Size effect on surface roughness and permeability of rock fracture

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Abstract. Size effect on surface roughness is critical in characterizing the permeable property of rock fracture. Based on the three-dimensional morphology of the fracture surface scanning in the field, the size effect on the fracture surface roughness and permeability were investigated. First, the morphology of the fracture surface was obtained, including asperity height distribution, surface roughness and anisotropic. It was found that the distributions of asperity height followed the normal distribution and the roughness of the scanning fracture surface is anisotropic. Then, the root-mean-square of the first derivative of asperity Z2 was adopted to represent the fracture surface roughness and the size effect on fracture surface roughness was analysed based on the asperity height data. As fracture size continued to increase, fracture surface roughness tended to move a relatively stable state. The roughness variation coefficient of 15% is taken as the criterion for the roughness to enter a stable state and the critical size for the stability of fracture surface roughness is 300 mm. Finally, the size effect on the fracture permeability was analysed. The size effect on the permeability of rock fracture was investigated, the results show that the fracture permeability tended to move to a relatively stable state with the increase of fracture size, which was the same as the fracture surface roughness. The critical fracture size of fracture permeability is less than 100 mm in the study, which is also less than that of fracture surface roughness with the same coefficient of variation. It indicated that a lower coefficient of variation may provide the best estimate of the critical fracture size of fracture permeability comparing with the fracture surface roughness.

Keywords. Surface roughness of rock fracture, permeability, size effect, Representative Volume Element (REV)

1. Introduction

Natural rock masses under geological processes and construction disturbance can result in abundant fractures that potentially dominate flow channels. A comprehensive underground of the fluid flow through fractures is significant in addressing oil/gas production and storage, grouting activities and other geophysical problems. The scale of natural fractures ranges from micrometres to several hundred kilometres. Fracture surface roughness and permeability exhibit a strong correlation with the size effect of fractures.

The roughness of the fracture surface is related to the fracture surface sizes. Fardin et al. [9, 10] adopted the fractal dimension D to characterize the fracture morphology and analysed the relation between the fractal dimension and fracture size. It was found that fractal dimension decreased with the fracture size. When the fracture size increased to a critical value, the fractal dimension tended to be constant. Chen et al. [11] used the sill value and variable range of the variogram to describe the fracture surface roughness and analysed the relation between the fracture surface roughness and size. The
results were shown that the fracture surface roughness decreased with the fracture surface size and the value remained constant when the size was larger than a threshold value. Yan et al. [12] proposed a new parameter AHD to characterize the fracture surface roughness and the relation between AHD and fracture surface size was the same as that between fracture surface roughness and the size by Chen et al. [11]. Nigon et al. [13] analysed the multiscale characterization of fracture surface roughness and it was found that the standard deviation of asperity height and characteristic length of fracture surface decreased with size.

The permeable property of fracture is related to fracture size [14, 15]. Raven and Gale [16] conducted tests about the fluid flow in a natural fracture under normal loading conditions. It was found that the flow rate decreased with the fracture size and the deviation from the cubic increased with the fracture size. Qian et al. [17] analysed the relation between fracture permeability and fracture size. The results showed that the permeability increased linearly with the fracture size, which may be due to two-dimensional tortuous flow. Except for the test method, many scholars used the numerical simulation method to study the size effect on fluid flow in a single fracture [18, 19]. Matsuki et al. [20] created several synthetic fractures based on the measured data and the influences of fracture size on the hydraulic aperture and conductivity were investigated by numerical simulation. It was found that the fracture size on the hydraulic aperture disappeared when the fracture size was larger than 0.2 m. Huang et al. [21] extracted the fracture surfaces ranged from 25 to 200 mm in size from four self-affine with the size of 256 mm and fluid flow through these fractures was simulated. The simulation results showed that the permeability changed significantly first and then tend to be stable with the size of the fracture model. At present, the size effect on fracture surface roughness and fracture hydraulic behaviour are independent, and few people are concerned with the relationship and difference between them.

The size effect on surface roughness and permeability of rock fracture was investigated in the study. Firstly, the process of 3D laser scanning fracture surface in the field and the method of data processing were introduced and the fracture morphology was obtained. Then, the root-mean-square of the first derivative of asperity $Z_2$ was adopted to characterize the fracture surface roughness and the relation between fracture surface roughness $Z_2$ and fracture surface size was analysed. Finally, the numerical simulation of fluid flow in fracture was carried out based on the fracture model established by scanning fracture surface data. The size effect on the permeable property was analysed and the results were compared with the size effect on fracture surface roughness.

