Reevaluation of the Density Dependence of Nucleon Radius and Mass in the Global Color Symmetry Model of QCD*

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October 29, 2018

Abstract

With the global color symmetry model (GCM) at finite chemical potential, the density dependence of the bag constant, the total energy and the radius of a nucleon in nuclear matter is investigated. A relation between the nuclear matter density and the chemical potential with the action of QCD being taken into account is obtained. A maximal nuclear matter density for the existence of the bag with three quarks confined within is given. The calculated results indicate that, before the maximal density is reached, the bag constant and the total energy of a nucleon decrease, and the radius of a nucleon increases slowly, with the increasing of the nuclear

*Work supported by the National Natural Science Foundation of China, the Major State Basic Research Development Program and the Foundation for University Key Teacher by the Ministry of Education
matter density. As the maximal nuclear matter density is reached, the mass of the nucleon vanishes and the radius becomes infinite suddenly. It manifests that a phase transition from nucleons to quarks takes place.

**PACS Numbers:** 24.85.+p, 11.10.Wx, 12.39.Ba, 14.20.Dh

**Keywords:** Effective field theory of QCD, Bag model, Nucleon, Density dependence
1 Introduction

It is well known that nucleons bound in nuclear medium alter their properties from those in free space. For instance, an effective mass ($< M_0$) should be introduced to the nucleon to describe the property of nuclear matter[1], and a swell hypothesis should be made to describe the EMC effect well (see for example Ref. [2]). Although how much those properties change is of fundamental interest in nuclear physics, it is still not clear now. Then, in the relativistic mean field theory (RMF) based on the $\sigma - \omega$ model, the effective nucleon mass in nuclear medium is determined with iteration on the fields of nucleon and mesons[1]. However, this model is valid only at the hadronic level, at which a nucleon is described as a point-like particle. Because of the importance of the substructure of nucleon observed in deep inelastic scattering experiments and predicted by quantum chromodynamics (QCD), it is imperative to employ the quark and gluon degrees of freedom to describe nuclear phenomena. Since it is now very difficult to make quantitative predictions with QCD at low and intermediate energy region, various phenomenological models based on the QCD assumptions, such as the bag models[3, 4], quark-meson coupling (QMC) model[5], and so on, have been developed. In bag models and the QMC model, the size and mass of a nucleon are represented by the radius and energy of the bag with quarks being confined, respectively. In order to take the modification of the volume energy on the mass of a nucleon into account, the bag constant is exploited in the models. To take into account the nuclear matter medium effect in the QMC model, a phenomenological density dependence of the bag constant in the medium has been introduced[6, 7, 8, 9, 10]. In the global color symmetry model (GCM) [11, 12, 13, 14] of QCD, even though the bag constant, the radius and the mass of a nucleon in nuclear matter have been evaluated[15] in a consistent way, a Fermi gas approximation was taken for the relation between the nuclear matter density and the chemical potential. The interaction among the nucleons which is believed to be the residual of the quark-quark interactions has not yet been taken into account. Then it is still necessary to investigate the nuclear matter density dependence of the radius, the mass and the bag constant of a nucleon sophisticatedly.

It has been shown that the global color symmetry model (GCM)[11, 12, 13, 14] is a quite successful effective field theory of QCD in describing hadron properties in free space(i.e., at temperature $T = 0$, chemical potential $\mu = 0$). With the global color symmetry model at finite chemical potential $\mu$, we will determine the relation between
the nuclear matter density and the chemical potential in the GCM and reevaluate
the nuclear matter density dependence of the bag constant, the total energy and the radius
of a nucleon in this paper.

The paper is organized as follows. In Section 2 we describe the formalism of the GCM
model at finite chemical potential $\mu$ and the relation between the chemical potential and
the baryon density in nuclear matter. In Section 3 we represent the calculation and the
obtained results of the bag constant, the bag radius and the bag energy as functions of
the nuclear matter density. In Section 4, a brief summary and some remarks are given.

