Sociological and Mathematical Models as Tools of Social Processes Applied Sociological Research

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Abstract: The methodological aspects of building sociological and mathematical models as tools of applied sociological research of social phenomena and processes in the conditions of the information society have been elaborated on in the article. The role of modelling method in social cognition, forms of social phenomena and processes, issues of building models of social processes, approaches to constructing their models, criteria for systematization of mathematical models of social processes have been considered. The analysis of the problem of using the modelling method in sociology has been carried out, the definition of the concept of "descriptive-information model" has been presented. The methods used to classify the use of mathematical means in sociology, including stochastic models for social processes, deterministic models for social processes, models of structure and models of human behavior, have been defined. The technology of computer modeling, which acts in modern conditions as an alternative to mathematical models in sociology, has been considered. There is a gap in mathematical and formal sociology between models and empirical analysis. A sociological and mathematical model of the process of information interaction based on fuzzy cognitive modeling has been presented, which allows to formalize and take into account the subjective characteristics of the participants of communication in social systems whose assessment has verbal expression.

Key words: sociological research, social systems, social processes, sociological modeling, multagent approach, fuzzy cognitive model, computer modeling technology, information interaction.

I. INTRODUCTION

Rationale. The hallmark of modern society is its constant modernization. Society as a whole and its individual spheres are constantly changing, and these changes are, as a rule, ambivalent: some are enhanced, others fall into disrepair. Therefore, there is a need to analyze and model various social processes. Today, traditional sociological methods are not enough for a comprehensive analysis of social processes. Fluid environment and constant dynamics of events that require consideration of the whole set of significant factors have an impact. The dynamics manifests itself not only in the speed but also in the direction of the processes, which deprives the forecast based on linear logic, intuition, experience, extrapolation model of the efficiency. The ambiguity of the possible interpretation, which puts the result of the study in the dependence of the ideological bias, material dependence, personal loyalty of the analyst and other subjective moments, also significantly affects. The need to take into account all aspects, actors, and factors of social processes requires a systematic approach. And the inability to reproduce the full range of social process conditions makes us resort to modeling the real situation. The models that emerge, reflecting complex social objects, themselves are so complex that their effective use is difficult to imagine beyond mathematical descriptions and applications of computer technology. Solving practical social problems, especially in times of social transformations, escalations, and crises, requires the use and serious analysis of the available range of mathematical methods used in Russia and in the world in conducting specific sociological studies, as well as a comprehensive discussion of the whole range of methodological problems of the application of mathematics and modelling on sociology.

Problem statement and analysis. The problematic situation of social and mathematical modelling is caused by the contradiction between the need to increase the model building time due to the accelerating complication of society and between the contracting time of the existence of a social phenomenon due to the shortened periods of realization of social cycles that form the simulated social reality. Therefore, issues of building of sociological and mathematical models of social processes that allow taking into account the relevant specificity, to pass some phases of their construction faster in terms of technology - thus solving the problem of timely building of a useful model of the social process, clearly reflecting the specificity of the growth of the complexity with the help of information technology. Traditionally, simulating social dynamics is the "prerogative" of social science, which has accumulated a solid amount of knowledge on this subject. However, sociologists themselves are often forced to admit that it is premature to say that there is a common methodology for constructing mathematical models as a tool for studying social processes. V.A. Shvedovsky writes: "The modelling that most sociologists study is of a theoretical nature, at best rising to the level of diagrams or graphs, tables and formulas of primary processing of empirical material" [38, p. 3-4]. The problems of the application of the modelling method in sociology, the study of social processes, the use of mathematical and formalized methods in the social sciences, modelling of social systems were reflected in the works of domestic scientists E.A. Abgaryan, E.P. Andreev, A.V. Borisov, Yu.N. Gavrilts, A.K. Gutz, P.G. Maslov, A.P. Mikhailov, G.V. Osipov, Yu.M. Plotinsky, V.M. Safronova, G.G. Tatarova, NP Tikhomirov, Yu.N. Tolstova, O.F. Shabrov, A.V. Shvedovsky, V.N. Shubkin and others. Despite the considerable volume and results of the research works of these authors, it should be noted that, as a whole, the problem of modelling in sociology has not been sufficiently studied in the entirety and variety in the scientific literature, as well as the rigorous consistency of the modelling method at all stages of sociological research.
The objective of the study is to develop methodological bases for building sociological and mathematical models as tools for applied sociological studies of social processes and patterns of information interaction in social systems.

II. RESEARCH METHODOLOGY

The theoretical and methodological basis of the study is the conceptual position of theoretical and applied sociology, as well as the general theory and practice of mathematical modelling of social processes using systems of dynamic models. Methods such as analysis, synthesis, modeling, abstraction, structural and functional, information and institutional and network were used in the work. The study used computer simulation technology for social processes. Computer modeling provides an opportunity to realize the idea of the birth of complex social behavior from the relatively simple actions of individuals.

Scheme. Technology of computer modeling of social processes

The study also used mathematical (computer) modeling of the process of information interaction in social systems.

Scheme 2. The technology of mathematical (computer) modelling of the process of information interaction in a social system.
An algorithm for modelling the process of information interaction in social systems has been developed.

