A geometrical estimation of saturation of partonic densities

N. Armesto$^a$ and C. A. Salgado$^b$

$^a$ Departamento de Física, Módulo C2, Planta baja, Campus de Rabanales, Universidad de Córdoba, E-14071 Córdoba, Spain
$^b$ Laboratoire de Physique Théorique, Université de Paris XI, Bâtiment 210, F-91405 Orsay Cedex, France

We propose a new criterium for saturation of the density of partons both in nucleons and nuclei. It is applicable to any multiple scattering model which would be used to compute the number of strings exchanged in $ep$ and $eA$ collisions. The criterium is based on percolation of strings, and the onset of percolation is estimated from expectations coming from the study of heavy ion collisions at high energies. We interpret this onset as an indication of saturation of the density of partons in the wave function of the hadron. In order to produce quantitative results, a particular model fitted to describe present HERA data and generalized to the nuclear case is used. Nevertheless, with the number of scatterings controlled by the relation between inclusive and diffractive processes, conclusions are weakly model-dependent as long as different models are tuned to describe the experimental data. This constitutes a new approach, based on the eikonal description of soft hadronic collisions, and different from others which employ either perturbative QCD ideas or semiclassical methods. It offers an alternative picture for saturation in the small $Q^2$ region.

PACS: 24.85.+p, 11.80.La, 13.60.Hb, 12.40.Nn.

Keywords: saturation; partonic densities; multiple scattering; percolation.
Much interest has recently been devoted to the saturation of partonic densities [1], i.e. the change in the increase of the partonic densities from power-like to logarithmic or constant with decreasing parton momentum fraction $x$, both in nucleons [2, 3, 4] and in nuclei [5, 6, 7]. From the point of view of experimental data on lepton-hadron scattering, the most striking feature was the change in the logarithmic slope of the proton structure function $dF_2/d \ln (Q^2)$ at $x \sim 10^{-4}$, the so-called Caldwell plot [8] (now known to be mainly due to a $Q^2 - x$ correlation), but the situation is not conclusive: Nucleon data can be described not only in approaches which consider saturation [2, 3, 4], but also satisfactorily accommodated in the usual global fits [9] (also available for nuclei [10]) that consider the standard QCD evolution or resummation [11], starting from initial conditions at low photon virtualities $Q^2$ which do not include saturation (see [12] for an application to the Caldwell plot and [13] for a discussion on the present situation).

From a theoretical point of view, the saturation regime is a very interesting one characterized by a small coupling constant and high occupation numbers, where a semiclassical description in terms of fields has been proposed [7]. Different models offer explanations based on multiple scattering (i.e. unitarization) or gluon interaction, both in the case of nucleons [1, 2, 3, 4] and nuclei [1, 5, 6, 7]. These two approximations to the problem are equivalent (see e.g. [14]) in different reference frames, but the models predict the onset of saturation in different kinematical regions and the saturation features are also diverse. In this short note we will essay another approach to the problem, inherited from multiparticle production in nucleus-nucleus ($AB$) collisions at high energies and applicable to any model formulated in terms of multiple scatterings.

The concept of saturation, not of the density of partons in the hadronic wave function but of the number of partons produced in the collision, was proposed some time ago [15] in $AB$ collisions at high energies and has been reconsidered recently in the context of the search of the Quark Gluon Plasma (QGP); such high partonic density should provide the initial condition for the possible thermalization of the created system. Several related ideas have been used to compute the multiplicity of produced particles in $AB$ collisions at the Relativistic Heavy Ion Collider (RHIC) at BNL and at the future Large Hadron Collider (LHC) at CERN [16, 17]. For example, in [18] perturbative QCD (pQCD) is used to compute the initial number of gluons, quarks and antiquarks, which are limited according to the simple geometrical criterium that the number of partons per unit of transverse space times their transverse dimension...
($\propto 1/p_1^2$) cannot be greater than 1 (see [19] for other attempts in this direction). Besides, the semiclassical methods used in [7] have also been employed to estimate the initial number of gluons in a heavy ion collision [20].

