Conformal Unification of General Relativity and Standard Model

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Abstract

The unification of general relativity and standard model for strong and electro-weak interactions is considered on the base of the conformal symmetry principle. The Penrose-Chernikov-Tagirov Lagrangian is used to describe the Higgs scalar field modulus and gravitation. We show that the procedure of the Hamiltonian reduction converts the homogeneous part of the Higgs field into the dynamical parameter of evolution of the equivalent reduced system. The equation of dynamics of the “proper time” of an observer with respect to the evolution parameter reproduces the Friedmann-like equation, which reflects the cosmological evolution of elementary particle masses. The value of the Higgs field is determined, at the present time, by the values of mean density of matter and the Hubble parameter in satisfactory agreement with the data of cosmological observations.

1. Introduction.

The Standard Model (SM) for electroweak and strong interactions is almost established for phenomena up to 100GeV; one only needs to observe the Higgs particle in experiment and manage to include gravity into the unified theory. The conventional scheme [1] of the minimal coupling of the scalar field with gravity supposes naive adding of the General Relativity (GR) and SM, each of these having own dimensional parameters. In this scheme, there is a number of difficulties connected with the existence of a scalar mode of a nonvanishing vacuum expectation value in cosmology [1].

Another more fundamental way of the unification of GR and SM is to propose that there is only one universal dimensional parameter for all interactions and all regions of energies. First attempt to describe the Newton coupling constant in GR as the vacuum averaging of the same Higgs field were made in 1974 [2] with the idea that spontaneous symmetry breaking forms simultaneously the scale of masses in both GR and SM. (In the context of the change of the Newton coupling constant in GR by the scalar field, we should also recall the Jordan-Brans-Dicke scalar tensor theory [3].)

The next essential step on the way of decreasing the number of dimensional parameters in the Lagrangian of the unified theory of GR and SM was made in paper [4], where the conformal invariant unified theory was considered without any dimensional parameters. The theory represents the standard
model of strong and electro-weak interactions in which gravitation and the modulus of the Higgs scalar field are described by the Penrose-Chernikov-Tagirov Lagrangian [5], which has no any dimensional parameters.

In the present paper, we investigate the dynamics of a scalar field in the conformal unified theory (CUT) GR and SM [4, 6]. We use the Dirac-ADM parametrization of the metric [6, 7, 8] and the Lichnerowicz conformal invariant variables [10] constructed with the help of the space scale component of the initial metric.

2. Conformal Unification of GR and SM

The action of the conformal invariant theory of GR and SM is a sum of these theories

$$W_{CUT} = W_{PCT} + W_{cSM}^\chi,$$

(1)

where the Penrose-Chernikov-Tagirov (PCT) action $W_{PCT}$$

$$W_{PCT}(\varphi_{PCT}, g) = \int d^4x [-\sqrt{-g}R(g)\frac{\varphi_{PCT}^2}{6} + \varphi_{PCT}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi_{PCT})]$$

(2)

describes the metric and the PCT scalar field $\varphi_{PCT}$, and

$$W_{cSM}[\varphi_H, n, V, \psi, g] = \int d^4x \left( \mathcal{L}_{0}^{SM} + \sqrt{-g}[-\varphi_{H}F + \varphi_{H}^2B - \lambda\varphi_{H}^4] \right)$$

(3)

is the conformally invariant part of the SM action (i.e. the conventional SM action without the “free” part for the modulus of the Higgs $SU(2)$ doublet $\varphi_H$ and without the Higgs mass term), $B$ and $F$ are the mass terms of the vector $V$ and fermion $\psi$ fields, respectively

$$B = Dn(Dn)^*; F = (\bar{\psi}_L n)\psi_R + h.c.; \quad n = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}; \quad n_1^* + n_2^* = 1,$$

(4)

$n$ is the angular component of the Higgs $SU(2)$ doublet

The conformal symmetry of the Lagrangian [1] means that it is invariant with respect to simultaneous transformations of all fields in the theory, according to the rule

$$(n)f'(x) = (n)f(x)\Omega^n(x),$$

(5)

where $(n)$ is the conformal weight of the field $f$.

