THE RELEVANCE OF POSITIVITY IN SPIN PHYSICS

Jacques Soffer
Physics Department, Temple University, Philadelphia, PA 19122-6082, USA

Xavier Artru
Université de Lyon, IPNL and CNRS/IN2P3, 69622 Villeurbanne, France

Mokhtar Elchikh
Université des Sciences et de Technologie d’Oran, El Menauoer, Oran, Algeria

Jean-Marc Richard
LPSC, Université Joseph Fourier, CNRS/IN2P3, INPG, Grenoble, France

Oleg Teryaev
Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Moscou region, Russia

Abstract

Positivity reduces substantially the allowed domain for spin observables. We briefly recall some methods used to determine these domains and give some typical examples for exclusive and inclusive spin-dependent reactions.

Key words: constraints, positivity domains, spin observables

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1 INTRODUCTION

Spin observables for any particle reaction, contain some unique information which allow a deeper understanding of the nature of the underlying dynamics and this is very useful to check the validity of theoretical assumptions. We emphasize the relevance of positivity in spin physics, which puts non-trivial model independent constraints on spin observables. If one, two or several observables are measured, the constraints can help to decide which new observable will provide the best improvement of knowledge. Different methods can be used to establish these constraints and they have been presented together with many interesting cases in a recent review article [1].

If $X, Y, Z$ are spin observables with the standard normalization $-1 \leq X \leq +1$, the domain for the pair of observables $(X,Y)$ is often smaller than the square $[-1, +1]^2$ and the domain for the triple of observables $(X,Y,Z)$ is often smaller than the cube $[-1, +1]^3$. Explicit inequalities are obtained, relating two or three spin observables, for instance $X^2 + Y^2 \leq 1$, corresponding to a disk, or $X^2 + Y^2 + Z^2 \leq 1$, corresponding to a sphere, but one is also led to triangles, tetrahedrons, etc...(see below).

2 EXCLUSIVE REACTIONS

i) $\pi N \rightarrow \pi N$ scattering

A scattering process involving spinning particles and described by $n$ complex amplitudes is fully determined by $(2n - 1)$ real functions, up to an over-all phase. Since there are $n^2$ possible measurements for this reaction, we must have $(n - 1)^2$ independent quadratic relations between the $n^2$ observables. First consider the simplest case of an exclusive two-body reaction with a spin 0 and a spin 1/2 particle, namely $0 + 1/2 \rightarrow 0 + 1/2$. As an example $\pi N \rightarrow \pi N$ is described in terms of two amplitudes, the non-flip $f_+$ and the flip $f_-$, so we have four observables, the cross section $d\sigma/dt$, the polarization $P_n$ and two rotation parameters $R$ and $A$. There is one well known quadratic relation, that is $P_n^2 + A^2 + R^2 = 1$. If all three are measured they should lie on the surface of the unit sphere, which is a consistency check of the data. Concerning the crossed reaction $\bar{p}p \rightarrow \pi\pi$, large spin effects have been seen at LEAR. One has an analogous identity as the one above for $\pi N$, namely $A_n^2 + A_{nm}^2 + A_{ml}^2 = 1$. Hence $|A_n| = 1$, implies $A_{nm} = A_{ml} = 0$. For $\bar{p}p \rightarrow KK$ similar results were obtained [1].
ii) \(- Spin1/2 + Spin1/2 \rightarrow Spin1/2 + Spin1/2 \)

For the reaction \(1/2 + 1/2 \rightarrow 1/2 + 1/2 \), for example \(pp\) elastic scattering, we have five amplitudes, therefore twenty-five observables and sixteen quadratic relations between them. For the derivation one considers a 5x5 matrix of the observables, which is positive Hermitian and the final results can be found in Ref. [2]. These relations are very useful to check the data and when one observable is not measured, it is set to zero and the equality becomes an inequality. In particular one finds the following very simple condition \(A_{LL}^2 + D_{NN}^2 \leq 1\), between the two-spin correlation parameters \(A_{LL}\) and \(D_{NN}\). This condition turns out to be very useful because if one of the parameters is close to one, the other one is bounded to be near zero. In the reaction \(\bar{p}p \rightarrow \Lambda\Lambda\), one has found in a certain kinematic region \(A_{LL} = -1\), so before making any measurement one can conclude that \(D_{NN} \sim 0\), which is also a no-go theorem for some theoretical considerations [3].

