Research on Constitutive Model of Unsaturated Sand

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Abstract. For unsaturated sand, due to the existence of suction, under low confining pressure, it has a large volume change during shearing and obvious dilatancy and peak characteristic points appear in the whole stress-strain behavior. Based on the above situation, the boundary surface constitutive model of unsaturated sand is established in general stress space and triaxial stress space based on phase transition state and hydraulic-mechanical coupling. The model has smooth transition among different saturation state and model parameters are easily calibrated via common triaxial tests and some other simple tests.

1. Water-force process of unsaturated sand

For saturated or dry medium-dense and dense sand, especially coarse grain sand, dilatancy is easy to occur under the action of shear stress. While the characteristics of dilatancy for loose sand or silty sand is not obvious that is related to the initial porosity ratio and the soil quality. But under the action of suction, additional "effective pressure", that is the surface tension generated at the meniscus of the particle contact point push the particles more tightly and with the effect of shear stress, the particles are more easily to be lifted and rotate which lead to dilatancy, Correspondly, the shear strength is also greatly improved.

As shown in Figure 1, the confining pressure is 20 kPa, 75kPa for ASU dense silty sand under different suction[1]. During the process of shearing, the shear stress continues to increase until the obvious peak point appears, and then softening stage appears, taking on obvious strain strengthening and softening characteristics; The volume is in the shrinking state at the beginning, and soon reaches the phase transition point and the dilatancy state afterwards; Under the same confining pressure, the greater the suction, the greater the shear stiffness, and more quickly the peak point is reached. Under the same suction but different confining pressure, the smaller the confining pressure, the easier it is to enter the dilatancy state.
Fig.1 The stress-strain curves of unsaturated dense sands with $\sigma_{ij}^u = 20.75kPa$ under triaxial tests$^{[1]}$

Above data$^{[1]}$ show that loose sand is just like transformed into dense sand with the action of suction that lead to the dilatancy characteristics more obvious, and the phase transition point more easily appears in the deformation process. Therefore, it is more appropriate for unsaturated sand to establish a constitutive relationship with the phase transition state as its starting point.

2. Elasto-plastic model

Based on the above characteristics, for the dilatancy of saturated sand and unsaturated fine sand, this paper proposes a constitutive model for unsaturated sand with different densities. The model must have the ability to reflect the hydraulic-mechanical behavior of unsaturated sand. To attain the smooth transition between saturated and unsaturated states, the unsaturated soil model should be transformed into the saturated soil model when the suction is less than the air intake value. Generally, the air intake value of sand is very small for the sand particles are very large relative to clay, and the air intake value is only a few kPa.

In this paper, the stress variables of unsaturated sand are:

$$\sigma'_{ij} = (\sigma_{ij} - u_a^e) + sS_0 \delta_{ij}; \tilde{s} = ns = n(u_a - u_w)$$

(1)

Here $\sigma_{ij}, u_a^e, \sigma'$ are respectively the net stress, the atmospheric pressure and the average stress of the soil skeleton, and $\sigma'$ is also called the effective stress of the soil skeleton, $\tilde{s} = ns$ is the modified suction, $n$ is for porosity.

2.1 Elastic incremental relations

Here elastic-plastic coupling is not considered for the elastic relationship with saturated sand, and as well as for the elastic modulus has no relationship with plasticity. Therefore, the elastic relationship between soil skeleton and water can be expressed as follows:

$$d\varepsilon' = \frac{dp'}{K}; \quad d\varepsilon' = \frac{dS'}{2G}; \quad dS' = -\frac{1}{\Gamma} \frac{d\tilde{s}}{\tilde{s}}$$

(2)
Here, $p'$ and $S$ are respectively the effective average stress and the deviatoric stress tensor of unsaturated sand, and $p' = \frac{(\text{tr}\sigma')}{3}$. Deviatoric stress tensor $S = \sigma - p'\mathbf{I}$, $\mathbf{I}$ is the unit tensor, $\varepsilon$, with $\varepsilon$ are the volumetric strain and deviatoric strain tensor, here $G$, $K$, $\gamma'$ are the elastic shear modulus, bulk modulus for soil skeleton and elastic modulus of water. The proposed equation[2], expressed as a function of the effective stress and the current pore ratio:

$$G = G_0 \left(\frac{(2.97-e^e)^2}{1+e}\right)(p'p_a)^\frac{1}{2}, K = G \frac{2(1+\nu)}{3(1-2\nu)}$$

Here $G_0$, $p_a$ are respectively the material constant, the current void ratio and the reference pressure, $p_a = 101$ kPa. In fact, it can also be set to other values. $\nu$ here is Poisson's ratio and set as a constant.

