Form Factors and Differential Branching Ratio of $B \to K\mu^+\mu^-$ in AdS/QCD

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The light-front distribution amplitudes (LFDAs) and the decay constant of the $K$ pseudoscalar meson are extracted within the framework of the AdS/QCD correspondence. For the LFDAs calculation, we consider a momentum-dependent (dynamical) helicity wave function which contains the dynamical spin effects. We use the LFDAs to predict the transition form factors of the semileptonic $B \to K\ell^+\ell^-$ decay. With the help of these form factors, the differential branching ratio of the $B \to K\mu^+\mu^-$ on $q^2$ is plotted. A comparison is made between our prediction in AdS/QCD and the results obtained from two models including the light-cons sum rules (LCSR) and lattice QCD as well as the experimental values.

I. INTRODUCTION

The flavor changing neutral current (FCNC) transitions have received remarkable attention, both experimentally and theoretically. The decay of a $b$ quark into an $s$ quark and lepton pairs, $b \to s\ell^+\ell^-$, is a good tool to study the FCNC processes; it is also a very good way to probe the new physics effects beyond the standard model (SM).

The $B \to K\ell^+\ell^-$ decay, which occurs by the $b \to s\ell^+\ell^-$ process at the quark level, is a suitable candidate for experimental researchers who study the FCNC transition. The differential branching ratio, the forward-backward, and the isospin asymmetries for this transition have been measured at the BaBar, Belle, and CDF Collaborations [1–4]. Researchers in the LHCb Collaboration have reported newer results for these observable quantities [5–7]. Recently, the updated results have been released for differential branching fraction and the angular analysis of the $B \to K\mu^+\mu^-$ decay [8]. On the other hand, physicists have tried to improve their results for this decay via the theoretical approaches [9]. Recently a new analysis has been made to estimate the transition form factors of the $B \to K\mu^+\mu^-$ decay by the lattice QCD [10].

To evaluate the branching ratio and the other observable, we need to describe the intended transition according to its form factors, which are defined in terms of the distribution amplitudes (DAs). The DAs for the $K$ pseudoscalar meson have been obtained, for the first time, from the LCSR [11, 12]. In recent years, a relatively new tool named the anti-de Sitter/Quantum Chromodynamics (AdS/QCD) correspondence has been used to obtain the DAs for the light mesons. In this approach, the wave function which describes the hadrons in the AdS space is mapped to the wave function used for the bound states in the light-front QCD. Both of them satisfy a Schrodinger-like wave function equation. The holographic light-front wave function (LFWF) is used to describe the light-front DAs (for instance, see [13–17]).

So far, the isospin asymmetry of $B \to K^*\mu^+\mu^-$ transition has been considered in the AdS/QCD correspondence [18]. Dynamical spin effects have been taken into account of the holographic pion wave function in order to predict its mean charge radius, decay constant, the space-like electromagnetic form factor, twist-2 DA, and the photon-to-pion transition form factor [19]. Our goal in this paper is to extract the twist-2, 3 and twist-4 DAs of the $K$ pseudoscalar meson and use this holographic DAs in order to compute the form factors and the differential branching ratio for the $B \to K\mu^+\mu^-$ transition.

Our paper is organized as follows: In section II, the light-front DAs for the $K$ pseudoscalar meson are calculated. In this section, the connection between the holographic LFWF and DAs is presented. Using these quantities, the transition form factors can be investigated. In section III, we analyze the light-front DAs and transition form factors of the FCNC $B \to K$ transitions. Then, the differential branching ratio of $B \to K\mu^+\mu^-$ decay on $q^2$ is plotted. Our prediction is compared with those made by the lattice QCD and LCSR approaches, as well as the experimental values.

