Mathematical model of underground structure-soil interaction

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Abstract. Based on existing data of experimental studies of underground structure-soil interaction, a condition for boundary interaction between the underground pipeline and soil was developed. The proposed condition (a model) of interaction at the contact boundary takes into account the structural destruction of the contact soil layer and the cyclical nature of interaction process. The authors have shown by parametric analysis of the interaction model, that under cyclic interaction of the underground pipeline with soil, the structural disturbance of the soil contact layer does not reach the peak shear stress; the applicability of the problems of underground structure-soil interaction is also shown. The model of interaction on the contact structure-soil surface in particular cases passes into the existing conditions of interaction, including Coulomb friction.

1. Introduction
In a theoretical study the conditions accepted on the contact surface are important [1-3] when assessing the strength of underground structures and pipelines that interact with soil under dynamic load. The importance is that these interaction conditions are the soil response to an underground structure or vice versa. In [4–9], the Coulomb friction law, the Coulomb-Moore condition, and other conditions were used at the contact boundary. According to the results of experiments [10] on the underground structure interaction with soil, the pattern of change in shear stresses at the boundary of contact surface does not completely coincide with the law of dry friction; the shear stress depends on the displacement of a rigid body relative to the displacement of soil particles (a relative displacement) [10] depending on strain properties and stress states of soils. Based on these results, various conditions for the interaction of an underground structure with soil were developed [3, 11-13]. These conditions take into account rheological properties and structural destruction of soil [14] on the contact surface and transition to the Coulomb law after complete structural destruction of the soil layer on the contact surface. These interaction conditions at the boundary of two media were tested in problems considered in [15-17].

In [15], a multiple increase in stresses was observed in an underground pipeline. This stress increase was explained by the transformation of friction force at the contact into an active force. In the case of the active force on the contact, the actual increase in stresses in cylindrical body was shown in [18].

The transition of friction force into an active one occurs along with a change in force direction at the underground structure-soil contact. When solving problems by numerical method, for each change in sign (direction) of shear stress action, it is necessary to mark those points where the sign changes. If at the initial stage of interaction the direction of shear stress action on the contact surface corresponding to a partial structural damage of soil changes, then those points should also be kept in mind. An account for all these factors leads to the formulation of a new interaction condition. The aim of the study is to construct the interaction conditions that take into account the changes in action direction and structural destruction without reaching the “peak” value of shear stress under cyclic change in the interaction direction.

2. Basic prerequisites for creating the interaction conditions
From the results of studies given in [15-17] we conclude that the Winkler-type conditions describe the principal features of underground structure interaction with soil. Consider the results of longitudinal interaction of an underground pipeline with soil using this model in a two-dimensional axisymmetric
The problem statement is similar to the problem considered in [18-19]. When solving such a problem, difficulties arise mainly when implementing the interaction conditions at the contact boundary. Figure 1 shows the changes in shear stress depending on time and on relative displacement on the contact surface in the process of solving the problem using numerical method given in [20]. Here, at the initial stage of interaction, a Winkler type model is taken in the form [3]. The parameters of this model are taken as follows: \( f = 0.3; \beta = 0; K_N = 10 \text{ m}^{-1} \) (curves a), \( K_N = 100 \text{ m}^{-1} \) (curves b), \( K_N = 500 \text{ m}^{-1} \) (curves c and Figure 1(a)). From \( \beta = 0 \) it follows that the external medium is structurally destroyed.

![Figure 1](image)

**Figure 1.** Change in shear stresses depending on time (a) and on relative displacement (b) on the contact surface. 1 – \( x = 0.4 \text{ m} \); 2 – \( x = 1.2 \text{ m} \); 3 – \( x = 2 \text{ m} \).