### 2.2 Yield, Dilatancy, Boundary Functions

Based on literature[3][4], the defined yield surface should consider both isotropic hardening and kinematic hardening. Here, only monotonous loading is considered. It is first described in triaxial space and then extended to general stress space, as shown in Figure 2.

**Fig. 2 Schematic illustration of the yield, dilatancy, critical state, bounding lines in the general stress space and $p', q$ space**

Therefore, solid-phase yielding formula uses a similar expression on saturated sand:

$$f = q - (\alpha \pm \varepsilon)p = 0$$

Here $p'$ is the effective average stress. Because triaxial compression and triaxial tensile conditions are the same, for ease of description, here only triaxial compression space is discussed. In the triaxial compression space, the principal stress $\sigma_1 > \sigma_2 = \sigma_3$ (axisymmetric condition), $p' = (\sigma'_1 + 2\sigma'_3)/3$, $q = \sigma' - \sigma'$, $\alpha$ is the back stress ratio, in general space, $\alpha$ denotes the back stress ratio tensor. In this paper, the parameters of phase transition state of unsaturated sand, dilatancy stress ratio, peak boundary stress ratio and back stress ratio are respectively introduced as $\beta$, $\beta' = (e(p')/e_p)'(p' - 1)$.

$M_{ptl}, M_{ptl} \alpha_{ptl}, \alpha_{ptl}$ which are denoted as

$$M_{ptl} = M_p \exp(k_p \beta)$$

$$\alpha_{ptl} = M_p - m = M_p \exp(k_p \beta) - m$$

$$M_{ptl} = M_p \exp(-k_p \beta)$$

$$\alpha_{ptl} = M_p - m = M_p \exp(-k_p \beta) - m$$

The above variables are described in the triaxial stress space. In order to describe the behavior of unsaturated sand in the general stress space, the yield surface should be converted from the triaxial stress space to the general stress space. Therefore, in the general stress space, the yield surface
equation is
\[ f = \sqrt{(S - p'\mathbf{a}) : (S - p'\mathbf{a})} - \frac{2}{3\lambda'} = 0 \]  
(6)

Here \( \mathbf{a} \) is the back stress tensor, by means of the modified Lord's angle \( \theta \) to complete the conversion of triaxial to general stress space, and the deviatoric stress ratio tensor is set \( \mathbf{r} = S/p' \), The loading direction of the yield surface at the following direction \( \mathbf{r} \) is:
\[ \mathbf{L} = \frac{\partial f}{\partial \mathbf{\sigma}} = \mathbf{n} - \frac{1}{3}(\mathbf{n} : \mathbf{r})\mathbf{I}; \mathbf{n} = \frac{\mathbf{r} - \mathbf{a}}{[(\mathbf{r} - \mathbf{a}) : (\mathbf{r} - \mathbf{a})]^{1/2}} \]  
(7)

We adopt the method similar to saturated sand, in the general stress space, the dilatancy and boundary surfaces, and the corresponding back stress tensor corresponding to them \( \mathbf{\alpha}_\mu \) are as follows:
\[ M'_\mu = g(\theta, c_\mu)M_\mu; \mathbf{\alpha}'_\mu = \sqrt{\frac{2}{3}}\mathbf{\alpha}_\mu^\varepsilon \mathbf{n}; \mathbf{\alpha}'_\mu = g(\theta, c_\mu)M_\mu - m \]  
(8)

The liquid phase yield function can be used as the soil-water characteristic curve [5], where the slope of the main drying line and the main soaking line is \( \lambda_w \), and the slope of the scanning line \( \kappa_w \).

