Molecular scale height in NGC 7331

Narendra Nath Patra$^1$ *

$^1$ NCRA-TIFR, Post Bag 3, Ganeshkhind, Pune 411 007, India

ABSTRACT

Assuming a vertical hydrostatic equilibrium between different baryonic disk components, Poisson’s equations were set up and solved numerically to obtain the molecular scale height in the spiral galaxy NGC 7331. The scale height of the molecular disk was found to vary between $\sim 100 - 200$ pc depending on radius and assumed velocity dispersion of the molecular gas. The solutions of the hydrostatic equation and the rotation curve were used to produce a dynamical model and a total intensity map of the molecular disk. The modelled and the observed molecular disk matches with each other to a large extent. However, the modelled molecular disk falls short in producing observed thickness, specially at the central region. The velocity dispersion of the molecular disk is found to have no detectable influence on the projected thickness and hence can not account for the discrepancy in the thickness. A change in the inclination also found to be incapable of eliminating the difference completely. The molecular disk of NGC 7331 was projected to an inclination of $90^\circ$ to estimate its observable edge-on thickness, which was found to be $\sim 2$ kpc. Our result indicates that a simple hydrostatic condition is capable of explaining thick molecular disks observed in external galaxies.

1 INTRODUCTION

Molecular gas plays a significant role in galaxy formation and evolution. Molecular clouds provide the sites of active star formation and hence host a suite of stellar activity e.g. stellar feedback, polluting the Inter Stellar Medium (ISM), supernova etc. The conversion of the gas into stars greatly regulated by this phase of the ISM. Hence, the abundance and distribution of the molecular gas in galaxies are of immense importance. Dynamically, the molecular gas in galaxies is expected to settle in a thin disk due to its low thermal pressure, however, its complete three dimensional distribution is still poorly understood.

It is very difficult to measure the three dimensional distribution of the molecular clouds in the Galaxy mainly due to distance ambiguities and extinction by dust. Out of the plane distribution of $^{12}$CO outside the solar circle is only studied by [Grabelsky et al. (1987)]. In the inner Galaxy, the extinction is large and the distances to the molecular clouds were estimated mainly from their latitude and hence are ambiguous (e.g., Sanders, Solomon & Scoville [1984], Scoville et al. [1987], Bronfman et al. [1988]) and one can only conclude that most of the molecular gas resides in a thin disk of scale height (FWHM) $\sim 150$ pc, flaring up to $350$ pc at its edges.

However, a number of recent studies provide mounting evidence of a much thicker molecular disk than what was expected earlier. For example, Garcia-Burillo et al. [1992] observed NGC 891 using IRAM 30 m telescope and found a thick molecular disk. They detected molecular gas at $\sim 8$ kpc above and below the mid-plane. A number of previous studies used vertical hydrostatic equilibrium condition to estimate the vertical structure of atomic gas in spiral and dwarf galaxies (Olling [1995], Becquert & Combes [1997], Narayan, Saha & Jog [2005], Banerjee & Jog [2008], Banerjee, Matthews & Jog [2010], Banerjee et al. [2011], Patra et al. [2014]). These studies neglected the contribution of the molecular gas due to lack of observational inputs. With recent surveys of molecular gas in nearby galaxies (e.g. HERACLE [Leroy et al. 2009]), it is possible to estimate the vertical scale height of the molecular gas in nearby spiral galaxies. However, detecting molecular gas in dwarf galaxies remains challenging and no sig-
significant detections were made in spite of large efforts \cite{Cormier2014,McQuinn2012,Leroy2006}.

In this paper, I set up the Poisson’s equation of hydrostatic equilibrium for a galactic disk with different components, e.g. stars, atomic gas, molecular gas etc for the galaxy NGC 7331. This galaxy is a part of THINGS survey \cite{Walter2008} and HERACLE \cite{Leroy2009} survey which makes the necessary data available for this study. Not only that this galaxy has an inclination of \( \sim 76^\circ \) which makes it suitable for the estimation of its rotation curve reliably and at the same time its gas disk produces an observed thickness which is sensitive to the vertical structure of the gas disk. I numerically solve the second order partial differential equations to obtain the vertical structure of the molecular disk. In §2 I present the recipe for modelling a galactic disk using vertical hydrostatic equilibrium and describe the method to solve the equation using required inputs. In §3 I discuss the results of the study. In §4 I conclude along with a brief overview of the future work.

