A note on the spin-$\frac{1}{2}$ XXZ chain concerning its relation to the Bose gas

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Abstract. By considering the one-particle and two-particle scattering data for the spin-$\frac{1}{2}$ Heisenberg chain at $T = 0$ we derive a continuum limit relating the spin chain to the 1D Bose gas. Applying this limit to the quantum transfer matrix approach to the Heisenberg chain we obtain expressions for the correlation functions of the Bose gas at arbitrary temperatures.

Keywords: correlation functions, integrable quantum field theory, integrable spin chains (vertex models), quantum integrability (Bethe ansatz)

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1. Introduction

Bethe’s famous investigation on the spectrum of the Heisenberg chain in 1931 [1] was performed by a method later termed the Bethe ansatz approach and was found, after suitable modifications, to be applicable to other many-particle systems, e.g. to the repulsive Bose gas [2] and the Hubbard model [3].

The algebraic reason for the exact solvability of these models was found in the Yang–Baxter relation [4] and allowed for rather powerful solution techniques on the basis of the quantum inverse scattering method [5]. The eigenstates of the Heisenberg Hamiltonian are obtained from a trivial vacuum by the action of certain operators satisfying the Yang–Baxter algebra, an algebra with quadratic relations and structure coefficients given by the $R$-matrix of the six-vertex model. Within this algebraic Bethe ansatz effective algebraic formulae for scalar products [6] and norms [7] were derived.

As a further development, Kitanine et al [8] were able to express local operators of the XXZ chain in terms of the algebra and to calculate static correlation functions at $T = 0$ by evaluating scalar products. Thus they rederived the multiple-integral representations for correlation functions of Jimbo et al [9,10] and generalized them to finite longitudinal magnetic fields [11,12].

The next logical step was to introduce finite temperatures with a first application to the generating function [13] of the $zz$-correlation. This was achieved by Göhmann et al in [14], where the algebraic structure of [6,12] underlying the treatment of ground state correlation functions was combined with the functional approach [15]–[17] for the quantum transfer matrix (QTM). This object describes an auxiliary spin chain whose leading eigenvalue is nothing but the partition function of the XXZ chain and the eigenvector encodes all thermal correlations.

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In the latest development, with respect to the Heisenberg chain, Sakai [18] succeeded in deriving a multiple-integral representation for the generating function of dynamical correlations for finite temperatures.

An alternative to the QTM approach, and actually a more traditional treatment of finite temperatures, is the so-called thermodynamical Bethe ansatz (TBA) approach introduced by Yang and Yang [19] for the 1D Bose gas. Knowing the spectrum in terms of the Bethe ansatz [2], a combinatorial expression for the entropy and a minimization of a free energy functional lead to the thermodynamics. For the Bose gas this approach yields an integral expression for the thermodynamical potential in terms of an auxiliary function to be determined from a non-linear integral equation. The same approach, however, for the Heisenberg model yields many more integral equations for the bound states of magnons underlying the ‘string hypothesis’.

The goal of this paper is the computation of correlation functions for the integrable Bose gas at finite temperature by use of technical tools developed for the spin-1/2 Heisenberg chain. Unfortunately, the direct application of the QTM approach fails as it is a concept that is most natural for lattice systems. Here we apply these techniques to a lattice system containing the Bose gas as a suitable continuum limit. This is where the anisotropic spin-1/2 Heisenberg chain and the Bose gas meet.

In section 2 we review the Bethe ansatz solution of the spin-1/2 Heisenberg chain with XXZ anisotropy. In section 3 we identify the continuum limit taking the one-particle spectral and the two-particle scattering properties of the Heisenberg chain to be those of the Bose gas. In section 4 we demonstrate how to rederive the famous TBA equations of the Bose gas from thermodynamical equations for the Heisenberg chain in the continuum limit. In section 5, a useful generating function for correlations is identified and results for this generating function in the case of the Heisenberg chain are quoted from the literature. These expressions are then translated in section 6 into (novel) results for the 1D Bose gas by use of the continuum limit. This completes our demonstration on how to obtain correlation functions for continuum models from those of lattice systems.

