The entropy of an acoustic black hole in neo-Newtonian theory

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In this paper we consider the metric of a 2+1-dimensional rotating acoustic black hole in the neo-Newtonian theory and applying the quantum statistical method, we calculate the statistical entropy using a corrected state density due to the generalized uncertainty principle (GUP). In our calculations we have shown that the obtained entropy is finite and correction terms are generated. Moreover, the computation of the entropy for this method does not present logarithmic corrections.

I. INTRODUCTION

The study to understand the entropy of black holes is one of the most important issues in theoretical physics. It has been proposed by Bekenstein and Hawking that the black hole entropy is proportional to its horizon area [1]. Since then, various methods have been proposed in the literature to explore the statistical origin of black holes entropy. One of such methods is the so-called brick-wall method introduced by G. ’t Hooft [2]. To calculate the entropy by this method it is necessary to introduce an ultraviolet cut-off in order to eliminate the divergences in the density of states near the horizon of the black hole. In Ref. [3], by applying the brick-wall method, one has been computed the entropy of an acoustic black hole in (1 + 1) - dimensions.

On the other hand, by considering models in which the Heisenberg uncertainty relation is modified the divergences that arise in the brick-wall model are eliminated [4]. The statistical entropy of various black holes has also been calculated via corrected state density of the GUP [5]. Moreover, considering the effects of the GUP in the tunneling formalism, the quantum-corrected Hawking temperature and entropy of black holes has been investigated [6], [7], [8], [9], [10]. The results have shown that near the horizon quantum state density and its statistical entropy are finite. In [11] a relation for the corrected states density by GUP has been proposed. The authors in [12] applying a new equation of state density due to the GUP [13], analyzed the statistical entropy of a 2+1-dimensional rotating acoustic black hole. It was shown that considering the effect due to the GUP on the equation of state density, no cut-off is needed [14] and the divergence in the brick-wall model disappears.

Since the seminal paper by Unruh [15], the study of analogue models of gravity [16–18] has been an important field to investigate the Hawking radiation as well as to improve the theoretical understanding of quantum gravity. In addition, the study of a relativistic version of acoustic black holes was presented in [19–22]. In the study of analogous models, in general, a classical approach is applied, such as a Newtonian treatment. However, the standard Newtonian approach is valid only for pressureless fluids. Nevertheless, the effect of pressure may suitably be introduced into the dynamics in a Newtonian framework. This is called neo-Newtonian theory which is a modification of the usual Newtonian theory by appropriately considering the effects of pressure.

Once the pressure is generally a parameter that can be easily adjustable and it can play the role of an external field, the neo-Newtonian theory might provide an interesting way to test analog effects, such as the Aharonov-Bohm (AB) effect due to an acoustic geometry of a vortex in the fluid [23–25]. As shown in [26], this indeed happens. The AB phase shift which depends on the pressure is analogous to a magnetic flux. McCrea in [27] deduced the neo-Newtonian equations that were later refined in [28]. In addition, in [29] was obtained a final expression for the equation of fluid considering a perturbative treatment of neo-Newtonian equations (see also [30, 33]). The authors in [34] studied acoustic black holes in the framework of neo-Newtonian hydrodynamics and in [35] was analyzed the effect of neo-Newtonian hydrodynamics on the superresonance phenomenon.

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In this paper, we use the quantum statistical method via corrected state density of the GUP and we calculate the entropy of the rotating acoustic black hole in the neo-Newtonian theory. In our calculations, we have obtained the Bekenstein-Hawking area entropy of acoustic black hole and its correction terms. Again, by considering the GUP on the equation of state density we found that the divergence in the brick-wall model disappears and no cut-off is required. Furthermore, using this method, terms of logarithmic corrections are not generated.

II. ACOUSTIC BLACK HOLES IN NEO-NEWTONIAN HYDRODYNAMICS

In this section we briefly review the neo-Newtonian hydrodynamics and introduce the acoustic black hole metric obtained in [34]. In neo-Newtonian formalism first we redefine the concept of energy density as follows:

$$\rho_i \rightarrow \rho + p.$$ (1)

Thus, the neo-Newtonian equations are given by [31–33]

$$\partial_t \rho_i + \nabla \cdot (\rho_i \vec{v}) + p \nabla \cdot \vec{v} = 0.$$ (2)

$$\dot{\vec{v}} + (\vec{v} \cdot \nabla) \vec{v} = -\frac{\nabla p}{\rho + p},$$ (3)

where $\rho_i$ is the fluid density, $p$ is the pressure and $\vec{v}$ the flow/fluid velocity. The expression (2) is the continuity equation and (3) is the Euler equation modified due to gravitational interaction. In [27] this result has been generalized in the presence of pressure. Moreover, in [28] this approach has been modified and leads to neo-Newtonian cosmology. Note that when $p = 0$ the Newtonian equations are obtained.

