Securer and Faster Privacy-Preserving Distributed Machine Learning

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Abstract. With the development of machine learning, it is difficult for a single server to process all the data. So machine learning tasks need to be spread across multiple servers, turning centralized machine learning into a distributed one. However, privacy remains an unsolved problem in distributed machine learning. Multi-key homomorphic encryption over torus (MKTFHE) is one of the suitable candidates to solve the problem. However, there may be security risks in the decryption of MKTFHE and the most recent result about MKFHE only supports the Boolean operation and linear operation. So, MKTFHE cannot compute the non-linear function like Sigmoid directly and it is still hard to perform common machine learning such as logistic regression and neural networks in high performance.

This paper first introduces secret sharing to propose a new distributed decryption protocol for MKTFHE, then designs an MKTFHE-friendly activation function, and finally utilizes them to implement logistic regression and neural network training in MKTFHE. We prove the correctness and security of our decryption protocol and compare the efficiency and accuracy between using Taylor polynomials of Sigmoid and our proposed function as an activation function. The experiments show that the efficiency of our function is 10 times higher than using 7-order Taylor polynomials straightly and the accuracy of the training model is similar to that using a high-order polynomial as an activation function scheme.

Keywords: Privacy-preserving machine learning · Distributed machine learning · Multi-key fully homomorphic encryption · Multi-key decryption · MKTFHE-friendly activation function.

1 Introduction

In the big data era, it is necessary to spread the machine learning tasks across multiple servers and transform centralized systems into distributed ones [31]. These distributed systems confront new challenges and one of the unsolved challenges is privacy [17].
Privacy computing is a kind of technique that performs data computation without specified information leakage. To outsource private computations, a cryptographic tool called fully homomorphic encryption (FHE) is exploited. FHE is a special form of encryption that permits users to perform computations on encrypted data without first decrypting it. FHE can be split into two categories: single-key fully homomorphic encryptions and multi-key fully homomorphic encryptions.

Single-key FHE only allows a server to perform addition and multiplication on data encrypted by the same key. In contrast, multi-key FHE (MKFHE) proposed in [23] enables users to encrypt their own data under their own keys, but during the decryption of MKFHE, all secret keys of all participants are used. It prevents conspiracy between a user and a server to steal the data of other users.

For multi-key fully homomorphic encryption over torus (MKTFHE), researchers care about the decryption algorithm and evaluation algorithm. As for the study on the evaluation algorithm, Chen et al. [6] developed the library to implement MKTFHE where the evaluation algorithm takes a NAND gate as input. As an improvement, Jiang et al. [18] made the evaluation algorithm can take arithmetic operators including adder, substracter, multiplier, and divider so that MKTFHE can support linear multi-key homomorphic evaluation. However, MKTFHE cannot evaluate the non-linear operations, such as the Sigmoid function, which means that more complex machine learning schemes like logistic regression and neural networks cannot be done. As for the study on decryption algorithm, Chen et al. provided a naive decryption algorithm in the origin MKTFHE which asked all the secret keys of users to input together. However, in a real scenario, anyone shouldn’t have the access to the secret key of others even during the decryption. Then, Lee et al. [21] proposed a distributed decryption algorithm that splits the decryption algorithm in [6] into two sub-algorithms: partial decryption and final decryption which each user only uses their own secret key in partial decryption.

However, with the ciphertext and a partial decryption of a user $u_i$, the existing MKTFHE scheme leaks information of the secret key $s_i$ of the user $u_i$. In detail, suppose a ciphertext $(a_1, \ldots, a_k, b)$ with $b = \frac{1}{4} m - \sum_{j=1}^{k} <a_j, s_i> + e$ where $k$ is the number of users in total, $m$ is a bit message, and $e$ is an error to randomize $b$. Then, a partial decryption $p_i = b + <a_i, s_i>$ and $b$ from the ciphertext gives out $<a_i, s_i>$ which leaks at least a bit information of $s_i$. In addition, if the external adversary obtains all partial decryption results, the computation results under multi-key encryption can be finally obtained by computing $\sum_{i=1}^{k} p_i - (k-1)b$. These problems can cause security concerns.

On the other hand, to better apply multi-key homomorphic encryption on more practical machine learning schemes, we first propose a distributed decryption protocol for MKTFHE, then design an MKTFHE-friendly activation function and utilize it to implement privacy-preserving logistic regression and neural networks.

In this paper, we make the following contributions:
1. We construct a secure distributed decryption protocol for MKTFHE by introducing a secret sharing scheme to solve the information leakage problem. We define our security goal for MKTFHE against a possible static adversary, and then prove the correctness of our protocol. Our idea is that each data provider uses their secret key to run partial decryption and the secret sharing is utilized to finish the final decryption so that each user does not have the access to the secret key of other users in the system and the external adversary cannot get the partial decryption result which we truly protect the decryption and users’ key from both internal and external adversaries.

2. We utilize MKTFHE with our proposed secure distributed decryption protocol to train and evaluate logistic regression and neural network models. We design a homogenizer to modify the length of bits of the operand in order to use operators of the less bits to reduce the operation time. In addition, to accelerate the computation of the activation function, we design a compare quads to implement a kind of MKTFHE-friendly activation function. Experimental results show that the efficiency of our function is 10 times higher than using 7-order Taylor polynomials straightly and the accuracy of the training model is similar to that using a high-order polynomial as an activation function scheme.

2 Related Work

There are numerous works on privacy-preserving machine learning prediction [2, 3, 13, 19, 29] and training [7, 8, 20]. These solutions are based on single-key FHE which cannot support the data participants using different secret keys to encrypt their own data. And the prediction can support more complex models such as logistic regression and even neural networks in the second level, but the training only focuses on simpler models such as logistic regression in the hour level or higher. Besides, we also note that there are other approaches based on secure multi-party computation (MPC), e.g. [24, 25] and comparing with the above works based on FHE, the performances of the solutions based on MPC is very impressive. But they need interactivity between the data participants and the computation parties which may lead to many problems such as network latency or high bandwidth usage. Considering the above downsides, we focus on FHE, especially multi-key FHE.

In 2012, the concept of multi-key fully homomorphic encryption was first proposed by Lopez et al. [23], which is intended to apply to on-the-fly multi-party computation based on NTRU. In 2015, the first LWE-based MKFHE was constructed by Clear et al. [12], and in 2016, was improved by Mukherjee and Wichs [26]. These schemes are single-hop MKFHE schemes, which means all the participants must be known in advance. In 2016, multi-hop MKFHE schemes was proposed by Peikert et al. [28] and Brakerski et al. [4], but their schemes are impractical and without implementation.

