From Chiral Mean Field to Walecka Mean Field 
And Kaon Condensation

G.E. Brown\textsuperscript{a} and Mannque Rho\textsuperscript{b,c}

\textsuperscript{a} Department of Physics, State University of New York, Stony Brook, N.Y. 11794, USA.
\textsuperscript{b} Service de Physique Théorique, CEA Saclay 91191 Gif-sur-Yvette Cedex, France
\textsuperscript{c} Institute for Nuclear Theory, University of Washington Seattle, WA 98195, U.S.A.

ABSTRACT

A unified treatment of normal nuclear matter described by Walecka's mean field theory and kaon condensed matter described by chiral perturbation theory is proposed in terms of mean fields of an effective chiral Lagrangian. The BR scaling is found to play a key role in making the link between the ground state properties of nuclear matter and the fluctuation into the strangeness-flavor direction. A simple prediction for kaon condensation is presented.

\textsuperscript{*}Supported by the Department of Energy under Grant No. DE-FG02-88ER 40388
Kaon-nuclear physics is probably one of the most exciting new directions in nuclear physics. The issues involved are strangeness productions in heavy-ion collisions and kaon condensation in compact star matter. In discussing kaon-nuclear (or in general pseudo-Goldstone boson) interactions appropriate for kaon condensation, chiral Lagrangians are found to be useful in describing the fluctuation in the strangeness direction. While chiral perturbation theory ($\chi$PT) is believed to be the most efficient way to implement chiral symmetry of QCD at low energy, there is a subtlety in applying it to nuclear or compact star matter which has not yet been satisfactorily addressed in the literature. This has to do with the consistency in the description of the ground state of the dense matter and of the fluctuation around that background defined by the ground state. In treatments to date, there is no consistency between the two sectors. For instance, the ground state – nuclear matter – is successfully described by Walecka mean-field theory with the parameters of an apparently non-chiral Lagrangian determined from nuclear matter properties while the kaon condensation phenomenon, the physics of which is lodged in an effective action (or potential), is described by low-order chiral perturbation theory with the parameters of a chiral Lagrangian determined mostly by free-space data. Communication between the ground-state sector and the fluctuation sector has been glaringly missing. What would be needed is a formalism that describes both sectors from a given chiral Lagrangian with common parameters operative in both sectors simultaneously. An attractive possibility is that a lump of nuclear matter arises as a nontopological soliton in the form of “chiral liquid” as suggested by Lynn around which fluctuations in various flavor directions could be described. Such a scheme would incorporate chiral symmetry in a self-consistent way in both sectors. Unfortunately such a strategy has not been yet formulated into a workable scheme.

In this Letter, we supply the missing link at mean field level with the help of the BR scaling introduced by us sometime ago.

Linking chiral and Walecka mean fields

We begin by restating the result of the in a form suitable for making contact with Walecka mean-field theory for nuclear matter. The starting point of Ref. is a chiral Lagrangian for large $N_c$ (where $N_c$ is the number of colors) implemented with the scale anomaly of QCD, written in terms of pseudo-Goldstone fields $U$, the scalar field $\chi$ of the scale anomaly and hidden-gauge fields $V_\mu$. With such a Lagrangian, baryons arise as topological solitons (skyrmions). Suppose we embed this chiral Lagrangian into a medium characterized by density $\rho$ and call it $\mathcal{L}_\chi$. We assume that $\mathcal{L}_\chi$ incorporates physics above the chiral scale $\Lambda_\chi \sim 1 \text{ GeV}$ into the parameters and counterterms of the Lagrangian and in a potential $V(U,\chi,V_\mu)$ of the Coleman-Weinberg type. The basic premise of the is that at a given
background density $\rho$, the chiral Lagrangian takes formally the same form dictated by QCD symmetries as in matter-free space with however the basic parameters of the Lagrangian, $f_\pi^*$ (pion decay constant), $g$ (hidden local symmetry constant) and the “vacuum condensate” $\langle \chi \rangle^*$ etc. determined at a given density by a Coleman-Weinberg-type mechanism. This leads at mean field level to the BR scaling

$$\frac{m_N^*}{m_N} \approx \frac{m_V^*}{m_V} \approx \frac{m_S^*}{m_S} \approx \cdots \approx \frac{f_\pi^*}{f_\pi}$$