2. The Heisenberg chain

The Hamiltonian of the anisotropic XXZ chain on $L$ lattice sites exposed to an external longitudinal magnetic field $h$,

$$H_L = J \sum_{j=1}^{L} \left[ \sigma^x_{j-1} \sigma^x_j + \sigma^y_{j-1} \sigma^y_j + \Delta (\sigma^z_{j-1} \sigma^z_j - 1) \right] - \frac{h}{2} \sum_{j=1}^{L} \sigma^z_j,$$

(1)

is closely related to the transfer matrix of a six-vertex model with local Boltzmann weights encoded in the $R$-matrix

$$R(\lambda, \mu) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & b(\lambda, \mu) & c(\lambda, \mu) & 0 \\ 0 & c(\lambda, \mu) & b(\lambda, \mu) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad b(\lambda, \mu) = \frac{\text{sh}(\lambda - \mu)}{\text{sh}(\lambda - \mu + \eta)}, \quad c(\lambda, \mu) = \frac{\text{sh}\eta}{\text{sh}(\lambda - \mu + \eta)},$$

(2)

The coupling strength of the Hamiltonian is denoted by $J$, $\Delta = \text{ch}\eta$ is the anisotropy and $\sigma^\alpha_j$, $\alpha = x, y, z$, denote the usual Pauli matrices. The eigenvalue problem of the transfer matrix of the six-vertex model can be solved for instance in the framework of the algebraic Bethe ansatz; see e.g. [20]. The energies $E_M$ of (1) and momenta $\Pi_M$ are...
given by
\[ E_M = 2J \text{sh} \eta \frac{\partial \ln \Lambda_M}{\partial \lambda}(\eta/2) - (L/2 - M)\hbar, \quad \Pi_M = -i \ln \Lambda_M(\eta/2) \]  
(3)
where the transfer matrix eigenvalue \( \Lambda_M(\lambda) \) is obtained from
\[ \Lambda_M(\lambda) = \left[ \prod_{l=1}^{M} \frac{\text{sh}(\lambda - \lambda_l - \eta)}{\text{sh}(\lambda - \lambda_l)} \right] + \left( \frac{\text{sh}(\lambda - \eta/2)}{\text{sh}(\lambda + \eta/2)} \right)^L \left[ \prod_{l=1}^{M} \frac{\text{sh}(\lambda - \lambda_l + \eta)}{\text{sh}(\lambda - \lambda_l)} \right] \]
(4)
provided that the Bethe equations
\[ \left( \frac{\text{sh}(\lambda_j - \eta/2)}{\text{sh}(\lambda_j + \eta/2)} \right)^L = \left[ \prod_{l \neq j}^{M} \frac{\text{sh}(\lambda_j - \lambda_l - \eta)}{\text{sh}(\lambda_j - \lambda_l + \eta)} \right], \quad j = 1, \ldots, M \]
(5)
for the \( M \) Bethe roots \( \lambda_j \) are satisfied.

The above listed Bethe ansatz equations allow for the computation of the entire spectrum of the system. For identifying a suitable scaling limit, in which the XXZ lattice system turns into the Bose gas in the continuum, it is sufficient to consider the one-particle properties and the two-particle scattering data. In particular, we are going to identify the ferromagnetic vacuum with the Fock vacuum for bosons, and flipped spins on the ferromagnetic background are considered as particles.

In the one-magnon sector \((M = 1)\) the energy–momentum dispersion simply reads (with \( E_0 = -hL/2 \))
\[ E_1 - E_0 = \frac{2J \text{sh}^2 \eta}{\text{sh}(\lambda_1 + \eta/2) \text{sh}(\lambda_1 - \eta/2)} + \hbar, \quad \Pi_1 = i \ln \frac{\text{sh}(\lambda_1 - \eta/2)}{\text{sh}(\lambda_1 + \eta/2)} \]
(6)
and can be considered as some one-particle excitation energy \( E \) over the vacuum \((M = 0)\) with a chemical potential \( \mu \), \( E_1 - E_0 = : E - \mu \).

