On the Unusual Effectiveness of Type-aware Mutations for Testing SMT Solvers

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Abstract. We propose type-aware mutation testing, a simple and effective approach for testing SMT solvers. The key idea is to mutate operators of same type in SMT formulas to generate type-correct mutant formulas. These mutant formulas are then used as the test cases for SMT solvers. Type-aware mutations enable us to fabricate a vast amount of type-correct formulas efficiently; independent of the seed formulas’ logic. We realized the type-aware mutations within a tool called OpFuzz and stress-tested two state-of-the-art SMT solvers Z3 and CVC4. Type-aware mutations are unusually effective: In six months of extensive testing with OpFuzz, we reported 743 bugs in Z3 and CVC4, out of which 521 bugs were confirmed and 457 of the confirmed bugs were fixed by the developers. Our bug findings are highly diverse: We found bugs of many different types (soundness bugs, invalid model bugs, crashes, etc.), logics and solver configurations. We found 400 out of 743 bugs in the default modes of the solvers and 101 soundness bugs. Perhaps most notably, OpFuzz found 10 critical soundness bugs in CVC4.

1 Introduction

Satisfiability Modulo Theory (SMT) solvers lie at the heart of many applications in verification research, such as (software) model checkers [10, 12], symbolic execution engines [11, 13] and neural network verifiers [14]. Incorrect results of SMT solvers invalidate the verification results of these tools which can be disastrous in safety critical domains. Hence, the SMT community has undertaken great efforts to make SMT solvers reliable. A prominent example of these efforts is the SMT-LIB initiative [1] which established standard input/output file formats for SMT solvers, semi-formal logic/theory specifications and provides extensive benchmark repositories. Furthermore, the community holds a yearly SMT Competition in which soundness bugs are heavily penalized. To date, there are several mature SMT solvers of which Z3 [16] (5.3k stars on GitHub) and CVC4 [2] (374 stars on GitHub) are the most popular. Both Z3 and CVC4 are very stable and reliable. In Z3, there have been fewer than 150 reported soundness bugs in more than

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Figure 1: Bug-triggering formulas in SMT-LIB format. (a) Formula triggers a soundness bug in Z3. (c) Formula causes CVC4 to crash. (e+g) CVC4 reports an invalid model.

Endnotes:

3 From April 2015 to October 2019.
4 From July 2010 to October 2019.
such as nonlinear arithmetic exist, it remains open whether there are bugs in
the more mature logics such as bit-vectors or uninterpreted functions.

To more effectively find bugs in SMT solvers, we present *type-aware mutation
testing*, a novel approach to stress-test SMT solvers. The key insight is that we
can fabricate diverse type-correct formulas by mutating their operators. The
mutated formulas are then used as test cases for SMT solvers. To motivate our
technique, we first examine four exemplary bugs.

Consider the formula in Figure 1a that realizes \( \frac{2a}{a} = \frac{a}{a} = 1 \). Here, division by
zero terms are meaningful according to the SMT-LIB standard. In fact, we can
set \( a = 0 \) to realize a model: Both \( \frac{2a}{0} \) and \( \frac{a}{0} \) can be chosen arbitrarily, *i.e.*, to be
1 to satisfy the assert. Hence, the formula in Figure 1a is satisfiable. However,
Z3 reports *unsat* on it, which is incorrect. The bug disappears if we mutate the
first division operator to a plus cf. Figure 1b.

Consider the formula in Figure 1e on which CVC4 returns the following
model: \( a = \frac{-3}{2} \) and \( b = \frac{-1}{2} \). This model is incorrect as \( a \cdot b \neq 1 \). Mutating the
greater operator \( > \) to the equals operator \( = \) hides this bug (see Figure 1f).

As another example, consider the formula in Figure 1g. CVC4 gives an invalid
model on this formula by setting \( f = g = false \) and crashes on the formula in
Figure 1c. Again in both cases, the bug disappears with a single operator change
(see Figure 1d and Figure 1h).

These examples show that mutating same operators of the same type features
a fine-grained control over SMT formulas. We call such mutations *type-aware*
mutations as they preserve the type of the mutated operator and so guarantee
diverse but type-correct formulas. Since late September 2019, we began stress-
testing SMT solvers using our realization \texttt{OpFuzz}. \texttt{OpFuzz} is unusually effective:
Although we only focused on the most basic function symbols for our mutations,
\texttt{OpFuzz} found bugs in many different SMT logics and solver modes. In summary,
we made the following contributions in this paper.

