Gravitational lensing of Type Ia supernovae

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1. INTRODUCTION

Gravitational lensing has become an increasingly important tool in astrophysics and cosmology. In particular, the effects of lensing has to be taken into account when studying sources at high redshifts. In an inhomogeneous universe, sources may be magnified or demagnified with respect to the case of a homogeneous universe with the same average energy density.

The effects of gravitational lensing has been studied numerically by a number of authors, see, e.g., [1–5]. The most common method traces light rays through inhomogeneous matter distributions obtained from N-body simulations. Lensing effects are accounted for by projecting matter onto lens planes, and using the thin-lens approximation (see, e.g., [1]).

Recently, Holz and Wald (HW [1]) have proposed another ray-tracing method for examining lensing effects in inhomogeneous universes. This method can be summarized as follows: First, a Friedmann-Lemaître (FL) background geometry is selected. Inhomogeneities are accounted for by specifying matter distributions in cells with energy density equal to that of the underlying FL model. A light ray is traced backwards to the desired redshift by being sent through a series of cells, each time with a randomly selected impact parameter. After each cell, the FL background is used to update the scale factor and expansion. By using Monte Carlo techniques to trace a large number of light rays, and by appropriate weighting [1], statistics for the apparent luminosity of the source is obtained.

The advantages with this method are that light rays are traced through a three-dimensional matter distribution without projection onto lens planes, thus avoiding any assumptions regarding the accuracy of the thin-lens approximation [1]. Furthermore, the method is flexible in the sense that cells may be taken to represent both galaxies and larger structures with different matter distributions, including non-spherical ones. For instance, HW have performed a number of tests to determine effects of clustering, and argue that this does not significantly affect statistical properties of magnification. They also investigate the case of substructure in the form of compact objects, and conclude that this can be adequately modelled by randomly distributed compact objects of arbitrary mass. It should be pointed out that the method is not well-suited to model clustering on scales larger than cell sizes. Still, galaxy clusters can be modelled by specifying appropriate masses with corresponding larger cells. Another drawback is that the method only considers infinitesimal ray bundles, making it impossible to keep track of multiple images. However, it is still possible to distinguish between primary images and images that have gone through one or several caustics [3].

HW considered pressure-less models with a cosmological constant, using the following matter distributions: point masses; singular, truncated isothermal spheres (SIS); uniform spheres; and uniform cylinders. The individual masses were determined from the underlying FL model using a fixed co-moving cell radius of \( R_c = 2 \) Mpc, reflecting typical galaxy-galaxy separation length-scales.

The aim of this paper is to allow for matter distributions more accurately describing the actual properties of galaxies. We will extend the list of matter distributions to include the density profile proposed by Navarro, Frenk and White (NFW; [9]) and we will use a distribution of galaxy masses. Also, other matter distribution parameters such as the scale radius of the NFW halo and the
cut-off radius of the SIS halo will be determined from distributions reflecting real galaxy properties. The method of HW has also been generalized in Bergström et al. [8] to allow for general perfect fluids with non-vanishing pressure.

Gravitational lensing effects may be of importance when, e.g., trying to determine cosmological parameters using observations of supernovae at high redshifts [4,10,11]. In this paper, we study the effect from lensing when, e.g., trying to determine cosmological parameters.

Thus, for each cell we obtain a random mass, $M$, from a galaxy mass distribution $dn/dM$, and calculate the corresponding radius from the condition that the average energy density in the cell should be equal to the average matter density of the universe at the redshift of the cell:

$$ M = \frac{4\pi}{3} \Omega_M \rho_{\text{crit}} R_c^3, $$

(1)

where $\Omega_M$ is the normalized matter density, and $\rho_{\text{crit}} = 3H^2/8\pi$ is the critical density. A galaxy mass distribution can be obtained, for example, by combining the Schechter luminosity function (see, e.g., Peebles [12], Eq. 5.129)

$$ dn = \phi_s y^\alpha e^{-y} dy, $$

(2)

$$ y = \frac{L}{L_*}, $$

(3)

with the mass-to-luminosity ratio (see, e.g., Peebles [12], Eq. 3.39) normalized to a “characteristic” galaxy with $L = L_*$ and $M = M_*$,

$$ \frac{M}{M_*} = y^{1/(1-\beta)}. $$

(4)

