RADIATIVE CORRECTIONS TO $e^+e^- \rightarrow \bar{f}f^*$

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The past ten years of physics with $e^+e^-$ colliding experiments at LEP and SLAC have shown the success of these experiments on not only impressively proving the theoretical predictions of the Standard Model (SM), but also to help provide stringent bounds on physics beyond the SM. With this experience in mind, there appear two equally fascinating opportunities for studying fermion-pair production processes at a future Linear Collider (LC).

On the one hand, performing high precision measurements to the SM, for example, when running with high luminosity at the Z boson resonance, could be a quick and feasible enterprise in order to pin down the symmetry breaking mechanism of the electroweak sector through indirectly determining the masses of a light SM or MSSM Higgs boson or supersymmetric particles via virtual corrections. On the other hand, looking for such particles in direct production or other ‘New Physics’ effects at energies between, for example, roughly 500 and 800 GeV will naturally be the main motivation to pursue the challenging endeavor of building and utilizing such a unique facility. These two scenarios for the LC shall be sketched here, with particular emphasis on the semi-analytical program ZFITTER for fermion-pair production in comparison with numerical programs like TOPAZ0, KK2f, and others.

1 Introduction

A future Linear Collider (LC) running with high luminosities at energies up to 500 . . . 800 GeV will demand a dedicated effort not only on its experimental realization, but also from the theoretical side on predicting observables under experimentally realistic conditions with an unprecedented precision. For this, theory has applied quantum field theory successfully to calculate quantum corrections in a perturbative approach, in order to accurately predict or confirm high energy observables in the past, which will be even more demanding for the special case of the LC with its high resolution power. The proof that this can be done on the solid ground of gauge theories as renormalizable, unitarity-conserving quantum field theories for all three microscopically observed forces in nature – the electromagnetic, weak, and strong interaction – was rewarded just recently, underlining the validity and practical applicability of our modern-day theoretical particle physics description. In practice, this

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also resulted in such pioneering work – just to show some examples – as giving limits on mass differences in the electroweak (EW) sector of the SM, resulting in indirect top mass bounds from LEP, calculating radiative corrections to the weak vector boson masses, shown at LEP, or providing first computational tools indispensable for today’s practically involved, Feynman-diagrammatic calculations using computer algebra techniques \[5\].

The goal of this paper in the context of an \(e^+e^-\) LC now is to briefly outline the, in this respect still interesting and fruitful physics potential of the ‘classical’ fermion-pair production processes, \(e^+e^- \to f\bar{f}\), at these energies and luminosities \[3\], but also the theoretical implications and resulting necessities stressed for an update of existing numerical programs, especially for the case of the semi-analytical code ZFITTER \[6,7,8,9,10,11,12,13\].

It is absolutely clear that the focus at such a machine will be on physics at these high energies, for which it is envisaged, but also a quick, high-luminosity run at the \(Z\) boson resonance could be an interesting extra option (Giga-Z), thus substantially increasing the experimental precision on \(Z\) lineshape observables and the SM parameters, on which they are sensitive, with a manageable extra effort \[14\][15][16]. Both situations – the Giga-Z option and the high-energy run – shall be discussed now for the fermion-pair production case, with special emphasis on what this means for the program ZFITTER \[18,8\] in comparison with other available 2-fermion codes \[19,20,21,22,23\].

1.1 High precision measurements to the SM and MSSM

First, the Giga-Z option: In \[14\] it was demonstrated that with a factor of 100 or so higher statistics than at LEP running on the \(Z\) boson resonance \[1\] – corresponding to a luminosity of \(\mathcal{L} \sim 5 \cdot 10^{33} \text{cm}^{-2}\text{s}^{-1}\) or roughly \(10^9\) hadronic \(Z\) boson decays after just a few months of running – especially fermion-pair production asymmetries like \(A_{LR}\) or the polarized \(b\bar{b}\) forward-backward asymmetry \(A_{FB}^b\) could be measured with very high precision when using one or both beams polarized. This latter condition together with good \(b\)-tagging techniques and the collected experiences at LEP and SLD should help to keep the systematic errors under control. The implication of this from the theoretical side on extracted SM parameters, like e.g. on the \(W\) boson mass, \(M_W\), or the effective weak mixing angle, \(\sin^2\theta_{eff}\), was nicely illustrated in \[16\], with expected experimental accuracies of \(\Delta M_W = 6\text{MeV}\) or \(\Delta \sin^2\theta_{eff} = 4 \times 10^{-5}\) at the Giga-Z. This is to be compared with the presently achievable total experimental errors by the end of LEP \[14\] of \(\Delta M_W = 40\text{MeV}\) or \(\Delta \sin^2\theta_{eff} = 1.8 \times 10^{-4}\). Due to loop corrections, \(M_W\) and \(\sin^2\theta_{eff}\) are sensitive to the mass of a light Higgs boson, \(M_h\), the top quark mass, \(m_t\), and in the supersymmetric (susy) case, also on the \(\text{susy}\) mass scale, \(M_{\text{susy}}\). These much improved experimental values for \(M_W\) and \(\sin^2\theta_{eff}\) thus allow at the Giga-Z together with the precise knowledge of \(m_t\) an indirect determination of the mass of a light Higgs boson in the SM at the 10% level or strong consistency checks on the SM/MSSM values of \(M_W\) and \(\sin^2\theta_{eff}\) with \(m_t\) and \(\text{susy}\) masses as input parameters \[16\].

