Two component dark matter with inert Higgs doublet: neutrino mass, high scale validity and collider searches

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ABSTRACT: The idea of this work is to investigate the constraints on the dark matter (DM) allowed parameter space from high scale validity (absolute stability of Higgs vacuum and perturbativity) in presence of multi particle dark sector and heavy right handed neutrinos to address correct neutrino mass. We illustrate a simple dark sector, consisting of one inert SU(2)L scalar doublet and a scalar singlet, both stabilised by additional $Z_2 \times Z_2$ symmetry, which aid to vacuum stability. We demonstrate DM-DM interaction helps achieving a large allowed parameter space for both the DM components by evading direct search bound. But, high scale validity puts further constraints on the model, for example, on the mass splitting between the charged and neutral component of inert doublet, which has important implication to its leptonic signature(s) at the Large Hadron Collider (LHC).

KEYWORDS: Dark matter, perturbativity, vacuum stability.
1 Introduction

Discovery of the ‘Higgs’ boson at LHC in 2012 [1, 2] strongly validates the Standard Model (SM) of particle physics as the fundamental governing theory of Strong, Weak and Electromagnetic interactions. However many unresolved issues persist. For example, the electroweak (EW) vacuum turns out to be metastable [3–9] with the present measured value of Higgs mass \( m_h \sim 125.09 \text{ GeV} [10] \) and top quark mass \( m_t \sim 173.1 \text{ GeV} [10] \). Large uncertainty in the measured value of \( m_t \) can even make EW vacuum unstable, questioning the existence of our universe. It is well known that inclusion of additional scalars in the theory can help stabilizing EW vacuum by compensating the negative contributions of
fermionic couplings in renormalization group (RG) running of Higgs quartic coupling \( \lambda_H \) [11, 12]. This motivates us to look for extended scalar sector.

On the other hand, the existence of Dark matter (DM) in the Universe is convincingly supported from the observations of various experiments around the globe, for example, WMAP [13] and Planck [14]. Extensions of SM to accommodate DM is therefore inevitable. The simplest of its kind is Weakly Interacting Massive Particle (WIMP) [15] and most economical is the presence of a singlet scalar dark matter [16–24] with Higgs portal interaction with SM. Non observation of DM in direct search experiments like LUX [25], XENON 1T [25–27], Panda X [28] puts a stringent bound on this model (with mass below 1 TeV getting disallowed) due to its prediction of large spin independent (SI) direct search cross section. As an alternative, multi-component DM scenarios [29–45] are proposed where DM-DM interactions (see for example, [37, 38]) play an important role to evade direct search bound. Therefore multipartite scalar dark sector can satisfy the required DM constraints easily, while can also stabilize the EW vacuum.

However, singlet DMs have limited colliders search possibilities, due to lack of interaction with visible sector. The search strategy for such DM is therefore limited to mono-\( X \) signature with missing energy, where \( X \) can be jet, \( W, Z \) or Higgs. Such signal arises out of initial state radiation (ISR) and suffers from huge SM background. Therefore, searching such singlet DM candidates turn out to be difficult. Higher multiplet in dark sector, being equipped with charge components have better possibilities of getting unravelled in future collider search experiments, but tighter constraints arise from dark sector. The simplest of its kind is to assume an inert Higgs \( SU(2)_L \) doublet (IDM). However, due to gauge coupling, a single component IDM is severely constrained and is not allowed between DM mass within \( 80 - 550 \) GeV, referred as the desert region for under abundance. However, it can produce hadronically quiet single and two lepton signatures with missing energy at LHC. Therefore, it is ideal to study a multipartite DM framework, involving an inert doublet to satisfy DM constraints as well as to have interesting phenomenological consequences.

Physics beyond the SM (BSM) is also strongly motivated by the presence of tiny neutrino masses (\( \sim eV \)), which suffers from huge fine tuning once assumed just to have a Dirac mass term like other SM fermions. Seesaw models have therefore been proposed based on the fact that right handed neutrinos can possess Majorana masses and one can have SM gauge invariant dimension five operator of the form \( \sim \frac{1}{\Lambda} LLHH \), where \( L \) represents SM lepton doublet, \( H \) represents SM Higgs doublet and \( \Lambda \) is an unknown heavy physics scale. They all necessarily predict BSM physics with important and interesting phenomenological consequences. Type-I seesaw is the simplest possibility [46, 47], which predict the presence of heavy right handed (RH) neutrinos to yield correct light neutrino mass through additional Yukawa coupling with SM Higgs. This in turn alter the Higgs vacuum stability condition at high scale larger than the RH neutrino masses.

Our model under scrutiny addresses all of the above features. We address a multipartite dark sector consisting of a \( SU(2)_L \) doublet (IDM) and a scalar singlet, both stabilized by additional \( Z_2 \times \hat{Z}_2 \) symmetry (for an earlier effort, see [39]) and provide a two component DM set up. The presence of DM-DM interactions enlarge the available parameter space significantly, while the inert DM can also produce leptonic collider signature at LHC. We
also augment the model with heavy RH neutrinos to address neutrino masses. However, the presence of RH neutrino Yukawa coupling tends to destabilize the EW vacuum \[48–58\] while the additional scalars tend to stabilize them. So, we take up an interesting exercise of validating the model from DM constraints, neutrino masses and high scale validity (absolute stability of the Higgs vacuum and perturbativity). This analysis provides some important conclusions, which are phenomenologically testable at LHC.

Let us finally discuss the plan of the paper. In section 2, we discuss the model construct in details. Section 3 presents possible theoretical and experimental constraints on the model parameters. Then in sections 4,5 and 6 subsequently, we discuss DM phenomenology. In section 7, we investigate the high scale validity of the model. Section 8 summarises collider signature(s) in context of the proposed set up. Finally we conclude in section 9. The high scale stability condition on the single component DM frameworks are chalked out in Appendix A; all the constraints together on the model parameter space along with different choices of RH neutrino mass and Yukawa couplings for the two-component set up are listed in Appendix B, tree level unitarity condition is elaborated in Appendix C.

2 The Model

The model is intended to capture the phenomenology of two already established DM frameworks involving that of a singlet scalar and that of an inert scalar doublet together with right handed neutrinos to address neutrino mass under the same umbrella. Therefore, we extend SM by an inert doublet scalar \((\Phi)\) and a real scalar singlet \((\phi)\) and include three RH Majorona neutrinos \(N_i (i = 1, 2, 3)\) in the set up. The lightest neutral scalar mode of the IDM and \(\phi\) are the DM candidates provided an appropriate symmetry in addition to that of SM stabilizes both of them. This is minimally possible by introducing an additional \(Z_2 \times Z_2'\) discrete symmetry under which all SM fields along with the right handed neutrinos transform trivially and the other additional fields transform non-trivially as tabulated in the Table 1. We also note the charges of SM Higgs \((H)\) explicitly in Table 1, as it will be required to form the scalar potential of the model. Note here, that charges of the two DM candidates \((\Phi\) and \(\phi\)) are complementary, i.e. odd under either \(Z_2\) or \(Z_2'\) for their stability. We also point out that the \(U(1)_Y\) hypercharge assignment of \(\Phi\) is identical to SM

| BSM and SM Higgs Fields | \(SU(3)_C \times SU(2)_L \times U(1)_Y \times Z_2 \times Z_2' \equiv \mathcal{G}\) |
|--------------------------|--------------------------------------------------|
| \(\Phi \equiv \left( \begin{array}{c} H^+ \\ \frac{1}{\sqrt{2}} (H^0 + i A^0) \end{array} \right) \) | \(1\) \(2\) \(+1\) \(-\) \(+\) |
| \(\phi \) | \(1\) \(1\) \(0\) \(+\) \(-\) |
| \(N_i \) \((i = 1, 2, 3)\) | \(1\) \(1\) \(0\) \(+\) \(+\) |
| \(H \equiv \left( \begin{array}{c} w^+ \\ \frac{1}{\sqrt{2}} (h + v + iz) \end{array} \right) \) | \(1\) \(2\) \(+1\) \(+\) \(+\) |

Table 1: Charge assignments of the BSM fields assumed in the model under \(\mathcal{G}\) as well as that of SM Higgs. The \(U(1)_Y\) hypercharge is chosen as \(Q = T_3 + Y/2\).
doublet $H$. Therefore the only $SU(2)_L \times U(1)_Y$ invariant terms are $H^\dagger H$, $\Phi^\dagger \Phi$, $H^\dagger \Phi$ and its conjugate.

The scalar Lagrangian reads as:

$$\mathcal{L}_{\text{scalar}} = |D^\mu H|^2 + |D^\mu \Phi|^2 + \frac{1}{2}(\partial^\mu \phi)^2 - V(H, \Phi, \phi),$$  
(2.1)

$$D^\mu = \partial^\mu - ig_2 \frac{\sigma^a}{2} W^{a\mu} - ig_1 \frac{Y}{2} B^\mu,$$

where $g_2, g_1$ denote $SU(2)_L$ and $U(1)_Y$ coupling respectively.

The most relevant renormalizable scalar potential in this case is given by,

$$V(H, \Phi, \phi) = -\mu_H^2 (H^\dagger H) + \lambda_H (H^\dagger H)^2 + V(H, \Phi) + V(H, \phi) + V(\Phi, \phi),$$  
(2.2)

where,

$$V(H, \Phi) = \mu_\Phi^2 (\Phi^\dagger \Phi) + \lambda_\Phi (\Phi^\dagger \Phi)^2 + \lambda_1 (H^\dagger H)(\Phi^\dagger \Phi)$$
$$+ \lambda_2 (H^\dagger \Phi)(\Phi^\dagger H) + \frac{\lambda_3}{2}((\Phi^\dagger \Phi)^2 + \text{h.c.}),$$  
(2.3)

$$V(H, \phi) = \frac{1}{2} \mu_\phi^2 \phi^2 + \frac{\lambda_\phi}{4!} \phi^4 + \frac{1}{2} \lambda_{\phi h} \phi^2 (H^\dagger H),$$  
(2.4)

$$V(\Phi, \phi) = \frac{\lambda}{2} (\phi^2) (\Phi^\dagger \Phi).$$  
(2.5)

The Lagrangian involving right handed neutrinos can be written as,

$$\mathcal{L}' = -(Y_\nu)_{ij} \bar{l}_i \tilde{H} N_j - \frac{1}{2} M_{N_{ij}} \bar{N}_i^C N_j,$$  
(2.6)

where $\tilde{H} = i \sigma_2 H^*$. We have considered three generations of RH neutrinos with $\{i, j\} = 1, 2, 3$, which can acquire Majorana masses and can possess Yukawa interactions with SM lepton doublet $l_L$. Note here, that the charge assignment of the $N$ fields then aid us to obtain neutrino masses through standard Seesaw-I \cite{46, 47} mechanism (as detailed later), while it also prohibits the operator like $\bar{l}_L \Phi N$ due to $Z_2$ charge assignment, and hence discards the possibility of generating the light neutrino mass radiatively. The ingredients and interactions of the model set up is described in the cartoon as in Fig. 1.

After spontaneous symmetry breaking, SM Higgs doublet acquires non-zero vacuum expectation value (VEV) as $H = (0 \frac{v+h}{\sqrt{2}})^T$ with $v=246$ GeV. Also note that neither of the added scalars acquire VEV to preserve $Z_2 \times Z_2'$ and act as DM components. After minimizing the potential $V(H, \Phi, \phi)$ along different field directions, one can obtain the following relations between the physical masses and the couplings involved:

$$\mu_H^2 = \frac{m_H^2}{2}, \quad \mu_\Phi^2 = m_\Phi^2 - \lambda_L v^2, \quad \lambda_3 = \frac{1}{v^2} (m_{A0}^2 - m_{A0}^2),$$
$$\lambda_2 = \frac{1}{v^2} (m_{H0}^2 + m_{A0}^2 - 2 m_{H^\pm}^2) \quad \text{and} \quad \lambda_1 = 2 \lambda_L - \frac{2}{v^2} (m_{H0}^2 - m_{H^\pm}^2),$$  
(2.7)

where $\lambda_L = \frac{1}{2}(\lambda_1 + \lambda_2 + \lambda_3)$ and $m_h, m_{H0}, m_{A0}$ are the mass eigenvalues of SM-like neutral scalar found at LHC ($m_h = 125$ GeV), heavy or light additional neutral scalar and the
Figure 1: A schematic diagram illustrating the different sectors of the model and their connection to SM. The dotted lines represent Higgs portal coupling, wavy line indicate gauge coupling, while the thin solid line indicates direct DM-DM coupling through $\frac{1}{2}\phi^2(\Phi^\dagger \Phi)$ term.

CP-odd neutral scalar respectively. $m_{H^\pm}$ denotes the mass of charged scalar eigenstate(s). The mass for $\phi$ DM will be rescaled as $m_\phi^2 = m_0^2 + \frac{1}{2}\lambda_{\phi h} v^2$. The independent parameters of the model, those are used to evaluate the DM, neutrino mass constraints are as follows:

Parameters : $\{m_{H^0}, m_{A^0}, m_{H^\pm}, m_\phi, \lambda_L, \lambda_{\phi h}, \lambda_\phi, Y_{\nu_{ij}}, M_{\nu_{ij}}\}$.

(2.8)

3 Theoretical and Experimental constraints

We would like to address possible theoretical and experimental constraints on model parameters here.

- **Stability:** In order to get the potential bounded from below, the quartic couplings of the potential $(H, \Phi, \phi)$ must have to satisfy following co-positivity conditions (CPC) following [59, 60],

\[
\text{CPC}\{1, 2, 3\} : \lambda_H(\mu) \geq 0, \quad \lambda_\phi(\mu) \geq 0, \quad \lambda_\phi h(\mu) \geq 0, \\
\text{CPC}\{4, 5\} : \left( \lambda_1(\mu) + \lambda_2(\mu) \pm \lambda_3(\mu) \right) + \sqrt{\lambda_H(\mu) \lambda_\phi(\mu)} \geq 0, \\
\text{CPC}\{6, 7\} : \lambda_1(\mu) + 2\sqrt{\lambda_H(\mu) \lambda_\phi(\mu)} \geq 0, \quad \lambda_\phi h(\mu) + \sqrt{\frac{2}{3} \lambda_H(\mu) \lambda_\phi(\mu)} \geq 0, \\
\text{CPC8} : \lambda_c(\mu) + \sqrt{\frac{2}{3} \lambda_\phi(\mu) \lambda_\phi(\mu)} \geq 0, \quad (3.1)
\]

where $\mu$ is the running scale. The above conditions show that the model offers to choose even negative $\lambda_{1,2,3,\phi h}$ satisfying the above conditions. However, as demonstrated in Eqn. 2.8, we use $\lambda_L$ and physical masses to be the parameters. Therefore, if we choose a specific mass hierarchy as: $m_{H^\pm} \geq m_{A^0} \geq m_{H^0}$ with positive $\lambda_L$, we are actually using $\lambda_1$ to be
positive while $\lambda_{2,3}$ negative abiding by the above conditions (see Eqn. 2.8).

