Phenomenology and Theory of Possible Light Higgs Bosons

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We study the implications of the absence of a direct discovery of a Higgs boson at LEP. First we exhibit 15 physically different ways in which one or more Higgs bosons lighter than the LEP limit could still exist. In the minimal supersymmetric standard model (MSSM) all of these, as well as the cases where \( m_h \geq 115 \) GeV, seem fine-tuned. We examine some interpretations of the fine tuning in high scale theories. The least fine-tuned MSSM outcome will have \( m_h \simeq 115 \) GeV, while approaches that extend the MSSM at the weak scale can naturally have larger \( m_h \).

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The Minimal Supersymmetric Standard Model is defined as the simplest supersymmetric extension of the Standard Model (SM). Every SM particle has a superpartner, the basic Lagrangian is supersymmetric, and the gauge group is the same \( SU(3) \times SU(2) \times U(1) \) as that of the SM. The full supersymmetry is softly broken by certain dimension two and three operators. There is considerable indirect evidence that this theory is likely to be part of the description of nature. If it is, a Higgs boson with mass less than about 130 GeV must exist, and superpartners must be found with masses not too much larger than those of the W, Z and top quark. This theory has three neutral Higgs boson physical states (denoted \( h \), \( H \), and \( A \) in the simplest CP conserving case) and a charged Higgs pair \( H^\pm \). Of course it could be an extension of the MSSM that describes nature at the weak scale, but most extensions behave similarly to the MSSM at low energies.

While the Higgs boson mass can be as heavy as 130 GeV in the MSSM, it has been known for some time that most models imply a lighter state, usually below about 110 GeV, when constraints from non-observation of superpartners (real or virtual) are imposed, and including the constraint that the indirect arguments for supersymmetry are valid without fine-tuning. Even allowing that perhaps LEP has seen a Higgs boson with \( m_h = 115 \) GeV, there is some fine-tuning present because the tree level Higgs mass is bounded by \( M_Z \) so the one loop corrections have to supply about \( \delta m_h^2 \simeq (70 \text{ GeV})^2 \) when added in quadrature. That is possible but requires some parameters to be larger than naively expected. Of course, discussing an issue such as fine-tuning can only be done in the context of a theory. Fine tuning in the low scale theory could be a clue to a natural structure of the high scale theory. Or it could be a clue that the low scale theory is effectively excluded. If the MSSM is extended to have a larger gauge group in such a way that new scalars mix significantly with the MSSM scalars then the tree level mass of the lightest Higgs can be larger than \( M_Z \) and no fine tuning is needed to have a heavier Higgs spectrum.

It is interesting to ask if the Higgs sector of the MSSM could be such that LEP would not have found a signal because of reduced Higgs production cross sections there, or reduced branching ratios. In the following we present in Table I 15 physically different scenarios where that is indeed the case, plus the minimal scenario with \( m_h = 115 \) GeV (No. 10), and the case with all Higgs heavier than about 115 GeV (No. 17). All are allowed by other data. All are detectable at the Fermilab Tevatron collider with sufficient luminosity. All satisfy the constraints for electroweak symmetry breaking, though sometimes in unconventional ways. Having found these we can then ask if any of them seem to be less fine-tuned, and thus may point towards an underlying theory.

The ranges of masses in Table I are representative ones since we think more detailed study is not appropriate at this stage. In some cases there could be two or even three light Higgs. LEP groups have reported as many as three possible signals (each about 2\( \sigma \)) \[ \text{1, 2} \], so for simplicity we identify possible signals as the masses where the reported LEP hints occur. We emphasize that we are not in any way claiming LEP signals exist – we are simply using these points to illustrate the opportunities. If there were observable light Higgs bosons, they are more likely to have masses where tentative signals have been recorded. We group the models according to whether three, two, or none of the light states could have given a signal at LEP. We assume that if they did give a signal it was at one of the 2\( \sigma \) reported masses 98 GeV or 115 GeV for one of the two CP-even eigenstates, or where the sum of two masses \( m_h + m_A = 187 \) GeV \[ \text{2, 4} \]. Three of the 16 light Higgs cases have parameters that can exist in the unified minimal supergravity (mSUGRA) paradigm; the others do not. Some require complex soft terms. We also give low-scale parameters for some cases.

