On the neutrino mass ordering and flavor mixing structure

Zhi-zhong Xing *
Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

Abstract

Whether the neutrino mass spectrum is normal ($m_1 < m_2 < m_3$) or abnormal ($m_3 < m_1 < m_2$) remains an open question, but we show that the latter possibility looks quite unnatural when it is related to the lepton flavor mixing matrix $U$ in a way similar to the reasonable correlation between the quark mass spectrum and the quark flavor mixing matrix. Taking into account the freedom in choosing the basis of weak interactions, we make a novel prediction for $|U_{\tau 1}|/|U_{\tau 2}|$ in terms of the three neutrino masses in the large tau mass limit. This result is testable, and it implies that the normal neutrino mass ordering is more likely to coincide with the observed structure of $U$.

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*E-mail: xingzz@ihep.ac.cn
In a straightforward extension of the standard electroweak model which allows its three neutrinos to be massive, a nontrivial mismatch between the mass and flavor eigenstates of leptons or quarks arises from the fact that lepton or quark fields can interact with both scalar and gauge fields, leading to the puzzling phenomena of flavor mixing and CP violation [1]. The $3 \times 3$ lepton and quark flavor mixing matrices appearing in the weak charged-current interactions are referred to, respectively, as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix $U$ [2] and the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V$ [3]:

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \left[ (e \mu \tau)_L \gamma^\mu U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W^-_\mu + (u \, c \, t)_L \gamma^\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W^+_\mu \right] + \text{h.c.} \quad (1)$$

with all the fermion fields being the mass eigenstates. By convention $U$ and $V$ are defined to be associated with $W^-$ and $W^+$, respectively. In Eq. (1) the charged leptons and the quarks with the same electric charges all have the normal mass hierarchies (i.e., $m_e \ll m_\mu \ll m_\tau$, $m_u \ll m_c \ll m_t$ and $m_d \ll m_s \ll m_b$ [4]). Yet it remains unclear whether the three neutrinos also have a normal mass ordering ($m_1 < m_2 < m_3$). Now that $m_1 < m_2$ has been fixed from solar neutrino oscillations [5], the only possible “abnormal” mass ordering is $m_3 < m_1 < m_2$. The neutrino mass ordering is one of the central concerns in particle physics, and it will be determined in the foreseeable future with the help of either an accelerator-based neutrino oscillation experiment (e.g., T2K and NOνA [6]) or a reactor-based antineutrino oscillation experiment (e.g., JUNO [7]), or both of them. Depending on the neutrino mass ordering to be normal or abnormal, a number of physical processes such as the neutrinoless double-beta decay will have remarkably different observable consequences. If $m_3 < m_1 < m_2$ turned out to be true in nature, one would have to explain what is behind it at a fundamental level.

On the other hand, the observed pattern of the PMNS matrix $U$ is very different from that of the CKM matrix $V$. The latter exhibits an approximate symmetry about its diagonal axis and an obvious hierarchy among its nine elements (i.e., $|V_{tb}| > |V_{ud}| > |V_{cs}| \gg |V_{us}| > |V_{cd}| \gg |V_{cb}| > |V_{ts}| \gg |V_{td}| > |V_{ub}|$ [8]), while the former is structurally less symmetrical and hierarchical. This difference might be attributed to a non-hierarchical neutrino mass spectrum, or distinct underlying flavor symmetries which are separately associated with leptons and quarks. In the lack of a full flavor theory, it is a big challenge to resolve this kind of problem. Some great ideas such as grand unifications, supersymmetries and extra dimensions are not very helpful to deal with most of the flavor puzzles, and current exercises of various group languages or flavor symmetries are too divergent to converge to something unique [9]. Although an access to the quark flavor mixing structure seems possible in certain quark mass limits [10], no prediction in regard to it has been ventured.