2 Formalism

The starting point of the global color symmetry model (GCM) is the action in Euclidean
matric\[11\]

$$S = \int d^4x d^4y \left[ \bar{q}(x) (\gamma \cdot \partial + m) \delta(x - y) q(y) - \frac{g^2}{2} j^a_\mu(x) D_{\mu\nu}(x - y) j^a_\nu(y) \right],$$

where $j^a_\mu = \bar{q}(x) \frac{\Lambda^a}{2} \gamma_\mu q(x)$ is the quark current, $D_{\mu\nu}(x - y)$ is an effective two-point gluon
propagator, $m$ is the current quark mass, $g$ is the quark-gluon coupling constant. Taking
the effective gluon propagator to be diagonal, i.e., $D_{\mu\nu}(x - y) = \delta_{\mu\nu} D(x - y)$ and applying
the transformation of Fierz reordering to the quark fields, one can rewrite the current-
current term as

$$\frac{g^2}{2} \int d^4x d^4y j^a_\mu(x) D(x - y) j^a_\nu(y) = -\frac{g^2}{2} \int d^4x d^4y j^\theta(x, y) D(x - y) j^\theta(y, x),$$

where $j^\theta(x, y) = \bar{q}(x) \Lambda^\theta q(y)$ with $\Lambda^\theta$ being the direct products of Lorentz, flavor and
color matrices which produce the scalar, vector, pseudoscalar and axial vector terms. It
is obvious that such a $j^\theta(x, y)$ is a bilical current. With two flavors $u$ and $d$ of quarks
being taken into account, each $\Lambda$ is either isoscalar or isovector. The color matrices in-
volved in the Fierz transformation contains color-singlet and color-octet terms. Taking
the bosonization procedure one can transfer the bilocal quark current structure into aux-
iliary Bose-fields carrying the quantum number $\theta$. The action of the GCM in free space
(i.e., at the chemical potential $\mu = 0$) for the zero-mass quark can then be rewritten\[11\]
in the Euclidean space as

$$S(B) = \int d^4x d^4y \bar{q}(x) \left[ \gamma \cdot \partial \delta(x - y) + \Lambda^\theta B^\theta(x, y) \right] q(y) + \int d^4x d^4y \frac{B^\theta(x, y) B^\theta(y, x)}{2g^2 D(x - y)},$$
where $B^\theta(x, y)$ is the bilocal Bose-field. As a consequence, the generating functional is given as

$$Z[\bar{\eta}, \eta] = N \int D\bar{q} Dq DB^\theta e^{[-S(B^\theta) + \int d^4x (\bar{\eta}q + \bar{q}\eta)]},$$

where $\bar{\eta}$ and $\eta$ are the quark sources. To extend the GCM to finite nuclear matter density (with finite chemical potential $\mu$), one should, in view of the statistical mechanics, take the partition function of the canonical (quark) ensemble into that of the grand ensemble with quarks and hadrons that are the solitons collecting quarks\[12\]. The quark field should be transformed under a constraint on the baryon number through the chemical potential $\mu$

$$q(x) \rightarrow q'(x) = e^{\mu x^4} q(x). \quad (1)$$

After some derivation, we have the action of the GCM in nuclear matter

$$S(B^\theta, \mu) = \int d^4xd^4y \bar{q}(x) \left[ (\gamma \cdot \partial - \mu \gamma_4) \delta(x-y) + e^{\mu x^4} \Lambda^\theta B^\theta(x, y) e^{-\mu y^4} \right] q'(y)$$

$$+ \int d^4xd^4y \frac{B^\theta(x, y) B^\theta(y, x)}{2g^2 D(x-y)},$$

and the generating functional is given as

$$Z[\mu, \bar{\eta}, \eta] = \int D\bar{q} Dq DB^\theta e^{[-S(B^\theta, \mu) + \int d^4x (\bar{\eta}q + \bar{q}\eta)]}. \quad (2)$$

