Application of Mathematical Model Based on Optimization Theory and Particle Swarm Algorithm in Radar Station Layout Optimization

Wang Xuefeng, Mimi Chen
Xi'an University of Science and Technology, China Xi'an City, Shaanxi Province, 710600
Mailbox: xxhe@dlps.ecnu.edu.cn

Abstract: The optimization problem of radar station layout was derived through its mathematical description to explore the application of the optimization theory and particle swarm optimization (PSO) in this optimization problem; after the optimization problem was solved, a disturbance factor was introduced to present the improved PSO algorithm (PSO-DF); in the end, the PSO-DF algorithm was used to construct an radar station layout optimization model. The simulation results of the algorithm performance and the radar station layout optimization model show that in comparison with the basic PSO algorithm, the improved PSO-DF algorithm has better local search performance; corresponding to different classical functions, the optimal value, the worst value, mean value and variance of the PSO-DF algorithm are all superior to those of the basic PSO algorithm; the fitness value corresponding to the simulation via the constructed radar station layout optimization model is about 0.8, and the network shows satisfying effects in the monitoring of different areas of responsibility. This research provides a certain reference for the application of the optimization theory and PSO algorithm in the field of radar station layout optimization.

1. Introduction
High-performance lightning monitoring networks have been established and developed in recent years, and the effectiveness of information acquired by radar system is of vital importance to defense system [1-3]. The countermeasures faced by radar mainly include radar signal detection and release of electronic interference, radar destroying by antiradar missile, use of stealth technology and low-altitude defense penetration. The anti-stealth goal can be reached by radar station layout, and the radar station layout optimization is rightly a part of radar station layout [4,5]. Genetic algorithm, particle swarm optimization (PSO), etc. are algorithm tools commonly used to achieve optimization, where PSO has become a research hotspot by virtue of good convergence performance and simple encoding characteristic [6-8]. As for the application of PSO algorithm in solving optimization problems, Liu et al. (2017) proposed a method which integrated the entropy weight grey relational analysis into the PSO algorithm, which shows great performance in multi-objective optimization with easy implementation and high convergence rate [9]; Yin et al. (2018) applied the intelligent optimization algorithm to the optimization of smart power grid energy management system, and verified its effectiveness in family environment [10]. In a word, many research results have been achieved by applying the PSO algorithm to the optimization problem solving, but scarce research has been done regarding the application of the improved PSO algorithm to radar station layout optimization.

In order to analyze and discuss the preliminary application of the optimization theory and improved
PSO algorithm in the radar station layout optimization, the basic PSO algorithm was improved and optimized, and then applied to the solving of radar station layout optimization model, aiming to provide a reference for the solving and development of the radar station layout optimization problem.

2. Method

2.1 Mathematical description of optimization problem
The optimization theory is an important branch of mathematics, with the main research content of seeking for the optimal scheme among multiple ones [11]. The optimization problem can be expressed as below:

\[
\text{min} = f(X)
\]

\[
s.t. X \in B = \{X | g_i(X) \leq 0 \quad i = 1, \ldots, m\}
\]

In the above equations, \(f(X)\) denotes the objective function; \(g_i(X)\) corresponds to the constraint function; \(B\) is the constrained domain; \(X\) is \(n\)-dimensional optimization variable.

Whether the above optimization problem is a linear programming problem depends on the attributes of objective function and constraint function. When one of the abovementioned functions is a linear function, the optimization problem can be considered as a nonlinear programming problem.

Following the solving idea of other problems, the optimization technology, which is used to solve related practical problems [12], needs to construct a mathematical model for the radar station layout optimization problem according to the practical situation, and then the model will be solved through a proper method to obtain the optimal solution, which, however, is of relativity. After the mathematical model is constructed, it should be solved by selecting a suitable algorithm tool. Intelligent optimization algorithms show superior performance in the solving of optimization model, among which genetic algorithm and PSO algorithm are commonly used. Out of comprehensive consideration, the PSO algorithm was chosen to handle the radar station layout optimization problem.

