Monopole, gluino and charge condensates in gauge theories with broken $\mathcal{N}=2$ supersymmetry$^1$

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Abstract

We consider chiral condensates in SU(2) gauge theory with broken $\mathcal{N}=2$ supersymmetry and one fundamental flavor in the matter sector. Matter and gaugino condensates are determined by integrating out the adjoint field. The only nonperturbative input is the Affleck-Dine-Seiberg one-instanton superpotential. The results are consistent with those obtained by the ‘integrating in’ procedure. We then calculate monopole, dyon, and charge condensates using the Seiberg-Witten approach. The key observation is that the monopole and charge condensates vanish at the Argyres-Douglas point where the monopole and charge vacua collide. We interpret this phenomenon as a deconfinement of electric and magnetic charges at the Argyres-Douglas point.

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1 Introduction

This talk is based on the work \cite{1} in which I have collaborated with Alexander Gorsky and Alexei Yung.

The derivation of exact results in $\mathcal{N}=1$ supersymmetric gauge theories from low energy effective superpotentials and holomorphy was pioneered in \cite{2,3}. Then a new wave of development was initiated by Seiberg, see \cite{4} for review. Additional input was provided by the Seiberg-Witten solution of $\mathcal{N}=2$ supersymmetric gauge theories with and without matter \cite{5}.

In the $\mathcal{N}=2$ theory the chiral $\mathcal{N}=1$ superfield $\Phi$ in the adjoint representation appears as a partner to the gauge fields in the $\mathcal{N}=2$ supermultiplet. The key feature of the $\mathcal{N}=2$ theory is the existence of the Coulomb branch where the vacuum expectation value of the lowest component of $\Phi$ serves as a modulus \cite{5}.

The simplest way to break $\mathcal{N}=2$ supersymmetry (SUSY) down to $\mathcal{N}=1$ amounts to giving a nonvanishing mass $\mu$ to the superfield $\Phi$. At small values of $\mu$ the theory is close to its $\mathcal{N}=2$ counterpart while at large $\mu$ the adjoint matter decouples and the pure $\mathcal{N}=1$ theory emerges. The emerging theory at large $\mu$ is close to supersymmetric QCD (SQCD) but does not coincide with it. A trace of the massive adjoint remains in the effective theory in the form of nonrenormalizable quartic terms \cite{6} in the superpotential which are suppressed by $1/\mu$. Although in the $\mathcal{N}=1$ theory the degeneracy on the Coulomb branch is lifted by the superpotential, memory of the structure of the Riemann surfaces remains. Namely, the vanishing of the discriminant of the Riemann surface defines the set of vacua in the corresponding $\mathcal{N}=1$ theory \cite{5–10}.

We consider an $\mathcal{N}=1$ theory with both adjoint and fundamental matter and limit ourselves to the most tractable case of SU(2) gauge group with one fundamental flavor and one multiplet in the adjoint representation. Our strategy is as follows. First, in Sec. 2 we integrate out the adjoint matter to get SQCD-like effective superpotential for the fundamental matter. The only nonperturbative input in this effective superpotential is given by the Affleck-Dine-Seiberg (ADS) superpotential generated by one instanton \cite{2}. Difference with pure SQCD is due to the mentioned above nonrenormalizable term generated by the level heavy adjoint exchange. Similarly to SQCD, the effective superpotential together with the Konishi relations unambiguously fixes condensates of fundamental and adjoint matter as well as the gaugino condensates in all three vacua of the theory.
Our results for matter and gaugino condensates are consistent with those obtained by the ‘integrating in’ method [8, 11, 12] and can be viewed as an independent confirmation of this method. What is specific to our approach is that we start from the weak coupling regime where the notion of an effective Lagrangian is well defined, and then use holomorphy to extend results for chiral condensates into strong coupling.

In Sec. 3 we determine monopole, dyon, and charge condensates following the Seiberg-Witten approach, i.e. considering effective superpotentials near singularities on the Coulomb branch of the \( N=2 \) theory. Again, holomorphy allows us to extend our results to the domain of the “hard” \( N=2 \) breaking. This extension includes not only the mass term of adjoint but also breaking of \( N=2 \) in Yukawa couplings.

