Low temperature transport in granular metals

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We investigate transport in a granular metallic system at large tunneling conductance between the grains, $g_T \gg 1$. We show that at low temperatures, $T \leq g_T \delta$, where $\delta$ is the single mean energy level spacing in a grain, the coherent electron motion at large distances dominates the physics, contrary to the high temperature ($T > g_T \delta$) behavior where conductivity is controlled by the scales of the order of the grain size. The conductivity of one and two dimensional granular metals, in the low temperature regime, decays with decreasing temperature in the same manner as that in homogeneous disordered metals, indicating thus an insulating behavior. However, even in this temperature regime the granular structure remains important and there is an additional contribution to conductivity coming from short distances. Due to this contribution the metal-insulator transition in three dimensions occurs at the value of tunnel conductance $g_T^C = (1/6\pi) \ln(E_C/\delta)$, where $E_C$ is the charging energy of an isolated grain, and not at the generally expected $g_T^C \propto 1$. Corrections to the density of states of granular metals due to the electron-electron interaction are calculated. Our results compare favorably with the logarithmic dependence of resistivity in the high-$T_c$ cuprate superconductors indicating that these materials may have a granular structure.

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A great deal of research in the current mesoscopic physics focuses on understanding properties of granular metals (see \cite{1, 2, 3}). The interest is motivated by the fact that while their properties are generic for a wealth of strongly correlated systems with disorder, granular metals offer a unique experimentally accessible tunable system where both the interaction strength and degree of disorder can be controlled.

The key phenomenon revealing the most of the underlying physics is transport, where the effects of interactions play a crucial role. The processes of electron tunneling from grain to grain that govern electron transfer, are accompanied by charging the grains involved after each electron hop to another grain. This may lead to a Coulomb blockade, and one justly expects this effect to be of the prime importance at least in the limit of weak coupling. It makes it thus clear, on a qualitative level, that it is the interplay between the the grain-to-grain coupling and the electron-electron Coulomb interaction that controls transport properties of granular metals; yet, despite the significant efforts expended, a quantitative theory of transport in metallic granular systems is still lacking.

A step towards formulation such a theory was made recently in Ref. \cite{3}. It was shown that depending on the dimensionless tunneling conductance $g_T$ one observes either exponential-, at $g_T \ll 1$, or logarithmic, at $g_T \gg 1$ temperature dependence of conductivity. The consideration in Ref. \cite{3} was based on the approach developed by Ambegaokar, Eckern and Schön (AES) \cite{4} for tunnel junctions. This technique however, as shown in Ref. \cite{4}, applies only at temperatures $T > g_T \delta$, where $\delta$ is the mean energy level spacing in a single grain, in this regime the electron coherence does not extend beyond the grain size. At low temperature region, $T \leq g_T \delta$, the effects of the electron coherent motion at distances much exceeding the single grain size $a$ must be included, thus this important regime is not described by the AES approach \cite{4}.

Although experimentally the low temperature regime is well within the experimental reach \cite{1, 2, 3}, it has never been addressed theoretically so far. The important question whether the system is a metal or becomes an insulator, in other words, whether the conductivity of the granular metals at large conductances remain finite in the limit of $T \to 0$ is still open.

In this Letter we investigate the low-temperature conductivity of granular samples focusing on the case of large tunneling conductance between the grains, $g_T \gg 1$. To this end we develop a technique that goes beyond the AES approach and includes effects of coherent electron motion at distances larger than the size of the grain. Without the Coulomb interaction the granular system would be a good metal in the limit, $g_T \gg 1$, and our task is to include the charging effects into the theory. We find that at temperatures, $T \leq g_T \delta$ properties of the granular metal depend on the dimensionality of the array, and corrections to the conductivity and density of states due to the effects of Coulomb interaction are similar to those obtained in Ref. \cite{3} for a homogeneous metal. Thus at low temperatures the systems behaves essentially as a homogeneous metal contrasting the case of large temperatures, $T \gg g_T \delta$ considered in Ref. \cite{3}.

