1D lattice model for binary growth and surface relaxation

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Abstract. Motivated by recent progress in producing metastable metallic binary alloy films and nanoclusters through atomic deposition techniques, we propose a simple, analytically solvable model of a growing Ising-type one-dimensional chain whose kinetics are determined by random addition of two atomic species and by compositional relaxation restricted to the surface. Solutions are presented for the influence of the external flux on both surface segregation and compositional short-range correlations in the bulk, which is frozen. Despite the simplicity of the model, these results can help to understand opposing trends in surface segregation and bulk short-range order, revealed in recent experiments and simulations for fcc-type nanoalloys.

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1. Introduction

Growth of ultrathin films and nanoclusters by the molecular beam epitaxy (MBE) technique opens a way to generate non-equilibrium solid structures, which can display novel physical properties distinctly different from the equilibrium bulk properties [1]. To suppress equilibration, the substrate temperature is chosen low enough that atomic rearrangement processes in the bulk are frozen. The resulting bulk structure then exclusively depends on the kinetics at the growing surface, driven by the deposition flux and by atomic surface relaxation steps within the outermost ‘active’ layers. Multicomponent systems, grown under codeposition of two or more atomic species, offer the possibility to tune atomic ordering effects with the help of the growth parameters. For example, in MBE-grown metallic magnetic nanoalloys it has become possible to induce an anisotropic short range order that entails perpendicular magnetic anisotropy, a feature of great current interest from a technological viewpoint [2, 3].

Theoretically, the relationship between the frozen atomic structure below the surface and the growth mechanism is only poorly understood, despite some studies in the recent past. The emergence of frozen short and long range atomic correlations in ordering alloys and their interplay with magnetic properties has been investigated recently by kinetic Monte Carlo simulations both for films [4] and nanoclusters [5]–[7]. Other recent studies of frozen-in correlations emerging from surface processes focused on the density correlation function below a surface that grows according to the non-conserved Kadar–Parisi–Zhang equation [8].

Here, we investigate a minimal model for unidirectional binary AB alloy growth. It consists of a linear atomic array that includes a surface relaxation mechanism by allowing AB-exchange between the two progressing outermost positions. This effect should mimic indirect and direct exchange processes between unlike atoms A and B in the outermost layers of realistic three-dimensional systems. Pairwise interatomic interactions imply that the chosen AB-exchange rate depends on the occupation of the third position from the top. Formally, this model is equivalent to a growing sequence of two-state systems: after addition of one particle only a newly formed AB two-state system is allowed to relax in a way that depends on the frozen state formed in previous steps. This generates a kind of linear kinetic Ising model with competing dynamics [9], represented by the AB-exchange and the external flux. Stationary state properties and transient structures can be derived from exact recursion relations for three-point correlators. When applied to measurements, this model should allow one to elucidate opposing trends in surface segregation and short-range order, and the dependence of both features on the deposition fluxes \( F_A \) and \( F_B \). Effects of this kind have been observed in kinetic Monte Carlo simulations of three-dimensional cluster and film growth, where they could be analyzed quantitatively within a specific energetic model for ordering fcc alloys [5, 6].

2. Growth model

Consider a finite one-dimensional array of A and B atoms at positions \( m \) above the substrate; \( 1 \leq m \leq n \). The array evolves in time according to the following elementary steps:

1. Growth at discrete times \( n \tau \); \( n = 1, 2, \ldots \); by adding an A or B atom with probability \( p_A \) or \( p_B = 1 - p_A \) to the top of the array.

2. In between two successive growth events, unlike atoms in the first and second position from the top are allowed to interchange positions. Specifically, if the top atom is A and the
second one is B, the exchange rate is denoted by $w_1$, while for the reverse order we denote it by $w_2$. This exchange process is described by a continuous time $2 \times 2$ master equation. The rates $w_1$ and $w_2$ depend on the occupation of the third position from the top, such that the energetics of the system is incorporated via the detailed balance condition. The occupation of the third and lower positions are frozen. Let $V_{\alpha\beta}$ be the nearest neighbor (NN) interaction energy between species $\alpha$ and $\beta$ ($\alpha, \beta = A$ or B) and $\epsilon_\alpha$ a surface potential that acts only on an $\alpha$-atom in the top position. Then,

$$w_1(\gamma)e^{-(V_{\beta\gamma}+\epsilon_\gamma)/k_B T} = w_2(\gamma)e^{-(V_{\alpha\gamma}+\epsilon_\gamma)/k_B T}$$

where $\gamma = A$ or B specifies the (frozen) occupation of the third position. For example, if $\gamma = A$, an AB-attraction will decrease the rate $w_1(A)$ but increase $w_2(A)$.

