Pairwise Fairness for Ranking and Regression

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Abstract

We present pairwise fairness metrics for ranking models and regression models that form analogues of statistical fairness notions such as equal opportunity, equal accuracy, and statistical parity. Our pairwise formulation supports both discrete protected groups, and continuous protected attributes. We show that the resulting training problems can be efficiently and effectively solved using existing constrained optimization and robust optimization techniques developed for fair classification. Experiments illustrate the broad applicability and trade-offs of these methods.

Introduction

As ranking models and regression models become more prevalent and have a greater impact on people’s day-to-day lives, it is important that we develop better tools to quantify, measure, track, and improve fairness metrics for such models. A key question for ranking and regression is how to define fairness metrics. As in the binary classification setting, we believe there is not one “right” fairness definition: instead, we provide a paradigm that makes it easy to define and train for different fairness definitions, analogous to those that are popular for binary classification problems.

One key distinction is between unsupervised and supervised fairness metrics: for example, consider the task of ranking restaurants for college students who prefer cheaper restaurants, and suppose we wish to be fair to French vs Mexican restaurants. Our proposed unsupervised statistical parity constraint would require that the model be equally likely to (i) rank a French restaurant above a Mexican restaurant, and (ii) rank a Mexican restaurant above a French restaurant. In contrast, our proposed supervised equal opportunity constraint would require that the model be equally likely to (i) rank a cheap French restaurant above an expensive Mexican restaurant, and (ii) rank a cheap Mexican restaurant above an expensive French restaurant.

Like some recent work on fair ranking [Beutel et al., 2019; Kallus and Zhou, 2019], we draw inspiration from the standard learning-to-rank strategy [Liu, 2011]: we reduce the ranking problem to that of learning a binary classifier to predict the relative ordering of pairs of examples. This reduction of ranking to binary classification enables us to formulate a broad set of statistical fairness metrics, inspired by analogues in the binary classification setting, in terms of pairwise comparisons. The same general idea can actually be applied more broadly: in addition to group-based fairness in the ranking setting, we show that the same overall approach can also be applied to (i) the regression setting, or (ii) the use of continuous protected attributes instead of discrete groups. In all three of these cases, we show how to effectively train ranking models or regression models to satisfy the proposed fairness metrics, by applying state-of-the-art constrained optimization algorithms.

Ranking Pairwise Fairness Metrics

We begin by considering a standard ranking set-up [Liu, 2011]: we’re given a sample $S$ of queries drawn i.i.d. from an underlying distribution $D$, where each query is a set of candidates to be ranked, and each candidate is represented by an associated feature vector $x \in \mathcal{X}$ and label $y \in \mathcal{Y}$. The label space can be, for example, $\mathcal{Y} = \{0, 1\}$ (e.g. for click data: $y = 1$ if a result was clicked by a user, $y = 0$ otherwise), $\mathcal{Y} = \mathbb{R}$ (each result has an associated quality rating), or $\mathcal{Y} = \mathbb{N}$ (the labels are a ground truth ranking). We adopt the convention that higher labels should be ranked closer to the top. Any of these choices of label space $\mathcal{Y}$ induce a partial ordering on examples, for which all candidates belonging to the same query are totally ordered, and any two candidates $(x, y)$ and $(x', y')$ belonging to different queries are incomparable.

Suppose that we have a set of $K$ protected groups $G_1, \ldots, G_K$ partitioning the space of examples $\mathcal{X} \times \mathcal{Y}$ such that every example belongs to exactly one group. We define the group-dependent pairwise accuracy $A_{G_i, G_j}$ as the accuracy of a ranking function $f : \mathcal{X} \rightarrow \mathbb{R}$ on those pairs for which the labeled “better” example belongs to group $G_i$, and the labeled “worse” example belongs to group $G_j$. That is:

$$A_{G_i, G_j} := P(f(x) > f(x') \mid y > y', (x, y), (x', y') \in G_i, (x', y') \in G_j),$$

where $(x, y)$ and $(x', y')$ are drawn i.i.d. from the distribution of examples, restricted to the appropriate protected
groups. Notice that this definition implicitly forces us to construct pairs only from examples belonging to the same query, since \(y\) and \(y'\) are not comparable if they belong to different queries—however, the probability is taken over all such pairs, across all queries. Given \(K\) groups, one can compute the \(K \times K\) matrix of all possible \(K^2\) group-dependent pairwise accuracies. One can also measure how each group performs on average:

\[
A_{G_i > G_j} \triangleq P(f(x) > f(x') \mid y > y', (x, y) \in G_i) \quad (2)
\]

\[
A_{i > G_j} \triangleq P(f(x) > f(x') \mid y > y', (x', y') \in G_i). \quad (3)
\]

The accuracy in (2) is averaged over all pairs for which the \(G_i\) example was labeled as “better,” and the “worse” example is from any group, including \(G_i\). Similarly, (3) is the accuracy averaged over all pairs where the \(G_i\) example should not have been preferred. Lastly, the overall pairwise accuracy \(P(f(x) > f(x') \mid y > y')\) is simply the standard AUC.

Next, we use the pairwise accuracies to define supervised pairwise fairness goals and unsupervised fairness notions.

### Pairwise Equal Opportunity

We construct a pairwise equal opportunity analogue of the equal opportunity metric [Hardt et al., 2016]:

\[
A_{G_i > G_j} = \kappa, \text{ for some } \kappa \in [0, 1], \text{ for all } i, j \quad (4)
\]

Equal opportunity for binary classifiers [Hardt et al., 2016] requires positively-labeled examples to be equally likely to be predicted positively regardless of protected group membership. Similarly, this pairwise equal opportunity for ranking problems requires pairs to be equally-likely to be ranked correctly regardless of the protected group membership of both members of the pair. By symmetry, we could equally well consider \(A_{G_i > G_j}\) to be a true positive rate or a true negative rate, so there is no distinction between “equal opportunity” and “equal odds” in the ranking setting, when all of the pairwise accuracies are constrained equivalently.

