Strings and Loops in Event Symmetric Space-Time

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Abstract

Open and Closed super-string field theories are constructed in an event-symmetric target space. The partition functions of Statistical and Quantum models are constructed in terms of invariants defined on Lie-algebra representations. An attractive feature of the closed string models is the elegant unification of the space-time symmetries with the gauge symmetries.

“It’s clear that there is a very deep and beautiful mathematical structure that underlies all the startling results that we are finding and that some very elegant and profound principle is there to be found. ... A concern one might have is that the mathematics which is required just gets so difficult that the human mind is unable to deal with it!”

John Schwarz

“In string theory there aren’t four or ten dimensions. That’s only an approximation. In the deeper formulation of the theory the whole notion of what we mean by a dimension of space-time will have to be altered. ... once the correct fundamental formulation of the theory is really understood, it will probably be something startlingly simple.”

Michael Green.

Quotes from “Superstrings: A Theory of Everything?” [24].

String Theories

Despite the lack of experimental data above the Electro-Weak energy scale, the search for unified theories of particle physics beyond the standard model has yielded many mathematical results based purely on constraints of high symmetry, renormalisability and cancellation of anomalies. In particular, space-time supersymmetry [5] has been found to improve perturbative behaviour and to bring the gravitational force into particle physics. One ambitious but popular line of research is superstring theory [5, 10, 11]. String
models were originally constructed in perturbative form and were found to be finite at each order but incomplete in the sense that the perturbative series were not Borel summable [20].

There has been some notable success in formulating both open [15] and closed [32] String Field Theories. There have also been some important steps taken towards background independent formulations of these theories [33, 39]. However, these formulations fail to provide an elegant unification of space-time diffeomorphism symmetry with the gauge symmetry. This is a significant failure because string theories are supposed to unify gravity with the other gauge forces.

A successful theory of Quantum Gravity should describe physics at the Planck scale [1]. It is likely that there is a phase transition in string theories at their Hagedorn temperature near \( kT = \text{Planck Energy} \) [5]. It has been speculated that above this temperature there are fewer degrees of freedom and a restoration of a much larger symmetry [22, 17, 18, 19].

There are also indications that string theory may be discrete in some sense at short distance scales. It is possible to calculate exact string amplitudes from a lattice theory with a non-zero spacing [25]. Furthermore, there are signs of a scaling duality which also limits measurement of distances smaller than the Planck length [12].

There is another non-perturbative approach to string theories which gives important insights. Random Matrix Models in the large N double scaling limit can be shown to be equivalent to two dimensional gravity, or equally, string theory in zero dimensional target space. The models can be extended to a one dimensional target space but not to critical dimension where string theory is perturbatively finite.

One interpretation of the present state of string theories is that it lacks a geometric foundation and that this is an obstacle to finding its most natural formulation. It is possible that our concept of space-time will have to be generalised to some form of “stringy space”. Perhaps such space-time must be dynamical and capable of undergoing topological or even dimensional changes [12, 15].

The Loop Representation of Quantum Gravity

Recently there has been some progress in attempts to quantise Einstein Gravity [4, 8] by canonical methods. A reformulation of the classical theory in which the connection takes the primary dynamic role instead of the metric
has led to the Loop Representation of Quantum Gravity \cite{23,24}. The fact that Einstein Gravity is non-renormalisable is considered to be not necessarily disastrous since gravity theories in 1+1 and 2+1 dimensions have been successfully quantised by various means \cite{37}.

A similarity between the loop representation and string theories is that there are attempts to understand them in terms of groups defined on loop objects \cite{8}. This and other similarities between the Loop Representation of Quantum Gravity and formulations of String Field Theories may be more than superficial \cite{41}. Superstring theory and the Loop Representation can not be equivalent since the former only works in ten or eleven dimensions while the latter only works in four. It is possible that they could be different phases of the same pre-theory provided that pre-theory allows changes of dimension.

One other notable aspect of the Loop Representation is that it has a discrete nature at scales smaller than the Planck length. The loop area is quantised in multiples of the Planck length squared. This has inspired renewed interest in discrete methods.

**Event Symmetric Space-Time**

Whichever approach to quantum gravity is taken the conclusion seems to be that the Planck length is a minimum size beyond which the Heisenberg Uncertainty Principle \cite{4} prevents measurement \cite{46}. Space-time may have to be viewed very differently to understand physics beyond the Planck scale. Until the geometric principles have been understood a consistent formulation of quantum gravity may be impossible. The increase of mathematical sophistication in physics which has emerged from these lines of research may suggest to some that the required mathematics to solve the problem has not yet been developed. However, the view taken here is that a discrete approach using a simple geometric principle may provide the solution.

In the light of what quantum gravity seems to have to say about the structure of space-time on small scales Wheeler and others have speculated that a pre-geometry theory is needed \cite{13}. In such a theory the space-time continuum would not be part of the fundamental formulation but would arise as a consequence of dynamics. Pre-geometry may have to have a discrete formulation.

The paradigm of Event Symmetric space-time is one such discrete approach \cite{47}. The exact nature of space-time in this scheme will only become
apparent in the solution. Even the number of space-time dimensions is not
set by the formulation and must by a dynamic result. It is possible that
space-time will preserve a discrete nature at very small length scales. Quantum
mechanics is reduced to a minimal form. The objective is to find a
statistical or quantum definition of a partition function which reproduces a
unified formulation of known and hypothesised symmetries in physics and
then worry about states, observables and causality later.

We can seek to formulate a lattice theory in which diffeomorphism in-
variance takes a simple and explicit discrete form. At first glance it would
seem that only translational invariance can be adequately represented in a
discrete form on a regular lattice but this overlooks the most natural gen-
eralisation of diffeomorphism invariance in a discrete system. Diffeomorphism
invariance requires that the action should be symmetric under any differ-
entiable 1-1 mapping on a $D$ dimensional manifold. This is represented by
the diffeomorphism group $\text{diff}(M_D)$. On a discrete space we could demand
that the action is symmetric under any permutation of the discrete space-
time events ignoring continuity altogether. Generally we will use the term
Event Symmetric whenever an action has an invariance under the Symmet-
ric Group $S(\mathbb{N})$ over an infinite number of discrete “events” (or some larger
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If a continuum is to be restored there must be a mechanism of sponta-
neous symmetry breaking in which event symmetry is replaced by a residual
diffeomorphism invariance. The mechanism will determine the number of
dimensions of space. It is possible that a model could have several phases
with different numbers of dimensions and may also have an unbroken event-
symmetric phase.

