Resonance contributions $\phi(1020, 1680) \to K\bar{K}$ for the three-body decays $B \to K\bar{K}h$

Ying-Ying Fan$^1$ and Wen-Fei Wang$^2$

$^1$College of Physics and Electronic Engineering, Xinyang Normal University, Xinyang 464000, China and
$^2$Institute of Theoretical Physics, Shanxi University, Taiyuan, Shanxi 030006, China

(Dated: June 16, 2020)

We study the contributions for the $K^+K^-$ and $K^0\bar{K}^0$ originated from the intermediate states $\phi(1020)$ and $\phi(1680)$ in the charmless three-body decays $B \to K\bar{K}h$, with $h = (\pi, K)$, in the perturbative QCD approach. The subprocesses $\phi(1020, 1680) \to K\bar{K}$ are introduced into the distribution amplitudes of $K\bar{K}$ system via the kaon electromagnetic form factor with the coefficients in which adopted from the fitted results. The predictions for the branching fractions of the decays $B \to \phi(1680)h \to K^+K^-\bar{h}$ are about 6%-8% of the corresponding results for the decays $B \to \phi(1020)h \to K^+K^-\bar{h}$ in this work, and the quasi-two-body decay mode with the subprocess $\phi(1680) \to K^0\bar{K}^0$ has the same branching fraction of its corresponding mode with $\phi(1680) \to K^+K^-\bar{h}$.

PACS numbers: 13.20.He, 13.25.Hw, 13.30.Eg

I. INTRODUCTION

Charmless three-body hadronic $B$ meson decays are very important for us to test the Standard Model (SM) and to explore the Quantum Chromodynamics (QCD). The decay amplitudes of these three-body processes can be described as the coherent sum of the resonant and nonresonant contributions in the isobar formalism [1–3]. The resonance contributions, which are always related to the low energy scalar, vector and tensor intermediate states and are associated with the various subprocesses of the three-body decays, could be isolated from the total decay amplitudes and studied in the quasi-two-body framework [4–6] when the three-body effects [7, 8] and rescattering effects [9] in the final states are neglected. The studies of the quasi-two-body decays could also help us to investigate the properties of different resonances and will lead us to understand the relationship among the different three-body processes with the same intermediate state.

The $P$-wave resonance contributions for the kaon pair in the charmless three-body decays $B \to K\bar{K}h$, with $h$ is pion or kaon, are originated from the resonances $\rho(770)$, $\omega(782)$, $\phi(1020)$ and their excited states [5]. The contributions from the resonance $\rho(1450)$ and from the tails of the Breit-Wigner (BW) formula [10] for the intermediate states $\rho(770)$ and $\omega(782)$ for $K^+K^-$ in the three-body decays $B^+ \to K^+K^-\pi^+$ have been discussed in Ref. [11]. In this work, we shall focus on the quasi-two-body decays $B \to \phi(1020, 1680)h \to K\bar{K}h$ within the perturbative QCD (PQCD) approach [12–15], with $K\bar{K}$ is the $K^+K^-$ or $K^0\bar{K}^0$ in the final state. One should note that the $K^0\bar{K}^0$ which comes from the $P$-wave intermediate states could form the $K_S$ plus $K_L$ but can not generate the $K_S$ pair in the final state because of the Bose-Einstein statistics.

The parameters such as mass and decay width for $\phi(1020)$, the ground state of $s\bar{s}$, have been measured quite well with the processes $e^+e^- \to K^+K^-\gamma$ and $e^+e^- \to K_SK_L$ [16–22]. The $K^+K^-$ and $K^0\bar{K}^0$ branching fractions for $\phi(1020)$ is consistent with the masses dependence in the two-body breakup momentum for the charged and neutral kaon as expected from a $P$-wave decay [23]. The structure-dependent radiative corrections to the $\phi(1020)$ decays into $K^+K^-$ and $K_SK_L$ can be found in [24]. The $2\frac{3}{2}S_1$ $s\bar{s}$ state $\phi(1680)$ was discovered in the processes of $e^+e^- \to K_SK_L\pi^+$ [25], with the decay dominant into $K^+K^*(892)$ [26, 27]. The $KK$ channel for $\phi(1680)$ was found to be about 7% of the $KK^*(892)$ for the branching fraction [26]. In Ref. [28], the contribution from the subprocess $\phi(1680) \to K^+K^-$ for the three-body decay $B^0 \to J/\psi K^+K^-$ was found to be $(4.0 \pm 0.3 \pm 0.3)\%$ of the total branching fraction by LHCb Collaboration recently, which is about 6% of the contribution from $\phi(1020) \to K^+K^-$ in the same decay channel. The detailed discussions of the general aspects for $\phi(1680)$ can be found in Ref. [29]. The $1^-\bar{\eta}$ resonance $\phi(2175)$ was found by BaBar Collaboration [30] and confirmed by different experiments [31–36]. In view of its ambiguous nature [37], we shall leave the possible subprocess $\phi(2175) \to K\bar{K}$ to the future studies.

*Electronic address: wfwang@sxu.edu.cn
The intermediate states of the quasi-two-body decays \( B \to \phi(1020, 1680)h \to K K h \) are generated in the hadronization of the quark-antiquark pair \( ss \) as demonstrated in the Fig. 1, in which the factorizable and nonfactorizable diagrams have been merged for the sake of simplicity, symbol \( B \) in the diagrams stands for the mesons \( B^+, B^0 \) and \( B_s^0 \), and the inclusion of charge-conjugate processes throughout this work is implied. The subprocesses \( \phi(1020, 1680) \to K K \) which can not be calculated in the PQCD approach, will be introduced into the distribution amplitudes of the \( K K \) system by the vector meson dominance kaon electromagnetic form factor. The PQCD approach has been adopted in Refs. [38–41] for the tree-body \( B \) decays, and the quasi-two-body framework based on PQCD has been discussed in detail in [4] which has been followed by the works [42–47] for the charmless quasi-two-body \( B \) meson decays recently. Parallel analyses of three-body \( B \) decays with the QCD factorization (QCDF) can be found in Refs. [48–58], and the relevant works within the symmetries are referred to Refs [59–68].

This work is organized as follows. In Sec. II, we give a brief introduction for the theoretical framework involves the vector time-like form factors for kaon, the \( P \)-wave \( K \bar{K} \) system distribution amplitudes and the differential branching fractions. In Sec. III, we provide numerical results for the concerned decay processes and give some necessary discussions. Summary of this work is presented in Sec. IV. The relevant quasi-two-body decay amplitudes are collected in the Appendix.

**II. FRAMEWORK**

In the light-cone coordinates, with the mass \( m_B \), the momenta \( p_B \) for the \( B \) meson and \( k_B \) for its light spectator quark are written as

\[
p_B = \frac{m_B}{\sqrt{2}}(1, 1, 0_T), \quad k_B = \left( \frac{m_B}{\sqrt{2}} x_B, 0, k_{BT} \right)
\]

(1)

in the rest frame of \( B \) meson. For the kaon pair which generated from the intermediate state \( \phi(1020) \) or \( \phi(1680) \) by the strong interaction, one has the momentum \( p = \frac{m_K}{\sqrt{2}}(\zeta, 1, 0_T) \) and the longitudinal polarization vector \( \epsilon_L = \frac{1}{\sqrt{2}}(\sqrt{1/\zeta}, 1/\sqrt{\zeta}, 0_T) \), with the variable \( \zeta = s/m_B^2 \) and the invariant mass square \( s = m_{KK}^2 \equiv p^2 \). The spectator quark comes out from \( B \) meson and goes into resonance in the hadronization as shown in Fig. 1 (a) has the momentum \( k = (0, \frac{m_B}{\sqrt{2}}z, k_T) \). For the bachelor final state pion or kaon and its spectator quark, we define their momenta \( p_3 \) and \( k_3 \) as

\[
p_3 = \frac{m_B}{\sqrt{2}}(1 - \zeta, 0, 0_T), \quad k_3 = \left( \frac{m_B}{\sqrt{2}} (1 - \zeta) x_3, 0, k_{3T} \right).
\]

(2)

The \( x_B, z \) and \( x_3 \) above, which run from zero to one, are the momentum fractions for the \( B \) meson, resonance and the bachelor final state, respectively.

