Neutralino Dark Matter in mSUGRA: 
Reopening the light Higgs pole window

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Abstract

The requirement that the lightest neutralino $\tilde{\chi}_1^0$ has the right thermal relic density to explain all Dark Matter in the universe strongly constrains the parameter space of supersymmetric models in general, and of the mSUGRA model in particular. Recently improved calculations of the mass of the light CP–even Higgs boson $h$ present in this model, and the increased central value of the mass of the top quark, have re–opened the possibility that $2m_{\chi^0_1} \lessgtr m_h$. In this “$h$–pole region” the LSP annihilation cross section is enhanced by near–resonant $h$ exchange in the $s$–channel, reducing the relic density to acceptable values. We delineate the corresponding region of mSUGRA parameter space, and explore its phenomenology. In particular, we find strong upper bounds on the masses of the gluino, lighter chargino and LSP.
1 Introduction

Supersymmetrizing the phenomenologically very successful Standard Model (SM) of particle physics has many advantages. In addition to solving the (technical aspect of) the hierarchy problem [1], it is also compatible with the Grand Unification of all gauge interactions [2]. In addition, if $R$ parity is conserved, the lightest superparticle (LSP) is stable, making it a possible candidate for the cold dark matter (DM) in the universe, the existence of which is inferred from cosmological observations [3], in particular from detailed analyses of the anisotropy of the cosmic microwave background [4].

The necessary breaking of supersymmetry in general introduces many unknown parameters. Most of these parameters are associated with flavor mixing and/or CP–violation, and are severely constrained by experiment. This motivates the study of models where additional sources of flavor changing neutral currents (FCNC) and CP–violation are automatically suppressed. These models have the additional advantage of being able to describe the entire superparticle and Higgs spectrum with a small number of free parameters, which dramatically increases their predictive power. The oldest such model [5, 6] goes under the name of minimal Supergravity (mSUGRA). Here one assumes that gaugino masses, soft breaking scalar masses, and trilinear soft breaking parameters all have universal values, $m_{1/2}$, $m_0$ and $A_0$, respectively, at the scale of Grand Unification $M_X \simeq 2 \cdot 10^{16}$ GeV. Unlike models with gauge [7] or anomaly [8] mediated supersymmetry breaking, mSUGRA allows the lightest neutralino $\tilde{\chi}_1^0$ as LSP in the visible sector to have the required thermal relic density for natural masses (in the range of tens to hundreds of GeV).

At least in the framework of standard cosmology, the Dark Matter density is by now quite well known, in particular from the WMAP data [4]. In our analysis we will use the 99% (confidence level) CL region

$$0.087 \leq \Omega_{\tilde{\chi}_1^0} h^2 \leq 0.138,$$

where $\Omega_{\tilde{\chi}_1^0}$ is the LSP mass density in units of the critical density, and $h$ is today’s Hubble constant in units of 100 km/(s·Mpc). Not surprisingly, this requirement greatly constrains the allowed parameter space. Here we work under the usual assumption that the LSP once was in thermal equilibrium; its relic density is then essentially inversely proportional to its annihilation cross section [3]. Recent analyses [9, 10] found four distinct “cosmologically acceptable” regions\(^1\) where these assumptions lead to a relic density in the range (1). Scenarios where both $m_0$ and $m_{1/2}$ are rather small (the “bulk region”) are most natural from the point of view of electroweak symmetry breaking, but are severely squeezed by lower bounds from searches for superparticles and Higgs bosons [12]. In the “co–annihilation” region one

\(^1\)An additional region, with co–annihilation of the LSP with top squarks [11] is in general disfavored in mSUGRA–type scenarios.
has $m_{\tilde{\chi}_1^0} \simeq m_{\tilde{\tau}_1}$, leading to enhanced destruction of superparticles since the $\tilde{\tau}_1$ annihilation cross section is about ten times larger than that of the LSP; this requires $m_{1/2} \gg m_0$. The “focus point” or “hyperbolical branch” region occurs at $m_0 \gg m_{1/2}$, and allows $\tilde{\chi}_1^0$ to have a significant higgsino component, enhancing its annihilation cross sections into final states containing gauge and/or Higgs bosons; however, if $m_t$ is near its current central value [13] of 178 GeV, this solution requires multi–TeV scalar masses. Finally, if the ratio of vacuum expectation values $\tan \beta$ is large, the $s$–channel exchange of the CP–odd Higgs boson $A$ can become nearly resonant, again leading to an acceptable relic density (the “$A$–pole” region).

