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Characterisation of minimal-span plane Couette turbulence with pressure gradients

Atsushi Sekimoto1, Callum Atkinson1, Julio Soria1,2
1 Laboratory for Turbulence Research in Aerospace and Combustion, Department of Mechanical and Aerospace Engineering, Monash University, Clayton 3800, AUSTRALIA
2 Department of Aeronautical Engineering, King Abdulaziz University Jeddah 21589, KINGDOM OF SAUDI ARABIA
† Present address: Department of Materials Engineering Science, Osaka University, Osaka 560-8531, Japan
E-mail: asekimoto@cheng.es.osaka-u.ac.jp

Abstract. The turbulence statistics and dynamics in the spanwise-minimal plane Couette flow with pressure gradients, so-called, Couette-Poiseuille (C-P) flow, are investigated using direct numerical simulation. The large-scale motion is limited in the spanwise box dimension as in the minimal-span channel turbulence of Flores & Jiménez (Phys. Fluids, vol. 22, 2010, 071704). The effect of the top wall, where normal pressure-driven Poiseuille flow is realised, is distinguished from the events on the bottom wall, where the pressure gradient results in mild or almost-zero wall-shear stress. A proper scaling of turbulence statistics in minimal-span C-P flows is presented. Also the ‘shear-less’ wall-bounded turbulence, where the Corrsin shear parameter is very weak compared to normal wall-bounded turbulence, represents local separation, which is also observed as spanwise streaks of reversed flow in full-size plane C-P turbulence. The local separation is a multi-scale event, which grows up to the order of the channel height even in the minimal-span geometry.

1. Introduction
The adverse-pressure-gradient (APG) effect on the turbulent boundary layer (TBL) has been a challenging problem for a long time, since the scaling law for the mean velocity profile and turbulence statistics is not fully understood. Increasing the computational resources would pave the way to understand the problem and for the development of a better turbulence model in complex geometries. Many studies have dealt with the self-similar aspect of an APG-TBL, in the sense that each of the terms in the governing equation has the same proportionality with streamwise position [1, 2, 3]. However, since the boundary layer with a strong APG grows rapidly, direct numerical simulation (DNS) of APG-TBL requires a very large computational domain with artificial inlet and outlet boundary conditions, so that the realisation of a healthy self-similar region is still limited [4].

In order to simplify the canonical problem of self-similar APG-TBLs, the present study investigates plane Couette flow with a pressure gradient, the so-called Couette-Poiseuille (C-P) flow, to achieve a zero-wall-shear stress on the stationary wall [5, 6]. As the pressure gradient is increased, the APG-TBL flow approaches that of a free-shear flow, and most of the large-scale coherent structures tend to be detached from the wall [7]. However, recent studies on
the coherent structures in plane channel flow turbulence \[8\] and in a statistically-stationary homogeneous shear turbulence (SS-HST) \[9, 10\] have revealed that the statistical structure of intense Reynolds stress regions is common whether eddies are attached or detached \[11\]. The effects of pressure gradients on the turbulence structures in plane channel flow can be similar to APG-TBLs.

Here, we consider a minimal-span geometry where the spanwise box dimension \(L_z\) determines the largest length scale \[12\]. Since the effects of large-scale motions of C-P flow in a full size simulation \[6\] are not fully understood, these effects are artificially filtered in the present study. Johnstone et al. \[5\] have reported that the mean turbulence statistics in the minimal-span simulations are quite different from those in the full simulation case. The effect of the wall-surface manipulation has also been investigated recently by MacDonald et al. \[13\]. They have shown that the roughness functions can be computed from the minimal-span simulations. Therefore, the basic near-wall mechanism and turbulence statistics in a minimal-span C-P can also provide useful information towards understanding the scaling of the ‘shear-less’ wall-bounded turbulence.

The aim of the present study is to characterise the minimal-span C-P flows and analyse them. At first, the numerical scheme is briefly introduced and the effect of the pressure gradient on a ‘pure’ Couette flow is presented in \$2\). The minimal-span C-P flows are characterised in \$3\), where the box dependency and the pressure gradient effect on the statistics are also discussed. In \$4\), the instantaneous reversed flow region over ‘shear-less’ walls is investigated, and the conclusion is provided in \$5\).

