Hunting the Higgs Boson(s)

JOHN F. GUNION

Davis Institute for High Energy Physics, Department of Physics
University of California at Davis, Davis CA, USA

I give a brief review of some of the opportunities and challenges that could arise in our quest to unravel the Higgs sector that very probably underlies electroweak symmetry breaking. In particular, I review scenarios with an extended Higgs sector that allow for a heavy SM-like Higgs boson and/or make discovery more difficult while at the same time maintaining consistency with current limits and precision electroweak constraints.

Presented at the
5th International Symposium on Radiative Corrections
(RADCOR–2000)
Carmel CA, USA, 11–15 September, 2000
and at
8th International Symposium on Particles, Strings and Cosmology
(PASCOS–2001)
University of North Carolina, Chapel Hill, 10–15 April, 2001

*Work supported in part by the U.S. Department of Energy and by the Davis Institute for High Energy Physics.
1 The Standard Model

The most important and immediate goal in our quest to understand nature at the microscopic level is the determination of the mechanism by which elementary particles acquire mass. One very attractive approach is to hypothesize the existence of a Higgs sector (for a review, see [1]) of scalar fields (some of which must have non-zero quantum numbers under the weak SU(2)×U(1) electroweak gauge group). The Higgs potential must be such that one or more of the neutral components of the Higgs fields spontaneously acquires a non-zero vacuum expectation value, thereby giving masses to the $W^\pm$ and $Z$ gauge bosons. In the minimal Standard Model (SM), mass generation is accomplished through the existence of a Higgs sector containing a single complex scalar field doublet (under weak isospin), $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$. When $\text{Re}\phi^0$ acquires a vacuum expectation value ($v/\sqrt{2}$), the $\phi^\pm$ and $\text{Im}\phi^0$ fields are absorbed by the hitherto massless $W^\pm$ and $Z$ fields which thereby acquire mass. At the same time, Yukawa couplings $\lambda_f \bar{f}f\phi$ lead to the generation of mass for the fermions, $m_f \propto \lambda_f v$. The quantum fluctuations of the remaining field $\text{Re}\phi^0$, correspond to a physical particle, the neutral Higgs boson, denoted $h_{\text{SM}}$. The couplings of the $h_{\text{SM}}$ to other SM particles are completely constrained. However, the mass of the $h_{\text{SM}}$ is completely unconstrained in the SM context without referencing physics at higher energy scales.

![Figure 1: Triviality and (meta)stability bounds for the SM Higgs boson as a function of the new physics scale $\Lambda$. From [2].](image)

If the SM is the correct description of electroweak symmetry breaking at current energies, it could still be that the SM is only an effective theory valid below some
higher energy scale $\Lambda$. Above $\Lambda$, new physics enters and a more complete/fundamental theory would emerge. One possibility is that there is no new physics between electroweak scales and the Planck scale, $M_P$. Or it could be that a theory such as supersymmetry emerges at a lower scale. Fig. 4 (from [2]) shows that the SM could remain valid as an effective theory all the way up to $M_P$ only for a very limited range of $m_{h_{\text{SM}}}$, roughly $140 < m_{h_{\text{SM}}} < 180$ GeV. For $m_{h_{\text{SM}}}$ outside this range, new physics would have to enter at a much lower scale. For example, if a $m_{h_{\text{SM}}} \sim 115$ GeV SM Higgs boson is discovered, then $\Lambda \lesssim 1000$ TeV. The upper bound shown in Fig. 4 derives from requiring that the coupling $\lambda$ appearing in the Higgs field quartic self-coupling term in the Higgs potential, $\propto \lambda \phi^4$, remain perturbative when ‘probed’ at energy scale $\Lambda$. Since $\lambda$ grows with energy scale, this bounds $\lambda(m_{h_{\text{SM}}})$, thereby bounding $m_{h_{\text{SM}}} \sim 2v^2\lambda(m_{h_{\text{SM}}})$. The lower bound shown derives from requiring stability of the potential. In particular, $\lambda$ is not allowed to be driven negative at energy scales below $\Lambda$ (by the large top quark contribution to the running of $\lambda$); i.e. we require $\lambda(\Lambda) > 0$. Without this constraint the universe would ultimately prefer to tunnel to a state in which the Higgs field $\phi$ has values with $|\phi| \gtrsim \Lambda$, yielding large negative $V(\Lambda) \propto \lambda(\Lambda)|\phi|^2 \Lambda^4$ if $\lambda(\Lambda) < 0$. The meta-stability condition, that the time scale for such tunneling be longer than the age of the universe, is only slightly less constraining.

Precision electroweak data suggests [3] the presence of a light SM-like $h$, the best single SM-like Higgs boson fit being obtained for $m_h \lesssim 100$ GeV. Recent LEP data [4] contain hints (at the roughly 2.9$\sigma$ level) that a SM-like Higgs boson might be present with mass $m_h \sim 115$ GeV. This same data could also be interpreted as providing weak evidence for a somewhat spread-out Higgs signal in the region $m_h \lesssim 115$ GeV,
such as might arise if there were a number of Higgs bosons with overlapping resonance shapes, each one having $ZZ$ coupling-squared that is a small fraction of the strength expected for the $h_{SM}$.

If the precision electroweak and LEP hints for a single light SM-like Higgs boson are correct, the Tevatron will have an excellent chance of detecting such an $h$ with $L = 15 \text{ fb}^{-1}$ of accumulated luminosity (per experiment). This is illustrated in Fig. 2 from [5]. If $90 \text{ GeV} \lesssim m_{h_{SM}} \lesssim 130 \text{ GeV}$ one employs $q^*q \rightarrow W^* \rightarrow W h_{SM} \rightarrow \ell \nu b \bar{b}$ and $q^*q \rightarrow Z^* \rightarrow Z h_{SM} \rightarrow \nu \bar{\nu} b \bar{b}$ and $\ell^+ \ell^- b \bar{b}$. If $130 \text{ GeV} \lesssim m_{h_{SM}} \lesssim 190 \text{ GeV}$ one uses $gg, W^* W^* \rightarrow h_{SM}$ as well as $q^*q \rightarrow W^* \rightarrow W h_{SM}$ and $q^*q \rightarrow Z^* \rightarrow Z h_{SM}$, all with $h_{SM} \rightarrow WW^*, ZZ^*$. Relevant final states for $h_{SM}$ decay would be $\ell^+ \ell^- jj X$ and $\ell^+ \ell^- \nu \bar{\nu}$. Currently, it is believed that $L = 15 \text{ fb}^{-1}$ can be accumulated by 2006-2007, i.e. just as LHC will start producing physics results.

Figure 3: The statistical significance in various channels for a Standard Model Higgs signal with $L = 100 \text{ fb}^{-1}$ of accumulated luminosity for the ATLAS detector at the LHC. Also shown is the net statistical significance after combining channels. From [6]. The CMS detector finds similar results [7].
The LHC and its detectors have been specifically designed to discover the $h_{SM}$ for any $m_{h_{SM}} \lesssim 1$ TeV or to see signs of a strongly interacting $W$ sector if the effective Higgs mass is even larger. The discovery modes are the following. For $m_{h_{SM}} \lesssim 130$ GeV, one employs $gg, W^*W^* \to h_{SM} \to \gamma\gamma$, $q\overline{q}, j \to W^\pm h_{SM}$ and $gg \to t\overline{t}h_{SM}$ with $h_{SM} \to \gamma\gamma$ and $h_{SM} \to b\overline{b}$. For $m_{h_{SM}} > 130$ GeV, the best signal is $gg, W^*W^* \to h_{SM} \to ZZ^{(*)} \to 4\ell$ ($gg, W^*W^* \to h_{SM} \to WW^* \to \ell\nu\ell\nu$ for $m_{h_{SM}} \sim 2m_W$). If $m_{h_{SM}} > 300$ GeV ($400$ GeV) the $gg, W^*W^* \to h_{SM} \to WW \to \ell\nu jj$ ($\to ZZ \to \ell\nu\ell\nu$) modes are very robust. The statistical significances for various channels are shown in Fig. 3. For $L = 100$ fb$^{-1}$, a signal of at least $10\sigma$ is achieved for all $m_{h_{SM}} < 1$ TeV.