An algorithm is a strictly defined sequence of actions for a researcher, leading to a goal or a given result in a finite number of steps. This article has applied such a way of representing an algorithm for modelling the process of information interaction in a social system as verbal recording (in natural language).

III. RESULTS ANALYSIS

During the study, the following results have been achieved.

1. Methodological aspects of sociological modeling of social processes have been studied and presented.

   An analysis of the literature that uses the term "model" shows that the term is used in two meanings: 1) in the meaning of theory (abstract model) and 2) in the meaning of the object (or process) that is reflected in this theory (specific model). At the same time models can be material and ideal, real and signed, have the form of a spatial analogue, image, mathematical or specially constructed verbal description.

   The Philosophical Encyclopedic Dictionary interprets modelling as a method of exploring objects of cognition on their models. In this context, the concept of "model" acts as "an analogue (scheme, structure, sign system) of a certain fragment of natural or social reality, the generation of human culture, conceptual and theoretical formation, etc. - the original of the model," [29, p. 381-382].

   Analysis of the principles of modelling allows establishing its general scientific nature. "Modelling implies the existence of clear methodological and theoretical prerequisites. The initial basis of modelling is the material unity of the world, the laws of its development, according to which same, especially structural and functional laws, laws of the organization are inherent to qualitatively different systems." [6, p. 31].

   On this basis, the idea of a model is something variable that depends on the purpose of the study (problem statement), experience, knowledge of the scientist and the choice of object (the level of complexity of the studied system). At the same time, the object of the study can be both the construction of scientific theory and the experimental verification of any hypothesis or the development of a practical method in the subject area of knowledge.

   From an epistemological point of view, modelling is a solution to the triune problem. First, the characteristics of the system-object obtained during its preliminary study are reproduced on the model. Then the model as a set of reflected or reproduced characteristics of the original system is thoroughly investigated, resulting in gaining new knowledge about these characteristics, and often about the system as a whole. Finally, the knowledge gained on the model is transferred to the original, resulting in the knowledge of the original system being enriched, deepened and expanded. This new knowledge, enriched through the study of the model, can be put into practice. The modelling process thus acts as a unity of three stages: studying the parameters of a real system and constructing its model on this basis (the first); model research (second); extrapolation of the studied properties of the model to its original (third).

   The analysis of modern literature shows that the following are distinguished by the scientists as the basic principles characterizing modelling [16]:
   - the principle of analogy of model and object of study;
   - the principle of interaction in the process of modelling of intuitive and content and formal aspects of knowledge;
   - the principle of obtaining new knowledge about the object of study;
   - the principle of experimental verification of new knowledge and some hypothetical ideas about objects of a certain kind.

   Modelling as a way of analyzing and designing different processes is used by many sciences. They study objects and processes from different perspectives and build different types of models. The application of this method in sociology has its own peculiarities. Thus, according to V.A. Yadov, modelling can be used in the accepted methodology of sociological research at the stage of preliminary systematic analysis of the object of research.

   "A preliminary system analysis of the subject of study is essentially a 'modelling ' of the problem" [39, p. 59]. Yu.M. Plotinsky believes that "it is the model approach that can cement the theoretical and applied sociological studies. Instead of fragmentary analysis of individual variables, consideration of their relationship, i.e. the model, the integrity of the approach will be ensured, since the model certainly has a degree of integrity and in this sense is a system." [32, p. 88]

   A model in sociology is, as a rule, a descriptive and informative analogue of a social object or process created with the purpose of studying, predicting and managing it.

   Descriptive and information model is a set of information characterizing the properties and states of an object, process, phenomenon, as well as their relationship with each other and with the outside world, as a rule, in the text-graphic form [37, p. 78].

   This definition shows that the model in sociology is necessary in order to:
   - understand how the real object is arranged, its composition and structure, basic properties, features of functioning and interaction with other subjects and with the environment;
   - predict the consequences (explicit and hidden) of different forms of impact on the object and different forms and directions of interactions;
   - manage the objects and processes of their interactions.

   Of particular interest to sociological science is the modelling of interaction processes.
The sociological model, as a rule, consists of the following formal elements [21]:

- component, the basic (if necessary, decomposed into components) structural element of the sociological model. In a simplified, "mechanistic" sense, it is a brick in a laying, an atom in a molecule. In the model it performs a simple function;
- module - a block in the sociological model, which consists of several components. It can be especially considered as a private sociological model. Performs a complex function;
- basic structure - the minimum basis of the model, its core. If any element is excluded from the basic structure, the model loses its working capacity and explanatory potential;
- contour - a closed circuit, which can both connect several components inside the module, or combine several modules into a single circuit. The convenience of using the contour is that, depending on the specific research task, you can build a descriptive and explanatory procedure, starting with any module (component) included in the contour.

As you know, models can be divided into statistical and dynamic. Statistical - describing the specific state of the object of interest to us. Dynamic - representing the process of state changes, that is, describing the process and result of interactions. Accordingly, in our opinion, it is advisable to use the so-called composition (or structure) model to describe objects, and a model of interaction (or functioning) to describe the process (relations between objects).