After integrating the quark fields, we obtain

$$S(B^\theta, \mu) = - \text{Tr} \ln \left[ (\gamma \cdot \partial - \mu \gamma_4) \delta(x-y) + e^{\mu x^4} \Lambda^\theta B^\theta(x, y) e^{-\mu y^4} \right] + \int d^4xd^4y \frac{B^\theta(x, y) B^\theta(y, x)}{2g^2 D(x-y)}. \quad (3)$$

Generally, the bilocal field $B^\theta(x, y)$ can be written as\[13\]

$$B^\theta(x, y) = B_0^\theta(x, y) + \sum_i \Gamma_i^\theta(x, y) \phi_i^\theta \left( \frac{x + y}{2} \right), \quad (4)$$

where $B_0^\theta(x, y) = B_0^\theta(x-y) = B^\theta(x-y)$ is the vacuum configuration of the bilocal field. $\Sigma_i \Gamma_i^\theta(x, y) \phi_i^\theta \left( \frac{x + y}{2} \right)$ correspond to the fields which can be interpreted as effective meson fields. In the lowest order approximation with only the Goldstone bosons being taken into account, the $\phi_i^\theta$ includes $\pi$ and $\sigma$ mesons. The corresponding width of the fluctuations are approximately the same as the vacuum configuration\[13\], i.e., $\Gamma_i^\theta = B_0^\theta$. Since the bilocal field arises from the bilocal current of quarks, the internal $\bar{q}q$ structure of the mesons can be well described in the GCM. The vacuum configuration can be determined by the
saddle-point condition $\frac{\partial S}{\partial B_0} = 0$. Then an equation of the translational invariant quark self-energy $\Sigma(q, \mu)$ is obtained as a truncated Dyson-Schwinger equation

$$\Sigma(p, \mu) = \int \frac{d^4q}{(2\pi)^4} g^2 D(p-q) \frac{t^a}{2} \gamma_\nu \frac{1}{i\gamma \cdot q - \mu \gamma_4 + \Sigma(q, \mu)} \gamma_\nu \frac{t^a}{2},$$

With $\tilde{q}_\mu = (\tilde{q}, q_4 + i\mu)$ being introduced, the above equation can be rewritten as

$$\Sigma(\tilde{p}) = \int \frac{d^4q}{(2\pi)^4} g^2 D(p-q) \frac{t^a}{2} \gamma_\nu \frac{1}{i\gamma \cdot \tilde{q} + \Sigma(\tilde{q})} \gamma_\nu \frac{t^a}{2}. \quad (5)$$

Considering the fact that the inclusion of the chemical potential breaks the O(4) symmetry in the four-dimensional space, one should rewrite the decomposition of the self-energy $\Sigma$ as

$$\Sigma(p, \mu) = i[A(p, \mu) - 1] \tilde{\gamma} \cdot \tilde{p} + i[C(p, \mu) - 1] \gamma_4 (p_4 + i\mu) + B(p, \mu), \quad (6)$$

where $B(p, \mu)$ is the counterpart of the vacuum configuration in the momentum space, i.e.,

$$B(p, \mu) = \int \frac{1}{(2\pi)^4} B_0^\theta(z) e^{i\tilde{p}z} dz.$$ 

It requires then that the vacuum configuration of $\sigma$ and $\pi$ should satisfy the restriction $\sigma^2 + \pi^2 = 1$. Combining Eqs. (5) and (6), one can obtain the equations to determine the $A(\tilde{p})$, $B(\tilde{p})$ and $C(\tilde{p})$ as follows

$$[A(\tilde{p}) - 1] \tilde{p}^2 = \frac{8}{3} \int \frac{d^4q}{(2\pi)^4} g^2 D(p-q) \frac{A(\tilde{q}) \tilde{q} \cdot \tilde{p}}{A^2(\tilde{q}) \tilde{q}^2 + C^2(\tilde{q}) \tilde{q}_4^2 + B^2(\tilde{q})}, \quad (7a)$$