**Perturbativity:** In order to maintain perturbativity, the quartic couplings of the scalar potential $V(H, \Phi, \phi)$, gauge couplings ($g_{i=1,2,3}$) and neutrino Yukawa coupling $Y_\nu$ should obey:

$$
|\lambda_H(\mu)| < 4\pi, \quad |\lambda_{\phi}(\mu)| < 4\pi, \quad |\lambda_{\phi}(\mu)| < 4\pi,
|\lambda_c(\mu)| < 4\pi, \quad |\lambda_{\phi h}(\mu)| < 4\pi,
|\lambda_1(\mu)| < 4\pi, \quad |\lambda_2(\mu)| < 4\pi, \quad |\lambda_3(\mu)| < 4\pi,
$$

$$
|g_{i=1,2,3}| < \sqrt{4\pi} \quad \text{and} \quad \text{Tr}\left[ Y^I_\nu(\mu) Y_\nu(\mu) \right] < 4\pi. \quad (3.2)
$$

**Tree Level Unitarity:** Next we turn to the constraints imposed by tree level unitarity of the theory, coming from all possible $2 \to 2$ scattering amplitudes as detailed in Appendix C follows as [61, 62]:

$$
|\lambda_H| < 4\pi, \quad |\lambda_{\phi}| < 4\pi,
|\lambda_c| < 8\pi, \quad |\lambda_{\phi h}| < 8\pi,
|\lambda_1| < 8\pi, \quad |\lambda_1 + 2(\lambda_2 + \lambda_3)| < 8\pi
$$

$$
|\lambda_1 + \lambda_2 + \lambda_3| < 8\pi, \quad |\lambda_1 - \lambda_2 - \lambda_3| < 8\pi,
| (\lambda_\phi + \lambda_H) \pm \sqrt{(\lambda_2 + \lambda_3)^2 + (\lambda_H - \lambda_\phi)^2} < 8\pi, \quad \text{and} \quad |x_{1,2,3}| < 16\pi. \quad (3.3)
$$

where $x_{1,2,3}$ be the roots of the following cubic equation as detailed in Appendix C:

$$
x^3 + x^2(-12\lambda_H - 12\lambda_{\phi} - \lambda_\phi) + x(-16\lambda_1^2 - 16\lambda_1\lambda_2 - 16\lambda_{1,3} - 4\lambda_2^2 - 8\lambda_2\lambda_3 - 4\lambda_3^2 - 4\lambda_c^2 + 144\lambda_H\lambda_{\phi} + 12\lambda_H\lambda_{\phi} + 12\lambda_{\phi}\lambda_\phi - 4\lambda_{\phi h}^2) + 16\lambda_1^2\lambda_\phi + 16\lambda_1\lambda_2\lambda_\phi + 16\lambda_{1,3}\lambda_\phi - 32\lambda_1\lambda_c\lambda_{\phi h} + 4\lambda_2^2\lambda_\phi + 8\lambda_2\lambda_3\lambda_\phi - 16\lambda_2\lambda_c\lambda_{\phi h} + 4\lambda_3^2\lambda_\phi - 16\lambda_3\lambda_c\lambda_{\phi h} + 48\lambda_2^2\lambda_H - 144\lambda_H\lambda_{\phi}\lambda_\phi + 48\lambda_{\phi h}^2 = 0 \quad (3.4)
$$

**Electroweak precision parameters:** There exists an additional $SU(2)_L$ doublet ($\Phi$) in our model in addition to a gauge singlet scalar ($\phi$). As the vev of $\Phi$ is zero, it does not alter the SM predictions of electroweak $\rho$ parameter [63]. However IDM, being an $SU(2)_L$ doublet makes a decent contribution to $S$ and $T$ parameters [64, 65] which we will identify as $\Delta S$ and $\Delta T$. The experimental bound from the global electroweak fit results on $\Delta S$ and $\Delta T$ using $\Delta U = 0$ are given by:

$$
\Delta S|_{\Delta U = 0} = 0.06 \pm 0.09, \quad \Delta T|_{\Delta U = 0} = 0.1 \pm 0.07, \quad (3.5)
$$

at 1$\sigma$ level with correlation coefficient 0.91 [66]. We show the constraint from $\Delta S$ and $\Delta T$ on the model parameter space in Fig. 2 using the standard formulae as presented in [64, 65]. In left plot, we scan 1$\sigma$ fluctuation on $\Delta S$ in $m_{A^0} - m_{H^0}$ versus $m_{H^{\pm}} - m_{H^0}$ plane for two different values of $m_{H^0} = \{80, 500\}$ GeV. We see that for smaller $m_{H^0}$, the constraint is larger. In right panel, we show 1$\sigma$ and 2$\sigma$ limits from $\Delta T$ in $m_{A^0} - m_{H^0}$ versus
Figure 2: Constraints from $\Delta S$ (left) and $\Delta T$ (right) in $m_{A^0} - m_{H^0}$ and $m_{H^\pm} - m_{H^0}$ plane. For $\Delta S$, we have taken $1\sigma$ limit for two different choices of $m_{H^0} = \{80, 500\}$ GeV. For $\Delta T$ scan, we show both $1\sigma$ and $2\sigma$ limits for a range of $m_{H^0} = \{80 - 500\}$ GeV.

$m_{H^\pm} - m_{H^0}$ plane for a range of IDM mass $m_{H^0} = \{80 - 500\}$ GeV. We can clearly see, that $\Delta T$ constrains the mass splitting much more than $\Delta S$.

- **Higgs invisible decay:** Whenever the DM particles are lighter than half of the SM Higgs mass, the Higgs can decay to DM and therefore it will contribute to Higgs invisible decay. Therefore, in such circumstances, we have to employ the bound on the invisible decay width of the 125 GeV Higgs as [10]:

\[
Br(h \rightarrow \text{Inv}) < 0.24
\]
\[
\frac{\Gamma(h \rightarrow \text{Inv})}{\Gamma(h \rightarrow \text{SM}) + \Gamma(h \rightarrow \text{Inv})} < 0.24.
\]

where

\[
\Gamma(h \rightarrow \text{Inv}) = \Gamma(h \rightarrow H^0 H^0) + \Gamma(h \rightarrow \phi \phi),
\]

and $\Gamma(h \rightarrow \text{SM} = 4.2$ MeV. In this analysis, we have mostly focused in the region where $m_\phi, m_{H_0} > m_h/2$, actually larger than $W$ mass, i.e. $m_\phi, m_{H_0} \geq m_W$, so that the above constraint is not applicable.

- **Collider search constraints:** Experimental searches for additional charged scalars and pseudoscalar in LEP and LHC provides bound on IDM mass parameters and coupling coefficients of IDM with SM particles.

  (i) **Bounds from LEP:** The observed decay widths of $Z$ and $W$ bosons from LEP data restrict the decay of gauge bosons to the additional scalars and therefore provide a bound on IDM mass parameters as $m_{A^0} + m_{H^0} > m_Z$, $2 m_{H^\pm} > m_Z$ and $m_{H^\pm} + m_{H^0, A^0} > m_W$. In addition, neutralino searches at LEP-II, provides a lower limit on the pseudoscalar Higgs ($m_{A^0}$) to 100 GeV when $m_{H^0} < m_{A^0}$ [67]. While the chargino search at LEP-II limits indicate a bound on the charged Higgs to $m_{H^\pm} > 70$ GeV [68].
(ii) **Bounds from LHC:** Due to the presence of SM Higgs and IDM interaction in the Lagrangian, charged scalars $H^\pm$ take part into the decay of SM Higgs to diphoton. Thus it contributes to Higgs to diphoton signal strength $\mu_{\gamma\gamma}$ which is defined as \cite{69–72}

$$\mu_{\gamma\gamma} = \frac{\sigma(gg \to h \to \gamma\gamma)}{\sigma(gg \to h \to \gamma\gamma)_{\text{SM}}} \approx \frac{\text{Br}(h \to \gamma\gamma)_{\text{IDM}}}{\text{Br}(h \to \gamma\gamma)_{\text{SM}}}.$$  

(3.7)

Now when ID particles are heavier than $m_h/2$, one can further write

$$\frac{\text{Br}(h \to \gamma\gamma)_{\text{IDM}}}{\text{Br}(h \to \gamma\gamma)_{\text{SM}}} = \frac{\Gamma(h \to \gamma\gamma)_{\text{IDM}}}{\Gamma(h \to \gamma\gamma)_{\text{SM}}}.$$  

(3.8)

The analytic expression of $\Gamma(h \to \gamma\gamma)_{\text{IDM}}$ can be obtained as \cite{69–72}:

$$\Gamma(h \to \gamma\gamma)_{\text{IDM}} = |A_{\text{SM}} + \alpha_e m_H^{3/2} \frac{\lambda_1 v}{m_H^{3/2}} m_{H^\pm}^2 F\left(\frac{m_h^2}{4m_{H^\pm}^2}\right)|^2,$$  

(3.9)

where $A_{\text{SM}}$ represents pure SM contribution (see \cite{69–72}). And $F(x) = -[x - f(x)]x^{-2}$ where

$$f(x) = \begin{cases} (\sin^{-1} x)^2, & x \leq 1 \\ -\frac{1}{4}\left[\ln\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}} - i\pi\right]^2, & x > 1 \end{cases}.$$  

(3.10)

Therefore it turns out that ID contribution to $\mu_{\gamma\gamma}$ is function of both mass of the charged Higgs ($m_{H^\pm}$) and the coefficient of the trilinear coupling $hH^+H^-$ i.e. $\lambda_1$. The measured value of $\mu_{\gamma\gamma}$ are given by $\mu_{\gamma\gamma} = 1.17\pm0.27$ from ATLAS \cite{73} and $\mu_{\gamma\gamma} = 1.14_{-0.23}^{+0.26}$ from CMS \cite{74}. In Fig. 3, we show the variation of $\mu_{\gamma\gamma}$ as function of $m_{H^\pm}$ for different values of $\lambda_1$.

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**Figure 3:** $\mu_{\gamma\gamma}$ as function of $m_{H^\pm}$ for different values of $\lambda_1$ as defined in the inset. 1σ and 2σ limits of $\mu_{\gamma\gamma}$ from ATLAS are also shown in blue and orange colours for comparison purpose.

We also present the experimental limits on $\mu_{\gamma\gamma}$ from ATLAS in Fig. 3. Excepting for the
resonance at $m_{h}/2$, we see that our choice of parameters are consistent with experimental bound. The larger is $m_{H\pm}$ GeV, the contribution to $\mu_{\gamma\gamma}$ → 1 i.e. to SM value. Also, we see that $\lambda_1 > 0$ diminishes $\mu_{\gamma\gamma}$, while $\lambda_1 < 0$ tends to enhance it. In this analysis, we mostly consider $m_{H\pm} > m_{h}/2$ and positive $\lambda_1$ within correct experimental limit.

- **Relic Density of DM:** The PLANCK experiment [14] provides the observed amount of relic abundance

$$0.1166 \leq \Omega_{DM}h^2(\equiv \Omega_{H^0}h^2 + \Omega_{\phi}h^2) \leq 0.1206 .$$  \hspace{1cm} (3.11)

Furthermore, strong constraints exist from direct DM search experiments. In our analysis we will consider the most recent bound on direct detection cross section provided by XENON 1T [26].

- **Neutrino observables:** The parameters associated to the neutrino sector should satisfy the bounds provided by different ongoing neutrino experiments. Limit on sum of light neutrino masses $\sum m_{\nu_i} \leq 0.17$ eV is provided by PLANCK data [14]. The present values of neutrino mass hierarchies and mixing angle can be found in [75, 76]. Due to the presence of RH neutrino in the set up, the constraint from lepton flavor violating decay (LFV) (dominantly from $\mu \rightarrow e\gamma$ ) will be applicable [77–79]. LFV constraint can be successfully evaded for $M_N \gtrsim 10^{3.5}$ GeV [80, 81] even for neutrino Yukawa coupling of $\mathcal{O}(1)$. It is important to note that in our model, the ID does not interact with SM leptons and thus plays no role in LFV.

4 Single component DM framework involving $\phi$ or $H^0$

**Figure 4:** Annihilation processes of real scalar singlet DM ($\phi$) to SM particles. SM in the last graph stands for $W^\pm, Z, h$ and SM fermions.

The model inherits two DM candidates inert DM (IDM) $H_0$ and singlet scalar DM $\phi$. Both the DM components have been studied extensively in literature as individual candidates to satisfy relic density and direct search bounds. Let us first revisit the single component frameworks for these two cases here. The relic density of scalar singlet ($\phi$) is obtained via thermal freeze out through annihilation to SM through the Feynman graphs shown in Fig. 4. The direct search constraint for $\phi$ comes from the $t$- channel Higgs portal interaction (turning the last graph of Fig. 4 upside down). The relevant parameters of the model are [17, 20, 23]:

$$\phi \text{ as single component DM : } \{m_\phi, \lambda_{\phi h}\}. \hspace{1cm} (4.1)$$
Figure 5: Relic density (red dots) and direct search/XENON1T (blue dots) allowed parameter space of the single component DM; scalar singlet ($\phi$) on left (in $m_\phi - \lambda_{\phi h}$ plane) and IDM ($H^0$) on right (in $m_{H^0} - \lambda_L$ plane). For the right hand side plot, we have used: $0 \leq m_{A^0} - m_{H^0} \leq 200$ GeV, $1 \leq m_{H^\pm} - m_{H^0} \leq 400$ GeV, and $\lambda_{\phi} = 0.001$.

The allowed parameter space of $\phi$ is depicted in left hand side (LHS) of Fig. 5 in $m_\phi - \lambda_{\phi h}$ plane by the red dots. Direct search allowed parameter space from XENON1T data [26, 27] using spin-independent DM-nucleon scattering cross section is shown by the blue dots. We therefore see that the model can only survive either in the Higgs resonance region ($\sim m_h/2$) or at a very heavy mass $\gtrsim 900$ GeV. The under abundant (shown in yellow) and over abundant regions are also indicated, which implies that the under-abundant region is further constrained from direct search due to larger annihilation and therefore direct search cross-section.

Figure 6: Annihilation processes of IDM ($H^0$) to SM particles. SM in the top right graph stands for $W^\pm, Z, h$ and SM fermions.

IDM ($H^0$) as a single component DM have annihilation and co-annihilation channels
for freeze-out due to both gauge and Higgs portal interactions as shown by the Feynman graphs in Figs. 6 and 7. IDM therefore suffers from large annihilation and co-annihilation cross-section particularly due to gauge interaction, which is difficult to constrain. The parameters, which govern the IDM phenomenology are [82]:

$$H^0 \text{ as single component DM : } \{m_{H^0}, m_{A^0}, m_{H^\pm}, \lambda_L \}.$$  

(4.2)

Relic density allowed parameter space for single component IDM is shown in right hand side (RHS) of Fig. 5 in $m_{H^0} - \lambda_L$ plane by red dots. Direct search (XENON1T data) allowed points are shown by blue dots. The scan is obtained using $1 \leq m_{A^0} - m_{H^0} \leq 200$ GeV, $1 \leq m_{H^\pm} - m_{H^0} \leq 400$ GeV with self coupling $\lambda_\Phi = 0.001$ kept constant. Here we see again that IDM allowed region from relic density and direct search constraint lies either in the small mass region $m_{H^0} \lesssim 80$ GeV or in the heavy mass region $m_{H^0} \gtrsim 550$ GeV. It is a well known result coming essentially due to too much annihilation and co-annihilation of IDM to SM through gauge interactions [82]. The disallowed region $80$ GeV $< m_{H^0} < 550$ GeV is often called desert region, which is obviously under abundant. Another important point of single component IDM is that the direct search allowed parameter space beyond resonance ($m_{H^0} \gtrsim 550$ GeV) have significant co-annihilation dependence with $m_{H^\pm} - m_{H^0} \leq 10$ GeV and $m_{A^0} - m_{H^0} \leq 10$ GeV, as co-annihilation doesn’t contribute to direct search cross-section.

5 Two component DM set-up with $\phi$ and $H^0$

5.1 Coupled Botzmann Equations and Direct search

In presence of two DM components ($\phi$ and $H^0$) DM-DM conversion plays a crucial role. The heavier DM can annihilate to the lighter component and thus contribute to the freeze-out of heavier DM. The conversion processes are shown in Fig. 8, which shows that they
are dictated by four point contact interactions as well as by Higgs portal coupling. It is clear that \( H^\pm, A^0 \) are not really DM, but annihilation to them is broadly classified within DM-DM conversion as none of them contribute to direct search. The two component DM set up therefore requires following parameters for analysis:

Two component DM : \( \{ m_{H^0} , m_{H^\pm}, m_{A^0} , \lambda_L , \lambda_{\phi h} \} \). \hspace{1cm} (5.1)

There are two self interacting quartic couplings present in the model; namely \( \lambda_L \) and \( \lambda_{\phi h} \), which do not play an important role in DM analysis, but appear in vacuum stability constraint that we discuss later.

When \( m_{\phi} > m_{H^0}, m_{H^\pm}, m_{A^0} \), then \( \phi \) can annihilate to all possible IDM components. The dominant \( s \)-wave DM-DM conversion cross-sections \( (\sigma v) \) of \( \phi \) in non-relativistic approximation are given by:

\[
(\sigma v)_{\phi \to H^0 H^0} = \frac{1}{64\pi m_{\phi}} \left[ \lambda_c + \frac{2\lambda_L \lambda_{\phi h} v^2}{(4m_{\phi}^2 - m_H^2)} \right]^2 \sqrt{1 - \frac{4m_{H^0}^2}{4m_{\phi}^2}} \Theta(m_{\phi} - m_{H^0})
\]

\[
(\sigma v)_{\phi \to A^0 A^0} = \frac{1}{64\pi m_{\phi}} \left[ \lambda_c + \frac{2\lambda_S \lambda_{\phi h} v^2}{(4m_{\phi}^2 - m_H^2)} \right]^2 \sqrt{1 - \frac{4m_{A^0}^2}{4m_{\phi}^2}} \Theta(m_{\phi} - m_{A^0})
\]

\[
(\sigma v)_{\phi \to H^+ H^-} = \frac{1}{32\pi m_{\phi}^2} \left[ \lambda_c + \frac{\lambda_1 \lambda_{\phi h} v^2}{(4m_{\phi}^2 - m_H^2)} \right]^2 \sqrt{1 - \frac{4m_{H^\pm}^2}{4m_{\phi}^2}} \Theta(m_{\phi} - m_{H^\pm})
\]

On the other hand, when \( m_{\phi} < m_{H^0} \), the conversion process will be as \( H^0 H^0(A^0) \to \phi \phi \) or \( H^+ H^- \to \phi \phi \). The corresponding cross-sections can easily be gauged from Eq. 5.2.