The vector time-like form factors \( F_{K^+}(s) \) and \( F_{K^0}(s) \) for the charged and neutral kaons are related to the electromagnetic form factors for \( K^+ \) and \( K^0 \), respectively, which are defined as [69]

\[
\langle K^+(p_1)K^-(p_2)|j^{em}_\mu|0 \rangle = (p_1 - p_2)_\mu F_{K^+}(s), \quad \langle K^0(p_1)\bar{K}^0(p_2)|j^{em}_\mu|0 \rangle = (p_1 - p_2)_\mu F_{K^0}(s),
\]

(3)

with the squared invariant mass \( s = (p_1 + p_2)^2 \), the constraints \( F_{K^+}(0) = 1 \) and \( F_{K^0}(0) = 0 \), and the electromagnetic current \( j^{em}_\mu = \frac{i}{2} \bar{u} \gamma_\mu u - \frac{i}{2} \bar{d} \gamma_\mu d - \frac{1}{3} \bar{s} \gamma_\mu s \) carried by the light quarks \( u, d \) and \( s \) [70]. The form factors \( F_{K^+} \) and \( F_{K^0} \) can be separated into the isospin \( I = 1 \) and \( I = 0 \) components as \( F_{K^+}(s) = F^{I=1}_{K^+} + F^{I=0}_{K^+}(0) \) and \( F_{K^0}(s) = F^{I=0}_{K^0} \) and \( F^{I=1}_{K^+} = -F^{I=1}_{K^0} \), and \( \langle K^+(p_1)\bar{K}^0(p_2)|\bar{u} \gamma_\mu d|0 \rangle = (p_1 - p_2)_\mu 2F^{I=0}_{K^+}(s) \) [5, 69].

**FIG. 1:** Typical Feynman diagrams for \( B \to \phi(1020, 1680)h \to K K h \) decays. The \( \times \) denotes the possible attachments for hard gluons, and the rectangle represents the resonances \( \phi(1020) \) and \( \phi(1680) \).
With the BW formula for the resonances $\omega$ and $\phi$ and the Gounaris-Sakurai (GS) model [71] for $\rho$, we have the electromagnetic form factors [69, 72]

$$F_{K^+}(s) = +\frac{1}{2} \sum_{\rho} c_{\rho}^K \text{GS}_{\rho}(s) + \frac{1}{6} \sum_{\omega} c_{\omega}^K \text{BW}_{\omega}(s) + \frac{1}{3} \sum_{\phi} c_{\phi}^K \text{BW}_{\phi}(s),$$  

(5)

$$F_{K^0}(s) = -\frac{1}{2} \sum_{\rho} c_{\rho}^K \text{GS}_{\rho}(s) + \frac{1}{6} \sum_{\omega} c_{\omega}^K \text{BW}_{\omega}(s) + \frac{1}{3} \sum_{\phi} c_{\phi}^K \text{BW}_{\phi}(s),$$  

(6)

where the $\sum$ means the summation for the resonances $\rho, \omega$ or $\phi$ and their corresponding excited states, the explicit expressions and auxiliary functions for BW and GS are referred to Refs. [71, 73].

Phenomenologically, the vector time-like form factor for kaon can also be defined by [54]

$$\langle K^+(p_1)K^-(p_2)|\bar{q}\gamma_{\mu}q|0\rangle = (p_1 - p_2)_\mu F_{K^+K^-}(s),$$

(7)

$$\langle K^0(p_1)K^0(p_2)|\bar{q}\gamma_{\mu}q|0\rangle = (p_1 - p_2)_\mu F_{K^0K^0}(s).$$

(8)

When considering only the resonance contributions, we have

$$F_{K^+K^-}^u = F_{K^0\bar{K}^0}^d = F_\rho + 3F_\omega,$$

(9)

$$F_{K^+K^-}^d = F_{K^0\bar{K}^0}^u = -F_\rho + 3F_\omega,$$

(10)

$$F_{K^+K^-}^s = F_{K^0\bar{K}^0}^s = -3F_\phi.$$  

(11)

Then the electromagnetic form factors can be expressed by $F_{K^+} = F_\rho + F_\omega + F_\phi$ and $F_{K^0} = -F_\rho + F_\omega + F_\phi$ [54]. The expressions for $F_\rho, F_\omega$ and $F_\phi$ and their parameters can be found in [54, 74]. It’s easy to check that

$$F_\phi = \frac{1}{3} \sum_{\phi} c_{\phi}^K \text{BW}_{\phi}(s), \quad F_{K^+K^-} = F_{K^0\bar{K}^0} = -\sum_{\phi} c_{\phi}^K \text{BW}_{\phi}(s).$$

(12)

We concern only the $\phi$ component of the vector kaon time-like form factors in this work. Rather, for simplicity, we employ $F_K$ to stands for $F_{K^+K^-}$ and $F_{K^0\bar{K}^0}$ in the following discussions.

For the subprocesses $\phi(1020, 1680) \to K\bar{K}$, the $P$-wave $K\bar{K}$ system distribution amplitudes are organized into [11, 75]

$$\phi_{K}\phi_{\bar{K}}^{P\text{-wave}}(z, s) = -\frac{1}{\sqrt{2N_c}} \left[ \sqrt{\pi} \xi_L \phi^0(z, s) + \xi_L \phi^1(z, s) + \sqrt{s} \phi^s(z, s) \right] ,$$

(13)

with the momentum $p = p_1 + p_2$. We have the distribution amplitudes

$$\phi^0(z, s) = \frac{3F_K(s)}{\sqrt{2N_c}} z(1 - z) \left[ 1 + a_2^s C_2^{3/2}(1 - 2z) \right] ,$$

(14)

$$\phi^s(z, s) = \frac{3F_K^s(s)}{2\sqrt{2N_c}} (1 - 2z),$$

(15)

$$\phi^1(z, s) = \frac{3F_K^1(s)}{2\sqrt{2N_c}} (1 - 2z)^2 ,$$

(16)

with the Gegenbauer polynomial $C_2^{3/2}(\chi) = 3 (5\chi^2 - 1)/2$ and $F_K^s(s) = (f_\phi^T/f_\phi) F_K(s)$ [4] with the ratio $f_\phi^T/f_\phi = 0.75$ at the scale $\mu = 2$ GeV [76]. The Gegenbauer moment $a_2^s$ for $\phi^0(z, s)$ are the same as it in the distribution amplitudes of the light vector meson $\phi$ in [75] for the two-body $\bar{B}$ meson decays.

The $CP$ averaged differential branching fractions ($B$) for the quasi-two-body decays $B \to \phi(1020, 1680)h \to K\bar{K}h$ are written as [11, 50, 77]

$$\frac{d\mathcal{B}}{d\zeta} = \tau_B \frac{q_h^3 q^3}{12\pi^3 m_B^3} |A|^2 ,$$

(17)

where $\tau_B$ being the $B$ meson mean lifetime. The magnitudes of the momenta $q$ and $q_h$ for the kaon and the bachelor $h$ in the rest frame of the resonances $\phi(1020, 1680)$ are written as

$$q = \frac{1}{2} \sqrt{s - 4m_K^2} ,$$

(18)

$$q_h = \frac{1}{2\sqrt{s}} \sqrt{(m_B^2 - m_h^2)^2 - 2(m_B^2 + m_h^2)s + s^2} ,$$

(19)
with $m_h$ the mass for the bachelor meson pion or kaon. The direct $CP$ asymmetry $A_{CP}$ is defined as

$$A_{CP} = \frac{B(B \rightarrow f) - B(B \rightarrow \bar{f})}{B(B \rightarrow f) + B(B \rightarrow \bar{f})}$$

(20)

The Lorentz invariant decay amplitudes for the quasi-two-body decays $B \rightarrow \phi(1020, 1680)h \rightarrow K\bar{K}h$ are collected in the Appendix.

