Lattice Spinor Gravity
Quantum gravity

- Quantum field theory
- Functional integral formulation
Symmetries are crucial

- Diffeomorphism symmetry
  (invariance under general coordinate transformations)

- Gravity with fermions: local Lorentz symmetry

Degrees of freedom less important:
- metric, vierbein, spinors, random triangles, conformal fields...

Graviton, metric: collective degrees of freedom in theory with diffeomorphism symmetry
Regularized quantum gravity

- For finite number of lattice points: functional integral should be well defined
- Lattice action invariant under local Lorentz-transformations
- Continuum limit exists where gravitational interactions remain present
- Diffeomorphism invariance of continuum limit, and geometrical lattice origin for this
Spinor gravity is formulated in terms of fermions
Unified Theory
of fermions and bosons

Fermions fundamental
Bosons collective degrees of freedom

- Alternative to supersymmetry
- Graviton, photon, gluons, W-,Z-bosons, Higgs scalar: all are collective degrees of freedom (composite)
- Composite bosons look fundamental at large distances, e.g. hydrogen atom, helium nucleus, pions
- Characteristic scale for compositeness: Planck mass
Massless collective fields or bound states – familiar if dictated by symmetries

for chiral QCD:

Pions are massless bound states of massless quarks!

for strongly interacting electrons:

antiferromagnetic spin waves
Gauge bosons, scalars …

from vielbein components in higher dimensions (Kaluza, Klein)

concentrate first on gravity
Geometrical degrees of freedom

- $\Psi(x)$: spinor field (Grassmann variable)
- vielbein: fermion bilinear
Possible Action

\[ S_E \sim \int d^d x \det (\tilde{E}^m_\mu (x)) \]

\[ \tilde{E} = \frac{1}{d!} \epsilon^{\mu_1 \ldots \mu_d} \epsilon_{m_1 \ldots m_d} \tilde{E}^{m_1}_{\mu_1} \ldots \tilde{E}^{m_d}_{\mu_d} = \det(\tilde{E}^m_\mu) \]

contains 2d powers of spinors, d derivatives contracted with \( \varepsilon \)-tensor
Symmetries

- **General coordinate transformations** (diffeomorphisms)
  - Spinor \( \psi(x) \): transforms as scalar
  - Vielbein \( \tilde{E}^m_\mu = i \overline{\psi} \gamma^m \partial_\mu \psi \): transforms as vector
  - Action \( S \): invariant

K.Akama,Y.Chikashige,T.Matsuki,H.Terazawa (1978)
K.Akama (1978)
D.Amati, G.Veneziano (1981)
G.Denardo,E.Spallucci (1987)
Lorentz- transformations

Global Lorentz transformations:

- spinor $\psi$
- vielbein transforms as vector
- action invariant

Local Lorentz transformations:

- vielbein does not transform as vector
- inhomogeneous piece, missing covariant derivative

$$\tilde{E}^\mu = i\bar{\psi}\gamma^\mu\Theta_\mu\psi$$
Two alternatives:

1) Gravity with **global** and not **local** Lorentz symmetry?
   Compatible with observation!

2) Action with **local** Lorentz symmetry?
   Can be constructed!
Spinor degrees of freedom

- Grassmann variables
- Spinor index $\gamma = 1 \ldots 8$
- Two flavors $a = 1, 2$
- Variables at every space-time point $x^\mu = (x^0, x^1, x^2, x^3)$

- Complex Grassmann variables

$$\varphi^a_\alpha(x) = \psi^a_\alpha(x) + i\psi^a_{\alpha+4}(x)$$
Action with local Lorentz symmetry

\[ S = \alpha \int d^4x A^{(8)} D + c.c. \]

