Improved version of the PHOBOS Glauber Monte Carlo

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“Glauber” models are used to calculate geometric quantities in the initial state of heavy ion collisions, such as impact parameter, number of participating nucleons and initial eccentricity. Experimental heavy-ion collaboration, in particular at RHIC and LHC, use Glauber Model calculations for various geometric observables. In this document, we describe the assumptions inherent to the approach, and provide an updated implementation (v2) of the Monte Carlo based Glauber Model calculation, which originally was used by the PHOBOS collaboration. The main improvement w.r.t. the earlier version (v1) \textsuperscript{4} are the inclusion of tritium, Helium-3, and Uranium, as well as the treatment of deformed nuclei and Glauber-Gribov fluctuations of the proton in p+A collisions. A users’ guide (updated to reflect changes in v2) is provided for running various calculations.

I. INTRODUCTION

In heavy-ion collisions, initial geometric quantities such as impact parameter and shape of the collision region cannot be directly determined experimentally. However, it is possible to relate the number of observed particles and number of spectator neutrons to the centrality of the collision. Using the percentile centrality of a collision, the initial geometric configuration can be estimated with models assuming the configuration nuclei from nucleons and the collision process of two nuclei.

These models fall in two main classes. (For a review, see Ref. \textsuperscript{2}). In the so called “optical” Glauber calculations, a smooth matter density is assumed, typically described by a Fermi distribution in the radial direction and uniform over solid angle. In the Monte Carlo (MC) based models, individual nucleons are stochastically distributed event-by-event and collision properties are calculated by averaging over multiple events. As discussed in Ref. \textsuperscript{2} and Ref. \textsuperscript{3}, the two type of models lead to mostly similar results for simple quantities such as the number of participating nucleons ($N_{\text{part}}$) and impact parameters ($b$), but give different results in quantities where event-by-event fluctuations are significant, such as participant frame eccentricity ($\epsilon_{\text{part}}$).

In this paper, we discuss several improvements of the Monte Carlo Glauber calculation originally implemented by the PHOBOS collaboration \textsuperscript{4}. The main improvement w.r.t. the earlier version (v1) \textsuperscript{4} are the inclusion of tritium, Helium-3, and Uranium, as well as the treatment of deformed nuclei and Glauber-Gribov fluctuations of the proton in p+A collisions. Another widely used implementation of a MC Glauber calculation can be found in Ref. \textsuperscript{4}. In section \textsuperscript{II} the MC approach is outlined and the assumptions that go into the calculation are introduced. In section \textsuperscript{IV} we discuss the implementation and the tutorial functions provided. Section \textsuperscript{IV} summarizes the paper.

II. THE MODEL

The MC Glauber model calculation is performed in two steps. First, the nucleon positions in each nucleus are stochastically determined. Then, the two nuclei are “collided”, assuming the nucleons travel in a straight line along the beam axis (eikonal approximation) such that nucleons are tagged as wounded (participating) or spectator.

A. Makeup of nuclei

The position of each nucleon in the nucleus is determined according to a probability density function. In a quantum mechanical picture, the probability density function can be thought of as the single-particle probability density and the position as the result of a position measurement. In the context of the MC Glauber model, one commonly requires a minimum inter-nucleon separation ($d_{\text{min}}$) between the centers of the nucleons for the determination of the nucleon positions inside a given nucleus.

The probability distribution for spherical nuclei is taken to be uniform in azimuthal and polar angles. The radial probability function is modeled from nuclear charge densities extracted in low-energy electron scattering experiments \textsuperscript{5}. The nuclear charge density is usually parameterized by a Fermi distribution with three parameters

$$\rho(r) = \rho_0 \frac{1 + w(r/R)^2}{1 + \exp\left(\frac{r-w}{a}\right)}, \quad (1)$$

where $\rho_0$ is the nucleon density, $R$ is the nuclear radius, $a$ is the skin depth and $w$ corresponds to deviations from a spherical shape. The overall normalization ($\rho_0$) is not relevant for this calculation. Values of the other parameters used for different nuclei are listed in Table \textsuperscript{VI}.

