Toward inertial sensing with a $2^3S$ positronium beam

Sebastiano Mariazzi$^{1,2,a}$, Ruggero Caravita$^{2,3}$, Michael Doser$^3$, Giancarlo Nebbia$^4$, and Roberto S. Brusa$^{1,2}$

$^1$ Department of Physics, University of Trento, via Sommarive 14, 38123 Povo, Trento, Italy
$^2$ TIFPA/INFN Trento, via Sommarive 14, 38123 Povo, Trento, Italy
$^3$ Physics Department, CERN, 1211 Geneva, Switzerland
$^4$ INFN Padova, via Marzolo 8, 35131 Padova, Italy

Received 20 November 2019/ Received in final form 13 February 2020
Published online 22 April 2020
© EDP Sciences / Società Italiana di Fisica / Springer-Verlag GmbH Germany, part of Springer Nature, 2020

Abstract. In this work, we discuss the possibility of inertial sensing with positronium in the $2^3S$ metastable state for the measurement of optical dipole, relativistic and gravitational forces on a purely leptonic matter-antimatter system. Starting from the characteristics of an available $2^3S$ beam, we estimate the time necessary to measure accelerations ranging from $\sim 10^5 \text{m/s}^2$ to $9.1 \text{m/s}^2$ with two different inertial sensitive devices: a classical moiré deflectometer and a Mach–Zehnder interferometer. The sensitivity of the Mach–Zehnder interferometer has been estimated to be several tens of times better than that of the moiré deflectometer, for the same measurement time. Different strategies to strengthen the $2^3S$ beam flux and to improve the sensitivity of the devices are proposed and analyzed. Among them, the most promising are reducing the divergence of the positronium beam through 2D laser Doppler cooling and coherent positronium Raman excitation from the ground state to the $2^3S$ level. If implemented, these improvements promise to result in the time required to measure an acceleration of $9.1 \text{m/s}^2$ of few weeks and $100 \text{m/s}^2$ of a few hours. Different detection schemes for resolving the fringe pattern shift generated on $2^3S$ positronium crossing the deflectometer/interferometer are also discussed.

1 Introduction

Positronium (Ps), the bound state of an electron and a positron with very short lifetime (125 ps in the singlet ground state, 142 ns in the triplet ground state), is one among the few neutral matter-antimatter/pure antimatter systems experimentally accessible for testing symmetries and fundamental interactions on antimatter. Since its discovery in 1951 [1], Ps was used as a probe for nano/mesoporosity in materials [2–4] and became a prime testing ground of bound-state Quantum Electrodynamics [5–7]. Ionic [8–10] and molecular [11] Ps have been also observed and are currently being investigated.

At present, Ps plays a central role in one of the antimatter community’s biggest efforts: measuring the gravitational force acting on antimatter bodies in the Earth’s gravitational field. The universality of the free-fall, i.e. the equivalence between inertial and gravitational masses, is a cornerstone of Einstein’s Equivalence Principle and General Relativity. Any observed discrepancy from the postulated equality would be an indication of new physics. On the one hand, Ps is being used by experiments in CERN’s Antiproton Decelerator as an (experimentally demonstrated [12]) intermediate tool to produce antihydrogen via a charge-exchange reaction with antiprotons. The goal of these experiments is to probe gravity using antihydrogen [13,14]. Both a classical deflectometer [15] and an interferometer [16] have been proposed for inertial sensing measurements with antihydrogen. On the other hand, several experimental schemes to directly measure gravity with long-lived Ps beams have been proposed (thus probing the gravitational interaction with a purely leptonic matter-antimatter system), either by letting Ps atoms free-fall in a drift tube and observing their vertical displacement with position-sensitive detector [17] or by adopting a matter-wave atom interferometer [18].

Ps has been also proposed as a probe to investigate forces other than gravity [18]. Indeed thanks to its low mass and high magnetic moment [7], Ps is expected to be the ideal system to study relativistic forces like the Anandan force [19]. Moreover, one can exert a force on Ps via electric dipole interactions with far-detuned laser light (optical dipole force). Optical dipole forces can exceed by up to several orders the magnitude of the gravitational force [20].

Such force-sensitive inertial experiments require a long-lived Ps beam. A widely adopted approach to increase the short lifetime of Ps is to excite it to Rydberg levels ($n = 15 – 30$) with a two-step laser excitation scheme, i.e.
either using \( n = 2 \) [21] or \( n = 3 \) [22] as intermediate levels. This approach is beneficial for the long lifetimes that can be achieved (>100 \( \mu s \)), yet exhibits some potential limitations in view of measuring tiny forces (such as gravity). Rydberg Ps has a non-negligible electrical polarizability (which scales as \( n^6 \)), making it sensitive to stray electric field gradients [7,17], and spontaneously decays towards lower excited states populating a large number of sublevels. Exciting Ps to circular Rydberg states would mitigate this issue [23], at the price of flux and a much more complex laser excitation stage [17]. An alternative pathway consists in laser-exciting the atoms to the long-lived \( 2^3S \) metastable level, either by a two-photon transition [5] or using a pulsed mixing electric field [24] or by spontaneous decay from \( n = 3 \) [25]. In the absence of strong electric fields, Ps in this level cannot decay optically (due to dipole selection rules) and has a relatively long annihilation lifetime (1.14 \( \mu s \)).

