TOWARDS AN AXIOMATIC MODEL OF FUNDAMENTAL INTERACTIONS
AT PLANCK SCALE

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By exploring possible physical sense of notions, structures, and logic in a class of noncommutative geometries, we try to unify the four fundamental interactions within an axiomatic quantum picture. We identify the objects and algebraic operations which could properly encode the formation and structure of sub-atomic particles, antimatter, annihilation, CP-symmetry violation, mass endowment mechanism, three lepton-neutrino matchings, spin, helicity and chirality, electric charge and electromagnetism, as well as the weak and strong interaction between particles, admissible transition mechanisms (e.g., $\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_\mu$), and decays (e.g., $n^0 \rightarrow p^+ + e^- + \bar{\nu}_\mu$).

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Introduction
In this paper we further our approach \cite{1, 2} to an axiomatic description of the four fundamental interactions; for this we explore a natural class of noncommutative geometries \cite{2, 3}. Our approach is to some extent non-orthodox: we try to recognize physical phenomena beyond a given set of mathematical objects and structures; their algebraic simplicity is an evidence that such logic of information processing could be admissible or even dominant at Planck scale. Let us emphasize that we view quantum phenomena in the Universe as interaction of information codes so that the algebraic constructions at hand are the objects of Nature.

This review is a sequel to \cite{2}, where we started to build the noncommutative model by considering a quasicrystal tiling of empty space. This tiling would yield an untwisted affine Lie algebra determined by the irreducible root system $A_2, B_3$, or $C_3$, see \cite{2} — if there were no defects in the crystal structure; note that the usefulness of the compactified fourth dimension which is associated by tadpoles $S^1 \times S^1$ with the algebra's null vector will be revealed in section \cite{11} of this paper. We interpreted the time as a process of local topological reconfigurations of the lattice — whence the defects do emerge — and we argued that contractions of edges in the 1-skeleton of the cell complex at hand shows up as the mass. The reverse process of spontaneous edge decontractions releases energy via $E = mc^2$. We thus noted that Hubble’s law reflects a steady self-generation of space, whence we conjectured that the emission of cosmic microwave background radiation (CMBR) is an immanent property of space as topological manifold. This implies that the CMBR cannot be altogether shielded, which suggests a possible (dis)verifying experiment checking our concept. By identifying the mass with reconfigurations in the topology of space, we revealed a candidate for the graviting but invisible ‘dark matter’ — in this picture, it appears not as any form of matter but as a specific, meta-stable state of vacuum. (Another algebraic notion of (synonyms to) zero-length cyclic words shows up as a scalar field of ‘dark energy,’ see section \cite{1} below.) In this paper we focus on the formation of matter and on the geometry of fundamental interactions in quantum space; we shall analyse the algebraic structures and logical operations which are immanent to that noncommutative world.

It must be noted that there co-exist many approaches to discretisation of the space-time and making it noncommutative; to the best of our knowledge, the paper \cite{2} was seminal (see also \cite{8}). A remarkable sample of axiomatisation trend is \cite{6}. Nowadays, various directions are represented, e.g., by \cite{10–15}, etc.; in contrast to loc. cit., the non-standard construction in \cite{10} is built on an explicit assumption of sufficient smoothness for all structures at hand. However, it is readily seen that a benefit of space discretisation is gained from the use of finite, lattice-dependent adjacency tables for points which mark the quantum domains. This makes the loop quantum gravity paradigm \cite{17} close to ours; yet we add the compactified, null dimension of the tadpoles $S^1 \times S^1$, which relates the picture to Kaluza–Klein models. Continuing a comparison of our reasoning with existing schemes, let us recall that in \cite{2} we analysed why a smooth, large-scale limit of the robust topological setup looks like the electroweak $U(1) \times SU(2)$-gauge theory. Furthermore, by introducing the noncommutative jet spaces and by re-establishing the entire calculus of variational multivectors in \cite{1, 3, 18}, we aim to properly encode the quarks, that is, confined building blocks of the strong interaction. Finally, we recall that by a use of cyclic word approach \cite{6} for coding sub-atomic particles one realizes them via closed strings of symbols written around the circles; the calculus of such necklaces then essentially amounts to familiar construction of topological pair of pants $S^1 \times S^1 \rightarrow S^1$, cf. \cite{17}.

We attempt to understand the Universe as an information processor. While we only hypothesise about the

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algorithms which are actually employed to encode and process information, still we ought to define what information is — at least in this context. Let us temporarily accept the following heuristic definition which appeals to a common-sense idea that information must be meaningful and in principle verifiable; hence our formula is non-rigorous, possibly incomplete or self-referencing, and maybe contradictory; we also remark that the quantitative measurement of information is a much more delicate issue (here we refer to the classical concepts of A. N. Kolmogorov, C. E. Shannon, etc.).

**Definition 1.** Information is a rule that inputs a message, that is, a sequence of 0’s and 1’s, and states a (non-)strict preference ≼ or ≳; by its output the rule attempts to predict the elementary event for the next digit of the message to be either 0 or 1. The non-strict preference 0 ≳ 1 says that 0 is not less likely than 1 whereas 0 ≼ 1 tells us that 1 is not less likely than 0. The act of comparison of the rule’s prediction and the actually available next elementary event (possibly, itself being the first in a longer sequence) is a non-obligatory act of verification.

We accept that the processing of information is serious: the rule states the same output on the same input data whenever an input is processed twice. If the rule exhibits its readiness to change opinion for repeated input, then the elementary events 0 and 1 are equally possible (for example, an oracle is asked about the result of throwing a coin outside the light cone of its past). The non-strictness of preference builds the idea of uncertainty into information messages; however, if the rule states an absolute preference 0 ≳ 1 or 0 ≼ 1 for zero and unit, respectively, then the information is called precise (yet it may be false; uncertain information can also be true or false).

A sample application of such information messages is a check whether a given (i.e., contained in the input) object belongs to an encoded class of objects possessing a given property. For example, such is the coding of a triangle in a graph (see Fig. 4 on p. 7): whenever two sides are given, the rule discards all offered objects except for edges and all test edges except for the one which closes the contour.

