Towards the Unification of Gravity and other Interactions: What has been Missed?

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Abstract.
Faced with the persisting problem of the unification of gravity with other fundamental interactions we investigate the possibility of a new paradigm, according to which the basic space of physics is a multidimensional space $C$ associated with matter configurations. We consider general relativity in $C$. In spacetime, which is a 4-dimensional subspace of $C$, we have not only the 4-dimensional gravity, but also other interactions, just as in Kaluza-Klein theories. We then consider a finite dimensional description of extended objects in terms of the center of mass, area, and volume degrees of freedom, which altogether form a 16-dimensional manifold whose tangent space at any point is Clifford algebra $\text{Cl}(1,3)$. The latter algebra is very promising for the unification, and it provides description of fermions.

1. Introduction
Our current physical theories are mostly formulated in terms of spacetime, a 4-dimensional manifold whose points correspond to ‘events’ associated with, e.g., collisions of particles. General relativity is based on the concept of curved spacetime which enables description of gravity. One possible generalization is to consider higher dimensional spacetimes which, besides 4D gravity, include other fundamental interactions. Another possible generalization is to assume that fundamental objects are not point particles but strings, whose excitations contain fundamental particles and interactions. Though very promising, those theories have encountered numerous difficulties, and currently we still do not have a generally accepted ‘unified theory’.

When considering extended objects, there is a possibility of formulating the theory in terms of the corresponding configuration space. Such idea has been considered before in numerous illuminating works by Barbour [1]. For some other approaches see refs. [2]. Here I will adopt the view that general relativity should be extended to configuration space $C$, a multidimensional manifold equipped with metric, connection and curvature. In this picture a 4-dimensional spacetime is just a subspace of $C$, associated with the degrees of freedom of a single point particle. In order to describe not only 4D gravity, but also other interactions, we can employ the extra dimensions of $C$. 
2. Generalizing relativity

2.1. Configuration space replaces spacetime

Let us consider a system of point particles whose coordinates are $X^\mu_i$, where $\mu = 0, 1, 2, 3$, $i = 1, 2, ..., N$, $N$ being the number of particles in the configuration. We can consider $X^\mu_i$ as coordinates in a $4 \times N$-dimensional configuration space $C$, therefore it is convenient to introduce a more compact notation:

$$\dot{X}^\mu_i \equiv \dot{X}^{(i\mu)} \equiv \dot{X}^M, \quad M = (i\mu)$$ (1)

Let us assume that the action for such system is

$$I[X^M] = M \int d\tau \ [\dot{X}^M \dot{X}^N G_{MN}(X^M)]^{1/2}$$ (2)

which is proportional to the length of a worldline in $C$. Constant $M$ has the role of mass in $C$, whereas $\tau$ is an arbitrary monotonically increasing parameter.

An action which is equivalent to (2) is the Schild action

$$I[X^M] = \int d\tau \ \dot{X}^M \dot{X}^N \frac{M}{K} G_{MN}(X^M)$$ (3)

From the equations of motion derived from (3) it follows that

$$\dot{X}^M \dot{X}^M G_{MN} = K^2$$ (4)

Therefore (3) is a gauge fixed form of the action (2).

We will assume that configuration space $C$ is a manifold equipped with metric $G_{MN}$, connection and curvature (that in general does not vanish).

In particular, for the block diagonal metric

$$G_{MN} \equiv G_{(i\mu)(j\nu)} = g_{(i\mu)(j\nu)} =
\begin{pmatrix}
g_{\mu\nu}(x_1) & 0 & 0 & \cdots \\
0 & g_{\mu\nu}(x_2) & 0 & \cdots \\
0 & 0 & g_{\mu\nu}(x_3) & \cdots \\
\vdots & \vdots & \vdots & \ddots
\end{pmatrix}$$ (5)

the action (3) becomes

$$I[\dot{X}^\mu_i] = \int d\tau \sum_i \dot{X}^\mu_i \dot{X}^\nu_i \frac{M}{K} g_{\mu\nu}(X_i^\nu)$$ (6)

Rewriting eq. (4) as

$$\frac{\dot{X}_1^2}{K^2} = 1 - \frac{\dot{X}_2^2}{K^2} - \frac{\dot{X}_3^2}{K^2} - \cdots - \frac{\dot{X}_N^2}{K^2}$$ (7)

and introducing a new constant $m_1^2 \equiv p_1^2 = M^2 - p_2^2 - p_3^2 - \cdots - p_N^2$, we find

$$\frac{M^2}{K^2} = \frac{m_1^2}{k_1^2} \quad \text{or} \quad \frac{M}{K} = \frac{m_1}{k_1}$$ (8)