In this paper, we investigate the possibility to perform force-sensitive inertial experiments with \( 2^3S \) Ps. We will discuss the time needed (that is related to the characteristics of the \( 2^3S \) Ps beam) to perform such experiments because it represents one of the main limitations with the present technologies. The work is organized as follows. In Section 2, the characteristics of the monochromatic \( 2^3S \) Ps source currently available (intensity, \( 2^3S \) velocity and angular divergence) are summarized [25–27]. In Section 3, the performances of a moiré and a Mach–Zehnder interferometer are compared. The feasibility of force-sensitive inertial experiments on \( 2^3S \) is discussed and the required time to measure forces of different intensity on this system is estimated. Section 4 is devoted to the analysis of the detection schemes of the fringe pattern generated by the deflectometer/interferometer. Finally, in Section 5, we discuss possible improvements to enhance the intensity and quality of the \( 2^3S \) beam and the sensitivity of the interferometer.

### 2 \( 2^3S \) positronium beam: state of the art

Ps in the \( 2^3S \) state can be produced via spontaneous decay from the \( 3^3P \) state previously populated from the ground state \( 1^3S \) with a 205 nm UV laser pulse [25,26].

The use of an efficient positron trapping and storage technology [28,29] allows to produce bunches of around \( 10^7 \) positrons compressed in less than 10 ns [30,31]. If 50 mCi \( ^{22}\text{Na} \) sources are employed, these bunches can be implanted every \( \sim 40 \) s in \( e^+/\text{Ps} \) nanochannelled silicon converters from which more than 30% of implanted positrons reemerge as Ps (when \( e^+ \) are implanted with an energy of 3–5 keV) [32,33]. Ps emission is expected to be roughly isotropic [26]. A fraction of the emitted Ps can then be excited to the \( 3^3P \) state [22]. The subsequent \( 3^3P \rightarrow 2^3S \) spontaneous decay has been found to occur with a branching ratio of \( \sim 10\% \), in good agreement with theoretical expectations [25,26]. The short duration of the laser pulse used to excite Ps to the \( 3^3P \) state allowed selecting a fraction of the Ps cloud with a quasi-monochromatic velocity distribution [26]. Moreover, by changing the laser pulse delay with respect to the positron implantation instant, it has been possible to address Ps populations emitted with different velocities and thus a tuning of the \( 2^3S \) velocity has been obtained [26]. With a laser pulse delay of 20 ns, we observe a velocity distribution with a mean of \( 10^6 \) m/s and a standard deviation of less than \( 10^4 \) m/s. The \( 2^3S \) production efficiency has been found to be up to 1.5% of the overall number of Ps emitted by a nanochanneled silicon target [25,26]. A further factor \( \sim 3 \) of increase in the \( 2^3S \) production efficiency has been achieved by employing a 1312 nm laser pulse to stimulate the transition from the \( 3^3P \) to the \( 2^3S \) level [27]. As summarized in Table 1, combining all these currently available techniques, around \( 1.3\times10^5 \) monochromatic \( 2^3S \) Ps atoms can be produced every \( \sim 40 \) s. The bandwidth of the 205 UV laser pulse (120 GHz), used to populate the \( 3^3P \) level, is narrower than the Doppler profile of Ps emitted from the converter [26]. As a consequence, the UV pulse performs a Doppler selection on the Ps along the laser propagation line (\( z \) axis in Fig. 1).

Table 1. Efficiency of \( e^+ \rightarrow 2^3S \) positronium conversion according to presently available techniques. Bunches of \( 10^7 e^+ \) every \( \sim 40 \) s have been considered for the final estimation of \( 2^3S \) production rate.

| Process | Efficiency | \( 2^3S \) production |
|---------|------------|-----------------------|
| \( e^+/\text{Ps} \) conversion efficiency | \( \sim 0.3 \) [32] | |
| \( 1^3S \rightarrow 3^1P \rightarrow 2^3S \) excitation efficiency | \( \sim 0.015 \) [26] | \( 10^7 \times 0.3 \times 0.045 \sim 1.3 \times 10^5 \) |
| \( 3^1P \rightarrow 2^3S \) stimulated decay | \( \times 3 \) [27] | per bunch |
| Overall \( 2^3S \) production efficiency | \( \sim 0.045 \) (0.015 \( \times 3 \)) | |

An example for the combination of interferometer and detector. The UV laser pulse excites the Ps to the \( 3^3P \) state, from which the Ps spontaneously decay to the \( 2^3S \) state and emit a laser pulse (120 GHz), used to populate the \( 3^3P \) level, is narrower than the Doppler profile of Ps emitted from the converter [26]. As a consequence, the UV pulse performs a Doppler selection on the Ps along the laser propagation line (\( z \) axis in Fig. 1).
As an example, a classical device is reported in Figure 3 sensitivity devices composed of two gratings and a detector. We discuss here deflectometry/interferometry inertial sen-

3 Deflectometry/interferometry systems

Fig. 2. Number of surviving $^{2}$S Ps atoms per bunch as a function of the travelled distance from the $e^{+}$/Ps converter in a beam with a vertical divergence of 17 mrad (black squares) and 5.5 mrad (open squares). A source intensity of $1.3 \times 10^{5}$ $^{2}$S Ps atoms and a velocity of $10^{5}$ m/s for the $^{2}$S Ps atoms in a $\pm 14^\circ$ wedge have been assumed.