II. THE HOLOGRAPHIC DISTRIBUTION AMPLITUDES FOR K MESON

The holographic DAs for the $K$ pseudoscalar meson are derived in this section. For this aim, we plan to obtain a connection between the DAs and the holographic LFWF of the $K$ meson. Using the definition of the DAs for
the $K$ meson introduced by the meson-to-vacuum matrix elements \cite{11 12 20 21}, and choosing $p^\mu = (p^+, \frac{m_K^2}{p^+}, \mathbf{0}_\perp)$ for the four-momentum of the $K$ meson, the following matrix elements can be written in the light-front coordinate, $x^\mu = (x^+, x^-, x_\perp)$, at equal light-front time, $x^+ = 0$, as

$$
\langle 0 | \bar{u}(0) \gamma^5 s(x^-) | K(p) \rangle = i \frac{f_K}{p^+} \int_0^1 du e^{i p^+ x^-} \phi_k(u, \mu),
$$

$$
\langle 0 | \bar{u}(0) \gamma^5 s(x^-) | K(p) \rangle = -i \frac{f_K m_K^2}{m_u + m_s} \int_0^1 du e^{i p^+ x^-} \phi_p(u, \mu),
$$

$$
\langle 0 | \bar{u}(0) \sigma^{\alpha \beta} (1 + \gamma^5) s(x^-) | K(p) \rangle = \frac{i}{6} \frac{f_K m_K^2}{m_u + m_s} p^{[\alpha x^\beta]} \int_0^1 du e^{i p^+ x^-} \phi_\sigma(u, \mu),
$$

$$
\langle 0 | \bar{u}(0) \gamma^\alpha s(x^-) | K(p) \rangle = i f_K (x^-)^2 p^+ \int_0^1 du e^{i p^+ x^-} \frac{g_1(u, \mu) - f_K (x^- - \frac{x^-}{p^+} \alpha)}{p^+},
$$

$$
\times \int_0^1 du e^{i p^+ x^-} g_2(u, \mu),
$$

where $\mu$ is the renormalization scale and $f_K$ is the decay constant of the $K$ pseudoscalar meson. In these relations, $\phi_k$ is twist-2, $\phi_p$ and $\phi_\sigma$ are twist-3, $g_1$ and $g_2$ are twist-4 DAs for the $K$ meson. To isolate $\phi_k$ and $\phi_p$, we take $\alpha = +$ and apply the Fourier transform of the Eqs. \eqref{1} and \eqref{2} with respect to $x^-$. It yields:

$$
\phi_k(\alpha, \mu) = -i \frac{f_K}{k} \int dx^- e^{i p^+ x^-} \langle 0 | \bar{u}(0) \gamma^5 s(x^-) | K(p) \rangle,
$$

$$
\phi_p(\alpha, \mu) = i \frac{(m_u + m_s)}{m_K^2} p^+ \int dx^- e^{i p^+ x^-} \langle 0 | \bar{u}(0) \gamma^5 s(x^-) | K(p) \rangle.
$$

Choosing $\sigma^{+-}$ in Eq. \eqref{3}, and using integration by parts with the boundary condition $\phi(u)|_0^1 = 0$, as well as performing the Fourier transform with respect to $x^-$, the derivative of the twist-3 $\phi_\sigma(\alpha, \mu)$ is obtained as

$$
\frac{\partial \phi_\sigma(\alpha, \mu)}{\partial \alpha} = \frac{6(m_u + m_s)}{m_K^2} p^+ \int dx^- e^{i p^+ x^-} \langle 0 | \bar{u}(0) \sigma^{+-} (1 + \gamma^5) s(x^-) | K(p) \rangle.
$$

Taking $\alpha = +$ (and afterwards $\alpha = -$) in Eq. \eqref{4}, and then using integration by part, the following relations are derived:

$$
\langle 0 | \bar{u}(0) \gamma^+ s(x^-) | K(p) \rangle = \frac{i f_K}{p^+} \left[ \int_0^1 du e^{i p^+ x^-} \frac{\partial^2 g_1(u, \mu)}{\partial u^2} - \int_0^1 du e^{i p^+ x^-} \frac{\partial g_2(u, \mu)}{\partial u} \right],
$$

$$
\langle 0 | \bar{u}(0) \gamma^- s(x^-) | K(p) \rangle = \frac{i f_K}{p^+} \left[ \frac{m_K^2}{(p^+)^2} \int_0^1 du e^{i p^+ x^-} \frac{\partial^2 g_1(u, \mu)}{\partial u^2} - \left( 1 - \frac{m_K^2}{(p^+)^2} \right) \right]

