Coulomb Distortion in Quasielastic ($e, e'$) Scattering on Nuclei: 
Effective Momentum Approximation and Beyond

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Abstract

The role of the effective momentum approximation to disentangle Coulomb distortion effects in quasielastic ($e, e'$) reactions is investigated. The separation of the cross section in longitudinal and transverse components is discussed including higher order DWBA corrections due to the focusing of the electron waves. The experimental studies performed, in the last few years, making use of different approximate treatments are shown to be sometime inconsistent. As a consequence some of the longitudinal and transverse responses, extracted from the inclusive cross sections cannot be considered reliable. A separation procedure based on the effective momentum approximation is discussed in connection with the recent experimental data on electron/positron quasielastic scattering on $^{12}$C and $^{208}$Pb.

Pacs: 25.30.Fj

Keywords: Quasielastic electron scattering, longitudinal/transverse responses, Coulomb corrections.
I. INTRODUCTION

The quasielastic electron scattering off nuclei has represented, in the last 20 years, one of the most successful tools to study nuclear and nucleon structure properties. Both inclusive \((e, e')\) and exclusive (single arm \((e, e', N)\) or double arm \((e, e', N, N)\)) contributed to a deeper understanding of the many-body structure of strongly interacting systems like light and heavier nuclei opening the possibility of investigating also the \textit{in medium} nucleon properties.

In particular the quenching of the longitudinal strength in inclusive reactions \cite{1} has been related to partial restoration of chiral symmetry in nuclei \cite{2} combined with effects due to many-body short-range correlations in dense matter \cite{3}. Similar results have been recently obtained within a relativistic RPA approach taking into account the \textit{in medium} modifications of the nucleon structure as described by a quark-meson coupling model \cite{4}.

However the experimental studies of inclusive and exclusive reactions induced by electrons have an intrinsic limitation in the case of target nuclei with a large number of protons. The strong Coulomb field induces a distortion of the wave front which modifies the structure of the \((e, e')\) cross section and induces sizable effects in the longitudinal/transverse separation of the electromagnetic responses \cite{5–8}.

The theoretical framework to investigate Coulomb corrections to the electron-nucleus cross sections is well established \cite{5} and is called Distorted Wave Born Approximation (DWBA) in contrast to the better known Plane Wave Born Approximation (PWBA) where the incoming and outgoing charged leptons are described by (Dirac) plane wave neglecting the effect of the Coulomb interaction between the projectile and the target nucleus. The application of the DWBA scheme to the quasielastic \((e, e')\) regime is in principle straightforward \cite{6}, even if the numerical complications are extremely time consuming. Moreover the DWBA cross section cannot be written in a (Rosenbluth) separable form to extract charge (longitudinal) and current (transverse) responses: a property valid in PWBA only. As a consequence the direct numerical application of the DWBA approach cannot help in the separation of the structure functions \cite{3–8}. 
The aim of the present work is to demonstrate that an Effective Momentum Approximation (EMA) can be defined and used, even in heavier nuclei, to disentangle Coulomb corrections from the experimental cross section once the effective value of the Coulomb interaction between the electron and the nucleus is experimentally determined. Focusing corrections are automatically included at the lowest order of the EMA and higher order corrections can be estimated both theoretically and experimentally.

In section II the concept of effective momentum transfer is reviewed emphasizing how the Mott cross section can be factorized out in quasielastic scattering. In section III higher order corrections are investigated and the mean value of the Coulomb interaction discussed in view of recent experimental results. Numerical approaches are discussed in section IV and the experimental analysis of \((e,e')\) quasielastic data revised in section V. Conclusions are drawn in section VI.

II. THE EFFECTIVE MOMENTUM TRANSFER

Since the \((e,e')\) DWBA cross section does not assume a separable form, longitudinal and transverse components can be extracted, in heavy nuclei, only approximately and with the help of theoretical assumptions. The milestones of this path have been indicated by several authors in the past and I would like to follow their main arguments to demonstrate that the (approximated) PWBA-like form of the DWBA cross section must have a structure related to a specific physical ingredient: the effective momentum transfer. This quantity can be defined only in a phenomenological way because it is connected to an asymptotic expansion of the cross section, and embodies leading corrections to the PWBA cross section, on top of which one has to consider higher order effects, if relevant.

A. The eikonal approximation

Czyż and Gottfried [9], in their seminal work, discussed the breakdown of the PWBA for heavy nuclei and concluded that “one may readily and quite reliably correct for this”.