Here $k_i^2 = X_i^2 = g_{\mu\nu}(x_i) \dot{X}^\mu_i \dot{X}^\nu_i$, and $p_i^2 \equiv p_i^\mu p_i^\nu g_{\mu\nu}$. Since the above derivation can be repeated for any $i = 1, 2, ..., N$, we have

$$\frac{M}{K} = \frac{m_i}{k_i}$$ (9)

1 Symbol $M$, of course, has a different meaning when occurring as an index.
Inserting the latter relation into the action (6) we obtain the Schild action for a system of
relativistic particles moving in a gravitational field $g_{\mu\nu}$, in fact the sum of Schild action for
individual particles. Thus the ordinary relativistic theory for a many particle system is just a
special case of the general action (2) or (3) for a particular, block diagonal metric (5).

By allowing for a more general metric, that cannot be transformed into the form (5) by a
choice of coordinates in $\mathcal{C}$, we go beyond the ordinary theory.

Configuration space $\mathcal{C}$ is the space of possible “instantaneous” configurations in $M_4$. Its points
are described by coordinates $x^M \equiv x^M_i$. A given configuration traces a worldline $x^M = X^M(\tau)$
in $\mathcal{C}$.

A dynamically possible worldline in $\mathcal{C}$ is a geodesic in $\mathcal{C}$, and it satisfies the variation principle
based on the action (2).

Instead of considering a fixed metric of $\mathcal{C}$, let us assume that the metric $G_{MN}$ is dynamical,
so that the total action contains a kinetic term for $G_{MN}$:

$$I[X^M, G_{MN}] = I_m + I_g$$

where

$$I_m = \int d\tau \; M \left( G_{MN} \dot{X}^M \dot{X}^N \right)^{(1/2)} = \int d\tau \; M \left( G_{MN} \dot{X}^M \dot{X}^N \right)^{(1/2)} \delta^D(x - X(\tau)) d^Dx$$

and

$$I_g = \frac{1}{16\pi G_D} \int d^Dx \sqrt{|G|} \mathcal{R}$$

Here $\mathcal{R}$ is the curvature scalar in $\mathcal{C}$. So we have general relativity in configuration space $\mathcal{C}$. We
have arrived at a theory which is analogous to Kaluza-Klein theory. Configuration space is a
higher dimensional space, whereas spacetime $M_4$ is a 4-dimensional subspace of $\mathcal{C}$, associated
with a chosen particle.

The concept of configuration space can be used either in macrophysics or in microphysics.
Configuration space associated with a system of point particles is finite dimensional. Later we
will discuss infinite dimensional configuration spaces associated with strings and branes.

2.2. Equations of motion for a configuration of point particles

The equations of motion derived from the action (10) are the Einstein equations in configuration
space $\mathcal{C}$. Let us now split the coordinates of $\mathcal{C}$ into 4-coordinates $X^\mu \equiv X^{i\mu}$, $\mu = 0, 1, 2, 3$
associated with position of a chosen particle, labeled by 1, and the remaining coordinates $X^\bar{M}$:

$$X^M = (X^\mu, X^\bar{M})$$

The quadratic form occurring in the action (2) can then be split—according to the well known
procedure of Kaluza-Klein theories—into a 4-dimensional part plus the part due to the extra
dimensions of configuration space $\mathcal{C}$:

$$\dot{X}^M \dot{X}^N G_{MN} = \dot{X}^\mu \dot{X}^\nu g_{\mu\nu} + \text{extra terms}$$

More precisely, if for the metric of $\mathcal{C}$ we take the ansatz

$$G_{MN} = \left( \begin{array}{cc}
  g_{\mu\nu} + A^\bar{M}_\mu A^\bar{N}_\nu \phi_{\bar{M}\bar{N}} & A^\bar{N}_\mu \phi_{\bar{M}\bar{N}} \\
  A^\bar{N}_\nu \phi_{\bar{M}\bar{N}} & \phi_{\bar{M}\bar{N}}
\end{array} \right)$$

then we obtain

$$\dot{X}^M \dot{X}^N G_{MN} = \dot{X}^\mu \dot{X}^\nu g_{\mu\nu} + \dot{X}_{\bar{M}} \dot{X}_{\bar{N}} \phi_{\bar{M}\bar{N}}$$
\[ \dot{X}_{\bar{M}} = G_{MN} \dot{X}^N = A_{\bar{M}} \dot{X}^\mu + \phi_{\bar{M} \bar{N}} \dot{X}^\bar{N} \] (17)

