Passivity based distributed tracking control of networked Euler-Lagrange systems

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Abstract: In this paper we present three distributed control laws for the coordination of networked Euler-Lagrange (EL) systems. We first reformulate the passivity-based control design method in Arcak (2007) by considering that each edge is associated with an artificial spring system instead of the usual diffusive coupling among the communicating agents. With this configuration, the networked EL system possesses a "symmetric" feedback structure which together with the strict passivity of both agents' and edges' dynamics lead to a strictly passive network dynamics. Subsequently we present the networked version of two different passivity-based tracking controllers that are particular cases of our method and the one in Arcak (2007). Numerical simulation is presented to show the performance of the proposed methods.

Keywords: Multi-agent systems, passivity-based control, Euler-Lagrange systems, distributed control.

1. INTRODUCTION

The use of collaborative robots (which include mobile robots and UAVs) and of networked electro-mechanical systems are pervasive in various application domains, such as, smart factories, smart logistic systems, intelligent buildings and smart grids. For instance, the collaborative robots can be deployed to solve a variety of different tasks by autonomously coordinating their movements and actions among themselves. As another example, a network of machines in the shop floor of a smart factory can reconfigure themselves cooperatively and autonomously to produce a variety of different products. Against the backdrop, the distributed control methods thereof have been an active area of research for the past decade, providing provably-correct control algorithms that can guarantee the completion of every given task by the group of robots or by the networked machines. These physical systems are typically belong to the class of Euler-Lagrange (EL) systems and the distributed tracking control method of networked EL systems is the focus of this paper.

For a single EL system, passivity-based control (PBC) techniques have become the de-facto control design framework for a single EL system due to their natural interpretation of the closed-loop systems, see, for example the books of Ortega et al. (1998) and Van der Schaft (2017). In this framework, physical energy variables of the EL systems are explicitly used in the control design, the systems' interconnection is translated to energy exchange among different systems and the total energy dissipation is shaped in such a way that it fulfills the transient behaviour requirement and the state asymptotically converge to the minimum of the total energy. The term passivity-based control was introduced in Ortega and Spong (1988). Recent results on PBC for a single EL system are the works of Reyes-Báez et al. (2018b), Reyes Báez et al. (2017), Reyes-Báez et al. (2018a) which incorporate recent advances in contraction theory and incremental stability, such as, Jouffroy and Fossen (2010) and Andrieu et al. (2016).

The passivity of EL mechanical systems is further exploited in the work of Slotine and Li (1987) where a so-called virtual system is used as target closed-loop system in the control design procedure. Remarkably, such virtual system inherits the passivity of the actual one and it has been shown recently that this virtual system has contraction properties in Jouffroy and Fossen (2010). This observation was recently exploited for differential passivity based control design in Reyes-Báez et al. (2018b), Reyes Báez et al. (2017) and Reyes-Báez et al. (2018a).

Recently, the passivity approach has been extended to solve the distributed control problem for networked EL systems.

The generalization of these PBC methods to the multi-agent setting has been well-studied in recent decade. The book of Arcak (2007), Bai et al. (2011), Van der Schaft (2017) and the article by Chopra and Spong (2006) provide a thorough exposition to the design of passivity-based distributed control methods where a number of coordination control problems can be solved through PBC approach, including, synchronization and formation control. For networked EL systems, some relevant works are the articles by Garcia de Marina Peinado et al. (2018), Nuño et al. (2013b), Nuño et al. (2013a) and Chung and Slotine (2009).
In the present paper, we reformulate the PBC design methodology in Arcak (2007) where, roughly speaking, the coordination control problem is solved by interconnecting (strictly passive) systems attached to the nodes of a graph via diffusive coupling that preserves the passivity of the network dynamics. As an alternative, we attach strictly passive artificial spring systems to each node and they are feedback interconnected to nodes dynamics. This results in dynamics protocols where the spring dynamics can be interpreted as a (nonlinear) integral action. Due to the strict passivity of the interconnected system, the asymptotic stability result can be established by using the total storage function as a strict Lyapunov function. We also present two other distributed control methods which use networked virtual systems and can be seen as a particular case of our proposed method, as well as, the one in Arcak (2007).

