EXPANSION LENSING (I) - A NEW PARADIGM FOR COSMOLOGICAL LUMINOSITY-ANGULAR DISTANCES RELATION

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ABSTRACT

The Einstein's General Relativity (GR) theory and the Friedmann-Lemaître-Robertson-Walker (FLRW) metric define the main equations that rule the history and future of the Universe. The standard model of cosmology collects this knowledge along with some other elements as cosmological distances. The link between the theory and observations is commonly performed through some basic cosmological distances definitions as transverse comoving distance $D_M$, luminosity distance $D_L$, angular diameter distance $D_A$ and its relation $D_L = D_A(1 + z)^2$. In this paper, Expansion Lensing (EL) is presented. EL is a novel paradigm revealing a new relationship between $D_L$ and $D_A$ in an expanding universe given by $D_L = D_A(1 + z)$. EL is demonstrated separately from the angular distance definition, from the FLRW metric and from the source point of view. On the other hand, a new surface brightness-luminosity relation —different from the one proposed by Tolman— is derived from Cosmic Microwave Background (CMB) equations, providing another independent derivation of the new paradigm. As a consequence, previous cosmological methods and results entrusting on luminosity as observational data must be reviewed. The expansion rate and the relative densities of the dark components of the Universe, as dark matter and dark energy, should be revised within the new paradigm.

Keywords: Cosmology: theory · Galaxies: distances and redshifts · cosmological parameters · cosmic background radiation · dark matter · dark energy

1 Introduction

During the 20th century were established the foundations of modern cosmology. The field equations of General Relativity (GR) were formulated by Einstein [1915], and its application to the Universe by Friedmann [1922], Lemaître [1927, 1931], Robertson [1933], Walker [1937] led to FLRW model. FLRW model describes the solutions to Einstein’s field equations for an expanding homogeneous and isotropic universe whose scale factor varies with time. According to GR and FLRW equations, the evolution and fate of the Universe depends on the nature of different density components, i.e., radiation, matter, curvature and dark energy. Contemporaneously to these achievements, observational evidences of the universe expansion were found by Hubble [Hubble [1929]: the correlation between redshifts and distances for extragalactic sources was considered the major evidence of the universe expansion.

Different cosmological tests were proposed to probe whether the Universe is expanding or remains static. A conclusive test for universe expansion is the time dilation of Type Ia supernovae light curves that was suggested by Wilson [1939]. The results obtained by Leibundgut et al. [1996] and Goldhaber et al. [2001] on this test, strongly supports the cosmological expansion and argues against alternative explanations. In another test, Tolman (1930, 1934) predicted that in an expanding universe, the surface brightness of a receding source with redshift $z$ will be dimmed by $\sim (1 + z)^{-4}$. Consequently to Tolman’s prediction, the equation $D_L = D_A(1 + z)^2$ was established between luminosity distance $D_L$ and angular diameter distance $D_A$, relation commonly known as Etherington distance-duality (Ellis [2007]). On the
other hand, the discovery of the Cosmic Microwave Background (CMB) by Wilson & Penzias [1967] supports strongly the universe expansion, and theoretical CMB considerations apparently agrees to above Etherington’s and Tolman’s relations.

In this paper Expansion Lensing (EL), a new paradigm for cosmological luminosity-angular distances relation is presented. EL revises the flux dilution for receding light sources within the FLRW metric —i.e., in the context of an expanding universe—. As a consequence of this inspection, a new luminosity-angular distances relation is deduced from FLRW metric given by $D_L = D_A(1 + z)$. In addition, a compatible surface-brightness relation —different from the one predicted by Tolman— is obtained from CMB equations. Expansion Lensing affects deeply to cosmology and the conclusions of many previous studies derived from luminosity distance should be revised.

The rest of the paper is organized as follows: in section 2 some basic distance definitions of the standard model are reviewed. The expansion lensing paradigm is unveiled in section 2.3 providing new relevant cosmological distance equations. Finally, the conclusions are presented in section 4.

