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Synergy of adaptive thresholds and multiple transmitters in free-space optical communication

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Abstract: Laser propagation through extended turbulence causes severe beam spread and scintillation. Airborne laser communication systems require special considerations in size, complexity, power, and weight. Rather than using bulky, costly, adaptive optics systems, we reduce the variability of the received signal by integrating a two-transmitter system with an adaptive threshold receiver to average out the deleterious effects of turbulence. In contrast to adaptive optics approaches, systems employing multiple transmitters and adaptive thresholds exhibit performance improvements that are unaffected by turbulence strength. Simulations of this system with on-off-keying (OOK) showed that reducing the scintillation variations with multiple transmitters improves the performance of low-frequency adaptive threshold estimators by 1-3 dB. The combination of multiple transmitters and adaptive thresholding provided at least a 10 dB gain over implementing only transmitter pointing and receiver tilt correction for all three high-Rytov number scenarios. The scenario with a spherical-wave Rytov number $\mathcal{R} = 0.20$ enjoyed a 13 dB reduction in the required SNR for BER’s between $10^{-5}$ to $10^{-3}$, consistent with the code gain metric. All five scenarios between 0.06 and 0.20 Rytov number improved to within 3 dB of the SNR of the lowest Rytov number scenario.

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1. Introduction

Laser communications offer tremendous advantages over radio frequency (RF) in bandwidth and security due to the ultra-high frequencies and point-to-point nature of laser propagation. In addition, optical transmitters and receivers are much smaller and lighter than their RF counterparts and operate at much lower power levels. Current airborne sensors are collecting data at an ever-increasing rate. With the advent of hyperspectral imaging systems, this trend will continue as two-dimensional data are replaced by three-dimensional data cubes at finer resolutions. RF communication systems cannot keep up with this trend. As a promising alternative, free-space optical communication (FSOC) systems can keep up as they are capable of transmitting at multi-gigabit per second rates [1].

The unfortunate reality is that FSOC is severely affected by clouds, dust, and atmospheric turbulence, causing deep, long fades at the receiver. The advantages are clear, but ultimately a hybrid approach which includes RF communication is necessary, since clouds, fog, or dust occasionally obstructs the path for laser communication. Even when the channel is clear, the same atmospheric turbulence effects that limit the resolution of imaging systems and make the stars twinkle can significantly reduce the received power. This atmospheric turbulence in the propagation path causes the laser beam to wander, spread, and break up; leading to such long, deep fades. These effects can cause the received signal power to drop below the receiver’s threshold for milliseconds at a time. For a 10 Gbit/s binary FSOC system, a millisecond fade produces about five million bit errors. Since the turbulence of an air-to-air link extends along the entire path and causes long, deep fades, simply increasing the transmitter power would not be effective. In addition, airborne FSOC systems require special considerations in size, complexity, power, and weight.
This research shows how multiple transmitters (Tx’s) reduce received signal variability and the length of fades caused by long propagation paths through extended turbulence in FSOC systems. Many researchers have studied how adaptive optics (AO) systems can improve FSOC system performance, especially in ground-to-space and space-to-ground communications [2–4]. These conventional AO systems correct for the phase only and cannot correct for strong scintillation, but here multiple transmitters “average out” strong scintillation effects by incoherently summing multiple beams at the receiver. Others have researched multiple-input-multiple output (MIMO) and multiple-transmitter FSOC systems [5–7], but have not analyzed the temporal considerations, adaptive thresholding, and fade statistics for the airborne regime. In this research, two mutually incoherent laser beams angularly separated by the irradiance independence (anisoplanatic) angle average out the scintillation effects and reduce the fade length, depth, and rate. In previous research, we determined this optimal separation between two transmitters for the air-to-air regime [8]. Here, wave-optics simulations show that a combination of transmitter diversity and adaptive thresholding significantly reduces the bit error rate (BER) even further. We also show the synergistic effect of multiple transmitters for low-bandwidth adaptive threshold systems when we compare the performance to high-bandwidth and ideal adaptive threshold systems.

