COLLECTIVE PROPERTIES OF NEUTRON-STAR X-RAY BINARY POPULATIONS OF GALAXIES. II. PRE-LOW-MASS X-RAY BINARY PROPERTIES, FORMATION RATES, AND CONSTRAINTS

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Received 2012 December 27; accepted 2014 February 2; published 2014 March 10

Abstract

We continue our exploration of the collective properties of neutron-star X-ray binaries in the stellar fields (i.e., outside globular clusters) of normal galaxies. In Paper I of this series, we considered high-mass X-ray binaries (HMXBs). In this paper (Paper II), we consider low-mass X-ray binaries (LMXBs), whose evolutionary scenario is very different from that of HMXBs. We consider the evolution of primordial binaries up to the stage where the neutron star just formed in the supernova explosion of the primary is in a binary with its low-mass, unevolved companion, and this binary has circularized tidally, producing what we call a pre-low-mass X-ray binary (pre-LMXB). We study the constraints on the formation of such pre-LMXBs in detail (since these are low-probability events), and calculate their collective properties and formation rates. To this end, we first consider the changes in the binary parameters in the various steps involved, viz., the common-envelope phase, the supernova, and the tidal evolution. This naturally leads to a clarification of the constraints. We then describe our calculation of the evolution of the distributions of primordial binary parameters into those of pre-LMXB parameters, following the standard evolutionary scenario for individual binaries. We display the latter as both bivariate and monovariate distributions, discuss their essential properties, and indicate the influences of some essential factors on these. Finally, we calculate the formation rate of these pre-LMXBs. The results of this paper will be used in a subsequent one to compute the expected X-ray luminosity function of LMXBs.

Key words: binaries: close – stars: evolution – stars: low-mass – stars: neutron – supernovae: general – X-rays: binaries – X-rays: galaxies

1. INTRODUCTION

In this series of papers, we have undertaken a first-principles exploration of the essential theoretical underpinnings of the observed distributions of the collective properties of accretion powered neutron-star X-ray binaries, in particular, of their X-ray luminosity functions (XLFs). Paper I (Bhadkamkar & Ghosh 2012) of the series dealt with high-mass (or massive) X-ray binaries (HMXBs). In the next two papers of the series, we are dealing with low-mass X-ray binaries (LMXBs). It is convenient to divide this LMXB study into two parts, viz., (1) that which deals with the evolution of a primordial binary up to the formation of a neutron star in a supernova (SN) explosion, which produces a binary consisting of this neutron star and a relatively low-mass companion, which we call a pre-low-mass X-ray binary (pre-LMXB), and (2) that which deals with the evolution of this pre-LMXB into an X-ray active LMXB, and the subsequent evolution of this LMXB through its accretion phase. Accordingly, we handle the first part in this paper, which is Paper II in this series. The second part will be handled in a later paper.

We briefly recount a few essential features of studies such as this. First, such studies have become meaningful only in recent years, when it became possible to construct robust and dependable distributions of the essential collective properties of X-ray binaries, e.g., their XLFs, after the accumulation of four decades of observational material (Grimm et al. 2002, 2003; Kim & Fabbiano 2004; Gilfanov 2004; Gilfanov et al. 2004a, 2004b; Liu et al. 2007; Kim & Fabbiano 2010). Second, in our study contained in this series of papers, we focus on X-ray binaries which are outside globular clusters, i.e., in the stellar field of the galaxy under study, which implies that the evolution of a given X-ray binary can be treated in isolation, without any significant perturbation from other stars or X-ray binaries. This is why the standard scenarios of individual X-ray binary evolution (van den Heuvel 1983, 1991, 1992, 2001) can be applied to the problem. Third, the multi-step evolutionary sequence from a primordial binary to an X-ray binary may involve both (1) steps in which only the initial and final states matter for our purposes, and (2) steps in which not only the initial and final states but also the entire intermediate process of evolution have to be considered for our purposes. Naturally, the second situation leads to more involved calculations. We showed in Paper I that the entire HMXB evolutionary process is described by the first situation. We shall show in this paper (Paper II) that the evolution from primordial X-ray binaries to pre-LMXBs is also almost described by the first situation, with one exception which is easily handled. By contrast, the subsequent LMXB evolution is almost entirely described by the second situation, and so has to be handled in a very different manner. This is a major reason why we shall give it in a subsequent paper, viz., Paper III.

Fourth, as with the HMXB work of Paper I, an effort of this type should be regarded only as a proof-of-principle demonstration that the observed X-ray-binary collective properties can be (at least) qualitatively accounted for by evolving well-known, plausible, collective properties of primordial binaries through the well-known and well-accepted scenarios for the evolution of an individual primordial binary into an X-ray binary.

Works on the expected collective properties of LMXBs began even before the first observational determinations of the XLFs of external galaxies were available (Kalogera & Webbink 1996, 1998). These works studied the constraints on the binary parameters at various stages of evolution, and presented the expected distributions of pre-LMXBs under these constraints. We have

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revisited some of these constraints in this work (see Section 4). A different line of previous approach was to construct a large library of individual LMXB models, each of which calculated the detailed evolution of that individual system (Podsiadlowski et al. 2002). This library was then combined with a population synthesis code to obtain the expected distribution of LMXBs as a function of accretion rate, orbital period, etc. (Pfahl et al. 2003). It is instructive to compare the pre-LMXB distributions obtained by us with those obtained in earlier calculations. Such comparisons will be presented later at appropriate places. With the construction of the StarTrack code, extensive population syntheses of many different categories of binaries involving compact objects (Belczynski et al. 2008) became possible. This code used suitably constructed analytical prescriptions for stellar evolution from Hurley et al. (2000), and the code was used in works aimed at understanding LMXB populations of individual elliptical galaxies (Fragos et al. 2008, 2009), wherein the authors extensively studied the effects of varying the parameters on the XLF shape, in order to constrain the allowed models. These authors also studied the effects of transient LMXBs on the overall results, as also the relative contributions of the different subtypes of LMXBs. In Paper III, where we give our XLF results, we shall compare these aspects of our results with those obtained in previous works.