2 MODELLING THE GALACTIC DISKS

2.1 Formulation of equation

To model the disk of a galaxy, I assume it to be a three component baryonic system consisting of stars, atomic gas and molecular gas settled under mutual gravity in the external potential field of the dark matter halo. All the disks of different components then would be in vertical hydrostatic equilibrium. For simplicity the disks of the different components were assumed to be co-planar, concentric and symmetric. The potential of the dark matter halo is considered to be fixed and can be determined observationally. Under these assumptions, the joint Poisson’s equation for the disks plus dark matter halo in cylindrical polar coordinate system can be given as

\[
\frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \Phi_{\text{total}}}{\partial R} \right) + \frac{\partial^2 \Phi_{\text{total}}}{\partial z^2} = 4\pi G \left( \sum_{i=1}^{3} \rho_i + \rho_0 \right) \tag{1}
\]

where \( \Phi_{\text{total}} \) is the total potential due to all the components and dark matter halo, \( \rho_i \) denotes the volume density of different components where \( i \) runs for stars, atomic gas and molecular gas. \( \rho_0 \) denotes the mass density of dark matter halo. As the dark matter halos of spiral galaxies are better described by an NFW profile as compared to an isothermal one, I chose to adopt an NFW profile to represent the dark matter halo of NGC 7331. The dark matter density profile of an NFW halo \cite{Navarro1997} can be given as

\[
\rho_R(r) = \frac{\rho_0}{r_\epsilon \left( 1 + \frac{r^2}{r_\epsilon^2} \right)^2} \tag{2}
\]

where \( \rho_0 \) is the characteristic density and \( r_\epsilon \) is the scale radius. These two parameters completely describe a spherically symmetric NFW dark matter halo.

The equation of hydrostatic equilibrium for individual components can be written as

\[
\frac{\partial}{\partial z} \left( \rho_i \langle \sigma_z^2 \rangle_i \right) + \rho_i \frac{\partial \Phi_{\text{total}}}{\partial z} = 0 \tag{3}
\]

where \( \langle \sigma_z \rangle_i \) is the vertical velocity dispersion of the \( i^{th} \) component, an input parameter.

Eliminating \( \Phi_{\text{total}} \) from Equation (1) and (4) one gets,

\[
\frac{\partial}{\partial z} \left( \rho_i \langle \sigma_z^2 \rangle_i \right) + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \rho_i}{\partial R} \right) = -4\pi G \rho_i + \rho_{H1} + \rho_{H2} + \rho_0 \tag{4}
\]

where \( \rho_i \), \( \rho_{H1} \) and \( \rho_{H2} \) are the mass density of stars, H1 and molecular gas respectively. Eq. (4) represents three second-order partial differential equations in the variables \( \rho_i \), \( \rho_{H1} \) and \( \rho_{H2} \). However they can be further simplified using the fact (see \cite{Banerjee2010} for more details)

\[
\frac{\partial \theta_{\text{total}}}{\partial R} = \frac{v_{\text{rot}}^2}{R} \tag{5}
\]

where \( v_{\text{rot}} \) is the circular rotation velocity. Assuming a negligible vertical gradient in \( v_{\text{rot}} \), one can approximate the \( v_{\text{rot}} \) by the observed rotation curve \( v_{\text{rot}} \), which is a function of \( R \) alone. Thus Eq. (4) reduces to

\[
\frac{\partial}{\partial z} \left( \rho_i \langle \sigma_z^2 \rangle_i \right) + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \rho_i}{\partial R} \right) = -4\pi G \rho_i + \rho_{H1} + \rho_{H2} + \rho_0 \tag{6}
\]

Eq. (6) represents three coupled, second-order ordinary differential equations in the variables \( \rho_i \), \( \rho_{H1} \) and \( \rho_{H2} \). The solution of Eq. (6) at any radius \( R \) gives the density of these components as a function of \( z \). Thus solutions of this equation at every \( R \) will provide the three dimensional density distribution of different disk components.