2. Turbulence conditions
This research simulates a 100 km air-to-air laser propagation path using a wave-optics simulation to show the advantages of using multiple transmitters in FSOC. First, knowledge of the spatial statistics of the turbulence effects is required to determine the simulation parameters and the separation distance for the transmitters. By adjusting the turbulence profile (i.e. the location and strength of turbulence phase screens with time) we were able to adjust the spherical-wave coherence diameter \( r_0 \), spherical-wave Rytov number \( \mathcal{R} \), and Greenwood frequency \( f_G \) to create the proper phase and amplitude correlation properties. The Rytov number \( \mathcal{R} \) is equal to the spherical-wave log-amplitude variance \( \sigma^2 \chi \) for weak turbulence and is a common measure of turbulence strength [9]. The parameters were chosen to emulate this air-to-air horizontal scenario with aircraft velocities between 56 and 280 m/s and altitudes between 4 and 15 km. These parameter ranges were chosen based on our previous research that showed a relatively small separation of about 31 cm is required to average atmospheric scintillation effects for a 100-km air-to-air communication link [8,10]. The experimental design fully investigates the different scenarios and conditions of an air-to-air scenario. Since \( r_0, \mathcal{R}, \) and \( f_G \) adequately describe the spatial and temporal turbulence effects, the simulated conditions consist of a one-half fractional factorial design of these three factors. Designing the test in this way enables the determination of the primary driving factors for fades and bit errors. Table 1 summarizes the atmospheric parameters for the simulations. There are five different scenarios with different altitudes, air velocities, sampling times, spherical-wave Rytov numbers \( \mathcal{R} \), Greenwood frequencies \( f_G \), and spherical-wave coherence diameters \( r_0 \). Scenarios 3 and 4 are non-physical scenarios and therefore do not have an altitude associated with them. The other three scenarios model horizontal propagation. In this table, the “H” refers to the relatively higher numerical value and the “L” refers to the relatively lower numerical value for each parameter (\( \mathcal{R}, f_G, \) and \( r_0 \)). The optical wavelength was \( \lambda = 1.55 \mu m \). The spatial statistics of the turbulence effects also determine how far apart the transmitters must be to get good averaging. Good averaging occurs when the turbulence effects of two or more paths are relatively uncorrelated [8]. The farther the two transmitters are separated, the less correlated the effects become (i.e. anisoplanatic). This is important when multiple transmitters are used to average out the turbulence effects. Next, the anisoplanatic phase and amplitude effects are considered.
Table 1. Atmospheric parameters for the scenarios used in the simulations.

| Scenario | Alt. (km) | Air Speed (m/s) | ∆t (µs) | Rytov Number $\mathcal{R}$ | $f_c$ (Hz) | $r_0$ (cm) |
|----------|-----------|-----------------|---------|-----------------------------|------------|------------|
| 1        | 15        | 113             | 133     | 0.060 (L)                  | 117 (L)    | 74 (H)     |
| 2        | 4         | 56              | 137     | 0.18 (H)                   | 113 (L)    | 38 (L)     |
| 3        | N/A       | 280             | 30      | 0.085 (L)                  | 518 (H)    | 38 (L)     |
| 4        | N/A       | 280             | 30      | 0.20 (H)                   | 518 (H)    | 55 (H)     |
| 5        | 4         | 225             | 30      | 0.18 (H)                   | 518 (H)    | 38 (L)     |

2.1. Anisoplanatic effects

For a multiple-Tx system, the phase and amplitude fluctuations of each path decorrelate as the beam separation increases in distance or angle. Starting at small separations, high-order and low-order phase effects are highly correlated for Tx separations less than or equal to the isoplanatic angle $\theta_0$. Most of the literature defines this as the angle at which the phase perturbation structure function is less than or equal to unity [8,11,12]. Applying the maximum value of the phase perturbation structure function determines the phase independence angle. At this separation, the high-order and low-order phase effects of multiple beams are relatively uncorrelated so that they spread and wander independently. This angle was defined in our previous work and shown here again as [8,10]

$$\theta_{\text{ind}} = 2\sigma_{\psi,\text{pl}}^2 \theta_0,$$

(1)

where $\sigma_{\psi,\text{pl}}^2$ is the phase variance for a plane-wave source and a point receiver.