It is unlikely that there would be any way to distinguish a space-time
with an uncountable number of events from space-time with a dense covering
of a countable number of events so it is acceptable to consider models in
which the events can be labelled with positive integers. The symmetry
group $S(\aleph_1)$ is replaced with $S(\aleph_0)$. In practice it may be necessary to
regularise to a finite number of events $N$ with an $S(N)$ symmetry and take
the large $N$ limit while scaling parameters of the model as functions of $N$.

Renormalisation and the continuum limit must also be considered but
it is not clear what is necessary or desired as renormalisation behaviour. In
quantum field theories with a lattice formulation such as QCD it is normally
assumed that a continuum limit exists where the lattice spacing tends to
zero as the renormalisation group is applied. In string theories, however,
the theory is perturbatively finite and the continuum limit of a discrete
model cannot be reached with the aid of renormalisation. It is possible that
it is not necessary to have an infinite density of events in space-time to
have a continuum or there may be some alternative way to reach it, via a
q-deformed non-commutative geometry for example [44].

It stretches the imagination to believe that a simple event symmetric
model could be responsible for the creation of continuum space-time and
the complexity of quantum gravity through symmetry breaking, however,
nature has provided some examples of similar mechanisms which may help
us accept the plausibility of this claim.

The simplest possibility is to model space-time as a critical solid formed
from randomly bonded points [36]. The points are assigned a set of $D$
real numbers and are analogous to molecules moving in a $D$ dimensional
space. For a suitable action symmetric in exchange of molecules they can
model a critical solid at a second order melting phase transition. This gives
rise dynamically to what might be interpreted as a $D$ dimensional curved
manifold. In this case the number of dimensions is predetermined and it is
difficult to see how the space-time could form different topologies.

There is another variant of this natural mechanism that has more flexibil-
ity. Consider the way in which soap bubbles arise from a statistical physics
model of molecular forces. The forces are functions of the relative positions
and orientations of the soap and water molecules. The energy is a function
symmetric in the exchange of any two molecules of the same kind. The sys-
tem is consistent with the definition of event symmetry since it is invariant
under exchange of any two water or soap molecules and therefore has an
$S(N) \otimes S(M)$ symmetry where $N$ and $M$ are the number of water and soap
molecules. Under the right conditions the symmetry breaks spontaneously
to leave a diffeomorphism invariance on a two dimensional manifold in which
area of the bubble surface is minimised.

Events in these models correspond to molecules rather than space-time
points. Nevertheless, they are perfect mathematical analogies of event-
symmetric systems where the symmetry breaks in the Euclidean sector to leave diffeomorphism invariance in two dimensions as a residual symmetry. Indeed the models illustrate a deep analogy between events in event symmetric space-time and many-particle systems. The models considered further are more sophisticated than the molecular models and do not pre-determine geometry in any way. However, the analogy between particles and space-time events remains useful.

A number of Event Symmetric models will be described in this paper. Some of these can best be understood as statistical theories with a partition function defined for a real positive definite action.

\[ Z = \int e^{-S} \]  

(2)

Others can only be considered as quantum theories for which the action need not be positive definite provided the partition function is well defined

\[ Z = \int e^{iS} \]  

(3)

It is not always clear when such an integral should be considered well defined. For example the action,

\[ S = x^2 - y^2 \]  

(4)

gives a well defined quantum partition function in the two variables \((x, y)\) but if the variables are transformed by a 45 degree rotation to \((u, v)\), the action becomes

\[ S = 2uv \]  

(5)

for which the integral is not well defined.

It might be safer to consider only positive definite actions and assume that in a physically valid theory, the only difference between the statistical event symmetric model and the quantum one should be a factor of \(i\) against the action in the exponential. We might expect that in the statistical version the Event Symmetry will break to give Riemannian space-time with a Euclidean signature metric while in the quantum version it breaks to give the physical Lorentzian theory with Minkowski signature metric.

It is not clear if this is realistic, after all, continuum lagrangian densities for field theories in Lorentzian space-time are made non-positive definite by the signature of the metric. It is not clear what conditions should be placed on the form of an event-symmetric action to ensure a well defined tachyon
free quantum theory which produces dynamically the correct Lorentz signature. Even in continuum theories this is an interesting question and it is believed that a Lorentzian signature is preferred for certain theories in 4 and 6 dimensions. [40].

**Random Matrix Models**

A basic type of event-symmetric model places field variables $A_{ab}$ on links joining all pairs of events $(a,b)$. A suitable action must be a real scalar function of these variables which is invariant under exchange of any two events.

The link variables $A_{ab}$ can be organised into the upper triangle of a matrix. If there are no self links the diagonal terms are zero so it is natural to extend the matrix to the lower half by making the matrix anti-symmetric.

A four link loop action is

$$S = \sum_{a,b,c,d} A_{ab} A_{bc} A_{cd} A_{da}$$  \hspace{1cm} (6)

This is equal to

$$S = Tr(A^4)$$  \hspace{1cm} (7)

which is an invariant under $O(N)$ similarity transformations on the matrix.

This suggests that we consider actions which are functions of the traces of powers of the matrix $A$. Then the symmetry group of the system is $O(N)$ which has $S(N)$ as a subgroup. The idea can be extended to unitary groups by using complex variables for hermitian matrices or symplectic groups by using quaternions.

This is an appealing idea since it naturally unifies the $S(N)$ symmetry, which we regard as an extension of diffeomorphism invariance, with gauge symmetries. If the symmetry broke in some miraculous fashion then it is conceivable that the residual symmetry could describe quantised gauge fields on a quantised geometry. For example a discrete gauge $SO(10)$ symmetry on a lattice of $M$ points would be,

$$SO(10)^N \subset O(10N)$$  \hspace{1cm} (8)

For the symmetry to break in the way we desire, i.e. leaving a finite dimensional topology, the events will have to organise themselves into some
arrangement where there is an approximate concept of distance between them perhaps defined by correlations between field variables. Matrix elements linking events which are separated by large distances would have to be correspondingly small. Only variables which are localised with respect to the distance could have significant values.

There are several possible generalisations to multi-matrix models, tensor models and models with fermions. In each case the action can be a function of any set of scalars derived from the tensors by contraction. For the action to reduce to an effective local action on this space the original action must be restricted to forms in which it is the sum of terms which are written as contractions over tensors and which do not separate into products of two or more such scalar quantities. For example if there are two matrices $A$ and $B$ defining the field variables then the action could contain terms such as,

$$tr(ABAB)$$ (9)

but not,

$$tr(AB)^2$$ (10)

or

$$tr(A)tr(B)$$ (11)

This locality condition is important when selecting suitable actions for models which might exhibit dimensional symmetry breaking.

