A new semiempirical formula for exotic cluster decays of nuclei

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(March 30, 2022)

A new semiempirical formula, with only three parameters, is proposed for cluster decay half-lives. The parameters of the formula are obtained by making a least squares fit to the available experimental data. The calculated half-lives are compared with an earlier proposed model-independent scaling law. Also, the calculated results of this formula are compared with the recent results of the preformed cluster model for \(^{12}\)C and \(^{14}\)C emissions from different deformed and superdeformed Nd and Gd parents. The results are in good agreement with the available experimental data. The calculated half-lives are obtained by making a least squares fit to the formula.

\[ 23.90.+w_{25.85}.Ca,23.60.+e \]

I. INTRODUCTION

In a radioactive decay series, the end product is reached not only via the emission of \(\alpha\) and \(\beta\) particles but also directly via heavy nuclei emission, the clusters, like Carbon, Oxygen, Florine, Neon, Magnesium and Silicon. Such a process is known as cluster decay, first proposed theoretically in 1980 by Sândulescu, Poenaru and Greiner [1], before its experimental realization in 1984 by Rose and Jones [2]. In this decay process a parent nucleus \((A,Z)\), with mass number \(A\) and charge number \(Z\), breaks into two fragments, \(viz.,\) the emitted cluster \((A_2,Z_2)\) and the associated daughter \((A_1,Z_1)\), where \(A=A_1+A_2, Z=Z_1+Z_2\). The light fragment \((A_2,Z_2)\) is a cluster, heavier than the \(\alpha\)-particle but lighter than the lightest fission fragment observed so far.

The detection of cluster-decay was hindered mainly due to large pile-up of \(\alpha\) particles, but with improved facilities these difficulties are removed and a large number of cluster-decays are observed from different radioactive nuclei. In the years that followed, the cluster decay process has been studied extensively using different theoretical models with different realistic nuclear interaction potentials. In general, two kinds of models are used for explaining the observed and/or for predicting new decay modes. In one kind of these models, the \(\alpha\)-particle as well as the heavy cluster(s) were assumed to be pre-born in a parent nucleus, before they could penetrate the barrier with the available Q-value. These models are called the preformed cluster models [3–7]. In such a model, the clusters of different sizes (mass and/or charge numbers) are considered to be preformed in the parent nucleus, with different probabilities. The Gamow-like barrier penetration is also taken into account in these models. In the other kind of models, only the Gamow’s idea of barrier penetration is used, without considering the cluster(s) being or not being preformed in the parent nucleus. In other words, in this kind of models, called the unified fission models [1,8–13], the cluster radioactivity is considered as a simple barrier penetration phenomenon, in between the \(\alpha\)-decay and the spontaneous fission. In this paper, we attempt to give a model-independent, semiempirical formula for studying this above mentioned process of exotic cluster decay.

Geiger and Nuttal [14] were the first who proposed a semiempirical law connecting the \(\alpha\)-decay half-life and its Q-value. Now, a large number of \(\alpha\)-decays are observed from medium mass to superheavy nuclei, and several attempts have been made to give a universal formula [15–17]. These formulae vary among themselves mainly in the number of parameters. One such scaling law, proposed recently by Horoi et al [15], accounts for both the \(\alpha\) and cluster decays, and is given as,

\[ \log T_{1/2} = (a_1\mu_1^x + b_1) \left[ \frac{(Z_1Z_2)^y}{\sqrt{Q}} - 7 \right] + (a_2\mu_2 + b_2). \]

(1)

This scaling law involves six parameters \(a_1 = 9.1, b_1 = -10.2, a_2 = 7.39, b_2 = -23.2, x = 0.416\) and \(y = 0.613\), which were obtained by fitting 119 \(\alpha\)-decays and 11 cluster-decays from various even-even parents. In the following, we propose a new semiempirical formula based on only three parameters that are least squares fitted to the cluster-decay data alone. So far, no attempt has been made to fit the \(\alpha\)-decay data.

The evolution of our proposed formula is presented in Section II, followed by a brief description of the preformed cluster model (PCM) of Gupta and collaborators [3–5] in Section III. The PCM is recently used by two of us and collaborators [18] to calculate the cluster-decay half-lives of some deformed and superdeformed Gd and Nd parents. Some of these results of Ref. [18] are used here for comparisons with the calculations based on the proposed new semiempirical formula. The results of our calculation are discussed in Section IV, and the summary and conclusions are presented in Section V.
II. THE NEW SEMIEMPIRICAL FORMULA

We base the new formula for cluster decay half-lives on the following three simple experimental facts:

(i) It is known from experiments that the cluster decay half-lives increase with the size (mass and/or charge) of the clusters. Hence, the empirical formula should contain terms showing direct dependence on the mass number and charge number of the cluster.