The main idea of the present paper (introduced in [4] and developed in [6]) is to identify the PCT scalar field with the modulus of the Higgs doublet within the rescaling factor $\chi$

$$\varphi_H = \chi\varphi_{PCT}.$$ 

(6)

The rescaling factor $\chi$ must be regarded as a new coupling constant, which coordinates weak and gravitational scales [4]. The value of rescaling factor is not predicted by the present theory and must be given by experiment.
3. Hamiltonian and Evolution Parameter

The Hamiltonian description, in general relativity, is achieved by the (3 + 1) foliation of the four-dimensional manifold \[ 5 \]
\[ (ds)^2 = g_{\mu\nu}dx^\mu dx^\nu = N^2dt^2 - (3) g_{ij}dx^i dx^j; \quad (dx^i = dx^i + N^i dt) \] (7)

Our model differs from the conventional Einstein theory by an additional local conformal symmetry \[ 3 \]. This symmetry allows us to eliminate one degree of freedom which is formally present in the Lagrangian but for which there is no dynamical equation of motion. We can choose the retransformation field parameter \( \Omega = \Omega_c \) in such a way that the space scale factor ||(3)\( g_c || \) is eliminated from the observable variables and the interval

\[ ||(3)g_c|| = \Omega_c^6||g|| = 1; \quad (ds)^2_c = N_c^2dt^2 - (3) g_{(c)ij}dx^i dx^j. \] (8)

Then, the space volume \( \int d^3x\sqrt{(3)g_c} = \int d^3x \) becomes an integral of motion. The new metric \( N_c, (3)g_c \) and new variables

\[ (n)f_c(x) = (n)f(x)\Omega^n_c(x); \quad \varphi_H_c = \varphi_H \Omega_c; \quad \varphi_c \overset{\text{def}}{=} (\varphi_{PCT})_c = \varphi_{PCT} \Omega_c \] (9)

coincide with the conformal variables introduced in GR by Lichnerowicz \[ 14 \], which are very convenient for studying the problem of initial data \[ 8, 9 \].

To extract physical information from the theory, we formulate the theory in terms of invariant dynamical variables. It is well known that as a result of such a formulation the angular components of the scalar fields (n) are absorbed by the physical vector fields \( V^p \) and \( \psi^p \) in the unitary gauge

\[ B^p = V_i^p \dot{Y}_i V_j^p; \quad F^p = \psi^p_\alpha \dot{X}_{\alpha\beta} \psi^p_\beta, \] (10)

where \( \dot{Y}, \dot{X} \) are the ordinary matrices of vector meson and fermion mass couplings in the WS theory multiplied by the rescaling parameters \( \chi^2 \) and \( \chi \), respectively.

In the first order formalism, the action \[ 11 \] in terms of the Lichnerowicz variables has the form

\[ W_{\text{CUT}}^E = [P_f, f; P_g, g^c, P_\varphi, \varphi_c] = \int \frac{t_2}{t_1} \int d^3x \left[ \sum_{f=g,\psi} \int_{g,\psi} P_f D_0 f - P_\varphi D_0 \varphi_c - N_c \mathcal{H} \right], \] (11)

where

\[ \mathcal{H} = -\frac{\dot{B}^2}{4} + \frac{g^2}{2\varphi_c^2} - \varphi_c^2 \dot{B} + \varphi_c F^p + \mathcal{H}_0^{SM} + \bar{\lambda} \varphi_c^4 \] (12)

is the Hamiltonian density, \( \dot{B} \) is a contribution of the potential part of bosonic fields \( (g_c, V^p) \)

\[ \dot{B} = B^p - \frac{1}{6}(3) R(g^{c}_{ij}) + 8\varphi_c^{-1/2} \Delta \varphi_c^{1/2}; \quad \Delta \varphi_c = \partial_k (g^c_{ij} \partial_j \varphi_c), \] (13)

\( B \) and \( F^p \) is given by \[ 10 \] and \( \bar{\lambda} = \chi^4 \lambda; \quad P_f, P_\varphi \) are the canonical momenta of the corresponding fields, for example

\[ D_0 \varphi_c = \partial_0 \varphi_c - \partial_k (N^k \varphi_c) + \frac{2}{3} \varphi_c \partial_k N^k, \quad D_0 g^{c}_{ij} = \partial_0 g^{c}_{ij} - \nabla_i N_j - \nabla_j N_i + \frac{2}{3} \partial_k g^{c}_{ij} N^k. \] (14)