Using Eq. (3), we find that

$$ \frac{dn}{dM} \propto y^\delta e^{-y}, $$

(5)

$$ \delta = \alpha - \frac{\beta}{1 - \beta}. $$

(6)

Assuming that the entire mass of the universe resides in galaxy halos we can write

$$ \int_{y_{\text{min}}}^{y_{\text{max}}} n(y) M(y) dy = \rho_m. $$

(7)

Using the Schechter luminosity function and the mass-to-luminosity fraction we get

$$ M_* = \frac{\Omega_M \rho_{\text{crit}}}{n_\alpha \int_{y_{\text{min}}}^{y_{\text{max}}} y^\alpha e^{-y} dy}. $$

(8)

Thus, by supplying values for $n_\alpha$, reasonably well-determined by observations, $y_{\text{min}}$ and $y_{\text{max}}$, from which the dependence of $M_*$ is weak, together with parameters $\alpha$ and $\beta$ we can obtain a $M_*$ consistent with $\Omega_M$. For the parameter values used in this paper (see Sec. V), we get

$$ M_* \approx 7.5 \Omega_M \cdot 10^{13} M_\odot. $$

(9)

### II. MASS DISTRIBUTION

Realistic modelling of galaxies calls for realistic mass distributions and number densities, i.e., one has to allow for the possibility of the cell radius, $R_c$, to reflect the actual distances between galaxies.

An advantage of the method of HW is that it is very easy to allow for any mass distribution and number density, including possible redshift dependencies, as long as the average density agrees with the underlying FL-model.

Thus, for each cell we obtain a random mass, $M$, from a galaxy mass distribution $dn/dM$, and calculate the corresponding radius from the condition that the average energy density in the cell should be equal to the average matter density of the universe at the redshift of the cell:

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### III. THE NAVARRO-FRENK-WHITE DISTRIBUTION

In the work of HW, the treatment of realistic galaxy models has been limited to the use of the singular, truncated isothermal sphere (SIS). Another often-used matter distribution is the one based on the results of detailed N-body simulations of structure formation by Navarro, Frenk and White [3]. The NFW density profile is given by

$$ \rho(r) = \frac{\rho_{\text{crit}} \delta_c}{(r/R_s) [1 + (r/R_s)]^2}, $$

(10)

where $\delta_c$ is a dimensionless density parameter and $R_s$ is a characteristic radius. The potential for this density profile is given by

$$ \Phi(r) = -4\pi \rho_{\text{crit}} \delta_c R_s^3 \frac{\ln(1 + x)}{x} + \text{const.}, $$

(11)

where $x = r/R_s$. The matrix $J_{\alpha\beta}$, describing the evolution of a light beam passing through a cell [see Eq. (37) in HW], can then be obtained analytically, see [3]. The mass inside radius $r$ of a NFW halo is given by

$$ M(r) = 4\pi \rho_{\text{crit}} \delta_c R_s^3 \left[ \ln(1 + x) - \frac{x}{1 + x} \right]. $$

(12)

Combining this expression with Eq. (3), i.e., setting $M = M(r)$, we obtain

$$ \delta_c = \frac{\Omega_M}{3} \frac{x_c^3}{\left[ \ln(1 + x_c) - \frac{x_c}{1 + x_c} \right]} $$

(13)

where $x_c = R_c/R_s$. That is, for a given mass $M$, $\delta_c$ is a function of $R_s$. From the numerical simulations of NFW we also get a relation between $\delta_c$ and $R_s$. This relation is computed numerically by a slight modification of a
FIG. 1. Luminosity distributions for 10,000 perfect standard candles at redshift \( z = 1 \) in a \( \Omega_M = 0.3, \Omega_{\Lambda} = 0.7 \) universe. The magnification zero point is the luminosity in the corresponding homogeneous (“filled-beam”) model. The full line corresponds to the point-mass case; the dashed line is the distribution for SIS halos, and the dotted line is the NFW case. This plot can be compared with Fig. 22 of HW.

