Spatial light modulation based on coherent population oscillation in semiconductor quantum dots

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Due to coherent population oscillation (CPO) effect, we theoretically examine the generation and manipulation of Laguerre-Gaussian beams in terms of phase and amplitude modulation respectively in semiconductor quantum dots (SQDs). The results indicate that both phase modulation with low absorption of probe field and amplitude modulation with high efficiency can be achieved. Thus the practical way demonstrates that the SQDs system can function as effective optically addressed spatial light modulator with which the transverse spatial properties of probe fields can be artificially modulated.

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I INTRODUCTION

A spatial light modulator (SLM) is an object that imposes some form of spatially-varying modulation on a beam of light. It can modulate the intensity as well as the phase of the light beam. Nowadays a great deal of research [1, 2, 3, 4] is focused on storing information imprinted in the transverse plane of the probe beam (i.e., images) and on reducing the effects of atomic diffusion. Shuker et al. pointed out that the immunity to diffusion can be improved by appropriately manipulating the phase of different points in the image [2]. In the process, the utilization of a SLM is necessary. In fact, SLMs are used extensively in holographic data storage and display setups. They can encode information into a laser beam and produce light fields with peculiar spatial structures.

On the other hand, electromagnetically induced transparency (EIT) has been intensively investigated for decades [5, 6]. Nevertheless, coherent population oscillation (CPO), which is also a phenomenon of quantum interference as EIT, has been proven to be another powerful technique that can eliminate the absorption at the pump-probe beam frequency and dramatically change the refractive index [7, 8, 9]. In systems based on EIT and CPO, the probe field can be manipulated coherently and all-optically by strong pump field. With the well-known properties, these types of systems are good candidates to realize velocity control of light, storage and retrieval of images and other information processing. Moreover, the slow light based on CPO can easily be achieved in a solid-state material at room temperatures, while for EIT, it is usually required to cool the medium to very low temperatures to obtain a large index slope [8]. Thus the CPO effect leads itself more readily towards realistic applications using solid-state devices, such as semiconductors.

In view of the advantages of CPO effect, in this paper, according to CPO, we propose a way to spatially modulate probe light beams with the use of optical patterns (e.g., images) with desired intensity distributions in the pump fields. The material we introduce to perform CPO consists of semiconductor quantum dots (SQDs) in which excitons behave as two-level systems. To exemplify our proposal, we examine the generation and manipulation of Laguerre-Gaussian (LG) beams. The theoretical results suggest the feasibility of the modulation of spatial light in terms of CPO in SQDs and our work may open an avenue to SLM technology in the future.

II THEORY

Our model consists of tens of layers of SQDs embedded in a GaAs matrix. We consider a spherical SQD in the presence of a strong pump field and a weak probe field. For the SQD, we assume a two-level model which consists of the ground state |0⟩ and the first excited state (single exciton) |1⟩. The SQD via exciton interacts with a probe field of frequency ωs and is coherently driven by a strong pump field of frequency ωc. As usual, this two-level system can be characterized by the pseudo-spin-1/2 operators Sz and Sz. Then the Hamiltonian of the system in a rotating frame at the pump field frequency ωc reads as follows:

\[
H = \hbar(\omega_{ex} - \omega_c)S_z - \hbar\Omega(S^+ + S^-)/2 - \mu(S^+ E_x e^{-i\delta t} + S^- E_x e^{i\delta t})/2,
\]

where \(\hbar\omega_{ex}\) is the energy of exciton binding, \(\Omega = \mu E_x/\hbar\) is the Rabi frequency, \(\mu\) is the interband dipole matrix element, \(E_x\) is the slowly varying envelope of the pump field, \(E_x\) is the slowly varying envelope of the probe field, \(\delta = \omega_s - \omega_c\) is the detuning of the probe and the pump field.

The temporal evolution of the exciton in the SQD are determined by the Heisenberg equation of motion. After replacing the operator by the mean values defined as classical variable \(\langle S^+ \rangle\) and \(\langle S^- \rangle\) and then setting \(w = 2\langle S^z \rangle\) and \(\rho = \mu \langle S^- \rangle\) [10, 11, 12], we have the generalized optical Bloch equations:
\[
\frac{dp}{dt} = -\left(\frac{1}{T_2} + i\Delta\right)p - \frac{i\mu^2}{2\hbar} w E, \tag{2}
\]

\[
\frac{dw}{dt} = -\frac{1}{T_1} (w + 1) + 2\text{Im}(pE^\ast)/\hbar. \tag{3}
\]

where \(\Delta = \omega_{ex} - \omega_c\) and \(E = E_c + E_s e^{-i\delta t}\). \(T_1\) is the exciton dephasing time and \(T_2\) is the exciton dephasing time.