The structure of the paper is as follows: In Section 2 we present the preliminaries of graph theory and mechanical systems in the EL framework that are necessary for the network dynamics modeling. Moreover, we recall two well-known passivity-based tracking controllers for a single mechanical system. In Section 3 we present the main contribution of our work, where instead of diffusive coupling we attach an artificial spring system to each node. The interconnection of the nodes and edges exhibits a symmetric structure that preserves passivity. Two other diffusive coupling protocols are presented in Section 4 which can be seen as particular cases of the proposed methodology exploiting the concept of virtual system for the network dynamics. Section 5 presents a simulation result that show the performance of our proposed approach in Section 3.

2. NETWORKED EULER-LAGRANGIAN SYSTEMS
PRELIMINARIES

In this work we consider a network of N EL systems (agents) which interact among themselves for solving tracking control problem in a coordinated manner. The interaction among agents in the network is represented by the links of a graph.

2.1 Graph theory tools

The following preliminaries of graph theory are taken from Bollobás (1998) and Van der Schaft and Maschke (2013).

Definition 1. (Graph). A graph $G$ is defined by a pair $(V,E)$ where $V$ is a finite set of $N$ vertices (also called nodes) and $E$ the finite set of $M$ edges (also called links). Furthermore, there is an injective mapping from $V$ to the set of unordered pairs $V$, identifying edges with unordered pairs of vertices.

The set $V$ is called the vertex set of the graph $G$, and the set $E$ is called the edge set. The graph $(V,E)$ is said to be directed if the edges are ordered pairs $e_{ij} = (w_i, w_j)$. Then $w_i$ represents the tail vertex and $w_j$ the head vertex of $e_{ij}$. If $e_{ij} = (w_i, w_j) \in E$, then the vertices $w_i$ and $w_j$ are called adjacent or neighboring vertices; and $w_i$ and $w_j$ are incident with the edge $e_{ij}$. Two edges are adjacent if they have exactly one common end-vertex. A path (weak path) of length $r$ in a directed graph is a sequence $w_1, \ldots, w_r$ of $r + 1$ distinct vertices such that for each $i \in \{1, \ldots, r\}$, $(w_i, w_{i+1}) \in E$ (respectively, either $(w_i, w_{i+1}) \in E$ or $(w_{i+1}, w_i) \in E$. A directed graph is strongly connected (weakly connected) if any two vertices can be joined by a path (respectively, weak path).

Given a graph $G$, we define its vertex space $\Lambda_0(G)$ as the vector space of all functions from $V$ to some linear space $R$. Furthermore, we define the edge space $\Lambda_1(G)$ as the vector space of all functions from $E$ to $R$. The dual spaces of $\Lambda_0$ and $\Lambda_1$ will be denoted by $\Lambda^0$ and $\Lambda^1$, respectively.

For a directed graph $G$, the incidence operator is a linear map $\mathcal{B} : \Lambda_1 \to \Lambda_0$ with matrix representation $\mathcal{B} \otimes I$, where

$$\mathcal{B} \otimes I := \begin{bmatrix} b_{11}I & \cdots & b_{1M}I \\ \vdots & \ddots & \vdots \\ b_{N1}I & \cdots & b_{NM}I \end{bmatrix}$$

with $I$ be the identity map and $b_{ik} := \begin{cases} -1 & \text{if node } i \text{ is at the tail of } k-\text{th edge} \\ 1 & \text{if node } i \text{ is at the head of } k-\text{th edge} \\ 0 & \text{otherwise} \end{cases}$

The adjoint operator of $\mathcal{B}$ is given by the map $\mathcal{B}^* : \Lambda^0 \to \Lambda^1$ with matrix representation $\mathcal{B}^* \otimes I$, and it is called the co-incidence operator. We will throughout use $B (B^T \otimes I)$ both for the incidence (respectively, co-incidence) matrix and for incidence (respectively, co-incidence) operator. The rank of $B$ is at most $N - 1$ due to the sum of its rows is zero. Indeed, the rank is $N - 1$ when the graph is connected. The columns of $B$ are linearly independent. We also introduce the Laplacian matrix given by $L := BB^T$.

