DeepPSL: End-to-end perception and reasoning with applications to zero shot learning

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Abstract

We introduce DeepPSL a variant of Probabilistic Soft Logic (PSL) to produce an end-to-end trainable system that integrates reasoning and perception. PSL represents first-order logic in terms of a convex graphical model – Hinge-Loss Markov random fields (HL-MRFs). PSL stands out among probabilistic logic frameworks due to its tractability having been applied to systems of more than 1 billion ground rules. The key to our approach is to represent predicates in first-order logic using deep neural networks and then to approximately back-propagate through the HL-MRF and thus train every aspect of the first-order system being represented. We believe that this approach represents an interesting direction for the integration of deep learning and reasoning techniques with applications to knowledge base learning, multi-task learning, and explainability. We evaluate DeepPSL on a zero shot learning problem in image classification. State of the art results demonstrate the utility and flexibility of our approach.

1 Introduction

Bridging the gap between reasoning and perception is a critical challenge for the future of Artificial Intelligence. Over the last decade deep learning has lead to remarkable advances in the field of perception at the expense of explainability, robust learning, and large data requirements. At the same time advances in probabilistic reasoning and graphical models have yet to be adequately integrated into Deep Learning as the pendulum has swung from incorporating prior knowledge to the primacy of data. Of course, we must leverage both prior knowledge and data and so integrating the capabilities of deep learning and probabilistic reasoning represents a critical challenge for the AI community. We tackle this challenge with DeepPSL an end-to-end integration of Deep Learning with Probabilistic Soft Logic (PSL) [1].

PSL provides a powerful formalism for representing and reasoning over the facts in a knowledge base while respecting the uncertainty associated with those facts. PSL uses Hinge-Loss Markov Random Fields (HL-MRFs) to represent first order logic. Inference is then a convex optimization problem which is tractable even for very large systems (billions of ground rules).

The first order expressions in PSL are built from predicates which capture the truth of some assertion “HasFur(KarlTheCat)” about some entity “KarlTheCat”. In PSL these predicates are associated with
soft truth values in the interval $[0, 1]$ which reflect a degree of belief in the predicate. Many of these values are obtained from a knowledge base.

Consider a knowledge base of facts about animals.

- Cats have claws
- Tigers are cats
- Tigers have stripes
- Zebras have stripes
- Zebras don’t have claws

These “facts” aren’t always true (consider a cat that has been declawed) but they should be useful in determining whether a previously unseen animal is a Zebra or not. If we knew that this animal had stripes but no claws then PSL could be used to infer that it is likely to be a zebra.

On the other hand deep learning can learn to identify animals directly from an image. However, we need training data for each concept we want to learn – perhaps large quantities of training data.

With DeepPSL we integrate these two approaches. Each of the concepts in our knowledge base is represented as a predicate, e.g., HasFur(x), HasClaws(x), IsTiger(x), IsZebra(x). Some of these predicates are associated with a deep neural network that can determine their truth based on an image. At inference time DeepPSL can combine all of these truth values to yield more confident and more accurate results, e.g., given that we see fur and stripes then the likelihood that its a Zebra increases. Furthermore it can be used to produce an explanation, e.g., “because it has stripes but no claws”. At training time DeepPSL combines the multiple learning tasks associated with each predicate in a rigorous way allowing us to learn concepts for which we might have minimal or no data.

We make two changes to PSL. First, we replace some predicates with deep networks. Second, we develop an approach to back-propagate through the inference of PSL to learn the parameters of these deep networks.

We demonstrate the effectiveness of this approach by evaluating on a zero shot learning problem in image classification. We connect a PSL based reasoner to the output of a deep learning model that recognises attributes of images. The PSL reasoner infers the classes based on the attributes recognised by the deep learning model. The integrated system is then trained end-to-end using the proposed algorithm.

While we demonstrate our method on zero shot learning, the method is general and can be applied to a variety of problems that integrate perception with reasoning.

2 Related Work

2.1 Integrating Reasoning and Deep Learning

Integrating probabilistic reasoning and Deep Learning has received considerable recent attention.

