1. INTRODUCTION

Early-type galaxies, ellipticals, and S0s form an impressively regular sequence with luminosity, as seen in the Faber–Jackson relation (Faber & Jackson 1976) and generalized in the fundamental plane relations (Djorgovski & Davis 1987; Binney & Merrifield 1998). The core radius and brightness are basic observational properties that hold clues to the origin of early-type galaxies. Galaxy merging, star-formation, the presence of black holes, and, in sufficiently dense cores, dynamical friction and two-body relaxation all play some role in creating the cores. One widely considered possibility for the formation and evolution of early-type galaxy cores is that they are largely a result of stellar dynamical processes in the core with gas and star formation playing minor roles.

The coarse grain phase space density can be defined as \( Q_0 \equiv \rho / \sigma^3 \). The coarse grain phase density of the galaxy core, \( Q_0 \), is a key dynamical quantity for assessing the relative roles of various stellar dynamical processes. The local phase density controls the rate of two-body relaxation and dynamical friction, along with the masses of the orbiting bodies (Binney & Tremaine 2008). Liouville’s theorem (Goldstein et al. 2002) requires that the fine grain phase density, \( F \), is a constant of the motion in a conservative Hamiltonian system, with the coarse grain phase density required to be \( Q_0 \leq F \) (with appropriate velocity space normalization).

The observational description of the central surface brightness distribution of early-type galaxies began with ground-based telescope data and used the physically well-motivated King model (King 1962, 1966, 1978). CCDs observations showed that some early-type galaxies, particularly those with lower luminosities, had central brightness distributions that rose above the constant brightness King model core (Kormendy 1985; Lauer 1985) although earlier photographic data showed this as indicating the presence of a stellar nucleus (Binggeli et al. 1984). The angular resolution of the Hubble Space Telescope (HST) showed that what became known as power-law cores (which we will describe as steep profiles) were increasingly common with decreasing galaxy luminosity. (Crane et al. 1993; Ferrarese et al. 1994; Kormendy et al. 1994).

The central density profiles of early-type galaxies are classified as being steep if the negative logarithmic slope is greater than 0.5 or shallow if the slope is shallower than 0.3 (Gebhardt et al. 1996; Ravindranath et al. 2001; Rest et al. 2001; Lauer et al. 2007b). However, the core parameters and the degree of bimodality of the central slope distribution depend on the surface brightness fitting model and the sample definition.

The purpose of this paper is to calculate the phase density of spheroidal systems, focusing on early-type galaxies, to consider its correlations with core properties. In particular, we examine to what degree phase density relates to the suggestions that early-type cores have a bimodal distribution in their brightness profile slopes. These relations are considered to be tests of stellar dynamical models for the formation and evolution of early-type galaxy cores.

2. THE OBSERVATIONAL DATA

There are two leading functional forms to fit a general core brightness profile. The Nuker formula (Lauer et al. 1995; Carollo et al. 1997; Lauer et al. 2007b) is designed to describe the central region of a galaxy. An alternative, whole galaxy brightness profile fitting approach is the core-Sérsic function (Graham & Guzmán 2003; Graham et al. 2003; Trujillo et al. 2004; Ferrarese et al. 2006; Turner et al. 2012; Dullo & Graham 2012). The core-Sérsic function provides good fits to the entire brightness profile of a galaxy at the relatively small cost of a single extra parameter. There are significant differences in the details between the two approaches and no clear consensus on the best approach has emerged. The extrapolated Nuker law does not match the galaxy profiles on large scales which can have an effect on the fit of the inferred central slope.

The slope of the brightness profile near the resolution limit of the data, under 0.1 depending on the HST instrument and sampling, defines the parameter \( \gamma' \) of the Nuker approach and is intended to catch central brightness cusps, if they exist, or simply to confirm that the nearly flat core brightness profile continues to small radii. The Nuker \( \gamma' \) is in general agreement with the core-Sérsic measurement of the comparable quantity,
although with some scatter and a systematic offset (Dullo & Graham 2012). The preferred measure of the physical size of the core is the quantity, $r_γ$, the radius at which the Nuker profile has a slope of 0.5 (Carollo et al. 1997; Lauer et al. 2007a). Dullo & Graham (2012) show that the core-Sérsic break radius, $r_{b,cS}$, is in good agreement with the Nuker $r_γ$.

Defining a representative sample of early-type galaxies is important when comparing trends within a population. The Lauer et al. (2007b) sample is the largest available uniform compilation and is approximately Magnitude-limited. A very wide ranging set of kinematic and dynamical data are available for the ATLAS3D Cappellari et al. (2011) sample, which is designed to be a complete volume-limited sample. Both samples report Nuker profile fit parameters and serve as two large, comparable, and complementary samples for our analysis.

