Effective Theories and Electroweak Precision Constraints

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This is a pedagogical and self-contained review on obtaining electroweak precision constraints on TeV scale new physics using the effective theory method. We identify a set of relevant effective operators in the standard model and calculate from them corrections to all major electroweak precision observables. The corrections are compared with data to put constraints on the effective operators. Various approaches and applications in the literature are reviewed.

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1. Introduction

Electroweak precision tests (EWPTs) performed in the last few decades have achieved great success in establishing the standard model (SM) as the correct description of electroweak physics. Nevertheless, we are certain the SM is incomplete and expect new physics to appear at the TeV scale. The lack of experimental deviations from the SM predictions does not rule out TeV scale new physics. Rather, it provides us guidance on the directions we should pursue. The purpose of this review is to provide readers quick access to necessary knowledge and tools for constraining various extensions of the SM.

In order to be model independent, we adopt the effective field theory (EFT) approach. EFT is a powerful tool when there is a distinctive scale separating the low energy and the high energy dynamics. The basic ideal is that the low energy physics can be described by an effective Lagrangian containing a few degrees of freedom which are not sensitive to extra degrees of freedom related to the details of the high energy physics. Fermi’s theory of weak interaction is a well known example.

For our purpose, given a TeV scale model, we first integrate out all new physical states and obtain a set of effective operators involving the SM fields. These operators are added to the SM Lagrangian and introduce new interactions among the SM particles. Consequently they contribute to electroweak observables (EWPOs) and cause deviations from the SM predictions. Since that the experiments agree with
the SM remarkably well, the deviations must be small. This is where the constraints come from. The key observation in favor of the effective theory approach is that the relevant effective operators are actually much fewer than the vast possibilities of physics beyond the SM. Therefore, it is natural to reverse the above procedure, namely, we first calculate constraints on all possible effective operators (and operator combinations) without specifying the model. Once this is done, the remaining work for constraining a given model is to obtain the operator coefficients in terms of parameters in that model. In this approach, one needs to make contact with experimental data only once. In this review, we will show how to calculate corrections to EWPOs from the effective operators, assuming arbitrary coefficients, while refer readers to Ref. 5 on how to obtain operator coefficients from an underlying theory.

Schematically, the effective theory can be written as

\[ \mathcal{L} = \mathcal{L}_{SM} + \sum a_i O_i, \]  

where \( \mathcal{L}_{SM} \) is the standard model Lagrangian and \( O_i \)'s are operators containing only the SM fields. Since we are interested in extensions of the SM, we assume that \( O_i \)'s respect the SM gauge symmetry. Below the new physics scale, all effects of the new physical states are encoded in \( O_i \)'s. Of course one can only work with operators to a given order and higher order operators have to be truncated. The power counting depends on whether the electroweak symmetry is linearly or non-linearly realized. For the former, \( \mathcal{L}_{SM} \) is renormalizable and the electroweak symmetry is broken by the vacuum expectation value (vev) of a Higgs doublet. The power counting is determined by the canonical dimensions of the operators \( O_i \),

\[ \mathcal{L} = \mathcal{L}_{SM} + \sum a_i O_i = \mathcal{L}_{SM} + \sum \frac{c_i}{\Lambda^{n_i-4}} O_i, \]  

where \( n_i \) is the dimension of \( O_i \), \( c_i \)'s are dimensionless parameters and \( \Lambda \) is an energy scale of \( O(\text{TeV}) \). For the non-linear case, the SM spectrum does not contain a light Higgs boson and \( \mathcal{L}_{SM} \) is a non-linear sigma model. In this review, I will concentrate on the linear case while also explain how to translate the results to the nonlinear case.

The effective theory breaks down if the new physics contains physical states much lighter than 1 TeV. For example, the mass of the lightest neutralino can be as low as \( \sim 100\text{GeV} \) in the minimal supersymmetric standard model (MSSM). In this case, there is not an energy scale that distinctly separates the new physics from the SM, and an EFT without the new states is not well defined. Nevertheless, the new states do not appear in initial or final states in any of the EWPT processes and their dominant contributions can often be captured in a few effective operators.

With the effective Lagrangian (2) defined, the theoretical prediction for a given observable \( X \) can be written as

\[ X_{th}(a_i) = X_{SM} + \sum a_i X_i, \]  

where \( X_{SM} \) is the SM prediction including loop corrections, which can be found in the literature. X_i is the correction to X from the operator O_i. The current
experimental precision allows us to keep only the corrections linear in the operator coefficients $a_i$. Most of the review will be devoted to the calculation of $X_i$, for all well measured EWPOs.

After obtaining $X_{th}$, we can compare the theoretical predictions with the measured values of $X$ and get constraints on the coefficients $a_i$. The constraints are conveniently given in the $\chi^2$ distribution,

$$\chi^2(a_i) = \sum_{X} \left( \frac{X_{th}(a_i) - X_{exp}}{\sigma_X} \right)^2,$$

(4)

where $X_{exp}$ is the experimental value of $X$, and $\sigma_X$ includes both experimental and theoretical errors. In practice, the above expression is complicated from correlations between different observables. At this step the advantage of the effective theory approach is manifest: all relevant experimental information has been collected in the $\chi^2$ distribution, which is a simple quadratic function of the operator coefficients $a_i$.

There have been many analyses using the effective theory approach in the literature\cite{6–15}. The oblique parameters\cite{16–22} are perhaps the best known example. The results are often given in different bases or conventions. In this review, we will adopt the basis in Ref.\cite{6} where all independent dimension-6 operators of the SM are included. They are discussed and trimmed according to our needs in the next section. In particular, we will identify the operators relevant to EWPTs. Section 3 contains a summary of the EWPOs included in this review. In Section 4 we calculate corrections to EWPOs from the effective operators. The constraints are then given in Section 5. The analysis largely follows Ref.\cite{14} Section 6 contains a few discussions, including translations between different bases and conventions.

2. Effective Operators

2.1. The standard model Lagrangian

For establishing the notation and later convenience, we give explicitly the SM Lagrangian,

$$\mathcal{L}_{SM} = -\frac{1}{4} G_{\mu\nu}G^{\mu\nu} - \frac{1}{4} W_\mu W^{\mu\nu} - \frac{1}{4} B_\mu B^{\mu\nu} + (D_\mu h)^\dagger (D^\mu h) + m^2 h^\dagger h - \frac{1}{2} \lambda (h^\dagger h)^2 + i\bar{d}Dl + i\bar{e}De + i\bar{q}Dq + i\bar{u}Dd + i\bar{d}Dd + (Ye\bar{e}h + Yq\bar{q}h + Yd\bar{d}h + h.c.),$$

(5)

where $G_{\mu\nu}$, $W_\mu$ and $B_\mu$ denote gauge field strength of the SM gauge groups $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$, respectively. For simplicity, we have assumed there is only one Higgs doublet, denoted by $h$. The number of Higgs doublets is irrelevant for corrections to EWPOs at tree level, since all their contributions are through the
electroweak breaking vev,
\[
\langle h \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}
\]
with \( v = 246 \text{GeV} \). The left-handed fermion doublets are denoted \( q \) and \( l \) and the right-handed singlets \( u, d \) and \( e \). For simplicity, we have omitted the corresponding subscripts \( L \) and \( R \) and also the flavor indices.

Variations of the SM Lagrangian with respect to the fields give us equations of motion, which we can use to eliminate redundant operators. Two of them will prove useful in our discussion later, which are listed below. They are obtained by varying \( W^\mu_a \) and \( B^\mu \) respectively.
\[
(D_\nu W^{\nu\mu})^a = -\frac{1}{2}g((ih^\dagger \overrightarrow{D}^\mu \sigma^a h + \bar{l} \gamma^\mu \sigma^a l + \bar{q} \gamma^\mu \sigma^a q),
\]
\[
\partial_\nu B^{\nu\mu} = -\frac{1}{2}g'(ih^\dagger \overrightarrow{D}^\mu h) - g' \sum_f Y_f \bar{f} \gamma^\mu f,
\]
where \( g \) and \( g' \) are gauge couplings of \( SU(2)_L \) and \( U(1)_Y \). \( Y_f \) is the hypercharge of fermion \( f \).

Substituting the Higgs vev in the SM Lagrangian, we arrive at the mass eigenstates,
\[
Z_\mu = cW^3_\mu - sB_\mu, \quad A_\mu = sW^3_\mu + cB_\mu, \quad W^{\pm}_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu),
\]
where
\[
s = \sin \theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}, \quad c = \cos \theta_W = \frac{g}{\sqrt{g^2 + g'^2}}.
\]
Then the couplings between gauge bosons and fermions can be read from the covariant derivative,
\[
D_\mu = \partial_\mu - igW^a_\mu T^a - ig'B_\mu,
\]
\[
= \partial_\mu - \frac{g}{\sqrt{2}}(W^+ T^+) - \frac{e}{2sc}Z_\mu(T^3 - s^2 Q) - iA_\mu Q,
\]
where \( Q \) is the fermion charge and
\[
e = \frac{gg'}{\sqrt{g^2 + g'^2}}.
\]
It is often convenient to write the \( Z \)-fermion couplings in terms of \( g_V^f \) and \( g_A^f \) as defined by
\[
\mathcal{L} = \frac{e}{2sc} \bar{f} \gamma^\mu (g_V^f - g_A^f \gamma^5) \gamma^5 f Z_\mu.
\]
The SM values for $g_{V}^{f}$ and $g_{A}^{f}$, denoted $g_{V0}^{f}$ and $g_{A0}^{f}$, are given by

$$
\begin{align*}
&f \\
\nu_e, \nu_\mu, \nu_\tau & + \frac{1}{2} \\
e, \mu, \tau & - \frac{1}{2} + 2s^2 - \frac{1}{2} \\
u, c, t & + \frac{1}{2} - \frac{3}{2}s^2 + \frac{1}{2} \\
d, s, b & - \frac{1}{2} + \frac{3}{2}s^2 - \frac{1}{2}
\end{align*}
$$

(14)

### 2.2. New operators

Usually, experiments considered as EWPTs conserve CP, as well as baryon and lepton numbers. Constraints on processes violating these symmetries are very stringent, indicating that they are irrelevant to TeV scale physics. For example, operators contributing to proton decay have to be suppressed by the grand unification scale or higher. CP violating operators that contribute to $K - \bar{K}$ mass difference or lepton electric dipole moment are typically suppressed by a scale of $10^{3}$ to $10^{4}$ TeV.

Therefore, it is convenient to restrict us to operators conserving CP, and baryon and lepton numbers.

Similarly, constraints on flavor violating processes are also stringent (with the exception of processes involving the third generation). In the lepton sector, the limits on $\mu \rightarrow e\gamma$ decay implies $10^{4}$ TeV suppression for contributing operators. In the quark sector, limits on new physics scale obtained from a variety of $\Delta F = 2$ processes are also above $10^{3}$ to $10^{4}$ TeV. Therefore, we will assume that the operators, and therefore the observables included in this review all conserve flavors.

The only dimension-5 operator that respects the SM gauge symmetry is the neutrino Majorana mass operator, which does not conserve lepton number and is irrelevant to EWPTs. Not counting flavors, there are 80 independent dimension-6 operators that conserve baryon and lepton numbers, out of which 28 violate CP. We will go through the remaining 52 operators in this section and identify those contributing to the EWPOs.

