High energy neutrino and tau airshowers in standard and new physics

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Abstract.
High Energy Neutrino may lead to a New High Energy Astronomy. Neutrino interaction in matter at PeVs- EeVs may behave differently from Standard Model predictions because possible TeV Gravity scenario. While traditional \( km^3 \) neutrino underground detector will be uneasy to disentangle the Gravity TeV scenario any new Horizontal Tau Air-Shower detectors at PeVs \( \tau \) energy, beyond Mountain Chains, will be greatly enhanced and it may probe fruit-fully the New TeV Physics imprint. Moreover observing upcoming and horizontal \( \tau \) air-showers (UPTAUS, HORTAUS) from high mountains toward Mount Chains or along widest Earth Crust crown masses at the Horizons edges, Ultra High Energy, UHE, \( \nu_\tau \) and its consequent \( \tau \) lepton decay in flight, will greatly amplify the single UHE \( \nu_\tau \) track and it will test, at highest energies, huge volumes comparable to future underground \( km^3 \) ones; observing UPTAUS and HORTAUS from higher balloons or satellites at orbit altitudes, at GZK energies, the Horizontal Crown effective Masses may even exceed 150 \( km^3 \). These Highest energies \((10^{19} eV)\) \( \nu_\tau \) astronomy are tuned to test the needed abundant \( \nu \) fluence in GZK Z-Showering model: in this scenario ZeV Ultra High Energy neutrino are hitting on relic light \((0.1 - 5 eV \text{ masses})\) anti-neutrinos, clustered in dark hot halos, creating UHE Z bosons whose decay in flight may be the hadronic secondaries observed on Earth atmosphere as UHECR, isotropically spread along the cosmic edges and clustered toward few correlated BL Lac sources.

1. Introduction: The need and the rise of a New UHE \( \nu \) astronomy

While Cosmic Rays astronomy is severely blurred by random terrestrial, solar, galactic and extragalactic magnetic lenses, the highest \( \gamma \) ray astronomy (above tens TeVs) became more or less blind because photon-photon opacity (electron pair production) at different energy windows. Indeed the Infrared- TeV opacity as well as a more severe BBR(2.75K')-PeV cut-off are bounding the TeVs -PeV \( \gamma \) ray astronomy in a very nearby cosmic ( or even galactic) volumes. Therefore rarest TeVs gamma signals are at present the most extreme trace of High Energy Astronomy. However we observe copious cosmic rays at higher \((\gg 10^{15} eV)\) energies almost isotropically spread in the sky. However UHECR astronomy must arise, because at largest energies \((\geq 10^{19} eV)\) the magnetic cosmic lenses bending are almost un-effective and UHECR point to their astronomical sources. Because of it also a parasite UHE \( \nu \) astronomy is expected to be present as a necessary consequence of UHECR interaction and cut-off by photopion production on cosmic 2.75 BBR, the well known Greisen, Zatsepin, Kuzmin GZK cut-off [22],[34]. Moreover in a different scenario, the so called Z-Burst or Z-Shower model, UHECR above GZK cut-off are originated by UHE \( \nu \) scattering onto relic light \( \nu \) clustered as a dark hot halos; in this scenario UHE Astronomy is not just a consequence but itself the cause of UHECR signals. Let us remind, among the \( \gamma \) TeVs discoveries, the signals of power-full Jets blazing to us from Galactic (Micro-Quasars) or extragalactic edges (BL Lacs). At PeVs energies astrophysical \( \gamma \) cosmic rays should also be present, but, excluded a very rare and elusive Cyg X3 event, they have not being up date observed; only upper bounds are known at PeV energies. The missing \( \gamma \) PeVs sources, as we mentioned, are very probably...
absorbed by their own photon interactions (electron pairs creation) at the source environment and/or along the photon propagation into the cosmic Black Body Radiation (BBR) or into other diffused background radiation. Unfortunately PeVs charged cosmic rays, easily bend and bounded in a random walk by Galactic magnetic fields, loose their original directionality and their astronomical relevance; their tangled trajectory resident time in the galaxy is much longer ($\geq 10^3 - 10^5$) than any linear neutral trajectory, as gamma rays, making the charged cosmic rays more probable to be observed by nearly a comparable length ratio. However astrophysical UHE neutrino signals at $10^{13}$eV-$10^{19}$eV (or even higher GZK energies) are unaffected by any radiation cosmic opacity and may easily open a very new exciting window toward Highest Energy sources. Being weakly interacting the neutrinos are ideal microscope to deeply observe in their accelerator (Jet,SN,GRB, Black Hole) cores. Other astrophysical $\nu$ sources at lower energies ($10^8$ eV - $10^{12}$ eV) should also be present, at least at EGRET fluence level, but their signals are very probably drowned by the dominant diffused atmospheric $\nu$, secondaries of muon secondaries, produced as pion decays by the same charged (and smeared) UHE cosmic rays (while hitting terrestrial atmosphere): the so called diffused atmospheric neutrinos. Indeed the astrophysical neutrinos signal is being observed and its modulation has inferred the first conclusive evidence for a neutrino mass and for a neutrino flavour mixing. At lowest (MeVs) $\nu$ energy windows, the abundant and steady solar neutrino flux, (as well as the prompt, but rare, neutrino burst from a nearby Super-Novae (SN 1987A)), have been, in last twenty years, successfully explored, giving support to neutrino mass reality. Let us mention that Stellar evolution, Supernova explosion but in particular Early Universe had over-produced and kept in thermal equilibrium neutrinos whose relic presence here today pollute the cosmic spaces either smoothly (lightest relativistic $\nu$) or in denser clustered Hot halos (eVs relic $\nu$ masses). A minimal tiny (above 0.07 eV) $\nu$ mass, beyond a Standard Model, are already making their cosmic energy density component almost two order of magnitude larger than the corresponding 2.75 K (Black Body Radiation) BBR radiation density. Moreover, as recently noted their relic presence may play a key role acting as a calorimeter of ZeV UHE neutrinos born at cosmic edges, solving the GZK cut-off puzzle: The so called Z-burst or Z-WW Shower model [8], [9], [33], [15]. Here we concentrate on the possibility to detect a component of the associated UHE neutrino flux astronomy (above PeVs-EeV up to GZK energies) by UHE $\nu$ interactions in Mountain chains or in Earth Crust leading to Horizontal or Upward Tau Air-Showers [10], [16], [17].