\times \int_0^1 du e^{i p^+ x^-} \frac{\partial g_2(u, \mu)}{\partial u}.
$$

Solving Eqs. \eqref{8} and \eqref{9} in terms of $\frac{\partial^2 g_1(u, \mu)}{\partial u^2}$ and $\frac{\partial g_2(u, \mu)}{\partial u}$, as well as performing the Fourier transform with respect to $x^-$, we obtain:

$$
\frac{\partial g_2(\alpha, \mu)}{\partial \alpha} = \frac{i}{f_K [2m_K^2 - (p^+)^2]} \int dx^- e^{i p^+ x^-}

\times \left[ \frac{m_K^2}{(p^+)^2} \langle 0 | \bar{u}(0) \gamma^+ s(x^-) | K(p) \rangle - \langle 0 | \bar{u}(0) \gamma^- s(x^-) | K(p) \rangle \right],
$$

$$
\frac{\partial^2 g_1(\alpha, \mu)}{\partial \alpha^2} = \frac{i}{f_K [2m_K^2 - (p^+)^2]} \int dx^- e^{i p^+ x^-}

\times \left[ \frac{m_K^2}{(p^+)^2} - 1 \right] \langle 0 | \bar{u}(0) \gamma^+ s(x^-) | K(p) \rangle - \langle 0 | \bar{u}(0) \gamma^- s(x^-) | K(p) \rangle.
$$

In order to evaluate the holographic DAs for the $K$ meson, the hadronic matrix elements should be determined in Eqs. \eqref{17} and \eqref{10\&11}. For this purpose, the Fock expansion of noninteracting 2-particle states is used for a hadronic
eigenstate |P⟩ as \[22\]:

\[
|P(p)⟩ = \sqrt{4πN_c} \sum_{h, h} \int \frac{dk^+ d^2k_⊥}{16π^3 \sqrt{k^+(p^+ - k^+)}} \Psi^P_{h, h}(k^+, k_⊥; p^+, -k^+, h, h),
\]

in which, \(\Psi^P_{h, h}(α, k_⊥)\) is the LFWF of the pseudoscalar meson, \(h \) and \(\bar{h}\) are the helicities of quark and anti-quark, respectively. By utilizing the expansion of Dirac fields (quark and anti-quark) in terms of particle creation and annihilation operators, and also the equal light-front time anti-commutation relations for these operators, the matrix element \(|\langle 0|u(0) Γ s(x^-)|P(p)⟩\) is obtained as

\[
|\langle 0|u(0) Γ s(x^-)|P(p)⟩⟩ = \sqrt{4πN_c} \sum_{h, h} \int \frac{dk^+ d^2k_⊥}{16π^3 \sqrt{k^+(p^+ - k^+)}} \Psi^P_{h, h}(α, k_⊥)
\]

\[
\times \bar{v}_h(p^+ - k^+, -k_⊥) \Gamma u_h(k^+, k_⊥) e^{-ik^+x^-},
\]

which \(u_h\) and \(v_h\) are light-front helicity spinors for quark and anti-quark, respectively. The renormalization scale \(μ\) is used as the ultraviolet cut-off on transverse momenta \[23\,\,24\]. In our work, \(Γ\) can be \(σ^−(1 + γ^5)\), \(γ^+\) or \(γ^−\). By integrating with respect to \(k^+\) and applying the Fourier transform to the left and right-hand-sides of Eq. \[13\], the following result is obtained:

\[
\int d^−e^{ik^+x^-}⟨0|u(0) Γ s(x^-)|P(p)⟩⟩ = \sqrt{4πN_c} \sum_{h, h} \int |k_⊥|<μ \frac{d^2k_⊥}{(2π)^3} \Psi^P_{h, h}(α, k_⊥)
\]

\[
\times \left\{ \bar{v}_h(α p^+, -k_⊥) \Gamma u_h(α p^+, k_⊥) \right\},
\]

where \(α = \frac{k^+}{p^+}\), and \(\bar{α} = 1 - α\). In the \(k\)-space, the holographic LFWF is given as \[22\]

\[
Ψ^P_{h, h}(α, k_⊥) = \frac{1}{\sqrt{4π}} S^P_{h, h}(α, k_⊥),\]

