Systematic uncertainties from mechanics and MDI in luminosity measurement at CEPC

I. Smiljanic\textsuperscript{a}, I. Bozovic Jelisavcic\textsuperscript{a}, G. Kacarevic\textsuperscript{a}

\textsuperscript{a}Vinca Institute of Nuclear Sciences, National Institute of the Republic of Serbia, University of Belgrade, Belgrade, Serbia

Abstract: The very forward region is one of the most challenging regions to instrument at a future $e^+e^-$ collider. At CEPC, machine-detector interface include, among others, a calorimeter dedicated for precision measurement of the integrated luminosity at a permille level or better. Here we review a feasibility of such precision, from the point of view of luminometer mechanical precision and positioning, beam-related requirements and physics background. A method of the effective center-of-mass determination due to the beam-spread, initially proposed for FCC, is also discussed for the CEPC beams.
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1. Introduction

The Circular Electron Positron Collider (CEPC) is a large international scientific facility proposed by the Chinese particle physics community in 2012 to test the validity scale of the Standard Model (SM) in precision measurements of in the Higgs, BSM and EW sector. These measurements should provide critical tests of the underlying fundamental physics principles of the Standard Model and are vital in exploration of new physics beyond the SM. In electron-positron collisions, the CEPC is designed to operate at around 91.2 GeV as a $Z$ factory, at around 160 GeV of the $WW$ production threshold and at 240 GeV as a Higgs factory. The vast amount of bottom quarks, charm quarks and $\tau$-leptons produced in $Z^0$ decays also makes the CEPC an effective $B$-factory and $\tau$-charm factory.

In order to achieve precision required for realization of the CEPC physics program, relative uncertainty of the integrated luminosity measurement should be of order of $10^{-4}$ at 91.2 GeV and of order of $10^{-3}$ at 240 GeV. Precision reconstruction of position and energy of electromagnetic showers generated by the Bhabha scattering at a high-energy $e^+e^-$ collider can be achieved with finely granulated luminometer. The method for integrated luminosity measurement at CEPC is described in [1 and 2]. However, the reconstruction precision doesn’t exhaust the long list of systematic uncertainties in integrated luminosity measurement, including detector related uncertainties, beam related uncertainties and uncertainties originating from physics and machine-related interactions (like beam-beam interactions, beam-gas scattering and physics background). In this paper we review the effects of the detector and beam related uncertainties, namely mechanical uncertainties of the luminometer position and size and uncertainties related to the beam energy, beam synchronization and interaction point displacements, as well as uncertainties originated from physics background. In addition, impact of the uncertainty of the effective center-of-mass energy on integrated luminosity measurement is studied, and a corrective method is proposed based on [3].

2. Forward region of CEPC

Luminometer at CEPC is proposed to cover the polar angle region between 26 mrad and 105 mrad (with fiducial volume between 53 mrad and 79 mrad) corresponding to the detector aperture of 25 mm for the inner radius and 100 mm for the outer, at 100 cm distance from the interaction point (IP). The most compact design currently proposed seems to be Si-W sandwich type of calorimeter that could provide over 20 $X_0$ in a

\[^1\] Corresponding author, i.smiljanic@vin.bg.ac.rs
longitudinal dimension not larger than 10 cm. Luminometer might be supplemented with an additional layer of tracker in order to improve e-γ separation and calibration of the device. Since the luminometer will be placed at $z = \pm 100 \text{ cm}$ that is a half a way of the tracking volume, shower leakage from the outer edge of the luminometer have been studied and proven to be negligible after absorption by a 5 mm iron filter positioned around the luminometer [1]. Layout of the very forward region at CEPC is given in Figure 1 [1].

![Figure 1: Layout of the very forward region at CEPC.](image)

### 3. Integrated luminosity measurement

Integrated luminosity measurement is a counting experiment based on Bhabha scattering. It is defined as $L = \frac{N_{bh}}{\sigma}$, where $N_{bh}$ is Bhabha count in the certain phase space and within the detector acceptance (fiducial) region and $\sigma$ is the theoretical cross-section in the same geometrical and phase space. However, in a real experiment there are several effects influencing Bhabha count. Here we list some of them that are addressed at the simulation level, assuming detector geometry as described in Sec.2 and the CEPC beams as in [1]:

- mechanics (positioning and alignment);
- center-of-mass energy, beam-energy asymmetry, beam synchronization, IP displacements;
- beam-spread related uncertainty and
- physics background from 2-photon processes.