**Contributions**

- **Approach & Tool**: We propose *type-aware mutation testing*, a simple, highly
effective approach for testing SMT solvers. We have realized type-aware mu-
tation testing within our tool \texttt{OpFuzz} in no more than 250 lines of code. \texttt{OpFuzz} helps SMT solver developers and practitioners to stress-test SMT
decision procedures regardless of the used logic and solver.

- **Bug findings**: We have stress-tested the two state-of-the-art SMT solvers Z3
and CVC4. We reported 743 bugs on the issue trackers of Z3 and CVC4, out
of which 521 bugs were confirmed and 457 of the confirmed bugs were fixed
by the developers. We found these bugs in various different logics and solver
configurations. We found 400 out of 743 bugs in the default modes of the
solvers and 101 soundness bugs. Perhaps most notably, we found 10 critical
soundness bugs in CVC4.

- **Towards understanding the Effectiveness**: We have done two experiments to
understand the reason for the unusual effectiveness of type-aware mutation
testing. In the first experiment, we measure whether OpFuzz can improve code coverage. In a second experiment, we examine the influence of type-aware mutations on the execution traces of the SMT solvers Z3 and CVC4.

The rest of the paper is structured as follows. We begin by a motivating example for type-aware mutation testing (Section 2). Then, we present type-aware mutation testing formally and show how we apply it to SMT solver testing with our realization OpFuzz (Section 3). We then present our empirical evaluation (Section 4) and show sampled bugs (Section 4.2). Finally, we give related work (Section 5) and conclude (Section 6).

2 Motivating Example

This section motivates type-aware mutation testing by an example and concisely explains its unique features. Our aim is stress-testing the decision procedures in SMT solvers. The key challenge is to fabricate a vast amount of diverse, type-correct test formulas.

A common way of testing programs with complex inputs is by mutation-based fuzzers. Given a set of test seeds, such mutation-based fuzzers usually perform their mutations at file-level, e.g., do bit-flips or replace bytes of input files. While this gives a fine-grained control over the test seeds, it is not guaranteed to produce syntactically correct formulas. In fact, many inputs generated by file-level mutations, would be rejected by an SMT solver’s parser. Another way would be to perform the mutations at logic level, e.g., by formulating conjunctions, disjunctions of test formulas. However, generating many diverse test seeds from a single (pair) of formulas at the logic level is challenging. How can we fabricate diverse type-correct test cases for SMT solvers while maintaining fine-grained control over the seed formulas? We argue that gradually mutating operators in SMT formulas at type-level is an effective way to do so. We next describe our approach to generating gradual type-aware mutations for a given SMT formula \( \varphi \).

1. **Formula skeletonization.** We first create the formula skeleton of formula \( \varphi \) which is a structure where each occurrence of an operator \( f_i \) in \( \varphi \) is holed out, i.e., replaced by a fresh function symbol \( \Box_i \).

2. **Type identification.** We define the type of each hole \( \Box_i \) by the type of the operator \( f_i \) that \( \Box_i \) has replaced.

3. **Gradual operator mutations.** We randomly chose a single hole \( \Box_r \) in \( \varphi \) and replace it with an operator \( f \) of the same type. We repeat this process \( n \) times \( (n \) is usually within 200 and 400) to fabricate \( n \) different formulas from the same skeleton where the consecutive mutations only differ with respect to one hole.

Consider formula \( \varphi \) in Figure 2a. The formula skeleton of \( \varphi \) is shown in Figure 2b. Each of \( \varphi \)’s operators is replaced by a hole \( \Box_i \). The types for the holes in the
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formula skeleton are defined as \( \alpha(\Box_1) = \text{Real}, \ldots, \Box_j \rightarrow \text{Real} \) and \( \alpha(\Box_j) = \text{Num}, \ldots, \Box_j \rightarrow \text{Num} \) for \( 2 \leq j \leq 4 \). We now mutate the operators to obtain four gradual type-aware mutations (see \( \varphi_1, \ldots, \varphi_4 \)). In formula \( \varphi_1 \) (Figure 2a), we filled the first hole with the equals operator = and left all the other holes unchanged (i.e., same as in \( \varphi \)). In formula \( \varphi_2 \) (Figure 2b), we additionally filled the third hole with the division operator \( / \); in formula \( \varphi_3 \) (Figure 2c), we filled the second hole with the plus operator +; in formula \( \varphi_4 \) (Figure 2d), we filled the forth hole with the minus operator -. Formulas \( \varphi_1, \ldots, \varphi_4 \) can be used as test cases for SMT solvers. In fact, \( \varphi_1 \) has triggered a soundness bug in Z3 which reports \( \text{unsat} \) on it while CVC4 reports \( \text{sat} \), the correct result. Z3 was incorrectly handled expression that simplify to a division by zero. The bug got promptly fixed by Z3’s main developer.