On the other hand and in the framework of string models for soft multiparticle production (see [21] and references therein), a simple geometrical criterion for saturation has been proposed. In these models particle production comes from string breaking, strings which are considered, in a first approximation, as formed and decayed independently. As the number of binary nucleon-nucleon collisions (each one producing 2 strings [21, 22]) increases with increasing centrality, energy or nuclear mass, this approximation should break down. The onset of this phenomenon can be estimated considering strings with a certain area in the transverse space of the collision, and taking into account the possibility of two-dimensional percolation of the strings when they overlap in this transverse space. Percolation is a second order phase transition which takes place when clusters of overlapping strings, with a size of the order of the total transverse area available, appear. This idea has been proposed in $AB$ collisions [23] and applied to signatures of QGP [24].

The purpose of this note is to use percolation of strings as an indication for the onset of saturation of the density of partons in nucleons and nuclei, a quantity which in our case is not directly related with the partonic densities measured in DIS, as such identification [3, 14] can only be done at high $Q^2$, and our approach will be devoted to the low $Q^2$ regime. For this, we need a multiple exchange model for $ep$ collisions which allows us to compute the number of binary collisions, generalize it to the nuclear case, translate the number of collisions to a number of strings and estimate the density of strings to compute whether percolation takes place or not. The method can be applied to any multiple scattering model, and the results in any of these models should be quite the same (within the uncertainties due to the extrapolation of the model to nuclei and to higher energies or smaller $x$) as long as the model is able to describe the fully inclusive and diffractive experimental data on $ep$ collisions, see comments below.

Let us give a brief description of the model developed in [4], which is the particular one we are going to use to compute the number of binary collisions and then of strings in order to give quantitative predictions. The goal of the model was the description of total and diffractive data on $ep$ scattering at low and moderate $Q^2$ and small $x$. This region is where unitarity corrections are more important and where the transition from non-perturbative to perturbative QCD takes place.
In the proton rest frame, the virtual photon coming from the lepton fluctuates into a $q\bar{q}$ state. Then this hadronic state interacts with the proton. Unitarity corrections are described by the multiple scattering of the $q\bar{q}$ fluctuation with the proton. In a quasieikonal approach the total cross section is given by

$$
\sigma_{\gamma^p}(s, Q^2) = 4g(Q^2) \int d^2 b \left(1 - \frac{\exp\{-C\chi(s, Q^2, b)\}}{2C}\right),
$$

(1)

where $g(Q^2)$ is the $\gamma^* - (q\bar{q})$ coupling, $2\chi(s, Q^2, b)$ is the elementary $(q\bar{q}) - p$ cross section at fixed impact parameter $b$, and $C = 1.5$ is a parameter taking into account the diffractive dissociation of the proton. For small sizes $r$ of the $q\bar{q}$ pair, $\chi \propto r^2$ from pQCD calculations. As $r^2 \propto 1/Q^2$, for these small sizes $\chi \propto 1/Q^2$. For large sizes of the fluctuation no $Q^2$-dependence is expected. In [4] two components, corresponding to small ($S$) and large ($L$) sizes of the $q\bar{q}$ pair, were taken into account, $\chi = \chi_L + \chi_S$. The fact that $\chi_S \propto 1/Q^2$ while $\chi_L$ does not depend on $Q^2$ makes the correction terms in Eq. (1) more important for the $L$ part than for the $S$ one. So, more scatterings are present – in average – for the $L$ than for the $S$ component; for this reason and also due to the fact that we will consider small $Q^2$, only the $L$ component, which is the dominant one [4] for $Q^2 \lesssim 2$ (GeV/c)$^2$, will be used in the actual computations. The energy dependence of these $\chi$'s is given by a single pomeron of intercept $\Delta = 0.2$,

$$
\chi_L = \frac{C_L}{\lambda_L} \exp \left\{ \Delta\xi - \frac{b^2}{4\lambda_L} \right\},
$$

(2)

Here, $\xi = \ln \frac{s+Q^2}{s_0+Q^2}$, $\lambda_L = R_L^2 + \alpha'_P\xi$ with $R_L^2 = 3$ GeV$^{-2}$, and $\alpha'_P = 0.25$ GeV$^{-2}$, the slope of the pomeron trajectory, gives the ln $s$ behavior of the total cross section for very large $s$; besides, $C_L = 0.56$ GeV$^{-2}$ and $s_0 = 0.79$ GeV$^2$. The variable $\xi$ is chosen so that $\chi \propto x^{-\Delta}$ for large $Q^2 \gg s_0$ and $\chi \propto (s/s_0)^{\Delta}$ for $Q^2 \to 0$; in this way, the model can be used for photoproduction. For the $S$ part, similar expressions were used in [4] with an extra $r^2$ factor in Eq. (1).