These covariant derivatives multiplied by the factor \( dt \) are invariant under kinemetric transformations \[ 16 \]
\[ t \to t' = t'(t); \quad x^k \to x'^k = x'^k(t, x^1, x^2, x^3), \quad N \to N'... \] (15)
This invariance means that GR and CUT represent an extended systems (ES) with constraints and “superfluous” variables [15, 14, 11]. To separate the physical sector of invariant variables and observables from the parameters of general coordinate transformations, one needs the procedure of the Hamiltonian reduction, which leads to an equivalent unconstraint system, where one of “superfluous” variables becomes the dynamical parameter of evolution [14, 11].

The Hamiltonian reduction requires the evolution parameter of reduced system to be point out as one of the initial (superfluous) variables of the extended system. Such an evolution parameter in GR can be the global homogeneous component of the scale space factor [8, 9, 16, 11] with a negative sign of its kinetic term.

In our theory the role of the scale space factor is played by the scalar (Higgs) field $\varphi_c = \varphi_H/\chi$ (see (3) and (6)). Therefore, we extract the evolution parameter by splitting the Higgs field and lapse function into two factors: homogeneous (global) and local

$$\bar{\varphi}_c(x,t) = \varphi_0(t)a(x,t); \quad N_c(x,t) = N_0(t)N(x,t);$$ \hspace{1cm} (16)

the second factor $a(t,x)$, by definition, is constrained by the relation

$$\int d^3x a(x,t) \frac{D_0a(x,t)}{N_c} = 0,$$ \hspace{1cm} (17)

which diagonalizes the kinetic term of the action (11). To get the conventional canonical structure for the new variables

$$\int d^3x (\bar{P}_\varphi D_0\bar{\varphi}_c) = \varphi_0 \int d^3x \bar{P}_\varphi a + \varphi_0 \int d^3x \bar{P}_\varphi D_0a = \varphi_0 P_0 + \int d^3x P_a D_0a,$$ \hspace{1cm} (18)

we define decomposition of $\bar{P}_\varphi$ over the new momenta $P_0$ and $P_a$ conjugated to the new variables (19)

$$\bar{P}_\varphi = \frac{P_a}{\varphi_0} + P_0 \frac{a}{NV_0}; \quad \left( \int d^3x a(x,t) P_a \equiv 0, \quad V_0 = \int d^3x \frac{a^2}{N} \right).$$ \hspace{1cm} (19)

The substitution of (19) into the Hamiltonian part of the action (11) extracts the “superfluous” momentum term

$$\int d^3x N_c H = N_0 \left[ -\frac{P_0^2}{4V_0} + H_f \right].$$ \hspace{1cm} (20)

Finally, the extended action (11) is

$$W_E[P_f, f; P_0, \varphi_0 | t] = \int_{t_1}^{t_2} dt \left( \int d^3x \sum_f P_f D_0f \right) - \dot{\varphi}_0 P_0 - N_0 \left[ -\frac{P_0^2}{4V_0} + H_f \right].$$ \hspace{1cm} (21)

4. Reduction and Dynamics of Proper Time

To remove arbitrariness connected with invariance of the theory with respect to time reparametrizations, we use the method of Hamiltonian reduction [14, 11] where one of the dynamical variables transforms into the evolution parameter.

The reduction means explicit resolving of the constraint

$$\int d^3x N_c \frac{\delta W}{\delta N_c} = 0 \Rightarrow \frac{P_0^2}{4V_0} = H_f \equiv V_0 \rho_{\text{CUT}}(\varphi_0).$$ \hspace{1cm} (22)
with respect to the momentum $P_0$. This equation has two solutions which correspond to two reduced systems with the actions

$$W^R_\pm (P_f, f|\varphi_0) = \int_{\varphi_1=\varphi_0(t_1)}^{\varphi_2=\varphi_0(t_2)} d\varphi_0 \left\{ \left( \int d^3 x \sum_f P_f D\varphi f \right) \mp 2\sqrt{V_0 H_f} \right\}$$

(23)

where $\varphi_0$ plays the role of the evolution parameter, and $D\varphi f = D_0 f/\dot{\varphi}_0$ is the covariant derivative with the new shift vector $N^k$ and vector field $V$, which differs from the old ones by the factor $(\dot{\varphi}_0)^{-1}$.