Figure 1 illustrates the deposition and exchange processes described in this way. It is convenient to introduce the parameters $J = (V_{AA} + V_{BB} - 2V_{AB})/4$ and $h = V_{BB} - V_{AA} + 2(\epsilon_A - \epsilon_B)$. The (negative) Ising interaction $J$ generates short-range correlations, while $h$ acts as a surface field. For example, for $J > 0$ and $h > 0$ AB-alternation and surface segregation of B-atoms will be favored. From (1) it follows that

$$w_2(A) = w_1(A)E^-, \quad w_1(B) = w_2(B)E^+$$

with

$$E^\pm = \exp[2(J \pm h/4)/k_B T].$$

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From the form of (7) (see below) it will become clear that the deposition time \( \tau \), which is related to the external fluxes by \( F_A = p_A/\tau; \) \( F_B = p_B/\tau \), enters the problem via the factors
\[
\Lambda_\alpha = \exp[-(w_1(\alpha) + w_2(\alpha))\tau],
\]
reflecting the competition between external flux and the internal dynamics. In fact, \( \Lambda_\alpha \) describes the relaxation during the deposition time \( \tau \) by exchange of two unlike atoms at first and second position from the surface, given that third position is occupied by an \( \alpha \)-atom.

3. Kinetic equations

Let us introduce the three-point probabilities \( \rho(n)(\alpha, \beta, \gamma) \) that an array of length \( n \geq 3 \) immediately after the addition of the \( n \)th atom (at time \( t = n\tau \)) contains atoms of species \( \alpha, \beta \) and \( \gamma \) in the first, second and third position from the top, respectively. Similarly \( \rho(n)(\alpha, \beta, \gamma) \) is the corresponding probability immediately before the addition of the \( (n+1) \)th atom. We now derive recurrence relations which provide complete information on the array evolution.

First of all, since like atoms do not interchange,
\[
\rho(n)(\alpha, \alpha, \gamma) = \rho(n)(\alpha, \alpha, \gamma).
\]
Interchange of unlike atoms leads to
\[
\begin{pmatrix}
\rho(n)(A, B, \gamma) \\
\rho(n)(B, A, \gamma)
\end{pmatrix} = U(\tau; \gamma) \begin{pmatrix}
\rho(n)(A, B, \gamma) \\
\rho(n)(B, A, \gamma)
\end{pmatrix},
\]
where \( U(t, \gamma) \) is the \((2 \times 2)\) time evolution operator describing AB-exchange,
\[
U(t; \gamma) = \frac{1}{w_1 + w_2} \left[ \begin{pmatrix} w_2 & w_2 \\ w_1 & w_1 \end{pmatrix} + \begin{pmatrix} w_1 & -w_2 \\ -w_1 & w_2 \end{pmatrix} e^{-(w_1+w_2)t} \right],
\]
with \( U(0; \gamma) = 1 \) and \( w_i \equiv w_i(\gamma) \).

Equations (6) and (7) provide the mapping of probabilities over one time interval \( \tau \). According to the deposition rule, matching with the subsequent time interval is expressed by
\[
\rho(n+1)(\alpha, \beta, \gamma) = \rho(n)(\alpha, \beta, \gamma),
\]
where we have used the notation
\[
\rho(n)(\beta, \gamma) = \sum_{\gamma'} \rho(n)(\beta, \gamma, \gamma')
\]
for the two-point probabilities of the two upper positions.