A: product of all eight spinors, maximal number, totally antisymmetric

\[ A^{(8)} = \frac{1}{8!} \epsilon_{\epsilon_1 \epsilon_2 \cdots \epsilon_8} \varphi_{\epsilon_1} \cdots \varphi_{\epsilon_8} = \frac{1}{(24)^2} \epsilon_{\alpha_1 \alpha_2 \alpha_3 \alpha_4} \varphi_{\alpha_1}^1 \cdots \varphi_{\alpha_4}^1 \epsilon_{\beta_1 \beta_2 \beta_3 \beta_4} \varphi_{\beta_1}^2 \cdots \varphi_{\beta_4}^2 = \varphi_1^1 \varphi_2^1 \varphi_3^1 \varphi_4^1 \varphi_1^2 \varphi_2^2 \varphi_3^2 \varphi_4^2 \]

D: antisymmetric product of four derivatives, L is totally symmetric Lorentz invariant tensor
Symmetric four-index invariant

Symmetric invariant bilinears

Lorentz invariant tensors

Symmetric four-index invariant

Two flavors needed in four dimensions for this construction
Weyl spinors

\[ \varphi_+ = \frac{1}{2} (1 + \bar{\gamma}) \varphi , \quad \varphi_- = \frac{1}{2} (1 - \bar{\gamma}) \varphi \]

\[ \bar{\gamma} = -\gamma^0 \gamma^1 \gamma^2 \gamma^3 \]

\[ = \text{diag} \left( 1, 1, -1, -1 \right) \]

\[ \gamma^0 = \tau_1 \otimes 1 , \quad \gamma^k = \tau_2 \otimes \tau_k . \]
Action in terms of Weyl - spinors

\[ S = \alpha \int d^4x \epsilon_{\mu_1 \mu_2 \mu_3 \mu_4} F_{\mu_1 \mu_2}^+ F_{\mu_3 \mu_4}^- + c.c. \]

\[ F_{\mu_1 \mu_2}^\pm = A_{\mu_1 \mu_2}^\pm D_{\mu_1 \mu_2}^\pm \]

\[ A^+ = \varphi_{+1}^1 \varphi_{+2}^1 \varphi_{+1}^2 \varphi_{+2}^2 \quad D_{\mu_1 \mu_2}^\pm = \partial_{\mu_1} \varphi_{\eta_1} S_{\eta_1 \eta_2}^\pm \partial_{\mu_2} \varphi_{\eta_2} \]

Relation to previous formulation

\[ A^{(8)} = A^+ A^- \quad D = \epsilon_{\mu_1 \mu_2 \mu_3 \mu_4} D_{\mu_1 \mu_2}^+ D_{\mu_3 \mu_4}^- \]

\[ S = \alpha \int d^4x A^{(8)} D + c.c. \]
SO(4,C) - symmetry

\[ \delta \varphi^a_\alpha(x) = -\frac{1}{2} \epsilon_{mn}(x) (\Sigma_{E}^{mn})_{\alpha\beta} \varphi^a_\beta(x) \]

\[ \Sigma^{mn}_E = -\frac{1}{4} [\gamma^m_E, \gamma^n_E], \{\gamma^m_E, \gamma^n_E\} = 2\delta^{mn} \]

Action invariant for arbitrary complex transformation parameters \( \epsilon \)!

Real \( \epsilon : SO(4) - transformations \)
Signature of time

Difference in signature between space and time:

only from spontaneous symmetry breaking, e.g. by expectation value of vierbein – bilinear!
Minkowski - action

\[ S = -iS_M, \quad e^{-S} = e^{iS_M} \]

Action describes **simultaneously** euclidean and Minkowski theory!

SO (1,3) transformations:

\[ \epsilon_{0k} = -i\epsilon^{(M)}_{0k} \]
\[ \epsilon^{(M)}_{kl} = \epsilon_{kl} \]

\[ \delta \varphi = -\frac{1}{2} \epsilon^{(M)}_{mn} \Sigma^{mn}_M \varphi, \]

\[ \Sigma^{mn}_M = -\frac{1}{4} [\gamma^m_M, \gamma^n_M], \quad \{\gamma^m_M, \gamma^n_M\} = \eta^{mn} \]

\[ \gamma^0_M = -i\gamma^0_E, \quad \gamma^k_M = \gamma^k_E \]
Emergence of geometry