2. Numerical methodology
2.1. Governing equations and numerical schemes
We consider the fluid motion between two walls as in Fig. 1(a): the top wall is moving in the streamwise direction, while the bottom wall is stationary. The governing equations are the Navier–Stokes and continuity equations,

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nabla \cdot (\nu \nabla \mathbf{u}) + \mathbf{G}, \quad \nabla \cdot \mathbf{u} = 0, \tag{1}
\]

where \(\mathbf{u} = (u, v, w)\) is the total instantaneous velocity in the streamwise \((x)\), wall-normal \((y)\) and spanwise \((z)\) directions, \(t\) is time, \(p\) is the pressure divided by the fluid density, which is unity throughout the paper, and \(\nu\) is the kinematic viscosity. The frame of reference is the velocity of the bottom wall, \(u|_{y=0} = 0\), and the velocity of the top wall \((y = \delta)\) is \(u|_{y=\delta} = U_w\). The boundary condition is no-slip for the walls \((y = 0, \text{ and } y = \delta)\) and periodic in the homogeneous directions \((x, z)\).

The time-dependent spatially-invariant forcing term, \(\mathbf{G} = G_x(t)i_x\), is introduced to fix the mean bulk velocity \[14\],

\[
u_b \equiv \frac{1}{\delta} \int_0^{\delta} U(y)dy = \text{const.}, \tag{2}
\]

where \(U(y) = \langle u \rangle\), and \(\langle \cdot \rangle\) is the velocity averaged over time and the homogeneous directions. The plane averaged values are defined as \(\langle \cdot \rangle_{xx}\), and \(\langle \cdot \rangle'\) is used for the turbulence intensities, like the root-mean-square values, \(u'\). The corresponding instantaneous vorticities are \(\omega = (\omega_x, \omega_y, \omega_z)\).

Eqs. (1) are reduced to the evolution equations for \(v, \omega_y\) and the mean velocities, \(\langle u \rangle_{xx}\) and \(\langle w \rangle_{zz}\) \[15\], and the time integration is performed using the third-order Runge–Kutta scheme of Sekimoto et al. \[9\] with the modification of the viscous term being semi-implicit. Fourier–Chebyshev spectral transformations are applied to the variables,

\[
\mathbf{u} = \sum_{k=0}^{N_y} \sum_{l=-N_x/2}^{N_x/2-1} \sum_{m=-N_z/2}^{N_z/2-1} \hat{\mathbf{u}}_{k,l,m} \exp(i(\alpha lx + \beta mz))T_k(y),
\]

\[
\mathbf{u}_j = \sum_{k=0}^{N_y} \sum_{l=-N_x/2}^{N_x/2-1} \sum_{m=-N_z/2}^{N_z/2-1} \hat{\mathbf{u}}_{k,l,m} J_k^{(j)}(\beta_mz) \exp(i(\alpha_l x)),
\]
Figure 1. (a) The computational domain. (b) The schematic picture of the total Reynolds stress. See the text for the definition of variables.

where $N_x$ and $N_z$ are the number of Fourier modes in the homogeneous directions after $2/3$ dealiasing, $k,l,m$ are integers, $\hat{u}_{k,l,m}$ are the Fourier–Chebyshev coefficients, $\alpha, \beta = (2\pi/L_x, 2\pi/L_z)$ are the fundamental wavenumbers, and $T_k$ are the Chebyshev polynomials. The corresponding 2nd- and 4th-order Helmholtz equations of the coefficients ($\omega_y$ and $\psi$) are solved using the Chebyshev–Legendre–Galerkin method [16], where the boundary condition in the wall-normal direction is imposed using basis functions based on Legendre polynomials. The basis functions which satisfy the boundary conditions for $\omega_y$ and $\psi$ are, respectively,

\[
\{\Psi_{\omega}\}_k = L_k - L_{k+2} \quad (k = 0, 1, ..., N_y - 2),
\]

and

\[
\{\Psi_{\psi}\}_k = d_k^{-1} (L_k + a_k L_{k+2} + b_k L_{k+4}) \quad (k = 0, 1, ..., N_y - 4),
\]

where

\[
a_k = -\frac{2(2k + 5)}{2k + 7}, \quad b_k = \frac{2k + 3}{2k + 7}, \quad d_k = \sqrt{2(2k + 3)^2(2k + 5)},
\]

and $N_y$ is the number of collocation points in $y$. The spatial derivatives are computed using Chebyshev polynomials, and this scheme requires Chebyshev-Legendre transformations before solving the Helmholtz equations. After solving them, the coefficients of the basis functions are passed into the Legendre coefficients by the inverse Chebyshev-Legendre transformations. This scheme has the advantages of both the Legendre– and Chebyshev–Galerkin methods [16]. The snapshots are stored in the Fourier ($x$–Fourier ($z$–Chebyshev ($y$) space. The DNS data of the plane channel turbulence was used as an initial condition for the present DNSs.

