Shape of the Equilibrium Profile of Watercourse Based on Theory of Minimum Energy Dissipation

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Abstract: There is certain orientation in the development of the fluvial geography, because the process is controlled by the Theory of Minimum Energy Dissipation. According to the theory of entropy and take equilibrium profile of watercourse as example, this paper discussed the theoretical shapes of the fluvial geography after analyzing the mathematics innate character of Theory Minimum Energy Dissipation. Studying result shows that the equilibrium profile of all watercourses should be a cycloid line. Thus point out a new thinking method to the study of the unity of topography in fluvial geography.

1. Introduction
Any dynamic geography appearance on the earth surface is a coefficient process and the result of dynamic action occurrence at external or internal of the Earth. The geography appearance under these actions will be changed, but the orientation and target of this kind of change is not arbitrary, the changing process is also not infinite and has no end. This feature not only show us some special conditions must be need in the process of every kind of geography type's development, and will have the unique external appearance on the end, but also shows that, after a certain stage in growth, corresponding to synthesize function caused by one or a few dynamic factors, the geography will shows a mutually appearance characteristic, and the consistency of this kind of appearance characteristic performance is widespread. For example, under the function of wind, the sand dune shapes throughout the world present an astonishing consistency. At the same time, glacier geography, coastal geography also has a unique appearance mode. Corresponding with the water action taking place on the earth's surface, most gullies and stream's longitudinal section present as a cave curve form, which with a sheer slope upstream then descend to be less in downstream [1-2].

So, what is the mechanism of the consistency of geographical appearance, especially in appearance consistency of fluvial geography? What is its deep essence? Does here has a widespread law in the process of geographic development? This paper tries to discuss this problem exactly.

2. Equilibrium profile and Theory of minimum Energy Dissipation

2.1. Concept of the Equilibrium profile
Aim at the widespread consistency of the shape of the watercourse longitudinal section, researchers put forward the conception of the" equilibrium profile", this can be found in Googerlier's one viewpoint(1597) on river bed's growth and changing regulation[3], Googerlier thought the balance represents equilibrium between erode dint and resistance power. Afterwards, Jeeberter gave equilibrium an explain in sediment dynamics, he said that the equilibrium equals to
erosion—deposition balance, then the outstanding geomorphologist, W.M. Davis (1850-1934), who put forward the idea that mutually stream's erosion—deposition balance is an important link of "erosion circle" in 1902[4].

Though scholars in the earlier years put forward the concept of the equilibrium profile, many disputing of the equilibrium profile still exists for a long time. Modern scholars do not deny the existence of the equilibrium profile, but in most time they consider it from the microscopic viewpoint, that is, from the sight of the sediment dynamics [5-6], this limit the achievement's extensive application in other geographic aspects.

Author think that the equilibrium profile is one of the most basic and important concept in external dynamic geography, substance of the equilibrium profile is an final product after flowing water reformed the primitive topography, its appearing tell us the end of flowing water's deep erosion process, means that the geography appearance adapted to the route shape requirement of the flowing water completely. Admitting the existing of equilibrium profile is to confirm the viewpoint that the external change in the Earth geography have clear and definite trend or orientation, and it is a preface process which having destination starting point and terminal point, research on it will benefit to translate geography science from a pre-science only with describing knowledge's accumulation to a modern science that can predict geographical developed orientation. Then the research should be carried out from the sight of macroscopic view, thus making the research result even more matching the macroscopic characteristics of the geography.

2.2. Theory of minimum Energy Dissipation

What kind of mechanism controls the equilibrium profile's development? What is the reason of the consisten
cy appeared in numerous external dynamic geographic appearance? As mentioned before, Googerlier think that it is the balance trends between water flow's erosion power and river bed's "resistance" power requirement cause equilibrium profile, after that time, most scholars approve this viewpoint, and engaged carrying out their study in the equilibrium profile from a view sight of balance among different physical forces.

After 20 centuries, along with a gradually development and formation of the systematic theory, scholars recognized that, any system's spatial characteristic developing trend is the reflection of energy changing trends. Longitudinal shape changing of the river bed, therefore, incline to make energy consumption of water flow along the watercourse lease and lease, therefore, Theory of minimum energy dissipation formed, and many people believed it should be a basic reason to cause equilibrium profile of all kind of watercourse, and it also can be expanded in explaining all external geography appearance's consistency.

