Theoretical Study and calculation The cold Reaction Rate of Deuteron Fusion In Nickel Metal Using Bose–Einstein Condensate Theory

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Abstract: In this paper, we focused on the investigated and studied the cold fusion reaction rate for D-D using the theory of Bose-Einstein condensation and depending on the quantum mechanics consideration. The quantum theory was based on the concept of single conventional of deuterons in Nickel-metal due to Bose-Einstein condensation, it has supplied a consistent description and explained of the experimental data. The analysis theory model has capable of explaining the physical behaviour of deuteron induced nuclear reactions in Nickel metals upon the five-star matter, it’s the most expected for a quantitative predicted of the physical theory. Based on the Bose-Einstein condensation theorem formulation, we calculation the cold fusion reaction rate for D-D transfer to Nickel-metal using the astrophysical S factors for d(d,p)T, d(d, n)3He reactions and for reaction. The results of the calculation for three reactions give rise a wide compatible with the other experimental works.

Keywords: Deuteron-Deuteron Fusion, Nickel Metal, Bose Einstein Condensates, reaction rate.

1. Introduction

Since the Bose-Einstein condensation (BEC) has been predicted in 1924 by Einstein and it leads to becoming a good new topic field in physics, it had been the attraction of interesting researcher to gate high attention of behaviour of condensate matter[1]–[3]. Furthermore, the atoms in ideal Bose gas could be occupied and condensed into certain quantum states as a consequence of Bose quantum statistical mechanics and its certain energy states[4]. Several physical quantities can be described utilizing the notation of BEC comprising liquid 4He, it exactions in semiconductors, pions and kaons in the condensed nuclear matter[5]. Particles that enjoy low temperature and high density subject to quantum statistics called Bose-Einstein and these particles are termed bosons[6], [7]. Bosons are included photons and helium 4He for instance shared one quantum state. It has been stated that cooling a gas of atoms under a critical temperature (approximates 106 times lower than the lowest temperature in the globe) leads atoms to condense and become indistinguishable. Although fermions (3He) obey Fermi-Dirac statistics, they
inhabit various quantum state at zero temperature[8]. Moreover, the phenomenon of BEC touches a several disciplines of physics, including statistical mechanics, thermodynamics, quantum mechanics, condensed matter physics, field theory, finally in nuclear physics[9]. Yeong E. Kim et al in (2000) accomplished the ground state solutions for trapped bosons in an isotropic trap by used (ELTB) mechanism. In order to obtain theoretical formulas for calculating the rates and probabilities of nuclear fusion for nuclei trapped in ion. The solutions have been used in the system. However, the formula of the fusion rate has been applied to the D-D fusion rates trapped by the ionic trap and micro trap[10]. M.W.Zwierlein et al in (2003) observed that B.E.C. on the molecular level, it was tabulated for fermionic ⁶Li atoms using evaporative cooling in an optical dipole trap. The lithium gas atoms turned into ⁶Li₂ molecules. By the sudden start of a bimodal density distribution, a Bose-Einstein condenser was estimated to reach 900 000 molecules at a temperature less than 600 nK[11]. Alexander L. and et al in (2006) were achieved the B.E.C. mechanism for one type of LENR operations in condensed substance to the case of a mix of two various types of the Bose nuclei in harmonic traps. Relying on the "ratio" of the parameters within, it seemed that both parts might cohabit in the same area of space, instead of the Coulomb repulsion among both types. This lead to getting a close selection rule including charges and nuclear masses of both types. For a mix types of D and Li, these anticipate that the reacting rate of (D+ ⁶Li) might be bigger than reacting rate of (D+D), that’s permeation to suggesting that the (D+ ⁶Li) reactions might control on the (D+D) reactions in experiments of low energy nuclear reaction(LENR) in condensed substances[12]. Yeong E. Kim in 2010 has developed a theory of nuclear fusion for condensing Bose-Einstein condensation to describe numerous diverse experimental data of the nuclear reaction caused by deuterons in metals, which noticed in "gas loading and electrolysis" experiments. The theory showed that fusion energy transferring to metal could be accomplished by the stopping power of metal[13]. Takeo Oku in 2018 described the conditions of nuclear fusion condensation devices and the possible applications of nanomaterials of the nuclear fusion devices. Catalyzed fusion by muon was considered as one of the ways used in nuclear fusion to allow the fusion to occur at very low temperatures. The charged muons were created by (heavy ions or protons) irradiated on metals like copper/ beryllium with high energy[14]. Djamel et al in (2019) proposed that the energy liberated from the "green cold fusion" be used to treat water through the distillation process. The green word refers to the processes which is not accompanied by pollution and nuclear dangers. On the other hand, the large energy released through cold fusion suggests its investment in the method of distillation to obtain active performance in terms of time and cost. The phenomenon of cold fusion was noticed during the study of substances that have a very high solubility in hydrogen isotopes such that palladium , some other materials and alloys[15]. In this present work , we
introduce a model for reaction rate fusion of trapping deuteron in metal. The reaction rate has calculated for D trapped in Nickel metal because the Ni has properties similar that palladium metal and easily solvable in hydrogen isotopes.