To proceed, it is most convenient to eliminate \( \rho(n) \) with the help of (8). Summing (5) and (6) over \( \gamma \) leads to recursion relations
\[
Q^{(n+1)}_i = \sum_{k=1}^{4} V_{i,k} Q^{(n)}_k
\]
among the quantities \( Q_i^{(n)} = q^{(n)}(\alpha, \beta), \) where \( i = 1, \ldots, 4 \) corresponds to the combinations \((\alpha, \beta) = (AB), (BA), (AA) \) and \((BB), \) respectively. Clearly, \( \sum_{i=1}^{4} Q_i^{(n)} = 1. \) The transition matrix of the Markov chain \( Q^{(n)} \) is explicitly given by

\[
(V_{i,k}) = \begin{pmatrix}
p_B B_{1,2} & p_A A_{1,1} & p_B A_{1,2} & p_A B_{1,1} \\
p_B B_{2,2} & p_A A_{2,1} & p_B A_{2,2} & p_A B_{2,1} \\
p_A & 0 & p_A & 0 \\
0 & p_B & 0 & p_B
\end{pmatrix}
\]

with abbreviations \( A_{i,k} = U_{i,k}(\tau; A); B_{i,k} = U_{i,k}(\tau; B) \) for the elements of the \((2 \times 2)\) matrices \( U(\tau, \gamma) \) in (6). Note that \( \det A = \Lambda_A, \det B = \Lambda_B, \) and \( \sum_{i=1}^{2} U_{i,k}(\tau_i; \gamma) = 1, \) hence \( \sum_{i=1}^{4} V_{i,k} = 1. \) Equation (10) is applicable to \( n \geq 2 \) under initial conditions \( Q_i^{(2)} = q^{(2)}(\alpha, \beta). \) Solving (10) we immediately can derive all quantities of interest. In particular, using (8) and (6) one can obtain all three-point correlators \( p^{(n)}(\alpha, \beta, \gamma) \) and \( q^{(n)}(\alpha, \beta, \gamma). \)

Physically, the most important properties of the chain are surface segregation and occupational correlations in the frozen 'bulk' below the growing surface. A measure of surface segregation in an array of length \( n \) is the probability\(^4\)

\[
q^{(n)}(\alpha) = \sum_{\beta} q^{(n)}(\alpha, \beta),
\]

for the topmost position being occupied by an \( \alpha \)-atom, right before deposition of the \((n + 1)\)th atom. Joint probabilities for the frozen occupations at positions \( m \) and \( m - 1 \) and frozen single-particle densities in arrays of length \( n \geq m + 2 \) are, respectively, given by

\[
C^{(m)}(\alpha, \beta) = \sum_{\gamma} q^{(m+1)}(\gamma, \alpha, \beta),
\]

and

\[
p^{(m)}(\alpha) = \sum_{\beta} C^{(m)}(\alpha, \beta).
\]

In terms of the quantities \( Q_i^{(n)} \) these relations can be rewritten as\(^5\)

\[
q^{(n)}(B) = 1 - q^{(n)}(A) = Q_2^{(n)} + Q_4^{(n)},
\]

\[
C^{(m)}(A, B) = (p_A + p_B B_{22}) Q_1^{(m)} + p_A B_{21} Q_4^{(m)},
\]

\[
p^{(m)}(B) = 1 - p^{(m)}(A) = Q_1^{(m+1)} + Q_4^{(m+1)}.
\]

\(^4\) Alternatively, surface segregation can be measured by averaging the time-dependent occupational probability of the top position over one period \( \tau. \)

\(^5\) Likewise, \( C^{(m)}(A, A) = (p_A + p_B A_{22}) Q_2^{(m)} + p_A A_{21} Q_2^{(m)}. \) Expressions for \( C^{(m)}(B, A) \) and \( C^{(m)}(B, B) \) can be obtained by interchanging \( A \leftrightarrow B \) and indices \( 1 \leftrightarrow 2; 3 \leftrightarrow 4. \)
4. Results