(15)

The structure of \(S^P_{h, h}(α, k_⊥)\) for the pseudoscalar mesons that includes the helicity-dependent wave function is as follows:

\[
S^P_{h, h}(α, k_⊥) = \frac{\bar{u}_h(α p^+, -k_⊥)}{\sqrt{\bar{α}}} \left[ (A p + B m_K) γ^5 \right] \frac{v_h(α p^+, k_⊥)}{\sqrt{α}},\]

(16)

where \(A\) and \(B\) are arbitrary constants. If \(B \neq 0\), the dynamical spin effects are allowed. We discuss how to choose the coefficients \(A\) and \(B\) in the next section.

Using the light-front spinors presented in Ref. \[22\], \(S^P_{h, h}\) is evaluated for \(K\) meson as:

\[
i S^P_{h, h}(α, k_⊥) = \frac{A}{α} \left\{ [α \bar{α} m_u^2 + m_u m_s + k^2] δ_{h^+, \bar{h}^±} + k [m_u e^{-iθ_k} δ_{h^+, \bar{h}^+} + m_s e^{iθ_k} δ_{h^−, \bar{h}^−}] \right\}
\]

\[
= \frac{B m_K}{α} \left\{ [α m_u + \bar{α} m_s] \mp k e^{-iθ_k} δ_{h^±, \bar{h}^±} \right\},
\]

(17)

where \(k e^{±iθ_k}\) is the complex form of the transverse momentum \(k_⊥\), in addition, \(h^+\) and \(h^−\) are used for positive and negative helicity, respectively.

The light-front spinors are also utilized to obtain the matrix elements in the right-hand-side of Eq. \[14\]. The final results can be written as:

\[
\frac{\bar{v}_h}{\sqrt{α}} γ^5 \frac{u_h}{\sqrt{α}} = 2p^+ δ_{h^±, \bar{h}^±},
\]

\[
\frac{\bar{v}_h}{\sqrt{α}} γ^5 \frac{u_h}{\sqrt{α}} = 2p^+ δ_{h^±, \bar{h}^±},
\]

\[
\frac{\bar{v}_h}{\sqrt{α}} γ^5 \frac{u_h}{\sqrt{α}} = ± 2p^± δ_{h^±, \bar{h}^±},
\]

\[
\frac{\bar{v}_h}{\sqrt{α}} γ^5 \frac{u_h}{\sqrt{α}} = \frac{1}{α} \left\{ k e^{±iθ_k} δ_{h^±, \bar{h}^±} \mp (α m_s + \bar{α} m_u) δ_{h^±, \bar{h}^±} \right\},
\]

\[
\frac{\bar{v}_h}{\sqrt{α}} γ^5 (1 + γ^5) \frac{u_h}{\sqrt{α}} = \frac{4i}{α} \left\{ k e^{±iθ_k} (1 - 2α) δ_{h^±, \bar{h}^±} + α m_u δ_{h^±, \bar{h}^±} + \bar{α} m_s δ_{h^±, \bar{h}^±} \right\}.
\]

(18)
Inserting Eqs. (17,18) in Eq. (14), the hadronic matrix elements in Eqs. (5,7) and (10,11) are determined. Therefore, the holographic DAs can be calculated for the $K$ meson in terms of $\phi(\alpha, k_\perp)$ in the $k$-space. Applying the Fourier transform to $r$-space and using relations such as $\int_0^{2\pi} e^{-ikr\cos \theta} d\theta = 2\pi J_0(kr)$, and $\int_0^{\mu} k J_0(kr) d(kr) = \mu/r J_1(\mu r)$, where $J_0$ and $J_1$ are Bessel functions, the following expressions are obtained for the holographic DAs in the $r$-space:

$$
\phi_K(\alpha, \mu) = \frac{\beta_1}{\alpha} \int dr J_1(\mu r) \left\{ 2A \left( \alpha \bar{m}^2_K + m_u m_s - \nabla^2 \right) + Bm_K \left( \alpha m_u + \alpha m_s \right) \right\} \phi(\alpha, r),
$$