### 2.1. Phase Density Calculation

Calculating the core coarse grain phase space density requires measurements of the core velocity dispersion and core radius, from which other quantities can be derived. The different types of systems have a range of density profiles and somewhat different approaches to measurement are used, meaning that the phase densities will have small systematic differences between them, although we expect these to be much smaller than the large range of $Q_0$ values present. To obtain a physical mass density in the core, we use the King radius relation (Binney & Tremaine 2008) of a non-singular isothermal sphere for all systems,

$$\rho_0 = \frac{9}{4\pi G} \frac{\sigma_v^2}{r_c^2}. \quad (1)$$

We recognize that few of the systems are particularly well described as an isotropic isothermal sphere, however, Equation (1) provides a uniform basis to calculate basis that will be correct within a factor of a few. To employ Equation (1), we need to identify a central velocity dispersion and a measure of the core radius for the systems that we consider. Although our primary interest is the early-type galaxies, in which core radii are measured in the same way, we will make comparisons to phase densities of nuclear star clusters of late-type galaxies and globular clusters in which the measures of the core radius are comparable but not identical, which will lead to systematic differences, but these will not affect our limited use.

Given our assumptions,

$$Q_0 = \frac{166.6}{\sigma_0 r_c^2} \quad (2)$$

for $\sigma_0$ in units of km s$^{-1}$ and $r_c$ in units of parsecs. $Q_0$ is the approximate phase space density at the core radius and normally increases inward from that location.

### 2.2. Early-type Galaxies

The Lauer et al. (2007a, 2007b) sample is a large compilation of available HST data, but as an approximately apparent magnitude-limited sample, it contains more luminous galaxies than a volume-limited sample (Lauer et al. 2007b; Côté et al. 2007). In total, there are 189 galaxies in the sample that have core profiles and central velocity dispersions.

The ATLAS3D sample (Krajnovič et al. 2013) is constructed to be an essentially complete sample of galaxies more massive than about $6 \times 10^9 M_\odot$ within 42 Mpc. Krajnovič et al. (2013) provides Nuker fits for the surface brightness profiles. We remove galaxies with upper limits for $r_c$ from the sample, which reduces the number of galaxies to 74. We equate $r_γ$ with $r_c$ below. For the ATLAS3D sample, we use the values of velocity dispersion at $R_e/8$ as tabulated in Cappellari et al. (2013).

For all early-type galaxies we calculate the core mass from the projected quantities, using the relationship $\Sigma_0 = 2\rho_0 r_c$,

$$M_c = 4\pi \rho_0 r_c^3 \int_0^{r_c} r(r/r_c)^{-\gamma'} dr. \quad (3)$$

This integrates to $M_c = 4\pi \rho_0 r_c^3 / (2 - \gamma')$.

Density profiles are classified as being shallow if $\gamma' < 0.3$ (red in plots) and steep if $\gamma' > 0.5$ (blue in plots), with an intermediate-type between (cyan in plots) (Lauer et al. 2007b).

### 2.3. Globular Clusters

Globular clusters may play a significant role in the formation of galactic cores (Tremaine et al. 1975). Accordingly, it is interesting to understand where their phase densities fits into the overall sequence (Walcher et al. 2005). In this work, we will assume that the Milky Way globular clusters are representative of the group as a whole. For these clusters, core radii and central velocity dispersions come from the 2010 edition of the compilation of Harris (1996). In the following figures, globular cluster data are plotted as solid black dots.

We use Equation (1) to calculate the core density. We approximate the core a constant density, so the core mass is

$$M_c = \frac{4\pi}{3} \rho_0 r_c^3. \quad (4)$$

### 2.4. Disk Galaxy Nuclear Star Clusters

The data for the nuclear star clusters in disk galaxies comes from the work of Böker et al. (2004) and Walcher et al. (2005), who also estimated phase space densities at the half-mass radius. There are six galaxies in Table 3 of Walcher et al. (2005), for which values of the profile fitted $r_c$ are available in the Böker et al. (2004) paper. Densities for these galaxies were calculated using Equation (1) above, but with $r_c \equiv r_e/0.75$ (that is, the $r_b$ of Walcher et al. 2005). This identification of the core radius effectively considers the entire NSC to be a core. Depending on the concentration of the core, the phase density can be several times higher. Under this assumption, $Q_0$ for each galaxy can then be calculated from the tabulated values of $\sigma_c$. In the figures, these six galaxies are plotted as solid green dots. For the remaining galaxies in Table 1 Böker et al. (2004) with $r_e$ values, core masses were calculated from the given luminosities and an assumed $M/L$ of 0.5, which is the median value in Table 3 of Walcher et al. (2005). Velocity dispersions were obtained using the following empirically determined relation by fitting to the data in Table 3 of Walcher et al. (2005),

$$\sigma_0^2 = 0.45 \frac{GM_c}{r_e} \quad (5)$$

The resulting empirically determined values of $Q_0$ are plotted as green squares in the figures.

