Supermassive Black Holes with High Accretion Rates in Active Galactic Nuclei. VIII. Structure of the Broad-line Region and Mass of the Central Black Hole in Mrk 142

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Abstract

This is the eighth in a series of papers reporting on a large reverberation mapping (RM) campaign to measure black hole (BH) mass in active galactic nuclei with high accretion rates. We employ the recently developed dynamical modeling approach for broad-line regions (BLRs) based on the method of Pancoast et al. to analyze the RM data set of Mrk 142 observed in the first monitoring season. In this approach, continuum variations are reconstructed using a damped random walk process, and BLR structure is delineated using a flexible disk-like geometry, in which BLR clouds move around the central BH with Keplerian orbits or inflow/outflow motion. The approach also includes the possibilities of anisotropic emission from BLR clouds, nonlinear response of the line emission to the continuum, and different long-term trends in the continuum and emission-line variations. We implement the approach in a Bayesian framework that is apt for parallel computation and use a Markov chain Monte Carlo technique to recover the parameters and uncertainties for the modeling, including the mass of the central BH. We apply three BLR models with different prescriptions of BLR cloud distributions and find that the best model for fitting the data of Mrk 142 is a two-zone BLR model, consistent with the theoretical BLR model surrounding slim accretion disks. The best model yields a BH mass of \( M_\odot \) of \( \log(M_\odot / M_\odot) = 6.23^{+0.26}_{-0.20} \), resulting in a virial factor of \( f = -0.36^{+0.33}_{-0.25} \) for the full width at half maximum of the H\( \beta \) line measured from the mean spectrum. The virial factors for the other measures of the H\( \beta \) line width are also presented.

Key words: black hole physics – galaxies: active – galaxies: individual (Mrk 142) – quasars: general

1. Introduction

Broad emission lines with widths of several thousands of kilometers per second are a hallmark feature of the spectra of active galactic nuclei (AGNs). The basic photoionization theory and the long-known fact that both the lines and continuum emissions of AGNs vary on quite short timescales, ranging from days to months (e.g., Burbidge 1967; Cromwell & Weymann 1970), spurred the proposal of the widely used technique of "reverberation mapping" (RM) by Blandford & McKee (1982). The principle underlying RM is that the broad-line region (BLR) is photoionized by the central ionizing continuum arising from the accretion disk and reproduces broad emission lines. The temporal behaviors of emission lines are thereby blurred echoes of continuum variations with light-crossing delays. The responses from different parts of the BLR have different time delays and Doppler shifting velocities due to the motion of BLR gas in the gravitational potential well of the central black hole (BH). Thus, by appropriately analyzing the varying properties of continuum and emission lines, one can place constraints on the geometry and kinematics of the BLR as well as the BH mass.

Owing to the insufficient cadences or signal-to-noise ratios of spectroscopic data, the major goals of most early RM experiments have largely focused on measuring characteristic time lags between the variations of emission lines and continuum, which correspond to the light-travel distance from the continuum source to the line-emitting gas. The measured time lags are thereby used to deduce the sizes of BLRs (e.g., Peterson 1993). The recognition that emission-line lags and line widths follow the "virial relationship" makes the RM technique a promising method for measuring BH mass in AGNs (e.g., Wandel 1997; Ho 1999; Peterson & Wandel 1999; Peterson et al. 2004). Specifically, based on the virial theorem, virial BH mass measurements are generally derived by combining the emission-line lags (\( \tau \)) and line widths (\( \Delta V \)) using a simple recipe (e.g., Peterson et al. 2004):

\[
M_{\text{vir}} = \frac{f \epsilon \tau (\Delta V)^2}{G},
\]

where \( \epsilon \) is the speed of light and \( G \) is the gravitational constant. To connect this virial mass with true BH mass, a virial factor \( f \) has to be invoked on account of our ignorance of the geometry and kinematics of BLRs. The virial factor is practically
calibrated by comparing RM AGNs with measured stellar velocity dispersion in the bulge against the well established $M_{\star} - \sigma_*$ relation of local quiescent galaxies (e.g., Onken et al. 2004; Ho & Kim 2014). Clearly, the virial factor determined in this way applies in a statistical sense and is subject to the intrinsic scatter of the $M_{\star} - \sigma_*$ relation, which is about 0.3 dex (e.g., Kormendy & Ho 2013). It is unknown yet whether the virial factor has a common value for various AGN populations.

Given the complicated structures of BLRs inferred from the previous velocity-binned RMs (e.g., Bentz et al. 2010; Denney et al. 2010; Grier et al. 2013b; Du et al. 2016a, hereafter Paper VI), the virial factor is most likely to vary from object to object. Recent recalibration of the virial factor by Ho & Kim (2014) using the comprehensively revised $M_{\star} - \sigma_*$ relation of Kormendy & Ho (2013) indeed showed that the virial factor seems to depend on the bulge type (classical or pseudo) of the host galaxies. Therefore, invoking of the virial factor in the traditional RM approach actually impedes by itself a further improvement in BH mass measurements (Krolik 2001).

The way around this weakness of the traditional RM approach is, similar to the BH mass measurements in quiescent galaxies through stellar or gas dynamics, to develop feasible dynamical models for BLRs and analyze velocity-resolved RM data to determine the BH mass self-consistently without the need to invoke the virial factor. Such an approach dates back to the early work of Bottorff et al. (1997), who applied the outflow model of Emmering et al. (1992) to the RM database of the well-monitored Seyfert galaxy NGC 5548, in an attempt to constrain the central BH mass and the BLR dynamics. With the aid of the development of the mathematical description of AGN variability (Kelly et al. 2009), Pancoast et al. (2011) constructed a Bayesian framework with a flexible BLR dynamical model for RM data analysis. In this new approach, BH mass and other BLR parameters (e.g., inclination angle and opening angle) are fully determined by comparing the model predictions with the observed time series of continuum and broad emission lines. Pancoast et al. (2014a) subsequently reinforced their previous framework by incorporating more complicated phenomenological treatments of the anisotropy of BLR emissions and inflows and outflows. Based on the model of Pancoast et al. (2011), Li et al. (2013) also carried out an independent implementation that, additionally, includes the nonlinear response of emission lines to the ionizing continuum. Although the dynamical modeling approach is still at an early stage, the application to several RM AGNs shows its remarkable capability for understanding BLR dynamics and measuring BH mass (Brewer et al. 2011b; Pancoast et al. 2012, 2014b, 2018; Grier et al. 2017; Williams et al. 2018).

In 2012, we began a large RM observational project using the Lijiang 2.4 m telescope at Yunnan Observatories, aiming at monitoring a sample of selected candidates for AGNs with high accretion rates (hereafter dubbed super-Eddington accreting massive BHs; SEAMBHs) with good cadences for measuring BH mass reliably and studying BLR physics. In particular, based on the model of a slim accretion disk that describes BH accretion at high accretion rates, Wang et al. (2014b) demonstrated that the geometrically thick funnel of an inner slim disk produces an anisotropic radiation field, which divides the surrounding BLR into two regions with distinct incident ionizing photon fluxes. Such BLR structures are different from those of sub-Eddington accreting BHs, which are powered by standard geometrically thin accretion disks (Shakura & Sunyaev 1973). By combining with the previous RM sample that mainly consists of sub-Eddington accreting AGNs (Bentz et al. 2013), this RM project provides important complementary objects to generate a homogeneous sample with a broad range of accretion rates.

The project has continued uninterrupted for six years and is still ongoing. In the first monitoring season (2012–2013), nine objects were finally verified to show statistically significant H$\beta$ lags through cross-correlation analysis. The resulting data sets and time-lag analysis between the optical continuum and H$\beta$ emission line and FeII emission have been reported in papers of this series by Du et al. (2014, Paper I), Wang et al. (2014a, Paper II), Hu et al. (2015, Paper III), and Xiao et al. (2018, Paper VII). The data sets and time-lag measurements from the other monitoring seasons have also been reported in papers of this series by Du et al. (2015, Paper IV; 2016b, Paper V; 2018, Paper IX). We make use of the spectroscopic data sets for the nine objects in the first monitoring season and employ our developed dynamical modeling for BLRs to study the structure and dynamics of the H$\beta$ BLRs and derive the BH mass for these nine objects. This paper reports the first application to Mrk 142, which has the highest data quality among the nine objects.

The paper is organized as follows. We briefly describe the properties of observation data for Mrk 142 in Section 2. Section 3 describes the method for reconstruction of the continuum light curve and Section 4 presents the methodology for BLR dynamical modeling and Section 5 the Bayesian framework for inferring the model parameters. Section 6 summarizes the results from the dynamical modeling including the obtained BH mass, and also compares our results with these from the cross-correlation analysis. The discussions and conclusions are given in Sections 7 and 8, respectively.

### 2. Observation Data

Mrk 142 was spectroscopically and photometrically monitored between 2012 October and 2013 June. The details of the observations and data reduction and analysis were presented in Papers I and III. All the spectra were taken by simultaneously placing a nearby comparison star in the slit to achieve high-accuracy flux calibration. However, since the host galaxies are extended and resolved, this observing strategy can result in apparent flux variations of the host galaxy contamination due to variable seeing and miscentering. A spectral decomposition scheme as described below helps to alleviate this effect (see Appendix A of Paper III).