This type of random matrix model has been extensively studied in the context where $N$ is interpreted as the number of colours or flavours. (see [26, 31]) The event-symmetric paradigm suggests an alternative interpretation in which $N$ is the number of events times the number of colours.

Symmetry breaking in one-matrix models appears to be limited to simple forms [34]. One interesting result is that the perturbation theory of an $SU(N)$ matrix model in the large $N$ double scaling limit is equivalent to a $c = 0$ string theory [6, 28].

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However, it seems likely that this type of model cannot break event symmetry in any useful way given the invariance and locality conditions described if there are a finite number of tensors involved.

### Event Supersymmetric Matrix Models

It would be an obvious next step to generalise to supersymmetric matrix models \[30\]. So far we have matrix models based a families of groups such as \(S(N), O(N), SU(N)\) or \(Sp(N)\). Tensor representations and invariants can be used to construct models with commuting variables, anticommuting variables or both. Similarly we can define models based on supersymmetry groups of which there are also several families such as \(SU(N/M)\) and \(OSp(N/M)\). For analysis of supergroups see \[27\].

Just one simple model will be described. The representation has an anti-hermitian matrix \(A\) of commuting variables

\[
A_{ab}^* = -A_{ba}
\]

and a vector \(\psi\) of anti-commuting variables. A suitable action could be,

\[
S = m(2i\psi^*_a\psi_a + A_{ab}A_{ba}) + \beta(3\psi^*_aA_{ab}\psi_b - iA_{ab}A_{bc}A_{ca})
\]

As well as \(U(N)\) invariance this is invariant under a super-symmetry transform with an infinitesimal anticommuting parameter \(\epsilon_b\),

\[
\delta A_{ab} = \epsilon_b^*\psi_a - \epsilon_a\psi^*_b\delta\psi_a = i\epsilon_b A_{ab}\delta\psi^*_a = i\epsilon_b^* A_{ba}
\]

It is encouraging that supersymmetric generalisations of matrix models can be so easily constructed on event symmetric space-time. Demanding supersymmetry helps reduce our choice of actions but not actually very much. There are still many different possibilities like the above which can be constructed from contractions over tensor representations of supersymmetry groups. These models are special cases of matrix or tensor models so they will not be more successful as a scheme for dimensional symmetry breaking.
Event Symmetric String Models

The fact that a large number of degrees of freedom are perhaps required to produce event symmetry breaking suggests that string theories might provide answers. The place to start is with string groups [21].

To define the string groups an open orientated string is first considered as a topological object independently of any target space. There is an abstract operator $L_C$ for each string $C$ and a Lie-product is defined for these operators by specifying the structure constants,

$$L_A \wedge L_B = \sum f_{AB}^C L_C$$ (18)

Three strings $(A, B, C)$ are said to form a triplet if the end of $A$ matches the start of $B$, the end of $B$ matches the start of $C$ and the end of $C$ matches the start of $A$. They must match in such a way that there is no part of any string unmatched. In other words they form three matching lengths radiating from one central point.

When $(A, B, C)$ is such a triplet then the structure constants are

$$f_{AB}^{CT} = f_{BC}^{TA} = f_{CA}^{TB} = 1$$ (19)

and

$$f_{BA}^{CT} = f_{CB}^{TA} = f_{AC}^{TB} = -1$$ (20)

$C^T$ is the transposed string formed by reversing its orientation.

All other structure constants are zero. It can be checked that this does define a Lie-algebra because the product is anti-symmetric and satisfies the Jacobi identity. This Lie-algebra is regarded as generating the gauge group of the open string field theories.

The algebra can be realised on a continuous $D$ dimensional manifold $M_D$ when the strings are continuous open orientated curve segments in the space. The group is then called the Universal String Group $\text{str}(M_D)$

Such string groups have been used to formulate both open and closed string field theories [31].

These formulations are, however, not completely satisfying. If string theory really unifies gravity with the gauge forces then the symmetry group of gravity, $\text{diff}(X)$ on a manifold $X$, should be unified with the string group, $\text{str}(X)$ on target space $X$. This is not achieved in the continuum string field theories. Furthermore the phenomenon of topology change suggests that it is not possible [38] because the string groups must be the same on topologically
different manifolds. Ideally the diffeomorphism groups should be contained in the full string groups

\[ \text{diff}(X) \subset \text{str}(X) \sim \text{str}(X') \]  

(21)

But then the string group must contain \( \text{diff}(X) \) for every possible topologically different manifold \( X \).

This seems quite unreasonable but in fact it is exactly what happens in event symmetric field theories which contain the full event symmetric group \( S(X) \). These groups are isomorphic for any two manifolds and contain the diffeomorphism groups

\[ \text{diff}(X) \subset S(X) \sim S(X') \]  

(22)

The solution will be to find a string group which contains the symmetric group

\[ \text{diff}(X) \subset S(X) \subset \text{str}(X) \]  

(23)

We shall see how it is possible to define string groups on an event symmetric target space of discrete points in such a way that the symmetric group is included as a subgroup.

### Discrete Open String Associative Algebras

To begin constructing event symmetric string models we will extend matrix algebras to discrete string algebras then use these to construct extensions of matrix models \[48\].

A basis for a discrete open string vector space is defined by the set of open ended oriented strings through an event symmetric space of \( N \) events. E.g. a possible basis element might be written,

\[ C = (1, 4, 3, 1, 7) \]  

(24)

Note that a string is allowed to intersect itself. In the example the string passes through the event 1 twice. A string must be at least two events long. A null string passing through zero events or Strings passing through just one event could be included but are not needed. The order in which the string passes through the events is significant e.g.

\[ (1, 4, 3) \neq (1, 3, 4) \]  

(25)
but the order in which the events themselves have been numbered is irrelevant since the models are to be event symmetric. A complete set of field variables in an event symmetric string model would be an element of this infinite dimensional vector space which could be written as a sum over strings $C$

$$\Phi = \sum \Phi^C C$$  \hfill (26)

This can define either a real or complex vector space. To avoid questions about convergence in some of the definitions that follow it is easiest to specify that only a finite number of the components can be non-zero. Other ways of regularising could be used or the sums could be regarded as just formal expressions.

An inner product can be defined.

$$\Phi_1 \cdot \Phi_2 = \sum \Phi_1^C \Phi_2^C$$ \hfill (27)

[An asterisk is used to denote complex conjugation.]

