1. INTRODUCTION

The need for physics beyond the Standard Model (SM) is clear at least from two experimental facts: non-zero neutrino mass and the presence of dark matter in the universe. The search for this new physics is one of the most active fields of modern physics and has three major directions: energy, cosmic and precision frontiers. The precision frontier we are interested in is driven by the indirect searches, looking for impact of new particles on observables such as scattering cross section asymmetry causing small deviations from original SM predictions. The high-precision electroweak experiments involving parity-violating (PV) Møller scattering, electron–positron collisions or electron–nucleon scattering can provide indirect access to new physics at multi-TeV scales and play an important complementary role to the LHC research program. These low-energy experiments tend to be less expensive than experiments at the high-energy colliders, but they do require a significant theoretical input, part of which we aim to provide in this paper.

One of such experiments, MOLLER [1], studying parity-violating Møller scattering, would allow a determination of the weak mixing angle with an uncertainty of 0.1%, an improvement of a factor of five in fractional precision compared with the previous E-158 measurement. At such precision, any inconsistency with the SM could signal new physics.

Starting in 2011, our group has been making a steady progress calculating major gauge invariant two-loop contributions to the Møller parity-violating asymmetry [6–9]. We divide the two-loop EWC to the Born cross section (\(\mathcal{M}_0\)) onto two classes: \(Q\)-part induced by quadratic one-loop amplitudes \(\sim \mathcal{M}_1\), and \(T\)-part—the interference of Born and two-loop amplitudes \(\sim 2\text{Re}\left[\mathcal{M}_0\mathcal{M}_2^*\right]\) (here index \(i\) in the amplitude \(\mathcal{M}_i\) corresponds to the order of perturbation theory). The \(Q\)-part was calculated exactly in [6] (using Feynman–t’Hooft gauge and the on-shell renormalization), where we show that the \(Q\)-part is much higher than the planned experimental uncertainty of MOLLER, i.e. the two-loop EWC are larger than was assumed in the past. The large size of the \(Q\)-part demands detailed and consistent treatment of \(T\)-part,
but this formidable task will require several stages. Our first step was to calculate the gauge-invariant double boxes [7]. In paper [9] we considered the EWC arising from the contribution of a wide class of the gauge-invariant Feynman amplitudes of the box type with one-loop insertions: fermion mass operators [or Fermion Self-Energies in Boxes], vertex functions [or Vertices in Boxes], and polarization of vacuum for bosons [or Boson Self-Energies in Boxes]. In this paper we do the next step—we calculate the insertions of two-loop vertices to vertices (VV), fermion self-energies to vertices (FSEV) and double vertices (DV).

The paper is organized as follows. In Section 2 we consider the asymmetry in Born approximation and introduce the basic notations. In Section 3 we calculate two Feynman diagrams with extra W and Z boson subgraphs (VV). Section 4 is devoted to the diagrams with lepton mass operators insertions (FSEV). We consider complex vertices (DV) in Section 5 and give numerical estimation of total effect of these contributions in Section 6.

2. BASIC NOTATIONS

We consider the process of electron-electron elastic scattering, i.e. Møller process:

\[ e(p_1, \lambda_1) + e(p_2, \lambda_2) \rightarrow e(p'_1, \lambda'_1) + e(p'_2, \lambda'_2), \]  \hspace{1cm} (1)

where \( \lambda_{1,2} \) (\( \lambda'_{1,2} \)) are the chiral states of initial (final) electrons and \( p_{1,2} \) are 4-momenta of initial electrons and \( p'_{1,2} \) are 4-momenta of final electrons. The first measurement of parity-violating (left-right) asymmetry

\[ A \frac{d\sigma}{d\Omega} = \frac{d\sigma^{−−} − d\sigma^{++}}{d\sigma^{−−} + d\sigma^{++} + d\sigma^{−+} + d\sigma^{++} + d\sigma^{−−} + d\sigma^{−+}}, \]

\[ = \frac{|M^{−−}|^2 − |M^{−+}|^2}{\sum_{\lambda} |M^{\lambda}|^2} \]  \hspace{1cm} (2)

in Møller scattering was made by E-158 experiment at SLAC [10–12]. In lowest order of perturbation theory in frames of QED the matrix element squared which is summed over polarization states of electrons has the following form:

\[ \sum_{\lambda} |M^{\lambda}|^2 = 8(4\pi\alpha)^2 \frac{s^4 + u^4 + t^4}{t^2 u^2}. \]  \hspace{1cm} (3)