3 Approach

In this section, we formally introduce type-aware mutation testing and propose OpFuzz, an algorithm for stress-testing SMT solvers.

3.1 Type-aware Mutations

We consider first-order logic formulas of the satisfiability modulo theories (SMT). Such a theory is a pair \( \langle \Sigma, M \rangle \) where \( \Sigma \) is a signature and \( M \) is a class of \( \Sigma \)-models. A formula \( \varphi \) over \( \Sigma \) is satisfiable if there is at least one model \( M \in M \) under which \( \varphi \) evaluates to true and unsatisfiable otherwise. For such formulas, we conveniently interchange prefix notation and infix notation whenever appropriate, i.e., we write \( a + (b - c) \) in infix notation and \( (+ a (~ b c)) \) for the same formula in prefix notation. Signature \( \Sigma = \langle F, \alpha \rangle \) consists of a set
of function symbols $F = \{f_1, \ldots, f_n\}$ and a typing function $\alpha$ mapping each function symbol to its type. We call these function symbols as operators in this paper. We abbreviate the type of an $r$-valued function $\alpha(f) = (\langle z_1, \ldots, z_r \rangle, z)$ by $f : z_1, \ldots, z_r \to z$. In this paper, we only consider the following primitive types: $\text{Bool}$, $\text{Real}$, $\text{Int}$, $\text{Quant}$ for the booleans, real numbers, integer numbers and quantifiers respectively. We define $F[t] = \{f \in F | \alpha(f) = t\}$ as the set of function symbols of type $t$. The formula skeleton $\phi^\square$ of $\phi$ is a structure where each occurrence of a function symbol $f_i \in F(\phi)$ in $\phi$ is replaced by a fresh function symbol $\square_i$. Formula skeleton $\phi^\square$ can be represented by a characteristic vector $c_\phi^\square = \langle \square_1, \ldots, \square_n \rangle$. The elements $\square_i$ of $c_\phi^\square$ are called holes e.g., $\langle f_1, \ldots, f_n \rangle$ realizes $\phi$ and $\langle f_1, \ldots, f_{n-1}, \square_n \rangle$ fills the first $n$ holes of $\phi$ but keeps the last hole $\square_n$ unfilled.

**Definition 1 (Type-aware mutation).** Let $\phi$ be a formula and $\langle f_1, \ldots, f_n \rangle$ its characteristic vector. A formula $\phi'$ is a type-aware mutant of $\phi$ if $\langle \square_1, \ldots, \square_n \rangle$ is also the formula skeleton of $\phi'$ and $\alpha(f_i) = \alpha(f'_i)$ for all $1 \leq i \leq n$, where $c_{\phi'} = \langle f_1, \ldots, f_n \rangle$ is the characteristic vector of $\phi'$. Transforming $\phi$ to $\phi'$ is called type-aware mutation.

Type-aware mutations have two unique advantages. Firstly, they give a fine-grained control on mutating operators in formulas. Secondly, a type-aware mutation of a formula is guaranteed to be type-correct. Figure 3 shows exemplary operators in the SMT-LIB languages categorized by their types. For simplicity, in this paper, we only consider the operators in Figure 3.

Example 1. Consider the formula $\phi = (= a (- b c))$ in SMT-LIB format. Formula $\phi$’s skeleton is $\langle \square_1 a \ (\square_2 b c) \rangle$ where $\alpha(\square_1) = \text{Type}$, $\alpha(\square_2) = \text{Num}$, $\alpha(\square_3) = \text{Type \to \Bool}$, and $\alpha(\square_4) = \text{Num \to \Num}$. The formulas $\langle * a \ (+ b c) \rangle$ and $\langle \text{distinct a } (* b c) \rangle$ are type-aware mutations of $\phi$ since $\alpha(\star) = \alpha(\ldots) = \star$.

