Abstract

We study two-body exclusive decays of the form $\bar{B} \to D^{(*)} L$ ($L = \pi, \rho$) in the heavy-quark limit. We perform a renormalon analysis of such processes to determine the order at which nonperturbative factorization-breaking power corrections enter the amplitude. We find that a class of leading power corrections to the color octet matrix element, of $O(\Lambda_{\text{QCD}}/m_b)$, vanish in the limit of a symmetric light meson parton distribution function. We discuss the phenomenological significance of this result.
I. INTRODUCTION

The weak decays of $B$ mesons into hadronic final states are important for an understanding of the CKM sector of the standard model, and particularly for the study of CP violation. These decays involve a mixture of calculable weak physics, perturbative QCD, and nonperturbative QCD. It is the latter component which contains the bulk of the theoretical difficulty, and it is primarily contained in the evaluation of low energy matrix elements of quark operators.

A common assumption used to simplify the nonperturbative component of hadronic decays is factorization, which specifies that the hadronic matrix element of a four-quark operator be factored into two matrix elements of simple currents. For example,

$$
\langle \pi^− D^+ | (\bar{c}b)_{V-A} (\bar{d}u)_{V-A} | \bar{B} \rangle \to \langle \pi^− | (\bar{d}u)_{V-A} | 0 \rangle \langle D^+ | (\bar{c}b)_{V-A} | \bar{B} \rangle.
$$

(1.1)

This prescription (which, following the authors of [1,2], we refer to as ‘naive factorization’) considerably simplifies matters because the factored structures on the right hand side may be parameterized in terms of decay constants and form factors. It amounts, however, to ignoring corrections which connect the $(\pi^−)$ to the $(\bar{B}D^+)$ system. Since these corrections are responsible for final-state rescattering and strong interaction phase shifts, leaving them out ignores important physics. Also, the left hand side of (1.1) is renormalization scale dependent, while the right-hand side is not — a clear indication that relevant physics is being lost.

Recently it was argued that, for certain $B$ decays to heavy-light final states $\bar{B} \to HL$, the strong interactions which break factorization are hard in the heavy quark limit [1,2], and can therefore be calculated perturbatively. This proposal has been explicitly verified to two-loop order [2]. This idea allows one to include perturbative corrections missing in (1.1) without introducing any new nonperturbative parameters. It thus provides a remarkably attractive means to study rescattering and strong phases in two-body hadronic decays. A generalization of this idea has also been proposed for decays to two light mesons [1,2], but in this article we will restrict our attention to final states with one heavy meson ($D, D^*$) and one light meson ($\pi, \rho$).

A potential problem with this proposal is that it is valid only in the strict heavy quark limit [1]. It receives power corrections of the form $(\Lambda_{QCD}/m_b)^n$ from a variety of sources: examples are hard spectator interactions, non-factorizable soft and collinear gluon exchange, and transverse momenta of quarks in the light meson. Unlike power corrections in inclusive $B$ decays, there is as yet no systematic way to compute these corrections for exclusive decays. By naive power counting one expects such corrections at $O(\Lambda_{QCD}/m_b)$, but situations are known where the naively expected corrections vanish. For instance, in the zero recoil $B \to D$ transition matrix element in Heavy Quark Effective Theory the leading $1/m_b$ corrections vanish [3]. Therefore rather than relying on the naive expectation, one would like to calculate the power corrections directly.

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$^1$Following the authors of [1,2], we assume that the physical $b$ quark mass is not so large that Sudakov form factors modify the power counting.
In the absence of a general theory of power corrections, we aim to determine at least the order at which power corrections enter. This may be done by carrying out a renormalon analysis, which involves calculating a subset of Feynman graphs at each order in perturbation theory. It exploits the fact that the perturbative series in quantum field theory is asymptotic, and permits one to extract from the large-order behavior of the theory information about the scaling behavior of nonperturbative power corrections. In this paper, we use renormalons to assess the parametric size of a subset of power corrections to the separation (1.1) for the class of $\bar{B} \to HL$ decays discussed in [2].