Our particular interest is the study of chiral condensates in the Argyres-Douglas (AD) points. These points were originally introduced in the moduli/parameter space of \( N=2 \) theories as points where two singularities on the Coulomb branch coalesce [13–15]. It is believed that the theory in the AD point flows in the infrared to a nontrivial superconformal theory. The notion of the AD point continues to make sense even when the \( N=2 \) theory is broken to \( N=1 \); in the \( N=1 \) theory it is the point in parameter space where two vacua collide.

In particular, we consider the AD point where the monopole and charge vacua collide at a certain value of the mass of the fundamental flavor. Our key result is that both monopole and charge condensates vanish at the AD point. We interpret this as deconfinement of both electric and magnetic charges at the AD point. It provides evidence that the theory at the AD point remains superconformal even after strong breaking of \( N=2 \) to \( N=1 \). Argyres and Douglas conjectured this in their consideration of SU(3) theory [13].

\[ N\text{-}1 \] theory with SU(2) gauge group where the matter sector consists of the adjoint field \( \Phi^\alpha_{\beta} = \Phi^a (\tau^a/2)^{\alpha}_{\beta} \) (\( \alpha, \beta = 1, 2; \ a = 1, 2, 3 \)), and

\[ 3 \] Vanishing of condensates for coalescing vacua was mentioned by Douglas and Shenker [16] in the context of SU(\( N \)) theories without fundamental matter for \( N \geq 3 \). Note, that it was even before the notion of the AD point was introduced in [13].
two fundamental fields $Q^f_\alpha (f = 1, 2)$ describing one flavor. The general renormalizable superpotential for this theory has the form,

$$W = \mu \text{Tr} \Phi^2 + \frac{m}{2} Q^\alpha_\alpha Q^f_\alpha + \frac{1}{\sqrt{2}} h^{fg} Q^\alpha_\alpha \Phi^\beta_\beta Q^g_\alpha .$$

(1)

Here the parameters $\mu$ and $m$ are related to the masses of the adjoint and fundamental fields, $m_\Phi = \mu/Z_\Phi$, $m_Q = m/Z_Q$, by the corresponding $Z$ factors in the kinetic terms. Having in mind normalization appropriate for the $\mathcal{N}=2$ case we choose for bare parameters $Z_\Phi^0 = 1/g_0^2$, $Z_Q^0 = 1$. The matrix of Yukawa couplings $h^{fg}$ is symmetric, and summation over color indices $\alpha, \beta = 1, 2$ is explicit. Unbroken $\mathcal{N}=2$ SUSY appears when $\mu = 0$ and $\text{det} \, h = -1$.

To obtain an effective theory similar to SQCD we integrate out the adjoint field $\Phi$ implying that $m_\Phi \gg m_Q$. In the classical approximation this integration reduces to the substitution

$$\Phi_\beta^\alpha = -\frac{1}{2\sqrt{2} \mu} h^{fg} \left( Q_\beta^\gamma Q^\alpha_\gamma - \frac{1}{2} \delta_\beta^\gamma Q_\gamma^f Q_\gamma^g \right),$$

(2)

which follows from $\partial W/\partial \Phi = 0$. What is the effect of quantum corrections on the effective superpotential? It is well known from the study of SQCD that perturbative loops do not contribute and nonperturbative effects are exhausted by the Affleck-Dine-Seiberg (ADS) superpotential generated by one instanton [2]. The effective superpotential then is

$$W_{\text{eff}} = m V - \left( -\text{det} \, h \right) V^2 + \frac{\mu^2 \Lambda_1^3}{4 V}$$

(3)

where the gauge and subflavor invariant chiral field $V$ is defined as

$$V = \frac{1}{2} Q^\alpha_\alpha Q^f_\alpha .$$

(4)

The first two terms in Eq. (3) appear on the tree level after substitution (2) into Eq. (1) while the third nonperturbative one is the ADS superpotential. The scale parameter $\Lambda_1$ is given in terms of the mass of Pauli-Villars regulator $M_{PV}$ and the bare coupling $g_0$ (plus the vacuum angle $\theta_0$) as

$$\Lambda_1^3 = 4 M_{PV}^3 \exp \left( -\frac{8\pi^2}{g_0^2} + i\theta_0 \right).$$

(5)

3
The coefficient $\mu^2\Lambda^3_3/4$ in the ADS superpotential is equivalent to $\Lambda^5_5$ in SQCD. The factor $\mu^2$ in the coefficient reflects four fermionic zero modes of the adjoint field.