Deep Logic Models (DLM) [2] and their sequel Relational Neural Machines (RNM) [3] are most closely related to this work. RNMs model reasoning using a Markov random field and backpropagate through that field to learn underlying neural models. Their work differs from ours in several key respects. First, they do not allow any learned values to be used directly in logical rules. Rather they add potentials that couple the learned values to output variables which must be either observed or latent in the Markov random field potentially resulting in a large increase in the number of latent variables. Second, they address latent variables by employing an EM procedure. Third, as a result of their formulation they do not need to backpropagate through an \[ \arg \min \] . DLM and RNM are related to Semantic-based Regularization [4], Logic Tensor Networks [5] and Neural Logic Machines [6] which allow logical constraints to constrain the learning of Deep networks.

DeepProbLog [7] augments the probabilistic logic programming language ProbLog [8] by incorporating neural predicates, i.e., predicates whose truth values are the output of Deep networks as we do here. DeepProbLog is similar to PSL in that inference is performed by first grounding all variables.
Inference in ProbLog and DeepProbLog is based on SLD-resolution while inference in PSL is based on convex optimization. PSL has demonstrated substantially greater scalability.

Markov Logic Networks (MLN) [9, 10] and Probabilistic Soft Logic (PSL) [11, 1] map probabilistic first order logic to a Markov network. Neural Markov Logic Networks [12] extend MLNs by defining the potential functions as neural networks.

Neural Theorem Prover [13] is an end-to-end differentiable prover. TensorLog [14] is a recent framework to reuse the deep learning infrastructure of TensorFlow to perform probabilistic logical reasoning. Neither of these methods model predicates using Deep learning.

[15] presents an iterative distillation method that transfers structured information of first-order logic rules into the weights of the neural networks. [16] generalized the approach to include rules built using PSL.

2.2 Differentiable Convex Optimization

End-to-end training of a DeepPSL model requires solving a bi-level optimization problem. The techniques discussed in [17–19] for solving a bi-level optimization computes gradient of the loss function which needs computation of inverse of the Hessian that is expensive to compute at each iteration.

[20–24] consider neural network layers consisting of a variety of optimization problems: arg min and arg max problems [22], quadratic programming problems [21, 23], convex problems [20], cone programs [24]. All of these methods require that the optimization functions have continuous derivatives and make use of second derivatives to allow backpropagation through the optimization problems. HL-MRFs do not have continuous derivatives and therefore are not amenable to the same approaches.

2.3 Zero Shot Learning

Zero shot learning (ZSL) has been addressed by many different methods. Attribute based methods include Direct Attribute Prediction (DAP) and Indirect Attribute Prediction (IAP) [25]. DAP learns probabilistic classifiers for each attribute through independent training. It utilises binary labels indicating the presence or absence of an attribute in a class. These attribute classifiers are used during test time to estimate the presence of attributes and later map them to test classes. IAP on the other hand first learns a probabilistic multi-class classifier for each of the training classes. During test time, the posterior distribution of training class labels induce a distribution over the labels of unseen classes using the attribute class relationship.

Recent methods use additional information in the form of word embeddings and hierarchical embeddings instead of or in addition to the attributes. These techniques operate by directly learning a cross-modal mapping from an image feature space to a semantic space. ALE [26], DeVise [27], and SJE [28] learn linear compatibility functions while methods such as LatEm [29] learn non-linear multi-modal embeddings. SSE [30] and SYNC [31] on the other hand adopt the approach of embedding both image and attributes into another common intermediate space. ESZSL [32] also learns a bilinear compatibility while regularising the objective with respect to the Frobenius norm. Methods such as GFZSL [33] use generative models.

Attribute based methods are most relevant here. We use attribute based zero shot learning to demonstrate the integration of deep learning models for each attribute with PSL reasoning to infer class labels.