2.2. Nondimensional parameters

In the minimal-span simulations, the largest-scale motion is limited by the spanwise box dimension [12, 9], so that we explicitly determine the largest scale as $L_z$. The box aspect ratios are important parameters for turbulence statistics [12], and here they are $A_x = L_x/L_z$ and $A_y = \delta/L_z$ as in [9].

The global Reynolds number in a full box, $Re_\delta = U_w \delta/\nu$ $(A_y < 1)$, is modified as the minimal-span Reynolds number, $Re_z = U_w L_z/\nu$, for a minimal-span box $(A_y > 1)$. The friction Reynolds numbers are defined respectively for the bottom and top walls as $L_{z,0}^+ = L_{z,0}^+$ and $L_{z,0}^+$, based on the friction velocity, $u_{\tau,0} = \text{sign}(\tau_0)|\tau_0|^{1/2}$ and $u_{\tau,w} = |\tau_w|^{1/2}$, where $\tau_0 = \nu \partial_y U|_{y=0}$ and $\tau_w = \nu \partial_y U|_{y=\delta}$ are the wall-shear stress on the bottom and top wall. Throughout the paper,
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\( (\cdot)_0^+ \) and \( (\cdot)_w^+ \) are values normalised using \( \nu, u_{r,0} \) and \( u_{r,w} \), respectively. When the mean flow has a reverse flow near the bottom wall, we use the negative values of \( \tau_0, u_{r,0} \) and \( L_{z,0}^+ \). Note that \( (\cdot)^+ \) is the average of the top and bottom wall for ‘pure’ Couette flows.

The displacement thickness, \( \delta_D \), is related to the mean bulk velocity in Eq. (2), and is defined in the conventional way analogous to that for a turbulent boundary layer,

\[
\delta_D = \int_0^\delta \left( 1 - \frac{U}{U_w} \right) \, dy = \delta \left( 1 - \frac{u_b}{U_w} \right).
\]

By fixing the mean bulk velocity, the present DNS explicitly sets the displacement thickness. We only consider the negative pressure gradient, \( G_x \), so that \( \beta \) has a reverse flow near the bottom wall, we use the negative values of \( \nu, u_{r,0} \) and \( L_{z,0}^+ \).

The role of the pressure gradient \( G_x \) is to redistribute the Reynolds stresses. By integrating the Reynolds stress equation, from \( y = 0 \) to \( y = \delta \),

\[
\frac{\partial (uv)_{xz}}{\partial y} = \nu \frac{\partial^2 U}{\partial y^2} + G_x,
\]

we have

\[
G_x \delta + (\tau_w - \tau_0) = 0.
\]

The non-dimensional pressure gradient parameter, \( \beta \), can also be calculated as in the turbulent boundary layers with adverse pressure gradients [1],

\[
\beta = \frac{\delta_D}{\tau_0} (-G_x) = \frac{\delta_D}{\delta} \frac{\tau_w - \tau_0}{\tau_0} = \left( 1 - \frac{u_b}{U_w} \right) \frac{\tau_w - \tau_0}{\tau_0}.
\]

To simplify notation, we will use from now on \( \sigma = -(uv)_{xz} \) for the tangential Reynolds stress. As shown in Fig. 1(b), the local total shear stress, \( \tau^* = \sigma + \nu \partial_y U = \tau_0 - G_x y \), can be written as

\[
\tau^* = \tau_0 \left[ 1 + \left( \frac{y}{\delta_D} \right) \beta \right] = \tau_0 \left[ 1 + \left( \frac{y}{L_z} \right) \beta_z \right],
\]

where \( \beta_z = (L_z/\tau_0) (-G_x) \).

