TESTING THE STANDARD MODEL AND BEYOND

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ABSTRACT

This paper is based on lectures presented to mathematical physicists and attempts to provide an overview of the present status of the Standard Model, its experimental tests, phenomenological and experimental motivations for going beyond the Standard Model via supersymmetry and grand unification, and ways to test these ideas with particle accelerators.
1 Introduction to the Standard Model and its (Non-Topological) Defects

When we phenomenological particle physicists talk of the Standard Model, we include QCD, our theory of the strong interactions and the Glashow-Weinberg-Salam electroweak theory \(^1\). Much of this lecture will be concerned with the following fundamental question: why are the masses of the force-carrying gauge bosons of the Standard Model so different, whilst their couplings to matter are so similar? Phenomenologists believe that the answer to this question is provided by some variant of the Higgs mechanism, but we do not yet have any direct experimental evidence for this belief. However, precision electroweak data are beginning to provide us with some indications on the nature of Higgs physics, as discussed in Section 3, and may be providing us with some experimental motivation for supersymmetry, as discussed in Section 4. The recent big news in the current catalogue of the elementary particle constituents of matter has been the confirmation of the discovery of the top quark \(^2\), with a mass close to predictions \(^3\) based on precision electroweak data \(^4\), as we shall review in Section 2. As you probably know, one of the first major facts established by the LEP accelerator was that there are no more light neutrino species. Within the Standard Model context, this limits the number of lepton doublets, and hence presumably means that there are no more charged leptons either, and (in order to cancel triangle anomalies) hence no more quarks in generations like the three which are now known.

The Standard Model outlined in the previous paragraph has been tested and verified, by experiments at LEP in particular, with a precision which is now better than 1 % \(^4\). Although the Standard Model has passed (almost) all these tests with flying colours, it has many (non-topological) defects which motivate going beyond it. Theoretically, the Standard Model is very unsatisfactory because it provides no explanation for the elementary particle quantum numbers (colour, electroweak isospin, hypercharge), and contains twenty or more arbitrary parameters. We would dearly love to reduce the number of these parameters!

The major classes of problem that motivate going beyond the Standard Model are three.

The Problem of Mass: What are the origins of the different particle masses? Is there an elementary Higgs boson? Why are all the particle masses so much smaller than the Planck mass, the only candidate we have for a fundamental scale in physics? Does supersymmetry play a role \(^3\) in answering this question? As discussed in Section 4, there are good reasons to expect that this set of questions may be answered by experiments performed at forthcoming accelerators, in particular the LHC as discussed in Section 6.

The Problem of Unification: Is there a simple gauge framework which includes all the interactions of the Standard Model? Does this yield novel phenomena such as proton decay and neutrino masses which can be detected, possibly by non-accelerator experiments?

The Problem of Flavour: Why are there so many different types of quarks and leptons? Why are the couplings of the \(W^\pm\) mixed? What is the origin of CP violation? Some phenomenologists suggest that these questions may be answered in a composite model of quarks and leptons.
Personally, I have never seen a composite model that I find convincing. Moreover, there is no experimental indication on the scale at which these flavour questions might be answered. I believe that obtaining the answer to this question will have to wait for a better understanding of string theory.

String theory is the only serious candidate we have for a Theory of Everything which includes gravity as well as the Standard Model interactions described above, reconciles gravity with quantum mechanics [3], explains the origin of the space-time, tells us why we live in four dimensions, etc. Since the scope of this lecture is purely phenomenological, I will not address here these fascinating problems.

2 Testing the Standard Model

The electroweak sector of the Standard Model has been tested in a large variety of experiments at a vast range of energies and distance scales. These extend from measurements of parity violation in atoms [7], with an effective invariant momentum transfer $Q^2$ of about $10^{-10}$ GeV$^2$, through neutrino-electron scattering at $Q^2 \sim 0.1$ GeV$^2$ [8], deep-inelastic electron-, muon- and neutrino-hadron scattering at $Q^2 \sim 1 - 100$ GeV$^2$, electron-positron collisions at $Q^2 \lesssim 10^4$ GeV$^2$ and proton-antiproton colliders at $Q^2 \sim 10^4$ GeV$^2$. The largest momentum transfers of all have been seen in deep-inelastic electron-proton collisions at HERA [9], but these are not yet of sufficient precision to provide sensitive tests.

The most sensitive tests of the Standard Model are those provided [4] by electron-proton collisions in the LEP accelerator at CERN, and the SLC accelerator at SLAC. Most of the data taken at these accelerators so far have been in the neighbourhood of the $Z^0$ peak [10], which is perhaps the most precisely studied Breit-Wigner peak in history. The following are the basic measurements performed on the $Z^0$ peak.

The Total Hadronic Cross Section: At the tree level, this is given by

$$\sigma_h^0 = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{had}}{\Gamma_Z^2}$$

where $M_Z$ and $\Gamma_Z$ are the mass and total decay rate of the $Z^0$ boson, respectively, and $\Gamma_e, \Gamma_h$ are its partial decay rates into electron-positron pairs and hadrons, respectively. After including electromagnetic radiative corrections, the cross section in Eq. (1) is reduced to about 30 mb [10]. The total event rate at LEP is given by the product of this cross section and the luminosity (collision rate) which is $\lesssim 2 \times 10^{31}$ cm$^{-2}$ s$^{-1}$, yielding almost one event per experiment per second.

The Total $Z^0$ Decay Rate: In the absence of exotic decay modes, this can be written in the form

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + N_{\nu} \Gamma_{\nu} + \Gamma_{had}$$

2
where the three leptonic decay rates are equal if one assumes universality, and \( N_\nu \) is the number of light neutrino species. In the Standard Model,

\[
\Gamma_\nu = 1.992 \pm 0.003 \Gamma_{\ell\ell} .
\]

(3)

Since the neutrinos are not seen directly in the experiment, they cannot be distinguished from other weakly-interacting neutral particles, so the total

\[
\Gamma_{\text{invisible}} \equiv N_\nu \Gamma_\nu
\]

(4)

may be parametrized by a non-integer value of \( N_\nu \)!