III. RESULTS

In the numerical calculation, we employ the decay constants $f_B = 0.189$ GeV and $f_{B_s} = 0.231$ GeV [78], the mean lives $\tau_{B^0} = (1.520 \pm 0.004) \times 10^{-12}$ s, $\tau_{B^{+}} = (1.638 \pm 0.004) \times 10^{-12}$ s and $\tau_{B_s^0} = (1.509 \pm 0.004) \times 10^{-12}$ s [27] for the $B^0$, $B^+$ and $B_s^0$ mesons, respectively. The masses and the decay constants for the relevant particles in this work, the full widths for $\phi(1020)$ and $\phi(1680)$, and the Wolfenstein parameters of the Cabbibo-Kobayashi-Maskawa (CKM) matrix are presented in Table I.

| TABLE I: Masses, decay constants, full widths of $\phi(1020)$ and $\phi(1680)$ (in units of GeV) and Wolfenstein parameters [27]. |
|---------------------------------------------------------------|
| $m_{B^0}$ = 5.280 | $m_{B^+}$ = 5.279 | $m_{B_s^0}$ = 5.367 | $m_{s\pi} = 0.140$ | $m_{s\phi} = 0.135$ |
| $m_{K^{*+}} = 0.494$ | $m_{K^{*0}} = 0.498$ | $f_K = 0.156$ | $f_{s} = 0.130$ | $m_{\phi(1680)} = 1.019$ |
| $\Gamma_{\phi(1020)} = 0.00425$ | $m_{\phi(1680)} = 1.680 \pm 0.020$ | $\Gamma_{\phi(1680)} = 0.150 \pm 0.050$ | $\lambda = 0.22453 \pm 0.00044$ | $A = 0.836 \pm 0.015$ | $\check{\rho} = 0.122^{+0.018}_{-0.017}$ | $\bar{\eta} = 0.355^{+0.012}_{-0.011}$ |

The coefficients $c^K_{\phi(1680)}$ and $c^K_{\phi(1020)}$ in the electromagnetic form factors $F_{K^+}$ and $F_{K^0}$, the Eqs. (5)-(6), have been fitted to the data in Refs. [69, 72, 79]. The results of the constrained and unconstrained fits in [69, 72] and the results of the Model I and Model II in [79] for $c^K_{\phi(1680)}$ agree with each other. While the values for $c^K_{\phi(1020)}$ are quite different in Refs. [69, 72, 79], with the results $-0.018 \mp 0.006$ (0.001 $\mp$ 0.007) and 0.0042 $\pm$ 0.0015 (0.0136 $\pm$ 0.0024) of the constrained (unconstrained) fits in [69] and [72], respectively, and $-0.117 \pm 0.020$ ($-0.150 \pm 0.009$) for the Model I (II) in [79]. But we noticed that the coefficient $c^K_{\phi(1450)}$ for the pion electromagnetic form factor $F_\pi$ in Refs. [73, 80-83] by different Collaborations are consistent with each other. With the relations [69]

$$|c^K_{\phi(1450)}| \approx \frac{f_{\rho(1450)}|g_{\rho(1450)}|}{\sqrt{2}m_{\rho(1450)}}$$

and the result $|c^K_{\phi(1450)}| = 0.178$ in [73], it’s easy to get $|c^K_{\phi(1680)}| \approx 0.160$ supposing the coupling constants $g_{\phi(1680)KK} \approx -g_{\rho(1450)\pi\pi}/\sqrt{2}$ [69] and the decay constants $f_{\rho(1450)}/f_{\rho(1680)} \approx f_{\rho(770)}/f_{\rho(1020)}$. With the partial width ratio [26]

$$\frac{\Gamma(KK)}{\Gamma(K\bar{K}^{*}(892))} \approx 0.073$$

(22)

and the rough branching ratio $B_{K\bar{K}^{*}(892)} \approx 0.7$ [35, 84] for the resonance $\phi(1680)$, one could estimate $|c^K_{\phi(1680)}| \approx 0.092$. While with the decay widths 19.8 $\pm$ 4.3 MeV in [85] and 17 MeV in [86] for $\phi(1680) \rightarrow K\bar{K}$, we estimate the coefficient $|c^K_{\phi(1680)}|$ at about 0.130-0.162. In view of our estimated values, we employ the fitted result $c^K_{\phi(1680)} = -0.150 \pm 0.009$ [79] in our numerical calculation in this work. As for the coefficient $c^K_{\phi(1020)}$ of the electromagnetic form factors $F_{K^+}$ and $F_{K^0}$, we adopt its fitted value 1.038 in the Model II in Ref. [79].

Utilizing the differential branching fraction the Eq. (17) and the decay amplitudes collected in Appendix A, we obtain the concerned direct $CP$ asymmetries and the $CP$ averaged branching fractions for the quasi-two-body decays $B \rightarrow \phi(1020)h \rightarrow K\bar{K}h$ in Table II and $B \rightarrow \phi(1680)h \rightarrow K^+\bar{K}^-$ in Table III. Only the modes $B^+ \rightarrow \phi(1020, 1680)K^+$ and $B^0_s \rightarrow \phi(1020, 1680)\pi^0$ with $\phi(1020, 1680)$ decay into $K^+\bar{K}^-$ or $K^0\bar{K}^0$, which contain the contributions from the current-current operators of the weak effective Hamiltonian [87], have the direct $CP$ asymmetries in Tables II, III. The first error of these results in Tables II, III comes from the uncertainty of the shape parameters $\omega_B = 0.40 \pm 0.04$ for $B^+$ and $B^0_s$ and $\omega_B = 0.50 \pm 0.05$ for $B^0_s$, the second error is induced by the chiral masses $m_{\pi}^0 = 1.40 \pm 0.10$ GeV, $m_{K}^0 = 1.60 \pm 0.10$ GeV and the Gegenbauer moment $a_2^{\tau,K} = 0.25 \pm 0.15$ for $\pi$ and $K$ as in [88], the third one is contributed by the Gegenbauer moment $a_2^{\phi} = 0.18 \pm 0.08$ [75] and the fourth error in Table III comes from the
variation of the coefficient $c^K_{\phi(1680)}$ of the form factor $F_K$, which will not change the direct $CP$ asymmetries. There are other errors come from the uncertainties of the masses and the decay constants of the initial and final states, the other parameters in the distribution amplitudes of the bachelor pion or kaon, the Wolfenstein parameters of the CKM matrix, etc. are small and have been neglected.

### TABLE II: PQCD predictions of the $CP$ averaged branching fractions and the direct $CP$ asymmetries for the $B \to \phi(1020)h \to K\bar{K}h$ decays.

| Decay modes | Quasi-two-body results |
|-------------|------------------------|
| $B^+ \to \phi(1020)K^+ \to K^+K^-K^+$ $\mathcal{B}(10^{-6})$ | $4.03 \pm 0.67 (\omega_B) \pm 0.49 (m_0^K + a_2^K) \pm 0.15 (a_2^K)$ |
| $A_{CP}$(%) | $-1.31 \pm 0.91 (\omega_B) \pm 2.63 (m_0^K + a_2^K) \pm 0.55 (a_2^K)$ |
| $B^+ \to \phi(1020)\pi^+ \to K^+K^-\pi^+$ $\mathcal{B}(10^{-9})$ | $3.58 \pm 1.17 (\omega_B) \pm 1.87 (m_0^\pi + a_2^\pi) \pm 0.34 (a_2^\pi)$ |
| $B^0 \to \phi(1020)K^0 \to K^+K^-K^0$ $\mathcal{B}(10^{-6})$ | $3.62 \pm 0.64 (\omega_B) \pm 0.59 (m_0^K + a_2^K) \pm 0.19 (a_2^K)$ |
| $B^0 \to \phi(1020)\pi^0 \to K^+K^-\pi^0$ $\mathcal{B}(10^{-9})$ | $1.74 \pm 0.53 (\omega_B) \pm 0.91 (m_0^\pi + a_2^\pi) \pm 0.14 (a_2^\pi)$ |
| $B^+ \to \phi(1020)\pi^0 \to K^+K^-\pi^0$ $\mathcal{B}(10^{-8})$ | $9.11 \pm 2.03 (\omega_B) \pm 0.14 (m_0^\pi + a_2^\pi) \pm 0.61 (a_2^\pi)$ |
| $B^+ \to \phi(1020)K^+ \to K^0\bar{K}^0K^+$ $\mathcal{B}(10^{-6})$ | $2.79 \pm 0.46 (\omega_B) \pm 0.34 (m_0^K + a_2^K) \pm 0.11 (a_2^K)$ |
| $A_{CP}$(%) | $2.47 \pm 0.81 (\omega_B) \pm 1.30 (m_0^\pi + a_2^\pi) \pm 0.24 (a_2^\pi)$ |
| $B^0 \to \phi(1020)\pi^0 \to K^0\bar{K}^0\pi^0$ $\mathcal{B}(10^{-6})$ | $2.50 \pm 0.44 (\omega_B) \pm 0.41 (m_0^K + a_2^K) \pm 0.13 (a_2^K)$ |
| $B^0 \to \phi(1020)K^0 \to K^0\bar{K}^0K^0$ $\mathcal{B}(10^{-6})$ | $1.20 \pm 0.37 (\omega_B) \pm 0.63 (m_0^\pi + a_2^\pi) \pm 0.10 (a_2^\pi)$ |
| $B^+ \to \phi(1020)\pi^0 \to K^0\bar{K}^0\pi^0$ $\mathcal{B}(10^{-8})$ | $5.76 \pm 0.33 (\omega_B) \pm 0.65 (m_0^K + a_2^K) \pm 1.44 (a_2^K)$ |
| $B^0 \to \phi(1020)K^0 \to K^0\bar{K}^0K^0$ $\mathcal{B}(10^{-8})$ | $6.30 \pm 1.40 (\omega_B) \pm 0.10 (m_0^\pi + a_2^\pi) \pm 0.43 (a_2^\pi)$ |