As for the amplitude effects, this independence or uncorrelated angle occurs at much smaller separations. The correlation width $\rho_{cw}$ is often used to determine how large receivers need to be to provide some degree of aperture averaging of the scintillation effects. The correlation width is defined as the 1/e² point of the normalized irradiance covariance function [12]. Since $\rho_{cw}$ for weak turbulence varies between 1 to 3 Fresnel zones $(Lk)^{1/2}$ depending on beam size [12], in this work we refer to the constant $\rho_c = (Lk)^{1/2}$. In recent work, the principle of reciprocity was used to illustrate that transmitter separations of $\rho_{cw}$ could provide adequate scintillation averaging in the receiver [8,10]. Due to angle-of-arrival considerations, the increase in off-axis irradiance variance, and negatively correlated amplitude effects near $\rho_{cw}$, very wide separations are not necessarily the optimal configuration [7,8,10,12,13]. Previously, we determined an angular separation of $2\theta_{\text{c}} = 2(Lk)^{1/2}$, or a separation of 2$\rho_c$ at the transmitters corresponding to about 30 cm for the 100 km air-to-air path was adequate [8].
3. Temporal considerations

Thus far, only spatial statistics have been used to describe the effects of atmospheric turbulence. In this section, the temporal statistics are considered to determine BER improvement afforded by multiple transmitters and adaptive thresholding. Taylor’s frozen flow hypothesis states that the turbulence structure is essentially frozen as it moves across the propagation path for short time intervals [14]. This idea was used to scroll the random phase screens across the propagation grid at different points along the path to generate a time series of the turbulence in the simulations performed in Section 4.

3.1. Frequency of the turbulence effects

The first thing to consider when building a temporal simulation is the sampling frequency. One approach is to determine the time duration $t_{irr}$ where the turbulence evolves so that the scintillation effects are only slightly different than the previous time slice (time difference for isoplanatic scintillation). These simulations use a conservative estimate of $t_{irr}$ to ensure they include all potential signal variations. For this case, the log-amplitude structure function value for the time separation used was only 2.2% of the structure function maximum. Using $f_G$ as a reference and varying the temporal sampling frequency of the simulations enables the determination of an adequate sampling rate. The power spectral density (PSD) of signal power was estimated for successively finer temporal resolutions until two sequential estimates were relatively similar from 0 Hz to the frequency at which the PSD is 20 dB below its maximum value. This determination is shown graphically in Fig. 1. These PSD estimates were consistent for different random realizations. The resulting sampling frequency is $f_s = 64f_G$.

3.2. Threshold determination

For a binary-symbol system like the one used here, once the signal is received a decision must be made whether a ‘1’ or a ‘0’ was sent based on a threshold. The transmitter modulates the intensity with on-off keying (OOK), where the laser turns on to transmit a ‘1’ and turns off to transmit a ‘0’. The transmission of a ‘1’ or ‘0’ is denoted by the event $H_1$ or $H_0$, respectively.
The likelihood ratio test (LRT) determines the optimal decision threshold based upon the probability density function (PDF) of the measured current level \( i_m \) of the transmission of a ‘1’ \( p(i_m|H_1) \) and transmission of a ‘0’ \( p(i_m|H_0) \). Using the LRT and the assumption that \( P(H_0) = P(H_1) \) (equally likely signaling) leads to the following two relations [15]:

\[
p(i_m|H_1) > p(i_m|H_0) \quad (2)
\]

the algorithm picks \( H_1 \) and if

\[
p(i_m|H_1) \leq p(i_m|H_0) \quad (3)
\]

the algorithm picks \( H_0 \). The optimum detection criteria can best be described graphically as the intersection of the PDF of the measurement of the transmission of a ‘1’ and the PDF of the measurement of a ‘0’ transmission. The turbulence conditions vary significantly over time, and thus the receiver performance could benefit from a threshold that varies with the optical signal level [16,17].

