Toward discovering the excited $\Omega$ baryons through nonleptonic weak decays of $\Omega$.

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The nonleptonic weak decay processes $\Omega_c \to \Omega \pi^+ / \Omega (1P) \pi^+ / \Omega (1D) \pi^+ / \Omega (2S) \pi^+$ are studied using the constituent quark model. The branching fraction of $\Omega_c \to \Omega \pi^+$ is predicted to be 1.0%. Considering the newly observed $\Omega(2012)$ resonance as a conventional 1P-wave $\Omega$ excite state with spin-parity $J^P = 3/2^-$, the newly measured ratio $\beta(\Omega_c \to \Omega(2012)\pi^+) / \beta(\Omega_c \to \Omega \pi^+)$ at Belle can be well understood. Besides, the production rates for the missing 1P-wave state $\Omega(1^2P_{1/2})$, two spin quartet 1D-wave states $\Omega(1^4D_{1/2})$, and $\Omega(1^4D_{3/2})$, and two 2S-wave states $\Omega(2^2S_{1/2})$ and $\Omega(2^4S_{3/2})$ are also investigated. It is expected that these missing excited $\Omega$ baryons should have large potentials to be discovered through the nonleptonic weak decays of $\Omega_c$ in forthcoming experiments by Belle II and/or LHCb.

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I. INTRODUCTION

Establishing a relatively complete hadron spectrum and understanding the properties of hadrons are important topics in hadron physics. The knowledge about the $\Omega$ baryon spectrum is very scarce. So far, the ground state $\Omega(1672)$ and its four possible excited states $\Omega(2012), \Omega(2250), \Omega(2380)$, and $\Omega(2470)$, have been observed in experiments [1]. The unambiguous discovery of $\Omega(1672)$ in both production and decay was by Barnes et al. in 1964 using the $K^-$-meson beam at the Brookhaven National Laboratory [2, 3]. In 1985, the $\Omega(2250)$ and $\Omega(2380)$ resonances decaying into $\Xi^- \pi^- K^-$ were observed in an experiment at the CERN SPS charged hyperon beam using incident $\Xi^-$ [4]. In 1987, the $\Omega(2250)$ resonance was produced in $K^- p$ interactions at SLAC [5]. In 1988, the $\Omega(2470)$ resonance was observed in the $\Omega^- \pi^+ \pi^-$ invariant mass spectrum with a signal significance claimed to be at least 5.5 standard deviations by using the $K^- p$ scattering at SLAC [6]. Since then, there was no progress toward searching for $\Omega$ resonances for as long as 30 years due to no effective production mechanisms. In order to promote the experiment, people proposed to produce $\Omega$ states on a proton target in CLAS12 through the photoproduction processes [7], or produce them by using a secondary kaon beam from the photoproduction processes at JLab etc. [8, 9].

In 2018, the first low-lying $\Omega(2012)$ resonance was observed by the Belle Collaboration in the $K^- \Xi^0$ and $K^0 \Xi^-$ invariant mass distributions by using a data sample of $e^+e^-$ annihilations [10]. The $\Omega(2012)$ resonance may fa-

vor the low-lying $P$-wave excited $\Omega$ state with $J^P = 3/2^-$ [11–15], although it may be a candidate of hadronic molecule state as discussed in the literatures [16–23]. Recently, the Belle Collaboration also discovered the $\Omega(2012)$ resonance by using the $\Omega_c$ weak decay process $\Omega_c \to \Omega(2012)\pi^+$ [24]. The measured branching fraction ratio $\beta(\Omega_c \to \Omega(2012)\pi^+) / \beta(\Omega_c \to \Omega \pi^+)$ is $0.220 \pm 0.059$(stat.) $\pm 0.035$(syst.) [24]. Such a large relative ratio indicates that the weak decay processes $\Omega_c \to \Omega(X)\pi^+$ may provide a new and ideal platform to investigate the low-lying excited states $\Omega(X)$ both theoretically and experimentally.

Theoretical studies on the $\Omega(X)$ resonances mainly focus on the mass spectrum within various approaches, such as nonrelativistic quark models [11, 25–29], relativistic quark models [30–33], Lattice QCD [34, 35], and the Skyrme model [36]. The predicted mass spectrum for the conventional $\Omega$ baryons are collected in Table I as a reference. It can be seen that most of the predicted masses for the $1P$-, $2S$- and $1D$-wave states lies in the mass ranges $\sim 2000 \pm 50$, $\sim 2200 \pm 50$, and $\sim 2300 \pm 50$ MeV, respectively. Additionally, in Refs. [37–39], the authors investigated the low-lying five-quark $\Omega$ configurations with negative parity and further considered their mixing combined the corresponding low-lying three-quark $\Omega$ configurations. Recently, stimulated by the newly observed resonance $\Omega(2012)$ at Belle, the strong decay behaviors of some low-lying $1P$, $2S$- and $1D$-wave $\Omega$ resonances were also systematically investigated using the chiral quark model [11, 12] and $\chi P_0$ model [40]. The results suggest that the $1P$, $2S$- and $1D$-wave $\Omega$ baryons have relatively narrow decay widths of less than 50 MeV, and they may be...
TABLE I: The predicted mass spectrum (MeV) of Ω baryons with principal quantum number N ≤ 2 in various quark models. The baryon states denoted as \([N_c, N_s, N, L, J^P]\), where \(N_c\) stands for the irreducible representation of spin-flavor SU(6) group, \(N_s\) stands for the irreducible representation of flavor SU(3) group, and \(N, L, J^P\) stand for the principal, spin, total orbital angular momentum, and spin-parity quantum numbers, respectively. In the \(L - \bar{S}\) coupling scheme, the \(\Omega\) states are also denoted by \(n^{3/2+}L_P\).

| \(n^{3/2+}L_P\) | \([N_c, N_s, N, L, J^P]\) | Ref. [36] | Ref. [30] | Ref. [31] | Ref. [26] | Ref. [27] | Ref. [25] | Ref. [34] | Ref. [11] | Observed mass |
|-----------------|-------------------|---------|---------|---------|---------|---------|---------|---------|---------|-------------|
| \(1^2S_{1/2}\)  | \([56^+, 10, 0, 0, \frac{1}{2}^+]\) | 1694   | 1635   | 1678   | 1675   | 1673   | 1656   | 1642(17) | 1672   | 1672.45     |
| \(1^2P_{1/2}\)  | \([70^+, 10, 1, 1, \frac{1}{2}^+]\) | 1837   | 1950   | 1941   | 2020   | 2015   | 1923   | 1944(56) | 1957   |             |
| \(1^2P_{3/2}\)  | \([70^+, 10, 1, 1, \frac{3}{2}^+]\) | 1978   | 2000   | 2038   | 2020   | 2015   | 1953   | 2049(32) | 2012   | 2012.5      |
| \(2^2S_{1/2}\)  | \([70^+, 10, 2, 0, \frac{1}{2}^-]\) | 2140   | 2220   | 2301   | 2190   | 2182   | 2191   | 2350(63) | 2232   |             |
| \(2^2P_{1/2}\)  | \([70^+, 10, 2, 0, \frac{1}{2}^-]\) | 2165   | 2173   | 2065   | 2078   | 2170   | 2159   |             |        |             |
| \(1^2D_{5/2}\)  | \([70^+, 10, 2, 2, \frac{5}{2}^-]\) | 2282   | 2345   | 2304   | 2265   | 2263   | 2194   | 2470(49) | 2245   |             |
|                  | \([56^+, 10, 2, 2, \frac{5}{2}^-]\) | 2282   | 2345   | 2401   | 2265   | 2260   | 2210   | 2303     |        |             |
| \(4^2D_{5/2}\)  | \([56^+, 10, 2, 2, \frac{5}{2}^-]\) | 2280   | 2346   | 2294   | 2225   | 2224   | 2178   | 2252     |        |             |
| \(1^4D_{5/2}\)  | \([56^+, 10, 2, 2, \frac{5}{2}^-]\) | 2295   | 2332   | 2210   | 2205   | 2183   | 2321   |          |        |             |

