Chemical potential response of pseudoscalar meson masses in the Nambu–Jona-Lasinio model

O. Miyamura and S. Choe

Dept. of Physics, Hiroshima University, Higashi-Hiroshima 739-8526, Japan

Using the Nambu–Jona-Lasinio (NJL) model we study chemical potential response of the pion and kaon masses as a function of temperature and chemical potential, i.e., \( \frac{\partial m}{\partial \mu}(T, \mu) \). First, we obtain the responses assuming the vector–axial vector coupling is zero \((g_V=0)\). Then, we include a non-zero \( g_V \) and study the effects of \( g_V \) on the responses. We find that the behavior of \( \frac{\partial m}{\partial \mu} \) for the pion is quite different from that for the kaon. It means that \( \frac{\partial m}{\partial \mu} \) is much dependent on the mass difference between the two quark species, i.e., the \( u \) and \( s \) quarks (or even between the \( u \) and \( d \) quarks). Our results may give a clue to future studies of \( \frac{\partial m}{\partial \mu} \) on the lattice.

1. Introduction

While the structure of QCD at finite temperature has been investigated in detail using lattice QCD, little is known about matter at finite baryon density due to the sign problem [1]. In order to avoid this difficulty one can consider 2–color QCD simulations, imaginary chemical potential simulations [8], the response of a hadron mass to changes in the chemical potential, \( \frac{\partial m}{\partial \mu} \) and \( \frac{\partial^2 m}{\partial \mu^2} \), at zero chemical potential \((\mu=0)\) [10], and so on.

Since the direct application of QCD at finite temperature and chemical potential is not available in the present lattice QCD simulations, effective models of QCD are commonly used. One of the most popular models is the Nambu–Jona-Lasinio (NJL) model [6]. This model has been widely used for describing hadron properties in hot and/or dense matter [7].

In this work we present NJL model calculations of \( \frac{\partial m}{\partial \mu} \) for the pion and kaon. The primary goal of our study is to get the same quantity which is simulated on the lattice, i.e., \( \frac{\partial m}{\partial \mu}(T, \mu=0) \). Of course, the direct comparison between the lattice data and the present NJL model calculations is rather difficult because \( \frac{\partial m}{\partial \mu} \) on the lattice means the variation of a screening mass, while it corresponds to the variation of a dynamical mass in the present work. Nevertheless, we can compare both results to each other qualitatively.

In contrast to the lattice simulations, we also get \( \frac{\partial m}{\partial \mu} \) at finite chemical potential within this model. Then, this may give information on the role of the light quark (the \( u \) and \( d \) quarks) chemical potential and/or the strange quark chemical potential in hot and/or dense matter. In addition, we consider two cases for the pion. One is the pion with the same \( u \) and \( d \) quark mass. The other is the pion with non-degenerate \( u \) and \( d \) quark masses. Since the present lattice QCD can not simulate the characteristics coming from slightly different \( u \) and \( d \) quark masses [8], the response of the pion mass with different \( u \) and \( d \) quark masses can be useful for future studies of \( \frac{\partial m}{\partial \mu} \) on the lattice.

Due to the limitation of space we present only the results with a non-zero vector–axial vector coupling \( g_V \) in this paper. Please refer to [9] for basic formulae to get \( \frac{\partial m}{\partial \mu} \) for the kaon and pion in the NJL model and the results without \( g_V \). Detailed derivations including small corrections in the figures in [10] will be given elsewhere [10].

2. The effects of a non-zero \( g_V \)

One of NJL model calculations showed that the \( K^- \) mass at finite density with \( g_V \neq 0 \) is quite different from that with \( g_V=0 \) [11]. In this section, we show the chemical potential response of the kaon mass by including a non-zero \( g_V \). In the case of the pion, the effect of the non-zero vector–axial coupling
We follow the formalism and the same parameters in [11]. First, in Fig. 1 let us show the responses of quark condensates with (and also without) $g_V$ at $\mu_u=\mu_d=0.02, 0.04$ GeV, respectively. The effect of a non-zero $g_V$ on the $s$ quark condensate is much larger than that on the $u$ quark condensate. This behavior can be understood from a large difference in the $s$ quark mass, i.e., $m_s=135.7$ MeV in [9] and $m_s=88.0$ MeV in the present work. Since the response of the $s$ quark condensate becomes much larger than before, one can expect that $\frac{\partial m_K}{\partial \mu_S}$ for the kaon will be quite different from the previous values.

