Computer simulation of special tomographic defectoscopy

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Abstract. The paper proposes an algorithm for the reconstruction of the defect of the product on the basis of non-destructive testing, based on the use of a reference sample of the studied object. It solves the two-dimensional problem of industrial tomography, which consists in the restoration of the image of the defect based on the use of a reference sample. Assuming that inside the object there is a defect of a flat elongated structure, the width of which is much smaller than the diameter of the object, it becomes possible to implement a fairly economical algorithm for defect reconstruction, taking into account only those elements of the projection matrix that correspond to the rays, which significantly pass through the defect.

1. Introduction
In worldwide practice, people constantly have to deal with the measurement, study of certain objects and phenomena those are very diverse in nature, in their manifestations, content, device, etc. At the same time, of course, it is good when the object of study is available for direct consideration and there are opportunities to study it both outside and inside. However, the situation becomes complicated in the case of remote access objects or when it is impossible to "look" inside such an object. To solve this problem, the researcher can use remote diagnostic methods and methods of introscopy, such methods include Computational (Computer) tomography (CT).

The method of computational (computer) tomography is a relatively young method of scientific research, which received its name at the same time as another relatively new scientific direction in applied mathematics. Outstanding scientific and practical achievements obtained by this method were awarded by the Nobel prize at the beginning of the second half of the last century and are associated with its implementation in medicine [1,2].

Intensive development of computerized tomography has led to the appearance of modern computer tomographs with unique potential that makes it possible to solve considerably more complicated problems, such as a problem of nondestructive testing of the quality of commercial products. However, it is rather difficult to provide quality processing of the testing data. It should be noted that the equipment is very complex and expensive. The volume of calculations and the amount of the measured data increase by several orders of magnitude with increase in resolution of one order of magnitude, as far as known methods are concerned.

The results obtained in industrial tomography are not more significant than those in medical tomography. Software for modern industrial tomographs in Russia is not satisfactory, whereas the specific character of the flaw detection problem requires new efficient algorithms of reconstruction.
2. Numerical simulation of a special problem of defectoscopy

In this problem, it is assumed that x-rays are used as penetrating radiation, and inside the product there is a defect of a flat elongated structure, the width of which is much smaller than the diameter of the object.

The radiation source is located on the border of the object, which is a circle of radius $R$. The initial problem is two-dimensional, the beams used as penetrating radiation represent a fan-shaped located chords, it is known the number of beams $N$ and the angle of the fan of rays $\Theta = (N-1)\delta\phi$, $0 < \Theta < \pi$, where $\delta\phi$ is the angular distance between adjacent rays. The observation system (source and receivers) rotates counterclockwise with a step along the boundary of the object $\delta\alpha = 2/M \pi$, where $M$ is the number of source locations.

A set of measurements made for a single source position is called a projection. By placing the projections as rows of the projection matrix, we compose the matrix $P$ of all of the projections with dimensions $M \times N$. It is assumed that the value of $N$ is chosen odd in order to avoid losses of the beam passing through the origin of coordinates. The path at the receiver output is measured by $P_{i,j}$ (beam-sum):

$$P_{i,j} = \mu(x,y) ds$$

where $\mu$ is the absorption coefficient, $ds$ is the element of the beam $l_{i,j}$ that connect the source with the $j$-th receiver in the $i$-th projection, $i = 1, 2, ..., M$, $j = 1, 2, ..., N$.

Denoting the area occupied by the defect as $D$, we assume to $\mu(x,y)$ the ratio

$$\mu(x,y) = \begin{cases} 1, & (x,y) \in D \\ 0, & (x,y) \notin D \end{cases}$$

This assumption allows us to model immediately the working projection matrix, which elements are the differences of the corresponding elements of the projection matrix $P_0$, obtained for the reference object and the corresponding elements of the projection matrix $P$, measured for the studying object. Thus, only those elements of the matrix $P$ that correspond to the rays passing through the defect will be nonzero.

It is required, when we have the information about the method of taking of projection data and the data itself in the form of a matrix $P$, to restore the defect by determining its location and geometric shape.

The method of solving the problem is based on splitting of the set of rays by their informativeness into 2 subsets, differing in the distance that the beam passes through the defect:

- informative rays those go through the defect sufficiently along its stretch;
- normal rays those go through the defect sufficiently across (in the cross stretch) and have no sence to the defect;

Of course, there is no clear boundary between these subsets, so there will be many more "transient" rays in the data, which cannot be clearly attributed to any of the classes.

It is also significant that in practice, due to the limited sensitivity of the recording equipment, normal rays will be absent, and "transient" rays referred to interference.

The proposed method of selection of informative rays is based on the fact that the rays with the maximal path length through the defect are informative in each projection. Thus, after selection of the maximum element in each row of the working matrix $P$, we obtain the sequence $R_i$, $i = 1, 2, ..., M$, where

$$R_i = \max_j \{P_{i,j}\}, \quad j = 1, 2, ..., N$$
Each element of the working vector $R$ is assigned by two indices, $i$ and $j$, which determine the corresponding ray. Arranging the elements of the vector $R(i,j)$ in descending order, we obtain the sequence, the elements of which, for sufficiently large values of $M$ and $N$ will not have the sharp "jumps" of values, but the first items will be significantly different from the last. This construction of the vector $R(i,j)$, which elements contain information about the projection numbers, leads to a one-parameter family of lines defined by the elements of the vector $R$.