The numerical parameters are summarised in Table 1. We consider three cases of zero, mild and strong pressure gradients with different box aspect ratios. For example, ‘CP43m’ represents the Couette-Poiseuille turbulence with a mild pressure gradient in the box \((A_x, A_y) = (4, 3)\), and ‘C20’ is a ‘pure’ Couette turbulence in the full box \((A_x = 2, A_y = 0.637)\). Note that the spanwise box dimension does not necessarily represent the characteristic outer scale in the full simulation, so that the nondimensional pressure gradient parameter, \( \beta = \beta_z (\delta_D/L_z) \), is used to distinguish the intensity of the pressure gradient, i.e. ‘CP30m’ \((\beta_z = 17.7)\) is defined as a full-box simulation with a mild pressure gradient \((\beta \approx 1.8)\).

2.4. The effect of pressure gradients
The effect of the pressure gradient in the full-size simulation is briefly shown using cases C20, CP30m and CP30s. The control parameter \( (u_b/U_w) \) is gradually decreased from 0.5 to 0.33, at which the wall-shear stress at the bottom wall is almost zero. By decreasing the mean bulk velocity, the mean velocity near the bottom wall behaves as in Fig. 2(a, b), similar to a turbulent boundary layer with an adverse pressure gradient [17]. Although, the wall-unit scaling cannot
Table 1. Numerical parameters for the Couette (C) and Couette-Poiseuille (CP) flows; the first and second numbers represent the streamwise and vertical aspect ratios, $A_x$ and $A_y$, respectively; and the last characters, ‘m’ and ‘s’ stand for the ‘mild’ and ‘strong’ pressure gradients.

| case   | $N_x$ | $N_y$ | $N_z$ | $Re_z$ | $A_x$ | $A_y$ | $u_b/U_w$ | $\delta_D/L_z$ | $\beta_z$ | $L_{\tau_0}^+$ | $L_{\tau_w}^+$ |
|--------|-------|-------|-------|--------|-------|-------|-----------|----------------|-----------|----------------|----------------|
| C20    | 340   | 257   | 254   | 70700  | 2.0   | 0.637 | 0.5       | 0.318         | 0         | 1734           | 1735           |
| C21    | 170   | 257   | 126   | 39300  | 2.0   | 1.27  | 0.5       | 0.637         | 0         | 882.8          | 887.5          |
| C22    | 84    | 257   | 62    | 22000  | 2.0   | 2.55  | 0.5       | 1.27          | 0         | 421.6          | 420.3          |
| CP30m  | 512   | 193   | 338   | 90500  | 3.0   | 0.159 | 0.42      | 0.093         | 17.7      | 1678           | 3280           |
| CP30s  | 512   | 193   | 338   | 70900  | 3.0   | 0.159 | 0.325     | 0.107         | 8300      | 90.9           | 3303           |
| CP21s  | 42    | 97    | 40    | 12600  | 2.0   | 1.27  | 0.333     | 0.85          | 1560      | 12.4           | 553.4          |
| CP22s  | 42    | 129   | 40    | 12600  | 2.0   | 1.91  | 0.333     | 1.27          | -33.8     | -60.9          | 493.6          |
| CP23s  | 42    | 193   | 40    | 12600  | 2.0   | 3.0   | 0.333     | 2.0           | -7.0      | -90.1          | 424.1          |
| CP24s  | 42    | 257   | 40    | 12600  | 2.0   | 4.0   | 0.333     | 2.67          | -42.6     | -89.0          | 375.3          |
| CP43m  | 128   | 257   | 40    | 12600  | 4.0   | 3.0   | 0.417     | 1.75          | 1.85      | 132.9          | 334.8          |
| CP53m  | 170   | 257   | 40    | 12600  | 5.0   | 3.0   | 0.417     | 1.75          | 1.81      | 131.7          | 333.7          |
| CP63m  | 170   | 257   | 40    | 12600  | 6.0   | 3.0   | 0.417     | 1.75          | 1.74      | 133.7          | 339.9          |
| CP83m  | 256   | 257   | 40    | 12600  | 8.0   | 3.0   | 0.417     | 1.75          | 1.85      | 131.2          | 335.7          |
| CP33s  | 84    | 257   | 40    | 12600  | 3.0   | 3.0   | 0.37      | 1.89          | 24.6      | 44.2           | 382.3          |
| CP43s  | 128   | 257   | 40    | 12600  | 4.0   | 3.0   | 0.37      | 1.89          | 20.5      | 48.9           | 386.8          |
| CP53s  | 170   | 257   | 40    | 12600  | 5.0   | 3.0   | 0.37      | 1.89          | 21.3      | 47.6           | 383.5          |
| CP63s  | 170   | 257   | 40    | 12600  | 6.0   | 3.0   | 0.37      | 1.89          | 22.7      | 46.0           | 382.0          |
| CP83s  | 256   | 257   | 40    | 12600  | 8.0   | 3.0   | 0.37      | 1.89          | 31.9      | 38.6           | 379.7          |
| CP24m  | 42    | 257   | 40    | 12600  | 2.0   | 4.0   | 0.417     | 2.33          | 1.70      | 108.9          | 304.3          |
| CP44m  | 84    | 257   | 40    | 12600  | 4.0   | 4.0   | 0.417     | 2.33          | 1.61      | 108.8          | 296.8          |
| CP64m  | 128   | 257   | 40    | 12600  | 6.0   | 4.0   | 0.417     | 2.33          | 1.73      | 106.2          | 296.8          |
| CP84m  | 170   | 257   | 40    | 12600  | 8.0   | 4.0   | 0.417     | 2.33          | 1.67      | 108.0          | 299.5          |
| CP24s  | 84    | 257   | 40    | 12600  | 2.0   | 4.0   | 0.37      | 2.52          | -42.9     | -25.8          | 348.6          |
| CP44s  | 170   | 257   | 40    | 12600  | 4.0   | 4.0   | 0.37      | 2.52          | -25.5     | -31.1          | 343.6          |
| CP64s  | 256   | 257   | 40    | 12600  | 6.0   | 4.0   | 0.37      | 2.52          | -35.8     | -28.8          | 342.1          |
| CP84s  | 340   | 257   | 40    | 12600  | 8.0   | 4.0   | 0.37      | 2.52          | -40.3     | -26.2          | 341.3          |

