Dijets with 2, 1 or 0 rapidity gap: Factorization breaking at the Tevatron

A. Bialas
M. Smoluchowski Institute of Physics
Jagellonian University, Krakow*

R. Peschanski
CEA/DSM/SPhT, Saclay
Unité de recherche associée au CNRS†

March 25, 2022

Abstract

Central production cross-sections of hard dijets with 2, 1 or 0 rapidity gap at Tevatron are analyzed in terms of diffractive (“a la Good-Walker”) and non diffractive fluctuations of the incident particles. The observed large factorization breaking and the unexpected high value of the 2 to 1 gap cross-section ratio are explained in terms of scattering with and between the incident particles.

1. Recent measurements of production of two large transverse momentum jets in the central rapidity region at Fermilab energies [1, 2] revealed a strong breaking of the Regge Factorization between the “no-gap”, “single-gap” and “two-gap” cross-sections. If the presence of the rapidity gap is interpreted as the colour singlet (cf. “Pomeron”) exchange and its absence as the octet exchange, Regge factorization appears to be broken if

\[
\frac{\Sigma_2}{\Sigma_1} \equiv R_{2/1} \neq \frac{\Sigma_1}{\Sigma_0} \equiv R_{1/0},
\]

where \( \Sigma_0, \Sigma_1 \) and \( \Sigma_2 \) denote the cross-sections for “no-gap”, “single-gap” and “two-gap” events, respectively, and \( R_{i/j} \equiv \Sigma_i/\Sigma_j \), by definition.

*Address: Reymonta 4, 30-059 Krakow, Poland; e-mail: bialas@th.if.uj.edu.pl.
†Address: F-91191 Gif-sur-Yvette Cedex, France; e-mail: pesch@spht.saclay.cea.fr.
Experimentally [1], the ratio \( R_{2/1} \) (resp. \( R_{1/0} \)) in (1) is estimated in practice by the ratio of dijet event rates per unit \( \xi_p \) (resp. \( \xi_{\bar{p}} \)), the fraction of momentum lost by the incident proton (resp. antiproton), measured as a function of \( x_{Bj} \). Averaged over (small) \( x_{Bj} \), the results are quoted to be \( R_{2/1} \sim 0.8 \pm 0.26 \) for the L.H.S of (1) and \( R_{1/0} \sim 0.15 \pm 0.02 \) for the R.H.S. [1]. The factorization breaking \( R_{1/0}/R_{2/1} \sim 0.19 \pm 0.07 \) [1] thus reaches a factor as small as 1 over 5. In itself, the high value of \( R_{2/1} = \mathcal{O}(1) \) is also a question to be understood. Our aim is to give a theoretical estimate of these ratios.

In the present note we argue that the origin of these effects is basically the same as that observed in measurements of diffractive (virtual) photon-induced and hadron-induced processes [3, 2]. We show that it can be described in the same framework as done in Ref. [4] as due to the influence on dijet diffractive production of the soft scattering of the diffracted initial particles (protons and antiprotons at Tevatron).

2. Let us recall briefly the argument given in [4] where the (single) diffractive production of two large transverse momentum jets was discussed. Following the idea of Good and Walker [5], one first expands the incident hadron state into a superposition of diffractive (eigen)states.

\[
|h> = c_1|\psi_1> + c_2|\psi_2> + ...
\] (2)

A diffractive state is an eigenstate (by definition, in the subspace of states spanned by diffractive interactions) of absorption, i.e. we have

\[
T|\psi_i> = \lambda_i|\psi_i> + \omega_i|\phi_i>
\] (3)

where \( T = 1 - S \) is the absorption operator and thus \( \lambda_i \) is the absorption coefficient\(^1\) of the state \( |\psi_i> \). \( |\phi_i> \) describes the inelastic states which are at the origin of absorption. They are multiparticle states which do not show any rapidity gap\(^2\). Unitarity of the S-matrix implies

\[
|1 - \lambda_i|^2 = 1 - |\omega_i|^2 = 1 - \sigma_{nondiff}^i,
\] (4)