The evolution of DM number density for both components (\( \phi \) and \( H^0 \)) in early universe as a function of time is obtained by coupled Boltzmann equations (CBEQ) as described in Eqn. 5.3:

\[
\frac{dn_{H^0}}{dt} + 3Hn_{H^0} = - \sum_X (\sigma v)_{H^0 X \to SM} (n_{H^0} n_X - n_{eq}^{H^0} n_{eq}^X) \Theta(m_{H^0} + m_X - 2m_{SM}),
\]

\[
- \sum_X (\sigma v)_{H^0 \phi \to \phi} \left( n_{H^0} n_X - n_{eq}^{H^0} n_{eq}^X n_{eq}^\phi \right) \Theta(m_{H^0} + m_X - 2m_{\phi}),
\]

\[
+ \sum_{X,Y} (\sigma v)_{\phi \phi \to X Y} \left( n_{\phi}^2 - n_{eq}^{\phi} n_{eq}^X n_{eq}^Y \right) \Theta(2m_{\phi} - m_X + m_Y);
\]
\[
\frac{dn_\phi}{dt} + 3H n_\phi = -\langle \sigma v \rangle_{\phi SM} \left( n_\phi^2 - n_\phi^{eq2} \right) \Theta(m_\phi - m_{SM}), \\
- \sum_{X,Y} \langle \sigma v \rangle_X \left( n_\phi^2 - \frac{n_\phi^{eq2}}{n_X n_Y} n_X n_Y \right) \Theta(2m_\phi - m_X + m_Y), \\
+ \sum_{X} \langle \sigma v \rangle_{H_0} \left( n_{H_0} n_X - \frac{n_{H_0}^{eq} n_X^{eq}}{n_\phi^{eq2}} n_\phi^2 \right) \Theta(m_{H_0} + m_X - 2m_\phi) \quad (5.3)
\]

where \( \{X, Y\} = \{H_0, A^0, H^\pm\} \). We can clearly spot DM-DM conversion contributions in second and third lines of each equation, which actually make the two equations ‘coupled’. The freeze-out of two component DM is therefore obtained by numerically solving the above CBEQ and yields relic density (for a detailed discussion see for example [37]). The total relic density (\( \Omega_{DM} \)) will then have contributions from both DM components as:

\[
\Omega_{DM} h^2 = \Omega_{H_0} h^2 + \Omega_\phi h^2. \quad (5.4)
\]

Now let us turn to direct search of two component DM set up. Both the DM candidates can be detected through the spin independent (SI) direct detection (DD) processes through \( t \)-channel Higgs mediation as depicted in Fig. 9. The SI DD cross section for \( H_0 \) and \( \phi \) turn out to be [20, 37]:

\[
\sigma_{eff}^{H_0} = \left( \frac{\Omega_{H_0} h^2}{\Omega_{DM} h^2} \right) \frac{\lambda_{H_0}^2 f_N^2}{m_{H_0}^2 m_{h}^2}, \quad \sigma_{eff}^{\phi} = \left( \frac{\Omega_{\phi} h^2}{\Omega_{DM} h^2} \right) \frac{\lambda_{\phi}^2 f_N^2}{m_{\phi}^2 m_{h}^2}, \quad (5.5)
\]

where \( \mu_{\phi,N} = \frac{m_\phi}{m_N} \) and \( \mu_{H_0,N} = \frac{m_{H_0}}{m_{H_0}} \) are the reduced masses. \( f_N = 0.2837 \) represents the form factor of nucleon [83, 84] and \( m_N = 0.939 \) GeV represents nucleon mass. Importantly, the effective direct search cross-section for each individual component is modified by the fraction with which it is present in the universe, given by \( \Omega_{f_N} h^2 \) for \( H_0 \) and \( \Omega_{\phi} h^2 \) for \( \phi \).

To obtain relic density and DD cross sections of both the DM candidates numerically, we have used MicrOmegas [85]. It is noteworthy, that version 4.3 of MicrOmegas is capable of handling two component DM and we have cross-checked the solution from the code to match very closely to the numerical solution of CBEQ in Eq. 5.3. For generating the model files compatible with MicrOmegas, we have implemented the model in LanHEP [86].

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig9.png}
\caption{Spin independent direct detection processes for IDM (left) and scalar singlet DM (right).}
\end{figure}
5.2 Role of DM-DM conversion

We first study the variation of relic density with respect to DM mass and other relevant parameters to extract the importance of DM-DM conversion in this two-component set up before elaborating on the relic density and direct search allowed parameter space of the model.

![Figure 10:](image)

**Figure 10**: Relic Density of IDM $H^0$ (left panel) and scalar DM $\phi$ (right panel) as a function of IDM mass $m_{H^0}$, with different choices of $\lambda_c$. We illustrate two different combinations of DM-SM couplings, in the top panel: $\{\lambda_{\phi h} = 10^{-5}, \lambda_L = 0.1\}$ and in bottom panel: $\{\lambda_{\phi h} = 0.1, \lambda_L = 10^{-5}\}$. Other parameters kept fixed, are mentioned in the inset of each figure.

In Fig. 10 we plot relic densities of two DM candidates: $\Omega_{H^0}h^2$ in left panel and $\Omega_{\phi}h^2$ in right panel figures as function of $m_{H^0}$ for different values of $\lambda_c$. We also keep $m_{\phi}$ fixed at 200 GeV here. Other parameters are chosen as mentioned in the inset of individual plots.

- Top left of Fig. 10: In this figure we have shown the variation of $\Omega_{H^0}h^2$ with $m_{H^0}$ for different values of $\lambda_c = 0, 10^{-5}, 0.01, 0.2$ by yellow, blue, purple and black solid lines respectively. Pure IDM case (in a single component framework) is depicted by red dotted line, where also evidently $\lambda_c = 0$. It is important to note the other parameters kept fixed for this plot are: $\lambda_{\phi h} = 10^{-5}, \lambda_L = 0.1, m_{H^0} - m_{A^0} = m_{A^0} - m_{H^0} = 5$ GeV. We see that for $m_{H^0} < m_{\phi}$, relic density of $H^0$ changes significantly with the variation of $\lambda_c$. We also see that $\lambda_c = 0$ (in two component set-up) is way above the pure IDM case yielding a large relic density. Now, with slight increase in $\lambda_c = 10^{-5}$, relic density goes further up and
then reduces significantly for larger $\lambda_c = 0.01, 0.2$. The interesting point is that the case of $\lambda_c = 0.2$ lies very close to the pure IDM case. It is therefore evident that the presence of the second DM component $\phi$ plays an important role in $\Omega_{\phi} h^2$ through the coupling $\lambda_c$. For $m_{H^0} < m_\phi$, relic density of the heavier component $\phi$ can easily inherit the annihilation cross-section to SM plus conversion to other DM components $\{X, Y\} = \{H^0, A^0, H^\pm\}$ as:

$$\Omega_{\phi} h^2 \approx \frac{0.1 \text{ pb}}{\langle \sigma v \rangle_{\phi \rightarrow \text{SM SM}} + \langle \sigma v \rangle_{\phi \rightarrow X Y}}. \quad (5.6)$$

However it is difficult to envisage the relic density for the lighter DM component due to such DM-DM conversion. This can be understood from the CBEQ for the two component DM as in Eqn. 5.3. Let us define the following notations first:

$$\langle \sigma v \rangle_{H^0 X \rightarrow \text{SM}} n_{H^0 X} = \beta_{H^0} ; \quad \langle \sigma v \rangle_{\phi \rightarrow X Y} n_\phi^2 = \beta_\phi \; ; \quad \langle \sigma v \rangle_{H^0 X \rightarrow \phi} n_{H^0 X} n_\phi = \beta_{H^0 \phi}. \quad (5.7)$$

Again $\{X, Y\} = H^0, A^0, H^\pm$; as earlier. The CBEQ with above notation, turns out to be (assuming $m_\phi > m_{H^0}$):

$$\frac{dn_{H^0}}{dt} + 3H n_{H^0} \approx -\beta_{H^0} + \beta_{\phi H} \; ; \quad \frac{dn_\phi}{dt} + 3H n_\phi \approx -\beta_\phi - \beta_{\phi H}. \quad (5.8)$$

In Eq. 5.8, we neglected the equilibrium number densities as they are tiny near freeze-out, where the dynamics is under study. With $\lambda_c = 0$, and $\lambda_{\phi H} = 10^{-5}$, annihilation cross-sections for $\phi$, $(\langle \sigma v \rangle_{\phi \rightarrow \text{SM SM}}$ and $\langle \sigma v \rangle_{\phi \rightarrow X Y}$) are very small. Hence $\phi$ freezes out early and the number density of $\phi$ turns out to be large since $n_\phi \propto 1/(\sigma v)^{\text{eff}}$ where $\langle \sigma v \rangle^{\text{eff}} \approx \left(\langle \sigma v \rangle_{\phi \rightarrow \text{SM SM}} + \langle \sigma v \rangle_{\phi \rightarrow X Y}\right)$ following Eqn. 5.6. Now, it is easy to appreciate that with $\lambda_c = 0$, and $\lambda_{\phi H} = 10^{-5}$, annihilation of $\phi$ to SM is larger than conversion to other DM ($H^0$), i.e. $\langle \sigma v \rangle_{\phi \rightarrow \text{SM SM}} \gg \langle \sigma v \rangle_{\phi \rightarrow X Y}$. However due to large $\phi$ ($n_\phi \sim 10^9$), $\beta_{\phi H}$ becomes comparable with $\beta_{H^0}$. As these two terms ($\beta_{\phi H}$ and $\beta_{H^0}$) appear in the evolution of $n_{H^0}$ (Eqn. 5.8) with opposite sign, it is quite evident that effective annihilation cross-section for $H^0$ becomes small and hence $n_{H^0}$ after freeze out turns out to be much larger than the pure IDM case. Next let us consider non zero but small $\lambda_c (= 10^{-5})$. Then $\langle \sigma v \rangle_{\phi \rightarrow X Y}$ increases compared to the earlier case of $\lambda_c = 0$. However due to smallness of the coupling $\lambda_c$, this does not make any significant change in the number density of $\phi$ and $n_\phi \sim 10^9$ remains the same (as $\phi \rightarrow \text{SM SM}$ still dominantly contributes to the total annihilation cross section of $\phi$). Therefore, $\beta_{\phi H}$ increases and reduces the separation with $\beta_{H^0}$. Hence the effective annihilation cross-section for $H^0$ turns out to be even smaller than $\lambda_c = 0$ case. Therefore, for $\lambda_c = 10^{-5}$ relic density increases further than that of $\lambda_c = 0$. For larger value of $\lambda_c = 0.01$, contribution from DM-DM conversion, $\langle \sigma v \rangle_{\phi \rightarrow X Y}$ significantly rises and therefore the number density of $n_\phi$ drops to $n_\phi \sim 10^5$, and therefore $\beta_{\phi H}$ becomes much smaller than $\beta_{H^0}$. This increases the effective annihilation for $H^0$ and reduces relic density. This trend continues for higher values of $\lambda_c$ and eventually leads to
a vanishingly small $\beta_{\phi H}$ to closely mimic the case of single component IDM. The case of $m_{H^0} > m_\phi$ can also be understood from the CBEQ in this limit:

\[
\frac{dn_{H^0}}{dt} + 3Hn_{H^0} \simeq -\beta_{H^0} - \beta_{H\phi}; \\
\frac{dn_\phi}{dt} + 3Hn_\phi \simeq -\beta_\phi + \beta_{H\phi},
\]

(5.9)

For $m_{H^0} > m_\phi$, relic density of $H^0$ can be written simply as

\[
\Omega_{H^0}h^2 \simeq \frac{0.1 \text{ pb}}{\langle \sigma v \rangle_{H^0 \to X \rightarrow SM} + \langle \sigma v \rangle_{H^0 \to \phi \phi}}.
\]

(5.10)

The annihilation to SM ($\langle \sigma v \rangle_{H^0 X \to SM SM}$) due to gauge coupling is much larger than the conversion cross-section $\langle \sigma v \rangle_{H^0 X \to \phi \phi}$ for all the choices of $\lambda_c$, so we do not find any distinction between all those cases.

• Top right of Fig. 10: In the top right panel, the same parameter space is used to show the variation of $\Omega_\phi$ with respect to $m_{H^0}$. The dynamics is much simpler for $m_{H^0} > m_\phi$ (see BEQ. 5.8) where $\Omega_\phi h^2 \simeq \frac{0.1 \text{ pb}}{\langle \sigma v \rangle_{\phi \to SM SM} + \langle \sigma v \rangle_{\phi \to X \rightarrow SM}}$. With larger $\lambda_c$, the conversion to other DM ($\beta_{\phi H}$) becomes larger and relic density drops accordingly. For $m_\phi < m_{H^0}$, we see from Eq. 5.9, that there is a competition between $\beta_\phi$ and $\beta_{H\phi}$. With increasing $m_{H^0}$, $\langle \sigma v \rangle_{H^0 X \to \phi \phi}$ decreases, therefore $\beta_{H\phi}$ decreases and eventually becomes vanishingly small for $m_{H^0} \gtrsim 400$ GeV. The equation for $n_\phi$ then becomes equivalent to the single component case of $\phi$ where $m_{H^0}$ is no more relevant for $\Omega_\phi$.

• The bottom panel figures of Fig. 10 essentially indicate that with larger $\lambda_{\phi h}$, annihilation of $\phi$ to SM becomes large, resulting a smaller $n_\phi$ after freeze out. Therefore $\beta_{\phi H}$ in Eq. 5.8 turns insignificant. On the other hand, $\beta_{H\phi}$ also becomes smaller than $\beta_\phi$ in Eq. 5.9. Together, relic density of the lighter DM component is not affected by the presence of a heavy DM component.

Before we move on, let us summarise the outcome of Fig. 10. We see here that relic density of lighter DM component is affected by the heavier one, when the annihilation cross-section to SM is tiny. As the relic density for the lighter component becomes too large, the feature is almost absent in relic density allowed parameter space.

There is an interesting feature of IDM relic density coming from co-annihilation channels that we illustrate next in Fig. 11. We plot the variation of IDM relic density as function of DM mass $m_{H^0}$ for different choices of $\Delta m = m_{A^0} - m_{H^0} : \{1 - 50\}$ GeV shown by different coloured lines. The other parameters are kept fixed and mentioned explicitly in the figure insets. Let us first focus on the left panel plot. We see that for $m_{H^0} < m_W$, with larger $\Delta m = m_{A^0} - m_{H^0}$, relic density is larger, which follows the usual convention of co-annihilation cross-section being reduced by Boltzmann suppression ($e^{-\Delta m/T}$) with larger splitting ($\Delta m$). However, for $m_{H^0} > m_W$, the phenomena is reverse and with larger mass splitting, relic density decreases. This is thoroughly unconventional, but can be understood by looking at the co-annihilation channels in Fig. 7. When $m_{H^0} < m_W$, co-annihilation occurs to SM only through the $s$-channel graph. However, for $m_{H^0} > m_W$, $W$ final state opens up including that of $t$-channel graphs. The $t$-channel contributions inherit a negative
Figure 11: $\Omega H_0 h^2$ vs $m_{H^0}$ for $\lambda_c = 0.001$ (left panel) and $\lambda_c = 0.1$ (right panel) with different choices of $m_{A^0} - m_{H^0}$ which are depicted by different coloured lines in the figure. The other parameters kept fixed are mentioned in the inset of each figure.

sign to that of s-channel or contact interaction. This therefore causes a destructive interference and reduces the co-annihilation cross-section significantly for small $\Delta m$. When the splitting $\Delta m$ increases, the $t$-channel term gets larger and reduces the effective destructive interference to increase the co-annihilation contribution even on top of the larger Boltzmann suppression. Therefore, we see that relic density in $m_{H^0} > m_W$ region becomes larger with larger $\Delta m$. The same feature prevails in the right panel figure. Here, for a larger $\lambda_c$, the relic density of $H^0$ goes further down due to annihilation to $\phi$ beyond $m_{H^0} > m_\phi$.

6 Relic density and Direct Search allowed parameter space

One of the important motivations of this analysis is to study interacting multi-component DM phenomenology for DM mass lying between $80 \leq m_{DM} \leq 500$ GeV for both the components in view of relic density ($0.1166 \leq \Omega_{DM} h^2 \leq 0.1206$ [14]) and direct detection (XENON 1T [26]) bounds. We would like to recall that for the individual scenarios, none of the DM components satisfy relic and direct detection bounds simultaneously within this mass range (see section 4). Now in order to find a consistent parameter space in the set up, we perform a numerical scan of the relevant parameters within the specified ranges as mentioned below.

$$80 \leq m_{H^0} \leq 500 \text{ GeV}, \quad 80 \leq m_\phi \leq 500 \text{ GeV},$$

$$0 \leq m_{A^0} - m_{H^0} \leq 200 \text{ GeV}, \quad 0 \leq m_{H^+_/-} - m_{A^0} \leq 180 \text{ GeV},$$

$$0.001 \leq \lambda_L \leq 0.30, \quad 0.001 \leq \lambda_{\phi h} \leq 0.20, \quad 0.01 \leq \lambda_c \leq 1.00 . \quad (6.1)$$

We also note here that as $\lambda_L$ and $\lambda_{\phi h}$ enters into direct search cross-sections for $H^0$ and $\phi$ DMs respectively, we keep those couplings in a moderate range, while $\lambda_c$ governs DM-DM interactions, but do not directly enter into direct search bounds, therefore we choose a larger range for scanning $\lambda_c$.

There exists two possible mass hierarchies for the two-component DM set up relevant for phenomenological analysis: (i) $m_\phi > m_{H^0}$ and (ii) $m_\phi \leq m_{H^0}$, which we address separately below.
Case I: $m_\phi > m_{H^0}$

Primarily, in such a scenario, the main physics arises due to annihilation of $\phi$ to $H^0$, on top of their individual annihilation to SM to govern the freeze-out.

**Figure 12:** Relic Density allowed parameter space is shown in $m_\phi - \lambda_{\phi h}$ plane (top left), $m_{H^0} - \lambda_L$ plane (top right), $m_\phi - m_{H^0}$ plane (bottom left) and $\Omega_\phi h^2/\Omega_{DM} h^2(\%) - \Omega_{H^0 h^2}/\Omega_{DM} h^2(\%)$ plane (bottom right) for the mass hierarchy $m_\phi > m_{H^0}$.