2 Standard model luminosity-angular distances relation

2.1 Cosmological distances

The Friedmann-Lemaitre-Robertson-Walker (FLRW) metric is a solution of Einstein’s field equation of General Relativity describing a homogeneous and isotropic expanding universe given by

$$-c^2 d\tau^2 = -c^2 dt^2 + a(t)^2 \left[ \frac{dr^2}{1 - k r^2} + r^2 d\Omega^2 \right]$$

being

$$d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$$

where $k$ describes the curvature while $a(t)$ is the scale factor accounting for the universe expansion. There is different distance ladders defined on FLRW metric and related to observable measurements. Let one to provide a brief summary of some distances and its relation to cosmological models described by the normalized densities $\Omega_M, \Omega_r, \Omega_\Lambda, \Omega_k$ for matter, radiation, cosmological constant and curvature respectively (Hogg [1999]). The first Friedmann equation can be expressed from the Hubble parameter $H$ at any time, and the Hubble constant $H_0$ today as

$$\frac{\dot{a}(t)^2}{a(t)^2} = H^2 = H_0^2 E(z)^2$$

where

$$E(z) = \sqrt{\Omega_K(1 + z)^2 + \Omega_\Lambda + \Omega_M(1 + z)^3 + \Omega_r(1 + z)^4}$$

By integrating Eq. 1 along with Eq. 3 one can obtain the line of sight comoving distance $D_C$ as

$$D_C = D_H \int_0^z \frac{dz'}{E(z')}$$

where $D_H = c/H_0 = 3000h^{-1} Mpc$ is the Hubble distance. From the same equations one can get the transverse comoving distance $D_M$ as

$$D_M = \begin{cases} 
D_H \frac{1}{\sqrt{\Omega_k}} \sinh[\sqrt{\Omega_k} D_C/D_H] & \text{for } \Omega_k > 0 \\
D_c & \text{for } \Omega_k = 0 \\
D_H \frac{1}{\sqrt{|\Omega_k|}} \sin[\sqrt{|\Omega_k|} D_C/D_H] & \text{for } \Omega_k < 0 
\end{cases}$$
With respect to observable quantities, the *angular diameter distance* $D_A$ is defined as the ratio between the object physical size $S$ and its angular size $\theta$

$$D_A = \frac{S}{\theta}$$  \hfill (7)

The *angular diameter distance* is related to the *transverse comoving distance* by

$$D_M = D_A (1 + z)$$  \hfill (8)

where $z$ is the redshift. On the other hand, the *luminosity distance* defines the relation between the bolometric flux energy $f$ received at earth from an object, to its bolometric luminosity $L$ by means of

$$f = \frac{L}{4\pi D_L^2}$$  \hfill (9)

or finding $D_L$

$$D_L = \sqrt{\frac{L}{4\pi f}}$$  \hfill (10)

The relation between $D_L$ and $D_M$ within the *standard view* is given by

$$D_L = D_M (1 + z)$$  \hfill (11)

and taking into account Eq. 8

$$D_L^2 = D_A^2 (1 + z)^2$$  \hfill (12)

There are four $(1+z)$ factors affecting to flux energy diminution (Fig. 1). Two come from the elongation of the initial distance $D_A$ by a factor of $(1 + z)$ due to universe expansion. It is assumed that such elongation dilutes the luminosity by $D_A^2 (1 + z)^2$ according to the inverse square law. Another factor comes from the time dilation due to universe expansion that reduces the photon emission/reception rate by $(1 + z)^{-1}$. The last factor comes from the cosmological wavelength redshift that decrease the energy of photons by $(1 + z)^{-1}$. Therefore, a relevant relation is established between the *angular diameter distance* and the *luminosity distance* in the *standard view* as

$$D_L = D_A (1 + z)^2$$  \hfill (13)

Eq. 13 is commonly known as Etherington distance-duality relation.