$$
\phi_\rho(\alpha, \mu) = -\frac{(m_s + m_u)\beta_1}{\alpha^2 \bar{m}^2_K} \int dr J_1(\mu r) \left\{ A \left[ (\alpha m_u - \bar{\alpha} m_s) \left( \alpha \bar{m}^2_K + m_u m_s - \nabla^2 \right) \right] - (m_u + m_s)\nabla^2 \right\} \phi(\alpha, r),
$$

$$
\frac{\partial \phi_\sigma(\alpha, \mu)}{\partial \alpha} = -\frac{24(m_u + m_s)\beta_1}{\alpha^2 \bar{m}^2_K} \int dr J_1(\mu r) \left\{ A \left[ (\alpha m_u - \bar{\alpha} m_s) \left( \alpha \bar{m}^2_K + m_u m_s - \nabla^2 \right) \right] - (2\alpha - 1)(m_u - m_s)\nabla^2 \right\} \phi(\alpha, r),
$$

$$
\frac{\partial \phi_\sigma(\alpha, \mu)}{\partial \alpha} = \frac{\beta_1}{\beta_3 \alpha \bar{m} m_K} \int dr J_1(\mu r) B \left[ \alpha m_u + \bar{\alpha} m_s \right] \phi(\alpha, r),
$$

$$
\frac{\partial^2 g_1(\alpha, \mu)}{\partial \alpha^2} = \frac{\beta_1(\beta_2 - 1)}{\beta_3 \alpha \bar{m} m_K} \int dr J_1(\mu r) B \left[ \alpha m_u + \bar{\alpha} m_s \right] \phi(\alpha, r),
$$

where $\sqrt{N_c}/(\pi f_K) = \beta_1$, $1 - m_\perp^2/(p_\perp^2) = \beta_2$ and $2 - (p_\perp^2)/m_\perp^2 = \beta_3$.

To specify $\phi(\alpha, r)$, which includes dynamical properties of $K$ in the LFWF, we are going to use the AdS/QCD. Based on a first semiclassical approximation to the light-front QCD, with massless quarks, function $\phi$ can be factorized as [20]

$$
\phi(\zeta, \alpha, \theta) = N \frac{\psi(\zeta)}{\sqrt{2\pi} \zeta} f(\alpha) e^{iL\theta},
$$

where $N$ is a normalization constant. In this relation, $L$ is the orbital angular momentum quantum number and variable $\zeta = \sqrt{\alpha(1-\alpha)} r$, where $r$ is the transverse distance between the quark and anti-quark forming the meson. Function $\psi(\zeta)$ satisfies the so-called holographic light-front Schroedinger-like equation as

$$
\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \psi(\zeta) = M^2 \psi(\zeta),
$$

where $M$ is the hadron bound-state mass and $U(\zeta)$ is the effective potential. It should be noted that all the interaction terms and the effects of higher Fock states on the valence ($N = 2$ for mesons) state are hidden in the confinement potential.

According to the AdS/QCD, the holographic light-front Schroedinger equation is mapped onto the wave equation for strings propagating in the AdS space if $\zeta$ is identified with the fifth dimension in AdS. To illustrate this issue, the invariant action (up to bilinear terms) is written for a scalar field in the $AdS_5$ space as:

$$
S = \frac{1}{2} \int d^4 x dz \sqrt{g} e^{\varphi(z)} \left( g_{MN} \partial_M \Phi \partial_N \Phi - \mu^2 \Phi^2 \right),
$$

where $g = (\hat{R})^{10}$ is the modulus of the determinant of the metric tensor $g_{MN}$. Moreover, $\Phi(x^\mu, z)$ is a scalar field. Mass $\mu$ in Eq. (22) is not a physical observable. In this action, the dilaton background $\varphi(z)$ is only a function of the holographic variable $z$ which vanishes as $z \to \infty$. Variation of Eq. (22) and making the ansatz $\Phi(x^\mu, z) = e^{-iP \cdot x} \Theta(z)$, which describe a free hadronic state with four-momentum $P$ in holographic QCD, the eigenvalue equation is obtained as

$$
\left[ -\frac{3}{e^{\varphi(z)}} \frac{d^3}{dz^3} \left( \frac{e^{\varphi(z)}}{z^3} \partial_z \right) + \left( \frac{\mu R}{z} \right)^2 \right] \Theta(z) = M^2 \Theta(z),
$$

