Estimate the Ranges of $\rho$ and $\eta$
only from the Kobayashi-Maskawa Matrix Elements
of the First Two Generations

Yong Liu

Lab of Numerical Study for Heliospheric Physics
Chinese Academy of Sciences, P. O. Box 8701, Beijing 100080, P.R.China

Abstract

Based on the relation between weak $CP$ phase and the other three mixing angles in
Cabibbo-Kobayashi-Maskawa (CKM) matrix postulated by us before, the ranges of $\rho$ and $\eta$
have been estimated by using the best known two KM matrix elements $V_{ud}$ and $V_{cd}$ (or $V_{us}$).
It is found that, the upper limit on $\eta$ is about 0.008 which is consistent with that
estimated by Wolfenstein more than ten years ago but far small than the present popular
estimation.

PACS number(s): 12.10.Ck, 13.25.+m, 11.30.Er

Email address: yongliu@ns.lhp.ac.cn
Quark mixing and \(CP\) violation is one of the most interesting and important problem in weak interaction [1-4]. In the Minimal Standard Model (MSM), They are described by the unitary Cabibbo-Kabayashi-Maskawa (CKM) matrix, which takes the following form [5-7]

\[
V_{KM} = \begin{pmatrix}
    c_1 & -s_1 c_3 & -s_1 s_3 \\
    s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\
    s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta}
\end{pmatrix}
\]  

(1)

with the standard notations \(s_i = \sin \theta_i\) and \(c_i = \cos \theta_i\) being used. In fact, we always can take \(0 < \theta_i < \frac{\pi}{2}\) and \(-\pi < \delta < \pi\) by a suitable choice of phase.

To make it convenient to use the CKM matrix in the actual calculation, Wolfenstein parametrized it as [8]

\[
V_W = \begin{pmatrix}
    1 - \frac{1}{2} \lambda^2 & \frac{\lambda}{2} & A \lambda^3 (\rho - i \eta + i n \frac{1}{2} \lambda^2) \\
    -\lambda & 1 - \frac{1}{2} \lambda^2 - i n A \lambda^4 & A \lambda^2 (1 + i n \lambda^2)^2 \\
    A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1
\end{pmatrix}
\]  

(2)

Actually, one can take different parametrization [7][9-13] in different cases. They are only for the convenience in discussing the different concrete question, but the physics does not change when adopting various parametrizations.

In Eq.(2), \(\lambda\) and \(A\) are the two better known parameters. But, due to the uncertainty of hadronic matrix elements and other reasons, we can not extract more information about \(\rho\) and \(\eta\) from experimental results. Up to now, we still know little about them. More than ten years ago, Wolfenstein estimated that the upper limit on \(\eta\) is about 0.1 [8], but the recent estimate on \(\rho\) is about 0 and \(\eta\) about 0.35 [9][14].

The center purpose of this short letter is to give a limit on the ranges of \(\rho, \eta\) and their dependence on each other by only use of the two best known KM matrix elements \(V_{ud}\) and \(V_{cd}\) or \(V_{us}\). According to the usual view point, this is impossible. But, by using the constraint on weak \(CP\) phase and the three mixing angles postulated by us before, we can do so.

In Ref.[15], we have found that the weak \(CP\) phase and the other three mixing angles satisfy the following relation

\[
\sin \frac{\delta}{2} = \sqrt{\frac{\sin^2 \theta_1 + \sin^2 \theta_2 + \sin^2 \theta_3 - 2 (1 - \cos \theta_1 \cos \theta_2 \cos \theta_3)}{2 (1 + \cos \theta_1) (1 + \cos \theta_2) (1 + \cos \theta_3)}}
\]  

(3)
where $\theta_i (i = 1, 2, 3)$ are the corresponding angles in the standard KM parametrization matrix Eq.(1). Here, we have taken $\delta$ presented in Eq.(1) as a certain geometry phase. In fact, the geometry meaning of Eq.(3) is as follows

$\delta$ is the solid angle enclosed by $\theta_1$, $\theta_2$ and $\theta_3$, or the area to which the solid angle corresponds on a unit sphere.

To make $\theta_1$, $\theta_2$ and $\theta_3$ enclose a solid angle, the condition

$$\theta_i + \theta_j > \theta_k \quad (i \neq j \neq k \neq i. \ i, j, k = 1, 2, 3)$$

must be satisfied.

Based on Eq.(3) and Eq.(4), we can proceed our discussion.