2. The UHE $\nu km^3$ detectors: AMANDA, ANTARES, ICECUBE.

The UHE $10^{13}$eV - $10^{16}$eV $\nu$ 's, being weakly interacting and rarer, may be captured mainly inside huge volumes, bigger than Super-Kamiokande ones; at present most popular detectors consider underground ones (Cubic Kilometer Size like AMANDA-NESTOR) or (at higher energy $10^{19}$eV - $10^{20}$eV) the widest Terrestrial atmospheric sheet volumes (Auger Array Telescope or EUSO atmospheric Detectors). Underground km$^3$ detection is based mainly on $\nu_\mu$ tracks above hundred TeVs energies, because of their high penetration in matter, leading to $\mu$ kilometer size lepton tails [28]. Rarest atmospheric horizontal shower are also expected by $\nu$ interactions in air (and, as we shall discuss, in the Earth Crust) with more secondary tails. While km$^3$ detectors are optimal for PeVs neutrino muons, the Atmospheric Detectors (AUGER-EUSO like) exhibit a minimal threshold at highest ($\geq 10^{19}$eV) energies. The km$^3$ sensibility is more tuned to Tens TeV -PeV astronomy while AUGER has wider acceptance above GZK energies. As we shall discuss $\tau$ Air-Shower detector exhibit also huge acceptance at both energy windows being competitive both at PeV as well as at EeVs energy range as well as GZK ones; we shall not discuss here the $Km^3$ detector as the ICECUBE project.
3. UHECR galactic EeV neutron and EeV-PeV neutrinos astronomy

Incidentally just around such EeV ($10^{18}$ eV) energies an associated Ultra High Energy Neutron Astronomy might be already observed in anisotropic clustering of UHECR data because of the relativistic neutrons boosted lifetime, up to galactic sizes, comparable to our distance from the Galactic center. Therefore UHE neutrons at EeV may be a source candidate of the observed tiny EeV anisotropy in UHECR data. Indeed a 4% galactic anisotropy and clustering in EeV cosmic rays has been recently emerged by AGASA[23] along our nearby galactic spiral arm. These data have been confirmed by a South (Australia) detector(SUGAR) [2]. Therefore AGASA might have already experienced a first UHECR-Neutron astronomy (UHENA) at a very relevant energy flux ($\sim 10 eV cm^{-2}s^{-1}$). This EeV-UHENA signals may and must also be source of at least a comparable parasite ($10^{17} \sim 10^{16}$ eV secondary tails of UHE decaying neutrino $\bar{\nu}_e$ from the same neutron beta decay in flight. Their flavour oscillations and mixing in galactic or extragalactic flights (analogous to atmospheric and solar ones) must guarantee the presence of all lepton flavours nearly at equal foot: $\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau$ [10]. The latter UHE $\bar{\nu}_\tau$ imprint (added to other local astrophysical UHE $\nu$ production) could be already recorded [10] as Upward and Horizontal Tau Air-Shower Terrestrial Gamma Flash (considered as secondaries $\gamma$ of Upward Tau Air-Shower and Horizontal Tau Air-Shower): UPTAUS and HORTAUS.

4. The GZK puzzle and $\nu$ astronomy by Z-shower onto relic neutrino halo

At highest energy edges ($\geq 10^{19} \sim 10^{20}$eV), a somehow correlated New UHE Astronomy is also expected for charged Cosmic Rays; indeed these UHECR have such a large rigidity to avoid any bending by random galactic or extragalactic magnetic fields; being nearly undeflected UHECR should point toward the original sources showing in sky a new astronomical map. Moreover such UHECR astronomy is bounded by the ubiquitous cosmic 2.75K BBR screening (the well known Greisen, Zat’sepin, Kuzmin GZK cut-off) limiting its origination inside a very local (\leq 20 Mpc) cosmic volume. Surprisingly, these UHECR above GZK (already up to day above 60 events) are not pointing toward any known nearby candidate source. Moreover their nearly isotropic arrival distributions underlines and testify a very possible cosmic origination, in disagreement with any local (Galactic plane or Halo, Local Group) expected footprint by GZK cut-off. A very weak Super-Galactic imprint seem to be present but at low level and already above GZK volume. This opened a very hot debate in modern astrophysics known as the GZK paradox. Possible solutions has been found recently beyond Standard Model assuming a non-vanishing neutrino mass. Indeed at such Ultra-High energies, neutrino at ZeV energies ($\geq 10^{21}$eVs) hitting onto relic cosmological light (0.1 – 4eV masses) neutrinos [8] nearly at rest in Dark Hot Halos (galactic or in Local Group) has the unique possibility to produce UHE resonant Z bosons (the so called Z-burst or better Z-Showering scenario). The different channel cross-sections in Z-WW-ZZ-Shower by scattering on relic neutrinos are in Fig.1 while the boosted Z-Shower secondary chain are in Fig.2,Fig.3: [8], [11], [12], [13]: for a more updated scenario see [15], [19]. Indeed UHE neutrinos are unaffected by magnetic fields and by BBR screening; they may reach us from far cosmic edges with negligible absorption. The UHE Z-shower in its ultra-high energy nucleonic secondary component may be just the observed final UHECR event on Earth. This possibility has been reinforced by very recent correlations (doublets and triplets events) between UHECR directions with brightest Blazars sources at cosmic distances (redshift $\geq 0.1$) quite beyond ($\geq 300 Mpc$) any allowed GZK cut-off [21], [23]. [30]. Therefore there might be a role for GZK neutrino fluxes, either at very high fluence as primary in the Z-Showering scenario or, at least, as (but at lower intensities) necessary secondaries of all those UHECR primary absorbed
in cosmic BBR radiation fields by GZK cut off. Naturally other solutions as topological defects or primordial relics decay may play a role as a source of UHECR, but the observed clustering seems to favor compact sources possibly overlapping far BL Lac sources. The most recent evidence for self-correlations clustering at $10^{19}, 2 \cdot 10^{19}, 4 \cdot 10^{19}$ eVs energies observed by AGASA (Teshima, ICRR26 Hamburg presentation 2001) maybe a first reflection of UHECR Z-Showering secondaries: $p, \bar{p}, n, \bar{n}$ [15]. A very recent solution beyond the Standard Model (but within Super-Symmetry) consider Ultra High Energy Gluinos as the neutral particle bearing UHE signals interacting nearly as an hadron in Terrestrial Atmosphere [3]; this solution has a narrow window for gluino masses allowable (and serious problems in production bounds), but it is an alternative that deserves attention. To conclude this brief Z-Shower model survey one finally needs to scrutiny the UHE $\nu$ astronomy and to test the GZK solution within Z-Showering Models by any independent search on Earth for such UHE neutrinos traces above PeVs reaching even EeVs-ZeVs extreme energies.