On the other hand, a sample precise message “the undetectable does exist” affirmatively states —apart from the by-product information that there exist messages in general and there are means to encode and transmit them—the existence of existence and perhaps the existence of the one who—or something which created that message; other aspects of this message’s meaning are non-verifiable hence non-informative.

Summarising, information is (1.1) a formalised input and (1.2) a rule that establishes a preference for the next elementary event. In turn, processing information is (2.1) the information itself and (2.2) a second-order rule that states a preference for the choice of new rules on the basis of input rules. For example, the decay of a free neutron is processing the rule that said ‘yes’ whenever it was asked whether there was a neutron and which described that neutron; the processor’s output states a preference that the new rules should ascertain the existence and describe the (motion of) proton, electron, and electron’s antineutrino, see 1 on p. 9.

The laws of Nature are rules of third order, consisting of (3.1) second-order (reaction-) rules and (3.2) the rules to balance or modify the former (e.g., by prescribing the relative velocities). Such are the conservation laws for electric charge, energy, momentum, or angular momentum, etc. Notice that the laws of Nature do not refer to the formalised input (1.1) by using which one encodes the actual presence and configuration of events or particles and their properties; these laws are universal.

It is perhaps appropriate to say that the set of fourth-order rules for (non-)modification of the laws of Nature is a choice of the Universe itself. Indeed, should the fine-structure constant α = e²/(hc) ≈ 1/137 be varied or the dimension and topological properties of space (such as orientation or orientability) be changed, this would produce a different Universe; likewise, a slow modification of parameters in the laws of Nature (e.g., a drift of Hubble’s constant in time or a change of half-life time for neutrons) would mean that the Universe itself is changing.

**Part I. Noncommutative geometry of particles**

For consistency, let us briefly recall the construction of quantum space; we refer to 2 for motivation and detail. To discretise space, we first consider its filling with a CW-complex: one may think that within sufficiently large domains — wide with respect to a count of edges, diam ≫ 1, along the 1-skeleton the topology of such graph is that of crystal structure produced from the root systems A₃, B₃, or C₃; their irreducibility prevents a slicing of space to lower-dimensional components. The choice of a root system from the set of these three is not definite so that superimposing local fragments of the respective CW-complexes co-exist; a transition between the adjacency tables for different systems means a difference in organisation of information channels (see sec. 1111 below). At the walls of domains with regular crystal filling, the near-by vertices are interconnected by edges in a less regular way.

The cell complex which is dual to the CW-complex at hand creates – by a Voronoï tiling of space – a quantum domain around each vertex in the old graph; the adjacency table for vertices becomes the table of neighbours for new domains. By construction, the old vertices act now as markers of quantum domains. In 2 we analysed the subtle aspects of taking continuous limits of such tilings by using an infinite bisection of edges. Let us recall also that a macroscopic notion of length is not defined in such topological, homeo-class geometry; the distance between points and its measurement make sense only at large scale in the smooth, diffeo-class realisation of the Universe.

The reconfigurations of a given lattice consist of elementary events of edge (de)contractions, in the course of which the adjacency tables for a pair of neighbouring
vertices are merged (so that the edge connecting them is destroyed) and the tadpoles, of which we speak in the next paragraph, are identified; the decontraction means: splitting of the table, insertion of the edge, and separation of the tadpoles. We noted that a count of faces for those cells in the dual complex which encapsulate the contraction defects could render the entropic origin of gravity (in conformity with [15]). At the same time, we shall put the instability of particles in the context of spontaneous modifications in the topology of space and Hubble’s law.

The other step in the construction of quantum space is an attachment of tadpole to every vertex of the CW-complex’ 1-skeleton; in other words, the marker of each quantum domain is endowed with an extra edge which starts and ends there, going within the fourth, compactified dimension.

Let us suppose that the resulting CW-complex is oriented (cf. sec. IIB below) so that \( S^{\pm 1} \) is the path which runs along a tadpole in positive direction and \( S^{-1} \) is a walk against the wind. Denote by \( \vec{x}_i \) the (vertex-dependent) collection of edges issued from a given vertex, and set \( \vec{x}_i^{-1} \) for the same edges whenever they are went in the reverse direction whence \( \vec{x}_i \vec{x}_i^{-1} = 1 \) is the null path. We postulate that spatial edges are fermionic: no path can run along an edge in the same direction twice. On the other hand, the tadpoles \( S^{\pm 1} \) are bosonic (why? see [2]. Note 1). It is clear now that words written in the alphabet \( S^{\pm 1}, \vec{x}_i^{\pm 1} \) encode paths, or walks along the lattice’s edges.

1. FORMATION AND STRUCTURE OF SUB-ATOMIC PARTICLES

Suppose that the graph, i.e., 1-skeleton of the CW-complex is given and \( S^{\pm 1}, \vec{x}_i^{\pm 1} \) is the alphabet associated with its vertices; we note that it can be vertex-dependent due to the on-going spontaneous splitting of edges, which creates the irregularity of the graph’s structure and its possible local deviation from the regular tiling given by root systems. Then every word in the alphabet(s) determines the path along the edges, starting at a given point; we shall consider the words of length zero and their synonyms separately at the end of this section.

In what follows, we view paths as equivalence classes of walks because of a possibility to insert, wherever possible, trivial paths \( (a \cdot a^{-1}) \) along the spatial component of the 1-skeleton or a trivial word \( (S^1 \cdot S^{-1}) = (1) = (S^{-1} \cdot S^1) \) by using the tadpole attached to every vertex. We recall that the spatial edges of the graph are fermionic and thus may be passed at most once in each of the two directions; this eliminates the risk of infinite loops.

We note that the paths (or walks) of positive proper length\(^1\) can happen to be closed, i.e., end at their starting point but not retract to it if one pulls by both ends of a thread that has been unrolled along the edges of the path. Notice further that such cycles can equivalently start and end at any other vertex along the contour; thus, the words encoding them are cyclic-invariant (see [1, 2] and [3]).

