K⁺-nucleus quasielastic scattering

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K⁺–nucleus quasielastic cross sections measured for a laboratory kaon beam momentum of 705 MeV/c are presented for 3–momentum transfers of 300 and 500 MeV/c. The measured differential cross sections for C, Ca and Pb at 500 MeV/c are used to deduce the effective number of nucleons participating in the scattering, which are compared with estimates based on the eikonal approximation. The long mean free path expected for K⁺ mesons in nuclei is found. Double differential cross sections for C and Ca are compared to relativistic nuclear structure calculations.

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Quasielastic scattering on complex nuclei probes the nuclear response near the kinematics for scattering from free nucleons, and is a powerful means of exploring aspects of nuclear dynamics. In general, nuclear interactions manifest themselves as departures from the quasielastic predictions for a non–interacting Fermi gas of nucleons. There have been many studies of (e, e′) quasielastic scattering, and interpretations of these have been aided by the well understood nature and the weakness of the fundamental interaction. Similar studies with hadronic probes including pions, in principle yield responses not accessible in (e, e′) scattering. However strong projectile–nucleon interactions can greatly complicate theoretical analyses of these studies.

In light of these considerations, the K⁺ meson is an promising projectile for nuclear quasielastic scattering studies since it probes responses other than those of the electron and because the K⁺–N interaction is weaker than those of the proton or pion for laboratory momenta below about 800 MeV/c. Consequently K⁺ quasielastic scattering should be more sensitive to the nuclear interior than is the scattering of any other hadron. For example, at a laboratory 3–momentum k of 705 MeV/c, the mean free path in nuclear matter is about 4 fm for K⁺ compared to about 2 fm for either π⁺ or protons with that momentum.

The weakness of the K⁺–N interaction has generated much interest in K⁺–nucleus elastic scattering, since it was anticipated that simple multiple scattering treatments of the process should be quite accurate. How-
ever, differential cross section data for $K^+–C$ and $K^+–Ca$ elastic scattering at $k = 800$ MeV/$c$ are systematically underestimated by multiple scattering calculations. Similarly, the observed ratio of C to D total cross sections exceeds predictions. Since multiple scattering calculations implicitly assume that the bulk of the important absorptive part of the $K^+–nucleus$ scattering potential is due to quasielastic scattering, direct measurement of this process is likely to be useful in unravelling the physics of $K^+–nucleus$ elastic scattering.

We have measured the quasielastic scattering of $K^+$ mesons from natural isotopic targets of C, Ca and Pb, and from D in a solid CD$_2$ target. Double differential cross sections with $k = 705$ MeV/$c$ were measured at laboratory scattering angles of 24°, 33° and 42°, corresponding to laboratory 3–momentum transfers $q$ of 300, 400 and 500 MeV/$c$. At $q = 500$ MeV/$c$ the kinematic conditions for incoherent quasielastic scattering are satisfied. The experiment was done on the C–6 beam line of Brookhaven National Laboratory using the hypernuclear spectrometer system (Moby Dick). Beam tuning gave nearly equal intensities of $\pi^+$ and $K^+$ on target, typically around $2 \times 10^5$ per 1.2 second beam spill each 3.8 seconds. Trajectories of incident and scattered particles were determined with position measurements from 15 drift chambers spaced before and after the target. Particle identification was carried out with four plastic scintillators, two before the target and two after, used for time of flight measurements. In addition, a Čerenkov counter was used to identify beam pions.

The momentum acceptance of Moby Dick was measured with 13 data points from elastic scattering of beam protons from hydrogen in a solid CH$_2$ target of thickness 1.16 g/cm$^2$, and was checked for consistency with 7 data points from elastic $K^+–p$ scattering from the hydrogen in a nylon target of thickness 2.89 g/cm$^2$, with known hydrogen composition. The targets ranged in thickness from 1 to 5 g/cm$^2$, introducing an energy spread in the incident and scattered particles. In combination with the approximate 3 MeV spectrometer resolution, this gave an overall energy resolution better than 5 MeV — quite adequate for the broad quasielastic peaks observed. At 24° the resolution permitted us to make a reliable subtraction of elastic scattering from the spectra.