(1)

where the subscripts $N$, $V$ and $\sigma$ stand, respectively, for the nucleon, the vector mesons ($\rho$, $\omega$) and the scalar meson (to be defined below). In doing this in a medium, we of course lose manifest Lorentz invariance but this is not a serious matter since we can work with Lorentz covariance as in heavy-quark QCD by using the four-velocity $v_\mu$. As explained in [1, 7], the scalar field $\chi$ that enters into the medium-dependent constants is the low-energy quarkish component with the gluonic component having been integrated out. A similar idea of separating the trace anomaly into a quarkish component and a gluonic component has recently been proposed by Furnstahl, Tang and Serot [8]. It should be noted that $L_\chi$ with (1) is to be used in tree order for describing fluctuations around the background state. In arriving at this result, we have used the notion of the baryons as chiral solitons and the consequences, i.e. low-energy theorems of hidden gauge symmetry [9] such as the KSRF relation.

But how is the ground state to be described in this framework? To answer this question, we approach the same effective theory from a different angle. For this, we shall exploit a recent development [10, 11] that indicates that the above description using a bosonic Lagrangian is equivalent at long wavelength to an effective Lagrangian that contains baryons explicitly as matter fields. The key idea can be illustrated with the scale-invariant part of the chiral Lagrangian written in terms of the Dirac nucleon $\psi$, the pion (or kaon) $U \equiv \xi^2$ and the scalar $\chi$ field:

$$L_0 = \bar{N} \left[ i\gamma^\mu D_\mu + ig_A \gamma^\mu \gamma_5 \Delta_\mu - m_N - g_\chi \chi \right] N + \frac{1}{2} (\partial_\mu \chi)^2 - \frac{1}{2} m_\chi^2 \chi^2 + \cdots$$

(2)

where the covariant derivatives $\Delta_\mu$ and $D_\mu$ are given by $\Delta_\mu = \frac{1}{2} \left\{ \xi^\dagger, \partial_\mu \xi \right\} - \frac{i}{2} \xi^\dagger (V_\mu + A_\mu) \xi + \frac{1}{2} \xi (V_\mu - A_\mu) \xi^\dagger$, $D_\mu = \partial_\mu + \Gamma_\mu$ with $\Gamma_\mu = \frac{1}{2} \left\{ \xi^\dagger, \partial_\mu \xi \right\} - \frac{i}{2} \xi^\dagger (V_\mu + A_\mu) \xi - \frac{i}{2} \xi (V_\mu - A_\mu) \xi^\dagger$, and $V_\mu$ and $A_\mu$ represent, respectively, the external vector and axial vector fields introduced specifically here for reasons that will be apparent shortly. The ellipsis in (2) denotes terms containing only meson fields and other meson-baryon couplings that do not concern us for the moment. As it stands, this Lagrangian can be interpreted as describing fluctuations around the “background” defined by the medium with the scalar field shifted around that...
background. We shall show later that the scalar excitation carries the scaled mass. For the moment, we do not worry about this matter and take the scalar excitation to be sufficiently massive so that we can integrate it out. We do this in heavy-baryon formalism useful for $\chi$PT applied to many-nucleon systems \[12\]:

$$
\delta_\chi \mathcal{L} = \frac{g_\chi^2}{2m_\chi^2} (\bar{B}B)^2 + \frac{g_\chi^2}{2m_\chi^2 m_N^2} \bar{B}B \Gamma \frac{1}{m_N} B \\
- \frac{g_\chi^2}{2m_\chi^2} (\bar{B}B) \partial^2 (\bar{B}B) + \cdots
$$

where $B$ is the heavy-baryon field and

$$
\Gamma_{\frac{1}{m_N}} = S^{\mu\nu} \Gamma_{\mu\nu} + 2g_A \{v \cdot \Delta, S \cdot D\} - D^2 + (v \cdot D)^2 + g_A^2 (v \cdot \Delta)^2 + \cdots
$$