Here we emphasize that a fifth cosmologically acceptable region of mSUGRA parameter space exists. In a significant region of parameter space one has $2m_{\tilde{\chi}_1^0} \lesssim m_h$, so that $s$–channel $h$ exchange is nearly resonant. This “$h$–pole” region featured prominently in early discussions of the Dark Matter density in mSUGRA [14], but seemed to be all but excluded by the combination of rising lower bounds on $m_h$ and $2m_{\tilde{\chi}_1^0}$ from searches at LEP [12]. However, in recent years improved calculations [15] of the mass of the light CP–even $h$ boson and the increase of the central value of the top mass to 178 GeV [13] have resurrected this possibility; improved calculations [18] of the neutralino and chargino mass spectrum also play a role. The relevant region of parameter space is delineated in Sec. 2 and the resulting phenomenology is discussed in Sec. 3. Sec. 4 contains a brief summary and some conclusions.

2 The $h$–pole region

The mSUGRA parameter space is defined by four continuous parameters and one sign,

$$m_0, m_{1/2}, A_0, \tan \beta, \text{sign}(\mu).$$

The common scalar soft breaking mass $m_0$, the common gaugino mass $m_{1/2}$ and the common trilinear soft term $A_0$ are all defined at the scale $M_X$ where the running ($\overline{\text{DR}}$) $U(1)_Y$ and $SU(2)$ gauge couplings meet. The ratio of vacuum expectation values $\tan \beta$ is defined at the weak scale, which we take to be the geometric mean of the masses of the two $\tilde{t}$ mass eigenstates. The sign of the supersymmetric higgs(ino) mass parameter $\mu$ is independent of the scale. The connection between the weak scale and $M_X$ is established by a set of coupled renormalization group equations (RGE) [19]. The RG evolution can drive one combination of squared Higgs boson masses to negative values, thereby triggering electroweak symmetry breaking (EWSB) [20]. The requirement that this leads to the correct mass of the $Z$ boson fixes the absolute value of $\mu$.

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2In the relevant region of parameter space, $m_{\tilde{\chi}_1^0} \simeq m_{\tilde{\chi}_1^\pm}/2$; lower bounds on the chargino mass therefore directly translate into lower bounds on the LSP mass.

3Ref. [16], which uses $m_t = 175$ GeV, finds a small $h$–pole region for $\tan \beta = 10$, but not for larger values of $\tan \beta$. Ref. [17], which uses $m_t = 180$ GeV, finds an $h$–pole region for $\tan \beta = 30$. Neither of these papers systematically analyzes the extent of this region.
We use the Fortran package SUSPECT [21] to solve the RGE and to calculate the spectrum of physical sparticles and Higgs bosons, following the procedure outlined in [10]. Of special interest to the present study is that this calculation includes leading one–loop “threshold” corrections to neutralino and chargino masses as well as two–loop corrections to the corresponding RGE. Also included are complete one–loop corrections and many two–loop corrections [22] to scalar Higgs boson masses. As mentioned in the Introduction, some of these corrections have only been calculated in the last few years [15]; they increase the mass $m_h$ of the light CP–even Higgs boson $h$ by several GeV.

In addition to leading to consistent EWSB, a given set of input parameters has to satisfy several experimental constraints. The ones relevant for this study are:

- The total cross section for the production of any pair of superparticles at the highest LEP energy (206 GeV) must be less than 20 fb. The only exception is the $\tilde{\chi}^0_1\tilde{\chi}^0_1$ final state, which is invisible.\(^4\) This is a rather aggressive interpretation of LEP sparticle search limits [12, 23]; the true limits are often somewhat weaker than this. However, since the cross sections near threshold increase very quickly with decreasing sparticle masses, the resulting limits match experimental results rather closely in most cases.

- Searches for neutral Higgs bosons at LEP [12, 24] impose a lower bound on $m_h$; note that in the region of mSUGRA parameter space of interest here, the couplings of $h$ to SM particles are very similar to that of the single Higgs boson of the SM. Allowing for a theoretical uncertainty [25] in the calculation of $m_h$ of about 3 GeV, we therefore require the calculated value of $m_h$ to exceed 111 GeV.