At this point, we consider that the fluid is barotropic, i.e. $p = p(\rho)$, inviscid and irrotational, being the equation of state $p = k\rho^n$, with $k$ and $n$ constants. We write the fluid velocity as $\vec{v} = -\nabla \psi$ where $\psi$ is the velocity potential. Thus, we linearise the equations (2) and (3) by perturbing $\rho$, $\vec{v}$ and $\psi$ as follows:

$$\rho = \rho_0 + \varepsilon \rho_1 + 0(\varepsilon^2),$$ (4)

$$\rho^n = \left[\rho_0 + \varepsilon \rho_1 + 0(\varepsilon^2)\right]^n \approx \rho_0^n + n\varepsilon \rho_0^{n-1} \rho_1 + ..., $$ (5)

$$\vec{v} = \vec{v}_0 + \varepsilon \vec{v}_1 + 0(\varepsilon^2),$$ (6)

$$\psi = \psi_0 + \varepsilon \psi_1 + 0(\varepsilon^2),$$ (7)

where $\rho$ is the fluid density, $p$ its pressure and $\vec{v}$ its flow/fluid velocity. Hence, the wave equation becomes [33]

$$-\partial_t \left\{c_s^{-2} \rho_0 \left[\partial_t \psi + \frac{1}{2} + \frac{\gamma}{2}\right] \hat{\vec{v}}_0 \cdot \nabla \psi \right\} + \nabla \cdot \left\{- c_s^{-2} \rho_0 \hat{\vec{v}}_0 \left[\frac{1}{2} + \frac{\gamma}{2}\right] \partial_t \psi + \gamma \hat{\vec{v}}_0 \cdot \nabla \psi \right\} + \rho_0 \nabla \psi = 0,$$ (8)

that can be given as

$$\partial_{\mu}(f^{\mu\nu} \partial_{\nu} \psi) = 0.$$ (9)

The Eq. (9) can also be rewritten as the Klein-Gordon equation for a massless scalar field in a curved (2+1)-dimensional spacetime as follows

$$\frac{1}{\sqrt{-g}} \partial_{\mu}(\sqrt{-g}g^{\mu\nu} \partial_{\nu} \psi) = 0.$$ (10)

So in terms of the inverse of $g^{\mu\nu}$ the effective line element in polar coordinates ($\vec{v} = v_r \hat{\vec{r}} + v_\phi \hat{\vec{\phi}}$ and $d\tilde{r} = d\tilde{r} + rd\phi \hat{\vec{\phi}}$) can be written as

$$ds^2 = \tilde{\rho} \left[-c^2_s - \gamma(v_r^2 + v_\phi^2) \right] dt^2 - (1 + \gamma)(v_r dr + v_\phi r d\phi)dt + (dr^2 + r^2 d\phi^2) + \frac{(\gamma - 1)^2}{4c^2_s} (v_\phi dr - v_r r d\phi)^2, $$ (11)

where $\tilde{\rho} = \sqrt{\rho_0 \left[c^2_s + (v_r^2 + v_\phi^2) \gamma^{-1} \right]^{-1/2}}$ and $\gamma = 1 + kn\rho_0^{-1}$. Now we apply the following coordinate transformations

$$d\tau = dt + \frac{(1 + \gamma)v_r dr}{2(c^2_s - \gamma v_r^2)}; \quad d\varphi = d\phi + \frac{\gamma(1 + \gamma)v_r v_\phi dr}{r(c^2_s - \gamma v_r^2)}.$$(12)
In this way the line element can be written as
\[ ds^2 = \tilde{\rho} \left\{ -\left[c_s^2 - \gamma(v_r^2 + v_\phi^2)\right] d\tau^2 + \frac{c_s^2}{c_s^2 - \gamma v_r^2} \left(1 + \frac{v_r^2 + v_\phi^2}{2c_s^2}\right) d\nu^2 - v_\phi(1 + \gamma) r d\tau d\phi \right\}. \] (13)