II. FRAMEWORK

A. The model

A unique feature of \(\Omega_c \rightarrow \Omega_c(X)\pi^+\) is that this decay proceeds only via external \(W\)-emission diagram [52], which is displayed in Fig. 1. We consider the simple quark-level transition \(c \rightarrow su\text{d}d\), which is relevant for the Cabibbo-favored decay process of \(\Omega_c \rightarrow \Omega(X)\pi^+\). The effective Hamiltonian for \(c \rightarrow su\text{d}d\) can be given by [53]

\[
H_W = \frac{G_F}{\sqrt{2}} V_{td}^\ast V_{ud}(C_1 O_1 + C_2 O_2),
\]

where \(G_F = 1.1663787 \times 10^{-5}\) GeV\(^{-2}\) is the Fermi constant [1], and \(C_1 = 1.26\) and \(C_2 = -0.51\) are the Wilson coefficients taken at the \(m_t\) scale [53]. The Cabibbo-Kobayashi-Maskawa matrix elements \(V_{ts} = 0.987\) and \(V_{td} = 0.974\) are taken from the Review of Particle Physics (RPP) [1], and the current-current operators are

\[
O_1 = \bar{\psi}_c \gamma_\mu (1 - \gamma_5) \psi_d, \quad O_2 = \bar{\psi}_c \gamma_\mu (1 - \gamma_5) \psi_d \gamma^a (1 - \gamma_5) \psi_d a, \quad (2)
\]

with \(\psi_{i,j} = (u/d/s/c, \delta = a/b)\) representing the \(j\)th quark field in a meson or baryon, and \(a\) and \(b\) being color indices.

According to its parity behavior, \(H_W\) can be separated into a parity-conserving part \((H_W^{PC})\) and a parity-violating part \((H_W^{PV})\) [48],

\[
H_W = H_W^{PC} + H_W^{PV}. \quad (4)
\]

With a non-relativistic expansion, the two operators can be

discovered in the \(\Xi\bar{K}\) and/or \(\Xi(1530)\bar{K}\) final states. Some previous studies of the decays can be found in the Refs. [41, 42].

On the other hand, there are only a few studies on the productions of \(\Omega\) and its excited states through the weak decays of \(\Omega_c\) in theory. For example, the productions of the ground state \(\Omega(1672)\) have been studied via semileptonic decays of \(\Omega_c\) using a constituent quark model [43] and the nonleptonic two-body decays of \(\Omega_c\) by the covariant confined quark model [44, 45] and the light-front quark model [46]. In Ref. [43], the author also studied the productions of the \(1P\)-wave excited states \(\Omega(1P)\), which are considered via the \(\Omega_c\) semileptonic weak decay processes using a quark model. On the other hand, the newly observed \(\Omega(2012)\) resonance as a dynamically generated state was theoretically studied in the nonleptonic weak decays of \(\Omega_c \rightarrow \pi^0 \Omega(2012) \rightarrow (\Xi\bar{K})\pi^+\pi^-\) and \(\Xi(1530)\pi^+\pi^-\) in Ref. [47]. So far, the productions of the \(1P\)-, \(2S\)- and \(1D\)-wave excited states \(\Omega(X)\) via the \(\Omega_c\) nonleptonic weak decay processes are not systematically studied in theory.

In this work, we systematically study the production of the low-lying \(1P\)-, \(2S\)- and \(1D\)-wave resonances \(\Omega(X)\) via the hadronic weak decays of \(\Omega_c \rightarrow \Omega(X)\pi^+\pi^-\) using the constituent quark model. Recently, this model has been developed to study the hadronic weak decays of \(\Lambda_c\), the heavy quark conserving weak decays of \(\Xi_c\), and hyperon weak radiative decay by Niu et al. [48-50]. This model is similar to that developed to deal with the semileptonic decays of heavy \(\Lambda_c\) and \(\Omega_c\) baryons in Refs. [43, 51].

This paper is organized as follows. We perform the detailed formalism of two-body nonleptonic weak decays of \(\Omega_c\) in Sec. II. Then, the theoretical numerical results and discussions are presented in Sec. III. Finally, a short summary is given in Sec. IV.
FIG. 1: The nonleptonic weak decay Feynman diagram for the processes of $\Omega \rightarrow \Omega(X)\pi^+$. 

approximately expressed as [48]

$$H^{PC}_W \approx \frac{G_F}{\sqrt{2}} V_{cc}V_{ud} \frac{C_i \phi_i \gamma}{(2\pi)^3} \delta^3(p_3 - p_3' - p_4 - p_5)(\langle s_2|I|s_3\rangle$$

$$- i\langle s_2'|s|s_3\rangle \times \left(\frac{p_1}{2m_3} + \frac{p_1}{2m_3'}\right)\langle s_2|s_3|s_3\rangle\langle s_3|I|0\rangle)\hat{a}_3,$$

$$H^{PV}_W \approx \frac{G_F}{\sqrt{2}} V_{cc}V_{ud} \frac{C_i \phi_i \gamma}{(2\pi)^3} \delta^3(p_3 - p_3' - p_4 - p_5)(-\langle s_2'|I|s_3\rangle$$

$$\langle s_2|s_3|0\rangle - \langle s_2|s|s_3\rangle\langle s_3|s_3|0\rangle)\hat{a}_3.$$ 

In the above equations, $p_j$ and $m_j$ stand for the momentum and mass of the $j$th quark, respectively, as shown in Fig. 1. The $\phi_i$ ($i = 1, 2$ and $\phi_1^c = 1, \phi_2^c = \frac{1}{2}$) are color factors, $I$ is the dimension-two unit matrix, and $\hat{a}_3$ is the flavor operator which transforms $c$ quark to $s$ quark. The $s_j$ and $s_\bar{d}$ stand for the spin of the $j$th quark and the fourth antiquark, respectively. $\gamma$ is a symmetry factor and equals to one for a direct pion emission process considering in present work.

In order to evaluate the spin matrix element $\langle s_2|s_3|I|0\rangle$ and $\langle s_2|s|s_3\rangle|\sigma|0\rangle$ including an antiquark, the particle-hole conjugation [54] should be employed. Within the particle-hole conjugation relation:

$$|j, -m\rangle \rightarrow (-1)^{j+m}|j, m\rangle,$$

the antiquark spin transforms as follows: $|\uparrow\rangle \rightarrow |\downarrow\rangle$ and $|\downarrow\rangle \rightarrow |\uparrow\rangle$. For instance:

$$\langle \frac{1}{\sqrt{2}}(\uparrow 5 \downarrow 4 - \downarrow 5 \uparrow 4)|I|0\rangle = \frac{1}{\sqrt{2}}((\downarrow s|l|\uparrow 4) - (\downarrow s|l|\downarrow 4)) = -\sqrt{2}.$$ 

For a given decay process $A \rightarrow BC$, the transition amplitude $M$ is calculated by

$$M_{J_1, J_2; J, I} = \langle C(P_1; J_1, J_2), B(q)|H_W|A(P_1; J, I)\rangle,$$

$$= \langle C(P_1; J_1, J_2), B(q)|H_W^{PC}|A(P_1; J, I)\rangle + \langle C(P_1; J_1, J_2), B(q)|H_W^{PV}|A(P_1; J, I)\rangle,$$

$$= M^{PC}_{J_1, J_2; J, I} + M^{PV}_{J_1, J_2; J, I}.$$ 

where $A(P_1; J_1, J_2)$, $B(q)$ and $C(P_1; J_1, J_2)$ stands for the wave functions of the initial baryon $A$, final baryon $B$ and final baryon $C$, respectively. $(P_1, P_2)$, $(J_1, J_2)$, and $(J, I)$ are the momentum, the total angular momentum and the third component of the total angular momentum of the initial baryon $A$ and the final baryon $C$, respectively. $q$ is the three-momentum of the final state meson in the initial state rest frame.