The description of diffraction is a very important ingredient of the model. It is given by quadratic and higher order terms in $\chi$ in the expansion of Eq. (1). Thus, the ratio $\sigma_{\text{diff}}/\sigma_{\text{tot}}$ controls the unitarity (multiple scattering) corrections, i.e. the number of scatterings and strings; this idea has been used to compute nuclear structure functions from a description of diffraction at HERA, see e.g. [25]. So, any multiple scattering model able to reproduce the experimental data on this ratio should produce roughly the same number of scatterings (strings) and, consequently, the same predictions for
the onset of percolation and saturation. A triple pomeron term was introduced in \cite{4} in order to reproduce large mass diffractive processes. This term is another source of shadowing corrections to the total cross section. It will be used in the actual computations, see \cite{4} for the full expressions and parameters. The reggeon contribution which appears in \cite{4} decreases with increasing energy and is negligible at the energies under consideration, so it has been ignored.

The model in \cite{4} has 9 free parameters that were fitted to experimental data on diffractive and total \(ep\) cross sections for \(0 \leq Q^2 < 10 (\text{GeV}/c)^2\) and \(10^{-6} < x < 10^{-2}\). Once the parameters of the model are fitted, it is possible to know the mean number of collisions \cite{22}:

\[
\bar{n} = \sum_{n=1}^{\infty} n \frac{\int d^2b \sigma_n(s, Q^2, b)}{\sum_{n=1}^{\infty} \int d^2b' \sigma_n(s, Q^2, b')} \frac{\int d^2b 2C\chi(s, Q^2, b)}{\int d^2b'[1 - \exp \{-2C\chi(s, Q^2, b')\}],}
\]

where, for \(n \geq 1\),

\[
\sigma_n(s, Q^2, b) = \frac{g(Q^2) [2C\chi(s, Q^2, b)]^n}{n!} \exp(-2C\chi).
\]

Notice that in these expressions, \(\chi(s, Q^2, b)\) contains the triple pomeron contribution, so cuts in different branchings of one single fan diagram (i.e. one single tree of triple pomeron couplings) are included in the same \(\sigma_n\), which thus corresponds to the exchange of \(n\) fan diagrams, each of them cut in one or more than one of its branches. Thus Eq. (\ref{eq:3}) is a conservative estimation, as these cuts could give rise to a larger number of strings. Besides, all our expressions are asymptotic ones, not considering energy-momentum conservation (which could reduce slightly the number of collisions at the lowest energies).

Neglecting isospin at high energies, the generalization of any multiple scattering model formulated for \(ep\) to the case of \(eA\) collisions is straightforward in the Glauber-Gribov approach \cite{26}: The number of \(q\bar{q}-\text{nucleon}\) collisions (the number of participating nucleons of \(A\)) in this case, is given \cite{27} in terms of the inelastic non-diffractive cross sections by

\[
\langle n_{\text{part}} \rangle = A \sigma_{in}^{\gamma^*p}/\sigma_{in}^{A} \propto A^{1/3},
\]

with

\[
\sigma_{in}^{A} = g(Q^2) \int d^2b \left(1 - \exp \left\{-AT_A(b)\sigma_{in}^{\gamma^*p}/g(Q^2)\right\}\right),
\]
\[ T_A(b) = \int_{-\infty}^{\infty} dz \rho_A(z, \bar{b}) \] the profile function normalized to 1 taken from \[ 28 \] and

\[
\sigma_{in}^{\gamma p} = \sum_{n=1}^{\infty} \int d^2 \bar{b} \sigma_n(s, Q^2, \bar{b}) = \frac{g(Q^2)}{C} \int d^2 \bar{b} \left[ 1 - \exp \left\{ -2C\chi(s, Q^2, \bar{b}) \right\} \right].
\]

So, the total number of collisions is given by \( \langle n_{coll} \rangle = \langle n_{part} \rangle \bar{n}. \) As previously commented, in the actual computations we will only use the \( L \) component, Eq. (2), as it is the dominant one \[ 4 \] for \( Q^2 \lesssim 2 \text{ (GeV/c)}^2 \) where our calculations will be done.

