Space Product Reliability Evaluation in Two-Stage Development Based on Scaling Factor

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ABSTRACT Space products have high reliability and will experience multi-stage development with respective tests after each stage. Due to the limit of budget and time-to-market, the tests usually have small sample sizes and short durations. A few or no failures are more likely to appear in such tests for high-reliability products. In practical engineering, reliability assessment will not be satisfactory if only depending on data from the current test. And this problem will be improved if multi-stage data is included. In this paper, a scaling factor method is adopted to deal with the conversion problem of the data in the previous stage. Then the Bayesian posteriori estimations of the reliability parameters are obtained by the Markov Chain Monte Carlo (MCMC) sampling method. After two-dimensional integral is calculated, the confidence lower limit of reliability can be given accurately. A case study is carried out to illustrate the effectiveness of the proposed method.

INDEX TERMS MCMC, multi-stage test, reliability evaluation, scaling factor.

I. INTRODUCTION
High reliability is required for space products, such as turbopump of the liquid rocket engine, as insufficient reliability for such products will lead to catastrophic failure of a rocket launch. Reliability is most achieved in product development, during which, reliability tests are often used to evaluate whether the product has reached the required reliability [1]. The reliability tests for space products are usually in small sample sizes and short duration, due to the pressure of budget and time-to-market. In these tests, a few or even no failures occur because of the high reliability of aerospace products, incomplete data cannot be evaluated for reliability using traditional methods. Meanwhile, products will experience multi-stages development with respective tests after each stage, by which, weaknesses are identified and corresponding modifications will be made in the next stage. This belongs to reliability growth, also is an important procedure in product development. As in respective stages, product design changes. The samples from reliability tests may be assumed to follow the same group of life distributions with different parameters. To improve the accuracy of reliability evaluation, it is necessary to make full use of available information, especially, the data from previous stages. Based on this motivation, it is reasonable to design a more appropriate evaluation method.

For multi-stage reliability growth and assessment, current methods are divided into two categories: Classical Parameter Reliability Model (CPRM) and Bayes Reliability Growth Models (BRGM). The representative CPRMs include Duane model [2], AMSAA model [3], [4], Fries model [5] and nonhomogeneous geometric model (NHM) [6]. The traditional CPRMs are unable to describe reliability growth in the case of small sample size and fail to use multi-stage data to improve the accuracy of reliability evaluation. The available BRGMs mainly include the Smith model [7], Barlow-Scheuer model [8], Logistic model [9], scaling factor method [10]. Generally, few of the literature considers prior-distribution or converted test information from the previous stage, which only gives the reliability estimation of the product based on the current stage test and cannot predict the reliability of the final products. Furthermore, Erkanli et al. [11] proposed a reliability growth model with a prior distribution, known as the Dirichlet distribution. The edge distribution and joint distribution characteristics of the Dirichlet distribution are used to better combine expert opinions, experience, and test information about similar products. It can also predict product reliability and manage reliability.
growth. However, the physical meaning of the parameters in the new Dirichlet distribution class is unclear. It is difficult to determine directly from the prior information, and operate in practical applications.

According to whether the prior information is needed, the reliability research methods with zero-failure data mainly include two aspects: the classical analytical calculation method and the Bayesian method. Classical analytical calculation methods such as the modified Maximum Likelihood Estimation method (MMLE) [12], [13], sample space sorting method, and so on, do not need prior information to process data with no failure so that the accuracy of estimation is unsatisfactory. The accuracies of Bayes, multilayer Bayes, and E-Bayes (expect-Bayesian) estimation methods are significantly improved due to the use of prior information. Huiyang [14] analyzed the reliability of bearings with zero-failure by confidence bounds method, matching distribution method, MLE (Maximum Likelihood Estimation), and sampling method based on the Weibull distribution model. Jia et al. [15] used the matching distribution method to obtain the distribution of failure probabilities by considering the convex and concave property of the function which was applied to calculate reliability at a given time. Furthermore, a large number of studies [16]−[18] have been published to determine the prior distribution based on existing information, and then related Bayesian methods are used to obtain the evaluation. Distinctly, Bayesian theory is a very nice method. The employment of that should combine with the specific experimental background to determine the type of prior distribution.

The methods proposed in previous studies are not applicable in the case of small sample size with a few or even no failures. Therefore, this paper proposes a Bayesian evaluation method based on information conversion, which not only solves problems mentioned above but also provides the confidence lower limit of the reliability. In Section III, a new scaling factor method under Weibull life distribution will be given. The prior distributions of parameters are determined by the expertise. Then Section IV shows the point estimate and interval estimate of reliability by applying of converted data. Section V is a case study to verify this method. The paper is summarized in Section VI.