$$[C(\tilde{p}) - 1] \tilde{p}_4^2 = \frac{8}{3} \int \frac{d^4q}{(2\pi)^4} g^2 D(p-q) \frac{C(\tilde{q}) \tilde{q}_4 \tilde{p}_4}{A^2(\tilde{q}) \tilde{q}^2 + C^2(\tilde{q}) \tilde{q}_4^2 + B^2(\tilde{q})}, \quad (7b)$$

$$B(\tilde{p}) = \frac{16}{3} \int \frac{d^4q}{(2\pi)^4} g^2 D(p-q) \frac{B(\tilde{q})}{A^2(\tilde{q}) \tilde{q}^2 + C^2(\tilde{q}) \tilde{q}_4^2 + B^2(\tilde{q})}. \quad (7c)$$

Basing on the solution of the Dyson-Schwinger equations, one can determine the bilocal field, and fix further the GCM action.

With a nontopological-soliton ansatz, the action of the bag models at finite chemical potential $\mu$ in terms of the GCM can be given by extending the formalism proposed in Refs. [11] and [12] as

$$S_B = \bar{q}_j \{ \gamma \cdot \partial - \gamma_4 \mu - \alpha[\sigma(x) - i\tilde{\pi}(x) \cdot \tilde{\gamma}_5] \} q_j + \hat{S}(\sigma, \pi, \mu), \quad (j = 1, 2, 3), \quad (8)$$

where $\hat{S}(\sigma, \pi, \mu)$ includes $\sigma$ and $\pi$ mesons and reads

$$\hat{S}(\sigma, \pi, \mu) = \int \left[ \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{f_\sigma^2}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} m_\pi^2 \pi^2 + \frac{f_\pi^2}{2} (\partial_\mu \pi)^2 - V(\sigma, \pi) \right] d^4z + \cdots \quad (9)$$
with
\[
V(\sigma, \pi) = -12 \int \frac{d^4 p}{(2\pi)^4} \left\{ \ln \left[ \frac{A^2(\bar{p})\bar{p}^2 + C^2(\bar{p})\bar{p}_4^2 + (\sigma^2 + \bar{\pi}^2)B^2(\bar{p})}{A^2(\bar{p})\bar{p}^2 + C^2(\bar{p})\bar{p}_4^2 + B^2(\bar{p})} \right] \right. \\
\left. - \frac{(\sigma^2 + \bar{\pi}^2 - 1)B^2(\bar{p})}{A^2(\bar{p})\bar{p}^2 + C^2(\bar{p})\bar{p}_4^2 + B^2(\bar{p})} \right\},
\]
and the quark meson coupling constant \( \alpha \) is given as
\[
\alpha(x) = \int \frac{d^4 p}{(2\pi)^4} B(\bar{p}) e^{-i\bar{p} \cdot x}.
\]
It is evident that such a quark meson coupling constant is just the vacuum configuration of the bilocal field and is independent of the meson fields.

Analyzing the stationary property of the bag and differentiating Eq. (8), one has the equations of motion for the quarks and mesons
\[
\{ \gamma \cdot \partial - \gamma_4 \mu - \alpha[\sigma(x) - i\bar{\pi}(x) \cdot \tilde{\tau} \gamma_5] \} q_j = 0 ,
\]
\[
\frac{\partial S_B}{\partial \sigma(x)} = 0 ,
\]
\[
\frac{\partial S_B}{\partial \pi(x)} = 0 .
\]
The quark field and \( \sigma, \pi \) meson fields in symmetric nuclear matter can be determined by solving the Eqs. (10-12) self-consistently. As a consequence, the corresponding energies can be obtained. It is apparent that the meson fields corresponding to the vacuum configuration can be simply taken as \( \sigma = 1, \pi = 0 \) due to the restriction \( \pi^2 + \sigma^2 = 1 \). In light of the nontopological-soliton ansatz\[4, 11\], one can approximately take the meson fields inside a bag (i.e., in a nucleon) as \( \sigma = 0 \) and \( \pi = 0 \). For the quarks in a bag, Eq. (10) can thus be rewritten approximately as
\[
[\gamma \cdot \partial - \gamma_4 \mu] q_j(x) = 0 .
\]
The lowest total energy of a single quark with respect to the radius \( R \) of the bag is given as
\[
\epsilon_j(R) = \frac{\omega_0 - \mu}{R} ,
\]
where \( \omega_0 = 2.04 \). For a free nucleon (i.e., with \( \mu = 0 \)), this result is consistent with those obtained in Refs\[3, 6, 7\]. And the bag constant \( B \) is obtained as

