A NONPERTURBATIVE CALCULATION OF BASIC CHIRAL QCD PARAMETERS WITHIN DYNAMICAL EQUATIONS APPROACH TO QCD AT LOW ENERGIES

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Abstract

Basic chiral QCD parameters (the pion decay constant, the quark and gluon condensates, the dynamically generated quark mass, etc) as well as the vacuum energy density have been calculated from first principles within a recently proposed dynamical equations approach to QCD at low energies. The zero modes enhancement (ZME) model of quark confinement and dynamical chiral symmetry breaking (DCSB) based on the solution to the Schwinger-Dyson (SD) equation for the quark propagator in the infrared (IR) domain was used for this purpose. There are only two independent quantities by means of which calculations should be done within our approach. Our unique input data was chosen to be the pion decay constant in the chiral limit given by the chiral perturbation theory at the hadronic level (CHPT). Phenomenological estimates of these quantities, as well as the vacuum energy density, are in good agreement with our numerical results. The nonperturbative vacuum structure which emerges from the ZME model, appears to be well suited to describe quark confinement, DCSB, current-effective (dynamical)-constituent, as well as constituent-valence quark transformations, the Okubo-Zweig-Iizuka (OZI) rule, dimensional transmutation, etc. The importance of the instanton-type fluctuations in the true QCD vacuum for the ZME model is also empha-
sized. This allows to predict new, more realistic values for the vacuum energy density (apart from the sign, by definition, the bag constant) and the gluon condensate.

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I. INTRODUCTION

Why is it so important to calculate properties of observed physical states such as hadrons from first principles? An answer is almost obvious: because that reproduces the dynamical structure and the symmetry properties of the underlying fundamental theory - QCD [1] with minimal assumptions and in a model independent way. At the same time this is a very difficult task. First of all, it is a problem of large distances (infrared (IR) or low energy region). In contrast to short distances, little is known about QCD at these distances. The IR region is responsible for the nonperturbative dynamics, which plays a vital role in the formation of the above mentioned physical states. In order to correctly describe nonperturbative dynamics, it is necessary to bring two important nonperturbative ideas in QCD together: quark confinement [2] and dynamical (or equivalently spontaneous) chiral symmetry breaking (DCSB) [3-8]. There is a close connection between these nonperturbative effects (see, for example, Refs. 9-16). At high energies QCD is under the control of the phenomenon of asymptotic freedom [17] which justifies the use of the perturbation theory in this limit. At low energies, however, QCD is governed by $SU_L(N_f) \times SU_R(N_f)$ chiral symmetry ($N_f$ is the number of different flavors) and its dynamical breakdown in the vacuum to the corresponding vectorial subgroup [18]. Thus to understand chiral limit physics means to correctly understand the dynamical structure of low energy QCD. A realistic calculation of various physical quantities in this limit becomes important. The extrapolation to the chiral limit is also important in lattice calculations [19]. DCSB and the chiral limit itself are fine tuning effects in QCD and they need nonperturbative methods for their analytical and numerical investigations.

Recently we have proposed a nonperturbative and microscopic approach to QCD at low energies [16, 20], the dynamical equations approach to QCD. It consists of two parts: the chiral perturbation theory at the quark level (CHPTq) and the dynamical (or nonperturbative) quark propagator approach to QCD at large distances (low energies). After the removal of the possible singularities in a self-consistent way [20] (see also the next section), the CHPTq
represents the elements of the $S$-matrix, reproducing the corresponding physical quantities, in terms of formal Taylor expansions, in powers of small external momentum (the momentum of a physical state, for example, pion). In principle a general scheme of this method is rather simple. It is worth drawing the quark diagram (or diagrams) of the corresponding elements of the $S$-matrix (see Fig. 2). Then the analytic expressions of these diagrams (written down in accordance with the Feynman rules for the full quark propagators and proper vertices) or the corresponding elements of the $S$-matrix themselves should be expanded in powers of the corresponding external momentum (if any). In other words, the CHPTq yields correct analytic expressions of the amplitudes for physical quantities in a systematic way: term by term in power series expansions of the external (physical) momentum. On the other hand, this makes also possible to factorize dependence on the physical (hadron) and unphysical (quark) momenta which, obviously, greatly simplifies further analysis. Certainly the convergence of these series is assumed. At this stage we have no ideas about convergence yet since that depends strongly on the solutions to the quark Schwinger-Dyson (SD) equations.

The coefficients of these expansions (which are functions of an unphysical momentum only because of the above mentioned factorization) supply the solutions to the corresponding SD equations for the quark propagator. So the CHPTq admits incorporation of the quark confinement phenomenon, in a self-consistent way, through realistic, nonperturbative and confinement-type solutions to the above mentioned quark SD equations. These solutions, which correctly describe especially the IR region in QCD, are required for a reliable calculation of various important low-energy physical QCD parameters. Some of these solutions have already been published [12,13,16]. They were obtained on the basis of the zero modes enhancement (ZME) effect, a consequence of the (possible) nonperturbative IR divergences in QCD (see Section 3). Owing to the CHPTq and dynamical quark propagator approaches, which complement each other, it is possible to analytically investigate, and calculate numerically, low energy QCD physics from first principles in a self-consistent way.

To perform realistic calculations of basic chiral QCD parameters is our main purpose. We confirm numerically our expectation that the dominant contributions to the values of all
chiral QCD parameters come from large distances, while the contributions from short and intermediate distances can only be treated as small perturbative corrections. In this context let us note that nonperturbative analytic calculations (advocated here) of the properties of the physical states do not contradict lattice calculations which also provide the opportunity to compute them from first principles. Moreover, analytic and lattice [21], and especially lattice SD equations [22], calculations should complement each other in order to get correct physical information for the description of the properties of the physical states.

The paper is organized as follows. In Section 2 a CHPTq solution of the flavour non-singlet, chiral axial-vector Ward-Takahashi (WT) identity in the framework of DCSB is briefly discussed. Section 3 is devoted to a summary of our dynamical quark propagator approach to QCD at large distances in the ZME model of quark confinement and DCSB. In Section 4, we derive explicit expressions for the quark and gluon contributions to the vacuum energy density. In Section 5, within our approach, we derive expressions for the pion decay constant in the current algebra (CA) representation, quark condensate, dynamical quark mass, etc, suitable for numerical calculations. We present our numerical results in Section 6. Section 7 is devoted to our main conclusions. A detailed discussion of a possible dynamical mechanism of quark confinement, DCSB and problems related to the ZME effect in the QCD vacuum is given in Section 8 as well as a possible dynamical explanation of the famous Okubo-Zweig-Iizuka (OZI) selection rule. The need of the instanton-type fluctuations for our model of the QCD vacuum is also qualitatively discussed in this section. Some related models and important problems are discussed in Sections 9 and 10, respectively.

II. CHPT AT THE QUARK LEVEL SOLUTION OF THE CHIRAL, AXIAL-VECTOR WT IDENTITY

For the sake of the reader’s convenience and for some other purpose as well, let us briefly describe the CHPTq approach here, applying it to the flavour non-singlet, chiral axial-vector WT identity in the framework of DCSB [20]. As it is well known, the Goldstone states
associated with the dynamically broken chiral symmetry should be considered as quark-antiquark bound-states [5-8, 20]. The wave functions of these bound-states can be restored from the axial-vector, chiral WT identity as the coefficient function at the pole $q^2 = 0$ of the corresponding axial-vector vertex, according to DCSB. Moreover, the correct treatment of this identity within the CHPTq provides complete information on its regular piece at zero momentum transfer ($q = 0$) (see below). The main problem involved in the analytical calculations of the bound-states amplitudes and axial vertices through the corresponding axial-vector WT identities is their dependence on arbitrary form factors. These form factors are usually neglected by formally assuming that they are of order $g^2$ in the coupling constant [5, 9, 23, 24]. It has been shown in [20] that these perturbative arguments are not valid for treating such a nonperturbative effect like DCSB is.

After these general remarks, let us consider now the flavour non-singlet, axial-vector WT identity in the chiral limit $m_0 = 0$, where $m_0$ denotes the current ("bare") mass of a single light quark.

\[ iq_\mu \Gamma_{5\mu}^i(p + q, p) = \left( \frac{\lambda^i}{2} \right) \left\{ \gamma_5 S^{-1}(p) + S^{-1}(p + q)\gamma_5 \right\}, \]  

(2.1)

where $q = p' - p$ is the momentum transfer (the external momentum) and the quark propagator is given by

\[ -iS(p) = \hat{p}A(-p^2) + B(-p^2). \]  

(2.2)

In connection with (2.1), one has to point out that, in general, $\lambda^i$ is a $SU(N_f)$ flavour matrix and, in the massless case, the quark propagator is proportional to the unit matrix in the flavour space. The inverse of the quark propagator of Eq. (2.2) is expressed as

\[ \{-iS(p)\}^{-1} = \hat{p}\overline{A}(-p^2) - \overline{B}(-p^2), \]  

(2.3)

where

\[ \overline{A}(-p^2) = A(-p^2)D^{-1}(-p^2), \]  

\[ \overline{B}(-p^2) = B(-p^2)D^{-1}(-p^2), \]  

(2.4)

\[ D(-p^2) = p^2 A^2(-p^2) - B^2(-p^2). \]
Using the standard decomposition of the inverse quark propagator Eq. (2.3), DCSB at the fundamental quark level can be implemented by the following condition

\[ \{ S^{-1}(p), \gamma_{5} \} = i\gamma_{5} 2\overline{B}(-p^2) \neq 0, \]

so that the \( \gamma_{5} \) invariance of the quark propagator is broken and the measure of this breakdown is the double of the dynamically generated quark mass function \( 2\overline{B}(-p^2) \). This condition leads to the zero mass boson (Goldstone state) in the flavour, axial-vector WT identity (2.1).

Indeed, it follows from (2.1) that one gets a nonzero dynamical quark mass, defined by (2.5), if and only if \( \Gamma_{5\mu} (p + q, p) \) has a pseudoscalar pole at \( q^2 = 0 \) (a dynamical singularity which determines Goldstone state) and vice versa [4, 5, 20].

The Lorentz invariance and parity allow one to write the axial-vector vertex in terms of twelve independent form factors [20, 25] as

\[
\Gamma_{5\mu} (p + q, p) = (\frac{\Lambda}{2}) \gamma_{5} \{ \gamma_{\mu} G_{1} + p_{\mu} G_{2} + p_{\mu} \hat{p} G_{3} + \hat{p} \gamma_{\mu} G_{4} + q_{\mu} G_{5} + \hat{q} q_{\mu} G_{6} + \hat{q} p_{\mu} G_{7} + \hat{q} \gamma_{\mu} G_{8} + \hat{p} q_{\mu} G_{9} + \hat{p} \hat{q} p_{\mu} G_{10} + \hat{p} \hat{q} q_{\mu} G_{11} + \hat{p} \hat{q} \gamma_{\mu} G_{12} \},
\]

where \( (j = 1, 2, 3...12) \).

\[
G_{j} \equiv G_{j}(-p^2, -p'^2, -q^2) = G_{j}(-p^2, -(p + q)^2, -q^2) \equiv G_{j}(-(p + q)^2),
\]

In order to avoid unphysical (kinematic) singularities at \( (p'q) = 0 \) in what follows, the external momentum \( q \) (transfer momentum) and the initial momentum of the quark \( p \) are chosen as independent momenta in the decomposition (2.6) (thus, \( pq = 0 \) only at \( q = 0 \), see Ref. 26). We will be especially interested in the behaviour of the form factors (2.7) as functions of the external momentum \( q \) (momentum of a massless pion). So all the above mentioned physical singularities will be singularities of these form factors at \( q = 0 \). Only the form factors \( G_{5}, G_{6}, G_{9}, \) and \( G_{11} \) can contribute to the dynamical pole-like structure at \( q^2 = 0 \) of the axial-vector vertex (2.6) because only these ones are multiplied by \( q_{\mu} \), whereas the other ones can also be singular at \( q = 0 \) (see below). Dependence on \( p^2 \) at this stage remains, in principle, arbitrary and will be determined, within the CHPT\( q \) approach,
by nonperturbative solutions of the corresponding quark SD equations. Observe that one should find a nonperturbative quark propagator which is regular at the zero point and approaches the free propagator at infinity (asymptotic freedom [17]).

Substituting the decomposition (2.6) into the WT identity (2.1), one obtains four important relations between the form factors $G_j$, namely

$$G_1 + q^2 G_6 + (pq) G_7 = \overline{A}(-(p+q)^2)$$

$$G_2 + q^2 G_5 + q^2 G_8 = \overline{B}(-(p+q)^2) + \overline{B}(-p^2)$$

$$G_3 + q^2 G_9 + q^2 G_{12} = \overline{A}(-(p+q)^2) - \overline{A}(-p^2)$$

$$G_4 + q^2 G_{11} + (pq) G_{10} = 0,$$

where $\overline{A}(-(p+q)^2)$ and $\overline{B}(-(p+q)^2)$ are defined by (2.4) with the substitution $p \to p + q$.

In order to calculate certain important physical parameters, e.g. the pion decay constant, the meson form factors, their charge radii, etc., one needs to know the bound-state BS meson wave function, restored from the WT identity (2.1) as a residue at pole in the axial-vector vertex (2.6) and its regular piece at small momentum transfer $q$ and especially at zero momentum transfer $q = 0$ (see below). For this reason it is necessary to transform the singular part of the axial-vector vertex in (2.6) into its regular counterpart, thus making the corresponding form factors free of singularities at $q = 0$. This can be done in a complete analogy with the Ball and Chiu method [26] to remove unphysical (kinematical) singularities from the corresponding vertex. For the sake of simplicity, in what follows, let us call this the "Ball and Chiu procedure".

Indeed, from the exact relations (2.8-2.11) it follows that the form factors $G_1$ and $G_4$ are regular functions of their arguments from the very beginning. The form factors $G_j$ ($j=2,3,7,10$) have singularities of the type $(pq)^{-1}$ at $q = 0$ and the form factors $G_8$ and $G_{12}$ may have singularities of the type $q^{-2}$ at $q = 0$, which may be mixed up with the dynamical singularity at $q^2 = 0$. However one can explicitly show that these form factors, as well as the form factors $G_1$ and $G_4$, are regular functions of $q$ at small $q$ from the very beginning [20]. As mentioned above, only the form factors $G_5, G_6, G_9$ and $G_{11}$ need to have
dynamical pole singularities at \( q^2 = 0 \), whereas the form factors \( G_8 \) and \( G_{12} \) can have also the same singularities at this point. For this reason, let us express these form factors as follows \((j=5,6,9,11 \text{ and also } j=8,12)\)

\[
G_j(p,q) = \frac{1}{q^2} R_j(p,p) + G_j^R(p,q),
\]

where \( R_j(p,p) \) and \( G_j^R(p,q) \) are the residues and regular parts of the corresponding form factors.

In the framework of CHPTq, let us now write down the regular form factors \( G_1 \) and \( G_4 \) in terms of formal Taylor series in powers of small external momentum \( q \) as follows \((j = 1, 4)\)

\[
G_j(-\langle p + q \rangle^2) = \sum_{n=0}^{\infty} \frac{h^n}{n!} G_j^{(n)}(-p^2)
\]

where

\[
h = -2(pq) - q^2.
\]

The same expansions for the quark invariant functions are in order

\[
\bar{A}(-\langle p + q \rangle^2) = \sum_{n=0}^{\infty} \frac{h^n}{n!} \bar{A}^{(n)}(-p^2)
\]

and

\[
\bar{B}(-\langle p + q \rangle^2) = \sum_{n=0}^{\infty} \frac{h^n}{n!} \bar{B}^{(n)}(-p^2)
\]

respectively. Here and in what follows, as well as in expansions (2.13-2.14), differentiation is understood with respect to the argumentum \((-p^2)\) and, for example, \( \bar{B}^{(0)}(-p^2) = \bar{B}(-p^2) \).

By completing the Ball and Chiu procedure (for details see our paper [20]), one is now able to decompose the initial axial-vector vertex (2.6) into pole (dynamical) and regular parts as follows

\[
\Gamma_{5\mu}(p + q, p) = -\frac{q_{\mu}}{q^2} F_5 G_5^i(p + q, p) + \Gamma_{5\mu}^R(p + q, p),
\]

where the BS bound-state amplitude is
\[ G_i^5(p + q, p) = -\frac{1}{F_\pi} \left( \frac{\lambda_i}{2} \right) \gamma_5 G(p + q, p), \tag{2.18} \]

with

\[ G(p + q, p) = 2\overline{B}(-p^2) + \hat{q}R_6(-p^2) + \hat{p}\hat{q}R_{11}(-p^2) \tag{2.19} \]

and the arbitrary form factors are the residues of the corresponding form factors (2.12). The regular part now is determined as follows

\[ \Gamma_{5 \mu}^i(p + q, p) = \left( \frac{\lambda_i}{2} \right) \gamma_5 \{ \gamma_\mu G_1 + p_\mu G_2 + p_\mu \hat{p}G_3 + \hat{p}\gamma_\mu G_4 + O_\mu(q) \} \tag{2.20} \]

where \( O_\mu(q) \) defines the terms of order \( q \) and they play no further role. At zero momentum transfer, the regular part also depends on the same form factors \( R_6 \) and \( R_{11} \) as in (2.19) and is given by

\[
\begin{align*}
G_1(-p^2) &= \overline{A}(-p^2) - R_6(-p^2) \\
G_2(-p^2) &= -2\overline{B'}(-p^2) \\
G_3(-p^2) &= -2\overline{A'}(-p^2) \\
G_4(-p^2) &= -R_{11}(-p^2). 
\end{align*}
\tag{2.21}
\]

The system (2.21) is nothing else but the conditions for cancellation of the dynamical singularities at \( q = 0 \) for the corresponding form factors [20]. Dependence on the same arbitrary form factors in (2.19) and (2.21) indicates that the self-consistent separation of the singular (pole) part from the regular one in this vertex is not yet finished. Indeed, it is easy to see that the terms multiplied by these arbitrary form factors also give rise to nonvanishing contributions to the axial-vector vertex when \( q \) goes to zero. There is no explicit reason for keeping these terms in the pole part of the corresponding axial-vector vertex in (2.17). Moreover, in order to completely untangle the pole and regular parts, they also should be transmitted from the pole part to the regular part of the vertex (transmission effect). Finally one obtains

\[ \Gamma_{5 \mu}^i(p + q, p) = \left( \frac{\lambda_i}{2} \right) \gamma_5 \frac{q_\mu}{q^2} \left[ 2\overline{B}(-p^2) \right] + \Gamma_{5 \mu}^i(p + q, p), \tag{2.22} \]
where now

\[
\Gamma_{5\mu}^R(p + q, p) = \left(\frac{\lambda^i}{2}\right)\gamma_5\{[\gamma_\mu G_1 + \frac{q_\mu \hat{q}}{q^2} R_6(-p^2)] + p_\mu G_2 + p_\mu \hat{p} G_3 \\
+ [\hat{p}\gamma_\mu G_4 + \frac{q_\mu \hat{q}}{q^2} R_{11}(-p^2)] + O_\mu(q)\} \tag{2.23}
\]

On account of (2.21) it is easy to check that the sum in the first brackets in (2.23) becomes simply \(\gamma_\mu \tilde{A}(-p^2)\), i.e. an exact cancellation of the unknown arbitrary form factor \(R_6(-p^2)\) occurs. In the same way, owing to (2.21), the contribution from the second brackets completely disappears too, i.e. the exact cancellation of the arbitrary form factor \(R_{11}(-p^2)\) takes place as well. Thus the axial-vector vertex \(\Gamma_{5\mu}^i(p + q, p)\) in (2.17) has, indeed, a dynamical Goldstone pole singularity at \(q^2 = 0\), explicitly shown in (2.22), which corresponds to the massless pion with residue

\[
G(p, p) = R_5(-p^2) = 2\tilde{B}(-p^2) \tag{2.24}
\]

proportional to the pion decay constant \(F_\pi\) (2.18) [5, 8, 9, 20, 23, 24]. The regular part of the vertex at zero momentum transfer \((q = 0)\) becomes

\[
\Gamma_{5\mu}^R(p, p) = \left(\frac{\lambda^i}{2}\right)\gamma_5\{\gamma_\mu \tilde{G}_1 + p_\mu \tilde{G}_2 + p_\mu \hat{p} \tilde{G}_3 + \hat{p}\gamma_\mu \tilde{G}_4\}, \tag{2.25}
\]

where, evidently, all form factors are functions of \((-p^2)\) only and expressed as follows

\[
\tilde{G}_1(-p^2) = \tilde{\Lambda}(-p^2) \\
G_2(-p^2) = -2\tilde{B}'(-p^2) \\
G_3(-p^2) = -2\tilde{\Lambda}'(-p^2) \\
\tilde{G}_4(-p^2) = 0. \tag{2.26}
\]

Here the primes denote differentiation with respect to the argumentum \((-p^2)\). This is the general CHPTq solution to the regular piece of the axial-vector vertex at zero momentum transfer (2.25) in the chiral limit and it does not depend on the arbitrary form factors at all.

Expressions (2.22-2.26) indicate that the self-consistent separation of dynamical pole singularity from the regular part in the corresponding axial-vector vertex now is completed.
We call attention to that the arbitrary form factors $R_6(-p^2)$ and $R_{11}(-p^2)$ can not be put "by hand" to zero in (2.21) in order to obtain (2.26). An exact cancellation, because of the above mentioned transmission effect, takes place between the same (arbitrary) form factors entering the bound-state amplitude up to terms of order $q$ (2.18-2.19) and the regular part of the axial-vector vertex at zero momentum transfer $q = 0$ (2.20-2.21). These form factors become necessarily the same in order to cancel dynamical singularities at $q = 0$ in the axial-vector vertex [20]. Only this allows to completely untangle the pole and the regular parts in it.