In Fig. 12, we show relic density allowed parameter space for the model in terms of different relevant parameters. In top left panel of Fig. 12, we have shown the relic density satisfied points in $m_\phi - \lambda_{\phi h}$ plane for different ranges of $\lambda_c$, as depicted in the figure with different colour codes. This particular graph essentially dictate the contribution of $\phi$ to total relic density. As seen from the plot, for a fixed $m_\phi$, there is a maximum $\lambda_{\phi h}$. All possible values less than the maximum $\lambda_{\phi h}$ is also allowed subject to different choices of $\lambda_c$. The larger is $\lambda_c$, the smaller is the required $\lambda_{\phi h}$ thanks to the conversion of $\phi \rightarrow H^0$ to yield relic density. It is also noted that for large $\lambda_c \gtrsim 0.5$, as the DM-DM conversion is very high, the DM mass $m_\phi$ has to lie in the high mass region ($\gtrsim 400$ GeV) to tame the annihilation cross-section within acceptable range. To summarise, this figure shows that due to the presence of second DM component, much larger parameter space (actually the over-abundant regions of the single component framework, compare Fig. 5) is allowed. Top right panel figure shows the relic density allowed parameter space in $m_{H^0} - \lambda_L$ plane again.
for different ranges of \( \lambda_c \) as in the left plot. Essentially, this plot banks on the contribution
from IDM. It naturally depicts that \( \lambda_L \) is insensitive to \( m_{H^0} \) as the annihilation and co-annihilation cross-section of \( H^0 \) is mainly dictated by gauge interaction. However, we see a mild dependence on \( \lambda_c \), such that when \( \lambda_c \gtrsim 0.5 \), the DM mass \( m_{H^0} \) has to be heavy \( \gtrsim 400 \) GeV. This is because with large \( \lambda_c \), DM-DM conversion is large; to achieve relic density of correct order, \( m_\phi \) requires to be large and the conversion can only be tamed down by phase space suppression, i.e. by choosing \( H^0 \) mass as close as possible to \( \phi \) mass \( (m_{H^0} \sim m_\phi) \). Bottom left figure correlates the DM masses to obtain correct density within \( m_\phi > m_{H^0} \). We see that for small \( \lambda_c \), particularly with higher \( m_\phi \), large \( m_{H^0} \) values are disfavoured in order to keep the DM-DM conversion in the right order. While for large \( \lambda_c \), mass degeneracy is required \( (m_{H^0} \sim m_\phi) \) to tame the DM-DM conversion. Bottom right figure shows the relative contribution of relic density of the two DM components. First of all, this shows that \( \phi \) contributes with larger share of relic density, while the relic density of \( H^0 \) can at most be limited to 40% of the total. For small \( \lambda_c \), contribution from \( H^0 \) is even smaller, as relic density contribution from \( \phi \) gets larger due to small DM-DM conversion. However, with large \( \lambda_c \), the DM-DM conversion for \( \phi \) becomes larger and therefore the relic density of \( \phi \) can easily span between 60 \( \sim \) 100%. With very high \( \lambda_c \sim 1 \), DM-DM conversion becomes too large, therefore to keep relic density in the correct ballpark, the DM mass \( (m_\phi) \) has to be heavy and almost degenerate with the heavier DM \( (m_{H^0} \sim m_\phi) \). \( \lambda_{\phi h} \) in such cases, requires to be very small, which are validated by some dark blue points with \( \Omega_\phi h^2/\Omega_{DM} h^2 \sim 60\% \).

Relic density allowed parameters space consistent with direct search constraints where both DMs \( \phi \) and \( H^0 \) simultaneously satisfy XENON 1T 2018 [87] bound (for different ranges of \( \lambda_c \)) are shown next in Fig. 13. This is illustrated in \( m_\phi - \lambda_{\phi h} \) plane (top left panel), \( m_{H^0} - \lambda_L \) plane (top right panel) and in \( m_\phi - m_{H^0} \) plane (bottom panel) similar to Fig. 12. We have already mentioned that spin independent (SI) DM-nucleon cross-section depends on square of Higgs portal couplings of the respective DM candidates, \( \lambda_{\phi h} \) for \( \phi \) and \( \lambda_L \) for \( H^0 \) scaled by a pre-factor of the relative number density \( \Omega_i h^2/\Omega_{DM} h^2 \) (\( i = \phi, H^0 \)). Since in this two component scenario, the dominant contribution is coming from \( \phi \) DM, the pre-factor \( \Omega_i h^2/\Omega_{DM} h^2 \sim 1 \). While for \( H^0 \), the pre-factor is small \( \Omega_{H^0} h^2/\Omega_{DM} h^2 \sim 1 \), and will help \( H^0 \) reducing the effective direct search cross-section. Therefore, portal coupling \( \lambda_{\phi h} \) is tightly constrained from XENON 1T bound to \( \lambda_{\phi h} \lesssim 0.1 \) for DM mass \( m_\phi \gtrsim 500 \) GeV, as shown in top left panel of Fig. 13 (compare it with the top left panel of Fig. 12). Similarly in top right panel, the direct search allowed \( m_{H^0} - \lambda_L \) plane for \( H^0 \) shows that a large region corresponding to higher \( \lambda_L \) is excluded as a function of \( m_{H^0} \) (again, compare it with top right panel of Fig. 12). A possible mass correlation after direct search bound are plotted in bottom panel of Fig. 13 in \( m_\phi - m_{H^0} \) plane. The main outcomes from this figure are: (i) For small \( \lambda_c \leq 0.1 \), small \( m_{H^0} \sim 200 \) GeV is favoured, (ii) for moderate values within the span of \( \lambda_c \sim \{0.1 \sim 0.5\} \), there is no correlation and (iii) for large \( \lambda_c \), only degenerate mass scenario \( (m_\phi \sim m_{H^0}) \) with large \( m_\phi \sim 400 \) GeV is allowed.
Figure 13: Relic Density and direct detection (XENON 1T 2018) allowed parameter space is shown in: $m_\phi - \lambda_{\phi h}$ plane (top left panel), $m_{H^0} - \lambda_L$ plane (top right panel) and $m_\phi - m_{H^0}$ plane (bottom panel). The scans are performed for for the mass hierarchy $m_\phi > m_{H^0}$.

Case II: $m_\phi \leq m_{H^0}$

Naturally the conversion of the heavier DM $H^0$ to the lighter component $\phi$ will mainly dictate the relic density of DM components on top their annihilations to SM.

Relic density allowed parameter space for $m_\phi < m_{H^0}$ is shown in Fig. 14. Again, this is illustrated in $m_\phi - \lambda_{\phi h}$ plane (top left), $m_{H^0} - \lambda_L$ plane (top right), $m_\phi - m_{H^0}$ plane (bottom left) and $\Omega_\phi h^2/\Omega_{DM} h^2(\%) - \Omega_{H^0} h^2/\Omega_{DM} h^2(\%)$ plane (bottom right). Different ranges of $\lambda_c$ are shown by the same colour code as in Fig. 12, 13. Let us first focus on the top left figure. It shows that for small values of $\lambda_c$, relic density allowed parameter space points lie in the vicinity of single component framework of $\phi$ (red points in figure). In absence of a lighter mode, the relic density of $\phi$ is essentially governed by its annihilation to SM and due to small conversion cross-section the production of $\phi$ is also not large enough to change the conclusion. However, the situation changes significantly with larger $\lambda_c$ (cyan and blue points), where we see again that the overabundant region of the single component scenario is getting allowed by relic density. In order to understand this let us remind ourself of the CBEQ for $m_\phi < m_{H^0}$ as depicted in Eqn. 5.9. In particular, the number density of $\phi$ is dictated by $n_\phi + 3H n_\phi \simeq -\beta_\phi + \beta_{H^0}$. With larger $\lambda_c$ and
Figure 14: Relic Density allowed parameter space is shown in $m_\phi - \lambda_{\phi h}$ plane (top left), $m_{H^0} - \lambda_L$ plane (top right), $m_\phi - m_{H^0}$ plane (bottom left) and $\Omega_{\phi h^2}/\Omega_{DM h^2}^2(\%) - \Omega_{H^0 h^2}/\Omega_{DM h^2}^2(\%)$ plane (bottom right) for the mass hierarchy $m_\phi < m_{H^0}$.

Larger conversion, $\beta_{H\phi}$ increases to reduce the effective $\beta_\phi$ that determines the relic of $\phi$. Therefore, to keep the relic density of $\phi$ to correct order, $\beta_\phi$ has to increase. Now recall, $\beta_\phi = \langle \sigma v \rangle_{\phi \phi \rightarrow SM \ SM} n_\phi/2 \sim 1/\langle \sigma v \rangle_{\phi \phi \rightarrow SM \ SM}$ as $n_\phi \sim 1/\langle \sigma v \rangle_{\phi \phi \rightarrow SM \ SM}$. Therefore, to increase $\beta_\phi$, one has to reduce the annihilation cross-section $\langle \sigma v \rangle_{\phi \phi \rightarrow SM \ SM}$. This is possible by reducing $\lambda_{\phi h}$ as we see here in the plot. Next let us discuss the top right figure. This figure in $m_{H^0} - \lambda_L$ plane essentially depicts that with larger $\lambda_c$, larger $m_{H^0}$ is favoured to tame the DM conversion as well as annihilation cross-section to keep the relic within limit. The dependence however is not that much significant due to the presence of large number of co-annihilation channels which remain unaffected by $\lambda_c$. In the bottom left panel, mass correlation has been plotted and carries no information. Lastly, bottom right figure shows the relative relic density contributions of these two components. It is well understood that an additional channel for annihilation of $H^0$ only reduces the possibility of bringing $\Omega_{H^0 h^2}$ in the correct ballpark due to already existing gauge mediated annihilation and co-annihilation channels. Therefore, for small $\lambda_c$, it is still possible to get a contribution from $\Omega_{H^0 h^2} \sim 40\%$, but that becomes harder with large $\lambda_c$, where the relic density contribution of $H^0$ is further limited to $\Omega_{H^0 h^2} \sim 20\%$. 
Figure 15: Relic Density and direct detection (XENON 1T 2018) allowed parameter space is shown in: $m_{\phi} - \lambda_{\phi h}$ plane (top left panel), $m_{H^0} - \lambda_L$ plane (top right panel) and $m_{\phi} - m_{H^0}$ plane (bottom panel). The scans are performed for for the mass hierarchy $m_{\phi} < m_{H^0}$.

Direct search constraints from XENON 1T 2018 on the relic density allowed points are shown in Fig. 15. To emphasise again, the demand of these plots are simultaneous satisfaction of XENON1T limit for both DM components. The main outcome of this plot is to see the absence of small $\lambda_c$ points (red dots) upto $\lambda_c \sim 0.26$. This is simply due to the fact that, with small $\lambda_c$, the required $\lambda_{\phi h}$ is high enough for $\phi$ DM to be discarded by XENON1T data. The other important feature is that with larger $\lambda_c$, larger DM masses are favourised. Lastly a very important conclusion comes from the bottom panel figure in the mass correlation plot. This shows, as only high $\lambda_c$ region is allowed, the conversion of $H^0$ to $\phi$ still needs to be restricted and therefore the mass difference between the two DM components ($m_{H^0} - m_{\phi}$) has to be very very small. These features are all distinct from that of $m_{\phi} > m_{H^0}$ region.

So far our discussion has been focused on DM mass region $m_{W^\pm} \leq m_{H^0}$, and $m_{\phi} \leq 500$ GeV. But if ($m_{H^0} < m_{W^\pm}$ or $m_{H^0} < m_{W^\pm}$), while other DM mass is heavier than $m_{W^\pm}$, the only region available for lighter DM ($m_{DM} < m_{W^\pm}$) is the resonance regions: $m_{H^0} \sim \frac{m_Z}{2}$, $\frac{m_W}{2}$ and $m_{\phi} \sim \frac{m_a}{2}$. It is important to remind that the resonance regions are already available in absence of second DM component and therefore brings no new
phenomenological outcome.

7 Electroweak Vacuum stability and High Scale Perturbativity

One of the motivations of this work is to show that the presence of right handed neutrinos to yield correct neutrino masses in presence of multipartite DM. Although the neutrino sector considered here seems decoupled from the dark sector, is not completely true. The effect of the RH neutrinos alter the allowed DM parameter space when the model is validated at high scale.

7.1 \( \beta \) functions and RG running

To study the high scale validity of the model including perturbativity and vacuum stability, we need to consider the RG running of the associated couplings. The framework contains few additional mass scales: one extra scalar singlet, one inert doublet and three RH neutrinos with mass \( M_{N_i} = 1, 2, 3 \). Hence in the study of RG running, different couplings will enter into different mass scales. In DM phenomenology, we have considered the physical masses of DM sector particles within \( \sim 500 \text{ GeV} \). Then for simplicity, we can safely ignore the small differences between the dark sector particle masses and identify one additional mass scale as \( m_{DM} \). We also assume that RH neutrinos are heavier than the scalars \( (M_{N_i} = 1, 2, 3 > m_{DM}) \) present in the model. Hence, for running energy scale \( \mu > m_{DM} \), we need to consider the couplings associated to DM sector in addition to SM. On the other hand, when \( \mu > M_{N_{i=1,2,3}} \), the neutrino couplings will additionally enter into the scenario. Below we provide the one loop \( \beta \) functions for \( \mu > M_{N} \) in our model.

\( \beta \) functions of gauge couplings [88]:

\[
\begin{align*}
\beta_{g_1} &= \frac{21}{5} g_1^3, \\
\beta_{g_2} &= -3 g_2^3, \\
\beta_{g_3} &= -7 g_3^3.
\end{align*}
\] (7.1)

\( \beta \) functions of Yukawa couplings [88, 89]:

\[
\begin{align*}
\beta_{y_t} &= \frac{3}{2} y_t^2 + y_t \left( 3 y_t^2 - 8 g_3^2 - \frac{17}{20} g_1^2 - \frac{9}{4} g_2^2 + y_t \text{Tr}[Y_\nu^\dagger Y_\nu] \right), \\
\beta_{\text{Tr}[Y_\nu^\dagger Y_\nu]} &= 3 \text{Tr}[(Y_\nu^\dagger Y_\nu)^2] + \text{Tr}[Y_\nu^\dagger Y_\nu] \left( -\frac{9}{10} g_1^2 - \frac{9}{2} g_2^2 + 6 g_3^2 + 2 \text{Tr}[Y_\nu^\dagger Y_\nu] \right).
\end{align*}
\] (7.2)

\( \beta \) functions of quartic scalar couplings [88]:

\[
\begin{align*}
\beta_{\lambda_H} &= \frac{27}{200} g_1^4 + \frac{9}{20} g_1^2 g_2^2 + \frac{9}{8} g_2^4 - \frac{9}{5} g_1^2 \lambda_H - 9 g_3^2 \lambda_H + 24 \lambda_H y_t^2 - 6 y_t^4 \\
&\quad + 2 \lambda_1^2 + 2 \lambda_1 \lambda_2 + \lambda_2^2 + \lambda_3^2 + \frac{1}{2} \lambda_{\Phi}^2 + 4 \lambda_H \text{Tr}[Y_\nu^\dagger Y_\nu] - 2 \text{Tr}[(Y_\nu^\dagger Y_\nu)^2], \\
\beta_{\lambda_3} &= -\frac{9}{5} g_1^2 \lambda_3 - 9 g_2^2 \lambda_3 + 8 \lambda_1 \lambda_3 + 12 \lambda_2 \lambda_3 + 4 \lambda_3 \lambda_H + 4 \lambda_3 \lambda_\Phi + 6 \lambda_3 y_t^2 + 2 \lambda_3 \text{Tr}[Y_\nu^\dagger Y_\nu],
\end{align*}
\] (7.3)
\[
\begin{align*}
\beta_{\lambda_2} &= +\frac{9}{5}g_1^2 g_2^2 - \frac{9}{5}g_3^2 \lambda_2 - 9g_2^2 \lambda_2 + 8\lambda_1 \lambda_2 + 4\lambda_2^2 + 8\lambda_3^2 + 4\lambda_2 \lambda_H + 4\lambda_2 \lambda \Phi + 6\lambda_2 y_t^2 + 2\lambda_2 \text{Tr}[Y_\nu^\dagger Y_\nu], \\
\beta_{\lambda_1} &= \frac{27}{100}g_1^4 - \frac{9}{10}g_1^2 g_2^2 + \frac{9}{4}g_2^4 - \frac{9}{5}g_3^2 \lambda_1 - 9g_2^2 \lambda_1 + 4\lambda_1^2 + 2\lambda_2^2 + 2\lambda_3^2 + 12\lambda_1 \lambda_H \\
&\quad + 4\lambda_2 \lambda_H + 12\lambda_1 \lambda \Phi + 4\lambda_2 \lambda \Phi + 6\lambda_1 y_t^2 + \lambda_c \lambda_{\phi h} + 2\lambda_1 \text{Tr}[Y_\nu^\dagger Y_\nu], \\
\beta_{\lambda_\phi} &= 24\lambda_3^2 + 2\lambda_1^2 + 2\lambda_1 \lambda_2 - 9g_2^2 \lambda_\Phi + \frac{27}{200}g_1^4 + \frac{9}{20}g_2^4 \left( -4\lambda_\Phi + g_3^2 \right) + \frac{9}{8}g_2^4 + \lambda_2^2 + \lambda_3^2 + \frac{1}{2}\lambda_c^2, \\
\beta_{\lambda_{\phi h}} &= -\frac{9}{10} g_1^2 \lambda_{\phi h} - \frac{9}{2} g_2^2 \lambda_{\phi h} + 4\lambda_1^2 + 12\lambda_{\phi h} \lambda_H + \lambda_{\phi h} \lambda_c + 6\lambda_{\phi h} y_t^2 \\
&\quad + 4\lambda_1 \lambda_c + 2\lambda_2 \lambda_c + 2\lambda_{\phi h} \text{Tr}[Y_\nu^\dagger Y_\nu], \\
\beta_{\lambda_c} &= -\frac{9}{2} g_1^2 \lambda_c - \frac{9}{2} g_2^2 \lambda_c + 12\lambda_1 \lambda_2 \Phi + 4\lambda_1 \lambda_{\phi h} + 2\lambda_2 \lambda_{\phi h} + \lambda_c \lambda_c + 4\lambda_c^2, \\
\beta_{\lambda_\phi} &= 3 \left( 4\lambda_c^2 + 4\lambda_2^2 + \lambda_3^2 \right).
\end{align*}
\]

The above $\beta$ functions are generated using the model implementation in the code SARAH [90]. The running of $\lambda_H$ as in Eq.(7.3) is influenced adversely mostly by top Yukawa coupling $y_t \sim O(1)$ and right handed neutrino Yukawa coupling as $\text{Tr}[Y_\nu^\dagger Y_\nu]$. On the other hand, multipartite scalar DM couplings present in the model help in compensating the strong negative effect from $y_t$ and $\text{Tr}[Y_\nu^\dagger Y_\nu]$.