The remainder of this paper is organized as follows: in Section II we review the theoretical framework of renormalon analyses. In Section III we discuss the phenomenological context and motivation of our study. In Section IV we describe the renormalon calculation and give the result. Section V contains our conclusions.

II. RENORMALONS

A typical amplitude in QCD perturbation theory may be expressed in the general form

$$R(\alpha_s) = \sum_{n=0}^{\infty} R_n \alpha_s^{n+1}. \quad (2.1)$$

Normally one only calculates a few terms in this series. However, one may ask about the general behavior of this series at large orders in perturbation theory. It has been argued [4] that quantum field theories of phenomenological interest have large order coefficients of the form

$$R_n \sim a^n n! n^b \quad (2.2)$$

for some constants $a$ and $b$. Clearly, such a series is factorially divergent. It might appear that, as a result, a sum for the series cannot be defined. However, one may define the Borel sum $\tilde{R}$ in the following way. Perform a Borel transformation on the series:

$$R(\alpha_s) = \sum_{n=0}^{\infty} R_n \alpha_s^{n+1} \implies B[R](t) = \sum_{n=0}^{\infty} \frac{R_n}{n!} t^n. \quad (2.3)$$

This series in terms of the Borel parameter $t$ is convergent and may be explicitly summed. One may then perform an inverse Borel transformation to obtain the Borel sum

$$\tilde{R} = \int_0^{\infty} dt \ e^{-t/\alpha_s} B[R](t). \quad (2.4)$$

The original series $R$ and the Borel sum $\tilde{R}$ have the same series expansion.

In some cases, the transformed series $B[R](t)$ has poles along the positive real axis [4]. When these poles are encountered in the inverse Borel transformation (2.4), one is forced to deform the integration contour either above or below the real axis. Nothing specifies which choice to make, yet the result of the integration depends on the choice. As a result, the Borel sum acquires an ambiguity. For a simple pole located at $t_0 > 0$, the ambiguity is
\[\delta \tilde{R} \sim e^{-t_0/\alpha_s(\mu)} \sim \left(\frac{\Lambda_{\text{QCD}}}{\mu}\right)^{2u_0} = \left(\frac{\Lambda_{\text{QCD}}}{\mu}\right)^{2u_0}\]

where we have defined \(u_0 = -\beta_0 t_0 > 0\). The ambiguity has the form of a nonperturbative power correction. For physical quantities, which cannot be ambiguous, there must be present power corrections to remove this ambiguity. Note that for \(\mu > \Lambda_{\text{QCD}}\) it is the pole nearest the origin which gives the leading power correction. This simple sketch illustrates how the large-order perturbative behavior of the theory reveals something about the nonperturbative sector of the theory [4].

In general, this approach only permits one to determine the order of the power corrections and not their coefficients or analytic form. Furthermore, while a pole at \(u_0\) in the Borel plane definitely indicates the presence of power corrections \(\sim (\Lambda_{\text{QCD}}/m_b)^{2u_0}\), the absence of a pole does not necessarily imply the absence of power corrections of that order. The absence of a pole is, rather, suggestive that power corrections of the corresponding order are absent [4]. The renormalon technique has been applied in a variety of contexts where a general theory of power corrections has not been available [5–10]. In Section [V] we study power corrections to factorization in this way.

In practice one cannot sum the entire perturbative series (2.1) to obtain an exact expression for the Borel transformed amplitude (2.3). Instead, one sums a subset of the Feynman diagrams at each order, implicitly taking the resulting analytic structure to be characteristic of the full result. Typically, all-orders contributions are obtained by inserting into graphs ‘bubble chain’ propagators of the kind shown in Figure 1. One may take the formal limit \(N_f \to \infty\) with \(\alpha_s N_f\) fixed, in which case the set of graphs with a single ‘bubble chain’ insertion are dominant [4]. This ‘bubble chain’ propagator, which we denote by a dashed gluon line, has the form [7,11]

\[D_{\mu\nu}(k) = \frac{i}{k^2} \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right) \sum_{n=0}^{\infty} \frac{(-u)^n}{n!} \left(\beta_0 N_f \alpha_s\right)^n \left(\ln\left(-k^2/\mu^2\right) + C\right)^n.\]  

