A Framework To Handle Linear Temporal Properties in (ω-)Regular Model Checking

Ahmed Bouajjani\textsuperscript{a}, Axel Legay\textsuperscript{b}, Pierre Wolper\textsuperscript{c},

\textsuperscript{a} LIAFA - Université Paris 7
175, rue du chevaleret
Paris, France
\textsuperscript{b} Université de Rennes 1
Institut d’informatique INRIA
Rennes, France
\textsuperscript{c} Université de Liège
Institut Montefiore, B28
Liège, Belgium

Abstract

Since the topic emerged several years ago, work on regular model checking has mostly been devoted to the verification of state reachability and safety properties. Though it was known that linear temporal properties could also be checked within this framework, little has been done about working out the corresponding details. This paper addresses this issue in the context of regular model checking based on the encoding of states by finite or infinite words. It works out the exact constructions to be used in both cases, and proposes a partial solution to the problem resulting from the fact that infinite computations of unbounded configurations might never contain the same configuration twice, thus making cycle detection problematic.

Key words: (ω-)regular model checking, transducer, semi-algorithm, simulation, rewrite systems, Büchi automata, framework paper.

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Email addresses: abou@liafa.jussieu.fr (Ahmed Bouajjani), alegay@irisa.fr (Axel Legay), pw@montefiore.ulg.ac.be (Pierre Wolper).
1 Introduction

At the heart of all the techniques that have been proposed for exploring infinite state spaces, is a symbolic representation that can finitely represent infinite sets of states. In early work on the subject, this representation was domain specific, for example linear constraints for sets of real vectors. For several years now, the idea that a generic finite-automaton based representation could be used in many settings has gained ground, starting with systems manipulating queues and integers [WB95, BEM97, BRW98], then moving to parametric systems [KMM+97], and, recently, reaching systems using real variables [BJW01, BHJ03].

Beyond the necessary symbolic representation, there is also a need to “accelerate” the search through the state space in order to reach, in a finite amount of time, states at unbounded depths. In acceleration techniques, the move has again been from the specific to the generic, the latter approach being often referred to as regular model checking. In (ω-)regular model checking (see e.g. [BJNT00, DLS02, BLW04a]), the transition relation is represented by a finite-state transducer and acceleration techniques aim at computing the iterative closure of this transducer algorithmically, though necessarily foregoing totality or preciseness, or even both. The advantages of using a generic technique are of course that there is only one method to implement independently of the domain considered, that multidomain situations can potentially be handled transparently, and that the scope of the technique can include cases not handled by specific approaches. Beyond these concrete arguments, one should not forget the elegance of the generic approach, which can be viewed as an indication of its potential, thus justifying a thorough investigation.

However, computing reachable states is not quite model-checking. For reachability properties model checking can be reduced to a state reachability problem, but for properties that include a linear temporal component, the best that can be done is to reduce the model-checking problem to emptiness of a Büchi automaton [VW86], which represents all the executions of the system that do not satisfy the property. If this automaton is empty, then the system satisfies the property, else the property is not satisfied. In this framework, one thus has to check for repeated reachability rather than reachability.

In this paper, we consider the specification and the verification of linear temporal properties in the (ω-)regular model checking framework\textsuperscript{1}. The objective of the paper is to provide generic analysis techniques covering various classes of systems that can be encoded in this framework.

\textsuperscript{1} In the rest of the paper, we use “(ω-)regular model checking” to denote either “regular model checking” or “ω−regular model checking”, depending on whether states are encoded by finite or infinite words.
We fully worked out how to augment the transducer representing the system transitions in order to obtain a transducer encoding the Büchi automaton resulting from combining the system with the property. Once the transition relation of the Büchi automaton has been obtained, checking the automaton for nonemptiness is done by computing the iterative closure of this relation, finding nontrivial cycles between states, and finally checking for the reachability of states appearing in such cycles. When dealing with systems where the number of successors of each state is bounded, an accepting execution of the Büchi automaton will always contain the same state twice and hence an identifiable cycle. However, when dealing with states whose length can grow or that are infinite, there might very well be an accepting computation of the Büchi automaton in which the same state never appears twice.

To cope with this, we look for states that are not necessarily identical, but such that one entails the other in the sense that any execution possible from one is also possible from the other. The exact notion of entailment we use is simulation. For that, we compute symbolically the greatest simulation relation on the states of the system.

The nice twist is that the computation of the symbolic representation of the simulation relation is in fact, the computation of the limit of a sequence of finite-state automata, for which the acceleration techniques introduced in [BLW03, BLW04a, Leg07] can be used. However, there are also several cases where this computation converges after a finite number of steps, which has the added advantage of guaranteeing that the induced simulation equivalence relation partitions the set of configurations in a finite number of classes, and hence that existing accepting computations will necessarily be found, which might not be the case when the number of simulation equivalence classes is infinite.

**Structure of the paper.** The paper is structured as follows. In Section 2, we recall the elementary definitions on automata theory that will be used throughout the rest of the paper. Section 3 presents the (ω-)regular model checking framework as well as a methodology to reason about infinite executions. In Sections 4, 5, 6, and 7, the verification of several classes of linear temporal properties in the (ω-)regular model checking framework is considered. Finally, Sections 8 and 9 conclude the paper with a comparison with other works on the same topic and several directions for future research, respectively.
2 Background on Automataa Theory

In this section, we introduce several notations, concepts, and definitions that will be used throughout the rest of this paper. The set of natural numbers is denoted by \( \mathbb{N} \), and \( \mathbb{N}_0 \) is used for \( \mathbb{N} \setminus \{0\} \).

2.1 Relations

Consider a set \( S \), a set \( S_1 \subseteq S \), and two binary\(^2 \) relations \( R_1, R_2 \subseteq S \times S \). The identity relation on \( S \), denoted \( R^S_id \) (or \( R^S_id \) when \( S \) is clear from the context) is the set \( \{(s, s) \mid s \in S\} \). The image of \( S_1 \) by \( R_1 \), denoted \( R_1(S_1) \), is the set \( \{s' \in S_1 \mid (\exists s \in S_1)((s, s') \in R_1)\} \). The composition of \( R_1 \) with \( R_2 \), denoted \( R_2 \circ R_1 \), is the set \( \{(s, s') \mid (\exists s'' \in S)((s, s'') \in R_1 \land (s'', s') \in R_2)\} \). The \( i \)th power of \( R_1 \) (\( i \in \mathbb{N}_0 \)), denoted \( R_1^i \), is the relation obtained by composing \( R_1 \) with itself \( i \) times. The zero-power of \( R_1 \), denoted \( R_1^0 \), corresponds to the identity relation. The transitive closure of \( R_1 \), denoted \( R_1^+ \), is given by \( \bigcup_{i=1}^{+\infty} R_1^i \), its reflexive transitive closure, denoted \( R^* \), is given by \( R_1^+ \cup \bigcup_{i=1}^{+\infty} R^i_id \). The domain of \( R_1 \), denoted \( \text{Dom}(R_1) \), is given by \( \{s \in S \mid (\exists s' \in S)((s, s') \in R_1)\} \).

2.2 Words and Languages

An \emph{alphabet} is a (nonempty) finite set of distinct symbols. A \emph{finite word} of length \( n \) over an alphabet \( \Sigma \) is a mapping \( w : \{0, \ldots, n - 1\} \rightarrow \Sigma \). An \emph{infinite word}, also called \( \omega \)-word, over \( \Sigma \) is a mapping \( w : \mathbb{N} \rightarrow \Sigma \). We denote by the term \emph{word} either a finite word or an infinite word, depending on the context. The \emph{length} of the finite word \( w \) is denoted by \( |w| \). A finite word \( w \) of length \( n \) is often represented by \( w = w(0) \cdots w(n - 1) \). An infinite word \( w \) is often represented by \( w(0)w(1) \cdots \). The sets of finite and infinite words over \( \Sigma \) are denoted by \( \Sigma^* \) and by \( \Sigma^\omega \), respectively. We define \( \Sigma^{\infty} = \Sigma^* \cup \Sigma^\omega \). A \emph{finite-word} (respectively \emph{infinite-word}) \emph{language} over \( \Sigma \) is a (possibly infinite) set of finite (respectively, infinite) words over \( \Sigma \). Consider \( L_1 \) and \( L_2 \), two finite-word (resp. infinite-word) languages. The \emph{union} of \( L_1 \) and \( L_2 \), denoted \( L_1 \cup L_2 \), is the language that contains all the words that belong either to \( L_1 \) or to \( L_2 \). The \emph{intersection} of \( L_1 \) and \( L_2 \), denoted \( L_1 \cap L_2 \), is the language that contains all the words that belong to both \( L_1 \) and \( L_2 \). The \emph{complement} of \( L_1 \), denoted \( \overline{L_1} \), is the language that contains all the words over \( \Sigma \) that do not belong to \( L_1 \).

We also introduce \emph{synchronous product} and \emph{projection}, which are two operations needed to define relations between languages.

\(^2\) The term “binary” will be dropped in the rest of the paper.
Definition 1 Consider $L_1$ and $L_2$ two languages over $\Sigma$.

- If $L_1$ and $L_2$ are finite-word languages, the synchronous product $L_1 \times L_2$ of $L_1$ and $L_2$ is defined as follows
  \[
  L_1 \times L_2 = \{ (w(0), w(0)', \ldots, w(n), w(n')) \mid w = w(0)w(1)\ldots w(n) \in L_1 \wedge w' = w(0)'w(1)'\ldots w(n)' \in L_2 \}.
  \]

- If $L_1$ and $L_2$ are $\omega$-languages, the synchronous product $L_1 \times L_2$ of $L_1$ and $L_2$ is defined as follows
  \[
  L_1 \times L_2 = \{ (w(0), w(0)')(w(1), w(1)')\ldots \mid w = w(0)w(1)\ldots \in L_1 \wedge w' = w(0)'w(1)'\ldots \in L_2 \}.
  \]

The language $L_1 \times L_2$ is defined over the alphabet $\Sigma^2$.

Definition 1 directly generalizes to synchronous products of more than two languages. Given two finite (respectively, infinite) words $w_1, w_2$ (with $|w_1| = |w_2|$ if the words are finite) and two languages $L_1$ and $L_2$ with $L_1 = \{w_1\}$ and $L_2 = \{w_2\}$, we use $w_1 \times w_2$ to denote the unique word in $L_1 \times L_2$.

Definition 2 Suppose $L$ a language over the alphabet $\Sigma^n$ and a natural $1 \leq i \leq n$. The projection of $L$ on all its components except component $i$, denoted $\Pi_{\neq i}(L)$, is the language $L'$ such that
\[
\Pi_{\neq i}(L) = \{ w_1 \times \ldots \times w_{i-1} \times w_{i+1} \times \ldots \times w_n \mid (\exists w_i)(w_1 \times \ldots \times w_{i-1} \times w_i \times w_{i+1} \times \ldots \times w_n \in L) \}.
\]

2.3 Automata

Definition 3 An automaton over $\Sigma$ is a tuple $A = (Q, \Sigma, Q_0, \Delta, F)$, where

- $Q$ is a finite set of states,
- $\Sigma$ is a finite alphabet,
- $Q_0 \subseteq Q$ is the set of initial states,
- $\Delta \subseteq Q \times \Sigma \times Q$ is a finite transition relation, and
- $F \subseteq Q$ is the set of accepting states (the states in $Q \setminus F$ are the nonaccepting states).