3 Background

3.1 HL-MRFs: Hinge Loss Markov Random Fields

HL-MRFs are defined with $k$ continuous potentials $\phi = \{\phi_1, \ldots, \phi_k\}$ of the form:

$$\phi_j(x, y) = (\max\{l_j(x, y), 0\})^{p_j} \quad (1)$$
where $\phi_j$ is a potential function of $n$ free random variables $y = \{y_1, \ldots, y_n\}$ conditioned on $n'$ observed random variables $x = \{x_1, \ldots, x_{n'}\}$, each random variable can take soft values between $[0, 1]$. The function $l_j$ is linear in $y$ and $x$ and $p_j \in \{1, 2\}$. Collecting the definitions from above, a Hinge-Loss energy function $f$ is defined as

$$f(x, y) = \sum_{j=1}^{k} \theta_j \phi_j(x, y)$$

where $\theta_j$ is a positive weight corresponding to the potential function $\phi_j$. A HL-MRF over random variables $y$ and conditioned on random variables $x$ is a probability density defined as

$$P(y|x) = \frac{1}{Z(x)} \exp(-f(x, y))$$

where $Z(x)$ is the partition co-efficient. Maximum a posteriori (MAP) inference finds the most probable assignment to the free variables $y$ given the observed variables $x$. MAP inference is done by maximizing the probability density $P(y|x)$ while satisfying the constraint that the random variable $y \in [0, 1]^n$. Since the normalizing function $Z$ in (3) is not a function of $y$, maximizing $P(y|x)$ is equivalent to minimizing the energy function $f$, i.e.,

$$\arg\max_{y \in [0, 1]^n} P(y|x) \equiv \arg\min_{y \in [0, 1]^n} f(x, y)$$

Critically the function $f$ is convex in $y$, for a given $x$, allowing for tractable inference even for very large HL-MRFs. The inference problem solved in this study are relatively small, and therefore, we employ stochastic gradient descent (SGD) algorithm to solve these inference problems. For large scale inference problems, one may employ distributed optimization algorithm, alternating direction method of multipliers (ADMM), as discussed in [1].

3.2 PSL rules

PSL uses first order logic as a template language for HL-MRFs. A PSL program defines a set of rules in first order logic. These rules relate the truth values of predicates which specify facts. For example, a rule $\text{HasFur}(X) \implies \text{IsCat}(X)$ is a rule that relates the truth values of the HasFur and IsCat predicates. The truth of some of these predicates is specified by a database (the observed predicates) and others are unobserved and are the target of inference. Inference is performed by first grounding the PSL program, i.e., substituting all variables $X$ with all values in their domain, e.g., KarlTheCat, RalphTheDog. Each ground rule is then translated into a weighted Hinge-Loss potential. The sum of these potentials defines a HL-MRF. Minimizing that potential infers values for the unobserved predicates.

In the sequel each grounded predicate is associated with a random variable with a value in $[0, 1]$ that represents the truth value of the fact. The random variables associated with observations are denoted by $x$. The unobserved random variables are denoted by $y$.

It is beyond the scope of this paper to provide a detailed description of how first order logic rules are used as a template language for HL-MRFs, see [1] for further details.

4 DeepPSL

4.1 Deep Learning based Predicates

The key difference between PSL and DeepPSL is that some predicates are replaced with deep neural networks (DNNs). Instead of the $x$ being available through a knowledge base they are computed from a set of features $u$ with the help of a deep neural network.

\footnote{2 is used in this work for quadratic Hinge-Loss}

\footnote{These weights are not central to the remaining discussion in this paper and so will be omitted for simplicity of presentation in most of the following. They may be learned using a straight-forward gradient descent procedure.}
4.2 Learning

As mentioned earlier, the key problem that needs to be solved is to determine how to train this system end to end. In the proposed DeepPSL framework, the features $u$ are first processed through a neural net with tunable weights $w$ to generate estimates of the attribute $x$ which are predicates for the PSL. The estimates of attributes (predicates) in the DeepPSL are modeled by a differentiable function $p(u; w)$. These attributes then go through PSL inference to produce the final values of the random variables $y$. For end to end training, we need to enable backpropagation through the PSL inference. Since PSL inference is a convex optimization problem, there is no direct way to backpropagate and update the weights of the attribute network. We now describe our solution to address this problem.

4.2.1 Optimization objective

The prime objective of training this end-to-end learning model is to determine weights $w$ such that the HL-MRFs inference yields variables $y$ which are closely related to their true values on the training data $\hat{y}$. These free variables $y$ represent the outputs of DeepPSL. For example, a $y$ might represent a belief that given an image contains a zebra.