2.2 Improvement of PSO algorithm
Introducing the concepts of “swarm” and “evolution”, the PSO algorithm realizes the corresponding operations based on the particle fitness [13]. With easy encoding and simple operation characteristics, it was mainly used to handle the continuous optimization problems at the beginning, but afterwards, it was extended to the handling of combinational optimization problems [14]. The PSO algorithm is expressed as below:

\[
F^{k+1}_{id} = w \times F^k_{id} + a_1 \times r_1 \times (p_{id} - x^k_{id}) + a_2 \times r_2 \times (p_{gd} - x^k_{id})
\]

\[
x^{k+1}_{id} = x^k_{id} + F^{k+1}_{id}
\]

where \(i\) is the \(i\)-th particle; \(d\) is the dimensionality of particles; \(k\) represents the \(k\)-th iteration; \(w\) is the inertia weight; \(a_1\) and \(a_2\) are acceleration constants; \(r_1\) and \(r_2\) are random numbers within the interval \((0,1)\); \(F\) represents the flight speed of particles; \(P_g\) is the best position of particles. The implementation flow of the PSO algorithm is shown in Figure 1.
Initialization position and speed

The fitness function values of all particles are evaluated to obtain the initial optimal position and initial global optimal position of particles

According to the new fitness function, the optimal position and the global optimal position are updated

The fitness function values of all particles were evaluated again

Update the position and velocity of individual particles

Y

Meet the requirements of iteration times or precision

N

End

Figure 1 Implementation Flow of PSO Algorithm

However, when handling optimization problems, the PSO algorithm is prone to local optimum. Given this, the traditional PSO algorithm was optimized in this research mainly by introducing a disturbance factor. The optimized PSO algorithm is as below:

\[
F_{id}^{k+1} = w \times F_{id}^k + a_1 \times r_1 \times (p_{id} - x_{id}^k) + a_2 \times r_2 \times (p_{gd} - x_{id}^k) + \alpha \times (r_3 - 0.5)
\]

(5)

\[
x_{id}^{k+1} = x_{id}^k + F_{id}^{k+1}
\]

(6)

where \(\alpha\) is the disturbance factor; \(r_3\) is a random number within the interval \((0,1)\), and other parameters have the same meanings as the basic PSO algorithm. The improved algorithm has good global searching ability due to high particle speed in the initial phase. In the middle and later phase, the particle speed is slowed down due to their homoplasy, but it is kept above 0 by the fourth term where the disturbance factor is introduced, so the local searching is continued, making it possible for the particles to skip out of local optimum. The particle swarm optimization algorithm carrying a disturbance factor is called PSO-DF.

2.3 Construction of radar station layout optimization model

The optimization of radar station layout aims to determine the position distribution of each detection station, further realize the minimization of comprehensive error for the monitoring network and position optimization of configured stations, so that the monitoring network can exert the best effect. A design variable \(d\) should be introduced in the construction of station layout optimization model based on the optimization theory. This variable is mainly used to reveal the comprehensive error level and some constraints related to the variable. Specific to radar positioning, the mathematical model for station layout optimization constructed in this research is expressed in the following formula:

\[
f = r_1^c + r_2^o + r_3^\zeta + r_4^\tau
\]

(7)

\[
s.t. \quad r(k_i, k_j) < r_{ij}
\]

(8)

wherein \(k_1 - k_4\) represent the first to fourth radars; \(c\) is airspace coverage coefficient; \(o\) is airspace overlap coefficient; \(\zeta\) is frequency interference coefficient; \(r(k_i, k_j)\) is the distance from the \(i\)-th radar to the \(j\)-th radar; \(\tau\) is the proportion of coverage area in the total area when the number of radars is 3 or more, and this variable is obtained based on the detection ranges of radars \(r_i\) and \(r_j\) themselves.
The improved PSO-DF algorithm was applied to the optimization of radar station layout in this research.

2.4 Algorithm performance test and the solving of radar station layout optimization model

For the performance test of the proposed PSO-DF algorithm, the values of $\alpha$, $a_1$, $a_2$, $w_{\text{max}}$ and $w_{\text{min}}$ are taken as 0.05, 2, 2, 0.8 and 0.5, respectively, and the particle swarm size is set as 1 and the maximum number of iterations as 100. On this basis, the performance of the improved PSO-DF algorithm is analyzed through the fluctuation and change of the search curve corresponding to the algorithm.

And then, the following functions are introduced to verify the effectiveness of the improved PSO-DF algorithm, where the formula of Rosenbrock function is as below:

$$f(x) = \sum_{i=1}^{n}(100(x_{i+1} - x_i^2) + (x_i - 1)^2) \quad x_i \in [-30, 30]$$

Griewank function is expressed by the following formula:

$$f(x) = \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos \left( \frac{x_i}{\sqrt{i}} \right) + 1 \quad x_i \in [-100, 100]$$

The formula of Rastrigrin function is as follow:

$$f(x) = \sum_{i=1}^{n}(x_i^2 - 10\cos(2\pi x_i) + 10) \quad x_i \in [-100, 100]$$

For all of the above functions, 30, 30 and 6000 are assigned to be the dimensionality, particle swarm size and maximum number of iterations, respectively. The inertia weight strategy is adopted for both basic PSO algorithm and improved PSO-DF algorithm. The values of $a_1$, $a_2$, $w_{\text{max}}$ and $w_{\text{min}}$ are set as 2, 2, 0.8 and 0.5, respectively. The disturbance factor $\alpha$ is taken as 0.005 for the improved PSO-DF algorithm.