2.2 Input parameters

To get the vertical structure of the molecular disk of any galaxy one need to solve Eq. (6) In this work I particularly solve Eq. (6) for the galaxy NGC 7331 to estimate its vertical molecular structure. This galaxy was observed in H1 as part of the THINGS survey \cite{Walter2008} and the molecular data comes from the HERACLE survey \cite{Leroy2009}. As it is discussed in later sections, an inclination of \( \sim 76^\circ \) of this galaxy favours in comparing the modelled and the observed molecular disk.

In Fig. 1 I show the gas surface density images of NGC7331. The left panel shows the surface density of H1 gas as observed by the VLA as part of the THINGS survey \cite{Walter2008}, whereas the middle panel shows the molecular surface density as observed by the 30 m IRAM telescope as part of the HERACLE survey \cite{Leroy2009}. The right panel of Fig. 1 shows the total gas surface density i.e. (H1 + H2). The observing beams are shown by the black dots at the left bottom corner of the respective panels. The H1 data was at higher resolution as compared to the H2 data, hence to get a total gas surface density map, I first smooth the H1 data with a Gaussian kernel to produce an output resolution same as the H2 map and then sum them together to get the total gas surface density map. The gray scale in each panel are in the unit of \( M_\odot pc^{-2} \). I adopt the same CO(2-1) to H2 conversion factor as given by \cite{Leroy2008}

\[
\Sigma_{H2}(M_\odot pc^{-2}) = 5.5 I_{CO}(2 \rightarrow 1) (K km s^{-1}) \tag{7}
\]

However, the atomic gas, the molecular gas and stars act as independent components in the hydrostatic equation and hence their individual surface densities are of particular interest. In
Fig. 1. The gas surface density maps of NGC 7331. Left panel: The HI surface density from THINGS survey (Walter et al. 2008). Middle panel: The total intensity map of the molecular gas as traced by CO from the HERACLES survey (Leroy et al. 2009). Right panel: The intensity distribution of total gas surface density i.e., HI + H₂. The color bars in each panel indicates the surface densities in the unit of M☉ pc⁻².

Figure 2. The surface density profiles of NGC7331. The black asterisks represent stellar surface density, the blue squares represent the surface density of molecular gas whereas the red circles represent the HI surface density. It can be noted that the H₂ disk extends only up to ~ 10 kpc, whereas the HI and the stellar disks extends up to a much larger radius. The data were taken from Leroy et al. (2008).

Fig. 2 I plot the surface densities of different disk components as a function of radius. These data were taken from Leroy et al. (2008). The black asterisks represent stellar surface density, whereas the blue squares represent the molecular surface density and the red solid circles represent the HI surface density. It can be noted that the molecular gas disk extends up to ~ 10 kpc from the centre whereas the HI and the stellar disk extends far out to a much larger radius. For details of the surface density calculations I refer the readers to Leroy et al. (2008). It should be noted that the surface densities are one of the primary inputs to the hydrostatic equation and is a proxy to the mass distribution along vertical direction.

The vertical velocity dispersions of different disk components are another important input to Eq. 6. Banerjee et al. (2011) show that the vertical structure of the gaseous components are marginally affected by the accuracy of the assumed stellar velocity dispersion (σ_s). Hence, the stellar velocity dispersion was calculated analytically by assuming an isothermal stellar disk as described in Leroy et al. (2008).

The HI velocity dispersion (σ_HI) in spiral galaxies were studied extensively through the HI spectral line observations. Early work by many authors suggest a velocity dispersion of 6-13 km s⁻¹ (Shostak & van der Kruit 1984; Kamphuis & Santisi 1993). Petric & Rupen (2007) studied the nearly face-on galaxy NGC 1058 to find that the σ_HI varies between 4-14 km s⁻¹ and decreases with radius. In an extensive analysis, Tamburro et al. (2009) studied the σ_HI in spiral galaxies from THINGS survey and found a mean HI velocity dispersion to be ~ 10 km s⁻¹ at r25. In a later study, Ianjamasimanana et al. (2012) applied spectral stacking method to the same data to estimate the σ_HI with higher confidence. They found a σ_HI = 12.5 ± 3.5 km s⁻¹ (σ_HI = 10.9 ± 2.1 km s⁻¹ for galaxies with inclination less than 60°).