In Fig. 2 we show $\frac{\partial m_K}{\partial \mu_S}$ for $K^+$ and $K^-$. 
For comparison we also show the previous results without \( g_V \). The comparison of \( \frac{\partial m}{\partial \mu} K \) at each point is rather meaningless because all the parameters including the quark masses are changed. We would like to show only a qualitative behavior of \( \frac{\partial m}{\partial \mu} K \) for \( K^- \) is much different from the previous one, where \( g_V = 0 \). This result confirms the previous NJL model calculations that the vector–axial vector interaction reduces the effects of the Fermi sea and the \( K^- \) mass is a smoothly decreasing function of density \([12,11]\).

### 3. Concluding Remarks

Using the NJL model we have calculated the chemical potential responses of the kaon and pion masses, \( \frac{\partial m}{\partial \mu} K \) and \( \frac{\partial m}{\partial \mu} \pi \), at zero and finite chemical potential, and found that their behaviors are quite different from each other. Our results show that \( \frac{\partial m}{\partial \mu} K \) is much dependent on the mass difference of two quarks, i.e., the mass difference between the \( u \) and \( s \) quarks, or even between the \( u \) and \( d \) quarks. Even at very small chemical potentials \( \frac{\partial m}{\partial \mu} \) is much different from \( \frac{\partial m}{\partial \mu} \).

Let us discuss some uncertainties in our calculations. First, the mass in our formalism means the dynamical mass, while the screening mass is simulated on the lattice. For the direct comparison we may need the formalism to calculate the screening mass in the NJL model as given in \([13]\). This may help us to properly interpret the lattice data. Second, while the lattice simulations of \( \frac{\partial m}{\partial \mu} \) show a large difference between in the confinement phase and in the deconfinement phase \([5]\), we cannot predict \( \frac{\partial m}{\partial \mu} \) above the Mott temperature in the present work. This is because we have excluded the imaginary part of the one-loop polarization function in the dispersion relation and considered only the region below the Mott temperature \([10]\). Thus, the imaginary part of the polarization function should be included for a detailed analysis above the Mott temperature.

As a final remark, we confirm that \( \frac{\partial^2}{\partial \mu^2} \langle \bar{q} q \rangle \) and \( \frac{\partial^2}{\partial \mu^2} \langle \bar{q} q \rangle \) both below and above the Mott temperature \([10]\). The second order response of the quark condensate \( \frac{\partial^2}{\partial \mu^2} \langle \bar{q} q \rangle \) at \( \mu = 0 \) can also be simulated on the lattice \([14]\), and its behavior is similar to the result from the present NJL model calculation.

### Acknowledgements

We thank A. Nakamura for valuable comments. The work of S.C. was supported by the Japan Society for the Promotion of Science (JSPS).

### REFERENCES

1. See, e.g., S. Hands, these proceedings, \texttt{hep-lat/0109034}.
2. S.J. Hands, J.B. Kogut, S.E. Morrison, and D.K. Sinclair, Nucl. Phys. B (Proc. Suppl.) 94 (2001) 457.
3. S. Muroya, A. Nakamura, and C. Nonaka, Nucl. Phys. B (Proc. Suppl.) 94 (2001) 469; and these proceedings.
4. A. Hart, M. Laine, and O. Philipsen, Phys. Lett. B 505 (2001) 141.
5. QCD-TARO Collaboration, Nucl. Phys. B (Proc. Suppl.) 63 A-C (1998) 460; Nucl. Phys. B (Proc. Suppl.) 73 (1999) 477; \texttt{hep-lat/0107002} and these proceedings.
6. Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345; \textit{ibid.} 124 (1961) 246.
7. See, e.g., U. Vogl and W. Weise, Prog. Part. Nucl. Phys. 27 (1991) 195; S.P. Klevansky, Rev. Mod. Phys. 64 (1992) 649; T. Hatsuda and T. Kunihiro, Phys. Rep. 247 (1994) 221; and references therein.
8. For a recent work on this issue, see T. Takaishi and Ph. de Forcrand, \texttt{hep-lat/0108014}.
9. O. Miyamura and S. Choe, \texttt{hep-ph/0105195}.
10. O. Miyamura and S. Choe, in preparation.
11. M.C. Ruivo, C.A. de Sousa, and C. Providência, Nucl. Phys. A 651 (1999) 59.
12. M. Lutz, A. Steiner, and W. Weise, Phys. Lett. B 278 (1992) 29.
13. W. Florkowski and B. Friman, Acta Phys. Pol. B 25 (1994) 49; Nucl. Phys. A 611 (1996) 409.
14. The last reference in \([8]\).