### TABLE III: PQCD predictions of the $CP$ averaged branching fractions and the direct $CP$ asymmetries for the $B \to \phi(1680)h \to K^+K^-h$ decays. The decay mode with the subprocess $\phi(1680) \to K^0\bar{K}^0$ has the same branching fraction and direct $CP$ asymmetry of its corresponding decay with $\phi(1680) \to K^+K^-$.  

| Decay modes | Quasi-two-body results |
|-------------|------------------------|
| $B^+ \to \phi(1680)K^+ \to K^+K^-K^+$ $\mathcal{B}(10^{-7})$ | $2.51 \pm 0.33 (\omega_B) \pm 0.42 (m_0^K + a_2^K) \pm 0.15 (a_2^K) \pm 0.30 (c^K_6)$ |
| $A_{CP}$(%) | $-1.39 \pm 0.85 (\omega_B) \pm 1.33 (m_0^\pi + a_2^\pi) \pm 0.67 (a_2^\pi) \pm 0.00 (c_8^K)$ |
| $B^+ \to \phi(1680)\pi^+ \to K^+K^-\pi^+$ $\mathcal{B}(10^{-10})$ | $2.84 \pm 0.98 (\omega_B) \pm 1.73 (m_0^\pi + a_2^\pi) \pm 0.26 (a_2^\pi) \pm 0.34 (c_8^K)$ |
| $B^0 \to \phi(1680)K^0 \to K^+K^-K^0$ $\mathcal{B}(10^{-7})$ | $2.39 \pm 0.36 (\omega_B) \pm 0.41 (m_0^K + a_2^K) \pm 0.18 (a_2^K) \pm 0.29 (c_8^K)$ |
| $B^0 \to \phi(1680)\pi^0 \to K^+K^-\pi^0$ $\mathcal{B}(10^{-10})$ | $1.46 \pm 0.45 (\omega_B) \pm 0.73 (m_0^\pi + a_2^\pi) \pm 0.16 (a_2^\pi) \pm 0.18 (c_8^K)$ |
| $B^0 \to \phi(1680)\pi^0 \to K^+K^-K^0$ $\mathcal{B}(10^{-9})$ | $6.25 \pm 0.40 (\omega_B) \pm 1.05 (m_0^K + a_2^K) \pm 1.40 (a_2^K) \pm 0.75 (c_8^K)$ |
| $B^0 \to \phi(1680)K^0 \to K^+K^-\pi^0$ $\mathcal{B}(10^{-9})$ | $6.47 \pm 1.38 (\omega_B) \pm 0.17 (m_0^\pi + a_2^\pi) \pm 0.31 (a_2^\pi) \pm 0.78 (c_8^K)$ |
| $A_{CP}$(%) | $5.16 \pm 1.37 (\omega_B) \pm 1.46 (m_0^K + a_2^K) \pm 0.38 (a_2^K) \pm 0.00 (c_8^K)$ |

The two-body branching fractions for $B \to \phi h$ can be extracted from the quasi-two-body predictions with the relation

$$
\Gamma(B \to \phi h \to K\bar{K}h) \approx \Gamma(B \to \phi h) \cdot \mathcal{B}(\phi \to K\bar{K}) .
$$

In Ref. [44], a parameter $\eta$ was defined to measure the violation of the factorization relation the Eq. (23) for the $B \to K^0(1430)h$ and $B \to K^0(1430)h \to K\pi h$ decays. For the decays $B \to \phi(1020)h$ and $B \to \phi(1020)h \to K\bar{K}h$ in
this work, we have the definition

\[
\eta = \frac{\Gamma(B \to \phi(1020)h \to K\bar{K}h)}{\Gamma(B \to \phi(1020)h) \cdot B(\phi(1020) \to KK)} \\
\approx \frac{m^4_{\phi(1020)}}{384\pi m^3_{B}f^2_{\phi}q_{h}} B(\phi(1020) \to K\bar{K}) \\
\times \int_{4m_{K}^2}^{(m_B-m_h)^2} \frac{ds}{s^3} \left( \frac{\lambda^{3/2}(m^2_B, s, m^2_h)}{(s-m^2_{\phi(1020)}L)} + (m_{\phi(1020)}\Gamma_{\phi(1020)}(s)) \right)^2, \tag{24}
\]

where \(\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc\), the \(q_{h}\) is the expression of Eq. (19) in the rest frame of \(B\) meson and fixed at \(s = m^2_h\). As an example, we have \(\eta \approx 1.07\) for the decays \(B^0 \to \phi(1020)K^0\) and \(B^0 \to \phi(1020)K^0 \to K^+K^-K^0\) with the branching fraction \(B(\phi(1020) \to K^+K^-) = 0.492\) \cite{27}. It means that the violation of the Eq. (23) is small when neglecting the effect of the squared invariant mass \(s\) in the decay amplitudes of the quasi-two-body decays. As the verification of Eq. (24), we calculate the decay \(B^0 \to \phi(1020)K^0\) in the two-body framework of the PQCD approach with the same parameters and obtain its branching fraction \(B(B^0 \to \phi(1020)K^0) \approx 7.21 \times 10^{-6}\), which is about 98.0\% of the result \(7.36 \times 10^{-6}\) in Table IV extracted with the corresponding quasi-two-body result in Table II with the factorization relation.

The comparison of the extracted PQCD predictions with the experimental measurements for the relevant two-body branching fractions are shown in Table IV. The branching ratio \(8.8^{+0.7}_{-0.6} \times 10^{-6}\) for the two-body decay \(B^+ \to \phi(1020)K^+\), which was averaged from the results in Refs. [89–92] presented BaBar, CDF, Belle and CLEO Collaborations, is consistent with the prediction (8.19±1.71)\times10^{-6}\) in this work. The data \(B = (7.3±0.7) \times 10^{-6}\) in [27] averaged from the results in [89, 93–95] for the decay \(B^0 \to \phi(1020)K^0\) agree well with our prediction (7.36±1.81)\times10^{-6}\) in Table IV. In [96], an upper limit \(1.5 \times 10^{-7}\) was set by LHCb at 90\% confidence level for the branching fraction of the decay \(B^0 \to \phi\pi^\pm\). Very recently, LHCb Collaboration presented a fit fraction (0.3 ± 0.1 ± 0.1)\% of the total branching fraction of \(B^0 \to \pi^+K^+K^-\) for the subprocess \(\phi(1020) \to K^+K^-\) in Ref. [97], meaning the two-body branching ratio \(B(B^0 \to \phi(1020)\pi^\pm) = (3.2 \pm 1.5) \times 10^{-8}\) [98], which is larger than the corresponding prediction in IV but both with large uncertainty.

| Two-body decays | This work | Data [27] |
|-----------------|-----------|-----------|
| \(B^+ \to \phi(1020)K^+\) | \(8.19 \pm 1.71 \times 10^{-6}\) | \(8.8^{+0.7}_{-0.6} \times 10^{-6}\) |
| \(B^+ \to \phi(1020)\pi^+\) | \(7.28 \pm 4.54 \times 10^{-9}\) | \(< 1.5 \times 10^{-7}\) |
| \(B^0 \to \phi(1020)K^0\) | \(7.36 \pm 1.81 \times 10^{-6}\) | \((7.3 \pm 0.7) \times 10^{-6}\) |
| \(B^0 \to \phi(1020)\pi^0\) | \(3.54 \pm 2.16 \times 10^{-9}\) | \(< 1.5 \times 10^{-7}\) |

The \(B^+ \to \phi(1020)\pi^+\) was studied in [99] with its branching ratio at about \(5 \times 10^{-9}\) within QCDF, which agree with our prediction (7.28 ± 4.54) \times 10^{-9}\) within errors. The predicted results in Table II for the decays \(B^0_s \to \phi(1020)K^0\) and \(B^0_s \to \phi(1020)\pi^0\) are consistent with the theoretical results in Refs. [75, 99–101] within errors by considering \(B(\phi(1020) \to K^+K^-) = 0.492\) [27]. The branching ratios for the two-body decays \(B \to \phi\pi\) were found can be enhanced by the \(\omega-\phi\) mixing effect in [102]. The \(\omega-\phi\) mixing effect for the quasi-two-body decays \(B \to \phi(1020, 1680)h \to K\bar{K}h\) is out of the scope of this work and will be left to the future studies. The penguin-dominated two-body decays \(B^\pm \to K^\pm\phi(1020)\) and \(B^0 \to K^0\phi(1020)\) have been studied in Refs. [103–105] within PQCD approach with the consistent results with our predicted values in Table IV.

