Development of the Lower Extremity Exoskeleton Dynamics Model Using in the Task of the Patient Verticalization

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Abstract. The object of the study is an exoskeleton of the lower extremities with a rigid structure of the power frame, which has 7 degrees of freedom. The movement of the exoskeleton in the sagittal plane is considered with the assumption of symmetrical movement of the right and left legs. The aim of the study is to develop a mathematical model of the dynamics of the exoskeleton, taking into account the forces of viscous friction in the joints. The equations of motion are obtained under the condition that there is no slippage of the points of contact with the supporting surface. Based on the results of numerical simulation, the control moments were obtained, which must be created by the drives to provide program movement.

1. Introduction

Currently, the development and application exoskeletons, i.e. devices designed to facilitate movement and increase human motor efficiency is a perspective direction in the development of robotics. The relevance of research devoted to the optimization of energy consumption, trajectory of motion, dynamics, control, circuitry and structural performance of exoskeletons is due to their increasingly widespread use in industry [1], in transport, in military affairs [2], in medicine [3] and in other fields.

The object of consideration in this work is the active rigid exoskeleton of the lower extremities of a person, used in the rehabilitation of patients with impaired functions of the musculoskeletal system [4].

When developing such exoskeletons, there is always a need to solve interrelated scientific and technical problems – the formation of a skeletal scheme, optimization of the movement of links on the basis of a mathematical model of kinematics and dynamics, rational construction of the motor system, development of optimization control algorithms, etc.

One of the central problems connecting all of the listed tasks is finding control actions for program movement, obtained from a dynamic model of the system, or the formation of control actions as a PID
controller used to stabilize programmed motion, as well as a combination of these methods (see, for example, works [5–6]).

Therefore, the development of mathematical models for solving direct and inverse problems of kinematics and dynamics of an active exoskeleton is highly relevant, which is reflected in the reviewed publications on this topic [7–15]. In particular, in [7] the object is presented as an automatic control system, but without a mathematical description of non-stationary processes. In [8–10], the equations of motion of the exoskeleton are presented in a general form, which complicates their direct practical application. At the same time, in [8] the dynamic model takes into account only inertial, Coriolis and gravitational forces acting on the exoskeleton, and in [9, 10], in addition to the above forces, centrifugal and dissipative forces are also taken into account. The works [11–13] contain a description of the control of the lower limb exoskeleton with compensation of gravitational forces, however, only positional forces are taken into account, without the necessary proof in such cases of the insignificance of inertial and speed load factors.

In the presented work, an attempt is made to develop a mathematical model for solving the problems of analysis and synthesis of the lower extremity exoskeleton in non-stationary modes, taking into account inertial, velocity and gravitational forces, as well as viscous friction forces in the joints. Taking into account the solution of the problem of verticalization of patients using this exoskeleton, the symmetry of the movement of both limbs - legs is taken as an assumption.

2. Description of the system

We consider the structure of the exoskeleton of the lower extremities presented in Fig. 1. This design models each leg as three links connected by cylindrical joints.

The following designations are introduced on the scheme:
- \( C_2, C_3 \) и \( C_4 \) – are centers of mass of the lower legs, thighs and body, respectively;
- \( A_1, A_2 \) и \( A_3 \) – are joints, connecting the links of the exoskeleton;
- \( \phi_2, \phi_3 \) и \( \phi_4 \) – are angles of rotation of the links of the exoskeleton, which are counted from the axis \( x \) in the counterclockwise direction;
- \( M_2, M_3 \) и \( M_4 \) – are control moments created by drives located in the joints \( A_1, A_2 \) and \( A_3 \) respectively;

To describe the motion, a coordinate system \( xyz \), is introduced in which the axis \( x \) is directed along the supporting surface, and the axis \( y \) is directed along the local vertical to the supporting surface. In this case, the direction of the axis \( z \) is chosen in such a way that \( xyz \) is the "right" Cartesian coordinate system.

3. Dynamical model of the exoskeleton

To construct the equations for the dynamics of the exoskeleton, we will use the Lagrange formalism [16, 17]. In this case, the equations of motion have the form:
where

- $\mathbf{q} = (\varphi_2, \varphi_3, \varphi_4)^T$ – is Lagrange coordinates vector;
- $T$ and $P$ – are kinetic and potential energies of the system;
- $\Phi$ – is Rayleigh dissipation function;
- $\mathbf{Q}_M = (M_2 - M_1, M_1 - M_3, M_4)^T$ – is generalized forces vector corresponding to control moments.