- Recent measurements [26] of the anomalous magnetic moment of the muon lead to the constraint on the supersymmetric contribution [27] to $a_\mu \equiv (g_\mu - 2)/2$

\[
-5.7 \cdot 10^{-10} \leq a_{\mu, SUSY} \leq 4.7 \cdot 10^{-9}.
\]  

(3)

This range has been constructed from the overlap of the $2\sigma$ allowed regions using data from $e^+e^-$ annihilation into hadrons and from $\tau$ decays, respectively, to estimate the (hadronic) SM contribution to $a_\mu$; see refs. [28] for discussions of this theoretical uncertainty.

- The calculated $\tilde{\chi}^0_1$ relic density has to be in the range [11]. Our calculation uses proper thermal averaging of the squared $s$–channel, in particular $h$–exchange, contribution near the resonance [29], while all other contributions are treated using the standard non–relativistic expansion. Note that the total $\tilde{\chi}^0_1$ annihilation cross section is completely dominated by $h$–exchange diagrams in the region of parameter space of interest to this analysis.

\(^4\)Note that we assume $R$–parity to be conserved, and that the gravitino mass is larger than that of the lightest sparticle in the visible sector.
Allowing for experimental and theoretical errors [12], the branching ratio for radiative $b$ decays should satisfy

$$2.65 \cdot 10^{-4} \leq B(b \to s\gamma) \leq 4.45 \cdot 10^{-4}.$$  \((4)\)

We evaluate this branching ratio, including contributions from $tH^\pm$ and $t\tilde{\chi}^\pm$ loops, using the results of ref. [30].

We consider this last constraint to be not as reliable as the other limits discussed above. First of all, we are not aware of any complete fit of the elements of the quark mixing (Kobayashi–Maskawa) matrix in the context of mSUGRA (or any other extension of the SM). We follow the usual practice of using the value of $V_{ts}$ as extracted in the framework of the SM when evaluating the branching ratio; note that $B(b \to s\gamma) \propto |V_{ts}|^2$. Moreover, this calculation would be affected significantly if one allowed small deviations from universality, or equivalently, small non–diagonal entries in the squark mass matrix, at the input scale [31]. This modification would leave all other results, including the sparticle spectrum and the LSP relic density, nearly unchanged.

We are now ready to present some numerical results. In Fig. 1 we show the relevant region of the $(m_0, m_{1/2})$ plane for $A_0 = 0$, $\tan\beta = 30$ and $m_t = 178$ GeV. The region $m_0 < 700$ GeV is excluded by Higgs searches at LEP. If the theoretical uncertainty of the calculation of $m_h$ is ignored, i.e. if we require the calculated $m_h$ to exceed 114 GeV, values $m_0 < 1150$ GeV would be excluded; this is coincidentally very close to the limit on $m_0$ from the constraint on $b \to s\gamma$ decays (green area). The (weak) evidence for an SM–like Higgs boson with mass $\sim 116$ GeV [24] favors the red region. Finally, in the yellow region the LSP relic density satisfies \([11]\). This region extends to very large $m_0$, and eventually merges with the focus point/hyperbolical branch region. At smaller $m_0$ it splits into two bands, with $\Omega_{\tilde{\chi}_1^0}$ being too low between the two branches.

The origin of these two branches can be understood from Fig. 2, where we plot the scaled relic density as a function of $m_{1/2}$ for several combinations of the remaining parameters. Recall that the relic density is basically inversely proportional to the thermally averaged $\tilde{\chi}_1^0$ annihilation cross section. This cross section reaches its maximum for $m_{\tilde{\chi}_1^0}$ very close to, but just below, $m_h/2$. It remains quite large for somewhat smaller $m_{\tilde{\chi}_1^0}$, since the finite kinetic energy of the LSPs still enables them to annihilate resonantly; note that decoupling occurs at temperature $T \simeq m_{\tilde{\chi}_1^0}/20$, where the kinetic energy is still significant. On the other hand, the thermally averaged cross section drops very quickly once $m_{\tilde{\chi}_1^0} > m_h/2$, since the (positive) kinetic energy can then only move the LSPs even further away from the $h$–pole. This explains [29] why the curves in Fig. 2 are not symmetric around their minimum.\(^5\)

\(^5\)We note in passing that the usual definition of finetuning [32] would predict that no finetuning of $m_{1/2}$ is
Figure 1: Constraints on the $m_{1/2}-m_0$ mSUGRA parameter space for $m_t = 178$ GeV, $A_0 = 0$, $\tan\beta = 30$ and $\mu > 0$. The dark area is ruled out by the requirement of EWSB and sparticle search limits, as discussed in the text. The violet and green areas are ruled out by, respectively, the LEP Higgs search and the $b \to s\gamma$ constraints. The red area corresponds to the LEP evidence for a light MSSM Higgs boson with $m_h \sim 116$ GeV. In the yellow bands the neutralino relic density falls in the range favored by WMAP, $0.087 \leq \Omega_{\tilde{\chi}_1^0} h^2 \leq 0.138$.