**Partial Decay Rates:** By looking at particular final states, it is possible to disentangle various partial decay rates of the \( Z^0 \). Particularly accurately measured are the \( \Gamma_{\ell\ell} \), which can be related to the ratios \( R_\ell \equiv \Gamma_{\text{had}}/\Gamma_{\ell\ell} \). Of special recent interest have been the partial decay rates into bottom and charm quarks, parametrized by \( R_b = \Gamma_{b\bar{b}}/\Gamma_{\text{had}} \).

**Forward-Backward Asymmetries:** At the tree level, it is possible to parametrize the angular distribution of \( f\bar{f} \) \((f \neq e)\) final states by

\[
\frac{d\sigma}{d\cos \theta} (e^+e^- \to \bar{f}f) \simeq (1 + \cos^2 \theta) \cdot F_1 + 2 \cos \theta \cdot F_2
\]

(5)

One can then define the forward-backward asymmetry

\[
A_{FB} \equiv \frac{f^1_f - f^0_{\bar{f}}}{f^1_f + f^0_{\bar{f}}} = \frac{3 \cdot F_2}{4 \cdot F_1}
\]

(6)

which has the value \( 3(1 - 4 \sin^2 \theta_W)^2 \) for \( \mu^+\mu^- \) and \( \tau^+\tau^- \). This measurement is particularly free of systematic detector effects and is limited essentially by statistics.

**Final-State \( \tau \) Polarization:** The heavy lepton \( \tau \) analyzes its own polarization when it decays, which can be measured in a number of hadronic and leptonic final states. At the tree level in the Standard Model, the \( \tau \) polarization is given by

\[
P_\tau = \frac{2(1 - 4 \sin^2 \theta_W)}{1 + (1 - 4 \sin^2 \theta_W)^2}
\]

(7)

This is a particularly sensitive way of measuring \( \sin^2 \theta_W \), though again limited by statistics.

**Polarized-Beam Asymmetry:** If a longitudinally-polarized electron beam is available, as at the SLC, one can measure the total cross-section asymmetry

\[
A_{LR} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \frac{2(1 - 4 \sin^2 \theta_W)}{1 + (1 - 4 \sin^2 \theta_W)^2}
\]

(8)

where \( L \) and \( R \) label the different electron helicities. The electron and positron beams circulating at LEP have a natural transverse polarization, which is useful for calibrating the
beam energy and hence measuring the Z mass and width, as discussed shortly, but there are no plans at CERN to rotate the beam polarization to the longitudinal direction.

The precision electroweak measurements from high-energy experiments at CERN [4], SLAC [12] and Fermilab [14], are summarized in Table 1. Particularly notable is the high precision \(2 \times 10^{-5}\) with which the Z mass is measured. The other LEP measurements are also considerably more precise than was thought possible before LEP started operation [10]. The latest value for the total number of neutrino species is [4]:

\[ N_{\nu} = 2.991 \pm 0.016 \]  

(9)

I had always hoped that this number would turn out to be non-integer, such as \(\pi\) or (even better) \(e\), reflecting the presence of exotic physics, but this was not to be.

| \(M_Z\)          | 91.1884 ± 0.0022 GeV |
|-------------------|-----------------------|
| \(\Gamma_Z\)      | 2.963 ± 0.032 GeV     |
| \(\sigma_h^0\)    | 41.488 ± 0.078 nb     |
| \(R_L\)           | 20.788 ± 0.032        |
| \(A_{FB}^+\)      | 0.0172 ± 0.0012       |
| \(A_r\)           | 0.1418 ± 0.0075       |
| \(A_\ell\)        | 0.1390 ± 0.0089       |
| \(R_\nu\)         | 0.2219 ± 0.0017       |
| \(R_c\)           | 0.1543 ± 0.0074       |
| \(A_{FB}^-\)      | 0.0999 ± 0.0031       |
| \(A_{FB}^\prime\) | 0.0725 ± 0.0058       |
| \(\sin^2 \theta_{\text{eff}}(Q_{FB})\) | 0.2325 ± 0.0013 |
| \(M_W\)           | 80.26 ± 0.16 GeV      |
| \(\sin^2 \theta_{\text{eff}}(A_{LR})\) | 0.23049 ± 0.00050 |

As already mentioned, the transverse polarization of the LEP beams is useful to calibrate the beam energy, because the polarization disappears at certain resonance energies which are determined by the electron’s anomalous magnetic moment. Using this technique, it has been possible to measure the LEP beam energy with a precision better than 1 MeV [15]. When this was first done, it was discovered that the beam energy varied systematically by 10 MeV or more, considerably more than the quoted error in the Z mass. Over time, these variations in the LEP beam energy have become better understood, and reveal many subtle and amusing effects, in addition to banal effects associated with the temperature and humidity in the LEP tunnel. For example, as seen in Fig. 1, the energy of the LEP beam is correlated with the positions of the Sun and Moon [13], which exert tidal effects on the rock in which the LEP ring is embedded, causing it to expand and contract, which the tuning of the machine converts into a variation in its energy. Even after this effect was taken into account, significant variations in the size of the LEP ring were detected, as seen in Fig. 1. Most of the variation in 1993...
turned out to be correlated with the height of the water table inside the Jura mountains [16]: water in the rock causes it to expand, carrying LEP with it. However, this was not responsible for the variations seen in the first part of 1994. These were largely explained by the Swiss policy of emptying Lake Geneva in the spring, to make room for the run-off water from the melting snows in the mountains. The rock surrounding LEP expands during the months after the burden of this water is released, much as Scandinavia is still rising after the Ice Age.

Another bizarre effect that has been identified very recently is that of electric trains on the nearly railway line from Geneva to France. Not all of the return current passes through the rails, but some passes through the earth, and in particular through the LEP ring, which is a relatively good conductor. This can produce changes in the LEP magnets corresponding to a shift of several MeV in the beam energy, as seen in Fig. 2 [17]. The “TGV effect” on the LEP determination of the $Z^0$ mass remains to be evaluated, but seems unlikely to affect significantly the LEP determination of the $Z^0$ decay width [18].