Equation (3) tells us two things. On the one hand, the first four-baryon interaction term is one of the four-Fermi interactions expected in chiral Lagrangians \[13\]. In mean field, it is equivalent to the scalar mean-field potential in Walecka theory \[14\]. We should emphasize that the scalar field that enters here is a chiral singlet, not the fourth component of the scalar quartet of the linear sigma model. On the other hand, the second term of (3) gives a medium correction to the single-particle axial-charge operator $A_0$ and magnetic moment operator $\mu$ in nuclei \[12\] that shifts the nucleon mass $m_N$ to $m_N^*$ (here and in what follows, we will affix $\star$ for quantities defined in the “background” field which in our case is characterized by density $\rho$)

$$
m_N^* = \frac{m_N^*}{m_N} = (1 + \eta \frac{\rho}{\rho_0})^{-1}
$$

giving a correction of the form

$$
\frac{\delta_\chi A_0^0}{A_0^0} = \frac{\delta_\chi \mu}{\mu} = \eta \frac{\rho}{\rho_0}
$$

with $\eta = g_\chi^2 \rho_0 / (m_N m_\chi^2)$. The two phenomena cited above represent the same physics through Ward identities. We should note however that while the correction (6) for the axial charge operator turns out to remain significant as shown in \[15\], the corresponding correction for the magnetic moment is nearly completely canceled by the “back-flow” correction imposed by Galilean invariance, i.e. the effect of the Fermi-liquid parameter $F_1$.

The other mean field that figures importantly in Walecka theory is the $\omega$ meson field, $\omega_\mu$, which is also a chiral singlet. In chiral Lagrangian, this couples to the baryon as

$$
\mathcal{L}_{\omega NN} = g_\omega \bar{\psi} \gamma_\mu \psi \omega^\mu \simeq g_\omega \bar{B}v_\mu B \omega^\mu.
$$
Again integrating out the $\omega$ field, one gets the additional piece in the Lagrangian

$$\delta \omega L = -\frac{g_\omega^2}{2m_\omega^2} (\bar{B}v_\mu B)^2 + \cdots$$  \hspace{1cm} (8)

Note that (8) is yet another chirally symmetric four-Fermi interaction allowed in chiral Lagrangians. Thus Walecka’s repulsive vector mean field can be identified by a four-Fermi chiral Lagrangian involving $v_\mu$.

The $\rho$ and $a_1$ mesons could be introduced similarly using hidden local symmetry and generate isospin-dependent mean fields which however do not figure in symmetric nuclear matter.

We should stress that for mean-field calculations, there is no need to integrate out the heavy fields: One could work equally well with the mean fields of the mesons directly as in Walecka’s formulation.

The next question we must address is how the coupling constants and meson masses are defined in the background field. Let us first look at the vector mean field. Invoking the KSRF relation $m_{\rho_\pi}^2 = 2f_\pi^2 g^2$ where $g$ is the hidden gauge coupling and $V = \rho, \omega$ and $g_\omega/3 = g_{\rho} = g/2$, we can write the mean-field potential given by (8) as

$$V_N = \frac{9}{8f_\pi^2} \rho.$$  \hspace{1cm} (9)

Analysis in Walecka theory indicates that at nuclear matter density $\rho = \rho_0$, the vector potential (9) has the value

$$V_N(\rho_0) \approx (0.6)^{-1} \frac{9\rho_0}{8f_\pi^2}$$  \hspace{1cm} (10)

with $f_\pi = 93$ MeV. Thus we see that

$$\frac{f_\pi^2}{f_\pi^2} \approx 0.6.$$  \hspace{1cm} (11)

One may understand this scaling from the Gell-Mann-Oakes-Renner (GMOR) relations. Assuming that the GMOR relation holds in medium, we have\(^{\text{#2}}\)

$$\frac{f_\pi^2}{f_\pi} \approx \frac{m_{\pi}^2}{m_\pi^2} \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle}. $$  \hspace{1cm} (12)

From Feynman-Hellman theorem, we have\(^{[16]}\)

$$\frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} \approx 1 - \frac{\Sigma_{\pi N\rho}}{f_\pi^2 m_\pi^2} + \cdots$$  \hspace{1cm} (13)

\(^{#2}\)In some approaches, e.g., chiral perturbation theory in medium, a distinction is made between the “time component” and “space component” of $f_\pi$. Here there is no such distinction; the fact that we are dealing, in medium, with Lorenz covariance rather than invariance would lead to an additional (noninvariant) term to (12).
where $\Sigma_{\pi N} = 45$ MeV is the $\pi N$ sigma term. The pion mass is known to change slightly as density increases, so putting into Eq.(12) the empirical value obtained from $\pi$-mesic atoms [17], $m^{*}_{\pi}(\rho_0)/m_\pi \approx 1.05$, one obtains [11]. Since the coupling $g_\omega$ related to the hidden gauge coupling $g$ does not scale at mean field [11], this result immediately implies that at $\rho \approx \rho_0$

$$\frac{m^{*}_\omega^2}{m^2_\omega} = \frac{m^{*}_V^2}{m^2_V} = \frac{f^*_\pi^2}{f^2_\pi} \approx 0.6.$$  