Assuming \( \tilde{s}_y \) as the liquid phase yield stress, when \( y = 1 \), the yield produced by the decrease of suction is expressed when the suction is reduced. The elastic variation of liquid saturation can be expressed as:
\[ \text{d} \tilde{s}_y = -\kappa_w \frac{d \tilde{s}_y}{\tilde{s}_y} \]  
(9)

The elastic modulus of the liquid phase \( \Gamma_e = 1/\kappa_w \), The plasticity change of saturation caused by the change of suction can be expressed as follows
\[ \text{d} S'_y = -(\lambda_w - \kappa_w) \frac{d \tilde{s}_y}{\tilde{s}_y} \]  
(10)

The liquid phase plastic modulus is: \( \Gamma_p = \frac{1}{\lambda_w - \kappa_w} \).

2.3 Plastic relationship

Dilatancy ratio \( D \)

Just like saturated sand, the mapping point on the dilatant surface for the back stress tensor \( \mathbf{a} \) is \( \mathbf{a}^\varepsilon \), and the corresponding dilatancy equation is as follows:
\[ D = \frac{\sqrt{2}}{2} D_v(\mathbf{a}^\varepsilon - \mathbf{a}) : \mathbf{n} = D_v(\mathbf{a}^\varepsilon - \frac{\sqrt{2}}{2}\mathbf{a} : \mathbf{n}) \]  
(11)

Back stress ratio tensor \( \mathbf{\alpha} \)

The back stress ratio tensor \( \mathbf{\alpha} \) is usually set as a function of the current stress and plastic deviatoric strain of the soil skeleton, \( \mathbf{\alpha} = \mathbf{a}(\mathbf{\sigma}', \mathbf{e}'^p) \). For unsaturated sand, both liquid phase modified suction \( \tilde{s} \) and saturation \( S_i \) together affect the random hardening by contributing to the effective stress \( \mathbf{\sigma}' \). \( \mathbf{\alpha} \) is generally considered to point to the boundary mapping point \( \mathbf{a} \), and \( \text{d} \mathbf{\alpha} \) depends on the distance \( \mathbf{a}' - \mathbf{a} \) \[6,7\], so the following equation shows the development of the back stress ratio tensor \( \mathbf{\alpha} \)
\[ \text{d} \mathbf{\alpha} = (L)h(\mathbf{a}^\varepsilon - \mathbf{a}) = \frac{\sqrt{2}}{2} h(\frac{2}{3} \mathbf{a}^\varepsilon \mathbf{e}'^p - \mathbf{d} \mathbf{e}^p \mathbf{a}) \]  
(12)

\( h \) here is a state variable function which is the same as the method used in saturated sand [8], but the effective stress here \( p' \) contains the effect of suction, so, \( h \) is expressed as
\[ h = \frac{G_a h_0 (1 - e)/(2.97 - e)^2 / (1 + e)(p_u / p')^{1/2}}{\sqrt{3/2}} (r - \alpha_n) : (r - \alpha_n) \]  

(13)

Here \( G_a \) is the material constant, \( \alpha_n \) is the initial value of the back stress tensor.

**Isotropic hardening**

As the isotropic parameter, \( m \) denotes the elasticity range of yield surface which is not only a function of stress state, but also the plastic function of volumetric strain \( \varepsilon_v' \) and plastic saturation \( S_v' \). The isotropic parameters are determined based on experiments. It is generally believed that \( m \) increases as the pore ratio decreases. At the same time, the change of suction also affects the yield surface of the soil skeleton, but the literature [5] pointed out that plastic saturation can directly reflect the change of the fluid in the pores, and its influence on curved liquid surface is more important than suction. Therefore, only the plastic volume strain of the soil skeleton \( d\varepsilon_v' \) and plastic saturation \( dS_v' \) will affect the isotropic hardening parameters. The change for \( m \) easygoing with the plastic volume strain \( d\varepsilon_v' \) can be defined as \( d m / d\varepsilon_v' = c_v (1 + e_v) \) [3], and following the change of the irreversible water content \( dS_v' \) is defined as \( d m / dS_v' = c_v (n s S_v / p_a) \). In general, following the changes of \( d\varepsilon_v' \) with \( dS_v' \), \( m \) can be defined as the sum of these two terms, namely

\[ d m = c_v (1 + e_v) d\varepsilon_v' + c_u (n s S_v / p_a) dS_v' \]  

(14)

Here \( e_v \) is the initial void ratio, \( c_v, c_u, c \) are all model constants. The hardening effect induced by suction is considered by the upper formula.

