Experimental excitation and propagation of nonlinear localized oscillations in an air-levitation-type coupled oscillator array

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Received July 13, 2016; Revised December 19, 2016; Published April 1, 2017

Abstract: A mechanical apparatus of coupled oscillator chains which are levitated on a track by air has been constructed with designing anew nonlinear springs. By use of the apparatus, excitation of mobile type of intrinsic localized modes has been demonstrated with driving sinusoidally at one end of the chains. The apparatus consists of twenty identical oscillators, nonlinear springs, a long and straight air track with a blower at one end and a driver unit for forcing the chains. The oscillators are connected with neighboring ones through the nonlinear springs and levitated above the air track. The relation of restoring force of the spring to deflection is piecewise linear but approximately cubic. In the apparatus with appropriate tensions, the curve of relation is to be symmetrical with respect to the equilibrium position. One end of the chains is fixed and the other is driven sinusoidally in the direction of the chains at a frequency. It has been observed that, driven with a frequency above the cutoff, localized oscillations can be excited intermittently at the driven end and they are propagated along the chains at a constant speed.

Key Words: intrinsic localized mode, localized oscillation, periodic chains, experiment, air-levitation

1. Introduction

It is now known that the intrinsic localized modes (ILMs) or the discrete breathers (DBs) are generic in spatially periodic, discrete and nonlinear systems (see, for example [1–5].) It is also known that in the system of celebrated Fermi-Pasta-Ulam β-type (FPU-β) chains [6], which spatially extends infinitely, stationary and mobile types of ILMs can be excited [7]. Theoretical studies on the ILMs have shown that there are two types of stationary modes: the odd and even modes [1,8] and that the former is linearly unstable while the latter stable on locating in the chains [9]. The mobile ILMs take both of the two stationary modes and even their hybrid ones in the course of propagations [7]. It is also known that the mobile ILMs can be excited in semi-infinite chains driven at one end sinusoidally...
at a frequency $[10–15]$. It may be interesting to realize and demonstrate these theoretical results by mechanical nonlinear chains, because the number of papers which report experimental studies on ILMs is few as compared with theoretical ones. We have actually succeeded in development of nonlinear springs, making a coupled oscillator chains of the FPU-$\beta$ type and experimental demonstrations of ILMs. (The results of the previous experiments will be published in a forthcoming paper.) In this study, we design nonlinear springs anew, set up a new type of mechanical coupled oscillator chains by adopting air levitation and experimentally observe excitation and propagation of the ILMs.

2. Apparatus

The apparatus for experiments consists of twenty identical oscillators, nonlinear springs, a long and straight air track (hollow square pipe with many fine air holes) with a blower and a driver unit for forcing the chains at one end (see Fig. 1). At first, we designed and made a new type of mechanical springs which response nonlinearly and symmetrically to deflections by push or pull with respect to equilibrium state in the apparatus. We used barrel-shaped and pull-type coil springs which have conical and cylindrical parts (Fig. 2). An ultrahigh-strength stainless steel wire EX-SUS with strand diameter 1.0 mm has been used for the spring. In the forming process of the spring, an initial tension has been applied only to the cylindrical part. Between two separated points in the cylindrical part, (two) rigid strings are slackly stretched. The nonlinear response of the spring to tensile force is realized by the additional contrivance. The mechanism is explained as follows (see Fig. 3): When tensile force gradually operates on the spring in the natural-length state, only the conical parts are linearly stretched at first ($(a)\rightarrow(b)$). Subsequently, when the tensile force exceeds a threshold given by the tension initially applied, not only the conical but also cylindrical part is stretched, therefore the spring becomes soft as a whole ($(b)\rightarrow(c)$). When the strings are tighten up, stretching of cylindrical part is restricted and only the conical parts could be stretched again, therefore the spring becomes hard ($(d)$). As a result, the spring hardness is three-stage switched to increasing tensile force. Figure 4 shows the property of one of the springs we made: the relation of load to deflection, which is measured by a digital force gauge system (IMADA CO., LTD.: ZTS-50N, SVH-1000N-L). It is seen that, although the distribution of measured data is piecewise linear, the distribution is well fit by a cubic curve and approximates the interaction which is in the FPU-$\beta$ chains. Attached in the apparatus, the spring will
Fig. 3. Mechanism of nonlinearity of the spring: the initial tension applied to the only cylindrical part in advance or strings in tighten state is to vary the hardness of the spring as tensile force increases.

Fig. 4. Property of one of the springs we made and used in the experiment: the data of the load to deflection with an approximate curve by linear plus cubic function, where A and B represent the thresholds given by the initial tension and tighten-up strings, respectively, and the cross represents center of the approximate curve.

Fig. 5. (Left) An oscillator made and used in the apparatus. (Right) Chains by the oscillators, which are connected through nonlinear springs, set on the air track with appropriate tension and to be air-levitated by operating blower attached to the air track.

behavior as a symmetrical push/pull type spring. We have succeeded in making a sufficient number of this type of springs with small variation. Actually, for this apparatus we made 100 springs, measured all of them and from among those chose 21 ones so that the dispersion is made minimized. The mean value of spring constants calculated at centers of the approximate curves is 27.9 [N/m], which is to be used in the next section to determine the values of eigen frequencies of the chains.