Exceptions from Eq. (1) are the deuteron ($^2\text{H}$), tritium ($^3\text{H}$), Helium-3 ($^3\text{He}$), and sulfur ($^{32}\text{S}$) nuclei.
For sulfur, a three parameter Gaussian form is used
\[ \rho(r) = \rho_0 \frac{1 + w(r/R)^2}{1 + \exp\left(\frac{r^2 - R^2}{a^2}\right)}. \] (2)

The values of \( R, a \) and \( w \) for sulfur are also given in Table I. For deuteron, three options are supported with the 3rd option being the one used in PHOBOS analyses.

1. The three parameter Fermi form with the values given in Table I
2. The Hulthén form
\[ \rho(r) = \rho_0 \left(\frac{e^{-ar} + e^{-br}}{r}\right)^2, \] (3)
where \( a = 0.457 \text{ fm}^{-1} \) and \( b = 2.35 \text{ fm}^{-1} \).
3. The proton is taken from the Hulthén form given above with the neutron placed opposite to it.

For \(^3\text{H}\) and \(^3\text{He}\), the configurations were computed (and stored in a database) from Green’s function MC calculations using the AV18 + UIX model interactions, which correctly sample the position of the three nucleons, including correlations, as in Ref. [5]. Finally, note that we also provide the rescaled values for the radius and skin depth of \(^{16}\text{O}\) (“AuHN”) and \(^{62}\text{Cu}\) (“CuHN”) to take into account the finite nucleon profile as derived in Ref. [10].

For deformed nuclei we use
\[ \rho(x, y, z) = \rho_0 \frac{1}{1 + \exp\left(\frac{(r-R\hat{1}+\beta_2 Y_{20}+\beta_4 Y_{40})}{a}\right)}, \] (4)
where \( Y_{20} = \sqrt{\frac{5}{16\pi}}(\cos^2(\theta) - 1) \), \( Y_{40} = \frac{3}{16}\sqrt{3}(35\cos^4(\theta) - 30\cos^2(\theta) + 3) \) with the deformation parameters \( \beta_2 \) and \( \beta_4 \) taken from Ref. [10]. The values used for different nuclei (Si, Cu, Au and U) are listed in Table I.

### B. Collision Process

The impact parameter of the collision is chosen randomly from a distribution \( dN/db \propto b \) up to some large maximum \( b_{\text{max}} \) with \( b_{\text{max}} \simeq 2R_A \). The centers of the nuclei are calculated and shifted to \((-b/2, 0, 0)\) and \((b/2, 0, 0)\). The reaction plane, defined by the impact parameter and the beam direction, is given by the \( x \) - and \( z \) -axes, while the transverse plane is given by the \( x \) - and \( y \) -axes. It is assumed that the nucleons move along a straight trajectory along the beam axis. The longitudinal coordinate does not play a role in the calculation.

The inelastic nucleon-nucleon cross section \( \sigma_{\text{NN}} \), which is only a function of the collision energy, is extracted from \( \text{p+p} \) collisions. A value of \( \sigma_{\text{NN}} = 42 \text{ mb} \) is used at top RHIC energy of \( \sqrt{s_{\text{NN}}} = 200 \text{ GeV} \), while at the LHC \( \sigma_{\text{NN}} = 64 \text{ mb} \) for \( \sqrt{s_{\text{NN}}} = 2.76 \text{ TeV} \) and \( \sigma_{\text{NN}} = 70 \text{ mb} \) for \( \sqrt{s_{\text{NN}}} = 5.02 \text{ TeV} \) are used [12].

When estimating the systematic uncertainties on calculated quantities like \( N_{\text{part}} \) one varies the \( \sigma_{\text{NN}} \) (along with the other parameters) by typically \( \pm 3 \text{ mb} \) and \( \pm 5 \text{ mb} \) at RHIC and LHC, respectively.

