A Review of Key Line Identification Methods in Complex Networks

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Abstract. This paper summarizes the key line identification method of the most representative power network in the complex networks. It is of great significance to identify the key line of the power transmission network to prevent the cascade failure of the power system. By analyzing and comparing the methods of identifying key line in recent years, this paper summarizes the power betweenness method, electrical betweenness method, comprehensive importance method, entropy theory and some other methods, and expounds their advantages and disadvantages. Based on this, the future research direction of critical line identification in complex power transmission networks is proposed.

Keywords: Complex Networks, Key Line, Power Transmission Network, Identification, Power Betweenness, Electrical Betweenness, Comprehensive Importance, Entropy Theory

1. Introduction

The U.S. suffered the biggest blackout in history that shocked the world on August 14, 2003 Eastern Time[1]. On March 7, 2019, most areas in Venezuela, including the capital Caracas, suffered a power outage for more than 24 hours, causing massive traffic congestion.

In recent years, frequent blackouts have caused severe challenges to the world's major power transmission networks. The power transmission network is the most complex and important network in modern life[2], and complexity science represented by complex networks has become an effective way to analyze complex power transmission networks. With the deepening of research in recent years, scholars have discovered that the power transmission network is a dynamic complex network with scale-free characteristics[3] and small-world characteristics[4]. The initial failure will cause a series of failures to spread through the connections between system components, spread in the complex network of the real world, and eventually evolve into an avalanche cascading failure[5, 6]. Failure of a complex power transmission network can affect the integrity of a wide range of infrastructure, natural systems and social cohesion[7], and the causes of these failures have many unknown factors, which make them extremely difficult to predict or control[8]. We can only focus on the protection of key line, by imposing additional protection or changing the topological structure, to obtain a strong robust and low-risk electrical system [9]. Accurately identifying the key line in the complex power transmission network that affect the safety and stability of the system is of great significance for improving the
efficiency of grid operation and maintenance and risk warning[10], so protection of critical line can ensure better survival of the network[11].

This article summarizes the methods of identifying key line of complex power transmission networks in recent years, and finally explains the future development direction, hoping to provide references for the research of complex power transmission networks.

2. Based on Betweenness

Previous scholars’ research only introduced some basic concepts based on complex network theory to study the topological characteristics of power transmission networks [12-15]. Later researchers introduced some indicators with electrical characteristics combined with the indicators of complex networks to replace the traditional purely complex network indicators, so that the complex network tool can better fit the power transmission network.

2.1 Power Betweenness

Ref.[16] comprehensively considered the power flow of the power transmission network and the transmission capacity of the line, and proposed a method to identify key line through the active power flow betweenness. The active power flow betweenness of line $(i,j)$ is defined as follows:

$$F_{ij} = \delta_i K_j$$

(1)

$$K_j = \sum_{i \in G} \sum_{m \in L} \min(P_m^i, P_n^j) I_{ij}(m,n)$$

(2)

$$\delta_i = \frac{P_i}{P_{i_{\text{max}}}}$$

(3)

Where $\min(P_m^i, P_n^j)$ is the weight of the generator-load node, representing the smaller value of the actual active power in the generator-load node; $P_{ij}$ is the actual active power of line $(i,j)$, $P_{i_{\text{max}}}$ is the power transmission limit, and $I_{ij}(m,n)$ is the generator-load node current flowing through $(m,n)$ after injecting unit current into $(i,j)$. The power flow coefficient $\delta_i$ reflects the usage of the transmission line under the current operating conditions of the power transmission network, and $K_j$ is the electrical betweenness index after considering the power directionality factor.

Ref.[17] based on the power flow tracking method, the importance of the line in the transmission of active and reactive power is quantified. The traditional method defines the power betweenness as follows:

$$F_{bij} = \sum_{m \in L} \sum_{n \in L} \min(S_m^i, S_n^j) \frac{P_{ij}(m,n)}{P(m,n)}$$

(4)

The author made improvements on this basis and defined active power betweenness and reactive power betweenness respectively. The definition of active power betweenness is as follows:

$$F_{P_{ij}} = \sum_{m \in L, n \in L} \min(P_m^i, P_n^j) \frac{P_{ij}(m,n)}{P(m,n)}$$

(5)

The definition of reactive power between is as follows:

$$F_{Q_{ij}} = \sum_{m \in L, n \in L} \min(Q_m^i, Q_n^j) \frac{Q_{ij}(m,n)}{P(m,n)}$$

(6)

Use the weighted root mean square method to integrate it into the comprehensive transmission betweenness, and combine the two to define the comprehensive transmission betweenness as follows:

$$B_{ij} = \sqrt{\omega_p \times F_{P_{ij}}^2 + \omega_q \times F_{Q_{ij}}^2}$$

(7)
In the formula, \( \omega_p, \omega_q \) is the active power coefficient and the reactive power coefficient respectively. Since active power is more important than reactive power, set \( \omega_p \) to 0.6 and \( \omega_q \) to 0.4. \( F_{pp}^2, F_{qq}^2 \) is the normalized value.