Let us now turn to the scalar meson sector. The role of a low-mass scalar meson figuring importantly in nuclear phenomenology is much less clear in the context of chiral perturbation theory. The scalar in the linear sigma model, namely the fourth component of $O(4)$ group, while relevant at the critical point of chiral phase transition, is not visible in low-density regime. The scalar associated with the trace anomaly is much too massive, $\sim 2$ GeV, to be relevant without substantial modification. Our proposal is to take the relevant (chiral singlet) scalar field to interpolate $2\pi$, $4\pi$ etc. excitations with an effective low mass $m^{*}_\sigma$ (where we have renamed the light (quarkish) $\chi$ field by $\sigma$ following the convention). We have support for this idea from the phenomenological meson-exchange model of Durso, Kim and Wambach [18]. As reviewed in [1], the correlation of the two pions (and other multipion correlations) responsible for the effective scalar exchange is chiefly accomplished by crossed-channel $\rho$-exchange. Inclusion of the density dependence of the $\rho$-meson mass leads to important modifications in the attraction [18]. From the $\pi\pi \rightarrow N\bar{N}$ helicity amplitudes $f^J_{=0}(t)$ supplied by the authors of [18], we can track the downward movement, in mass, of the (distributed) scalar strength as the density increases. These $f^J_{=0}(t)$ helicity amplitudes show a relatively sharp resonance at $m^{*}_s \approx 500 - 600$ MeV for $\rho \approx \rho_0$. It seems clear that the strength in the effective scalar degree of freedom from correlated pion exchange originates in the $\gtrsim 900$ MeV range at zero density, and that the exchanged scalar, which is fictitious at $\rho = 0$, materializes as real resonance by $\rho \sim \rho_0$.

What is determined in the phenomenology of Walecka’s mean-field theory is the ratio $(g_\sigma^2/m^2_\sigma)$ in (3):

$$S_N = -\frac{g^*_\sigma^2}{m^*_\sigma^2} \rho_s$$

where $\rho_s$ is the scalar density. Numerically it is determined at $\rho = \rho_0$ to be [19]

$$g^*_\sigma^2/m^*_\sigma^2 \approx 5/m^2_\pi.$$  

To proceed, let us assume as in [1] that the nonlinear chiral Lagrangian “linearizes” at nuclear matter density. Briefly what this means is as follows. At zero density and low energies, the strong interaction is governed by a nonlinear chiral Lagrangian implemented
with the trace anomaly of QCD. As density increases or at higher energy, the scalar of the trace anomaly tends to be “dilatonic” and moves towards the Goldstone pions to form a quartet, thereby approaching an $O(4)$ symmetry. Even if the mass of the scalar is not degenerate with that of the pions, the symmetry is “mended” in the sense described by Beane and van Kolck [20], with the physics of the quartet described effectively by a linear sigma model. Note, however, that in a strict sense, the scalar will remain chiral singlet all the way to the chiral restoration point at which the scalar will merge into the multiplet of $O(4)$. (See [21] on this matter.) Assuming this, we use the fact that $g_A^* \approx 1$ at $\rho \approx \rho_0$ and the in-medium Goldberger-Treiman relation to arrive at

$$g_A^* \approx 10.$$  

Therefore (16) gives

$$m_\sigma^* \approx 600 \text{ MeV}$$

essentially the mass used in mean field calculations. This implies that

$$m_\sigma^*/m_\sigma \approx f_\pi^*/f_\pi.$$  

It is not obvious how the nucleon scaling (5) is related to the scaling of the pion decay constant. In the skyrmion description, one finds [22] $m_N^*/m_N \approx \sqrt{g_A^* f_\pi^*}$. Now up to $\rho \approx \rho_0$, in-medium loop corrections lead to $g_A^* \approx 1$, and beyond $\rho_0$, $g_A^*$ remains unscaled. Thus at mean field level, it seems reasonable to take $m_N^*/m_N \approx f_\pi^*/f_\pi$, modulo an overall constant.