We use the notation for the kinematic invariants neglecting of electron mass \( m \):

\[ s = 2p_1p_2, \quad t = −2p_1p'_1, \quad u = −2p_1p'_2, \quad s + t + u = 0. \]  \hspace{1cm} (4)

Thus here and further we neglect the terms of order \( O(m^2/s) \) since in MOLLEKR experiment it is expected that beam energy is \( E_{\text{beam}} = 11 \text{ GeV} \), that is \( s = 2mE_{\text{beam}} \approx 0.011244 \text{ GeV}^2 \). Sometimes we retain electron mass at intermediate steps of calculations, but in final results it survives only as the arguments of col-linear logarithm (in form \( \ln(s/m^2) \)). Within the Standard Model one has additional contribution in Born approximation with Z-boson exchange which gives rise to polarization asymmetry \( A^0 \):

\[ A^0 = \frac{s}{2m_w^2} A_{(0)} \frac{a}{s_w}, \quad A_{(0)} = \frac{y(1−y)}{1+y^4+(1−y)^4}, \]

\[ y = \frac{t}{s} = \frac{1−c}{2}, \]  \hspace{1cm} (5)

where \( c = \cos \theta \) is the cosine of scattering angle \( \theta = \left( p_1, p'_1 \right) \) in the system of center-of-mass of electrons, \( m_w \) is the W-boson mass and \( a \) is the so-called “weak electron charge” \( a = 1−4s_w^2 \).  \hspace{1cm} (6)

Now let’s recall that \( s_w(c_w) \) is the sine (cosine) of the Weinberg angle expressed in terms of the Z- and W-boson masses according to the Standard Model rules:

\[ s_w = \sqrt{1−c_w^2}, \quad c_w = m_w/m_Z. \]  \hspace{1cm} (7)

Thus, the factor \( a \) is just \( a \approx 0.109 \) and the asymmetry is therefore suppressed by both \( s/m_w^2 \) and \( a \). Even at Central Region (CR) of MOLLEKR (at \( \theta \rightarrow 90^\circ \), i.e. \( t \approx u \approx −s/2 \)), where the Born asymmetry is maximal, this asymmetry is extremely small:

\[ A^0 \approx \frac{s}{9m_w^2 s_w} a \approx 9.4968 \times 10^{-8}. \]  \hspace{1cm} (8)

It is the main aim of this paper to estimate the contribution of some classes of two-loop contributions, which have some logarithmical enhancement. As for the non-enhanced ones—they have an order of \( (−t/m_Z^2)(\alpha/\pi)^2 \approx 10^{-11} \) for the CR of MOLLEKR. Below we consider contribution to the vertex function \( \Delta V_\mu \) for on-mass-shell electrons \( p_i^2 = p_i'^2 = m^2 \) and the space-like 4-momentum of the virtual photon \( Q^2 = −q^2 = −(p_i − p_i')^2 \approx m^2 \) in two-loop level from the class of Feynman diagrams containing the intermediate states with \( W \)- and \( Z \)-bosons (see Fig. 1). Due to vertex renormalization condition \( \Delta V_\mu |_{\theta=0} = 0 \) the corresponding contribution is proportional to \( Q^2/m_{W,Z}^2 \). Thus we restrict ourselves by the condition

\[ \rho_i = Q^2/m_i^2 \ll 1, \quad i = W, Z. \]  \hspace{1cm} (9)
3. VERTEX SUBGRAPHS WITH EXTRA W AND Z BOSONS

The one loop expression for contribution of $W \gamma \gamma$ vertex to $e(p_1) \rightarrow e(p_1 - k) + \gamma(k)$ vertex (see Fig. 1a) has a form:

$$V_{\mu}^a(p_1, k) = -ie\bar{u}(p_1 - k)\gamma_{\mu}\omega_u(p_1) \frac{g^2}{32\pi^2} I^a(k^2), \quad (10)$$

where $\omega_{\pm} = 1 \pm \gamma_5$, $g = e/s_W$ and $e$ is the electron charge value ($e = 1 > 0$). Here and below we use the common notation $\overline{x} = 1 - x$, $\overline{y} = 1 - y$, etc. Analogously, one-loop vertex with one additional $Z$-boson (see corresponding subgraph in Fig. 1b) looks like:

$$V_{\mu}^b(p_1, k) = -ie\bar{u}(p_1 - k)\gamma_{\mu}(a \pm \gamma_5)\omega_u(p_1) \frac{g^2}{16\pi^2} I^b(k^2), \quad (12)$$