The regular part at zero momentum transfer is now determined involving neither arbitrary form factors nor the BS bound-state amplitude which coincides with the residue at the pole. The form factor $G_9$ could, but did not, contribute to the pole-like structure of the vertex from the very beginning. The form factors $G_6$ and $G_{11}$, as shown above, give rise to the same (finite) contributions to the pole, as well as to the regular parts of the vertex, so they cancel each other at zero momentum transfer (transmission effect). The form factor $G_5$ alone determines the pole-like structure at $q^2 = 0$ of the axial-vector vertex. Its residue coincides with the BS amplitude at zero momentum transfer, Eq. (2.24). This result is well-known [5, 8, 9, 20, 23, 24], and we only recast it within the CHPTq approach in terms of our form factors in the general decomposition of (2.6-2.7). A new exact result appears in Eqs. (2.25-2.26), which makes possible to express the regular part of the axial-vector vertex at zero momentum transfer in terms of the dynamical quark propagator variables alone. As mentioned above, these form factors ($G_2, G_3$ and $G_4$) had previously been neglected by assuming that they were of order $g^2$ in the coupling constant [9, 23, 24]. This solution (2.25-2.26) leads to the new nonperturbative expression for the pion decay constant in the Jackiw-Johnson (JJ) representation [4, 20]. Needless to say, there is no hope for an exact solution of the BS-type integral equation for the regular part of the axial-vector vertex even at zero external momentum transfer.

Note that the quantities of physical interest (bound-state amplitudes and regular parts) can be restored from the corresponding axial WT identity up to terms of $O(q^0)$ only. This
is in agreement with the general situation when the WT identity can provide nontrivial information only on quantities at zero momentum transfer (longitudinal terms) and the terms of order $q$ and higher (transverse terms) always remain undetermined. In other words, the correct treatment of this identity provides exact information on the first terms of the corresponding Taylor series (in powers of the external momentum) for the bound-state amplitudes and the regular parts of the corresponding vertices, etc. In many cases of physical interest this information is completely sufficient for calculating important physical parameters from low-energy physics in a self-consistent way. In order to find terms of order $q$ and higher in the CHPTq expansion of the BS bound-state amplitude (2.18-2.19), it is necessary to develop the CHPTq in the framework of the corresponding BS integral equation (which is now in progress) or investigate the corresponding BS integral equation in some other approximation as it was done, for example, in Ref. 27.

Incidentally, let us note that from (2.1) and (2.17-2.22), in general, it follows

$$ i q^i_\mu \Gamma^{iR}_{5\mu}(p + q, p) = \left( \frac{\lambda_i}{2} \right) \left\{ \gamma_5 S^{-1}(p) + S^{-1}(p + q) \gamma_5 \right\} + i F_\pi G^i_5(p + q, p), \quad (2.27) $$

so differentiation with respect to $q_\nu$, on account of Eq. (2.19), and then setting $q = 0$ gives

$$ i \Gamma^{iR}_{5\mu}(p, p) = \left( \frac{\lambda_i}{2} \right) \partial_\mu S^{-1}(p) \gamma_5 + i \Delta^i_{5\mu}(p, p), \quad (2.28) $$

where

$$ \Delta^i_{5\mu}(p, p) = \left( \frac{\lambda_i}{2} \right) [\gamma_\mu R_6(-p^2) - \hat{p}\gamma_\mu R_{11}(-p^2)] \gamma_5. \quad (2.29) $$

Thus, in order to find these arbitrary form factors, it is necessary to solve the BS integral equation for the regular part (2.28) at zero momentum transfer ($q = 0$) in some appropriate approximation for the BS scattering kernel or to solve the BS integral equation up to terms of order $q$ for the pion bound-state amplitude (2.18-2.19)

$$ G^i_5(p + q, p) = -\frac{1}{F_\pi} \left( \frac{\lambda_i}{2} \right) \gamma_5 [2\overline{B}(-p^2) + \hat{q}_6 R_6(-p^2) + \hat{p}\hat{q}_R_{11}(-p^2)] \quad (2.30) $$

in the same approximation. The above mentioned CHPTq within the BS formalism is the most appropriate tool for this purpose. It makes sense also to assume some very similar
nonperturbative ansatz for the arbitrary form factors $R_6(-p^2)$ and $R_{11}(-p^2)$ (see Subsection A in Section 5 below).

Concluding this section a few remarks are in order. In our paper [20] it has been already explained why the above mentioned CHPTq formal Taylor expansions, in powers of small external momentum $q$, are valid when the quark momentum $p$ is also small. Introduction of the large mass scale parameter $\mu$ is necessary in the CHPTq approach to make sense of the formal Taylor expansions at small $p$. So, even in the domain of small $p$, contributions from small external momentum $q$ in (2.13-2.16) are suppressed by a factor of $\Lambda_\chi/\mu \ll 1$, where $\Lambda_\chi$ is the scale of DCSB at the hadronic level - the scale of the effective field theory (CHPT) [28]. It is worth noting that if one formally identifies the large mass scale parameter $\mu$ with the mass of the heavy quark $m_Q$ then the same factor of suppression appears as in the heavy quark expansion [29].

III. A DYNAMICAL QUARK PROPAGATOR APPROACH TO QCD AT LOW ENERGIES

As it was already pointed out in the introduction, the IR or low energy region (large distances) is responsible for the nonperturbative effects in QCD. The most important ones of these effects are quark confinement and DCSB. In order to study these and other interesting effects one should develop a nonperturbative approach to QCD at large distances. The system of the SD equations with the corresponding Slavnov-Taylor (ST) identities for Green’s functions [1, 30] can serve as an adequate and effective tool for a nonperturbative approach to QCD (see also recent reviews in Refs. 31). One must keep in mind that an infinite system of nonlinear integral SD equations, complemented by the corresponding ST identities contains, in principle, the full dynamical information of quantum field theory. One of the major and long-standing problems in quantum field theory is to find a gauge-invariant approximation scheme to the SD equations and the corresponding ST identities since one holds no hope for an exact solution. It seems to us that our approach to QCD at large distances [12, 13, 16]
provides a reasonable solution to this essential problem though in the quark sector only.

The central role in our approach belongs to the quark propagator. A correct description and the numerical calculation of the properties of the physical particles (e.g. pions) is extremely difficult because one needs to know the quark propagator that satisfies some necessary conditions. A quark propagator must not be an explicitly gauge-dependent quantity. It should be nonperturbative and preferably regular at the zero point; it should have no poles (confinement-type solution), and it should correspond to the dynamical breakdown of chiral symmetry (see below). A quark propagator must also be free of ghost complications, provided it was obtained in any covariant gauge and, finally, it must have a correct asymptotic behaviour at infinity, i.e. it must asymptotically approach the free propagator at infinity (asymptotic freedom).

Making only one widely accepted dynamical assumption that the full gluon propagator becomes IR singular like (hereafter referred to as the enhancement of the zero modes in the QCD true vacuum (see the next subsection))

\[ D_{\mu\nu}(q) \sim (q^2)^{-2}, \quad q^2 \to 0 \] (3.1)
in the covariant gauge, such a quark propagator has recently been found \[12, 13, 16\]. There exist direct and indirect arguments in favour of this asymptotically IR singular behaviour of the full gluon propagator (3.1). It makes sense to remind the reader only of a few of them. These have nothing to do with the potential concept relevant for the constituent quark model (CQM) \[32\] despite its phenomenological success \[33\] for single-hadron states.

I. In the above mentioned case the Wilson loop defined as

\[ W(c) = \langle TrP_c \exp i g \oint A_\mu dx_\mu \rangle = Tr \exp \left\{-g^2 \oint dx_\mu dy_\nu D_{\mu\nu}(x-y) + \ldots\right\} , \] (3.2)
obeys an area law, indicative of quark confinement. Indeed, if the full gluon propagator is singular like (3.1) as the momentum goes to zero, then \[D_{\mu\nu}(x-y) \sim \ln(x-y)\] and

\[ W(c) \sim \exp[-\sigma A(c)] , \] (3.3)
where $A(c)$ is the minimal area of some surface bounded by curve $c$. This is the Wilson criterion for quark confinement [2, 34]. For the free gluon propagator, $D_{\mu\nu}(q) \sim (q^2)^{-1}$, then $D_{\mu\nu}(q) \sim (x - y)^{-2}$ and the Wilson loop behaves as follows

$$W(c) \sim \exp[-\mu P(c)], \quad (3.4)$$

where $P(c)$ is the perimeter of curve $c$. Therefore no statement about confinement can be deduced.

II. The cluster property of the Wightman functions in QCD fails and this allows such a singular behaviour like (3.1) for the full gluon propagator in the IR domain [35].

III. The form factor of the full gluon propagator (see, (3.6)-(3.7) below) is nothing else but the running coupling constant. Then the ansatz (3.1) leads to the strong coupling behaviour of the Callan-Symanzik-Gell-Mann-Low (CS-GML) function $\beta(g)$ at large distances ($q^2 \to 0$). In this case, the CS-GML function is always negative, i.e. there is no IR stable fixed point at all. This behaviour of the strong coupling regime ($g \to \infty$) is usually refered to as IR slavery, indicative of confinement [1, 36].

IV. After the pioneering works of Baker, Ball and Zachariasen in the axial gauge and Mandelstam in the covariant (Landau) gauge [37], the consistency of the singular asymptotics (3.1) with the direct solution of the full gluon propagator in the IR domain was repeatedly confirmed (see, for example, Refs. 38, our paper [16] and references therein). Despite many problems and inconsistencies due to the complicated mathematical structure and the highly nonlinear nature of the SD equation for the full gluon propagator in QCD, the deep IR singular asymptotics (3.1) should be considered as well-established.

A. The zero modes enhancement model of quark confinement and DCSB

Today there are no doubts that the dynamical mechanisms of quark confinement and DCSB are closely related to the complicated topological structure of the QCD nonperturbative vacuum [1, 15, 39] but, unfortunately, a detailed dynamical picture of these nonperturbative effects is not yet known. Also it becomes clear that the nonperturbative IR
divergences, are closely related, on one hand, to the above mentioned nontrivial vacuum structure, on the other hand, they are important as far as the large scale behaviour of QCD is concerned [1, 37, 39]. If it is true that QCD is an IR unstable theory (has no IR stable fixed point) then the low-frequency modes of the Yang-Mills fields should be enhanced due to the nonperturbative IR divergences. So the gluon propagator diverges faster than $(q^2)^{-1}$ at small $q$, in accordance with (3.1) - the zero modes enhancement (ZME) effect in QCD. If, indeed the low-frequency components of the virtual fields in the true vacuum have a larger amplitude than those of the bare (perturbative) vacuum [37], then the Green function for a single quark should be reconstructed on the basis of this effect. It is important that the possible effect of the ZME (3.1) is our primary dynamical assumption. We will consider this effect as a very similar confining ansatz for the full gluon propagator in order to use it as input information for the quark SD equation.

In what follows, for the convenience of the reader, we present the above mentioned reconstruction in its most general important features, leaving additional details for our paper [16]. Let us begin from some general remarks of mathematical nature. Such a singular behaviour of the full gluon propagator requires the introduction of a small IR regulation parameter $\epsilon$ in order to define the initial SD equations and ST identities in the IR region (postponing the precise definition of the distribution $(q^2)^{-2}$ in $n$-dimensional Euclidean space). Because of this, the quark propagator and other Green’s functions become dependent, in general, on this IR regulation parameter $\epsilon$, which is to be set to zero at the end of the computation ($\epsilon \to 0^+$). For the sake of brevity, this dependence will be always understood but it will not be indicated explicitly.

There exist only two different types of behaviour of the quark propagator with respect to $\epsilon$ in the $\epsilon \to 0^+$ limit. If the quark propagator does not depend on the $\epsilon$ - parameter in the $\epsilon \to 0^+$ limit then one obtains the IR finite (from the very beginning) quark propagator. In this case quark confinement is understood as the disappearance of the quark propagator pole on the real axis at the point $p^2 = m^2$, where $m$ is the quark mass. Such an interpretation of quark confinement comes, apparently, from Preparata’s massive quark model (MQM) [40]
in which quarks were approximated by entire functions. A quark propagator may or may not be an entire function, but in any case the first order pole disappears (see references in our paper [16]). On the other hand, a quark propagator can vanish after the removal ($\epsilon \rightarrow 0^+$) of the IR regulation parameter $\epsilon$. A vanishing quark propagator is also a direct manifestation of quark confinement. Apparently, this understanding of quark confinement follows from two-dimensional QCD with $N_c$ large limit [41].

Closing these general remarks, let us consider the exact, unrenormalized SD equation for the quark propagator in momentum space (Fig. 1)

$$S^{-1}(p) = S_0^{-1}(p) + g^2 C_F \int \frac{d^n q}{(2\pi)^n} \Gamma_\mu(p, q) S(p - q) \gamma_\nu D_{\mu\nu}(q),$$  

where $C_F$ is the eigenvalue of the quadratic Casimir operator in the fundamental representation. Other notions are obvious. From now on we will suppose that the IR region is effectively decoupled from the UV region, following the paper of Pagels [36]. The standard UV renormalization program should be performed after completion of our program.

The full gluon propagator in an arbitrary covariant gauge is

$$D_{\mu\nu}(q) = -i \left\{ g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right\} \frac{1}{q^2} d(-q^2, a) + a \frac{q_\mu q_\nu}{q^4},$$  

where $a$ is a gauge fixing parameter ($a = 0$, Landau gauge).

Assuming that in the IR region

$$d(-q^2, a) = \left( \frac{\mu^2}{-q^2} \right) + \beta(a) + O(q^2), \quad q^2 \rightarrow 0,$$

where $\mu$ is the appropriate mass scale parameter, we obtain the above mentioned generally accepted form of the IR singular asymptotics for the full gluon propagator (3.1) (enhancement of the zero modes). In accordance with (3.7), hereafter we keep only the leading and next-to-leading terms of the corresponding Green’s functions expansions in the IR region under control, omitting always the terms of order $q^2$.

In connection with (3.6) and (3.7), let us note that despite the approximation of the full gluon propagator (3.6) with its deep IR singular asymptotics (3.7), formally in the whole
range \([0, \infty)\), such kind of behaviour may occur only in the nonperturbative (IR) region. This means that correct solutions to the quark SD equations should manifest the existence of a scale at which nonperturbative effects become essential. As we will show explicitly below, our solutions to the quark SD equation really contain such a characteristic scale intrinsically.

In order to actually define an initial SD equation (3.5) in the IR region (at small momenta) let us apply the gauge-invariant dimensional regularization method of 't Hooft and Veltman [42] in the limit \(n = 4 + 2\epsilon\), \(\epsilon \to 0^+\). Here and below \(\epsilon\) is the above mentioned small IR regulation parameter. Now we consider the SD equations and the corresponding quark-gluon ST identity in Euclidean space \((d^nq \to id^nq_E, \quad q^2 \to -q_E^2, \quad p^2 \to -p_E^2\), but for simplicity the Euclidean subscript will be omitted).

Let us use, in the sense of distribution theory, the relation [43]

\[
(q^2)^{-2+\epsilon} = \frac{\pi^2}{\epsilon} \delta^4(q) + (q^2)^{-2} + O(\epsilon), \quad \epsilon \to 0^+, \tag{3.8}
\]

which implies that the full gluon propagator (3.6) in the IR region behaves like

\[
D_{\mu\nu}(q) = \epsilon^{-1} \bar{D}_{\mu\nu}(q), \quad \epsilon \to 0^+, \tag{3.9}
\]
in the \(\epsilon \to 0^+\) limit and \(\bar{D}_{\mu\nu}(q)\) exists as \(\epsilon \to 0^+\). Here and below \((q^2)^{-2}\) is the functional acting on the main (test) functions according to the so-called ”plus prescription” standard formulae [43] (see below).

The singularity (3.8-3.9) is the only initial singularity in our approach to QCD at large distances. All other Green’s functions will be considered as regular functions of their arguments. In the distribution theory their singular dependence leads to more complicated SD equations for the IR finite (renormalized) quark propagator and therefore requires special treatment. With these caveats let us proceed to the realization of the renormalization program in order to evaluate nonperturbative IR divergences, provided by the strongly singular confining ansatz (3.8-3.9).

Substituting (3.8-3.9) into the SD equation (3.5), we obtain the quark propagator expansion in the IR region (in four-dimensional Euclidean space)
\[ S^{-1}(p) = S_{0}^{-1}(p) + \frac{1}{\epsilon} \tilde{g}^2 \Gamma_{\mu}(p,0) S(p) \gamma_{\mu} \]
\[ + c_1 \int \frac{d^4 q}{(2\pi)^4} \Gamma_{\mu}(p,q) S(p-q) \gamma_{\nu} t_{\mu\nu}(q)(q^2)^{-2} \]
\[ + c_2 \int \frac{d^4 q}{(2\pi)^4} \Gamma_{\mu}(p,q) S(p-q) \gamma_{\nu} T_{\mu\nu}(q,a)(q^2)^{-1} \]
\[ + O(\epsilon), \quad \epsilon \to 0^+, \]  \hfill (3.10)

where \( \tilde{g}^2 = C_F \frac{3}{4} g^2 \mu^2 \pi^2 (2\pi)^{-4} \) and

\[ t_{\mu\nu}(q) = [g_{\mu\nu} - \frac{q_{\mu} q_{\nu}}{q^2}] \]  \hfill (3.11)

is the transverse tensor and the tensor

\[ T_{\mu\nu}(q,a) = \beta(a)t_{\mu\nu}(q) + a \frac{q_{\mu} q_{\nu}}{q^2} \]  \hfill (3.12)

explicitly depends on a gauge-fixing parameter \( a \).

Within the Gelfand’s and Shilov’s distribution theory [43] one has

\[ \int \frac{d^4 q}{(2\pi)^4} \Gamma_{\mu}(p,q) S(p-q) \gamma_{\nu} t_{\mu\nu}(q)(q^2)^{-2} \]
\[ = \int \frac{d^4 q}{(2\pi)^4} t_{\mu\nu}(q) \{ \Gamma_{\mu}(p,q) S(p-q) - \Gamma_{\mu}(p,0) S(p) \} \gamma_{\nu}(q^2)^{-2}. \]  \hfill (3.13)

So, expanding in powers of \( q \) and keeping the terms of order \( q^{-2} \) (the Coulomb order terms), in agreement with (3.7), from (3.10) and on account of (3.13), one finally obtains

\[ S^{-1}(p) = S_{0}^{-1}(p) + \frac{1}{\epsilon} \tilde{g}^2 \Gamma_{\mu}(p,0) S(p) \gamma_{\mu} \]
\[ + c_1 \int \frac{d^4 q}{(2\pi)^4} t_{\mu\nu}(q)(q^2)^{-2} \left\{ -q^\sigma q^\lambda \Gamma_{\mu}^{\sigma}(p,0) S^{\lambda}(p) + \frac{1}{2} q^\sigma q^\beta \Gamma_{\mu}^{\sigma\beta}(p,0) S(p) \right\} \gamma_{\nu} 
\[ + \int \frac{d^4 q}{(2\pi)^4} \left\{ \frac{c_1}{2} t_{\mu\nu}(q)(q^2)^{-2} q^\lambda q^\tau \Gamma_{\mu}(p,0) S^{\lambda\tau}(p) + c_2 T_{\mu\nu}(q,a)(q^2)^{-1} \Gamma_{\mu}(p,0) S(p) \right\} \gamma_{\nu} \]
\[ + O(\epsilon), \quad \epsilon \to 0^+, \]  \hfill (3.14)

In derivation of this expansion, we have used formal Taylor series for Green’s functions as

\[ \Gamma_{\mu}(p,q) = \Gamma_{\mu}(p,0) + \sum_{n=1}^{\infty} \frac{q^n}{n!} \left\{ \frac{d^n \Gamma_{\mu}(p,q)}{dq^n} \right\}_{q=0} \]  \hfill (3.15)

and
\[ S(p - q) = S(p) + \sum_{n=1}^{\infty} \frac{(-1)^n q^n}{n!} \frac{d^n S(p)}{dp^n}, \]  
(3.16)

where \( q^n \) and \( dq^n \) stand for
\[ q^n = \prod_{m=1}^{n} q^{\alpha_m} \quad \text{and} \quad dq^n = \prod_{m=1}^{n} dq^{\alpha_m}, \]  
(3.17)

respectively. The same notation stands for \( dp^n \) as well. In (3.14), for example, \( \Gamma_\mu^\alpha(p,0) = \left\{ \frac{d\Gamma_\mu(p,q)}{dq^n} \right\}_{q=0} \) and so on.

It is worth making a few remarks in advance. We will show, by completing our renormalization program (see below), that the finite terms (next-to-leading terms), coming from the IR region and explicitly depending on a gauge fixing parameter, in any case, always become terms of order \( \epsilon \). For this reason they vanish as \( \epsilon \to 0^+ \), like the finite term does (which is not written down) containing the logarithm of the coupling constant and arising because of the dimensional regularization method [42].