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5 We emphasize that the selection of operators and types is arbitrary. In principle, we could extend type-aware mutations by other operators and types, e.g., from bit-vector or string logic.
Procedure OpFuzz $(Φ, S_1, S_2)$:

\[
\text{bugs} \leftarrow \emptyset \\
\text{while no interrupt do} \\
\quad \varphi_0 \leftarrow \text{random.choice}(Φ) \\
\quad \text{for } i \leftarrow 1 \text{ to } n \text{ do} \\
\quad\quad \varphi_i \leftarrow \text{mutate}(\varphi_{i-1}) \\
\quad\quad \text{if } \text{val}(\varphi_i, S_1, S_2) \text{ then} \\
\quad\quad\quad \text{bugs} \leftarrow \text{bugs} \cup \{\varphi_i\} \\
\quad\text{end} \\
\text{end} \\
\text{end}
\]

Function \text{mutate}(\varphi):

\[
\begin{align*}
&f_j \leftarrow \text{random.choice}(F(\varphi)) \\
&f'_j \leftarrow \text{random.choice}(F[α(f_j)]) \\
&\varphi' \leftarrow \langle f_1, \ldots, f_{j-1}, f'_j, f_{j+1}, \ldots, f_n \rangle \\
&\text{return } \varphi'
\end{align*}
\]

Figure 4: Left: OpFuzz’s main process iterates formula mutation (\text{mutate}) with validation (\text{val}). Function \text{val} differentially tests the SMT solvers $S_1$ and $S_2$. Right: Function \text{mutate} replaces a randomly chosen function symbol $f_j$ in $\varphi$ by a function symbol $f'_j$ of the same type.

\[\alpha(*) \text{ and } \alpha(=) = \alpha(\text{distinct}).\] However, the formulas $(\Rightarrow a \ (+ \ b \ c))$ and $(\Rightarrow \text{div} \ (+ \ b \ c))$ are no type-aware mutations of $\varphi$ as $\alpha(=) \neq \alpha(\Rightarrow)$ and $\alpha(*) \neq \alpha(\text{div})$.

### 3.2 OpFuzz’s Pseudocode

Having presented type-aware mutations, we next propose OpFuzz, a tool to stress-test SMT solvers. For the algorithm of OpFuzz see Figure 4. OpFuzz takes a set of seed formulas $Φ$ and two SMT solvers $S_1$ and $S_2$ as its input. OpFuzz collects bugs in the set $\text{bugs}$ which is initialized to the empty set. The main process runs inside a while loop until an interrupt is detected, e.g., by the user or by a time or memory limit that is reached. We first choose a random formula $\varphi_0$ from the set of formulas $Φ$ for initialization. In the while loop, we then perform a type-aware mutation on $\varphi_{i-1}$ realized by the \text{mutate} function. In the \text{mutate} function, we first randomly choose a function symbol $f_j$. Then, we randomly choose a function symbol from $f'_j \in F[α(f_j)]$, i.e., of the same as $f_j$. The choices for $f'_j$ result from Figure 3. After we obtained $\varphi'$ by type-aware mutation on $\varphi_{i-1}$, we call the function \text{val}. It tests the two SMT solvers $S_1$ and $S_2$ differentially on input formula $\varphi_i$. First, it checks whether either of the solvers has crashed on solving $\varphi_i$. If that is the case, the function returns $\text{false}$. Otherwise it checks whether the results of the solvers are different and returns $\text{false}$ if so, else \text{val} returns $\text{true}$ indicating that $\varphi$ has not exposed a bug in neither of the solvers $S_1$ and $S_2$. OpFuzz realizes an $n$-times repeated type-aware mutation on every seed formula.
4 Empirical Evaluation

This section presents our evaluation. Between late September 2019 and early April 2020, we were running OpFuzz to stress-test the SMT solvers Z3 [16] and CVC4 [2]. We have chosen Z3 and CVC4 since they (1) both are popular and widely used in academia and industry (2) support a rich set of logics, and (3) adopt an open-source development model. During our testing period, we have filed numerous bugs on the issue trackers of Z3 and CVC4. This section describes the outcome of our efforts.