Integrating over 4-momenta of photon $k$ we get the contribution of (10) into full two-loop vertex $V_{\mu}$ (see Fig. 1a) as

$$\Delta V_{\mu}^a = -ie\frac{g^2}{16\pi^2} I_{1\mu}^a, \quad (14)$$

$$\Delta V_{\mu}^b = -ie\bar{u}(p_1 - k)\gamma_{\mu}(a \pm \gamma_5)\omega_u(p_1) \frac{g^2}{16\pi^2} I_{2\mu}^b, \quad (15)$$

where two terms $I_{1\mu}^a$ in (14) correspond to two terms in (11). Omitting the terms of order $O(\rho^{-2})$ we write down the contribution $I_{1\mu}^a$ as

$$I_{1\mu}^a = 6\int_0^1 d\overline{y} \int_0^1 d\overline{x} \left( \ln \frac{m_{W}^2}{b^2} - 1 \right) + \text{UVD-part}, \quad (16)$$

Using Feynman parameters trick one can integrate over loop momenta $k$ and obtain the result as a sum of ultra-violet finite (UV-finite) and ultra-violet divergent (UVD) parts:

$$Q_{\mu}^2 \int_0^1 d\overline{x} \left( \ln \frac{m_{W}^2}{b^2} - 1 \right) dx + \text{UVD-part},$$

where $b^2 = (p_1 x + p_1 \overline{x})^2 = m^2 + Q^2 \overline{x}$. Here and below we use the same notations for unrenormalized and renormalized quantities. Thus after renormalization of $I_{1\mu}^a$ we obtain expression

$$I_{1\mu}^a = \frac{Q^2}{m_{W}^2} c_{\mu}^a \bar{u}(p_1)\gamma_{\mu}\omega_u(p_1), \quad (17)$$

$$c_{\mu}^a = \frac{1}{5} \ln \frac{m_{W}^2}{Q^2} + \frac{7}{18} = 5.0406.$$
Here and everywhere below the number value corresponds to CR of MOLLER.

Second term $I_{2a}^{\mu}$ in (14) can be written as

$$I_{2a}^{\mu} = \int_{y=0}^{y=1} \int_{x=0}^{x=1} \frac{1-2x}{1-x} dy dx \times \int \frac{d^4k}{i\pi^2} \frac{N^a_\mu}{\left(k^2 - \sigma^2\right)\left(k^2 - 2lpk\right)}$$

where $\sigma^2 = m_w^2 y/(x\bar{x})$. Again using standard manipulations we arrive to

$$I_{2a}^{\mu} = \bar{\nu}(p_1)\gamma_\mu\omega\nu(p_1) \int_{y=0}^{y=1} \int_{x=0}^{x=1} \frac{1-2x}{1-x} dy dx \times \left[\ln \frac{\Lambda^2}{D_a} - \frac{3}{2} \frac{Q^2}{D_a} \left(1 - y_{i1}\right)\left(1 - y_{i1}\right)\right].$$

where $D_a = y_{i1}^2 b_{i1}^2 + \sigma^2 y_{i1}$, $b_{i1}^2 = \left[x_{i1}p_1 + \bar{x}_{i1}p_1\right]^2 = m^2 + x_{i1}\bar{x}Q^2$ and $\Lambda$ is the UV-regularization parameter. Applying the renormalization procedure we obtain

$$I_{2a}^{\mu} = \frac{Q^2}{m^2} c^a_i\bar{\nu}(p_1)\gamma_\mu\omega\nu(p_1),$$

$$c^a_i = -\int_{x=0}^{x=1} \int_{y=0}^{y=1} \frac{1-2x}{1-x} dy dx \times \left[\rho_w \ln \left(1 + \frac{1}{\rho_w}\frac{x_{i1}\bar{x}y_{i1}\bar{x}}{y_{i1}}\right) + \frac{\rho_w x_{i1}}{\rho_w y_{i1} + y_{i1}^2 x_{i1}\bar{x}_1}\right] = -0.0930.$$