Let $A = (Q, \Sigma, Q_0, \Delta, F)$ be an automaton and $a \in \Sigma$. If $(q_1, a, q_2) \in \Delta$, then we say that there is a transition from $q_1$ (the origin) to $q_2$ (the destination) labeled by $a$. We sometimes abuse the notations, and write $q_2 \in \Delta(q_1, a)$ instead of $(q_1, a, q_2) \in \Delta$. Two transitions $(q_1, a, q_2), (q_3, b, q_4) \in \Delta$ are consecutive if $q_2 = q_3$. Given two states $q, q' \in Q$ and a finite word $w \in \Sigma^*$, we write $(q, w, q') \in \Delta^*$ if there exist states $q_0, \ldots, q_n$ and $w(0), \ldots, w(n-1) \in \Sigma$ such that $q_0 = q, q_n = q', w = w(0)w(1)\ldots w(n-1)$, and $(q_i, w(i), q_{i+1}) \in \Delta$ for all $0 \leq i < n - 1$. Given two states $q, q' \in Q$, we say that the state $q'$ is
reachable from \( q \) in \( A \) if \( (q, a, q') \in \Delta^* \). The automaton \( A \) is complete if for each state \( q \in Q \) and symbol \( a \in \Sigma \), there exists at least one state \( q' \in Q \) such that \( (q, a, q') \in \Delta \). An automaton can easily be completed by adding an extra nonaccepting state.

A finite run of \( A \) on a finite word \( w : \{0, \ldots, n-1\} \rightarrow \Sigma \) is a labeling \( \rho : \{0, \ldots, n\} \rightarrow Q \) such that \( \rho(0) \in Q_0 \), and \( (\forall 0 \leq i \leq n-1)( (\rho(i), w(i), \rho(i+1)) \in \Delta) \). A finite run \( \rho \) is accepting for \( w \) if \( \rho(n) \in F \). An infinite run of \( A \) on an infinite word \( w : \mathbb{N} \rightarrow \Sigma \) is a labeling \( \rho : \mathbb{N} \rightarrow Q \) such that \( \rho(0) \in Q_0 \), and \( (\forall 0 \leq i)((\rho(i), w(i), \rho(i+1)) \in \Delta) \). An infinite run \( \rho \) is accepting for \( w \) if \( \inf(\rho) \cap F \neq \emptyset \), where \( \inf(\rho) \) is the set of states that are visited infinitely often by \( \rho \).

We distinguish between finite-word automata that are automata accepting finite words, and Büchi automata that are automata accepting infinite words. A finite-word automaton accepts a finite word \( w \) if there exists an accepting finite run on \( w \) in this automaton. A Büchi automaton accepts an infinite word \( w \) if there exists an accepting infinite run on \( w \) in this automaton. The set of words accepted by \( A \) is the language accepted by \( A \), and is denoted \( L(A) \). Any language that can be represented by a finite-word (respectively, Büchi) automaton is said to be regular (respectively, \( \omega \)-regular).

The automaton \( A \) may behave nondeterministically on an input word, since it may have many initial states and the transition relation may specify many possible transitions for each state and symbol. If \( |Q_0| = 1 \) and for all state \( q_1 \in Q \) and symbol \( a \in \Sigma \) there is at most one state \( q_2 \in Q \) such that \( (q_1, a, q_2) \in \Delta \), then \( A \) is deterministic. In order to emphasize this property, a deterministic automaton is denoted as a tuple \( (Q, \Sigma, q_0, \delta, F) \), where \( q_0 \) is the unique initial state and \( \delta : Q \times \Sigma \rightarrow Q \) is a partial function deduced from the transition relation by setting \( \delta(q_1, a) = q_2 \) if \( (q_1, a, q_2) \in \Delta \). Operations on languages directly translate to operations on automata, and so do the notations.

One can decide whether the language accepted by a finite-word or a Büchi automaton is empty or not. It is also known that finite-word automata are closed under determinization, complementation, union, projection, and intersection [Hop71]. Moreover, finite-word automata admit a minimal form, which is unique up to isomorphism [Hop71].

Though the union, intersection, synchronous product, and projection of Büchi automata can be computed efficiently, the complementation operation requires intricate algorithms that not only are worst-case exponential, but are also hard to implement and optimize (see [Var07] for a survey). The core problem is that there are Büchi automata that do not admit a deterministic/minimal form. To working with infinite-word automata that do own the same properties as finite-word automata, we will restrict ourselves to weak automata [MSS86].
defined hereafter.

**Definition 4** For a Büchi automaton $A = (\Sigma, Q, q_0, \delta, F)$ to be weak, there has to be a partition of its state set $Q$ into disjoint subsets $Q_1, \ldots, Q_m$ such that for each of the $Q_i$, either $Q_i \subseteq F$, or $Q_i \cap F = \emptyset$, and there is a partial order $\leq$ on the sets $Q_1, \ldots, Q_m$ such that for every $q \in Q_i$ and $q' \in Q_j$ for which, for some $a \in \Sigma$, $q' \in \delta(q, a)$ ($q' = \delta(q, a)$ in the deterministic case), $Q_j \leq Q_i$.

A weak automaton is thus a Büchi automaton such that each of the strongly connected components of its graph contains either only accepting or only non-accepting states.

Not all $\omega$-regular languages can be accepted by deterministic weak Büchi automata, not even by nondeterministic weak automata. However, there are algorithmic advantages to working with weak automata: deterministic weak automata can be complemented simply by inverting their accepting and non-accepting states; and there exists a simple determinization procedure for weak automata [Saf92], which produces Büchi automata that are deterministic, but generally not weak. Nevertheless, if the represented language can be accepted by a deterministic weak automaton, the result of the determinization procedure will be inherently weak according to the definition below [BJW01] and thus easily transformed into a weak automaton.

**Definition 5** A Büchi automaton is inherently weak if none of the reachable strongly connected components of its transition graph contain both accepting (visiting at least one accepting state) and non-accepting (not visiting any accepting state) cycles.

This gives us a pragmatic way of staying within the realm of deterministic weak Büchi automata. We start with sets represented by such automata. This is preserved by union, intersection, synchronous product, and complementation operations. If a projection is needed, the result is determinized by the known simple procedure. Then, either the result is inherently weak and we can proceed, or it is not and we are forced to use the classical algorithms for Büchi automata. The latter cases might never occur, for instance if we are working with automata representing sets of reals definable in the first-order theory of linear constraints [BJW01].

A final advantage of weak deterministic Büchi automata is that they admit a minimal form, which is unique up to isomorphism [Löd01].
2.4 Transducers

In this paper, we will consider relations that are defined over sets of words. We use the following definitions taken from [Nil01]. For a finite-word (respectively, infinite-word) language $L$ over $\Sigma^n$, we denote by $[L]$ the finite-word (respectively, infinite-word) relation over $\Sigma^n$ consisting of the set of tuples $(w_1, w_2, \ldots, w_n)$ such that $w_1 \times w_2 \times \ldots \times w_n$ is in $L$. The arity of such a relation is $n$. Note that for $n = 1$, we have that $L = [L]$. The relation $R_{id}$ is the identity relation, i.e., $R_{id} = \{(w_1, w_2, \ldots, w_n) | w_1 = w_2 = \ldots = w_n\}$. A relation $R$ defined over $\Sigma^n$ is $(\omega)$-regular if there exists a $(\omega)$-regular language $L$ over $\Sigma^n$ such that $[L] = R$.

We now introduce transducers that are automata for representing $(\omega)$-regular relations over $\Sigma^2$.

**Definition 6** A transducer over $\Sigma^2$ is an automaton $T$ over $\Sigma^2$ given by $(Q, \Sigma^2, Q_0, \Delta, F)$, where

- $Q$ is the finite set of states,
- $\Sigma^2$ is the finite alphabet,
- $Q_0 \subseteq Q$ is the set of initial states,
- $\Delta : Q \times \Sigma^2 \times Q$ is the transition relation, and
- $F \subseteq Q$ is the set of accepting states (the states that are not in $F$ are the nonaccepting states).

Given an alphabet $\Sigma$, the transducer representing the identity relation over $\Sigma^2$ is denoted $T_{id}^\Sigma$ (or $T_{id}$ when $\Sigma$ is clear from the context). All the concepts and operations defined for finite automata can be used with transducers. The only reason to particularize this class of automata is that some operations, such as composition, are specific to relations. In the sequel, we use the term “transducer” instead of “automaton” when using the automaton as a representation of a relation rather than as a representation of a language. We sometimes abuse the notations and write $(w_1, w_2) \in T$ instead of $(w_1, w_2) \in [L(T)]$. Given a pair $(w_1, w_2) \in T$, $w_1$ is the input word, and $w_2$ is the output word.

The transducers we consider here are often called structure-preserving, which means that when following a transition, a symbol of the input word is replaced by exactly one symbol of the output word.

Given two transducers $T_1$ and $T_2$ over the alphabet $\Sigma$ that represents two relations $R_1$ and $R_2$, respectively. The composition of $T_1$ by $T_2$, denoted $T_2 \circ T_1$ is the transducer that represents the relation $R_2 \circ R_1$. We denote by $T_i^i$ ($i \in \mathbb{N}_0$) the transducer that represents the relation $R_i^i$. The transitive closure of $T$ is $T^+ = \bigcup_{i=1}^\infty T_i^i$; its reflexive transitive closure is $T^* = T^+ \cup T_{id}$. The transducer
$T$ is reflexive if and only if $L(T_{id}) \subseteq L(T)$. Given an automaton $A$ over $\Sigma$ that represents a set $S$, we denote by $T(A)$ the automaton representing the image of $A$ by $T$, i.e., an automaton for the set $R(S)$.

Let $T_1$ and $T_2$ be two finite-word (respectively, Büchi) transducers defined over $\Sigma^2$ and let $A$ be a finite-word automaton (respectively, Büchi) automaton defined over $\Sigma$. We observe that $T_2 \circ T_1 = \pi_2[(T_1 \times T_{id}^2) \cap (T_{id}^2 \times T_2)]$ and $T(A) = \pi_1[(A^E \times \Sigma) \cap T]$, where $A^\Sigma$ is an automaton accepting $\Sigma^\ast$ (respectively, $\Sigma^\omega$). As a consequence, the composition of two finite-word ((weak) Büchi) transducers is a finite-word transducer. However, the composition of two deterministic weak Büchi transducer is a weak Büchi transducer whose deterministic version may not be weak. A same observation can be made about the composition of a transducer with an automaton.

3 Systems models and ($\omega$)-Regular Model Checking

3.1 The Framework

In this section, we recall the definition of state-transition system, that is the abstraction formalism which is generally used to describe programs. We then present an automata-based encoding of state-transition systems. Finally, the properties of this encoding are discussed.

3.1.1 State-transition Systems

Systems are often modeled as state-transition systems.

Definition 7 A state-transition system is a tuple $(S, S_0, R)$, where

- $S$ is a (possibly infinite) set of states,
- $S_0 \subseteq S$ is a (possibly infinite) set of initial states, and
- $R \subseteq S \times S$ is a (possibly infinite) reachability relation that describes the transitions between the states of the system.

Let $T = (S, S_0, R)$ be a state-transition system. If $(s, s') \in R$, then we say that there is a transition from $s$ (the origin) to $s'$ (the destination). Given two states $s, s' \in S$, we write $s \rightarrow_R s'$ if and only if $(s, s') \in R$. A state $s' \in S$ is said to be reachable from a state $s \in S$ if there exists $k > 0$ and states $s_0, s_1, s_2, \ldots, s_{k-1} \in S$ such that $s_0 = s$, $s_{k-1} = s'$ and $s_i \rightarrow_R s_{i+1}$, for all $0 \leq i < k - 1$. The fact that $(s, s')$ belongs to the reflexive transitive closure $R^\ast$ of $R$ is denoted by $s \rightarrow_R^\ast s'$. A state $s \in S$ is reachable if
it is reachable from a state in $S_0$. The set of all reachable states of $T$ is denoted $S_T^R$. The state space $(S_T^R, R_T^R)$ of $T$ is the (possibly infinite) graph whose nodes are the reachable states of $T$, and whose edges $R_T^R$ are given by $R \cap (S_T^R \times S_T^R)$. We say that $T$ is finite if $S_T^R$ is finite, it is infinite otherwise. $T$ is said to be locally-finite if and only if any executions from any state in $S_T^R$ can only go through a finite number of distinct states. A finite execution $\pi$ of $T$ is a mapping $\pi : \{0, \ldots, n-1\} \to S$ such that $\pi(0) \in S_0$ and for all $0 \leq i < n-1$, $\pi(i) \rightarrow_R \pi(i+1)$. A finite execution is often represented by $\pi = \pi(0)\pi(1)\pi(2)\ldots\pi(n-1)$. An infinite execution $\pi$ of $T$ is a mapping $\pi : \mathbb{N} \to S$ such that $\pi(0) \in S_0$ and for all $i \geq 0$, $\pi(i) \rightarrow_R \pi(i+1)$. An infinite execution is often represented by $\pi = \pi(0)\pi(1)\pi(2)\ldots$. In the rest of this paper, we consider systems whose executions are all infinite.