They defined the effective momentum transfer in their eq.(2.10)

\[
q_{\text{eff}} = k_{i,\text{eff}} - k_{f,\text{eff}} = q + \left( \hat{k}_i - \hat{k}_f \right) \bar{V}_C, \tag{1}
\]

where \( q = k_i - k_f \) is the kinematical momentum transfer as measured in the laboratory frame where the initial (final) electron momentum \( k_i \) (\( k_f \)) is determined. \( \bar{V}_C \) represents the Coulomb interaction energy between the electron and the target nucleus so that its effective momentum in the vicinity of the nucleus becomes: \( k_{i,f,\text{eff}} = k_{i,f} - \hat{k}_{i,f} \bar{V}_C. \) (\( E_{i,f} = |k_{i,f}| \) are the energies of the incoming and outgoing electrons as measured in the lab frame and whose masses are neglected in the high-energy limit).

The DWBA cross section as calculated in ref. [9] (eq.(2.11)) is found "identical to the cross section in Born Approximation, except for the displacement

\[
q^2 \rightarrow q_{\text{eff}}^2 = \omega^2 + 4 \left( E_i - \bar{V}_C \right) \left( E_f - \bar{V}_C \right) \sin^2 \theta/2. \tag{2}
\]

By means of a Taylor expansion the Coulomb interaction energy \( \bar{V}_C \) is approximated by \( \bar{V}_C(0) \), i.e. the energy at the center of the nucleus \( (\bar{V}_C(0) = -3/2 Ze^2/R \) for a hard sphere model of the nucleus with charge \( Ze \) and radius \( R \)).

The conclusions of Czyż and Gottfried are questionable as well as their definition of EMA. In particular one can notice that the displacement \( [2] \) implies a modification of the Mott cross section

\[
\sigma_{\text{Mott}} = 4 \alpha^2 \frac{E_i^2}{q^4} \cos^2 \theta/2 \rightarrow 4 \alpha^2 \frac{E_i^2}{q_{\text{eff}}^4} \cos^2 \theta/2, \tag{3}
\]

as can be seen from their eq.(2.11). This is an artifact originating from the form of the eikonal approximation assumed by Czyż and Gottfried as will be discussed in the next section [11B2].

B. High-Energy analytical solutions
1. The approach of Yennie, Boos and Ravenhall

Yennie, Boos and Ravenhall [10] derived a three-dimensional approximation to high-energy electron scattering on nuclei extracting an analytical expression valid in the vicinity of the nucleus. The method employed an asymptotic expansion in inverse powers of $qR$; the electron wave function does not keep the plane wave form and both amplitude and phase contain several contributions. In particular current conservation introduces a factor $k_{\text{eff}}/k$ which modifies the electron wave functions at lowest order, an effect not considered by Czyż and Gottfried and which has deep consequences on the modifications of the cross section as I am going to illustrate.

2. The synthesis of Rosenfelder and the Mott cross section

Rosenfelder [11], in his comprehensive paper on quasielastic electron scattering, discussed also Coulomb corrections making use of a high-energy electron wave function [12] based on the formulation due to Yennie et al. Referring to their own work [12], he wrote that “for high-energy electrons the distorted wave can be approximated by

$$
\psi_k(r) = \frac{|k_{\text{eff}}|}{|k|} e^{i k_{\text{eff}} \cdot r} \text{ with } k_{\text{eff}} = k - \hat{k}V_C ,
$$

where $V_C$ is a mean value of the electrostatic potential of the nucleus ($V_C \approx -3 Z\alpha/2 R$ with $R = (5/3)^{1/3} \langle r^2 \rangle^{1/2}$ for a nucleus with charge $Z$ and rms-radius $\langle r^2 \rangle^{1/2}$). The net effect is the replacement $q \rightarrow q_{\text{eff}}$ as argument in the structure functions. Note that the amplitude factor $|k_{\text{eff}}|/|k|$ makes sure that the Mott cross section remains unchanged.