\[
B = V(0,0) - V(1,0) = 12\int \frac{d^4p}{(2\pi)^4} \left\{ \ln \left[ \frac{A^2(\tilde{p})\tilde{p}^2 + C^2(\tilde{p})\tilde{p}_4^2 + B^2(\tilde{p})}{A^2(\tilde{p})\tilde{p}^2 + C^2(\tilde{p})\tilde{p}_4^2} \right] - \frac{B^2(\tilde{p})}{A^2(\tilde{p})\tilde{p}^2 + C^2(\tilde{p})\tilde{p}_4^2 + B^2(\tilde{p})} \right\}. \tag{15}
\]

With the correction from the motion of center-of-mass, the zero-point effect and the color-electronic and color-magnetic interactions being taken into account, the total energy of a bag is given as

\[
E = 3\epsilon_j(R) + \frac{4}{3}\pi R^3B - \frac{Z_0}{R} = \frac{3(\omega_0 - \mu) - Z_0}{R} + \frac{4}{3}\pi R^3B, \tag{16}
\]

where \( \frac{Z_0}{R} \) denotes the corrections of the motion of center-of-mass, zero-point energy and other effects.

Just as the same as that in Ref.[11], the bag is identified as a nucleon in the present work. It satisfies the equilibrium condition

\[
\frac{dE(R)}{dR} = 0.
\]

From this condition, we get

\[
R = \left( \frac{a}{4\pi B} \right)^{1/4}, \tag{17}
\]

where \( a = 3(\omega_0 - \mu) - Z_0 \). As a consequence, Eq. (16) can be rewritten as

\[
E = \frac{4a}{3} \left( \frac{4\pi B}{a} \right)^{1/4}. \tag{18}
\]

It is apparent that, with the solutions of Dyson-Schwinger equations (Eqs. (7a), (7b) and (7c)) being taken as the input for Eqs. (15), (17) and (18), the properties of nucleons (i.e., bags) in nuclear matter can be obtained.

In the practical calculation, since the knowledge about the exact behavior of \( g^2 \) and \( D(p - q) \) in low energy region is still lacking, one has to take some approximations or phenomenological form to solve the Dyson-Schwinger equations. For simplicity, we adopt the infrared dominative form\[16, 11\]

\[
g^2D(p - q) = \frac{3}{16} \eta^2 \delta(p - q), \tag{19}
\]
where \( \eta \) is an energy-scale parameter and can be fixed by experimental data of hadrons. Although this form does not include the contribution from the ultraviolet energy region, it maintains the main property of QCD in the low energy region. With Eqs. (7a), (7b), (7c) and (19), one has

\[
A(\tilde{p}) = C(\tilde{p}) = 2, \quad B(\tilde{p}) = (\eta^2 - 4\tilde{p}^2)^{1/2}, \quad \text{for } Re(\tilde{p}^2) < \frac{\eta^2}{4}, \quad (20a)
\]

\[
A(\tilde{p}) = C(\tilde{p}) = \frac{1}{2} \left[ 1 + \left( 1 + \frac{2\eta^2}{\tilde{p}^2} \right)^{1/2} \right], \quad B(\tilde{p}) = 0, \quad \text{for } Re(\tilde{p}^2) > \frac{\eta^2}{4}, \quad (20b)
\]

which describes the phase where chiral symmetry is spontaneously broken and the dressed-quarks are confined\[12, 17, 18, 19, 20\]. Meanwhile one has also the Wigner solution

\[
A(\tilde{p}) = C(\tilde{p}) = \frac{1}{2} \left[ 1 + \left( 1 + \frac{2\eta^2}{\tilde{p}^2} \right)^{1/2} \right], \quad B(\tilde{p}) = 0, \quad (21)
\]

which characterizes a phase in which chiral symmetry is not broken and the dressed-quarks are not confined.

