Analysis of $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay within family non–universal $Z'$ model

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Abstract

We perform a comprehensive analysis of the rare $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay in the framework of family non–universal $Z'$ model. It is shown that $Z'$ gives considerable contribution to the decay width. Zero positions of the forward–backward asymmetry and $\alpha_\theta$ parameter are shifted to the left compared to the Standard Model result. The obtained results could be tested in near future at LHC–b.

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1 Introduction

Recently, CDF Collaboration has reported the first observation of the baryonic flavor-changing neutral current (FCNC) decay $\Lambda_b \to \Lambda \mu^+\mu^-$ [1]. At quark level this decay is described by the $b \to s \mu^+\mu^-$ transition, which is forbidden in SM at tree level and take place only at loop level. Therefore this transition represents an excellent channel in searching new physics beyond the Standard Model (SM).

The new physics effects in rare decays manifest themselves either through the Wilson coefficients which are different compared to the one in the SM counterpart, or via the new operator structures in an effective Hamiltonian which are absent in the SM.

In this regard, the study of the baryonic flavor changing neutral currents is quite promising, since they could maintain the helicity structure of the effective Hamiltonian, in contrary to the mesonic case where it is lost through hadronization [2]. Following the observation of rare $\Lambda_b \to \Lambda \ell^+\ell^-$ decay by CDF Collaboration, the next step is a comprehensive study of various weak, electromagnetic and strong decays of the $\Lambda_b$ baryons. Note that $\Lambda_b \to \Lambda \ell^+\ell^-$ decay is planned to be investigated in detail at Large Hadron Collider (LHC). Having the present experimental motivation, it is timely now to study the properties of the heavy baryons theoretically.

Rare baryonic $\Lambda_b \to \Lambda \ell^+\ell^-$ decay has been investigated in the SM in numerous works (see for example [3] and references therein). The branching ratio of the $\Lambda_b \to \Lambda \mu^+\mu^-$ decay is found to have the value $Br(\Lambda_b \to \Lambda \mu^+\mu^-) = (4.0 \pm 1.2) \times 10^{-6}$. Many physical observables such as the branching ratio, forward–backward asymmetry $A_{FB}$, lepton polarization induced by the $b \to s \ell^+\ell^-$ transition are very sensitive to the existence of new physics.

In addition to these observables in the $\Lambda_b \to \Lambda \ell^+\ell^-$ decay, measurement of the of the polarizations $\Lambda_b$ and $\Lambda$ are very useful in this respect.

In the present work, we study the rare baryonic $\Lambda_b \to \Lambda \ell^+\ell^-$ decay within non–universal $Z'$ model. The non–universal $Z'$ models appear in certain string construction [4] and $E_6$ models [5] by introducing family non–universal $U(1)$ gauge symmetry. The family non–universal $Z'$ model has comprehensively been developed in [6].

The possible manifestation of non–universal $Z'$ bosons in various $B$ meson sectors has been investigated in detail in many works [7–9] (for a recent review, see [10]).

The plan of the work is as follows. In section 2, the effective Hamiltonian describing $b \to s \ell^+\ell^-$ transition is presented in both standard and $Z'$ models. Using this effective Hamiltonian, the matrix element for the $\Lambda_b \to \Lambda \ell^+\ell^-$ decay is then obtained. In this section we also present the expressions of various physical observables in $Z'$ model. Section 3 is devoted to the numerical analysis of the obtained physical observables. We present our conclusions in section 3.
2 $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay in the SM and family non–universal $Z'$ model

At quark level, $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay is described in the SM by the $b \rightarrow s \ell^+ \ell^-$ transition. The effective Hamiltonian for this transition in the SM is [11, 12]

$$\mathcal{H} = -\frac{4G}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i \mathcal{O}_i,$$

where $\mathcal{O}_i(\mu)$ are the local operators and $C_i$ are the corresponding Wilson coefficients. The expressions of all local operators can be found in [11, 12]. Here, the terms proportional to $V_{ub} V_{us}^*$ have been neglected, since $|V_{ub} V_{us}^*/V_{tb} V_{ts}^*| \leq 0.02$. In further discussion we need the operators $\mathcal{O}_7$, $\mathcal{O}_9$ and $\mathcal{O}_{10}$, whose expressions are given as follows:

$$\mathcal{O}_7 = \frac{e}{g_s^2} m_b \bar{s} \sigma_{\mu \nu} P_R b \bar{\ell} \gamma_{\mu} \ell,$$

$$\mathcal{O}_9 = \frac{e^2}{g_s^2} \bar{s} \gamma_\mu P_L b \bar{\ell} \gamma_{\mu} \ell,$$

$$\mathcal{O}_{10} = \frac{e^2}{g_s^2} \bar{s} \gamma_\mu P_L b \bar{\ell} \gamma_{\mu} \gamma_5 \ell,$$

where $P_{L,R} = (1 \mp \gamma_5)/2$. The values of the Wilson coefficients at $\mu = m_b$ scale at next–to–next leading logarithm (NNLL) are calculated in many works (for example see [13] and references therein).