In 2019, the first implementation of the MKFHE scheme was achieved by Chen et al. [6], named MKTFHE which is the variant of TFHE [9, 11]. Their
scheme only provided a bootstrapped NAND gate to evaluate. Then, Lee and Park [21] first formalized the distributed decryption for MKFHE and improved the decryption part of MKTFHE, but a passive adversary still can recover the decryption result through the partial results. In 2021, Jiang et al. [18] designed other bootstrapped gates, utilized them to build arithmetic operators including adder, subtractor, multiplier, and divider, and then implemented a privacy-preserving linear regression in the GD method.

However, only using arithmetic operators cannot directly compute the non-linear activation function like the Sigmoid function. So, there is still a gap between MKTFHE and the implementation of more complex privacy-preserving machine learning such as logistic regression and neural networks.

3 Preliminaries

**Notation:** In the rest of this paper, \( \mathbb{R} \) denotes the real numbers, \( \mathbb{Z} \) denotes the integers, and \( \mathbb{T} \) indicates \( \mathbb{R}/\mathbb{Z} \), the torus of real numbers modulo 1. We use TLWE to denote the (scalar) binary Learning With Error problem over Tours, and TRIWE for the ring mode. We define \( \text{params} \) as the parameter set in TFHE, \( \text{mkparams} \) in MKTFHE and our scheme. Besides, \( k \) is used to represent the number of the participants in MKTFHE and \( l \) is used to represent the bit length of a message or ciphertext. Then we use bold letter, e.g. \( \mathbf{a} \), to denote vector and use \( \langle \mathbf{a}, \mathbf{b} \rangle \) to represent the inner product between vector \( \mathbf{a} \) and vector \( \mathbf{b} \).

### 3.1 Multi-key Fully Homomorphic Encryption over Torus

MKTFHE scheme is the multi-key version of TFHE scheme. TFHE, constructed by Chillotti et al. [9][11], is a fast fully homomorphic encryption (FHE) scheme over the torus, which generalizes and improves the FHE based on GSW [15] and its ring variants. In the TFHE scheme, bootstrapped binary gates are designed to represent the functions developers need.

The main idea of TFHE is to bootstrap after every binary gate evaluation to refresh the ciphertext in order to make it usable for the following operations, resulting in that arbitrarily deep circuits can be homomorphically evaluated. The entire homomorphic evaluation of the circuits will take time proportional to the number of the binary gates used.

The message space of TFHE bootstrapping gates is \( \mathbb{T} \). A TLWE ciphertext \( (\mathbf{a}, \mathbf{b}) \in \mathbb{T}^{n+1} \) encrypted a message \( \mu \in \mathbb{T} \) with noise parameter \( \alpha \).

In the TFHE scheme, the homomorphic evaluation of a binary gate is achieved with operations between TLWE samples and a gate-bootstrapping just following (expect NOT gate). By using this approach, all the basic gates can be evaluated with a single gate bootstrapping (GB) process:

- TFHE.NAND \( (ct_1, ct_2) = \text{GB} \left( \left( 0, \frac{\alpha}{2} \right) - ct_1 - ct_2 \right) \)
- TFHE.AND \( (ct_1, ct_2) = \text{GB} \left( \left( 0, -\frac{\alpha}{2} \right) + ct_1 + ct_2 \right) \)
common distributed decryption for MKFHE \cite{27} has been defined below: The participants to jointly decrypt a multi-key ciphertext is more practical. The most key sk in practical use, for security reasons, the decryptor should not hold any secret is the decryptor holds a set of secret keys \{sk_i\}_{i \in \mathbb{Z}}.

3.2 Distributed Decryption

The TFHE scheme has the advantages of fast bootstrapping, efficient homomorphic logic circuit evaluation, and so on. Its multi-key version, named MKTFHE, was constructed by Chen et al. \cite{6} in 2019. MKTFHE is the first attempt in the literature to implement an MKFHE scheme in codes.

In the MKTFHE scheme, the ciphertext length increases linearly with the number of users, and a homomorphic NAND gate with bootstrapping is given. The MKTFHE scheme is comprised of the following algorithms:

- \text{mkparams} \leftarrow \text{MKTFHE.SETUP}(1^\lambda): Take a security parameter \lambda as input, and output the public parameter set \text{mkparams}.
- \{sk_i, pk_i\} \leftarrow \text{MKTFHE.KEYGEN}(\text{mkparams}): Take the \text{mkparams} as input, and output secret key sk_i and public key pk_i for a single participant i.
- ct \leftarrow \text{MKTFHE.ENC}(\mu): Encrypt an input bit \mu \in \{0, 1\} and output a TLWE ciphertext ct with the scaling factor \frac{1}{4}. The output ciphertext ct = (a, b) \in \mathbb{Z}^{n+1}, satisfying b + (a, s) \approx \frac{1}{4}\mu.
- \mu \leftarrow \text{MKTFHE.DEC}(ct, \{sk_i\}_{i \in \mathbb{Z}}): Input a TLWE ciphertext ct = (a_i, \ldots, a_k, b) \in \mathbb{T}^{kn+1} and a set of secret keys \{sk_i\}_{i \in \mathbb{Z}}, and output the message \mu \in \{0, 1\} which satisfies b + \sum_{i=1}^{k} (a_i, sk_i) \approx \frac{1}{4}\mu \mod 1.
- ct \leftarrow \text{MKTFHE.NAND}(ct_1, ct_2): Input two TLWE ciphertexts ct_1 = \text{MKTFHE.ENC}(\mu_1) \in \mathbb{T}^{n+1}, ct_2 = \text{MKTFHE.ENC}(\mu_2) \in \mathbb{T}^{n+1}, where ct_1, ct_2 can be constructed by different participants respectively, and output the multi-key ciphertext result ct = \text{MKTFHE.ENC}(\mu_1 \oplus \mu_2) \in \mathbb{T}^{kn+1}:
  - Extend ct_1 and ct_2 to ct'_1 and ct'_2 to make them encrypted under the multi-key \{sk_i\}_{i \in \mathbb{Z}} by putting zero in the empty extending slots.
  - Evaluate GB \((0, \ldots, 0, \frac{1}{8}) - ct'_1 - ct'_2\) and return the result.

For the GB part, we will not discuss it in this paper and refer to the original paper. And we call the evaluated ciphertext as multi-key ciphertext whose dimension is the number of participants in the MKTFHE scheme.

