On Self Sustained Photonic Globes\textsuperscript{a)}

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In this paper we consider a classical treatment of a very dense collection of photons forming a self-sustained globe under its own gravitational influence. We call this a “photonic globe”. We show that such a dense photonic globe will have a radius closely corresponding to the Schwarzschild radius. Thus lending substance to the conjecture that the region within the Schwarzschild radius of a black hole contains only pure radiation.

As an application example, we consider the case of a very large photonic globe whose radius corresponds to the radius of the universe and containing radiation of the frequency of the microwave background (160.2 GHz). It so turns out that such a photonic globe has an average density which closely corresponds to the observed average density of our universe.

I. INTRODUCTION: PROBLEM STATEMENT

The possible existence of a self-sustained radiation existing as a spherical or near spherical region was first conjectured by J.A. Wheeler \cite{1}, who gave it the name Geon. Subsequently many researchers \cite{2-6} have investigated this possibility and have studied two types of Geons — gravitational and electromagnetic. All the studies made were to find out if such structures can exist and be consistent with the field equations of general relativity. The conclusion arrived at was that such structures are essentially unstable and at best are not of long duration. In addition, Teo \cite{7} has investigated the possibility of photons forming a stable orbit under the influence of a black hole and has concluded that such stable orbits are possible at a fixed distance which is exactly equal to 1.5 times the Schwarzschild radius of the black hole. Such structures were then called "photonic spheres", which actually is a thin layer of photons (like a thin balloon) at 1.5 times the distance of the Schwarzschild radius (SR).

We in this paper consider the possibility of photons existing under its own gravitational field, and investigate under what conditions such a collection of photons occupying a finite region including the origin (photonic globe) can exist and be stable.

In order to do so we make the following assumptions:

(i) The treatment of the problem is classical
(ii) The velocity of each photon is the speed of light: \( c \)
(iii) We assume that each photon is gravitationally attracted by another photon, according to Newton’s Gravitational law and behaves for this purpose as having a “mass” proportional to \( h\nu/c^2 \) (\( h \) and \( \nu \) being Planck’s constant, frequency of the photon resp.)
(iv) In this brief study it will be assumed that the photons all have the same frequency, and that the photons form a self sustained globe of radius \( R \), the number density of the photons \( \sigma_v(r) \), will be assumed to be a function of \( r \) alone, \( r \) being the radial distance from the centre 0.

It will be shown that by imposing conditions of stability of the system one can show that the radius \( R \) of the photonic globe corresponds to the Schwarzschild radius, an expression for the number density \( \sigma_v(r) \) is also obtained.

II. BRIEF DETAILS OF CALCULATION

We herewith assume that a photonic globe, of radius \( R \), consisting solely of photons under its own gravitational field and centered about the origin is extant. We define \( M(r) \) to be the “mass” of an imaginary sphere of radius \( r \), \( 0 < r \leq R \), then

\[
M(r) = \int_0^r 4\pi r^2 \sigma_v(r)(h\nu/c^2)dr
\]

(1)

Now consider a point P at a distance \( r \) from the centre, O, and surrounded by a volume element (in polar coordinates), the mass \( m_\Delta = \sigma_v(r)(h\nu/c^2)^2 dr \sin\theta d\theta d\phi \)

The gravitational force, \( F \), on this small element \( m_\Delta \) is given by:

\[
F = \frac{Gm_\Delta M(r)}{r^2}
\]

(2)

Now imagine the photons at P are moving inward with a velocity \( c \) and making an acute angle \( \psi \), with respect to the radial line drawn from P to the origin O. Then the tangential velocity, \( v_t \), of the photons in this volume element will be \( v_t = c \sin \psi \).

The centrifugal force (c.f.) on this volume element will be given by \( c.f. = m_\Delta v_t^2 / r \), but \( v_t^2 = c^2 \sin^2 \psi \); substituting the average value of \( \sin^2 \psi \) over 0 to \( \pi \) as 1/2, we see that \( v_t^2 = c^2 / 2 \), hence the centrifugal force on the photons in the volume element will be:

\[ F \]

\[ \frac{Gm_\Delta M(r)}{r^2} \]
$$c.f. = \frac{m\Delta c^2}{2r}$$ \hspace{2cm} (3)

The condition of stability requires that $F = c.f$, hence by equating (2) and (3), we have:

$$M(r) = \frac{c^2}{2G} r$$ \hspace{2cm} (4)

Substituting for $M(r)$, from (1), we have:

$$\int_0^r r^2 \sigma_\nu(r) \, dr = \left( \frac{c^4}{8\pi G \hbar \nu} \right) r$$ \hspace{2cm} (5)

Since eq. (5) must be true for all r, we can see that this is not possible unless the number density, $\sigma_\nu(r)$, is given by the following expression:

$$\sigma_\nu(r) = \left( \frac{c^4}{8\pi G \hbar \nu} \right) \frac{1}{r^2}$$ \hspace{2cm} (6)

It may be noted that though the number density seems to become infinite as $r$ tends to zero, the number of photons in a very small sphere of radius $\epsilon$ will be $\sigma_\nu(\epsilon) \frac{4}{3} \pi \epsilon^3$ which is finite.