The composition model displays which parts (elements) the studied object consists of. The main difficulty in constructing the composition model is that the division of the whole object into parts is relative, conditional, depending on the goals of the modelling. In addition, the definition of the smallest part - the element - is also relative.

The internal structure of the object can also be diverse, heterogeneous and consist of many functional elements. In order to get an idea of the properties of the studied object, it is often necessary to identify certain connections (relationships) between its internal elements. The set of relationships of elements with each other, ensuring the integrity of the object, is called its structure. The structure model in its simplest form is a list of relationships within an object that are essential for solving a specific problem. Thus, the composition (or structure) model of the studied object is the identification of the constituent elements of the object and their relationships among themselves within this object.

As for the models of interaction (functioning) between objects, the basis can be taken as an agent-based model of interactions widely used in economic science [20]. In agent-based modelling, the basic unit of the model is an agent that operates in a specific environment. Each agent acts independently and interacts with other agents based on certain rules. An agent is "a certain entity that has activity, autonomous behavior, can make decisions in accordance with a certain set of rules, can interact with the environment and other agents, and can also change" [19, p. 292].

It is important that the result of the interaction of agents is the behavior of the system as a whole. Accordingly, within the framework of this approach to modelling it becomes necessary to correctly display the mechanism of behavior and interaction of system elements, that is, "agents", especially since even a small change in the rules of behavior and interaction can lead to significant changes.

In general, for the condition of social objects and processes using modelling, the following approaches can be used: the object under study - a hypothesis - an experiment - a descriptive and informational model. Another option can be offered: the object of study - a theoretical model - an experiment. Or: formalization - modelling itself - interpretation.

There are the following signs of the classification of models of social processes [25, p. 248]:

1. The type of mathematical apparatus by which the process is formalized. The main difference is related to whether the model is random, that is, the dynamics of change cannot be accurately predicted, or deterministic (definite, causal). Other signs relate to the type of variables used: constant or specific time; whether the dependent variable is constant or whether it is represented by discrete states.

2. The main function of process models in abstract and empirical research. Based on this basic function, the models are divided into abstract and empirical.

3. The content of the processes under consideration: processes in small and large groups, individual and group decision-making processes, group structure dynamics, etc.

4. Whether the given process or phenomenon is analyzed as a process without control or as a controlled process.

Modelling of social processes involves three stages in the study: 1) formalization of the phenomenon under study and the construction of an appropriate analogue; 2) searches for a solution to the problem using operations with an analogue; 3) an explanation of the result in relation to the studied society.

2. The technology of computer modelling of social processes has been highlighted and presented.

As already noted, modelling is the main method for carrying out research in all spheres of human activity and a scientifically sound method for evaluating heterogeneous systems used in management in various spheres of social activity. Existing and developed systems can be effectively studied using mathematical models (analytical and modelling), applied using computer technology, which in this case act as a tool for conducting an experiment with a system model.

Mathematical modelling of social processes is the creation of structures, graphic images, diagrams, and other abstractions, mathematically ordered and based on mathematical principles. The relationships between the constituent parts (factors) of the scheme, structure, graph, or mathematical equation can be characterized as factor dependencies based on the degree of preference by the expert of certain factors in the process under study. [31, p. 19].
Four classes of mathematical models are distinguished in sociology: stochastic models for social processes, deterministic models for social processes, structure models and human behavior models [27]. A distinctive feature of the application of mathematics in modern sociology is that it becomes increasingly difficult to consider the process, structure and behavior separately.

Computer mathematical modelling in its various manifestations uses almost the entire apparatus of modern mathematics. Under the computer model the following is understood [7]:

- a conventional image of an object or some system of objects (or processes) described using interconnected computer tables, flowcharts, diagrams, graphs, drawings, animated fragments, hypertexts and showing the structure and relationships between elements of the object. Computer models of this kind are called structural and functional;
- a separate program, a set of programs, a software package that allows using a sequence of calculations and graphical displays from the results to reproduce (simulate) the functioning processes of an object, a system of objects, provided that various, usually random, factors are exposed to the object. Such models are called simulation models.

Computer modelling is a method for solving the problem of analysis or synthesis of a complex system based on the use of its computer model. The essence of computer modelling is to obtain quantitative and qualitative results based on the existing model. Qualitative outcomes obtained from the results of the analysis make it possible to detect previously unknown properties of a complex system: its structure, development dynamics, stability, integrity, etc. Quantitative outcomes are mainly of the nature of forecasting some future ones or explaining past values of the variables characterizing the system. Computer modelling provides an opportunity to realize the idea of creating complex social behavior from the relatively simple actions of individuals [23].

The technology of computer modelling of social phenomena and processes contains the following steps [35, p. 25-26]:

1. The study of sociological theory, on the basis of which the model is determined.
2. The search for the main components of the structure of the object, the relationship of determining factors.
3. The definition of the information model and analytical visual schemes based on the sociological theory of the object of modelling.
4. The study of the theoretical basis of the finished information model and the construction of a mathematical model (choice of mathematical apparatus, formalization of structure, relationships and elements).
5. Building a mathematical model using a computer (selection of a computer simulation method and a modelling algorithm).
6. Application of a ready-made computer model (work with computer models as objects of study: entering initial data, obtaining results in the form of graphs and diagrams, analyzing and interpreting the data obtained, changing the initial conditions based on available results to find the optimal solution).

