Simulation analysis of control system for six degrees of freedom damping platform based on MATLAB

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Abstract. On the basis of relevant mathematical theory inference and the observability analysis of sensor arrangement and the controllability analysis of actuator arrangement, the simplified model of the six-degree-of-freedom air spring active vibration isolation system is established. Considering the structure of the air spring damping platform and the environment interference around the vibration isolation platform, we use the simulation software named MATLAB to establish three control models: PID control, Fuzzy control and Fuzzy PID control. The simulation analysis results show that the Fuzzy PID control consequent is obviously benefit to the damping platform, in addition, the composite feedback can also greatly promote the control consequent.

1. Introduction
In the laboratory, there are various sources of interference. In addition to short-range interference sources indoor, long-distance interference sources [1] that their frequencies are mainly concentrated between 0 and 100 Hz need to be considered. Plenty of multiplex equipment that is related to tiny dimensions [2] smaller than 100 nm, such as transmission electron microscopes (TEM), normally demand sufficiently silent laboratories to attain the performance guaranteed. Therefore, it is obviously essential to design an active vibration isolation system with excellent vibration isolation capability for low frequency vibration signals. In this paper, according to the structural characteristics and control requirements of the vibration isolation system, the Fuzzy PID [3] control model is established which based on the air spring vibration isolation platform. In the simulation process, we have compared the control effect of the Fuzzy PID model, the traditional PID model and the fuzzy control and analyzed the control effects under different feedback signals. The results show that the vibration isolation effect of the active control system based on Fuzzy PID controller is obviously better than the simple fuzzy control and PID control.

2. Controllability analysis of vibration isolation system
In order to measure the optimal value of the six degrees of freedom [4] data of the damping platform, the number of sensors and the placement position are shown in Figure 1(a). The sensors are placed at the positions where the displacement of the platform stiffness is large, which not only improves the sensor data quality but also circumvents the nodes of the platform modality [5].
Figure 1. Schematic diagram of the placement position.
(a) $s_{z1}$, $s_{z2}$, $s_{z3}$, $s_{x1}$, $s_{x2}$ and $s_{x3}$ are positions of sensors. (b) $F_a$, $F_b$, $F_c$, $F_{z1}$, $F_{z2}$ and $F_{z3}$ are six output forces.

We can assume that the load locates at the centroid position of the platform, in other words, it is at the intersection of the x-axis and the y-axis. At the same time, it is assumed that the load is completely in a rigid state, and there is no error in the positioning of the sensor. The amount of speed at each sensor is: $\dot{Z}_1$, $\dot{Z}_2$, $\dot{Z}_3$, $\dot{X}_1$, $\dot{X}_2$, $\dot{Y}_1$. Establish an ideal logical axis coordinate system at the position of the center mass of the vibration isolation platform [6]: (x, y, z, α, β, γ). Then we can calculate the linear variation of the absolute displacement of the sensor and the center of the platform with time.

\[
\begin{align*}
\dot{X}_1 &= x - cy \\
\dot{X}_2 &= -x - cy \\
\dot{Y}_1 &= y - dy \\
\dot{Z}_1 &= z + ca - dy \\
\dot{Z}_2 &= z + ca + dy \\
\dot{Z}_3 &= z - ca + dy
\end{align*}
\]

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z} \\
\dot{\alpha} \\
\dot{\beta} \\
\dot{\gamma}
\end{bmatrix} =
\begin{bmatrix}
0.5 & -0.5 & 0 & 0 & 0 & 0 \\
\frac{1}{2c} & -\frac{b}{2c} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.5 & 0 & 0.5 \\
0 & 0 & 0 & \frac{1}{2c} & \frac{1}{2d} & \frac{1}{2d} \\
0 & 0 & -\frac{1}{2d} & \frac{1}{2d} & 0 & 0 \\
-\frac{1}{2c} & -\frac{1}{2c} & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{X}_1 \\
\dot{X}_2 \\
\dot{Y}_1 \\
\dot{Z}_1 \\
\dot{Z}_2 \\
\dot{Z}_3
\end{bmatrix}
\]

