Toward 2-D and 3-D simulations of core-collapse supernovae with magnetic fields

John C Hayes and Stephen W Bruenn

1 Center for Astrophysics and Space Sciences, University of California at San Diego, La Jolla, CA 92093
2 Department of Physics, Florida Atlantic University, Boca Raton, FL 33432
E-mail: jchayes@ucsd.edu, bruenn@fau.edu

Abstract. We describe a code development and integration effort aimed at producing a numerical tool suitable for exploring the effects of stellar rotation and magnetic fields in a neutrino-driven core-collapse supernova environment. A one-dimensional, multi-energy group, flux-limited neutrino diffusion module (MGFLD) has been integrated with ZEUS-MP, a multi-dimensional, parallel gas hydrodynamics and magnetohydrodynamics code. With the neutrino diffusion module, ZEUS-MP can simulate the core-collapse, bounce, and explosion of a stellar progenitor in two and three space dimensions in which multidimensional hydrodynamics are coupled to 1-D neutrino transport in a ray-by-ray approximation. This paper describes the physics capabilities of the code and the technique for implementing the serial MGFLD module for parallel execution in a multi-dimensional simulation. Because the development and debugging of the integrated code is not yet complete, we provide a current status report of the effort and identify outstanding issues currently under investigation.

1. Introduction
While roughly forty years of active research have not yet produced a definitive picture of the core-collapse supernova explosion mechanism, a general consensus on several key issues has emerged. The decay of the prompt shock into a standing accretion shock appears to be an inevitable consequence of nuclear photodissociation and enervating neutrino losses. The modern neutrino-driven explosion paradigm was born with the hypothesis that subsequent neutrino reheating behind the stalled shock, on timescales of order 100 ms or longer, could re-energize the shock and lead to an explosion. Despite the initial success of pioneering work in 1-D [1, 2], subsequent 1-D efforts with improved neutrino transport and modern equations of state have failed to produce explosions. While later 2-D simulations employing grey neutrino diffusion showed explosions [3, 4], very recent simulations coupling 2-D gas dynamics to multigroup diffusion [5] or boltzmann transport [6] have not reproduced this behavior.

That a self-consistent explosion mechanism remains to be identified leads one to ask whether success lies in improved descriptions of physical processes currently included, other processes heretofore ignored (or treated very approximately at best), or a combination of both. The new effort we describe aims to incorporate two such additional processes, magnetic fields and stellar rotation, into models featuring multi-dimensional hydrodynamics and multigroup flux-limited neutrino diffusion (MGFLD). Our goal is to investigate whether rotation and magnetic fields, acting in concert via the magnetorotational instability, can alter the dynamics of a post-bounce explosion.
core-collapse environment in a manner facilitating explosion. Because our code development and debugging is incomplete, this is a progress report rather than a science discussion. In what follows we describe the tools we have integrated into a new simulation vehicle, we identify key diagnostic measures assisting us in gauging the quality of our results, and we note hurdles remaining to be overcome.

2. Numerical tools

We compute the hydrodynamic evolution of our models with ZEUS-MP, a massively parallel 3-D implementation of the ZEUS code [7, 8]. While a public version of ZEUS-MP is available on the World Wide Web, this version is not suitable for core-collapse studies, owing both to lack of flexibility in the algorithms and a number of errors in the source code. Our version has been completely re-engineered at the software level and features many new capabilities, including: (1) 1-D and 2-D hydro modules augmenting the 3-D module, (2) an MHD module valid in both 2-D and 3-D, (3) multispecies advection, (4) accommodation of general equations of state, (5) a Poisson solver written for 2-D and 3-D problems on general orthogonal coordinate meshes, and (6) a moving grid algorithm. Items (1)-(5) have never been present in any public ZEUS code. The moving mesh option, a central feature of the ZEUS advection algorithm, was omitted from the public version of ZEUS-MP but restored in our version. We use this algorithm to allow radial grid lines to follow the fluid during infall in an approximately Lagrangean fashion in order to minimize diffusivity of advected quantities prior to core-bounce. Because we are studying models which include rotation, the infall will not be spherical. We adopted a procedure described by Symbalisty [9] to equate grid velocity to a momentum-weighted average of radial velocity over angle along a given radial grid line.