Then, the partial decay width for a given decay process $A \rightarrow BC$ can be expressed as

$$\Gamma = \frac{\Phi(ABC)}{2J_1 + 1} \sum_{\text{spins}} |M|^2,$$

where $\Phi(ABC)$ is the phase-space factor for the decay.

The choice of phase space is not clear. For the phase space factor $\Phi(ABC)$, there are three typical options adopted in the literature [55–58]. The usual option is the relativistic phase-space factor (RPF)

$$\Phi(ABC) = 8\pi^2 |q|E_B E_C \frac{M_B M_C}{M_A},$$

where $M_A$ is the mass of the initial hadron $A$, while $E_B$ and $E_C$ stand for the energies of final hadrons $B$ and $C$, respectively.

To match the transition matrix element calculated nonrelativistically, a fully nonrelativistic phase-space factor (NRPF) is used, that is

$$\Phi(ABC) = 8\pi^2 |q|\frac{M_B M_C}{M_A},$$

where $M_B$ and $M_C$ is the mass of the final hadron $B$ and $C$, respectively.

However, in many cases the momenta of the final hadrons are quite large so that the relativistic phase space is significantly different from the nonrelativistic limit. In Ref. [55], Kokoski and Isgur suggested a “mock-hadron” phase-space factor (MHPPF),

$$\Phi(ABC) = 8\pi^2 |q|\frac{\bar{M}_B \bar{M}_C}{\bar{M}_A},$$

in their calculation of meson decay widths. The $\bar{M}_B$, $\bar{M}_C$ and $\bar{M}_A$ are effective hadron masses of hadron $A$, $B$ and $C$, respectively. They are evaluated with a spin-independent inter-quark interaction. In the weak-binding limit, the mass of $\pi$ meson is degenerate with that of $\rho$ meson.
B. Wave functions

To work out the decay amplitude $M$, we need the wave functions of the initial and final states. Here, the initial state is the ground $\Omega_c$ baryon, the final states are the $\pi^+$ meson and the $\Omega_c^{+}(X)$ states. These wave functions are constructed within the non-relativistic constituent quark model. For simplicity, the spatial wave functions of the baryons and mesons are adopted the harmonic oscillator form in our calculations.

The spatial wave function for a baryon with principal quantum number $N$, total orbital angular momentum quantum numbers $L$, and $M_L$ is a product of the $\rho$-oscillator part and the $\lambda$-oscillator part. In momentum space, the baryon spatial wave function is given by [11]

$$\Psi_{NLM_L}(p_\rho, p_\lambda) = \sum_{N,M_L} \langle n_\rho, l_\rho, m_\rho, s_\rho | n_\lambda, l_\lambda, m_\lambda, s_\lambda \rangle \psi_{n_\rho, l_\rho, m_\rho, s_\rho}(p_\rho) \psi_{n_\lambda, l_\lambda, m_\lambda, s_\lambda}(p_\lambda),$$

with $N = 2(n_\rho + n_\lambda) + I_\rho + I_\lambda$, $M_L = m_\rho + m_\lambda$, and

$$\psi_{n_\rho, l_\rho, m_\rho, s_\rho}(p_\rho) = \left(\frac{i}{2}\right)^{1/2} \frac{(p_\rho^2)}{\alpha_\rho^{3/2}} Y_{l_\rho}^{m_\rho}(p_\rho).$$

$$\psi_{n_\lambda, l_\lambda, m_\lambda, s_\lambda}(p_\lambda) = \left(\frac{i}{2}\right)^{1/2} \frac{(p_\lambda^2)}{\alpha_\lambda^{3/2}} Y_{l_\lambda}^{m_\lambda}(p_\lambda).$$

Here, $Y_{lm}(p) = |p|^l Y_{lm}(\hat{p})$ is the solid harmonic polynomial. $p_\rho$ and $p_\lambda$ are the internal momenta of the $\rho$- and $\lambda$-oscillator wave functions, respectively. They can be expressed as functions of the quark momenta $p_j$ ($j = 1, 2, 3$):

$$p_\rho = \frac{\sqrt{2}}{2} (p_1 - p_2),$$

$$p_\lambda = \frac{\sqrt{6}}{2} \left[ (p_1 + p_2) - (m_1 + m_2)p_3 \right] / (m_1 + m_2 + m_3).$$

The $n_\rho$ and $n_\lambda$ are the principal quantum numbers of the $\rho$- and $\lambda$-mode oscillators, respectively. $(l_\rho, m_\rho)$ and $(l_\lambda, m_\lambda)$ are the orbital angular momentum quantum numbers of the $\rho$- and $\lambda$-mode oscillators, respectively. $s = s_\rho, s_\lambda, \cdots$, stand for different excitation modes with different permutation symmetries. $\alpha_\rho$ and $\alpha_\lambda$ are two oscillator parameters. For the $\Omega$ baryons, we have $\alpha_\rho = \alpha_\lambda$, while for the charmed $\Omega_c$ baryons, we have

$$\alpha_\lambda = \left(\frac{3m_c}{2m_x + m_c}\right)^{1/4} \alpha_\rho,$$

where $m_x$ and $m_c$ stand for the masses of the strange and charmed quarks, respectively. The flavor and spin wave functions of the $\Omega_c$- and $\Omega$-baryon wave functions have been given in our previous works [59, 60]. The product of spin, flavor, and spatial wave functions of the heavy baryons must be symmetric since the color wave function is antisymmetric. The details about the quark model classifications for the $\Omega_c$ spectrum can be found in the works of [59, 61, 62], while for the $\Omega$ baryon spectrum can be found in Refs. [11, 60].

Finally, the wave function of the $\pi^+$ meson is constructed by

$$\phi(p_\pi, p_\sigma) = \phi_{\pi^+} \chi^\sigma \phi(p_\rho, p_\lambda),$$

where the spin wave function $\chi^\sigma$ is

$$\chi^\sigma = \frac{1}{\sqrt{2}} (\uparrow \downarrow - \downarrow \uparrow),$$

and the flavor wave function $\phi_{\pi^+}$ is

$$\phi_{\pi^+} = ud\bar{s}.$$

The spatial wave function in the momentum space is adopted the simple harmonic oscillator form

$$\psi(p_\rho, p_\lambda) = \frac{1}{\sqrt{\pi^{3/2} \beta^3}} \exp \left[ -\frac{(p_\rho - p_\lambda)^2}{2 \beta^2} \right],$$

where $\beta$ is a size parameter of the meson wave function. The $p_\rho$ and $p_\lambda$ stand for the quark momenta of the $\pi^+$ meson as shown in Fig. 1.