### 2.2 Cosmic Microwave Background

According to the standard cosmology [Peebles 1993, Weinberg et al. 2008], about 300,000 years after the Big Bang, the Universe was formed by a soup of protons, electrons and photons. When the temperature of the Universe fell down to 3000K, electrons were linked to protons to form hydrogen atoms. At this moment, the decoupling was produced and the Universe became transparent to radiation since the scattering between free electrons and photons dropped drastically. At this epoch, radiation and matter were in thermal equilibrium and the released radiation formed a perfect back body. The Cosmic Microwave Radiation detected nowadays has the same black body feature, and thus it is assumed to be the relic of this radiation cooled and redshifted due to expansion.

A key question to support this theory is whether the black body spectrum shape can be maintained uniquely by the effect of the expansion or it is required some thermalization process. Let $N d\nu$ be the number of photons emitted at decoupling at temperature $T$ with photon frequency between $\nu$ and $\nu + d\nu$ given by
Figure 1: Standard view luminosity-angular distances relation: Angular diameter distance \( D_A \), comoving distance \( D_C \) and luminosity distance \( D_L \) for a flat universe. \( D_A \) is the distance at emission, \( D_C \) is the distance at reception and \( D_L \) account for the distance elongation due to universe expansion \( \sim (1+z) \), time dilation and wavelength redshifting \( \sim (1+z) \). The relation \( D_L = D_A(1+z)^2 \) can be deduced from the figure.

![Diagram of distances](image)

\[
N d\nu = \frac{8\pi V}{c^3} \frac{\nu^2}{e^{\frac{h\nu}{kT}} - 1} d\nu
\]  

(14)

and let \( N' d\nu' \) be the number of photons measured today at temperature \( T' \) with photon frequency between \( \nu' \) and \( \nu' + d\nu' \) given by

\[
N' d\nu' = \frac{8\pi V'}{c^3} \frac{\nu'^2}{e^{\frac{h\nu'}{kT'}} - 1} d\nu'
\]  

(15)

Dividing Eq. 14 by Eq. 15 and substituting the relationships \( V'/V = (1+z)^3 \), \( \nu = (1+z)\nu' \), \( d\nu = (1+z)d\nu' \) and \( T = (1+z)T' \), all \( 1+z \) factors are cancelled out obtaining

\[
N d\nu = N' d\nu'
\]  

(16)

Thus, the number of photons emitted at decoupling at temperature \( T \) with photon frequencies between \( \nu \) and \( \nu + d\nu \), corresponds to the number of photons measured today at temperature \( T' \) with photon frequencies between \( \nu' \) and \( \nu' + d\nu' \). Thus, it is plausible the CMB to be the relic of the universe black body radiation at early states.

2.3 Standard Tolman surface brightness derived from CMB (st-\( \mu \))

In this section the common misleading derivation of the surface brightness from CMB is shown. Let \( l \) be the CMB bolometric energy emitted per unit time per unit surface. According to the Stephan-Boltzmann law one has

\[
l = \sigma T_{em}^4
\]  

(17)
where \( \sigma \) is the Stephan-Boltzmann constant and \( T_{em} \) the blackbody temperature at emission. Let \( \mu \) be the observed CMB bolometric energy per unit surface. According to the Stephan-Boltzmann law one has

\[
\mu = \sigma T_{em}^4
\]  

(18)

where \( T_{obs} \) is the blackbody temperature at reception.

Dividing both quantities,

\[
\frac{\mu}{l} = \frac{T_{obs}^4}{T_{em}^4}
\]

(19)

Since the relation between the temperature at observation and emission in the expanding universe is

\[
T_{em} = T_{obs}(1 + z)
\]

(20)

Equation (19) becomes

\[
\frac{\mu}{l} = (1 + z)^{-4}
\]

(21)

that apparently agrees with Tolman’s surface brightness-luminosity.

In section 3.4 a more adequate surface brightness-luminosity relation that take into account the differential surface elements is derived from the same CMB equations.

3 Expansion Lensing: A new paradigm for the luminosity-angular distances relation

The main support for an expanding universe comes from the observed redshift of extragalactic sources. The wavelength of light is stretched out by the universe expansion while traveling from galaxies to the earth, displacing the wavelength to the red. While much attention has been paid to the effect on the light produced by the expansion in the radial direction —driving to the concept of cosmological redshift—, a relevant observational property on the traversal direction remains unnoticed. Let one to unveil this feature.