2.2 Agents and network dynamics

We consider agents evolving on a configuration manifold $Q_i$ of dimension $n$ for all $i \in \{1, \ldots, N\}$. The position of the $i$-th robot is given by the vector $q_i \in Q_i$, and velocity $\dot{q}_i \in T_{Q_i}Q_i$. The equation of motion of the $i$-th agent are given by the Euler-Lagrange equations

$$\dot{q}_i = v_i$$

where $\tau_i \in T_{Q_i}Q$ are external forces $g_i(q_i) = g_{Q_i}(q_i)$ and $C_i(q_i, v_i)$ is any matrix satisfying

$$C_i(q_i, v_i)v_i = M_i(q_i)v_i - \frac{\partial}{\partial q_i} \left( \frac{1}{2} q_{i}^T M_i(q_i)v_i \right).$$

When $C_i(q_i, v_i)$ is expressed in terms of the so-called Christoffel symbols, it leads to the following property

Lemma 1. (Van der Schaft (2017)). The matrix

$$N_i(q_i, v_i) = M_i(q_i) - 2C_i(q_i, v_i),$$

is linear in $v_i$ and skew-symmetric for every $q_i$ and $v_i$. This property is a clear expression that the forces $N_i(q_i, v_i)$ are workless. Indeed, if we take the total coenergy $\dot{H}_i(q_i, v_i) = \frac{1}{2} q_{i}^T M_i(q_i)v_i + P_i(q_i)$ as storage function, then system (3) is passive (lossless), i.e., $\dot{H}_i(q_i, q_i) = \dot{q}_{i}^T N_i(q_i, \dot{q}_i) + y_i^T \tau_i = y_i^T \tau_i$ where the output is $y_i = v_i$. Notice that, also in case that $C_i(q_i, v_i)$ is not parametrized by the Christoffel symbols, by energy conservation, matrix $N_i(q_i, v_i)$ still satisfies condition $v_i^T N_i(q_i, v_i)v_i = 0$ for every $q_i$ and $v_i$. 

\footnote{Notice that, also in case that $C_i(q_i, v_i)$ is not parametrized by the Christoffel symbols, by energy conservation, matrix $N_i(q_i, v_i)$ still satisfies condition $v_i^T N_i(q_i, v_i)v_i = 0$ for every $q_i$ and $v_i$.}
The EL network dynamics can be expressed in a compact form by using the concatenated vectors, 
\[ q := [q_1^T, \ldots, q_N^T]^T, \quad v := [v_1^T, \ldots, v_N^T]^T, \]
\[ \tau := [\tau_1^T, \ldots, \tau_N^T]^T, \quad g := [g_1^T, \ldots, g_N^T]^T, \]
together with the block diagonal matrices
\[ M(q) := \text{diag}(M_1(q_1), \ldots, M_N(q_N)), \]
\[ C(q, v) := \text{diag}(C_1(q_1, v_1), \ldots, C_N(q_N, v_N)), \]
such that the dynamics of the network with state \((q, v)\) is given by
\[ \dot{q} = v, \quad M(q)\dot{v} + C(q, v)v + g(q) = \tau, \quad (6) \]
where, as before, \(g(q) = \frac{\partial P}{\partial q}(q)\) with the potential function \(P(q) := \sum_{i=1}^N P_i(q_i)\). Notice that network dynamics (6) preserves the structure of the \(i\)-th agent in (3), e.g., the matrix \(N(q, v) := M(q) - 2C(q, v)\) is skew-symmetric, and the map from \(\tau\) to \(y = v\) is lossless with storage function
\[ H^*(q, v) = \sum_{i=1}^N H_i^*(q_i, v_i) = \frac{1}{2} v^T M(q)v + P(q). \quad (7) \]

2.3 Passivity based tracking controller for single agent

The skew-symmetry of \(N_i(q_i, v_i)\) is crucial in the definition of a virtual system associated to (3), see e.g. Van der Schaft (2017) and Ortega et al. (1998).

Definition 2. A virtual system associated to (3), in the state \((\bar{r}_i, \bar{s}_i) \in \mathbb{T}\), is defined as the system
\[ \begin{align*}
\dot{r}_i &= s_i, \\
M_i(q_i)\dot{s}_i + C_i(q_i, v_i)s_i + g_i(r_i) &= \pi_i, \\
y_i &= s_i,
\end{align*} \quad (8) \]
parametrized by \((q_i, v_i)\) with input \(u_i\) and output \(y_i\).