Inserting expression (16) into the action (11), we have

\[ I_m[X^\mu, X^{\bar{M}}] = M \int d\tau \left[ \dot{X}^\mu \dot{X}^\nu g_{\mu \nu} + \phi^\bar{M} \bar{N} (A_{\bar{M}} \dot{X}^\mu + \phi_{\bar{M} \bar{J}} \dot{X}^\bar{J}) (A_{\bar{N} \nu} \dot{X}^\nu + \phi_{\bar{N} \bar{K}} \dot{X}^\bar{K}) \right]^{1/2} \] (18)

Let us now assume that the “internal” subspace of \( \mathcal{C} \) admits isometries given by the Killing vector fields \( k^\alpha_\alpha \). Index \( \alpha \) runs over the independent Killing vectors, whereas \( \bar{J} \), like \( \bar{M}, \bar{N} \), runs over the “internal” coordinates. Then, as it is customary in Kaluza-Klein theories, we write

\[ A^\bar{J} = k^\alpha_\alpha A^\alpha_\mu \] (19)

The metric \( \phi^{\bar{M} \bar{N}} \) of the internal space can be rewritten in terms of a metric \( \varphi^{\alpha \beta} \) in the space of isometries:

\[ \phi^{\bar{M} \bar{N}} = \varphi^{\alpha \beta} k^\alpha_\alpha k^\beta_\beta + \phi^{\text{extra}} \] (20)

Here \( \phi^{\text{extra}} \) are additional terms due to the directions that are orthogonal to isometries. For particular internal spaces \( \bar{C} \), those additional terms may vanish.

The projections of momenta onto Killing vectors are

\[ p_{\alpha} \equiv k^\alpha_\alpha p_\beta \] (21)

We may chose a coordinate system in \( \mathcal{C} \) in which

\[ k^\alpha_\alpha = \left(k^\mu_\alpha, k^{\bar{M}}_\alpha\right), \quad k^\mu_\alpha = 0, \quad k^{\bar{M}}_\alpha \neq 0 \] (22)

Variation of the action (18) gives

\[ \frac{1}{\lambda} \frac{d}{d\tau} \left( \frac{X^\mu}{\lambda} \right) + (4) \Gamma^\mu_{\rho \sigma} \frac{\dot{X}^\rho X^\sigma}{\lambda^2} + \frac{p_\alpha}{m} \Gamma^\alpha_\mu \frac{\dot{X}^\nu}{\lambda} + \frac{1}{2m^2} \left( \varphi^{\alpha \beta} - \varphi^{\alpha \beta} k^\beta_\beta A^\alpha_\mu \right) p_\alpha p_\beta + \frac{1}{\lambda m} \frac{dm}{d\tau} = 0 \] (23)

where \( \lambda = \left( X^\mu \dot{X}^\nu g_{\mu \nu} \right)^{1/2} \).

In the above derivation we have used the following relation between the 4-dimensional and the higher dimensional mass

\[ \frac{m}{M} = \left( \frac{X^\mu \dot{X}^\nu g_{\mu \nu}}{X^M \dot{X}^N G_{MN}} \right)^{1/2} \] (24)

which, for the special case of the block diagonal metric \( G_{MN} \) has been already given in eq. (8).

From eq. (23), in which \( p_\alpha \) have the role of gauge charges, we see that \( m \) has the role of inertial mass in 4-dimensions. The 4-dimensional mass \( m \) is given by higher dimensional mass \( M \) and the contribution due to the extra components of momentum \( P_{\bar{M}} \):

\[ m^2 = g^{\mu \nu} p_\mu p_\nu = M^2 - \phi^{\bar{M} \bar{N}} p_\bar{M} p_\bar{N} = M^2 - \varphi^{\alpha \beta} p_\alpha p_\beta \] (25)

These extra components \( P_{\bar{M}} \) are in fact momenta of all other particles within the configuration. In general \( m \) is not constant, but in configuration spaces with suitable isometries it may be
constant. In the last step in eq. (25), for simplicity, we have considered the case in which the terms with $\rho_{\mu\nu}^{MN}$, occurring in eq. (20), vanish.

A configuration under consideration can be the universe. Then, according to this theory, the motion of a subsystem, approximated as a point particle, obeys the law of motion (23). Besides the usual 4-dimensional gravity, there are extra forces. They come from the generalized metric, i.e., the metric of configuration space. Since the inertial mass of a given particle depends on momenta of other particles and their states of motion (their momenta), the Mach principle is automatically incorporated in this theory. Such approach opens a Pandora’s box of possibilities to revise our current views on the universe. Persisting problems, such as the horizon problem, dark matter, dark energy, the Pioneer effect, etc., can be examined afresh within this theoretical framework based on the concept of configuration space.

