Orthogonal Compactly Supported Near-Symmetric Wavelets in Denoising Satellite Images

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Abstract

Objectives: This is to investigate the performance of two orthogonal, compactly supported, near symmetric wavelet families’ viz., coiflets and symlets in denoising satellite images. Methods/Statistical Analysis: We use the Stationary Wavelet Transform (SWT) which is a shift-invariant transform and three levels of image decomposition. Additive White Gaussian Noise (AWGN) is used to produce the corrupted images used for the study. Findings: The study identified the most suitable wavelet in each of the above two wavelet families, for denoising satellite images. A comparison between the denoising performances of these wavelet families has also been made. It has been found that the denoising performance of wavelets belonging to both the families decrease as the wavelet order increases. Applications/Improvements: The study finds application in the denoising of satellite images and images obtained by space exploration and such images corrupted by a combination of several types of noise distributions.

Keywords: Compact Support, Denoising, Gaussian, Near-Symmetry, Orthogonal

1. Introduction

‘Noise’ in an image refers to unwanted signals that corrupt the image. Noise in digital images, fall under various categories like Poisson noise, Gaussian noise, Salt and Pepper noise, Rician noise and Speckle noise. Each type of noise is characterized by the Probability Density Function (pdf) peculiar to it. Image denoising intends to create an image with no noise at all, from a noisy image. It is a major topic of research in the area of present-day signal processing. Images may become noisy at any or all of the stages viz., image acquisition, and transmission or processing. As a result of contamination due to noise the message or data conveyed by the image becomes erroneous. This leads to major drawbacks like wrong interpretation of the images from surveillance radar or mishaps like erroneous diagnosis of a brain tumor. Noisy satellite images are mostly distorted and hence they cannot be studied properly³. Denoising of space exploration images is also an important concern. Therefore image denoising continues to be a vital area of research. There have been various jumps in the area of research in image denoising; these include the change from spatial filtering to frequency domain filtering and to wavelet domain denoising. For image denoising the use of wavelet transform is found to be better than spatial domain techniques or the Fourier Transform (FT), there are abundant numbers of wavelets belonging to several wavelet families, with different features.

A lot of research has been carried out in this field. Coiflet Wavelet is better than Daubechies wavelet for image denoising². But coiflets are near – symmetric wavelets while Daubechies wavelets are far from being symmetric.Symlet gives better denoising and visual quality than Haar or Db2 wavelets for medical images¹. Sym5 gives best denoising for natural images⁴.

The present study unveils image denoising characteristics of wavelets belonging to two families’ viz., the Coiflet family and the Symlet family, in denoising satellite images.

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The major desirable features of wavelets include symmetry or nearness to symmetry, orthogonality, compact support and availability of good number of vanishing moments. If a wavelet is orthogonal the computation of wavelet coefficients is extremely simple. ‘Near-symmetry’ implies closeness to linear phase and results in visually pleasant reconstructed image that closely resembles the original. When the number of vanishing moments is high it enables a more sparse representation of the signal. When a wavelet is ‘compactly supported’ its filters are of the Finite Impulse Response (FIR) type, hence always stable and easily realizable. Coiflets and Symlets possess these good features. It is reasonably expected that these good qualities will reflect in the desirability of these families of wavelets in denoising images. Investigation of their denoising characteristics hence gains importance. We also compare the image denoising characteristics of these wavelet families and also study the variation in their denoising performance with variation in the wavelets’ order.

2. Materials and Methods

2.1 Type of Noise Used

We used Additive White Gaussian Noise (AWGN) of ‘0’ mean value and variance 0.5 in a 0 - 1 scale. AWGN is the most common type of noise present in satellite images. It is also the type of noise present in most natural images. This type of noise gets its name ‘Gaussian’ because it has a Gaussian pdf. The Gaussian pdf is described by the expression:

$$F(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$  \hspace{1cm} (1)

Where $x$ is the gray level, $\sigma$ is the noise standard deviation, and $\mu$ is the mean value.

The random values of the noise get directly added to the image pixels since the noise is ‘additive.’ The term ‘white’ implies that it contains all frequencies. A Gaussian pdf can effectively approximate a large number of random variables of different pdfs as what is postulated by the central limit theorem. This implies that the results obtained in this study can be effectively applied to situations in which an image contains noise of several types of distributions.

