On possibility of local suppression of natural vibrations of elastic structures under shock effects

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Abstract. Manifestations of shock effects in the form of subsequent natural oscillations of structures for various purposes can be different in intensity, in directions of oscillatory processes (forms of natural oscillations) and in frequencies involved in oscillatory processes. At the same time, the tasks of suppressing or reducing the intensity of oscillations in certain specified directions have obvious relevance. Suppression of forced stationary oscillations in certain specified directions (obtaining zero amplitudes of oscillations in these directions) is possible by forming dynamic quenching effects implemented as the systems that satisfy a number of formalized requirements for the design parameters of such systems. In earlier papers of the authors it was shown, that the realization of such effects in the processes of natural oscillations is associated with the formation of conditions for the multiplicity of natural oscillations of elastic systems. In the proposed paper the conditions for the formation of the multiplicity of natural frequencies of multiply connected finite-dimensional systems of small dimension are considered, reflecting the specific manifestation of the multiplicity of natural oscillation frequencies of finite-dimensional dynamic systems of arbitrary dimension, the properties of such systems are applied to the suppression of natural oscillations in specified directions.

1. Introduction
Building structures, ship and aircraft structures are subjected to pulsed and shock effects [1,2]. In elastically deformable structures, shock impacts are accompanied by subsequent natural vibrations [3]. For the structures whose design schemes in dynamic manifestation are multidimensional, multiply connected dynamic systems vibration processes are distinguished by complexity and inhomogeneous intensity distribution over structural elements [4,5]. The arising tasks related to determining the intensity of vibrations in the given computational areas can be caused by the need to place precision and measuring equipment in them [6], the need for expertise or other numerous reasons. Since admissible simplifications of dynamic models of such deformable systems – up to one-dimensional in the overwhelming majority of cases – are impossible [4], inevitably we have to introduce into consideration internal elastic constraints of dynamic systems, which are displayed in off-diagonal coefficients of stiffness matrices in a formalized presentation [7,8].

In the proposed work, we consider the manifestation of the properties of natural vibration processes in a finite-dimensional multiply connected dynamic system [8, 9] with different variants of impulse effects. To study such properties, we consider the variants of dynamic responses of a multidimensional system to the effects of instantaneous pulses in various directions of the degrees of freedom of a
dynamic system [10,11]. The manifestations of the properties of such an interaction are formalized by representing the elastic dynamical system in the space of natural vibration forms (eigenvectors) with the display of the impulse effect vector in the coordinate system of the eigenvectors of the dynamical system [10,11]. The coordinate system of eigenvectors (normal coordinate system) is formed by solving the complete eigenvalue problem for the dynamic stiffness matrix of the original multiply-connected elastic dynamic system [12,13]. The possibilities of eliminating vibrations of finite-dimensional linear dynamic systems in given directions when exposed to an instantaneous pulse are considered. The properties of finite-dimensional systems with short frequencies of natural vibrations and the necessary conditions for the formation of systems of limiting frequency multiplicity are studied. It is proved that, for finite-dimensional systems, the conditions of limiting multiplicity lead to the necessity of eliminating cross elastic bonds.

2. Relevance
Finite-dimensional dynamic models currently have an overwhelming prevalence of use in practical calculations and research of various kinds. One of the reasons for such popularity is the possibility of forming dynamic models based on the formation of finite-element design schemes [14-16], which allow flexible approximation of the boundaries of computational regions and heterogeneous boundary conditions. By themselves, finite element computational schemes, describing the stiffness properties of the object being calculated, make it possible to form dynamic models by concentrating inertial parameters at the nodes of the finite element model [6] using various discretization methods. Being formed in such a way, dynamic models, being finite-dimensional, allow using a well-developed apparatus of linear algebra [17], using the advantages of matrix operations [7,17] in computer implementation of calculation algorithms [18,19]. Given the combination of these circumstances, it would be very useful to solve the problem of the possibility of suppressing natural oscillations in specified directions of oscillations within the framework of finite-dimensional approximations of dynamic models of structures and structures. Previously it was shown that the solution of this problem, as well as the solution of the problem of the intensity of pulse formation in a given direction, is associated with studies of oscillatory processes with multiple frequencies of natural oscillations. It is obvious that the study of such processes is also of independent interest.

3. Method of determining the parameters of natural oscillations of a finite-dimensional dynamic system
In previous works [10,11], it was shown that when an instantaneous pulse is applied to a finite-dimensional linear dynamic model of dimension \( n \) in the absence of natural vibrations in a certain direction \( j \) that does not coincide with the direction of influence \( i \), it is necessary and sufficient to provide \( n \) the multiplicity of the eigenvalues of the dynamic matrix stiffnesses \( D=M^{-1}R \), where \( M \) is the matrix of inertial parameters, \( R \) is the stiffness matrix of the dynamic model.

Named directions \( i, j \) can be visually represented when the matrix \( R \) is formed by the displacement method [4, 7] by means of numbering the connections according to possible directions of kinematic mobility or when this matrix \( R \) is formed by the finite element method [9,14,16] by possible directions of nodal displacements of finite elements in the global system coordinates of an ensemble of finite elements.