There are several mechanisms for a given path to be closed (apart from being a tadpole \( S^{\pm 1} \) hence closed by definition). First, there is a glossary, i.e., a list of cyclic-invariant words (more precisely, a point-dependent gallery of drawn contours); we shall quote from that source in sec. III and in Part II from p. 8 onwards. Second, there could be an additional list of (formal sums of) paths which themselves are not closed but which link to a contour whenever attached consecutively in suitable order, see sec. V. Thirdly, one can proclaim that a given path is closed by manually contracting a set of edges between its loose ends; this may require a considerable or infinite energy (recall that the topology of the CW-complex is trivial and one may not return to the starting point by walking in one direction and still coming back around the entire Universe).

Next, there is a mechanism that generates cycles in the course of decontraction of edges. Namely, consider the synonyms \( (1) = (\vec{x}_i \vec{x}_i^{-1}) \) and suppose that the vertex which the null path does not leave is a pair of vertices connected by a contracted edge, see Fig. 1. The formation of cycle is completed by matching the direction in which the new, decontracted edge is passed with the new face’s orientation induced from the oriented CW-complex; note that energy is released in this process.

Finally, the energy-consuming scenario of cycle formation is an emission of two closed contours which are walked in the opposite directions (see Fig. 6b) on p. 6: the there-and-back-again path visits its starting point

\(^1\) The proper length of a path is the minimal number of edges in this path and in all its synonyms that may differ from it by synonyms of zero-length word inserted at any vertex along the way.
thrice and is torn exactly in the middle, which creates two mutually inverse replicas; energy is spent on the disrup-
tion of the synonym of trivial word $\langle 1 \rangle = \langle a \cdot a^{-1} \rangle$ in
two nontrivial cyclic words $\langle a \rangle$ and $\langle a^{-1} \rangle$.

We postulate that particles are the meaning of cyclic
words that encode contours along the graph; such words
contain the mark-up of contracted edges (this has noth-
ing to do with the contractions configuration for edges
where the path does not run). We have noted in [2]
that the dynamics of contraction configurations deter-
mines the evolution of curvature and hence mass-energy.
We let this be the mass endowment mechanism for par-
ticles: if there is at least one contracted edge along the
contour, the particle is massive; otherwise it is massless
(see Fig. 3.4 on p. 7). Note that a given continuous con-
tour with its contractions mark-up could have different
masses with respect to the continuous limits of different
tilings of space.

To formalise this approach in algebraic terms and make
applicable the formalism of $\mathbb{R}$, we say that a particle is a
functional

$$
\mathcal{H} = \sum_{\text{words}_x} \int \langle \text{cycle}_x(S^{\pm 1}, \vec{x}^{\pm}) \rangle \, d\mu(x),
$$

where we use the following notation: The cycle is a closed
contour that starts and ends at a point $x$ of continuous
space (a reference of the particle to a point $x$ is equiv-
lent to referring it to any other point $y$ which lies on
the contour passing through $x$; still this amounts to a re-
placement of the contour’s cyclic word by its equivalent,
starting the walk now at $y$, which leaves the contour in-
tact); the measure, which refers to sets of points in space
but does not exploit the notion of length, allows us to refer
the particle to just one point – or create a cloud of matter by spreading the contour in space over a given set;² the units of measurement for the functional $\mathcal{H}$ are
those of energy.

Remark 1. In terms of the diffeo-class geometry of $\mathbb{R}$,
the functionals $\mathcal{H} = \sum_{\text{words}_x} \int \rho(x, t) \langle \text{cycle}_x \rangle \, d\mu(x)$ are Hamilton-
ionians defined on the noncommutative space of maps
from space to a free associative algebra’s quotient over
relation of linear equivalence under cyclic permutations,
or on the total space of infinite jet bundle over such space
of maps. The values of such functionals are (formal sums
of) possibly massive contours; particles interact by using
algorithms and structures which we discuss in Part II (see
also $\mathbb{R}$). Interactions between particles form a chain of
events which contributes to a pace of time for a local
observer.

If a particle is referred to only one point of space, the
discrete measure realises Dirac’s $\delta$-distribution; note that
it is the information about the contour and its prop-
ties which is ascribed to one point — still nothing is
“compressed.” In particular, it is impossible to split
an electron in fragments and assemble it by traspor-
ting such fragments from the spatial infinity to a given
point, spending infinite energy to overcome the repulsion
potential of the would-be fractions of the charge $–e$; this
approach also resolves the difficulty with an infinite den-
sity of electron’s mass. Indeed, the particle is proclaimed
existing at a certain point of space.

On the other hand, for dimensionful particles like pro-
ton or neutron the measure is concentrated on a larger
set; typically, its support is a connected bounded set. We
thus exclude from further consideration rapidly decreas-
ing distributions with –juridically speaking, unbounded–
supports.

Remark 2. The interactions operate with the values of
the functionals that encode particles but not with the
functionals themselves; in effect, particles interact as
indivisible entities so that the processes refer to the
particles’ existence but not to their instant “shapes,”
for those are undefined due to the Heisenberg uncer-
tainty principle. For example, in the course of decay
$\nu^0 \rightarrow p^+ + e^- + \nu_e$ the free neutron stops existing in
space at all its points when proton and the other two
particles are formed or start to form.

Remark 3. The edges which are contracted break the
symmetry of any contour; they also produce the irre-
ularity of the graph’s structure near the merging ver-
tices, not necessarily at points of the contour. Such
defects induce the particle to interact with other ob-
jects; conversely, massless point particles with simple
contours without contracted edges or marked vertices
(e.g., not carrying electric charge) demonstrate very low
cross-sections for interaction with matter.

The scalar field of zero-length words is the density of
vacuum energy. It does not show up in the form of energy
communicated to any particles chiefly because of absence
of those particles; it just is. However, we already know
that the trivial word $\langle 1 \rangle$ is synonymic to paths $\langle a \cdot a^{-1} \rangle$ walked twice, there and back again; in particular, it is
synonymic to a closed cycle $a$ and then $a^{-1}$ walked in the
opposite directions. By spending some extra energy on
rupturing the contour $a$ from the anticontour $a^{-1}$, which
creates two cyclic-invariant words of positive length, we
convert a part of the spare vacuum energy into the mat-
ter-antimatter pair of particles.