We want to obtain good outputs or predictions where "good" is measured by a loss relative to their true values on training data $\hat{y}$. We restrict our analysis here to HL-MRFs in which all $y$ correspond to outputs.

In order to measure if the inferred values $y$ are close enough to the true values $\hat{y}$; let us consider a differentiable convex loss function:

$$\mathbb{R}^n \times \mathbb{R}^n \ni (\hat{y}, y) \mapsto L(\hat{y}, y) \in \mathbb{R}$$

The DeepPSL inference problem (4) is approximated with soft constraints as

$$y^* = \arg \min_y \tilde{f}(u, w, y)$$

where $\tilde{f}(u, w, y) = f(p(u; w), y) + \sum_{i=1}^n \gamma_i (\max\{0, -y_i\})^2 + \sum_{i=1}^n \gamma_i (\max\{0, y_i - 1\})^2$ with fixed $\gamma_i, \gamma_i > 0$. Therefore, the weight training problem is set up as a nonlinear optimization

$$\min_{w, y} L(\hat{y}, y)$$

subject to $y = \arg \min_y \tilde{f}(u, w, \hat{y})$

(7)

4.2.2 Gradient Following Algorithm

We develop a gradient descent procedure for solving the nonlinear optimization (7). This task is challenging because we need to back-propagate through the $\arg \min$. The most direct approach involves inverting the Hessian $\nabla_{yy} \tilde{f}(u, w, y)$ which is not well-defined for HL-MRFs which do not have continuous derivatives.

We take an alternative approach which avoids this pitfall by taking advantage of the convexity of $L$ and $\tilde{f}$ and assuming that $p$ is Lipschitz continuous.

We produce a sequence of convex optimization problems

$$y_{t+1} = \arg \min_y \tilde{f}(u, w_t, y)$$

such that $L(\hat{y}, y_{t+1})$ is decreasing. At each step we alternate between two optimization problems. First, we solve $y_t = \arg \min_y \tilde{f}(u, w_{t-1}, y)$ to determine the value of the outputs $y_t$ and calculate the gradient of the loss $\nabla_y L(\hat{y}, y_t)$ with respect to these outputs. Next we update $w$ to modify the convex optimization problem so that its minimum is moved "in the negative direction" of this gradient. Proceeding iteratively in this way we obtain incrementally better parameterizations of the HL-MRF resulting in decreasing values of the loss.

The key step is to move "in the negative direction" of the gradient. We formulate this as a second optimization problem:

$$w_t = \arg \min_w \tilde{f}(u, w, y_t - \alpha \nabla_y L(\hat{y}, y_t)) - \tilde{f}(u, w, y_t)$$

(9)
The objective is to modify the HL-MRF objective so that its minimum \( y_{t+1} = \arg \min_y \tilde{f}(u, w, y) \) yields a lower value of the loss. Recall that \( \lim_{h \to 0} \frac{g(v + hz) - g(v)}{h} = \nabla g(v) \cdot z \) so \( (9) \) is equivalent to:

\[
\begin{align*}
  w_t &= \arg \max_w \nabla_y \tilde{f}(u, w, y_t) \cdot \nabla_y L(\hat{y}, y_t)
\end{align*}
\]  

(10)

for sufficiently small \( \alpha \). If \( \nabla_y \tilde{f}(u, w, y_t) \cdot \nabla_y L(\hat{y}, y_t) > 0 \) then \( -\nabla_y \tilde{f}(u, w, y_t) \) is a descent direction of both \( \tilde{f} \) and \( L \) as is any \( \hat{y} \) for which \( \nabla_y \tilde{f}(u, w, y_t) \cdot \hat{y} \) is sufficiently large. In particular since \( p \) is Lipschitz continuous, \( \tilde{f} \) is convex\(^3\) and \( \nabla_y \tilde{f} \) is Lipschitz continuous then a sufficiently small change in \( w \) yields a small change in \( \arg \min_y \tilde{f}(u, w, y) \). So by making only small changes in \( w \) we can ensure that \( y_{t+1} = \arg \min_y \tilde{f}(u, w_t, y) \) results in \( L(\hat{y}, y_{t+1}) < L(\hat{y}, y_t) \).

