Controlled light storage in a double lambda system

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Abstract

It is shown theoretically that after light storing in a medium of four-level atoms it is possible to release a new pulse of a different frequency, the process being steered by another driving beam. It is also possible to store one pulse and to release two different ones, with their time separation and heights being controlled.

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It has been shown both theoretically and experimentally (see, e.g., Refs [1, 2] and a review [3]) that a light pulse, propagating in a medium composed of three-level atoms in a Λ configuration, suitably driven by another pulse, can be stopped and later released in a controlled way. The process is interpreted in terms of inducing a transient Raman coherence between two lower atomic states or, in the language of quasi-particles, in terms of an adiabatic evolution of the so-called dark-state polariton [4, 5]. Natural questions arise whether it is possible to convert the stored light in a controlled way into a pulse of a frequency different from that of the stopped one, or into more pulses of different frequencies, or what happens if the Raman coherence is due to a transient absorption of more than one pulse. Some experimental results concerning light frequency conversion during the process of its storing and retrieving in a four-level system have recently been mentioned by Matsko et al. [6]. In this paper we present a complete theoretical analysis of the situations when a four-level-atom medium is driven by two control laser fields and, after stopping one or two probe pulses, one or two pulses can be released. We also point out the possibility of controlling the process by a relative time shift of the control fields.

Consider a quasi one-dimensional medium of four-level atoms with two lower metastable states |b⟩ and |c⟩ and two upper states |a⟩ and |d⟩ (Fig. 1). The position of an atom is described by the variable z, which is considered continuous. The weak signal fields 1 and 3 couple the initial state |b⟩ with |a⟩ and |d⟩, while the control fields 2 and 4 couple |c⟩ with |a⟩ and |d⟩, respectively. The interaction Hamiltonian is 

\[ V = -\hat{d}\sum_{j=1}^{d} \epsilon_{j} \cos \phi_{j}, \]

with \( \phi_{j} = \omega_{j} t - k_{j} z \), \( \epsilon_{j} = \epsilon_{j}(z, t) \) being slowly varying envelopes and all the fields having the same linear polarization. The matrix elements of the dipole moment \( \hat{d}_{1} = (\hat{d})_{ab}, \hat{d}_{2} = (\hat{d})_{ac}, \hat{d}_{3} = (\hat{d})_{bd}, \hat{d}_{4} = (\hat{d})_{cd} \) are taken real. Resonant conditions concerning all the couplings are assumed, i.e. \( h\omega_{1} = E_{a} - E_{b}, h\omega_{2} = E_{a} - E_{c}, h\omega_{3} = E_{d} - E_{b}, h\omega_{4} = E_{d} - E_{c} \).

The evolution equation \( i\hbar \dot{\rho} = [H, \rho] \) for the density matrix \( \rho = \rho(z, t) \) for an atom at position \( z \), after making the rotating-wave approximation, transforming-off the rapidly oscillating factors: \( \rho_{ab} = \sigma_{ab} \exp(-i\phi_{1}), \rho_{ac} = \sigma_{ac} \exp(-i\phi_{2}), \rho_{bc} = \sigma_{bc} \exp[i(\phi_{1} - \phi_{2})], \rho_{db} = \sigma_{db} \exp(-i\phi_{3}), \rho_{dc} = \sigma_{dc} \exp(-i\phi_{4}), \rho_{ad} = \sigma_{ad} \exp[i(\phi_{3} - \phi_{1})], \rho_{ii} = \sigma_{ii} \), and after adding relaxation terms describing the spontaneous emission within the system, takes the form