This effective Hamiltonian leads to the following result for the matrix element of the $b \rightarrow s \ell^+ \ell^-$ transition,

$$\mathcal{M} = \frac{G}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ C_9^{\text{eff}} \bar{s} \gamma_\mu P_L b \bar{\ell} \gamma_{\mu} \ell + C_{10}^{\text{eff}} \bar{s} \gamma_\mu P_L b \bar{\ell} \gamma_{\mu} \gamma_5 \ell - 2C_7^{\text{eff}} \frac{1}{q_2^2} \bar{s} \gamma_\mu P_R b \bar{\ell} \gamma_{\mu} \ell \right\}.$$ (3)

The Wilson coefficient $C_9^{\text{eff}}$ contains short and long distance contributions whose expression is given as,

$$C_9^{\text{eff}} = \frac{4\pi}{\alpha_s} C_9 Y_{SD}(z, \hat{s}) + Y_{LD}(z, \hat{s}).$$

In this expression $z = m_c/m_b$, $\hat{s} = q^2/m_b^2$. The term $Y_{SD}(z, \hat{s})$ represents the contributions coming from local four–quark operators. The Wilson coefficient $C_9$ receives also long distance contributions $Y_{LD}$ due to the real $\bar{c}c$ intermediate states, i.e., $J/\psi$, $\psi'$, etc. Explicit expression of $Y_{LD}(z, \hat{s})$ and detailed discussion about it can be found in [14].

Let us now discuss how non–universal $Z'$ effects modify the effective Hamiltonian. For this aim we will follow the model presented in [6]. In this model interactions of $Z'$ with the right–handed quarks are flavor diagonal. The effective Hamiltonian for the $b \rightarrow s \ell^+ \ell^-$ transition in the presence of $Z'$ is modified as [7]

$$\mathcal{H}^{Z'}_{\text{eff}}(b \rightarrow s \ell^+ \ell^-) = -\frac{2G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ -\frac{B_{tb}^L B_{tt}^L}{V_{tb} V_{ts}^*} (\bar{s}b)_{V-A}(\ell\ell)_{V-A} \right. \left. -\frac{B_{tb}^L B_{tt}^R}{V_{tb} V_{ts}^*} (\bar{s}b)_{V-A}(\ell\ell)_{V+A} \right] + h.c.,$$ (4)
where $B^L_{sb}$ and $B^R_{\ell\ell}$ correspond to the chiral $Z'$ couplings with quarks and leptons.

Assuming that there is no considerable running effects between $m_{Z'}$ and $m_W$ scales, $Z'$ contribution leads to the modification of the Wilson coefficients, i.e., $C'_{9,10}(m_W) = C_{9,10}^{SM}(m_W) + \Delta C_{9,10}(m_W)$. In other words, the $Z'$ part of the effective Hamiltonian for the $b \to s\ell^+\ell^-$ transition can be written as

$$\mathcal{H}^{Z'} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \Delta C_9 \mathcal{O}_0 + \Delta C_{10} \mathcal{O}_{10} \right] + \text{h.c.} ,$$

where

$$\Delta C_9^{Z'} = -\frac{g_s^2}{\alpha} \frac{B^L_{sb}}{V_{tb} V_{ts}^*} (B^L_{\ell\ell} + B^R_{\ell\ell}) ,$$

$$\Delta C_{10}^{Z'} = \frac{g_s^2}{\alpha} \frac{B^L_{sb}}{V_{tb} V_{ts}^*} (B^L_{\ell\ell} - B^R_{\ell\ell}) .$$

From $m_W$ to $m_b$ scale the running effects are the same as in SM [15]. In further numerical analysis we shall use

$$C_9(m_b) = 0.0682 - 28.82 \frac{B^L_{sb}}{V_{tb} V_{ts}^*} (B^L_{\ell\ell} + B^R_{\ell\ell}) ,$$

$$C_{10}'(m_b) = -0.0695 + 28.82 \frac{B^L_{sb}}{V_{tb} V_{ts}^*} (B^L_{\ell\ell} - B^R_{\ell\ell}) .$$

In result, in order to implement the effects coming from $Z'$ boson it is enough to make the replacements

$$C_9(m_b)^{SM} \rightarrow C_9(m_b) ,$$

$$C_{10}^{SM}(m_b) \rightarrow C_{10}'(m_b) .$$

Hence, in the considered version of flavor non–universal $Z'$ model there does not appear any new operator structure other than those in SM.