3.2 Distributed Decryption

The decryption algorithm of existing MKTFHE is a single decryptor case, which is the decryptor holds a set of secret keys \{sk_i\}_{i \in \mathbb{Z}} of all participants. However, in practical use, for security reasons, the decryptor should not hold any secret key sk_i of participants. Therefore, the distributed decryption which involved all participants to jointly decrypt a multi-key ciphertext is more practical. The most common distributed decryption for MKFHE \cite{27} has been defined below:

- \pi \leftarrow \text{PartDec}(ct, sk_i): Input a multi-key ciphertext ct under a set of secret keys \{sk_i\}_{i \in \mathbb{Z}} of all participants, and the i-th secret key sk_i, output a partial decryption result \pi_i;
- m \leftarrow \text{FinDec}(\pi_1, \ldots, \pi_k): Input a set of partial decryption results \{\pi_i\}_{i \in \mathbb{Z}} of all participants and output the plaintext of the multi-key ciphertext.
3.3 Homomorphic Gates and Operators Based on MKTFHE

Based on TFHE scheme and its multi-key variant, Jiang et al. [18], designed other binary gates with the same efficiency as NAND gates in MKTFHE, and used their designed binary gates to implement the k-bit complement arithmetic operators, so that the addition, subtraction, multiplication, and division of both positive and negative numbers can be evaluated in MKTFHE. They also used our proposed integer operation of MKTFHE to implement the training of a naive linear regression model by gradient descent method. The experiments show that the time consumption of linear operators such as adders and subtracters increase linearly with the increase of bit-number, and the time consumption of array operators such as multipliers and dividers increase exponentially with the increase of bit-number. Besides, all the bit-number of the above operators need be predefined, so do operands.

The definition of binary gates and operators in MKTFHE is as follows:

- \( ct \leftarrow \text{MKAND}(ct_1, ct_2) \): Input two TLWE ciphertexts \( ct_1 = \text{MKTFHE.ENC}(\mu_1) \) and \( ct_2 = \text{MKTFHE.ENC}(\mu_2) \), extend \( ct_1 \) and \( ct_2 \) to multi-key ciphertexts \( ct'_1 \) and \( ct'_2 \), then evaluate \( GB \left( (0, \cdots, 0, -\frac{1}{8}) + ct'_1 + ct'_2 \right) \) and return the evaluated result \( ct = \text{MKTFHE.ENC}(\mu_1 \land \mu_2) \).

- \( ct \leftarrow \text{MKOR}(ct_1, ct_2) \): Input two TLWE ciphertexts \( ct_1 = \text{MKTFHE.ENC}(\mu_1) \) and \( ct_2 = \text{MKTFHE.ENC}(\mu_2) \), extend \( ct_1 \) and \( ct_2 \) to multi-key ciphertexts \( ct'_1 \) and \( ct'_2 \), then evaluate \( GB \left( (0, \cdots, 0, \frac{1}{8}) + ct'_1 + ct'_2 \right) \) and return the evaluated result \( ct = \text{MKTFHE.ENC}(\mu_1 \lor \mu_2) \).

- \( ct \leftarrow \text{MKNOT}(ct_1) \): Input a TLWE ciphertext \( ct_1 = \text{MKTFHE.ENC}(\mu_1) \), extend \( ct_1 \) to multi-key ciphertext \( ct'_1 \), then evaluate \( (0, \cdots, 0, \frac{1}{8}) - ct_1 \) and return the evaluated result \( ct = \text{MKTFHE.ENC}(\neg \mu_1) \).

- \( c[l] \leftarrow \text{MKENC}(l, m) \): Input bit length \( l \) and a \( l \)-bit plaintext integer \( m \), encode \( m \) as complement \( m' [l] \in \{0, 1\}^l \) and call \( \text{MKTFHE.ENC}(m' [l]_{i \in [l]}) \) \( k \) times to construct \( k \)-bit TLWE ciphertext \( c[l] \) to return.

- \( ct[l] \leftarrow \text{MKADD}(ct_1[l], ct_2[l]) \): Input two \( l \)-bit TLWE ciphertexts \( ct_1[l] \) and \( ct_2[l] \), which are from \( m_1 \) and \( m_2 \) complement encoded and call \( \text{MKTFHE.ENC} \) to encrypt, and output a \( l \)-bit TLWE ciphertext \( ct[l] \) which is a multi-key ciphertext of \( m_1 + m_2 \).

- \( ct[l] \leftarrow \text{MKSUB}(k, ct_1[l], ct_2[l]) \): Input two \( l \)-bit TLWE ciphertexts \( ct_1[l] \) and \( ct_2[l] \), which are from \( m_1 \) and \( m_2 \) complement encoded and call \( \text{MKTFHE.ENC} \) to encrypt, and output a \( l \)-bit TLWE ciphertext \( ct[l] \) which is a multi-key ciphertext of \( m_1 - m_2 \).

- \( ct[2l] \leftarrow \text{MKMUL}(k, ct_1[l], ct_2[l]) \): Input two \( l \)-bit TLWE ciphertexts \( ct_1[l] \) and \( ct_2[l] \), which are from \( m_1 \) and \( m_2 \) complement encoded and call \( \text{MKTFHE.ENC} \) to encrypt and output a \( 2l \)-bit TLWE ciphertext \( ct[2l] \) which is a multi-key ciphertext of \( m_1 \times m_2 \).

- \( ct[l] \leftarrow \text{MKDIV}(k, ct_1[2l], ct_2[l]) \): Input a \( 2l \)-bit TLWE ciphertext \( ct_1[2l] \) and an \( l \)-bit TLWE ciphertext \( ct_2[l] \), which are from \( m_1 \) and \( m_2 \) complement encoded and call \( \text{MKTFHE.ENC} \) to encrypt and output an \( l \)-bit length TLWE ciphertext \( ct[l] \) which is multi-key ciphertext of \( m_1 \div m_2 \).
Note that the input of the gate circuit is a single bit ciphertext, while the input of the operator is a multi-bit ciphertext. Therefore, before inputting a multi-bit integer, first encode it as complement, and then encrypt it by bit. In addition, the bit-number of the multiplication and division input data and output data are different, which is prone to data overflow or the bits of the input data do not match the operator.

3.4 Secret Sharing Based on Arithmetic Circuit

The secret sharing protocol on arithmetic circuit is carried out on a finite field. In the secure 2-party computation, the $l$-bit value $x$ is shared by the participants into two elements on $\mathbb{Z}_2^l$ ring which are sent to two computing parties $P_0$ and $P_1$ respectively. Make $[x]_0^A$, $[x]_1^A$ represents the sub secret owned by the computing party $P_i$, and the superscript $A$ represents the secret share on the arithmetic circuit. Secret sharing on arithmetic circuits is in $\mathbb{Z}_2^l$ which satisfies $[x]_0^A + [x]_1^A = x \mod 2^l$ where $[x]_0^A$, $[x]_1^A \in \mathbb{Z}_2$. So, the participants can share and reconstruct the secret $x$ by the following algorithms:

- $\{[x]_0^A, [x]_1^A\} \leftarrow Share(x)$: Input an $l$-bit secret $x$, randomly choose $r \in \mathbb{Z}_2^l$, set $[x]_0^A = x - r$, $[x]_1^A = r$ and then output $[x]_0^A$, $[x]_1^A$.
- $x \leftarrow Rec([x]_0^A, [x]_1^A)$: Input $[x]_0^A, [x]_1^A$, compute $[x]_0^A + [x]_1^A$, then output the result.