If Ps atoms with average velocity of $10^{5}$ m/s are considered, the Doppler selected $^{2}$S Ps expand within a spherical wedge of around $\pm 14^\circ$ with respect to the normal plane to the target ($x$-$y$ plane in Fig. 1) [26]. By using a slit, it is possible to select a collimated fraction in a given direction ($y$ direction in Fig. 1) of this spherical wedge, at the price of a reduction in the flux. In Figure 2 the number of $^{2}$S Ps atoms still alive at a given distance from the source in a beam selected with a slit fixing the vertical divergence to 17 mrad (5.5 mrad) is reported. The numbers are for a source of $1.3 \times 10^{5}$ $^{2}$S Ps atoms (see Tab. 1).

3.1 Moiré deflectometer

The moiré deflectometer is a classical device where the gratings are physical gratings with a periodicity $d$ much larger than the de Broglie wavelength ($\lambda_{dB}$) of the test particles [34].

$$\lambda_{dB} = \frac{h}{mv},$$ (1)

where $m$ is the inertial mass of the test particle, $v$ its velocity and $h$ the Planck constant. The classical trajectories defined by the two gratings lead to a fringe pattern with the grating periodicity at the longitudinal distance $L$ from the second grating where the detector is positioned [15]. If the transit time of the particles through the device is known, absolute force measurements are possible by employing Newton’s second law of dynamics. As indicated in Figure 3, the position of the moiré pattern is shifted in presence of a force with respect to the geometric shadow by:

$$\Delta_y = \frac{F t^2}{m},$$ (2)

where $F$ represents the force component along the grating period, $a$ is the acceleration and $t$ is the time of flight between the two gratings spaced a length $L$ apart. The time $t$ is called interrogation time. Thus, in the case of a monochromatic beam with known velocity $v$, $t$ is simply $L/v$. The sensitivity of the system (i.e. the minimum detectable acceleration, $a_{min}$) is given by:

$$a_{min} = \frac{d}{2\pi V t^2 \sqrt{N}},$$ (3)

where $N$ is the number of detected particles and $V$ is the visibility of the fringes [15]. For particles with very long lifetimes, by increasing the distance $L$ between the gratings, it is possible to enhance the sensitivity simply by increasing the interrogation time $t$. However, for particles with a limited lifetime -like metastable Ps- the number of particles reaching the detector scales as equation (4):

$$N = N_0 T e^{-\frac{t}{\tau}},$$ (4)

where $N_0$ is the number of particles at the entrance of the device, $\tau$ is the particle lifetime (1142 ns for $^{2}$S Ps) and $T$ is a factor of transparency of the gratings. A value of 1 corresponds to the case that no particles are stopped. Combining equations (3) and (4), one can estimate the behavior of the minimum detectable acceleration for $^{2}$S Ps as a function of the interrogation time (Fig. 4).

The plot shows that the minimum acceleration sensitivity is reached for $\sim 2 \mu s$ i.e. 2 times the $^{2}$S lifetime. This means that, once the velocity of the beam is defined, the best distance between the gratings is also set to $L \sim v * 2 \mu s$. The moiré deflectometer offers the advantage of working even for a divergent source of particles [35] allowing the use of a relatively large solid angle of...
the beam without sacrificing particles. On the other hand, the number of particles reaching the detector is limited by the transparency of the material gratings that is far from 1 [35]. Moreover, the dimension of the grating period sets an important constraint on the minimum measurable acceleration with this device (Eq. (3)). With a divergence of 17 mrad (consistent both with the geometry of our experimental set-up of the $2^3 S$ beam [31] and the angular acceptance of a moiré deflectometer [15]), with a grating period of the order of 40 µm (a slit width of 12 µm) and a transparency of the gratings of ~0.3, a visibility of the fringes of ~0.3 can be obtained [15,35]. These grating dimensions are currently feasible [15].

With the characteristics of the $2^3 S$ beam reported in Section 2, taking atoms with a velocity of $10^5$ m/s (i.e. $L \sim v \times 2\mu s = 20$ cm), the vertical divergence indicated above and the first grating at 3 cm from the target, a minimum acceleration of $6.1 \times 10^3 \text{g}$, $6.1 \times 10^2 \text{g}$ and $61 \text{g}$ can be measured in $4 \times 10^2$ shots (~4.5 h), $4 \times 10^4$ shots (~2.6 weeks of measurement) and $4 \times 10^6$ shots (60 months) (Eqs. (2) and (3)). The sensitivity estimations are reported in Table 2.

The discussion above does not take into account the presence of possible systematics such as external field gradients or longitudinal asymmetries and rotational misalignments of the gratings. The effect of such eventual systematics would be to degrade the quality of the pattern, reducing the visibility $V$ of the fringes [36] and consequently increasing the measurement time (Eq. (3)). As our experiment can be done in a free field region (see Refs. [26,31] for details), the effect of field gradients is expected to be limited. On the opposite, the precision in the alignment of the gratings is of great importance. According to reference [36], the effect of a longitudinal displacement that asymmetrize the position of the two gratings and/or the fringe pattern plane as well as the effect of a rotational misalignment between the gratings are directly proportional to the periodicity $d$ and inversely proportional to the divergence of the beam. For the deflectometer and the beam considered in this section, a visibility of the fringes higher than ~0.15 can be obtained with a longitudinal misplacement of the gratings smaller than ~0.6 nm and a rotational misalignment smaller than ~3 mrad [36].