This in particular means that at large conductances the 3D system is a good metal. On the other hand, at $g_T \ll 1$ a granular sample is in the insulating state. Therefore a 3D system should exhibit a metal-insulator transition at the critical value of the conductance $g_T$, such that
samples with conductances $g_T > g_T^C$ are metals and their conductivity remains finite at $T \to 0$ while samples with $g_T < g_T^C$ are insulators and their conductivity vanishes at $T \to 0$.

The main results of our work are as follows: (i) We find the critical value $g_T^C$ of the tunnel conductance at which the metal-insulator transition in 3D occurs

$$g_T^C = (1/6\pi) \ln(E_C/\delta),$$

where $E_C$ is the charging energy of an isolated grain. (ii) We find the expression for the conductivity of a granular metal that includes corrections due to Coulomb interaction and holds for all temperatures as long as these corrections are small. The corresponding answer can be conveniently written separating the correction due to the contribution from the large energy scales $\varepsilon > g_T\delta$ from that coming from the low energy scales $\varepsilon < g_T\delta$. Denoting corrections as $\delta\sigma_1$ and $\delta\sigma_2$ respectively we have

$$\sigma = \sigma_0 + \delta\sigma_1 + \delta\sigma_2,$$  \hspace{1cm} (2a)

where $\sigma_0 = 2e^2g_Ta^{2-d}$, with $a$ being the size of the single grain is the classical Drude conductivity for a granular metal (spin included). Correction $\delta\sigma_1$ in Eq. (2a) contains the dimensionality of the array $d$ only as a coefficient and is given by the following expression

$$\frac{\delta\sigma_1}{\sigma_0} = -\frac{1}{2\pi g_T} \ln \left( \frac{g_T E_C}{\max(T,g_T\delta)} \right).$$  \hspace{1cm} (2b)

On the contrary the correction $\delta\sigma_2$ in Eq. (2a) that is important only at temperatures $T < g_T\delta$ strongly depends on the dimensionality of the array

$$\frac{\delta\sigma_2}{\sigma_0} = \begin{cases} \frac{\alpha}{12\pi^2 g_T} \sqrt{T/g_T\delta} & D = 3, \\ \frac{\beta}{4\pi^2} \ln T/g_T\delta & D = 2, \\ \frac{\beta}{4\pi} \sqrt{T/g_T\delta} & D = 1. \end{cases}$$  \hspace{1cm} (2c)

Here $\alpha = \int_0^\infty dx x^{-1/2} \left[ 1 - \coth(x) + x/\sinh^2(x) \right] \approx 1.83$ and $\beta = \int_0^\infty dx x^{-3/2} \left[ \coth(x) - x/\sinh^2(x) \right] \approx 3.13$ are the numerical constants. For a 3D granular system a temperature independent term of the order $1/g_T$ has been subtracted in the first line in Eq. (2c).

Corrections $\delta\sigma_1$ and $\delta\sigma_2$ are of a different origin: the correction $\delta\sigma_1$ comes from the large energy scales, $\varepsilon > g_T\delta$ where the granular structure of the array dominates the physics. The fact that this correction is essentially independent of the dimensionality $d$ means that the tunneling of electrons with energies $\varepsilon > g_T\delta$ can be considered as incoherent. On the other hand, correction $\delta\sigma_2$ in Eq. (2c) is similar to that obtained for homogeneous metals long ago, and comes from the low energy scales, $\varepsilon \leq g_T\delta$, where the coherent electron motion on the scales larger than the grain size $a$ dominates the physics.

It is important to note that in the low temperature regime all temperate dependence of conductivity comes from the correction $\delta\sigma_2$. At the same time, in this regime the correction $\delta\sigma_1$, though being temperature independent, still exists and can be even larger than $\delta\sigma_2$.