(2.6)

The factors in the sum arise from the fermion loops depicted in Figure 1. These loops have been renormalized in an MS-like scheme, and \(C\) is a scheme dependent constant. In the MS scheme, \(C = -5/3\). We discuss renormalization of amplitudes containing the renormalon propagator in more detail in Section [IV]. The Borel transform of this propagator with respect to \(\alpha_s N_f\) is

\[B[D_{\mu\nu}(k)](u) = \frac{1}{\alpha_s N_f} \frac{i}{k^2} \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right) \sum_{n=0}^{\infty} \frac{(-u)^n}{n!} \left(\ln\left(-k^2/\mu^2\right) + C\right)^n = \frac{1}{\alpha_s N_f} \left(\frac{\mu^2}{e^C}\right)^u \frac{i}{(\mu^2)^{2+u}} (k_\mu k_\nu - k^2 g_{\mu\nu}).\]  

(2.7)

The limit \(u \to 0\) of this expression, equivalent to retaining only the first term in the expansion depicted in Figure 1, reduces to the usual gluon propagator as expected. For Feynman graphs in which the \(\alpha_s\) dependence arises only from gluon exchange, the Borel transformed graph is obtained by replacing the gluon propagator by this renormalon propagator.
III. FACTORIZATION AND TWO-BODY HADRONIC B DECAYS

We work in an effective theory where the weak bosons and top quark have been integrated out. The relevant part of the effective Hamiltonian, valid below $M_W$, is

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub} V_{cb} (C_1 O_1 + C_8 O_8)$$

where the singlet and octet operators are, respectively,

$$O_1 = \bar{c} \gamma_\mu (1 - \gamma_5) b \bar{d} \gamma_\mu (1 - \gamma_5) u,$$

$$O_8 = \bar{c} \gamma_\mu (1 - \gamma_5) T^a b \bar{d} \gamma_\mu (1 - \gamma_5) T^a u.$$ 

The running of the Wilson coefficients $C_i$ has been calculated at next-to-leading order [12,13].

The computation of any Feynman amplitude in this theory involves evaluating matrix elements of the operators $O_{1,8}$. In general such matrix elements contain both hard and soft physics. One would like to disentangle these energy scales; the hard physics could be calculated directly and the soft physics parameterized by form factors and decay constants. Though it is not obvious that the physics can be disentangled in this way, it has been argued [1,2] that in certain situations it is possible to do so.

More specifically, consider a decay of the form $\bar{B} \to HL$, where $H$ is a heavy meson ($H = D, D^*$) and $L$ is a light meson ($L = \pi, \rho$). We require that the topology of the decay be such that the light quark in the initial state is transferred to the heavy final state meson. In this case, and in the heavy quark limit, it is argued that 'non-factorizable' corrections are perturbative and may be calculated. These statements are summarized in the factorization equation [1,2]

$$\langle HL | O_i | \bar{B} \rangle = F_{\bar{B} \to H}(m_L^2) f_L \int_0^1 dx T^I_i(x) \Phi_L(x) + \cdots$$

where the ellipsis denotes contributions suppressed by powers of $\Lambda_{\text{QCD}}/m_b$. In this expression, the $B$ decay form factor $F_{\bar{B} \to H}$ and the light meson decay constant $f_L$ are the nonperturbative parameters present in the case of naive factorization (1,3). The 'non-factorizable' physics is contained in the convolution of the perturbatively calculable hard-scattering kernel $T^I_i(x)$ with the light-cone momentum distribution of the leading Fock state (quark-antiquark) of the light meson $\Phi_L(x)$. The parameter $x$ is the momentum fraction of one of the quarks inside the light meson.

To leading order in $\alpha_s$, one finds [2,14]
\[ T^I_1(x) = 1 + \mathcal{O}(\alpha_s^2), \quad T^I_8(x) = 0 + \mathcal{O}(\alpha_s). \quad (3.5) \]

Given that the light meson distribution function \( \Phi_L(x) \) is normalized to unity, naive factorization \((1.1)\) is restored as the leading term in a perturbative expansion.

