Study of $\Xi^-$-atom based on the $\Xi N$ interaction from QCD on lattice

Takashi Inoue for HAL QCD Collaboration
Nihon University, College of Bioresource Sciences, Fujisawa 252-0880, Japan
E-mail: inoue.takashi@nihon-u.ac.jp

Abstract. We report our preliminary theoretical study on $\Xi^-$-atom based on a $\Xi N$ interaction extracted from QCD on lattice. First, we study $\Xi N$ interaction in nuclear matter. Then, we make a $\Xi$-nucleus potential in a simple folding procedure. We study $\Xi^-$-$^{60}$Ni system as an example, and find that energy levels of the atom are shifted downward due to the $\Xi N$ interaction.

1. Introduction

Hyperon-nucleon (YN) scattering experiment is difficult due to the short life time of hyperons because of their weak decay. Therefore, the old bubble chamber data of $\Lambda N$ and $\Sigma N$ scattering in the 1960s, have been an essential experimental information of hyperon-nucleon interactions [1]. Further experimental information of the interaction has been obtained through deliberate studies of hypernuclei. Thanks to the rich data of $\Lambda$-hypernuclei, $\Lambda N$ interaction is revealed relatively well nowadays. On the other hand, experimental data on the $\Sigma$-nucleus and $\Xi$-nucleus system are limited, and hence $\Sigma N$ interaction and $\Xi N$ interaction are still unclear. Especially, nature of $\Xi N$ interaction have been a mystery until very recently. Significant experimental effort have been devoted to study these interactions at many facilities. The first evidence of the $\Xi$-hypernucleus, $^{14}\Xi N$, was found recently at KEK in Japan [2]. Some other $\Xi$-hypernuclei seem to be observed and will be reported before long. These discovery of $\Xi$-hypernuclei certify that there is an attractive component in $\Xi N$ interaction.

Because $\Xi^-$ hyperon has a negative charge, there exist $\Xi^-$-nucleus atomic bound states. Energy levels of $\Xi^-$-atom should be influenced by the strong $\Xi N$ interaction and shifted [3]. In principle, the shifts can be observed experimentally by measuring X-ray accompanying transition of the $\Xi^-$-atom. In fact, such experiments are planed at some facilities. This means that there is a possibility which we can obtain information of $\Xi N$ interaction through studies of $\Xi^-$-atom, or we can check theoretical prediction of $\Xi N$ interaction through $\Xi^-$-atom, in near future. Therefore, in this paper, we study $\Xi^-$-atom theoretically.

2. $\Xi N$ interaction in a vacuum

Traditionally, hyperon interactions have been investigated in quark models, meson exchange models, and so on [4, 5, 6]. Because of the shortage of experimental data, there are some uncertainties in those interactions. In 2006, a new method was invented to extract nucleon-nucleon interaction from QCD on lattice [7]. This HAL QCD method has been extended and improved continuously, and applied to various multi-hadron systems successfully [8, 9, 10].
Essence of the HAL QCD method for a baryon-baryon (BB') interaction is following. We calculate the so-called 4-point correlation functions in lattice QCD

\[
\phi_{(BB')\langle J\rangle}(\vec{r}, t) = \frac{1}{\sqrt{Z_B Z_{B'}}} \sum_{\vec{x}} \langle 0 | B(\vec{x} + \vec{r}, t) B'(\vec{x}, t) J(t_0) | 0 \rangle
\]

where \(B(\vec{y}, t) B'(\vec{x}, t)\) is a product of baryon field operators at sink, \(J(t_0)\) is a source operator which creates two baryon at \(t_0\), and \(Z_B\) is a renormalization factor of the baryon field. It was proved that the 4-point correlation function \(\phi_{(BB')\langle J\rangle}(\vec{r}, t)\) contains scattering observables in the same way as a quantum mechanical wave function dose [11, 12]. Then, we define a interaction kernel or potential of the BB' interaction so that it reproduce scattering observables contained in a set of \(\phi_{(BB')\langle J\rangle}(\vec{r}, t)\), and extract it by taking inversion of the set of Schrödinger equations in the Euclidean space-time. Thus, we can derive potential of any baryon-baryon interaction in lattice QCD numerical calculation.

In this HAL QCD method, we do not need to separate or suppress two-body excitation in the 4-point functions. This is a significant advantage of this method over the conventional direct one in which separation or suppression of two-body excitation is crucial but impossible to achieve practically for multi-baryon systems [13].