In this work we show that the abnormal neutrino mass ordering $m_3 < m_1 < m_2$ looks quite unnatural when it is related to the observed pattern of lepton flavor mixing in a way similar to the reasonable correlation between the quark mass spectrum and the CKM matrix. Taking account of the freedom in choosing the basis of weak interactions, we predict

$$\left| \frac{U_{\tau 1}}{U_{\tau 2}} \right| = \sqrt{\frac{m_1 \cdot m_3 + m_1 \cdot m_3 + m_2}{m_2 \cdot m_3 - m_1 \cdot m_3 - m_2}} \quad (2)$$

in the large tau mass limit. This novel relationship is more likely to coincide with the present and future experimental data if the neutrinos have a normal mass ordering $m_1 < m_2 < m_3$. 

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Given the abnormal neutrino mass ordering $m_3 < m_1 < m_2$, one may “renormalize” it to $m'_1 < m'_2 < m'_3$ by setting $m'_1 = m_3$, $m'_2 = m_1$ and $m'_3 = m_2$. In this case the corresponding neutrino mass eigenstates are $\nu'_1 = \nu_3$, $\nu'_2 = \nu_1$ and $\nu'_3 = \nu_2$. Then the weak charged-current interactions in Eq. (1) can be rewritten as

$$-\mathcal{L}_{cc}' = \frac{g}{\sqrt{2}} \left[ (e - \mu - \tau)_L \gamma^\mu \begin{pmatrix} \nu'_1 \\ \nu'_2 \\ \nu'_3 \end{pmatrix}_L W^-_\mu + (u - c - t)_L \gamma^\mu V \begin{pmatrix} d \\ s \end{pmatrix}_L W^+_\mu \right] + \text{h.c.}$$

(3)

in the normal neutrino mass ordering (i.e., $m'_1 < m'_2 < m'_3$) basis, where $U'$ comes from $U$ through the reordering transformation $(\nu_1, \nu_2, \nu_3) \rightarrow (\nu'_2, \nu'_3, \nu'_1)$. As a result,

$$U' = \begin{pmatrix} U'_{e1} & U'_{e2} & U'_{e3} \\ U'_{\mu1} & U'_{\mu2} & U'_{\mu3} \\ U'_{\tau1} & U'_{\tau2} & U'_{\tau3} \end{pmatrix} = U \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}. \tag{4}$$

A global $\chi^2$ analysis of those currently available neutrino oscillation data [11] allows us to obtain the magnitudes of nine elements of $U'$ at the 3$\sigma$ confidence level:

$$|U'| = \begin{pmatrix} 0.126 \rightarrow 0.178 & 0.795 \rightarrow 0.846 & 0.513 \rightarrow 0.585 \\ 0.579 \rightarrow 0.808 & 0.205 \rightarrow 0.543 & 0.416 \rightarrow 0.730 \\ 0.567 \rightarrow 0.800 & 0.215 \rightarrow 0.548 & 0.409 \rightarrow 0.725 \end{pmatrix}, \tag{5}$$

in which the unitarity of $U'$ has been assumed and thus its elements are correlated with one another. The smallest element of $U'$ is located on its upper left corner, while that of $U$ is located on its upper right corner. This structural difference implies that the flavor mixing angles of $U'$ must be very different from those of $U$ in a given parametrization. Let us adopt the “standard” parametrization advocated by the Particle Data Group [4],

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{23}e^{i\delta} & s_{13}c_{23}e^{-i\delta} \\ -s_{12}c_{13} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P_\nu, \tag{6}$$

where $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$ (for $ij = 12, 13, 23$), and $P_\nu = \text{Diag}\{e^{i\nu}, e^{i\nu}, 1\}$ denotes the Majorana phase matrix provided three massive neutrinos are the Majorana particles. $U'$ takes the same parametrization with $P'_\nu = \text{Diag}\{e^{i\nu}, e^{i\nu}, 1\}$ coming from $P_\nu$ through the reordering transformation $(\nu_1, \nu_2, \nu_3) \rightarrow (\nu'_2, \nu'_3, \nu'_1)$. The exact analytical relations between the two sets of flavor mixing parameters of $U$ and $U'$ are given by