II. LIFE DISTRIBUTION AND TEST DESIGN

A. LIFE DISTRIBUTION AND ASSUMPTION

Weibull distribution, proposed in the mid-20th century for manufacturing and failure analysis, is probably the most widely used distribution to describe product life. The reason is Weibull distribution has great adaptability by the variation of parameters. Therefore, it is assumed that the product life obeys Weibull distribution in this paper. Then the cumulative distribution function (CDF) and probability density function (PDF) of the two-parameter Weibull distribution are

\[
F(t, m, \eta) = 1 - \exp\left(-\frac{t}{\eta}\right)^m, \quad (1)
\]

\[
f(t, m, \eta) = \frac{m}{\eta^m} t^{m-1} \exp\left(-\frac{t}{\eta}\right)^m, \quad (2)
\]

where \( m \) is the shape parameter which is a material-related parameter and \( \eta \) is the scale parameter, also known as characteristic life. The corresponding reliability function is

\[
R(t, m, \eta) = \exp\left(-\frac{t}{\eta}\right)^m. \quad (3)
\]

In order to describe the problem explicitly, we assume:

1) The shape parameter in the 1st stage is predetermined based on previous studies or expertise.

2) The improvement has been implemented before every next stage. Therefore, the parameters of Weibull distribution are changed. Namely, there is a reliability growth between the adjacent two stages.

B. TEST BACKGROUND

1) TEST DESIGN OF THE FIRST STAGE

There is a reliability test that stops at \( t_a \) and the number of test sample is \( n_1 \). This type of reliability test as described above are called time-censored test or type-I censoring test. If there are \( r \) failures, the failure data is \( t_1, t_2, \ldots, t_r \), then \( t_{r+1} = t_{r+2} = \ldots = t_{n_1} = t_a \) are the right-censored data.

Various methods have been proposed for reliability evaluation of Weibull distribution with failure data. According to the assumptions above, the MLE method [19] is adopted here to estimate the characteristic life \( \eta_1 \). As the likelihood function is

\[
L(D|m, \eta) = \frac{n_1!}{(n_1 - r)!} \prod_{i=1}^{r} f(t_i) R(t_a)_{\eta_1 - r}. \quad (4)
\]

By substituting the known shape parameter \( m_1 \), the characteristic life of the 1st stage is

\[
\eta_1 = \left(\frac{1}{r} \sum_{i=1}^{n_1} \frac{m_1}{t_i}\right)^{1/m_1}. \quad (5)
\]

2) TEST DESIGN OF THE SECOND STAGE

In the second stage, the sample size is recorded as \( n_2 \) under censored time \( t_b \). There is no failure occurred until the end of this test.

For the second stage, the only known information is the sample size and censored time, which makes it improper in engineering projects to evaluate reliability by the 2nd stage data alone. Therefore, multi-source information is proposed and utilized to obtain more precise reliability evaluations.

III. DETERMINE THE SCALING FACTOR

Taking the design improvement into consideration, the previous stage data should be converted into prior information when the prior distribution of the current stage is determined. The conversion is implemented by introducing a scaling factor.

Jinhuai [20] introduced the classical scaling factor of the binomial distribution. The failure probability is obtained by
TABLE 1. The data of two-stage and converted data after processing.

| Stage | \( t_1 \) | ... | \( t_{n_1} \) | Stage | \( t_b \) | ... | \( t_b \) | Converted | \( t_{11} \) | ... | \( t_{1n_1} \) | \( t_b \) | ... | \( t_b \) |
|-------|---------|-----|----------------|-------|---------|-----|----------------|---------|-------|---------|-----|----------------|---------|-----|----------------|

MLE where \( r \) is the number of failures and \( n \) is the sample size.

\[
p = \frac{r}{n}.
\]  

Then the scaling factor is defined as

\[
C = \frac{p_i}{p_{i+1}},
\]  

where \( p_i \) is failure probability of the \( i \)th stage. The converted data of previous stage that can be utilized in current stage is obtained by

\[
(n_i, r_i) \rightarrow (n_iC, r_i).
\]  

However, as the life distribution of space product is assumed to obey Weibull distribution, it is improper to define the scaling factor by (7). Moreover, (8) only converts the sample size between two stages, while ignoring the important lifetime. Therefore, inspired by this method, a modified scaling factor method is introduced here in the two-stage test.