1.2 Virtual Corrections and New Physics Phenomena

Probably one of the most fascinating applications of fermion-pair production processes at higher energies is then the search for ‘New Physics Phenomena’ (NPP), i.e. observed effects not described by the SM \[24\]. This is of course quite actively pursued already at existing

\[a\text{In comparison to SLD at SLAC, it would be even a factor of roughly 2000.}\]
\begin{enumerate}
\item $e^+e^-$ facilities, giving quite stringent bounds on masses and couplings of exchanged ‘exotic’ particles or minimal interaction scales of NPP. With a future LC, however, reaching much higher energies at or nearly at the TeV scale and using high luminosities, there is the justified hope of touching this ‘beyond the SM’ domain of particle physics and really uncovering NPP. Examples of such investigations\cite{25,26,27} are e.g. setting lower limits on four-fermion contact interaction scales or on masses and couplings of extra heavy neutral or charged gauge bosons, \textit{Z}' and \textit{W}', of susy particles in \textit{R} parity violating supersymmetric models, or for interaction-unifying models (GUTs), searches for excited leptons, leptoquarks, preons etc., or looking at effects of spin-2 boson exchanges on e.g. angular cross section distributions in string-inspired, low-scale quantum gravity models. With a LC, the so far checked energy region for NPP from LEP/SLD can be extended from typically \textit{O}(few TeV) up to several tenths of TeV at a LC. For a summary of these activities also refer to\cite{24,26}.

Another quite interesting application in the context of the Giga-Z option could be looking for lepton flavor number violating \textit{Z} decays like \textit{Z} $\rightarrow \mu\tau$, \textit{e\tau}, or \textit{e\mu} when heavy neutrinos are exchanged in virtual corrections (Dirac or Majorana type). The estimated branching ratios in the case of \textit{Z} $\rightarrow$ \textit{e\tau} or \textit{\mu\tau} could be large enough in some models to be observable at the Giga-Z. There is a vast literature on this topic, and also some preliminary studies for the LC were presented at this workshop\cite{28}.

\section{ZFITTER and other programs at the \textit{Z} boson resonance}

The semi-analytical program ZFITTER\cite{6,7,8} calculates observables for fermion-pair production like total cross sections and asymmetries and contains analytical formulae with the exact one-loop radiative corrections and important higher order effects for SM applications, or alternatively, allowing a model-independent approach e.g. for ‘New Physics’ searches (see Section\cite{12}). In the program, the analytical formulae only need to be numerically integrated over the final state invariant mass squared, \(m_f^2\), making the code numerically fast and stable with the inclusion of a limited number of experimentally relevant, kinematical cuts to the final state. After a recent update of the ZFITTER code for the special case of combined cuts on energies, acollinearity, and acceptance angle of leptonic final states\cite{7,8} and comparisons with other numerical programs\cite{19,21,23}, the situation for LEP 1 energies can be stated as quite satisfactory\cite{12,14,15,16}: total cross sections and forward-backward asymmetries are now treated better than at the per mil level for both cut options – the invariant mass and the acollinearity cut – around the \textit{Z} boson resonance, and even better than \(10^{-4}\) at the \textit{Z} peak itself, illustrated in Table\cite{12} below (from Table 7 in\cite{10} and Table 2 in\cite{12,13}).