The first two items are discussed in the next section as uncertainties from mechanics and MDI.

In order to control luminosity at the required level of $10^{-4}$ ($10^{-3}$), both $N_{bh}$ and $\sigma$ should be known at the same level, which means that all these effects should be controlled with the same precision.

### 4. Systematic uncertainties of integrated luminosity from mechanics and MDI

Systematic uncertainties from detector and machine-detector interface (MDI) related effects have been quantified through a simulation study, assuming $10^7$ Bhabha scattering events generated using BHLUMI Bhabha event generator [4], at two CEPC center-of-mass energies: 240 GeV and $Z^0$ production threshold. Detector fiducial volume, where the showers are fully contained and thus the sampling term constant, is between 50 mm and 75 mm radial distance from the detector axis that is assumed to be set at the outgoing beam. The crossing-angle at CEPC is 33 mrad [1]. The effective Bhabha cross-section in this angular range is

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[1] Reference to cited sources is provided if necessary.
of order of a few nb. Final state particles are accepted in the polar angle range from 45 mrad to 85 mrad that is within 8 mrad margin outside of the detector fiducial volume to allow events with non-collinear FSR to contribute. Close-by particles are summed up to imitate cluster merging. We assume that the shower leakage from the luminometer is negligible.

Furthermore, we have applied event selection that is asymmetric in polar angle acceptance on the left and right arm of the detector, as it has been done at OPAL [5]. That is, at one side we consider the full fiducial volume, while at the other side we shrink the radial acceptance for ∆r. This has been done subsequently to the left (L) and right (R) side of the luminometer, on event by event basis. In addition, we require high-energy electrons carrying above 50% of the available beam energy. Against this type of event selection for luminosity measurement, we compare the selection based of the full fiducial volumes on both sides of the detector. An example is given in Figure 2, illustrating the cancelation of systematics uncertainties caused by the assumption of L-R symmetry in an event, when asymmetric selection in polar angle is applied. It is clear that asymmetric selection is advantageous, requiring a luminometer placed at the outgoing beams.

**Figure 2:** Luminosity uncertainty from the longitudinal IP displacements w.r.t. the luminometer, for symmetric (circled) and asymmetric selection with a radial shrink of the fiducial volume ∆r.

Comparison of results against the full fiducial volume counting is given in Table 1, where systematic uncertainties at 240 GeV center-of-mass energy and at the $Z^0$ pole are summarized. We assumed $10^{-3}$ and $10^{-4}$ contribution to the relative uncertainty of integrated luminosity from each individual effect, at 240 GeV and $Z^0$ pole respectively. Considered detector-related uncertainties arising from manufacturing, positioning and alignment are:

- uncertainty of the luminometer inner radius ($∆r_{in}$),
- spread of the measured radial shower position w.r.t. to the true impact position on the luminometer front plane ($σ_r$),
- uncertainty of the longitudinal distance between left and right halves of the luminometer ($Δd$),
- mechanical fluctuations of the luminometer position with respect to the IP caused by vibrations and thermal stress, radial and axial ($σ_{xIP}, σ_{zIP}$),
- twist of the calorimeters corresponding to different rotations of the left and right detector axis with respect to the outgoing beam ($Δφ$).

The MDI related effects list as follows:

- uncertainty of the average net center-of-mass energy ($ΔE_{CM}$),
- uncertainty of the asymmetry in energy of the $e^+$ and $e^-$ beams, ($σ$),
- IP position displacements with respect to the luminometer, radial and axial ($Δx_{IP}, Δz_{IP}$), caused by the finite beam transverse sizes and beam synchronization, respectively.
Results from Table 1. are published in [1], and they rely on previous work of I. Bozovic Jelisavcic and S. Lukic within the CEPC LumiCal group.