Because the one loop top/stop radiative correction to the Higgs potential is rather large, a large phase (specif-
TABLE I: Possible explanations consistent with LEP Higgs search results. Ranges of neutral and charged Higgs masses consistent with background only hypotheses as well as one, two or three “signal” hypotheses are listed. The column headed by “Signals” indicates what signals might have appeared for a given model. Qualitative tan $\beta$ and Higgs coupling ranges for each individual parameter space is given. All ranges should be understood as indicative of the allowed region at the roughly 10% accuracy level: fine scans of the parameter space have not been performed. For Higgs state $\varphi_i$, the ZZ$\varphi_i$ coupling is $(g_2 M_Z / \cos \theta_W) C_i$, approximate values are given in the table. The column marked $\phi$ indicates a non-trivial phase $\phi_{\mu A_i}$ is needed. When there is nontrivial phase, $m_A$ is understood as the mass of the neutral higgs with smallest $C_{ZZH}$ coupling. The column $\mu$ indicates the presence of a large $\mu$ term. The column marked U indicates this scenario is compatible with a unified SUSY breaking scenario such as mSUGRA. We think all other such scenarios effectively reduce to one of these.

| No. | $m_h$ | $m_A$ | $m_H$ | $m_{H^\pm}$ | Signals | tan $\beta$ | $C_h^2$ | $C_A^2$ | U | $\mu$ | $\phi$ |
|-----|-------|-------|-------|------------|---------|------------|---------|---------|---|------|-------|
| 1   | 98    | 89    | 115   | 112-123    | 98,115,187 | 6-12       | 0.2     | 0.8     | Y | Y    |       |
| 2   | 98    | < $m_h$ | 115   | 106-127    | 98,115     | 4-13       | 0.2     | 0.8     | Y | Y    |       |
| 3   | 98    | $\approx m_h$ | 115   | 121-136    | 98,115     | 5-50       | 0.2     | 0.8     | Y | Y    |       |
| 4   | 98    | 115-130 | 115   | 112-124    | 98,115     | 10-24      | 0.2     | 0.8     | Y |       |       |
| 5   | 70-91 | 96-116 | 115   | 110-140    | 115,187    | 10-50      | 0.0     | 1.0     | Y |       |       |
| 6   | 98    | 89    | > 115  | 118-127    | 98,187     | 6-10       | 0.2     | 0.8     | Y | Y    |       |
| 7   | 82-110 | < $m_h$ | 115   | 7-50       | 115        | 7-50       | 0.0     | 1.0     | Y | Y    |       |
| 8   | 82-110 | $\approx m_h$ | 115   | $\sim m_A$ | 115$^a$    | 5-50       | 0.0     | 1.0     | Y |       |       |
| 9   | 82-110 | 115-140 | 115   | $\sim m_A$ | 115        | 6-24       | 0.0     | 1.0     | Y |       |       |
| 10  | 115   | $m_A \approx m_H > 115$ | 115   | $\sim m_A$ | 115$^a$    | 3-50       | 1.0     | 0.0     | Y |       |       |
| 11  | 98    | 100-130 | 120-130 | $\sim m_A$ | 98         | 5-50       | 0.20    | 0.80    | Y |       |       |
| 12  | 98    | < 98   | 120-130 | 106-128    | 98         | 4-13       | 0.20    | 0.80    | Y |       |       |
| 13  | 65-93 | 94-120 | 116-125 | 110-140    | 187        | 8-50       | 0.0     | 1.0     | Y |       |       |
| 14  | 80-100 | 25-40  | 133-154 | 109-130    | None$^b$   | 2.5       | 0.5-0.8 | 0.2-0.5 | Y |       |       |
| 15  | 111-114.4 | $m_A \approx m_H > 114.4$ | 115   | $\sim m_A$ | None$^b$   | 2.4-4      | 1.0     | 0.0     | Y |       |       |
| 16  | 70-114.4 | 90-140 | > 114.4 | $\sim m_A$ | None$^b$   | 4-50      | 0.0     | 1.0     | Y |       |       |
| 17  | > 114.4 | $m_A \approx m_H > 114.4$ | 115   | $\sim m_A$ | None$^b$   | 4-50      | 1.0     | 0.0     | Y |       |       |

$^a$ Dominant decay is CP violating process $H_2 \to H_1 H_1$. This case was studied in Ref. $\beta$.