The inner product will prove useful when we wish to define a positive definite form for an action since,

$$\Phi \cdot \Phi = \sum |\Phi^C|^2 \geq 0$$ \hfill (28)

To add the structure of an associative algebra the product $AB$ of two strings in the space is defined by joining them when the end of $A$ matches the beginning of $B$ reversed. It is necessary to add together all the ways in which this can be done e.g.

$$(1, 4, 3, 1, 7)(7, 1, 5, 1) = (1, 4, 3, 5, 1) + (1, 4, 3, 1, 1, 5, 1)$$ \hfill (29)

In the case where the last point of the first string is not the first point of the second the product is always zero. This ensures that string models are local, i.e. strings which do not intersect should not interact directly. E.g.

$$(1, 4, 3, 1, 7)(1, 6) = 0$$ \hfill (30)

To ensure associativity this rule must be balanced with another rule that if the whole of one of the strings in a product matches, the last event is not cancelled e.g.

$$(1, 4, 3, 1, 7)(7, 1) = (1, 4, 3, 1, 1)$$ \hfill (31)

It can now be checked that these rules define an associative multiplication, and that it is closed over strings of length 2 and greater.

$$A(BC) = (AB)C$$ \hfill (32)
The multiplication of strings extends immediately to multiplication on the vector spaces.

\[ \Phi_1(\Phi_2 \Phi_3) = (\Phi_1 \Phi_2) \Phi_3 \]  
(33)

The algebra has an identity

\[ I = \sum_{i=1}^{N} (i, i) \]  
(34)

Examining the local string algebra it is observed that there is a sub-algebra spanned by the rank two bases,

\[ (a, b)(c, d) = \delta_{bc}(a, d) \]  
(35)

The sub-algebra is isomorphic to multiplication of \( N \times N \) Real or Complex matrices.

A notation for the string algebras will be adopted which reflects this relationship between the string algebras and matrix algebras. The string algebras are extensions of the algebras \( M(N, \mathbb{R}) \) and \( M(N, \mathbb{C}) \) and will be written \( \text{Open}(M(N, \mathbb{R})) \) and \( \text{Open}(M(N, \mathbb{C})) \).

There is an alternative non-local algebra in which two strings which do not share common points have a non-zero product, e.g.

\[ (1, 4, 3, 1, 7)_E(1, 6)_E = (1, 4, 3, 1, 7, 1, 6)_E(1, 4)_E(4)_E = (1)_E + (1, 4, 4)_E \]  
(36)

In this algebra it is necessary to include a null string and strings of length one. The null string is the identity. The real and complex algebras on the vector spaces spanned by these base strings can be denoted by \( \text{Open}(N, \mathbb{R}) \) and \( \text{Open}(N, \mathbb{C}) \). These non-local (global) algebras turn out to be (isomorphic to) sub-algebras of the local ones. This is realised by summing over strings with first and final events equal.

\[ (1, 4, 3, 1, 7)_E = \sum_{i=1}^{N} (i, 1, 4, 3, 1, 7, i) \]  
(37)

On further examination it emerges that the local algebra factorises into the tensor product of the matrix algebra acting on the two end points and the global algebra acting on the rest.

\[ \text{Open}(M(N, \mathbb{C})) = \text{Open}(N, \mathbb{C}) \otimes M(N, \mathbb{C}) \]  
(38)
This leads to some more general definitions. Firstly there is no need for $N$ in either of the two factors to be the same so define,

$$\text{Open}(N, M(L, \mathbb{Bbb}C)) = \text{Open}(N, \mathbb{Bbb}C) \otimes M((L, \mathbb{Bbb}C))$$  \hfill (39)

The open string extension of a general associative algebra $A$ can be defined as,

$$\text{Open}(N, A) = \text{Open}(N, \mathbb{Bbb}C) \otimes A$$  \hfill (40)

For the moment we return to the specific case of the extended matrix algebras to define the generalised trace and adjoints. The trace is best defined as the trace from the matrix part. I.e. there is a contribution from the components of length two strings on the matrix diagonal only.

$$\text{Tr}(C) = 1 \text{ if } C = (i, i) \text{ and } 0 \text{ otherwise}$$ \hfill (41)

There is an extended trace defined as the sum over components of any even length string which is palindromic, i.e. the same when reversed e.g.,

$$OTr(1, 4, 4, 1) = OTr(3, 3) = 1OTr(2, 3) = OTr(1, 2, 1) = 0$$ \hfill (42)

Both these traces behave like traces should and in particular,

$$\text{Tr}(\Phi_1 \Phi_2) = \text{Tr}(\Phi_2 \Phi_1) \text{ OTr}(\Phi_1 \Phi_2) = \text{OTr}(\Phi_2 \Phi_1)$$ \hfill (43)

The orientation reversal of strings will be used to define transposition denoted with a $T$. e.g.

$$(1, 5, 4)^T = (4, 5, 1)$$ \hfill (44)

The adjoint of a general element of the space, denoted by a dagger, is defined by transposing each base element and in the case of the complex space the complex conjugate of the components is also taken. I.e.

$$\Phi^\dagger = \sum \Phi^{C*} C^T$$ \hfill (45)

The usual relation between adjoints and multiplication holds

$$(AB)^T = B^T A^T (\Phi_1 \Phi_2)^\dagger = \Phi_2^\dagger \Phi_1^\dagger$$ \hfill (46)

Finally the inner product can be written in terms of these operations.

$$\Phi_1 \bullet \Phi_2 = \text{Tr}(\Phi_1^\dagger \Phi_2)$$ \hfill (47)
The Open String Lie-Algebras

From the associative algebra \( \text{Open}(M(N, Bbb C)) \) an infinite dimensional Lie algebra can be defined with the Lie product being given by the anticommutator,

\[ A \wedge B = AB - BA \quad (48) \]

This product automatically satisfies the Jacobi identity because of the associativity of the original algebra product,

\[ A \wedge (B \wedge C) + B \wedge (C \wedge A) + C \wedge (A \wedge B) = 0 \quad (49) \]

With the wedge product the algebra is an infinite dimensional Lie Algebra and in principle it defines a group by exponentiation. To avoid complications in this process only the lie-algebras will be considered.