The final expression for contribution of $W$-vertex subgraph to the vertex function (see Fig. 1a) is

$$\Delta V^{\text{MO}}_\mu = -ie \frac{Q^2}{m^2} \frac{g_2^2 \pi\alpha}{\left(16\pi^2\right)^2} \left(c^a_1 + c^a_2\bar{\nu}(p_1)\gamma_\mu\omega\nu(p_1)\right).$$

Contribution of $Z$-vertex subgraph (see Fig. 1b) has a form

$$\Delta V^{Z}_\mu = -ie \frac{g_2^2 \pi\alpha}{\left(4c_w\right)^2 \left(16\pi^2\right)^2} I^{b}_\mu,$$

$$I^{b}_\mu = \int \frac{d^4k}{i\pi^2} \frac{N^b_\mu}{k^2 - 2lpk - \left(k^2 - 2lpk\right)} - I^{b}(k^2),$$

where

$$N^b_\mu = \bar{\nu}(p_1)\gamma_\lambda(\bar{p}_1 - \bar{k} + m) \times \gamma_\mu(\bar{p}_1 - \bar{k} + m)\gamma_\lambda(a + \gamma_5)^2 u(p_1).$$

In the similar way we obtain for contribution of $Z$-vertex subgraph to the vertex function

$$\Delta V^{Z}_\mu = -ie \frac{Q^2}{m^2} \frac{g_2^2 \pi\alpha(1 + a)^2}{\left(4c_w\right)^2 \left(16\pi^2\right)^2} \left(c^b_1 + c^b_2\bar{\nu}(p_1)\gamma_\mu\omega\nu(p_1)\right),$$

where

$$c^b_1 = \frac{2}{9} \frac{\ln m^2}{Q^2} + \frac{7}{27} = 3.4164,$$

$$c^b_2 = -\int_{x=0}^{x=1} \int_{y=0}^{y=1} \int_{z=0}^{z=1} \frac{1-2x}{1-x} dy dx \times \left[\rho_z \ln \left(1 + \frac{1}{\rho_z}\frac{x_{i1}\bar{x}y_{i1}y_{i1}}{y_{i1}}\right) + \frac{\rho_z x_{i1}y_{i1}}{\rho_z y_{i1} + y_{i1}^2 x_{i1}\bar{x}_1}\right] = -0.0944.$$

4. ELECTROWEAK ELECTRON MASS OPERATOR INSERTION TO THE VERTEX FUNCTION

Now let’s consider the set of Feynman diagrams of vertex type containing electron Mass Operator (MO) with internal $Z$ or $W$ bosons insertions (see Fig. 1c, 1d). The relevant contribution to the vertex function has a form

$$\Delta V^{\text{MO}}_\mu = -ie \frac{\alpha}{2\pi} \left[V^{Z}_\mu + V^{W}_\mu\right],$$

$$V^{i}_\mu = \int \frac{d^4k}{i\pi^2} \frac{N^i_\mu}{k^2 - 2lpk},$$

where $i = Z, W$ and the numerator $N^i_\mu$ is:

$$N^i_\mu = \bar{\nu}(p_1)\gamma_\lambda(\bar{p}_1 - \bar{k} + m) \times \gamma_\mu(\bar{p}_1 - \bar{k} + m)\gamma_\lambda(a + \gamma_5)^2 u(p_1) M^i(k, p_1),$$

with

$$c^{(Z)} = \frac{g_2^2(a + \gamma_5)^2}{\left(4c_w\right)^2 \left(16\pi^2\right)^2}, \quad c^{(W)} = \frac{g_2^2 a}{\left(4c_w\right)^2 \left(16\pi^2\right)^2}.$$

Mass operator $M(k, p_1)$ of the off-mass-shell electron looks like:

$$M(k, p_1) = \int_{x=0}^{x=1} dx_1 \int_{z=0}^{z=1} dz \int_{y=0}^{y=1} dy_1 \int_{x=0}^{x=1} dx_2 \frac{1}{m_j - x_1 z(k^2 - 2lpk).}$$
Standard Feynman procedure of joining the denominators and the loop momentum integrating gives us

\[ V'_\mu = \int_0^1 \left[ \bar{\psi}_1(d x_1) \int_0^1 d z \int_0^1 d x \int_0^1 d y \left( \ln \frac{D_1}{D_1} - \frac{D_2}{D_2} \right) \right] \]

which can be simplified to the form:

\[ V'_\mu = \frac{1}{N_\mu} \int_0^1 \bar{\psi}(p_i) \gamma_\mu(u(p_i)) \psi(p_i) V', \]

where \( D_i = b^2 y^2 + x y \sigma^2 \). After renormalization and expansion on powers of \( Q^2/m^2 \) we obtain

\[ V_i = -\frac{Q^2}{m_i} \int_0^1 \left[ x y \bar{\psi}(p_i) \gamma_\mu(u(p_i)) \right] \]