(1)

(2)

The air spring is placed under the platform, in order to ensure that the platform can achieve real-time control in six degrees of freedom, seeking the best space pose. There should be an actuator on each degree of freedom of the controlled platform [7], and the position of the actuator should be as far as
possible from the platform centroid. The layout is shown in Figure 1(b). The force and torque required to complete the vibration isolation of the center of the platform are \( (F_X, F_Y, F_Z, M_X, M_Y, M_Z) \). In order to find the execution force of the output required by each actuator, we can list the equilibrium force equation of the platform and solve it.

\[
\begin{align*}
F_X &= -\frac{\sqrt{3}}{2}x_4 + \frac{\sqrt{3}}{2}x_6 \\
F_Y &= -\frac{1}{2}x_4 + x_5 - \frac{1}{2}x_6 \\
F_Z &= 2x_4 + 5x_5 - 2x_6 \\
M_X &= -2cx_1 + 2cx_3 - \frac{1}{2}hx_4 + hx_5 - \frac{1}{2}hx_6 \\
M_Y &= -dx_1 + 2dx_2 - dx_3 + \frac{\sqrt{3}}{2}hx_4 - \frac{\sqrt{3}}{2}hx_6 \\
M_Z &= -Lx_4 - Lx_5 - Lx_6
\end{align*}
\]

(3)

\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix} =
\begin{bmatrix}
\frac{h}{6d} & \frac{h}{6c} & \frac{1}{6} & -\frac{1}{4c} & -\frac{1}{6d} & 0 \\
\frac{h}{3d} & 0 & \frac{1}{6} & 0 & \frac{1}{3d} & 0 \\
\frac{h}{6d} & -h & \frac{1}{6} & \frac{1}{4c} & -\frac{1}{6d} & 0 \\
-\frac{\sqrt{3}}{3} & \frac{1}{6} & 0 & 0 & 0 & -\frac{1}{3L} \\
0 & \frac{2}{3} & 0 & 0 & 0 & -\frac{1}{3L} \\
\frac{\sqrt{3}}{3} & -\frac{1}{3} & 0 & 0 & 0 & -\frac{1}{3L}
\end{bmatrix}
\begin{bmatrix}
F_X \\
F_Y \\
F_Z \\
M_X \\
M_Y \\
M_Z
\end{bmatrix}
\]

(4)

It can be clearly seen from the above matrix that: the six free absolute acceleration values at the center of mass of the platform are measurable and the actuator is theoretically controllable under this layout.

3. Dynamic model of air spring vibration isolation system

In order to facilitate the study of the air spring vibration isolation platform [8], we consider the interference and active control of the platform in the z direction, and establish a simplified model of the relevant structure as shown in Figure 2.

**Figure 2.** Vibration isolation platform simplified model diagram.
The dynamic equation corresponding to the passive control system is:

\[ m\ddot{z}(t) + c\dot{z}(t) + kz(t) = c\ddot{z}_0(t) + kz_0(t) + f_\text{p}(t) \]  

(5)

We assume that the feedback amount is the amount of speed and b is the feedback gain, the dynamic equation corresponding [9] to the active control system is:

\[ m\ddot{z}(t) + c\dot{z}(t) + kz(t) = c\ddot{z}_0(t) + kz_0(t) + f_\text{p}(t) - b\dot{z}(t) \]  

(6)

Its general solution is:

\[ z = Ae^{jw_1t} + Be^{jw_2t} \]  

(7)

\[ A = \frac{1 + j2\tilde{\omega}_1 / \omega_0}{1 - (\omega_1 / \omega_0)^2 + j2\tilde{\omega}_1 / \omega_0(1 + \frac{b}{c})}Z_0 \]  

(8)

\[ B = \frac{1 + j2\tilde{\omega}_2 / \omega_0}{k(1 - (\omega_2 / \omega_0)^2 + j2\tilde{\omega}_2 / \omega_0(1 + \frac{b}{c}))}F_\text{p} \]  

(9)

\[ s = jw_i (i = 1, 2) \]  