Pairwise equal opportunity can be relaxed either by requiring all pairwise accuracies (i) to only be within some quantity of each other (e.g. \(\max_{i \neq j} A_{G_i > G_j} - \min_{i \neq j} A_{G_i > G_j} \leq 0.1\)), or (ii) only requiring the minimum pairwise accuracy \(A_{G_i > G_j}\) to be as big as possible (i.e. maximize \(\min_{i \neq j} A_{G_i > G_j}\)), in the style of robust optimization [e.g. Chen et al., 2017]. We will later show how models can be efficiently trained subject to both these types of pairwise fairness constraints using existing algorithms.

### Within-Group vs. Cross-Group Comparison

We have observed that labels for within-group comparisons \((i = j)\) are sometimes more accurate and consistent across raters than labels for cross-group comparisons \((i \neq j)\) can be noisier and less consistent. This especially arises when the labels are coming from experts that are more comfortable with rating candidates from certain groups. For example, consider a video ranking system where group \(i\) is sports videos and group \(j\) is cooking shows. If our experts can choose which videos they rate (as in most consumer recommendation systems with feedback), sports experts are likely to rate sports videos and do so accurately, cooking experts are likely to rate cooking shows and do so accurately, but on average we may not get as accurate ratings on pairs with a sports and cooking video.

Thus one may wish to separately constrain cross-group pairwise equal opportunity:

\[
A_{G_i > G_j} = \kappa, \text{ for some } \kappa \in [0, 1] \text{ for all } i \neq j \quad (5)
\]

and within-group pairwise equal accuracy:

\[
A_{i > G_j} = \kappa', \text{ for some } \kappa' \in [0, 1], \text{ for all } i. \quad (6)
\]

In certain applications, particularly those in which cross-group comparisons are rare or do not occur, we might want to constrain only pairwise equal accuracy (6). For example, we might want a music ranking system to be equally accurate at ranking jazz as it is at ranking country music, but avoid trying to constrain cross-group ranking accuracy because we may not have confidence in cross-group ratings.

### Marginal Equal Opportunity

The previous pairwise equal opportunity proposals are defined in terms of the \(K^2\) group-dependent pairwise accuracies. This may be too fine-grained, either for statistical significance reasons, or because the fine-grained constraints might be infeasible. To address this, we propose a looser marginal pairwise equal opportunity criterion that asks for parity for each group averaged over the other groups:

\[
A_{G_i >} = \kappa \text{ for some } \kappa \in [0, 1], \text{ for } i = 1, \ldots, K. \quad (7)
\]

### Statistical Parity

Our pairwise setup can also be used to define unsupervised fairness metrics. For any \(i \neq j\), we define pairwise statistical parity as:

\[
P(f(x) > f(x') \mid (x, y) \in G_i, (x', y') \in G_j) = \kappa. \quad (8)
\]

A pairwise statistical parity constraint requires that if two candidates are compared from different groups, then on average each group has an equal chance of being top-ranked. This constraint completely ignores the training labels, but that may be useful when groups are so different that any comparison is too apples-to-oranges to be legitimate, or if raters are not expert enough to make useful cross-group comparisons.

### Regression Pairwise Fairness Metrics

Consider the standard regression setting in which \(f : \mathcal{X} \rightarrow \mathcal{Y}\) attempts to predict a regression label for each example. For most of the following proposed regression fairness metrics, we treat higher scores as more (or less) desirable, and we seek to control how often each group gets higher scores. This asymmetric perspective is applicable if the scores confer a benefit, such as regression models that estimate credit scores or admission to college, or if the model scores dictate a penalty to be avoided, such as getting stopped by police. This asymmetry assumption that getting higher scores is either preferred (or not-preferred) is analogous to the binary classification case where a positive label is assumed to confer some benefit.

We again propose defining metrics on pairs of examples. This is not a ranking problem, so there are no queries—instead, given a training set of \(N\) examples, we compute pairwise metrics over all \(N^2\) pairs. One can sample a random subset of pairs if \(N^2\) is too large.
**Regression Equal Opportunity**

One can compute and constrain the pairwise equal opportunity metrics as in [5], [8], [9], and [7] for regression models. For example, restricting [5] constrains the model to be equally likely for all groups \( G_i \) and \( G_j \) to assign a higher score to group \( i \) examples over group \( j \) examples, if the group \( i \) example’s label is higher.

**Regression Equal Accuracy**

Promoting pairwise equal accuracy as in (6) for regression requires that, for every group, the model should be equally faithful to the pairwise ranking of any two within-group examples. This is especially useful if the regression labels of different groups originate from different communities, and have different labeling distributions. For example, suppose that all jazz music examples are rated by jazz lovers who only give 4-5 star ratings, but all classical music examples are rated by critics who give a range of 1-5 star ratings, with 5 being rare. Simply minimizing MSE alone might cause the model training to over-focus on the classical music score examples, since the classical errors are likely to be larger and hence affect the MSE more.

**Regression Statistical Parity**

For regression, the pairwise statistical parity condition described in [6] requires, “Given two randomly drawn examples from two different groups, they are equally likely to have the higher score.” One sufficient condition to guarantee pairwise statistical parity is to require the distribution of outputs \( f(X) \) for a random input \( X \) to be the same for each of the protected groups. This condition can be enforced approximately by histogram matching the output distributions for different protected groups [e.g. [Agarwal et al., 2019]].

**Regression Symmetric Equal Accuracy**

For regression problems where each group’s goal is to be accurate (rather than to score high or low), one can define symmetric pairwise fairness metrics as well, for example, the symmetric pairwise accuracy for group as \( G_i \) is \( A_{G_i,G_i} := A_{G_i} \), and one might constrain these accuracies to be the same across groups.

**Continuous Protected Features**

Suppose we have a continuous or ordered protected feature \( Z \); e.g. we may wish to constrain for fairness with respect to age, income, seniority, etc. The proposed pairwise fairness notions extend nicely to this setting by constructing the pairs based on the ordering of the protected feature, rather than protected group membership. Specifically, we change [1] to the following continuous attribute pairwise accuracies:

\[
A_\geq := P(f(x) > f(x') \mid y > y', z \geq z'),
\]

\[
A_\leq := P(f(x) > f(x') \mid y > y', z < z'),
\]

where \( z \) is the protected feature value for \((x,y)\) and \( z' \) is the protected feature value for \((x',y')\). For example, if the protected feature \( Z \) measures height, then \( A_\geq \) measures the accuracy of the model when comparing pairs where the candidate who is taller should receive a higher score.