An empirical method consists of generating randomly six complex amplitudes, for \(\bar{p}p \rightarrow \Lambda\Lambda\), to compute the spin observables and look at the constraints (see Fig. 1). The empirical method was extended to the case of triple of spin observables. Several remarkable shapes were discovered for the allowed domain: sphere, cylinder, cone, pyramid, tetrahedron, octahedron, intersection of two cylinders, intersection of three orthogonal cylinders, coffee filter, etc...[1].

iii) \(- Pseudoscalar meson photoproduction \)

Consider the reactions \(\gamma N \rightarrow KY\), with \(Y = \Lambda, \Sigma\), where the incoming photon beam is polarized, the nucleon target is polarized and the polarization of the outgoing \(\Lambda, \Sigma\) is measured. Need to determine four complex amplitudes, \textit{i.e.} seven real numbers. One can perform sixteen different experiments: the unpolarized cross section, three single spin asymmetries, the linearly polarized photon asymmetry \(\Sigma^\gamma\), the polarized target asymmetry \(A_N\) and the recoil baryon polarization \(P_Y\). We have the linear relations \(|A_N \pm P_Y| \leq 1 \pm \Sigma^\gamma\), which give the tetrahedron shown in Fig. 2. The allowed volume is 1/3 of the entire cube \([-1, +1]^3\). Finally, there are four double correlations between the target and the recoil baryon spins in the scattering plane with unpolarized photons, four double spin correlations with linearly polarized photons and four double spin correlations with circularly polarized photons. Clearly, these sixteen measurements must be constrained by nine quadratic relations, for example, \(P_Y\) and the double correlation parameters between the circularly polarized photon and the recoil baryon spin along the directions \(\hat{x}\) and \(\hat{z}\) in the scattering plane, \(C_x^Y\) and \(C_z^Y\), \textit{i.e.} \((P_Y)^2 + (C_x^Y)^2 + (C_z^Y)^2 \leq 1\). Following
the analysis of the CLAS data at Jefferson Lab., it is almost saturated and this implies the saturation of other quadratic constraints with three or more observables [1].

3 INCLUSIVE REACTIONS

i) - Asymmetries in DIS
A bound on the DIS transverse asymmetry $A_2$ has been established long time ago and reads $|A_2| \leq \sqrt{R}$ where $R$ is the standard ratio $\sigma_L/\sigma_T$. There is an improved version of this positivity constraint, namely, $|A_2| \leq \sqrt{R(1 + A_1)/2}$, which is very relevant when the DIS longitudinal asymmetry $A_1$ is negative. This is the case for $A^p_1$, with a neutron target in a certain kinematic range.

ii) - Spin-transfer observables
Consider a parity conserving inclusive reaction of the type, $a(spin1/2) + b(unpol.) \rightarrow c(spin1/2) + X$. One can define eight observables, which must satisfy

$$\left(1 \pm D_{NN}\right)^2 \geq \left(P_{cN} \pm A_{aN}\right)^2 + \left(D_{LL} \pm D_{SS}\right)^2 + \left(D_{LS} \mp D_{SL}\right)^2$$ (1)

If we concentrate for the moment on the case where the particle spins are normal to the scattering, for example for $p^\uparrow p \rightarrow \Lambda^\uparrow X$, one has $1 - D_{NN} \geq |P_{\Lambda} \pm A_N|$. The corresponding allowed domains are displayed on Fig. 3.

iii) - Quark Transversity Distribution $\delta q(x, Q^2)$
In addition to $q(x, Q^2)$ and $\Delta q(x, Q^2)$, there is a new distribution function $\delta q(x, Q^2)$, chiral odd, leading twist, which decouples from DIS and has been indirectly extracted recently for the first time. It must satisfy the following positivity bound [4], $q(x, Q^2) + \Delta q(x, Q^2) \geq 2|\delta q(x, Q^2)|$, which survives up to NLO corrections. The corresponding allowed triangle occurred also for $\bar{p}p \rightarrow \Lambda\Lambda$, as seen above.

\footnote{NOTE: The eight transverse momentum dependent quark distributions obey the same constraints since they are related to $nucleon(p, S) \rightarrow quark(k, S') + X$.}
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Figure 1: Random simulation for $\bar{p}p \rightarrow \bar{\Lambda}\Lambda$ of $P_n$ versus $A_n$ (square), $A_n$ versus $D_{mm}$ (disk) and $P_n$ versus $C_{nn}$ (triangle).

Figure 2: Tetrahedron domain limited by inequalities $|A_N \pm P_Y| \leq 1 \pm \Sigma^\gamma$, for the photoproduction observables $x = A_N$, $y = P_Y$ and $z = \Sigma^\gamma$. 
Figure 3: The allowed domain corresponding to the constraints Eq. (1) (left). The slice of the full domain for $D_{NN} = 0$ (middle) and for $D_{NN} = 1/3$ (right).