Ant colony system algorithm for generalized trapezoidal fuzzy capacitated vehicle routing problem

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Abstract. Traffic congestion causes late arrivals at customers and long travel times resulting in large transport costs. Traffic congestion also can result in the travel time of the vehicle from one place to another cannot be determined precisely even though the distance travelled is the same. In this paper, fuzzy capacitated vehicle routing problem (FCVRP) with travel time expressed by generalized trapezoidal fuzzy numbers is addressed. The fuzzy model is reduced to corresponding crisp one using fuzzy ranking method. Then, we propose ant colony system (ACS) algorithm with the status transition rule, global pheromone trail update and local pheromone trail update for constructing the optimal vehicle routes of the reduced problem. A numerical example of Bright gas 5.5 kg distribution is demonstrated to find the optimal solution of the proposed method. Travel time are obtained basing on real data measured by an electronic system at ten different times. The proposed method is simpler and more computationally efficient when compared to existing methods.

1. Introduction
One of problems faced by the agency distribution of goods is to determine the optimal route that departs from a depot to some consumers and returns to the depot with the aim of minimizing the cost of distribution. The problem of determining routes or vehicle routing problem (VRP) is a common form of transportation problem which was first introduced by Dantzig in 1959 [1]. VRP has evolved into several types, and the main focus of this article is capacitated vehicle routing problem (CVRP), namely the problem of determining the minimum distribution costs and consider the capacity of the vehicle.

Distribution costs from one place to another can be declared as the travel time. However, traffic jam can cause travel time at a time from one place to another can be different when the traffic is normal despite the same distance. Thus, the cost of distribution is not specified explicitly. Fuzzy numbers can be used to declare the uncertain travel time [2].

Zimmerman was the first who develop a method of fuzzy optimization and manage to show that the solution of the method is always efficient [3]. Then, fuzzy optimization method was developed by others to solve transportation problem [4,5]. Fuzzy optimization method was also developed by some researchers to solve VRP. Zarandi assess CVRP with fuzzy numbers travel time [6]. While Ghannadpour develop fuzzy optimization method to solve the problem of VRP with multi-objective [7], which was later developed for the VRP with multi-depot [8].

Although many research on the fuzzy vehicle routing problem (FVRP) have been done, but there are still many issues that need to be addressed further. One important issue that concern the researchers is complicated calculation process and requires a lot of time to find the optimal solution of a simple
problem FVRP. Some researchers have tried to examine this problem by developing new methods, but the enormous level of computational difficulty is still faced with increasing number of variables used [5,9,10]. The above problem clearly inhibits the application of the FVRP method. Therefore we propose a new method for solving a special type of capacitated fuzzy vehicle routing problems in which the travel time are represented by generalized trapezoidal fuzzy numbers. The FCVRP is converted into CVRP, which are then solved using Ant Colony System algorithms. As far as the author's knowledge, articles that discuss the use of generalized trapezoidal fuzzy numbers on VRP problems are still limited also.

The remainder of this paper is organized as follows. Section 2 gives basic definitions, the ranking function and fuzzy distribution. In section 3, a FCVRP models is formulated where travel time coefficients are represented as generalized trapezoidal fuzzy numbers. Then we use the ranking function fuzzy to reduce FCVRP into CVRP models which then were solved by the algorithm Ant Colony System (ACS), which is one of the development of ACO algorithms. An examples of route problem of LPG 5.5 kg distribution in one distribution company in Indonesia will also be discussed as an illustration.

2. Preliminaries
In this section, some basic definitions and existing method for constructing trapezoidal fuzzy number are presented.

2.1. Basic definitions

Definition 1. Fuzzy set A in X is defined as \( A = \{ (x, \mu_A(x)) | x \in X \} \), with the membership function \( \mu_A(x) \) maps X into closed interval \([0, 1]\) [3].

The core set of fuzzy set A is defined as \( B = \{ (x, \mu_B(x) = 1) | x \in A \} \). If \( B \) is non-empty set then A is normal. And, if \( \mu_A(\delta x + (1-\delta)y) \geq \min \{ \mu_A(x), \mu_A(y) \} \), for any \( x, y \in X \) and \( \delta \in [0, 1] \) then \( A \) in \( X \) is convex. The normal and convex fuzzy set of the real numbers \([0, 1]\) is called fuzzy number [3]. Further, generalized trapezoidal fuzzy number is one types of fuzzy number defined as follows.

