Unconventional superconducting states induced in a ferromagnet by a $d$-wave superconductor

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We develop a quasi-classical theory for the superconducting proximity effect in a ballistic ferromagnetic layer in contact with a $d$-wave superconductor. In agreement with recent experiments we find that the density of states oscillates around the normal state value with varying the thickness of the ferromagnetic layer. We show that the phase, the amplitude, and the period of these oscillations depend on the orientation of the superconductor. This effect reveals spatial oscillations and anisotropy of the induced superconducting correlations in the ferromagnet.

Proximity effects in hybrid structures of superconductors and ferromagnets provide the possibility for the controlled studies of the coexistence of ferromagnetism and superconductivity. Superconducting correlations penetrate into normal metals by means of a special scattering process at the normal metal-superconductor (NS) interface known as Andreev reflection: an electron propagating in the normal metal can cross the NS boundary by reflecting as a hole and transferring a Cooper pair into the superconductor. The Andreev reflected hole is correlated with the incident electron and carries information about the phase of the order parameter of the superconductor. The existence of correlated electron-hole pairs corresponds to a non-vanishing superconducting pair amplitude (order parameter), which gives to the normal metal the typical characteristics of the superconducting states.

The peculiarity of this proximity effect in ferromagnetic metals comes from the fact that the Andreev reflection from a singlet pairing superconductor is accompanied by an inversion of the spin direction, and consequently the correlated electron-hole occupy opposite spin bands. The spin splitting exchange field of the ferromagnet causes a phase shift of the correlated Andreev electron-hole pairs, which results in a spatially oscillatory behavior of the induced pair amplitude. Such an inhomogeneous superconducting state coexisting with ferromagnetism is strikingly similar to an FFLO state in a bulk superconductor with spin splitting originally predicted by Fulde and Ferrell and by Larkin and Ovchinnikov.

One of the most interesting manifestations of the oscillations in the proximity pair amplitude is the oscillatory behavior of the density of states (DOS) in ferromagnet-superconductor bilayers as a function of the ferromagnetic layer thickness. Kontos et al. have observed this effect experimentally in thin ferromagnetic layers connected to a conventional $s$-wave superconductor. Theoretically, the superconducting proximity effect in ferromagnetic layers has been studied extensively. It has been shown that the theory based on the quasi-classical formalism is in a quantitative agreement with the experiments.

In parallel, there has been much attention to the proximity of a ferromagnet to high-$T_c$ superconductors, which are widely believed to have a dominant $d$-wave pairing symmetry. Most of the studies have concentrated on the influence of the exchange field on the zero bias conductance peak (ZBCP) as the most remarkable feature in the tunneling spectroscopy of a junction between a high-$T_c$ superconductor and a normal metal. ZBCP is the result of a zero-energy Andreev peak (ZEAP) in the DOS, which originates from change in the sign of the $d$-wave order parameter under a $\pi/2$ rotation. Many effects including a splitting of the ZBCP and the developing of a zero bias conductance dip (ZBCD) have been found and attributed to the exchange splitting.

Most recently, Freamat and Ng have performed tunneling spectroscopy measurement on multilayered junctions of $d$-wave superconductor and ferromagnets. They observed an oscillatory behavior in the tunneling conductance spectra with the thickness of the ferromagnetic domain. This effect has been considered as the $d$-wave analog of the effect observed in and explained as the signature for an FFLO state induced in the ferromagnetic layers by the $d$-wave superconducting layer. However one would expect that the anisotropy and the sign change of the $d$-wave superconducting order parameter leads to an anisotropy and phase effects in the oscillation of the DOS, which distinguish it from the case of a conventional $s$-wave superconductor. The goal of the present work is to study this effect theoretically.

In this letter we study the proximity effect at the interface between a ferromagnetic metal and a $d$-wave superconductor. We model the ferromagnet as a thin ballistic film with rough boundaries which is connected to the superconductor through a disordered interface of finite transparency. We make use of the quasi-classical formalism to calculate the density of states in the ferromagnetic layer. In correspondence with the recent experiments, we find that at sufficiently strong exchange field the DOS oscillates as a function of the thickness of the ferromagnetic layer. We show that the phase, the amplitude and the period of the DOS oscillations are affected by changing the orientation of the $d$-wave order parameter. This effect, which reveals the highly anisotropic nature of the induced superconducting correlations in the ferromagnet, can also be used as a further test of the $d$-wave scenario in high-$T_c$ superconductors.