5. A UHE $\nu$ Astronomy by $\tau$ Air-Shower: An ideal amplifier

Recently [14], [10] it has been proposed a new competitive UHE $\nu$ detection based on ultra high energy $\nu_\tau$ interaction in matter and its consequent secondary $\tau$ decay in flight while escaping from the rock (Mountain Chains, Earth Crust) or water (Sea,Ice) in air leading to Upward or Horizontal $\tau$ Air-Showers (UPTAUs and HORTAUs). [12], [13]. In a pictorial way one may compare the UPTAUs and HORTAUs as the double bang processes expected in $km^3$ ice-water volumes [29] : the double bang is due first to the UHE $\nu_\tau$ interaction in matter and secondly by its consequent $\tau$ decay in flight. Here we consider a (hidden) UHE $\nu$-N Bang in (the rock-water within a mountain or the Earth Crust) and a $\tau$ bang out in air, whose shower is better observable at high altitudes. A similar muon double bang amplifier is not really occurring because of the extreme decay lenght of ultarelativistic ($\gtrsim 10^{13}$ eV
Figure 2. Z-Showering Energy Flux distribution for different channels assuming a light (fine tuned) relic neutrino mass $m_\nu = 0.4\,\text{eV}$ \cite{11}, \cite{15}, \cite{19}. Note that the nucleon injection energy fit the present AGASA data as well as the very recent evidence of a corresponding tiny Majorana neutrino mass \cite{25}. Lighter neutrino masses are able to modulate UHECR at higher energies \cite{15}.

Figure 3. Z-Showering Energy Flux distribution for different channels assuming a light (fine tuned) relic neutrino mass $m_\nu = 1.2\,\text{eV}$ able to partially fill the highest $10^{20}\,\text{eV}$ cosmic ray edges. Note that this value lead to a Z-Knee cut-off, above the GZK one, well tuned to present Hires data; more details on a very light non degenerated neutrino mass ($\leq 0.1\,\text{eV}$) are discussed in the last figure \cite{15}, \cite{24}.
Figure 4. The Horizontal Tau on front of a Mountain Chain; different interaction lengths will reflect in different event rate [4], [10].

The main power of the UPTAUs and HORTAUs detection is the huge amplification of the UHE neutrino signal, which may deliver almost all its energy in numerous secondaries (Cherenkov lights, gamma, X photons, electron pairs, collimated muon bundles) in a wider cone volume. Indeed the multiplicity in \(\tau\) Air-showers secondary particles, \(N_{opt} \simeq 10^{12}(E_{\tau}/PeV)\), \(N_{\gamma}(<E_{\gamma}> \sim 10 MeV) \simeq 10^8(E_{\tau}/PeV)\), \(N_{\mu^{-}e^{+}} \simeq 2 \cdot 10^7(E_{\tau}/PeV)\), \(N_{\mu} \simeq 3 \cdot 10^5(E_{\tau}/PeV)^{0.85}\) makes easy the UPTAUs-HORTAUs discover. These HORTAUs, also named Skimming neutrinos [18], studied also in peculiar approximation in the frame of AUGER experiment, in proximity of Ande Mountain Chains (see Fig. 4) [14], [5], maybe also originated on front of large Vulcano [14], [27] either by \(\nu_{\tau} N, \bar{\nu}_{\tau} N\) interactions as well as by \(\bar{\nu}_{e} \rightarrow W^{-} \rightarrow \bar{\nu}_{\tau} \tau\). Also UHE \(\nu_{\tau} N, \bar{\nu}_{\tau} N\) at EeVs may be present in rare AGASA Horizontal Shower (one single definitive event observed used as un upper bound) facing Mountain Chain around the the Akeno Array (see Fig.4). This new UHE \(\nu_{\tau}\) detection is mainly based on the oscillated UHE neutrino \(\nu_{\tau}\) originated by more common astrophysical \(\nu_{\mu}\), secondaries of pion-muon decay at PeVs-EeVs-GZK energies. These oscillations are guaranteed by Super Kamiokande evidences for flavour mixing within GeVs atmospheric neutrino data [20] as well as by most solid and recent evidences of complete solar neutrino mixing observed by SNO detector [2]. Let us remind that HORTAUs (see Fig. 4) from mountain chains must nevertheless occur, even for no flavour mixing, as being inevitable \(\bar{\nu}_{e}\) secondaries of common pion-muon decay chains \((\pi^{-} \rightarrow \mu^{-} + \bar{\nu}_{\mu} \rightarrow e^{-} + \bar{\nu}_{e}\) near the astrophysical sources at Pevs energies. These Pevs \(\bar{\nu}_{e}\) are mostly absorbed by the Earth and are only rarely arising as UPTAUS (see Fig. 5 and cross-section in Fig.7). Their Glashow resonant interaction allow them to be observed as HORTAUs only within a very narrow and nearby horizons at horizon (not to be discussed here).

At wider energies windows \((10^{14}eV - 10^{20}eV)\) only neutrino \(\nu_{\tau}, \bar{\nu}_{\tau}\) play a key role in UPTAUS and HORTAUS. These Showers might be easily detectable looking downward the Earth’s surface from mountains, planes, balloons or satellites observer. Here the Earth itself acts as a ”big mountain” or a wide beam dump target (see Figs.5-6). The present upward \(\tau\) at horizons should not be confused with an independent and well known, complementary (but rarer) Horizontal Tau Air-shower originated inside the same terrestrial atmosphere: we may
referee to it as the Atmospheric Horizontal Tau Air-Shower. These rare events are responsible for very rare double bang in air. Their probability to occur, as derived in detail in next paragraph (summarized in last row Table 1 below, labeled as ratio R between the event the ground over air event rate) is more than two order magnitude below the event rate of HORTAUs. The same UPTAUS (originated in Earth Crust) have a less competitive upward showering due to $\nu_e, \bar{\nu}_e$ interactions within atmosphere, showering in thin upward air layers [4]: this atmospheric Upward Tau presence is as a very small additional contribute, because rock is more than 3000 times denser than air, see Table 1. Therefore at different heights we need to estimate (See for detail next paragraph) the UPTAUS and HORTAUs event rate occurring along the thin terrestrial crust below the observer, keeping care of their correlated variables: from a very complex sequence of functions we shall be able to define and evaluate the effective HORTAUs volumes keeping care of the thin shower beaming angle, atmosphere opacity and detector thresholds. At the end of the study, assuming any given neutrino flux, one might be easily able to estimate at each height $h_1$ the expected event rate and the ideal detector size and sensibility for most detection techniques (Cherenkov, photo-luminescent, gamma rays, X-ray, muon bundles)(see Fig.18). The Upcoming Tau Air-Shower and Horizontal ones may be already recorded as Terrestrial Gamma Flashes as shown by their partial Galactic signature shown in Fig.8 (over EGRET celestial background) and in Fig.9 (over EeV anisotropy found by Agasa).
6. The Ultra High Energy $\nu_\tau$ astronomy by UPTAUs-HORTAUs detection