The only term in the superpotential (3) which differentiates it from the SQCD case is the second term which is due to tree level exchange by the adjoint field. At $h = 0$ it vanishes and we are back to the known SQCD case with two vacua and a Higgs phase for small $m$.

When $\det h$ is nonvanishing we have three vacua, marked by the vevs of the lowest component of $V$,

$$v = \langle V \rangle.$$  (6)

These vevs are roots of the algebraic equation $dW_{\text{eff}}/dv = 0$ which has the form

$$m - \frac{(-\det h) v}{2 \mu} - \frac{\Lambda^3_3}{4} \left( \frac{\mu}{v} \right)^2 = 0.$$  (7)

This equation shows, in particular, that although the second term in the superpotential (3) seems to be suppressed at large $\mu$ it turns out to be of the same order as the ADS term. From Eq. (7) it is also clear that the dependence on $\mu$ is given by the scaling $v \propto \mu$.

To see the dependence on the other parameters let us substitute $v$ by the dimensionless variable $\kappa$ defined by the relation

$$v = \mu \sqrt{\frac{\Lambda^3_3}{4m}} \kappa.$$  (8)

Then Eq. (7), when rewritten in terms of $\kappa$,

$$1 - \sigma \kappa - \frac{1}{\kappa^2} = 0$$  (9)

is governed by the dimensionless parameter $\sigma$,

$$\sigma = \frac{(-\det h)}{4} \left( \frac{\Lambda^3_3}{m} \right)^{3/2}.$$  (10)

We see that the two parameters $m$ and $\det h$ enter only as $m (-\det h)^{-2/3}$. The dependence of $v$ on $\mu$ is linear as we discussed above.

The particular dependence of condensate $v$ on the parameters $\mu$, $m$ and $\det h$ follows from the $R$ symmetries of the theory. Following Seiberg [17] one can consider $\mu$, $m$ and $\det h$ as background fields and identify two nonanomalous $R$ symmetries which prove the dependence discussed above. The charges
Table 1: Nonanomalous U(1) symmetries

| Fields/parameters | $\Phi$ | $Q$ | $W$ | $\theta$ | $m$ | $\mu$ | $h$ |
|-------------------|--------|-----|-----|---------|-----|-------|-----|
| $U_J(1)$ charges  | 0      | 1   | 1   | 1       | 0   | 2     | 0   |
| $U_R(1)$ charges  | 1      | −1  | 1   | 1       | 4   | 0     | 3   |

of the fields and parameters of the theory under these two U(1) symmetries are shown in Table 1. The first of these symmetries $U_J(1)$ is a subgroup of the global $SU_R(2)$ group related to the $\mathcal{N} = 2$ superalgebra \cite{5}. The second nonanomalous symmetry $U_R(1)$ is similar to the $R$ symmetry of Ref. \cite{2} extended to include the adjoint field. As a consequence, for a given chiral field $X$

$$
\langle X \rangle = \mu^{Q_J/2} m^{Q_R/4} \Lambda_1^{d_X - (Q_J/2) - (Q_R/4)} f_X(\sigma),
$$

where $Q_J, Q_R$ are the $U_J(1), U_R(1)$ charges of the field $X$, $d_X$ is its dimension, and $f_X$ is an arbitrary function of the dimensionless parameter $\sigma$ defined by Eq. \cite{10}.

The important benefit of the consideration above is that in a theory with $\mathcal{N}=2$ SUSY strongly broken by large $\mu$ and $\det h \neq -1$ we can still relate chiral condensates with those in softly broken $\mathcal{N}=2$ where $\det h = -1$ and $\mu$ is small.

Here is an example. When $\sigma \to 0$ two roots of Eq. \cite{10} are $\kappa_{1,2} = \pm 1$ and the third one goes to infinity as $\kappa_3 = 1/\sigma$. For two finite roots one can suggest dual interpretations. Firstly, taking $h = 0$, one can relate them to two vacua of SQCD in the Higgs phase. Second, for $\det h = -1$ (which is its $\mathcal{N}=2$ value) one can make $\sigma$ small by taking the limit of large $m$. But this limit should bring us to the monopole and dyon vacua of softly broken $\mathcal{N}=2$ SYM. The naming of vacua refers to the particle whose mass vanishes in the corresponding vacuum.