As already stated before, we employ type-I seesaw mechanism to generate the light neutrino mass, for which three RH neutrinos are included in the set up. We now describe the strategy in order to study their impact on RG evolution. For simplicity, the RH neutrino mass matrix $M_N$ is considered to be diagonal with degenerate entries, i.e. $M_{N_{i=1,2,3}} = M_R$. We have already seen that $\text{Tr}[Y_\nu^\dagger Y_\nu]$ enters in the $\beta$ function of the relevant couplings. In order to extract the information on $Y_\nu$, we use the type-I seesaw formula for neutrino mass $m_\nu = Y_\nu^T Y_\nu \frac{v^2}{2M_R}$. Then, naively one would expect that large Yukawa couplings are possible with even heavier RH neutrino masses. For example with $M_R \sim 10^{14}$ GeV, $Y_\nu$ comes out to be 0.3 in order to obtain $m_\nu \simeq 0.05$ eV. However, contrary to our naive expectation, it can be shown that even with smaller $M_R$ one can achieve large values of $\text{Tr}[Y_\nu^\dagger Y_\nu]$, but effectively reducing $Y_\nu^T Y_\nu$ using a special flavor structure of $Y_\nu$ through Casas-Ibarra parametrisation [51].

Note that our aim is to study the maximum effect coming from the right handed neutrino Yukawa i.e large value of $\text{Tr}[Y_\nu^\dagger Y_\nu]$ for vacuum stability. For this purpose, we use the parametrisation by [91] and write $Y_\nu$ as

\[
Y_\nu = \sqrt{2} \sqrt{\frac{M_R}{v}} R \sqrt{m_\nu^d} U_{\text{PMNS}}^\dagger,
\]

where $m_\nu^d$ is the diagonal light neutrino mass matrix and $U_{\text{PMNS}}$ is the unitary matrix diagonalizing the neutrino mass matrix $m_\nu$ such that $m_\nu = U_{\text{PMNS}}^* m_\nu^d U_{\text{PMNS}}^\dagger$. Here $R$ represents a complex orthogonal matrix which can be written as $R = O \exp(iA)$ with $O$ as real orthogonal matrix and $A$ as real antisymmetric matrices. Using above parametrisation, then one gets,

\[
\text{Tr}[Y_\nu^\dagger Y_\nu] = \frac{2M_R}{v^2} \text{Tr} \left[ \sqrt{m_\nu^d} e^{2iA} \sqrt{m_\nu^d} \right].
\]
Note that the real antisymmetric matrix $\mathcal{A}$ however does not appear in the seesaw expression for neutrino mass as $m_\nu = \frac{Y_\nu^T M_R Y_\nu}{2 M_R}$. Therefore with any suitable choice of $\mathcal{A}$, it would actually be possible to have relatively large Yukawa even with light $M_R$. This on the contrary, following Eq. 7.6 can affect the RG evolution of $\lambda_H$ significantly in adverse way. As an example, let us consider magnitude of all the entries of $\mathcal{A}$ to be equal, say $a$ with all diagonal entries to be zero. Then using the best fit values of neutrino mixing angles and considering the mass of lightest neutrino zero, one can find for $M_R = 1$ TeV, $\text{Tr}[Y_\nu^T Y_\nu]$ can be as large as 1 with $a = 8.1$[91, 92]. Therefore, it is legitimate to study Higgs vacuum stability in presence of RH neutrinos even with moderate values.

With this, let us analyse the SM Higgs vacuum stability at high energy scale. Below we provide the stability and metastability criteria.

- **Stability criteria**: The stability of Higgs vacuum can be ensured by the condition $\lambda_H > 0$ at any scale. However, we have multiple scalars (SM Higgs doublet, one inert doublet and one gauge real singlet) in the model. Therefore we should ensure the boundedness or stability of the entire scalar potential in any field direction. This can be guaranteed by using the co-positivity criterion in Eqn. 3.1. Note that once $\lambda_H$ becomes negative the other copositivity conditions no longer remain valid.

- **Metastability criteria**: On the other hand, when $\lambda_H$ becomes negative, there may exist another deeper minimum other than the EW one. Then the estimate of the tunnelling probability $P_T$ of the EW vacuum to the second minimum is essential to confirm the metastability of the Higgs vacuum. Obviously, the Universe can be in a metastable state only, provided the decay time of the EW vacuum is longer than the age of the Universe. The tunnelling probability is given by [4, 5],

$$P_T = T_U^4 \mu_B^4 e^{-\frac{8 a^2}{3 \lambda_H \mu_B^2}},$$

where $T_U$ is the age of the Universe, $\mu_B$ is the scale at which tunnelling probability is maximized, determined from $\beta \lambda_H = 0$. Therefore, solving above equation, we see that metastable Universe requires [4, 5]:

$$\lambda_H(\mu_B) > \frac{-0.065}{1 - \ln \left( \frac{\nu}{\mu_B} \right)}.$$ (7.8)

As noted in [4], for $\mu_B > M_P$, one can safely consider $\lambda_H(\mu_B) = \lambda_H(M_P)$. One should also note, that even with metastability of Higgs vacuum, the instability in other field direction can also occur. The conditions to avoid the possible instability along various field directions for $\lambda_H < 0$ are listed below [93].

(i) $\lambda_\Phi > 0$ to avoid the unboundedness of the potential along $H^0$, $A^0$ and $H^\pm$ directions.

(ii) $\lambda_1 > 0$ to ensure the stability of the potential along some direction between $H^\pm$ and $h$.

(iii) $\lambda_L > 0$, otherwise the potential will be unbounded along a direction in between $H^0$ and $h$. 

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(iv) $\lambda_S > 0$, otherwise the potential will be unbounded along a direction in between $A^0$ and $h$.

(v) $\lambda_\phi > 0$, otherwise the potential will be unbounded along $\phi$ direction.

(v) $\lambda_{\phi H} > 0$, otherwise the potential will be unbounded along a direction between $\phi$ and $h$.

Now to begin the vacuum stability analysis in the scenario, let us choose two benchmark points (BP1, BP2) as tabulated in Table 2 which satisfy both relic density and direct detection bounds. For BP1, both DM masses are on lower side and for BP2 they are a little higher. Also for BP1, $\lambda_L$ is smaller while $\lambda_{\phi h}$ is larger. It is other way for BP2. There is also an important distinction between the two benchmark points in terms of mass difference of IDM with charged components, its very small for BP1, while it is larger in BP2. $\lambda_1, \lambda_2, \lambda_3$ although do not enter into DM phenomenology directly, but they importantly alter the mass differences, which crucially controls the relic density and direct search outcome of IDM in particular. These parameters are mentioned in the caption of Table 2 for both benchmark points. We also show the value of electroweak parameters and $\mu_{\gamma\gamma}$ for these two benchmark points in Table 3.

| BPs | $m_{H^0}$ | $m_\phi$ | $m_{A^0}$ | $m_{H^\pm}$ | $\lambda_L$ | $\lambda_{\phi h}$ | $\lambda_c$ | $\Omega_{H^0} h^2$ | $\Omega_{\phi h^2}$ | $\sigma^{2f}_{H^0}$ (cm$^2$) | $\sigma^{2f}_{\phi}$ (cm$^2$) |
|-----|-----------|----------|-----------|-------------|-------------|-----------------|-------------|----------------|----------------|----------------|----------------|
| BP1 | 330       | 343      | 333       | 339         | 0.043       | 0.065           | 0.2         | 0.033          | 0.086          | $1.6 \times 10^{-46}$ | $2.2 \times 10^{-46}$ |
| BP2 | 427       | 449      | 438       | 440         | 0.086       | 0.017           | 0.3         | 0.027          | 0.088          | $3.3 \times 10^{-46}$ | $1.0 \times 10^{-47}$ |

**Table 2**: Benchmark points used to analyse EW vacuum stability in our model. All masses are in GeVs. The other couplings used in these benchmark points play an important role; they are: BP1: $\{\lambda_1 = 0.285, \lambda_2 = -0.17, \lambda_3 = -0.033\}$, BP2: $\{\lambda_1 = 0.544, \lambda_2 = -0.215, \lambda_3 = -0.157\}$.

| BPs | $\Delta S$ ($10^{-4}$) | $\Delta T$ ($10^{-4}$) | $\mu_{\gamma\gamma}$ ($10^{-5}$) |
|-----|----------------|----------------|----------------|
| BP1 | -12           | 5.1           | 3             |
| BP2 | -9            | 2.5           | 3             |

**Table 3**: Estimate of electroweak precision parameters and $\mu_{\gamma\gamma}$ for the two benchmark points as chosen in Table 2.

We run the three loop RG equations for all the SM couplings and one loop RG equations for the other relevant couplings in the model from $\mu = m_t$ to $M_P$ energy scale. We use the initial boundary values of all the SM couplings as provided in [4]. The boundary values have been evaluated at $\mu = m_t$ in [4] by taking various threshold corrections and mismatch between top pole mass and $\overline{MS}$ renormalised couplings into account. In Fig. 16, we show the running of $\lambda_H$ for BP1 as a function of energy scale $\mu$. The left panel shows running of $\lambda_H$ for different values of RH neutrino mass $M_R$, considering top quark mass $m_t = 173.1$
Figure 16: RG running of $\lambda_H$ with energy scale for BP1. In left panel, we have shown the effect of different right handed neutrinos masses, $M_R$ (indicated by different colours and mentioned in figure inset) for a fixed top quark mass $m_t = 173.1$ GeV. The black dotted line corresponds to the case when right handed neutrinos are absent in the scenario. In right panel, we choose a specific $M_R = 10^8$ GeV and consider top mass in $2\sigma$ range: $m_t = 173.1 \pm 0.9$ GeV. $\text{Tr}[Y^\dagger_Y Y_Y] = 0.5$ is kept constant in both plots.

GeV, Higgs mass $m_h = 125.09$ GeV and $\text{Tr}[Y^\dagger_Y Y_Y] = 0.5$. As it is visible that for low value of $M_R \sim 10^4$ GeV, $\lambda_H$ enters into unstable region at very early stage of its evolution (blue line in the figure). In contrary, for large value of $M_R$, although $\lambda_H$ becomes negative at some high energy scale, however it stays in metastable region till $M_P$ energy scale (violate line). Green region here describes stable, white region describes metastable (see Eq. 7.8) and the red region indicates unstable solution for the potential. For comparison, we also display the evolution of $\lambda_H$ (black dotted curve) in absence of RH neutrinos in the theory. This clearly shows that in absence of RH neutrinos, EW vacuum could be absolutely stable till $M_P$ energy scale. In right panel of Fig. 16, we study the evolution of $\lambda_H$ considering $2\sigma$ uncertainty of measured top mass $m_t$, keeping $m_h$, $M_R = 10^8$ GeV and $\text{Tr}[Y^\dagger_Y Y_Y] = 0.5$ fixed. It is trivial to find that with the increase of top mass, $\lambda_H$ crosses zero at earlier stage in its evolution.

Next we show the effect of $\text{Tr}[Y^\dagger_Y Y_Y]$ in the RG evolution of $\lambda_H$ in Fig. 17 for BP1 (left) and BP2 (right). For the purpose we mix the RH neutrino mass scale $M_R = 10^8$ GeV. It can be seen from left panel that large value of $\text{Tr}[Y^\dagger_Y Y_Y] \sim 0.7$ brings down $\lambda_H$ towards the negative values at earlier energy scale. This is obvious from the $\beta$ function of $\lambda_H$. With comparatively lesser value of $\text{Tr}[Y^\dagger_Y Y_Y] \sim 0.3$, the EW vacuum might be in metastable state provided other conditions $((i) - (v))$ are satisfied as shown in left panel of Fig. 18. If we further reduce the value of $\text{Tr}[Y^\dagger_Y Y_Y] \sim 0.2$ the EW vacuum might be absolutely stable. For the stability case we also show the satisfaction of all the copositivity criterias (Eq.(19)) in left panel of Fig. 19. This ensures the boundness of the scalar potential in any arbitrary field direction. The analysis for BP2 turns out to be similar as observed from right panels of Fig. 17-19.
Figure 17: RG running of $\lambda_H$ with energy scale $\mu$ for different values of $\text{Tr}[Y_L^\dagger Y_R]$ (shown in different colours and values are mentioned in the figure inset) for the benchmark points BP1 (left) and BP2 (right). Here we have chosen $M_R = 10^8$ GeV for illustration. The black dotted line here corresponds to the case when right handed neutrinos are absent in this scenario.

Figure 18: RG running of all the quartic couplings in metastability criteria for BP1 (left) and BP2 (right) to ensure the boundedness of the scalar potential when $\lambda_H < 0$ in various field directions with energy scale $\mu$ for $\text{Tr}[Y_L^\dagger Y_R] = 0.3$ (left) and 0.85 (right). The choices of $\text{Tr}[Y_L^\dagger Y_R]$ are demonstrated in cyan (0.3) and in orange (0.85) in left and right panels of Fig. 17 to yield metastability.

Based on the inputs from above analysis, now we constrain $\text{Tr}[Y_L^\dagger Y_R] - M_R$ parameter space using the stability, metastability and instability criteria (green, white and red regions respectively) for BP1 (left panel) and BP2 (right panel) in Fig. 16. The criteria has been set at Planck scale. We use $\alpha_S = 0.1184$ GeV and $m_h = 125.09$ GeV for both the plots. The solid lines indicate the contour for $m_t = 173.1$ GeV while the dotted lines denote 1σ uncertainty of the measured value of $m_t$. It can be concluded from Fig. 20, that to have a stable/metastable EW vacuum, smaller values of $M_R$ requires low $\text{Tr}[Y_L^\dagger Y_R]$ and
Figure 19: RG running of all copositivity criteria in Eqn. 3.1 for BP1 (left) and BP2 (right) to ensure the boundedness of the scalar potential in any field direction with energy scale $\mu$ for $\text{Tr}[Y^T\nu Y_\nu] = 0.2$ (left) and 0.7 (right) The choices of $\text{Tr}[Y^T\nu Y_\nu]$ are shown in dark blue and purple colours in left and right panels of Fig. 17 respectively to yield stability.

The main important distinction between the left and right panel figure arises from two different benchmark points used for the analysis. BP2 has significantly larger parameter space available from high scale stability. This is because of the larger values of $\lambda_{1,2,3}$ parameters used in BP2 compared to BP1 (see Table 4 for details). Therefore, it can be concluded, that larger is the mass splitting in IDM sector, the more favourable it is from the stability point of view. However there is an important catch to this statement, which we will illustrate next.

Figure 20: Stability, metastability and instability regions plot on $M_R - \text{Tr}[Y^T\nu Y_\nu]$ plane for the benchmark point BP1 (left panel) and BP2 (right panel). For illustration we have considered top mass variation in 1$\sigma$ range : $173.1 \pm 0.9$ GeV.

In Fig. 21 we plot the running of all the individual couplings present in the set up. We see that (fortunately) for the two benchmark points used in the analysis, we are still okay
Figure 21: RG running of relevant coupling parameters for BP1 (left panel) and BP2 (right panel). $M_R = 10^8$ GeV, $\text{Tr}[Y_0^\dagger Y_0] = 0.5$ and $m_t = 173.1$ GeV have been kept fixed in both plots.

with the perturbative limit at the high scale, i.e. all the couplings obey the perturbative limit, $\lambda_i < 4\pi$, $\text{Tr}[Y_0^\dagger Y_0] < 4\pi$ at Planck scale. However, for BP1, with $M_R = 10^8$ GeV, and $\text{Tr}[Y_0^\dagger Y_0] = 0.5$ as shown in the left panel yields unstable solution to EW vacuum with $\lambda_H$ running negative. On the other hand, BP2 with same choices of right handed neutrino mass and Yukawa yields a stable vacuum. Therefore, once again we find that larger splitting in IDM sector is more favourable for EW vacuum stability, as we have also inferred before from Fig. 20. But, it turns out that as larger splitting in the IDM sector also uses larger values of $\lambda_{1,2,3}$, it is possible, that many of those points become non-perturbative i.e. $\lambda_i > 4\pi$ when run up to Planck scale. We will show in the next section, that this very phenomena disallows many of the relic density and direct search allowed parameter space of model. Another point to end this section is to note that our available parameter space from DM constraints heavily depend on large DM-DM conversion, which naturally comes from large choices of the conversion coupling $\lambda_c$. It is natural, that some of those cases will also be discarded by the perturbative limit $\lambda_c < 4\pi$, when we evaluate the validity of the model at high scale.