From three to five spectrometer settings were required to cover the wide range of excitation energy $\omega$ of each nuclear target. The finite spectrometer angular acceptance of 3° gave $q$ constant to within approximately 20 MeV/$c$ across the full range of the spectra. Measurements of elastic $K^+–p$ scattering using the nylon target, double–checked using another CH$_2$ target of thickness 0.94 g/cm$^2$, and checked in an additional run using yet another CH$_2$ target of thickness 2.35 g/cm$^2$, were normalized to the SP88 phase shift solution to obtain the cross section scale for the final spectra. Sample spectra showing the double differential cross section $d^2\sigma/d\Omega d\omega$ versus $\omega$ for C and Ca at $q = 500$ and 300 MeV/$c$ are displayed in Figs. 1 and 2. Only statistical uncertainties
are shown in the figures.

To assess the extent to which the K⁺ probes the nuclear interior we have extracted A_{\text{eff}} from our 500 MeV/c data. A_{\text{eff}} is the experimentally determined effective number of nucleons "seen" by the K⁺ in quasielastic scattering. Experimental values have been found by integrating the double differential cross sections over the quasielastic region. The resulting \(d\sigma/d\Omega\) values appear in Table I. The quoted errors in Table I include both normalization and systematic uncertainties totalling 12% and the statistical uncertainty of 5%. A_{\text{eff}} has been extracted from \(d\sigma/d\Omega\) by dividing by the appropriate combination of K⁺–p and K⁺–n differential cross sections. These cross sections at 42° were taken to be 2.00 and 1.21 mb/sr, respectively [9]. The measured H cross section is consistent with the K⁺–p cross section and the D cross section agrees with the summed K⁺–p and K⁺–n cross sections.

Table I also lists A_{\text{eff}}^E, the effective number of nucleons, estimated using the following expression based on the eikonal approximation:

\[
A_{\text{eff}}^E = \int d^2b T(b) e^{-\sigma_T T(b)},
\]

where

\[
T(b) = \int_{-\infty}^{\infty} dz \rho(\sqrt{b^2 + z^2}),
\]

in which \(\rho(r)\) is the nuclear density taken from the ground state proton matter distributions of Ref. [10], and where the neutron density is assumed to have the same shape as the proton density. Also \(\sigma_T\) is the appropriately isospin averaged K⁺–N total cross section determined from the K⁺–p and K⁺–n total cross sections of 12.40 and 15.76 mb, respectively [9]. The experimental values of A_{\text{eff}} for C, Ca and Pb are consistently about 30% larger than the corresponding eikonal predictions A_{\text{eff}}^E. These results imply, in the context of Eq. (1), that a value of \(\sigma_T\) smaller than that given by the phase shift solution is required by experiment. Now consider the eikonal expression for the K⁺–nucleus total cross section

\[
\sigma_{\text{tot}} = 2 \int d^2b [1 - e^{-\frac{1}{2} \sigma_T T(b)}].
\]

As mentioned above, multiple scattering theory underestimates nuclear elastic scattering data, including the total cross sections, which indicates that a larger value of \(\sigma_T\) is required in Eq. (3). At present we can offer no explanation of this apparent contradiction with the quasielastic results.

Independent of their physical origin, the large values of A_{\text{eff}} found for C, Ca and Pb confirm the nuclear penetration anticipated for K⁺. For example we find that the effective number of protons in Pb is 18.1 out of 82, while for pion single charge exchange (SCX) on Bi the effective number of protons was found to be only 9.3 out
Shown also in Table I are the radius $R$ and the fraction of the central density $\rho(R)/\rho(0)$ for which

$$A_{\text{eff}}/A = \int_R^\infty d^3r \rho(r).$$  \hspace{1cm} (4)$$

Using this simple model we find that K$^+$'s reach a nuclear density in Pb which is 55% of the central density, while for pion SCX only 25% of the central density is reached.

We now compare the data with both nuclear matter and full finite nucleus calculations based on quantum hadrodynamics (QHD), a relativistic theory of nuclear dynamics \cite{12}. Further, we present calculations with and without random phase approximation (RPA) treatment of long range correlations. Details of our RPA method are found in Refs. \cite{13,14}. We have specifically used the relativistic Hartree approximation (RHA)--RPA in which 1--loop vacuum polarization effects are included via a local--density--approximation. Mean field theory (MFT)--RPA calculations, which ignore vacuum polarization effects, give results quite similar to the RHA--RPA results. Our RHA calculations employ the meson masses and coupling constants of Ref. \cite{15} while the MFT results use the parameters of Ref. \cite{16}. In the plane wave impulse approximation, the K$^+$--nucleus double differential cross section is