If this minimum corresponds to a relic density below the lower limit in \(1\), one has two allowed ranges of $m_{1/2}$. The asymmetry of the thermally averaged cross section implies that the range to the right of the minimum is very narrow; indeed, the scan used for Fig. 1 often failed to find this region. This explains the rather ragged nature of the thin yellow strip at $m_{1/2} \simeq 142$ GeV and $m_0 < 2$ TeV. On the other hand, if the minimum of the relic density falls in the range \(1\), a single allowed range of $m_{1/2}$ results. The depth of this minimum is determined essentially by the strength of the $h\tilde{\chi}_1^0\tilde{\chi}_1^0$ coupling. Note that this coupling requires higgsino–gaugino mixing, which generally is suppressed if $|\mu|^2 \gg M_Z^2$. Expanding to first order in small quantities, one finds that the coupling of the LSP, which is Bino–like required to obtain the right relic density if one happens to sit at the minimum of one of the curves in Fig. 2, assuming the value of $\Omega_{\tilde{\chi}_1^0} h^2$ in the minimum falls in the range \(1\), since here the derivative $d(\Omega_{\tilde{\chi}_1^0} h^2)/dm_{1/2}$ vanishes. This is rather counter–intuitive, given that the $h$–pole region only extend over a narrow range in $m_{1/2}$. 
Figure 2: The lightest neutralino relic density $\Omega_{\tilde{\chi}_1^0} h^2$ as a function of $m_{1/2}$ in four mSUGRA scenarios: i) $m_t = 178$ GeV, $m_0 = 1.5$ TeV, $A_0 = -1$ TeV, $\tan \beta = 30$ leading to $m_h \approx 117$ GeV; ii) $m_t = 182$ GeV, $m_0 = -A_0 = 1$ TeV, $\tan \beta = 10$ leading to $m_h \approx 115$ GeV; iii) $m_t = 185$ GeV, $m_0 = 1$ TeV, $A_0 = 0$, $\tan \beta = 20$ leading to $m_h \approx 116$ GeV; iv) $m_t = 174$ GeV, $m_0 = -A_0 = 1.5$ TeV, $\tan \beta = 30$ leading to $m_h \approx 116$ GeV. In all cases $\mu > 0$ is assumed. The area favored by WMAP, $0.087 \leq \Omega_{\tilde{\chi}_1^0} h^2 \leq 0.138$, is also indicated.

here, to the light CP–even Higgs boson scales like

$$g_{h\tilde{\chi}_1^0\tilde{\chi}_1^0} \propto \frac{M_Z (2 \mu \cos \beta + M_1)}{\mu^2 - M_1^2},$$

where we have assumed $\sin \beta \approx 1$ and $M_1 \cos \beta \ll |\mu|$, $M_1 \approx 0.4 m_{1/2}$ being the soft breaking Bino mass. For the examples shown in Fig. 2, $\mu \sim 400$ to 600 GeV $\gg M_1$. Eq. (5) then shows that the $h\tilde{\chi}_1^0\tilde{\chi}_1^0$ coupling decreases with increasing $\tan \beta$ and increasing $|\mu|$. In turn, $|\mu|$ increases with increasing $m_t$, increasing $|A_0|$ (if $A_0 < 0$ or $A_0 > m_{1/2}$), and increasing $m_0$. This latter behavior explains why the two yellow strips in Fig. 1 merge into one if $m_0 \geq 2.1$ TeV.

The dependence on $m_0$ and $A_0$ is further explored in Figs. 3. The region where the LSP relic density falls in the range $[1]$ is again indicated in yellow; in between these regions $\Omega_{\tilde{\chi}_1^0} h^2$ is too small. We again see that the (minimum of the) relic density in the $h$–pole region falls with increasing $m_0$ and increasing $\tan \beta$, leading to a merging of the two yellow strips at $\tan \beta = 30$, $m_0 \geq 1.8$ TeV. The slope of the right yellow strip comes about since the increase of $m_h$ with increasing $m_0$ has to be compensated by a reduction of $A_0$, in order to keep the difference $m_h - 2 m_{\tilde{\chi}_1^0}$ approximately constant.
Figure 3: Constraints on the $A_0 - m_0$ mSUGRA parameter space for $m_{1/2} = 140$ GeV, $\mu > 0$, $\tan \beta = 10$ (left) and 30 (right); the top mass is fixed to $m_t = 178$ GeV. The notation is as in Fig. 1 but the $b \to s\gamma$ constraint is partly covered by the Higgs mass constraint. Note that between the red and violet areas, there is an area in which $111 \text{ GeV} < m_h < 114 \text{ GeV}$ corresponding to the assumed 3 GeV theoretical error on the calculation of $m_h$.

