HOMOM/PE Hybrid Algorithm Based Calculation on Ground-Wave of the HF Antennas over Irregular Terrain

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Abstract. The parabolic equation model usually uses a gaussian pattern to fit the actual antenna's pattern in free space to set the initial field, and then uses the image theory to deal with the effect of ground on the antennas. However, this method of setting the initial field cannot consider the specific HF antenna, and the image theory is only applicable to the radio wave propagation of antennas working in the microwave bands and above. The HF antenna is erected low, and the irregular terrain or the buildings around the antenna and their media parameters will affect the current distribution on the antenna, resulting in a large difference with the radiation pattern in the free space. The traditional method of fitting the pattern is no longer accurate. Aiming at the problem of radio wave propagation of HF antennas in the actual installation environment by parabolic equation method, a hybrid algorithm based on higher order-method of moment and parabolic equation method (HOMOM-PE) is proposed. The hybrid algorithm divides the whole calculation region into two regions, which is the inner zone and the outer zone. The inner zone is the zone that has a great influence on the antenna current distribution. In this area, HOMOM is used to model the antenna and the terrain together, and the influence of the complex terrain near the antenna on the antenna is strictly calculated. The PE algorithm is used to calculate the long-distance propagation, and the initial field data is derived from the electromagnetic vector distribution calculated in the inner region. The hybrid algorithm combines the antenna radiation and the wave propagation directly, which solves the problem of the initial field setting of the actual HF antenna under the complicated terrain conditions.

1. Introduction
Radio wave propagation are related to many parameters, such as operating frequency, geological parameters, terrain fluctuations, the position of the transmitting and receiving antennas, the curvature of the Earth, the refractive index of the atmosphere, etc., which complicates the parameters of the wave propagation between the transmitting and receiving antennas. Therefore, an efficient algorithm that can comprehensively consider the influence of various parameters is needed. The parabolic equation model is not limited by wavelength, and can better deal with the influence of atmospheric refractive index distribution, irregular terrain and geological parameters on radio wave propagation. One of the main algorithms for solving the problem of radio wave propagation is split-step Fourier algorithm. The split-step Fourier algorithm is used as a stepping algorithm for parabolic equations. It can be solved step by step to obtain the field distribution on the propagation path once the initial field is got. Therefore, the field distribution at the initial position needs to be obtained first. The commonly used method is the inverse Fourier transform of the antenna pattern in the free space, which is the aperture field distribution on the initial position, and then the effect of the ground on the antenna is
processed by the image theory. However, this method is only applicable to electromagnetic waves with a frequency above 100 MHz and a high erection[1]. The HF antenna is erected low, and the antenna carrier, the nearby buildings and the ground have an influence on the current distribution on the antenna, thereby changing the radiation characteristics of the antenna, resulting in a quite different antenna pattern with that in free space. So, this method of acquiring initial field is not suitable for HF antennas. In the paper [2], the current distribution under the influence of the ground is taken as the initial field to consider the influence of the ground on the antenna. However, this method can only consider the antenna under the flat ground condition and the integral involved in the solution process is very complicated, which is not applicable to engineering applications. In order to consider the influence of complex terrain near the antenna, this paper proposes an algorithm combining high-order moment of method and parabolic equation model to solve the ground wave field of HF antenna over irregular terrain.

2. Fourier Split-step Parabolic Equation Method

In the two-dimensional Cartesian coordinate system, assuming that the electromagnetic field time harmonic factor is $e^{i \omega t}$, where $\omega$ is the angular frequency, the Maxwell equation can be used to derive $\varphi(z, x)$, the field component of the electromagnetic wave during propagation (for horizontally polarized waves, the electric field has only one non-zero component $E_y$; for vertically polarized waves, the magnetic field has only one non-zero component $H_y$, which $\varphi$ is convenient for writing and uniform for replacement.), and the following two-dimensional scalar equations need to be satisfied.

$$
\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k_0^2 n^2 \varphi(z, x) = 0
$$

(1)

Among them $k_0$ is the wave number in vacuum, $n$ is the corrected atmospheric refractive index. To remove the effects of rapidly changing phase terms, introduce a harmonic of the direction $z$:

$$
\varphi(z, x) = e^{ik_0 z} u(z, x)
$$

(2)

Equation (1) can be rewritten as:

$$
\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + 2ik_0 \frac{\partial}{\partial z} + k_0^2 \left( n^2 - 1 \right) \right] u(z, x) = 0
$$

