Reliability and Sensitivity assessment of a Thermal power plant using Boolean function technique

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Abstract: The aim of aforementioned paper is to investigate the terminal reliability, mean time to failure and sensitivity analysis for a thermal power plant based on minimizing Boolean expression approach. A thermal power plant be expressed by four major subsystems namely conveyer, boiler, turbine and generator arranged in mixed configuration. Conveyer and boiler both individually have two units in parallel redundancy. Throughout the task thermal power plant may work in basically three different states namely good, degraded and failed state. The various results regarding reliability, mean time to failure (MTTF) and sensitivity analysis are authenticated with the help of graphs.

Keywords: Boolean expression approach; Thermal power plant; Sensitivity analysis, Weibull distribution;

1. Introduction
The improved approach of science, technology and the requirements of modern civilization are daring each other. As a result of this strife, focus of industries is to announce more working capability in their industrial actions in pursuance to achieve the requirements of the civilization. The complication of industrial systems along with their outcomes is accumulating regularly. The development in effectiveness of such difficult system has therefore, captured special engrossment in recent years. The system’s potency is understood to mean the appropriateness of the system for completion of the assigned task and the potency of utilizing the means put into it and increasing performance of the system. The appropriateness of performing assigned tasks is essentially determined by the reliability of the systems. The requirement of having more reliable systems has acquired special importance along with the development of the already existing technology as well as in existing approach. Reliability engineering has many applications in various industrial systems including sugar mill, nuclear power plant, thermal power plant etc. Reliability plays a vital role at the planning, designing and working stages of various industrial systems. Gupta and Aggarwal, 1983 investigated the reliability parameter using Boolean function technique. An analytical examination of reliability reveals the actuality that reliability of system decreases rapidly when failure follows Weibull distribution but not in case of exponential distribution. In this study author restrict their self up to reliability. Aupperle et al., 1989 investigate a fault-tolerant system in which assignments and fault arrivals are not time-homogeneous. The method used for analytic techniques is based on Markov processes and stochastic activity networks. Aggarwal et al., 2015 proposed a method (by using Markov process) to compute RAMD indices to measure the improvement of the working in skim milk powder production system of a dairy
plant. They also introduced the dependability concept and dependability ratio. Gupta et al., 2005 evaluated the reliability and availability of the butter-oil manufacturing plant. This manufacturing plant consists of eight components in series configuration. Reliability, availability, and MTBF (RAM) of the serial process were calculated for various failure rates and repair rates which help in improving the production quality for the same. Smotherman and Zemoudeh, 1989 have taken assumptions of Markov modeling to compute reliability of phased-missions systems for the identification of representing limit flexibility. In this paper they generalize the representation of state-dependent behavior and handle phases of random duration using the globally-time-dependent distributions of time changing phase, and have done the modeling for globally-time-dependent failure and repair rates. The proposed method is based on a single non-homogeneous Markov model in which the extension of the concept of state transition is used to include globally-time-dependent phase changes. Phase change times are described using non-overlapping distributions with PDF's that are zero outside chosen time intervals. A comparison between a numerical solution of the model and simulation established the result that the numerical solution can be numerous times faster than that of simulation. Prescott et al., 2009 presented the modeling of non-repairable phased missions using behaviour driven development (BDD). This method form the basis of an anticipating capability in a decision making planning for autonomous systems, but this method is not suitable for real-time analysis. The approach also includes how components of the system with multiple failure modes that represent external effects on the system might not be appropriate to BDD analysis. Burdick et al., 1977 presents both exact and approximate methods of unreliability of a non-repairable system in the phase mission. The approximation methods for unreliability evaluated for phased missions have industrial applications. Fratta and Montanari, 1973 used set of all simple paths in the middle of two nodes in a network; the terminal reliability can be significantly calculated by converting a Boolean sum of products into an equivalent form and it includes all disjoint terms. This approach is overcoming the current methods both for exact as well as approximate calculation of the terminal reliability. Rhyne et al., 1977 presented a new minimization technique that allows switching functions involve in many variables which shortened the calculations. This technique named as the direct search algorithm which is relevant to both manual as well as computer programming. Bansal et al., 2010 used Boolean algebra approach to detect the terminal reliability of the whole system. But other reliability parameters like sensitivity analysis of MTTF is ignored due to which the effect of manifold choices of failure rates are not revealed. Rao and Naikan, 2014 proposed a composite approach known as MSD approach that has been used to study the progressive behavior of the systems. This approach can be used for all types of failure and repair rates. Abraham, 1979 used modified method of factoring for calculating the reliability. Locks, 1979 used an efficient technique for flipping minimal paths of reliability logic diagram or fault tree and then minimizing to get minimal cuts. The technique is generalized for some more system also. By keeping in mind the above literature here author authors gets an idea for apply the Boolean function technique in some of the industrial system. Since a thermal power plant is one of the industrial systems which play vital role in power production. Basically a thermal power plant is a system which is responsible for power production. It consists of many components but some of the key components
are conveyer, boiler, turbine and generator. These components are connected in mixed configuration. A flow diagram for the thermal power plant is shown in following Figure 1.