Note also that both $m_h$ and $m_{\tilde{\chi}_1^0}$ fall if $m_0$ is kept fixed and $A_0$ is reduced from large negative to large positive values. Increasing $A_0$ from large negative values means both a reduction of $L - R$ mixing in the $\tilde{t}$ sector and a reduction of the DR top mass, both of which reduce the corrections to $m_h$; $\tilde{t}$ mixing reaches a minimum at $A_0 \sim m_{1/2}$, but the corrected top mass keeps decreasing. In turn, the LSP mass is reduced by gaugino–higgsino mixing if $\mu$ is reduced; $\mu$ also reaches a minimum near $A_0 \sim m_{1/2}$. In addition the two–loop RGE for $M_1$ contains a term which reduces (increases) the weak–scale Bino mass for positive (negative) $A_0$. We see that for both $m_h$ and $m_{\tilde{\chi}_1^0}$ two effects contribute with equal sign as $A_0$ is increased from large negative values to $A_0 \sim m_{1/2}$, while the two effects tend to compensate each other if $A_0$ is increased further. The overall $A_0$ dependence turns out to be somewhat faster for the LSP mass, so that the crucial difference $m_h - 2m_{\tilde{\chi}_1^0}$ increases from slightly negative to significantly positive values as $A_0$ is increased. The left yellow strips in Figs. 3 therefore correspond to the solution to the right of the minimum in Fig. 2; the
extremely strong dependence of $\Omega_{\tilde{\chi}_1^0}$ on $m_h - 2m_{\tilde{\chi}_1^0}$ in this region explains why the left strip is significantly narrower than the right one. We note that, in addition to the two-loop terms in the RGE for the gaugino masses, the finite (threshold) corrections to these masses [18] are also important here; without them, significant parts of the parameter space shown in Figs. 3 would be excluded by the LEP chargino searches.

3 Phenomenology of the $h$–pole region

The results of the previous section indicate that $h$–exchange can lead to an acceptable LSP relic density for quite wide ranges of $m_0$ and $A_0$, whereas $m_{1/2}$ is constrained to be close to 140 GeV. In this section we discuss the phenomenology of this region of parameter space.

To this end we first quantify the upper and lower bounds on sparticle and Higgs boson masses in this region. To be conservative, we again allow a 3 GeV theoretical uncertainty in $m_h$ when interpreting the LEP Higgs search limits. Moreover, we allow $m_t$ to lie anywhere between 171 and 185 GeV; this corresponds to the current 90% CL range [13].

In view of the theoretical uncertainty of the SM prediction for $g_\mu - 2$, as well as the strong model dependence of the prediction for $B(b \to s\gamma)$, we performed two different scans, as shown in the second and third column of the Table. The first scan employs our standard set of constraints, including the requirements (3) and (4). In contrast, the second scan requires a positive MSSM contribution to $a_{\mu}$,

$$1.06 \cdot 10^{-9} \leq a_{\mu,\text{SUSY}} \leq 4.36 \cdot 10^{-9},$$

(6)
corresponding to the 90% CL allowed region when only using data from $e^+e^-$ annihilation into hadrons for the evaluation of the SM contribution. In the $h$–pole region scenarios with such a large $a_{\mu,\text{SUSY}}$ and sufficiently heavy Higgs spectrum can be found only for $\tan \beta \gtrsim 15$ and not too large $m_0$. The combination of rather small squark masses and large $\tan \beta$ leads to large $\tilde{t} - \tilde{\chi}_\pm$ loop contributions to $B(b \to s\gamma)$. In fact, in our minimal model, i.e. for strict squark mass universality and no flavor mixing at scale $M_X$ and a value of $V_{ts}$ essentially equal to that in the SM, the requirements (3) and (4) are incompatible in the entire $h$–pole region. In our second scan we have therefore ignored the constraint (4); as discussed in Sec. 2, this is justified if some $\tilde{b} - \tilde{s}$ mixing is present, or if significantly different values of $|V_{ts}|$ turn out to be allowed in the context of mSUGRA.