The previously proposed pairwise fairness constraints for discrete groups have analogous definitions in this setting by replacing [1] with [9]. Pairwise equal opportunity becomes

\[
A_\geq = A_\leq.
\]

This requires, for example, that the model be equally accurate when the taller or shorter candidate should be higher ranked.

**Training for Pairwise Fairness**

We show how one can use the pairwise fairness definitions to specify a training objective, and how to optimize these objectives. We formulate the training problem for ranking and cross-group equal opportunity, but the formulation and algorithms can be applied to any of the pairwise metrics.

**Proposed Formulations**

Let \( A_{G_i,G_j}(f) \) be defined by [1] for a ranking model \( f: \mathcal{X} \to \mathbb{R} \). Let \( A_{G}(f) \) be the overall pairwise accuracy. Let \( \mathcal{F} \) be the class of models we are interested in. We formulate training with fairness goals as a constrained optimization with an allowed slack \( \epsilon \):

\[
\max_{f \in \mathcal{F}} A_{G}(f)
\]

s.t. \( A_{G_i,G_j}(f) - A_{G_k,G_l}(f) \leq \epsilon \) \( \forall i \neq j, k \neq l \). (12)

Or one can pose the robust optimization problem:

\[
\max_{f \in \mathcal{F}, \xi} \xi
\]

s.t. \( \xi \leq A_{G}(f), \xi \leq A_{G_i,G_j}(f) \) \( \forall i \neq j \). (13)

For regression problems, we replace AUC with MSE.

**Optimization Algorithms**

Both the constrained and robust optimization formulations can be written in terms of rate constraints [Goh et al., 2016] on score differences. For example, we can re-write each pairwise accuracy term as a positive prediction rate on a subset of pairs:

\[
A_{G_i,G_j}(f) = \mathbb{E} \left[ I_{f(x) - f(x') > 0} \mid ((x,y),(x',y')) \in S_{ij} \right],
\]

where \( I \) is the usual indicator function and \( S_{ij} = \{(x,y),(x',y') \mid y > y', (x,y) \in G_i, (x',y') \in G_j\} \). This enables us to adopt algorithms for binary fairness constraints to solve the optimization problems in (12) and (13).

In fact, all of the objective and constraint functions that we have considered can be handled out-of-the-box by the proxy-Lagrangian framework of [Cotter et al., 2019] and [Cotter et al., 2019a,b]. Like other constrained optimization approaches [Agarwal et al., 2018; Kearns et al., 2018], this framework learns a stochastic model that is supported on a finite set of functions in \( \mathcal{F} \). The high-level idea is to set up a min-max game, where one player minimizes over the model parameters, and the other player

\[1\]Similar to the pairwise ranking metrics, \( A_\leq \) is the true negative rate for pairs \((x,y),(x',y')\) where \( y > y' \), and by symmetry, \( A_\geq \) is also equal to the true positive rate for pairs where \( y < y' \):

\[
A_\leq = P(f(x) < f(x') \mid y < y', z > z'),
\]

\[
A_\geq = P(f(x) > f(x') \mid y > y', z < z') = TPR_{z < z'}.
\]

Therefore, [1] equates both the TPR and the TNR for both sets of pairs, and specifies both equalized odds and equal opportunity.
maximizes over a weighting \( \lambda \) on the constraint functions. \cite{cotter2019fair} use a no-regret optimization strategy for minimization over the model parameters, and a swap-regret optimization strategy for maximization over \( \lambda \), with the indicators \( \mathbb{1} \) replaced with hinge-based surrogates for the first player only. They prove that, under certain assumptions, their optimizers converge to a stochastic model that satisfies the specified constraints in expectation. In the Appendix, we present more details about the optimization approach and re-state their theoretical result for our setting.

**Related Work**

We review related work that we build upon in fair classification, and then related work on the problems addressed here: fair ranking, fair regression, and handling continuous protected attributes.

**Fair Classification**

Many statistical fairness metrics for binary classification can be written in terms of rate constraints, that is, constraints on the classifier’s positive (or negative) prediction rate for different groups \cite{goh2016fairness, narasimhan2018fairness, cotter2019fairb}. For example, the goal of demographic parity \cite{dwork2012fairness} is to ensure that the classifier’s positive prediction rate is the same across all protected groups. Similarly, the equal opportunity metric \cite{hardt2016equality} requires that true positive rates should be equal across all protected groups. Many other statistical fairness metrics can be expressed in terms of rates, e.g. equal accuracy, no worse off and no lost benefits \cite{cotter2019fairb}. Constraints on these fairness metrics can be added to the training objective for a binary classifier, then solved using constrained optimization algorithms or relaxations thereof \cite{goh2016fairness, zafar2017fairness, donini2018fairness, agarwal2018reductions}. Here, we extend this work to train ranking models and regression models with pairwise fairness constraints.

**Fair Ranking**

A majority of the previous work on fair ranking has focused on list-wise definitions for fairness that depend on the entire list of results for a given query \cite[e.g.][]{zehlike2017fair, celis2018fair, biega2018fair, singh2018fairness, zehlike2018fair, singh2019fair}. These include both unsupervised criteria that require the average exposure near the top of the ranked list to be equal for different groups \cite[e.g.][]{singh2018fairness, celis2018fair, zehlike2018fair, zehlike2019fair}, and supervised criteria that require the average exposure for a group to be proportional to the average relevance of that group’s results to the query \cite{biega2018fair, singh2018fairness, biega2019fair}. Of these, some provide post-processing algorithms for re-ranking a given ranking \cite{biega2018fair, celis2018fair, singh2018fairness, singh2019fair}, while others, like us, learn a ranking model from scratch \cite{zehlike2018fair, singh2019fair}.\cite{beutel2019fair}

**Pairwise Fairness**

\cite{beutel2019fair} propose ranking pairwise fairness definitions equivalent to those we give in (1), (2) and (3). Their work focuses on ranking and on categorical protected groups, whereas we generalize these ideas to capture a wider variety of statistical fairness notions, and generalize to regression and continuous protected features.

