Stability, dark energy parameterization and swampland aspect of Bianchi Type-\textit{V}\textsubscript{h} cosmological models with f(R, T)-gravity

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Abstract

Stability, dark energy (DE) parameterization and swampland aspects for the Bianchi form-\textit{V}\textsubscript{h} universe have been formulated in an extended gravity hypothesis. Here we have assumed a minimally coupled geometry field with a rescaled function of \(f(R, T)\) replaced in the geometric action by the Ricci scalar \(R\). Exact solutions are sought under certain basic conditions for the related field equations. For the following theoretically valid premises, the field equations in this scalar-tensor theory have been solved. It is observed under appropriate conditions that our model shows a decelerating to accelerating phase transition property. Results are observed to be coherent with recent observations. Here, our models predict that the universe’s rate of expansion will increase with the passage of time. The physical and geometric aspects of the models are discussed in detail. In this model, we also analyze the parameterizations of dark energy by fitting the EoS parameter \(\omega(z)\) with redshift. The results obtained would be useful in clarifying the relationship between dark energy parameters. In this, we also explore the correspondence of quintessence dark energy with swampland criteria. The swampland criteria have been also shown the nature of the scalar field and the potential of the scalar field.

Keywords: Bianchi type-\textit{V}\textsubscript{h} space-time; \(f(R, T)\) gravity; dark energy; EoS parameterization.

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1 Introduction

Our view of cosmology has been revolutionized by recent cosmological observations. This appears that there is an exponential expansion of the currently observable universe \cite{1-3}. The source that drives this acceleration is termed as ‘dark energy’, whose beginning in present-day cosmology is still a mystery. This is a result of the way that we do not have, up until now, a predictable hypothesis of quantum gravity. New prove from astronomy and cosmology has recently revealed a rather surprising image of the Universe. Our new datasets from different sources, for example, Cosmic Microwave Background Radiation (CMBR) and Supernovae, imply that the universe’s vitality spending plan resembles as: 4% typical baryonic matter, 22% dark matter, and 74% dark energy \cite{4-7}.

One of the hot topic issues between cosmologists and astrophysicists is the accelerated problem of expansion of the universe. The common view suggests that it is dark energy and updated alternative theories to explain the expansion of the universe. Big alternative theories are \(f(R)\) \cite{8} theory, \(f(G)\) \cite{9} theory, \(f(R, T)\) \cite{10} theory, and so on. Author \cite{10} proposes the concept of \(f(R, T)\) to describe accelerated universe expansion. They come up with three models to overcome the argument. They also constructed a remodelled theory of F(R, T) gravity in which the gravitational Lagrangian is described by an arbitrary function of The Ricci scalar \(R\) and the stress energy tensor trace \(T\). Bianchi type models with anisotropic spatial segments are fascinating as in they are more broad than the Friedman models. Despite the fact that there is a solid discussion going on the reasonability of Bianchi type models \cite{11}, these models can be helpful in the portrayal of early inflationary stage and with appropriate component can be diminished to isotropic conduct at late occasions. In the system of f(R) gravity, numerous authors have examined various parts of Bianchi type models \cite{12-14}. As of late anisotropic cosmological arrangements in \(f(R)\) gravity have gotten in \cite{15}. 


Bianchi type models are anisotropic and homogeneous. Such models are nine altogether, yet their grouping permits them to be divided into two classes. Bianchi (I, II, VII, VIII and IX) models are in class A and Bianchi (III, IV, V, VI and VII) are in class B. Spatially homogeneous cosmological models assume a significant job in clarifying the structure and space properties of all Einstein field conditions and cosmological arrangements. Bianchi type-I axially symmetrical cosmological models with a magnetic field investigated\(^{16}\) and string cosmology in Bianchi III and VI\(_0\) has discussed in\(^{17-18}\). Bianchi type-III cosmological model in \(f(R,T)\) hypothesis of gravity have acquired in\(^{19}\). Bianchi types-I and V cosmological models in \(f(R,T)\) gravity have acquired careful arrangements of\(^{20}\) and\(^{21}\) have considered another class of cosmological models in \(f(R,T)\) gravity. Recently, Bianchi type VI\(_h\) universe model in \(f(R,T)\) gravity has been studied by\(^{22}\). There are not many studies in modified theories in the literature on the Bianchi form VI\(_h\) metric. Then in this analysis we investigated the distribution of MSQM in \(f(R,T)\) gravitation theory for Bianchi VI\(_h\) universe\(^{23}\). Many experiments have been carried out in this theory to study the dynamical aspects of anisotropic cosmological models\(^{24}-28\).

Bianchi type VI\(_h\) cosmological model with perfect fluid as an origin of gravitational field is given in the theory of self-creation for various cases of matter discussed by\(^{29-30}\). This addresses the model’s physical and mathematical properties. The objective of the paper is to study the Bianchi type VI\(_h\) model as suggested by Harko et al.\(^{10}\) as part of an extended theory of gravity. In the present work, our inspiration is to build up a general formalism to research these anisotropic models with a simulated time-differing deceleration parameter. In earlier works\(^{31-33}\), the idea of this scale factor was already conceived. Time varying \(DP\) is required to explain a progress of the universe from a decelerated stage to a accelerated stage at an ongoing age. The other purpose of this paper is to study the swampland criteria in Bianchi models. In this way, the Swampland criteria can be utilized to oblige the quintessence \(DE\) models that begin from a scalar field theory. In literature there are lots of works that address the Swampland criteria and quintessence models\(^{34-37}\). Here now we investigate whether the dark energy complies with the swampland criteria. The string swampland criteria for a powerful field theory to be reliable with the string theory.