where \( \sigma_{nondiff} \) is the non-diffractive cross-section. (4) is a special case of the generic relations for diffraction

\[
\begin{align*}
|1 - \lambda|^2 &= 1 - \sigma_{inel} = 1 - \sigma_{nondiff} - \sigma_{dd} \\
1 - \omega^2 &= 1 - \sigma_{nondiff} = 1 - \sigma_{inel} + \sigma_{dd}
\end{align*}
\] (5)

where \( \sigma_{dd} \) is the diffractive dissociation cross-section. Indeed, for a diffractive state, as seen from (3), \( \sigma_{dd} = 0 \).

\(^1\)At high energy \( \lambda_i \) is dominantly real and positive.

\(^2\)The first term in (3) is usually interpreted as “Pomeron exchange” where no colour is exchanged between \( |\psi_i> \) and the target. The second term would then describe all colour exchanges and thus \( |\omega_i|^2 \) represents the probability of this colour-exchange interaction.
Using (2) and (3), it is not difficult to express the diffractive transitions between different hadronic states in terms of the absorption ($\lambda_n$) and expansion ($c_n$) coefficients:

$$< h'|t|h> = \sum_n \lambda_n (c_n')^* c_n .$$  \hspace{1cm} (6)

The physical content of the expansion (2) and of the formula (6) depends, of course, on the physical meaning one ascribes to the “diffractive” states $|\psi_i >$. Following [4] we could take them as states with a fixed number and transverse positions (in impact parameter space) of partons [6, 7]. In a modern language, we could equivalently consider QCD dipole states [8].

3. We are interested in three\(^3\) processes:

\begin{align*}
(i) & \quad p + \bar{p} \rightarrow p + (C) + \bar{p} \\
(ii) & \quad p + \bar{p} \rightarrow p' + (C) + \bar{p}' \\
(iii) & \quad p + \bar{p} \rightarrow p' + (C) + \bar{p}' ,
\end{align*}

(7)

where \( (C) \) denotes a centrally produced system (at rapidity \( y \)) of two large transverse momentum jets and a “background” of the soft particles nearby. \( p' \), \( (\bar{p}') \) represent the states which have no rapidity gap between the \( p \) (\( \bar{p} \)) and \( (C) \). Thus (i) corresponds to “two-gap” events, (ii) to “single-gap” events and (iii) to “no-gap” events, see Fig.1.

![Figure 1: Diffractive dijets with and without gaps.](image)

Figure 1: Diffractive dijets with and without gaps.

Now we have to write down the expansions of (7) into the diffractive states. Since we are interested in \textit{hard} diffraction, i.e., in the process whose probability is small, we follow [4] and assume that the expansion is quasi-diagonal. Consequently we write

\(^3\)It will also be useful to consider the process \((ii)^*\) symmetric to \((ii)\) in the interchange of left and right moving projectiles, namely

\[(ii)^* \quad p + \bar{p} \rightarrow p + (C) + \bar{p}' .\]
\[ |p\rangle = |g\rangle + \epsilon |g + D\rangle + \epsilon_P |p + D\rangle \]
\[ |p' + C\rangle = -\epsilon^* |g\rangle + |g + D\rangle + \epsilon' |p + D\rangle \]
\[ |p + C\rangle = -\epsilon_P^* |g\rangle - \epsilon^* |g + D\rangle + |p + D\rangle \]

(8)

where \(\epsilon, \epsilon_P\) and \(\epsilon'\) are small (and thus will be kept only up to first order)\(^4\). Here \(|g\rangle\) denote a superposition of diffractive states representing a bunch of soft partons (close to their distribution in the left moving proton) and \((D)\) is a superposition of partonic states consisting of a hard \(i.e.\) small in transverse space dipole at rapidity \(y\) and a number of soft partons with rapidities nearby (this system eventually decays predominantly into two large transverse momentum jets and the background of soft particles).