(3)

Factoring equation (3):

$$
\left( \frac{\partial}{\partial z} + ik_0 (1 - Q) \right) \left( \frac{\partial}{\partial z} + ik_0 (1 + Q) \right) u(z, x) = 0
$$

(4)

Where $Q$ is the pseudo-differential operator, it can be defined as equation (5):

$$
Q = \left( 1 + k_0^2 \frac{\partial^2}{\partial x^2} + \left( n^2 - 1 \right) \right)^{1/2}
$$

(5)

The two parts of the equation (4) represent the forward and backward propagation waves respectively, ignoring the backward propagation, then the equation (4) can be simplified as:

$$
\left( \frac{\partial}{\partial z} + ik_0 (1 - Q) \right) u(z, x) = 0
$$

(6)
According to the definition of differential, the equation (6) can be rewritten as:

\[ u(z + \Delta z, x) = \exp(-ik_0z(1 - Q))u(z, x) \quad (7) \]

There are many methods for dealing with \( Q \) factors. Different processing methods will cause the different propagation angles which the parabolic equation model can calculate accurately. Because the lobes of the short-wave antenna are wide and the influence of the ground, the actual lobes of the antenna are upturned. The narrow-approximation (which is limited to a propagation angle of less than 10-15 degrees) is no longer accurate, so a wide-angle approximation is required and \( Q \) will be spread according to the Feit-Fleck model (propagation angle less than 30-40 degrees)[3].

\[ Q \approx Q_{Feit-Fleck} = \left(1 + k_0^{-2} \frac{\partial^2}{\partial z^2}\right)^{\frac{1}{2}} + n - 1 \quad (8) \]

The numerical solution of PE at large scale is commonly used in SSPE algorithm. The algorithm can be solved step by step, and the step size is not restrained by the wavelength of the electromagnetic wave radiated by the antenna. It is especially suitable for the fast calculation of electromagnetic field under large scale. The expression with the wide-angle parabolic equation of the algorithm adopted is equation (9):

\[
\begin{align*}
\exp(-ik_0^2(n - 1)\Delta z) 
\times F^{-1} \left\{ \exp\left[ -ik_0^2 \frac{\Delta z}{k_0 + (k_0^2 - k_n^2)^\frac{1}{2}} \right] F[u(z, x)] \right\}
\end{align*}
\]

Where \( F \) and \( F^{-1} \) represents the Fourier transform and the inverse Fourier transform, respectively, and satisfies the transform domain variable (which is the angle between the electromagnetic wave propagation direction and the horizontal direction). It can be known from equation (9) that after the initial field distribution is given, all field distributions can be calculated by stepping solves.

3. Higher Order-Method of Moment

HO-MOM[4][5], mainly composed of high order hook face and high order basis function[6], is an electromagnetic calculation method of decreasing unknown numbers without reducing calculation accuracy.

![Figure 1. The bilinear surface](image)

High order hook face can be used for better fitting precision and larger facet, therefore reducing numbers of facet and unknown. Its modeling is represented by the bilinear surface[6][7], as shown in Figure 1.

Bilinear surface is a non-planar curved quadrilateral and is uniquely determined by four vertexes. The parametric equation for this quadrilateral is expressed in equation (10).
\[
r(p, s) = \frac{1}{\Delta p \Delta s}[r_{11}(p_2 - p)(s_2 - s) + r_{12}(p_2 - p)(s - s_1)]
+ r_1(p - p_1)(s_2 - s) + r_22(p - p_1)(s - s_1)]
\]
\[
\Delta p = p_2 - p_1; \Delta s = s_2 - s_1; \Delta s = s_2 - s_1; p_1 \leq p \leq p_2; s_1 \leq s \leq s_2
\]

where \(r_{11}, r_{12}, r_{21},\) and \(r_{22}\) are position vectors of the four vertexes, \(p\) and \(s\) are local coordinates. Equation (11) is the transformation of equation (10):

\[
r(p, s) = r_c + r_p p + r_s s + r_{ps} p s
\]

where \(r_c, r_p,\) and \(r_s\) are the vectors expressed in the following equation (12) \~ (15).

\[
r_c = \frac{1}{\Delta p \Delta s}(r_{11}p_2s_2 - r_{12}p_2s_1 - r_{21}p_1s_2 + r_{22}p_1s_1)
\]

\[
r_p = \frac{1}{\Delta p \Delta s}(-r_{11}s_2 + r_{12}s_1 + r_{21}s_2 - r_{22}s_1)
\]

\[
r_s = \frac{1}{\Delta p \Delta s}(-r_{11}p_2 + r_{12}p_2 + r_{21}p_1 - r_{22}p_1)
\]

\[
r_{ps} = \frac{1}{\Delta p \Delta s}(r_{11} - r_{12} - r_{21} + r_{22})
\]

Changing \(r_c, r_p,\) and \(r_{ps}\) brings forth new bilinear surface.

