Deeply virtual vector meson electroproduction at small Bjorken-\(x\)

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Abstract. It is reported on an analysis of vector meson electroproduction at small Bjorken-\(x\) within the handbag approach. Using a model for the generalized parton distributions (GPDs) and calculating the partonic subprocess, electroproduction off gluons, within the modified perturbative approach, cross sections and spin density matrix elements (SDME) are evaluated. The numerical results of this analysis agree fairly well with recent HERA data.

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It has been shown \[1\] that, at large photon virtuality \(Q^2\), meson electroproduction factorizes in a partonic subprocess, electroproduction off gluons or quarks, \(\gamma^* g(q) \rightarrow V g(q)\), and GPDs, representing soft proton matrix elements (see Fig. 1). At small \(x_{\text{Bj}}\) the quark subprocesses can be ignored. In the following I am going to report on an analysis of vector meson electroproduction within this handbag approach \[2\] in the kinematical regime of large \(Q^2\) and large energy \(W\) in the photon-proton c.m.s. but small \(x_{\text{Bj}}\) and Mandelstam \(t\). An exploratory study of the longitudinal cross section \(\sigma_L\) for \(\gamma^* p \rightarrow V p\) has been performed by Mankiewicz et al. \[3\] within this approach. Effects of the GPDs have been estimated by Martin et al \[4\].

The structure of the proton is rather complex. In correspondence to its four form factors there are four gluon GPDs \(H^g, E^g, \tilde{H}^g\) and \(\tilde{E}^g\) and four for each quark flavour. All GPDs are functions of three variables, \(t\), skewness \(\xi\) and the average momentum fraction \(\bar{x}\), the latter two are defined by

\[
\xi = \frac{(p-p')^+}{(p+p')^+}, \quad \bar{x} = \frac{k^+}{p^+}.
\]

These parameters are related to the usual momentum fractions the gluons carry with respect to their parent proton, by \(x^{(i)} = (\bar{x} \pm \xi)/(1 \pm \xi)\). The skewness is kinematically fixed to \(\xi \simeq x_{\text{Bj}}/2\) in a small \(x_{\text{Bj}}\) approximations. Hence, \(x \neq x'\). This is to be contrasted with the leading \(\log(1/x_{\text{Bj}})\) approximation \[5\] where \(x \simeq x' \simeq x_{\text{Bj}}\) is assumed and the GPD replaced by the usual gluon distribution \(g(x)\).

The handbag approach leads to the following proton helicity non-flip amplitude

\[
M_{\mu+,\mu+}^V = \frac{e^2}{2} \sigma_V \int_0^1 \frac{d\bar{x}}{\bar{x} + \xi} \left[ H_{\mu+,\mu+}^V (\bar{x} + \xi + i\epsilon) [H_{\mu',\mu+}^V + H_{\mu',\mu-}^V] H^g(\bar{x}, \xi, t) \right].
\]

Contributions from other GPDs can be neglected at small \(x_{\text{Bj}}\) and for unpolarized protons. The photon and meson helicities are denoted by \(\mu\) and \(\mu'\), respectively. The
FIGURE 1. Left: The handbag diagram for meson electroproduction off protons. The large blob represents a GPD while the small one stands for the subprocess. The momenta of the involved particles are specified. Right: Model results for the GPD $H^g$ at $t \approx 0$ and for the case $n = 1$. The results for $n = 2$ are similar. The solid (dashed, dash-dotted) line represents the GPD at $\xi = 5 \ (1, 0.5) \cdot 10^{-3}$ and at a scale of 2 GeV.

explicit labels in the full (subprocess) amplitude, $M^V (H^V)$ refer to the helicities of the protons (gluons).

The GPDs are controlled by non-perturbative QCD. In the absence of an GPD analysis in analogy to those of the usual PDFs (see however [6]) one has to rely on a model. Its construction is however not an easy matter since the GPDs are functions of three variables. Factorising the $t$ dependence from the $\bar{x}, \xi$ one is probably incorrect. We therefore restrict ourselves to the forward direction and exploit the ansatz for a double distribution proposed in Ref. [7] ($n = 1, 2$)

$$f(\beta, \alpha, t \approx 0) = g(\beta) \frac{\Gamma(2n+2)}{2^{2n+1} \Gamma^2(n+1)} \frac{[(1 - |\beta|)^2 - \alpha^2]n}{(1 - |\beta|)^{2n+1}}. \quad (3)$$

The GPDs is then obtained by an integral over $f$

$$H^g(\bar{x}, \xi) = \left[ \Theta(0 \leq \bar{x} \leq \xi) \int_{x_3}^{x_1} d\beta + \Theta(\xi \leq \bar{x} \leq 1) \int_{x_2}^{x_1} d\beta \right] \frac{\beta}{\xi} f(\beta, \alpha = \frac{\bar{x} - \beta}{\xi}). \quad (4)$$

Using the NLO CTEQ5M [8] result as input we obtain the GPD $H^g$ shown in Fig. [1].