\section{EW and QED corrections at LEP and LC energies - a comparison}

As already mentioned above, the semi-analytical program ZFITTER was originally developed for SM predictions of cross sections and asymmetries at LEP 1 energies. Observables like total cross sections and asymmetries can be calculated in an \textit{effective Born description}, as is done in the ZFITTER approach: EW and QCD corrections are described as effective couplings in \textit{effective Born observables} which are convoluted with the photonic corrections as flux functions. Higher order QED effects can then partly be described by resumming finite soft and virtual corrections (\textit{soft-photon exponentiation}). This was illustrated e.g. in\cite{6,7,8,10,12,13}.
while at $\theta_{acol} < 10^\circ$ and $M_Z - 3 \ M_Z - 1.8 \ M_Z \ M_Z + 1.8 \ M_Z + 3$

| $\theta_{acc} = 0^\circ$ | $M_Z - 3$ | $M_Z - 1.8$ | $M_Z$ | $M_Z + 1.8$ | $M_Z + 3$ |
|--------------------------|-----------|-------------|-------|-------------|-----------|
| **TOPAZO**               | 0.21932   | 0.46287     | 1.44795 | 0.67725     | 0.39366   |
|                          | -7.16     | -4.43       | -0.07  | +2.49       | +3.17     |
| **ZFITTER**              | 0.21928   | 0.46284     | 1.44780 | 0.67721     | 0.39360   |
|                          | -7.16     | -4.40       | -0.03  | +2.60       | +3.27     |

| $\theta_{acc} = 0^\circ$ | $M_Z - 3$ | $M_Z - 1.8$ | $M_Z$ | $M_Z + 1.8$ | $M_Z + 3$ |
|--------------------------|-----------|-------------|-------|-------------|-----------|
| **TOPAZO**               | -0.28450  | -0.16914    | 0.00033 | 0.11512     | 0.16107   |
|                          | -0.28158  | -0.16665    | 0.00088 | 0.11385     | 0.15936   |
|                          | +2.92     | +2.49       | +0.55  | -1.27       | -1.71     |
| **ZFITTER**              | -0.28497  | -0.16936    | 0.00024 | 0.11496     | 0.16083   |
|                          | -0.28222  | -0.16710    | 0.00083 | 0.11392     | 0.15926   |
|                          | +2.75     | +2.27       | +0.60  | -1.03       | -1.56     |

Table 1: A comparison of predictions from ZFITTER v.6.11 and TOPAZO v.4.4 for muonic cross sections and forward-backward asymmetries around the $Z$ peak. First row is without initial-final state interference, second row with, third row the relative effect of that interference in per mil [10,12,13].

While at LEP 1 the EW and QCD corrections can in general be considered as small in comparison to the QED Bremsstrahlung, this observation is not necessarily valid anymore at higher energies, where EW and QED corrections can grow to comparable magnitudes. In order to underline this, we compared in Fig. 1 for muon-pair production cross sections $\sigma_T$ as an illustrative example the effect of virtual $ZZ$ and $WW$ box corrections as important EW corrections with corrections from the QED initial-final state interference.

In Fig. 1a, we switched off the $ZZ$ and $WW$ box corrections in order to visualize a positive effect. Correspondingly, the QED interference was switched on and off in the right-hand plot (Fig. 1b). The net effect of these EW box corrections grows with increasing c.m. energy roughly up to per cent level at LEP 2 energies, with the QED interference corrections being slightly larger depending on the cut applied. At LC energies, however, the EW contributions can even surpass the QED interference contribution by roughly a factor of 2, while the effect from the QED interference approaches a more or less constant value of 2 to 3%. For this comparison the ZFITTER code, version v.6.22 [8], was run ‘blindly’ as it stands, i.e. without considering possible extra effects above the $tt$ threshold due to the top quark mass. For an estimate of the EW situation at energies up to 1 TeV also consult for example [30].

4 Photonic Corrections above the $Z$ resonance

We want to focus now on the QED radiative corrections at higher energies. The status of the description of QED radiative corrections in the ZFITTER code in comparison with other programs now at LEP 2 and higher energies below the $tt$ threshold was examined in [12,13]: For the option with invariant mass cut the deviation of the codes ZFITTER...
v.6.22 \cite{8} and TOPAZ v.4.4 \cite{21} is typically not more than few per mil at LEP 2 energies and is under control with respect to the experimentally demanded accuracy \cite{29}. For the acollinearity cut branch, the deviation of the codes increases to few per cent, even with stringent hard-photon cuts \cite{11,12,13}. This is depicted in Fig. 2 for forward-backward asymmetries $A_{FB}$ of muon-pairs with different cuts (see also Fig. 16b. of \cite{10}).

