Super-Enhancement Focusing of Teflon Spheres

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A Teflon (polytetrafluoroethylene) sphere can be used as a focusing lens in the applications of imaging and sensing due to its low-loss property in the terahertz band. Herein, field intensities and focusing parameters are analytically calculated for Teflon spheres at different low-loss levels and then a super-enhancement focusing effect in the spheres with particular size parameters was discovered, which can simulate about 400 times stronger field intensity than that for incident radiation as well as the great potential of overcoming diffraction limit despite high sensitivity to the magnitude of Teflon loss. A subsequent analysis of scattering amplitudes proves that the strong scattering of a single-order mode in the internal electric or magnetic field is the main factor causing this phenomenon.

1. Introduction

Lord Rayleigh stepped forward to scattering study of small particles and found that a small sферical dielectric particle with a radius, $a$, which is much smaller than the wavelength of incident light, $\lambda$ ($a/\lambda \ll 1$), can symmetrically scatter light in the forward and backward directions like a point electric dipole. Lorentz–Mie theory fully described the optical absorption and scattering of light through a homogeneous sphere with a size that was similar to the wavelength of incident light ($a/\lambda \approx 1$) thereafter and involved spherical multipole partial waves to solve the Maxwell equations. Besides, an asymmetrical jet-like near-field focus situated near the shadow surface of a dielectric sphere sized in a certain range of $a/\lambda$ (mesoscale dimension, $1 < a/\lambda < 30$) has been well studied with a popular name “photonic jet” since 2000. Its typical features include strong enhancement of field intensity, little divergence for the propagation of several wavelengths, transverse dimension smaller than diffraction limit, etc. Many approaches, e.g., metals, assembly of nanofibers, and addition of a pupil-mask, were carried out to further improve these features for the photonic jet, but most of them were based on the geometrical or morphological changes of the sphere and difficult to realize using a relatively easy experimental setup.

Meanwhile, lossless ideal materials were used in the majority of theoretical research of dielectric particle focusing, which might be acceptable and less influential to the materials working in the optical band; however, there is a different criterion on that for the materials used in the applications of terahertz (THz) and millimeter wave radiation. In fact, a material possessing a loss tangent, $\tan \delta$, at the order of magnitude of 1e-4 can be called a lossless material in the THz band. The Mie resonance effect and focusing characteristics of a sphere made from a two-component TiO$_2$-PE (polyethylene) composition with a TiO$_2$ volume fraction of 0.75 and the effective refractive index—$n = 2.5$—and extinction coefficient—$k = 1.985e-2$—were numerically studied in the wavelength range of 150–600 µm (THz band) by Storozhenko et al., but this material is difficult to synthesize and expensive. Teflon is such a kind of material with an extremely small extinction coefficient, $k$, in the wavelength range of 30 µm to 1 mm or the frequency range of 300 GHz to 10 THz. The lenses and probes made of it have not only the strength of low loss but also the advantages on cost and focusing capability compared to those from composite dielectric materials and noble metals.

In this article, we extend our research scope to investigate focusing of the Teflon spheres with multiple size parameters determined from Lorentz–Mie theory while taking into account loss impact. Minin et al. studied similar focus in THz band by a non-resonant dielectric cube particle and found it can be formed in both transmission and reflection modes. The characteristics of resonance modes were investigated in a low-loss, high-index dielectric sphere made of alumina ceramic (permittivity $\varepsilon_r = 10.7$ and tan $\delta = 0.9e-3$) to highlight high-gain and high-radiation efficiency achieved in an antenna design based on it by Dey and Hesselbarth. Yue et al. and Wang et al. studied focusing of specifically sized dielectric spheres in their publications and...
they thought the unusual Poynting vector circulation and Fano resonance could result in the extraordinary near-field focus with a large field intensity by a high-index sphere. In the aspect of the experiment, Peppernick et al. used to successfully observe a strong focus near the shadow surface of a polystyrene (PS) sphere (3 μm diameter) on a platinum/palladium (Pt/Pd) substrate angularly illuminated by the 400 and 800 nm lasers with a photoemission electron microscopy (PEEM). However, the refractive index of PS sphere at the wavelength of 400 and 800 nm (n = 1.63 and 1.58) is much higher than that for Teflon in THz band (n = 1.43); moreover, the additional factors, such as Pt/Pd substrate to enhance the collection of emitted electrons for PEEM and angular incidence of laser, complicated the for-