One distinguishes between two types of properties.

(1) **Reachability properties.** We assume that a reachability property $\varphi$ is described as a set of states $S_\varphi \subseteq S$. The system $T$ satisfies $\varphi$ if and only if $S_T^R \subseteq S_\varphi$. Verifying reachability properties thus reduces to computing the set of reachable states.

(2) **Linear temporal properties.** We assume that a linear temporal property $\varphi$ is described as a set of executions $\pi_\varphi$, which are often represented by a Büchi automaton. The system $T$ satisfies $\varphi$ if and only if each of its executions belongs to $\pi_\varphi$. In general, the verification of linear temporal properties does not reduce to the computation of the set of reachable states of the system.

### 3.1.2 (\(\omega\))-Regular Model Checking

In this paper, we suppose that states of state-transition systems are encoded by words over a fixed alphabet. If the states are encoded by finite words, then sets of states can be represented by finite-word automata and relations between states by finite-word transducers. This setting is referred to as regular model checking [KMM+97, WB98]. If the states are encoded by infinite words, then sets of states can be represented by deterministic weak Büchi automata and relations between states by deterministic weak Büchi transducers. This setting is referred to as \(\omega\)-regular model checking [BLW04a]. Formally, a finite automata-based representation of a state-transition system can be defined as follows.

**Definition 8** A (\(\omega\)-)regular system for a state-transition system $T = (S, S_0, R)$ is a triple $M = (\Sigma, A_{S_0}, T_R)$, where
• $\Sigma$ is a finite alphabet over which the states are encoded as finite (respectively infinite) words;
• $A_{S_0}$ is a deterministic finite-word (respectively deterministic weak Büchi) automaton over $\Sigma$ that represents $S_0$;
• $T_R$ is a deterministic finite-word (respectively deterministic weak Büchi) transducer over $\Sigma^2$ that represents $R$.

States being represented by words, the notion of set of states, initial states, reachability relation, computation, reachable state, locally-finite for ($\omega$-)regular systems are defined identically to those of the corresponding state-transition system. There are many state-transition systems whose sets of states cannot be encoded by ($\omega$)-regular languages\footnote{Indeed, there are uncountably many subsets of an infinite set of states, but only countably many finite strings of bits.}. Consequently, there are many state-transition systems for which there exists no corresponding ($\omega$)-regular system.

In the finite-word case, an execution of the system is an infinite sequence of same-length finite words over $\Sigma$. The regular model checking framework was first used to represent parametric systems [AJMd02,KMM+97,ABJN99]. The framework can also be used to represent various other models, which includes linear integer systems [WB95,WB00], FIFO-queues systems [BG96], XML specifications [BHRV06,Td06], and heap analysis [BHMV05,BHRV06].

We now give insight about how to represent parametric systems. Let $P$ be a process represented by a finite state-transition system. A parametric system for $P$ is an infinite family $S = \{S_n\}_{n=0}^{\infty}$ of networks where for a fixed $n$, $S_n$ is an instance of $S$, i.e. a network composed of $n$ copies of $P$ that work together in parallel. In the regular model checking framework, the finite set of states of each process is given as an alphabet $\Sigma$. Each state of an instance of the system can then be encoded as a finite word $w = w(0)\ldots w(n-1)$ over $\Sigma$, where $w(i-1)$ encodes the current state of the $i$th copy of $P$. Sets of states of several instances can thus be encoded together by finite-word automata. Observe that the states of an instance $S_n$ are all encoded with words of the same length. Consequently, relations between states in $S_n$ can be represented by binary finite-word relations, and eventually by transducers.

**Example 9** Consider a simple example of parametric network of identical processes implementing a token ring algorithm. Each of these processes can be either in idle or in critical mode, depending on whether or not it owns the unique token. Two neighboring processes can communicate with each other as follows: a process owning the token can give it to its right-hand neighbor. We consider the alphabet $\Sigma = \{N,T\}$. Each process can be in one of the two following states : $T$ (has the token) or $N$ (does not have the token). Given a word $w \in \Sigma^*$ with $|w| = n$ (meaning that $n$ processes are involved in the
execution), we assume that the process whose states are encoded in position \( w(0) \) is the right-hand neighbor of the one whose states are encoded in position \( w(n - 1) \). The transition relation can be encoded as the union of two reachability relations that are the following:

- \((N, N)^*(T, N)(N, T)(N, N)^*\) to describe the move of the token from \( w(0) \) to \( w(n - 1) \), and
- \((N, T)(N, N)^*(T, N)\) to describe the move of the token from \( w(n - 1) \) to \( w(0) \).

The set of all possible initial states where the first process has the token is described by \( TN^* \).

In the infinite-word case, an execution of the system is an infinite sequence of infinite words over \( \Sigma \). The \( \omega \)-regular model checking framework has been used for handling systems with both integer and real variables [BW02,BJW05], such as linear hybrid systems with a constant derivative (see examples in [ACH+95] or in [BLW04b,Leg07]).

Verifying reachability properties of state-transition systems using their (\( \omega \)-)regular representation can easily be conducted with simple automata-based manipulations, assuming the existence of finite-word (respectively weak Büchi) automata for representing both the set of reachable states and the property. Computing an automaton that represents the set of reachable states can be reduced to the (\( \omega \)-)regular reachability problems defined hereafter.

**Definition 10** Let \( A \) be a deterministic finite-word (respectively weak Büchi) automaton, and \( T \) be a deterministic finite-word (respectively weak Büchi) transducer. The (\( \omega \))-regular reachability problems for \( A \) and \( T \) are the following:

1. **Computing** \( T^*(A) \): the goal is to compute a finite-word (respectively weak Büchi) automaton representing \( T^*(A) \). If \( A \) represents a set of states \( S \) and \( T \) a relation \( R \), then \( T^*(A) \) represents the set of states that can be reached from \( S \) by applying \( R \) an arbitrary number of times;
2. **Computing** \( T^* \): the goal is to compute a finite-word (respectively weak Büchi) transducer representing the reflexive transitive closure of \( T \). If \( T \) represents a reachability relation \( R \), then \( T^* \) represents its closure \( R^* \).

Being able to compute \( T^*(A) \) is clearly enough for verifying reachability properties. On the other hand, we will see that the computation of \( T^* \) is generally incontrovertible when considering the verification of temporal properties. In the rest of this paper, we propose techniques that reduce the verification of several classes of linear temporal properties to the resolution of the (\( \omega \)-)regular reachability problems over an augmented system.
3.2 On Solving (\(\omega\))Regular Reachability Problems

Among the techniques to compute \(T^*(A)\) and \(T^*\), one distinguishes between domain specific and generic techniques. Domain specific techniques exploit the specific properties and representations of the domain being considered and were for instance obtained for systems with FIFO-queues in [BG96,BH97], for systems with integers and reals in [Boi99,BW02,BHJ03], for pushdown systems in [FWW97,BEM97], and for lossy queues in [AJ96]. Generic techniques consider automata-based representations and provide algorithms that operate directly on these representations, mostly disregarding the domain for which it is used. There are various generic techniques to computing \(T^*(A)\) and \(T^*\) when considering \(T\) and \(A\) to be finite-word automata (e.g. [BJNT00,DLS02,BLW03]). The \(\omega\)-regular reachability problems can be addressed with the technique introduced in [BLW04a].

3.3 Convention, Concepts, and Observations

This section introduces some concepts and observations that will be used throughout the rest of the paper. We first introduce Büchi (\(\omega\))regular systems.

**Definition 11** A Büchi (\(\omega\))regular system is a tuple \((M, F)\), where \(M = (\Sigma, A_{S_0}, T_R)\) is a (\(\omega\))regular system, and \(F\) is a deterministic finite-word (resp. deterministic weak Büchi) automaton called the Büchi acceptance condition.

The notions of set of states, initial states, reachability relation, computation, reachable state, and locally-finite for Büchi (\(\omega\))regular system \((M, F)\) are defined exactly as those of its underlying (\(\omega\))regular system \(M = (\Sigma, A_{S_0}, T_R)\). An infinite computation \(\pi = \pi(0)\pi(1)\ldots\) of \((M, F)\) is accepting if and only if there are infinitely many \(i\) such that \(\pi(i) \in L(F)\). We say that \((M, F)\) is empty if all its infinite executions are non-accepting. In the rest of the paper, we abuse the notations and write \((\Sigma, A_{S_0}, T_R, F)\) instead of \((M, F)\).

We now reason on infinite executions. Consider a (\(\omega\))regular system \(M = (\Sigma, A_{S_0}, T_R)\) that encodes a state-transition system \(T = (S, S_0, R)\). The fact that \(T_R\) is structure-preserving does not imply that \(M\) is locally-finite. Indeed, as it is illustrated with the following example, each state of \(T\) can potentially be associated to an infinite set of encodings.

**Example 12** Following the framework of [WB00], the digit 5 can be encoded in base 2 as 0101, or as 00101, or as 000101, ..., and in fact by any word in the set \(0^+101\).
By definition, parametric systems are always locally-finite. Indeed, the number of finite-state processes is fixed during the whole execution. This makes it impossible to visit an infinite number of different states. Most other classes of infinite-state systems can either be locally-finite or not, depending on their specifications.

**Example 13** An integer system that continuously adds 1 to a variable \(x\) up to a constant value is locally-finite. However, if there is no bound on the value of \(x\), then the system is not locally-finite.

Unfortunately, testing whether a system is locally-finite is an undecidable problem. As a consequence only partial solutions can be proposed. In the rest of this section, we propose such a solution that is based on a reduction to the \((\omega)\)-regular reachability problems over an augmented system. Our solution is formalized with the following theorem.

**Theorem 14** Consider a state-transition system \(T = (S, S_0, R)\) and the following sets

- \(S_0^a = S_0 \times \{0\}\),
- \(R^a = \{(s, i), (s', i + 1) | (i \in \mathbb{N})((s, s') \in R \setminus R_{id})\}\),
- \(S_{lf} = \{s \in S | \exists i, \forall (j > i), \neg \exists s'((s, 0), (s', j)) \in (R^a)^*\}\).

If \(S_0^a \subseteq S_{lf}\), then \(T\) is locally-finite.

**PROOF.** Direct by construction.

The procedure sketched above requires to compute the set \(S_{lf}\). In the \((\omega)\)-regular model checking framework, this computation can easily be performed when both \((R^a)^*\) and \(S_0^a\) represent solutions of Presburger arithmetic formulas [WB00,BJW05].

### 4 Linear Temporal Properties in Regular Model Checking

#### 4.1 Definitions

In this section we propose a methodology to verify linear temporal properties of state-transition systems that are represented in the regular model checking framework. Our first step is a symbolic representation for linear temporal properties in this framework. We propose the following definitions.
Definition 15  Given an alphabet $\Sigma$, a state property is a set $\text{cop} \subseteq \Sigma^*$ that can be represented by a finite-word automaton.

Definition 16  Let COP be a finite set of state properties. A global system property over COP is a set $\text{gsp} \subseteq (2^\text{COP})^\omega$, i.e. a set of infinite sequences of state properties, that can be represented by a Büchi automaton.

Assume a set of state properties COP, and a global system property gsp defined over COP. An execution $\pi = w_0w_1w_2w_3w_4 \ldots$ of a regular system $M$ satisfies gsp, denoted $\pi \models \text{gsp}$, if and only if $\text{cop}(w_0) \text{cop}(w_1) \cdots \in \text{gsp}$, where $\text{cop}(w) = \{\text{cop}_i \in \text{COP} | w \models \text{cop}_i\}$. We say that $M$ satisfies gsp, denoted $M \models \text{gsp}$, if and only if all its executions satisfy the property.