Final expression for the contribution to the vertex function (see Fig. 1c, 1d) is

\[ \Delta V'_\mu^{MO} = i e \alpha \frac{Q^2}{2m_w} \left( 1 + a \right)^2 \bar{u}(p_i) \gamma_\mu(u(p_i)) \]

with

\[ c_\bar{z} = \frac{1}{6} \ln \frac{m_w^2}{Q} + z = 3.0345, \]

\[ c_w = \frac{1}{6} \ln \frac{m_w^2}{Q} + w = 2.9925. \]

### 5. CONTRIBUTION OF DIAGRAMS CONTAINING \( WW'\gamma, WW'\gamma' \) VERTICES

Below we consider diagrams containing \( WW'\gamma, WW'\gamma' \) vertices only because their contributions are associated with logarithmic enhancement. Let’s consider the Feynman diagram with virtual photon which is emitted from the initial electron and absorbed by the final electron (see Fig. 1e). The relevant contribution to the vertex function is

\[ \Delta V'_\mu = \frac{i e}{2(16\pi^2)} \left( \frac{Q^2}{m_w} \right)^2, \]

\[ \Delta V'_\mu = \left( \frac{Q^2}{m_w} \right)^2, \]

where \( \lambda \) is the photon mass and

\[ V_{\sigma\eta} = g_{\sigma\eta}(2p_i - p_i - k - k_1) \eta \]

\[ + g_{\mu\sigma}(2p_i - p_i - k - k_1)_{\eta} \]

\[ + g_{\nu\sigma}(2p_i - p_i - k - k_1_{\eta} \mu), \]

\[ N_{\eta\sigma} = \bar{u}(p_i) \gamma_\sigma(p_i - k) \]

\[ \times \gamma_\eta \hat{k}_\eta \gamma_{\omega} u(p_i). \]

Doing the similar treatment as it was done above one can integrate over loop momentum \( k_1 \), renormalize the amplitude of this subgraph and obtain

\[ V'_\mu = \frac{3Q^2}{2m_w} \]

\[ \times \int \left[ \bar{u}(p_i) \gamma_\sigma(p_i - k) \gamma_{\omega} u(p_i) \right] \]

After integration over \( k \) one gets:

\[ V'_\mu = \frac{3Q^2}{2m_w} \]

\[ \times \left[ \ln \frac{m_w^2}{D_e} - \frac{Q^2}{D_e} (y^2 + z^2) \right] \]

where \( D_e = y^2 b^2 + \lambda^2 y^2 \). Further we use simple integrals

\[ \int_0^1 2y dy \left[ \ln \frac{m_w^2}{D_e} - \frac{Q^2}{D_e} (y^2 + z^2) \right] = \frac{1}{b^2} \ln \frac{b^2}{\lambda^2}, \]

\[ \int_0^1 \frac{Q^2}{D_e} dx = 2 \int \frac{Q^2}{m^2}, \]

\[ \int_0^1 \frac{Q^2}{D_e} dx = 1, \]

\[ \int_0^1 \frac{Q^2}{D_e} dx = \ln \frac{Q^2}{m^2} - \pi^2 / 3, \]
and obtain

$$\Delta V^{f}_{\mu} = ie \frac{3g^2}{2m_w^2} \bar{u}(p_1) \gamma_\mu u(p_1) \frac{4\pi\alpha g^2}{2(16\pi^2)^2} I^f,$$

$$I^f = 2 + \frac{\pi^2}{3} + \ln \frac{m_w^2}{\Lambda^2}$$

(38)