7.2 Allowed parameter space of the model from high scale validity

Finally, we come to the point where we can assimilate all the constraints together, from DM constraints to high scale validity. In order to do that, we choose relic density and direct search allowed parameter space of the model as discussed in Section 6 and impose the high scale validity of the model till some energy scale $\Lambda$ by demanding:

- Stability of the total scalar potential (Eq.(3.1) determined by the satisfaction of the copositivity conditions for any energy scale $\mu$.
- Non-violation of perturbativity and unitarity of all the relevant couplings present in the model as defined in Eqns. 3.2 and 3.3.

Note that the high scale validity of the models does not depend on the structure of mass hierarchy of the DM candidates (i.e. on the sign of mass difference $m_{H_0} - m_{\phi}$). To study the
EW vacuum stability we demand the positivity of $\lambda_H$ at each energy scale from EW to high scale $\Lambda$. As evident from $\beta_{\lambda_H}$ in Eqn. 7.3, a particular combination of the scalar couplings in the form of $2\lambda_1^2 + \lambda_2^2 + \lambda_3^2 + 2\lambda_1\lambda_2 + \frac{1}{2}\lambda_{ph}^2$ determines the positivity of $\lambda_H$ during its running. However the factor $\lambda_{ph}$ is strongly constrained from the SI DD cross section bound for $m_{\phi} < 500$ GeV. Hence, we can assume safely that the factor $2\lambda_1^2 + \lambda_2^2 + \lambda_3^2 + 2\lambda_1\lambda_2$ without $\lambda_{ph}$ effectively determines the stability of Higgs vacuum in relic density and direct search allowed parameter space. It turns out that when $\lambda_H > 0$, all other copositivity conditions for all relic and DD cross section satisfied points in our model stays positive from $\mu = m_t$ to $\mu = M_P$.

In Fig. 22, we constrain relic density and direct search allowed points of the model in $2\lambda_1^2 + \lambda_2^2 + \lambda_3^2 + 2\lambda_1\lambda_2$ plane to additionally satisfy perturbativity and vacuum stability conditions following Eqn. 3.1. Orange points satisfy relic density and DD bounds, while the green, blue and red points on top of that satisfy perturbativity and vacuum stability conditions about high energy scales $\mu = 10^{10}$ GeV, $10^{16}$ GeV and $10^{19}$ GeV respectively. This very figure essentially shows that all those points with either small values of $\lambda_1$ (i.e. small $m_{H^\pm} - m_{H^0}$) are discarded due to stability of EW vacuum, while those with large $\lambda_1$ (i.e. large $m_{H^\pm} - m_{H^0}$) are discarded by perturbative limits of the coupling at high scale.

In Fig. 23 we study the correlation between the individual scalar couplings to satisfy the DM constraints, perturbativity limits and vacuum stability criteria. In left panel of Fig. 23, we show the DM relic density and DD cross section satisfied points in $\lambda_2 - \lambda_3$ plane for different values of $\lambda_1$. In right panel we first identify the relevant parameter space in the same plane which satisfy the DM constraints, the perturbativity bound and vacuum stability criteria till EW energy scale. Then we further impose perturbativity bound and
Figure 23: Relic density and direct search allowed points in $\lambda_2 - \lambda_3$ plane for different values of $\lambda_1$ (left). Allowed points in the same parameter space from DM constraints (orange), stability and perturbativity conditions following Eqns. 3.1-3.3 considering the high scale $\mu = \mu_{EW}$, $10^{10}$ GeV (green), $10^{16}$ GeV (blue) and $10^{19}$ GeV (purple) (right).

Vacuum stability conditions considering the high scale as $\mu = 10^{10}$, $10^{6}$ GeV and $10^{19}$ GeV in addition to the DM constraints. It is seen that the lower portion of the available parameter space gets discarded by vacuum stability or high value of $\lambda_c$ while perturbativity bounds constrain the higher values of the couplings. In this plot also we kept $M_R = 10^8$ GeV, $\text{Tr}[Y^\dagger \nu Y \nu] = 0.5$ with $m_t = 173.1$ GeV. We must also note that with larger $M_R$ and smaller Yukawa $\text{Tr}[Y^\dagger \nu Y \nu]$, we could obtain a larger available parameter space from high scale validity.

In Fig. 24, we show the high scale validity of relic density and direct search allowed parameter space of the model in $m_{H^0} - m_{A^0}$ (top left), $m_{H^0} - m_{H^\pm}$ (top right) and $\Delta m = m_{H^0} - m_{H^\pm}$ planes with $M_R = 10^8$ GeV and $\text{Tr}[Y^\dagger \nu Y \nu] = 0.5$ at different high energy scales $\mu = \{10^{10}, 10^{16}, 10^{19}\}$ GeVs denoted by light blue, dark blue and red points. The orange points are corresponding to relic and direct search allowed parameter space at EW scale. We see that larger mass difference between inert higgs components, $\Delta m - \Delta M$ which are related with quartic couplings, $\lambda_{1,2,3}$ (see Eqn. 2.7), are discarded from perturbativity conditions mentioned in Eqn. 3.2. While the small mass differences between inert components are also excluded from stability criteria of Higgs potential.

Till now, while discussing the effect of stability and high scale validity of the proposed set up, we have considered fixed right handed neutrino mass, $M_R = 10^8$ GeV and corresponding Yukawa coupling $\text{Tr}[Y^\dagger \nu Y \nu] = 0.5$. For sake of completeness we extend our study for few different values of RH neutrino mass and $\text{Tr}[Y^\dagger \nu Y \nu]$ and find out the allowed ranges of the relevant parameters considering both the hierarchies $m_{\phi} > m_{H^0}$ and $m_{\phi} \leq m_{H^0}$ separately. We note the corresponding results in Table 9 and Table 10 of Appendix B.
We further apply stability and perturbativity conditions following Eqns. 3.1-3.3 at different energy scales $\mu = 10^{10}$ GeV (light blue), $10^{16}$ GeV (dark blue) and $10^{19}$ GeV (red).

**Figure 24:** Relic density and direct search allowed points (orange) in $m_{A^0} - m_{H^0}$ (top left), $m_{H^\pm} - m_{H^0}$ (top left) and $\Delta m(= m_{A^0} - m_{H^0}) - \Delta M(= m_{H^\pm} - m_{H^0})$ (bottom). We further apply stability and perturbativity conditions following Eqns. 3.1-3.3 at different energy scales $\mu = 10^{10}$ GeV (light blue), $10^{16}$ GeV (dark blue) and $10^{19}$ GeV (red).

8 Collider signature of Inert doublet DM at LHC

Inert doublet has been an attractive DM framework, due to the possibility of collider detection [94, 95]. Here, we relook into the possible collider search strategies of IDM at LHC in presence of a second scalar singlet DM component. It is also worth mentioning here that the real scalar singlet, which interacts with SM only through Higgs portal coupling, does not have any promising collider signature excepting mono-$X$ signature arising out of initial state radiation (ISR), where $X$ stands for $W, Z, \text{jet}$. Such signals are heavily submerged in SM background due to weak production cross-section of DM in relic density and direct search allowed parameter space [96]. The charge components $H^+, H^-$ of inert DM can be produced at LHC via Drell-Yan $Z$ and $\gamma$ mediation as well as through Higgs mediation. Further decay of $H^\pm$ to DM ($H^0$) and leptonic final states through on/off shell $W^\pm$ yields hadronically quiet opposite sign di-lepton plus missing energy (OSDL+$E_T$), as shown in left panel of Fig. 25. In this study, we focus on this particular signal of inert dark matter

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1There are other possible signatures (for example, three lepton final state) of inert DM arising from the the combination of $H^\pm, A^0$ production and their subsequent decays, for a detailed list see [42, 95].
as detailed below:

\[
\text{Signal} \ni \text{OSDL} + \hat{E}_T \equiv \ell^+\ell^- + (\hat{E}_T) : \ p\ p \rightarrow H^+H^-, \ (H^- \rightarrow \ell^- \nu_{\ell} H^0), \ (H^+ \rightarrow \ell^+ \nu_{\ell} H^0); \quad \text{where } \ell = \{e, \mu\}.
\]

\[\sqrt{s} = 14 \text{ TeV as a function of } \sqrt{s} = 14 \text{ TeV at LHC.} \]

\[\text{Figure 25: [Left] Feynman graph for OSDL+} \hat{E}_T \text{ signature of IDM at LHC. [Right] Variation of production cross-section } \sigma_{pp \rightarrow H^+H^-} \text{ (in fb) with } m_{H^\pm} (= m_{H^0} + \Delta M) \text{ in GeV for different choices of } m_{H^\pm} - m_{H^0} \text{ for center- of mass energy } \sqrt{s} = 14 \text{ TeV at LHC.} \]

| BPs | \{ m_{H^0}, m_\phi, m_{A_0}, m_{H^\pm}, \lambda_L, \lambda_{\phi h}, \lambda_c \} | \Omega_{H^0}h^2 | \Omega_\phi h^2 | \sigma_{H^0}^eff (cm^2) | \sigma_\phi^eff (cm^2) |
|-----|-------------------------------------------------|-----------------|-----------------|----------------|----------------|
| BPC1 | \{ 177, 309, 193, 207, 0.039, 0.011, 0.11 \} | 0.0100 | 0.1097 | 1.4 \times 10^{-46} | 1.0 \times 10^{-47} |
| BPC2 | \{ 122, 421, 167, 173, 0.0019, 0.032, 0.13 \} | 0.003 | 0.116 | 2.1 \times 10^{-47} | 4.9 \times 10^{-47} |
| BPC3 | \{ 81, 499, 90, 162, 0.012, 0.101, 0.12 \} | 0.006 | 0.111 | 3.8 \times 10^{-47} | 3.3 \times 10^{-46} |
| BPC4 | \{ 74, 482, 132, 143, 0.002, 0.153, 0.17 \} | 0.0645 | 0.0496 | 1.4 \times 10^{-47} | 3.7 \times 10^{-46} |

| BPs | \{ m_{H^\pm} - m_{H^0} \} | \{ \lambda_1, \lambda_2, \lambda_3 \} | \{ M_R, \text{ Tr}[Y_0^TY_0] \} | \text{Validity Scale (}\mu\text{)} |
|-----|-----------------|-----------------|----------------|----------------|
| BPC1 | 30 \ (< m_W) \ | \{ 0.459, \ -0.283, \ -0.098 \} | \{ 10^8, \ 0.5 \} | 1.22 \times 10^{19} (M_{pl}) |
| BPC2 | 51 \ (< m_W) \ | \{ 0.535, \ -0.282, \ -0.215 \} | \{ 10^8, \ 0.5 \} | 1.22 \times 10^{19} (M_{pl}) |
| BPC3 | 81 \ (~ m_W) \ | \{ 0.674, \ -0.625, \ -0.025 \} | \{ 10^8, \ 0.5 \} | \sim 10^{16} (\text{GUT}) |
| BPC4 | 69 \ (< m_W) \ | \{ 0.499, \ -0.2974, \ -0.197 \} | \{ 10^8, \ 0.5 \} | 1.22 \times 10^{19} (M_{pl}) |

\[\text{Table 4: DM masses, quartic couplings, relic densities and spin independent effective DM-}\]
\[\text{neucleon cross-section of selected benchmark points for collider study. All benchmark points}\]
\[\text{chosen here have } m_{H^\pm} - m_{H^0} < m_{W^\pm} \text{ for off-shell production of } W^\pm. \text{ The maximum scale}\]
\[\text{(}\mu\text{)} \text{ of Higgs vacuum satability and peturbativity in presence of right handed neutrinos are}\]
\[\text{also noted. All masses and scales are in GeV.} \]

In the right panel of Fig. 25, we show variation of charged pair \((H^+ H^-)\) production cross-section at LHC for center-of-mass energy \(\sqrt{s} = 14 \text{ TeV as a function of } m_{H^\pm} = \)
Table 5: DM masses, quartic couplings, relic densities and spin independent effective DM-nucleon cross-section of selected benchmark points for collider study. All benchmark points chosen here have $m_{H^\pm} - m_{h^0} > m_{W^\pm}$ for on-shell production of $W^\pm$. The maximum scale ($\mu$) of Higgs vacuum stability and perturbativity in presence of right handed neutrinos are also noted. All masses and scales are in GeV.

$m_{h^0} + \Delta M$, where $\Delta M$ indicates the mass difference with the inert DM and serves as a very important variable for the signal characteristics. The plot on RHS show that production cross-section is decreasing with larger charged scalar mass $m_{H^\pm}$, where we have demonstrated three fixed values of $m_{H^\pm} - m_{h^0} = 5, 50$ and 100 GeV. Around $m_{H^\pm} \sim m_h/2$, there is a sharpe fall of production cross-section. This is because, for $m_{H^\pm} \leq m_h/2$, there is a significant contribution arising from Higgs production and its subsequent decay to the charged scalar components, which otherwise turns into an off-shell propagator to yield a subdued contribution to Drell-Yan production. Following [97], a conservative bound on the charge scalars is applied here as $m_{H^\pm} \geq 70$ GeV, as indicated in the RHS plot of Fig. 25.

Table 6: Signal cross-section for BPC1-BPC4 after the selection cuts are employed.
We next choose a set of benchmark points (BPs) allowed from DM relic, direct search constraints as well as from Higgs invisible decay constraints for performing collider simulation, shown in Table 4 and Table 5. The BPs are also allowed from absolute Higgs vacuum stability and perturbativity limits in presence of right handed neutrinos, valid upto scale $\mu$ as mentioned in the tables. The benchmark points are divided into two categories: (BPC1-BPC4) in Table 4 correspond to $\Delta M = m_{H^\pm} - m_{H^0} \lesssim m_{W^\pm}$ where the charged scalar, $H^\pm$ decay through off-shell $W^\pm$. On the other hand, benchmark points (BPD1-BPD4) in Table 5 correspond to $\Delta M = m_{H^\pm} - m_{H^0} > m_{W^\pm}$, where the charged scalar $H^\pm$ decay through on-shell $W^\pm$. Each table (Table 4 and Table 5) consists of two parts: the first part contains all the relevant dark sector masses, couplings, relic density and direct search cross-sections of both DM components. The second part demonstrates the mass difference ($\Delta M = m_{H^\pm} - m_{H^0}$), choice of right handed neutrino mass, neutrino Yukawa and the maximum scale of validity ($\mu$) of the Higgs vacuum.

The simulation technique adopted here is as follows. We first implemented the model in FeynRule [98] to generate UFO file which is required to feed into event generator Madgraph[99]. Then these events are passed to Pythia [100] for hadronization. All parton level leading order (LO) signal events and SM background events$^2$ are generated in Madgraph at $\sqrt{s} = 14$ TeV using cteq6l1 [101] parton distribution. Leptons ($\ell = e, \mu$) isolation, jet and unclustered event formation to mimic to the actual collider environment

| BPs | $\sigma_{p,p\rightarrow H^+ H^-}$(fb) | $E_T$(GeV) | $H_T$(GeV) | $\sigma^{\text{OSD}}$(fb) | $N^{\text{OSD}}_{\text{eff}}@L=10^2 \text{ fb}^{-1}$ |
|-----|----------------------------------|------------|------------|-----------------|-----------------------------------|
| BPD1 | 16.70 | $>100$ | $>150$ | 0.002 | $<1$ |
|      |      | $>150$ | $>150$ | 0.0002 | $<1$ |
|      |      | $>200$ | $>150$ | 0.0002 | $<1$ |
| BPD2 | 11.75 | $>100$ | $>150$ | 0.008 | 1 |
|      |      | $>150$ | $>150$ | 0.004 | $<1$ |
|      |      | $>200$ | $>150$ | 0.0009 | $<1$ |
| BPD3 | 18.65 | $>100$ | $>150$ | 0.015 | 2 |
|      |      | $>150$ | $>150$ | 0.008 | 1 |
|      |      | $>200$ | $>150$ | 0.003 | $<1$ |
| BPD4 | 12.00 | $>100$ | $>150$ | 0.010 | 1 |
|      |      | $>150$ | $>150$ | 0.005 | 1 |
|      |      | $>200$ | $>150$ | 0.003 | $<1$ |

Table 7: Signal cross-section for BPD1-BPD4 after the selection cuts are employed.

$^2$There are several SM process which contribute to the chosen $\ell^+\ell^-+(E_T)$ signal, dominant processes are: $t\bar{t}$, $W^+W^-$, $ZZ$ and $W^+W^-Z$. 


are performed as follows:

- Lepton isolation: The minimum transverse momentum required to identify a lepton \((\ell = e, \mu)\) has been kept as \(p_T > 20\) GeV and we also require the lepton to be produced in the central region of detector followed by pseudorapidity selection as \(|\eta| < 2.5\). Two leptons are separated from each other with minimum distance \(\Delta R \geq 0.2\) in \(\eta-\phi\) plane. To separate leptons from jets we further imposed \(\Delta R \geq 0.4\).