2. Theory

Under assume the N-bosons system in Hilbert space associated with wave function.

\[ |\varphi(r)\rangle = \rho_{(r)}^{\frac{3N-1}{2}} |\phi(\rho)\rangle \]  

(1)

Where \( \rho \) is the density of bosons at system, the fusion rate for deuterons trap (FRDT) in metal is[16].

\[ FRDT = -2\Omega \frac{\hbar}{h} \frac{\int_{0}^{\infty} \phi_{(r)}^{*} \sum_{F} \phi_{(r)} d\rho}{\int_{0}^{\infty} \phi_{(\rho)}^{*} \phi_{(\rho)} d\rho} \]  

(2)

Where \( \Omega \) is the probability of the ground state occupation, \( h \) is the Dirac constant and \( \sum_{F} \) is the Fermi potential is[17].

\[ \sum_{F} = -\frac{Ah}{2} \delta(r) \]  

(3)

Substituting Eq.(3) in Eq.(2) and integrated to results.

\[ FRDT = -2\Omega \frac{\hbar^{2}}{h} \frac{\rho_{(r)}^{\frac{3N-1}{2}}}{\rho_{(r)}^{\frac{3N-1}{2}}} \int_{0}^{\infty} \phi_{(r)}^{*} \sum_{F} \phi_{(r)} d\rho \]  

(4)

Since the potential operator for wave function \( \sum_{F} \phi_{(\rho)} \) in Eq.(4) with Eq.(3) leads to.

\[ \sum_{F} \phi_{(\rho)} = \frac{-Ah N(N - 1) \Gamma(3N/2)}{2^{\frac{3N-1}{2}} \sqrt{\pi} \Gamma\left(3N-3/2\right)} \frac{1}{2\pi\rho^{3}} \int_{0}^{\infty} \phi_{(r)} \delta(r-\rho) d\rho = \frac{-Ah N(N - 1) \Gamma(3N/2)}{2^{\frac{3N-1}{2}} \sqrt{\pi} \Gamma\left(3N-3/2\right)} \frac{\phi_{(\rho)}}{\rho^{3}} \]  

(5)

Inserting Eq.(5) in Eq.(4) to obtained
\[ FRDT = -\frac{2}{\hbar} \Omega_{\omega} \frac{\Delta h N(N-1)}{2N(N-3/2)} \frac{\Gamma(3N/2)}{\Gamma(3N-3/2)} \int_{0}^{\infty} \frac{\phi(\rho) \phi(\rho) d\rho}{\rho^3} \]  

\[ \text{Assume that } \rho^3 = \left(\frac{\hbar}{m_0}\right)^{3/2} \tilde{\rho} , \text{then the wave function in Eq.(1) become.} \]

\[ |\Phi(\tilde{\rho})\rangle = \sum c_i \rho^{3N-1/2} e^{-\frac{(\tilde{\rho})^2}{2}}, \]  

\[ m_0 \text{ is the quantum electro dynamic "fine structure "}[18]. \text{Then inserting Eq.(7) in Eq.(6) and simply to} \]

\[ \int_{0}^{\infty} \tilde{\rho}^{3N-1} e^{-\frac{(\tilde{\rho})^2}{2}} d\tilde{\rho} = \frac{1}{2} \alpha^{3N-3} \Gamma\left(\frac{3N-3}{2}\right) \]  

\[ \text{And} \]

\[ \int_{0}^{\infty} \tilde{\rho}^{3N-1} e^{-\frac{(\tilde{\rho})^2}{2}} d\tilde{\rho} = \frac{1}{2} \alpha^{3N} \Gamma\left(\frac{3N}{2}\right) \]  

\[ \text{Substituting Eq.(9) and Eq.(10) in Eq.(8) to results.} \]