Using (3) and (8) we thus obtain

\[
\langle p|t|p\rangle = \lambda_p = \lambda_g \\
\langle p' + C|t|p\rangle = \epsilon (\lambda_{(g+D)} - \lambda_g) \\
\langle p + C|t|p\rangle = \epsilon_P (\lambda_{(p+D)} - \lambda_g),
\]

(9)

where the transition matrix \(t\) is to be considered diagonal in both rapidity and impact parameter space. All quantities on the R.H.S. are to be understood as linear combinations of the corresponding quantities defined for truely diffractive parton states.

Following [4] we now assume that \(\lambda_{(g+D)}\) and \(\lambda_{(p+D)}\) for diffractive states can be calculated assuming an independent scattering of their components [9], i.e.

\[
1 - \lambda_{(g+D)} = (1 - \lambda_g)(1 - \lambda_D); \quad 1 - \lambda_{(p+D)} = (1 - \lambda_p)(1 - \lambda_D).
\]

(10)

When this is inserted into (9) we have

\[
\langle p' + C|t|p\rangle = \epsilon \lambda_D(1 - \lambda_g) \\
\langle p + C|t|p\rangle = \epsilon_P \lambda_D(1 - \lambda_g).
\]

(11)

4. To obtain the cross-sections \(\Sigma_{1,2}\) for one-gap and two-gap events, see (1), we have to square the corresponding amplitudes, sum over the final hadronic states and integrate over the suitable rapidity and impact parameter intervals. In a first stage we shall discuss the differential cross sections \(\sigma_{1,2}\), before integration over rapidity and impact parameter variables. For given rapidity \(y\) and impact parameter position \(\vec{s}\), the summation extends for \(\sigma_{1}\) over all states \((p')\) and \((C)\) as the detailed final state is not measured. For \(\sigma_{2}\) one sums only over all states of \((C)\), because the final proton is identified. These differences imply different

\(^4\)The relations between expansion coefficients follow from orthonormality of the states.
averaging procedures for the diffractive states. Taking this into account, we obtain

\[
\sigma_1(\vec{s}, y) = \left[|\epsilon(\vec{s}, y)|^2|\lambda_D(\vec{b} - \vec{s}, Y - y)|^2\right]_{av} \left[|1 - \lambda_g(\vec{b}, Y)|^2\right]_{av} ;
\]

\[
\sigma_2(\vec{s}, y) = \left[|\epsilon_P(\vec{s}, y)|^2|\lambda_D(\vec{b} - \vec{s}, Y - y)|^2\right]_{av} |1 - \left[\lambda_g(\vec{b}, Y)\right]_{av}|^2 ,
\]

where we have explicitly indicated the averaging procedures. \(Y, \vec{b}\) are the rapidity and impact parameter distances between the incident particles. At this point one can observe that, as seen from (4), (5),

\[
\left[|1 - \lambda_g(\vec{b}, Y)|^2\right]_{av} = 1 - \left[\omega_g^2(\vec{b}, Y)\right]_{av} = 1 - \omega^2_P(\vec{b}, Y) = \left(1 - \left[\lambda_g(\vec{b}, Y)\right]_{av}\right)^2 + \sigma_{dd}. (13)
\]

It is important to remember that - since the averaging concerns only one vertex - \(\sigma_{dd}\) here corresponds to the single diffractive dissociation in one vertex. All this implies that

\[
\sigma_1(\vec{s}, y) = \left[|\epsilon(\vec{s}, y)|^2|\lambda_D(\vec{b} - \vec{s}, Y - y)|^2\right]_{av} \left[1 - \left[\lambda_g(\vec{b}, Y)\right]_{av}\right]
\]

so that in the following one can omit the index \(av\) without running into confusion.

One sees that the formula (12) for \(\sigma_2\) is not symmetric with respect to the interchange of the projectile and the target. To restore the symmetry we have to require that

\[
|\lambda_D(\vec{b} - \vec{s}, Y - y)|^2 = V_P \left|\epsilon_P(\vec{b} - \vec{s}, Y - y)\right|^2
\]

where, \textit{a priori}, \(V_P\) is a vertex function of \((\vec{s}, y; \vec{b} - \vec{s}, Y - y)\) which must be symmetric with respect to exchange of the corresponding arguments. However, since the L.H.S. of (15) depends only on \(\vec{b} - \vec{s}\) and \(Y - y\), \(V_P\) can depend neither on \(\vec{s}\) nor on \(y\). Symmetry implies that it must be a constant, depending only on the internal variables of the vertex.