C. Parameters

For self consistency, the quark model parameters are taken the same as those adopted in our previous work [59]. The constituent masses for the $u/d$, $s$ and $c$ quarks are taken to be $m_{u/d} = 330$ MeV, $m_s = 450$ MeV and $m_c = 1480$ MeV, respectively. For the initial state $\Omega_c$, the harmonic oscillator parameter $\alpha_\rho$ is taken to be $\alpha_\rho = 440$ MeV, the other harmonic oscillator parameter $\alpha_\lambda$ is related to $\alpha_\rho$ by $\alpha_\lambda = [3m_c/(2m_x + m_c)]^{1/4} \alpha_\rho$. For the final state $\Omega_c^+(X)$, a unified harmonic oscillator parameter is adopted, i.e., $\alpha_\lambda = \alpha_\rho = 440$ MeV. For the $\pi^+$ meson, the size parameter is taken to be $\beta = 280$ MeV as that adopted in Ref. [48]. The masses for the $\pi^+$, $\Omega$ and $\Omega_c$ are taken the RPP average values 140 MeV, 1672 MeV and 2695 MeV, respectively [1]. In the MHPF defined in Eq. (13), we need determine the effective masses of the mock hadrons. For the process $\Omega_c \rightarrow \Omega(\pi^+)\pi$, we adopt $M_{\pi} = 0.72$ GeV, consistent with Kosokos and Isgur [55], and $M_{\Omega} = M_{\Omega_c}$ and $M_{\Omega^0_{\pi^+}} = M_{\Omega^0_{\pi^+}}$.

III. NUMERICAL RESULTS AND DISCUSSION

In this work, considering the uncertainties from the relativistic effect, we perform our calculations with the three typical phase space options, RPF, NRPF and MHPF. Our results are listed in Table III. It is seen that the nonleptonic weak decay properties of $\Omega_c$ have a significance dependence on the options of the phase space factor. The results from RPF and MHPF are comparable with each other. However, the predicted partial widths with NRPF are a factor of $\sim 2-6$ smaller those calculated with RPF and MHPF.

The Wilson coefficients $C_1$ and $C_2$ are usually taken to be $C_1 = 1.26$ and $C_2 = -0.51$ at the $m_c$ scale [53]. These coefficients have some uncertainties due to their scale dependencies. To see the effects of the uncertainties of $C_1$ and $C_2$ on our results, as an example in Fig. 2 we plot the partial width of $[\Omega_c^0 \rightarrow \Omega^0 \pi^+]$ as a function of the $C_1$ and $C_2$ in the range of $C_1 \in (1.0, 1.5)$ and $C_2 \in (-0.64, -0.38)$. From the figure,
one can see that considering a 20% uncertainty for the Wilson coefficients $C_1 = 1.26$ and $C_2 = -0.51$ at the $m_c$ scale, the partial decay width of $\Gamma[\Omega^0 \to \Omega^- \pi^+]$ lies in the range of $(1.3, 4.0) \times 10^{-14}$ GeV, which shows a sizeable decay rate on the Wilson coefficients.

A. $\Omega^0_c \to \Omega^- \pi^+$

First, we study the weak decay process $\Omega^0_c \to \Omega^- \pi^+$. This weak decay process, as an important process, has been widely studied by the Belle, BaBar, CLEO, SELEX, FOCUS collaborations [63–69]. With the RPF, the partial decay width of $\Omega^0_c \to \Omega^- \pi^+$ is predicted to be

$$\Gamma[\Omega^0_c \to \Omega^- \pi^+] \approx 2.6 \times 10^{-14} \text{ GeV}. \quad (23)$$

By using the measured lifetime $\tau = 2.68 \times 10^{-13}$ s of $\Omega^0_c$ [1], we further predict the branching fraction

$$B[\Omega^0_c \to \Omega^- \pi^+] \approx 1.05 \%. \quad (24)$$

If adopting the MHPF, there is a ~ 20% correction to the results of RMF. However, when adopting the NRPF the results are about a factor of ~ 6.6 smaller than that predicted with RMF. From Table II, it is found that our predicted branching fraction with both RPF and MHPF is close to the predictions in Refs. [46, 70]. While, if adopting the NRPF, our predicted branching fraction $B[\Omega^0_c \to \Omega^- \pi^+] \approx 0.16 \%$ is consistent with that from the covariant confined quark model [45].

| $\Omega^0$ branching fraction compared with that of other theoretical works. |
|----------------------------------|------------------|------------------|------------------|------------------|
| $\text{RPF/MHPF/NRPF}$ | Ref. [45] | Ref. [70] | Ref. [46] | Ref. [44] |
| 1.05%/0.82%/0.16% | 0.2% | 1.0% | 0.5% | 2.3% |

Furthermore, combined the predicted branching fraction of $B[\Omega^0_c \to \Omega^- \pi^+]$ with the measured relative branching ratios $\Gamma[\Omega^0 \to \Xi^- \bar{K}^0 \pi^+] = 1.20 \pm 0.24$ and $\Gamma[\Omega^0 \to \Xi^- \bar{K}^0 \pi^+] = 2.12 \pm 0.38$, the branching fractions for the three-body weak decay processes $\Omega^0 \to \Xi^- \bar{K}^0 \pi^+ / \Xi^- \bar{K}^0 \pi^+$ can be obtained easily. With the RPF, we have

$$B[\Omega^0 \to \Xi^- \bar{K}^0 \pi^+] \approx (1.26 \pm 0.27) \times 10^{-2}. \quad (25)$$

$$B[\Omega^0 \to \Xi^- \bar{K}^0 \pi^+] \approx (2.23 \pm 0.43) \times 10^{-2}. \quad (26)$$

While when adopting the NRPF, we have small branching fractions

$$B[\Omega^0 \to \Xi^- \bar{K}^0 \pi^+] \approx (0.19 \pm 0.04) \times 10^{-2}. \quad (27)$$

$$B[\Omega^0 \to \Xi^- \bar{K}^0 \pi^+] \approx (0.33 \pm 0.06) \times 10^{-2}. \quad (28)$$

due to the small nonrelativistic phase space factor.

B. $\Omega^0 \to \Omega^- (1P)\pi^+$

In the $\Omega$ family, there are two 1P-wave states $\Omega(1^2P_{1/2})$ and $\Omega(1^2P_{3/2})$ with spin-parity $J^P = 1^+ / 2^+$ and $J^P = 3^– / 2–$, respectively. The newly observed $\Omega(2012)$ resonance may favor the assignment of $\Omega(1^2P_{3/2})$ state, since both the measured mass and width are consistent with the quark model predictions [11–15]. The masses of the unestablished $\Omega’ (X)$ states are taken the predictions in Ref. [11], which have been collected in Table I. However, the $\Omega(1^2P_{1/2})$ classified in the quark model is still missing.