$$t'_{12} = \frac{U'_{e2}}{U'_{e1}} = \frac{U_{e1}}{U_{e2}} = \frac{c_{12}c_{13}}{s_{13}},$$

$$s'_{13} = \frac{U'_{e3}}{U_{e3}} = \frac{s_{12}}{s_{13}},$$

$$t'_{23} = \frac{U'_{\mu2}}{U'_{\tau2}} = \frac{U_{\mu2}}{U_{\tau2}} = \frac{\sqrt{c_{12}^2c_{23}^2 - 2c_{12}s_{12}s_{13}c_{23}^3c_\delta + s_{12}^2s_{13}^2s_{23}^2}}{\sqrt{c_{12}^2c_{23}^2 + 2c_{12}s_{12}s_{13}c_{23}c_\delta + s_{12}^2s_{13}^2s_{23}^2}},$$

$$s'_{\delta} = \frac{\text{Im}(U'_{e2}U'_{\mu2}^*U_{e3}U_{\mu3}^*)}{c_{12}^2s_{13}^2c_{23}^2s_{13}^2s_{23}} = \frac{\text{Im}(U_{e1}U_{\mu2}U'_{e3}U'_{\mu1}^*)}{c_{12}^2s_{13}^2c_{23}^2s_{13}^2s_{23}^2s_{\delta}} = \frac{s_{12}^2c_{13}s_{13}c_{23}s_{23}^2}{c_{13}^2s_{13}^2c_{23}^2s_{13}^2s_{23}}, \tag{7}$$

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where \( t'_{ij} \equiv \tan \theta'_{ij} \), \( c_\delta \equiv \cos \delta \), \( s_\delta \equiv \sin \delta \) and \( s'_\delta \equiv \sin \delta' \). In view of Eq. (5) or Ref. [11] with \( \delta \in [0^\circ, 360^\circ] \) and \( \delta' \in [0^\circ, 360^\circ] \), we arrive at the 3\( \sigma \) ranges of three flavor mixing angles as

\[
\begin{align*}
\theta'_{12} &= 77.4^\circ \rightarrow 81.5^\circ, \quad \theta'_{13} = 30.9^\circ \rightarrow 35.8^\circ, \quad \theta'_{23} = 29.8^\circ \rightarrow 60.7^\circ; \\
\theta_{12} &= 31.1^\circ \rightarrow 35.9^\circ, \quad \theta_{13} = 7.2^\circ \rightarrow 10.0^\circ, \quad \theta_{23} = 35.8^\circ \rightarrow 54.8^\circ.
\end{align*}
\]

(8)

In comparison, the three quark flavor mixing angles in the same parametrization of the CKM matrix \( V \) read \( \vartheta'_{12} = 13.023^\circ \pm 0.038^\circ \), \( \vartheta_{13} = 201^\circ \pm 0.068^\circ \) and \( \vartheta_{23} = 2361^\circ \pm 0.068^\circ \), extracted from current experimental data as given by [4].

\[
|V| = \begin{pmatrix}
0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351 \pm 0.00015 \\
0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412 \pm 0.00011 \\
0.00867 \pm 0.00029 & 0.0404 \pm 0.00011 & 0.99914 \pm 0.000021
\end{pmatrix}.
\]

(9)

The CP-violating phase of \( V \), denoted as \( \delta_q \) in this parametrization, has also been determined to a good degree of accuracy: \( \delta_q = 69.21^\circ \pm 2.55^\circ \) [4]. It seems very hard to find any potentially interesting numerical correlation between the three lepton mixing parameters \( (\theta'_{12}, \theta'_{13}, \theta'_{23}) \) and the three quark mixing parameters \( (\vartheta_{12}, \vartheta_{13}, \vartheta_{23}) \), and the previous phenomenological conjectures \( \theta_{12} + \vartheta_{12} = 45^\circ \) and \( \theta_{23} + \vartheta_{23} = 45^\circ \) are subject to the parametrization (or the flavor basis) itself and maybe unstable against the renormalization-group running effects [12]. Although one may similarly make the conjectures like \( \theta'_{12} + \vartheta_{12} = 90^\circ \) and \( \theta'_{23} + \vartheta_{23} = 45^\circ \), they are unlikely to provide us with any enlightening information about the underlying dynamics of lepton and quark flavor mixing.