The predictions for the branching fractions of the decays \(B \to \phi(1680)h \to K^+K^-h\) in Table III are about 6\%-8\% of the corresponding results for \(B \to \phi(1020)h \to K^+K^-h\) in Table II. The main portion of these branching fractions for \(B \to \phi(1020, 1680)h \to K^+K^-h\) lies in the region around the pole masses of the intermediate states \(\phi(1020)\) and \(\phi(1680)\), which could be concluded from the differential branching fractions for the decays \(B^0 \to \phi(1020)K^0 \to K^+K^-K^0\) and \(B^0 \to \phi(1680)K^0 \to K^+K^-K^0\) shown in Fig. 2. In Ref. [28], the contributions from the subprocesses \(\phi(1020) \to K^+K^-\) and \(\phi(1680) \to K^+K^-\) were fitted by LHCb to be (70.5 ± 0.6 ± 1.2)\% and (4.0 ± 0.3 ± 0.3)\%, respectively, of the total branching fraction for the three-body decay \(B^0_s \to J/\psi K^+K^-\), implying a ratio at about 0.06 between the branching fractions of the quasi-two-body decays \(B^0_s \to J/\psi(1680) \to J/\psi K^+K^-\) and \(B^0_s \to J/\psi(1020) \to J/\psi K^+K^-,\) which is consistent with the results 6\%-8\% in this work for \(B \to \phi(1680, 1020)h \to K^+K^-h\).
The ratio $R_{\phi(1680)}$ between branching fractions of the decays $\phi(1680) \to K^0\bar{K}^0$ and $\phi(1680) \to K^+K^-$ is defined as

$$R_{\phi(1680)} = \frac{B(\phi(1680) \to K^0\bar{K}^0)}{B(\phi(1680) \to K^+K^-)} \approx \frac{g^2_{\phi(1680)K^0\bar{K}^0}(m_{\phi(1680)}^2 - 4m_{K^0}^2)^{3/2}}{g^2_{\phi(1680)K^+K^-}(m_{\phi(1680)}^2 - 4m_{K^+}^2)^{3/2}} \approx 1$$

by considering the coupling constants $g_{\phi(1680)K^0\bar{K}^0} = g_{\phi(1680)K^+K^-}$ \cite{69} and

$$m_{\phi(1680)}^2 - 4m_{K^0}^2 \approx m_{\phi(1680)}^2 - 4m_{K^+}^2. \quad (25)$$

It tells us that the decay mode with the subprocess $\phi(1680) \to K^0\bar{K}^0$ has the same branching fraction of its corresponding process with $\phi(1680) \to K^+K^-$ for $B \to \phi(1680)h \to K\bar{K}h$. While for the decays $\phi(1020) \to K^0\bar{K}^0$ and $\phi(1020) \to K^+K^-$, one has $R_{\phi(1020)} \approx 0.66$, which is consistent with the ratio 0.69 between the branching fractions in \cite{27} for these two decays, with the coupling constants $g_{\phi(1020)K^0\bar{K}^0} = g_{\phi(1020)K^+K^-}$ \cite{16, 69} and the replacement $\phi(1680) \to \phi(1020)$ for the Eq. (25). The results in Table II for the subprocess $\phi(1020) \to K^0\bar{K}^0$ are deduced from $B(\phi(1020) \to K^0\bar{K}^0) = 34.0\%$ \cite{27} along with the results in the same table for the decays with the subprocess $\phi(1020) \to K^+K^-$. With the decay amplitude for $B^+ \to \phi(1020)K^0 \to K^+K^-\phi$, we calculate the branching fraction and direct CP asymmetry for the decay $B^+ \to \phi(1020)K^+ \to K^0\bar{K}^0\phi$, and obtain the central values $B = 2.83 \times 10^{-6}$ and $A_{CP} = -1.25\%$ for it, which are agree well with the results in Table II for this process.

IV. SUMMARY

In this work, we studied the contributions for the $K^+K^-$ and $K^0\bar{K}^0$ which originated from the intermediate states $\phi(1020)$ and $\phi(1680)$ in the charmed three-body decays $B \to K\bar{K}h$ in PQCD approach. The subprocesses $\phi(1020, 1680) \to K\bar{K}$ were introduced into the distribution amplitudes of $K\bar{K}$ system via the kaon electromagnetic form factor with the coefficients $c^K_{\phi(1020)}$ and $c^K_{\phi(1680)}$, which are adopted from the fitted results. With $c^K_{\phi(1020)} = 1.038$ and $c^K_{\phi(1680)} = -0.150 \pm 0.009$ we predicted the branching fractions for the quasi-two-body decays $B \to \phi(1020)h \to K\bar{K}h$ and $B \to \phi(1680)h \to K^+K^-h$ and the direct CP asymmetries for the decay modes $B^+ \to \phi(1020, 1680)K^+$ and $B^0 \to \phi(1020, 1680)\pi^0$ with $\phi(1020, 1680)$ decay into $K^+K^-\phi$ or $K^0\bar{K}^0\phi$.

The predictions for the branching fractions of the decays $B \to \phi(1680)h \to K^+K^-h$ are about 6%-8% of the corresponding results for $B \to \phi(1020)h \to K^+K^-h$ in this work. The branching fraction for the decay $\phi(1680) \to K^0\bar{K}^0 \phi$ is equal to that for $\phi(1680) \to K^+K^-\phi$, and the decay mode with the subprocess $\phi(1680) \to K^0\bar{K}^0$ has the same branching fraction of its corresponding mode with $\phi(1680) \to K^+K^-\phi$ for $B \to \phi(1680)h \to K\bar{K}h$. We defined a parameter $\eta$ to measure the violation of the factorization relation for the decays $B \to \phi h$ and $B \to \phi h \to K\bar{K}h$ and found the violation is quite small. With the factorization relation, we extracted the branching fractions for the two-body decays $B^{0,+} \to \phi(1020)K^{0,+}$ and $B^{0,+} \to \phi(1020)\pi^{0,+}$. The predictions for the decays $B^0 \to \phi(1020)K^0$ and $B^+ \to \phi(1020)K^+$ are agree with the existing data. And our results for $B^{0,+} \to \phi(1020)\pi^{0,+}$ consistent with the theoretical results in literature.
Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grants No. 11505148, No. 11547038 and No. 11575110. Y.Y. Fan was also supported by the Nanhu Scholars Program for Young Scholars of XYNU. W.F. Wang thank Ai-Jun Ma for valuable discussions.

Appendix A: DECAY AMPLITUDES

The Lorentz invariant decay amplitude $A$ for the quasi-two-body processes $B \rightarrow \phi(1020, 1680)h \rightarrow K\bar{K}h$ is given by $A = \Phi_B \otimes H \otimes \Phi_h \otimes \Phi_{KK}$ [4, 38] in the PQCD approach, according to Feynman diagrams the Fig. 1. The symbol $\otimes$ means convolutions in parton momenta, the hard kernel $H$ contains one hard gluon exchange at the leading order in strong coupling $\alpha_s$. The distribution amplitudes $\Phi_B, \Phi_h$ and $\Phi_{KK}$ absorb the nonperturbative dynamics in the relevant processes. The $\Phi_B$ and $\Phi_h$ for $B$ meson and the bachelor final state $h$ in this work are the same as those widely employed in the studies of the hadronic $B$ meson decays in the PQCD approach, one can find their expressions and parameters in the Appendix of [44] and the references therein.