When deriving Eqs. (2) we neglected possible weak localization corrections that may originate from quantum interference of electron waves. This approximation is legitimate if a magnetic field is applied as in Ref. 3 or dephasing is strong due to inelastic processes.

Now we turn to the description of our model and the derivation of Eqs. (2). We consider a $d$-dimensional array of metallic grains with the Coulomb interaction between electrons. The motion of electrons inside the grains is diffusive and they can tunnel from grain to grain. In principle, the grains can be clean such that electrons scatter mainly on grain surfaces. We assume that the sample in the absence of the Coulomb interaction would be a good metal. For large tunneling conductance we may also neglect the nonperturbative charging effects (discreteness of the electron charge), which give an exponentially small (as $\exp(-\#g_T)$) contribution to the conductivity. Although we assume that the dimensionless tunneling conductance $g_T$ is large, it should be still smaller than the grain conductance, $g_0$, such that $g_T < g_0$. This inequality means that the granular structure is still important and the main contribution to the macroscopic resistivity comes from the contacts between the grains.

The system of weakly coupled metallic grains can be described by the Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{H}_c + \sum_{ij} t_{ij} \left[ \hat{\psi}^\dagger(r_i) \hat{\psi}(r_j) + \hat{\psi}^\dagger(r_j) \hat{\psi}(r_i) \right],$$

where $t_{ij}$ is the tunneling matrix element corresponding to the points of contact $r_i$ and $r_j$ of $i$-th and $j$-th grains. The Hamiltonian $\hat{H}_0$ in Eq. (3) describes noninteracting isolated disordered grains. The term $\hat{H}_c$ describes the Coulomb interaction inside and between the grains. It has the following form

$$\hat{H}_c = \frac{e^2}{2} \sum_{ij} \hat{n}_i C_{ij}^{-1} \hat{n}_j,$$

where $C_{ij}$ is the capacitance matrix and $\hat{n}_i$ is the operator of electrons number in the $i$-th grain. In the regime under consideration one can neglect the coordinate dependence of a single grain diffusion propagator. The electron hopping between the grains can be included using the diagrammatic technique developed in Refs. 3, 4, 8, which we outline below.

Electron motion in a random impurity potential within a single grain can be considered using the standard diagrammatic techniques described, for example in Ref. 3. Electron hoppings between the grains can be considered in a similar way assuming that tunneling matrix elements between the grains are random variables obeying the Gaussian statistics and correlated as

$$\langle t_{k_1,k_2} t_{k_3,k_4} \rangle = t^2 (\delta_{k_1,k_3} \delta_{k_2,k_4} + \delta_{k_1,k_4} \delta_{k_2,k_3}),$$

(5)
where \( t \) is related with the average intergranular conductance as \( g_T = 2\pi t^2/\delta^2 \). The average Green function is defined by the Dyson equation where self energy, shown on Fig. 1 has two contributions: The first contribution (a) corresponds to scattering inside a single grain while the second (b) is due to processes of scattering between the neighboring grains. Both this processes result in a similar contribution \( \sim \text{sign}(\omega) \) to the electron self-energy thus on the level of single particle electron Green function intergranular scattering results only in small renormalization of the relaxation time \( \tau \)

\[
\tau^{-1} = \tau_0^{-1} + 2dg_T\delta,
\]

where \( \tau_0 \) is the electron mean free time in a single grain.

The next step is to consider the diffusion motion of electron through a granular metal: Diffusion motion inside a single grain is given by the usual ladder diagram that results in the diffusion propagator

\[
D_0(\Omega) = \frac{1}{\tau|\Omega|},
\]

where coordinate dependence was neglected since we assume the zero dimensional limit for a single grain. Tunneling between the grains is accounted for in a similar way, such that the total diffusion propagator is given by the ladder diagrams shown on Fig 2a. This results in the following expression:

\[
D(\omega, q) = \frac{1}{\tau |\Omega| + \delta \varepsilon_q},
\]

where \( \varepsilon_q = 2g_T \sum_a(1 - \cos qa) \) with \( a \) being the lattice vectors. For small quasimomenta \( q \ll a^{-1} \) we have \( \varepsilon_q \to g_T\delta a^2 q^2 \) such that the propagator \( \delta \varepsilon_q \) describes the diffusion motion on the scales much larger than \( a \) with effective diffusion coefficient \( D = g_T a^2 \delta \).