We see that the first set of constraints favors a rather heavy sfermion and Higgs spectrum (with the exception of the light CP–even scalar $h$, of course). The reason is that large values of $m_0$ are needed to satisfy both the LEP Higgs and $b \to s\gamma$ limits, given that $m_{1/2}$ is quite small in the $h$–pole region. In fact, sfermion and heavy Higgs boson masses can exceed 3 TeV, the upper end of the range we scanned; the upper bounds on their masses are then

9
| Quantity          | Range I          | Range II         |
|------------------|------------------|------------------|
| $m_{\tilde{\ell}_R} \simeq m_{\tilde{\mu}_R}$ [GeV] | [708, –]         | [299, 1300]      |
| $m_{\tilde{\ell}_L} \simeq m_{\tilde{\mu}_L}$ [GeV] | [713, –]         | [311, 1300]      |
| $m_{\tilde{\tau}_1}$ [GeV]                | [627, –]         | [98.8, 934]      |
| $m_{\tilde{\tau}_2}$ [GeV]                | [714, –]         | [306, 1130]      |
| $m_{\tilde{\nu}_\tau}$ [GeV]              | [708, –]         | [281, 1130]      |
| $m_{\tilde{\chi}_1^\pm}$ [GeV]            | [105, 122]       | [105, 115]       |
| $m_{\chi_2^0}$ [GeV]                       | [279, 1820]      | [276, 580]       |
| $m_{\tilde{\chi}_1^0}$ [GeV]              | [52.9, 60.7]     | [53.4, 58.4]     |
| $m_{\tilde{\chi}_2^0}$ [GeV]              | [105, 122]       | [105, 115]       |
| $m_{\tilde{\chi}_3^0}$ [GeV]              | [280, 1820]      | [280, 574]       |
| $m_{\tilde{\chi}_4^0}$ [GeV]              | [293, 1820]      | [294, 578]       |
| $m_{\tilde{\ell}_R} \simeq m_{\tilde{\mu}_R}$ [GeV] | [383, 482]       | [365, 433]       |
| $m_{\tilde{\ell}_L} \simeq m_{\tilde{\mu}_L}$ [GeV] | [774, –]         | [431, 1340]      |
| $m_{\tilde{d}_1}$ [GeV]                    | [607, –]         | [302, 920]       |
| $m_{\tilde{d}_2}$ [GeV]                    | [772, –]         | [408, 1030]      |
| $m_{\tilde{b}_1}$ [GeV]                    | [110, –]         | [102, 791]       |
| $m_{\tilde{b}_2}$ [GeV]                    | [645, –]         | [417, 930]       |
| $m_{h}$ [GeV]                               | [114, 122]       | [114, 119]       |
| $m_{H}$ [GeV]                               | [228, –]         | [216, 825]       |
| $m_{H^\pm}$ [GeV]                          | [246, –]         | [234, 830]       |
| $\sigma(\tilde{\chi}_1^0p \rightarrow \tilde{\chi}_1^0p)$ [pb] | [3.1 \cdot 10^{-11}, 1.4 \cdot 10^{-7}] | [6.6 \cdot 10^{-10}, 2.0 \cdot 10^{-7}] |

Table 1: Allowed ranges of sparticle and Higgs masses and of the LSP–proton scattering cross sections in the $h$–pole region of mSUGRA under two different sets of assumptions. Range I is based on the loose $g_\mu – 2$ constraint and also uses the $b \rightarrow s\gamma$ constraint, whereas Range II is based on the more aggressive $g_\mu – 2$ constraint but does not impose any constraint on $b \rightarrow s\gamma$ decays. A dash (–) means that values well in excess of 3 TeV are possible, which we consider to be quite unnatural.

essentially set by finetuning arguments. The upper bounds on the masses of the higgsino–like states $\tilde{\chi}_{3,4}^0$ and $\tilde{\chi}_2^\pm$ might also be increased if values of $m_0 > 3$ TeV are permitted. In contrast, the lower bound on the mass of the lighter chargino is set by searches at LEP. Due to gaugino mass unification, this implies lower bounds on the mass of the (Bino–like) LSP, the (Wino–like) second neutralino, and the gluino. The upper limits on all these masses are set by the requirement that $h$–exchange leads to an acceptable LSP relic density; we saw in the previous Section that this is possible only if $m_{1/2}$ is around 140 GeV. This restriction, as well as the limits on $|A_0|$ that follow from the $b \rightarrow s\gamma$ constraint, also reduce the upper bound.