Using the structure constants for the algebra the Lie product can be written

\[ A \wedge B = \sum f_{ABC} C \quad (50) \]

The three strings \( C, A \) and \( B \) are said to form a triplet if \( f_{ABC} \) is plus one, and an anti-triplet if it is minus one. They are a triplet if and only if they are all different and the end of \( A \) matches the beginning of \( B \), the end of \( B \) matches the beginning of \( C \) and the end of \( C \) matches the beginning of \( A \) without any events being left out or used twice. Anti-triplets are triplets with two of the strings interchanged. It follows that the structure constants are fully antisymmetric.

\[ f_{ABC} = f_{BCA} = f_{CAB} = -f_{ACB} = -f_{CBA} = -f_{BAC} \quad (52) \]

The following important relation is also valid

\[ A \bullet (B \wedge C) = (A \wedge B) \bullet C = f_{ABC} \quad (53) \]

From this description of the Lie-product the relationship with the Universal String Group is clear. The only essential difference is that the group is now defined on an event-symmetric space-time rather than a continuous one.
Because the Lie-product was defined as the anticommutator on the string extended matrix algebra $\text{Open}(M(N,BbbC))$ we know that the Lie-algebra must be an extension of the general linear Lie-algebra. This is confirmed by the relation,

\[(a, b) \wedge (c, d) = \delta_{bc}(a, d) - \delta_{ad}(c, b)\]  

(54)

The algebra is therefore given the name $\text{open}(gl(N,BbbC))$. A number of other extended Lie-algebras follow immediately by using the anti-commutator of the appropriate extended associative matrix algebras. e.g. $\text{open}(gl(N,BbbR))$, $\text{open}(gl(N,BbbH))$, $\text{open}(N, gl(L, BbbC))$ and of course $\text{open}(N, BbbC)$.

The trace of the matrix algebras can also be used to define the special subgroups because,

\[Tr(A \wedge B) = 0\]  

(55)

The sub-algebra of traceless elements of $\text{open}(gl(N,BbbC))$ will be denoted by $\text{open}(sl(N,BbbC))$. The Open trace can also be used to define subgroups. I.e. the elements of $\text{open}(gl(N,BbbC))$ for which

\[OTr(\Phi) = 0\]  

(56)

form the sub-algebra $s\text{open}(gl(N,BbbC))$. There is also an algebra $s\text{open}(N, BbbC)$ defined in this way and if both traces are used we have $s\text{open}(sl(N,BbbC))$.

There is an important alternative definition of the special groups for matrix algebras. Given a Lie Algebra $\mathcal{L}_0$ a subalgebra is defined as those elements which are formed from the Lie-product.

\[\mathcal{L}_1 = \mathcal{L}_0 \wedge \mathcal{L}_0\]  

(57)

If $\mathcal{L}_0$ is $gl(N,BbbC)$ then $\mathcal{L}_1$ is $sl(N,BbbC)$. For the string extended algebras there are many linear invariant operators $O$ which have the trace-like property.

\[O(\Phi_1 \wedge \Phi_2) = 0\]  

(58)

so applying the same technique to $\text{open}(gl(N,BbbC))$ will give a sub-algebra of $s\text{open}(sl(N,BbbC))$. For the present only the traceless definition will be used.

Of more importance to event symmetric string theories are the open string extensions to the families of Lie-algebras of compact matrix groups.
so(N), u(N) and sp(N). These are easy to define with the adjoint operator which has the property,
\[(\Phi_1 \wedge \Phi_2)^\dagger = -(\Phi_1^\dagger \wedge \Phi_2^\dagger)\] (59)

The algebra open(u(N)) is defined as the sub-algebra of open(gl(N, BbbC)) containing all elements for which
\[\Phi^\dagger = -\Phi\] (60)

The algebras open(so(N)) and open(sp(N)) are the similarly defined sub-algebras of open(gl(N, BbbR)) and open(gl(N, BbbH)).

There are also compact groups derived from the non-local groups Open(N, BbbR) etc which will be denoted by Comp(N, BbbR), Comp(N, BbbC) etc.

Statistical and Quantum Models for Open Strings

To define a model or theory which incorporates the group structures defined in the previous sections we need to choose a representation and an invariant action. The obvious representation to choose is the fundamental representation which takes elements of the lie-algebra
\[\Phi = \sum \phi_{ab}(a,b) + \sum \phi_{abc}(a,b,c) + \ldots .\] (61)

The infinitesimal transformations are generated by an element \(\epsilon\) of the algebra as follows,
\[\delta \Phi = \Phi \wedge \epsilon\] (62)

There are many alternative representations formed from tensor products, direct sums etc but the fundamental representation has the advantage that there is exactly one component field variable for each degree of symmetry.

The action should be real and must satisfy a certain locality principle. It will take a polynomial form in the components of the representation and in no term must there appear a product of two components of strings which do not pass through the same event. This rules out the non-local groups Open(N, BbbC), Comp(N, BbbC) etc since they have very few invariants which are local in this sense. The special groups will also be ruled out since constraints such as \(Tr(\Phi) = 0\) can be considered non-local.

The trace is a source of invariants since
\[\delta Tr(\Phi) = Tr(\Phi \wedge \epsilon) = 0\] (63)
Furthermore the associative product can be used since the lie-algebra acts like a differential operator on the extended matrix algebra according to the Leibnitz rule,

\[(\Phi_1 \Phi_2) \land \epsilon = (\Phi_1 \land \epsilon)\Phi_2 + \Phi_1 (\Phi_2 \land \epsilon)\]  (64)

So there is an infinite sequence of invariants given by,

\[I_n = Tr(\Phi^n), (n = 1, \ldots)\]  (65)

Another sequence of invariants can be defined using the extended trace and there are many other possible invariants but for simplicity only these will be considered. Any action which is written as a sum of these invariants is consistent with the locality condition.

\[S = \sum g_n I_n\]  (66)

A statistical model has a partition function defined on a real action which is positive definite or at least bounded below. For the string extended general linear groups the trace invariants are not positive definite. This problem is resolved in the same way as it is for matrix models by using the Lie-algebras of the compact groups for which

\[\Phi^\dagger = -\Phi\]  (67)

Then the even trace invariants can be written,

\[I_{2n} = tr(\Phi^{2n}) = (-1)^n \Phi^n \bullet \Phi^n\]  (68)

The simplest non-trivial action for a statistical model is therefore

\[S = m \Phi \bullet \Phi + \beta \Phi^2 \bullet \Phi^2\]  (69)

[It is important to recognise that the model has an infinite number of degrees of freedom even for finite \(N\). It would be necessary to demonstrate that it can give a well defined model despite this.]

There are many other possibilities but this is the most immediately interesting bosonic open string statistical model. It is also possible to construct fermionic models using representations such as

\[\Psi = \sum \Psi^C C\]  (70)
Where the components $\Psi^C$ are anticommuting Grassman variables. an action for this model can be written,

$$S = im\Psi \bullet \Psi + \beta (\Psi \wedge \Psi) \bullet (\Psi \wedge \Psi) \quad (71)$$

The extended trace can also be used to define positive definite actions because $OTr(\Phi^2)$ is bounded even though it contains such non-square terms as,

$$\sum \phi_{ab} \phi_{bd} \quad (72)$$

For quantum models the conditions can be relaxed a little since the action does not have to be positive definite to give a well defined partition function. The general linear groups are still ruled out but extended Poincare groups might be considered as well as the compact groups and the odd trace invariants could also be valid terms in the action.