Theory of minimum Energy Dissipation is a widespread principle that exists in all nature matter's movement. Any systems in movement and changing process attempt to attain the energy consumption to be minimum, Yang Chih Ted express this principle as Theory of minimum rate of energy dissipation [7-8], think, while flowing water system tend to approach its balance state, each factors (include factors of current's flowing state and factors of the boundary condition) of system will adjust itself automatically to make the system's energy consumption rate least, at this time:

\[ \frac{dE}{dt} = 0 \]  

In a unit river length, \( E \) can be expressed as

\[ E = \gamma \int \int \int UJdxdydz \]  

(Three dimension ) \hspace{1cm} (2)

\[ E = \gamma \int QJdx \]  

(one dimension ) \hspace{1cm} (3)

\( \gamma \) — heavy volume of flowing water, \( U \) — velocity of flow; \( J \) — water course's longitudinal slop; \( Q \) — volume of flowing water

Zhang Haiyan also put forward the Minimum Unit Energy Consumption rate Theory[9], compare
thermodynamics entropy with surface channel current's water level Y, consider that when the system attains a good balance, here should has

$$\frac{dy}{dt} = \text{Minimum}$$  \hspace{1cm} (4)

And here

$$\frac{dy}{dt} = (\frac{dx}{dt}) \times (\frac{dy}{dx}) = U \cdot J$$  \hspace{1cm} (5)

Then, in a unit river length, multiplication value of current velocity $U$ and longitudinal slop ratio $J$ attempt to attain the least value as it approach to a balance state.

Obviously, from the viewpoint of energy, particularly from the Theory of minimum Energy Dissipation, carrying out the research on balance phenomenon have got more macroscopic character than from the view sight of the force, and could more matching the systematic theory which is widespread at present time, and the research result will have more extensive applicability in fluvial geography study.

3. Systematic Theory’s origin of Theory of minimum Energy Dissipation

All geography process and geography form are take place and develop in a certain system, fluvial geography too is not an exception. Therefore, only when bring the shape growth of the flowing water into the concept of the system, can we get the right conclusion.

Under the certain external condition, the fluvial geography system is an open system that lies near equilibrium lineal area, as to this system, its automatic non-revising process will take system into equilibrium state, but, this process requests the entropy creation inside the system attain the least value gradually[10], that is:

$$\frac{dp}{dt} = 0$$  \hspace{1cm} (6)

Entropy creation $p$ means an energy variety in fluvial geography system, $dp$ can be replaced by energy difference between up section and down section in unit river segment, namely

$$dp = E_{up} - E_{down} = \Delta E = dE$$  \hspace{1cm} (7)

Thus, the Principle of Minimum Entropy Creation can immediately be expressed in a specific way in fluvial geography system as Theory of minimum Energy Dissipation:

$$\frac{dE}{dt} = 0$$  \hspace{1cm} (8)

Here E includes all kinetic energy and power.

Therefore, the essence of Theory of minimum Energy Dissipation is the specific express type of Principle of Minimum Entropy Creation, because of the difficult measurement of entropy value, after being replaced with easy measured energy value, research of the characteristic of fluvial geography system become more easy to be understand, and, this kind of analogy is suitable to any researches on geography appearance.

4. Theoretical Deduction of Equilibrium profile base on Theory of Minimum Energy Dissipation

According to Principle of Minimum Entropy Creation, fluvial geography system is an open system which near but still not enter equilibrium lineal area, while approach to its balance state(permanent or stable state),the system require to attain the state of minimum entropy creation by all means, that is

$$\frac{dp}{dt} = 0$$  \hspace{1cm} (9)

It is to say, while reaching balanced state, water current lost its energy in flowing process as less as it can, it also means that the development of fluvial geography system must follow the Theory of Minimum Energy Dissipation.

Obey Orra's thinking way, the water current movement can be view as numerous fluid drops' movement, under the condition of uniform stratum and permanent flowing volume, we can simplify the process of equilibrium profile's formation into a water drop's movement from the first place A in the upper to the second lower place B ,two point are not in a same perpendicular line ( see figure l).
Requirement of water drop consume minimum energy in its movement process is equal to make the power least; water drop's energy equation can be shown as:

$$ E_{power} = E_{kinetic} + W $$

(10)

Suppose the position of A and B is determined, then the $E_{power}$ is determined too, the end purpose of water drop's movement is to make $W$ minimum, as for $W$, we know its expression is:

$$ W = \int_{A}^{B} f ds $$

(11)

here $f$ means friction resistance, $s$ is the arc length.