The training methodology is also very different. \cite{beutel2019fair} propose adding a fixed regularization term to the training objective that measures the correlation between the residual between a clicked and unclicked item and the group memberships of the items. In contrast, we enable explicitly specifying any desired pairwise fairness constraints, and then directly enforce the desired pairwise fairness criterion using constrained optimization. Their approach is parameter-free, but only because it does not give the user any way to control the trade-off between fairness vs. accuracy.

Second, \cite{beutel2019fair} consider only two protected groups, whereas we enable the user to constrain any number of groups, with the constrained optimization algorithm automatically determining how much each group must be penalized in order to satisfy the fairness constraints. A straightforward extension of the fixed regularization approach of \cite{beutel2019fair} to multiple groups would have no hyperparameters to specify how much to weight each group. One could introduce separate weighting hyperparameters to weight each group’s penalty, but then they would need to be tuned manually. The approach we propose does this tuning automatically to achieve the desired fairness constraints.

Finally, there are major experimental differences to \cite{beutel2019fair}: they provide an in-depth case study of one real-world recommendation problem, whereas we provide a broad set of experiments on public and real-world data illustrating the effectiveness on both ranking and regression problems, for categorical or continuous protected attributes.

In other recent works, \cite{kallus2019fairness} provide pairwise fairness metrics based on AUC for bipartite ranking problems, and \cite{kuhlman2019fairness} provide metrics analogous to our pairwise equal opportunity and statistical parity criteria for ranking. Both these works only consider categorical groups, whereas we also handle regression problems and continuous protected attributes. \cite{kallus2019fairness} additionally propose a post-processing optimization approach to optimize the specified metrics by fitting a monotone transformation to an existing ranking model. In contrast, we provide a more flexible approach that enables optimizing the entire model by including explicit constraints during training.

**Pinned AUC**

Pinned AUC is a fairness metric introduced by \cite{dixon2018fair}. With two protected groups, pinned AUC works by resampling the data such that each of the two groups make up 50% of the data, and then calculating the ROC AUC on the resampled dataset. Based on the well-known equivalence between ROC AUC and average pairwise accuracy, \cite{borkan2019fairness} demonstrate that pinned AUC, as well as their proposed weighted pinned AUC metric, can be decomposed as a linear combination of within-group and cross-group pairwise accuracies. In other words, both pinned AUC and weighted pinned AUC can be written as linear combinations of different pairwise accuracies \( A_{G_i,G_j} \) in (4). In our experiments, we compare against (a version of) the sampling-based approach of \cite{dixon2018fair}.
We illustrate our proposals on five ranking problems and two regression problems. We report expectations over random draws of the scoring function $f$ from the stochastic model\footnote{Code available at: https://github.com/google-research/google-research/tree/master/pairwise_fairness}.

### Fair Regression
Defining fairness metrics in a regression setting is a challenging problem, and has been studied for many years in the context of standardized testing \cite[e.g.][]{Hunter and Schmidt, 1976, Komiyama et al., 2018} consider the unfairness of a regressor in terms of the correlation between the output and a protected attribute. Pérez-Suáy et al.\footnote{[2017]} regularize to minimize the Hilbert-Schmidt independence between the protected features and model output. These definitions have the “flavor” of statistical parity, in that they attempt to remove information about the protected feature from the model’s predictions. Here, we focus more on supervised fairness notions.

Berk et al.\footnote{[2017]} propose regularizing linear regression models for the notion of fairness corresponding to the principle that similar individuals receive similar outcomes \cite{Dwork et al., 2012}. Their definitions focus on enforcing similar squared error, which fundamentally differs from our definitions in that we assume each group would prefer higher scores, not necessarily more accurate scores.

Agarwal et al.\footnote{[2019]} propose a bounded group loss definition which requires that the regression error be within an allowable limit for each group. In contrast, our pairwise equal opportunity definitions for regression do not rely on a specific regression loss, but instead are based on the ordering induced by the regression model within and across groups.

### Continuous Protected Features
Most prior work in machine learning fairness has assumed categorical protected groups, in some cases extending those tools to continuous features by bucketing \cite{Kearns et al., 2018}. Fine-grained buckets raise statistical significance challenges, and coarse-grained buckets may raise unfairness issues due to how the lines between bins are drawn, and the lack of distinctions made between element within each bin. Raff et al.\footnote{[2018]} considered continuous protected features in their tree-growing criterion that addresses fairness. Kearns et al.\footnote{[2018]} focused on statistical parity-type constraints for continuous protected features for classification. Komiyama et al.\footnote{[2018]} controlled the correlation of the model output with protected variables (which may be continuous). Mary et al.\footnote{[2019]} propose a fairness criterion for continuous attributes based on the Rényi maximum correlation coefficient. Counterfactual fairness \cite{Kusner et al., 2017, Pearl et al., 2016} requires that changing a protected attribute, while holding causally unrelated attributes constant, should not change the model output distribution, but this does not directly address issues with ranking fairness.

### Experiments
We illustrate our proposals on five ranking problems and two regression problems. We implement the constrained and robust optimization methods using the open-source Tensorflow constrained optimization toolbox of Cotter et al.\footnote{[2019a,b]}.

The datasets used are split randomly into training, validation and test sets in the ratio 1/2:1/4:1/4, with the validation set used to tune the relevant hyperparameters. For datasets with queries, we evaluate all metrics for individual queries and report the average across queries. For stochastic models, we report expectations over random draws of the scoring.

### Pairwise Fairness for Ranking
We detail the comparisons and ranking problems.

#### Comparisons
We compare against: (1) an adaptation of the debiasing scheme of Dixon et al.\footnote{[2018]} that optimizes a weighted pairwise accuracy, with the weights chosen to balance the relative label proportions within each group; (2) the recent non-pairwise ranking fairness approach by Singh and Joachims\footnote{[2018]} that re-ranks the scores of an unconstrained ranking model to satisfy a disparate impact constraint; (3) the post-processing pairwise fairness method of Kallus and Zhou\footnote{[2019]} that fits a monotone transform to an unconstrained model; and (4) the fixed regularization pairwise approach of Beutel et al.\footnote{[2019]} that like us incorporates the fairness goal into the model training. See Appendix for more details.