Any solution \(q_i(t)\) of (3) for a certain input \(r_i(t)\) produce the solution of \((r_i(t), s_i(t)) = (q_i(t), \dot{q}_i(t))\) to the virtual system (8) for \(\pi_i = \tau_i\), but not every pair \((r_i(t), s_i(t))\) is a solution to (3). This is only the case if additionally \((r_i(t), s_i(t)) = (q_i(t), \dot{q}_i(t))\). Remarkably, system (8) keeps the (structural) lossless property, with the storage function
\[ H^*(r_i, s_i) = \frac{1}{2}s_i^T M_i(q_i)s_i + P_i(r_i) \quad (9) \]
parametrized by \(q_i\). The time derivative a long (8) is
\[ H^*(r_i, s_i) = \frac{1}{2}s_i^T M(q)s_i + s_i^T (u_i - C_i(q_i, v_i)s_i), \]
\[ = s_i^T u_i + s_i^T N_i(q_i, v_i)s_i, \]
\[ = y_i^T u_i. \quad (10) \]

With the feedback \(\pi_i = g_i(r_i) - K_i s_i + u_i\) in (8) with \(K_i > 0\) and \(\pi_i\), an external input, position and velocity dynamics are decoupled. The velocity dynamics results in
\[ M_i(q_i)\dot{s}_i + C_i(q_i, v_i)s_i + K_i s_i = u_i, \]
\[ y_i = s_i, \quad (11) \]
which is output strictly passive with storage function \(S_{s,i}(s_i, q_i) = \frac{1}{2}s_i^T M_i(q_i)s_i\) parametrized by \(q_i\). From a design point of view, this fact makes (11) a suitable of target closed-loop system.

In fact this is the seminal tracking controller in Slotine and Li (1987), where a sliding manifold is made invariant and attractive by designing the sliding (error) variable such that it preserves the velocity dynamics structure as in the virtual system (11).

Theorem 1. Consider a smooth reference trajectory \(q_d(t)\) together with the change of variables \(s_i = v_i - v_{i,r}\), where \(v_{i,r} = \dot{q}_i - \Pi_i(q_i - q\bar{a})\) is an artificial reference velocity with \(\Pi_i = \Pi_i^T > 0\). Then, velocity dynamics of (3) in closed-loop with the control law
\[ \tau_i = M_i(q_i)\dot{v}_{i,r} + C_i(q_i, \dot{q}_i)v_{i,r} + g_i(q_i) - K_i s_i + u_i \quad (12) \]
is given by virtual system (11) and \(s(t) \rightarrow 0, \dot{q}(t) - \dot{q}_d(t) \rightarrow 0\) and \(q(t) - q_d(t) \rightarrow 0\) exponentially as \(t \rightarrow \infty\).

This control law gives exponential stability as shown in Spong et al. (1990). A backstepping redesign is proposed in Fossen and Berge (1997) where the closed-loop system performance is further improved as given in the following theorem.

Theorem 2. Consider the change of coordinates \(\bar{q}_i = q_i - q_d\) and all hypothesis and variables of Theorem 1. Then, for system (3) in closed-loop with the control law
\[ \tau_i = M_i(q_i)\dot{v}_{i,r} + C_i(q_i, \dot{q}_i)v_{i,r} + g_i(q_i) - K_i s_i - \Pi_i \bar{q}_i + u_i \quad (13) \]
the origin \((\bar{q}_i, s_i) = (0, 0)\) is globally uniformly exponentially stable equilibrium point.

3. DISTRIBUTED NODE & EDGE DYNAMIC CONTROLLER DESIGN

3.1 Group coordination problem formulation

In this paper, we are interested in the following group coordination problem: For a network of EL systems in (3), design a distributed control law on each node and edge based only on local information\(^3\) such that

- Each agent's velocity \(v_i(t)\) in (3) tracks a common velocity reference \(v_r(t)\) prescribed for the network; that is
  \[ \lim_{t \to \infty} \|v_i(t) - v_r(t)\| = 0 \quad \text{for all} \quad i \in \{1, \ldots, N\}. \quad (14) \]

- The interaction variable \(\zeta_{ik}\) (defined on the edge and denotes the relative displacement between agents \(i\) and \(j\) interconnected through an artificial spring that will be defined shortly below) converges to a nonempty compact set \(\mathcal{A}_k \subset \mathbb{Q}\), for all \(k \in \{1, \ldots, M\}\).

3.2 Node & edge dynamic control design method

Our proposed design procedure is as follows:

(1) For each EL agent (3), consider the nodal feedback control (12) such that the closed-loop system is passive from an external input \(u_i\) to the velocity "error" \(y_i := s_i = v_i - v_r\). Denote this local closed-loop system as \(y_i = H_i(u_i)\).