be defined for the ‘shear-less’ case (CP30s), the C-P turbulence with a mild pressure gradient exhibits a logarithmic-like profile as in Fig. 2(b). The logarithmic profile is, however, shifted slightly from the zero-pressure gradient case (C20). This behaviour is also observed in strong APG-TBLs [18]. As shown in Fig. 2(c), the pressure redistributes the tangential Reynolds stress as discussed in the previous section. The inclined profiles of $\sigma$ for CP30m and CP30s happen to cross with the zero-pressure gradient case (C20) at the same wall-normal position, $y/\delta = 0.27$. There is, however, no reason for $\sigma$ to cross at this point, and it is likely to be dependent on the Reynolds number.
Figure 2. (a) Mean velocity profile. (b) The same as (a), but in wall units based on the stationary wall. (c) Tangential Reynolds stress. ——, ‘pure’ Couette turbulence (C20); —·— (in blue), C-P turbulence with mild pressure gradient (CP30m); - - - - (in red), C-P turbulence with strong pressure gradient (CP30s).

3. Minimal-span simulations

3.1. ‘Pure’ Couette turbulence
Here, DNS results of plane Couette turbulence, i.e. without pressure gradient ($u_b/U_w = 0.5$), are presented. For the cases C20, C21, and C22 (see Table 1), the streamwise aspect ratio, $A_x$, is fixed, and the friction Reynolds number, $\delta^+ \approx 1100$, is also adjusted to the same value as Avsarkisov et al. [19] ($A_x = 3.33, A_y = 0.106$). By increasing the vertical box aspect ratio $A_y$, the large-scale motion is constrained by the narrow computational domain, and the turbulence statistics are affected. The effective large length scale is limited by $L_z$, which is the same as the minimal-span plane Poiseuille channel by Flores & Jiménez [12] and also the statistically-stationary homogeneous shear turbulence [9]. The mean velocity profile of C20 does not differ from that of Avsarkisov et al. (not shown). As $A_y$ is increased, the mean velocity profile departs from the logarithmic profile (see Fig. 3(b)), indicating that the net Reynolds number decreases as $A_y$ increases. The departing point is roughly $y_d = L_z/3$ from the present minimal-span Couette turbulence with good agreement with ‘pure’ Poiseuille turbulence [12]. The mean velocity fluctuations are shown in Fig. 3(c) for the full simulation [19] and the present minimal-span cases (C20, C22). There are differences from the buffer to the outer layer. As the net Reynolds number $L_z^+$ is decreased, the buffer layer peaks of $u'w$ and $w'u$ become less marked.