**Fluctuations in the kaon sector**

Consider the S-wave $KN$ interaction relevant to $K$-nuclear interactions in symmetric nuclear matter. We shall focus on the Weinberg-Tomozawa term and the sigma term

$$\mathcal{L}_{KN} = -\frac{6i}{8f^2}(\bar{B}\gamma_0 B)K\partial_t K + \frac{\Sigma_{KN}}{f^2}(\bar{B}B)KK \equiv \mathcal{L}_\omega + \mathcal{L}_\sigma$$

where $K^T = (K^+K^0)$. The constant $f$ in (20) can be identified in free space with the pion decay constant $f_\pi$. In medium, however, it can be modified as we shall see shortly. In chiral perturbation expansion, the first term corresponds to $O(Q)$ and the second term to $O(Q^2)$. There is one more $O(Q^2)$ term proportional $\partial_t^2$ which we will discuss later.

One can associate the first term of (20) arising from integrating out the $\omega$ meson as in the baryon sector. The resulting $K^-N$ vector potential in medium can then be deduced in the same way as for $V_N$:

$$V_{K^\pm} = \pm \frac{3}{8f_\pi^2}\rho.$$  

(21)
Thus in medium, we may set \( f \approx f_\pi^* \) and obtain

\[ V_{K^\pm} = \pm \frac{1}{3} V_N. \]  

(22)

One way of understanding this result is that the constituent quark description with the \( \omega \) meson coupling to the kaon as a *matter field* (rather than a Goldstone) becomes applicable: the \( \omega \) coupling to the kaon which has one nonstrange quark is \( 1/3 \) of the \( \omega \) coupling to the nucleon which has three nonstrange quarks.

Now what about the second term of (20)?

As in the baryon sector, one may couple the scalar \( \chi \) field to the kaon field. But with the kaon considered as a pseudo-Goldstone field, the only way to couple them without using derivatives is through the symmetry breaking term, thus the appearance of the sigma term. On the other hand, if the kaon is treated as a *matter field* as suggested by its coupling to the \( \omega \)-meson field – a reasonable assumption in medium in view of the fact that *all* non-strange hadron masses other than Goldstone bosons drop – then we can use the coupling

\[ \mathcal{L}_\sigma = \frac{1}{3} 2m_K g^*_{\sigma} \overline{K} \chi \]  

(23)

where the factor \( 1/3 \) accounts for one non-strange quark in the kaon as compared with three in the nucleon. When the \( \chi \) field is integrated out as above, we will get, analogously to the nucleon case,

\[ \mathcal{L}_\sigma = 2m_K \frac{1}{3} g^2_{\sigma} \overline{B} \overline{K} \chi. \]  

(24)

Comparing with the second term of (20), we find

\[ \frac{\Sigma_{KN}}{f^2} \approx \frac{2}{3} \frac{m_K g^2_{\sigma}}{m^2_{\sigma}}. \]  

(25)

This somewhat strange relation may be understood by recalling the dual role that the kaon plays in the structure of hyperons: On the one hand, the kaon is light enough to be considered as a pseudo-Goldstone boson so the skyrmion picture with \( SU(3) \) collective coordinates applies; on the other hand, it can be considered as heavy enough so that it can be treated as a matter field (rather than a Goldstone field) as in the Callan-Klebanov model [4]. Reference [23] describes how these two descriptions can be joined smoothly in the case of the kaon and heavier mesons. It is shown there that as the mass of a pseudoscalar boson \( \Phi \) (such as the \( D \) meson) increases above the chiral scale \( \sim 1 \) GeV, the heavy-meson field can be taken to transform like a massive matter field, eventually exhibiting heavy-quark symmetry. We believe it is this dual character of the kaon that is reflected in (25) in the background where the masses of other hadrons scale down as proposed in [4]: the Goldstone boson nature of the kaon gives the left-hand side of Eq. (25) while the massive matter field
character of the kaon in medium supplies the right-hand side. The situation resembles the overlapping in the strange-quark hadron sector of the “top-down” approach starting from heavy-quark symmetry and the “bottom-up” approach starting from chiral symmetry as discussed in [23]. In the present case, the overlapping region seems to be located between $\sim \rho_0$ and $\sim 2\rho_0$ in density.