There are other decay topologies (penguin, annihilation) for which certain assumptions leading to \((3.4)\) are invalid. However, detailed arguments show that these topologies are suppressed by powers of \( \Lambda_{QCD}/m_b \), and are therefore irrelevant in the heavy quark limit \([2]\).

In the next section, we present the renormalon analysis of the ‘non-factorizable’ corrections. This will involve studying the large-order perturbative properties of \((3.5)\) as a means of determining the order at which power corrections enter the factorization equation \((3.4)\).

**IV. THE CALCULATION**

For a particular heavy final state \( H (D, D^*) \) and light final state \( L (\pi, \rho) \) we must calculate the matrix elements

\[ \langle O_{1,8} \rangle \equiv \langle H(p')L(q)|O_{1,8}|\bar{B}(p)\rangle. \quad (4.1) \]

![FIG. 2. The factorization-breaking corrections with renormalon propagators.](image)

The matrix element for the singlet operator is the simplest, so we consider it first. At leading order in \( \alpha_s \) the operator factorizes cleanly into a product of currents. The leading factorization-breaking QCD corrections are shown in Figure 2. In order to create a color singlet structure from the resulting graph, one would have to consider the light meson to be in a Fock state higher than \((q'\bar{q})\); this situation, however, is power suppressed by \( \Lambda_{QCD}/m_b \). Alternatively, one could retain the leading Fock state and consider the exchange of two gluons rather than just one; this situation is suppressed by \( \alpha_s \) relative to the graphs in Figure 4. In our calculation, then, the singlet matrix element is simply
\[ \langle O_1 \rangle = \langle L(q) | \bar{d} \gamma^\mu (1 - \gamma_5) u | 0 \rangle \langle H(p') | \bar{c} \gamma_\mu (1 - \gamma_5) b | B(p) \rangle \]
\[ = i f_L \left( \langle J_V \rangle - \langle J_A \rangle \right) \]  
(4.2)

where we define two matrix elements \( \langle J_V \rangle = \langle H(p') | \bar{c} \gamma_\mu q^\mu b | B(p) \rangle \) and \( \langle J_A \rangle = \langle H(p') | \bar{c} \gamma_\mu q^\mu \gamma_5 b | B(p) \rangle \).

The octet matrix element \( \langle O_8 \rangle \) has a richer structure. It vanishes at leading order, but the color structures of the graphs shown in Figure 2 are such that \( \langle O_8 \rangle \) receives perturbative corrections at all higher orders. Let us denote the individual amplitudes of the diagrams in Figure 2 as \( A_i \), and the corresponding Borel transformed amplitudes as \( B[A_i] \). Defining \( x \) to be the momentum fraction carried by the up quark of the light meson, \( \bar{x} = 1 - x \) to be the momentum fraction of the down quark, and \( z = m_c / m_b \), the individual Borel transformed amplitudes of the diagrams shown in Figure 2 may be written as

\[ B[A_1](u) = \frac{i f_L C_F}{2 N_c} \left( \frac{\mu^2}{e^2 m_b^2} \right)^u (F_1(x, z, u) \langle J_V \rangle - F_1(x, -z, u) \langle J_A \rangle) \]

\[ B[A_2](u) = \frac{i f_L C_F}{2 N_c} \left( \frac{\mu^2}{e^2 m_b^2} \right)^u (F_2(\bar{x}, z, u) \langle J_V \rangle - F_2(\bar{x}, -z, u) \langle J_V \rangle) \]

\[ B[A_3](u) = \frac{i f_L C_F}{2 N_c} \left( \frac{\mu^2}{e^2 m_c^2} \right)^u (F_2(x, 1/z, u) \langle J_V \rangle - F_2(x, -1/z, u) \langle J_A \rangle) \]

\[ B[A_4](u) = \frac{i f_L C_F}{2 N_c} \left( \frac{\mu^2}{e^2 m_c^2} \right)^u (F_1(\bar{x}, 1/z, u) \langle J_V \rangle - F_1(\bar{x}, -1/z, u) \langle J_A \rangle) \]