![Flow diagram of Thermal power plant](image1)

**Figure 1 Flow diagram of Thermal power plant**

The Conveyer consists of two parallel units namely \( y_1 \) and \( y_2 \). Boiler also consists parts in parallel namely \( y_3 \) and \( y_4 \). The Turbine is denoted by \( s \) and generator is denoted \( y_6 \).

The function of Thermal power plant is to convert heat energy to electricity with the help of heat sources like steam and fossil fuels. Thermal power plant relies on conveyer to transport coal. In the first stage, water is feed into a boiler at a high pressure. After this high pressurized water is heated into a boiler which helps to converts it into high pressurized superheated steam. This high energized steam passes through Turbine and rotates it. The rotations of the blades of turbine generate mechanical energy which converted into electrical energy with the aid of generator. Finally this electricity is distributed to the source stations, from where the distribution of the same is started. A mathematical model of the problem is formulated in Figure 2.

![Block diagram of Thermal Power Plant](image2)

**Figure 2 Block diagram of Thermal Power Plant**

2. **Assumptions**
   The trailing assumptions are taken in to consideration.
   - At initial stage assume that the system is in perfect working state.
   - Various failures involved in the process are taken to be constants.
   - Each component in the considered system is either in good stage or in failed stage.
3. Notations
The following notations are followed throughout the calculation process.

| $y_i$; $i = 1,2,3,4,5,6$ | Components of the system. |
|------------------------|----------------------------|
| $y_1, y_2$            | Conveyor (for two parallel units). |
| $y_3, y_4$            | Boilers (for two parallel units). |
| $y_5$                 | Turbine. |
| $y_6$                 | Generator. |
| $y_i'$                | Negation of $y_i \forall i$. |
| $\land / \lor$        | Conjunction/Disjunction. |
| $R_i$                 | Reliability of component of the system, |
| $R_S$                 | Reliability of the complete system. |
| $Q_i$                 | $1^{-}$ Probability of failure of the system. |
| $R_{SW} (t)$          | Reliability of the complete system when Weibull Distribution is followed by the failures. |
| $R_{SE} (t)$          | Reliability of the complete system when Exponential Distribution is followed by the failures. |
| $MTTF$                | Mean time to failures. |
| $\lambda_i$; $i = 1,2,3,4,5,6$ | Failure rate of first unit of conveyer/second unit of conveyer/first unit of boiler/second unit of boiler/turbine/generator. |

4. Mathematical Formulation and solution of the problem
By using the Boolean function approach, the probability of successful working of the whole system in premises of logical matrix is shown as (by considering Figure2):

$$F(y_1, y_2, y_3, y_4, y_5, y_6) = \begin{vmatrix} y_1 & y_3 & y_5 & y_6 \\ y_1' & y_4 & y_5' & y_6' \\ y_2 & y_3 & y_5 & y_6 \\ y_2' & y_4 & y_5' & y_6' \end{vmatrix}$$

(1)

By making the use of algebra of logics, we can rewrite the above equation (1) as

$$F(y_1, y_2, y_3, y_4, y_5, y_6) = [y_5 \land y_6 \land f(y_1, y_2, y_3, y_4)]$$

(2)

