Testing a Model with additional Vector Fermions at the LEP2 Collider

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Abstract

Our aim is to test a model recently presented, motivated by the reported $R_b$ and $R_c$ “crisis”, which contains extra vector fermions. We suggest an alternative indirect test of the possible existence of new heavy quark flavours at the LEP2 collider, which turns out to give the clearest signal. We calculate $q\bar{q}$ cross sections within this framework, including one loop corrections, and find measurable differences compared to the Standard Model predictions.

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1 Introduction

Motivated by the so-called $R_b, R_c$ crisis that arose during the last couple of years, several models have been invoked, to explain this puzzle using different kinds of extensions to the Standard Model (SM). Besides (leptophobic) $Z'$ models and supersymmetric corrections, there were several attempts which introduce new heavy vector fermions [1, 2]. Even though the latest publications by the LEP collaborations [3] show a value of $R_c = 0.1715 \pm 0.0056$, which hence lies within the predictions of the SM, and the $R_b$ value is reported to be $R_b = 0.2178 \pm 0.0011$, yet in excess by only 1.9$\sigma$ of the SM, the suggested models cannot be entirely excluded and it seems worthwhile at this stage to find an alternative test of these models, such that a picture of consistency may (or may not) emerge. There are now many predictions on how to test the $Z'$ model [4] in a more general framework independently from $R_b$, $R_c$ measurements, but there is still lacking information about the testability of the new vector fermion models beyond their genuine motivation.

In this paper we are considering the model suggested by Chang et al. [2], which introduces a new heavy vector quark triplet $X$ mixing with the quarks of the SM. As a consequence of this mixing, there are effects to be expected, showing up in different processes as stated below. Adopting especially this model, we can simultaneously solve for possible excesses of $R_b$ as well as $R_c$, without introducing anomalies.

The possible tests for this model could be:

(a) the calculation of, e.g., heavy flavour production, analogue to the $Z'$ at hadron colliders, to judge, whether a significant change in the cross section can be observed,

(b) the study of flavour–changing neutral current (FCNC) effects in, e.g., decay modes of some mesons like $K^0_L$ or $B^0$,

(c) low energy physics experiments, e.g., $\nu N$ scattering,

(d) quark–antiquark production at $e^+e^-$ colliders, especially at the CERN LEP2 collider.

An analogous treatment to the $Z'$ model like in (a) is not possible because the new vector and axial–vector couplings induced by the mixing with the new vector fermions are still of order $\alpha_W$ instead of order $\alpha_S$, as they were for the $Z'$ coupling [3]. The introduction of new vector fermions produces new small FCNC effects (b) at the tree–level, which are, however, not in disagreement with the relevant experiments [3]. The limits, imposed by FCNC experiments, exclude the possibility of low energy measurements (c), like $\nu N$ neutral current scattering, as discussed, e.g., in [3] for the case of the $Z'$.

We employ in this Letter the effect of the mixing between the new quark vector triplet and the SM quark flavours $q$ on the production of $q\bar{q}$ pairs at the LEP2 collider. We do not claim that that our $q\bar{q}$ studies are the unique testing ground: further studies must follow to find the best signal/background ratio for this new vector fermions model.
To give a brief outline of this Letter: we first give some details about the model we employ. In Section 3 we fit the model parameters to recent values of $R_b$ and $R_c$ and present numerical results for various $q\bar{q}$ cross sections at LEP2 energies. We conclude with some critical remarks concerning the detectability of our predicted effects in Section 4.