(4.3)

where

\[ F_1(x, z, u) = \frac{\Gamma(1 - 2u) \Gamma(u - 1)}{\Gamma(2 - u) (x(1 - z^2))^u} \left\{ \frac{(1 - 2u)(3u - 1)}{u} f_1(x, z, u) - \left[ 1 - u - 2 \left( 1 - \frac{1}{x(1 - z^2)} \right) \right] f_2(x, z, u) \right\} \]

(4.4)

and

\[ F_2(x, z, u) = \frac{\Gamma(2 - 2u) \Gamma(u - 1)}{\Gamma(3 - u) (x(1 - z^2))^u} \left\{ \left[ \frac{2 - u + u^2}{u} + \frac{2uz}{1 - x(1 - z^2)} \right] f_1(x, z, u) + \left[ \frac{2 - u}{1 - 2u} \left( \frac{2}{x(1 - z^2)} - (1 + u) \right) - \frac{2uz}{1 - x(1 - z^2)} \right] f_2(x, z, u) \right\}. \]

(4.5)

We have written our result in terms of the hypergeometric functions

\[ f_1(x, z, u) \equiv \begin{pmatrix} 1-u, u; 2-u; 1-\frac{1}{x(1 - z^2)} \end{pmatrix} \]

(4.6)

\[ f_2(x, z, u) \equiv \begin{pmatrix} 1-u, 1+u; 2-u; 1-\frac{1}{x(1 - z^2)} \end{pmatrix}. \]

(4.7)

We have evaluated each of these graphs in \( d = 4 \) dimensions, as the divergences which would normally be present are regulated by the Borel parameter \( u \) in the renormalon propagator \( L(q) \). In the limit \( u \to 0 \), however, each of the \( B[A_i](u) \) contain both infrared and
ultraviolet divergences. It was a central result of \cite{1,2} to show that in the heavy quark limit the sum

\[ B_0[A](u) = \sum_{i=1}^{4} B[A_i](u) \]  

(4.8)

is infrared finite for amplitudes of the type we are considering. This cancellation may be seen explicitly from our amplitudes (4.3): in the vicinity \( u \sim 0 \) we have

\[
B[A_1](u) = \frac{i f_L C_F}{2 N_c} \langle J_{V-A} \rangle \left[ \frac{1}{u^2} + \frac{1}{u} \left( -2 \log x (1 - z^2) - C + \log \frac{\mu^2}{m_b^2} - 2 \right) \right] + \cdots
\]

\[
B[A_2](u) = \frac{i f_L C_F}{2 N_c} \langle J_{V-A} \rangle \left[ \frac{1}{u^2} + \frac{1}{u} \left( 2 \log \bar{x} (1 - z^2) + C - \log \frac{\mu^2}{m_c^2} - 1 \right) \right] + \cdots
\]

\[
B[A_3](u) = \frac{i f_L C_F}{2 N_c} \langle J_{V-A} \rangle \left[ \frac{1}{u^2} + \frac{1}{u} \left( 2 \log \bar{x} (1 - 1/z^2) + C - \log \frac{\mu^2}{m_c^2} - 1 \right) \right] + \cdots
\]

\[
B[A_4](u) = \frac{i f_L C_F}{2 N_c} \langle J_{V-A} \rangle \left[ \frac{1}{u^2} + \frac{1}{u} \left( -2 \log \bar{x} (1 - 1/z^2) - C + \log \frac{\mu^2}{m_c^2} - 2 \right) \right] + \cdots
\]

(4.9)

where the ellipses denote terms finite as \( u \to 0 \), and we introduce the shorthand \( \langle J_{V-A} \rangle = \langle J_V \rangle - \langle J_A \rangle \). The collinear divergences cancel in pairs among the even and odd numbered amplitudes, while the remaining soft divergences cancel in the manner prescribed by Bjorken’s color transparency argument \cite{2,15}.