Where $f(y_1, y_2, y_3, y_4)$ is defined as

$$f(y_1, y_2, y_3, y_4) = \begin{vmatrix} y_1 & y_3 \\ y_1 & y_4 \\ y_2 & y_3 \\ y_2 & y_4 \end{vmatrix}$$

(3)

Where,
By making use of orthogonalization in (3), we get

\[ f(y_1, y_2, y_3, y_4) = \begin{vmatrix}
T_1 \\
T'_1 & T_2 \\
T'_1 & T'_2 & T_3 \\
T'_1 & T'_2 & T'_3 & T_4
\end{vmatrix} \]

(8)

Now,

\[ T'_1T_2 = \begin{vmatrix}
y'_1 \\
y_1 \\
y_3 \\
y_4
\end{vmatrix} \begin{vmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4
\end{vmatrix} = \begin{vmatrix}
y_1 \\
y_3 \\
y_4
\end{vmatrix} \]

(9)

\[ T'_1T'_2T_3 = \begin{vmatrix}
y'_1 \\
y_1 \\
y_3 \\
y_4
\end{vmatrix} \begin{vmatrix}
y'_1 \\
y_1 \\
y_4 \\
y_3
\end{vmatrix} \begin{vmatrix}
y_2 \\
y_3
\end{vmatrix} = \begin{vmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4
\end{vmatrix} \]

(10)

\[ T'_1T'_2T'_3T_4 = \begin{vmatrix}
y'_1 \\
y_1 \\
y_3 \\
y_4
\end{vmatrix} \begin{vmatrix}
y'_1 \\
y_1 \\
y_4 \\
y_3
\end{vmatrix} \begin{vmatrix}
y_2 \\
y_3
\end{vmatrix} \begin{vmatrix}
y_2 \\
y_3
\end{vmatrix} = \begin{vmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4
\end{vmatrix} \]

(11)

Using all these equations in equation (8), we get

\[ f(y_1, y_2, y_3, y_4) = \begin{vmatrix}
y_1 \\
y_1 \\
y_3 \\
y_1
\end{vmatrix} \begin{vmatrix}
y_3 \\
y_4 \\
y_3 \\
y_4
\end{vmatrix} = \begin{vmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4
\end{vmatrix} \]

(12)

Using equation (12) in equation (2), we get becomes

\[ f(y_1, y_2, y_3, y_4, y_5, y_6) = \begin{vmatrix}
y_1 \\
y_1 \\
y_3 \\
y_1
\end{vmatrix} \begin{vmatrix}
y_3 \\
y_4 \\
y_5 \\
y_6
\end{vmatrix} = \begin{vmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4 \\
y_5 \\
y_6
\end{vmatrix} \]

(13)

Here equation (12) is the disjunction of dismember conjunctions; therefore, the reliability of the considered system is given by the following expression

\[ R_s = Pr \{ F(y_1, y_2, y_3, y_4, y_5, y_6) = 1 \} = R_5R_6[R_1R_3 + Q_1Q_3 R_4 + Q_1R_2 R_3 + Q_1R_2Q_3 R_4 ] \]
where $R_j$ is the reliability corresponding to system state. Thus $y_i$ and $Q_i = 1 - R_i$,

$$R_S = R_5 R_6 [R_1 R_3 + R_1 R_4 - R_1 R_3 R_4 + R_2 R_3 - R_1 R_2 R_3 R_4 + R_1 R_2 R_3 R_4 ]$$

(14)

5. Some special cases

Case 1: If reliability of each component is taken as equal that is equal R. Then equation (14) becomes

$$R_S = R^2 (4R^2 - 4R^3 + R^4)$$

$$= 4R^4 - 4R^3 + R^4$$

(15)

Case 2: If Weibull distribution is followed:

Weibull distribution is frequently used to find the reliability. By choosing the parameters properly, the curve behave a diversity of results. We consider this distribution to find the reliability because the needed information for reliability assessment comes from trials. Let $\lambda_i$ be the failure rate of the $i^{th}$ component $y_i$, $\forall i = 1, 2, ..., 6$, then the reliability of the system as a whole at any instant 't' is given by

$$R_{SW}(t) = \sum_{i=1}^{5} \exp\{-\alpha_i t^p\} - \sum_{j=1}^{6} \exp\{-\beta_j t^p\}$$