The following method of defect localization is possible. A graphical image of the elements of the vector $R$ is constructed, then the parameter $L$ is visually determined by this graph as the number of elements of the vector $R$ used to restore the image of the defect.

While processing the data from a priori information, the resolution and sensitivity threshold used in the computational experiment of the measuring system are known. This ensures that we know the value $\Delta$ – the minimum value of the thickness (width) of the defect diagnosed with this measuring system. In this case, the value $L$ can be chosen as the result of evaluating the number of ray pathways, for which the values of the beam-sums amounts after subtracting the corresponding value, related to the reference product, would satisfy the inequality $R_i > k\Delta$, $k = \text{const}$, $i = 1, 2, ..., M$. The constant $k$ is chosen based on a priori information about the geometric conditions of measurements of the values $P_{i,j}$ and of the parameters of the desired defect.

After determining the value of $L$, the image of the defect in the rays is constructed (i.e., the selected rays are drawn, the antinodes of which form the image of the defect). The image of the defect in the beams can be used for further computational processing and visualization of the defect.

During the computational experiment it was considered the class of the defects described by the ratio $f(x)=a+b\cdot\sin(c\cdot x+d)\pm w$, where $a$, $b$, $c$, $d$ are adjustable parameters, $w$ is the thickness of the defect.

It was implemented a software application in HTML5 + Javascript for the simulation of the problems of tomographic radiography. It is important to note that, since the automated construction of the envelope to the resulting family of straight lines was not currently developed by objective, including mathematical, difficulties, the human factor is necessary when visualizing the defect. Obviously, in the case of a well-chosen model of the defect and parameters of the observation system, a very visual image of the sample can be obtained.

3. Note

We obtain an estimation of the ratio of the length of the possible path of the beam through the defect and its characteristic width (thickness). Consider the defect model shown in Figure 1. Here the defect is formed by two concentric circles with radii $R$ and $R+\varepsilon$, where $\varepsilon$ is the characteristic opening of the defect, in this case constant, $l$ is the part of the beam belonging to the defect. Then it is possible to write:

$$\frac{l}{2} = \sqrt{(R + \varepsilon)^2 - R^2} = \sqrt{R^2 + \varepsilon^2 - 2R\varepsilon - R^2},$$

or

$$\frac{l}{2} = \sqrt{2R\varepsilon + \varepsilon^2}.$$  

Omitting $\varepsilon^2$ we obtain

$$\frac{l}{2} = \sqrt{\varepsilon} \sqrt{2R}.$$  

This means that among the rays intersecting the domain and disposed between the mentioned circles (curves) there are rays with length of order $\sqrt{\varepsilon}$ this is essential for the sufficiently small values of $\varepsilon$. Thus, if $\varepsilon \sim 10^{-4} \div 10^{-6}$ m, the path length of the probing radiation beam will be $\sim 10^{-2} \div 10^{-3}$ m, which is a sufficient value for measuring the working functionals of the response of the object of study.
to the probing effect, taking into account the length of the minimum spatial period reproduced by the tomograph.

Figure 1. Defect model: 1, 2 – concentric circles centered at O; l – path of the beam on the defect; ε – characteristic opening (width, thickness) of the defect.

Geometric illustration to the determination of the ratio of the length of the possible path of the probing radiation beam through the defect to its characteristic opening.

4. Conclusion
The proposed method can significantly reduce the computational cost in the processing of experimental information. It is designed for rapid detection of thin extended defects having a type of delamination or cracks, but the approach based on the use of a phantom reference sample can have applications in all areas of defectoscopy, where the construction of such a phantom is possible. For example, one of the non-traditional tomographic directions of interest is the so-called "radon transformation [14] in the band". Here the system of observations is located on two parallel lines, in the limit theoretically investigated case the segments (location of sources and receivers of the probing signal) are infinite, i.e. (- ∞, + ∞). In our approach, infinite limits are not required.

Another possible application of the computational method of the standard model should be mentioned. There is an important problem of detection of knots in the process of tree trunks sawing. It is required to organize the sawing procedure in an economic way from the material consumption viewpoint and, at the same time, to obtain the timber planks without knots [15]. This is especially important for the elite carpentry when working with valuable, unique samples of wood. For this case it is crucial knowledge of the internal state of the sawn or milled workpiece, not to mention the optimal initial sawing logs. Close research can be found in [16-18]. It is clear that if you know the breed of wood, the density of knots and other hidden internal formations, the density of the host wood, then it would not be difficult to create a reference phantom. At the same time, as noted above, the reconstruction can be carried out in the band along the studied object. It should be noted an important fact that the intensity of x-ray radiation is comparable to the medical case. In contrast to traditional methods of two-dimensional tomography, which require transmission of the entire cross-section and the construction of a full two-dimensional tomogram, this approach allows to reconstruct only the most important parts of the full cross-section of the defect as a result of their local transmission.

It is important to note that the combination of techniques of local reconstruction and replenishment of projection data in case of their incompleteness may be of great importance [19-21].

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