The $\tau$ airshowers are observable at different height $h_1$ leading to different underneath observable terrestrial areas and crust volumes. HORTAUs in deep valley are also relate to the peculiar geographical morphology and composition \[10\] and more in detail as discussed below. We remind in this case the very important role of UHE $\bar{\nu}_e e \rightarrow W^- \rightarrow \bar{\nu}_\tau \tau^-$ channels which may be well observable even in absence of any $\nu_\tau$, $\nu_\tau$ UHE sources or any neutrino flavour mixing: its Glashow peak resonance make these neutrinos unable to cross all the

| $\rho$ | 1 | 2.65 | 1 | 2.65 | 1 | 2.65 | 1 | 2.65 |
|--------|----|------|----|------|----|------|----|------|
| $h_1$ (eV) | 2 | 2 | 5 | 5 | 25 | 25 | 500 | 500 |
| $E_{th}$ (km$^2$) | $3.12 \cdot 10^{18}$ | $3.12 \cdot 10^{18}$ | $4.67 \cdot 10^{18}$ | $4.67 \cdot 10^{18}$ | $8 \cdot 10^{18}$ | $8 \cdot 10^{18}$ | $1.08 \cdot 10^{18}$ | $1.08 \cdot 10^{18}$ |
| $A_{TOT}$ (km$^2$) | $8 \cdot 10^4$ | $8 \cdot 10^4$ | $2 \cdot 10^5$ | $2 \cdot 10^5$ | $10^6$ | $10^6$ | $1.8 \cdot 10^7$ | $1.8 \cdot 10^7$ |
| $l_\tau$ (km) | 21.7 | 11 | 24.3 | 12.1 | 27.5 | 13.1 | 29.4 | 13.8 |
| $\delta \theta$ | $1.31^o$ | $0.97^o$ | $1.79^o$ | $1.07^o$ | $2.36^o$ | $1.08^o$ | $2.72^o$ | $1.07^o$ |
| $l_\tau \sin\delta \theta$ (km) | 0.496 | 0.186 | 0.76 | 0.225 | 1.13 | 0.247 | 1.399 | 0.257 |
| $A_R$ (km$^2$) | $7.9 \cdot 10^4$ | $7.2 \cdot 10^4$ | $1.9 \cdot 10^5$ | $1.45 \cdot 10^5$ | $7.16 \cdot 10^5$ | $3.83 \cdot 10^5$ | $4.3 \cdot 10^6$ | $1.75 \cdot 10^6$ |
| $d\Omega/\Omega$ | $2.5 \cdot 10^{-5}$ | $2.5 \cdot 10^{-5}$ | $2.5 \cdot 10^{-5}$ | $2.5 \cdot 10^{-5}$ | $2.5 \cdot 10^{-5}$ | $2.5 \cdot 10^{-5}$ | $2.5 \cdot 10^{-5}$ | $2.5 \cdot 10^{-5}$ |
| $\Delta V$ (km$^3$) | $3.95 \cdot 10^4$ | $1.34 \cdot 10^4$ | $1.45 \cdot 10^5$ | $3.2 \cdot 10^4$ | $8.12 \cdot 10^5$ | $9.5 \cdot 10^4$ | $6 \cdot 10^6$ | $4.5 \cdot 10^5$ |
| $V_{eff}$ | 0.987 | 0.335 | 3.64 | 0.82 | 20.3 | 2.4 | 150.6 | 11.3 |
| $\Delta M$ (km$^3$) | 0.987 | 0.89 | 3.64 | 2.17 | 20.3 | 6.3 | 150.6 | 30 |
| $R=\frac{M_T}{M_{ATM}}$ | 49.6 | 49.2 | 75 | 59.6 | 113 | 65.45 | 140 | 68 |
Figure 7. Different interaction lengths for Ultra High Energy Neutrino that will reflect in different event rate either for Horizontal Shower from Mountain Chains as well as from Upward and Horizontal ones from Earth Crust; a severe suppression in the neutrino interaction length, $R_{\text{New}}$, due to any New TeV gravity will increase by $2 - 3$ order of magnitude the neutrino birth probability leading to an expected tens of thousand event a year (respect to a hundred a year) by a ten-km length Array detector on front of a Mountain Chain, assuming a $10^{3} \text{eV cm}^{-2}\text{s}^{-1}$ cosmic neutrino fluence (see previous figures) [10].

Earth across but it may be observable beyond mountain chain [10]; while testing $\tau$ air-showers beyond a mountain chain one must consider the possible amplification of the signal because of a possible New TeV Physics (see cross-section in Fig 7) [10]. In the following we shall consider in general the main $\nu_{\tau} - N$, $\bar{\nu}_{\tau} - N$ nuclear interaction on Earth crust. It should be kept in mind also that UPTAUs and in particular HORTAUS are showering at very low densities and their geometrical escaping opening angle from Earth (here assumed at far distances $\theta \sim 1^\circ$ for rock and $\theta \sim 3^\circ$ for water) is not in general conical (like common down-ward showers) but their ending tails are more shaped in a thin fan-like twin Jets (like the observed 8 shaped horizontal Air-Showers) bent and split in two thin elliptical beams by the geo-magnetic fields. These fan shape are not widely opened by the Terrestrial Magnetic Fields while along the North-South magnetic field lines. These UPTAUs-HORTAUs duration time are also much longer than common down-ward showers because their showering occurs at much lower air density and they are more extended: from micro (UPTAUS from mountains) to millisecond (UPTAUs and HORTAUs from satellites) long gamma-flashes. Indeed the GRO did observe upcoming Terrestrial Gamma Flashes are possibly correlated with the UPTAUs [10]; these events show the expected millisecond duration times. In order to estimate the rate and the fluence for of UPTAUs and HORTAUs one has to estimate the observable crown terrestrial crust mass, facing a complex chain of questions, leading for each height $h_1$, to an effective observable surface and volume from where UPTAUs and HORTAUs might be originated. From this effective volume it is easy to estimate the observable rates, assuming a given incoming UHE $\nu$ flux model for galactic or extragalactic sources. Here we shall only refer to the Masses estimate unrelated to any UHE $\nu$ flux models. These steps are
Figure 8. The Terrestrial Gamma Flash arrival map over the EGRET (hundred MeVs-GeV) data in celestial coordinate. It is manifest the partial galactic signature and the crowding of repeater events toward the Galactic Center. Also some relevant repeater event are observed toward anti-galactic direction and to well known extragalactic source (see next map) and [10].

Figure 9. The Terrestrial Gamma Flash arrival map over the EeV anisotropic map \((10^{18}eV)\) data in celestial coordinate. Some relevant X-\(\gamma\)-TeV sources are also shown in the same region; see [10].
Figure 10. The Upward Tau Air-Shower Ring or Crown Areas, either labeled UPTAUs, the Horizontal Tau Air-Shower Ring Area, labeled by HORTAU, where the $\tau$ is showering and flashing toward an observer at height $h_1$. The HORTAU Ring Areas are described both for water and rock matter density.

linking simple terrestrial spherical geometry and its different geological composition, high energy neutrino physics and UHE $\tau$ interactions, the same UHE $\tau$ decay in flight and its air-showering physics at different quota within terrestrial air density. Detector physics threshold and background noises, signal rates have been kept in mind [10], but they will be discussed and explained in forthcoming papers.