With the subprocesses $\phi \rightarrow K^+K^-$, $\phi \rightarrow K^0\bar{K}^0$, and $\phi$ is the $\phi(1020)$ or $\phi(1680)$, the concerned quasi-two-body decay amplitudes are given as follows:

\[
A(B^+ \rightarrow \phi K^+) = \frac{G_F}{\sqrt{2}} \left\{ V_{tb}^*V_{us}[a_1 F_{LL}^{LL} + C_1 M_{LL}^{LL}] - V_{tb}^*V_{ts}[a_3 + a_4 + a_5 - \frac{a_7 + a_9 + a_10}{2}] \right\} F_{LL}^{LL} + \frac{1}{2} \left( a_6 + a_8 \right) F_{SP}^{LL} + \left( C_3 + C_6 - \frac{C_7}{2} \right) M_{TH}^{LL} + \left( C_5 - \frac{C_8}{2} \right) M_{TH}^{SP} + \left( C_9 \right) M_{AH}^{LL} + \left( C_4 + C_7 \right) M_{AH}^{SP}, \tag{A1}\]

\[
A(B^+ \rightarrow \phi \pi^+) = \frac{G_F}{\sqrt{2}} \left\{ -V_{tb}^*V_{td}[a_3 + a_5 - \frac{a_7 + a_9}{2}] F_{TH}^{LL} + \left( C_4 - \frac{C_7}{2} \right) M_{TH}^{LL} + \left( C_5 - \frac{C_8}{2} \right) M_{TH}^{SP} \right\} , \tag{A2}\]

\[
A(B^0 \rightarrow \phi K^0) = \frac{G_F}{\sqrt{2}} \left\{ -V_{tb}^*V_{ts}[a_3 + a_5 - \frac{a_7 + a_9}{2}] F_{TH}^{LL} + \left( a_6 - \frac{a_8}{2} \right) F_{TH}^{LL} + \left( C_4 - \frac{C_7}{2} \right) M_{TH}^{LL} + \left( C_5 - \frac{C_8}{2} \right) M_{TH}^{SP} \right\} , \tag{A3}\]

\[
A(B^+ \rightarrow \phi \pi^0) = \frac{-1}{\sqrt{2}} A(B^+ \rightarrow \phi \pi^+), \tag{A4}\]

\[
A(B^0_s \rightarrow \phi K^0) = \frac{G_F}{\sqrt{2}} \left\{ -V_{tb}^*V_{ts}[a_1 - \frac{a_10}{2}] F_{T\phi}^{LL} + \left( a_6 - \frac{a_8}{2} \right) F_{T\phi}^{LL} + \left( a_3 + a_5 - \frac{a_7 + a_9}{2} \right) F_{T\phi}^{LL} \right\} + \left( C_4 - \frac{C_7}{2} \right) M_{TH}^{LL} + \left( C_5 - \frac{C_8}{2} \right) M_{TH}^{SP} + \left( C_3 + C_6 - \frac{C_7}{2} \right) M_{AH}^{LL} + \left( C_5 - \frac{C_8}{2} \right) M_{AH}^{SP}, \tag{A5}\]

\[
A(B^0_s \rightarrow \phi \pi^0) = \frac{G_F}{2} \left\{ V_{ub}^*V_{us}[a_2 F_{T\phi}^{LL} + C_2 M_{LL}^{LL}] - V_{tb}^*V_{ts}\frac{3}{2} \left[ a_9 - a_7 \right] F_{T\phi}^{LL} + \frac{3}{2} C_10 M_{T\phi}^{LL} + \frac{3}{2} C_6 M_{T\phi}^{SP} \right\} , \tag{A6}\]

where $G_F$ is the Fermi coupling constant, $V$’s are the CKM matrix elements. The combinations $a_i$ for the Wilson coefficients are defined as

\[
a_1 = C_2 + \frac{C_3}{3}, \quad a_2 = C_1 + \frac{C_3}{3}, \quad a_3 = C_4 + \frac{C_4}{3}, \quad a_4 = C_4 + \frac{C_3}{3}, \quad a_5 = C_5 + \frac{C_6}{3}, \tag{A7}\]

\[
a_6 = C_6 + \frac{C_5}{3}, \quad a_7 = C_7 + \frac{C_8}{3}, \quad a_8 = C_8 + \frac{C_7}{3}, \quad a_9 = C_9 + \frac{C_10}{3}, \quad a_{10} = C_{10} + \frac{C_9}{3}. \tag{A8}\]

It should be understood that the Wilson coefficients $C_i$, the amplitudes $F$ and $M$ for the factorizable and nonfactorizable Feynman diagrams, respectively, appear in convolutions in momentum fractions and impact parameters $b$.

The amplitudes from Fig. 1 (a) are written as

\[
F_{T\phi}^{LL} = 8\pi C_F m_{b}^4 f_{h}(\zeta - 1) \int dx_B dz \int b_bdb_Bbd\phi_B(x_B,b_B) \left\{ [(1+z)\phi^0 + \sqrt{z}(1-2z)(\phi^* + \phi') \right\} \times \left\{ E_{a12}(t_{a1})h_{a1}(x_B,z,b_B) + [\phi^0 + 2\sqrt{z}\phi^*]E_{a12}(t_{a2})h_{a2}(x_B,z,b_B) \right\} , \tag{A9}\]

\[
F_{T\phi}^{RR} = -F_{T\phi}^{LL}, \tag{A10}\]
\[ F_{\psi}^{SP} = 16\pi C f m_B^4 f_B \int dx_B dz \int b dB \cdot \Phi_B(x_B, B) \left\{ \left[ (2z - 1) + 1 \right] \phi \phi + \sqrt{\phi} \right\} \times E_{a12}(t_{a1}) h_{a1}(x_B, z, B, b) + [x_B \phi + 2\sqrt{\phi}(z - 1) - 2\phi] E_{a12}(t_{a2}) h_{a2}(x_B, z, B, b) \}, \quad (A11) \]

\[ M_{\psi}^{LL} = 32\pi C f m_B^4 / \sqrt{2N_c} (\zeta - 1) \int d x_B d z d x_3 \int b dB \cdot \Phi_B(x_B, B) [ \left\{ (1 - \zeta)(1 - x_3) - x_B \left \{ \phi + \sqrt{\phi} \right \} \right] E_{a34}(t_{a3}) h_{a3}(x_B, z, x_3, B, b) + [x_3(1 - 1) - x_B - z \phi^0 + z \sqrt{\phi} \phi^0 + \phi^0] E_{a34}(t_{a4}) h_{a4}(x_B, z, x_3, B, b) \}, \quad (A12) \]

\[ M_{\psi}^{LR} = 32\pi C f m_B^4 / \sqrt{2N_c} (\zeta - 1) \int d x_B d z d x_3 \int b dB \cdot \Phi_B(x_B, B) [ \left\{ (1 - \zeta)(1 - x_3) - x_B \phi + \sqrt{\phi} \phi^0 - \phi^0 \right\} \right] E_{a34}(t_{a3}) h_{a3}(x_B, z, x_3, B, b) + [x_3(1 - 1) - x_B - z \phi^0 + z \sqrt{\phi} \phi^0 + \phi^0] E_{a34}(t_{a4}) h_{a4}(x_B, z, x_3, B, b) \}, \quad (A13) \]

\[ M_{\psi}^{SP} = 32\pi C f m_B^4 / \sqrt{2N_c} (\zeta - 1) \int d x_B d z d x_3 \int b dB \cdot \Phi_B(x_B, B) [ \left\{ (1 - \zeta)(1 - x_3) - x_B \phi^0 + \sqrt{\phi} \phi^0 - \phi^0 \right\} \right] E_{a34}(t_{a3}) h_{a3}(x_B, z, x_3, B, b) + [x_3(1 - 1) - x_B - z \phi^0 + z \sqrt{\phi} \phi^0 - \phi^0] E_{a34}(t_{a4}) h_{a4}(x_B, z, x_3, B, b) \}, \quad (A14) \]

where the color factor \( C_F = 4/3 \) and the ratio \( r = m_Q^2 / m_B \). The amplitudes from Fig. 1 (b) are written as