2.2 Wavelets

A wavelet $\psi(t)$ is a small wave with zero mean value, rapidly decaying to zero and having no constant frequency. Wavelets are generated from a single function $\psi(x)$ called the ‘mother wavelet’ by means of its dilations and translations according to the relation:

$$\psi_{u,v}(x) = |u|^{-\frac{1}{2}} \psi\left(\frac{x-v}{u}\right), \hspace{0.5cm} u \neq 0, \hspace{0.5cm} u, v \in \mathbb{R}$$  \hspace{1cm} (2)

Where $\psi_{u,v}(x)$ the wavelets generated, $u$ is the dilation parameter and $v$ is the translation parameter. Wavelets help us to get multi resolution of an image, i.e., to get the minute as well as coarse details in the image. No other technique so far invented can provide this unique feature. This technique of Multi Resolution Analysis (MRA) is made possible by the inherent capability of wavelets to be dilated and translated.

2.2.1 Coiflets

Coiflets are compactly supported orthogonal wavelets having the highest number of vanishing moments for both wavelet function and scaling function for a given support width. They have a filter length of 6N where N is the order of the wavelet and are nearly symmetric. Figure 1 shows the first five wavelets in the coiflet family, viz., coif 1, coif 2, coif 3, coif 4 and coif 5 which we consider for the study.

2.2.2 Symlets

These are compactly supported orthogonal wavelets having the least asymmetry and highest number of vanishing moments for a given support width. Symlets have filter length 2N where N is the order of the wavelet. Figure 2 shows the first five wavelets in the symlet family used in this study, viz., sym 2, sym 3, sym 4, sym 5 and sym 6.

Figure 1. (a) coif 1, (b) coif 2, (c) coif 3, (d) coif 4 (e) coif 5.

Figure 2. (a) Sym 2, (b) sym 3 (c) sym 4 (d) sym 5 (e) sym 6.
2.3 Wavelet Transform

A transform making use of a wavelet as its basis function is termed ‘wavelet transform’. For a long time the FT has been the tool for image processing. The FT has the basis function $e^{j\omega t}$ which is the combination of two infinitely oscillating sinusoids. With FT we can know which all frequency components are present in the signal. But since its basis function extends over all time, it is impossible to know the time instant at which any particular frequency component exists. In other words, with FT we get frequency resolution but not time resolution. But a wavelet transform can provide both time and frequency resolutions. We can choose any wavelet from an abundant set of wavelets with different desirable features, as the basis function in a wavelet transform whereas such a choice of basis function is impossible with FT. Also, a sparse representation of a function is possible using wavelet transform. This fact along with the ability for MRA increases the usefulness of wavelet transform. A wavelet transform is also useful for effective denoising of images.

The classical wavelet transform which is the Discrete Wavelet Transform (DWT) involves down sampling and hence it is prone to appearance of artifacts in the reconstructed image. This undesirable effect can be prevented if down sampling is avoided. The Stationary Wavelet Transform (SWT) is a transform which avoids down sampling; we use SWT for this study.

2.4 Algorithm for Denoising

The algorithm for denoising involves the following steps:
1. Decompose the noisy image to an optimum number of levels using the desired wavelet transform employing a chosen wavelet.
2. Apply a chosen thresholding rule to the coefficients that result in step 1.
3. Reconstruct the image from the coefficients that succeed step 2.

2.5 Threshold Rule

The major threshold rules are (i) hard threshold and (ii) soft threshold. Hard threshold results in abrupt discontinuities whereas soft threshold precludes occurrence of such discontinuities. In this work we adopt the soft fixed form threshold rule. Soft threshold follows the formula:

$$W_b = \begin{cases} \text{sgn}(w)(|w| - t), & |w| \geq t \\ 0, & |w| < t \end{cases}$$

Where $\text{sgn}(w)$ is the signum function defined as

$$\text{sgn}(w) = \begin{cases} 1, & w > 0 \\ 0, & w = 0 \\ -1, & w < 0 \end{cases}$$

$t$ is the threshold value, and $w$ and $W_b$ represents the wavelet coefficients before thresholding and after thresholding respectively.

We adopt the fixed form value given by the ‘universal threshold’ for $t$, i.e.,

$$t = s \times \sqrt{2 \log MN}$$

Where $s$ is the standard deviation of noise and $M$ and $N$ represent the number of rows and columns respectively in the digital image.