The synthesis proposed by Rosenfelder has many practical consequences, namely:

i) the leading form of the electron wave function (4) incorporates, in addition to the effective momentum, the change in amplitude due to the focusing of the wave front also discussed by Yennie et al.;

ii) both incoming and outgoing leading focusing corrections to the electron wave functions concur to preserve the Mott cross section in its classical form. In fact the two terms
factorize in calculating the \((e,e')\) cross section and are absorbed in the Mott expression: an important difference with respect the simplified assumptions of Czyż and Gottfried (cf. eq.(3)). In detail:

\[
d\sigma = 4\alpha^2 \frac{\cos^2 \theta/2}{q^2} \left( \frac{|k_{i,eff}|}{|k_i|} \right)^2 d\mathbf{k}_f \sum_n \frac{1}{2} \sum_{\lambda_i,\lambda_f} |W_{n0}|^2 \delta (E_{n0} - \omega) = \\
= 4\alpha^2 \frac{\cos^2 \theta/2}{q^4} E_f^2 dE_f d\Omega_f \sum_n \frac{1}{2} \sum_{\lambda_i,\lambda_f} |W_{n0}|^2 \delta (E_{n0} - \omega) = \\
\equiv \sigma_{\text{Mott}} dE_f d\Omega_f \sum_n \frac{1}{2} \sum_{\lambda_i,\lambda_f} |W_{n0}|^2 \delta (E_{n0} - \omega) ,
\]

where

\[
W_{n0} = \frac{1}{\cos \theta/2} \int d\mathbf{x} e^{i\mathbf{q}_{eff} \cdot \mathbf{x}} \bar{u}_{\lambda_i}(\mathbf{k}_{i,eff}) \gamma_{\mu} u_{\lambda_f}(\mathbf{k}_{f,eff}) \langle n|J_{\mu}(\mathbf{q}_{eff})|0\rangle
\]

is the usual matrix element of the transition current for free electrons with momenta \(k_{i,f,eff}\).

On the contrary the Effective Momentum Approximation procedure proposed by Czyż and Gottfried would imply a modification of the Mott cross section which must be further compensated by considering the renormalization of the incoming and outgoing electron flux before comparing theory with data. The emphasis I am giving to this point is not academic; the confusion on that specific aspect is at the origin of incorrect experimental analysis as I will discuss in section V C.

iii) Rosenfelder mentions, as interaction energy to be used in the definition of the effective electron momentum and energy, a \textit{mean} value of the Coulomb potential; a choice which differs from the popular assumption of the value at the origin \(V_C(0)\) (a formal mathematical consequence of the expansion of the wave function). The practical value he assumes is again the central value of an hard sphere model (cf. his discussion after eq.(3)), but the intuition is basically correct and I will discuss this aspect again in section III C.

III. DWBA CROSS SECTION
A. Higher Order effects

The contribution we gave to the problem of finding an approximate expression for the DWBA cross section is strongly based on the path summarized in the previous points. The step forward made is the inclusion of the relevant additional focusing terms beyond the simple leading factors $|k_{i,f,\text{eff}}|/|k_{i,f}|$ of eq.(4), terms which modify the phase of the electron waves as discussed by Yennie et al. \[10\] and Lenz and Rosenfelder \[12\] (cf. sections II B 1 and II B 2). Generalizing a method proposed by Knoll \[14\] for the investigation of the transition form factors to discrete states, the analytic solution of ref. \[12\] has been used to expand the DWBA matrix elements in terms of the Born solution and its derivative with respect the momentum transfer and applied to exclusive $(e,e',p)$ as well as inclusive $(e,e')$ quasielastic scattering \[7,8\]. The approach has been developed up to second order in $Z\alpha$ and leads to an approximated but transparent way of writing the DWBA $(e,e')$ cross section, namely:

$$
\frac{d\sigma}{dE_d\Omega_f} \approx \sigma_{\text{Mott}} \left\{ \left( \frac{q_{\text{eff}}^2}{q^2} \right)^2 S_L(q_{\text{eff}},\omega) \left[ 1 + \Delta_L(q_{\text{eff}},\omega,E_{i,\text{eff}}) \right] + \left[ -\frac{q_{\text{eff}}^2}{2q_{\text{eff}}^2} + \tan^2 \frac{\theta}{2} \right] S_T(q_{\text{eff}},\omega) \left[ 1 + \Delta_T(q_{\text{eff}},\omega,E_{i,\text{eff}}) \right] + S_{\text{int}}(q_{\text{eff}},\omega,E_{i,\text{eff}}) \right\} .
$$

(7)

Equation (7) is, as a matter of fact, close to the Rosenfelder’s conclusions because $\sigma_{\text{Mott}}$ assumes its classical form (cf. eq.(3)) and the effective momentum transfer $q_{\text{eff}}$ replaces the kinematical momentum $q$ as argument in the structure functions. However additional modifications appear and they are embodied in the terms $\Delta_L$, $\Delta_T$ and in a Longitudinal-Transverse interference contribution $S_{\text{int}}(q_{\text{eff}},\omega,E_{i,\text{eff}})$. All these terms derive from higher order focusing contributions in the high-energy expansion of the electron waves and prevent the separability of the DWBA cross section. The size of their contribution is crucial to understand the limit of the PWBA approximation and the role of the effective momentum transfer.