Right now explore the consistency of the dark energy with swampland measure. Also we investigate the variation of dark energy condition of state parameter. Here we compute the variations of \(\omega\) with \(z\) for dark energy model. A comparable test for swampland parameters for the scalar quintessence model has been performed in a prior work\(^{38}\). They have been considered a core of this research scope and investigated the swampland criteria for the quintessence dark energy scope. In doing so they have taken the trial limits by composing the varieties of dark energy condition of state in the standard CPL parameterization shape and afterward make an interpretation of it to acquire an upper bound of a recreated condition of \(\omega(z)\). Here we specified two parameters \(\omega_0\) and \(\omega_s\), dark energy parameters, with the recent available SNe Ia, BAO and plank (2018) observation data. Chevallier-Polarski-Linder (CPL) parameterization\(^{39-42}\), Jassal-Bagla-Padmanabhan (JBP)\(^{43}\), Barboza-Alcaniz parameterization\(^{44}\) are PADE-I and PADE-II\(^{45}\). These are some well known and most used dark energy parameterization in this series. We used these five well-known parameterization of dark energy, namely CPL, JBP, BA, PADE-I and PADE-II in our model and also find the swampland correspondence with our related model.

The paper is structured as follows: For Sec. 2, in the context of \(f(R,T)\) gravity, we have developed the basic field equations and derived the related geometric parameters. Section 3 describes the dynamics of the model. Stability and physical acceptability of the solution are discussed in Sec. 4. Parameterization of dark energy is addressed in Sec. 5. Section 6 shows the correspondence of swampland criteria of dark energy. In the last section 7, the description and conclusion are given.

## 2 Metric and Field Equations

Right now in this section we talk about the formalism created in an insignificantly coupling theory of \(f(R,T)\) to investigate certain models. We consider a Bianchi VI\(_h\) space time

\[
\begin{align*}
    ds^2 = dt^2 - A^2 dx^2 - B^2 e^{2x} dy^2 - C^2 e^{2hx} dz^2,
\end{align*}
\]

where \(A(t), B(t)\) and \(C(t)\) are functions of the cosmic time \(t\). The space-time exponent \(h\) is taken the value as \(-1,0,1\). Here, we considered the \(h = -1\) because of the significance of the metric that envisages an isolated universe with invalid total energy and momentum\(^{46-47}\).
\[ T_{ij} = (p + \rho)u_i u_j - \rho_B x_i x_j - p g_{ij}. \]  

Here \( u^i x_i = 0 \) and \( x^i x_i = -u^i u_i = -1 \). The four velocity vector of the fluid in a co-moving model is \( u^i = \delta_i^0 \). \( x^i \) is anisotropic fluid direction and orthogonal to \( u^i \). Such as the Perfect fluid and anisotropic fluid (\( \rho_B \)), with energy density (\( \rho \)). Harko et al. [10] proposed the principle of \( f(R, T) \) theory on the interaction of matter and geometry. The four-dimensional Einstein-Hilbert movement is composed as:

\[ S = \frac{1}{16\pi} \int d^4 x \sqrt{-g} f(R, T) + \int d^4 x \sqrt{-g} L_m, \]  

where \( f(R, T) \) is a function of \( T(= g_{ij} T^{ij}) \) and Ricci scalar \( R \) in the operation, \( T^{ij} \) is the energy- momentum tensor. It is possible to take Lagrangian \( L \) as \([25]-[48]\) after our earlier works. The four-dimensional Einstein-Hilbert movement is composed as:

\[ x \]

From Eq. (5), however, is a rescaled generalization of GR equations. The field Eq. (5) can derive from a specific choice of \( f(R, T) = \lambda (R + T) \)

\[ G_{ij} = \left( \frac{8\pi + \lambda}{\lambda} \right) T_{ij} + \Delta(T) g_{ij}, \]  

where the corresponding partial differentiations are \( f_R = \frac{\partial f(R)}{\partial R} \) and \( f_T = \frac{\partial f(T)}{\partial T} \). The field equations [48] derive from a specific choice of \( f(R, T) = \lambda (R + T) \)

\[ \dot{H}_x + H_x^2 \dot{H}_y + \dot{H}_y + H_y H_z + \frac{1}{A^2} = - \left( \frac{16\pi + 3\lambda}{2\lambda} \right) (p - \rho_B) + \frac{\rho B}{2}, \]  

\[ \dot{H}_x + H_x^2 \dot{H}_y + \dot{H}_y + H_y H_z - \frac{1}{A^2} = - \left( \frac{16\pi + 3\lambda}{2\lambda} \right) p + \left( \frac{\rho_B + \rho}{2} \right), \]  

\[ \dot{H}_x + H_x^2 \dot{H}_y + \dot{H}_y + H_y H_z - \frac{1}{A^2} = - \left( \frac{16\pi + 3\lambda}{2\lambda} \right) p + \left( \frac{\rho_B + \rho}{2} \right), \]  

\[ H_x H_y + H_y H_z + H_z H_z - \frac{1}{A^2} = \left( \frac{16\pi + 3\lambda}{2\lambda} \right) \rho - \left( \frac{p - \rho_B}{2} \right), \]  

\[ H_y - H_z = 0. \]  

Here \( \lambda \) is the non-zero scale factor. Directional Hubble parameters for the anisotropic model are \( H_x = \frac{\dot{A}}{A} \), \( H_y = \frac{\dot{B}}{B} \) and \( H_z = \frac{\dot{C}}{C} \). In view of Eq. (10) we have \( H_y = H_z \). Assuming \( H_x = m H_z \) for \( m \neq 1 \), \( H \) is the mean Hubble parameter can be defined as:

\[ H = \frac{\dot{a}}{a} = \frac{1}{3} (H_x + H_y + H_z) = \frac{1}{3} \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right). \]  

