Tunneling between de Sitter and anti de Sitter black holes in a noncommutative $D_3$-brane formalism

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We obtain dS and AdS generalized Reissner-Nordstrom like black hole geometries in a curved $D_3$-brane frame-work, underlying a noncommutative gauge theory on the brane-world. The non-commutative scaling limit is explored to investigate a possible tunneling of an AdS vacuum in string theory to dS vacuum in its low energy gravity theory. The Hagedorn transition is invoked into its self-dual gauge theory to decouple the gauge nonlinearity from the dS geometry, which in turn is shown to describe a pure dS vacuum.

I. INTRODUCTION

In the recent years, considerable amount of interest has been devoted to explore the possibility of de Sitter (dS) vacua in quantum gravity [1]-[2]. Contrary to the well understood anti de Sitter (AdS) spaces [3], the dS geometries are usually hard to perceive in a quantum theory. The primary reason lies in the quantum tunneling of dS to AdS, which assures metastable dS vacua. The fact that the complete event horizon in an hyperbolic geometry is not accessible to an observer make dS in a different footing than AdS and Minkowski vacua. Interestingly, the construction of dS vacua has been achieved by taking into account a small number of $D_3$-branes along with the AdS vacua in a type IIB string theory [4].

Among the recent developments, the nonlinear electromagnetic (EM-) field on a $D_3$-brane turns out to be a potential candidate to address some of the quantum aspects of gravity [5]. In fact, consistent noncommutative deformations of Einstein gravity has been the subject of interest in the recent literature [6]. In the context, a very recent review may be found in ref. [7].

In this paper, we obtain generalized $dS_4$ and $AdS_4$ Reissner-Nordstrom (RN-) like black hole geometries in a curved $D_3$-brane frame-work [8], underlying the noncommutative gauge theory on the brane-world [9]. We investigate the gravity decoupling regime initiated by the Hawking radiation phenomenon from the black holes. A noncommutative scaling [10] limit generated in the frame-work, is explored to obtain the low energy gravity regime. A priori, the theory may be seen to describe $2D$ extremal dS black hole geometry, which may alternately be viewed as a combination of AdS and dS geometries. However, the presence of three extra large dimensions in the regime is argued to elevate a near horizon $dS_2$ geometry to an appropriate $dS_5$. The Hagedorn phase in the self-dual gauge-string theory is exploited to show that the extremal black hole Hawking radiates the EM-nonlinearity [11] and is described by a pure dS space. The analysis incorporates a series of tunnelings among AdS and dS vacua and may provide a clue to our present day metastable brane-world.

II. CURVED $D_3$-BRANE AND SMALL $\Lambda$

A $D$-brane governs the boundary $\partial \mathcal{M}$ dynamics of an open string. The induced fields on the brane ($g_{\mu\nu}$ and antisymmetric $b_{\mu\nu}$) are the pull-back of the respective dynamical background fields in the string bulk.
In principle, the gravity dynamics can be incorporated into a curved brane frame-work along with the gauge dynamics of a $D_3$-brane. The formulation inspires one to seek for a fundamental theory $^{12}$ in presence of a three brane, such as D=12 constructions $^{13}$. However our starting point, in this paper, lies in the bosonic string theory.

$$S_{\text{string}} = - \int d^4x \sqrt{G}E \left( \frac{1}{16\pi G_N} R - 2(\partial \phi)^2 - \frac{1}{2} F_1^{(k)} C_{kl} F_1^{(l)} - \frac{1}{2 \cdot 2!} F_2^{(i)} D_{ij} F_2^{(j)} - \frac{Z}{2 \cdot 4!} L_{mn} F_4^{(m)} F_4^{(n)} \right), \quad (1)$$

where $(C_{kl}, D_{ij}, L_{mn})$ govern the appropriate moduli coupling to the gauge field of various ranks and $Z$ is a normalization constant. Then, the four form energy density becomes nontrivial and can be given by a potential in the moduli space

$$V_4(\phi) = \frac{Z}{48} F_4^{(m)} L_{mn} F_4^{(n)}. \quad (2)$$

On the other hand, the $D_3$-brane dynamics has been worked out, explicitly, for constant induced fields only. The Minkowski inequality in the theory enforces a self-duality of the EM-fields, in the $D_3$-brane dynamics. Then, the noncommutative gauge theory on the $D_3$-brane can be approximated by the Dirac-Born-Infeld dynamics. It is given by