As seen from Figure 1, the shear stress on the contact surface develops with the development of relative displacement. Between the wave fronts in the pipeline and soil, these stresses have positive values, which are the resistance force for the pipeline, and the force involved into soil motion. Further, behind the wave front in soil, due to a change in relative velocity sign, the direction of shear stress action changes. At the beginning of the change, the relative displacement decreases and after some time the soil displacement exceeds the structure displacement, therefore the relative displacement changes the sign and begins to grow. Here, the direction of shear stress action is determined by the sign of relative velocity. The correctness of application of shear stress sign is beyond doubt, since the experiments described in [10] confirm this. With a change in the action direction, the shear stress due to the high normal stress begins to increase, however, due to the continuity of pipe and soil particles displacements, at some point in time, the relative displacement passes through the ordinate axis. In this case, the shear stress also instantly vanishes, then, with an increase in relative displacement, the shear stress continues to act as a resistance force for soil and as an active force for the pipeline. Such a course of shear stress variation is especially observed with the distance from the initial section, i.e. with the distance between the wave fronts in the pipeline and soil (curves 2 and 3, Figure 1(a)). Such a process depending on shear stress and relative displacement, leads to a “loop” (a bow) at the origin \((\tau, \bar{u})\) at high values of the interaction coefficient (curve 3, option c, Figure 1(b)). As seen from Figure 1(b), the value of the interaction coefficient determines the inclination in shear stress variation from the relative displacement and the transition to the next (second) stage of interaction. It is interesting to mention that in the first stage of interaction, the dependences \( \tau(\bar{u}) \) of different points on the third quarter of coordinates \((\tau, \bar{u})\) coincide.

The main assumption regarding the application of the interaction condition [3] in the problem under consideration is not the determination of the shear stress action direction, but the consequence of not accounting for the “new” reference of relative displacements. Therefore, in the case of applying the interaction conditions, which consist of the relationship between shear stress and relative displacement, it is necessary to specify the course of these curves, similar to the strain theory of plasticity. This conclusion is facilitated by the results of experimental studies of underground structure interaction with soil under dynamic load [10].

3. Interaction condition formulation

In the process of applying the interaction condition [3], the question to implement the condition under “unloading” arises as well as the question - what is the meaning of this “unloading”. As is known from the analysis of the studies mentioned in the Introduction and [19], the interaction conditions essentially
reflect the properties of the near-contact soil layer; however, in the process of interaction under “unloading” it reflects the changes in the direction of shear stress action. Indeed, according to the results of experiments [10], the shear stress instantly drops to zero and then changes continuously depending on relative displacement. Moreover, if the sign of shear stress is changed, not reaching the "peak" value, then this value is observed in the opposite direction of its action for structurally undisturbed soil. In general, for multi-cycle interaction, the structural destruction of the contact layer should depend on the history of interaction and on the state of the layer itself. An account for this factor leads to destruction of the contact layer under friction, not reaching the “peak” value of shear stress, which is important for many applied problems (for example, when pulling out piles buried in the ground).

Thus, to solve problems with the interaction conditions [3,11-13], it is necessary to specify the trajectory and direction of change of the boundary conditions on the contact surface. It is obvious that the change in shear stress depends not only on the state of structure-soil system, but also on the previous history of their change, therefore, in the general case, the formulation of the interaction conditions (a model) requires the use of step procedures.

Consideration of the above factors leads to the following ratio of Winkler-type interaction conditions:

\[
\frac{dL}{dt} + \lambda_S \tau = (1 - \alpha_s I_S) K_N \sigma_N \exp\left(\alpha_n \left(1 - I_0\right)\right) \frac{d\sigma}{dt},
\]

(1)

where

\[
\alpha_n = \begin{cases} 
\alpha_S + \alpha_n (1 - I_{S,n-1})^{n-1} & \text{at } \alpha_{s,n-1} > \alpha_S \text{ or } n = 1 \\
\alpha_S & \text{at } \alpha_{s,n-1} = \alpha_S,
\end{cases}
\]