By using Eq. (10), we get

\[ H = \frac{\dot{a}}{a} = \frac{1}{3} (H_x + 2H_y) = \left( \frac{m + 2}{3} \right) H_z. \]  

Here \( q \) is as a linear function of \( H \) then
\[ q = -\frac{\ddot{a}}{a^2} = \frac{\beta}{\sqrt{2\beta t + k}} - 1. \]  

(13)

In addition, as indicated by SNe Ia’s despise resent observation, the present universe is extending and the estimation of DP is in the scope of \(-1 < q < 0\) sooner or later. This is the reason our model is perfect with resent observation, in every one of the three circumstances.

\[ q_0 = -1 + \beta H_0 = -1 + \frac{\beta}{\sqrt{2\beta t_0 + k}}. \]  

(14)

Since the present estimation of declaration parameter can be taken as here \(k\) and \(\beta\) are positive constants.

From Eq. (13) we observed that \(q > 0\) for \(\frac{\beta}{\sqrt{2\beta t + k}} < 1\) and \(q < 0\) for \(\frac{\beta}{\sqrt{2\beta t + k}} > 1\). so we can choose a value of \(\beta\) and \(k\).

Integrating Eq. (13) then we get a scale factor \((a)\) which is depends on time.

\[ a(t) = e^{\frac{3}{2}\sqrt{2\beta t + k}}. \]  

(15)

We can express the directional Hubble parameters as \(H_x = e^{-\frac{3m}{m+2}\sqrt{2\beta t + k}}\) and \(H_y = H_z = e^{-\frac{3m}{m+2}\sqrt{2\beta t + k}}\), so that the metric (11) can be written as

\[ ds^2 = dt^2 - (e^{\frac{3m}{m+2}\sqrt{2\beta t + k}})dx^2 - (e^{\frac{3m}{m+2}\sqrt{2\beta t + k}})e^{2x}dy^2 - (e^{\frac{3m}{m+2}\sqrt{2\beta t + k}})e^{2hx}dz^2. \]  

(16)

The spatial volume \(V\) is given as

\[ V = (AB^2) = a^3(t). \]  

(17)

The Hubble parameter is defined as:

\[ H = \frac{1}{3}(H_x + 2H_y) = \frac{1}{\sqrt{2\beta t + k}}. \]  

(18)

The scalar expansion in the universe is

\[ \theta = 3H = (H_x + 2H_y) = \frac{3}{\sqrt{2\beta t + k}}. \]  

(19)

The shear scalar is

\[ \sigma^2 = \frac{3}{m+2} \left( \frac{m-1}{m+2} \right)^2 \frac{1}{\sqrt{2\beta t + k}}. \]  

(20)

The cosmic jerk parameter \(j\) in cosmology is characterized as the third derivative of the scale factor concerning the astronomical time is given as

\[ j = \frac{1}{H^3} \left( \frac{\dddot{a}}{a} \right) = \left( q + 2q^2 - 2 \frac{\dot{q}}{H} \right). \]  

(21)

In cosmology the jerk parameter is used to describe the models close to \(\Lambda\)CDM. It is acknowledged that for models with negative estimation of the deceleration parameter and positive estimation of the jerk parameter, the transition of the universe from decelerated stage to accelerated stage happens. The \(\Lambda\)CDM jerk parameter has consistent with \(j = 1\). Right now, we get

\[ j = 1 - \frac{3\beta}{\sqrt{2\beta t + k}} + \left( \frac{3\beta^2}{2\beta t + k} \right). \]  

(22)

Here we take three observational data and find the value of \((\beta, k)\) to be the best fit with the latest observations and considered for drawn all figures.
3 Dynamics of the Model

In this section we can obtain physical and kinematic parameters from the Eq. (6)-(11).

\[ \rho = \frac{6}{1 - 4\alpha^2} \left[ \frac{2}{m + 2} \left( \dot{H} + H^2 \right) + \frac{(5 - 2m) - 6\alpha(2m + 1)}{(m + 2)^2} H^2 \right] + \frac{2a - \frac{6m}{1 - 2\alpha}}{1 - 2\alpha} \]  

(23)

\[ \rho_B = \frac{6}{1 - 2\alpha} \left[ \frac{m - 1}{m + 2} \left( \dot{H} + 3H^2 \right) \right] - \frac{4a - \frac{6m}{1 - 2\alpha}}{1 - 2\alpha} \]  

(24)

\[ p = \frac{6}{1 - 4\alpha^2} \left[ \frac{(m - 1) + 2\alpha(m + 1)}{m + 2} \left( \dot{H} + H^2 \right) + \frac{(2m^2 - 4m - 7) + 2\alpha(2m^2 + 1)}{(m + 2)^2} H^2 \right] - \frac{2a - \frac{6m}{1 - 2\alpha}}{1 - 2\alpha} \]  

(25)

The effective cosmological constant \( \Lambda \) and EoS parameter \( (\omega) \) are other complex features of the model. Such parameters are calculated using the scale function.

\[ \omega = \frac{(1 + 2\alpha)(3m^2 + 3m + 2)(\dot{H} + H^2) + (6m^2 - 18m - 6)H^2}{6(m + 2)(\dot{H} + H^2) - 3(2m - 5)H^2 + (m + 2)^2a - \frac{6m}{1 - 2\alpha}} \]  

(26)

\[ \Lambda = \frac{6}{(1 + 2\alpha)(m + 2)} \left( \dot{H} + 3H^2 \right) \]  

(27)

Here we use \( \alpha = (16\pi + 3\lambda)/2\lambda \). All the physical parameters are expressed in terms of Hubble parameter \( (H) \) in the above equations.