**Liquid phase hardening**

For the hardening of liquid phase, the influence of solid deformation on soil-water characteristic curve is usually considered, and the soil-water characteristic curve characterizes the change of the liquid phase, so the hardening equation of the liquid phase is adopted with the same form as in literature [9]

\[ \frac{d\bar{s}_s}{s_s} = -\Gamma_{p'} dS_v' + \Gamma_{p} d\varepsilon_v', \Gamma_{p'} = \frac{\kappa_{w}(1 + e_v)}{\lambda - k} \]  

(15)

In this case, \( \bar{s}_s \) denotes the liquid drying or humidifying modified suction, \( \lambda \) \( k \) for the slope of normal consolidation line and rebound line of \( \nu\ln p' \), \( \nu = 1 + e \), for liquid phase plastic modulus \( \Gamma_{p'} = 1/(\lambda - k) \), \( \Gamma_{p} \) indicates the effect of the solid on the liquid phase, \( \kappa_{w} \) shows the coupling coefficient reflecting the amplitude of the movement of the liquid phase yield surface when the solid phase yields [5] which control the position of the intersection of the two yield surfaces in the stress space. According to the article, which is usually assumed to be constant and can be determined experimentally or empirically. According to the above description, the change of air intake value with volumetric strain is

\[ \frac{d\bar{s}_s}{s_s} = \Gamma_{p'} d\varepsilon_v' = \kappa_{w}(1 + e_v) \ln p'/p_0 \]  

(16)

Among them, \( \bar{s}_s \) denotes air intake value of the material, \( p' \) and \( p_0 \) are respectively corresponding to the effective stresses for different air intake values.
2.4 Consistency conditions and plastic modulus

Consistency conditions for solid phase

In order to ensure the stress still on the yield surface during the plastic loading process, the yield surface should meet the uniform conditions, \( df = 0 \), and the following formula is obtained:

\[
\frac{\partial f}{\partial \sigma} : d\sigma' + \frac{\partial f}{\partial \alpha} : \frac{\partial \sigma}{\partial \alpha} + \frac{\partial f}{\partial m} : D = \frac{\partial f}{\partial \alpha} : \frac{\partial h(\alpha^0 - \alpha)}{\partial \alpha} (L) + \frac{\partial f}{\partial \sigma} D(q) + \frac{\partial f}{\partial \sigma} : \frac{\partial S^p}{\partial \sigma^p} dS^p
\]

\[= \frac{\partial f}{\partial \sigma} : d\sigma' - K_p < L > - K_p dS^p = 0 \tag{17} \]

Here \( K_p, K_p \) represent two kinds of plastic modulus of two mechanical mechanisms (hydraulic, mechanical) related to the yield surface. \( K_p \) is the plastic modulus of the soil skeleton, \( K_p \) is a new plastic modulus indicating the effect of hydraulic mechanism on the yield surface of the solid phase. It can be clearly seen that the loading index \( L \) is affected by the net stress and suction. According to plastic theory, the strain tensor of the soil skeleton is

\[d\epsilon^p = \gamma \leq L > \gamma \leq n > D : d\epsilon^p \leq \gamma \leq L > n: d\epsilon^p \leq \gamma L > D \tag{18} \]

Integrated with equation (17), we can obtain

\[\frac{\partial f}{\partial S} = \frac{\partial f}{\partial S} + \frac{\partial f}{\partial p} : \frac{\partial \sigma}{\partial p} = \frac{\partial f}{\partial \sigma} : \frac{\partial \sigma}{\partial \sigma} \frac{\partial \sigma}{\partial \sigma} + \frac{\partial f}{\partial \sigma} : \frac{\partial S^p}{\partial \sigma^p} \frac{\partial S^p}{\partial \sigma^p} \tag{19} \]