Given the $KN$ sigma term, we could determine independently what $f$ is. However since the strange-quark mass is not well-known, we cannot determine the sigma term accurately even if the $\bar{s}s$ content of the nucleon could be measured on lattice. Suppose we take it from lattice gauge calculations [24] (or from chiral perturbation estimate coming from $KN$ scattering [25]), $\Sigma_{KN} \approx 3.2m_\pi$. Then the left- and right-hand sides of (25) are equal if we set

$$f \approx f_\pi^*.$$  

(26)

This is consistent with the scaling for the $\omega$-meson exchange leading to the relation (22). It follows from (24) with $f$ replaced by $f_\pi^*$ that the scalar kaon-nuclear potential is

$$S_{K^\pm} = \frac{1}{3}S_N.$$  

(27)

**Kaon-nuclear potential**

Given Walecka mean fields for nucleons, we can now calculate the corresponding mean-field potential for $K^-$-nuclear interactions in symmetric nuclear matter. From the results obtained above, we have

$$S_{K^-} + V_{K^-} \approx \frac{1}{3}(S_N - V_N).$$  

(28)

Phenomenology in Walecka mean-field theory gives $(S_N - V_N) \lesssim -600$ MeV for $\rho = \rho_0$ [2]. This leads to the prediction that at nuclear matter density

$$S_{K^-} + V_{K^-} \lesssim -200 \text{ MeV}.$$  

(29)

This seems to be consistent with the result of the analysis in K-mesic atoms made by Friedman, Gal and Batty [26] who find attraction at $\rho \approx 0.97\rho_0$ of

$$S_{K^-} + V_{K^-} = -200 \pm 20 \text{ MeV}.$$  

(30)

**Kaon condensation**

In applying the above mean-field argument to kaon condensation in compact-star matter to chiral $O(Q^2)$, we need to make two additions to the Lagrangian [21]. The first is that the compact-star matter relevant to the problem consists of roughly 85% neutrons and
15% protons, so we need to take into account the $\rho$-meson exchange in addition to the $\omega$ exchange. The $\rho$ exchange between $K^-$ and neutrons (protons) gives repulsion (attraction), equal in magnitude to $1/3$ of the attraction from the $\omega$-exchange. The second correction has to do with an $O(Q^2)$ term in the Lagrangian proportional to $\omega_K^2$ (where $\omega_K$ is the kaon frequency) associated with “$1/m$” corrections in chiral expansion. The effect of this term is to cut down the scalar exchange given by the second term of (20) by the factor $F \simeq \left(1 - 0.37 \frac{\omega_K^2}{m_K^2}\right)$.

We are now in position to estimate the critical density for kaon condensation using the resulting mean-field theory. Let us start with Walecka mean fields for nucleons in symmetric nuclear matter. We take for illustration the following values from (10) and (15):

$$V_N(\rho = \rho_0) \approx 275 \text{ MeV}, \quad S_N(\rho = \rho_0) \approx -350 \text{ MeV}. \quad (31)$$

These are, in fact, the values calculated from the Bonn two-body potential in a relativistic Brueckner-Hartree-Fock approach \[19\] and, thus, have some grounding in a microscopic theory. As Brockmann and Machleidt point out, these are more or less central values for those used in Walecka mean-field calculations. Now at $\rho \approx \rho_0$, $F \simeq (1 - 0.37 \frac{\omega_K^2}{m_K^2}) \approx 0.86$ and so

$$S_{K^-}(\rho = \rho_0) \approx -\frac{1}{3} \times 350 \times 0.86 \text{ MeV} \approx -100 \text{ MeV}. \quad (32)$$

This is valid for $K^-$ independently of the isospin content of the matter. For symmetric nuclear matter, our scaling gives

$$V_{K^-}(\rho = \rho_0) \approx -\frac{1}{3} \times 275 \text{ MeV} \approx -92 \text{ MeV}. \quad (33)$$

Thus for $K^-$ in nuclear matter, we get $(S_{K^-} + V_{K^-}) \approx -192 \text{ MeV}$ which is consistent with the K-mesic atom data. For a matter of 85% neutrons and 15% protons appropriate in compact stellar matter, we get instead

$$V_{K^-}(\rho = \rho_0) \approx -92 \times (0.85 \times \frac{2}{3} + 0.15 \times \frac{4}{3}) \text{ MeV} \approx -71 \text{ MeV}. \quad (34)$$