Figure 1: (a) Plot of DP \( (q) \) versus \( t \)  (b) Plot of DP \( (q) \) versus redshift \( z \)

Table 1: According to three observed data's, the corresponding values of of DP \( (q_0) \) and Hubble parameter \( (H_0) \), we find the values of \( \beta \) and \( k \).

| Case   | Data               | \( q_0 \) | \( H_0 \) | \( \beta \)     | \( k \)          | Reference |
|--------|--------------------|-----------|-----------|-----------------|------------------|-----------|
| Case-I | Supernova type Ia Union | -0.73     | 73.8      | 0.0036          | 0.000084         | [49]      |
| Case-II| BAO and CMB        | -0.54     | 73.8      | 0.0062          | 0.000016         | [50]      |
| Case-III| OHD+JLA           | -0.52     | 69.2      | 0.000069364     | 0.000008394      | [51]-[52] |
Figure 1(a) shows the assortment of deceleration parameter \( q \) with astronomical time \( t \) as indicated by Eq. (13). We just observe our design model is an acceleration stage \( (q < 0) \) for \( k = 0.000084 \) and \( \beta = 0.0036 \) (case I). In (case-II) \( k = 0.000016 \) and \( \beta = 0.0062 \) our model depicts the phase transition from +ve to -ve deceleration parameter \( q \). This shows that our model is changing from \( (q > 0) \) deceleration to \( (q < 0) \) acceleration. \( t_c = \frac{\beta^2 - c}{2\beta} \) is the critical time at which the phase transaction took place. Similarly, [51, 52] used the value of the joint dataset (OHD+JLA) in which \( k = 0.0069364 \) and \( \beta = 0.000008394 \) we find decelerating-accelerating phase transition (case III) (Table-1).

The average scale factor \( a(t) \) as far as redshift \( z \) is given by \( a(t) = \frac{a_0}{1+z^2} \). From Eq. (15), we get \( \frac{\sqrt{H_0H_0}}{a} = \ln(a) \), then \( \ln(a) = \ln(a_0) - \ln(1+z) \). Substituting the above in Eq. (13), we shows \( q \)-parametrization given by

\[
q(z) = -1 + \frac{1}{\ln(a_0) - \ln(1+z)},
\]
where \( a_0 \) is present value of scale factor, for three cases of \( (\beta, k) \) \( (\beta = 0.0036, k = 0.000084, \beta = 0.0062, k = 0.000016, \beta = 0.00006, k = 0.00000) \) for plot the graphs. In the figure 1(b) we have discovered that \( q(z) \) expansion is a smooth progress from a decelerated stage to accelerated period of development and \( q \rightarrow -1 \) as \( z \rightarrow -1 \). As of late [53, 54] have discovered the change redshift from decelerating to accelerated in modified gravity cosmology. SNe type Ia dataset has given the progress from past deceleration to ongoing increasing speed at \( \Lambda \)CDM. More recently, in 2004 (HZSNS) team have identified \( z_t = 0.46 \pm 0.13 \) at \( (1\sigma) \) c.l. [2] which is again improved to \( z_t = 0.43 \pm 0.07 \) at \( (1\sigma) \) c.l. [7]. SNLS [55], and recently compiled by [56], provide a progress redshift \( z_t + 0.6(1\sigma) \) in improved concurrence with the flat \( \Lambda \)CDM.

In addition, the \( q(z) \) reconstruction is performed by the Joined \( (SNIa + CC + H_0) \), which have acquired the change redshift \( z_t = 0.69^{+0.09}_{-0.07}, 0.65^{+0.07}_{-0.07} \) and \( 0.61^{+0.12}_{-0.08} \) inside \( (1\sigma) \) [57], which are seen as well predictable with past outcomes [58-62] including the \( \Lambda \)CDM expectation \( z_t = 0.7 \). Another constraint of change redshift is \( 0.60 \leq z_t \leq 1.18 \) (2\sigma joint examination) [63]. From the \( H(z) \) data we find evidence as \( z_t = 0.720 \pm 0.14 \) for redshift, which is in acceptable concurrence with the [61] assurance of \( z_t = 0.72 \pm 0.05 \) as well as [65] assurance of \( z_t = 0.82 \pm 0.08 \) at 1\sigma error. This is again improved as \( z_t = 0.72 \pm 0.14 \) at 68% c.l. which is in acceptable understanding. From the combination of \( H(z) \) and SNIa datasets, we note that the transition from deceleration to acceleration in the BI expansion process takes place at a redshift of \( z_t = 0.57 \pm 0.0037 \) which is in acceptable concurrence with the outcomes acquired [66, 67]. Therefore, with reference to resent observation of SNe Ia, the present universe is expanding and at some stage the value of DP is within the range of \(-1 < q < 0 \). So, we observe that in all three cases, our model is aligned with resent findings.

The transition redshift for our derived models for two cases (iii) \( \beta = 0.000008394, k = 0.0069364 \) and (ii) \( \beta = 0.0062, k = 0.000016 \) are found to be \( z_t = 1.954 \) and \( z_t = 2.28 \) respectively (Fig. 1(b)) which are in good agreement with observational values.
Figure 2: Plot of energy density $\rho$ versus $t$ and redshift. Here $\lambda = 1$, $\alpha = 26.6$, $m = 0.004$.

Figure 2(a, b) which is related to the Eq. (28), depict the energy density $\rho$ with time $t$ and redshift $z$ for three cases. It is seen that $\rho$ remains positive during the infinite development. Here we additionally noticed that $t \to 0$, $\rho \to \infty$ showing the big-bang situation. Our model is likewise steady with ongoing perceptions. We conclude that in fig.2(a) & fig.2(b) the $\rho$ is a positive diminishing function of time and increasing function of $z$. It approaches to zero as $t \to \infty$. Which is consistent with the well establishment scenario.