In order to investigate the dependence of nucleon properties on the nuclear matter density \( \rho \) explicitly, we must transfer the above obtained \( \mu \)-dependence to that of the \( \rho \)-dependence. It has been known that the baryon number in nuclear matter can be related with the generating functional \( Z(B^\theta, \mu) \) of the system\[12, 21, 22\] as

\[
n = \frac{\partial}{\partial \mu} \ln Z(B^\theta, \mu). \quad (22)
\]

Combining Eqs. (2), (3) and (8), and accomplishing a Legendre transformation along the way described in Ref.\[12\], we have

\[
n \approx -\frac{\partial}{\partial \mu} S(B^\theta, \mu) = \frac{\partial}{\partial \mu} Tr \ln G^{-1}(B^\theta, \mu), \quad (23)
\]

where \( G^{-1}(B^\theta, \mu) \) is the inverse of the quark propagator in the medium, i.e.,

\[
G^{-1}(B^\theta, \mu) = (\not{\phi} - \gamma_4 \mu) \delta(x - y) + e^{\mu x_4} \Lambda^\theta B^\theta(x - y)e^{-\mu y_4}. \quad (24)
\]

Extending \( \ln G^{-1}(B^\theta(x - y), \mu) \) in a Taylor series, we have

\[
\ln G^{-1}(B^\theta(x - y), \mu) = \ln[(\not{\phi} - \gamma_4 \mu) \delta(x - y) + e^{\mu x_4} \Lambda^\theta B^\theta(x - y)e^{-\mu y_4}]
\]

\[
= \ln[(\not{\phi} - \gamma_4 \mu) \delta(x - y)] + \left\{ \frac{1}{\not{\phi} - \gamma_4 \mu} \Lambda^\theta B^\theta(x - y) e^{\mu(x_4 - y_4)}
\]

\[
- \frac{1}{2} \left[ \frac{1}{\not{\phi} - \gamma_4 \mu} \Lambda^\theta B^\theta(x - y) e^{\mu(x_4 - y_4)} \right]^2 + \ldots \right\}.
\]
We get then
\[ n \approx \frac{1}{2} \frac{\partial}{\partial \mu} \text{tr} \int d^4 y d^4 x \frac{1}{\phi - \gamma_4 \mu} \Lambda^\theta B^\theta(x - y) e^{\mu(x_4 - y_4)} \frac{1}{\phi - \gamma_4 \mu} \Lambda^\theta B^\theta(y - x) e^{\mu(y_4 - x_4)}. \]

Considering the Fourier transformation
\[ \frac{1}{\phi - \gamma_4 \mu} \Lambda^\theta B^\theta(x - y) e^{\mu(x_4 - y_4)} = \int \frac{d^4 q}{(2\pi)^4} \frac{\Lambda^\theta}{i \vec{q}} B^\theta(\vec{q}, q_4 + i\mu) e^{i\vec{q}(x - y)}, \]
we obtain finally
\[ n \approx 2 \frac{\partial}{\partial \mu} \int \frac{d^4 x}{(2\pi)^4} \int d^4 p \left[ B^\theta(\vec{p}, p_4 + i\mu) \right]^2 \frac{p^2}{p^2}. \]

The baryon number density can thus be given as
\[ \rho_n = \frac{n}{\int d^4 x} \approx \frac{2}{(2\pi)^4} \frac{\partial}{\partial \mu} \int d^4 p \left[ B^\theta(\vec{p}, p_4 + i\mu) \right]^2 \frac{p^2}{p^2}. \tag{25} \]

Combining Eqs. (15), (17), (18), (20) and (25), we can obtain the dependence of the bag constant \( B \), the total energy \( E \) and the radius \( R \) of a nucleon (i.e., those of a bag) on the nuclear matter density \( \rho_n \).

