Enhancement of the Gilbert damping constant due to spin pumping in non-collinear ferromagnet / non-magnet / ferromagnet trilayer systems

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We analyzed the enhancement of the Gilbert damping constant due to spin pumping in non-collinear ferromagnet / non-magnet / ferromagnet trilayer systems. We show that the Gilbert damping constant depends both on the precession angle of the magnetization of the free layer and on the direction of the magnetization of the fixed layer. We find the condition to be satisfied to realize strong enhancement of the Gilbert damping constant.

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There is currently great interest in the dynamics of magnetic multilayers because of their potential applications in non-volatile magnetic random access memory (MRAM) and microwave devices. In the field of MRAM, much effort has been devoted to decreasing power consumption through the use of current-induced magnetization reversal (CIMR) \cite{1,2,3,4,5,6,7}. Experimentally, CIMR is observed as the current perpendicular to plane-type giant magnetoresistivity (CPP-GMR) of a nano pillar, in which the spin-polarized current injected from the fixed layer exerts a torque on the magnetization of the free layer. The torque induced by the spin current is utilized to generate microwaves.

The dynamics of the magnetization \(M\) in a ferromagnet under an effective magnetic field \(B_{\text{eff}}\) is described by the Landau-Lifshitz-Gilbert (LLG) equation

\[
\frac{dM}{dt} = -\gamma M \times B_{\text{eff}} + \alpha_0 \frac{M}{|M|} \times \frac{dM}{dt},
\]

where \(\gamma\) and \(\alpha_0\) are the gyromagnetic ratio and the Gilbert damping constant intrinsic to the ferromagnet, respectively. The Gilbert damping constant is an important parameter for spin electronics since the critical current density of CIMR is proportional to the Gilbert damping constant \cite{8,9}. Several mechanisms intrinsic to ferromagnetic materials, such as phonon drag \cite{11} and spin-orbit coupling \cite{12}, have been proposed to account for the origin of the Gilbert damping constant. In addition to these intrinsic mechanisms, Mizukami et al. \cite{15} and Tserkovnyak et al. \cite{16} showed that the Gilbert damping constant in a non-magnet (N) / ferromagnet (F) / non-magnet (N) trilayer system is enhanced due to spin pumping. Tserkovnyak et al. \cite{17} also studied spin pumping in a collinear F/N/F trilayer system and showed that enhancement of the Gilbert damping constant depends on the precession angle of the magnetization of the free layer.

On the other hand, several groups who studied CIMR in a non-collinear F/N/F trilayer system in which the magnetization of the free layer is aligned to be perpendicular to that of the fixed layer have reported the reduction of the critical current density \cite{10,21}. Therefore, it is intriguing to ask how the Gilbert damping constant is affected by spin pumping in non-collinear F/N/F trilayer systems.

In this paper, we analyze the enhancement of the Gilbert damping constant due to spin pumping in non-collinear F/N/F trilayer systems such as that shown in Fig. 1. Following Refs. \cite{16,17,18,19}, we calculate the spin current induced by the precession of the magnetization of the free layer and the enhancement of the Gilbert damping constant. We show that the Gilbert damping constant depends not only on the precession angle \(\theta\) of the magnetization of a free layer but also on the angle \(\rho\) between the magnetizations of the fixed layer and the precession axis. The Gilbert damping constant is strongly enhanced if angles \(\theta\) and \(\rho\) satisfy the condition \(\theta = \pi - \rho\).

The system we consider is schematically shown in Fig. 1. A non-magnetic layer is sandwiched between two ferromagnetic layers, \(F_1\) and \(F_2\). We introduce the unit
vector $\mathbf{m}_i$ to represent the direction of the magnetization of the $i$-th ferromagnetic layer. The equilibrium direction of the magnetization $\mathbf{m}_1$ of the left free ferromagnetic layer $F_1$ is taken to exist along the $z$-axis. When an oscillating magnetic field is applied, the magnetization of the $F_1$ layer precesses around the $z$-axis with angle $\theta$. The precession of the vector $\mathbf{m}_1$ is expressed as $\mathbf{m}_1 = (\sin \omega t, \sin \omega t, \cos \omega t)$, where $\omega$ is the angular velocity of the magnetization. The direction of the magnetization of the $F_2$ layer, $\mathbf{m}_2$, is assumed to be fixed and the angle between $\mathbf{m}_2$ and the $z$-axis is represented by $\rho$. The collinear alignment discussed in Ref. [17] corresponds to the case of $\rho = 0, \pi$.