The data are plotted in the $r_c$ vs $\sigma_0$ observational plane in Figure 1. Cores with steep density profiles are plotted as blue dots and shallow cores as red dots. We note that in this plot the various types of objects substantially overlap when projected onto any one axis. There is also substantial mass overlap between the various object types.
3. Q0 DEPENDENCE OF CORE SLOPE

In Figure 2, we plot the core phase densities as a function of their inner brightness profile slope, $\gamma'$, for all early-type galaxies for both the Lauer et al. (2007b) and Krajnović et al. (2013) samples. A strong correlation between the log of $Q_0$ and $\gamma'$ is readily visible. The Pearson linear correlation coefficients for the $Q_0$–$\gamma'$ correlation is 0.78 for the Lauer et al. (2007b) sample and 0.83 for the Krajnović et al. (2013) sample. That is, the core phase space density explains an impressive 69% ($r^2$) of the variance in $\gamma'$. The correlation of $\gamma'$ with luminosity is fairly weak in the ATLAS3D sample, $r = -0.51$, i.e., only about 25% of the variance, where we use the log of the r-band luminosities, $L_r$, of Cappellari et al. (2013). The correlation of log $Q_0$ and either log of the stellar mass or r-band luminosity is also comparably weak, $r = -0.49$ and $-0.50$, respectively. The weak correlation with the total mass or luminosity of the galaxy indicates that although early-type cores evolve within their host galaxy, the host galaxy does not completely control the resulting core properties.

To further explore the correlation of the core density profile and the bimodality proposals, Figures 3 and 4 show for the two early-type samples the distribution over $Q_0$ of shallow and steep core types, $\gamma' < 0.3$ and $\gamma' > 0.5$, respectively, as a function of their phase space density. Remarkably, $Q_0$ fairly cleanly separates the shallow and steep cores into two almost non-overlapping distributions. The relative numbers of shallow and steep cores are comparable in both samples. Of the 25 shallow and 26 steep cores in the Krajnović et al. (2013) sample, only 1 from each falls into an overlapping region of $Q_0$ in Figure 4.
The intermediate core types, those with $0.3 \leq \gamma' \leq 0.5$, fill in the $Q_0$ region between the shallow and steep core types. The intermediate core slope population is relatively large in the Krajnović et al. (2013) sample, 22 of 74, with 11 of 189 in the Lauer et al. (2007b) sample, likely reflecting the difference between the sample selection criteria. Although it is true that shallow and steep cores do form two nearly non-overlapping distributions in $Q_0$, a more physical interpretation is that $\gamma'$ is a continuous variable that is strongly ordered with the core $Q_0$.

We note that the transition from steep to shallow cores occurs near $Q \simeq 0.003 M_\odot \text{pc}^{-3} \text{km}^{-3} \text{s}^3$.

4. DYNAMICAL QUANTITIES AND $Q_0$

4.1. Core Mass

The core phase densities of early-type galaxies, nuclear star clusters, and globular clusters are shown in Figure 5 as a function of the calculated core mass. As a group, the objects appear to form a single sequence in this particular dynamical space. There is a distinct break in the slope between shallow and steep cores.

We know what to expect for a $Q_0-M_c$ relation. Equations (2) and (4) give $Q_0 \propto \sigma_0^3 M_c^{-2}$. If we take a Faber–Jackson relation for the core to be $\sigma_0 \propto M_c^{\beta}$ and use Equation (1) to eliminate the density, then

$$Q_0 \propto M_c^{-2+3\beta}.$$  \hspace{1cm} (6)