To verify this interesting mapping we need to determine the vev

$$
u = \langle U \rangle = \langle \text{Tr} \Phi^2 \rangle ,
$$

which can be accomplished using the set of Konishi anomalies. Generic
equation for an arbitrary matter field \( Q \) looks as follows:
\[
\frac{1}{4} \bar{D}^2 J_Q = Q \frac{\partial W}{\partial Q} + T(R) \frac{\text{Tr} W^2}{8\pi^2}, \tag{13}
\]
where \( T(R) \) is the Casimir in the matter representation. The left hand side is a total derivative in superspace so its average over any supersymmetric vacuum vanishes. In our case this results in two relations for the condensates,
\[
\left\langle \frac{m}{2} Q^f Q^f + \frac{1}{\sqrt{2}} h^{fg} Q_{\alpha f} \Phi_{\beta g} Q_{\beta g} + \frac{1}{2} \frac{\text{Tr} W^2}{8\pi^2} \right\rangle = 0
\]
\[
\left\langle 2 \mu \text{Tr} \Phi^2 + \frac{1}{\sqrt{2}} h^{fg} Q_{\alpha f} \Phi_{\beta g} Q_{\beta g} + 2 \frac{\text{Tr} W^2}{8\pi^2} \right\rangle = 0 \tag{14}
\]
From the first relation, after the substitution in (2) and comparing with Eq. (7), we find an expression for gluino condensate
\[
\langle \text{Tr} \lambda^2 \rangle = \langle \text{Tr} W^2 \rangle = \frac{\mu^2 \Lambda_1^3}{4v} \tag{15}
\]
This is consistent with the general expression \([T_G - \sum T(R)] \langle \text{Tr} \lambda^2 \rangle/16\pi^2\) for the nonperturbative ADS piece of the superpotential (3), see [18]. Combining the two relations in (14) we can express the condensate \( u \) in terms of \( v \),
\[
u = \frac{1}{2\mu} (m v + 3 s) = \frac{1}{2\mu} \left( m v + \frac{3}{4} \frac{\mu^2 \Lambda_1^3}{v} \right) = \sqrt{\frac{m \Lambda_1^3}{4 v}} \left( \kappa + \frac{3}{\kappa} \right). \tag{16}\]
Now we see that in the limit of large \( m \) two vacua \( \kappa = \pm 1 \) are in perfect correspondence with \( u = \pm \Lambda_0^2 \) for the monopole and dyon vacua of \( \mathcal{N} = 2 \) SYM. Indeed, \( \Lambda_0^4 = m \Lambda_1^3 \) is the correct relation between the scale parameters of the theories.

The opposite limit of massless fundamentals \( m \to 0 \) corresponds to \( \sigma \to \infty \). In this limit the three vacua are related by a \( Z_3 \) symmetry [3],
\[
v = \frac{\mu \Lambda_1}{(2 \det h)^{1/3}} e^{2\pi i k/3}, \quad u = \frac{3}{8} \frac{\Lambda_1^2}{(2 \det h)^{1/3}} e^{-2\pi i k/3}. \tag{17}\]
where \( k = 0, \pm 1 \) marks different vacua. Note that the massless limit exists due to the nonvanishing Yukawa coupling. When \( h \to 0 \) we are back to the runaway vacua of massless SQCD.
For the third vacuum at large \( m \) the value \( u = m^2/(-\det h) \) corresponds on the Coulomb branch to the so called charge vacuum, where some fundamental fields become massless. Moreover, the correspondence with \( \mathcal{N}=2 \) results can be demonstrated for the three vacua at any value of \( m \).

To this end we use the relation (16) and Eq. (9) to derive the following equation for \( u \),

\[
(-\det h) u^3 - m^2 u^2 - \frac{9}{8} (-\det h) m \Lambda_1^3 u + m^3 \Lambda_1^3 + \frac{27}{28} (-\det h)^2 \Lambda_1^6 = 0. \tag{18}
\]

The three roots of this equation are the vevs of \( \text{Tr} \Phi^2 \) in the corresponding vacua.