As a result of the analysis of the computer model, a conclusion is drawn about its suitability for a particular social process. Then the following is decided: either to modify the structure of the constructed model to improve it, or to conduct an additional analysis of the selected sociological object, or to collect the missing information about the studied social process.

The mathematical models built using computer modeling methods are implemented in the form of algorithms used to create software tools.

Computer modelling methods are divided into two levels according to the object of modelling social processes. Each of them is divided into several different types, respectively, characterized by the selected mathematical apparatus used to construct the mathematical model.

The main methods of computer modelling:

a) micromodelling: multi-agent modelling; modelling of cellular automata; queue implementation;
b) macromodelling: modelling of dynamic systems; expert systems modelling [35, p. 20].

Computer models have many advantages over other approaches to the study of social processes. In particular, they make it possible to take into account a large number of variables, to predict the development of nonlinear processes, the occurrence of synergistic effects (qualitative transformation of the structure of the studied object as a result of an unpredictable change in state). In addition, they allow not only to obtain a forecast, but also to find with the help of computational experiments which control actions will lead to the most favorable course of events.

Thus, there are many technologies that make it possible to simulate social systems, as well as obtain conclusions and forecasts. At the same time, it should be specially noted that for each of the studied systems any techniques can be selected, however, it should not be assumed that the conclusions and forecasts obtained using this or that type of modelling do not depend on expert assessments and the analyst’s interpretation of the available data.

3. The sociological and mathematical model of the process of information interaction in the social system has been developed.

The social system is a holistic formation, the main element of which are people, their interactions, relationships, and communications. Today, social communication channels, and first of all, the global information space of the Internet with its social services, are actively used to organize various actions of information and psychological influence on the individual, social groups and society as a whole [18]. A negative manifestation of this practice is the propaganda of the ideology of extremism, terrorism and other types of illegal activities. Another serious threat to the stability of society in recent years has been the spread of information resources promoting suicide.

Various forms of information warfare have actually turned into the stage of information warfare. Therefore, to identify and counter the acts of destructive influences,
it is necessary to know the features of the process of disseminating information, taking into account the capabilities of modern mass communication means.

**Features of the process of disseminating information in social systems.** Unlike traditional media (newspapers, radio, television), modern digital communication channels provide active participants in the process of information interaction the opportunity to influence the political, economic and cultural life of society, changing and transforming public opinion [30].

On the one hand, the opinion expressed by any participant in communication has an impact on the process of information interaction. On the other hand, each person decides on the need for further dissemination of information that has become known to him. Therefore, when modelling the process of information interaction, it is necessary to take into account individual characteristics of each of the parties to the process.

Understanding the features of the process of disseminating information allows decision-makers to carry out information management actions in the social system with various effects - from organizing and supporting individual legitimate information flows to combating malicious ones. In this case, information management is understood as an implicit indirect mechanism of influence, when the control unit is provided with information blocks that direct and stimulate it to choose a certain line of behavior [26].

A number of works by domestic and foreign researchers is devoted to an analysis of the features of information interaction in social systems: D.A. Novikova, D.A. Gubanov, A.G. Chkhartishvili, V.V. Breer, A.D. Rogatkin, I.N. Barabanov, M. Granovetter, T. Schelling, A. Kofman, S. Wasserman, C. Faust and others [9, 11, 12, 40-43].

However, in the simulation models developed by these and other authors [13], not all the specific features of social systems that are of interest in applied sociological research are reflected. For instance, they do not always take into account the subjective personal characteristics of communication participants that affect the process of information interaction, for example, various degrees of sociability, willingness to disseminate information and the degree of external influence. Also, these models do not allow to explore and assess the speed of dissemination of information in social systems, which is of great importance in planning management activities. In addition, existing mathematical models are based on a number of fairly strong assumptions, the presence of which complicates their use in applied research on real social interactions.

Today, one of the promising directions of modelling processes occurring in social systems is the multi-agent approach [24]. Within its framework, system elements are analyzed as separate units interacting with each other. This allows you to explore the properties of the social system as a whole, based on the properties of its constituent elements and the rules of interaction between them.

It should be borne in mind that a person, being a key element of social systems, introduces a factor of subjective uncertainty into the processes of their analysis, which turns modelling processes in such systems into a poorly formalized problem [3]. Therefore, when modelling the process of information interaction, in addition to the methods of probability theory and mathematical statistics, special tools are needed, which should be used as an apparatus of the theory of fuzzy sets and fuzzy cognitive modelling [36]. Fuzzy cognitive modelling allows us to more accurately assess and take into account the subjective characteristics of interaction agents that do not have the correct quantitative expression and formalize information that is verbal in nature [1]. To formalize subjective data, it is necessary to determine the linguistic “Factor Level” variable and determine the term - set of values, consisting generally of several elements belonging to the negative and positive range of values [33].