High order basis function, defined in parametrized coordinates, is a polynomial combination function. It fits real current distribution on the target surface by fewer unknown, with better calculation accuracy, efficiency and better convergence. Representing the change of current and magnetic current basing on logical adjusting the order can extended grid length (subdivision size) under traditional MOM to about one wavelength. Generally, only 20 basis functions can fully express the change of current and magnetic current in a square wavelength, which reduces the number of the matrix.

4. The Hybrid SSPE\~HOMOM Algorithm

The hybrid algorithm divides the calculation area into two regions, which are region I and region II as shown in the figure 2. Region I is the terrain near the antenna including the antenna. The terrain or buildings on this region has a great influence on the current distribution of the antenna. According to the calculation, it is sufficient for the conventional situation to take \(3 \sim 5\) wavelengths. The specific range needs to be determined by the calculation result. Region II is a long-distance terrain that does not contain an antenna. The region can be up to several tens of kilometers depending on actual needs. The terrain data of the calculation regions can be obtained by digital map or field measurement.
In the region I, the antenna and terrain modeling are solved by the high-order moment of method, and the electromagnetic vector distribution at the interface between region I and region II is obtained as the initial field of the parabolic equation algorithm in the region II. According to the polarization mode of the electromagnetic wave radiated by the antenna, the relationship between the field vector and the equation (1) is also different. If the field component is not relevant to $y$, the component $E_y$ is corresponding to the horizontally polarized wave and the component $H_y$ is corresponding to the vertically polarized wave.

To verify the accuracy of the hybrid algorithm against the results recommended by ITU-R, establish the same conditions as ITU-R and use the hybrid algorithm to calculate, that is: a short vertical monopole at the surface of the region I, a radiated power of 1 kW; The ground of region I and region II are flat and the media parameters of it is $\varepsilon_r=40, \sigma=0.03 \text{ s/m}$; Region II is calculated under standard atmospheric conditions.

The distance-dependent field strength distribution calculated using the hybrid algorithm is compared with the data of ITU p.368. As shown in the figure 3, the curves in the figure 3 are the comparison results at frequencies of 5MHz, 10MHz, 15MHz and 20MHz, respectively. The calculation results are roughly the same as the field strength data recommended by the ITU, which verifies the accuracy of the hybrid algorithm.
5. Examples

The purpose of the hybrid algorithm is to accurately calculate the influence of the terrain of the inner zone (region I) on the radiation characteristics of the HF antenna by the higher-order moment of method, and then provide an accurate initial field for the parabolic equation method to calculate the characteristic of the propagation of the antenna over long distances. The actual irregular terrain is different, which is not representative. In order to facilitate the analysis of the results, a simple ‘long slope’ terrain is designed. Taking the HF sanxian antenna as an example, the antenna is erected in the middle of the long slope, the height of the antenna is 10 m and the slope is at an angle of 0 degrees (flat terrain), 5 degrees, 10 degrees, 15 degrees and 20 degrees, respectively. The sanxian antenna and long slope terrain model are shown in the figure 4 and figure 5, respectively. The radiant power of the sanxian antenna is set to 1 kW and the frequency is 12 MHz. The ground media parameters of dry soil and wet ground are selected (dry soil: \( \varepsilon_r = 4, \sigma = 0.001 \text{s/m} \); wet ground: \( \varepsilon_r = 30, \sigma = 0.01 \text{s/m} \)). The region I is modeled by the high-order moment method, and the field strength distribution at the boundary is obtained as shown in the figure 6 and figure 7.