These two optimizations \( (8) \) and \( (9) \) are executed alternatively until convergence in Algorithm 1.

Algorithm 1 Joint optimization: backpropagating loss to the neural network

1: Initialization: \( t = 1, \alpha > 0, \eta > 0, \delta > 0, N \geq 1. \)
2: Neural network weights \( w_0 \) are initialized using standard techniques.
3: while \( \delta > \epsilon \) do \( \triangleright \) outer iterations
4: \( y_t = \arg \min_y \tilde{f}(u, w_{t-1}, y) \) \( \triangleright \) MAP inference
5: \( w_t = w_{t-1} \)
6: for \( i = 1, \ldots, N \) do \( \triangleright \) Training the neural network weights \( w_t \)
7: \( L_z(u, w_t, y_t) = \tilde{f}(u, w_t, y_t - \alpha \nabla_y L(\hat{y}, y_t)) - \tilde{f}(u, w_t, y_t) \)
8: \( w_{t+1} = \eta \nabla_{w_t} L_z(u, w_t, y_t) \)
9: \( \delta = ||w_t - w_{t-1}|| \)
10: \( t = t + 1 \)

4.3 Limitations

DeepPSL has some limitations several of which follow from the limitations of PSL. The inference performed by PSL is more closely related to Łukasiewicz logic than to probabilistic reasoning. PSL can scale to very large sets of ground formulae but efficient inference can take some care in setting up the optimization.

Because DeepPSL uses Deep networks to model predicates learning is not convex and may suffer from local minima. While Deep networks have been shown to be insensitive to local minima the same may not be true for DeepPSL due to the final PSL stage.

The proposed technique currently applies only to systems without latent variables. However, we believe that a straightforward modification building on the work of \([33]\) will address this limitation.

5 Experimental Evaluation

5.1 Problem Description

To demonstrate the training of DeepPSL we consider a zero shot learning problem. Similar to \([35]\) and \([32]\), we formally define the zero shot learning problem as: Let \( U \in \mathbb{R}^{d \times m} \) be a matrix containing \( m \) training instances with each instance being a feature vector \( u \) of dimensions \( d \), and \( Y \in \{0, 1\}^{m \times z} \) denote the ground truth label of each training instance belonging to one of \( z \in C \) classes. The goal of zero shot learning is to construct a classifier using this training data and then classify a data set with the same feature space but with classes in a set of \( z' \in C' \) where \( C \cap C' = \emptyset \).

\(^3\)Please note that, in general, convexity of \( f \) is not sufficient to ensure continuous dependence of \( \arg \min_y \tilde{f}(u, w, y) \) on \( w \). However, \( \arg \min_y \tilde{f}(u, w, y) + \nu ||y - y_t||^2 \) for any \( \nu > 0 \) is augmented to ensure uniqueness of the solution and so continuous dependence on \( w \).
To evaluate DeepPSL, we examine the attribute based classification approach to ZSL. We assume attribute information is available for the classes in the training as well as the test data. Specifically, we have a matrix $A \in [0,1]^{z \times a}$ and a matrix $A' \in [0,1]^{z' \times a}$. The matrix may contain boolean entries indicating presence or absence of an attribute or it could contain continuous value in $[0,1]$ indicating a soft link between attribute and class.

5.2 Dataset for evaluation

We evaluate DeepPSL for the zero shot learning problem on two benchmark datasets for image classification: AWA2 and CUB. Along with images with associated class labels, each of these data sets has a class-attribute matrix that provides attribute information for each class. These attributes have continuous values between 0 and 1 that represent the strength of association between the attributes and classes.

AWA2 dataset [35] consists of 50 animal classes and 85 numeric attribute values for each class. It also provides 2048 dimensional feature representations for each image extracted from a ResNet 101 model [36] pre-trained on ILSVRC2012. CUB-200-2011 [37] is a fine grained data set consisting of 200 categories of birds with 312 binary attributes annotated for each image.

The authors of [35] have proposed new splits on the above mentioned datasets to ensure that the test classes do not overlap with the classes used for pre-training the ResNet and have supplied the ResNet101 features for both datasets. They have re-evaluated various state of the art (SOTA) approaches using features from pre-trained ResNet as common inputs to obtain a fair comparison. In our experiments, we use the same proposed splits and ResNet features to facilitate comparison with other approaches.