2. Data acquisition

2.1. 3D laser scanning of natural fracture surface

The scanning of fracture surfaces was selected from Gongchangling District, Liaoyang City, Liaoning Province, China. The scanning fracture surfaces are from in three areas, numbered A, B and C, as shown in Fig. 1. The lithology of the three areas is quartz sandstone and the rock weathering in area A is more serious than that in the other two areas.

![Figure 1. The natural fracture surface selected in the study](image-url)
fracture surface was about 300×300 mm². The 3D point cloud images of fracture surfaces were obtained after calibration and denoising. Then the topography point cloud image of the fracture surface was reconstructed by interpolation function in MATLAB. These processes are shown in Fig. 2.

Figure 2. Data acquisition process of fracture surface

A total of 104 fracture surfaces with the size of 300×300 mm² were obtained, including 43 fracture surfaces in area A, 12 fracture surfaces in area B and 49 fracture surfaces in area C. The number of point clouds on each fracture surface after calibration and denoising is about 150000-2200000 and the mean sampling interval is estimated to be about 0.64-0.77 mm. Because of the complexity of fracture surface and the application of multi-view scanning automatic splicing technology, the point cloud of splicing position is denser than other positions. In other words, the sampling interval of 3D point cloud images of fracture surface after calibration and denoising is not equal, which is not conducive to subsequent research. It is necessary to use the interpolation function to reconstruct the topography point cloud image of fracture surface in MATLAB to obtain equidistant point cloud data. The sampling interval in MATLAB is 1mm in this study.

2.2. The roughness of scanning fracture surface

The 9 adjacent fracture surfaces were selected and spliced into a larger fracture surface with the size of 900×900 mm², which are numbered C3. The reconstructed topography point cloud image of the fracture surface was shown in Fig. 3.

Figure 3. Reconstructed topography point cloud image of the fracture surface C3.

3. Characterization of large scale fracture surface morphology

3.1. Distribution of asperity height of large scale fracture surface

The parameters characterizing fracture surface morphology include the asperity height [22] and roughness Z² [23]. The spatial and statistical distributions of asperity height of fracture surface C3 were shown in Fig.4. It shows that the spatial distribution of asperity height is uneven and the statistical results show that they follow the normal distribution. The range of asperity height is range -24.80 to 28.60 mm. The mean and standard deviation of asperity height are -0.634 and 3.953 mm, respectively.
3.2. Anisotropy of fracture surface roughness

The morphology of rock fracture surface is complex and the roughness along different directions is different, i.e., the roughness of fracture surface has directionality. The root-mean-square of the first derivative of asperity $Z_2$ is used to characterize the roughness of two-dimensional fracture surface, which is expressed as [24]:

$$Z_2 = \frac{1}{(n-1)(\Delta x)^2} \sqrt{\sum_{i=1}^{n} (z_{i+1} - z_i)^2}$$  \hspace{1cm} (1)

where, $z_i$ is the asperity height at point $i$, $n$ is the number of point clouds, $\Delta x$ is the distance in the axial direction between point $i+1$ and $i$. The distance is the sampling interval and it is constant in the study. The roughness $Z_2$ in different directions [25,26] was calculated according to Eq. (1), as shown in Fig. 5. The direction interval is $10^\circ$ and the roughness of each fracture surface was calculated in 36 directions.

![Figure 5](image_url)

**Figure 5.** The roughness $Z_2$ of fracture surface C3 in different directions.

Fig. 5 shows that the roughness of different directions on the same fracture surface is different. A dimensionless parameter, discontinuity anisotropic coefficient (DAC), is selected to quantitatively describe the anisotropy of fracture surface roughness, which is expressed as [26]:

$$DAC = 1 - \frac{1}{e^{CV_{Z_2}^2}}$$  \hspace{1cm} (2)

where, $CV_{Z_2}$ is the variation coefficient [27] of roughness $Z_2$ in 36 directions. When DAC=0, the roughness of the fracture surface is isotropic. When $0<$DAC$<1$, the roughness of the fracture surface is anisotropic. The larger the DAC value is, the more anisotropic the fracture surface is. The DAC value was obtained according to Eq. (3) and the DAC of fracture surface C3 is 0.042.

4. Size effect on fracture surface roughness

To analyse the size effect on fracture surface roughness, the “base” fracture surface was discretized into sampling fracture surfaces of various sizes in rectangular coordinate systems, which was shown in Fig. 6 [2,28]. The “base” and sampling fracture surface are all squares. The size of the fracture surface is represented by side length for convenience in the following study. The size of the
“base” fracture surface is 900 mm and the sizes of the sampling fracture surfaces range from 100 to 800 mm with an interval of 100 mm. 10 sampling fracture surfaces of each size are obtained at different positions to eliminate the particularity.