The formula (15) shows that the two-gap diffractive interaction can be equivalently understood as the elastic interaction of the projectile with a Good-Walker (Pomeron-like) fluctuation of the target.

From (12) and (15) we deduce that

\[
\sigma_2(\vec{s}, y) = V_P \left|\epsilon_P(\vec{s}, y)^2 \left|\epsilon_P(\vec{b} - \vec{s}, Y - y)\right|^2|1 - \lambda_g(\vec{b}, Y)|^2\right.
\]

\[
\quad = V_P^{-1} \left|\lambda_D(\vec{s}, y)^2 \left|\lambda_D(\vec{b} - \vec{s}, Y - y)\right|^2|1 - \lambda_g(\vec{b}, Y)|^2\right. .
\]

5. We have to discuss now the no-gap events and thus to go beyond the Good-Walker argument, as the latter refers only to diffractive interactions. To this end we observe that the probability of a no-gap event to occur can be written as the product of (a) the probability of fluctuation of the projectile into \(|g' + D>\), i.e.
\[ |\epsilon|^2 \], and (b) the probability of non-diffractive interaction of the central system \((D)\) with the target, i.e. \(\omega_D^2\):

\[ \sigma_0 = |\epsilon(s, y)|^2 \omega_D^2(\vec{b} - \vec{s}, Y - y) \]  

(17)

Since the same argument can also be used by exchanging the roles of the target and projectile we also have

\[ \sigma_0 = |\epsilon(\vec{b} - \vec{s}, Y - y)|^2 \omega_D^2(s, y) \]  

(18)

which implies that

\[ [\omega_D(s, y)]^2 = V |\epsilon(s, y)|^2 \]  

(19)

where \(V\) is another vertex function, symmetric with respect to exchange of the corresponding arguments. For the similar reasons as those following Eq.(15) we deduce that it is actually a constant. Thus we finally obtain

\[ \sigma_0 = V |\epsilon(s, y)|^2 |\epsilon(\vec{b} - \vec{s}, Y - y)|^2 \]

\[ = V^{-1} \omega_D^2(s, y) \omega_D^2(\vec{b} - \vec{s}, Y - y) . \]  

(20)

A closer look shows that the vertex functions \(V\) and \(V_P\) are identical. This can be seen by observing that the cross-section for one-gap events, \(\sigma_1\), given in (12), can be also calculated as a product of (a) the probability of fluctuation of the target into \((\bar{p} + D)\), i.e. \(|\epsilon_P|^2\), (b) the probability of non-diffractive interaction of \((D)\) with the projectile, i.e. \(\omega_D^2\) and (c) the probability that no non-diffractive interaction of the final antiproton with the projectile took place\(^5\), i.e. \((1 - \omega_P^2)\) (c.f. (5)). Thus we can write \((\omega_P = \omega_p)\)

\[ \sigma_1(s, y) = |\epsilon_P(\vec{b} - \vec{s}, Y - y)|^2 \omega_D^2(\vec{s}, y) [1 - \omega_p^2(Y)] . \]  

(21)

Comparing (21) and (14) we have

\[ |\epsilon_P(\vec{b} - \vec{s}, Y - y)|^2 [\omega_D(\vec{s}, y)]^2 = |\epsilon(\vec{s}, y)|^2 [\lambda_D(\vec{b} - \vec{s}, Y - y)]^2 . \]  

(22)

This equation has a simple physical meaning. It says that the probability of a fluctuation of the projectile into the projectile+gap+D configuration is proportional to the elastic interaction in the projectile-D system, whereas the probability of fluctuation into no-gap configuration is proportional to the non-diffractive interaction in this system. It thus provides the interpretation, in the Good-Walker language, of the “Pomeron exchange” mechanism for gap creation.