If the water drop has come to a point $p(x, y)$ along with the route line, its force analysis still be shown in Fig 1, base on Newton's second law, then

$$ F - f = ma = m \frac{d^2 s}{dt^2} $$

(12)

Here $F$ is the force of slipping down.

Obviously, to a given length arc route (namely $s$ has known), acceleration rate of water drop (quantity $m$ is certain)

$$ a = m \frac{d^2 s}{dt^2} $$

(13)

will increase while time interval decrease, but when $a$ become great, value of $(F - f)$ will increase, namely compare to $F$, $f$ is becoming smaller, thus

$$ W = \int_{A}^{B} f ds $$

(14)

will also become smaller along with this, we can then approaching the target of consuming the minimum energy. Therefore, deduction on water drop's minimum energy consuming process turns into a question on how to look for a path line connect A and B(Fig 1), in order to make the water drop move from a start point to terminal point along this line only cost least time .This is a question of Toboggan-slide with Least Time, a classical functional question in mathematics.

As we all know, line of Toboggan-slide with Least Time is a cycloid line, though deduction on equilibrium profile's shape has some non-matching conditions with deduction on Toboggan-slide with Least Time, because of their substantial consistency, we can completely certain that ideal line form of equilibrium profile is a cycloid line, too, same as line of Toboggan-slide with Least Time .And, this conclusion also coincide with most scholars' standpoint of equilibrium profile’s form should be a conic line with downward sag.
5. Utilization of the Equilibrium Profile
The equilibrium profile of a water course is a cycloid line, so the cycloid correlation degree of a real watercourse's profile can be regarded as one of the indicators on telling us the grade of maturity of it. But a single indicator of correlation cannot solve all the problems, such as whether the development of the profile has not yet reached the equilibrium profile (means erode down will continue), or, has it exceeded the standard shape of the equilibrium profile (means silting)? Thus author prefer to the concept of under-concave deficiency or under-concave excess, but that involves complex conditions designing and computation, so, various methods are designed out by comparing the measured geographical parameters of a water course with the standard cycloid parameters. Here are two examples of this kind of index system [11]:

5.1. Index of cycloid correlation \( R_c \)
To calculate the correlation degree between the real longitudinal profile of a water course and the theoretic cycloid line, then to judge the development potential and development stage of a watercourse according to this correlation degree. The concrete method is to measure the elevation of source area and estuarine area, the total length of a watercourse. At the same time, to get the river bed’s elevation of each point with a same interval distance in the channel according to the horizontal distance. Based on the above two kinds of data, the correlation degree between the theoretical cycloid and the measured longitudinal profile curve of the watercourse has been calculated under the condition as long as the source-estuary position has been determined.

5.2. Index of geometric shape parameters of the longitudinal section of water course
It is found that when the longitudinal profile of the watercourse reaches an equilibrium state, the geometric parameters, including the average longitudinal ratio \( J \), the concave degree \( D \) and the height difference of the watercourse \( H \), will all obtain a certain value. When the tangent depth of the longitudinal section is greater than the standard cycloid line, channel’s \( J \) will be less than this value, and \( D \) will be greater than this value. When the elevation of the source region changed, value of \( H \) of the channel will be smaller than the theoretic height difference of a standard cycloid. When the profile is underdeveloped, the opposite situation will be seen. Based on these relationships, we can further predict the potential of watercourse erosion and development in the future.

6. Summary and Conclusions
The consistency of various external dynamic geography appearance, especially the exists of the equilibrium profile in fluvial geography, all verify that there has a widespread macroscopic regulation - Theory of minimum Energy Dissipation, it is a concrete expression of the minimum principle of entropy creation, make the principle of minimum entropy become measured and applied concrete theories in fluvial geography research.

The equilibrium profile problem is a most basic and important problem in flowing water geography, set from Theory of minimum Energy Dissipation a conclusion can be deduced that its form is similar to the line of toboggan-slide with least time, it is a cycloid line.

Study the geography appearance, particularly the fluvial geographical development characteristic from the energy view angle, can make us grasp the essence that inter-adaption and inter-moderation between geographic form and dynamic factors thus constitute the geographic form's consistency from macroscopic view sight. Of course, this paper is just a beginning. On the other hand, research on equilibrium profile has great theoretical and practical value. In riverbed development and its dredge prediction of reservoir sedimentation, gully erosion harness, this study all can find its way of use. So, it is worthwhile to put more power to study the related problem.

Acknowledgements
The authors thank Prof. Bao Zhongmo for assistance with partly theoretical work. The research was supported by a National Natural Science Foundation of China (41271295).
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