As mentioned above, all Green’s functions become dependent generally on the IR regularization parameter \( \epsilon \). In order to extract the finite Green’s functions in the IR region, we introduce the renormalized (IR finite) quark-gluon vertex function at zero momentum transfer and the quark propagator as follows
\[ \Gamma_\mu(p,0) = Z_1(\epsilon)\Gamma_\mu(p,0), \]
\[ S(p) = Z_2(\epsilon)\bar{S}(p), \quad \epsilon \to 0^+. \]  
(3.18)

Here \( Z_i(\epsilon) \) \((i = 1, 2)\) are the corresponding IR renormalization constants. The \( \epsilon \)-parameter dependence is indicated explicitly to distinguish them from the usual UV renormalization constants. In all relations, here and below, containing the IR renormalization constants, the \( \epsilon \to 0^+ \) limit is always assumed. There are no restrictions on the \( \epsilon \to 0^+ \) limit behaviour of the IR renormalization constants, apart from the regular \( \epsilon \) dependence of the IR renormalization constant \( Z_2(\epsilon) \) of the quark wave function. This is required by the quark confinement definition (mentioned above). \( \Gamma_\mu(p,0) \) and \( \bar{S}(p) \) are the renormalized (IR finite) Green’s functions and they therefore do not depend on \( \epsilon \) in the \( \epsilon \to 0^+ \) limit, i.e. they exist as \( \epsilon \to 0^+ \).
It is obvious that relations (3.18) and the analogous relations for other Green’s functions [16] are the most general expressions, relating the unrenormalized Green’s functions to the renormalized (IR finite) ones, if one assumes (in analogy with UV multiplicative renormalizability (MR)) the property of MR of the IR nonperturbative divergences, arising in the theory due to nonperturbative confining ansatz (3.8-3.9) for the full gluon propagator in the IR domain. This property was one of the main topics investigated and proven in our paper [16].

In connection with relations (3.8-3.9), Eq.(3.10) and relations (3.18), let us make a few remarks in advance. The correct treatment of such a strong singularity like (3.8-3.9) within distribution theory [43] by the gauge-invariant dimensional regularization method of t’Hooft and Veltman [42] enabled us to extract the required class of test functions in the renormalized quark SD equation. The test functions do consist of the quark propagator and the corresponding quark-gluon vertex function, Eq.(3.10). By the renormalization program we have found the regular solutions for the quark propagator (see below). For that very reason relation (3.8) is justified, it is multiplied by the appropriate smooth test functions.

From (3.14) and on account of (3.18) a cancellation of nonperturbative IR divergences takes place if and only if (iff)

\[ Z_1(\epsilon)Z_2^2(\epsilon) = \epsilon Y_1, \quad \epsilon \to 0^+, \quad (3.19) \]

with \( Y_1 \) being an arbitrary finite constant. It is evident that this very condition and the similar ones below, govern the concrete \( \epsilon \)-dependence of the IR renormalization constants which, in general, remain arbitrary. From (3.19) it follows that the behaviour of the vertex IR renormalization constant \( Z_1(\epsilon) \) is determined completely by the quark IR renormalization constant \( Z_2(\epsilon) \). Carrying out our renormalization program, we show that all other IR renormalization constants can also be expressed through the quark wave function IR renormalization constant \( Z_2(\epsilon) \) only.

Expression (3.19) is a convergence condition for the quark propagator. Because of (3.19), the explicitly gauge-dependent terms (the next-to-leading terms) in the SD equation (3.14)
become $\epsilon$-order terms. For this reason these noninvariant terms vanish in the $\epsilon \to 0^+$ limit. From the formal Taylor expansions (3.15) and (3.16) it follows that IR renormalization properties of the lower (quark propagator) and higher (quark-gluon vertex) Green’s functions are different. Indeed, if the derivatives of the quark propagator are renormalized like the quark propagator itself then one cannot say anything about the derivatives of the quark-gluon vertex function since we know its renormalization only at zero momentum transfer (3.18). This is a principle difference between the nonperturbative IR renormalization of any lower and higher Green’s functions in our model. At the same time it is easy to see that these terms are always accompanied by proper powers of $q$, making the terms containing, in general, the derivatives of the Green’s functions free from nonperturbative IR divergences. Obviously, the finite next-to leading terms, coming from the IR region and not containing the derivatives of the higher Green’s functions become, terms of order $\epsilon$. For this reason they vanish in the $\epsilon \to 0^+$ limit, as well as the above mentioned finite terms explicitly depending on a gauge fixing parameter. Thus the leading and next-to-leading terms of our expansion for the quark propagator in the IR domain always become not explicitly dependent on a gauge-fixing parameter $a$. This is a general feature of our expansions in the IR region for various Green’s functions. The next-to-leading terms coming from the IR region, which remain finite by completing our renormalization program (the ones containing the derivatives of higher Green’s functions), should be considered as a good approximation for the intermediate region, which still remains terra incognita in QCD. Apparently, the Gromes constant [33], which occurs in the derivation of the quark-antiquark static potential in the Wilson loop approach [44, 45], is the nonrelativistic counterpart of these fully relativistic terms. These terms (second line in Eq. (3.14)) are of the same Coulomb order of magnitude ($\sim q^{-2}$ in contrast to the initial singularity (3.1)) as the corrections coming from the UV region and they should be investigated, along with them, elsewhere. The correct treatment of the enhancement of the zero modes, given by (3.8) within the distribution theory (3.12-3.16), nevertheless effectively converts the initial strong IR singularity ($\sim q^{-4}$) into a Coulomb-like behaviour ($\sim q^{-2}$) in the intermediate and UV regions, which is compatible with asymptotic freedom. Also, these
terms are gauge invariant in comparison with the terms displayed in the third line of Eq. (3.16) which disappear after completing the nonperturbative renormalization program.

An important general observation is appropriate here. The ZME model requires the full Green’s functions, not the free ones, in particular only the full quark-gluon vertex is relevant for our approach. Also, from now on we will be interested only in the leading terms of the corresponding Green’s functions expansions in the IR domain. The renormalization scheme to remove all nonperturbative IR divergences, on a general ground, depends only on these terms. So one has (as $\epsilon \to 0^+$)

$$
\bar{S}^{-1}(p) = Z_2(\epsilon)S_0^{-1}(p) + \bar{g}^2 Y_1 \bar{\Gamma}_\mu(p, 0) \bar{S}(p) \gamma_\mu,
$$

(3.20)
as the deep IR content of the quark SD equation in our model. To conclude, let us stress once more that the mechanism described above is a general one, which shows clearly how to untangle the explicitly gauge-independent and the explicitly gauge-dependent parts of the IR piece of the quark propagator. The next-to-leading terms which remain finite by completing the nonperturbative IR renormalization program, because of the much less singular behaviour in the IR, are expected to produce only small corrections to the contributions determined by the leading term.

**B. The ground solution**

Absolutely in the same way should be reconstructed the ghost self-energy and the corresponding ST identity for the quark-gluon vertex [16]. We develop a method for the extraction of the IR-finite Green’s functions in QCD. The IR finiteness of the Green’s functions means that they do not depend on the IR regulation parameter $\epsilon$ as $\epsilon \to 0^+$. For this purpose, we have worked out a renormalization program in order to cancel all the IR nonperturbative divergences. It was possible to obtain a closed set of the SD equations and the corresponding ST identity in the quark sector. By completing our renormalization program, we have explicitly shown [16] that all Green’s functions are IR MR. In other words, multiplication by
the quark IR renormalization constant only makes possible to remove all nonperturbative IR
divergences from the theory. It is a general feature of our approach that the behaviour of all
Green’s functions in the IR region depends only on the quark wave function IR renormaliza-
tion constant. We also discover that our approach implies the existence of a characteristic
scale at which confinement and other nonperturbative effects begin to play a dominant role.
If QCD confines then such a characteristic scale must certainly exist. We have shown that
for the covariant gauges the complications due to ghost contributions can be considerable
in our approach. We are now able to formulate the main result of our approach in terms of
the following theorem [16].

**Quark confinement theorem**

Let us assume that the full gluon propagator in the covariant gauge becomes infrared
singular like \((q^2)^{-2}\) at small momentum (enhancement of the zero modes). The quark SD
equation has then three and only three confinement-type solutions for the quark propagator
in the infrared region. Two of them are infrared vanishing solutions after the removal of the
infrared regulation parameter. The third solution does not depend on the infrared regulation
parameter at all, but it has no pole on the real axis in the complex momentum plane and
it implies dynamically chiral symmetry breakdown (DCSB).

Basic chiral QCD parameters will be calculated from the IR finite solution to the quark
SD equation. The closed set of equations in this case, obtained in our paper [16], should
read

\[
S^{-1}(p) = S_0^{-1}(p) + \tilde{g}^2 \Gamma_\mu(p,0)S(p)\gamma_\mu, \tag{3.21}
\]

\[
\frac{1}{2} \dot{b}(0) \Gamma_\mu(p,0) = i\partial_\mu S^{-1}(p) - \frac{1}{2} \dot{b}(0) S(p) \Gamma_\mu(p,0) S^{-1}(p). \tag{3.22}
\]

where

\[
S_0^{-1}(p) = -i(\hat{p} - m_0), \tag{3.23}
\]

is the free quark propagator with \(m_0\) being the current ("bare") mass of a single quark and
\(\tilde{g}^2\) includes the mass scale parameter \(\mu^2\), determining the validity of the above mentioned
deep IR singular asymptotic behaviour of the full gluon propagator (3.7). Let us recall also that

$$\partial_{\mu} S^{-1}(p) = \frac{\partial}{\partial p_{\mu}} S^{-1}(p)$$

(3.24)
as well as that the IR finite quark renormalization constant (which should multiply the free quark propagator in Eq.(3.21)) is to be set to unity without losing generality (multiplicative renormalizability) [16]. It is worth noting here that Eq.(3.21) and Eq.(3.22) describe the leading terms of the corresponding expansions of the quark SD equation and ST identity in the IR region, respectively [16].

In order to solve the system (3.21-3.22), it is convenient to represent the quark-gluon vertex function at zero momentum transfer as follows

$$\bar{\Gamma}_\mu(p, 0) = F_1(p^2)\gamma_\mu + F_2(p^2)p_\mu + F_3(p^2)p_\mu \hat{p} + F_4(p^2)\hat{p}\gamma_\mu.$$  

(3.25)

Substituting this representation into the ST identity (3.22), one can express the scalar functions $F_i(p^2)(i = 1, 2, 3, 4)$ in terms of the quark propagator scalar functions $A$ and $B$.

$$F_1(p^2) = -\overline{A}(p^2),$$
$$F_2(p^2) = -2\overline{B}'(p^2) - F_4(p^2),$$
$$F_3(p^2) = 2\overline{A}'(p^2),$$
$$F_4(p^2) = \frac{A^2(p^2)B^{-1}(p^2)}{E(p^2)}.$$  

(3.26)

Here the prime denotes differentiation with respect to the Euclidean momentum variable $p^2$ and

$$\overline{A}(p^2) = A(p^2)E^{-1}(p^2),$$
$$\overline{B}(p^2) = B(p^2)E^{-1}(p^2),$$
$$E(p^2) = p^2A^2(p^2) + B^2(p^2).$$

(3.27)

For the sake of convenience the ghost self-energy at zero point $\bar{b} \equiv \bar{b}(0)$ will be included into the definition of a new coupling constant (see below (3.31)).
Proceeding now to the dimensionless variables

\[ A(p^2) = \mu^{-2}A(t), \quad B(p^2) = \mu^{-1}B(t), \quad t = p^2/\mu^2 \]  

(3.28)

and doing some algebra, the initial system (3.21-3.22) can be rewritten as follows (normal form)

\[ A' = -2\left(1 + \frac{1}{\lambda}t\right)A - \frac{2}{\lambda t}m_0B, \]  

(3.29)

\[ B' = -\frac{3}{2}A^2B^{-1} + \frac{2}{\lambda}(m_0A - B), \]  

(3.30)

where \( A \equiv A(t), \ B \equiv B(t), \) and here the prime denotes differentiation with respect to the Euclidean dimensionless momentum variable \( t. \) A new coupling constant is

\[ \lambda = g^2[b(0)]^{-1}(2\pi)^{-2}, \]  

(3.31)

where \( \bar{b}(0) \) is the finite ghost self-energy at the zero point.

Thus our system (3.29-3.30) does not depend on the parameter \( \bar{b}(0) \) in explicit form. Instead of two different parameters, i.e. initial coupling constant \( g^2 \) and \( \bar{b}(0), \) we have only one parameter - the coupling constant \( \lambda \) (3.31) and we therefore avoid the difficulties associated with the unknown ghost contributions in the covariant gauge. In (3.31) one can recognize the effect of finite renormalization of the initial coupling constant with the help of the ghost self-energy, finite at zero point. This means that the ghosts, having played their important role, retire from the scene.

Let us note here that a dimensionless initial coupling constant \( g^2, \) as well as \( \lambda \) of (3.31), plays no independent role in the presence of the initial mass scale parameter \( \mu, \) which characterizes the nonperturbative region (see the next section). However, here we did not include it into \( \mu \) because of two reasons: 1) to retain the correspondence with contributions to the quark SD equation, coming from the UV region, where this possibility is lacking. 2) to explicitly demonstrate the nonperturbative nature of our solutions.

As it will be shown later, this system always has a chiral symmetry breaking solution \( (m_0 = 0, B(t) \neq 0). \) Our system (3.29-3.30) excludes the trivial solution \( A = B = 0. \) Any
nontrivial solution automatically breaks the $\gamma_5$ invariance of the quark propagator and it therefore leads to spontaneous chiral symmetry breakdown ($m_0 = 0, B(t) \neq 0$, dynamical quark mass generation). In the general case ($m_0 \neq 0$), it is possible to show that the solutions of this system cannot have pole-like singularities at any finite point on the real axis in the whole complex $t$-plane [16]. This is a direct manifestation of quark confinement.

Let us consider the chiral limit of this system, $m_0 = 0$. In this case the initial system (3.29-3.30) can be exactly solved. The regular solutions are

$$A(t) = -\left(\frac{A}{2}\right) t^{-2} \left\{ \exp \left( -\frac{2}{\lambda} t \right) - 1 + \frac{2}{\lambda} t \right\}.$$  \hspace{1cm} (3.32)

and

$$B^2(t_0, t) = 3 \exp \left( -\frac{4}{\lambda} t \right) \int_t^{t_0} \exp \left( \frac{4}{\lambda} t' \right) A^2(t') dt'.$$  \hspace{1cm} (3.33)

respectively, where $t_0$ is an arbitrary constant of integration.

The exact solutions (3.32) for $A(t)$ and (3.33) for dynamically generated quark mass function $B(t_0, t)$ are not entire functions. The functions $A(t)$ and $B(t_0, t)$ have removable singularities at zero. In addition, the dynamically generated quark mass function $B(t_0, t)$ also has algebraic branch points at $t = t_0$ and at infinity. Apparently, these unphysical singularities are due to ghost contributions. The quark propagator may or may not be an entire function, but in any case the pole-type singularities should disappear. This is a general feature of quark confinement and holds in any gauge. Our solutions are of the confinement-type since they have no pole singularities in the whole complex $t$-plane on the real axis at any finite points. Thus they satisfy the quark confinement criterion - disappearance of the pole-type singularities in the corresponding quark Green’s functions.

It is easy to see that $A(t)$ automatically has a correct behaviour at infinity (it approaches the free quark propagator). In order to reproduce automatically the correct behaviour of the dynamically generated quark mass function at infinity, it is necessary to put $t_0 = \infty$ in (3.33) from the very beginning. Obviously in this case solution (3.33) cannot be accepted at zero point ($t = 0$), so that one needs to keep the constant of integration $t_0$ arbitrary, but finite, in order to obtain a regular, finite solution at zero point.
The region $t_0 > t$ can be considered as nonperturbative, whereas the region $t_0 \leq t$ can be considered as perturbative. By approximating the full gluon propagator by its deep IR asymptotics such as $(q^2)^{-2}$ in the whole range $[0, \infty)$, we nevertheless obtain that our solution for the dynamical quark mass function $B(t_0, t)$ manifests the existence of the boundary value momentum (dimensionless) $t_0$, which separates the IR (nonperturbative) region from the intermediate and UV (perturbative) regions. If QCD confines then a characteristic scale, at which confinement and other nonperturbative effects become essential, must exist. The arbitrary constant of integration $t_0$ should be related to a scale at which nonperturbative phenomena (such as confinement and DCSB) dominate.

On one hand, playing a role of the UV cut-off ($t_0 = \Lambda$), the arbitrary constant of integration $t_0$ thereby prevents our quark propagator from having an imaginary part. This is consistent with the idea that a confined particle should have no imaginary part [9]. On the other hand, this makes it possible to eliminate the influence of the above mentioned unphysical singularities which come from the solutions to the quark SD equations (due to necessary ghost contributions) on the $S$-matrix elements reproducing physical quantities.

Thus, in order to obtain numerical values of any physical quantity, e.g. the pion decay constant (see below), the integration over the whole range $[0, \infty]$ reduces to the integration over the nonperturbative region $[0, t_0]$, which determines the range of validity of the deep IR asymptotics (3.1) of the full gluon propagator and consequently the range of validity of the corresponding solutions (3.32) and (3.33) for the IR piece of the full quark propagator.

Here we ought to emphasize that the main contribution to DCSB, as well as to the values of the pion decay constant and other physical quantities, comes from the nonperturbative region (large distances), whereas the contributions from the short and intermediate distances (perturbative region) can only be treated as perturbative corrections. We have confirmed this physically reasonable assertion numerically in the present paper. Despite having a formally correct asymptotic behaviour at infinity, solutions (3.32) and (3.33) for $B(t_0, t)$ (at $t_0 = \infty$) cannot be responsible for the UV asymptotic behaviour of the exact quark propagator. It is better to say that they do not spoil the UV asymptotics of the exact
quark propagator rather than they reproduce it. In order to reproduce correctly the UV asymptotics of the exact quark propagator, it is necessary to make renormalization group improvements over the decomposition of the full gluon propagator (3.6). In other words, one needs to take into consideration the short distance contributions to the full gluon propagator. As it was mentioned above, they are not important for the numerical calculations, but from a conceptual point of view they are certainly important. Unfortunately, the way this can be done self-consistently, i.e. taking into account gauge covariance, ghost contributions, etc. is not yet known. We will discuss this problem briefly at the end of this paper (Section 10).

From Eqs.(3.32-3.33) it can easily be seen that our quark propagator is nonperturbative and ”gauge-invariant”, it is free of ghost complications (since it does not depend explicitly on a gauge-fixing parameter and the ghost self-energy at zero point) and it has no pole at any finite point on the real axis in the complex t-plane (confinement solution). Our quark propagator is also regular at zero point and approaches the free propagator at infinity. So the proposed quark propagator has only one inevitable defect, it explicitly reproduces unphysical singularities due to ghost contributions. A solution to this problem, in our approach, is to integrate the \( S \)-matrix elements of physical quantities over the finite nonperturbative region \((t < t_0)\) whose size is determined by the confinement scale \(\Lambda_c \) [16, 46] (see also next sections).

Now come the main conclusions. Firstly, if the enhancement of the zero modes of vacuum fluctuations (3.7-3.9) takes place indeed then the quark Green’s function, reconstructed on the basis of this effect, has no poles. In other words, the enhancement of the zero modes at the expense of the virtual gluons alone removes nevertheless a ”single” quark from the mass-shell (quark confinement theorem). Secondly, a chiral symmetry violating part of the quark propagator in this case is automatically generated. From the obtained system (3.29-3.30) explicitly follows that it does not allow a chiral symmetry preserving solution \((m_0 = B(t) = 0, A(t) \neq 0)\). So a chiral symmetry violating solution \((m_0 = 0, B(t) \neq 0, A(t) \neq 0)\) for the quark SD equation is required and this is in intrinsic connection with the absence of the pole singularities in this solution. As was it emphasized above, this solution (3.32-3.33) (the ”ground solution”) will be used to compute basic chiral QCD parameters, as well as
the vacuum energy density, in the remaining part of this paper.

IV. THE VACUUM ENERGY DENSITY AND GLUON CONDENSATE

Any correct nonperturbative model of quark confinement and DCSB necessarily becomes a model of the ground state of QCD, i.e., the nonperturbative vacuum. The effective potential method for composite operators \([5, 6, 47]\) allows one to investigate the vacuum of QCD since in the absence of external sources the effective potential is nothing but the vacuum energy density. In this section we will evaluate the vacuum energy density within the ZME effect (3.1) in QCD.

The effective potential at one-loop is \([47]\)

\[
V(S,D) = V(S) + V(D) = -i \int \frac{d^np}{(2\pi)^n} Tr\{\ln(S_0^{-1}S) - (S_0^{-1}S) + 1\} \\
+ i\frac{1}{2} \int \frac{d^np}{(2\pi)^n} Tr\{\ln(D_0^{-1}D) - (D_0^{-1}D) + 1\},
\]

(4.1)

where \(S(p)\) (2.2), \(S_0(p)\) (3.23) and \(D(p)\) (3.6), \(D_0(p)\) are the full, free quark and gluon propagators, respectively. The trace over space-time and colour group indices is implied to be taken but they are suppressed for simplicity’s sake in this equation. Let us recall now that the free gluon propagator can be obtained from (3.6) by setting simply \(d(-q^2, a) = 1\). The effective potential is normalized as follows

\[
V(S_0, D_0) = V(S_0) = V(D_0) = 0.
\]

(4.2)

In order to evaluate the effective potential, let us use the well-known expression

\[
Tr\ln(S_0^{-1}S) = 3 \times \ln det(S_0^{-1}S) = 3 \times 2\ln p^2 \left[ p^2 A^2(-p^2) - B^2(-p^2) \right],
\]

(4.3)

where

\[
p^2 A^2(-p^2) - B^2(-p^2) = \sqrt{det[-iS(p)]}.
\]

(4.4)
The factor 3 comes from the trace over quark colour indices. Going over to Euclidean space \((d^4p \rightarrow id^n p, \quad p^2 \rightarrow -p^2)\) and dimensionless variables (3.28), we finally obtain, after some algebra, \((n = 4)\)

\[
V(S) = \frac{6\mu^4\pi^2}{(2\pi)^4} \int_0^{t_0} dt \, t \{ \ln t \left[ tA^2(t) + B^2(t_0, t) \right] + 2tA(t) + 2 \}, \tag{4.5}
\]

where we introduced the UV cutoff \((\Lambda)\) which should be identified with the arbitrary constant of integration \(t_0\), as it was discussed in the previous section [48]. The explicit expressions for \(A(t)\) and \(B^2(t_0, t)\) are given by (3.32) and (3.33) respectively.