$$S_{D_3} = - \int_{\partial \mathcal{M}} d^3x \sqrt{G} \left( \lambda_b - \frac{1}{4} G^\mu{}^\lambda G_{\nu}{}^\rho \tilde{F}_{\mu \nu} \ast \tilde{F}_{\lambda \rho} \right), \quad (3)$$

where $\lambda_b$ is the brane tension and $G \equiv \det G_{\mu \nu}$. The Moyal $\ast$-product accounts for the nonlocality arise due to the infinite number of derivatives there. Importantly, the gravitational back reaction has been incorporated into the effective theory, which is apparent from the definition of the modified metric $G_{\mu \nu} = (g_{\mu \nu} - [bg^{-1}b]_{\mu \nu} + [bg^{-1}b bg^{-1}b]_{\mu \nu} + \ldots)$. Now, the curved $D_3$-brane dynamics is obtained by coupling the noncommutative $D_3$-brane $^8$ to an effective string theory $^{11}$. In a static gauge, the complete dynamics of a curved $D_3$-brane can be given by

$$S = - \int d^4x \sqrt{G} \left( \frac{1}{16\pi G_N} (R - 2\Lambda) - 2(\partial \phi)^2 - \frac{1}{2} F_1^{(k)} C_{kl} F_1^{(l)} - \frac{1}{4} F_2^{(p)} D_{pq} F_2^{(q)} \right), \quad (4)$$

where $\Lambda(\phi) = 8\pi G_N (V_4(\phi) - \lambda_b)$. \quad (5)

$\lambda_b$ can take a large constant value as it can be seen to be controlled by an $U(1)$ gauge non-linearity in the theory. The multiple four forms in the theory together with the brane tension, redefine the vacuum energy $^{6}$. Since an explicit membrane dynamics is absent in the frame-work $^{4}$, the (multiple) four form equations of motion are worked out to yield $\partial \mu \left( \sqrt{G} L_{mn} F_4^{(m)} F_4^{(n)} \right) = 0$. For a stable minima in $V_4(\phi)$, the $L_{mn}$ takes a constant value. Then, the solution(s) to the equation(s) of motion are given by $F_4^{(n)}_{\mu \nu \lambda \rho} = \lambda^{(n)} \epsilon_{\mu \nu \lambda \rho}$, where $\lambda^{(n)}$ are constants and $\epsilon_{\mu \nu \lambda \rho}$ is a totally antisymmetric tensor. Thus at a local minima, the $\Lambda(\phi)$ takes a constant value

$$\Lambda(\phi) \rightarrow 8\pi G_N \left( \frac{Z}{2} \sum_{n=1}^{\infty} \left[ \lambda^{(n)} \right]^2 - \lambda_b \right). \quad (6)$$

It implies that the multiple four-forms along with the gauge non-linearity could possibly reduce the effective cosmological constant $^{2}$ to a small value in 4D, which lies along the idea of dynamical neutralization $^{14}$. The potential, between moduli and second rank gauge fields in $^{11}$, becomes $V_2(\phi) = - [\tilde{Q}_{\text{eff}}^2 + Q^{(i)} D_{ij} Q^{(j)}]$, where $Q_{\text{eff}}$ and $Q^{(i)}$ denote the electric (or magnetic) charges, respectively, on the brane and in the effective string theory.
With a gauge choice $G_{\alpha\beta} = 0$, for $(\alpha, \beta) \equiv (x^1, x^1)$ and $(i, j) \equiv (x^2, x^3)$, the action \( I \) is simplified using a noncommutative scaling \( \{10\} \). The scaling incorporates vacuum field configurations for some of the field components: $\partial_\alpha \varphi^{(m)} = 0$ and $F_{\alpha\beta}^{(p)} = 0$. Then, the relevant curved brane dynamics can be governed by its on-shell action. It is given by

$$ S = - \int d^2 x^{(i)} d^2 x^{(i)} \sqrt{h} \sqrt{\gamma} \left[ \frac{1}{16\pi} (R_h - 2\Lambda) + \frac{1}{64\pi} h^{ij} \partial_i h_{\alpha\beta} \partial_j h_{\gamma\delta} r^\alpha r^\beta - 2h^{ij} C_{mn} \partial_i \varphi^{(m)} \partial_j \varphi^{(n)} - \frac{1}{2} h^{\alpha\beta} h^{ij} D_{pq} F_{\alpha i}^{(p)} F_{\beta j}^{(q)} \right], \quad (7) $$

where $C_{mn}$ and $D_{pq}$ for $p, q = (1, 2, \ldots, i, i + 1)$ are the appropriate moduli couplings and $\varphi^{(m)}$ take into account the dilaton and axions in the theory. A most general static, spherically symmetric ansatz, for the metric in the frame-work is given by

$$ ds^2 = f dt^2_E + f^{-1} dr^2 + h^2 d\Omega^2, \quad (8) $$

where $f$ and $h^2$ are arbitrary functions of $r$.