### 3.2 Mach–Zehnder interferometer

An alternative scheme is constituted by the Mach–Zehnder interferometer (see for example [38–40]). Such a scheme has been proposed for positronium in reference [18]. As depicted in Figure 5, it requires generating standing waves of light where the diffractive optical elements are thicker than the Talbot length ($L_{\text{Talbot}}$):

$$L_{\text{Talbot}} = \frac{2d^2}{\lambda dB},$$

for the considered Ps with a velocity of $10^5$ m/s, $\lambda dB = h/(mv) = 3.6 \text{nm}$. In this configuration an atom incident at the Bragg angle $\theta_B$ traverses more than one grating plane and a fraction of the incident particles is deviated with an angle of:

$$\theta = 2\theta_B = 2\frac{\lambda dB}{2d}.$$  

A standing light wave with wavelength $\lambda$ leads to a periodic potential for the atoms with $d = \lambda/2$. Following the original scheme proposed in reference [18], where $\lambda = 1312 \text{nm}$ is used, the diffraction angle $\theta = 5.5 \text{mrad}$.

In order to be able to distinguish the interference fringes, the angular collimation of the beam has to be smaller than this diffraction angle given by the standing light wave of 5.5 mrad.

The expected number of $2^3 S$ Ps in the slit with a vertical divergence of 5.5 mrad as a function of the distance from the target is shown in Figure 2. If the first light grating is placed at 3 cm from the source, around 680 $2^3 S$ Ps atoms are expected to reach the entrance of the interferometer with vertical divergence of 5.5 mrad. The reduction of the beam flux, required by the Mach–Zehnder interferometer, negatively affects the sensitivity of the device (Eqs. (3) and (4)). However, this effect is compensated by the reduction of the grating period $d$ and -more importantly- by the very high transparency of the light gratings that approaches unity. As a result, this device would allow to detect an acceleration of $10^2 \text{g}$, $10 \text{g}$ and $1 \text{g}$ in $4 \times 10^2$ shots (~4.5 h of measurement), in $4 \times 10^4$ shots (~2.6 weeks of measurement) and $4 \times 10^6$ shots (~60 months of measurement), assuming a visibility of the fringes of 0.3 and a dimension of the device like in Section 3.1 (for the feasibility of a Mach–Zehnder interferometer with $L$ of the order of tens of centimeters

1 Such a scheme could be more difficult for Rydberg states. Indeed, the light crystal needs a wavelength very close to a given transition and for Rydberg states the adjacent levels are very close. For instance, for Ps in $n = 15$, the standing light wave for the Mach–Zehnder interferometer should be of the order of 340 µm (corresponding to the gap with $n = 16$). If one decides to use a different transition (for instance $n = 2 - n = 15$) the possibility of photoionization of the level has to be taken into account.

![Fig. 4. Sensitivity estimate for a $2^3 S$ interferometer. The graph shows the behavior of the minimum detectable acceleration in arbitrary units as a function of the interrogation time.](image-url)
Table 2. Comparison of the minimum measurable acceleration in 4.5 h, 2.6 weeks and 60 months with a moiré deflectometer and a Mach–Zehnder interferometer by employing the present e⁺/Ps converters and tested strategies for the production of monochromatic 2³S.

| Deflectometer/interferometer | Angular divergence at the entrance of the device per shot | Number of 2³S | \(a_{\text{min}}\) in 4.5 hours (\(\sim 4 \times 10^2\) shots) | \(a_{\text{min}}\) in 2.6 weeks (\(\sim 4 \times 10^4\) shots) | \(a_{\text{min}}\) in 60 months (\(\sim 4 \times 10^6\) shots) |
|-----------------------------|----------------------------------------------------------|----------------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|
| moiré                       | 17 mrad                                                   | \(\sim 2100\) | \(\sim 6.1 \times 10^5\) g                      | \(\sim 6.1 \times 10^2\) g                      | \(\sim 61\) g                                    |
| Mach–Zehnder                | 5.5 mrad                                                  | \(\sim 680\)  | \(\sim 10^5\) g                                 | \(\sim 10^2\) g                                 | \(\sim 1\) g                                     |

Fig. 5. Schematic of a matter wave Mach–Zehnder interferometer. Two standing light waves act like diffractive optical elements deflecting a fraction of the beam and creating the interference fringes.

4 Fringe pattern detection

Two different approaches can be used to detect the fringe pattern of the deflectometer/interferometer. The first method consists in resolving the pattern with a position sensitive detector. The second one is to probe the fringe pattern with a third grating followed by a detector with no spatial resolution, measuring the particle flux [35].