\[ F_{A_\phi}^{LL} = 8\pi C f m_B^4 f_B \int d x_B d z d x_3 \int b dB \cdot \Phi_B(x_B, B) \left\{ [(1 - \zeta)(1 - z) \phi^0 + 2\sqrt{\phi} \phi^0] + [(1 - x_3) \phi^0 + 2\sqrt{\phi} \phi^0] \right\} E_{a12}(t_{a1}) h_{a1}(x_B, z, x_3, B, b) + [(1 - x_3)(1 - \zeta) - x_B \phi^0 + \sqrt{\phi} \phi^0 - \phi^0] E_{a12}(t_{a2}) h_{a2}(x_B, z, B, b) \}, \quad (A15) \]

\[ F_{A_\phi}^{LR} = - F_{A_\phi}^{LL} \]

\[ F_{A_\phi}^{SP} = 16\pi C f m_B^4 f_B \int d x_B d z d x_3 \int b dB \cdot \Phi_B(x_B, B) \left\{ [(1 - \zeta)(1 - z) \phi^0 + 2\sqrt{\phi} \phi^0] - 2\sqrt{\phi} \phi^0\right\} E_{a12}(t_{a1}) h_{a1}(x_B, z, x_3, B, b) + [(1 - x_3)(1 - \zeta) - x_B \phi^0 + \sqrt{\phi} \phi^0 - \phi^0] E_{a12}(t_{a2}) h_{a2}(x_B, z, B, b) \}, \quad (A16) \]

\[ M_{A_\phi}^{LL} = 32\pi C f m_B^4 / \sqrt{2N_c} \int d x_B d z d x_3 \int b dB \cdot \Phi_B(x_B, B) \left\{ [(1 - \zeta)(1 - z) \phi^0 + 2\sqrt{\phi} \phi^0] - 2\sqrt{\phi} \phi^0\right\} E_{a12}(t_{a1}) h_{a1}(x_B, z, x_3, B, b) + [(1 - x_3)(1 - \zeta) - x_B \phi^0 + \sqrt{\phi} \phi^0 - \phi^0] E_{a12}(t_{a2}) h_{a2}(x_B, z, B, b) \}, \quad (A17) \]

\[ M_{A_\phi}^{LR} = 32\pi C f m_B^4 / \sqrt{2N_c} \int d x_B d z d x_3 \int b dB \cdot \Phi_B(x_B, B) \left\{ [(1 - \zeta)(1 - z) \phi^0 + 2\sqrt{\phi} \phi^0] - 2\sqrt{\phi} \phi^0\right\} E_{a12}(t_{a1}) h_{a1}(x_B, z, x_3, B, b) + [(1 - x_3)(1 - \zeta) - x_B \phi^0 + \sqrt{\phi} \phi^0 - \phi^0] E_{a12}(t_{a2}) h_{a2}(x_B, z, B, b) \}, \quad (A18) \]

\[ M_{A_\phi}^{SP} = 32\pi C f m_B^4 / \sqrt{2N_c} \int d x_B d z d x_3 \int b dB \cdot \Phi_B(x_B, B) \left\{ [(1 - \zeta)(1 - z) \phi^0 + 2\sqrt{\phi} \phi^0] - 2\sqrt{\phi} \phi^0\right\} E_{a12}(t_{a1}) h_{a1}(x_B, z, x_3, B, b) + [(1 - x_3)(1 - \zeta) - x_B \phi^0 + \sqrt{\phi} \phi^0 - \phi^0] E_{a12}(t_{a2}) h_{a2}(x_B, z, B, b) \}, \quad (A19) \]
The amplitudes from Fig. 1 (d) are written as

\[
F_{TL}^{LL} = 8\pi C_F m_B^4 F_K \int dx_B dx_3 \int b_B db_B b_3 db_3 \phi_B(x_B, b_B) \left\{ \left[ (1 - \zeta)(x_3(\zeta - 1) - \phi^A + r(2x_3 - 1)\phi^P) - r(1 + \zeta - 2x_3(1 - \zeta))\phi^T \right] E_{c12}(t_{c1}) h_{c1}(x_B, x_3, b_B, b_3) + [x_B(1 - \zeta)\phi^A - 2r(1 - \zeta(1 - x_B))\phi^P] E_{c12}(t_{c2}) \times h_{c2}(x_B, x_3, b_B, b_3) \right\}, \tag{A21}
\]

\[
F_{TR}^{LR} = F_{TL}^{LL}, \tag{A22}
\]

\[
M_{TL}^{LL} = 32\pi C_F m_B^4 \sqrt{2N_c} \int dx_B dx_3 \int b_B db_B b_3 db_3 \phi_B(x_B, b_B) \phi^0 \left\{ [(x_B + z - 1)(1 - \zeta)^2 \phi^A + r[\zeta(x_B + z) \times (\phi^P + \phi^T) + x_3(1 - \zeta)(\phi^P - \phi^T) - 2\zeta\phi^T]] E_{c34}(t_{c3}) h_{c3}(x_B, x_3, b_B, b_3) + [(1 - \zeta)x_3(\zeta - 1) + x_B - z]\phi^A + r[x_3(x_3(\zeta - 1)(\phi^P + \phi^T) - (x_B - z)\zeta(\phi^P - \phi^T))] E_{c34}(t_{c4}) h_{c4}(x_B, x_3, b_B, b_3) \right\}, \tag{A23}
\]

\[
M_{TL}^{LP} = 32\pi C_F m_B^4 \sqrt{2\zeta}/\sqrt{2N_c} \int dx_B dx_3 \int b_B db_B b_3 db_3 \phi_B(x_B, b_B) \phi^0 \left\{ [(x_B + z - 1)(1 - \zeta)(\phi^A - r(x_B - z)\phi^T + 2\zeta\phi^T] E_{c34}(t_{c3}) h_{c3}(x_B, x_3, b_B, b_3) + [(z - x_B)(1 - \zeta)(\phi^A - r(x_B - z)\phi^T + 2\zeta\phi^T] E_{c34}(t_{c4}) h_{c4}(x_B, x_3, b_B, b_3) \right\}. \tag{A24}
\]

\[
M_{SP}^{LP} = 32\pi C_F m_B^4 \sqrt{2\zeta}/\sqrt{2N_c} \int dx_B dx_3 \int b_B db_B b_3 db_3 \phi_B(x_B, b_B) \phi^0 \left\{ [(z - x_B)(1 - \zeta)(\phi^A - r(x_B - z)\phi^T + 2\zeta\phi^T] E_{c34}(t_{c3}) h_{c3}(x_B, x_3, b_B, b_3) + [(z - x_B)(1 - \zeta)(\phi^A - r(x_B - z)\phi^T + 2\zeta\phi^T] E_{c34}(t_{c4}) h_{c4}(x_B, x_3, b_B, b_3) \right\}. \tag{A25}
\]

The amplitudes from Fig. 1 (d) are written as

\[
F_{LH}^{LR} = 8\pi C_F m_B^4 F_B \int dz dx_3 \int bdbdb_3 \phi_B(x, z, b_B, b_3) \left\{ [(x_3(1 - \zeta) - 1)(\zeta^2 - 1)\phi^A + 2r\sqrt{\zeta}\phi^A \phi^P + 2\phi^P] E_{d12}(t_{d1}) h_{d1}(x, x_3, b_b, b_3) + [z(\zeta - 1)\phi^A - 2r\sqrt{\zeta} \phi^A \phi^P + (1 - \zeta)(\phi^A - \phi^P)] E_{d12}(t_{d2}) \times h_{d2}(x, x_3, b_b, b_3) \right\}, \tag{A26}
\]

\[
F_{LR}^{LR} = -F_{LH}^{LR}, \tag{A27}
\]

\[
F_{SP}^{LP} = 16\pi C_F m_B^4 F_B \int dz dx_3 \int bdbdb_3 \phi_B(x, z, b_B, b_3) \left\{ [(\zeta - 1)(\phi^A - \phi^P)] E_{d12}(t_{d1}) h_{d1}(x, x_3, b_b, b_3) + [z\sqrt{\zeta} \phi^A \phi^P + 2r(z_3(\zeta) - 1)\phi^P] E_{d12}(t_{d2}) h_{d2}(x, x_3, b_b, b_3) \right\}. \tag{A28}
\]