In a rather similar way, for the gluon part one has

\[
Tr \ln(D^{-1}_0 D) = 8 \times \ln \det(D^{-1}_0 D) = 8 \times 4 \ln \left[ \frac{3}{4} d(-p^2) + \frac{1}{4} \right], \tag{4.6}
\]

where the factor 8 is due to the trace over the gluon colour indices. After doing some similar algebra and on account of (3.7), one obtains in Euclidean space

\[
V(D) = -\frac{16\mu^4\pi^2}{(2\pi)^4} \int_0^{t_0} dt \, t \{ \ln(\frac{3\lambda}{t} + 1) - \frac{3\lambda}{4t} + b \}, \tag{4.7}
\]

where \(b = \frac{3}{4} - 2 \ln 2\). In derivation of (4.7) we extracted the coupling constant \(\lambda\) from \(d(-p^2)\) in (3.7).

Because of the normalization of the effective potential (4.2), the vacuum energy density now should be defined as follows

\[
\epsilon = V(S_0, D_0) - V(S, D) = -V(S, D). \tag{4.8}
\]

This means that the perturbative vacuum is normalized to zero because of (4.2) and hence

\[
\epsilon = \epsilon_q + \epsilon_g \tag{4.9}
\]

with

\[
\epsilon_q = -V(S), \quad \epsilon_g = -V(D), \tag{4.10}
\]

where \(V(S)\) and \(V(D)\) are given by (4.5) and (4.7), respectively.
For the numerical calculation of the vacuum energy density (4.9), along with other chiral QCD quantities below, it is convenient to define some new variable and parameters by the relations
\[
\frac{2}{\lambda} t = z, \quad \frac{2}{\lambda} t_0 = z_0, \quad t_0 = \frac{k_0^2}{\mu^2}, \tag{4.11}
\]
in terms of which we recast (4.5), on account of (4.10), as
\[
\epsilon_q = -\frac{3}{8\pi^2} k_0^4 z_0^{-2} \int_0^{z_0} dz' z' \left\{ \ln z' \left[ zg^2(z') + B^2(z_0, z') \right] - 2zg(z) + 2 \right\}, \tag{4.12}
\]
where
\[
g(z) = z^{-2} [\exp(-z) - 1 + z] \tag{4.13}
\]
and
\[
B^2(z_0, z) = 3 \exp(-2z) \int_z^{z_0} \exp(2z') g^2(z') \, dz'. \tag{4.14}
\]
so that
\[
\frac{\lambda}{2} A(t) = -g(z), \quad \frac{\lambda}{2} B^2(t_0, t) = B^2(z_0, z). \tag{4.15}
\]
This expression will be used for the numerical calculation of the vacuum energy density contribution of the confining quarks with dynamically generated masses. It is instructive to write down the initial system of equations (3.29-3.30) in terms of the new variables and parameters defined in (4.11). In this case it reads as
\[
g'(z) = -\left[ \frac{2}{z} + 1 \right] g(z) + \frac{1}{z} + \tilde{m}_0 B(z), \tag{4.16}
\]
\[
B'(z) = -\frac{3}{2} g^2(z) B^{-1}(z) - [\tilde{m}_0 g(z) + B(z)], \tag{4.17}
\]
where \( \tilde{m}_0 = m_0(\frac{2}{\lambda})^{1/2} \). It is easy to check that (4.13) and (4.14) are solutions to this system in the chiral limit \( \tilde{m}_0 = 0 \).

The vacuum energy density of the nonperturbative gluon contributions (4.7), on account of (4.10), in the same variables, is
\[ \epsilon_g = \frac{1}{\pi^2} k_0^4 z_0^{-2} \times I_g(z_0) \]  

(4.18)

where

\[ I_g(z_0) = \int_0^{z_0} dz z \left\{ \ln(1 + \frac{6}{z}) - \frac{3}{2z} + b \right\} \]

\[ = \frac{1}{2} z_0^2 \ln(1 + \frac{6}{z_0}) - 18 \ln(1 + \frac{z_0}{6}) + \frac{3}{2} z_0 + \frac{1}{2} b z_0^2. \]  

(4.19)

Here one important remark is in place. The vacuum energy density, \( \epsilon_g \), does not vanish at \( z_0 \to \infty \) as it should because of (4.2). Thus it needs an additional regularization in this limit. From (4.19) it follows that the term containing the constant \( b \) should be subtracted from this expression. So the regularized vacuum energy density should be calculated with the relation (4.18) which now becomes

\[ \epsilon_g = -\frac{1}{\pi^2} k_0^4 z_0^{-2} \times \left[ 18 \ln(1 + \frac{z_0}{6}) - \frac{1}{2} z_0^2 \ln(1 + \frac{6}{z_0}) - \frac{3}{2} \ln(z_0) \right]. \]  

(4.20)

The vacuum energy density (4.12) automatically disappears at \( z_0 \to \infty \), so it does not require an additional regularization, neither do the other chiral QCD parameters (see below).

The vacuum energy density is important in its own right as the main characteristics of the nonperturbative vacuum of QCD. Futhermore it assists in estimating such an important phenomenogical parameter as the gluon condensate, introduced within the QCD sum rules approach to resonance physics [49]. Indeed, because of the Lorentz invariance,

\[ \langle 0 | \Theta_{\mu \nu} | 0 \rangle = 4 \epsilon \]  

(4.21)

holds where \( \Theta_{\mu \nu} \) is the trace of the energy-momentum tensor and \( \epsilon \) is the sum of all possible contributions to the vacuum energy density, in particular is the sum of \( \epsilon_q \) and \( \epsilon_g \) at least. According to QCD this is equal to

\[ \Theta_{\mu \nu} = \frac{\beta(\alpha_s)}{4 \alpha_s} G_{\mu \nu}^a G_{\mu \nu}^a \]  

(4.22)

in the chiral limit (\( G_{\mu \nu}^a \) being the gluon field strength tensor) [50]. The CS-GML function \( \beta(\alpha_s) \), up to terms of order \( \alpha_s^3 \), is:
\begin{equation}
\beta(\alpha_s) = -\frac{9\alpha_s^2}{2\pi} + O(\alpha_s^3). \tag{4.23}
\end{equation}

Sandwiching (4.22) between vacuum states and on account of (4.23), one obtains

\begin{equation}
\langle 0 | \Theta_{\mu\nu} | 0 \rangle = -\frac{9}{8} \langle 0 | \frac{\alpha_s}{\pi} G^{a}_{\mu\nu} G^{a}_{\mu\nu} | 0 \rangle \tag{4.24}
\end{equation}

where \( \langle 0 | \frac{\alpha_s}{\pi} G^{a}_{\mu\nu} G^{a}_{\mu\nu} | 0 \rangle \) is nothing but the gluon condensate [49]. From (4.21) it follows

\begin{equation}
\langle 0 | \frac{\alpha_s}{\pi} G^{a}_{\mu\nu} G^{a}_{\mu\nu} | 0 \rangle = -\frac{32}{9} \epsilon \tag{4.25}
\end{equation}

The weakness of this derivation is, of course, relation (4.23) which holds only in the perturbation theory. Unfortunately, a calculation of the CS-GML function \( \beta(\alpha_s) \) beyond the perturbation theory is not yet known. In any case, it would be enlightening to estimate the gluon condensate with the help of (4.25) and on account of (4.9), which is the sum of the contribution from the confining quarks with dynamically generated masses (4.12) and the nonperturbative gluons contribution (4.20). This can also be expressed as a function of the dynamically generated quark mass (see (5.16) below). To this end, all is needed is to replace the combination \( k^4_0 z^{-2}_0 \) in (4.12) and (4.20) by the combination \( m^4_d B^4(z_0,0) \).

\section{V. THE BASIC CHIRAL QCD PARAMETERS}

Because of its especially small mass, the pion is the most striking example of the Goldstone realization of the chiral symmetry \( SU(2)_L \times SU(2)_R \). The pion decay constant \( F_\pi \) is an important constant in Nature. On one hand it determines the rate of the pion semileptonic decays \( \pi^\pm \rightarrow e^\pm \nu_e, \mu^\pm \nu_\mu \), on the other hand, beside the quark condensate and the dynamically generated quark mass, it is one of the three most important chiral QCD parameters that determine the scale of chiral dynamics. All other parameters in chiral QCD can be expressed in terms of these independent basic parameters.
A. The pion decay constant in the current algebra representation

In order to show explicitly how the CHPTq works, and for further aims, let us apply the CHPTq directly to the axial-vector matrix element which determines such an important physical parameter as the pion decay constant. As it is well known, the pion decay constant $F_\pi$ is defined in the current algebra (CA) as

$$\langle 0|J_{5\mu}(0)|\pi^i(q)\rangle = iF_\pi q_\mu \delta^{ij}. \quad (5.1)$$

(Here the normalization $F_\pi = 92.4 MeV$ is used [51]). Clearly, this matrix element can be written in terms of the pion- quark-antiquark proper vertex and quark propagators as

$$iF_\pi q_\mu \delta^{ij} = \int \frac{d^4p}{(2\pi)^4} \text{Tr}\left\{\left(\frac{\lambda^i}{2}\right)\gamma_5\gamma_\mu S(p+q)G^{ij}_5(p+q,p)S(p)\right\}. \quad (5.2)$$

The trace is understood over the Dirac and colour indices. To get an expression for $F_\pi$ one has to differentiate Eq.(5.2) with respect to the external momentum $q_\nu$ and then set $q = 0$ (Fig. 2). Let us denote the first term in Fig. 2 by $D^{ij}_\mu\nu(a)$ and the second one by $D^{ij}_\mu\nu(b)$, respectively. Thus, in general, one has

$$iF_\pi = D(a) + D(b) = \delta^{ij}g_{\mu\nu}[D^{ij}_\mu\nu(a) + D^{ij}_\mu\nu(b)], \quad (5.3)$$

By taking into account the BS pion wave function, up to terms of order $q$ given by (2.18-2.19) and (2.2) with the substitution $p \to p + q$ and also using the Taylor expansions (2.15-2.16), this expression can be easily evaluated. Now it is clear that in order to investigate analytically, and calculate the pion decay constant (and many other important physical parameters) numerically, our parametrization of the full quark propagator (2.2) is technically much more convenient than (2.3) is. This parametrization is also preferable for investigations of the confinement properties of the quark propagator [12, 13, 16]. Going over to the Euclidean space ($d^4p \to id^4p$, $p^2 \to -p^2$) and using dimensional variables (3.28), one finally obtains that the sum of two diagrams (5.3) should read as follows

$$F^2_{CA} = \frac{12\pi^2}{(2\pi)^4}\mu^2 \int_0^\infty \frac{dt}{t} \{-\overline{B}(t)[AB + \frac{1}{2}t(A'B - AB')] - \frac{3}{4}tARB_{11}(t) + \frac{1}{4}R_6(t)(E - 3B^2)\}, \quad (5.4)$$
The first line of this equation describes the contribution coming from the first diagram in Fig. 2, while the second line is the contribution from the second diagram. The contribution from the second diagram completely disappears if one uses the pion BS bound-state amplitude, restored from the axial-vector WT identity in a self-consistent way, as a residue at pole (2.22-2.26). It is the contribution of the first diagram in Fig. 2 that leads to the well-known expression of Pagels-Stokar-Cornwall (PSC) for the pion decay constant in the chiral QCD [5, 8, 23, 24] (see our paper [20]). Here and below the primes denote differentiation with respect to the dimensionless Euclidean momentum variable $t$, $A = A(t)$, $B = B(t)$ and quantities with overline are shown in (3.27) on account of (3.28). We denote the pion decay constant $F_\pi$, in the chiral limit of the CA representation, by $F_{CA}$. This is a general CHPTq expression for the pion decay constant in the CA representation, obtained from the BS bound-state amplitude, which is restored from the corresponding axial-vector WT identity in a self-consistent way, up to terms of order $q$.

The main problem now is to find a good nonperturbative ansatz (mentioned above) for the arbitrary form factors $R_j(t)(j = 6, 11)$ in the IR region. In nonperturbative calculations these terms cannot be ignored by saying formally they are of order $g^2$, as it was done in the perturbative treatments [5, 8, 23, 24]. In connection with this, let us point out that the difference between the vector and axial-vector currents disappears in the chiral limit. For this reason let us assume [46] that the IR finite quark-gluon vector vertex function at zero momentum transfer (3.25-3.26) is a good approximation to the regular piece of the axial-vector vertex at zero momentum transfer in the chiral limit (2.20-2.21), which in Euclidean space becomes

\begin{align}
G_1(p^2) &= -\overline{A}(p^2) - R_6(p^2) \\
G_2(p^2) &= 2\overline{B}(p^2) \\
G_3(p^2) &= 2\overline{A}(p^2) \\
G_4(p^2) &= -R_{11}(p^2).
\end{align}

(5.5)

A fortunate feature that admits to explore a partial analogy between the vector and axial-
vector currents in the chiral limit for the flavor non-singlet channel is that the contribution to the pion decay constant in the CA representation (5.4), coming from the second diagram in Fig. 2, does not depend on the form factor $G_2(p^2)$ at all. In this case the analogy between (3.26) and (5.5) becomes complete and one obtains

$$R_6(p^2) = 0, \quad R_{11}(p^2) = -\frac{A^2(p^2)B^{-1}(p^2)}{E(p^2)}.$$  (5.6)

Certainly we cannot prove these relations, though it will be shown later (Section 6) that this dynamical assumption (nonperturbative ansatz) numerically works very well and it leads to a self-consistent calculation scheme for all chiral QCD parameters thereby, justifying it once again.

Using the quark SD equation of motions (3.29-3.30) for the ground solution, taking (5.6) into account, after some algebra one finally arrives at the following result for the pion decay constant in the CA representation

$$F_{CA}^2 = \frac{12\pi^2}{(2\pi)^4} \mu^2 \int_0^{t_0} dt \frac{tB^2(t_0, t)}{\{tA^2(t) + B^2(t_0, t)\}},$$  (5.7)

where we introduced a cut-off which should be identified with the arbitrary constant of integration $t_0$, as it was discussed in Section 3.

In terms of new parameters and variable (4.11), we recast (5.7) as follows

$$F_{CA}^2 = \frac{3}{8\pi^2} m_d^2 k_0 z_0^{-1} \int_0^{z_0} dz \frac{zB^2(z_0, z)}{\{zg^2(z) + B^2(z_0, z)\}}.$$  (5.8)

The expression (5.8) will be used for the numerical calculation of the pion decay constant in the CA representation. This can also be expressed as a function of the dynamically generated quark mass $m_d$ (see the next section) and the dimensionless parameter $z_0$, i.e.

$$F_{CA}^2 = \frac{3}{8\pi^2} m_d^2 B^2(z_0, 0) \int_0^{z_0} dz \frac{zB^2(z_0, z)}{\{zg^2(z) + B^2(z_0, z)\}}.$$  (5.9)

where $B^2(z_0, 0)$ is given by (4.14) at $z = 0$. This expression clearly shows that a nonzero pion decay constant in the chiral limit is possible only if dynamical quarks exist depending, in turn, on the formation of quark condensates (see below).
Let us note that in QCD there exists an exact representation of the pion decay constant $F_\pi$, derived by Jackiw and Johnson (JJ) in Ref. 4 (see also Refs. 8, 20, 23, 24). In comparison with the CA representation their representation is free from overlapping divergences but it requires an explicit investigation of the corresponding BS scattering kernel. A numerical investigation of the chiral QCD in the JJ representation will be performed elsewhere.

**B. The dynamically generated quark mass**

It is worthwhile to discuss the relation between the effective quark mass $m_{\text{eff}}$ and the dynamically generated quark mass $m_d$ in our model. Let us define $m_{\text{eff}}$ as the inverse of the full quark propagator (2.3) at the zero point \[9, 16, 20\]

\[m_{\text{eff}} = [iS(0)]^{-1} = \overline{B}(0). \quad (5.10)\]

Let us define the dynamically generated quark mass $m_d$ in a similar way, namely

\[m_d = [iS_{\text{ch}}(0)]^{-1} = \overline{B}_0(0), \quad (5.11)\]

where $S_{\text{ch}}(0)$ denotes the full quark propagator in the chiral limit $m_0 = 0$. Evidently, here $\overline{B}(0)$ is the constant of integration of the full quark SD equation and $\overline{B}_0(0)$ is the same constant in the chiral limit.

From the quark SD equation

\[[iS(p)]^{-1} = [iS_0(p)]^{-1} + \Sigma(p), \quad (5.12)\]

it follows that

\[m_{\text{eff}} = m_0 + m_d. \quad (5.13)\]

In the chiral limit ($m_0 = 0$), $m_{\text{eff}} = m_d$ and thus $m_d$ arises only because of the interaction between quarks and gluons (quark mass generation). So, by our definitions, an effective chiral quark means simply a dynamical quark.
These definitions allow to exploit an analogy with the free quark propagator at zero point. They are also consistent with the renormalization group analysis in the IR region [1, 16]. These definitions, on one hand, generalize the constituent quark model propagator

\[ S(p) = \frac{i}{\not{p} - m_q}, \tag{5.14} \]

where \( m_q \) denotes constituent quark mass. On the other hand, they are consistent with the solutions to the quark SD equation which have no poles (confinement-type solutions), reconstructed on the basis of the ZME effect due to the nonperturbative IR divergences, leading to the both quark confinement and DCSB phenomena in QCD (see preceding section, our paper [16] and references therein). In connection with (5.14) it makes sense to note in advance that from our numerical results (Section 6), it follows that \( m_q \) differs little from \( m_d \), i.e. \( m_q = m_d + \Delta \) in the chiral limit. We are able also to propose an explanation of the dynamical origin of this \( \Delta \) on account of the ZME effect in bound-state problems (see Section 8).

Here a few remarks are in order. In gauge theories to extract gauge-invariant physical information is not a straightforward procedure. For example, in QED the electron Green’s function is an explicitly gauge-dependent quantity but the position of the pole is gauge-invariant. In QCD this fails because of the quark confinement phenomenon which, as it was mentioned above, is understood as the disappearance of the pole singularities in the quark Green’s function. Obviously definitions (5.10) and (5.11) assume regularity also at the zero point. Though the effective quark mass \( m_{eff} \) is not an experimentally observable quantity, it is desirable to find such kind of solutions to the quark SD equations in which the explicit dependence on a gauge-fixing parameter disappears. In this sense \( m_{eff} \) and \( m_d \), defined by (5.10) and (5.11) respectively, become gauge-invariant. As it was discussed in Section 3, such a nonperturbative quark propagator has been found within our approach to QCD at large distances [12, 13, 16].

As we mentioned in Section 2, the \( \gamma_5 \) invariance of the quark propagator is broken, as shown by relation (2.5), and the measure of this breakdown is the double of the dynamically
generated quark mass function $2\mathcal{B}(-p^2)$. This quantity at zero, $2\mathcal{B}(0)$, can be defined as a scale of DCSB at the fundamental quark level [20]. In accordance with (5.11), let us denote it as

$$\Lambda_{CSBq} = 2\mathcal{B}(0) = 2m_d,$$

(5.15)

while recalling that in the chiral limit $m_{ef} = m_d$. The definitions $m_{ef}$, $m_d$ and $\Lambda_{CSBq}$ have now direct physical sense within the above mentioned solutions to the quark SD equation.

Let us write down also the final result for the dynamically generated nonperturbative quark mass (5.11), expressed in terms of new parameters and variable (4.11) within the ground solution (3.33)

$$m_d = k_0\{z_0B^2(z_0,0)\}^{-1/2},$$

(5.16)

where $B^2(z_0,0)$ is given by (4.14) at the zero point. It should be clear now that the dynamically generated quark mass is a constant of integration of the corresponding SD equation and is, at the same time, one of the two independent parameters of our treatment (see Section 6). Let us also express the dimensionful parameter $k_0$ as function of $m_d$ and $z_0$:

$$k_0 = m_d\{z_0B^2(z_0,0)\}^{1/2},$$

(5.17)

emphasizing that precisely this parameter determines a scale at which nonperturbative effects take the leading role.

C. The quark condensate

As it is well known, the order parameter of DCSB - quark condensate can also be expressed in terms of the quark propagator scalar function $B(-p^2)$ (2.2). The definition is

$$\langle \overline{q}q \rangle = \langle 0|\overline{q}q|0 \rangle = \int \frac{d^4p}{(2\pi)^4} Tr S(p).$$

(5.18)

Let us write down the final result expressed in the new variables (4.11) as follows
\[ \langle \overline{q}q \rangle_0 = -\frac{3}{4\pi^2} k_0^3 z_0^{-3/2} \int_0^{z_0} dz \, z B(z_0, z), \quad (5.19) \]

and as a function of \( m_d \) as

\[ \langle \overline{q}q \rangle_0 = -\frac{3}{4\pi^2} m_d^3 B^3(z_0, 0) \int_0^{z_0} dz \, z B(z_0, z), \quad (5.20) \]

where for light quarks in the chiral limit

\[ \langle \overline{q}q \rangle_0 \equiv \langle 0 | \overline{u}u | 0 \rangle_0 = \langle 0 | \overline{d}d | 0 \rangle_0 = \langle 0 | \overline{s}s | 0 \rangle_0 \quad (5.21) \]

by definition. Eq. (5.20) clearly shows that in our model the existence of the dynamically generated quark mass, which is a consequence of the enhancement of the zero modes, should be certainly accompanied by the formation of quark condensates and vice versa. Quark condensates lead to the creation of the effective (dynamical) quark mass.