A future linear $e^+e^-$ collider would also be certain to detect the SM $h_{SM}$ unless $m_{h_{SM}} > \sqrt{s}$. Comprehensive reviews are found in \cite{8} and \cite{9}. If $m_{h_{SM}} < \sqrt{s} - m_Z$, $e^+e^- \to Z^* \to Z h_{SM}$ production would allow both an inclusive recoil mass determination of $\sigma(Zh_{SM})$ and exclusive final state determinations of $\sigma(Zh_{SM})B(h_{SM} \to X)$ for various final states $X$. The ratio of the latter to the former gives a result for $B(h_{SM} \to X)$. The power of this approach and of the LC detectors to separate the various channels $X$ is illustrated by the fact that for $L = 500$ fb$^{-1}$ of accumulated luminosity (1 or 2 years of operation) one can even obtain an accurate determination of $B(h_{SM} \to WW^*)$ if $m_{h_{SM}} \gtrsim 120$ GeV and of $B(h_{SM} \to \gamma\gamma)$ if $m_{h_{SM}} \lesssim 130$ GeV. Even the very narrow width of the light Higgs can be determined quite accurately by indirect means. For instance, by isolating $e^+e^- \to e^+e^- W^*W^* \to e^+e^- h_{SM}$ events one can extract $\Gamma_{tot}^{h_{SM}} = \frac{\sigma(W^*W^* \to h_{SM} \to WW^*)}{[B(h_{SM} \to WW^*)]^2}$. The $\gamma\gamma$ collider option at the LC can also play an important role. In particular, the $\gamma\gamma \to h_{SM}$ coupling can be determined from the ratio $\sigma(\gamma\gamma \to h_{SM} \to b\overline{b})/B(h_{SM} \to b\overline{b})$ (the latter determined using the $Zh_{SM}$ techniques). This coupling is very sensitive to the presence of loops containing heavy particles whose mass is acquired via the Higgs mechanism. In addition, at low masses such that the $W^*W^*$ technique for total width determination is not very accurate, $\gamma\gamma \to h_{SM}$ allows \cite{10} extraction of $\Gamma_{tot}^{h_{SM}} = \frac{\sigma(\gamma\gamma \to h_{SM} \to b\overline{b})}{B(h_{SM} \to \gamma\gamma)B(h_{SM} \to b\overline{b})}$. The process $\gamma\gamma \to h_{SM}$ also allows determination of the CP nature of the $h_{SM}$ by studying the cross section dependence upon relative orientation of the (transverse) polarizations of the colliding $\gamma$'s \cite{11,12,13}. CP=+ (−) implies $\gamma\gamma \to h_{SM}$ cross section proportional to $\hat{e}_1 \cdot \hat{e}_2$ ($\hat{e}_1 \times \hat{e}_2$). Finally, by studying angular distributions of the $t$, $\overline{t}$ and $h$ in $e^+e^- \to t\overline{t}h$ it is possible to determine the CP of the resonance eigenstate \cite{14,15}.

2 Non-Exotic Extensions of the SM Higgs Sector

Even within the SM effective field theory context, the Higgs sector need not consist of just a single doublet; one should consider extended Higgs sector possibilities. Indeed, typical models in which the Higgs sector is the result of a new strong interaction at a higher scale $\Lambda$ produce an effective field theory below $\Lambda$ that contains at least two doublets and/or extra singlets \cite{16}. Higher representations are also a
possibility. String models also often yield quite a number of Higgs representations at low energy [7]; singlets, doublets and higher representations are all possible.

Addition of singlets poses no particular theoretical problems (or benefits). Addition of one or more extra doublet representation(s) has both attractive and unattractive aspects. On the unattractive side is the fact that the squared-mass(es) of the additional charged Higgs boson(s) become new parameter(s) that must be chosen to be positive definite in order to avoid breaking of electromagnetic symmetry. This unfavorable aspect is, in the view of many, more than compensated by the fact that a multi-Higgs-doublet model allows for the possibility of explaining all CP-violation phenomena as a result of explicit or spontaneous CP violation in the Higgs sector. Triplet representations and higher are deemed ‘exotic’ in that \( \rho \) is no longer computable when they participate in EWSB (\( i.e. \) when the vev of the neutral member of the representation is non-zero); instead, \( \rho \) becomes infinitely renormalized and must be treated as an input parameter to the model [18]. In this section, we focus our attention on singlet and doublet extensions. In both cases, detection and simulation considerations change dramatically. Triplets will be discussed very briefly in the next section.

The new considerations that arise for an extended SM Higgs sector are brought most immediately into focus by discussing the discovery prospects for Higgs bosons at an \( e^+e^- \) collider; other colliders will encounter even greater difficulty in ensuring discovery of at least one Higgs boson of an extended sector.

2.1 A Continuum Signal

As stated above, it is not entirely unreasonable to consider a case in which there are many singlets and/or extra doublets, possibly even triplets, in addition to the original doublet Higgs field \( \phi \). Each complex neutral field results in an extra scalar and extra pseudoscalar degree of freedom. The former will generally mix with \( \text{Re}\phi^0 \) and the interesting question is the extent to which this could make Higgs discovery difficult. The worst case scenario is that in which the physical eigenstates share the \( WW/ZZ \) coupling-squared and are spread out in mass in such a way that their separation is smaller than the \( \sim 10 \text{ GeV} \) or so mass resolution of the detector. The result could be a very spread out and diffuse signal that could only be searched for as a broad excess in the \( M_X \) recoil mass spectrum in \( e^+e^- \rightarrow ZX \) production, where \( M_X \) is computed from \( p_X = p_{e^+} + p_{e^-} - p_Z \) for events in which the \( Z \) decays to \( e^+e^- \), \( \mu^+\mu^- \) (and possibly jets). As noted earlier, LEP2 data is consistent with a small spread-out excess of events at high \( M_X \) (in the \( M_X \sim 100 - 110 \text{ GeV} \) region) beyond the number predicted by background computations; this excess could be interpreted in terms of such a diffuse spread-out signal.

Fortunately, there are constraints on this scenario. First, defining \( C_i \) to be the strength of the \( h_iVV \) coupling relative to that of the \( h_{\text{SM}} \), unitarity for \( WW \) scattering, as well as the general structure of the theory, imply the sumrule \( \sum_i C_i^2 \geq 1 \);
if only singlet and doublet representations are present \( \sum_i C_i^2 = 1 \). Second, precision electroweak constraints imply that the value of \( \langle M^2 \rangle \) appearing in

\[
\sum_i C_i^2 m_{h_i}^2 = \langle M^2 \rangle.
\]

(1)

would not exceed about \((200 \text{ GeV})^2\). For the most general supersymmetric model Higgs sector, imposing the requirement of perturbativity of couplings after evolving up to the GUT scale yields this same result for the maximum possible \( \langle M^2 \rangle \) [19].

To illustrate the consequences [20], assume \( C_i^2 \) is constant from \( m_{h_i}^\text{min} \) to \( m_{h_i}^\text{max} \), using continuum notation, \( C^2(m_h) \geq 1/(m_{h_i}^\text{max} - m_{h_i}^\text{min}) \) for \( \int dm_h C^2(m_h) \geq 1 \), while Eq. (1) implies \( \frac{1}{2}([m_{h_i}^\text{max}]^2 + m_{h_i}^\text{max} m_{h_i}^\text{min} + [m_{h_i}^\text{min}]^2) \leq \langle M^2 \rangle \). Let us also suppose that LEP data can be used to show that \( C^2(m_h) \) is very small for \( m_h < 70 \text{ GeV} \) (this is being examined currently). Then if \( C^2(m_h) \) is constant from \( m_{h_i}^\text{min} = 70 \text{ GeV} \) out to \( m_{h_i}^\text{max} = 300 \text{ GeV} \) the sumrule will be saturated.

Clearly, LEP2 would have had great difficulty confirming the presence of such a broad excess. One needs to have \( e^+e^- \) collisions at high enough energy to avoid kinematic suppression over the bulk of the \( M_X \) region in question. A \( \sqrt{s} = 500 \text{ GeV} \) collider would be more or less ideal. In Ref. [21], the backgrounds in the recoil \( M_X \) spectrum for \( ZX \) production were examined for \( \sqrt{s} = 500 \text{ GeV} \) over the interval 70 GeV to 200 GeV. For the \( m_{h_i}^\text{min} = 70, m_{h_i}^\text{max} = 300 \text{ GeV} \) case described, a fraction \( f \sim 0.57 \) of the continuum signal resides in this region. In order to avoid the large \( ZZ \) background, it is actually best to restrict consideration to the 100 – 200 GeV range in which a fraction \( f \sim 0.43 \) of the signal resides. For \( L = 500 \text{ fb}^{-1} \), the excess signal event rate after cuts would be \( S \sim 1350 f \sim 580 \) with a background of \( B = 2700 \).