$^b$ The “invisible” decay $h \to N_1 N_1$ and $h \to bb$ decays are comparable.

$^c$ These scenarios were studied in Ref. $\mu$.

physically the relative phase of $\mu$ and $A_i$ can enter, and lead to a relative phase between the Higgs vevs at the minimum of the potential. This phase is physical and cannot be rotated away. It leads to mixing between the mass eigenstates, and affects the production rates and decay branching ratios $\phi$. The column headed by $\phi$ has a $Y$ if a non-trivial phase (not zero or $\pi$) plays a role for a given model. When CP is conserved we call the states by the usual names $h$, $H$ and $A$; one can show here that $m_h$ is always less than max$(M_Z, m_A)$, even allowing one-loop corrections for $m_h$, so any model with $m_A$ and $M_Z < m_h$ requires a non-trivial phase. This conclusion does not include loop effects for $m_A$, so one can have $m_A$ a few GeV less than $m_h$ for certain parameters if tan $\beta$ is large. The column headed by $\mu$ has a $Y$ if $\mu$ is very large, say well above several hundred GeV; this is particularly relevant because of the question of fine-tuning needed to obtain electroweak symmetry breaking.

In most cases the charged Higgs mass $m_{H^\pm}$ is less than the top quark mass, so the decay $t \to b + H^\pm$ is allowed. Existing data from D0 excludes $m_{H^\pm}$ below about 125 GeV for tan $\beta$ larger than about 50 with mild model dependence, so no model is fully excluded – though parts of the parameter range of some models are probably excluded by non-observation of $H^\pm$. With more and better data from Run II the $H^\pm$ of most of these models could be observed or excluded $[10]$. These small values for $m_{H^\pm}$ can also exceed limits from $Br(b \to s\gamma)$, but using light chargino and gluino contributions provides significant flexibility. However, cases 8 and 16 exceed the limits on $Br(b \to s\gamma)$ by more than a factor of two and are thus likely to be excluded, though we should note that this is based on using a unified mSUGRA model for these cases and may not hold when departures from universality are entertained.

For completeness we exhibit a set of low-energy parameters that determine the resulting Higgs sector of the model for three entries in Table I in a longer paper we will provide this information for all of them and give more details on the ranges $[11]$. Entry No. 1 has $m_h = 98$ GeV, $m_H = 115$ GeV and $m_b + m_A = 187$ GeV. Its parameters are tan $\beta = 6$, $|\mu| = 1700$, $|A_t| = |A_b| = 400$, $\approx 115$ GeV and $m_{H^\pm}$ is needed.
\( \phi_\mu = -130^\circ, \ M_1 = 100, \ M_2 = 200, \ M_3 = 600 \) and 
\( m_{\tilde{Q}_3} = m_{\tilde{b}_R} = m_{\tilde{\tau}_R} = 500 \), with all parameters in GeV. This gives \( m_{H^\pm} = 120 \) GeV. The masses of the three mass eigenstates are \( m_1 = 89.1, \ m_2 = 97.4, \) and 
\( m_3 = 115.0 \) GeV, with \( C_i^2 \) respectively of 0.024, 0.229, and 0.747. All three states have \( \text{BR}(\phi_i \rightarrow b \bar{b}) \approx 0.91 \). These give about 2\( \sigma \) signals at 98 and 115 GeV. Since \( m_A \approx M_Z \) the \( Zh \) and \( Ah \) channels add to give an apparent 187 GeV signal.

Entry No. 8 has \( m_H = 115 \) GeV with the other neutral Higgs states having smaller masses. Its parameters are 
\( \tan \beta = 46.9, \ \mu = -540, \ A_t = -758, \ A_b = -882, \ M_1 = 188, \ M_2 = 351, \ M_3 = 1015, \ m_{\tilde{Q}_3} = 860, \ m_{\tilde{b}_R} = 848 \) and 
\( m_{\tilde{\tau}_R} = 776 \), with all masses in GeV. This gives \( m_{H^\pm} = 113.5 \) GeV. The masses of the three mass eigenstates are 
\( m_h = 82.7, \ m_A = 86.2 \) and \( m_H = 114.7 \) GeV, with \( C_i^2 \) respectively of 0.013, 0 and 0.987. All three states have 
\( \text{BR}(\phi_i \rightarrow b \bar{b}) \approx 0.934 \) which yields an apparent 2\( \sigma \) signal at 115 GeV.