In expanded form, matrices \( M \) and \( R \) and have the form:

\[
M = \begin{bmatrix}
m_1 & 0 & \cdots & 0 \\
0 & m_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & m_n
\end{bmatrix}
\]
The oscillatory process can be described by a system of ordinary differential equations
\[ M \cdot \ddot{X}(t) + R \cdot \dot{X}(t) + G \cdot X(t) = 0, \]
where \( X(t) \) is the function vector of nodal displacements, \( G \) is the dissipation matrix of the dynamic system.

In [11], it was shown that the necessary condition for achieving the zero value of the amplitudes of the vibrations \( x_j(t) \), when acting in some arbitrary direction (let it be the direction with number 1) is the fulfillment of equalities:
\[
\frac{\sin \omega_1 t}{\omega_1} = \frac{\sin \omega_2 t}{\omega_2} = \cdots = \frac{\sin \omega_n t}{\omega_n}
\]
where \( x_j(t) \), \( (j = 1 \div n) \) – the function of moving a certain model node in the direction \( j \); \( \omega_i \) – the natural frequency; \( t \) is a time parameter.

The validity of this statement follows from the condition of orthogonality of the vectors \( A_j \) and \( B_1 \), \( (j \neq 1) \), where \( A_j \) – the vector row of the matrix \( F \); \( B_1 \) – vector column of matrix \( F^{-1} \); \( F \) – the matrix of eigenvectors of the matrix \( D \) [13], which can be represented as:
\[
F = \begin{bmatrix} A_1 \ A_2 \ \cdots \ A_j \ A_{n} \end{bmatrix},
\]
where \( a_{ij} \) are elements of vectors \( A_j \) and \( b_{11} \) are elements of vectors \( B_1 \).

For a system with a dynamic stiffness matrix \( D=M^{-1}R \), the eigenvalues \( \lambda_i \) are determined from the equality:
\[
|D - \lambda I| = 0,
\]
where \( I \) – the identity matrix. Let \( r_{ij} \) – elements of the matrix \( R \); \( m_i \) is the inertia parameter when moving in the direction with the number \( i \) \( (i = 1 \div n) \).

For a system with two degrees of freedom, the finding of this condition is determined by the multiplicity of the roots \( \lambda_i \) of equation (2), which can be written in the form:
\[
\begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} + \begin{bmatrix} f_{11} \\ f_{21} \end{bmatrix} \begin{bmatrix} m_1 \ m_2 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} - \frac{r_{12}^2}{m_1 m_2} = 0
\]

The multiple roots of equation (3) are multiple eigenvalues, and if the determinant of equation (3) is equal to zero:
\[
\left(\frac{f_{11}}{m_1} - \frac{f_{12}}{m_2}\right)^2 + 4 \cdot \frac{r_{12}^2}{m_1 m_2} = 0
\]
By virtue of the non-negativity of the components of the left-hand side of equality (4), it is true:
Equalities (5) show that the condition for the frequency multiplicity of natural vibrations for second-order systems makes it necessary to form two uncoupled first-order systems. A dynamic system that has elastic constraints in the form \( r_{12} = r_{21} \neq 0 \) cannot have multiple frequencies.

For a system with thirds of freedom, it is also assumed that the instantaneous impulse is applied in direction number 1. Then the displacement function of a node in direction 2 can be written as:

\[
x_2(t) = \sum_{i=1}^{3} a_{i2} b_{i1} \sin(\omega_i t) \frac{t}{\omega_i},
\]

the identical equality \( x_2(t) = 0 \) is attainable when

\[
\omega_1 = \omega_2 = \omega_3
\]

(6)

In this case, fair

\[
a_{21} b_{11} \frac{\sin(\omega_1 t)}{\omega_1} + a_{22} b_{21} \frac{\sin(\omega_2 t)}{\omega_2} + a_{23} b_{31} \frac{\sin(\omega_3 t)}{\omega_3} = (A_{21} R_i) \frac{\sin\omega_i t}{\omega_i}.
\]

Since the right-hand side of this equality is the scalar product of different vectors of mutually inverse matrices \( F \) and \( F^{-1} \), it is identically equal to zero.

Consider the conditions for the validity of equality (6). Expanded equation equation \(|D - \lambda E| = 0\) for the third-order system has the form:

\[
f(\lambda) = -\lambda^3 + \lambda^2 \left( \frac{r_{11} + r_{22} + r_{33}}{m_1} \right) + \lambda \left( \frac{r_{12}^2}{m_1 m_2} + \frac{r_{13}^2}{m_1 m_3} + \frac{r_{23}^2}{m_2 m_3} - \frac{r_{11} r_{22} - r_{12} r_{21} - r_{13} r_{33} + r_{23} r_{32}}{m_1 m_2 m_3} \right) + \frac{r_{12} r_{23} r_{31} + 2 r_{12} r_{13} r_{23} - r_{11} r_{22} r_{33} - r_{22} r_{32} r_{13} - r_{33} r_{23} r_{12}}{m_1 m_2 m_3} = 0
\]