#### Simulated Ranking Data
For this toy ranking task with two features, there are 5,000 queries, and each query has 11 candidates. For each query, we uniformly randomly pick one of the 11 candidates to have a positive label $y = +1$ and the other 10 candidates receive a negative label $y = -1$, and we randomly assign each candidate’s protected attribute $z$ i.i.d. from a Bernoulli(0.51) distribution. Then we generate two features simulated to score how well the candidate matches the query, from a Gaussian distribution $N(\mu_{y,z}, \Sigma_{y,z})$, where $\mu_{-1} = [-1,1]$, $\mu_{1,1} = [-2,1]$, $\mu_{1,0} = [1,0]$, $\mu_{0,1} = [-1.5, 0.75]$, $\Sigma_{-1,1} = \Sigma_{1,0} = \Sigma_{-1,0} = \Sigma_{1,1} = I_2$ and $\Sigma_{1,0} + \Sigma_{-1,0} = 0.5 I_2$.

We train linear ranking functions $f : \mathbb{R}^2 \to \mathbb{R}$ and impose a cross-group equal opportunity with constrained optimization by constraining $|A_{0>1} - A_{1>0}| \leq 0.01$. For the robust optimization, we implement this goal by maximizing $\min\{A_{0>1}, A_{1>0}, AUC\}$. We also train an unconstrained model that optimizes AUC. Table 1 gives the test ranking accuracy, and the test pairwise fairness violations, measured as $|A_{0>1} - A_{1>0}|$. Only constrained optimization achieves the fairness goal, with robust optimization coming a close second. Figure 1(a)–(b) shows the $2 \times 2$ pairwise accuracy matrices. Constrained optimization satisfies the fairness constraint by lowering $A_{0>1}$ and improving $A_{1>0}$.

We also generate a second dataset with 3 groups, where the first two groups follow the same distribution as groups 0 and 1 above, and the third group examples are drawn from a Gaussian distribution $N(\mu_{y,z}, \Sigma_{y,z})$ where $\mu_{-2} = [-1,1,1]$ and $\mu_{1,2} = [1.5, 0.5, 0.5]$, and $\Sigma_{-1,2} = \Sigma_{1,2} = I_2$. We use the same number of queries and candidates as above, and assign the protected attribute $z$ to 0, 1, or 2 with probabilities 0.45, 0.1, and 0.45 respectively. We impose the marginal equal opportunity fairness goal on this dataset in two different ways: (i) constraining $\max_{i \neq j} |A_{i>2} - A_{j>2}| \leq 0.01$ with constrained optimization, and (ii) optimizing $\min\{AUC, A_{0>1}, A_{1>2}, A_{2>1}\}$ with robust optimization. We show each group’s row-marginal test accuracy in Figure 1(c)–(d). While robust optimization maximizes the minimum of the three marginals, constrained optimization...
yields a lower difference between the marginals (and does so at the cost of lower accuracies for the three groups). This is consistent with the two optimization problem set-ups: you get what you ask for.

We provide further results and an additional experiment with an in-group equal opportunity criterion in the appendix.

**Business Matching** This is a proprietary dataset from a large internet services company of ranked pairs of relevant and irrelevant businesses for different queries, for a total of 17,069 pairs. How well a query matches a candidate is represented by 41 features. We consider two protected groups, chain (C) businesses and not chain (NC) businesses. We define a candidate as a member of the chain group if its query is seeking a chain business and the candidate is a chain business. We define a candidate as a member of the non-chain group if its query is not seeking a chain business and the candidate is a non-chain business. A candidate does not belong to either group if it is chain and the query is non-chain-seeking, or vice-versa.

We experiment with imposing a marginal equal opportunity constraint: \( |A_{A_{c>}, A_{c=}}| - |A_{A_{nc>}, A_{nc=}}| \leq 0.01 \). This requires the model to be roughly as accurate at correctly matching chains as it is at matching non-chains. With robust optimization, we maximize \( \min\{A_{A_{c>}, A_{A_{nc>}}} \}, \min\{A_{A_{c=}, A_{A_{nc=}}} \}, \min\{A_{A_{c<}, A_{A_{nc<}}} \}, \min\{A_{A_{c<}, A_{A_{nc<}}} \}, \min\{A_{A_{c=}, A_{A_{nc=}}} \} \} \) with robust optimization. All methods trained a two-layer neural network model with 10 hidden nodes. As seen in Table 1 compared to the unconstrained approach, constrained optimization yields very low fairness violation, while only being marginally worse on the test AUC. The post-processing approach of [Kallus and Zhou, 2019] also achieves a similar fairness metric, but with a lower AUC. [Singh and Joachims, 2018] could not be applied to this dataset as it did not have the required query-candidate structure.

**Wiki Talk Page Comments** This public dataset contains 127,820 comments from Wikipedia Talk Pages labeled with whether or not they are toxic (i.e. contain "rude, disrespectful or unreasonable" content) [Dixon et al., 2018]. This is a dataset where debiased weighting has been effective in learning fair, unbiased classification models [Dixon et al., 2018]. We consider the task of learning a ranking function that ranks comments that are labeled toxic higher than the comments that are labeled non-toxic, in order to help the model’s users identify toxic comments. We consider the protected attribute defined by whether the term ‘gay’ appears in the comment. This is one of the many identity terms that Dixon et al. [2018] consider in their work. Among comments that have the term ‘gay’, 55% are labeled toxic, whereas among comments that do not have the term ‘gay’, only 9% are labeled toxic. We learn a convolutional neural network model with the same architecture used in Dixon et al. [2018].

We consider a cross-group equal opportunity criteria. We impose \( |A_{A_{Other>Gay}} - A_{A_{Gay>Other}}| \leq 0.01 \) with constrained optimization and maximize \( \min\{A_{A_{Other>Gay}}, A_{A_{Gay>Other}}, A_{UC}\} \) with robust optimization. The results are shown in Table 1 and Figure 1(e)-(g). Among the cross-group errors, the unconstrained model is more likely to incorrectly rank a non-toxic comment with the term ‘gay’ over a toxic comment without the term. By balancing the label proportions, debiased weighting reduces the fairness violation considerably. Constrained optimization yields even lower fairness violation (0.010 vs. 0.014), but at the cost of a slightly lower test AUC. Singh and Joachims [2018] also achieves a similar fairness metric, but with a lower AUC. Singh and Joachims [2018] failed to produce feasible solutions for this dataset, we believe because there were very few pairs per query.