(ii) The same cluster is emitted by different parents and hence the formula should contain dependence on the mass and charge asymmetries

\[ \eta = \frac{A_1 - A_2}{A}, \quad \eta_z = \frac{Z_1 - Z_2}{Z}, \]

respectively.

(iii) Since the \( \alpha \)-decay and cluster-decay are physically similar processes, the Q-dependence is taken to be the same as in Geiger-Nuttal law for \( \alpha \)-decay, i.e.,

\[ \log T_{1/2} \propto Q^{-1/2}. \]

Combining the above three results, we get

\[ \log T_{1/2}^{AZ} = \frac{a A_2 \eta + b Z_2 \eta_z}{\sqrt{Q}} + c, \]

where the constants \( a = 10.603, b = 78.027 \) and \( c = -80.669 \) are obtained by making a least squares fit of the available experimental half-lives for exotic cluster decays alone, with an rms deviation \( d_{rms} = 0.89(s) \), defined as

\[ d_{rms} = \sqrt{\sum_{i=1}^{n} \left( \frac{y_i - f(x_i, \alpha)}{\sigma_i} \right)^2}, \]

with \( f(x_i, \alpha) \) denoting the function in Eq. (3), \( n \) the number of measurements and \( y_i \) the experimentally observed values. The \( \sigma_i^2 = \frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y})^2 \) is the variance, which gives the standard deviation \( \sigma_i \), with \( \bar{y} = \frac{1}{n} \sum_{i=1}^{n} b_i \) giving the arithmetic average of the experimentally measured quantities. We refer to this formula as the AZ-formula (AZF). Note that the dependence on both \( \eta \) and \( \eta_z \) must be included in the formula (3) since these are separately measurable quantities, though the coupling between them is known to be weak [19–21], as is also evident when we consider the \( \eta \) and \( \eta_z \) dependence separately. These special cases of AZ-formula, i.e., \( b = 0 \) or \( a = 0 \) in (3), respectively, are expected to give reasonably good results, though poorer than the AZF results. These truncated expressions are referred to as the AF and ZF formulae in the following, whose constants are also obtained directly by the least squares fit to data. These are: \( a = 30.568 \) and \( c = -51.348 \) for AF and \( b = 112.197 \) and \( c = -89.025 \) for ZF, with rms deviations \( d_{rms} = 1.652 \) and 1.112 (s), respectively. Apparently, the rms deviations for truncated expressions are larger, and hence the fits are poorer, compared to the total expression (3). This will also be evident when we compare the results of these expressions with experimental data in Section IV.

III. THE PREFORMED CLUSTER MODEL

In the prefomed cluster model (PCM) of Gupta and collaborators [3–5], the decay half-life is given in terms of three factors, the \( P_0 \), \( P \) and \( \nu \), as

\[ \log T_{1/2} = \frac{\ln 2}{P_0 P \nu}, \]

where \( P_0 \) is the preformation probability of the two fragments (the cluster and daughter nuclei) in their respective ground states, \( P \) the probability to tunnel the confining nuclear interaction barrier and \( \nu \) as an assault frequency.

For calculating \( P_0 \) and \( P \), Gupta et al. introduced the coupled motion in dynamical collective coordinates of mass asymmetry \( \eta \) and relative separation \( R \) via a stationary Schrödinger equation

\[ H(\eta, R)\psi(\eta, R) = E\psi(\eta, R), \]

with the potential in it defined by the sum of experimental binding energies [22], the Coulomb and the proximity [23] potentials, as

\[ V(\eta, R) = \sum_{i=1}^{2} B_i (A_i, Z_i) + \frac{Z_i Z e^2}{R} + V_p. \]

Here the charges \( Z_i \) are fixed by minimizing the potential (without \( V_p \)) in the charge asymmetry co-ordinate \( \eta \). Equation (5) is solved in the decoupled approximation, which gives

\[ P_0 \propto |\psi(\eta)|^2 \]

(7)

\[ P \propto |\psi(R)|^2. \]

(8)

Only the ground state solution is relevant for the cluster decay to occur in the ground state of the daughter nucleus. Then, for \( \eta \)-motion, the properly normalized fractional preformation probability for, say, cluster \( A_2 \) at a fixed \( R = R_0 = C_t = C_1 + C_2, \) \( C_i \) being the Süssmann central radii) is

\[ P_0(A_2) = |\psi(\eta)|^2 \sqrt{\frac{B |\eta|^2}{A}}. \]