From (15), (19) and (22) one sees that indeed the vertex functions \(V\) and \(V_P\) are identical:

\[ V = V_P . \]  

(23)

\(^5\)Diffractive excitation of the projectile is allowed.
Using now (22) and (21) we obtain

$$\sigma_1(\vec{s}, y) = V \frac{|\epsilon(\vec{s}, y)|^2|\epsilon_p(\vec{b} - \vec{s}, Y - y)|^2[1 - \omega_p(\vec{b}, Y)]^2}{|\epsilon(\vec{s}, y)|^2} \frac{|\omega_D(\vec{s}, y)|^2|\lambda_D(\vec{b} - \vec{s}, Y - y)|^2[1 - \omega_p(\vec{b}, Y)]^2}{|\omega_D(\vec{s}, y)|^2}.$$ 

(24)

6. Putting together the formulae Eqs. (16),(20) and (24) for the differential cross-sections, and even before evaluating directly the observable quantities mentioned in section 1, it is already possible to get important qualitative hints on the pattern of factorization breaking in diffractive dijet production at the Tevatron.

Considering for instance, the differential ratios \(r_{i/j} \equiv \sigma_i / \sigma_j\), we find

\[
\begin{align*}
   r_{2/1} &= \frac{|\epsilon_P(\vec{s}, y)|^2}{|\epsilon(\vec{s}, y)|^2} \frac{|1 - \lambda_g(\vec{b}, Y)|^2}{|1 - \omega_p^2(Y)|^2} = \frac{|\lambda_D(\vec{s}, y)|^2}{|\omega_D(\vec{s}, y)|^2} \frac{|1 - \lambda_g(\vec{b}, Y)|^2}{|1 - \omega_p^2(Y)|^2} \\
   r_{1*/0} &= \frac{|\epsilon_P(\vec{s}, y)|^2}{|\epsilon(\vec{s}, y)|^2} \frac{|1 - \omega_p^2(Y)|}{|1 - \omega_p^2(Y)|^2} = \frac{|\lambda_D(\vec{s}, y)|^2}{|\omega_D(\vec{s}, y)|^2} \frac{|1 - \lambda_g(\vec{b}, Y)|^2}{|1 - \omega_p^2(Y)|}.
\end{align*}
\]

(25)

where we make use of \(\sigma^*_1\), (cf. footnotes 3, 6) instead of \(\sigma_1\) in the second row for formal convenience, the qualitative conclusions being unchanged.

Formulae (25) show explicitly that the factorization formula (1) is violated:

$$\frac{\sigma^*_1(\vec{s}, y)}{\sigma_0(\vec{s}, y)} = \frac{\sigma_2(\vec{s}, y)}{\sigma_1(\vec{s}, y)} \times \frac{1 - \sigma_{nondiff}(\vec{b}, Y)^2}{1 - \sigma_{inel}(\vec{b}, Y)}.$$ 

(26)

We have expressed the coefficients \(\lambda_g, \omega_p\) in terms of their physical interpretation (4),(6). One sees that the factorization violating factor has the same origin as that which was shown in [4] to be responsible for violation of factorization between single diffractive processes at HERA and at FERMILAB. The numerical value may be, however, somewhat different, depending on the size of the cross-section for the soft diffractive dissociation in \(p\bar{p}\) collisions (giving the difference between \(\sigma_{inel}\) and \(\sigma_{nondiff}\)).

Another interesting remark individually concerns the ratios \(r_{2/1}\) and \(r_{1*/0}\). Rewriting expression (25) as

\[
r_{2/1} = \frac{|\epsilon_P|^2(\vec{s}, y)}{|\epsilon|^2} \frac{1 - \sigma_{inel}}{1 - \sigma_{nondiff}}
\]

6We can equivalently consider the symmetric \(\sigma_i^*\) of \(\sigma_1\) by interchange of left and right moving particles, namely

\[
\sigma_i^*(\vec{s}, y) \equiv \sigma_1(\vec{b} - \vec{s}, Y - y).
\]

7Not to be confused with the integrated ones (1).