The steep density profile cores in Figure 5 have a $Q_0-M_c$ slope from 72 points $-1.109 \pm 0.064$, whereas the 105 shallow cores have $-1.641 \pm 0.025$. Kormendy & Bender (2013) find Faber–Jackson relationships, $L_v \propto \sigma_0^n$, where their $n = 1/\beta$, or $n \simeq 3.74$ for power-law galaxies and 8.33 for cores, assuming that the core mass is directly proportional to the total luminosity, which is approximately true, as shown in Figure 6. Accordingly, we would expect $Q_0-M_c$ slopes of $-1.20$ and $-1.64$, consistent with the fitted relations, for steep and shallow cores, respectively. The correlation between $Q_0$ and $M_c$ has $r = 0.95$. However, $Q_0$
It is straightforward to undertake a principal component analysis (PCA) using the R project software (http://www.r-project.org/). We remove strongly correlated variables, leaving the PCA with $y'$, the r-band luminosity, and the parameters reported in Cappellari et al. (2013), the flattening at $r_c/2$, the rotation parameter within $r_c/2$, the kinematic/photometric misalignment angle, $\psi$, and $f_{DM}$, the dark matter fraction inside $r_c$. The PCA finds that the first component provides 49% of the variance, and three components of seven are required to provide 80% of the variance. The PCA confirms that $Q_0$ is the single most strongly correlated variable with $y'$. However, the next most correlated variable is the dark matter fraction, $f_{DM}$, as evaluated at $r_c$. A general linear model fit finds $y' = (0.138 \pm 0.015) \log Q_0 - (0.32 \pm 0.12) f_{DM} + (0.25 \pm 0.10) \lambda_{r_c/2} + 0.745$. The $f_{DM}$ term has a probability of only 0.8% of chance occurrence and the rotation term $\lambda_{r_c/2}$ has a probability of 1.6% of chance occurrence. However, log $Q_0$ removes about 69% of the variance, whereas both $f_{DM}$ and rotation each account for only about 2.3%, leaving 26% unaccounted. The negative correlation with $y'$ indicates that a lower dark matter fraction is associated with steep cores. This is suggestive that somewhat more dissipated galaxies, hence lower dark matter fraction, are more likely to have steep core profiles at the same $Q_0$, but it is a small effect.

4.4. Central Super-massive Black Holes

Super-massive black holes are closely connected to galaxy cores although the evolutionary dependence is not entirely clear (Kormendy & Ho 2013). There are well-known correlations between the velocity dispersion of ellipticals and their black holes (Magorrian et al. 1998; Ferrarese & Merritt 2000; Gebhardt et al. 2000). Black hole masses are available in Graham & Scott (2013) and McConnell & Ma (2013). Although the galaxies with black hole masses are a subset of those with core density measurements, there is no clear bias other than the size and distance considerations needed to allow a black hole mass measurement. Figure 8 shows the ratio of the derived black hole mass to our estimated core masses as a function of their core phase densities. The high $Q_0$ steep density profile cores have core masses comparable to the black hole mass, with a median $M_{BH}/M_c \simeq 0.5$. That is, the cores of most galaxies with $Q_0 > 0.003 M_\odot \, \text{pc}^{-3} \, \text{km}^{-3} \, \text{s}^2$ are within the black holes sphere of gravitational influence (Milosavljević & Merritt 2001). Shallow core galaxies have much more massive cores on the average, with a median value $M_{BH}/M_c \simeq 0.1$.

5. IMPLICATIONS OF CORE PHASE DENSITIES

5.1. Dynamical Times in Dense Cores

The phase space density determines the time for two-body relaxation (Binney & Tremaine 2008). We adopt the parameter choices of Merritt (2013), his Equation (5.61), to find

$$t_2 = \frac{1.1 \times 10^9 \, M_\odot}{Q \, m \, \ln \lambda^{\gamma \text{yr}}}.$$  
(7)

A relaxation time of $10^9$ yr for a population of $m = 1 M_\odot$ stars occurs for $Q = 1.1$. The steep cores containing black holes shown in Figure 8, have median $Q_0 \simeq 0.1$, in which case interaction with two-body relaxation of stars around the black hole likely plays a significant role in building and maintaining a steep core (Merritt & Szell 2006).
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Figure 8. Ratio of the central black hole mass to the core mass as a function of $Q$.

The timescale for dynamical friction depends linearly on the phase space density and the mass of the inspiraling satellite (Binney & Tremaine 2008). We again use the parameters of Merritt (2013), his Equation (5.32), to give the radial dependence of the dynamical friction timescale,

$$t_{df}(r) = \frac{1.4 \times 10^{10} \, 0.35 \, M_\odot \, 10 \, \text{yr}}{Q(r)} \frac{\ln \Lambda}{G(x) \, m}. \tag{8}$$

where $G(v/\sigma)$ is a slowly varying function with a value of 0.347 for $v/\sigma = \sqrt{2}$, roughly as expected for an inspiraling satellite. For a somewhat heavy $3 \times 10^8 M_\odot$ globular cluster to spiral in within $10^{10}$ yr requires that it orbits in a region with a local $Q > 4 \times 10^{-6} M_\odot \, \text{pc^{-3} km^{-3} s^{-1}}$. Therefore, the most massive early-type galaxies, those with cores more massive than about $10^6 M_\odot$ (Figure 5), only allow globular clusters more massive than $3 \times 10^5 M_\odot$ to spiral into the center.