There are also two chiral QCD parameters closely related to the quark condensate. Let us consider the coupling constant of the pseudoscalar density to the pion which is defined as

\[ \langle 0 | \overline{q}i\gamma_5 \tau_i q | \pi^j(0) \rangle = G_\pi \delta^{ij}. \quad (5.22) \]

Similarly to the pion decay constant (5.2), the analytical expression for this matrix element is

\[ G_\pi \delta^{ij} = \int \frac{d^4p}{(2\pi)^4} Tr \{ i\gamma_5 \left( \frac{\tau_i}{2} \right) S(p) G_3^j(p, p) S(p) \}, \quad (5.23) \]

where the trace is understood over Dirac and color indices. Taking into consideration (2.2) for the quark propagators, (2.18-2.19) for the pion wave function \( G_3^j(p, p) \) at zero momentum transfer, one can easily evaluate this expression by substitution \( \lambda^j \rightarrow \tau^j \), as

\[ G_\pi \times F_\pi = 12i \int \frac{d^4p}{(2\pi)^4} B(-p^2) = -\int \frac{d^4p}{(2\pi)^4} Tr S(p). \quad (5.24) \]

Comparing this with the definition of the quark condensate (5.18), one obtains the well-known current algebra result

\[ G_\pi \times F_\pi = -\langle \overline{q}q \rangle \quad (5.25) \]
and in the chiral limit, we have

\[ G \times F = -2 \langle \bar{q}q \rangle_0, \quad (5.26) \]

where we denote \( G_\pi \) in the chiral limit by \( G \), similarly like it was done above with \( F \). This formula will be used for the numerical calculation of this constant.

In the chiral perturbation theory at the hadronic level (CHPT, or, equivalently, the effective field theory) \([52, 53]\) there is a second low energy constant \( B \), determined by

\[ \langle \bar{q}q \rangle_0 = -F^2 B. \quad (5.27) \]

This measures the vacuum expectation value of the scalar densities in the chiral limit. It is just this constant that determines the meson mass expansion in the general case. Indeed, in leading order (in powers of quark masses and \( e^2 \)) from the CHPT, one obtains \([52, 53]\)

\begin{align*}
M_{\pi}^2 &= (m_u^0 + m_d^0)B \quad (5.28) \\
M_{K^+}^2 &= (m_u^0 + m_s^0)B \quad (5.29) \\
M_{K^0}^2 &= (m_d^0 + m_s^0)B. \quad (5.30)
\end{align*}

Calculating independently the constant \( B \) from (5.27), one then will be able to estimate the current quark masses \( m_u^0, m_d^0 \) and \( m_s^0 \) by using the experimental values of meson masses \([54]\) in (5.28-5.30).

**D. The Goldberger-Treiman relations at the quark and hadronic levels**

The famous Goldberger-Treiman (GT) relation (identity) at the fundamental quark level reads as follows

\[ F_\pi \mathcal{G}(p, p) = 2B(-p^2), \quad (5.31) \]

where \( \mathcal{G}(p, p) \) denotes the left-hand-side (lhs) of (2.18) in the limit \( q = 0 \). We omitted the minus sign which appears because of the presence of \( \gamma_5 \gamma_\mu \) instead of the standard \( \gamma_\mu \gamma_5 \). This
is an unrenormalized (for a unit axial-vector coupling constant of the constituent quark, i.e. $g_A = 1$) GT relation at the quark level [7-9, 24] and it is exact in the chiral limit.

On account of (5.11) at zero momentum, one obtains, by simply identifying $G(0,0)$ with $2g_{\pi qq}^o$, as well as by denoting here, for the sake of convenience, the pion decay constant in the chiral limit as $F_\pi^o$, instead of previous $F$, the GT relation (5.31) at the quark level as

$$F_\pi^o g_{\pi qq}^o = m_d,$$

(5.32)

where $g_{\pi qq}^o$ denotes pion-quark coupling constant in the chiral limit. This is a standard form of the unrenormalized ($g_A = 1$) GT relation at the quark level. In order to recover the renormalized GT identity, it is necessary only to multiply the rhs of relation (5.32) by $g_A^o$, which is the above mentioned corresponding axial-vector coupling constant of the constituent quark in the chiral limit, and replace $m_d$ by $m_q$ i.e.,

$$F_\pi^o g_{\pi qq}^o = m_q g_A^o,$$

(5.33)

where we retained the same notations for the renormalized quantities. This is a renormalized GT relation at the fundamental quark level and it becomes exact in the chiral limit [55]. For the numerical calculation of $g_{\pi qq}^o$ this relation (5.33) will be used, even though it requires knowledge of $g_A^o$ which remains still unknown in our calculation scheme at this stage. However, let us put this problem aside until the end of this subsection.

The standard GT identity [56, 57] for pion-nucleon physics reads as

$$F_\pi g_{\pi NN} = M_N G_A.$$

(5.34)

Experimental values of the pion-nucleon coupling constant $g_{\pi NN}$ and the nucleon axial-vector coupling $G_A$, as well as of the pion decay constant and nucleon mass, are well known [51, 54, 57], namely

$$g_{\pi NN} = 13.04 \pm 0.06, \quad G_A = 1.257 \pm 0.003$$

(5.35)

$$F_\pi = 92.4189 \ MeV, \quad M_N = \frac{1}{2}(M_p + M_n) = 938.92 \ MeV$$

(5.36)
Note that just these values, denoted as $g_{\pi p n}$ and $\sqrt{2}f_\pi = 2F_\pi$, are quoted in Ref.57. From these experimental values it follows that (in units of MeV)

$$F_\pi g_{\pi NN} = 1205.14 > M_N G_A = 1180.22 \quad (5.37)$$

Following the paper of Marciano and Sirlin [57], let us consider the discrepancy between the rhs and lhs of the GT relation (5.34) as

$$\Delta_\pi = -1 + \frac{F_\pi g_{\pi NN}}{M_N G_A} = 0.02111, \quad (5.38)$$

where we used numerical values from (5.35-5.36). So, in the non-chiral case, the GT relation is not exact and it is expected to hold up to (1-2)% accuracy [57].

In the chiral limit the GT relation becomes exact. This means that the quantities entering the GT relation (5.34) slightly change their numerical values in the chiral limit. Denoting, in addition, the pion-nucleon coupling constant in the chiral limit as $g_{\pi NN}^o$, as well as nucleon mass as $M_N^o$, one obtains

$$F_\pi^o g_{\pi NN}^o = M_N^o G_A^o. \quad (5.39)$$

This is the exact GT relation at the hadronic level (for pion-nucleon physics) in the chiral limit. The above mentioned discrepancy (5.38) should vanish in this case. In agreement with the statement that the numerical values of the physical quantities (those considered in this paper) in the chiral limit should not exceed their experimental values, and on account of (5.37), it follows that the deviation of the lhs in the GT relation (5.34) is larger than the deviation of the rhs from their experimental values.

Unfortunately, at this stage, we are not able to independently calculate the pure nucleonic degrees of freedom in the chiral limit, in particular $M_N^o$ and $G_A^o$. Though we might have taken these values from their lattice calculation reported in Refs. 58 but we are not going to proceed this way because that would undesirably complicate our calculation scheme and would not give new information either. Note that some serious problems with the chiral limit itself ("hard" chiral logarithms), recently discovered in quenched lattice calculations
[59], prompted us not to use these results. For the sake of transparency of our calculation scheme, it completely suffices to use the chiral value of the pion decay constant as obtained by the CHPT in Ref. 60, namely

$$F^o_\pi = (88.3 \pm 1.1) \text{ MeV.} \quad (5.40)$$

This value is chosen as unique input data in our numerical investigation of chiral QCD. The pion decay constant is a good experimental number since it is a directly measurable quantity in contrast, for example, to the quark condensate or the above mentioned pion-quark coupling constant. For this reason our choice (5.40) as input data opens up the possibility of reliably estimating the deviation of the chiral values from their "experimental" phenomenologically determined values of various physical quantities which can not be directly measured. Thus to assign definite values to the physical quantities in the chiral limit is a rather delicate question. At the same time it is a very important theoretical limit which determines the dynamical structure of low-energy QCD. Note that from now on we will use only the central value of the pion decay constant in the chiral limit as given by (5.40).

Let us now return to the GT relation (5.33). In QCM [32, 61] the following relation holds true on account of (5.35).

$$g_A = 3 \times G_A = 0.7542. \quad (5.41)$$

This value is usually considered as the phenomenological ("experimental") value of this quantity (see, however, the second paper in Refs. 55). Let us assume that relation (5.41) is valid in the chiral limit too. Then from (5.33) and (5.39) one obtains

$$g^o_{\pi qq} = 3 \frac{m_q}{5 M_N^o} g^o_{\pi NN}. \quad (5.42)$$

Having approximated the chiral values of $M_N^o$ and $g^o_{\pi NN}$ with their experimental values (5.36) and (5.35) respectively, this expression yields the value for the pion-quark coupling constant.
VI. NUMERICAL RESULTS

Let us begin this section with the discussion of one of the most interesting features of DCSB. As it was explicitly shown above, there are only five independent quantities by means of which all other chiral QCD parameters can be calculated. For the sake of convenience, let us write down them together.

\[ F_{CA}^2 = \frac{3}{8\pi^2 k_0^2 z_0^{-1}} \int_0^{z_0} dz \frac{z B^2(z_0, z)}{\{z g^2(z) + B^2(z_0, z)\}} , \]  
\[ m_d = k_0 \{z_0 B^2(z_0, 0)\}^{-1/2} , \]  
\[ \langle \bar{q}q \rangle_0 = -\frac{3}{4\pi^2 k_0^3 z_0^{-3/2}} \int_0^{z_0} dz \ z B(z_0, z) , \]  
\[ \epsilon_q = -\frac{3}{8\pi^2 k_0^4 z_0^{-2}} \int_0^{z_0} dz \ z \ \ln \left[ z g^2(z) + B^2(z_0, z) \right] - 2 z g(z) + 2 \]  
\[ \epsilon_g = -\frac{1}{\pi^2 k_0^4 z_0^{-2}} \times \left[ 18 \ln(1 + \frac{z_0}{6}) - \frac{1}{2} z_0^2 \ln(1 + \frac{6}{z_0}) - \frac{3}{2} z_0 \right] . \]

recall that \( B^2(z_0, z) \) and \( g(z) \) are given by (4.14) and (4.13), respectively. The definition (5.15) for the DCSB scale is

\[ \Lambda_{CSBq} = 2 m_d . \]

These final expressions are going to be used to calculate the chiral QCD parameters. They depend only on two independent quantities, namely the mass scale parameter \( k_0 \) and the constant of integration of dynamical quark SD equation of motion \( z_0 \). However, from (6.2) we get that information on the parameter \( z_0 \) must be extracted from \( m_d \) and the initial mass scale parameter \( k_0 \), which characterizes the region where confinement, DCSB and other nonperturbative effects are dominant. The second independent parameter of the ground solution \( z_0 \) is given in terms of \( k_0 \) and \( m_d \). Despite the fact that in our consideration the initial mass scale parameter \( \mu \) (characterizing the scale of nonperturbative effects) has
been introduced "by hand", such a transformation of pair of independent parameters \( k_0 \) and \( z_0 \) into the pair of \( k_0 \) and \( m_d \) is also a direct manifestation of the phenomenon of the "dimensional transmutation" [62]. This phenomenon occurs whenever a massless theory acquires masses dynamically. It is a general feature of spontaneous symmetry breaking in field theories.

Our calculation scheme is self-consistent because we calculate \( n = 5 \) independent physical quantities by means of \( m = 2 \) free parameters, which possess clear physical sense. So condition of self-consistency \( n > m \) is satisfied. The general behaviour of all our parameters, given by relations (6.1-6.5), are shown in Figs. 3-7. In our model one may calculate all the chiral QCD parameters (not only those considered here but others as well!) at any requested combination of \( m_d \) and \( k_0 \). However, in order to analyse our numerical results, it is necessary to set a scale at which it should be done. We set a scale by two, at first sight different, ways but these lead to almost the same numerical results.

A. Analysis of the numerical data at a scale of DCSB at the quark level

There is a natural scale in our approach to DCSB. At the fundamental quark level the chiral symmetry is spontaneously broken at a scale \( \Lambda_{CSBq} \) defined by (6.6). We may then analyse our numerical data at a scale at which DCSB at the fundamental quark level occurs. For this aim, what is needed is only to simply identify mass scale parameter \( k_0 \) with this scale \( \Lambda_{CSBq} \), i.e. to put

\[
k_0 \equiv \Lambda_{CSBq} = 2m_d. \tag{6.7}
\]

Now one can uniquely determine the constant of integration of the quark SD equation. Indeed, from (6.2) and on account of (6.7), then it immediately follows that this constant is equal to

\[
z_0 = 1.34805. \tag{6.8}
\]
From the pion decay constant in the chiral limit (5.40), chosen as input data, and on account of (6.8) and (6.7), from (6.1) one obtains a numerical value for $k_0$ (see Table 1). This means that all physical parameters considered in our paper are uniquely determined. The results of our calculations are displayed in Table 1.

It is plausible that our approach makes it possible to intrinsically compare numerical results of different approaches with each other. For example, of the CHPT [52, 53, 60] with those of QCD sum rules [49, 63] and the CQM [32, 61, 64] as well as lattice approach [58] and vice versa. In the most simplest way this can be done by setting a scale based on definition (6.7) (calculation scheme A). To achieve this goal one should chose input data from the corresponding approach and then proceed as it was described above. We do not present these calculations. It will be instructive to display our numerical results when the chiral value of the pion decay constant is approximated by its experimental value, as advocated in Refs. 51 and 57, namely

$$F^0_\pi = 92.42 \text{ MeV},$$  \hfill (6.9)

as well as by the standard value

$$F^0_\pi = 93.3 \text{ MeV}.$$  \hfill (6.10)

In accordance with the above mentioned, this allows for estimating the deviation of the chiral values of the physical parameters, which can not be directly measured, from their phenomenologically estimated (“experimental”) values. For the above calculated parameters these results are also shown in Table 1.

The estimate of quark condensate in Refs. 49 and 63,

$$\langle \bar{q}q \rangle_0^{1/3} = -(225 \pm 25) \text{ MeV}$$  \hfill (6.11)

is in good agreement with our values. QCD sum rules give usually the numerical values of physical quantities, in particular the quark condensate, approximately, within an accuracy of (10-20)\% (see, for example Ref. 65).
Our values for the current quark masses are also in good agreement with recent estimates from hadron mass splittings [66]

\[ m_u^0 = (5.1 \pm 0.9) \text{ MeV}, \]
\[ m_d^0 = (9.0 \pm 1.6) \text{ MeV}, \]
\[ m_s^0 = (161 \pm 28) \text{ MeV} \] (6.12)

and QCD sum rules [67]

\[ m_u^0 = (5.6 \pm 1.1) \text{ MeV}, \]
\[ m_d^0 = (9.9 \pm 1.1) \text{ MeV}, \]
\[ m_s^0 = (199 \pm 33) \text{ MeV}, \] (6.13)

see also reviews [68]. Note that the agreement of our values (Table 1) with the QCD sum rules values (6.13) is slightly better than with those of (6.12) obtained from hadron mass splittings.

In order to obtain the numerical values of the pion-quark coupling constant (5.42), we approximated \( m_q \) in (5.42) by the numerical values of \( m_d \) (Table 1). Here it is worth noting in advance that from our model of quark confinement (see discussion in Section 8) it follows that \( m_q \) differs little from \( m_d \); so, making no big mistake, one can simply use \( m_d \) instead of \( m_q \) in (5.42). Moreover, doing so one arrives at the conclusion that the chiral perturbation theory with (5.40) and the CQM with the value for the constituent quark mass \( m_q = 362 \text{ MeV} \) advocated by Quigg [64] are nearly in one-to-one correspondence in our calculation scheme A (see Table 1).

The phenomenological analysis of QCD sum rules [49] for the numerical value of the gluon condensate implies

\[ \langle 0 | \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{\mu\nu}^a | 0 \rangle \simeq 0.012 \text{ GeV}^4, \] (6.14)

and using then again (4.25), one obtains the vacuum energy density

\[ \epsilon \simeq -0.003375 \text{ GeV}^4 \] (6.15)
with QCD sum rules [49]. In the random instanton liquid model (RILM) [69] of the QCD vacuum, for a dilute ensemble, one has

\[ \epsilon = -\frac{9}{4} \times 1.0 \, fm^{-4} \simeq -0.003411 \, GeV^4. \] (6.16)

The estimate, with QCD sum rules, of the gluon condensate can be changed within a factor of two [49]. We trust our numerical results for the vacuum energy density much more than those of the gluon condensate. The former estimate was obtained on the basis of the completely nonperturbative ZME model of the vacuum of QCD while the latter one was obtained on account of the perturbative solution for the CS-GML \( \beta \)-function (4.23). In order to reliably calculate gluon condensate from (4.22), it is necessary to calculate the CS-GML \( \beta \)-function in the nonperturbative ZME model of quark confinement and DCSB. This calculation is not straightforward but we undertake the task. Let us also emphasize the important fact that our calculation of the vacuum energy density is a calculation from first principles while in the RILM the parameters characterizing the vacuum, the instanton size \( \rho_0 = 1/3 \, fm \) and the "average separation" \( R = 1.0 \, fm \) were chosen to precisely reproduce traditional (phenomenologically estimated from QCD sum rules) values of quark and gluon condensates, respectively.

We reproduce values (6.14-6.16), which are due to the instanton-type fluctuations only, especially well when the pion decay constant in the chiral limit was approximated by its experimental value. Moreover, our numerical results clearly show that the contribution to the vacuum energy density by the confining quarks (with dynamically generated masses), \( \epsilon_q \), is approximately equal to \( \epsilon_g \), which comes from the nonperturbative gluons. It is well known that in the chiral limit (massless quarks) tunneling is totally suppressed, i.e. the contribution to the vacuum energy density of the instanton-type fluctuations vanishes. It will be restored again in the presence of DCSB [70, 71]. Thus, in general, the total vacuum energy density should be the sum of these three quantities (see discussion in Section 8).

To set a scale by the way described in this subsection has the advantage that it is based on the exact definition (6.7) for a scale at which analysis of the numerical data must be
done. This is a scale responsible for DCSB at the fundamental quark level. In general it is
not obvious at all that this scale $\Lambda_{CSBq}$ and the scale at which quark confinement occurs,
$\Lambda_c$, should be of the same order of magnitude. Moreover the information about $\Lambda_c$, at which
quark confinement occurs at the fundamental microscopic level, is hidden in this scheme of
calculation. In order to clear up the origin of $\Lambda_c$ and its relation to $\Lambda_{CSBq}$, let us set a scale
in the way described in the next subsection.

B. Analysis of the numerical data at the confinement scale

As noted above, there exists only one scale in our model, denoted first by $\mu$ and then by
$k_0$ (to distinguish between the nonperturbative phase and the perturbative one), which is
responsible for all nonperturbative effects in QCD at large distances. If there is a close con-
nection between quark confinement and DCSB (and we believe that this is so) then the scale
of DCSB at the fundamental quark level $\Lambda_{CSBq}$ (6.6) and the confinement scale $\Lambda_c$ should be,
at least, of the same order of magnitude. This is in agreement with Monte Carlo simulations
on the lattice which show that the deconfinement phase transition and the chiral symmetry
restoring phase transition occur approximately at the same critical temperature [72, 73],
confirming thereby the close intrinsic link between these nonperturbative phenomena.

Unfortunately, the exact value of neither $m_d$ or of $k_0$ is known. For this reason, let us
first reasonably assume that for the dynamically generated quark masses

$$300 \leq m_d \leq 400 \, (MeV)$$

(6.17)
is imposed but they otherwise remain arbitrary. We believe that this interval covers all
possible realistic values used for and obtained in various numerical calculations. The second
independent parameter $k_0$ should be varied in the region of 1 GeV - the characteristic scale of
low energy QCD. Varying now independently this pair of parameters $m_d$ and $k_0$ numerically,
one can calculate all chiral QCD parameters using the above derived formulae (6.1-6.5).

From the value of the pion decay constant in the chiral limit (5.40) as well as from
the range selected first for $m_d$ (6.17) and on account of (6.1) and (6.2), it follows that the momentum $k_0$ should always satisfy the upper and lower boundary value conditions, namely

$$691.32 \leq k_0 \leq 742.68 \ (MeV).$$

(6.18)

See Fig. 3. In the ranges determined by (6.17) and (6.18) the vacuum energy density of the nonperturbative gluons (6.5) changes its sign (Fig. 6) and becomes positive which is undesirable. It is easy to show that this is a result of that that the lower bound chosen for the dynamical generated quark mass in (6.17) is too low. Indeed, the vacuum energy density (6.5) vanishes at the critical point $z_0^{cr} = 1.45076$ (Fig. 8). Then from (6.2), calculated at this point, it follows that

$$k_0 \leq 2.26 m_d.$$  

(6.19)

Because of this inequality the vacuum energy density (6.5) will always be negative as it should be and only at the critical value

$$k_0^{cr} = 2.26 m_d$$

(6.20)

becomes zero (worst case). From the chosen interval for $m_d$ (6.17) and the obtained interval for $k_0$ (6.18) it follows that the ratio between the corresponding lower bounds $k_0/m_d = 691.32/300 = 2.3044$ does not satisfy the relation (6.19) while this ratio for the corresponding upper bounds $k_0/m_d = 742.68/400 = 1.8567$ does satisfy it. This explicitly shows that the lower bound for $m_d$ in (6.17) was incorrectly chosen. It is, as mentioned above, too low. The exact lower bound for $m_d$ can be found from the relation (6.20) as

$$742.68 = 2.26 m_d,$$

(6.21)

and (6.17) becomes

$$328.62 \leq m_d \leq 400 \ (MeV).$$

(6.22)

In the ranges determined by (6.18) and (6.22), the vacuum energy density (6.5) will be always negative because any combination (ratio) of $k_0$ and $m_d$ from these intervals will satisfy the
relation (6.19). However, this is not the whole story. A new lower bound for $m_d$ leads to a new lower bound for $k_0$ as well. Combining now this new lower bound (6.22) with the chiral value of the pion decay constant (5.40), one obtains a new lower bound for $k_0$ as well.