The local equations of motion of the systems (23) reproduce the invariant sector of the initial extended system and determine the evolution of all variables $(P_f, f)$ with respect to the parameter $\varphi_0$

$$(P_f(x, t), f(x, t), \ldots) \rightarrow (P_f(x, \varphi_0), f(x, \varphi_0), \ldots).$$

(24)

The reduced action (23) is completed by the equations of global dynamics:

$$\frac{\delta W^E}{\delta N_0} = 0 \Rightarrow (P_0)_\pm = \pm 2V_0 \sqrt{\rho_{\text{CUT}}(\varphi_0)}; \quad (\rho_{\text{CUT}} = \frac{H_f}{V_0})$$

(27)

$$\frac{\delta W^E}{\delta \varphi_0} = 0 \Rightarrow P_0' = V_0 \frac{d}{d\varphi_0} \rho_{\text{CUT}}(\varphi_0); \quad (f' = \frac{d}{d\eta} f)$$

(28)

$$\frac{\delta W^E}{\delta P_0} = 0 \Rightarrow \left( \frac{d\varphi_0}{d\eta} \right)_\pm = \frac{(P_0)_\pm}{2V_0} = \pm \sqrt{\rho_{\text{CUT}}(\varphi_0)}$$

(29)

where the effective Hamiltonian density functional can be decomposed over powers of $(\varphi_0)$

$$\rho_{\text{CUT}} = \frac{k^2_A}{\varphi_0^2} + h^2_R + \mu_F^2 \varphi_0 + \Gamma_B^{-2} \varphi_0^2 + \Lambda \varphi_0^4,$$

(30)

where the coefficients of the decomposition are the functionals of the local fields.

5. Cosmic Higgs vacuum

Equations (27), (28), and (29) lead to the Friedmann-like evolution of global conformal time of an observer

$$\eta(\varphi_0) = \int_{0}^{\varphi_0} \rho_{\text{CUT}}^{-1/2}(\varphi),$$

(31)

and to the conservation law

$$\left( \frac{k^2_A}{\varphi_0^2} \right)' + (h^2_R)' + (\mu_F^2)' \varphi_0 + (\Gamma_B^{-2})' \varphi_0^2 + (\Lambda)' \varphi_0^4 = 0.$$  

(32)
The red shift and the Hubble law in the conformal time version

\[ z(D_c) = \frac{\varphi_0(\eta_0)}{\varphi_0(\eta_0 - D_c)} - 1 \simeq D_c H_{Hubb}, \quad H_{Hubb} = \frac{1}{\varphi_0(\eta_0)} \frac{d}{d\eta} \varphi_0(\eta) \]  

(33)

reflect the alteration of the size of atoms in the process of evolution of masses \[\text{[13, 14]}\].

In the dependence on the value of \(\varphi_0\), there is dominance of the kinetic or the potential part of the Hamiltonian (30), (32), and different stages of evolution of the Universe (31) can appear: anisotropic \((k^2 A \neq 0)\) and radiation \((h^2 R \neq 0)\) (at the beginning of the Universe), dust \((\mu^2 F \neq 0; R^2 B)\) and De-Sitter \(\Lambda \neq 0\) (at the present time).

In perturbation theory, the factor \(a(x, t) = 1 + \delta_a\) represents the potential of the Newton gravity \(\delta a\). Therefore, the Higgs-PCT field, in this model, has no particle-like excitations (as it was predicted in paper [4]).

For an observer, who lives in the Universe, a state of “vacuum” is the state of the Universe at the present time: \(|\text{Universe} > = |\text{Lab. vacuum} >\), as his unified theory pretends to describe both observational cosmology and any laboratory experiments.