8In [4] this difference was neglected. If taken into account, the corresponding factor between the HERA and FERMILAB cross-sections should be \(1 - \sigma_{nondiff}\) rather than \(1 - \sigma_{inel}\). At small impact parameters, however, which are most important for numerical estimates, this difference is expected to be small.
\[ r_{1*0} = \frac{|\epsilon_p|^2}{|\epsilon|^2} (\vec{s}, y) \left(1 - \sigma_{\text{nondiff}}\right). \]  

(27)

In fact, we have to take into account that the specific factor \(|\epsilon_p|^2/|\epsilon|^2\) is factorizable since it reflects the probability ratio for a color singlet over an octet to be coupled to the projectile. Hence, we see that the essential factorization breaking factor is concentrated in the second ratio \(r_{1*0}\) and it has the same content as the corresponding factor between the HERA and FERMILAB cross-sections [4]. On contrary, the factorization breaking factor in \(r_{2/1}\) is mild, since it depends only on the difference between \(1 - \sigma_{\text{inel}}\) and \(1 - \sigma_{\text{nondiff}}\) due to the diffraction dissociation contribution in the total cross-section. These conclusions will be confirmed by the following phenomenological application.

7. To obtain numerical estimates of the cross-sections \(\Sigma_i\) and of the measured ratios \(R_{ij}\), cf. section 1, one has to perform integration over the rapidity interval \(y\) and impact parameters \(\vec{b}\) and \(\vec{s}\).

For rapidity intervals, the integration is, in some sense, taken into account by the measurement itself, since event densities by unit of \(\xi\) are given. Hence, even if a full simulation would be welcome to match precisely the experimental conditions, the rapidity dependence is considered to be already integrated out.

The impact parameter dependence has to be taken into account. This is difficult because several necessary elements are not known, so that -at best- only a rough estimate is possible. Only the elastic \(p\bar{p}\) amplitude is known with some precision:

\[ \lambda_p(\vec{b}, Y) = a_0 e^{-b^2/2B_{el}}, \quad a_0 = \frac{\sigma_{\text{tot}}}{4\pi B_{el}}. \]  

(28)

At \(\sqrt{s} = 1800\) GeV, \(\sigma_{\text{tot}} = 71.71 \pm 2.02\) mb and \(B_{el} = 16.3 \pm 5\) GeV\(^{-2}\) [10], so that \(a_0 \approx 0.85\). There is also the inequality (Pumplin bound [12])

\[ \lambda_D^2 \leq \omega_D^2. \]  

(29)

To simplify the discussion we assume tentatively that the shapes (in impact parameter space) of \(\lambda_D^2\) and \(\omega_D^2\) are similar and take the Gaussian forms for easy integration:

\[ [\lambda_D(\vec{s}, y)]^2 = \eta \omega_D^2(\vec{s}, y) = A \exp[-s^2/B_D]. \]  

(30)

where \(B_D\) is the slope in the elastic scattering of the central system (\(D\)). Guided by the results from hard diffraction of virtual photons [11], we estimate \(B_D\) to be in the region around 4 GeV\(^{-2}\).

Using (16), this allows to determine \(\sigma_2\). The integration over \(d^2s\) and \(d^2b\) gives

\[ \frac{\sigma_2}{\sigma_0} = R_{2/0} = \eta^2 \left(1 - 2 \frac{a_0}{1 + \zeta} + \frac{a_0^2}{1 + 2\zeta}\right). \]  

(31)
where $\zeta = B_D/B_{el}$. Using the obvious identity

$$R_{2/1} = R_{2/0} R_{0/1} = \frac{R_{2/0}}{R_{1/0}}, \quad (32)$$

one can thus calculate $R_{2/1}$ from (31) and the measured ratio $R_{1/0}$.

The Fermilab measurements [1] give $R_{1/0} \approx 0.15 \pm 0.02$. Substituting all this into (32) and using the measured ratio $R_{2/1} = 0.8 \pm 0.26$ we deduce that for $\zeta \approx 0.25$, $\eta \approx 1$ (with an error of about 30%). Although this value of $\eta$ is a perfectly acceptable one\(^9\), it should be noted that it saturates the unitarity limit. More precise data are needed, however, to consider this interesting property as really established.