The rest of this paper is arranged as follows. In Section 2, we briefly recount the formation and evolution scenarios of pre-LMXBs and LMXBs. In Section 3, we describe the changes in the binary parameters in various steps of primordial-binary evolution leading up to pre-LMXB formation, taking in turn the common-envelope (CE) phase, the SN, and the tidal evolution. In Section 4, we describe the constraints on the pre-LMXB parameters. In Section 5, we describe our calculation of the distribution of pre-LMXB parameters. We start from a summary of canonical primordial-binary parameter distribution, and we describe how we transform this distribution to obtain that for pre-LMXBs. We show the latter as both bivariate and monovariate distributions, discuss their properties, and show the influence of some essential factors on these. In Section 6, we present our calculation of the formation rate of pre-LMXBs. We discuss our results in Section 7.

2. LMXB FORMATION AND EVOLUTION SCENARIOS

We briefly recount here the standard formation scenario for neutron-star LMXBs. Their progenitors are primordial binaries of two main sequence stars, which are much more disparate in mass than those in HMXB progenitors. This extreme mass ratio changes the course of LMXB formation and evolution completely from that of HMXBs. The more massive star, i.e., the primary, completes its main sequence life faster, on timescales \(\sim 10^6 \text{–} 10^7 \text{ yr}\). On the giant branch, it fills its Roche lobe and starts transferring mass to the secondary, which, in such a progenitor, is so much less massive than the primary, and hence has so much longer of a thermal timescale, that it is unable to accept the transferred matter, which then forms an envelope around the two stars—the CE. The primary keeps losing mass to form this CE until it is completely stripped of its H-envelope. The CE, which engulfs the resulting binary of the He-core of the primary and the secondary, exerts a strong frictional drag on this binary, so that the two stars spiral in toward each other. The orbital energy released due to this spiral-in is deposited into the CE, heats it, and attempts to expel it. Systems which were very close to begin with now go into a merger and so do not survive this phase. However, those which were sufficiently wide do have enough energy to expel the CE altogether, and so survive the CE phase, emerging as a very compact binary consisting of the He-core of the primary and the secondary, the latter remaining practically unchanged during this whole process.

The post-CE system is typically detached and the He-core evolves as if it were a single He-star, eventually exploding as an SN. A neutron star of typical mass \(1.4 M_\odot\) is left after the SN explosion of the He-core, the rest of the mass being lost from the system. Due to the large mass loss from the system during the previous CE phase, the mass loss in this SN event is not always destructively large, and some of these systems do survive, but often with rather large eccentricity. The rest of the systems are disrupted. Natai kicks are given to the neutron star when the SN explosion is asymmetric, and such kicks further influence the survival probability of these post-CE binaries. The resultant binary, consisting of the neutron star and its low-mass companion, now evolves tidally on timescales \(\sim 10^7 \text{–} 10^9 \text{ yr}\) (Zahn 1977; Bhadkamkar & Ghosh 2009), circularizing the binary. Thus, the whole sequence of events outlined above occurs over a total time \(\sim 10^7 \text{ yr}\) or less, which is very short compared to the nuclear timescale of the secondary, which for a typical secondary mass of \(1 M_\odot\) is \(> 10^9 \text{ yr}\). Therefore, the secondary undergoes little nuclear evolution during the above time, and can still be considered almost a zero age main sequence star. We call this binary system of a neutron star and its low-mass unevolved companion a pre-LMXB.

LMXB formation and evolution can thus be thought of as a two-step process. The first step is the rapid one outlined above, occurring over timescales of \(\sim 10^7 \text{ yr}\) or less. The second step is a very slow one, occurring over timescales \(\sim 10^9 \text{ yr}\), during which (1) the above pre-LMXB evolves through its detached binary phase to the onset of Roche-lobe overflow and mass transfer, at which point the system turns on as an LMXB, and (2) this LMXB lives out its X-ray emission phase and turns off, after which the system evolves into a binary consisting of a recycled neutron star with a white dwarf companion. We treat the first step in detail in the rest of this paper (Paper II), deferring a detailed treatment of the second step to the next paper of this series, and giving only a very brief outline of that step in the next paragraph.

The pre-LMXB evolves through the angular momentum loss which causes its orbit to shrink, the two major mechanisms for such loss being gravitational radiation and magnetic braking. The first is always in operation, though not always dominant. Magnetic braking is operational when the companion has a sufficiently large and sustained magnetic field, and a magnetically coupled stellar wind carries off significant angular momentum. This is thought to require both a sizable radiative core (to anchor the magnetic field) and a sizable convective envelope (to run the dynamo that produces the magnetic field) in the companion, a point to which we return later. Both mechanisms have strengths which increase rapidly with decreasing orbital radii, and so are important only at small radii. Accordingly, these mechanisms can bring the system into Roche-lobe contact within the main-sequence lifetime of the low-mass companion only if the initial orbital radius of the pre-LMXB is sufficiently small. For such systems, Roche-lobe overflow is established while the companion is still on the main-sequence, and the subsequent mass transfer is driven by angular-momentum loss. For wider systems, however, Roche-lobe contact becomes possible only after the companion finishes its main-sequence lifetime, gets on the giant branch and expands to fill its Roche lobe. Subsequent
mass transfer is driven dominantly by the nuclear evolution and expansion of the companion. The crossover point between the above two possible evolutionary paths occurs at a critical initial orbital period called the bifurcation period, whose value is \( \sim 14-18 \) hr (Plyusniner & Savonije 1988, 1989). In either case, the matter transferred to the neutron star’s Roche-lobe forms an accretion disk and eventually reaches the surface of the neutron star, the energy released in the process being emitted in X-rays, making the system a bright LMXB. This LMXB phase ends when accretion stops and the companion becomes a degenerate white dwarf, the final product being a recycled neutron star that will evolve through the X-ray phase as the dominant mechanism for the formation of systems of this kind.