5.2. Core Phase Density Evolution Pathways

A successful core formation and evolution scenario needs to offer a way to understand the correlations presented here between the core phase density and its brightness profile, rotation, and black hole mass. High-redshift progenitor galaxies may largely form with steep cores through dissipative star formation processes (Loose et al. 1982), although current specific models appear to create too steep a Faber–Jackson relation (McLaughlin et al. 2006; Antonini 2013). A widely investigated approximation, which we will follow, is to assume that the stars in the cores of early-type galaxies were largely created elsewhere and brought to the core through merging and dynamical friction and possibly had their density profile altered through two-body relaxation. Here, we briefly examine to what degree these stellar dynamical processes can account for the phase space correlations of core properties. Two widely discussed and quite distinct ideas are either that early-type galaxies began with large low-density cores which were subsequently built up, or they were formed with power-law cores, which were subsequently scoured out in the more massive galaxies. Both mechanisms may be at work.

The slope of the steep cores in the $Q_0-M_c$ diagram, Figure 5, appears to be a continuation of the trend line from globular clusters through nuclear star clusters. Precisely the same data are multiplied with its core mass squared to remove the basic mass dependence of Equation (6). The rescaled results are displayed in Figure 9, which helps make the idea of two sequences clearer. Shown on the plot is a line of constant density, under the self-gravitating virial assumption, which gives $\sigma^2 \propto M_c^{2/3} \rho^{1/3}$ and therefore $Q_0 M_c^2 \propto \rho^{1/3} M_c$.

N-body simulations for the bulk of the galaxy find $\beta \simeq 0.3$ for equal mass mergers and $\beta \simeq -0.3$ for minor mergers (Naab et al. 2009; Hilz et al. 2012), although these values will not necessarily strictly apply to the cores. Recall that we have translated the Faber–Jackson relation to $Q_0 M_c^2 \propto M_c^{10}$. The shallow trend of the shallow core early-types in Figure 9 is highly suggestive that major merging dominates the evolution of these cores, with some admixture of minor merging to flatten out the relation, as has been noted from their Faber–Jackson relations (Kormendy & Bender 2013). If the merging were predominantly minor mergers, it would cause $Q_0 M_c^2$ to decline with increasing core mass, which is not seen. It is also suggestive that a steeper trend continues through steep cores as has been previously noted (Walcher et al. 2005; Côté et al. 2007; Glass et al. 2011). The globular cluster core densities are set through

![Figure 9. Phase space density scaled with the core mass squared.](image)
star formation processes and subsequent dynamical evolution. The globular cluster inspiral process will dissolve the clusters when the mean interior density of galaxy core and the cluster are equal, that is, the mass buildup will tend to push the increasing mass objects along a roughly constant density line (Capuzzo-Dolcetta & Miocchi 2008; Hartmann et al. 2011). A sequence of infall events leads to a modest increase in central density (Antonini et al. 2012). The presence of a central black hole complicates the process but the general trends remain (Antonini 2013).

5.2.1. The Steep Core Buildup Scenario

Although not necessarily the only mechanism which creates shallow cores, one interesting possibility is that most of the stars in early-type galaxies may have been formed in galactic disks, either isolated or in star bursts associated with merging. Subsequent merging of the stellar components leads to a core phase density that is no higher than the phase density of the central regions of the merging disks (Toomre & Toomre 1972; Barnes 1988; Cox et al. 2006).

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Genzel et al. (2011, 2014) presents observational measurements of star forming galaxies near redshift two. The phase space density of a disk is \( Q_d = \Sigma/(2\sigma^3) \), using our normalization and where \( \Sigma \) is the local surface density, \( h \) the scale height, and \( \sigma \) the velocity dispersion which we will take to be isotropic for simplicity. Taking high-redshift disk parameters of \( \Sigma \approx 2-3 \times 10^3 M_\odot \text{pc}^{-2} \) (with some fraction as stars), \( \sigma_0 \approx 300 \text{pc} \), and \( \sigma \approx 80 \text{km s}^{-1} \) (Genzel et al. 2011), which gives a representative central disk \( Q_0 \approx 3 \times 10^{-6} M_\odot \text{pc}^{-3} \text{km}^{-3} \text{s}^3 \). The Milky Way has a comparable central phase density. Merging such stellar disks would create an initial core \( Q_0 \approx 3 \times 10^{-6} M_\odot \text{pc}^{-3} \text{km}^{-3} \text{s}^3 \), if about one-third of the gas eventually turns into stars and the rest of the central gas is driven away. Dissipationless merging of the stellar components of such disks would create galaxies with core phase densities comparable to the most massive, lowest \( Q_0 \), early-type galaxies (Carlberg 1986). For disks in the same potential but with lower surface mass densities, that phase density will be higher. That is, in a plane parallel sheet \( \pi G \delta = \sigma^2, Q_d \propto \Sigma^2 \sigma^{-3} \). To keep the Toomre disk stability parameter (Toomre 1964) near unity requires that the velocity dispersion adjust in proportion to the surface mass density, \( \sigma \propto \Sigma \). Therefore, \( Q_d \propto \sigma^{-3} \). The minimum velocity dispersion of a galactic disk is approximately \( 10 \text{ km s}^{-1} \), due to stirring of the molecular clouds and internal motions of dissolving star clusters so the highest phase density that merging disks would create would be \( \approx 10^{-3} M_\odot \text{pc}^{-3} \text{km}^{-3} \text{s}^3 \). Therefore, a pure disk merger scenario could account for the core phase densities of all the shallow core, \( \gamma' < 0.3 \), early-type galaxies but cannot account for the higher phase densities of the early-type galaxies with steep cores. This is not to dismiss the important role that core scouring likely also plays, as discussed below.