For our purpose, it is enough to consider dimension-6 operators only. Dimension-7 operators do not contribute to EWPTs at tree level, and we do not need to go to dimension-8 or higher. The reason is the following: compared with the SM contribution, the corrections from operator $O_i$ is suppressed by $(v/\Lambda)^{n_i-4}$ or $(E/\Lambda)^{n_i-4}$ where $E \leq O(200)$GeV is the energy scale involved in the EWPTs. For dimension-6 operators, this suppression is roughly equal to the experimental precision of the EWPOs when $\Lambda$ is a few TeV. The suppression factor is much smaller for the dimension-8 operators which makes their contributions to EWPOs negligible unless they have extraordinarily large coefficients.

The above reasoning indicates that when calculating the corrections to the EWPOs from the dimension-6 operators, we only need to work to the linear order in $a_i = c_i/\Lambda^2$, since the quadratic corrections are much smaller and therefore can
be neglected. This means we only need to work at tree level for those operators. Moreover, when calculating from the matrix element to the cross-section, only the interferences between the SM Feynman diagrams and diagrams induced by new operators are important. This important observation will be used to reduce the number of operators.

The linear-order assumption also helps us understand why we only need the vev of the Higgs doublet: the physical Higgs boson contributes to EWPOs only at loop levels, any corrections to the Higgs mass or coupling from dimension-6 operators will be suppressed by an extra loop factor when calculating the corrections to EWPOs and therefore negligible.

The 52 dimension-6 operators that conserve CP are enumerated below. The naming scheme follows Ref. [6] with small changes in the notation. We will identify those that are relevant to EWPTs by general arguments in this section. More detailed calculations are presented in Sec. 4. If the analysis here is too sketchy for the reader, it should become clear later on.

(1) Vectors only

\[ O_G = \epsilon^{abc} G^a_{\mu\nu} G^b_{\nu\lambda} G^c_{\lambda\mu} , \]
\[ O_W = \epsilon^{abc} W^a_{\mu\nu} W^b_{\nu\lambda} W^c_{\lambda\mu} . \] (15)

The two operators affect triple gauge couplings. At tree level, the former only affects pure hadronic processes, which are not measured as well as EWPOs. Therefore, only the operator \( O_W \) is interesting to us.

(2) Fermions only

(a) \( \overline{\text{LLL}}L \) operators

\[ O_{\overline{l}l}^{(1)} = \frac{1}{2} (\overline{\gamma}^\mu l)(\overline{\gamma}^\mu l) , \quad O_{\overline{l}l}^{(3)} = \frac{1}{2} (\overline{\gamma}^\mu \sigma^a l)(\overline{\gamma}^\mu \sigma^a l) , \]
\[ O_{qq}^{(1,1)} = \frac{1}{2} (\overline{\gamma}^\mu \sigma^a q)(\overline{\gamma}^\mu \sigma^a q) , \quad O_{qq}^{(8,1)} = \frac{1}{2} (\overline{\gamma}^\mu \lambda A q)(\overline{\gamma}^\mu \lambda A q) , \]
\[ O_{qq}^{(1,3)} = \frac{1}{2} (\overline{\gamma}^\mu \sigma^a q)(\overline{\gamma}^\mu \sigma^a q) , \quad O_{qq}^{(8,3)} = \frac{1}{2} (\overline{\gamma}^\mu \lambda A \sigma^a q)(\overline{\gamma}^\mu \lambda A \sigma^a q) , \]
\[ O_{lq}^{(1)} = \frac{1}{2} (\overline{\gamma}^\mu l)(\overline{\gamma}^\mu q) , \quad O_{lq}^{(3)} = \frac{1}{2} (\overline{\gamma}^\mu \sigma^a l)(\overline{\gamma}^\mu \sigma^a q) . \] (16)

\(^a\)The exception is muon \((g-2)\) discussed later. Due to the high precision of this measurement, loop level diagrams are also important.
(b) $\bar{R}R\bar{R}R$ operators

\begin{align*}
O_{ee} &= \frac{1}{2} (\bar{e} \gamma^{\mu} e)(\bar{e} \gamma_{\mu} e), \\
O_{uu}^{(1)} &= \frac{1}{2} (\bar{u} \gamma^{\mu} u)(\bar{u} \gamma_{\mu} u), \quad O_{uu}^{(8)} = \frac{1}{2} (\bar{u} \lambda^{A} \gamma^{\mu} u)(\bar{u} \lambda^{A} \gamma_{\mu} u), \\
O_{dd}^{(1)} &= \frac{1}{2} (\bar{d} \gamma^{\mu} d)(\bar{d} \gamma_{\mu} d), \quad O_{dd}^{(8)} = \frac{1}{2} (\bar{d} \lambda^{A} \gamma^{\mu} d)(\bar{d} \lambda^{A} \gamma_{\mu} d), \\
O_{eu} &= (\bar{e} \gamma^{\mu} e)(\bar{u} \gamma_{\mu} u), \quad O_{ed} = (\bar{e} \gamma^{\mu} e)(\bar{d} \gamma_{\mu} d), \\
O_{ul}^{(1)} &= \frac{1}{2} (\bar{u} \gamma^{\mu} u)(\bar{d} \gamma_{\mu} d), \quad O_{ul}^{(8)} = \frac{1}{2} (\bar{u} \lambda^{A} \gamma^{\mu} u)(\bar{d} \lambda^{A} \gamma_{\mu} d). 
\end{align*}

(17)

(c) $\bar{L}L\bar{R}R$ operators

For convenience, we have changed the form from $\bar{L}R\bar{R}L$ in Ref. 6 to $\bar{L}L\bar{R}R$ by using the Fierz transformation

\begin{align*}
(\bar{\psi}_1 L \psi_2 R)(\bar{\psi}_3 R \psi_4 L) &= \frac{1}{2} (\bar{\psi}_1 L \gamma^{\mu} \psi_4 L)(\bar{\psi}_3 R \gamma_{\mu} \psi_2 R).
\end{align*}

\begin{align*}
O_{le} &= (\bar{\ell} \gamma^{\mu} l)(\bar{e} \gamma_{\mu} e), \\
O_{lu} &= (\bar{\ell} \gamma^{\mu} l)(\bar{u} \gamma_{\mu} u), \\
O_{ld} &= (\bar{\ell} \gamma^{\mu} l)(\bar{d} \gamma_{\mu} d), \\
O_{qe} &= (\bar{q} \gamma^{\mu} q)(\bar{e} \gamma_{\mu} e), \\
O_{qu}^{(1)} &= (\bar{q} \gamma^{\mu} q)(\bar{u} \gamma_{\mu} u), \quad O_{qu}^{(8)} = (\bar{q} \gamma^{\mu} \lambda^{A} q)(\bar{u} \gamma_{\mu} \lambda^{A} u), \\
O_{qd}^{(1)} &= (\bar{q} \gamma^{\mu} q)(\bar{d} \gamma_{\mu} d), \quad O_{qd}^{(8)} = (\bar{q} \gamma^{\mu} \lambda^{A} q)(\bar{d} \gamma_{\mu} \lambda^{A} d), \\
O_{qde} &= (\bar{q} \gamma^{\mu} q)(\bar{d} \gamma_{\mu} e). 
\end{align*}

(18)

(d) $\bar{L}R\bar{R}L$ operators

\begin{align*}
O_{qq}^{(1)} &= \epsilon^{ab}(\bar{q}^{a} u)(\bar{q}^{b} d), \quad O_{qq}^{(8)} = \epsilon^{ab}(\bar{q}^{a} \lambda^{A} u)(\bar{q}^{b} \lambda^{A} d), \\
O_{lq} &= \epsilon^{ab}(\bar{l}^{a} e)(\bar{q}^{b} u). 
\end{align*}

(19)

Again we can safely ignore those operators containing four quarks. For the operators $O_{qde}$ and $O_{lq}$, they contain pieces $\bar{\tau}_{L} e_{R} d_{R} d_{L}$ and $\bar{\tau}_{L} e_{R} \bar{\tau}_{L} d_{R} d_{L}$ that contribute to the process $e^{+}e^{-} \rightarrow \text{hadrons}$. However, the corresponding SM contribution are highly suppressed by electron and quark Yukawas, which are negligible. Since we are only interested in interferences between the SM and the new physics, the contributions from $O_{qde}$ and $O_{lq}$ are negligible too. Thus the interesting operators are: $O_{l1}^{(1)}$, $O_{l1}^{(3)}$, $O_{lq}^{(1)}$, $O_{lq}^{(3)}$, $O_{ee}$, $O_{eu}$, $O_{ed}$, $O_{ul}$, $O_{ld}$, $O_{qe}$.

(3) Scalars only

\begin{align*}
O_{h} &= \frac{1}{3} (h \lambda h)^{3}, \\
O_{ah} &= \frac{1}{2} \partial_{\mu} (h^{\dagger} h) \partial^{\mu} (h^{\dagger} h).
\end{align*}

(20)
The two operators affect the Higgs mass and self-coupling, which are negligible as explained earlier.

(4) Fermions and vectors

\[ O_{1W} = i\bar{\theta}^a \gamma^\mu D^\nu l W^a_{\mu\nu}, \quad O_{1B} = i\bar{\theta}^a \gamma^\mu D^\nu l B^a_{\mu\nu}, \]
\[ O_{2W} = i\bar{\psi} \gamma^\mu D^\nu e B^a_{\mu\nu}, \quad O_{2B} = i\bar{\psi} \gamma^\mu D^\nu q B^a_{\mu\nu}, \]
\[ O_{3G} = i\bar{\psi} \gamma^\mu D^\nu q G^a_{\mu\nu}, \quad O_{3B} = i\bar{\psi} \gamma^\mu D^\nu q B^a_{\mu\nu}, \]
\[ O_{4G} = i\bar{\psi} \gamma^\mu D^\nu q G^a_{\mu\nu}, \quad O_{4B} = i\bar{\psi} \gamma^\mu D^\nu q B^a_{\mu\nu}, \]
\[ O_{5G} = i\bar{\psi} \gamma^\mu D^\nu e G^a_{\mu\nu}, \quad O_{5B} = i\bar{\psi} \gamma^\mu D^\nu e B^a_{\mu\nu}, \]
\[ O_{6G} = i\bar{\psi} \gamma^\mu D^\nu u G^a_{\mu\nu}, \quad O_{6B} = i\bar{\psi} \gamma^\mu D^\nu u B^a_{\mu\nu}, \]
\[ O_{7G} = i\bar{\psi} \gamma^\mu D^\nu d G^a_{\mu\nu}, \quad O_{7B} = i\bar{\psi} \gamma^\mu D^\nu d B^a_{\mu\nu}. \]

At first sight, these operators alter couplings between the gauge bosons and the fermions. However, the couplings are imaginary compared with the SM couplings. Therefore the SM diagrams do not interfere with the new diagrams. In this case, corrections from these operators can be ignored.

(5) Scalars and vectors

\[ O_{hG} = \frac{1}{2} (h^\dagger h) G^A_{\mu\nu} G^{A\mu\nu}, \]
\[ O_{hW} = \frac{1}{2} (h^\dagger h) W^a_{\mu\nu} W^{a\mu\nu}, \]
\[ O_{hB} = \frac{1}{2} (h^\dagger h) B^a_{\mu\nu} B^{a\mu\nu}, \]
\[ O_{hW} = (h^\dagger h) W^a_{\mu\nu} B^{a\mu\nu}, \]
\[ O_{hB} = (h^\dagger h) B^a_{\mu\nu} B^{a\mu\nu}, \]
\[ O_{h}^{(1)} = (h^\dagger h) (D^\mu h)(D^\mu h), \quad O_{h}^{(2)} = (O_h) = (h^\dagger D^\mu h)(D^\mu h). \]

When \( h \) acquires a vev, the operators \( O_{hG}, O_{hW}, O_{hB}, O_{h}^{(1)} \) yield corrections to the kinetic terms for \( G^A_{\mu\nu}, W^a_{\mu\nu}, B^a_{\mu\nu} \) and \( h \), which can be absorbed by field redefinitions. Therefore, they do not contribute to EWPOs at the order \( 1/\Lambda^2 \).