3.2.1. Fixed Threshold

The optimal fixed threshold calculation takes into account the PDF of the signal \( p(s) \) due to variations caused by channel conditions. In this case, the channel conditions are dictated by the atmospheric turbulence. As mentioned in the previous section, the optimal threshold depends upon the PDF’s of the measurement of ‘1’ and a ‘0’. The measurement noise of a ‘1’ can be broken into the sum of the thermal, shot, and amplifier noise, defined by

\[
\sigma^2_t = \sigma^2_{elec} + \sigma^2_{shot} + \sigma^2_{ASE},
\]

where \( \sigma^2_{elec} \) is the electronic thermal noise, \( \sigma^2_{shot} \) is the shot noise due to the random arrival of photons, and \( \sigma^2_{ASE} \) is the amplified spontaneous emission (ASE) noise associated with an Erbium-doped fiber amplifier (EDFA) [8,18]. Since the shot noise and ASE noise are signal-dependent, they depend upon the signal PDF \( p(s) \). To simplify the notation, we let

\[
p_1(i_m) = p(i_m|H_1) \quad \text{and} \quad p_0(i_m) = p(i_m|H_0).
\]

Then the LRT for this scenario is [17,19]

\[
P(H_0)p_0(i_T) = P(H_1)\int_0^{\infty} p_1(i_m|s)p(s)ds
\]

\[
\frac{1}{\sigma_{elec}} \exp \left( \frac{-i_T^2}{2\sigma^2_{elec}} \right) = \int_{-\infty}^{\infty} p(s) \exp \left\{ -\frac{[i_T - i_m(s)]^2}{2\sigma^2_t(s)} \right\} ds.
\]

The threshold current \( i_T \) (in µA) can be solved for numerically whether the PDF of the turbulence-induced power fluctuations \( p(s) \) is analytic, measured, or calculated from the histogram of the simulated received power before the measurement noise is applied. Since we want to compare the adaptive threshold approaches to the best possible fixed threshold performance, we used the PDF estimate \( p(s) \) of the simulated received power. The noise associated with measuring a ‘0’ is primarily due to thermal noise (a.k.a. Johnson noise). The probability of an error \( P_e \) is the probability of a missed detection \( P_{md} \) plus the probability of a false alarm \( P_{fa} \) so that

\[
P_e = P(H_1)P_{md} + P(H_0)P_{fa} = \frac{P_{md}}{2} + \frac{P_{fa}}{2},
\]

where

\[
P_{md} = \frac{1}{2} \int_{0}^{\infty} \erfc \left( \frac{i_m(s) - i_T}{\sqrt{2}\sigma_t(s)} \right) p(s)ds,
\]

\[
P_{fa} = \frac{1}{2} \int_{0}^{\infty} \erfc \left( \frac{i_T - i_m(s)}{\sqrt{2}\sigma_t(s)} \right) p(s)ds.
\]
\[ P_{fa} = \frac{1}{2} \text{erfc} \left( \frac{i_r}{\sqrt{2}\sigma_{elec}} \right). \]  

In Eqs. (7) and (8), \( \text{erfc}(\cdot) \) is the complementary error function.

### 3.2.2. Adaptive optimal threshold

For temporally varying turbulence, an *ideal* optimal adaptive threshold results in the lowest probability of error for each instant in time [15,16]. Since the threshold is determined for each current level, the PDF of the received signal level \( p(s) \) is not required for this calculation. Only the estimates of the means (\( \mu_i \) and \( \mu_0 \)) and the standard deviations (\( \sigma_i \) and \( \sigma_0 \)) of the two conditions are required to set the threshold. Solving for the optimal adaptive threshold current assuming Gaussian distributions for \( p_1(i_m) \) and \( p_0(i_m) \) yields [17,20]

\[
i_r = \frac{\mu_0 \sigma_i^2 - \mu_i \sigma_0^2}{\sigma_i^2 - \sigma_0^2} + \frac{\sigma_i \sigma_0}{\sigma_i^2 - \sigma_0^2} \sqrt{(\mu_i - \mu_0)^2 + 2(\sigma_i^2 - \sigma_0^2) \ln \left( \frac{\sigma_i}{\sigma_0} \right)}.
\]  

(9)