Now if we substitute $r = R$, and noting that $M(R) = M$, the mass of the photonic globe, we have

$$R = \frac{2GM}{c^2}$$ \hspace{2cm} (7)

It may be noted that the rhs of (7) is nothing but the Schwarzschild radius.

III. ON THE PROPERTIES OF THE PHOTONIC GLOBE

From the above calculation, it so turns out that the radius $R$ of a photonic globe, eq.(7), is nothing but the Schwarzschild radius an expression for which radius was derived by Schwarzschild in 1916, for a spherically symmetric body by using equations of general relativity for regions outside this radius. We have derived the same expression for the Schwarzschild radius by using completely different arguments for regions inside this radius by considering a collection of photons and using some assumptions detailed above. It is well known that the event horizon for a black hole occurs at a radius equal to the Schwarzschild radius. The physics within this radius is not well known and can only be guessed at. Also when eq(4) written as $r = 2GM(r)/c^2$, is a valid equation for any radius $r$ centered around the origin, $M(r)$ being the mass of the imaginary sphere of this radius, we see that $r$ is the Schwarzschild radius for this sphere, so every point P at an arbitrary distance $r$ in the globe lies on an “event horizon”. The photon number density $\sigma_\nu(r)$, as a function of $r$, of such a photonic globe is given by eq.(6).

The above calculation seems to lead to an interesting conjecture: That the region within the Schwarzschild radius of a black hole consists of pure radiation, a stable photonic globe, sustained within itself by its own “gravitational” field.

IV. APPLICATION REGARDING THE DENSITY OF THE UNIVERSE

In this section, we will consider a very large photonic globe which contains photons corresponding to 160.2 GHz, the frequency of the background radiation and assume the radius of the globe to be the radius of the universe. If we start from the expression, Eq(6), for the number density of photons at frequency $\nu$, and substitute $\nu = 160.2$ GHz, where $\nu_B$ is the frequency of the background radiation of the universe (which corresponds to a wave length $\lambda = 0.1872$ cms), using the value of the Gravitational constant $G = 6.673 \times 10^{-8}$ cgs(cm-gram-second ) units and Planck’s constant $\hbar = 6.626 \times 10^{-27}$ erg-sec (cgs units ) , and the value of the velocity of light $c = 3.0 \times 10^{10}$ cms per sec; we see that at this frequency $\nu_B$ we can write

$$\sigma_\nu(r) = 4.55 \times 10^{62} \frac{1}{r^2}$$ \hspace{2cm} (8)

which we denote for convenience as $\sigma_\nu_B(r) = \beta/r^2$, where $\beta = 4.55 \times 10^{62}$.

Now to calculate the total number of Photons $N_R$ inside a sphere of radius $R$, we need to integrate the above and obtain

$$N_R = \int_0^R 4\pi r^2 \sigma_\nu_B(r) \, dr$$

$$= 4\pi \beta R$$ \hspace{2cm} (9)

The total energy, $E_{\text{total}}$, of radiation is then $E_{\text{total}} = N_R \hbar \nu_B$. The equivalent mass will be $M = E_{\text{total}}/c^2$. Hence the average “mass density” inside this sphere will be $\rho_{\text{av}} = M/(\frac{4}{3} \pi R^3)$. That is

$$\rho_{\text{av}} = \frac{3\beta \hbar \nu_B}{R^2 c^2}$$ \hspace{2cm} (10)

Now if we take $R$ as the the radius of the visible universe $R=13.5$ billion light years, ie. $R = 1.227 \times 10^{28}$ cms.

Substituting these values for $R$, $\beta$, $\nu_B$, and $c$, we get an average mass density for the universe as $\rho_{\text{av}} = 9.809 \times 10^{-30}$ grams/cc. Which is very close to the actual
estimated mass density, by the WMAP,[8], as may be gathered from the following quotation in the NASA, article [8]:

“WMAP determined that the universe is flat, from which it follows that the mean energy density in the universe is equal to the critical density (within a 0.5% margin of error). This is equivalent to a mass density of \(9.9 \times 10^{-30}\) g/cm\(^3\), which is equivalent to only 5.9 protons per cubic meter.”

It is well known that only about 4 percent of the mass of the universe consists of Baryonic matter. So, if we, (for a stating approximation), adopt the hypothesis that the universe is a photonic globe containing photons of frequency of the CMB, then the above calculations give the correct average density, and obviously the correct Mass for the universe. However, if one considers the number density of photons of frequency \(\nu_B\), it turns out to be grossly over estimated, (as can be easily calculated from the above equations) from the actual value of 400 photons per cc (near earth). So here we have a situation where the mass and mass-density are correct but the number of photons estimated are far too much. So where have all the extra photons gone? It could then be conjectured that some unknown physical process has converted all this extra radiation into dark matter and dark energy, thus keeping the total energy (mass) unchanged. Only further experimentation and research can resolve such issues.

V. CONCLUSION

In this paper, we have considered the possibility of the existence of a stable a selfsustained photonic globe and have arrived at the following: (i) that such a globe must have its radius equal to the Schwarzchild radius and (ii) if we consider a photonic globe which contains photons of frequency equal to 160.2 GHZ and a radius equal to the radius of the universe then the average mass (energy) density of such a photonic globe is very close to the latest estimate of the average mass density of the universe by NASAs WMAP team[7].

VI. REFERENCES

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