Based on the above equation, we get

\[L = \frac{1}{K_p} \left( \frac{\partial f}{\partial S} : dS + \frac{\partial f}{\partial p} : d\sigma \right) \frac{K_p}{K_p} \frac{dS^p}{\partial \sigma^p} \tag{20} \]

\[K_p = p^h \left( \frac{\alpha}{3} - \frac{\alpha^0}{3} \right) + \sqrt{\frac{2}{3} m_c (1 + e_0) D_h (\alpha^0 - \frac{\alpha}{2})} \tag{21} \]

From formula (21), it can be seen that the plastic modulus \( K_p \) depends on the distance of stress ratio \( (\frac{\alpha^0}{3} - \frac{\alpha}{3} : n) \) and \( (\frac{\alpha^0}{3} - \frac{\alpha}{3} : n) \), the sign of \( K_p \) is completely determined by \( (\frac{\alpha^0}{3} - \frac{\alpha}{3} : n) \) with \( (\frac{\alpha^0}{3} - \frac{\alpha}{3} : n) \). If the isotropic hardening of the yield surface is neglected (for dense sand, \( c_s \) is very small and can be set to a constant value, i.e. \( c_s = 0 \)), Then the plastic modulus will be a very simple form \( K_p = p^h (\frac{\alpha^0}{3} - \frac{\alpha}{3} : n) \).

Consistency conditions on liquid phase

It also accords with the classical plastic theory for the liquid phase, that is, the liquid yield surface function \( f_w = 0 \) which meet the consistency conditions, i.e. \( df_w = 0 \)

\[df_w = \frac{\partial f}{\partial S} : dS + \frac{\partial f}{\partial S} : dS^p + \frac{\partial f}{\partial S} : dS^p = 0 \tag{22} \]

Integrated with formula (15) can get

\[\frac{\partial S}{\partial \sigma} = \Gamma \frac{\partial S}{\partial \sigma} \frac{\partial S}{\partial \sigma} = - \Gamma \frac{\partial S}{\partial \sigma} \tag{23} \]

Combined with formula (17), (22), (23) and available
\[ K_p(L) + K_p' dS_p' = \frac{\partial f}{\partial \sigma'} : d\sigma' + \frac{\partial f}{\partial \sigma} : \tilde{\sigma} + D(L) + \frac{\partial f}{\partial \tilde{\sigma}} : \tilde{\sigma} = \frac{\partial f}{\partial \tilde{\sigma}} = \frac{\partial f}{\partial \tilde{s}} \]  

(24)

Also:

\[ \frac{\partial f}{\partial s} = 1, \quad \frac{\partial f}{\partial \tilde{s}} = -1, \quad \frac{\partial \tilde{s}}{\partial \rho'} = \Gamma_p, \quad \frac{\partial \tilde{s}}{\partial s} = -\Gamma_p \]  

(25)

(25) Bring formula (25) into formula (24) and simplify it:

\[ K_p(L) + K_p' dS_p' = \frac{\partial f}{\partial \sigma'} : d\sigma' + \tilde{\sigma} = \tilde{s} \]  

(26)

Combined with the above equation, the plastic increment of liquid phase is as follows:

\[ dS_p' = \frac{\Gamma_p D}{\Pi_p} : \frac{\partial \sigma'}{\partial \sigma} : d\sigma' - \frac{K_p}{s, \Pi_p} ds = \Pi_p = K_p \Gamma_p D + K_p \Gamma_p \]  

(27)

Substituting equation (27) into equation (24), then solid-phase loading index \( L \) can be expressed as

\[ L = \frac{B}{K_p} \frac{\partial f}{\partial \sigma'} : d\sigma' + C d\tilde{s} \]  

(28)

Here \( B = 1 - K_p \Gamma_p D / \Pi_p, C = K_p / s, \Pi_p \).