The same ladder diagrams describe the dressing of interaction vertex as it shown on Fig. 2b. The dressed vertex can be used to obtain the polarization operator, that defines effective dynamically screened Coulomb interaction (Fig 2c):

\[
V(\Omega, q) = \left[ \frac{C(q)}{e^2} + \frac{2\varepsilon_q}{|\Omega| + \delta \varepsilon_q} \right]^{-1}.
\]

The conductivity of the granular metals is given by the analytical continuation of the Matsubara current-current correlator. In the absence of the electron-electron interaction the conductivity is represented by the diagram (a) in Fig. 3 that results in high temperature (Drude) conductivity \( \sigma_0 \) which is defined below Eq. (2b). First order interaction corrections to the conductivity are given by the diagrams (b-e) in Fig. 3. These diagrams are analogous to ones considered in Ref. 6 for the correction to the conductivity of homogeneous metals. We consider the contributions from diagrams (b,c) and (d,e) separately: The sum of the diagrams (b,c) results in the following correction to the conductivity

\[
\frac{\delta \sigma_1}{\sigma_0} = -\frac{1}{2\pi dg_T} \text{Im} \sum_q \int d\omega \gamma(\omega) \varepsilon_q \tilde{V}(\omega, q).
\]

where \( \gamma(\omega) = \frac{d\omega}{d\varepsilon} \coth \frac{\varepsilon}{2T} \), and the potential \( \tilde{V}(\omega, q) \) is the analytic continuation of the Screened Coulomb potential with dressed interaction vertices included attached at both ends

\[
\tilde{V}(\omega, q) = \frac{2EC(q)}{(\varepsilon_q\delta - i\omega)(4\varepsilon_qEC(q) - i\omega)}.
\]

The above expression was simplified using that the charging energy \( EC(q) = e^2/2C(q) \), expressed in terms of the Fourier transform of the capacitance matrix \( C(q) \) is much larger than \( \delta \). Performing the integration over the frequency and summing over the quasimomentum \( q \) in Eq. (10) with the logarithmic accuracy we obtain the correction \( \frac{\gamma(\omega)}{\sigma_0} \). One can see from Eq. (10) that the contribution \( \delta \sigma_1 \) in Fig. 2c comes from the large energy scales, \( \varepsilon > g_T \delta \) such that at low temperatures the logarithm is cut off on the energy scale \( g_T \delta \).

To obtain the total correction to the conductivity of granular metal the two other diagrams, (d) and (e) in

![FIG. 2: These diagrams represent (a) Dyson equation for diffusion propagator (b) interaction vertex dressed by impurity and intergranular scattering (c) Screened Coulomb interaction.](image)
has the following asymptotic form

\[ C_q \] contribution to the sum over the quasimomentum \( q \) correction scribe effective screened electron-electron propagator. The metals due to electron-electron interaction. The solid lines denote the propagator of electrons and the dashed lines describe effective screened electron-electron propagator. The sum of the diagrams (b) and (c) results in the conductivity correction \( \delta \sigma_1 \) in Eq. (2a). The other two diagrams, (d) and (e) result in the correction \( \delta \sigma_2 \).

FIG. 3: Diagrams describing the conductivity of granular metals; the diagram (a) corresponds to \( \sigma_0 \) in Eq. (2a) and it is the analog of Drude conductivity. Diagrams (b)-(e) describing first order correction to the conductivity of granular metals due to electron-electron interaction. The solid lines denote the propagator of electrons and the dashed lines describe effective screened electron-electron propagator. The sum of the diagrams (b) and (c) results in the conductivity correction \( \delta \sigma_1 \) in Eq. (2a). The other two diagrams, (d) and (e) result in the correction \( \delta \sigma_2 \).