5.3 DeepPSL architecture

The information available in the class-attribute matrix was used to create first order rules in PSL for both the training and test classes. Each attribute corresponds to a predicate in DeepPSL and the rules on each attribute were created as below:

$$
\theta_i: A_i(img) \rightarrow Label(img, "c")
$$

$$
\theta'_i: !A_i(img) \rightarrow !Label(img, "c")
$$

where, $A_i$ is a predicate corresponding to the $i_{th}$ attribute and $\theta_i$ represents the weight of the rule. The weights for a rule can be set to the association between the attribute $A_i$ and class $c$. Alternately, the $\theta_i$ can be converted into a binary quantity (1 or 0) representing the presence or absence of attribute $A_i$ in class $c$. These discrete weights can be set by thresholding on the global mean of the attribute matrix as suggested in [25].

The DeepPSL architecture consists of a neural network that infers the predicates in the PSL program described above. We used a neural net with one hidden layer of 512 nodes for both data sets. The neural net takes in a 2048 dimensional feature vector of an image as input and produces a vector with dimensionality equal to the number of attributes, i.e. 85 for AWA2 and 312 for CUB respectively. We use ELU activation at the hidden layer and sigmoid activation at the output layer. We call this neural network the Attribute Extraction Network and the produced output the Attribute Vector. Each element in the attribute vector corresponds to an attribute in the image. These attribute predictions serve as observed random variables that are fed into the PSL program. The class label of an image is inferred from the attributes by minimising the HL-MRF potential.

5.4 Training and Results

The DeepPSL system is trained end-to-end using Algorithm[1]. The training was performed on a MacBook Pro with 2.6 GHZ Intel i7 processor with 2 of the 6 cores being used during training.
In accordance with the argument presented in [27], the error in inferred classes per training image is computed with Hinge rank loss:

\[
L_1 = \sum_{j=1, j \neq \text{label}}^{z} \max(\text{margin} - y_{\text{label}} + y_j, 0.0)
\]

(11)

Where \(y_{\text{label}}\) is the inferred value for the target class and the \(y_j\) are the inferred values for the other classes. The ranking loss margin used was 0.3 for AWA2 data and 0.1 for CUB data set. In our implementation, we process the training samples in batches. We accumulate \(L_1\) losses over all the training samples in a batch and backpropagate on the accumulated loss to update the weights in the Attribute Extraction Network using Adam optimizer. From experimentation, it was found that using \(N = 1\) in Algorithm 1 gave the best results for both data sets. The rule weights were set using the continuous attribute information. Further details of the hyper-parameters used during training on various datasets can be found in Table 1. For these conditions, average training time for the AWA2 data set was 33 minutes and 89 minutes for the CUB data set.

Table 1: Hyper-parameters used for training

| Global Execution Parameters | |
|----------------------------|---|
| Batch size                 | 32 |
| Num Epochs                 | 10 |

| Inference Parameters       | |
|----------------------------|---|
| Optimizer                  | SGD |
| Optimizer LR               | 5e-3 |
| Optimizer Loss change threshold | 1e-6 |
| Optimizer Max iterations   | 5000 |

| Training Parameters        | |
|----------------------------|---|
| Alpha                      | 1e-4 |
| Optimizer                  | Adam |
| Adam Learning rate (\(\eta\)) | 1e-3 |
| Adam \((\beta_1, \beta_2), \epsilon\) (used default in pytorch) | (0.9, 0.999), 1e-8 |
| Adam weight decay          | 0 |
| Num update steps \((N)\)   | 1 |

To evaluate the trained DeepPSL model on the test data, we replace the PSL program with rules corresponding to the test classes and process the feature vector of each test image through the DeepPSL model. We compute the top 1 class-averaged accuracy and compare against the accuracy of DAP and IAP as published in [35]. For DeepPSL we report the 95% confidence interval of the mean accuracy on the test set calculated from 10 repetitions of the same experiment. For the other approaches we do not provide the confidence intervals as it is not available in [35].