2.2. Super-Enhancement

The statistics of all focusing parameters are illustrated as the functions of q with the curves of loss levels of k = 0, 1.40e-4, 5.95e-4, and 1.7e-3 in Figure 1. Figure 1a summarizes the |E| peak field enhancements for all spheres and they manifest an increasing tendency with the regular oscillations for all four curves. Several super-enhancement giant peaks appear in the curves of k = 0 and 1.40e-4 at the particular q values, especially for the ideal case of lossless Teflon with k = 0. It has the most super-enhancement q positions and the highest peak with a maximum |E| field enhancement of 3950 presents at the position of q = 28.64159. This magnitude is much larger than that reported in the previous literature investigating Teflon lens focusing. Another similar super-enhancement peak is at the position of q = 46.54159 and reaches |E| field intensity of 2952. The increase of Teflon loss level effectively weakens this super-enhancement, which leads to two less fluctuant curves of k = 5.95e-4 and 1.70e-3 in Figure 1a. Figure 1b exhibits all focal positions normalized to the particle radius, a, (distance between the focus and the sphere center divided by the sphere radius) with a dashed line at the position of 1.0 referring to the lower boundary of the sphere. The majority of the markers are above the dashed line of position 1.0, which means these focuses are located below the lower boundary and outside the sphere. By contrast, it is noted that a few markers of the cases of k = 0 and k = 1.40e-4 are below the dashed line of position 1.0 and in the shaded area. The size parameters of these markers also correspond to the q positions of the super-enhancement giant peaks shown in Figure 1a, which proves that all super-enhancement focuses are inside the sphere and mainly distributed in the area between position 0.8 and position 1.0 across the z-axis, as shown in Figure 1b.

Figure 1c,d demonstrated the normalized transverse dimensions of the focuses to sphere radius in FWHM as a function of q for TE mode (FWHM_{TE}/a) and TM mode (FWHM_{TM}/a),
respectively. Here, we compare these dimensions to the normalized Rayleigh criterion, $RC/a$, and diffraction limit, $DL/a$, as well (based on Equations (2) and (3)), and then display them as two dashed curves in Figure 1c,d. It is demonstrated that $FWHM_{TE}/a$ and $FWHM_{TM}/a$ have a general relationship with the Rayleigh criterion as $FWHM_{TE}/a < RC/a < FWHM_{TM}/a$, which results in an elliptical focus profile with a major axis in x-axis direction and is in accordance with Heisenberg’s uncertainty principle applied in the scattering of photons.[33] However, this tendency cannot apply to the spheres stimulating super-enhancement effect. Several of their $FWHM_{TE}/a$ can overcome the diffraction limit, as shown in Figure 1c, and offer the great potential of approaching or overcoming the diffraction limit for these spheres.

Figures 2 and 3 manifest the distributions of electric and magnetic field intensities ($|E|^2$ and $|H|^2$) on xy plane (TM mode) at four loss levels for two super-enhancement Teflon spheres with the size parameters of 22.24159 and 28.64159, respectively. Their positions in Figure 1a are marked by two black dashed lines. It should be mentioned that the near-field distributions of the super-enhancement effect occurring in the spheres with $k = 0$ and 1.4e-4 are generally in the form of hotspots situated around the sphere poles in both electric and magnetic fields. The differences between these two super-enhancement examples are the shape and number of the hotspots around the pole and the layout in the magnetic field. For the sphere with $q = 22.24159$, the shape of hotspots is more circular, and the jet-shape focuses can be found in the magnetic field layouts besides the polar features. However, the sphere with $q = 28.64159$ has the longitudinally symmetric oval hotspots at the poles in both electric and magnetic fields pairing with the patterns of Whispering-gallery mode. Meanwhile, Figures 2 and 3 show that the increase of loss level can inhibit this super-enhancement effect, which reflects on the drop of the field intensities of the hotspots with the growth of $k$ value, especially for the hotspot at the upper pole. Therefore, the super-enhancement of $|E|^2$ and $|H|^2$ field intensities only can be stimulated at the loss levels of $k = 0$ and 1.4e-4 with the existence of the upper polar hotspots in this study. At the loss levels of $k = 5.95e-4$ and 1.70e-3, the near-field distributions in two examples transition into the classic photonic jet layout, while decreasing peak enhancements. The near-field distributions of the normal spheres with the neighboring $q$ values of 22.14159 and 28.74159 are included in Figures S1 and S2, Supporting Information, of the article. They present low field enhancements and typical photonic jets at all loss levels in both electric and magnetic fields.
3. Discussion