We consider the proximity system shown in Fig. 1. A thin ferromagnetic layer ($F$) of thickness $d$ is contacted by a $d$-wave superconductor ($S$) on one side and covered on the other side by an insulator. We consider a clean structure at both of the S and F sides of the FS-interface. At the same time the FS-interface itself may contain disorder and band mismatch which enhance backscattering of quasi-particles from
this boundary. F is characterized by a mean exchange field $h$ which is assumed to be constant in the F-layer and vanishes in S. S is characterized by a $d_{x^2-y^2}$-wave order parameter of the form $\Delta_d(\theta) = \Delta \cos[2(\theta - \chi)]$, where $\theta$ is the angle that the Fermi velocity $v_F$ makes with the normal to the interface and $\chi$ is the angle between the crystallographic $a$ axis of S and the interface normal. For calculation we adopt the Eliashberg equations (1) in the clean limit for this system. In the absence of spin-flip scattering in F, transport of quasi-particles with spin $\sigma (= \pm 1)$ is described by an independent equation for the corresponding matrix Green’s function $\hat{g}_\sigma(E, v_F, r)$, which reads

$$-i v_F \nabla \hat{g}_\sigma(E, v_F, r) = [(E + \sigma h(E))\hat{\tau}_3$$
$$-i \hat{\tau}_2 \Delta(r, v_F), \hat{g}_\sigma(E, v_F, r)]. \quad (1)$$

Here $\hat{\tau}_i$ denote the Pauli matrices and $\Delta(r, v_F)$ is the superconducting order parameter, which is taken to be real. We neglect the spatial variation of the order parameter close to the FS-interface and take $\Delta(r, v_F) = \Delta_d(v_F)$ in all points of S.

Equation (1) can be conveniently solved along a classical trajectory. A typical trajectory is shown in Fig. 1. It starts at S and extends into the F-layer making an angle $\theta_1$ with the normal to the FS-interface. After several reflections from the insulator and the FS interface, which makes a path of total length $l$ in F, the trajectory returns into S at an angle $\theta_2$.

The superconducting order parameter that an electron feels in the beginning and at the end of the trajectory are respectively $\Delta_d(\theta_1)$ and $\Delta_d(\theta_2)$. We have solved Eq. (1) and found that the DOS on a given trajectory only depends on $\theta_1$, $\theta_2$, and $l$ and given by

$$N(E, \theta_1, \theta_2, l) = \frac{N_0}{4} \sum_{\sigma=\pm 1} \text{Re}[\text{tr} \hat{\tau}_3 \hat{g}_\sigma(E, v_F, r)] = \frac{N_0}{2} \sum_{\sigma=\pm 1} \text{Re}\{ -i \tan[\kappa_{\sigma} l/v_F + \alpha(\theta_1) + \alpha(\theta_2) + \eta(\theta_1, \theta_2)/2] \}, \quad (2)$$

where $N_0$ is DOS at the Fermi level in the normal state, $\alpha(\theta) = \arcsin[E/|\Delta_d(\theta)|]/2$, $\kappa_{\sigma} = (E + \sigma h)/v_F$ and $\eta(\theta_1, \theta_2) = \pi[1 - \text{sign}(\Delta_d(\theta_1)\Delta_d(\theta_2))]/2$ is the $\pi$ phase shift resulting from the possible change in the sign of the order parameter at the beginning and the end of the trajectory.

The total density of states $N(E)$ is obtained by averaging the DOS per trajectory Eq. (2) over all the possible values of $\theta_1, \theta_2$, and $l$:

$$N(E) = \int dl d\theta_1 d\theta_2 \bar{p}(l, \theta_1, \theta_2) N(E, l, \theta_1, \theta_2), \quad (3)$$

where $\bar{p}(l, \theta_1, \theta_2)$ is the distribution function of the trajectories.

To determine $\bar{p}(l, \theta_1, \theta_2)$, we model F as a weakly disordered thin layer bounded by a rough surface and a rough FS-interface with a constant transparency $t$. We introduce $g(\theta_1, \theta_2)$ as the correlation function between the incoming and outgoing directions $\theta_1$ and $\theta_2$. This function determines the specularity of the scattering from the boundaries. For perfectly flat boundaries with purely specular scattering, $\theta_1$ and $\theta_2$ are completely correlated and $g \sim \delta(\theta_1 + \theta_2)$. For the boundaries with strong roughness the dominantly diffusive scattering destroys any correlation between $\theta_1$ and $\theta_2$, and $g(\theta_1, \theta_2)$ is constant. In the general case of an arbitrary strength of the roughness we assume $g = C^{-1} \exp(-(\theta_1 + \theta_2)^2/(\pi z)^2)$, where $C = \int_{-\pi/2}^{\pi/2} d\theta_1 d\theta_2 g(\theta_1, \theta_2)$ is a normalization factor and $z$ measures the degree of the roughness. The limits of $z \ll 1$ and $z \sim 1$ correspond to weak and strong roughness respectively. As we will see below the parameter $z$ play a crucial role to reveal the anisotropic nature of the induced superconducting correlations in the F-layer.