The “ball diameter” defined as
\[ D = \sqrt{\sigma_{\text{NN}}/\pi} \] (5)

parameterizes the interaction strength of two nucleons: Two nucleons from different nuclei are assumed to collide if their relative transverse distance is less than the ball diameter. If no such nucleon–nucleon collision is registered for any pair of nucleons, then no nucleus–nucleus collision occurred. Counters for determination of the total (geometric) cross section are updated accordingly.

| Name | R [fm] | a [fm] | w [fm] |
|------|--------|--------|--------|
| \(^d\text{He}^b\) | 0.01 | 0.5882 | 0 |
| \(^{16}\text{O}\) | 2.608 | 0.513 | -0.51 |
| \(^{28}\text{Si}\) | 3.34 | 0.580 | -0.233 |
| \(^{32}\text{S}\) | 2.54 | 2.191 | 0.16 |
| \(^{40}\text{Ca}\) | 3.766 | 0.586 | -0.161 |
| \(^{58}\text{Ni}\) | 4.309 | 0.517 | -0.1308 |
| \(^{62}\text{Cu}\) | 4.2 | 0.596 | 0 |
| \(^{62}\text{CuHN}\) | 4.28 | 0.5 | 0 |
| \(^{186}\text{W}\) | 6.58 | 0.480 | 0 |
| \(^{197}\text{Au}\) | 6.38 | 0.535 | 0 |
| \(^{197}\text{AuHN}\) | 6.42 | 0.44 | 0 |
| \(^{207}\text{Pb}^c\) | 6.62 | 0.546 | 0 |

\(^a\)Several different parameterizations for the deuteron \((^2\text{H})\) are implemented. Please refer to the code.
\(^b\)The configurations for \(^3\text{H}\) and \(^3\text{He}\) were obtained from Green’s function MC calculations as in Ref. [5].
\(^c\)These values are also used for \(^{208}\text{Pb}\) for which Fermi parameters are not available. It has been noted that Bessel-Fourier coefficients for the two nuclei are similar [5].
\(^d\)Implementation as done in Ref. [11].
The version described here is 2.0. Three classes, TGlauNucleon, TGlauNucleus and TGlauMC and three example functions runAndSaveNtuple(), runAndSaveNucleons(), and runAndSmearNtuple() are defined in the provided macro. Note that for generating events with $^3$H or $^3$He you will need the text files called “h3_plaintext.dat” or “he3_plaintext.dat” in the current path. While the functionality is essentially complete for known applications of the Glauber approach, users are encouraged to write their own functions to access results of the Glauber simulation or to modify the code.

The main classes are the following:

- **TGlauNucleon** is used to store information about a single nucleon. The stored quantities are the position of the nucleon, the number of binary collisions that the nucleon has had and which nucleus the nucleon is in, “A” or “B”. For every simulated event, the user can obtain an array containing all nucleons (via TGlauMC::GetNucleons()).

- **TGlauNucleus** is used to generate and store information about a single nucleus. The user is not expected to interact with this class.

- **TGlauMC** is the main steering class used to generate events and calculate event-by-event quantities such as the number of participating nucleons.

The steering class TGlauMC has one constructor

\[
\text{TGlauMC::TGlauMC(Text_t* NA, Text_t* NB, Double_t xsect, Double_t xsectsigma)}
\]

where NA and NB are the names of the colliding nuclei and xsect and xsectsigma are the nucleon-nucleon cross section mean and width given in mb. The defined nuclei names are given in Tables I and II. For deuteron, the names “d”, “dhh” and “dh” correspond to the three options described in section II A respectively. Units are generally given in fm for distances, while in mb for cross sections. If a finite value ($>0$) for the width of the cross section is given, the calculation will use the Glauber-Gribov model for fluctuations of the proton. This option currently is only implemented for p+A collisions. In this case, as described in Ref. [15] $\sigma_{NN}$ will be distributed according to $1/\lambda P(\sigma/\lambda)$ with

\[
P = \rho \frac{\sigma}{\sigma_0 + \sigma} \exp\left(-\frac{\sigma - \sigma_0}{\Omega}\right)^2
\]

where $\sigma_0$ denotes the mean $\sigma_{NN}$ and $\Omega$ the width. The normalization $\rho$ and the rescaling parameter $\lambda = \sigma_0 / \langle \sigma_0 \rangle$ are computed from the provided input. Example distributions for p+Pb collisions at the LHC are given in Fig. 2.
A. Running the Code