Next, we denote the oscillators and air track we made. Figure 5 shows one of the oscillators (left)
and the oscillators set on the air track (right). An oscillator is composed of an aluminum strip with metal hooks for connecting to springs and pedestal made of hard resin with $1.0 \times 10^2$ mm in length and L-shaped cross-section and weighs 81.8 grams in total. The air track is a hollow square aluminum pipe with $4.0 \times 10^3$ mm in length, 40 mm in one outer side length and 2.0 mm in thickness and has many fine through-holes with diameter of 0.70 mm bored on the upper faces along the longitudinal direction at 5.0 mm interval. The air track is supported horizontally by stands and to its one end a high-power blower (BOSCH: GBL800E) is attached. Operating the blower, a higher air pressure is maintained inside of the pipe and the oscillators set on the track are levitated by the constant air
Fig. 8. Excitation of mobile type of ILMs to driving with a frequency, 38.9 rad/s, higher than the cutoff one: (a) spatial and temporal profile of the displacements of the oscillators from the quiescent state, (b) temporal profile of the relative displacement of the oscillator nearest the driver and (c) frequency spectrum of the oscillations of the 10th oscillator.

blowing off through the fine holes of the pipe. Because the holes are regularly and closely spaced, the oscillators can be displaced linearly in the direction of track with less friction and damping. Air volume of the blower has been adjusted so that the damping is homogeneous with respect to the distance from the blower.

Finally, we explain the method for excitation and observation of oscillating modes. A driver unit is attached to one end of the chains, which is forced to be displaced along the air track. The other end of the chains is fixed. The unit is constituted of a brushless motor (ORIENTAL MOTOR CO., LTD.: BLM230HP-5S) and Scotch yoke, via which the rotary motion of the motor is converted into linear reciprocating motion to force the adjacent spring sinusoidally with an amplitude and frequency. In the studies we set the amplitude to be 15.0 mm. Excited modes in the chains are observed by a set of motion capture system, where 3 fixed optical cameras can track about 10 oscillators with markers and time series data of their 3D position are obtained every $10^{-2}$ second (OptiTrack: TrackingTools, Flex3) (Fig. 6). From the data we can calculate the velocities and relative displacements from equilibrium positions of the oscillators.

3. Experiments

Motions of the chains are described by the set of equations of motion for oscillations with boundary conditions. As is well known, there are $N$ eigenfrequencies $\omega_n$ ($n = 1, \cdots, N$) in the linearized system with both-end fixed boundary condition given by

$$\omega_n = 2\sqrt{\frac{kL}{m}} \sin \frac{n\pi}{2(N + 1)}$$
Evanescent mode to driving with a much higher frequency, 39.1 rad/s, where oscillations are confined near the driven end.

and they lie below the value $\omega_\infty = 2\sqrt{k_L/m}$. Using the values of parameters $m = 81.8\,[g]$ and $k_L = 27.9\,[N/m]$, the cutoff angular frequency is calculated to be $\omega_\infty = 36.8\,[rad/s]$. It is expected that driving with a frequency higher than the cutoff can excite ILMs. After switching on the blower to levitate the chains, the oscillator which is nearest the driver and initially fixed in equilibrium state is gently released. We have verified excitations of eigen modes predicted by linear analysis to the driving with a frequency lower than cutoff frequency.

In the case that the driving force has a slightly higher frequency than the cutoff one, we have observed that localized oscillations are excited intermittently at the driving end, propagated down the array at a constant speed to the other end, reflected there and propagated back and forth in the chains, subject to nonlinear interactions between them. A typical case is shown in Fig. 7, where the driving frequency is chosen to be 37.7 rad/s. Figure 7(a) shows the spatial and temporal profile of the oscillators from the quiescent state. Figure 7(b) shows the temporal profile of the relative displacement of the oscillator nearest the driver. It is seen that the amplitude of oscillation exceeds the threshold intermittently, at which the spring hardness changes (see Fig. 4) and which is shown by the broken lines in the figure, and at that time the localized oscillations are excited. The FFT shows that the peak of the spectrum of the oscillations is located in the region where linear wave propagations are prohibited (Fig. 7(c)), therefore, the localized oscillations we observed can be regarded as the mobile type of ILMs.

Figure 8 shows the excitations and propagations of moving ILMs in the case of driving with a higher frequency, 38.9 rad/s. Experimental results for excitations of ILMs show that the higher the frequency is, the wider the time interval generating localized oscillations becomes. In the case that driving frequency is much higher and exceeds a value, ILMs tend not to be excited but to be evanescent. The value gives the upper limit for exciting ILMs. The results for the case of 39.1 rad/s are shown in Fig. 9.
4. Conclusions

We made another type of mechanical nonlinear coupled oscillator chains, where the oscillators can move smoothly on the long and straight track by air levitation. Improvements on the nonlinear springs are also made, as a result, the concern about buckling of the array of coupled chains for large-amplitude deflections is dispelled. It is observed that driving sinusoidally with a higher frequency than the cutoff can excite movable ILMs. It is also observed that above the cutoff there is a bandwidth of frequency where ILMs could be excited. In the band the higher driving frequency is, the wider the time interval between excitations of localized oscillations becomes and beyond the threshold ILMs are no longer excited. The results obtained by the new apparatus correspond to the ones by our previous apparatus.

Acknowledgments

This work was supported by JSPS KAKENHI Grant Numbers JP24654124, JP26400394. With regard to making springs, the authors got advice and help from HOKUYO SPRING CO., LTD.

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