From the two best known matrix elements

$$V_{ud} = c_1$$

and

$$V_{cd} = s_1 c_2$$

in KM parametrization, we can get $\theta_1$ and $\theta_2$. According to Eq.(4), we have

$$| \theta_1 - \theta_2 | < \theta_3 < \theta_1 + \theta_2.$$  

When we let $\theta_3$ vary in the above domain, we can obtain a permitted domain of $\delta$ from Eq.(3). Considering the relation between KM parametrization and Wolfenstein’s parametrization [16]

$$\rho = \frac{\sin \theta_3}{(\sin^2 \theta_2 + \sin^2 \theta_3 + 2 \sin \theta_2 \sin \theta_3 \cos \delta)^{1/2}}$$

$$\eta = \frac{\sin \theta_2 \sin \theta_3 \sin \delta}{(\sin^2 \theta_2 + \sin^2 \theta_3 + 2 \sin \theta_2 \sin \theta_3 \cos \delta)}$$

taking use of Eq.(8) and Eq.(9), we can finally get the ranges of $\rho$, $\eta$ and their dependence on each other.

Take the experimental values [7]

$$V_{ud} = 0.975 \pm 0.001 \quad V_{cd} = 0.2205 \pm 0.0018$$

as input, when fixing $V_{ud}$ and $V_{cd}$ at their central values, the dependence of $\eta$ on $\rho$ is shown in Fig.(1). Taking the errors of $V_{ud}$ and $V_{cd}$ into account, we can get the permitted ranges of $\rho$ and $\eta$, the result is given in Fig.(2).
From Fig.(2), we find that a relative large $\eta$ corresponds to a relative narrow window of $\rho$ around 0.55.

In conclusion, we find that, the upper limit on $\eta$ is about 0.008, this is consistent with the prediction given by Wolfenstein fifteen years ago [8], but very lower than the present estimate accepted by most people. According to Wolfenstein [8][17], the smaller $|\epsilon'|/|\epsilon|$, the smaller $\eta$, so, if we can see a small $|\epsilon'|/|\epsilon|$ in the future experiments, especially, if $|\epsilon'|/|\epsilon|$ can be small to the order of about $10^{-5}$ or less, we will win a strong support to our viewpoint.

References

[1] L.L.Chau, Phys.Rept. 95(1983)1.
[2] A.Pich, Preprint CP violation, CERN-TH.7114/93.
[3] CP Violation Ed. C.Jarlskog. World Scientific Publishing Co.Pte.Ltd 1989.
[4] CP Violation Ed. L.Wolfenstein, North-Holland, Elsevier Science Publishers B.V. 1989.
[5] M.Kobayashi and T.Maskawa, Prog.Theor.Phys. 42(1973)652.
[6] N.Cabibbo, Phys.Rev.Lett. 10(1963)531.
[7] Particle Data Group, Phys.Rev.D. 54(1996)94.
[8] L.Wolfenstein, Phys.Rev.Lett. 51(1983)1945.
[9] Z.Z.Xing, Phys.Rev.D. 51(1995)3958. A.Ali and D.London, Z.Phys.C 65(1995)431. A.J.Buras, Phys.Lett.B 333(1994)476.
[10] A.J.Buras, M.E.Ladtenbacher and G.Ostermaier, Phys.Rev.D. 50(1994)3433.
[11] L.-L.Chau and W.-Y.Keung, Phys.Rev.Lett. 53(1984)1802.
[12] L.Maiani, Phys.Lett.B 62(1976)183.
[13] H.Fritzsch and J.Plankl, Phys.Rev.D 35(1987)1732.
[14] J.L.Rosner, Top Quark Mass, [hep-ph/9610222], CERN-TH/96-245. M.Schmidtler and K.R.Schubert, Z.Phys.C 53(1992)347.
[15] Yong Liu and Jing-Ling Chen, New Constraint on the Parameters in Cabibbo-Kobayashi-Maskawa Matrix of Wolfenstein’s Parametrization, hep-ph/9711293. Jing-Ling Chen, Mo-Lin Ge, Xue-Qian Li and Yong Liu, New Viewpoint to the Source of Weak CP Phase, hep-ph/9711330.

[16] E.A.Paschos and U.Turke, Phys.Rept. 4(1989)145.

[17] J.S.Hagelin and F.Gilman, SLAC Report No. SLAC PUB-3087, 1983. F.J.Gilman and M.B.Wise, Phys.Lett.B 83(1979)83.
Fig. (1) The dependence of $\eta$ on $\rho$
where $V_{(ud)} = 0.975$ and $V_{(cd)} = 0.2205$