In Fig. 3 we present an earlier analysis \cite{31} for the branch with acollinearity cut up to energies of 300 GeV for codes ALIBABA v.1 \cite{19} and ZFITTER v.4.5 \cite{18}, together with an update of the comparison in \cite{10,11} for codes ALIBABA v.2 \cite{19} and ZFITTER v.6.22 \cite{8}. The per cent level discrepancy of the codes from LEP 2 energies onwards has not changed much since then, although it seems now clear that the large deviation is due to higher order QED corrections, contained in the code ALIBABA v.1 and v.2, but not in ZFITTER for the acollinearity cut branch \cite{12,13}. The reason for these much larger deviations compared to the invariant mass cut is believed to be due to hard-photon effects from a radiative return to the $Z$ boson, not completely suppressed through an acollinearity cut and surviving at higher energies. This is still under investigation.

We have now extended the comparison of total cross section predictions by codes ZFITTER v.6.22, TOPAZ v.4.4, and KK2f v.4.12 \cite{23} up to typical LC energies of 500 to 800 GeV for the invariant mass cut option. This is shown in Figures \ref{fig:fig1} and \ref{fig:fig2}.

The general result of our analysis is that the deviation of the cross section predictions by the three codes is not more than 5 per mil for the complete energy range for the TOPAZ–ZFITTER comparison. This observation also holds for the case of initial state QED Bremsstrahlung (ISR) alone when comparing code KK2f with ZFITTER, applying sufficiently

\footnote{For this, a sufficiently large invariant mass cut preventing the radiative return to the $Z$ boson resonance is applied.}
strong invariant mass cuts and taking into account different higher order corrections (Fig. 4). Including QED initial-final state interference (IFI), our comparisons with KK2f delivered a maximal deviation of roughly 1% (Fig. 5).

In detail, this meant: The numerical precision of TOPAZ0 and ZFITTER was better than $10^{-5}$ everywhere, while the accuracy of the Monte Carlo (MC) event generator KK2f was necessarily restricted due to limited CPU time: Calculating ISR with an accuracy of at least $10^{-3}$ required samples of 100000 events for each energy point. When including the (resummed) IFI, we had to use smaller samples of 30000 events, resulting in a precision of only roughly $2 \times 10^{-3}$. For ISR only, the typical CPU time per MC data point on e.g. an HP-UX 9000 workstation was about 25 minutes, increasing to roughly 100 minutes if IFI is added for the event samples stated above. In comparison, TOPAZ0 calculated one cross section value in a few minutes, while ZFITTER with its semi-analytical approach calculated all 32 cross section values for one cut in a few seconds. On the other hand, when interested in more complex setups, i.e. calculating multi-differential observables, using a wider variety of cuts, or including extra higher order effects to the initial-final state interference, which ZFITTER cannot or only partly provide, the numerical programs TOPAZ0, or respectively KK2f, clearly have their advantages.

We compared the effect of ISR solely (Fig. 4) or of ISR together with IFI (Fig. 5) for three different cut values, $\sqrt{s'/s} > 0.6441$, 0.8397, and 0.9164, in the case of the TOPAZ0–ZFITTER comparison, and $\sqrt{s'/s} > 0.9164$ when comparing with KK2f with $s'$ defined here as the invariant mass squared of the $\gamma$ or $Z$ propagator after ISR, which is equal to the final

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The cut values correspond approximately to a relatively strong cut on the maximal final state leptons' acollinearity angle of 25°, 10°, and 5° respectively.
state invariant mass squared including the emitted final state photons. For this, final state radiation (FSR) was treated in form of a global correction factor. Alternatively, cutting on the final state invariant mass squared, $m_{f\bar{f}}^2$, after FSR, though it slightly worsened the good agreement at LEP 1 energies, it did not change the overall agreement in Figures 4 and 5 substantially. For a recent discussion on this issue, defining kinematical cuts with radiative corrections for the experimental and computational situation – e.g. when including mixed QED and QCD contributions from photonic and gluonic emission in the case of hadronic final states – please consult [29,32]. In particular, at LEP 2 energies the predictions of the codes lie well inside the estimated experimental accuracies of e.g. $\Delta \sigma_{\mu\mu} \approx 1.2\%$, $\Delta \sigma_{had} \approx 0.5\%$ for sufficiently strong cuts [29,32].

