Homotopy perturbation method: when infinity equals five

Francisco M. Fernández

INIFTA (UNLP,CCT La Plata-CONICET), División Química Teórica,
Diag. 113 y 64 (S/N), Sucursal 4, Casilla de Correo 16,
1900 La Plata, Argentina

Abstract

I discuss a recent application of homotopy perturbation method to a heat transfer problem. I show that the authors make infinity equal five and analyze the consequences of that magic.

There has recently been great interest in the application of several approximate procedures, like the homotopy perturbation method (HPM), the Adomian decomposition method (ADM), and the variation iteration method (VIM), to a variety of linear and nonlinear problems of interest in theoretical physics [1–15]. From now on I will refer to those variation and perturbation approaches as VAPA. In a series of papers I have shown that most of the VAPA results are useless, nonsensical, and worthless [16–20]. In many of those papers the authors try to solve nonlinear problems by means of elaborated VAPA implementations that merely produce the Taylor expansion of the solutions. Of course, such approximate expressions do not give the overall picture of the dynamics, and the authors are merely content with a description of the initial stages of the evolution which do not tell us anything relevant about the

1 e-mail: fernande@quimica.unlp.edu.ar

Preprint submitted to Elsevier

21 October 2008
process. Other authors solve the Schrödinger equation and obtain trivial un-
physical solutions that are not square integrable. As an example I mention two
great feats of the VAPA users: the expansion of exponential functions of the
form $e^{i\alpha t}$ [15] and a prey–predator model that predicts a negative population
of rabbits [14] (see also my comments [18, 19]).

However, my criticisms have not been welcome because they lack “the qualities
of significant timeliness and novelty that we are seeking in this journal” and
for that reason they remain unpublished outside arXiv.

The purpose of this article is the analysis of a recently published paper that
certainly meets the criterion of timeliness and novelty sought in that journal.
Esmaeilpour and Ganji [5] applied homotopy perturbation method (HPM) to
the solution of the problem of forced heat convection over an horizontal flat
plate. After some algebraic manipulation of the Navier–Stokes equations they
obtained two coupled nonlinear differential equations: [5]

$$f'''(\eta) + \frac{1}{2} f(\eta) f''(\eta) = 0$$
$$\varepsilon \theta''(\eta) + \frac{1}{2} f(\eta) \theta'(\eta) = 0$$

with the boundary conditions

$$f(0) = f'(0) = 0, \ f'(\infty) = 1$$
$$\theta(0) = 1, \ \theta(\infty) = 0$$

(1)

The HPM yields series of the form

$$f = \sum_{j=0}^{\infty} f_j p^j, \ \theta = \sum_{j=0}^{\infty} \theta_j p^j$$

(3)

where the dummy perturbation parameter $p$ is set equal to unity at the end
of the calculation. Esmaeilpour and Ganji [5] choose the boundary conditions
\[ f_j(0) = 0, \frac{d f_j}{d \eta}(0) = 0, \frac{d^2 f_j}{d \eta^2}(\infty) = \delta_{j0} \]
\[ \theta_j(0) = \delta_{j0}, \theta_j(\infty) = 0 \] (4)

Surprisingly, the perturbation corrections \( f_j(\eta) \) and \( \theta_j(\eta) \) are polynomial functions of \( \eta \) [5] which cannot satisfy the boundary conditions at infinity (4) although the authors appear to state otherwise [5]. In fact, the approximate function

\[ f_{\text{HPM}}(\eta) = \frac{1348969}{7741440} \eta^2 - \frac{4867}{10752000} \eta^5 + \frac{451}{322560000} \eta^8 - \frac{1}{532224000} \eta^{11} \] (5)

corrected to third order \( (j \leq 3) \) does not satisfy the boundary conditions (2). However, the figures shown by Esmaeilpour and Ganji [5] exhibit a reasonable agreement between the exact and approximate solutions for \( 0 \leq \eta \leq 5 \).