\[
p_{ij} = P(T < t_j) = 1 - \exp\left(-\frac{t_j^{m_1}}{\eta_1}\right), \quad j = 1, \ldots, n_1,
\]  

where, \( p_{ij} \) is the failure probability when \( t = t_j \) in the 1st stage. Bailey [21] proposed a new estimate method of failure probability with zero-failure data. It performs better than existing methods. So, the failure probability of the 2nd stage is

\[
p_2 = 1 - 0.5 \frac{n_1^{m_1}}{\eta_1^{m_1}}.
\]  

Based on the above definition, the modified scaling factor is defined as

\[
C_j = \frac{p_{ij}}{p_2}.
\]  

And then, the converted data is obtained by

\[
t_j \rightarrow t_{ij} = t_j \times C_j,
\]  

where \( t_{ij} \) is the converted lifetime of the 1st stage which can be used in the 2nd stage and listed in Table 1. This means that the reliability of the 2nd stage will be evaluated next by combining the two-stage data.

The Bayesian method is the most suitable method for processing small sample size data. In many engineering cases, multi-source information is fused by the Bayesian method as the prior distribution. Then it is updated by the sample data to obtain posterior distribution. Therefore, Section IV will show the process of reliability assessment using the Bayesian method.

IV. RELIABILITY ESTIMATION BY BAYESIAN METHOD

A. BAYESIAN PRIOR DISTRIBUTION

The prior distribution is the most important procedure in Bayesian assessment. The selection of that determines the quality of Bayesian assessment, so it should be carried out prudently. In this paper, the prior distribution combines the information of previous stage and expertise to make the estimates more precise. The reliability \( R_e \), at given time \( \tau \), can be calculated by Weibull parameters \((m, \eta)\). Usually, the reliability at \( \tau \) in the 2nd stage is greater than that in the 1st stage due to the reliability growth. So, the lower limit of \( R_e \) can be defined as

\[
R_L = \exp\left(-\frac{\tau}{\eta_1}\right)^{m_1}.
\]  

Ronghua et al. [22] and Yan and Bo [23] selected logarithmic inverse gamma distribution as the prior distribution of \( R_e \), as such distribution functions exhibit different shapes with the changes of parameters. Thus, this method can approximate various distributions. However, there is no enough information to determine the parameters of Gamma distribution, except the lower limit by (13) and upper limit which must be less than 1. So, it is reasonable and practical to assume that the prior distribution of \( R_e \) is uniform.

\[
\pi(R_e) = \frac{1}{1 - R_L}.
\]  

Besides, the prior distribution of the scale parameter \( m_2 \) also needs to be determined. In our case, there are two options. When it is noninformative prior distribution, the function is

\[
\pi(m_2) \propto m_2^{-1}, \quad m_2 \geq 0.
\]  

Then the function can be written as (16) if a uniform distribution is assumed.

\[
\pi(m_2) = \frac{1}{m_b - m_a}, \quad m_a \leq m_2 \leq m_b,
\]  

where the upper and lower limits \( m_a \) and \( m_b \) are given by expertise.

There is an improvement in reliability between the two stages as the assumption. However, the change of the shape parameters \( m \) is not predictable. In engineering practice, an interval that contains the true value can be provided by experience. Due to this reason, uniform distribution in (16) is a better option of the shape parameter \( m_2 \).

B. BAYESIAN ESTIMATION BY METROPOLIS-HASTINGS(M-H) ALGORITHM

The reliability assessment of 1st stage has been accomplished in Section II, which provides prior information for the second stage, and helps to produce the prior distribution in (14). At the same time, the converted data of the 1st stage is supplemented to the 2nd stage, making the available data change from a single group of data without failure into multiple groups of data with failure records. This is a very meaningful multi-source information fusion method, which
can improve the accuracy of reliability assessment to some extent. Employing multiple groups of failure data is reflected in (17).

According to the data in Table 1, the likelihood function is

\[ L(D,m_2, \eta_2) = \frac{(n_1 + n_2)!}{(n_1 + n_2 - r)!} \prod_{i=1}^{r} f(t_{1i}) R(t_{1i})^{n_1} \eta_2 R(t_{2i})^{n_2}, \]

(17)

where \( t_{1i} (i = 1, 2 \ldots r) \) are converted failure data, \( t_{1i} \) is the converted truncation time of the first stage.