III. dS AND AdS BLACK HOLES

A. Constant moduli

We consider constant moduli in the theory \( \{11\} \) and restrict the EM-field on the brane only, i.e. $\hat{Q} \neq 0$ and $Q^{(i)} = 0$. The anstaz for $\hat{A}_\mu \equiv (\hat{A}_1, \hat{A}_2, 0, 0)$ becomes $\hat{A}_t = -\hat{Q}_{eff} \sin \theta \cos \phi$ and $\hat{A}_r = \hat{Q}_{eff} \sin \theta \sin \phi$. The non-vanishing components of the self-dual EM-field are $E_\theta = B_\theta = (\hat{Q}_{eff}/r) \cos \phi$ and $E_\phi = B_\phi = -(\hat{Q}_{eff}/r) \sin \phi$. Then, the independent components of Ricci tensor in the theory can be expressed in terms of $f(r)$, $h(r)$ and $V_2(\phi)$. The metric components are worked out to yield

$$ f_\pm = \left( 1 \mp \frac{r^2}{b^2} - \frac{2M_{eff}}{r} \right) \left( 1 \pm E^2 \right) = \left( 1 - \frac{2M_{\Theta}}{r} - \frac{\Lambda}{3} r^2 \right) \quad \text{and} \quad h(r) = \left( r^2 - \frac{2M_{eff} \hat{Q}_{eff}^2}{r} \right)^{1/2}, \quad (9) $$

where $f_+$ and $f_-$ signify the appropriate geometries $f(r)$, respectively, for the dS and AdS spaces. The effective mass parameter $M_{\Theta}$ takes into account the noncommutative $\Theta$-corrections, from the boundary string dynamics \( \{12\} \), to the ADM mass and charge of a black hole. Explicitly, it can be expressed as

$$ M_{\Theta} = M_0 \left[ 1 - \frac{\Theta}{2r^2} + \mathcal{O}(\Theta^2) + \ldots \right] = \left( M_{eff} + \frac{\hat{Q}_{eff}^2}{2r} \right), \quad \text{where} \quad M_0 = G_N \left( M + \frac{\hat{Q}_{eff}^2}{2r} \right). \quad (10) $$

In the case, $M_{eff}$ and $\hat{Q}_{eff}$, respectively, denote the ADM mass and the charge of a generalized black hole. To order $\mathcal{O}(G_N)$, the explicit geometry corresponding to dS and AdS RN-like black holes are given by

$$ ds^2 = - \left( 1 + \frac{r^2}{b^2} - \frac{2M_{eff}}{r} \pm \frac{\hat{Q}_{eff}^2}{r^2} \right) dt^2 + \left( 1 + \frac{r^2}{b^2} - \frac{2M_{eff}}{r} \pm \frac{\hat{Q}_{eff}^2}{r^2} \right)^{-1} dr^2 + r^2 d\Omega^2, \quad (11) $$

where $b$ is the dS (or AdS) radius as appropriate to a geometry. The generalized black hole geometries are characterized by three parameters $(\Lambda, M_{eff}$ and $\hat{Q}_{eff})$. The horizon equation $f(r) = 0$ can be solved to obtain three physical horizons in $dS_2 \times S^2$. In the decreasing order of their radius, they are characterized by a cosmological horizon $r_c$, an event horizon $r_+$ and an inner horizon $r_-$. Interestingly, for $(M = 0 = \hat{Q})$ and $\Lambda \neq 0$, the black hole geometry reduces to a pure dS with a horizon at $r_c = b$. Similarly, for $\Lambda = 0$, the geometry corresponds to a generalized dS RN-like black hole with horizons at $r_{dS}^\pm = (M_{eff} \pm [M_{eff}^2 - \hat{Q}_{eff}^2]^{1/2})$. On the other hand, the AdS radius incorporates a periodicity in time coordinate $t_E \to t_E + 2\pi b$. For $\Lambda = 0$, there is only one event horizon which is unlike to that of dS black hole. The radius of the event horizon though resembles to that of a typical Schwarzschild black hole $r_{AdS}^H \simeq 2M_{eff}$, it governs a regular geometry there.
B. Arbitrary moduli