The other two operators \( O_{hW} \) and \( O_{h}^{(3)} \) (simplified as \( O_h \) in the rest of the review) modify the gauge boson propagators. Indeed, they correspond to the well-known oblique \( S \) and \( T \) parameters. One also notices that \( O_{hW} \) contains corrections to triple gauge couplings.

(6) Fermions and scalars

\[ O_{e h} = (h^\dagger h) (\bar{\ell} e h), \]
\[ O_{u h} = (h^\dagger h) (\bar{q} u h), \]
\[ O_{e h} = (h^\dagger h) (\bar{\nu} d h). \]
The effects of these operators can be absorbed in the SM Yukawa couplings and therefore do not make any contribution.

(7) Fermions, gauge bosons and scalar

(a) One derivative

\[ \begin{align*}
O^{(1)}_{hl} & = i(h^\dagger D_\mu h)(\bar{l}_\tau \gamma^\mu l) + h.c., \\
O^{(3)}_{hl} & = i(h^\dagger D_\mu \sigma^a h)(\bar{l}_\tau \gamma^\mu \sigma^a l) + h.c., \\
O_{he} & = i(h^\dagger D_\mu h)(\bar{\nu}_e \gamma^\mu e) + h.c., \\
O^{(1)}_{hq} & = i(h^\dagger D_\mu h)(\bar{q}_s \gamma^\mu q) + h.c., \\
O^{(3)}_{hq} & = i(h^\dagger D_\mu \sigma^a h)(\bar{q}_s \gamma^\mu \sigma^a q) + h.c., \\
O_{hu} & = i(h^\dagger D_\mu h)(\bar{u}_R \gamma^\mu u) + h.c., \\
O_{hd} & = i(h^\dagger D_\mu h)(\bar{d}_R \gamma^\mu d) + h.c., \\
O_{hh} & = i(h^T \epsilon D_\mu h)(\bar{\nu}_e \gamma^\mu \nu) + h.c. 
\end{align*} \]

When \( h \) gets a vev, these operators modify the gauge-fermion couplings and therefore are important for our calculation. It is worth noting that the operator \( O_{hh} \) induces a coupling in the form \( W^+_{\mu} \mathcal{R}^\mu d_R + h.c. \) which is absent in the SM. In other words, there is no interference for diagram containing this coupling and the SM diagram. Therefore, we expect loose bound on the operator \( O_{hh} \).

(b) Two derivatives

\[ \begin{align*}
O_{De} & = (\bar{l} D_\mu e) D^\mu h, & O_{\overline{D}e} & = (\bar{D}_\mu \overline{e}) D^\mu h, \\
O_{Du} & = (\bar{q} D_\mu u) D^\mu h, & O_{\overline{D}u} & = (\bar{D}_\mu \overline{u}) D^\mu \tilde{h}, \\
O_{Dd} & = (\bar{q} D_\mu d) D^\mu h, & O_{\overline{D}d} & = (\bar{D}_\mu \overline{d}) D^\mu \tilde{h}, \\
O_{eW} & = (\bar{l}_s \sigma^{\mu \nu} \sigma^a e) h W^a_{\mu \nu}, & O_{eB} & = (\bar{l}_s \sigma^{\mu \nu} e) h B_{\mu \nu}, \\
O_{uG} & = (\bar{q}_s \lambda^A \lambda^A u) \tilde{h} G^A_{\mu \nu}, \\
O_{uW} & = (\bar{q}_s \sigma^{\mu \nu} \sigma^a u) \tilde{h} W^a_{\mu \nu}, & O_{uB} & = (\bar{q}_s \sigma^{\mu \nu} u) \tilde{h} B_{\mu \nu}, \\
O_{dG} & = (\bar{q}_s \lambda^A \lambda^A d) h G^A_{\mu \nu}, \\
O_{dW} & = (\bar{q}_s \sigma^{\mu \nu} \sigma^a d) h W^a_{\mu \nu}, & O_{dB} & = (\bar{q}_s \sigma^{\mu \nu} d) h B_{\mu \nu}. 
\end{align*} \]

Like operators in Eqs. (24), Hermitian conjugate should be added to each operator above. Most of these operators do not have a tight bound from EWPTs. They all produce couplings absent in the SM in the tree level and therefore do not interfere with the SM diagrams. The exception is \( O_{eW} \) and \( O_{eB} \), which contribute at tree level to the lepton \( (g-2) \). Indeed, the precision for electron and muon \( (g-2) \) measurements are so good that if we naively take the operator coefficients to be \( O(1/\Lambda^2) \), the bound on \( \Lambda \) will be larger than 100 TeV. Usually, the contribution from TeV scale physics is suppressed by both the loop factor and the lepton mass, which
makes it too small to compare with the experiments for the electron $g - 2$ and the bounds from muon $g - 2$ comparable to the other EWPOs. Other dimension-6 operators can also make sizable contributions to muon $(g - 2)$ at one loop level. We will come back to this observable later.

The relevant operators are summarized below. Following Ref. [14] we have simplified the notation by replacing superscripts (1) and (3) with $s$ and $t$, denoting singlets and triplets respectively.

1. Operators that modify gauge boson propagators
   
   \begin{equation}
   O_{WB}, O_h.
   \end{equation}

2. Operators that affect tree level SM gauge-fermion couplings
   
   \begin{equation}
   O^s_{ht}, O^t_{ht}, O_{he}, O^s_{hq}, O^t_{hq}, O_{hu}, O_{hd}.
   \end{equation}

3. Four-fermion operators
   
   \begin{equation}
   O^s_{ll}, O^t_{ll}, O^s_{lq}, O^t_{lq}, O_{le}, O_{lu}, O_{ld}, O_{ee}, O_{eu}, O_{ed}.
   \end{equation}

4. Operators that modify triple gauge boson couplings
   
   \begin{equation}
   O_W, (O_{WB}).
   \end{equation}

5. Operators that modify lepton $g - 2$
   
   \begin{equation}
   O_{eW}, O_{eB}.
   \end{equation}

We have not specified the flavor structure for the above operators except the assumption that they conserve flavors. Flavors can be conserved separately for each generation. In this case, each of the $O_i$’s actually corresponds to several operators with different flavors and independent coefficients. This is the most general case. However, it requires the quarks in the operators to be mass eigenstates. Therefore the new physics must be aligned with the Yukawa matrices, which seems unnatural and is difficult to realize. A simpler option is imposing flavor universality for the effective operators. This is natural for flavor-blind new physics, for example, a $Z'$ gauge boson that couples to all generations universally. In this case, each $O_i$ corresponds to several operators with different flavors but the same coefficient $a_i$. The operators should be understood to be contracted over flavor indices (in the 4-fermion operators, flavor indices are contracted over fermions (12) and (34) respectively. In the following analysis, unless specified, we will assume flavor universality. Nevertheless, calculations for flavor universal and non-universal cases are very similar and it is straightforward to translate from one case to the other.

Flavor universality is assumed in the analysis of Ref. [14]. It is realized through a global $U(3)^5$ symmetry, with each $U(3)$ corresponds respectively to each of $q, l, u, d$ and $e$. Under the $U(3)$, fermions and antifermions transform as fundamental

\(^c\)Electron $g - 2$ is one of the measurements from which the input parameter $\alpha$ is determined.
and anti-fundamental representations. This is the largest symmetry of the fermion kinetic terms in the SM, i.e., the symmetry when the Yukawa couplings are turned off. This symmetry not only guarantees flavor universality, but also forbids some of the operators discussed above, for example, $O_{qde} = \left( \bar{l} \gamma^\mu q \right) \left( \bar{d} \gamma^\mu e \right)$, because left-handed fields and right-handed fields transform under different $U(3)$’s. Most of those operators do not interfere with the SM, and therefore have already been excluded from the list Eq. (26)-(30). The only exception is the operators $O_{eB}$ and $O_{eW}$, which contribute to the muon $g - 2$, but is excluded in Ref. [14]. In this review, we will quote results from Ref. [14] whenever is appropriate, and also include discussions on the muon $g - 2$. An alternative flavor symmetry is discussed in Section 6.

Following a summary of the electroweak observables in the next section, we will consider one by one the operators in Eqs. (26)-(30) and calculate their corrections to the EWPOs in Section 4.

3. Experiments

As mentioned before, measurements included in this review do not contain those violating CP or flavor symmetries. The observables, together with their experimental values and SM predictions are summarized in Table 1. The experimental values and the SM predictions are excerpted from Ref. [1] except LEP 2 measurements. Multiple measurements for the same observable have been combined. We will give more information for each observable when calculating corrections from the effective operators. The information included in this review is minimal for understanding the calculations. It is strongly recommended that readers consult more comprehensive reviews for further details [11-13].

Not shown in Table 1 are three most precisely measured observables: the fine structure constant $\alpha$, Fermi constant $G_F$ determined from muon lifetime and the Z boson mass measured at LEP 1. The experimental values are [1]

$$\alpha = 1/137.03599911(46), \quad G_F = 1.16637(1) \times 10^{-5}\text{GeV}^{-2},$$
$$M_Z = 91.1876 \pm 0.0021\text{GeV}. \quad (31)$$

Not counting Higgs and fermion masses and mixings, the SM Lagrangian has three parameters $g, g'$ and $v$. However, they are not directly measurable. Therefore we take the above three observables as the input parameters, from which the SM predictions are calculated. The SM values also depend on the top quark mass $m_t$, the Higgs mass $m_H$, and the strong coupling constant $\alpha_s$, for which the best fit values are [1]

$$m_t = 172.7 \pm 2.8\text{GeV}, \quad m_H = 89^{+38}_{-29}\text{GeV}, \quad \alpha_s(M_Z) = 0.1216 \pm 0.0017. \quad (32)$$

It is interesting that the data favors a light Higgs, with an upper bound of $\sim 200\text{GeV}$ for the mass at 95% confidence level (CL).

The SM predictions with the above input parameters agree with the experimental values very well. The $\chi^2$ per degree of freedom is 1.1 (excluding LEP 2 measurements), and only two observables have deviations exceeding 2 $\sigma$, -2.4 for $A_{fb}^{0,b}$ and -2.7 for $g_L^2$. 