FIG. 2. Luminosity distributions for 10,000 sources at redshift \( z = 1 \) in a \( \Omega_M = 0.3, \Omega_{\Lambda} = 0.7 \) universe. This is the same situation as depicted in Fig. 1, only that we have added an intrinsic luminosity dispersion of the sources with \( \sigma_m = 0.16 \) mag. (corresponding to the case of Type Ia supernovae).

Fortran routine kindly supplied by Julio Navarro. Of course, one wants to find a \( R_s \) compatible with both the average density in each cell and the numerical simulations of NFW. Hence, we iteratively determine a value of \( R_s \) consistent with both expressions for \( \delta_c \). Generally, \( R_s \) will be a function of mass \( M \), the Hubble parameter \( h \), the density parameters \( \Omega_M, \Omega_{\Lambda} \), and the redshift \( z \). However, we will use the result from Del Popolo [13] and Bullock et al. [14] that \( R_s \) is approximately constant with redshift. We will compute \( R_s \) for a variety of \( M, h \) and \( \Omega_M \) (all at \( z = 0 \)) in both open and flat cosmologies and interpolate between these values to obtain \( R_s \) for any combination of parameter values.

IV. TRUNCATION RADII FOR SIS-LENSES

In their calculations for SIS halos, HW use a fix truncation radius \( d \). However, using a realistic mass distribution, the cut-off should depend on the mass of the galaxy. Here we derive an expression for \( d \).

The SIS density profile is given by

\[
\rho_{\text{SIS}}(r) = \frac{\sigma^2}{2\pi r^2},
\]

where \( \sigma \) is the line-of-sight velocity dispersion of the mass particles. The mass of a SIS halo truncated at radius \( d \) is then given by

\[
M(d) = \int_0^d \rho(r)\,dV = 2\sigma^2d.
\]

IV. RESULTS

As an application of the method, we investigate lensing effects on observations of distant supernovae. In Fig. 2, we compare the luminosity distributions obtained with
FIG. 3. Luminosity distributions for 10 000 perfect standard candles at redshift $z = 1$ in a $\Omega_M = 1, \Omega_\Lambda = 0$ universe. The magnification zero point is the luminosity in the corresponding homogeneous (“filled-beam”) model. The full line corresponds to the point-mass case; the dashed line is the distribution for SIS halos, and the dotted line is the NFW case. This plot can be compared with Figs. 18 and 20 of HW.

In Fig. 4 we have added an intrinsic luminosity dispersion represented by a Gaussian distribution with $\sigma_m = 0.16$ mag., due to the fact that Type Ia supernovae are not perfect standard candles. The effect is to make the characteristics of the luminosity distributions even less pronounced, since the form of the resulting luminosity distributions predominantly is determined by the form of the intrinsic luminosity distribution. It is still possible to observationally distinguish whether lenses consist of compact objects or smooth galaxy halos, as has been pointed out in [17,18]. Generating several samples containing 100 supernova events at $z = 1$ in a $\Omega_M = 0.3, \Omega_\Lambda = 0.7$ cosmology filled with smooth galaxy halos, we find that for 98 % of the samples one can rule out a point-mass distribution with a 99 % confidence level. Furthermore, for a similar sample containing 200 supernovae, the confidence level is increased to 99.99 %.