Figure 3: Plot of anisotropic fluid energy density ($\rho_B$) versus $t$ and redshift. Here $\lambda = 1$, $\alpha = 26.6$, $m = 0.004$.

Figure 3 displays anisotropic fluid energy density in terms of cosmic time $t$ and redshift $z$. Eq. (29) corresponding to anisotropic fluid energy density ($\rho_B$) is a declining time function and remains positive during the cosmic assessment. First, we can see in the figure 3(a), it fell sharply, then gradually, and at the present epoch, it approached to a small positive value. Here ($\rho_B$) tends to 0 as $t \to \infty$. But in Figure 3(b) anisotropic fluid energy density increases with redshift because density decreases imply the volume increase, the expansion of the universe. In all three cases that the anisotropic density of the dark matter decreases with time and tends to zero at late times.
Figure 4: Plot of pressure $p$ versus $t$ and redshift. Here $\lambda = 1$, $\alpha = 26.6$, $m = 0.004$.

Figure 4 represents the expansion for fluid pressure $p$ for the model and corresponding to (25). We observed that for all three cases pressure is negative increasing function of time $t$ and redshift $z$. From the figures 4(a) and 4(b) we observed for the homogeneous and isotropic model, pressure is consistently negative and approaches zero at late time.

Figure 5: Plot of EoS parameter $\omega$ versus $t$ and redshift. Here $\lambda = 1$, $\alpha = 26.6$, $m = 0.004$.

Figs. 5(a) and 5(b) depict the behavior of EoS parameter $\omega$ with respect to time $t$ and redshift $z$ respectively. The EoS parameter may lies in phantom region ($\omega < -1$). Both figures represent the same scenario of the universe for all cases. It is clearly shows in fig. 5(a) that first equation of state parameter decreased sharply and approach to a small negative values at the present epoch. The fig. 5(b) shows that the EoS parameter is a increasing function of $z$, the rapidity of its growth at the early stage deponds on the types of the universe.
Figure 6: Plot of cosmological constant $\Lambda$ versus $t$ and redshift. Here $\lambda = 1$, $\alpha = 26.6$, $m = 0.004$.

Figure 6(a) and 6(b) depicts the cosmological constant with respect to time and redshift $z$. In all cases $\Lambda$ is decreasing function of time and increasing function of redshift $z$. We expect that in the universe, the positive value of $\Lambda$ the expansion will tends to accelerate. In fig. 6(a) we observed that the $\Lambda$ decreases more sharply as time increases in empty universe as compared to radiating dominated and stiff fluid universe. Here fig.6(b) shows that $\Lambda$ increases with high redshift. A positive cosmological constant resists attractive gravity of matter due to its negative pressure.

4 Physical Acceptability and Stability of the Outcomes

Now, we shall test by means of some recently used diagnostic tools, whether our models are stable or not?

4.1 Energy Condition

In this subsection, for our derived model, we are checking the energy conditions. The three energy conditions (NES)($\rho + p \geq 0$), (SEC)($\rho + 3p \geq 0$) and (DEC)($\rho - p \geq 0$) are given. In Figs. 7(a), 7(b) and 7(c), we have plotted the energy conditions with respect to cosmoc time for all three cases I, II & III. From these figures we observe that energy conditions violet in all cases which is aspected in the case of DE.

Figure 7: Plot of energy conditions (NEC,SEC,DEC) with $t$. Here $\lambda = 1$, $\alpha = 26.6$, $m = 0.004$.

4.2 Velocity of Sound

Numerous authors [68-70] have researched the requirements on sound speed of dynamic DE models with time-changing EoS ($\omega$) and presumed that imperative on the sound speed of DE is exceptionally weak. DE parameters
just as other cosmological parameters are independent of the compelling sound speed of DE, there is no limitation from present astronomical information on the effective sound speed. The system is unstable if squared speed of sound \( v_s^2 < 0 \). It is necessary that the squared speed of sound should be less than the velocity of light \( c \). As we work with unit speed of light in gravitational units, i.e. sound velocity exists within the range \( 0 \leq v_s^2 = \left( \frac{dp}{d\rho} \right) \leq 1 \).

We get the velocity of sound as,

\[
v_s^2 = \frac{6}{1-4\alpha^2} \frac{(m-1) + 2\alpha(m+1)}{m+2} \left( \frac{-2\beta}{(2\beta t + k)^2} + \frac{3(\beta)^2}{(2\beta t + k)^2} \right) - 2\beta \frac{(2m^2 - 4m - 7) + 2\alpha(2m^2 + 1)}{(m+2)^2(2\beta t + k)^2}
\]

\[+ 12m \frac{4e^{-6m\sqrt{2\beta t + k}}}{(m+2)^2(2\beta t + k)} \times \frac{1}{1 - 2\alpha \left( \frac{m+1}{m+2} \right) \left( \frac{-2\beta}{(2\beta t + k)^2} + \frac{3(\beta)^2}{(2\beta t + k)^2} \right) - 2\beta \frac{(-2m + 5) + 6\alpha(2m + 1)}{(m+2)^2(2\beta t + k)^2}}
\]

\[- 12m \frac{4e^{-6m\sqrt{2\beta t + k}}}{(m+2)^2(2\beta t + k)} \times \frac{1}{1 - 2\alpha \left( \frac{m+1}{m+2} \right) \left( \frac{-2\beta}{(2\beta t + k)^2} + \frac{3(\beta)^2}{(2\beta t + k)^2} \right) - 2\beta \frac{(-2m + 5) + 6\alpha(2m + 1)}{(m+2)^2(2\beta t + k)^2}}
\]

\[(29)\]

Figure 8: Plot of velocity of sound versus \( t \). Here \( \lambda = 1, \alpha = 26.6, m = 0.004 \).