The definition of global system properties is illustrated in Figure 1.

Remark 17  Any Linear Temporal Logic property\textsuperscript{4} (LTL in short) whose atomic propositions are represented by sets of states is thus a global system property. The set of LTL properties whose atomic propositions are represented by sets of states is a strict subset of the set of global system properties.

4.2 Verification

Assume a regular system $M = (\Sigma, A_{S_0}, T_R)$, a set of state properties COP = \{cop$_1$, \ldots cop$_k$\}, and a global system property gsp defined over COP. Suppose

\textsuperscript{4} We assume the reader is familiar with the syntax, the semantic, and the notations of the linear temporal logic introduced in [Pnu77]. We recall the shortcuts for the temporal operators that are $\Box$ for “always”, $\diamond$ for eventually, and $\bigcirc$ for “next”.

15
that each $\text{cop}_i \in \text{COP}$ is represented by a complete deterministic finite-word automaton $A_{\text{cop}_i} = (Q_{\text{cop}_i}, \Sigma, q_{0_{\text{cop}_i}}, \delta_{\text{cop}_i}, F_{\text{cop}_i})$. We extend the automata theoretic approach of [VW86] towards a semi-algorithm to test whether $M$ satisfies $gsp$. Our approach consists in three successive steps that are the following:

1. Computing a complete Büchi automaton $A_{\neg gsp} = (Q_{\neg gsp}, 2^{\text{COP}}, q_{0_{\neg gsp}}, \Delta_{\neg gsp}, F_{\neg gsp})$ representing the negation of the property $gsp$, i.e. $(2^{\text{COP}} \setminus gsp)$.
2. Building a Büchi regular system $M_{\neg gsp}^a = (\Sigma^a, A_{\neg gsp}, T_R^a, F^a)$ whose accepting executions correspond to those of $M$ that are accepted by $A_{\neg gsp}$.
3. Testing whether $M_{\neg gsp}^a$ is empty or not. By construction, $M$ satisfies $gsp$ if and only if $M_{\neg gsp}^a$ is empty.

The property $gsp$ being (by definition) representable by a Büchi automaton, one can always compute the automaton $A_{\neg gsp}$. We now focus on the two other problems. The system $M_{\neg gsp}^a$ can be built by taking the product between the states of $M$ and those of $A_{\neg gsp}$. Given $w, w' \in \Sigma^*$ and $q_{\neg gsp}, q'_{\neg gsp} \in Q_{\neg gsp}$, the product must ensure that one can move from the pair $(w, q_{\neg gsp})$ to the pair $(w', q'_{\neg gsp})$ if and only if (1) $(w, w') \in T_R$, and (2) $(q_{\neg gsp}, \text{cop}(w), q'_{\neg gsp}) \in \Delta_{\neg gsp}$. Since the set of states of $M$ may be infinite, we have to work with a symbolic representation of $\text{cop}$. We propose to represent $\text{cop}$ implicitly by associating to each pair $(w, q)$ the set $\text{COP}_i$ such that $\text{cop}(w) = \text{COP}_i$. Hence a state of the product is now a triple $(w, q_{\neg gsp}, \text{COP}_i)$ such that (1) $w \in \Sigma^*$, (2) $q_{\neg gsp} \in Q_{\neg gsp}$, and (3) $\text{cop}(w) = \text{COP}_i$. Each triple $(w, q_{\neg gsp}, \text{COP}_i)$ has to be encoded by a finite word over an extended alphabet. The solution is to label the last symbol of $w$ with $\text{COP}_i$ and $q_{\neg gsp}$, and the other symbols by $\bot$. Hence, we define the augmented alphabet to be

$$\Sigma^a = \Sigma \times (Q_{\neg gsp} \cup \{\bot\}) \times (2^{\text{COP}} \cup \{\bot\}).$$

Given a word $w^a \in (\Sigma^a)^*$, we denote by $\Pi_{\Sigma}(w^a)$, the word $w \in \Sigma^*$ obtained from $w^a$ by removing all the symbols that do not belong to $\Sigma$. As an example, given $w^a = (w(0), \bot, \bot)(w(1), \bot, \bot) \cdots (w(n-1), q, \lambda)$ with $q \in Q_{\neg gsp}$, $\lambda \in 2^{\text{COP}}$, $\Pi_{\Sigma}(w^a) = w(0)w(1) \cdots w(n-1)$.

An execution $\pi^a = w^a_0 w^a_1 w^a_2 \cdots$ of $M_{\neg gsp}^a$ is an infinite sequence of finite words over $\Sigma^a$. This sequence has to satisfy the following four requirements:

1. For each $i \geq 1$ $(\Pi_{\Sigma}(w^a_{i-1}), \Pi_{\Sigma}(w^a_i)) \in T_R$, which ensures that the transitions of $M_{\neg gsp}^a$ are compatible with the transition relation of $M$;
2. For each $i \geq 0$, $w^a_i \in (\Sigma \times \bot \times \bot)^* (\Sigma \times Q_{\neg gsp} \times 2^{\text{COP}})$;
3. For each $i \geq 0$ and $w^a = (w_i(0), \bot, \bot)(w_i(1), \bot, \bot) \cdots (w_i(n-1), q_{\neg gsp}, \text{COP}_i)$, $\text{cop}(\Pi_{\Sigma}(w^a_i)) = \text{COP}_i$;
4. For each $i \geq 1$ and $w^a_{i-1} = (w_{i-1}(0), \bot, \bot)(w_{i-1}(1), \bot, \bot) \cdots (w_{i-1}(n-1), \bot, \bot)$,
1), \(q_{i-1\text{gsp}}\), \(COP_{i-1}\), and \(w^n = (w_i(0), \perp, \perp)(w_i(1), \perp, \perp) \cdots (w_i(n-1), q_{i\text{gsp}}, COP_i)\), we have \((q_{i-1\text{gsp}}, COP_{i-1}, q_{i\text{gsp}}) \in \Delta_{gsp}\), this to ensure that the infinite sequence of labellings from \(2^{\text{COP}}\) and \(Q_{gsp}\) form a run of the automaton \(A_{gsp}\).

We have to build automata for \(A_{S_0}^a\), \(F^a\), and \(T_R^a\) in such a way that the four requirements above are satisfied.

Let \(T_R = (Q_R, \Sigma, q_{0R}, \delta_R, F_R)\), the transducer \(T_R^a = (Q_R', (\Sigma^a)^2, q_{0R}^a, \Delta_R^a, F_R^a)\) is built as follows:

- The set of states \(Q_R'\) is \(Q_R = Q_R \times \prod_{1 \leq i \leq k} Q_{cop_i} \times \{0, 1\}\), the last Boolean being used to remember if non \(\perp\) labellings have been seen and \(\prod_{1 \leq i \leq k} Q_{cop_i}\) is used to run the automata representing the state properties, this to ensure that each state of \(M^a_{gsp}\) is associated to the set of state properties it satisfies;
- The initial state is \(q_{0R}^a = (q_{0R}, q_{0\text{cop}_1}, \ldots, q_{0\text{cop}_k}, 0)\);
- The transition relation \(\Delta_R^a\) is defined by

\[
(q_R', q_{\text{cop}_1}', \ldots, q_{\text{cop}_k}', b') \in \Delta_R^a((q_R, q_{\text{cop}_1}, \ldots, q_{\text{cop}_k}, b), ((a_1, \alpha_1, \lambda_1), (a_2, \alpha_2, \lambda_2)))
\]

if and only if
- \(q_R' \in \delta_R(q_R, (a_1, a_2))\) and \(q_{\text{cop}_i}' = \delta_{\text{cop}_i}(q_{\text{cop}_i}, a_1)\), for \(1 \leq i \leq k\),
- \(b' = 1\) if and only if \(\lambda_1, \alpha_1, \lambda_2, \) and \(\alpha_2\) are not equal to \(\perp\) and, in this case, \(\alpha_2 \in \Delta_{gsp}(a_1, \lambda_1)\), which checks that we have a run of \(A_{gsp}\) and for \(1 \leq i \leq k\), \(q_{\text{cop}_i}' \in F_{cop_i}\) if and only if \(\text{cop}_i \in \lambda_1\), which checks that the label \(\lambda_1\) matches the result of running the automata \(\text{cop}_i\) on the state (this justify the need for each \(\text{cop}_i\) to be deterministic and complete);
- The set of accepting states \(F_R^a\) is defined as \(F_R \times \prod_{1 \leq i \leq k} Q_{cop_i} \times \{1\}\).

The definition of \(T_R^a\) ensures that requirements (1) (3) and (4) are satisfied.

The set of initial states of \(M^a_{gsp}\) contains states of the following form:

\[
(w(0), \perp, \perp) \cdots (w(n-2), \perp, \perp)(w(n-1), q_{0gsp}, \lambda),
\]

where \(w(0) \cdots w(n-1) \in L(A_{S_0})\) and \(\lambda\) is any element of \(2^{\text{COP}}\). This definition combined with the one of \(T_R^a\) ensures that the second requirement on the executions of \(M^a_{gsp}\) is always satisfied. The set of initial states can be represented by a finite-word automaton \(A_{S_0}^a\) that is given by \(A_{S_0} \times A_{\perp}\), where \(A_{\perp}\) is the automaton representing the set \((\perp \times \perp)^*(q_{0gsp} \times 2^{\text{COP}})\).

The set of accepting states of \(M^a_{gsp}\) is defined as follows:

\[
(\Sigma \times \{\perp\} \times \{\perp\})^*(\Sigma \times F_{gsp} \times 2^{\text{COP}}).
\]

We directly see that this set can be represented by a finite-word automaton \(F^a\).
Theorem 18 The Büchi regular system $M_{a-gsp}^a$ has an accepting execution of the form $π^a = π^a(0)π^a(1)\ldots$ if and only if the execution $π = w_0w_1w_2\ldots$ of $M$, where $w_i = ΠΣ(π^a(i)) (\forall i)$, does not satisfy $gsp$.

PROOF. Follows from the construction above.

The next step is to test whether $M_{a-gsp}^a$ is empty or not. If $M$ is locally-finite, then $M_{a-gsp}^a$ is also locally-finite and checking the emptiness of $M_{a-gsp}^a$ can be reduced to solving the regular reachability problems. We have the following result.

Proposition 19 If $M_{a-gsp}^a$ is locally finite, then it is empty if and only if

\[ L((T^a_R)^*(A^a_{S_0}) \cap F^a \cap Π\varphi_2((T^a_R)^+ \cap T_{id})) = \emptyset. \]

PROOF. Directe by observing that since $M_{a-gsp}^a$ is locally-finite, any of its accepting execution must repeatedly reach a given state in $F^a$.

If $M_{a-gsp}^a$ is not locally-finite, then we cannot reduce the problem of deciding if it has an infinite accepting execution to the one of finding reachable accepting loops. Indeed, in this case, an infinite execution could never visit the same state twice. Therefore, our approach is to search for a reachable state $w$ from which it is possible to nontrivially reach some state $w'$ such that (1) the path from $w$ to $w'$ visits a repeating state of $A_{-gsp}$, and (2) $w'$ has at least the same execution paths as $w$. To check the condition (2), we check actually for a stronger condition which is the fact that $w'$ must simulate $w$.