In addition, the reliability at time \( \tau \) is

\[ R_\tau = \exp(-\frac{\tau}{\eta_2}), \]

(18)

\[ \eta_2^{-m_2} = \frac{-\ln R_\tau}{m_2}. \]

(19)

(19) is another form of (18) and then introduced into (17). Consequently, the likelihood function of \( R_\tau \) and \( m_2 \) is showed in (20).

\[ L(D,m_2, R_\tau) = \left( \frac{-m_2 \ln R_\tau}{m_2} \right)^r \prod_{i=1}^{r} t_{1i}^{n_1} R_\tau \frac{\tau^{m_1 - 1}}{\tau^{m_2} + \eta_2 n_2 \tau \tau^{m_2}}. \]

(20)

By combining with Bayesian theory, the joint posterior distribution of \( R_\tau \) and \( m_2 \) is

\[ \pi(m_2, R_\tau | D) = \frac{\pi(m_2)\pi(R_\tau)L(D,m_2, R_\tau)}{\int \int \pi(m_2)\pi(R_\tau)L(D,m_2, R_\tau)dm_2dR_\tau}. \]

(21)

Moreover, the kernel of the posterior distribution is represented after substituting (14), (16) and (20) into (21).

\[ \pi(m_2, R_\tau | D) \propto \left( \frac{-m_2 \ln R_\tau}{m_2} \right)^r \prod_{i=1}^{r} t_{1i}^{n_1} R_\tau \frac{\tau^{m_1 - 1}}{\tau^{m_2} + \eta_2 n_2 \tau \tau^{m_2}}. \]

(22)

The performance to obtain estimates of \((m_2, R_\tau)\) is the multivariate distribution sampling method. MCMC method is a promising sampling method for complex distribution without an analytical solution [24]. M-H sampling and Gibbs sampling are two widely used sampling plans in MCMC. What’s more, the Markov process constructed by the M-H sampling algorithm satisfies the meticulous and stationary conditions.

The basic principle of the M-H algorithm: suppose that it needs to sample from the target probability density function \( p(\theta) \) while \( \theta \) satisfies the condition that \(-\infty < \theta < \infty\). M-H sampling method generates a sequence based on Markov chain: \( \theta^{(1)} \rightarrow \theta^{(2)} \rightarrow \ldots \theta^{(t)} \rightarrow \), where \( \theta^{(t)} \) expresses the state of the Markov chain at the \( t \)-th generation. In the process of this sampling, the state value \( \theta^{(1)} \) is initialized first. Then a distribution \( q(\theta | \theta^{(t-1)}) \) is used to form a new candidate state \( \theta^{(s)} \). A probability \( \alpha \) is used to judge whether to accept the new value or reject it. This process continues until the sampling converges. After convergence, \( \theta^{(t)} \) is the generated sample in the target distribution \( p(\theta) \).

For multivariate sampling, the M-H sampling algorithm usually has the following two strategies: Block-wise and Component-wise. Since the given distribution has the same dimension as the target function, the Block-wise M-H algorithm is superior. For the above-mentioned updating strategy, vector updating strategy is adopted in the Block-wise M-H sampling algorithm, namely, \( \Theta = (\theta_1, \theta_2, \ldots, \theta_N) \) \( \theta_i \) represent the status in MCMC). The algorithm procedure can be described as:

(1) Initialize time \( t = 1 \);

(2) Set the value of \( u \), and initializes the initial state \( \Theta^{(t)} = u \) (the initial values of \( m_2 \) and \( R_\tau \) are random numbers in each uniform distribution in our case);

(3) Repeat the following process:

(3.1) Set \( t = t + 1 \);

(3.2) Choose a candidate state \( \Theta^{(s)} \) from the given distribution \( q(\theta | \Theta^{(t-1)}) \);

(3.3) Calculate the probability of acceptance:

\[ \alpha = \min \left( \frac{p(\Theta^{(s)} | D) q(\Theta^{(t-1)} | \Theta^{(s)})}{p(\Theta^{(t-1)} | D) q(\Theta^{(s)} | \Theta^{(t-1)})} \right); \]

(3.4) Generate a random number \( a \) from the uniform distribution \( U \sim (0, 1) \);

(3.5) If \( a \leq \alpha \), the newly generated value is accepted and \( \Theta^{(t)} = \Theta^{(s)} \), otherwise, \( \Theta^{(t)} = \Theta^{(t-1)} \).

(4) Until \( t = T \).