The work here assumes \( \mu_0 = 0 \) (i.e. zero dark current) and \( \sigma_0 = \sigma_{elec} \), since \( \sigma_{shot} = \sigma_{ASE} = 0 \) when a ‘0’ is sent. This *ideal* adaptive threshold system calculates the optimal adaptive threshold for each time slice with the corresponding raw received signal level \( s \) in the simulation and implements that threshold to determine whether it is a ‘1’ or ‘0’. For the adaptive threshold case, the probability of a missed detection and the probability of false alarm now have a threshold that varies with the signal level along with all of the other signal-dependent terms. Accordingly, \( P_{md} \) becomes

\[
P_{md} = \frac{1}{2} \int_0^\infty \text{erfc} \left( \frac{i_r(s) - i_r(s)}{\sqrt{2}\sigma_i(s)} \right) p(s)ds,
\]  

where the threshold now becomes a function of the received power \( s \). The \( P_{fa} \) also becomes a function of \( s \) given by

\[
P_{fa} = \frac{1}{2} \int_0^\infty \text{erfc} \left( \frac{i_r(s)}{\sqrt{2}\sigma_i(s)} \right) p(s)ds.
\]  

(11)

A *realistic* system requires an estimator to determine what threshold \( i_r \) is used for the next particular time slice. The performance of this estimator is driven by the measurement noise and the estimator’s sampling frequency. Since the mean and variance of the transmission of a ‘0’ are relatively constant (for a fixed temperature), Eq. (10) becomes a function of the mean and variance of the signal level of a ‘1’. Because the variance \( \sigma_i^2 \) is signal-dependent and the signal variation is slow compared to the data rate, the variation in the adaptive threshold is only a function of the signal level for the transmission of a ‘1’. In addition, since the transmission of a ‘1’ or ‘0’ is equally likely, the mean signal level for the transmission of a ‘1’ can be determined by multiplying the mean received signal value \( \mu_{rcvd} \) by two and subtracting the mean signal level of the transmission of a ‘0’ (i.e. \( \mu = 2\mu_{rcvd} - \mu_0 \)). This approach is valid if averaged over a short period of time with respect to the turbulence. Therefore, the estimated optimal adaptive threshold can be deduced in a simulation by using the estimate of the current signal level \( i_r \) and the estimated measurement noises \( \sigma_{elec} \) and \( \sigma_{i,m} \).

In this work, the received power is split into two branches with 99% of the power used in the digital Rx and 1% used in the estimator. The estimator measures the current in the previous time slice \( i_{Em} \) and the differential signal in the previous two measurements \( i_{Em} \) to determine the estimated signal level. To further refine the estimate, the differential of the measured signal \( \Delta_+ = (i_{m+}) - (i_{m-}) \) determines the trend in the previous two estimator
measurements $i_{Em}$ and $i_{Em-}$. Figure 2 depicts this process. In these simulations the temperature and bandwidth are constant, so $\sigma^2_{elec}$ remains constant. The estimated current $\hat{i}$ is determined by

$$\hat{i} = 99[i_{Em} + n(i_{Em-}) + \Delta_m]$$

where $i_{Em}$ is the raw actual estimator value and the noise $n(i_{Em-})$ in the measurement $i_{Em-}$ is a zero-mean Gaussian random variable with a variance equal to

$$\sigma^2 = \sigma^2_{elec} + \sigma^2_{sot} + \sigma^2_{ASE}.$$  

The estimator bandwidth and the signal level drive these noise sources in Eq. (13), but since the estimator bandwidth need only be in the kHz range to keep up with the turbulence, the noise power is relatively low. Reducing bandwidth of the estimator further increases the latency of the estimator and degrades the performance of the estimator. If $\mu_1$ in Eq. (9) is set to equal the estimated signal $\hat{i}$ and $\mu_0 \approx 0$, the equation becomes

$$\hat{i} = \frac{\hat{i} \sigma^2_{elec}}{\sigma^2_{elec} - \sigma^2_i} + \frac{\sigma^2_{elec} \hat{s}}{\sigma^2_{elec} - \sigma^2_i} \sqrt{\frac{\hat{s}^2}{\sigma^2_{elec}} + 2(\hat{s} - \sigma^2_{elec}) \ln \left(\frac{\hat{s}}{\sigma_{elec}}\right)}, \quad (14)$$

where $\hat{s}$ is the estimate of $\sigma$ using $\hat{i}$.