As we emphasized several times, $k_0$ should be treated as a momentum which separates the nonperturbative phase (region) from the perturbative one (region), so that in a new region obtained for $k_0$ the nonperturbative effects, such as quark confinement and DCSB, begin to play a dominant role. This is the region where confinement occurs. From now on, let us call this scale for $k_0$ a confinement scale (in the chiral limit) and denote it $\Lambda_c$. So the final numerical limits for the confinement scale are

$$707 \leq \Lambda_c \leq 742.68 \text{ (MeV)}.$$  \hspace{1cm} (6.23)

In the intervals determined by (6.22) and (6.23) the vacuum energy density due to the nonperturbative gluon contributions, $\epsilon_g$, will be always negative (see Fig. 9).

It is important that any value from the obtained interval (6.23) is allowed but not each combination of $\Lambda_c$ from interval (6.23) and $m_d$ from interval (6.22) will automatically yield the value of the pion decay constant given by (5.40). Therefore, it is necessary to adjust values of $m_d$ from (6.22) and $\Lambda_c$ from interval (6.23), and vice versa, as it is seen in Fig. 10. This means that $m_d$ is in a close relationship with $\Lambda_c$. Moreover, completing the above mentioned adjusting procedure, one finds that $\Lambda_c$ is nearly double the generated quark mass $m_d$, i.e.

$$\Lambda_c \approx 2m_d.$$  \hspace{1cm} (6.24)

This confirms that $\Lambda_c$ and $\Lambda_{CSBq}$, defined by (5.15) or (6.6), are nearly the same indeed. In the previous calculation scheme the adjusting procedure was automatically fulfilled because of relation (6.7). There is a close relationship between $\Lambda_{CSBq}$ and $\Lambda_c$ on one hand, and the double generated quark mass $m_d$, on the other.

Evidently, interval (6.23) for $\Lambda_c$, along with the new range for $m_d$ (6.22), uniquely determines the upper and lower bounds for all other chiral QCD parameters considered here.
like in the previous case, our numerical results are shown in Table 2, where the shorthand notation \( \langle 0|G^2|0 \rangle \) stands for the gluon condensate \( \langle 0|\frac{2\pi}{\alpha} G_{\mu\nu}^2 G_{\mu\nu}|0 \rangle \). Our numerical bounds for the vacuum energy density \( \epsilon \) need some additional remarks. We emphasize that bounds for \( \epsilon \) is not the sum of bounds for \( \epsilon_q \) and \( \epsilon_g \). Indeed, the upper and lower bounds for \( \epsilon_q \) are achieved at the upper and lower bounds for \( m_d (\Lambda_c) \) while for \( \epsilon_g \) they are achieved at the lower and upper bounds of \( m_d (\Lambda_c) \). At the same time, the intervals for \( \epsilon \) differ very little from each other in either case (see Table 2).

Let us now prove the relation (6.24). We have already learnt that correct values of \( k_0 \) belongs to the interval for \( \Lambda_c \) (6.23). Then identifying \( k_0 \) with \( \Lambda_c \) in (6.19), one obtains

\[
\Lambda_c \leq (2 + 0.26)m_d = \Lambda_{CSBq} + 0.26m_d,
\]

so

\[
\Delta = \pm(1 + \frac{\Lambda_c}{\Lambda_{CSBq}}) \leq 0.13,
\]

where the positive sign corresponds to \( \Lambda_c \geq \Lambda_{CSBq} \) and the negative one holds when \( \Lambda_c \leq \Lambda_{CSBq} \). In derivation of both these relations we used definition (6.6). The maximum deviation will be achieved only at the critical point, i.e. when the contribution to the vacuum energy density of the nonperturbative gluons vanishes. Of course, this is not the case and these two scales are very close indeed.

It is worth underlining once more that besides good numerical results obtained in this subsection (see Table 2), we have established the existence of a realistic lower bound for the dynamically generated quark masses. In each calculated case their numerical values are shown in Table 2. Thus one concludes that the vacuum energy density of the nonperturbative gluons is sensitive to the lower bound for \( m_d \). The second important result is that we have clearly shown that the confinement scale \( \Lambda_c \) and DCSB scale \( \Lambda_{CSBq} \) are nearly the same indeed.
VII. CONCLUSIONS

In the theoretical part of our paper we have seen that the ZME model of both the quark confinement and DCSB reveals several desirable and promising features. The quark propagator has indeed no poles (quark confinement theorem). A single quark (heavy or light) is already off mass-shell by virtue of the ZME effect in the ground state of QCD - true vacuum. Let us recall that a ”single” quark is understood by means of a propagation described by the solution to the quark SD equation. Our model also implies DCSB at the fundamental quark level. The system of equations (3.29-3.30) for the ground solution (3.32-3.33) of the quark propagator (2.2) forbids a chiral symmetry preserving solution \( m_0 = B(t) = 0, \ A(t) \neq 0 \). Moreover, a chiral symmetry violating solution \( (m_0 = 0, \ B(t) \neq 0, \ A(t) \neq 0) \) for the quark SD equation is required. Thus chiral symmetry is certainly dynamically (spontaneously) broken in QCD because of the ZME effect. The finite corrections, coming from the intermediate and UV regions, to the solution for the quark propagator, because of the much less singular behaviour in the IR, cannot destroy these features of the ground solution. The absence of the pole singularities in the quark propagator (quark confinement) and DCSB at the fundamental quark level are closely related. The intrinsic link between these two important phenomena is a direct consequence of the nonperturbative, IR singular structure of the QCD ground state - true vacuum. Thus the ZME effect alone provides the dynamical mechanism for both DCSB and quark confinement phenomena in QCD.

From our theoretical considerations, which were numerically confirmed in the second part of our paper, it follows that the formation of the dynamical quark, as well as the quark and gluon condensates, at a certain scale becomes inevitable if the zero modes are indeed enhanced in true vacuum of QCD. Dynamical quarks must be formed and this should be accompanied by the formation of quark and gluon condensates. The formation of quark and gluon condensates necessarily leads to the dynamical quarks. Moreover the existence of the dynamical quarks is the first necessary step for the pion to be a Goldstone state. Its decay constant, in the chiral limit, Eq. (5.9), depends on the mass of the dynamical quark. The
Goldstone nature of the pion was also independently confirmed by the investigation of the structure of the dynamical singularities of the corresponding axial-vector WT identity in the chiral limit (see Section 2 and our paper [19]).

Thus the dynamical quark, defined as the inverse of the quark propagator at zero point (5.11), is one of the key objects of our model [74]. Our numerical results clearly show that its definition in the chiral limit (5.11) is in fair agreement with the definition of a scale responsible for DCSB at the quark level (5.15) and consequently the confinement scale as well. One may conclude that the nonperturbative ansatz (5.6), which allows to take into account nonperturbative contributions to the pion decay constant coming from the second diagram in Fig. 2, works in the right way. The vacuum energy density calculated through the effective potential for composite operators, on one hand, numerically is in a self-consistent relation with all other chiral QCD parameters. On the other hand, it agrees very well with values provided by QCD sum rules and the RILM (see, however, the next section). Moreover we find that contributions to the vacuum energy density, coming from the confining quarks with dynamically generated masses, $\epsilon_q$, and the nonperturbative gluons, $\epsilon_q$, are nearly the same indeed.

As it was previously noted, the QCD coupling constant, in the massless quark case, plays no independent role. We remind the reader that the ZME effect leads to the strong regime for the effective (running) coupling constant (item III of the Section 3). For this reason, and because of the above mentioned dimensional transmutation phenomenon [62], analysis, based on the existence of a critical value of the strong coupling constant (at which DCSB and quark confinement occur), should be replaced by the analysis based on the existence of a certain scale at which these phenomena become essential. This was one of the highlights in our paper.

Let us emphasize independently that the numerical investigation of the chiral sector in QCD, in our calculation schemes, shows clearly that the scales of both nonperturbative phenomena (quark confinement and DCSB) are nearly the same, i.e.
\[ \Lambda_c \approx \Lambda_{CSBq} = 2m_d. \]  

The scale (7.1) has a clear physical meaning in our model. It determines a scale at which transition between nonperturbative and perturbative phases occurs. The definition (7.1) is relevant in the case of the light quarks only (\(u, d\) and \(s\)). Let us note also that in our paper [20], on the basis of naive counting arguments, it was explained why the scale \(\Lambda_{CSBq}\) should be approximately the half of the corresponding scale of DCSB at the hadronic level \(\Lambda_{\chi}\) - the scale of effective field theory (CHPTth) [28, 52, 53].

For reasonable ranges obtained for \(m_d\) (see Tables 1 and 2) and for the choice (6.7), one certainly comes to the self-consistent numerical picture of the nonperturbative dynamics in our general approach to QCD at large distances. Our model depends only on two independent (free) parameters which have a clear physical sense and therefore it provides a well-controlled calculation scheme (in various forms). In principle, we are able to numerically calculate integrals in both schemes to any requested accuracy. It is also worthwhile to compare the relative simplicity of our calculations with the indisputable complexity of the lattice ones. The proposed dynamical quark propagator approach complemented by the CHPTq can be applied with the same success for investigating analytically and computing numerically from first principles any other low energy physical quantities in a self-consistent way.

VIII. DISCUSSION

In the preceding two sections we have obtained and summarized our results which are direct (quantitative) consequences of the zero modes enhancement (ZME) effect in the framework of our general approach to low energy QCD. Until now we have not made any attempts to interpret the above mentioned effect from the viewpoint of dynamics. Let us make now a few detailed remarks of semi-intuitive, as well as semi-speculative, character which, will, nevertheless, shed some light on our understanding of the actual dynamical mechanism of quark confinement and DCSB might be interpreted with the help of the ZME effect. People
who do not like this way of thinking are recommended to skip this section. Still, we think, this line of thought illustrates the rich possibilities of this effect in QCD. Moreover, the section contains predictions for more realistic values of the vacuum energy density (the bag constant, apart from the sign) and the gluon condensate.

A. A possible dynamical picture of quark confinement and DCSB within the ZME effect in QCD

"Once upon a time in America", Susskind and Kogut [39] have noticed that "the absence of quarks and other colored objects could only be understood in terms of an IR divergences in the self-energy of a color bearing object". In our approach gluons remain massless, only zero modes are enhanced. We will discuss in more detail contributions to the self-energy of the colored quark leading first to the dynamical and then to constituent quarks in the context of the ZME which is caused by the nonperturbative IR divergences in the true vacuum of QCD. In other words, we will try to explain below that the ZME model of quark confinement and DCSB is a direct realization and development of Susskind's and Kogut's program.

In order to clarify the dynamical picture which may lie at the heart of our model, let us introduce, following Mandelstam [37], two sorts of gluons. The actual (external) gluons are those which are emitted by a quark and absorbed by another one, while the virtual (internal) gluons are those which are emitted and absorbed by the same quark. At first sight this separation seems to be a simple convention but we will show below that it has a firm dynamical ground, thus it makes our understanding of the above mentioned picture more transparent.

Let us consider now all the possible contributions to the self-energy of a single quark. The most simplest one is shown in Fig. 11. Let us recall that the same self-energy diagram occurs also in the quantum electrodynamics (QED). In contrast to QED, there is an infinite number of additional contributions to the self-energy of a single quark because of the non-
abelian nature of QCD, i.e. because of the direct interaction between virtual gluons which is absent in QED. Some of these are shown in Fig. 12. So, from the point of view of the contributions to the self-energy of a single quark, the zero modes are indeed enhanced in QCD in comparison with the electron self-energy in QED. The self-interaction of virtual gluons alone removes a single quark from the mass shell, making it an effective (dynamical in the chiral limit) object. This was the context of the quark confinement theorem described in Section 3. The effective mass of this object is defined as the inverse of the full quark propagator at zero point (Eq. (5.13)).

But this is not the whole story yet in QCD because up to now we took into account only contributions induced by the virtual gluons alone. The actual gluons emitted by one quark can contribute to the self-energy of another quark and vice versa. The simplest diagrams of this process are shown in Fig. 13. Moreover contributions shown in Fig. 14 are also inevitable and they describe the process of the conversion (transformation) of virtual gluons into actual ones and the other way around. Thus we consider diagrams, of these type, not as corrections to the cubic and quartic gluon vertices but rather as additional contributions to the self-energy of the quarks. Contributions to the self-energy of each quark will be essentially enhanced in the presence of another quark. In other words each quark additionally enhances the interaction with the vacuum (zero modes) of another quark. It seems to us that the IR strong singular asymptotics of the full gluon propagator (3.1) or equivalently (3.8) effectively describes this phenomenon in QCD correctly, i.e. enhancement of the zero modes by virtue of self-interaction of virtual (internal) and actual (external) gluons.

It is quite plausible that, at large distances between quarks, actual gluons emitted by each quark do repeatedly succeed to convert into virtual ones and vice versa. This leads to a multiple enhancement of the zero modes of each quark. We think that these additional contributions to the self-energy of each effective quark makes it a constituent object in our model. The mass of the constituent (heavy or light) quark becomes the sum of three terms, namely
\[ m_q = m_{\text{eff}} + \Delta = m_0 + m_d + \Delta, \tag{8.1} \]

where we used definition (5.13). All terms on the right hand side have clear physical sense. The first term is, obviously, the current mass of a single quark. The second one \(m_d\) describes contributions to the constituent quark mass induced by the self-interaction of virtual gluons alone, while the third term, \(\Delta\), describes contributions to the constituent quark mass which come from the process of the conversion of actual gluons into virtual ones as it was discussed above. This way our model provides a natural dynamical foundation of the current-effective (dynamical)-constituent transformation [75] of the quark degrees of freedom on the basis of non-abelian character of the gluon fields. The existence of a nonzero \(\Delta\) is principal for our model but numerically it should not be large, even for light quarks. Our intuition (based on the obtained numerical results) tells us that it is only of the order of a few per cent of the displayed values of \(m_d\) (Tables 1 and 2).

Remember that this naive formula shows only the nature of the contributions to the constituent quark mass. The contributions are so mixed up in reality that they can not be separated from each other. There exists an infinite number of possible, topologically complicated, configurations of the vacuum fluctuations of the non-abelian gauge (gluon) fields contributing to the self-energy of each quark while making them constituent objects. The true vacuum of QCD, however, is not settled by these fluctuations alone, its structure is much more richer [70] than that (see also the discussion below).

Certainly, a finite number of favorable, topologically distinct vacuum configurations, which minimize the energy of the bound states, should exist. It is hard to believe that in the real world of four dimensions, the favorable topologically complicated configurations are strings or planar ones. In this context, it is important to comprehend that the linearly rising quark-antiquark potential, at large distances, nicely showed by recent lattice calculations [76], is not a privilege of the planar or string configurations only. Though the above mentioned linear potential does not contradict the ZME effect, nevertheless the potential concept in general is a great simplification of the real dynamical picture which emerges from
our model. As it was mentioned in Section 3, the enhancement of zero modes necessary leads to full vertices while the potential concept of the CQM [32, 33, 61, 64] is based on point-like ones.

One may say that at large distances interaction between quarks proceeds mainly through the vacuum and thus leads first to the formation of the constituent quarks and then to the formation of bound states of these constituent quarks. According to the ZME effect this process should be accompanied by the condensation of the quark and gluon loops - quark and gluon condensates. Hadrons are bound-states of the constituent quarks strongly interacting with the vacuum of QCD at the expense of each other. At the same time, direct interaction (through the actual gluons) between constituent colored objects becomes weaker. Also, they should be surrounded by quark condensates playing the role of some external field (see Subsection C below). From our dynamical picture it follows that hadrons are composed of heavy objects (constituent quarks) because of the enhanced contributions to their self-energy, and because of this, the quarks interact weakly. The reason of the relatively week interaction between the constituent quarks is that the biggest part of the actual gluons is expected, at large distances, to be converted into virtual ones which contribute to the self-energy of each quark.

There exists an interaction between electrons and positrons in QED which goes through the vacuum in the sense described above, but it is strongly suppressed because of the small magnitude of the coupling constant (fine structure constant) and the absence of the direct interaction between photons. Consequently, there does not exist such an object as the “constituent electron” in QED. In other words the perturbative vacuum of QED can produce only small corrections to the self-energy of the electron leading to a ”clothed or dressed” electron, while the nonperturbative vacuum of QCD produce not only corrections but substantial contributions to the self-energy of the quarks which make them constituent objects. Thus a single electron can be in a free state and remain ”clothed” but a constituent quark can not be in a free state as a ”clothed” electron can because it becomes constituent in the presence, at least, of another quark which itself becomes constituent too.
It is plausible that the energetically advantageous configurations of vacuum fluctuations (leading to the formation of the bound-state of the constituent quarks) occur at a certain distance between constituent quarks. This configuration will be completely deformed (or even destroyed) if one attempts to separate these constituent quarks further from each other. The nonperturbative vacuum of QCD is filled with quark-antiquark virtual pairs which consist of various components of quark degrees of freedom (light, heavy, constituent, dynamical, etc). This is an inevitable consequence of the ZME effect. As a result of the above mentioned nontrivial topological deformation, at least one quark loop will be certainly "cut" (see also Subsection C below). We may, for convenience, think of this as such a topological deformation which allows for quarks from the loop to recombine with the initial constituent quarks. It is evident that the breaking of the gluon line is not so important as the above mentioned cut of the quark loop which is equivalent to the creation of the corresponding quark-antiquark pairs from the vacuum. The vacuum of QCD will be immediately rearranged and, instead of "free" constituent quarks, new hadron states will occur.

Here let us remind the reader that the constituent quark can not go on mass-shell (quark confinement theorem, Section 3). Apparently, at zero temperature to confine such quarks in order to generate new hadrons states is much more energetically profitable than to keep them always "free" and off mass-shell. At nonzero temperature the transition from hadron matter to quark-gluon plasma is possible [73, 77]. In both cases, at zero temperature (confining quarks inside hadrons) and at nonzero temperature ("free" in quark-gluon plasma) quarks always remain off mass-shell. In other words, in the ZME model of the vacuum of QCD, deconfinement should be interpreted as the screening of the confining potential and the dissociation of bound states; nevertheless, quarks can never go on mass-shell. This clearly shows once again the restricted sense of the potential concept. Thus our model admits the existence of the quark-gluon plasma because, by melting more and more hadron matter, it is impossible to remove quarks from off mass-shell.

At short distances the situation is completely different from the above described. Indeed, in this case the actual gluons emitted by each quark do not repeatedly succeed to convert into
virtual ones. So, at these distances, interaction between quarks proceeds mainly through the exchange of actual gluons. This means that the interaction of the constituent quarks with the vacuum, due to the above mentioned process of conversion, is essentially decreased. So they become valence quarks. The intensity of the process of conversion determines the constituent-valence transformation. One can say that the constituent (valence) quarks are ” valence (constituent)-in-being quarks”. At low energies a valence quark becomes constituent by absorbing self-interacting gluons and sea quarks with the help of another constituent quark. At high energies these degrees of freedom will be lost and all constituent quarks become valence ones.

The potential concept (based on point-like vertices) becomes relevant in this case. At the experimental large momentum scale, the virtual photon sees the hadron as made of valence quarks, sea quarks and gluons. The Coulomb-type behaviour is a good approximation for the interaction between quarks which, at this scale, become rather simple objects (almost massless). Contrary to the small momentum scale, at which complicated objects are quarks, at this scale gluons themselves play a major role. At large distances quarks interact strongly with the vacuum, and therefore interaction between them becomes weak, while at short distances gluons interact strongly with the vacuum which produces a strong anti-screening effect in it, leading to asymptotic freedom [17].

B. The Okubo-Zweig-Iizuka selection rule

In the context of the ZME model of quark confinement and DCSB it becomes clear that the topological rearrangement of the vacuum by means of the direct annihilation of the initial (final) constituent quarks, entering the same hadrons, is hardly believable. In fact, what does the above mentioned direct annihilation mean? This would mean that the initial (final) constituent quarks, emitting (absorbing) a number of gluons, can annihilate with each other without the break-up of the corresponding quark loops in our model. The initial (final) constituent quarks always emit and adsorb gluons in each preferable configuration. Nothing
interesting should happen during these processes. This is a normal phase of each preferable configuration and it describes only its trivial rearrangement. Any nontrivial rearrangement of the vacuum can only begin with cutting the quark loop. As it was mentioned above, this is equivalent to the creation of a quark-antiquark pair from the vacuum. Then the annihilation of the initial (final) constituent quarks with the corresponding quarks, liberated from the loop, becomes possible. Diagramatically this looks like a direct annihilation (see Fig. 15). The probability to create the necessary pair, in order to annihilate the initial (final) constituent quarks, is rather small, so, in general, the annihilation channel must be suppressed. It is much more probable for the quarks liberated from the loop to recombine with the initial (final) constituent quarks in order to generate new hadron states. In more complicated cases (when many quark loops are cut) the annihilation of the initial (final) constituent quarks becomes more probable and this process will compete with the process of the recombination of the initial (final) constituent and liberated- from-the-vacuum quarks to generate new hadron states directly. Ending these general remarks, first let us analyse mesons consisting of heavy quarks, for example $c\bar{c}$ systems.