A detailed calculation performed in a simple model of quasielastic scattering \[15\], suggests that the interference contribution $S_{\text{int}}$ is negligible in the whole kinematical range of interest also for nuclei as large as $^{208}\text{Pb}$ ($\lesssim 0.01\%$ with respect to $S_L$ and $S_T$) and also
the contributions $\Delta_L$ and $\Delta_T$ are rather small (remaining within 0.5% in the quasielastic peak region and reaching 2 - 4% for the high-$\omega$ region and forward angles or for low-$\omega$ and backwards angles). These small deviations can play some minor role in the longitudinal transverse separation of the cross section and for a discussion I refer the reader to the papers of ref. [15]. Of course the estimation of the absolute values of $\Delta_L$, $\Delta_T$ and $S_{\text{int}}$ are model dependent and they can differ for more sophisticated models of ($e,e'$) reactions. However the relative sizes are much more independent and the conclusion on their tiny contributions can be considered reliable.

**B. EMA: the result of an asymptotic expansion**

The marginal role of higher order corrections reduces the cross section (7) to a simplified and separable form valid (in particular) for medium-weight nuclei:

$$\left. \frac{d\sigma}{dE_2d\Omega_f} \right|_{\text{DWBA}} \approx \left. \frac{d\sigma}{dE_2d\Omega_f} \right|_{\text{EMA}} = \sigma_{\text{Mott}} \left\{ \left( \frac{q_{\text{eff}}^2}{q_{\text{eff}}^2} \right)^2 S_L(q_{\text{eff}},\omega) + \left[ - \frac{q_{\text{eff}}^2}{2q_{\text{eff}}^2} + \tan^2 \frac{\theta}{2} \right] S_T(q_{\text{eff}},\omega) \right\}. $$

(8)

I will call the approximation (8) Effective Momentum Approximation (EMA) in analogy with my previous works [8,15]. However let me stress that the explicit form of the electron wave function responsible for the reduction (8) contains also the flux renormalization factors of eq.(4) in order to preserve current conservation, a factor which also preserves the classical form of the Mott cross section (cf. eq.(5)). Another interesting point must be kept in mind: the expansion which produces the analytical result (7) is an asymptotic expansion. The effective momentum transfer $q_{\text{eff}}$ has to be chosen close to the ”real” momentum transfer (which differs from the kinematical momentum $q = k_i - k_f$ as measured in the laboratory) in such a way that the transition matrix elements of the nuclear current become smooth functions in the neighbourhood of $r = 0$ and the coefficients of the expansion tend soon to zero [14]. Since the effective momentum is a phenomenological quantity, its value has to be deduced from experimental evidences and eventually justified, from a theoretical point
of view, only á posteriori. That is why most of the authors followed the mathematical
guide, due to the expansion around \( r = 0 \), by choosing \( V_C(0) \) as correction terms in the
definitions (I) and (II). The way to know the ”real” momentum transfer in quasieelastic
scattering off heavy nuclei is to measure it so that eq.(8) assumes all its relevance only after
the experimental determination of \( \bar{V}_C \).

C. The effective momentum from experiments

Guèye et al. reported on a dedicated experiment [16] performed at the Saclay linear
accelerator and recently published [17]. Inclusive quasielastic \((e,e')\) cross sections on \(^{12}\text{C}\) and
\(^{208}\text{Pb}\) have been measured using electron and positron beams in order to investigate charge
dependent Coulomb corrections. Guèye et al. have been able to measure both the lowest
order correction (determining the Coulomb interaction energy \( \bar{V}_C \)) and higher order effects.
These last contributions turn out to be quite small (\( \sim 3\% \)) once the effective kinematics is
extracted from the data and the EMA of eq.(8) used to determine the total response. At the
same time the experiment shows that the Coulomb potential energy related to the effective
kinematics is quite close to the average

\[
\bar{V}_C = \frac{\int d^3r V_C(r) \rho_{\text{charge}}(r)}{\int d^3r \rho_{\text{charge}}(r)}
\]  

an observation which definitely substantiates the mean value idea proposed by Rosenfelder
(cf. section II B 2). In particular, in the case of for \(^{208}\text{Pb}\), \( |\bar{V}_C| = 18.9 \pm 1.5 \) MeV from the
experiment and \( |\bar{V}_C| = 20.1 \) MeV from eq.(9), (while \( |V_C(0)| = 25.9 \) MeV).