(2)

\[
\lambda_S = -\frac{\sigma_N}{\sigma_N} \text{ and } I_S = \left|\frac{\bar{u} - \bar{u}_{s,n-1}}{\bar{u}_{s,n-1} - \bar{u}}\right|, \text{ at } \bar{u} - \bar{u}_{s,n-1} \leq \bar{u},
\]

(3)

\[
\lambda_S = \frac{1 - \alpha_s}{u_s} \frac{d\bar{u}}{dt} - \frac{\sigma_N}{\sigma_N} \text{ and } I_S = 1 \text{ at } \bar{u} - \bar{u}_{s,n-1} > \bar{u}.
\]

(4)

Here \( n \) is the number of sign changes of relative velocity; \( \alpha_S \) is the dimensionless parameter characterizing the variation range of the interaction function in a structurally disturbed contact layer; \( \alpha_n \) is the same parameter corresponding to the interaction state of cycle \( i \). The fulfillment of conditions \( \bar{u} - \bar{u}_{s,n-1} \leq \bar{u} \) means the first stage of interaction, in opposite case - the second stage of interaction occurs, which consists in a proportional relationship between shear and normal stresses on the contact surface. Therefore, in this case, relation (1) should reflect the law of interaction given in [3,11-13]. Under this assumption, a value of \( \lambda_S \) is defined in [sec^{-1}] and it reflects the same meaning as the Grigoryan’s strain model [16-17].

The independent parameters of interaction conditions (1) are the coefficient \( K_N \) characterizing the rigidity of the soil-pipeline interaction, the critical relative displacement \( \bar{u}_r \), and the parameters \( \alpha_s \) corresponding to the disturbed structure and \( \alpha_n = \alpha_S + \alpha_n \), reflecting the initial state of disturbed structure of the contact layer. The values of these parameters for various types of soil are defined in [3,10], where \( \bar{u}_r \) varies from 0.002 m to 0.065 m; \( K_n = 100-400 \text{ m}^4/\text{m}^3 \); \( \alpha_n = 0.5-2.5 \), and the value of \( \alpha_S \) corresponding to soils of disturbed structure is always \( \alpha_S \leq 1 \).

Now we conduct a parametric analysis of the constructed law of interaction. First, consider the special cases of interaction:

- Let the interaction of a rigid body with structurally undisturbed soil occur without a change in action direction of shear stress. In this case, \( I_S = \left|\bar{u}/\bar{u}_r\right| \) holds in the first stage of interaction, \( \alpha_n = \alpha_S + \alpha_n \), characterizes the range of change in interaction function in undisturbed soil. The integrated expression (1) with zero initial conditions (integration is carried out in parts) completely coincides with the interaction model [3,11-13]. In the case of \( \bar{u} \rightarrow 0 \), condition (1) becomes the modified law of friction [2-5], and if structural disturbance of soil is not taken into account \( (\alpha_n = \alpha_S = 0) \), relation (1) describes the behavior of the near-contact soil layer, which coincides with the strain model given in [19].

- Let the normal pressure values remain constant on the pipe-soil contact surface. In this case, in the first stage of interaction from the first relation (3) we have \( \lambda_S = 0 \), and equation (1) takes the form which coincides with the interaction model [4-5,18], when soil disturbance is not taken into account:
\[ \frac{d\tau}{dt} = (1-\alpha_{i} I_{s}) K_{s} \sigma_{v} \exp(\alpha_{i}(1-I_{s})) \frac{d\tau}{dt} \].

(4)

- Assume that the structural destruction of soil contact layer occurs in the \(i\)-th cycle of interaction. Then, according to (3), in subsequent cycles, the values of \(\alpha_{i}\) do not change, and the interactions in both directions have the same form. In general, in a disturbed soil structure, the implementation of the interaction condition is greatly simplified.

- A special case is observed in the interaction of rigid bodies with certain types of loamy soils, at \(\alpha_{s} = 1\). The special feature here is that after a complete destruction of the contact layer, both stages of interaction are expressed by one law:

\[ \frac{d\tau}{dt} - \frac{d\sigma_{v}}{\sigma_{v}} \tau = (1-\alpha_{i} I_{s}) K_{s} \sigma_{v} \exp(\alpha_{i}(1-I_{s})) \frac{d\tau}{dt} \text{ at } \frac{d\tau}{dt} \neq 0 \].