This result shows two points. First that our analysis can naturally explain a rather large value of the ratio $R_{2/1}$ measured by the CDF coll. This seems to be a rather non-trivial result. As a sort of by-product we obtain the second point, namely the strong violation of the Regge factorization.

To obtain the theoretical value of the ratio

$$R \equiv R_{2/1}/R_{1/0} = R_{2/0}/R_{1/0}^2, \quad (33)$$

one needs additional information about the soft single diffractive dissociation cross-section in one vertex (c.f. Eq. (4)). For a rough estimate we take

$$\langle \lambda^2_0(b) \rangle = a_0^2 e^{-b^2/B_{diff}}, \quad (34)$$

which is a simplified version of the analysis by Miettinen and Pumplin [7]. From this and (28) we deduce the diffraction dissociation cross-section to be

$$\sigma_{dd} = (\kappa - 1)\sigma_{el}, \quad (35)$$

where\(^{10}\) $\kappa = B_{diff}/B_{el}$. This gives

$$R_{1/0} = \eta \left(1 - 2 \frac{a_0}{1 + \zeta} + \frac{a_0^2}{1 + 2\zeta/\kappa}\right) \quad (36)$$

\(^9\)The fact that we can take $\eta \sim \mathcal{O}(1/2 - 1)$ is related to the fact that the central system D contains some soft partons apart from the hard dipole, e.g. inclusive diffractive production [13]. Therefore we would predict that in an experiment which will measure the “elastic” two jets e.g. exclusive diffractive production without accompanying soft hadronic radiation [14], one shall expect $\eta \ll 1$ and falling with increasing transverse momentum of the jets.

\(^{10}\)The ratio $\kappa > 1$ does not imply that the slope in diffractive dissociation must be larger that in elastic scattering. This was explained already in Ref.[7]. The parameter $B_{diff}$ is responsible for the behaviour of the average of sum of the squares of the diffractive amplitudes (including elastic amplitude) in impact parameter space. From this information it is not possible to deduce directly the behaviour of diffractive dissociation channels in momentum transfer. As shown by Ref.[7], although the diffractive dissociation is more peripheral in $b$ space than elastic scattering, its slope in momentum transfer is smaller than that of elastic scattering. As explicitly derived and clarified in the parton picture of Ref.[7], the slope in momentum transfer depends strongly on degrees of freedom which are other than the transverse ones and thus is not given by the fourier transform of (34) (cf. the comparison of Fig. 2 (for $d\sigma/d^2b$) and Fig. 5 (for $d\sigma/dt$) in Ref.[7]).
One sees that the ratio $R$ is independent of $\eta$ and thus completely determined by the value of $\kappa$. To estimate $\kappa$ we observe that the total soft diffractive dissociation cross-section is approximately equal to the elastic one. Assuming the approximate factorization between the single-diffractive and double-diffractive cross-sections we can write for the sum of all contributions to the total soft diffractive cross-section

$$\sigma_{diff,tot}/\sigma_{el} = 2(\kappa - 1) + (\kappa - 1)^2 = (\kappa^2 - 1) \approx 1$$

leading to $\kappa \approx \sqrt{2}$. For $\zeta = 0.25$ this gives $R \approx 4$, in agreement with the experimental value $5.3 \pm 1.8$.

8. In conclusion, we have analyzed the factorization breaking observed in dijet diffractive production at Tevatron, using the same framework which allowed to explain the factorization breaking between HERA and Tevatron single diffractive processes. In both cases, the long distance diffractive interactions between the proton and the antiproton are the cause of the factorization breaking mechanisms.

Interestingly enough, the pattern of factorization breaking in this framework is quite dependent of the number of rapidity gaps (“no-gap”, “single-gap” and “two-gap”), in agreement with the experimental findings. The factorization breaking is strong for the comparison between “no-gap” and “single-gap” cross-sections, since it is related to the total $p\bar{p}$ absorption factor (4), while it is weak$^{11}$ for “single-gap” vs. “two-gap” ones, where it is related to the fraction of the inelastic diffraction to the total inelastic cross-section which is small.