### A. Inertial and Velocity Terms of the Equations of Motion

For the considered exoskeleton scheme, the kinetic energy of the system has the form:

$$T = \frac{1}{2} \left( m_2 V_{c2}^2 + J_2 \dot{\varphi}_2^2 + m_3 V_{c3}^2 + J_3 \dot{\varphi}_3^2 + m_4 V_{c4}^2 + J_4 \dot{\varphi}_4^2 \right),$$

where $m_1, m_2, m_4$ – are masses of the lower legs, thighs and body for the "human+exoskeleton" system; $J_2, J_3, J_4$ – are moments of inertia of the lower legs, thighs and body for the "human+exoskeleton" system relative to the axis $z$; and the expressions for the velocities of the centers of mass $V_{c2}, V_{c3}, V_{c4}$ in projections on the axis $x$ and $y$ have the form:

\[
V_{c2} = \begin{pmatrix}
-\varphi_2 a_2 \sin \varphi_2 \\
\varphi_2 a_2 \cos \varphi_2
\end{pmatrix},
\quad
V_{c3} = \begin{pmatrix}
-\varphi_3 l_2 \sin \varphi_2 - \varphi_1 a_1 \sin \varphi_3 \\
\varphi_3 l_2 \cos \varphi_2 + \varphi_1 a_1 \cos \varphi_3
\end{pmatrix},
\quad
V_{c4} = \begin{pmatrix}
-\varphi_4 l_2 \sin \varphi_2 - \varphi_3 l_3 \sin \varphi_3 - \varphi_4 a_4 \sin \varphi_4 \\
\varphi_4 l_2 \cos \varphi_2 + \varphi_3 l_3 \cos \varphi_3 + \varphi_4 a_4 \cos \varphi_4
\end{pmatrix}.
\]

Here $l_1 = |\mathbf{A}_2|, l_2 = |\mathbf{A}_3|, l_3 = |\mathbf{A}_4|, l_4 = |\mathbf{A}_5|, l_5 = |\mathbf{A}_6|$ – are length of lower legs and thighs, and $a_2 = |\mathbf{A}_2|, a_3 = |\mathbf{A}_3|, a_4 = |\mathbf{A}_4|, a_5 = |\mathbf{A}_5|$ – are distances that characterize the location of the centers of mass of the links.

To find the terms of the equations of motion that correspond to the inertial and velocity forces, we substitute expressions (3) into formula (2) and calculate the derivatives included in the left side of equation (1). As a result of these actions, we obtain the left side of the Lagrange equations (1) in the form:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \mathbf{\dot{q}}} \right) - \frac{\partial T}{\partial \mathbf{q}} = \mathbf{A}(\mathbf{q}) \mathbf{\ddot{q}} + \mathbf{F}(\mathbf{q}) \mathbf{\dot{q}}^2,$$

where $\mathbf{q}^2 = (\varphi_2^2, \varphi_3^2, \varphi_4^2)^T$ – is the vector of squares of generalized velocities and the notation for the matrices have the form:

\[
\mathbf{A}(\mathbf{q}) = \begin{bmatrix}
\mathbf{J}_{22} & \mathbf{J}_{23} c(\varphi_2 - \varphi_3) & \mathbf{J}_{24} c(\varphi_2 - \varphi_4) \\
\mathbf{J}_{23} c(\varphi_2 - \varphi_3) & \mathbf{J}_{33} & \mathbf{J}_{34} c(\varphi_3 - \varphi_4) \\
\mathbf{J}_{24} c(\varphi_2 - \varphi_4) & \mathbf{J}_{34} c(\varphi_3 - \varphi_4) & \mathbf{J}_{44}
\end{bmatrix},
\]

\[
\mathbf{F}(\mathbf{q}) = \begin{bmatrix}
0 & \mathbf{J}_{23} s(\varphi_2 - \varphi_3) & \mathbf{J}_{24} s(\varphi_2 - \varphi_4) \\
-\mathbf{J}_{23} s(\varphi_2 - \varphi_3) & 0 & \mathbf{J}_{34} s(\varphi_3 - \varphi_4) \\
-\mathbf{J}_{24} s(\varphi_2 - \varphi_4) & -\mathbf{J}_{34} s(\varphi_3 - \varphi_4) & 0
\end{bmatrix}.
\]
Here are introduced abbreviations for writing trigonometric functions: \( s(x) = \sin x \) and \( c(x) = \cos x \); as well as designations for the given moments of inertia:

\[
J_{22} = J_2 + m_2 a_2^2 + (m_1 + m_4) l_2^2, \quad J_{24} = m_4 a_4 l_2,
\]

\[
J_{33} = J_3 + m_3 a_3^2 + m_1 l_3^2, \quad J_{23} = (m_1 a_1 + m_4 l_1) l_2,
\]

\[
J_{44} = J_4 + m_4 a_4^2, \quad J_{34} = m_4 a_4 l_3.
\]

### B. Positional, Dissipative Forces and Control Moments in the Equations of Motion

For the considered scheme of the exoskeleton, the potential energy of the system has the form:

\[
P = m_2 g a_2 \sin \varphi_2 + m_3 g (l_2 \sin \varphi_2 + a_3 \sin \varphi_3) + m_4 g (l_2 \sin \varphi_2 + l_3 \sin \varphi_3 + a_4 \sin \varphi_4).
\]

Taking into account the linearity of the dependences of the viscous friction forces in the joints, the Rayleigh dissipation function takes the form:

\[
\Phi = \frac{1}{2} \left( \mu_2 \dot{\varphi}_2^2 + \mu_3 \dot{\varphi}_3^2 + \mu_4 \dot{\varphi}_4^2 \right),
\]

where \( \mu_2, \mu_3, \mu_4 \) – are viscous friction coefficients.