The efficiency parameter \( \alpha \) (see Equation (1)) is usually taken as

\[
\alpha(E_{b1} - E_{b2}) = E_{\text{c} - \text{e}}.
\]

The binding energy \( E_c \) can be written as \( G M_1 M_2 / 2a \) where the masses and the distance are understood to be taken appropriately for the initial (subscript \( b1 \)) and final (subscript \( b2 \)) states. Calculation of \( E_{\text{c} - \text{e}} \) requires a knowledge of the detailed structure of the star, which depends on its evolutionary stage, and the result is usually parameterized (Dewi & Tauris 2000 and references therein) as

\[
E_{\text{c} - \text{e}} = \frac{G M_{p,e} M_{p,e}}{\lambda R},
\]

where \( M_{p,c} \) and \( M_{p,e} \) are the masses of the core and the envelope of the primary, respectively, and \( R \) is the radius of the star. Details of the structure and evolutionary stage of the primary are contained in the parameter \( \lambda \), on which we elaborate below.

The relation between the initial and final core-envelope separations \( (a_0 \) and \( a_{\text{CE}} \), respectively) can be written as

\[
a_{\text{CE}} = \frac{M_{p,e} M_g}{M_p - 2 + M_{p,e}(\alpha \lambda r_L)^{-1}}.
\]

The relation between \( M_{p,c} \) and \( M_p \) is often taken as a power-law approximation to the results of detailed stellar-structure calculations, given by

\[
M_{p,c} = M_0 M_p^{1/3},
\]

where \( M_{p,c} = M_p - M_{p,c} \) (Ghosh 2007). Here, \( r_L \) is the effective Roche lobe radius of the primary. It depends only on the mass ratio \( q \equiv M_1 / M_2 \), and analytical approximations to the numerical results are available in the literature. We adopt here the widely used Eggleton approximation (Eggleton 1983), which is

\[
r_L = \frac{0.49 q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})},
\]

From Equation (3), it is clear that the orbital shrinkage during the CE phase depends on the product \( \alpha \lambda \) of the two essential parameters of the problem. Consequently, a modeling of the CE process alone cannot give independent handles on these two parameters, but can only constrain their product. In all our calculations reported here, we thus work only with the product \( \alpha \lambda \), calling it the CE parameter.

We note first, however, that the concepts underlying the parameters \( \alpha \) and \( \lambda \) are different, and we briefly discuss them here, also indicating the suggested values of these parameters. The efficiency parameter \( \alpha \) (see Equation (1)) is \( \leq 1 \) by definition (unless one argues that sources of energy other than orbital binding energy are available for ejecting the CE), but its actual
The transformation connecting the parameters of the primordial binary to those of the post-CE binary are given by Equations (3) and (4) along with the definition $M_\ast = q M_p$. The inverse transformation can also be derived from these, and the Jacobian for that transformation works out, after a little algebra, to be

$$J_{\text{CE}} = \frac{\xi}{M_\ast} \frac{a_0}{a_{\text{CE}}}.$$  \hfill (6)

3.2. Supernova

The detached post-CE system evolves at first without affecting the binary parameters. Then the He-core of the primary finishes its evolution like a single He-star and explodes as an SN, which suddenly changes the binary characteristics. Throughout this paper, we assume that the neutron star formed in the SN explosion has a mass of $1.4 M_\odot$, as we did in Paper I, and that the rest of the mass of the He-core is lost from the system. If this mass loss is sufficiently large (which it can be, depending on the details of the primordial binary, despite the heavy mass loss from the primary during the previous CE phase), the binary is disrupted. If the SN explosion is symmetric, a mass loss of more than half of the total mass of the system will unbind the binary. For smaller mass losses, the parameters of the post-SN binary are related in a simple way to those of the pre-SN binary. However, natal kicks given to the neutron star due to asymmetric SN explosions alter this simple picture considerably. A quantitative general treatment of the binary-parameter changes including all of the above points has been given in Section 3.2 of Paper I, with the details of the computation (particularly those of the averaging process over the distribution of natal kicks) being given in Appendix A of that paper.

We continue here with the notation introduced in the above places in Paper I. In particular, in averaging over the 3D isotropic Maxwellian distribution $\propto v^2 \exp(-v^2/2\sigma^2)$ of the natal kick velocities $v$, we introduced an upper limit $v_{\text{up}}$ where we truncated the processes of averaging over this kick-velocity distribution (see Equation (A2) of Paper I): this limit corresponds to the point at which the post-SN system becomes just unbound (see Equation (A1) of Paper I), so that extending the integration above this limit would incorrectly include the unbound systems also, which, in reality, must be excluded. Further, for algebraic convenience, we expressed this upper limit in the form $v_{\text{up}} \equiv (\sqrt{2}\sigma) f$, and worked with the parameter $f$. As shown in Appendix A of Paper I, the distribution-averaged kick-velocity square $v^2_{\text{f}}$ could then be expressed in the simple form $v^2_{\text{f}} = \sigma^2 h(f)$, where $h(f)$ was a function of $f$ alone. As detailed there and in Figure 13 of Paper I, two regimes of behavior were clearly shown by $h(f)$: for $f < f_c$, $h(f) \propto f^2$, and for $f > f_c$, $h(f) \approx$ constant, with a critical value $f_c \approx \sqrt{\frac{2}{5}}$. The transition region between the two regimes is very narrow. This clear separation of regimes made the calculation of $v^2_{\text{f}}$ easy.