### 2.2 Electrical Betweenness

The electrical betweenness \( B_i(m,n) \) defined by Ref.[18, 19] reflects the utilization of the power flow between the power generation-load node pair on the line, also quantifies the contribution of the branch to the power flow propagation of the entire network. The electrical betweenness is defined as follows:

\[
B_i(m,n) = \sum_{j=\Gamma(m,n)} \sqrt{W_iW_j} |l^q(m,n)|
\]  

(8)

Where \( l^q(m,n) \) is the current induced on line \((i,j)\) after adding unit injection current element between generation-load node pair \((m,n)\); \( W_i \) is the weight of the generation node \(i\); \( W_j \) is the weight of the load node \(j\).

Ref.[20] considered the influence of transmission power when defining electrical betweenness, and defined electrical betweenness as follows:

\[
B(l) = \sum_{j=N,i\in \Gamma(l)} |P_{ij}P_{ji}| \frac{P_{ij}(l)}{P_{ij}}
\]  

(9)

Where \( P_{ij} \) is the output power of generator \(i\), \( P_{ij} \) is the active power of load node \(j\), \( P_{ij}(l) \) is the power from generator \(i\) to load node \(j\) on line \(l\), and \( P_{ij} \) is the total power transmitted from generator \(i\) to load node \(j\). The second indicator is the transmission power of the transmission line. Once a fault occurs, other line must accept the power flow of the faulty line. Therefore, a faulty line with a larger transmission power is more likely to cause a greater power flow and a greater risk of overload.

Ref.[21] improved on the basis of traditional electrical betweenness. Based on the definition of the current correlation coefficient matrix, the power sensitivity matrix is defined. Combined with the different types of nodes, the improved electrical betweenness of the nodes is calculated to identify the key line of the power transmission network. Define the current correlation coefficient matrix:

\[
I_{k,B} = \lambda_k,1 \cdot I_{k,1} + \cdots + \lambda_k,N \cdot I_{k,N} + \cdots + \lambda_k,W \cdot I_{k,W} \quad (10)
\]

\[
\beta_{k,l} = \frac{\lambda_k}{U_{k,B}} \left( \cos \varphi_{k,B} \cos \varphi_{l,N} + \sin \varphi_{k,B} \sin \varphi_{l,N} \right) \quad (11)
\]

Based on the power sensitivity matrix and the traditional electrical betweenness, define the improved electrical betweenness \( B_k \) of line \(k\) as:

\[
B_k = \frac{P_k}{P_{k,max}} \sum_{m=I\in \Gamma(l)} \sqrt{W_mW_k} \beta_{k-\cdot,m} \quad (12)
\]

\[
\beta_{k-\cdot,m} = \beta_{k,m} - \beta_{k,k} \quad (13)
\]

Where \( \beta_{k-\cdot,m} \) is the increased power of line \(m\) when generator node \(m\) increases unit output and load node \(k\) increases unit load. This method focuses on the role of active power transmission in the system, and does not comprehensively consider the influencing factors of voltage and reactive power.

### 3. Based on Comprehensive Importance

Ref.[22] defines the global index, system load rate index, system overload degree index, bus voltage fluctuation degree index, reactive power flow transfer degree index and loss load index according to the overall and local characteristics of the power transmission network. The method calculates the weight of the index and constructs a comprehensive key index.

Global index:
$$A_i = |k_i|$$

Where $|k_i|$ is the absolute value of the slope of the distribution curve of the load rate of the remaining components of the system after the line $i$ is disconnected.

System load rate index:

$$L_{si} = \frac{\sum_{j=1}^{m} |F_{ij}|}{\sum_{j=1}^{m} |F_{ij,\text{max}}|}$$

Where $F_{ij}$ is the active power flow on line $j$ when line $i$ is disconnected, and $F_{ij,\text{max}}$ is the maximum active power limit on line $j$.

System overload index:

$$L_{so} = \frac{m_i}{n}$$

In the formula, $m_i$ is the number of overloaded line caused by the disconnection of line $i$. The greater the degree of system overload, the greater the probability of a major blackout.

Bus voltage fluctuation degree index:

$$oU_i = \sum_{j=1}^{n} \frac{|U_{ij} - U_{\text{avg},j}|}{U_{\text{avg},j}}$$

In the formula, the larger the value of $oU_i$, the greater the voltage fluctuation of the system after the line is disconnected, and the more unstable the system.

