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Information Technologies Based on Noise-like Signals: I. Discrete Chaotic Algorithms
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Abstract: Perspective directions of using information technologies based on dynamic chaos for the transmission, processing, storage and protection of information are considered. On the basis of nonlinear systems with chaotic dynamics, finite-dimensional generating mathematical algorithms have been developed for the synthesis of chaotic encoding signals with increased structural complexity. The analysis of structural and fractal complexity of pseudo-random integer and binary sequences has been carried out. It is shown that complex coding signals of this type have a high information capacity and, in terms of statistical, correlation, and fractal properties, practically coincide with the parameters of random sequences and can be effectively used in various multi-user radio engineering systems where high noise immunity, protection against unauthorized access, and cryptographic strength are required.

Keywords: information technologies, chaotic dynamics, pseudo-random sequences, information coding

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of information coding, or in the form of a sequence of electrical impulses – a digital form of coding. With analog coding, the necessary information is transmitted with appropriate modulation of the amplitude, frequency or phase of oscillations of a continuous electrical signal. In digital form, information is represented in the form of a binary code 0, 1. Due to good protection against errors and interference, high processing speeds in computer systems and high transmission density over communication channels, digital codes are predominantly common in modern radio relay systems.

The trend observed in recent years of the global propagation of various open telecommunication systems and a sharp increase in the number of subscribers lead to the need to protect information not only at the level of government agencies, special services or business groups, but also at the level of almost every individual user. In information networks, this problem is associated not so much with the confidentiality of information as with the loss of information due to the low noise immunity of various communication channels [1].

The saturation of the frequency range with traditional multi-channel communications based on the principle of frequency division of channels has led to the development of new coding methods for the so-called code division of channels, in which coding streams of random (pseudo-random) numbers (CDMA-technology) are used as codes. When streaming information coding using continuous random coding streams with a uniform distribution function, maximum noise immunity is ensured, and hence the maximum cryptographic strength of the information channel. It should be noted that in telecommunication systems with code division of subscribers, all subscriber communication channels operate in a common wide frequency band [2].

Currently, a promising direction is actively developing in radio-relay communication systems, and especially in radar, representing wideband and ultra-wideband technologies. Within the framework of this direction, it seems possible to move to a qualitatively new level of solving problems of noise immunity and information protection in communication channels, as well as remote radar detection of objects. For example, in addition to the standard radar energy criterion (at the "yes"/"no" level) of detecting an object against the background of noise and the underlying surface, one can proceed to the formation of a radar portrait of an object and the development of systems for automatically recognizing an object by his portrait, which qualitatively increases the information capabilities radar systems. Radar signals with a wide spectrum of frequencies allow for high-precision, informative measurements of the parameters of reflecting objects in difficult electromagnetic conditions under the influence of active and passive interferences.

Increasing the accuracy and resolution of radar measurements is associated with the complication of the structure and the expansion of the frequency band of the probing signal. Such broadening can be achieved either by pulse shortening, or by using frequency or phase modulation of a continuous or quasi-continuous carrier. The limiting case of a continuous wideband probing signal is the so-called white noise with a uniform spectrum, i.e. a signal having an uncertainty function of the δ-function type. Such a signal provides high-precision, unambiguous measurements of both the range to the target and the radial component of the target's velocity. An additional advantage of continuous wideband noise is the ability to provide a good signal-to-noise ratio at the input.
of the receiver compared to pulsed signals. In the case of ultrashort pulses, to obtain a satisfactory signal-to-noise ratio, a huge pulse signal power is required, while in continuous operation, the required signal-to-noise ratio is easily achieved at a power that is much lower than the power per pulse.

Wideband noise-like signals (WBS) due to their specific features, such as low spectral density, high noise immunity in relation to stationary and organized interference of high power, the ability to separate by code, high resistance in multipath propagation, high resolution in measuring distances, all more widely used in various radioengineering systems. WBSs are used in the construction of satellite communication and navigation systems, cellular mobile radio communication systems, local radio networks, communication systems inside buildings, and in a number of other systems [3].

Recently, in connection with the development of multiuser communication systems, such a class of wideband signals as signals with code spreading spectrum [4]. The frequency band of the transmitted signal with code spreading spectrum can be much wider than the frequency band of the information message. For many communication systems, it is important to be able to transmit information simultaneously to several users over the same communication line due to code division of subscriber channels (CDMA technology).

Wideband signals are formed by expanding the frequency band of the information signal and (or) by expanding the carrier. Broadening the signal frequency band is achieved by modulating the carrier wave according to the law of transmitted messages, for example, frequency modulation with a large index, phase shift keying using a pseudo-random sequence of short binary symbols. Frequency band expansion is also characteristic of digital signals with additional, noise immunity coding, since the introduction of redundant symbols while maintaining a constant messages rate leads to the need to reduce the duration of each symbol. In this case, the frequency band of the transmitted encoded signal is extended [5].

An effective extension function must satisfy certain requirements regarding frequencies bandwidth, receiver structure, and message transmission method. The extension function should be deterministic over a relatively large time interval and have a uniform noise-like spectrum in a wide frequency band (large base), hence a narrow autocorrelation function with small side ejections [6].

An ensemble of extension functions used by different systems or a single multi-channel system should have good cross-correlation and group properties. The expansion function can be continuous analog or discrete digital. The formation of wideband pseudo-random signals is most promising to be carried out by digital signal processing methods. In this case, the extension functions are formed on the basis of digital code sequences. In some cases, it is possible to simultaneously expand the spectrum of the signal due to various modulation methods, when, for example, along with the expanding function, digital, noise immunity coding of messages by recovery codes is used.

A number of fairly stringent requirements are imposed on the type and quality of signals in radio engineering systems with WBS:

1) the signal must be sufficiently wideband: signal base \( B \), i.e. the product of the signal duration \( T \) and its bandwidth \( F \) must be much greater than one;

2) the noise spectral density in the transmission channel band must be uniform;
3) the autocorrelation function (ACF) of the signal should have one narrow peak and small side ejections in the interval $T$;
4) the signal must be reproducible in the receiving device in the case of a correlation method of reception.