Figure 3 shows the implications of some of the precision measurements in Table 1 for the couplings of the $Z^0$ to charged leptons. We see that $g_A$ is close to the value -1/2 predicted in the Standard Model at the tree level, while $g_V$ is significantly different from zero, as expected in the Standard Model with $\sin^2 \theta_W < 1/4$. Figure 3 also shows predictions in the Standard Model for different values of the top-quark and Higgs-boson masses $m_t$ and $M_H$. As was pointed out by Veltman [19] in particular, the precision electroweak measurements in Table 1 are sensitive to quantum corrections associated with unseen particles. For example, at the one-loop level, the $W$ and $Z$ masses are given by [10]

$$m_W^2 \sin^2 \theta_W = m_Z^2 \cos^2 \theta_W \sin^2 \theta_W = \frac{\pi \alpha}{\sqrt{2} G_\mu} (1 + \Delta r)$$

(10)

The radiative correction $\Delta r$ receives an important corrections from the massive top quark. If it were absent, the third quark isospin doublet of the Standard Model would be incomplete, breaking gauge symmetry and destroying the renormalizability of the Standard Model. The quantity $m_t^2 - m_b^2$ is a measure of electroweak isospin breaking, which is sensed by precision electroweak measurements through the vacuum-polarization (oblique) diagram shown in the first part of Fig. 4. These make a contribution [19], [10]

$$\Delta r \gtrsim \frac{3G_\mu}{8\pi^2\sqrt{2}} m_t^2 \quad \text{for} \quad m_t \gg m_b.$$  

(11)

The Higgs boson also contributes to $\Delta r$. Again, the Standard Model would not be renormalizable if the gauge symmetry were broken explicitly, rather than spontaneously. In agreement with a screening theorem proved by Veltman [19], the sensitivity to the physical Higgs-boson mass provided by the last two diagrams in Fig. 4 is only logarithmic

$$\Delta r \gtrsim \frac{\sqrt{2} G_\mu}{16\pi^2} m_W^2 \left\{ \frac{11}{3} \ln \frac{M_H^2}{m_Z^2} \ldots \right\} \quad \text{for} \quad M_H \gg m_W$$

(12)

but experiments are now also sensitive [20] to the parameter $M_H$. 

5
Figure 5 shows the numerical sensitivity of $\Delta r$ to $m_t$ and $M_H$ \[10\]. A measured value of $\Delta r$ does not determine uniquely both $m_t$ and $M_H$, since a trade-off between their contribution is possible, but a combination of many different precision electroweak measurements does allow $m_t$ and $M_H$ to be disentangled. Global fits to the precision electroweak data now use many calculations \[21\] of higher-order effects going considerably beyond \((10), (11), (12)\).

Combining all the available precision electroweak data from LEP, SLC, Fermilab and low-energy $\nu q, eq, \mu q$ and $\nu e$ interactions, a global fit with $M_H$ left as a free parameter predicts \[20\]

$$m_t = 155 \pm 14 \text{ GeV}$$

(13)
as seen in Fig. 6. The contributions of the different sectors to the $\chi^2$ function are shown in Fig. 7. A somewhat higher value is obtained if the LEP data alone are used in the fit, and the central value of $m_t$ is increased substantially if $M_H$ is not left free, but is fixed at 300 GeV \[4\]. The indirect determination \((13)\) is consistent (within errors) with the mean value of the published CDF and D0 measurements \[3\]:

$$m_t = 181 \pm 12 \text{ GeV}.$$ (14)

It is therefore appropriate to make a combined fit of the direct and indirect measurements (which have comparable weights), yielding \[20\]

$$m_t = 172 \pm 10 \text{ GeV}.$$ (15)

With the recent discovery of the top quark at Fermilab, the “Mendelev table” of elementary matter constituents is apparently now complete. Now the fun starts, namely solving the problem of mass and finding the Higgs boson, or whatever replaces it.

3 The Electroweak Vacuum

It is generally accepted by theorists that generating the masses of the particles in the Standard Model requires a spontaneous breakdown of its gauge symmetry

$$m_{W,Z} \neq 0 \iff <0|X_{I_s}|0> \neq 0$$ (16)

where $X$ is some field with non-trivial isospin and a non-zero vacuum expectation value. Measurements of the $W$ and $Z$ masses

$$\rho = \frac{m_W^2}{m_Z \cos^2 \theta_W} \simeq 1$$ (17)

indicate that the field $X$ mainly has isospin $I = 1/2$ \[22\]. This is also what is required to give masses to the quarks and leptons in the Standard Model:

$$\lambda_f H_{I=1/2} \bar{f}_L f_R \Rightarrow m_f \bar{f}_L f_R$$ (18)
There is a general consensus on the above statements: however, you can start an argument when you discuss whether $X$ is elementary or composite.

The option chosen in the original formulation of the Standard Model by Weinberg and Salam \[1\] was that of an elementary Higgs boson: $<0|H^0|0> \neq 0$. This is fine at the classical tree level, but yields problems when you calculate quantum loops. Each individual one of the diagrams shown in Fig. 8 yields a quantum correction

$$\delta M_H^2 \simeq 0 \left( \frac{\alpha}{\pi} \right) \Lambda^2$$

(19)

where $\Lambda$ represents a cut-off in momentum space, above which the Standard Model is modified or replaced. As discussed in the next section, these quantum corrections may be reduced to $\lesssim m_H^2$ if one invokes \[5\] supersymmetry at an energy scale below 1 TeV.

The alternative option is to postulate that $X$ is composite, presumably a condensate of strongly-interacting fermion-antifermion pairs:

$$<0|\bar{FF}|0> \neq 0$$

(20)

by analogy with quark condensation in QCD, and the condensation of Cooper pairs in conventional superconductivity. Possible candidates for the strongly-interacting fermion $F$ include the top quark \[23\], which could be bound by strong Yukawa couplings if it were sufficiently heavy, or techniquarks $T$ \[24\] bound by new technicolour interactions at an energy scale of order 1 TeV.

