We propose a massless nonminimally coupled scalar field as a mechanism for stabilizing the size of the extradimensional in the Randall-Sundrum I scenario. Without needing to introduce self interactions terms we obtain a potential for the modulus field that sets the size of the fifth dimension. The minimum of this potential yields appropriate values of the compactification scale for small values of the coupling $\xi$.

1. introduction

An approach to the hierarchy problem has been proposed in which large compactified extradimensions may provide an alternative solution [1,2]. In these models the observed Planck mass $g_P$ is related to $M$, the fundamental scale of the theory, by $M_P^2 = M^{n+2} V_n$, where $V_n$ is the volume of the compactified dimensions. In $V_n$ is large enough, $M$ can be of the order of the weak scale. This properties rely on a factorizable geometry, namely the metric of the four familiar dimensions is independent of coordinates in the extradimensions. In the works [3,4], Randall and Sundrum have proposed a higher dimensional scenario which allow the existence of $4 + n$ non-compact dimensions in perfect compatibility with experimental gravity. In this model the solution to the hierarchy problem rely on a non-factorizable geometry, and consists (in the five dimensional case) of a single $S^1/Z_2$ orbifold extradimension with two 3-branes of opposite tension, residing at the orbifold fixed points [3,4]. Introducing a bulk cosmological constant and solving the Einstein’s equations, gives the solution with non-factorizable geometry, that respects the four dimensional Poincare invariance

$$ds^2 = e^{-2kr_c|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2d\phi^2$$ (1)

where $-\pi \leq \phi \leq \pi$ is the extradimensional coordinate and $r_c$ is the interbrane distance, called compactification radius. This solution holds only when the brane tensions and the bulk cosmological constant are related by the so called fine tuning conditions [3]. This condition ammounts to setting the four dimensional cosmological constant to zero, which is not a desired situation. A similar scenario to the one proposed in [3], is that of Horava-Witten [5], which arises within the context of supergravity nd M theory (see also [6]for supergravity solutions).

Due to the exponential factor in the spacetime metric, a field confined to the brane at $\phi = \pi$ with mass $m$, will have physical mass $me^{-kr_c\pi}$ and for $kr_c \approx 12$, the fundamental Planck scale $M$ is reduced to the weak scale (1Tev). However, as is well known, the dynamics does not determine the value of $r_c$, leaving it a free parameter. A solution to this so called radion stabilization problem, has been found by adding a bulk scalar field which generates the potential to stabilize the value of $r_c$. This potential can appear from the presence of a bulk scalar field with interaction terms that are localized to the two 3-branes [7]. Alternative stabilization mechanisms have been proposed in [8,13,10,11]. The solution presented in the work [12], fixes the interbrane distance by adding to the brane tension matter density and pressure. Depending on the kind of matter, such solutions can be either stable or unstable under small perturbations.

In this paper we present a solution to the radion stabilization problem, by considering a massless bulk scalar field, non-minimally coupled to 5-dimensional gravity. We find that, without introducing self interaction terms on the branes, we obtain an effective potential, and the minima can be arranged to yield a value of $kr_c \approx 12$, for small
value of the bulk coupling constant $\xi$. We present our solution in section II and conclusions in section III.

2. Effective potential for the coupled scalar field

We denote the spacetime coordinates by $x^A = \{x^\mu, y\}$, where $x^\mu$, $\mu = 0, ..., 3$ are Lorentz coordinates on the 3-branes and $y = y_c, -\pi \leq \phi \leq \pi$. The orbifold fixed points are at $\phi = 0$ and $\phi = \pi$ and $r_c$ is the size of the extradimension.

The action for the bulk scalar field is given by

$$S = \frac{1}{2} \int d^4x \sqrt{-G} \left( G^{AB} \partial_A \Phi \partial_B \Phi - \xi R \Phi^2 \right),$$

(2)

where $G = \text{det}[G_{AB}]$, $R$ is the bulk curvature for the metric (1) and is given by Eq.

$$R = 20 \sigma'^2 - 8 \sigma''$$

(3)

where $\sigma' = \partial_y \sigma$. The $\phi$-dependent vacuum expectation value $\Phi(\phi)$ is determined by solving the equation of motion

$$-\frac{1}{r_c^2} \partial_y (e^{-4\sigma(\phi)} \partial_y \Phi) + \xi e^{-4\sigma} (20 \sigma'^2 - 8 \sigma'') \Phi = 0$$