$$\frac{d^2\sigma}{d\Omega d\omega} = \kappa \left( |f_s|^2 S_{SS} + |f_v|^2 \left( \frac{E + E'}{2m} \right)^2 \left( \frac{Q^4}{q^4} S_{00} + \frac{Q^2}{q^2} S_{11} \right) \right) - \left( 1 + \frac{Q^2}{4m^2} \right) S_{11} + 2\text{Re}(f_s f_v^*) \frac{E + E' Q^2}{2m q^2} S_{00},$$  \hspace{1cm} (5)$$

where $\kappa$ is a kinematical factor arising from the transformation of the solid angle from the center--of--momentum (c.m.) frame to the laboratory frame, $Q^2 = q^2 - \omega^2$, $E(E')$ is the initial (final) K$^+$ total energy, $m$ is the K$^+$ mass and $f_s$ and $f_v$ are relativistic invariants. These are related to the on-shell, isospin averaged, c.m. frame K$^+$--N scattering amplitude via

$$f = F + \sigma_n G = \bar{u}_f \left[ f_s + \frac{\gamma \cdot K}{m} f_v \right] u_i,$$  \hspace{1cm} (6)$$

where the $u_i$ ($u_f$) are the initial (final) nucleon Dirac spinors, $F$ and $G$ are the non--spin--flip and spin--flip scattering amplitudes, respectively, taken from Ref. \cite{17}. $K^\mu$ is the average K$^+$ 4--momentum and $\sigma_n = \bar{\sigma} \cdot \vec{K} \times \vec{q}/|\vec{K} \times \vec{q}|$. Finally, the nuclear responses are

$$S_{ij} = -\frac{1}{\pi} \text{Im} \text{Tr} \left[ \theta_i \Pi(\omega, q) \theta_j \right],$$  \hspace{1cm} (7)$$

where $\Pi$ is the nuclear polarization insertion. For $i \rightarrow s$, $\theta_i \rightarrow 1$, while for $i \rightarrow 0$, $\theta_i \rightarrow \gamma_0$ and for $i \rightarrow 1$, $\theta_i \rightarrow \vec{\gamma} \cdot \hat{e}_T$, where $\hat{e}_T$ is a unit vector in the scattering plane and perpendicular to $\vec{q}$. All calculations have been scaled by the values of $A_{\text{eff}}$ listed in Table I.

While the responses $S_{00}$ and $S_{11}$ enter in the Coulomb and transverse ($e, e'$) responses, the scalar--scalar and mixed scalar--vector responses $S_{SS}$ and $S_{S0}$ do not appear there. In addition, due to the detailed nature of
the $K^+ - N$ interaction and the fact that the responses $S_{00}, S_{00}$ and $S_{00}$ are typically very similar in magnitude, there are strong cancellations between the sum of the first two terms and the third term in the expression for the double differential cross section. Such cancellations make it possible that relatively small kinematic or nuclear structure effects might cause strong departures from naive descriptions of quasielastic scattering.

Figure 1 presents the $q = 500 \text{ MeV}/c$ data for Ca and C, clearly displaying the characteristic shape of a quasielastic response. Uncorrelated calculations for Ca appear in the upper section of the figure. The dotted curve is a relativistic Fermi gas prediction while the dashed curve includes the effects of a reduced nucleon effective mass $M^*$ in the nucleus due to the strong attractive scalar potentials which characterize QHD. The exact value of the effective mass used ($M^*/M = 0.8576$ for Ca in RHA) has been determined self-consistently at the average nuclear density at which the $K^+ - N$ interaction occurs. We note that, while the individual Fermi gas responses each peak near the $\omega$ given by free $K^+ - N$ kinematics, the corresponding double differential cross section peaks at a considerably lower $\omega$ due to the effects of kinematical factors and cancellations, as mentioned above. The considerable influence of such effects on the shape of the summed response persists in all the calculations discussed here. The reduced effective mass used to compute the dashed curve in the upper panel gives a greatly improved agreement with the data compared to the Fermi gas result. In the middle panel RHA–RPA calculations for Ca are displayed. As shown by the dot-dashed curve, RPA correlations in nuclear matter have little effect at this momentum transfer. The solid curve is a full finite nucleus RHA–RPA calculation which is remarkably similar to the nuclear matter result, as well as being in excellent agreement with data. The bottom panel shows similar calculations for C with correspondingly satisfactory accord with experiment. MFT calculations are quite similar except that the full finite nucleus results for C are too low by about 25%.