Considering $\Omega(2012)$ as the $\Omega(1^2P_{3/2})$ assignment, we have studied the $\Omega^0 \to \Omega^- (2012)\pi^+$ process, the results are listed in Table III. It is found that the $\Omega^-$ baryon has a fairly large decay rate into $\Omega(2012)^- \pi^+$, with the RPF or MHPF the branching fraction is predicted to be

$$B[\Omega^0 \to \Omega^- (2012)\pi^+] \approx 2.2 \times 10^{-3}. \quad (29)$$

Combining it with the branching fraction of $B[\Omega^0_c \to \Omega^- \pi^+]$ obtained in Eq.(24), we predict the relative ratio

$$R^1_{\text{th}} = \frac{B[\Omega^0_c \to \Omega^- \pi^+]}{B[\Omega^0 \to \Omega^- (2012)\pi^+]} \approx 0.22, \quad (30)$$

which is in good agreement with experimental value $R^1_{\text{exp}} = 0.220 \pm 0.059$ (stat.) $\pm 0.035$ (syst.) that was recently measured by the Belle Collaboration [24]. According to the strong decay properties of $\Omega(2012)$ predicted using the constituent quark model in Refs. [11, 12], branching fractions of $\Omega(2012)$ decaying into $\Xi^- K^-$ and $\Xi^- \bar{K}^0$ are predicted to be $B[\Omega(2012) \to \Xi^- K^-] \approx 52 \%$ and $B[\Omega(2012) \to \Xi^- \bar{K}^0] \approx 48 \%$, respectively. Combining these strong branching fractions of $\Omega$ with our predicted branching fractions for the weak decay processes $B[\Omega^0_c \to \Xi^- \bar{K}^0 \pi^+ / \Xi^- \bar{K}^0 \pi^+ / \Omega(2012)\pi^+]$ in Eqs. (27)-(29), one can obtain

$$R^2_{\text{th}} = \frac{B[\Omega^0_c \to \Xi^- \bar{K}^0 \pi^+ / \Omega(2012)\pi^+] \times B[\Omega(2012) \to \Xi^- K^-]}{B[\Omega^0 \to \Xi^- K^-] \times B[\Omega(2012) \to \Xi^- K^-]} \approx 0.09, \quad (31)$$

$$R^3_{\text{th}} = \frac{B[\Omega^0 \to \Xi^- \bar{K}^0 \pi^+ / \Omega(2012)\pi^+] \times B[\Omega(2012) \to \Xi^- \bar{K}^0]}{B[\Omega^0 \to \Xi^- \bar{K}^0 \pi^+]} \approx 0.05, \quad (32)$$
TABLE III: Predicted decay properties of the $\Omega_c \rightarrow \Omega^{(i)}(X)^{-}\pi^+$ processes within three options of the phase space. RPF, NRPF and MHPF, respectively. $\Gamma_i$ stands for the partial decay width, $B$ for the branching fraction, and $M_f$ stands for the mass of the final state $\Omega^{(i)}(X)$. The total width of $\Omega_c$ is $\Gamma = 2.47 \times 10^{-13}$ GeV (corresponding to life time $\tau = 2.68 \times 10^{-13}$ s [1]). The units for decay width $\Gamma_i$ and branching ratio $B$ are $10^{-13}$ GeV and $10^{-6}$, respectively.

| final state | $M_f$ (MeV) | $\Gamma_i$ | $B$ | $\prod_\Omega^{(i)}(X)^{-}\pi^+$ | $\Gamma_i$ | $B$ | $\prod_\Omega^{(i)}(X)^{-}\pi^+$ | $\Gamma_i$ | $B$ | $\prod_\Omega^{(i)}(X)^{-}\pi^+$ |
|-------------|-------------|-----------|-----|---------------------|-----------|-----|---------------------|-----------|-----|---------------------|
| $\Omega(1^{+}_{3/2})^{-}\pi^+$ | 1672 | 26 | 10.5 | 1.0 | 3.8 | 1.6 | 1.0 | 21 | 8.2 | 1 |
| $\Omega(1^{+}_{2+})^{-}\pi^+$ | 1957 | 9.5 | 3.8 | 0.38 | 2.0 | 0.80 | 0.50 | 8.7 | 3.6 | 0.44 |
| $\Omega(1^{+}_{2+})^{-}\pi^+$ | 2012 | 5.4 | 2.2 | 0.22 | 1.2 | 0.49 | 0.31 | 5.2 | 2.1 | 0.26 |
| $\Omega(2^{+}_{3/2})^{-}\pi^+$ | 2232 | 1.2 | 5.0$\times 10^{-1}$ | 0.05 | 3.9$\times 10^{-1}$ | 0.16 | 0.01 | 1.5 | 6.3$\times 10^{-1}$ | 0.08 |
| $\Omega(2^{+}_{3/2})^{-}\pi^+$ | 2159 | 3.0 | 1.2 | 0.12 | 0.8 | 0.34 | 0.21 | 3.3 | 1.4 | 0.17 |
| $\Omega(1^{+}_{2+})^{-}\pi^+$ | 2245 | 2.1$\times 10^{-1}$ | 8.4$\times 10^{-2}$ | 0.008 | 6.7$\times 10^{-2}$ | 2.7$\times 10^{-2}$ | 0.002 | 2.6$\times 10^{-1}$ | 1.1$\times 10^{-1}$ | 0.01 |
| $\Omega(1^{+}_{2+})^{-}\pi^+$ | 2303 | 1.3$\times 10^{-2}$ | 5.0$\times 10^{-3}$ | 5.0$\times 10^{-4}$ | 5.4$\times 10^{-3}$ | 2.0$\times 10^{-3}$ | 1.0$\times 10^{-3}$ | 1.9$\times 10^{-2}$ | 7.7$\times 10^{-3}$ | 9.4$\times 10^{-4}$ |
| $\Omega(1^{+}_{2+})^{-}\pi^+$ | 2141 | 3.3 | 1.3 | 0.13 | 8.8$\times 10^{-1}$ | 0.36 | 0.23 | 3.6 | 1.5 | 0.18 |
| $\Omega(1^{+}_{2+})^{-}\pi^+$ | 2188 | 2.3 | 0.95 | 0.09 | 6.8$\times 10^{-1}$ | 0.28 | 0.18 | 2.7 | 1.1 | 0.13 |
| $\Omega(1^{+}_{2+})^{-}\pi^+$ | 2252 | 3.3$\times 10^{-3}$ | 1.3$\times 10^{-2}$ | 1.3$\times 10^{-4}$ | 1.2$\times 10^{-3}$ | 4.5$\times 10^{-4}$ | 2.8$\times 10^{-4}$ | 4.2$\times 10^{-3}$ | 1.7$\times 10^{-3}$ | 2.1$\times 10^{-4}$ |
| $\Omega(1^{+}_{2+})^{-}\pi^+$ | 2321 | 3.2$\times 10^{-3}$ | 1.3$\times 10^{-2}$ | 1.3$\times 10^{-4}$ | 1.3$\times 10^{-3}$ | 5.1$\times 10^{-4}$ | 3.2$\times 10^{-4}$ | 4.7$\times 10^{-3}$ | 1.9$\times 10^{-3}$ | 2.3$\times 10^{-4}$ |

which are also consistent with the experimental values $R_5^{\text{Exp}} = 0.096 \pm 0.032$(stat.) ± 0.018(syst.) and $R_6^{\text{Exp}} = 0.055 \pm 0.028$(stat.) ± 0.007(syst.) recently measured by the Belle Collaboration [24], respectively. It should be mentioned that these predicted relative ratios $R_i^{\Omega_i}$ ($i = 1, 2, 3$) are nearly independent on the options of phase space factor in the calculations.