The diagrams in Figs. 1f, 1g, 1h has a general enhancement factor which is associated with the collinear photon emission in vertex. Let’s demonstrate this in general. The common structure for all three diagrams contains the emission of photon with momentum $k$ from initial electron. This leads to the following structure of the amplitude:

$$V^{f,g,h}_{\mu} = \frac{e}{16\pi^2} \left( \int \frac{d^4 k}{i\pi^2} \bar{u}(p_1) O^{\gamma}_{\mu}(\bar{p}_1 - k + m) \gamma_\mu u(p_1) \right),$$

(39)

where $O^{\gamma}_{\mu}$ corresponds to the remaining part of Feynman diagram and different for each diagram. We note that the dominant contribution to this integral comes from the integration region of small photon momentum (i.e. $k^2 \ll m_w^2$) and thus we can omit $k$ in the remaining part of vertex amplitude, containing the momenta of a $W$-boson. Joining the denominators we have

$$V^{f,g,h}_{\mu} \approx \frac{e}{16\pi^2} \left( \int \frac{dx}{x^2} \bar{u}(p_1) p_{\mu} O^{\gamma}_{\mu} \right)_{x \approx x_{m_w}},$$

where we approximated the traces of the amplitude by taking it in collinear region (i.e. putting $k \rightarrow x_{p_1}$). This gives us the possibility to integrate over $d^4 k$ and obtain

$$V^{f,g,h}_{\mu} \approx \frac{e}{16\pi^2} \left( \int \frac{dx}{x^2} \bar{u}(p_1) p_{\mu} O^{\gamma}_{\mu} \right)_{x \approx x_{m_w}} \ln \frac{m_w^2}{m^2} - 1.$$  

(40)

The diagram containing the $WW\gamma\gamma$ vertex (see Fig. 1f) gives

$$\Delta V^{f}_{\mu} = 2ie R \frac{g^2}{2m_w} \bar{u}(p_1) \gamma_\mu u(p_1),$$

$$\times \int \frac{d^4 k_1}{i\pi^2} \frac{N^{\gamma}_{f\sigma}}{k_1^2(k_1^2 - 2p_1 k_1 - m_w^2)(k_1^2 - 2p_1 k_1 - m_w^2)},$$

$$N^{\gamma}_{f\sigma} = \bar{u}(p_1) \gamma_\mu \gamma_\sigma \gamma_\omega u(p_1),$$

$$S_{\mu\nu\lambda\sigma} = 2g_{\mu\nu} g_{\lambda\sigma} - g_{\mu\lambda} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\lambda}.$$  

(41)

The loop momentum integral do not have ultraviolet as well as infrared divergences. Standard manipulations lead to

$$\Delta V^{f}_{\mu} = 2ie \frac{Q^2}{4m_w^2} \frac{4\pi\alpha g^2}{2(16\pi^2)^2} \frac{L \bar{u}(p_1) \gamma_\mu \gamma_\omega u(p_1)}{L = \ln \frac{m_w^2}{\Lambda^2}}.$$  

(42)

For the Feynman diagram with $WW\gamma$ vertex shown in Fig. 1g we have

$$\Delta V^{g}_{\mu} = -2i R \frac{g^2}{2m_w^2} \bar{u}(p_1) \gamma_\mu u(p_1),$$

$$\times \int \frac{d^4 k_1}{i\pi^2} \frac{N^{\gamma}_{g\sigma}}{k_1^2(k_1^2 - 2p_1 k_1 - m_w^2)}$$

$$\times \bar{u}(p_1) \gamma_\mu \gamma_\sigma \gamma_\omega u(p_1),$$

$$N^{\gamma}_{g\sigma} = \bar{u}(p_1) \gamma_\mu \gamma_\sigma \gamma_\omega u(p_1).$$

(43)

where vertices have the form

$$V^{\gamma\nu}_{1} = (2p_1 - k_1)^\nu \gamma_\omega,$$

$$+ (2k_1 - p_1)^\nu \gamma_\omega,$$

$$V^{\gamma_\nu}_{2\mu} = (p_1 - k_1)^\nu \gamma_\omega,$$

$$+ (2p_1 - k_1)^\nu \gamma_\omega,$$

$$+ (2p_1 - k_1)^\nu \gamma_\omega.$$  

(44)