\(^3\) The \(i\)-th agent can use the information \(y_j - y_j\) if the \(j\)-th agent is a neighbor, where \(y_i\) is the passive output of the \(i\)-th agent in (3).
(2) For every edge \( k \), we assign a (spring) system
\[
\Sigma^{z}_{\mu,k} : \begin{cases} 
\dot{\zeta}_{k} = \mu_{k} - \phi_{\zeta,k}(\zeta_{k}), \\
\tau_{k} = \frac{\partial P_{\zeta,k}}{\partial \zeta}(\zeta_{k}), \\
k \in \{1,\ldots,M\},
\end{cases}
\]
to the \( k \)-th edge if \((i,j) \in E_{k}\), where \( \phi_{\zeta,k} \) is a potential force to be designed and \( P_{\zeta,k} : D_{k} \subseteq \mathbb{R}_{+}^{n} \rightarrow \mathbb{R}_{+}^{n} \) is a \( C^{2} \) convex artificial potential energy function defined on the open set \( D_{k} \) with minimum at \( \zeta_{d} \) where \( \zeta_{d} \) is the vector of desired relative displacement between communicating agents. Note that, in this case, we assume that \( A_{k} = \{\zeta_{d}\} \).

(3) We interconnect all node systems \( y_{i} = \mathcal{H}_{i}(u_{i}) \), \( i = 1,\ldots,N \) and all edge spring system \( \Sigma^{z}_{\mu,k}, k, k = 1,\ldots,M \) through the following passivity preserving interconnection
\[
\mu_{k} := \sum_{i=1}^{N} b_{ik} y_{i}, \quad u_{k} = -\sum_{k=1}^{M} b_{ik} \tau_{k},
\]
for all \( k \in \{1,\ldots,M\} \) and all \( i \in \{1,\ldots,N\} \), respectively.

The function \( P_{\zeta,k}(\zeta_{k}) \) is designed to render the target sets \( A_{k} \) invariant and asymptotically stable for \( k \in \{1,\ldots,M\} \). Interconnection laws in (16) satisfy the following relations
\[
\mu = (B^{\top} \otimes I_{n}) y \quad \text{and} \quad u = -(B \otimes I_{n}) y,
\]
with \( y = [y_{1}^{\top},\ldots,y_{N}^{\top}]^{\top} \), and \( \mu = [\mu_{1}^{\top},\ldots,\mu_{M}^{\top}]^{\top} \). This means that \( \mu \) must be in the image space \( \text{Im}(B^{\top} \otimes I_{n}) \).

Remark 1. We make the following observations with respect to the design procedure in Arcak (2007).

- If \( \phi_{\zeta,k}(\zeta_{k}) = 0 \) for all \( k \in \{1,\ldots,M\} \) and \( y = v \) in (17) then we recover the procedure in Arcak (2007).
- Due to relations in (17), the “symmetric” interconnection structure is preserved.

Similar to the first step in the design procedure in Arcak (2007), we also impose that the system \( y_{i} = \mathcal{H}_{i}(u_{i}) \) to be strictly passive from \( u_{i} \) to \( y_{i} \) with a \( C^{1} \), positive definite, radially unbounded storage function
\[
S_{i}(r_{i},s_{i}) = \frac{1}{2} s_{i}^{\top} M_{i}(q_{i}) s_{i} + \frac{1}{2} \rho_{i}^{\top} \Pi \rho_{i},
\]
satisfying
\[
\dot{S}_{i}(q_{i},s_{i}) \leq W_{q,i}(s_{i}) + y_{i}^{\top} u_{i},
\]
for the positive definite function \( W_{q,i}(s_{i}) = s_{i}^{\top} K_{S_{i}} s_{i} \).

Likewise, for the spring system we also require that its dynamics (15) to be strictly passive. To this end, take \( \phi_{\zeta,k}(\zeta_{k}) = \tau_{k} \). Indeed, with \( P_{\zeta,k}(\zeta_{k}) \) as storage function, the map \( \mu_{k} \mapsto \tau_{k} \) is strictly passive. This is easily seen from the time derivative of \( P_{\zeta,k}(\zeta_{k}) \) along system (15),
\[
\dot{P}_{\zeta,k} = \frac{\partial P_{\zeta,k}^{\top}}{\partial \zeta_{k}}(\zeta) \mu_{k} - \frac{\partial P_{\zeta,k}^{\top}}{\partial \zeta_{k}}(\zeta) \frac{\partial P_{\zeta,k}}{\partial \zeta_{k}}(\zeta).
\]
In particular, in this preliminary work we consider
\[
P_{\zeta,k}(\zeta_{k}) = \frac{1}{2} (\zeta_{k} - \zeta_{d})^{\top} K(\zeta_{k} - \zeta_{d}).
\]