Let us first focus on the stationary solution of (10), which corresponds to an eigenvector of the matrix (11) with eigenvalue unity. For the stationary probabilities $Q^\text{st}_i$ we readily find

$$Q^\text{st}_i = \frac{1}{Z}(p_A A_{i,i} + p_B B_{i,i}), \quad \text{for} \quad i = 1, 2,$$

$$Q^\text{st}_3 = (p_A / p_B)Q^\text{st}_1; \quad Q^\text{st}_4 = (p_B / p_A)Q^\text{st}_2,$$

where $Z$ is determined from $\sum_i Q^\text{st}_i = 1$. Inserting (18), (19) into (15)–(17), straightforward algebra yields the stationary state quantities $\Delta q^\text{st}(\alpha) = q^\text{st}(\alpha) - p_\alpha$; $\Delta C^\text{st}(\alpha, \beta) = C^\text{st}(\alpha, \beta) - p_\alpha p_\beta$, with $p^\text{st}(\alpha) = p_\alpha$.

The main features of our model become already apparent by setting $p_A = p_B = 1/2$. Then, from equations (15) and (16)

$$\Delta q^\text{st}(B) = \frac{1}{2K} \left[ (E^+ E^- - 1) (\Lambda_A - \Lambda_B) + (E^+ - E^-) (2 - \Lambda_A - \Lambda_B) \right],$$

$$\Delta C^\text{st}(A, B) = \frac{1}{4K} \left[ (E^+ E^- - 1) (1 - \Lambda_A \Lambda_B) + (E^+ - E^-) (\Lambda_A - \Lambda_B) \right]$$

with

$$K = (1 + E^+) (1 + E^-) (2 + \Lambda_A + \Lambda_B).$$

Moreover, $\Delta C^\text{st}(B, A) = \Delta C^\text{st}(A, B)$ and $\Delta C^\text{st}(A, A) = -\Delta C^\text{st}(A, B)$. These relations together with the normalization conditions $\sum_\alpha \Delta q^\text{st}(\alpha) = \sum_{\alpha, \beta} \Delta C^\text{st}(\alpha, \beta) = 0$ determine all quantities $\Delta q^\text{st}(\alpha)$ and $\Delta C^\text{st}(\alpha, \beta)$. Under a sign change of $h$ (or $J$) it holds that $\Delta q^\text{st}(\alpha) \rightarrow (-1)^\tau \Delta q^\text{st}(\alpha)$ and $\Delta C^\text{st}(\alpha, \beta) \rightarrow (-1)^\tau \Delta C^\text{st}(\alpha, \beta)$.

To facilitate our further discussion, we adopt symmetrical rates $w_{1,2}(\alpha) = \text{vexp}(\Delta E_{\alpha}/2k_B T)$, where $\Delta E_{\alpha}$ is the difference in energy before and after the respective exchange jump and $\nu$ is some prefactor. Two special cases of (20) and (21) with $J = 0$ and $h = 0$, respectively, already illustrate the influence of the external flux on surface segregation and ordering,

$$\Delta q^\text{st}(B) = \frac{1 - \lambda(\tau; h/2k_B T)}{2(1 + \lambda(\tau; h/2k_B T))} \tanh(h/4k_B T); \quad J = 0,$$

$$\Delta C^\text{st}(A, B) = \frac{(1 - \lambda(\tau; J/k_B T))}{8} \tanh(J/k_B T); \quad h = 0,$$

where

$$\lambda(\tau; x) = \exp(-2\nu \tau \cosh x).$$

Evidently, both quantities (23) and (24) diminish upon increasing the flux or decreasing $\tau$. In the limit where deposition is much faster than exchange, $\tau \rightarrow 0$, we have $\lambda \rightarrow 1$, corresponding to random incorporation of A and B into the array with $\Delta q^\text{st}(\alpha) = 0$; $\Delta C^\text{st}(\alpha, \beta) = 0$. Generally, the correlated occupation of a pair of NN sites is built by a succession of three interdependent equilibration steps within that pair and with the adjacent sites. Because of the associated constraints the frozen NN correlator (24) is smaller than the corresponding quantity in the fully equilibrated NN Ising chain even in the limit $\tau \rightarrow \infty$ or $\lambda \rightarrow 0$.  