It is clear that due to the $1/\theta^3$ dependence of the Bhabha cross-section from the polar angle, inner aperture of the luminometer is one of the most demanding parameters to control. Another challenge comes from the effective center-of-mass energy, needed to be controlled at the level of $\sim 10^{-5}$ with respect to the beam energy, what is smaller than the foreseen beam-spread. This issue will be discussed further in the next section.

| parameter          | limit@240 GeV symmetric sel. | limit@240 GeV asymmetric sel. | limit@91 GeV asymmetric sel. |
|--------------------|-------------------------------|-------------------------------|-------------------------------|
| $\Delta ECM$ (MeV) | 120                           | 120                           | 5                             |
| (MeV)              | 120                           | 240                           | 11                            |
| $\Delta x_{IP}$ (mm) | 0.1                          | 1.0                           | 0.5                           |
| $\Delta z_{IP}$ (mm) | 1.4                          | 10.0                          | 2.0                           |
| beam synch. (ps)   | 1                            | 15                            | 3                             |
| $\Delta r_{in}$ (mm) | 13                           | 10                            | 1                             |
| $\sigma_x$ (mm)    | 0.15                         | 1.00                          | 0.20                          |
| $\Delta d$ (mm)    | 1.00                         | 1.00                          | 0.08                          |
| $\sigma_{x_{IP}}$ (mm) | 0.1                          | 1.0                           | 0.5                           |
| $\sigma_{z_{IP}}$ (mm) | 1                            | 10                            | 7                             |
| $\Delta \phi$ (mrad) | 6.0                          | 6.0                           | 0.8                           |

Table 1: Summary of the systematic uncertainties from mechanics and MDI in the integrated luminosity measurement at 240 GeV and 91 GeV CEPC.

5. Effective center-of-mass energy

According to [1], beam energy spread at CEPC will not exceed 0.134% of the beam energy at 240 GeV center-of-mass and 0.08% of the beam energy at 91.2 GeV, and its shape will ideally be Gaussian. That implies that the difference in energy of colliding particles can be as large as 322 MeV for the Higgs factory and up to 73 MeV for the $Z^0$ factory, which gives a rise to a longitudinal boost of the center-of-mass (CM) frame of colliding particles with respect to the lab frame, $\beta_Z$:

$$|E_{e^+} + E_{e^-}| = \Delta E \rightarrow \beta_Z = \frac{\Delta E}{E_{CM}}$$

The above further leads to the counting loss in luminometer, due to acolinearity of Bhabha final states, as shown at Figure 3. Uncertainty of count of $10^{-5}$ implies knowledge of the asymmetry in beam energies at the level of 12.5% of the beam-spread at the $Z^0$ pole. As this requirement is below the natural energy-spread of the beam, a dedicated method has been applied in order to determine the effective center-of-mass energy ($s'$) in a relatively short time interval.

![Figure 3](image-url) Figure 3. Counting loss in the luminometer due to longitudinal boost of the CM frame. $\Delta r$ values correspond to different polar angle acceptance of detector left and right arms. Dashed line indicates $10^{-3}$ uncertainty in count.
It is interesting to note that other EW observables critically depend on the knowledge of the beam-spread at $Z^0$ pole, like the cross-section for $Z^0$ production, $Z^0$ total width and mass. We have found the following requirements on the beam spread at the CEPC $Z^0$ pole: 0.5%, 0.2% and 10%, respectively.

As illustrated in Figure 4 [3], non-zero beam-spread will result in accolinearity of final state muons produced in $e^+e^- \rightarrow \mu^+\mu^-$. According to the expected performance of the central tracker at CEPC [1], muon polar angle resolution over the whole tracking volume should be 0.1 mrad, which corresponds to 100 $\mu$m position resolution in TPC. The effective center-of-mass energy $s'$ can be calculated from the reconstructed muons’ polar angles:

$$s' = \frac{\sin\theta^+ + \sin\theta^- - |\sin(\theta^+ + \theta^-)|}{\sin\theta^+ + \sin\theta^- + |\sin(\theta^+ + \theta^-)|}$$

where $s$ stands for the nominal CM energy and $\theta^+$ and $\theta^-$ are polar angles of outgoing $\mu^+$ and $\mu^-$. As already mentioned, the method is originally proposed for FCCee [3].

![Figure 4](image)

**Figure 4.** Process $e^+e^- \rightarrow \mu^+\mu^-$ without beam energy spread (red) and with beam energy spread (green).