As a result, including the $Z'$ contribution by making the replacements given in Eq. (7), the matrix element responsible for $b \to s\ell^+\ell^-$ transition coincides with Eq. (3), i.e.,

$$\mathcal{M} = \frac{G_F}{2\sqrt{2}\pi} V_{tb} V_{ts}^* \left\{ C_9' \bar{s}\gamma_\mu(1 - \gamma_5)b \ell \gamma_\mu \ell + C_{10}' \bar{s}\gamma_\mu(1 - \gamma_5)b \ell \gamma_\mu \gamma_5 \ell 
- 2m_b C_7 \frac{1}{q^2} \bar{s}\sigma_{\mu\nu} q^{\nu}(1 + \gamma_5) b \ell \gamma_\mu \ell \right\} .$$

The amplitude of exclusive $\Lambda_b \to \Lambda^+\ell^-\ell^-$ decay, which is described at quark level by the $b \to s\ell^+\ell^-$ transition can be obtained from Eq. (8) by replacing it between the initial and final baryon states. These matrix elements are parametrized in terms of the form factors as follows:

$$\langle \Lambda | \bar{s}\gamma_\mu(1 - \gamma_5)b | \Lambda_b \rangle = \bar{u}_\Lambda(p) \left[ \gamma_\mu f^T_1(q^2) + i\sigma_{\mu\nu} q^{\nu} f^T_2(q^2) + q_\mu f^T_3(q^2) 
- \gamma_\mu \gamma_5 g_1(q^2) - i\sigma_{\mu\nu} \gamma_5 q^{\nu} g_2(q^2) - q_\mu \gamma_5 g_3(q^2) \right] u_{\Lambda_b}(p_b) ,$$

$$\langle \Lambda | \bar{s}\sigma_{\mu\nu} q^{\nu}(1 + \gamma_5) b | \Lambda_b(p_b) \rangle = \bar{u}_\Lambda(p) \left[ \gamma_\mu f^T_1(q^2) + i\sigma_{\mu\nu} q^{\nu} f^T_2(q^2) + q_\mu f^T_3(q^2) q_\mu 
+ \gamma_\mu \gamma_5 g^T_1(q^2) + i\sigma_{\mu\nu} \gamma_5 q^{\nu} g^T_2(q^2) + q_\mu \gamma_5 g^T_3(q^2) \right] u_{\Lambda_b}(p_b) .$$
The matrix element for exclusive $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay can easily be obtained in terms of twelve form factors from Eqs. (8–9), and we find that

$$\mathcal{M} = \frac{G_F}{8\sqrt{2}\pi} V_{tb} V_{ts}^\ast \left\{ \bar{\ell} \gamma^\mu(1 - \gamma_5)\ell \bar{u}_\Lambda(p) \left\{ (A_1 - D_1)\gamma_\mu(1 + \gamma_5) + (B_1 + E_1)\gamma_\mu(1 - \gamma_5) 
\right.ight.
+ i\sigma_{\mu\nu} q'\left[(A_2 - D_2)(1 + \gamma_5) + (B_2 - E_2)(1 - \gamma_5)\right]
+ q_\mu\left[(A_3 - D_3)(1 + \gamma_5) + (B_3 - E_3)(1 - \gamma_5)\right]\biggr\} u_\Lambda(p_b)
\left. \left. + \bar{\ell} \gamma^\mu(1 + \gamma_5)\ell \bar{u}_\Lambda(p) \left\{ (A_1 + D_1)\gamma_\mu(1 + \gamma_5) + (B_1 + E_1)\gamma_\mu(1 - \gamma_5)
\right.ight.
+ i\sigma_{\mu\nu} q'\left[(A_2 + D_2)(1 + \gamma_5) + (B_2 + E_2)(1 - \gamma_5)\right]
+ q_\mu\left[(A_3 + D_3)(1 + \gamma_5) + (B_3 + E_3)(1 - \gamma_5)\right]\biggr\} u_\Lambda(p_b) \right\},
$$

where

$$A_1 = -\frac{2m_b}{q^2} C_7 (f_1^T + g_1^T) + C_9 (f_1 - g_1),
A_2 = A_1 (1 \to 2),
A_3 = A_1 (1 \to 3),
B_i = A_i \left(g_i \to -g_i; g_i^T \to -g_i^T\right),
D_1 = C_{10} (f_1 - g_1),
D_2 = D_i (1 \to 2),
D_3 = D_i (1 \to 3),
E_i = D_i (g_i \to -g_i).$$