### 3. Continuum limit

In the desired continuum limit, the ferromagnetic point \( \Delta = -1 \) is approached from the region \( |\Delta| < 1 \) in a well defined way. Therefore we specify the anisotropy of the interaction as \( \Delta = \chi \eta \) with \( \eta = i\pi - i\varepsilon \). Inserting this into the Bethe equations (5) as well as the reparametrization \( \lambda_j = x_j/L \) of the Bethe roots leads to
\[
\text{LHS} = \left( \frac{\text{sh}(\lambda_j - \eta/2)}{\text{sh}(\lambda_j + \eta/2)} \right)^L \approx \left( \frac{1 + i\varepsilon x_j/2L}{1 - i\varepsilon x_j/2L} \right)^L \rightarrow e^{i\varepsilon x_j} \]
(7)
\[
\text{RHS} = \left[ \prod_{l \neq j}^{M} \frac{\text{sh}(\lambda_j - \lambda_l - \eta)}{\text{sh}(\lambda_j - \lambda_l + \eta)} \right] \approx \left[ \prod_{l \neq j}^{M} \frac{x_j - x_l + iL\varepsilon}{x_j - x_l - iL\varepsilon} \right] \]
(8)
where we have used the formula \( \lim_{n \to \infty}(1 + x/n)^n = e^x \) for the ‘LHS’ and anticipated \( L \to \infty \) and \( \varepsilon x_j \) finite. Introducing a lattice spacing \( \delta \) such that the chain has the physical length \( \ell = \delta L \), the rescaling \( \varepsilon x_j = \ell \nu_j \) reveals the Bethe equations to be exactly those of
the Bose gas [2],
\[ e^{i \nu_j} = \left[ \prod_{l=1 \atop l \neq j}^M \frac{\nu_j - \nu_l + ic}{\nu_j - \nu_l - ic} \right], \quad j = 1, \ldots, M, \]
(9)
where the repulsion strength can be identified as \( c = \varepsilon^2/\delta \). Furthermore, the single-particle energy–momentum dispersion is
\[ E = 2J(\delta \nu)^2 \]
(10)
with mass \( m_B = 1/4J\delta^2 \) (implying \( J \to \infty \)) and a chemical potential \( \mu = 2J\varepsilon^2 - h \) directly connected to the magnetic field. For convenience we set \( m_B = 1/2 \) as Lieb and Liniger [2] already did.

3.1. Parameters for the spin-$\frac{1}{2}$ Heisenberg chain and the Bose gas

Let us recapitulate the parameters of the spin chain and the Bose gas with lattice spacing \( \delta \) and the anisotropy parametrized by \( \Delta = -\cos \varepsilon, \varepsilon \ll 1: \)

| XXZ chain | Bose gas |
|-----------|-----------|
| Interaction strength \( J > 0 \) | Particle mass \( m_B = 1/4J\delta^2 = 1/2 \) |
| \# lattice sites \( L \), lattice constant \( \delta \) | Physical length \( \ell = L\delta \) |
| Magnetic field \( h > 0 \) | Chemical potential \( \mu = 2J\varepsilon^2 - h \) |
| Anisotropy \( \Delta = \varepsilon^2/2 - 1 \) | Repulsion strength \( c = \varepsilon^2/\delta \) |

This allows for a well defined continuum limit for the five lattice parameters while keeping the four continuum parameters fixed. In the next section we are going to apply the above continuum limit, established at \( T = 0 \), to the finite temperature formalism of the Heisenberg chain.