(23)
where \( P_\mu P^\mu = M^2 \) is the invariant mass. Factoring out the scale \( (1/3)^{-2} \) and dilaton factors from the AdS field as \( \Theta = (\frac{1}{3})^{-2} e^{-\Theta(z)/2} \), and using a substitution as \( z \rightarrow \zeta \), the light-front Schrödinger equation (Eq. (21)) is modified with the effective potential \( U(\zeta) = \frac{1}{2} \varphi''(\zeta) + \frac{1}{2} \varphi'(\zeta)^2 - \frac{1}{2} \varphi(\zeta) \), and the AdS mass \( (\mu R)^2 = L^2 - 1 \). In this correspondence, \( \varphi(\zeta) \) and \( (\mu R)^2 \) are related to the effective potential and the internal orbital angular momentum \( L \), respectively.

Choosing \( \varphi(\zeta) = \kappa^2 \zeta^2 \) in the soft-wall model \( [27] \) leads to \( U(\zeta) = \kappa^2 \zeta^2 - 2 \kappa^2 \). Solving Eq. (23) with this potential and comparing the equation with the quantum mechanical oscillator in the polar coordinates, the results are obtained for eigenfunctions \( (\psi_{n,L}(\zeta)) \) and eigenvalues \( (M^2(n, L, S)) \).

Therefore, \( \phi(r, \alpha) \) for the K meson with massless quarks, and \( n = 0, L = 0 \) is obtained as:

\[
\phi^K(\alpha, \zeta) = N \frac{\kappa}{\sqrt{\pi}} \sqrt{2\alpha} \exp \left( -\frac{\kappa^2 \zeta^2}{2} \right),
\]

where \( \kappa \) is the AdS/QCD scale. It should be noted that the condition \( \int_0^1 d\alpha \frac{f(\alpha)^2}{\alpha^2} = 1 \) is used to determine the function \( f(\alpha) \) in Eq. (20). To include the mass of quarks in Eq. (24), first, a Fourier transform is applied to \( k \)-space as \( \phi^K(\alpha, k_\perp) = \int d^2r e^{-ikr \cos \theta_k} \phi^K(\alpha, \zeta) \); it yields:

\[
\tilde{\phi}^K(\alpha, k_\perp) = N \frac{2}{\sqrt{\alpha \alpha}} \frac{\sqrt{\pi}}{\kappa} \exp \left( -\frac{k^2}{2\alpha \kappa^2} \right).
\]

After substituting this into the wave function and Fourier transforming back to the transverse position-space, the final form of the AdS/QCD wave function is obtained as:

\[
\phi^K(\zeta, \alpha) = N \frac{\kappa}{\sqrt{\pi}} \sqrt{\alpha \alpha} \exp \left( -\frac{\kappa^2 \zeta^2}{2} \right) \exp \left\{-\frac{\alpha m^2_\perp - \alpha m^2_\parallel}{2\alpha \kappa^2} \right\}.
\]

In position-space, \( N \) can be fixed by this normalization condition \( [22] \):

\[
\int d^2r \, d\alpha \left[ \sum_{h,h} |\psi^K_{h,h}(r, \alpha)|^2 \right] = 1.
\]

### III. NUMERICAL ANALYSIS

In this section, we present our numerical analysis for the light-front holographic DAs of the K meson, the \( B \rightarrow K \ell^+ \ell^- \) transition form factors, as well as the differential branching ratio of the \( B \rightarrow K \mu^+ \mu^- \) transition on \( q^2 \).