Summary

- Many confirmed bugs: In six months of stress-testing Z3 and CVC4, we have reported 743 bugs on the GitHub issue trackers of Z3 and CVC4 based on OpFuzz’s findings. Out of these bugs, 521 bugs were confirmed and 457 bugs were fixed by the developers. We found significantly more bugs in Z3 (386 confirmed bugs) as compared to CVC4 (142 confirmed bugs).

- Many logics affected: OpFuzz found bugs in several logics: bit-vector logic (QF_BV, BV), (non-linear) arithmetic (QF_NRA, QF_NIA, NRA), uninterpreted functions (QF_UF, UF), array logic (QF_AX), floating point logic (QF_FP), string logic (QF_S) and combinations of these logics (QF_ABVFP, UFLIA, UFLRA, QF_SLIA).

- Diverse bug types: OpFuzz found soundness bugs, invalid models, and crashes. Most bugs were found in default mode of the solvers. In addition OpFuzz found bugs in incremental mode, different string (e.g., z3str3) and arithmetic solvers in Z3, syntax-guided inference in CVC4, and many other solver configurations.

Setup. We have realized OpFuzz in a total of 212 lines of Python 3.7 code. OpFuzz can be run in parallel mode, which significantly increases its throughput. Users can customize OpFuzz’s command-line interface to test specific solvers and/or configurations.

We have only used OpFuzz with Z3 and CVC4, but in principle it can be used with any SMT solver that takes SMT-LIB v2.6 files as its input. We have implemented the type-aware function mutations according to Table 3. As test seeds, we have used the SMT-LIB benchmarks [6]. In addition, we have used the regression test suites of Z3 and CVC4. We have run OpFuzz on an AMD Ryzen Threadripper 2990WX processor with 32 cores and 32GB RAM on an Ubuntu 18.04 64-bit. When a bug is found, we reduce the bug-triggering formula to a small enough size for reporting. We use C-Reduce [17], a C code reduction tool, which also works for the SMT-LIB language. We implemented a pretty printer to help with the bug reduction process, e.g., when C-Reduce has converged to a still very large formula or hangs. The pretty-printer makes simple modifications to the

6 http://smtlib.cs.uiowa.edu/benchmarks.shtml
abstract syntax tree of a formula, e.g., flattens nestings of the same operator, removes additions and multiplications with neutral elements and returns the modified formula in a human-readable format.

We have tested Z3 and CVC4 in many frequently used configurations of which we will detail a subset in this section. For both solvers, we have tested with model validation (model.validate for Z3 and --check-models for CVC4). While incremental mode, model production and string support are by default enabled in Z3, we had to supply the options --incremental, --check-models and --strings-exp to CVC4 to enable support for these features. We tested different arithmetic solvers of z3: smt.arith_solver=x with $x \in \{1, 2, 3\}$.

Despite these widely used and stable configurations, we have also tested some recently popular/promising configurations to investigate OpFuzz’s bug finding capabilities on less stable features which are still under active development. For example, the string solver z3str3 [4] (smt.string_solver=z3str3) has shown promising performance on QF_S problems, CVC4’s syntax-guided synthesis procedure [18] (--sygus-inference) performs well on quantified problems.

### Bug types

We have encountered many different kinds of bugs and issues while testing SMT solvers. We distinguish them by the following categories:

- **Soundness bug**: Formula $\varphi$ triggers a soundness bug if solvers $S_1$ and $S_2$ both do not crash and give different satisfiabilities for $\varphi$.
- **Invalid model bug**: Formula $\varphi$ triggers an invalid model bug if the model returned by the solver cannot satisfy $\varphi$.
- **Crash bug**: Formula $\varphi$ triggers a crash bug if the solver throws out an assertion violation or a segmentation fault while solving $\varphi$.

OpFuzz detects soundness bugs by comparing the standard outputs of the solvers $S_1$ and $S_2$. OpFuzz detects invalid model bugs by internal errors when using the SMT solver’s model validation configuration. A crash bug is detected whenever a solver returns a non-zero exit and no timeout occurred.

### 4.1 Quantitative Evaluation

Having defined the setup and bug types, we continue with the quantitative evaluation which is divided into three parts: (1) bug/logic/configuration counts (2) coverage measurements and (3) solver trace comparisons.