We define the greatest simulation relation over $M_{a-gsp}^a$ which is compatible with the set of state properties $COP$ to be the relation $Sim$ defined as the limit of the (possibly infinite) decreasing sequence of relations $Sim_0, Sim_1, Sim_2, \ldots$ with

\[
\begin{align*}
Sim_0 &= \{(w_1^a, w_2^a) \mid w_1^a \times w_2^a \in (\Sigma^a \times \Sigma^a)^* \land \text{cop}(ΠΣ(w_1^a)) = \text{cop}(ΠΣ(w_2^a))\} \quad (1) \\
Sim_{k+1} &= Sim_k \cap \{(w_1^a, w_2^a) \in Sim_k \mid \forall w_3^a, ((w_1^a, w_2^a) \in T^a_R \Rightarrow \exists w_4^a, (w_2^a, w_4^a) \in T^a_R \land (w_3^a, w_4^a) \in Sim_k), \forall k \in \mathbb{N}\} \quad (2)
\end{align*}
\]

The complement of $Sim$, denoted $\neg Sim$, is the set $\{(w_1^a, w_2^a) \mid (w_1^a \times w_2^a) \in (\Sigma^a \times \Sigma^a)^* \land (w_1^a, w_2^a) \notin Sim\}$. The greatest simulation equivalence over $M_{a-gsp}^a$ which is compatible with $COP$ is the relation $\sim Sim = Sim \cap Sim^{-1}$. Observe that
Sim can be represented by a finite-word transducer over the alphabet \((\Sigma^a)^2\). Since, for each \(k \geq 0\), the relation \(\text{Sim}_k + 1\) is defined in terms of the relations \([L(T^a_R)]\) and \(\text{Sim}_k\) using Boolean operations and projections (needed to apply the quantifiers), it can be represented by a finite-word transducer over the alphabet \((\Sigma^a)^2\). Moreover, if \(\text{Sim}_k\) can be represented by a transducer, then its complement and inverse can also be represented in the same way.

Assume that \(\text{Sim}\) and \((\Sigma^a)^*\) are respectively represented by a transducer \(T_{\text{Sim}}\) and an automaton \(A^{(\Sigma^a)^*}\). We have the following result.

**Proposition 20** If

\[
L\left((T^a_R)^*\left(A^a_{S_0}\right) \cap \Pi_{\neq 2}((T^a_R)^+ \cap (A^{(\Sigma^a)^*} \times F^a) \cap T_{\text{Sim}})\right) \neq \emptyset,
\]

then \(M_{\text{gsp}}^a\) has an infinite execution that does not satisfy \(\text{gsp}\).

**PROOF.** The set \(L\left((T^a_R)^*\left(A^a_{S_0}\right) \cap \Pi_{\neq 2}((T^a_R)^+ \cap (A^{(\Sigma^a)^*} \times F^a) \cap T_{\text{Sim}})\right)\) is the set of states \(w\) from which it is possible to reach an accepting state \(w'\) such that \(w'\) simulates \(w\). Since \(w'\) simulates \(w\), one can reach from \(w'\) an other accepting state \(w''\) that simulates \(w'\) and, inductively, there exists an execution that infinitely often goes through an accepting state.

The main issue is now to determine whether the iterative computation of \(\text{Sim}\) terminates and can be represented by an automaton. We consider the two following cases.

### 4.2.1 Exact Analysis

We say that \(M_{\text{gsp}}^a\) has a finite-index simulation if the simulation equivalence \(\tilde{\text{Sim}}\) has a finite number of equivalence classes. The following lemma is quite straightforward.

**Lemma 21** The iterative computation of the simulation relation \(\text{Sim}\) terminates if and only if \(M_{\text{gsp}}^a\) has a finite-index simulation.

If \(M_{\text{gsp}}^a\) has a finite-index simulation equivalence, then every infinite execution of \(M_{\text{gsp}}^a\) must visit infinitely often some of the equivalence classes. Therefore, we have the following proposition.

**Proposition 22** Assume that the system \(M_{\text{gsp}}^a\) has a finite-index simulation. \(M_{\text{gsp}}^a\) has an accepting execution if and only if

\[
L\left((T^a_R)^*\left(A^a_{S_0}\right) \cap \Pi_{\neq 2}((T^a_R)^+ \cap (A^{(\Sigma^a)^*} \times F^a) \cap T_{\text{Sim}})\right) \neq \emptyset.
\]
However, the system $M^a_{\neg gsp}$ is in general not finite-index simulation. Moreover this property is undecidable. Therefore, we adopt an approach based on the use of over/lower approximations of $Sim$.

4.2.2 Using lower approximations:

Instead of computing the decreasing sequence of relations $(Sim_i : i \in \mathbb{N})$, we can compute the increasing sequence of their negations $(\neg Sim_i : i \in \mathbb{N})$. Then, the computed sequence of relations is actually an increasing sequence of relations $(N_i : i \in \mathbb{N})$ such that for every $i \geq 0$, $N_i = \neg Sim_i$. Since each $N_i$ can be represented by a transducer, we can use the extrapolation-based technique of [BLW03,BLW04a,Leg07]. The technique can compute an automaton that represents an extrapolation $N^{e_*}$ of the limit $\bigcup_{i=0}^{+\infty} N_i$ by observing finite prefixes of the sequence $N_0, N_1, N_2, \ldots$. A sufficient criterion to test whether this extrapolation is safe (does it contain the limit?) consists in applying one more time the construction that builds $Sim_{k+1}$ from $Sim_k$ to the complement of $N^{e_*}$, and then check if the complement of the result we obtain is included in $N^{e_*}$. We can use the technique of [BLW03,BLW04a,Leg07] to compute an upper approximation $N^{e_*}$ of the limit of the sequence $(N_i : i \in \mathbb{N})$. The negation of $N^{e_*}$, denoted $\neg N^{e_*}$, is a lower approximation of $S$. Let $T_{\neg N^{e_*}}$ be the transducer representing $\neg N^{e_*}$. If the following condition holds

$$L((T^a_R)^+(A^{\omega}_{\pi_0}) \cap \Pi_{\neq 2} \cap (T^a_{R} \cap A^{(\Sigma^a)^*} \times F^a) \cap T_{\neg N^{e_*}}) \neq \emptyset,$$

then we can deduce that $M^a_{\neg gsp}$ has an infinite accepting execution, which means that $M^a_{\neg gsp}$ does not satisfy the property $gsp$.

5 Linear Temporal Properties in $\omega$-Regular Model Checking

5.1 Definitions

We extend the concept of global system properties from regular to $\omega$-regular systems. For this, we simply encode state-properties as sets of infinite words rather than sets of finite words. We propose the following definitions.

Definition 23 Given an alphabet $\Sigma$, a $\omega$-state property is a set $\text{cop} \subseteq \Sigma^\omega$ that can be represented by a deterministic weak Büchi automaton.

The choice of using deterministic weak automata to represent $\omega$-state properties is for technical reasons that will be clarified in the next section.
Definition 24 Let COP be a finite set of \( \omega \)-state properties defined over an alphabet \( \Sigma \). An \( \omega \)-global system property over COP is a set \( gsp \subseteq (2^{\text{COP}})^\omega \), i.e. a set of infinite sequences of \( \omega \)-state properties, that can be represented by a Büchi automaton.

Assume a set of \( \omega \)-state properties COP, and a global system property gsp defined over COP. An execution \( \pi = w_0w_1w_2w_3w_4 \cdots \) of an \( \omega \)-regular system \( M \) satisfies gsp, denoted \( \pi \models gsp \), if and only if \( \text{cop}(w_0)\text{cop}(w_1)\cdots \in gsp \), where \( \text{cop}(w) = \{ \text{cop}_i \in COP \mid w \models \text{cop}_i \} \). We say that \( M \) satisfies gsp, denoted \( M \models gsp \), if and only if all its executions satisfy the property.

5.2 Verification

Assume an \( \omega \)-regular system \( M = (\Sigma, A_{S_0}, T_R) \), a set of \( \omega \)-state properties COP = \{\text{cop}_1, \ldots, \text{cop}_k\}, and an \( \omega \)-global system property gsp defined over COP. Suppose that the negation of gsp can be represented by a Büchi automaton \( A_{\neg gsp} = (Q_{\neg gsp}, 2^{\text{COP}}, \text{q}_0_{\neg gsp}, \text{F}_{\neg gsp}) \), and that each \( \text{cop}_i \in COP \) can be represented by a complete deterministic weak Büchi automaton \( A_{\text{cop}_i} = (Q_{\text{cop}_i}, \Sigma, \text{q}_0_{\text{cop}_i}, \delta_{\text{cop}_i}, \text{F}_{\text{cop}_i}) \).

To test whether \( M \) satisfies gsp, we proceed as in Section 4.2 and build a Büchi \( \omega \)-regular system \( M^a_{\neg gsp} = (\Sigma^a, A^a_{S_0}, T^a_R, F^a) \) whose executions correspond to those of \( M \) that do not satisfy gsp. We then check whether \( M^a_{\neg gsp} \) is empty or not. We already provided partial solutions to test whether a Büchi regular system is empty or not, and those solutions directly extend to Büchi \( \omega \)-regular systems. In the rest of this section, we mainly focus on the construction of \( M^a_{\neg gsp} \).

The main difference between the present case and the one in Section 4.2 is that since we are working with infinite-words, we cannot encode the current state of the Büchi automaton \( A_{\neg gsp} \) and the current set of \( \omega \)-state properties satisfied only in one position of each word of \( M \). Therefore, we include this information everywhere (in each position) of the word. We must also ensure that this information is the same for each position (which is needed to be coherent with the definition of product between \( M \) and \( A_{\neg gsp} \)). We use the following augmented alphabet:

\[
\Sigma^a = \Sigma \times Q_{\neg gsp} \times 2^{\text{COP}}.
\]

Let \( T_R = (Q_R, (\Sigma^a)^2, Q^a_0, \delta_R, F_R) \), the possibly nondeterministic transducer \( T^a_R = (Q^a_R, (\Sigma^a)^2, Q^a_0, \Delta^a_R, F^a_R) \) is built as follows:
• The set of accepting states $Q_R^a$ is $Q_R^a = Q_R \times \prod_{1 \leq i \leq k} Q_{cop_i} \times Q_{-gsp} \times 2^{COP}$. Instead of the Boolean variable, we have to store the state $A_{-gsp}$ and the set $COP_i \in 2^{COP}$ in each state of $T_R^a$.

• The set of initial states $Q_{0R}$ contains elements of the form $(q_{0R}, q_{0cop_1}, \ldots, q_{0cop_k}, q_{0-gsp}, \lambda)$, where $\lambda$ is any element in $2^{COP}$.

• The transition relation $\triangle_R^a$ is defined by $(q_R', q_{cop_1}', \ldots, q_{cop_k}', \alpha_1, \lambda_1) \in \triangle_R^a((q_R, q_{cop_1}, \ldots, q_{cop_k}, \alpha_1, \lambda_1), ((a_1, \alpha_1, \lambda_1), (a_2, \alpha_2, \lambda_2)))$ if and only if

  - $q_R' \in \delta_R(q_R, (a_1, a_2))$ and $q_{cop_i}' = \delta_{cop_i}(q_{cop_i}, a_1)$, for $1 \leq i \leq k$,
  - $\alpha_2 \in \delta_{-gsp}(\alpha_1, \lambda_1)$, which checks that we have a run of $A_{-gsp}$.

• The set of accepting states $F_R^a$ contains states of the form $(q_R, q_{cop_1}, \ldots, q_{cop_k}, \alpha_1, \lambda_1)$ with for $1 \leq i \leq k$, $q_{cop_i}' \in F_{cop_i}$ iff $cop_i \in \lambda_1$.

Observe that, since $T_R$ is deterministic weak and the $\omega$-state properties are represented by deterministic weak automata, the transducer $T_R^a$ is also deterministic weak.

The initial states of $M_{-gsp}^a$ are those of the following form:

$$(w(0), q_{0-gsp}, \lambda)(w(1), q_{0-gsp}, \lambda) \cdots$$

where $w(0)w(1) \cdots \in L(A_{\Sigma_0})$, and $\lambda$ is any element of $2^{COP}$.

The set of accepting states of $M_{-gsp}^a$ are those of the following form:

$$(\Sigma \times q_{-gsp} \times COP_i)(\Sigma \times q_{-gsp} \times COP_i) \cdots$$

where $COP_i \in 2^{COP}$ and $q_{-gsp} \in F_{-gsp}$. We directly see that the sets of initial and accepting states can be represented by deterministic weak automata.