In this protocol, if one party does not abide by the rules and sends the wrong value during the final reconstruction, the honest party cannot reconstruct the secret, while the fraudulent party can reconstruct the real secret. Therefore, this agreement is semi-honest and the participants need to abide by the rules of the agreement.

Besides, the addition operation of this protocol is free and both computing parties can directly perform the calculation locally which follows below:

- $\{[z]_0^A, [z]_1^A\} \leftarrow Add(x, y)$: Input the secret shares of $x, y$, compute $[z]_0^A = [x]_0^A + [y]_0^A$, $[z]_1^A = [x]_1^A + [y]_1^A$, and output $[z]_0^A, [z]_1^A$.

The addition of this protocol satisfies the below equation:

$$[z]_0^A + [z]_1^A = \left([x]_0^A + [y]_0^A\right) + \left([x]_1^A + [y]_1^A\right)$$

3.5 Machine Learning

Gradient Descent Gradient descent (GD) is a classic approximation method for obtaining a minimum of a function $J(\Theta)$ by iteration which is widely used in
the training of machine learning. With the training data set, people can obtain an excellent model after many iterative calculations. The iterative equation is described as follows:

$$\Theta^{(n+1)} = \Theta^{(n)} - \alpha \nabla \Theta,$$

where $\alpha$ is a learning rate which means the gradient descent minimum in each iteration, $\nabla \Theta$ means the gradient of coefficient $\Theta$ and coefficient $\Theta$ can be initialized as a vector of random values.

In practical application, there are different GD methods derived. For example, the most classic Batch Gradient Descent (BGD) uses all the training data to update the coefficient in each iteration, and makes full use of all the data in each iteration. So, it can be more accurate towards the direction of the extreme value. Assuming that the sample size of the training data set is $n$, the gradient expression of BGD is as follows:

$$\nabla \Theta = \frac{1}{n} \sum_{i=1}^{n} \frac{\partial J_i(\Theta)}{\partial \Theta}.$$

However, BGD needs to calculate all training data set in each iteration. When the sample size of the training set is large, it will consume more time. Therefore, Stochastic Gradient Descent (SGD) is introduced which is to update the coefficients by calculating a single training data in each iteration. The gradient expression of SGD is as follows:

$$\nabla \Theta = \frac{\partial J_i(\Theta)}{\partial \Theta}.$$

But SGD is greatly affected by a single sample of the training set, which will produce a concussion effect and reduce the efficiency. For this reason, Gradient Descent with Momentum (GDM) is proposed which considers the previous gradient and updates the coefficient in a weighted way. The gradient expression of GDM is as follows:

$$\nabla \Theta^{(n+1)} = \beta \nabla \Theta^{(n)} + (1 - \beta) \frac{\partial J_i(\Theta)}{\partial \Theta},$$

where $\nabla \Theta^{(0)}$ can be initialized to zero. Since each weighted average gradient contains the information of the previous gradient, the oscillation amplitude of the coefficient is less than SGD, so the general optimization effect is better than SGD.

**Logistic Regression** Logistic regression is a generalized linear regression model. It is a classical method to solve the binary classification problem by using the activation function. The binary classification problem refers to that the output
of the predicted value $y$ has only two values (0 or 1). E.g., in a spam filtering system, input the features of the mail, and predict whether the mail is spam or normal. Therefore, the activation function is used to bound the output of prediction between 0 and 1. In the traditional logistic regression, the activation function is defined as Sigmoid function, function $f(x) = \frac{1}{1 + e^{-x}}$. As shown in Fig. 1, the two tails of the logistic function converge to 0 and 1. At the same time, although introducing the activation function to logistic regression, the BGD method can still obtain the global optimal solution instead of the local optimal solution. The BGD method for logistic regression updates the coefficients in each iteration as follows:

$$z = \theta_0 + \theta_1 x_1 + \cdots + \theta_n x_n = \theta^T x$$

$$h_\theta (x) = f(\theta^T x) = \frac{1}{1 + e^{-\theta^T x}}$$

$$\theta_j = \theta_j - \alpha \frac{1}{m} \sum_{i=1}^{n} (h_\theta (x_i) - y_i) x_i^j.$$

The phase to calculate the predicted output $h_\theta (x_i)$ is called forward propagation, and the phase to calculate the gradient $\alpha \frac{1}{m} \sum_{i=1}^{n} (h_\theta (x_i) - y_i) x_i^j$ is called backward propagation.

**Neural Networks** Neural networks are a more generalized regression model compared by logistic regression to learn more complex relationships between high dimensional input data and multiple output label. Fig. 2 shows an example of a neural networks. Each node (also called neurons) in the hidden layer and the output layer is an instance of logistic regression and is associated with an activation function and a coefficient vector. Traditional activation functions are like Sigmoid function $f(x)$ or RELU function.

Standard error Back Propagation (BP) neural networks can be trained by GDM method so that the coefficients convergence will be faster than BGD and
more stable than SGD. By the chain rule, the coefficients are updated as follows:

\[ a_i = f(\sum_{i=1}^{m} w_{ji} x_i^k), \hat{y}_j^k = \sum_{i=1}^{n} a_i v_{ji}, \]

\[ \Delta v_{ji}^{(n+1)} = \beta_1 \Delta v_{ji}^{(n)} + (1 - \beta_1)(\hat{y}_j^k - y_j^k)a_i, \]

\[ \Delta w_{ij}^{(n+1)} = \beta_2 \Delta w_{ij}^{(n)} + (1 - \beta_2) a_i (1 - a_i) x_j^k \sum_{j=1}^{p} (\hat{y}_j^k - y_j^k) v_{ji}, \]

\[ v_{ji}^{(n+1)} = v_{ji}^{(n)} - \alpha_1 \Delta v_{ji}^{(n+1)}, w_{ij}^{(n+1)} = w_{ij}^{(n)} - \alpha_2 \Delta w_{ij}^{(n+1)}, \]

where \( m, n \) and \( p \) are the number of neurons in the input layer, hidden layer, and output layer respectively, \( x_i^k \) and \( y_j^k \) is the \( i \)-th feature and \( j \)-th label of \( k \)-th input data, \( a_i \) is the output of \( i \)-th neuron in the hidden layer, \( \hat{y}_j^k \) is the output of \( j \)-th neuron in output layer from \( k \)-th input data, \( \alpha_1, \alpha_2, \beta_1, \beta_2 \) are the four decimal parameters in GDM, \( v_{ji}^{(n)} \) is the \( i \)-th coefficient of \( j \)-th neuron in the output layer in the \( n \)-th iteration and \( w_{ij}^{(n)} \) is the \( j \)-th coefficient of \( i \)-th neuron in the hidden layer in the \( n \)-th iteration.