Here in the case of pre-LMXBs, $v_{\text{up}}$ has a relatively large range $\sim 50$–$400$ km s$^{-1}$. However, the values $\sigma = 26.5$ km s$^{-1}$, and $f = 265$ km s$^{-1}$, corresponding to electron capture supernova (ECSN) and iron core collapse supernova (ICCSN), respectively, still give values of $f$ which are on opposite sides of $f_c$. Due to the extremely narrow transition region in $f$, these two cases can, therefore, still be treated in a way similar to that used in the treatment used for HMXBs in Paper I. Detailed calculations show that $\sigma = 26.5$ km s$^{-1}$ gives results extremely close to the no-kick case. Therefore, for our LMXB work here we need only calculate the large-kick ICCSN case, i.e.,

value is uncertain and difficult to calculate. The range of values adopted in the literature (Dewi & Tauris 2000; Willems & Kolb 2004; de Marco et al. 2011; Ivanova 2011) is generally $0.5 \leq \alpha \leq 1$ (but see below). The structure parameter $\lambda$ can be either above or below unity (notice its definition in Equation (2)), depending on the details of the physics included, and the stellar evolutionary stage. Early calculations of $\lambda$ included only the gravitational binding energy, yielding values $\lambda \sim 0.3$–0.5. In the 1990s and 2000s, it was realized that the internal thermodynamic energy (which includes thermal energies of the gas and the radiation, ionization energies of atoms, dissociation energies of molecules, as well as the Fermi energy of degenerate electron gas, where appropriate) of the envelope must also be included in these calculations (Han et al. 1994, 1995; Dewi & Tauris 2000 and references therein). The results showed that the resultant total value of $\lambda$ was in the range $\lambda \sim 0.5$–10 for the stellar evolutionary stages (red giant and asymptotic giant) and primary masses relevant for the CE evolution of pre-LMXBs, thus demonstrating how the internal energy dominated over the gravitational for the higher values of $\lambda$ in the above range (Dewi & Tauris 2000), which occurred when the envelope underwent rapid expansion and cooling in the giant phase.

We now summarize the values of the CE parameter $\alpha \lambda$ used in previous literature and indicate the values we use in this work. In the original work where the parameter $\lambda$ was introduced in its present form, de Kool (1990) adopted a range 0.1–1 for the CE parameter. While exploring model evolutionary systems for some binary millisecond pulsars, van den Heuvel (1994) and Tauris (1996) adopted a CE parameter $\approx 2$, and so were faced with the paradoxical conclusion that $\alpha$ would have to be $\approx 4$ for an assumed value $\lambda \approx 0.5$, say. This paradox was subsequently resolved by the results from the Han et al. and Dewi–Tauris work described above, as it became clear that $\lambda$ could easily be in the range 2–4 for the evolutionary states relevant for the systems under consideration, giving the required value of the CE parameter for $\alpha \sim 0.5$–1. Dewi & Tauris (2000) also considered specific binary pulsar systems, for which they inferred CE parameter values of 3–4 (see their Figure 5 and related discussion). Most recently, Fragos et al. (2013) have presented results from a large-scale population synthesis study of the evolution of X-ray binary populations over cosmic time, in which they studied CE parameter values in the range 0.1–0.5, and found that a value $\sim 0.1$ gave the best agreement with the semi-analytical galaxy catalog generated from the results of the Millennium II simulation.

Keeping the above results in mind, here we have studied the range of values 0.5–2 of the CE parameter. As we describe in Section 4, lower values of this parameter seem to underproduce pre-LMXBs in our straightforward scheme, while higher values seem to overproduce them, so that this range seems optimal for the study reported in the present paper. Indeed, CE-parameter values $\sim 0.1$–0.3 appear to produce a severe shortage of pre-LMXBs, in agreement with the earlier Fragos et al. (2008) result that values less than $\sim 0.2$ produce hardly any system at all. We note that there is no contradiction between these Fragos et al. (2008) results and the Fragos et al. (2013) results summarized above, since the former work investigated a neutron-star-LMXB dominated population, as we have done here, while the latter work investigated a black-hole-X-ray-binary dominated population, and massive black-hole progenitor stars are known to have significantly lower values of $\lambda$, which would naturally lead to lower optimal values of the CE parameter.
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\[ \sigma = 265 \text{ km s}^{-1} \], explicitly in addition to the no-kick case, since the latter serves as an excellent description of the small-kick ECSN case. The Jacobian \( J_{\text{SN}} \) calculated in Appendix A.2 of Paper I can be used directly in this work for appropriate values of the relevant parameters.

### 3.3. Tidal Evolution

The post-SN binary of the neutron star and its low-mass companion is typically very eccentric, as we show in Section 5.2. Such a system evolves due to tidal interaction on a timescale of \( \sim 10^{4} - 10^{6} \) yr (see Section 2). Tidal interaction has three effects on the binary: (1) circularization, (2) synchronization, and (3) change in the semimajor axis. Tidal evolution of binaries has been much studied in the literature, (Zahn 1977; Hut 1981), particularly in the context of LMXBs (Kalogera 1996). We treat tidal effects in pre-LMXBs in this work with the aid of the prescription given by Hut, which can be used in case of large eccentricities. Since the orientation of the orbit is immaterial for our purposes here, we only consider the changes in the eccentricity and the semimajor axis of the orbit and those in the rotation of the companion. Further, note that we are interested here not in the detailed evolution of these parameters, but rather in the relations between their initial and final values. Such relations can be obtained by noting that the tidal evolution roughly preserves the total angular momentum of the system (Kalogera 1996), since the tidal-evolution timescale \( \sim 10^{4} - 10^{6} \) yr is shorter than either the timescale \( \sim 10^{7} - 10^{9} \) yr on which angular momentum is lost from the system through gravitational radiation, or the timescale \( \sim 10^{7} - 10^{9} \) yr on which it is lost through magnetic braking (see, e.g., Banerjee & Ghosh 2006). We include the effects of exchange of angular momentum between orbital motion and spin of the companion by writing the constancy of the total angular momentum schematically as \( J_{\text{orb}} + J_{s,\text{spin}} = \text{constant} \). Using standard expressions for orbital and spin angular momenta, this relation can then be re-expressed as

\[ F[\gamma^{-1/3}(1 - e^2)^{1/2} - 1] = n_f^{1/3}(1 - \gamma k), \quad (7) \]