\[
\begin{align*}
\dot{\sigma}_{ab} &= -\frac{1}{2} \epsilon_{1} d_{1}(\sigma_{ba} - \sigma_{ab}) - \frac{1}{2} \epsilon_{2} d_{2}(\sigma_{ca} - \sigma_{ac}) - i\hbar(\Gamma_{b}^{a} + \Gamma_{c}^{a})\sigma_{ab}, \\
\dot{\sigma}_{bb} &= -\frac{1}{2} \epsilon_{1} d_{1}(\sigma_{ab} - \sigma_{ba}) - \frac{1}{2} \epsilon_{3} d_{3}(\sigma_{db} - \sigma_{bd}) + i\hbar\Gamma_{b}^{a}\sigma_{ba} + i\hbar\Gamma_{b}^{d}\sigma_{dd}, \\
\dot{\sigma}_{cc} &= -\frac{1}{2} \epsilon_{2} d_{2}(\sigma_{ac} - \sigma_{ca}) - \frac{1}{2} \epsilon_{4} d_{4}(\sigma_{dc} - \sigma_{cd}) + i\hbar\Gamma_{c}^{a}\sigma_{ca} + i\hbar\Gamma_{c}^{d}\sigma_{dd}, \\
\dot{\sigma}_{dd} &= -\frac{1}{2} \epsilon_{3} d_{3}(\sigma_{bd} - \sigma_{db}) - \frac{1}{2} \epsilon_{4} d_{4}(\sigma_{cd} - \sigma_{dc}) - i\hbar(\Gamma_{b}^{d} + \Gamma_{c}^{d})\sigma_{dd}, \\
\dot{\sigma}_{ab} &= -\frac{1}{2} \epsilon_{1} d_{1}(\sigma_{bb} - \sigma_{aa}) - \frac{1}{2} \epsilon_{2} d_{2}(\sigma_{ca} - \sigma_{ac}) + \frac{1}{2} \epsilon_{3} d_{3}\sigma_{ad} - \frac{i\hbar}{2}(\Gamma_{b}^{a} + \Gamma_{c}^{a})\sigma_{ab}, \\
\dot{\sigma}_{ac} &= -\frac{1}{2} \epsilon_{1} d_{1}\sigma_{bc} - \frac{1}{2} \epsilon_{2} d_{2}(\sigma_{cc} - \sigma_{aa}) + \frac{1}{2} \epsilon_{3} d_{3}\sigma_{ad} - \frac{i\hbar}{2}(\Gamma_{b}^{a} + \Gamma_{c}^{a})\sigma_{ac}, \\
\dot{\sigma}_{ad} &= -\frac{1}{2} \epsilon_{1} d_{1}\sigma_{bd} - \frac{1}{2} \epsilon_{2} d_{2}(\sigma_{cd} - \sigma_{ac}) + \frac{1}{2} \epsilon_{3} d_{3}\sigma_{ab} - \frac{i\hbar}{2}(\Gamma_{b}^{a} + \Gamma_{c}^{a} + \Gamma_{b}^{d} + \Gamma_{c}^{d})\sigma_{ad}, \\
\dot{\sigma}_{bc} &= -\frac{1}{2} \epsilon_{1} d_{1}\sigma_{ac} + \frac{1}{2} \epsilon_{2} d_{2}\sigma_{ba} - \frac{1}{2} \epsilon_{3} d_{3}\sigma_{dc} + \frac{1}{2} \epsilon_{4} d_{4}\sigma_{bd},
\end{align*}
\]
\[ i\hbar \dot{\sigma}_{bd} = -\frac{1}{2} \epsilon_1 d_1 \sigma_{ad} - \frac{1}{2} \epsilon_3 d_3 (\sigma_{dd} - \sigma_{bb}) + \frac{1}{2} \epsilon_4 d_4 \sigma_{bc} - \frac{i\hbar}{2} (\Gamma_b^d + \Gamma_c^d) \sigma_{bd}, \]
\[ i\hbar \dot{\sigma}_{cd} = -\frac{1}{2} \epsilon_2 d_2 \sigma_{ad} + \frac{1}{2} \epsilon_3 d_3 \sigma_{cb} - \frac{1}{2} \epsilon_4 d_4 (\sigma_{dd} - \sigma_{cc}) - \frac{i\hbar}{2} (\Gamma_b^d + \Gamma_c^d) \sigma_{cd}, \]

where \( \Gamma_b^a \) is the decay rate of the state \( |a\rangle \to |b\rangle \)

each.