7. The skin crown Earth volumes as a function of $h$ observation height

Let us therefore define, list and estimate below the sequence of the key variables whose dependence (shown below or derived in Appendices) leads to the desired HORTAUs volumes (useful to estimate the UHE $\nu$ prediction rates) summarized in Table 1 and in Conclusions. These Masses estimate are somehow only a lower bound that ignore additional contribute by more penetrating or regenerated $\tau$ [26]. Let us now show the main functions whose interdependence with the observer altitude lead to estimate the UPTAUs and HORTAUs equivalent detection Surfaces, (See Fig. 10-11-13), Volumes and Masses (see Table 1).

(i) The horizontal distance $d_h$ at given height $h_1$ toward the horizons (see Fig. 12):

$$d_h = \sqrt{(R_\odot + h_1)^2 - (R_\odot)^2} = 113 \sqrt{\frac{h_1}{km}} \cdot km \sqrt{1 + \frac{h_1}{2R_\odot}}$$  

The corresponding horizontal edge angle $\theta_h$ below the horizons ($\pi/2$) is (see Fig. 11):

$$\theta_h = \arccos \frac{R_\odot}{(R_\odot + h_1)} \simeq 1^\circ \sqrt{\frac{h_1}{km}}$$  

(All the approximations here and below hold for height $h_1 \ll R_\odot$)

(ii) The consequent characteristic lepton $\tau$ energy $E_{\tau_h}$ making decay $\tau$ in flight from $d_h$ distance just nearby the source:

$$E_{\tau_h} = \left(\frac{d_h}{c\tau_0}\right) m_\tau c^2 \simeq 2.2 \cdot 10^{18} eV \sqrt{\frac{h_1}{km}} \sqrt{1 + \frac{h_1}{2R_\odot}}$$
Figure 11. The geometrical disposal and the main parameters, as in the text, defining the UPTAU and HORTAU Ring (Crown or Coronas) Areas; the distances are exaggerated for simplicity.

At low quota \( (h_1 \leq \text{a few kms}) \) the air depth before the Tau decay necessary to develop a shower corresponds to a Shower distance \( d_{Sh} \sim 6 \text{kms} \ll d_h \). More precisely at low quota \( (h_1 \ll h_o, \text{where } h_o \text{ is the air density decay height= 8.55 km.}) \) one finds:

\[
d_{Sh} \simeq 5.96 \text{km}[1 + \ln \left( \frac{E_\tau}{10^{18} \text{eV}} \right)] \cdot e^{h_1 / h_o}
\]

So we may neglect the distance of the final shower respect to the longest horizons ones. However at high altitude \( (h_1 \geq h_o) \) this is no longer the case (see Appendix A). Therefore we shall introduce from here and in next steps a small, but important modification, whose physical motivation is just to include the air dilution role at highest quota: \( h_1 \rightarrow h_1 + h_1 / H_o \), where \( , \text{as in Appendix A, } H_o = 23 \text{ km.} \) Therefore previous definition (at height \( h_1 < R_{\oplus} \)) becomes:

\[
E_\tau \simeq 2.2 \cdot 10^{18} \text{eV} \sqrt{\frac{h_1}{1 + h_1 / H_o}} \sqrt{1 + \frac{h_1}{2R_{\oplus}}}
\]

This procedure, applied tacitly everywhere, guarantees that there we may extend our results to those HORTAUs at altitudes where the residual air density must exhibit a sufficient slant depth. For instance, highest \( \gg 10^{19} \text{eV} \) HORTAUs will be not easily observable because their \( \tau \) life distance exceed (usually) the horizons air depth lengths. The parental UHE \( \nu_\tau, \bar{\nu}_\tau \) or \( \bar{\nu}_e \) energies \( E_{\nu_\tau} \) able to produce such UHE \( E_\tau \) in matter are:

\[
E_{\nu_\tau} \simeq 1.2E_\tau \simeq 2.64 \cdot 10^{18} \text{eV} \cdot \sqrt{\frac{h_1}{1 \text{km}}}
\]

(iii) The neutrino (underground) interaction lengths at the corresponding energies is \( L_{\nu_\tau} \):
The same distance projected cord

The corresponding opening angle observed from height

The underground chord

The maximal neutrino depth $h_2(h_1)$, see Fig. 11 above, under the chord along the UHE neutrino-tau trajectory of length $L_\nu(h_1)$ has been found:

$$h_2(h_1) = \frac{L_{\nu h}^2}{2} \cdot \frac{L_{\nu h}^2}{2(R - h_2)} \simeq \frac{L_{\nu h}^2}{8R_\oplus} \simeq 1.81 \cdot km \cdot \left( \frac{h_1}{km} \right)^{-0.863} \cdot \left( \frac{\rho_{\text{rock}}}{\rho_r} \right)^2$$

See Figure 11, for more details. Because the above $h_2$ depths are in general not too deep respect to the Ocean depths, we shall consider respectively either sea (water) or rock (ground) materials as Crown matter density.

(iv) The corresponding opening angle observed from height $h_1$, $\delta_{1h}$ encompassing the underground height $h_2$ at horizons edge (see Fig. 11) and the nearest UHE $\nu$ arrival directions $\delta_1$:

$$\delta_{1h}(h_2) = 2 \arctan \frac{h_2}{2d_h} = 2 \arctan \left[ \frac{8 \cdot 10^{-3} \cdot \left( \frac{h_2}{km} \right)^{-0.863} \cdot \left( \frac{\rho_{\text{rock}}}{\rho_r} \right)^2}{\sqrt{1 + \frac{h_1}{2R_\oplus}}} \right]$$

$$\simeq 0.91^\circ \left( \frac{\rho_{\text{rock}}}{\rho_r} \right)^2 \cdot \left( \frac{h_1}{km} \right)^{-0.863}$$

(v) The underground chord $d_{u_1}$ (see Fig. 11 - 13) where UHE $\nu_r$ propagate and the nearest distance $d_1$ for $\tau$ flight (from the observer toward Earth) along the same $d_{u_1}$ direction, within the angle $\delta_{1h}$ defined above, angle below the horizons (within the upward UHE neutrino and HORTAUs propagation line) is:

$$d_{u_1} = 2 \cdot \sqrt{\sin^2(\theta_h + \delta_{1h})(R_\oplus + h_1)^2 - d_h^2}$$

Note that by definition and by construction $L_\nu \equiv d_{u_1}$. The nearest HORTAUs distance corresponding to this horizontal edges still transparent to UHE $\tau$ is:

$$d_1(h_1) = (R_\oplus + h_1) \sin(\theta_h + \delta_{1h}) - \frac{1}{2} d_{u_1}$$

Note also that for height $h_1 \geq km$ :

$$\frac{d_{u_1}}{2} \simeq (R_\oplus + h_1) \sqrt{\delta_{1h} \sin 2\theta_h} \simeq 158 \sqrt{\frac{\delta_{1h}}{1^\circ}} \sqrt{\frac{h_1}{km}} \cdot km.$$