The equation (18) at \( \det h = -1 \) coincides on the \( \mathcal{N}=2 \) side with the condition of vanishing discriminant of the Seiberg-Witten curve,

\[
y^2 = x^3 - u x^2 + \frac{1}{4} \Lambda_1^3 m x - \frac{1}{64} \Lambda_1^6. \tag{19}
\]

Moreover, Eq. (18) with \( \det h \neq -1 \) can be reduced to the case \( \det h = -1 \) by the rescaling

\[
u = (-\det h)^{1/3} u', \quad m = (-\det h)^{2/3} m', \quad v = (-\det h)^{-1/3} v'. \tag{20}
\]

This is in agreement with the master parameter \( \sigma \) which contains the product \( m^{-3/2} \det h \) and the nonanomalous U(1) symmetries we discussed above. In other words, breaking of \( \mathcal{N}=2 \) by Yukawa couplings does not influence consideration of the chiral condensates modulus the rescaling (20).

The consideration above shows that the only nonperturbative input needed to determine the chiral condensates is provided by the one-instanton ADS superpotential. This means that any reference to the \( \mathcal{N}=2 \) limit is not crucial at all, i.e. in regard to these condensates the exact Seiberg-Witten solution of \( \mathcal{N}=2 \) is equivalent to the ADS superpotential.

The relations for the condensates we have derived are not new, they were obtained in [8] by the ‘integrating in’ procedure introduced in [12]. Our approach which is based on ‘integrating out’, plus the Konishi relations, can be viewed as an independent proof of the ‘integrating in’ procedure.

What we see as an advantage of our approach it is that, within a certain range of parameters, the superpotential (3) gives a complete description of the low energy physics. Indeed, when the mass \( m_V \) of the field \( V \),

\[
m_V = 2m (2 - 3\sigma \kappa), \tag{21}
\]
is much less than the other masses, such as $m_\Phi = g^2 \mu$ and $m_W = |g^2 v|^{1/2}$, we are in the weakly coupled Higgs phase and enjoy full theoretical control. The Konishi relations help to determine the condensates of heavy fields in this phase. Holomorphy then allows for continuation of these results for the condensates to strong coupling.

At strong coupling the superpotential (3), like other versions of the Veneziano-Yankielowicz Lagrangians [20], does not describe the low energy physics. It can be viewed as a shorthand equation that gives the correct values of the condensates, an equivalent of ‘integrating in’ procedure [8].

One comment to add is about the photino condensate. The gaugino condensate $\langle \text{Tr} \lambda^2 \rangle$ we found above can be viewed as a sum of the condensates for charged gauginos and the photino,

$$\langle \text{Tr} \lambda^2 \rangle = \langle \lambda^+ \lambda^- \rangle + \frac{1}{2} \langle \lambda^3 \lambda^3 \rangle$$  \hspace{1cm} (22)

In gauge invariant form the photino condensate can be associated with $\langle (\text{Tr} W \Phi)^2 \rangle$. It was shown in [21] that $\mathcal{N}=2$ is preserved in the effective QED even when the breaking parameter $\mu$ is nonvanishing. An immediate consequence of this observation is that the photino condensate vanishes, it is not the lowest component in the corresponding $\mathcal{N}=2$ supermultiplet. So, the gaugino condensate is solely due to the charged gluino.

2.1 Argyres-Douglas points

When the mass $m$ changes from large to small values we interpolate between the two quite different structures of vacua shown above. Let us consider this transition when, for definiteness, $\det h = -1$ and $m$ is real and positive and changes from large to small values. At large positive $m$ all the vacua are situated at real values of $u$, the dyon vacuum is at negative $u$, the monopole vacuum is at positive $u$, and the charge vacuum is also at positive, but much larger, values of $u$. When $m$ diminishes then at some point the monopole and charge vacua collide on the real axis of $u$ and subsequently go more off to complex values producing the $Z_3$ picture at small $m$.