The propagation equations for the signal fields 1 and 3 are written as usual in the slowly varying envelope approximation [3] after rejecting the second space and time derivatives of \( \epsilon_j \). In the conditions of the resonance they read

\[ \frac{\partial \epsilon_1}{\partial z} + \frac{1}{c} \frac{\partial \epsilon_1}{\partial t} = iN d_1 \frac{\omega_1}{\epsilon_0 c} \sigma_{ab}, \]
\[ \frac{\partial \epsilon_3}{\partial z} + \frac{1}{c} \frac{\partial \epsilon_3}{\partial t} = iN d_3 \frac{\omega_3}{\epsilon_0 c} \sigma_{bd}, \]

(2)

where \( N \) is the atom density, \( \epsilon_0 \) is the vacuum electric permittivity and use has been made of the fact that in the resonance conditions \( \sigma_{ab} \) and \( \sigma_{db} \) are imaginary numbers. Similarly as in earlier papers, we have neglected propagation effects for the driving fields, i.e. \( \epsilon_{2,4} = \epsilon_{2,4}(t) \).

Eqs (1) and (2) have been solved numerically in the moving window frame of reference: \( t' = t - z/c, z' = z \) using the method described by Shore [3]. Switching the driving fields on and/or off was modeled by a hyperbolic tangent. The initial probe pulse was taken as the sine square shape \( \epsilon_1(0,t) = \epsilon_{10} \sin^2[\pi(t - t_2)/(t_2 - t_1)]\Theta(t - t_1)\Theta(t_2 - t) \), while the initial condition for the atomic part was \( \sigma_{bb}(z,0) = 1 \), with other matrix elements equal to zero.

We have performed simulations for somewhat arbitrarily chosen data, being however of realistic orders of magnitude. The model atomic energies were \( E_a = -0.10 \) a.u., \( E_b = -0.20 \) a.u., \( E_c = -0.18 \) a.u., \( E_d = -0.05 \) a.u., all the relaxation rates due to the spontaneous emission were taken \( \Gamma_{a,b,c} = 2.4 \times 10^{-9} \) a.u., from which the corresponding transition dipole moments were calculated. The length of the atomic sample was \( 3 \times 10^7 \) a.u. (1.6 mm) and its density \( 3 \times 10^{-13} \) a.u. (2 \times 10^{12} \text{ cm}^{-3} ). The initial signal pulse length was \( 10^{11} \) a.u. (2.4 \mu s) and \( \epsilon_{10} = 10^{-10} \) a.u. (which corresponds to the power density of \( 3.5 \times 10^{-4} \text{ W cm}^{-2} \)); the maximum value of the control field amplitudes was \( 1.2 \times 10^{-9} \) a.u. (50 mWcm^{-2}).

The upper part of Fig. 2 shows for comparison the results obtained by solving Eqs (1) and (2) in the case corresponding to the recently studied light storage in a single \( \Lambda \) system (cf. the results of Ref. [4]). The pulse’s electric field is shown as a function of the ”local” time \( t' \) (which for our data is almost equal to \( t \)). The left peak of the signal pulse is the untrapped fraction of the incoming pulse, i.e. its fraction transmitted by the medium before the control pulse has been switched off. The rest of the pulse is trapped and later released (the right peak) due to turning the control pulse back on. We have checked that, as expected, the time interval between the left and right peaks is the same as that between the instants of switching the control pulse off and on.

The lower part of Fig. 3 shows the new result predicted in this paper. As before, the signal pulse \( \epsilon_1 \) is trapped due to switching the control field \( \epsilon_2 \) off and its untrapped part is observed. However, in this case, after some time the other control field \( \epsilon_4 \) is turned on, which results in generating a new pulse \( \epsilon_3 \) of frequency different from that of the original trapped signal pulse. Also in this situation the instant of appearing of the released pulse (now the field 3) is controlled by choosing the moment of the switch-on of the driving field (now the field 4). Numerical calculations show that the height and shape of this new released pulse
strongly depend on the process parameters, in particular on the maximum value and slope of the control field $\epsilon_4$.

From our observations it follows that it should be possible to trap a single pulse and then to release two or more pulses. Such a possibility is demonstrated in Figs 3a and 3b, which show the released pulses in the cases in which the driving field $\epsilon_4$ was switched on, respectively, before and after the driving field $\epsilon_2$. The difference of the pulses heights shows that by choosing the order of the turn-on of the driving pulses and their time delay we can influence the fraction of the atomic coherence taken over by each of the released pulses. We have also obtained the results (not shown here) which prove that it is possible to simultaneously store two pulses and release one or two new ones.