Recently, we have extracted potentials of S-wave interaction for all the octet-baryon pairs from lattice QCD at almost physical point [14]. The full QCD gauge configuration set we have used for that configuration is called K-configuration [15], where quark masses are almost physical, namely pion mass is 146 MeV, kaon mass is 525 MeV, nucleon mass is 958 MeV, and \(\Lambda\)-hyperon mass is 1140 MeV, \(\Sigma\)-hyperon mass is 1223 MeV, and \(\Xi\)-hyperon mass is 1354 MeV. Extracted BB' interaction potentials for strangeness \(S=\pm 2\) sector, including N interaction, can be found in reference [16] for example.

3. \(\Xi N\) interaction in nuclear matter
In order to study \(\Xi^-\)-atom (and \(\Xi\)-hypernucleus), first we consider \(\Xi N\) interaction in nuclear matter. Here, nuclear matter is a hypothetical uniform matter consisting of infinite number of nucleons interacting each other through the nuclear force. The Brueckner-Hartree-Fock (BHF) theory is a traditional framework to deal with nuclear matter.

In the BHF theory, the so called G-matrix which describes scattering of two baryons in the matter, plays essential role. The G-matrix is obtained by solving the Bethe-Goldstone equation

\[
G_{a,b}(\omega) = V_{a,b} + \sum_c \sum_{k,k'} V_{a,c} \langle k,k' \rangle \frac{Q_c(k,k')}{\omega - E_c(k,k') + i\epsilon} \langle k,k' \rangle G_{c,b}(\omega)
\]

where \(Q(k,k')\) is the angle-averaged Pauli exclusion factor, and \(E_c(k,k')\) is energy of an intermediate baryon pair which depends on the G-matrix through the single-particle potentials of baryons [17, 18, 19]. Therefore, the BHF equations are highly coupled and solved in the iteration procedure.

By combining our lattice QCD YN potentials and the BHF theory, we have studied the single-particle potentials of hyperons \(U_\gamma(k)\) in nuclear matter [16]. We have used \(U_p(k)\) and \(U_n(k)\) obtained in the BHF with the AV18 phenomenological two-nucleon force [20] supplemented by the Urbana-type three-nucleon force [21]. We obtained, for the hyperons stopping in the symmetric nuclear matter at the normal nuclear matter density \(\rho_0\),

\[
U_\Lambda(0) = -28 \text{ MeV}, \quad U_\Sigma(0) = +15 \text{ MeV}, \quad U_\Xi(0) = -4 \text{ MeV}
\]

with a statistical error approximately \(\pm 2\) MeV associated with our lattice QCD Monte Carlo simulation. These results are consistent with experimental indications, in particular with the discovery of the \(\Xi\)-hypernucleus.
The G-matrix is a kind of effective interaction in the nuclear matter. In order to apply the obtained G-matrix to finite systems, we convert it to the so-called G-matrix potential, which is a local potential for each partial wave made so that it simulates the corresponding G-matrix on-shell elements [22].

\[
\frac{\pi}{2} \int_0^{\infty} r^2 j_L(kr) G_{\Xi N}^{S L J}(\omega, r) j_L(kr) \simeq G_{\Xi N}^{S L J}(\omega, k, k) \tag{4}
\]

We chose value of \(\omega\) so that \(\Xi\) hyperon almost stops in the nuclear matter, and drop it hereafter. Imaginary part of the \(\Xi N\) G-matrix is almost zero at small \(k\). This is because nucleon in the nuclear matter receive strong attraction from the matter, and hence there is no phase space to decay for stopped \(\Xi\) hyperon. Moreover, the \(\Xi N\)-\(\Lambda\Lambda\) off-diagonal interaction is predicted very weak in our lattice QCD calculation. Therefore, in this paper, we consider only the real part of the \(\Xi N\) G-matrix. We express \(\tilde{G}_{\Xi N}^{S L J}(r)\) as a three-range Gaussian and fit its parameters to the G-matrix elements through the equation (4).

Fig 1 shows obtained \(\Xi N\) G-matrix potentials in the symmetric nuclear matter at five representative densities. We see that the \(\Xi N\) repulsion at short distance becomes much weaker than the corresponding original potential shown as a broken line. We notice also that the density dependence of the G-matrix potential is very complicated, especially in the spin singlet channels.