Fig. 2. Adaptive threshold estimator.

### 4. Results

#### 4.1. Simulation set-up

The turbulence effects explored subsequently in simulated scenarios were generated using ten Fourier-series-based random phase screens with the correct statistics placed along the path [21,22]. The layered analytic spherical-wave coherence diameter $r_{0ph}$, spherical-wave Rytov number $R$, and isoplanatic angle $\theta_0$ matched within 1% of the full path continuous atmospheric turbulence parameters.

Computer simulations of airborne single-Tx and double-Tx FSOC systems were performed for the scenarios described in Table 1. The separation distance for all five scenarios for the double-Tx system was $2\rho_c = 31$ cm. The simulations propagate either one or two collimated Gaussian beams depending upon the Tx configuration with a 1/e field radius of $W_0 = 2.5$ cm using a split-step Fresnel propagation to a 20 cm diameter receiver aperture.
Great care was taken to adequately sample the Fresnel propagation between the screens to avoid aliasing in the beam as well as the quadratic phase term [23]. At the Rx, the light is coupled into a single-mode fiber to be amplified by an EDFA with a spontaneous emission factor of $n_{sp} = 4$ and a gain of 30, factoring into the ASE noise $\sigma_{ASE}^2$ [18]. The simulations modeled the fiber coupling by projecting the field onto the guided mode of a single-mode 4 $\mu$m-radius fiber. For this single mode fiber, the efficiency of the coupling was modeled by the LP01 mode field using the Bessel functions of the first and second kind. The mode field diameter of the fiber was 10.5 $\mu$m with an index of refraction of $n_1 = 1.45$ and a V number of 2.405. See Refs [24], and [25] for a description of the calculation of the coupling of the fundamental guided mode.

A transmitter pointing system was used to center the centroid of the irradiance on the receiving aperture, and a tracking system removed the tilt at the receiver. The tilt corrector simulates the tracker by centering the centroid of the irradiance at the pupil plane. In order to emulate the Tx and Rx trackers, temporal errors are incorporated in the tilt correctors by adjusting the loop gain of the feedback controller. In previous work, we compared the performance of different trackers for multiple transmitter systems for fixed and ideal adaptive threshold systems [24,27]. Here, the simulated residual beam wander and beam angle-of-arrival (AOA) jitter are one quarter of the open-loop beam wander for the Tx and AOA jitter for the Rx, respectively.

Equation (4) describes the noise in the measurement of a ‘1’ which factors into the probability of accurately detecting a ‘1’. The primary factors that drive the shot noise and thermal noise are the signal bandwidth, the temperature, and the input resistance. The bandwidth used for the calculation of the fade statistics was 10 GHz and for the BER rate calculations it was 1 GHz. All other parameters were fixed including a temperature of 300 Kelvin, input resistance of 1000 Ohms, and the wavelength of 1.55 $\mu$m. The SNR for all calculations was varied by adjusting the transmission loss for the link and keeping all other factors constant.

Section 4.2 describes the resulting temporal fade statistics of the detected signal. Section 4.3 compares the PDF’s of the received signal for single and multiple transmitter systems. Finally, Section 4.4 plots the BER for all five scenarios and for different techniques used to improve their performance.