Now for a static and position independent density, the flow/fluid velocity (which is a solution obtained from the continuity equation (2)) is given by
\[ \vec{v} = A \hat{r} + B \hat{\phi}, \] (14)
and the velocity potential is
\[ \psi(r, \phi) = -A \ln r - B \phi. \] (15)
Thus, considering \( c_s = 1 \) and substituting (14) into the metric (13) we obtain, up to an irrelevant position-independent factor, the acoustic black hole in neo-Newtonian theory which is given by [26]
\[ ds^2 = \beta_1 \left[ - \left(1 - \tilde{r}_e^2/r^2\right) d\tau^2 + (1 + \beta_2) \left(1 - \frac{\tilde{r}_h^2}{r^2}\right)^{-1} d\nu^2 - \frac{2B\beta_3}{r^2} r dr d\phi + \left(1 + \frac{\beta_4}{r^2}\right) r^2 d\phi^2 \right]. \] (16)
where
\[ \beta_1 = (1 + \beta_2)^{-1/2}, \quad \beta_2 = r^2 \left(\frac{\gamma - 1}{2}\right)^2, \]
\[ \beta_3 = \frac{(1 + \gamma)}{2}, \quad \beta_4 = \left(\frac{A(\gamma - 1)}{2}\right)^2, \] (17)
being \( \tilde{r}_e \) the radius of ergo-region and \( \tilde{r}_h \) the event horizon, i.e.,
\[ \tilde{r}_e = \sqrt{\gamma(A^2 + B^2)} = \sqrt{\gamma r_e}, \quad \tilde{r}_h = \sqrt{\gamma |A|} = \sqrt{\gamma r_h}. \] (19)
Thus, the metric (16) can be now written in the form
\[ g_{\mu\nu} = \beta_1 \begin{bmatrix} -f_1 & 0 & 0 & -B \beta_3 \beta_4 \frac{\gamma - 1}{2} \left(\frac{A(\gamma - 1)}{2}\right)^2 \end{bmatrix}, \]
\[ Q = 1 - \frac{\tilde{r}_h^2}{r^2}. \] (21)
Now we obtain the Hawking temperature of the acoustic black hole as
\[ \tilde{T}_h = \frac{k}{2\pi} = \frac{1}{2\pi r_h} \left[ \frac{\gamma}{1 + \left(\frac{\gamma - 1}{2}\right)^2 \left(1 + \frac{\gamma B^2}{r_h^2}\right)} \right]^{-1/2}. \] (22)
Since the equation of state of the present fluid is \( p = k\rho^n \) then \( \gamma = 1 + kn\rho_0^n = 1 + np_0/\rho_0 \), for \( np_0/\rho_0 \ll 1 \) and so Hawking temperature can be written as follows
\[ \tilde{T}_h = \left[ 1 - \frac{np_0}{2\rho_0} + \frac{1}{4} \left(\frac{np_0}{\rho_0}\right)^2 \right] T_h - \frac{\pi^2 B^2}{2} \left(\frac{np_0}{\rho_0}\right)^2 T_h^3 + \cdots, \] (23)
where \( T_h = 1/2\pi r_h \), while the Unruh temperature for an observer at a distance \( r \) is

\[
T = \frac{a}{4\pi} = \frac{f'(\tilde{r}_h)}{4\pi} F^{-1/2}(r). \tag{24}
\]

They satisfy the following relation

\[
T_h = \sqrt{F(r)} T = \frac{f'(\tilde{r}_h)}{4\pi}. \tag{25}
\]

where \( f(r) = \beta_1 Q \) and

\[
F(r) = -\tilde{g}_{tt} = -\frac{g_{tt} g_{\phi\phi} - \tilde{g}_{\phi\phi}^2}{g_{\phi\phi}} = \beta_1 \left( f_1 + \frac{B^2 \beta_2}{r^2 + \beta_4} \right) = \beta_1 \left[ Q - \frac{B^2}{r^2} \left( \gamma - \frac{\beta_2^2}{1 + \beta_4/r^2} \right) \right]. \tag{26}
\]

### III. THE STATISTICAL ENTROPY

In this section we consider the quantum statistical mechanics via density of states corrected by the GUP to calculate the entropy of a rotating acoustic black hole in neo-Newtonian theory. So, we will consider the following partition function for a system of bosons:

\[
\ln Z_0 = - \sum_i g_i \ln \left( 1 - e^{-\beta \epsilon_i} \right), \tag{27}
\]