4 Distributed Decryption Protocol

4.1 Our Security Goal

In MKFHE scheme, participants generally generate their own secret keys independently, encrypt their own data by their own secret key, and decrypt the multi-key ciphertext by all secret keys jointly. Therefore, our security goal of MKFHE decryption is to protect individual single-key encrypted message and common multi-key encrypted message.

We already know that MKFHE is IND-CPA secure \[19\], so the multi-key ciphertext is guaranteed to be secure for both adversaries. However, in the existing MKTFHE scheme, the participant can obtain partial decryption results of other participants \( p_i \) (still encrypted by other’s secret key), but they can get \( \langle a, s_i \rangle \) by computing \( p_i - b \). Then after obtaining multiple partial decryption results, the participants can compute the secret keys of other participants, so as to obtain the clear message of other participants. In addition, if the external adversary obtains all partial decryption results, the computation results under multi-key encryption can be finally obtained by computing \( \sum_{i=1}^{k} p_i - (k - 1)b \).

In order to solve the above problems, we introduce secret sharing technique and propose a new distributed decryption protocol.

So far, we can observe that there are at least two kinds of static (passive) adversaries: an internal adversary and an external adversary. The internal adversary is one of the participants in the scheme, but the external adversary is not. Both adversaries want to know any information about each participant’s
message and the external adversary also hopes to obtain the computation result. To simplify, we take both of them together as semi-honest adversaries $A$. And we assume the adversary $A$ can corrupt any subset of the participants and servers (at least two participants and the server is uncorrupted) which only the data of the corrupted participants but nothing else about the remaining honest participants’ data can be learned by the adversary $A$. We define the security using the framework of Universal Composition (UC) [5].

### 4.2 Our Distributed Decryption Protocol for MKTFHE

Denote the multi-key ciphertext by $\hat{ct} = (a_1, \ldots, a_k, b) \in \mathbb{T}^{kn+1}$, which satisfies $b = \frac{1}{4}m - \sum_{j=1}^k \langle a_j, s_j \rangle + e \pmod{1}$ [6], where $m \in \{0, 1\}$ is the plaintext after decryption of the ciphertext, $k$ is the number of participants, $s_j$ is the private key of the $j$-th participant. The secure distributed decryption algorithm based on MKTFHE is defined as follows:

- **Partial decryption algorithm:**
  
  $p_i \leftarrow \text{Part}\_\text{Dec}(\hat{ct}, s_i)$: The input is the multi-key ciphertext $\hat{ct}$ and the secret key $s_i$ of the $i$-th participant. The output $p_i$ is the partial decryption result of the $i$-th participant, and the computation is $p_i = b + \langle a_i, s_i \rangle$.

- **Final decryption algorithm:**
  
  $\frac{1}{4}m + \bar{e} \leftarrow \text{Fin}\_\text{Dec}(\{p_i\}_{i \in k})$: The input $\{p_i\}_{i \in k}$ is the partial decryption result of all participants, and the output is plaintext with noise before rounding, and the computation is $\frac{1}{4}m + \bar{e} = \sum_{i=1}^k p_i - (k-1)b$.

The protocol can be divided into four steps, as shown in Fig 3, take two participants as an example:

- **Step 1** Partial decryption: each participant $P_1, \ldots, P_k$ uses their own secret key $s_i$ to compute partial decryption $\text{Part}\_\text{Dec}(\hat{ct}, s_i)$ to obtain their personal partial decryption results $p_i$;

- **Step 2** Secret sharing: each participant run secret share for its own partial decryption result $p_i$, and get secret shares $[p_i]^A_0$ and $[p_i]^A_1$, and send them to the cloud server and the decryption party respectively;

- **Step 3** Offline computing: the cloud server and the decryption party receive secret shares received from participants, and then offline compute final decryption $\text{Fin}\_\text{Dec}(\{[p_i]^A_0\}_{i \in k})$ and $\text{Fin}\_\text{Dec}(\{[p_i]^A_1\}_{i \in k})$ respectively, so as to obtain the secret share of final decryption result $[\frac{1}{4}m + \bar{e}]^A_0$ and $[\frac{1}{4}m + \bar{e}]^A_1$.

- **Step 4** Secret reconstruction: the decryption party $DP_1, DP_2$ send the secret share of the final decryption result $[\frac{1}{4}m + \bar{e}]^A_0$ and $[\frac{1}{4}m + \bar{e}]^A_1$ to participants and the participants use the secret recovery protocol to reconstruct the final decryption result and obtain the final decryption result $\frac{1}{4}m + \bar{e}$.
Correctness Proof The correctness of the protocol follows:

\[
\left( \sum_{i=1}^{k} [p_i]_0^A - (k-1) [b]_0^A \right) + \left( \sum_{i=1}^{k} [p_i]_1^A - (k-1) [b]_1^A \right) \\
= \left( \sum_{i=1}^{k} [p_i]_0^A + \sum_{i=1}^{k} [p_i]_1^A \right) - \left( (k-1) [b]_0^A + (k-1) [b]_1^A \right) \\
= \sum_{i=1}^{k} p_i - (k-1)b = \sum_{i=1}^{k} (b + \langle a, s_i \rangle) - (k-1)b \\
= b + \sum_{i=1}^{k} \langle a, s_i \rangle = \frac{1}{4} m + e
\]

If the error \(e\) is less than \(\frac{1}{8}\), the decryption will work correctly.

Security Proof In the UC framework, security is defined by comparing the real world and ideal world. The real world is involved in the protocol, adversary \(\mathcal{A}\), and honest participants. And the ideal world includes the trusted party to represent the protocol, the simulator \(\mathcal{S}\) to simulate the ideal world, and honest participants. If the view of real world and ideal world is indistinguishable, the protocol is secure.

We consider security in the semi-honest model in which all participants and servers follow the protocol exactly. We assume that the two servers are non-colluding. We choose the adversary \(\mathcal{A}\) who corrupts a server \(DP_1\) and all but two of the participants \(\{P_1, \ldots, P_{k-2}\}\) as an example that can cover all scenarios in our security goal. Simulator \(\mathcal{S}\) is to simulate the above in ideal world which submits
the partial decryption of the participants and receives the final decryption from the trusted party.

During the simulating, on behalf of the honest participants $S$ sends a randomized partial decryption share $[p_i]_0^A$, $[p_i]_1^A$ in $\mathbb{Z}_{2^l}$ to $DP_1$ and $DP_2$. This is the only phase where participants are involved. Then each server evaluates independently until the final decryption is recovered.