**A model for disseminating information in social systems.** When describing the process of disseminating information in social systems, personality parameters are taken into account characterizing the members of social systems on the basis of using the results of the theory of fuzzy sets and fuzzy cognitive modelling [17]:

1) The "conservatism level" indicator, which is the ability to maintain one’s opinion under the influence of the information background. The “high conservatism” value corresponds to the tendency to always stick to one's opinion, while the “low” one corresponds to a high degree of conformity (the tendency to change one’s point of view, adapting to the opinion of others). The inverse indicator of conservatism can be called the coefficient of receptivity of someone else's opinion (an indicator of conformism). In some cases, it is more convenient for use in calculations.

2) The likelihood that a person will disseminate information that has become known to him largely depends on the level of his sociability. Various psychological tests can be used to determine the level of sociability. For example, the famous test "Assessment of the level of sociability" by V.F. Ryakhovsky, consisting of 16 questions, allows you to assess sociability according to the points obtained [10]. At the same time, the degree of sociability is classified into seven categories: from obvious non-communicability to its painful character.

3) It is also necessary to identify a list of topics, the discussion of which is of interest to members of the studied social system (or, conversely, will not find a response among the participants of the social system in the process of disseminating information). This allows us to draw conclusions about the level of interest, as well as the distribution of initial opinions regarding the information disseminated.

4) The degree of trust of a participant in information interaction to sources is a characteristic that determines the weight of opinions of those information exchange agents through which information is received.

If we imagine the information interaction in a social system consisting of N members, in the form of a graph $S = (M, D)$, the top elements of which are people $M = \{M1; M2; ...; MN\}$, then the set of edges $D = \{Dij\}$, which reflects the potential information exchange between them, forms the "information exchange matrix":
with $M_j$;

$$D_{ij} = \begin{cases} 1, & \text{if } M_i \text{ communicates with } M_j, \\ 0, & \text{if } M_i \text{ doesn’t communicate with } M_j. \end{cases}$$

The most important characteristic of such a graph is the distribution of units by the number of connections - the number of "contacts" of the participant in information interaction. Information interaction in social systems is also characterized by the TI confidence matrix for the topic under study, the elements of which can be expressed by fuzzy numbers. At the same time, both the trust matrix and the information exchange matrix are not necessarily symmetric: it does not follow from the fact that the j-th participant shares information with the i-th or trusts him, that the latter will do the same in response. In particular, a vivid example of such a case is the situation when certain information is disseminated by the media. In this case, information interaction is strictly one-sided.

The asymmetry of these matrices is caused, first of all, by the difference in the levels of "conservatism" and "sociability" of the participants in information interaction.

An important dynamic indicator of the process of information interaction is the change in the number of informed participants, which is reflected by the awareness vector. When modelling this process, obtaining information by the i-th participant is reflected by changing from 0 to 1 the corresponding coordinate in the awareness vector $Z (Z_1; Z_2; \ldots; Z_n)$:

$$Z = \begin{cases} 1, & \text{if } M_i \text{ possess information;} \\ 0, & \text{if } M_i \text{ doesn’t possess information}. \end{cases}$$

The "information transfer indicator" by the i-th agent ("repost indicator") indicates his readiness to disseminate information. This dynamic indicator depends on both the degree of sociability of the communication participant Oi and the level of expression of his current opinion. The repost indicator is 0 if the degree of sociability is estimated by the value of H (Low) from the term-set (1), or if the agent has a weakly expressed negative (C-) neutral (H) or weakly expressed positive (C+) attitude to information I not leading to its further distribution. The set of indicators of information transfer is represented by the repost vector $R (R_1; R_2; \ldots; R_N)$ [5, p. 149].

The process of information interaction in the social system is as follows. Information block $I$, which has a certain subject, is introduced into the social environment at the initial time $t = 0$ by a finite number of its representatives $M = \{M_l\} (l = 1; L, L < N)$ with a deliberately positive (or negative) attitude towards $I$. The set $M$ is called the initiating set. Interpersonal information exchange ensures the dissemination of information between participants. At the same time, the j-th participant with information I brings it to the attention of the i-th participant along with his opinion about this information. Based on the foregoing, to evaluate the current (at a discrete time $t = t + 1$) relationship $V_{it+1}$ of the i-th representative of the social system to information I, after exchanging opinions with other participants, it is proposed to use the following formulas:

$$\hat{V}_{it+1} = C_{i} \cdot F_{it+1} + (1 - C_{i}) \cdot CSP_{it+1},$$

$$V_{it+1} = \begin{cases} 1, & \text{atDef } [\hat{V}_{it+1}] \geq 1; \\ \text{Def } [\hat{V}_{it+1}], & \text{at } -1 < \text{Def } [\hat{V}_{it+1}] < 1; \\ -1, & \text{atDef } [\hat{V}_{it+1}] \leq -1. \end{cases}$$

