Application of MATLAB in the teaching of probability theory and mathematical statistics

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Abstract: "Probability theory and mathematical statistics" is an important public compulsory course of arithmetical foundation. MATLAB software is introduced into the teaching process to make some abstract concepts and theorems expressed in the form of graphics to achieve visualization effect. This method makes it easy for students to understand, improve their learning enthusiasm and achieve twice the result with half the effort.

1. Introduction
Probability theory and mathematical statistics is a compulsory subject for most science and engineering majors. It is a mathematics course to study uncertainty and its statistical laws. It is widely used in natural science, engineering technology and social science. Through the study of probability theory and mathematical statistics, students can master the basic concepts, ideas and methods of random phenomena, and improve their ability to analyze and resolve practical problems.

Because this course studies the phenomenon of uncertainty, which is quite different from the deterministic content and thinking mode that students have learned before, and the students' reaction is difficult. Therefore, in the teaching process, the mathematical software MATLAB is introduced to carry out the auxiliary teaching, and the MATLAB software is easy to learn, the numerical calculation function and the visualization ability are strong. The drawing function of MATLAB software can express the concepts, theorems and methods in probability theory and mathematical statistics with graphs, so as to effectively improve the innovativeness of the teaching of probability theory and mathematical statistics, greatly reduce the students' difficulties in understanding concepts and theorems in the learning process, stimulate students' learning enthusiasm, and cultivate students' knowledge of probability theory and mathematical statistics. The ability of MATLAB software to analyze and solve some practical problems can improve the teaching efficiency and improve the teaching quality.

2. Case study
Application examples of MATLAB software in the teaching of probability theory and mathematical statistics are as follows.

Case 1 in the teaching of probability theory and mathematical statistics, the distribution law or density function of random variables has many parameters, such as Poisson distribution, binomial distribution [3], normal distribution [3] [4] [5], exponential distribution. In the textbook, only one or a few graphics are displayed statically, which is not interactive and vivid. Matlab software has powerful drawing function, it can make the curve of distribution law or density function different parameters...
display in the same plane coordinates, so that students can intuitively summarize the role of parameters, deepen students' memory and understanding of this part of the concept and theory. Arouse students' strong interest in learning, so as to improve the teaching effect.

After starting MATLAB, you can refer to the software interface after opening. In the middle is the command line window. We need to start programming after the two greater than signs indicated by the arrow, as follows:

clc

clear all

x=[0:20];

figure(4)

x4=0:0.1:20;

y4=exppdf(ones(3,1)*x4, [3;5;10]);

plot(x4,y4(1,:),x4,y4(2,:),x4,y4(3,:))

title('Exponential distribution probability density function')

ylim([0,0.35])

legend('
\lambda=3','\lambda=5','\lambda=10')

% text(6,0.3,'\lambda=1')

ylabel('pdf')

figure(5)

% y5=binopdf(ones(4,1)*x,[20;5;10;30],[0.5;0.3;0.3;0.3]);

y51=binopdf(x,20,0.5); y52=binopdf(x,5,0.3);

y53=binopdf(x,10,0.3); y54=binopdf(x,30,0.3);

subplot(2,2,1)

bar(x,y51)

xlim([0,20])

subplot(2,2,2)

bar(x,y52)

xlim([0,6])

subplot(2,2,3)

bar(x,y53)

xlim([0,11])

subplot(2,2,4)

bar(x,y54)

xlim([0,20])

figure(6)

% y6=poisspdf(ones(3,1)*x,[1;4;10]);

y61=poisspdf(x,1); y62=poisspdf(x,4);

y63=poisspdf(x,10);

plot(x,y61,'o-',x,y62,'o-',x,y63,'o-')

legend('
\lambda=1','\lambda=4','\lambda=10')

figure(7)

x5=-5:0.01:5;

y7=normpdf(ones(4,1)*x5,[0;0;0;-2],[sqrt(0.2);1;sqrt(5);sqrt(0.5)]);

plot(x5,y7(1,:),x5,y7(2,:),x5,y7(3,:),x5,y7(4,:))

legend('
\mu=0,\sigma^2=0.2', '\mu=0,\sigma^2=1.0', '\mu=0,\sigma^2=5.0', '\mu=-2,\sigma^2=0.5')

Running the above program, we can get Figure 1 - Figure 4.
Figure 1 (binomial distribution)

From Figure 1, it is easy to see the graphical characteristics of the binomial distribution law: for fixed $n$ and $p$, when $k$ increases, $P\{X = k\}$ the first monotonic increase reaches the maximum, and then monotonously decreases.

