Final State Interaction within the Bethe-Salpeter Approach in Charge Exchange $pD \rightarrow n(pp)$ process.

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Abstract

The exclusive charge exchange reaction $pD \rightarrow n(pp)$ at intermediate and high energies is studied within the Bethe-Salpeter formalism. The final state interaction in the detected at zero-near excitation energy $pp$-pair is described by the $^1S_0$ component of the Bethe-Salpeter amplitude. Results of numerical calculations of polarization observables and differential cross-section persuade that, as in the non-relativistic case, this reaction can be utilized for a relativistic deuteron tensor polarimeter and as a source of information about the elementary nucleon-nucleon charge exchange amplitude.

1 Introduction

Nowadays large programs of experimental study of processes with polarized particles are in progress. The setups with deuteron targets (beams) occupy the leading place [1, 2, 3, 4]. For an investigation of the $NN$ interaction in the deuteron at short distances the three deuteron form factors, magnetic, electric and quadrupole, should be determined. In the elastic $eD$-scattering with unpolarized particles one can measure only two quantities, e.g. the magnetic form factor and the deuteron function $A(Q^2)$, which is a kinematical combination of all three form factors. Even these two quantities provide important information about the quark physics and dynamics at short distances (see, for instance recent measurements [1] at TJNAF). However, for a full determination of the deuteron form factors, one needs measurements with polarized particles. For example, measurements of the tensor analysing power $T_{20}$ of recoil deuterons in elastic $eD$-scattering allow the determination of the charge form factor $G_c$ at high transferred momenta. The hadron-deuteron processes
can be considered as complementary tools in investigation of phenomena at short distances and also as a source of unique information unavailable in electromagnetic reactions (study of nucleon resonances, checking the non relativistic effective models, \(NN\) potentials etc.). Experimental and theoretical investigation of the proton-deuteron processes at intermediate and high energies has started some decades ago by studying elastic \(pD\) scattering \([5]\), exclusive and inclusive break-up \([6, 7]\). In \(pD\) processes it is possible to completely restore the reaction amplitude by measuring a full set of polarization observables (see, e.g. \([8, 9, 10]\)). Hence, as in the electromagnetic case one needs to measure different polarizations of the recoil deuteron. Since polarization observables can be studied only by an additional secondary scattering of the reaction products (in polarimeter), it is obvious that second process must possess a high enough cross section to assure a good efficiency of the polarimeter. In ref. \([11]\) Bugg and Wilkin have proposed as an effective deuteron polarimeter the process \(p\bar{D} \rightarrow (pp)n\) where the final \(pp\)-pair is detected with extremely low excitation energy (see also ref. \([12]\)). Later investigations \([13, 11, 14, 15, 16]\) confirmed the theoretical predictions and the charge exchange processes were suggested also for use in investigation of a number of reactions with deuterons, e.g. \(pp \rightarrow D\pi^+\) \([17]\), \(NN\pi\)-systems, inelastic \((\bar{D}, \bar{D}')\)-reactions off heavy nuclei to study isoscalar transitions \(\Delta T = 0, \Delta S = 1\) \([18]\) etc. The direct consequence of these facts is that nowadays the interest in investigation of charge exchange processes does not abate. In our previous paper \([19]\) we investigated the process \(pD \rightarrow n(pp)\) within the impulse approximation. The goal of the present work is to consider theoretically the effects of the final state interaction in this reaction at relativistic initial energies (COSY, Dubna) and to confess whether in this case the non relativistic predictions \([11]\) hold and the reaction can be still regarded as a deuteron polarimeter tool. We propose a covariant generalization of the spectator mechanism \([11]\) based on the Bethe-Salpeter formalism and on numerical solution of the Bethe-Salpeter (BS) equation with a realistic one-boson exchange kernel \([21, 22]\). Here we focus our attention in calculations of the cross section and the tensor analysing power \(T_{20}\) within the COSY kinematical conditions at, as in non relativistic case, zero vector analyzing powers of the deuteron.

### 2 Kinematics and the invariant amplitude

We select those processes which, in the deuteron center of mass system, correspond to final states with one fast neutron and a slowly moving proton-proton pair, i.e. reactions
of the type
\[ p + \vec{D} = n + (p_1 + p_2). \] (1)

The transferred momentum from the proton to the neutron is low, hence the main mechanism of the reaction can be described as a charge exchange process of the incoming proton off the internal neutron whereas the proton in the deuteron remains merely as a spectator. If so, then the resulting \( pp \)-pair will be detected with low total and relative momenta. In Fig. 1 the diagram of such processes is schematically depicted. The following notations are adopted: \( p = (E_p, \mathbf{p}) \) and \( n = (E_n, \mathbf{n}) \) are the 4-momenta of the incoming proton and outgoing neutron, \( P' \) is the total 4-momentum of the \( pp \)-pair, which is a sum of the corresponding 4-momenta of detected protons, \( p_1 = (E_1, \mathbf{p}_1), p_2 = (E_2, \mathbf{p}_2) \): \( P' = p_1 + p_2 \).