Figure 8 depicts that a positive value of squared speed of sound \( v_s^2 > 0 \) represents a stable model. Here we notice that in case I, our model is unstable as \( v_s^2 < 0 \) throughout the evolution of the Universe but in case II and case III, our model is stable in early phase of the Universe whereas unstable in present phase.

5 Dark Energy Parameterization

Given a large number of scalar field models with a range of potentialities, testing all individual models is always difficult. Alternatively, we often use a parameterization of the evolution of dark energy that broadly describes a large number of DE models in the scalar region. In parameterizing \( \omega = p/\rho \) dark energy equation of state is the most common practice. So, as to evaluate the dynamical parts of the model, we have plotted the EoS parameter \( \omega \) as an element of redshift and the literature[71] also contains a large number of parameterizations for \( \omega \). The conduct of the EoS parameter of the model has been contrasted and that of some notable EoS parameterization like CPL[39-42], JBP[43], BA[44], PADE-I and PADE-II[45] for our measurements of the evidence together with the regular \( \Lambda CDM \) and the constant dark energy equation of state model.
Now, in this segment we consider the five well known DE parameterization. The first is the Chevallier-
Polarski-Linder model (CPL), where $\omega_0$ is the present EoS value and its overall time evolution $\omega_a$ is the CPL model written as follows.

$$\omega = \omega_0 + \omega_a \frac{z}{(1+z)}.$$  \hfill (30)

The CPL parameterization issue at high redshift $z$ has been discussed in [71, 73]. To point out this behavior, the authors [71, 73] proposed CPL parameterization where parameters of $\omega_0$ and $\omega_a$ have the same definitions as those defined for parameterization of the CPL. Another we consider that Jassal-Bagla-Padmanabhan (JBP) parameterization of DE as

$$\omega = \omega_0 + \omega_a \frac{z}{(1+z)^2}.$$  \hfill (31)

proposed in [43], this model represents a dark energy component in both low and high redshift regions with rapid variation at low $z$ where the parameterization of the CPL cannot be generalized to the whole universe. Here, the parameters $\omega_0$ and $\omega_a$ have the same meanings as those defined for the above two models. Barboza-Alcaniz parameterization proposed in [44], that this model presents a step forward in redshift areas where the CPL parameterization cannot be reached out to the whole history of the universe. It is useful the structure is given by:

$$\omega = \frac{\omega_0 + \omega_a z}{1 + \omega_b (1 + z)}.$$  \hfill (32)

which is well-behaved at $z \to -1$. The other parameterization of DE include PADE-I and PADE-II. The EoS parameter can be written in terms of $z$ as:

$$\omega = \frac{\omega_0 + \omega_a z}{1 + \omega_b (1 + z)}.$$  \hfill (33)

Here the EoS parameter with $\omega_b \neq 0$ maintains a strategic distance from the dissimilarity at $a \to \infty$ (or proportionally at $z = -1$) [45].

$$\omega_z = \begin{cases} \frac{\omega_0 + \omega_a}{1 + \omega_b} & \text{for } a \to 0 \quad (z \to \infty \text{ early time}) \\ \omega_0 = 0 & \text{for } a = 1 \quad (z = 0 \text{ present time}) \\ \frac{\omega_a}{\omega_b} & \text{for } a \to \infty \quad (z \to -1 \text{ future time}) \end{cases}$$

Here we have to set $\omega_b \neq 0$ and $-1$. In this manner we conclude that the PADE (I) formula is a well-behaved function in the scope of $\leq a \leq \infty$ (or comparably at $-1 \leq z \leq \infty$). Clearly PADE (I) estimation has three free parameters: $\omega_0$, $\omega_a$, and $\omega_b$, to evade singularities in the cosmic extension, $\omega_b$ necessities to lie in the range $-1 < \omega_b < 0$. Here the present parameterization is composed as a component of $lna$. Right now, EoS parameter can be composed as

$$\omega = \frac{\omega_0 + \omega_a \ln \frac{1}{1+z}}{1 + \omega_b \ln \frac{1}{1+z}},$$  \hfill (34)

where $\omega_0$, $\omega_a$, and $\omega_b$ are constants [45]. In PADE (II) parameterization, to keep away from singularities at these ages, we need to force $\omega_b \neq 0$

$$\omega_z = \begin{cases} \frac{\omega_0 + \omega_a}{1 + \omega_b} & \text{for } a \to 0 \quad (z \to \infty \text{ early time}) \\ \omega_0 = 0 & \text{for } a = 1 \quad (z = 0 \text{ present time}) \\ \frac{\omega_a}{\omega_b} & \text{for } a \to \infty \quad (z \to -1 \text{ future time}) \end{cases}$$
Table 2: Dark energy parameterization with best fit values of $\omega_0$ and $\omega_a$.