For applying the PSO algorithm to the solving of the radar station layout optimization model, a fitness function should be constructed firstly, and the function value should be an important basis for the extremum updating in the PSO algorithm. The fitness function established based on the above station layout optimization model is expressed as follow:

$$g = \max(f) = \max\left(\frac{k_c c + k_o o + k_{\zeta} \zeta + k_x x}{4}\right)$$

Next, the constraints are processed, and based on the above station layout optimization model, one constraint is described as below:

$$r(k_i, k_j) < r_{ij}$$

wherein $r(k_i, k_j)$ is the distance from the $i$-th radar to the $j$-th radar. Hereby is another constraint:

$$g(x, y) \neq 0, \forall x, y$$

The geographical location is considered in the above constraint, and the position $(x, y)$ represents can be deployed with radar.

Under the practical situation of radar station layout optimization, all constraints used for problem handling are nonlinear constraints. On this basis, the death penalty function is used to process the above constraints. Given the fact that they are all inequality constraints, the formula of individual fitness in the algorithm is as below:
To sum up, the mathematical model for applying the PSO algorithm to the optimization problem of radar station layout is:

\[
F = \begin{cases} 
  f(x) \\
  0 
\end{cases}
\quad (15)
\]

This research mainly aims to analyze the performance of the improved PSO-DF algorithm on the optimization of radar station layout.

3. Results and Discussion

3.1 Comparison of PSO-DF algorithm and PSO algorithm in search performance

The fluctuation and variation trend of both the improved PSO-DF algorithm and basic PSO algorithm are displayed in Figure 2.

By analyzing the distribution of local search results in the above figure, both basic PSO algorithm and improved PSO-DF algorithm showed high speed in the initial iteration phase, and the corresponding global search abilities of particles were favorable. However, it was evident that when the number of iterations was set at 20, the particle speed of the basic PSO algorithm was significantly slowed down, and it declined to 0 as the number of iterations increased to 50, indicating the termination of the whole search process. Comparatively speaking, the particle deceleration of the improved PSO-DF algorithm was low because the disturbance factor was introduced, and not all particle speeds were 0 when the number of iterations became 80, so the whole local search process could be continued. Therefore, the introduction of the disturbance factor endowed the PSO-DF algorithm a more effective performance than the basic PSO algorithm in the local search.

3.2 Calculation results of PSO and PSO-DF algorithms under different functions

The calculation results of the PSO-DF algorithm based on the three classical functions – Rosenbrock function, Griewank function and Rastrigrin – are shown in Figure 3.
Figure 3 Calculation Results of PSO Algorithm and PSO-DF Algorithm Based on Three Classical Functions

It can be found by analyzing the curves in the figure that under the three classical functions, the optimal value, worst value, mean value and variance of the PSO-DF algorithm were all better than those of the basic PSO algorithm, making it more effective than the original PSO-DF algorithm.

To figure out the reason for the better performance of the improved PSO-DF algorithm, the problem that the basic PSO algorithm could be easily stuck in local optimum was solved by introducing the disturbance factor, thus making it more possible for the algorithm particles to skip out of the local optimum.

3.3 Implementation of radar station layout optimization model
Based on the related parameter setting for the above improved PSO-DF algorithm, the simulation analysis of the radar station layout optimization model was carried out under Visual C++6.0 environment. The corresponding fitness values and the change of distribution with operating time are shown in Figure 4. The practical simulation effect of the constructed radar station layout optimization model corresponding to a map of a northwest city is manifested in Figure 5.
Figure 4 Fitness Values under Radar Station Layout Optimization Model

A: Small area of responsibility
B: Appropriate area of responsibility
C: Large area of responsibility

By combining different radar station layout optimization projects and the simulation effect of the model on the city, the airspace coverage redundancy displayed by the constructed radar station layout optimization model was large in the small area of responsibility; when the area of responsibility involved was large, the detection range of the established model was obviously expanded, and the coverage area became broader; in appropriate area of responsibility, the model also showed a broad detection range.
On the whole, the fitness values of the radar station layout optimization model under different circumstances were largely distributed at around 0.8.

The radar detection range is a key index reflecting the radar performance during radar monitoring. Meanwhile, it also plays a significant role in the optimized deployment of radar network [15]. By applying the improved PSO algorithm to the optimization, the constructed radar station layout optimization model showed satisfying performance under all circumstances, namely, small, appropriate and large areas of responsibility. Being able to reach the effect of seamless detection, this optimization model can play an effective role on coping with four threats: radar signal detection and release of electronic interference, radar destroying by antiradar missile, use of stealth technology and low-altitude defense penetration; meanwhile, it also strengthened the radar sensing, and reduced the dead zones covered, which contributed to collaborative communication and target discovery; the performances of the constructed radar station layout optimization model in detection range and its resistance against “four threats” verified the effectiveness of the improved PSO algorithm in the optimization of radar station layout.