Figure 2 shows the $q = 300 \text{ MeV}/c$ data for C and Ca. In this case the quasielastic peak is greatly overshadowed by the strength concentrated at much lower $\omega$. The top panel compares uncorrelated finite nucleus RHA calculations with the C data. The solid curve in each panel is the sum of the isoscalar ($\Delta T = 0$, dotted) and isovector ($\Delta T = 1$, dashed) contributions. Agreement for $\omega < 25 \text{ MeV}$ is poor. Inclusion of RPA correlations (middle panel) and the resulting collectivity in the isoscalar response at low $\omega$ corrects this discrepancy to some extent (somewhat stronger collectivity is seen in the MFT–RPA results). However for both C and Ca (bottom panel) collectivity at $\omega < 25 \text{ MeV}$ is significantly underestimated. In contrast the data are well described at higher $\omega$.

We conclude that, at the present beam momentum of 705 MeV/c, the $K^+$ meson does probe the nuclear interior more deeply than other hadrons. In fact, the effective number of nucleons seen by $K^+$s at a momentum trans-
fer of 500 MeV/c is substantially greater than estimates based on the eikonal approximation and employing K+–N interactions taken from phase shift solutions. This finding is in conflict with studies of K+–nucleus elastic scattering. Microscopic QHD calculations of the K+ nuclear response at $q = 500$ MeV/c show that $M^*$ effects as well as RPA correlations and full treatment of finite nuclear size effects are important in obtaining a quantitative description of the prominent quasielastic peak. At $q = 300$ MeV/c collective strength at low excitation energies dominates the response and is not fully accounted for by the calculations. At excitation energies above about 25 MeV, full finite nucleus RHA–RPA calculations agree well with the largely featureless data.

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FIG. 1. Quasielastic spectra for Ca and for C in 4 MeV bins at $q = 500$ MeV/c are displayed. Only statistical uncertainties are shown. In the top panel the Ca spectrum is compared to uncorrelated RHA nuclear matter calculations using $M^*/M = 1$ (dotted) and $M^*/M = 0.8576$ (dashed). In the middle panel the Ca spectrum is compared to nuclear matter (dot–dashed) and to finite nucleus (solid) RHA–RPA calculations. A similar comparison with the C spectrum appears in the bottom panel. Arrows indicate the location of free $K^+–$nucleon scattering.

FIG. 2. Quasielastic spectra for C and for Ca in 3 MeV bins at 300 MeV/c are displayed. Only statistical uncertainties are shown and elastic peaks have been removed. Finite nucleus RHA calculations for C appear in the top panel. Finite nucleus RHA–RPA calculations for C are shown in the middle panel. The solid lines are the sums of the isoscalar (dotted) and isovector (dashed) responses in each panel. Finite nucleus RHA–RPA calculations for Ca are presented in the bottom panel. Arrows indicate the location of free $K^+–$nucleon scattering.
TABLE I. The measured differential cross sections for $K^+$–nucleus quasielastic scattering at 500 MeV/c are listed. $A_{\text{eff}}$ is the experimentally determined effective number of nucleons participating in the scattering, and is compared to $A_{\text{eff}}^E$, deduced from the eikonal approximation for C, Ca and Pb. Also the radius $R$ and fraction of the central nuclear density reached by the $K^+$ are shown for C, Ca and Pb.

|       | $d\sigma/d\Omega$ (mb/sr) | $A_{\text{eff}}$ | $A_{\text{eff}}^E$ | $A_{\text{eff}}/A_{\text{eff}}^E$ | $R$(fm) | $\rho(R)/\rho(0)$ |
|-------|--------------------------|-------------------|---------------------|-----------------------------------|---------|-------------------|
| H     | 2.10 ± 0.27              | 1.05 ± 0.14       |                     |                                   |         |                   |
| D     | 3.38 ± 0.44              | 2.10 ± 0.27       |                     |                                   |         |                   |
| C     | 13.6 ± 1.8               | 8.47 ± 1.1        | 6.7                 | 1.27 ± 0.17                       | 1.7     | 0.90              |
| Ca    | 33.7 ± 4.4               | 21.0 ± 2.7        | 16.0                | 1.31 ± 0.17                       | 3.2     | 0.64              |
| Pb    | 69.5 ± 9.0               | 45.7 ± 5.9        | 36.0                | 1.27 ± 0.17                       | 6.5     | 0.55              |