At this point, it could be argued that using the model in \[ 4 \] (or any other multiple scattering model) there is a possibility to study saturation of the density of partons, both in nucleons and in nuclei, simply looking at the point in which amplitudes in impact parameter space become energy independent, or alternatively the point in which cross sections reach a regime in which their energy behavior becomes identical to that of the size of the target (expanding in the case of a nucleon). Nevertheless, the generalization of \[ 4 \] to nuclei is not so obvious: ours is a very simple one, but more rigorous generalizations \[ 28 \] also rely in simplifications of the exact Gribov calculus \[ 30 \] or Glauber-Gribov theory \[ 26 \]. So we think that an estimation as the one we perform, based on geometrical criteria, is worthy, of simple and general applicability, and may provide, as in the case of nucleus-nucleus collisions, an indication of the onset of a high density, non-linear regime.

Let us establish now our criterium for saturation of the density of partons in the wave function of the target. As it was said, percolation is a non-thermal second order phase transition, which takes place when clusters of overlapping objects acquire a size comparable to the total size available \[ 31 \]. In our case the space is the transverse dimension available for the collision, and the overlapping objects are strings. The parameter which controls the onset of percolation is the dimensionless string density

\[ \eta = \frac{N t}{T} \]

(which may be related \[ 17 \] with the dimensionless density of gluons found in semiclassical models), with

\[ N = 2 \langle n_{coll} \rangle \]

the number of strings exchanged in the collision (each collision gives rise to two strings due to the pomeron dominance at high energies, see \[ 21, 22 \]), \( t = \pi r_0^2 \) is the transverse
dimension of the string with \( r_0 \simeq 0.20 \div 0.25 \text{ fm} \) as extracted from phenomenology \([23],[24]\), and \( T \) the total transverse area available for the collision. This last quantity is not known and could depend on the virtuality of the fluctuation \( Q^2 \); however, for small and moderate \( Q^2 \), it can be estimated to be the typical size of the vector meson in which the virtual photon fluctuates (as this is the smaller object in the interaction). So, we will use \( T = 1 \text{ fm}^2 \) (a radius \( \sqrt{T/\pi} \simeq 0.56 \text{ fm} \)). Also, a size varying with the energy, in the spirit of an expanding proton, could be explored; for example, a size increasing with increasing energy would slow the corresponding increase of the density of strings but, for simplification, we will employ a fixed size.

The critical value for \( \eta \) where percolation takes place, has been computed using different methods and depends quite strongly on the profile of the nucleus (i.e. on the distribution of the overlapping objects inside the available transverse space). For continuum two-dimensional percolation and from \([31],[32]\), we take \( \eta_c \simeq 1.12 \div 1.50 \). Defining the string density as \( n = N/T \) and allowing for the different values of \( r_0 \) and \( \eta_c \), we find a critical string density

\[
n_c \simeq 6 \div 12 \text{ strings/fm}^2.
\]  

With this critical string density, it is tempting to estimate the behavior of the \( Q^2 \) at which, for a fixed \( s \), saturation of the density of partons takes place in this approach, \( Q^2_{\text{sat}} \) \([1],[2],[3],[5],[6],[7],[14]\). However, in our case, being \( \chi_L \) almost independent on \( Q^2 \), the density of strings is also almost \( Q^2 \)-independent for fixed \( s \); besides, the model is only valid for small \( Q^2 \) and has not been designed for \( Q^2 \)-evolution. More significant in our model is the value of \( x \) or \( s \) where, for fixed \( Q^2 \) and \( A \), saturation takes place, \( x_{\text{sat}} \) or \( s_{\text{sat}} \) respectively. Considering neither triple pomeron nor reggeon contributions and approximating in Eq. (3) \( \sigma_{\text{in}}^{tr} \simeq g(Q^2) \pi R_0^2 A^{2/3} \), with \( R_0 \simeq 1.2 \text{ fm} \), it is found that \( x_{\text{sat}} \propto [Q^2/(s_0+Q^2)] A^{1/(3\Delta)} \) and \( s_{\text{sat}} \propto (s_0+Q^2) A^{-1/(3\Delta)} \), \( 1/(3\Delta) = 5/3 \). So, for \( Q^2 \to 0 \), \( x_{\text{sat}} \) increases linearly with increasing \( Q^2 \) while \( s_{\text{sat}} \) is roughly constant; these qualitative features will be observed in the numerical results.