| Model   | Parameterization                                           | best fit parameter using SNe Ia LA | Reference |
|---------|------------------------------------------------------------|-----------------------------------|-----------|
| CPL     | $\omega = \omega_0 + \omega_a \frac{1 + z}{1 + z^2}$     | $\omega_0 = -0.991 \pm 0.036$, $\omega_a = 0.297 \pm 0.779$ | [43], [55] |
| JBP     | $\omega = \omega_0 + \omega_a \frac{1 + z}{1 + z}$       | $\omega_0 = -1.013 \pm 0.070$, $\omega_a = -0.297 \pm 4.306$ | [43], [55] |
| BA      | $\omega = \omega_0 + \omega_a \frac{1 + z}{1 + z^2}$     | $\omega_0 = -0.997 \pm 0.034$, $\omega_a = -0.245 \pm 0.545$ | [44], [55] |
| P ADE - I | $\omega = \omega_0 + \omega_a \ln \frac{1 + z}{1 + z^2}$ | $\omega_0 = -0.825 \pm 0.091$, $\omega_a = -0.683 \pm 0.040$ | [45], [55] |
| P ADE - II | $\omega = \omega_0 + \omega_a \ln \frac{1 + z}{1 + z^2}$ | $\omega_0 = -0.889 \pm 0.080$, $\omega_a = 0.297 \pm 0.779$ | [45], [55] |

Figure 9 shows the behaviour of the EoS parameter verses redshift $z$ for five DE parameterization. All the parameterization contain two parameters $\omega_0$ and $\omega_a$ and the the parameterization (PADE-1,PADE-II) contains three parameters. Here $\omega_0$ is related to the present value of the equation of state for the dark energy and $\omega_a$ & $\omega_b$ determines its evolution with time. In this framework, using the best-fit values, we found that only the PADE-II parameterization remains in the quintessence regime ($1 < \omega < 1/3$) and at high redshifts, while they enter in the phantom regime. The rest of the parameterizations (PADE-1, CPL, BA, JBP) evolve in the phantom region ($\omega < -1$) at high redshifts.

![Figure 9: Plot of EoS parameter $\omega$ versus $z$ for five DE parameterization](image)

6 Correspondence of Quintessence Dark Energy Model with Swampland Criteria

Swampland criteria may also be examined for alternative Gravity theories [83, 84]. It is particularly interesting because some modified theories are a useful paradigm to cure shortcomings of General Relativity at ultraviolet and infrared scales, due to the lack of a full quantum gravity theory [85]. In this direction Bebetti and Capozzilla [85] worked on the Noether Symmetry method with Swampland conjecture in f(R) gravity. The selected f(R) models, satisfying the swampland conjecture, are consistent, in principle, with both early and late-time cosmological behaviors. In this section we are discussing the consequences of swampland’s DE principles in the framework of current and prospective cosmological observations. The swampland refers to a set of low-energy theories which look consistent from low-energy perspective but fail to be UV completed with quantum gravity.

The swampland Conjecture concerning the effective potential $V(\phi)$ of the scalar field $(\phi)$ are provided by subsequent inequalities:

$$Conjecture \; 1 \; : \; M_{Pl} \left| \nabla V \right| \geq c_1 V$$  \hspace{1cm} (35)
Here $M_{p1}$ is the reduced planck mass. The field should not, additionally, fluctuate more than around one planck unit over the entire universe’s history.

Conjecture 2 : $\triangle \phi \lesssim c_2 M_{p1}$, 

(36)

where $c_1$ and $c_2$ are the constants of first order has become significant, or otherwise light fields become important and the effective field theory becomes invalid. The swampland criteria have started expansions proposed in [63]. Recently lots of activity, due to their suggestions for inflation are discussed in [86]-[95].

The gravitational field and the scalar field are depicted by the action

$$S = \int d^4x \sqrt{-g} \left( \frac{M_{p}^2}{2} (R) + L_m - \frac{1}{2} g^{ij} \partial_i \phi \partial_j \phi - V(\phi) \right),$$

(37)

where $R$ stands for Ricci scalar, and $V(\phi)$ is a potential and $\phi$ is the scalar field.

In this section we focus on the exponential quintessence model with a potential of the form $V(\phi) = V_0 e^{\lambda_1 \phi}$

We compute the variations of dark energy equation of state for $\lambda_1 = M_{p1} V' / V$ and compare them with the dark energy parametrisation. The behavior for the the pressure and the energy density of the quintessence field

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi)$$

(38)

Therefore the string theory criteria can be used to constrain the dark energy models that originate from a scalar field theory. Numerous works in literature that address Swampland criteria and quintessence models [87]-[92]. The string Swampland criteria for an effective field theory to be consistent with the string theory. The quintessence models of dark energy were significantly compelled by the string theory of swampland criteria.

The behaviour of $\phi$ and $V(\phi)$ are shown in Figs. 10 and 11. It is obvious that the scalar field $\phi$ is the increasing function verses $z$ and potential field $V(\phi)$ in the terms of the scalar field $\phi$ are also a positive decreasing function associated with swampland criteria 1 & 2.

The swampland criteria gives tight limitations to the dark energy models of the late time acceleration of the Universe just as on in stationary models of the early universe [97]-[99] right now, the basis of present and future cosmological perceptions.

![](image)

Figure 10: Plot of scalar field ($\phi$) versus $z$. Here $\lambda = 1$, $\alpha = 26.6$, $m = 0.004$. 
A discussion on the significance of exponential type of potential to address the validity of swampland models is given in [100]. Dark energy has likewise been considered in [94] for dark energy equation of state investigation with CPL parametrization with regards to examining swampland measures. In figure 12 we plot equation of state as a function of red-shift. We further analyze our dark energy with the 1σ, 2σ, 3σ upper limits of ω, developed by CPL parameterization, with the recent cosmological results given in [100]. We are calculating our results of ω vs z for standard quintessence dark energy model for values of λ1 where λ1 = 0.6, 0.8, 1.0, 1.2, 1.4. In this figure if λ1 = 0.8 the quintessence DE model lies below in the 2σ and 3σ upper bounds and λ1 = 1, 1.6 quintessence DE model lies below in the 3σ upper bounds and and satisfied the Swampland criteria. Therefore the quintessence dark energy model is fulfills and satisfied the swampland criteria for recent data. Hence we have examined the results of swampland parameters on scalar DE field models, to be specific general core.