The limits of the application of a position sensitive detector in the case of 2³S Ps are evident from the comparison of equation (2) and Figure 4. Indeed, the figure tells us that the minimum detectable acceleration can be obtained for interrogation times of the order of 2–5 µs. According to equation (2), this means that shifts of \(\Delta y < 0.3\) nm (alternatively, 3 µm) should be resolved to detect an acceleration of \(g\) (alternatively, \(\sim 10^4\) g). Thus far, the best resolution in the determination of the annihilation position of a Ps atom on a plane is of the order of 90 µm [42]. This resolution has been achieved with a detector based on the ionization of Ps in a strong homogeneous magnetic field and the detection of the photopositrons with a microchannel plate (MCP)+phosphor screen assembly [42]. In the present context, a position sensitive detector could be realized by employing a continuous laser to photoionize 2³S Ps in the plane where the fringes have to be resolved. Extracted e⁺ can then be guided by a homogeneous magnetic or electric field (taking care that the fringe field does not penetrate in the deflectometer/interferometer) towards the MCP assembly. Thanks to the guiding field, the charges conserve the information of the Ps impact position within the photoionizing plane [42]. The limits to the spatial resolution are the cyclotron radius of the charges in the magnetic field, that can be reduced by increasing the intensity of the magnetic field (for a free e⁺ with an energy of some tens of meV, the cyclotron radius is of the order of 1 µm in a field of 1 T), and -more importantly- the spatial resolution of the MCP assembly. A reduction of the spatial resolution to values lower than 10 µm could be obtained by employing MCPs with small pores and by replacing the phosphor screen with Timepix chips (see for example Ref. [43]), which allows to implement methods of centroid reconstruction [44]. Even better spatial resolutions, in such a Ps detection scheme, could be obtained by using an emulsion detector (see for example [45]) placed downstream of the photoionizing plane. Photo-detached positrons can be accelerated to the emulsion and imaged with a spatial resolution down to around 1 µm [45]. To summarize, the use of the available spatial resolution detectors would allow to measure accelerations down to some \(10^4\) g.

On the opposite, the probing of the fringe pattern with a third grid requires only a counting detector with a relatively low spatial distribution in order to distinguish between the 2³S atoms crossing the last grating (and annihilating on a following stopper) and the ones annihilating on it (see Fig. 6). While, as seen in Section 3, the first and the second gratings should be standing waves to maximize the transparency of the device, the last grid can be a physical grating with the same period because also the stopped atoms can be detected and used as confirmation of the signal generated by the crossing ones
not an absolute necessity and, if the shape of the expected fringe pattern (or of the passing/stopping probability distribution) is known, the measurement in a fixed position with and without acceleration (along the direction \( y \) of Fig. 6) could already give a good estimate of the investigated force.

5 Improvements of the interferometer sensitivity

According to the considerations above, with the present \( 2^3S \) production efficiency the measurement of accelerations ranging between 10 \( g \) and \( g \) on Ps would require, with a Mach–Zehnder interferometer and with a third grating detection scheme, between 2.6 \( \times \) 2 weeks and 60 \( \times \) 2 months of continuous data taking. A shortening of the measurement time is highly desirable, especially for small accelerations, and looks feasible by increasing the intensity of the \( 2^3S \) beam. Further improvements to the sensitivity can come from the use of light gratings with different periodicity with respect to the 1312 nm originally proposed in reference [18].

5.1 Laser collimation of the \( 2^3S \) beam

An important boost to the intensity of the \( 2^3S \) Ps beam could come from Ps laser cooling. Although Ps laser cooling has been discussed for many years [49,50], it has not yet been experimentally demonstrated even if the atomic structure of Ps does not present any fundamental limitations to Ps cooling [7,51]. Indeed, the most efficient transition in terms of the energy removed from the system per cooling cycle is the \( 1^3S - 2^2P \) transition; therefore a 1D laser cooling scheme requires a long duration counter-propagating \( \sim 243 \) nm laser beam crossing the expanding Ps cloud in the direction in which the cooling is needed [49]. In order to collimate the Ps beam, two laser beams, perpendicular to each other and perpendicular to the axial diffusion direction of Ps, have to be employed. The duration of these pulses should be limited to few tens of ns. Indeed, as demonstrated in reference [26], a longer delay between the positron implantation in the \( e^+ \) Ps converter and the excitation to \( 3^P \) level implies a reduction of the \( 2^3S \) production efficiency. However, Monte Carlo simulations reported in reference [52], show that this time appears sufficient to cool an important fraction of the Ps in the laser direction. There, 25 ns long laser pulses with 5 kW power, 50 GHz spectral bandwidth and 75 GHz and 120 GHz detunings from the \( 1^3S - 2^2P \) transition resonance where considered. When applied to the 1D Doppler distribution of \( \sim 1000 \) K Ps (like in [22]), the effect of 75 GHz laser pulses is to increase by a factor 1.75 the amount of Ps travelling with a velocity lower than \( \sim 10^4 \) m/s or to enhance by a factor 1.2 the amount of Ps travelling with a velocity lower than \( \sim 2.5 \times 10^4 \) m/s if the 120 GHz detuned pulse is used. As the average modulus of the velocity of Ps at 1000 K is of the order of \( 10^3 \) m/s, the atoms with a velocity component along the laser beam below \( \sim 10^4 \) m/s and \( \sim 2.5 \times 10^4 \) m/s expand with an angle

---

1. The short distance between 3rd grating and the stopper is required to minimize the number of self-annihilations in the stopper. In the case of \( 2^3S \) atoms with a velocity of \( 10^3 \) m/s, the number of self-annihilations in the indicated distance of 6 mm is less the 5%.
2. https://www.aerotech.co.uk/product-catalog/piezo-nanopositioners/qnp-1-series.aspx; https://www.piusa.us/en/products/piezo-flexure-nanopositioners/x-linear-piezo-flexure-nanopositioning-stages/p-752-high-precision-nanopositioning-stage-200800/
lower than ±7° and ±14° with respect to the axial direction of the beam, respectively. If the two laser beams are employed simultaneously with the 75 GHz-detuned laser beam perpendicular to the long side of the slit (vertical direction in Fig. 1) and the 120 GHz-detuned laser perpendicular to the first one (horizontal direction in Fig. 1), the number of atoms expanding in the solid angle defined by the slit increases by 1.75 × 1.2 ~ 2.1 times. Thus, the following excitation to 2^3S level will result in a 2.1 times more intense beam reaching the interferometer (see Tab. 3).