(9)

For R-motion, the potential \( V(R) \) is obtained from Eq. (6) for fixed \( \eta \) and, instead of solving the corresponding radial Schrödinger equation, the WKB approximation is used for calculating the penetrability \( P \). Finally, the assault frequency \( \nu \) in PCM is defined by considering that the total kinetic energy, shared between the two fragments, is the positive Q-value:

\[ \nu = v/R_0 = \sqrt{2Q/m A_2}/R_0. \]

(10)

Here \( R_0 \) is the spherical radius of the parent nucleus and \( m \), the nucleon mass.
IV. RESULTS AND DISCUSSION

Figure 1 and Table I give our calculated logarithms of decay half-lives by using the AZ-formula (3) for different clusters emitted from various radioactive parents, compared with the experimental data. In Fig. 1, we have also plotted the results of our calucations for AF \((b = 0)\) and ZF \((a = 0)\) versions of the AZ-formula. It is evident that the AZF fit to the data is better than the AF and ZF fits, as expected from our discussion above in Section II.

In Table I, we have also added the experimental data on Q-values and the results of another calculation using Eq. (1) of Horio et al. [15]. The comparison between the experiments and formulae (1) and (3) are also displayed in Fig. 2 for the illustrative cases of \(^{14}\)C and \(^{24}\)Ne cluster decays (respectively, the upper and lower panel). Apparently, our semiempirical AZ-formula is much closer to experiments, as compared to the other formula due to Horio et al. [15].

Finally, in order to compare the results of our semiempirical formula with the results of a model-dependent theoretical cluster decay calculation, we use a recent calculation [18] of two of us (MB and RKG) and collaborators based on PCM for the emission of \(^{12}\)C and \(^{14}\)C clusters from \(^{133-137}\)Nd and \(^{144-158}\)Gd parents. It may be mentioned here that these cluster-decay calculations are of interest only for nuclear structure information since their experimental observation may not be feasible in the near future [18]. Figs. 3(a) to (d) show the results of the PCM calculations, compared with those from the semiempirical AZ-formula (3) and the scaling law (1) of Horio et al. [15]. We notice that, in general, the predictions of our semiempirical law lie higher than those of the PCM and Horio et al. This is more so for Nd parents than for Gd parents where the predictions of the three calculations are nearly similar. The interesting point is that the three calculations predict an exactly the same structure for \(\log T_{1/2}\) vs. \(A\), the parent mass number. For quantitative comparisons, it may be reminded here that the constants \(a\), \(b\) and \(c\) of our semiempirical formula are obtained by fitting the data from trans-actinide region, which may or may not be good for this trans-tin region.

ACKNOWLEDGMENTS

One of the authors (M.B.) acknowledges with thanks the partial financial support by the Department of Science and Technology (DST), vide Grant NO. SR/FTP/PSA-02/2002. Also, the support by DST under the FIST programme vide letter No. SR/FST/PSI-005/2000 to the Department of Physics, M.S. University, Tirunelveli, India, is gratefully acknowledged.

Figure Captions

Fig. 1 The logarithms of decay half-lives for different clusters emitted from various radioactive parents, calculated by using AZ-formula (AZF) and compared with experimental data. Also, the results of calculations for AF \((b=0)\) and ZF \((a=0)\) truncations of AZF are shown for comparisons. The parents are labelled at the top X-axis and the corresponding clusters emitted by these parents are labelled at the bottom X-axis.

Fig. 2 The experimental data on logarithms of decay half-lives for the emission of \(^{14}\)C and \(^{24}\)Ne clusters from different radioactive parents, compared with the results of calculations using AZ-formula (AZF) proposed here and the scaling law of Ref. [15].