Our equation of state is a tabular form of the Lattimer-Swesty nuclear equation of state [10]. The LS EOS code has enjoyed widespread use in core-collapse calculations; our tabular version employs an interpolation procedure which preserves thermodynamic consistency of the computed quantities.

Transport of all 3 neutrino species and their antiparticles is computed with a neutrino transport module based upon the method described by Bruenn [11]. Since the method was first published, a number of extensions and improvements have been made. The flux-limiter currently in use has been calibrated against 1-D Boltzmann transport and described in [12]. While not yet in use in our simulations, general relativity has been added as described in [13], and additional neutrino processes, such as neutrino-nucleon elastic scattering [14] and nucleon-nucleon bremsstrahlung [15], have been added.

3. Parallel implementation

ZEUS-MP uses the Message Passing Interface (MPI) library to perform parallel simulations on distributed-memory computing platforms. The 2-D (and eventually 3-D) simulations comprising our target application proceed by subdividing the problem domain along each coordinate axis and assigning unique chunks of problem data to each processor. MGFLD is a 1-D code written for serial execution; the desire to deploy MGFLD for ray-by-ray transport in a parallel multi-D environment presented a non-trivial design challenge. We found an effective means for interfacing the two codes in the Global Arrays Toolkit (GAT) [16], a parallel library which simulates global shared-memory access on distributed-memory computers. The GAT allows us to define global arrays of density, temperature, $Y_e$, and other needed field variables that are accessible to all processors in a calculation. GAT functions allow each processor to select a unique angular grid line (a “ray”) and extract 1-D radial distributions of the required data from that ray. With the ray data in hand, all processors may execute 1-D MGFLD transport solutions simultaneously. A second call to the GAT library then performs the inverse mapping of updated ray data back into the appropriate multi-D arrays defined by ZEUS-MP.
Employing the serial MGFLD module in a parallel calculation in the manner described above requires extensive data communication between processors in a parallel calculation, the management of which is the responsibility of the GAT-powered interface. The scalability of this interface is therefore an important consideration. Preliminary testing of the interface in the core-collapse context is quite encouraging. Table 1 shows timing results for 100 cycles of a core-collapse calculation coupling hydrodynamics to MGFLD transport with neutrino-electron scattering accompanying standard absorption, emission, and isoenergetic scattering processes. 256 radial zones are used in each calculation, and the number of angular zones matches the number of processors, thus in this comparison perfect scalability would correspond to a constant measure of CPU wall clock time. The GAT interface is not used in the single-processor baseline calculation. The data show agreeably flat timing results among the multi-processor calculations, as well as a fairly uniform overhead in CPU relative to the single-processor baseline. The excellent scaling results indicate that the CPU cost of the MGFLD update is more than sufficient to amortize the communication costs incurred by the GAT interface.

Table 1. Timing results for ZEUS-MP+MGFLD with the GAT interface.

| # Processors | CPU Wall Clock (sec) |
|--------------|----------------------|
| 1            | 172                  |
| 8            | 193                  |
| 16           | 196                  |
| 32           | 198                  |
| 64           | 197                  |
| 128          | 200                  |

4. Initial models
To test and evaluate our integrated code, we have adopted the Woosley and Weaver [17] “s15s7b” 15 $M_\odot$ progenitor for core-collapse calculations. To test the effects of stellar rotation, we map the s15s7b model onto a 2-D grid and superimpose an angular velocity law published by Buras et. al [6] for their 2-D ray-by-ray calculations with a similar progenitor model. For tests with magnetic fields, we print a uniform magnetic field aligned with the polar axis on the mesh.