Here $V_{it+1}$ is the defuzziﬁed (clear) value of the attitude of $M_i$ to information I at a discrete point of time $(t+1)$. For defuzziﬁcation (Def) of a fuzzy value, the "center of gravity" method is used:

$$C_{i} = \text{the conservatism coefficient of participant } M_i, \ \text{reflecting how much he relies on his own opinion within the framework of topic } I. \ \text{It should be noted that the closer the average value } C_{i} = \frac{1}{N} \sum_{l=1}^{N} C_{il} \text{ to } 1, \ \text{the more conservative "collective opinion" in the framework of the topic } I. \ \text{The contribution to the change in } M_i \text{ agent's opinion of the information that became known to him from those around him at the time } (t+1) \text{ is reflected in the so-called "Cumulative Social Power (CSP)" (the term was introduced in [15]). To determine it, it is proposed to use the following formula:}$$

$$CSP_{it+1} = \frac{W_{it}^{+} + W_{it}^{-}}{g},$$

where $W_{it}^{+}$ and $W_{it}^{-}$ - weighted by the degree of trust in the source sums of positive and negative opinions expressed by other participants at a time $(t+1)$; $G = N^{+}\cdot G_{it}^{+} + N^{-}\cdot G_{it}^{-}$ - the number of positive and negative reviews about information I received at the time point $(t+1)$, respectively:

$$W_{it}^{+} = \sum_{j=1}^{N^{+}} (T_{ij}^{+} \cdot W_{ij}^{+}) \cdot T_{jk},$$

$$W_{it}^{-} = \sum_{k=1}^{N^{-}} (T_{ik}^{-} \cdot W_{ik}^{-}) \cdot T_{jk},$$

where $W_{ij}^{+}$-positive opinion received by $M_i$ from the j-th participant at the point of time $(t+1)$; $W_{ik}^{-}$ - negative opinion received by $M_i$ from the k-th participant at the point of time $(t+1)$.

The obtained clear values of $V_{it+1}$ are interpreted according to the Harrington scale as follows [22]:

- $0 \leq V_{it+1} \leq 0.64$ - a pronounced positive attitude that encourages the dissemination of information I, together with their negative opinion (negative rating) (state of the participant in communication $S = S_{+}$);
- $0.64 < V_{it+1} < 0.64$ - weakly pronounced negative (-0.64 < $V_{it+1} < 0$) or weakly pronounced positive (0 < $V_{it+1} < 0.64$) attitudes toward I, which do not lead to further dissemination of information (state $S = S_{0}$);
- $0.64 \leq V_{it+1} \leq 1$ - a strongly pronounced positive attitude that encourages the dissemination of information I, together with my positive opinion (positive assessment) (state $S = S_{+}$).
Aggregate of $V^{t+1} = (V^{t+1})$ reflects the “spectrum of opinions” of members of the social system regarding information $I$ at the time point $(t + T)$. The statistical parameters of the distribution of opinion spectra for various information blocks reflect the sentiments in social systems and allow their monitoring. The information exchange process is terminated if the Hamming distance $\rho_H$ between the current vector $V^{t+1}$ and the vector $V_i$, obtained at the previous time step does not exceed a certain value $N^*$ (set before the simulation):

$$\rho_H(V^{t+1}; V_i) \leq N^*.$$  \hfill (8)

Hamming distance $\rho_H \in [0; N]$ corresponds to the number of communication participants who have changed their mind between two consecutive time steps. When $\rho_H$ becomes insignificant, the discussion of information block $I$ in the social environment is considered complete [8].

**Algorithm for modelling the process of information interaction.** Based on the foregoing, the simulation algorithm of the process of information interaction in the social system can be formulated as follows [28, p. 25]:

1) the creation of information block $I$, related to a specific topic, for distribution in the course of information interaction;
2) determination of the initial vector of opinions of participants $\tilde{V}^0_i$ at step $t = 0$;
3) specifying the size $L$ of the initiating set, consisting of participants with a deliberately strongly pronounced positive (or negative) attitude to information $I$;
4) the formation of the initial awareness vector $Z$, in which the awareness indices of the members of the initiating set $M$ are 1, for the remaining participants - 0;
5) launch of the iteration of information exchange at a time step $t=t+1$;
6) creation of the repost vector $R$;
7) transfer of information $I$ from agents with repost indicator equal to 1 to other participants according to the information exchange matrix $D$; calculation of the current awareness vector $Z$;
8) calculation of the current vector of opinions $\tilde{V}^{t+1}_i$;
9) determination of the Hamming distance between the current vector $V^{t+1}$ and the vector $V_i'$ obtained at the previous modelling step;
10) Hamming distance check: if the condition $\rho_H \leq N^*$ is fulfilled - termination of the simulation and output of the obtained data for analysis; otherwise, return to step 5.

When modelling, the time interval necessary for the single implementation of all communication relationships reflected in the exchange matrix $D$ is taken as one step (tact).

**The specifics of modelling the process of information interaction in large social systems.** The application of the above algorithm allows us to estimate the speed of information dissemination in social systems (increase in the number of informed participants), as well as to track the dynamics of changes in the opinions of agents regarding the information distributed. However, for this, baseline data on the composition and structure of the social system and data on the individual properties of individuals - participants in information interaction should be known.

In relatively small social systems, the number of members in which does not exceed several hundred, obtaining initial data is usually not fundamentally difficult. Moreover, such information is deterministic (non-probabilistic) in nature. For example, in small organizations, the information exchange matrix, the employee’s interest in the information disseminated, and the degree of trust in its source within certain topics can be obtained directly from the analysis of the information provided by the personnel management service. [4].