Caldú-Primo et al. (2013) studied the σ_HI and σ_CO using stacking technique in a sample of 12 nearby spiral galaxies. They found σ_HI/σ_CO = 1.0 ± 0.2 with a median σ_HI = 11.9 ± 3.1 km s⁻¹. However, Mogotsi et al. (2016) studied the same sample by analysing individual high SNR spectra to find σ_HI/σ_CO = 1.4 ± 0.2 with median σ_HI = 11.7 ± 2.3 km s⁻¹ and σ_CO = 7.3 ± 1.7 km s⁻¹. As can be seen from these studies, the σ_HI in spiral galaxies could be assumed to be ~ 12 km s⁻¹.

It can be noted though, the velocity dispersion of HI does not influence the molecular scale height considerably.

However, the velocity dispersion of the molecular gas (σ_H₂) directly influences the vertical structure of the molecular disk. Stark (1984) observed molecular clouds in the Galaxy and found that the velocity dispersion of low mass clouds are higher than the high mass clouds. The low mass clouds have a σ_H₂ ~ 9.0 km s⁻¹ whereas the high mass clouds have σ_H₂ ~ 6.6 km s⁻¹. Combes & Becquart (1997b) studied two nearly face-on galaxies to find σ_H₂ ~ 6 - 8.5 km s⁻¹. They also found that the σ_H₂ is almost constant over the whole galaxy and comparable to the velocity dispersion of HI (σ_HI). Stark & Lee (2005) used observations of 12CO J = 1 → 0 in 1400 molecular clouds in the Galaxy to find that the velocity dispersion of small clouds are higher than that of the Giant Molecular Clouds (GMCs). I assume a primary velocity dispersion of molecular gas, σ_H₂ to be ~ 7 km s⁻¹, along with a variation between 6-10 km s⁻¹ to explore the observed molecular disk in more details.

The second term on the right hand side of Eq. 6 represents the contribution of the centripetal acceleration against the gravity, however, the vrot in Eq. 6 is an observable quantity. In Fig. 3 the observed rotation curve of NGC 7331 is plotted. The red solid circles are from de Blok et al. (2008) whereas the blue asterisks are from Begeman (1987), de Blok et al. (2008) used relatively high resolution HI data from THINGS survey. As in Eq. 6 we need to use the derivatives of the rotation velocity (V_r), it is useful to parametrise the rotation curve instead of using actual
is used to parametrise the rotation curve and the data were fitted to estimate the parameters. A $V_{\text{max}} = 262.2 \pm 0.8$ km s$^{-1}$, $R_{\text{max}} = 6.1 \pm 0.1$ kpc and $n = 0.67 \pm 0.06$ were found for de Blok et al. (2008) data and $V_{\text{max}} = 257.5 \pm 1.0$ km s$^{-1}$, $R_{\text{max}} = 6.7 \pm 0.1$ kpc and $n = 0.89 \pm 0.07$ were found for Begeman (1987) data. As can be seen that the fit parameters of both the data matches very well with each other. I chose to work with the parameters found with Begeman (1987) data as it is smoother than the THINGS data, however, I note that this does not make any fundamental difference to the results.

The dark matter halo is another important input to the hydrostatic equilibrium equation. For NGC 7331 I used the dark matter halo parameters from de Blok et al. (2008) (Table 4). The dark matter halo of NGC 7331 can be described both by the isothermal and NFW profile well. However, as the NFW profile in general describe the dark matter halo of spiral galaxies better than isothermal one, I choose to use an NFW profile as given by Eq. (8) with $\rho_0 = 1.05 \times 10^{-3}$ $M_\odot$ kpc$^{-3}$ and $r_s = 60.2$ kpc (see de Blok et al. (2008) for more details).