## 3 Calculation and Results

By calibrating the nucleon mass \( M_0 = 939 \text{ MeV} \) as the total energy of a bag and radius \( R_0 = 0.8 \text{ fm} \) in free space (i.e., \( \mu = 0, \rho = 0 \)), we get the energy-scale \( \eta = 1.220 \text{ GeV} \), \( Z_0 = 3.303 \). Such a best fitted energy-scale \( \eta \) fits well the value 1.37 GeV, which was fixed by a good description of \( \pi \) and \( \rho \) meson masses\[18, 19\], and is much more close to the Bjorken-scale 1.0 GeV (see Ref.\[23\] and the references therein). The obtained \( Z_0 \) is larger than the originally fitted value 1.84\[24\]. However what we refer to here includes all the effects but not only the zero-point energy. Meanwhile other investigations (see for example Ref.\[25\]) have shown that the zero-point energy parameter can be larger than 1.84, even though the other effects are taken into account separately. With the above parameters \( \eta, \ Z_0 \) and Eq.(15), we get at first the bag constant in free space as \( B_0 = (172 \text{ MeV})^4 \). It is evident that the presently obtained value \( B_0 \) is quite close to the result given in Ref.\[7\].

By varying the chemical potential \( \mu \), we obtain the relation between the nuclear matter density and the chemical potential from Eqs. (20) and (25)
\[ \rho_n = \frac{1}{\pi^2} \left[ \mu^3 + \frac{\eta^2}{4\mu} \right]. \]
It is obvious that the nuclear matter density changes monotonously with respect to the chemical potential $\mu$. Such a result shows also that, besides the global coefficient, the chemical potential dependence of the baryon density in the GCM differs from that in the Fermi gas approximation ($\rho_n = \frac{2}{3\pi^2} \mu^3$) in a linear term which arises from the self-interaction of quarks (the constituents of nucleons). In the case of small chemical potential (or low density), this difference can be quite large since the $\frac{\eta^2}{4}$ may be much larger than $\mu^2$. Furthermore, we get the variation behavior of the ratios of bag constant, the nucleon radius and the total energy of the bag in nuclear matter to the corresponding value in free space against the nuclear matter density. The results are illustrated in Figs. 1-3, respectively. To show the influence of the quark self-interaction, we show also the results without the interaction being taken into account (i.e., in the Fermi gas approximation) in the figures.

From the figures, one may easily realize that, as the density of nuclear matter increases, the bag constant and the total energy of the bag (i.e., the mass of a nucleon) decrease monotonously. Meanwhile, the radius of a nucleon increases. When the nuclear matter density reaches the value about 12 times the normal nuclear matter density $\rho_0$ (corresponding to a chemical potential $\mu \approx 0.316$ GeV), the bag constant and the total energy of a bag vanish simultaneously, and the radius of a nucleon becomes infinite. Such behaviors indicate that nucleons can no longer exist as bags consisting of quarks, i.e., a phase transition from hadrons to quarks happens. It manifests that a critical nuclear matter density $\rho_c \approx 12\rho_0$ exists for the quark deconfinement to take place. Since the density $12\rho_0$ is approximately close to the maximal density in the center area of neutron stars and larger than the average (which is commonly believed to be about $10\rho_0$), the presently obtained results show that the nuclear matter would change to quark matter as the density of nuclear matter gets beyond the maximal average density of neutron stars. It provides then a clue that there may be hybrid matter with both hadrons and quarks, even pure quark matter in the center part of neutron stars. Such a feature of neutron star is quite similar to the previous result (see for example Ref. [26]). Furthermore, pure bare quark stars can also exist if the matter density is very large (at least $>12\rho_0$). It is also worth mentioning that the increase of the nucleon radius induces naturally a swell of nucleons. The nucleon swell hypothesis to the EMC effect (see for example Ref. [2]) has then a quite solid QCD foundation.