**Figure 6.** Method of obtaining sampling fracture surface with different sizes from “base” fracture surface

The sampling fracture surface roughness along the x-axis and y-axis were calculated, respectively. $Z_2$ is calculated according to a fracture profile on the fracture surface. The mean value $\bar{Z}_2$ of all fracture profiles along a certain direction of the fracture surface is the roughness in the direction. The roughness of sampling fracture surfaces along the x-axis and y-axis were obtained, as shown in Fig. 8. The roughness of fracture surfaces varies remarkably at the small fracture surface size. As fracture surface size continues to increase, fracture surface roughness tends to move to a relatively stable state. The roughness variation coefficient of 15% is taken as the criterion for the roughness to enter a stable state [29]. The REV sizes of roughness are all 300 mm. The equivalent roughness of fracture surface along the x-axis and y-axis are 0.178 and 0.153, respectively.

**Figure 7.** The relation between roughness and fracture surface sizes

**5. Size effect on fracture permeability**

**5.1. Fracture models and boundary conditions**

The fracture model was obtained by translating the scanned fracture surface to a certain distance using the FEM software COMSOL Multiphysics, as shown in Fig. 16. The rough fracture model in Fig. 8 was obtained by translating the fracture surface C3 1 mm along the z-axis and its size was 900×900×1 mm$^3$. To be noted, the fracture size is represented by the side length of the fracture surface for the convenience of comparison with the fracture surface results, i.e., the fracture size can be expressed as 900 mm. The size of the “base” fracture model was 900 mm. The sampling fracture models with different sizes were obtained in the same way as sampling fracture surface, which is shown in Fig. 6. The aperture of sampling fracture models was all 1 mm. The permeability in two
directions needs to be obtained and two different boundary conditions were considered: unidirectional flow along the x-axis and y-axis. For all sampling fracture models with different sizes, a constant pressure gradient was maintained between the inlet and out boundaries, and other boundaries were fixed with the imperious condition \[21,28\].

![Fracture model](image)

**Figure 8.** The rough fracture model by translating the fracture surface C3 1 mm along the z-axis.

### 5.2. Size effect on the permeability

The flow rate at the outlet can be easily obtained by numerical simulation and the equivalent permeability can be calculated according to Reynolds equation \[21\]. The results were shown in Fig. 9.

![Permeability](image)

**Figure 9.** The relation between equivalent permeability along (a) x-axis and (b) y-axis and fracture model size.

The equivalent permeability along the x-axis and y-axis with different model sizes were shown in Fig. 9. The permeability of fracture varies remarkably at the small fracture size. As fracture size continues to increases, the permeability tends to move to a relatively stable state. The results are the same as that of the fracture surface roughness. As the result of the roughness size effect, the variation coefficient of 15% is used as the criterion for permeability to enter the stable state \[29\]. It finds that the variation coefficient of permeability with different sizes is less than 15% and even less than 5%. The RES size for permeability is less than 100 mm under the criterion of variation coefficient. The REV sizes for permeability along the x-axis and y-axis are all 300 mm. The REV sizes for permeability using 2.5% CV criterion are equal to that for roughness using 15% CV criterion. The equivalent permeability of fracture along the x-axis and y-axis are \(7.23 \times 10^{-8}\) and \(7.39 \times 10^{-8}\) m\(^2\), respectively.

### 6. Conclusion

Based on the scanning fracture surface, the size effect on the morphology of fracture surface and fracture permeability was investigated. The asperity height data of fracture surface was scanned by 3D laser scanning in the field and the morphology of fracture surfaces was obtained. It was found that the distributions of asperity height followed the normal distribution and the roughness of the scanning
The fracture surface is anisotropic. Based on the asperity height data, the roughnesses of fracture surfaces with different fracture surface sizes were calculated and the size effect on fracture surface roughness was obtained. The results showed that the roughness of fracture surfaces varies remarkably at the small fracture surface size and the roughness tended to move to a relatively stable state when the fracture surface continued to increase. The critical size for the stability of fracture surface roughness is 300 mm and the equivalent roughness of fracture surface C3 along the x-axis and y-axis are 0.178 and 0.153. A rough single fracture model was established based on the scanned fracture surface data and the relation between fracture permeability and fracture size was analysed. It was found that the fracture permeability tended to move to a relatively stable state with the increase of fracture size, which was the same as the fracture surface roughness. The critical fracture size of fracture permeability is less than 100 mm in the study, which is also less than that of fracture surface roughness with the same coefficient of variation. It indicated that a lower coefficient of variation may provide the best estimate of the critical fracture size of fracture permeability comparing with the fracture surface roughness.

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