Figure 2 (Poisson distribution)      Figure 3 (exponential distribution)

Figure 2 shows the density function of Poisson distribution when the parameters are 1, 4, 10. It is easy to get the relationship between the parameters and the probability of events from the graph: the smaller the parameter $\lambda$, the greater the probability of the event $P\{X \leq x\} (x > 0)$.

Figure 3 shows the density function of exponential distribution when the parameters are 3, 5 and 10. It is easy to get that the parameter in the density function is the reciprocal of the ordinate of the intersection point of the density curve and the Y axis.
Figure 4 shows four groups of density functions with normal distribution parameters of 0, 0.2, 0, 1, 0, 5, 2, 0.5. The characteristics of density function of normal distribution can be easily obtained from the comparison of several figures: the curve of normal distribution density function is symmetrical about $x = \mu$, axis $x$ is the asymptote of curve, when fixed $\sigma$ and change the size of $\mu$, the shape of the curve remains unchanged, only translation transformation is made along the $x$ axis. When fixing $\mu$ and changing the size of $\sigma$, the shape of the curve is changing. The larger $\sigma$ is, the higher the figure is, the thinner it is; the smaller $\sigma$ is, the shorter and fatter the figure is [1] [6].

Case 2 Poisson distribution, binomial distribution and the normal distribution are three important distributions learned from probability theory and mathematical statistics. The relationship among them is as follows: the parameter sum of binomial distribution exists and has limit. Then the limit of binomial distribution is Poisson distribution with parameter. If it tends to infinity (if it is a constant value), then according to the Demoivre-Laplace central limit theorem, the binomial distribution tends to be normal distribution. These relationships can be proved in theory, but the theoretical proof is not intuitive and difficult for students to understand. Next, we use MATLAB software to visualize these relationships [2].

Theorem 1 (Poisson's theorem) [1] let $\lambda > 0$ be a constant and $n$ a positive integer. If $np_n = \lambda$, then for any fixed nonnegative integer, we have

$$\lim_{n \to +\infty} \binom{n}{k} p_n^k (1 - p_n)^{n-k} = \frac{\lambda^k e^{-\lambda}}{k!}$$

Similar to the above operation in MATLAB programming, figure 5 is obtained.
Figure 5(Binomial distribution and Poisson distribution)

Figure 5 is composed of two graphs, which are the distribution law of binomial distribution of \( n = 10, p = 0.2 \) and Poisson distribution of \( \lambda = 2 \), binomial distribution of \( n = 100, p = 0.2 \) and Poisson distribution of \( \lambda = 20 \). When \( n \) is large enough and \( P \) is very small, Poisson distribution can be used to approximate binomial distribution. When the binomial distribution parameter \( n = 10, p = 0.2 \) in Figure 5, the approximation effect is better.

Theorem 2 (deMoivre-Laplace central limit theorem) [1] suppose that random variable \( X \sim B(n, p)(0 < p < 1, n = 1, 2, \ldots) \), is for any \( x \), we can get

\[
\lim_{n \to \infty} P\left(\frac{X - np}{\sqrt{np(1-p)}} \leq x\right) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = \Phi(x)
\]

Similar to the above operation in MATLAB programming, figure 6 is obtained.

Figure 6(Binomial distribution and Normal distribution)

Figure 6 is composed of two graphs, which are the distribution law and the normal approximation of binomial distribution with parameter \( n = 10, p = 0.2 \) and the distribution law and the normal approximation of binomial distribution with parameter \( n = 100, p = 0.2 \). According to the deMoivre-Laplace central limit theorem, if \( np \) tends to infinity (if \( p \) is a fixed value), the binomial distribution tends to be normal distribution. From Fig. 6, it can be seen that the error of binomial distribution approximating normal distribution is large when the parameter is \( n = 10, p = 0.2 \), and the approximation effect of binomial distribution and the normal distribution is better when parameter is \( n = 100, p = 0.2 \).

3. main conclusion

Through the above teaching examples, it is fully explained that the MATLAB software is introduced
into the classroom teaching of probability theory and mathematical statistics. The powerful drawing function of Matlab can enhance the visual effect of classroom teaching, make the teaching more vivid, make the students easier to understand the learning content, arouse the students' learning enthusiasm, and achieve twice the result with half the effort.

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