Here \( \gamma \equiv n_i/n_f \), \( k \equiv \Omega_i/\Omega_f \), and \( n_i \) and \( n_f \) are the average angular velocities in the initial and final states, given by \( n = \sqrt{G(M_{\text{NS}} + M_f)/a^3} \), with the masses and semimajor axes appropriate for the initial and final states, respectively. Since the masses do not change during tidal evolution, \( n_i \) and \( n_f \) differ only through the values of semimajor axes, so that their relation also gives that between the initial and final semimajor axes, provided that other quantities are specified. \( \Omega_i \) is the initial spin angular velocity of the companion. It is assumed that \( \Omega_f = n_f \) and \( e_f = 0 \), i.e., the binary is circular and the companion is rotating synchronously at the end of the tidal evolution. \( \Omega_i \), and therefore \( k \), can be treated as a free parameter in our calculations, with an allowed range \( 0 < k < k_{\text{max}} \), where \( k_{\text{max}} \) represents a maximally spinning companion just after the SN. Our calculations show that value of \( k \) does not change \( \gamma \) by a large amount. The most likely value of \( k \) is \( k_{\text{CE}} = \sqrt{1 - x^2}/(1 - e^2) \). Since the CE process applies a large frictional drag on the binary, we can safely assume that the companion’s spin period equals the orbital period at the end of the CE phase. Thus \( n_{\text{CE}} = \Omega_{\text{CE}} = \Omega_i \), where \( \Omega_{\text{CE}} \) and \( n_{\text{CE}} \) are the spin and the orbital angular velocities at the end of the CE phase, respectively. \( F \) in Equation (7) is a function of the two masses and hence is constant for a given system. It is given by

\[ F = \frac{G^{2/3}M_{\text{NS}}M_f}{(M_{\text{NS}} + M_f)^{1/3}I_s}. \quad (8) \]

where \( I_s \) is the moment of inertia of the secondary. It can be written in terms of solar units as \( I_s = i_s M_\odot R_\odot^2 \). The scaling of \( i_s \) with the stellar mass is given by Rucinski (1988) in a tabular form. In this work, we use an analytical fit to their data which is given by \( i_s = 0.084M_\odot^2 \). This analytical fit matches the tabulated values within 3% accuracy.

Solving Equation (7) for \( \gamma \) gives the relation between orbital separations before and after the tidal evolution. The companion mass \( M_f \) is of course unchanged during the process, and the initial eccentricity is taken as the third dummy parameter for calculational ease. Thus the transformation is essentially only in one parameter, namely, the orbital separation. The Jacobian for the inverse transformation is then given by

\[ J_{\text{id}} = \sqrt{a(1 - e^2)} \left[ 1 - \frac{3}{f} n_f^{1/3} \right]. \quad (9) \]

### 4. CONSTRAINTS ON PRE-LMXB PARAMETERS

Formations of LMXBs are very low-probability events. Various system parameters need to have values in very narrow ranges for the binary to survive through the various phases of evolution. Constraints on the allowed ranges of these binary parameters at different stages of evolution can be translated into those on primordial binary parameters, using the transformations described in Section 3. Such constraints were first discussed by Kalogera & Webbink (1998). Let us first study the constraints which are relevant at the pre-LMXB stage, identify the allowed zones, and calculate the probability of pre-LMXB formation. These constraints can be expressed as follows.

1. **The primary must fill its Roche lobe.** This ensures the formation of a CE, which is an essential step toward pre-LMXB formation. In this work, we adopt a very conservative limit for this by setting \( a_{\text{UL}}(1/q) = R_{\text{BAGB}} \), where \( R_{\text{BAGB}} \) is the radius of the primary at the base of the asymptotic giant branch. Analytical fits given by Hurley et al. (2000) are used to calculate \( R_{\text{BAGB}} \). This gives an upper limit on \( a_0 \) as a function of the metallicity of the primary for given primary and secondary masses. Figure 1 (upper panel) shows this upper limit as a function of the mass of the secondary, for \( M_f = 12M_\odot \). The lower limit on \( a_0 \) for a CE-parameter value of \( \alpha \lambda = 1 \) is also shown for comparison. It can be easily seen that higher metallicity allows higher phase-space area, thus producing a larger number of systems, although this condition is not very restrictive.

2. **The post-CE system must be detached.** This ensures uninterrupted evolution of the He-core of the primary, resulting in the formation of a neutron star. This is a twofold constraint, requiring that the radii of both the secondary and the He-core must be less than their respective Roche-lobe radii. Although this constrains the post-CE orbital separation from below, we can convert it into a constraint on \( a_0 \) using Equation (3). This constraint depends, of course, on the value of the CE parameter, as shown in Figure 1 (lower panel). An upper limit on \( a_0 \), corresponding to \( z = 0.02 \), is repeated in this panel for comparison.

It is clear from the figure that very low values of the CE parameter prohibit most of the parameter space, since for such values the lower limit is larger than the upper limit over a large range of \( M_f \). For values of the CE parameter above \( > 0.5 \), however, a considerable range is allowed and the constraints are not very restrictive. We also note that
changes in the CE parameter make larger changes in the allowed range of $a_0$, making it the more dominant parameter in determining the probability distribution.

3. The binary must survive the SN explosion. Disruption of the binary due to sudden mass loss is discussed above in Section 3.2, with reference to detailed calculations in Paper I. This requirement puts a lower limit on the allowed values of $M_\star$, or equivalently on those of $q$, as functions of $M_p$. We set an absolute lower limit on $M_\star$ as 0.1 $M_\odot$. Thus the limit obtained from the condition of SN survival is applicable only above a certain value of $M_p$, below which the entire range is allowed. We set the upper limit on $M_\star$ as 4.0 $M_\odot$ matching with the constraints used by Pfahl et al. (2003) for stable mass transfer. Figure 2 depicts these constraints. It can be seen that for neutron-star LMXBs, the progenitor primary must be less massive than $\approx 25 M_\odot$.

5. PRE-LMXB PARAMETER DISTRIBUTION

5.1. Distribution of Primordial Binary Parameters

Primordial binary distributions for LMXB progenitors are characterized by three parameters, viz., the primary mass $M_p$, the secondary-to-primary mass ratio $q$, and the orbital separation $(a_0)$, as for the HMXB progenitors considered in Paper I. These three parameters essentially follow the same distribution as in case of HMXBs, i.e., an IMF describing the distribution of $M_p$, a power-law (flat or falling) distribution for $q$ and Opik’s law for $a_0$. However, the allowed ranges for some of these parameters are drastically different from those that apply to HMXB progenitors. Specifically, we note that the $q$-ranges for the two cases are completely disjointed, as expected from the thermal timescale arguments. Also, the range of $a_0$ is more tightly constrained in the case of LMXBs due to stricter conditions of survival through various phases, as described in Section 4.