The use of 1-D progenitor models to initiate multi-D simulations involving either rotation or magnetic fields merits further discussion. Imposing rotation laws or magnetic fields on progenitors which were evolved without such effects introduces a physical inconsistency into our calculations. While 1-D progenitors which include rotation and magnetic fields in an approximate way are available (e.g. [18]), we have found that mapping such models onto a 2-D mesh is numerically problematic, a finding reported by other researchers [19]. Furthermore, a detailed understanding of the effects of rotation and magnetic fields on pre-collapse massive star evolution is not yet at hand, thus large uncertainties remain regarding the true structure of core-collapse progenitors (see, for example, the discussion in [19]). These issues, coupled with our desire to connect our research results to those previously published, justifies in our minds the approach of adding simple but physically plausible prescriptions for rotation and magnetic fields to existing models on which the science community heavily relies.
5. Development milestones

As of this writing, we are unable to present post-bounce results from 2-D core-collapse simulations, the current roadblock being unresolved parallel-implementation errors in the hydrodynamics module. Nonetheless, a number of key milestones in the project have been achieved, and we present a short summary of early results here.

Figure 1 presents a time series of plots showing entropy vs radius during the first 100 ms of post-bounce evolution from a 1-D radiation hydrodynamic (RHD) core-collapse simulation. Extensive testing in 1-D has revealed a shortcoming with regard to energy conservation: immediately prior to and just after core bounce, we observe a transient in the measured value of the total energy of our evolving model. This transient has an amplitude on the order of one foe ($10^{51}$ erg) and is followed by a net loss in total energy of a foe or more. Because this is comparable to the expected explosion energy of a successful model, this is an unacceptably large error. Further study has indicated several contributing factors to the problem, including inconsistencies resulting from our procedure for operator-splitting the neutrino evolution and the hydrodynamics; errors in the evolution of the neutrino energy spectrum due to fluid motions, and issues with timestep control. We are currently implementing new procedures to correct the problems we have identified to date.

While we have identified energy conservation as an area in which improvement is required, our anticipated science agenda prompts us to advertise a strength of ZEUS-MP critical for studies of rotating core-collapse: conservation of angular momentum. A fluid advection scheme which conserves globally-integrated angular momentum is not guaranteed to conserve angular momentum locally unless particular care is taken to advect velocities across the mesh consistently with the mass flux, a point demonstrated quantitatively by Norman et. al [20] in the evaluation of hydrodynamics methods for the problem of protostellar collapse. As these authors note, a precise measure of angular momentum conservation is provided by the specific angular momentum spectrum, $M(K)$, where $M(K)$ is the total mass in the system with specific angular momentum $K$. The specific angular momentum spectrum is shown in Figure 2 along with the total mass $M$ as a function of the specific angular momentum $K$ at two different times, $t = 0$ (solid line) and $1$ ms before bounce (open circles).
\((J/\rho)\) less than or equal to \(K\):

\[
M(K) = \int_0^K dM(k). \quad (1)
\]

For a system with no external torques, this quantity is a constant of the motion, and computed values should therefore remain invariant during collapse. Figure 2 shows the calculation of \(M(K)\) by ZEUS-MP for the adiabatic collapse of our 15 \(M_\odot\) progenitor augmented by the initial rotation profile used in Buras et. al [6]. Integrated mass is computed in solar units; \(K\) is expressed in CGS units. The solid line shows the spectrum computed at the start of the simulation; the open circles correspond to the time at which the central density reaches \(3 \times 10^{14}\) g cm\(^{-3}\), roughly 1 ms prior to core bounce. 256 radial zones and 32 angular zones were used in this demonstration calculation. The horizontal portion of the curve at low values of angular momentum (found near the poles and origin of the grid) is an artifact of grid resolution.

6. Summary

We have coupled ZEUS-MP, a multi-dimensional hydrodynamics code, to MGFLD, a 1-D radial neutrino transport code, to produce a simulation tool for studying 2-D and 3-D core-collapse evolution with rotation and magnetic fields. Current work includes 1-D studies aimed at improving energy conservation and 2-D experiments used to test and debug the hydrodynamics module in parallel simulations.

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