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Figure 2. B-atom segregation under stationary growth ($p_A = p_B = 1/2$). The quantity plotted, $\Delta q^{st}(B) = q^{st}(B) - 1/2$, is given by the probability $q^{st}(B)$ of the top position to be occupied by a B-atom immediately before attachment of a new atom. (a) $J$-dependence for different surface fields $h$ at $T = 1$ in the limit $\tau \to \infty$ (b) $T$-dependence with $J = 1$, $h = 4$ and $E_t = 5$, for different $\tau$. Maxima of curves indicate the blocking temperature $T_b(\tau)$. ($\tau$-values are in units of $v_0^{-1}$ and $k_B = 1$.)

Figure 3. NN AB-correlator $\Delta C^{st}(A, B)$ in the frozen bulk under stationary growth ($p_A = p_B = 1/2$). (a) Dependence on the surface field $h$ for different $J$ at $T = 1$ in the limit $\tau \to \infty$. (b) $T$-dependence; other parameters as in figure 2(b).

The combined effect of the parameters $J$ and $h$ is displayed in figures 2 and 3. An increasing $J$ impedes surface segregation, as seen from figure 2(a) for $\tau \to \infty$. Figure 2(b) shows the temperature-dependence of $\Delta q^{st}(B)$ for fixed $J/h = 4$. Following recent work on a realistic, three-dimensional growth model [5, 6] we assume here that $\nu$ is thermally activated, $\nu = \nu_0 \exp(-E_t/k_B T)$, with $E_t$ a transition state energy and $\nu_0$ an attempt frequency. It follows that for low temperatures, $\nu \tau \ll 1$, surface segregation becomes kinetically suppressed ($\Lambda_\alpha \sim 1$), whereas at higher temperatures with $\Lambda_\alpha \ll 1$ results of the type as in figure 2(a) are recovered. Thus a non-monotonic temperature dependence of $\Delta q^{st}(B)$ emerges with a maximum at some $\tau$-dependent blocking temperature $T_b(\tau)$ with respect to surface
Occupation probabilities

0.350
0.375
0.400
0.425
0.450
0.475
0.500
0.525
0.550
0.600
0.625
0.650

of atomic layers, one needs to consider the small- 

T = 1 and τ → ∞. For an array of length n relevant data points are limited to 
m ≤ n − 2. (a) Occupational profiles \( p^{(m)}(α) \). (b) NN-correlator \( C^{(m)}(A, B) - p^{(m)}(A)p^{(m−1)}(B) \), identical to \( C^{(m)}(B, A) - p^{(m)}(B)p^{(m−1)}(A) \).

These findings agree with the behavior of surface segregation found in KMC-simulations of fcc alloy cluster growth [5, 6].

Complementary effects in the NN correlator are seen in figure 3(a): by increasing the surface field \( h \), \( ∆C^{st}(A, B) \) decreases. Despite the one-dimensionality of our model allowing only for short-range order, this observation can help to understand the effect, shown by simulations [6], that surface segregation suppresses superstructure formation in three-dimensional nanoclusters of ordering alloys. That both \( J \) and \( h \) influence \( Δq^{st}(B) \) and \( ∆C^{st}(A, B) \) in opposite directions becomes most apparent for large \( τ \) and low temperatures such that \( Δ_α \ll 1 \), \( \exp(J/k_BT) \gg 1 \) and \( \exp(h/k_BT) \gg 1 \). Then one can show that \( Δq^{st} = f(x)/2 \) and \( ∆C^{st}(A, B) = f(−x)/8 \), where \( f(x) = (1 + \exp x)^{-1} \) is the Fermi function and \( x = 2(J − h/4)/k_BT \). Hence, in that limit, the weighted sum \( Δq^{st}(B) + 4ΔC^{st}(A, B) \) is independent of the interaction parameters. Upon varying the temperature at finite \( τ \), \( ∆C^{st}(A, B) \) behaves again non-monotonically and displays a maximum between blocked and surface-equilibrated states, see figure 3(b). The overshoot of the different curves over the asymptote for \( τ \to ∞ \) indicates a kind of stochastic resonance phenomenon, occurring under conditions \( [w_1(α) + w_2(α)]τ ≃ 1 \).