Table 1. Relevant measurements

| Notation | Value | SM prediction |
|----------|-------|---------------|
| Atomic parity $Q_W(Cs)$ | $-72.62 \pm 0.46$ | $-73.17 \pm 0.03$ |
| $Q_W(Tl)$ | $-116.6 \pm 3.7$ | $-116.78 \pm 0.05$ |
| $\mu$-nucleon scattering $g_1^\mu$ | $0.30005 \pm 0.00137$ | $0.30378 \pm 0.00021$ |
| $g_2^\mu$ | $0.03076 \pm 0.00110$ | $0.03096 \pm 0.00003$ |
| $\nu$-$e$ scattering $g_1^{\nu e}$ | $-0.940 \pm 0.015$ | $-0.0390 \pm 0.0003$ |
| $g_2^{\nu e}$ | $-0.507 \pm 0.014$ | $-0.5064 \pm 0.0001$ |
| $e^+e^- \rightarrow ff$ at Z-pole $|F_2| [\text{GeV}]$ | $2.4952 \pm 0.0023$ | $2.4968 \pm 0.0011$ |
| $\sigma^{\nu}_f [\text{nb}]$ | $41.541 \pm 0.037$ | $41.467 \pm 0.009$ |
| $R_1$ | $20.804 \pm 0.050$ | $20.756 \pm 0.011$ |
| $R_2$ | $20.785 \pm 0.033$ | $20.756 \pm 0.011$ |
| $R_3$ | $20.764 \pm 0.045$ | $20.801 \pm 0.011$ |
| $R_\theta$ | $0.21629 \pm 0.000066$ | $0.21578 \pm 0.000010$ |
| $A_{tb}$ | $0.1721 \pm 0.0030$ | $0.17230 \pm 0.000004$ |
| $A_{tu}$ | $0.0145 \pm 0.0025$ | $0.01622 \pm 0.00025$ |
| $A_{tb,\mu}$ | $0.0169 \pm 0.0025$ | $0.01622 \pm 0.00025$ |
| $A_{tb,\tau}$ | $0.0188 \pm 0.0017$ | $0.01622 \pm 0.00025$ |
| $A_{tb,\nu}$ | $0.0992 \pm 0.0016$ | $0.1031 \pm 0.0008$ |
| $A_{tb,c}$ | $0.0707 \pm 0.0035$ | $0.0737 \pm 0.0006$ |
| $|\sin^2 \theta_{eff}^{\nu}(Q_{fb})|$ | $0.2319 \pm 0.0012$ | $0.23152 \pm 0.00014$ |
| $A_{\tau}$ | $0.1514 \pm 0.0019$ | $0.1471 \pm 0.0011$ |
| $A_{\mu}$ | $0.142 \pm 0.015$ | $0.1471 \pm 0.0011$ |
| $A_{e}$ | $0.1433 \pm 0.0041$ | $0.1471 \pm 0.0011$ |
| $W$ pair $d\sigma_W / d\cos \theta$ | Ref. [24] | Ref. [28] |
| $W$ mass $M_W [	ext{GeV}]$ | $80.410 \pm 0.032$ | $80.376 \pm 0.017$ |

Note that the observables listed in Table 1 are often “pseudo-observables”, in the sense that they are not the directly measured quantities in the experiments. For example, the LEP 1 experiments measured the cross sections for $e^+e^- \rightarrow f\bar{f}$ at a few different center mass energy around the Z-pole. They have all been combined and translated to a few quantities at Z-pole. For LEP 2 measurements at various center of mass energies, there is no such simplification and therefore the numerical values of SM predictions are not listed in Table 1. Instead, we have given the corresponding references.

The Z-pole observables and several low-energy observables have the best precision. They dominate the constraints whenever the considered operators contribute to them. Therefore, in the literature, some of the observables in Table 1 are often omitted, which does not significantly alter the bounds on the models in consideration. Nevertheless, there are also operators that cannot be constrained by the Z-pole and low energy measurements. As we will see, some of the 4-fermion operators are only constrained by the LEP 2 measurements. The LEP 2 measurement for $e^+e^- \rightarrow W^+W^-$ cross sections also provides us unique constraints on triple gauge couplings.
4. Calculations

4.1. General consideration

We now start to calculate the corrections from operators (26) through (30) to the EWPOs listed in Table [1]. In other words, we want to calculate the values of $X_i$ in Eq. (3), which is the leading order contribution to observable $X$ from the operator $O_i$. Before obtaining $X_i$ for each observable $X$, let us consider in more detail how those operators contribute:

(1) Operators that modify gauge boson propagators

(a) $O_{WB}$

Substituting in the vev of $h$, we have

$$a_{WB}h^\dagger W^{\alpha \mu \nu} \sigma^\alpha h B_{\mu \nu}$$

$$= -a_{WB} \frac{v^2}{2} [sc(A^{\mu \nu} A_{\mu \nu} - Z^{\mu \nu} Z_{\mu \nu}) + (c^2 - s^2) A^{\mu \nu} Z_{\mu \nu}]$$

$$- a_{WB} v^2 g f^{abc} W^{b \mu} W^{c \nu} \partial_\mu B_\nu,$$

(33)

where we have used Eq. (9). From Eq. (33), we can identify the corrections to the gauge boson propagators

$$\Pi'_{Z\gamma} = -a_{WB} v^2 (c^2 - s^2), \quad \Pi'_{\gamma \gamma} = -2a_{WB} v^2 sc, \quad \Pi'_{ZZ} = 2a_{WB} v^2 sc.$$  (34)

After field redefinitions

$$Z_\mu \rightarrow Z_\mu (1 - \Pi'_{ZZ})^{1/2}, \quad A_\mu \rightarrow A_\mu (1 - \Pi'_{\gamma \gamma})^{1/2},$$  (35)

the kinetic terms become canonical and the physical $Z$ mass is shifted to

$$M^2_{Z} = M^2_{Z0} (1 + \Pi'_{ZZ}) = M^2_{Z0} (1 + 2a_{WB} v^2 sc),$$  (36)

where $M_{Z0} = \sqrt{g^2 + g^2 v^2/2}$. Another effect of the field redefinition is that the couplings between the physical gauge bosons, $\gamma$ and $Z$, and the fermions are modified by a factor $(1 + \Pi'_{\gamma \gamma})^{1/2}$ and $(1 + \Pi'_{ZZ})^{1/2}$ respectively. In particular, the fine structure constant becomes

$$\alpha = \frac{e^2}{4\pi} (1 + \Pi'_{\gamma \gamma}) = \frac{e^2}{4\pi} (1 - 2a_{WB} v^2 sc).$$  (37)

The mixing term $\Pi'_{\gamma Z}$ makes another correction to the $Z$-fermion couplings in the form

$$\Delta g^f_\nu = 2sc \Pi'_{Z\gamma} Q_f,$$

(38)

where $Q_f$ is the electric charge of the fermion $f$.

(b) $O_h$

$$a_h |h^\dagger D_\mu h|^2 = a_h \frac{v^4}{16} g^2 Z_\mu Z^\mu.$$  (39)
This is a direct contribution to the $Z$ mass:

$$\Delta M_Z^2 = \Pi_{ZZ}(0) = a_h \frac{v^2 g^2}{8 c^2},$$

\(\text{(40)}\)

(2) Operators that modify gauge-fermion couplings

These operators all contain 2 Higgs doublets and 2 fermions and a covariant derivative which affect the $Z$-fermion couplings $g^f_V$ and $g^f_A$ defined in Eq. (13). For example, from

$$a_h l(h^1 D_\mu h)(\bar{l}\gamma^\mu l) + h.c. = g f V 2 c Z_\mu (\bar{\nu}\gamma^\mu \nu + \bar{e}_L\gamma^\mu e_L),$$

\(\text{(41)}\)

we read

$$\Delta g^e_V = \Delta g^e_A = \Delta g^\nu_V = \Delta g^\nu_A = - a_h \frac{v^2}{2}. \quad \text{(42)}$$

The corrections from the other operators are similar. Combining all contributions, we obtain

$$\Delta g^e_V = \Pi'_{Z\gamma} Q e 2 s c - \frac{v^2}{2} a_h^e - \frac{v^2}{2} a_h^l - \frac{v^2}{2} a_h^e,$$

$$\Delta g^e_A = - \frac{v^2}{2} a_h^e + \frac{v^2}{2} a_h^l,$$

$$\Delta g^\nu_V = - \frac{v^2}{2} a_h^e + \frac{v^2}{2} a_h^l,$$

$$\Delta g^\nu_A = - \frac{v^2}{2} a_h^e + \frac{v^2}{2} a_h^l,$$

$$\Delta g^d_V = \Pi'_{Z\gamma} Q d 2 s c - \frac{v^2}{2} a_h^d - \frac{v^2}{2} a_h^q - \frac{v^2}{2} a_h^d,$$

$$\Delta g^d_A = - \frac{v^2}{2} a_h^d + \frac{v^2}{2} a_h^q,$$

$$\Delta g^u_V = \Pi'_{Z\gamma} Q u 2 s c - \frac{v^2}{2} a_h^u - \frac{v^2}{2} a_h^q - \frac{v^2}{2} a_h^u,$$

$$\Delta g^u_A = - \frac{v^2}{2} a_h^u - \frac{v^2}{2} a_h^q + \frac{v^2}{2} a_h^u,$$

\(\text{(43)}\)

where we have added the contributions from $\Pi'_{Z\gamma}$ given in \(\text{(34)}\). As mentioned, there is also a contribution from the field redefinition of $Z$ for each coupling,

$$\Delta g^f_{V,A} = \frac{1}{2} g_{V0,A0} \Pi'_{ZZ} = g_{V0,A0} a_{WB s c} v^2.$$

\(\text{(44)}\)

The operators $a_h^l$ and $a_h^q$ also modify the $W$-fermion couplings:

$$g \rightarrow g \left(1 + v^2 a_h^l\right), \quad \text{(W – lepton couplings)},$$

$$g \rightarrow g \left(1 + v^2 a_h^q\right), \quad \text{(W – quark couplings)}.$$ 

\(\text{(45)}\)

(3) 4-fermion operators

These operators simply introduce new vertices involving four fermions.
(4) Operators that modify triple gauge boson couplings

Obviously, $O_W$ is such an operator. The only other one is $O_{WB}$, which contains

\begin{equation}
-a_{WB} v^2 g f^{abc} W^{\mu} W^{\nu} \partial_{\mu} B_{\nu}.
\end{equation}

These operators alter the cross-sections for $e^+ e^- \rightarrow W^+ W^-$ which will be
discussed later.

(5) $O_{eB}$ and $O_{eW}$ are discussed later.

4.2. Direct and indirect corrections

Now we consider the corrections to the EWPOs. The corrections can be divided
to two categories, direct and indirect. To understand the two different kinds of
corrections, let us write Eq. (3) in a different way:

\begin{equation}
X_{th} = X_{SM}(g, g', v) + \sum_i a_i X'_i.
\end{equation}

Here $X_{SM}(g, g', v)$ is the SM prediction calculated from the Lagrangian parameters
$g$, $g'$ and $v$. At loop levels, it also depends on a few other parameters, $m_t$, $m_h$ and
$\alpha_s$ which we take as fixed.

$X'_i$ is the “direct” correction from new Feynman diagrams induced by the opera-
tor $O_i$. For example, the four fermion operator $(\overline{\tau} \gamma^\mu \tau)(\overline{q} \gamma_\mu q)$ introduces a 4-fermion
vertex and therefore a diagram for $e^+ e^- \rightarrow \overline{q} q$. A correction to the $Z$-boson coupling
can also be viewed as a new diagram with a new coupling.