Fig. 3 should be taken into account. These diagrams result in the following contribution to the conductivity

\[
\frac{\delta \sigma_2}{\sigma_0} = -\frac{2g_T \delta}{\pi d} \sum_q dE(\omega) \Im \tilde{V}(\omega, q) \sum_a \sin^2 \left( \frac{q a}{2} \right).
\]

In contrast to the contribution \( \delta \sigma_1 \) in Eq. (10), the main contribution to the sum over the quasimomentum \( q \) in Eq. (12) comes from the low momenta, \( q \ll a^{-1} \). In this regime the capacitance matrix, \( C(q) \) in Eqs. (11) and (12) has the following asymptotic form

\[
C^{-1}(q) = \frac{2}{a^d} \left\{ \begin{array}{ll}
\ln(1/a q) & D = 1, \\
\pi/q & D = 2, \\
2\pi/q & D = 3.
\end{array} \right.
\]

Using Eqs. (11) and (13), we obtain the result for the correction \( \delta \sigma_2 \) in Eq. (24). This correction has a physical meaning similar to that of the Altshuler-Aronov correction \( \tilde{\delta} \) derived for homogeneous disordered metals.

Comparing our results in Eqs. (24) with those obtained in Ref. [2] using the AES functional we see that the correction to the conductivity obtained in Ref. [2] is equivalent to the correction \( \delta \sigma_1 \) in Eq. (2a), which corresponds in our approach to the sum of diagrams (b) and (c) in Fig. 3. The correction \( \delta \sigma_2 \) in Eq. (24) becomes important only at low temperatures, \( T < g_T \delta \) where AES functional is not applicable. While in our approach both corrections to the conductivity must be small \( \delta \sigma_1, \delta \sigma_2 \ll \sigma_0 \) the method of Ref. [2] gives a possibility to show that for \( T \gg g_T \delta \) the dependence of the conductivity is logarithmic so long as \( \sigma_0 / a^d \gg 1 \).

It follows from Eq. (24) that at low temperatures, \( T < g_T \delta \), for a 3D granular array, there are no essential corrections to the conductivity coming from the low energies since the correction \( \delta \sigma_2 \) is always small. This means that the result for the renormalized conductance, \( \tilde{\gamma}_T \) of Ref. [2] (see also [2]) for 3D samples within the logarithmic accuracy can be written in the following form
that the temperature (energy) dependence of the DOS for $\varepsilon \ll g_T \delta$ given by Eq. (16) coincides up to the constant term with the result for the correction to the DOS of the homogeneous metal $\text{Bi}_2$. The logarithmic behavior $\sigma \propto T^{-\delta}$ of the conductivity is in a good agreement with experimental findings $\text{La}_{2-y}\text{Sr}_y\text{CuO}_4$ and $\text{Bi}_{2}\text{Sr}_{2-x}\text{La}_x\text{CuO}_{6+\delta}$ in a very strong magnetic field $\text{La}_{2-y}\text{Sr}_y\text{CuO}_4$ and $\text{Bi}_{2}\text{Sr}_{2-x}\text{La}_x\text{CuO}_{6+\delta}$ $\text{Bi}_2$. A possible granularity of these samples was suggested in Ref. [3]. Recently the microscopic granularity was directly experimentally observed in the superconducting state of $\text{Bi}_{2}\text{Sr}_{2}\text{CaCu}_2\text{O}_8$ by the STM probe $\text{La}_{2-y}\text{Sr}_y\text{CuO}_4$ and $\text{Bi}_{2}\text{Sr}_{2-x}\text{La}_x\text{CuO}_{6+\delta}$ $\text{Bi}_2$. If we accept that samples studied in $\text{La}_{2-y}\text{Sr}_y\text{CuO}_4$ and $\text{Bi}_{2}\text{Sr}_{2-x}\text{La}_x\text{CuO}_{6+\delta}$ $\text{Bi}_2$ are indeed microscopically granular, we can compare the results of the experiments with our predictions. When doing so it is convenient to scale three dimensional conductivity to the conductivity of CuO planes, $\sigma_{\text{plane}}$. According to our predictions