Except where otherwise stated, we used the default settings of the programs, thus taking into account the $O(\alpha^2)$ photonic initial state corrections with the leading logarithmic $O(\alpha^3)$ corrections together with the exact $O(\alpha)$ IFI contribution. In ZFITTER and TOPAZO, the $O(\alpha^2)$ corrections are complete. All three programs have installed resummed higher order corrections to ISR, exponentiating the finite soft and virtual photonic corrections. In contrast to codes ZFITTER and TOPAZO, KK2f also possesses a procedure to exponentiate IFI corrections with its newly implemented coherent exclusive exponentiation (CEEX) [33,34,35,23].

To be more precise, CEEX does not include $O(\alpha^3)$ contributions up to now, while the EEX option in KK2f does not contain the second order, subleading $O(\alpha^2 L)$ corrections, so

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\footnote{The Yennie-Frautschi-Suura (EEX) prescription was used, resumming soft and virtual photonic contributions to all orders, correctly removing all infrared divergences [23].}

\footnote{The initial-final state interference contribution in KK2f is only available with the CEEX option; for EEX it is neglected.}
both options are complementary to each other when interested in estimating these higher order effects for the initial state Bremsstrahlung with KK2f. In both cases KK2f lacks the NNLL $O(\alpha^2 L^0)$ terms which, however, are estimated to be of the order $10^{-5}$ and so do not play a visible role in this comparison [23].

In Fig. 4, we compared the predictions by KK2f with those by ZFITTER for ISR alone, first for the $EEX$ option with the LL $O(\alpha^3 L^3)$ and $O(\alpha^2 L^2)$ corrections (ZFITTER flag values: FOT2 = 3, 5). Then we used the $CEEX$ option for KK2f and compared the $O(\alpha^2)$ results (ZFITTER: FOT2 = 2). The cross section ratios and the maximally 5 per mil deviation of the codes do not change considerably with values calculated with an uncertainty of $0.4 \times 10^{-3}$. At the $Z$ peak, and $1 \times 10^{-3}$ overall.

In Fig. 5 we give the cross section ratios, now with the IFI contribution, for $CEEX O(\alpha^1)$ and $CEEX O(\alpha^2)$. There is roughly a 2 per mil shift of the central values – always having in mind calculational uncertainties of $2 \times 10^{-3}$ – when going to the $O(\alpha^2)$ calculation, but they stay inside the overall $\pm 1\%$ margin.

Other higher order corrections due to initial state pair creation, implemented in codes
Figure 5: Cross section ratios for muon-pair production with different \( s' \) cuts for codes ZFITTER v.6.22, TOPAZ0 v.4.4, KK2f v.4.12 (1999) from 60 to 800 GeV c.m. energy with initial-final state interference (INI PP: initial state pair production; LL: leading logarithmic terms).

ZFITTER and TOPAZ0, only had minor effects on the cross section ratios at higher energies (see Fig. 4 and 5). It must be emphasized again that all three programs were run as they are, i.e. without considering perhaps necessary, later updates of the codes for electroweak corrections above the \( t \bar{t} \) threshold.

Another important, in ZFITTER recently updated contribution are initial state pair corrections if the Bremsstrahlung photon dissociates into a light fermion pair [36]: TOPAZ0 and ZFITTER versions contain the \( O(\alpha^2) \) leptonic and hadronic initial state pairs and a realization for simultaneous exponentiation of the photonic and pair radiators [37]. According to [23], initial state pair corrections are not included in the KK2f code. Since ZFITTER v.6.20, also the exact \( O(\alpha^3) \) and the LL \( O(\alpha^4) \) initial state pair corrections can be taken into account through convolution of the photonic and pair flux functions [38]. The effect of the pair corrections is e.g. with strong cuts roughly 2.5 per mil at the \( Z \) peak, slightly decreases to approximately 2 per mil at LEP 2 energies, and is not more than roughly 1 per mil at 500 to 800 GeV c.m. energy. In Fig. 4 and 5, switching on the pair corrections for different cuts, does not change the level of agreement between ZFITTER v.6.22 and TOPAZ0 v.4.4 substantially. One
interesting feature at lower energies between roughly 100 and 150 GeV – just where the $Z$ radiative return is not prevented anymore by the applied cuts – is the fact that the several per mil deviation of the two codes there disappears when the pair corrections are switched off. From Fig. 4 and 5 it can also be seen that such deviations can also be prevented by a sufficiently large $s'/s > 0.9$ if initial state pair corrections shall be included.