To isolate the AGN continuum and H$\beta$ line, a spectral decomposition was performed by including a featureless AGN power-law continuum, FeII blends, various emission lines (H$\beta$, [O III], HeII, HeI, and several coronal lines), and a host galaxy template (see Paper III for details). The FeII blends were fitted by the template from Boroson & Green (1992) and the host galaxy template was chosen to be a single stellar population model with an instantaneous burst of 11 Gyr and a metallicity of $Z = 0.05$. The AGN continuum flux was measured as the flux at 5100 Å from the decomposed featureless power-law component. The H$\beta$ line profile is obtained by subtracting the above-listed components from each spectrum. The narrow H$\beta$ line component, which originates from the narrow-line region, is not subtracted for two reasons: first, Mrk 142 has fairly weak narrow [O III] lines, indicating that the narrow H$\beta$ lines are also...
weak; second, the spectral resolution is about 500 km s⁻¹ in terms of the FWHM, too low to reliably decompose the narrow component (Paper III). Figure 1 shows an example of decomposition of a spectrum from a single night for Mrk 142. Paper III also presented the Hβ lag with respect to the 5100 Å continuum through the cross-correlation method. Table 1 lists the overall properties of the observation data of Mrk 142.

Three points merit emphasis regarding the data sets used for our dynamical modeling. First, the spectra are aligned using [O III] λ5007 as the wavelength reference to correct the night-to-night wavelength shifts. Second, the wavelength range of the Hβ line is adopted to be about three times the FWHM. Third, by comparing the observed spectrum of the comparison stars with stellar templates, Paper VI determined the spectral resolution of the spectra at each individual epoch. We use the mean values for Mrk 142 as the input of the instrumental resolution to our dynamical modeling procedure (see below).

To correct for the redshift effect, we reduce the observation dates of the series by a factor (1 + z), where z = 0.0449 is the redshift of Mrk 142. Since the analysis depends only on the time differences between data points rather than the absolute times, this manipulation equivalently converts the time series to the rest frame.

### 3. Continuum Modeling

We use the damped random walk (DRW) model to describe the variability of the continuum fluxes (e.g., Kelly et al. 2009; Zu et al. 2013 and references therein), which allows us to interpolate and extrapolate the continuum light curve in a statistical way (Pancoast et al. 2011). In the DRW model, the covariance function between any two points at times t₁ and t₂ is given by

\[
S(t_1, t_2) = \sigma_d^2 \exp \left( -\frac{|t_1 - t_2|}{\tau_d} \right),
\]

where \( \sigma_d \) is the long-term standard deviation of the variation and \( \tau_d \) is the typical timescale of variation. In this prescription, the variation of light curves on a short timescale (\( t \ll \tau_d \)) is \( \sigma_d \sqrt{t/2\tau_d} \). To relax the correlation between \( \sigma_d \) and \( \tau_d \), a new parameterization of \( \tilde{\sigma}_d = \sigma_d / \sqrt{\tau_d} \) is used to replace \( \sigma_d \).

Let \( s \) denote the real underlying signal of the continuum variations to be inferred from observations. A set of measurements \( y \) for a continuum light curve in a monitoring campaign can be written as

\[
y_c = s + Lq + n_c,
\]

where \( n_c \) represents the measurement noises and the term \( Lq \) represents a linearly varying trend in the light curve. Here \( L \) is a matrix of known coefficients and \( q \) is a vector of unknown linear coefficients (see Rybicki & Press 1992 for details). Assuming that the measurement noise \( n_c \) is Gaussian and uncorrelated, the likelihood probability for \( y \) is (Rybicki & Press 1992; Li et al. 2013)

\[
P(y_c | \tau_0, \tau_d, q) = \frac{1}{\sqrt{(2\pi)^m|C|}} \times \exp \left( -\frac{(y_c - Lq)^T C^{-1} (y_c - Lq)}{2} \right),
\]

where superscript “T” denotes transposition, \( m \) is the number of data points, \( C = S + N \), \( S \) is the covariance matrix of \( s \) given by Equation (2), and \( N \) is the covariance matrix of \( n_c \). In our calculations, we include by default the zero-order linear trend. In this case, \( L \) is a vector with all unity elements and \( q \) is the long-term mean value of the light curve. This helps to remove the bias in modeling the light curve at epochs far from any data points (see discussion in Rybicki & Press 1992).
Given parameters \((\sigma_a, \tau_a, \mathbf{q})\), the probability of a signal \(s\) underlying a set of measurements \(\mathbf{y}\) is (e.g., Rybicki & Press 1992; Zu et al. 2011)

\[
P(s|\mathbf{y}) \propto \exp \left( -\frac{(s - \tilde{s})^T \mathbf{Q}^{-1}(s - \tilde{s})}{2} - \frac{(\mathbf{q} - \tilde{\mathbf{q}})^T \mathbf{C}_q^{-1}(\mathbf{q} - \tilde{\mathbf{q}})}{2} - \frac{(\mathbf{y} - L\mathbf{q})^T \mathbf{C}_L^{-1}(\mathbf{y} - L\tilde{\mathbf{q}})}{2} \right),
\]

(5)

where

\[
\begin{align*}
\tilde{s} &= \mathbf{S}^{-1}(\mathbf{y} - L\mathbf{q}), \\
\tilde{\mathbf{q}} &= \mathbf{C}_q L^T \mathbf{C}^{-1} \mathbf{y}, \\
\mathbf{C}_q &= (L^T \mathbf{C}^{-1} L)^{-1}, \\
\mathbf{Q} &= (S^{-1} + N^{-1})^{-1}.
\end{align*}
\]

Equation (5) indicates that the signal \(s\) is a Gaussian process with mean \(\tilde{s}\) and covariance matrix \(\mathbf{Q}\) (Rybicki & Press 1992). One can thereby generate a signal \(s\) by adding to \(\tilde{s}\) a Gaussian process with zero mean and covariance matrix \(\mathbf{Q}\) given by Equation (9). Similarly, the probability of \(q\) is a Gaussian with mean \(\tilde{q}\) and covariance matrix \(\mathbf{C}_q\). As a result, a typical realization for the observed continuum light curve is given by

\[
\tilde{\mathbf{y}} = (\mathbf{u}_s + \tilde{s}) + L(\mathbf{u}_q + \tilde{\mathbf{q}}),
\]

(10)

where \(\mathbf{u}_s\) and \(\mathbf{u}_q\) are Gaussian processes with zero mean and covariance matrices \(\mathbf{Q}\) and \(\mathbf{C}_q\), respectively. Note that given \((\sigma_a, \tau_a, \mathbf{q})\), \(\tilde{s}\) and \(\tilde{\mathbf{q}}\) are uniquely determined by Equations (6) and (7). In the following analysis, we use \(\mathbf{u}_s\) and \(\mathbf{u}_q\) as free parameters, which are further constrained by additional measured data on broad emission lines.

4. BLR Modeling

As mentioned above, Pancoast et al. (2011) developed a Bayesian approach for BLR dynamical modeling, which was first applied to the RM data of Arp 151 (Brewer et al. 2011b) and Mrk 50 (Pancoast et al. 2012). Li et al. (2013) carried out an independent implementation of this approach by additionally including the nonlinear response of the emission lines to the continuum and detrending of light curves. Pancoast et al. (2014a) further improved their BLR model by including more complicated treatments on anisotropy of line emissions and on cloud kinematics so as to generate highly asymmetric line profiles. This improved approach was subsequently applied to several RM objects by Pancoast et al. (2014b, 2018), Grier et al. (2017), and Williams et al. (2018). The BLR models used in this paper are based on Pancoast et al. (2014a) and Li et al. (2013), but with several new modifications. Here, for the sake of completeness, we list all the essential details.

The basic scenario of BLR dynamical modeling is that BLRs are composed of a large number of discrete, point-like clouds, which orbit around the central BH (e.g., Netzer 1990). These clouds are exposed to the central ionizing source and instantaneously re-radiate emission lines by absorbing the ionizing continuum. Owing to the lack of UV/X-ray monitoring data, we use 5100 Å fluxes as a surrogate for the ionizing continuum. This may lead to the nonlinear response of BLR cloud emission (Gaskell & Sparke 1986; Goad & Korista 2014). To avoid confusion, hereafter we use the term “particles” to represent units of emissions from these BLR clouds. The effect of inverse square decline of the incident continuum flux density is assumed to be implicitly included in the radial distribution of BLR particles. Below we construct three BLR models \(M1, M2, \) and \(M3\). Model \(M1\) is the same as that of Pancoast et al. (2014a). \(M2\) differs from \(M1\) in the prescription of the radial distributions of BLR particles. \(M3\) is a two-zone model motivated by the theoretical model of Wang et al. (2014b). Throughout the calculations, a spherical coordinate frame \((r, \theta, \varphi)\) is used and the BH is placed at the origin.

4.1. Geometry

The distribution of BLR particles is assumed to be axisymmetric and follows a flexible disk-like geometry, which can yield a variety of shapes with suitable parameters, including shells, spheres, and rings. The BLR has an inclination angle \(\theta_{inc}\) to the observer, which is defined by the angle between the line of sight and the axis of symmetry of the BLR. The BLR particles subtend an opening angle \(\theta_{opn}\), which is defined by \(\theta_{opn} = \pi/2\) for a spherical BLR and \(\theta_{opn} = 0\) for an infinitely thin disk-like BLR (see the schematic Figure 1 of Li et al. 2013). Within the opening angle, particles are distributed uniformly in the \(\varphi\)-direction. In the \(\theta\)-direction, particles are distributed with a prescription (Pancoast et al. 2014a)

\[
\theta = \cos^{-1}[\cos \theta_{opn} + (1 - \cos \theta_{opn}) \times U^{\gamma}],
\]

(11)

where \(U\) is a random number from a uniform distribution between 0 and 1 and \(\gamma\) is a free parameter that controls the extent to which particles are clustered over the outer face of the BLR disk.

