A refractive index in bent fibre optics and curved space

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Abstract. The refractive index in the bent fibre optics and in curved space of the gravitational field are studied. We obtain the results that refractive index in bent fibre optics is similar with refractive index in curved space of the gravitational field. Both results show that refractive index increases when light traverses through the bent fibre optics and curved space of the gravitational field. Based on analogy reasoning taken from fibre optics bending loss phenomena, we propose that if light traverses through curved space of the gravitational field, light will lose its energy. We calculate this energy loss numerically using bending loss model of Faustini-Martini. We find that light loses its energy around 467.7073 dB/m, when it traverses through curved space with radius of curvature $R = 0.01$ m due to massive object around $8.7 \times 10^{20}$ kg.

1. Introduction

The refractive index of a material medium is an important optical parameter since it exhibits the optical properties of the material [1]. The refractive index is one of the physicochemical properties of optical medium. It is defined as velocity of light of a given wavelength in empty space or vacuum ($c$) divided by its velocity in a substance ($v$), $n = c/v$. Maxwell’s equations relate the permittivity and the permeability to the refractive index as $n = \pm \sqrt{\varepsilon \mu}$. The sign of the refractive index is often taken as positive, but in 1968 Veselago shows that there are substrates with negative permittivity and negative permeability. In these substrates, refractive index have a negative value [2].

The refractive index is a scalar because it only depends on a property of incident wave (i.e. dispersion) [3]. Mathematically, refractive index is a tensor of rank-0 (scalar) and it can not be tensor of rank-1 (vector), but it can be tensor of rank-2, even a tensor of rank-3 and higher rank (which is well known as non-linear phenomena of order-2,3, etc) [4]. In the most substrates, the refractive index decreases by increasing of the temperature [2]. A denser material generally tends to have a larger refraction index [5]. The refractive index in an fibre optic can be changed due to external forces (tensile force, bending force) [6].

The refractive index has the large number of applications. It is mostly applied for identify a particular substance, confirm its purity or measure its concentration. It can be used also in determination of drug concentration in pharmaceutical industry, to calculate the focusing power...
of lenses and the dispersive power of prisms. Also it is applied for estimation of thermophysical properties of hydrocarbons and petroleum mixtures [2].

2. The Refractive Index in the Bent Fibre Optics

In fibre optics, the pure bending loss for a single mode fibre with a bending length \( L \), has the form [7]

\[
L_s = 10 \log_{10} e^{2\alpha L} = 8.686 \alpha L
\]  

(1)

where \( \alpha \) is an bending loss factor which is determined by the fibre structure, bending radius and input wavelength.

A simple formula for the bend loss factor \( \alpha \) was presented by Hagen Renner [7]

\[
\alpha = \alpha_0 \frac{2\sqrt{Z_2 Z_1}}{(Z_2 + Z_1) - (Z_2 - Z_1) \cos(2\theta_0)}
\]  

(2)

where \( Z_q = -\left(\frac{2k^2 n_q^2}{R}\right)^{2/3} X_q(x_2, 0); q = 1, 2; \theta_0 = \frac{2}{3}[-X_1(x_2, 0)]^{3/2} + \frac{\pi}{4} \) and

\[
X_q(x_2, 0) = \left(\frac{R}{2k^2 n_q^2}\right)^{2/3} \left[\beta^2 - k^2 n_q^2 \left(1 + \frac{2x}{R}\right)\right]
\]  

(3)

where \( R \) is radius of curvature (winding radius) of fibre optics, \( n_q \) is a refractive index in region \( q \) and \( \left(1 + \frac{2x}{R}\right) \) is correction factor of \( n_q \) due to geometry [8], [9], \( k \) is vacuum wave number at wavelength \( \lambda \) i.e. \( k = \frac{2\pi}{\lambda} \) and \( \beta \) is the complex propagation constant. For illustration of fibre optics bending, see Figure 1 below [9]

![Figure 1. Typical geometry of a bend fiber](image)

From eq.(3), we can define [8]

\[
n_{e,f}^2(x, y) \equiv n_q^2(x, y) \left(1 + \frac{2x}{R}\right)
\]  

(4)

where \( n_{e,f}(x, y) \) is the effective refraction index distribution in the bent fibre and \( n_q(x, y) \) is the refractive index distribution in the straight fibre. \( R \) denotes the "effective" bend radius which differs from the actual (experimental) bend radius \( R_{exp} \) by a material-dependent elastooptical correction factor. The bending of fibre optics will change the refractive index, due to photoelastic effect [10]. This eq.(4) relates the radius of curvature of fibre optics \( R \) with refractive index.