The inclusion of 4-fermion final states in this context, e.g. from final state pair creation, with their rather large, per cent level corrections at LEP 2 and higher energies [33] naturally constitutes one of the next tasks which have to be approached for an update of the codes for the LC. Especially, the definition of background and signal diagrams in the hadronic case together with kinematical cuts – experimentally and theoretically – will be one of the major obstacles to overcome [33]. For a general summary of the present status of different available codes at higher energies on 2-fermion, 4-fermion, WW etc. physics see also [40].

5 ZFITTER above the $t\bar{t}$ threshold - a brief note

The ZFITTER code contains the complete one-loop virtual EW corrections to the $(\gamma, Z)f\bar{f}$ vertex from v.5.12 onwards [1,2,3]. While at LEP 1, the off-resonant $WW$ box corrections and the $\gamma b\bar{b}$ vertex corrections are negligible compared to the $Z b\bar{b}$ vertex, they become more and more important with increasing c.m. energy, especially at LEP 2 in the case of the $WW$ and $ZZ$ box corrections, leading to maximal effects between 2 and 4%, taking into account the $s$ dependency of the vertices and the angular dependency of the box corrections [3,4]. At higher energies, all virtual corrections will start to become equally relevant introducing large gauge cancellations. Corrections due to logarithmic and double logarithmic “Sudakov-type” contributions from collinear and soft gauge boson exchange could lead to measurable 1% or larger effects at a LC for $\sigma_T(b\bar{b})$ and $R_b$ at 500 GeV or higher, but only to per mil level modifications for different $b\bar{b}$ asymmetries [13].

The possibility of top quark pair production at a LC then was naturally one of the key fields of interest and discussions at this workshop [4]. For the ZFITTER code, this is one of the still missing, but urgently needed branches in order to make quick and reliable estimates for $t\bar{t}$ cross sections with radiative corrections, at least for perturbative predictions sufficiently above the $t\bar{t}$ threshold. A crucial role, of course, also plays the correct inclusion of mass effects at the available energies. While massive SM and MSSM calculations to $e^+ e^- \rightarrow t\bar{t}$ with virtual and real QED [17], EW [18], and QCD [19] corrections are already available, work still has to be done concerning a description in the context of EW form factors or the inclusion of hard QED and QCD corrections for the massive case [50].

6 Conclusions

Except for a possible and quite useful quick later run at a LC in the Giga-Z mode, the era of high precision measurements at the $Z$ boson resonance appears to be coming to an end. These high precision measurements have not only impressively confirmed the SM predictions at the up-to-now available energies, but also led to many constraints on non-SM physics, ranging from narrowing estimates on the mass of a light Higgs boson in the SM or MSSM to lower limits on new interaction scales. This is irrevocably also connected to the success of numerical codes for fermion-pair production like ZFITTER, TOPAZO, KORALZ/KK2f, and others, predicting experimentally measured quantities like cross sections and asymmetries with
steadily increasing theoretical precision in order to extract the interesting physics information from experimental data.

While the codes appear in ‘good shape’ in the $Z$ boson resonance region with predictions at the per mil level or better, still a lot has to be done for the higher energy domain at a $LC$ ranging e.g. from roughly 350 to 800 GeV, if one takes into account the expected experimental precisions with high luminosities, polarized beams, and improved experimental analysis techniques. Having this in mind, we have listed below some examples of updates we expect to become necessary for the semi-analytical program $ZFITTER$ used at LC energies:

- For $A_{FB}$: LLA $O(\alpha^2)$ corrections from initial state pair production;
- Exponentiation of the initial-final state QED interference and reexamination of the initial state exponentiation for different cuts;
- A $ZFITTER$–$GENTLE$ merger: final state pair corrections and other $4f$ contributions (e.g. neutral current processes NC08, NC32);
- Further comparisons between $TOPAZ0$–$ZFITTER$–$KK2f$ etc., especially for LEP 2, and then LC precisions;
- $tt$ production with real and virtual radiative corrections including final state masses;
- Further options like inclusion of beamstrahlung effects etc.

With the combined effort of the different programming groups and constant interaction with the experimental community, this seems to be a tedious, but solvable ‘request list’ of tasks with the rewarding promise of delivering e.g. stringent mass bounds on a light Higgs boson, with a possible distinction between the SM and MSSM case, or giving hints for the mentioned, other extensions to the SM – or for even more ‘exotic, unasked-for’ new physics.

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