Euclidean vierbein bilinear
\[ \tilde{E}_\mu^m = \varphi^a C \gamma^m \partial_\mu \varphi^b V^{ab} = -\partial_\mu \varphi^a C \gamma^m \varphi^b V^{ab} \]

Minkowski - vierbein bilinear
\[ \tilde{E}_\mu^{(M)m} = \varphi V C \gamma^m_M \partial_\mu \varphi \]
\[ \tilde{E}_\mu^{(M)0} = -i \tilde{E}_\mu^0, \quad \tilde{E}_\mu^{(M)k} = \tilde{E}_\mu^k \]

Global Lorentz - transformation
\[ \delta \tilde{E}_\mu^{(M)m} = -\tilde{E}_\mu^{(M)n} \epsilon_n^{(M)m} \]

vierbein
\[ \langle \tilde{E}_\mu^{(M)m} \rangle = \langle (\tilde{E}_\mu^{(M)m})^* \rangle = e_\mu^m / \Delta \]

metric
\[ g_{\mu\nu} = e_\mu^m e_\nu^n \eta_{mn} \]
Action can be reformulated in terms of vierbein bilinear

\[ S = \alpha \int d^4 x W \det(\tilde{E}^m_\mu) + c.c. \]
How to get gravitational field equations?

How to determine geometry of space-time, vielbein and metric?
Functional integral formulation of gravity

- Calculability
  (at least in principle)
- Quantum gravity
- Non-perturbative formulation

\[ Z = \int \mathcal{D}\psi g_f \exp(-S)g_{in}, \]

\[ \int \mathcal{D}\psi = \prod_x \prod_{a=1}^2 \left\{ \int d\psi_1^a(x) \ldots \int d\psi_8^a(x) \right\} \]

\[ \langle A \rangle = Z^{-1} \int \mathcal{D}\psi g_f A \exp(-S)g_{in} \]
Vierbein and metric

\[ E^m_\mu(x) = \langle \tilde{E}^m_\mu(x) \rangle \]
\[ g_{\mu\nu}(x) = E^m_\mu(x)E^m_{\nu m}(x) \]

Generating functional

\[ Z[J] = \int \mathcal{D}\psi \exp \left\{ - (S + S_J) \right\} \]
\[ S_J = - \int d^d x J^\mu_\mu \tilde{E}^m_\mu \]
\[ E^m_\mu(x) = \langle \tilde{E}^m_\mu(x) \rangle = \frac{\delta \ln Z}{\delta J^\mu_\mu(x)} \]
If regularized functional measure can be defined (consistent with diffeomorphisms)

Non-perturbative definition of quantum gravity

\[ Z[J] = \int D\psi \exp \left\{ - (S + S_J) \right\} \]
Effective action

\[ \Gamma[E_\mu^m] = -W[J_\mu^m] + \int d^d x J_\mu^m E_\mu^m \]

\[ W = \ln Z \]

Gravitational field equation for vierbein

\[ \frac{\delta \Gamma}{\delta E_\mu^m} = J_\mu^m \]

similar for metric
Symmetries dictate general form of effective action and gravitational field equation

diffeomorphisms!

Effective action for metric:

\[ \text{curvature scalar } R + \text{ additional terms} \]
Lattice regularization

- Hypercubic lattice
- Even sublattice
- Odd sublattice

Spinor degrees of freedom on points of odd sublattice
Lattice action

- Associate cell to each point $y$ of even sublattice
- **Action**: sum over cells

\[ S = \tilde{\alpha} \sum_y \mathcal{L}(y) + c.c. \]

- For each cell: twelve spinors located at nearest neighbors of $y$ (on odd sublattice)

\[ \tilde{z}^\mu \left( \tilde{x}_j(\tilde{y}) \right) = \tilde{y}^\mu + V_j^\mu \]

\[
V_1 = (-1, 0, 0, 0), \quad V_5 = (0, 0, 0, 1) \\
V_2 = (0, -1, 0, 0), \quad V_6 = (0, 0, 1, 0) \\
V_3 = (0, 0, -1, 0), \quad V_7 = (0, 1, 0, 0) \\
V_4 = (0, 0, 0, -1), \quad V_8 = (1, 0, 0, 0)
\]
Local SO (4,C) symmetry