2 Short Description of the Model

We shall be very cursory in the description of this model as it was originally motivated and discussed in Ref. \[2\]. Introducing the following vector quark triplet

$$X^{(Q,T^3)}_{L,R} = \begin{pmatrix} x_1^{(5/3,+1)} \\ x_2^{(2/3,0)} \\ x_3^{(-1/3,-1)} \end{pmatrix}_{L,R},$$

with mass $M_X$ and the stated quantum numbers for charge ($Q$) and third component of weak isospin ($T^3$), we allow a mixing of $(x_3)_L$ with the $d$–type quarks ($d, s, b$)$_L$ and $(x_2)_L$ with the $u$–type quarks ($u, c, t$)$_L$. In the context of this model it is expected that the $M_X$ is much bigger than the top–quark mass. However, we are exclusively interested in the mixing effects this new quark triplet shows with the quark flavours of the SM. The mixing of the left–handed components are proportional to $1/M_X$, while for the right–handed couplings a $1/M_X^2$ dependence can be found, assuming a large $M_X$ approximation, and therefore the latter can be neglected (cf. Refs. \[2, 8\]). The neutral–current Lagrangian, including $\gamma$ and $Z$ exchange, reads

$$\mathcal{L}_{NC} = eJ_{\mu}^{em} A^\mu + \sqrt{2} \left( \frac{G_F}{\sqrt{2}} \right)^{1/2} M_Z \sum_q \bar{\Psi}_q \gamma_\mu (v_q - a_q \gamma_5) \Psi_q Z^\mu,$$

where the $\Psi_q$ are meant to be gauge–eigenstates. One can immediately see that taking into account the unitary transformation matrix that shuffles mass– into gauge–eigenstates, $\Psi_{q,L,R}^\text{gauge} \rightarrow U_{L,R} \Psi_{q,L,R}^\text{mass}$, modifies the isospin matrices $T^3_{q,L,R} \rightarrow \overline{U}_{L,R} T^3_{q,L,R} U_{L,R}$. As a result, we finally conclude that the vector and axial–vector couplings defined by

$$v_q = g_q^L + g_q^R = T^3_{q,L} + T^3_{q,R} - 2Q_q \sin^2 \Theta_W,$$

$$a_q = g_q^L - g_q^R = T^3_{q,L} - T^3_{q,R},$$

are directly influenced by the presence of this new vector triplet. Consider, e.g., the new mass matrix $M_D$ for $d$–type quarks

$$M_D = \begin{pmatrix} m_d & 0 & 0 & J_1 \\ 0 & m_s & 0 & J_2 \\ 0 & 0 & m_b & J_3 \\ 0 & 0 & 0 & M_X \end{pmatrix} = \begin{pmatrix} \tilde{M}_D & J \\ 0 & M_X \end{pmatrix},$$

where $J_1, J_2, J_3$ are diagonal matrices.

2
where the matrix elements $J_i$ measure the relative strength of the mixing between $(x_3)_L$ and the corresponding $d$–type quarks. $U_L$ diagonalises the matrix product $M_D M_D^\dagger$. Assuming the most general form

$$U_L = \begin{pmatrix} K & R \\ S & T \end{pmatrix},$$  

we find in the large $M_X$ approximation [3]

$$R = \frac{1}{M_X} J, \quad S = -\frac{1}{M_X} J^\dagger K,$$  

with $K$ being the $3 \times 3$ unity matrix.

In complete analogy we can study the $u$–type quark mixing mediated by the matrix $U'_L$ and it can be shown that $S' = S K'$, with $K' \approx V_{\text{CKM}}^\dagger$, where $V_{\text{CKM}}$ denotes the standard Cabibbo–Kobayashi–Maskawa matrix.

Consequently we derive for the new vector and axial–vector couplings in the case of the $u$– and $d$–type quarks

$$\tilde{v}_u = v_u + (T_{3u}^L - T_u^L) |U'_{xu}|^2, \quad \tilde{a}_u = a_u + (T_{3u}^L - T_u^L) |U'_{xu}|^2,$$  

$$\tilde{v}_d = v_d + (T_{3d}^L - T_d^L) |U_{xd}|^2, \quad \tilde{a}_d = a_d + (T_{3d}^L - T_d^L) |U_{xd}|^2.$$

