Probing QCD-instanton induced effects in deep inelastic scattering at HERA

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After a brief introduction into the phenomenology of instantons, results from instanton perturbation theory applied to deep-inelastic electron proton scattering are summarized. Based on event signatures derived from Monte Carlo studies, strategies for enhancing instanton induced events relative to Standard Model deep-inelastic events are developed. Preliminary results for a search for such events performed with the H1 detector at HERA are given.

1 Introduction

Localized solutions of non-linear field equations known from classical cases as solitons were discovered also in gauge theories. In 2 spatial dimensions, these are the Nielsen-Olesen-vortices and in 3 spatial dimensions, the Polyakov-'t Hooft-monopoles. In 4 dimensional Euclidean space (with $x_4 = -ix_0$) such localized solutions of classical Yang-Mills fields were discovered by Belavin, Polyakov, Schwartz and Tyupkin; since these solutions are localized in space as well as in time, they were given the name pseudo-particles or instantons

For non-abelian gauge fields with finite action, the vanishing of the Lagrange density i.e. $G_{\mu\nu}$ at $|x| \to \infty$ does not imply $A_\mu \to 0$ but only $A_\mu \to U^{-1} \partial_\mu U$ ("pure gauge"). The corresponding classical vacuum states are then classified according to a topological number $n$: the winding number resp. Chern-Simons number. For topological reasons, there exists an infinite number of non-equivalent vacua.

While classical transitions between these different vacua are forbidden, quantum-mechanical tunneling transitions are possible. Since tunneling corresponds to a classical path in Euclidean space, solutions with minimum action are favored. The Euclidean action is minimal for self-/anti-self-dual fields i.e. $G_{\mu\nu}^a = \pm \tilde{G}_{\mu\nu}^a$, where $\tilde{G}_{\mu\nu}$ is the dual field strength tensor. The tunneling then is characterized by the topological charge

$$Q = \frac{1}{32\pi^2} \int d^4x G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a. \tag{1}$$

One finds $Q = n_{cs}(t = \infty) - n_{cs}(t = -\infty)$, showing that the tunneling connects different topological vacua. The tunneling process itself is given the name instanton

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1 Talk given at the CICHEP 2001 in Cairo, Egypt
For the tunneling probability, one finds the characteristic exponential suppression \( P \sim \exp(-S_E) \) with the Euclidean action \( S_E = 8\pi^2 |Q|/g^2 \). As was noticed by Ringwald and Espinosa, at high energies the exponential suppression of instanton effects in scattering processes can be overcome by multiple emission of gauge bosons. Explicitly, the BPST-solution takes the form

\[
(G^a_{\mu\nu})^2 = \frac{192\rho^4}{(x^2 + \rho^2)^2}
\]

where \( \rho \) is a parameter characterizing the size of the instanton.

\[\text{Figure 1: Potential energy of classical non-abelian gauge field as function of the topological charge (Chern-Simons number).}\]

’t Hooft made the important step of coupling light fermions, i.e. massless quarks, to the gauge field. From the Adler-Bell-Jackiw axial triangle diagram anomaly, one then has for the axial current \( j_5^\mu \)

\[
\partial_\mu j_5^\mu = \frac{1}{16\pi^2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}.
\]

Using (1) it is seen that instantons induce transitions between states of different axial charge \( Q_5(t) = \int d^3x j_5^\mu(x, t) \) with

\[
\Delta Q_5 = Q_5(\infty) - Q_5(-\infty) = \int_{-\infty}^{+\infty} dt \int d^3x \partial_0 j_5^\mu(x) = \pm 2.
\]

This result holds separately for each quark flavor and for \( N_f \) flavors one, therefore, has \( \Delta Q_5 = \pm 2N_f \). These topological results, though derived in Euclidean space, also hold in Minkowski space. Instantons thus induce non-perturbative hadronic processes violating chirality.
2 Instantons in deep-inelastic scattering

Balitsky and Braun[5] first showed that the contribution of instanton ($I$)-induced processes to deep-inelastic electron proton scattering from a real gluon calculated in Euclidean space and continued to Minkowski space, rises very rapidly with decreasing Bjorken-$x$. Their calculations were restricted, however, to $x > 0.3 - 0.35$. A detailed theoretical[6, 7, 8] and phenomenological[9, 10] investigation of deep-inelastic scattering at HERA has been pursued by Ringwald, Schrempp and collaborators.