Thus, considering the metric (16), the partition function of the system is given by

\[
\ln Z = - \int 2\pi \sqrt{g_{\phi\phi} g_{rr}} dr \sum_i g_i \ln \left( 1 - e^{-\beta \epsilon_i} \right)
= - \int \sqrt{g_{\phi\phi} g_{rr}} dr \int_0^\infty dp \left( pe^{-\lambda p^2} \right) \ln \left( 1 - e^{-\beta \omega} \right)
\approx \int \sqrt{g_{\phi\phi} g_{rr}} dr \int_{m \sqrt{\gamma}}^{\infty} dp \frac{\beta_0 e^{-\lambda p^2} p^2 d\omega}{2 \left( e^{\beta \omega} - 1 \right)}, \tag{28}
\]

here \( \beta = \beta_0 \sqrt{-\tilde{g}_{tt}}, \omega = \omega_0 \sqrt{-\tilde{g}_{tt}} \) and \( -\tilde{g}_{tt} = -\frac{g_{tt} g_{\phi\phi} - \tilde{g}_{\phi\phi}^2}{g_{\phi\phi}} \). Next, we can now determine the free energy of the system as following

\[
F = -\frac{1}{\beta_0} \ln Z = \int \sqrt{g_{\phi\phi} g_{rr}} dr \int_{m \sqrt{\gamma}}^{\infty} \frac{e^{-\lambda p^2} p^2 d\omega}{2 \left( e^{\beta \omega} - 1 \right)}. \tag{29}
\]

Now we can find the entropy of the system in the following way

\[
S = \beta_0^2 \frac{\partial F}{\partial \beta_0} = \beta_0^2 \int \sqrt{g_{\phi\phi} g_{rr}} dr \int_{m \sqrt{\gamma}}^{\infty} \frac{\omega e^{\beta \omega} e^{-\lambda p^2} p^2 d\omega}{2 \left( e^{\beta \omega} - 1 \right)}
= \frac{1}{2} \int \sqrt{g_{\phi\phi} g_{rr}} dr \int_{m \sqrt{\beta}}^{\infty} \frac{x e^x}{(e^x - 1)^2} e^{-\lambda \frac{x^2}{2}} \frac{1}{e^{\beta \omega} - 1} \left( \frac{x^2}{\beta^2} - m^2 \right) dx, \tag{30}
\]

in the equation above we have defined the relationship \( x = \beta \omega_0 = \beta_0 \omega \). Also, we made use of the relation among energy, momentum and mass \( \omega_0^2 = \frac{\beta_0^2}{\gamma} = p^2 + m^2 \), being \( m \) the static mass of particles. Thus, near the horizon, \( \tilde{g}_{tt}(\tilde{r}_h) \to 0 \), we integrate (30) with respect to \( r \)

\[
S = \frac{1}{2} \int \sqrt{g_{\phi\phi} g_{rr}} dr \int_0^{\infty} \frac{x^3 e^x}{\beta^2 (e^x - 1)^2} e^{-\lambda \frac{x^2}{2}} dx = \frac{1}{2\beta_0^2} \int_0^{\infty} \frac{dx}{4 \sinh^2(x/2)} I(x, c), \tag{31}
\]
where

\[
I(x, \epsilon) = \int \frac{\sqrt{g_{\phi\phi}g_{rr}}}{-\tilde{g}_{tt}} x^3 e^{-\lambda x^2/2r^2} dr
= \int \frac{\sqrt{(1 + \beta_2)(1 + \beta_4/r^2)}}{Q} \left[ Q - \frac{B^2}{r^2} \left( \gamma - \frac{\beta_4^2}{1 + \beta_4/r^2} \right) \right]^{-1} x^3 e^{-\lambda x^2/2r^2} \frac{dr}{r^2}.
\]

(32)

Since we only consider the quantum field near the black hole horizon, we take \([\tilde{r}_h, \tilde{r}_h + \epsilon]\) as the integral interval with respect to \(r\), where \(\epsilon\) is a positive small constant. Initially, let us consider the particular case where \(B = 0\). So, when \(r \to \tilde{r}_h\), \(f(r) = \beta_1 Q(r) \approx 2\kappa (r - \tilde{r}_h)\), so we have

\[
I(x, \epsilon) = \sqrt{\frac{\alpha_2}{\alpha_1}} \int_{\tilde{r}_h}^{\tilde{r}_h + \epsilon} \frac{(r - \tilde{r}_h) + \tilde{r}_h}{(2\kappa(r - \tilde{r}_h))^3/2} x^3 e^{-\lambda x^2/[2\kappa(r - \tilde{r}_h)\beta_0^2]} dr,
\]