\[
M_{LR}^{LR} = 32\pi C_F m_B^4 \sqrt{2N_c} \int dx_B dz dx_3 \int bdbdb_3 \phi_B(x, b_B) \left\{ [(x_B + z - 1)(\phi^A - r(1 - x_3)\phi^P - r(1 - \zeta)\phi^T)] E_{d34}(t_{d3}) h_{d3}(x_B, x_3, b_b, b_3) + [(1 - \zeta)x_3(\zeta - 1)(\phi^A - r(1 - \zeta)\phi^P - r(1 - \zeta)\phi^T)] E_{d34}(t_{d4}) h_{d4}(x_B, x_3, b_b, b_3) \right\} \tag{A29}
\]

\[
M_{SP}^{LP} = 32\pi C_F m_B^4 \sqrt{2N_c} \int dx_B dz dx_3 \int bdbdb_3 \phi_B(x, b_B) \left\{ [(\zeta - 1)(\phi^A - \phi^P)] E_{d34}(t_{d3}) h_{d3}(x_B, x_3, b_b, b_3) + [(1 - \zeta)(\phi^A - \phi^P)] E_{d34}(t_{d4}) h_{d4}(x_B, x_3, b_b, b_3) \right\}. \tag{A30}
\]

\[
M_{SP}^{SP} = 32\pi C_F m_B^4 \sqrt{2N_c} \int dx_B dz dx_3 \int bdbdb_3 \phi_B(x, b_B) \left\{ [(\zeta - 1)(x_3 - 1)(\phi^A - \phi^P)] E_{d34}(t_{d3}) h_{d3}(x_B, x_3, b_b, b_3) + [(1 - \zeta)(x_3 - 1)(\phi^A - \phi^P)] E_{d34}(t_{d4}) h_{d4}(x_B, x_3, b_b, b_3) \right\}. \tag{A31}
\]

For the errors induced by the parameter $\mathcal{P} \pm \Delta \mathcal{P}$ for the $B$ and $A_{CP}$ in the numerical calculation of this work, we
employ the formulas \[\text{[44]}\]

\[\Delta B = \left| \frac{\partial B}{\partial P} \right| \Delta P, \quad \Delta A_{CP} = \frac{2(B\Delta B - B\Delta B)}{(B + \bar{B})^2}\]  \hspace{1cm} (A32)

The PQCD functions which appear in the factorization formulas, the Eqs. (A9)-(A31), can be found in the Appendix B of [44].

[1] G.N. Fleming, Phys. Rev. 135, B551 (1964)
[2] D. Morgan, Phys. Rev. 166, 1731 (1968)
[3] D. Herndon, P. Soding, R.J. Cashmore, Phys. Rev. D 11, 3165 (1975)
[4] W.F. Wang, H.N. Li, Phys. Lett. B 763, 29 (2016)
[5] D. Boito et al., Phys. Rev. D 96, 113003 (2017)
[6] J.H. Alvarenga Nogueira et al., arXiv:1605.03889 [hep-ex]
[7] K.S.F.F. Guimarães et al., Nucl. Phys. Proc. Suppl. 199, 341 (2010)
[8] P.C. Magalhães et al., Phys. Rev. D 84, 094001 (2011)
[9] I. Bediaga, P.C. Magalhães, arXiv:1512.09284 [hep-ph]
[10] G. Breit, E. Wigner, Phys. Rev. 49, 519 (1936)
[11] W.F. Wang, Phys. Rev. D 101, 111901(R) (2020)
[12] Y.Y. Keum, H.N. Li, A.I. Sanda, Phys. Lett. B 504, 6 (2001)
[13] Y.Y. Keum, H.N. Li, A.I. Sanda, Phys. Rev. D 63, 054008 (2001)
[14] C.D. Liu, K. Ukai, M.Z. Yang, Phys. Rev. D 63, 074009 (2001)
[15] H.N. Li, Prog. Part. Nucl. Phys. 51, 85 (2003)
[16] E.A. Kozyrev et al., Phys. Lett. B 779, 64 (2018)
[17] M.N. Achasov et al., Phys. Rev. D 94, 112006 (2016)
[18] E.A. Kozyrev et al. [CMD-3 Collaboration], Phys. Lett. B 760, 314 (2016)
[19] J.P. Lees et al. [BaBar Collaboration], Phys. Rev. D 88, 032013 (2013)
[20] R.R. Akhmetshin et al., Phys. Lett. B 695, 412 (2011)
[21] R.R. Akhmetshin et al. [CMD-2 Collaboration], Phys. Lett. B 669, 217 (2008)
[22] M.N. Achasov et al., Phys. Rev. D 63, 072002 (2001)
[23] B. Dey et al. [CLAS Collaboration], Phys. Rev. C 89, 055208 (2014). Addendum: [Phys. Rev. C 90, 019901 (2014)]
[24] F.V. Flores-Báez, G. López Castro, Phys. Rev. D 78, 077301 (2008)
[25] F. Mane et al., Phys. Lett. B 112, 178 (1982)
[26] J. Buon et al., Phys. Lett. B 118, 221 (1982)
[27] M. Tanabashi et al. [Particle Data Group], Phys. Rev. D 98 030001 (2018)
[28] R. Aaij et al. [LHCb Collaboration], JHEP 1708, 037 (2017)
[29] T. Barnes, N. Black, P.R. Page, Phys. Rev. D 68, 054014 (2003)
[30] B. Aubert et al. [BaBar Collaboration], Phys. Rev. D 74, 091103(R) (2006)
[31] B. Aubert et al. [BaBar Collaboration], Phys. Rev. D 76, 012008 (2007)
[32] B. Aubert et al. [BaBar Collaboration], Phys. Rev. D 77, 092002 (2008)
[33] M. Ablikim et al. [BES Collaboration], Phys. Rev. Lett. 100, 102003 (2008)
[34] C.P. Shen et al. [Belle Collaboration], Phys. Rev. D 80, 031101(R) (2009)
[35] J.P. Lees et al. [BaBar Collaboration], Phys. Rev. D 86, 012008 (2012)
[36] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. D 91, 052017 (2015)
[37] J. Ho, R. Berg, T.G. Steele, W. Chen, D. Harnett, Phys. Rev. D 100, 034012 (2019)
[38] C.H. Chen, H.N. Li, Phys. Lett. B 561, 258 (2003)
[39] C.H. Chen, H.N. Li, Phys. Rev. D 70, 054006 (2004)
[85] M. Piotrowska, C. Reisinger, F. Giacosa, Phys. Rev. D 96, 054033 (2017)
[86] D. Ebert, R.N. Faustov, V.O. Galkin, Phys. Lett. B 744, 1 (2015)
[87] G. Buchalla, A.J. Buras, M.E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996)
[88] W.F. Wang, Z.J. Xiao, Phys. Rev. D 86, 114025 (2012)
[89] J.P. Lees et al. [BaBar Collaboration], Phys. Rev. D 85, 112010 (2012)
[90] D. Acosta et al. [CDF Collaboration], Phys. Rev. Lett. 95, 031801 (2005)
[91] A. Garmash et al. [Belle Collaboration], Phys. Rev. D 71, 092003 (2005)
[92] R.A. Briere et al. [CLEO Collaboration], Phys. Rev. Lett. 86, 3718 (2001)
[93] K.F. Chen et al. [Belle Collaboration], Phys. Rev. Lett. 91, 201801 (2003)
[94] B. Aubert et al. [BaBar Collaboration], Phys. Rev. Lett. 87, 151801 (2001)
[95] B. Aubert et al. [BaBar Collaboration], Phys. Rev. D 69, 011102 (2004)
[96] R. Aaij et al. [LHCb Collaboration], Phys. Lett. B 728, 85 (2014)
[97] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 123, 231802 (2019)
[98] P.A. Zyla et al. [Particle Data Group], Prog. Theor. Exp. Phys. 2020, 083C01 (2020)
[99] M. Beneke, M. Neubert, Nucl. Phys. B 675, 333 (2003)
[100] W. Wang, Y.M. Wang, D.S. Yang, C.D. Lü, Phys. Rev. D 78, 034011 (2008)
[101] H.Y. Cheng, C.K. Chua, Phys. Rev. D 80, 114026 (2009)
[102] Y. Li, C.D. Lü, W. Wang, Phys. Rev. D 80, 014024 (2009)
[103] H.N. Li, S. Mishima, Phys. Rev. D 74, 094020 (2006)
[104] S. Mishima, Phys. Lett. B 521, 252 (2001)
[105] C.H. Chen, Y.Y. Keum, H.N. Li, Phys. Rev. D 64, 112002 (2001)