**Theorem 25** The Büchi regular system $M_{-gsp}^a$ has an accepting execution of the form $\pi^a = \pi^a(0)\pi^a(1) \cdots$ if and only if the execution $\pi = w_0w_1w_2 \cdots$ of $M$, where $w_i = \Pi_{\Sigma}(\pi^a(i)) \ (\forall i)$, does not satisfy $gsp$.

**Proof.** Follows from the construction above.

As already mentioned, testing the emptiness of $M_{-gsp}^a$ can be done with the techniques developed in Section 4.2. Recall that the definition of the greatest simulation relation over $M_{-gsp}^a$ is given by the limit of the (possibly infinite) decreasing sequence of relations $Sim_0, Sim_1, \ldots$ defined as follows:

$$Sim_0 = \{(w^a_1, w^a_2) | w^a_1 \times w^a_2 \in (\Sigma^a \times \Sigma^a)^{\omega} \land \text{cop}(\Pi_{\Sigma}(w^a_1)) = \text{cop}(\Pi_{\Sigma}(w^a_2)) \}$$

$$Sim_{k+1} = Sim_k \cap \{(w^a_1, w^a_2) \in Sim_k | \forall w^a_3, ((w^a_1, w^a_2) \in T^a_R \Rightarrow \exists w^a_4, (w^a_2, w^a_4) \in T^a_R \land (w^a_3, w^a_4) \in Sim_k)\}, \forall k \in \mathbb{N}$$
A lower approximation of the limit of this sequence can be computed with the techniques introduced in [BLW04a,Leg07]. In the present case, the technique requires that each of the $Sim_k$ can be represented by a deterministic weak automaton. It is easy to see that $Sim_0$ can be represented by a deterministic weak Büchi automaton. However, the fact that $Sim_k$ is represented by a deterministic weak Büchi automaton does not necessarily imply that $Sim_{k+1}$ can be represented in the same way. Indeed, building $Sim_{k+1}$ from $Sim_k$ requires projection operations, and there is no theoretical guarantee that the resulting automaton can be turned to a weak deterministic one.

6 Linear Temporal Properties for Parametric Systems: Parametrization

Suppose that we are working with a regular system representing a parametric system. Global system properties allow to express communal temporal properties of parametric systems, i.e. properties such as “if a process is in a state $s_1$, then finally some (possibly different) process will reach a state $s_2$. However, global system properties cannot express individual temporal properties, i.e. properties such as “if the process $i$ is in a state $s_1$, then finally the process $i$ (the same process) will reach a state $s_2”$. Indeed, global system properties can only reason on the whole execution of a system, while individual temporal properties require to reason on the execution of one of the processes. In this section, we define a new class of temporal properties that allows to express individual temporal properties of parametric systems.

6.1 Definitions

In our model, an execution of a parametric system is represented by an infinite sequence of identical length finite words. Each position in these words corresponds to the state of a process, also called a local state, and the infinite sequences of identically positioned letters in an execution represents a process execution. We thus use the following notations and definitions.

**Definition 26** Consider an execution $\pi = w_0w_1w_2w_3\ldots$ of a regular system $M = (\Sigma,A_{S_0},T_R)$. The $j$th local projection $\Pi_j(\pi)$ is the infinite word $w_0(j)w_1(j)w_2(j)\ldots$.

Given an execution $\pi = w_0w_1w_2w_3\ldots$ of a parametric system, the $j$th local projection $\Pi_j(\pi)$ corresponds to the execution of the $j$th process.

**Definition 27** Given an alphabet $\Sigma$, a local execution property is a set $\ellep \subseteq$
Fig. 2. Local-oriented system properties: an illustration.

$\Sigma^\omega$ that can be represented by a Büchi automaton.

A local execution property $\ellep$ is satisfied by an execution $\pi$ of a parametric system at position $j$, denoted $\Pi_j(\pi) \models \ellep$, if and only if $\Pi_j(\pi) \in \ellep$.

We are now ready to define a logic suited for parametric systems.

**Definition 28** Given a set of local execution properties $\text{LEP} = \{\ellep_1, \ldots, \ellep_k\}$, a local-oriented system property is a set $\text{losp} \subseteq (2^{\text{LEP}})^*$, i.e. a set of finite sequences of subsets of $\text{LEP}$, that can be represented by a finite-word automaton.

Assume a local-oriented system property $\text{losp}$ defined over $\text{LEP}$. An execution $\pi$ of a parametric system $M$ satisfies $\text{losp}$, denoted $\pi \models \text{losp}$, if and only if $\text{lep}(\Pi_1(\pi))\text{lep}(\Pi_2(\pi))\cdots\text{lep}(\Pi_n(\pi)) \in \text{losp}$, where $n$ is the common length of the words in $\pi$, and $\text{lep}(\Pi_i(\pi)) = \{\ellep_i \in \text{LEP} \mid \Pi_i(\pi) \models \ellep_i\}$. We say that $M$ satisfies $\text{losp}$, denoted $M \models \text{losp}$, if and only if all its executions satisfy the property.

The definition of local-oriented system properties is illustrated in Figure 2.

**Example 29** Consider the parametric system defined in Example 9. Given a natural $i$ and a state $N$, the Boolean proposition $N[i]$ is true if and only if the $i$-th process involved in the computation (i.e. the one whose state is encoded in
the i-th letter of the word describing the global state) is in state $N$. The fact that whenever a process $i$ is in state $N[i]$, it will eventually move to state $T[i]$ ($\Box(N[i] \Rightarrow \Diamond T[i])$ using the well-known notations for LTL) is a local execution property. That this property holds for each process ($\forall i(\Box(N[i] \Rightarrow \Diamond T[i]))$) is then a local-oriented system property. It is easy to see that this property is trivially satisfied by the system. Indeed, the transition relation does not allow for a process to keep the token indefinitely.

6.2 Verification

Consider a regular system $M = (\Sigma, A_{S_0}, T_R)$ that represents a parametric system, a set of local execution properties $LEP = \{\ell_{ep_1}, \ldots \ell_{ep_k}\}$, and a local-oriented system property $\ell_{osp}$ defined over $LEP$. Suppose that for $1 \leq i \leq k$, $\ell_{ep_i}$ is represented by a Büchi automaton $A_{\ell_{ep_i}} = (Q_{\ell_{ep_i}}, \Sigma, q_{0_{\ell_{ep_i}}}, \Delta_{\ell_{ep_i}}, F_{\ell_{ep_i}})$, which is assumed to be complete. We extend the automata theoretic approach of [VW86] towards a semi-algorithm to test whether $M$ satisfies $\ell_{osp}$. Our approach consists in three successive steps that are the following:

1. Computing a deterministic finite-word automaton $A_{\neg \ell_{osp}} = (Q_{\neg \ell_{osp}}, 2^{LEP}, q_{0_{\neg \ell_{osp}}}, \delta_{\neg \ell_{osp}}, F_{\neg \ell_{osp}})$, which is the finite-word automaton accepting the finite sequences that do not satisfy $\ell_{osp}$, i.e. sequences in $\neg \ell_{osp} = (2^{LEP})^* \setminus \ell_{osp}$.
2. Building a Büchi regular system $M^a_{\neg \ell_{osp}} = (\Sigma^a, A^a_{S_0}, T^a_R, F^a)$ whose accepting executions correspond to those of $M$ that are accepted by $A_{\neg \ell_{osp}}$.
3. Testing whether $M^a_{\neg \ell_{osp}}$ is empty or not.

The property $\ell_{osp}$ being (by definition) representable by a finite-word automaton, one can always compute the automaton $A_{\neg \ell_{osp}}$. Computing $M^a_{\neg \ell_{osp}}$ is a much harder endeavor for which we propose the following solution.

For each automaton $A_{\ell_{ep_i}}$, we assume the existence of a complete automaton $A_{\neg \ell_{ep_i}} = (Q_{\neg \ell_{ep_i}}, \Sigma, q_{0_{\neg \ell_{ep_i}}}, \Delta_{\neg \ell_{ep_i}}, F_{\neg \ell_{ep_i}})$ whose accepted language is the complement of the one of $A_{\ell_{ep_i}}$. Consider an execution $\pi^a$ of $M^a_{\neg \ell_{osp}}$. Since, a priori, we do not know which local execution property will be satisfied by which process, each of the automata $A_{\ell_{ep_i}}$ and $A_{\neg \ell_{ep_i}}$ has to be run in parallel with the local executions of the processes involved in $\pi$. So, we need to extend the alphabet of $M$ in such a way that each local state is now also labeling by a state of each of the $A_{\ell_{ep_i}}$ and $A_{\neg \ell_{ep_i}}$. For each $1 \leq i \leq k$, running $A_{\neg \ell_{ep_i}}$ is necessary since the automaton $A_{\ell_{ep_i}}$ being nondeterministic, the fact that it has a nonaccepting run does not indicate that the corresponding property does not hold.

\footnote{This can be achieved since the automata are complete.}
Furthermore, in each position, each property \( \ell e p_i \in LEP \) might be satisfied (\( A_{\ell e p_i} \) has an accepting run), or might not be satisfied (\( A_{\neg \ell e p_i} \) has an accepting run). We make a note of these facts by also labeling each position by an element of \( 2^{LEP} \) corresponding exactly to the properties \( \ell e p_i \) that are satisfied. This labeling will remain unchanged from position to position and will enable us to run the automaton \( A_{\neg \ell e p} \). The next step is to check whether there is an execution of \( M_{a \neg \ell e p} \) that is accepting for suitable automata \( A_{\ell e p_i} \) and \( A_{\neg \ell e p_i} \). Precisely, at a given position \( j \) in the state, the run of the automaton \( A_{\ell e p_i} \) has to be accepting if \( \ell e p_i \in \ell e p_j \) and the run of \( A_{\neg \ell e p_i} \) has to be accepting if \( \ell e p_i \not\in \ell e p_j \), where \( \ell e p_j \) is the element of \( 2^{LEP} \) labeling that position. We face thus with the problem of checking not one, but several Büchi conditions, i.e. a generalized Büchi condition. To do this, we use the fact that a generalized Büchi automaton has an accepting run exactly when it has an accepting run that goes sequentially through each of the accepting sets. We now define \( M_{a \neg \ell e p} \). The augmented alphabet is

\[
\Sigma^a = \Sigma \times \prod_{1 \leq i \leq k} Q_{\ell e p_i} \times \prod_{1 \leq i \leq k} Q_{\neg \ell e p_i} \times 2^{LEP} \times 2^{LEP} \times \{ \text{reset, noreset} \}.
\]

We thus have two subsets of \( LEP \), the second being used to remember if suitable automata checking for properties \( \ell e p_i \) (or \( \neg \ell e p_i \)) have seen an accepting state; the last component of the labeling indicates whether the second of these subsets has just been reset of not. We denote by \( \Pi_{\Sigma}(w^a) \), the word \( w \in \Sigma^* \) obtained from \( w^a \) by removing all the symbols that do not belong to \( \Sigma \).

An execution \( \pi^a = w_0^a w_1^a w_2^a \ldots \) of \( M_{a \neg \ell e p} \) is an infinite sequence of finite words over \( \Sigma^a \) that has to satisfy three requirements:

1. For each \( i \geq 1 \) (\( \Pi_{\Sigma}(w_{i-1}^a), \Pi_{\Sigma}(w_i^a) \) \( \in T_R \), which ensures that the transitions of \( M_{a \neg \ell e p} \) are compatible with the transition relation of \( M \);
2. For each position in a state, the labeling by states of the \( A_{\ell e p_i} \) form a run of these automata;
3. The labeling of each position by elements of \( 2^{LEP} \) stays the same when moving from one state to the next one.

We have to build \( A_{S_0}^a, F^a \), and \( T_R^a \) in such a way that the three requirements above are satisfied.

