NEW COSMIC LOW ENERGY STATES OF NEUTRINO

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A field theory is studied where the consistency condition of equations of motion dictates strong
correlation between states of "primordial" fermion fields and local value of the dark energy.
In regime of the fermion densities typical for normal particle physics, the primordial fermions
split into three families identified with regular fermions. When fermion energy density is
comparable with dark energy density, the theory allows transition to new type of states. The
possibility of such Cosmo-Low Energy Physics (CLEP) states is demonstrated in a model of
FRW universe filled with homogeneous scalar field and uniformly distributed nonrelativistic
neutrinos. Neutrinos in CLEP state are drawn into cosmological expansion by means of
dynamically changing their own parameters. One of the features of the fermions in CLEP
state is that in the late time universe their masses increase as $a^{3/2}$ ($a = a(t)$ is the scale
factor). The energy density of the cold dark matter consisting of neutrinos in CLEP state
scales as a sort of dark energy; this cold dark matter possesses negative pressure and for the
late time universe its equation of state approaches that of the cosmological constant. The
total energy density of such universe is less than it would be in the universe free of fermionic
matter at all.

1 Two Measures Theory (TMT)

The TMT is a generally coordinate invariant theory where the action has the form $S = \int L_1 \Phi \sqrt{-g} d^4x$ including two Lagrangians $L_1$ and $L_2$ and two measures of integration:
the usual one $\sqrt{-g}$ and another $\Phi = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\mu \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d$ built of four scalar fields $\varphi_a$ $(a = 1, 2, 3, 4)$; it is also a scalar density. It is assumed that the Lagrangians $L_1$ and $L_2$ are functions of the matter fields, the dilaton field $\phi$, the metric, the connection but not of the "measure fields" $\varphi_a$. Solving equation that results from variation of $\varphi_a$, if $\Phi \neq 0$, we get $L_1 = M^4$ where $M$ is a constant of integration with the dimension of mass. Important feature of TMT that is responsible for many interesting and desirable results of the field theory models studied so far consists of the assumption that all fields, including also metric, connection (or vierbein
and spin-connection) and the measure fields \( \phi_a \) are independent dynamical variables.

In TMT there is no a need to postulate the existence of three species for each type of fermions (like three neutrinos, three charged leptons, etc.) but rather this is achieved as a dynamical effect of TMT in normal particle physics conditions. The matter content of our model includes the dilaton scalar field \( \phi \), two so-called primordial fermion fields (the neutrino primordial field \( \nu \) and the electron primordial field \( E \)) and electromagnetic field \( A_\mu \). Generalization to the non-Abelian gauge models including Higgs fields and quarks is straightforward. To simplify the presentation of the ideas we ignore also the chiral properties of neutrino; this can be done straightforward and does not affect the main results.

Keeping the general TMT structure of the action it is convenient to represent it in the following form:

\[
S = \int d^4x e^{\alpha \phi/M_p} \left[ \Phi + b\sqrt{-g} \right] \left[ -\frac{1}{k} R(\omega, e) + \frac{1}{2} g^{\mu \nu} \phi_{,\mu} \phi_{,\nu} \right] - \int d^4x e^{2\alpha \phi/M_p} \left[ \Phi V_1 + \sqrt{-g} V_2 \right] - \int d^4x e^{\alpha \phi/M_p} - \frac{1}{4} g^{\alpha \beta} g^{\mu \nu} F_{\alpha \mu} F_{\beta \nu} + \int d^4x e^{\alpha \phi/M_p} \left[ \Phi + k \sqrt{-g} \right] \left( \gamma^\alpha e_\alpha \nabla^{(i)} - \nabla_\alpha e^{\phi} \right) \Psi_i - \int d^4x e^{2\alpha \phi/M_p} \left[ \Phi + h \sqrt{-g} \right] \mu_\nu \sigma \nu + \left[ \Phi + h_E \sqrt{-g} \right] \mu_E \eta \phi_4 \right]
\]