The requirement of a viable formation of CE constrains $q$ from above as $q < 0.3$, which is an absolute upper limit. This allows us to have binaries with companion masses such that the some of the final products may be more appropriately called intermediate mass X-ray binaries (IMXBs), in addition to LMXBs. The upper limit for the companion mass is generally taken to be in the range 1–3 $M_\odot$ if the binary is to be classified as an LMXB. We take this upper limit as 4.0 $M_\odot$ following Pfahl et al. (2003). It must be noted here that this is the upper limit on the initial mass of the companion at pre-LMXB stage. Systems with more massive companions are expected to have very large mass-loss rates and so are much less likely to be observed in this phase. Indeed, systems which are most likely to be observed would have companion masses in the canonical range of $M_\star < 1.0 M_\odot$ established observationally.) This puts another upper limit on $q$, given by $4.0/M_p$. We find that, over nearly the entire range of $M_p$ ($M_p > 13.3 M_\odot$), this limit is always lower than the limit imposed by CE formation given above. However, the distribution of $q$ is poorly constrained within this range. It has been suggested by Sana & Evans (2010) that the uniform $q$-distribution observed for $< 0.2$ can be extended to the lower $q$-values. For LMXB/IMXB population studies, this uniform $q$-distribution has been indeed been used by Pfahl et al. (2003). By contrast, a $q^{-2.7}$-distribution, i.e., a rather steeply falling power-law, was used in the comprehensive LMXB work of Kaloger & Webbink (1998). In this work, we use the general form $\propto q^\beta$, which can be used to explore both $\beta = 0$ and $\beta = -2.7$ cases, in order to accommodate the two approaches mentioned above, as also to explore any other power-law, as and when necessary.

The distribution of the orbital separation is taken as Opik’s law, (Opik 1924) as per current norm. We note, however, that the tight constraints in the LMXB-formation process described in Section 4 severely limit the allowed range of $a_0$ which can eventually produce viable pre-LMXBs. It is not impossible that fluctuations within this small range may have been overlooked while deriving Opik’s law observationally from a much wider
range of separations. However, due to a lack of close coverage of data over small ranges of separations, we find it most prudent to assume at this time that the wide-range log-uniform distribution (i.e., Opik’s law) also applies over the smaller range relevant for our purposes here. At a minimum, it certainly gives an indication of the average trend.

The distribution of the primary mass is given by the IMF (Salpeter 1955; Kroupa et al. 1993; Kroupa & Weidner 2003). In this work, we use the IMF given by Kroupa & Weidner. The probability density function (PDF) of the primordial binaries can therefore be written as

\[ f_{\text{prim}}(M_p, q, a_0) = \frac{1}{N} \text{IMF}(M_p) \frac{q^\beta}{a_0}. \] (10)

Here, \( N \) is the normalization parameter, defined such that when the above PDF is integrated over the relevant ranges of all the parameters, it yields unity.

5.2. Transformation of PDF to pre-LMXB Stage

The Jacobian formalism described in Paper I can be used to transform the distribution of primordial binaries to the distribution of pre-LMXBs. Formation of pre-LMXBs proceeds through the three stages of parameter changes described in Section 3. The binary at each stage is defined by three parameters, which can be connected to the parameters of the next stage. We start with the transformation from primordial binary to post-CE binary. The three parameters describing the primordial binary are \((M_p, q, a_0)\) and those describing the post-CE binary are \((M_p, q, a_{CE})\). The transformation relations are given by Equations (3) and (4), and the relation between \(q \) and \( M_p \) is of course \( M_p = q M_p \). The Jacobian for the inverse transformations is given by Equation (6).

The next stage of parameter change occurs at the SN explosion. The post-SN binary is described by \((M_p, a_{\text{psn}}, e)\). We first note that \( M_p \) is unchanged during this transformation. The inverse transformations for the other two parameters and the Jacobian for that are described in Appendix A of Paper I. As noted in Section 3.2, we perform the calculations for the ICCSN \( \sigma = 265 \text{ km s}^{-1} \) case and for the no-kick case, which closely represents the ECSN case with \( \sigma = 26.5 \text{ km s}^{-1} \).

For the post-tidal binary, only two parameters, viz., \((M_p, a)\) are sufficient to describe the system. To describe the immediate post-SN system, however, we also need the orbital eccentricity \( e \), i.e., a total of three parameters. We do the transformation of parameters and distributions between pre- and post-SN systems as described, and then integrate over the eccentricity in order to arrive at the post-tidal binary parameters. The distribution of the immediate post-SN eccentricity is shown in Figure 3. The small kicks characteristic of ECSN (whose effects are essentially identical to those for no kicks, as explained above) give an \( e \)-distribution which peaks at low eccentricities, while the large kicks characteristic of ICCSN give high eccentricities with the \( e \)-distribution peaking at a rather large value, as expected. We note that the positions of the peaks of the \( e \)-distributions obtained here for both ECSN and ICCSN are similar to those obtained for HMXBs (see Paper I). This is particularly true in the latter case, although the distribution for LMXBs appears to be much wider than that for HMXBs. We emphasize again here that the immediate post-SN \( e \)-distribution is not amenable to observations, though indirect clues on eccentricities of pre-LMXBs can be obtained from observations of individual X-ray binaries if they can be plausibly identified as pre-LMXBs, as described in Bhadkamkar & Ghosh (2009).