We use two types of radial distribution for BLR particles as follows:

1. \(M1\). The radial distribution is parameterized by a gamma distribution, the same as in Pancoast et al. (2014a). Specifically, the radial location of a particle is assigned by

\[
r = F\mu + (1 - F)\mathcal{R},
\]

(12)

where \(\mathcal{R}\) is a random number drawn from the gamma distribution with a mean \(\mu\) and a standard deviation \(\beta\mu\), and \(F\) is a fraction to account for the possibility that within an inner edge \((F\mu)\), clouds are completely ionized so that they do not reverberate to the continuum.

2. \(M2\). The radial distribution is parameterized by a double power law (Stern et al. 2015) as

\[
f(r) \propto \begin{cases} \ r^\alpha, \ & \text{for } F_{in} \leq r/R_0 \leq 1, \\ 1/r, \ & \text{for } 1 \leq r/R_0 \leq F_{out}, \end{cases}
\]

(13)

where \(\alpha\) is the slope of the power law, \(R_0\) is the characteristic radius, and \(F_{in}\) and \(F_{out}\) are fractions to describe the inner and outer radii.

Hereafter, we also denote the BLR model with the gamma distribution as \(M1\) and that with the power-law distribution as \(M2\).

4.2. Emissivity

We assume that the ionizing continuum is isotropic, and is proportional to the optical 5100 Å continuum. Self-shadowing among clouds is not considered for the present simple
modeling. As in Li et al. (2013), we relax the usual assumption of linear responses of emission lines to the continuum and adopt a power-law index $\delta$ to describe the nonlinearity as

$$
\epsilon(t) \propto f_c^{1+\delta}(t - \tau),
$$

where $\epsilon$ is the emissivity of the particle at time $t$ irradiated by the ionizing continuum with a flux of $f_c$ at time $t - \tau$.

To account for the possibility that BLR clouds are optically thick so that their emission is anisotropic, we use a simple parameterization by assigning a weight to each particle as (Blandford & McKee 1982)

$$
w = \frac{1}{2} + \kappa \cos \phi,
$$

where $\kappa$ is a free factor in the range $[-1/2, 1/2]$ and $\phi$ is the angle between the observer’s and particle’s lines of sight to the central ionizing source.

It is possible that the particles below the equatorial plane are partially obscured by some material in the equatorial plane. As in Pancoast et al. (2014a), we use a parameter $\xi$ to describe the transparency of this equatorial material. For $\xi \rightarrow 0$, the entire half of the BLR below the equatorial plane is obscured, whereas for $\xi \rightarrow 1$, that half becomes transparent.

### 4.3. Dynamics

The motion of particles is assumed to be completely dominated by the gravity of the central BH. Following Pancoast et al. (2014a), three kinematic components are considered: bound elliptical orbits, and bound and unbound inflow and outflow. The fraction of bound elliptical orbits is described by parameter $f_{\text{ellip}}$, and the remaining fraction $1 - f_{\text{ellip}}$ of BLR particles is thus either inflowing or outflowing. A parameter $f_{\text{flow}}$ is used to determine whether BLR particles are inflowing ($0 < f_{\text{flow}} < 0.5$) or outflowing ($0.5 < f_{\text{flow}} < 1$).

Velocities of particles are first assigned in the particles’ orbital planes and then converted into real three-dimensional velocities through coordinate rotations. For bound elliptical orbits, radial and tangential velocities are drawn from Gaussian distributions centered around the point $(v_r, v_\theta) = (0, v_{\text{circ}})$ of an ellipse in the $v_r$-$v_\theta$ plane (see Figure 2 in Pancoast et al. 2014a), where $v_{\text{circ}} = \sqrt{GM_\star/r}$. The ellipse has a semiminor axis $v_{\text{circ}}$ in the $v_\phi$-direction and a semimajor axis $\sqrt{2}v_{\text{circ}}$ in the $v_r$-direction. The widths of Gaussian distributions for radial and tangential velocities are controlled by parameters $\sigma_{v_r, \text{circ}}$ and $\sigma_{v_\theta, \text{circ}}$, respectively, where $\rho$ and $\Theta$ are the radial and angular coordinates in the $v_r$-$v_\theta$ plane.

For inflowing or outflowing particles, velocities are assigned the same as for elliptical orbits, except that the Gaussian distributions are centered around points $(v_r, v_\theta) = (\pm \sqrt{2}v_{\text{circ}}, 0)$ in the $v_r$-$v_\theta$ plane, where “$+$” corresponds to outflow and “$-$” to inflow. In addition, the Gaussian distributions are allowed to rotate around the ellipse by an angle $\theta_c$ considering that real clouds may have a combination of Keplerian inflow/outflow motion. When $\theta_c = 0$, inflowing or outflowing velocities are centered around the escape velocity $v_e = \pm \sqrt{2}v_{\text{circ}}$. As $\theta_c \rightarrow 90^\circ$, inflowing or outflowing particles approach the same motion as the elliptical orbits.

Macroturbulence is included by adding a random velocity to the line-of-sight velocity of particles as (Pancoast et al. 2014a)

$$
v_{\text{turb}} = \mathcal{N}(0, \sigma_{\text{turb}})v_{\text{circ}},
$$

where $\mathcal{N}(0, \sigma_{\text{turb}})$ is a random number drawn from a Gaussian distribution with a zero mean and standard deviation $\sigma_{\text{turb}}$.

### 4.4. A Two-zone BLR Model M3

At high accretion rates, accretion disks are usually categorized into the slim-disk regime (Abramowicz et al. 1988), in which a geometrically thick funnel is formed in the inner disks due to the strong radiation pressure. Wang et al. (2014b) showed that the self-shadowing effect of such a funnel feature produces anisotropy of the ionizing radiation field that leads to two distinct BLR regions. Mrk 142 was identified to be an SEAMBH with a dimensionless accretion rate of $\dot{\mathcal{M}} = 45$ using the BH mass derived from cross-correlation function (CCF) analysis and an assumed inclination angle of $\cos \theta_{\text{inc}} = 0.75$ (Paper V), where $\dot{\mathcal{M}} = M\dot{c}/L_{\text{Edd}}$, $M$ is mass accretion rate, and $L_{\text{Edd}}$ is the Eddington luminosity. Motivated by the above scenario, we construct a two-zone BLR model. Figure 2 shows a schematic of the two-zone BLR geometry for Mrk 142. Zone I is ionized by radiation emitted within the funnel whereas zone II is ionized by radiation emitted outside the funnel. For simplicity, we assume that the ionizing emissions received by zone I and zone II are correlated and we thereby apply the observed 5100 Å continuum light curve for both regions. In addition, we neglect the obscuration that zone I causes to zone II.

The configuration of this two-zone model is set as follows. The structure and dynamics of zone I are described with the same parameters as in model M1 in Section 4.1. For zone II, the radial distribution of BLR particles follows a gaussian distribution but with distinct parameters. Meanwhile, zone I also has new dynamical parameters $f_{\text{ellip},1}$ and $f_{\text{flow},1}$. Hereafter, we denote this two-zone model as M3. In a nutshell, compared to model M1, M3 has new additional parameters $(\mu_1, \beta_1, P_1, \Theta_{\text{inc},1}, \rho_1, f_{\text{ellip},1}, \text{and } f_{\text{flow},1})$. The meanings of these parameters are also explained in Table 2.

### 4.5. Different Long-term Trends of Continuum and Emission Line

There is incident detection in previous RM observations that the variations of continuum and emission line undergo different long-term (compared with RM timescales) secular trends (e.g., Denney et al. 2010; Li et al. 2013; Peterson et al. 2014), which are irrelevant to RM analysis and therefore should be appropriately accounted for. A low-order polynomial was usually used to detrend the light curves of continuum and emission line (Welsh 1999), and this generally leads to improvements in the RM analysis.

In the present framework, we use a linear polynomial to model the difference in the long-term trends of continuum and emission line. We add this linear trend to the reconstructed continuum light curve so that the new light curve has the same secular trend as that of the emission line. To keep the mean flux of the continuum light curve unchanged, only a free parameter is needed to delineate the slope of the linear polynomial. Visual inspection reveals no apparently different trends in the light curves of Mrk 142, therefore we do not include this procedure in our calculations. However, there are indeed a few objects in our monitored sample showing different long-term trends. We describe the procedure for including different long-term trends here for the sake of completeness. We stress that here the linear trend serves the purpose of accounting for the different long-term trends in continuum and emission line, and is distinct from
the trend defined in Section 3, which only refers to the continuum itself.

5. Bayesian Framework

5.1. Formulations

It is now trivial to calculate the intrinsic emission-line profile at time $t$ and velocity $v$ by summing up the emissions from BLR particles with a line-of-sight velocity $v$:

$$ f_{i, \text{int}}(v, t) = \sum_i \epsilon_i(v, t) = A \sum_i \delta(v - u_i) w_i f_i^{1+\gamma} (t - \tau_i), $$

(17)

where $\delta(x)$ is the Dirac function, $A$ is the response coefficient, and $w_i$, $u_i$, $r_i$, and $\tau_i$ are, respectively, the weight of emissivity (given by Equation (15)), the line-of-sight velocity, the distance to the central source, and the time lag of re-radiation from the $i$th particle. The continuum flux $f_c$ includes the different long-term trends in continuum and emission, as described in Section 4.5. Here the subscript “int” refers to intrinsic line profile, to distinguish it from the observed line profile, which suffers additional broadening due to the seeing and instrument effects. The so-called transfer function reads

$$ \Psi_{\text{int}}(v, \tau) = A \sum_i w_i \delta(v - u_i) \delta(\tau - \tau_i). $$

(18)

This simplifies Equation (17) into a generalized integral form for RM with a nonlinear response,

$$ f_{i, \text{int}}(v, t) = \int \Psi_{\text{int}}(v, \tau) f_i^{1+\gamma} (t - \tau) d\tau. $$

(19)

For long time series, manipulating time-averaging upon both sides of the above equation yields the delay integral of $\Psi(v, \tau)$:

$$ \Psi_{\text{int}}(v) = \int \Psi_{\text{int}}(v, \tau) d\tau = \frac{\langle f_{i, \text{int}}(v, t) \rangle}{\langle f_i(t) \rangle}, $$

(20)

where the angle brackets denote time-average. The velocity integral of $\Psi(v, \tau)$,

$$ \Psi(\tau) = \Psi_{\text{int}}(\tau) = \int \Psi_{\text{int}}(v, \tau) dv, $$

(21)

yields the usual velocity-unresolved delay map.