Such signals are usually formed on the basis of pseudo-random code sequences.

Binary pseudo-random sequences (PRS) must satisfy three randomness criteria:

- a balanced of the binary code;
- for a binary code, the probability of a block of $k$ identical symbols appearing should be close to $1/2^k$;
- the result of summing the code modulo 2 with its cyclic shift should also give a balanced code.

### 1.1. Pseudorandom Coding Sequences

It is known from information theory that stochastic signals generated by random processes have the highest information capacity [7]. The main problem in the development of information carriers in digital telecommunication channels is the difficulty in generating random binary sequences using a short master key. Mathematical algorithms that form pseudo-random sequences (PRS) of numerical values based on a key must have a number of necessary properties:

1) an arbitrarily long period of the non-periodic segment of the resulting PRS;
2) statistical semblance of the resulting sequence of numbers to the properties of a purely random sample;
3) the possibility of software and hardware implementation of a random number generator for use in a communication channel with the appropriate speed.

It should be specially noted that when generating pseudo-random sequences, one of the main problems is the need to form long implementations when using a short master key that determines the initial conditions.

In the software implementation of algorithms for generating pseudo-random processes, the computer operates with discrete numbers in a binary representation with a finite number of discharges. Taking into account this limitation on the finite word capacity of numbers in a computer, the total volume of the phase space (PS), any point of which corresponds to the unambiguous state of the system, is limited, and, accordingly, any algorithmic method of formation must sooner or later come to the periodic repetition of the same segments of the generated sequence, that is, to come to the cycle, although its period can be very large and even infinite from the point of view of a number of practical applications [8].

The requirements for the properties of sequences of pseudo-random numbers depend on specific applications and, as a rule, one algorithm is not able to satisfy all these requirements. In the general case, we can formulate the main requirements for PRS [9]:

- high quality: PRS according to statistical criteria should be close to a random process and have the longest possible period;
- efficiency: the algorithm should be fast and occupy as little memory as possible;
- reproducibility: with exact reproduction of the initial conditions of the algorithm, the same SRP should be formed on implementations of any duration, and minor changes in the initial procedure should lead to the generation of qualitatively different sequences;
- simplicity: the formula of the algorithm should be easy to implement and use.

All of the above emphasizes the relevance of the search for new deterministic algorithms that provide the formation of streams of pseudo-random numbers that satisfy various systems of requirements.
In the public literature, there is practically no information about the methods for developing algorithms for generating pseudo-random numbers. The development of new algorithms requires an understanding of the regularities in the formation of the PRS of numbers with certain given statistical properties.

From the point of view of practical application in digital information technologies, algorithms defined on a closed interval of integers number are of interest. Their advantage is related to the absence of the need to use any rounding in the process of calculating the terms of the sequence. Accordingly, the calculation results in this case will not depend on the bit depth data bus in a particular computer and the number of significant digits in the representation of fixed-point numbers [10].

Despite the fact that quite a lot of algorithms for generating PRSs are known, in practice, to generate binary PRSs, as a rule, a recurrent algorithm is used, when, based on a linear recurrent relation and some initial values, an infinite sequence is constructed, each subsequent term of which is determined from the previous ones. Binary sequences based on recurrence relations are quite easily implemented on a computer in the form of programs and circuitry based on high-speed multi-bit binary shift registers.

Attempts to adapt operations on real numbers for digital algorithms ended in failure, since replacing a real number with its approximate value greatly changes the statistics of the resulting sequence. The rounding operation introduces an unpredictable perturbation into the generating algorithm, and the resulting sequence ceases to be statistically independent, and therefore random.

The main method for obtaining PRS at present is the formation of M-sequences (sequences of the maximum period) based on shift registers, when the numerical value of the sequence is currently determined by linear relationships with a certain weight (code) in relation to the previous members of the sequence. In this case, the weight coefficients are selected in such a way as to ensure a rapid drop in the correlation function to values of the order of $1/\sqrt{N}$, where $N$ is the length of the period of the M-sequence. The biggest disadvantage of this method is the lack of a mathematical apparatus that allows one to obtain algebraic polynomials that generate sequences of the maximum period of an arbitrarily large degree, moreover, information about high degree polynomials suitable for noise immunity coding is extremely secret [11].

The known classes of RPSs, both linear (M-sequences, sequences of Hadamard, Gold, Kasami, etc.) and non-linear (Legendre sequences, bent-sequences, etc.), have certain disadvantages and do not satisfy some of the requirements listed above. A certain solution to the problem is provided by the use of noise-like signals generated by nonlinear systems with dynamic chaos. Such noise-like signals, having correlation properties no worse than those of M-sequences, have a practically unlimited set of lengths, can form ensembles of signals of large volumes and are non-linear, which makes it difficult to recognize them for subsequent reproduction.

All known dynamical systems with a small number of degrees of freedom that have dynamic chaos ("strange attractor") – Lorentz attractor, Ressler attractor, Chua systems, ring systems with delay and purely amplitude nonlinearity – also do not provide correlation functions with the necessary parameters [12].

Good statistical properties are possessed by dynamic systems in which both dissipative (amplitude) nonlinearity and reactive (phase) nonlinearity are present. In self-oscillatory
systems with phase nonlinearity and delay, as a result of the existence of phase nonlinearity, the phase balance conditions and mode synchronization conditions are violated, and in the process of chaotization of oscillations, the intraspectral connections are weakened and the correlations in the generated signal split faster (compared to other autostochastic systems). Signals with good correlation properties can be obtained in the class of nonlinear ring systems with delay, in which both active (amplitude) and reactive (phase) nonlinearities are simultaneously present.

At the same time, the fundamental feature of the algorithms that describe a system with dynamic chaos is their non-linearity, and the feature of the generated time process is its non-periodicity.