Assuming that all early-type galaxies do begin with fairly flat cores, a widely discussed mechanism for subsequent core buildup is that globular clusters can be dragged into the centers of galaxies (Tremaine et al. 1975). Although the current globular cluster population is insufficient to provide the required mass, it is likely that at high-redshift significantly more high-mass clusters were present, which evolved under the action of various dynamical processes that have been studied and tested extensively (Murali & Weinberg 1997a, 1997b; Fall & Zhang 2001; McLaughlin & Fall 2008; Gieles 2009; Larsen 2009; Chandar et al. 2010; Fall & Chandar 2012). Allowing for the evolution of the globular cluster population can build up all of the nuclear star cluster mass in galaxies with stellar masses below about \( 10^{11} M_\odot \) (Antonini et al. 2012; Antonini 2013). The phase density can be used for a rough calculation of the accreted globular cluster mass. As discussed with Equation (8), the phase density at the maximum radius from which globular clusters can spiral in is \( Q \gtrsim 4 \times 10^{-6} M_\odot \text{pc}^{-3} \text{km}^{-3} \text{s}^3 \), at which phase density the interior stellar mass of \( \approx 2 \times 10^{10} M_\odot \), which can be seen in Figure 5. For the greatly enhanced progenitor globular cluster to stellar mass fraction of 0.04 that Gnedin et al. (2014) suggest, the accreted mass will be \( 8 \times 10^8 M_\odot \), which spirals down the center to build a core of that mass. Reverting to Figure 5, we see that this mass is approximately where the transition value of \( Q_0 \approx 0.003 \) that separates steep from shallow cores occurs. The globular cluster population enhancement, which only needs to apply to the inner kiloparsec or so, is about a factor of 10 over current epoch galaxies have an average of approximately 0.003 of their stellar mass in globular clusters (Harris et al. 2013). Outside the core, the stellar density distribution that is providing the friction generally falls faster than the core mass—core radius relation so smaller cores, which have higher \( Q_0 \), will bring in fewer clusters. Since the available globular cluster mass is proportional to the galaxy mass, the inscribed globular cluster mass will scale in proportion to the core mass, Figure 6. A second constraint on the progenitor globular cluster population is that it must have significant net rotation to produce the rotation of power-law cores; Figure 7.

The detailed calculations of Gnedin et al. (2014) predict that the excess mass in the core relative to the galaxy mass should be proportional to the inverse square galaxy mass, comparable to the excess mass—velocity dispersion relation of Antonini (2013). To test this prediction, we calculate \( \Delta M_c \) as the difference between Equation (4) and a constant density core. We define \( \Delta M_c = M_c(\gamma' = 0) - M_c(\gamma' = \infty) \). The correlation between the logarithms of \( \Delta M_c/M_\star \) and \( M_\star \), the stellar mass of the galaxy, is a very weak \( r = -0.33 \) for all the ATLAS3D galaxies. Restricting the fit to the galaxies with \( \gamma' > 0.3 \), i.e., all non-shallow core galaxies, the slope of the log \( \Delta M_c/M_\star \) vs log \( M_\star \) relation is \( -0.5 \pm 0.1 \), in accord with the predicted inverse square root relation (Gnedin et al. 2014) but the correlation rises to only a marginal \(-0.49 \). Although it seems inevitable that a substantial number of globular clusters have spiraled into galactic cores, the predicted relation between \( \Delta M_c \) and \( M_\star \) accounts for only about 11% of the variance. Of course, galaxy to galaxy differences in formation history, central globular cluster numbers, and the specific properties of the globular clusters from one galaxy to another could account for the scatter.