6The upper bound on $m_h$ is that given by SUSPECT. If we allow an increase by 3 GeV to reflect the theoretical uncertainty [25], the upper limits on $m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0} \simeq m_{\tilde{\chi}_2^0}$ and $m_{\tilde{\ell}}$ would increase by about 1.5, 3 and 9 GeV, respectively.
on $m_h$ significantly, relative to general mSUGRA (without Dark Matter constraint) [22].

Due to the large sfermion masses, prospects of Tevatron experiments to probe such a scenario are not very good [33]. The gluino mass is above the range that can be probed in inclusive missing $E_T$ searches at Run 2. The cross section for $\tilde{\chi}_2^0 \tilde{\chi}_1^\pm$ production would exceed 100 fb; however, the branching ratio of $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \ell^+ \ell^-$ decays ($\ell = e$ or $\mu$) would be at best 6% (for very large $m_0$), and often smaller. Even here one would need $\sim 2$ fb$^{-1}$ of accumulated luminosity to exclude the model at 99% CL; a $5\sigma$ discovery would need even higher luminosity [17, 33, 34]. Similarly, with the currently foreseen integrated luminosity, Tevatron Higgs searches might exclude some of the range of $m_h$ shown in the Table, but a $5\sigma$ discovery seems unlikely. The best hope might therefore be searches for $\tilde{t}_1$ production, either in pairs or from top quark decays. However, this can only probe part of the parameter space with $\tan \beta \lesssim 5$; at larger $\tan \beta$ the lower bound on $m_{\tilde{t}_1}$ lies well above 200 GeV, largely because of the $b \rightarrow s\gamma$ constraint.

In contrast, the cross section for gluino pair production at the LHC [35] would be guaranteed to exceed 10 pb, leading to more than $10^5$ gluino pair events per year even at low luminosity. The cross section for $\tilde{\chi}_1^\pm \tilde{\chi}_1^\mp$ production would exceed 1 pb, so over most of the parameter space these particles should be detectable, in principle, both directly and in $\tilde{g}$ decays (a detailed analysis is, however, required to assess to which extent this can be done). However, slepton searches seem hopeless even at the LHC, and the searches for heavy Higgs bosons would be promising only for $\tan \beta \gtrsim 50$; note that we didn’t find any solutions with $\tan \beta > 53$, whereas values as large as 60 are allowed in other mSUGRA scenarios. Finally, the cross section for the production of a first generation squark together with a gluino is sizable over much of the parameter space; however, it is not clear whether it will be detectable on top of the large “background” from gluino pair production. Moreover, detection of the heavier, higgsino–like neutralino and chargino states would be very difficult.

In this scenario sfermions would also be too heavy to be produced at the next linear $e^+e^-$ collider [36], now called ILC for International Linear Collider, again with the possible exception of $\tilde{t}_1$. Searches for heavy Higgs bosons would here also only be able to probe parts of the parameter space with $\tan \beta \gtrsim 50$. In contrast, discovery of $\tilde{\chi}_1^\pm$ pair production would be guaranteed already at center–of–mass energy $\sqrt{s} = 300$ GeV, where associate $\tilde{\chi}_1^\mp \tilde{\chi}_2^0$ production should also be detectable over much of the parameter space. The very large cross section for $Zh$ production at this energy might also allow to detect invisible $h \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$ there; this would be a “smoking gun” signature for our scenario. However, with a rough analysis using the program HDECAY [37], we find that in the allowed region of parameter space the branching ratio for this decay never exceeds 1%. At larger $\sqrt{s}$ the production of the heavier chargino and neutralino states together with one of the light states should be feasible for a significant fraction of the parameter space.
We also show the range of the elastic $\tilde{\chi}_1^0$ proton scattering cross section from spin-independent interactions [38]. This cross section essentially determines if relic neutralinos will be detectable in direct Dark Matter search experiments. We find cross sections well below current sensitivity.\footnote{In the relevant region $m_0 \gg m_{1,2}$ we find somewhat smaller cross sections than those reported in [39]. This is probably due to the somewhat larger values of $|\mu|$ predicted by SUSPECT relative to ISASUSY. The precise calculation of $|\mu|$ in this region of parameter space is notoriously difficult [40].} The upper end of this range can be probed in the near future; note that our LSP mass is close to the value where current experiments have maximal sensitivity (for given cross section). Unfortunately such a relatively large cross section is only possible at large $\tan \beta$; for example, in the region $\tan \beta \leq 30$, we find $\sigma(\tilde{\chi}_1^0 p \to \tilde{\chi}_1^0 p) \leq 1.3 \cdot 10^{-9}$ pb, which is close to the limit of sensitivity even for experiments of the next–to–next generation [41].