(10)

\[ A = \frac{cs_1 + k}{ms_1^2 + cs_1 + k(1 + \frac{b}{c})} \]  

(11)

\[ B = \frac{1}{k} \frac{cs_2 + k}{ms_2^2 + cs_2 + k(1 + \frac{b}{c})} \]  

(12)

Then the transmission rate of the vibration isolation system can be defined as:

\[ G(s) = \left| \frac{cs_1 + k}{ms_1^2 + cs_1 + k(1 + \frac{b}{c})} + \frac{1}{k} \frac{cs_2 + k}{ms_2^2 + cs_2 + k(1 + \frac{b}{c})} \right| \]  

(13)

In the \( G(s) \) formula, \( m \) is the mass of platform, then \( \dot{z}(t), \dot{z}_0(t) \) are the velocity of the platform and the ground. The force \( F_Z \) applied in the direction of the platform \( z \) can be simplified to \( f(t) \). \( f_\text{p}(t) \) is the interference force acting directly on the platform. Lastly, \( Z_0(t) = Z_0e^{jw_1t}, f_\text{p}(t) = F_\text{p}e^{jw_2t} \) \((Z_0 \text{ and } F_\text{p} \text{ are the amplitudes of the basic interference and direct interference) [6].} \)
4. Model simulation and analysis

Based on the classical PID control, the velocity deviation \( (K_e) \) and the velocity deviation change rate \( (K_{ec}) \) are taken as the input of the fuzzy controller [10], and the \( K_p \), \( K_i \) and \( K_d \) of PID are taken as the output of the fuzzy controller. In addition, the scaling factors \( U_p \), \( U_i \) and \( U_d \) are equally taken 1, the quantization factor \( K_e \) is taken 0.4 and the quantization factor \( K_{ec} \) is taken 2. We take advantage of Matlab to frame the corresponding simulation model by means of integrating the transfer functions \( G(s) \). In order to better compare and analyze the control effects of the three, we are supposed to put the three control methods into the same simulation model, as shown in Figure 3.

![Figure 3. Three control scheme simulation diagrams](image)

Through combining the actual situation of the damping platform, the relevant parameters selected are as follows: the mass \( (m=4300\text{kg}) \), the damping coefficient \( (c=4\times10^6\text{N}\cdot\text{s}/\text{m}) \), the stiffness coefficient \( (k=6.4\times10^4\text{N}/\text{m}) \), the feedback gain coefficient \( (b=2) \). Signal 1 and signal 2 respectively represent sinusoidal signals with amplitudes of 30 μm/s and 80 μm/s, as shown in Figure 4 (a) result of Fuzzy control. (b) result of Fuzzy control. (c) result of Fuzzy control. In order to further explore the effect of speed feedback and speed-shift composite feedback on the control effect of Fuzzy PID controller, some simulations have been done under two sinusoidal signals. The specific simulation results are shown in Figure 4 (d) and Table 1.
Figure 4. Simulation results under different schemes

Table 1. Three Scheme comparing.

| Vibration Interference Velocity (m/s) | PID Control Velocity (m/s) | Fuzzy Control Velocity (m/s) | Fuzzy PID Control Velocity (m/s) |
|--------------------------------------|---------------------------|-----------------------------|---------------------------------|
| 3E-5                                 | 1.5E-6                    | 7.6E-4                      | 7.2E-8                          |
| 8E-5                                 | 3.3E-6                    | 7.5E-4                      | 1.85E-7                         |

5. Conclusion
The actuator layout scheme mentioned in this paper is theoretically controllable and is a non-redundant output that can achieve six degrees of freedom. The control efficiency in the control process is higher than the redundant output and there is no additional Conditional. For the two sinusoidal signals of amplitude 30μm/s and 80μm/s, the vibration isolation effect of the active control system of Fuzzy PID controller is obviously better than the simple fuzzy control and PID control. The simple fuzzy control is evidently unstable to the damping effect of the damping platform, and the time required for the adjustment is relatively long. For the Fuzzy PID controller, the composite feedback is more perfect than the speed feedback.

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