A nonconstant moduli in the theory retains nontriviality in the effective potential. In addition to the nonlinear $U(1)$ gauge potential on the brane, there are non-vanishing multiple $U(1)$ potentials in the case. We consider an appropriate anstaz for the multiple gauge fields $A^{(i)}$. The non-vanishing components of $A^{(i)}$ are the $⊥$-components and we consider them as $A_{\phi}^{(i)} = Q^{(i)}/r$ and $A_{\theta}^{(i)} = Q^{(i)} \cos \theta$. The corresponding EM-field(s) are given by $E^{(i)} = Q^{(i)} [(1/r^2) dt \wedge d\theta + \sin \theta d\phi \wedge d\phi]$. The non-vanishing electric or magnetic field components are given by $E_r^{(i)} = B_r^{(i)} = Q^{(i)}/r^2$. Interestingly, with an orthogonal rotation, the arbitrary function $f(r)$ can be represented by eq. (12). However the $E^2$ there, receives correction due to the multiple gauge fields at $O(G_N^2)$. It becomes

$$E^2 = \frac{1}{r^2} \left( \hat{Q}_{\text{eff}}^2 + \frac{G_N^2}{r^2} Q^{(i)} D_{ij} Q^{(j)} \right).$$

(12)

On the other hand, the moduli significantly modifies the radius of $S^2$. It is computed to yield

$$h = r e^{\phi(r)}, \quad \text{where} \quad e^{\phi(r)} = \left( e^{2\phi_h} - \frac{G_N}{r r_h} Q^{(i)} D_{ij} Q^{(j)} \right)^{1/2},$$

(13)

where the constant $\phi_h$ is the value of $\phi$ at the event horizon $r_h$. The effective mass $M_{\phi}$, in the case, can be seen to accommodate higher order terms $O(G_N^4)$. Explicitly, the $G^{tt}$ component is given by

$$f_\pm = \left( 1 + \frac{r^2}{b^2} - \frac{2M_{\text{eff}}}{r} \pm \frac{\hat{Q}_{\text{eff}}^2}{r^2} \right) \pm \frac{G_N^2}{r^2} Q^{(i)} D_{ij} Q^{(j)} \left( 1 - \frac{r^2}{b^2} \right) + O(G_N^3).$$

(14)

Then, the generic dS and AdS RN-like black hole geometries, to $O(G_N)$ in the theory, are given by

$$ds^2 = - \left( 1 + \frac{r^2}{b^2} - \frac{2M_{\text{eff}}}{r} \pm \frac{G_N \hat{Q}^2}{r^2} \mp \frac{\Theta \hat{Q}_{\text{eff}}^2}{r^4} \right) dt^2$$

$$+ \left( 1 + \frac{r^2}{b^2} - \frac{2M_{\text{eff}}}{r} \mp \frac{G_N \hat{Q}^2}{r^2} \mp \frac{\Theta \hat{Q}_{\text{eff}}^2}{r^4} \right)^{-1} dr^2 + e^{2\phi(r)} r^2 d\Omega^2.$$  

(15)

It implies that the area of the event horizon tend to shrink in presence of moduli in the theory. Unlike to the dS black hole, the AdS geometry possesses only one horizon. Our analysis suggests that the moduli corrections along $(t, r)$-space to dS and AdS black hole geometries begin at $O(G_N^3)$. However, the shrinking radius of the event horizon is reconfirmed even at $O(G_N)$. In absence of $D_3$-brane and with $\Lambda = 0$, the dS geometry reduces to the one obtained in an effective string theory (15).

IV. TUNNELING: AdS $\leftrightarrow$ dS

A. Gravity decoupling limit

In presence of nonlinear EM-charges, i.e. $\Theta$ corrections, the interesting feature of the gravity decoupling limit $g \to 0$ can be exhibited in the formalism. The limit describes the low energy aspects of an effective curved brane and essentially governs a semi-classical regime. It can be checked that the usual extremal limit $M \to \hat{Q}$ can be reached by taking $M \to 0$ instead. In the limit, the generic dS$_4$ black hole (16) is governed by $dS_2 \times S^2$. The extremal dS and AdS black hole geometries are given by

$$ds^2 = - \left( 1 + \frac{r^2}{b^2} \pm \frac{\hat{Q}_{\text{eff}}^2}{r^2} \right) dt^2 + \left( 1 + \frac{r^2}{b^2} \mp \frac{\hat{Q}_{\text{eff}}^2}{r^2} \right)^{-1} dr^2 + \left[ r_h^2 e^{2\phi_h} - G_N Q^{(i)} D_{ij} Q^{(j)} \right] d\Omega^2,$$  