To mock real observations, we need to take into account line broadening caused by the seeing and instruments. The observed line profile can be deemed to be a convolution between the predicted intrinsic line profile and line-broadening function. Equation (19) is recast as

$$ f_i(v, t) = f_{i, \text{int}}(v, t) \otimes \xi(v, t) $$

$$ = \int \Psi_{\text{int}}(v, \tau) \otimes \xi(v, t) f_i^{1+\gamma} (t - \tau) d\tau, $$

(22)

where “$\otimes$” denotes a convolution operation over the velocity axis and $\xi(v, t)$ is the line-broadening function, which generally depends on the instrument and seeing conditions. We again manipulate time-averaging upon both sides of the above equation and note that $\xi(v, t)$ and $f_i(t)$ are usually uncorrelated. For long time series, we have

$$ \langle f_i(v, t) \rangle = \langle f_i(t) \rangle \int \Psi_{\text{int}}(v, \tau) \otimes \langle \xi(v, t) \rangle d\tau. $$

(23)
If we denote

$$\Psi(v) = \int \Psi_{\text{int}}(v, \tau) \otimes \langle \xi(v, t) \rangle d\tau, \quad (24)$$

we obtain exactly the same form for $\Psi(v)$ as Equation (20),

$$\Psi(v) = \frac{\langle f_i(v, t) \rangle}{\langle f_i(t) \rangle} \propto \langle f_i(v, t) \rangle. \quad (25)$$

This implies that the delay integral of the (broadened) transfer function has the same shape as the mean observed profile of the emission line (Blandford & McKee 1982; Perry et al. 1994). Unless stated otherwise, transfer functions shown in figures throughout the paper include by default the broadening effect, namely, convolution with the line-broadening function.

A set of measurements for an emission line in real observations is a sum of the predicted line profiles and measurement noises. Written in a concise form of tensors,

$$y_i = f_i + n_i = f_{\text{int}} \otimes \xi + n_i, \quad (26)$$

where $n_i$ are measurement noises. For simplicity, we parameterize the line-broadening function $\xi$ by a Gaussian and adopt the dispersion from the mean value derived in Paper VI. The value of the dispersion is fixed throughout the period of the RM data (see Table 1). Again, we assume that the measurement noises $n_i$ are Gaussian and uncorrelated along

### Table 2

Parameters for Models $M_1$, $M_2$, and $M_3$

| Parameter | Model | Prior | Range          | Unit | Implication                                      |
|-----------|-------|-------|----------------|------|-------------------------------------------------|
| $\delta_0$ | $M_1, M_2, M_3$ | Logarithmic | $(10^{-3}, 10^{-1})$ | ... | Long-term standard deviation of DRW variation |
| $\tau_d$  | $M_1, M_2, M_3$ | Logarithmic | $(1, 10^4)$ | Day | Typical timescale of DRW variation               |
| $n_q$     | $M_1, M_2, M_3$ | Gaussian | ... | ... | Deviations of long-term trend of continuum fluxes |
| $n_t$     | $M_1, M_2, M_3$ | Gaussian | ... | ... | Deviations of continuum light curve              |

**Continuum**

| Parameter | Model | Prior | Range          | Unit | Implication                                      |
|-----------|-------|-------|----------------|------|-------------------------------------------------|
| $A$       | $M_1, M_2, M_3$ | Logarithmic | $(0.1, 10)$ | ... | Response coefficient of BLR                      |
| $\beta$   | $M_1, M_2, M_3$ | Uniform | $(-1, 3)$ | ... | Power-law index for nonlinear response of BLR    |
| $\mu$     | $M_1, M_3$   | Logarithmic | $(0.1, 100)$ | lt-day | Mean radius of BLR region I                     |
| $\sigma$  | $M_1, M_3$   | Uniform | $(0, 2)$ | ... | Shape of radial distribution of BLR particles    |
| $\tilde{F}$ | $M_1, M_3$ | Uniform | $(0, 1)$ | ... | Inner edge of BLR                                |
| $\mu_1$   | $M_3$       | Logarithmic | $(0.1, 100)$ | lt-day | Mean radius of BLR region I                     |
| $\beta_1$ | $M_3$       | Uniform | $(0, 2)$ | ... | Shape of radial distribution of BLR particles for region I |
| $\rho$    | $M_3$       | Uniform | $(0, 1)$ | ... | Inner edge of BLR region I                      |
| $\alpha$  | $M_2$       | Uniform | $(1, 3)$ | ... | Fraction of BLR particles in region I            |
| $R_0$     | $M_2$       | Logarithmic | $(0.1, 100)$ | lt-day | Characteristic radius of double power law        |
| $F_{\text{in}}$ | $M_2$ | Uniform | $(0, 1)$ | ... | Inner radius of double power law                 |
| $F_{\text{out}}$ | $M_2$ | Logarithmic | $(1, 10)$ | ... | Outer radius of double power law                 |
| $\theta_{\text{incl}}$ | $M_1, M_2, M_3$ | Uniform | $(0, 90)$ | deg | Inclination angle of BLR to the light of sight   |
| $\theta_{\text{p}}$ | $M_1, M_3$  | Uniform | $(0, 90)$ | deg | Opening angle of BLR region I                    |
| $\theta_{\text{p,1}}$ | $M_3$  | Uniform | $(0, 90)$ | deg | Opening angle of BLR region I                    |
| $\gamma$  | $M_1, M_2, M_3$ | Logarithmic | $(-0.5, 0.5)$ | ... | Anisotropy of particle emission                  |
| $\xi$     | $M_1, M_2, M_3$ | Uniform | $(1, 5.5)$ | ... | Transparency of equatorial material              |
| $M_0$     | $M_1, M_2, M_3$ | Logarithmic | $(10^9, 10^{13})$ | $M_\odot$ | BH mass                                         |
| $f_{\text{in}}$ | $M_1, M_2, M_3$ | Uniform | $(0, 1)$ | ... | Fraction of bound elliptical orbits              |
| $f_{\text{out}}$ | $M_1, M_2, M_3$ | Uniform | $(0, 1)$ | ... | Fraction of bound elliptical orbits for region I |
| $f_{\text{in,1}}$ | $M_3$ | Uniform | $(0, 1)$ | ... | Flag for determining inflowing or outflow orbits |
| $f_{\text{out,1}}$ | $M_3$ | Uniform | $(0, 1)$ | ... | Flag for determining inflowing or outflow orbits for region I |
| $\sigma_{\text{in,inc}}$ | $M_1, M_2, M_3$ | Logarithmic | $(0.001, 0.1)$ | ... | Radial standard deviation around circular orbits |
| $\sigma_{\text{in,circ}}$ | $M_1, M_2, M_3$ | Logarithmic | $(0.001, 1.0)$ | ... | Angular standard deviation around circular orbits |
| $\sigma_{\text{rad}}$ | $M_1, M_2, M_3$ | Logarithmic | $(0.001, 0.1)$ | ... | Radial standard deviation around radial orbits   |
| $\sigma_{\text{rad,1}}$ | $M_1, M_2, M_3$ | Logarithmic | $(0.001, 1.0)$ | ... | Angular standard deviation around radial orbits |
| $\tau_k$  | $M_1, M_2, M_3$ | Uniform | $(0, 0.90)$ | deg | Rotation angle of inflow or outflow orbits       |
| $\sigma_{\text{ turb}}$ | $M_1, M_2, M_3$ | Logarithmic | $(0.001, 0.1)$ | ... | Standard deviation of macroturbulent velocities |
| $\alpha$  | $M_1, M_2, M_3$ | Uniform | $(-0.1, 0.1)$ | ... | Slope for different long-term trends in continuum and emission line |
| $\sigma_d$ | $M_1, M_2, M_3$ | Uniform | $(1.0, 10.0)$ | ... | Additive noise for emission-line data            |

**Notes.** Parameters in bold are composed of an array. The unit of $\alpha_p$ is the unit of flux $\times$ day$^{-1}$. The prior ranges of $A$ and $\alpha_p$ are assigned in terms of the mean fluxes of the light curves normalized to unity. The prior for $\sigma_d$ is set as $P(x) = 1/(1 + x)$, where $x = \sigma_d/\bar{\sigma}$ and $\bar{\sigma}$ is the mean measurement error of the emission line. $P(x)$ behaves like a uniform prior when $x \ll 1$ and like a logarithmic prior when $x \gg 1$ (Gregory 2011).

$^a$ This column indicates whether the parameters are included in the three models $M_1$, $M_2$, and $M_3$.  

:::
both wavelength and time axes. This results in a Gaussian likelihood probability for $y_i$ as

$$P_i(y_i|\Theta) = \prod_i \frac{1}{\sqrt{2\pi} \sigma_{ij}} \exp\left(-\frac{(y_{ij} - f_{ij})^2}{2\sigma_{ij}^2}\right),$$  

(27)

where $\Theta$ denotes the whole set of involved model parameters listed in Table 2, $i$ and $j$ represent the epoch and wavelength bin, and $\sigma_{ij}$ is the measurement noise.