For open string models there appear to be many possible gauge groups, many possible representations and many possible invariants. There are several ways to generate many more possibilities than have been described here. For example models of charged strings can be constructed from algebras such as $open(N, so(10N))$. Some further criterion would be needed to select a good theory. It is possible to speculate that only a small number of these models would have the desired symmetry breaking features to identify them as good theories. This might be considered unsatisfactory since it would be better to have a kinematic reason for selecting the right model rather than a dynamic one.

Another feature of the open string models which is unsatisfactory is that the event symmetry is not unified with the gauge group. It is true that the extended matrix models include the symmetric group as a subgroup of the matrix group. However, true event symmetry is invariance under permutation of events and although the models above possess this invariance it is not the same as the symmetric subgroup of the matrix group which acts only on the ends of the strings.

**Alternative Open String Groups**

Before moving on it is necessary to mention some alternative groups which could be used to construct similar theories to the open string models described above.

A question that might be asked is “Is it necessary to use an event symmetric target space rather than, say, a regular lattice?”. For the groups as
constructed above the answer is that the space must be event symmetric. If you try to restrict to strings which only follow links on a regular lattice you find that the group cannot be closed.

However, there is an alternative string group which does close on any lattice. For this group the associative multiplication rule is modified to keep the last event common to the two strings. E.g.

\[(1, 2, 3, 4)(4, 3, 5) = (1, 2, 3, 4, 3, 5) + (1, 2, 3, 5)\] (73)

This can be used to define models with symmetries very similar to the models already described. In this case the strings restricted to follow the links of any lattice form a closed sub-algebra of the full event-symmetric algebra.

These groups will therefore be called lattice string groups. They have all the useful properties described for the open string groups but are not extensions of matrix algebras. These algebras will not be discussed further.

Another algebra which could replace \(\text{Open}(N, \mathbb{Bbb} R)\) has a simpler multiplication rule in which the strings are simply joined without adding any terms where part of the string is cancelled. E.g

\[(1, 2, 3, 4) \circ (5, 6, 7) = (1, 2, 3, 4, 5, 6, 7)\] (74)

This is a non-local algebra but it can be used to define another class of string extensions for matrix algebras which are local. What makes it interesting, however, is that the corresponding group on a continuous target space has recently been identified as interesting in the context of the loop representation of quantum gravity where it is known as the Extended Loop Group [35].

This group is less suitable for string theories since it does not correspond to the Universal String Group in the same way as the event symmetric Open string groups and the lattice groups do.

**Supersymmetric String Groups**

An attractive feature of the discrete string groups on event symmetric spacetime is that supersymmetric versions can be constructed in a very natural way.

The matrix algebras \(M(N, \mathbb{Bbb} R)\) and \(M(N, \mathbb{Bbb} C)\) can be generalised to superalgebras \(M(L/K, \mathbb{Bbb} R)\) and \(M(L/K, \mathbb{Bbb} C)\) [27]. From these super algebras a number of families of super Lie-algebras can be constructed of
which the most important include $gl(L/K, \mathbb{Bbb R})$, $gl(L/K, \mathbb{Bbb C})$, $u(L/K)$, $osp(L/K)$.

It is possible to apply the string extension methods for ordinary algebras to these superalgebras to construct $Open(N, M(L/K, \mathbb{Bbb C}))$, $open(N, u(L/K))$ etc. This can be improved by first generalising $Open(N, \mathbb{Bbb C})$ to the super-symmetric algebra $Open(L/K, \mathbb{Bbb C})$. To define this algebra it is sufficient to describe a consistent grading of the base strings into odd and even strings. To do this the events themselves are given parity so that event-supersymmetric space-time contains $L$ even events and $K$ odd events. For notational convenience even events will be labelled with even integers and odd events with odd integers.

The parity of a string is defined to be the total parity of the events it passes through. The parity of a string $C$ written $\text{par}(C)$ is zero for even strings and one for odd strings. This defines a grading of the vector space which is consistent with the associative multiplication since the parity of the product of two strings is the sum of their parities modulo two

$$\text{par}(AB) = \text{par}(A) + \text{par}(B) - 2\text{par}(A)\text{par}(B)$$  \hspace{1cm} (75)

The components of the vectors must be taken from a Grassman algebra with even (commuting) variables for components of even strings and odd (anti-commuting) variables for components of odd strings i.e.

$$\Phi = \sum \Phi^C C \Phi^A B = (-1)^{\text{par}(A)\text{par}(B)} \Phi^B \Phi^A$$  \hspace{1cm} (76)

The real and complex algebras defined in this way are denoted by $Open(L/K, \mathbb{Bbb R})$ and $Open(L/K, \mathbb{Bbb C})$. Note that while $Open(L/0, \mathbb{Bbb R})$ is isomorphic to $Open(L, \mathbb{Bbb R})$, the algebra $Open(0/L, \mathbb{Bbb R})$ is a super-algebra in which the parity of a string is the parity of its length. This is in contrast to the matrix algebras for which $M(L/0, \mathbb{Bbb R})$ is the same as $M(0/L, \mathbb{Bbb R})$.

It is now possible to define local super-matrix algebras $Open(L/K, M(P/Q, \mathbb{Bbb C}))$ using the tensor product prescription. In the case $P = L$ and $Q = K$ we write simply $Open(M(L/K, \mathbb{Bbb R}))$ for consistency the indices of the matrix algebra are also taken as odd and even.

The adjoint operator must fulfill the usual relation

$$(\Phi^1 \Phi^2)\dagger = \Phi^2_1 \Phi^1\dagger$$  \hspace{1cm} (77)
This is achieved by modifying the previous definition to include a factor of $i$ when taking the adjoint of an odd element. This restricts us to the complex version of the model.

$$\Phi^\dagger = \sum i^{\text{par}(C)} \Phi C^* C^T$$  \hspace{1cm} (78)

When generalising the definition of trace and extended trace extra sign factors are needed corresponding to the parity of half the even string. E.g.

$$Tr(2,2) = 1, Tr(3,3) = -1 OTr(3,5,5,3) = 1, OT(1,4,4,1) = -1$$  \hspace{1cm} (79)

String extended super Lie-algebras can also be constructed for each of the supersymmetric families of matrix lie-algebras. From the super-algebra $\text{Open}(M(L/K, \mathbb{Bbb}C))$ a Lie-product is defined using the anticommutator,

$$\Phi_1 \wedge \Phi_2 = \Phi_1 \Phi_2 - \Phi_2 \Phi_1$$  \hspace{1cm} (80)

Then the lie product for elements of the representation will be anticommuting.

$$\Phi_1 \wedge \Phi_2 = -\Phi_2 \wedge \Phi_1$$  \hspace{1cm} (81)

But because of the commutation/anti-commutation relations on the components the Lie product of two odd base elements must be symmetric instead of anti-symmetric. I.e.

$$A \wedge B = AB - (-1)^{\text{par}(A)\text{par}(B)} BA$$  \hspace{1cm} (82)

This defines $\text{open}(gl(L/K, \mathbb{Bbb}C))$.