II. MATTER VERSUS ANTIMATTER

The orientation of the CW-complex distinguishes be-
tween two directions to walk around a given contour (e.g.,

² A typical macroscopic diameter of such set of reference points
would be the diameter of proton, which is $\approx 10^{-18}$ m, making
$\sim 10^{18}$ Planck units.
a face of a cell). To pass a contour backwards, the relay is this:

- replace each letter $S^{\pm 1}$ or $x_i^{\pm 1}$ with its inverse, resp., $S^\mp 1$ and $x_i^\mp 1$;
- read the word backwards, i.e., in the right-to-left order.

This mechanism tells matter from antimatter; the same principle is applicable literally to formal sums of non-closed paths, see sec. III B; after its minor part is spent on the disruption of contours, the exact amount of released energy depends on the mass-energy of the two vanishing enantomorphs.

The fact that the CW-complex is oriented not only distinguishes between Left (L) and Right (R) but also motivates a possible violation of the CP-symmetry, which itself is the mirror-reflection $Left \rightleftharpoons Right$ in the orientation of space and a substitution matter $\rightleftharpoons$ antimatter (i.e., reading backwards all the words from the glossary).

Explanation. Let us take a –now-existing– tetrahedron in the spatial part of the graph and contract it along three bold edges as in Fig. 2 doing this in two mirror-reflected ways (note that the orientation swap $Left \rightleftharpoons Right$ is local so that space is not turned inside-out; the labels of vertices remain what they are because it is only the edge contraction scenario which is reflected in the mirror). Let us recall now that a catalogued particle itself and the processor which handles particles –e.g., by disrupting contours and reconfiguring the available edges– is an automaton: it reads the (cyclic) words from the glossary and crawls along the contracted graph by running a program like this:

R: move right, then right, then right again

![FIG. 2: Mirror-reflected contractions.](image)

Suppose for definition that the tetrahedra in Fig. 2 b-c) are the only contracted edges in the Universe and it is these two objects which encode the choice of its orientation. The mechanism of CP-symmetry violation is that the mirror-reflected contractions of the tetrahedron produce unequal configurations of the tadpoles, see Fig. 3. Thus, the R-automaton that runs the R-program and, for definition, reaches the vertex 1 after its first step terminates the program at the point two steps to the right from the vertex 2. Reshape now the tetrahedron’s contraction and switch to the antiparticles and L-program; the L-automaton starts at the R-automaton’s endpoint, runs the L-program, and terminates at the point 1 (but not at the starting point of the R-automaton) because the edge $2 \rightarrow 1$ becomes a tadpole in the antiworld instead of being contracted in the right-oriented world (the

![FIG. 3: Formation of tadpoles in Fig. 2](image)

3 We emphasize that the order in which one passes the edges when reading the contour’s cyclic word is not the same as a choice of the contour’s orientation (either matching or reverse with respect to the orientation induced from the CW-complex); we reserve that choice for the definition of spin.

4 The catalogue of matter and antimatter is the glossary of equivalence classes of contour-determining cyclic words with a mark-up of the edges to-contract; the glossary is independent from the actual configuration of contractions in the graph — it is indeed a list of words.

5 Notice that the order of reading letters in catalogued words and the arising precedence “before” and antecedence “after” have nothing to do with the time as physical process; it is not appropriate to postulate that antimatter flies backwards in time.
orientation is determined by the order $1 \prec 2 \prec 3 \prec 4$ in Fig. 2(b)).

In particular, it may happen that by running its $R$-program the $R$-automaton crawled along the closed contour while reading the right words from the glossary but the path of the $L$-automaton appears to be not closed, after the orientation was reversed by switching from Fig. 2(c) to Fig. 2(b). Recall that a reaction is information-processing the output of which is a word, and the Left and Right automata attempt to create the particles’ contours by reading this word in one of the two possible directions. We conclude that, specifically to the scenario which we have discussed, the $L$-channel either is suppressed altogether (by an earlier convention that spatial edges are fermionic) or is possible at an expense of energy needed to contract extra edge(s) in order to close the $L$-path and thus bring the $L$-automaton to its starting point.

This produces the CP-asymmetry if the preference of Right over Left is implanted in the Universe since the moment of decontraction of the first tetrahedron in its history.

Corollary 1. The masses of particles and respective antiparticles can be (slightly) unequal.

Corollary 2. If Right has been prevailing over Left since the moment when creation of particles became possible, there is an imbalance between matter and antimatter nowadays.

Remark 4. The risk of CP-symmetry violation is built into the automaton which (1) reads from the glossary dogmatically and (2) attempts, by disrupting a word in between two letters and by having disrupted another given cyclic word, to paste that open word from the glossary in between, or immediately before or after the letters of the given word, and then (3) attempts to realise the output as a route along the graph — not taking into account the actual configuration of contractions.

On the contrary, the functionality of a different-type automaton that handles already-existing closed contours in space is stable. Indeed, the already paved routes remain closed if they had this property before the CP-transformation (note that the decontracting edges are pasted into the contours), see sec. V for details.

III. PROPERTIES AND EXAMPLES OF ELEMENTARY PARTICLES

The definition of quantum numbers and each act of their measurement channels information between tiling(s) of the homeo-class topological manifold by a CW-complex and the diffeo-class, smooth and commutative visible space. There is no surprise in that the available data which can be transmitted in this way must be very rough and appeal to topological invariants only.

A. The spin

From now on let us suppose that the contours which determine particles are oriented; note that a choice of orientation for the contour’s edges within each of its loops (e.g., consider a bouquet of circles) is in general not correlated with the order in which these edges are passed when one reads the particle’s cyclic word. We notice also that a contour’s orientation is thus not necessarily inherited from the orientation of the CW-complex.

Definition 2. The spin of a given particle is a quantum number which is equal to the sum

$$s = \sum_{\text{loops}} \pm \frac{1}{2} \hbar$$

over all the loops in the spatial component of the closed contour. (The tadpoles $S^{\pm 1}$ at each vertex are dealt with separately in the next section). The contribution of a loop is $+\frac{1}{2} \hbar$ if the choice of its orientation coincides with the order in which these edges are written in the cyclic word, and equals $-\frac{1}{2} \hbar$ otherwise.