3.1 Expansion Lensing demonstration from the angular distance definition

Let us set the origin of coordinates at the observer O, and consider a extended cosmological object (galaxy) initially located at angular distance \( D_A \) from O (Fig. 2). The angular distance is defined in cosmology as

\[
D_A = \frac{S}{\theta}
\]

(22)

where \( S \) corresponds to the size of the object and \( \theta \) the angle subtended by the object. Note that \( D_A \) is defined identically for static and expanding universes. Since \( D_A \) corresponds to the distance at emission its value is the same for both universes, so does \( S \) and therefore \( \theta \). Thus, since \( \theta \) is the same for static and expanding universes, the objects are observed as they were at time of emission. The apparent image size (i.e., subtended angle) remains unaltered to the observer from the emission to reception in spite of the expansion. The unique effect of expansion is the scaling of the light rays or light cone. It produces a flux dilution of \((1 + z)\) due to time dilation of elongated light rays and another \((1 + z)\) factor due to wavelength redshifting. The luminosity distance \( D_L \) accounts for all the factors that contribute to reduce the flux \((f)\) received from the emitted luminosity \((L)\). Thus, for a static universe we have

\[
f = \frac{L}{4\pi D_L^2} = \frac{L}{4\pi D_A^2}
\]

(23)
and for a expanding universe

$$f = \frac{L}{4\pi D_L^2} = \frac{L}{4\pi D_A^2(1+z)^2}$$

(24)

Note that the dispersion by the inverse square law is produced at time of emission, i.e. $\propto D_A^{-2}$. Once emitted, the light rays maintain their direction towards the observer and the unique dilution come from their elongation and wavelength redshifting as explained above, but there is no additional flux dispersion given by $(1+z)^{-2}$ as assume the standard view.

### 3.2 Expansion Lensing demonstration from FLRW metric

The cosmological principle in an expanding universe requires that the effect of the expansion on light rays be homogeneous and isotropic both on radial and traversal directions. Consequently, the scale factor $a(t)$ of FLRW metric—responsible of the universe expansion— affects to both the radial and traversal directions (Eq. 1). The best way to assess the luminosity distance $D_L$ is to examine the metric for which it is defined. Let us to focus on *FLRW* metric (Eq. 1). According to General Relativity, light rays follow null geodesics where $d\tau^2 = 0$. Substituting this value in Eq. 25, light rays follow the equation

$$c^2 dt^2 = a(t)^2 \left[ \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right]$$

(25)

Let us set the origin of coordinates at the observer O and consider a extended cosmological object (galaxy) initially located at a distance $D_A$ from O at time of emission $t_e$ (Fig. 2). Assume for a moment we live in a static universe, i.e., $a(t)=1 \forall t$. According to Eq. 25 the light rays that will arrive to O in the future are those pointing initially towards the observer at time of emission $t_e$. These rays will maintain the same direction up to arriving to the observer, i.e., $\Omega = cte$, $d\Omega = 0$ in Eq. 25 (leaving apart other possible cosmological effects as gravitational lensing or astrophysical events). Substituting $d\Omega = 0$ in Eq. 25, the light rays that will arrive to the observer meet

$$c^2 dt^2 = a(t)^2 \frac{dr^2}{1-kr^2}$$

(26)

The same reasoning can also be applied to an expanding universe responding to any scaling function $a(t)$. The light rays of the galaxy that will arrive in the future to the observer are those pointing initially to the observer at time of emission. Those rays will maintain the same direction all the time in spite of expansion due to the scaling of the light cone. The unique effect of expansion is the scaling (i.e., preserving the angles) of the light rays lengths with respect to the static universe (Fig. 2). Thus, integrating Eq. 26 from $t_e = 0$ we have
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\[ \int_0^t \frac{cdt}{a(t)} = \int_0^{D_M} \frac{dr}{\sqrt{1 - kr^2}} \]  

(27)

where

\[ \delta t' = \int_0^t dt/a(t) \]  