3.2. Box dependency
The effect of the aspect ratios of the box, $A_x$ and $A_y$ is investigated in this section. In the full simulation, CP30s in Table 1, an almost zero wall-shear stress at the bottom wall has been realised by using $u_b/U_w = 0.33$. The cases of CP21s, CP22s, CP23s and CP24s in Table 1 are preliminary tests to investigate the $A_y$-dependency, and it appears that the net pressure gradient parameter, $\beta_z$, decreases as $A_y$ is increased. Reverse flows are observed for $A_y > 2$ near the bottom wall as shown in Fig. 4(a). Here, the mean velocity is scaled using $U_w/A_y$ for clarity. It turns out this reference velocity scales well the velocity fluctuations and the tangential Reynolds stress in the minimal-span simulations (see in Fig. 4 b). It is confirmed that the reversed flows are observed even at higher Reynolds numbers with the same aspect ratios and $u_b/U_w$ (not shown), indicating that $\beta_z$ (or $\beta$) is independent of the Reynolds numbers, at least when the turbulence is fully-developed. As it is apparent from Eq. (9), the nondimensional pressure gradient parameter is dependent only on the boundary condition ($u_b/U_w$) and the ratio of the wall shear stresses of the bottom and top walls. The tangential Reynolds stress is not significantly affected by the
Figure 3. (a) Mean velocity for the ‘pure’ Couette turbulence, C20, C21 and C22. (b) The same as (a), but in wall units. The dashed lines are \( u^+ = y^+ \) and \( u^+ = \kappa^{-1} \ln y^+ + 5.1 \) (\( \kappa = 0.41 \)). The vertical dash-dotted lines are \( y = L_z/3 \) for C21 and C22. (c) The velocity fluctuations for the reference [19] (- - - -), C20 (———), C22 (——): (blue lines) \( u'^+ \); (black lines) \( v'^+ \); (red lines) \( w'^+ \). The arrows are in the sense of increasing \( A_y \).

Figure 4. (a) Mean velocity. (b) Tangential Reynolds stress: - - - -(red), CP21; —○—, C22; —△—, C23; —▽—, C24.

reversed flow in the range of \( |L_{z,0}^+| < 100 \). It is probably because the minimal flow unit [20] can not fit in this spanwise length, and turbulence structures are not energetic. As \( A_y \) increases, the profile of \( \sigma \) changes roughly in a self-similar way. The effect of the streamwise box aspect ratio, \( A_x \), does not produce significant differences for the minimal-span box simulations with \( A_y = 3, 4 \) (see Table 1). This is true both for the mild and strong pressure gradient cases.

The mean velocity profiles for relatively long-box simulations, CP83m, CP83s, CP84m and CP84s, are shown in Fig. 5(a–c). The wall-unit scaling on the bottom wall is not useful both for the mild and strong pressure gradient cases, while the mean velocity near the top wall is the same as in minimal-span channel turbulence. Note that CP84s has a slightly negative wall-shear stress on the bottom wall, so that the corresponding line is not shown in Fig. 5(b). Going back to the effect of \( A_y \), the mild pressure-gradient cases with \( A_y = 3, 4 \) have the same mean velocity profile near the bottom wall, while the strong cases have slightly shifted profiles as shown in Fig. 5(a). It indicates that the logarithmic-like mean velocity profile is more robust when the minimal flow unit of turbulence is captured, \( |L_{z,0}^+| > 100 \), probably because it has some resilience to the sweeps from the outer layer. Such resilience disappears when \( |L_{z,0}^+| < 100 \), and the near-wall viscous layer is directly affected by the sweep.
Figure 5. Mean velocity for CP84s (- - - , in blue), CP84m (---○---), CP83s (—△—) and C83m (—∇—). (a) In outer units using $U_w/A_y$ and $L_z$. (b,c) In wall units based on the bottom (b) and the top wall (c). The dashed lines are $u^+=y^+$ and $u^+=k^{-1}\ln y^++5.1$ ($k=0.41$).

Figure 6. (a) $u'/(U_w/A_y)$. (b) $\sigma/(U_w/A_y)^2$. (c) $v'/(U_w/A_y)$. (d) $u'/(U_w/A_y)$: - - - -(in blue), CP84s; - -○-, CP84m; —△—, CP83s; and —∇—, C83m.

The low-order turbulence statistics for relatively long-box simulations, CP83m, CP83s, CP84m and CP84s, are shown in Fig. 6(a–d). An interesting feature is that the velocity fluctuations do not decrease linearly near the wall, even when the wall-shear stress linearly approaches zero on the bottom wall (CP83s, CP84s). The origin of these velocity fluctuations is from above the outer layer. In a strong APG-TBL, more sweep events are observed near the
wall than in zero-pressure gradient TBLs [21].