The change in the semimajor axis is obtained by solving Equation (7) numerically for \( \gamma \), which gives \( a/a_{\text{SN}} \). The Jacobian for this transformation is given by Equation (9). The primordial-binary PDF \( f_{\text{prim}} \) given by Equation (10) can now be transformed into the post-tidal PDF \( f_{\text{ptid}} \) with the aid of the Jacobian transformation, which can be schematically written as

\[ f_{\text{ptid}}(M_p, a, e) = f_{\text{prim}}(M_p, q, a_0) J_{\text{CE}} J_{\text{in}} f_{\text{ptid}}. \] (11)

The pre-LMXB PDF \( f_{\text{plm}} \) is finally obtained by integrating \( f_{\text{ptid}} \) over eccentricity:

\[ f_{\text{plm}}(M_p, a) = \int_0^1 f_{\text{ptid}} \, d\varepsilon. \] (12)

5.3. Properties of the pre-LMXB PDF

The pre-LMXB PDF is a bivariate function of \( M_p \) and \( a \). We display this bivariate PDF in Figures 4 and 5 for the four possible cases arising out of the two \( q \)-distributions (i.e., flat or steeply falling power-law) and the two SN-kick situations (i.e., no-kick/small ECSN kick or large ICCSN kick) that we explore in this work. (Note that in these 3D displays, we have used the optimal viewing angle, as far as possible, for bringing out the most essential features of the distribution in each panel, so that this viewing angle is not exactly the same in all panels.)

The individual (monovariate) PDF for a given variable can then be obtained by further integration of the above bivariate PDF over the other variable. Since \( f_{\text{prim}} \) has been normalized over its allowed range for viable pre-LMXB formation, \( f_{\text{plm}} \) is normalized automatically. However, any further integration requires a knowledge of the appropriate limits for the two parameters of pre-LMXB systems. The range of \( M_p \) comes from various constraints described in Section 4. The distribution of \( a \) is obtained by integrating \( f_{\text{plm}} \) over this range of values of \( M_p \). The integration range of \( a \) is chosen as \([0–20]\) for obtaining the PDF as a function of \( M_p \). Note that the lower \( a \)-limit is not very restrictive, as the PDF must fall to zero at \( a \approx 1.5 R_e \), so that Roche lobe contact does not occur just after the SN. The upper \( a \)-limit comes from the fact that wider systems will not come into Roche-lobe contact in a Hubble time.

5.3.1. Companion-mass Distribution

The distribution of the companion mass is not directly affected by the CE parameter and the metallicity. These effects

![Figure 3](image-url)
enter indirectly through modifications in the allowed phase-space area, since these parameters affect the allowed range of \(a\). By contrast, the primordial mass-ratio distribution exponent \(\beta\) affects the distribution of the companion mass directly. Figure 6 shows the companion-mass PDF for \(\beta = 0\) (flat distribution) and \(\beta = -2.7\) (falling-power-law distribution), clearly demonstrating that a falling-power-law distribution makes the \(M_c\) distribution peak at much lower values of the companion mass and so makes the rise to this peak faster. On the other hand, the shape of the PDF changes little between the different kick-scenarios, particularly in the \(\beta = -2.7\) case. The shapes of these PDFs are in agreement with those given in earlier works in the subject, i.e., Pfahl et al. (2003) for the \(\beta = 0\) case, and Kalogera & Webbink (1998) for the \(\beta = -2.7\) case. In the former case, the agreement is particularly striking. In the latter case, the authors gave their PDFs as bivariate 3D-plots, so that it is easier to compare our corresponding plots displayed in Figure 5. The nearly flat top region in our \(M_c\)-distribution for \(\beta = -2.7\) and large ICCSN kicks shows up as a “ridge” in our bivariate distribution of Figure 5, which can be compared with the corresponding feature in Figure 6 of Kalogera & Webbink (1998).

Figure 7 shows the variation of the companion-mass PDF for different values of the CE parameter. Values of 0.5 and 2 give dramatically different results. In the former case, the PDF rises almost linearly, while in the latter case it saturates quickly after an initial rise, followed by a decay. A value of 1 gives an intermediate case, as expected. Again, we see that the inclusion of kicks does not change the shape of the PDF drastically.

Variations in the metallicity affect the \(M_c\)-PDF very weakly, as seen in Figure 8, and the kick-scenario has almost no effect.

5.3.2. Distribution of Orbital Separation

The distribution of the orbital separation \(a\) is also affected by the above parameters. Figure 9 shows the variation of the \(a\)-distribution with the CE parameter for the two SN-kick scenarios (note that this and the next two figures show the PDF plotted on a logarithmic \(a\)-scale, i.e., PDF = \(dP/d\ln a\) is displayed as a function of \(a\) plotted on a logarithmic scale). The peak of the distribution shifts to smaller \(a\)-values as the
CE parameter increases, reflecting an increase in the allowed zone. The \( a \)-distribution is clearly not log-uniform for either kick-scenario.

Figure 10 shows the variation of \( a \)-distribution with metallicity. Little change is seen for different values of the metallicity, with the peak shifting slightly to the left at lower metallicities. Again, we see little change in the shape of the PDF after the inclusion of SN-kicks.

Figure 11 shows the effect of varying \( \beta \), i.e., the exponent of the \( q \)-distribution, on the \( a \)-distribution, which is quite strong. The steeply falling power law with \( \beta = -2.7 \) shifts the distribution’s peak to much smaller values of \( a \) compared to the situation for the flat (\( \beta = 0 \)) \( q \)-distribution. Again, here we find close similarities to previous results in the literature, i.e., Pfahl et al. (2003) for the \( \beta = 0 \) case, and Kalogera & Webbink (1998) for the \( \beta = -2.7 \) case, and again the former similarity is particularly striking. We emphasize that the general rise-and-fall shape of the \( a \)-distribution for pre-LMXBs seems both quite generic and confirmed by all previous calculations known to us, and that this shape stands in contrast to the generically flat or nearly flat shape at intermediate \( a \)'s that we found for pre-HMXBs and HMXBs in Paper I.

6. FORMATION RATE

In this section, we describe our recipe for calculating the formation rate of pre-LMXBs, which will serve as an input to the calculation of the LMXB XLF. The first point to notice is that the various steps in the formation process of pre-LMXBs described in previous sections do not take equal amounts of time. The three timescales that are of relevance here are (1) \( \tau_{\text{prim}} \), the timescale for the primordial binary to reach the CE phase, (2) \( \tau_{\text{postCE}} \), the timescale of evolution of the post-CE binary up to the SN explosion, and, (3) \( \tau_{\text{tid}} \), the timescale of tidal evolution of the post-SN binary. We first note that \( \tau_{\text{postCE}} \), which essentially is the timescale of evolution of a He-star, is much shorter than the other two timescales. \( \tau_{\text{tid}} \) can have a considerable range, depending on the initial eccentricity of the system just after the SN. Tidal timescales given in literature are \( \sim 10^4-10^6 \) yr (Zahn 1977; Bhadkamkar & Ghosh 2009). Thus, \( \tau_{\text{tid}} \) is typically an order of magnitude smaller than \( \tau_{\text{prim}} \), which is the timescale of the evolution of the primary, \( \sim 10^9-10^{10} \) yr. The dominant timescale in pre-LMXB evolution is thus \( \tau_{\text{prim}} \).