4.2. Bit error rate (BER) fade statistics

First, a fade definition is required to determine the fade statistics. In previous papers, we defined a fade as when the signal due to the turbulence in the channel caused it to cross the optimal fixed threshold [8,24,26,27]. Here, we have devised a definition that is more practical for our purposes rather than selecting an arbitrary received power threshold. Each signal level for each time slice has a particular BER associated with it given the measurement noise. Figure 3 illustrates the definition used in this work by showing the calculated probability of a bit error versus time and the fade threshold. In this paper, we define that a fade occurs when the instantaneous BER estimate crosses above a BER of $10^{-3}$. Since we are comparing the performance of an adaptive threshold to a fixed threshold, we chose to use this definition to accurately illustrate the advantages of our hybrid technique.
Scenarios 1 and 3 did not experience any fades for the double-Tx cases according to the definition given above so the fades are not plotted. However, these scenarios did experience bit errors, and their performance are shown in Section 4.4. For the other three scenarios, Figs. 4–6 show that the double-Tx cases, denoted with dashed line, have shorter and less frequent fades than the single-Tx cases denoted by solid line for all cases. This is due to the fact that the fade depths are reduced when the multiple Tx’s “smooth out” the variation in the received power. The only difference between Figs. 4 and 6 for these two cases is the Greenwood frequency of the turbulence. For the low-frequency case in Fig. 4, the fade length is about 4.4 times as long, but there are fewer fades than in the high-\(f_G\) case in Fig. 6. All three adaptive threshold systems performed much better than the fixed threshold system. The higher-fidelity estimator with \(f_s = 64 \times f_G\) performed almost as well as the ideal adaptive threshold system. The low-frequency adaptive threshold estimator performed worse than the high-frequency estimator in almost all cases. The two estimators did perform comparably for an SNR above 20 dB for the two transmitter scenario 4 (HHH) case. Since the scintillation indicated by \(\rho = 0.1979\) for this scenario (HHH) was higher than the other four scenarios, the improvement in the low-frequency case was most likely due to the reduction in scintillations when using two transmitters. The peaks in each of the fade rates for Figs. 4–6 can be graphically explained by varying the threshold for a particular signal like the one shown in Fig. 3. As the threshold successively drops, more and more fades are encountered until the fades get so long that they merge to make longer fades, thereby reducing the number of threshold crossings and number of fades. In each case, as the SNR decreases, the mean fade length increases.
Fig. 4. BER fade statistics for Scenario 2 (HLL). (a) & (c) The mean fade length for a fade above an error rate of $10^{-3}$. (b) & (d) The number of fades per second above an error rate of $10^{-3}$. Dashed line is double-Tx case and solid line is single-Tx case.

Fig. 5. BER fade statistics for Scenario 4 (HHH). (a) & (c) The mean fade length for a fade above an error rate of $10^{-3}$. (b) & (d) The number of fades per second above an error rate of $10^{-3}$. Dashed line is double-Tx case and solid line is single-Tx case.
4.3. Probability density function (PDF) estimates of the received signal

First, the normalized histograms of the raw received signal $p(i_s)$ with variations caused by atmospheric turbulence are used to estimate the PDF of the received signal. These PDF’s were compared with their double-transmitter cases for all five scenarios listed in Table 1. If the PDF estimate is heavily weighted to the left, the chances of a missed detection are greater, as it might not reach above the threshold. Figure 7 shows that the PDF’s of the received signal for all of the scenarios shifted to the right when two transmitters were used. Even for the low-$\mathcal{R}$ cases, the PDF’s markedly shifted to the right thereby improving performance. This shift to the right reduces the probability of error specifically by reducing the probability of a missed detection. It also shifts the optimal threshold to the right, thereby reducing the probability of a false alarm. These performance improvements are quantified with the BER calculations in the next subsection.
Fig. 7. This plot uses the normalized histograms of the raw received power to estimate the PDFs of the received signals due to turbulence $p(i_s)$ for each of the scenarios for the single-Tx (solid line) and double-Tx (dashed line) cases. Subplots (a)-(d) were calculated for scenarios 1, 2 & 3, and 4, respectively.

4.4. Bit error rate (BER)

Next, the BER's of ideal and realistic adaptive thresholds are compared to the optimal fixed case. Recall this optimal fixed case takes into account the PDF of the received signal $p(i_s)$ over the ensemble of the runs calculated [see Eq. (5)]. There were 10 independent realizations with 1000 time slices for each realization. The time increments were determined by $\tau_s = 1/(64f_G)$ for each of the scenarios. Therefore, each independent realization covered a time frame of over 15 Greenwood time constants, resulting in well over 150 relatively independent realizations per scenario.