Less obvious are the “indirect” corrections from the shifts to the input param-
eters: the SM parameters $g$, $g'$ and $v$ and therefore $X_{SM}(g, g', v)$ are not directly
accessible. Instead, we derive them from the most precisely measured observables
($\alpha$, $M_Z$, $G_F$). When calculating the SM predictions $X_{SM}(\alpha, M_Z, G_F)$, the SM rela-
tions between $(g, g', v)$, and $(\alpha, M_Z, G_F)$ are assumed. These relations are altered
when the new operators are included. For example, the operator $O_h$ contributes to
the physical $Z$ mass. Taking into consideration the shifts, we can express $(g, g', v)$
as functions of both $(\alpha, M_Z, G_F)$, and the operator coefficients $a_i$’s. Since $a_i$’s are
small, we use the linear approximation to obtain

\begin{equation}
X_{SM}[g(M_Z, a_i)] = X_{SM}[g(M_Z)] + \frac{\delta X_{SM}}{\delta a_i} a_i = X_{SM}[g(M_Z)] + \frac{\delta X_{SM}}{\delta g} \frac{\delta g}{\delta a_i} a_i.
\end{equation}

For illustration, I have simplified the formula by keeping only one parameter and
one operator coefficient. Generalizing to three parameters and all operator coeffi-
cients are straightforward. Note that $X_{SM}[g(M_Z)]$ is the SM prediction given in
the literature, including loop corrections. The quantity $\frac{\delta X_{SM}}{\delta g} \frac{\delta g}{\delta a_i} a_i$ is the “indirect”
corrections to the SM prediction due to the presence of the new operator $O_i$. When
calculating the indirect corrections, tree level results for $\frac{\delta X_{SM}}{\delta g}$ and $\frac{\delta g}{\delta a_i}$ are sufficient,
which simplify our calculations. Combining the indirect corrections with the direct
correction $X'_i$, we arrive at Eq. (49) given in the introduction.
4.3. Corrections to EWPOs

We start our calculation from indirect corrections, by establishing the relations between the parameters in the Lagrangian \((g, g', v)\) and the input parameters \((\alpha, M_Z, G_F)\). For convenience, we define

\[
M_{Z0} = \frac{\sqrt{g^2 + g'^2}v^2}{2}, \quad G_{F0} = \frac{1}{\sqrt{2}v^2}, \quad \alpha_0 = \frac{e^2}{4\pi}.
\]

(49)

Gathering the corrections to \(\alpha\) and \(M_Z\) from Eq. (34) and (39), we have

\[
\alpha = \frac{e^2}{4\pi}(1 - 2v^2sc\alpha_{WB}),
\]

(50)

\[
M_Z = M_{Z0}(1 + v^2sc\alpha_{WB} + \frac{v^2}{4}a_h).
\]

(51)

\(G_F\) is obtained from muon lifetime, which is determined by the effective Lagrangian

\[
\mathcal{L} = -2\sqrt{2}G_F \bar{e}_L \gamma^\mu \nu_{e_L} \bar{P}_\mu \gamma_\mu L.
\]

There are two operators that contribute to the effective Lagrangian, one from the 4-fermion operator \(O^{hl}_{ll}\), and the other one given by the corrections to \(W\)-fermion couplings from \(O^{hl}_{hl}\). Therefore,

\[
G_F = G_{F0} + \frac{1}{\sqrt{2}} \left(2a^{hl}_{hl} - a^{ll}_{ll}\right).
\]

(52)

We can easily solve Eqs. (50), (51) and (52) for \(g, g', v\). For a given observable, we then substitute the solutions in the corresponding SM expression and expand to linear order in \(a_i\)’s. For example, from

\[
M_W = \frac{gv^2}{2},
\]

(53)

we obtain the corrections to \(M_W\):

\[
\Delta M_W = 10^6 \left(-1.73a_h - 2.08a^{hl}_{hl} + 1.04a^{ll}_{ll} - 3.80a_{WB}\right),
\]

(54)

where \(a_i\) is in GeV\(^{-2}\). Then the theoretical prediction for \(M_W\) is

\[
M_{W,th} = M_{W,SM} + \Delta M_W = 80.376 \text{ GeV} + \Delta M_W.
\]

(55)

Note that the SM prediction \(M_{W,SM}\) is the one given in Table I which contains SM loop corrections. It is not calculated with the tree level expression, Eq. (53), which is only used to calculate corrections from new physics.

There is no direct correction to \(M_W\), so Eq. (55) contains all corrections to \(M_W\). The experimental value of \(M_W\) is obtained from both LEP [58] and Tevatron [29].

For other observables, we should add the direct corrections as well. In the following, we will give both direct corrections and the SM tree level formulae for calculating indirect corrections (but omit the obvious step of expanding to linear order in \(a_i\)).
The process is $e^+e^- \rightarrow f \bar{f}$, which was studied around the $Z$-pole at SLC and LEP 1. SLC also had longitudinal beam polarization. The measured cross-sections and asymmetries for different center of mass energies have been translated to pseudo-observables at the $Z$-pole, which can all be derived from two basic quantities: the partial decay width of $Z \rightarrow f \bar{f}$, $\Gamma_{f\bar{f}}$, and the polarized asymmetry $A_f$. They are related to the $Z$-fermion couplings $g_f^V$ and $g_f^A$ defined in eq. (1):  

$$\Gamma_{f\bar{f}} = \frac{e^2 M_Z}{48 \pi s^2 c^2} (g_f^V + g_f^A)^2, \quad (56)$$

$$A_f = 2 \frac{g_f^V / g_f^A}{1 + (g_f^V / g_f^A)^2}. \quad (57)$$

In the above equations, $g_V = g_{V0} + \Delta g_V$ and $g_A = g_{A0} + \Delta g_A$, where $g_{V0}$ and $g_{A0}$ are given in (14), and $\Delta g_V$ and $\Delta g_A$ are given in (43). Note that $g_V$ and $g_A$ contain both direct and indirect corrections. From $\Gamma_{f\bar{f}}$ and $A_f$, we derive the following observables:

(a) Total decay width $\Gamma_Z = \sum_f \Gamma_{f\bar{f}}$.

(b) Total hadronic cross-section

$$\sigma^0_h = \frac{12 \pi \Gamma_{ee} \Gamma_{had}}{M_Z^2 \Gamma_Z}, \quad (58)$$

where $\Gamma_{had}$ is the partial decay width to hadrons and $\Gamma_{ee}$ is the partial decay width to $e^+e^-$.  

(c) The ratios

$$R^0_e = \Gamma_{had}/\Gamma_{ee}, \quad R^0_\mu = \Gamma_{had}/\Gamma_{\mu\mu}, \quad R^0_\tau = \Gamma_{had}/\Gamma_{\tau\tau}. \quad (59)$$

(d) The pole asymmetries $A^0_{f\bar{b}}, (f = e, \mu, \tau)$ related to $A_f$ by

$$A^0_{f\bar{b}} = \frac{3}{4} A_e A_f. \quad (60)$$

(e) The polarized asymmetries $A_e, A_\mu, A_\tau$ from both LEP 1 and SLC.

(f) Heavy flavor observables including

$$R^0_b = \Gamma_{bb}/\Gamma_{had}, \quad R^0_c = \Gamma_{cc}/\Gamma_{had}, \quad A^0_{f\bar{b}}, A^0_{f\bar{b}}, A_{f\bar{b}}, A_e. \quad (61)$$

(g) Hadronic charge asymmetry $\langle Q_{fb} \rangle$. The result is translated to the leptonic effective electroweak mixing angle,

$$\sin^2 \theta_{\text{lept eff}} = \frac{1}{4} \left(1 - \frac{g_V^2}{g_A^2} \right). \quad (62)$$

Note that at the $Z$-pole, the denominator of the $Z$ propagator is dominated by $i M_Z \Gamma_Z$, which is imaginary relative to the 4-fermion operator contributions. Therefore, 4-fermion operators do not interfere with the $Z$-pole measurements and there is no contributions linear in the corresponding $a_i$’s.
This includes a variety of deep inelastic experiments performed at CDHS, CHARM, CCFR and NuTeV. The experiments measured cross sections for both the neutral current (NC), $\nu_\mu + N \rightarrow \nu_\mu + N$, and the charged current (CC) $\nu_\mu + N \rightarrow \mu + X$. The experimental results are usually translated to the effective couplings between the $Z$ boson, and the up and down quarks: $g_{u,L}^{\text{eff}}, g_{u,R}^{\text{eff}}, g_{d,L}^{\text{eff}}$ and $g_{d,R}^{\text{eff}}$. They are effective in the sense that they only apply to the $\nu$-nucleon scattering observables. Since that these experiments were performed at energies far below the $Z$-pole, the effective couplings receive contributions from both 4-fermion operators and operators modifying gauge-fermion couplings. In order to cancel out uncertainties from the strong interaction, what is measured is actually the ratio between the NC and the CC cross sections. Therefore, both corrections to the NC and CC need to be considered.

For the CC, comparing the effective Lagrangian in the SM

$$\mathcal{L}_{\text{CC}}^{\text{eff}} = -2\sqrt{2}G_F(\bar{\nu}_\mu \gamma^\mu \nu_\mu)(d_L \gamma_\mu u_L),$$

and the one with new operators

$$\mathcal{L}_{\text{CC}}^{\text{eff}} = \left[-2\sqrt{2}G_F (1 + a_h t^2 + a_t q^2) \right] (\bar{\nu}_\mu \gamma^\mu \nu_\mu)(d_L \gamma_\mu u_L),$$

we see that it is modified by an overall factor

$$F_{cc} = \frac{G_{F0}}{G_F} \left[ (1 + a_h t^2 + a_t q^2) - \frac{a_t q^2}{2\sqrt{2}G_F} \right].$$

The effective Lagrangian for the NC is

$$\mathcal{L}_{\text{NC}}^{\text{eff}} = -\frac{e^2}{2\alpha^2 c^2 M_Z} \bar{\nu}_\mu \gamma^\mu (g_V' - g_A' \gamma_5) \nu_\mu \sum_q (\bar{q} \gamma_\mu \nu_\mu q + \bar{q} R \gamma_\mu q R)$$

$$+ a_h \bar{q} \gamma_\mu \nu_\mu (\bar{u} L \gamma_\mu u L + \bar{d} L \gamma_\mu d L)$$

$$+ a_t \bar{q} \gamma_\mu \nu_\mu (\bar{u} L \gamma_\mu u L - \bar{d} L \gamma_\mu d L)$$

$$+ a_t \bar{q} \gamma_\mu \nu_\mu (\bar{u} R \gamma_\mu u R)$$

$$+ a_t \bar{q} \gamma_\mu \nu_\mu (\bar{d} R \gamma_\mu d R),$$

where

$$g_V' = \frac{g_V + g_A}{2}, \quad g_A' = \frac{g_V - g_A}{2}.$$ 

Note that $g_V' = g_V^d$ holds for all new operator contributions. Therefore, all new physics contributions can be absorbed in the effective couplings $g_{u,L}^{\text{eff}}$ and $g_{d,L}^{\text{eff}}, g_{u,R}^{\text{eff}}, g_{d,R}^{\text{eff}}$

$$\mathcal{L}_{\text{NC, eff}} = -2\sqrt{2}G_F \bar{\nu}_\mu \gamma^\mu \nu_\mu \sum_{q=u,d} \left( g_{L,\text{eff}}^{\mu} \bar{q} L \gamma_\mu q L + g_{R,\text{eff}}^{\mu} \bar{q} R \gamma_\mu q R \right).$$

However, note that the results have been renormalized to the scale of $M_Z$. 