The notion of the invariant mass of the pair \( s_f \), \( s_f^2 = \left(P'_f\right)^2 = (2m + E_x)^2 \), where \( m \) stands for the nucleon mass and \( E_x \) for the excitation energy of the pair, is also explored in what follows. The excitation energy \( E_x \) ranges from zero to few MeV, \( E_x \sim 0 \rightarrow 8 \) MeV. At such low values of \( E_x \) the main contribution to the final state of the \( pp \) pair in the continuum comes from the the \( ^1S_0 \)-configuration [15]. The differential cross section for the reaction (1) reads [19]:
\[ \frac{d^2\sigma}{dt \, ds_f} = \frac{1}{2} \frac{1}{64\pi \lambda(p, D)} \sqrt{1 - \frac{4m^2}{s_f}} \frac{1}{(2\pi)^2} |M_{fi}|^2. \] (2)

The experimental data from Saclay [16] are binned into the following intervals of excitation energy:

- \( I : \) \( 0 \leq E_x \leq 1 \) MeV,
- \( II : \) \( 1 \) MeV \( \leq E_x \leq 4 \) MeV,
- \( III : \) \( 4 \) MeV \( \leq E_x \leq 8 \) MeV.

To compare the cross section (2) with the experimental data it is necessary to integrate over the invariant mass of the pair in the regions \( I, II \) and \( III \) according to
\[ \left( \frac{d\sigma}{dt} \right)_{I,II,III} = \frac{1}{(8\pi)^3\lambda} \int_{I,II,III} ds_f \sqrt{1 - \frac{4m^2}{s_f}} |M_{fi}|^2. \] (3)

By using the Mandelstam technique [23] the covariant matrix element corresponding to the diagram on Fig. 1 can be written in the form
\[ T^{M}_{rr'} = \sum_{ss'} \frac{1}{(2m)^2} \int d^4k \, f_{r'\,s',sr} \times \]
\[ \times \, \bar{u}^s(p_n) \Psi_M(k) \left( \frac{\hat{D}}{2} - \hat{k} + m \right) \bar{\Psi}_{P'}(k - \frac{q}{2}) \, u^{s'}(p_p). \] (4)
The elementary charge-exchange amplitude $A^e$ is incorporated in the matrix element (4) by the on-shell amplitudes $f_{r's',sr}$ and the Dirac spinors. In doing so the off-shell effects are neglected and the elementary subprocess is considered as real process with on-shell particles. In our numerical calculations we use the helicity amplitudes of pn scattering, resulting from both the Nijmegen partial wave analysis [28, 26] and the well-known results of SAID [27]. For the final $^1S_0$-state within the $\rho$-classification the BS amplitude $\Psi_{P'}$ in the center of mass of the $NN$ pair is represented by four partial amplitudes $^1S_0^{++}$, $^1S_0^{--}$, $^3P_0^{+-}$ and $^3P_0^{-+}$ [24], which for the sake of brevity are denoted as $\phi_1, \ldots, \phi_4$. The partial amplitudes $\phi_i$ may be found from the BS equation, which, in the simplest case of pseudo scalar exchanges reads as

$$\Psi_{P'}(p) = \Psi^0_{P'}(p) + ig^{2}_{\pi NN} \int \frac{d^4p'}{(2\pi)^4} \Delta(p-p')S(p'2)\gamma_5\Psi_{P'}(p')\gamma_5S(p),$$

(5)

where $\Delta$ and $S$ are the scalar and spinor propagators, respectively, $\tilde{S} \equiv U_C^* S U_C^{-1}$, and $\Psi^0_{P'}(p)$ is the free amplitude corresponding to two non interacting nucleons (the relativistic plane wave). The solution of eq. (5) may be presented as a Neuman-like series, the first term of which is the free term from eq. (5):

$$\Psi_{P'}(p) = \Psi^0_{P'}(p) + \Psi^i_{P'}(p).$$

(6)

The second part in eq. (6) is entirely determined by the interaction and may be symbolically referred to as scattered wave. To determine the scattered wave in eq. (6) it is necessary to solve the BS equation of the type (5). Solving the BS equation in the continuum is a much more cumbersome procedure than, e.g. for the homogenous equation. Besides difficulties encountered in solving the latter (singularities of amplitudes, poles in propagators, cuts etc.) the former even does not allow the Wick rotation [25] to the Euclidian space, and in the Minkowsky space there are no rigourous mathematical methods of finding solutions. However, an approximate solution of eq. (5) may be obtained by applying the so-called ”one-iteration approximation” [10]. Within that one may obtain a rather good estimate of the interaction term.