Compared with the normal mass hierarchies of up- and down-type quarks which are associated with a hierarchical structure of the CKM matrix \( V \), the “normalized” neutrino mass ordering \( m'_{1} < m'_{2} < m'_{3} \) corresponds to a quite unnatural structure of the PMNS matrix \( U' \) as illustrated in Eq. (5). The normal neutrino mass ordering \( m_{1} < m_{2} < m_{3} \) is favored in this connection, because its corresponding PMNS matrix \( U \) has a somewhat more natural structure [11].

\[
|U| = \begin{pmatrix}
0.795 \rightarrow 0.846 & 0.513 \rightarrow 0.585 & 0.126 \rightarrow 0.178 \\
0.205 \rightarrow 0.543 & 0.416 \rightarrow 0.730 & 0.579 \rightarrow 0.808 \\
0.215 \rightarrow 0.548 & 0.409 \rightarrow 0.725 & 0.567 \rightarrow 0.800
\end{pmatrix}.
\]

(10)

This kind of “naturalness” might come from a comparison between \( U \) and \( V \), in contrast to a comparison between \( U' \) and \( V' \). However, one should keep in mind that the definitions of \( U \) and \( V \) are related to \( W^- \) and \( W^+ \), respectively. If \( \mathcal{L}_{cc} \) in Eq. (1) is rewritten as

\[
-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \begin{pmatrix}
\mu & \tau & d & s & b
\end{pmatrix}_L \gamma^\mu \begin{pmatrix}
U & 0 \\
0 & V^\dagger
\end{pmatrix} \begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3 \\
c \\
t
\end{pmatrix}_L W^- + \text{h.c.},
\]

then it seems more reasonable to compare between \( U \) and \( V^\dagger \) instead of \( V \) itself. But the magnitudes of six off-diagonal elements of \( V \) exhibit an approximate symmetry about its
diagonal axis, and therefore $|V| \simeq |V'|$ is actually a good approximation. Defining $V' \equiv V^\dagger$ and taking the same “standard” parametrization for it, we obtain $\vartheta'_{12} = 13.015^\circ \pm 0.038^\circ$, $\vartheta'_{13} = 0.497_{-0.018^\circ}^{+0.016^\circ}$, $\vartheta'_{23} = 2.315_{-0.028^\circ}^{+0.064^\circ}$ and $\delta'_{q} = -22.23_{-1.11^\circ}^{+1.33^\circ}$. It is again difficult to see any meaningful correlation between $U$ and $V'$ or between $U'$ and $V''$.

Now let us turn to the mass matrices of leptons and quarks so as to look at their possible relations with the observed structures of $U$ and $V$. Assuming massive neutrinos to be the Majorana particles, we write the lepton and quark mass terms as

$$-\mathcal{L}_{\text{mass}} = \mathbf{E}^\dagger M_\ell \mathbf{E} + \frac{1}{2} \mathbf{N}_L^\dagger M_\nu N_L + \mathbf{U}_L^\dagger M_u U_R + \mathbf{D}_L^\dagger M_d D_R + \text{h.c.},$$