When VAPA does not fit the problem the users make the problem fit VAPA. In this case Esmaeilpour and Ganji [5] do some kind of magic and make infinity equal five. Consequently, their approximate solutions satisfy the following boundary conditions

\[ f_j(0) = 0, \frac{d f_j}{d \eta}(0) = 0, \frac{d^2 f_j}{d \eta^2}(5) = \delta_{j0} \]
\[ \theta_j(0) = \delta_{j0}, \theta_j(5) = 0 \] (6)

Unfortunately, the authors forgot to say how they did this miracle. Since I am not that smart and still think that there is something else beyond that shrunk infinity I produced Fig. 1 that shows the actual behaviour of \( f'_{\text{HPM}}(\eta) \) in a wider interval.

When solving the differential equation for \( f \) one has to determine the value of \( f''(0) \) that is consistent with the boundary condition at infinity. Esmaeilpour and Ganji [5] do not discuss the calculation of this unknown parameter although they obtained the numerical solution by a standard software. Our straightforward approximate calculation based on trial and error suggests that
\( f''(0) \approx 0.3320574 \) and the HPM function (5) yields \( f''_{HPM}(0) = 0.349 \). The discrepancy is probably due to the fact that I have not been able to enter the shrunk infinity discovered by the authors. I suppose that for this very reason my contribution cannot be considered to carry the qualities of significant timeliness and novelty.

It is my opinion that VAPA have produced one of the greatest concentrations of bad papers I have ever seen. If the reader proves me wrong I will certainly apologize.

**References**

[1] M. Rafei, H. Daniali, D. D. Ganji, and H. Pashaei, Appl. Math. Comput. 188 (2007) 1419-1425.

[2] M. S. H. Chowdhury, I. Hashim, and O. Abdulaziz, Phys. Lett. A 368 (2007) 251-258.

[3] A. Yildirim and T. Özis, Phys. Lett. A 369 (2007) 70-76.

[4] M. S. H. Chowdhury and I. Hashim, Phys. Lett. A 365 (2007) 439-447.

[5] M. Esmaeilpour and D. D. Ganji, Phys. Lett. A 372 (2007) 33-38.

[6] D. D. Ganji, G. A. Afrouzi, H. Hosseinzadeh, and R. A. Talarposhti, Phys. Lett. A 371 (2007) 20-25.

[7] A. Sami Bataineh, M. S. M. Noorani, and I. Hashim, Phys. Lett. A 371 (2007) 72-82.

[8] M. S. H. Chowdhury and I. Hashim, Phys. Lett. A 372 (2008) 1240-1243.

[9] B.-G. Zhang, S.-Y. Li, and Z.-R. Liu, Phys. Lett. A 372 (2008) 1867-1872.

[10] A. Sami Bataineh, M.S.M. Noorani, and I. Hashim, Phys. Lett. A 372 (2008) 4062-4066.
[11] I. Mustafa, Phys. Lett. A 372 (2008) 356-360.

[12] A. Sami Bataineh, M.S.M. Noorani, and I. Hashim, 372 (2008) 613-618.

[13] A. Rafiq, M. Ahmed, and S. Hussain, Phys. Lett. A 372 (2008) 4973-4976.

[14] E. Yusufoglu and B. Erbas, Phys. Lett. A 372 (2008) 3829-3835.

[15] A. Sadighi and D. D. Ganji, Phys. Lett. A 372 (2008) 465-469.

[16] F. M. Fernández, Perturbation Theory for Population Dynamics, arXiv:0712.3376v1

[17] F. M. Fernández, On Some Perturbation Approaches to Population Dynamics, arXiv:0806.0263v1

[18] F. M. Fernández, On the application of homotopy-perturbation and Adomian decomposition methods to the linear and nonlinear Schrödinger equations, arXiv:0808.1515v1

[19] F. M. Fernández, On the application of the variational iteration method to a prey and predator model with variable coefficients, arXiv.0808.1875v2

[20] F. M. Fernández, On the application of homotopy perturbation method to differential equations, arXiv:0808.2078v2

[21] C. M. Bender and S. A. Orszag, Advanced mathematical methods for scientists and engineers, (McGraw-Hill, New York, 1978).
Fig. 1. Numerical (dashed line) and HPM (solid line) values of $f'(\eta)$