( I_- \) and \( I_+ \). In order to regenerate the values of \( (\theta, \phi) \) at \( t_+ \), knowing their initial values \( (\theta, \phi) \) at \( t_- \),

\[ t_+ \text{ and } t_- \text{ are the times of arrival for } I_+ \text{ and } I_-, \text{ respectively.} \]

\[ \text{we proceed in the following way. The angles } (\theta, \phi) \text{ are defined in a local detector's coordinate system, see Figure 2. For simplicity we assume that it is a cardinal coordinate system of the ET located at latitude } 45^\circ \text{ N, longitude } 0^\circ \text{ (Abernathy et al. 2011). Then we transform the coordinate of the GW signal to the center of the Earth with } z \text{ axis coincident with the Earth’s rotation axis. In this new nonrotating coordinate system, the direction of the GW signal is fixed.} \]

\[ \text{The angles } (\beta, \alpha) \text{ are used to describe the location of the ET: } \beta = \text{const.} = 45^\circ, \text{ while } \alpha = \alpha_0 + \Omega_{e} t, \text{ where } \Omega_{e} \frac{2\pi}{T} \text{ is the rotational angular velocity of the Earth. Consequently, } \alpha_+ = \alpha_+ - \Omega_{e} \Delta t, \text{ and knowing the time delay} \]

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Δt, we can transform the coordinate of the GW signal back to the detector’s frame. Furthermore, we assume the time delay \( Δt \) follows a uniform distribution as \([0, 24\ hr]\). Of course what matters here is \( Δt \mod 24\ hr \).

Knowing \( ρ_{\text{intr.}} \), we obtain the optical depth \( τ_{\text{opt}}(z, ρ_{\text{intr.}}) \) for “\( I_+ \)” which enables one to count the number of events having \( ρ_+ > ρ_0 = 8 \) across all the redshift bins. Finally, we count the events for which both the \( I_− \) and \( I_+ \) could be detected by the ET. Note that our procedure of considering the lensed image of \( I_− \) first and then \( I_+ \) does not mean that the \( I_− \) arrives earlier than \( I_+ \). Actually, the \( I_+ \) arrives first. However, the identification of lensed GW requires that both \( I_− \) and \( I_+ \) are detected. In this work, we consider detection of \( I_− \) as the prior condition.

We performed \( 10^5 \) realizations of Monte Carlo simulation for all DCO systems and averaged the results.

### 3. Results and Discussion

Table 1 shows the expected yearly rates of lensed GW, based on the standard DCO scenario. Cases where only the \( I_− \) image is detected and when both \( I_− \) and \( I_+ \) are detected by the ET, are shown. One can see that the event rate of “\( I_− \) and \( I_+ \)” is smaller than the event rate of “only \( I_− \)” This is the result of rotation of the Earth affecting the prediction of lensed event rates. Table 1 demonstrates that, compared to previous works, our updated event rates decrease by factors of \(~40\%\), \(~20\%\), \(~10\%\), for NS–NS, BH–NS, and BH–BH systems, respectively. The reason why different DCO systems are affected in a distinct way can be understood from the distribution of the intrinsic S/N for different types of binaries (see Figure 1 of Ding et al. 2015). In the case of NS–NS systems, detectable lensed events (i.e., with \( ρ > 8 \)) correspond to the upper tail of the distribution, while the events with \( ρ < 8 \) dominate in NS–NS systems. Therefore, these systems are more affected by the Earth’s rotation than BH–BH systems, which are dominated by \( ρ > 8 \) cases. Threshold value \( ρ = 8 \) splits the distribution of BH–NS systems into two approximately equal parts, which explains the intermediate value of their lensing rate reduction due to rotation of the Earth. However, because the event rate is dominated by BH–BH systems, total GW lensing rate decreases by a factor of \(~10\%\). Despite the above described reduction of yearly rates, this means that if the \( I_− \) image of the lensed GW event is detected, then the \( I_+ \) image is very likely to have been detected as well. One can expect that because the effect of the Earth’s rotation has also been neglected in Li et al. (2018) and Ng et al. (2018), their predictions of the total lensed event rate are overestimated by \(~10\%\). Figure 3 shows the relationship between the yearly lensed event rates of “\( I_− \) and \( I_+ \)” and the source redshift. This relationship looks very similar to previous results (Ding et al. 2015, right panel at Figure 2), which indicates that our Monte Carlo simulation achieved results consistent with numerical calculations.

In this work, we conservatively considered the SIS model, although considering the SIE model (or the power-law profile) would be more realistic and would enable one to study the quadruple lensing systems. However, for the purpose of predicting the lensed GW event rate, the SIS model is sufficient, considering the fact that the quad fraction is only about \(~10\%\) of all strong lensing systems (Oguri & Marshall 2010; Li et al. 2018). Of course, the predictions taking into account the Earth’s rotation would be more difficult and challenging for the SIE model because there would be three mutual time delays between images. However, the Monte Carlo simulation similar to that used in this paper, would then be the only reasonable approach.

The results obtained in this work are applicable to the ground-based detectors only. They are not valid for the next generation of space-based detectors like LISA, which will be placed on an Earth-trailing heliocentric orbit (Amaro-Seoane et al. 2013, 2017) with three satellites forming an equilateral triangle. Since the satellites follow their own orbits, the triangle will rotate as well, but the rotation period would be about 1 yr. This means that the rotation of the LISA detector would have less of an effect on the lensed GW detection rate than the rotation of the Earth has in the case of ground-based detectors.

### 4. Conclusion

In this work, we used the Monte Carlo simulation technique to evaluate the effect of the Earth’s rotation on the prediction of GWs strong lensing rates for the third generation ground-based detector—the ET. All previous works concerning GW lensing ignored this effect and assumed that detecting the fainter image \( I_− \) guarantees that the \( I_+ \) image could also be observed. This was a wrong, unjustified assumption. Our results show that the rate of detecting GW signals from both \( I_− \) and \( I_+ \) images is less than the rate of detecting \( I_− \) only. In particular, for NS–NS systems, the event rate of both \( I_− \) and \( I_+ \) images is \(~40\%\) less than the rate of detecting only one of them. For BH–BH systems, the impact of the Earth’s rotation is the smallest \(~10\%\). Furthermore, we have shown that the Monte Carlo method provides reliable and more accurate results compared to previous work (Piórkowska et al. 2013; Biesiada et al. 2014; Ding et al. 2015) using numerical calculation.

Since the total event rate is dominated by BH–BH systems, the impact of the Earth’s rotation on the total rate is at the level of \(~10\%\). Therefore, one should not worry much about this effect making cosmological inference, such as determination of the Hubble constant or cosmic equation of state, using catalogs of inspiral events. One should keep in mind, however, that this effect is the highest \(~40\%\) for NS–NS systems, which are accompanied by electromagnetic counterparts. Therefore, the Earth’s rotation should be taken into account in all considerations concerning lensed GW signals from coalescing NS binaries.
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