3 The one-iteration approximation

For a consistent relativistic analysis of the reaction (1) one should solve the BS equation for both bound state and scattering state with the same interaction kernel. We have

1Actually there is one realistic solution of the inhomogeneous BS equation in the ladder approximation, obtained by Tjon [20].
found a numerical solution for the deuteron bound state with a realistic one-boson exchange potential [21]. The Bete-Salpeter equation, after a partial decomposition, has been solved numerically by using an iteration method. We found that the iteration procedure converges rather quickly if the trial function is properly chosen. In such case even after the first iteration the BS solution coincides with the exact one up to relative momentum $p \sim 0.6 - 0.7 \, GeV/c$. This circumstance becomes useful if one needs an approximate solution of the BS equation at not too large momenta $p \leq 0.5 - 0.7 \, GeV/c$. This is just our case, since in reaction (1) the relative momentum of the $pp$-pair is expected to be rather small and the scattering part of the amplitude (6) can be obtained from the equation (5) by one iteration. To solve eq. (5) we proceed as follows (see also ref. [10]): i) for simplicity, in the inhomogeneous BS equation we leave only the pseudo scalar isovector exchanges ($\pi$-mesons) ii) by disregarding the dependence upon $p_0$ in the meson propagator in eq. (5) and then using the standard representation of propagators via generalized Legendre polynomials $Q_l$, one obtains for the main partial amplitude

$$\phi_{0S^+}^0(p_0, |p|) = \phi_{0S^+}^0(p_0, |p|) - \frac{g_{\pi NN}^2}{4\pi} \frac{1}{\left(\frac{\sqrt{s_f}}{2} - E_p\right)^2 - p_0^2} \times$$

$$\int_0^\infty \frac{d|p'|}{2\pi} \frac{|p'|}{|p|} \frac{1}{E_p E_{p'}} \left[ (E_p E_{p'} - m^2)Q_0(\tilde{y}_\mu) - |p||p'|Q_1(\tilde{y}_\mu) \right] u_{1S^0}(s_f, |p'|),$$

(7)

where $\tilde{y}_\mu = \frac{p^2 + p'^2 + \mu^2}{2|p||p'|}$. In obtaining (7) the integration over $p'_0$ has been carried out in the residium $\tilde{p}_0 = \frac{\sqrt{s_f}}{2} - E_{p'}$ and, by definition, the BS wave function in the continuum is

$$u_{1S^0}(s_f, |p'|) = \frac{g_{1S^0_0}^0(\tilde{p}_0, |p'|)}{\sqrt{s_f} - 2E_{p'}},$$

(8)

where $g_{1S^0_0}^0$ is the vertex function (for more details see [10, 24]). Now if we restrict ourself to only one iteration in (7) and take the trial function (8) as a non relativistic solution of the Schrödinger equation, e.g. the Paris wave function $u_{1S^0_0}^{NR}(s_f, |p'|)$, the BS amplitude is obtained by formula

$$\phi_{1S^+}^0(p_0, |p|) = \phi_{1S^+}^0(p_0, |p|) - \frac{G^{\alpha,i}(\tilde{p}_0, |p|)}{\left(\frac{\sqrt{s_f}}{2} - E_p\right)^2 - p_0^2}$$

(9)

where the "one-iteration" BS vertex $G^{\alpha,i}(\tilde{p}_0, |p|)$ is defined as

$$G^{\alpha,i}(\tilde{p}_0, |p|) = \frac{1}{\pi} \frac{g_{\pi NN}^2}{4\pi} \left\{ 1 - \frac{E_p}{m} \right\} \int_0^\infty dr e^{-\mu r} j_0(np) u_{1S^0_0}^{NR}(r) +$$

where $j_0(np)$ is the Bessel function of the first kind of order zero and $u_{1S^0_0}^{NR}(r)$ is the non relativistic wave function.
Now from eqs. (9) and (10) one may easily find the non relativistic analogue of the obtained formulae. So, the free term in eq. (9) together with the first term in eq. (10) reflect the non relativistic equation for the $^1S_0$ wave function, while the second term in (10) turns out to be corrections of pure relativistic origin.