2.6 Levels of Decomposition

The denoising operation involves decomposition of the image using the wavelet transform, in to two types of coefficients viz., the approximation coefficients which consist of the output of the low pass filter and detail coefficients in the horizontal, vertical and diagonal orientations which constitute outputs of the high pass filter, at each level of decomposition. Noise manifests mostly as high frequency components; hence thresholding is applied only to detail coefficients. In this work we carry out three such levels of decomposition and thresholding to remove as much noise as possible. In every thresholding operation a small portion of the signal also gets removed, hence if we further increase the number of levels of decomposition the reconstructed image gets blurred.

2.7 Metrics of Performance

The denoising performance is assessed in terms of the Mean Squared Error (MSE) and the Peak Signal to Noise Ratio (PSNR). These are defined as

$$\text{MSE} = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} (x(i,j) - y(i,j))^2$$

Where $x(i,j)$ represents the pixel at the $i^{th}$ row and $j^{th}$ column in the original image and $y(i,j)$ represents the pixel at the corresponding position in the denoised image, $M$ and $N$ represent the number of rows and columns in either image

And

$$\text{PSNR} = 10 \log_{10} \left( \frac{255^2}{\text{MSE}} \right)$$
3. Results and Discussions

For our study we use the images of (i) a valley in Mars and (ii) a volcanic mountain called Arsia Mons in Mars, both captured by India’s Mangalyan Mars Mission satellite, and the image of (iii) polar region of Moon, captured by India’s Chandrayan. Each of these images has a size of 632x576. The original images are shown in Figures 3(a), Figure 3(b) and Figure 3(c) and the corresponding noisy images are shown in Figures 3(d), Figure 3(e) and Figure 3(f). For convenience we represent the noisy images as J1, J2 and J3 respectively and the corresponding denoised images as N1, N2 and N3. Table 1 shows the values of MSE and PSNR of the noisy images and the MSE and PSNR values of the images obtained on denoising these images using the symlets sym 2, sym 3, sym 4, sym 5 and sym 6. Table 2 shows the values of MSE and PSNR of the denoised images obtained using the coiflets coif 1, coif 2, coif 3, coif 4 and coif 5. The plots of MSE vs. the different symlets / coiflets for the images N1, N2 and N3 are shown in Figures 4, Figure 5 and Figure 6 respectively.

Figure 3. Original images and images corrupted by AWGN of ‘0’ mean and variance 0.05: (i) (a),(d) mars valley; (ii) (b),(e) Arsia mons; (iii) (c),(f) polar region of moon.

Figure 4. Plots of MSE vs. symlets / coiflets for image N1.

The X-axes of all these plots show five divisions. These divisions represent the five coiflet wavelets viz., coif 1, coif 2, coif 3, coif 4 and coif 5. These divisions also stand for the five symlet wavelets viz., sym 2, sym 3, sym 4, sym 5 and sym 6. The Y-axes of these figures show the values of MSE of the respective denoised images got by denoising using symlets /coiflets.
From Figure 4 we can make the following observations:

1. the lowest MSE for image N1 is given by sym 2
2. coif 1 gives an MSE which is only slightly higher than the MSE given by sym 2
3. the other symlets give better denoising compared to the coiflets; and
4. as the order of the symlet or coiflet wavelet increases, the denoising gets poor as represented by increase in MSE values obtained with those wavelets.

The plots in Figure 5 and Figure 6 confirm these observations. It is also noted that as seen in Figure 6 the increase in MSE for image N3 got by denoising image J3 with sym 6, from the MSE got by denoising with sym

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**Table 1.** MSE and PSNR values of noisy images and denoised images obtained with symlets

| Wavelet | Denoised images | MSE  | PSNR  | MSE  | PSNR  | MSE  | PSNR  |
|---------|----------------|------|-------|------|-------|------|-------|
| Noisy image 118.9666 | J1 | 27.3766 | 96.9346 | J2 | 28.2660 | 88.1997 | J3 | 28.6761 |
| sym 2 | N1 | 34.4954 | 32.7532 | N2 | 31.8813 | 33.0954 | N3 | 44.6382 | 31.6337 |
| sym 3 | N1 | 35.3696 | 32.6445 | N2 | 32.3830 | 33.0276 | N3 | 44.7307 | 31.6247 |
| sym 4 | N1 | 35.8683 | 32.5837 | N2 | 32.6559 | 32.9912 | N3 | 44.8876 | 31.6095 |
| sym 5 | N1 | 36.2714 | 32.5352 | N2 | 32.8523 | 32.9651 | N3 | 45.0543 | 31.5934 |
| sym 6 | N1 | 36.4982 | 32.5081 | N2 | 32.9968 | 32.9461 | N3 | 45.0673 | 31.5922 |