(vi) The same distance projected cord $x_1(h_1)$ along the horizontal line (see Fig. 11):

$$x_1(h_1) = d_1(h_1) \cos(\theta_h + \delta_{1h})$$

The total terrestrial underneath any observer at height $h_1$ is $A_T$ (see Figs. 10-13):

$$A_T = 2\pi R_\oplus^2 (1 - \cos \tilde{\theta}_h) = 2\pi R_\oplus h_1 \left( \frac{1}{1 + \frac{h_1}{R_\oplus}} \right) A_T = 4 \cdot 10^4 km^2 \left( \frac{h_1}{km} \right) \left( \frac{1}{1 + \frac{h_1}{R_\oplus}} \right)$$

Where $\tilde{\theta}_h$ is the opening angle from the Earth along the observer and the horizontal point whose value is the maximal observable one. At first sight one may be tempted to consider
all the Area $A_T$ for UPTAUs and HORTAUs but because of the air opacity (HORTAUs) or for its paucity (UPTAUs) this is incorrect. While for HORTAUs there is a more complex Area estimated above and in the following, for UPTAUs the Area Ring (or Disk) is quite simpler to derive following very similar geometrical variables summarized in Appendix B.

(vii) The Earth Ring Crown crust area $A_R(h_1)$ delimited by the horizons distance $d_h$ and the nearest distance $d_1$ still transparent to UHE $\nu_\tau$ (see Figs.10-13-16-17). The ring area $A_R(h_1)$ is computed from the internal angles $\delta \tilde{\theta}_h$ and $\delta \tilde{\theta}_1$ defined at the Earth center (Fig.13) (note that $\delta \tilde{\theta}_h = \delta \theta_h$ but in general $\delta \tilde{\theta}_1 \neq \delta \theta_1$).

\[ A_R(h_1) = 2\pi R_\oplus^2 (\cos \tilde{\theta}_1 - \cos \tilde{\theta}_h) = 2\pi R_\oplus^2 \left( \sqrt{1 - \left( \frac{x_1(h_1)}{R_\oplus} \right)^2} - \frac{R_\oplus}{R_\oplus + h_1} \right) \]  

(11)

Here $x_1(h_1)$ is the cord defined above.

(viii) The characteristic interaction lepton tau length $l_\tau$ defined at the average $E_{\tau_1}$, from interaction in matter (rock or water). These lengths have been derived by a analytical equations keeping care of the Tau lifetime, the photo-nuclear losses, the electro-weak losses [10]. See Fig.14 below. The tau length along the Earth Skimming distance $l_{\tau_2}$ is vertically projected along the $\sin(\delta \tilde{\theta}_h)$:

\[ \delta \tilde{\theta}_{h_1} \equiv \tilde{\theta}_h - \arcsin \left( \frac{x_1}{R_\oplus} \right) \]  

(12)

The same quantity in a more direct approximation:

\[ \sin \delta \tilde{\theta}_{h_1} \simeq \frac{L_{\nu}}{2R_\oplus} = \frac{304km}{2R_\oplus} \left( \frac{\rho_{rock}}{\rho} \right) \frac{h_1}{km}^{-0.1815} \]
Figure 13. Total and Ring (Crown) Areas and Angles for UPTAUS-HORTAUS observed at different height.

Figure 14. Lepton $\tau$ (and $\mu$) Interaction Lengths for different matter density: $R_{\tau_0}$ is the free $\tau$ length, $R_{\tau_{\text{new}}}$ is the New Physics TeV Gravity interaction range at corresponding densities, $R_{\tau_{\text{new+p}}}$, see also [11], [13], is the combined $\tau$ Ranges keeping care of all known interactions and lifetime and mainly the photo-nuclear one. There are two slightly different split curves (for each density) by two comparable approximations in the interaction laws. $R_{\text{Weakp}}$ is the electro-weak Range at corresponding densities (see also [28]), [10].
Figure 15. The $\delta \tilde{\theta}_{h_1}$ opening angle toward Ring Earth Skin for density $\rho_{water}$ and $\rho_{rock}$ (see Figs. 11-15).

From highest ($h \gg H_o = 23km$) altitude the exact approximation reduces to:

$$\delta \tilde{\theta}_{h_1} \approx 1^o \left( \frac{\rho_{rock}}{\rho} \right) \left( \frac{h_1}{500 \cdot km} \right)^{-0.1815}$$

Therefore the penetrating $\tau$ skin depth $l_{\tau_1}$ is

$$l_{\tau_1} = l_\tau \cdot \sin \delta \tilde{\theta}_{h_1} \approx 0.0462 \cdot l_\tau \left( \frac{\rho_{water}}{\rho} \right) \frac{h_1}{km}^{-0.1815}$$

(13)

Where the $\tau$ ranges in matter, $l_\tau$ has been calculated and shown in Fig.14.

(ix) The final analytical expression for the Earth Crust Skin Volumes and Masses under the Earth Skin inspected by HORTAU$\bar{s}$ are derived combining the above functions on HORTAU$\bar{s}$ Areas with the previous lepton Tau $l_{\tau_1}$ vertical depth depths:

$$V_{h_1} = A_R(h_1) \cdot l_{\tau_1};$$  \hspace{1cm} (14)

$$M_{h_1} = V_{h_1} \cdot \left( \frac{\rho}{\rho_{water}} \right)$$  \hspace{1cm} (15)

(x) A More approximated but easy to handle expression for Ring area for high altitudes ($h_1 \gg 2km, h_1 \ll R_\oplus$) may be summarized as:

$$A_R(h_1) \approx 2\pi R_\oplus^2 \sin \theta_\nu \delta \tilde{\theta}_{h_1} \propto \rho^{-1}$$

$$\approx 2\pi R_\oplus^2 \sqrt{\frac{2h_1}{R_\oplus}} \left( \frac{1 + \frac{h_1}{2R_\oplus}}{1 + \frac{h_1}{R}} \right) \left( \frac{L_\nu}{2R_\oplus} \right)$$

(16)
Figure 16. Total Area $A_T$ and Ring (Crown or Coronas) Areas for two densities $A_R$ at low altitudes.