Adopting the same convention and notation as in [16], the helicity amplitudes are given by the following expressions:

$$\mathcal{M}_{+1/2}^{++} = 2m_\ell \sin\theta \left(H_{+1/2, +1}^{(1)} + H_{+1/2, +1}^{(2)}\right) + 2m_\ell \cos\theta \left(H_{+1/2, 0}^{(1)} + H_{+1/2, 0}^{(2)}\right)$$
$$+ 2m_\ell \left(H_{+1/2, +1}^{(1)} + H_{+1/2, +1}^{(2)}\right),$$

$$\mathcal{M}_{+1/2}^{+-} = -\sqrt{q^2} (1 - \cos\theta) \left[(1 - v)H_{+1/2, +1}^{(1)} + (1 + v)H_{+1/2, +1}^{(2)}\right] - \sqrt{q^2} \sin\theta \left[(1 - v)H_{+1/2, 0}^{(1)}
\right.\right.
$$
$$+ (1 + v)H_{+1/2, 0}^{(2)}\right),$$

$$\mathcal{M}_{+1/2}^{-+} = \sqrt{q^2} (1 + \cos\theta) \left[(1 + v)H_{+1/2, +1}^{(1)} + (1 - v)H_{+1/2, +1}^{(2)}\right] - \sqrt{q^2} \sin\theta \left[(1 + v)H_{+1/2, 0}^{(1)}
\right.\right.
$$
$$+ (1 - v)H_{+1/2, 0}^{(2)}\right),$$

$$\mathcal{M}_{+1/2}^{--} = -2m_\ell \sin\theta \left(H_{+1/2, +1}^{(1)} + H_{+1/2, +1}^{(2)}\right) - 2m_\ell \cos\theta \left(H_{+1/2, 0}^{(1)} + H_{+1/2, 0}^{(2)}\right)$$
$$+ 2m_\ell \left(H_{+1/2, +1}^{(1)} - H_{+1/2, +1}^{(2)}\right),$$

$$\mathcal{M}_{-1/2}^{++} = -2m_\ell \sin\theta \left(H_{-1/2, -1}^{(1)} + H_{-1/2, -1}^{(2)}\right) + 2m_\ell \cos\theta \left(H_{-1/2, 0}^{(1)} + H_{-1/2, 0}^{(2)}\right)$$
$$+ 2m_\ell \left(H_{-1/2, -1}^{(1)} + H_{-1/2, -1}^{(2)}\right),$$

$$\mathcal{M}_{-1/2}^{+-} = -\sqrt{q^2} (1 + \cos\theta) \left[(1 - v)H_{-1/2, -1}^{(1)} + (1 + v)H_{-1/2, -1}^{(2)}\right] - \sqrt{q^2} \sin\theta \left[(1 - v)H_{-1/2, 0}^{(1)}
\right.\right.$$

$$+ (1 + v)H_{-1/2, 0}^{(2)}\right),$$

$$\mathcal{M}_{-1/2}^{-+} = -\sqrt{q^2} (1 + \cos\theta) \left[(1 - v)H_{-1/2, -1}^{(1)} + (1 + v)H_{-1/2, -1}^{(2)}\right] - \sqrt{q^2} \sin\theta \left[(1 - v)H_{-1/2, 0}^{(1)}
\right.\right.$$

$$+ (1 + v)H_{-1/2, 0}^{(2)}\right),$$

$$\mathcal{M}_{-1/2}^{--} = -\sqrt{q^2} (1 - \cos\theta) \left[(1 - v)H_{-1/2, -1}^{(1)} + (1 + v)H_{-1/2, -1}^{(2)}\right] - \sqrt{q^2} \sin\theta \left[(1 - v)H_{-1/2, 0}^{(1)}
\right.\right.$$

$$+ (1 + v)H_{-1/2, 0}^{(2)}\right).$$

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+ (1 + v) H^{(2)}_{-1/2,0} , \\
\mathcal{M}^{+}_{-1/2} = \sqrt{q^2 (1 - \cos \theta)} [(1 + v) H^{(1)}_{-1/2,-1} + (1 - v) H^{(2)}_{-1/2,-1}] - \sqrt{q^2} \sin \theta [(1 + v) H^{(1)}_{-1/2,0} + (1 - v) H^{(2)}_{-1/2,0}] , \\
\mathcal{M}^{-}_{-1/2} = 2m_\ell \sin \theta \left( H^{(1)}_{-1/2,-1} + H^{(2)}_{-1/2,-1} \right) - 2m_\ell \cos \theta \left( H^{(1)}_{-1/2,0} + H^{(2)}_{-1/2,0} \right) + 2m_\ell \left( H^{(1)}_{-1/2,t} - H^{(2)}_{-1/2,t} \right) , \quad (12)