The last item of the amplitude (2) to be discussed is the subprocess amplitude. Its treatment is rather standard, it only differs in detail from versions to be found in the literature [4, 9]. In the modified perturbative approach invented by Sterman and collaborators [10], in which quark transverse momenta are retained and gluonic radiative corrections in the form of a Sudakov factor are taken into account, it reads

$$\mathcal{M}^V = \int \frac{d\tau dk^2}{\sqrt{216\pi^2}} \Psi_V \text{Tr} \left\{ (\gamma^\mu + m_V)\gamma^\nu T_0 - \frac{k^2 g_{\alpha\beta}}{2M_V} \{ (\gamma^\mu + m_V)\gamma^\nu, \gamma_\alpha \} \Delta T_\beta \right\}. \quad (5)$$

Higher order terms in this expansion are not shown. Gaussians for the meson’s wavefunctions, $\Psi_V = \Psi(\tau, k^2_\perp)$, are used which may depend on the polarization of the vector
There are two parameters specifying the wavefunction, the meson’s decay constant and a transverse size parameter. For longitudinally polarized vector mesons these parameters are fairly well-known. The first term in Eq. (5) dominates for $V_L$ while it is approximately zero for transversally polarized vector mesons ($V_T$). In the latter case the second term is the dominant one. Note that the soft physics parameter $M_V$ in this term is of order of the vector meson mass $m_V$. As can be seen from Eq. (5) the $L \rightarrow L$ transition is dominant while the $T \rightarrow T$ one is of relative order $\langle k^2 \rangle_1^{1/2}/Q$ and the $T \rightarrow L$ one of $\sqrt{-t}/Q$. The latter amplitude is tiny and only noticeable in some of the SDMEs. All other transitions are negligible. Eventual infrared singularities that may occur for transitions to $V_T$, are regularized in the modified perturbative approach.

Before comparing the results to experiment I have to comment on the $t$ dependence of the amplitudes. Exponentials in $t$ are assumed with slopes $B_{LL(TT)}^V$ taken from experiment. Combined with the calculated forward amplitudes one can evaluate the integrated cross sections and the SDME at small $t$. From (5) one sees that the size of the $T \rightarrow T$ amplitude is controlled by the following product of parameters

$$|M_{TT}^V| \propto \left( \frac{f_T^V}{M_V} \right)^2 \frac{1}{B_{TT}^V}. \quad (6)$$

Without precise $t$-dependent data at disposal only this product is probed. One can therefore, for instance, assume $B_{TT}^V \simeq B_{LL}^V/2$. Combined with $M_V = m_V$ and $f_T^0 = 250$ MeV this assumption provides reasonable results for vector meson electroproduction, see Fig. 2. An alternative choice is $B_{TT}^V \simeq B_{LL}^V$, $M_V = m_V$, $f_T^0 = 170$ MeV which leads to practically the same results for the cross sections. Only the $t$ dependence of the SDME differs in both the cases. Given the accuracy of the present data [11,12] both the scenarios are in agreement with experiment.

In Fig. 2 the cross section $\sigma_L$ and the ratio $R = \sigma_L/\sigma_T$ are displayed. The data on $R$ are extracted from the SDME measurements. This extraction is problematic if the slopes are different. For comparison the ratio of the corresponding differential cross sections is also shown in Fig. 2 (at $t \simeq -0.15$ GeV$^2$). Results for the SDME of $\rho$ and $\phi$ mesons are also presented in [2] in fair agreement with experiment.

Finally I want to comment on the $W$ dependence of the dominant longitudinal cross section. It is given by the imaginary part of the $L \rightarrow L$ amplitude with a correction of about 10% from the real part. The cross section is therefore approximately proportional to $|H^g(\xi, \xi)|^2$. Through the model (4) the low-x behaviour of the PDF $\bar{x}g(\bar{x}) \sim \bar{x}^{-\delta(Q^2)}$ is transferred to the GPD and one finds

$$\sigma_L \propto W^{-4\delta(Q^2)}. \quad (7)$$

The $Q^2$ dependence of $\delta$ is a consequence of evolution. Comparison with experiment reveals that this behaviour is in remarkable agreement with the data within admittedly large errors. A last remark: The expression for $\sigma_L$ the GPD approach provides, is also obtained in the leading log $1/\xi_{BJ}$ approximation given that the subprocess is treated equally and that $H^g(\xi, \xi)$ is replaced by $2\xi g(2\xi)$. The quality of this approximation is rather good for $\xi \lesssim 10^{-2}$, there is only an enhancement of the GPD by about 18%, the skewing effect [4]. For increasing $\xi$ the approximation becomes gradually worse.
FIGURE 2. Left: The integrated cross section for $\gamma p \rightarrow \rho p$ versus $Q^2$ at $W \simeq 75$ GeV. Right: The ratio of longitudinal and transverse cross sections for $\rho$ production versus $Q^2$ at $W \simeq 75$ GeV. Data taken from [11] (filled squares) and [12] (open symbols). The solid (dashed) lines are the results of Ref. [2] for the ratio of differential (integrated) cross sections.

I summarize: Vector meson electroproduction off unpolarized protons at small $x_{Bj}$ and small $t$ probes the GPD $H^g$. Calculating the partonic subprocess within the modified perturbative approach (using gaussian wavefunctions) fair agreement with HERA data on the integrated cross sections for longitudinally and transversally polarized virtual photons and the spin density matrix elements are obtained for electroproduction of $\rho$ and $\phi$ mesons. It is to be stressed that only the forward amplitudes are calculated within the GPD approach. Their $t$ dependencies are assumed to be exponentials with slopes taken from experiment. The present data do, however, not fix the slope of the $T \rightarrow T$ amplitude precisely. This treatment of the $t$ dependence is unsatisfactory and improvements are required. In principle the GPD approach has the potential to do better but the GPDs as a function of $t$ are needed for that. It is also possible to go to larger values of $x_{Bj}$ with it. Some results on $\phi$ production at COMPASS kinematics are presented in [2].

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