Let us turn to numerical evaluations. Using Eqs. (3)-(7), (9) and (10), we can now compute the string density for different hadronic targets, \( Q^2 \), and \( x \) or \( s \). When this density becomes larger than the critical value, Eq. (10), percolation will take place, which we will interpret as a signal of the onset of saturation of the density of partons

\[1\] In this approach this transverse dimension plays the rôle of an intrinsic scale of soft physics, \( Q^2 \to 0 \), where a description in terms of pQCD degrees of freedom becomes dubious.
in the target. In Fig. 1 we present results for the string density in $\gamma^* - A$ collisions versus $x$ for $Q^2 = 0.1$ and 1 (GeV/c)$^2$, with $A = 1, 9, 56$ and 207 (corresponding to $p$, $Be$, $Fe$ and $Pb$ respectively). In Fig. 2 the same quantity is presented versus $s$ for $Q^2 = 0$ and 2 (GeV/c)$^2$. Some comments are in order: First, from Fig. 1 it looks as if saturation (percolation) is favored by a higher $Q^2$, apparently in contradiction to what is commonly expected. This is due to the fact that, as we have said, the number of rescatterings is hardly dependent on $Q^2$ – as can also be observed in Fig. 2, and that $s$ is the variable which controls this number (indeed, in Fig. 2 it can be seen that $s_{sat}$ increases slightly with increasing $Q^2$, as expected). So, for a fixed $s$ at which percolation occurs, the higher the $Q^2$ the higher the $x_{sat}$ (in agreement with the naive expectations in the previous paragraph). Second, the dependence of $s_{sat}$ on $A$, parametrized as $A^{-\alpha}$, is found to be stronger in the numerical computations ($\alpha \simeq 5/2$) than the power $5/3$ estimated in the previous paragraph. This discrepancy is due to the triple pomeron contribution included in the numerical computations, which appear as a denominator in Eq. (2), diminishing the 'effective' $\Delta$ which appears in $s_{sat}$ and thus making $\alpha = 1/(3\Delta)$ larger.

To summarize, a criterium for saturation in the small $Q^2$ region applicable to any multiple scattering model, has been presented. To produce quantitative results, a multiple scattering model for $\gamma^* - p$ collisions in this $Q^2$ region [4] has been generalized to the nuclear case, and used to compute the number of exchanged strings. As multiple scattering (and thus the number of produced strings) is controlled by the ratio $\sigma^{diff}/\sigma^{tot}$, this number is related to experimental data on diffraction and the actual realization of the model is not crucial to compute the string densities as long as it reproduces the experimental data. Employing the ideas of percolation of strings taken from Heavy Ion Physics, the kinematical regions for the onset of percolation, which has been interpreted as saturation of the density of partons in the target [2], have been calculated. This constitutes a new approach, based on Regge phenomenology and different from others which use either pQCD ideas or semiclassical methods; it offers an alternative picture, based in hadronic degrees of freedom (strings), for saturation in the small $Q^2$ region. In view of the results presented in the Figures for the onset of percolation, which for large nuclei may appear at not so small $x$, saturation could be observed in future $eA$ colliders [33], and the effects of this second order phase transition

\footnote{As commented previously, we are working in the low $Q^2$ regime, so this quantity cannot be identified with the partonic densities measured in DIS.}
visible in correlations (as proposed in $AB$ collisions\cite{34}).

**Acknowledgments:** The authors express their gratitude to A. Capella for a critical reading of the manuscript, to K. J. Eskola for useful discussions on the model of saturation in\cite{38}, and to C. Pajares for his interest in this work and constant encouragement. N. A. acknowledge financial support by CICYT of Spain under contract AEN99-0589-C02 and by Universidad de Córdoba, and C. A. S. a postdoctoral grant by Ministerio de Educación y Cultura of Spain. Laboratoire de Physique Théorique is Unité Mixte de Recherche – CNRS – UMR no 8627.