### Bug Findings

Figure 5a shows the bug counts. We have reported a total of 743 bugs on Z3’s and CVC4’s respective issue trackers. Among these, 521 were confirmed and 457 were fixed. Strikingly, although we devoted equal testing effort to both solvers, we found more than twice as many bugs in Z3 as in CVC4. The development models of the solvers are different. While Z3 is essentially maintained by one core developer and a couple of assisting developers, CVC4 is more of a community effort. CVC4 has mandatory code reviews while, to the
| Status | Z3 | CVC4 | Total |
|--------|----|------|-------|
| Reported | 562 | 181 | 743 |
| Confirmed | 379 | 142 | 521 |
| Fixed | 360 | 97 | 457 |
| Duplicate | 51 | 16 | 67 |
| Won’t fix | 61 | 6 | 67 |

| Type | Z3 | CVC4 | Total |
|------|----|------|-------|
| Crash | 236 | 129 | 365 |
| Soundness | 91 | 10 | 101 |
| Invalid model | 52 | 3 | 55 |

Figure 5: Bugs counts in Z3 and CVC4 found by OpFuzz. a) bug status and b) bug type among the confirmed bugs.

best of our knowledge, Z3 does not. However, the bugs in Z3 are usually fixed very fast while the CVC4 developers must sometimes wait for their fixed code to be reviewed. Among the bug types of the confirmed bugs, crash bugs were most frequent (365), followed by soundness bugs (101) and invalid model bugs (55).

Out of the 743 bugs we have found, 400 bugs were found in the default modes of the solvers (i.e., without supplying any options to the solvers) while 343 bugs were found with options.

|          | Z3          | CVC4         |
|----------|-------------|--------------|
|          | lines | functions | branches | lines | functions | branches |
| Φ_{1000} | 33.2% | 36.2% | 13.7% | 28.5% | 47.1% | 14.3% |
| OpFuzz   | 33.5% | 36.4% | 13.8% | 28.8% | 47.4% | 14.4% |

Figure 6: Line, function and branch coverage achieved by Φ_{1000} and OpFuzz on Z3 and CVC4’s source code.

**Code Coverage.** Code coverage is a reference for the sufficiency of software testing. This experiment aims to answer whether the mutants generated by OpFuzz can achieve higher coverage than the seed formulas. *Hypothesis: OpFuzz’s bug finding can be explained by significantly increased code coverage.* We randomly sampled 1,000 formulas (Φ_{1000}) from all formulas that we used for stress-testing SMT solvers. We view the absolute line/function and branch coverage achieved by Benchmark to be the baseline for OpFuzz. We instantiated OpFuzz with \( n = 300 \), run OpFuzz on the seeds Φ_{1000} and then measure the cumulative line/function/branch coverage over all formulas and runs.\(^7\)

The results show that OpFuzz increases the code coverage upon Φ_{1000} (Figure 6). Z3 and CVC4 have over 436K LoC and 238k LoC respectively, so that

\(^7\) This makes a total of 300k runs.
0.1% improvement already translate to hundreds of additionally covered lines. However, although noticeable, the coverage increments are not significant (less than 0.5%). We therefore reject our hypothesis. This experiment also provides further evidence that standard coverage metrics (e.g., statement and branch coverages), although useful, are insufficient for measuring the thoroughness of testing. Indeed, such small coverage increases led to 167 new bugs in two of the most mature, widely-used solvers.

**Execution Trace Comparisons.** Since code coverage could not thoroughly explain the effectiveness of OpFuzz, we examine the internals of the solvers by investigating the similarity of the solver’s execution traces. What is the relative similarity of the execution traces with respect to the seed? In the following experiment, we approach this question.

In Z3 and CVC4, we obtain an execution trace by setting the flags `TRACE=True` and `--trace theory` respectively. Before describing the experiment, we first show the format of Z3’s and CVC4’s respective traces via an example. Consider formula \( \varphi \) and a type-aware mutation \( \varphi_{\text{mutant}} \) of \( \varphi \) (see Figure 7a and d). Figures 7b and c show the Z3 and CVC4 traces of \( \varphi \), Figures 7e and f show the Z3 and CVC4 traces of \( \varphi_{\text{mutant}} \).