(16)

where $p$ is defined as a positive parameter and

$\alpha_1 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6; \alpha_2 = \lambda_1 + \lambda_4 + \lambda_5 + \lambda_6; \alpha_3 = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6; \alpha_4 = \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6; \alpha_5 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6; \alpha_6 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6; \alpha_7 = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6; \alpha_8 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6; \alpha_9 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6; \alpha_{10} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6.$

Case 3: If the Exponential distribution is followed:

Exponential distribution is extensively used in reliability. When the failures follow the exponential distribution. So, the reliability of the considered system as a whole at any instant is given by

$$R_{SE}(t) = \sum_{i=1}^{5} \exp\{-\alpha_i t\} - \sum_{j=1}^{6} \exp\{-\beta_j t\}$$

(17)

where $\alpha_i$ and $\beta_j; i = 1, 2, 3, 4, 5; j = 1, 2, 3, 4$ are mentioned above.

Numerical Computation

For numerical the values of various parameters are taken as

$\lambda_1 = 0.05; \lambda_2 = 0.05; \lambda_3 = 0.02; \lambda_4 = 0.09; \lambda_5 = 0.03; \lambda_6 = 0.01$. Now, by using these failures, one can obtain $\alpha_i$'s and $\beta_j$'s as follows:

| $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $\alpha_4$ | $\alpha_5$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\beta_4$ |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 0.11      | 0.18      | 0.11      | 0.18      | 0.25      | 0.16      | 0.23      | 0.2       | 0.2       |
6.1 Reliability
The reliability of the considered system is retrieved as given in following Table 1 and corresponding Figure 3 and Figure 4 respectively (by using equation (16 and (17)) when failure rates follow two different distributions namely Exponential time distribution or Weibull time distribution.

Table 1 Reliability vs. Time using two different distributions

| Time(t) | R_{SE} | R_{SW} |
|---------|--------|--------|
| 0       | 1.000000 | 1.000000 |
| 1       | 0.956869 | 0.956869 |
| 2       | 0.908849 | 0.804988 |
| 3       | 0.857729 | 0.550637 |
| 4       | 0.804988 | 0.118619 |
| 5       | 0.751818 | 0.117029 |
| 6       | 0.699164 | 0.017353 |
| 7       | 0.647746 | 0.008906 |
| 8       | 0.59811 | 0.001730 |
| 9       | 0.550637 | 0.000538 |
| 10      | 0.505598 | 0.000033 |

Figure 3 Reliability of the system when failures follow Exponential time distribution

Figure 4 Reliability of the system when failures follow Weibull time distribution

6.2 Mean Time to Failure (MTTF)
MTTF for the considered system is given by

\[
MTTF = \int_{0}^{\infty} R_{SE}(t) dt
\]

\[
= \sum_{i=1}^{5} \frac{1}{\alpha_i} - \sum_{j=1}^{8} \beta_j
\]
Now by using the values of various parameters (from section 6) in (18) and then varying various failure rates, we obtain the MTTF of considered system as given in Table 2 and in corresponding Figure 5.

### Table 2 MTTF vs. Failure rates

| Failure rate | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ | $\lambda_6$ |
|--------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 0.01         | 15.990442  | 15.990449  | 13.736969  | 14.480837  | 16.054175  | 12.695103  |
| 0.02         | 14.783925  | 14.783925  | 12.695103  | 14.034576  | 14.192935  | 11.466307  |
| 0.03         | 13.895084  | 13.895084  | 11.869943  | 13.693575  | 12.695103  | 10.441854  |
| 0.04         | 13.220205  | 13.417833  | 11.205139  | 12.669968  | 11.466307  | 9.5759884  |
| 0.05         | 12.695103  | 12.695103  | 10.661617  | 12.458703  | 10.441854  | 8.8354812  |
| 0.06         | 12.278185  | 12.097025  | 10.211360  | 12.288234  | 9.5759884  | 8.1956521  |
| 0.07         | 11.941465  | 11.941465  | 9.8347136  | 12.140952  | 8.8354812  | 7.6378075  |
| 0.08         | 11.665517  | 11.665517  | 9.5160351  | 12.033844  | 8.1956521  | 7.1475442  |
| 0.09         | 11.436491  | 11.436491  | 9.2441626  | 11.937527  | 7.6378075  | 6.7135962  |
| 0.10         | 11.244290  | 11.244290  | 9.0103800  | 11.856235  | 7.1475442  | 6.3270321  |