So far we considered only the stationary solution of (10), reached in the limit \( n \to ∞ \). In practice, regarding structural properties of nanoclusters with a height of only a small number of atomic layers, one needs to consider the small-\( n \), transient behavior of equation (10), where initial conditions \( Q_{i}^{(2)} \) are determined by the specific interactions between adatoms and the substrate. To illustrate possible transient effects, we evaluate equations (10), (17) and (16) by choosing random initial conditions \( Q_{i}^{(2)} = 1/4 \). As seen from figure 4(a), occupational profiles show a decay of \( p^{(m)}(A) \) towards the stationary value \( p_α = 1/2 \), and a corresponding increase in \( p^{(m)}(B) \). Superimposed are oscillations that reflect short range correlations due to the effective AB-attraction. The reason for the A-atom excess and B-atom depletion in the transient zone getting buried as the system approaches stationary growth, is the surface

6 In principle, one can define two blocking temperatures through the conditions \( [w_1(α) + w_2(α)]τ ≃ 1 \) for \( α = A \) and B, but this issue will not be addressed further.
field $h$ which drives the enrichment of B-atoms at the free surface. In fact, as expected
from conservation of A- and B-particles, one can show that for large $n$ the quantity $\Delta q^\text{st}(B)$
exactly balances the B-depletion in the transient zone. Figure 4(b) displays the approach,
including oscillations, of the corresponding NN-correlator $C^{(m)}(A, B) - p^{(m)}(A) \, p^{(m-1)}(B) \equiv C^{(m)}(B, A) - p^{(m)}(B) \, p^{(m-1)}(A)$ to its stationary value.

5. Conclusions

As a soluble model for binary alloy growth on a substrate we studied a linear atomic array
with Ising-type interactions that grows by adding A- and B-atoms and includes surface
relaxation, while the bulk remains frozen. With these ingredients it appears to be the simplest
atomistic model which can describe structural properties emerging solely from processes at
the growing surface. For stationary growth we find that bulk short range order gets suppressed
when the surface field $h$ is increased, a feature consistent with three-dimensional simulations
for fcc nanoalloy growth [6], and that surface segregation diminishes upon increasing the
ordering interaction $J$. Although part of these features are reminiscent of the competition
between surface segregation and ordering in equilibrated alloys [10], their physical origin
here is fundamentally different, because of the frozen bulk structure. Moreover, the transition
of the surface from equilibrated to kinetically hindered states with decreasing $T$ leads to
a non-monotonic $T$-dependence in the segregation parameter $\Delta q^\text{st}(B)$ and the short-range
order parameter $\Delta C^\text{st}(A, B)$, again consistent with those three-dimensional simulations. There,
because of kinetic hindrance, a drop in the B-atom segregation and in the degree of AB-ordering
with decreasing temperature was found, in analogy to figures 2(b) and 3(b), respectively. Thus it
appears that our model can serve as a basis for understanding important aspects in the far-from
equilibrium structural evolution of more realistic systems.

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References

[1] Michely T and Krug J 2004 Islands, Mounds and Atoms (Berlin: Springer)
[2] Shapiro A L, Rooney P W, Tran M Q, Hellman F, Ring K M, Kavanagh K L, Rellinghaus B and Weller D
1999 Phys. Rev. B 60 12826
[3] Albrecht M, Maier A, Treubel F, Maret M, Poinsot P and Schatz G 2001 Europhys. Lett. 56 884
[4] Maranville B B, Schuermann M and Hellmann F 2006 Phys. Rev. B 73 104435
[5] Heinrichs S, Dieterich W and Maass P 2006 Europhys. Lett. 75 167
[6] Heinrichs S, Dieterich W and Maass P 2007 Phys. Rev. B 75 085437
[7] Einax M, Heinrichs S, Maass P, Majhofer A and Dieterich W 2007 J. Phys.: Condens. Matter 19 086227
   Einax M, Heinrichs S, Maass P, Majhofer A and Dieterich W 2007 Mater. Sci. Eng. C 27 1325
[8] Katsav E, Edwards S F and Schwartz M 2006 Europhys. Lett. 75 29
[9] Rácz Z 1997 Nonequilibrium Statistical Mechanics in One Dimension ed V Privman (Cambridge: Cambridge
   University Press) p 73
[10] Polak M and Rubinovich L 2000 Surf. Sci. Rep. 38 127

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