In perturbation theory, the leading contribution to electron-proton scattering is photon-gluon fusion (Fig. 2). The instanton induced part of the inclusive electron-proton cross section $\sigma_{ep}^I$ is obtained by an integration of the total quark-gluon cross section $\sigma_{q'g}^I$ over the photon flux, the gluon density and the quark flux in the instanton field, $\sigma_{q'g}^I$, which contains all instanton dynamics, is given by an integral over the “collective coordinates” of the instantons/anti-instantons, i.e. size $\rho$ with the distribution function $D(\rho)$, separation $R$ and relative color orientation $U$:

$$\sigma_{q'g}^I \sim \int d^4Re^i(p+q')R \int d\rho D(\rho)d\hat{\rho}D(\hat{\rho})e^{-(\rho+\hat{\rho})^2} \int dU\epsilon^{-\frac{4\pi}{\alpha_s}\Omega(R^2/\rho,\rho/R,U)}\{...\} \quad (3)$$

with less important contributions collected in {...}. $\Omega(...,U)$, describing the instanton-interaction and incorporating final state gluon effects, and $D(\rho)$ are known in principle in instanton-interaction theory for $\alpha_s(\mu_r)\ln \mu_r/\rho \ll 1$ and $R^2/\rho\bar{\rho} \gg 1$. $D(\rho)$ has a power law behavior

$$D(\rho) \sim \rho^{6-(2/3)nf+0(\alpha_s)}$$
which is infrared divergent unless $\rho$ is constrained. In DIS, the exponential in (3) introduces an effective cut-off $\rho < 1/Q'$ ensuring a finite integral.

For large $Q^2$ and the most attractive relative color orientation, the integral is dominated by a unique saddle point in $\rho \sim 1/Q'$ and $(R/\rho)^2 \sim 4(x'/1 - x')$ which defines a fiducial region in $x'$ and $Q'$. Taking the limits on $\rho$ and $R/\rho$ from the UKQCD lattice calculations[11] with $\Lambda_{\overline{MS}}^{n_f}$ from the ALPHA collaboration[12], the fiducial region is given by

\[
\rho \lesssim 0.35\text{fm} \quad \frac{R}{\rho} \gtrsim 1.1 \quad \frac{Q'}{\Lambda_{\overline{MS}}} \gtrsim 30.8 \quad x' \gtrsim 0.35
\]

which defines the following “standard cuts” to be applied to the experimental data:

\[x \geq 10^{-3}, \quad x' \gtrsim 0.35, \quad 0.1 \leq y \leq 0.9, \quad Q = Q' \geq 30.8\Lambda_{\overline{MS}}^{n_f} .\]

In order to suppress contributions from non-planar graphs which are hard to control, a further cut $Q^2_{\text{min}} = Q'^2_{\text{min}}$ has been applied. With these cuts, a HERA cross section $\sigma_{ep} \simeq 30 \pm 10$ pb is obtained. Such a cross section is well within the reach of HERA experiments.

Figure 3: Distribution of the transverse energy $E_T$ in pseudo rapidity ($\eta$)-azimuthal ($\phi$)-plane in the hadronic CMS for a typical instanton induced HERA-event generated by QCDINS ($x = 0.0012, Q^2 = 66$ GeV$^2, p_T(\text{Jet}) = 3.6$ GeV) after typical detector cuts. Clearly recognizable are the current jet at $\phi = 160^\circ, \eta \simeq 3$ and the instanton band at $0 \lesssim \eta \lesssim 2$.

This instanton model has been converted into the Monte Carlo QCDINS[13]. A typical event, shown in Fig. 3, exhibits the following characteristic features:
• a quark-jet \( (q'' \text{ in Fig. 2}) \)
• a hadron band in the pseudo rapidity-azimuthal angle plane in the hadronic CMS which is flat in \( \phi \)
• high total \( E_t \)
• enrichment in heavy flavors \( (K, \Lambda, \ldots) \).

As pointed out above, the hadronic final state is expected to violate helicity conservation.

3 Monte Carlo Studies

Monte Carlo generated events have been used to select observables which discriminate instanton induced events from DIS events. By means of combining several observables, an enhancement of instanton induced events is achieved\(^{[10]}\).

Standard DIS events have been simulated by two rather different models, i.e. a matrix element-parton shower model (MEPS) and a color dipole model (CDM) in order to obtain some information about systematic uncertainties of Monte Carlos in this extreme region of the phase space. MEPS is based on RAPGAP which incorporates the \( O(\alpha_s) \) QCD matrix element and models parton emission to all orders in the leading-log approximation by means of parton showers; the hadronization is performed using the LUND string model as implemented in JETSET. CDM is using ARIADNE in which gluon emission is simulated by independently radiating color dipoles and the hadronization is performed using JETSET.

Instanton induced events have been simulated with the Monte-Carlo QCDINS which consists of the hard instanton process generator embedded in HERWIG. The QCDINS 2.0 version with its default values \( x' < 0.35, Q'^2 > 113 \text{ GeV}^2 \) and \( n_f = 3 \) with CTEQ 4L parton densities has been used. The hadronization is performed using the cluster fragmentation model of HERWIG.