(33)

where \(\alpha_1 = \sqrt{1 + (\gamma - 1)^2/4}\), \(\alpha_2 = 1 + (\gamma - 1)^2/4\gamma\) and \(\kappa = 2\pi \beta_0^{-1}\) is the surface gravity of the acoustic black hole and by variable substitution \(t = \frac{\lambda x^2}{4\pi(r - \tilde{r}_h)\beta_0}\), we have

\[
I(x, \epsilon) = \sqrt{\frac{\alpha_2}{\alpha_1}} \int_{\delta}^{\epsilon} \left[ \frac{\beta_0 x^4 \sqrt{\lambda}}{(4\pi)^2} t^{-3/2} + \tilde{r}_h \beta_0^2 x^2 4\pi \sqrt{\lambda} t^{-1/2} \right] e^{-t} dt
= \sqrt{\frac{\alpha_2}{\alpha_1}} \left[ \frac{\beta_0 x^4 \sqrt{\lambda}}{(4\pi)^2} \Gamma \left( -\frac{1}{2}, \delta \right) + \tilde{r}_h \beta_0^2 x^2 4\pi \sqrt{\lambda} \Gamma \left( \frac{1}{2}, \delta \right) \right],
\]

(34)

where \(\delta = \frac{x^2}{2\pi \beta_0}\) and \(\Gamma(z) = \int_{\delta}^{\infty} t^{z-1} e^t dt\) is the incomplete Gamma function.

The \(\epsilon\) can be obtained by the smallest length given by generalized uncertainty principle

\[
\Delta X \Delta P = \frac{1}{2} e^{(\Delta P)^2 + (2\gamma)^2},
\]

(35)

and the least uncertainty of location \(\sqrt{\epsilon\lambda/2}\) can be determinate. Now, if we consider it as a least length of pure space line element, we have

\[
\sqrt{\epsilon\lambda/2} = \int_{\tilde{r}_h}^{\tilde{r}_h + \epsilon} \sqrt{g_{rr}} dr \approx \int_{\tilde{r}_h}^{\tilde{r}_h + \epsilon} \frac{dr}{\sqrt{2\kappa(r - \tilde{r}_h)}} = \sqrt{\frac{2\epsilon}{\kappa}}.
\]

(36)

Thus, from (36), we have \(\delta = \frac{x^2}{2\pi \epsilon}\) and

\[
S = \frac{1}{2 \beta_0^2} \sqrt{\frac{\alpha_2}{\alpha_1}} \int_{0}^{\infty} dx \frac{dx}{4 \sinh^2(x/2)} \left[ \frac{\beta_0 x^4 \sqrt{\lambda}}{(4\pi)^2} \Gamma \left( -\frac{1}{2}, \delta \right) + \tilde{r}_h \beta_0^2 x^2 4\pi \sqrt{\lambda} \Gamma \left( \frac{1}{2}, \delta \right) \right],
\]

(37)

with \(x \to 2x\), we have \(\delta = \frac{2x^2}{\pi \epsilon}\) and we obtain

\[
S = \sqrt{\frac{\alpha_2}{\alpha_1}} \left[ \frac{4 \sqrt{\lambda}}{(4\pi)^2} \beta_0 \delta_1 + \frac{\tilde{r}_h}{4\pi \sqrt{\lambda}} \delta_2 \right],
\]

(38)

being

\[
\delta_1 = \int_{0}^{\infty} \frac{x^4}{\sinh^2(x)} \Gamma \left( -\frac{1}{2}, \delta \right) dx, \quad \delta_2 = \int_{0}^{\infty} \frac{x^2}{\sinh^2(x)} \Gamma \left( \frac{1}{2}, \delta \right) dx.
\]

(39)

For \(\sqrt{\lambda} = \delta_2/(2\pi^2)\), we find

\[
S = \frac{1}{4} \sqrt{\frac{\gamma \alpha_2}{\alpha_1}} (2\pi \tilde{r}_h) + \frac{\alpha_2}{\alpha_1} \frac{\delta_1 \delta_2}{8\pi^2} \tilde{r}_h,
\]