On making the ansatz \(13\): \(p = p_0 + p_1 e^{-i\delta t} + p_{-1} e^{i\delta t}\), \(w = w_0 + w_1 e^{-i\delta t} + w_{-1} e^{i\delta t}\), obtaining and solving the equations in the steady state, the dimensionless susceptibility is given by

\[
\chi^{(1)}(\omega_c) = \frac{p_1}{E_s \mu^2 T_2} = \frac{-i w_0}{1 + i(\Delta_c - \delta_c)} \times (1 - \frac{2\Omega^2}{D(\delta_c)} (1 + i\Delta_c)(1 - i(\Delta_c + \delta_c))(2 - i\delta_c)), \tag{4}
\]

where \(D(\delta_c) = (\frac{\delta_c}{T_1} - i\delta_c)(1 + \Delta_c^2) + (1 - i\delta_c)^2 + \Delta_c^2 + 4\Omega^2(1 - i\delta_c)(1 + \Delta_c^2)\), \(\delta_c = \delta T_2\), \(\Omega_c = \mu E_0 T_2\), \(\Delta_c = \Delta T_2\), and the population inversion of exciton \(w_0\) is determined by \(w_0 = \frac{-4\Omega^2 w_0 T_2}{1 + 2\Delta_c^2} - 1\). Also, we can have \(\Delta_c = (\omega_s - \omega_{ex}) T_2\) which is the frequency detuning between the probe field and exciton. Note that the Rabi frequency \(\Omega_c\) and the detunings are all normalized with respect to the exciton dephasing time \(T_2\).

![FIG. 1: The real part of the linear susceptibility for the probe field induced by the azimuthal intensity distribution for three cases: a fixed detuning \(\Delta_s = 0.05\) (solid line); Gaussian distribution: \(\sigma_c = 0.05\) (dash-dotted line), \(\sigma_c = 0.15\) (dashed line). The frequency detuning between the probe field and exciton is \(\Delta_s = 0\). The inset shows the real part of the linear susceptibility for the probe field as a function of \(\Delta_s\) with dashed line showing \(\text{Re} \chi^{(1)}\) for zero pump field, with solid line showing \(\text{Re} \chi^{(1)}\) for the Rabi frequency of the pump field \(\Omega_c = 0.3\). \(\Delta_s\) is fixed as 0.05.](image1.png)

![FIG. 2: The imaginary part of the linear susceptibility for the probe field induced by the azimuthal intensity distribution for three cases: a fixed detuning \(\Delta_s = 0.05\) (solid line); Gaussian distribution: \(\sigma_c = 0.05\) (dash-dotted line), \(\sigma_c = 0.15\) (dashed line). The detuning \(\Delta_s = 0\). The inset shows the imaginary part of the linear susceptibility for the probe field as a function of \(\Delta_s\) with dashed line showing \(\text{Im} \chi^{(1)}\) for zero pump field, with solid line showing \(\text{Im} \chi^{(1)}\) for the Rabi frequency of the pump field \(\Omega_c = 0.3\). \(\Delta_s\) is fixed as 0.05.](image2.png)

### III RESULTS AND DISCUSSION

Since the dimensionless linear optical susceptibility of our system has been derived, the refractive index (associated with \(\text{Re} \chi^{(1)}\)) and the absorption spectrum (associated with \(\text{Im} \chi^{(1)}\)) as functions of the detuning \(\Delta_s\) can be obtained and are shown in the insets of Fig. 1 and Fig. 2 respectively. Here and in the following calculation, for the relaxation time in the SQD in the room-temperature regime, we take \(T_2 = 3 \times 10^{-13}\) s, \(T_1 = 1.5 \times 10^{-11}\) s \([14]\), and choose \(\Omega_c = 0.3\), \(\Delta_s = 0.05\). It can be seen clearly that when the pump field is turned on, the slope of the refractive index is dramatically changed at the resonant condition \(\Delta_s = 0\). In addition, a non-absorbing hole appears at \(\Delta_s = 0\), suggesting the system becomes transparent for the probe field in the presence of pump field. Now let’s consider the generation of LG modes by means of phase modulation. Firstly we give a chief introduction of the possible experimental setup. The pump beam is shaped as a Gaussian beam and is in exact resonance with the transition \(|0\rangle \leftrightarrow |1\rangle\) (i.e., \(\Delta_s = 0\)). The pump beam passes through a image mask. The plane of the image mask is imaged onto the center of the GaAs matrix by using lens. The pump and probe beams are combined on a beam splitter and copropagate through the SQDs. After that, they are divided by a second beam splitter. To generate LG modes, an azimuthal phase winding \(e^{i\phi}\) should be imprinted onto the wave front of the incident Gaussian probe field, where \(l\phi\) is the helical phase, \(l\) is the integer winding number and \(\phi\) is the azimuthal angle around optical axis \([13]\). The phase imprinting leads to an azimuthal variation of the refractive index \(n(\phi)\) in our system. To realize it, an azimuthally dependent Rabi frequency \(\Omega_c(\phi)\) of the pump field is required which
can make $\chi^{(1)}$ vary with $\phi$ according to Eq.(4). Thus we assume $16$

$$\Omega_c(\phi) = \frac{\mu E_c}{h} T_2 = \sqrt{\frac{a}{\beta c + c}},$$  

(5)