3.3 Interconnected system stability analysis

Proposition 1. Consider agent’s dynamics (3) which is in closed-loop with the control law (13) for all \( i \in \{1,\ldots,N\} \), combined with (15) and (17). Then the equilibrium point \( (\bar{q},\bar{s},\zeta) = (0,0,\zeta_{d}) \) is uniformly exponentially stable with rate \( \beta = k_{2}/k_{1} \), where
\[
k_{2} = \max\{\lambda_{\text{max}}(\Pi),\lambda_{\text{max}}(M(q)),\lambda_{\text{min}}(K_{S_{i}})\},
k_{1} = \min\{\lambda_{\text{min}}(\Pi^{\top}),\lambda_{\text{min}}(K),\lambda_{\text{min}}(K_{S_{i}}^{\top})\}.
\]

Proof. First notice that the closed-loop system is
\[
\ddot{q} + \Pi \dot{q} = s,
\]
\[
M(q)\ddot{s} + C(q,\dot{q})s + Ks = -\Pi \dot{q} - (B \otimes I_{n}) \frac{\partial P_{\zeta,k}}{\partial \zeta}(\zeta),
\]
(22)
\[
\dot{\zeta} + \frac{\partial P_{\zeta,k}}{\partial \zeta}(\zeta) = (B^{\top} \otimes I_{n}) s
\]
where in this case \( \Pi := \text{diag}[\Pi_{1},\ldots,\Pi_{N}] \) and \( K = \text{diag}[K_{1},\ldots,K_{N}] \). Take as a candidate Lyapunov function
\[
S(q,s,\zeta,q) = \frac{1}{2} \dot{q}^{\top} \Pi M(q) \dot{q} + P_{\zeta}(\zeta),
\]
which satisfies the bounds
\[
k_{1} \left\| \begin{bmatrix} \dot{q} \\ s \end{bmatrix} \right\|^{2} \leq S(q,s,\zeta,q) \leq k_{2} \left\| \begin{bmatrix} \dot{q} \\ s \end{bmatrix} \right\|^{2}
\]
where \( k_{1} = \min\{\lambda_{\text{min}}(\Pi^{\top}),\lambda_{\text{min}}(M(q)),\lambda_{\text{min}}(K_{S_{i}}^{\top})\} \).

The time derivative of (23) along (22) is given by
\[
\dot{S}_{q,s} = -\dot{q}^{\top} \Pi q - s^{\top} K s - (\zeta - \zeta_{d})^{\top} K_{S_{i}}^{\top} (\zeta - \zeta_{d}) < 0.
\]
That is, (23) is a strict Lyapunov function for system (22). Furthermore, with the bounds in (24) it is easy to see that \( \dot{S}_{q,s} < -\delta S_{q,s} \), \( \zeta = \zeta_{d} \), and the function (23) is radially unbounded. This completes the proof.

4. PASSIVITY-BASED SYNCHRONIZED TRACKING CONTROLS

In this section we present two alternative distributed trajectory tracking control laws which can be seen as particular cases of the method described in Section 3 and that also fit with approach in Arcak (2007) where virtual systems structure is exploited.

4.1 Slotine-Li synchronized tracking control

Recall that the networked EL dynamics (6) has the same structure as the individual agent dynamics as in (3). This motivates us to introduce a similar controller construction as the one in Theorem 1 for a single agent. However, in this case, the control objective is not only tracking to a reference signal \( q_{d}(t) \in \mathbb{Q} \) but synchronized tracking to a common reference signal \( q_{d}(t) \) for all the agents in the network. To this end, we propose the following sliding manifold for system (6) given by
\[
\Omega = \{(q,v) : \dot{q} + (\Pi \otimes I_{n}) \dot{q} + (B \Delta B^{\top} \otimes I_{n}) q = 0\}
\]
where \( \ddot{q} = q - (\Pi_{N} \otimes q_{d}(t)) \) with \( \Pi_{N} \in \mathbb{R}^{N} \) the vector of all ones, \( \Pi = \Pi^{T} \in \mathbb{R}^{N \times N} \) and \( \Delta = \Delta^{T} \in \mathbb{R}^{M \times M} \) are positive definite diagonal matrices. Since \( (B \Delta B^{\top} \otimes I_{n}) (\Pi_{N} \otimes q_{d}(t)) = 0_{N} \), the dynamics of (6) in the manifold \( \Omega \) in (26) satisfies
\[
\dot{q} = -((\Pi + B \Delta B^{\top}) \otimes I_{n}) \ddot{q}.
\]
Thus if \( \Pi \) and \( \Delta \) are chosen such that they satisfy
\[
P((\Pi + B \Delta B^{\top}) \otimes I_{n}) + ((\Pi + B \Delta B^{\top}) \otimes I_{n})^{T} P = -Q
\]
where \( P, Q \in \mathbb{R}^{N \times N} \) are symmetric and positive definite matrices, then \( q(t) \to (1_N \otimes q_d(t)) \) exponentially as \( t \to \infty \). Hence all the agents track \( q_d(t) \) in a synchronized fashion. Thus, by defining