Systems with chaotic nonlinear dynamics differ from traditional self-oscillatory systems, which are imaged in the phase space as limit cycles in the form of closed non-intersecting curves on a plane or multidimensional tori in the case of a large number of degrees of freedom of the system. The trajectories of a system with chaotic dynamics contract in the phase space not to limit cycles, but to complex multidimensional surfaces, which are commonly called "strange attractors" and which are Cantor sets with a fractal (fractional) dimension [13].

Dynamic systems have different attractors, and, consequently, the generated processes corresponding to them and the signal systems built on their basis will have different properties [14]. An algorithmic approach based on the use of the phenomenon of dynamic chaos will make it possible to purposefully form systems of noise-like signals with the desired properties.

The application of this approach makes it possible to create a new class of pseudo-random sequences for use in radio engineering information transmission systems – wideband chaotic signals, which fully meet all of the above requirements.

The purpose of this work is to develop and study the properties of pseudo-random coding signals generated by generative recurrent algorithms based on systems with chaotic dynamics to create expanding functions in Spread Spectrum wideband information technologies.

2. WIDEBAND SIGNALS BASED ON CHAOTIC DISCRETE ALGORITHMS WITH NONLINEAR DYNAMICS

At present, the most promising method for forming pseudorandom sequences is the use of chaotic algorithms that describe the complex nonequilibrium behavior of nonlinear dynamic systems. Nonlinear dynamic systems that generate chaos have extremely high information content, which makes it possible to implement many different types of oscillations with a wide spectrum in the same analog or digital circuit.

For application in radio engineering systems, a new class of random sequences is proposed, which are formed on the basis of algorithms that describe the behavior of self-oscillating systems with delay, having dynamic chaos modes. A feature of such systems is their non-linearity and non-periodicity of the time process generated by them. By changing the parameters of such a dynamic system and the initial conditions, it is possible to change the nature of its behavior over a wide range and thereby purposefully control the type and properties of the generated chaotic signal.

The proposed algorithms for generating a chaotic signal simulate the behavior of ring self-oscillatory systems with delayed feedback and strong amplitude-phase nonlinearity [15]. When the signal circulates through the feedback loop, the non-linearity of the system
leads to an expansion of the signal spectrum. The width of this spectrum is limited by the filtering properties of the self-oscillatory system. The relationship between these two competing factors – non-linearity that spreads the spectrum and filtering that narrows the spectrum – allows you to create a chaotic signal with a given spectral width. The signals generated in this case belong to the class of wideband chaotic signals. The scheme of such a system can be represented as a ring of three blocks:

\[ \rightarrow (1 \text{ non-linearity}) \rightarrow [\hat{F}] \rightarrow, \ (2 \text{ delay}) \rightarrow [T] \rightarrow, \ (3 \text{ filter operator}) \rightarrow [\hat{\Phi}] \rightarrow \]

A block diagram of such a system is shown in Fig. 1. The mechanism of self-oscillations in such a system, accompanied by stochastization, can be described by an integral equation, where the action of all three of these functional blocks is sequentially taken into account [16]:

\[ \hat{x}(t) = \int_{-\infty}^{\infty} g(t-\tau) \hat{F}(\tau-T) d\tau, \quad (1) \]

which can be converted to a discrete form if we introduce a rectangular filtering of the signal, represent the functions \( g \) and \( \hat{F} \) in the form of orthogonal Kotelnikov series, and perform some transformations [17]:

\[ \hat{x}_k = [1-\exp(-h)]\hat{F}_k + \exp(-h)\hat{x}_{k-1}, \quad (2) \]

where: \( x = a\exp(i\varphi) \), \( a \) – amplitude, \( \varphi \) – signal phase, \( F_k = F(a_k)\exp\{i[\varphi_k + \Phi(a_k)]\} \), \( F(a) \) and \( \Phi(a) \) – nonlinear transformation functions amplitude and phase of the signal, \( b \) – discretization, selected in accordance with the Kotelnikov theorem, \( N \) – delay parameter (the number of samples in the interval of the delay duration).

The form of the functions \( F(a) \) and \( \Phi(a) \) together with the values of the parameters \( b \) and \( N \) determines the nature of the formed chaotic process and its statistical properties. Nonlinear functions for converting the amplitude and phase of the signal \( F(a) \) and \( \Phi(a) \), which determine the process of stochastization of oscillations in a given dynamic system, depending on the choice of the type of nonlinear amplifier, can be quite complex. The determining factor for obtaining the desired statistical properties of the signal is the presence of a steep slope of the phase characteristic with respect to the value of the signal at the input of the nonlinear element.

The phase space of a dynamic system with delay is \( n \)-dimensional, where \( n \) is the number of values that uniquely determine the behavior of the system at each next step. For a system with a delay, the dimension of the phase space is determined by the number of dynamic variables and the duration of the feedback delay, presented in a discrete form.

A special place among the algorithms for the formation of random sequences is occupied by the algorithms for the formation of integer sequences. Usually they are defined on a finite set of integers, which is due to the bit depth limitation used to represent integers in digital technology. The advantage of integer sequences is that they are identically reproduced on various types of computing devices and, when implemented in hardware, are easily reproduced circuitry.

In the practical implementation of a new class of signals in digital communication technology, which is based mainly on a binary

Fig. 1. Block diagram of a nonlinear ring system with delay, in which both active (amplitude) and reactive (phase) nonlinearity are simultaneously present.
code, there are two possibilities for obtaining binary signals. The first method is associated with clipping multilevel signals obtained as a result of calculations. This method is associated with a large loss of information embedded in the original multilevel signal, but, fortunately, the correlation properties of the signals wherein practically do not deteriorate.

The second method is a direct construction of discrete self-oscillatory systems. The algorithm for obtaining a binary signal in a self-oscillatory system has the form:

\[
x_k = (1 - e^{-x}) \text{sign}[F(x_{k-1})] + e^{-x}x_{k-1},
\]

\[
F(x_k) = \text{sign}[x_k],
\]

The ratio is obtained directly from equation (2).