(16)
where \( r_h \) is the radius of \( S^2 \) at the event horizon in absence of moduli. It implies that the moduli at the event horizon can be expressed in terms of the \( U(1) \) gauge charges, which in turn can shrink the effective event horizon radius to a small value in the regime. On the other hand \( f(r) = 0 \) relates the effective event horizon radius to the nonlinear \( U(1) \) charges. The radius of \( S^2 \) can be checked to satisfy

\[
r_h^2 e^{2\phi_h} = \frac{b^2}{2} \left[ \pm 1 + \left( 1 + \frac{4\tilde{Q}_{\text{eff}}^2}{b^2} \right)^{1/2} \right] + G_N Q^{(i)} D_{ij} Q^{(j)} .
\]  

(17)

For a fixed \( \tilde{Q}_{\text{eff}} \), the radius square of \( S^2 \) becomes \( |b^2 - V_2(\phi_h)| \), for the dS geometry and \( |V_2(\phi_h)| \), for the AdS there. It implies that the effective radius of the event horizon is not fixed in general, rather it is governed by \( V_2(\phi) \) in the moduli space. In the decoupling regime, the moduli moves to its local minima along the potential, \( i.e. V_2(\phi) \to V_2(\phi_h) \), and hence decouples the \( S^2 \) from the effective 4D geometry. The emerging 2D large black hole geometry is given by

\[
ds^2 = -r^2 dt^2 + \frac{dr^2}{r^2} .
\]  

(18)

Naively, \( dS_4 \) and \( AdS_4 \) black holes appear to govern different geometries. However, in the gravity decoupling limit they can be argued to overlap and describe a new \( dS \) black hole in two dimensions. In particular, the event horizon \( r_h \) of the \( AdS \) describes the curvature singularity of the new \( dS \) black hole.

**B. Extra dimensions and \( \Theta \)-decoupling**

In the extremal limit, the generic black hole solutions \(^{15}\) can be seen to provide hint for the existence of three large extra dimensions. In fact, the nonlinear EM-charges remain in the gravity decoupled regime govern an effective theory of gravity there. Naively, \( \tilde{Q}_{\text{eff}}^2 \) can be seen to correspond to a light mass for the black hole in the effective theory. The fact that \( 1/r^4 \) term is associated with a mass term in the extremal black hole geometry \(^{16}\) favors the assertion of three extra large dimensions in addition to the 2D black hole geometry within the curved \( D_3 \)-brane frame-work. In addition, the assertion can further be re-confirmed in presence of \( \Theta \)-terms there in \( dS \) and \( AdS \) solutions. In particular, its association with the \( 1/r^4 \) term, a priori, predicts some appropriate seven dimensional theory. However, two of the noncommutative constraints from the boundary theory, make the effective space-time dimension to five. In other words the formulation urges for an underlying 5D effective theory of gravity instead of that in 4D. Very recently in a collaboration \(^3\), the presence of a fifth dimension in the extremal limit has been exploited by using a noncommutative scaling on the brane. The orthogonality in coordinates make the fifth dimension \( \perp \)- to a generic \( D_3 \)-brane world-volume. In otherwords, the curvature in the curved brane theory becomes appreciable along the extra dimensions. Incorporating the required large extra dimensions into the curved \( D_3 \)-brane formalism, one may alternately view the 2D extremal black hole \(^{15}\), either as an \( dS_2 \times S^1 \) or as an \( AdS_2 \times S^3 \) geometry. The result is in agreement with the \( dS_5/CFT \) \(^2\) and precisely with the \( AdS_5/CFT \) correspondence in string theory \(^3\).