4. Non-linear integral equation and free energy

The free energy \( F_{XXZ}/L \) per lattice site for the Hamiltonian (1) follows within the quantum transfer matrix approach [17] as
\[ \frac{F_{XXZ}}{L} = -\frac{h}{2} - T \int_C \frac{d\omega}{2\pi i} \frac{\text{sh} \eta}{\text{sh} \omega} \ln(1 + a(\omega)), \]
(11)
where the auxiliary function \( a(\lambda) \) is calculated from the non-linear integral equation
\[ \ln a(\lambda) = -\frac{h}{T} + \frac{2J \text{sh}^2(i\varepsilon)}{T \text{sh} \lambda \text{sh}(\lambda - i\varepsilon)} + \int_C \frac{d\omega}{2\pi i} \frac{\text{sh}(2i\varepsilon)}{\text{sh}(\lambda - \omega + i\varepsilon) \text{sh}(\lambda - \omega - i\varepsilon)} \ln(1 + a(\omega)). \]
(12)

Here the temperature is denoted by \( T \) and the external magnetic field \( h \) in the continuum limit (with the fully polarized state in positive direction corresponding to the vacuum) takes positive values. In the integrals above, the contour \( C \) is a rectangular path centred around zero. In the critical regime under consideration, with \( \eta = i\pi - i\varepsilon \), its height is restricted by \( \varepsilon \) and the width tends to infinity as depicted in figure 1.

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5
Applying the low-\(T\) limit (note that \(J/T \to \infty\)) to the integral equation the contribution from the upper line of the contour at \(\omega = i\varepsilon/2\) vanishes and one can restrict both the free argument \(\lambda\) and the integration variable \(\omega\) to the lower part of the remaining contour. A shift of this line to imaginary part \(-\eta/2\) without crossing any poles of the inhomogeneity results in the line integral

\[
\ln a(\lambda - \eta/2) = -\frac{h}{T} - \frac{2J \sinh^2 \eta}{T \sinh(\lambda - \eta/2) \sinh(\lambda + \eta/2)} + \int_{\mathbb{R}} \frac{d\omega}{2\pi i} \frac{\sinh(2i\varepsilon)}{\sinh(\lambda - \omega + i\varepsilon) \sinh(\lambda - \omega - i\varepsilon)} \ln(1 + a(\omega - \eta/2))
\]

over the real axis. Observing that in the continuum limit the inhomogeneity of (13) is nothing but the one-particle excitation energy with some chemical potential, cf (6) and (10), the new auxiliary function reads

\[
\ln \tilde{a}(\nu) = -\frac{E - \mu}{T} + \int_{\mathbb{R}} \frac{dw}{2\pi} \frac{2c}{(\nu - w)^2 + c^2} \ln(1 + \tilde{a}(w)).
\]

For the free energy we proceed in the same way. First, we note that the Bose gas grand canonical potential per unit length \(\phi\) corresponds to the free energy of the Heisenberg chain per lattice constant \(\delta\), in detail \(\phi = (F_{XXZ}/L + h/2)/\delta\). Second, starting from the integral representation (11) at low temperatures only the lower part of the contour contributes. Shifting this integration line to imaginary part \(-\eta/2\) without crossing poles from the kernel we get in the continuum limit

\[
\phi = -T \int_{\mathbb{R}} \frac{dw}{2\pi} \ln(1 + \tilde{a}(w)).
\]

Looking up the TBA results \cite{19} of Yang and Yang provides the same integral representations (14) and (15) describing the thermodynamics.

Figure 1. Depiction of the canonical contour \(C\) surrounding the real axis in anticlockwise manner within the strip \(-\varepsilon/2 < \text{Im} \lambda < \varepsilon/2\).
5. Finite temperature correlation functions

The repulsive 1D Bose gas is captured by the quantum non-linear Schrödinger equation. The Hamiltonian for a system of the physical length $\ell$ then reads

$$H_B = \int_0^\ell \frac{dz}{d} \left[ \partial_z \psi^\dagger(z) \partial_z \psi(z) + c \psi^\dagger(z) \psi^\dagger(z) \psi(z) \psi(z) - \mu \psi^\dagger(z) \psi(z) \right],$$  \hspace{1cm} (16)

where the operators $\psi^\dagger$ and $\psi$ are canonical Bose fields and the mass of the bosons is set to $m_B = 1/2$ for convenience. Aiming at the number of particles in the interval $[0, x]$ the corresponding operator $Q_1(x)$ has the explicit representation

$$Q_1(x) = \int_0^x \frac{dz}{d} \psi^\dagger(z) \psi(z)$$  \hspace{1cm} (17)

and the ground state expectation value of $\exp(\varphi Q_1(x))$ can be used to compute the density–density correlation function [13] reading

$$\langle \psi^\dagger(x) \psi(x) \psi^\dagger(0) \psi(0) \rangle = \frac{1}{2} \frac{\partial^2}{\partial \varphi^2} \frac{\partial^2}{\partial x^2} \exp(\varphi Q_1(x)) \bigg|_{\varphi=0}. \hspace{1cm} (18)$$