### 2.3 Solving the hydrostatic equilibrium equation

With all the above mentioned inputs, Eq. (6) was solved to obtain the vertical structure of different disk components. The coupled second order ordinary differential equations were solved numerically using 8th order Runge Kutta method as implemented in scipy package. As each equation is a second order differential equation, one needs two initial conditions to solve it. I use two initial conditions given as

$$(\rho_i)_{z=0} = \rho_{i,0} \quad \text{and} \quad \frac{d \rho_i}{dz} = 0$$

However, the midplane density i.e. $\rho_{i,0}$ is not a directly measurable quantity, instead, the surface density $\Sigma_i$ can be used as a representative of it. Assuming a trial value for $\rho_{i,0}$, Eq. (6) was solved and the solutions were obtained. Then the density solution for a particular $R$ was integrated to get the surface density of that component at radius $R$. This surface density then compared with the observed one to update the next trial value until it converges. I note that using this method, the surface densities found to converge with better than 0.1% accuracy in less than few hundred iterations.

A similar strategy as described in Banerjee & Jog (2008); Patra et al. (2014) was adopted to solve the coupled differential equations. Each equation was solved independently in the first iteration assuming no coupling. In the consequent iterations, the solutions from the $i^{th}$ iteration for two components (for example HI and H$_2$) were substituted in Eq. (6) (in the first term of the RHS) while solving the other component (e.g., star). At every iteration, the solutions were checked for convergence (better than 0.1% accuracy). I note that, for NGC 7331, the solutions were converged in less than 10 iterations at any radius. It takes about a few minutes to solve the coupled equation at any radius in a normal workstation. However, as the hydrostatic condition at any radius is independent of other radii, Eq. (6) can be solved in parallel. Hence, for fast computation the hydrostatic equation solver was implemented using MPI based parallel code which can solve Eq. (6) simultaneously for different radius.

As can be seen from Fig. 2 the surface density of the molecular gas do not extend beyond $R \sim 10$ kpc and hence Eq. (6) was solved at $R \leq 10$ kpc. From Fig. 2 it can also be noted that the stellar surface density is very high at the central region. Not only that, but also the assumed dark matter halo density profile (NFW) sharply peaks at the central region. To avoid any divergence due to these factors, a central region of 1 kpc was avoided and not solved. Thus, Eq. (6) was solved within $1 \leq R \leq 10$ kpc with an interval of 100 pc. The linear spatial resolution of the molecular data is $\sim 1$ kpc. Hence, an radial interval of 100 pc is expected to be more than enough to sample the molecular disk well. However, it is well known that the vertical thickness of the molecular disk is much smaller as compared to its radial extent, and hence a much higher resolution is needed to sample the molecular disk in vertical direction. To achieve this, an adaptive resolution depending on the scale-height was used and found to be always better than a few pc.
3 RESULTS AND DISCUSSION

In Fig. 5 I plot the HWHM profiles of the molecular disk as a function of radius. For comparison, the HWHM profiles of the atomic gas is also plotted in the figure. The solid lines represent the scale heights of the atomic disks whereas the broken lines represent the scale heights of the molecular disks. The legends in the figure indicate different HWHM profiles for different assumed \( \sigma_{H_2} \) values. It can be noted that the molecular scale height in NGC 7331 varies between 50 pc - 200 pc depending on the assumed \( \sigma_{H_2} \) and radius. For an assumed 7 km s\(^{-1}\), \( \sigma_{H_2} \), the scale height varies between 60 - 100 pc which is \( \sim \) a factor of 2 smaller than what is found in the Galaxy. It can also be seen that the molecular scale height changes by a factor of \( \sim 2 \) as one changes the \( \sigma_{H_2} \) from 6 km s\(^{-1}\) to 10 km s\(^{-1}\). However, the atomic scale height is rather insensitive to the change in \( \sigma_{H_2} \).