Basic SO(4,C) invariant building blocks

\[ \mathcal{H}_\pm^k(\tilde{x}) = \varphi^a_\alpha(\tilde{x})(C^\pm)_\alpha\beta(\tau_2\tau_k)^{ab}\varphi^b_\beta(\tilde{x}) \]

Lattice action

\[ \mathcal{L}(y) = \frac{1}{6} \{ \mathcal{F}^{1,2,8,7}_+\mathcal{F}^{3,4,6,5}_- + \mathcal{F}^{1,3,8,6}_+\mathcal{F}^{7,4,2,5}_- \\
+ \mathcal{F}^{1,4,8,5}_+\mathcal{F}^{3,7,6,2}_- + (\mathcal{F}_+ \leftrightarrow \mathcal{F}_-) \}. \]

\[ \mathcal{F}_{\pm}^{abcd} = \frac{1}{24} \epsilon^{klm} [\mathcal{H}_\pm^k(\tilde{x}_a)\mathcal{H}_\pm^l(\tilde{x}_b)\mathcal{H}_\pm^m(\tilde{x}_c) \\
+ \mathcal{H}_\pm^k(\tilde{x}_b)\mathcal{H}_\pm^l(\tilde{x}_c)\mathcal{H}_\pm^m(\tilde{x}_d) + \mathcal{H}_\pm^k(\tilde{x}_c)\mathcal{H}_\pm^l(\tilde{x}_d)\mathcal{H}_\pm^m(\tilde{x}_a) \\
+ \mathcal{H}_\pm^k(\tilde{x}_d)\mathcal{H}_\pm^l(\tilde{x}_a)\mathcal{H}_\pm^m(\tilde{x}_b) ] \]
Lattice symmetries

- Rotations by $\pi/2$ in all lattice planes
  \[ \mathcal{F}_{abcd} = \mathcal{F}_{bcda} = \mathcal{F}_{cdab} = \mathcal{F}_{dabc} \]

- Reflections of all lattice coordinates
  \[ \mathcal{F}_{cbad} = \mathcal{F}_{dacb} = -\mathcal{F}_{abcd} \]

- Diagonal reflections e.g. $z_1 \leftrightarrow z_2$
Lattice derivatives

\[
\hat{\partial}_0 \varphi(y) = \frac{1}{2\Delta} (\varphi(\tilde{x}_8) - \varphi(\tilde{x}_1)) \\
\hat{\partial}_1 \varphi(y) = \frac{1}{2\Delta} (\varphi(\tilde{x}_7) - \varphi(\tilde{x}_2)) \\
\hat{\partial}_2 \varphi(y) = \frac{1}{2\Delta} (\varphi(\tilde{x}_6) - \varphi(\tilde{x}_3)) \\
\hat{\partial}_3 \varphi(y) = \frac{1}{2\Delta} (\varphi(\tilde{x}_5) - \varphi(\tilde{x}_4))
\]

and cell averages

\[
\bar{\varphi}_0(y) = \frac{1}{2} (\varphi(\tilde{x}_1) + \varphi(\tilde{x}_8)) , \quad \bar{\varphi}_1(y) = \frac{1}{2} (\varphi(\tilde{x}_2) + \varphi(\tilde{x}_7)) \\
\bar{\varphi}_2(y) = \frac{1}{2} (\varphi(\tilde{x}_3) + \varphi(\tilde{x}_6)) , \quad \bar{\varphi}_3(y) = \frac{1}{2} (\varphi(\tilde{x}_4) + \varphi(\tilde{x}_5))
\]