\[ FRDT = \frac{A N(N-1)}{2} \Delta h N \left(\frac{3N}{2}\right) \frac{\Gamma(3N-3/2)}{\Gamma(3N-3/2)} \left(\frac{m_0}{h}\right)^{3/2} \frac{\Gamma\left(\frac{3N-3}{2}\right)}{\Gamma\left(\frac{3N}{2}\right)} \frac{3}{2} = \frac{A N(N-1)\left(\frac{m_0}{h}\right)^{3/2}}{2} \]  

\[ \text{By satisfy strength of trapping is } \alpha_t = \left(\frac{\zeta}{\zeta}\right)^{3/2} \text{ and introduce a parameter } \zeta = 2 \left(\frac{m c^2}{\hbar \omega_{\omega0}}\right)^{3/2} \alpha N \]  

\[ \text{Where } m \text{ is the mass of Bosons, } c \text{ is light speed and } \omega \text{ is the frequency. Substituting Eq.(12) with } \alpha_t \text{ in Eq.(11) to results.} \]
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\[
FRDT = \frac{A \Omega N (N-1)}{2 (2\pi)^{3/2} \left( \frac{m \omega}{\hbar} \right)^{3/2} \left( \frac{2\pi \hbar \omega}{mc^2} \right)^{1/2}} \frac{1}{2 \alpha N}
\]  

(13)

For large \(N\) then \((N-1) \sim N\), we can simplify Eq.(13) to .

\[
FRDT = \frac{3A}{8 \pi a \hbar c} N \Omega \omega \left( \frac{m \omega}{\hbar} \right)^{1/2}
\]  

(14)

By assume that,

\[
B = \frac{3A}{8 \pi a} \frac{m}{\hbar c} \text{ and } A = \frac{2S \, r_B}{(\pi \, \hbar)}
\]  

(15)

Where \(S\) is S-factor for nuclear fusion reaction ,then reaction rate in Eq.(14) with Eq.(15) can written as .

\[
FRDT = BN \, \Omega \omega^2
\]  

(16)

Where

\[
\omega^2 = \sqrt{\frac{3}{4\pi}} \alpha \left( \frac{bc}{m} \right) \frac{N}{(r)^3}
\]  

(17)

\[
FRDT = A \Omega N^2 \frac{1}{2} \sqrt{\frac{3}{4\pi}} \frac{1}{(r)^3}
\]  

(18)

\[
D = 2(r)
\]

\[
N = n_D \left( \frac{\pi}{6} \right) D^3
\]  

(19)

\[
n_D
\]

\[
FRDT = \frac{1}{4} \sqrt{\frac{3}{\pi}} A \Omega N \, n_D
\]  

(20)
\[
R_t = FRDT N_{trap} = FRDT \frac{V n_D}{N}
\]

\[
N_{trap} = \frac{v n_D}{N} n_D
\]

\[
R_t = \frac{1}{4} \sqrt{\frac{3}{\pi}} A \Omega V n_D^2
\]

\[
e^{-2m_\gamma}
\]

\[
R_t
\]

3. Result and Discussion

\[D-D\ fusion\ reaction\]

\[
(22)\quad S = 110 \times 10^6
\]

\[
\Omega \leq 10^{-22} \quad V = 1 = 1.54 \times 10^{-16}\ \text{cm}^3\text{sec}^{-1}
\]

\[
n_D = 9.1 \times 10^{22}\ \text{cm}^{-3}
\]

\[
D + D \rightarrow ^4\text{He} + 23.8\text{MeV}
\]

10\[6\] and \[S = 110 \times 10^{13}\).
\[
\begin{array}{|c|c|c|}
\hline
 & d + d \rightarrow p + t & d + d \rightarrow ^{4}\text{He} \\
\hline
d + d \rightarrow n + ^{3}\text{He} & & \\
\hline
\end{array}
\]

Figure 1:
three factors: Probability of BE ground state occupation ($\Omega$) Which increases with lower temperatures, $R_t$ and $n^2_f$. 

Figure 2: $S = 110 \times 10^6$. 

Figure 3: $S = 110 \times 10^{13}$. 

\[ g_1 = 110 \times 10^\gamma g_2 \Phi g_3 \]
$$(R_e)$$

, i.e. $n_D = n_{Ni}$

$$D + D \rightarrow ^4He + 23.8 MeV$$

4. Conclusions

$$D + D \rightarrow ^4He + 23.8 MeV$$

$$R_e > R_3$$ and $$R_6 > R_4$$ about $\geq 10^6$

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