The precision electroweak data reviewed in the previous section already provide us some indications on $M_H$ \[20\], which seem to disfavour the available composite Higgs scenarios. The correlation between $m_t$ and $M_H$ seen in Fig. 5 is weakened when one makes a global fit to all the high- and low-energy data, as seen in Fig. 9. Indeed, as seen in Fig. 10, a global fit provides a $\chi^2$ function which looks Gaussian as a function of $\log M_H$, even before the direct CDF and $D0$ measurements of $m_t$ are included. As also seen in Fig. 9, the data prefer a relatively light Higgs boson. If we do not include the direct measurements of $m_t$, we find \[20\]

$$M_H = 36^{+56}_{-22} \text{ GeV}$$

(21)

which becomes

$$M_H = 76^{+152}_{-50} \text{ GeV}$$

(22)

if the direct Fermilab measurements \[2\] are included. The range in Eq. (22) can be rephrased as

$$\log_{10} \left( \frac{M_H}{M_Z} \right) = -0.08^{+0.48}_{-0.46}$$

(23)

which is perhaps more appropriate in view of the logarithmic sensitivity to $M_H$. As seen in Fig. 11, this preference for a relatively light Higgs boson has been a consistent trend for several years \[23\]. Moreover, it is now confirmed by several other recent global fits to the available electroweak data \[1\], \[24\]. As discussed in more detail in the next section, the preferred value of $M_H$ is highly consistent with the range predicted in the minimal supersymmetric extension
of the Standard Model (MSSM). Independently of this theoretical prejudice, I now offer 3-to-1 odds that $M_H < 300$ GeV.

The relatively low value Eq. (21) and (22) bodes ill for a composite Higgs model. Minimal scenarios for $t\bar{t}$ condensation based on a Nambu-Jona Lasinio model

$$\mathcal{L}_{NJL} = \bar{\psi}a D\psi^a + \frac{1}{2} G \left[ (\bar{\psi}a\psi^a)^2 - (\bar{\psi}a\gamma^5\psi^a)^2 \right]$$

(24)

correspond to a reformulation of the Standard Model with constraints that lead to

$$M_H \simeq (1 \text{ to } 2)m_t : m_t \sim 200 \text{ to } 250 \text{ GeV} \quad (25)$$

Neither of these predictions agrees well with experiment, in particular the top quark appears to be too light. This has not completely discouraged would-be top-quark condensers, some of whom are postulating epicycles such as supersymmetry and/or an extension of the Standard Model gauge group [27].

Technicolour [24] would be able to provide masses for the $W$ and $Z$ with just one isospin doublet of techniquarks $T$, whose new gauge interactions would become strong at an energy scale

$$\Lambda_{TC} \simeq 3000\Lambda_{QCD} \quad (26)$$

However, this minimal model requires some extension if it is to provide fermion masses, and the conventional scenario [28] discussed is a model with one technigeneration:

$$(\nu, e) \quad (u, d) \quad (N, E)_{1,\ldots, N_{TC}} \quad (U, D)_{1,\ldots, N_{TC}} \quad (27)$$

This model has long had potential problems with light charged technipions and a possible flavour-changing neutral interactions, which have motivated variants such as “walking” technicolour [29]. The miseries of this model have been compounded by recent precision electroweak data.

The quantum effects of a large class of extensions of the Standard Model which add new isospin representations, including the above-mentioned technicolour model, can largely be characterized by their effects on three combinations of bosonic vacuum polarizations [30]:

$$T \equiv \frac{\epsilon_1}{\alpha} = \Delta\rho \quad \Delta\rho = \frac{\pi_{XX}(0)}{m_Z^2} - \frac{\pi_{WW}(0)}{m_W^2} - \tan \theta_W \frac{\pi_Z(0)}{m_Z^2}$$

$$S \equiv \frac{4\sin^2\theta_W}{\alpha} \epsilon_3 , \quad U \equiv -\frac{4\sin^2\theta_W}{\alpha} \epsilon_2 \quad (28)$$

in the Standard Model, the leading behaviours of $T$ and $S$ are

$$T = \frac{3}{16\pi} \frac{1}{\sin^2\theta_W \cos^2\theta_W} \frac{m_t^2}{m_Z^2} - \frac{3}{16\pi \cos^2\theta_W} \ln \left( \frac{M_H^2}{M_Z^2} \right) + \ldots$$

$$S = \frac{1}{12\pi} \ln \left( \frac{M_H^2}{M_Z^2} \right) + \ldots \quad (29)$$
as functions of $m_t$ and $M_H$. The previous constraints on $m_t$ and $M_H$ may be regarded, alternatively, as bounds on new physics contributions to $S, T, U$ \cite{30, 31}. Figure 12 shows as an example one analysis of the constraints on these variables found \cite{32} in a global fit, in which a fourth parameter $\epsilon_b$ is introduced to parametrize quantum corrections to the $Zb\bar{b}$ vertex \cite{31}. Also shown in Fig. 12 are the Standard Model predictions, shown as a grid for different values of $m_t$ and $M_H$, and the range of possible predictions in a minimal one-family technicolour model with $N_{TC} = 2$. The technicolour model is apparently disfavoured, but some possible modifications of its predictions could be envisaged \cite{32}, as indicated by the arrows in Fig. 12. Discarding for the moment these possibilities, and disregarding the uncalculable possibility of “walking” technicolour \cite{29}, Fig. 13 shows the price that one must pay in order to reconcile a minimal technicolour model with precision electroweak data.

It seems that the Higgs boson is likely to be relatively light, in apparent conflict with the available strongly-interacting models. The indications on $M_H$ presently available are likely to become strengthened during the coming decade \cite{33}, as seen in Fig. 14. We may even discover the Higgs! As seen in Fig. 15, the LEP2 accelerator now starting to provide data should enable us to explore Higgs masses up to about 95 GeV \cite{34}. This already covers much of the range favoured by the present data shown in Fig. 9, and also explores much of the MSSM parameter space, as discussed in the next section.

4 Motivations for Supersymmetry

Supersymmetry \cite{35} is a beautiful theory, but the motivations for it to appear at accessible energies are related to the problem of mass mentioned above, namely the origin of the hierarchy of mass scales in physics, and its naturalness in the presence of radiative corrections \cite{5}. The question why $m_W$ is much less than $m_{Planck}$ or $m_{GUT}$ can be rephrased as the question: why is $G_F \lesssim G_N$, or even why the Coulomb potential inside an atom is much stronger than the Newtonian potential:

$$\frac{e^2}{r} \lesssim G_N \times \frac{m^2}{r}$$

This hierarchy is valuable to radiative corrections. We say that a theory is natural if the radiative corrections are not much larger than the physical values of observable quantities. For example, the leading one-loop correction to a fermion mass takes the form

$$\delta m_f = 0 \left(\frac{\alpha}{\pi}\right) m_f \ln \left(\frac{\Lambda}{m_f}\right)$$

which is not much larger than $m_f$ for any reasonable cut-off $\Lambda \gtrsim m_P$.