The third column in the Table shows that ignoring the constraint on $B(b \to s\gamma)$ dramatically reduces the lower bounds on sfermion masses in the $h$–pole region. The reason is that much larger values of $|A_0|$ are now allowed, constrained mostly by the required absence of weak–scale minima of the scalar potential where charge and/or color are broken [42], leading to a much reduced lower bound on $m_0$. At the same time, the requirement of a significant positive contribution to $a_\mu$ from sparticle loops imposes significant upper bounds on the masses of all superparticles and Higgs bosons.\footnote{In a general MSSM the requirement of a positive, nonzero $a_\mu_{\text{SUSY}}$ imposes upper bounds on both gaugino and slepton masses [43]. In the context of mSUGRA this translates into upper bounds on both $m_0$ and $m_{1/2}$, and hence on the masses of all new (s)particles. Of course, these constraints became strengthened when focusing on the $h$–pole region of interest to us.} The upper bound on $m_0$ also leads to a reduced upper bound on $m_h$. This leads to a reduction of the upper bounds on the masses of all gaugino–like states; the reduced loop corrections to these masses, due to reduced sfermion masses, also play a role here.

Unfortunately the reduced slepton masses tend to reduce the leptonic branching ratios of $\tilde{\chi}_2^0$, making the detection of $\tilde{\chi}_1^0 \tilde{\chi}_2^0$ production at hadron colliders even more difficult. On the other hand, for some part of the parameter space slepton pair production will now be possible at the ILC; squark production should be detectable quite easily at the LHC; and the associate production of a light, gaugino–like neutralino or chargino together with a heavy, higgsino–like state should also be detectable over the entire allowed parameter space at the second stage of the ILC operating at $\sqrt{s} \gtrsim 750$ GeV. Note also that the lower bound on the LSP–proton scattering cross section has increased by more than a factor 20, relative to the range derived from the first set of constraints.

4 Summary and Conclusions

We have discussed the $h$–pole region as cosmologically viable region of mSUGRA parameter space, in addition to the bulk, $\tilde{\tau}$ co–annihilation, focus point and $A$–pole regions. Here $\tilde{\chi}_1^0$ is
the LSP and annihilates efficiently through the exchange of a nearly on–shell light CP–even Higgs boson $h$. The resurrection of this region is due to a larger central value for the top mass, as well as the calculation of additional loop corrections to $m_h$ and to the masses of the light gaugino–like states.

We saw that this region extends over large ranges of $m_0$ and $A_0$, but requires $m_{1/2} < 145$ GeV. This leads to strong upper limits on the masses of all gaugino–like sparticles, in particular $m_{\tilde{\chi}_1^\pm} \simeq m_{\tilde{\chi}_2^0} < 125$ GeV and $m_{\tilde{g}} < 500$ GeV. Discovery of some superparticles would therefore be trivial at the LHC or at the ILC; the best bet for the Tevatron would be searches for light $\tilde{t}_1$ production, which can however only probe a small fraction of the allowed parameter space. Moreover, sfermions, higgsino–like charginos and neutralinos as well as the heavy Higgs bosons could all be beyond the reach even of CLIC, if the supersymmetric contribution to the anomalous magnetic moment of the muon is small, as indicated by SM predictions for this quantity based on $\tau$ decay data. On the other hand, if a sizable supersymmetric contribution is required, as indicated by SM analyses based on $e^+e^- \rightarrow$ hadrons data only, squarks and the higgsino–like states should also be in easy striking range of the LHC and the second stage of the ILC, respectively.

We conclude that the $h$–pole region will soon be covered by sparticle searches at the LHC. This distinguishes it from the $A$–pole, focus point/hyperbolical branch and $\bar{\tau}$ co–annihilation regions, which are difficult to probe comprehensively at the LHC or ILC. If light charginos and gluinos as well as a light $h$ boson are found at the LHC, one will have to measure $m_{\tilde{\chi}_1^0}$ and $m_h$ very precisely, in order to check whether $h$–exchange can indeed lead to the correct $\tilde{\chi}_1^0$ relic density; some information on the $h\tilde{\chi}_1^0\tilde{\chi}_1^0$ coupling will also be required. These measurements will probably be difficult at the LHC, but should be straightforward at the ILC.

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