We weak charge obtained by considering the effective Lagrangian:

\[
W = \text{only need the corrections to}
\]

The measured quantities are expressed in terms of the effective couplings:

\[
\text{Then the weak charge is given by}
\]

The measured quantities are expressed in terms of the effective couplings:

\[
g_L^2 = g_{L,\text{eff}}^2 + g_{L,\text{eff}}^2,
\]

(3) \(\nu - e\) scattering

\(\nu - e\) scattering was performed at CHARM II. Similarly, the corrections are obtained by considering the effective Lagrangian:

\[
\mathcal{L}_{\nu e,\text{eff}} = \sqrt{2}G_F \bar{\nu}_L \gamma^\mu \nu_L \bar{\gamma}_\mu (g_{\nu e,\text{eff}}(g_{\nu e,\text{eff}} + g_{\nu e,\text{eff}}^5)\nu),
\]

where \(g_{\nu e,\text{eff}}^5\) and \(g_{\nu e,\text{eff}}^5\) are measured in the experiment and related to our operators as

\[
g_{\nu e,\text{eff}}^5 = \frac{M^2_{Z_0}}{M^2_Z} \frac{G_F}{G_F} 2g_{\nu e}^5 \left( g_{\nu e}^5 - \frac{a_t^5}{2\sqrt{2}G_F} + \frac{a_l^5}{2\sqrt{2}G_F} - \frac{a_e}{2\sqrt{2}G_F} \right),
\]

\[
g_{\nu e,\text{eff}}^5 = \frac{M^2_{Z_0}}{M^2_Z} \frac{G_F}{G_F} 2g_{\nu e}^5 \left( g_{\nu e}^5 - \frac{a_t^5}{2\sqrt{2}G_F} + \frac{a_l^5}{2\sqrt{2}G_F} + \frac{a_e}{2\sqrt{2}G_F} \right).
\]

(4) Weak charge

The weak charges for Cs and Tl are measured in the atomic parity violation experiments. The effective Lagrangian is conventionally written in terms of \(C_{1q}\) and \(C_{2q}\):

\[
\mathcal{L} = \frac{G_F}{\sqrt{2}} \sum_q \left[ C_{1q}(\bar{e} \gamma^\mu \gamma^5 e)(\bar{q} \gamma_\mu q) + C_{2q}(\bar{e} \gamma^\mu e)(\bar{q} \gamma_\mu \gamma^5 q) \right].
\]

We only need the corrections to \(C_{1u}\) and \(C_{1d}^d\):

\[
C_{1u} = \frac{G_F M^2_{Z_0}}{G_F M^2_Z} 2g_{\nu e}^5 \frac{1}{2\sqrt{2}G_F} (-a_t^q + a_l^q + a_e + a_e - a_t),
\]

\[
C_{1d} = \frac{G_F M^2_{Z_0}}{G_F M^2_Z} 2g_{\nu e}^5 \frac{1}{2\sqrt{2}G_F} (-a_t^q - a_l^q + a_e + a_e - a_t).
\]

Then the weak charge is given by

\[
Q_W(Z,N) = -2[(2Z + N)C_{1u} + (Z + 2N)C_{1d}],
\]

where \(Z\) and \(N\) are respectively proton number and neutron number of the atom.
(5) $e^+ e^- \rightarrow f \bar{f}$ at LEP 2

The observables are differential cross-sections for fermion pair production. In the following, we will give the formulae for the matrix elements from which the cross-sections can be calculated. Unlike LEP 1, LEP 2 experiments were performed above the $Z$-pole, therefore, 4-fermion operators also contribute.

(a) $e^+ e^- \rightarrow f \bar{f}$ ($f \neq e$) \[76\]

$$
\mathcal{M} = \mathcal{M}_Z + \mathcal{M}_g + \mathcal{M}_{4-fermi}$$

$$+ \frac{e^2}{(p+p')^2 - M_Z^2 + i\Gamma_Z M_Z} \left[ \frac{1}{4e^2s^2} \bar{v}(p')\gamma^\mu (g_{\nu} - \gamma^5 g_{\nu})u(p)\bar{u}(k)\gamma_\mu (g_{\nu} - \gamma^5 g_{\nu})v(k') \right]$$

$$+ \frac{e^2Q}{(p+p')^2} [\bar{v}(p')\gamma^\mu u(p)] [\bar{u}(k)\gamma_\mu v(k')]$$

$$+ a_1 [\bar{v}(p')\gamma^\mu u(p)] [\bar{u}(k)\gamma_\mu v(k')]$$

$$+ a_2 [\bar{v}(p')\gamma^\mu \gamma^5 u(p)] [\bar{u}(k)\gamma_\mu v(k')]$$

$$+ a_3 [\bar{v}(p')\gamma^\mu u(p)] [\bar{u}(k)\gamma_\mu \gamma^5 v(k')]$$

$$+ a_4 [\bar{v}(p')\gamma^\mu \gamma^5 u(p)] [\bar{u}(k)\gamma_\mu \gamma^5 v(k')]$$.

where $p, p', k, k'$ are the momenta of the incoming $e^+ e^-$ and the outgoing fermions. $\mathcal{M}_Z$ and $\mathcal{M}_g$ are s-channel contributions from $Z$ and photon exchange. $\mathcal{M}_{4-fermi}$ denotes contributions from 4-fermion operators, where depending on the fermion $f$, the couplings $a_1, a_2, a_3, a_4$ are related to the operator coefficients by

$$a_1 = 1/4(4i\xi - 4i\eta + 2a_{ie} + a_{ee}),$$

$$a_2 = 1/4(-4i\xi + 4i\eta + a_{ee}),$$

$$a_3 = 1/4(-4i\xi + 4i\eta + a_{ee}),$$

$$a_4 = 1/4(4i\xi - 4i\eta - 2a_{ie} + a_{ee});$$ \[77\]

$$a_1 = 1/4(4i\xi - 4i\eta + a_{ie} + a_{ee}),$$

$$a_2 = 1/4(-4i\xi + 4i\eta + a_{ee}),$$

$$a_3 = 1/4(-4i\xi + 4i\eta + a_{ee}),$$

$$a_4 = 1/4(4i\xi - 4i\eta - 2a_{ie} + a_{ee});$$ \[78\]

(b) $e^+ e^- \rightarrow e^+ e^-$ \[79\]

For $f = e$, there are extra contributions from $t$-channel diagrams. The
matrix element is
\[ \mathcal{M} = \mathcal{M}_\gamma + \mathcal{M}_Z + \mathcal{M}_{4-fermi} \]
\[= \frac{e^2}{(p-k)^2} \bar{u}(k) \gamma'^\mu u(p) \bar{v}(p') \gamma_\mu v(k') \]
\[- \frac{e^2}{(p'+p)^2} \bar{v}(p') \gamma'^\mu u(p) \bar{u}(k) \gamma_\mu v(k') \]
\[+ \frac{e^2}{(p-k)^2 - M_Z^2 + i\Gamma_Z M_Z/4} \frac{1}{2} \bar{u}(k) \gamma'^\mu (g'_V - \gamma^5 g'_A) u(p) \bar{v}(p') \gamma_\mu (g'_V - \gamma^5 g'_A) v(k') \]
\[- \frac{e^2}{(p+p')^2 - M_Z^2 + i\Gamma_Z M_Z/4} \bar{v}(p') \gamma'^\mu (g'_V - \gamma^5 g'_A) u(p) \bar{u}(k) \gamma_\mu (g'_V - \gamma^5 g'_A) v(k') \]
\[+ a_1 [\bar{u}(k) \gamma'^\mu u(p) \bar{v}(p') \gamma_\mu v(k') - \bar{v}(p') \gamma'^\mu u(p) \bar{u}(k) \gamma_\mu v(k')] \]
\[+ a_2 [\bar{u}(k) \gamma'^\mu u(p) \bar{v}(p') \gamma_\mu v(k') - \bar{v}(p') \gamma'^\mu u(p) \bar{u}(k) \gamma_\mu v(k')] \]
\[+ a_3 [\bar{u}(k) \gamma'^\mu u(p) \bar{v}(p') \gamma_\mu \gamma^5 v(k') - \bar{v}(p') \gamma'^\mu u(p) \bar{u}(k) \gamma_\mu \gamma^5 v(k')] \]
\[+ a_4 [\bar{u}(k) \gamma'^\mu \gamma^5 u(p) \bar{v}(p') \gamma_\mu v(k') - \bar{v}(p') \gamma'^\mu \gamma^5 u(p) \bar{u}(k) \gamma_\mu v(k')] \] \hspace{1cm} (80)

where
\[ a_1 = 1/4(a_1^e + a_1^l + 2a_{le} + a_{ee}) \]
\[ a_2 = 1/4(-a_1^e + a_1^l + a_{ee}) \]
\[ a_3 = 1/4(-a_1^e - a_1^l + a_{ee}) \]
\[ a_4 = 1/4(a_1^e + a_1^l - 2a_{le} + a_{ee}) \] \hspace{1cm} (81)

(6) \( e^+e^- \to W^+W^- \) at LEP 2

There are two diagrams contributing to this process, one with \( t \)-channel neutrino exchange and the other one with \( s \)-channel \( \gamma \) or \( Z \) exchange. The former involves \( W \)-fermion couplings and therefore receives corrections from (45). The \( \gamma \) or \( Z \) exchange diagram involves triple gauge boson couplings, which are modified by the operators \( O_{WB} \) and \( O_W \). The relevant couplings are given by
\[ \Delta \mathcal{L} = ia_{WB} v^2 g W^+_{\mu} W^-_{\nu} (c A^{\mu\nu} - s Z^{\mu\nu}) + 6ia_W W^{-\mu}_{\nu} W^{+\nu}_{\mu}\lambda (s A^\lambda_\chi + c Z^\lambda_\chi). \] \hspace{1cm} (82)

The tree-level differential cross-section for \( e^+e^- \to W^+W^- \) is calculated in Ref. [37] for arbitrary triple gauge boson couplings, which are parametrized as
\[ \frac{\mathcal{L}_{WWV}}{g_{WWV}} = ig_V^V (W^+_{\mu} W^-_{\nu} - W^-_{\mu} W^+_{\nu} V^{\mu\nu}) + i\kappa_V W^+_{\mu} W^-_{\nu} V^{\mu\nu} + \frac{i\lambda_V}{\Lambda^2} W^+_{\mu} W^-_{\rho} V^{\mu\rho} \]
\[- g_4 V^+_{\mu} W^-_{\nu} (\partial^\mu V^\nu + \partial^\nu V^\mu) + g_6 c_V^\nu \lambda \left[ W^+_{\mu} (\partial_\rho W_{\nu}) - (\partial_\rho W^+_{\nu}) W^-_{\mu} \right] V_\lambda \]
\[+ i\tilde{\kappa}_V W^+_{\mu} W^-_{\nu} \hat{V}^{\mu\nu} + \frac{i\tilde{\lambda}_V}{\Lambda^2} W^+_{\mu} W^-_{\rho} \hat{V}^{\mu\rho}, \] \hspace{1cm} (83)

where \( V = \gamma, Z \) and \( g_{WWV} = -e, g_{WWZ} = -ec/s \). Our effective couplings, Eq. [82], correspond to the terms multiplying \( \kappa_V \) and \( \lambda_V \). To obtain the cross-
section we substitute
\[
\Delta \kappa_\gamma = \frac{v^2 c}{s} a_{WB},
\]
\[
\Delta \kappa_Z = -\frac{v^2 s}{c} a_{WB},
\]
\[
\Delta \lambda_\gamma = \Delta \lambda_Z = \frac{3 v^2 g}{2} a_{W}.
\]
(84)
where \(\Delta\)'s denote the deviations from the SM values.

(7) Muon \(g - 2\)\textsuperscript{38}

As mentioned, the operators \(O_{eB}\) and \(O_{eW}\) contribute to \(g - 2\) at tree level,
\[
a_\mu = 2 \sqrt{2} v m_\mu \frac{g}{g'} a_{eW} - 2 \sqrt{2} v m_\mu \frac{g}{g'} a_{eB}.
\]
(85)
Due to the high precision of this measurement, these corrections certainly cannot be introduced at tree level for TeV scale physics. Therefore, one should assume \(a_{eB}\) and \(a_{eW}\) vanish and consider loop corrections from the other operators. These include corrections to the SM loop diagram from shift of input parameters, and corrections from new loop diagrams induced by the effective operators. This has been done in Ref.\textsuperscript{25} In this review, we will simply assume that all corrections have been absorbed into \(a_{eB}\) and \(a_{eW}\) and put constraints on them using Eq. (85).