To generate Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV ($\sigma_{NN} = 42$ mb) one has to construct a TGlauberMC object by issuing the commands:

```root
root[0] gSystem->Load("libMathMore")
root[1] .L runglauber_X.Y.C+
root[2] TGlauberMC glauber("Au","Au",42);
```

where the first ROOT command loads the needed library libMathMore, the second compiles, links and loads the compiled macro (where you have to replace X.Y with the current version number of the code, for example 2.0) including the Glauber code as explained in chapter 2 of the ROOT users’ guide, while the third commands sets up the Glauber simulation for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

Events can be generated interactively using the two functions:

- **TGlauberMC::NextEvent(Double_t bgen)**, which is used to run an event at a specified impact parameter, or over a range of impact parameters (if `bgen=-1`, the default value) as described in section II.B.
- **TGlauberMC::Run(Int_t nevents)** which is used to run a large event sample by invoking `NextEvent` many times.

Other important public member functions are:

- **TGlauberMC::SetMinDistance(Double_t d)**, which is used to set minimum nucleon separation within a nucleus, $d_{\text{min}}$ (default is 0.4 fm)
- **TGlauberMC::SetBmin(Double_t bmin)** and **TGlauberMC::SetBmax(Double_t bmax)**, which can be used to set the range of impact parameter values generated in `Run()`.
- **TGlauberMC::GetTotXSect()** which returns the total nucleus-nucleus cross section, calculated when the function `Run()` is called.
- **TGlauberMC::Draw()** which draws the current event in the current pad.

B. Example functions

Three example functions (macros) are provided to demonstrate how to run the model:

1. **runAndSaveNtuple**

The macro `runAndSaveNtuple()` generates a number of Monte Carlo events and saves some event-by-event quantities. It takes as parameters, the number of events to be generated, the collision system, the nucleon-nucleon cross section, its width to simulate Glauber-Gribov fluctuations (for p+A), the minimum separation distance and the output file name. It creates and stores an ntuple in the output file with the following event-by-event quantities:

- $N_{\text{part}}$: Number of participating nucleons.
- $N_{\text{coll}}$: Number of binary collisions.
- $B$: Generated impact parameter.
- $\{x\}$: Mean of $x$ for wounded nucleons.
- $\{y\}$: Mean of $y$ for wounded nucleons.
- $\{x^2\}$: Mean of $x^2$ for wounded nucleons.
- $\{y^2\}$: Mean of $y^2$ for wounded nucleons.
- $\{xy\}$: Mean of $xy$ for wounded nucleons.
- $\sigma_x^2$: Variance of $x$ for wounded nucleons.
- $\sigma_y^2$: Variance of $y$ for wounded nucleons.
• $\text{VarXY}$: Covariance of $x$ and $y$ for wounded nucleons, $\sigma_{xy} \equiv \{xy\} - \{x\}\{y\}$.
• $\text{MeanXSystem}$: Mean of $x$ for all nucleons.
• $\text{MeanYSystem}$: Mean of $y$ for all nucleons.
• $\text{MeanXA}$: Mean of $x$ for nucleons in nucleus A.
• $\text{MeanYA}$: Mean of $y$ for nucleons in nucleus A.
• $\text{MeanXB}$: Mean of $x$ for nucleons in nucleus B.
• $\text{MeanYB}$: Mean of $y$ for nucleons in nucleus B.
• $N_{\text{partA}}$: $N_{\text{part}}$ from nucleus A.
• $N_{\text{partB}}$: $N_{\text{part}}$ from nucleus B.
• $\Psi_N$: Event plane angle of Nth harmonic.
• $\text{EccN}$: Participant eccentricity for Nth harmonic.

It is important to note that for each of these event-by-event quantities a “getter” function is implemented providing the users the option to write their own event loop (using TGlauberMC::NextEvent()).

2. runAndSaveNucleons

The macro runAndSaveNucleons() generates a number of Monte Carlo events and saves an array of TGlauberNucleon objects for each event. It is also possible to use this function to print out the values stored in the nucleons by setting the verbosity parameter. The function takes as parameters the number of events to be generated, the collision system, the nucleon-nucleon cross section, the minimum separation distance, the verbosity flag and the output file name.

3. runAndSmearNtuple

The macro runAndSmearNtuple() generates a number of Monte Carlo events and saves a reduced set of event-by-event quantities, where the position of the nucleon is smeared similar to what is done in Ref. [8]. It takes the same arguments as the runAndSaveNtuple() macro.