Here

\begin{align*}
H^{(1)}_{\pm 1/2,\pm 1} &= H^{(1)}_{1/2,1} \pm H^{(1)}_{1/2,1} , \\
H^{(2)}_{\pm 1/2,\pm 1} &= H^{(2)}_{1/2,1} \pm H^{(2)}_{1/2,1} , \\
H^{(1,2)}_{1/2,0} &= H^{(1,2)}_{1/2,1} \pm H^{(1,2)}_{1/2,1} , \\
H^{(1,2)}_{1/2,t} &= H^{(1,2)}_{1/2,1} \pm H^{(1,2)}_{1/2,1} , \quad (13)
\end{align*}

where \( \theta \) is the angle of the positron in the rest frame of the intermediate boson with respect to its helicity axes. The superscripts in \( \mathcal{M} \) correspond to the helicities of leptons and subscript denotes the helicity of the \( \Lambda \) baryon. The amplitudes \( H^{V,A}_{\lambda,\lambda_b} \) are defined as:

\begin{align*}
H^{(1)}_{1/2,1} &= -\sqrt{Q_-} \left[ F^V_1 - (m_{\lambda_b} + m_\lambda) F^V_2 \right] , \\
H^{(1)}_{1/2,1} &= -\sqrt{Q_+} \left[ F^A_1 + (m_{\lambda_b} - m_\lambda) F^A_2 \right] , \\
H^{(2)}_{1/2,1} &= H^{(1)}_{1/2,1} \left( F^V_1 \to F^V_3 , F^V_2 \to F^V_4 \right) , \\
H^{(2)}_{1/2,1} &= H^{(1)}_{1/2,1} \left( F^A_1 \to F^A_3 , F^A_2 \to F^A_4 \right) , \\
H^{(1)}_{1/2,0} &= -\frac{1}{\sqrt{q^2}} \left\{ \sqrt{Q_-} \left[ (m_{\lambda_b} + m_\lambda) F^V_1 - q^2 F^V_2 \right] \right\} , \\
H^{(2,2)}_{1/2,0} &= -\frac{1}{\sqrt{q^2}} \left\{ \sqrt{Q_+} \left[ (m_{\lambda_b} - m_\lambda) F^A_1 + q^2 F^A_2 \right] \right\} , \\
H^{(2)}_{1/2,0} &= H^{(1)}_{1/2,0} \left( F^V_1 \to F^V_3 , F^V_2 \to F^V_4 \right) , \\
H^{(2)}_{1/2,0} &= H^{(1)}_{1/2,0} \left( F^A_1 \to F^A_3 , F^A_2 \to F^A_4 \right) , \\
H^{(1)}_{1/2,t} &= -\frac{1}{\sqrt{q^2}} \left\{ \sqrt{Q_+} \left[ (m_{\lambda_b} - m_\lambda) F^V_1 + q^2 F^V_2 \right] \right\} , \\
H^{(1)}_{1/2,t} &= -\frac{1}{\sqrt{q^2}} \left\{ \sqrt{Q_-} \left[ (m_{\lambda_b} + m_\lambda) F^A_1 - q^2 F^A_2 \right] \right\} , \\
H^{(2)}_{1/2,t} &= H^{(1)}_{1/2,t} \left( F^V_1 \to F^V_3 , F^V_2 \to F^V_4 \right) , \\
H^{(2)}_{1/2,t} &= H^{(1)}_{1/2,t} \left( F^A_1 \to F^A_3 , F^A_2 \to F^A_4 \right) , \quad (14)
\end{align*}

where

\begin{align*}
Q_+ &= (m_{\lambda_b} + m_\lambda)^2 - q^2 , \\
Q_- &= (m_{\lambda_b} - m_\lambda)^2 - q^2 ,
\end{align*}
and