In order to determine $s'$ sensitivity to the beam-spread, we generated between 100K and 250K $e^+e^- \rightarrow \mu^+\mu^-$ events at 91.2 GeV and 240 GeV. Events are generated using WHIZARD 2.6.2 [6], in polar angle ranged from $8^\circ$ to $172^\circ$, which is the angular acceptance of the TPC at CEPC [1]. Events are generated without any additional effects, in order to study individual effects of ISR, beamstrahlung and muon angular resolution competing with the beam-spread. Detector energy resolution is simulated by performing Gaussian smearing of the muons’ polar angles in case of a few different central tracker reconstruction capabilities.

Figure 5. illustrates $s'$ distribution in the presence of ISR and the beam-spread. The beam-spread is assumed in accordance with the nominal CEPC beam parameters: 0.134% of the beam energy at 240 GeV and 0.08% of the beam energy at 91.2 GeV. Tracker (muon polar angle) resolution is here assumed to be infinitely accurate. It can be seen that at both energies the beam-spread dominates the $s'$ shape at energies close to the nominal center-of-mass energy. In order to rely on this method, excellent theoretical description of ISR effect is required. In Figure 6, the effect of muon polar angle resolutions of 0.1 mrad and 1 mrad are illustrated on top of ISR and the beam-spread. From Figure 6 is clear that 0.1 mrad tracker resolution of muons polar angles reconstruction does not affect the $s'$ sensitivity to the beam spread. On the other hand, tracker resolution of 1 mrad significantly influences the method. The same stands for 120 GeV beam.
Then it can be asked how far one may go in deterioration of the central tracker performance. The answer is shown at Figure 7, illustrating that central tracker polar angle (positioning) resolution should stay within the range of 0.1 - 0.2 mrad.

Finally, the energy asymmetry of the colliding beams corresponding to the effective beam-spread can be demined from the population of the top-part of the $s'$ distribution. Beam-spread values are varied around the nominal ones at 240 GeV and at the $Z^0$ pole. To reduce statistical uncertainties, 250K events is generated for each beam-spread value at 91.2 GeV and 100K events at 240 GeV. Results are shown at Figure 8. As expected, increase of the beam-spread leads to increase of acollinearity of outgoing muons, and the corresponding reduction of the center-of-mass energy available for a collision. The muon count dependence on the beam-spread can be fitted using a simple linear fit. This fit enables us to calculate the effective beam-spread in the experiment, simply by counting muons, as shown in Table 2. With a statistics of 250K events at

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**Figure 5.** $s'$ sensitivity to the beam spread at 240 GeV (left) and 91.2 GeV CEPC (right).

**Figure 6.** $s'$ sensitivity to the beam spread and tracker resolution at 91.2 GeV.

**Figure 7.** $s'$ sensitivity to the different tracker resolutions in muon polar angle, simulated at 91.2 GeV.

**Figure 8.** $s'$ dependence on the beam-spread at 91.2 GeV.
$Z'$ pole and 100K events at 240 GeV, relative variations of the nominal beam-spread of 2.5 % (15 %) can be measured, respectively.

![Beam spread + ISR + tracker resolution 0.1rad](image1.png)

**Figure 8.** Dependence of the most energetic muons count on the beam-spread, at 91.2 GeV (left) and 240 GeV CEPC

![Beam spread + ISR + tracker resolution 0.1 rad](image2.png)

For such relative precision in determination of the effective beam-spread, only 4 minutes of collecting the most energetic muons are needed at the $Z^0$ pole, with the CEPC nominal luminosity, which corresponds to the relative uncertainty of the integrated luminosity uncertainty of $2.5 \cdot 10^{-5}$. This also corresponds to the uncertainty of the effective center-of-mass energy, $\Delta E_{cm}^{eff} = 2.5 \cdot 10^{-2} \cdot \delta E_{beam} \approx 900 keV$. Figure 9 shows dependence of uncertainties of the $Z^0$ cross-section $\sigma_z$, $Z'$ total width $\Gamma_z$ and mass $m_z$, on uncertainty

![Uncertainties of $\delta(\sigma_z)1.5 \cdot 10^{-4}$](image3.png)

![Precision on Z width (MeV) $\Delta \Gamma_z 300keV$](image4.png)

![Precision on mass (MeV) $\Delta m_z 100keV$](image5.png)

**Figure 9.** Uncertainties of $\sigma_z$ (up left), $\Gamma_z$ (up right) and $m_z$ (down) as a function of the uncertainty of the effective center-of-mass energy determined as 2.5% of the beam-spread (~900 keV) at the $Z'$ pole, after 4 minutes of data taking.
of the effective center-of-mass energy, and for $\Delta E_{cm}^{eff} \approx 900$ keV, the following uncertainties are found: $\delta(\sigma_z) \sim 1.5 \cdot 10^{-4}$, $\Delta\Gamma_z \sim 300$ keV and $\Delta m_z \sim 100$ keV.