This explains the former special choice for the isospin components of the vector triplet. As $\Gamma(e^+ e^- \to q\bar{q}) \propto \left(\tilde{v}_q^2 + \tilde{a}_q^2\right)$, $T_{3u}^L = 0$ will reduce the hadronic width of the $u$–type quarks, whereas $T_{3d}^L = -1$ enhances the hadronic width of the $d$–type quarks and therefore lower the SM value of $R_c$ and simultaneously raise $R_b$. In terms of the above formalism, the modified couplings of, e.g., the $b$ and $c$ quarks read now

$$\tilde{v}_c = v_c - \frac{1}{2} |S'_2|^2, \quad \tilde{a}_c = a_c - \frac{1}{2} |S'_2|^2,$$  

$$\tilde{v}_b = v_b - \frac{1}{2} |S_3|^2, \quad \tilde{a}_b = a_b - \frac{1}{2} |S_3|^2,$$

where $S$ and $S'$ are to be identified with the matrix elements used in the definition of $U_L$ (cf. Eq.(6)) and $U'_L$. Thus the model depends on 6 parameters $(S_{1,2,3}, S'_{1,2,3})$. But because of the relation $S' = S V_{\text{CKM}}^\dagger$, it is sufficient to constrain 3 parameters in order to calculate the full set. The input to constrain $S_2$ and $S_3$ will be $R_c$ and $R_b$, respectively. After determining all modified vector and axial–vector couplings $\tilde{v}_q$ and $\tilde{a}_q$, we proceed to answer the question of how the total $q\bar{q}$ cross section at LEP2 is affected by the presence of the additional vector fermions.

The total cross section for the subprocess $e^+ e^- \to Z q\bar{q}$, where $q$ stands for one quark flavour, via $Z$ and $\gamma$ exchange, reads in the finite–quark–mass Born approximation

$$\sigma^0(e^+ e^- \to Z q\bar{q}) = \frac{\beta}{2} (3 - \beta^2) \sigma^0_V + \beta^3 \sigma^0_A,$$  

(12)
\[ \sigma_V^0 = \frac{4\pi\alpha^2}{s}Q_e^2Q_q^2 + \frac{4\alpha}{\sqrt{2}}G_F Q_e Q_q v_e \tilde{v}_q \frac{M_Z^2(s - M_Z^2)}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2} \]
\[ + \frac{G_F^2 v_q^2}{2\pi} (a_e^2 + v_e^2) \frac{sM_Z^4}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2}, \] 
\[ \sigma_A^0 = \frac{G_F^2 a_q^2}{2\pi} (a_e^2 + v_e^2) \frac{sM_Z^4}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2}, \] 

with the quark velocity \( \beta = \sqrt{1 - 4m_q^2/s} \) and the electron couplings \( v_e = -1/2 + 2\sin\Theta_W \) and \( a_e = -1/2 \). For further details we refer to, e.g., [9]. We implemented in our calculations the following corrections:

- QCD \( \mathcal{O}(\alpha_S) \) corrections due to real and virtual gluon emission [9],
- universal QED \( \mathcal{O}(\alpha) \) corrections \( 3\alpha Q_e^2/(4\pi) \),
- initial state radiation up to \( \mathcal{O}(\alpha^2) \) in QED and soft photon exponentiation [10],
- universal electroweak corrections due to \( tt \) and \( tb \) loop corrections to the \( Z \) propagator and \( \gamma-Z \) mixing as well as vertex corrections for \( b \) quarks due to virtual \( t \) quark exchanges [11]. The box–graph contributions turn out to be unimportant for our purposes [12].

### 3 Numerical Results

For our numerical analysis we used the on–shell scheme addressing the following electroweak data: \( \alpha(M_Z) = 1/128.8, M_Z = 91.188 \text{ GeV}, M_W = 80.33 \text{ GeV} \) with the strong coupling constant \( \alpha_S(M_Z) = 0.123 \). Furthermore we fixed the top quark mass to be \( m_t = 175 \text{ GeV} \). For the CKM–matrix elements we used the averaged values given in Ref. [13].