So integrated with formula (18), the plastic strain increment of the soil skeleton can be obtained as

\[ d\epsilon_p = (L) R = \frac{BR}{K_p} \frac{\partial f}{\partial \sigma'} : d\sigma' + CRds = \frac{B}{K_p} \frac{1}{3} (n + 1) D)I : \frac{\partial f}{\partial \sigma'} : d\sigma' + C(n + 1) D)I d\tilde{s} \]

\[ = \frac{B}{K_p} - \frac{n}{3} \frac{\partial f}{\partial \sigma'} : d\sigma' + Cn + \frac{1}{3} DB \frac{1}{3} K_p \frac{\partial f}{\partial \sigma'} : d\sigma' + 1 \frac{C}{3} D)I d\tilde{s} \]

\[ = \left( \frac{B}{K_p} \right) \frac{\partial f}{\partial \sigma'} : d\sigma' + C n + \frac{B}{3} \frac{\partial f}{\partial \sigma'} : d\sigma' + C d\tilde{s} \) \]  

(29)

2.5 Constitutive Equation

To sum up, the strain increment of unsaturated soil is as follows:

\[ d\epsilon = d\epsilon_p + d\epsilon_c \]

\[ d\epsilon = d\epsilon_p + d\epsilon_c \]

(30)

The strain increment relationship of soil skeleton is as follows

\[ d\epsilon = d\epsilon_p + d\epsilon_c \]

\[ d\epsilon = d\epsilon_p + d\epsilon_c \]

(31)

The liquid phase increment relationship is

\[ dS_p = dS_p' + dS_p'' \]

\[ dS_p = dS_p' + dS_p'' \]

(32)

In order to make the description simple, we will transform the general stress space to give the constitutive relationship of each phases for unsaturated sand in the triaxial stress space, and express it as a matrix.
Observed from formula (33), when saturation $S_s = 100\%$, the constitutive relation of saturated sand or dry soil, respectively, realizes the smooth transition of unsaturated, saturated and dry soil.

In practical engineering applications, it is generally known that the external total stress, water pressure and air pressure, so it is necessary to transform the stress variables, according to the definition of effective stress and modified suction, we can get

$$
\begin{align*}
\{d\varepsilon\} &= \begin{bmatrix}
\frac{1}{K + K_r} DB \frac{\partial f}{\partial p'} + DB \frac{\partial f}{\partial q} CD \\
\frac{1}{K_r} \frac{\partial p'}{\partial q} + \frac{1}{C} & 3G & \frac{1}{K_r} \frac{\partial q}{\partial q} \\
\frac{1}{3G} & \frac{1}{K_r} & \frac{1}{C} \\
\frac{1}{\Pi_r} & \frac{1}{\Pi_r} & \frac{1}{\Pi_s} \\
\end{bmatrix}
\begin{bmatrix}
\{dp\}'
\{dq\}'
\{ds\}'
\end{bmatrix}
\end{align*}
$$

(33)

By substituting formula (34) into (33), the stress increment and strain variable in the equation become the function of the stress increment $dp$, $dq$, $du_a$, $du_u$ respectively. The atmospheric pressure is usually kept constant $u_a$ in the test, and only tree other quantities can be directly controlled in the triaxial test. Therefore, as long as the stress increment is known, the corresponding strain increment can be obtained by using the formula (33).

3. Model validation

3.1 Determining of model parameters

For medium or dense sands, dilatancy is its most significant feature, but difficult to measure its critical state. Then another aspect of dilatancy "phase transition " is considered to build the soil skeleton part of the model. There are in all 15 parameters and all relevant parameters are as follows:

Elastic constant: $G$, $\nu$; Phase transition state parameters: $M_{\phi}$, $k_w$, $k_s$, $\gamma$; Hardening parameters: $h_0$, $m$, $c_\phi$, $c_w$, $c$; Dilatancy parameters: $D_0$; SWCC parameters: $\lambda_w$, $k_w$, $\kappa_{sw}$.