5.2 Use of slower 2^3S along the beam direction

The dependency of both the deflection induced by a given acceleration (Eq. (2)) and the minimum detectable acceleration (Eqs. (3) and (4)) from the interrogation time t -determined by the lifetime of the test particle- looks to close any possibility to increase the beam intensity by acting on the 2^3S speed. Indeed, every change in the 2^3S velocity along the beam direction requires a rescale of the device length L to keep t in the optimum detection range (see Fig. 4). However, the reduction of the velocity has the advantage of increasing the de Broglie wavelength of the test particle (Eq. (1)) and, as a consequence, to enhance the diffraction angle in the Mach–Zehnder interferometer (Eq. (6)). This allows working with a larger angular acceptance of the beam. For instance, the halving of the Ps speed from 10^7 m/s to 5 × 10^6 m/s, keeping the same intensity of the 2^3S source and halving the dimensions of the device (the distance of the first grating from the source and the length L), doubles the angular acceptance of the beam and thus its intensity (see Tab. 3).

The realization of quasi- monochromatic 2^3S Ps with velocity lower than 10^7 m/s has been already achieved by increasing the delay of the 1^3S → 3^1P laser pulse; i.e. addressing Ps that has spent more time in the nanochannels [26]. Unfortunately, with this procedure, the amount of the produced 2^3S decreases by increasing the pulse delay. This problem could be circumvented by sending the 1^3S → 3^1P laser pulse without further delays (20 ns from the positron implantation in the target) but keeping the e^+/Ps converter at a temperature lower than room temperature. Indeed, the production of slower 2^3S Ps without sacrificing the produced amount has been observed in similar e^+/Ps converters held at cryogenic temperatures [33, 53, 54].

5.3 Raman excitation scheme

Stimulated decay from 3^3P level allows to have a ~4.5% efficiency of 2^3S production with respect to the overall Ps emitted into vacuum by the converter (See Sect. 2). The use of a more efficient monochromatic 2^3S production scheme would permit to enhance the beam intensity with important benefits for interferometry measurements. A promising alternative to the incoherent two-step 1^3S → 3^1P → 2^3S is a coherent Raman excitation scheme involving an intermediate virtual level (see for example [55]). In order to achieve the required transition, the Raman excitation scheme could use simultaneously a 212 nm laser pulse (5th YAG harmonic) and a 1700 nm pulse. The advantage of this scheme is that in Raman excitation absorption and stimulated emission occur simultaneously. Thus, a π-pulse transfers all the addressable population from the 1^3S level to 2^3S [55] without the distribution of the population on the different involved levels typical of the two-step scheme. The addressable fraction of Ps that one could excite via coherent Raman excitation is determined by the limited geometrical overlap of the laser spots on the Ps cloud and the large Doppler profile of Ps emitted from the target that is not entirely covered by the limited spectral width of the lasers. As shown in reference [22], the first step (1^3S → 3^3P) of the two-step transition has an efficiency equal to ~16% of the overall population of Ps emitted by the target. As the maximum theoretical 1^3S → 3^3P efficiency is expected to be ~93%, it means that the addressable fraction of Ps is 16/93 ~ 17% [22]. Thus, if in the Raman scheme, laser pulses with similar band width and spot dimension to the lasers used in reference [22] are employed, up to 17% of the overall Ps could be transferred to the 2^3S level. This would be an increment of 3.77 times in the 2^3S beam intensity with respect to the 1^3S → 3^3P → 2^3S two-step procedure (Tab. 3).

5.4 Light gratings with periodicity smaller than 1312 nm

The sensitivity estimations for a Mach–Zehnder interferometer reported in Section 3.2 are for the use of light gratings with a wavelength of ~1312 nm and a small detuning from the 2 → 3 transition, according to the original proposal of reference [18]. However, transitions from n = 2 to levels higher than n = 3 can be used to reduce the period of the gratings. Ultimately, the only upper limit is set by the level at which photoionization of the 2^3S Ps starts to become effective (below λ = 730 nm for Ps in n = 2), killing a part of the beam. To avoid this undesirable effect one could sit close to the 2 → 17 transition whose energy gap is 1.679 eV, corresponding to ~730 nm. If standing waves with this wavelength are used, their period is reduced to d ~ 365 nm. According to equation (3), the decrease of d proportionally reduces the minimum detectable acceleration. Going from d ~ 1312/2 nm to d ~ 730/2 nm, the sensitivity of the interferometer directly increases by a factor ~1.8. The reduction of the grating period has another positive effect on the interferometry measurement. Indeed, as pointed out by equation (6), the reduction of the grating period increases the diffraction angle in the Mach–Zehnder interferometer. Once again, this allows having a larger angular acceptance of the beam that could be increased up to 20 μrad. This 1.8 enhancement in angular acceptance of the beam implies an identical increase of its intensity (Tab. 3).