7 Conclusions

In this paper we have presented anisotropic Bianchi Type−VIh space-time filled with anisotropic fluid in the frame work of $f(R,T)$ gravity propose by [35]. The motive is to obtain a set of field equations with a time-
dependent \( \Lambda \).

The main features of the models are as follows:

- We have discussed our theoretical models based on three data sets: (i) supernova type Ia union data [49], (ii) BAO and CMB data [50] and (iii) current data in combination with HOD and LA observations [51, 52].

- The solution of the corresponding field equations is obtained by assuming a time-dependent DP \( q(t) = -1 + \sqrt{\frac{\beta}{2k + \beta}} \), where \( k \) and \( \beta \) are arbitrary integrating constants. In plotting all figures, we have used three sets of \( (\beta, k) \): (i) \( \beta = 0.0036, k = 0.000084 \), (ii) \( \beta = 0.0064, k = 0.000016 \) and (iii) \( \beta = 0.000008394, k = 0.0069364 \) respectively (Table-1). These three sets of values of \( (\beta, k) \) are obtained by using three observational data sets [49-52].

- The transition redshift for our derived models for two cases (iii) \( \beta = 0.000008394, k = 0.0069364 \) and (ii) \( \beta = 0.0062, k = 0.000016 \) are found to be \( z_t = 1.954 \) and \( z_t = 2.28 \) respectively (Fig. 1(b)) which are in good agreement with observational values [101-106]. The current \( H(z) \) data redress a big redshift range, \( 0.7 \leq z \leq 2.36 \) [101-106] larger than that covered by Type Ia supernovae \( (0.01 \leq z \leq 1.30) \). The \( H(z) \) data can be used to trace the cosmological deceleration-acceleration transition [60, 107].

- For two cases (ii) \( \beta = 0.0062, k = 0.000016 \), and (iii) \( \beta = 0.000008394, k = 0.0069364 \), the DP varies from deceleration (early time) to acceleration (present time) whereas for another case (i) \( \beta = 0.0036, k = 0.0000084 \), we find only accelerating universe (Fig. 1(a)).

- For all the three cases, the energy density \( (\rho) \) is found to be a decreasing function of time and remains always positive in the whole evolution of the Universe (Fig. 2(a)). The Fig. 2(b) shows the variation of \( \rho \) with redshift \( (z) \). From this Fig. 2(b) we observe that \( \rho \) is an increasing function of \( z \) which is consistent with well established law.

- The anisotropic fluid energy density \( (\rho_B) \) is declining time function and remains always positive (Figs. 3(a)). We also observe that \( \rho_B \) is an increasing function of redshift (Figs. 3(b)) for all three cases.

- The fluid pressure \( (p) \) is negative increasing function of cosmic time \( t \) and remains always negative (Figs. 4(a)) which show the existence of dark energy (negative pressure). The \( p \) is decreasing function of redshift (Figs. 4(b)).

- We observe that for one case (ii) the universes are varying in quintessence \( \omega \geq -1 \) region [108] through of evolution, while later on crosses PDL \( (\omega = -1) \) and finally approaches to phantom region \( \omega \leq -1 \) [109] (Figs. 5(a) & (b)). In other two cases (i) & (iii) the universe is varying only in phantom region. Therefore, we conclude that we are living in phantom scenario.

- The latest cosmological model confirm the existence of a decaying vacuum energy density of \( \Lambda(t) \). These results on type Ia supernova’s magnitude and redshift suggested that our universe could extend through the cosmological \( \Lambda \)-term with induced cosmological density. From Figs. 6(a) & (b), we observe that the \( \Lambda(t) \) is a positive decreasing function of cosmic time \( (t) \) in all three cases models of the Universe which is consistent with observations.

- We have found that all energy conditions i.e. WEC, SEC and DEC are not satisfied for all the universes for all three values of \( (k, \beta) \) (see Figs. 7a, b & c). This is consistent with dark energy scenario.

- We have observed that speed of sound remains less than light velocity \( (c = 1) \) throughout the universe evolution (Figs. 8) in the three cases (i), (ii) & (iii). This prove the physical acceptability of our solutions.

- The parameterization of DE EoS \( (\omega) \) is significant to evaluate the dynamical part of the models. In Figs. (9), some renowned EoS parameterization like CPL, JBP, BA, PADE-I and PADE-II have been graphed. The results of our analysis of DE parameterization with the best-fit values of \( \omega_0 \) and \( \omega_1 \) have been given in Table-2. Based on this analysis, we put constraints on the model parameters and found that the expansion data. In the present framework, using best-fit values, we found that only PADE-II and JBP parameterization remains in the quintessence regime and the rest CPL, BA and PADE-I evolve in the phantom region.
We have discussed the outcomes of swampland’s DE precepts in the reference of current cosmological observations. The behaviour of $V(\phi)$ and $\phi$ are shown in Figs. 10 & 11 respectively. We observe that potential is positive declines from finite value and disappears at late time (present era). The scalar field $\phi$ increases with time and always positive. In figure 12 we plot equation of state as a function of red-shift, in this figure we have analyze the dark energy with the $1\sigma$, $2\sigma$, $3\sigma$ upper limits of $\omega$, developed by CPL parameterization, with the recent cosmological results.

Thus, our newly constructed models and their solutions are physically acceptable. Therefore, for the better understanding of the characteristics of Bianchi type-$V_{Ih}$ cosmological models in our universe’s evolution within the framework of $f(R,T)$ gravity theory and confrontation observational data, may be helpful.

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