3.1. Scattering Amplitude

The regular oscillation of the peak field enhancement displayed in Figure 1a is considered to be caused by the optical resonance in a spherical cavity. In this case, the scattering of a plane wave radiation is a collective effect contributed by the independent order modes. The scattering amplitude of a single order mode is determined by the complex scattering wave coefficients—$B_l^e$ for electric field and $B_l^m$ for magnetic field. These two coefficients are expressed as follows:

$$B_l^e = i^{l+1} \frac{2l + 1}{l(l + 1)} \hat{\eta} \psi_l' (\hat{n} q) \psi_l (\hat{n} q) - \psi_l' (\hat{n} q) \psi_l (\hat{n} q)$$

$$B_l^m = i^{l+1} \frac{2l + 1}{l(l + 1)} \hat{\eta} \zeta_l^{(e)} (\hat{n} q) \psi_l (\hat{n} q) - \zeta_l^{(e)} (\hat{n} q) \psi_l' (\hat{n} q)$$

where $l$ is the number of order mode, $\hat{n}$ is the complex refractive index of Teflon sphere relative to the surrounding medium, air, and $\psi_l(q)$, and $\zeta_l^{(e)}(q)$ are defined by

$$\psi_l(q) = \sqrt{\frac{\pi q}{2}} J_{\frac{l-1}{2}}(q)$$

$$\zeta_l^{(e)}(q) = \psi_l(q) - i \chi_l(q) = \sqrt{\frac{\pi q}{2}} H_{\frac{l}{2}}^{(1)}(q)$$

$$\chi_l(q) = -\sqrt{\frac{\pi q}{2}} N_{\frac{l-1}{2}}(q)$$

Figure 2. Distributions of a–d) $|E^2|$ and e–h) $|H^2|$ field intensities for the sphere with $q = 22.24159$ at the loss levels of $k = 0, 1.40e-4, 5.95e-4,$ and $1.70e-3$ in TM mode.
Figure 3. Distributions of a–d) $|E|^2$ and e–h) $|H|^2$ field intensities for the sphere with $q = 28.64159$ at the loss levels of $k = 0, 1.40e-4, 5.95e-4, \text{and } 1.70e-3$ in TM mode.

where $J_{\ell-\frac{1}{2}}(q)$, $J_{\ell+\frac{1}{2}}(q)$, and $N_{\ell+\frac{1}{2}}(q)$ are the Bessel functions, the Hankel functions, and the Neumann functions, respectively.\[^{[15]}\]

The size parameter, $q$, and number of order mode, $l$, as the only two variables are highly involved with the complex functions as the denominators in the equations of $B_{\ell}^e$ and $B_{\ell}^m$ (Equations (4) and (5)). Hence, these two variables with particular values can make the denominator in Equation (4), $B_{\ell}^e$, or Equation (5), $B_{\ell}^m$, very small, even approach 0, in the mathematical circumstance of a similar magnitude for the terms of $\tilde{n} \xi_{\ell}^{(0)}(q) \psi_{\ell}(\tilde{n}a)$ and $\xi_{\ell}^{(0)}(q) \psi_{\ell}'(\tilde{n}a)$, or the terms of $\tilde{n} \xi_{\ell}^{(0)}(q) \psi_{\ell}(\tilde{n}a)$ and $\xi_{\ell}^{(0)}(q) \psi_{\ell}'(\tilde{n}a)$, based on their subtractive relationship. This could explain the principle of the discovered super-enhancement effect and its strong dependence on the size parameter. Also, it is known that scattering amplitude of a single-order mode decreases and approaches 0 with an increase of order mode number, $l$, and the number of that order mode with 0 contribution, $l_0$, can be expressed by size parameter, $q$. Its empirical formula is given by \[^{[16]}\]

$$l_0 \approx q + 4.3q^{\frac{1}{3}} + 1$$  \hspace{1cm} (9)

The algorithm calculates the collective field intensity enhancement for every sphere using an iteration of all order modes until $l_0$ is reached. Consequently, field intensity enhancement of a sphere with a large-size parameter (large $q$) should be larger than that for a sphere with a small-size parameter (small $q$) due to more contributions from high-order modes \[^{[37,38]}\] which leads to a general increasing tendency for all curves in Figure 1a. Surprisingly, the super-enhancement effect can break this tendency and let the sphere with a relatively small size parameter generate the largest field intensity enhancement within the scanning range, e.g., the sphere with $q = 28.64159$ shown in Figures 1a and 3.
3.2. External and Internal Fields

According to the Lorenz-Mie theory, the collective scattering of radiation is spatially decided by both external and internal fields of a sphere (outside and inside of the sphere). This is represented by four factors of scattering amplitudes: $a_l$ for the external electric field, $b_l$ for the external magnetic field, $c_l$ for the internal magnetic field, and $d_l$ for the internal electric field. Their formulas are expressed as follows:

$$a_l = \frac{F^{(a)}_l}{F^{(a)}_l + iG^{(a)}_l},$$
$$b_l = \frac{F^{(b)}_l}{F^{(b)}_l + iG^{(b)}_l},$$
$$c_l = \frac{i\mu l}{F^{(b)}_l + iG^{(b)}_l},$$
$$d_l = \frac{i\mu l}{F^{(a)}_l + iG^{(a)}_l}.$$