4. \(\Xi^{-}\)-atom with a QCD \(\Xi N\) interaction

In this section, we consider atomic bound state of \(\Xi^{-}\) hyperon and \(^{60}\text{Ni}\) nucleus as an example. We chose \(^{60}\text{Ni}\) merely so that number of proton \(i.e.\) charge of nucleus is around 30, which is similar to setting of coming experiment.
We construct a \( \Xi \)-nucleus optical potential by folding the \( \Xi N \) G-matrix potential \( \tilde{G}^{S\ell J}_{\Xi N}(r) \) with a proton and neutron density of the nucleus, \( \rho_p^{\Xi Ni} \) and \( \rho_n^{\Xi Ni} \).

\[
U(\vec{r})_{\Xi-Ni} = \int d^3\vec{r}' \rho_p^{\Xi Ni}(\vec{r}'), \tilde{G}_{\Xi p}(|\vec{r} - \vec{r}'|, \vec{\rho}) + \int d^3\vec{r}' \rho_n^{\Xi Ni}(\vec{r}'), \tilde{G}_{\Xi n}(|\vec{r} - \vec{r}'|, \vec{\rho}) 
\]

(5)

Because iso-spin symmetry is broken in the nucleus, we need to use \( \Xi N \) potentials in the charge base. Since spin is saturated in the nucleus, we use \( \Xi N \) potentials averaged over spin singlet and triplet. For the proton and neutron density of the nucleus, we use a theoretical one obtained in the density dependent Hartree-Fock theory with the Skyrme-III parameter set. Because the density dependence of \( \Xi N \) G-matrix potential is very complicated as shown in Fig 1, we carry out a very primitive folding in this paper, namely we fix \( \vec{\rho} = 0.5 \rho_0 \).

Fig 2 Left shows the obtained \( ^{60}\text{Ni-}\Xi \) nucleus potential based on our lattice QCD \( \Xi N \) interaction. By including this potential in addition to the Coulomb potential, we solve the Schrödinger equation of the \( ^{60}\text{Ni} \) nucleus system. Fig 2 Right shows the obtained energies of bound states with the angular momentum \( L=0,1,2,3, \) and \( 4 \). There, the blue bars represent energy of states without \( \Xi N \) interaction, while red bars represent energy of states with our lattice QCD \( \Xi N \) interaction. We see that lower energy levels are shifted downward due to the strong \( \Xi N \) interaction. The magnitude of the shifts decrease as \( L \) increases, and become invisible at \( L=4 \) with the scale of this figure.

5. Summary and outlook
In this paper, we have studied effect of the \( \Xi N \) interaction on the \( \Xi^- \)atom theoretically. We have started from a \( \Xi N \) interaction extracted from QCD on lattice. First, we have prepared an effective \( \Xi N \) interaction in nuclear matter in the BHF theory. Then, we have made a \( \Xi^{-^{60}}\text{Ni} \) nucleus potential tentatively in a primitive folding procedure. We have solved the Schrödinger equation of the system for the angular momentum \( L \) up to 4, and obtained energy of bound states. We have found that energy levels of \( \Xi^{-^{60}}\text{Ni} \) atom are sifted downward order of MeV due to the \( \Xi N \) interaction.

In this paper, we have not considered absorption of \( \Xi \)-hyperon at all. In reality, \( \Xi \)-hyperon will be absorbed into nucleus through the strong interaction, and hence the energy levels of actual \( \Xi^- \)atom will have a certain width. Accordingly, deeply bound levels will not be accessible in experiments [3]. For example, it is planned to measure X-ray accompanying transition from \( L=7 \) bottom to \( L=6 \) bottom in one experiment. This means that, in order to check a theoretical \( \Xi N \)
interaction through $\Xi^-$-atom, we have to predict much smaller energy shifts (order of keV) at larger $L$ state than ones we obtained in this study. For this purpose, we need to make more precise $\Xi$-nucleus potential. At least, we need to carry out a sophisticated folding procedure by taking the density dependence of the effective $\Xi N$ interaction in matter into account. This is what we are trying now. For experimental study of $\Xi^-$-atom, width of the energy levels are important for its feasibility. Therefore, we want to estimate the width based on the lattice QCD $\Xi N$ interaction. We plan to estimate width perturbatively by using the small imaginary part of the $\Xi N$ G-matrix.

In this paper, we have considered $\Xi N$ interaction in only S-wave. This is because extraction of P-wave interaction in lattice QCD is very expensive [23], and we have not carried out it at near the physical point. Since P-wave hyperon interactions are important in physics of hypernuclei, neutron stars, and so on, we hope we can obtain the interactions in lattice QCD and confirm them through experiment in near future.