(28)

is the time dilation. Therefore, once integrated, \( D_M \) depends on the time dilation and wavelength redshift, and accounts for all the luminosity dilution taking the role of \( D_L \). Consequently, we can express the expansion lensing equations as

\[ D_L = D_M \]  

(29)

and since

\[ D_M = D_A \frac{\lambda_0}{\lambda_e} = D_A (1 + z) \]  

(30)

we have

\[ D_L = D_A (1 + z) \]  

(31)

The phenomenon can also be explained from the light-cone produced by the bundle of rays from the galaxy to the observer (Fig. 2). The scaling of the light-cone produced by the universe expansion, preserves the light-cone shape and hence the angle \( \theta \) subtended by the imaging of the galaxy. The angular size conservation of the image while universe expands, maintains the image focused on the observer preventing from flux dispersion and hence from flux dilution. The focusing of the images towards the observer, produces an increment on the flux received with respect to what is commonly expected. Thus, the received flux by the observer resembles the one received in a non-expanding universe, but only dimmed by the time dilation and wavelength redshifting due to the path elongation (Eq. 32).

\[ f = \frac{L}{4\pi D_L^2} = \frac{L}{4\pi D_M^2} = \frac{L}{4\pi D_A^2 (1 + z)^2} \]  

(32)

We can see that the inverse square law is now preserved in the FLRW geometry. Note that the results presented are derived for any scale factor function \( a(t) \). Therefore, taking into account Eq. 3 and Eq. 4, the results are valid for any feasible combination of \( \Omega_M, \Omega_r, \Omega_A, \Omega_k \).

### 3.3 Expansion lensing deduction from the source point of view

The luminosity-angular distance relation is usually explained from the source point of view according to Fig. 3 (left). A point source stated at the origin emits a bundle of rays towards the detector located at a distance \( D_A \). As long as the distance between the source and the detector increases due to expansion, the flux is diluted according to the inverse square law (in addition to the effect of time dilation and wavelength redshift). The key point of the fault of this view comes from considering a bundle of rays departing from a point source rather than from an extended source. With this view, the source traversal image information is missing in spite it is also affected by the expansion. While it has not impact in static environments as shows the Gauss’s theorem, the situation changes in an expanding universe. The effect of working with extended sources in an expanding universe is clarified in Fig. 3 (right). It shows how the apparent image becomes focused to the observer in spite of the expansion. The focusing of the image flux towards the observer while travelling to earth, prevent any additional luminosity dilution beyond that due to the initial distance between the source and the target at time of emission (\( \sim 1/D_A^2 \)). Therefore, contrary to what is expected by the current standard view, the expansion of the universe does not dilute the flux due to photon dispersion by distance elongation. Nevertheless, the universe expansion still decreases the flux by \( \sim (1 + z)^{-2} \) due to time dilation and wavelength redshifting.
3.4 Expansion Lensing surface brightness derived from CMB

In section 2.3 the Tolman surface brightness-luminosity relation was derived from CMB considerations. Let one to revise this derivation taking into account the differential surface elements at each epoch, i.e., $dS$ (at emission) and $dS'$ (at reception).

Let $dl/dS$ be the CMB bolometric energy emitted per unit time per unit surface. According to the Stephan-Boltzmann law one has

$$\frac{dl}{dS} = \sigma T_{em}^4$$  \hspace{1cm} (33)

where $\sigma$ is the Stephan-Boltzmann constant and $T_{em}$ the blackbody temperature at emission. Let $d\mu/dS'$ be the observed CMB bolometric energy per unit surface. According to the Stephan-Boltzmann law one has

$$\frac{d\mu}{dS'} = \sigma T_{obs}^4$$  \hspace{1cm} (34)

where $T_{obs}$ is the blackbody temperature at reception.

Dividing both quantities,

$$\frac{d\mu}{dl} = \frac{T_{obs}^4}{T_{em}^4} \frac{dS'}{dS}$$  \hspace{1cm} (35)

The relation between the differential surface elements at observation and emission is given by

$$dS' = dS(1 + z)^2$$  \hspace{1cm} (36)

Taking into account the relation between temperatures (Eq. 20) and differential surface elements (Eq. 36), Equation 35 becomes
\[ \frac{d\mu}{dl} = (1 + z)^{-2} \] (37)

that provides the expansion lensing surface-brightness/luminosity relation.