We have performed simulations for various cosmologies, and found a substantial difference between SIS halos and NFW halos only in a matter-dominated universe, $\Omega_M = 1, \Omega_\Lambda = 0$, where the luminosity distribution using NFW halos is shifted towards the point-mass case (see Fig. 3). However, adding an intrinsic source dispersion as in Fig. 4, we see that even with the phenomenal statistics of 10 000 sources, it would be a difficult task to distinguish between the two density profiles. The increased number of high-magnification events for NFW halos would probably be the only way to make such a discrimination.

In these calculations, we have used the following parameter values (see further [8]):

- $\beta = 0.2$
- $\alpha = -0.7$

\footnote{However, in 1 % of the samples, we will erroneously rule out the halo distribution with the same confidence level.}
• $y_{\text{min}} = 0.5$
• $y_{\text{max}} = 2.0$
• $n_\ast = 1.9 \cdot 10^{-2} h^3 \text{ Mpc}^{-3}$
• $\sigma_\ast = 220 \text{ km/s}$
• $\lambda = 0.25$

A more extensive discussion of the luminosity distributions of perfect standard candles obtained with the different halo models at different source redshifts can be found in Bergström et al. [8], where also some analytical fitting formulas for the probability distributions are given.

VI. DISCUSSION

In this paper, the method of Holz and Wald [1] has been generalized to allow for matter distributions reflecting the actual properties of galaxies, including the density profile proposed by Navarro, Frenk and White [9]. In order to make matter distributions as realistic as possible, all parameter values in the lens models are obtained from reasonable probability distributions, as derived from observations and N-body simulations. This includes the mass of the galaxies, the truncation radius of SIS lenses and the characteristic radius of NFW halos. One of the virtues of this method is that it can be continuously refined as one gains more information about the matter distribution in the universe from observations.

The motivation for these generalizations is to use this method as part of a model for simulation of high-redshift supernova observations. In this paper, we have considered lensing effects on supernova luminosity distributions. Results for different mass distributions in smooth dark matter halos was found to be very similar, making lensing effects predictable for a broad range of density profiles. Furthermore, given a sample of 100 supernovae at $z \sim 1$, one should be able to discriminate between the case with smooth dark matter halos and the (unlikely) case of having a dominant component of dark matter in point-like objects.

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[1] D.E. Holz, and R.M. Wald, Phys. Rev. D 58 (1998) 063501.
[2] J. Wambsganss, R. Cen and J.P. Ostriker, Astrophys. J. 494 (1998) 29.
[3] P. Premadi, H. Martel and R. Matzner, Astrophys. J. 493 (1998) 10.
[4] S. Marri and A. Ferrara, Astrophys. J. 509 (1998) 43.
[5] B. Jain, U. Seljak and S. White, Astrophys. J. 530 (2000) 547.
[6] P. Schneider, J. Ehlers and E.E. Falco, Gravitational Lenses (Springer Verlag, Berlin, 1992).
[7] T.P. Kling, E.T. Newman and A. Perez, preprint gr-qc/0003057.
[8] L. Bergström, M. Goliath, A. Goobar, and E. Mörtssell, to appear in Astron. Astrophys.
[9] J.F. Navarro, C.S. Frenk, and S.D.M. White, Astrophys. J. 490 (1997) 493.
[10] J. Wambsganss, R. Cen, G. Xu and J.P. Ostriker, Astrophys. J. 475 (1997) L81.
[11] R.B. Metcalf, MNRAS 305 (1999) 746.
[12] P.J.E. Peebles, Principles of physical cosmology (Princeton University Press, Princeton, 1993).
[13] A. Del Popolo, preprint astro-ph/9908195.
[14] J.S. Bullock, et al., preprint astro-ph/9908159.
[15] S. Perlmutter et al., Astrophys. J. 517 (1999) 565.
[16] A.G. Riess et al., Astronom. J. 116 (1998) 1009.
[17] R.B. Metcalf and J. Silk, Astrophys. J. 519 (1999) L1.
[18] U. Seljak and D. Holz, Astronom. Astrophys. 351 (1999) L10.