\[ F_1^V = A_1 - D_1 + B_1 - E_1, \]
\[ F_1^A = A_1 - D_1 - B_1 + E_1, \]
\[ F_2^V = F_1^V (1 \rightarrow 2), \]
\[ F_2^A = F_1^A (1 \rightarrow 2), \]
\[ F_3^V = A_1 + D_1 + B_1 + E_1, \]
\[ F_3^A = A_1 + D_1 - B_1 - E_1, \]
\[ F_4^V = F_3^V (1 \rightarrow 2), \]
\[ F_4^A = F_3^A (1 \rightarrow 2), \]
\[ F_5^V = F_1^V (1 \rightarrow 3), \]
\[ F_5^A = F_1^A (1 \rightarrow 3), \]
\[ F_6^V = F_3^V (1 \rightarrow 3), \]
\[ F_6^A = F_3^A (1 \rightarrow 3). \] (15)

Rest of the helicity amplitudes entering into Eq. (12) can be obtained from the following relations,

\[ H_{\lambda, \lambda W}^{V(A)} = H_{-\lambda, -\lambda W}^{V(A)}. \] (16)

The square of the matrix element for the \( \Lambda_b \rightarrow \Lambda \ell^+ \ell^- \) decay is given as

\[ |M|^2 = |M_+^{+1/2}|^2 + |M_{+1/2}^{-+}|^2 + |M_{+1/2}^{++}|^2 + |M_{+1/2}^{-+}|^2 + |M_{+1/2}^{+-}|^2 + |M_{+1/2}^{--}|^2. \] (17)

In further discussions, we shall study the following observables:

1) \( q^2 \) dependence of the differential branching ratio \( d\mathcal{B}/dq^2 \). The expression of the differential branching ratio can be obtained by integrating the double differential branching ratio over \( x = \cos \theta \) whose explicit form is presented in the appendix.

2) Forward–backward asymmetry, \[ \mathcal{A}_{FB} = \frac{\int_0^1 \frac{d\Gamma}{dsdx} dx - \int_0^0 \frac{d\Gamma}{dsdx} dx}{\int_0^1 \frac{d\Gamma}{dsdx} dx + \int_0^0 \frac{d\Gamma}{dsdx} dx}, \]

and again, its explicit form can easily be obtained from \( d\Gamma/dq^2 dx \).

3) The polar angle \( \theta_\Lambda \) distribution of the cascade \( \Lambda \rightarrow a + b \) decay. This distribution is given by

\[ \frac{d\Gamma}{dq^2 d\cos \theta_\Lambda} \sim 1 + \alpha \alpha_\Lambda \cos \theta_\Lambda. \]

4) The polar angle distribution of the cascade \( V^* \rightarrow \ell^+ \ell^- \) decay, whose explicit form is given as
\[
\frac{d\Gamma}{dq^2 \, dcos \theta} \sim 1 + 2\alpha \cos \theta + \beta \cos^2 \theta .
\]

5) The polarization asymmetry parameter \(\alpha_{\Lambda_b}\), when the polarization of the initial \(\Lambda_b\) is considered. The parameter \(\alpha_{\Lambda_b}\) has the following form

\[
\frac{d\Gamma}{dq^2 \, dcos \theta_{\Lambda_b}} \sim 1 - \alpha_{\Lambda_b} \mathcal{P} \cos \theta_{\Lambda_b},
\]

where \(\mathcal{P}\) is the polarization of \(\Lambda_b\) and \(\theta_{\Lambda_b}\) is the angle between the polarization of \(\Lambda_b\) with its momentum.

### 3 Numerical analysis

In this section we present our numerical calculations of the physical observables given in the previous section, in family non–universal \(Z'\) model. As has already been mentioned, in this considered version version of the family non–universal \(Z'\) model no new operators appear compared to the SM, and hence the effect of \(Z'\) contribution is implemented by making modifications in the new Wilson coefficients \(C_9\) and \(C_{10}\). These modifications are described by four new parameters \(|B_{sb}^L|, \phi_s^L, B_{\ell\ell}^L\) and \(B_{\ell\ell}^R\). Constraints to \(|B_{sb}^L|\) and \(\phi_s^L\) coming from \(\bar{B}_s - B_s\) mixing, \(B \to \pi K^(*)\), \(\rho K^(*)\) are studied in detail in [9]. Moreover, in these work, restrictions to the parameters \(B_{\ell\ell}^L\) and \(B_{\ell\ell}^R\) that come from \(B \to X_s \mu^+ \mu^-\), \(B \to K^(*) \mu^+ \mu^-\) and \(B_s \to \mu^+ \mu^-\) decays are obtained. Recently, more new data on the above–mentioned decays have been accumulated in Tevatron and LHC, and therefore some of the constraints on these parameters might might be changed. In Table 1 we present the numerical results of these parameters, where S1 and S2 correspond to UT–fit collaboration’s two fitting results for the \(\bar{B}_s - B_s\) mixing [17].

