STUDY OF THE $\eta - ^7{\text{Be}}$ INTERACTION NEAR THRESHOLD IN
$p - ^6{\text{Li}}$ FUSION

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We present a calculation for $\eta$ production in the $p - ^6{\text{Li}}$ fusion near threshold including
the $\eta - ^7{\text{Be}}$ final state interaction (FSI). We consider the $^6{\text{Li}}$ and $^7{\text{Be}}$ nuclei as $\alpha - d$
and $\alpha - ^3{\text{He}}$ clusters respectively. The calculations are done for the lowest states of
$^7{\text{Be}}$ with $J = \left(\frac{3}{2}^{-}, \frac{1}{2}^{-}\right)$ resulting from the $L = 1$ radial wave function. The $\eta - ^7{\text{Be}}$
interaction is incorporated through the $\eta - ^7{\text{Be}}$ $T$-matrix, constructed from the medium
modified matrices for the $\eta - ^3{\text{He}}$ and $\eta - \alpha$ systems. These medium modified matrices
are obtained by solving few body equations, where the scattering in nuclear medium is
taken into account.

1. Introduction

The search for exotic $\eta$–nucleus quasi-bound states is guided by the strong and
attractive nature of the $\eta - N$ interaction in the $s$–wave. With this motivation,
measurements for the $^6{\text{Li}}(p, \eta)^7{\text{Be}}$ reaction were carried out by the Turin group
in 1993 at an incident energy of 683 MeV. A theoretical study of this reaction was made and it was summarized that the information available was not sufficient
to conclude the formation of an $\eta - ^7{\text{Be}}$ quasi-bound state. The interest in this
reaction has been revived by the recent sophisticated study of this reaction at the
COSY Laboratory at an incident energy of 673 MeV. Using the experience gained
by studying the interaction of $\eta$ with lighter nuclei like the deuteron and $^3{\text{He}}$
by solving few body equations, we extend our study to a heavier nucleus, $i.e.$, the
$^7{\text{Be}}$. 

2. The Formalism

2.1. The production mechanism

We consider $^6\text{Li}$ and $^7\text{Be}$ nuclei as $\alpha - d$ and $\alpha - ^3\text{He}$ clusters. The reaction mechanism is, hence, built by assuming a collision of the beam proton with the deuteron in $^6\text{Li}$ producing an $\eta$ and $^3\text{He}$. Combining with the spectator $\alpha$, $^3\text{He}$ produces $^7\text{Be}$ in a particular state (see Fig. 1). The production $T-$matrix for a relative angular momentum $L$ between the $^3\text{He}$ and $\alpha$ is written as

$$
\langle \vec{k}_p, m_p | T_{p^6\text{Li} \rightarrow \eta^7\text{Be}} | \vec{E} \rangle = \frac{1}{\mu} \left( \frac{1}{2}, L, \mu, M_L | J, m_J \right)
$$

where, $F_L(Q) = \int_0^\infty r^2 dr (\psi^*_L(r) j_L(Qr) \psi^0_{6\text{Li}}(r))$ is the transition form factor for $^6\text{Li} \rightarrow ^7\text{Be}$, with $\vec{Q} = \frac{1}{2} \vec{k}_p - \frac{2}{3} \vec{k}_\eta$. Here, $\psi^0_{6\text{Li}}(r)$ and $\psi^*_L(r)$ are the radial wave functions for $^6\text{Li}$ and $^7\text{Be}$ respectively. The $T-$matrix for the elementary process, $\langle \mid T_{p^6\text{Li} \rightarrow \eta^7\text{Be}} \mid \rangle$, is written in a two-step model from our earlier work. The calculations have been carried out by neglecting the effect of Fermi motion on $\langle \mid T_{p^6\text{Li} \rightarrow \eta^7\text{Be}} \mid \rangle$.

The transition matrix for the $p^6\text{Li} \rightarrow \eta^7\text{Be}$ reaction, including the $\eta - ^7\text{Be}$ FSI is given as

$$
T = \langle \vec{k}_\eta, m_7 \mid T_{p^6\text{Li} \rightarrow \eta^7\text{Be}} \mid \vec{k}_p \rangle + \sum_{m_6} \int \frac{d\vec{q}}{(2\pi)^3} \langle \vec{k}_\eta, m_7 \mid T_{\eta^7\text{Be}} \mid \vec{q} \rangle \langle \vec{q}, m_p \mid T_{p^6\text{Li} \rightarrow \eta^7\text{Be}} \mid \vec{k}_p \rangle
$$

where $\vec{k}_p$ and $\vec{k}_\eta$ are the initial and final momenta in the centre of mass system. $m_p$, $m_6$ and $m_7$ are the spin projections for the proton, $^6\text{Li}$ and $^7\text{Be}$ respectively.