On the basis of a mathematical model of a ring self-oscillatory system with strong amplitude-phase nonlinearity, filtering and delay, a discrete generating algorithm for a chaotic signal has been developed and studied, which belongs to the class of algorithms of the recurrent-parametric type with delay. The general form of the algorithm of this class has the form of a discrete functional transformation (mapping) of the form

\[
x_n = f(x_{n-1}, x_{n-2}, ..., x_{n-N_z}),
\]

where \( x_n \) and \( x_{n-1} \) are, respectively, the newly calculated member of the generated pseudo-random sequence at the \( n \)-th step and the previous member of this sequence at the \( (n - 1) \)-th step, \( N_z \) is the delay parameter that determines the number of sequence members on delay interval \( x_{n-1}, x_{n-2}, ..., x_{n-N_z} \), which completely determine the new value of \( x_n \) and which must be set as the initial condition at the first step, and the function \( f(x) \) reflects the amplitude and phase transformations in the generating ring self-oscillating system in chaos mode. The algorithm is defined on the set \( M \) of integers of the natural series belonging to the closed numerical interval \([M_1, M_2]\), \((M_2 > M_1, M = M_2 - M_1 + 1)\), and forms a practically uncorrelated pseudo-random sequence of integers with a probability distribution close to uniform, and correlation characteristics that meet the requirements for coding signals.

A feature of algorithms with delay is that the mapping formula given by them can take out a new value \( x \) beyond the domain of the algorithm \([M_1, M_2]\). Therefore, the formula of algorithm (4) must be supplemented with a special operation that ensures that the value of each newly calculated member of the sequence will returned to the given numerical interval if it is outside its boundaries. Transformations of this kind with a mapping of a numerical set "into itself" have been known for a long time. The well-known "baker" transformation [18] can serve as an example. Other types of transformations are also possible, but among them it is necessary to highlight those that do not significantly change the probability distribution of the generated numbers.

The cardinality of the used set of integers is much less than the cardinality of the continuum of the continuous set on which the dynamical system is defined. As a result of this limitation, in the process of algorithmic formation of such sequences, with an increase in the number of their members, an inevitable exit to a cycle takes place, which is an analogue of the limit cycle of dynamical systems defined on a continuous numerical set. At the same time, it is important that, on the interval before reaching the repetition period corresponding to this cycle, the sequences implemented algorithmically have statistical properties close to those of truly random sequences.

The used algorithm with a delay has the property that in order to uniquely determine
the entire subsequent “trajectory”, it is necessary to specify all \( Nz \) values on the delay interval. Hence it follows that if in the sequence generated by the algorithm two non-overlapping sections (segments) of length \( Nz \) completely coincide, separated by a distance \( L \) of algorithm computation steps between the beginnings of the segments \( L > Nz \), then the sequence will be periodic with period \( T = L \). The probability of occurrence such an event for an algorithm defined on the integer interval \([0, 255]\) of the order of the reciprocal value of the volume of the phase space

\[
P(256, n) \approx 1/(256)^n = 3 \times 10^{-39} \text{ for } n = Nz = 16.
\]

The result obtained can be interpreted as an estimate of the possible period of the sequence generated by the algorithm. The value of the latter, therefore, can be \( T \approx 10^{38} \) (for \( Nz = 16 \)) sequence members. This estimate should be considered as the probable value of the existence of a period in the generated sequence at \( M = 256 \) and \( Nz = 16 \). It follows that as the delay \( Nz \) increases, the probability of the occurrence of a period in the sequence generated by the algorithm can be made negligible.

The chaotic signal algorithm under consideration generates a multilevel integer signal \( \{x_n\} \in [0, 255] \). In practice, systems of binary signals are also widely used. Such a signal can be obtained from a multilevel one using the clipping operation.

The most complete information about the statistical properties of discrete sequences is given by the analysis of the probability distributions of the numbers \( p(x) \) and the distributions of conditional probabilities \( p(i + j, x_n / ix_k), j = 1, 2, 3, ..., N, n, k = 1, 2, 3, ..., M \), i.e. the probability of generating the number \( x_n \) at the \((i+j)\)-th step of the algorithm, if the number \( x_k \) was obtained at the \(i\)-th step. In this case, the definition domain of the discrete algorithm is an arbitrary closed integer interval \([M_1, M_2]\), \( M = M_2 - M_1 + 1, x_n \in [M_1, M_2] \).

If the distribution of conditional probabilities for any \( j \) practically coincides with the uniform distribution, then it follows that all transition probabilities \( p(i + j, x_n / ix_k) \approx 1/M, j = 1, 2, 3, ... \) for an arbitrary choice of \( i \). At the same time, if the probability distribution of the generated numbers \( p(x) \) is close to uniform, then the probability of the value \( x_n \) is practically also equal to \( 1/M \). Thus, the transition probabilities to the state \( x_n \) at the \(j\)-th step coincide with the probability of this value at this step, regardless of the values of the sequence at the previous steps of the algorithm, which is typical for random sequences in independent trials. Moreover, the pseudo-random sequence formed by such an algorithm will be close in its probabilistic characteristics to the sequence of independent equally probable numbers from the interval \([M_1, M_2]\). In the latter case, the given sequence can be expected to have the best statistical properties. The establishment of such a fact emphasizes the importance of studying the distributions of conditional probabilities for a priori judgments about the quality of the generated pseudo-random sequences.

To characterize the conditional distributions \( p(x_{i+j} / x_i) \), the position of the points \( (x_{i+j}, x_i) \) on the plane for the map \( x_{i+j} = \text{func}(x_i) \) defined by the discrete algorithm is of great importance for the corresponding values \( j = 1, 2, 3, ..., N \). Obtaining the scatter of points \( (x_{i+j}, x_i) \) and visualizing it on the screen does not require large computational resources compared to the direct calculation of conditional probabilities, and although the nature of this scatter does not directly give the shape of the distribution of conditional probabilities, nevertheless, the visualization of
the scatter indicates the degree of regularity of these distributions, the presence of functional connections, the existence of forbidden transitions, and even entire forbidden zones, which inevitably affects the correlation and other statistical properties of the sequence.