- Jet formation: For jet formation, we used cone algorithm PYCELL in built in Pythia. All partons within a cone of \(\Delta R \leq 0.4\) around a jet initiator with \(p_T > 20\) GeV is identified to form a jet. It is important to identify jets in our case because we require the final state signal to be hadronically quiet i.e. to have zero jets.

- Unclustered Objects: All final state objects with \(0.5 < p_T < 20\) GeV and \(2.5 < |\eta| < 5\) are considered as unclustered objects. Those objects neither form jets nor identified as isolated leptons and they only contribute to missing energy.

\[\text{Figure 26: Distribution of missing energy (}\hat{E}_T\text{), invariant mass of opposite sign Dilepton (}\ell^+\ell^-\text{) and transverse mass (}\hat{H}_T\text{) for signal events }\ell^+\ell^- + (\hat{E}_T)\text{ and dominant SM background events at LHC with }\sqrt{s} = 14\text{ TeV}.\]

The main idea is to see if the signal events rise over SM background. For that there are three key kinematic variables where the signal and background show different sensitivity. They are:
Figure 27: Distribution of missing energy ($\vec{E}_T$), invariant mass of opposite sign Dilepton ($m_{\ell^+\ell^-}$) and transverse mass ($H_T$) for signal events $\ell^+\ell^- + (\vec{E}_T)$ and dominant SM background events at LHC with $\sqrt{s} = 14$ TeV.

• **Missing Energy ($\vec{E}_T$):** The most important signature of DM being produced at collider. This is defined by a vector sum of transverse momentum of all the missing particles (those are not registered in the detector); this in turn can be estimated form the momentum imbalance in the transverse direction associated to the visible particles. Thus missing energy (MET) is defined as:

$$\vec{E}_T = -\sqrt{\left(\sum_{\ell,j} p_x\right)^2 + \left(\sum_{\ell,j} p_y\right)^2}, \quad (8.1)$$

where the sum runs over all visible objects that include the leptons, jets and the unclustered components.

• **Transverse Mass ($H_T$):** Transverse mass of an event is identified with the scalar sum of the transverse momentum of objects reconstructed in a collider event, namely lepton and jets as defined above.

$$H_T = \sum_{\ell,j} \sqrt{(p_x)^2 + (p_y)^2}. \quad (8.2)$$
• **Invariant mass** \((m_{\ell\ell})\): Invariant mass of opposite sign dilepton hints to the parent particle, from which the leptons have been produced and thus helps segregating signal from background. This is defined as:

\[
m_{\ell^+\ell^-} = \sqrt{\left(\sum_{\ell^+\ell^-} p_x\right)^2 + \left(\sum_{\ell^+\ell^-} p_y\right)^2 + \left(\sum_{\ell^+\ell^-} p_z\right)^2}.
\]  

(8.3)

The distribution of missing energy \((E_T)\), invariant mass of opposite sign dilepton \((m_{\ell^+\ell^-})\) and transverse mass \((H_T)\) for the BPs along with dominant SM background events are shown in Fig. 26 top left, top right and bottom panel respectively. All BPs depicted in Fig. 26 correspond to \(m_{H^\pm} - m_{H^0} \equiv \Delta M < m_{W^\pm}\) where opposite sign di-lepton are produced from off-shell \(W^\pm\) mediator. All the distributions are normalised to one event. Missing energy (as well as \(H_T\)) distributions of BPs (BPC1-BPC4) show that the peak of distribution for the signal is on the left of SM background. This is because the benchmark points are characterised by small \(\Delta M\), where the charged scalars and inert DM have small mass splitting. Therefore, such situations are visibly segregated from SM background by MET and \(H_T\) distribution. Clearly, when \(\Delta M\) becomes \(m_{W^\pm}\) (for example, BPC3) the distribution closely mimic SM background. Therefore the signal events for this class of benchmark points can survive for a suitable upper \(E_T\) and \(H_T\) cut while reducing SM backgrounds. It is important to take a note that OSDL events coming from \(ZZ\) background naturally peaks at \(m_Z\) in \(m_{\ell\ell}\) distribution. Therefore, we use invariant mass cut in the \(Z\) mass window to get rid of this background.

The situation is reversed for larger splitting between the charged scalar component with DM, i.e \(\Delta M \equiv m_{H^\pm} - m_{H^0} > m_{W^\pm}\) corresponding to BPs (BPD1-BPD4) as in Table 5. The distributions of \(E_T\), \(m_{\ell\ell}\) and \(H_T\) therefore become flatter and peak of the distribution shifts to higher value as shown in Fig. 27. In such cases the signal events for large \(\Delta M\) can be separated from SM background at a suitable lower end cut of \(E_T\) and \(H_T\).

Therefore the selection cuts used in this analysis are summarised as follows:

- **Invariant mass** \((m_{\ell\ell})\) cuts: \(m_{\ell\ell} < (m_Z - 15)\) GeV and \(m_{\ell\ell} > (m_Z + 15)\) GeV.
- **\(H_T\) cuts:**
  - \(H_T < 70\) when \(m_{H^\pm} - m_{H^0} < m_{W^\pm}\).
  - \(H_T > 150, 200\) when \(m_{H^\pm} - m_{H^0} > m_{W^\pm}\).
- **\(E_T\) cuts:**
  - \(E_T < 30, 40\) when \(m_{H^\pm} - m_{H^0} < m_{W^\pm}\).
  - \(E_T > 100, 150\) when \(m_{H^\pm} - m_{H^0} > m_{W^\pm}\).

We next turn to signal and background events that survive after the selection cuts are employed. The signal events are listed in Table 6 for BPC1-BPC4. We can see that BPC4 (where IDM lies within \(m_W\)) have the better chance due to large production cross section, other points like BPC2, also has some possibility of showing its presence at high
| SM Bkg. | $\sigma_{p \rightarrow SM}$ (fb) | $E_T$ (GeV) | $H_T$ (GeV) | $\sigma_{\text{OSD}}$ (fb) | $N_{\text{eff}}^{\text{OSD}}@\mathcal{L} = 10^3 \text{fb}^{-1}$ |
|--------|-----------------|-------------|-------------|-----------------|--------------------------|
| $t \bar{t}$ | $814.78 \times 10^3$ | $< 30$ | $< 70$ | 13.04 | 1304 |
| | | $< 40$ | | 24.44 | 2444 |
| | | $> 100$ | $> 150$ | 10.59 | 1059 |
| | | | $> 200$ | 2.44 | 244 |
| | | $> 150$ | $> 150$ | 2.44 | 244 |
| | | | $> 200$ | 0.00 | $< 1$ |
| $W^+ W^-$ | $100.06 \times 10^3$ | $< 30$ | $< 70$ | 104.56 | 10456 |
| | | $< 40$ | | 160.09 | 16009 |
| | | $> 100$ | $> 150$ | 18.01 | 1801 |
| | | | $> 200$ | 10.01 | 1001 |
| | | $> 150$ | $> 150$ | 6.00 | 600 |
| | | | $> 200$ | 6.00 | 600 |
| | | $> 200$ | $> 150$ | 2.00 | 200 |
| | | | $> 200$ | 2.00 | 200 |
| $Z Z$ | $14.00 \times 10^3$ | $< 30$ | $< 70$ | 0.70 | 70 |
| | | $< 40$ | | 0.98 | 98 |
| | | $> 100$ | $> 150$ | 0.14 | 14 |
| | | | $> 200$ | 0.14 | 14 |
| | | $> 150$ | $> 150$ | 0.14 | 14 |
| | | | $> 200$ | 0.14 | 14 |
| $W^+ W^- Z$ | $0.16 \times 10^3$ | $< 30$ | $< 70$ | 0.03 | 3 |
| | | $< 40$ | | 0.04 | 4 |
| | | $> 100$ | $> 150$ | 0.13 | 13 |
| | | | $> 200$ | 0.09 | 9 |
| | | $> 150$ | $> 150$ | 0.076 | 8 |
| | | | $> 200$ | 0.056 | 6 |
| | | $> 200$ | $> 150$ | 0.042 | 4 |
| | | | $> 200$ | 0.036 | 4 |

Table 8: Dominant SM background contribution to $\ell^+ \ell^- + (E_T)$ signal events for $\sqrt{s} = 14$ TeV at LHC. The variation of effective number of final state background events for luminosity $\mathcal{L} = 100 \text{ fb}^{-1}$ with $E_T$, $H_T$ and $m_{\ell\ell}$ cut-flow are tabulated. To incorporate the Next-to-Leading order (NLO) cross section of SM background we have used appropriate K-factors [102].

We would like to make a remark here that the point like BPC4 is already available in the single component IDM framework. However, the coupling ($\lambda_L$) requires to be smaller in single component framework to address total relic density. The presence of second DM component as in our model, enhances the DM-SM coupling ($\lambda_L$) for under abundance and therefore enhances the visibility of such points at collider. It is obvious that
a two component framework also allows points like BPC1, BPC2 and BPC3 where inert DM mass is larger than $m_W$ and is accessible for collider detection with large luminosity.

Similarly, signal events for BPD1-BPD4 are listed in Table 7 using a lower cut on $\slashed{E}_T$ and $H_T$. The signal cross-section and event numbers for this class of points are much smaller due to the fact that the charged scalar masses are on higher side as for all of the cases $\Delta m > m_W$ independent of DM masses. However, the SM background events get even more suppressed with the set of $\slashed{E}_T$, $H_T$ cuts. The SM background cross-section and event numbers after cut flow is mentioned in Table 8. Therefore, in spite of smaller signal events in this region of parameter space with BPD benchmark points, the discovery potential of the signal requires similar luminosity to that of BPC cases.

![Figure 28](image_url)

**Figure 28**: Signal significance of some select benchmark points at LHC for $\sqrt{s} = 14$ TeV, in terms of Luminosity ($fb^{-1}$). $3\sigma$ and $5\sigma$ lines are shown. Left: Points with $\Delta M < m_W$; Right: Points with $\Delta M > m_W$.

Finally we present the discovery reach of the signal events in terms of significance $\sigma = \frac{S}{\sqrt{S+B}}$, where $S$ denotes signal events and $B$ denotes SM background events in terms of luminosity. This is shown in Fig. 28. This shows that the benchmark points that characterise the two component DM framework, can yield a visible signature at high luminosity with $\mathcal{L} \sim 10^5 - 10^6 fb^{-1}$ depending on the charged scalar mass and its splitting with DM.

## 9 Summary and Conclusions

We have studied a two component scalar DM model in presence of right handed neutrinos that address neutrino mass generation through type I seesaw. The DM components are (i) a singlet scalar and (ii) an inert scalar doublet, both studied extensively as single component DM in literature. We show that the presence of second component enlarges the available parameter space significantly considering relic density and direct search constraints. In particular, the inert scalar DM will now be allowed in the so called ‘desert region’: \{m_W - 550\} GeV. Also for singlet scalar, we can now revive it below TeV, which is otherwise discarded (except Higgs resonance) from direct search in single component framework. The
results obtained for DM analysis crucially depends on DM-DM conversion, which have been demonstrated in details.

We also study the high scale perturbativity and vacuum stability of the Higgs potential by analysing two loop RGE β functions. This in turn puts further constraints on the available DM parameter space of the model. One of the important conclusions obtained are that the mass splitting of the charged scalar component to the corresponding DM component of inert doublet is crucially tamed depending on the absolute stability scale of the scalar potential, coming from the perturbativity constraint on the quartic and Yukawa coupling. For example, we find:

- Validity scale ($\mu$) $\sim$ intermediate scale ($10^{10}$ GeV): $\Delta M = m_{H^\pm} - m_{H^0} \sim \{7 - 120\}$ GeV,

- Validity scale ($\mu$) $\sim$ Planck scale ($10^{19}$ GeV): $\Delta M = m_{H^\pm} - m_{H^0} \sim \{11 - 70\}$ GeV,

with RH neutrino mass $M_R = 10^8$ GeV and Yukawa $\text{Tr}[Y^\dagger \nu Y \nu] = 0.5$. The presence of RHNs in the model not only helps us addressing the neutrino masses but also controls the high scale validity of the model parameters, for example, low $\Delta M$ regions. This is how the neutrino and dark sector constraints affect each other.

Inert Higgs having charged scalars have collider detectability. We point out that the collider search prospect of the charged components are not only limited to low DM masses ($< m_W$), but is open to a larger mass range in presence of the second DM component, even after taking the high scale validity constraints. We exemplified this at LHC for hadronically quiet dilepton channel with missing energy, where $\Delta M$, turns out to be a crucial kinematic parameter, constrained from DM, high scale validity and neutrino sector. LHC, due to its $t\bar{t}$ background, can only probe the high $\Delta M$ regions of this model, where $e^+e^-$ annihilation have the possibility to explore low $\Delta M$ regions. Here, we would like to comment that there are several studies that have been done in this direction, but the high scale validity constraint may alter the conclusion significantly as we demonstrate.

Finally, we would like to mention that the analysis performed here, although focus on a specific model set up, but there are some generic conclusions that can be borrowed. For example, if the two DM components have sufficient interaction in between, the available parameter space will be enlarged significantly from both relic density and direct search. The conversion of one DM into the other may also affect the collider outcome of the DM significantly. It is obvious that richer signal is obtained when we have larger multiplets in dark sector (as scalar doublet produces two lepton final state in the analysis). It is also possible that the dark sector and neutrino sector although may not inherit a common origin, the high scale validity of the model can bring them together.

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A Vacuum stability and High Scale Validity of Single component DM

Here we show the allowed parameter space for single component DM when the vacuum stability conditions are taken into account in presence of RH neutrinos. In left panel of Fig. 29 the parameter space for scalar singlet DM ($\phi$) is shown considering $M_R = 10^8$ GeV and $\text{Tr}[Y_d^TY_u] = 0.3$, while in right panel we present the same for IDM ($H^0$) with $M_R = 10^8$ GeV and $\text{Tr}[Y_d^TY_u] = 0.5$. We see that for $\phi$, the DM mass is allowed beyond 900 GeV considering the absolute stability of the EW vacuum upto $10^{10}$ GeV [81]. However for IDM, we notice that the absolute EW vacuum stability can be extended even upto Planck scale (with simillar Yukawa) due to the presence of several scalar degrees of freedom.

![Figure 29](image.png)

**Figure 29**: Parameter space scan for (left) scalar singlet DM and (right) IDM considering satisfaction of relic density bound, direct detection cross section limit and high scale validity in presence of RH neutrinos.
### B Constraining relevant parameters from relic density, direct detection and high scale validity

| $m_\phi > m_{H_0}$ (in GeV) | RH Neutrinos | Relic + DD (XENON 1T) + Stability + Perturbativity valid range | $\mu = 10^{10}$ GeV | $\mu = 10^{10}$ GeV (GUT) | $\mu = 10^{10}$ GeV ($M_{1/2}$) |
|-----------------------------|--------------|---------------------------------------------------------------|-------------------|--------------------------------|---------------------------------|
| $m_{H_0} > 80$ GeV          | $m_{H_0} \sim (80 - 500)$ | $m_{H_0} \sim (81 - 496)$ | $m_{H_0} \sim (81 - 496)$ | $m_{H_0} \sim (81 - 496)$ |
| $m_{H_0} \sim (171 - 500)$ | $m_{H_0} \sim (218 - 500)$ | $m_{H_0} \sim (218 - 500)$ | $m_{H_0} \sim (218 - 500)$ | $m_{H_0} \sim (218 - 500)$ |
| $\Delta m \sim (0 - 90)$    | $\Delta m \sim (0 - 52)$ | $\Delta m \sim (0 - 52)$ | $\Delta m \sim (0 - 50)$ | $\Delta m \sim (0 - 50)$ |
| $\Delta M \sim (3 - 120)$  | $\Delta M \sim (3 - 81)$ | $\Delta M \sim (3 - 81)$ | $\Delta M \sim (3 - 69)$ | $\Delta M \sim (3 - 69)$ |
| $\lambda_s \sim (0.066 - 0.7)$ | $\lambda_s \sim (0.066 - 0.52)$ | $\lambda_s \sim (0.066 - 0.52)$ | $\lambda_s \sim (0.066 - 0.52)$ | $\lambda_s \sim (0.066 - 0.52)$ |
| $\lambda_L \sim (0.001 - 0.12)$ | $\lambda_L \sim (0.001 - 0.09)$ | $\lambda_L \sim (0.001 - 0.09)$ | $\lambda_L \sim (0.001 - 0.076)$ | $\lambda_L \sim (0.001 - 0.076)$ |
| $\lambda_{ab} \sim (0.001 - 0.14)$ | $\lambda_{ab} \sim (0.001 - 0.14)$ | $\lambda_{ab} \sim (0.001 - 0.14)$ | $\lambda_{ab} \sim (0.001 - 0.14)$ | $\lambda_{ab} \sim (0.001 - 0.14)$ |

| $m_{H_0} > 80$ GeV          | $m_{H_0} \sim (80 - 500)$ | $m_{H_0} \sim (81 - 496)$ | $m_{H_0} \sim (81 - 496)$ | $m_{H_0} \sim (81 - 496)$ |
| $m_{H_0} \sim (171 - 500)$ | $m_{H_0} \sim (218 - 500)$ | $m_{H_0} \sim (218 - 500)$ | $m_{H_0} \sim (218 - 500)$ | $m_{H_0} \sim (218 - 500)$ |
| $\Delta m \sim (0 - 90)$    | $\Delta m \sim (0 - 52)$ | $\Delta m \sim (0 - 52)$ | $\Delta m \sim (0 - 50)$ | $\Delta m \sim (0 - 50)$ |
| $\Delta M \sim (3 - 123)$  | $\Delta M \sim (3 - 81)$ | $\Delta M \sim (3 - 81)$ | $\Delta M \sim (3 - 69)$ | $\Delta M \sim (3 - 69)$ |
| $\lambda_s \sim (0.066 - 0.69)$ | $\lambda_s \sim (0.066 - 0.52)$ | $\lambda_s \sim (0.066 - 0.52)$ | $\lambda_s \sim (0.066 - 0.52)$ | $\lambda_s \sim (0.066 - 0.52)$ |
| $\lambda_L \sim (0.001 - 0.119)$ | $\lambda_L \sim (0.001 - 0.09)$ | $\lambda_L \sim (0.001 - 0.09)$ | $\lambda_L \sim (0.001 - 0.076)$ | $\lambda_L \sim (0.001 - 0.076)$ |
| $\lambda_{ab} \sim (0.001 - 0.14)$ | $\lambda_{ab} \sim (0.001 - 0.14)$ | $\lambda_{ab} \sim (0.001 - 0.14)$ | $\lambda_{ab} \sim (0.001 - 0.14)$ | $\lambda_{ab} \sim (0.001 - 0.14)$ |