### 3.5 Expansion Lensing deduced from surface brightness reasoning

Expansion lensing surface-brightness/luminosity relation can also be easily deduced reasoning with different sources. Assume two sources of luminosity per surface unit \( l \) in a static universe at distances \( d_1 \) and \( d_2 \). Since \( z = 0 \) in a static universe we have

\[ \mu = l(1 + z)^n = l \] (38)

whatever standard view \((n = -4)\) or expansion lensing \((n = -2)\) equation is considered. In fact, it occurs in local universe.

Now assume the sources depicted above correspond to the position of the galaxy at emission \( d_1 \) and at reception \( d_2 \) in an expanding universe. The unique drop in surface-brightness due to expansion with respect to the static case comes from one \((1 + z)^{-1}\) factor due to time-dilation and another \((1 + z)^{-1}\) factor coming from wavelength redshifting obtaining

\[ \frac{\mu}{l} = (1 + z)^{-2} \] (39)

that corresponds to the expansion lensing surface-brightness relation.

Therefore, to be a consistent theory, the standard view should explain the origin of the additional unexplained \((1 + z)^{-2}\) factor it predicts.

### 3.6 Agreement with Etherington equation

The goal of Etherington [1933] paper was to relate distances computed from apparent size \( \Delta' \) (i.e., \( D_A \)) to distances computed from apparent brightness \( \Delta \) (i.e., \( D_L \)), assuming the existence of a redshift from the source to the observer. The relation obtained by Etherington [1933]-Eq.23, is reproduced here

\[ \Delta = (1 + \frac{\delta \lambda}{\lambda}) \Delta' = (1 + z) \Delta' \] (40)

That is, the original Etherington equation agrees perfectly with the expansion lensing luminosity-angular distance relation (Eq. 31). Even more, among the conclusions of this equation, Etherington wrote:

“(ii) For spherical objects of the same absolute size and brightness, moving variously at varying distances from the observer, the ratio of the apparent diameter to the square root of the apparent brightness is proportional to \( \delta \lambda/\lambda \).”

That is, for objects of the same absolute size \( S \) and brightness \( L \), the ratio of the apparent diameter \( \theta \) to the square root of apparent brightness \( \sqrt{\frac{L}{4\pi}} \) is proportional to \((1+z)\). Thus, from Eq. [7] and Eq. [10] one has

\[ \frac{D_L}{D_A} = \frac{\sqrt{\frac{L}{4\pi}}}{S} \frac{\theta}{\sqrt{\frac{L}{4\pi}}} = (1 + z) \] (41)

that again agrees with expansion lensing.

Unfortunately, Etherington equation has long been misinterpreted, an unnecessary \((1+z)\) factor has been added to his equation (probably by confusion with the mainstream), in a formula commonly known as Etherington distance-duality relation (Eq. [13]).
4 Conclusions

The standard model compile the current knowledge of the Universe based on Einstein equations and FLRW metric, along with the definition of basic cosmological distances related by fundamental laws as the distance-duality and the Tolman’s surface brightness relations.

In this paper, the expansion lensing paradigm is presented. Expansion lensing unveil an unready effect of extragalactic images in an expanding universe: the flux focusing. The flux focusing produces an amplification on the received flux from extragalactic sources by a factor \((1 + z)^2\) with respect to what is expected by the standard model. The phenomenon is demonstrated independently from the angular distance definition, the FLRW metric, CMB formulas, from the source point of view and from surface-brightness considerations. As a consequence, new equations are established to reflect the cosmological luminosity-angular distances relationship.

Expansion lensing affects deeply to cosmological equations based on luminosity measurements. A revision of luminosity cosmological probes within the new paradigm is required, since it may affects to some cosmological parameters as the Hubble constant, dark energy and dark matter content.

Empirical evidences of the new paradigm are shown in [De Vicente-Albendea 2022].

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