express spinors in derivatives and averages

\[
\varphi(\tilde{x}_j) = \sigma_{\mu}^j \bar{\varphi}_\mu + V_{j}^\mu \Delta \hat{\partial}_\mu \varphi \\
\sigma_{j}^\mu = (V_{j}^\mu)^2
\]
Bilinears and lattice derivatives

\[ \mathcal{H}_\pm^k(x_j) = \sigma_j^\mu \bar{\mathcal{H}}_{\pm \mu}^k(y) + 2\Delta V_j^\mu \bar{\mathcal{D}}_{\pm \mu}^k(y) + \Delta^2 \sigma_j^\mu \mathcal{G}_{\pm \mu}^k(y) \]

\[ \hat{\mathcal{D}}_{\pm \mu}^k = (\varphi^a_{\mu})_{\alpha}(C_\pm)_{\alpha \beta}(\tau_2 \tau_k)^{ab} \hat{\partial}_\mu \varphi^b_{\beta} \]

\[ \hat{\mathcal{G}}_{\pm \mu}^k = \hat{\partial}_\mu \varphi^a_{\alpha}(C_\pm)_{\alpha \beta}(\tau_2 \tau_k)^{ab} \hat{\partial}_\mu \varphi^b_{\beta} \]

\[ \hat{\mathcal{H}}_{\pm \mu}^k = \mathcal{H}_{\pm \mu}^k + \Delta^2 \hat{\mathcal{G}}_{\pm \mu}^k, \quad \mathcal{H}_{\pm ab}^k = \frac{1}{2}(\hat{\mathcal{H}}_{\pm a}^k + \hat{\mathcal{H}}_{\pm b}^k). \]

\[ \mathcal{F}_{1,2,8,7}^\pm = \frac{2\Delta^2}{3} \epsilon^{klm} \mathcal{H}_{+01}^k(\mathcal{D}_{+0}^l \mathcal{D}_{+1}^m - \mathcal{D}_{+1}^l \mathcal{D}_{+0}^m) \]

\[ \mathcal{F}_{01}^\pm = -\mathcal{F}_{10}^\pm = \mathcal{F}_{\pm}^{1,2,8,7} \]

\[ \mathcal{F}_{\mu \nu}^\pm = \frac{2\Delta^2}{3} \epsilon^{klm} \mathcal{H}_{\pm \mu \nu}^k(\mathcal{D}_{\pm \mu}^l \mathcal{D}_{\pm \nu}^m - \mathcal{D}_{\pm \nu}^l \mathcal{D}_{\pm \mu}^m) \]

\[ \mathcal{L}(y) = \frac{1}{24} \epsilon^{\mu_1 \mu_2 \mu_3 \mu_4} \mathcal{F}_{\mu_1 \mu_2}^+ \mathcal{F}_{\mu_3 \mu_4}^- \]
Continuum limit

\[ \mathcal{L}(y) \rightarrow \frac{32}{3} \Delta^4 \varepsilon_{\mu_1 \mu_2 \mu_3 \mu_4} F^+_{\mu_1 \mu_2} F^-_{\mu_3 \mu_4} \]

\[ \Delta^4 \Sigma_y = \frac{1}{2} \int_y \]

Lattice distance \( \Delta \) drops out in continuum limit!

\[ S = \frac{16}{3} \tilde{\alpha} \int_y \varepsilon^{\mu_1 \mu_2 \mu_3 \mu_4} F^+_{\mu_1 \mu_2} F^-_{\mu_3 \mu_4} + c.c \]

\[ \tilde{\alpha} = \frac{3\alpha}{16} \]
Regularized quantum gravity

- For finite number of lattice points: functional integral should be well defined.
- Lattice action invariant under local Lorentz-transformations.
- Continuum limit exists where gravitational interactions remain present.
- Diffeomorphism invariance of continuum limit, and geometrical lattice origin for this.
Conclusions

- Unified theory based only on fermions seems possible
- Quantum gravity – functional measure can be regulated
- Does realistic higher dimensional unified model exist?
Gravitational field equation and energy momentum tensor

\[ \frac{\delta \Gamma}{\delta E^m_{\mu}} = J^\mu_m \]

\[ T^{\mu\nu} = E^{-1} E^m_{\mu} J^\nu_n \]