To control the relative uncertainty of the integrated luminosity at the level of $10^{-4}$, only two minutes of collecting muons are needed. Such a strict control of the beam-spread variation is not possible at 240 GeV CEPC. However, it is neither needed since for the luminosity uncertainty of $10^{-3}$, asymmetry in energy of the colliding beams should be known within 150% of the nominal beam-spread. The last row in Table 2 is given for comparison between CEPC and FCCee and shows the time needed to determine beam-spread variation at 91.2 GeV FCCee [6]. It reflects the combination of two compensating facts: instantaneous luminosity at FCCee is approximately an order of magnitude larger than at CEPC, while at CEPC $Z^0$ pole the beam-spread is almost two times smaller than at FCCee.

| CEPC       | Luminosity @ IP (cm$^{-2}$ s$^{-1}$) | Nominal beam-spread (%) | Number of events | Cross-section $e^+e^- \rightarrow \mu^+\mu^-$ | Collecting time | Beam-spread variation $\delta E_b$(900 keV) |
|------------|----------------------------------|-------------------------|------------------|-------------------------------------------|-----------------|-----------------------------------------|
| $Z^0$ pole | $3.2 \cdot 10^{35}$              | 0.080                   | 250 KEvt.        | 1.5 nb                                    | ~ 4 min         | ~2.5 $\cdot 10^{-2} \delta E_b$         |
| Higgs factory | $3.0 \cdot 10^{34}$              | 0.134                   | 100 KEvt.        | 4.1 pb                                    | ~ 10 days       | ~0.15 $\delta E_b$ (~24 MeV)            |
| FCCee $Z^0$ pole | $2.3 \cdot 10^{36}$              | 0.132                   | 540 KEvt.        | 1.5 nb                                    | ~ 3 min         | ~2 $\cdot 10^{-3} \delta E_b$ (~120 keV) |

Table 2. Beam-spread variations experimentally accessible at CEPC and FCCee.

Method requires further refinements to be applied: effect of ISR (theoretical) uncertainty, full detector simulation and impact of similar final states backgrounds and presence of beamstrahlung should be included in the future. Also, different choices of the fit function describing beam-spread dependence of the high-energy muons count, lead to the systematic uncertainty of the method, as well as the fact that the beam energy spread is not ideally Gaussian.

### 6. Physics processes as a background to the Bhabha count

In $e^+e^-$ collisions there are several 4-fermion processes (multiperipheral, annihilation, brehmstrahlung and conversion) representing possible background for the Bhabha scattering.

![Figure 10. Multiperipheral process for Bhabha scattering in $e^+e^-$ colliders](image)

However, the multiperipheral (Landau-Lifshitz) processes, shown at Figure 10, have the biggest impact, due to its large cross-section (~nb), saturating on higher center-of-mass energies, and due to the fact that spectator electrons are emitted at very small polar angles. Even though the most of high-energy electron spectators from these processes go below the luminometer’s angular acceptance region, some of them can still be misidentified as Bhabha electrons. In order to correct for this miscount, the uncertainty of the
theoretical cross-section for $e^+e^- \rightarrow e^+e^-\bar{f}f$ has to be known at CEPC energies with the same precision as the integrated luminosity.

In order to estimate the B/S ratio in luminosity measurement at 240 GeV CEPC, we simulated $10^5 e^+e^- \rightarrow e^+e^+\mu^+\mu^-$ background events using WHIZARD V2.6 [6] in the polar angle range $|\cos\theta| < 0.999$. For these events, the effective cross-section, $\sigma_{eff} = 0.3 pb$ in the fiducial volume of the luminometer and $\delta(\sigma) = 1\%$ is found. Signal (Bhabha events) is simulated using BHLUMI V4.04 [4]. We simulated $10^7$ Bhabha events in the polar angle range $\theta > 3 mrad$, with $\sigma_{eff} = 3.3 nb$ in the fiducial volume of the luminometer and relative uncertainty of the cross-section $\delta(\sigma) = 1.7 \cdot 10^{-4}$. The luminometer is placed 95 cm from the interaction point. All results are normalized to 5.6 ab$^{-1}$, which corresponds to 7 years of data taking at 240 GeV CEPC. Results are shown in Figures 11 and 12.