Extrapolating the interpretation concerning a single $\Lambda$ system one may look, analogously as in Ref. [1], for the solutions of Eqs (1) and (2), assuming a perturbative, adiabatic and relaxationless evolution. It follows then that in particular

$$\sigma_{bc} = -\frac{\epsilon_1 d_1}{\epsilon_2 d_2} = -\frac{\epsilon_3 d_3}{\epsilon_4 d_4}. \tag{3}$$

Those approximations allow one to find a shape-preserving solution of the corresponding Maxwell-Bloch equations $\Psi(z, t) = \Psi(z - \int_0^t v(t') dt', t = 0)$, $\Psi$ being a combination of field and atomic variables

$$\Psi = \sqrt{\frac{d^2 \omega_1}{d^2 \omega_3} + \frac{2N \hbar \omega_1}{\epsilon_0} + \frac{d^2 \omega_1}{d^2 \omega_3} \epsilon_4^2}. \tag{4}$$

The velocity $v$ is given by

$$v = \frac{d^2 \epsilon_2}{d^2 \epsilon_1} + \frac{2N \hbar \omega_1}{\epsilon_0} + \frac{d^2 \omega_1}{d^2 \omega_3} \epsilon_4^2. \tag{5}$$

However, after switching the field 4 on, the solution $\Psi$ tends to $\sqrt{\frac{\omega_1}{\omega_3}} \epsilon_3$ instead of $\epsilon_3$, which means that this solution cannot fully characterize the pulse release phase of the evolution. The latter would be described by an adiabatic evolution of $\Psi' = \sqrt{\frac{\omega_1}{\omega_3}} \Psi$. This means that light storing and releasing in a double $\Lambda$ system must include a nonadiabatic phase; the measure of nonadiabaticity is the ratio of the frequencies of the two signals. This has been checked numerically: we have compared the computed coherence $\sigma_{bc}(t')$ with its adiabatic approximations given by both parts of Eq. (3). While the evolution of $\sigma_{bc}$ in the storage phase was well reproduced by the part of Eq. (3) including the fields 1 and 2, this was not the case for the part including the fields 3 and 4 in the release phase. The coherence calculated from the two parts of Eq. (3) exhibited a discontinuity, which could be reduced by artificially introducing the factor $\sqrt{\frac{\omega_1}{\omega_3}}$. The failure of the adiabatic approximation in this case is connected with the fact that a four level atom in the conditions of resonance, dressed by the four interactions (1-4) satisfying the second equality in Eq. (3), has a double real eigenvalue (equal to the bare energies). Thus the assumptions of the adiabatic theorem are not satisfied. Considerations analogous to those presented above but performed in the formalism of second quantization would allow an interpretation of the process in terms of
quasiparticles. As in the case of a single Λ both the storage and release phases can be seen as an adiabatic evolution of a dark state polaritons, but here the two polaritons in the two phases are not identical and a nonadiabatic transformation of one into another must take place.

The above results suggest new interesting possibilities of controlling light propagation effects. A systematic quantitative analysis of the dependence of the pulse shapes, as well as of the time evolution of the atomic properties, on the numerous accessible control parameters will be the subject of a future work.
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FIGURES

FIG. 1. The double Λ scheme of levels and couplings; the indices 1 and 3 refer to signal fields and 2 and 4 - to control fields.

FIG. 2. The shape of the signal pulses at the end of the sample as the function of the "local" time $t'$; also shown are the control pulses (for pictorial reasons their values were reduced by the factor of 0.03); (a) the case of a single Λ system: $\epsilon_{3,4} \equiv 0$, $\epsilon_1$, solid line; $\epsilon_2$, dashed line; for comparison is also shown the shape of initial pulse, dotted line; (b) the case of a double Λ system: the transmitted part of the original signal pulse $\epsilon_1$, solid line; the newly created signal pulse $\epsilon_3$, dashed line; $\epsilon_2$, short-dashed line; $\epsilon_4$, dotted line.

FIG. 3. The shape of the released pulses depending on the order of switching on of the control pulses: (a) the pulse (4) switched on before the pulse (2); (b) the pulse (4) switched on after the pulse (2). The line styles as in Fig. 2b.
field amplitude (10^{-11} \text{ a.u.})

(a)

(b)