However, when studying the process of information interaction in larger social systems (for example, on VK or Odnoklassniki social networks), one has to deal with a much larger amount of data [2]. In addition, in most of these cases, the personal data of participants in information interaction is confidential information, obtaining of which is legally impossible without their consent.

Thus, unlike the case of information dissemination within small social systems, it is difficult to obtain exact values for modelling the process of information interaction in large social systems. The initial data in such cases are depersonalized and are only statistical in nature (the nature of statistical distributions). Partially, they can be found in analytical reports on the structure and composition of social networks used as communication channels (see, for example, [34]).

Since the initial data for the simulation of the process of information interaction in large social systems are only statistical in nature, then to evaluate the number of informed participants at each modelling step, as well as to assess the number of different opinions of participants regarding the disseminated information, some additions to the above simulation modeling algorithm are required.

First of all, since the number of their connections is of great importance in calculating the total number of informed agents at each modelling step (discussion step), it is necessary to determine the statistical distribution of the number of contacts (“friends”) with which participants in the social system interact from the results of a study of a representative sample.

At the same time, it is advisable to identify several ranges of the possible number of connections $\{bimin... bimax\}$ $i=1, n$, and accordingly, n categories of participants in information interaction having such communication boundaries. The distribution of the number of connections in the social system is expressed by the shares of agents in each category $\beta_i, i=1, n$.

The process of information exchange between agents follows the following rules:
1) only agents with a high coefficient of sociability $O$ share information;
2) only agents with a strongly expressed attitude to it (positive or negative) disseminate information together with their assessment.

Initially, the number of informed participants is assumed to be equal to the volume of the initialization set $L$ from the general aggregate $N$. Moreover, if information is disseminated for the purpose of managerial impact,
then the initial set is formed so that its members have a predominant positive (or negative) attitude towards the information block being implemented, extensive connections, were quite sociable and enjoyed confidence among the participants in interaction.

Then, after the first iteration (first step) of the discussion, the number of participants with information I is determined as follows:

\[ K_1 = L + \left( \frac{N-K}{N} \right) \cdot L \cdot \sum_{i=1}^{n} b_i \beta_i, \tag{9} \]

where \( \beta_i \) is the average value of the number of connections in the i-th range.

The multiplier \( (N-L)/N \) reflects the proportion of the remaining uninformected participants in the information interaction at this step. This allows the calculation not to take into account participants who receive information repeatedly in the process of exchanging opinions (although the opinions of agents may vary).

At the time step \( t = t+1 \), the number of informed participants in the social system is determined by the formula:

\[ K_{s+1} = K_s + q_i \left( \frac{N-K}{N} \right) \cdot (K_{++} + K_{--}) \cdot \sum_{i=1}^{n} b_i \beta_i, \tag{10} \]

where \( K_{++} \) or \( K_{--} \) is the number of agents with strongly expressed positive and negative opinions at the previous time step; \( q_i \) is the share of agents in step \( t \) who are ready to further disseminate information (make a repost).

The value of the \( q_i \) coefficient, which is included in formula (10), depends on the proportion of participants in information interaction with a high level of sociability - Com, as well as on the relevance of the information disseminated at the time step \( t \) - Acte.

Since the level of sociability is an immanent (intrinsic) property of participants in the social system, we can consider the share Com of participants in information interaction with a high level of sociability to be a constant value (independent of time \( t \)).

On the other hand, the relevance of information disseminated in the process of information interaction decreases over time. Based on this, the following formula is used to determine \( q_i \):

\[ q_i = \text{Com} \cdot \text{Act}_t = \text{Com} \cdot \text{Act}_{t0} \cdot e^{-\tau \text{act}}, \tag{11} \]

where \( \text{Act}_{t0} \) - the initial value of the relevance of the information (at \( t = 0 \)) - is usually taken equal to 1; coefficient of falling relevance (according to numerous studies, for example, [14] \( \alpha = 2.3 \)); \( \tau \text{act} \) - maximum time for maintaining the relevance of the distributed information (information life cycle time).

It should be noted that formula (10) defines the upper bound of the awareness function. In a real social system, the number of newly informed at each step may be less due to the fact that some participants in the information interaction will receive information simultaneously from different sources.

Thus, the number of informed members of the social system is a function

\[ K_{s+1} = K_{s+1}(L, q_i, b, K_{00}), \quad b, \tag{12} \]

where \( b \) is the coefficient of connectivity of the social system (the average number of connections between participants in information interaction in the system).

The formation of opinions during the exchange of information depends on the sensitivity of the participants to the information background. The indicator "coefficient of susceptibility to the opinions of others" (the coefficient of conformity, suggestibility) for participants in information interaction can be estimated by the value from the term set \{low (H); medium (C); high (B)\}. Accordingly, the shares of agents with varying degrees of susceptibility are denoted as \( \omega^H, \omega^C, \omega^B \).