For the sum of the four amplitudes we find

\[
B_0[A](u \sim 0) = -\frac{i f_L C_F}{2 N_c} \langle J_{V-A} \rangle \frac{6}{u} + \text{finite}.
\]  

(4.10)

This remaining divergence is ultraviolet in origin, and may be renormalized in a manner consistent with MS-like subtraction schemes. To this end, we follow the prescription of \cite{7,16} by defining a renormalized amplitude

\[
B[A](u) = B_0[A](u) + S_A(u)
\]  

(4.11)

where \( S_A(u) \) contains a divergence which cancels that in \( B_0[A](u) \) at the origin but is finite elsewhere. This regulating function may be written as

\[
S_A(u) = \frac{1}{u} \sum_{n=0}^{\infty} \frac{g_n}{n!} u^n
\]  

(4.12)

where the coefficients \( g_n \) are the expansion coefficients of another function \( G(\epsilon) = \sum_{n=0} g_n \epsilon^n \) related to the amplitudes computed in \( d = 4 - 2\epsilon \) dimensions. We refer the reader to the relevant literature for an explanation of this method \cite{7,16}. For the sum of amplitudes \( \mathcal{A} \) we find

\[
G(\epsilon) = \frac{i f_L C_F}{2 N_c} \frac{2(1 + \epsilon)(1 + 2\epsilon)(3 + 2\epsilon)\Gamma(4 + 2\epsilon)}{3\Gamma(1 - \epsilon)\Gamma^2(2 + \epsilon)\Gamma(3 + \epsilon)} \langle J_{V-A} \rangle
\]

\[
= \frac{i f_L C_F}{2 N_c} \langle J_{V-A} \rangle (6 + 23 \epsilon + \frac{127}{6} \epsilon^2 + \cdots).
\]  

(4.13)
The first term in this expansion cancels the ultraviolet divergence remaining in (4.11); $B[A](u)$ is infrared and ultraviolet finite.

In this notation, the Borel transform of the factorization equation (3.4) is

$$B[(O_8)](u) = \int_0^1 dx B[A](u) \Phi_L(x).$$

(4.14)

Comparing to (3.4), we see that $B[A](u)$ is proportional to the Borel transform of the hard scattering kernel $T^I_8(x)$.

### A. Borel poles and power corrections

Recall from (2.5) that it is the pole nearest the origin on the positive real axis in the Borel plane which indicates the leading power correction. A pole at the origin would indicate an $O(1)$ correction, but the renormalization procedure outlined in the previous section ensures that no such pole is present. We find that the first pole is located at $u = 1/2$, corresponding to a power correction of $O(\Lambda_{\text{QCD}}/m_b)$. This is not a very surprising result, as power corrections of this order are known to be present from a variety of sources [2]. However, in the vicinity of this pole the Borel transformed amplitude has the form

$$B[(O_8)](u \sim 1/2) \propto \frac{1}{u - 1/2} \int_0^1 dx \Phi_L(x) \frac{(x - \bar{x})}{x \bar{x}} \langle J_{V-A} \rangle + \cdots$$

(4.15)

where $\bar{x} = 1 - x$, and the ellipsis denotes nonsingular terms.

Before interpreting this result, we must specify the form of the light meson momentum distribution $\Phi_L(x)$. It is customary to write it as [17]

$$\Phi_L(x) = 6x(1 - x) \left[1 + \sum_{n=1}^{\infty} \alpha_n^L(\mu) C_n^{3/2}(2x - 1)\right]$$

(4.16)

where the Gegenbauer polynomials $C_n^{3/2}(y)$ are given by

$$C_n^{3/2}(y) = \frac{1}{n!} \frac{d^n}{dh^n} \left[(1 - 2hy + h^2)^{-3/2}\right] \bigg|_{h=0}.$$ 

(4.17)

The distribution function $\Phi_L(x)$ is a nonperturbative object for which the Gegenbauer moments $\alpha_n^L(\mu)$ are unspecified. It is known, however, that $\alpha_n^L(\mu \to \infty) = 0$ [17]. For $\mu \sim m_b \gg \Lambda_{\text{QCD}}$, one may take the distribution function to have the asymptotic form $\Phi_L^0(x) = 6x(1 - x)$ up to power corrections of $O(\Lambda_{\text{QCD}}/m_b)$. In this case the wavefunction is symmetric under $x \to 1 - x$, and the integration over $x$ in (4.15) vanishes, removing the pole.