Before studying spin pumping in non-collinear systems, we shall give a brief review of the theory of spin pumping in a collinear $F/N/F$ trilayer system [17]. Spin pumping is the inverse process of CIMR where the spin accumulation is the counter part of the spin current induced by the precession of the magnetization. The spin current due to the precession of the magnetization in the $F_1$ layer is given by

$$I_{\text{pump}}^s = \frac{h}{4\pi} g^{1+} \mathbf{m}_1 \times \frac{d\mathbf{m}_1}{dt},$$

where $g^{1+}$ is a mixing conductance [18] [19] and $h$ is the Dirac constant. Spins are pumped from the $F_1$ layer into the $N$ layer and the spin accumulation $\mu_N$ is created in the $N$ layer. Spins also accumulate in the $F_1$ and $F_2$ layers. In the ferromagnetic layers the transverse component of the spin accumulation is assumed to be absorbed within the spin coherence length defined as $\lambda_{\text{tra}} = \pi/|k_{F_1} - k_{F_2}|$, where $k_{F_i}^{1+}$ is the spin-dependent Fermi wave number of the $i$-th ferromagnet. For ferromagnetic metals such as Fe, Co and Ni, the spin coherence length is a few angstroms. Hence, the spin accumulation in the $i$-th ferromagnetic layer is aligned to be parallel to the magnetization, i.e., $\mu_F = \mu_F, \mathbf{m}_i$. The longitudinal component of the spin accumulation decays on the scale of spin diffusion length, $\lambda_{\text{sd}}$, which is of the order of $10$ nm for typical ferromagnetic metals.

The difference in the spin accumulation of ferromagnetic and non-magnetic layers, $\Delta \mu_i = \mu_N - \mu_F, \mathbf{m}_i$ $(i = 1, 2)$, induces a backflow spin current, $I_{\text{back}}(i)$, flowing into both the $F_1$ and $F_2$ layers. The backflow spin current $I_{\text{back}}(i)$ is obtained using circuit theory [18] as

$$I_{\text{back}}^s = \frac{1}{4\pi} \left\{ \frac{2g^{1+}g^{1+}(\mathbf{m}_i \cdot \Delta \mu_i)\mathbf{m}_i}{g^{1+} + g^{1+}} + g^{1+} \mathbf{m}_i \times (\Delta \mu_i \times \mathbf{m}_i) \right\},$$

where $g^{1+}$ and $g^{1+}$ are the spin-up and spin-down conductances, respectively. The total spin current flowing out of the $F_1$ layer is given by $I_{\text{pump}}^s = I_{\text{pump}}^s - I_{\text{back}}^{(1)}$ [17]. The spin accumulation $\mu_N$ in the $F_1$ layer is obtained by solving the diffusion equation. We assume that spin-flip scattering in the $N$ layer is so weak that we can neglect the spatial variation of the spin current within the $N$ layer, $I_{\text{pump}}^s = I_{\text{back}}^{(2)}$. The torque $\mathbf{t}_1$ acting on the magnetization of the $F_1$ layer is given by

$$\mathbf{t}_1 = I_{\text{pump}}^s - (\mathbf{m}_1 \times I_{\text{pump}}^s) \mathbf{m}_1 = \mathbf{m}_1 \times \frac{d\mathbf{m}_1}{dt},$$

where $\nu = (g^{1+} - g^*)/(g^{1+} + g^*)$ is the dimensionless parameter introduced in Ref. [17]. The Gilbert damping constant in the LLG equation is enhanced due to the torque $\mathbf{t}_1$ as $\alpha_0 \rightarrow \alpha_0 + \alpha'$ with

$$\alpha' = \frac{g_{NN} g_{NN}^{1+}(1 - \nu \sin^2 \theta)}{8\pi M_1 d_{F_1} S} \left( \frac{1}{1 - \nu^2 \cos^2 \theta} \right),$$

where $\nu$ is the Landé $g$-factor, $\mu_B$ is the Bohr magneton, $d_{F_1}$ is the thickness of the $F_1$ layer and $S$ is the cross-section of the $F_1$ layer.

Next, we move on to the non-collinear $F/N/F$ trilayer system with $\rho = \pi/2$, in which the magnetization of the $F_2$ layer is aligned to be perpendicular to the $z$-axis. Following a similar procedure, the LLG equation for the magnetization $\mathbf{M}_i$ in the $F_1$ layer is expressed as

$$\frac{d\mathbf{M}_1}{dt} = -\gamma_{\text{eff}} \mathbf{M}_1 \times \mathbf{B}_{\text{eff}} + \frac{\gamma_{\text{eff}}}{\gamma} \left( \alpha_0 + \alpha' \right)^\prime \frac{\mathbf{M}_1}{|\mathbf{M}_1|} \times \frac{d\mathbf{M}_1}{dt},$$