Naturalness is, however, a problem for an elementary Higgs boson, which in the electroweak sector of the SM must have a mass

$$m_H = m_W \times 0 \left(\frac{\alpha}{\pi}\right)^{0\pm1}.$$
As already mentioned, the one-loop diagrams shown in Fig. 8 lead to “large” radiative corrections of the form
\[ \delta m_{H,W}^2 \simeq 0 \left( \frac{\alpha}{\pi} \right) \Lambda^2. \] (33)
These are much larger than the physical value \( m_H^2 \) for a cut-off \( \Lambda \), representing the scale at which new physics appears, of order \( m_P \) or \( m_{\text{GUT}} \).

Supersymmetry solves the naturalness problem of an elementary Higgs boson by virtue of the fact that it has no quadratic divergences and fewer logarithmic divergences than non-supersymmetric theories. The diagrams shown in Fig. 8 have opposite signs, so that their net result is
\[ \delta m_{W,H}^2 \simeq - \left( \frac{g_F^2}{4\pi^2} \right) (\Lambda^2 + m_F^2) + \left( \frac{g_B^2}{4\pi^2} \right) (\Lambda^2 + m_B^2). \] (34)
The leading divergences cancel if there are the same numbers of bosons and fermions, and if they have the same couplings \( g_F = g_B \), as in a supersymmetric theory. The residual contribution is small if supersymmetry is approximately valid, i.e., if \( m_B \simeq m_F \):
\[ \delta m_{W,H}^2 \simeq 0 \left( \frac{\alpha}{\pi} \right) |m_B^2 - m_F^2| \] (35)
which is no larger than \( m_{W,H}^2 \) if
\[ |m_B^2 - m_F^2| \lesssim 1 \text{ TeV}^2 \] (36)
This property provides the first motivation for supersymmetry at low energies. However, it must be emphasized that this is a qualitative argument which should be regarded as a matter of taste. After all, an unnatural theory is still renormalizable, even if it requires fine tuning of parameters. A second supersymmetric miracle is the absence of many logarithmic divergences: for many Yukawa couplings and quartic terms in the effective potential,
\[ \delta \lambda \propto \lambda \] (37)
which vanishes if the rare coupling \( \lambda = 0 \). The combination of Eqs. (35) and (37) means that if \( M_W \leq M_P \) at the tree level, it stays small in all orders of perturbation theory, solving the naturalness problem and providing a context for attacking the hierarchy problem [5].

The latter is particularly acute in theories with both large and small scales, in which the former may “leak” and contaminate the latter [36]. Consider for example a Grand Unified Theory with two sets of Higgs bosons, \( H \) with a large vacuum expectation value \( V_{\text{GUT}} \) and \( h \) with a small vacuum expectation value \( v_{\text{EW}} \). In a generic Grand Unified Theory, there will be a quartic coupling \( \lambda hhHH \), which yields
\[ \delta m_H^2 \simeq \lambda \cdot V_{\text{GUT}}^2 \] (38)
which is a large and potentially disastrous contribution to the light Higgs mass. Even if \( \lambda = 0 \) at the tree level (why? this is the hierarchy problem), radiative corrections will regenerate a non-zero coupling, so that
\[ \delta m_H^2 \simeq 0 \left( \frac{\alpha}{\pi} \right)^2 V_{\text{GUT}}^2 \] (39)

Such contributions need to be suppressed to many orders of perturbation theory, which requires a powerful symmetry, such as supersymmetry. It is also worth pointing out that it has been argued that quantum gravity effects may also generate a large shift in the mass of an elementary Higgs boson

\[ \delta m^2_H = 0(m_P^2) \] (40)

although to be sure of this, one needs a consistent quantum theory of gravity. Effects such as (40) are likely to be absent in a supersymmetric theory, and, in any case, the only consistent quantum theory of gravity which we possess is string theory, which is difficult or impossible to formulate consistently without supersymmetry.

5 Model Building

Now that we are motivated to construct a supersymmetric model, the first question is whether the known fermions \((q, \ell)\) could be the supersymmetric partners of the “known” bosons \((\gamma, W, Z, H, g)\)? As was first pointed out by Fayet, the answer is not for phenomenology, since their quantum numbers do not match. For example, the quarks \(q\) appear in \(3\) representations of \(SU(3)_c\), whereas the bosons appear in \(1\) and \(8\) representations. Likewise, the leptons \(\ell\) have non-zero lepton number, whereas all the bosons have zero lepton number. As a result, one must introduce supersymmetric partners for all the known particles, as shown in Table 2. You may not appreciate the economy in particles, but you should appreciate the economy of the supersymmetric principle.

| J particle | J sparticle | J squark |
|------------|-------------|---------|
| \(q_{L,R}\) | \(1/2\) \(\tilde{q}_{L,R}\) | 0 |
| \(\ell_{L,R}\) | \(1/2\) \(\tilde{\ell}_{L,R}\) | 0 |
| \(\gamma\) | 1 \(\tilde{\gamma}\) | 1/2 |
| \(Z\) | 1 \(\tilde{Z}\) | 1/2 |
| \(W^{\pm}\) | 1 \(\tilde{W}^{\pm}\) | 1/2 |
| \(H^{\pm,0}\) | 0 \(\tilde{H}^{\pm,0}\) | 1/2 |

- Table 2 -

You may wonder whether, if \(N = 1\) supersymmetry is good, perhaps \(N > 1\) supersymmetry is better? The answer is: not for phenomenology, because such a theory cannot accommodate chiral fermions. The available \(N = 2\) supermultiplets are

\[
\begin{pmatrix}
\frac{1}{2} \\
0, 0 \\
-\frac{1}{2}
\end{pmatrix}
\oplus
\begin{pmatrix}
\frac{1}{2} \\
1, 1 \\
-\frac{1}{2}
\end{pmatrix}
\]

in which the fermions of helicity \(+1/2\) have the same internal quantum numbers as the fermions of helicity \(-1/2\), making it impossible to accommodate the parity violation seen in the electroweak interactions.
The starting point for any discussion of supersymmetric phenomenology is the minimal supersymmetric extension of the Standard Model \[39\], which has the same gauge interactions as the Standard Model, and whose Yukawa interactions are derived from the following superpotential, which is written as a holomorphic function of left-handed superfields:

\[
W = \sum_{L,E} \lambda_L LE^c H_1 + \sum_{Q,U} \lambda_U QU^c H_2 \\
+ \sum_{Q,D} \lambda_C QD^c H_1 + \mu H_1 H_2
\]  