Almost all known particles do have spin and most of the stable particles (either massive or without mass) are coded by simple contours –the unknots– so that their spins are $\pm \frac{1}{2} \hbar$.

Remark 5. The orientation of a loop in a contour can instantly switch from $+\frac{1}{2} \hbar$ to $-\frac{1}{2} \hbar$, making no effect on the orientation of other loops in the contour, so that the overall value of the spin changes by unit steps $\pm \hbar$ between its minimal nonpositive and maximal nonnegative values.

Remark 6. In view of a possibility for an sudden swap of orientation of a given loop, it is still meaningful to say that products of a weak reaction —in the course of which the loops are instantaneously disrupted and recombined— can at that moment retain the orientation of edges from loops in the input particles. We therefore expect that the spins of free neutrons and protons which are emitted in the $\beta$-decay $n^0 \rightarrow p^+ + e^- + \nue$, are correlated (though for none of them we can measure for certain the projection of spin to a given direction in the macroscopic space at the moment of decay, and though neither before nor after the reaction two measurements of the same particle’s spin would necessarily coincide).

B. The electric charge

Let us recall from [2] that each vertex of the CW-complex (and every point in the continuous limit of a tiling) carries

\[6\] Experiments report that the electron neutrinos $\nu_e$ usually have

left helicity, that is, the orientation of their contours makes left-handed helix propagating in the direction of their macroscopic instant velocity; likewise, the electron antineutrinos $\nueb$ tend to have right helicity, see Fig. [2] on p. 7.
the tadpole(s) \( S^1 \) starting and ending at that point.

**Definition 3.** The electric charge of a given particle is a quantum number which is equal to the difference

\[
\begin{align*}
  c &= \sum_{\langle \text{words} \rangle} \left( \frac{\sharp S^1}{\sharp S^{-1}} \right) \cdot e
\end{align*}
\]

of the numbers of time that particle’s contour passes the tadpoles –as they are written in the particle’s word(s)– in positive and negative directions, each loop in the compactified dimension thus contributing with the respective elementary charge \( \pm e \).

**Corollary 3.** The electric charges of particles and their antiparticles are equal by absolute value and have opposite signs.

**Remark 7.** In the beginning, the initial electric charge of the Universe was equal to zero because the contours did not exist yet (for all the tadpoles were contracted).

**Corollary 4.** Nowadays, the Universe is overall electrically neutral because all separately existing electric charges in it were obtained by disruption of neutral contours (indeed, all the contours were obtained from copies of the trivial word \( \langle 1 \rangle \) by decontraction of edges, see Fig. 1 on p. 3 or by disruptions, see Fig. 5 on p. 9).

**Remark 8.** There is a temporary shortage of magnetic charges in this Universe.

**C. The photon**

By introducing the following definition we resolve a delicate issue that the photon is a bosonic particle that can exist in two possible states (or polarisations), which usually requires that one manually removes the spin-zero state from the triplet \( s \in \{ -h, 0, h \} \).

**Definition 4.** The polarised photons \( \gamma \) are massless point particles whose cyclic words are

\[
\begin{align*}
  \gamma_\circ &= \langle S^1_m S^{-1}_n \rangle, \\
  \gamma_\bullet &= \langle S^{-1}_m S^1_n \rangle, \quad m < n,
\end{align*}
\]

where \( m, n \in I \) belong to the ordered indexing set at a point of quantum space (see [2, Note 1]); photons carry no electric charge \( (\pm e \mp e = 0) \).

**Corollary 5.** The polarised antiphotons \( \overline{\gamma} \),

\[
\begin{align*}
  \overline{\gamma}_\circ &= \langle S^{-1}_m S^1_n \rangle^{-1} = \langle S^1_m S^{-1}_n \rangle = \langle S^{-1}_m S^1_n \rangle = \gamma_\circ, \\
  \overline{\gamma}_\bullet &= \langle S^{-1}_m S^1_n \rangle^{-1} = \langle S^1_m S^{-1}_n \rangle = \langle S^1_m S^1_n \rangle = \gamma_\bullet,
\end{align*}
\]

(here \( m < n \)) are identical to the photons with opposite polarisations.

**Definition 5.** The electron \( e^- \), antielectron (or positron) \( e^+ \), electron neutrino \( \nu_e \), and electron antineutrino \( \overline{\nu}_e \) are point particles whose contours are drawn –schematically and up to homeomorphisms– in Fig. 4; the contracted edges are marked by dotted lines. Each of the four particles has spin \( \pm \frac{1}{2} \hbar \) due to the two possibilities to choose an orientation of these unknots in \( \mathbb{E}^3 \). The electron \( e^- \) has negative electric charge \( -e \) and that of positron \( e^+ \) equals \( +e \).

**Corollary 6.** Apart from carrying no electric charge and probably having no mass, the electron (anti)neutrino is in all other respects “indistinguishable” from the (anti)electron.

The mass of (anti)electron is approximately 0.511 MeV/c\(^2\); by using Fig. 1 on p. 3 we argued that there are chargeless massless spin-\( \frac{1}{2} \hbar \) particles and now we identify them with electron (anti)neutrinos; however, our reasoning does not forbid the existence of particles with very similar properties yet with a tiny mass (hence travelling slower than light). If being massless and thus having no naturally marked vertex or

**Remark 9.** Photons travel\(^7\) in space with invariant light speed \( c \). However, when a photon is stopped by a material object in continuous space at a given point, the indexed set of fermionic tadpoles at that point reduces to a unique circle \( S^1 \) so that the photon \( (S^{1\pm 1}\leftarrow S^{1\pm 1}) \) immediately becomes synonymic to the zero-length word \( \langle 1 \rangle \). Thus, photons \( \gamma \) play the rôle of energy carriers.

**D. Lepton–neutrino matchings**

Let us use the tiling associated with the root system \( A_3 \); this defines the configuration of local information channels for the rules which process information about particles in the course of reactions.