where \( \Psi_i \) (\( i = \nu, E \)) is the general notation for the primordial fermion fields \( \nu \) and \( E \); \( V_1 \) and \( V_2 \) are constants; \( F_{\alpha \beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha \); \( \mu_\nu \) and \( \mu_E \) are the mass parameters; \( \nabla^{(i)} = \frac{\partial}{\partial \mu} + \frac{1}{2} \omega^\mu_{\alpha \beta} \sigma_{\alpha \beta} \); \( \nabla_{\mu}^{(E)} = \frac{\partial}{\partial \mu} + \frac{1}{2} \omega^\mu_{\alpha \beta} \eta_{\alpha \beta} + i e A_\mu \); \( R(\omega, e) = e^\alpha e^{\nu \lambda} R_{\mu \nu \lambda \rho}(\omega) \) is the scalar curvature; \( e_\mu \) and \( \omega^\mu_{\alpha \beta} \) are the vierbein and spin-connection; \( g^{\mu \nu} = e^\mu_\alpha e^{\nu \beta} \eta_{\alpha \beta} \) and \( R_{\mu \nu \lambda \rho}(\omega) = \frac{\partial \nu}{\partial \lambda} \omega^{\rho}_{\nu \lambda} + \omega^{\rho}_{\mu \nu} \omega_{\nu \lambda} - (\mu \leftrightarrow \nu) \); constants \( b, k, h \) are real dimensionless parameters.

The action (1) is invariant under the global scale transformations

\[
e^\alpha_\mu \rightarrow e^{\nu \lambda}/e^{\alpha}_\mu, \quad \omega^\mu_{\alpha \beta} \rightarrow \omega^\mu_{\nu \lambda}, \quad \phi \rightarrow \lambda_a \phi_a, \quad where \quad \Pi \lambda_a = e^2 \theta \quad A_\alpha \rightarrow A_\alpha, \quad \phi \rightarrow \phi - \frac{M_p}{\alpha} \theta, \quad \Psi_i \rightarrow e^{-\theta/4} \Psi_i, \quad \nabla_i \rightarrow e^{-\theta/4} \nabla_i.
\]

Except for a few special choices providing positivity of the energy and the right chiral structure in the Einstein frame, Eq. (1) describes the most general TMT action satisfying the formulated above symmetries.

The appearance of a nonzero integration constant \( M^4 \) in the mentioned above equation \( L_1 = M^4 \) spontaneously breaks the scale invariance (2). One can show that the measure \( \Phi \) degrees of freedom appear in all the equations of motion only through dependence on the scalar field \( \zeta = \frac{\Phi}{\sqrt{-g}} \). In particular, the gravitational and all matter fields equations of motion include noncanonical terms proportional to \( \partial_\mu \zeta \). It turns out that with the set of the new variables \( \bar{e}_{\alpha \mu} = e^{1/2} e_{\alpha \mu} \), \( \bar{\Psi}_i = e^{-1/2} \bar{\Phi} (\zeta + k)^{1/2} (\zeta + b)^{-1/2} \Psi_i \) (\( \phi \) and \( A_\mu \) remain the same) which we call the Einstein frame, the spin-connections become those of the Einstein-Cartan space-time and the noncanonical terms proportional to \( \partial_\mu \zeta \) disappear from all equations of motion.

The gravitational equations in the Einstein frame take the standard GR form \( G_{\mu \nu}(\bar{\bar{\alpha} \beta}) = \frac{8}{\lambda} T_{\mu \nu}^{eff} \) where \( T_{\mu \nu}^{eff} = K_{\mu \nu} + \bar{g}_{\mu \nu} V_{eff}(\bar{\bar{\alpha} \beta}; \zeta) + T_{\mu \nu}^{(can)} + T_{\mu \nu}^{(noncan)} + T_{\mu \nu}^{(f, can)} \). Here \( K_{\mu \nu} = \phi_{,\mu} \phi_{,\nu} - \frac{1}{2} \bar{g}_{\mu \nu} \bar{\Phi}_{,\alpha} \bar{\Phi}_{,\beta} \); \( G_{\mu \nu}(\bar{\bar{\alpha} \beta}) \) is the Einstein tensor in the Riemann space-time with the metric \( \bar{g}_{\mu \nu} \); \( V_{eff}(\bar{\bar{\alpha} \beta}; \zeta) = (\zeta + b)^{-2} \left[ b \left( s M^4 e^{-2\alpha \phi/M_p} + V_1 \right) - V_2 \right] \); \( T_{\mu \nu}^{(can)} \) is the canonical energy momentum tensor for the electromagnetic field; \( T_{\mu \nu}^{(f, can)} \) is the canonical energy momentum
tensor for (primordial) fermions $\nu'$ and $E'$ in curved space-time including also standard electromagnetic interaction of $E'$. $T_{\mu \nu}^{(\text{noncan})} = -\tilde{g}_{\mu \nu} \sum_i F_i(\zeta) \bar{\Psi}_i \Psi_i' \equiv \tilde{g}_{\mu \nu} \Lambda_{dyn}^{(\text{ferm})}$ $(i = \nu', E')$ is the noncanonical contribution of the fermions into the energy momentum tensor, and $F_i(\zeta) \equiv \mu_i 2^{-1/2}(\zeta + k)^{-2}(\zeta + b)^{-1/2}[\zeta^2 + (3h_i - k)\zeta + 2b(h_i - k) + kh_i]$. The structure of $T_{\mu \nu}^{(\text{noncan})}$ shows that it is originated by fermions but behaves as a sort of variable cosmological constant. This is why we will refer to it as dynamical fermionic $\Lambda$ term $\Lambda_{dyn}^{(\text{ferm})}$.

