Analytical and experimental methods for evaluation stress concentration in attachment point

A S Arutyunova1, A N Polilov2 and N A Tatus3

1National Research Nuclear University MEPhI (Moscow Engineering Physics Institute)
31 Kashirskoe shosse, Moscow, 115409, Russia
2Blagonravov Mechanical Engineering Research Institute of Russian Academy of Science
4 Maly Kharitonyevsky Pereulok, Moscow, 101990, Russia
3Moscow State University of Civil Engineering,
26, Yaroslavskoye Shosse, Moscow, Russia
nasty99.90@mail.ru

Abstract. The article considers a flat specimen made of composite material deposited on a part of its side surface. The article presents calculations of stress concentration factors close to grips by the simplified model of shear analysis. The dependence of the strength on the modules ratio, on the thickness of the specimen, on the lengths of the grip and of the working part is investigated.

Composites are widely used in aviation and rocket and space technology due to their high tensile strength and low specific gravity. Usage of composites allows us to solve three main problems: to reduce operating costs, to improve technical characteristics and their service properties. The hardest thing in the experiment is to determine the strength along the fibers of unidirectional reinforced plastic. The main problem is the complexity of uniform loading of a smooth specimen over its entire cross section and length. It is caused by the fact that any type of fixing leads to the creation of stress concentration. In order to obtain reliable results on rectangular specimen, it is necessary to reduce and somehow take into account the effect of stress concentration near the grips on the realization of strength. 1. The article depicts the dependence of the effective stress concentration coefficient (strength reduction coefficient) on various parameters. The composite specimen has the form of a strip loaded with shear stress along the gripping part of the side surface. For polymer fiber composites, specimens in the form of rectangular stripes are standardized. A strip of a layered composite with alternating layers of fibers and a matrix is considered (lower indices f and m, respectively). The strip length is 2L, the thickness is 2h, the width is w, and uniformly distributed shear stresses with intensity τ0 are applied on a part of the side surface of length a.
For a layered composite consisting of alternating hard and soft layers, taking into account the Hooke's law in displacements, we obtain the equilibrium equation in finite differences, from which, assuming smallness of thickness of the fiber layers \( h_f \) and the matrix \( h_m \), a differential equation of the second order is derived (and not the fourth, as in the usual theory of elasticity).

\[
\frac{\partial^2 u}{\partial x^2} + \beta^2 \cdot \frac{\partial^2 u}{\partial y^2} = 0, \quad \text{where} \quad \beta^2 = \left( h_f + h_m \right)^2 \cdot G_m \cdot \left( h_f \cdot h_m \cdot E_f \right)^{-1}. \tag{1}
\]

This equation is further simplified from the conditions of smallness of the Young's modulus and the shear modulus of the matrix material in comparison with the modulus of the fibers and the application of Voigt and Reuss hypotheses of the equality of deformations and shear stresses in the layers of fibers and matrix: \( \beta = \left( G_c \cdot E_c^{-1} \right)^{1/2} \).

For an infinite strip with repeating sections, the periodic solution is constructed by the method of separation of variables:

\[
u(x, y) = \sum_{n=1}^{\infty} u_n(x, y) = \sum_{n=1}^{\infty} X_n(x) \cdot Y_n(y). \tag{2}
\]

Substitution of particular solutions in (1), which are sought in the form of sine functions, leads to an ordinary differential equation, the solution of which is sought from the parity conditions, and the coefficients are determined from the boundary conditions on the surfaces.

The distribution of stresses from the obtained expression for stresses can be found from Hooke's law. The obtained solution (3) is valid for a strip with infinitely repeating sections 2L:

\[
\sigma(x, y) = \sum_{n=1}^{\infty} \left\{ 2\tau_0 \frac{\cos \left( \frac{n\pi L}{L} \right) - \cos n\pi}{\beta_n \cdot \pi \cdot \sin \left( \frac{n\pi h}{\beta L} \right)} \cdot \frac{n\pi y}{L} \cdot \sin \left( \frac{n\pi h}{\beta L} \right) \right\}.
\tag{3}
\]

The expression for stresses satisfies the boundary conditions at only one boundary. To satisfy the condition of the absence of efforts at the ends of the specimen, we determine by means of integration (3) the average tensile stress, which is added to the general solution with the opposite sign. Similarly, the distribution of displacements is determined as the sum of a series and additional displacements from balancing stresses at the ends. The average stress in the working part is determined as the ratio of the product of shear stresses and capture length to the thickness of the specimen.