The formation rate of pre-LMXBs is related to the star formation rate (SFR). Since the SFR evolves on timescales that are much longer than \( \tau_{\text{prim}} \) (typically \( \sim 10^9 \) yr), we can treat the SFR to be quasistatic during pre-LMXB formation, and say that the formation rate of pre-LMXBs is roughly equal to the SFR at a (small) time lag of \( \tau_{\text{prim}} \). Since we are ultimately interested in the evolution of these systems to the LMXB phase, we need to take into account the formation rates over a long span of time when computing the evolution of pre-LMXBs into LMXBs. The typical evolutionary timescale of LMXB systems is \( \sim 10^9 \) yr, so that we need to consider the evolutionary history.
of the SFR over timescales of Gyrs. The evolution of the SFR has been studied in great detail in the literature over the last fifteen years or so, using multiwavelength studies of galaxies as well as theoretical investigations of the underlying evolutionary processes (Madau et al. 1996, 1998; Pettini et al. 1998; Blain et al. 1999; Hartwick 2004). The SFR profiles fitted to the data can be generally divided into two classes: peak-type and anvil-type (see Blain et al. 1999 for a detailed summary of the models in each class). These models generally provide the SFR as a function of the redshift $z$, which needs to be converted into the lookback time for our purposes here. The relation $z \equiv z(t)$ is dependent on the details of the cosmology assumed: we do not go into the details of cosmological models here, but rather assume the standard modern prescription in the literature.

Here we present a general method of calculating the formation rate of pre-LMXBs, applicable to any form of the SFR profile. We note first that $\tau_{\text{prim}}$ is roughly equal to the main sequence lifetime of the primary, which is a function of the mass of the primary, and can be approximated by $\tau_{\text{prim}} \approx 12 M_p^{-2.5} \text{Gyr}$ for the primary mass range of interest in this work (see Appendix C of Ghosh 2007). $M_p$ can be calculated using the inverse transformations, but we need all three parameters describing the final stage to calculate $M_p$. The total formation rate of the pre-LMXBs at any given time is thus given by SFR($t - \tau_{\text{prim}}$).

The formation rate given by Equation (13) will be used as a starting point for studying the evolution of LMXBs in a subsequent paper. As noted above, one needs to know $R(M_p, a; t)$ for at least a few Gyr back from present epoch if integrating the product over all eccentricities:

$$R(M_p, a; t) = \int_0^1 \text{SFR}(t - \tau_{\text{prim}}) f_{\text{fid}}(M_p, a, e) de. \quad (13)$$
one wants to calculate the properties of the current population of LMXBs. The importance of cosmic star formation history on collective properties of LMXBs has been studied for more than a decade now (White & Ghosh 1998; Ghosh & White 2001). The work of these authors was a first step in this direction, which described the cosmic evolution of the total number of these systems without considering the distributions of the system parameters. Our scheme of calculations presented here and in subsequent papers enables us to study the cosmic evolution of various collective properties of LMXB populations, such as distributions of luminosity (i.e., XLF) and orbital period.

7. Discussion

In this paper, we have described a method for calculating the formation rate of pre-LMXBs from given distributions of primordial binaries and given SFRs. We studied the pre-LMXB PDF as a function of the companion mass and the orbital separation for various values of the CE parameter and other parameters, and for various SN-kick scenarios. The main conclusions of this work can be summarized as follows.

1. LMXB formation is a very tightly constrained process. We showed that the constraints suggested by Kalogera & Webbink can be transformed into constraints on primordial binaries and so demonstrated how only a small allowed region in the phase space of primordial binaries is able to produce pre-LMXBs and then possibly LMXBs (if further conditions are satisfied, e.g., attainment of Roche lobe contact within a Hubble time). We showed that the CE parameter was a major factor affecting the allowed phase space, so that a good understanding of the CE process was essential for modeling the collective properties of LMXBs.

2. The PDF of pre-LMXBs was studied in a bivariate form, i.e., as a function of $M_*$ and $a$, as well as in monovariate forms for each of these variables. These PDFs were shown to agree with the results of earlier studies in this field. It was shown that a power-law distribution of the primordial mass ratio $q$ of the form $q^{-\beta}$ can lead to very different PDFs for different values of the exponent $\beta$. $\beta = 0$ leads to a larger number of wider systems, and a companion mass distribution skewed toward the higher end. This would naturally lead to a larger number of LMXB systems harboring giant companions. On the contrary, $\beta = -2.7$ leads to a larger number of compact systems with smaller-mass companions. This would lead to a larger number of LMXB systems with main-sequence companions. We shall take up these questions in more detail elsewhere.

3. The effects of the metallicity of the primordial primary and of the inclusion of natal SN-kicks were also studied. The former effects are generally small. For the latter, significant effects come only for ICCSN-kicks, as the ECSN-kicks are so small as to give results essentially identical to those for the no-kick scenario.

In the next paper of this series, we shall proceed from the pre-LMXB formation rate found in this paper to a computation of the expected LMXB XLF, which we shall compare with the observed LMXB XLF with a view to understanding and constraining the essential processes of pre-LMXB and LMXB formation and evolution. We re-emphasize that this effort should be regarded as a proof-of-principle type of exercise to understand the basic physics underlying the LMXB XLF, similar to what we did in Paper I for the HMXB XLF.

It is a pleasure to thank M. Gilfanov, E. P. J. van den Heuvel, V. Kalogera, L. Stella, R. A. Sunyaev, and T. Fragos for stimulating discussions, and the referee for helpful comments.

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