The heavy constituent quarks inside hadrons are much closer to each other (the distance between them is of the order $m_h^{-1} \ll m_q^{-1}$, where $m_h$ and $m_q$ denote the masses of the heavy and light constituent quarks, respectively) than their light counterparts. At first sight this should promote the annihilation of the initial (final) constituent quarks. However at short distances the interaction between them proceeds mainly not through the vacuum fluctuations but via the exchange of the actual gluons as this was explained above. This means, in turn, that the number of virtual loops which should be cut is small. Also the probability to cut heavy quark loop, or equivalently to create a heavy quark-antiquark pair from the vacuum, is much less than to create, for example, a light pair. Apparently, the fluctuations in the density of instantons (see next section) and condensates during the vacuum’s rearrangement also come into play. Thus the process of the rearrangement of the vacuum, leading to the transition between hadrons (their strong decays) on the basis of the annihilation of the initial (final) constituent quarks, as shown in Fig. 15, should be strongly suppressed in
comparison with the process of the recombination of the initial (final) constituent quarks with the quarks liberated from the vacuum. This is shown in Fig. 16. Thus one comes to a dynamical explanation of the famous Okubo-Zweig-Iizuka (OZI) selection rule [78]. In the case of heavy quarks it is in agreement with the standard explanation of the OZI rule which states that the QCD coupling constant becomes weak at short distances and suppresses the annihilation channel. However this argument fails to explain why the violation of the OZI rule for the pseudoscalar octet is bigger than for the vector one.

Let us analyse this problem in our approach. Light constituent quarks inside pseudoscalar and vector mesons are at relatively large distances ($\sim m_q^{-1}$ from each other) than their heavy counterparts, for example in $c\bar{c}$ systems. This means that interaction between them is mediated mainly by the vacuum fluctuations which provide plenty of various quark loops to be cut during the process of the vacuum rearrangement. So the annihilation channel should not be strongly suppressed for these octets. Indeed, the violation of the OZI rule in the pseudoscalar mesons is not small, but for the vector mesons it is again small, i.e. comparable to the violation in the $c\bar{c}$ systems. Our model provides the following explanation for this problem: The same quark-antiquark pair in pseudoscalar and vector mesons is in the same $S$-state. The only difference between them is in the relative orientation of the quark spins. Quark and antiquark spins are oriented in the same direction in the vector mesons, while in the pseudoscalar mesons their orientation is opposite. This is schematically shown in Figs. 17 and 18. For light constituent quarks spin effects become important, while for heavy constituent ones such a relativistic effect as spin and, in particular, its orientation is not so important. As it was repeatedly mentioned above, a nontrivial rearrangement of the vacuum in our model always starts from the cut of the quark loops. In order to analyse the violation of the OZI rule, from the point of view of annihilation of spin degrees of freedom, let us think of quark loops as ”spin loops”. In pseudoscalar mesons at least one spin-antispin liberated-from-the-vacuum-pair is needed to annihilate the initial pair. This is schematically shown in Fig. 17. In vector mesons at least two spin-antispin pairs are needed for this purpose, but, in addition, an intermediate meson (exited) state certainly appears,
see Fig. 18. This means that the annihilation channel for the vector mesons, unlike for the pseudoscalars ones, is suppressed. It is worth noting that the OZI rule is a selection rule and it is not a conservation law of some quantum number. So its breakdown is always possible and suppressed processes may proceed through the appropriate intermediate states [77]. Exactly this is shown in Fig. 18 schematically. All this explains the violation of the OZI rule in the pseudoscalar channel in comparison with the vector one in our model. For vectors mesons the decays $\phi \to 3\pi, \rho \pi$, proceeding through the annihilation channels, are suppressed in comparison with the decay $\phi \to K^+ K^-$ which occurs via the recombination channel; while for pseudoscalar mesons, say, the decay $\eta \to 3\pi$ is not suppressed [54].

To conclude this topic, let us underline that our model of the QCD vacuum on the basis of the ZME effect provides an explanation of the OZI rule on a general ground for all mesons ($q\bar{q}$ systems), in particular, for $c\bar{c}$, pseudoscalar, vector, etc ones. In order to confirm our qualitative explanation of the OZI selection rule quantitatively, it is necessary to calculate the strong decay widths of mesons in our model, which is beyond the scope of the present paper and will be done elsewhere.

C. Instantons

The main ingredients of the QCD vacuum, in our model, are quark and gluon condensates, quark-antiquark virtual pairs (sea quarks) and self-interacting nonperturbative gluons. The vacuum of QCD has, of course, much more remarkable (richer) topological structure than this. It is a very complicated medium and its topological complexity means that its structure can be organized at various levels and it can contain perhaps many other components [1, 70] besides the above mentioned. There are a few models of the nonperturbative vacuum of QCD which are suggestive of what a possible confinement mechanism might be like (see recent paper [80] and references therein). We will not discuss these models; let us only mention that the monopole vacuum of Mandelstam and ’t Hooft as well as the mechanism of the confining medium recently proposed by Narnhofer and Thirring [81] also invokes
the ZME effect [37, 80, 82]. Let us ask one of the main questions now.

What is a mechanism like which initiates a topologically nontrivial rearrangement of the vacuum? It is already known, within our model, that this may begin with the cut, at least, of one quark loop and therefore, at least, one quark-antiquark pair emerges from the vacuum. Why can the quark loop be cut at all and what prevents the quarks from the cut loop to annihilate again with each other? The fluctuations in the nonperturbative vacuum of QCD must exist which do the above mentioned job. This is an inevitable consequence of our model of the vacuum. We see only one candidate for this role in four dimensional QCD, namely instantons and anti-instantons and their interactions [83]. In this sense let us discuss the possible role of instantons in our picture of quark confinement. At this stage of our knowledge of the detailed mechanism of confinement, one may only make a few remarks which will be necessarily qualitative.

Instantons are classical (Euclidean) solutions to the dynamical equation of motion of the nonabelian gluon fields and represent topologically nontrivial fluctuations of these fields [1, 83]. Self-interaction of gluons should be important for the existence of the instanton-like fluctuations even at classical level. In the above mentioned RILM [69], light quarks can propagate over large distances in the QCD vacuum by simply jumping from one instanton to the next one. In contrast, in our model the propagation of all quarks is determined by the corresponding SD equations (due to the ZME effect) so that they always remain off mass-shell. Thus we do not need the picture of jumping quarks. As opposed to the RILM, we think that the main role of the instanton-like fluctuations is precisely to prevent the quarks and gluons from freely propagating in the QCD vacuum. Running against instanton-like fluctuations, the quarks undergo difficulties in their propagation in the QCD vacuum which is a very complicated inhomogenous medium. Along with the instanton component, it consists of a confining quark component, the nonperturbative gluon component, etc. As we have already learned the confining quark component, in turn, consists of various quark degrees of freedom such as dynamical, light, heavy, constituent ones, etc. At some critical value of the instanton density the free propagation of the virtual quarks from the loops,
apparently, become impossible so they never annihilate again with each other. Obviously, this is equivalent to the creation of the quark-antiquark pairs from the vacuum. From this moment the nontrivial rearrangement of the vacuum may start. The role of the instanton-like fluctuations appears to be ”cutting” the quarks loops and preventing them from the immediate annihilation of quarks and antiquarks liberated from the loops. Their main task is to promote transitions between hadrons, i.e. in our terminology they destabilize energetically advantageous (dominant) configurations of the vacuum fluctuations which lead to hadron states. One of the main features of the instanton-induced effects is tunneling between topologically distinct vacuums in Minkovski space [1, 84]. It seems to us that our understanding of their role in the QCD vacuum structure does not contradict this.

Being classical (not quantum!) fluctuations, instantons can cut the quark loops in any points even in the quark- gluon vertices. A simple cut of the quark loops, as it was repeatedly emphasized above, is equivalent to the creation of the corresponding quark-antiquark pairs from the vacuum. Let us ask now what happens if all the external (actual) gluon lines will be cut from the quark loops by the instantons. Well, in this case each quark loop becomes a closed system. Because of the vacuum pressure they immediately collapse and one obtains nothing else but quark condensate if, of course, all internal gluon lines can be included into the quark self-energy. Another scenario is also possible when not all the internal gluon lines can be included into the quark or gluon self-energy (see, for example, the two-particle irreducible (2PI) vacuum diagrams, sketched in Fig. 19). In general, the presence of the internal gluon lines in the vacuum diagrams prevent them from collapsing (because they counterbalance the vacuum pressure) and consequently they should contribute to the vacuum energy density.

It would be suggestive to conclude that the same mechanism works in order to produce gluon condensates despite the much more complicated character of the gluon self-interactions. But this is not the case indeed, since there is a principal difference between quark and gluon condensates. The former ones do not contribute to the vacuum energy density and, in this sense, they play the role of some external field. While the latter ones,
as was shown first in Ref. 49 and numerically confirmed in our paper, are closely related to the vacuum energy density. Like we noted previously, nothing interesting should be happen if the instantons cut the gluon line in one point. Let us imagine that many gluon lines will be cut by the instantons in many points. The vacuum becomes filled up with gluon pieces (segments) which, due to the existence of the gluon strong self coupling, can recombine, in principle, as some colour singlet bound-states – gluonia or glueballs. Unlike quark condensates, glueballs should have internal pressure because of the strong self-interaction of composite gluon segments which prevents them from collapsing. In turn, this means that the glueballs should be heavy enough. At this stage we see no reason why the ZME model of the QCD vacuum should forbid non-quark-antiquark bound-states, such as glueballs, multiquark states or even hybrids \((qar{q}g)\) [54].

Our numerical results were obtained in the chiral limit but our qualitative discussion was a general one. We are mainly concerned with realistic (nonchiral) QCD. Let us now briefly discuss the important case of massless quarks (i.e. the chiral limit). The pseudoscalar mesons (consisting of light quarks) are Goldstone states so their masses remain zero in the chiral limit even in the presence of DCSB. From our model it follows that the existence of the instanton-type fluctuations in the true vacuum promote strong meson decays by preventing quarks and gluons from freely propagating in it. One can conclude in that instanton-type fluctuations should be totally suppressed in this case in order to provide stability for the massless Goldstone states since massless particles cannot decay. This feature of the instanton physics in the massless quark case was discovered by t’ Hooft [71]. In the presence of DCSB, however, the instanton-type fluctuation are restored [70, 71] but, apparently, the contribution of the instanton component to the vacuum energy density still remains small. So the dilute gas approximation for the instanton component seems to be relevant in this case. Not going into details of the instanton physics (well described by Callan, Dashen and Gross in Ref. 70), let us only emphasize that at short distances the density of small size instantons should rapidly decrease and conversely increase at large distances where large scale instantons, anti-instantons and their interactions also come into play. Otherwise it
would be difficult to understand the role which we would like to assign to the instantons in our model. This is in agreement with the behaviour of the instanton component of the QCD vacuum at short and large distances described in the above mentioned paper [70] as well as with our understanding of the actual dynamical mechanism of quark confinement and DCSB described in this section.

The nontrivial rearrangement of the vacuum can start only when the density of instantons achieves some critical values, different for all distinct vacuums. For this reason, despite being a classical phenomena, instantons should nevertheless contribute to the vacuum energy density through the above mentioned quantum tunneling effect which is known to lower the energy of the ground-state. We have already calculated the contributions to the vacuum energy density of confining quarks with dynamically generated masses and nonperturbative gluons (Tables 1 and 2). So the total vacuum energy becomes (as minimum) the sum of three quantities now, namely

$$\epsilon_I = \epsilon_I + \epsilon_g + N_f \epsilon_q$$

where $\epsilon_I$ describes the instanton component of the vacuum. We introduce also the explicit dependence on the number of different quark flavors $N_f$ since $\epsilon_q$ itself is the contribution of a single confining quark. Of course, this should be valid for the non-chiral (realistic) case as well. The distinction will be in the actual values of each component, apart from, maybe, $\epsilon_g$.

Up to now our discussion of a possible role of the instanton fluctuations in our model was quite qualitative. Let us now run a risk and make a few quantitative predictions. As it was noticed, $\epsilon_I$ should be small in the chiral limit and in the presence of DCSB, i. e. for light quarks with dynamically generated masses. However, the same conclusion seems to be valid for heavy quarks as well in our model. Heavy quarks are at short distances from each other (in mesons) at which the nonperturbative effects, such as instantons and enhancement of zero modes, are suppressed. Thus the dilute gas approximation for the instanton component seems to be applicable to light quarks with dynamically generated masses as well as to heavy quarks. In other words, $\epsilon_I$ for light quarks with dynamically generated masses and for heavy
quarks is nearly the same in our model. If this is so then it is worth assuming, following
the authors of Ref. 49, that the light and heavy quarks match smoothly. This allows one
to choose the average value between (6.15) and (6.16) for the instanton component of the
vacuum energy density $\epsilon_I$ in (8.2), i.e. namely $\epsilon_I \simeq 0.0034$ GeV$^4$. Then our predictions
for the total vacuum energy $\epsilon_t$ (in the case of $N_f$ light confining quarks with dynamically
generated masses) and the corresponding values of the gluon condensate are listed in Tables
3 and 4, 5 for both calculation schemes A and B, respectively.

It is worth reproducing explicitly some interesting particular values of the total vacuum
energy density and the corresponding values of the gluon condensate. Thus for a pure
gluodynamics ($N_f = 0$) one has

$$\epsilon_t \simeq -0.005 \text{ GeV}^4,$$

$$\langle 0|\frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu}|0 \rangle \simeq 0.0177 \text{ GeV}^4,$$  \hspace{1cm} (8.3)

and

$$-0.00661 \leq \epsilon_t \leq -0.003837 \ (\text{GeV}^4),$$

$$0.0136 \leq \langle 0|\frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu}|0 \rangle \leq 0.0235 \ (\text{GeV}^4).$$  \hspace{1cm} (8.4)

Here and below the numbers correspond to the approximation of the pion decay constant in
the chiral limit by its standard value.

For the most realistic case $N_f = 2$ one obtains

$$\epsilon_t \simeq -0.008 \text{ GeV}^4,$$

$$\langle 0|\frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu}|0 \rangle \simeq 0.0283 \text{ GeV}^4,$$  \hspace{1cm} (8.5)

and

$$-0.00933 \leq \epsilon_t \leq -0.00724 \ (\text{GeV}^4),$$

$$0.0256 \leq \langle 0|\frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu}|0 \rangle \leq 0.0331 \ (\text{GeV}^4).$$  \hspace{1cm} (8.6)

There exist already phenomenological estimates of the gluon condensate [85] as well as
lattice calculations of the vacuum energy density [86] pointing out that the above mentioned
standard values (6.14) and (6.15-6-16) are too small. Our numerical predictions are in agreement with these estimates though we think that their numbers for the gluon condensate [85] are too big. At the same time, it becomes quite clear why the standard values are so relatively small, because they are due to the instanton component of the vacuum only.

As it was mentioned above, a nontrivial rearrangement of the QCD vacuum will occur if the density of the instanton-like fluctuations achieves some critical value. This critical value can be reached when $\epsilon_I \simeq \epsilon_q + \epsilon_g$, i.e. when at least one sort of virtual quarks is presented in the true QCD vacuum. On one hand, this assumption is supported by our numerical results for $\epsilon = \epsilon_q + \epsilon_g$. On the other hand, the numerical results for $\epsilon_I$, as given by (6.15) or (6.16), also confirm this. In the realistic (non-chiral) case the instanton part, along with other contributions, may substantially (but not surely) differ from those shown in Tables 1, 2, 3, 4 and 5.

**D. Summary**

One of our main conclusions is that in QCD, unlike in QED, the self-energy and bound-state problems are closely related to each other. The propagation of the constituent quark necessarily becomes a many-body problem. Apparently, the ZME effect correctly describes this feature of QCD at large distances and therefore it correctly reflects the nonperturbative structure of the QCD vacuum, at least, at first approximation. Following Susskind and Kogut [39], we also would like to especially emphasize the importance of quantum fluctuations of the IR degrees of freedom and the non-abelian character of QCD. Let us additionally underline the role of the ghost degrees of freedom in the IR structure of the QCD vacuum. Neglecting them “by hand”, one necessarily comes to the almost trivial ST identities for cubic and quartic gluon vertices which are responsible for the direct self-interaction of gluons [30].

From our model it follows that the non-abelian character of QCD alone provides quark confinement and DCSB at large distances through the strong interaction of quarks with the
nonperturbative vacuum (enhancement of the zero modes) and at short distances it leads to the asymptotic freedom phenomenon because of the strong interaction with the vacuum of the gluon degrees of freedom. This confirms the expectation of Marciano and Pagels expressed in their excellent review on QCD "The gluon field, unlike the photon, carries color charge and has self-interactions. This distinction, we believe, leads to confinement; but we will not see this property using ordinary perturbation techniques" [1]. If our QCD inspired nonperturbative model of quark confinement is correct, at least in first approximation, then confinement is not an additional external requirement to complete QCD in order to be a self-consistent theory of strong interactions. It seems to us that QCD is a complete theory and provides a unique description of the hadron dynamics at high and low energies without any additional assumptions.

It is well known that instantons themselves can produce a rather complicated mechanism for DCSB in the QCD vacuum [70, 87, 88] but they do not explain confinement. In contrast, our model explains quark confinement and DCSB (and many other things) on the same basis of the enhancement of the zero modes because of the possible nonperturbative IR divergences in the true vacuum of QCD. From our discussion it clearly follows that our model needs instanton-like fluctuations in the vacuum of QCD in order to provide transitions between hadrons. We understand the role of instantons in quite a different way [89] than in the RILM while, nevertheless, it seems to us that our model is able to reconcile the instantons with the quark confinement phenomenon, at least qualitatively. Generally speaking, it would be of a great interest to find instanton-like solutions to the classical equations, on account of the enhancement of the zero modes, if it is possible at all, since this is essentially a quantum effect.

**IX. COMPARISON WITH RELATED MODELS**
A. The CQM

From a QCD theoretical field point of view, the CQM is nothing but an approximation of the full quark propagator by the constituent quark propagator (5.14) and the full constituent-quark-gluon vertices by point-like ones. The bound-state amplitudes are approximated by the corresponding coupling constants, for example, the pion BS amplitude (2.18-2.19) should be approximated by the pion-quark coupling constant (5.32). Choosing a reasonable value for a possible UV cutoff, one arrives at a calculation scheme which works surprisingly well (despite its simplistic structure) in the description of static hadron properties. It is well known that the CQM agrees with experiment within an accuracy of 20% [90].

It has been already emphasized above that the ZME model of quark confinement and DCSB necessarily leads to the concept of the constituent quark. Our numerical results clearly show that the dynamical (and consequently constituent, too, because of the relation (8.1)) quark really plays an important role. From the discussion in the preceding section about the dynamical mechanism of quark confinement and DCSB, within the ZME effect, it follows that the hadrons are relatively weak bound states of the constituent quarks. A current quark in our quantum field theory model, because of the above mentioned effect, becomes first effective and then constituent (i.e. heavier) only at the expense of the quantum fluctuations of the gluon fields, i.e. it becomes constituent one without any additional admixture of more complicated states. Having become constituent, then its interaction with actual gluons can be described as point-like, at least, in first approximation. On one hand, this justifies the point-like character of the constituent quark-gluon vertex in the CQM. On the other hand, it confirms Shuryak’s observation [91] in the QCM that the coupling constant of the constituent quark to the bare or ”current” quark is equal to one within error limits. The constituent quark-gluon vertex does not differ much from the point-like current quark-gluon one, at least, in first approximation. This is shown in Fig. 20.

As a side remark, recently Fritzsch [55] proposed to interpret a constituent quark inside hadrons as a quasi-particle which has a non-trivial internal structure on its own, i.e.
consisting of a valence quark, many quark-antiquark pairs and of gluons - in short, it looks like one third of a nucleon. As was it discussed above, nearly the same picture emerges from our model which is based on the ZME effect of QCD. The definition of the effective quark mass, as a solution of the full quark propagator at zero (5.10), should be appropriate for an effective mass of Fritsch’s constituent quark. This definition incorporates a complicated dynamical structure of a constituent quark and, in a self-consistent way, relates it to such nonperturbative effects in QCD like quark confinement and DCSB.

Thus, in many aspects, our approach, with the constituent quark playing such a significant role, looks like as a generalization of the CQM [32, 61, 64, 90]. However our approach is directly derived from QCD. Therefore it can be considered as a theory giving strong arguments in favour of the reconciliation of the CQM with QCD and thereby it puts the CQM on a firm theoretical basis. In other words our model makes it possible to understand how the CQM may be derived from QCD. However, see remarks in the next section.

B. The bag model

A nontrivial relation between our model, on one hand, and the bag [92-94] and string [95] models, on the other hand, would not be surprised. In this connection some dynamical aspects of our model should be underlined. From the above consideration it follows that, from a dynamical point of view, maybe the ZME effect does not lead to string configurations of flux tube type between quarks. Nevertheless, there is no doubt that this dynamical process works like a string preventing quarks to escape from each other. It takes place in the finite volume of the QCD vacuum but it does not require the introduction of an explicit surface. The finiteness of the cut-off $z_0$ results in unphysical singularities (at this point $z_0$) of the solutions to the quark SD equation which are due to inevitable ghost degrees of freedom in QCD. This has nothing to do with the bag fixed boundary. Numerically it depends on a scale at which nonperturbative effects become essential in our model. We treat a hadron as a dynamical process which takes place in some finite volume of the vacuum rather than
as an extended object with an explicitly fixed surface in the vacuum. The ZME model remains a local field theory. However, the existence of the vacuum energy per unit volume – the bag constant $B$ – is important in our model as well. The inward positive pressure $B$ counterbalances the vacuum energy density needed for generating the vacuum fluctuations, inspired by the enhancement of the zero modes in our model, i.e., the sum of the bag constant and the nonperturbative vacuum energy density must be zero. In agreement with the authors of paper [96], we consider the bag constant as a universal one which characterizes the complex nonperturbative structure of the QCD vacuum itself and it does not depend on the hadron matter. Moreover, being an important characteristic of the QCD vacuum, it greatly influences the quark-gluon plasma (QGP) equation of state [77], bridging the gap between hadron matter and the QGP phase.