According to the light-front holographic prediction, the mass squared of mesons composed of light quarks is given as \( M^2(n, L, S) = 4 \kappa^2 \left(n + L + \frac{S}{2} \right) \), where the quantum numbers \( L \) and \( n \) describe the orbital angular momentum and excitations of the meson spectrum, respectively. By fitting this mass relation to the experimentally measured Regge slopes, the AdS/QCD scale \( \kappa \) is reported to be 590 MeV for pseudoscalar mesons \( [29] \). By considering the nonzero quark masses, the mass relation is improved by \( \Delta M^2 \) as \( M^2 = 4 \kappa^2 \left(n + L + \frac{S}{2} \right) + \Delta M^2 \). Considering this correction in the mass of mesons and fitting it to the Regge slopes yields \( \kappa = 523 \) MeV \( [30] \). In this paper, we choose these two values of \( \kappa \) in our analysis.

Using the experimental values of the decay constants, \( f_\pi \) and \( f_K \), and choosing the value of \( \kappa \), we can obtain the mass of the light quarks related to our analysis; they are in fact, the effective quark masses used in the holographic LFWFs \( [31] \). The decay constant for a pseudoscalar meson, which contains \( q \) and \( q' \) quarks, can be defined as:

\[
\langle 0 | \bar{q}(0) \gamma^\alpha \gamma^5 q'(0) | S(p) \rangle = i f_S p^\alpha.
\]

After expanding the left-hand-side of Eq. (29) in the procedure described in previous section, the decay constant formula for pion and kaon in the AdS/QCD correspondence is calculated as:

\[
f_S = \frac{\sqrt{N_c}}{\pi} \int_0^1 d\alpha \left[ \frac{B(\alpha m_\perp - \alpha m_\parallel)}{MS} + 2A (\alpha \alpha m_\perp^2 + m_\perp m_\parallel - \nabla^2) \right] \frac{\phi(\alpha, r)}{\alpha \alpha} \bigg|_{r=0}.
\]

Here, \( N_c \) is the number of colors, \( A \) is the constant, and \( B \) is the effective potential.
The effective masses for two light quarks, $u$ and $d$, are equal in the AdS/QCD. So, by inserting $m_u = m_d$, in addition to the experimental value $f_\pi = 130 \pm 0.26$ MeV, and $A = B = 1$ \cite{17} in Eq. (30), we can plot $m_u$ with respect to $\kappa$ (see Fig. 1). By having the values of $m_u$ according to $\kappa$, as well as the experimental value $f_K = 156 \pm 0.49$ MeV, and applying them in Eq. (30), we can also display $m_s$ based on $\kappa$. According to Fig. 1 for $\kappa = 590$ MeV, the mass of quarks $[m_{u,d}, m_s]$ is obtained in MeV as [200, 350]. As can be seen in Fig. 1 there are two values for $m_s$ in $\kappa < 575$ MeV; therefore, we can not use this plot for obtaining $m_s$ for $\kappa = 523$ MeV. Now, we choose $A = 0$ and $B = 1$ \cite{32, 33}, and repeat the previous steps and obtain the mass of quarks $[m_{u,d}, m_s]$ is [160, 310] in MeV for $\kappa = 523$ MeV.

Fig. 2 shows $\phi_K(\alpha, \mu)$ with respect to $\alpha$, obtained form Eqs. (19), on which red and blue lines show the results for $\kappa = 523$ MeV and 590 MeV in $\mu = 1$ GeV, respectively.

To evaluate the differential branching ratio of the $B \to K\mu^+\mu^-$ transition on $q^2$, we need to calculate the transition form factors. The explicit expressions of these form factors in terms of light-cone DAs are given in Ref. \cite{34}. We use these expressions and replace the holographic DAs in them; then we convert the obtained form factors based on the following definitions, which are more conventional \cite{10}:

\begin{align}
\langle K(p)|\bar{s}\gamma_\mu b|B(p_B) \rangle &= P_\mu f_+(q^2) + q_\mu \frac{m_B^2 - m_K^2}{q^2} [f_-(q^2) - f_+(q^2)], \\
\langle K(p)|\bar{s}i\sigma_{\mu\nu} q^\nu (1 + \gamma_5) b|B(p_B) \rangle &= [P_\mu q^2 - (m_B^2 - m_K^2) q_\mu] \frac{f_T}{m_B + m_K}. \tag{31}
\end{align}
In these definitions, $p$ and $p_B$ refer to the momentums of the $K$ and $B$ meson, respectively; $q = p_B - p$ is the momentum carried by leptons and $P = p_B + q$.