4 Numerical results

In Figs. 2 and 3 we present the results of numerical calculations of the cross section $d\sigma/dt$ and tensor analysing power $T_{20}$. The elementary charge exchange amplitude has been taken from ref. [29] and the non relativistic trial function $u_{^1S_0}^{NR}(r)$ is the solution of the Schrödinger equation with the Paris potential [30]. The Bethe-Salpeter amplitudes are those from the numerical solution [21] obtained with a realistic one-boson exchange interaction. The dashed lines in Figs. 2 and 3 correspond to results within the relativistic impulse approximation [19], while the solid lines denote results with taking into account the final state interaction in one-iteration approximation (as it is described above). It is seen that in all three energy bins the agreement with data for the cross section is essentially improved. Especially it concerns the range $1 \leq E_x \leq 4$. For the energy bin close to zero there is still a disagreement with data at low transferred momenta which, probably may be addressed to the fact that in our calculations we have not taken into account the Coulomb interaction in the $pp$-pair. For higher excitation energies ($E_x \sim 8$ MeV), other partial waves (e.g. triplet state) in the $pp$-final state contribute and, within the adopted assumptions one may expect only qualitative agreement with data. From the Fig. 2 one may conclude that at low excitation energies the supposed mechanism for the reaction (1) (charge exchange subprocess with interaction in $^1S_0$ state of the $pp$-pair in the continuum) seems to be correct. Moreover, from a comparison of the left and right panels in Fig. 2 one may expect that for the higher initial energy there is larger kinematical region where the mechanism holds. Figure 3 demonstrates that the tensor analysing power is less sensitive to final state interaction effects. As a matter of fact, the tensor analysing power, being a ratio of non diagonal products of partial amplitudes to the diagonal ones, serves as a measure of the quality of parametrization of partial amplitudes and their mutual relative phases. This has been pointed out in a series of publications (see e.g. refs.[6, 10]), where a good simultaneous description of cross sections and $T_{20}$ in reactions of the deuteron
break-up or elastic scattering from protons, is still lacking. Nevertheless, since in the
process (1) the behaviour of the partial amplitudes is mostly governed by the elementary
charge exchange ones, an experimental investigation of the tensor analysing power $T_{20}$
in reactions of the type (1) can essentially supplement data on the $N N$ charge exchange
amplitudes at high energies. In Figs. 4 and 5 we present the predicted cross section and
tensor analysing power at high energies (COSY, Dubna). It is immediately seen that the
cross section is substantially decreasing with the energy increasing, nevertheless remains
large enough to be experimentally investigated. Another peculiarity of the studied process
at relativistic energies is that the tensor analysing power $T_{20}$ does not change the sign
remaining positive in a large kinematical region, in contrast to the lower energies (cf. Fig.
3). Note again, that in the above calculations the vector analyzing power of the deuteron
is strictly zero.

From the performed analysis one can conclude that there is a kinematical region for
the excitation energy, $E_x < 5 \text{ MeV}$, and transferred momentum, $|q| \leq 0.3 \div 0.4 \text{ GeV/c}$
(the COSY [4] kinematics), for which the mechanism of the reaction (1) is fairly well
described within the spectator approach by an elementary $pn$ charge exchange subprocess,
for active nucleons, with detection of the $pp$-pair in the $^1S_0$ final state. Our covariant
approach agrees with previous non relativistic calculations and allows predictions of the
cross sections and polarization observables at intermediate and relativistic energies, in
particular, for kinematical conditions achievable at COSY. The predicted cross sections
and tensor analysing power $T_{20}$ are large enough for the process (1) to be used, in a large
range of initial energies, for the determination of properties of polarized deuteron, provided
experimentally one simultaneously detects vanishing vector polarization of deuterons.

5 Summary

In summary, the performed covariant analysis of the reactions $\vec{D}(p, n)pp$ with two protons
in the $^1S_0$ final state allows to conclude that, as in the non relativistic limit, such process
can be used as an effective deuteron polarimeter also at relativistic energies, achieved at
COSY and Dubna. The effects of final state interaction are found to be substantial and
essentially improve the agreement with data.
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Figure 1: The spectator mechanism for the charge-exchange process $Dp \rightarrow (pp)n$. The Bethe-Salpeter amplitudes for the deuteron bound state and the $pp$-pair in the continuum are denoted as $\Psi$ and $\bar{\Psi}$, respectively. The elementary $pn$ charge-exchange amplitude is symbolically represented by $\mathcal{A}$. 
Figure 2: Results of full calculations of the differential cross section (3) with taking into account the effects of final state interaction in $^1S_0$ state (solid lines). Experimental data are those from SATURN-II [16], the elementary amplitude has been taken from ref.[27, 29]. The dashed lines reflect the results of calculations within the pure impulse approximation (cf. [19]).
Figure 3: Results of full calculations of the tensor analysing power with taking into account the effects of final state interaction in $^1S_0$ state. Notation as in Fig. 2.
Figure 4: Results of full calculations of the differential cross section (3) with taking into account the effects of final state interaction in $^1S_0$ state (solid lines). Kinematical conditions correspond to those proposed at COSY [4]. The elementary amplitude has been taken from ref.[27, 29]. The dashed lines reflect the results of calculations within the pure impulse approximation (cf. [19]).
Figure 5: Results of full calculations of the tensor analysing power with taking into account the effects of final state interaction in $^1S_0$ state within the COSY kinematical conditions [4]. Other notation as in Fig. 4.