Entry No. 15 has no signal at LEP and a lightest Higgs boson mass below 115 GeV. Its parameters are 
\( \tan \beta = 2.4, \ \mu = 190, \ A_t = A_b = 4000, \ M_1 = 55, \ M_2 = 250, \ M_3 = 700 \) and 
\( m_{\tilde{Q}_3} = m_{\tilde{b}_R} = m_{\tilde{\tau}_R} = 2000 \), with all masses in GeV. This gives \( m_{H^\pm} = 505 \) GeV. The masses of the three mass eigenstates are \( m_h = 111.2, \ m_A = 499.6 \) and \( m_H = 504.3 \) GeV, with \( C_i^2 \) respectively of 0.999, 0 and 0.001. The branching ratios of the lightest state are 
\( \text{BR}(h \rightarrow b \bar{b}) = 0.296 \) and \( \text{BR}(h \rightarrow N_1 \bar{N}_1) = 0.021 \), where \( N_1 \) is the stable lightest superpartner and is 
a good candidate for the cold dark matter of the universe. In the case presented here, \( m_{\tilde{Q}_3} = 43.5 \) GeV.

It would be very nice if one or more of the models pointed to a simple high scale model which we could then study and perhaps motivate. Unfortunately, this does not seem to occur. The low energy values given above do not completely specify the MSSM soft Lagrangian. Thus, translating these values to a high energy boundary condition scale \( \Lambda_{\text{UV}} \) through renormalization group (RG) evolution will involve some arbitrariness – for example, in choosing the low-energy values of slepton and second-generation squark masses. In some instances, such as entry No. 8 described above, the necessary low scale values could be obtained from a unified mSUGRA model at the high scale. For the remaining cases we have chosen representative points in the low-energy allowed parameter space and evolved them from the bottom upwards. Some subset of these results for the soft terms at 
\( \Lambda_{\text{UV}} = \Lambda_{\text{cut}} = 1.9 \times 10^{16} \) are given in Table II including those of Entries 1, 8 and 15.

| Entry | 1 | 3 | 4 & 9 | 8 | 15 | 16 |
|-------|---|---|-------|---|----|----|
| \( \tan \beta \) | 6 | 10 | 11.3 | 46.9 | 2.4 | 46.41 |
| \( M_1 \) | 242 | 291 | 726 | 450 | 133 | 560 |
| \( M_2 \) | 243 | 292 | 365 | 450 | 304 | 560 |
| \( M_3 \) | 210 | 245 | 349 | 450 | 245 | 560 |
| \( A_t \) | 3266 | 3596 | -3835 | 0 | 26288 | 0 |
| \( A_b \) | 1577 | 1799 | -840 | 0 | 8535 | 0 |
| \( m_{\tilde{Q}_3} \) | -529^2 | 935^2 | -(1018)^2 | 450^2 | (10059)^2 | (300)^2 |
| \( m_{\tilde{b}_R} \) | -682^2 | 1397^2 | -(1225)^2 | 450^2 | (14101)^2 | (300)^2 |
| \( m_{\tilde{\tau}_R} \) | -(196)^2 | -(355)^2 | -(759)^2 | 450^2 | (1908)^2 | (300)^2 |
| \( m_{\tilde{Q}_{1,2}} \) | -(246)^2 | -(417)^2 | -(772)^2 | 450^2 | (1890)^2 | (300)^2 |
| \( m_{\tilde{b}_{1,2}} \) | -(180)^2 | -(371)^2 | -(756)^2 | 450^2 | (1902)^2 | (300)^2 |
| \( m_{\tilde{\tau}_{1,2}} \) | -(182)^2 | -(366)^2 | -(732)^2 | 450^2 | (1902)^2 | (300)^2 |
| \( m_{\tilde{b}_R} \) | -(1800)^2 | -(1684)^2 | -(2295)^2 | 450^2 | (17114)^2 | (300)^2 |
| \( m_{\tilde{\tau}_R} \) | -(1690)^2 | -(525)^2 | -(1989)^2 | 450^2 | (412)^2 | (300)^2 |
| \( \mu \) | -1687 | -492 | 1971 | -660 | 212 | -794 |

Table II: Relevant soft term and \( \mu \) values at the GUT scale for selected entries in Table I

at the low scale by the Higgs sector, we have found no instances where the patterns of severe hierarchies and negative scalar mass-squareds can be alleviated. Note that these non-universal cases are particularly perverse in that both charge and color symmetries are radiatively restored in these models as the parameters are evolved towards the electroweak scale.