Usually, the numerical results for the form factors calculated via different methods in QCD have a cut-off. So, to evaluate the form factors for the whole physical region $m_2^2 \leq q^2 \leq (m_B - m_K)^2$, we look for a good parametrization of the form factors in such a way that, in the large values of $q^2$, this parametrization can coincide with the lattice predictions [10]. Our numerical calculations show that the sufficient parametrization of the form factors with respect to $q^2$ is as follows:

$$F(q^2) = \frac{1}{1 - \left(\frac{q^2}{m_B^2}\right)^2} \sum_{r=0}^{2} b_r \left[z^r + (-1)^r \frac{r}{3} z^4\right],$$

(32)

where $z = \sqrt{\frac{t_+ - q^2 - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2 + \sqrt{t_+ - t_0}}}}$, $t_+ = (m_B + m_K)^2$ and $t_0 = (m_B + m_K)(\sqrt{m_B^2 - m_K^2})^2$ [35]. Table I shows the values of $b_r$ ($r = 0, ..., 2$) for the form factors.

| $\kappa$ | $b_0$ | $b_1$ | $b_2$ | $\kappa$ | $b_0$ | $b_1$ | $b_2$ |
|----------|-------|-------|-------|----------|-------|-------|-------|
| 523 MeV  | 0.39  | -1.56 | -0.85 | 590 MeV  | 0.43  | -1.13 | -0.21 |
| 523 MeV  | 0.24  | -0.28 | -0.99 | 590 MeV  | 0.27  | 0.08  | -0.25 |
| 523 MeV  | 0.40  | -1.47 | -0.39 | 590 MeV  | 0.45  | -0.99 | 0.12  |

Fig. 3 shows the results for $f_+, f_-$ and $f_T$ form factor in two values of $\kappa = 523$ MeV and 590 MeV. In this figure, circles show the lattice predictions in the large values of $q^2$.

FIG. 3: The form factor $f_+, f_-$ and $f_T$ of the $B \to K$ decay on $q^2$. Circles show the lattice data in large $q^2$.

Now, we can evaluate the differential branching ratio of the $B \to K\mu^+\mu^-$ transition on $q^2$. The expressions of the differential decay width $d\Gamma/dq^2$ for the $B \to K l^+ l^-$ can be found in [33]. This expression contains the CKM matrix elements, Wilson coefficients, and the form factors related to the definitions in Eq. (31). In this paper, we use $C_7^{\text{eff}} = -0.313$ and $C_{10} = -4.669$ [30]. The dependency of the differential branching ratio for the $B \to K\mu^+\mu^-$ decay on $q^2$ is presented in Fig. 4. In this figure, the results obtained by the LCSR [34] and lattice QCD [10] approaches are shown with yellow and green lines, respectively. Also, the experimental values [8] with their errors are plotted in this figure. As can be seen, our results for the differential branching ratio of the $B \to K\mu^+\mu^-$ transition on $q^2$ with the AdS/QCD correspondence for the two values of $\kappa$ are in a good agreement with the experimental values.

To summarize, based on the dynamical spin effects, we extracted the twist-2, 3 and 4 DAs of the $K$ pseudoscalar meson in the AdS/QCD correspondence approach. The AdS/QCD scale $\kappa = 523$ MeV and 590 MeV were used in our analysis; these values were provided by fitting them to the Regge slopes. For a better analysis, we calculated the masses of the light quarks with the help of the experimental values for the decay constants of pion and kaon pseudoscalar mesons. By evaluating the transition form factors, the differential branching ratio for the $B \to K\mu^+\mu^-$ decay on $q^2$ was plotted for the two values of $\kappa$. A comparison with the experimental values showed that our predictions with AdS/QCD were good.
FIG. 4: The differential branching ratios of the semileptonic $B \rightarrow K \mu^+ \mu^-$ decays on $q^2$.

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