![Figure 5 MTTF of the considered system vs. Failure rates](image)

#### 6.3 Sensitivity analysis

A technique which is used to decide how the distinct values of an independent factor influence a fixed dependent factor under some constraints is known as sensitivity analysis. Also one can define it by the factor by which one can analyzed that which factor out of all the factors effects the system’s performance most. In the present study sensitivity analysis is performed for considered system’s MTTF for finding that failure of which unit affects the system MTTF most.

#### 6.3.1 Sensitivity of MTTF

Sensitivity surveillance of the examined system with respect to MTTF is executed by partially differentiating the MTTF expression with respect to distinct failure rates individually and thereupon placed the values of distinct failure rates as given in section 6 in these partial derivatives. Now wavering the failure rates individually in these partial derivatives, and so one can obtain the following Table 3 and interrelated Figure 6 respectively.
Table 3 Sensitivity of MTTF vs. Failure rates

| Failure rate | $\lambda(\text{MTTF})_{A_1}$ | $\lambda(\text{MTTF})_{A_2}$ | $\lambda(\text{MTTF})_{A_3}$ | $\lambda(\text{MTTF})_{A_4}$ | $\lambda(\text{MTTF})_{A_5}$ | $\lambda(\text{MTTF})_{A_6}$ |
|--------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.01         | -141.57000      | -141.57000      | -117.51500      | -51.268742      | -12.624459      | -208.51763      |
| 0.02         | -102.58192      | -102.58192      | -92.232656      | -38.747758      | -10.502444      | -166.05799      |
| 0.03         | -76.862539      | -76.862539      | -73.728238      | -29.946827      | -8.824803       | -135.05155      |
| 0.04         | -59.156754      | -59.156754      | -59.874104      | -23.590108      | -7.481976       | -111.76878      |
| 0.05         | -46.342733      | -46.342733      | -49.287903      | -18.890722      | -6.3951070      | -93.874104      |
| 0.06         | -37.299250      | -37.299250      | -41.057273      | -15.345893      | -5.506493       | -79.846478      |
| 0.07         | -30.363664      | -30.363664      | -34.580374      | -12.824459      | -4.732777       | -68.662165      |
| 0.08         | -25.053925      | -25.053925      | -29.363023      | -10.502444      | -4.163192       | -59.612455      |
| 0.09         | -20.917675      | -20.917675      | -25.155586      | -8.824803       | -3.651644       | -52.194659      |
| 0.10         | -17.646363      | -17.646363      | -21.713083      | -7.481976       | -3.219671       | -46.044475      |

Figure 6 Sensitivity of MTTF vs. Failure rates

6. Results discussion and Conclusion

Thermal power plant is investigated to investigate different parameters of system performance by Boolean function and logical algebra technique. Here author’s do a comparison between reliability when failure time follow exponential distribution and Weibull distribution (which was not done in past). Figure 3 and Figure 4 shows the reliability of thermal power plant when failure time follows exponential distribution and Weibull distribution respectively. It can also be observed from the graphs that the system reliability with exponential distribution/Weibull distribution at 10 units of time is 0.505598/0.000033 respectively. The difference between these two graphs reflects that the system reliability is more when failure time follows exponential distribution whiles in Weibull distribution it is almost zero.

Figure 5 gives the MTTF of thermal power plant with variation in its failure rates. It is observed that the system MTTF is highest with respect to failure rate of boiler and lowest with respect to failure rate of generator. So in order to reduce the MTTF these two failures need more attention. Sensitivity analysis of the thermal power plant for the MTTF is shown in Figure 6. It reflects that the system MTTF is most sensitive with respect to failure rate of generator. Overall by this one can observe that the system reliability is improved when its failure follow exponential distribution. In future the same can be used to compare system reliability when failure follows some other distribution.
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