where $a$, $b$, $c$ are the adjustable parameters. This pump field can be constructed by the images of amplitude masks. To illustrate clearly, the intensity distribution $I(\phi) \propto E^2_c(\phi)$ of the pump field is shown in terms of images in case of $l = 0, 1, 2, 4$ respectively in Fig. 3. One can see uniform illumination when $l = 0$ because of disappearance of phase shift. One can also find the fold of intensity distribution with higher winding number as compared to the image with $l = 1$. In what follows, we assume $a = 1$, $b = c = 1$ to make the pump field strong: $\Omega_c(\phi) \gg 1/T_1$, take the detuning $\Delta_c = 0.05$. The solid lines in Fig. 1 and Fig. 2 show the real part and imaginary part of $\chi^{(1)}$ for the probe field in the SQDs as functions of azimuthal angle $\phi$, respectively. We can see that the absorption of the system is suppressed evidently within a $2\pi$ phase regime. In the theoretical calculations above, the detuning $\Delta_c$ is a fixed value, which can only stand for the condition of one quantum dot with a certain dot size. To describe the ensemble of SQDs and reflect the size distribution of the dots and the inhomogeneous broadening of the transitions, we assume a Gaussian distribution function for the pump field-exciton detuning $\Delta_g$:

$$G(\Delta_g) = \frac{1}{\sqrt{2\pi} \sigma_c} \exp\left[-\frac{(\Delta_g - \Delta_s)^2}{2\sigma_c^2}\right],$$  

(6)

where $\Delta_s$ is the center of Gaussian wave-packets and $\sigma_c = \sigma T_2$ is the half-width of the distribution. Thus, more precise results can be obtained for $\text{Re}\chi^{(1)}$ and $\text{Im}\chi^{(1)}$ and they are plotted with dash-dotted and dashed lines in Fig. 1 and Fig. 2 to present a comparison with the former results.

Furthermore, since a $2\pi$ phase difference between the maximum and minimum should be imprinted onto the wave front of the probe field, we can obtain $16$: $\Delta n \cdot d = [n(2\pi) - n(0)]d = \Delta l \cdot \lambda$, where $n$ is the refractive index, $d$ is the thickness of the SQDs medium, and $\Delta l = 1$ because we discuss, for simplicity, the generation of LG modes in the CPO system. As it is easy to obtain $n \approx 1 + \frac{\pi}{2} \chi$, which can result in $d = \frac{\text{Re}\chi^{(1)}}{\lambda}$, where $\lambda \approx \frac{2\pi c h}{E} = 530$ nm, and $E = 2.34$ eV is the interband transition energy $14$. Thus we obtain the thickness of the solid medium $d \approx 71$ μm. With the thickness $d$, we can calculate the transmission probability $T(= e^{-\alpha d})$, where the absorption coefficient $\alpha = 2\pi \text{Im}\chi^{(1)}/\lambda$. In most cases, the transmission probability of the probe field in our system is above 80% within a $2\pi$ phase regime.

Secondly comes the discussion on amplitude modulation. Beams with phase singularities such as LG beams have been generated at optical wavelengths by the diffraction of a plane wave using specially synthesized holograms or masks. Among the holograms or masks, the forked binary grating is one kind of practical application $17, 18$. The boundaries between the transparent and opaque regions are given by a special instance of the general formula, which can be expressed in polar coordinates $(r, \phi)$ as $18$:

$$p \frac{\phi}{\pi} = n + \frac{2r}{D} \cos \phi,$$  

(7)

where $p$ is the topological charge, $D$ is the period of the grating far away from the forked center and $n = 0, \pm 1, \pm 2, \ldots$. Thus, to generate LG modes by means of amplitude modulation, the aforementioned images with azimuthal intensity distributions should be replaced by images with forked binary patterns $16$. As a result, in the dark fringes of the forked pump field (i.e., $\Omega_c = 0$), with the same detunings as those in phase modulation (i.e., $\Delta_c = 0.05, \Delta_s = 0$), we can obtain $\text{Im}\chi^{(1)} = 0.4 \Rightarrow T = e^{-\alpha d} = e^{-3.3 \times 10^2} \rightarrow 0$, where $d = 71$ μm is obtained through the previous calculation. Nevertheless, in the bright fringes of the forked pump field where the strong pump field is introduced, the outcome is $T \rightarrow 1$, which demonstrates the probe field can transmit with low absorption in the system. Thus a tunable forked binary amplitude grating is formed in the SQDs system for the probe field. The transmission grating is made by first photographing a computer-generated pattern and has been successfully demonstrated with high efficiency to generate beams with phase singularities at optical wavelengths $18$. Thus the theoretical results obtained above indicate the possibility of realizing amplitude modulation of spatial light in SQDs medium.
IV CONCLUSION

In conclusion, the generation of LG modes from Gaussian beam has been demonstrated in SQDs due to CPO. Both phase modulation with low absorption of probe field and amplitude modulation with high efficiency can be achieved, indicating that the system can be used as effective optically addressed SLMs. Thus the SQDs system is a promising candidate to realize spatial light modulation in a practical way and may be useful for applications in holographic technology and quantum information processing.

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