It was shown that with an appropriate choice of parameters, discrete algorithms with delay form long non-periodic segments of pseudo-random sequences with a uniform probability distribution, which, in terms of statistical and correlation parameters, are close to the characteristics of a random equiprobable process.

3. STRUCTURAL AND FRACTAL COMPLEXITIES OF CHAOTIC DISCRETE SIGNALS

For the effective implementation of chaotic signals in radio engineering complexes, telecommunication systems, as well as for their use as an information carrier in new generation information technologies, it is necessary to develop methods for assessing the structural complexity and fractal dimension these signals.

For this purpose, in this paper, we analyzed the simplest algorithms for generating pseudo-random sequences of integers \( \{x_n\} \) with delay, using the Fibonacci mapping and its modifications:

- **Algorithm \( F-1 \)**: 
  \[
  \tilde{x}_n = x_{n-1} + (-1)^{Kz} x_{n-Nz} \tag{1}
  \]

- **Algorithm \( F-2 \)**: 
  \[
  \tilde{x}_n = x_{n-1} + (-1)^{Kz} x_{n-Nz} \tag{2}
  \]

- **Algorithm \( F-3 \)**: 
  \[
  \tilde{x}_n = x_{n-1} + x_{n-Nz} \tag{3}
  \]

where \( Nz \) and \( Kz \) are algorithms parameters, \( 2 \leq Kz \leq (Nz - 1) \). In contrast to [6], the sign in front of the retarded term in \( F-1, F-2 \) does not change randomly independently, but is determined by the internal dynamics of the system. The feedback parameter \( Nz \) determines the dimension of the phase space of the algorithm and, accordingly, the dimension of the radius vector \( R_{n, Nz} (x_{n-1}, x_{n-2}, \ldots, x_{n-Nz}) \) of the state of the discrete dynamical system at each step.

The phase space (PS) of the Fibonacci mapping of dimension \( Nz \) is not limited. For the practical application of PRS algorithms in radio engineering systems and the formation of modulating digital signals of finite bit capacity, it is necessary to set the domain of definition of the algorithm on a finite set of numbers of a closed interval of the natural series \([1, M]\), where \( M > 1 \). For this, mappings (1-3) must be supplemented by the operation of converting the numerical interval \([1, M]\) into itself, for example, of the following form:

- \( x_n = \tilde{x}_n, \quad \text{if} \quad \tilde{x}_n \in [1, M] \)
- \( x_n = \tilde{x}_n - M, \quad \text{if} \quad \tilde{x}_n > M \)
- \( x_n = \tilde{x}_n + M, \quad \text{if} \quad \tilde{x}_n < 1 \).

This transformation, corresponding to the contraction of the segment \([1, M]\) into a ring, plays an important role in the mechanism of the chaotic behavior of these dynamical systems. Firstly, this operation limits the volume of the phase space, making it finite, equal to \( V_{ps} = M^{Nz} \) state points, and, secondly, it provides additional mixing of trajectories in the phase space. It should be noted that one transformation of a numerical interval into itself is not enough for effective mixing of trajectories in the phase space. A certain mechanism of chaotization should already be contained in the mapping function. In this case, this is provided by the properties of the Fibonacci mapping. These two conditions – the limited volume of the phase space and the presence of a powerful mixing mechanism – are necessary conditions for the chaotic behavior of any dynamic system.

Algorithm (\( F-4 \)) based on the Fibonacci mapping (3) was also considered as an alternative, but with a different operation of converting the numerical interval \([1, M]\) into itself – the type of a reflecting border [19]:

\[
\text{Algorithm } F-4: \quad \tilde{x}_n = x_{n-1} + \begin{cases} 
  -1 & \text{if } x_n < 1 \\
  1 & \text{if } x_n > M
\end{cases}
\]
Depending on the choice of initial conditions, the radius vector $R_n$ describes a trajectory in the phase space of the algorithm, which is a successive discrete transitions from one point of the state of a dynamic system (DS) to another according to a random law. These \"trajectories\" of the motion of a discrete DS in the PS, due to the limited volume of the PS, form closed cycles, which, due to the univocal of the transformations, do not intersect and have no common points. In addition, cycle pools and isolated points can exist in the PS. So, for example, when $N_\zeta = 4$, $K_\zeta = 3$ and $M = 5$, the PS of the $F$-1 algorithm has three single cycles with periods of $526, 27,$ and $8$ and one isolated point, the PS of the $F$-2 algorithm contains one 13-cycle cycle with the trajectory pool of 611 points leading to this cycle, plus one isolated point, the PS of the $F$-3 algorithm consists of two cycles with a period of 312 and one isolated point. The spectrum of periods of the $F$-4 algorithm (the cycle multiplicity is indicated in brackets): $T = 36(1)$, $15(3)$, $5(1)$, $1(1)$ and $538$ points of the pool of cycles.

The cycles of the $F$-1, $F$-2, $F$-3, $F$-4 algorithms have an important distinctive feature: the behavior of the dynamic system before the cycle is closed (and also on the trajectory of the pool, if it exists) is chaotic, and the non-periodic sequence, which generated herewith by the algorithm, is pseudo-random. The set of points in the PS united in a cycle is called a pseudo-random cycle if the non-periodic process formed by the algorithm before the cycle is closed has a chaotic character, in contrast to the regular cycle, which corresponds to a regular process before the cycle closes. A pseudo-random cycle corresponds to an irregular motion in the phase space, and a regular cycle corresponds to a regular one. Of course, in both cases the behavior of the dynamical system on the cycle is completely determined. The trajectory of a pseudorandom cycle is a deterministic set of randomly following one after another points of states of a discrete dynamical system in the entire volume of the phase space of the algorithm. An analogue of the pseudo-random cycle of a discrete system is a strange attractor of a continuous dynamical system.