We calculate eq.(4) numerically. We assume that \( x = r \) i.e. radius of fibre optics (from origin to a point where refractive index is calculated) and \( R \) is radius of curvature/mandrel. We obtain the result as below
In Figure 2, we see that the refractive index of fibre optics is affected by radius of curvature, $R$. If $R$ tends to decrease (curvature $\kappa = 1/R$ \cite{11} increases) then the refractive index, $n_{\text{eff}}(x, y)$, tends to increase and vice versa.

3. The Refractive Index in Curved Space of the Gravitational Field

In the space of the gravitational field, the radius of curvature for the curved path traversed by light, $R$, is related to the space dependent refractive index, $n(r)$, by the relation \cite{12} or eq.(85.9) of \cite{13}

$$\frac{1}{R} = \mathbf{\hat{N}} \cdot \nabla \ln n(r) \quad (5)$$

where $\mathbf{\hat{N}}$ is an unit vector along the principal normal and \cite{12}

$$n(r) = \left(1 - \frac{2GM}{c^2 r}\right)^{-1} \quad (6)$$

Let us analyse eq.(5). First, we define that

$$\mathbf{\hat{N}} \equiv \frac{\nabla n}{|\nabla n|} \quad (7)$$

Substitute eq.(7) into eq.(5), we obtain

$$\frac{1}{R} = \frac{\nabla n}{|\nabla n|} \cdot \nabla \ln n = \frac{dn}{dr} \hat{r} \cdot \left( \frac{d}{dr} \hat{r} \right) \int \frac{1}{n} \frac{dn}{dr} = \frac{1}{n} \left( \frac{dn}{dr} \right)^2 \left( \frac{|dn|}{|dr|} \right)^{-1} \quad (8)$$

due to $n$ is depend on radius $r$ only. So,

$$\nabla n(r) = \frac{dn(r)}{dr} \hat{r} \quad (9)$$

and $\hat{r} \cdot \hat{r} = |\hat{r}| = 1$, $\hat{r}$ is a unit vector which its magnitude i.e. $|\hat{r}|$ is 1.

From (6), let us calculate derivative of $n(r)$ to $r$. We obtain

$$\frac{d}{dr} n(r) = - \left(1 - \frac{2GM}{c^2 r}\right)^{-2} \left( \frac{2GM}{c^2 r^2} \right) \quad (10)$$
Substitute (6), (10) into (8), we obtain
\[ R = \frac{c^2 r^2}{2GM} - r \]  
(11)
or
\[ M = \frac{c^2 r^2}{2G(R + r)} \]  
(12)

Eq.(11) is calculated numerically, using parameters: a gravitational constant, \( G = 6.67408 \times 10^{-11} \text{m}^3\text{kg}^{-1}\text{s}^{-2} \) and speed of light \( c = 299792458 \text{m/s} \). Here, \( r \) is radius from origin to the point where \( n \) is calculated and \( R \) is radius of curvature. We obtain

Figure 3. Numerical result of eq.(11)

In Figure 3, we see that the radius of curvature, \( R \), decreases if the mass of object, \( M \), increases and vice versa.

If we substitute \( M \) from eq.(11) into (6), we obtain
\[ n(r) = \left( 1 - \frac{2GM}{c^2r} \right)^{-1} = \left[ 1 - \frac{2G}{c^2r} \left( \frac{c^2 r^2}{2G(R + r)} \right) \right]^{-1} = 1 + \frac{r}{R} \]  
(13)
If eq.(13) is calculated numerically using the same parameters for calculating eq.(11) above, then we obtain

Figure 4. Numerical result of eq.(13)
In Figure 4, we see that the refractive index, $n$, increases when the radius of curvature, $R$, decreases and vice versa.