Recently Kitanine et al [21] derived a multiple-integral representation for the density–density correlation function for the Bose gas at $T = 0$, similar to the generating function $\exp(\varphi Q_1,m)$ of the $zz$-correlation for the XXZ chain. By identifying the operator $Q_1(x)$ with $Q_{1,m}$ involving $m$ consecutive sites, the definition for the spin chain reads

$$Q_{1,m} = \frac{1}{2} \sum_{j=1}^m (1 - \sigma_j^z). \hspace{1cm} (19)$$

We already showed [14] that within the quantum transfer matrix approach the thermal expectation value $\langle \exp(\varphi Q_{1,m}) \rangle_T$ has the multiple-integral representation

$$\langle \exp(\varphi Q_{1,m}) \rangle_T = \sum_{n=0}^m \frac{1}{(n!)^2} \prod_{j=1}^n \int_C \frac{d\omega_j}{2\pi i} \frac{a(\omega_j)}{1 + a(\omega_j)} \left( \frac{\sinh(\omega_j + \eta)}{\cosh(\omega_j + \eta)} \right)^m \times \prod_{j=1}^n \frac{\sinh(\omega_j - \omega_k + \eta)}{\cosh(\omega_j - \omega_k + \eta)} \times \det_n M(\omega_j, \omega_k) \det_n G(\omega_j, \omega_k) \hspace{1cm} (20)$$

with the $n \times n$ matrices

$$M(\omega_j, \omega_k) = -\frac{\sinh(\omega_j - \omega_k)}{\sinh(\omega_j - \omega_k) \sinh(\omega_j - \omega_k - \eta)} \prod_{i=1}^n \frac{\sinh(\omega_j - \omega_i - \eta)}{\cosh(\omega_j - \omega_i - \eta)}$$

$$-\frac{\sinh(\omega_j - \omega_k) e^\sigma}{\sinh(\omega_j - \omega_k) \sinh(\omega_j - \omega_k + \eta)} \prod_{i=1}^n \frac{\sinh(\omega_j - \omega_i + \eta)}{\cosh(\omega_j - \omega_i + \eta)} \hspace{1cm} (21)$$
and $G(\omega_j, z_k)$, where $G(\lambda, z)$ is defined by the linear integral equation

$$G(\lambda, z) = -\frac{\text{sh} \eta}{\text{sh}(\lambda - z) \text{sh}(\lambda - z + \eta)} \int_C \frac{d\omega}{2\pi i} \frac{\text{sh}(\lambda - \omega + \eta) \text{sh}(\lambda - \omega - \eta)}{1 + a(\omega)} G(\omega, z).$$  (22)

The integration path $\Gamma$ surrounds the origin in an anticlockwise manner and has to be enclosed by the canonical contour $C$ occurring already in (12).

6. Density–density correlations of the Bose gas

For applying the Bose limit to the integral representation (20), let us note that the contour $C$ can be chosen to consist of two horizontal lines with imaginary parts $\pm i\varepsilon/2$, and $\Gamma$ similarly, but inside $C$.

Proceeding with the low-$T$ limit of (20), only the lower integration lines of $C$ remain due to the factors $a/(1 + a)$. As the upper line of the path $\Gamma$ is then no longer bounded by the contour $C$ it can be shifted to the imaginary part $i\varepsilon/2$. Now in (20), the $z_j$-integral over this line vanishes in the continuum limit as the factor

$$\left(\frac{\text{sh}(z_j + \eta)}{\text{sh} z_j}\right)^m \to \left(\frac{-v_j - ic/2}{v_j + ic/2}\right)^{x/\delta}$$  (23)

in the integrand becomes rapidly oscillating as $m = x/\delta \to \infty$ for fixed $x$.