where $\gamma_{\text{eff}}$ and $\alpha'$ are the effective gyromagnetic ratio and the enhancement of the Gilbert damping constant, respectively. The effective gyromagnetic ratio is given by

$$\gamma_{\text{eff}} = \gamma \left( 1 - \frac{g_L \mu_B (\pi \cot \theta \cos \psi \sin \omega t)}{8\pi M d_{F_1} S t} \right)^{-1},$$

where $\cos \psi = \sin \theta \cos \omega t = \mathbf{m}_1 \cdot \mathbf{m}_2$ and

$$\epsilon = 1 - \nu^2 \cos^2 \psi - \nu (\cot^2 \theta \cos^2 \psi - \sin^2 \psi + \sin^2 \omega t).$$

The enhancement of the Gilbert damping constant is expressed as

$$\alpha' = \frac{g_{NN} g_{NN}^{1+} \nu \cot \theta \cos^2 \psi}{8\pi M d_{F_1} S} \left( 1 - \frac{\nu \cot^2 \theta \cos^2 \psi}{\epsilon} \right).$$

It should be noted that, for non-collinear systems, both the gyromagnetic ratio and the Gilbert damping constant are modified by spin pumping, contrary to what occurs in collinear systems. The modification of the gyromagnetic ratio and the Gilbert damping constant due to spin pumping can be explained by considering the pumping spin current and the backflow spin current [See Figs. 2(a) and 2(b)]. The direction of the magnetic moment carried by the pumping spin current $I_{\text{pump}}^s$ is parallel to the torque of the Gilbert damping for both collinear and non-collinear systems. The Gilbert damping constant is enhanced by the pumping spin current $I_{\text{pump}}^s$. On the other hand, the direction of the magnetic moment carried
is parallel to the precession axis. However, for the non-collinear system. The arrows in $\mathbf{I}_{\text{pump}}$ and $\mathbf{I}_{\text{back}}^{(1)}$ represent the magnetic moment of spin currents. (b) The back flow $\mathbf{I}_{\text{back}}^{(1)}$ has components aligned with the direction of the precession and the Gilbert damping. The back flow spin current $\mathbf{I}_{\text{s}}^{\text{back}(1)}$ depends on the direction of the magnetization of the F2 layer. As shown in Eq. 3, the backflow spin current in the F2 layer $\mathbf{I}_{\text{back}}^{(2)}$ has a projection on $\mathbf{m}_2$. Since we assume that the spin current is constant within the N layer, the backflow spin current in the F1 layer $\mathbf{I}_{\text{s}}^{\text{back}(1)}$ also has a projection on $\mathbf{m}_2$. For the collinear system, both $\mathbf{I}_{\text{pump}}$ and $\mathbf{I}_{\text{back}}^{(1)}$ are perpendicular to the precession torque because $\mathbf{m}_2$ is parallel to the precession axis. However, for the non-collinear system, the vector $\mathbf{I}_{\text{back}}^{(1)}$ has a projection on the precession torque, as shown in Fig. 2(b). Therefore, the angular momentum injected by $\mathbf{I}_{\text{back}}^{(1)}$ modifies the gyromagnetic ratio as well as the Gilbert damping in the non-collinear system.

Let us estimate the effective gyromagnetic ratio using realistic parameters. According to Ref. 17, the conductances $g^{11}$ and $g^\ast$ for a Py/Cu interface are given by $g^{11}/S = 15[\text{nm}^{-2}]$ and $\nu \simeq 0.33$, respectively. The Landé $g$-factor is taken to be $g_L = 2.1$, magnetization is $4\pi M = 800[\text{Oe}]$ and thickness $d_F = 5[\text{nm}]$. Substituting these parameters into Eqs. 4 and 8, one can see that $\gamma_{\text{eff}}/\gamma \simeq 0.001$. Therefore, the LLG equation can be rewritten as

$$\frac{dM_1}{dt} \simeq -\gamma M_1 \times B_{\text{eff}} \left( a_0 + \alpha' \frac{M_1}{|M_1|} \right) \times \frac{dM_1}{dt}. \quad (10)$$

The estimated value of $\alpha'$ is of the order of $0.001$. However, we cannot neglect $\alpha'$ since it is of the same order as the intrinsic Gilbert damping constant $a_0$. 22 23