A few comments are in order:

i) the experiment of Guèye et al. corroborates the EMA scheme discussed in the previous
sections;

ii) the information on the ”real” value of the momentum transfer or, equivalently, of the
average Coulomb interaction energy, validates eq.(8) as the approximated separable

\[ \bar{V}_C = \bar{V}_C(0) \]
form of the \((e,e')\) cross section for medium-weight and heavy nuclei as long as few percent residual effects (due to higher order focusing effects) can be neglected.

IV. COULOMB CORRECTIONS: THE NUMERICAL APPROACH

A rigorous treatment of Coulomb distortion can be performed by means of a direct numerical calculation of the DWBA matrix elements of the nuclear current.

A. The DWBA calculation of Co’ and Heisenberg

The first complete numerical attempt for quasielastic scattering is due to Co’ and Heisenberg [1] and they conclude that the separability of the cross section is definitely lost in DWBA, in particular for the transverse response. A reliable model of the nuclear excitation in the quasielastic region is needed in order to extract the correction factors due to Coulomb distortions. This pessimistic conclusion on the model independence of the response functions is due to the comparison of their complete calculation with the PWBA results and to the idea that focusing contributions, added by the DWBA description, cannot be disentangled with the necessary precision in a model independent way. In fact Co’ and Heisenberg assume that the eikonal approximation \(a la\) Czyż and Gottfried (cf. section II A) is the only analytic approximation one can make. The relevance of the flux renormalization effects, included in the DWBA and not in the eikonal approach, prevents the definition of an effective momentum approximation while the nature of the numerical solution cannot manifest the structure of cross section in DWBA.

However the contribution of Co’ and Heisenberg remains fundamental. The conclusion that the Rosenbluth plot of the DWBA appears to be linear despite the non separability of the cross section is illuminating. At that time such information was known [7], but within an approach including terms up to \((Z\alpha)^2\) only and not for a complete DWBA calculation. The fact that the usual Rosenbluth plot of a complete DWBA calculation shows a rather close linearity [8-10] demonstrates that experimental evidence of linearity is not a sufficient
condition for the separation of the cross section. A large effect of the Coulomb distortion on the Rosenbluth representation is a rotation of the straight line whose intercept and slope are no longer connected with the longitudinal and transverse responses.

B. 1996: the approach of Kim et al.

Also Kim et al. [18] have the advantage of an exact DWBA numerical solution, but they have a more ambitious project: extracting an approximated form of the cross section which is not as time consuming as the complete DWBA procedure but able to reproduce the exact results keeping, at the same time, a separable form. The main ingredient is the local momentum transfer. In fact the momentum of the electron, in the external Coulomb field, can be rigorously defined as a local ($r$-dependent) quantity only. Consequently a Local Effective Momentum Approximation (LEMA) is considered in ref. [18] as leading approximation instead of the EMA which involves an average over the nuclear volume. To transform this scheme into a simple form of DWBA cross section the authors need a certain number of ad hoc assumptions such as:

i) the cross section is $\ddagger$ a priori assumed to have a separable form of Rosenbluth type. Interference contributions like those included in eq.(7), are simply not considered. Moreover the structure functions, which depend, to a good approximation, on the effective momentum and energy transfer only, (cf. eq.(8)) are charged of an artificial dependence on the kinematical conditions (incident energies and angles) due to the choice of factorizing the Rosenbluth terms $\left(\frac{q^2}{q_T}\right)^2$ and $\left[-\frac{q^2}{2q_T} + \tan^2 \frac{\theta}{2}\right]$ instead of the averaged quantities $\left(\frac{q^2_{\text{eff}}}{q_T}\right)^2$ and $\left[-\frac{q^2_{\text{eff}}}{2q_{\text{eff}}} + \tan^2 \frac{\theta}{2}\right]$;

\[1\] The analytic approach of ref. [15] shows that the specific error is, in practice, irrelevant because the interference contributions are generally negligible. However this conclusion cannot be drawn from a numerical calculation and is, in any case, valid $\ddagger$ a posteriori only.