Table 2 lists the performance of DAP, IAP and DeepPSL on AWA2 and CUB datasets. DeepPSL has better performance than either of these approaches for AWA2. For CUB data, DeepPSL is comparable to DAP and outperforms IAP. While using the same information on attributes, DeepPSL provides significant gain in accuracy. More importantly, it demonstrates that the DeepPSL is effectively trained using the proposed algorithm and is capable of learning intermediate features (in the form of predicates) required for the final task of ZSL. For sake of completeness, Table 2 also includes performance of other approaches as reported in [35]. We note that using extremely simple attribute based PSL ruleset, the proposed method outperforms many state of the art approaches. Notably, our method competes well with approaches that use additional information such as word embeddings, hierarchical embeddings or use special regularisers to enhance attribute based learning.

We further perform an ablation study where we compare DeepPSL with a two stage approach. The attribute extraction network is trained independently with the target vector defined by the discrete attribute information for each class. For evaluating on the test images, attributes are predicted using the attribute extraction network and then passed into the PSL ruleset to infer the class label. The
Table 2: Comparison of results on Zero Shot Learning with various SOTA approaches

| Method  | AWA2 | CUB  |
|---------|------|------|
| DAP     | 46.1 | 40.0 |
| IAP     | 35.9 | 24.0 |
| **DeepPSL** | **52.3 ± 2.2** | **41.2 ± 2.1** |
| CONSE   | 44.6 | 33.6 |
| CMT     | 37.9 | 34.6 |
| SSE     | 61.0 | 43.9 |
| LATEM   | 55.8 | 49.6 |
| ALE     | 62.5 | 54.9 |
| DEVISE  | 59.7 | 52.0 |
| SJE     | 61.9 | 53.9 |
| ESZSL   | 58.6 | 51.9 |
| SYNC    | 49.3 | 56.0 |
| SAE     | 54.1 | 33.3 |
| GFZSL   | 63.8 | 49.3 |

Table 3: Comparison of ZSL results with different setups on AWA2 and CUB

| Method                               | AWA2     | CUB     |
|--------------------------------------|----------|---------|
| Separate Att. classifier + PSL       | 40.7 ± 1.1 | 23.9 ± 0.6 |
| **DeepPSL**                          | **52.3 ± 2.2** | **41.2 ± 2.1** |

A neural network architecture used is the same as in the DeepPSL experiment. We refer to this system as Separate Att. classifier + PSL. From the results in Table 3 we can see that DeepPSL performs significantly better than the method using independent training of attribute extraction model despite the same deep learning architecture being used in both cases.

6 Conclusions and Future Work

We introduced DeepPSL a variant of Probabilistic Soft Logic (PSL) to produce an end-to-end trainable system that integrates reasoning and perception. In DeepPSL, some of the PSL predicates are replaced by deep learning models. We highlighted the challenge in training such a system and proposed a novel algorithm to enable end-to-end training. Experimental results demonstrate successful training of an attribute based DeepPSL system for zero shot learning. Further DeepPSL outperformed classical attribute based methods for zero shot learning and achieved competitive performance with other state of the art methods that use additional information such as embeddings. Ablation studies show that these benefits are not just from constraints applied at inference time but are directly attributable to the end-to-end training.

PSL has demonstrated excellent performance on collective labeling and link prediction tasks in which predicted labels are not independent (unlike the zero shot learning case). Our preliminary work demonstrates that DeepPSL is also effective in these settings.

The work described in this paper does not address latent variables. Future work will report on extensions of DeepPSL to the latent variable case.

7 Broader Impact

DeepPSL promises to allow the integration of prior knowledge into Deep learning at scale. We are optimistic that this can reduce both the data and computational needs to train effective AI systems with the potential to decrease the associated environmental impact. On the other hand DeepPSL itself imposes additional computational burdens which may nullify this benefit or exacerbate the environmental impact.
DeepPSL could be used as a final "control layer" in deep networks to enforce constraints on models behavior. For example, it could be used to limit the likelihood of high-risk outputs or it could be used to directly impose rules on the behavior of the deep network. This mechanism could be used to mitigate biased or unfair outcomes from such systems. Alternately these same mechanisms could be used to introduce biases where none previously existed.

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