Table 9: Allowed ranges of relevant parameters considering $m_\phi > m_{H_0}$ for different values of RH neutrino mass and $\text{Tr}[Y_d^T Y_d]$. 
| $m_\nu \leq m_{\nu}\nu$ (in GeV) | RH Neutrinos | Relic + DD (XENON 1T) allowed range $M_H$ | Relic + DD (XENON 1T) + Stalitity + Perturbativity valid range $\mu = 10^{16}$ GeV | $\mu = 10^{18}$ GeV (GUT) | $\mu = 10^{19}$ GeV (M$A_0$) |
|---|---|---|---|---|---|
| $m_{\nu} \sim \{ 95 - 479 \}$ | $m_{\nu} \sim \{ 95 - 479 \}$ | $m_{\nu} \sim \{ 205 - 452 \}$ | $m_{\nu} \sim \{ 205 - 452 \}$ | $m_{\nu} \sim \{ 205 - 225 \}$ |
| $m_{\nu} \sim \{ 93 - 571 \}$ | $m_{\nu} \sim \{ 93 - 471 \}$ | $m_{\nu} \sim \{ 201 - 451 \}$ | $m_{\nu} \sim \{ 201 - 224 \}$ | $m_{\nu} \sim \{ 201 - 224 \}$ |
| $\Delta m \sim \{ 0 - 57 \}$ | $\Delta m \sim \{ 0 - 67 \}$ | $\Delta m \sim \{ 1 - 11 \}$ | $\Delta m \sim \{ 4 - 11 \}$ | $\Delta m \sim \{ 4 - 11 \}$ |
| $\lambda_\nu \sim \{ 0.32 - 0.10 \}$ | $\lambda_\nu \sim \{ 0.43 - 0.81 \}$ | $\lambda_\nu \sim \{ 0.42 - 0.59 \}$ | $\lambda_\nu \sim \{ 0.42 - 0.59 \}$ | $\lambda_\nu \sim \{ 0.42 - 0.59 \}$ |
| $\lambda_{\nu4} \sim \{ 0.001 - 0.10 \}$ | $\lambda_{\nu4} \sim \{ 0.008 - 0.085 \}$ | $\lambda_{\nu4} \sim \{ 0.024 - 0.052 \}$ | $\lambda_{\nu4} \sim \{ 0.024 - 0.052 \}$ | $\lambda_{\nu4} \sim \{ 0.024 - 0.052 \}$ |
| $\lambda_{\nu5} \sim \{ 0.001 - 0.11 \}$ | | $\lambda_{\nu5} \sim \{ 0.002 - 0.062 \}$ | | $\lambda_{\nu5} \sim \{ 0.002 - 0.062 \}$ |
| $m_{\nu} \sim \{ 96 - 455 \}$ | $m_{\nu} \sim \{ 96 - 455 \}$ | | No parameter space available | No parameter space available |
| $m_{\nu} \sim \{ 93 - 450 \}$ | $m_{\nu} \sim \{ 93 - 450 \}$ | $m_{\nu} \sim \{ 126 - 452 \}$ | $m_{\nu} \sim \{ 126 - 313 \}$ | $m_{\nu} \sim \{ 126 - 313 \}$ |
| $\Delta m \sim \{ 0 - 57 \}$ | $\Delta m \sim \{ 0 - 57 \}$ | $\Delta m \sim \{ 1 - 19 \}$ | $\Delta m \sim \{ 4 - 19 \}$ | $\Delta m \sim \{ 4 - 19 \}$ |
| $\lambda_\nu \sim \{ 0.32 - 0.98 \}$ | $\lambda_\nu \sim \{ 0.42 - 0.81 \}$ | $\lambda_\nu \sim \{ 0.42 - 0.61 \}$ | $\lambda_\nu \sim \{ 0.42 - 0.61 \}$ | $\lambda_\nu \sim \{ 0.42 - 0.61 \}$ |
| $\lambda_{\nu4} \sim \{ 0.001 - 0.15 \}$ | $\lambda_{\nu4} \sim \{ 0.004 - 0.085 \}$ | $\lambda_{\nu4} \sim \{ 0.004 - 0.084 \}$ | | |
| $\lambda_{\nu5} \sim \{ 0.001 - 0.096 \}$ | $\lambda_{\nu5} \sim \{ 0.002 - 0.062 \}$ | $\lambda_{\nu5} \sim \{ 0.002 - 0.062 \}$ | | |
| $m_{\nu} \sim \{ 96 - 479 \}$ | $m_{\nu} \sim \{ 96 - 479 \}$ | | No parameter space available | No parameter space available |
| $m_{\nu} \sim \{ 93 - 471 \}$ | $m_{\nu} \sim \{ 93 - 471 \}$ | $m_{\nu} \sim \{ 122 - 451 \}$ | $m_{\nu} \sim \{ 122 - 313 \}$ | $m_{\nu} \sim \{ 122 - 313 \}$ |
| $\Delta m \sim \{ 0 - 57 \}$ | $\Delta m \sim \{ 0 - 57 \}$ | $\Delta m \sim \{ 1 - 19 \}$ | $\Delta m \sim \{ 4 - 19 \}$ | $\Delta m \sim \{ 4 - 19 \}$ |
| $\lambda_\nu \sim \{ 0.32 - 0.98 \}$ | $\lambda_\nu \sim \{ 0.42 - 0.81 \}$ | $\lambda_\nu \sim \{ 0.42 - 0.61 \}$ | $\lambda_\nu \sim \{ 0.42 - 0.61 \}$ | $\lambda_\nu \sim \{ 0.42 - 0.61 \}$ |
| $\lambda_{\nu4} \sim \{ 0.001 - 0.15 \}$ | $\lambda_{\nu4} \sim \{ 0.004 - 0.085 \}$ | $\lambda_{\nu4} \sim \{ 0.004 - 0.084 \}$ | | |
| $\lambda_{\nu5} \sim \{ 0.001 - 0.096 \}$ | $\lambda_{\nu5} \sim \{ 0.002 - 0.062 \}$ | | | |
| $m_{\nu} \sim \{ 96 - 479 \}$ | $m_{\nu} \sim \{ 96 - 479 \}$ | | No parameter space available | No parameter space available |
| $m_{\nu} \sim \{ 93 - 471 \}$ | $m_{\nu} \sim \{ 93 - 471 \}$ | $m_{\nu} \sim \{ 122 - 451 \}$ | $m_{\nu} \sim \{ 122 - 313 \}$ | $m_{\nu} \sim \{ 122 - 313 \}$ |
| $\Delta m \sim \{ 0 - 57 \}$ | $\Delta m \sim \{ 0 - 57 \}$ | $\Delta m \sim \{ 1 - 19 \}$ | $\Delta m \sim \{ 4 - 19 \}$ | $\Delta m \sim \{ 4 - 19 \}$ |
| $\lambda_\nu \sim \{ 0.32 - 0.98 \}$ | $\lambda_\nu \sim \{ 0.42 - 0.81 \}$ | $\lambda_\nu \sim \{ 0.42 - 0.61 \}$ | $\lambda_\nu \sim \{ 0.42 - 0.61 \}$ | $\lambda_\nu \sim \{ 0.42 - 0.61 \}$ |
| $\lambda_{\nu4} \sim \{ 0.001 - 0.15 \}$ | $\lambda_{\nu4} \sim \{ 0.004 - 0.085 \}$ | $\lambda_{\nu4} \sim \{ 0.004 - 0.084 \}$ | | |
| $\lambda_{\nu5} \sim \{ 0.001 - 0.096 \}$ | $\lambda_{\nu5} \sim \{ 0.002 - 0.062 \}$ | | | |

Table 10: Allowed ranges of relevant parameters considering $m_\phi \leq m_{H_0}$ for different values of RH neutrino mass and $\text{Tr}[Y_\nu Y_\nu]$.}

C Tree Level Unitarity Constraints

In this section, we perform the analysis to find the tree level unitarity limits on quartic couplings present in our model at high energy. The scattering amplitude for any $2 \rightarrow 2$ process can be expressed in terms of the Legendre polynomial as $[103]$

$$\mathcal{M}^{2 \rightarrow 2} = 16\pi \sum_{l=0}^{\infty} a_l (2l + 1) P_l(\cos \theta), \quad (C.1)$$

where $\theta$ is the scattering angle and $P_l(\cos \theta)$ is the Legendre polynomial of order $l$. In the high-energy limit, only the s-wave ($l = 0$) partial amplitude $a_0$ will determine the leading energy dependence of the scattering processes. The unitarity constraint turns out to be
[61, 62, 103]

\[
\text{Re } |a_0| < \frac{1}{2}.
\] (C.2)

The constraint in Eq. (C.2) can be further converted to a bound on the scattering amplitude \( \mathcal{M} \) [61, 62, 103]:

\[
|\mathcal{M}| < 8\pi.
\] (C.3)

In the present set up, we have multiple possible \( 2 \rightarrow 2 \) scattering processes. Therefore, we need to construct a matrix \( (M_{ij}^{2 \rightarrow 2} = M_{i \rightarrow j}) \) by considering all possible two particle states. Finally, we calculate the eigenvalues of \( \mathcal{M} \) and employ the bound as in Eq. (C.3). In the high-energy limit, we express the SM Higgs doublet as \( H^T = \left( w^+ \frac{h + iz}{\sqrt{2}} \right) \). Then, the scalar potential in Eq.(2.2) gives rise to 19 neutral combinations of two particle states:

\[
w^+w^-, \ H^+H^-, \ \frac{hh}{\sqrt{2}}, \ \frac{zz}{\sqrt{2}}, \ \frac{H^0 H^0}{\sqrt{2}}, \ \frac{A^0 A^0}{\sqrt{2}}, \ \frac{\phi \phi}{\sqrt{2}}, \ h \ z, \ H^0 \ A^0, \ w^+H^-, \ H^+w^-, \ h \ H^0, \ h \ A^0, \ z \ H^0, \ z \ A^0, \ h \ \phi, \ z \ \phi, \ H^0 \ \phi, \ A^0 \ \phi.
\] (C.4)

and 10 singly charged two-particle states:

\[
h \ w^+, \ z \ w^+, \ H^0 H^+, \ A^0 H^+, \ h \ H^+, \ z \ H^+, \ H^0 \ w^+, \ A^0 \ w^+, \ \phi \ w^+, \ \phi \ H^+.
\] (C.5)

Therefore, we can write the scattering amplitude matrix \( (M) \) in block-diagonal form by decomposing it into a neutral \((NC)\) and singly charged sector \((SC)\) as

\[
M = \begin{pmatrix} M^{NC}_{19 \times 19} & 0 \\ 0 & M^{SC}_{10 \times 10} \end{pmatrix}.
\] (C.6)

where the sub-matrices are given by

\[
M^{NC}_{19 \times 19} = \begin{pmatrix} (M_1^{NC})_{7 \times 7} & 0 & 0 & 0 \\ 0 & (M_2^{NC})_{2 \times 2} & 0 & 0 \\ 0 & 0 & (M_3^{NC})_{6 \times 6} & 0 \\ 0 & 0 & 0 & (M_4^{NC})_{4 \times 4} \end{pmatrix}
\] (C.7)

with

\[
M_1^{NC} = \begin{pmatrix} 4\lambda_H & \lambda_1 + \lambda_2 + \lambda_3 & \sqrt{2}\lambda_H & \sqrt{2}\lambda_H & \sqrt{2}\lambda_\phi & \sqrt{2}\lambda_\phi & \lambda_{\phi h} & \lambda_{\phi h} \\ \lambda_1 + \lambda_2 + \lambda_3 & 4\lambda_H & \lambda_\phi & \sqrt{2}\lambda_H & \sqrt{2}\lambda_H & \sqrt{2}\lambda_\phi & \sqrt{2}\lambda_\phi & \lambda_{\phi h} \\ \sqrt{2}\lambda_H & \lambda_\phi & 3\lambda_H & \lambda_H & 2(\frac{\lambda_1 + \lambda_2 + \lambda_3}{2}) & 2(\frac{\lambda_1 + \lambda_2 + \lambda_3}{2}) & \lambda_\phi & \lambda_{\phi h} \\ \sqrt{2}\lambda_H & \lambda_\phi & 3\lambda_H & \lambda_H & 2(\frac{\lambda_1 + \lambda_2 + \lambda_3}{2}) & 2(\frac{\lambda_1 + \lambda_2 + \lambda_3}{2}) & \lambda_\phi & \lambda_{\phi h} \\ \lambda_1 + \lambda_2 + \lambda_3 & \sqrt{2}\lambda_H & \sqrt{2}\lambda_H & \sqrt{2}\lambda_\phi & \sqrt{2}\lambda_\phi & \lambda_{\phi h} & \lambda_{\phi h} & \lambda_{\phi h} \\ \sqrt{2}\lambda_\phi & \sqrt{2}\lambda_\phi & 2(\frac{\lambda_1 + \lambda_2 + \lambda_3}{2}) & 2(\frac{\lambda_1 + \lambda_2 + \lambda_3}{2}) & \lambda_\phi & \lambda_{\phi h} & \lambda_{\phi h} & \lambda_{\phi h} \\ \sqrt{2}\lambda_\phi & \sqrt{2}\lambda_\phi & 2(\frac{\lambda_1 + \lambda_2 + \lambda_3}{2}) & 2(\frac{\lambda_1 + \lambda_2 + \lambda_3}{2}) & \lambda_\phi & \lambda_{\phi h} & \lambda_{\phi h} & \lambda_{\phi h} \\ \lambda_\phi & \lambda_\phi & \lambda_\phi & \lambda_\phi & \lambda_\phi & \lambda_\phi & \lambda_\phi & \lambda_\phi \end{pmatrix},
\] (C.8)

\[
M_3^{NC} = \begin{pmatrix} \lambda_1 + \lambda_2 + \lambda_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_1 + \lambda_2 + \lambda_3 & \frac{\lambda_2}{2} + \frac{\lambda_3}{2} & -\frac{\lambda_3}{2} + \frac{\lambda_2}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{\lambda_2}{2} + \frac{\lambda_3}{2} & 0 & -\frac{\lambda_3}{2} + \frac{\lambda_2}{2} & 0 & 0 & 0 & 0 \\ \frac{\lambda_2}{2} - \frac{\lambda_3}{2} & \frac{\lambda_3}{2} - \frac{\lambda_2}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\lambda_2}{2} + \frac{\lambda_3}{2} & \frac{\lambda_3}{2} + \frac{\lambda_2}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\lambda_2}{2} - \frac{\lambda_3}{2} & -\frac{\lambda_3}{2} + \frac{\lambda_2}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.
\]
where \(x_{1,2,3}\) be the roots of following cubic equation

\[
x^3 + x^2(-12\lambda_H - 12\lambda_\Phi - \lambda_\phi) + x(-16\lambda_H^2 - 16\lambda_1\lambda_2 - 16\lambda_1\lambda_3 - 4\lambda_2^2 - 8\lambda_2\lambda_3 - 4\lambda_3^2 - 4\lambda_\phi^2 - 144\lambda_H\lambda_\Phi + 12\lambda_H\lambda_\phi + 12\lambda_\Phi\lambda_\phi - 4\lambda_\phi^2) + 16\lambda_H^2\lambda_\phi + 16\lambda_1\lambda_2\lambda_\phi + 16\lambda_1\lambda_3\lambda_\phi - 32\lambda_1\lambda_\phi\lambda_\phi\]

\[
+ 4\lambda_2^2\lambda_\phi + 8\lambda_2\lambda_3\lambda_\phi - 16\lambda_2\lambda_\phi\lambda_\phi + 4\lambda_3^2\lambda_\phi - 16\lambda_3\lambda_\phi\lambda_\phi + 48\lambda_\phi^2\lambda_H - 144\lambda_H\lambda_\Phi\lambda_\phi + 48\lambda_\phi^2\lambda_\phi
\]

\[
= 0
\]  

(C.13)

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