|      | \(|B_{sb}^L| \times 10^{-3}\) | \(\phi_s^L[0]\) | \(B_{\mu\mu}^L \times 10^{-2}\) | \(B_{\mu\mu}^R \times 10^{-2}\) |
|------|------------------|----------------|-----------------|----------------|
| S1   | 1.09 ± 0.22      | -72 ± 7        | -4.75 ± 2.44    | 1.97 ± 2.24    |
| S2   | 2.20 ± 0.15      | -82 ± 4        | -1.83 ± 0.82    | 0.68 ± 0.85    |

Table 1: The values of the input parameters for the \(Z'\) couplings.

In order to maximize the effects of \(Z'\), we choose the maximum values of these parameters. In the case of S1, we use \(B_{sb}^L = 1.31 \times 10^{-3}\), \(\phi_s^L = -79^0\), \(B_{\ell\ell}^L + B_{\ell\ell}^R = -6.7 \times 10^{-2}\), \(B_{\ell\ell}^L - B_{\ell\ell}^R = -9.3 \times 10^{-2}\).

Other input parameters that are essential in performing numerical analysis are the form factors. All form factors responsible for the \(\Lambda_b \to \Lambda\) transition within the light cone QCD sum rules are calculated in [3] and these results have been used in the present numerical analysis.
In Figs. 1–4, we present the $q^2$ dependence of the differential branching ratio, forward–backward asymmetry, the polar angle $\theta_\Lambda$ distribution of the cascade $\Lambda \rightarrow a + b$ decay, the polar angle $\alpha_\theta$ distribution of the cascade $V^* \rightarrow \ell^+\ell^-$ decay, respectively.

From these figures we get the following results.

- The effect of S1 set of parameters to the differential branching ratio is larger compared to the S2 case. Branching ratio in both cases exceeds the SM prediction. For example, in the “low” $q^2$ region branching ratio is enhanced about 100% in S1 and 40% in S2 cases for the central values of the parameters, compared to the SM predictions.

- Zero position of the forward–backward asymmetry is shifted to left compared to the SM case. Therefore, experimental determination of the zero position is quite important for establishing new physics beyond the SM.

- In contrast to the differential branching ratio $d\mathcal{B}/dq^2$ and forward–backward asymmetry $\mathcal{A}_{AB}$, the parameter $\alpha_\Lambda$ does practically coincide with the $Z'$ model prediction for both S1 and S2 set of parameters.

- Similar to the forward–backward asymmetry case, the zero position of the asymmetry parameter $\alpha_\theta$ is very sensitive to the new physics effects and it is shifted to the left compared to the SM prediction.

- The parameters $\alpha_{A_b}$ and $\beta_\theta$ are not sensitive to the new physics effects.

As a result, the physical observables $d\mathcal{B}/dq^2$, $\mathcal{A}_{FB}$ and $\alpha_\theta$ are very sensitive to the new physics contributions. All these results can be checked in new future at LHCb.

Our final remark is as follows. Investigation of the lepton polarizations is quite an efficient tool for searching new physics. There immediately follows then the following question: how sensitive lepton polarizations are to the new flavor–non diagonal $Z'$ effects. We are planning to discuss this issue elsewhere in future.

In conclusion, the effects of new family non–universal $Z'$ model contributions have been studied for the $\Lambda_b \rightarrow \Lambda\ell^+\ell^-$ decay. We found that the contributions of the family non–universal $Z'$ model to the differential branching ratio is quite significant and enhances the SM predictions considerably. We also observe that the analysis of the zero positions of the forward–backward asymmetry and parameter $\alpha_\theta$ are both shifted to the left compared to the SM prediction and can serve as a very efficient tool for establishing new physics beyond the SM.

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Appendix

In this appendix, we present the double differential decay rate with respect to the angle $\theta$ between $\ell^-$ and $\Lambda_b$, and dimensional invariant mass of the leptons. Using the same conventions and notation given in [18], the double differential decay rate is given as:

$$
\frac{d\Gamma}{ds\,dx} = \frac{G^2\alpha^2 m_{\Lambda_b}}{16384\pi^5} |V_{ts}|^2 v\sqrt{\lambda(1, r, s)} \left[ T_0(s) + T_1(s)x + T_2(s)x^2 \right].