Figure 17. Total Area $A_T$ and Ring (Crown) Areas for two densities $A_R$ at high altitudes.
At high altitudes the above approximation corrected accordingly to the exact one shown in Figure, becomes:

\[ A_R(h_1) \simeq 2\pi R_\oplus d_{h_1} \delta \theta h_1 \simeq 4.65 \cdot 10^6 \sqrt{\frac{h_1}{500\text{km}}} \left( \frac{\rho_{\text{water}}}{\rho} \right) \text{km}^2 \]  \hspace{1cm} (17)

Within the above approximation the final searched Volume \( V_{h_1} \) and Mass \( M_{h_1} \) from where HORTAUs may be generated is:

\[ V_{h_1} = \frac{\pi}{2} \sqrt{\frac{2h_1}{R_\oplus}} \left( \frac{\sqrt{1 + \frac{h_1}{2R_\oplus}}}{1 + \frac{h_1}{R_\oplus}} \right) L_\nu l_r \propto \rho^{-3} \]  \hspace{1cm} (18)

\[ M_{h_1} = \frac{\pi}{2} \sqrt{\frac{2h_1}{R_\oplus}} \left( \frac{\sqrt{1 + \frac{h_1}{2R_\oplus}}}{1 + \frac{h_1}{R_\oplus}} \right) L_\nu l_r \rho \propto \rho^{-2} \]  \hspace{1cm} (19)

(xi) The effective observable Skin Tau Mass \( M_{\text{eff.}}(h_1) \) within the thin HORTAU or UPTAUs Shower angle beam \( \simeq 1^\circ \) is suppressed by the solid angle of view \( \frac{\delta \Omega}{\Omega} \simeq 2.5 \cdot 10^{-5} \).

\[ \Delta M_{\text{eff.}}(h_1) = V_{h_1} \cdot \left( \frac{\rho}{\rho_{\text{water}}} \right) \frac{\delta \Omega}{\Omega} \]  \hspace{1cm} (20)

The lower bound Masses \( M_{\text{eff.}}(h_1) \) exactly estimated , with no approximation as in eq.19, from eq.15, at different realistic high quota experiments, are discussed in the Conclusion below and summarized in Table 1.

8. Summary and conclusions

The discover of the expected UHE neutrino Astronomy is urgent and just behind the corner. Huge volumes are necessary. Beyond underground \( \text{km}^3 \) detectors a new generation of UHE neutrino calorimeter lay on front of mountain chains and just underneath our feet: The Earth itself offers huge Crown Volumes as Beam Dump calorimeters observable via upward Tau Air Showers, UPTAUs and HORTAUs. Their effective Volumes as a function of the quota \( h_1 \) has been derived by an analytical function variables in equations above and Appendix B. These Volumes are discussed below and summarized in the last column of Table 1 and displayed as bounds in Fig. 18. At a few tens meter altitude the UPTAUs and HORTAUs Ring are almost overlapping. At low altitude \( h_1 \leq 2 \text{Km} \) the HORTAUs are nearly independent on the \( \rho \) matter density: \( \Delta M_{\text{eff.}}(h_1 = 2\text{Km})(\rho_{\text{water}}) = 0.987\text{km}^3 \); \( \Delta M_{\text{eff.}}(h_1 = 2\text{Km})(\rho_{\text{Rock}}) = 0.89\text{km}^3 \). These volumes are the effective Masses expressed in Water equivalent volumes. On the contrary at higher quotas, like highest Mountain observations sites, Airplanes, Balloons and Satellites, the matter density of the HORTAUs Ring (Crown) Areas play a more and more dominant role asymptotically proportional to \( \rho^{-2} \); \( \Delta M_{\text{eff.}}(h_1 = 5\text{Km})(\rho_{\text{water}}) = 3.64\text{km}^3 \); \( \Delta M_{\text{eff.}}(h_1 = 5\text{Km})(\rho_{\text{Rock}}) = 2.17\text{km}^3 \). From Air-planes or balloons the effective volumes \( M_{\text{eff.}} \) increases and the density \( \rho \) plays a relevant role. \( \Delta M_{\text{eff.}}(h_1 = 25\text{Km})(\rho_{\text{water}}) = 20.3\text{km}^3 \); \( \Delta M_{\text{eff.}}(h_1 = 25\text{Km})(\rho_{\text{Rock}}) = 6.3\text{km}^3 \). Finally from satellite altitudes the same effective volumes \( M_{\text{eff.}} \) are reaching extreme values:

\[ \Delta M_{\text{eff.}}(h_1 = 500 \text{ km})(\rho_{\text{water}}) = 150.6 \text{ km}^3 \]

\[ \Delta M_{\text{eff.}}(h_1 = 500 \text{ km})(\rho_{\text{Rock}}) = 30 \text{ km}^3 \]
These Masses must be compared with other proposed $km^3$ detectors, keeping in mind that these HORTAUs signals conserve the original UHE $\nu$ direction information within a degree. One has to discriminate HORTAUS (only while observing from satellites) from Horizontal High Altitude Showers (HIAS) [[13]], due to rare UHECR showering on high atmosphere. While wide (RICE) one might also remind the UPTAUs (at PeVs energies) volumes as derived in Appendix B and in [[10]] whose values (assuming an arrival angle $\approx 45^\circ - 60^\circ$ below the horizon) are nearly proportional to the $\rho$ density:

$$\Delta M_{eff}(h_1 = 500\,km)(\rho_{Water}) = 5.9 \, km^3$$

$$\Delta M_{eff}(h_1 = 500\,km)(\rho_{Rock}) = 15.6 \, km^3$$

These widest Masses values, here estimated analytically for main quota, are offering an optimal opportunity to reveal UHE $\nu$ at PeVs and EeVs-GZK energies by crown array detectors (scintillators, Cherenkov, photo-luminescent) facing vertically the Horizontal edges, located at high mountain peaks or at air-plane low sides and finally on balloons and satellites. As it can be seen in last row of Table1, the ratio $R$ between HORTAUs events and Showers over atmospheric UHE $\nu$ interaction is a greater and greater number with growing height, implying a dominant role (above two order of magnitude) of HORTAUS grown in Earth Skin Crown over Atmospheric HORTAUs. These huge acceptance may be estimated by comparison with other detector thresholds (see fig.18 adapted to present GZK-$\nu$ models).

9. Event rate of upward and horizontal Tau air-showers

The event rate for HORTAUs are given at first approximation by the following expression normalized to any given neutrino flux $\Phi_\nu$:

$$\dot{N}_{year} = \Delta M_{eff} \cdot \Phi_\nu \cdot \dot{N}_o \cdot \frac{\sigma_{E_\nu}}{\sigma_{E_{\nu o}}}$$

(21)

Where the $\dot{N}_o$ is the UHE neutrino rate estimated for $km^3$ at any given (unitary) energy $E_{\nu o}$, in absence of any Earth shadow. In our case we shall normalize our estimate at $E_{\nu o} = 3$ PeVs energy for standard electro-weak charged current in a standard parton model [[28]] and we shall assume a model-independent neutrino maximal flux $\Phi_\nu$ at a flat fluence value of nearly $\Phi_{\nu o} \approx 3 \cdot 10^3 eV cm^{-2} \cdot s^{-1} \cdot sec^{-1} \cdot sr^{-1}$ corresponding to a characteristic Fermi power law in UHE $\nu$ primary production rate decreasing as $\frac{dN_\nu}{dE_\nu} \approx E^{-2}_\nu$ just below present AMANDA bounds. The consequent rate becomes:

$$\dot{N}_{year} = 29 \cdot \frac{\Delta M_{eff}}{km^3} \cdot \frac{\Phi_\nu}{\Phi_{\nu o}} \cdot \frac{\sigma_{E_\nu}}{\sigma_{E_{\nu o}}}$$