**W3C Experts Search** We also evaluate our methods on the W3C Experts dataset, previously used to study disparate
We treat the percentage of black population in a community as a continuous protected attribute. We impose a cross-group equal opportunity constraint: $|A_{\text{Female}>\text{Male}} - A_{\text{Male}>\text{Female}}| \leq 0.01$. For robust optimization, we maximize $\min \{A_{\text{Female}>\text{Male}}, A_{\text{Male}>\text{Female}}, AUC\}$. As seen in Table 1, the unconstrained ranking model incurs a huge fairness violation. This is because the unconstrained model treats gender as a strong signal of expertise, and often ranks female candidates over male candidates. Not only do the constrained and robust optimization methods achieve significantly lower fairness violations, they also happen to produce higher test metrics due to the constraints acting as regularizers and reducing overfitting. On this task, Beutel et al. [2019] achieves the lowest fairness violation and the highest AUC.

The method of Singh and Joachims [2018] performs poorly because the LP that it solves per query tends to be infeasible for most queries in this dataset. Thus, to run this baseline, we extended their approach to have a per-query slack in their optimization approach which is the most flexible and direct of the strategies considered, and is very effective for achieving low pairwise fairness violations. The closest competitor to our proposals considered, and is very effective for achieving low pairwise fairness violations. The closest competitor to our proposals considered, and is very effective for achieving low pairwise fairness violations.

### Conclusions

We showed that pairwise fairness metrics can be intuitively defined to handle supervised and unsupervised notions of fairness, for ranking and regression, and for discrete and continuous protected attributes. We also showed how pairwise fairness metrics can be incorporated into training using state-of-the-art constrained optimization solvers.

Experimentally, the different methods compared often produced different trade-offs between fairness and accuracy. We do not use robust optimization as the squared error is not necessarily comparable with the regression pairwise metrics. The results are shown in Table 2.

#### Law School:
This dataset contains details of 27,234 law school students, and we predict the undergraduate GPA for a student from the student’s LSAT score, family income, full-time status, race, gender and the law school cluster the student belongs to, with gender as the protected attribute. We impose a cross-group equal opportunity constraint: $|A_{\text{Female}>\text{Male}} - A_{\text{Male}>\text{Female}}| \leq 0.01$. The constrained optimization approach successfully massively reduces the fairness violation compared to the unconstrained MSE-optimizing model, at only a small increase in MSE. It also performs strictly better than Beutel et al. [2019].

#### Communities and Crime:
This dataset has continuous labels for the per capita crime rate for a community. Once again, we treat the percentage of black population in a community as a continuous protected attribute and impose a continuous attribute equal opportunity constraint: $|A_{\text{Female}>\text{Male}} - A_{\text{Male}>\text{Female}}| \leq 0.01$. The constrained optimization approach yields a huge reduction in fairness violation, though at the cost of an increase in MSE.
The key way one specifies pairwise fairness metrics is by the selection of which pairs to consider. Here, we focused on within-group and cross-group candidates. One could also bring in side information or condition on other features. For example, in the ranking setting, we might have side information about the presentation order that candidates for each query were shown to users when labeled, and this position information could be used to either select or weight candidate pairs. In the regression setting, we could assign weights to example pairs based on their label differences.

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Appendix

Table 3: Example group-dependent pairwise accuracy matrix of \( K = 3 \) groups, and the frequency-weighted row mean and column mean for the restaurant ranking example in the introduction.

|       | French | Mexican | Chinese | Row Mean |
|-------|--------|---------|---------|----------|
| Cheap | 0.76   | 0.68    | 0.56    | 0.70     |
|       | 0.61   | 0.62    | 0.55    | 0.60     |
|       | 0.62   | 0.61    | 0.56    | 0.60     |
| Col. Mean | 0.69  | 0.65    | 0.56    | 0.65     |

Proxy-Lagrangian Optimization

For completeness, we provide details of the proxy-Lagrangian optimization approach for solving the constrained optimization in (12). For ease of exposition, we consider a setting with two protected groups, i.e. \( K = 2 \). Denoting \( \Delta_{ij}(f) = \Delta_{G_{i}>G_{j}}(f) - \Delta_{G_{j}>G_{i}}(f) \), we can restate (12) for two protected groups as:

\[
\max_{\theta \in \Theta} \text{AUC}(f) \quad \text{s.t.} \quad \Delta_{01}(f) \leq \epsilon, \quad \Delta_{10}(f) \leq \epsilon,
\]

where the ranking model \( f \) is parameterized by \( \theta \) and \( \Theta \) is a set of model parameters we wish to optimize over.

Since the pairwise metrics are non-continuous in the model parameters (due to the presence of indicator functions \( I \)), the approach of Cotter et al. [2019a] requires us to choose a surrogate for the AUC objective that is concave in \( \theta \) and satisfies:

\[
\text{AUC}(f) \leq \tilde{\text{AUC}}(f),
\]

and surrogates for the constraint differences \( \Delta_{ij} \) that are convex in \( \theta \) and satisfy:

\[
\Delta_{ij}(f) \geq \tilde{\Delta}_{ij}(f).
\]

In our experiments, we construct these surrogates by replacing the indicator functions \( I \) in the pairwise metrics with hinge-based upper/lower bounds. Cotter et al. [2019a] then introduce Lagrange multipliers or weighting parameters for the objective and constraints \( \lambda \) and defines two proxy-Lagrangian functions:

\[
\mathcal{L}_\theta(f, \lambda) := \lambda_1 \tilde{\text{AUC}}(f) + \lambda_2 (\tilde{\Delta}_{01}(f) - \epsilon) + \lambda_3 (\tilde{\Delta}_{10}(f) - \epsilon); \quad \mathcal{L}_\lambda(f, \lambda) := \lambda_2 (\Delta_{01}(f) - \epsilon) + \lambda_3 (\Delta_{10}(f) - \epsilon),
\]

where \( \lambda \in \Delta^3 \) is chosen from the 3-dimensional simplex. The goal is to maximize \( \mathcal{L} \) over the model parameters, and minimize \( \mathcal{L}_\lambda \) over \( \lambda \), resulting in a two-player game between a \( \theta \)-player and a \( \lambda \)-player.