Special case: effective action depends only on metric

\[ \Gamma'[E^m_{\mu}] = \Gamma'[g_{\nu\rho}[E^m_{\mu}]] \]

\[ g_{\mu\nu} = E^m_{\mu} E_{\nu m} \]

\[ T^{\mu\nu}_{(g)} = -\frac{2}{\sqrt{g}} \frac{\delta \Gamma'[g]}{\delta g_{\mu\nu}} \]

\[ T^{\mu\nu} = -E^{-1}E^m_{\mu} \frac{\delta \Gamma'[g]}{\delta g_{\rho\sigma}} \frac{\delta E^m_{\nu}}{\delta E^r_{\nu}} = T^{\mu\nu}_{(g)} \]
Unified theory in higher dimensions and energy momentum tensor

- Only spinors, no additional fields – no genuine source
- $J^\mu_m$: expectation values different from vielbein and incoherent fluctuations
- Can account for matter or radiation in effective four dimensional theory (including gauge fields as higher dimensional vielbein-components)
Time space asymmetry from spontaneous symmetry breaking

Idea: difference in signature from spontaneous symmetry breaking

With spinors: signature depends on signature of Lorentz group

- Unified setting with complex orthogonal group:
- Both euclidean orthogonal group and minkowskian Lorentz group are subgroups
- Realized signature depends on ground state!
Complex orthogonal group

d=16, ψ: 256-component spinor, real Grassmann algebra

\[ \delta \psi = \begin{pmatrix} \rho, & -\tau \\ \tau, & \rho \end{pmatrix} \psi \]

\[ \rho = -\frac{1}{2} \varepsilon_{mn} \hat{\Sigma}^{mn}, \quad \tau = \frac{1}{2} \varepsilon_{mn} \hat{\Sigma}^{mn} \]

\[ \Sigma_E^{mn} = \hat{\Sigma}^{mn} 1, \quad B^{mn} = -\hat{\Sigma}^{mn} I, \]

\[ I = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad I^2 = -1 \]

**SO(16,C)**

\( \mathfrak{q}, \tau: \) antisymmetric
128 x 128 matrices

Compact part: \( \mathfrak{q} \)
Non-compact part: \( \tau \)
vielbein

\[ \tilde{E}_\mu^0 = \psi_\alpha \partial_\mu \psi_\alpha , \quad \tilde{E}_\mu^k = \psi_\alpha (\hat{a}^k I)_{\alpha \beta} \partial_\mu \psi_\beta \]

\[ \{\hat{a}^k, \hat{a}^l\} = -2\delta^{kl} , \quad k, l = 1 \ldots 15 \]

\[ \hat{\Sigma}^{kl} = \frac{1}{4}[\hat{a}^k, \hat{a}^l] , \quad \hat{\Sigma}^{0k} = -\frac{1}{2} \hat{a}^k \]

\[ E^m_\mu = \delta^m_\mu : \]

SO(1,15) - symmetry

however:

*Minkowski signature not singled out in action!*
Formulation of action invariant under SO(16,C)

- Even invariant under larger symmetry group SO(128,C)
- Local symmetry!
complex formulation

so far real Grassmann algebra
introduce complex structure by

\[ \varphi_{\hat{\alpha}} = \psi_{\dot{\alpha}} + i\psi_{128+\dot{\alpha}}, \quad \varphi^*_{\hat{\alpha}} = \psi_{\dot{\alpha}} - i\psi_{128+\dot{\alpha}} \]

\[ \delta \varphi_{\hat{\alpha}} = \sigma_{\hat{\alpha}\dot{\beta}} \varphi_{\dot{\beta}}, \quad \sigma = \rho + i\tau \]

\(\sigma\) is antisymmetric 128 x 128 matrix, generates SO(128,C)
Invariant action