Let \( T_R = (Q_R, \Sigma^2, q_0R, \delta_R, F_R) \). The possibly nondeterministic transducer \( T_R^a = (Q_R^a, (\Sigma^a)^2, q_0^aR, \Delta_R^a, F_R^a) \) is built as follows:

- Its set of states and accepting states are \( Q_R^a = Q_R \) and \( F_R^a = F_R \), respectively; its initial state is \( q_0^aR = q_0R \);
• The transition relation is defined by (assuming nondeterministic automata)

\[(q_R^a) \in \Delta(q_R^a, (a_1, q_0 \_lep_1, \ldots, q_1 \_lep_k, q_0 \_\neg \_lep_1, \ldots, q_1 \_\neg \_lep_k, \lep_1, \lep_F, \rho_1), (a_2, q_2 \_lep_1, \ldots, q_2 \_\neg \_lep_k, q_2 \_\neg \_lep_1, \ldots, q_2 \_\neg \_lep_k, \lep_2, \lep_F, \rho_2))\]

if and only if

- for \(1 \leq i \leq k\), \((q_R^a) \in \Delta_R(q_R^a, (a_1, a_2))\) and \(q_2 \_lep_i \in \delta \_lep_i(q_1 \_lep_i, a_1), q_2 \_\neg \_lep_i \in \delta \_\neg \_lep_i(q_1 \_\neg \_lep_i, a_1), \)
- \(\lep_1 = \lep_2, \)
- if \(\lep_F = LEP\), then \(\lep_F = \emptyset \) and \(\rho_2 = reset\), or \(\lep_F = \lep_F\) and \(\rho_2 = noreset\), otherwise, \(\lep_F = \lep_F \cup \{\lep_i \in \lep_1 | q_\_lep_i,1 \in F_\_lep\} \cup \{\lep_i \notin \lep_1 | q_\_\neg \_lep_i,1 \in F_\_\neg \_lep\}\) and \(\rho_2 = noreset.\)

Note that at a given position, when all required accepting conditions have been satisfied, the choice to reset or not is nondeterministic, which makes it possible to wait until the required acceptance conditions have been satisfied at each position and then to reset everywhere simultaneously;

• The set of accepting states \(F_R^a\) is \(F_R\).

The initial states of \(M^a_{\_gsp}\) are those of the following form:

\[
(w(0), q_0 \_lep_1, \ldots, q_0 \_lep_k, q_0 \_\neg \_lep_1, \ldots, q_0 \_\neg \_lep_k, \lep_1, \emptyset, noreset)\]

\[
(w(1), q_0 \_lep_1, \ldots, q_0 \_lep_k, q_0 \_\neg \_lep_1, \ldots, q_0 \_\neg \_lep_k, \lep_2, \emptyset, noreset)\]

\[
\ldots\]

\[
(w(n-1), q_0 \_lep_1, \ldots, q_0 \_lep_k, q_0 \_\neg \_lep_1, \ldots, q_0 \_\neg \_lep_k, \lep_n, \emptyset, noreset),\]

where \(w(0) \ldots w(n-1) \in L(A_{S_0})\) and \(\lep_1 \lep_2 \ldots \lep_n \in \neg \_osp.\)

The accepting states in the language of the automaton \(F^a\) are those in which for every position the last part \(\rho\) of the label is \(reset\), which implies that all relevant automata have seen an accepting state since the last “reset”.

**Theorem 30** The Büchi regular system \(M_{\_osp}^a\) has an accepting execution of the form \(\pi^a = \pi^a(0)\pi^a(1)\ldots \) if and only if the execution \(\pi = w_0w_1w_2\ldots\) of \(M\), where \(w_i = \Pi_{\Sigma}(\pi^a(i)) \forall i\), does not satisfy \(\neg \_osp.\)

**PROOF.** Follows from the construction above.

The system \(M\) being locally-finite, \(M_{\_osp}^a\) is also locally-finite. We thus have the following result that shows that checking the emptiness of \(M_{\_osp}^a\) can be reduced to solving the regular reachability problems.
Proposition 31  The Büchi regular system $M_{\text{tosp}}^a = (\Sigma^a, A_{S_0}^a, T_R^a, F^a)$ is empty if and only if

$$L((T_R^a)^*(A_{S_0}^a) \cap F^a \cap \Pi_{\neq 2}((T_R^a)^+ \cap T_{id})) = \emptyset.$$ 

**PROOF.** Same as Proposition 19.

7 Boolean Combinations and Multiple Alternations for Parametric Systems

7.1 Boolean Combinations

It is easy to see that one can verify Boolean combinations of global and local-oriented system properties (each property being a literal). Indeed, any Boolean combination can be turned into another combination that only uses the connectors for the disjunction ($\lor$) and the negation ($\neg$). Properties being defined by finite-word and Büchi automata, one can always compute their negation. Verifying the disjunction of several properties is direct by definition.

7.2 Multiple Alternations for Parametric Systems

In some situations, it is also interesting to consider properties with multiple alternations between local-oriented and global system properties. By multiple alternations, we mean local-oriented properties that reference global system properties and vice-versa. We will not formally characterize the way alternations can occur, but rather illustrate the concept with several examples. Multiple-alternation properties will be specified by combining the notations introduced in Sections 4.1 and 6.1. The semantics of multiple-alternation properties easily follows from those notations.

We now propose several examples that illustrate how multiple-alternation properties can be reduced to properties with a simple alternation on an augmented system, a problem for which this paper provided verification procedures. We consider a parametric system, and assume that each of its processes can be in one of the two following states $\{C, T\}$. The following property is a local-oriented system property:

$$\forall i \Box (C[i] \Rightarrow \Diamond T[i]).$$  (5)
Indeed, we could think that this property is a local oriented system property. However, due to the presence of the ∃ quantifier, □(C[i] ⇒ ◻(∃j ≠ i)T[j]) can reference several processes and is thus not a local execution property.

The solution we propose is to reduce the property above to a local execution property over an augmented system. This is done by introducing new Boolean variables in the specification of each process. Those variables can be arbitrarily true or false in any moment of an execution. Let us go back to our example and assume that we add to each process a Boolean variable "a" that behaves as described above. We use a[i] to denote that the variable a is true for the process i in the current state, and ¬a[i] to denote that it is false. In this case Property 13 can be rewritten as

\[ \forall i □(C[i] ⇒ ◻a[i]) \land \]
\[ □∀i (a[i] ⇔ (∃j ≠ i)T[j]) \land \]
\[ □∀i (a[i] ∨ ¬a[i]). \]

Clearly, \( \phi_1 \equiv ∀i □(C[i] ⇒ ◻a[i]) \) is a local-oriented system property, and \( \phi_2 \equiv □∀i (a[i] ⇔ (∃j ≠ i)T[j]) \) and \( \phi_3 \equiv □∀i (a[i] ∨ ¬a[i]) \) are global system properties.

We now give two other illustrating examples.

**Example 32** Consider the following property:

\[ □(∀i ◻T[i] \land (∃jC[j])). \]

This property cannot be expressed neither by a local-oriented system property nor by a global system property. The solution is again to reduce the extended state property to a state property over an augmented system. We introduce a Boolean variable "a" that can be either true or false in each state. Using variable "a", Property 9 can be rewritten as a conjunction of local-oriented and global system properties.

---

6 When we add a Boolean variable, we extend the alphabet on which processes’s states are encoded. As an example, if the set of states was given by \( \Sigma = \{C, T\} \) before the variable a is added, it becomes \( \Sigma_{\{a\}} = \Sigma \times \{-a, a\} = \{(C, -a), (C, a), (T, -a), (T, a)\} \) after the addition occurs. As a consequence, any automaton defined over \( \Sigma \) must take this extension into account, which is done by duplicating each of its transitions. As an example, a transition labeled by T is duplicated into two transitions, one labeled by (T, a) and the other one by (T, ¬a). To not lengthen the presentation, we will assume this translation to be implicit, and we write a[i] for (T, a)[i] ∨ (C, a)[i] and T for (T, a)[i] ∨ (T, ¬a)[i].
Of course, we can have several alternations in the same formula. In such situations, construction has to be applied for each alternation. Consider the following example.

**Example 33** Consider the following property $\varphi_1$:

$$\forall \forall i \Box (C[i] \Rightarrow \Box(\exists j \neq i) \text{Buchi}_\varphi[j]),$$

where $\text{Buchi}_\varphi[j]$ is a Büchi modality which is true if and only if the $j$-th process satisfies the local execution property $\varphi$ described by a Büchi automaton $\text{Buchi}_\varphi$.

Property $\varphi_1$ cannot be expressed neither by a local-oriented system property nor by a global system property. The solution is to introduce two Boolean variables “a” and b. Using those variables, $\varphi_1$ can be rewritten as the property $\varphi_2$ defined as follows:

$$\forall \forall i \Box (C[i] \Rightarrow \Box a[i]) \land (14)$$

$$\Box \forall i (a[i] \Rightarrow (\exists j \neq i) b[j]) \land (15)$$

$$\forall \forall i (b[i] \Rightarrow \text{Buchi}_\varphi[i]) \land (16)$$

$$\Box \forall i (a[i] \land \neg a[i]) \land (17)$$

$$\Box \forall i (b[i] \land \neg b[i]). (18)$$

By observing that $\forall \forall i \Box (b[i] \Rightarrow \text{Buchi}_\varphi[i])$ is a local-oriented property (The set of executions that satisfy $b$ can easily be described with a Büchi automaton), we conclude that $\varphi_2$ is a Boolean combination of local-oriented and global system properties.

There are also alternations that we have not been able to handle. As an example, we cannot treat a property that has two free-variables or a second order variable under the scope of a temporal LTL operator. Such an observation was made for a similar logic in [AJN+04,AJNS04].
8 Related Work on Verifying Temporal Properties in (ω-)Regular Model Checking

The problem of verifying linear temporal properties in the framework of regular model checking has been first addressed in [BJNT00,PS00,Sha01]. However, the treatment of this problem in these papers was preliminary and somewhat adhoc for very particular kinds of properties of parametric systems.

In [AJN+04,AJNS04], Abdulla et al. independently proposed an approach based on a specification logic called LTL(MSO), which combines the monadic second order logic MSO and the linear temporal logic LTL. Properties written in the LTL(MSO) logic are local-oriented system properties, where the local system properties are LTL properties that can make assumptions on the executions of the other processes up to some restrictions. The LTL(MSO) logic has been designed for parametric systems and is not suited (and sometimes not powerful enough) to express very simple properties of many other interesting classes of systems such as systems with integer variables (when considering a non-unary encoding). The verification procedure in [AJN+04,AJNS04] is only dedicated to regular systems that are locally-finite and the ω-regular framework is not considered. Finally, unlike our local-oriented properties, the LTL(MSO) logic cannot be used to express properties which are Boolean combinations of properties written in logics that are more expressive/concise than LTL (e.g. PTL [GO03,LPZ85], ETL [Wol82], or µTL [Var88]).

In [VSVA05], Agha et al. proposed to use learning-based algorithms [Ang87] to verify global system properties of regular systems. The technique they proposed relies on the computation of several fixed point operators which are used to test whether a Büchi regular system is empty or not. The use of learning algorithms to make fixed point computation terminating requires to enrich the systems with two extra variables. This is a clear restriction since it is known that there are many systems for which the set of reachable states is regular before the variables have been introduced, but not after. The work in [VSVA05] also lacks of a clear description of the encoding of linear temporal properties in the regular framework, which is one of the main contribution of our work. Finally, we mention that [VSVA05] does not consider the ω-regular framework.

\footnote{The approach in [AJN+04,AJNS04] has been proposed in the same period of time as our early work [BLW04b], whose present paper is an extension of.}
9 Conclusion and Future Work

We have presented a general framework for specifying and verifying a large class of linear temporal properties for systems represented in the (ω)-regular model checking framework. The verification techniques we provide are based on reductions to the (ω-)reachability problems.

Our objective was not performances evaluation. A next step will thus be to implement our constructions in several regular model checking tools (e.g. T(O)RMC [Leg08], LEVER [VV06], or RMC [RMC]) and compare the performances. Another direction for future work is to extend our results to the verification of computational tree logics properties. It would also be of interest to propose criteria to check whether the extrapolation of the simulation with the technique of [BLW03,BLW04a,Leg07] is precise. Developing a methodology to decide whether FIFO-Queue and pushdown systems are locally-finite is another topic of interests. We would also like to give a formal characterization of what are the allowed alternations between local-oriented and global system properties.