5.2.2. The Core Scouring Scenario

All early-type galaxies could be created with fairly steep density profile cores, which then requires a mechanism to hollow out the cores in larger galaxies. Super-massive black hole pairs that come together after two progenitor galaxies merge will quickly sink into the core (Begelman et al. 1980). Stars that encounter the pair can be ejected from the core to “scour” out of a power-law core and produce a much shallower central density profile (Ebisuzaki et al. 1991; Faber et al. 1997; Milosavljević & Merritt 2001). The process preferentially depletes stars on radial orbits to leave a relatively tangential velocity ellipsoid in the core (Quinlan & Hernquist 1997; Milosavljević & Merritt 2001; Antonini et al. 2012). Kinematic observations of a sample of shallow core early-type galaxies do show the expected signature (Thomas et al. 2014). Simulations have shown that a single
binary black hole pair ejects only a few times the resulting merged black hole mass, so the small $M_{\text{BH}}/M_\star$ ratios of shallow core galaxies require multiple mergers to successfully scour the core (Faber et al. 1997; Milosavljević & Merritt 2001; Merritt 2006; Gualandris & Merritt 2008; Dullo & Graham 2013).

Kormendy & Bender (2013) present observational arguments for a scenario in which subsequent nearly dissipationless major merging dominates the formation of the most massive early-type galaxies, as indicated by the very slow rise of central velocity dispersion with increasing mass. Their Faber–Jackson relations for shallow and steep core galaxies can be translated into the core phase density–core mass relation which we present. The core scouring model at present does not address the strong correlation between the core profile slope $\gamma'$ and the core phase density. Since steep cores also contain black holes, they too should have been subject to core-scouring, but many of them have sufficiently high core phase densities that a power-law core can be maintained or rebuilt (Bahcall & Wolf 1976; Merritt & Szell 2006; Merritt 2009). Core scouring certainly plays some role which can vary from galaxy between creating the core to largely one of maintaining a shallow profile.

6. CONCLUSIONS

The coarse grain core phase density, $Q_0$, of early-type galaxies provides dynamical insight into the properties of the galactic cores and serves as a dynamical ordering parameter. The core phase density and the slope of the core brightness profile, $\gamma'$, are very strongly correlated, $r = 0.83$. We also find that for the standard definitions of shallow and steep density profile cores, $\gamma' < 0.3$ and $\gamma' > 0.5$, respectively, then $Q_0$ does a very good job of separating the two distributions, although there is a substantial intermediate core slope population between the two. The transition from steep to shallow cores occurs around $Q_0 \approx 0.003 M_\odot \text{pc}^{-3} \text{km}^{-3} \text{s}^{-2}$. $Q_0$ correlates significantly but less strongly with the galaxy rotation, the ratio of the central black hole mass to the core mass, and quite weakly with the fraction of dark matter inside the effective radius.

Primordial cores could have been built as steep cores through dissipative star formation or as relatively low density shallow core profiles through disk merging. The two main models for subsequent core evolution, globular cluster inspiral and core scouring by black hole binary pairs, are considered relative to the phase space density correlations. We find that the predicted relation between excess mass above a flat core and galaxy stellar mass is present in the steep and intermediate slope cores but with considerable scatter. In a diagram of $Q_0$ weighted with the $M^2$ as a function of $M_\star$, the steep cores are the high-mass end of a sequence from globular clusters through nuclear star clusters. The sequence has slowly rising core density with mass, roughly as simulations of tidal dissolution of an initial distribution globular clusters have found. The shallow cores are on a much shallower relation with mass, about as would be expected from major mergers. The transition $Q_0$ between steep and shallow cores can be understood as the maximum possible steep profile core that can be built from the greatly enhanced globular cluster population at high redshift present in the largest shallow core early-type galaxy. Core scouring clearly plays a role in maintaining fairly flat core profiles, but offers no ready explanation for the fairly abrupt disappearance of shallow core galaxies at a phase density of $Q_0 = 0.003 M_\odot \text{pc}^{-3} \text{km}^{-3} \text{s}^{-2}$.

We conclude that no single model completely explains the core slope-phase density relationship, although a combination of stellar dynamical effects seems likely to be the mechanism which leaves an indicative signature in the strong correlation with core phase density.