The point in the parameter manifold where the two vacua coincide is the AD point [13]. In the SU(2) theory these points were studied in [14]. Mutually non-local states, say charges and monopoles, becomes massless at these points. On the Coulomb branch of the $\mathcal{N}=2$ theory these points correspond to a non-trivial conformal field theory [14].
Here we study the $\mathcal{N}=1$ SUSY theory, where $\mathcal{N}=2$ is broken by the mass term for the adjoint matter as well as by of the Yukawa coupling. Collisions of two vacua still occur in this theory. We find the values of $m$ at which AD points appear generalizing the consideration in [14].

Coalescence of two roots for $v$ means that together with Eq. (7) the derivative of its left-hand-side should also vanish,

$$m - \frac{(-\det h)}{2} \frac{v}{\mu} - \frac{A_1^3}{4} \left(\frac{\mu}{v}\right)^2 = 0, \quad -(-\det h) + A_1^3 \left(\frac{\mu}{v}\right)^3 = 0. \quad (23)$$

This system is consistent only at three values of $m = m_{AD}$,

$$m_{AD} = \frac{3}{4} \omega \Lambda_1 (-\det h)^{2/3}, \quad \omega = e^{2\pi in/3}, \quad (n = 0, \pm 1), \quad (24)$$

related by $Z_3$ symmetry. The condensates at the AD vacuum are

$$v_{AD} = \omega \frac{\mu \Lambda_1}{(-\det h)^{1/3}}, \quad u_{AD} = \omega^{-1} \frac{3}{4} \Lambda_1^2 (-\det h)^{1/3},$$

$$s_{AD} = \omega^{-1} \frac{1}{4} \mu \Lambda_1^2 (-\det h)^{1/3}. \quad (25)$$

### 3 Dyon condensates

In this section we calculate various dyon condensates at the three vacua of the theory. As discussed above, holomorphy allows us to find these condensates starting from a consideration on the Coulomb branch in $\mathcal{N}=2$ near the singularities associated with a given massless dyon. Namely, we calculate the monopole condensate near the monopole point, the charge condensate near the charge point and the dyon $(n_m, n_e) = (1, 1)$ condensate near the point where this dyon is light. Although we start with small values of the adjoint mass parameter $\mu$, our results for condensates are exact for any $\mu$ as well as for any value of $\det h$.

#### 3.1 Monopole condensate.

Let us start with calculation of the monopole condensate near the monopole point. Near this point the effective low energy description of our theory can be given in terms of $\mathcal{N}=2$ dual QED [4]. It includes a light monopole...
hypermultiplet interacting with a vector (dual) photon multiplet in the same
way as electric charges interact with ordinary photons. Following Seiberg
and Witten [5] we write down the effective superpotential in the following
form,
\[ W = \sqrt{2} \tilde{M} M A_D + \mu U, \] (26)
where \( A_D \) is a neutral chiral field (it is a part of the \( \mathcal{N}=2 \) dual photon
multiplet in the \( \mathcal{N}=2 \) theory) and \( U = \text{Tr} \Phi^2 \) considered as a function
of \( A_D \). The second term breaks \( \mathcal{N}=2 \) supersymmetry down to \( \mathcal{N}=1 \).

Varying this superpotential with respect to \( A_D, M \) and \( \tilde{M} \) we find that
\( A_D = 0 \), i.e. the monopole mass vanishes, and
\[ \langle \tilde{M} M \rangle = -\frac{\mu}{\sqrt{2} \partial A_D} \bigg|_{A_D=0}. \] (27)

The non-zero value of the monopole condensate \( \langle \tilde{M} M \rangle \) ensures U(1) con-
finement for electric charges via the formation of Abrikosov-Nielsen-Olesen
vortices.

Let us work out the r.h.s. of Eq. (27) to determine the \( \mu \) and \( m \) de-
pendence of the monopole condensate. From the exact Seiberg-Witten solu-
tion [4], we have
\[ \frac{\partial A_D}{\partial u} = \frac{\sqrt{2}}{8\pi} \oint_{\gamma} \frac{dx}{y(x)}. \] (28)
Here for the Seiberg-Witten curve \( y(x) \) given by Eq. (13) we use the form
\[ y^2 = (x - e_0)(x - e_-)(x - e_+). \] (29)