**References**

[1] L. V. Gribov, E. M. Levin and M. G. Ryskin, Phys. Rept. **100**, 1 (1983); A. H. Mueller and J.-W. Qiu, Nucl. Phys. **B268**, 427 (1986).

[2] K. Golec-Biernat and M. Wüsthoff, Phys. Rev. **D59**, 014017 (1999); **D60**, 114023 (1999).

[3] E. Gotsman, E. M. Levin, U. Maor and E. Naftali, Nucl. Phys. **B539**, 535 (1999); E. M. Levin and U. Maor, preprint TAUP-2643-2000 (hep-ph/0009217); M. B. Gay Ducati and V. P. Gonçalves, Phys. Lett. **B466**, 375 (1999).

[4] A. Capella, E. G. Ferreiro, A. B. Kaidalov and C. A. Salgado, Nucl. Phys. **B593**, 336 (2001); Phys. Rev. **D63**, 054010 (2001).

[5] A. H. Mueller, Nucl. Phys. **B335**, 115 (1990); **B558**, 285 (1999); Yu. V. Kovchegov and A. H. Mueller, Nucl. Phys. **B529**, 451 (1998); Yu. V. Kovchegov, Phys. Rev. **D54**, 5463 (1996); **D55**, 5445 (1997); E. M. Levin and K. Tuchin, Nucl. Phys. **B573**, 383 (2000); E. M. Levin and M. Lublinsky, preprint TAUP-2670-2001 (hep-ph/0104108).

[6] M. A. Braun, Eur. Phys. J. **C16**, 337 (2000); hep-ph/0010041; N. Armesto and M. A. Braun, Eur. Phys. J. **C20**, 517 (2001).

[7] L. McLerran and R. Venugopalan, Phys. Rev. **D49**, 2233 (1994); 3352; **D50**, 2225 (1994); J. Jalilian-Marian, A. Kovner, L. McLerran and H. Weigert, Phys.
Rev. **D55**, 5414 (1997); E. Iancu, A. Leonidov and L. McLerran, preprint Saclay-T00/166 and BNL-NT-00/24 [hep-ph/0011241]; E. Iancu and L. McLerran, Phys. Lett. **B510**, 145 (2001).

[8] A. Caldwell at the *DESY Theory Workshop* (Hamburg, Germany, October 1997); ZEUS Collaboration: J. Breitweg *et al.*, Eur. Phys. J. **C7**, 609 (1999).

[9] A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Eur. Phys. J. **C4**, 463 (1998); M. Glück, E. Reya and A. Vogt, Eur. Phys. J. **C5**, 461 (1998); L. Lai *et al.*, Eur. Phys. J. **C12**, 375 (2000).

[10] K. J. Eskola, V. J. Kolhinen and C. A. Salgado, Eur. Phys. J. **C9**, 61 (1999); D. Indumath and W. Zhu, Z. Phys. **C74**, 119 (1997); M. Hirai, S. Kumano and M. Miyama, Phys. Rev. **D64**, 034003 (2001).

[11] G. Altarelli, R. D. Ball and S. Forte, Nucl. Phys. **B599**, 383 (2001); R. S. Thorne, Nucl. Phys. **B512**, 323 (1998); M. Ciafaloni, D. Colferai and G. P. Salam, Phys. Rev. **D60**, 114036 (1999).

[12] A. B. Kaidalov, C. Merino and D. Pertermann, Eur. Phys. J. **C20**, 301 (2001).

[13] E. Gotsman, E. Ferreira, E. M. Levin, U. Maor and E. Naftali, Phys. Lett. **B500**, 87 (2001).

[14] A. H. Mueller, in *Proceedings of the XVII Autumn School: QCD: Perturbative or Nonperturbative?*, Eds. L. S. Ferreira, P. Nogueira and J. I. Silva-Marcos, World Scientific, Singapore 2001, p. 180.

[15] J. P. Blaizot and A. H. Mueller, Nucl. Phys. **B289**, 847 (1987); A. H. Mueller, Nucl. Phys. **B572**, 227 (2000).

[16] S. A. Bass *et al.*, Nucl. Phys. **A661**, 205c (1999).

[17] N. Armesto and C. Pajares, Int. J. Mod. Phys. **A15**, 2019 (2000).