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REFERENCES

Antonini, F. 2013, ApJ, 763, 62
Antonini, F., Capuzzo-Dolcetta, R., Mastrobuono-Battisti, A., & Merritt, D. 2012, ApJ, 750, 111
Barnes, J. E. 1988, ApJ, 331, 699
Bahcall, J. N., & Wolf, R. A. 1976, ApJ, 209, 214
Begelman, M. C., Blandford, R. D., & Rees, M. J. 1980, Natur, 287, 307
Binggeli, B., Sandage, A., & Tarenghi, M. 1984, AJ, 89, 64
Binney, J., & Merrifield, M. 1998, Galactic Astronomy (Princeton, NJ: Princeton Univ. Press)
Binney, J., & Tremaine, S. 2008, Galactic Dynamics (2nd ed.; Princeton, NJ: Princeton Univ. Press)
Böker, T., Sarzi, M., McLaughlin, D. E., et al. 2004, AJ, 127, 105
Cappellari, M., Emsellem, E., Krajnović, D., et al. 2011, MNRAS, 413, 813
Cappellari, M., McDermid, R. M., Alatalo, K., et al. 2013, MNRAS, 432, 1862
Cappellari, M., Scott, N., Alatalo, K., et al. 2013, MNRAS, 432, 1709
Capuzzo-Dolcetta, R., & Miocchi, P. 2008, ApJ, 681, 1136
Carlb erg, R. G. 1986, ApJ, 310, 593
Carollo, C. M., Franx, M., Illingworth, G. D., & Forbes, D. A. 1997, ApJ, 481, 710
Chandar, R., Fall, S. M., & Whitmore, B. C. 2010, ApJ, 711, 1263
Côté, P., Ferrarese, L., Jordán, A., et al. 2007, ApJ, 671, 1456
Cox, T. J., Dutta, S. N., Di Matteo, T., et al. 2006, ApJ, 650, 791
Crane, P., Stiavelli, M., King, I. R., et al. 1993, AJ, 106, 1371
Djorgovski, S., & Davis, M. 1987, ApJ, 313, 59
Dullo, B. T., & Graham, A. W. 2012, ApJ, 755, 163
Dullo, B. T., & Graham, A. W. 2013, ApJ, 768, 36
Ebisuzaki, T., Makino, J., & Okumura, S. K. 1991, Natur, 354, 212
Emsellem, E., Cappellari, M., Krajnović, D., et al. 2011, MNRAS, 414, 888
Emsellem, E., Cappellari, M., Peletier, R. F., et al. 2004, MNRAS, 352, 721
Faber, S. M., & Jackson, R. E. 1976, ApJ, 204, 668
Faber, S. M., Tremaine, S., Ajhar, E. A., et al. 1997, AJ, 114, 1771
Fall, S. M., & Chandar, R. 2012, ApJ, 752, 96
Fall, S. M., & Zhang, Q. 2001, ApJ, 561, 751
Ferrarese, L., Côté, P., Jordán, A., et al. 2006, ApJS, 164, 334
Ferrarese, L., & Merritt, D. 2000, ApJL, 539, L9
Ferrarese, L., van den Bosch, F. C., Ford, H. C., Jaffe, W., & O’Connell, R. W. 1994, AJ, 108, 1598
Gebhardt, K., Bender, R., Bower, G., et al. 2000, ApJL, 539, L13
Gebhardt, K., Richstone, D., Ajhar, E. A., et al. 1996, AJ, 112, 105
Genzel, R., Förster Schreiber, N. M., Lang, P., et al. 2014, ApJ, 785, 75
Genzel, R., Newman, S., Jones, T. et al. 2011, ApJ, 733, 101
Gieles, M. 2009, MNRAS, 394, 2113
Glass, L., Ferrarese, L., Côté, P., et al. 2011, ApJ, 726, 31
Gnedin, O. Y., Ostriker, J. P., & Tremaine, S. 2014, ApJ, 785, 71
Goldstein, H., Poole, C., & Safko, J. 2002, Classical Mechanics (3rd ed.; San Francisco, CA: Addison-Wesley)
Graham, A. W., Erwin, P., Trujillo, I., & Asensio Ramos, A. 2003, AJ, 125, 2951
Graham, A. W., & Guzmán, R. 2003, AJ, 125, 2936
Graham, A. W., & Scott, N. 2013, ApJ, 764, 151
Gualandris, A., & Merritt, D. 2008, ApJ, 678, 780
Harris, W. E. 1996, AJ, 112, 1487
Harris, W. E., Harris, G. L. H., & Alvesi, M. 2013, ApJ, 772, 82
Hartmann, M., Debbattista, V. P., Seth, A., Cappellari, M., & Quinn, T. R. 2011, MNRAS, 418, 2697
Hilz, M., Naub, T., Ostriker, J. P., et al. 2012, MNRAS, 425, 3119
King, I. 1966, AJ, 71, 64
King, I. R. 1978, ApJ, 222, 1
Kormendy, J. 1985, ApJL, 292, L9
Kormendy, J., & Bender, R. 2013, ApJL, 769, L5
Kormendy, J., Dressler, A., Byun, Y. I., et al. 1994, in European Southern Observatory Conference and Workshop Proceedings, Vol. 49, ESO/OHP Workshop on Dwarf Galaxies, ed. G. Meylan & P. Prugniel (Providence: ESO), 147
Kormendy, J., & Ho, L. C. 2013, ARA&A, 51, 511