(42)

Here \(L\) and \(Q\) denote left-handed quark and lepton doublets, respectively and \(E^c\), \(U^c\), \(D^c\) denote the conjugate lepton and quark singlets. The first term in Eq. (42) provides masses for the charged leptons:

\[
m_L = \lambda_L v_1
\]

and the next two provide masses for the charged 2/3 and charged -1/3 quarks, respectively:

\[
m_u = \lambda_u v_2, \quad m_d = \lambda_d v_1
\]

(44)

Notice that two Higgs doublets \(H_{1,2}\) are needed in order to preserve the holomorphy of the superpotential \(W\), and to cancel out axial \(U(1)\) current anomalies, and that the fourth term in \(W\) accommodates mixing between the two Higgs doublets. The Yukawa and gauge couplings in the MSSM make a supersymmetric contribution to the effective potential of the form:

\[
V = \sum_i |F_i|^2 + \frac{1}{2} \sum_a (D^a)^2
\]

(45)

where

\[
F_i^* = \frac{\partial W}{\partial \phi^i}, \quad D_a = g_a \phi^i (T^a)^i_0 \phi^j
\]

(46)

are the conventional \(F\) and \(D\) terms, respectively. The fact that the quartic terms in the effective potential are so constrained provides restrictions on the supersymmetric Higgs boson masses, as will be discussed later.

As you may well imagine, all the searches for supersymmetric particles have been unsuccessful so far, and have provided the following approximate lower limits on some of their masses

\[
m_{\tilde{\ell}, \tilde{\mu}, \tilde{\tau}, \tilde{W}} \gtrsim 45 \text{ GeV} \quad [40], \quad m_{\tilde{q}, \tilde{g}} \lesssim 150 \text{ GeV} \quad [4]
\]

(47)

The LEP2 energy upgrade will provide us with access to a new range of sparticle masses, and continuation of the Fermilab proton-antiproton collider will increase the search range for squarks and gluinos. Why do we phenomenologists keep the faith that supersymmetric particles will eventually be found, despite the lack of direct experimental evidence?

In addition to the theoretical motivations for supersymmetry, there are two tentative and indirect experimental motivations provided by the precision electroweak data, which come mainly from LEP. One is provided by the previously-mentioned indication that the Higgs boson is “probably” light: \(M_H \lesssim 300 \text{ GeV}\), which is consistent with the MSSM expectation that

\[
m_h \simeq m_Z \pm 40 \text{ GeV}
\]

(48)
As mentioned above, the MSSM contains two Higgs doublets

\[ H_2 = \left( \begin{array}{c} H_2^+ \\ H_2^0 \end{array} \right), \quad H_1 = \left( \begin{array}{c} H_1^0 \\ H_1^- \end{array} \right) \]  

which contain a total of eight real degrees of freedom. Three of these are “eaten” by the \( W^\pm \) and the \( Z^0 \) to yield their masses, leaving five physical Higgs bosons to be discovered by experiment. Three of these are neutral, the scalars \( h, H \) and the superscalar \( A \), and two are charged \( H^\pm \). At the tree level, all the masses and couplings of these Higgses are controlled by two parameters, which may be taken as \( (m_A, \tan \beta \equiv v_2/v_1) \):

\[
\begin{align*}
m_h^2 + m_H^2 &= m_A^2 + m_Z^2 \\
m_{H^\pm}^2 &= m_A^2 + m_W^2 \\
m_{h,H}^2 &= \frac{1}{2} \left[ m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 + m_Z^2)^2 - 4m_A^2m_Z^2\cos^22\beta} \right]
\end{align*}
\]  

(50)

In particular, the lighter scalar Higgs \( h \) was guaranteed to be lighter than the \( Z \), which was good news for LEP2. However, radiative corrections associated in particular with the heavy top quark \[42\]

\[
\delta m_h^2 \propto \frac{m_t^4}{m_W^2} \ln \left( \frac{m_Z^2}{m_t^2} \right)
\]  

(51)

increase the upper limit to

\[ m_h \gtrsim 130 \text{ GeV} \]  

(52)

As seen in Fig. 16, it is still true that much of the \( h \) mass range will be explored at LEP2 \[34\], but, alas, not all of it.

As already mentioned, the range \(52\) is highly consistent with the indirect indications from the precision electroweak data on the possible mass of the Higgs. One can even go further, and argue that the LEP data slightly favour the MSSM over the Standard Model \[20\]. In the latter, the requirement that all the couplings remain finite in the energy range \( E \ll \Lambda_P \) impose an upper limit on the Higgs mass. If the Standard Model is to remain valid all the way up to \( \Lambda_P \sim m_{\text{GUT}} \) or \( M_P \), then \( M_H \lesssim 200 \text{ GeV} \) as seen in Fig. 17 \[13\]. On the other hand, the (meta)stability of the electroweak vacuum imposes a lower limit which depends on the scale \( \Lambda_V \) up to which the effective Higgs potential is assumed to be reliable, as also seen in Fig. 17 \[44\]. Thus, the Standard Model as we know it is consistent with only a small range of Higgs masses

\[ 116 \text{ GeV} \lesssim M_H \lesssim 190 \text{ GeV} \]  

(53)

for \( \Lambda_P = \Lambda_V = 10^{19} \text{ GeV} \) and \( m_t \simeq 172 \text{ GeV} \) as found in the previous global fit. This is to be contrasted with the range

\[ 50 \text{ GeV} \lesssim m_h \lesssim 124 \text{ GeV} \]  