Like in section 4, a simultaneous shift of all remaining (lower) integration lines to the imaginary part $-\eta/2$ results in

$$\langle e^{iQ_1m}\rangle_T = \sum_{n=0}^{\infty} \frac{1}{(n!)^2} \left[ \prod_{j=1}^{n} \int_{\mathbb{R}} d\omega_j \frac{a(\omega_j - \eta/2)}{1 + a(\omega_j - \eta/2)} \left(\frac{\text{sh}(\omega_j - \eta/2)}{\text{sh}(\omega_j + \eta/2)}\right)^m \right]$$

$$\times \left[ \prod_{j=1}^{n} \int_{\mathbb{R} + i0} dz_j \frac{\text{sh}(z_j + \eta/2)}{\text{sh}(z_j - \eta/2)} \right] \left[ \prod_{j,k=1}^{n} \text{sh}(\omega_j - z_k + \eta) \text{sh}(z_j - z_k + \eta)\right] \det_n^\chi M(\omega_j - \eta/2, z_k - \eta/2) \det_n M(\omega_j - \eta/2, z_k - \eta/2)$$  (24)

for the Bose limit. Suitably reparametrizing the variables of integration we find the final form

$$\langle e^{iQ_1}\rangle = \sum_{n=0}^{\infty} \frac{1}{(n!)^2} \left[ \prod_{j=1}^{n} \int_{\mathbb{R}} dw_j \sum_{a(w_j)} \frac{dp_j}{2\pi} e^{-i(p_j - w_j)x} \right]$$

$$\times \left[ \prod_{j,k=1}^{n} \text{det}_n \sum_{a(w_j)} \text{det}_n \rho(w_j, p_k) \right].$$  (25)

As above, we set the physical length of the $m$ consecutive sites to be $x = m\delta$ and the limit $\delta \to 0$ led to $m \to \infty$ and to the infinite series on the RHS of the last equation.
The matrix
\[
\tilde{M}(w_j, p_k) = -\frac{c}{(w_j - p_k)(w_j - p_k + ic)}
\]
\[
- \frac{c e^{i\phi}}{(w_j - p_k)(w_j - p_k + ic)} \prod_{l=1}^{n} \frac{p_l - w_j + ic}{w_j - p_l + ic} \frac{w_j - w_l + ic}{w_l - w_j + ic}
\]
(26)
follows from (21) and as a direct consequence of (22) the density \(\rho(w, p)\) is the solution of the linear integral equation
\[
2\pi \rho(\nu, p) = -\frac{c}{(\nu - p)(\nu - p - ic)} + \int_{\mathbb{R}} \frac{2c \, dw}{(\nu - w)^2 + c^2} \frac{\tilde{a}(w)\rho(w, p)}{1 + \tilde{a}(w)}.
\]
(27)

By use of means rather different from those of [21] we found a new way to derive correlations of the Bose gas with even wider applicability, namely to the finite temperature case.

7. Conclusion

At the level of the Bethe ansatz equations for \(T = 0\) we showed how to relate the XXZ chain near the ferromagnetic point to the 1D repulsive Bose gas. Applying this to the QTM approach reproduces the known TBA thermodynamics of Bose particles. Then, and most importantly, we were able to derive a multiple-integral representation for the generating function of the density–density correlations of the Bose gas, valid for arbitrary temperatures.

Unfortunately, the multiple-integral representation is an infinite series with an infinite number of integrals to be calculated. In this respect it resembles the Fredholm determinant representation [22, 23] but avoids dual quantum fields. Apart from the difficulties lying in the infinite series, the derivation of explicit results on the temperature dependence of the correlation lengths is feasible by considering the next-to-leading eigenvalues of the quantum transfer matrix.

Furthermore it is interesting to see how to interpret and how to treat other correlation functions of the XXZ chain in the continuum limit considered.

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A note on the spin-$1/2$ XXZ chain

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