**Definition 5.** The electron \( e^- \), antielectron (or positron) \( e^+ \), electron neutrino \( \nu_e \), and electron antineutrino \( \overline{\nu}_e \) are point particles whose contours are drawn –schematically and up to homeomorphisms– in Fig. 4. Each photon \( \gamma \) conveys a single photon at the other end. The count of such events is the pace of time.

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\(^7\) In [2] we understood the propagation of a photon as the automaton that creates unit events by destroying the photon at one end of an edge in the graph and creating the same photon at the other end. The count of such events is the pace of time.
edge —unlike the (anti)electron— neutrinos symptomise a general disinclination to interaction of any kind with any type of matter.

However, we owe almost everything to these simplest neutrinos because it was the contour $\nu_e$ which was first created in the spatial component of the CW-complex in the course of decontraction of the first oriented face, see Fig. 1; the first photon $\gamma$ appeared at about the same time, after the decontraction of the first tadpole; we see no reason to debate which word was first.

The two heavier point leptons are the (anti)muon $\mu^\pm$ ($m_\mu \approx 105.7$ MeV/c$^2$) and (anti)tau $\tau^\pm$ ($m_\tau \approx 1776.8$ MeV/c$^2$); these four particles are unstable (their proper life-times are approximately $2 \mu$s and $3 \cdot 10^{-13}$ s, respectively); in the course of decay they are reported to produce the respective (anti)neutrinos $\nu_\mu$, $\bar{\nu}_\mu$, $\nu_\tau$, and $\bar{\nu}_\tau$, which is quite logical: after decontraction of heavy leptons’ contours, their electric charges are decoupled —by spending minor part of the released energy on detachment of the charge tadpoles— and re-attached to the newly-generated, then separated mutually inverse pairs of contours for lighter leptons and their antineutrinos.

Example 1. Consider the Michel decay $\mu^- \rightarrow \nu_\mu + e^- + \pi^-$. The decontraction of edges which endow the muon $\mu^-$ with mass and deprivation of its contour from its negative electric charge, which is encoded by the tadpole $S^{-1}$, produces the neutrino $\nu_\mu$ and a store of dispensable energy, which has been partly used to detach the charge and which is used then to disrupt the trivial word issued from the point where the charge is located,

\[
\langle 1 \rangle = (\nu_e \cdot \nu_e^{-1}) \rightarrow (\nu_e \cdot S^{-1}) + (\pi^-).
\]

A contraction of edge(s) in the charged first term of the right-hand side yields the second and third particles in the reaction’s output. The remainder of energy (left from the excess of muon’s mass) is communicated to the three particles $\nu_\mu$, $e^-$, and $\pi^-$ as their kinetic energy. (Note the overall conservation of electric charge and a likely preservation of muon’s spin by the muon neutrino $\nu_\mu$.)

However, we see that the discarded contour for $\nu_\mu$ in Michel’s reaction and the mutually reverse prototypes for $e^-$ and $\pi^-$ could co-exist on different CW-complexes which fill in a unique continuous homeo-class Universe; we notice that the trivial word (1) to-expand at the location of the charge does not refer to a choice of the alphabet, whereas the store of energy and the loop along $S^{-1}$, not leaving the point of physical space, are logically transportable between the schemes of information processing.

If so, the decays of the heaviest $\tau^\pm$ and less heavy $\mu^\pm$ could be second-order phase transitions, in the course of which the information channels are locally reconfigured (i.e., one crystal structure transforms into another —no less well ordered— so that the order parameter is constant) and the energy and electric charge are pumped into the new logical processor of information; the old processor calculates the receding of muon’s neutrino.

Conjecture 7. There are, and there are exactly three types of leptons $-e$, $\mu$, and $\tau$— because there are exactly three types of irreducible lattices $-A_3$, $B_3$, and $C_3$— in Euclidean space $E^3$, open domains in which are homeomorphic to the spatial components of domains in homeo-class realisation of the Universe.

Part II. Geometry of fundamental interactions

Let us focus now on the algebraic structures of the four fundamental interactions; we assume for definition that the processes of reconfiguration and interaction occur on the same lattice so that the alphabet $X\pm, S\pm$ is common for all particles. We first consider the weak and strong forces and then we discuss the long-range electromagnetism and gravity.

We recall that the algebraic operations which we usually do with words are

- writing them consecutively,
- disrupting a word in between two letters in accordance with the rules of hyphenation.

Notice that the first option prevails in frequency over the second whenever a sufficiently long fragment of text is already written; however, Nature first constructed the hyphenation table for its glossary.

IV. THE WEAK FORCE

The defining property of a weak process by which it is recognised at once is an arbitrary combination of the following very unlikely events:

- a zero-length word, which means just one vertex in terms of paths, expands to a synonymic cycle which is walked twice in opposite directions;
- a path, being either a previously existing contour from the reaction’s input or a newly-produced synonym of (1), is torn.

Then the available collection of paths’ fragments, which themselves may be not cyclic words but their separate letters or syllables, recombine and join the loose ends, forming new contours and thus creating new particles. Note that the output of a weak reaction is an anagram of letters belonging to the original text — which was extended with extra letters and their negations by using the add-subtract arithmetic trick.

Example 2. Let us consider the weak decay of a free neutron $n^0$; having spin $\pm h/2$ and parity $-1$ (i.e., the orientation-matching of the unknot’s word and the orientation of the lattice is “yes”)”, neutron’s contour resembles the one drawn in Fig. 1(a). The primary channel
V. THE STRONG FORCE

The strong interaction does not beg, borrow, or steal the contours which did not belong to reaction’s input but which could be created at the expense of zero-length words of energy; the output of a strong reaction looks much the same as the original text, only the order of sentences in it is mixed. For example, several protons and neutrons link to a bouquet or form a chain mail, that is, a nucleous. We emphasize that the strong interaction handles the already-existing contours (i.e., not just words encoded in the glossary via route instructions and edge contraction configurations).