\[
v_r := (1 \otimes q_d(t)) - ([I + B\Delta B^T] \otimes I_n) \dot{q},
\]

as an artificial velocity reference signal for the network dynamics (6), the sliding variable is given by \( s = \dot{q} - v_r \).

**Corollary 1.** Consider a strongly connected graph \( \mathcal{G} \) with incidence matrix \( B \) and a reference trajectory \( (1_N \otimes q_d(t)) \) for system (6) with \( q_d(t) \in \mathcal{Q} \). Let \( s, v_r \) be the sliding variable and artificial velocity reference signal as defined before and the control law be given by

\[
\tau = M(q)\dot{v}_r + C(q, \dot{q})v_r + g(q) - Ks + u
\]

where \( K = K^T \in \mathbb{R}^{N \times N} \) is a positive definite gain matrix. Then the closed-loop system of (6) and (30) \( \dot{q} + ([I + B\Delta B^T] \otimes I_n)\dot{q} = s \)

defines a strictly passive map \( u \mapsto y = s \) with with respect to the storage function

\[
S_s(s, q) = \frac{1}{2} s^T M(s, q) s
\]

parametrized by \( q \). Moreover, by taking \( u = \alpha(s) \) with a passive map \( \alpha \), we have that \( s(t) \to 0 \) and \( \dot{q}(t) \to 0 \) as \( t \to \infty \).

Corollary 1 can be seen as a particular case of the result in (Bai et al., 2011, Theorem 6.3). The only difference is that we consider the (artificial) reference velocity \( v_r \) defined above (that is not common to all the agents), instead of a pure time-varying signal. The structure of \( v_r \) in this case implies that the external contains also a diffusive velocity coupling, i.e.,

\[
\tau_r = -(B\Delta B^T \otimes I_n)(q + v) + u.
\]

Nevertheless, Corollary 1 can be proved exactly in the same way as in (Bai et al., 2011, Theorem 6.3). The existence of \( \Omega \) in (26) is guaranteed with \( v_r \) as defined above. This in turn implies that both \( (1_N \otimes q_d(t)) \) and \( v_r \) are attractive trajectories for (36). Furthermore, the control scheme (30) can be split into the so-called equivalent control \( 4 \) with \( \tau_q = M(q)\dot{v}_r + C(q, \dot{q})v_r + g(q) \) and a feedback term \( \tau_{s} = -Ks \); the first ensures invariance once constrained to \( \Omega \) and the later ensures that the off-manifold "distance" \( s \) converges to zero, i.e., attractivity.

The sliding dynamics is given in (27), modulo a change of coordinates.

### 4.2 Backstepping synchronized tracking control

In the previous distributed control approach, passivity of the closed-loop dynamics (31) as a whole is not used. This can be further exploited for performance improvement. To this end, notice that the position error dynamics is passive from the input \( \tau_q = s \) to the output \( \dot{q}_s = \dot{q} \) with the storage function

\[
S_{\dot{q}}(\dot{q}) = \frac{1}{2} \dot{q}^T ([I + B\Delta B^T] \otimes I_n) \dot{q}.
\]

It is hierarchically interconnected to the passive dynamics of \( s \) in (31) via \( \tau_q = s \), that is, \( \dot{q} \)-dynamics does not influence \( s \)-dynamics. The above observations motivate the following proposition where we apply a backstepping redesign for the protocol in Corollary 1, which can be seen as the networked version of (13).