The resulting \( \sim 50\% \) excess over background would be readily detected, and would yield \( S/\sqrt{B} \sim 26 f \sim 11 \). Allowing for some extra weighting of the signal into the \( M_X = m_Z \) region, it still seems safe to say that \( S/\sqrt{B} > 5 \) would be achieved for \( L \gtrsim 200 \text{ fb}^{-1} \).

Obviously, detection of this type of signal would be very difficult, if not impossible, at a hadron collider due to the inability to reconstruct the recoil mass in a \( ZX \) or \( WX \) event (the energies of the colliding quarks being unknown). In this scenario, the LHC would have good evidence that \( WW \) scattering at high-\( m_{WW} \) was perturbative, but the continuum of Higgs bosons responsible for this perturbativity would probably not be directly detected. Detection of the Higgs bosons would require that only a few of the Higgs bosons decayed to some particular identifiable final state (e.g. \( b\bar{b} \)) and that these same Higgs bosons were sufficiently well separated in mass that the individual mass peaks could be reconstructed. This latter is a possibility if some of the Higgs bosons give mass to some fermions and not others, rather than all Higgs bosons contributing roughly equally to the various fermionic masses. Of course, this type of channel separation would make resonance peak reconstruction possible at the LC as well.
2.2 The General Two-Higgs-Doublet Model

This is a particularly useful model to consider since it already displays many features that would be present in still more complex Higgs sectors (see [1] for a review and references). We will confine our attention to a type-II two-doublet model (in which one Higgs doublet, $\phi_u$, gives mass to up quarks while the second, $\phi_d$, gives mass to down quarks and leptons). Of course, the MSSM Higgs sector is a constrained type-II two-doublet model. If CP is conserved in the Higgs sector, then there are two CP-even eigenstates, $h^0$ and $H^0$, one CP-odd eigenstate, $A^0$ and a charged Higgs pair, $H^\pm$. If CP is violated, the $h^0, H^0, A^0$ would mix to form a trio of mixed-CP eigenstates, $h_i^{i=1,2,3}$. One of the most important parameters of a 2HDM model is $\tan\beta = v_u/v_d$, the ratio of vacuum expectation values for the neutral field components of $\phi_u$ and $\phi_d$; $v_u^2 + v_d^2 = v_{SM}^2$ is required to obtain the correct $W$ and $Z$ masses.

The most pressing question is again whether or not we are guaranteed that there should be at least one light Higgs boson with properties such that we can detect it at the various colliders. The answer is model dependent.

As we shall review later, in the general 2HDM context (i.e. without the constraints of supersymmetry), it is possible to satisfy precision electroweak constraints even if the only Higgs boson that has substantial $WW/ZZ$ coupling is quite heavy (but, at most $\sim 1$ TeV). Precision constraints are most easily satisfied if there is one light Higgs boson (with no $WW/ZZ$ coupling), all others being quite heavy. Would we discover this light $\hat{h}$?

Again we use the $e^+e^-$ collider to illustrate. There the relevant production processes would be $e^+e^- \rightarrow t\bar{t}\hat{h}$, $e^+e^- \rightarrow b\bar{b}\hat{h}$, $e^+e^- \rightarrow Z^* \rightarrow Z\hat{h}\hat{h}$ and $e^+e^- \rightarrow \nu\nu\hat{h}\hat{h}$. As regards the fermion processes, there are sumrules that guarantee that the $b\bar{b}$ and $t\bar{t}$ couplings cannot both be suppressed [22]. In particular, for a $\hat{h}$ of a general type-II 2HDM with no $VV$ coupling one finds $g_{\tilde{t}\tilde{t}}/g_{\tilde{t}\tilde{t}}^{SM} = \cot\beta$ and $g_{\tilde{b}\tilde{b}}/g_{\tilde{b}\tilde{b}}^{SM} = \tan\beta$. The $Z^* \rightarrow Z\hat{h}\hat{h}$ and $WW \rightarrow \hat{h}\hat{h}$ processes are dominated by the quartic coupling which is determined purely by the covariant gauge derivative structure, $(D_\mu \Phi)\mathbb{I}(D^\mu \Phi)$, responsible for the relevant interactions. We will now outline why these processes are not necessarily sufficient to guarantee $\hat{h}$ discovery.

Yukawa processes

Because of the $\tan\beta$ dependence of the couplings, $e^+e^- \rightarrow t\bar{t}\hat{h}$ will always yield an observable event rate if $\tan\beta$ is small enough (and the process is kinematically allowed) while $b\bar{b}\hat{h}$ will be observable for large enough $\tan\beta$. However, even for $L = 2500$ fb$^{-1}$ there is a wedge of $\tan\beta$, beginning at $m_{\hat{h}} \sim 50$ GeV (80 GeV) for $\sqrt{s} = 500$ GeV (800 GeV), for which neither process will have as many as 50 events [22,23], deemed the absolute minimum number of events for which detection would be possible at the LC. Of course, the upper limit limit for the wedge illustrated up to 400 GeV rises further for still larger $m_{\hat{h}}$ values, while the lower line delimiting the
For $\sqrt{s} = 500$ GeV (dashes) and $\sqrt{s} = 800$ GeV (solid), we plot the maximum and minimum tan $\beta$ values between which $t\bar{t}h$ and $b\bar{b}h$ both have fewer than 50 events assuming (a) $L = 1000$ fb$^{-1}$ or (b) $L = 2500$ fb$^{-1}$.

Wedge disappears once $t\bar{t}h$ is kinematically forbidden. In short, the fermionic coupling sum rules do not yield any guarantees. They only restrict the problematical region.

Double Higgs production processes

In Fig. 5 we plot the cross section for $e^+e^- \rightarrow Z^* \rightarrow Zh\bar{h}$. We see that this process can probe up to $m_{\tilde{h}} = 150$ GeV (250 GeV) for a 20 event signal with $L = 1000$ fb$^{-1}$ (50 events for $L = 2500$ fb$^{-1}$). Similar results are obtained for $WW \rightarrow hh$ fusion production. Thus, even after combining these process with the Yukawa processes, there is a large range of $m_{\tilde{h}}$ and tan $\beta$ values for which the only Higgs boson light enough to be produced in $e^+e^-$ collisions cannot be detected.

Precision electroweak constraints in the 2HDM

In this subsection, we demonstrate how a 2HDM can give good agreement with precision electroweak constraints, even if there is only one Higgs boson with $VV$ decoupling and it has mass $\sim 1$ TeV [23]. As noted earlier, a heavy SM-like $h$ gives a large $\Delta S > 0$ and large $\Delta T < 0$, as illustrated by the locations of the stars in Fig. 6. The key is to compensate the negative $\Delta T$ from the 1 TeV SM-like Higgs with a large $\Delta T > 0$ from a small mass non-degeneracy (weak isospin breaking) of heavier Higgs. For example, for a light $h = A^0$, the $h^b$ would be taken heavy and SM-like and the

Figure 4: For $\sqrt{s} = 500$ GeV (dashes) and $\sqrt{s} = 800$ GeV (solid), we plot the maximum and minimum tan $\beta$ values between which $t\bar{t}h$ and $b\bar{b}h$ both have fewer than 50 events assuming (a) $L = 1000$ fb$^{-1}$ or (b) $L = 2500$ fb$^{-1}$.
For $\sqrt{s} = 500$ GeV and 800 GeV and for $\hat{h} = h^0$ and $\hat{h} = A^0$, we plot as a function of $m_{\hat{h}}$ the maximum and minimum values of $\sigma(e^+e^- \rightarrow \hat{h}\hat{h}Z)$ found after scanning $1 < \tan\beta < 50$ taking all other Higgs masses equal to $\sqrt{s}$. For $\hat{h} = h^0$, we require $\sin(\beta - \alpha) = 0$ during the scan. The 20 event level for $L = 1$ ab$^{-1}$ is indicated.

The value of $\Delta \rho$ would be approximately given by:

$$
\Delta \rho = \frac{\alpha}{16\pi m_W^2 c_W^2} \left\{ \frac{c_W^2 m_{H^\pm}^2 - m_{H^0}^2}{s_W^2} - 3m_W^2 \left[ \log \frac{m_{H^0}^2}{m_W^2} + \frac{1}{6} + \frac{1}{s_W^2} \log \frac{m_W^2}{m_Z^2} \right] \right\}
$$

(2)

From this formula, it is clear that one can adjust $m_{H^\pm} - m_{H^0} \sim$ few GeV (both $m_{H^\pm}$ and $m_{H^0}$ being large) so that the $S,T$ prediction moves to the location of the blobs shown.