The binary algebraic operation that creates the strong force is the multiplication × for cyclic words; we described it in detail in [1, 5] (see also [3]). Essentially, this is the standard unlock-and-join technique of the topological pair of pants $S^1 \times S^1 \to S^1$. By definition, the value of operation × at two paths is calculated in three steps:

1. both contours are unlocked at one vertex each;
2. the unlocked paths are transported along the lattice such that the two (un)locks coincide;
3. the loose ends of disrupted contours are recombined in such a way that the left-to-right order of reading all words is preserved.

The structure × takes the sum over all possible (or preferred) locations of the locks on each of the contours; if a certain summand is forbidden (recall that oriented spatial edges of the graph are viewed as fermions), then it is omitted; the result is normalised by the actual number of contributing terms.

Notice that the multiplication of particles’ contours is commutative but not associative; indeed, the contours [1] and [2] are always adjacent in the product $([1] \times [2]) \times [3]$ but they can be separated by edges of [3] in some of the terms in $[1] \times ([2] \times [3])$. Yet we recall that the strong interaction is not associative at the level of nuclear fusion and fission: e.g., the channel $p^+ \times (p^+ \times n^0) \to ^3\text{He}$ is not realised via $(p^+ \times p^+) \times n^0$ by a would-be intermediate two-proton $^3\text{He}$; likewise, the equally possible (50%-50%) processes $^3\text{T} \to ^3\text{He} + p^+$ or $^3\text{He} + n^0$ do not amount to a bare assembly of two protons and two neutrons.

Finally, we recall that the full power of noncommutative calculus is revealed by the introduction of noncommutative bundles $\pi^nC$ whose base is the noncommutative tangent bundle over space, i.e., the homeo-class space itself (see [5]). Namely, let us consider open, positive-length words (containing the mark-up of contracted edges) which we view as noncommutative fields over space; the new structures are auxiliary in a description of the full quantum geometry of the strong force and may determine no particles existing as independent objects, yet they could be helpful. These fields are sections of the noncommutative bundle $\pi^nC$ over the homeo-class space.
At each point of the base, a local basis of such fields extends the alphabet $S^\pm_1, x^\pm_1$ of generators of the lattice; not without insight we denote by $(u, d, s, c, t, b)$ the elements of such bases introduced pointwise.\(^8\) We foresee that the supports of such (anti)sections are finite in space in all inertial reference frames yet such supports do not amount to single points but contribute to construction of clouds of matter, that is, dimensionful particles.

**Corollary 8.** Provided that the information encoding leptons is referred to one point in space at each instant of time, and under the hypothesis that the sections of $\pi^{nC}$ are piecewise-continuous with respect to the continuous limits of space tilings, the leptons may not consist of such auxiliary building blocks.

**Remark 10.** Because the auxiliary blocks $(u, d, s, c, t, b)$ are introduced to encode formal sums of paths which are open, they in practice can not be isolated physically and registered as objectively existing particles. (The same argument applies now, long after the Big Bang, to each generator $x_i$ of a lattice in space.)

### VI. ELECTROMAGNETISM

Now we address the effective long-range interactions: electromagnetism first and then gravity. It must be noted that both concepts involve the same idea of edge contraction (in disguise, formation of tadpoles), that is, a prescription that the endpoints of an edge become one vertex. (The difference between the two concepts is that a spatial edge vanishes altogether, not forming a loop, whereas the generators $S^\pm_1$ are tadpoles for granted.)

Therefore, it is logical to expect that macroscopic properties of the two forces are much alike in their classical description: a charged massive point locally produces the Newtonian electric and gravitational potentials inverse-proportional to distance in space.

**Remark 11.** The count of electric charge, which is by definition the difference $2S^+1 - 2S^-1$ of the numbers of loops that a path winds in the fourth, tadpole dimension, does not interfere with disruptions or rearrangements of path’s spatial edges. Consequently, the electric charge is conserved but in other respects it does not influence weak or strong processes.

The large-scale alikeness of the long-range forces is readily seen from the coding of electron on the equilateral triangular lattice, see Fig. 6: the particle consists of indivisible mass and indivisible charge which are held close to each other but always stay separated by emptiness; the information about particle’s closed contour does not take shape of a faintly shimmering rope or cord. Note that the contraction of the edge which endows electron with mass creates the asymmetry of space surrounding the point where the mass is located; on the other hand, the formation of negative charge $-e$ by the tadpole $S^{-1}$ is fully symmetric with respect to space.

**Remark 12.** It is logical also that, whenever accelerating in electromagnetic field, the electron radiates. Indeed, Lorentz’ force acting on its charge, the electron becomes an oscillator in which space itself plays the rôle of elastic spring between the point charge and the point mass; the oscillations then amount to periodic elastic deformations of the space-time structure, pulling or slowing the mass as it retards or overtakes the charge.

In conclusion, electromagnetism is a very famous example of cyclic-invariant theory; outside point particles, its gauge description could be exact.

### VII. GRAVITY: THE BIG BANG LOGISTICS

(We owe this term to Yu. I. Manin who coined it.)

Let us attempt to track logically the scenario of events in the early Universe, taking into account the assertions which we have made so far.

1. Initially, the Universe consisted (according to its topology) of only one point; in other words, the CW-complex was fully contracted, i.e., formed by only one vertex and no edges. It is possible that the initial point was also assigned an extra number that indicated the energy surplus over its store in the contracted edges.

2. The fully contracted lattice was released from hold and its edges began to decontract; each event of edge decontraction released energy which took shape of the zero-length word $⟨1⟩$. The time started; it first amounted to the count of decontraction events and derivative events of reconfigurations in the lattice defects’ portrait; still no particles were formed yet and there was no light.

A decontraction of the first spatial edge with tadpoles at each of its ends created the possibility of existence of light viewed as the automaton that propagates the null photon’s path $(x^+_{m1}, x^-_{m1})$ from a vertex to its neighbour; this does not imply that the ready-to-work automaton did actually start to work at once. The decontraction of the first triangle did create the first electron neutrino (see Fig. 7); we think that it is scholastic to debate whether the first photon preceded the first neutrino or vice versa. For the first tetrahedron to decontract, a coin was thrown and this determined whether that was a Left or Right
tetrahedron (see Fig. [2]); space was thus oriented and Nature chose Right.