Note that the \( \theta \)-player’s objective \( \mathcal{L}_\theta \) is defined using the surrogates \( \tilde{\text{AUC}} \) and \( \tilde{\Delta}_{ij} \)'s (this is needed for optimization over model parameters), while the second player’s objective \( \mathcal{L}_\lambda \) is defined using the original pairwise metrics (as the optimization over \( \lambda \) does not need the indicators to be relaxed).

The proxy-Lagrangian approach uses a no-regret optimization strategy with step-size \( \eta_t \) to optimize \( \mathcal{L}_\theta \) over model parameters \( \theta \), and a swap-regret optimization strategy with step-size \( \eta_t \) to optimize \( \mathcal{L}_\lambda \) over \( \lambda \). See Algorithm 2 of Cotter et al. [2019a] for specific details of this iterative procedure. The result of \( T \) steps of this optimization is a stochastic model \( \tilde{\mu} \) supported on \( T \) models, which under certain assumptions, is guaranteed to satisfy the specified cross-group constraints. We adapt and re-state the convergence guarantee of Cotter et al. [2019a] for our setting:

**Theorem 1** (Cotter et al. [2019a]). Let \( \Theta \) be a compact convex set. For a given choice of step-sizes \( \eta_0 \) and \( \eta_T \), let \( \theta_1, \ldots, \theta_T \) and \( \lambda_1, \ldots, \lambda_T \) be the iterates returned by the proxy-Lagrangian optimization algorithm (Algorithm 2 of Cotter et al. [2019a]) after \( T \) iterations. Let \( \tilde{\mu} \) be a stochastic model with equal probability on each of \( f_{\theta_1}, \ldots, f_{\theta_T} \) and let \( \lambda = \frac{1}{T} \sum_{t=1}^{T} \lambda_t \). Then there exists choices of step sizes \( \eta_0 \) and \( \eta_T \) for which \( \tilde{\mu} \) thus obtained satisfies with probability \( \geq 1 - \delta \) (over draws of stochastic gradients by the algorithm):

\[
E_{f \sim \tilde{\mu}} [\tilde{\text{AUC}}(f)] \geq \max_{f : \tilde{\Delta}_{01}(f) \leq \epsilon, \tilde{\Delta}_{10}(f) \leq \epsilon} \tilde{\text{AUC}}(f) - \tilde{O}\left( \frac{1}{\sqrt{T}} \right)
\]

and

\[
E_{f \sim \tilde{\mu}} [\Delta_{01}(f)] \leq \epsilon + \tilde{O}\left( \frac{1}{\lambda_1 \sqrt{T}} \right); \quad E_{f \sim \tilde{\mu}} [\Delta_{10}(f)] \leq \epsilon + \tilde{O}\left( \frac{1}{\lambda_1 \sqrt{T}} \right),
\]

where \( \tilde{O} \) only hides logarithmic factors.

Under some additional assumptions (that there exists model parameters that strictly satisfy the specified constraints with a margin), Cotter et al. [2019a] show that for a large \( T \), the term \( \lambda_1 \) in the above guarantee is essentially lower-bounded by a constant, and the returned stochastic model is guaranteed to closely satisfy the desired pairwise fairness constraints in expectation.
We provide further details about the experimental set-up and present additional results.

Setup We use Adam for gradient updates. For the Wiki Talk Pages dataset and the Law School dataset, we compute stochastic gradients using minibatches of size 100 to better handle the large number of pairs to be enumerated. For all other datasets, we enumerate all pairs and compute full gradients. The datasets are split into training, validation and test sets in the ratio 1/2:1/4:1/4, with the validation set used to tune the learning rate, the number of training epochs for the unconstrained optimization

We give more details about the debiased-weighting baseline. This is an imitation of the debiasing scheme of Dixon et al. [2018] by optimizing a weighted pairwise accuracy (without any explicit constraints):

$$\frac{1}{n_+ n_-} \sum_{i,j:y_i > y_j} \alpha_{z_i, y_i} \alpha_{z_j, y_j} \mathbb{1}(f(x_i) > f(x_j)),$$

where $n_+, n_-$ are the number of positively labeled and negatively labeled training examples, and $\alpha_{z,y} > 0$ is a non-negative weight on each label and protected group are set such that the relative proportions of positive and negative examples within each protected group are balanced. Specifically, we fix $\alpha_{0,-1} = \alpha_{0,+1} = \alpha_{1,+1} = 1$ and set $\alpha_{1,-1}$ so that

$$\frac{\{x_i \mid z_i=0, y_i=-1\}}{\{x_i \mid z_i=0, y_i=+1\}} = \alpha_{1,-1}, \frac{\{x_i \mid z_i=1, y_i=-1\}}{\{x_i \mid z_i=1, y_i=+1\}} = \alpha_{1,+1}.$$ 

This mimics Dixon et al. [2018] where they sample additional negative documents belonging group 1, so that the relative label proportions within each group are similar.
Table 6. This approach again failed to produce feasible solutions for the Business matching dataset as it had a very small number of pairwise examples (the o group). The hyperplanes learned by the constrained and robust optimization methods work more equally well for both groups.

| Dataset                      | Prot. Group | Unconstrained | Constrained | Robust |
|------------------------------|-------------|---------------|-------------|--------|
| Sim. Marginal                | {0, 1, 2}   | 0.95 (0.28)   | 0.72 (0.03) | 0.91 (0.07) |

**Hyperparameter Choices.** The unconstrained approach for optimizing AUC (MSE) is run for 2500 iterations, with the step-size chosen from the range \( \{10^{-3}, 10^{-2}, \ldots, 10\} \) to maximize AUC (or minimize MSE) on the validation set. The proxy-Lagrangian algorithm that we use for constrained and robust optimization is also run for 2500 iterations. We fix the step-sizes \( \eta \) and \( \theta \) for this solver to the same value, and choose this value from the range \( \{10^{-3}, 10^{-2}, \ldots, 10\} \). We use a heuristic provided in Cotter et al. [2019b] to pick the step-size that best trades-off between the objective and constraint violations on the validation set. The final stochastic classifier for the constrained and robust optimization methods is constructed as follows: we record 100 snapshots of the iterates at regular intervals and apply the “shrinking” procedure of Cotter et al. [2019b] to construct a sparse stochastic model supported on at most \( J + 1 \) iterates, where \( J \) is the number of constraints in the optimization problem.