It is clear from Fig. 8 that the BER significantly decreases when two transmitters are used (3-10 dB depending upon the scenario). The ideal adaptive threshold systems improved performance by up to 5 dB for the high-Rytov cases in plots (c) and (d). This is substantial since this was compared to the optimal fixed threshold case for the particular scenario which used the actual PDF of the received signal to determine the optimal threshold. This a priori knowledge of the turbulence resulted in an optimistic BER for the fixed threshold. In most cases, the fixed threshold is not chosen in such an accurate manner. The double-Tx systems outperformed all other techniques even though improvements due to the adaptive threshold technique were up to 5 dB. As expected, the system with an ideal adaptive threshold and two transmitters performed the best.

The realistic estimators simulated in this study did, in fact, improve the performance in all cases. The performances of three different adaptive threshold systems are compared in Fig. 8;
an ideal adaptive threshold, an adaptive threshold with an estimator operating at $f_s = 64 f_G$, and another system with an estimator operating at $f_s = 16 f_G$. For a single transmitter, the performance for the $f_s = 16 f_G$ estimator was the poorest for the highest-Rytov case in scenario 4 (HHH). For this and all other cases, this lower-sampling-rate estimator performance greatly improved when two transmitters were implemented. The single-transmitter cases have more variability in the received irradiance and require a higher fidelity estimator to keep up with the turbulence. This trickle-down effect indicates multiple transmitters can enable the use of cheaper, lower-sampling-rate estimators.

Finally, the turbulence effects for each of the scenarios are compared using the BER performance to determine causality. The only difference between scenarios 2 (HLL) and 5 (HHL) is the speed of the turbulence and therefore, as expected, their BER's are identical. The three scenarios with high Rytov numbers (scenarios 2, 4, and 5) have the worst performance, but the improvement provided by multiple transmitters is much greater for these scenarios. The combination of multiple transmitters and adaptive thresholding provided at least a 10 dB gain over implementing none of them for all three high-Rytov number scenarios. Scenario 5 with a Rytov number $\mathcal{R} \approx 0.20$ enjoyed a 13 dB overall improvement. Due to the improvements afforded by these multiple techniques, the high-Rytov cases of $\mathcal{R} \approx 0.20$ were on par with Rytov numbers of $\mathcal{R} \approx 0.06$ without these techniques. All five scenarios tested $0.06 < \mathcal{R} < 0.20$ were within 3 dB of each other when all of the improvement techniques were implemented.
5. Conclusion

In all scenarios tested, the coherence diameter $r_0$ was greater than the diameter of the receiver $D$, therefore changes in the Rytov number had a much larger effect than $r_0$ changes. If $D/r_0 > 1$, the phase effects due to the turbulence would likely have had a larger effect on the BER.

Adaptive thresholding systems provide significant improvement over optimal fixed thresholds for both single-Tx and double-Tx systems, providing an additional 3-4 dB over both systems. As long as the estimator kept up with the turbulence, the realistic estimators performed well. As the scintillation effects were stronger for the single-Tx high-Rytov scenario (HHH), the lower-bandwidth estimator performance lagged behind the high-bandwidth estimator and the ideal adaptive threshold system. The performance degradation in the lower-bandwidth estimator cases was mitigated by using two transmitters to reduce the scintillation. Lower scintillation causes less variability in the signal, allowing the lower-bandwidth estimator to keep up with the turbulence.

The improvement due to implementing two transmitters can be scaled to a small degree for multiple transmitters. A limit to this improvement does exist, since separating beams much greater than $2\rho_c$ results in diminishing improvement [8,10]. A trade-study should be performed to determine the optimal number of transmitters. The impact of non-uniform turbulence profile (such as slant path) would also be an interesting extension on this research. If further error reduction is required, an interleaver/FEC receiver could be implemented. An interleaver spreads out the errors randomly in time so the FEC code can be used more effectively. The averaging effect of two transmitters not only reduced the BER, it also reduced the length of a fade. Shorter fade lengths require shorter interleavers, reducing data latency and making shorter interleavers more effective.

The next logical step in this research is to perform hardware-in-the-loop tests to simulate the air-to-air regime. Future research into aero-optic effects dealing with turbulence around particular air-frames and gimbals could further refine the estimates on future system performance. Fixed ground FSOC experiments could provide an adequate feasibility experiment prior to flight tests. Finally, flight tests involving multiple transmitter systems would validate this as a viable approach.

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