(54)

allowed in the MSSM for the same value of \( m_t \). According to Fig. 18, which is deduced from the \( \chi^2 \) function in Fig. 9, the apparent probabilities of these mass ranges are about 18 and 36 \%, respectively \[20\]. Therefore I offer another bet: I offer 2-to-1 odds on the MSSM!
The second indirect indication in favour of supersymmetry is provided by the well-publicized consistency of the measurements at the Standard Model gauge couplings $\alpha_{1,2,3}$ with the predictions of minimal supersymmetric GUTs [45]. Ever since 1987 [46], [47], but with a statistical strength which has increased greatly with the advent of LEP data, the prediction for $\sin^2 \theta_W$ in a minimal non-supersymmetric GUT [48]:

$$\sin^2 \theta_W(m_Z) \bigg|_{\text{MS}} = 0.208 + 0.004(N_H - 1) + 0.006 \ln \left( \frac{400 \text{ MeV}}{\Lambda(\text{MS})} \right)$$

$$= 0.214 \pm 0.004 \text{ for } \Lambda(\text{MS}) = 200 \text{ to } 800 \text{ MeV}$$

has been in conflict with data, which now indicate [49]

$$\sin^2 \theta_W(m_Z) \bigg|_{\text{MS}} = 0.2312 \pm 0.0003$$

The prediction for $\sin^2 \theta_W$ is less precise, even in the minimal supersymmetric GUT, because it contains more parameters, and it is not possible at present to use this consistency to provide meaningful constraints on the possible masses of supersymmetric particles [50]. Nevertheless, we are encouraged to believe that supersymmetry may lie “just around the corner”, which means either at LEP2 or at the LHC, as we now discuss.

6 Physics with the LHC

This accelerator [51] provides us with our best prospect for exploring the 1 TeV energy region, where we may expect to find the Higgs boson and supersymmetry. The LHC offers several possibilities for colliding different types of particle. Of most interest for new particle searches is its proton-proton collider mode, which will have a centre-of-mass energy of up to 14 TeV, and a luminosity of up to $10^{34} \text{ cm}^{-2} \text{ sec}^{-1}$. Also possible are heavy-ion collisions with nuclei up to lead: used as a lead-lead collider, the LHC would have a centre-of-mass energy up to 1.2 PeV and a luminosity of up to $10^{27} \text{ cm}^{-2} \text{ sec}^{-1}$, whilst the luminosity could be higher if lighter calcium is used. It will also be possible to use the LHC as an electron-proton, proton-nucleus or electron-nucleus collider, if the mood so takes us. The LHC was approved by the CERN Council at the end of 1994, to start doing physics in the 2004. For reasons of cash flow, the initial approval was for a machine with fewer magnets, able to reach a centre-of-mass energy of 10 TeV to start with. However, if non-member states contribute significantly, it may be possible to start immediately at the full design energy of 14 TeV: the final machine schedule and energy will be decided at a review in 1997.

The initial LHC experimental programme is expected to include the following four experiments: ATLAS [52] and CMS [53], which are large general-purpose experiments for discovery physics in proton-proton collisions, ALICE [54], which is primarily intended for heavy-ion experiments searching for the quark-gluon plasma, though it may also be used to look for diffractive scattering, and LHC-B [55], an experiment designed primarily to look for CP violation in the decays of B mesons produced in proton-proton collisions.
The LHC accelerator will benefit fully from the existing CERN infrastructure, since it will be built inside the existing LEP tunnel, and will receive particles which have been pre-accelerated by the other CERN accelerators. The LHC magnets are of a very ambitious design, with a high magnetic field above 9 Tesla and two magnetic channels carrying beams circulating in opposite directions. Successful tests have been made with the first magnetic prototypes, indicating that the maximum design energy should be reachable, and may even be exceeded. The ALICE and LHC-B experiments will be placed in underground pits which have already been dug for two of the LEP experiments, but the ATLAS and CMS experiments will require two very large new pits. These and tunnels for transferring the proton and heavy-ion beams from the lower-energy SPS accelerator are the main pieces of civil engineering that will be required. It just so happens that one of the beam transfer lines points in the direction of Italy and Greece, where neutrino detectors are now being built that could be used for long baseline neutrino experiments [56], using neutrinos produced by a CERN proton beam, but it has not yet been decided whether this will be included in the LHC programme.

Top of the LHC physics agenda will be the search for the Higgs boson, which should have a mass below about 1 TeV, indeed below about 300 GeV if one believes the indirect indications from precision electroweak experiments [20], even 90 ± 40 GeV if the MSSM is correct [12]. A Standard Model Higgs boson will be detectable at LEP2 as soon as the centre-of-mass energy is increased a few GeV above \( M_Z + M_H \) [34]. Since the maximum LEP2 centre-of-mass energy is expected to be about 192 GeV, this means that a Higgs weighing more than about 95 GeV will be prey for the LHC. The Standard Model Higgs boson will be detectable at the LHC by its decay into \( \gamma \gamma \) if \( M_H < 140 \) GeV, by its decay into \( 4\ell^\pm \) if \( 130 \) GeV \( \leq M_H \leq 700 \) GeV, and by its decay into \( \ell^+\ell^-\nu\bar{\nu} \) if \( 700 \) GeV \( \leq M_H \leq 1 \) TeV. Figure 19 summarizes the Higgs discovery significance that is expected by combining the ATLAS and CMS experiments [52], [53]. The vertical axis is the number of signal events \( S \) divided by the statistical fluctuation in background \( \sqrt{B} \). Discovery can be claimed if \( S/\sqrt{B} > 5 \), which is seen from Fig. 19 to be the case for the full range 90 GeV \( \leq M_H \leq 1 \) TeV.

The search for one or more MSSM Higgs bosons is more complicated, because the product of the production cross-section and the observable decay branching ratio is often smaller than in the Standard Model, and there are several Higgs bosons to be found with a number of different signatures. Figure 20 summarizes incomprehensibly the overall prospects for the supersymmetric Higgs search at the LHC [54], [55], [57]. Regions on the shaded side of each solid line can be explored by the LHC. Diligent examination of the two-dimensional parameter space will not reveal any region where discovery is impossible. We therefore conclude that the LHC will be able to prove or disprove the MSSM via its Higgs sector alone.

Next on the LHC physics agenda will be the search for supersymmetric particles, which may well turn out to be the biggest banana of all. Figure 21 exhibits the expected cross-sections for producing pairs of gluinos and/or squarks at the LHC. These are expected to—have a high probability for decays with large missing transverse energy carried away by the lightest supersymmetric particle, which is expected to be a weakly-interacting neutral particle analogous to the neutrino, but heavier. This missing transverse-energy signature is expected to be observable even if the squarks and gluinos decay in a cascade through various intermediate states before
arriving at the lightest supersymmetric particle. Figure 22 shows that the supersymmetric signal is expected to stick out above the irreducible total Standard Model background and the background due to experimental imperfections, if the gluino and squark each weigh 1.5 TeV and one looks for events with more than 300 GeV of missing transverse energy. The ATLAS and CMS collaborations have concluded that they should be able to detect squarks and gluinos weighing anything up to about 2 TeV, which includes all the range motivated by the naturalness and hierarchy arguments presented previously.