### 5.2. Bayesian Inference

The RM data ($D$) at hand are the time series of continuum $\chi$ and emission line $y$, and their respective associated measurement errors. We first use Equation (10) to generate realizations for the continuum time series, which are then used as input for deriving time series of emission lines with Equations (17) and (22). In this regard, the continuum data are treated as a prior for BLR modeling (A. Pancoast, private communications; Pancoast et al. 2011, 2014a). From this paradigm, the likelihood probability for $D$ is thereby

$$P(D|\Theta) = P(y|\Theta).$$  

(28)

According to Bayes’ theorem, the posterior probability distribution for the parameter set $\Theta$ is

$$P(\Theta|D) = \frac{P(\Theta)P(D|\Theta)}{P(D)},$$  

(29)

where $P(\Theta)$ is the prior for the parameter set and $P(D)$ is the Bayesian evidence that just plays the role of normalization factor and is important for model selection. The priors for $u_s$ and $u_q$ are Gaussian, as described in Section 3. The priors for the other parameters are listed in Table 2, and are assigned following the convention that a uniform prior is assigned for parameters whose typical ranges are known, but a logarithmic prior is assigned if the parameter information is completely unknown (Sivia & Skilling 2006, Chapter 5). For all the priors, we set a reasonably broad but still finite range to avoid the posterior improperity.

We include an extra noise parameter $\sigma_{ij}$, added in quadrature to the measurement noises of emission-line data. This is based on two considerations: the present model for BLRs is simple so it is unlikely to fit all the features of data; on the other hand, there are probably additional noises beyond the known measurement uncertainties. Another additional advantage of using an extra noise parameter is that this provides a very useful annealing operation that benefits the convergence of Markov chains (see below) when the initial parameter values are far from the best-fitting values.

In Table 2, we list the overall free parameters for continuum and BSR modeling. We use 200 points to describe the continuum light curve, leading to a total of hundreds of free parameters. Meanwhile, there are strong correlations among BSR parameters, such as inclination and BH mass. This requires sophisticated algorithms that can handle massive and highly correlated parameters. We use the Markov chain Monte Carlo (MCMC) method to construct samples from the posterior distribution and determine the best-fitting estimate for the parameters. We employ the diffusive nested sampling (DNS) algorithm proposed by Brewer et al. (2011a) to generate the Markov chains. The DNS algorithm is effective at exploring multimodal distributions and strong correlations between parameters. It also allows us to calculate the Bayesian evidence, which can be used for subsequent model selection. Moreover, the DNS algorithm is inherently parallel and is easy to implement on parallel computing interfaces. We write our own DNS code in C language using the standardized message passing interface so that the code is portable to a wide range of supercomputer clusters without any reliance on special features of proprietary compilers. We develop a code named BRAINS to implement the above BLR dynamical modeling and Bayesian inference, which is publicly available at https://github.com/LiyrAstroph/BRAINS. Unless stated otherwise, throughout the calculations, the best estimates for the parameters are taken to be the median values of their posterior distributions and the uncertainties are determined from the 68.3% confidence intervals.

### 6. Results

#### 6.1. Overview

Figures 3–5 show the results of fitting BSR models $M_1$, $M_2$, and $M_3$ respectively to the RM data of Mrk 142. In each figure, the top three panels plot the observed $H\beta$ spectral time series, an exemplary model fit, and the residuals between the data and the fit. The bottom panels plot the recovered $H\beta$ profiles at two selected epochs and the reconstructed light curves of the continuum and $H\beta$ fluxes. The three models can generally reproduce the continuum and $H\beta$ flux light curves well, but show varying degrees of success in reproducing the detailed $H\beta$ spectral time series. The residuals obtained between the data and model fit illustrate that the fits of model $M_2$ have systematic deviations around $\sim -500$ km s$^{-1}$ and $\pm 1500$ km s$^{-1}$ of $H\beta$ profiles. Models $M_1$ and $M_3$ give similar fitting to the data, with $M_3$ slightly better by visual inspection. Figure 6 shows examples of the inferred geometry of the BSR for the three models. In Figure 7, we plot the posterior distributions of BH mass obtained by the three models. The best inferred BH mass is log$(M_*/M_\odot) = 5.90^{+0.37}_{-0.25}$ and $6.23^{+0.28}_{-0.25}$ for $M_1$, $M_2$, and $M_3$, respectively. These values are consistent with each other to within uncertainties. The inferred values for the major parameters of the three models are tabulated in Appendix A.

We use the standard approaches of model comparison to determine the best model. In Table 3, we calculate the maximum likelihood ($\ln L_{\text{max}}$), the Bayesian information criterion (BIC$^{10}$), the Akaike information criterion (AIC$^{11}$), and the Bayes factor$^{12}$ for the three models. The best model is chosen to be the one that maximizes $L_{\text{max}}$ and the Bayes factor

$^{10}$ The BIC is defined by (Schwarz 1978)

$$\text{BIC} = k \ln n - 2 \ln L_{\text{max}},$$  

(30)

where $k$ is the number of model parameters, $n$ is the sample size, and $L_{\text{max}}$ is the maximum value of the likelihood function.

$^{11}$ The AIC is defined by (Akaike 1973)

$$\text{AIC} = 2k - 2 \ln L_{\text{max}},$$  

(31)

where $k$ is the number of model parameters. The original form of the AIC is only strictly valid asymptotically. Hurvich & Tsai (1989) proposed a correction to AIC for finite sample size, defined as

$$\text{AIC} = 2k - 2 \ln L_{\text{max}} + \frac{2k(k + 1)}{n - k - 1},$$  

(32)

where $n$ is the sample size. We use this corrected AIC in our calculations.

$^{12}$ The Bayes factor is defined by the ratio of the posterior probabilities (Sivia & Skilling 2006). For two models, say $M_1$ and $M_2$ with equal priors, the Bayes factor is equal to the ratio of the corresponding Bayesian evidence:

$$K = \frac{P(M_2|D)}{P(M_1|D)} = \frac{P(D|M_2)}{P(D|M_1)}.$$  

(33)
and minimizes the AIC and BIC. All the approaches rank model M3 as the best model for fitting the RM data of Mrk 142. In the following, we study the BLR structure in Mrk 142 based only on the results from model M3. Figure 8 plots the obtained transfer functions and the reconstructed Hβ light curves over selected velocity bins. For each velocity bin, the transfer function peaks at zero lag and then gradually decreases, features typically seen in transfer functions of inclined disk-like BLRs (e.g., Goad & Wanders 1996; Pancoast et al. 2014b). We note that the variability characteristic for all the velocity bins is generally small (Fvar ∼ 7%). Compared with AGNs at sub-Eddington accretion rates, low variation amplitudes are a major challenge for monitoring SEAMBH objects (Rakshit & Stalin 2017). An example of a two-dimensional transfer function is shown in the top left panel of Figure 9. It is slightly asymmetric with longer response on the red side of the Hβ profile. Such an asymmetric feature is more clearly seen in the velocity-binned delay map in the top right panel. This asymmetry is mainly caused by the anisotropic parameter κ, which means that particles’ emissions depend on their locations, and also by the dynamical parameter fflow < 0.5 or fflow > 0.5 motion (see Figure 10). The bottom left panel of Figure 9 plots the delay integral of the transfer function vY(), in good agreement with the scaled mean Hβ profile as expected from Equation (25).

6.2. The Structure and Dynamics of the Two-zone BLR
The best model M3 indicates that the BLR in Mrk 142 consists of two regions, consistent with the self-shadowing effects of slim accretion disk models (Wang et al. 2014b). In Figure 10, we plot the inferred posterior distributions of several selected main parameters in model M3. The top panels show the common parameters for both zones I and II (see the schematic in Figure 2). The bottom panels show the distributions of the parameters for zone I in red and zone II in blue. The inclination angle is 41° ± 11°. The anisotropic parameter κ peaks at either −0.5 or 0.5, which means that the
observer sees the majority of emissions from either the near side or the far side of the BLR. The parameter $\gamma$ has a broad distribution over $\left(1, 5\right)$, but tends to peak at $\gamma = 5$, indicating that the particles tend to concentrate near the outer face of the BLR disks. The distribution of $\xi$ is also broad and peaks around $\xi = 0$, which corresponds to a completely obscured half of the BLR below the equatorial plane.

From the bottom panels of Figure 10, we see that the distributions of the dynamical parameters $f_{\text{ellip}}$ and $f_{\text{flow}}$ are roughly similar for zones I and II. However, the mean radius $\mu$ and the opening angle $\theta_{\text{opn}}$ are different. Remarkably, the mean radius of zone I is clearly larger than that of zone II, in agreement with the theoretical model proposed by Wang et al. (2014b). Because of the self-shadowing effects of the inner funnel of geometrically thick slim disks, the ionizing continuum flux received by zone II is significantly lower than that received by zone I, leading to a shrunken BLR in zone II. The ratio of the size scales of the two zones can be approximated by

$$\frac{R_1}{R_2} \approx 2 \times \left(\frac{\dot{M}}{50}\right)^{0.3}. \quad (36)$$

Using the accretion rate $\log \dot{M} = 2.4_{-0.6}^{+0.3}$ for Mrk 142 obtained below, the anticipated ratio is $\log((R_1)/(R_2)) = 0.5_{-0.2}^{+0.4}$. Our results give $\log((R_1)/\text{lt-day}) = 1.22_{-0.20}^{+0.21}$ and $\log((R_2)/\text{lt-day}) = 0.09_{-0.06}^{+0.44}$, marginally consistent with the anticipated value to within uncertainties.