In correspondence with this definition, the Hamiltonian \(H_f\) can be split into the large (cosmological – global) and small (laboratory – local) parts

\[ H_f[\varphi_0] \overset{\text{def}}{=} \rho_0 V_0 + (H_f - \rho_0 V_0) = \rho_0(\varphi_0)V_0 + H_L \]  

(34)

where the global part of the Hamiltonian \(\rho_0(\varphi_0)V_0\) can be defined as the “Universe” averaging so that the “Universe” averaging of the local part of the Hamiltonian (34) is equal to zero

\[ <\text{Universe}|H_f|\text{Universe}> = \rho_0 V_0, \quad <\text{Universe}|H_L|\text{Universe}> = 0. \]  

(35)

Let us suppose that the local dynamics \((H_L)\) can be neglected if we consider the cosmological sector of the proper time dynamics \((H_f, H_L)\). In this case, eqs. (29) and (33) give the relation between the present-day value of the scalar field and the cosmological observations

\[ \bar{\varphi}(\eta = \eta_0) = \sqrt{\frac{\rho_0(\eta_0)}{H_0(\eta_0)}}. \]  

(36)

The present-day mean matter density

\[ \rho_b = \Omega_0 \rho_{cr}; \quad (\rho_{cr} = \frac{3H_0}{8\pi}M_{Pl}^2) \]  

(37)

is estimated from experimental data on luminous matter \((\Omega_0 = 0.01)\), the flat rotation curves of spiral galaxies \((\Omega_0 = 0.1)\) and others data [19] \((0.1 < \Omega_0 < 2)\).

We should also take into account that these observations reflects the density at the time of radiation of light from cosmic objects \(\Omega(\eta_0 - \text{distance/c})\), which was less than at the present-day density \(\Omega(\eta_0) = \Omega_0\) due to an increasing mass of the matter. This effect of retardation can be roughly estimated by the averaging of \(\Omega(\eta_0 - \text{distance/c})\) over distances (or proper time) \(\gamma = \eta_0 \Omega_0 \int_0^{\eta_0} d\eta \Omega(\eta)\).

For the dust stage the coefficient of the increase is \(\gamma = 3\). Finally, we get the relation of the cosmic value of the Planck “constant” and the GR one

\[ \frac{\bar{\varphi}(\eta = \eta_0)}{M_{Pl}} \sqrt{\frac{8\pi}{3}} = \sqrt{\frac{\gamma \Omega_0(\text{exp})}{h}} = \omega_0, \]  

(38)

where \(h = 0.4 \div 1\) is the observational bounds for the Hubble parameter.
From data on $\Omega_0$ we can estimate $\omega_0$: $\omega_0 = 0.04$ (luminous matter), $\omega_0 = 0.4$ (flat rotation curves of spiral galaxies), and $0.4 < \omega_0 < 9$ (others data [19]) for lower values of $h$ ($h = 0.4$).

The second term of the decomposition of the reduced action (23) over $V^{-1}_0$ defines the action for local excitations

$$P_0 d\varphi_0 = 2V_0 \sqrt{\rho_0(\varphi_0)} d\varphi_0 + H_L(\varphi_0) d\eta + o\left(\frac{1}{V_0}\right) \tag{39}$$

in terms of the measurable time $\eta$.

Really, an observer uses the action for description of laboratory experiments in a very small interval of time in comparison with the lifetime of the Universe $\eta_0$: $\eta_1 = \eta_0 - \xi; \eta_2 = \eta_0 + \xi; \xi \ll \eta_0$, and during this time-interval $\varphi_0(\eta)$ can be considered as constant $\varphi_0(\eta + \xi) \approx \varphi_0(\eta_0) = M_{Pl}\sqrt{3/8\pi}$. Thus, we got the $\sigma-$model version of the standard model [4].

6. Conclusion

The conformal unified theory (CUT) of strong, electroweak and gravitation interactions from a physical point of view identifies the Higgs scalar field in SM with the determinant of the space metric (i.e. scale factor) in the theory of gravity. This identification leads to important physical consequences:

CUT doesn’t need the Higgs potential for the formation of the homogeneous part of the Higgs field. The homogeneous part is extracted by the Hamiltonian reduction as an evolution parameter of the reduced system. The proper time of an observer becomes the dynamical variable with respect to the evolution parameter with the Friedmann-like cosmological equation. In contrast with the conventional Higgs effect (where the Higgs field is determined by parameters of the vacuum state), the Higgs field in CUT is determined by the integrals of motion and initial data of the state of the Universe (its density $\rho$ and time of life $H^{-1}_0$): $\phi_0 = \sqrt{\rho_0}/H_0$, in satisfactory agreement with observational data.

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