Initial value $m$ and coefficient $c_\phi$ are related to isotropic hardening. $c_\phi = 0$ means the yield surface size is independent of the plastic volume strain of the soil skeleton. In general, the condition for saturated sand is satisfied for the constant $m$, but for unsaturated sand, the coupling between the solid phase and the liquid phase needs to be considered. The corresponding isotropic yield surface also changes as $c_{sw}$, $c$ change. However, from the triaxial drainage test of unsaturated sand, the effect of suction on the yield surface is not obvious.

Determination for SWCC model parameters $\lambda_w$, $k_w$, $\kappa_{sw}$. They are readily available from any drying or wetting process. This paper uses bilinear soil-water characteristic curve model. The soil-water characteristic curve is simple for silty dense sand, and the air intake value is very small which can be easily determined. $\lambda_w$, $k_w$, $\kappa_{sw}$ are the coupling parameters indicating the effect of soil skeleton deformation on the soil-water characteristic curve, and at least two sets of soil-water characteristic curves under different confining pressures are required to determine them.

3.2 Test data

To verify the validity of the proposed model, ASU silt and dense sand (data from Natalia Perez (2006) [1] -ASU sand) is used as the soil sample in this paper. According to ASTM standard, ASU sand
belongs to dense clayless silt sand, and with a specific gravity of 2.71, \( c' \) with \( \phi \) are 2.3kPa and 36.1°, respectively, it contains 67% sandy soil, 19.6% silt, 7.4% clay and maximum dry density 19.5kN/m³, optimal water content 10.5%, initial porosity ratio \( e_0 = 0.44 - 0.46 \).

Test results \(^1\) show that the soil-water characteristic curve is not sensitive to changing confining pressures, so the bilinear soil-water characteristic curve model shown in Figure 3 is adopted.

![Fig. 3 SWCC Schematic illustration of ASU sands](image)

| Table 1 Experimental data for ASU sands |
|----------------------------------------|
| sample | davg cm | havg cm | Sample weight | W% | \( \gamma_d \) | \( m_s \) | \( m_w \) | \( \nu \) | Sample weight | \( \Delta w\% \) | \( \rho \) | \( e_0 \) | w% |
| 75-125 | 10.16 | 22.3 | 3731.98 | 11.56 | 1.8 | 3342.97 | 389.00 | 1807.93 | 3648.05 | 0.091257 | 2.017 | 0.465 | 0.0893 |
| 250-250 | 10.15 | 22.4 | 3733.16 | 11.64 | 1.8 | 3340.49 | 392.66 | 1800.82 | 3648.13 | 0.092093 | 2.025 | 0.460 | 0.0871 |
| 75-360 | 10.11 | 22 | 3736.32 | 11.53 | 1.8 | 3334.46 | 401.85 | 1772.01 | 3602.49 | 0.080379 | 2.032 | 0.440 | 0.0769 |
| 250-355 | 10.12 | 22.2 | 3731.6 | 11.16 | 1.8 | 3341.18 | 390.41 | 1799.82 | 3601.37 | 0.077871 | 2.000 | 0.450 | 0.0789 |
| 75-23 | 10.12 | 22.2 | 3732.97 | 11.3 | 1.8 | 3353.59 | 379.37 | 1816.04 | 3693.23 | 0.104904 | 2.048 | 0.461 | 0.1038 |
| 250-25 | 10.13 | 22.4 | 3732.94 | 11.55 | 1.8 | 3345.38 | 387.55 | 1804.37 | 3696.33 | 0.104904 | 2.048 | 0.461 | 0.1038 |

Seen from Table 1 that water content is only slightly changed in the triaxial test under the same initial conditions. For the suction 95-135 group, the maximum change in water content is 0.1%; For the suction in the 320-355 group, the maximum change of water content is 0.1%. According to the water content formula \( \epsilon_S = wG_s \), the effect of the whole shear strain on the movement of the liquid yield surface is not obvious.

3.3 Model parameter determination

All model parameters are determined as follows. Analyzed from the table2, when \( \sigma - u_s = 7.5kPa, 250kPa, k_d, k_s, \gamma \) is very close and almost the same (except for very small suction) which can more accurately reflect the characteristics of the phase change point and peak point of dense sand, and it is concluded that the phase transition stress ratio increases with the increase of suction at the same confining pressure. When the suction is more than 300kPa, the change of the phase stress ratio is very small, indicating that the suction has a certain contribution to the confining pressure.
When the suction exceeds a certain value, the contribution of the suction to the normal stress is weakened.