(Complex orthogonal group, diffeomorphisms)

\[ S = \alpha \int d^d x W[\varphi] R(\varphi, \varphi^*) + c.c., \]

\[ W[\varphi] = \frac{1}{16!} \varepsilon^{\mu_1 \cdots \mu_{16}} \partial_{\mu_1} \varphi \hat{\alpha}_1 \cdots \partial_{\mu_{16}} \varphi \hat{\alpha}_{16} L^{\hat{\alpha}_1 \cdots \hat{\alpha}_{16}} \]

\[ L^{\hat{\alpha}_1 \cdots \hat{\alpha}_{16}} = \text{sym} \{ \delta^{\hat{\alpha}_1 \hat{\alpha}_2} \delta^{\hat{\alpha}_3 \hat{\alpha}_4} \cdots \delta^{\hat{\alpha}_{15} \hat{\alpha}_{16}} \} \]

\[ R(\varphi, \varphi^*) = T(\varphi) + \tau T(\varphi^*) + \kappa T(\varphi) T(\varphi^*), \]

\[ T(\varphi) = \frac{1}{128!} \varepsilon^{\hat{\beta}_1 \cdots \hat{\beta}_{128}} \varphi_{\hat{\beta}_1} \cdots \varphi_{\hat{\beta}_{128}} \]

For \( \tau = 0 \) : Local Lorentz-symmetry !!

Invariants with respect to \( SO(128,\mathbb{C}) \)

and therefore also

with respect to subgroup \( SO(16,\mathbb{C}) \)

contractions with \( \delta \) and \( \varepsilon - \)tensors

no mixed terms \( \varphi \varphi^* \)
Generalized Lorentz symmetry

- Example $d=16$ : $SO(128,\mathbb{C})$ instead of $SO(1,15)$

- Important for existence of chiral spinors in effective four dimensional theory after dimensional reduction of higher dimensional gravity

S. Weinberg
Unification in $d=16$ or $d=18$?

- Start with irreducible spinor
- Dimensional reduction of gravity on suitable internal space
- Gauge bosons from Kaluza-Klein-mechanism
- 12 internal dimensions: $SO(10) \times SO(3)$ gauge symmetry: unification + generation group
- 14 internal dimensions: more $U(1)$ generator symmetry

$(d=18$: anomaly of local Lorentz symmetry $)$

L. Alvarez-Gaume, E. Witten
Ground state with appropriate isometries:
guarantees massless gauge bosons and graviton in spectrum
Chiral fermion generations

- Chiral fermion generations according to chirality index
  C.W., Nucl.Phys. B223,109 (1983);
  E. Witten, Shelter Island conference, 1983

- Nonvanishing index for brane geometries
  (noncompact internal space)
  C.W., Nucl.Phys. B242,473 (1984)

- and warping
  C.W., Nucl.Phys. B253,366 (1985)

- $d=4 \mod 4$ possible for ‘extended Lorentz symmetry’ (otherwise only $d = 2 \mod 8$)
Rather realistic model known

- $d=18$: first step: brane compactification
- $d=6$, SO(12) theory: (anomaly free)
- second step: monopole compactification
- $d=4$ with three generations, including generation symmetries
- SSB of generation symmetry: realistic mass and mixing hierarchies for quarks and leptons (except large Cabibbo angle)

C.W., Nucl.Phys. B244,359 (1984); B260,402 (1985); B261,461 (1985); B279,711 (1987)
## Comparison with string theory

| Requirement                                                  | SStrings | Sp.Grav. |
|--------------------------------------------------------------|----------|----------|
| Unification of bosons and fermions                           | ok       | ok       |
| Unification of all interactions (d > 4)                      | ok       | ok       |
| Non-perturbative (functional integral) formulation           | -        | ok       |
| Manifest invariance under diffeomorphisms                    | -        | ok       |
## Comparison with string theory

| Feature                                      | SStrings | Sp.Grav. |
|----------------------------------------------|----------|----------|
| Finiteness/regularization                    | ok       | ok       |
| Uniqueness of ground state/predictivity      | -        | ?        |
| No dimensionless parameter                   | ok       | ?        |