The obtained expressions for the distribution of stresses in an infinite strip, the average stress at the ends, and the average stress in the working part of the specimen allow us to calculate the theoretical stress concentration factor (TSCF) at any point. It is determined by the ratio of local stress to average. At the edge of the grips, the TSCF is maximum and tends to infinity, since series (3) diverges.

To determine the true strength, it is not the TSCF that is important, but the effective strength reduction factor ESRF, which can be determined in several ways, discussed below.
Figure 2. GFRP specimen after fracture in the grips under tension.

The highest stress concentration, and, consequently, rupture occurs at the edge of the grips. To assess the strength reduction factor, it is necessary to compare the tensile strength of a smooth specimen with a certain stress averaged over a layer of thickness $\delta$. An analytical study was conducted in the Mathcad software package. The dependences of the effective stress concentration factor $K_\sigma$ on the characteristic material size $\delta$ are obtained. The following notations are accepted in the article: load - $P$, width - $w$, gripper length - $a$, half thickness of the sample - $h$, half length of the sample - $L$, Young's modulus - $E_c$, shear modulus - $G_c$. The parameters of the specimen for constructing the dependences: $P=80000$ N, $w=15$ mm, $a=56$ mm, $L=125$ mm, $E_c=142$ GPa, $G_c=30$ GPa, $\delta=0.5; 0.3; 0.15$.

\[
K_\sigma = \frac{\sigma_{\text{eff}}(\delta)}{\bar{\sigma}} = \frac{1}{\delta \bar{\sigma}} \int_{h-\delta}^{h} \sigma^* dy =
\]

\[
= -\frac{\sigma_0}{\bar{\sigma}} + \frac{2\tau_0 L}{\pi^2 \delta \bar{\sigma}} \sum_{n=1}^{\infty} \left\{ \frac{1}{n^2} \cos \frac{\pi n (L-a)}{L} \times
\right.\]

\[
\left. \times \left[ 1 - \frac{sh(\pi n(h-\delta))}{sh(\pi nh/\beta L)} \right] \left( \cos \left( \frac{\pi n (L-a)}{L} \right) - \cos \pi n \right) \right\}. \quad (4)
\]

After analyzing this dependence, we can conclude that the effective stress concentration factor decreases with an increase in the characteristic size of the material and increases with an increase in the thickness of the specimen, and, therefore, the realization of strength decreases. A comparison of two materials of carbon fiber reinforced plastic and fiberglass is given. The fibers layup in the selected materials are unidirectional. Parameters for comparing the ESRF for various materials: fiberglass ($E_c=56$ GPa, $G_c=5$ GPa) and carbon fiber ($E_c=142$ GPa, $G_c=30$ GPa).

Figure 3. The dependence of the effective stress concentration factor on the root of the ratio of the modules of the material $\beta$ - (a) and on the length of the grips $a$ - (b).

The ESRF increases with a decrease in the shear modulus of the composite and with a decrease in the length $a$ of the grips, which is illustrated in Figure 3.
To reduce the stress concentration during the test, a new method for fixing the specimen is presented, shown in Figure 4. The method of fastening in a ten on with a small cone may allow a more even distribution of stresses over the thickness of the material. To maintain equal structural strength, the width of the fastening zone must be increased by approximately 30%. After specimen preparation, every two layers of reinforcement and one matrix layer remain.

The specimen with grooves in the end region is fixed in the grip, consisting of a set of plates. Between the plates are inserts, the thickness of which corresponds to the thickness of the layers of the specimen between the grooves. The grooves can be made mechanically, but better - by introducing a Teflon film in the manufacture of specimens, which eliminates the bond between the layers. The entire construction of the grippers is compressed by external mounting bolts.

These plates are fixed on both sides in bearing blocks to which the load from the testing machine is applied. A general view of the proposed design of the capture assembly is shown in Figure 4. The plates and inserts are compressed with bolts with significant preload. With the same total force, it is supposed to halve the level of surface shear stresses, and this means that the stress concentration factor decrease by about half. This is a trial (but difficult) way - to try to get closer to obtaining the real strength of the composite, by reducing the stress concentration.

Conclusions
1. The simplest analytical estimates show that an increase in the surfaces along which shear forces are applied to hold the specimen in the grip will increase the maximum tensile load that the specimen can withstand.
2. An increase in the length of the clamped part of the specimen contributes to a decrease in the stress concentration near the grips, and an increase in thickness leads to the opposite effect: the stress concentration near the grips increases.
3. In the given in Figure 4 scheme suggests only the capture concept. A detailed calculation of the thickness and number of gripper plates, the dimensions of the fasteners and the required forces in the bolts is required. A calculation of the stress distribution in the proposed type of specimens will also be required. Some issues are related to manufacturing technology and groove sizes.

References
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