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ii) the local effective momentum transfer is assumed along the kinematical momentum transfer, a choice valid for elastic scattering only. More precisely, the inelastic effective momentum of eq. (11) can be written

\[
q_{\text{eff}} = q + \left( \hat{k}_i - \hat{k}_f \right) \vec{V}_C = q \left( 1 - \frac{\vec{V}_C}{E_i} \right) + \frac{\omega}{E_i} \vec{V}_C \hat{k}_f ,
\]

and becomes \( q_{\text{eff}} = q \left( 1 - \frac{\vec{V}_C}{E_i} \right) \) only for \( \omega = 0 \).

The assumption of Kim et al. introduces deviations comparable with higher order focusing corrections (in particular at backward angles). As an example, for \( \theta = 140^\circ \) and momentum transfer as high as 400 MeV/c, the cross section is enhanced by \( \approx 4\% \) on top of the quasielastic peak and reduced by \( \approx 20\% \) at higher energy transfer (when its value reaches \( 1/3 \) of the maximum);

iii) several ad hoc factors are introduced in the approximated LEMA expression to reproduce the DWBA results. Most of them are tuned on the DWBA electron cross sections and they can induce quite different effects in the case of positron scattering partially explaining the discrepancy of the LEMA calculation by Kim et al. with the experimental results of ref. [17].

C. faiblesse of the numerical approach

The merit of a complete numerical approach is obvious; nevertheless there is a point de faiblesse, already visible in the paper by Co’ and Heisenberg. It is determined by the intrinsic difficulty in separating higher order (focusing) effects from the simple flux renormalization due to the factors \( |k_{i,f,eff}|/|k_{i,f}| \) of eq. (14) (both the effects are of course included in the complete solution). The natural leading reduction of a full DWBA numerical calculation appears to be the eikonal approximation of Czyż and Gottfried (cf. section IA) with the consequence of a change of the Mott cross section and the need of a renormalization of incoming and outgoing flux. This is a complicated procedure which can become source of errors in the analysis of the experimental data, as I will show in the next section.
V. ANALYSIS OF THE EXPERIMENTAL DATA

Some of the experimental data on quasielastic electron scattering for medium weight and heavy nuclei have been analyzed including Coulomb distortion effects. In particular the Bates experiment on $^{238}\text{U}$ $^{[19]}$, the Saclay data on $^{208}\text{Pb}$ $^{[20]}$, the reanalysis of Jourdan $^{[21]}$, and the $^{40}\text{Ca}$ experiment at Bates $^{[22]}$. In this section I will summarize the situation to ask for new analysis which include Coulomb effects in a more consistent and/or more reliable way.

A. Bates data on $^{238}\text{U}$

The first attempt of obtaining quasielastic $(e,e')$ data on an heavy nucleus dates back to an experiment performed at Bates on $^{238}\text{U}$ $^{[19]}$. The data have been analyed by means of an effective momentum transfer. However the approximation adopted was just a generalization of the scheme known for elastic scattering and the effective momentum was chosen to be along the kinematical momentum transfer, a choice valid for elastic scattering only as discussed already in section $^{[13]}$ (cf. eq. (10)).

Also focusing corrections were included by adapting a phase shift code used for elastic scattering. The details of the procedure are discussed neither in the article nor in the PhD thesis of Blatchley. The approach, however, has the merit of a first attempt even if manifestly insufficient for a complete analysis.

B. Saclay data on $^{208}\text{Pb}$

The Saclay data on $^{208}\text{Pb}$ $^{[20]}$ have been analyzed including the quasielastic effective momentum and higher order corrections systematically. In particular the EMA in the form given by eq.$^8$ is used, for the first time, as leading order approximation to disentangle Coulomb corrections. Higher order effects are also discussed and included within the approximations of ref. $^8$. These approximations are, however, too severe to give a quantitative
account of higher order contributions and a more rigorous treatment of the transition matrix elements shows [15] that the effects are smaller. Also in the case of $^{208}$Pb a reanalysis is, therefore, useful even if the expected modifications cannot change the qualitative conclusions drawn in 1994.