(40)
where $2\pi r_h$ is the horizon area of the acoustic black hole. The second term is a correction term to the area entropy and is proportional to the radiation temperature,

$$\tilde{T}_h = T_h \left[ \gamma + \frac{\gamma(\gamma - 1) \delta_1 \delta_2}{2} \right]^{-1/2},$$

(41)

of acoustic black hole. For $\frac{n p_0}{\rho_0} \ll 1$ the entropy becomes

$$S = \frac{1}{4} \left( 1 + \frac{n p_0}{2 \rho_0} + \cdots \right) (2\pi r_h) + \left( 1 - \frac{n p_0}{2 \rho_0} + \cdots \right) \frac{\delta_1 \delta_2}{8\pi^4} T_h.$$  

(42)

Now, for the case $B \neq 0$ we have

$$S = \frac{1}{4} \sqrt{\frac{\alpha_2}{\alpha_1}} (2\pi r_h) + \sqrt{\frac{\alpha_2 \delta_1 \delta_2}{\alpha_1}} \frac{\tilde{T}_h}{8\pi^4},$$

(43)

where

$$\alpha_1 = \sqrt{1 + \frac{(\gamma - 1)^2}{4} \left( 1 + \frac{\gamma B^2}{r_h^2} \right)},$$

(44)

so that for $\frac{n p_0}{\rho_0} \ll 1$ we obtain

$$S = \frac{1}{4} \left( 1 + \frac{n p_0}{2 \rho_0} - \frac{(n p_0)^2}{16 \rho_0^2} \right) (2\pi r_h) - \frac{\pi^2 B^2 \delta_1 \delta_2}{16 \rho_0^2} T_h + \left( 1 - \frac{n p_0}{2 \rho_0} + \frac{5}{16} \frac{(n p_0)^2}{\rho_0^2} \right) \frac{\delta_1 \delta_2}{8\pi^4} T_h$$

$$- \frac{3B^2 \delta_1 \delta_2}{32 \pi^2} \frac{(n p_0)^2}{\rho_0^2} T_h^3 + \cdots.$$  

(45)

Note that for the first term we have obtained corrections to the area of the horizon of the acoustic black hole in neo-Newtonian theory. The second term is a correction term that arises from the contribution due to the presence of pressure and $B$ parameter associated with the rotation of the acoustic black hole. The third term is a correction term to the area entropy and is proportional to the radiation temperature of acoustic black hole. The fourth term is a new term correction to the entropy of a rotating acoustic black hole due to the GUP. Moreover, the calculation of entropy for this method does not generate logarithmic corrections. On the other hand, it has been shown in [6] that by considering the GUP in tunneling formalism these corrections were also obtained.

### IV. CONCLUSIONS

In summary, in the present study we calculate the entropy of an acoustic black hole in the context of neo-Newtonian hydrodynamics. Applying quantum statistical method we obtain corrections to the entropy due to the GUP. The use of the GUP in the equation of state density allows us to solve the partition function without the requirement of a cut-off and the divergences that arise when we apply the brick-wall method are eliminated. In our results we found that the leading term in Eq. (45) is proportional to horizon area of the acoustic black hole and terms of corrections are proportional to $T_h$ and $T_h^3$. In addition, we have obtained a new term correction that is proportional to $T_h^3$ that arises due to the presence of pressure and $B$ parameter associated with the rotation of the acoustic black hole.

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[1] S.W. Hawking, Phys. Rev. Lett. 25, 1344 (1971); J.D. Bekenstein, Phys. Rev. D 7, 2333 (1973); S.W. Hawking, Commun. Math. Phys. 43, 199 (1975); J.D. Bekenstein, Phys. Rev. D 9, 3292 (1974).
[33] H. Velten, D. J. Schwarz, J. C. Fabris and W. Zimdahl, Phys Rev D 88, 103522 (2013).
A. M. Oliveira, H. E. S. Velten, J. C. Fabris, I. G. Salako, Eur. Phys. J. C 74 3170 (2014).
[34] J. C. Fabris, O. F. Piattella, I. G. Salako, J. Tossa, H. E. S. Velten, Mod. Phys. Lett A 28, 1350169 (2013), [arXiv:1308.1859 [gr-qc]].
[35] I. G. Salako and A. Jawad, arXiv:1503.08714 [gr-qc].