After programming through Matlab or Python, the point estimate of \((m_2, R_\tau)\) will be obtained. Then, the characteristic life of the Weibull distribution is calculated according to (19). Furthermore, by substituting (21) into (23), the lower confidence limit \( R_{LT} \) at a given confidence level \( \gamma \) is obtained by (24). This complex two-dimensional integral problem can be solved by MATLAB packages.

\[ \pi(R_\tau | D) = \int_{\eta_1}^{\eta_2} \pi(m_2, R_\tau | D)dm_2. \]

(24)

V. A STUDY CASE

A. RELIABILITY ASSESSMENT

A key component of liquid rocket engine is under two-stage development. The mission time \( \tau \) is 500s. In designing engineers, most of the reliability test time probably based on the real working duration for turbopump. All test data are listed in Table 2. There is only one failure in the 1st stage and no failure occurred in the 2nd stage. According to the experts, the shape parameter of the 1st stage is 1.5.

Using the method mentioned in Sections II and III, \( \eta_1 \) can be calculated by (25) and the scaling factors are listed in Table 3. For the 2nd stage test, the information we have is sample size \( n_2 \), censored time \( t_b \), and no failure occurred.

| Sample size | Failure time | Censored time |
|-------------|--------------|---------------|
| First stage | 10           | 316           | 500           |
| Second stage| 10           | -             | 500           |

**TABLE 2. The test data of two-stage.**
TABLE 3. The scaling factor between the two-stage test.

| $p_{ij}$ | $p_2$ | $C_f$ | $t_{ij}$ |
|---------|-------|-------|---------|
| 0.0515  | 0.0611| 0.8431| 266.39  |
| 0.0999  |       | 1.6357| 817.84  |

FIGURE 1. MCMC sampling results of $m_2$ and $R_\tau$.

In this case, the empirical formula (10) is used to calculate $p_2$ which is only related to the sample size $n_2$. Therefore $p_2$ is a constant in Table 3, $p_{ij}$ is treated as time-dependent because the data in the 1st stage is sufficient and the Weibull distribution is assumed. The parameter $\eta_1$ has been obtained by the MLE method in (25). Next, the failure probability at different times are known by (9). So there are two sets in $p_{1j}$, as well as $C_j$ and $t_{1j}$ in Table 3.

$$\eta_1 = \left( \sum_{i=1}^{10} t_i \right)^{1.5} = 2243.1.$$  \hspace{1cm} (25)

By referring to (13), (14) and (16), the prior distributions of $R_\tau$ and $m_2$ are

$$\pi(R_\tau) = \frac{1}{1-R_L}(R_L = 0.9).$$ \hspace{1cm} (26)

$$\pi(m_2) = \frac{m_b - m_a}{m_a = 1, m_b = 4},$$ \hspace{1cm} (27)

where $R_L$ is calculated by (13), and $m_a$ and $m_b$ are given by experts. Then the kernel of the posterior distribution is

$$\pi(m_2, R_\tau|D) \propto \frac{-m_2 \ln R_\tau}{t_{1j}} \cdot t_{1j}^{m_2-1} \cdot R_\tau^{m_2+\alpha_0+\beta_0/2} \cdot e^{-\frac{m_2}{2}}.$$ \hspace{1cm} (28)

Variables are estimated by an algorithm and the results are shown in Fig.1 and 2. Expected values of $(m_2, R_\tau)$ are (1.99, 0.96).

The confidence lower limits of reliability at different confidence levels $\gamma$ are listed in Table 4. $\eta_2$ is obtained by substituting $m_2$ and $R_\tau$ into (19). Then a comparison between the two stages is shown in Fig.3.

B. COMPARISON OF METHODS

There are no applicable methods that can solve this proposed problem directly. To illustrate the validity of the proposed method in this paper (called “Paper method-CI”), some methods from published papers are implemented here for comparison [25]. Another scaling factor method by Jinhuai [20] illustrated in Section III (called “Method-C”) is employed for comparison. Then Li mentioned the classical confidence bounds methods (called “Classical Method”) with no failure data in [26].

The CDFs under various methods are compared and plotted in Fig.4. Method-C and Classical Method both have fatal flaws. As the product follows Weibull life distribution, it is improper to use the binomial distribution method to handle data conversion. In (25), $\eta_1 = 2243.1$, and there is a reliability growth between two stages, therefore, $\eta_2 > \eta_1$ should behold. But from Table 5, both estimates are less than 2243.1. This indicates that they are not proper to be used in our case. The Classical Method also has been unsuitable because the shape parameter $m_2$ is assumed to be 1.5 before the estimation of $\eta_2$. While the MLE CDF curve is very close to our proposed method, but the solution of MLE is unstable when two parameters are estimated at the same time. It is the artificiality that makes it stable when $m_2$ is approximately equal to Paper method-CI ($m_2 = 1.99$).