Definition 2. \( A = (a, b, c, d; w) \) for \( 0 < w \leq 1 \) is said to be generalized trapezoidal fuzzy number in \( [0, 1] \) if its membership function is given by [5]:

\[
\mu_A(x) = \begin{cases} 
\frac{x-a}{b-a}, & a \leq x < b \\
\frac{b-x}{c-b}, & b \leq x < c \\
\frac{c-d}{d-c}, & c < x \leq d \\
0, & \text{otherwise}
\end{cases}
\tag{1}
\]

2.2. Fuzzy Distribution

Fuzzy distribution is a method used to construct a trapezoidal fuzzy number \( A = (a, b, c, d) \) based on historical data with \( a = \) minimum value, \( b = (\) minimum value + mean value)/2, \( c = (\) mean value + maximum value)/2, and \( d = \) maximum value [11]. In this article we propose a simple method for construction of generalized trapezoidal fuzzy number of travel time. \( B = (a, b, c, d; w) \) is defined as generalized trapezoidal fuzzy number of \( A = (a, b, c, d) \) with \( w \) is equal to the percentage of historical data that lies in the interval \( b \leq x \leq c \).

2.2.1. An example. If the 10-day travel time from consumer to consumer \( i \) to \( j \) is 2, 1.8, 1.85, 1.9, 1.85, 1.6, 1.54, 1.66, 1.7, and 1.78 hours, then mean value = 1.768 hours. Then \( a = 1.54, b = 1.654, c = 1.884, \)
and \( d = 2 \). Therefore, trapezoidal fuzzy number of 10-day travel time from consumer to consumer \( i \) to \( j \)
\[ c_{ij} = (1.54, 1.654, 1.884, 2) \]. And since there are 6 travel time data that lies in the interval \( 1.654 \leq x \leq 1.884 \),
then generalized trapezoidal fuzzy number of 10-day travel time from consumer to consumer \( i \) to \( j \) is
\[ d_{ij} = (1.54, 1.654, 1.884, 2; 0.6) \].

2.3. Ranking function

Ranking function is one of efficient methods to compare fuzzy numbers. It is a function which maps each of members of fuzzy number with a real number.

**Definition 3.** Ranking function \( R : F(\mathbb{R}) \rightarrow \mathbb{R} \) maps \( F(\mathbb{R}) \) a set of generalized trapezoidal fuzzy numbers defined on set of real line. Let \( A = (a, b, c, d; w) \) be a generalized trapezoidal fuzzy number [5], then
\[
R(A) = w \frac{(a+b+c+d)}{4} \quad (2).
\]

Subsequently, arithmetic operations properties of generalized trapezoidal fuzzy numbers, defined on universal set of real numbers, are presented as follows [12],

Let \( c_j = (a, b, c, d; w) \) and \( x_j = (a, b, c, d; w) \) be generalized trapezoidal fuzzy numbers, then
\[
R\left( \sum_{j=1}^{n} c_j \otimes x_j \right) = \sum_{j=1}^{n} R\left( c_j \otimes x_j \right) = \sum_{j=1}^{n} R\left( c_j \right) R\left( x_j \right) \quad (3)
\]
and
\[
R\left( \sum_{j=1}^{n} a_j \otimes x_j \right) = \sum_{j=1}^{n} R\left( a_j \otimes x_j \right) = \sum_{j=1}^{n} a_j R\left( x_j \right) \quad (4).
\]

2.3.1. An example. Let \( c_1 = (8, 9, 12, 26; 0.5) \) and \( x_1 = (3, 5, 8, 13; 0.7) \) be a generalized trapezoidal fuzzy number. According to equation (2)–(4), then
\[ R\left( c_1 \right) = 6.875, \quad R\left( x_1 \right) = 5.075, \]
\[ R\left( c_1 \otimes x_1 \right) = R\left( c_1 \right) + R\left( x_1 \right) = 11.95 \quad \text{and} \quad R\left( c_1 \otimes x_1 \right) = R\left( c_1 \right) R\left( x_1 \right) = 34.891. \]