Locality, as we know it in the usual 4-dimensional relativity, no longer holds in this new theory, at least not in general. But in particular, when the metric of $C$ assumes the block diagonal form (5), we recover the ordinary relativity (special and general), together with locality. However, it is reasonable to expect that metric (5) may not be a solution of the Einstein equations in $C$. Then the ordinary relativity, i.e., the relativity in $M_4$, could be recovered as an approximation only. Even before going into the intricate work of solving the equations of general relativity in $C$, we already have a crucial prediction, namely that locality in spacetime holds only approximately. When considering the universe within this theory, we have to bear in mind that the concept of spacetime has to be replaced by the concept of configuration space. Locality in $M_4$ has thus to be replaced by locality in $C$. More technically this means that, instead of differential equations in $M_4$ (e.g., the Einstein equations), we have differential equations in $C$: a given configuration (a point in $C$) can only influence a nearby configuration (a nearby point in $C$). Only in certain special cases this translates into the usual notion of locality in $M_4$ (a subspace of $C$). The so called ‘horizon problem’ does not arise in this theory.

3. Configuration spaces associated with strings and branes

Instead of a system of point particles we can consider extended objects such as strings and, in general, branes. A brane configuration is described by the set of functions $X^\mu(\xi^a)$, where $\xi^a$, $a = 1, 2, ..., n$ is a set of parameters on the brane. We will consider a brane configuration as a point in an infinite dimensional configuration space, called brane space $M$. Following refs. [4, 5, 3], we will therefore use a condensed notation

$$X^\mu(\xi^a) \equiv X^{\mu(\xi)} \equiv X^M \quad (26)$$

We assume that the branes within classes of tangentially deformed branes are in principle physically distinct objects. All such objects are represented by different points of $M$-space.

Instead of one brane we can take a 1-parameter family of branes $X^\mu(\tau, \xi^a) \equiv X^{\mu(\xi)}(\tau) \equiv X^M(\tau)$, i.e., a curve (trajectory) in $M$. In principle every trajectory is kinematically possible. A particular dynamical theory then selects which amongst those kinematically possible branes and trajectories are dynamically possible. We assume that dynamically possible trajectories are geodesics in $M$ determined by the minimal length action [4, 5]:

$$I[X^M] = \int d\tau \ (\rho_{MN} \dot{X}^M \dot{X}^N)^{(1/2)} \quad (27)$$

Here $\rho_{MN}$ is the metric of $M$.

In particular, if metric is

$$\rho_{MN} = \rho_{\mu(\xi^a)\nu(\xi')} = \kappa \frac{\sqrt{|f(\xi)|}}{\sqrt{\dot{X}^2(\xi)}} \delta(\xi' - \xi'') \eta_{\mu\nu} \quad (28)$$
as a manifold whose tangent space at any point is Clifford algebra. The line element in $M$-dimensional space $C$ is determined at a point in a 16-dimensional space $G$ where $f$ freedom is not a string but a 2-brane, then we can also consider the corresponding volume degrees of freedom. Let the action be a worldline $x^M = X^M(\tau)$ in $C$ be associated with the motion of the extended object, e.g., a brane. Let the action be $I = \int d\tau \left( G_{MN} \dot{X}^M \dot{X}^N \right)^{1/2}$.
If $G_{MN} = \eta_{MN}$ is Minkowski metric, then the equations of motion are

$$\ddot{X}^M = \frac{d^2X^M}{d\tau^2} = 0$$ \hspace{1cm} (32)

They hold for tensionless branes. For the branes with tension one has to replace $\eta_{MN}$ with a generic metric $G_{MN}$ with non vanishing curvature. Eq. (32) then generalizes to the corresponding geodesic equation

$$\frac{1}{\sqrt{X^2}} \left( \frac{\dot{X}^M}{\sqrt{X^2}} \right) + \Gamma^M_{JK} \frac{\dot{X}^J \dot{X}^K}{X^2} = 0$$ \hspace{1cm} (33)

Such higher dimensional configuration space, associated with branes, enables unification of fundamental interactions à la Kaluza-Klein [10, 8].