(4)

where $\sigma(\phi) = kr_c |\phi|$ and we take into account that according to the solution of the Randall-Sundrum set up [3], $\sigma'' = 2kr_c (\delta(\phi) - \delta(\phi - \pi))$. The solution to this equation excluding the fixed points $\phi = 0, \pi$ is of the form [7]

$$\Phi(\phi) = Ae^{(2+\nu)\phi} + Be^{(2-\nu)\phi},$$

(5)

where $\nu = 2\sqrt{1 + 5\xi}$. After replacing this solution into the scalar field action and integrating over $\phi$, we obtain an effective potential which depends on $r_c$ and $\xi$

$$V(r_c, \xi) = \frac{2A^2k}{\nu r_c} (2 + \nu + 10\xi + 8\xi \nu)(e^{2\nu kr_c} - 1) + \frac{2B^2k}{\nu r_c} (2 - \nu + 10\xi - 8\xi \nu)(1 - e^{-2\nu kr_c})$$

(6)

The coefficients $A$ and $B$ can be determined by imposing boundary conditions,

$$A + B = \Phi(0) = V_h$$

$$A e^{(2+\nu)\pi kr_c} + B e^{(2-\nu)\pi kr_c} = \Phi(\pi) = V_v$$

(7,8)

where the subindices $h$ and $v$ stand for hidden and visible branes respectively. From Eqs. (7,8) we obtain, after neglecting $e^{-\pi kr_c}$ compared with $e^{\pi kr_c}$, in the large $kr_c$ limit

$$A = V_v e^{-(2+\nu)\pi kr_c} - V_h e^{-2\nu kr_c}$$

$$B = V_h (1 + e^{-2\nu kr_c}) - V_v e^{-(2+\nu)\pi kr_c}$$

(9,10)

Replacing the above given values of $A$ and $B$ into the Eq. (6) for the effective potential, and neglecting terms of order $\xi^2$, one obtains

$$V_{eff}(r_c) = -(4 + 21\xi)[e^{-4\pi kr_c}$$

$$+ \left(\frac{V_h}{V_v}\right)^2 e^{-2\nu kr_c} - 2 \frac{V_h}{V_v} e^{-(2+\nu)\pi kr_c}] + 11\xi[e^{-2(2+\nu)\pi kr_c}$$

$$+ \left(\frac{V_h}{V_v}\right)^2 - 2 \frac{V_h}{V_v} e^{-(2+\nu)\pi kr_c}]$$

(11)

where $V_{eff}(r_c) = V(r_c)/kV_v^2$ and we assumed $e^{-\nu kr_c}\ll 1$. Considering $V_h/V_v = 2$ this potential has a minimum at

$$kr_c = \frac{1}{(\nu - 2)\pi} \ln(\nu),$$

(12)

with $\nu$ given by $\nu = 2\sqrt{1 + 5\xi}$. Taking $\xi = 1/265$ yields $kr_c \approx 12$ which is the expected magnitude for $kr_c$ if we want to reduce the scale of quantum gravity effects to the weak scale. The positive value of $d^2V(r_c)/dr_c^2$ signals the stability of the effective radion potential. Here we have not considered the back reaction of the scalar field on the background geometry. This follows from the conditions $\xi \ll 1$ and $V_v, V_h \ll M^{3/2}$, which allows the stress tensor for the scalar field to be neglected in comparison to the stress tensor induced by the bulk cosmological constant [3,7].

3. Conclusions

We have seen that a massless bulk scalar field, non minimally coupled to 5-dimensional curvature, can generate a potential to stabilize $r_c$ for
very small values of the coupling constant $\xi$. Thus the large value of $k_r$ arises not from small bulk scalar mass [7], but from the small value of the coupling constant, and in addition, this stabilized modulus is obtained without introducing interaction terms on the branes. If we consider quantum effects arising from bulk scalars, then non trivial vacuum energy appears. This bulk Casimir effect should play a remarkable role in the stabilization mechanism, as it gives contribution to both, the brane and the bulk cosmological constants (see [13] for works in this direction). It would be worthwhile to consider the effect of the bulk scalar field on the background geometry, and also to explore the role of this field in cosmological models with accelerating expansion.

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