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References

[ABJN99] P. A. Abdulla, A. Bouajjani, B. Jonsson, and M. Nilsson. Handling global conditions in parameterized system verification. In Proc. 11th Int. Conference on Computer Aided Verification (CAV), volume 1633 of Lecture Notes in Computer Science, pages 134–145. Springer, 1999.

[ACH+95] R. Alur, C. Courcoubetis, N. Halbwachs, T. A. Henzinger, P.-H. Ho, X. Nicollin, A. Olivero, J. Sifakis, and S. Yovine. The algorithmic analysis of hybrid systems. Theoretical Computer Science, 138(1):3–34, 1995.

[AJ96] P. A. Abdulla and B. Jonsson. Verifying programs with unreliable channels. Information and Computation, 127(2):91–101, June 1996.

[AJMd02] P. A. Abdulla, B. Jonsson, P. Mahata, and J. d’Orso. Regular tree model checking. In Proc. 14th Int. Conference on Computer Aided Verification
[AJN+04] P. A. Abdulla, B. Jonsson, M. Nilsson, J. d’Orso, and M. Saksena. Regular model checking for ltl(mso). In Proc. 16th Int. Conference on Computer Aided Verification (CAV), volume 3114 of Lecture Notes in Computer Science, pages 348–360. Springer, 2004.

[AJNS04] P. A. Abdulla, B. Jonsson, M. Nilsson, and M. Saksena. A survey of regular model checking. In Proc. 15th Int. Conference on Concurrency Theory (CONCUR), volume 3170 of Lecture Notes in Computer Science, pages 35–48. Springer, 2004.

[Ang87] D. Angluin. Learning regular sets from queries and counterexamples. Information and Computation, 75(2):87–106, 1987.

[BEM97] A. Bouajjani, J. Esparza, and O. Maler. Reachability analysis of pushdown automata: Application to model-checking. In Proc. 8th Int. Conference on Concurrency Theory (CONCUR), volume 1243 of Lecture Notes in Computer Science, pages 135–150. Springer, July 1997.

[BG96] B. Boigelot and P. Godefroid. Symbolic verification of communication protocols with infinite state spaces using qdds (extended abstract). In Proc. 8th Int. Conference on Computer Aided Verification (CAV), volume 1102 of Lecture Notes in Computer Science, pages 1–12. Springer, 1996.

[BH97] A. Bouajjani and P. Habermehl. Symbolic reachability analysis of fifo channel systems with nonregular sets of configurations (extended abstract). In Proc. 24th Int. Colloquium on Automata, Languages and Programming (ICALP), volume 1256 of Lecture Notes in Computer Science, pages 560–570. Springer, 1997.

[BHJ03] B. Boigelot, F. Herbreteau, and S. Jodogne. Hybrid acceleration using real vector automata (extended abstract). In Proc. 15th Int. Conference on Computer Aided Verification (CAV), volume 2725 of Lecture Notes in Computer Science, pages 193–205. Springer, 2003.

[BHMV05] A. Bouajjani, P. Habermehl, P. Moro, and T. Vojnar. Verifying programs with dynamic 1-selector-linked structures in regular model checking. In Proc. 11th Int. Conference on Tools and Algorithms for the Construction and Analysis of Systems (TACAS), volume 3440 of Lecture Notes in Computer Science, pages 13–29. Springer, 2005.

[BHHRV06] A. Bouajjani, P. Habermehl, A. Rogalewicz, and T. Vojnar. Abstract regular tree model checking of complex dynamic data structures. In Proc. 13th Int. Symposium on Static Analysis (SAS), volume 4134 of Lecture Notes in Computer Science, pages 52–70. Springer, 2006.

[BJNT00] A. Bouajjani, B. Jonsson, M. Nilsson, and T. Touili. Regular model checking. In Proc. 12th Int. Conference on Computer Aided Verification
(CAV), volume 1855 of Lecture Notes in Computer Science, pages 403–418. Springer-Verlag, 2000.

[BJW01] B. Boigelot, S. Jodogne, and P. Wolper. On the use of weak automata for deciding linear arithmetic with integer and real variables. In Proc. Int. Joint Conference on Automated Reasoning (IJCAR), volume 2083 of Lecture Notes in Computer Science, pages 611–625, Siena, Italy, June 2001. Springer-Verlag.

[BJW05] B. Boigelot, S. Jodogne, and P. Wolper. An effective decision procedure for linear arithmetic over the integers and reals. ACM Transactions on Computational Logic, 6(3):614–633, 2005.

[BLW03] B. Boigelot, A. Legay, and P. Wolper. Iterating transducers in the large (extended abstract). In Proc. 15th Int. Conference on Computer Aided Verification (CAV), Lecture Notes in Computer Science, pages 223–235. Springer, 2003.

[BLW04a] B. Boigelot, A. Legay, and P. Wolper. Omega-regular model checking. In Proc. 10th Int. Conference on Tools and Algorithms for the Construction and Analysis of Systems (TACAS), volume 2988 of Lecture Notes in Computer Science, pages 561–575. Springer, 2004.

[BLW04b] A. Bouajjani, A. Legay, and P. Wolper. Handling liveness properties in (omega-)regular model checking. In Proc. 6th Int. Workshop on Verification of Infinite State Systems (INFINITY), volume 138(3) of Electronic Notes in Theoretical Computer Science. Elsevier Science Publishers, 2004.

[Boi99] B. Boigelot. Symbolic Methods for Exploring Infinite State Spaces. Collection des publications de la Faculté des Sciences Appliquées de l’Université de Liège, Liège, Belgium, 1999.

[BRW98] B. Boigelot, S. Rassart, and P. Wolper. On the expressiveness of real and integer arithmetic automata (extended abstract). In Proc. 25th Int. Colloquium on Automata, Languages and Programming (ICALP), volume 1443 of Lecture Notes in Computer Science, pages 152–163. Springer, 1998.

[BW02] B. Boigelot and P. Wolper. Representing arithmetic constraints with finite automata: An overview. In Proc. 18th Int. Conference on logic Programming (ICLP), volume 2401 of Lecture Notes in Computer Science, pages 1–19. Springer, 2002.

[DLS02] D. Dams, Y. Lakhnech, and M. Steffen. Iterating transducers. Journal of Logic and Algebraic Programming (JLAP), 52-53:109–127, 2002.

[FWW97] A. Finkel, B. Willems, and P. Wolper. A direct symbolic approach to model checking pushdown systems. In Proc. 2nd Int. Workshop on Verification of Infinite State Systems (INFINITY), volume 9 of Electronic Notes in Theoretical Computer Science. Elsevier Science Publishers, 1997.
[GO03] P. Gastin and D. Oddoux. Ltl with past and two-way very-weak alternating automata. In Proc. Mathematical Foundations of Computer Science 2003, 28th International Symposium (MFCS), volume 2747 of Lecture Notes in Computer Science, pages 439–448. Springer, 2003.

[Hop71] J. E. Hopcroft. An $n \log n$ algorithm for minimizing states in a finite automaton. Theory of Machines and Computation, pages 189–196, 1971.

[KMM+97] Y. Kesten, O. Maler, M. Marcus, A. Pnueli, and E. Shahar. Symbolic model checking with rich assertional languages. In Proc. 9th Int. Conference on Computer Aided Verification (CAV), volume 1254 of Lecture Notes in Computer Science, pages 424–435. Springer, 1997.

[Leg07] A. Legay. Generic Techniques for the Verification of Infinite-state Systems. Collection des publications de la Faculté des Sciences Appliquées de l’Université de Liège, Liège, Belgium, 2007. to appear.

[Leg08] A. Legay. T(o)rmc: A tool for (omega-)regular model checking. In Proc. 20th Int. Conference on Computer Aided Verification (CAV), volume XXX of Lecture Notes in Computer Science, page XXXX. Springer, 2008. to appear.

[Löd01] C. Löding. Efficient minimization of deterministic weak $\omega-$automata. Information Processing Letters, 79(3):105–109, 2001.

[LPZ85] O. Lichtenstein, A. Pnueli, and L. D. Zuck. The glory of the past. In Proc. Int. Conference on Logics of Programs, volume 193 of Lecture Notes in Computer Science, pages 196–218. Springer, 1985.

[MSS86] D. E. Muller, A. Saoudi, and P. E. Schupp. Alternating automata, the weak monadic theory of the tree and its complexity. In Proc. 13th Int. Colloquium on Automata, Languages and Programming, pages 275–283, Rennes, 1986. Springer-Verlag.

[Nil01] M. Nilsson. Regular model checking. Master’s thesis, Uppsala University, 2001.

[Pnu77] A. Pnueli. The temporal logic of programs. In Proc. 18th Annual Symposium on Foundations of Computer Science (FOCS), pages 46–57, 1977.

[PS00] A. Pnueli and E. Shahar. Liveness and acceleration in parameterized verification. In Proc. 12th Int. Conference on Computer Aided Verification (CAV), volume 1855 of Lecture Notes in Computer Science, pages 328–343. Springer, 2000.

[RMC] The regular model checking tool (RMC). Available at http://www.it.uu.se/research/docs/fm/apv/rmc.

[Saf92] S. Safra. Exponential determinization for $\omega$-automata with strong-fairness acceptance condition. In Proceedings of the 24th ACM Symposium on Theory of Computing, Victoria, May 1992.
[Sha01] E. Shahar.  *Tools and Techniques for Verifying Parametrized Systems*. PhD thesis, Weizmann Institute of Science, 2001.

[Td06] T. Touili and J. d’Orso. Regular hedge model checking. In *Proc. 4th Int. IFIP Conference on Theoretical Computer Science (TCS06)*, 2006.

[Var88] M. Y. Vardi. A temporal fixpoint calculus. In *Proc. 15th Int. Symposium on Principles of Programming Languages (POPL)*, pages 250–259. ACM, 1988.

[Var07] M. Y. Vardi. From church and prior to psl, 2007. Available at [http://www.cs.rice.edu/~vardi/papers/index.html](http://www.cs.rice.edu/~vardi/papers/index.html).

[VSVA05] A. Vardhan, K. Sen, M. Viswanathan, and G. Agha. Using language inference to verify omega-regular properties. In *Proc. 11th Int. Conference on Tools and Algorithms for the Construction and Analysis of Systems (TACAS)*, volume 3440 of *Lecture Notes in Computer Science*, pages 45–60. Springer, 2005.

[VV06] A. Vardhan and M. Viswanathan. Lever: A tool for learning based verification. In *Proc. 18th Int. Conference on Computer Aided Verification (CAV)*, volume 4144 of *Lecture Notes in Computer Science*, pages 471–474. Springer, 2006.

[VW86] M. Y. Vardi and P. Wolper. An automata-theoretic approach to automatic program verification (preliminary report). In *Proc. 2nd IEEE Symposium on Logic in Computer Science (LICS)*, pages 332–344. IEEE Computer Society, 1986.

[WB95] P. Wolper and B. Boigelot. An automata-theoretic approach to presburger arithmetic constraints (extended abstract). In *Proc. 2nd Int. Symposium on Static Analysis (SAS)*, volume 983 of *Lecture Notes in Computer Science*, pages 21–32. Springer, 1995.

[WB98] P. Wolper and B. Boigelot. Verifying systems with infinite but regular state spaces. In *Proc. 10th Int. Conference on Computer Aided Verification (CAV)*, volume 1427 of *Lecture Notes in Computer Science*, pages 88–97. Springer-Verlag, 1998.

[WB00] P. Wolper and B. Boigelot. On the construction of automata from linear arithmetic constraints. In *Proc. 6th Int. Conference on Tools and Algorithms for the Construction and Analysis of Systems (TACAS)*, volume 1785 of *Lecture Notes in Computer Science*, pages 1–19. Springer, 2000.

[Wol82] P. Wolper. *Synthesis of Communicating Processes from Temporal Logic Specifications*. PhD thesis, Stanford University, 1982.
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