(1)
$$

where $s = q^2/m_{\Lambda_b}^2$, $r = m_{\Lambda}/m_{\Lambda_b}^2$, $\lambda(1, r, s) = 1 + r^2 + s^2 - 2r - 2s - 2rs$, $v = \sqrt{1 - 4m_{\ell}^2/q^2}$ is the lepton velocity, $x = \cos \theta$. The expressions for $T_0(s)$, $T_1(s)$ and $T_2(s)$ are:

$$
T_0(s) = 32m_{\ell}^2 m_{\Lambda_b}^4 s(1 + r - s) \left( |D_3|^2 + |E_3|^2 \right) + 64m_{\ell}^2 m_{\Lambda_b}^3 (1 - r - s) \text{Re}[D_1^* E_3 + D_3 E_1^*] + 64m_{\Lambda_b}^2 \sqrt{r} \text{Re}[D_1^* E_1] + 64m_{\Lambda_b}^2 \sqrt{r} \{2m_{\Lambda_b} s \text{Re}[D_3^* E_3] + (1 - r + s) \text{Re}[D_1^* D_3 + E_1^* E_3]\} + 32m_{\Lambda_b}^2 (2m_{\ell}^2 + m_{\Lambda_b}^2) \{ (1 - r + s) m_{\Lambda_b} \sqrt{r} \text{Re}[A_1^* A_2 + B_1^* B_2] - m_{\Lambda_b} (1 - r - s) \text{Re}[A_1^* B_1 + A_2^* B_2] - 2\sqrt{r} (\text{Re}[A_1^* B_1] + m_{\Lambda_b} s \text{Re}[A_2^* B_2]) \} + 8m_{\Lambda_b}^2 \{ 4m_{\ell}^2 (1 + r - s) + m_{\Lambda_b}^2 (1 - r - s) - 2\} \left( |A_1|^2 + |B_1|^2 \right) + 8m_{\Lambda_b}^2 \{ 4m_{\ell}^2 \left[ \lambda + (1 + r - s) \right] + m_{\Lambda_b}^2 \left[ (1 - r - s) - 2\right] \left( |A_2|^2 + |B_2|^2 \right) - 8m_{\Lambda_b}^2 \{ 4m_{\ell}^2 (1 + r - s) - m_{\Lambda_b}^2 \left[ (1 - r - s) - 2\right] \left( |D_1|^2 + |E_1|^2 \right) + 8m_{\Lambda_b}^2 s v^2 \{ -8m_{\Lambda_b} \sqrt{r} \text{Re}[D_1^* E_2] + 4(1 - r + s) \sqrt{r} \text{Re}[D_1^* D_2 + E_1^* E_2] - 4(1 - r + s) \text{Re}[D_1^* D_2 + E_1^* E_2] + m_{\Lambda_b} \left[ (1 - r - s) - 2\right] \left( |D_2|^2 + |E_2|^2 \right) \} \},
$$

$$
T_1(s) = -32m_{\Lambda_b}^4 s v \sqrt{\lambda} \{ \text{Re}[A_1^* D_1] - \text{Re}[B_1^* E_1]\} + m_{\Lambda_b} \text{Re}[B_1^* D_2 - B_2^* D_1 + A_2^* E_1 - A_1^* E_2]\ - m_{\Lambda_b} \sqrt{r} \text{Re}[A_1^* D_1 + A_2^* D_2 - B_2^* E_1 - B_1^* E_2]\},
$$

$$
T_2(s) = -8m_{\Lambda_b}^4 v^2 \lambda \left\{ |A_1|^2 + |B_1|^2 + |D_1|^2 + |E_1|^2\right\} - m_{\Lambda_b}^2 s \left\{ |A_2|^2 + |B_2|^2 + |D_2|^2 + |E_2|^2\right\}.\]
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Figure captions

Fig. (1) The dependence of the differential branching ratio for the $\Lambda_b \to \Lambda \mu^+ \mu^-$ decay on $q^2$ for two different parameter sets of the scenarios $S1$, $S2$. For a comparison we also present the $SM$ result.

Fig. (2) The same as in Fig. (1), but for the forward–backward asymmetry.

Fig. (3) The dependence of the asymmetry parameter $\alpha_A$ on $q^2$ for two different parameter sets of the scenarios $S1$, $S2$ and $SM$.

Fig. (4) The same as in Fig. (3), but for the asymmetry parameter $\alpha_\theta$. 
Figure 1:

\[ \frac{d\mathcal{B}}{dq^2}(\Lambda_b \to \Lambda\mu^+\mu^-) \times 10^6 \]

Figure 2:
Figure 3:

Figure 4: