Reversible optical–microwave quantum conversion assisted by optomechanical dynamically dark modes

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Abstract
We propose a dynamically dark-mode (DDM) scheme to realize the reversible quantum conversion between microwave and optical photons in an electro-optomechanical (EOM) model. It is shown that two DDMs appear at certain times during the dynamical evolution of the EOM model. It is demonstrated that the DDMs can induce two kinds of reversible and highly efficient quantum conversion between the microwave and optical fields, the conditional quantum conversion (CQC) and the entanglement-assisted quantum conversion (EAQC). The CQC happens at the condition of vanishing of the initial-state mean value of one of the microwave and optical fields and only depends on the coupling ratio of the system under consideration. The EAQC occurs in the presence of the initial-state entanglement between the microwave and optical fields. It is found that the EAQC can be manipulated by engineering the initial-state entanglement and the coupling ratio. It is indicated that it is possible to realize the entanglement-enhanced (or suppressed) quantum conversion through controlling the phase of the initial-state parameter. Our work highlights the power of generating reversible and highly efficient quantum conversion between microwave and optical photons by the DDMs.

Keywords Optical–microwave quantum conversion · Dynamically dark modes · An electro-optomechanical converter · Quantum entanglement
1 Introduction

In recent years, much attention has been paid to quantum conversion between microwave and optical photons due to its importance in quantum technologies [1–3]. Such quantum conversion is of special significance to realize a quantum internet [4–8] and distributed quantum tasks including computing or sensing [9,10]. A number of systems such as atomic, molecular, and solid-state impurity spins [11–18], magnons in ferromagnetic materials [19], electro-optic modulators [20–23], and mechanical oscillators [24–32] have been proposed as suitable candidates for mediating interaction between microwave and optical fields. In particular, the electro-optomechanical (EOM) system based on quantum cavity optomechanics is regarded as a promising and versatile platform with several experiments demonstrating efficient conversion between the microwave and optical fields [33–36].

The mechanical modes in cavity optomechanical systems [37–42] can couple with the optical and microwave modes. The coupling between the mechanical mode and the optical and microwave modes has been demonstrated in recent experiments [43–55]. Such systems can hence serve as an interface in quantum networks to connect optical and microwave photons [56]. It is well known that the optomechanical dark mode exists in a lot of cavity optomechanical systems [27,57–59]. The optomechanical dark mode was experimentally demonstrated by coupling two optical whispering gallery modes to a mechanical breathing mode in a silica resonator in the regime of weak optomechanical coupling [57]. Since the optomechanical dark mode can protect the system from mechanical dissipation, it can be employed for the realization of high-efficient quantum state conversion [27,58], the reservoir-engineered entanglement [60, 61], quantum illumination [26], optomechanically induced transparency [62,63], and wider quantum applications [64–66].

In this work, we theoretically investigate the reversible optical-to-microwave quantum conversion arising from the optomechanical dynamically dark modes (DDMs), which are dynamically decoupled from the mechanical resonator at some specific times in an EOM quantum conversion model. The presence of the DDMs results in a bidirectional and highly efficient quantum conversion between microwave and optical fields. The remainder of the paper is organized as follows: In Sect. 2, we introduce the electro-optomechanical (EOM) quantum conversion model, present an approximately analytical solution to the EOM model, and demonstrate the existence of the dynamically dark mode. In Sect. 3, we show reversible and highly efficient quantum conversion between the optical and microwave fields. In Sect. 4, we study the entanglement-assisted quantum conversion between the optical and microwave fields. We show that the DDMs can induce two kinds of bidirectional and highly efficient quantum conversion between the microwave and optical fields, the conditional quantum conversion and the entanglement-assisted quantum conversion. Finally, the concluding section, Sect. 5, summarizes and discusses our main results.
2 The electro-optomechanical model and dynamically dark modes

In this section, we introduce the cavity EOM model and present an approximately analytical solution of this model in terms of the Senm–Mandal method [67–69]. The cavity EOM model under our consideration is a EOM quantum converter, which consists of a driven superconducting microwave cavity of resonant frequency $\omega_w$, a driven a Fabry–Pérot optical cavity with resonant frequency $\omega_o$, and a mechanical resonator with resonant frequency $\omega_m$ [70,71]. The mechanical resonator (annihilation operator $\hat{b}$) is capacitively coupled on the one side to a driven superconducting microwave cavity (annihilation operator $\hat{a}_w$) and on the other side to a driven Fabry–Perot optical cavity (annihilation operator $\hat{a}_o$) [33–35,70,71]. In the frame rotating at the frequencies of the microwave and optical driving fields, the Hamiltonian of the EOM model [70] is given by:

$$\hat{H} = \hbar \omega_m \hat{b}^\dagger \hat{b} + \hbar \sum_{j=w,o} \left[ \Delta_j + g_j (\hat{b} + \hat{b}^\dagger) \right] \hat{a}_j^\dagger \hat{a}_j + \hbar \sum_{j=w,o} E_j (\hat{a}_j^\dagger - \hat{a}_j),$$  \hspace{1cm} (1)

where $g_j$ is the coupling constant between the mechanical resonator and cavity $j$, $\Delta_j = \omega_j - \omega_{d,j}$ are the detunings from their resonant frequencies $\omega_j$ with $j = w, o$ denoting the microwave and optical cavities, $E_o$ and $E_w$ are the optical and microwave driving field amplitudes [71], respectively.

We can treat cavity modes with semiclassical description and linearize the Hamiltonian by expanding the cavity modes around their steady-state field amplitudes $\hat{c}_j = \hat{a}_j - \sqrt{N_j}$ with the $N_j = |E_j|^2/(\kappa_j^2 + \Delta_j^2) \gg 1$ being the mean numbers of intracavity photons induced by the microwave or optical pumps [71] with $\kappa_j$ are the total cavity decay rates, since the single photon coupling constants between the optical and microwave fields and the mechanical resonator $g_j$ are small in current experiments [26]. When we choose the effective cavity detunings $\Delta_w = -\Delta_o = \omega_m$ and neglect the terms at $\pm \omega_m$, under rotating wave approximation, the linearized Hamiltonian of the EOM model is given by:

$$\hat{H} = \hbar G_o (\hat{c}_o \hat{b} + \hat{b}^\dagger \hat{c}_o^\dagger) + \hbar G_w (\hat{c}_w \hat{b} + \hat{b}^\dagger \hat{c}_w^\dagger),$$  \hspace{1cm} (2)

where the multiphoton coupling rate is $G_j = g_j \sqrt{N_j}$. We have described the above interaction Hamiltonian in Fig. 1.
In what follows, we solve the above linearized Hamiltonian in the Heisenberg picture by means of the Senm–Mandal method [67–69]. From Hamiltonian (2), we can obtain the Heisenberg operator equations of motion involving the field operators

\[
\begin{align*}
\dot{b}(t) &= -i G_o \hat{c}_o^\dagger(t) - i G_w \hat{c}_w(t), \\
\dot{c}_o(t) &= -i G_o \hat{b}(t), \\
\dot{c}_w(t) &= -i G_w \hat{b}(t).
\end{align*}
\tag{3}
\]

Senm and Mandal developed the approximate method to solve Heisenberg equations of motion. Note that the field operators in the Heisenberg picture have the following formal solution

\[
\begin{align*}
\hat{b}(t) &= \exp(i \hat{H}t) \hat{b}(0) \exp(-i \hat{H}t), \\
\hat{c}_o(t) &= \exp(i \hat{H}t) \hat{c}_o(0) \exp(-i \hat{H}t), \\
\hat{c}_w(t) &= \exp(i \hat{H}t) \hat{c}_w(0) \exp(-i \hat{H}t),
\end{align*}
\tag{4}
\]

which can be expanded as:

\[
\begin{align*}
\hat{b}(t) &= \hat{b}(0) + it \left[ \hat{H}, \hat{b}(0) \right] + (it)^2 \left[ \hat{H}, \left[ \hat{H}, \hat{b}(0) \right] \right] + \cdots \\
\hat{c}_o(t) &= \hat{c}_o(0) + it \left[ \hat{H}, \hat{c}_o(0) \right] + (it)^2 \left[ \hat{H}, \left[ \hat{H}, \hat{c}_o(0) \right] \right] + \cdots \\
\hat{c}_w(t) &= \hat{c}_w(0) + it \left[ \hat{H}, \hat{c}_w(0) \right] + (it)^2 \left[ \hat{H}, \left[ \hat{H}, \hat{c}_w(0) \right] \right] + \cdots
\end{align*}
\tag{5}
\]

For the mechanical mode, the related commutators are obtained as follows:

\[
\begin{align*}
\left[ \hat{H}, \hat{b}(0) \right] &= -G_o \hat{c}_o^\dagger(0) - G_w \hat{c}_w(0), \\
\left[ \hat{H}, \left[ \hat{H}, \hat{b}(0) \right] \right] &= -G_o^2 \hat{b}(0) + G_w^2 \hat{b}(0), \\
\left[ \hat{H}, \left[ \hat{H}, \left[ \hat{H}, \hat{b}(0) \right] \right] \right] &= (G_o^2 - G_w^2)[G_o \hat{c}_o^\dagger(0) + G_w \hat{c}_w(0)].
\end{align*}
\tag{6}
\]

It is straightforward to show that all higher-order commutators in the right-hand side of the first equation in Eq. (5) contain only the three initial operators \( \hat{b}(0), \hat{c}_w(0) \) and \( \hat{c}_o^\dagger(0) \). Then, the operator of the mechanical mode \( \hat{b}(t) \) is a linear superposition of the three operators \( \hat{b}(0), \hat{c}_w(0) \) and \( \hat{c}_o^\dagger(0) \). Similarly, we find that the operator of the optical mode \( \hat{c}_o(t) \) is a linear superposition of the three initial operators \( \hat{c}_o(0), \hat{c}_w^\dagger(0) \) and \( \hat{b}^\dagger(0) \), while the operator of the microwave mode \( \hat{c}_w(t) \) is a linear superposition of
the three initial operators $\hat{c}_w(0), \hat{c}_w^\dagger(0)$ and $\hat{b}(0)$. Therefore, the approximate analytical solution of the Heisenberg equation of motion can be directly expressed as:

\[
\begin{align*}
\dot{b}(t) &= f_1(t)\hat{b}(0) + f_2(t)\hat{c}_w(0) + f_3(t)\hat{c}_w^\dagger(0), \\
\dot{c}_o(t) &= g_1(t)\hat{c}_o(0) + g_2(t)\hat{c}_w^\dagger(0) + g_3(t)\hat{b}(0), \\
\dot{c}_w(t) &= h_1(t)\hat{c}_w(0) + h_2(t)\hat{c}_o(0) + h_3(t)\hat{b}(0),
\end{align*}
\]

(7)

where the time-dependent coefficient functions $f_i(t), g_i(t), h_i(t), (i = 1, 2, 3)$ should satisfy the following initial conditions:

\[
\begin{align*}
f_1(0) &= g_1(0) = h_1(0) = 1, \\
f_2(0) &= g_2(0) = h_2(0) = 0, \\
f_3(0) &= g_3(0) = h_3(0) = 0.
\end{align*}
\]

(8)

The self-consistency of the solutions leads to:

\[
\begin{align*}
\left[\dot{b}(t), \dot{c}_w^\dagger(t)\right] &= |f_1(t)|^2 + |f_2(t)|^2 - |f_3(t)|^2 = 1, \\
\left[\dot{c}_o(t), \dot{c}_o^\dagger(t)\right] &= |g_1(t)|^2 - |g_2(t)|^2 - |g_3(t)|^2 = 1, \\
\left[\dot{c}_w(t), \dot{c}_w^\dagger(t)\right] &= |h_1(t)|^2 - |h_2(t)|^2 + |h_3(t)|^2 = 1,
\end{align*}
\]

(9)

which indicates that at any time all of the three isochronal commutation relations satisfy commutation relations of the harmonic oscillator to ensure the self-consistency of the three mode solutions.

Substituting Eq. (7) into Eq. (3) and comparing the coefficients of equations, we obtain the differential equations of the coefficient functions:

\[
\begin{align*}
\dot{f}_1(t) &= -iG_o b_3^*(t) - iG_w h_3(t), \\
\dot{f}_2(t) &= -iG_o b_2^*(t) - iG_w h_1(t), \\
\dot{f}_3(t) &= -iG_o b_1^*(t) - iG_w h_2(t), \\
\dot{g}_1(t) &= -iG_o f_3(t), \\
\dot{g}_2(t) &= -iG_o f_2^*(t), \\
\dot{g}_3(t) &= -iG_o f_1^*(t), \\
\dot{h}_1(t) &= -iG_w f_2(t), \\
\dot{h}_2(t) &= -iG_w f_3(t), \\
\dot{h}_3(t) &= -iG_w f_1(t).
\end{align*}
\]

(10)

For the above differential equations, when $G_w \neq G_o$, we can obtain the following solution:

\[
\begin{align*}
f_1(t) &= \cos \Omega t, \\
f_2(t) &= -i \frac{\sin \Omega t}{\sqrt{1 - k^2}}, \\
f_3(t) &= k f_2(t), \\
g_1(t) &= -\frac{k^2 \cos \Omega t}{1 - k^2} + \frac{1}{1 - k^2}, \\
g_2(t) &= -\frac{k \cos \Omega t}{1 - k^2} + \frac{k}{1 - k^2}, \\
g_3(t) &= -i \frac{k \sin \Omega t}{\sqrt{1 - k^2}}, \\
h_1(t) &= \frac{\cos \Omega t}{1 - k^2} - \frac{k^2}{1 - k^2}, \\
h_2(t) &= \frac{k \cos \Omega t}{1 - k^2} - \frac{k}{1 - k^2},
\end{align*}
\]
\[ h_3(t) = -i \frac{\sin \Omega t}{\sqrt{1-k^2}}, \]  

(11)

where we have introduced the ratio of two coupling strengths \( k \) and \( \Omega \) defined by:

\[ k = \frac{G_o}{G_w}, \quad \Omega = \sqrt{G_w^2 - G_o^2} = \sqrt{1-k^2G_w}. \]  

(12)

According to the solution given by Eq. (11), it is interesting to note that the optical-wave and microwave can be decoupled with mechanical vibrator at some special times. In fact, in equation (13) if we choose special moments

\[ t = t_n = \frac{n\pi}{\Omega}, \quad n = 0, 1, 2, 3, \ldots \]  

(13)

the solution given by Eq. (11) becomes

\[
\begin{align*}
 f_1(t_n) &= -1, \\
 f_2(t_n) &= f_3(t_n) = 0, \\
 g_1(t_n) &= \frac{1+k^2}{1-k^2}, \\
 g_2(t_n) &= \frac{2k}{1-k^2}, \\
 g_3(t_n) &= 0, \\
 h_1(t_n) &= -\frac{1+k^2}{1-k^2}, \\
 h_2(t_n) &= -\frac{2k}{1-k^2}, \\
 h_3(t_n) &= 0.
\end{align*}
\]  

(14)

Obviously, these coefficient functions satisfy the following relationship:

\[
|g_1(t_n)|^2 - |g_2(t_n)|^2 = 1, \quad |h_1(t_n)|^2 - |h_2(t_n)|^2 = 1.
\]  

(15)

By substituting Eq. (14) into Eq. (7), we can rewrite the solution of the EOM model as:

\[
\begin{align*}
 \hat{c}_o(t_n) &= g_1(t_n)\hat{c}_o(0) + g_2(t_n)\hat{c}_w(0), \\
 \hat{c}_w(t_n) &= h_1(t_n)\hat{c}_w(0) + h_2(t_n)\hat{c}_o(0),
\end{align*}
\]  

(16)

which indicate that the optical-wave (microwave) field mode at the time \( t_n \) only involves the initial optical-field operator and the initial microwave-field operator. Hence, the optical-wave and microwave modes are well decoupled with the mechanical oscillator at the moments \( t = t_n \); in this sense, we call the two modes \( \hat{c}_o(t_n) \) and \( \hat{c}_w(t_n) \) as dynamically dark modes with respect to the mechanical mode.

These DDMs are mechanically dark modes, which are superpositions of the optical and microwave modes. Although they are decoupled from the mechanical oscillator, they can still mediate an effective optomechanical coupling between the optical and microwave modes. In the following, we will investigate the DDM-assisted quantum conversion between the optical and microwave fields.
3 Reversible and highly efficient optical-to-microwave quantum conversion

In this section, we investigate the DDM-assisted quantum conversion between optical and microwave fields in the EOM converter. In general, a full description of the quantum conversion system should include the inputs and outputs of the optical, microwave and mechanical resonators [34]. However, we only need to pay our attention to the inputs and outputs of the optical and microwave fields in the DDM case since the optical and microwave fields are completely decoupled to the mechanical resonator. In order to characterize the EOM converter performance, we introduce the quantum conversion rate between the optical (microwave) and microwave (optical) fields defined by:

\[
\eta_{ow} = \left| \frac{\langle \hat{c}_w(t_n) \rangle}{\langle \hat{c}_o(0) \rangle} \right|^2, \quad \eta_{wo} = \left| \frac{\langle \hat{c}_o(t_n) \rangle}{\langle \hat{c}_w(0) \rangle} \right|^2, \tag{17}
\]

where \( \eta_{ow} (\eta_{wo}) \) describes the quantum conversion capability of the EOM converter from the initial-state optical (microwave) field to the transient-state microwave (optical) field at time \( t_n \).

Substituting Eq. (16) into Eq. (17), we obtain

\[
\eta_{ow} = \left| h_1(t_n) \frac{\langle \hat{c}_w(0) \rangle}{\langle \hat{c}_o(0) \rangle} + h_2(t_n) \frac{\langle \hat{c}_o(0) \rangle}{\langle \hat{c}_w(0) \rangle} \right|^2, \quad \eta_{wo} = \left| g_1(t_n) \frac{\langle \hat{c}_o(0) \rangle}{\langle \hat{c}_w(0) \rangle} + g_2(t_n) \frac{\langle \hat{c}_w(0) \rangle}{\langle \hat{c}_w(0) \rangle} \right|^2, \tag{18}
\]

which indicates that when the initial-state mean value of the output mode vanishes, i.e., \( \langle \hat{c}_w(0) \rangle = 0 (\langle \hat{c}_o(0) \rangle = 0) \), the quantum conversion rate from the input optical (microwave) to output microwave (optical) fields is simply reduced to:

\[
\eta_{ow} = |h_2(t_n)|^2 = \frac{4k^2}{(1-k^2)^2}, \quad \eta_{wo} = |g_2(t_n)|^2 = \frac{4k^2}{(1-k^2)^2}. \tag{19}
\]

It is worthwhile to note that the optical–microwave quantum conversion corresponding to Eq. (19) is a conditional quantum conversion (CQC) between the optical and microwave fields in which the highly efficient quantum conversion from the optical (microwave) to microwave (optical) fields is at the condition of the vanishing initial-state mean value of the microwave (optical) field. The CQC is independent of the initial state of the converted optical (microwave) field. The quantum conversion rate of the CQC depends on only the ratio of the two coupling strengths between the optical and microwave fields and the mechanical resonator under the condition of the initial-state mean value vanishing for the output mode. From Eq. (14), we can see that the conversion rate of the CQC from the optical field to the microwave field is equal to that from the microwave field to the optical field. Therefore, we can conclude that the CQC between the optical and microwave fields is a reversible quantum conversion.
Fig. 2 (Color online) The optical-to-microwave quantum conversion rate $\eta_{ow}$ with respect to the coupling ratio $k$ for the conditional quantum conversion between the optical and microwave fields.

In this sense, the CQC can be regarded as a universal quantum conversion between the optical and microwave fields.

In order to see how to manipulate the quantum conversion rate between the optical (microwave) and microwave (optical) fields by varying the coupling ratio $k$, we have plotted the quantum conversion rate with respect to the coupling ratio in Fig. 2. From Fig. 2, we can see that the quantum conversion rate between the optical (microwave) and microwave (optical) fields increases with the increase of the coupling ratio.

It should be pointed out that the quantum conversion rate given by Eq. (17) in the present DDM scheme is different from the quantum conversion efficiency in conventional quantum conversion schemes between the microwave and optical photons [2,34,57,72,73]. The conventional quantum conversion schemes are steady-state schemes. The quantum conversion efficiency in the steady-state schemes is defined as the ratio of the input photon flux for the optical (microwave) field with respect to the output photon flux for the microwave (optical) field. The quantum conversion efficiency can be obtained by the use of the scattering matrix approach [27,74,75]. The output photon flux can be obtained by using the well-known input–output relation, which relates the intra-cavity field to the incident and outgoing fields. The quantum conversion efficiency in the steady-state schemes is generally smaller than unit. However, the DDM scheme is a transient-state quantum conversion scheme; the quantum conversion rate defined by Eq. (17) is a transient-state conversion rate between the initial-state optical (microwave) field and the transient-state microwave (optical) field. It is the ratio of the transient microwave (optical) field at time $t_n$ over the initial optical (microwave) field. The quantum conversion rate in the DDM scheme is not a normalized function, so it may take values larger than one.

4 Entanglement-assisted optical-to-microwave quantum conversion

In this section, we investigate the influence of the initial entanglement between the optical and microwave fields on the quantum conversion rate. We will show that entanglement-assisted optical-to-microwave quantum conversion can happen in the present DDM scheme of the optical-to-microwave quantum conversion. Without loss of generality, we consider the following optical–microwave initial entangled coherent state:

$$|\psi(0)\rangle_{ow} = N \left[ \cos \theta |\alpha \rangle_o \otimes |0\rangle_w + \sin \theta |0\rangle_o \otimes |\beta \rangle_w \right],$$ (20)
where $|\alpha\rangle_o$ and $|\beta\rangle_w$ are the Glauber coherent states; the normalization constant $N$ is given by:

$$N^{-2} = 1 + \sin(2\theta)e^{-\frac{1}{2}(|\alpha|^2+|\beta|^2)}.$$  

(21)

The degree of quantum entanglement of a bipartite two-component entangled state can be described by the quantum concurrence [76–79]. In general, for the following bipartite two-component entangled state

$$|\psi\rangle = N\left[\mu|\eta\rangle \otimes |\gamma\rangle + \nu|\xi\rangle \otimes |\delta\rangle\right].$$  

(22)

where $N$ is the normalization constant, and the quantum concurrence is given by:

$$C = 2|\mu||\nu|N^2\sqrt{(1 - |p_1|^2)(1 - |p_2|^2)},$$  

(23)

where the two state-overlapping functions $p_1$ and $p_2$ are defined by:

$$p_1 = \langle \mu | \gamma \rangle, \quad p_2 = \langle \xi | \delta \rangle.$$  

(24)

Making use of Eq. (23), we can obtain the entanglement amount of the initial entangled coherent state (20) with the following expression:

$$C = \frac{|\sin(2\theta)|\sqrt{(1 - e^{-|\alpha|^2})(1 - e^{-|\beta|^2})}}{[1 + \sin(2\theta)e^{-\frac{1}{2}(|\alpha|^2+|\beta|^2)/2}]},$$  

(25)

which indicates that for given values of $\alpha$ and $\beta$, when $\theta = 0$ or $\pi/2$, the entanglement amount of the initial entangled coherent state (20) vanishes, while when $\theta = -\pi/4$ the initial entangled coherent state (20) has the largest entanglement amount

$$C_{max} = \frac{\sqrt{(1 - e^{-|\alpha|^2})(1 - e^{-|\beta|^2})}}{[1 - e^{-(|\alpha|^2+|\beta|^2)/2}]},$$  

(26)

which means that the initial entangled coherent state (20) is a maximally entangled state with $C_{max} = 1$ when $\alpha = \pm \beta$.

For the initial entangled coherent state (20), we can obtain the mean values of the optical and microwave field operators

$$\langle \hat{c}_o(0) \rangle = \alpha N^2\left[\cos^2\theta + \frac{1}{2}\sin(2\theta)e^{-\frac{1}{2}(|\alpha|^2+|\beta|^2)/2}\right],$$

$$\langle \hat{c}_w(0) \rangle = \beta N^2\left[\sin^2\theta + \frac{1}{2}\sin(2\theta)e^{-\frac{1}{2}(|\alpha|^2+|\beta|^2)/2}\right].$$  

(27)
For simplicity, we consider the case of $\alpha = \beta$. Substituting Eq. (27) into (18), we can obtain the quantum conversion rate between the optical and microwave fields

$$\eta_{ow}(\theta, \varphi) = \left| h_1(t_n) \left[ \frac{2 \sin^2 \theta + \sin(2\theta) e^{-|\alpha|^2}}{2 \cos^2 \theta + \sin(2\theta) e^{-|\alpha|^2}} \right] + h_2(t_n) e^{-2i\varphi} \right|^2,$$

$$\eta_{wo}(\theta, \varphi) = \left| g_1(t_n) \left[ \frac{2 \cos^2 \theta + \sin(2\theta) e^{-|\alpha|^2}}{2 \sin^2 \theta + \sin(2\theta) e^{-|\alpha|^2}} \right] + g_2(t_n) e^{-2i\varphi} \right|^2,$$

(28)

where we have set $\alpha = |\alpha| e^{i\varphi}$, and $h_1(t_n), h_2(t_n), g_1(t_n)$ and $g_2(t_n)$ are given by Eq.(14).

From Eqs. (14) and (28), we can find that

$$\eta_{ow}(\theta, \varphi) = \left( \frac{1 + k^2}{1 - k^2} \right)^2 \left[ \frac{2 \sin^2 \theta + \sin(2\theta) e^{-|\alpha|^2}}{2 \cos^2 \theta + \sin(2\theta) e^{-|\alpha|^2}} \right] + \frac{2k^2 e^{-2i\varphi}}{1 + k^2} \right|^2,$$

$$\eta_{wo}(\theta, \varphi) = \left( \frac{1 + k^2}{1 - k^2} \right)^2 \left[ \frac{2 \cos^2 \theta + \sin(2\theta) e^{-|\alpha|^2}}{2 \sin^2 \theta + \sin(2\theta) e^{-|\alpha|^2}} \right] + \frac{2k^2 e^{-2i\varphi}}{1 + k^2} \right|^2,$$

(29)

which indicate that the optical-to-microwave quantum conversion rate depends on the initial-state parameters ($|\alpha|, \varphi$) and the coupling ratio $k$. In Fig. 3, we have plotted the optical-to-microwave quantum conversion rate with respect to the phase of the initial-state parameter when the coupling ratio $k = 0.1, 0.2, 0.6,$ and 0.9, respectively. From Fig. 3, we can see that the optical-to-microwave quantum conversion rate sensitively depends on the phase of the initial-state parameter and the coupling ratios. Hence, we can conclude that the optical-to-microwave quantum conversion rate can be efficiently manipulated by varying the phase of the initial-state parameter and the coupling ratios.

From Eq. (29), we can find that without initial entanglement, i.e., $\cos \theta = 0$ or $\sin \theta = 0$, the previous quantum conversion rates given by Eq. (19) are recovered. When the initial state (20) is an maximally entangled state with $\theta = \pi/4$, from Eq. (29) we can find that
\[ \eta_{ow}(\pi/4, \varphi) = \eta_{wo}(\pi/4, \varphi) = \left| \frac{1 + k^2 + 2ke^{-2i\varphi}}{1 - k^2} \right|, \quad 0 \leq k < 1, \quad (30) \]

which means that the quantum conversion rate from the optical to microwave field equals to that from the microwave to optical field. Once again, we observe the reversible quantum conversion between the optical and microwave fields.

In particular, when \( \varphi = 0 \) from Eq. \((30)\) we have

\[ \eta_{ow}(\pi/4, \varphi = 0) = \eta_{wo}(\pi/4, \varphi = 0) = \left( \frac{1 + k}{1 - k} \right)^2, \quad 0 \leq k < 1, \quad (31) \]

which indicates that the optical–microwave conversion rate can be sensitively enhanced with the increase of the coupling ratio \( k \). Hence, we can observe the reversible entanglement-enhanced quantum conversion between the optical and microwave fields in this situation.

In order to assess the effect of the initial entanglement on the optical–microwave quantum conversion, we introduce a characteristic parameter, the entanglement-affecting factor (EAF), which is defined by the following expression:

\[ R(\varphi) = \frac{\eta_{ow}(0, \varphi)}{\eta_{ow}(\pi/4, \varphi)} = \frac{\eta_{wo}(0, \varphi)}{\eta_{wo}(\pi/4, \varphi)}, \quad (32) \]

which is the ratio of the optical–microwave conversion rate without the initial entanglement with respect to that with maximal initial entanglement. \( R(\varphi) < 1 \) implies that the initial optical–microwave entanglement can enhance the optical–microwave conversion rate, while \( R(\varphi) > 1 \) means that the initial entanglement can suppress the optical–microwave conversion. When \( R(\varphi) = 1 \), the optical–microwave conversion rate with the initial entanglement is equal to that without the initial entanglement, so that the initial entanglement does not affect the optical–microwave conversion rate in this situation.

The optical–microwave conversion rate with the maximal initial entanglement \( \eta_{wo}(\theta = \pi/4, \varphi) \) is given by Eq.\((30)\), while the optical–microwave conversion rate without initial entanglement can be obtained from Eq. \((28)\) by setting \( \theta = 0 \) or \( \pi/2 \) with the following expression:

\[ \eta_{ow}(0, \varphi) = \eta_{wo}(0, \varphi) = \left( \frac{2k}{1 - k^2} \right)^2, \quad (33) \]

which means that the optical–microwave conversion rate without initial entanglement is independent of the phase of the initial-state parameter \( z \).

When the initial-state parameter \( z \) is a real number with \( \varphi = 0 \), from Eqs. \((31)\) and \((33)\) we can find that the EAF is given by:

\[ R(0) = \frac{\eta_{ow}(0, 0)}{\eta_{ow}(\pi/4, 0)} = \left( \frac{\sqrt{2k}}{1 + k} \right)^4 < 1, \quad (34) \]
which indicates that the initial-state entanglement enhances the quantum conversion rate in the whole regime of $0 \leq k < 1$.

When the initial-state parameter $z$ is a pure imaginary number with $\varphi = \pi/2$, from Eq. (30) we obtain

$$\eta_{ow}(\pi/4, \pi/2) = \left( \frac{1-k}{1+k} \right)^2.$$  \hspace{1cm} (35)

Then, we can get the following expression of the EAF

$$R(\pi/2) = \frac{\eta_{ow}(0, \pi/2)}{\eta_{ow}(\pi/4, \pi/2)} = \left( \frac{\sqrt{2k}}{1-k} \right)^4.$$  \hspace{1cm} (36)

In Fig. 4, we have plotted the EAF $R(\varphi)$ with respect to the coupling ratio $k$. The solid and dashed lines correspond to the two cases with and without the initial entanglement ($\varphi = 0, \pi/2$), respectively. From Fig. 4, we can find that the initial entanglement in the case of $\varphi = \pi/2$ can exhibit a different influence on the optical–microwave conversion in different regimes of the coupling ratio $k$. From Eq. (36), it is easy to see that there exists a critic point of the coupling ratio $k_c$. The initial entanglement can enhance the optical–microwave conversion in the regime of $k < k_c$ with $R(\varphi) < 1$, while the initial entanglement can suppress the optical–microwave conversion in the regime of $k > k_c$ with $R(\varphi) > 1$. However, the initial entanglement does not affect the optical–microwave conversion rate at the critic point $k = k_c$. Making use of Eq. (36), we can find the critic point value $k_c = 2 - \sqrt{3}$.

Above analyses indicate that the quantum conversion between the optical wave and microwave can be efficiently manipulated by controlling the initial-state parameters. In particular, we find that the entanglement-assisted quantum conversion is a kind of the phase-sensitive quantum conversion under certain conditions.

Finally, it should be mentioned that the present DDM scheme requires the accurate time control. This is because the quantum conversion between the microwave and optical fields in the DDM scheme happens only at times that the dark mode occurs, while the quantum conversion is time-independent in the conventional steady-state quantum conversion schemes. Below we discuss the experimental feasibility to realize the accurate time control in the DDM scheme. From Eq. (12), we can see that the DDM appears periodically, and the dark-mode evolution period is given by:
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\[ T = \frac{\pi}{\sqrt{G_w^2 - G_o^2}} = \frac{\pi}{\sqrt{g_w^2 N_w - g_o^2 N_o}}, \quad (37) \]

where \( N_w / N_o \) can be expressed as:

\[ \frac{N_w}{N_o} = \frac{P_w/\hbar \omega_w}{P_o/\hbar \omega_o} = \frac{\omega_o}{\eta \omega_w}, \quad (38) \]

where \( P_w(o) \) is the microwave (optical) power, and \( \eta \) is the power conversion rate from the optical to microwave photons, and \( \omega_w(o) \) is the microwave (optical) photon frequency.

Then, the dark-mode evolution period given by Eq. (37) becomes:

\[ T = \frac{\pi}{\sqrt{N_o}} \left( \frac{\omega_o g_w^2}{\omega_w \eta} - g_o^2 \right), \quad (39) \]

which indicates that the dark-mode evolution period can be manipulated through changing the optical and microwave parameters, the optomechanical and electromagnetic coupling rates \( g_o \) and \( g_w \).

Taking the experimental parameters in Ref. [71], \( g_w/2\pi = 0.327 \) Hz, \( g_o/2\pi = 115.512 \) Hz, \( \omega_w/2\pi = 10 \) GHz, and the laser wavelength at 1064nm. Assuming \( \eta = 0.2 \), in a typical measurement with \( N_o < 10^{10} / cm^2 \), we can estimate the dark-mode evolution period \( T = 115 \) ns. This implies that the dark mode happens no sooner than every 115 ns, which is reachable by a typical quantum-optical sensing technique. Therefore, it is experimentally feasible to realize the accurate time control in the DDM scheme.

5 Concluding remarks

We have proposed the DDM scheme to realize reversible and highly efficient quantum conversion between the microwave and optical fields in terms of the EOM quantum conversion model. The DDM scheme is the transient-state quantum conversion scheme. Comparing with the conventional steady-state schemes, the DDM scheme can exhibit advantages on quantum manipulation. For instance, the quantum conversion in the DDM scheme can be controlled by changing the initial-state parameters of the microwave and optical fields. In particular, it has been indicated that the entanglement-assisted quantum conversion can occur in the DDM scheme. However, the conventional steady-state schemes describe the quantum conversion during a long-time evolution, they are independent of initial states of the microwave and optical fields, and thus one cannot realize the initial-state control of the quantum conversion between the microwave and optical fields.

We have obtained an analytical solution of the EOM model by means of the Senm–Mandal approach. It has been demonstrated that two optomechanical DDMs appear at some specific moments during the dynamical evolution of the EOM model. It has been found that the DDMs can induce two kinds of bidirectional and highly efficient
quantum conversion. The first one is the microwave–optical CQC which happens at the condition of vanishing of the initial-state mean value of one of the microwave and optical fields. The CQC is reversible since the bidirectional quantum conversion has the same conversion rate in this case. In some sense, the DDM-assisted reversible quantum conversion is universal because the bidirectional conversion rate is independent of the initial state of the converted field; it only depends on the coupling ratio between the microwave and optical fields and the mechanical resonator. The second one is the EAQC, which occurs in the presence of the initial-state entanglement between the microwave and optical fields. We have demonstrated that the EAQC between microwave and optical fields can be manipulated by varying the initial-state entanglement and the coupling ratio of related interaction strengths. Especially, it has been indicated the microwave–optical EAQC is the phase-sensitive conversion. It has been indicated that it is possible to realize the entanglement-enhanced or entanglement-suppressed quantum conversion through controlling the phase of the initial-state parameter. It should be mentioned that the DDM scheme is robust against the mechanical noise since the DDMs can protect the system from mechanical dissipation through the two DDMs decoupling with the mechanical mode of the system under consideration. It should be noted that the reversible and highly efficient quantum conversion is a DDM effect essentially. It could be expected that the DDMs may lead to more novel quantum phenomena, which deserve to be further explored. The DDM-assisted quantum conversion scheme proposed in the present paper provides a versatile route to manipulate the microwave–optical quantum conversion with the DDMs. The ability to coherently convert information between microwave and optical fields opens new possibilities for quantum information, in particular, quantum-coherent connection between microwave and optical photons mediated by the mechanical resonator.

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**References**

1. Zeuthen, E., Schliesser, A., Sørensen, A.S., Taylor, J.M.: Figures of merit for quantum transducers. Quantum Sci. Technol. 5, 034009 (2020)
2. Lambert, N.J., Rueda, A., Sedlmeir, F., Schwefel, H.G.L.: Coherent conversion between microwave and optical photons-An overview of physical implementations. Adv. Quantum Technol. 3, 1900077 (2019)
3. Lauk, N., Sinclair, N., Barzanjeh, S., Covey, J.P., Saffman, M., Spiropulu, M., Simon, C.: Perspectives on quantum transduction. Quantum Sci. Technol. 5, 020501 (2020)
4. Kimble, H.J.: The quantum internet. Nature 453, 1023 (2008)
5. Castelvecchi, D.: The quantum internet has arrived (and it hasn’t). Nature 554, 289 (2018)
6. Reiserer, A., Rempe, G.: Cavity-based quantum networks with single atoms and optical photons. Rev. Mod. Phys. 87, 1379 (2015)
7. Wehner, S., Elkouss, D., Hanson, R.: Quantum internet: Avision for the road ahead. Science 362, eaam9288 (2018)
8. Dong, C., Wang, Y., Wang, H.: Optomechanical interfaces for hybrid quantum networks. Natl. Sci. Rev. 2, 510 (2015)
9. Pirandola, S., Bardhan, B.R., Gehringer, T., Weedbrook, C., Lloyd, S.: Advances in photonic quantum sensing. Nat. Photonics 12, 724 (2018)
10. Maccone, L., Ren, C.L.: Quantum radar. Phys. Rev. Lett. 124, 200503 (2020)
11. Sørensen, A.S., van der Wal, C.H., Childress, L.I., Lukin, M.D.: Capacitive coupling of atomic systems to mesoscopic conductors. Phys. Rev. Lett. 92, 063601 (2004)
12. Tian, L., Rabl, P., Blatt, R., Zoller, P.: Interfacing quantum-optical and solid-state qubits. Phys. Rev. Lett. 92, 247902 (2004)
13. Rabl, P., DeMille, D., Doyle, J.M., Lukin, M.D., Schoelkopf, R.J., Zoller, P.: Hybrid quantum processors: molecular ensembles as quantum memory for solid state circuits. Phys. Rev. Lett. 97, 033003 (2006)
14. O’Brien, C., Lauk, N., Blum, S., Morigi, G., Fleischhauer, M.: Interfacing superconducting qubits and telecom photons via a rare-earth-doped crystal. Phys. Rev. Lett. 113, 063603 (2014)
15. Xia, K., Twamley, J.: Solid-state optical interconnect between distant superconducting quantum chips. Phys. Rev. A 91, 042307 (2015)
16. Das, S., Elfving, V.E., Faez, S., Sørensen, A.S.: Interfacing superconducting qubits and single optical photons using molecules in waveguides. Phys. Rev. Lett. 118, 140501 (2017)
17. Gard, B.T., Jacobs, K., McDermot, R., Saffman, M.: Microwave-to-optical frequency conversion using a cesium atom coupled to a superconducting resonator. Phys. Rev. A 96, 013833 (2017)
18. Lekavicius, I., Golter, D.A., Oo, T., Wang, H.: Transfer of phase information between microwave and optical fields via an electron spin. Phys. Rev. Lett. 119, 063601 (2017)
19. Hisatomi, R., Osada, A., Tabuchi, Y., Ishikawa, T., Noguchi, A., Usami, K., Nakamura, Y.: Bidirectional conversion between microwave and light via ferromagnetic magnons. Phys. Rev. B 93, 174427 (2016)
20. Tsang, M.: Cavity quantum electro-optics. Phys. Rev. A 81, 063837 (2010)
21. Tsang, M.: Cavity quantum electro-optics. II. Input-output relations between traveling optical and microwave fields. Phys. Rev. A 84, 043845 (2011)
22. Javerzac-Galy, C., Plekhanov, K., Bernier, N.R., Toth, L.D., Feofanov, A.K., Kippenberg, T.J.: On-chip microwave-to-optical quantum coherent converter based on a superconducting resonator coupled to an electro-optic microresonator. Phys. Rev. A 94, 053815 (2016)
23. Rueda, A., Sedlmeir, F., Collodo, M.C., Vogl, U., Stiller, B., Schunk, G., Strekalov, D.V., Marquardt, C., Fink, J.M., Painter, O., Leuchs, G., Schwefel, H.G.L.: Efficient microwave to optical photon conversion: an electro-optical realization. Optica 3, 597 (2016)
24. Stannigel, K., Rabl, P., Sørensen, A.S., Zoller, P., Lukin, M.D.: Optomechanical transducers for long-distance quantum communication. Phys. Rev. Lett. 105, 220501 (2010)
25. Taylor, J.M., Sørensen, A.S., Marcus, C.M., Polzik, E.S.: Laser cooling and optical detection of excitations in a LC electrical circuit. Phys. Rev. Lett. 107, 273601 (2011)
26. Barzanjeh, S., Abdi, M., Milburn, G.J., Tombesi, P., Vitali, D.: Reversible optical-to-microwave quantum interface. Phys. Rev. Lett. 109, 130503 (2012)
27. Tian, L.: Adiabatic state conversion and pulse transmission in optomechanical systems. Phys. Rev. Lett. 108, 153604 (2012)
28. Wang, Y.-D., Clerk, A.A.: Using interference for high fidelity quantum state transfer in optomechanics. Phys. Rev. Lett. 108, 153603 (2012)
29. Clader, B.D.: Quantum networking of microwave photons using optical fibers. Phys. Rev. A 90, 012324 (2014)
30. Yin, Z.-Q., Yang, W.L., Sun, L., Duan, L.M.: Quantum network of superconducting qubits through an optomechanical interface. Phys. Rev. A 91, 012333 (2015)
31. Černotík, O., Hammerer, K.: Measurement-induced long-distance entanglement of superconducting qubits using optomechanical transducers. Phys. Rev. A 94, 012340 (2016)
32. Okada, A., Oguro, F., Noguchi, A., Tabuchi, Y., Yamazaki, R., Usami, K., Nakamura, Y.: Cavity enhancement of anti-Stokes scattering via optomechanical coupling with surface acoustic Waves. Phys. Rev. Appl. 10, 024002 (2018)
33. Bochmann, J., Vainsencher, A., Awschalom, D.D., Cleland, A.N.: Nanomechanical coupling between microwave and optical photons. Nat. Phys. 9, 712 (2013)
34. Andrews, R.W., Peterson, R.W., Purdy, T.P., Cicak, K., Simmonds, R.W., Regal, C.A., Lehnert, K.W.: Bidirectional and efficient conversion between microwave and optical light. Nat. Phys. 10, 321 (2014)
35. Bageci, T., Simonsen, A., Schmid, S., Villanueva, L.G., Zeuthen, E., Appel, J., Taylor, J.M., Sørensen, A.S., Usami, K., Schlissker, A., Polzik, E.S.: Optical detection of radio waves through a nanomechanical transducer. Nature 507, 81 (2014)
36. Balram, K.C., Davanço, M.I., Song, J.D., Srinivasan, K.: Coherent coupling between radiofrequency optical and acoustic waves in piezo-optomechanical circuits. Nat. Photonics 10, 346 (2016)
37. Bowen, W.P., Milburn, G.J.: Quantum Optomechanics. CRC Press, Taylor and Francis Group, Boca Raton (2016)
38. Aspelmeyer, M., Kippenberg, T.J., Marquardt, F.: Cavity optomechanics. Rev. Mod. Phys. 86, 1391 (2014)
39. Kippenberg, T.J., Vahala, K.J.: Cavity optomechanics: back-action at the mesoscale. Science 321, 1172 (2008)
40. Jiao, Y.F., Zhang, S.D., Zhang, Y.L., Miranowicz, A., Kuang, L.M., Jing, H.: Nonreciprocal optomechanical entanglement against backscattering losses. Phys. Rev. Lett. 125, 143605 (2020)
41. Tan, Q.S., Yuan, J.B., Liao, J.Q., Kuang, L.M.: Supersensitive estimation of the coupling rate in cavity optomechanics with an impurity-doped Bose-Einstein condensate. Opt. Express 28, 22867 (2020)
42. Zhai, C.L., Huang, R., Jing, H., Kuang, L.M.: Mechanical switch of photon blockade and photon-induced tunneling. Opt. Express 28, 22867 (2020)
43. Gröblacher, S., Hammerer, K., Vanner, M.R., Aspelmeyer, M.: Observation of strong coupling between a micromechanical resonator and an optical cavity field. Nature (London) 460, 724 (2009)
44. Weis, S., Riviere, R., Deleglise, S., Gavartin, E., Schliesser, A., Kippenberg, T.J.: Optomechanically induced transparency. Science 330, 1520 (2010)
45. Safavi-Naeini, A.H., Alegre, T.P.M., Chan, J., Eichenfield, M., Winger, M., Lin, Q., Hill, J.T., Chang, D.E., Painter, O.: Electromagnetically induced transparency and slow light with optomechanics. Nature (London) 472, 69 (2011)
46. Chan, J., Alegre, T.P.M., Safavi-Naeini, A.H., Hill, J.T., Krause, A., Gröblacher, S., Aspelmeyer, M., Painter, O.: Laser cooling of a nanomechanical oscillator into its quantum ground state. Nature (London) 478, 89 (2011)
47. B rahms, N., Botter, T., Schreppler, S., Brooks, D.W.C., Stamper-Kurn, D.M.: Optical detection of the quantization of collective atomic motion. Phys. Rev. Lett. 108, 133601 (2012)
48. Verhagen, E., Deleglise, S., Weis, S., Schliesser, A., Kippenberg, T.J.: Quantum-coherent coupling of a mechanical oscillator to an optical cavity mode. Nature (London) 482, 63 (2012)
49. Massel, F., Cho, S.U., Pirkkalainen, J.-M., Hakonen, P.J., Heikkilä, T.T., Sillanpää, M.A.: Multimode circuit optomechanics near the quantum limit. Nat. Commun. 3, 987 (2012)
50. Agarwal, G.S., Huang, S.: Electromagnetically induced transparency in mechanical effects of light. Phys. Rev. A 81, 041803(R) (2010)
51. Zhou, X., Hocke, F., Schliesser, A., Marx, A., Huebl, H., Gross, R., Kippenberg, T.J.: Slowing, advancing and switching of microwave signals using circuit nanoelectromechanics. Nat. Phys. 9, 179 (2013)
52. O’Connell, A.D., Hofheinz, M., Ansmann, M., Bialczak, R.C., Lenander, M., Lucero, E., Neeley, M., Sank, D., Wang, H., Weides, M., Wenner, J., Martinis, J.M., Cleland, A.N.: Quantum ground state and single-phonon control of a mechanical resonator. Nature (London) 464, 697 (2010)
53. Teufel, J.D., Li, D., Allman, M.S., Cicak, K., Sirois, A.J., Whittaker, J.D., Simmonds, R.W.: Circuit cavity electromechanics in the strong-coupling regime. Nature (London) 471, 204 (2011)
54. Riviere, R., Deleglise, S., Weis, S., Gavartin, E., Arcizet, O., Schliesser, A., Kippenberg, T.J.: Optomechanical sideband cooling of a micromechanical oscillator close to the quantum ground state. Phys. Rev. A 83, 063835 (2011)
55. Thompson, J.D., Zwickl, B.M., Jayich, A.M., Marquardt, F., Girvin, S.M., Harris, J.G.E.: Strong dispersive coupling of a high-finesse cavity to a micromechanical membrane. Nature (London) 452, 72 (2008)
56. Cirac, J.I., Zoller, P., Kimble, H.J., Mabuchi, H.: Quantum state transfer and entanglement distribution among distant nodes in a quantum network. Phys. Rev. Lett. 78, 3221 (1997)
57. Dong, C., Fiore, V., Kuzyk, M.C., Wang, H.: Optomechanical dark mode. Science 338, 1609 (2012)
58. Wang, Y.-D., Clerk, A.A.: Using interference for high fidelity quantum state transfer in optomechanics. Phys. Rev. Lett. 108, 153603 (2012)
59. Zhang, X., Zou, C.-L., Zhu, N., Marquardt, F., Jiang, L., Tang, H.X.: Magnon dark modes and gradient memory. Nat. Commun. 6, 8914 (2015)
60. Wang, Y.-D., Clerk, A.A.: Reservoir-engineered entanglement in optomechanical systems. Phys. Rev. Lett. 110, 235301 (2013)
61. Tian, L.: Robust photon entanglement via quantum interference in optomechanical interfaces. Phys. Rev. Lett. 110, 233602 (2013)
62. Lai, D.G., Wang, X., Qin, W., Hou, B.P., Nori, F., Liao, J.Q.: Tunable optomechanically induced transparency by controlling the dark-mode effect. Phys. Rev. A 102, 023707 (2020)
63. Lake, D.P., Mitchell, M., Sanders, B.C., Barclay, P.E.: Two-colour interferometry and switching through optomechanical dark mode excitation. Nat. Commun. 11, 2208 (2020)
64. Kuzyk, M.C., Wang, H.: Controlling multimode optomechanical interactions via interference. Phys. Rev. A 96, 023860 (2017)
65. Sommer, C., Genes, C.: Partial optomechanical refrigeration via multimode cold-damping feedback. Phys. Rev. Lett. 123, 203605 (2019)
66. Ockeloen-Korppi, C.F., Gely, M.F., Damskägg, E., Jenkins, M., Steele, G.A., Sillanpää, M.A.: Sideband cooling of nearly degenerate micromechanical oscillators in a multi-mode optomechanical system. Phys. Rev. A 99, 023826 (2019)
67. Sen, B., Mandal, S.: Squeezed states in spontaneous Raman and in stimulated Raman processes. J. Mod. Opt. 52, 1789 (2005)
68. Sen, B., Giri, S.K., Mandal, S., Raymond Ooi, C.H., Pathak, A.: Intermodal entanglement in Raman processes. Phys. Rev. A 87, 022325 (2013)
69. Giri, S.K., Sen, B., Pathak, A., Jana, P.C.: Higher-order two-mode and multimode entanglement in Raman processes. Phys. Rev. A 93, 012340 (2016)
70. Barzanjeh, S., Guha, S., Weedbrook, C., Vitali, D., Shapiro, J.H., Pirandola, S.: Microwave quantum illumination. Phys. Rev. Lett. 114, 080503 (2015)
71. Barzanjeh, S., Vitali, D., Tombesi, P., Milburn, G.J.: Entangling optical and microwave cavity modes by means of a nanomechanical resonator. Phys. Rev. A 84, 042342 (2011)
72. Hill, J.T., Safavi-Naeini, A.H., Chan, J., Painter, O.: Coherent optical wavelength conversion via cavity optomechanics. Nat. Commun. 3, 1196 (2012)
73. Jiang, W., Sarabalis, C.J., Dahmani, Y.D., Patel, R.N., Mayor, F.M., McKenna, P.T., Laer, R.V., Safavi-Naeini, A.H.: Efficient bidirectional piezo-optomechanical transduction between microwave and optical frequency. Nat. Commun. 11, 1166 (2020)
74. Safavi-Naeini, A.H., Painter, O.: Proposal for an optomechanical traveling wave phonon-photon translator. New J. Phys. 13, 013017 (2011)
75. Wang, Y.D., Clerk, A.A.: Using interference for high fidelity quantum state transfer in optomechanics. Phys. Rev. Lett. 108, 153603 (2012)
76. Kuang, L.M., Zhou, L.: Generation of atom-photon entangled states in atomic Bose–Einstein condensate via electromagnetically induced transparency. Phys. Rev. A 68, 043606 (2003)
77. Wang, X.G.: Bipartite entangled non-orthogonal states. J. Phys. A: Math. Gen. 35, 165 (2002)
78. Hill, S., Wootters, W.K.: Entanglement of a pair of quantum bits. Phys. Rev. Lett. 78, 5022 (1997)
79. Wootters, W.K.: Entanglement of formation of an arbitrary state of two qubits. Phys. Rev. Lett. 80, 2245 (1998)

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