However, the HWHM profile of the molecular disk is not a directly observable quantity even for an edge-on galaxy. Rather the total intensity map is what is obtained through an observation. To better compare the density distribution of the molecular gas, a three dimensional dynamical model of the molecular disk was made using the solutions of Eq. 6 and the rotation curve. This 3D model then inclined to the observed inclination of 76° and convolved with the telescope beam to produce a model total intensity (MOMNT0) map. This map then can be compared with the observed one to check the consistency of the assumed inputs or the hydrostatic assumption itself.

In Fig. 6 top panel, the simulated total intensity map (MOMNT0) of the molecular gas in NGC 7331 was shown as was obtained using the solutions of Eq. 6. In the bottom panel, this simulated MOMNT0 map was compared with the observed one. The color scale represents the observed \( I_{CO} \) distribution in the unit of K km s\(^{-1}\), whereas the contours represent the simulated map. The successive contours are at 90% level of the previous contour starting from the maximum (\( \sim 24 \) K km s\(^{-1}\)). However, one should exclude central 1-2 kpc region for comparison as the Eq. 6 was not solved within 1 kpc and the convolution with the telescope beam of size \( \sim 1 \) kpc will introduce error. A deeper inspection of the bottom panel of the figure reveals an extra emission features at both the upper and lower half of the galaxy which was not accounted by the model observation (contours) properly.

To capture it in more details, instead of simple overplot of total intensity, the observed projected thickness of the galaxy as a function of radius was estimated and compared with the modelled one. To estimate the projected thickness, a vertical cut along the minor axis was taken and an one dimensional vertical profile was extracted. In Fig. 7 the vertical profiles of the projected molecular disk were plotted as a function of height.
Each panel represents the vertical profiles at different galactocentric radii as quoted at the top left corner of the panel. The black and the red solid lines in each panel represent the observed vertical profiles of two different halves at a particular galactocentric radius. The blue dashed line represents the vertical profile of the modelled molecular disk.

In Fig. 8, I plot the observed and the modelled projected thickness for different radial distances. The solid red and black curves represent the observed vertical profiles of two different halves at a particular galactocentric radius. The blue dashed line represents the vertical profile of the modelled molecular disk.

Figure 7. The vertical profiles of the observed and simulated molecular disk at different radial distances. The solid red and black curves represent the observed vertical profiles of two different halves at a particular galactocentric radius. The blue dashed line represents the vertical profile of the modelled molecular disk.

Figure 8. The projected thickness of the molecular disc as a function of radius. The red triangles with errorbars represent the observed thickness whereas the black dots indicates the modelled one.

Figure 9. The projected thickness for different assumed $\sigma_{H_2}$ as mentioned in legend in the unit of km s$^{-1}$.

As can be seen, the projected thickness of the molecular disk do not show any detectable change for different $\sigma_{H_2}$. This could be due to two reasons. Firstly, due to increase in $\sigma_{H_2}$, though the scale height of the density profile increases, the mass densities decreases to keep the surface density constant. This results in a thicker but fainter disk. When a constant threshold applied, that probably a low intensity diffuse emission at the outer disk is what which could not be modelled properly. Or alternatively there could be a possible error in the assumed inclination of NGC 7331 (76°) which leads to a mismatch in the observed and the modelled projected thickness. This possibility is explored in more details in the later part of this section.

To solve Eq. 6 and generate the model molecular intensity maps as shown in Fig. 8 a $\sigma_{H_2} = 7$ km s$^{-1}$ was used. However, as discussed earlier, a higher velocity dispersion is quite possible to exist in molecular disks and hence one should explore if an assumption of higher velocity dispersion can produce the observed projected thickness. To do that moment maps were built from the solutions of Eq. 6 assuming different $\sigma_{H_2}$ and the projected thickness were estimated for each assumed $\sigma_{H_2}$. In Fig. 9 I plot the estimated projected thickness for different $\sigma_{H_2}$. Different points represent different $\sigma_{H_2}$ as indicated in the legends.