Thanks to the identification of a factorization breaking mechanism, an outcome of our approach is the numerical evaluation of the ratios $R_{i/j}$ and in particular $R_{2/1}$ which is found experimentally to be $O(1)$, that is surprisingly high. With quite reasonable values for the absorption parameters we find results in nice agreement with the data. Stimulating dijet production results, expected coming soon [16] from the Tevatron, Run II, will allow us to perform a more differential analysis of the mechanism we propose.

Note added. While completing this study, the paper Ref. [17] has appeared, treating the same question within the rapidity gap formalism. We note an agreement between the two approaches.

Acknowledgements

Discussions with Dino Goulianos are highly appreciated. A.B. thanks the Theory Department of the Saclay Centre for a kind hospitality. This investigation was supported in part by the by Subsydium of Foundation for Polish Science NP 1/99 and by the Polish State Committee fo Scientific Research (KBN) Grant No 2 P03 B 09322 (2002-2004).

$^{11}$Note that our derivation gives an interpretation of the empirical “gap probability renormalization” proposed in Ref.[15]. In particular, only little price (see e.g. formula (27)) is payed for the formation of a second rapidity gap once the first is present.
References

[1] T. Affolder et al., CDF Coll., *Phys. Rev. Lett.* **85** (2000) 5043.

[2] For a review of CDF results on Diffraction: K. Goulianos, *Nucl. Phys. Proc. Suppl.* **99A** (2001) 37.

[3] T. Affolder et al., CDF Coll., *Phys. Rev. Lett.* **84** (2000) 4215.

[4] A. Bialas, *Acta Phys. Polon.* **B33** (2002) 2635.

[5] M.L. Good and W.D. Walker, *Phys. Rev.* **120** (1960) 1857.

[6] K. Fialkowski and L. Van Hove, *Nucl. Phys.* **B107** (1976) 211.

[7] H. Miettinen and J. Pumplin, *Phys. Rev.* **D18** (1978) 1696.

[8] A. Bialas, R. Peschanski, *Phys. Lett.* **B378** (1996) 302; **B387** (1996) 405.

[9] R.J. Glauber, *Phys. Rev.* **100** (1955) 242.

[10] Data for elastic scattering:
    Total cross-section: E811 Coll., C. Avila *et al.*, *Phys. Lett.* **B445** (1999) 419;
    Elastic slope: N.A. Amos *et al.*, *Phys. Rev. Lett.* **63** (1989) 2784.

[11] For a recent review: K. Voss, *Vector meson production at HERA*, XXXVII-Ith Rencontres de Moriond *QCD and Hadronic interactions at high energy*, March 23rd-29th 2003, hep-ex/0305052.

[12] J. Pumplin, *Phys. Rev.* **D8** (1973) 2899.

[13] M. Boonekamp, R. Peschanski, C. Royon, *Phys. Rev. Lett.* **87** (2001) 251806; hep-ph/0301244, to appear in *Nucl. Phys. B*.

[14] A. Bialas and P. V. Landshoff, *Phys. Lett.* B256 (1990) 540. V. Khoze, A. Martin, M. Ryskin, *Eur. Phys. C24* (2002) 459 and references therein.

[15] K. Goulianos, J. Montanha, *Phys.Rev.* **D59** (1999) 114017; General review and Refs. in K. Goulianos, *Diffraction in QCD*, presented at CORFU-2001, Corfu, Greece, hep-ph/0203141.

[16] M. Gallinaro (representing the CDF collaboration), *QCD Results from the CDF Experiment at $\sqrt{s} = 1.96 \ TeV$, XXXVIIIth Rencontres de Moriond *QCD and Hadronic interactions at high energy*, March 23rd-29th 2003, hep-ph/030421.

[17] A.B. Kaidalov, V.A. Khoze, A.D. Martin, M.G. Ryskin, *Phys. Lett.* **B559** (2003) 235.