(7)

When equality (6) is fulfilled, equation (7) has the only real solution of the multiplicity

\[
\frac{df(\lambda)}{d\lambda} = -3\lambda^2 + 2\lambda \left( \frac{r_{11} + r_{22} + r_{33}}{m_1} \right) + \left( \frac{r_{12}^2}{m_1 m_2} + \frac{r_{13}^2}{m_1 m_3} + \frac{r_{23}^2}{m_2 m_3} - \frac{r_{11} r_{22} - r_{12} r_{21} - r_{13} r_{33} + r_{23} r_{32}}{m_1 m_2 m_3} \right) + 6 \left( \frac{r_{12}^2}{m_1 m_2 m_3} + \frac{r_{13}^2}{m_1 m_2 m_3} + \frac{r_{23}^2}{m_1 m_2 m_3} \right)
\]

(8)

Consider the determinant \( \Delta_f \) of the equation \( f(\lambda) = 0 \), we obtain:

\[
\Delta_f = 2 \left[ \left( \frac{r_{11} - r_{22}}{m_1 m_2} \right)^2 + \left( \frac{r_{11} - r_{33}}{m_1 m_3} \right)^2 + \left( \frac{r_{22} - r_{33}}{m_2 m_3} \right)^2 + 6 \left( \frac{r_{12}^2}{m_1 m_2 m_3} + \frac{r_{13}^2}{m_1 m_2 m_3} + \frac{r_{23}^2}{m_1 m_2 m_3} \right) \right]
\]

(9)

The multiplicity of solutions of the equation \(|D| = 0 (f(\lambda) = 0) \) occurs when \( \Delta_f = 0 \).

By virtue of non-negativity, \( \Delta_f \) is true:

\[
\left\{ \begin{array}{l}
r_{11} = r_{22} = r_{33} \\
m_1 = m_2 = m_3 \\
r_{12} = r_{13} = r_{23} = 0
\end{array} \right.
\]

(9)

Equalities (9) show that the condition for the multiplicity of natural frequencies for third-order systems leads to the need to form uncoupled systems. A dynamic system that has elastic constraints in the form of \( r_{ij} \neq 0 \) with \( i \neq j \) multiple frequencies cannot have.
For a system with dimension $n \geq 4$, the use of the previous approach is impossible. Since the matrix $D$ is asymmetric for different values of the inertia parameters, the multiplicity conditions are determined by solving the asymmetric eigenvalue problem [13].

From the condition for solving the problem of eigenvalues, we have

$$ F^{-1} \cdot D \cdot F = \Lambda \tag{10} $$

where $\text{diag} (\lambda_i) = \Lambda, i = 1..n$.

If the matrix $D$ has all multiple eigenvalues $\lambda_k$, then $\Lambda$ can be described as $\text{diag} (\lambda_k) = \Lambda = \lambda_k I$, where $I$ is the identity matrix. Then equality (11) can be written as

$$ F^{-1} \cdot D \cdot F = \lambda_k I \tag{11} $$

from here

$$ F^{-1} \cdot \frac{1}{\lambda_k} D \cdot F = I . \tag{12} $$

From the condition $F^{-1} F = I$, we have

$$ \frac{1}{\lambda_k} D \cdot F = F . $$

In expanded form, this equality can be transformed in the form:

$$ \begin{bmatrix}
    r_{11} & r_{12} & \ldots & r_{1n} \\
    \lambda_k \cdot m_1 & \lambda_k \cdot m_1 & \ldots & \lambda_k \cdot m_1 \\
    r_{21} & r_{22} & \ldots & r_{2n} \\
    \lambda_k \cdot m_2 & \lambda_k \cdot m_2 & \ldots & \lambda_k \cdot m_2 \\
    \ldots & \ldots & \ldots & \ldots \\
    r_{n1} & r_{n2} & \ldots & r_{nn} \\
    \lambda_k \cdot m_n & \lambda_k \cdot m_n & \ldots & \lambda_k \cdot m_n \\
\end{bmatrix}
\begin{bmatrix}
    1 \\
    0 \\
    1 \\
    0 \\
    \ldots \\
    0 \\
\end{bmatrix}
= 
\begin{bmatrix}
    0 \\
    1 \\
    0 \\
    0 \\
    \ldots \\
    1 \\
\end{bmatrix} .
$$

Thus, it is true:

$$ \begin{cases}
    r_{ij} = \lambda_k \text{ at } i = j \, , \, i = 1..n \\
    \frac{r_{ij}}{m_i} = 0 \text{ at } i \neq j
\end{cases} \tag{13} $$

4. **Conclusion**

1. The condition of the frequency multiplicity of natural vibrations for systems with dimension $n$ leads to the need to form $n$ uncoupled first-order systems.
2. A dynamic system having elastic constraints in the form of $r_{ij} \neq 0$ with $i \neq j \, n$ cannot have multiple frequencies.
3. Unlike forced stationary vibrations, the dynamic quenching effect [20] (zero amplitude) cannot exist in systems with non-zero internal connections.

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