The full distribution function $\bar{p}(l, \theta_1, \theta_2)$ can be written in terms of $g(\theta_1, \theta_2)$ as

$$\bar{p}(l, \theta_1, \theta_2) = g(\theta_1, \theta_2)[l \delta(l - \frac{d}{\cos \theta_1} - \frac{d}{\cos \theta_2})$$
$$+r \rho(l - \frac{d}{\cos \theta_1} - \frac{d}{\cos \theta_2})], \quad (4)$$

where $p(l)$ is the length distribution of the trajectories regardless of the values of $\theta_1$ and $\theta_2$, and $r = 1-t$. For $t = 1$ the delta function in Eq. (4) assure that the total length of the trajectory is given by $l = d/\cos \theta_1 + d/\cos \theta_2$. In the limit of small transparency of the FS-interface $t \ll 1$, $p(l)$ can be

![FIG. 1: Sketch of a ferromagnetic layer (F) in connection with a $d$-wave superconductor (S).](Image)
approximated by the exponentially decaying function

\[ p(l) = \frac{1}{\bar{\ell}} e^{-l/\bar{\ell}}\Theta(l - 2d), \tag{5} \]

where \( \bar{\ell} \approx 2d \ln(\ell_{\text{imp}}/d)/t = 2d \ell_t \) is the mean trajectory length in the limit of small \( d/\ell_{\text{imp}} \) (\( \ell_{\text{imp}} \) is the elastic mean free path).

Combining Eqs. 2, 3, 4, and 5 we find the following result for the total DOS

\[ N(E) = \frac{N_0}{2} \sum_{\sigma = \pm 1} \sum_{n = -\infty}^{\infty} [t + rp(2nk_d)] J_n, \tag{6} \]

where \( p(k) = e^{-i2kd}/(ik\bar{\ell} + 1) \) is the Fourier transform of \( p(l) \) and

\[ J_n(k) = \int_{-\pi/2}^{\pi/2} d\theta_1 d\theta_2 g(\theta_1, \theta_2) f_n(\theta_1)f_n(\theta_2) e^{-i\pi n(\theta_1, \theta_2)}, \]

\[ f_n(\theta) = \begin{cases} e^{-i2nk_d/d \cos \theta + i2n \phi(\theta)} & \text{for } |E| \leq |\Delta_d(\theta)|, \\ e^{-i2nk_d/d \cos \theta - 2|n|\beta(\theta)} & \text{for } |E| > |\Delta_d(\theta)|. \end{cases} \]

Here \( \phi(\theta) = \arccos [E/|\Delta_d(\theta)|]/2 \) is the Andreev phase and \( \beta(\theta) = \arccos[E/|\Delta_d(\theta)|]/2 \) is the Andreev phase.

Equation 6 expresses the total DOS in terms of known functions. In real samples the F-layer is expected to have a nonuniform thickness due to the large scale roughness of the boundaries. To take this into account we average 6 over a Gaussian distribution of the thicknesses around a mean value \( d \). This leads to a smearing of the sharp features in the DOS which arise from the lower cutoff in the length distribution. The qualitative behavior, however, will not be changed. In practice, we have taken the width of the distribution to be of order 10%, which corresponds to the conditions of the experiments 3, 13.

Let us start by analyzing the effect of a weak exchange field on the proximity DOS of a thin F-layer with \( d \ll v_F/\Delta \) and \( t \ll 1 \). For the orientation \( \chi = \pi/4 \) the main feature in the DOS of a normal layer (\( h = 0 \)) is a sharp ZEAP for any strength of the roughness \( z \). In Fig. 2a the normalized DOS \( N(E)/N_0 \) versus \( d \hbar/v_F \) is plotted for different energies and values \( \chi = \pi/4, z = 0.5 \). The effect of the exchange field is to split the ZEAP into distinct peaks at finite energies. Increasing \( h \) further leads to the shifting of the splitted peaks toward higher energies, which is associated with decreasing the height of these peaks. The DOS at \( E = 0 \) decreases with \( h \) and goes through a minimum as \( d \hbar/v_F \) is increasing. Thus the exchange field can suppress the ZEAP in the DOS and develop a dip at \( E = 0 \). This will lead to a ZBCD in the conductance spectra at low temperatures, which also has been observed in the experiments 12.