The small grating period and the increased divergence of the beam in this measurement scheme will set stricter requirements on the precision of the alignment of the standing waves of light in order to preserve the visibility of the fringes. With the interferometer and the beam considered here, a visibility >0.15 will require a longitudinal asymmetry of the gratings smaller than ~4 μm and a rotational misalignment smaller than ~20 μrad [36].
Table 3. Expected enhancements in the beam intensity and in the sensitivity of the interferometer given by laser collimation of the beam, use of slower Ps, of a Raman excitation scheme and use of light gratings with periodicity of 730/2 nm. The effects on the measurement time to perform tests of $g$ and $10g$ are reported.

| Beam/interferometer characteristics | Angular acceptance | $2^3S$ at the entrance of the device per shot | $a_{\text{min}} = g$ | Needed shots to reach $g$ (10g) | Notes |
|------------------------------------|-------------------|---------------------------------------------|-----------------|-----------------------------|-------|
| Present $2^3S$ beam               | 5.5 mrad          | $\sim 680$                                   | $\sim 4 \times 10^6$ | $\sim 4 \times 10^4$         |       |
|                                    |                   |                                              | (60 months)     | (2.6 weeks)                 |       |
| Laser collimation                  | 5.5 mrad          | $\sim 1430$                                   | $\sim 1.9 \times 10^6$ | $\sim 1.9 \times 10^4$        | $\times 2.1$ beam intensity with 2D laser collimation |
|                                    |                   |                                              | (29 months)     | (1.2 weeks)                 |       |
| Use of slower $2^3S$ beam          | 11 mrad           | $\sim 2860$                                   | $\sim 10^6$     | $\sim 10^5$                 |       |
| along the beam direction           |                   |                                              | (14.3 months)   | (4.3 days)                  |       |
| Raman scheme                       | 11 mrad           | $\sim 10780$                                  | $\sim 2.6 \times 10^5$ | $\sim 2.6 \times 10^4$        | $\times 3.77$ beam intensity |
| $n = 2 \rightarrow n = 17$        | 11 mrad           | $\sim 10780$                                  | $\sim 8 \times 10^4$ | $\sim 8 \times 10^2$         | $\times 1.8$ in the sensitivity of the interferometer corresponding to a reduction of 1.8$^2$ in the measurement time |
| $d = 365$ nm                       |                   |                                              | ($\sim 5$ weeks) | (8.5 hours)                 |       |
| $n = 2 \rightarrow n = 17$        | 20 mrad           | $\sim 19400$                                  | $\sim 4.5 \times 10^4$ | $\sim 4.5 \times 10^2$        | $\times 1.8$ in the angular acceptance and consequently in the beam intensity |
| $d = 365$ nm                       |                   |                                              | ($\sim 2.8$ weeks) | (4.7 hours)                 |       |

6 Conclusion

In this work, we have analyzed the possibility to conduct force-sensitive inertial experiments on a matter/antimatter and purely leptonic system like metastable Ps in the $2^3S$ state. Two possible inertial sensitive devices have been considered: a classical moiré deflectometer and a Mach–Zehnder interferometer. In spite of the need imposed by the Mach–Zehnder interferometer to work with a quite collimated beam, the very high transpareny offered by its light gratings and their short period make the sensitivity of this device up to several tens of times better than that of a moiré deflectometer. Thus a Mach–Zehnder interferometer is preferable in the case of the measurement of very weak forces, like the gravitational interaction. Adopting already-demonstrated techniques for the production of a $2^3S$ beam, the Mach–Zehnder interferometer offers the possibility to measure accelerations of the order of 100 g in a few hours and 10 g in a few weeks. This sensitivity level would be sufficient for the first observation of Ps atoms’ deflection by a laser dipole force. The measurement of acceleration of the order of g, with the present $2^3S$ beam, would require very long data taking (60 x 2 months).

The shift of the fringe pattern due to an optical dipole force or to gravity on $2^3S$ Ps is expected to be in the $\mu$/sub-nm range. The spatial resolution necessary to resolve the smallest shifts can be reached with a three gratings scheme where the third mechanical grating is placed in the plane where the fringes are expected. The fraction of crossing and stopped $2^3S$ on the third grating can then be recorded in a fixed position of the grating with and without the investigated acceleration. The stopping/crossing probability can then be compared to extract the information about the strength of the investigated force.

Different strategies to increase the $2^3S$ beam flux and to improve the sensitivity of the devices have been proposed and analyzed. Among them, the most promising are Ps beam collimation through 2D laser Doppler cooling in the transverse direction and the coherent Ps Raman excitation from the ground state to the $2^3S$ level. In combination, the two techniques would increase the flux of the beam by almost a factor eight. Other improvements to the sensitivity can come from the use of slower $2^3S$ Ps and the realization of standing wave gratings with a period of $\sim 365$ nm corresponding to the $n = 2 \rightarrow n = 17$ transition. By implementing all these improvements, the data taking to measure the gravitational acceleration on Ps would decrease to few weeks, yielding a first test of the weak equivalence principle for a leptonic matter/antimatter system. With the same improvements, the measurement of an acceleration of 10 g (achievable by employing an optical dipole force) would require only a few hours.

We are grateful to P. Yzombard and D. Comparat for the useful feedbacks about laser cooling. This work was financially supported by the European Union’s Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No. 754496 – FELLINI.