C. Jourdan’s data analysis and the Bates experiment on $^{40}$Ca

I discuss the two analysis together for two reasons: i) they refer to medium-weight nuclei; ii) they both make use of the Coulomb distortion analysis proposed by Kim et al., i.e. the use of a numerical approach. In the first case the method and the corrections suggested by Kim et al. have been applied to existing data on $^{12}$C [23], $^{40}$Ca [24,25] and $^{56}$Fe [25,26], while in the latter Jin, Wright and Onley coauthored the paper on the experiment.

Both papers discuss first the problem of flux renormalization due to leading order focusing effects (i.e. the factors $|k_{i,\text{f,eff}}|/|k_{i,\text{f}}|$ I discussed in section II B 2). The complications induced by a numerical approach appear immediately: the correction factor chosen to renormalize the cross section data is $(|k_i|/|k_{i,\text{eff}}|)^2$ a choice which involves the incident energies only and it has no theoretical justification. In fact if the EMA of eq.(8) is accepted as leading approximation, the Mott cross section should be kept unchanged and higher order effects estimated. On the contrary if one prefers to obtain the cross section in the eikonal approximation of Czyż and Gottfried because this is the leading part of the numerical calculation, both the incoming and outgoing electron waves must be renormalized and not the incoming flux only (cf. section II B 2). Actually a possible origin of the incorrect normalization procedure is in the way of writing the Mott cross section in both papers, namely $\left(\frac{\alpha \cos \theta/2}{2E_i \sin \theta/2}\right)^2$.  

More precisely the two papers describe the application of two opposite procedures: the data of the $(e,e')$ cross section are reduced by a factor $(|k_i|/|k_{i,\text{eff}}|)^2$ in the Jourdan analysis and increased by the same quantity in the Bates paper: however a closer look at the data analysis supports the idea of a misprint in the paper on $^{40}$Ca [27].
The correct expression which distinguishes the contributions of two different regions (the interaction volume and the detector) reads \[ \left( \frac{2\Delta E \alpha \cos \theta/2}{q^2} \right)^2 \] and it reduces to the previous form only if Coulomb corrections are negligible. In fact the term \( E^2 \) in the numerator of eq.(3) originates from the detection volume \( d\mathbf{k}_f = E^2 d\Omega_f dE_f \) in the Lab, while the propagator of the virtual photon, \( \sim \frac{1}{q^2} \), involves the interaction region where the motion of the electron is influenced by the Coulomb potential and is to be modified to \( \sim \frac{1}{q_{\text{eff}}^2} \) as stated by Czyż and Gottfried (see discussion in section II A). The inclusion of the leading focusing terms of eq.(4) compensate such a modification as already discussed (cf. eq.(3)).

The manifest inconsistent treatment of Coulomb distortions influences the conclusions of the analysis of refs. [21,22]; a reanalysis would be welcome.

VI. FINAL REMARKS AND CONCLUSION

The structure of the DWBA cross section can be reduced (up to order \((Z\alpha)^2\)) to the form (7). The contributions \( \Delta_{L,T} \) and \( S_{\text{int}} \) are due to higher order focusing effects of the electron waves in the proximity of the nucleus, in particular to its phase deformation. On the contrary the renormalization of the electron waves due to current conservation is a leading order effect and can be incorporated in a simple form (the expression (9)) known as Effective Momentum Approximation (EMA) where only higher order effects are neglected. The fact that in eq.(8) the Mott cross section keeps its classical expression is just a byproduct of current conservation. The Effective Momentum Approximation is a good scheme to interpret inclusive data as experimentally verified in the recent analysis of electron and positron quasielastic scattering [17] and theoretically predicted in a series of papers [8,15].

Quasielastic data should be reanalyzed within such scheme in a consistent way to include those Coulomb distortion effects which give sizable contributions in the separation of the cross section in longitudinal and transverse components. The recent application [21,22] of more complete numerical DWBA results [18] shows a clear inconsistency and the data on longitudinal/transverse structure functions cannot be considered reliable, a remark con-
firmed by the comparison of the calculation by Kim *et al.* with the experimental data on Coulomb corrections measured comparing quasielastic scattering by electrons and positrons off $^{12}$C and $^{208}$Pb [17].

**ACKNOWLEDGMENTS**

Interesting discussions with J. Morgenstern and a useful correspondence with C. Williamson and J. Jourdan are gratefully acknowledged. I thank G. Orlandini and V. Vento for a critical reading of the manuscript.
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