It can be seen from the two figures that as the slope angle of the long slope increases, the direction of the maximum field strength at the boundary line between region I and region II has a tendency to shift to the ground, and the bigger the degree of inclination, the more obvious the phenomenon. The electromagnetic wave radiated to the ground is affected by the inclination of the ground, and the reflected wave is lower than the flat elevation angle. The elevation angle of the lobe of the antenna on the side of the slope surface is decreased, and the degree of decline is positively correlated with the inclination angle of the slope. Comparing the field strength distribution under different ground types, the field strength under the wet ground is larger than that of the dry ground. This is because the conductivity of the wet ground is higher than that of the dry ground. For the sanxian antenna, the polarization is horizontal. Horizontally polarized waves are greatly affected by the ground conductivity, and the loss under wet ground is smaller than that of dry land. Figure 6 and figure 7 confirms that the irregular terrain and ground media parameters near the antenna affect the radiation characteristics of the antenna, indicating the importance of accurately solving the initial field.

In order to verify the accuracy of the hybrid algorithm, the high-order moment method is used to solve the region I and the region II together. Since the region II modeling of the irregular terrain is too large for the computer memory and computing power, the region II is set to be flat. Because the high-order moment of method cannot handle the curvature of the earth, the field strength calculation results are chosen not far from the antenna. The hybrid algorithm is used to calculate the propagation characteristics of the two ground conditions and different slope angles. Field strength, which is distributed with height at range 10 km, is shown in the figure 8 and figure 9.

The figure 8 and figure 9 show that the hybrid algorithm and the pure HOMOM calculation result are in good agreement, and the field strength distribution increases with the slope inclination angle, which is caused by the increase of the antenna with the low elevation angle of the antenna as the slope angle increases. In line with the actual situation; the field strength under the wet ground is enhanced
compared with the dry ground surface under the same conditions, but it is not obvious. This is because
the maximum radiation direction of the three-wire antenna is not in the horizontal direction, and the
lobes rise due to the influence of the ground. When the long-distance propagating, the lobe is far away
from the ground, and the distribution of the field strength at a higher altitude is less affected by the
ground medium parameters.

The parabolic equation model treats obstacles such as buildings as just irregular terrain, while the
actual antenna or building antenna itself is usually installed near the roof of the building. The media
parameters of the building itself are quite different from the ground. In this case, the traditional
parabolic equation method cannot accurately model the building, nor can it consider the influence of
the building closer to the antenna on the antenna current distribution, so the calculation result is not
accurate.

And the parabolic equation method uses an approximation method that assumes that the
electromagnetic wave propagates in the axial direction, although the Feit-Fleck wide-angle
approximation can relax the accurately calculated propagation angle limit to 40 degrees, but for
buildings that are closer to the antenna or the terrain is still beyond the range of the propagation angle,
resulting in a parabolic equation model that relies on the stepping algorithm and the results are far
from the actual results. For this case, the hybrid algorithm is a good solution. The antenna or the
antenna near the building or the steep terrain is solved by the high-order moment method, and the
near-field distribution is substituted into the region II. This is the case: At a distance of 75 meters from
the antenna, there is a building 25 meters high, 75 meters long and 40 meters wide. The medium
parameters of the building are \( \varepsilon_r=3.28, \sigma=6.15\times10^{-7}/s/m \).

It can be seen from the figure 10 that the building has a great influence on the radiation
characteristics of the antenna. When there is occlusion of the building and no occlusion of the building,
the field strength distribution differs by about 7-8 dB, and the influence of different ground
characteristics on the propagation characteristics is not the same, it can be seen that the building has a
great influence on the radiation characteristics of the antenna, especially in buildings close to the
antenna, the influence of the building must be considered in the calculation of the wave propagation.

6. Conclusions
In this paper, a hybrid algorithm based on the higher-order moment of method and the Fourier step-by-
step algorithm of parabolic equation is proposed. Firstly, the calculation region is divided into two
parts: the inner zone and the outer zone. The inner zone uses the higher-order moment method to
obtain the field vector distribution at the interface. It is used as the initial field of the parabolic
equation method in the outer region, and then the ground wave field in the outer region is calculated
by the parabolic equation method. The method overcomes the problem of excessive memory and
computational power when solving the large-scale electromagnetic radiation problem by the high-
order moment method, and also solves the problem that the traditional parabolic equation method
cannot accurately solve the initial field problem of the short-wave antenna, and the existence of the
surrounding antenna. Radiation calculation defects of complex terrain or building conditions achieve a
balance between the accuracy and efficiency of the algorithm, and can effectively solve the problem of
solving the far field of the actual HF antennas in the undulating terrain.

The algorithm can also combine digital maps or field-measured topographic data to obtain a
plurality of two-dimensional topographic data according to azimuth splitting. Parallel calculation of
the coverage distribution by parabolic equations in each section can be used as the focus of further
research.
Figure 10. The effect of buildings on field strength

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