**Possible evidence from $a_\mu$ for a light $\hat{h} = A^0$**

The latest BNL result [25] for $a_\mu$ differs by 2.6$\sigma$ from the SM prediction (for a standard set of inputs for low energy $\sigma(e^+e^- \rightarrow \text{hadrons})$):

$$
\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 426(165) \times 10^{-11}.
$$

(3)
Figure 6: The outer ellipses indicate the current precision electroweak 90% CL region in the $S, T$ plane for $U = 0$ and $m_{h_{SM}} = 115$ GeV. The innermost (middle) ellipse show the size of the 90% (99.9%) CL region for $m_{h_{SM}} = 115$ GeV after new precision electroweak measurements (especially of $\sin^2 \theta_{\text{leptonic}}$ at a Giga-Z factory and a $\Delta m_W <\sim 6$ MeV threshold scan measurement. The blobs indicate the $S, T$ predictions for points with $\tan \beta > 2$ that lie within the no-discovery wedges illustrated in Fig. 5, adjusting other model parameters so that the $\Delta \chi^2$ of the precision electroweak fit is minimized while keeping all but one Higgs boson heavier than $\sqrt{s}$. Stars show $S, T$ predictions for the SM taking $m_{h_{SM}} = 500$ or 800 GeV.

Taking the above numbers at face value, the range of $\Delta a_{\mu}$ at 95% C.L. ($\pm 1.96\sigma$) is given by $10.3 \times 10^{-10} < \Delta a_{\mu} < 74.9 \times 10^{-10}$. A light $A^0$ ($h^0$) gives a positive (negative) contribution to $a_{\mu}$ dominated (for all but a very light Higgs boson) by the two-loop Bar-Zee graph. If we use a light $A^0$ as the entire explanation for $\Delta a_{\mu}$, Fig. 6 shows that this leads to constraints such that $\tan \beta > 15$ is required with $m_{A^0} < 100$ GeV (smaller values for smaller $\tan \beta$) [26]. For $\tan \beta > 17$ and $m_{A^0} < 100$ GeV, the $A^0$
will be found at a LC for sure, but discovery of such a light state primarily decaying into two (soft) jets will be hard at the LHC. If the size of $\Delta a_\mu$ should decline with analysis of the final data set, or with alternative input for $\sigma(e^+e^- \rightarrow \text{hadrons})$ at low energy, higher $m_A$ and/or smaller $\tan \beta$ would be needed to explain $\Delta a_\mu$. Thus, for smaller $\Delta a_\mu$ the $A^0$ might not be observable at either the LC or the LHC. Of course, there are many other new-physics explanations for $\Delta a_\mu$. Possibly a piece could come from the Higgs sector and a piece from these other sources.

![Figure 7: Explanation of new BNL $a_\mu$ value via light 2HDM $A^0$. (Cheung, Chou, Kong)](image)

### 3 Triplet Representations

It is certainly easy to construct models in which the Higgs sector contains one or more triplet representations (see [1] for a review of models). Most interesting would be the presence of a complex $|Y| = 2$ triplet representation. One can use a $2 \times 2$ matrix notation for such a representation:

$$\Delta = \begin{pmatrix} \Delta^+ / \sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+ / \sqrt{2} \end{pmatrix}.$$

(4)
The most dramatic new features of Higgs representations containing a complex triplet are the presence of the doubly-charged Higgs bosons, $\Delta^{--}$ and $\Delta^{++}$ and the possibility of lepton-number-violating Majorana-like couplings which take the form

$$\mathcal{L}_M = i h_{ij} \psi_i^T C \tau_2 \Delta \psi_j + \text{h.c.},$$

where $i,j = e, \mu, \tau$ are generation indices. This coupling will give rise to a see-saw mass term if $\langle \Delta^0 \rangle \equiv v_{\Delta} \neq 0$. However, if $v_{\Delta} \neq 0$, then we lose predictivity for $\rho$; $\rho$ is renormalized and becomes an input parameter for the model [18]. Whether or not $v_{\Delta} \neq 0$, $\mathcal{L}_M$ gives rise to $e^- e^- \rightarrow \Delta^{--}$ and $\mu^- \mu^- \rightarrow \Delta^{--}$ couplings. Left-right (L-R) symmetric models combine the best of both worlds. They introduce right-handed electroweak isospin in addition to the left-handed isospin of the SM and contain a left-triplet $\Delta_L$ with $\langle \Delta^0_L \rangle = 0$ (so that $\rho = 1$ is natural) and a right-triplet $\Delta_R$ with $\langle \Delta^0_R \rangle \neq 0$ so as to generate a Majorana neutrino mass. L-R symmetry requires that if the Majorana $\mathcal{L}_M$ is present for $\Delta_R$, then it must also be present for $\Delta_L$. In what follows, we discuss the phenomenology of the $\Delta_L$; that for the Higgs in $\Delta_R$ is very different [27]. We will drop the $L$ subscript in the following. Limits on the $h_{ij}$ by virtue of the $\Delta^{--} \rightarrow \ell^- \ell^-$ couplings include: Bhabha scattering, $(g-2)_{\mu}$, muonium-antimuonium conversion, and $\mu^- \rightarrow e^- e^- e^+$. Writing

$$|h^{\Delta^{--}}_{\ell\ell}|^2 \equiv c_{\ell\ell} m_{\Delta^{--}}^2 \text{ (GeV)},$$

$c_{ee} < 10^{-5}$ (Bhabha) and $\sqrt{c_{ee} c_{\mu\mu}} < 10^{-7}$ (muonium-antimuonium) are the strongest of the limits. There are no limits on $c_{\tau\tau}$.

If $v_{\Delta}$ is small or 0, the $\Delta^{--}$ width would be quite small, which can lead to very large $s$-channel production rates for $\Delta^{--}$ in $e^- e^-$ and $\mu^- \mu^-$ collisions [28]. The strategy for $\Delta^{--}$ detection is the following. For small or zero $v_{\Delta}$, we would discover the $\Delta^{--}$ in $p\overline{p}, pp \rightarrow \Delta^{--} \Delta^{++}$ with $\Delta^{--} \rightarrow \ell^- \ell^-, \Delta^{++} \rightarrow \ell^+ \ell^+$ ($\ell = e, \mu, \tau$) at the Tevatron or LHC for $m_{\Delta^{--}} \lesssim 1 \text{ TeV}$ [29]. This is precisely the mass range for which $s$-channel production of the $\Delta^{--}$ would be possible at a $\sqrt{s} \leq 1 \text{ TeV} e^+ e^-$ LC or $\mu^- \mu^-$ collider. Event rates can be enormous for $c_{\ell\ell}$ near the current upper limits [28]. Equivalently, $e^- e^-$ and $\mu^- \mu^-$ collisions probe very small $c_{\ell\ell}$. For small beam energy spread ($\delta E/E \equiv R$), the number of $\Delta^{--}$ produced in $\ell^- \ell^-$ collisions is

$$N(\Delta^{--})_{L=50 \text{ fb}^{-1}} \sim 3 \times 10^{10} \left( \frac{c_{\ell\ell}}{10^{-5}} \right) \left( \frac{0.2\%}{R} \right),$$

where $R \sim 0.2\%$ is reasonable in $e^- e^-$ collisions and $R \lesssim 0.01\%$ is possible in $\mu^- \mu^-$ collisions. If 100 events (of like sign dilepton pairs of definite known invariant mass) constitute a viable signal, Eq. (7) implies we can probe

$$c_{\ell\ell}|_{100 \text{ events}} \sim 3.3 \times 10^{-14} \left( \frac{R}{0.2\%} \right) \left( \frac{50 \text{ fb}^{-1}}{L} \right),$$

(8)
independent of $m_{\Delta^{-}}$. This is dramatic sensitivity — at least a factor of $10^8 - 10^9$ improvement over current limits at an $e^-e^-$ collider. If $\Delta^{-} \rightarrow \mu^-\mu^-$ primarily, then 10 events might constitute a viable signal and sensitivity would be further improved.

As a final remark, we note that if triplets are present in a SUSY model, the triplet Higgs field(s) will destroy coupling constant unification if intermediate scale matter is not included; but, this is not a severe problem since such matter is natural in L-R symmetric supersymmetric models.

4 Extra Dimensions and Higgs Physics

This is a very large area of recent research and I will say only a few words about a variety of interesting possibilities.