4. Conclusion

The PSO algorithm has been extensively applied to the solving of optimization problems. By optimizing the basic PSO algorithm and establishing the radar station layout optimization model, the improved PSO-DF algorithm was used to solve the radar station layout optimization model. It was then found that the PSO-DF algorithm showed better performances than the basic PSO algorithm in both local search and the calculation of classical functions, and moreover, based on the fitness function, the established radar station layout optimization model also has good performance in radar network monitoring. This research provides a potential application and development direction for the optimization thought and PSO algorithm in the field of station layout optimization. However, as the research is still at an exploratory stage and restricted by objective experimental conditions, the constructed radar station layout optimization model still remains to be adjusted and optimized, with various problems like error correction and data fusion in radar network, which need to be further explored in the future research.

Author:

First author: Wang Xuefeng (1963-), male, Xi’an University of Science and Technology, Xi’an, Shaanxi Province, Master, Professor, engaged in optimization theory and algorithm teaching.

The second introduction: Chen Mimi (1994-), Xi’an University of Science and Technology, female, Yulin, Shaanxi, Master, dedicated to the study of optimization theory and algorithm.

References:

[1] Delmarco S P, Dasgupta N, Tom V, et al. Modeling Radar Probability of Detection for a Randomly-Located Mover on an Arbitrary Road Network. IEEE Systems Journal, 2017, 9(4):1208-1217.
[2] Dymond K F, Nicholas A C, Budzien S A, et al. Ionospheric-thermospheric UV tomography: 2. Comparison with incoherent scatter radar measurements. Radio ence, 2017, 52(3-4):357-366.
[3] Hu Lei, Zhang Fengkui, Liu Baoxin, et al. American anti-missile early warning system construction thinking. Airborne Missiles, 2018, 405(09):69-72.
[4] Liu Jieyi, Zhang Lin JIan, Zhao Shanshan, et al. Analysis and simulation of optimal station placement method for radar under spoofing jamming. Journal of the University of Electronic Science and Technology, 2017, 46(004):513-519.
[5] Cui Shulin, Zhang Xiangyu, Zhang Guangyi. Research on cooperative tracking technology of multi-flight missile target. Journal of the Naval Aviation Engineering Institute, 2018, 033(002):210-216.
[6] Wu Y, Liu G, Guo X, et al. A self-adaptive chaos and Kalman filter-based particle swarm optimization for economic dispatch problem. Soft Computing, 2017, 21(12):1-13.
[7] Suresh S, Lal S. Multilevel Thresholding based on Chaotic Darwinian Particle Swarm Optimization for Segmentation of Satellite Images. Applied Soft Computing, 2017, 55:503-522.
[8] Li Fei, Liu Jianchang, Shi Huaitao, et al. multi-objective particle swarm optimization algorithm
based on decomposition and differential evolution. Control and Decision-making, 2017, 032(003):403-410.

[9] Liu H, Zhang Q, Yao L. D2D particle swarm optimization power control algorithm based on interference indication. Computer Engineering & Design, 2017, 22(3):265-274.

[10] Yin Zhongdong, Lin Zhi, Li Dezhi, et al. Intelligent home energy management strategy based on hybrid particle swarm optimization. Journal of North China Electric Power University (Natural Science Edition), 2018, 45(04):29-37.

[11] Andrea P, Alessio Z, Luca S. Solving Fractional Polynomial Problems by Polynomial Optimization Theory. IEEE Signal Processing Letters, 2018, 25(10):1540-1544.

[12] Zhao Wanjin, Gao Haiyan, Yan Guoliang, et al. TOC prediction techniques based on optimization estimation and bayesian statistics. Lithologic reservoirs, 2020, 032(001):86-93.

[13] Shi Xudong, Gao Yuelin, Han Junru. Particle swarm optimization algorithm based on fuzzy reasoning. Journal of Henan normal University: natural Science Edition, 2017(45):108-118.

[14] Jiang Fengli, Zhang Yu, Wang Yonggang. an adaptive particle swarm optimization algorithm based on bootstrap strategy. Computer Applied Research, 2017, 034(012):3599-3602.

[15] Li Z, Chen H, Chu H, et al. Monitoring wildfire using high-resolution compact X-band dual-polarization radar: A case study in southern China. Atmospheric Research, 2019, 225:165-171.