6.3. Comparison with the Cross-correlation Analysis

In Figure 9, we compare the velocity-binned time lags from our model fitting with those from CCF analysis over velocity bins chosen to be the same as those in Paper VI. We only use the velocity bins with the maximum correlation coefficients $r_{\text{max}} \geq 0.7$. For each velocity bin, the cross-correlation is calculated using the standard interpolated CCF method (Gaskell & Peterson 1987). The time delay is determined by measuring either the location $\tau_{\text{peak}}$ of the CCF peak ($r_{\text{max}}$) or the centroid $\tau_{\text{cent}}$ of the points around the peak above the threshold $r \geq 0.8r_{\text{max}}$. As for model fitting, we calculate $\tau_{\text{peak}}$ and $\tau_{\text{cent}}$ by cross-correlating the observed continuum light curve with the reconstructed H$\beta$ light curves interpolated to the observed epochs. There is a tendency for the time lags from model fitting to be slightly shorter than those from CCF analysis. We ascribe such a discrepancy to the fact that the CCFs (between the observed continuum and H$\beta$ light curves) are broad and possibly multimodal, as seen from the bottom right panel of Figure 9. Nevertheless, the time lags from the two approaches are consistent within uncertainties (at $2\sigma$ confidence level). The velocity-binned time lags show an
log 6.59 ± 0.07

consistent with the anticipated value.

with an approach similar to this work and obtained an H\textsc{\textbeta} spectrum and a virial factor of

14 However, Li et al. (2013) reanalyzed the same data (velocity-unresolved) with an approach similar to this work and obtained an H\textsc{\textbeta} lag of ~15 days, consistent with the anticipated value.

Mrk 142 was previously monitored by the LAMP project (Bentz et al. 2009). The obtained H\textsc{\textbeta} centroid lag is as short as 2.74\textpm0.83 days, in contrast to the anticipated lag of ~20 days\textsuperscript{14}, making it a significant outlier in the BLR size–luminosity relation (Bentz et al. 2013). The estimated BH mass is log(M_*/M_☉) = 6.23\textpm0.13 using the H\textsc{\textbeta} line dispersion of 859 \pm 102 km s\textsuperscript{-1} from the rms spectrum and a virial factor of f_\text{rms,σ} = 5.5 (Bentz et al. 2009). In our new observations of Mrk 142, the measured H\textsc{\textbeta} centroid lag is 7.9\textpm1.1 days through CCF analysis (Paper III) and the estimated BH mass is log(M_*/M_☉) = 6.59\textpm0.07 using the H\textsc{\textbeta} FWHM from the mean spectrum and a virial factor of f_\text{mean,FWHM} = 1.

Our model M3 yields a BH mass of log(M_*/M_☉) = 6.23\textpm0.26 for Mrk 142, in remarkable agreement with the measurement of Bentz et al. (2009). The resulting virial factor is log(f_\text{mean,FWHM} = −0.36\textpm0.13 and log(f_\text{rms,FWHM} = −0.40\textpm0.34 for the H\textsc{\textbeta} FWHM measured from the mean and rms spectra, respectively, and log(f_\text{mean,σ} = 0.07\textpm0.31) and log(f_\text{rms,σ} = −0.06\textpm0.30) for the H\textsc{\textbeta} line dispersion measured from the mean and rms spectra, respectively. There have been a number of f calibrations reported in the literature, mainly based on H\textsc{\textbeta} line dispersion measured from rms spectra (e.g., Onken et al. 2004; Graham et al. 2011; Park et al. 2012; Grier et al. 2013a; Woo et al. 2013). The calibrated value ranges from log(f_\text{rms,σ} = 0.45\textpm0.10 (Graham et al. 2011) to log(f_\text{rms,σ} = 0.77\textpm0.13 (Woo et al. 2013). Ho & Kim (2014) calibrated f factors for AGNs with classical and pseudo bulges separately based on the notion that classical and pseudo bulges obey different M_–σ relations (Kormendy & Ho 2013). They presented f factors for classical and pseudo bulges separately in the cases of four widely used measures of H\textsc{\textbeta} line widths, namely FWHM and line dispersion from mean and rms spectra. Table 4 summarizes the measurements of f factor in the literature. Despite the large uncertainties (\textsim0.4 dex), our obtained f factors tend to coincide with the f factors for pseudo bulges calibrated by Ho & Kim (2014). The host galaxy of Mrk 142 is a late-type spiral galaxy and shows a strong bar in the nucleus (Ohta et al. 2007). Also, the surface brightness decomposition of the Hubble Space Telescope image does not detect a notable bulge component (Bentz et al. 2013; Paper I), probably implying that Mrk 142 may not host a classical bulge.

Meanwhile, Pancoast et al. (2014b), Grier et al. (2017), and Williams et al. (2018) applied the BLR dynamical modeling

asymmetric pattern with slightly longer lags on the red side, which is usually regarded as a signature of an outflowing BLR (see the discussion in Paper VI). Our modeling results in Figure 10 show that the values of the dynamical parameter f_{\text{flow}} for both zones I and II is distributed over a broad range, meaning that both inflow and outflow can fit the RM data.

6.4. The BH Mass in Mrk 142

Mrk 142 was a late-type spiral galaxy and shows a strong bar in the nucleus (Ohta et al. 2007). The calibrated value ranges from log(f_\text{rms,σ} = 0.45\textpm0.10 (Graham et al. 2011) to log(f_\text{rms,σ} = 0.77\textpm0.13 (Woo et al. 2013). Ho & Kim (2014) calibrated f factors for AGNs with classical and pseudo bulges separately based on the notion that classical and pseudo bulges obey different M_–σ relations (Kormendy & Ho 2013). They presented f factors for classical and pseudo bulges separately in the cases of four widely used measures of H\textsc{\textbeta} line widths, namely FWHM and line dispersion from mean and rms spectra. Table 4 summarizes the measurements of f factor in the literature. Despite the large uncertainties (\textsim0.4 dex), our obtained f factors tend to coincide with the f factors for pseudo bulges calibrated by Ho & Kim (2014). The host galaxy of Mrk 142 is a late-type spiral galaxy and shows a strong bar in the nucleus (Ohta et al. 2007). Also, the surface brightness decomposition of the Hubble Space Telescope image does not detect a notable bulge component (Bentz et al. 2013; Paper I), probably implying that Mrk 142 may not host a classical bulge.

Meanwhile, Pancoast et al. (2014b), Grier et al. (2017), and Williams et al. (2018) applied the BLR dynamical modeling
analysis developed by Pancoast et al. (2014a) to a sample of AGNs. By combining the \( f \)-factor measurements obtained in the three studies, Williams et al. (2018) reported the mean \( f \) factors: \( \log f_{\text{mean}, \text{FWHM}} = 0.00 \pm 0.14 \), \( \log f_{\text{mean}, \sigma} = 0.43 \pm 0.09 \), and \( \log f_{\text{max}} = 0.57 \pm 0.09 \) (see Table 4). Our measured \( f \) factors for Mrk 142 are marginally consistent with these results to within uncertainties.

We estimate the dimensionless accretion rate according to the equation (Paper I)

\[
\dot{\lambda} = 20.1 \times \left( \frac{L_{5100}}{10^{44} \cos \theta_{\text{inc}}} \right)^{3/2} \left( \frac{M_*}{10^8 M_\odot} \right)^{-2}.
\]  

Using the 5100 Å luminosity \( \log L_{5100} = 43.56 \pm 0.06 \) (Paper IV), the inclination angle \( \theta_{\text{inc}} = 41^{1.0}_{-1.1} \), and the BH mass \( \log (M_*/M_\odot) = 6.23^{+0.23}_{-0.45} \), we obtain \( \log \dot{\lambda} = 2.4^{+1.3}_{-1.0} \). This confirms the BH in Mrk 142 to be an SEAMBH accreting at a super-Eddington rate.

7. Discussions

7.1. Continuum Reconstruction by the DRW Process

The continuum light curve is reconstructed using the DRW process, which is found to be adequate for large samples of AGN light curves on timescales of weeks to years (e.g., Kelly et al. 2009; MacLeod et al. 2010; Zu et al. 2011, 2013; Andrae et al. 2013; Kozłowski 2016a). However, on short timescales of days, there is evidence for deviations from the DRW process for high-cadence AGN light curves monitored by the Kepler telescope (Mushotzky et al. 2011; Kasliwal et al. 2015; Kozłowski 2016b). Kelly et al. (2014) proposed to use the generic continuous-time autoregressive moving average (CARMA) models to characterize the variability features of a broad range of stochastic light curves. The DRW process is a special case of CARMA processes with autoregressive order \( p = 0 \) and moving average order \( q = 0 \). Using the package CARMAPACK\(^\text{15}\) developed by Kelly et al. (2014), we can choose the best order of \( p \) and \( q \) for CARMA processes by minimizing the AIC. We find that the DRW process is still a more favorable model than high-order CARMA processes for the data of Mrk 142. In addition, using the Bayesian framework proposed by Li & Wang (2018), we perform a comparison between the DRW model and the power spectral density (PSD) model with a single power law. Note that the DRW model has a PSD \( \propto 1/[1 + (f/f_0)^2] \), where \( f \) is the frequency and \( f_0 = 1/2\pi \tau_d \). We confirm that the DRW model is slightly preferable.

On the other hand, different continuum models mainly affect the short-timescale variability of the reconstructed continuum between measurement points (e.g., see Li & Wang 2018). Such an effect will finally influence the amplitudes of the inferred parameter uncertainties. However, the inherent convolution operation in RM analysis (see Equation (19)) will largely smooth the short-timescale variations. We therefore expect that the estimated uncertainties should not be significantly affected by the details of the chosen continuum models (see also discussions in Skielboe et al. 2015 and Fausnaugh et al. 2018).