A representation of a reduced Lie sub-algebra $\text{open}(u(L/K))$ is defined as those elements which satisfy,

$$\Phi^\dagger = -\Phi$$  \hspace{1cm} (83)

The scalar product is now defined by

$$\Phi_1 \bullet \Phi_2 = Tr(\Phi_1^\dagger \Phi_2)$$  \hspace{1cm} (84)

This product is an invariant for the group $\text{open}(u(L/K))$ but is not positive definite because of the extra minus sign in the trace. Only a quantum model can be defined.

Actions for a model based on this representation are also the same as before. In general the action may contain any powers in the algebra squared with the scalar product.

$$S = g_1 \Phi \bullet \Phi + g_2 \Phi^2 \bullet \Phi^2 + g_3 \Phi^3 \bullet \Phi^3 + \ldots$$  \hspace{1cm} (85)
This supersymmetric generalisation is an analogue of the supersymmetric generalisation of matrix models already described.

It is possible that interesting physics exists in these models in a large $L, K$ double scaling limit with the constants $g_i$ scaled as functions of $N$.

**Discrete Closed String Groups**

In continuum string theory the closed string field theories are often considered to be of more physical interest but are also harder to construct. The same applies to event symmetric closed string models.

It is possible to construct Closed String algebras in which the base elements are cyclically symmetric. The extensions use a basis of closed discrete strings which will be written with square brackets to distinguish them from the open strings. When they are shifted cyclically a sign is introduced if they are even length i.e.,

\[
[a, b] = -[b, a] \\
[a, b, c] = [a, b, c] \\
[a, b, c, d] = -[b, c, d, a] \text{ etc.}
\]

(86) (87) (88)

Some strings of odd length must be excluded because of this sign rule e.g.

\[
[1, 1] = [1, 2, 3, 1, 2, 3] = 0
\]

(89)

The base elements are multiplied by identifying common sequences in opposite sense within them. E.g.

\[
[1, 2, 3, 4][5, 3, 2, 7] = -[1, 2, 2, 7, 5, 4] + [1, 7, 5, 3, 3, 4] - [1, 7, 5, 4]
\]

(90)

The sign for such a multiplication is chosen so that when the matching segments are moved to the end of the first string and the beginning of the second it is positive. For locality, when two strings have no points in common the product is zero and the whole of a string is not cancelled against part of another. This algebra is non-associative e.g.

\[
[1, 2]([2, 3][3, 4, 1]) = [1, 4, 1] - [2, 2, 4]([1, 2][2, 3])[3, 4, 1] = [1, 4, 1] - [3, 3, 4]
\]

(91)

Because of this non-associativity we can not be sure that defining a Lie-product as the anticommutator will satisfy the Jacobi identity.
If a string $A$ contains a piece $X$ and a string $B$ contains the same piece reversed, i.e. $X^T$ then we can write,

$$A = a \circ XB = X^T \circ b$$  \hspace{1cm} (92)

The circle symbol is used to mean joining pieces of strings. The term in the multiplication which involves the cancellation of $X$ can be written,

$$(AB)_X = ((a \circ X)(X^T \circ b))_X = a \circ b$$  \hspace{1cm} (93)

The full product can be written

$$AB = \sum_X (AB)_X$$  \hspace{1cm} (94)

It can be checked that,

$$(AB)_X = (-1)^{\text{len}(A)\text{len}(B)+\text{len}(X)}(BA)_X^T$$  \hspace{1cm} (95)

Where $\text{len}(A)$ is the number of events in a string or piece of string. We can try to find a Lie-product which might take the form

$$A \wedge B = \sum_X s(A, B, X)(AB)_X$$  \hspace{1cm} (96)

Where $s(A, B, X)$ are some form factors which must be determined to fulfill the graded commutation and Jacobi identities which are,

$$A \wedge B = (-1)^{\text{par}(A)\text{par}(B)} B \wedge A(-1)^{\text{par}(A)\text{par}(C)}(A \wedge B) \wedge C + [\text{cycle} A, B, C] = 0$$  \hspace{1cm} (97)

(The definition of parity of a string $\text{par}(A)$ has not yet been given.) One way to ensure the correct commutation relations is to take

$$s(A, B, X) = (1-(-1)^{\text{len}(A)\text{len}(B)+\text{len}(X)+\text{par}(A)\text{par}(B)})t(A, B)t(A, B) = t(B, A)$$  \hspace{1cm} (98)

The Jacobi Identity is more difficult. In general the three strings will have various pieces in common but could be decomposed as,

$$A = a \circ Y \circ b \circ XB = e \circ X^T \circ d \circ ZC = e \circ Z^T \circ f \circ Y^T$$  \hspace{1cm} (99)

With this decomposition the double product breaks into two terms and we can write,

$$(A \wedge B) \wedge C = \sum_{YX} s(A, B, X)s(AB, C, Y)((AB)_X C)_Y + \sum_{XZ} s(A, B, X)s(AB, C, Z)((AB)_X C)_Z$$  \hspace{1cm} (100)
The following identities can be established

\[(AB)_X C)_Z = (A(BC)_Z)X((AB)_X C)_Y = (-1)^{\text{len}(B)\text{len}(C)}((AC)_Y B)_X\]  \hfill (101)

With this substituted into the Jacobi identity it is evident that a clear solution is given by identifying the parity of a string with the parity of its length and defining,

\[s(A, B, X) = 1 - (-1)^{\text{len}(X)}\]  \hfill (102)

In other words only terms with cancellation of odd length pieces are included in the Lie-Product. (The only obvious way to get a non-abelian non-super algebra is to restrict to the algebra generated by even length strings.)

This defines a Lie-product satisfying the super-algebra Jacobi Identity in a non-trivial way. The Lie-product can be written as the (anti-)commutator of the algebraic product.

\[A \wedge B = AB - (-1)^{\text{par}(A)\text{par}(B)} BA\]  \hfill (103)

The product is not associative and the lie-product does not act as a differential operator on the algebra satisfying the graded Leibnitz rule. i.e.

\[(AB) \wedge C \neq (-1)^{\text{par}(B)\text{par}(C)}(A \wedge C)B + A(B \wedge C)\]  \hfill (104)

A counter example is

\[A = [1, 2] B = [3, 4, 5] C = [5, 4, 1]\]  \hfill (105)

This is unfortunate since it means that it is a little more difficult to construct invariants using the product in the same way as was done for open strings.