There is a region where $\sigma/q^2$ ($q^2 = u'^2 + v'^2 + w'^2$), is constant [22] for APG-TBLs, and it is true for the present minimal-span C-P turbulence. Fig. 7 shows $u'$ and $v'$ scaled by the local total shear stress, $\tau^*$, which is given in Eq.(10). The constant region for $y/L_z > 1$ indicates that there is a common mechanism which produces turbulence intensity as in the zero-pressure gradient case. The near-wall peaks increase as $\beta$ increases, eventually going to infinity when a reversed flow with negative $\tau^*$ is realised. In Fig. 7(b), the corresponding friction length scale, $l_\tau^* = \nu/\tau^*)^{1/2}$, is used, rather than the conventional friction velocity and length. It appears that the peak positions approach the bottom wall asymptotically as $\beta \to \infty$, while $u^* \approx 1.8$, $v^* \approx w^* \approx 1.4$ for $y/l_\tau^* \gtrsim 100$. These values agree with those at around the centre plane of the zero-pressure gradient case, i.e. minimal-span ‘pure’ Couette flow (see also Fig. 3c), and also with statistically-stationary homogeneous shear turbulence [9].

In the end, the large length scale and local shear are investigated. Fig. 8(a) shows the Corrsin length scale, $L_c = (\varepsilon/(\partial_y U_i)^2)^{1/2}$ [23], where $\varepsilon$ is the dissipation rate. The effect of the pressure gradient is limited near the wall (roughly $y < L_z$), and $L_c/L_z$ is almost constant in the bulk

**Figure 7.** The streamwise and wall-normal velocity fluctuations scaled by the analytical local friction velocity, $u^*_\tau = \tau^*/2$, as functions of (a) $y/L_z$ and (b) $y/l_\tau^*$: --- (in red), C22; - - - - (in blue), CP84s; ---○---, CP84m; —△—, CP83s; and —▽—, C83m. Note that the near-wall profiles for CP84s with $\tau^*/\overline{\tau} < 0.01$, where $\overline{\tau} = (\tau_w - \tau_0)/2$ are truncated to eliminate the singular behaviour ($\tau^* = 0$).

**Figure 8.** (a) The Corrsin length scale, $L_c$. (b) The nondimensional Corrsin shear parameter, $S^*$. Lines are as in Fig. 7.
uniformly sheared region in minimal-span Couette-Poiseuille flows. The ‘pure’ Couette flow has $L_c/L_z \approx 0.1$, which agrees well with that in SS-HST [9], while Couette-Poiseuille have a tendency to decrease roughly by 20–30% from $y/L_z = 1$ to the end of the uniformly sheared region ($1 < y/L_z \lesssim A_y - 1/3$). The mean velocities in the bulk region of C-P flows are not perfectly constant as already shown in Fig. 5, however it is not significant difference. The nondimensional Corrsin shear parameter, $S^* = \partial_y U q^2/\epsilon$, is similar between Couette and C-P flows (Fig. 8). The statistically-homogeneous shear turbulence [9] also represents $S^* \approx 7–8$, which agrees with both ‘pure’ Couette and C-P flows for $y/l_\tau > 100$. As $\beta_z$ approaches $\infty$, the near-wall peak of $S^*$ drastically decreases, and the local shear would not produce active turbulent structures at the buffer layer ($y/l_\tau \approx 10$).

4. Locally reversed flows
Focusing on the ‘shear-less’ wall-bounded turbulence, the dynamics of the region of the reversed flow ($u/U_w < 0$) is investigated. Figure 9 shows reversed flow regions, which are visualised by the isosurface of $u/U_w = 0.0$. In the full C-P simulation (CP20s), there are two typical structures: streamwise elongated large-scale ejections (‘A’ in Fig. 9(a)); and near-wall spanwise reversed flows (‘B’ in Fig. 9(a)). The large-scale motions span the whole channel height and interact with both bottom- and top-wall events. In the minimal-span simulations, such large-scale events are artificially filtered, therefore only the spanwise reversed flow regions exist as
Figure 10. (a, b) The spanwise-averaged wall-shear stress, $\langle \tau_0 \rangle_z/(U_w/A_y)^2$, as a function of time ($t$) and space ($x$) for (a) CP63s and (b) CP84s. (c) P.d.f.s of $\langle \tau_0 \rangle_z(t,x)/(U_w/A_y)^2$ for CP43m (——, in red), CP63s (——, in black), and CP84s (- - - - - , in blue). (d) Time–space correlations of $\langle \tau_0 \rangle_z$ for local $\tau_0/(U_w/A_y)^2$; — o — , CP63s; — △ — , CP84s. The contour levels are [-0.1 0.1 0.3 0.5 0.7] and the filled symbols represent the negative correlation.