**Proposition 2.** Consider a strongly connected graph \( \mathcal{G} \) with incidence matrix \( B \), and a reference trajectory \( (1_N \otimes q_d(t)) \) for system (6) with \( q_d(t) \in \mathcal{Q} \), s, \( v_r \), and \( \tau_r \) in (29) together with the control law given by

\[
\tau = M(q)\dot{v}_r + C(q, \dot{q})v_r + g(q) - Ks
\]

where \( K = K^T \in \mathbb{R}^{N \times N} \) is a positive definite gain matrix. Then the closed-loop system of (6) with the control law (35) given by

\[
\dot{q} + ([I + B\Delta B^T] \otimes I_n)\dot{q} = s
\]

is strictly passive from \( u \) to \( s \) with the storage function

\[
S_{\dot{q}}(\dot{q}, s, q) = S_{\dot{q}}(\dot{q}) + S_s(s, q)
\]

parametrized by \( q \). Moreover the origin of (36) is uniformly globally exponentially stable of the rate \( \beta = k_3/k_2 \) where

\[
k_2 = \max \{ \lambda_{\text{max}}([I + B\Delta B^T] \otimes I_n), \lambda_{\text{max}}(M(q)) \},
\]

\[
k_3 = \min \{ \lambda_{\text{min}}([I + B\Delta B^T]^2 \otimes I_n), \lambda_{\text{min}}(K) \},
\]

with \( \lambda_{\text{min}}(\cdot) \) and \( \lambda_{\text{max}}(\cdot) \) are the minimum and maximum eigenvalue of their argument, respectively.

**Proof.** We will show that (37) is a strict Lyapunov function for the nonautonomous system (36) following (Khalil, 2002, Theorem 4.10). First, we notice that (37) satisfies

\[
k_1 \left\| \dot{q} \right\|_2^2 \leq S_{\dot{q}}(\dot{q}, s, q) \leq k_2 \left\| \dot{q} \right\|_2^2
\]

where \( k_1 = \min \{ \lambda_{\text{min}}([I + B\Delta B^T] \otimes I_n), \lambda_{\text{min}}(M(q)) \} \), and \( k_2 \) as in (38). The time derivative of (37) along the trajectories of (36) is

\[
\dot{S}_{\dot{q}}(\dot{q}, s, q) = -\beta S_{\dot{q}}(\dot{q}, s, q) \leq -\beta S_{\dot{q}}(\dot{q}, s, q),
\]

uniformly in \( q \). Hence, the storage function (37) qualifies as a strict Lyapunov function. Using the bounds of the Lyapunov function in (39), it is straightforward to see that \( \dot{S}_{\dot{q}}(\dot{q}, s, q) \leq -\beta S_{\dot{q}}(\dot{q}, s, q) \), which completes the proof.

The extra term in the protocol (35) can be understood as a feedforward action to the closed-loop dynamics of the sliding variable \( s \) that preserves the passivity.

**Remark 2.** Since system (36) is linear in the state \( (\dot{q}, s) \), invoking contraction analysis arguments, it can be shown that the matrix

\[
\begin{bmatrix}
1 + B\Delta B^T \otimes I_n & 0 \\
0 & M(q)
\end{bmatrix}
\]

is a Riemannian metric, for details see Jouffroy and Fossen (2010) and references therein. Thus, the gain matrices \( \Pi, \Delta \) and \( K \) can be optimally tuned by taking (37) as a cost function subject to the network dynamics (36).
5. SIMULATIONS

To show the performance of the coordination protocol obtained in Section 3, consider a network of one degree of freedom systems \( n = 1 \) with \( N = 6 \) on an undirected graph \( G \) with the following specifications:

| \( q_i \) | 0.36 | \( K_i \) | 3.5 |
| \( z_i \) | 12 | \( K \) | 5 |

Due to space limitations, we show only the position tracking behavior in Figure 5, where in fact, the positions reach agreement for the given conditions.

![Position q_i vs time for i ∈ {1, 2, 3, 4, 5, 6}](image)

Fig. 1. Position \( q_i \) vs time for \( i \in \{1, 2, 3, 4, 5, 6\} \).

6. CONCLUSIONS

In this paper we have reformulated the design procedure in Arcak (2007), where an artificial spring system is designed at each edge in the graph, instead of diffusing information through the relative positions and velocities, as commonly adopted. In our proposed approach, we require that all the nodes’ and edges’ dynamics to be strictly passive such that via a passivity preserving interconnection, the total storage function can be used as a strict Lyapunov function to show exponential convergence to the desired trajectory.

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