5. Constraints

The constraint on each individual operator is given in Table\textsuperscript{2}. The bounds on \(O_{eB}\) and \(O_{eW}\) is calculated from muon \(g - 2\) with experimental values given in Table\textsuperscript{1}. Bounds on the other operators are taken from Ref.\textsuperscript{14}

| \(O\)     | \(\Lambda_i\) (in TeV) | \(O_i\) | \(\Delta_i\) | \(\Lambda_i\) | \(O_i\) | \(\Delta_i\) | \(\Lambda_i\) | \(O_i\) | \(\Delta_i\) | \(\Lambda_i\) | \(O_i\) |
|----------|-------------------|------|---------|------------|------|---------|------------|------|---------|------------|------|
| \(O_{WB}\) | 12.6              | \(O_h\) | 6.8      | \(O_{W}\) | 3.6  | \(O_{W}\) | 8.8        |      |         |            |      |
| \(O_{tq}\) | 5.4               | \(O_{tq}\) | 6.2      | \(O_{tc}\) | 4.3  | \(O_{tq}\) | 5.5        |      |         |            |      |
| \(O_{lu}\) | 3.8               | \(O_{ld}\) | 4.0      | \(O_{lo}\) | 3.5  | \(O_{lu}\) | 4.3        |      |         |            |      |
| \(O_{ed}\) | 3.9               | \(O_{hl}\) | 11.6     | \(O_{he}\) | 11.5 | \(O_{ed}\) | 6.4        |      |         |            |      |
| \(O_{lu}\) | 9.2               | \(O_{lu}\) | 4.5      | \(O_{hd}\) | 4.3  | \(O_{lu}\) | 9.7        |      |         |            |      |
| \(O_{W}\)  | 0.9               | \(O_{eB}\) | 0.4      | \(O_{AW}\) | 0.4  | \(O_{W}\) |             |      |         |            |      |

From Table\textsuperscript{2} we see that many of the operators are tightly constrained. At 90% CL, there are still quite a few bounds around 10 TeV, which manifest the “LEP paradox”\textsuperscript{39}.
The operator $O_W$ is not constrained as well as others, due to the low statistics in the measurement of $e^+e^- \rightarrow W^+W^-$ cross sections. Therefore, in the literature, this measurement is often neglected from the list of EWPOs. Nevertheless, it gives the unique constraint of $O(\text{TeV})$ if the new physics only modifies the triple gauge couplings.

We also see from the bounds on $a_{eB}$ and $a_{eW}$ that, the muon $g-2$ measurement does not give us stringent constraints if the new physics contribution is suppressed by a loop factor and the muon mass. See Ref. [25] for exceptions, as well as an analysis for the non-linear case.

Although it is clear from Table 2 how well each of the operators is constrained, in practice, constraints on individual operators are not useful. The reason is that we usually obtain multiple operators when the new physical states are integrated out. Their contributions to the observables are correlated. Therefore, in order to obtain the bounds on a new model, the full $\chi^2$ distribution should be used. Including experimental correlations, Eq. (4) is modified to

$$\chi^2(a_i) = \sum_{p,q} (X_{\text{th}}^p(a_i) - X_{\text{exp}}^p)(\sigma^2)^{-1}_{pq}(X_{\text{th}}^q(a_i) - X_{\text{exp}}^q),$$

where $\sigma^2$ is the error matrix, defined from the standard deviations $\sigma_p$ and the correlation matrix $\rho_{pq}$ as

$$\sigma^2_{pq} = \sigma_p \rho_{pq} \sigma_q.$$ (87)

The numerical result of the $\chi^2$ distribution, for operators (26)-(29), is given in Ref. [14]. After obtaining the $\chi^2$, it is straightforward to calculate constraints for a given model once the heavy particles are integrated out and the coefficients $a_i$’s are obtained. See Ref. [41-43] for a few examples on the little Higgs models [40] and models with a warped extra-dimension.

6. Discussions

6.1. Oblique corrections

The oblique corrections are modifications to the gauge boson propagators. We discuss in this section the relations between the oblique parameters and the effective operators.

6.1.1. S, T and U

The well known $S$, $T$ and $U$ parameters[17] are defined by

$$\frac{\alpha}{4s^2c^2} S = \Pi_{ZZ} - \frac{c^2 - s^2}{cs} \Pi_{Z\gamma} - \Pi'_{\gamma\gamma},$$

$$\alpha T = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2},$$

$$\frac{\alpha}{4s^2}(S + U) = \Pi_{WW}' - \frac{c}{s} \Pi_{Z\gamma}' - \Pi'_{\gamma\gamma}. $$ (88)
where the corrections only include new physics contributions. In our framework, the operators $O_h$ and $O_{WB}$ modify gauge boson propagators. Comparing with Eqs. (34) and (40), we have

$$S = \frac{4scv^2a_{WB}}{\alpha}, \quad T = -\frac{v^2}{2\alpha}a_h, \quad U = 0.$$  \quad (89)

We see that except for some constant factors, $S$ and $T$ are equivalent to $a_{WB}$ and $a_h$. Moreover, there is no dimension-6 operator corresponding to the $U$ parameter. It turns out that the dimension-8 operator $(h^\dagger W^a \mu \nu h)(h^\dagger W^a_{\mu \nu} h)$ corresponds to the $U$ parameter. By our power counting, its contribution is negligible. This explains why $U$ parameter is usually not significant in weakly coupled extensions of the SM. Nevertheless, as we will see, in the non-linearly realized electroweak symmetry case, $U$ can be important due to a different power counting. Therefore, we discuss below the corrections from the $U$ parameter to the EWPOs.

From Eqs. (88), we have

$$\frac{\alpha}{4s^2}U = \Pi_{WW}' - \Pi_{W3W3}'.$$  \quad (90)

When $\Pi_{WW}' = \Pi_{W3W3}'$, which corresponds to multiplying the kinetic term $-W^\mu \nu W_{\mu \nu}/4$ in the SM Lagrangian by an overall factor, there is no visible correction. Therefore, we can always trade $\Pi_{W3W3}'$ for $\Pi_{WW}'$, which can be absorbed into field redefinition of $W^+$ and $W^-:

$$W^+ \rightarrow W^+(1 - \Pi_{WW}')^{\frac{1}{2}}.$$  \quad (91)

The field redefinition affects the $W$ mass,

$$\Delta M_W^2 = M_W^2 \Pi_{WW}' = M_W^2 \frac{\alpha U}{4s^2},$$  \quad (92)

and $W$-fermion couplings

$$g \rightarrow g(1 + \frac{\alpha U}{8s^2}).$$  \quad (93)

Note that $U$ does not affect low energy experiments such as muon lifetime $(G_F)$ or charged current in DIS experiments because the matrix element is proportional to $g^2/M^2_W$, where $U$ cancels out.

As shown in Ref. 1, the $S$, $T$ and $U$ parameters are related to other parameters ($S_i, h_i, \hat{\epsilon}_i$)\(18\)\(20\) as

$$T = h_V = \hat{\epsilon}_1/\alpha,$$

$$S = h_{AV} = S_Z = 4s^2 \hat{\epsilon}_3/\alpha,$$

$$U = h_{AW} - h_{AZ} = S_W - S_Z = -4s^2 \hat{\epsilon}_2/\alpha,$$  \quad (94)

which can be used to relate our operators to these parameters as well.
6.1.2. W and Y

The list of oblique parameters is extended in Ref. [22] to include two more important parameters: W and Y. In the effective theory language, these parameters correspond to the dimension-6 operators

\[ O_{BB} = \frac{1}{2} (\partial_\mu B_{\mu\nu})^2, \quad O_{WW} = \frac{1}{2} (D_\mu W^{\mu\nu})^2. \]  

(95)

By Bianchi identities such as \[ \partial_\rho B_\mu B_\nu + \partial_\mu B_\rho B_\nu + \partial_\nu B_\rho B_\mu = 0, \] they are equal to \( (\partial_\nu B_\nu) \) and \( (D_\nu W^{\mu\nu}) \) respectively. These operators are not independent from other operators defined in Ref. [6]. They can be related by the equations of motion. Squaring Eqs. (7) and (8), we see that each of \( O_{WW} \) and \( O_{BB} \) is equivalent to a combination of operators already included in our list.

6.1.3. Disguise the oblique corrections

Another interesting consequence of the equations of motion is that, they can be used to “disguise” the oblique correction [44], that is, transfer oblique corrections to equivalent non-oblique corrections. Multiplying Eq. (7) by \( (ih^\dagger \sigma^a D_\mu h + h.c.) \) and Eq. (8) by \( (ih^\dagger D_\mu h + h.c.) \), we have

\[
-\frac{g}{2} O_{WB} + 2g' O_h + g' O_{hf}^Y = 2i B_{\mu\nu} D_{\mu} h^\dagger D_{\nu} h,
\]

\[
-g' O_{WB} + g(O_{hl} + O_{hq}) = 4i W^{a}_{\mu\nu} D_{\mu} h^\dagger \sigma^a D_{\nu} h,
\]

(96)

where we have neglected operators that do not contribute to EWPOs, and \( O_{hf}^Y = \sum_f Y_f O_{hf} = \frac{1}{6} O_{h^d} - \frac{1}{2} O_{h^d} - \frac{2}{3} O_{hl} - \frac{1}{2} O_{he} \). The left-hand side of Eqs. (96) is the two oblique operators corresponding to S and T, plus operators that modify gauge-fermion operators. The right-hand side is two operators that modify triple gauge boson couplings, which is only constrained by \( e^+ e^- \rightarrow W^+ W^- \) cross sections. Therefore, the combinations of operators on the left-hand side of Eqs. (96) are relatively weakly constrained. If \( e^+ e^- \rightarrow W^+ W^- \) data is neglected, there are no constraints at all. Similar observations for the non-linear case is made in Ref. [44].

The merit of the oblique parameters is that they are all stringently constrained by the experiments. When the new physics corrections are all oblique or the non-oblique corrections are sub-dominant, it is sufficient to consider oblique parameters only. Nevertheless, we see from Table 2 that some non-oblique operators are constrained as tightly as the oblique parameters, which are indispensable if appearing in the effective Lagrangian. In this case, an analysis using the full set of operators is required. Alternatively, one may use a subset which contains the most constrained operators (or operator combinations) and still get sensible constraints [45].

6.2. Relax the flavor symmetry

So far, we have always assumed flavor universality for the effective operators. Although it is convenient and arises in many models beyond the SM, it is not necessary.
Bounds on flavor violation for the first two generations are very stringent, therefore it is hard to break universality for the first two generations without introducing conflicts with the experiments. However, it is possible to break flavor universality for the third generation. This is due to two reasons. First, the precisions of experiments involving the third generation is not as good as the first two generations. Second, in the CKM matrix, the (13) and (23) components are much smaller than the (12) component. Therefore the gauge eigenstate is approximately aligned with the flavor eigenstate in the third generation.