Having obtained an intuition of the execution traces, we now get to the actual experiment. Our aim is to measure the relative change in the execution traces.
Figure 8: Left column: (a) seed formula \( \phi \) (b) Z3 trace snippet of \( \phi \) and (c) CVC4 trace of \( \phi \). Right column: (d) type-aware mutation \( \phi_{\text{mutant}} \) (e) Z3 trace snippet of \( \phi_{\text{mutant}} \) (f) CVC4 trace of \( \phi_{\text{mutant}} \). Differences in the execution traces are highlighted by shading.
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of Z3 and CVC4. We therefore perform 40 mutation steps for every formula in \( \Phi_{1000} \) and record the execution trace triggered in each step. To quantify the similarity of two traces \( t_1 \) and \( t_2 \) we compute a similarity metric \( \text{sim}(t_1, t_2) \) with

\[
\text{sim}(t_1, t_2) = \frac{2 \cdot \text{matching lines}(t_1, t_2)}{\text{lines}(t_1) + \text{lines}(t_2)}
\]

where \( \text{matching lines}(t_1, t_2) \) corresponds to the number of matching lines between \( t_1 \) and \( t_2 \), \( \text{lines}(t_1) \) and \( \text{lines}(t_2) \) are the number of lines in \( t_1 \) and \( t_2 \) respectively. As an example consider again Figure 8. \( \varphi \)'s Z3 trace and \( \varphi_{\text{mutant}} \)'s Z3 trace match in 10 out of 11 lines and therefore their similarity score is \( \frac{10}{11} \). For the trace pair of CVC4, the number of matching lines is 3, so the similarity of the Z3 trace is \( \frac{1}{2} \).

In our experiment, we fix the trace by the original formula to be \( t_1 \) and \( t_2 \) corresponds to the trace triggered by the mutant. Figure 7 shows the similarity of the corresponding mutation step averaged over all formulas in \( \Phi_{1000} \). The results of Z3 and CVC consistently show that along with the mutation step increase, the similarity between the traces triggered by the mutant and the original formula gradually decreases. The result indicates that OpFuzz can generate diverse test cases that trigger different execution traces via operator mutation.

Summary. In this section, we designed three quantitative evaluations to measure and understand the effectiveness of OpFuzz. First, the quantitative evaluation strongly supports that OpFuzz can find a significant number of bugs in various logics. Second, to understand why OpFuzz can find so many bugs, we designed a coverage evaluation. The evaluation result shows that OpFuzz can increase the coverage, but the increment is minor. As the coverage evaluation did not fully answer why OpFuzz is effective, we further designed the third evaluation investigating the similarity of execution traces. The trace evaluation shows that OpFuzz can gradually change the execution traces of the solvers, which partially explains the effectiveness of OpFuzz.

4.2 Assorted Bug Samples

This section details our diverse bug findings during the extensive stress-testing of the SMT solvers Z3 and CVC4 from September 2019 - April 2020. The bugs shown are reduced by C-Reduce since the unreduced formulas are too large to be presented.

Figure 8a shows a soundness bug in Z3’s bit-vector logic. The formula is clearly unsatisfiable as the nested \texttt{bvxnor} expression equals the unnested \texttt{bvxor} expression. However Z3 reports \texttt{unsat} on it which is incorrect.

Figure 8b shows a soundness bug in the implementation of the symbolic square root in CVC4. The formula can be satisfied by assigning an arbitrary negative real to variable \( x \). CVC4 incorrectly reported \texttt{unsat} on this formula.
Figure 9: Selected bug samples in Z3 and CVC4.
Figure 8c is a soundness bug in Z3. The formula has two asserts. It is reported sat by Z3 although the second assert is unsatisfiable. The root cause was a scoping issue: Z3 accidentally dropped the second assert and only solved for the first one.

Figure 8d shows an invalid model bug in CVC4. The two uninterpreted functions f and g should be distinct. CVC4 correctly reports sat on this formula, but returns an invalid model in which f and g have the same assignment. This is because CVC4 silently uses a high-order reasoning optimization.

Figure 8e presents an assertion violation in the Z3’s floating point logic. Z3 uses 64-bit integers to represent the exponents for floating point numbers. Z3 is hence unable to accept exponent bits greater than 63 which causes an assertion violation.

Figure 8f shows a soundness in CVC4’s string logic. The intuition behind this formula is the following. The index of string y in x after position 1 should be equal to the length of string x and x should contain y. The formula can be satisfied by setting y to the empty string and x to a string of length 1. However, CVC4 incorrectly reports unsat.

Figure 8g depicts a soundness bug in Z3’s string QF_<SLIA logic. The formula is unsatisfiable since the second assertion cannot be satisfied since b is strictly positive. However, Z3 reports sat on it.