For the orientation \( \chi = 0 \) the ZEAP is absent for perfectly specular boundaries (\( z = 0 \)). The ZEAP, however, appears when \( z \) is finite. This is the result of diffusive scattering caused by the roughness at the boundaries, which allows for the sign change of the superconducting order parameter at two sides of a sizable fraction of trajectories even when \( \chi = 0 \). In Fig. 2b we plot the normalized DOS \( N(0)/N_0 \) at \( E = 0 \) versus \( d \hbar/v_F \) when \( \chi = 0 \) and for different values of \( z \). For the normal layer (\( h = 0 \)) the height of the ZEAP increases with increasing \( z \). The effect of the exchange field for a given value of \( z \) is to suppress the superconducting features (zero DOS for \( z = 0 \) and ZEAP for finite \( z \) ) in the DOS. At higher exchange fields when \( d \hbar/v_F \gtrsim 1 \), the zero energy DOS for all values of \( z \) approach each other and becomes close to the normal state values \( N_0 \).

At strong exchange fields when \( dh/v_F \gtrsim 1 \), the DOS at all energies oscillates around the normal state values \( N_0 \) as a function of \( dh/v_F \). This oscillatory behavior of induced su-

![FIG. 2: Effect of weak exchange fields on the density of states when the transparency of the FS-interface is small (here \( t = 0.1 \)). Normalized DOS \( N(E)/N_0 \) as a function of \( d \hbar/v_F \) at different energies when \( \chi = \pi/4 \) and \( z = 0.5 \), and \( \beta \) zero energy when \( \chi = 0 \) and for different strength of the roughness of boundaries \( z \). The exchange field suppresses the typical superconducting features of the DOS (the zero energy Andreev peak when \( \chi = \pi/4 \), and also when \( \chi = 0 \) if the boundaries are rough, and the vanishing DOS at \( E = 0 \) when the boundaries are specular and \( \chi = 0 \)).](image)

![FIG. 3: Oscillations of the proximity density of states with the F-layer thickness \( d \). Dependence of the normalized DOS \( N(E)/N_0 \) on \( d \) when the interface is highly transparent (\( t = 1 \)) and \( h = 3\Delta \), \( \chi = \pi/4 \), \( z = 1 \).](image)
perconducting correlations has been observed in the experiments [13]. Fig. 3 represents these oscillations resulting from our calculations. The energy dependence of the normalized DOS $N(E)/N_0$ is plotted for different values of the F-layer thickness when the FS-interface is highly transparent ($t = 1$), $h = 3\Delta$ and $\chi = \pi/4$. We have taken $z = 1$, modeling rough boundaries. In this case the period of the DOS oscillations is roughly $\pi/4$.

Let us now analyze the effect of the $d$-wave symmetry of the superconducting order parameter on the oscillations of the DOS in F. For this we examine the oscillatory behavior of the DOS at the Fermi level ($E = 0$) for different orientations of S. We consider the case of $t \ll 1$ where for $h \gtrsim v_F/d$ the deviations of the DOS from the normal state value $N_0$ are of order $t$. In Fig. 4 we have plotted the reduced DOS $[N(0)/N_0 - 1]/t$ versus $dh/v_F$ for the case of specular boundaries ($z = 0$) at different $\chi$. The variation of $\chi$ affects the phase and the amplitude of the oscillations. Varying $\chi$ from 0 to $\pi/4$, the phase is shifted by $\pi$ and the amplitude is decreased. The $\pi$ phase shift originates from the sign change of the order parameter, which occurs for all the trajectories in specular boundaries if $\chi = \pi/4$. We note that the period of the oscillations is almost the same for different $\chi$ and equals to $\pi/2$, the period for the case where S has a $s$-wave order parameter [7].

The effect of an anisotropic order parameter is more pronounced for the case of diffusive boundaries. This is shown in Fig. 4 where the DOS oscillations at $E = 0$ are presented for the case of $z = 1$. Now, in addition to the variation of the phase and the amplitude, by changing the S orientation from $\chi = 0$ to $\chi = \pi/4$ the period of the oscillations is also changed. Fig. 4: shows how the change in the period occurs when the strength of the roughness $z$ varies for $\chi = \pi/4$. We can understand these effects by noting that the spatially oscillating order parameter in the F-layer has a direction-dependent amplitude and phase resulting from the anisotropy and the sign change of the $d$-wave order parameter of S. The change in the phase, amplitude and the period of the DOS oscillations is the result of unconventional induced superconducting correlations.

In conclusion, we have investigated theoretically the superconducting proximity effect in a thin ferromagnetic layer in contact with a $d$-wave superconductor. In correspondence with recent experiments [15] we have found that at sufficiently strong exchange fields the density of states in the ferromagnet oscillates around the normal state value as a function of its thickness. The phase, the amplitude, and the period of the oscillations depend on the orientation of the superconductor. This direction-dependence is the signature of an unconventional oscillatory superconducting state induced in the ferromagnet by the proximity to the $d$-wave superconductor.

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