![Figure 11. Normalized polar angle distribution of signal (black line) and background (red line) events.](image1.png)

![Figure 12. Normalized energy distribution of signal (black line) and background (red line) events in the fiducial volume (53 mrad – 79 mrad) of the luminometer. Left: energy of individual particles; Right: energy of the $e^+e^-$ pair normalized to the beam center-of-mass energy](image2.png)

We can see that the most of the spectators go below the luminometer. Initial contamination of the detector volume by $e^+e^- \rightarrow e^+e^+\mu^+\mu^-$ background (without any selection) is of order of $10^{-4}$ with respect to the signal. Background to signal ratio is approximately 10 times smaller than at 500 GeV ILC [3]. This is mostly due to the Bhabha cross-section dependence as $1/s$, while 2-photon cross-section is scaling like $ln^2(s)$.

In this study we’ve looked into spectator electrons from $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ process. The total amount of background should be conservatively scaled by a factor $\leq 3$ with flavor integration amounting to $B/S \leq 3 \cdot 10^{-4}$ without any selection. Energy cut on relative energy $E_{rel} = |E_{e^+} + E_{e^-}| > 0.8\sqrt{s}$ rejects ~30% of background, but is also important in a treatment of radiative Bhabha events and off-momentum background. Further refinements are possible with the coplanarity request between left and right detector arms ($|\varphi_{e^+} + \varphi_{e^-}|$), also useful to suppress off-momentum particles. Finally, physics background can be taken as a
correction to the count in which case luminosity uncertainty comes from the x-section uncertainty of 2-photon processes.

7. Discussion and summary

It is clear that the uncertainty of the luminometer inner radius at the micron level together with the uncertainty of the available center-of-mass energy are posing the most challenging requirements on integrated luminosity systematics control. Permille precision of the integrated luminosity seems to be feasible from the point of view of the existing technologies with respect to the other considered sources of systematic uncertainties. Luminosity uncertainty of $10^{-4}$ at the $Z^0$ pole seems to be more demanding, in particular, having in mind the requirement on the average center-of-mass uncertainty at the level of a few MeV.

The method of experimental determination of the effective center-of-mass energy based on muon reconstruction from $e^+e^- \rightarrow \mu^+\mu^-$ nicely works at CEPC $Z^0$ pole, due to the high cross-section for di-muon production and high instantaneous luminosity. At $Z^0$ pole CEPC, 2.5% relative accuracy of the beam spread is feasible (i.e. < 1 MeV) after 4 minutes of data collection, while only 2 minutes of running are required to meet the relative precision of integrated luminosity uncertainty of $10^{-4}$. At 240 GeV, beam-energy asymmetry within the existing beam-spread is satisfactory for $10^{-3}$ precision goal on integrated luminosity.

First estimates of the contribution of physics background to the luminosity systematics at 240 GeV CEPC has been done. Physics background is estimated to be present in the luminometer fiducial volume at the level of $\leq 3 \cdot 10^{-4}$ with respect to the signal. Other refinements are possible in terms of detector simulation, simulation of off-momentum background, application of the asymmetric acceptance in $\theta$ (needed to suppress other sources of L-R symmetric systematics) and introduction of the coplanarity requirement it the selection. All of these refinements should additionally improve background to signal ratio. The ultimate contribution to the luminosity uncertainty, if miscount from physics background is taken as a correction, will come from the uncertainty of the cross-section of 4-fermion processes. For that, some theoretical effort is needed.

Another very important source of systematic uncertainty in integrated luminosity measurement, in particular at lower CM energies (i.e. at the $Z^0$ pole) comes from electromagnetic-deflection (EMD) of the final states interacting with the bunch with opposite charge [7]. This effect is of greater importance at circular electron-positron machines, in comparison to the also present effect of beamstrahlung which at linear colliders requires particular treatment to correct for Bhabha counting loss as discussed in [8, 9]. There is ongoing effort of the authors to address EMD at $Z^0$ pole CEPC.

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