In a first step, the jet with maximal \( E_t \) is searched (using a cone algorithm with \( R = 0.5 \)) and identified as the current jet \( (q'' \text{ in Fig. 2}) \), allowing to estimate the 4-momentum \( q'' \). \( Q'^2 \) can be obtained from \( q'' \) and from the momentum of the scattered electron. After removal of the current jet from the event, an instanton band is defined with \( \bar{\eta} \pm 1.1 \), where \( \bar{\eta} = \sum_h E_{th}\eta_h / \sum_h E_{th} \). The charged multiplicity \( n_b \) in the instanton band is determined. For simplicity, the variables \( E_{tjet}, Q'^2 \) and \( n_b \), on which cuts will be applied, are named “primary observables”.

Furthermore, for the particles in the instanton band the transverse energy \( E_{th} \), the sphericity \( Sph \) in the rest-frame of the band and an observable \( \Delta_b \) characterizing the azimuthal anisotropy are defined. We define \( \Delta_b = 1 - (E_{out,b}/E_{in,b}) \) where \( E_{out} = \ldots \)
min ∑ₜ |Pₜ · i| and Eᵢᵣ = max ∑ₜ |Pₜ · i| with respect to an axis i. For pencil-like 2 jet-DIS events in photon-gluon-fusion, one expects Δₜ → 1 and for spherical instanton induced events Δₜ → 0. The variables Eᵢᵣ, Sph and Δₜ on which no cuts are applied are named “secondary variables”.

4 Experimental results

The experimental results presented in the following have been taken with the H1 detector at the HERA electron proton collider in 1997 and correspond to an integrated luminosity of 15.8 pb⁻¹. All results are preliminary.

Figure 4: Observables used to cut (“primary observables”): comparison of data with Monte Carlo before cuts (see text).

In the H1 detector, a central track detector is surrounded by a liquid argon calorimeter which consists of electromagnetic and hadronic sections and is covering the forward (with respect to the incoming proton) and central part of the solid angle. Scattered electrons (positrons actually) are detected with a backward (with respect to the incoming proton) electromagnetic calorimeter. Details of the H1 detector can be found elsewhere.

The event selection (real and Monte Carlo events) required a scattered electron with
$E > 10$ GeV in the backward calorimeter, a vertex within $-30 \text{ cm} < z < +30 \text{ cm}$ of the nominal interaction point and longitudinal energy conservation within $35 < \sum E - P_Z < 70$ GeV, where $+z$ is the proton beam axis coordinate and the sum is to be taken over all hadrons.

From the scattered electron $Q^2$ and $x$ are reconstructed. The analysis was restricted to the phase space defined by the electron polar angle $\Theta_e > 156^\circ$, $0.1 < y < 0.6$ and $x > 10^{-3}$. The resulting data sample contains 275k events.

![Figure 5: Observables not used for cuts (“secondary observables”): comparison of data with Monte Carlo before cuts on primary observables (see text).](image)

Figs. 4 and 5 compare H1 data with the Monte Carlos for the different observables. While the agreement in the primary observables is very good, some discrepancies are to be noted in the secondary variables $E_{tb}$ and $E_{tjet}$ at higher values of the variables. The effect of cuts on the primary variables on the efficiency $\epsilon_{ins}$ for retaining instanton induced events, the efficiency $\epsilon_{dis}$ for retaining DIS background events, and the separation power $\epsilon_{ins}/\epsilon_{dis}$ for DIS and instanton events has been systematically investigated. For the combined cuts $n_b > 8$, $105 \text{ GeV}^2 < Q'^2 < 200 \text{ GeV}^2$ and $Sph > 0.5$ one obtains $\epsilon_{ins} = 11.2\%$ and $\epsilon_{dis} = 0.13 - 0.16\%$ corresponding to a separation power of almost 100. After cuts on the primary variables 549 events are obtained in the data, while $363^{+22}_{-20}$ events are expected from CDM and $435^{+36}_{-22}$ events from MEPS. The distribution of these remaining events in the secondary variables is shown in Fig. 6. The excess of
data over the DIS expectations has to be contrasted with the difference between the two different DIS models which is of the same magnitude.

![Graphs showing observables](image)

**Figure 6:** Observables not used to cut (“secondary observables”) after cuts on “primary observables”

## 5 Summary

Instantons are a fundamental topological feature of non-pertubative QCD. Instanton perturbation theory supported by lattice QCD calculations predicts a measurable rate of instanton induced events in deep-inelastic scattering at HERA.

A strategy for enhancing instanton induced events in DIS events, which is based on cutting on selected event observables, has been employed by the H1 collaboration. An excess of data relative to DIS expectation is observed which is, however, of the same magnitude as systematic differences of the different DIS model predictions.

The increased data sample available in near future will allow more detailed investigations.
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