7.2. Anisotropic Emission from the Central Ionizing Source

We only take into account the possibility that the anisotropic ionizing emission from the geometrically thick funnel in the inner region of slim accretion disks produces two-zone BLRs. Indeed, there are two additional anisotropic effects for accretion disks. First, the ionizing emissions depend strongly on the angle between the axis of symmetry of disks and the direction toward BLR particles. Second, the size of the ionizing source may be no longer negligible, particularly when using the 5100 Å continuum as a surrogate for the ionizing continuum (see also Section 7.3). The first effect will cause the BLR to be thicker to compensate the \( \cos \theta \) dependence of the disk emission. To include the second effect, one needs to solve the structure of accretion disks and obtain the radial distribution of emissions. This will make the present model more complicated and the MCMC sampling more inefficient. We are thus content with the present simple treatments of the ionizing sources and defer the inclusion of these two effects to a separate paper.

7.3. Point-like Geometry of the Central Ionizing Source

We implicitly assume that the emission region of the 5100 Å continuum is point-like. However, multiwavelength RM observations on a handful of AGNs indeed detected time lags of the optical continuum variations with respect to the X-ray/UV variations (e.g., Edelson et al. 2015, 2017; Cackett et al. 2016).

\(^\text{15}\) Accessible at \url{https://github.com/brandonckelly/carma_pack}. 
This indicates that the emission region at 5100 Å could be spatially extended. We can estimate the characteristic radius for emission of the 5100 Å continuum using the standard accretion disk model. The local effective temperature of the accretion disk is written (e.g., Laor & Davis 2011) as

\[ T(r) = f(r, a) \left( \frac{3c^6}{8\pi G^2}\sigma \right)^{1/4} \frac{M^1/4}{M_*^{1/2}} r^{-3/4}, \]

where \( r = R/R_g \), \( R_g \) is the gravitational radius, \( a \) is the BH spin, \( M = \dot{M}_{\text{Edd}}/c^2 \) is the mass accretion rate, \( M_* \) is the BH mass, \( \sigma \) is the Stefan–Boltzmann constant, and \( f(r, a) \) is a dimensionless factor of the order of unity that is set by the inner boundary condition and the relativistic effects. Regardless of \( f(r, a) \) and using \( \log \dot{M} = 2.4^{+1.3}_{-0.6} \) and \( \log(M_* / M_\odot) = 6.23^{+0.23}_{-0.45} \), the corresponding radius for the 5100 Å emission is \( R_{5100} = 0.14^{+0.09}_{-0.11} \) lt-day. Note that for a slim disk the presence of prominent radial advection reduces the effective temperature (Abramowicz et al. 1988; Wang & Zhou 1999), making the above estimate conservative. Considering that the disk size inferred from multiwavelength RM observations is about three times that predicted by the standard disk model (e.g., Edelson et al. 2015), the 5100 Å emission radius \( R_{5100} \) would be comparable with the mean radius of zone I, but much smaller than the mean radius of zone II (see Table 5). The spatial extent of the 5100 Å emission region may lead the obtained BH mass to be underestimated. The influence of a spatially extended 5100 Å emission region on BH mass measurement is worth a detailed study. As discussed in the preceding section, for the sake of simplicity, we keep the assumption of point-like geometry of the 5100 Å emission region and defer the detailed study to a future paper.

### 7.4. Model Dependence of the Results

In the present BLR models, the prescriptions for BLR properties are purely phenomenological and adopted only for the sake of simplicity. This raises an issue as to whether the inferred results depend on the adopted model. To address this issue, we need: (1) independent measurements from alternative approaches, and (2) model selections to evaluate the most probable model for BLRs. A major challenge to performing model selection is that the existing BLR models (see the summary in Table 1 of Wang et al. 2012) invoke complicated physical processes, impeding an efficient MCMC inference. The results from the three BLR models indeed imply that the obtained BH masses appear to be slightly different, although they reproduce the RM data with different degrees of success. Recently, Czerny et al. (2017) developed a self-consistent BLR model based on the failed radiatively accelerated dusty outflow model (Czerny & Hryniewicz 2011), which only invokes the basic physical parameters, such as BH mass and spin, and accretion rate. The model is purely analytic and therefore apt for MCMC realization. A comparison of the inferred parameters from this model and the present dynamical modeling will shed light on the model dependence of the results.

### 7.5. Parameter Degeneracy

The significant degeneracy in the present models is among the BH mass, the inclination angle, and the opening angle. There are two causes of this degeneracy (Grier et al. 2017). The first comes from the model itself, such as the strong correlation between BH mass and inclination angle or opening angle, ascribed to the disk-like geometry adopted for the BLR (Collin et al. 2006; Li et al. 2013). The other cause is constraints by observation data.
Similar to Pancoast et al. (2014b) and Grier et al. (2017), we also find a tight correlation between inclination and opening angles. Moreover, the values of these two angles are approximately equal. As pointed out by Grier et al. (2017), the interpretations of such behavior are twofold: first, to generate single-peaked line profiles, the opening angle should be larger than the inclination angle; second, as the opening angle increases, the generated profiles tend to be flat in the core (e.g., Netzer & Marziani 2010), which is apparently incompatible with the observed line shapes. Therefore, the observations require the opening angle to be as small as possible yet still large enough to produce single-peaked line profiles. As a result, the opening angle is approximately equal to the inclination angle.

7.6. BLR Dynamical Modeling

The present dynamical model does not include possible systematic errors in the model assumptions. This issue can be overcome by comparing mass measurements against those from the other independent techniques, such as stellar dynamics and gas dynamics widely used in quiescent galaxies. Unfortunately, there are still extremely few objects with both RM monitoring and the other independent measurements (Peterson 2014). On the other hand, new techniques such as spectro-astrometry (Gnerucci et al. 2010; Stern et al. 2015) and spectro-interferometry (Kraus 2012; Petrov et al. 2012) are in the process of development with the purpose of spatially resolving gas dynamics surrounding the central BHs using the current ground-based 10 m class telescopes (Gnerucci et al. 2011, 2013; Rakshit et al. 2015). Hopefully, in the near future, there will be a sufficient data sample with independent mass measurements to allow us to explore the systematic errors of our dynamical models.

7.7. Comparison with the Maximum Entropy Method

The maximum entropy method (MEM, Horne 1994) is also widely used to derive transfer functions of BLRs and probe the structure and dynamics of BLRs. Paper VII presented the transfer function for Mrk 142 by applying MEM to the same RM data used in this paper. The obtained transfer function exhibits a major response around 5–10 days (see Figure 11 in Paper VII), seemingly distinct from the transfer function derived from our dynamical modeling analysis (see the top right panel of Figure 9). However, this is not the case because of the following reasons. First, MEM solves a modified equation compared to Equation (19) (regardless of the nonlinear response),

\[ f_t(v, \tau) = \tilde{f}_t(v) + \int \Psi_{MEM}(v, \tau)[\tilde{f}_t(t - \tau) - \tilde{f}_t]d\tau, \quad (39) \]

where \( \tilde{f}_t(v) \) and \( \tilde{f}_t \) are considered to be the constant background terms (Horne 1994). MEM employs maximum entropy regularization to find the smoothest solutions \( \Psi_{MEM}(v, \tau) \), \( \tilde{f}_t(v) \), and \( \tilde{f}_t \) that best fit the observed data. In real implementation, the derived \( \tilde{f}_t(v) \) and \( \tilde{f}_t \) usually include contributions from the nonvariable part of the broad emission line and continuum, which cannot be attributed to the background (Wanders 1995). As a result, \( \Psi_{MEM} \) is

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**Figure 8.** Left panels: the obtained transfer functions based on model M3 for selected wavelength bins marked in the right panels. Gray shaded areas represent the 1σ error band. Right panels: the reconstructed light curves. Each thin gray line represents one random reconstruction. Points with error bars are the observed data. The variability characteristic \( F_{var} \) of each observed light curve is also shown.
sensitive to changes in the responses of the BLR but insensitive to its total responses. In this sense, it is more appropriate to call $\Psi_{\text{MEM}}$ a “marginal transfer function.”

Second, MEM uses a free parameter to control the trade-off between smoothness of the solutions and goodness of fitting to the data. In practice, the value of this free parameter is chosen by eye to achieve the best compromise. Sharp features in transfer functions will generally be smeared out by MEM (see also discussions in Pancoast et al. 2018), so it is not straightforward to perform a direct, quantitative comparison with the results from dynamical modeling.

8. Conclusions

We employ the recently developed dynamical modeling for BLRs to analyze the RM data on the broad H$\beta$ line and 5100 Å continuum for Mrk 142 monitored between 2012 and 2013. The BH mass is self-consistently measured without resort to the virial factor required in the traditional RM analysis through the cross-correlation method. The main results are as follows.

1. We apply three BLR models to fit the RM data of Mrk 142 and find that the best model is a two-zone model (see the schematic in Figure 2), consistent with the theoretical BLR model proposed by Wang et al. (2014b). The two zones may be caused by anisotropic ionizing emission due to the self-shadowing of the slim accretion disk. Interestingly, the obtained mean size of zone I is larger than that of zone II, also in agreement with the theoretical model. It is possible that a much more complicated one-zone BLR model can also fit the data of Mrk 142. Still, our results are illustrative and application to other SEAMBH objects is required to reinforce the scenario of the two-zone BLR model.

2. The general geometry of H$\beta$ BLRs for Mrk 142 is described by an inclined disk with an inclination angle of...
Figure 10. Inferred posterior distributions of the selected main parameters for model M3. Top panels are for the common parameters of BLR zones I and II (see the schematic in Figure 2). Bottom panels show the distributions of the parameters for zone I in red and zone II in blue.