The bag constant is defined as the difference between the energy density of the perturbative and the nonperturbative QCD vacuums. We normalized the perturbative vacuum to zero, Eq. (4.8). In our notations the bag constant becomes

$$B = -\epsilon_t. \quad (9.1)$$

(Not to be mixed with the CHPT constant B which measures the vacuum expectation value of the scalar densities in the chiral limit, see Subsection C of Section 5 and Tables 1 and 2). Our predictions are listed in Tables 3 and 4, 5 for each calculation scheme A and B, respectively. In fact, our values for the bag constant overestimate the initial MIT bag [93] volume energy by one order of magnitude. Nevertheless, we think that the introduction of this quantity into physics was a main achievement of the bag model.

As in previous case, let explicitly reproduce some interesting particular values of the bag constant. For a pure gluodynamics ($N_f = 0$) it is:

$$B \simeq 0.005 \text{ GeV}^4 \simeq (266 \text{ MeV})^4 \simeq 0.651 \text{ GeV}/\text{fm}^3. \quad (9.2)$$

For the most realistic case $N_f = 2$ the bag constant becomes

$$B \simeq 0.008 \text{ GeV}^4 \simeq (300 \text{ MeV})^4 \simeq 1 \text{ GeV}/\text{fm}^3. \quad (9.3)$$
For simplicity’s sake we reproduced its values obtained within the calculation scheme A only. It has been noticed in [97] that nobody knows yet how big the bag constant might be, but generally it is thought it is about $1 \, GeV/fm^3$. The predicted value for $N_f = 2$ is in fair agreement with this expectation.

C. Some other models

The Nambu-Iona-Lasinio model [98] of QCD does not possess confinement phenomenon at all, so its relation to our model remains still unclear. It is not without sense to mention also the quark pair condensation model of chiral symmetry breaking in QCD [99] because in many aspects our model is a covariant generalization and extension of that model. Our numerical program and conclusions, concerning especially DCSB, are almost the same as in that model which approximates the QCD vacuum by a condensate of flavour and colour singlet quark-antiquark pairs. The important role of the constituent quark concept is also emphasized. The differences between our model and theirs are: 1) Our model is formulated in a relativistically invariant approach to QCD at low energies on the basis of the enhancement of zero modes due to the self-interactions of gluons fields.. 2) We established a close intrinsic link between DCSB and quark confinement. In their approach quark confinement was not considered at all. 3) As a result of our calculation scheme, we have established that the scales of both nonperturbative phenomena (quark confinement and DCSB) are nearly the same indeed, while in their model these two scales may be different.

An interesting approach to nonperturbative QCD, based on a weak-coupling treatment on the light front, was recently proposed in Ref. 100. As claimed by the authors, it can bridge the gap between QCD and the QCM. They also expect light-front IR divergences to be the sources of confinement and DCSB. The relation between our approaches is not clear for us at this stage, neither with the gauged nonlocal constituent quark model (GNCQM) [101].
X. SOME IMPORTANT PROBLEMS

Let us begin with the important question whether the confinement mechanism for heavy quarks is the same as for light ones or not. From the quark confinement theorem (Section 3) it follows that the propagation of a single off-mass-shell heavy or light quark is the same. However the propagation of heavy constituent quarks inside hadrons differs from the propagation of light constituent quarks. First of all, the chiral limit makes no sense for heavy quarks. This means that the quantum fluctuations (important for the light constituent quarks) do not influence heavy constituent quarks greatly, so the difference (described by $\Delta$ in (8.1)) between heavy constituent and effective quarks should become small. This is also true for the instanton-like fluctuations which influence heavy quarks from the loops much less than light ones (it is possible to say that, like in the RILM [102], in our model heavy (static) quarks essentially ignore instantons). This explains why, in general, ground-states of mesons consisting of heavy quarks (for example, $c \bar{c}$ systems) should have a much narrow width than mesons consisting of light ones [54], as well as why the potential concept should work much better for heavy constituent quarks than for light constituent ones [44]. The spin-flavor symmetry, discovered in the heavy quark effective theory [103], clearly shows that there might be a difference in the dominant configurations of quantum fluctuations between heavy constituent quarks and those of light constituent ones. In this context let us note that recently color-singlet glueballs were suggested (described as a loop of one quantum of color magnetic flux) to produce the heavy quark -confining force, even though it is difficult to distinguish a glueball from a color and flavour singlet quark-antiquark pair [104].

Thus a difference between the confinement of heavy and light quarks does exist but it is not principal. We see no principal distinction in the process of topologically nontrivial vacuum rearrangement for heavy-heavy, light-light and heavy-light constituent quarks from which corresponding mesons are build.

The quarks become constituent in the presence of each other in our model of the QCD vacuum, i.e. they are bound-state objects. While in the CQM for the description of the
bound-states a linearly rising (imitating contributions of the virtual gluons) plus a Coulomb-like (imitating contributions of the actual gluons) potential is used along with the corresponding corrections [44, 45]. In our approach this problem should be considered within the BS formalism for the bound-states (Fig. 21). So the potential concept of the QCM, based on the point-like vertices, is too naive an approximation of the real dynamical picture which emerges in the framework of our model. The same conclusion, namely, that, in fact, quark interactions are much more complicated than just simple universal confining forces, follows from the study of correlation functions in the QCD vacuum [91]. Moreover we think that the CQM, based on the potential concept, has already played its useful role and now it should be retired from the scene like the Bohr orbits were after the creation of the true theory of atoms - quantum mechanics [105].

To investigate the bound-state problem, one should begin from the first skeleton diagram (the ”generalized ladder approximation”, Fig. 22a) of the BS scattering kernel. The full gluon propagator has to be replaced by (3.6-3.8), reproducing the effect of the enhancement of the zero modes. Quark propagators should also be replaced by the solutions reconstructed on the basis of this effect (Section 3) together with the quark-gluon vertices. These vertices should be independently taken from the investigation of the corresponding WT identities (see our paper [16] and Section 3). The kernel of the BS integral equation for mesons obtained this way, in general, will contain the derivatives of the corresponding vertices (see, for example, equation (3.14)). This complicated kernel will be responsible for the interaction between the constituent quarks in our model in the generalized ladder approximation. This has nothing to do with the above mentioned potential (linear plus Coulomb-like) of the QCM. A much more complicated object appears if one takes into account the second diagram of the skeleton expansion of the BS scattering kernel (Fig. 22b), etc. We hope that this complication of the interaction between constituent quarks, that emerges in our model, will make it possible to resolve an old-standing problem of the QCM, namely the unphysical inverse-power color analog van der Waals potentials between separated hadrons are in substantial contradiction with experimental data (see Ref. 106 and references therein). The BS equation for single-
hadron states and the aspects of hadron-hadron interaction are of great important and will be investigated elsewhere. In our paper [19] the Goldstone nature of the pion was confirmed by the investigation of the corresponding axial-vector WT identity (see also Section 2). This should be supported by the investigation of the BS equation in the chiral limit for the pseudoscalar bound-states in our model of confinement and DCSB, too. This topic for further work is also important.

Let us now proceed to more technical problems related to our approach. Note that the non-chiral limit is neither a dynamical or a technical problem for our model. The first essential technical problem is that gauge invariance is important for numerical calculations of various low energy physical parameters. The quark SD equation depends explicitly on a gauge fixing parameter. This has two reasons. Firstly, it comes from the full gluon propagator, secondly from the corresponding quark-gluon vertex function. In both cases we have shown [16] that after the completion of our renormalization program, to remove all nonperturbative IR divergences, the explicitly gauge-dependent terms (the next-to-leading terms, i.e. the third line in Eq. (3.14)) disappear. The final form of the renormalized (IR finite) quark SD equation (3.29-3.30) does not depend explicitly on a gauge fixing parameter. The "gauge invariance" of the quark propagator (more precisely of its nonperturbative IR piece) is understood in this sense. Within the general scheme of our approach [16], the explicit dependence on a gauge-fixing parameter may appear again if one takes into account contributions coming from the ultraviolet (UV) region. This is beyond the scope of the present calculations. In any case, nobody knows yet how to take into account the UV piece of the quark SD equation in a gauge-invariant way.

Also, the UV renormalization program becomes trivial in our dynamical quark propagator approach [16]. Indeed, the ground solution (3.32-3.33) in the chiral limit at $t_0 = \infty$ automatically approaches the free quark propagator at infinity (asymptotic freedom) like it should. Renormalization was effectively taken into account by identifying the cut-off with the constant of integration of the equation of motion as described in Section 3. So, as far as dealing with the IR piece of the quark SD equation (automatically satisfying asymptotic
freedom), one can completely ignore the UV renormalization. A non-trivial UV renormalization should be performed when one will take into account contributions from the UV region in the quark SD equation, what, as mentioned above, is beyond the scope of this paper. Incidentally, let us stress once more that we have shown explicitly that the numerical values of basic chiral QCD parameters are determined mainly by the contributions coming from the IR region, while the contributions from short distances (for which UV renormalization is essential) can only be treated as small perturbative corrections.

Our treatment of the strong IR singularity (3.1), in the sense of distribution theory, within the system of equations in the quark sector, was carried out in Euclidean space. The corresponding expansion (3.8) takes place in this space since in Minkovskian domain it becomes much more complicated and may contain, for example, derivatives of the $\delta$-function [43]. This means that the final system of equations will be completely different from that obtained in Euclidean space and, therefore, requires separate consideration. However there may be a vital problem in the continuation of the obtained Euclidean solutions of the quark SD equation to the time-like region (Wick rotation). As it was pointed out in our paper [16], the analytical continuation of the nonperturbative solutions (with a typical exponential non-analiticity in the coupling constant (3.32-3.33)) could not be performed without additional conditions. For example, our solutions for the IR finite quark propagator (3.32-3.33) can, in principle, be analytically continued to the time-like region ($t \to -\infty$) if one changes simultaneously the sign of the ghost self-energy at zero point in (3.31), too. This can be done formally because the SD equation for the ghost self-energy at zero point has two solutions: a positive and a negative one [16]. This once more emphasizes the role of the ghost fields in the IR nonperturbative structure of the QCD ground state - i.e. true vacuum. However, as it turns out, we have no problems with the Wick-rotation within our low energies QCD model. In order to calculate physical quantities we always perform an integration over the nonperturbative region whose finite size is determined by the confinement scale $\Lambda_c$ (Section 6). Ending the discussion of these important problems, let us ask a, perhaps, heretic, but relevant question. Does it make sense at all to worry about Wick rotation of the solutions
for the unobservable quarks and gluons? The numerical values of observable characteristics of hadrons (for example, masses, decay constants, etc), calculated from first principles, do not depend, of course, on the metrics used.

There are many problems to be solved and questions to be answered in our model of quark confinement and DCSB. It is quite possible that our interpretation of the enhancement of the zero modes (3.1) or (3.8) as the enhancement of the contributions to the quark self-energy, due to self-interaction of gluon fields is not completely correct, but, we think, it works in the right direction. By all means, we believe that our model (even without the proposed interpretation) is a first step in that direction.

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    scope of the present paper. In any case, we know that \( \Lambda \) should be identified with \( t_0 \)
    in order to avoid the influence of unphysical singularities (due to the necessary ghost
    degrees of freedom) on the elements of the \( S \)-matrix, i.e. their contributions to the
    numerical values of the physical quantities. In our previous publication (Ref. 16) this
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TABLE I. Calculation scheme A

| $F^0_{\pi}$ | $88.3$ | $92.42$ | $93.3$ | MeV  |
|------------|-------|--------|--------|------|
| $\Lambda_{CSBq}$ | 724.274 | 758.067 | 765.284 | MeV   |
| $m_d$ | 362.137 | 379.0335 | 382.642 | MeV   |
| $\langle \bar{q}q \rangle_0$ | $(-208.56)^3$ | $(-218.29)^3$ | $(-220.36)^3$ | MeV$^3$ |
| $\epsilon_q$ | $-0.0012$ | $-0.00143$ | $-0.0015$ | GeV$^4$ |
| $\epsilon_g$ | $-0.0013$ | $-0.00157$ | $-0.0016$ | GeV$^4$ |
| $\epsilon$ | $-0.0025$ | $-0.0030$ | $-0.0031$ | GeV$^4$ |
| $\langle 0| \frac{\alpha_s}{\pi} G^{a}_{\mu\nu} G^{a}_{\mu\nu} |0 \rangle$ | 0.009 | 0.0106 | 0.011 | GeV$^4$ |
| B | 1163.51 | 1217.78 | 1229.23 | MeV   |
| $m^0_{u}$ | 6.65 | 6.36 | 6.30 | MeV   |
| $m^0_{d}$ | 10.08 | 9.63 | 9.54 | MeV   |
| $m^0_{s}$ | 202.85 | 193.75 | 191.94 | MeV   |
| $g_{\pi qq}^0$ | 3.0177 | 3.1584 | 3.1885 | MeV$^0$ |
| G | $2.05 \times 10^5$ | $2.25 \times 10^5$ | $2.29 \times 10^5$ | MeV$^2$ |
| \(F_\pi^0 = 88.3\) | \(F_\pi^0 = 92.42\) | \(F_\pi^0 = 93.3\) |
|---|---|---|
| \(707 \leq \Lambda_c \leq 742.68\) | \(737.9 \leq \Lambda_c \leq 768.4\) | \(744.4 \leq \Lambda_c \leq 773.86\) |
| \(328.62 \leq m_d \leq 400\) | \(340 \leq m_d \leq 400\) | \(342.416 \leq m_d \leq 400\) |
| \((-210.34)^3 \leq \langle \bar{q}q \rangle_0 \leq (-206.9)^3\) | \((-219.3)^3 \leq \langle \bar{q}q \rangle_0 \leq (-216.34)^3\) | \((-221.2)^3 \leq \langle \bar{q}q \rangle_0 \leq (-218.33)^3\) |
| \(-0.00135 \leq \epsilon_q \leq -0.00096\) | \(-0.0016 \leq \epsilon_q \leq -0.00128\) | \(-0.0017 \leq \epsilon_q \leq -0.00136\) |
| \(-0.0024 \leq \epsilon_g \leq -0.00045\) | \(-0.00226 \leq \epsilon_g \leq -0.00044\) | \(-0.00221 \leq \epsilon_g \leq -0.000437\) |
| \(-0.00336 \leq \epsilon \leq -0.0018\) | \(-0.00354 \leq \epsilon \leq -0.002\) | \(-0.00356 \leq \epsilon \leq -0.0021\) |
| \(0.0064 \leq \langle 0|G^2|0 \rangle \leq 0.0128\) | \(0.007 \leq \langle 0|G^2|0 \rangle \leq 0.0192\) | \(0.00746 \leq \langle 0|G^2|0 \rangle \leq 0.0199\) |
| \(1135.95 \leq B \leq 1193.56\) | \(1185.44 \leq B \leq 1234.76\) | \(1195.57 \leq B \leq 1243.34\) |
| \(6.48 \leq m^0_u \leq 6.81\) | \(6.27 \leq m^0_u \leq 6.53\) | \(6.22 \leq m^0_u \leq 6.47\) |
| \(9.83 \leq m^0_d \leq 10.33\) | \(9.5 \leq m^0_d \leq 9.89\) | \(9.43 \leq m^0_d \leq 9.81\) |
| \(197.67 \leq m^0_s \leq 207.7\) | \(191 \leq m^0_s \leq 199\) | \(189.76 \leq m^0_s \leq 197.34\) |
| \(2.74 \leq g^0_{\pi qq} \leq 3.33\) | \(2.83 \leq g^0_{\pi qq} \leq 3.33\) | \(2.89 \leq g^0_{\pi qq} \leq 3.33\) |
| \(2.0 \times 10^5 \leq G \leq 2.1 \times 10^5\) | \(2.19 \times 10^5 \leq G \leq 2.28 \times 10^5\) | \(2.23 \times 10^5 \leq G \leq 2.32 \times 10^5\) |
TABLE III. Calculation scheme A. Predictions

|    | $F^0_\pi$ | 92.42 | 93.3 | MeV | $GeV^4$ |
|----|-----------|-------|------|-----|---------|
| $\epsilon_t = \epsilon_I + \epsilon_g + N_f \epsilon_q$ | $-0.00497 - N_f 0.00143$ | $-0.005 - N_f 0.0015$ | $N_f 0.0015$ | $N_f 0.0015$ |
| $\langle 0 | \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{\mu\nu}^a | 0 \rangle$ | $0.01767 + N_f 0.00508$ | $0.01777 + N_f 0.00533$ | $N_f 0.00533$ | $N_f 0.00533$ |
| B | $0.00497 + N_f 0.00143$ | $0.005 + N_f 0.0015$ | $N_f 0.0015$ | $N_f 0.0015$ |
TABLE IV. Calculation scheme B. Predictions

|                      |               |
|----------------------|---------------|
| $F_\pi^0 = 92.42$    |               |
| $-0.00566 - N_f 0.00128 \leq \epsilon_t \leq -0.00384 - N_f 0.0016$ |               |
| $0.0136 + N_f 0.00568 \leq \langle 0|G^2|0 \rangle \leq 0.02 + N_f 0.0045$ |               |
| $0.00384 + N_f 0.0016 \leq B \leq 0.00566 + N_f 0.00128$ |               |
| Calculation scheme B. Predictions |
|-----------------------------------|
| $F_\pi^o = 93.3$                   |
| $-0.00661 - N_f 0.00136 \leq \epsilon_t \leq -0.003837 - N_f 0.0017$ |
| $0.0136 + N_f 0.006 \leq \langle 0|G^2|0 \rangle \leq 0.0235 + N_f 0.0048$ |
| $0.003837 + N_f 0.0017 \leq B \leq 0.00661 + N_f 0.00136$ |
FIGURES

FIG. 1. The SD equation for the quark propagator in momentum space.

FIG. 2. The exact expression for the pion decay constant $F_\pi$ in the current algebra (CA) representation. Here $G^j_5$, $S$ and $J^i_q$ are the pion-quark bound-state wave function, the quark propagator and the axial-vector current, respectively. The slash denotes differentiation with respect to momentum $q_\nu$ and setting $q = 0$.

FIG. 3. The pion decay constant $F_{CA}$ as a function of $k_0$, drawn only for the most reasonable region, selected first for the dynamically generated quark masses (6.17). The interval (6.18) is also explicitly shown.

FIG. 4. Quark condensate $\langle q\bar{q}\rangle_0^{1/3}$ as a function of $k_0$, drawn only for the most reasonable region, selected first for the dynamically generated quark masses (6.17).

FIG. 5. The vacuum energy density, due to confining quarks with dynamically generated masses, $\epsilon_q$ as a function of $k_0$, drawn only for the most reasonable region, selected first for the dynamically generated quark masses (6.17).

FIG. 6. The vacuum energy density of the nonperturbative gluons $\epsilon_g$ as a function of $k_0$, drawn only for the most reasonable region, selected first for the dynamically generated quark masses (6.17). The interval (6.18) is also explicitly shown.

FIG. 7. The vacuum energy density $\epsilon$ as a function of $k_0$, drawn only for the most reasonable region, selected first for the dynamically generated quark masses (6.17).

FIG. 8. The vacuum energy density, due to the nonperturbative gluons contribution, $\epsilon_g$ as a function of $z_0$.

FIG. 9. The vacuum energy density, due to the nonperturbative gluons contribution, $\epsilon_g$ as a function of $k_0$. $\Lambda_c$ is the confinement scale. A newly obtained interval for $m_d$ (6.22) is also shown. A similar figure can be drawn for the case when the pion decay constant is approximated by the experimental value.

FIG. 10. The pion decay constant $F_{CA}$ as a function of $k_0$. $\Lambda_c$ is the confinement scale. A newly obtained interval for $m_d$ (6.22) is also shown. A similar figure can be drawn for the case when the pion decay constant is approximated by the experimental value.
FIG. 11. The simplest contribution to the quark self-energy induced by a virtual (internal) gluon.

FIG. 12. The simplest contributions to the quark self-energy induced by the self-interactions of virtual (internal) gluons.

FIG. 13. The simplest contributions to the quark self-energy induced by an actual (external) gluons emitted by another quark.

FIG. 14. The simplest contributions to the quark self-energy due to the processes of the conversion (transformation) of the virtual gluons into the actual ones and vice versa.

FIG. 15. The quark diagram for the decay of the $J/\Psi$ meson. The dashed line schematically shows the $c\bar{c}$ pair emerged from the nonperturbative QCD vacuum. The true number of intermediate gluons is dictated by colour and charge conservation.

FIG. 16. The quark diagram for the decay of the $\Psi'$ meson. The dashed line schematically shows the $u\bar{u}$ pair emerged from the nonperturbative QCD vacuum.

FIG. 17. The quark diagram for the decay of the pseudoscalar (P) particle. The arrows show the reciprocal orientation of the spins. The dashed line schematically shows the pair emerged from the QCD vacuum. The true number of intermediate gluons is dictated by colour and charge conservation.

FIG. 18. The quark diagram for the decay of the vector (V) meson. The arrows show the reciprocal orientation of the spins. The dashed lines schematically show the pairs emerged from the QCD vacuum. The true number of intermediate gluons is dictated by colour and charge conservation. The intermediate vector state is inevitable.

FIG. 19. Two-particle irreducible (2PI) quark, ghost and gluon vacuum diagrams.

FIG. 20. The interaction of a constituent quark (thick line) and current quark (thin line) with actual gluon.

FIG. 21. The Bethe-Salpeter (BS) integral equation for flavored mesons.

FIG. 22. The BS scattering kernel.