Experimentally, the Gilbert damping constant is measured as the width of the ferromagnetic resonance (FMR) absorption spectrum. Let us assume that the F1 layer has no anisotropy and that an external field $\mathbf{B}_{\text{ex}} = B_0 \hat{z}$ is applied along the $z$-axis. We also assume that the small-angle precession of the magnetization around the $z$-axis is excited by the oscillating magnetic field $\mathbf{B}_1$ applied in the $xy$-plane. The FMR absorption spectrum is obtained as follows 24:

$$P = \frac{1}{T} \int_0^T dt \frac{\alpha \gamma M \Omega^2 B_1^2}{(\gamma B_0 - \Omega)^2 + (\alpha \gamma B_0)^2}, \quad (11)$$

where $\Omega$ is the angular velocity of the oscillating magnetic field, $T = 2\pi/\Omega$ and $\alpha = a_0 + \alpha'$. Since $\alpha$ is very small, the absorption spectrum can be approximately expressed as $P \propto a_0 + \langle \alpha' \rangle$ and the highest point of the peak proportional to $\langle 1/(a_0 + \alpha') \rangle$, where $\langle \alpha' \rangle$ represents the time-averaged value of the enhancement of the Gilbert damping constant. In Fig. 3(a), the time-averaged value $\langle \alpha' \rangle$ for a non-collinear system in which $\rho = \pi/2$ is plotted by the solid line as a function of the precession angle $\theta$. The dotted line represents the enhancement of the Gilbert damping constant $\alpha'$. Contrary to the collinear system, $\langle \alpha' \rangle$ of the non-collinear system in which $\rho = \pi/2$ takes its maximum value at $\theta = \pi/2$.

As shown in Fig. 2(b), the backflow spin current gives a negative contribution to the enhancement of the Gilbert damping constant. This contribution is given by the projection of the vector $\mathbf{I}_{\text{s}}^{\text{back}(1)}$ onto the direction of the torque of the Gilbert damping, which is represented by the vector $\mathbf{m}_1 \times \dot{\mathbf{m}}_1$. Therefore, the condition to realize the maximum value of the enhancement of the Gilbert damping is satisfied if the projection of $\mathbf{I}_{\text{s}}^{\text{back}(1)}$ onto $\mathbf{m}_1 \times \dot{\mathbf{m}}_1$ takes the minimum value; i.e., $\theta = \rho = \pi - \rho$.

We can extend the above analysis to the non-collinear system with an arbitrary value of $\rho$. After performing the appropriate algebra, one can easily show that the LLG equation for the magnetization of the F1 layer is given by Eq. 9 with

$$\gamma_{\text{eff}} = \gamma \left[ 1 - \frac{g_{\mu_B}g^{11}}{8\pi M dS} \nu \sin \rho \sin \omega t (\cot \theta \cos \tilde{\psi} - \csc \theta \cos \rho) \right]^{-1}, \quad (12)$$

$$\alpha' = \frac{g_{\mu_B}g^{11}}{8\pi M dS} \left\{ 1 - \frac{\nu (\cot \theta \cos \tilde{\psi} - \csc \theta \cos \rho)^2}{\tilde{\varepsilon}} \right\}, \quad (13)$$

where $\cos \tilde{\psi} = \sin \theta \sin \rho \cos \omega t + \cos \theta \cos \rho = \mathbf{m}_1 \cdot \mathbf{m}_2$ and

$$\tilde{\varepsilon} = 1 - \nu^2 \cos^2 \tilde{\psi} - \nu \left( (\cot \theta \cos \tilde{\psi} - \csc \theta \cos \rho)^2 - \sin^2 \tilde{\psi} + \sin^2 \rho \sin^2 \omega t \right). \quad (14)$$

Substituting the realistic parameters into Eqs. 12 and 14, we can show that the effective gyromagnetic ratio...
FIG. 3: (Color online) (a) The time-averaged value of the enhancement of the Gilbert damping constant $\alpha'$ is plotted as a function of the precession angle $\theta$. The solid line corresponds to the collinear system derived from Eq. (9). The dashed line corresponds to the non-collinear system derived from Eq. (13). (b) The time-averaged value of the enhancement of the Gilbert damping constant $\alpha'$ of the non-collinear system is plotted as a function of the precession angle $\theta$ and the angle $\rho$ between the magnetizations of the fixed layer and the precession axis.

In summary, we have examined the effect of spin pumping on the dynamics of the magnetization of magnetic multilayers and calculated the enhancement of the Gilbert damping constant of non-collinear F/N/F trilayer systems due to spin pumping. The enhancement of the Gilbert damping constant depends not only on the precession angle $\theta$ of the magnetization of a free layer but also on the angle $\rho$ between the magnetizations of the fixed layer and the precession axis, as shown in Fig. 3(b). We have shown that the $\theta$- and $\rho$-dependence of the enhancement of the Gilbert damping constant can be explained by analyzing the backflow spin current. The condition to be satisfied to realize strong enhancement of the Gilbert damping constant is $\theta = \rho$ or $\theta = \pi - \rho$.

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