Functions $X^M(\tau)$ are just like functions describing the worldline of a point particle. The four values of the index $M = \mu = 0, 1, 2, 3$ in fact correspond to the motion of a point particle in 4-dimensional spacetime $M_4$. The other values of the index $M = \mu_1...\mu_r$ for $r = 0$ (a scalar), $r = 2$ (bivector), $r = 3$ (3-vector) and $r = 4$ (4-vector) are associated with the particle’s ‘thickness’. From the point of view of spacetime $M_4$ we have thus a thick particle, and not a point particle. The thickness is encoded in 16 coordinates $X^M$. The elegance of this approach is in the fact that a thick particle in 4-dimensional spacetime $M_4$ can be described as a point particle in 16-dimensional Clifford space $C$.

Besides a point particle in Clifford space $C$ that sweeps a wordline $X^M(\tau)$ in $C$, we can envisage a string in $C$ that sweeps a worldsheet $X^{\mu}(\tau, \sigma)$ in $C$. The four values of the index $M = \mu = 0, 1, 2, 3$ correspond to a string in spacetime $M_4$. The rest of the indices $M = \mu_1...\mu_r$, $r = 0, 2, 3, 4$ label those coordinates which encode the object’s thickness. From the point of view of spacetime $M_4$ the object is a thick string. Usual strings are infinitely thin objects. Despite being called ‘extended objects’, they are in fact not fully extended. Instead of infinitely thin strings we thus consider thick strings. Their thickness is encoded in coordinates $X^M \equiv X^{\mu_1...\mu_r}$.

We thus have objects which are strings in a 16-dimensional space $C$. They are described by the Nambu-Goto action, or, equivalently, by the Polyakov action which, in the conformal gauge, takes the form

$$I = \frac{K}{2} \int d\tau d\sigma (\dot{X}^M X^N - X^M \dot{X}^N)G_{MN}$$ \hspace{1cm} (34)

where $\dot{X}^M = \frac{dX^M}{d\tau}$ and $X'^M = \frac{dX^M}{d\sigma}$.

If the signature of the underlying space time is $+----$, i.e., $(1, 3)$, then the signature of $C$ is $(8, 8)$ [11]. By taking the Jackiw-Kim-Noz definition of vacuum [12, 14], one finds [11] that there are no central terms in the Virasoro algebra, if the $D$-dimensional space in which the string lives has signature $(D/2, D/2)$.

Instead of adding extra dimensions to spacetime, we can thus start from 4-dimensional spacetime $M_4$ with signature $(+----)$ and consider the Clifford space $C$ whose dimension is 16, and signature $(8, 8)$. The necessary extra dimensions for consistency of string theory are in $C$-space. This also automatically brings spinors into the game. It is an old observation that spinors are the elements of left or right ideals of Clifford algebras [15]–[17] (see also, e.g., refs. [18]–[20]). In other words, spinors are particular sort of Clifford numbers [4]. With the string coordinates $X^M = (X, X^\mu, X^{\mu\nu}, ...)$ we can associate the basis of Clifford algebra $\gamma_M = (1, \gamma_\mu, \gamma_{\mu\nu}, ...)$, and consider the Clifford numbers $X^M \gamma_M$ that we call ‘polyvectors’.

2 It customary to describe position in a flat space by vectors. If the space is a “curved” $D$-dimensional manifold, then its points can be described in terms of vectors belonging to a vector space $\mathbb{R}^D$ which, in particular can be the tangent space at a chosen point of the manifold. In the case of a curved Clifford space $C$ its points are described in terms of ‘polyvectors’ which are the elements of a Clifford algebra $\mathbb{C}(8, 8)$, a tangent space at a chosen point—“an origin”—of $C$. 

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Therefore, the string coordinate polyvectors $X^{M\gamma_M}$ contain spinors. This is an alternative way of introducing spinors into the string theory [4, 11]. Attempts to achieve a deeper understanding of the structure of supersymmetry within the context of Clifford algebras have been undertaken in refs. [20, 21].

5. Conclusion
In this paper we have considered the concept of the space of all possible matter configurations. If one assumes that positions of all particles are fixed and only the position of one particle is variable, then one has the space of all possible positions of the single particle. This is just the 4-dimensional spacetime. But the latter space is not the whole story, since it is only a subspace of a more general configuration space. What we have missed so far is to employ this more general space in our theoretical considerations. It is true that certain researchers have considered configuration spaces, but the idea has not yet been generally accepted. So far we have been stuck by the fact that we as observers, when moving around, explore only four dimensional spacetime. Such, intuitively reasonable notion, that physical space is associated with the degrees of freedom of a single point particle, has to be revised. There are many particles around, and all their degrees of freedom count; moreover, the particles are actually not point-like, but extended. Thus we have a multidimensional space with the prospect for the unification of gravity with other interactions.

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