The real and complex Lie-algebras are given the names \textit{closed}(0/N, Bbb R) and \textit{closed}(0/N, Bbb C) respectively. The sub-algebras generated by the length two strings are \textit{so}(0/N, Bbb R) and \textit{so}(N, Bbb C).

An encouraging feature of these closed groups is that the event symmetry is included in the algebra in a sense that was not true for the open string algebras. This is because the matrix sub-algebra acts on all parts of the string in the appropriate fashion for the representation to be considered as a family of tensor representations of the matrix algebra.

\[\Phi = \sum \phi_{ab}(a, b) + \sum \phi_{abc}(a, b, c) + \ldots \phi_{ab} - \phi_{ba}\phi_{abc} = \phi_{bca} = \phi_{cab} etc.\]  \hfill (106)
The components of odd length strings are, of course, anticommuting Grassmann variables. A small change generated by the matrix sub-algebra gives e.g.

$$
\delta \phi_{abc} = \sum_d (\phi_{dbc} \phi_{da} + \phi_{adc} \phi_{db} + \phi_{abd} \phi_{dc})
$$

(107)

This is the correct transformation law for $\phi_{abc}$ as a third rank tensor under the group $SO(N)$ generated by $\phi_{dc}$. The corresponding transformation for the open string lacks the middle term. The higher rank components also transform correctly for the closed strings. The Alternating group $A(N)$ is a sub-group of $SO(N)$ and acts to permute events. Because of this the closed string algebra can be said to unify space-time symmetries and gauge symmetries in a unique and powerful way.

In the event-symmetric open string models this unification appears to be absent. This can be corrected by defining string groups which include the closed strings and open strings together. This observation is perhaps related to the fact that continuum open string theories must necessarily include closed strings.

The adjoint operator can be defined in the usual way for supersymmetric adjoints on the complex algebra.

$$
\Phi^\dagger = \sum_i \epsilon^{i \text{par}(C)} \Phi^C C^T
$$

(108)

The transpose of a string is its reversal. There is no ambiguity about which event it is transposed because of the cyclic relations. The sub-algebra of elements which are equal to minus their adjoints can be taken and will be denoted by simply $\text{closed}(0/N)$. This is an algebra of non-orientated closed discrete super-strings.

To complete the construction of an event-symmetric closed string field theory some invariants must be found. Because the components of the fundamental representation transform as a family of tensors under the orthogonal matrix subgroup it is necessary that any invariant must be written as contractions over indices of tensor products. This condition is not sufficient however.

First of all we should look for a quadratic invariant and can try,

$$
I(\Phi) = \sum q(\text{len}(C)) \Phi^C C^T \Phi^C
$$

(109)

with the a form factor $q(r)$ depending only on the length of the strings (i.e. the rank of the tensor) to be determined. However, the odd terms are
identically zero due to anti-commutivity and the even part with \( q(l) = 1 \) is only invariant for the bosonic sub-group generated by even length strings.

The problem of finding invariants can be solved by using the adjoint matrix representation. For each element \( \Phi \) an infinite matrix \( M(\Phi) \) acting on the graded vector space of the algebra is defined with components,

\[
M(\Phi)_{B}^{A} = \sum \Phi^{C} f_{BC}^{A}
\]  

(110)

These matrices form a representation of the super Lie-algebra with the graded anti-commutator as the Lie-product. Invariants can therefore be constructed using trace and product.

\[
Tr(M) = \sum M_{C}^{C} I_{n}(\Phi) = Tr(M^{n})
\]  

(111)

The first invariant \( I_{1}(\Phi) \) can be defined to be the extended trace,

\[
CTr(\Phi) = I_{1}(\Phi) = Tr(M(\Phi))
\]  

(112)

This receives contributions from even length palindromic strings e.g.,

\[
CTr[1, 2, 2, 1] = 1CTr[1, 2, 3, 3, 2, 1] = -1
\]  

(113)

With these invariants it is possible to define event symmetric quantum closed string field theories.

**Signature Groups**

Another class of groups closely related to the string groups is based on sets of discrete events where the order does not matter accept for a sign factor which changes according to the signature of permutations,

\[
[a|b|c] = -[b|a|c] etc.
\]  

(114)

Multiply by cancelling out any common events with appropriate sign factors. To get the sign right, permute the events until the common ones are at the end of the first set and at the start of the second in the opposite sense. The elements can now be multiplied with the same rule as for the open string. The same parity rules as for closed string apply. I.e. only cancellations of an odd number of events is permitted.

The representations of these groups are families of fully antisymmetric tensors. The Lie algebras are finite dimensional but it is not immediately obvious how to define models with well behaved action invariants other than the fermionic set with a similar action to the fermionic closed string above.
Multi-loop String Groups

The closed string group and the signature group are both sub-groups of the larger multi-loop group. The base elements of this group represent sets of closed loops. The closed loop group is the subgroup of single loops in the multi-loop group and the signature groups correspond to sets of loops each containing exactly one point.

The notation is chosen to be consistent with the loop and signature groups. For example a double loop base element with one loop of length three and one of two would be,

\[
[a, b, c|d, e] = -[a, b, c|e, d] = [b, c, a|d, e] = [d, e|a, b, c] etc.
\] (115)

The sign factor is always the signature of the permutation on the events in the string.

Antisymmetric multiplication is the obvious generalisation of multiplication on the closed and signature groups.

Conclusions

We started from a simple principle that physics could be described by an event symmetric model and considered open and closed string field theory on event-symmetric space-time as a possibility. The models which result unifies space-time symmetries and string gauge groups in a simple elegant way. Furthermore they can be recognised as natural extensions of random matrix models which are known to be of interest in the non-perturbative study of string theories.

It is possible that techniques used to study matrix models may also be applicable to event-symmetric string theories and that their study may provide further insight towards understanding the nature of superstring theories and of space-time.

According to the classification of Isham [43] the Event Symmetric approach to quantum gravity would be a type IV scheme. A new perspective is proposed from which, it is hoped, continuous space-time, particle physics and quantum gravity arise. Any such scheme is necessarily ambitious yet the concept of event symmetric space-time is both simple and in keeping with previous attempts to quantise gravity and unify particle physics.

The author welcomes all comments and corrections by e-mail to phil@galilee.eurocontrol.fr
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