$$
\frac{d\sigma_{\text{plane}}}{d\ln T} = (e^2/\pi\hbar) k, 
$$

where the coefficient $k = 1/2\pi$ in the low-temperature and $k = 1/d$ in the high temperature regimes. While in the low temperature regime the application of Eq. $\text{Bi}_{2}\text{Sr}_{2-x}\text{La}_x\text{CuO}_{6+\delta}$ $\text{Bi}_2$ is legitimate only under the assumption that electrons in different CuO plane are incoherent, in the high temperature regime the behavior of conductivity according to Eq. $\text{La}_{2-y}\text{Sr}_y\text{CuO}_4$ is logarithmic for any dimension. In this regime the real dimensionality $d$ should be replaced by $d = Z/2$, where $Z$ is the (average) number of the contacts of each grain with all the adjacent grains. Describing the data shown in Fig. 3 of Ref. $\text{La}_{2-y}\text{Sr}_y\text{CuO}_4$ our log dependencies at temperature $T \approx 5K$ we extract $k \approx 0.4$ for $\text{Sr}$ concentration of $y = 0.08$ for $\text{La}_{2-y}\text{Sr}_y\text{CuO}_4$ $\text{Bi}_{2}\text{Sr}_{2-x}\text{La}_x\text{CuO}_{6+\delta}$ $\text{Bi}_2$ concentration of $y = 0.08$ for $\text{La}_{2-y}\text{Sr}_y\text{CuO}_4$ $\text{Bi}_{2}\text{Sr}_{2-x}\text{La}_x\text{CuO}_{6+\delta}$ $\text{Bi}_2$ compound we find $k \approx 0.2$ for $x = 0.8$ $\text{La}$ concentration, and $k \approx 0.3$ for $x = 0.76$. For each particular curve the values $k$ extracted from Fig. 3 of Ref. $\text{Bi}_{2}\text{Sr}_{2-x}\text{La}_x\text{CuO}_{6+\delta}$ $\text{Bi}_2$ increase with temperature (especially in case of LSCO), this is in a complete agreement with our results provided that the “coherent-incoherent” crossover occurs at about $T \sim 5K$. At higher temperatures $k$ noticeably exceeds $1/2\pi$, supporting the idea of a granularity of doped cuprates.

In conclusion, we have investigated transport properties of granular metals at large tunneling conductance and obtained corrections to the conductivity, Eqs. $\text{Bi}_{2}\text{Sr}_{2-x}\text{La}_x\text{CuO}_{6+\delta}$ $\text{Bi}_2$ and DOS, Eqs. $\text{La}_{2-y}\text{Sr}_y\text{CuO}_4$ $\text{Bi}_{2}\text{Sr}_{2-x}\text{La}_x\text{CuO}_{6+\delta}$ due to electron-electron interaction. We have shown that at temperatures, $T > g_T \delta$ the granular structure of the array dominates the physics. On the contrary at temperatures, $T \leq g_T \delta$ the large-scale coherent electron motion is crucial. Comparison our results with experimental data supports the assumption about a granular structure of doped high-$T_c$ cuprates.

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[13] This curve for $\text{Bi}_{2}\text{Sr}_{2-x}\text{La}_x\text{CuO}_{6+\delta}$, $x = 0.15$ shows a very wide crossover at about 20K extending to low temperatures. Thus there are just a very few points in the narrow temperature interval 1-3K where our approach is applicable. This invalidates the comparison the data with our results for this particular curve.