Then we consider the weak decay rate of $\Omega_c$ into the other $1P$-wave state $\Omega(1^{+}_{2+})$ by emitting a $\pi^+$ meson. The mass of $\Omega(1^{+}_{2+})$ is predicted to be ~ 1950 MeV within the Lattice QCD [34] and the relativized quark models [30, 31]. Experimentally, there seems to be a weak enhancement around 1950 MeV in the $\Xi K^-$ invariant mass distributions from the Belle observations [10, 24], which may be a hint of $\Omega(1^{+}_{2+})$. Hence, in the calculations the mass of $\Omega(1^{+}_{2+})$ is taken to be 1957 MeV. If adopting the RPF or MHPF, the branching fraction is predicted to be

$$B[\Omega^0_c \rightarrow \Omega(1^{+}_{2+})^{-}\pi^+] \approx 2.4 \times 10^{-3},$$

which is about a factor of 5 larger than that predicted by NRPF. The predicted branching fraction $B[\Omega^0_c \rightarrow \Omega(1^{+}_{2+})^{-}\pi^+]$ should be slightly larger than that of the $\Omega(2012)$ $\pi^+$ final states. The branching fraction ratio between $\Omega^0_c \rightarrow \Omega(1^{+}_{2+})^{-}\pi^+$ and $\Omega^0_c \rightarrow \Omega^{-}\pi^+$ is predicted to be

$$\frac{B[\Omega^0_c \rightarrow \Omega(1^{+}_{2+})^{-}\pi^+]}{B[\Omega^0_c \rightarrow \Omega^{-}\pi^+]} \approx 0.38 - 0.50,$$

which is insensitive to options of the phase space factor. Such a large relative branching ratio indicates that the other missing $1P$-wave state $\Omega(1^{+}_{2+})$ has a good potential to be observed in the weak decay process $\Omega^0_c \rightarrow \Omega(1^{+}_{2+})^{-}\pi^+$.

According to the strong decay analysis in Refs. [11, 12, 40], the decays of $\Omega(1^{+}_{2+})$ should be nearly saturated by the $\Xi^0 K^−$ and $\Xi^- K^0$ channels. Combined the strong decay properties predicted within the chiral quark model in Refs. [11, 12], we can estimate the ratios

$$\frac{B[\Omega^0_c \rightarrow \Omega(1^{+}_{2+})^{-}\pi^+][B[\Omega(1^{+}_{2+})^{-}\pi^+] \rightarrow \Xi^0 K^-]}{B[\Omega^0_c \rightarrow \Xi^- K^0]} \approx 16\%,$$

$$\frac{B[\Omega^0_c \rightarrow \Omega(1^{+}_{2+})^{-}\pi^+][B[\Omega(1^{+}_{2+})^{-}\pi^+] \rightarrow \Xi^- K^0]}{B[\Omega^0_c \rightarrow \Xi^- K^0]} \approx 8\%,$$

which may provide useful references for future experiments.

To further explain the results of the $\Omega(1^{+}_{2+})$ and $\Omega(1^{+}_{2+})$ states, we fit the $(\Xi K)$ invarian mass spectrum of the process $\Omega_c \rightarrow \pi^+ \Omega^0(X) \rightarrow \pi^+ (\Xi K)$ measured by Belle Collaboration [24]. In our analysis, we adopt a relativistic Breit-Wigner function to describe the event distribution [1, 71–73]

$$\frac{dN}{dM(\Xi K^-)} = f_{BG} + C \sum_R M^2_{(K \Xi)} \Gamma_{\pi^0 \Omega(\chi)}(M_{(K \Xi)}) \Gamma_{(K \Xi)}(M_{(K \Xi)}),$$

where $M_{(K \Xi)}$ and $m_R$ stands for the invariant mass of $(\Xi K)$ and the resonance mass of $\Omega^0(X)$, respectively. $\Gamma_{\pi^0 \Omega(\chi)}(M_{(K \Xi)})$ and $\Gamma_{(K \Xi)}(M_{(K \Xi)})$ are the partial decay widths of $\Omega^0_c \rightarrow \Omega^0(X)^{-}\pi^+$ and $\Omega(1^{+}_{2+}) \rightarrow \Xi^- K^0$, respectively. The total decay width $\Gamma_R$ are adopted as the predictions obtained in Ref. [11], while $f_{BG}$ stands for the background contributions. In this work a linear background $f_{BG} = 18.5$ (MeV/c$^2$)$^{-1}$ is adopted, which is determined by fitting the backgrounds taken in Ref. [24]. Finally $C_R$ is a global parameter related to the resonance production rates.

In Fig. 3, we show our theoretical results for the $(\Xi K)^-$ invariant mass distributions of the decay $\Omega_c \rightarrow \pi^+ \Omega^0(X) \rightarrow \pi^+ (i(\Xi K)^-)$. The red curve has been adjusted to the strength...
FIG. 3: The $K\Xi^-$ invariant mass measured of the decay $\Omega_\circlearrowright \rightarrow \pi^+\Omega(X) \rightarrow \pi^+K^-\Xi^0$ by Belle Collaboration [24] (solid squares) compared to the theoretical description with two possible $\Omega^+$ (1P)-wave states, $\Omega(1^2P_{1/2})$ and $\Omega(1^2P_{3/2})$. Results 1 and 2 are fitting results of $\Omega(1^2P_{1/2})$ with widths about 12.4 MeV and 20.0 MeV, respectively.

of the experimental data of Belle Collaboration [24] at the peak around 2012 MeV by taking $C_R = 0.064$. Furthermore, the dashed curve stands for the resonance contribution of $\Omega(1^2P_{3/2})$ with $M_R = 2012$ MeV and $\Gamma_R = 5.7$ MeV, while the dash-dotted curve stands for the $\Omega(1^2P_{1/2})$ contribution with $M_R = 1957$ MeV and $\Gamma_R = 12.4$ MeV. From Fig. 3 one can easily find that the $\Omega(1^2P_{1/2})$ state has a significant contribution around 2012 MeV and the experimental data around that energy can be well reproduced. However, the contribution of the $\Omega(1^2P_{3/2})$ state is overestimated comparing with the experimental data around 1957 MeV. Yet, the quark model predicted widths for the $\Omega(1^2P_{1/2})$ and $\Omega(1^2P_{3/2})$ states have uncertainties, we perform a new calculation with a slightly large width $\Gamma_R = 20.0$ MeV for $\Omega(1^2P_{1/2})$ state, while we take the experimental value of 6.4 MeV for $\Omega(1^2P_{3/2})$. The new theoretical results are also shown in Fig. 3 with blue curve, where we see that the signal of the $\Omega(1^2P_{1/2})$ is much suppressed. It is expected that more precise experimental data can be used to pin down the contribution of the $\Omega(1^2P_{1/2})$ state in future.

On the other hand, the $\Omega_0^0 \rightarrow \pi^+\Omega(2012) \rightarrow \pi^+K^-\Xi^0$ decay was investigated within the picture that the $\Omega(2012)$ is a molecular state in Ref. [74], where the numerical results are also consistent with the experimental data. Indeed, we need further efforts to understand the nature of $\Omega(2012)$ state [75, 76].

C. $\Omega_0^0 \rightarrow \Omega^- (1D)\pi^+$

There are six 1D-wave states, $\Omega(1^2D_{3/2, 5/2})$ and $\Omega(1^1D_{1/2, 3/2, 5/2, 7/2})$ according to the quark quark model classification. Most of the predicted masses for the 1D-wave states lies in the mass range $\sim 2200 \pm 50$ MeV in various quark models. Taking the mass recently predicted in Ref. [11], we calculate the weak decay properties for the $\Omega_0^0 \rightarrow \Omega^- (1D)\pi^+$ processes. Our results are listed in Table III. It is seen that $\Omega_0^0$ has significant branching fractions decaying into the spin quartet states $\Omega(1^1D_{1/2})$ and $\Omega(1^1D_{3/2})$. The predicted branching fractions $\mathcal{B}[\Omega_0^0 \rightarrow \Omega(1^1D_{1/2})\pi^-]$ can reach up to the order of $\sim \mathcal{O}(10^{-3}) - \mathcal{O}(10^{-4})$. With the RPF, their relative ratios to $\mathcal{B}[\Omega_0^0 \rightarrow \Omega^- \pi^+]$ are predicted to be