Refer to eq.(6), we can find relation between the refractive index and mass. Using the same parameters for calculating eq.(11), our numerical calculation gives the result as below

![Figure 5. Numerical result of eq.(6)](image)

In Figure 5, we see that the refractive index, $n$, increases when the mass, $M$, increases and vice versa.

4. Does Light Loss Its Energy when It Traverses through the Gravitational Space?

In analogy with the fibre optics, we propose that if the light traverses through the space of the gravitational field then it will loss its energy. Experimental evidence shows that the light will loss its energy if it traverses through a fibre medium (non-vacuum medium). This phenomena is called "attenuation". For example fibre attenuation is 0.12 dB/km for wavelength 1550 nm. Attenuation also occurs when the light traverses through air and gas [10].

More realistic model of fibre optics bending loss is given by Faustini-Martini [9] as below

$$2\alpha = \frac{\kappa^2 \exp\left(\frac{2\gamma^3 R}{3k^2 n_2^2}\right)}{\gamma V^2 R_1^2 (a\gamma)} \int_0^\infty \frac{\chi_2^{1/2} \chi_3^{1/2} \exp\left(\frac{2R^2}{\kappa^2 n_2^2}\right)}{\chi_2 \cos^2 \theta(\zeta) + \chi_3 \sin^2 \theta(\zeta)} d\zeta$$

(14)

We calculate eq.(14) numerically using parameters $n_2 = 1.4444$, $n_3 = 1.5$, $\pi = 3.14159$, $\lambda = 1.550e-006$ m, $k = 2\pi/\lambda$, $a = 2.4e - 6$ m, Number Aperture (NA) = 0.137, $V = (2\pi/\lambda) a$ $NA$, $n_1 = [(NA)^2 + (n_2)^2]^{1/2} = 1.4509$, $R_{exp} = 0.010$ m, $R = R_{exp} \times 1.325$ m, $b = 0.0000625$ m. We obtain that light losses its energy around 467.7073 dB/m. This result is related with light which traverses through curved space with a radius of curvature, $R$, due to massive object around $8.7 \times 10^{20}$ kg.
5. Discussions and Conclusions

The refractive index changes its value due to some reasons. In fact, in fibre optics experiment, the refractive index changes its value due to bending of fibre optics. Usually, in fibre optics bending loss experiment, this fibre optics is bend using mandrel. If the radius of curvature decreases (the curvature increases) then the refractive index increases. This fact raises our curiosity that if space is bend as curved space e.g. by mass and light travels through this curved space: is the refractive index of light increasing?

In cosmology, especially in general theory of relativity, it is well known that the existence of mass will warp space surrounding the mass [11, 14, 15]. Does it mean that the refractive index of light which traverses through the curved space will increase similar with the refractive index of light which traverses through bent fibre optics? We define (7) and analyse eqs.(5),(6), using definition of (7). We obtain the results as shown in eqs.(11), (13). This eq.(13) looks similar with the eq.(4). It shows that the refractive index increases when the light traverses through bent fibre optics and curved space of the gravitational field. We also calculate eqs.(4), (6), (11), (13), (14) numerically and find the results as shown in Fig.1-Fig.6.

In fibre optics bending loss phenomena, light losses its energy when light traverses through the bent fibre optics. Analog with this phenomena, we propose that light will loss its energy when light traverses through curved space. We calculate the energy loss of light when it traverses through curved space using the bend loss model of Faustini-Martini (14). We find numerically that light losses its energy around 467.7073 dB/m, when it traverses through curved space with radius of curvature \( R = 0.01 \, \text{m} \) due to massive object around \( 8.7 \times 10^{20} \, \text{kg} \). In future, this work can be extended to search the refractive index of light due to any astrophysical/topological objects like cosmic strings or extremely massive objects like black holes.

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