For ranking settings, the fixed regularization approach of Beutel et al. [2019] optimizes a sum of a hinge relaxation to the AUC and their proposed correlation-based regularizer; for regression setting, it optimizes a sum of the squared error and their proposed regularizer. We run this method for 2500 iterations and choose its step-size from the range \( \{10^{-3}, 10^{-2}, \ldots, 10\} \), picking the value that yields lowest regularized objective value on the validation set. For the post-processing approach of Kallus and Zhou [2019], as prescribed, we fit a monone transform \( \phi(z) = \frac{1}{1 + \exp(-\alpha z + \beta)} \) to the unconstrained AUC-optimizing model, and tune \( \alpha \) from the range \( \{0, 0.05, \ldots, 5\} \) and \( \beta \) from the range \( \{-1, -2, -5, -10\} \), choosing values for which the transformed scores yield lowest fairness violation on the validation set. We implement the approach of Singh and Joachims [2018] by solving an LP per query to post-process and re-rank the scores from the unconstrained AUC-optimizing model to satisfy their proposed disparate impact criterion. We include a per-query slack to make the LPs feasible.

**Experiment Constraining All Matrix Entries** For the simulated ranking task with two groups, we present results for a second experiment, where we seek to enforce both cross-group equal opportunity and in-group equal accuracy criteria by constraining both \( |A_{0>1} - A_{1>0}| \leq 0.01 \) and \( |A_{0>0} - A_{1>1}| \leq 0.01 \). For the robust optimization, we implement this goal by maximizing \( \min\{|A_{0>1}, A_{1>0}| + \min\{|A_{0>0}, A_{1>1}| \) \). Table 4 gives the test ranking accuracy and pairwise fairness goal violations. The pairwise fairness violations are measured as \( \max\{|A_{0>1} - A_{1>0}|, |A_{0>0} - A_{1>1}| \) for this experiment.

As expected the unconstrained algorithm yields the highest overall ranking objective, but incurs very high fairness violations. The debiased weighting approach also gives a similar performance (as the relative proportions of positives and negatives are the same in expectation for the two protected groups since the protected group was independent of the label). Both the constrained and robust optimization approaches significantly reduce the fairness violations. In terms of the ranking objective, constrained optimization suffers a considerable decrease in the overall objective when constraining all entries of the accuracy matrix, whereas robust optimization incurs only a marginal decrease in objective.

Figure 3 shows the \( 2 \times 2 \) pairwise accuracy matrix for each method. From Figure 3b, one sees that constrained optimization satisfies the fairness constraints by lowering the accuracies for all four group-pairs. In contrast, Figure 3c, shows robust optimization maximizes the minimum entry in the fairness matrix. These results are consistent with the two different optimization problems: you get what you ask for.

Figure 2 shows the hyperplanes (dashed line) for the ranking functions learned by the different methods. Note that the quality of the learned ranking function depends on the slope of the hyperplane and is unaffected by its intercept. The hyperplane learned by the unconstrained approach ranks the majority examples (the + group) well, but is not accurate at ranking the minority examples (the o group). The hyperplanes learned by the constrained and robust optimization methods work more equally well for both groups.

**Additional Results for Simulated Ranking Task with 3 Groups** In Table 5, we provide the test ranking accuracy and pairwise fairness goal violations for the simulated ranking experiment with three groups, with a marginal equal opportunity fairness criterion.

**Additional Singh and Joachims [2018] Comparison** We also ran comparisons with the post-processing LP-based approach of Singh and Joachims [2018], with their proposed disparate treatment criterion as the constraint. The results are shown in Table 6. This approach again failed to produce feasible solutions for the Business matching dataset as it had a very small number of pairs/query, and could not be applied to the Wiki Talk Pages and Communities & Crime datasets as they did not have the required query-candidate structure. Because this approach seeks to satisfy a non-pairwise exposure-style fairness constraint, in doing so, it fails to perform well on our proposed pairwise metrics.
Table 6: Test AUC (higher is better) with test pairwise fairness violations (in parentheses) for comparisons with the approach of Singh and Joachims [2018] that enforces disparate treatment. For fairness violations, we report $|A_{G_0, G_1} - A_{G_1, G_0}|$.

| Dataset               | Prot. Group | Unconstrained | Constrained | S & J (DT) |
|-----------------------|-------------|---------------|-------------|------------|
| Sim. CG               | 0/1         | 0.92 (0.28)   | 0.86 (0.01) | 0.84 (0.05) |
| W3C Experts           | Gender      | 0.53 (0.96)   | 0.54 (0.10) | 0.38 (0.92) |

(a) Unconstrained
(b) Robust

Figure 4: Test row-based matrix averages on business match (ranking) data.

| Expert | Male | Female |
|--------|------|--------|
| Male   | 0.534| 0.028  |
| Female | 0.991| 0.573  |

(a) Unconstrained
(b) Robased
(c) Constrained/Cross-groups
(d) Robust/Cross-groups

Figure 5: Test pairwise accuracy matrix for W3C experts (ranking) data.

| Expert | Low | Male | Female |
|--------|-----|------|--------|
| Male   | 0.655| 0.490|
| Female | 0.795| 0.654|

(a) Unconstrained
(b) Constrained/Cross-groups

Figure 6: Pairwise accuracy matrix for law school (regression).

| Expert | High | Male | Female |
|--------|------|------|--------|
| Male   | 0.652| 0.647|
| Female | 0.666| 0.655|

(a) Unconstrained
(b) Constrained/Cross-groups

Figure 7: Continuous attribute pairwise accuracies for crime (regression) with % of black population as protected attribute.

**Pairwise Accuracy Matrices**  We also present the pairwise accuracy matrices for different methods applied to the Business matching task (see Figure 4), to the W3C experts ranking task (see Figure 5), to the Law School regression task (see Figure 6), and to the Crime regression task with continuous protected attribute (see Figure 7). In the case of the Business matching results, notice that robust optimization maximizes the minimum of the two marginals. Despite good marginal accuracies, robust optimization’s overall accuracy is not as good because of poorer behavior on the examples that were not covered by the two groups. Debiasing produces a negligible reduction in fairness violation, but yields better row marginals than the unconstrained approach.