More generally, note that the Gegenbauer polynomials $C_n^{3/2}(2x - 1)$ in (4.16) with $n$ even are, like $\Phi_L^0(x)$, even under $x \to 1 - x$, while those with $n$ odd are odd under the same replacement. The result of the integration (4.15) may then be written as

$$B[(O_8)](u \sim 1/2) \propto \frac{6}{u - 1/2} \langle J_{V-A} \rangle \left(\alpha_1^L(\mu) + \alpha_3^L(\mu) + \alpha_5^L(\mu) + \cdots\right).$$

(4.18)
Only the asymmetric terms of $\Phi_L(x)$ contribute to the residue. Thus the renormalon analysis indicates the presence of $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$ factorization-breaking power corrections only for asymmetric light meson wavefunctions.

For certain final states of interest ($L = \pi, \rho$), SU(2) symmetry ensures that the wavefunction is symmetric [18,19]. In these cases the pole at $u = 1/2$ disappears, and the renormalon analysis gives no indication of factorization-breaking $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$ corrections, suggesting that such power corrections are likely absent. This result implies that power corrections due to the ‘non-factorizable’ vertex diagrams in Figure 2 should be suppressed in $\bar{B} \rightarrow D^{(*)}\pi^-$ and $\bar{B} \rightarrow D^{(*)}\rho^-$ decays relative to decays into a light meson with an asymmetric distribution function, such as $\bar{B} \rightarrow D^{(*)}K^-$. In assessing this result, one should keep in mind that there are other sources of power corrections to these decays which our analysis does not address. For example, we have not considered soft interactions with the spectator quark, annihilation diagrams, or interactions with sub-leading Fock states involving additional soft partons [1,2]. Our conclusions about the vanishing $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$ corrections in the symmetric limit apply only to the class of diagrams we have considered.

V. CONCLUSIONS

In this paper we have carried out a renormalon analysis of factorization-breaking effects in $\bar{B}$ meson decays to certain hadronic heavy-light final states. The renormalon approach, which probes the theory at high orders in perturbation theory, allows us to learn about nonperturbative power corrections. In the low energy effective theory governing the $\bar{B}$ decays there are two operators, a color singlet and a color octet. The factorization-breaking corrections to the singlet operator are, however, suppressed by powers of $\alpha_s$ or $\Lambda_{\text{QCD}}/m_b$ relative to the octet operator; for this reason we focus our analysis on the octet.

We find that the renormalon analysis of ‘non-factorizable’ corrections to the octet matrix element indicates the presence of power corrections of $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$. We also find that these leading power corrections are sensitive only to asymmetries in the light meson light-cone parton distribution function. Thus for a symmetric distribution function the leading renormalon pole vanishes, suggesting the absence of $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$ ‘non-factorizable’ corrections in such cases. The next power corrections are present at $\mathcal{O}(\Lambda_{\text{QCD}}^2/m_b^2)$.

As it is natural to expect the light meson wave function to be symmetric for certain final states ($L = \pi, \rho$), the potential vanishing of the $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$ power corrections in the symmetric limit is an interesting result that warrants a few additional comments. First, it should be noted that a particular decay $\bar{B} \rightarrow HL$ of the type we have considered in this paper always involves a combination of the singlet and octet operator matrix elements. Our result, however, applies only to the octet matrix element — we are unable to conclude from our analysis whether the leading power corrections to the singlet matrix element vanish in an analogous way.
Second, the power corrections we can probe through the renormalon analysis are physically due to soft and collinear ‘non-factorizable’ gluon exchange, and our comments about suppression of the leading corrections should be understood as referring to these effects only. There are, however, other $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$ corrections originating from diagrams which were not considered in our analysis. A more detailed examination of these effects would have to be undertaken to accurately assess the size of $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$ corrections to a physical decay amplitude.

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