3. Proposed Methods

FCVRP problem that will be discussed in this article are CVRP with fuzzy travel time and can be formulated as follows,

minimize
\[
Z = \sum_{i=1}^{n} \sum_{j=0}^{n} c_{ij} \otimes x_{ij} \quad (5)
\]

subject to,
\[
\sum_{j=0}^{n} x_{ij}^{k} = 1 \quad \text{for all } k \quad (6),
\]
\[
\sum_{k=0}^{n} x_{ij}^{k} = 1 \quad \text{for all } k \quad (7),
\]
\[
\sum_{i=0}^{n} \sum_{k=1}^{n} x_{ij}^{k} = 1 \quad \text{for all } j \quad (8),
\]
\[
\sum_{i=0}^{n} \sum_{k=1}^{n} x_{ij}^{k} - \sum_{i=0}^{n} \sum_{j=0}^{n} x_{ij}^{k} = 0 \quad \text{for all } j \quad (9),
\]
and
\[
\sum_{i=0}^{n} \sum_{j=0}^{n} d_{ij} x_{ij}^{k} \leq D \quad (10).
\]
with $x_{ij}^k$ is a decision variable that is worth 1 if there is a vehicle $k$ travels from $i$ to $j$ and 0 otherwise.

Fuzzy number of travel time from consumer $i$ to consumer $j$ is denoted by $c_{ij}$, where $m$ is the number of vehicles and $n$ is the total number of depots and consumers to be visited.

This article propose a simple method for construction of fuzzy generalized trapezoidal fuzzy number of travel time. And then applies ranking method to reduce FCVRP into an ordinary crisp CVRP problem. We also propose ACS to find the optimal solution of the crisp problem and give numerical example for it.

3.1. Reduced model

Using fuzzy optimization method [3], arithmetic operations properties of generalized trapezoidal fuzzy numbers [12], and ranking function for generalized trapezoidal fuzzy number [5], FCVRP model in equation (5)-(10) above can be reduced into an ordinary crisp CVRP problem as follows, minimize

$$Z = \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=1}^{m} R(c_{ij})x_{ij}^k,$$

subject to equation (6)-(10).

3.2. ACS algorithm

CVRP categorizes of NP-hard problems, as a result many heuristic algorithms developed to solve them. Ant colony algorithm system is method that will be applied in this article with the following steps:

[Step 1] Input the travel time data from any point to any other point $c_{ij}$ and the demand of each customer $d_j$ then initialize the ACS parameters $\alpha$, $\beta$, $\xi$ and $0 \leq \xi \leq 1$, $q_0$ and $0 \leq q_0 \leq 1$, $\tau_0 = \frac{1}{nC_{ww}}$ with $C^m$ is the best route length of nearest neighbour, $It_{max}$, $D$.

[Step 2] Place each ant in the starting position i.e., point $i = 1$ or depot in the taboo list of each ant. The next point $j$ will be visited is chosen according to the pseudorandom proportional rule with $j = \arg\max_{j \in N_i(k)} \{ \tau_{ij}^\alpha \eta_{ij}^\beta \}$ for $q \leq q_0$ and $j = s$ otherwise, where $s$ is a selected customer which has the greatest probability with $p_{ij}^s = \frac{[\tau_{ij}^\alpha] [\eta_{ij}^\beta]}{\sum_{j \in N_i(k)} [\tau_{ij}^\alpha] [\eta_{ij}^\beta]}$ if $s \in N_i(k)$ and 0 otherwise. $\tau_{ij}$ and $\eta_{ij}$ = $\frac{1}{d_j}$ represent the pheromone level and the visibility of customer $j$ from customer $i$ respectively. $N_i(k)$ is defined as set of remain customers to be visited and $q \in [0,1]$ is a random number. The local Pheromone is updated on each path passed from $i$ to $j$ refers to equation $\tau_{ij}^{\text{new}} = (1-\xi)\tau_{ij}^{\text{old}} + \xi\tau_0$. Remove customer $j$ from $N_i(k)$ and set $j$ as the current customer, continue the search process for the point to be visited next by using the updated pheromone until all the customers have been visited. Repeat steps 2 through $It_{max}$ achieved.

[Step 3] Choose a route with minimum travel time as the optimal solution of the problem, then update global pheromone on routes with minimum travel time refers to equation $\tau_{ij}^{\text{new}} = (1-\rho)\tau_{ij}^{\text{old}} + \rho\Delta\tau_{ij}$ with $\Delta\tau_{ij} = \frac{1}{C_{ij}}$ if $(i,j)$ is the best route length and 0 otherwise. And $C_{ij}$ is the best route length and constant evaporation global ant trail $\rho \in [0,1]$.