The integration contour \( \gamma \) in the \( x \) plane circles around two branch points \( e_+ \)
and \( e_- \) of \( y(x) \). At the monopole vacuum, when \( u = u_M \), two branch points
\( e_+ \) and \( e_- \) coincide, \( e_+ = e_- = e \) and the integral (28) is given by the residue
at \( x = e \).
\[ \frac{\partial A_D}{\partial u}(u_M) = \frac{i \sqrt{2}}{4\sqrt{e - e_0}}. \] (30)
The value of \( e - e_0 \) (equal at \( u = u_M \) to \( (1/2) \partial^2(y^2)/dx^2 \) ) is fixed by the
equation \( \partial(y^2)/\partial x = 0 \),
\[ e - e_0 = \sqrt{u_M^2 - \frac{3}{4} m \Lambda^4}. \] (31)
Substituting this into the expression for the monopole condensate \( \langle \tilde{M}M \rangle \) we get finally

\[
\langle \tilde{M}M \rangle = 2i\mu \left( u_M^2 - \frac{3}{4}m\Lambda_1^3 \right)^{1/4}.
\] (32)

To test the result let us consider first the limit of a large masses \( m \) for the fundamental matter. As in Sec. 2 this limit can be viewed as a RG flow to pure Yang-Mills theory with the identification \( \Lambda_0^4 = m\Lambda_1^3 \), where \( \Lambda_0 \) is the scale of the \( \mathcal{N}=2 \) Yang-Mills theory. In this theory we have \( u_M = \Lambda_0^2 \). Then Eq. (32) gives \( \langle \tilde{M}M \rangle = \sqrt{2}i\mu\Lambda_0 \), which coincides with the Seiberg-Witten result \[5\]. This ensures monopole condensation and charge confinement in the monopole point at large \( m \).

Notice, that in the derivation above \( \mathcal{N}=2 \) was not broken by the Yukawa coupling, i.e. we assume \( \det h = -1 \). The result, however, can be easily generalized to arbitrary \( \det h \) by means of \( U(1) \) symmetries considered above, Eq. (32) for the monopole condensate remains valid for arbitrary \( \det h \).

### 3.2 Deconfinement in the Argyres-Douglas point

Now let us address the question: what happens with the monopole condensate when we reduce \( m \) and approach the AD point? The AD point corresponds to a particular value of \( m \) which ensures coalescence of the monopole and charge singularities in the \( u \) plane. Near the monopole point we have condensation of monopoles and confinement of charges while near the charge point we have condensation of charges and confinement of monopoles. As shown by ’t Hooft these two phenomena cannot happen simultaneously \[22\]. The question is: what happens when monopole and charge points collide in the \( u \) plane?

The monopole condensate at the AD point is given by Eq. (32). When \( m \) and \( u \) are substituted by \( m_{AD} \) and \( u_{AD} \) from Eqs. (24) and (25), we get

\[
\langle \tilde{M}M \rangle_{AD} = 0.
\] (33)

We see that the monopole condensate goes to zero at the AD point. Our derivation makes it clear why it happens. At the AD point all three roots of \( y^2 \) become degenerate, \( e_+ = e_- = e_0 \), so the monopole condensate which is proportional to \( \sqrt{e - e_0} \) naturally vanishes.

In the next subsection we calculate the charge condensate in the charge point and show that it also goes to zero as \( m \) approaches its AD value \( 24 \).
Thus, we interpret the AD point as a deconfinement point for both monopoles and charges.

### 3.3 Charge and dyon condensates

In this subsection we use the same method to calculate values for the charge and dyon condensates near the charge and dyon points respectively. We first consider $m$ above its AD value \(m_{AD}\) and then continue our results to values of $m$ below $m_{AD}$. In particular, in the limit $m = 0$ we recover $Z_3$ symmetry.

Let us start with the charge condensate. At $\mu = 0$, \(\det h = -1\) and large $m$ the effective theory near the charge point

\[
a = -\sqrt{2} m
\]

on the Coulomb branch is $\mathcal{N}=2$ QED. Here $a$ is the neutral scalar, the partner of photon in the $\mathcal{N}=2$ supermultiplet. Half of the degrees of freedom in color doublets become massless whereas the other half acquire a large mass $2m$. The massless fields form one hypermultiplet $\tilde{Q}^+, Q^+$ of charged particles in the effective electrodynamics. Once we add the mass term for the adjoint matter the effective superpotential near the charge point becomes

\[
\mathcal{W} = \frac{1}{\sqrt{2}} \tilde{Q}^+ Q^+ A + m \tilde{Q}^+ Q^+ + \mu U
\]