The dilaton $\phi$ field equation in the new variables reads $(-\tilde{g})^{-1/2}\partial_{\mu}(\sqrt{-\tilde{g}}\tilde{g}^{\mu \nu}\partial_{\nu}\phi) - 2\alpha\zeta(\zeta + b)^{-2}M_p^{-1}M^4e^{-2\alpha\phi/M_p} = 0$. Equations for the primordial fermions in terms of the new variables take the standard form of fermionic equations in the Einstein-Cartan space-time where the standard electromagnetic interaction presents also. All the novelty consists of the form of the $\zeta$ depending ”masses” $m_i(\zeta)$ of the primordial fermions $\nu'$, $E'$:

$$m_i(\zeta) = \frac{\mu_i(\zeta + h_i)}{(\zeta + k)(\zeta + b)^{1/2}} \quad i = \nu', E'$$

(3)

It should be noticed that change of variables we have performed provide also a conventional form of the covariant conservation law of fermionic current $j^\mu = \bar{\Psi}^a \gamma^\mu \tilde{x}^*_a \Psi'$.

The scalar field $\zeta$ in the above equations is defined by the constraint which is the consistency condition of equations of motion. In the Einstein frame it takes the form

$$-\frac{1}{(\zeta + b)^2} \left\{ (\zeta - b) \left[ sM^4e^{-2\alpha\phi/M_p} + V_1 \right] + 2V_2 \right\} = \sum_i F_i(\zeta) \bar{\Psi}_i \Psi_i'$$

(4)

Generically the constraint (4) determines $\zeta$ as a very complicated function of $\phi$, $\nu'$ and $E'$. However, there are a few very important particular situations where the theory allows exact solutions of great interest [5, 6].

2 Fermion Vacuum, Regular Fermions, Fifth Force Problem

In the case of the complete absence of massive fermions $\zeta = b - 2V_2/(V_1 + sM^4e^{-2\alpha\phi/M_p})$ and the effective $\phi$-potential is

$$V_{eff}^{(0)}(\phi) \equiv V_{eff}(\phi; \zeta)|_{\bar{\Psi} \Psi' = 0} = \frac{(V_1 + sM^4e^{-2\alpha\phi/M_p})^2}{4b(V_1 + sM^4e^{-2\alpha\phi/M_p}) - 2V_2}$$

(5)

Assuming $bV_1 > 2V_2$ one can see that $V_{eff}^{(0)}$ monotonically decreases (as $\phi \to \infty$) to the positive cosmological constant $\Lambda^{(0)} = \frac{V_1^2}{V_2(V_1 - 2V_2)}$.

In a typical particle physics situation, say detection of a single fermion, the measurement implies a localization of the fermion which is expressed in developing a very large value of $|\bar{\Psi} \Psi'|$. According to the constraint (4) this is possible if $F_i(\zeta) \approx 0$, $i = \nu', E'$ (which gives two constant solutions for $\zeta$) or $\zeta \approx -b$. These solutions allow to describe the effect of splitting of the primordial fermions into three generations of the regular fermions (for details see [5, 6]). It is interesting also that for the first two generations (which we associate with the solutions where $F_i(\zeta) \approx 0$) their coupling to the dilaton $\phi$ is automatically strongly suppressed which provides a solution of the fifth force problem.