We conclude optimistically that, within ten years or so, experiments at the LHC will be able to confirm or refute supersymmetry, if this has not already been done by LEP2 and/or the Fermilab proton-antiproton collider. Therefore we may soon know the answer to the question whether supersymmetry is just a beautiful holomorphic theory, or also a part of physics.

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Figure Captions

Fig. 1 Sensitivity of the LEP beam energy to (a) tides [13]: the solid lines are due to a tidal model, (b) the water table in the Jura mountains and (c) the level of Lake Geneva [16].

Fig. 2 (a) The “TGV effect” on the LEP beam energy [17], due (b) to the vagabond current from electric trains returning via the LEP ring.

Fig. 3 The values of the vector and axial couplings of leptons ($g_V, g_A$), as extracted from LEP data [4].

Fig. 4 Vacuum-polarization (oblique) diagrams contributing to the one-loop radiative corrections.

Fig. 5 The numerical sensitivity of $\Delta r$ (10) to $m_t$ (11) and $M_H$ (12). A determination of $\Delta r$ alone cannot fix both $m_t$ and $M_H$.

Fig. 6 The dependence of the $\chi^2$ function on $m_t$ for various assumed values of $M_H$, from a recent global fit [20].

Fig. 7 The contributions of the different sectors of precision electroweak data to the $\chi^2$ function shown in Fig. 6 [20].

Fig. 8 Quadratically-divergent contributions to $\delta M^2_H$.

Fig. 9 Combined fit to all precision electroweak data in the ($M_H, m_t$) plane, including (solid lines) or not (dashed lines) the direct determination of $m_t$ by CDF/D0 (error bar on the left) [2]. The contours correspond to $\Delta \chi^2 = 1, 4$ around the minimum (small circle) in either case. Notice that $M_H$ is significantly below 300 GeV at the 1\(\sigma\) level, and below 1 TeV at the 2\(\sigma\) level [20].

Fig. 10 The values of $\chi^2$ as functions of $M_H$ for the various indicated values of $m_t$ [20].

Fig. 11 $\Delta \chi^2 = 1$ ranges for $m_H$ in a series of global fits to the available precision electroweak data [20].

Fig. 12 Comparison of the Born approximation (stars), projections of the $\Delta \chi^2 = 1, 4$ ellipsoid (solid ellipses), the SM (grid) and the predictions of a one-generation TC model with $N_{TC} = 2$, a Dirac technineutrino, $M_U = M_D$, 100 GeV $< M_E < 600$ GeV, 50 GeV $< M_N < M_E$ and the technicolour parameter [24] $\xi > 1/2$ (scattered dots). The TC predictions are added to the SM radiative corrections, using the reference values $m_t = 170$ GeV and $M_H = M_Z$. Noted that the TC predictions are further than the SM from the experimental data. The bold arrows labelled TQ and B indicate possible shifts in the TC predictions of definite sign, and the other (thin) arrows labelled B and NC indicate shifts that are less certain.
Fig. 13 Contours of \( \sigma \equiv \sqrt{\Delta \chi^2} \) for one-generation models with either Dirac technineutrinos (a), (b) or Majorana technineutrino (c), (d). Note that \( \sigma \gtrsim 4.5 \) in all of the TC parameter space, to be compared with \( \sigma = 2.6 \) in the SM at the reference point \((m_t = 170 \text{ GeV}, M_H = M_Z)\). In the case of techniquark mass degeneracy \((M_U = M_D)\), the Dirac and Majorana models fits are comparable; in the case \(M_U > M_D\), however, the Dirac model becomes highly disfavoured. In all cases, \( \xi = 1/2 \) is assumed.

Fig. 14 Possible improvements in the precision with which \( M_H \) can be estimated by global fits to future precision electroweak data [33].

Fig. 15 The range of \( M_H \) accessible to LEP2 with a centre-of-mass energy of 192 GeV [34].

Fig. 16 Reach for Higgs bosons in the MSSM at LEP2 with a centre-of-mass energy of 192 GeV. The dark shaded regions are excluded theoretically [34].

Fig. 17 Comparison of combined top-Higgs mass fits in the Standard Model (SM, upper plot) and in its Minimal Supersymmetric extension (MSSM, lower plot), at \( \Delta \chi^2 = 1 \). The continuation of the \( \Delta \chi^2 = 1 \) contour below the LEP direct limit \( M_H > 65 \text{ GeV} \) [4] is shown dashed. Also shown in the SM plot are the lower limits on \( M_H \) from vacuum metastability [41] as a function of the “new physics” scale \( \Lambda_V = 10^4–10^{19} \text{ GeV} \) [43], and the upper limits that come from requiring the SM couplings to remain perturbative up to a scale \( \Lambda_P = 10^3–10^{19} \text{ GeV} \). In the MSSM plot, we show the intrinsic upper limits on the lightest Higgs mass for two values (2 and 16) of \( \tan \beta = v_2/v_1 \).

Fig. 18 The cumulative probability distribution calculated from the \( \chi^2 \) function in the SM shown in fig. 4, obtained after integrating appropriately over \( m_t \), including the direct measurements from CDF and D0 [4]. This may be used to estimate the relative probabilities of different Higgs mass ranges in the SM and the MSSM, as discussed in the text [20].

Fig. 19 The expected significance for a Standard Model Higgs boson in the ATLAS and CMS experiments at the LHC [52], [53]. The higher-mass range is also accessible up to about 1 TeV.

Fig. 20 Capability of ATLAS [51] and CMS [52] to explore the MSSM Higgs sector. The regions with shaded edges can be explored with the channels indicated. Also shown is the region accessible to LEP2. Between LHC and LEP2, essentially the entire plane is covered.

Fig. 21 Cross-sections for squark and gluino production at the LHC.

Fig. 22 Expected missing transverse energy signal for squarks and gluinos at the LHC, compared with the Standard Model and experimental backgrounds in ATLAS [52].
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