Given the above observation, it is possible that the new physics couples to the third generation differently from the first two generations, see Ref. 46 and 47 for a few examples. Accordingly, the flavor symmetry must be relaxed, which alters the counting of the effective operators. We remind the reader that in Ref. 14, flavor universality is guaranteed by imposing a $U(3)^5$ symmetry. This flavor symmetry is relaxed in Ref. 15 to $[U(2) \times U(1)]^5$, with the $U(2)$’s acting on the first two generations and $U(1)$’s the third. Correspondingly, the operators in (26)-(29) are still present, but the fermions $q, l, u, d$ and $e$ are contracted over the first two generations only. In addition, there are new operators involving only the third generation. Denoting the third generation fermions by $Q, L, b$ and $\tau$, these are

(1) Four-fermion operators:

\[
\begin{align*}
O_{LL}^i &= (\bar{L}_\gamma^\mu L)(\bar{L}_\gamma^\mu L), \quad O_{LL}^t = (\bar{L}_\gamma^\mu \sigma^a l)(\bar{L}_\gamma^\mu \sigma^a L), \\
O_{QQ} &= (\bar{Q}_\gamma^\mu Q)(\bar{Q}_\gamma^\mu Q), \quad O_{Qe} = (\bar{Q}_\gamma^\mu Q)(\bar{e}_\gamma^\mu e), \\
O_{Le} &= (\bar{L}_\gamma^\mu L)(\bar{e}_\gamma^\mu e), \quad O_{l\tau} = (\bar{l}_\gamma^\mu l)(\bar{\tau}_\gamma^\mu \tau), \\
O_{Qe} &= (\bar{Q}_\gamma^\mu Q)(\bar{e}_\gamma^\mu e), \quad O_{Qb} = (\bar{Q}_\gamma^\mu Q)(\bar{b}_\gamma^\mu b) \\
O_{e\tau} &= (\bar{e}_\gamma^\mu e)(\bar{\tau}_\gamma^\mu \tau), \quad O_{eb} = (\bar{e}_\gamma^\mu e)(\bar{b}_\gamma^\mu b).
\end{align*}
\]

(97)

(2) Operators modifying gauge-fermion couplings:

\[
\begin{align*}
O_{hL}^i &= i(h\dagger D^\mu h)(\bar{L}_\gamma^\mu L) + \text{h.c.}, \quad O_{hL}^t = i(h\dagger \sigma^a D^\mu h)(\bar{L}_\gamma^\mu \sigma^a L) + \text{h.c.}, \\
O_{hQ}^i &= i(h\dagger D^\mu h)(\bar{Q}_\gamma^\mu Q) + \text{h.c.}, \quad O_{hQ}^t = i(h\dagger \sigma^a D^\mu h)(\bar{Q}_\gamma^\mu \sigma^a Q) + \text{h.c.}, \\
O_{h\tau}^i &= i(h\dagger D^\mu h)(\bar{\tau}_\gamma^\mu \tau) + \text{h.c.}, \quad O_{hb} = i(h\dagger D^\mu h)(\bar{b}_\gamma^\mu b) + \text{h.c.}
\end{align*}
\]

(98)

There are fewer experiments involving the third generation than the first two. In particular, top quark never appears in the final state of any EWPTs. There is no $\nu_\tau$-nucleon or $\nu_\tau$-lepton scattering experiment either. Consequently, there exist “flat directions”, that is, combinations of operators that do not contribute to EWPOs and therefore are not constrained. These flat directions are expressed as the relations between the operator coefficients:

\[
\begin{align*}
\alpha_{QQ}^i &= -\alpha_{QQ}^t, \quad \alpha_{Le} = -\alpha_{l\tau}, \\
\alpha_{LL}^i &= -\alpha_{LL}^t, \quad \alpha_{hQ}^i = -\alpha_{hQ}^t.
\end{align*}
\]

(99)
6.3. Non-linear case

If electroweak symmetry is broken by new strong dynamics, the SM model is described by the non-linear chiral Lagrangian

\[ \mathcal{L} = -\frac{1}{4} B^{\mu\nu} B_{\mu\nu} - \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{v^2}{4} \text{Tr}[(D^\mu U)^\dagger (D_\mu U)], \]

where \( U \) is a dimensionless unitary unimodular matrix field transforming as a bi-fundamental under \( SU(2)_L \times SU(2)_C \). Here \( SU(2)_C \) denotes the custodial \( SU(2) \). The covariant derivative of \( U \) is defined by

\[ D_\mu U = \partial_\mu U + ig \sigma^a \frac{1}{2} W^a_\mu U - ig' U \sigma^3 \frac{1}{2} B_\mu. \]

For convenience, we also define

\[ V_\mu = (D_\mu U) U^\dagger, \quad T = U \sigma^3 U, \]
\[ \hat{V}_\mu = (D_\mu U^\dagger) U, \quad \hat{T} = \sigma^3. \]

Compared with the linear case, the light Higgs is absent from the Lagrangian. Therefore, the SM predictions are different from those listed in Table 1, but can be estimated by taking the Higgs mass to a large value in the linear case. To the leading order, the corrections from a different Higgs mass are given by

\[ S \approx \frac{1}{12\pi} \log \left( \frac{m_h^2}{m_{h,\text{ref}}^2} \right), \quad T \approx -\frac{3}{16\pi c^2} \log \left( \frac{m_h^2}{m_{h,\text{ref}}^2} \right). \]

For more accurate results, one can use the program GAPP, dedicated to the calculations of the SM predictions.

The difficulty of a strongly coupled electroweak sector is that, the coefficients of the effective operators below the electroweak symmetry breaking scale is generally not calculable. Nevertheless, once we do obtain the coefficients, which should be small enough to not conflict with the experiments, the rest of the calculation is analogous to the linear case.

Assuming CP conservation, we enumerate effective operators in the chiral Lagrangian, up to dimension-6, and consider their contributions to the EWPOs. The discussion follows Ref. 9 and 44.

There is one dimension-2 operator in addition to the kinetic term of \( U \) in (100):

\[ \mathcal{L}'_1 = \frac{v^2}{4} \beta_1 \text{Tr}(TV_\mu)^2. \]

Going to unitary gauge by setting \( U = \text{diag}(1, 1) \), we obtain a shift to the \( Z \) mass

\[ \Delta M_Z^2 = -\frac{1}{2} \beta_1 v^2 (g^2 + g'^2). \]

Therefore, \( \beta_1 \) is equivalent to the \( T \) parameter:

\[ \alpha T = 2\beta_1. \]
At dimension-4, there are 11 independent operators involving only gauge bosons and $U$:

\[
\mathcal{L}_1 = \frac{1}{2} \alpha_1 g' B_{\mu \nu} \text{Tr}(W^{\mu \nu} T), \\
\mathcal{L}_2 = \frac{i}{2} \alpha_2 g' B^{\mu \nu} \text{Tr}(T[V_\mu, V_\nu]), \\
\mathcal{L}_3 = i \alpha_3 g \text{Tr}(W_{\mu \nu}[V^\mu, V^\nu]), \\
\mathcal{L}_4 = \alpha_4 [\text{Tr}(V_\mu V_\nu)]^2, \\
\mathcal{L}_5 = \alpha_5 [\text{Tr}(V_\mu V^\mu)]^2, \\
\mathcal{L}_6 = \alpha_6 \text{Tr}(V_\mu V_\nu) \text{Tr}(TV^\mu) \text{Tr}(TV^\nu), \\
\mathcal{L}_7 = \alpha_7 \text{Tr}(V_\mu V^\mu) \text{Tr}(TV^\nu) \text{Tr}(TV_\nu), \\
\mathcal{L}_8 = \frac{1}{4} g^2 [\text{Tr}(TW_{\mu \nu})]^2, \\
\mathcal{L}_9 = \frac{i}{2} \alpha_9 g \text{Tr}(TW_{\mu \nu}) \text{Tr}([V^\mu, V^\nu] T), \\
\mathcal{L}_{10} = \frac{1}{2} \alpha_{10} [\text{Tr}(TV^\mu) \text{Tr}(TV_\mu)]^2, \\
\mathcal{L}_{11} = \alpha_{11} g^{\epsilon_{\mu \nu \rho \lambda}} \text{Tr}(TV_\mu) \text{Tr}(V_\nu W_{\rho \lambda}).
\]

(107)

These operators can modify the gauge propagators, as well as gauge boson self couplings. We identify the operators corresponding to $S$ and $U$:

\[
S = -16 \pi \alpha_1, \quad U = -16 \pi \alpha_8.
\]

(108)

Unlike the linear case, the $U$ operator arises at the same order as the $S$ parameter.

The triple gauge couplings defined in Eq. (83) are modified as

\[
g_1^Z = 1 - \frac{1}{c^2 - s^2} \beta_1 + \frac{1}{c^2(c^2 - s^2)} c^2 \alpha_1 + \frac{1}{s^2 c^2} s^2 \alpha_3, \\
g_7^Z = 1 - 0, \\
\kappa_Z - 1 = \frac{1}{c^2 - s^2} \beta_1 + \frac{1}{c^2(c^2 - s^2)} c^2 \alpha_1 + \frac{1}{c^2} c^2(\alpha_1 - \alpha_2) + \frac{1}{s^2} s^2(\alpha_3 - \alpha_8 + \alpha_9), \\
\kappa_\gamma - 1 = \frac{1}{s^2} c^2(- \alpha_1 + \alpha_2 + \alpha_3 - \alpha_8 + \alpha_9), \\
g_5^Z = \frac{1}{s^2 c^2} s^2 \alpha_{11}, \quad g_7^Z = 0.
\]

(109)

There are 6 dimension-4 operators containing two fermions:

\[
\mathcal{L}_1^f = i \bar{f}_L \gamma^\mu V_\mu f_L, \\
\mathcal{L}_2^f = i \bar{f}_L \gamma^\mu (V_\mu T + TV_\mu) f_L, \\
\mathcal{L}_3^f = i \bar{f}_L \gamma^\mu TV_\mu T f_L, \\
\mathcal{L}_4^f = i \bar{f}_R \gamma^\mu \hat{V}_\mu f_R, \\
\mathcal{L}_5^f = i \bar{f}_R \gamma^\mu (\hat{V}_\mu \hat{T} + \hat{T} \hat{V}_\mu) f_R, \\
\mathcal{L}_6^f = i \bar{f}_R \gamma^\mu \hat{V}_\mu \hat{T} f_R.
\]

(110)
where \( f = l, q \). Obviously, these operators affect gauge-fermion couplings, analogous to the operators in Eq. (27) for the linear case.

6.4. Light states and loop corrections

Due to the stringent constraints shown in Table 2 if the new physical states contribute to EWPOs at tree level, the constraints on their masses are usually more than a TeV. In this kind of models, there is a clear energy scale that separates the SM fields and the heavy states and therefore an effective theory below that scale is well defined. This is where the effective theory method finds their best use for.

There are also a variety of models that new physics contributions to EWPOs do not appear at tree level. The most extensively studied minimal supersymmetric standard model (MSSM) is one example. In recent years, the list has been extended to include universal extra dimensions with KK parity, little Higgs models with T parity and so forth. In these models, interactions involving new particles must include even number of such particles and therefore only contribute to EWPOs at loop levels. The loop factor suppression makes it possible to have very light new particles comparable to the \( Z \) mass, and still pass the electroweak precision constraints.

Although stringent constraints from EWPTs have been avoided, sometime one is interested in how light the new states are allowed. In principle, the effective theory breaks down because the new physics does not decouple from the SM. Nevertheless, since the EWPTs do not have new particles in the initial or final states, the loop corrections can often be written in the form of effective operators. In some cases, there are a few such operators dominating the loop corrections, which greatly simplifies the calculations. We have already seen an example: the corrections of a different Higgs mass can be absorbed to the \( S \) and \( T \) parameters. As a more complicated example, in Ref. 51 it is shown that the major corrections for certain supersymmetric models can be encoded in the oblique parameters \( S, T, W \) and \( Y \). For another examples, in Ref. 52 the loop corrections to the \( S, T, U \) and \( \Delta q_L^b \) parameters and 4-fermion operators are calculated for the littlest Higgs model with T-parity.

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