The first important point is that large extra dimensions are associated with much lower Planck scales, possibly as low as $M_S \sim 1$ TeV [30]. This reduces and can even eliminate the naturalness and hierarchy problems. In particular, the quadratic divergence in the Higgs mass loop calculation would be cutoff at $M_S$. As a result, this particular motivation for low-energy supersymmetry is greatly reduced. (Of course, in most such models one must view the MSSM unification of gauge couplings at the GUT scale in the usual four-dimensional theories as being totally accidental.) Other useful possibilities with large extra dimensions include various explanations of the small size of most Yukawa couplings. In one approach [31], the brane on which the SM particles live has significant width, and the Higgs is centered at one location while the weakly coupled fermions are located with significant separations from the Higgs centrum.

Extra dimensions can also provide new contributions to the precision electroweak observables [32]. These can shift expectations for the mass of the SM-like Higgs, in particular allowing it to be much heavier than the light $m_{h_{\text{SM}}} \sim 100$ GeV values required in the pure SM context. Just as in the general 2HDM case, the extra dimension theory only needs to give a small $\Delta S$ contribution and a large $\Delta T > 0$ contribution.

Extra dimensions can also be the source of electroweak symmetry breaking. In one approach [33], the Kaluza Klein (KK) modes mix with Higgs in such a way that the full effective potential takes the form $V_{\text{tot}} = V(\phi) - D V^2(\phi)$, with $D < 0$ from the KK summation. If $D < 0$, as for instance if the number of extra dimension is $\delta = 1$, then the minimum of this potential is at $V(\phi) = \frac{1}{2D}$, independent of the form of $V(\phi)$. In fact, even if $V(\phi)$ has no quartic term, the $-D V^2(\phi)$ term generates the quartic interactions and EWSB takes place. The physical Higgs boson is a complicated mixture of the usual Higgs field and a sum of KK modes. The main phenomenological implication is that such a Higgs might not have significant Yukawa couplings and invisible decays into KK modes could be dominant.
It is also the case that the Lagrangian could contain a term of form $-\frac{\xi}{2}R(g)\phi^{\dagger}\phi$, where $R(g)$ is the usual Ricci scalar. This term introduces mixing between the Higgs bosons and the KK excitations associated with the extra dimensions. The result is a large invisible decay width of the Higgs boson \cite{34}.

In the Randall-Sundrum model, there is only one graviscalar. It mixes with the Higgs boson, yielding two mixed physical states with properties that are intermediate between those of the radion and of the Higgs boson \cite{34}.

5 Detecting an Invisible Higgs Boson

Aside from invisible KK mode decays, there are also the possibilities of Higgs decays to Majorans, to $\tilde{\chi}_1^0\tilde{\chi}_1^0$ (in supersymmetric models), and to 4th generation neutrinos (with $m_{\nu_4} > m_Z/2$ to avoid $Z$ invisible width limits). If the Higgs decays are dominated by the invisible channel(s), alternative Higgs detection strategies are necessary. At a LC, there is no difficulty in seeing an invisibly decay Higgs in Higgsstrahlung production, $e^+e^- \rightarrow Z^* \rightarrow Zh$, by looking for a peak in the recoil mass $M_X$ in the $ZX$ final state, with $Z \rightarrow e^+e^-$ and $Z \rightarrow \mu^+\mu^-$. At hadron colliders, detection will be more difficult. The key is to be able to tag the Higgs event using particle(s) produced in association with the Higgs boson. The modes $t\bar{t}h$ production \cite{35} and $Wh, Zh$ production \cite{36,37} were identified early on as being very promising, but detailed experimental evaluation/simulation has only recently been begun. The latter modes might even be useful at the Tevatron \cite{35}. More recently, $WW \rightarrow h$ fusion using double tagging of highly energetic forward jets at the LHC has been proposed \cite{39}. It should be noted that the $Wh, Zh$ and $WW$ fusion modes all rely on the $VV$ coupling of the Higgs boson, whereas the $t\bar{t}h$ mode relies on the fermionic couplings and would be relevant even for the Higgs bosons of an extended Higgs sector that have small or zero tree-level $VV$ couplings.

For a SM-like Higgs, it was estimated that the $Wh + Zh$ and $t\bar{t}h$ modes have discovery reach at the LHC up to about 200 GeV and 250 GeV, respectively, with $L = 100 \text{ fb}^{-1}$ of accumulated luminosity. At the Tevatron, the $Wh + Zh$ modes will only exceed the limits for an invisibly decaying SM-like Higgs boson already established at LEP2 ($m_h > 100 \text{ GeV}$) when $L > 5 \text{ fb}^{-1}$. These discovery reaches are substantially less than those for the $h_{SM}$ with normal decays. A roughly equal mixture of invisible and normal decays would reduce the reach of both the invisible decay and normal decay detection techniques and possibly make Higgs detection all but impossible at the hadron colliders. A careful study is needed.
6 Supersymmetric Model Higgs Bosons

A good summary of the MSSM Higgs sector is found in [1]. At least two doublets are required in supersymmetry in order to give mass to both up quarks and down quarks and leptons. An even number of doublets, plus their higgsino partners, are also required for cancellation of anomalies. The MSSM contains exactly two doublets ($Y = +1$ and $Y = -1$) with type-II Yukawa couplings. TeV scale supersymmetry as embodied in the MSSM is the most popular cure for the naturalness and hierarchy problems for good reason. First, for two (and only two) doublets one finds perfect coupling constant unification at the GUT scale if the SUSY scale is $m_{\text{SUSY}} \sim 1$ TeV (actually, a significant number of sparticles with masses nearer 10 TeV gives better $\alpha_s$ unification with $\alpha_2$ and $\alpha_1$). If there are more doublets, triplets, etc. then coupling unification generally requires intermediate scale matter between the TeV and $M_U$ scales. If there are extra dimensions, unification would not necessarily be relevant (although it can be maintained by putting the SM particles in the bulk [40]). In short, the MSSM without extra large dimensions has very strong motivation.

The only extension to the MSSM Higgs sector that does not destroy gauge unification is to add one or more singlet Higgs fields. The model in which one singlet Higgs field is included is called the NMSSM (next-to-minimal supersymmetric model) [41] (see [1] for a review). This is an extremely attractive model in that it provides the most natural explanation for having a $\mu$ parameter that has TeV scale magnitude. The parameter $\mu$ is that appearing in the MSSM superpotential $\mu \hat{\phi}_u \hat{\phi}_d$, where the $\hat{\phi}_{u,d}$ are the superfields containing the scalar $\phi_{u,d}$ Higgs fields of the type-II Higgs sector. In the NMSSM, this interaction is replaced by the superpotential form $\lambda_S \hat{S} \hat{\phi}_u \hat{\phi}_d$, which generates an electroweak scale effective $\mu = \lambda_S s$ when the scalar component of $\hat{S}$ acquires an electroweak scale vacuum expectation value, $s \equiv \langle S^0 \rangle$, as is easily and naturally arranged. In addition, the NMSSM can contain a superpotential piece of the form $\frac{1}{3} \kappa \hat{S}^3$.

As is well known, there is a strong bound on the mass of the lightest Higgs boson $h^0$ of supersymmetric models. In the MSSM, if $m_{\tilde{t}} \leq 1$ TeV then $m_{h^0} \lesssim 130 - 135$ GeV after including stop loop corrections with $A_t \neq 0$. ($A_t$ is the magnitude of the trilinear soft supersymmetry breaking term.) This bound is so strong because at tree-level one finds $m_{h^0} \leq m_Z$ due to the fact that all Higgs self couplings are given in terms of gauge couplings, $g$ and $g'$. However, the choice above of $m_{\tilde{t}} \leq 1$ TeV is a bit arbitrary. As noted earlier, having some SUSY matter nearer 10 TeV actually improves coupling constant unification. For stop masses in this latter range, the upper bound on $m_{h^0}$ would be larger. Also, increasing the top mass within the current experimental error increases the upper limit on $m_{h^0}$. In the NMSSM, the upper bound is less constrained because of the new $\lambda_S$ parameter introduced. One finds $m_{h^0} \leq 150$ GeV assuming perturbativity for $\lambda_S$ up to $M_U$. If one adds more doublet Higgs superfields, this actually lowers the mass bound. Adding triplet Higgs superfields increases the mass...
bound (assuming perturbativity up to \( M_U \) again) to \( m_{h^0} \leq 200 \text{ GeV} \) [42]. This is the maximal value employed earlier in the sum rule of Eq. (4).