$\frac{\mathcal{B}[\Omega_0^0 \rightarrow \Omega(1^1D_{1/2})\pi^-]}{\mathcal{B}[\Omega_0^0 \rightarrow \Omega^- \pi^+]} \approx 0.13$, \hspace{1cm} (38)

$\frac{\mathcal{B}[\Omega_0^0 \rightarrow \Omega(1^1D_{3/2})\pi^-]}{\mathcal{B}[\Omega_0^0 \rightarrow \Omega^- \pi^+]} \approx 0.09$, \hspace{1cm} (39)

which are close to the results predicted with RPF and MHPF. The predicted branching fractions and ratios are comparable with those of $\Omega_0^0$ decaying into the $\Omega(2012)\pi^+$ and $\Omega(1^2P_{1/2})\pi^+$ channels. However, the decay rates of $\Omega_0^0$ into the other four 1D-wave states $\Omega(1^2D_{3/2, 5/2})$ and $\Omega(1^2D_{5/2, 7/2})$ are $\sim 1 \sim 3$ orders of magnitude smaller. The relatively large decay rates indicate that both $\Omega(1^1D_{1/2})$ and $\Omega(1^1D_{3/2})$ has good potentials to be established by using the weak decay processes $\Omega_0^0 \rightarrow \Omega(1^1D_{1/2, 3/2})\pi^-$. We further analyze the reasons of the small decay rates of $\Omega_0^0 \rightarrow \Omega(1^2D_{3/2, 5/2})\pi^- / \Omega(1^2D_{5/2, 7/2})\pi^+$ compared with that of $\Omega_0^0 \rightarrow \Omega(1^1D_{1/2, 3/2})\pi^+$ as follows. We note that the helicity transition amplitudes

$M_{J_{\pi}, L, S, \eta, \delta} \propto \sum_{M_{1', S', \eta', \delta'}} \langle LM_{1', S', \eta', \delta'} | J_J J_J \rangle \langle \Psi_{\Omega LM} \chi_{S'} | \bar{\Omega} \Psi_{\Omega} \chi_{S'} \rangle$,

(40)

where $\Psi_{\Omega}$ ($\Psi_{\Omega LM}$) and $\chi_{S'}$ ($\chi_{2S'}$) are the spacial and spin wave functions of the initial (final) baryons, respectively. For the decay processes involving the spin quartet states $\Omega(1^2D_{1/2, 3/2, 5/2, 7/2})$, the decay amplitude is the sum of $c_1(\Psi_{21\chi_{-3/2}} | \bar{\Omega} \Psi_{\Omega} \chi_{-3/2} \rangle$ and $c_2(\Psi_{22\chi_{-1/2}} | \bar{\Omega} \Psi_{\Omega} \chi_{-1/2} \rangle$. These two terms have strong constructive and destructive interference for the $\Omega_0^0 \rightarrow \Omega(1^1D_{1/2, 3/2})\pi^+$ and $\Omega_0^0 \rightarrow \Omega(1^1D_{3/2, 5/2})\pi^+$, respectively. Thus, the decay rates of $\Omega_0^0 \rightarrow \Omega(1^1D_{1/2, 3/2})\pi^+$ are strongly suppressed by the destructive interference between the two terms of the helicity transition amplitude. While for the decay processes involving the spin doublet $\Omega(1^2D_{3/2, 5/2})$, the decay amplitudes are proportional to $\langle \Psi_{22 \chi_{-1/2}} | \bar{\Omega} \Psi_{\Omega} \chi_{-1/2} \rangle$. In this term, the contribution from the part of the spin wave functions is about a factor of $2 \sim 4$ smaller than that for the spin quartet states. Thus, the decay rates of $\Omega_0^0 \rightarrow \Omega(1^1D_{3/2, 5/2})\pi^+$ is suppressed by the relative small overlapping of the spin wave functions of the initial and final states.

According to the analysis of the strong decay properties [11, 12], the $\Omega(1^1D_{1/2})$ state has a width of $\Gamma \approx 42$ MeV, and dominantly decays into the $\Xi K$ channel with a branching fraction $\sim 94\%$. While the $\Omega(1^1D_{3/2})$ has a width of $\Gamma \approx 31$ MeV, and dominantly decays into $\Xi K$ with a branching fraction $\sim 64\%$. Thus, the $\Xi^0 K^-$ and $\Xi^- K^0$ final states can be used to look for the $\Omega(1^1D_{1/2})$ and $\Omega(1^1D_{3/2})$ states if they are produced by the $\Omega_0^0$ weak decays. For the $\Omega(1^1D_{1/2})$ state, by combining the results of RPF we can estimate the following
ratios
\[ \frac{\mathcal{B}[\Omega_c^0 \to (1^D1_{1/2} \pi^+)] \mathcal{B}[\Omega(1^D1_{1/2}) \to \Xi^0 K^-]}{\mathcal{B}[\Omega_c^0 \to \Xi^0 K^- \pi^+]} \approx 5\%, \quad (41) \]
\[ \frac{\mathcal{B}[\Omega_c^0 \to (1^D1_{1/2} \pi^+)] \mathcal{B}[\Omega(1^D1_{1/2}) \to \Xi^0 K^-]}{\mathcal{B}[\Omega_c^0 \to \Xi^- K^0 \pi^+]} \approx 3\%, \quad (42) \]
while for the \( (1^D3_{3/2}) \) state, we can estimate the following ratios
\[ \frac{\mathcal{B}[\Omega_c^0 \to (1^D3_{3/2} \pi^+)] \mathcal{B}[\Omega(1^D3_{3/2}) \to \Xi^0 K^-]}{\mathcal{B}[\Omega_c^0 \to \Xi^0 K^- \pi^+]} \approx 2\%, \quad (43) \]
\[ \frac{\mathcal{B}[\Omega_c^0 \to (1^D3_{3/2} \pi^+)] \mathcal{B}[\Omega(1^D3_{3/2}) \to \Xi^- K^0]}{\mathcal{B}[\Omega_c^0 \to \Xi^- K^0 \pi^+]} \approx 1\%. \quad (44) \]
The above predicted ratios are less dependent on the options of the phase space factor.

Finally, it should be pointed out that the predicted masses of the 1D-wave \( \Omega \) states have some model dependencies. To see the effects from the mass uncertainties of the 1D-wave \( \Omega \) states on our predicted weak decay properties, we plot the weak branching fractions of \( \Omega_c^0 \) to \( \pi^+ \Omega(X) \) as functions of the masses of the 1D-wave \( \Omega \) excited state in their possible range \( M \in (2.1 - 2.3) \text{ GeV} \) in Fig. 4. It is seen that in the most possible mass range \( \sim 2200 \pm 50 \text{ MeV} \), the upper limit of our predicted partial widths is about a factor of 2 larger than that of the lower limit.

\[ \text{FIG. 4: The branching fraction of the } \Omega_c^0 \rightarrow \Omega(1^D1_{1/2}, 1^D3_{3/2}, 2^D1_{1/2}, 2^D3_{3/2}) \pi^+ \text{ as a function of mass of the final state } \Omega(X). \text{ It should be noted that since the results of } 1^D1_{1/2} \text{ and } 1^D3_{3/2} \text{ are the same, we omit the results of } 1^D3_{3/2}, \text{ here.} \]

