Support Equalities Among Ribbon Schur Functions

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Abstract of Poster Presentation: Schur functions are certain symmetric polynomials that are indexed by partitions; they form what is often considered the most important linear basis for the space of symmetric functions and are of particular importance to the fields of algebraic combinatorics and representation theory. Relatedly, a skew Schur function corresponds to a pair of partitions $\lambda$ and $\mu$ (which determine a skew shape $\lambda/\mu$). A skew Schur function $s_{\lambda/\mu}$ can be written as a linear combination of some Schur functions $s_{\nu_1}, s_{\nu_2}, \ldots, s_{\nu_n}$, and the set $\{s_{\nu_1}, s_{\nu_2}, \ldots, s_{\nu_n}\}$ is called the Schur support of $s_{\lambda/\mu}$. A result of P.R.W. McNamara implies that two connected ribbons (i.e. connected skew shapes without $2 \times 2$ shapes as subdiagrams) can have the same Schur support only if one is obtained by permuting row lengths of the other. We present substantial progress towards classifying when a permutation $\pi \in S_m$ of row lengths of a connected ribbon $\alpha$ produces a ribbon $\alpha_\pi$ with the same Schur support as $\alpha$; when this occurs for all $\pi \in S_m$, we say that $\alpha$ has full equivalence class. Our main results, the proofs of which incorporate the celebrated Littlewood-Richardson rule, include a sufficient condition for a connected ribbon $\alpha$ to have full equivalence class. Additionally, we prove a separate necessary condition, which we conjecture to be sufficient.

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