Now, the gauge theory perspective of extremal \( dS \) black hole \(^{15}\) is investigated for its near horizon geometry. The Hawking radiation leading to an extremal geometry essentially describes a typical \( D_3 \)-brane, which corresponds to the event horizon of an \( dS_5 \) black hole. It can be given by

\[
ds^2 = -\left( 1 - \frac{r^2}{b^2} + \frac{G_N \tilde{Q}^2}{r^2} - \frac{\Theta \tilde{Q}_{\text{eff}}^2}{r^4} \right) dt^2 + \left( 1 - \frac{r^2}{b^2} + \frac{\tilde{Q}_{\text{eff}}^2}{r^4} - \frac{\Theta \tilde{Q}_{\text{eff}}^2}{r^4} \right)^{-1} dr^2 + r^2 \, d\Omega_3^2 .
\]  

(19)

Here \( r \) is the radius of \( S^3 \). The event horizon of \( dS_5 \) RN-black hole is at \( r_h = (b^2 + \tilde{Q}_{\text{eff}}^2)^{1/2} \) and the curvature singularity is at \( r = \tilde{Q}_{\text{eff}} \). Due to the non-trivial \( U(1) \) charges in the solutions, they are often referred as monopole black hole in literature \(^{14}\). However, the no hair conjecture \(^{17}\) for the nonlinear charged black hole, make it unstable. In other words, the extremal black hole undergoes Hawking radiation to decouple the
The nonlinear gauge decoupling can be seen to describe a second order phase transitions and leads to the Hagedron phase in the framework. In the regime, the critical phase can be described by \((E^+ - E^-)\), where the nonzero string modes in the regime undergo an exchange between its real and imaginary components. As pointed out, it results in the decoupling of nonlinear gauge charges and lead to a RN-black hole solution in 5D. Finally, the Hawking radiation ceases with a stable remnant of \(U(1)\) gauge charge and the geometry may be described by

\[
dS^2 = - \left( 1 - \frac{r^2}{b^2} + \frac{G_N \hat{Q}^2}{r^2} \right) dt^2 + \left( 1 - \frac{r^2}{b^2} + \frac{G_N \hat{Q}^2}{r^2} \right)^{-1} dr^2 + r^2 d\Omega_3^2,
\]

where the radius of the event horizon is \(r_h = b\) and the curvature singularity can be seen to be at \(r = 0\). The monopole black hole solution seems to possess two interesting features. For large \(r\), it reduces to a pure dS geometry, possibly describing our 4D brane-world on its boundary. On the other hand, at Planck scale the dS black hole reduces to a precise monopole solution with asymptotically flat geometry, \(i.e. \Lambda = 0\). It possibly re-assures our earlier assertion that a nonlinear EM-field gives rise to a cosmological constant in a curved brane theory.

V. CONCLUDING REMARKS

To conclude, dS and AdS (generalized) RN-like black hole geometries were obtained in a curved \(D_3\) framework underlying a noncommutative \(U(1)\) gauge theory on its brane-world. The small value of cosmological constant was argued in the frame-work following a dynamical neutralization technique. The frame-work was shown to accommodate multiple \(U(1)\) gauge fields coupled to moduli, which are in addition to the noncommutative gauge field. While the nonlinear EM-field was shown to incorporate the back reaction into the metric, the multiple gauge fields there was shown to shrink the event horizon radius. In the regime, the reduced horizon radius was shown to be due to a large number of charges arising out of all the gauge fields in the frame-work. The emerging notion of a 2D extremal monopole black hole governing a dS geometry was shown in the regime. On the other hand, at Planck scale the event horizon of the extremal AdS there transformed to a curvature singularity in dS space. The effective gravitational potential associated with the reduced mass of extremal dS black hole was analyzed to confirm the presence of three extra large dimensions in the regime. The Hagedron transitions in the near horizon geometry of dS\(_5\) monopole black hole was analyzed to decouple the \(\Theta\)-terms. The new dS\(_5\) vacuum with a nontrivial cosmological potential possibly describes our brane-world, \(i.e.\) a \(D_3\)-brane, at its boundary. The potential was argued to be at its local minima on the brane-world and describes a small positive constant \(\Lambda\).

Finally, a careful analysis may reveal that two different topologies representing dS\(_2\) and AdS\(_2\) geometries are interchanged, \(i.e.\) \(\mathbb{R} \times S^1 \leftrightarrow S^1 \times \mathbb{R}^1\) in the gravity decoupling regime. Intuitively, it incorporates an interchange of 2D geometries, such as hyperbolic \(\leftrightarrow\) cylindrical. The change in topology is significant to an emerging two dimensional aspect of space-time within a 4D effective string theory. It provides an evidence to the notion of signature change discussed in a collaboration with Majumdar. Nevertheless, the topology of the 4D effective spacetime remains unchanged. The tunneling between dS\(_2\) and AdS\(_2\) vacua is a potential candidate and may possess deeper implications in quantum gravity. Though, it may be illuminating to view the tunneling analogous to the established closed \(\leftrightarrow\) open string duality, it remains to explore a concrete geometric relation in higher dimensions.

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