We can briefly argue that the view of the real world and ideal world is indistinguishable because of the security of the arithmetic secret sharing. The share of partial decryption is generated by participants randomly. Particularly, all messages sent, received, and reconstructed in our protocol are generated using uniformly random shares in both real world our protocol involved and ideal world simulator simulated, so the view of both identically distributed concludes our argument.

5 Privacy-Preserving Distributed Machine Learning

5.1 Pre-Work

Extract Sign Bit Thanks to the encryption and evaluation of MKTFHE is bit by bit, we can easily extract any bit in a multi-bit ciphertext. In complement coding, the highest bit can represent the sign of operand, which is that the highest bit is 0 and the operand is positive, the highest bit is 1 and the operand is negative. Therefore, we can use this property to extract the sign bit of any ciphertext. The operation of extracting the sign bit is defined as follows:

- $ct_{\text{sign}} \leftarrow \text{Extract\_Sign}(ct\ [l])$: Input a $l$-bit length TLWE ciphertext $ct\ [l]$, and output the sign bit $ct_{\text{sign}}$ of $ct\ [l]$.

Cut off and Expand Considering the bit-length of the existing arithmetic operators in MKTFHE is predefined, and the larger the bit-length, the more time consumption. Therefore, flexibly adjusting the bit-number of ciphertext and selecting the less bit-length arithmetic operators can improve the efficiency and accuracy of the overall scheme of machine learning based on MKTFHE.

We continue to use the encoding method in MKTFHE arithmetic operators [18] which is to use complement to encode both positive and negative integers. When the large-bit of ciphertext operands is input the little-bit arithmetic operators, the operands will be automatically cut off the rest bits of the input ciphertext operands, in other words, only the part of data with the same bit-number as the arithmetic operators will be calculated. For example, we can get a 16-bit ciphertext product from an 8-bit multiplier with couple 8-bit ciphertext operands input. And if this 16-bit ciphertext product doesn’t overflow 8-bit size, it can be put into the next 8-bit arithmetic operator directly with a little time of computation. But if the 16-bit ciphertext product overflows the size of 8-bit, it must be put into the 16-bit arithmetic operator in the next calculation with more time consumption and its corresponding the other 8-bit operand must be
expanded from 8-bit to 16-bit. Until the operand recovers the 8-bit size by subsequent operations such as division, we can continue to use the less bit-number operator.

In order to flexibly expand the bit-number of the operand and keep its sign, we design and implement a device named homogenizer, as shown in Fig. 4 and the specific design is as follows:

MKTFHE does not allow the ciphertext to be copied directly (it is considered unsafe), so we use the trivial TLWE(0) and the sign bit $ct_{sign}$ of the original small-bit ciphertext operand to calculate $\text{MKAND}(\text{TLWE}(0), ct_{sign})$, and fill the ciphertext results into the high bits. The operation of expanding is defined as follows:

$ct[l'] \leftarrow \text{Homogenizer}(ct[l], l')$: Input the $l$-bit length TLWE ciphertext $ct[l]$ and the bit-length $l'$, output the $l'$-bit length ciphertext $ct[l']$, the plaintext of which is the same as $ct[l]$.

![Fig. 4. Expand the bit-length of ciphertext](image)

**Compare** In the practical machine learning scheme, the comparison operation is usually required, but the existing MKTFHE scheme cannot support the comparison operation without decryption. We believe that the comparison can be divided into two categories. One is to need to know the comparison results, such as the millionaire problem, and the other is to determine the next calculation through the comparison result, which is similar to branch selection. At present, the comparison in the machine learning scheme is mainly the second category. Therefore, we utilize the Boolean operation and arithmetic operations in MKTFHE to design and implement the basic elements of the comparison operation, named Compare Quads, which is used to pick one from two ciphertext operands based on the results of comparison between the other two ciphertext operands, as shown in Fig. 5. A Compare Quads can be constructed in the following step:

1. Calculate $c_a[l] - c_b[l]$ by calling $c_{\text{mid-result}}[l] \leftarrow \text{MKSUB}(c_a[l], c_b[l])$;
2. Extract the sign bit $c_{\text{sign}}$ of $c_{\text{mid-result}}[l]$ by calling $c_{\text{sign}} \leftarrow \text{Extract_Sign}(c_{\text{mid-result}}[l])$;
3. Reverse $c_{\text{sign}}$ by call $\neg c_{\text{sign}} \leftarrow \text{MKNOT}(c_{\text{sign}}, \text{para})$;
4. Calculate $c_{\text{sign}} \land c_c[n]$ and $\neg c_{\text{sign}} \land c_d[n]$ bit by bit, call $\text{MKAND}(c_{\text{sign}}, c_c[t])$ and $\text{MKAND}(-c_{\text{sign}}, c_d[t])$ which $t$ is from 1 to $n$ and can compute in parallel;
5. Calculate $c_{\text{sign}} \land c_c[n] + -c_{\text{sign}} \land c_d[n]$ by calling $\text{MKADD}(c_{\text{sign}} \land c_c[t], -c_{\text{sign}} \land c_d[t])$ and return the result.

We use Algorithm 1 to evaluate $\text{Compare Quads}$. And the operation of comparison is defined as follows:

- $c_{\text{result}}[l] \leftarrow \text{Compare Quads}(c_a[l], c_b[l], c_c[l], c_d[l])$: Input four $l$-bit length ciphertext $c_a[l], c_b[l], c_c[l], c_d[l]$, if the plaintext of $c_a[l]$ is bigger than $c_b[l]$, then output $c_d[l]$, else output $c_c[l]$.

![Diagram](image)

**Fig. 5. Select a ciphertext by Compare Quads**

---

**Algorithm 1 Compare Quads**

**Input:** MKTFHE parameter set $\text{mkparams}$, four $l$-bit ciphertext $c_a[l] = \text{MKENC}(a, l), c_b[l] = \text{MKENC}(b, l), c_c[l] = \text{MKENC}(c, l), c_d[l] = \text{MKENC}(d, l)$, and the public keys of all participants $\{p_k_i\}_k$

**Output:** $l$-bit ciphertext $c_{\text{result}}[l] = \text{MKENC}((a \geq b) : d : c, l)$

1. Calculate $c_a[l] - c_b[l]$
2. Extract sign bit $c_{\text{sign}}$ of $c_a[l] - c_b[l]$
3. Calculate the result $c_{\text{result}}[l] = c_{\text{sign}} \land c_c[l] + (\neg c_{\text{sign}}) \land c_d[l]$

---

5.2 MKTFHE Friendly Activation Function

At present, the existing MKTFHE only supports integer linear operation and Boolean operation, so how to compute the Sigmoid function in logical regression and neural networks has become a main additional challenge. Prior work shows that polynomials can be used to fit Sigmoid function [1], and high-degree polynomials can achieve very high accuracy [22]. Hence, it is obvious that we can use the above method to implement Sigmoid function, but high-degree polynomials...
will seriously reduce the efficiency, and using low-degree polynomials will lose a lot accuracy.