Even allowing for the possibility that some of the high-scale values in Table I which appear similar can, in fact, be made to unify with the appropriate adjustment of low scale values, we are still confronted with a large number of unrelated parameters in the soft Lagrangian. Most models of supersymmetry breaking (such as mSUGRA) are studied for their simplicity; they tend to involve very few free parameters. The traditional models of minimal gravity, minimal gauge and minimal anomaly mediation, as studied in the Snowmass Points and Slopes Models have too few parameters to possibly describe these non-universal cases even when all three are combined in arbitrary amounts. Nor do string-based models generally provide sufficient flexibility, whether they be heterotic based or intersecting brane constructions such as Type IIB orientifold models. While having sufficient free parameters in the model is, strictly speaking, neither necessary nor sufficient to potentially generate one of the entries in Table I we feel it is a good indication of the theoretical challenge faced by models that cannot come from mSUGRA or other simple benchmark models. This is particularly true when the number of free parameters within, say, the scalar sector and the number of hierarchies in the soft Lagrangian are considered.

That many of the entries in Table I imply high scale soft supersymmetry breaking patterns with such
TABLE III: Measures of fine tuning with respect to high scale parameters in selected entries from Table I. For example, the entries for model 1 imply that a 1% shift in high scale parameters leads to a 95.3% shift in the value of $m^2_{\tilde{Z}}$.

| Entry | 1 | 3 | 4 & 9 | 8 | 10 | 15 | 16 |
|-------|---|---|-------|---|----|----|----|
| $\delta_Z$ | 1003 | 365 | 1250 | 89 | 83 | 28600 | 135 |
| $\delta_A$ | 953 | 135 | 640 | 80 | 1.4 | 275 | 93 |

* Does not include the specification of $\tan \beta$ and the sign of the $\mu$ parameter.

unattractive features (and no discernable theoretical structure) can be considered one element of the fine-tuning in such cases. In Table III we have counted the number of apparent free parameters for these sample cases, allowing for a generous interpretation of which parameters could be made to unify. It is not an automatic corollary, however, that the models that admit a unified explanation are necessarily less fine-tuned. In Table III we also provide two additional quantitative measures of the fine-tuning in these same cases. We have also included entry No. 10 from Table I for comparison, with high scale input values $\tan \beta = 25$, $m_0 = 500$ GeV, $m_{1/2} = 300$ GeV, $A_0 = -750$ GeV and positive $\mu$ term. The numbers $\delta_Z$ and $\delta_A$ are the sensitivities of $M_Z$ and $m_A$, respectively, to small changes in the values of the independent high-scale values $a_i$; i.e. $\delta = \sqrt{\sum (\delta_i)^2}$ where $\delta_i = (a_i/m) \Delta m/\Delta a_i$ [20]. In order to treat unified and non-unified models equally we have used the average scalar mass squared, gaugino mass and trilinear coupling as free variables in computing these sensitivities.

As far as we can see, all models with $m_A \sim m_b$, or equivalently $C_H \sim 1$ are significantly fine-tuned. This is not clear from the low-scale parameters, but seems to emerge when one examines the high-scale models that give rise to small $m_A$. Models which require specifying multiple soft parameters quite precisely also imply additional tuning costs relative to the mSUGRA models. This should be seen as evidence of the difficulty in finding areas of the low-energy parameter space capable of producing many of the entries in Table I. While the fine-tuning “price” of the LEP results for the MSSM has been often discussed [21], it is apparent from Table III that the least fine-tuned result continues to be the case with $m_b \simeq 115$ GeV.