As can be seen, the projected thickness of the molecular disk do not show any detectable change for different $\sigma_{H_2}$. This could be due to two reasons. Firstly, due to increase in $\sigma_{H_2}$, though the scale height of the density profile increases, the mass densities decreases to keep the surface density constant. This results in a thicker but fainter disk. When a constant threshold applied,
the disk could appear thinner due to its faintness even though the molecular gas extends to higher heights. The other possibility could be, the change in the scale height/thickness due to different $\sigma_{H_2}$ is too small as compared to the telescopic beam and hence washed away when the simulated map is convolved with the telescope beam. I investigated the second possibility with a much narrow beam of $\sim 5$ pc and found no significant detectable change in projected thickness due to change in $\sigma_{H_2}$ which strengthen the first possibility.

As mentioned earlier, the discrepancy between the observed and the modelled projected thickness could arise due to an error in the assumed inclination for the molecular disk as well. To check this, model thickness profiles for different inclinations were produced to compare with observation. It should be noted that, once one adopts a different inclination, the face-on surface densities as well have to be corrected before solving Eq. 6. With the corrected face-on surface densities, Eq. 6 were solved and the model intensity maps were produced for inclinations ranging from $70^\circ$ to $76^\circ$ in step of $1^\circ$. These moment maps were then used to generate projected thickness profiles and compared with the observed thickness profile.

In Fig. 10 I plot the projected thickness profiles for different inclinations. To capture thickness at different depths, the projected thickness profiles for different intensity thresholds were plotted in different panels. The threshold values used in each panel are quoted in the top right corner of the respective panel. The red up-triangles with errorbars represent the observed projected thickness profile whereas the modelled thickness profiles are shown by the points without errorbars. The green hollow pentagons and the empty black squares in each panel represent the projected thickness for inclination of $70^\circ$ and $71^\circ$ whereas the inclinations of $72^\circ$, $73^\circ$ and $74^\circ$ were shown by blue ‘x’, red empty circles and by black ‘+’ respectively. The inclination of $75^\circ$ and $76^\circ$ were represented by cyan empty down-triangles and black solid filled circles. In each panel (for a particular intensity threshold) the thickness profiles of the modelled molecular disks were compared with the observed one to find the inclination which explains the data best. A $\chi^2$ minimization was used to determine the inclination for which the modelled thickness profile matches best to the observed one. In Fig. 10 each panel, the solid lines represent the best fit modelled thickness profiles which matches best to the observed data. The colors of the lines are the same as the color of the points representing the best matched inclination. For 5-sigma and 10-sigma thresholds, an inclination of $72^\circ$ was found to fit the data best, whereas for 20 and 30-sigma threshold, an inclination of $74^\circ$ and $75^\circ$ respectively explains the data best. As can be seen, any one inclination doesn’t explain the observation for all intensity thresholds, which indicates that probably inclination alone can not account for the observed projected thickness. However, it can be noted that the change in the required inclination to match the observation is small and probably can be adjusted within the measurement errors.

As discussed in § II a thick molecular disk extending up to a few kpc from the mid-plane were observed in many edge-on galaxies (Garcia-Burillo et al. 1992). While it is puzzling that what could produce such a thick molecular disk and whether these disks are in hydrostatic equilibrium, are still open questions. To check if NGC 7331 can produce an observationally detectable thick molecular disk, the density distribution of the
molecular disk of NGC 7331 was projected to an inclination of 90°. In Fig. 11 top panel, I show the molecular disk of NGC 7331 at an edge-on projection, whereas the bottom panel shows the detectable thickness as measured from the mid-plane. A 5-sigma threshold (same rms was used as in the HERACLE map) was used to measure the projected thickness. As can be seen from the figure, though the scale height of the molecular gas is small (maximum \( \sim 100 - 200 \) pc as shown in Fig. 5) it can produce an observable thickness of \( \sim 1 \) kpc order when viewed edge-on. The thickness decreases as a function of radius and acquires a value of \( \sim 0.5 \) kpc at the outer edge of the molecular disk. Though the thickness of the molecular disc found in NGC 7331 is not as thick as found in NCG 891 which is a few kpc from the mid-plane (Garcia-Burillo et al. 1992), this results indicate that it is probably not very difficult to produce a thick molecular disk under the assumption of hydrostatic equilibrium.