**Table 2.** MSE and PSNR of denoised images

| Wavelet | Denoised images | MSE  | PSNR  | MSE  | PSNR  | MSE  | PSNR  |
|---------|----------------|------|-------|------|-------|------|-------|
| coif 1 | N1 | 34.5084 | 32.7516 | N2 | 31.9314 | 33.0886 | N3 | 44.6339 | 31.6342 |
| coif 2 | N1 | 35.9894 | 32.5691 | N2 | 32.6880 | 32.9869 | N3 | 44.9115 | 31.6072 |
| coif 3 | N1 | 36.5793 | 32.4985 | N2 | 33.0756 | 32.9357 | N3 | 45.1557 | 31.5837 |
| coif 4 | N1 | 36.9056 | 32.4599 | N2 | 33.3070 | 32.9054 | N3 | 45.3561 | 31.5644 |
| coif 5 | N1 | 37.1437 | 32.4320 | N2 | 33.4842 | 32.8824 | N3 | 45.5028 | 31.5504 |

**Figure 5.** Plots of MSE vs. symlets / coiflets for image N2.

**Figure 6.** Plots of MSE vs. symlets / coiflets for image N3.
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5 is very small. This shows up as a marked deviation in the plot. The pixels of the particular noisy image J3 when convolved with the 12 filter tap values of sym 6 give coefficients whose values after thresholding result in an MSE on reconstruction, which is very close to the MSE resulting with identical operations using the 10 filter values of sym 5. The pixels of the particular image on convolution with the filter values of the other symlets do not result in MSE values which are at such close quarters.

From Table 1 we find that the lowest MSE obtained with symlets for image N1 got by denoising image J1 is 34.4954 obtained with sym 2; this means a reduction in input MSE by 71 % and an increase in PSNR by 5.3766 dB. This is a good result and shows the high suitability of sym 2 in denoising a satellite image.

From Table 1, we can also see that the lowest MSE values obtained for denoised images N2 and N3 got by denoising images J2 and J3 are 31.8813 and 44.6382 respectively. Both of these are obtained with sym 2 and represent 67.11% and 49.39% reductions in MSE values respectively. These values correspond to increases in PSNR by 4.8294 dB and 2.9576 dB respectively.

From Table 2 we can observe the following facts regarding the results got on denoising the above images using coiflets:

The values of lowest MSE got for denoised images N1, N2 and N3 are 34.5084, 31.9314 and 44.6339 respectively. These values correspond to 70.99 %, 67.06 % and 49.39 % reductions in the respective input MSE values and maximum increases in PSNR values by 5.3750 dB, 4.8226 dB and 2.9581 dB respectively. The 70.99 % in MSE reduction and the equivalent increase in PSNR by 5.3750 dB are obtained on denoising with coif 1. The performance of coif 1 approaches that of sym 2 and it is seen that like sym 2 coif 1 is also very good in denoising a satellite image. The poor performance values obtained for N3 is attributed to the large number of craters which provide a highly irregular surface for the noisy image J3. Figure 7 shows the images with lowest MSE values got on denoising with symlets and coiflets.

![Figure 7. Denoised images of lowest MSE values](image)

(a) N1 with sym 2 (b) N2 with sym 2, (c) N3 with sym 2 (d) N1 with coif 1 (e) N2 with coif 1 (f) N3 with coif 1.
4. Conclusions

In denoising satellite images with symlets, sym 2 is found to give the lowest MSE values for the denoised images. When the image has a comparatively smooth surface sym2 gives very good denoising performance giving an increase in PSNR by a value as high as 5.3766 dB corresponding to a reduction in input MSE by 71%. For satellite images coif 1 also is found to give good denoising performance. For an image with a smooth surface a decrease in input MSE by a value as large as 70.99% could be obtained with coif1. This corresponds to an increase in PSNR by 5.3750 dB. The denoising performance of coif 1 is only slightly lower than that of sym 2. When the first five symlets and the first five coiflets in the respective wavelet families are considered the denoising performance of the symlets are found to be better than the denoising performance of the coiflets. Also it is seen that as the order of the wavelet increases the denoising gets poor for both symlets and coiflets. The results obtained in this study can be effectively applied to situations in which an image contains noise which is a combination of several types of noise distributions, since we have used Gaussian noise for this study.

5. References

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