where $\mathbf{E}$, $\mathbf{U}$ and $\mathbf{D}$ are the column vectors of charged leptons, up- and down-type quarks, respectively; $\mathbf{N}_L^\dagger$ represents the charge conjugation of the column vector of three left-handed neutrinos $\mathbf{N}_L$; and the effective Majorana neutrino mass matrix $M_\nu$ is symmetrical. Because the standard weak interactions do not involve any flavor-changing right-handed currents, the relevant physics keeps unchanged if each right-handed column vector in Eq. (12) undergoes an arbitrary unitary transformation. Without loss of generality, this kind of freedom allows us to do two things: (a) making $M_\ell$, $M_u$ and $M_d$ all Hermitian [13]; (b) obtaining three zeros for $M_u$ and $M_d$ in a suitable flavor basis, and similarly three zeros for $M_\ell$ and $M_\nu$ (by convention, a pair of off-diagonal zeros in a Hermitian or symmetrical fermion mass matrix is always counted as one zero) [14]. Here we work in the basis where both $M_\nu$ and $M_d$ take the Fritzsch texture [15], while $M_\ell$ and $M_u$ are arbitrary Hermitian matrices:

$$M_u = \begin{pmatrix} E_u & C_u & F_u \\ C_u^* & D_u & B_u \\ F_u^* & B_u^* & A_u \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & C_d & 0 \\ C_d^* & 0 & B_d \\ 0 & B_d^* & A_d \end{pmatrix},$$

$$M_\ell = \begin{pmatrix} E_\ell & C_\ell & F_\ell \\ C_\ell^* & D_\ell & B_\ell \\ F_\ell^* & B_\ell^* & A_\ell \end{pmatrix}, \quad M_\nu = \begin{pmatrix} 0 & C_\nu & 0 \\ C_\nu^* & 0 & B_\nu \\ 0 & B_\nu^* & A_\nu \end{pmatrix}.$$

(13)

It is worth stressing that the three zeros in either $M_d$ or $M_\nu$ come from a proper choice of the flavor basis and do not have any physical content by themselves, but any more texture zeros in the lepton or quark sector must be subject to a phenomenological assumption or some kind of model dependence [16]. Defining $\hat{H}_x \equiv M_x M_x^\dagger$ (for $x = u, d$ or $\ell$), one may diagonalize it by means of a unitary transformation $O_x^\dagger H_x O_x = \hat{H}_x$ (i.e., $\hat{H}_u = \text{Diag}\{m_u^2, m_c^2, m_t^2\}$, $\hat{H}_d = \text{Diag}\{m_d^2, m_s^2, m_b^2\}$ or $\hat{H}_\ell = \text{Diag}\{m_e^2, m_\mu^2, m_\tau^2\}$). Furthermore, the Majorana neutrino mass matrix $M_\nu$ can be diagonalized via the transformation $O_\nu^\dagger M_\nu O_\nu = \tilde{M}_\nu = \text{Diag}\{m_1, m_2, m_3\}$. Thanks to the Fritzsch form of $M_d$ or $M_\nu$, the elements of $O_d$ or $O_\nu$ can be exactly calculated in terms of the mass ratios of three down-type quarks or three neutrinos [18]. The main issue for now is how to deal with $M_\ell$ or $M_u$, such that the elements of the PMNS matrix $U = O_\ell^\dagger O_\nu$ and those of the CKM matrix $V = O_u^\dagger O_d$ can be fully or partially calculated via

$$U_{\alpha i} = \sum_{k=1}^{3} (O_\ell)_{k\alpha} (O_\nu)_{ki}, \quad V_{\alpha i} = \sum_{k=1}^{3} (O_u)_{k\alpha}^* (O_d)_{ki},$$

(14)

in which the subscripts $\alpha$ and $i$ run over $(e, \mu, \tau)$ and $(1, 2, 3)$ for $U$ or over $(u, c, t)$ and $(d, s, b)$ for $V$, respectively. The possibilities of assuming extra zeros in or imposing a certain flavor
symmetry on the textures of $M_\ell$ and $M_u$ have already been discussed in the literature [16]. But is there any other way to proceed?