6.1 Experimental limits from LEP2 on MSSM Higgs bosons

Limits from LEP2 in the MSSM context are quite significant. Roughly, \( m_{h^0}, m_{A^0} < \sim 91 \text{ GeV} \) are excluded [43] for maximal mixing in the stop squark sector (a certain choice of \( X_t \equiv A_t - \mu \cot \beta \)) and \( m_{\text{SUSY}} = 1 \text{ TeV} \). Using the theoretical upper bound on \( m_{h^0} \) as a function of \( \tan \beta \), this translates to exclusion of the region \( 0.5 < \tan \beta < 2.4 \) at 95% CL. Higher \( m_{\text{SUSY}} \) means that Higgs masses at a given \( \tan \beta \) increase with the result that less of the \( [m_{A^0}, \tan \beta] \) parameter space is excluded.

The above limits on \( m_{h^0}, m_{A^0} \) assume absence of CP violation in the Higgs sector and invisible decays of the \( h^0, A^0 \) are not allowed for. CP violation arises in the MSSM through phases of the \( \mu \) and \( A_t \) parameters. This CP violation leads to CP violation in the MSSM two-doublet Higgs sector brought in via the one-loop corrections sensitive to these phases. The two new parameters are: \( \phi_\mu + \phi_A \) and \( \theta \), the latter being the phase of one of the Higgs doublet fields relative to the other. Studies [44,45] suggest that MSSM Higgs mass limits will be weakened significantly, implying that the disallowed \( \tan \beta \) region is still allowed when CP violation is present.

Allowing for \( h^0 \) and \( A^0 \) to have some, perhaps substantial, invisible decays would considerably weaken the constraints on the \( h^0A^0 \) cross section. As a result, the \( ZX \) recoil mass analysis would have to be relied upon more heavily. I would guess that the limits on \( m_{h^0} \) and \( m_{A^0} \) deteriorate substantially. This deserves study by the LEP experimental groups.

6.2 Discovery prospects for MSSM Bosons at the Tevatron

We recall that the \( H^0 (h^0) \) has most of the \( WW, ZZ \) coupling when \( m_{A^0} \lesssim m_Z \) \( (m_{A^0} \gtrsim 150 \text{ GeV}) \). For, \( m_Z \lesssim m_{A^0} \lesssim 150 \text{ GeV} \), the \( h^0 \) and \( H^0 \) will share the \( WW, ZZ \) coupling strength to a greater or lesser extent depending upon other details of the input parameters. The useful production processes are \( q\bar{q} \rightarrow Vh^0, VH^0 \) with \( h^0, H^0 \rightarrow b\bar{b} \) being dominant for \( m_{A^0} \) values such that \( h^0, H^0 \) has substantial \( VV \) coupling, respectively. The decoupled Higgs boson, \( h^0 \) at low \( m_{A^0} \) or \( H^0 \) at high \( m_{A^0} \), will have \( b\bar{b} \) coupling that is enhanced by a factor of \( \tan \beta \) (relative to the SM-like value) and the processes \( gg, q\bar{q} \rightarrow b\bar{b}h^0 \) or \( b\bar{b}H^0 \), respectively, will be enhanced; \( gg, q\bar{q} \rightarrow b\bar{b}A^0 \) is always enhanced at high \( \tan \beta \). Obviously, a Higgs which decouples from \( VV \) and has enhanced \( b\bar{b} \) coupling will decay primarily to \( b\bar{b} \). A careful study, including a parameterized simulation of detector effects, was performed to study prospects at the Tevatron using these channels [3]. Except for some very special parameter configurations, the results are summarized by Fig. 8. One sees that \( L > 15 \text{ fb}^{-1} \) is needed to guarantee \( 5\sigma \) discovery at lower \( m_{A^0} \). For larger \( m_{A^0} \), as typically needed for successful generation of EWSB via the RGE running, much higher \( L \) will
be needed. Except in the upper left corner of low \(m_{A^0}\) and high \(\tan \beta\), only the \(h^0\) is observed. The small white region is where the \(h^0\) and \(H^0\) are sharing the \(VV\) coupling and neither is produced strongly enough for detection. If the root-mean squark mass, \(m_{\text{SUSY}}\), is increased above 1 TeV, the \(h^0\) mass increases and discovery prospects deteriorate.

![Figure 8: (a) 95% CL exclusion regions and (b) 5\(\sigma\) discovery regions in the \([m_{A^0}, \tan \beta]\) plane, for the maximal mixing scenario (using \(m_{\text{SUSY}} = 1\) TeV) and two different search channels: \(gg, qq \rightarrow V\phi \ [\phi = h^0, H^0], \phi \rightarrow b\bar{b}\) (shaded regions) and \(gg, qq \rightarrow b\bar{b}\phi \ [\phi = h^0, H^0, A^0], \phi \rightarrow b\bar{b}\) (region in the upper left-hand corner bounded by the solid lines). The region below the solid black line is excluded by \(e^+e^- \rightarrow Z\phi\) events at LEP2.](image)

### 6.3 Discovery Prospects for MSSM Higgs Bosons at the LHC

Focusing on large \(m_{A^0}\), discovery of the SM-like \(h^0\) will typically be possible using the same production/decay modes as for a light \(h_{\text{SM}}\) at the LHC. At high \(\tan \beta\) and large \(m_{A^0}\), the decoupled \(H^0\) and \(A^0\) can be found using \(gg, qq \rightarrow b\bar{b}H^0, b\bar{b}A^0\), with \(H^0, A^0 \rightarrow \tau^+\tau^-\) or \(\mu^+\mu^-\) and \(gb \rightarrow H^\pm t\) with \(H^\pm \rightarrow \tau^\pm\nu\). These are the main modes of importance since LEP2 limits pretty much exclude \(\tan \beta < 2.5\), for which other modes could be dominant. The contours for 5\(\sigma\) discovery are shown in Fig. 8. Discovery of at least one of the MSSM Higgs bosons is guaranteed for \(L = 300\) fb\(^{-1}\). If \(m_{A^0} \gtrsim 200\) GeV and \(\tan \beta\) is not large enough, it could happen that only the SM-like \(h^0\) will be observable.
Figure 9: 5σ discovery contours for MSSM Higgs boson detection in various channels are shown in the \([m_{A^0}, \tan \beta]\) parameter plane, assuming maximal mixing, \(m_{\text{SUSY}} = 1\) TeV and an integrated luminosity of \(L = 300\) fb\(^{-1}\) for the ATLAS detector. This figure is preliminary [46].

6.4 Discovery Prospects for MSSM Higgs Bosons at the LC

Recent reviews of this topic include [8] and [9]. Any Higgs boson with even very modest \(VV\) coupling can be detected using the Higgsstrahlung \(e^+e^- \to Z^* \to Zh\) process. For \(m_{A^0} \gtrsim 150\) GeV (as probable for RGE driven EWSB), decoupling has set in and it is the \(h^0\) that will be detected in this way. In particular, the upper limit of \(m_{h^0} \lesssim 135\) GeV guarantees that a \(\sqrt{s} = 350\) GeV LC would suffice. For the \(H^0, A^0, H^\pm\), the production mechanisms \(e^+e^- \to H^0 + A^0\) and \(e^+e^- \to H^+ + H^-\) would be full strength. However, at large \(m_{A^0}\), one finds \(m_{H^0} \sim m_{A^0} \sim m_{H^\pm}\) so that pair production requires \(m_{A^0} < \sqrt{s}/2\). If \(m_{A^0}\) exceeds \(\sqrt{s}/2\), then one must turn to \(e^+e^- \to b\bar{b}A^0, b\bar{b}H^0, btH^\pm\). As we have already discussed, the event rates for these processes are not large enough for observation unless \(\tan \beta\) is quite large. In the problematical moderate \(\tan \beta\) wedge, where the LHC will also not find the \(H^0, A^0, H^\pm\), observation might be possible using \(\gamma\gamma \to H^0, A^0\). In particular, this
will be possible if the value of $m_{A^0}$ is constrained to within $\pm 50$ GeV, since then the expected yearly luminosity would be such that an appropriately designed scan, using a peaked $\gamma\gamma$ luminosity spectrum, would reveal a $H^0, A^0$ signal when $E_{\gamma\gamma}^{\text{peak}} \sim m_{A^0}$ [49].