D. \( \Omega_c^0 \rightarrow \Omega(2S_1) \pi^+ \)

In the constituent quark model, there are two 2S-wave states \( \Omega(2^2S_{1/2}) \) and \( \Omega(2^4S_{3/2}) \). There are large uncertainties in the predictions of their masses in various quark models. The predicted masses scatter in the range of \( \sim 2.10 - 2.30 \) GeV. In Fig. 4, by using the RPF we plot the weak decay widths of the \( \Omega_c^0 \rightarrow \Omega(2^2S_{1/2}, 2^2S_{3/2}) \pi^+ \) processes as functions of the masses of the 2S-wave \( \Omega \) states. It is seen that in the mass range 2100 ~ 2300 MeV, for the weak decay process \( \Omega_c^0 \rightarrow \Omega(2^2S_{1/2}) \pi^+ \), the partial decay width is predicted to be \( \Gamma[\Omega_c^0 \rightarrow \Omega(2^2S_{1/2}) \pi^+] = (1.2 \pm 0.45) \times 10^{-15} \text{ GeV} \), the branching fraction can reach up to \( \frac{\mathcal{B}[\Omega_c^0 \rightarrow \Omega(2^2S_{1/2}) \pi^+]}{\mathcal{B}[\Omega_c^0 \rightarrow \Omega^- \pi^+]} \approx (0.50 \pm 0.18) \times 10^{-3}. \quad (45) \)

Combined with the predicted branching fraction \( \mathcal{B}[\Omega_c^0 \rightarrow \Omega^- \pi^+] \approx 10\% \), we obtain the relative branching ratio
\[ \frac{\mathcal{B}[\Omega_c^0 \rightarrow \Omega(2^2S_{1/2}) \pi^+]}{\mathcal{B}[\Omega_c^0 \rightarrow \Omega^- \pi^+]} \approx 0.05 \pm 0.02 \quad (46) \]

The production rate of \( \Omega(2^2S_{1/2}) \) via the \( \Omega_c \) weak decay is about a factor of 5 ~ 6 smaller than that of \( \Omega(2012) \). Due to the large decay rate into the \( \Xi(1530)K \) channel \([11, 12]\), the \( \Omega(2^2S_{1/2}) \) state is suggested to be searched in the decay chain \( \Omega_c^0 \rightarrow \Omega(2^2S_{1/2}) \pi^+ \rightarrow (\Xi(1530)K) \pi^+ \rightarrow (\Xi K) \pi^+ \) in future experiments.

For the other weak decay process \( \Omega_c^0 \rightarrow \Omega(2^4S_{3/2}) \pi^+ \), by using the RPF the partial decay width is predicted to be \( \Gamma[\Omega_c^0 \rightarrow \Omega(2^4S_{3/2}) \pi^+] = (3.0 \pm 1.6) \times 10^{-15} \text{ GeV} \), the branching fraction can reach up to \( \frac{\mathcal{B}[\Omega_c^0 \rightarrow \Omega(2^4S_{3/2}) \pi^+]}{\mathcal{B}[\Omega_c^0 \rightarrow \Omega^- \pi^+]} \approx (1.2 \pm 0.6) \times 10^{-3}. \quad (47) \)
Similarly, the relative branching ratio is predicted to be
\[ \frac{\mathcal{B}[\Omega_c^0 \rightarrow \Omega(2^4S_{3/2}) \pi^+]}{\mathcal{B}[\Omega_c^0 \rightarrow \Omega^- \pi^+]} \approx 0.12 \pm 0.06. \quad (48) \]

The production rate of \( \Omega(2^4S_{3/2}) \) via the \( \Omega_c \) weak decay is comparable with that of \( \Omega(2^2S_{1/2}) \). The dominant decay mode of \( \Omega(2^4S_{3/2}) \) is the \( \Xi(1530)K \) channel, and one can look for it in the decay chain \( \Omega_c^0 \rightarrow \Omega(2^4S_{3/2}) \pi^+ \rightarrow (\Xi(1530)K) \pi^+ \rightarrow (\Xi K) \pi^+ \).

IV. SUMMARY

In this work, we calculate the Cabibbo-favored weak decay processes \( \Omega_c \rightarrow \Omega(2^1P_{1/2}) \pi^+ \) within a constituent quark model. Our predicted branching fraction \( \mathcal{B}[\Omega_c^0 \rightarrow \Omega^- \pi^+] \approx 1.0\% \) which is in agreement with the early predictions in orders in Refs. \([44, 70]\). Considering the newly observed \( \Omega(2012) \) resonance as the conventional \( \Omega(1^P_{1/2}) \) state, it is found that the measured ratio \( \frac{\mathcal{B}[\Omega_c \rightarrow \Omega(2012) \pi^+ \rightarrow (\Xi K) \pi^+]}{\mathcal{B}[\Omega_c \rightarrow \Omega^- \pi^+] = 0.220 \pm 0.059 \text{ (stat.)} \pm 0.035 \text{ (syst.)}} \) at Belle can be well understood within our model calculations here. The production potentials of the missing low-lying \( 1P_- \), \( 2S_\pm \), and \( 1D \)-wave resonances \( \Omega(X) \) via the hadronic weak decays of \( \Omega_c \) are discussed as well. Our main conclusions are summarized as follows.

i) The missing \( 1P_- \)-wave state \( \Omega(1^2P_{1/2}) \) has a large potential to be observed in the decay chain \( \Omega_c^0 \rightarrow \Omega(1^2P_{1/2}) \pi^+ \rightarrow (\Xi K) \pi^+ \). The production rate of \( \Omega(1^2P_{1/2}) \) via the hadronic weak decays of \( \Omega_c \) is even slightly larger than that of \( \Omega(2012) \).
ii) For the 1D-wave $\Omega$ states, we find that both $\Omega(1^D_{1/2}^-)$ and $\Omega(2^D_{3/2}^-)$ have fairly large production rates via the $\Omega_c \rightarrow \Omega(1^D_{1/2}^-)\pi^+$ and $\Omega_c \rightarrow \Omega(2^D_{3/2}^-)\pi^+$ processes, respectively. Their production rates via the hadronic weak decays of $\Omega_c$ are comparable with those of the 1P-wave $\Omega$ states. Both $\Omega(1^D_{1/2}^-)$ and $\Omega(2^D_{3/2}^-)$ are most likely to be observed in the process $\Omega^0 \rightarrow \Omega(1^D_{1/2}^-)\pi^+ \rightarrow (\Xi K)^-\pi^+$. iii) The $2S$ states $\Omega(2^S_{1/2}^-)$ and $\Omega(2^S_{3/2}^-)$ also have fairly large production rates via the hadronic weak decays of $\Omega_c$. Their production rates are about a factor of $5 - 6$ smaller than that of $\Omega(2\Omega_c)$. Both $\Omega(2^S_{1/2}^-)$ and $\Omega(2^S_{3/2}^-)$ dominantly decay into the $\Xi(1530)\bar{K}$ channel, thus, they can be looked for in the decay chains $\Omega^0 \rightarrow \Omega(2^S_{1/2}^-)\pi^+ / \Omega(2^S_{3/2}^-)\pi^+ \rightarrow (\Xi(1530)\bar{K})^-\pi^+ \rightarrow (\Xi\pi\bar{K})^-\pi^+$.

Finally, it should be mentioned that our predicted partial widths for the weak decay processes $\Omega_c \rightarrow \Omega(0^+)\pi^+$ may have a large uncertainties due to relativistic effects. To roughly see the uncertainties from the relativistic corrections, we perform our calculations with the three typical phase space options, the relativistic phase space, the nonrelativistic phase space and the “mock-hadron” phase space. The predicted partial widths with the nonrelativistic phase space are a factor of $\sim 2 - 6$ smaller those calculated with the usual relativistic phase space and the “mock-hadron” phase space.

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