We refer to the idea in SecureML \cite{25} which discusses that the piecewise function can also achieve high accuracy. Hence, we design a new MKTFHE friendly activation function $g(x)$. In addition, in order to improve the accuracy, we fit the sigmoid function in the form of the tangent at the origin, as shown in Fig. \ref{fig:6}. Considering that MKTFHE only supports integers, we have zoomed the new activation function by 16 times. The function description is as follows:

$$g(x) = \begin{cases} 
16, & x > 2 \\
4x + 8, & -2 \leq x \leq 2 \\
0, & x < -2 
\end{cases}$$

Fig. 6. Our function $g(x)$ and Sigmoid function $f(x)$

Note that the comparison in our proposed activation function belongs to the second category of comparison in the compare part of Subsection 5.1 we have discussed before, which is the comparison results are used for the next calculation instead of the need of knowing the comparison results. Therefore, we can use two \texttt{CompareQuads} in Subsection 5.1 to implement the activation function can be evaluated by Algorithm 2 and Fig. \ref{fig:7}.

\begin{algorithm}
\caption{New activation function}
\textbf{Input:} MKTFHE parameter set $mkparams$, ciphertext $ct[l] = \text{MKENC}(x,l)$ and the public keys of all participants $\{pk_i\}_k$
\textbf{Output:} Ciphertext $c_{result}[l] = \text{MKENC}(g(x),l)$
1. Prepare the ciphertext $c(-2), c(16), c(0)$ and $c(4x + 8)$
2. Calculate $c_{mid-result}[l] = \text{CompareQuads}(ct[n], c(-2), c(0), c(4x + 8))$
3. Calculate $c_{result}[l] = \text{CompareQuads}(c_{mid-result}[l], c(16), c_{mid-result}[l], c(16))$
\end{algorithm}
5.3 Privacy-Preserving Machine Learning based on MKTFHE

After we propose the new MKTFHE friendly activation function, we will utilize it to compute the back propagation in the logistic regression and neural networks. Considering the accuracy, we continue to use the Sigmoid function to calculate the partial derivative and maintain the structure of the iterative equation. And prior research also shows that if we change to compute the partial derivative of the linear activation function, the cross-entropy function is no longer convex, and the accuracy of training will incur more losses [25].

In addition, since MKTFHE is for integer and Boolean, it is necessary to zoom the learning rate and other parameters into integers, and modify the relevant calculation equation, so as to ensure that the model coefficients after training are also enlarged in proportion.

We set the expansion factor $q$, use the integer learning rate $\alpha'$ and the new iterative computation for logistic regression is below:

$$h_{\theta}(x) = g(x)$$
$$\theta_j = \theta_j q - \alpha' \frac{1}{m} \sum_{i=1}^{m} (h_{\theta}(x_i) - y_i) x_{ij}$$

There are $m$ neurons in the input layer, $n$ neurons in the hidden layer and $p$ neurons in the output layer. Like the above logistic regression, we set the expansion factor $q$, use the integer learning rate $\alpha_1'$, $\alpha_2'$, $\beta_1'$, $\beta_2'$ and the rest definition is the same as Section [3.5] the iterative computation for neural networks is below:

$$o_i = g \left( \sum_{i=1}^{m} w_{ji} x_i q \right), \hat{y}_j = \sum_{i=1}^{n} o_i v_{ji} q$$

$$\Delta v_j^{(n+1)} = \beta_2' \Delta v_j^{(n)} q - \alpha_1' \Delta v_j^{(n+1)}$$

$$\Delta w_{ij}^{(n+1)} = \beta_2' \Delta w_{ij}^{(n)} + (q - \beta_2') (q - o_i) x_j q \sum_{j=1}^{p} (\hat{y}_j - y_j) v_{ji}$$

$$v_{ji}^{(n+1)} = v_{ji}^{(n)} q - \alpha_1' \Delta v_{ji}^{(n+1)}, w_{ij}^{(n+1)} = w_{ij}^{(n)} q - \alpha_2' \Delta w_{ij}^{(n+1)}$$
5.4 Our Framework

After implementing the privacy preserving logical regression and neural network training, we replace the original decryption with our proposed distributed decryption protocol, and finally propose a distributed privacy preserving machine learning framework based on MKTFHE, including four types of entities: participants, a cloud server, a CRS server, and a decryption party. The participants want to outsource computation and each of them holds their own part of data for model training which should not be learnt by a cloud server and other participants; The cloud servers are usually composed of one or more high-performance servers, which do not have their own data and only provide computing power. The CRS server is only responsible for generating the public parameters (that is, common reference string) of the framework which can be included in the cloud server. The decryption party only joins in the distributed decryption and does not need too much computing power, which can be acted by a participant or a single server.

We take two participants as examples. The steps of the whole scheme are shown in the Fig. 8:

Step 1 Set up parameters: The CRS server call MKTFHE.SETUP(1\(^\lambda\)) to generate the mkparams and communicates with the participants on the expansion factor \(q\), then sends the set of parameters and factors \{mkparams, q\} to each participant and the cloud server.

Step 2 Preprocess and encrypt data: Each participant uses the expansion factor \(q\) to zoom or round the origin data and uses the mkparams to call MKTFHE.KEYGEN(mkparams) to generate their own secret key and public key, then utilizes their own secret key to encrypt the preprocessed data bit by bit by calling MKTFHE.ENC(m) and finally sends the public key and the ciphertext to the cloud server.

Step 3 Train ML models: The cloud server first expands the single key ciphertext from each participant to multi-key ciphertext, then uses the arithmetic operators like MKADD, MKSUB, MKMUL, MKDIV, and our new designed Homogenizer, Compare Quads to train the model. In the end, the cloud server sends the ciphertext model to decryption party and all participants.

Step 4 Decrypt data: All the participants, the decryption party and the cloud server run the distributed decryption protocol together and the participants will get the plaintext in the end.

6 Implementation and Experiment

6.1 Implementation of Distributed Decryption Protocol

The Implementation and Experimental Environment Our code for the distributed decryption protocol is written in C++, mainly using the arithmetic
Fig. 8. Framework of privacy-preserving machine learning

secret sharing of ABY [14] which is a very efficient two-party secure computing scheme. Considering that the final decryption only involves addition and subtraction without multiplication, this experiment neither needs to use oblivious transfer (OT) and other operations that require online interaction, nor needs a semi honest third party (STP) to assist in generating multiplication triples.

Noted that the ABY library only uses unsigned number and negative number cannot be directly computed. Therefore, we naturally regard the unsigned numbers as the number encoded by complement code, and convert them into signed original code data after the final operation. In addition, since the finite field used in MKTFHE scheme is a 32-bit tours, we straightly use the 32-bit arithmetic secret sharing in ABY library for computation.