A model-dependent constraint on $m_{A^0}$ of this type might be possible. If one assumes a non-conspiratorial MSSM parameter scenario, $h^0$ vs. $h_{SM}$ branching ratio differences reflect the value of $m_{A^0}$ rather accurately. Further, expected LC precisions for the branching ratios are such that these differences could be measured with sufficient accuracy to determine $m_{A^0}$ within $\pm 50$ GeV if $m_{A^0} \lesssim 400$ GeV [17,18]. This is precisely the range of relevance for a $\gamma\gamma$ collisions at a $\sqrt{s} = 500$ GeV LC. Alternatively, if the properties of the observed light Higgs are found not to deviate from those predicted for the $h_{SM}$, then the most natural conclusion would be that $m_{A^0}$ is substantially heavier than 500 GeV. In this case, a $\gamma\gamma$ scan over the $E_{\gamma\gamma} \lesssim 400$ GeV range would not be useful. However, if one does not accept this model-dependent indirect determination of the magnitude of $m_{A^0}$, a full scan, say from $m_{A^0} \sim 250$ GeV up to $m_{A^0} \sim 400$ GeV would be called for. However, luminosity expectations for the NLC design might not suffice [49] to find the $H^0, A^0$ if one has to scan such a large range of mass. Much higher $L_{\gamma\gamma}$ luminosity is claimed by TESLA. This might be a rather crucial difference [48,50]. Once the mass of any of the $h^0, H^0, A^0$ is known, we can run with $E_{\gamma\gamma}^{\text{peak}}$ equal to the Higgs mass and determine the CP nature of the Higgs boson by adjusting the linear polarization orientations of the initial laser beams [11,12,13]. In particular, we can separate $A^0$ from $H^0$ when these are closely degenerate (as typical for $\tan\beta \gtrsim 4$ and $m_{A^0} > 2m_Z$).

### 6.5 Special Cases in the MSSM

As already noted, the above summaries assume relatively canonical MSSM parameter choices and absence of CP violation in the Higgs sector. These expectations need not apply. If there are substantial $h^0 \to \tilde{\chi}\tilde{\chi}$ decays, as still possible even given LEP2 lower bounds on $m_{\tilde{\chi}}$, observation of the $h^0$ at hadron colliders (but not the LC) would be more difficult. For low stop masses, corrections to the one-loop induced $gg h^0$ and $\gamma\gamma h^0$ couplings would be substantial. The stop and top loops negatively interfere leading to reduction of $gg$ fusion production and some increase in $B(h^0 \to \gamma\gamma)$ [1,51].

There can be substantial radiative corrections to the tree-level couplings. This would be especially important for $b\bar{b}$ decays of the $h^0$ when the $h^0$ is SM-like. In particular, after including radiative corrections, for the $b\bar{b}$ Yukawa Lagrangian one obtains $L \simeq \lambda_b \phi_d^\dagger b\bar{b} + \Delta \lambda_b \phi_u^\dagger b\bar{b}$. The coupling $\Delta \lambda_b$ is one-loop and arises from $\bar{b} - \tilde{g}$ and $\tilde{t} - \tilde{\phi}_u,d$ loops. Typically, $\Delta \lambda_b \approx 0.01$ (either sign). Further, $\Delta \lambda_b$ does not vanish in the limit of large SUSY masses (there is no decoupling). The result for the full
The $h^0 \to b\bar{b}$ coupling takes the form:

$$
\lambda_b^{h^0} \simeq -\frac{m_b \sin \alpha}{v \cos \beta} \left[ 1 + \frac{\Delta \lambda_b}{\lambda_b} \tan \beta \right] \left[ 1 - \frac{\Delta \lambda_b}{\lambda_b} \tan \alpha \right],
$$

implying that if $\tan \alpha \simeq \frac{\Delta \lambda_b}{\lambda_b}$ then $\lambda_b^{h^0} \simeq 0$. In particular, this can happen when $m_{A^0} \to \infty$ if $\Delta \lambda_b / \lambda_b < 0$, since, at large $m_{A^0}$, $\alpha \to \pi/2 - \beta$ and $\tan \alpha \to -1 / \tan \beta$ is small. Conversely, for $\Delta \lambda_b / \lambda_b > 0$, substantial enhancement of $\lambda_b^{h^0}$ is possible.

If parameters are such that the $h^0$ decouples from $b$'s (i.e. $h^0 \simeq \text{Re} \phi_u$), discovery strategies could not rely on the $b\bar{b}$ decay mode. However, since $\Gamma(h^0 \to \gamma\gamma)$ is dominated by $W$ and $t$ loops, small or vanishing $\lambda_b^{h^0}$ will affect the $h^0 \to \gamma\gamma$ partial width very little. There is also little impact on the $gg$ partial width. Thus, suppressed $\Gamma(h^0 \to b\bar{b})$ implies enhanced $B(h^0 \to \gamma\gamma), B(h^0 \to WW^*)$. In fact, the $\gamma\gamma$ mode can be viable for some range of $m_{h^0}$ at the Tevatron if $h^0 \sim \phi_u$. More generally, allowing for either suppressed or enhanced $\lambda_b^{h^0}$, LHC $gg \to h^0 \to \gamma\gamma$ and Tevatron $Wh^0[\to WW^*]$ modes improve when LHC, Tevatron $W, Z h^0[\to b\bar{b}]$ modes deteriorate. One also finds that the Tevatron and the LHC are complementary as $\lambda_b^{h^0}$ and $m_{h^0}$ vary in that $h^0$ discovery will occur at one or the other machine, even if not both.

Turning next to the $H^0, A^0, H^\pm$, discovery will typically become more difficult if these Higgs bosons have substantial branching ratios for decay to pairs of neutralinos, or charginos or sleptons, .... Such decays will, however, only be significant if $\tan \beta$ is in the low to moderate range, a significant part of which has already been excluded by LEP2 data. For larger $\tan \beta$, the $H^0, A^0 \to b\bar{b}$ and $H^+ \to t\bar{b}$ decay modes and their $\tau$ analogues are sufficiently enhanced that sparticle pair channels will have small branching ratios.

### 6.6 Discovery of NMSSM Higgs Bosons

The addition of the singlet superfield results in a third CP-even Higgs boson and a second CP-odd Higgs boson. The CP-even bosons mix, as do the CP-odd bosons. There is still a strong constraint of $m_{h_1^0} \leq 150$ GeV on the mass of the lightest CP-even physical state. If it does not have substantial coupling to $VV$, then it can be shown that one of the other two states ($h_2^0$ or $h_3^0$) must have at least moderate $VV$ coupling and must be relatively light. As a result, discovery of one (or more) of the CP-even Higgs bosons of the NMSSM is guaranteed at a LC with $\sqrt{s} > 350$ GeV. An important question is whether the sharing of the $VV$ coupling that is possible in the NMSSM means that discovery of one of the NMSSM Higgs bosons at the LHC cannot be guaranteed. A study for Snowmass 96 showed that parameters could be chosen so that no Higgs boson would be observed employing the experimentally verified modes available at that time, even for $L = 600$ fb$^{-1}$. For example, for $m_{h_1^0} = 105$ GeV and $\tan \beta = 5$, event rates in the $h_1,2,3 \to \gamma\gamma, ZZ^*$,
WW\textdagger, etc. final states could all be suppressed by virtue of a shared VV coupling configuration, while tan $\beta$ was too small to sufficiently enhance the $b\overline{b}h_{1,2,3}$ and $b\overline{b}a_{1,2}$ (with $h_{1,2,3}, a_{1,2} \to \tau^+\tau^-$) modes to the 5$\sigma$ level. What was missing in 1996 was a discovery mode based on the $b\overline{b}$ decays, especially those of the lightest Higgs bosons, $h_{1,2}$. Recently, the $t\overline{t}h \to t\overline{t}b\overline{b}$ mode (originally discussed in \cite{56}) has been shown to be viable for a SM-like Higgs by the ATLAS and CMS groups \cite{57}. Rescaling these results to the NMSSM, a preliminary study \cite{58} finds that all points for which discovery was found to be impossible without this mode would allow > 5$\sigma$ discovery of $h_1$ or $h_2$ in the $t\overline{t}b\overline{b}$ final state. Further study is needed, but it now appears that there is a ‘no-lose’ theorem for NMSSM Higgs discovery at the LHC once both ATLAS and CMS have each accumulated $L \geq 300$ fb$^{-1}$ of luminosity.

7 Conclusions

This brief overview of discovery prospects for Higgs bosons necessarily omitted many interesting topics, and almost completely ignored the very interesting programs for precision measurements of the properties of the Higgs bosons at the LHC and LC and how such measurements impact our ability to determine, for instance, the MSSM or NMSSM soft-supersymmetry-breaking parameters. It is these latter which are needed to connect TeV scale physics to the GUT scale physics that we ultimately hope to probe.

Acknowledgments

This work was supported in part by the U.S. Department of Energy and by the Davis Institute for High Energy Physics.

References

[1] J.F. Gunion, H.E. Haber, G. Kane and S. Dawson, The Higgs Hunters Guide, Addison-Wesley.

[2] T. Hambye and K. Riesselmann, Phys. Rev. D55 (1997) 7255.