4. Results and Discussion

Route problem of LPG 5.5 kg distribution to 6 customers of one distribution company in Indonesia is given as an illustration. Data collected are number of requests from 6 clients, namely $d_1 = 4$, $d_2 = 5$, $d_3 = 8$, $d_4 = 6$, $d_5 = 9$, and $d_6 = 5$. One vehicle is available with maximum capacity of 40 tubes. The travel
time from the depot to all customers and between customers taken from Google Maps at 10 different times by observing the intensity of the traffic in the period of March 1, 2018 to September 1, 2018. One of the matrix data travel time from one customer to the depot in 10 different times as shown in Table 1.

Table 1. Tabular representation of time travel

| Time | Depot → Customer 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------|-------------------|---|---|---|---|---|---|---|---|----|
| 21   | 21                | 21| 20| 28| 19| 19| 20| 21| 21|    |

4.1. Generalized trapezoidal fuzzy capacitated vehicle routing problem

FCVRP model (5)–(10) starts to be constructed by forming generalized trapezoidal fuzzy numbers time travel as the objective function coefficients are given in Table 2.

Table 2. Tabular representation of generalized trapezoidal fuzzy time travel

| Depot | Consumer 1 | Consumer 2 | Consumer 3 | Consumer 4 | Consumer 5 | Consumer 6 |
|-------|------------|------------|------------|------------|------------|------------|
|       | (19, 20:05, 24, 26.6, 33.1, 23, 25.26, 20, 21.8, 27.3, 32, 34.85, 32, 34/25, 32, 34/25, 32, 34/25) | (24, 26.6, 33.1, 23, 25.26, 20, 21.8, 27.3, 32, 34.85, 32, 34/25, 32, 34/25, 32, 34/25) | (37, 0.9) | (32, 34, 36, 0.9) | (31, 0.7) | (41, 85, 46, 0.8) |
|       | (32, 34/25, 39, 25, 42, 0.7) | (32, 34/25, 39, 25, 42, 0.7) | ... | ... | ... | ... | ... |
| Customer 6 | (13, 14, 16, 17.6, 9, 9.65, 11:15, 9, 9.9, 11, 13, 17.5, 2, 3.3, 6.3, 0, 0) | (19, 0.5) | (12, 0.5) | (12, 4, 14, 0.2) | (20, 0.1) | (8, 0.5) | 0 |

4.1.1. Reduced model. FCVRP model is reduced to an ordinary crisp CVRP model (11) and (6)-(10) as the objective function coefficients are given in Table 3.

Table 3. Tabular representation of ranking generalized trapezoidal fuzzy time travel

| Depot | Customer 1 | Customer 2 | Customer 3 | Customer 4 | Customer 5 | Customer 6 |
|-------|------------|------------|------------|------------|------------|------------|
| 0     | 16.03      | 27.16      | 26.28      | 17.52      | 30.94      | 18.44      |
| ...   | ...        | ...        | ...        | ...        | ...        | ...        |
| Customer 6 | 18.44 | 8.02 | 3.13 | 5.66 | 1.54 | 2.45 | 0 |

4.1.2. ACS solution method. The CVRP model obtained is then solved using the ACS algorithm by initializing the parameters used namely \( \alpha = 1, \beta = 2, \rho = 0.1, \xi = 0.1, q_0 = 0.9, \tau_0 = 0.1, I_{t_{\text{max}}} = 100, D = 40, m = 5 \). The optimal route which also consider vehicle capacity is 1 → 5 → 4 → 3 → 7 → 6 → 2 → 1 with a travel time for 46.68 minutes. The route generated by the ACS algorithm is better than the nearest neighbour (NN) algorithm.

5. Conclusion

Completion FCVRP by reducing the problem becomes CVRP using fuzzy ranking method, which is then resolved by ACS algorithm can reduce a lot of computing time compared to other methods and have better results than the algorithm nearest neighbor (NN). The total distribution time generated using ACS is smaller than NN. The proposed approach is very easy to understand and apply to decision makers to solve distribution problems in real life. Based on the research that has been carried out, further research on methods of ranking function and fuzzy distribution can be done to FCVRP with multi-objective and multi-depot.
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