To evaluate the opinions of participants, the values from the term set (3) are used [5]. We introduce the following notation:

- \( K^{++} \) - the number of agents with a strongly expressed positive (positive) opinion (their share is \( v_{t}^{++} = K_{t}^{++}/K_{t} \));
- \( K^{+} \) - the number of agents with a positive opinion (their share is \( v_{t}^{+} = K_{t}^{+}/K_{t} \));
- \( K^{0} \) - number of agents with a neutral attitude (\( v_{t}^{0} = K_{t}^{0}/K_{t} \));
- \( K^{−} \) - number of agents with a pronounced negative attitude (\( v_{t}^{−} = K_{t}^{−}/K_{t} \)).

As a rule, the initial distribution of opinions in a social system is determined by a representative sample.

The following rules have been developed that affect the dynamics of changes in opinions in the social system [28, p. 29]:

1) the opinion of participants with low susceptibility does not change;
2) participants with medium and high susceptibility, receiving emotionally colored reviews, change their minds;
3) people with medium and high susceptibility who receive reviews that are contrary to their beliefs "come out" (are subtracted) from the categories of participants with a strongly expressed opinion. Participants with an average sensitivity form just a positive or negative opinion, with a high susceptibility become neutral;
4) participants from the "neutral" group with high susceptibility form a strongly expressed opinion (positive or negative), receiving appropriate feedback;
5) "neutral" participants with an average susceptibility gain simply a positive or negative opinion in accordance with the feedback received;
6) participants with a positive opinion and medium or high susceptibility, receiving positive reviews, form a strongly expressed positive opinion;
7) participants with a negative opinion and medium or high susceptibility, receiving negative reviews, form a strongly expressed negative opinion;
8) participants with a positive opinion and average susceptibility, receiving negative reviews, become neutral. If the susceptibility is high - they form a negative opinion;
9) similarly, the opinion of participants with a negative opinion and average susceptibility,
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who received positive feedback, becomes neutral. If the susceptibility is high, they form a positive opinion.

Then, taking into account the formulated rules, the number of agents with a strongly expressed positive opinion at the modeling step \( t = t + 1 \) is determined as follows:

\[
K^++_{i+1} = K^+ + (K_{i+1} - K_i) \cdot (\nu^+_t - \nu^{++}_t) \\
\times \left( \frac{\nu^+_t \cdot (\omega^C + \omega^B) \cdot (\frac{K^+}{K^+ + K^-} + \frac{K^{++}}{K^{++} + K^-}) + \nu^{++}_t \cdot \omega^C \cdot (\frac{K^{++}}{K^{++} + K^-})}{\frac{K^+}{K^+ + K^-} + \frac{K^{++}}{K^{++} + K^-}} \right) \\
\times \left( \nu^+_t \cdot (\omega^C + \omega^B) \cdot (\frac{K^-}{K^+ + K^-} + \frac{K^{--}}{K^{--} + K^-}) + \nu^{--}_t \cdot \omega^C \cdot (\frac{K^{--}}{K^{--} + K^-}) \right).
\]

The number of participants in a social system with a negative and strongly negative opinion is determined by formulas similar to (13) and (14).

Thus, the proposed model of the process of information interaction allows us to study the patterns of information dissemination and the dynamics of the formation of opinions in social systems.

**IV. CONCLUSION**

Modelling provides the sociologist with the opportunity to test a complex, unstructured or poorly structured condition without experimenting on people. In this case, the starting point of modelling is the whole range of structural relationships, which are analyzed as a kind of unity. In connection with the improvement and more active use of models, as well as in connection with the increasing difficulties of directly obtaining new knowledge, modelling becomes one of the most important methods of sociological research. Further development of modelling in sociology involves: the creation of more advanced models for the description, transformation and structuring of social information; building models more appropriate to the studied phenomena and processes and capable of becoming the basis for constructing theoretical schemes of the corresponding objects; implementation of the transition from models of individual phenomena and processes to complexes of models constructed from them, which will allow studying various spheres of society’s life inextricably linked with solving management problems.

**RECOMMENDATIONS.**

Mathematical modelling in sociology makes it possible to replace the direct analysis of the basic properties of social phenomena with an analysis of the properties and characteristics of mathematical objects (models). Computer models of social processes and computational experiments with these models are an important means of managing social processes. The very procedure of translating theoretical representations in the form of a model allows a deeper understanding of the essence of simulated phenomena, and verification of the model is actually a test of the theory laid down by the researcher as the basis of the model. In applied sociological studies of real social systems, knowledge of the laws of information dissemination and the dynamics of forming opinions, as well as the specifics of social communication processes in large systems, makes it possible to monitor the information background, make forecasts, plan and implement information management acts. New discoveries in sociology can be obtained simply by applying mathematics in a new way. The achievement of basic and methodological progress in economic sociology and sociology of organizations was made possible through the use of mathematics in order to increase understanding of social phenomena. Long-term and ever-growing interest in the structural analysis of social networks, the spread of innovation and debate about the concept of a rational person are compelling examples of this approach.

At the same time, it would be wrong not to admit that statistical modelling is a field of sociology in which mathematics has the strongest effect on sociology as a whole. A number of statistical tools currently available for network analysis, historical events analysis, and hierarchical linear modeling take modern sociology to a new level of complexity, in part because of easy-to-use software packages that even sociologists who are weak in mathematics can identify and evaluate statistical models.
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