shown in Fig 9(b,c). Here, the spanwise-averaged local wall-shear stress, $\langle \tau_0 \rangle_z(t,x)$, reasonably detects such local separation events due to the narrow domains. Some examples of the time–space maps of $\langle \tau_0 \rangle_z$ are shown in Fig. 10(a, b) for strong pressure-gradient cases, CP63s and CP84s. These two cases have almost-zero wall-shear stress but also quite different probability density function of $\tau_0$. Note that the mean $\langle \tau_0 \rangle$ of CP84s is slightly negative. There are intermittent strong negative wall-shear-stress regions, which are observed as the dark blue spots in Fig. 10(a, b) and which make the tails of the p.d.f. wider for CP84s. Such intermittent wall-shear stress can be scaled by $U_w$ rather than $U_w/A_y$, indicating that local separations grows until the order of the box height, $\delta$. Figure 10(d) shows the time–space correlation of $\langle \tau_0 \rangle_z$ for the ‘shear-less’ cases (CP63s, CP84s), exhibiting a good agreement for small-scale separation events of the size of $L_z$. The negative correlation at $\delta x/L_z = 0.5A_y$ also indicate the existence of large-scale separation (or sweep) events. The advection velocity is roughly $0.2U_w$, which is estimated from the slope of the time–space correlation.

5. Conclusion

The minimal-span Couette-Poiseuille (C-P) turbulence has been investigated using newly developed direct numerical simulations (DNSs), targeting ‘shear-less’ wall-bounded turbulence,
as found in adverse-pressure gradient turbulent boundary layer (APG-TBL) flow at the verge of separation. A minimal-span simulation eliminates the upstream history effects, which complicates the analysis in TBLs, and drastically reduces the computational cost. It can be used as a fast method to characterise zero wall-shear (‘shear-less’) wall-bounded turbulence.

Since the large-scale motions are artificially limited by the narrow computational domain, the box-dependency on the Couette-Poiseuille flow has been revealed. The nondimensional numerical parameters are four: the Reynolds number, $Re_z$, box aspect ratios, $(A_x, A_y)$ and the nondimensional pressure gradient parameter $\beta$, which is defined as for APG-TBL flows. The minimal-span ‘pure’ Couette turbulence features a limited logarithmic layer until $y \approx L_z/3$, which agrees with that in the minimal-span channel turbulence [12]. In this minimal Couette flow, the homogeneously sheared region which exists above the limited logarithmic layer is similar to the statistically-stationary homogeneous shear turbulence (SS-HST) [9].

The effects of mild and strong pressure gradient on the turbulence statistics have been investigated. The mean velocity in the minimal-span C-P flow also results in a roughly uniform shear region. The latter is considered to be relevant to the outer layer in the APG-TBLs. As expected, the standard pressure-driven channel flow is realised over the top wall. It has been shown that the almost ‘shear-less’ case still has strong turbulent intensity near the wall, although the tangential Reynolds stress is linearly approaching zero. Such region can be considered as ‘inactive’, since the Corrsin’s nondimensional shear parameter is very small.

It is considered that the sweeps (or bursts when we consider the rotational symmetry) from the upper layer are the primary ingredient to generate such local separation and velocity fluctuations in the ‘inactive’ layer. The dynamics of the local separation events in the minimal-span domain have also been investigated. Although the effect of large-scale motions in a full simulation is eliminated from the dynamics of local separation events, it appears that the reverse flow (separating region) is not limited by the spanwise box dimension, $L_z$. The local reversed flow is mostly attached to the wall and occasionally grows up to the order of the channel height, $\delta$, being advected with the local advection velocity even in the minimal-span simulations. The wall-attached regions of the large-scale reversed flow advect with roughly $0.2U_w$, independently of the aspect ratios, $A_x$ and $A_y$. The local separation events are found to be multi-scale phenomena even in minimal-span simulations, with the effect of large-scale motions in a full simulation also requiring further investigation.

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