Depending on the values of the parameters $N_\zeta \geq 3$, $K_\zeta$ and $M$, in the phase space of the $F$-1, $F$-2, $F$-3 algorithms, there are a number of cycles of different periods, of which each long ($N \sim V_{ps}$) cycle before its closure corresponds to a non-periodic PRS with practically uniform distribution of generated numbers in a given interval of domain of definition $p(x) \approx 1/M$ and with uniform distributions of conditional probabilities. Only for the $F$-2 algorithm, in the distributions of conditional probabilities $p(i + 1, x_n/i, x_k)$, there are forbidden transitions for even or odd numbers, depending on the parity of the number at the previous step.

We will consider processes only up to the closure of cycles, i.e. non-periodic segments formed by the PRS algorithm. These segments can be of any length (arbitrarily large) with an appropriate choice of algorithm parameters and initial conditions. Thus, for the $F$-1 algorithm, with the parameter values $N_\zeta = 3$, $M = 63$, the length of the non-periodic PRS is $N = 7.8317 \cdot 10^4$ ($N/V_{ps} = 0.31$), with $N_\zeta = 5$, $M = 63$, the length of the non-periodic PRS is $N = 3.3174 \cdot 10^8$ ($N/V_{ps} = 0.33$), at $N_\zeta = 7$, $M = 63$ $N = 1.676 \cdot 10^{12}$ ($N/V_{ps} = 0.425$), at $N_\zeta = 9$, $M = 63$, the length of the non-periodic PRS is more than $5 \cdot 10^{12}$ steps of the algorithm, in the latter case the volume of
the phase space is \( V_{ps} = 1.56 \cdot 10^{16} \). Long and extra-long coding sequences are needed to ensure the operation of complex navigation systems such as NAVSTAR and GLONASS.

For comparison, as an example of a PRS with a non-uniform probability distribution function of the generated numbers, the results of a study of the \( F-4 \) algorithm are given. It is shown that for the PRS formed by the \( F-4 \) algorithm, the probability distribution density \( p(x) \) decreases monotonically towards the beginning of interval of domain of definition \([1, M]\).

To characterize the fractal properties of a chaotic set of points on the PRS, we confine ourselves to an analysis of the geometric (Euclidean) and correlation dimensions. The numerical experiment was carried out for small values of the parameters of the algorithms \( N_\xi = 4, M = 11 \), the length of the studied PRS of \( N = 500 \) numbers, which is of fundamental importance for estimating the majority properties of pseudorandom cycles. With an increase in the dimension of algorithms, the nature of the behavior of a discrete DS becomes much more complicated and the statistical characteristics of the formed PRSs improve.

An estimate of the correlation dimension \( D_2 \) of the pseudorandom cycle under study can be given based on the calculation of the correlation integral \( C(l) \) given on the set of distances \( l \) between all pairs of state vectors on the cycle in the PS, plotting the dependence \( \log_2 C(l) = f(\log_2 l) \) shown in Fig. 2, and determining the slope of a straight section on it [20].

For the \( F-1 \) algorithm with parameters \( N_\xi = 4, K_\xi = 2, M = 11 \), the correlation dimension of the set of points on the cycle with the initial vector \( R_0(8, 6, 7, 1) \) (curve 1) is equal to \( D_2 = 3.3 \). The obtained value agrees with the geometric dimension \( D_0 = 4, D_2/D_0 = 0.83 \). The value of the latter ratio can serve as a characteristic of the degree of uniformity of the filling of the full volume of the PS with cycle points. As the analysis showed, the studied cycle with the initial vector \( R_0(8, 6, 7, 1) \) corresponds to a non-periodic PRS of length \( N = 14030 \) with a distribution of generated numbers close to uniform.

The linear section of the graph (curve 2) obtained for the set of points of the basin trajectory and the cycle in the phase space of the \( F-2 \) algorithm \( (N_\xi = 4, M = 11, R_0(1, 1, 1, 1), N = 500) \) has several a smaller slope, which corresponds to the value of the correlation dimension about \( D_2 = 3.0 \). Curve 3 in Fig. 2 corresponds to the logarithm of the correlation integral for the pseudorandom cycle with \( R_0(1, 6, 6, 7) \) of the tested \( F-3 \) algorithm, \( N_\xi = 4, M = 11, N = 500 \). Graphs 1 and 3 of the function \( \log_2 C(l) = f(\log_2 l) \) in Fig. 2 almost exactly repeat each other and have an extended rectilinear section with a slope \( D_2 = 3.3 \), which makes it possible to obtain a quantitative estimate of the uniformity of space filling with DS state.

**Fig. 2.** Dependence of \( \log_2 C(l) \) on \( \log_2 l \) for algorithms \( F-1, F-2, F-3 \) and \( F-4 \).
points on pseudorandom cycles. We note that to the F-1 and F-3 algorithms correspond to the PRS with good statistical and correlation properties, especially when the delay $N_{Z}$ increases more than 5.

For the F-4 algorithm cycle with parameters $N_{Z} = 4$, $M = 17$, sequence length $N = 500$, initial radius vector $R_{0} = (7, 14, 6, 15)$, period $T = 613$, dependency $\log_{2} C(l) = f(\log_{2} l)$ (curve 4 in Fig. 2) does not have a clearly defined straight section. This means that the correlation integral has significant deviations from the $C(l) \sim l^{-D}$ law and, consequently, the points of this pseudorandom cycle are unevenly located in the PS.

To estimate the degree of complexity of the chaotic process generated by the algorithm, it is necessary to determine the homogeneity of the attractor in the PS on all discrete time scales. Determining the correlation dimension of attractors requires a large amount of computational resources, especially in the case of a high-dimensional DS, so it makes sense to study the structural properties of the implementation of a pseudo-random process, which is a projection of the DS motion trajectory in the PS onto one of the directions in this space.