Author contribution statement

The original idea of this work was from RC. It was expanded togheter with SM, MD, GN and RSB gave important suggestions. All the authors were involved in the preparation of the manuscript. All the authors reviewed and approved the manuscript.
References

1. M. Deutsch, Phys. Rev. 82, 455 (1951)
2. C.L. Wang, M.H. Weber, K.G. Lynn, K.P. Rodbell, Appl. Phys. Lett. 81, 4413 (2002)
3. L. Liszklay, F. Guilleminot, C. Corbel, J.P. Boilot, T. Gacoin, E. Barthel, P. Perez, M.F. Barthe, P. Desgardin, P. Crivelli, U. Gentodi, A. Rubbia, New J. Phys. 14, 065009 (2012)
4. M. Zanatta, G. Baldi, R.S. Brusa, W. Egger, A. Fontana, E. Gilioli, S. Mariazzi, G. Monaco, L. Ravelli, F. Sacchetti, Phys. Rev. Lett. 112, 5 (2014)
5. M.S. Fee, S. Chu, A.P. Mills Jr., R.J. Chichester, D.M. Zuckerman, E.D. Shaw, K. Danzmann, Phys. Rev. A 48, 192 (1993)
6. S.G. Karshenboim, Phys. Rep. 422, 1 (2005)
7. D.B. Cassidy, Eur. Phys. J. D 72, 53 (2018)
8. A.P. Mills, Phys. Rev. Lett. 46, 717 (1981)
9. Y. Nagashima, Phys. Rep. 545, 95 (2014)
10. K. Michishio, T. Kanai, S. Kuma, T. Azuma, K. Wada, I. Mochizuki, T. Hyodo, A. Yagishita, Y. Nagashima, Nat. Commun. 7, 11060 (2016)
11. D.B. Cassidy, A.P. Mills, Nature 449, 195 (2007)
12. C.H. Storry, A. Speck, D. Sage, N. Guise, G. Gabrielse, D. Grzonka, W. Oelert, G. Schepers, T. Sefzick, H. Pittner, M. Herrmann, J. Walz, T.W. Hänsch, D. Comeau, E.A. Hessels, Phys. Rev. Lett. 93, 263401 (2004)
13. M. Doser et al., (AEgIS collaboration), Classical Quantum Gravity 29, 183009 (2012)
14. P. Perez, Y. Sacquin, Classical Quantum Gravity 29, 184008 (2012)
15. S. Aghion et al., (AEgIS collaboration), Nat. Commun. 5, 4538 (2014)
16. P. Hamilton, A. Zhmoginov, F. Robicheaux, F. Hessels, T. Juffmann, J. Kotakoski, C. Mangler, A. Winter, A. Turchanin, J. Meyer, O. Cheshnovsky, M. Arndt, Nat. Nanotechnol. 10, 845 (2015)
17. A. Miffre, M. Jacquey, M. Bichier, G. Trenec, J. Vigue, Phys. Scr. 74, C15 (2006)
18. A.D. Cronin, J. Schmiedmayer, D.E. Pritchard, Rev. Mod. Phys. 81, 1051 (2009)
19. D.M. Giltner, R.W. McGowan, S.A. Lee, Phys. Rev. Lett. 75, 2638 (1995)
20. C. Ansler et al., (AEgIS collaboration), Nucl. Instrum. Methods Phys. Res., Sect. B 457, 44 (2019)
21. A. Miffre, M. Jacquey, M. Bichier, G. Trenec, J. Vigue, Phys. Scr. 74, C15 (2006)
22. P. Kowalski et al., (J-PET collaboration), Phys. Med. Biol. 63, 165008 (2018)
23. H. Iijima, T. Asonuma, T. Hirose, M. Irako, T. Kumita, M. Kajita, K. Matsuzawa, K. Wada, Nucl. Instrum. Methods Phys. Res. Sect. B 455, 104 (2000)
24. T. Kumita, T. Hirose, M. Irako, K. Kadoya, M. Kobayashi, M. Kajita, Nucl. Instrum. Methods Phys. Res. Sect. B 192, 171 (2002)
25. K. Shu, X. Fan, T. Yamazaki, T. Namba, S. Asai, K. Yoshioka, M. Kowalski, T. Kumita, T. Hirose, M. Irako, T. Kumita, M. Kajita, K. Matsuzawa, K. Wada, Nucl. Instrum. Methods Phys. Res. Sect. A 831, 12 (2016)
26. A.S. Tremssin, J.V. Vallerga, R.R Raffanti, J. Instrum. 13, C11005 (2018)
27. S. Aghion et al., (AEgIS collaboration), J. Instrum. 8, P08013 (2013)
28. C. Brand, M. Sclafani, C. Knobloch, Y. Lilach, T. Juffmann, J. Kotakoski, C. Mangler, A. Winter, A. Turchanin, J. Meyer, O. Cheshnovsky, M. Arndt, Nat. Nanotechnol. 10, 845 (2015)
29. A. Miffre, M. Jacquey, M. Bichier, G. Trenec, J. Vigue, Eur. Phys. J. D 39, 99 (2006)
30. P. Kowalski et al., (J-PET collaboration), Phys. Med. Biol. 63, 165008 (2018)
31. H. Iijima, T. Asonuma, T. Hirose, M. Irako, T. Kumita, M. Kajita, K. Matsuzawa, K. Wada, Nucl. Instrum. Methods Phys. Res. Sect. B 455, 104 (2000)
32. T. Kumita, T. Hirose, M. Irako, K. Kadoya, B. Matsumoto, K. Wada, N.N. Mondal, H. Yabu, K. Kobayashi, M. Kajita, Nucl. Instrum. Methods Phys. Res. Sect. B 192, 171 (2002)
33. K. Shu, X. Fan, T. Yamazaki, T. Namba, S. Asai, K. Yoshioka, M. Kowalski, T. Kumita, T. Hirose, M. Irako, T. Kumita, M. Kajita, K. Matsuzawa, K. Wada, Nucl. Instrum. Methods Phys. Res. Sect. B 455, 104 (2000)