The experiment of this subject runs on the Linux environment based on the following configuration.

(1) Cloud server $S_0$ and $S_1$, is configured as: Intel Xeon gold 5220 @2.2GHz processor, 256GB memory, and the operating system is Ubuntu 18.04 LTS;
(2) The client is Windows PC, Intel Core i7-8750H@2.20GHz processor, 16GB memory, and the operating system is Windows 10.

In the experiment of LAN, we use two servers located in the same area. The network bandwidth is 512MB/s and the network delay is 0.35ms.

Accuracy and Efficiency Analysis In this experiment, we compare with the original decryption scheme in MKTFHE, and set the participants $k$ to 2, 4 and 8 respectively. We test in 10 groups for both decryption scheme and each group
includes 1000 bits ciphertext. Then we record the average time of decryption. Note that in the specific implementation, in the distributed decryption protocol, we use the SIMD technique in ABY library for parallel optimization to improve the efficiency. The experimental results are shown below:

| Participants $k$ | MKTFHE/s | Our protocol/s | Accuracy |
|------------------|----------|----------------|----------|
| 2                | 0.024    | 0.261          | 100%     |
| 4                | 0.050    | 0.268          | 100%     |
| 8                | 0.112    | 0.263          | 100%     |

The result in Table 1 shows that compared with the original MKTFHE, the efficiency of our scheme is relatively lower, but it’s still acceptable. We think the reason is mainly in the establishment of secret sharing scheme and ciphertext transmission because the original scheme does not involve additional schemes and transmissions. By using the SIMD technique in ABY library, the decryption time basically remains unchanged with the increase of participants, while the decryption time of the original MKTFHE scheme increases linearly with the increase of participants. Therefore, in the case of multi-party participation, our scheme has more advantages in both security and efficiency.

### 6.2 Implementation of Privacy Preserving Distributed Machine Learning

**Data preprocessing** Considering that MKTFHE only supports integer and Boolean, we need to preprocess the input data. We have two methods to preprocess, one is rounding and the other is zooming. The input data of logistic regression is in a rather large range while the input data of neural networks is relatively small, so we apply the rounding method on logistic regression and the zooming method on neural networks to keep the data precise. We store the zooming factor for the following computation to guarantee accuracy in an acceptable range.

**Implementation of Privacy Preserving Logistic Regression** The input data are generated by ourselves which are several sets of linear data with small random noise, and we mainly use 16-bit and 32-bit operators in this implementation.

We first use the 7-order Taylor polynomial (high enough order) formed Sigmoid function and our proposed activation function as the activation function in logistic regression to train the models with plaintext of integer and float data. The result shows in Table 2 that in both integer and floating numbers, the accuracy of using a 7-order Taylor polynomial as an activation function is the
highest, and using our proposed activation function can be close to that of a 7-order Taylor polynomial.

Then, we utilize the operators and other tools in MKTFHE to train the logistic regression models with the above different types of activation functions in ciphertext. In addition to recording the accuracy and time in training in Table 3, we also compare the computation time of different activation functions under MKTFHE in Table 4. The result of the experiments shows that our scheme has no accuracy loss which means that the model trained in ciphertext is the same as that in plaintext, and the loss only occurs in the integer transfer stage. Using our proposed activation function can shorten the computing activation function time in ciphertext by 10 times and significantly shorten the training time compared with the 7-order Taylor polynomial and the accuracy is close to it. Note that we also compare the 3-order Taylor polynomial with our proposed function in both plaintext and ciphertext and the result shows that comparing with the 3-order Taylor polynomial we can also shorten the computing activation time by 5 times with much better accuracy.

Table 2. Logistic regression accuracy in plaintext

| Data type     | 7-order Taylor polynomial | 3-order Taylor polynomial | Our function |
|---------------|---------------------------|---------------------------|--------------|
| Floating data | 98%                       | 85%                       | 95%          |
| Integer data  | 95%                       | 80%                       | 92%          |

Table 3. Logistic regression accuracy in ciphertext

| Activation function       | Accuracy | Training time/iter/piece/s |
|---------------------------|----------|---------------------------|
| 7-order Taylor polynomial | 95%      | 4049                      |
| 3-order Taylor polynomial | 80%      | 2549                      |
| Our function              | 92%      | 611                       |

Table 4. Computing activation function in ciphertext

|                     | 7-order Taylor polynomial | 3-order Taylor polynomial | Our function |
|---------------------|---------------------------|---------------------------|--------------|
| Time/s              | 1440                      | 549                       | 130          |

Implementation of Privacy Preserving Neural Networks We use the Iris data set in sklearn [30] as input data for neural networks, half of them for
training and the rest for prediction. Like the above logistic regression, we also implement neural networks in both plaintext and ciphertext.

In plaintext, we use the above different kinds of functions as activation functions to train the model with both integer data and floating data, and the same in ciphertext. Note that we also compute every neuron in the same layer in parallel to optimize the code. The result of the experiments is shown in Table 5.

As the result shown in Table 6 using a 7-order Taylor polynomial as an activation function is more accurate and costs more time, but using our proposed activation function can greatly reduce the training time with close accuracy to it. Note that we also compare our function with 3-order Taylor polynomial and the result shows that we shorten the training time with much better accuracy as well.

| Data type      | 7-order Taylor polynomial | 3-order Taylor polynomial | Our function |
|---------------|---------------------------|----------------------------|--------------|
| Floating data | 96.23%                    | 72.17%                     | 95.46%       |
| Integer data  | 94.67%                    | 68.12%                     | 94.15%       |

| Activation function | Accuracy | Training time/iter/piece/s |
|---------------------|----------|---------------------------|
| 7-order Taylor polynomial | 94.67%   | 7301                       |
| 3-order Taylor polynomial | 68.12%   | 6736                       |
| Our function         | 94.15%   | 4654                       |

Parameters We choose the same parameters in MKTFHE. The achievement estimated security level of our scheme is 110-bit while the dimension of the TLWE problem is $k = 1$.

7 Conclusion and Discussion

In this paper, we propose privacy preserving logistic regression and neural networks with distributed decryption protocol based on MKTFHE. Firstly, we introduce secret sharing to protect the partial decryption and final decryption. Secondly, we design homogenizer and compare quads to implement our proposed MKTFHE friendly activation function. Then, we utilize them to train privacy preserving logistic regression and privacy preserving neural networks.
Finally, we formalize our distributed privacy preserving machine learning framework. The experimental results show that the efficiency of our distributed decryption protocol is acceptable. Compared with using Sigmoid function, with our activation function, the efficiency is greatly improved and the accuracy is basically unchanged.

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