We find that it should be a reasonable approximation to deal with $H_u$ (or $H_\ell$) in the large top (or tau) mass limit, simply because $m_t$ (or $m_\tau$) is the largest mass among the six quark (or lepton) masses. In the $m_t \rightarrow \infty$ limit the top quark is expected to be essentially decoupled from the up and charm quarks in $H_u$ [17], and a similar situation exists in $H_\ell$ in the $m_\tau \rightarrow \infty$ limit. Therefore, we have

$$\lim_{m_t \rightarrow \infty} H_u \propto \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & \infty \end{pmatrix}, \quad \lim_{m_t \rightarrow \infty} O_u \propto \begin{pmatrix} x & x & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\lim_{m_\tau \rightarrow \infty} H_\ell \propto \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & \infty \end{pmatrix}, \quad \lim_{m_\tau \rightarrow \infty} O_\ell \propto \begin{pmatrix} x & x & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

(15)

where the symbol “$x$” denotes a nonzero matrix element. A combination of Eqs. (14) and (15) leads us to the novel predictions

$$\lim_{m_t \rightarrow \infty} \left| \frac{V_{td}}{V_{ts}} \right| = \frac{|(O_d)_{3d}|}{|(O_d)_{3s}|} = \sqrt{\frac{m_t \cdot m_b + m_d \cdot m_s}{m_s \cdot m_b - m_d \cdot m_b - m_s}}$$

(16)

for the lower left corner of the CKM matrix $V$ and

$$\lim_{m_\tau \rightarrow \infty} \left| \frac{U_{\tau 1}}{U_{\tau 2}} \right| = \frac{|(O_\nu)_{31}|}{|(O_\nu)_{32}|} = \sqrt{\frac{m_1 \cdot m_3 + m_1 \cdot m_3 + m_2}{m_2 \cdot m_3 - m_1 \cdot m_3 - m_2}}$$

(17)

for the lower left corner of the PMNS matrix $U$. In obtaining Eqs. (16) and (17), we have used the exact analytical expressions of $O_d$ and $O_\nu$ given in Ref. [18] for the Fritzsch texture of $M_d$ and $M_\nu$. Taking account of the central values of $m_d = 2.82 \pm 0.48$ MeV, $m_s = 57^{+18}_{-12}$ MeV and $m_b = 2.86^{+0.16}_{-0.06}$ GeV at the electroweak scale [19], we arrive at $|V_{td}|/|V_{ts}| = 0.227$. Although its error bar is rather appreciable due to uncertainties of the quark masses (in particular, the big error associated with $m_s$), this result is in good agreement with the accurate experimental value $|V_{td}|/|V_{ts}| = 0.215^{+0.010}_{-0.013}$ [4]. The phenomenological success of Eq. (16) encourages us to conjecture that Eq. (17) is very likely to make sense in revealing an underlying correlation between the neutrino mass spectrum and the lepton flavor mixing structure. We illustrate the dependence of $|U_{\tau 1}|/|U_{\tau 2}|$ on the smallest neutrino mass $m_1$ (normal mass ordering or NMO) or $m_3$ (abnormal mass ordering or AMO) in FIG. 1, where we have input $\Delta m^2_{21} \equiv m^2_2 - m^2_1 = (7.00 \rightarrow 8.09) \times 10^{-5}$ eV$^2$ and $\Delta m^2_{31} \equiv m^2_3 - m^2_1 = (2.276 \rightarrow 2.695) \times 10^{-3}$ eV$^2$ (NMO) or $\Delta m^2_{32} \equiv m^2_3 - m^2_2 = (-2.649 \rightarrow -2.242) \times 10^{-3}$ eV$^2$ (AMO) at the 3$\sigma$ level [11]. Because of the unknown CP-violating phase $\delta$ and some uncertainties associated with the flavor mixing angles, the present experimental bound on the left-hand side of Eq. (17) remains quite loose: $|U_{\tau 1}|/|U_{\tau 2}| = 0.30 \rightarrow 1.34$ as indicated by Eq. (10). In this case either the NMO (i.e., $m_1 < m_2 < m_3$) or the AMO (i.e., $m_3 < m_1 < m_2$) can coincide with current neutrino oscillation data, but the former is apparently favored by the observed pattern of lepton flavor mixing, as shown in FIG. 1. In the foreseeable future more precise experimental data will test the validity of Eq. (17) and single out the correct neutrino mass ordering. Some comments and discussions are in order.
• In the AMO case \( m_1 \) and \( m_2 \) are nearly equal, and thus \(|U_{\tau 1}|/|U_{\tau 2}| \gtrsim 1\) holds, as one can clearly see from FIG. 1. If the experimental result of \(|U_{\tau 1}|/|U_{\tau 2}|\) turns out to be less than one, then Eq. (17) will definitely lead us to the NMO. FIG. 1 also tells us that the possibility of \( m_1 \to 0 \) has been ruled out by Eq. (17), and the possibility of \( m_3 \to 0 \) is still allowed but seems not to be favored.