Starting from the latest set of values for \( R_b \) and \( R_c \) (as discussed above), we first performed a general fit of the parameter space, including \( |S'_2|^2 \) and \( |S'_3|^2 \), as they directly govern the values for \( \Gamma_{cc} \) and \( \Gamma_{bb} \). Furthermore, following Ref. [2], we set \( |S_1|^2 \simeq 0 \). This is invoked by taking into account the FCNC of the process \( K_0^0 \rightarrow \mu^+\mu^- \), which does not support any sizeable \( d-s \) quark mixing. The missing values \( |S'_1|^2, |S'_3|^2 \) and \( |S_2|^2 \) are then calculated, consistent the relation \( S' = SV_{\text{CKM}}^\dagger \). Although we found a weak dependence of the various widths \( (\Gamma_Z, \Gamma_{Z\text{rad}}) \) on the input parameters, the cross sections remain quite insensitive to the widths compared to the modified coupling–dependence, as can easily be deduced from the formal expression of the total cross section in Eqs. (12)–(14).

Figure (1) shows our fitted values for \( |S'_2|^2 \) and \( |S'_3|^2 \), in particular the edges of the \( 1\sigma \) (68.3% confidence level of the normal distribution) and the \( 2\sigma \) (95.4% c.l.) regions
of \( R_b \) and \( R_c \). We find \(|S'_2|^2 = 0.01245 \) and \(|S'_3|^2 = 0.00922 \) for 1σ (corresponding to \( R_b = 0.2189 \) and \( R_c = 0.1659 \)) and \(|S'_2|^2 = 0.02528 \) and \(|S'_3|^2 = 0.01284 \) for the 2σ case (\( R_b = 0.2200 \) and \( R_c = 0.1603 \)).

With these two sets of parameters deduced from \( R_b \) and \( R_c \) we first give predictions for the subprocess cross sections \( \sigma(e^+e^- \rightarrow s\bar{s}, c\bar{c}, b\bar{b}) \), as they turn out to give the most significant signal. Specifically, Fig. (2a) shows the \( c\bar{c} \) production cross section as a function of the centre–of–mass energy. The contribution of the additional vector fermions to this cross section is negative, as it is for all \( u \)–type quarks, which can easily be checked from Eq. (8), as the former motivation is to decrease the SM value of \( R_c \). The result is very sensitive to the values of \( R_b \) and \( R_c \), as, e.g., the 1σ input and the 2σ input differ by a factor of roughly 2. Moreover it can be observed that in the energy region of the LEP2 collider (160 GeV–190 GeV), the contribution is nearly insensitive to \( \sqrt{s} \). The observed gaps in all figures which appear around the \( Z \) mass \( M_Z = 91.188 \) GeV, are due to the resonant behaviour of the total cross section.

| \( \sqrt{s} \) | \( c\bar{c} \) \(|\sigma_{q\bar{q}}| [\text{pb}] \) | \( \delta\sigma_{q\bar{q}}/\sigma_{q\bar{q}} [\%] \) | \( b\bar{b} \) \(|\sigma_{q\bar{q}}| [\text{pb}] \) | \( \delta\sigma_{q\bar{q}}/\sigma_{q\bar{q}} [\%] \) |
|----------------|----------------|----------------|----------------|----------------|
| 175 GeV  | 36.69 | -5.31 | 12.75 | +3.83 |
| 190 GeV  | 29.75 | -5.26 | 10.16 | +3.82 |

Table 1: The numerical values for \( \sigma_{q\bar{q}} = \sigma(e^+e^- \rightarrow q\bar{q}) \) and \( \delta\sigma_{q\bar{q}}/\sigma_{q\bar{q}} \) in the vector fermion model, for \( q \) being either a \( c \) or a \( b \) quark. The calculations were performed for two typical values of \( \sqrt{s} \) at LEP2, and for the two confidence levels as discussed in the text.

