Lattice QCD at finite isospin density.

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We have simulated QCD at a finite chemical potential $\mu_I$ for isospin ($I_3$) to probe part of the phase diagram for nuclear matter. Preliminary results suggest that for $\mu_I > \mu_c$, this theory forms a charged pion condensate which spontaneously breaks $I_3$, and the isospin density is non zero.

1. Introduction

Nuclear matter has finite baryon number density, finite (negative) isospin density and, under certain conditions, finite (negative) strangeness density. Since having a finite chemical potential for baryon number makes the fermion determinant complex and simulations intractable, we restrict ourselves to zero baryon-number chemical potential. QCD at finite chemical potential $\mu_I$ for isospin ($I_3$) and zero chemical potential for baryon number, has a real positive fermion determinant and can thus be simulated. This describes a surface in the phase diagram for nuclear matter. This theory turns out to be very similar to 2-colour QCD at finite quark-number density which we are also studying \cite{1,2}. We will also use various devices to include the effects of finite chemical potential $\mu_S$ for strangeness.

QCD at finite chemical potential $\mu_I$ for isospin has been studied by Son and Stephanov \cite{3} using effective (chiral) lagrangians. This analysis indicates that it undergoes a transition at $|\mu_I| = \mu_c = m_\pi$ to a state with a charged pion condensate, while the orthogonal charged pion becomes a true Goldstone boson. This condensate breaks $I_3$ and parity spontaneously.

Section 2 describes our simulations of 2 flavour QCD with a finite $\mu_I$. A brief discussion of how to include a finite $\mu_S$ is included. Section 3 gives our conclusions.

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2. Lattice QCD at finite isospin chemical potential

The lattice fermion action (staggered quarks) for QCD with isospin chemical potential, $\mu_I$ is

$$ S_f = \sum_{\text{sites}} \left[ \bar{\chi} \left[ D / (\tau_3 \mu_I) + m \right] \chi + i \lambda \epsilon \tau_2 \chi \right] $$ (1)

where $D(\mu)$ is the standard staggered $D$ with links in the $+t$ direction multiplied by $e^{+i \mu}$ and those in the $-t$ direction multiplied by $e^{-i \mu}$. The term proportional to $\lambda$, which explicitly breaks the third component of isospin, $I_3$, is introduced to allow us to observe spontaneous breaking of this symmetry from finite lattice simulations. We will be interested in the limit $\lambda \to 0$. The Dirac operator is

$$ M = \begin{pmatrix} D(\mu_I) + m & \lambda \epsilon \\ -\lambda \epsilon & D(-\mu_I) + m \end{pmatrix} $$ (2)

whose determinant

$$ \det \mathcal{M} = \det [\mathcal{A}^1 \mathcal{A} + \lambda^2] $$ (3)

where

$$ \mathcal{A} \equiv D(\mu_I) + m. $$ (4)

Since this determinant is strictly positive, standard simulation methods work. We use hybrid molecular dynamics simulations to tune the number of flavours down from 8 to 2.

We measure the chiral condensate

$$ \langle \bar{\psi} \psi \rangle \leftrightarrow \langle \bar{\chi} \chi \rangle, $$ (5)
the isospin condensate
\[ i \langle \bar{\psi} \gamma_5 \tau_2 \psi \rangle \Leftrightarrow i \langle \bar{\chi} \epsilon \tau_2 \chi \rangle \] (6)
and the isospin density
\[ j_0^3 = \left\langle \frac{\partial S_f}{\partial \mu_I} \right\rangle \] (7)

At \( \mu_I = \lambda = 0 \), there are 4 pseudo-Goldstone pions corresponding to
\[ i \langle \bar{\chi} \epsilon \tau_i \chi \rangle \] (8)
\[ i \langle \bar{\chi} \epsilon \chi \rangle \] (9)

Only 2 of these (those with \( i = 1, 2 \)) correspond to pseudo-Goldstone pions of the \( N_f = 2 \) theory — the third \( N_f = 2 \) pion is a massive 3-link object at finite lattice spacing. However, the other 2 Goldstone bosons should mimic the behaviour of the third \( N_f = 2 \) pion (\( \pi_0 \)) in the continuum limit and are thus useful to consider. As \( \mu_I \) increases one expects the \( i = 1 \) mass to fall, while the \( i = 2 \) mass should rise. The other 2 masses are expected to remain unchanged for small \( \mu_I \).

One expects a phase transition at \( \mu_I = \mu_c \approx m_{\pi} \), above which isospin(\( I_3 \)) is spontaneously broken and the \( i = 1 \) state becomes a true Goldstone boson. This is only part of the story. Once \( j_0^3 \) gains an expectation value, scalar mesons are also candidate pseudo-Goldstone bosons.

We are simulating this theory at \( \beta = 5.2, m = 0.025, 0.05, \lambda = 0.005, 0.0025 \) on an \( 8^4 \) lattice. We are also performing simulations at \( m = 0.05, \lambda = 0.005 \) on an \( 8^3 \times 4 \) lattice to study the finite temperature transition.

The pion condensate is shown in figure 1. This strongly suggests that there is some \( \mu_I = \mu_c \) above which isospin(\( I_3 \)) is spontaneously broken by a charged pion condensate. However, we will need to finish our simulations at \( \lambda = 0.0025 \) and extrapolate to \( \lambda = 0 \) to validate this observation.

The chiral condensate shown in figure 2 is approximately constant for \( \mu_I < \mu_c \). For \( \mu_I > \mu_c \) it starts to fall, indicating that the condensate is rotating from the chiral to the isospin-breaking direction — it does not appear to be a simple rotation which is suggested by the tree-level effective Lagrangian approach, however.

The isospin density, shown in figure 3 is consistent with zero for \( \mu_I < \mu_c \) and commences to rise for \( \mu_I > \mu_c \). It reaches its saturation value (3) for \( \mu_I \sim 2 \).

We will measure the spectrum of Goldstone and pseudo-Goldstone mesons on a larger \( (12^3 \times 24) \) lattice. In addition we will store configurations for offline calculations of other hadron spectra (\( N, \Delta, \rho \ldots \)) which are also expected to exhibit interesting behaviour.

3. Conclusions

QCD at finite chemical potential for isospin(\( I_3 \)) appears to spontaneously break isospin with a charged pion condensate, for \( \mu_I \) large enough. We need another \( \lambda \) to verify this and to show that \( \mu_c > 0 \). We will repeat these simulations on a larger \( (12^3 \times 24) \) lattice to measure the spectrum, in particular the Goldstone boson. Measurement of the instanton distribution is of interest. Is there any additional structure at high \( \mu_I \), where the effective Lagrangian approach is suspect, perhaps related to baryon thresholds? What we re-
ally would like to know is do such pion condensates form at finite baryon-number density (i.e. in nuclear matter) for \( \mu_I \) large enough?

Preliminary indications from our finite temperature \( (8^3 \times 4) \) simulations are that the evaporation of the pion condensate for large enough temperature occurs at a first order transition. This needs further study.

Simulations with finite chemical potentials for isospin and strangeness will study the competition between pion and kaon condensation. Two methods will be used to avoid a complex determinant. The first is to use a partially quenched approach where only the \( u \) and \( d \) quarks are dynamical. The second is to include a \( c \) quark with \( m_c = m_s \) and a chemical potential for the 3rd component of the \( SU(2) \) flavour group which mixes \( c \) and \( s \).

QCD at finite isospin, and QCD at finite isospin and strangeness should give sensible quenched results. In fact our earlier work on quenched QCD at finite baryon number density can be reinterpreted as quenched QCD at finite isospin density.

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