Figure 8h presents an unsatisfiable integer logic formula in Z3. Z3 reports sat on this formula due to an incomplete implementation of the mod operator.

Figure 8i shows an invalid model bug in Z3. Z3 correctly reports sat on this formula, but gives an invalid model $M = \{ e \mapsto true, c \mapsto false, d \mapsto true, s \mapsto 0, b \mapsto true \}$. This model does not satisfy the formula since it violates the constraint $(\star b (\star e c d))$. Z3’s main developer made a major change in 13 files when fixing this bug.

Figure 8j presents a formula that triggers a check failure in the non-linear arithmetic of CVC4. The check violation is caused by a value inference for arithmetic terms. CVC4 does not update its cache accordingly, which leads to an assertion violation.

5 Related Work

We are not the first work on testing SMT solvers. Roughly ten years ago, the fuzzing tool FuzzSMT [6] has been proposed, which is based on differential testing and targeted bit-vector logic. Unlike OpFuzz, FuzzSMT uses a grammar to generate the SMT formulas for testing. FuzzSMT totally found 16 solver defects
in five solvers, however, none in Z3. The efforts of the SMT-LIB initiative [3] have resulted in formalized SMT theories and common input/output file formats. In addition, the yearly solver competition SMT-COMP [9] heavily penalized solvers with soundness issues. Consequently, SMT solvers have robustified and finding bugs in SMT solvers became more difficult. Researchers have hence targeted the less mature logics such as the recently proposed unicode string theory. Blotsky et al. [5] proposed StringFuzz which focuses on performance issues in string logic. StringFuzz generates test cases in two ways, one is mutating and transforming the benchmarks, another one is randomly generating valid formulas from a grammar. StringFuzz found 3 performance and implementation bugs in z3str3. Bugariu and Müller [7] proposed a formula synthesis approach that generates formulas in String logic which can be specified to be by construction satisfiable or unsatisfiable. They showed that their approach can detect many existing bugs in String solvers and they found 5 new soundness/incorrect model bugs in z3 and z3str3. However, it remained an open question whether automated testing tools could find bugs in theories except the unicode string theory in Z3 and CVC4. Recently, semantic fusion [20] has been proposed which is an approach to stress-test SMT solvers by fusing formula pairs that are by construction either satisfiable or unsatisfiable. Their tool YinYang found ~ 40 bugs in Z3 and CVC4, in string and nonlinear logic. However, it remained unclear whether Z3 and CVC4 have bugs in the more mature theories such as bit-vector and uninterpreted functions.

Type-aware mutation testing is inspired by skeletal program enumeration (SPE) [22], an approach for validating compilers. Similar to type-aware mutation testing, program skeletons are generated from a set of seed programs. The holes in these skeletons are then systematically filled by exhaustive enumeration. SPE provides relative guarantees with respect to the input seed programs. On a purely technical level, the grammar-aware grey-box fuzzer Superion [19] is related to our approach. Superion is based on the AFL fuzzing engine and uses code coverage to guide the grammar-aware mutations. As a key difference to type-aware mutations, grammar-aware mutations do not guarantee type-correct inputs. Type-aware mutations on the other hand are by definition type-correct. Finally, our work is loosely related to research on testing software that crucially relies on SMT solvers. Examples are works on testing software model checkers [15, 21] and on testing symbolic execution engines [13]. Our approach indirectly benefits the tested tools by addressing the SMT solvers’ bugs.

6 Conclusion

We introduced type-aware mutation testing, a simple and effective approach for stress-testing SMT solvers. We realized type-aware mutation testing in our testing tool OpFuzz in little more than 200 LoC supporting only the most basic operators of the SMT-LIB language. Despite this, OpFuzz found 521 confirmed bugs (457 fixed) in Z3 and CVC4. These bug findings are highly diverse, ranging over various types, logics and solver configurations in both state-of-the-art SMT solvers. Among these were many critical bugs. Maybe most notably and
in difference to previous works, OpFuzz found critical bugs in logics such as bit-vector logic and uninterpreted functions that are widely believed to be stable. Our bug findings further show that SMT solvers are not yet reliable enough, even the most popular and stable, such as Z3 and CVC4. Our highly practical tool OpFuzz can help SMT solver developers making their solvers more reliable. For future work, we want to explore the full potential of type-aware mutation testing by invoking more sophisticated type-aware mutations.

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