ACKNOWLEDGMENTS

We thank the PACS Collaboration [15] for generating and providing gauge configurations, and the JLDG team [24, 25] for providing storage to save our data. Numerical computation of this work was carried out on the K computer at RIKEN R-CCS (hp120281, hp130023, hp140209, hp150223, hp150262, hp160211, hp170230), the HOKUSAI GreatWave at RIKEN Wako (G15023, G16030, G17002), and the HA-PACS at University of Tsukuba (14a-20, 15a-30). This research is supported in part by the JSPS Grant-in-Aid for Scientific Research (C)18K03628.

References

[1] C. B. Dover and H. Feshbach, Annals Phys. 198, 321 (1990).
[2] K. Nakazawa et al., PTEP 2015, no. 3, 033D02 (2015).
[3] C. J. Batty, E. Friedman and A. Gal, Phys. Rev. C 59, 295 (1999).
[4] M. Oka, K. Shimizu and K. Yazaki, Nucl. Phys. A 464, 700 (1987).
[5] Y. Fujiiwara, C. Nakamoto and Y. Suzuki, Phys. Rev. Lett. 76, 2242 (1996).
[6] T. A. Rijken, V. G. J. Stoks and Y. Yamamoto, Phys. Rev. C 59, 21 (1999).
[7] N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett. 99, 022001 (2007); S. Aoki, T. Hatsuda and N. Ishii, Prog. Theor. Phys. 123, 89 (2010); N. Ishii et al. [HAL QCD Coll.], Phys. Lett. B 712 (2012) 437.
[8] H. Nemura, N. Ishii, S. Aoki and T. Hatsuda, Phys. Lett. B 673, 136 (2009).
[9] T. Inoue et al. [HAL QCD Collaboration], Prog. Theor. Phys. 124, 591 (2010); Phys. Rev. Lett. 106, 162002 (2011); Nucl. Phys. A 881, 28 (2012); Phys. Rev. Lett. 111, 112503 (2013).
[10] K. Sasaki et al. [HAL QCD Collaboration], PTEP 2015, no. 11, 113B01 (2015).
[11] N. Ishizuka, PoS LATTICE 2009, 119 (2009).
[12] S. Aoki, B. Charron, T. Doi, T. Hatsuda, T. Inoue and N. Ishii, Phys. Rev. D 87, no. 3, 034512 (2013).
[13] T. Iritani et al. [HAL QCD Collaboration], JHEP 1610, 101 (2016), Phys. Rev. D 96, no. 3, 034521 (2017).
[14] T. Doi et al. [HAL QCD Collaboration], EPJ Web Conf. 175, 05009 (2018), H. Nemura et al. [HAL QCD Collaboration], EPJ Web Conf. 175, 05030 (2018), K. Sasaki et al. [HAL QCD Collaboration], EPJ Web Conf. 175, 05010 (2018), N. Ishii et al. [HAL QCD Collaboration], EPJ Web Conf. 175, 05013 (2018).
[15] K.-I. Ishikawa et al. [PACS Collaboration], PoS LATTICE 2015, 075 (2016).
[16] T. Inoue for HAL QCD Collaboration, PoS INPC 2016, 277 (2016), Takashi Inoue for HAL QCD Collaboration, AIP Conference Proceedings 2130, 020002 (2019).
[17] M. Baldo, G. F. Burgio and H. J. Schulze, Phys. Rev. C 61, 055801 (2000).
[18] M. Kohno and Y. Fujiiwara, Phys. Rev. C 79, 054318 (2009).
[19] Y. Yamamoto, T. Furumoto, N. Yasutake and T. A. Rijken, Phys. Rev. C 90, 045805 (2014).
[20] R. B. Wiringa, V. G. J. Stoks and R. Schiavilla, Phys. Rev. C 51, 38 (1995).
[21] J. Carlson, V. R. Pandharipande and R. B. Wiringa, Nucl. Phys. A 401, 59 (1983).
[22] Y. Yamamoto, T. Motoha and T. A. Rijken, Prog. Theor. Phys. Suppl. 185, 72 (2010).
[23] N. Ishii et al. [HAL QCD Collaboration], PoS LATTICE 2013, 234 (2014).
[24] Japan Lattice Data Grid, http://www.jldg.org/jldg/.
[25] T. Amagasa et al., J. Phys. Conf. Ser. 664, no. 4, 042058 (2015).