The dimension of the CW-complex became positive, and the initial point split to a set of points (depending on the convention about homeo-class continuous space or quantum space as a lattice, to continua or to finite sets through a finite number of decontraction events, respectively); space began to expand.

3. While the edges kept on decontracting in the replicas of adjacency table for each newly-produced copy of the original vertex, such events became independent and uncorrelated. Because of this, the values of noncommutative Ricci and scalar curvature were (almost) random at points of the early Universe. However, let us recall from [21] that in this case the Jacobi field connecting two infinitesimally close geodesics issued from a point grew exponentially — yet, paradoxically, for almost certain there appeared an arbitrarily large number of those geodesics’ focus points, which resulted in clashes and information exchange. (Here we use the assumption of space’s continuity and use light to introduce the smooth structure of space at the expense of infinite energy.) The Universe experienced the inflation.

4. It took certain time for space to expand and reach a configuration with relatively small fraction of defect edges that remained contracted in large finite neighbourhoods of many points. (Because the Universe continues expanding now, we may not refer to all its points but operate with sufficiently spacious regions.) Simultaneously, those lattice defects could assemble due to the reasons of entropy. However, the preceding decontraction of a major part of edges in such domains released a colossal amount of vacuum energy; it shaped into a scalar field over space.

5. Although the initial point of the Universe split (its descendants still continue splitting at the outer periphery of the Universe, sending us photons and neutrino flows; throughout Cosmos, the edges between its descendants continue splitting and so contribute to the validity of Hubble’s law), there remained or there re-appeared sets of “conservative-minded” vertices in the lattice which proclaimed themselves one-point-forever and merged the adjacency tables — but their neighbours refused to join the coalition. This created the singularities of the first generation of black holes. (Those were the times when a lack of curvature necessary to form the event horizon was out of dispute.) The eldest black holes produced considerable irregularities of the space-(time) geometry, which triggered the formation of matter from an otherwise still meta-stable state of the Universe already filled with neutrinos and photons (cf. [22]).

6. The excess of vacuum energy, the decontractions which created massless neutrino cycles as in Fig. [1] photons, and possible entropy-based gradient flows of the dark matter caused the formation of elementary particles via weak processes of contour disruption and reconfiguration (in particular, relatively near—in cosmic sense—to black holes). The Left–Right asymmetry, which had been built into the Universe since the decontraction of the first tetrahedron, implied the domination of matter over antimatter in the course of particle creation. The add-subtract mechanism for creating, distributing, and counting the electric charge kept the initially pathless and hence chargeless Universe neutral en masse.

7. The world became what we know it now; only a small part of its mass and energy shaped into particles. Nevertheless, those were enough to form galaxies around the eldest black holes which called that matter from unbeing. The start of chain reactions of nuclear fusion and ignition of the first star heralded the end of the beginning in the history of the Universe.

Conclusion
In this review, preceded by [2], we outlined a possible axiomatic quantum picture of fundamental interactions in quantum space and time; we hope that it will help us to resolve a part of known difficulties or at least offer us a good reformulation of paradoxes in the existing paradigm.

We have shown that the chosen set of postulates implies the following statements:
1. At (sub-)Planck scale, gauge theory is insufficient for a description of the interactions; one should use a theory that does not appeal to the locally-linear, diffeo-structure of the space-time but operate with geometry of the Universe at the topological, homeo-class level.
2. Vacuum, i.e., a domain in space which is known to contain no particles of any kind, can have mass-energy and, via its curvature, nonetheless produce gravity force; moreover, such vacuum does contain a store of energy which can be released and transmute into (anti)matter.
3. The (anti)matter is the meaning of information which is stored in space (specifically, within its topology and the loops paved through its homeo-realisation).
4. Having begun to expand from its initial state with the topology \( T = \{ \emptyset, \text{Universe} \} \), the early Universe experienced an inflation phase of exponential growth.
5. The Universe is electrically neutral. In the quantum world, electromagnetism does contribute to the processing of information with the quantum number charge but plays no dominant rôle in the interactions and decays (e.g., protons and neutrons form the nuclei of atoms). Outside the particles and under the ad hoc assumption of differentiability for the gauge transformations, the U(1)-gauge model is exact.

We argued in favour of the following possibilities:
6. The CP-symmetry violation in weak processes is a consequence of the Left \( \neq \) Right asymmetry which has been being built into the Universe since it became oriented; matter prevails now over antimatter. The masses (whenever both are nonzero) of the respective (anti)particles can be slightly unequal.

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9 The worst idea here would be that of averaging, see [23].
7. The cosmic microwave background radiation is an immanent property of space itself, so it cannot be shielded altogether by using any macroscopic medium.

8. The polarised antiphotons are cross-identical to the respective photons.

9. The weak processes are much less likely to occur than the strong ones.

10. A decay of the unstable leptons $\mu^\pm$ and $\tau^\pm$ and the (anti)neutrino oscillations $\nu_\tau \leftrightarrow \nu_\mu \leftrightarrow \nu_e$ are second order phase transitions.

In conclusion, it is quite remarkable that matter is, even if it constitutes only about 16% of the should-be mass (and possibly 4% of the mass-energy) of the Universe. The main store of mass is contained in the dark matter, i.e., in graphs’ contracted edges where no non-trivial contours run, and there is a huge store of the vacuum energy in zero-length words or their synonyms.

We conjecture that the organisation of (anti)matter in galaxies around very massive objects, as we observe it now, summarises the history of the Universe itself. Namely, we view the singularities of black holes in the centres of galaxies as the eldest remnants of the Universe’s One Point that did not expand to continua but centres of galaxies as the eldest remnants of the Universe.

Likewise, we expect that giant voids in the large-scale structure of the Universe, the voids bounded by walls formed by galaxies, are relatively poor in black holes and therefore are presently filled with the still dark matter and vacuum energy (however, in meta-stable state). This is why the voids (or each galaxy’s halo) look so empty even if they are so massive. Nevertheless, the superclusters, filaments, and walls in large-scale organisation of the Cosmos stem from a topological configuration of vertices, edges, and cells in Planck-scale neighbourhoods of all its points.

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