4 SUMMARY AND FUTURE WORK

In summary, assuming a vertical hydrostatic equilibrium between different disk components in a galaxy, the joint Poisson’s equations were set-up and solved to calculate the vertical structure of the molecular disk in the galaxy NGC 7331. Three coupled second-order partial differential equation (Eq. 6) were solved numerically using scipy and was implemented using MPI based code for fast computation. For NGC 7331, the hydrostatic equation was solved at \( 1 \leq R \leq 10 \) kpc to obtain the vertical structure of the molecular disk. The molecular scale height was found to be \( \sim 50 - 100 \) pc at the centre which increases to \( \sim 100 - 200 \) pc at the outer edge. The molecular scale height is sensitive to the assumed \( \sigma_{HI} \) and found to change by a factor of \( \sim 2 \) when \( \sigma_{HI} \) changes from \( 6 \) km s\(^{-1}\) to \( 10 \) km s\(^{-1}\). However, \( \sigma_{HI} \) do not have any significant influence on the scale height of the atomic gas.

Using the solutions of hydrostatic equations and the observed rotation curve, a three dimensional dynamical model of the molecular disk was made. This model was then inclined to the observed inclination and convolved with the telescope beam to produce a model intensity map. This model intensity map was compared with the observed one (Fig. 5) to find that the model largely matches with the observation except some low intensity excess emission features at largest heights. To investigate this extra emission feature in more details, the projected thickness of the molecular disk as a function of radius was estimated and compared with observation. To probe thickness of the molecular disk at different depths, different intensity thresholds were used to define the projected thickness. The model molecular disk was found to be thin as compared to the observed one, however, the discrepancy decreases as one moves to larger radius or probe higher depths. This indicates that it is the low intensity emission at the largest heights which was not captured properly in the model.

Assuming \( \sigma_{HI} \) a possible reason for this discrepancy, a number of total intensity maps were made with different \( \sigma_{HI} \) and projected thickness profiles were estimated consequently. However, no detectable change in the thickness profile were observed due to change in \( \sigma_{HI} \), even when observed with a \( \sim 5 \) pc beam. Given this result, an error in the assumed inclination was considered and explored. A number of total intensity maps were generated assuming inclinations from 70° to 76° and consequently the thickness profiles were made. These thickness profiles were then compared with the observed one to find an inclination for which the model matches the data best. The thickness profiles obtained through different intensity threshold were compared separately. However, it was found that any particular inclination could not produce the best fit to the observation at all intensity thresholds. The inclination varies between 72° - 75° which produces best fit to the observation at different intensity thresholds. This indicates that either the outer parts of the molecular disk have a lower inclinations as compared to the central parts or the extra bit of gas is not in hydrostatic equilibrium with the disk. With this modelling however, it is difficult to estimate the reason of this discrepancy.

Finally, to see if the the molecular disk of NGC 7331 can produce a thick observable disk in edge-on orientation, the molecular density distribution of NGC 7331 was projected to an inclination of 90°. Assuming similar observational uncertainties, the observed thickness of the molecular disk at 5-sigma threshold were estimated. It was found that at the central region, the observed thickness could reach up-to \( \geq 1 \) kpc (from the midplane) which then drops to \( \sim 500-700 \) pc at the outer edges. With this result, it appears that a simple vertical hydrostatic model of the molecular gas can produce a few kilo-parsec thick observed disk and hence, producing a thick molecular disk in external galaxies might not be as difficult as it was thought before (e.g., NGC 891 (Garcia-Burillo et al. 1992)).

In this work I assumed the molecular disk to be a single component system with a single \( \sigma_{HI} \). However, as discussed in a number of recent studies points towards the possibility of a two component molecular disk with a thin disk residing close to the midplane and a diffuse thick disk extending up to a few kpc. The \( \sigma_{HI} \) of these disks are expected to be different. In these scenarios, the assumption of a simple single component molecular
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disk will fail and one needs to add an extra component to the hydrostatic equilibrium equation. In future work, a detailed study with two-component molecular disk can be carried out to see if it can explain the observed discrepancy in the projected thickness at low intensity thresholds.

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