Table 4
A Summary for f-factor Measurements

| f factor | Value | Reference | Note |
|----------|-------|-----------|------|
| log$f_{\text{rms},\sigma}$ | $-0.74^{+0.12}_{-0.18}$ | 1 | ... |
| $-0.45^{+0.10}_{-0.09}$ | 2 | ... |
| $-0.71^{+0.11}_{-0.11}$ | 3 | ... |
| $-0.77^{+0.13}_{-0.13}$ | 4 | ... |
| $-0.63^{+0.09}_{-0.12}$ | 5 | ... |
| $-0.80^{+0.09}_{-0.12}$ | 6 | Classical bulges |
| $-0.51^{+0.09}_{-0.11}$ | 6 | Pseudo bulges |
| $-0.57^{+0.07}_{-0.07}$ | 7 | Dynamical modeling |
| $-0.06^{+0.30}_{-0.32}$ | 8 | Dynamical modeling |

| log$f_{\text{rms,FWHM}}$ | $-0.18^{+0.10}_{-0.13}$ | 6 | Classical bulges |
| $-0.15^{+0.09}_{-0.15}$ | 6 | Pseudo bulges |
| $-0.40^{+0.34}_{-0.56}$ | 8 | Dynamical modeling |

| log$f_{\text{mean,}\sigma}$ | $-0.75^{+0.09}_{-0.11}$ | 6 | Classical bulges |
| $-0.28^{+0.14}_{-0.20}$ | 6 | Pseudo bulges |
| $-0.43^{+0.09}_{-0.09}$ | 7 | Dynamical modeling |
| $-0.07^{+0.31}_{-0.52}$ | 8 | Dynamical modeling |

| log$f_{\text{mean,FWHM}}$ | $-0.11^{+0.12}_{-0.16}$ | 6 | Classical bulges |
| $-0.30^{+0.15}_{-0.22}$ | 6 | Pseudo bulges |
| $-0.00^{+0.14}_{-0.14}$ | 7 | Dynamical modeling |
| $-0.36^{+0.33}_{-0.54}$ | 8 | Dynamical modeling |

References. (1) Onken et al. (2004), (2) Graham et al. (2011), (3) Park et al. (2012), (4) Woo et al. (2013), (5) Grier et al. (2013a), (6) Ho & Kim (2014), (7) Williams et al. (2018), and (8) this work.

The opening angles for zones I and II are $30^{+14}_{-11}$ and $10^{+29}_{-5}$, respectively, corresponding to a thick disk with a total height aspect of $h/r \sim 0.6$.

3. The obtained BH mass is $\log(M/M_\odot) = 6.23^{+0.26}_{-0.33}$, resulting in a virial factor of $\log f_{\text{mean,FWHM}} = -0.36^{+0.33}_{-0.54}$ and $\log f_{\text{rms,FWHM}} = -0.40^{+0.34}_{-0.56}$ for the Hβ FWHM measured from the mean and rms spectra, respectively, and $\log f_{\text{mean,}\sigma} = 0.07^{+0.23}_{-0.52}$ and $\log f_{\text{rms,}\sigma} = -0.06^{+0.23}_{-0.52}$ for the Hβ line dispersion measured from the mean and rms spectra, respectively. These values are consistent to within uncertainties with previous measurements by similarly applying dynamical modeling to a dozen AGNs (Pancoast et al. 2014b; Grier et al. 2017; Williams et al. 2018).

Our obtained factors appear to coincide with the calibrations by Ho & Kim (2014) using the $M_\bullet-\sigma_*$ relation for pseudo bulges. If taking into account the intrinsic scatter ($\sim 0.3$ dex) of the $M_\bullet-\sigma_*$ relation (Kormendy & Ho 2013), our obtained factors are also marginally consistent with other calibrations that did not explicitly make a distinction between different morphologies of host bulges (e.g., Onken et al. 2004; Park et al. 2012; Woo et al. 2013; Grier et al. 2013a; see Table 4). The resulting dimensionless accretion rate is $\dot{M}/M_\odot = 2.4^{+2.3}_{-0.6}$, confirming that the BH in Mrk 142 is an SEAMBH accreting at a super-Eddington rate.

We end by remarking that the present dynamical modeling for BLRs is still at an early stage. Nevertheless, our application to Mrk 142 along with previous applications to a dozen AGNs (Brewer et al. 2011b; Pancoast et al. 2012, 2014b; Grier et al. 2017; Williams et al. 2018) is enlightening. Compared with the
traditional CCF approach, direct modeling of the BLR structure and dynamics can reveal much more information in the RM data and most importantly offers an approach for BH mass measurements without the need to invoke the virial factor. Future improvements of the dynamical modeling should address the issue of the associated systematic errors and incorporate physical processes (such as photoionization and radiation pressure).

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Appendix A

Inferred Parameter Values for the Three BLR Models

In Table 5, we summarize the inferred values of the major parameters for all three models. The best estimates are taken to be the median values of the posterior distributions, and the uncertainties are taken from the 68.3% confidence intervals.

| Parameter       | Inferred Parameters for Models M1, M2, and M3 |
|-----------------|------------------------------------------------|
| $b$             | $-0.11^{+0.10}_{-0.10}$                        |
| $\log(\mu_{i}/\text{lt-day})$ | $-0.92^{+0.31}_{-0.27}$                        |
| $\beta$         | $1.7^{+0.1}_{-0.1}$                            |
| $F$             | $0.09^{+0.02}_{-0.02}$                         |
| $\log(\mu_{i}/\text{d})$ | $...$                                       |
| $j_{1}$         | $1.1^{+0.2}_{-0.3}$                            |
| $\gamma$        | $4.4^{+0.4}_{-1.3}$                            |
| $\xi$           | $0.06^{+0.10}_{-0.06}$                         |
| $\log(M_{i}/M_{\odot})$ | $5.90^{+0.31}_{-0.31}$                        |
| $\delta_{\text{ellip}}$ | $0.02^{+0.04}_{-0.01}$                        |
| $f_{\text{flow}}$ | $0.76^{+0.16}_{-0.20}$                        |
| $\beta_{0}$     | $19^{+11}_{-12}$                              |
| $\sigma_{\text{inh}}$ | $-2.2^{+0.5}_{-0.8}$                          |

Appendix B

A Validity Test of the Code BRAINS

To test the validity of our code, we generate mock data with the same cadence and spectral resolution as the RM data of Mrk 142 using model M3 (the two-zone BLR model). Figure 11 shows the fitting results, and Figure 12 shows a comparison between the posterior distributions of major parameters of M3 and the input...
Figure 11. Same as Figure 3, but for fits to mock data generated using BLR model M3.
As can be seen, the posterior distributions are generally consistent with the input values.

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**References**

Abramowicz, M. A., Czerny, B., Lasota, J. P., & Szuszkiewicz, E. 1988, ApJ, 332, 646

Akaike, H. 1973, in Proc. Second Int. Symp. Information Theory, ed. B. Petrov & F. Csaki (Budapest: Akademiai Kiado), 267

Andrae, R., Kim, D.-W., & Bailer-Jones, C. A. L. 2013, A&A, 554, A137

Aoki, K., Kawaguchi, T., & Ohta, K. 2005, ApJ, 618, 601

Barth, A. J., Bennett, V. N., Canalizo, G., et al. 2015, ApJS, 217, 26

Bentz, M. C., Denney, K. D., Grier, C. J., et al. 2013, ApJ, 767, 149

Bentz, M. C., Walsh, J. L., Barth, A. J., et al. 2009, ApJ, 705, 199

Bentz, M. C., Walsh, J. L., Barth, A. J., et al. 2010, ApJ, 716, 993

Blandford, R. D., & McKee, C. F. 1982, ApJ, 255, 419

Boroson, T. A. 2002, ApJ, 565, 78

Boroson, T. A., & Green, R. F. 1992, ApJS, 80, 109

Bottorff, M., Korista, K. T., Shlosman, I., & Blandford, R. D. 1997, ApJ, 479, 200

Brewer, B. J., Piatat, L. B., & Csányi, G. 2011a, Stat. Comput., 21, 649

Brewer, B. J., Treu, T., Pancoast, A., et al. 2011b, ApJL, 733, L33

Burbridge, E. M. 1967, ARA&A, 5, 399

Cackett, E. M., Chiang, C.-Y., McHardy, I., et al. 2018, ApJ, 857, 53

Collin, S., Kawaguchi, T., Peterson, B. M., & Vestergaard, M. 2006, A&A, 456, 75

Condon, J. J., Hutchings, J. B., & Gower, A. C. 1985, AJ, 90, 1642

Cromwell, R., & Weymann, R. 1970, ApJ, 159, L147

Czerny, B., & Hryniewicz, K. 2011, A&A, 525, L8

Czerny, B., Li, Y.-R., Sredzinska, J., et al. 2017, ApJ, 846, 154

Denney, K. D., Peterson, B. M., Pogge, R. W., et al. 2010, ApJ, 721, 715

Du, P., Hu, C., Lu, K.-X., et al. 2014, ApJ, 782, 45, (Paper I)

Du, P., Hu, C., Lu, K.-X., et al. 2015, ApJ, 806, 22, (Paper IV)

Du, P., Lu, K.-X., Hu, C., et al. 2016a, ApJ, 820, 27, (Paper VI)

Du, P., Lu, K.-X., Zhang, Z.-X., et al. 2016b, ApJ, 825, 126, (Paper V)

Du, P., Zhang, Z.-X., Wang, K., et al. 2018, ApJ, 856, 6, (Paper IX)

Edelson, R., Gelbord, J., Cackett, E., et al. 2017, ApJ, 840, 41

Edelson, R., Gelbord, J. M., Horne, K., et al. 2015, ApJ, 806, 129

Emmering, R. T., Blandford, R. D., & Shlosman, I. 1992, ApJ, 385, 460

Fabian, A. C., Rees, M. J., Stella, L., & White, N. E. 1989, MNRAS, 238, 729