Fractal analysis can be applied not only to a chaotic set of points in a multidimensional PS, but also to a one-dimensional set of implementation numbers of the PRS. Definition by the standard method of the correlation dimension applied to a one-dimensional ($D_{0} = 1$) chaotic array of $N = 1000$ PRS numbers, formed by the F-1, F-2, F-3, F-4 algorithms with parameters $N_{Z} = 16$, $M = 21$ gave the following results. For all tested algorithms, the value of the correlation dimension is within $D_{2} = D_{2}/D_{0} = 0.91 \pm 0.96$, including for the random number generator RND(Maple) ($M = 21$). The obtained values of the ratio $D_{2}/D_{0}$ indicate a fairly good uniformity of filling the interval $[1, M]$ with generated numbers. This is confirmed by the analysis of the one-dimensional probability distribution of the numbers in the sequence. But on the basis of these data, nothing can be said about the structural complexity of the PRS and, most importantly, how close it is to a sequence of independent random events, which can be considered as a standard of chaotic behavior. To this end, we study the local structure of the PRS based on the analysis of fractal geometry.

A random sequence of integers can be viewed as a discrete topology of a complex geometric relief ("coastline"). To estimate the geometric structural complexity, we study changes in the distance successively between neighboring points of such a relief in a window of a given scale. In other words, based on the implementation data of the PRS of length $N$, we proceed to the analysis of the algebraic sequence

$$\{y_{n} = |x_{n} - x_{n+1}|, n = 1, 2, ..., (N - 1), y_{n} \in [0, (M - 1)].$$

Following the method of calculating the correlation integral, we calculate the number $N(l)$ of the occurrence of identical events $y_{n} = l$, $l = 0, 1, 2, ..., (M - 1)$ in a sequence of $(N - 1)$ terms and plot the frequency of occurrence of such events $p(l) = N(l)/(N-1)$ depending on $l$ (Fig. 3).

![Fig. 3. Probabilities of differences of numbers $l = |x_{n} - x_{n+1}|$ in sequence implementations.](image)
Calculations were performed for the PRS of length \( N = 50000 \). Curve 1 corresponds to the \( F-1 \) algorithm with parameters \( N_\gamma = 16, K_\gamma = 9, M = 21 \). This plot almost exactly repeats the corresponding theoretical dependence for a sequence of statistically independent equiprobable numbers – \( p_x(l) \), which is taken as the standard. As the latter, one can also use the experimentally obtained values of \( p(x) \) for the PRS in the case of their closeness to the theoretical ones, for example, for the PRS formed by the RND random number generator or the \( F-1 \) algorithm. The total deviation module of the values \( p_i = p(l) \) obtained for the analyzed process from the reference values – \( s = \sum |p_i - p_x| \). The value \( S = 1/(s + 1) \) can be taken as a measure of the structural complexity of this process \( \{x_n\} \).

Curves 2, 3, and 5, obtained by analyzing the \( F-2 \) and \( F-3 \) algorithms with the same large PS dimension (\( N_\gamma = 16, M = 21 \)) and the random number generator RND(Maple) with \( M = 21 \), also differ little from reference graph. Curve 4 corresponds to the \( F-4 \) algorithm with a non-uniform distribution of the generated numbers \( p(x) \).

Graphs 6, 7, 8 and 9 are constructed for the modified PRS of the \( F-1 \) algorithm in order to model discrete processes with different types of distribution function \( p(x) \) of numbers (average value \( x_{av} \), rms deviation \( \sigma \), skewness coefficients \( \gamma_1 \) and kurtosis \( \gamma_2 \) and autocorrelation interval \( \tau_{cor} \). Curves 4, 6, 7, 8, and 9 differ noticeably from the reference one, which indicates the high informativeness of the proposed method for estimating the structural complexity of algorithms by plotting the relative frequency of observation of the magnitude of the difference between neighboring numbers in the implementation of the PRS \( p(l) = f(l) \). This method does not require large amounts of computing resources compared to the methods of statistical, correlation and fractal analysis.

Table 1 summarizes the results of numerical experiments for all tested algorithms (parameters of Fibonacci type algorithms: \( N_\gamma = 16, M = 21 \), implementation length \( N = 50000 \)).

It can be seen from the given data that all three algorithms \( F-1, F-2 \) and \( F-3 \) based on the Fibonacci mapping, as well as the certified random number generator RND, demonstrate a fairly high structural quality of the generated sequences. When the distribution function of the generated numbers \( p(x) \) and the correlation coefficient change, the proposed method for estimating the degree of structural complexity effectively captures the corresponding change in the statistical properties of the PRS.

### 4. STRUCTURAL COMPLEXITY OF CHAOTIC BINARY SEQUENCES

Almost all digital radio systems use binary signals. Therefore, the problem of preserving all the features of pseudo-random sequences of binary numbers obtained by clipping integer sequences generated by the studied Fibonacci-type algorithms is quite relevant.

It is known that the probability of a block of \( k \) identical symbols appearing in a
random binary sequence must obey the law \( p(k) = 1/2^k \). In this case, this binary sequence has good correlation properties. For a random process of statistically independent equally probable events, the probability of the appearance of any fragment of \( k \) binary symbols (not necessarily identical) must obey the same law.

The purpose of the numerical experiment was to verify the validity of this regularity for binary PRSs formed by various algorithms with a delay of the Fibonacci type, and on this basis to establish a quantitative criterion for estimating the structural complexity of the corresponding binary PRS.

The goal was achieved by checking the following provisions:

- whether all possible fragments of a binary code with a length of \( k \) symbols are present in the implementation of the PRS;
- what is the probability of their occurrence in the implementation of the sequence in comparison with the law \( p(k) = 1/2^k \), valid for an ideal random process. Note that it is precisely such a check, in particular, that is provided by the Advanced Encryption Standard (AES) encryption standard, designed for statistical testing of code sequences used to ensure confidentiality in the transmission of information;
- evaluation of the structural complexity of binary PRSs generated by Fibonacci type algorithms with delay.