(5)

- Determination of shear stress in the second stage of interaction. In the general case (1) and in the discussed partial cases (4) - (5), the problem of interaction strength determination is reduced to integrating the following equation

\[ d(\ln \tau)/dt - d(\ln \sigma_{v})/\sigma_{v} = 0 \].

(6)

The integral of equation (6) reflects the proportional dependence of shear stress on normal pressure on the contact surface (\(\tau / \sigma_{v} \propto \text{const}\)). The proportionality coefficient is determined from the transition conditions to the second stage of interaction; it is easy to see that it reflects the coefficient of friction between the particles. It should also be noted that the transition from one stage of interaction to another is accompanied by a discontinuity of the first kind, with the exception of the case \(\alpha_{s} = 1\).

4. Numerical analysis

Now we carry out a numerical analysis of the interaction model (1)-(4). Suppose, an underground pipeline interacts with soil from a state of rest, and the interaction rate is described by \(d\tau/dt = \phi(t)\). Define the changes in shear stresses for a particular type of function \(\phi(t)\).

4.1. Constant rate interaction

In this case, \(\phi(t) = \bar{U} = \text{const} > 0\), hence \(\bar{\tau} = \bar{U} t\). Figure 2 shows the changes in shear stresses from relative displacements at constant normal pressure values. The initial data are taken as follows \(\sigma_{v} = 0.2\) MPa; \(K_{s} = 100\) m\(^{-1}\); \(\bar{\tau} = 0.5\) cm. Curves 1-5 in Figure 2(a) correspond to the values \(\alpha_{i} = 0; 0.5; 1; 1.5\) and 2, respectively. As seen from Figure 2(a), the pattern of changes in shear stresses from relative displacement completely coincides with the results of the study [10]. The “peak” value of shear stress is observed under undisturbed state of the near-contact layer of soil (\(\alpha_{i} > 1\)). Here, \(\alpha_{i}\) plays the same role as the parameter \(\beta\) in [3,11-13]. This parameter affects the “peak” value of shear stress. Its increase leads to an increase in shear stress "peak" value. The influence of other parameters on the run of the curves \(\tau(\bar{\tau})\) is shown in Figure 2(b).

![Figure 2](image-url)  
**Figure 2.** Dependence of shear stress on relative displacement.
Curves 1-3 in Figure 2(b), refer to the altered data \( K_N=200 \text{ m}^{-1}; \sigma_N=0.3 \text{ MPa}; \overline{u}=0.4 \text{ cm}, \) respectively. The remaining parameters are not changed in calculations. The solid curves correspond to \( \alpha_i=2, \) and the dashed curves to \( \alpha_i=1. \) Figure 2(b) shows that changes in rigidity coefficient, normal pressure, and critical relative displacement also affect the dependence \( \tau(\overline{u}) \). An increase in \( K_N \) or \( \sigma_N \) leads not only to an increase in shear stress, but also delays the transition to the second stage of interaction. Therefore, a decrease in \( \overline{u} \) reduces the maximum value of shear stress. In general, the obtained data are in good agreement with the experimental results [10].