All the argumentation drawn from Fig. (2a) also holds for the discussion of the \( s\bar{s} \) and \( b\bar{b} \) cross sections, presented in Figs. (2b)–(2c). The main difference is that the total contribution of the additional vector fermions is positive for \( d \)–type quarks. Again we refer to Eq. (9). Even though the absolute value of the \( s\bar{s} \) contribution is comparable to Fig. (2a), there is no hope to isolate this cross section in a LEP2 measurement. The \( b\bar{b} \) cross section, however, will be measurable and according to Ref. [14] the experimental uncertainty in \( \sigma_{b\bar{b}} \) is reported to be 2.5% for \( \sqrt{s} = 190 \) GeV and an assumed luminosity of 500 \( \text{pb}^{-1} \), which allows for a clear signal. Although former studies at the LEP2 showed a general lower accuracy for the tagging of \( c\bar{c} \) events, it still might be sufficient to detect our
calculated 5% effect in $\sigma_{ee}$ as shown in Fig. (2a). All numerical results are summarised in Table 1 for two fixed LEP2 centre–of–mass energies of $\sqrt{s} = 175$ GeV and 190 GeV.

Finally we present in Fig. (3) the total cross sections for $d$– and $u$–type quarks. Again, we can see the overall tendency that the SM cross section is being lowered under the presence of the vector fermions for $u$–type quarks, whereas we find a proper enhancement for $d$–type quarks, which is the characteristic feature of this model. We can not expect a tagging of these individual cross sections at this level of accuracy at LEP2, but for reasons of completeness we want to mention it at this stage, especially to demonstrate that there will be no signal to be expected in the total cross section, as individual subprocess contributions will cancel each other to an almost zero level.

4 Remarks and Conclusion

In this Letter we studied the impact of a model with additional vector fermions at the LEP2 collider. We made predictions for various quark production cross sections and discussed their possible detectability based on a recent phenomenological analysis given in Ref. [14]. However, there is probably no evidence for new physics at this stage, especially after the values of $R_b$ and $R_c$ are approaching the SM predictions, although, one can argue that exploiting the idea of additional vector fermions at energies beyond the $Z$ pole is of considerable interest. A remarkable feature of the model we were dealing with throughout our studies is that it is anomaly free, in contrast to alternative ideas emerging from the reported “crisis”, and therefore seems to be more physically sound.

Even though our predictions yield relatively small effects, within the scope of accuracy at LEP2 they might be measureable. Present studies concerning statistical and systematic errors at LEP2 are already underway. Especially taking into account that the overall corrections are different in sign for $u$– and $d$–type quarks, the simultaneous tagging of $c\bar{c}$ and $b\bar{b}$ pairs might further reduce statistical errors. We believe that this will yield, as a unique feature, a clear signal due to the existing splitting around the SM predictions, as we have already discussed in Section 3.

As a final remark we want to mention that we also checked a possible influence on the forward–backward asymmetries $A_{FB}^q$. At LEP1 energies the modified asymmetries are within the experimental errors, as already discussed in Ref. [2] and by Ma in Ref. [1]. The accuracy at LEP2 for the corresponding measurements is expected to be even lower, such that it is difficult to draw any conclusions from forward–backward asymmetry measurements.
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Figure 1: We show the allowed regions in the $|S'_2|^2$–$|S_3|^2$ plane (as discussed in the text) obtained by fitting the recent $R_b$, $R_c$ values [3]. We present the 1σ (68.3% c.l.) and 2σ (95.4% c.l.) regions, from which we read off our input parameters $|S'_2|^2$ and $|S_3|^2$. 

$R_b=0.2178\pm0.0011$, $R_c=0.1715\pm0.0056$

central values: $|S'_2|^2 = 0.00004$, $|S_3|^2 = 0.00588$

$|S'_2|^2$ $|S_3|^2$
Figure 2: We present the relative differences between the predictions of the vector fermion (VF) model and the SM ($\delta \sigma_q/\sigma_q := (\sigma_{VF}^{qq} - \sigma_{SM}^{qq})/\sigma_{SM}^{qq}$) in per cent as a function of $\sqrt{s}$, for three different flavours.
Figure 3: The total cross section for $u$- (a) and $d$-type (c) quarks in the SM calculation and the two vector fermion fits as discussed in the text, including the relative changes (b) and (d) as already shown in Fig. 2.