Comparing the results with and without the self-interaction among quarks, one can
easily recognize that the deconfinement phase transition from hadrons to quark-gluons can happen in the nuclear matter with a density only little larger than the normal density if the self-interaction has not been taken into account (i.e., in the Fermi gas approximation). It indicates that the self-interaction plays vital role for hadrons to exist as bags collecting quarks in nuclear matter with rather high density.

Looking over the figures 1-3 more cautiously, one may recognize that, as the nuclear matter density is much smaller than the critical density \( \rho_c \), the radius of the bag is almost independent of the variation of the nuclear matter density, and the total energy does not change drastically with respect to the density either. As the nuclear matter density is close to the critical value, the total energy and the radius change against the density abruptly. Furthermore the total energy becomes zero and the radius gets to be infinite suddenly as the density reaches the critical value. It shows that the phase transition from hadron matter to quark matter is the first order phase transition.

It is also worthy mentioning that the presently obtained changing feature of the radius, the mass and the bag constant of a nucleon with respect to the nuclear matter density is qualitatively consistent with that given in the QMC model\[6, 7, 8, 9\]. Such a behavior indicates that, dealing the bag constant and the bag radius with a phenomenological dependence on the medium density in the QMC model is reasonable.

4 Summary and Remarks

In summary we have investigated the density dependence of the bag constant of nucleons, the nucleon radius and the total energy of the bag in nuclear matter in the global color symmetry model, an effective field theory model of QCD. A relation between the nuclear matter density and chemical potential is fixed with the GCM action. A maximal density of nuclear matter, which is about 12 times the normal nuclear matter density, for the nucleon to exist as bags consisting of quarks is obtained. The calculated results indicate that the bag constant and the total energy of the bag decreases with the increasing of the nuclear matter density before the maximal density is reached. Meanwhile the size of nucleons swells. As the maximal density is reached, a phase transition from nucleons to quark-gluons takes place. On the other hand, the calculated results show that the self-interaction among quarks plays dominant role for baryons to exist as bags consisting of quarks in the nuclear matter at rather high density. Furthermore, the presently obtained
changing features agree quite well with those obtained in QMC. In this sense, it provides a clue of the QCD foundation to the QMC with a simple effective field theory model of QCD.

In the present calculation, the $g^2D(p-q)$ is taken to be proportional to a $\delta-$function. However, the detailed effects of the running coupling constant, the gluon propagator $D(p-q)$ and the other degrees of freedom on the changing feature have not yet been included. Especially, since the meson fields were taken to be $\sigma = 0$ and $\pi = 0$ in the bag with respect to those of the vacuum configuration $\sigma = 1$, $\pi = 0$, the self-consistent interaction and adjustment between the quarks and the meson fields have not yet been taken into account. Then a more sophisticated investigation is necessary and under progress.

This work is supported by the National Natural Science Foundation of China under contract No. 10075002, 10135030 and the Major State Basic Research Development Program under contract No.G2000077400. One of the author (Liu) thanks also the support of the Foundation for University Key Teacher by the Ministry of Education, China. The authors are also indebted to Professors Fan Wang, Ru-keng Su and Xiao-fu Lü for their helpful discussions.
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Figures and Figure Captions:

Figure 1: Calculated ratio of the bag constant in nuclear matter to that in free space as a function of the nuclear matter density (solid curve). The dashed curve illustrates the ratio in Fermi gas approximation.
Figure 2: Calculated ratio of the radius of nucleon in nuclear matter to that in free space as a function of the nuclear matter density (solid curve). The dashed curve represents the ratio in Fermi gas approximation.
Figure 3: Calculated ratio of the total energy of a bag in nuclear matter to that in free space as a function of the nuclear matter density (solid curve). The dashed curve displays the ratio in Fermi gas approximation.