(22)

For highest satellites and for a characteristic UHE GZK energy fluence $\Phi_{\nu o} \approx 3 \cdot 10^3 eV cm^{-2} \cdot s^{-1} \cdot sr^{-1}$ (as the needed Z-Showering one), the consequent event rate observable $\dot{N}_{year}$ above the Sea is:

$$\dot{N}_{year} = 24.76 \cdot \left( \frac{E_\nu}{10^{10} \cdot GeV} \right)^{-0.637} \cdot \frac{h}{500 km} \cdot \frac{\Phi_\nu}{\Phi_{\nu o}}$$

(23)

This event rate is comparable to UPTAUS one (for comparable fluence) and it may be an additional source of Terrestrial Gamma Flashes already observed by GRO in last
These event rate are considered as the detector thresholds for UPTAUS and HORTAUS and they are summarized in Table 1 and in Fig. 18 with other present and future experimental thresholds.

10. Appendix A

As soon as the altitude $h_1$ and the corresponding energy $E_{rh_1}$ increases the corresponding air density decreases. At a too high quota there is no more $X$ slant depth for any Air-Showering to develop. Indeed its value is:

$$X = \int_{d_1 + \frac{d_2}{2} + c\tau \gamma_t}^{d_1 + \frac{d_2}{2}} n_0 e^{-\frac{R_\oplus}{n_0} \left[ \sqrt{\left(1 - \frac{h_1}{R_\oplus}\right)^2 + \left(\frac{h_1}{R_\oplus}\right)^2} - 1 \right]} dx$$

$$\simeq \int_{\frac{d_1 + d_2}{2} + c\tau \gamma_t}^{d_1 + \frac{d_2}{2}} n_0 e^{-\frac{2R_\oplus h_0^2}{x}} dx \leq n_0 h_0$$

In order to find this critical height $h_1$ where the maximal energy HORTAU terminates we remind our recent approximation. The transcendental equation that defines the Tau distance $c\tau$ has been more simplified in:

$$\int_0^{+\infty} n_0 e^{-\frac{\sqrt{(c\tau + x)^2 + R_\oplus^2} - R_\oplus}{n_0}} dx \simeq n_0 h_0 A$$

(25)

$$\int_0^{+\infty} n_0 e^{-\frac{(c\tau + x)^2}{2R_\oplus^2 n_0}} dx \simeq n_0 h_0 A$$

(26)

$$c\tau = \sqrt{2R_\oplus h_0 \left[ \ln \left( \frac{R_\oplus}{c\tau} \right) - \ln A \right]}$$

(27)

Here $A = A_{\text{Had.}}$ or $A = A_{\gamma}$ are slow logarithmic functions of values near unity; applying known empirical laws to estimate this logarithmic growth (as a function of the $X$ slant depth) we derived respectively for hadronic and gamma UHECR showers [10], [12]:

$$A_{\text{Had.}} = 0.792 \left[ 1 + 0.02523 \ln \left( \frac{E}{10^{19} eV} \right) \right]$$

(28)

$$A_{\gamma} = \left[ 1 + 0.04343 \ln \left( \frac{E}{10^{19} eV} \right) \right]$$

(29)

The solution of the above transcendental equation leads to a characteristic maximal UHE $c\tau$ = 546 km flight distance, corresponding to $E \leq 1.1 \cdot 10^{19} eV$ energy whose decay occurs at height $H_0 = 23$ km; nearly 600 Km far from the horizon it was originated from there on the HORTAUS begins to shower. At higher quotas the absence of sufficient air density lead to a suppressed development or to a poor particle shower, hard to be detected. At much lower quota the same air opacity filter most of the electromagnetic shower allowing only to muon bundles and Cherenkov lights to survive at low a somehow ($\leq 10^{-3}$) level.
11. Appendix B: The UPTAUS area

The Upward Tau Air-Showers, mostly at PeV energies, might travel a minimal air depth before reaching the observer in order to amplify its signal. The UPTAUS Disk Area $A_U$ underneath an observer at height $h_1$ within a opening angle $\tilde{\theta}_2$ from the Earth Center is:

$$A_U = 2\pi R_\oplus^2 (1 - \cos \tilde{\theta}_2)$$

(30)

Where the $\sin \tilde{\theta}_2 = (x_2/R_\oplus)$ and $x_2$ behaves like $x_1$ defined above for HORTAUs. In general the UPTAUs area are constrained in a narrow Ring (because the mountain presence itself or because the too near observer distances from Earth are encountering a too short air slant depth for showering or a too far and opaque atmosphere for the horizontal UPTAUs)[16],[17]:

$$A_U = 2\pi R_\oplus^2 (\cos \tilde{\theta}_3 - \cos \tilde{\theta}_2)$$

(31)

An useful Euclidean approximation is:

$$A_U = \pi h_1^2 (\cot \theta_2^2 - \cot \theta_3^2)$$

(32)

Where $\theta_2, \theta_3$ are the outgoing $\tau$ angles on the Earth surface [10].

For UPTAUs (around $3 \cdot 10^{15}$eV energies) these volumes have been estimated in [10], assuming an arrival values angle $\simeq 45^\circ - 60^\circ$ below the horizons. For two characteristic densities one finds respectively:

$$\Delta M_{eff.}(h_1 = 500\, km)(\rho_{Water}) = 5.9\, km^3;$$

$$\Delta M_{eff.}(h_1 = 500\, km)(\rho_{Rock}) = 15.6\, km^3$$

Their detection efficiency is displayed in last Figure (Fig. 18), and it exceed by more than an order of magnitude, the future ICECUBE threshold.

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Figure 18. UPTAUS (lower bound on the center) and HORTAUS (right parabolic curves) sensitivity at different observer heights $h$ ($2, 5, 25, 500 km$) assuming a $km^3$ scale volume (see Table above) adapted over a present neutrino flux estimate in Z-Shower model scenario [24], [2] for light ($0.1 - 0.04 eV$) neutrino masses $m_\nu$; two corresponding density contrast has been assumed [15]; the lower parabolic bound thresholds are at different operation height, in Horizontal (Crown) Detector facing toward most distant horizons edge; these limits are fine tuned (as discussed in the text); we are assuming a duration of data records of a decade comparable to the BATSE record data (a decade). The paraboloid bounds on the EeV energy range in the right sides are nearly un-screened by the Earth opacity while the corresponding UPTAUS bounds in the center below suffer both of Earth opacity as well as of a consequent shorter Tau interaction length in Earth Crust, that has been taken into account. [10], [16], [17].
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