[18] K. J. Eskola, K. Kajantie, P. V. Ruuskanen and K. Tuominen, Nucl. Phys. **B570**, 379 (2000); K. J. Eskola, K. Kajantie and K. Tuominen, Phys. Lett. **B497**, 39 (2001); K. J. Eskola, P. V. Ruuskanen, S. S. Räsänen and K. Tuominen, preprint JYFL-3/01 [hep-ph/0104010].
[19] D. Kharzeev and M. Nardi, Phys. Lett. B507, 121 (2001); H.-J. Pirner and F. Yuan, Phys. Lett. B512, 297 (2001).

[20] A. Krasnitz and R. Venugopalan, Phys. Rev. Lett. 84, 4309 (2000); 86, 1717 (2001).

[21] A. Capella, U.-P. Sukhatme, C.-I. Tan and J. Tran Thanh Van, Phys. Rept. 236, 225 (1994).

[22] A. B. Kaidalov, Sov. J. Nucl. Phys. 45, 902 (1987); Yu. M. Shabelsky, Z. Phys. C57, 409 (1993).

[23] N. Armesto, M. A. Braun, E. G. Ferreiro and C. Pajares, Phys. Rev. Lett. 77, 3736 (1996).

[24] M. Nardi and H. Satz, Phys. Lett. B442, 14 (1998); H. Satz, Nucl. Phys. A642, 130 (1998); J. Dias de Deus, R. Ugoccioni and A. Rodrigues, Eur. Phys. J. C16, 537 (2000).

[25] A. Capella, A. B. Kaidalov, C. Merino, D. Pertermann and J. Tran Thanh Van, Eur. Phys. J. C5, 111 (1998).

[26] R. J. Glauber, in Lectures in Theoretical Physics, Vol. 1, ed. W. E. Brittin and L. G. Duham (Interscience, New York, 1959); V. N. Gribov, Sov. Phys. JETP 29, 483 (1969); 30, 709 (1970).

[27] A. Bialas, M. Bleszyński and W. Czyz, Nucl. Phys. B111, 461 (1976).

[28] C. W. De Jager, H. De Vries and C. De Vries, Atom. Data Nucl. Data Tabl. 14, 479 (1974).

[29] A. Schwimmer, Nucl. Phys. B94, 445 (1975); L. Caneschi, A. Schwimmer and R. Jengo, Nucl. Phys. B108, 82 (1976); A. Capella, A. B. Kaidalov and J. Tran Thanh Van, Heavy Ion Phys. 9, 169 (1999); S. Bondarenko, E. Gotsman, E. M. Levin and U. Maor, Nucl. Phys. A683, 649 (2001).

[30] V. N. Gribov, Sov. Phys. JETP 26, 414 (1968).

[31] M. B. Isichenko, Rev. Mod. Phys. 64, 961 (1992).

[32] A. Rodrigues, R. Ugoccioni and J. Dias de Deus, Phys. Lett. B458, 402 (1999).
[33] M. Arneodo et al., in *Proceedings of the Workshop on Future Physics at HERA* (Hamburg, Germany, September 1995); H. Abramowicz et al., *TESLA Technical Design Report, Part VI, Chapter 2*, Eds. R. Klanner, U. Katz, M. Klein and A. Levy.

[34] M. A. Braun and C. Pajares, Phys. Rev. Lett. **85**, 4864 (2000); Yu. V. Kovchegov, E. M. Levin and L. McLerran, Phys. Rev. **C63**, 024903 (2001).
Figure captions:

**Fig. 1.** String density in $\gamma^* - p$, Be, Fe and Pb collisions versus $x$ for $Q^2 = 0.1$ (GeV/c)$^2$ (solid lines) and $Q^2 = 1$ (GeV/c)$^2$ (dashed lines). Dotted lines are the bounds on the critical string density for percolation of strings.

**Fig. 2.** String density in $\gamma^* - p$, Be, Fe and Pb collisions versus $s$ for photoproduction (solid lines) and $Q^2 = 2$ (GeV/c)$^2$ (dashed lines). Dotted lines are the bounds on the critical string density for percolation of strings.
Figures:

![Graph with labels Pb, Fe, Be, and p]
Fig. 2