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2.2. Final state interaction

The $\eta - ^7\text{Be}$ interaction is incorporated through a half-off-shell $\eta - ^7\text{Be}$ $T-$matrix,

$$T_{\eta^7\text{Be}}(k'kz) = \int d\vec{x}_1 |\psi_{\alpha L\eta}^3\text{He}(x_1)|^2 \{T_3(k', k, a_1x_1, z) + T_\alpha(k', k, a_2x_1, z)\}$$

(3)

where, $T_3(k', k, a_1x_1, z)$ and $T_\alpha(k', k, a_2x_1, z)$ represent $\eta - ^3\text{He}$ and $\eta - ^4\text{He}$ scattering matrices, respectively. They are written as,

$$T_3(k', k, a_1x_1, z) = t_3(k', k, a_1x_1, z) + \frac{1}{2\pi^2} \int_0^\infty q^2 dq \frac{t_3(k', q, a_1x_1, z)}{(z - \frac{q^2}{2\mu})}$$

$$\times T_\alpha(k', k, a_2x_1, z)$$

(4)

$$T_\alpha(k', k, a_2x_1, z) = t_\alpha(k', k, a_2x_1, z) + \frac{1}{2\pi^2} \int_0^\infty q^2 dq \frac{t_\alpha(k', q, a_2x_1, z)}{(z - \frac{q^2}{2\mu})}$$

$$\times T_3(k', k, a_1x_1, z)$$

(5)

where, $x_1$, the initial Jacobi co-ordinate, is related to the position vector, $\vec{r}_i$ in the $^3\text{He} - \alpha - \eta$ centre of mass system of each of the constituents by $\vec{r}_i = a_i \vec{x}_i$. $z = E - |\epsilon_0| + i0$ with $|\epsilon_0|$ being the energy required for the break up of $^7\text{Be}$ into $^3\text{He}$ and $\alpha$. The $t_3(k', k, a_1x_1, z)$ and $t_\alpha(k', k, a_2x_1, z)$ matrices have been calculated as in Ref. [7].

To begin with, we assume that the $\eta$-meson scatters only once from each of the $^7\text{Be}$ constituents, i.e., $^3\text{He}$ and $\alpha$. The required single scattering terms, the $t_3(k', k, a_1x_1, z)$ and $t_\alpha(k', k, a_2x_1, z)$ in Eq. (4) and Eq. (5) are constructed using the factorized impulse approximation (FIA) [12][13]. In this approximation, the momentum in the $\eta^3\text{He}$ and $\eta\alpha$ centre of masses are obtained by boosting their momenta in the $\eta^-^7\text{Be}$ centre of mass. This includes the Fermi momenta of $^3\text{He}$ and $\alpha$ in $^7\text{Be}$.

2.3. $^6\text{Li}$ and $^7\text{Be}$ wave functions

We have used two prescriptions for $\psi_0^6\text{Li}$ and $\psi_1^7\text{Be}$ cluster wave functions, namely (1) Cluster model wave function generated using Wood-Saxon potential [10][10] and (2) Green’s function Monte Carlo (GFMC) Variational wave function generated using Urbana potential [11].

Since the energy spacing between the first four low-lying levels of the $^7\text{Be}$ is very small, it is important to include the contribution from these levels. In the present calculation we consider the lowest states of $^7\text{Be}$, $(\frac{3}{2}^+, \frac{1}{2}^+)$ doublet split by 0.48 MeV, resulting from the $L = 1$ radial wave function coupled to spin $\frac{1}{2}$ of the $^3\text{He}$.

3. Results and Discussion

In Fig. 2, we show the total cross sections as a function of the excess energy. In this figure we show the plane wave results and those obtained after including the
We find that effect of inclusion of the FSI changes the shape and the magnitude of the total cross section curve drastically. In this plot we also show sensitivity of the results to different choice of wave functions. The FSI matrix has been calculated using an \( \eta N t \)–matrix which corresponds to \( a_{\eta N} = 0.88 + i 0.41 \) fm.

![Figure 2](image1.png)

**Fig. 2.** Total cross section as a function of \( Q \) for \( a_{\eta N} = 0.88 + i 0.41 \) fm. The results are shown for two different prescriptions of cluster wave functions using the FIA approach.

![Figure 3](image2.png)

**Fig. 3.** Form factor, \( F_L(Q) \) as a function of \( Q \) for \( L = 1 \) state for two different prescriptions of cluster wave functions.

In Fig. 3 we show the form factor, \( F_L(Q) \), calculated using two different models. The region between the dotted lines corresponds to the range of \( Q \), at which \( F_L(Q) \) in Eq. (1) is getting calculated. This range of \( Q \) is shown in Fig. 2 by hashed region. It can be seen that two form factors differ in this region which gives rise to the difference in the total cross sections calculated using the two different wave functions.

In Fig. 4, we show the angular distributions for three different excess energies such that it spans region from close to threshold up to 20 MeV above threshold.
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We show results without FSI and with FSI for cluster model wave functions. We find that even after including FSI, the angular distribution is isotropic in nature. At excess energy $\sim 19$ MeV, the differential cross section is $\sim 1$ nb/sr for low-lying states of $^7\text{Be}$ resulting from the $L = 1$ radial wave function, in comparison with the experimental number $3$, i.e., $4.6 \pm 3.8$ nb/sr.

A calculation of contributions from the excited states of the $^7\text{Be}$ nucleus to the total cross section and that of more detailed FSI is in progress and shall soon be reported.

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