Table 2 Model parameters

| elastic constants | hardening parameters | SWCC parameters |
|-------------------|----------------------|----------------|
| $G_0$             | $\nu$                | $m$ $c_s$ $c_u$ $C$ | $\lambda_w$ $k_w$ $\kappa_{sw}$ |
| 125               | 0.15                 | 0.01 0.5 10 | 0.1 0.074 0.1 |

Table 3 Model parameters under different confining pressure

| $(\sigma_{1}-u_r)$~$x$(kPa) | $k_d$ | $D_i$ | $k_h$ | $\gamma$ | $h_i$ | $M_{pl}$ |
|-----------------------------|-------|-------|-------|----------|-------|--------|
| 75-23                       | 0.9   | 1.55  | 3.5   | 0.89     | 2.5   | 1.35   |
| 75-125                      | -3.5  | 1.55  | 3.5   | 0.89     | 3.0   | 1.40   |
| 75-360                      | -3.5  | 1.55  | 3.5   | 0.89     | 5.5   | 1.44   |
| 75-600                      | -3.5  | 1.55  | 3.5   | 0.89     | 5.9   | 1.45   |
| 250-25                      | -3.5  | 1.55  | 8.0   | 0.96     | 2.0   | 1.48   |
| 250-95                      | -1.5  | 1.55  | 8.0   | 0.96     | 1.75  | 1.49   |
| 250-353                     | -1.5  | 1.55  | 8.0   | 0.92     | 1.25  | 1.49   |

3.4 Comparison of calculation results and test results

A comparison between the experimental and simulated results of shear stress and volumetric strain with axial strain under two confining pressures of 75,250 kPa and different suction forces are shown in Figure 4-5 (see below). The results show that the two have a good fit and the model in this paper can well describe the stress-strain behavior of silty dense sand, and capture the salient characteristics of its phase transition and peak. Seen from the results for silty sand, from the beginning of shearing, just like saturated dense sand, it is always compressed first to attain the phase transition point. When the confining pressure is small, such as 75kPa in this article, it is quickly to reach the phase transition point (the corresponding axial strain is 3%, 2%), but when the confining pressure is 250kPa, the phase strain will only occur at 8% and 6% respectively. During the process of shearing, the soil sample further dilates until to the maximum dilatancy value, that is the peak strength after which the soil sample will enter the softening stage until it breaks. The results also clearly indicate that dilatancy increases with increasing suction and accompanies volume increment at a given confining pressure, and the shear strength also increases. Furthermore, the larger the suction, the easier it is to reach the phase transition, and dilatancy is very obvious under low stress and high suction. For silty sand belongs to clayless soil, then the coupling effect between solid and liquid is not obvious that can be explained by their soil-water characteristic curve, that is to say, its air intake value has remained almost unchanged, all staying at 2kPa under different confining pressures (75, 250Pa), and only a slight change in water content occurs when the suction is kept constant throughout the shearing process.

4. Summary

For unsaturated sand, under low confining pressure, due to the existence of suction, silty dense sand has a large volume change during shearing process and obvious dilatancy and peak characteristics appear throughout the stress-strain behavior, and is often destroyed before reaching the critical state. Then based on the phase transition state and considering the hydraulic-mechanical coupling, model parameters are introduced in this paper to establish the boundary surface of unsaturated sand in general stress space and triaxial stress space. Analyzed from the experimental data, it is shown that for unsaturated dense sand or clayless soil, the coupling has little effect on its stress-strain properties, but this is a supplement and improvement to the constitutive model.
Fig. 4 Shear stress, volumetric strain versus axial strain for ASU sands with $\sigma_3 - u_a = 75kPa$, $s = 23,125kPa$

Fig. 5 Shear stress, volumetric strain versus axial strain for ASU sands with $\sigma_3 - u_a = 250kPa$, $s = 25,95kPa$

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