Minimizing this superpotential we get condition (34) as well as

\[
\langle \tilde{Q}^+ Q^+ \rangle = -\sqrt{2} \mu \frac{du}{da} \bigg|_{a = -\sqrt{2} m}.
\]

Now, following the same steps which led us from (27) to (32), we get

\[
\sqrt{-\det h} \langle \tilde{Q}^+ Q^+ \rangle = 2 \mu (u_C^2 - \frac{3}{4} m \Lambda_i \lambda)^{1/4},
\]

where we include a generalization to arbitrary $\det h$. We choose to consider $\sqrt{-\det h} \langle \tilde{Q}+ Q^+ \rangle$ because it has the $U_R(1)$ charge equal to one, similar to the $\langle \tilde{MM} \rangle$ condensate considered above. By $u_C$ we denote the position of the charge vacuum in the $u$ plane.

Holomorphy allows us to extend the result (37) to arbitrary $m$ and $\det h$. So we can use Eq. (37) to find the charge condensate at the AD point. Using
Eqs. (24) and (25) we see that the charge condensate vanishes at the AD point in the same manner the monopole condensate does. As it was mentioned we interpret this as deconfinement for both charges and monopoles.

To write results for the charge, monopole and dyon condensates together let us introduce the dyon field $D_i$, $i = 1, 2, 3$, which stands for the charge, monopole and (1,1) dyon field,

$$D_i = \{(-\text{det } h)^{1/4} Q_+, \, M, \, D\}. \quad (38)$$

The arguments of the previous subsection which led us to the result (32) for monopole condensate give for $\langle \tilde{D}_i D_i \rangle$

$$\langle \tilde{D}_i D_i \rangle = 2 i \zeta_i \mu \left( u_i^2 - \frac{3}{4} m \Lambda_1^3 \right)^{1/4}, \quad (39)$$

where $u_i$ is the position of the i-th point in the $u$ plane and the $\zeta_i$ are phase factors.

For the monopole condensate at real values of $m$ larger than the $m_{AD}$ Eq. (32) gives $\zeta_M = 1$, while for the charge condensate from Eq. (37) we have $\zeta_C = -i$. For the dyon the phase factor is $\zeta_D = i$.

At the particular AD point we have chosen the monopole and charge condensates vanish, while the dyon condensate remains non-zero, see (39). Below the AD point, condensates are still given by Eq. (39), but the charge and monopole phase factors can change. The dyon phase factor does not change when we move through the AD point because the dyon condensate does not vanish at this point.

In the limit $m = 0$ we should recover the $Z_3$-symmetry for the values of condensates. From Eq. (39) it is clear that the absolute values of all three condensates are equal because the values of the three roots $u_i$ are on the circle in the $u$ plane, see (17). Imposing the requirement of $Z_3$ symmetry at $m = 0$ we can fix the unknown phase factors $\zeta_C$ and $\zeta_M$ below the AD point using the value $\zeta_D = i$ for dyon. This gives $\zeta_C = i, \quad \zeta_M = -i$.

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4Note that the quantum numbers of the “charge” and “monopole” are also transformed, see [23]
4 Conclusions

• In the $\mathcal{N}=1$ theory the chiral condensates of matter and gaugino fields are fixed by the ADS superpotential as the only nonperturbative input. In the limit of small adjoint mass we find for condensates a complete matching with the $\mathcal{N}=2$ Seiberg-Witten solution. Although the bulk of our results for matter and gaugino condensates overlaps with what is known in the literature we think that our approach clarifies some aspects of duality in $\mathcal{N}=1$ theories.

• Using the Seiberg-Witten approach of the broken $\mathcal{N}=2$ we determine the monopole, charge, and dyon condensates in the $\mathcal{N}=1$ theory.

• The Argyres-Douglas points exist in the $\mathcal{N}=1$ theories. When the monopole and charge vacua collide at the AD point both the monopole and charge condensates vanish. It results in the deconfinement of electric and magnetic charges at the AD point.

• Vanishing of condensates signals existence of new nontrivial $\mathcal{N}=1$ superconformal theories.
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