Meanwhile, $F^{(a)}_l$ and $G^{(a,b)}_l$ can be written in the Bessel and Neumann functions:

$$F^{(a)}_l = n\psi_l'(nq)\psi_l(q) - \psi_l(nq)\psi_l'(q),$$
$$G^{(a)}_l = n\chi_l'(nq)\psi_l(q) - \psi_l'(nq)\chi_l(q),$$
$$G^{(b)}_l = n\psi_l'(nq)\chi_l(q) - \psi_l'(nq)\psi_l(q).$$

where $\psi_l(q)$ and $\chi_l(q)$ are defined by Equations (6) and (8).

Using Equations (4)–(13), scattering contribution of each order mode can be quantified by these four factors and this information is useful to identify which field and order mode plays the most important role in the super-enhancement effect.

Figure 4 demonstrates the scattering amplitudes of all four factors against order mode, $l$, for the spheres with $q = 22.24159$ and $q = 22.14159$ on 4 loss levels.

![Figure 4](image-url)

Figure 4. Scattering amplitudes for factors of a) $a_l$, b) $b_l$, c) $d_l$, and d) $c_l$ against order mode, $l$, for the spheres with $q = 22.24159$ and $q = 22.14159$ on 4 loss levels.
amplitude of 250 is at the order mode of \( l = 27 \) for the curves of the super-enhancement sphere in Figure 4c-i, by contrast, the maximum amplitude of the normal sphere is less than 3.5 in Figure 4c-ii. This indicates that the super-enhancement effect in the sphere with \( q = 22.24159 \) is driven by the factor of \( d_i \) for the scattering at the order mode of \( l = 27 \) in the internal electric field. Also, the scattering amplitudes of \( a_i, b_i, c_i, \) and \( d_i \) against order mode, \( l \), are illustrated in Figure 5 for analysis of the super-enhancement effect in another sphere with \( q = 28.64159 \) in comparison with its neighboring normal sphere with \( q = 28.74159 \) (\( l_0 = 43 \)). Several extra amplitude peaks can be seen on the curves of the super-enhancement sphere in Figure 5, including those at \( l = 35 \) on \( a_i \) curves in Figure 5a, \( l = 31 \) on \( b_i \) curves in Figure 5b, \( l = 31 \) on \( d_i \) curves in Figure 5c, and \( l = 35 \) on \( c_i \) curves in Figure 5d. It should be pointed out that the factor of \( c_i \) representing the scattering in the internal magnetic field dominates the contributions to the super-enhancement effect in this sphere because of an amplitude peak approximately achieving 1500 at the order mode of \( l = 35 \), as shown in Figure 5d.

Apart from the above two examples, the identical analyses of scattering amplitudes were applied to all super-enhancement spheres manifested in Figure 1 and validated that the super-enhancement effect is driven by the scattering of a single-order mode, while it can be stimulated in either internal electric field or internal magnetic field. It is consistent with the field intensity distributions of the super-enhancement spheres because all polar hotspots are inside the sphere. Based on the statistical data of these analyses, it is noted that only two extreme enhancements (maximum \( |E| \) field intensity of 3950 at \( q = 28.64159 \) and 2952 at \( q = 46.54159 \)) are stimulated by the internal magnetic fields of the spheres, and the others are all from internal electric fields. This could conclude that the super-enhancement focusing effect induced by internal magnetic field is rare in Teflon spheres, but able to provide much stronger enhancement than that for electric field super-enhancements. Meanwhile, all amplitude peaks at the super-enhancement order modes shown in Figures 4 and 5 are highly sensitive to the loss increase and only exist at the loss levels of \( k = 0 \) and 1.40e-4. It could be related to the field intensity decline of the hotspots at the top pole of the sphere with an increase of Teflon loss, as shown in Figures 2 and 3, which moreover proves that a tiny growth of loss magnitude can adequately offset this super-enhancement effect through the inhibition of scattering from the particular order modes.
An analysis of their scattering amplitudes demonstrates that this super-enhancement effect should be caused by the internal scattering of a single-order mode in either electric or magnetic field, but enhancement from internal magnetic field is much stronger. Besides, it was found that this super-enhancement focusing effect can be weakened by the increase of Teflon intrinsic loss but inducing a focus whose size is beyond the diffraction limit and smaller than that for an ideal case of lossless Teflon.

### Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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### Conflict of Interest

The authors declare no conflict of interest.

### Keywords

dielectric particles, low-loss materials, Mie theory, near-field focusing, Teflon spheres

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