Similar to the bus voltage fluctuation index, the reactive power flow transfer degree index is defined as follows:

$$oQ_i = \sum_{j=1}^{n} \frac{|Q_{ij} - Q_{\text{avg},j}|}{Q_{\text{avg},j}}$$

The greater the volatility of reactive power flow, the greater the impact of the line on the system.

The author uses the entropy method to calculate the weights of the above five indicators to construct a comprehensive importance index:

$$V_i = \sum_{j=1}^{m} \omega_j x_{ij}$$

Where $x_{ij}$ is the $j$-th normalized index value of the $i$-th line; $\omega_j$ is the weight of the $j$-th normalized index value.

4. Based on Entropy Theory

Entropy theory is an important method in the field of complex systems. It can evaluate the chaos of the internal molecular motion of a system more deeply and comprehensively. Entropy has long been extended to the power system, mainly in the following aspects: utility risk entropy[23],power flow distribution entropy[24],weighted power flow entropy[25].

Ref.[26] improved the traditional power flow betweenness index, considering the influence of the line transmission margin, and proposed an improved power flow betweenness index. Entropy theory can well evaluate the stability of the entire system after a certain line fails. On this basis, a power flow transfer entropy index that weights the line load factor is proposed to overcome the shortcomings of the original index that only considers the uniformity of the transfer power flow distribution. Established comprehensive importance evaluation indicators. Define the improved power flow betweenness index $T_i^j$ as follows:
The author has made improvements on the basis of the traditional flow transfer entropy, which is defined as follows:

\[ H_f = -\sum_{a=0}^{k} \mu_{a(f)} p_{a(f)} \ln p_{a(f)} \]  

The weighted calculation is carried out on the average value of the load rate interval, and the load rate expansion multiple \( H \) is introduced, which overcomes the shortcoming of only considering the uniformity of the flow transfer distribution in the traditional power flow transfer entropy.

Ref.[27] starts with branch breaking entropy and load flow entropy, and proposes a method to identify key line based on improved load flow entropy. The traditional power flow entropy can reflect the power flow distribution of the system to a certain extent, but the power flow entropy of the system will increase after some line failures with too large or too small power flow, and cannot reflect the physical characteristics of the power flow entropy, so the active power After normalization, the power flow entropy is weighted, and the formula is:

\[ P_k = \frac{P_i - P_{\text{min}}}{P_{\text{max}} - P_{\text{min}}} \]  

After improvement, the load flow entropy is obtained:

\[ H_i^* = -P_k p_i \ln (p_i) \]  

Introducing the weighting coefficient \( P_k \) in the formula, distinguishing the heavy-load branch from the light-load branch leads to the larger entropy. The breaking transfer entropy is improved. Based on the weighted topology of the power transmission network, the weighted equivalent index \( S_k \) of line transmission capacity is defined:

\[ S_k = |W_i| P_k \]  

Construct the total weighted equivalent transmission ratio of the system to represent the transmission power of all line. The formula is as follows:

\[ \lambda^i = \frac{1}{m} \sum_{i=1}^{m} \lambda_i = \frac{1}{m} \sum_{i=1}^{m} \frac{1}{U_i \theta_i} \left| \theta_i - \theta_j \right| \]  

In the formula, \( \lambda_i = |S_i/S_{\text{max}}| \), \( \lambda_i \in [0,1] \) represent the relative size of the transmission power. The larger the value, the greater the impact of fault removal on the system. The breaking entropy index of each line is

\[ H_i^* = (\lambda_i - \lambda^0) \ln (\lambda_i - \lambda^0) \]  

Construct different index adjustment weight factors \( \alpha \), calculate the normalized entropy value of the above two indexes: \( H_i^*, H_i^{\alpha} \), and obtain the comprehensive entropy index by weighted sum:

\[ H_i = (1-\alpha) H_i^* + \alpha H_i^{\alpha} \]  

5. Conclusion
This article summarizes and evaluates typical key line identification methods to improve our in-depth understanding of complex power transmission networks. Even though many scholars have developed more mature key line identification methods, there are still many problems to be solved. The most
important thing is that researchers lack more real data. Based on real data, a more realistic and complex power transmission network model can be established. The future methods of identifying key line in complex power networks tend to the following aspects:

1) The method summarized in the article takes into account the influence of electrical characteristics and topological structure comprehensively, but does not consider the influence of frequency in the electrical characteristics on the entire network. There are certain limitations.

2) When assigning weights to various indicators, one should avoid subjective guesswork and use methods in the field of operations research: Analytic Hierarchy Process[28, 29], TOPSIS method[30] and other methods to assign reasonable weights to indicators to make them more theoretically supported.

In summary, identifying the key line in a complex power transmission network has great theoretical and practical significance. Through research in this field, people will have a deeper understanding and cognition of complex power transmission networks.

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