TABLE 4. The reliability estimates at different confidence levels $\gamma$.

| $\gamma$ | 0.95 | 0.90 | 0.75 |
|---------|------|------|------|
| $R_{LL}$| 0.9108| 0.9190| 0.9380 |

TABLE 5. Comparisons of different methods.

| Methods          | $m_2$ | $\eta_2$ |
|------------------|-------|----------|
| Paper method-CI  | 1.99  | 2942.0   |
| Method-C         | 2.45  | 1845.0   |
| Classical Method | 1.50 (Pre-set data) | 1866.7 |
| MLE              | 1.95  | 3020.8   |
| Zero-Failure 1.5 | 1.50 (Pre-set data) | 1989.1 |
| Zero-Failure 1.99| 1.99 (Pre-set data) | 1415.8 |
| MMLE 1.5         | 1.50 (Pre-set data) | 5325.5 |
| MMLE 1.99        | 1.99 (Pre-set data) | 2974.3 |
To highlight the advantages of Paper method-CI more clearly, CDFs obtained by [27] (called “Zero-Failure”) and MMLE method are compared in Fig.5. The MMLE method showed in (29) is an improvement of MLE, and it is a method that can perform reliability assessment with no-failure.

$$\eta_M = \left( \sum_{\xi=1}^{k} n_{\xi} t_{\xi}^{m} \right)^{-1} \ln(1 - \alpha),$$  \hspace{1cm} (29)

where $k$ is the number of time-censored tests, $t_{\xi}$ is the censored time and there is no failure occurred in every tests. Confidence $\alpha$ is same as Zero-Failure method, with a value of 0.25.

Both of Zero-Failure and MMLE are capable to calculate $\eta_2$ in this case. To facilitate the comparisons with other methods, we should pre-set the shape parameter $m_2$ with 1.5 and 1.99 respectively to calculate $\eta_2$. From Table 5, the Zero-Failure method does not support $\eta_2 > \eta_1$, so it is improper in this case. This pessimistic result is due to the ignorance of the data from the 1st stage. The result of MMLE when $m_2 = 1.5$ is over-optimistic. While $m_2 = 1.99$ is predetermined, the CDF curve is very close to Paper method-CI. These comparisons illustrate the superiority of Paper method-CI which not only obtains the estimates of two Weibull distribution parameters but makes full use of the data in two stages to produce more accurate results.

**C. STUDY ON SAMPLE SIZE WITHOUT FAILURE**

Next, the influence of the sample size $n_2$ on the evaluates $m_2$, $\eta_2$ and $R_T$ will be discussed. The results tendencies with the increasing sample size from 1 to 50 are shown in Fig.6.
The vertical axes of the three graphs in Fig.6 are shape parameter \( m_2 \), mission reliability \( R_{\tau} \), and characteristic life \( \eta_2 \). There is a little change of \( m_2 \) because of its material correlation in engineering practice. Reliability goes up slightly interpreted as that the more products are put into test with no-failure \( (n_2) \), the higher reliability exhibits. That is consistent with the actuality and failure mechanism. Similarly, the characteristic life \( \eta_2 \) shows a remarkable increase. And due to the transfer of the evaluated variable, there are some fluctuations.

With the increase of the sample size \( n_2 \), the estimates are changed and show an obvious trend. It explains the stability and applicability of the proposed method, which can be used both in large sample size and small sample size.

**VI. CONCLUSION**

The proposed scaling factor method exhibits better performance due to the advantage of using multi-stage test data. Failure data of previous stage are converted. Then the Bayesian method is applied to reliability assessment by the mixed current stage data and converted data. The proposed method has the following merits:

1. This method has a wide range of applications: it can be used whether the test sample size is small or not and whether there are failures in the tests. Besides, it can be applied in the same manner when multi-stage (more than two stages) development is involved.

2. To solve engineering problems, the scaling factor method based on Weibull distribution is more suitable than the Binomial scaling method.

3. Comparing with the MLE method, this method is stability and applicability because it provides a robust and accurate solution.

4. The confidence lower limit of reliability is calculated by this method, which provides a basis for further experimental design in space products. A solution to calculate the complicated two-dimensional integral problem is presented, which makes our method feasible and can be extended to other engineering fields.

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