[3] A. Freitas, W. Hollik, W. Walter and G. Weiglein, Phys. Lett. B495 (2000) 338 [hep-ph/0007091]; A. Freitas, S. Heinemeyer, W. Hollik, W. Walter and G. Weiglein, Nucl. Phys. Proc. Suppl. 89 (2000) 82 [hep-ph/0007129]; hep-ph/0101260.

[4] R. Barate et al. [ALEPH Collaboration], Phys. Lett. B495 (2000) 1 [hep-ex/0011043]; P. Abreu et al. [DELPHI Collaboration], Phys. Lett. B499 (2001)
23 [hep-ex/0102036]; M. Acciarri et al. [L3 Collaboration], [hep-ex/0012019]
G. Abbiendi et al. [OPAL Collaboration], Phys. Lett. B499 (2001) 38 [hep-
ex/0101014].

[5] M. Carena, J.S. Conway, H.E. Haber and J.D. Hobbs et al., hep-ph/0010338
[FERMILAB-CONF-00/270-T].

[6] For a recent review, see V.A. Mitsou, ATLAS-CONF-2000-002.

[7] K. Lassila-Perini, ETH Dissertation thesis No. 12961 (1998).

[8] TESLA Technical Design Report, R. Heuer, D. Miller, F. Richard, A. Wagner
and P.M. Zerwas (editors), obtainable from www.desy.de/~1cnotes/tdr/.

[9] The NLC Orange Book, . . . .

[10] J.F. Gunion and P.C. Martin, hep-ph/9610417; J. F. Gunion and P. C. Martin,
Phys. Rev. Lett. 78 (1997) 4541 [hep-ph/9607360].

[11] B. Grzadkowski and J.F. Gunion, Phys. Lett. B294 (1992) 361 hep-ph/9206262.

[12] J.F. Gunion and J.G. Kelly, Phys. Lett. B333 (1994) 110 [hep-ph/9404313].

[13] M. Krämer, J. Kühn, M.L. Stong and P.M. Zerwas, Z. Phys. C64 (1994) 21
[hep-ph/9404280].

[14] J.F. Gunion and X.G. He, Proceedings of Snowmass96, hep-ph/9609453.

[15] J.F. Gunion, B. Grzadkowski and X. He, Phys. Rev. Lett. 77 (1996) 5172 [hep-
ph/9605326].

[16] B.A. Dobrescu, hep-ph/9903407 and hep-ph/0103038.

[17] M. Cvetic, H. Lu, C. N. Pope, A. Sadrzadeh and T.A. Tran, Nucl. Phys. B586
(2000) 275 [hep-th/0003103].

[18] J.F. Gunion, R. Vega and J. Wudka, Phys. Rev. D43 (1991) 2322.

[19] J.R. Espinosa and M. Quirós, Phys. Rev. Lett. 81 (1998) 516 [hep-ph/9804235].

[20] J.R. Espinosa and J.F. Gunion, Phys. Rev. Lett. 82 (1999) 1084 [hep-
ph/9807273].

[21] J. F. Gunion, T. Han and R. Sobey, Phys. Lett. B429 (1998) 79 [hep-
ph/9801317].
[22] B. Grzadkowski, J.F. Gunion and J. Kalinowski, *Phys. Lett.* **B480** (2000) 287 [hep-ph/0001093].

[23] T. Farris, B. Grzadkowski, J.F. Gunion, J. Kalinowski and M. Krawczyk, *Phys. Lett.* **B496** (2000) 195 [hep-ph/0009271].

[24] T. Farris and J.F. Gunion, LC note in preparation.

[25] H. N. Brown *et al.* [Muon g-2 Collaboration], *Phys. Rev. Lett.* **86** (2001) 2227 [hep-ex/0102017].

[26] K. Cheung, C. Chou and O. C. Kong, [hep-ph/0103183].

[27] R.N. Mohapatra, *Fortsch. Phys.* **31** (1983) 185; J.F. Gunion, J. Grifols, A. Mendez, B. Kayser and F. Olness, *Phys. Rev.* **D40** (1989) 1546; N.G. Deshpande, J.F. Gunion, B. Kayser and F. Olness, *Phys. Rev.* **D44** (1991) 837.

[28] J.F. Gunion, *Int. J. Mod. Phys.* **A11** (1996) 1551 [hep-ph/9510350]; **A13** (1998) 2277 [hep-ph/9803222]. See also: P.H. Frampton, *Int. J. Mod. Phys.* **A15** (2000) 2455 [hep-ph/0002017]; F. Cuypers and M. Raidal, *Nucl. Phys.* **B501** (1997) 3 [hep-ph/9704224].

[29] J. F. Gunion, C. Loomis and K. T. Pitts, Snowmass96 Proceedings, [hep-ph/9610237].

[30] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B429** (1998) 263 [hep-ph/9803315].

[31] E. A. Mirabelli and M. Schmaltz, *Phys. Rev.* **D61** (2000) 113011 [hep-ph/9912265].

[32] T. G. Rizzo and J. D. Wells, *Phys. Rev.* **D61** (2000) 016007 [hep-ph/9906234].

[33] B. Grzadkowski and J. F. Gunion, *Phys. Lett.* **B473** (2000) 50 [hep-ph/9910456].

[34] G. F. Giudice, R. Rattazzi and J. D. Wells, *Nucl. Phys.* **B595** (2001) 250 [hep-ph/0002178].

[35] J. F. Gunion, *Phys. Rev. Lett.* **72** (1994) 199 [hep-ph/9309216].

[36] S. G. Frederiksen, N. Johnson, G. Kane and J. Reid, *Phys. Rev.* **D50** (1994) 4244.

[37] D. Choudhury and D. P. Roy, *Phys. Lett.* **B322** (1994) 368 [hep-ph/9312347].

[38] S. P. Martin and J. D. Wells, *Phys. Rev.* **D60** (1999) 035006 [hep-ph/9903253].
[39] O. J. Eboli and D. Zeppenfeld, *Phys. Lett.* **B495** (2000) 147 [hep-ph/0009158].

[40] K. R. Dienes, E. Dudas and T. Gherghetta, *Nucl. Phys.* **B537** (1999) 47 [hep-ph/9806292].

[41] J. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski and F. Zwirner, *Phys. Rev.* **D39** (1989) 844.

[42] J. R. Espinosa and M. Quiros, *Phys. Lett.* **B302** (1993) 51 [hep-ph/9212305].

[43] The ALEPH, DELPHI, L3 and OPAL Collaborations and the LEP Higgs Working Group, LHWG note 2001-2 (26 March 2001).

[44] G. L. Kane and L. Wang, *Phys. Lett.* **B488** (2000) 383 [hep-ph/0003198].

[45] M. Carena, J. Ellis, A. Pilaftsis and C. E. Wagner, *Phys. Lett.* **B495** (2000) 155 [hep-ph/0009212].

[46] The results of Fig. [3] were provided by F. Gianotti on behalf of the ATLAS collaboration. They are the preliminary results available as of March 3, 2001.

[47] J.F. Gunion, L. Poggioli, R. Van Kooten, C. Kao, P. Rowson et al., “Higgs boson discovery and properties,” in *New Directions for High Energy Physics*, Proceedings, Snowmass ’96 pp. 541–587 [hep-ph/9703330].

[48] See, for example, J.F. Gunion, [hep-ph/9705282].

[49] D. Asner, J. Gronberg, J. Gunion and T. Hill, in preparation.

[50] M.M. Mühlleitner, M. Krämer, M. Spira and P.M. Zerwas, [hep-ph/0101083].

[51] A. Djouadi, *Phys. Lett.* **B435** (1998) 101 [hep-ph/9806313].

[52] S. Mrenna and J. Wells, *Phys. Rev.* **D63** (2001) 015006 [hep-ph/0001224].

[53] M. Carena, S. Mrenna and C. E. Wagner, *Phys. Rev.* **D62** (2000) 055008 [hep-ph/9907422].

[54] U. Ellwanger and C. Hugonie, [hep-ph/9909260].

[55] J. F. Gunion, H. E. Haber and T. Moroi, Proceedings, Snowmass ’96 [hep-ph/9610337].

[56] J. Dai, J. F. Gunion and R. Vega, *Phys. Rev. Lett.* **71** (1993) 2699 [hep-ph/9306271].
[57] ATLAS Collaboration, “Detector and Physics Performance Technical Design Report”, CERN/LHCC/99-15 (1999); CMS Collaboration, Technical Design Reports, CMS TDR 1-5 (1997/98).

[58] U. Ellwanger, J.F. Gunion and C. Hugonie, in preparation.