C. Sample Results

As an example application of this code, 100k events were generated for Cu+Cu, Au+Au and U+U at RHIC energies ($\sqrt{s_{NN}} = 42$ mb), and Pb+Pb at LHC beam energy ($\sqrt{s_{NN}} = 64$ mb) using the runAndSaveNtuple() function. The resulting ntuples were used to plot the distributions of $N_{\text{part}}$ and $N_{\text{coll}}$, shown in Fig. 3.

![Figure 3: Distributions of $N_{\text{part}}$ (top panel) and $N_{\text{coll}}$ (bottom panel) for Cu+Cu, Au+Au and U+U collisions at RHIC, and Pb+Pb collisions at the LHC.](image)

![Figure 4: Reaction-plane eccentricity $\epsilon_{RP}$ (open symbols) and participant-plane eccentricity $\epsilon_{part}$ (closed symbols) as a function of $N_{\text{part}}$ for Cu+Cu, Au+Au and U+U collisions at RHIC and Pb+Pb collisions at the LHC.](image)
Using the event-by-event quantities, one can construct combinations of moments like [3]:

- Reaction-plane eccentricity $\epsilon_{RP}$
  \[
  \epsilon_{RP} = \frac{\text{Var}Y - \text{Var}X}{\text{Var}Y + \text{Var}X}
  \]

- Participant-plane eccentricity $\epsilon_{\text{part}}$
  \[
  \epsilon_{\text{part}} = \sqrt{\left(\text{Var}Y - \text{Var}X\right)^2 + 4\text{Var}XY^2} / \left(\text{Var}Y + \text{Var}X\right)
  \]

which are shown in Fig. 4 for different systems.

IV. SUMMARY

This document describes the updated implementation (v2) of the Monte Carlo based Glauber Model calculation, which originally was used by the PHOBOS collaboration. The main improvement w.r.t. the earlier version [1] are the inclusion of tritium, Helium-3, and Uranium, as well as the treatment of deformed nuclei and Glauber-Gribov fluctuations of the proton in p+A collisions. The code, accessible online, can be used within user code or in a standalone mode allowing analysis of various distributions. The authors welcome comments on the code and suggestions on how to make it more useful to both experimentalists and theorists.

[1] B. Alver, M. Baker, C. Loizides and P. Steinberg (2008), arXiv:0805.4411
[2] M.L. Miller, K. Reygers, S.J. Sanders, and P. Steinberg (2007), arXiv:nucl-ex/0701025
[3] B. Alver et al. [PHOBOS Collaboration], Phys. Rev. C 77 (2008) 014906, arXiv:0711.3724 [nucl-ex].
[4] M. Rybczynski, G. Stefanek, W. Broniowski and P. Bozek, Comput. Phys. Commun. 185, 1759 (2014), arXiv:1310.5475 [nucl-th].
[5] H. De Vries, C.W. De Jager, and C. De Vries, Atomic Data and Nuclear Data Tables 36 495 (1987).
[6] L. Hulthen and M. Sugawara, Handbuch der Physik 39 1 (1957).
[7] S.S. Adler et al. [PHENIX Collaboration], Phys. Rev. Lett. 91 072303 (2003).
[8] J. L. Nagle et al., arXiv:1312.4568 [nucl-th].
[9] T. Hirano and Y. Nara, Phys. Rev. C 79, 064904 (2009) arXiv:0904.4080 [nucl-th].
[10] P. Möller, J.R. Nix, W.D. Myers and W.J. Swiatecki, Atomic Data and Nuclear Data Tables 59 185 (1995).
[11] U. W. Heinz and A. Kuhlman, Phys. Rev. Lett. 94, 132301 (2005)
[12] See https://twiki.cern.ch/twiki/bin/view/Main/LHCGLauberBaseline.
[13] See http://root.cern.ch for installation files and documentation.
[14] See the TGlauberMC page on HepForge (http://www.hepforge.org/downloads/tglaubermc) for the most recent TGlauberMC release (currently version 2.0).
[15] ATLAS-CONF-2013-096, https://cds.cern.ch/record/1599773.