3 Cosmo-Low Energy Physics (CLEP) states

It turns out that besides the normal fermion vacuum where the fermion contribution to the constraint is totally negligible, TMT predicts possibility of so far unknown states which can
be realized, for example, in astrophysics and cosmology. Let us study a toy model where in addition to the homogeneous scalar field \( \phi \), the spatially flat universe is filled also with uniformly distributed nonrelativistic neutrinos as a model of dark matter. Spreading of the neutrino wave packets during their free motion lasting a long time yields extremely small values of \( \Psi \Psi^\dagger = u^\dagger u \) (\( u \) is the large component of the Dirac spinor \( \Psi \)). There is a solution where the decaying fermion contribution \( u^\dagger u \sim \frac{\text{const}}{a^3} \) to the constraint is compensated by approaching \( \zeta \to -k \). Then solving (4) for \( \zeta \) we have to take into account both sides of the constraint. After averaging over typical cosmological scales (resulting in the Hubble law), the constraint (4) reads

\[
-(k + b) \left( s M^4 e^{-2\alpha \phi/M_p} + V_1 \right) + 2 V_2 + (b - k)^2 n_0^{(\nu)} F_\nu(\zeta) \big|_{\zeta \to -k} = 0
\]

where \( F_\nu(\zeta) \big|_{\zeta \to -k} = \mu_\nu(h_\nu - k)(b - k)^{1/2}(\zeta + k) - 2 + O((\zeta + k)^{-1}) \) and \( n_0^{(\nu)} \) is a constant determined by the total number of the cold neutrinos.

The crucial role in the CLEP solutions of the cosmological equations belongs to the neutrino \( \Lambda_{\text{dyn}}^{(\text{ferm})} \) term. We assume here that \( V_1 > 0, V_2 > 0 \) and \( b > 0, k < 0, h_\nu < 0, h_\nu - k < 0, b + k < 0 \). In the late time universe, the pressure and density of the uniformly distributed neutrinos in the CLEP state

\[
-P_{\text{clep}} = \rho_{\text{clep}} = \frac{2 V_2 + |b + k| V_1}{(b - k)^2} + \frac{|b + k|}{(b - k)^2} M^4 e^{-2\alpha \phi/M_p}
\]

are typical for the dark energy sector including both a cosmological constant and an exponential \( \phi \)-potential. The total energy density and the total pressure in the same regime are \( \rho_{\text{dark}}^{(\text{total})} = \frac{1}{2} \dot{\phi}^2 + U_{\text{dark}}^{(\text{total})}(\phi), \quad P_{\text{dark}}^{(\text{total})} = \frac{1}{2} \dot{\phi}^2 - U_{\text{dark}}^{(\text{total})}(\phi) \), where \( U_{\text{dark}}^{(\text{total})}(\phi) = \Lambda + \frac{|k|}{(b - k)^2} M^4 e^{-2\alpha \phi/M_p} \) and \( \Lambda = \frac{V_2 + |b| V_1}{(b - k)^2} \). The remarkable result is that \( V_{\text{eff}}^{(\text{total})}(\phi) > U_{\text{dark}}^{(\text{total})}(\phi) \) which means that (for the same values of \( \phi^2 \)) the universe in "the CLEP state" has a lower energy density than in the fermion vacuum state. One should emphasize that this result does not imply at all that \( \rho_{\text{clep}} \) is negative.

For illustration of what kind of solutions one can expect, let us take the particular value for the parameter \( \alpha \), namely \( \alpha = \sqrt{3/8} \). Then the cosmological equations allow the following analytic solution at the late time universe: \( \phi(t) = \frac{M_p}{2\alpha} \varphi_0 + \frac{M_p}{2\alpha} \ln(M_p t), \quad a(t) \propto t^{1/3} e^{\lambda t} \), where \( \lambda = \frac{1}{M_p} \sqrt{\frac{3}{8}} e^{-\varphi_0} = \frac{2(b - k)M_p^2}{\sqrt{3}|k|M^4} \sqrt{\Lambda} \). The mass of the neutrino in such CLEP state increases exponentially in time \( m_\nu|_{\text{CLEP}} \sim a^{3/2}(t) \sim t^{1/2} e^{\frac{\lambda}{2}} \).

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