Yet even this outcome tends to require some superpartner masses heavier that one might naively expect, in order to obtain the (75 GeV)$^2$ radiative correction. In the standard mSUGRA-based studies [22, 23] one typically needs here either squarks or gluinos in excess of 1 TeV in mass at the low energy scale, with the latter being a much more serious problem for fine-tuning than the former [24, 25]. Most of these studies assume vanishing trilinear A-terms, however. The degree of tuning can be reduced substantially if the so-called “maximal mixing” scenario can work [26]. In this case, the need for large superpartner masses is mitigated by maximizing the loop correction to the lightest Higgs boson mass from the $m^2_{\tilde{Z}_R}$ entry of the stop mass matrix. In models whose scalar sector is well approximated by an overall universal scalar mass $m_0$, this tends to occur when $A_t \simeq -2m_0$ at the GUT scale [27]. In models with small departures from universality this relation remains approximately correct.

This relation is therefore an alluring goal for high-energy models, though few well-motivated models seem to naturally predict this relation without simultaneously re-introducing some heavy superpartners, and often the hierarchies or negative mass-squareds mentioned above still occur here. Minimal supergravity and minimal anomaly mediation effectively treat both trilinears and scalar masses as independent variables so no such relation is predicted. Minimal gauge mediation predicts trilinear terms which are small relative to scalar masses. In string-based examples some similar relations are predicted. For example, the dilaton domination model with tree-level Kähler potential [28] for the dilaton provides a relation $A_0 = -\sqrt{3}m_0 \simeq -1.7m_0$ at the SUSY breaking scale. On the other hand, since $M_{1/2} = -A_0$ in this model a sufficiently heavy Higgs state will require a gluino mass that is also rather large. In the generalized dilaton domination scenario [18, 29, 30] gauginos masses are parametrically small relative to scalars, but so too are the trilinears. Allowing some coupling of the Higgs fields to the Green-Schwarz counterterm in such theories can help generate larger A-terms, but generally suffer from light moduli problems [31, 52].

In heterotic models where moduli fields participate in supersymmetry breaking the tree-level trilinear A-terms vanish in the limit where the Yukawa couplings and positive $\mu$ term. This is not clear from the low-scale parameters, but seems to emerge when one examines the high-scale models that give rise to small $m_A$. Models which require specifying multiple soft parameters quite precisely also imply additional tuning costs relative to the mSUGRA models. This should be seen as evidence of the difficult in finding areas of the low-energy parameter space capable of producing many of the entries in Table I. While the fine-tuning “price” of the LEP results for the MSSM has been often discussed [21], it is apparent from Table III that the least fine-tuned result continues to be the case with $m_b \simeq 115$ GeV.

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\[ \sqrt{2m_{\tilde{q}}} \text{ and } A_\ell = -\sqrt{2m_{\tilde{q}}} \text{ can be obtained, with } m_{\tilde{q}} \text{ being the typical squark mass at the high scale. Non-integer and non-negative effective modular weights have recently been shown to be possible in weakly-coupled heterotic constructions as well, whenever MSSM fields are charged under an anomalous } U(1) \text{ factor and fields which get large vevs to cancel the Fayet-Iliopolous term are integrated out in a modular invariant manner.} \]

Thus we see that the ability to imply a result like \( A_0 \approx -2m_0 \) may be a significant discriminant among string-based models and mechanisms of transmitting supersymmetry breaking. Further analysis of many of these string-based approaches to achieving a large Higgs mass without fine-tuning will be presented elsewhere.

Given the above, the “best” answer for the MSSM remains that LEP did produce 115 GeV Higgs states, and that this mass was the result of some realization of what we will call the “constrained maximal mixing” scenario, with \( A_\ell \approx -2m_{\tilde{q}} \) and \( M_3, \mu \) small enough so that radiative electroweak symmetry breaking occurs without large cancellations. How to generate this constrained maximal mixing scenario in realistic high-scale theories is not clear, though some approaches may be good starting points. For instance, we find it interesting that the minimal SO(10) model of Raby et al. arrive at the same relationship between the A-terms and squark masses as a necessary condition for natural third-generation Yukawa unification. Alternatively, perhaps the LEP results suggest that what is needed is an extension to the usual MSSM electroweak sector, such as an additional \( U(1) \) sector whose associated scalars mix strongly with the MSSM Higgs sector. Such an extension must therefore arise at or near the weak scale. Perhaps the data is pointing us towards a still (relatively) unexplored, but crucial, region of theoretical parameter space whose theoretical simplicity we cannot yet see.

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