Fig. 3 The logarithms of half-lives for the emission of \(^{12,14}\)C clusters from different deformed and superdeformed Nd and Gd parents. Our calculations, using AZ-formula (AZF), are compared with the calculations based on PCM and the scaling law of Horio et al. [15].
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| Parent Cluster | Daughter | $Q^{\text{Expt.}}$ (MeV) | $\log T_{1/2}^{\text{AZF}}$ (s) | $\log T_{1/2}^{\text{Ref.}[15]}$ (s) | $\log T_{1/2}^{\text{Expt.}}$ (s) | Ref. |
|---------------|---------|--------------------------|-------------------------------|---------------------------------|---------------------------------|-----|
| $^{221}$Fr  | $^{14}$C | $^{207}$Tl          | 31.28                         | 14.54                           | 13.56                           | 14.52 | [24] |
| $^{221}$Ra  | $^{14}$C | $^{207}$Pb          | 32.39                         | 12.96                           | 12.28                           | 13.39 | [24] |
| $^{222}$Ra  | $^{14}$C | $^{208}$Pb          | 33.05                         | 11.98                           | 11.00                           | 11.01 | [25] |
| $^{223}$Ra  | $^{14}$C | $^{209}$Pb          | 31.85                         | 13.81                           | 13.38                           | 15.20 | [25] |
| $^{224}$Ra  | $^{14}$C | $^{210}$Pb          | 30.54                         | 15.94                           | 16.13                           | 15.68 | [26] |
| $^{225}$Ac  | $^{14}$C | $^{211}$Bi          | 30.48                         | 16.20                           | 17.26                           | 17.16 | [27] |
| $^{226}$Ra  | $^{14}$C | $^{212}$Pb          | 28.21                         | 20.08                           | 21.50                           | 21.19 | [28] |
| $^{228}$Th  | $^{20}$O | $^{208}$Pb          | 44.72                         | 21.90                           | 21.20                           | 20.72 | [29] |
| $^{230}$U   | $^{22}$Ne | $^{208}$Pb         | 61.59                         | 21.78                           | 19.28                           | 20.14 | [30] |
| $^{231}$Pa  | $^{23}$F  | $^{208}$Pb         | 51.84                         | 24.30                           | 23.85                           | 26.02 | [31] |
| $^{230}$Th  | $^{24}$Ne | $^{208}$Tl         | 57.78                         | 25.77                           | 23.88                           | 24.61 | [32] |
| $^{231}$Pa  | $^{24}$Ne | $^{207}$Tl         | 60.42                         | 23.62                           | 21.30                           | 23.23 | [33] |
| $^{232}$U   | $^{24}$Ne | $^{208}$Pb         | 62.31                         | 22.24                           | 19.94                           | 21.08 | [34] |
| $^{233}$U   | $^{24}$Ne | $^{209}$Pb         | 60.5                           | 23.87                           | 22.53                           | 24.83 | [32] |
| $^{234}$U   | $^{24}$Ne | $^{210}$Pb         | 58.84                         | 25.42                           | 25.01                           | 25.92 | [35,36] |
| $^{235}$U   | $^{24}$Ne | $^{211}$Bi         | 57.36                         | 26.87                           | 27.31                           | 27.42 | [35,37] |
| $^{231}$U   | $^{25}$Ne | $^{208}$Pb         | 60.75                         | 24.15                           | 22.98                           | 24.83 | [32] |
| $^{235}$U   | $^{25}$Ne | $^{210}$Pb         | 57.83                         | 26.94                           | 27.49                           | 27.42 | [35,37] |
| $^{231}$U   | $^{26}$Ne | $^{208}$Pb         | 59.47                         | 25.85                           | 25.75                           | 25.92 | [35,36] |
| $^{234}$U   | $^{28}$Mg | $^{208}$Hg          | 74.13                         | 26.24                           | 24.74                           | 27.54 | [37] |
| $^{236}$Pu  | $^{28}$Mg | $^{208}$Hg          | 79.67                         | 23.01                           | 20.83                           | 21.67 | [38] |
| $^{238}$Pu  | $^{28}$Mg | $^{208}$Hg          | 71.69                         | 28.18                           | 27.98                           | 27.58 | [39] |
| $^{238}$Pu  | $^{28}$Mg | $^{210}$Pb          | 75.93                         | 25.70                           | 25.39                           | 25.70 | [40] |
| $^{238}$U   | $^{30}$Mg | $^{208}$Hg          | 72.51                         | 28.36                           | 28.36                           | 27.58 | [39] |
| $^{238}$Pu  | $^{30}$Mg | $^{208}$Pb          | 77.03                         | 25.71                           | 25.41                           | 25.70 | [40] |
| $^{238}$Pu  | $^{32}$Si | $^{206}$Hg          | 91.21                         | 25.99                           | 25.68                           | 25.27 | [40] |
Fig. 1 "A new semiempirical formula for..." by M. Balasubramaniam et al.
Fig. 2 "A new semiempirical formula for..." by M. Balasubramaniam et al.
Fig. 3 "A new semiempirical formula for..." by M. Balasubramaniam et al.