Calculating the derivatives entering into equation (1), by functions (6) and (7), the right-side of the Lagrange equation (1) is written as follows:

\[
Q_M - \frac{\partial \Phi}{\partial \dot{q}} = \begin{bmatrix}
M_2 - M_4 - \mu_2 \dot{\varphi}_2 - M_{g2} \cos \varphi_2 \\
M_3 - M_4 - \mu_3 \dot{\varphi}_3 - M_{g3} \cos \varphi_3 \\
M_4 - \mu_4 \dot{\varphi}_4 - M_{g4} \cos \varphi_4
\end{bmatrix}.
\]

Here we use notation: \( M_{g2} = g (m_2 a_2 + m_3 l_2 + m_4 l_2) \), \( M_{g3} = g (m_3 a_3 + m_4 l_3) \) and \( M_{g4} = gm_4 a_4 \).

### C. Equations of the Motion

Calculating the left (4) and right (8) parts of the equations of dynamics (1), we obtain a dynamic model in the form:

\[
A(q) \ddot{q} + F(q) \dot{q} = \begin{bmatrix}
M_2 - M_4 - \mu_2 \dot{\varphi}_2 - M_{g2} \cos \varphi_2 \\
M_3 - M_4 - \mu_3 \dot{\varphi}_3 - M_{g3} \cos \varphi_3 \\
M_4 - \mu_4 \dot{\varphi}_4 - M_{g4} \cos \varphi_4
\end{bmatrix}.
\]

This mathematical model makes it possible to estimate the values of the control torques necessary for the implementation of the movement. The indicated estimates of the moments can be used for the selection of motors driving the links of the system [18, 19, 20].

### 4. Exoskeleton motion simulation

Let us consider an example of the numerical parameters of the system corresponding to a person 175 cm tall and the mass of the “person + exoskeleton” system equal to 80 kg. In this case, the values of the parameters of the mathematical model will be equal:

\[
a_2 = 0.22 \text{ m}; \quad a_3 = 0.30 \text{ m}; \quad a_4 = 0.40 \text{ m};
\]

\[
m_2 = 12.8 \text{ kg}; \quad m_3 = 16 \text{ kg}; \quad m_4 = 51.2 \text{ kg};
\]

\[
l_2 = 0.44 \text{ m}; \quad l_3 = 0.50 \text{ m}; \quad J_2 = 0.2065 \text{ kg} \cdot \text{m}^2;
\]

\[
J_3 = 0.3333 \text{ kg} \cdot \text{m}^2; \quad J_4 = 2.7307 \text{ kg} \cdot \text{m}^2;
\]

\[
\mu_2 = \mu_3 = \mu_4 = 1.2 \cdot 10^{-3} \text{ H} \cdot \text{m} \cdot \text{c}.
\]
The values of the angles in the joints at different times for the considered movement are given in Table I. Here, for the transition from the previous state \( \varphi_i(t_{k-1}) \) to the next \( \varphi_i(t_k) \), the law of change of the angle \( \varphi_i \) is used in the form [18] (at \( t \in [t_{k-1}, t_k] \)):

\[
\varphi_i(t) = \varphi_i(t_{k-1}) + \left( \varphi_i(t_k) - \varphi_i(t_{k-1}) \right) \cdot \left[ \frac{t}{t_k - t_{k-1}} - \frac{1}{2\pi} \sin \left( \frac{2\pi}{t_k - t_{k-1}} \cdot t \right) \right]
\]

(9)

| Time, s | Joint angles |
|---------|--------------|
|         | \( \varphi_2, ^\circ \) | \( \varphi_3, ^\circ \) | \( \varphi_4, ^\circ \) |
| 0       | 85           | 180          | 90          |
| 3       | 75           | 180          | 90          |
| 6       | 70           | 180          | 70          |
| 9       | 70           | 180          | 60          |
| 12      | 70           | 140          | 50          |
| 15      | 90           | 90           | 90          |

The graphs of the change in the angles of the exoskeleton links are shown in Fig. 2.

Substituting the dependences of the angles at the joints into the dynamic model (9), we calculate the control moments necessary to ensure the programmed motion. Graphs of control moments are shown in Fig. 3.

5. Conclusion
This article presents a mathematical model of the dynamics of movement of the lower extremities exoskeleton used to solve the problem of verticalization of patients. The developed model takes into account the inertia of the links, viscous friction in the joints and gravity acting on the links. The model is supposed to be used in solving direct and inverse optimization problems for the active exoskeleton of the human lower extremities. A numerical simulation of lifting a patient equipped in an exoskeleton from a sitting position to a standing position with a good degree of correspondence to reality has been performed. Based on the simulation results, estimates of the control moments on the programmed motion were obtained.
6. References

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