In a numerical experiment, the frequency of occurrence in the implementation formed by algorithms of \( N \) members of all possible fragments of length \( k \) from the system of a complete code of volume \( V(k) = 2^k \), where \( k = 2, 3, \ldots, 12 \), was successively determined. The frequencies of each \( i \)-th fragment of the full code \( n_i(k)/N \), \( i = 1.2\ldots2^k \) obtained in the experiment were compared with the probability \( p(k) = 1/2^k \) of a fragment of length \( k \) symbols in a sequence of independent equiprobable trials. Dispersion was determined

\[
\sigma^2(k) = \frac{1}{2^k} \sum_{i=1}^{2^k} \left( \frac{n_i}{N} - \frac{1}{2^k} \right)^2
\]

and root-mean-square deviation from this level is \( 1/2^k \) for a given value of \( k \). In the experiment, the size of the analyzed segments of the complete code was successively changed from the value \( k = 2 \) to \( k = 12 \). The lengths of the analyzed implementations of the sequences for all considered algorithms were determined by the relation \( N = A \cdot 2^k \), where \( A \) was chosen to ensure the necessary statistical representativeness of the samples. In the results presented below, \( A = 100 \) was assumed, so that there would be at least 100 trials per segment of the complete code in the implementation.

In the numerical analysis, the following values of the parameters of the algorithms \( F-1, F-2, F-3, F-4 \) were used: \( N_S = 16, M = 255, K_S = 9 \). Standard random number generators used in various software packages were also considered for comparison: Maple7, Mathcad and Pascal.

On Fig. 4 graphs of the rms deviation of the frequency of occurrence of all \( 2^k \) variants

![Fig. 4. Root-mean-square deviation from the law \( p(k) = 1/2^k \) of the probabilities of segments of the complete code in the implementations of binary PRSs.](image)
of segments of a complete code of length \( k \) from the law \( p(k) = 1/2^k \) for algorithms \( F-1 \) (curve 1), \( F-2 \) (2), \( F-3 \) (3), \( F-4 \) (4), random number generators RND Maple (5), Mathcad (6) and Pascal (7).

From the data shown in the Fig. 4, we can conclude that in the sequences generated by the \( F-1 \) and \( F-3 \) algorithms, the scatter of the experimentally observed frequencies of the appearance of segments of the full code relative to the level \( p(k) = 1/2^k \) practically coincides with the corresponding characteristics of the sequences generated by the sensors random numbers of software packages Maple, Mathcad and Pascal.

All these algorithms correspond to a uniform probability distribution of the generated numbers. The \( F-2 \) algorithm, which generates an integer sequence with a uniform probability distribution, but with forbidden transitions for even or odd numbers at one step of the algorithm, has a spread in the probabilities of the appearance of fragments of the full code 2-3 times greater. The uneven distribution of the generated numbers in the sequence, as is the case for the \( F-4 \) algorithm, leads to a significant (by an order of magnitude) deviation from uniformity appearance of fragments of the complete code in the implementation of the generated binary process.

Based on the results obtained in the numerical experiment, we determine the average value of rms deviations for all analyzed systems of the complete code \( (k = 2, 3, ..., 12) \):

\[
\sigma_{rp} = \frac{1}{11} \sum_{i=2}^{12} \sigma_i(k).
\]

Let us define \( K_{sc} = 1/(1 + \sigma_{rp}) \) as a coefficient characterizing the structural complexity of the PRS with respect to the complexity of a purely random binary sequence.

The obtained quantitative values of the coefficient \( K_{sc} \) for all analyzed algorithms are shown in Table 2. These data show that the structural complexity of pseudo-random binary sequences generated by integer algorithms with delay based on Fibonacci-type mappings with subsequent clipping, which have an almost uniform distribution of probabilities of generated numbers \( p(x) \) and distributions of conditional probabilities (transition probabilities) that are close to uniform, does not differ significantly from the structural complexity of purely random sequences. Just like from the complexity of sequences generated by certified random number generators.

To algorithms with high structural complexity should correspond correlation characteristics close to the corresponding characteristics of a random process. For all the analyzed Fibonacci-type algorithms with the operation of returning the generated numbers to the interval of the domain of definition, Table 2 shows estimates of the level \( R_{max} \) of lateral outliers of aperiodic auto- and cross-correlation functions. In a numerical experiment, aperiodic correlation functions were determined from 100 non-overlapping segments of length \( N_{\text{cod}} = 128 \) (which corresponds to the IS-95 standard for telecommunication CDMA systems).
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sequentially generated by algorithms without any selection, including selection by code balance. The given levels of lateral outliers of correlation functions for the segments of binary PRSs generated by the $F^{-1}$, $F^{-2}$, and $F^{-3}$ algorithms correspond quite well to the lateral outliers of the correlation functions of random sequences.

Thus, the structural complexity of the sequences generated by the developed chaotic algorithms practically coincides with the complexity of random sequences. Such sequences can be used as expanding signals in radio engineering and navigation systems with noise-like signals.

5. CONCLUSION

The applied application of information technologies involves the physical implementation of a specific coding process in the transmission, processing and storage of information in telecommunication systems and computer networks. The paper considers promising areas for the use of information technologies based on dynamic chaos for the transmission, processing, storage and protection of information. On the basis of nonlinear systems with chaotic dynamics, finite-dimensional generating mathematical algorithms have been developed for the synthesis of chaotic encoding signals with increased structural complexity. The analysis of structural and fractal complexity of pseudo-random integer and binary sequences has been carried out. It is shown that complex coding signals of this type have a high information capacity and, in terms of statistical, correlation, and fractal properties, practically coincide with the parameters of random sequences and can be effectively used in various multi-user radio engineering systems where high noise immunity, protection against unauthorized access, and cryptographic strength are required.

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