4.2. Cyclic interaction

Let the relative displacement on the contact surface change according to a harmonic law: \( \overline{u}=\overline{U}_\circ \sin(\omega t) \) in the process of pipe-soil interaction. Then \( \varphi(t)=\overline{U}_\circ \omega \cos(\omega t) \). Taking \( \omega=\pi, \) we consider the formation of shear stress over time from the initial point to 2.5 seconds. The results are shown in Figure 3. Here, Figure 3(a) refers to the parameter values \( S\alpha_i=0.5 \) and \( \alpha_i=1.5, \) and Figure 3(b) - to \( \alpha_i=1 \) and \( \alpha_i=1.5. \) The remaining data is taken unchanged. The duration of the considered moment of time allows realizing three cycles of changes in dependence \( \tau(\overline{u}) \). From figure 3 it is seen that at amplitudes \( \overline{U}_\circ=0.1; 0.15 \) and 0.2 cm of relative displacement, only the first stage of interaction occurs in the system (curves 1-3). In the case of a transition to another cycle, without reaching a structural destruction of soil, the manifestation of the "peak" value of shear stress is observed in the new cycle (curves 1-3, Figure 3(b)). In this case, according to (3), the value of parameter \( \alpha_i \) decreases, and therefore, the "peak" amplitude of shear stress decrease as well. In the case of \( \overline{U}_\circ=0.3 \) cm, the second stage of interaction is realized in the second cycle of interaction. Here, the value of the shear stress even at the first cycle reaches its "peak" value, therefore, in the next cycle, such a "peak" is not observed (curve 4). Complete structural destruction of the contact layer in the first interaction cycle occurs at \( \overline{U}_\circ=0.6 \) cm. In this case, the further interaction reflects a process similar to the one with structurally disturbed soil and does not depend on the interaction cycle (curve 5).

![Figure 3](image-url)

Figure 3. Change in shear stress as a function of relative displacement. 1- \( \overline{U}_\circ=0.1 \text{ cm}, 2- \overline{U}_\circ=0.15 \text{ cm}, 3- \overline{U}_\circ=0.2 \text{ cm}, 4- \overline{U}_\circ=0.3 \text{ cm}; 5- \overline{U}_\circ=0.6 \text{ cm.}

Figure 4 shows the dependences \( \tau(\overline{u}) \) in the case of a change in relative displacement according to the law \( \overline{u}=\overline{U}_\circ t \sin(\omega t). \) In this case, the relative velocity has the form: \( \varphi(t)=\overline{U}_\circ (\omega \cos(\omega t)+\sin(\omega t)) \). Here the solid curves refer to the values \( \alpha_i=1 \) and \( \alpha_i=1.5, \) and the dashed curves to \( \alpha_i=0.5 \) and \( \alpha_i=1.5. \) Here, an excess of shear stress value over its limiting value is observed in the first stage of interaction in the first two cycles. A decrease in the slope of the curves \( \tau(\overline{u}) \) from cycle to cycle is clearly seen.

5. Discussion

The formulated interaction law (1) - (4) allows us to directly apply it when solving the problems that have a varying direction of the interaction force on the contact surface. From the analysis performed, it can be noted that relations (1) - (4) do not take into account the influence of the interaction rate on the behavior
of shear stress. Instead of formulating such a condition for multi-cycle interaction, the authors consider it appropriate to develop and study the models of shear strain of soils under cyclic loads as in [14].

Note that in case of soil structure destruction, the “peak” value of shear stress is observed at the beginning of loading or before the transition to the limiting state. Such an effect can be taken into account by using its argument $I_s = I(u/\bar{u})$ for the functional dependence or by assuming the variability of values $\alpha_i$ within each cycle. We considered both cases, the first - using a power function, and the second - using $I_s$ instead of $I_{s,n-1}$. Both cases gave effective results, however, they practically did not affect the calculation result, and in addition their implementation became cumbersome.

![Figure 4. Change in shear stress as a function of relative displacement. 1 - $\bar{U}_0 = 0.1$ cm, 2 - $\bar{U}_0 = 0.2$ cm, 3 - $\bar{U}_0 = 0.3$ cm.](image)

6. Conclusions

Based on the existing experimental data, the conditions (a model) for the interaction of underground structures with soil have been developed. These conditions take into account the cyclical nature of interaction and structural changes in the contact layer of soil. Partial cases of interaction were shown that describe existing conditions of interaction. A parametric analysis of the developed interaction model was carried out and its applicability to the problems of underground structure-soil interaction was shown.

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