At $\rho \approx \rho_0$, therefore, the attraction in the stellar matter of the given composition would be

$$S_{K^-} + V_{K^-} \approx -171 \text{ MeV}. \quad (35)$$

Normally, a linear extrapolation with density, although commonly employed in Walecka mean field calculations, is dangerous, because correlations, Pauli blocking, etc. tend to cut down attraction progressively with increasing density. However, our effective coupling constant, starting from chiral Lagrangians, is $1/f\pi^2$, which steadily increases with density, at least in linear approximation (12) - (13). Given the decrease from many-body effects, and
the increase from the decreasing order parameter $f_x^*$ of the broken chiral symmetry mode, it is simplest to assume that these effects roughly cancel each other, and that the mean fields extrapolate linearly in density. Some empirical evidence for this exists in relativistic heavy-ion experiments, as we review in the next section.

Given our estimate (35), we see that

$$\omega_{K^+}(2\rho_0) \approx 495 - 342 \approx 153 \text{ MeV.}$$

From the equation of state for dense stellar matter available in the literature (e.g., PAL [27]), we have the chemical potential for electrons

$$\mu_e(2\rho_0) \approx 173 \text{ MeV}$$

for either $K_0 = 180$ MeV or 240 MeV. Since kaon condensation occurs when $\omega_{K^+} = \mu_K = \mu_e$, we expect $\rho_c \approx 2\rho_0$. This agrees well with the $O(Q^3)$ chiral perturbation result $\rho_c \approx 2.3\rho_0$ in [28] when the BR scaling is incorporated. Note, however, that by using the above linear extrapolation in the mean fields, we have implicitly assumed a definite parametrization for BR scaling for $\rho > \rho_0$. We will now argue that there is some justification for this.

**Heavy ion processes**

To conclude, we review results from analyses of heavy ion reactions. First of all, in the $Au + Au$ collisions at 1 GeV/N studied at SIS, there is an enhancement in subthreshold $K^+$ production by a factor of $\sim 3$ due to the attractive scalar potential. Fang et al. [28] obtain the enhancement using the scalar interaction (20), leaving out the scaling in $f$ (i.e., setting $f = f_\pi$) and the factor $F \simeq (1 - 0.37\omega_{K^+}^2/m_{K^+}^2)$. According to the estimate in this note, these two modifications would very nearly cancel each other. Maruyama et al. [29] apply the same scalar mean field to the $\Lambda$-particle as to the nucleon, none to the kaon. However, in determining the threshold for $K^\pm$ production, which is crucial in determining the number of subthreshold kaons produced [30], this is equivalent to applying $2/3$ of the scalar meson field to the $\Lambda$, $1/3$ to the kaon which we find is a reasonable procedure. The scalar mean field applied to the kaon is $S_K(\rho_0) \approx -73$ MeV. This is rather small compared with our recommended value (32), $-100$ MeV. However the factor $F \simeq (1 - 0.37\omega_{K^+}^2/m_{K^+}^2)$, with $\omega_{K^+} \gtrsim m_K$, would bring them close together. We thus see that there is some confirmation of our suggestion of a linear scaling in the mean fields up to $\rho \lesssim 3\rho_0$, the densities relevant for the threshold $K^+$ production.

Another interesting application of the scaling idea is to the recent dilepton experiments by the CERES collaboration [31]. There the scaling of $f_x^\pi$ can be related to a medium-dependent vector meson mass accounting for enhancement of dilepton pairs observed in 200
GeV/N S on Au collisions at the CERN-SPS. The idea of scaled vector masses has been used successfully in a recent analysis of the CERES process by Li, Ko and Brown [32].

The detailed analysis of the SIS and CERES experiments in terms of our mean-field connection (between the Walecka sector and the kaonic, chiral sector) will be given in a longer paper [33].

Acknowledgments

We would like to thank David Kaplan, Chang-Hwan Lee and Tae-Sun Park for helpful discussions. Part of this work was done while the authors were participating in the INT95-1 program on “Chiral dynamics in hadrons and nuclei” at the Institute for Nuclear Theory, University of Washington. We would like to acknowledge the hospitality of the INT and the Department of Energy for partial support.

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