• In the standard parametrization of \( U \), Eq. (17) establishes a relationship between the unknown neutrino mass scale and the unknown CP-violating phase \( \delta \):

\[
\frac{m_1 \cdot m_3 + m_1}{m_2 \cdot m_3 - m_1} \cdot \frac{m_3 + m_2}{m_3 - m_2} = \frac{s_{12}^2 s_{23}^2 - 2 c_{12} s_{12}^2 s_{13} c_{23} s_{23} c_\delta + c_{12}^2 s_{13}^2}{c_{12}^2 s_{23}^2 + 2 c_{12}^2 s_{12} s_{13} c_{23} s_{23} c_\delta + s_{12}^2 s_{13}^2 c_{23}}.
\]

(18)

Although this kind of correlation is subject to the large tau mass limit and more likely a leading-order approximation, it deserves an experimental test. A similar correlation in the quark sector described by Eq. (16) does survive the current experimental test.

Finally, let us emphasize that any kind of model building on the flavor structures of leptons and quarks depends on the choice of a specific flavor basis. The basis chosen in Eq. (13) is just such a case, but it has nothing to do with the phenomenological assumptions. It is the large top or tau mass limit taken in Eq. (15) that makes the novel predictions in Eqs. (16) and (17) possible. If the latter also proves to be a success in phenomenology, then one will be well motivated to search for the underlying flavor dynamics.

In summary, we have tried to explore some possible implications of a normal or abnormal neutrino mass ordering on the lepton flavor mixing structure. We show that \( m_3 < m_1 < m_2 \) looks quite unnatural after it is “renormalized” and related to the PMNS matrix in a way similar to the reasonable correlation between the quark mass spectrum and the CKM matrix. With the help of the freedom in choosing the basis of weak interactions, we have made the nontrivial predictions for \(|V_{td}|/|V_{ts}|\) in terms of the three down-type quark masses in the large top mass limit and \(|U_{\tau 1}|/|U_{\tau 2}|\) in terms of the three neutrino masses in the large tau mass limit. We find that the former is in good agreement with current experimental data, and the latter provides us with a preliminary hint that the normal neutrino mass ordering \( m_1 < m_2 < m_3 \) is more likely to coincide with the observed structure of \( U \).

It is certainly a theoretical challenge to pin down what is behind the normal or abnormal neutrino mass ordering [20] and whether there is a definite correlation between the fermion mass spectra and flavor mixing patterns. Our present attempt in this connection remains quite limited, but it has led us to some encouraging and interesting results. The underlying flavor theory, which might be related to a certain flavor symmetry and its spontaneous or explicit breaking mechanism, should finally give us a dynamical reason for the phenomena of lepton and quark flavor mixing and CP violation. But we are now following the opposite way to look for such a fundamental theory from the bottom up.

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FIG. 1. The dependence of $|U_{\tau 1}|/|U_{\tau 2}|$ on the smallest neutrino mass $m_1$ (normal mass ordering) or $m_3$ (abnormal mass ordering), as predicted by Eq. (17). Here the red and blue regions of the two curves arise from the 3σ ranges of two neutrino mass-squared differences, and the grey band stands for the range of $|U_{\tau 1}|/|U_{\tau 2}|$ allowed by current neutrino oscillation data [11].