Retaining in the numerator the terms quadratic over loop momenta one gets

$$\Delta V^{g}_{\mu} = -i R \frac{g^2}{2m_w^2} \bar{u}(p_1) \gamma_\mu u(p_1),$$

$$\times \int \frac{1}{0} \frac{1}{0} \frac{d^4 x^\prime}{d^4 y} \frac{Q^2}{D_8} \left( \frac{13}{4} \right),$$

where $D_8 \approx y m_w^2$. Finally we have for the contribution of Feynman diagram shown in Fig. 1g:

$$\Delta V^{g}_{\mu} = -ie \frac{Q^2}{2m_w^2} \frac{4\pi\alpha g^2}{2(16\pi^2)^2} \bar{u}(p_1) \gamma_\mu \gamma_\omega u(p_1),$$

(45)

$$\Delta V^{g}_{\mu} = -ie \frac{Q^2}{2m_w^2} \frac{4\pi\alpha g^2}{2(16\pi^2)^2} \bar{u}(p_1) \gamma_\mu \gamma_\omega u(p_1),$$

(46)
and the diagram shown in Fig. 1h gives the similar result
\[
\Delta V^h_\mu = - ie \frac{Q^2}{m^2} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2} \bar{\psi}(p) \gamma_\mu \sigma \cdot u(p).
\]  

**6. NUMERICAL CONTRIBUTION TO THE LEFT-RIGHT ASYMMETRY**

Collecting the result of considered two-loops contributions one can put the total result in the form
\[
\Delta V^{a+b} + \Delta V^{MO} + \Delta V^{f+g+h} = B^Z K^Z_\mu + B^W K^W_\mu,
\]  
where
\[
K^Z_\mu = i e \frac{Q^2}{m^2} (\pm a)^2 \frac{\alpha g^2}{(4\pi)^2} \int \frac{d^3 p}{(2\pi)^3} \bar{u}(p) \gamma_\mu \gamma_5 u(p),
\]  
and the coefficients look like
\[
B^Z = -8(c_1^b + c_2^b) + 32 c_3^Z = 70.5285,
\]
\[
B^W = -4(c_1^a + c_2^a) + 8c_3^W + 3f^{i}\gamma_0 f^{i}
\]  
\[
+ L \left(1 - \frac{26}{9} - \frac{67}{9}\right) = -312.382.
\]

Let’s note by index \(C\) the contributions investigated here, i.e. \(C = a, b, \ldots, h\). As specific corrections to observable parity-violating asymmetry induced by contribution \(C\) we choose the contribution to the asymmetry \((\Delta A)_C\) and the relative corrections \(D^C_A\),
\[
(\Delta A)_C = \frac{M_C^0 - M_C^{++}}{\sum |M_C|^2},
\]  
\[
D^C_A = \frac{(\Delta A)_C}{A^0} = \frac{|M_C^0|^2 - |M_C^{++}|^2}{|M^0_0|^2 - |M^{++}_0|^2}.
\]  

The physical effect of radiative effects from contribution \(C\) to observable asymmetry is determined by the relative correction (see [9] for more details):
\[
\delta^C_A = \frac{A^C - A^0}{A^0} = \frac{D^C_A - \delta^C}{1 + \delta^C},
\]  
where the relative correction to unpolarized cross section is \(\delta^C = \sigma^C_{00}/\sigma_{00}^0\). For two-loop effects (where \(\delta^C\) is small) the approximate equation for relative correction to asymmetry takes place: \(\delta^C_A \approx D^C_A\).

Contributions to asymmetry of \(Z\) and \(W\) types are
\[
(\Delta A)_Z = -16aB^Z Q^2 \left(\overline{\alpha g^2} \frac{\pi}{m^2} \right) \frac{2^4 \pi}{(4\pi)^2} \frac{1}{(16\pi^2)^3},
\]  
\[
(\Delta A)_W = 4B^W Q^2 \left(\overline{\alpha g^2} \frac{\pi}{m^2} \right) \frac{2^4 \pi}{(4\pi)^2} \frac{1}{(16\pi^2)^3},
\]  
which give the relevant numerical values:
\[
(\Delta A)_Z = -2.5410 \times 10^{-12},
\]  
\[
(\Delta A)_W = -3.1983 \times 10^{-10}.
\]

Taking into account that in CR of MOLLER the Born asymmetry \(A^0 = 94.97\) ppb the numbers for relative corrections \(D^C_A\) are
\[
D^Z_A = -0.0000267,
\]  
\[
D^W_A = -0.0033677.
\]

We can see that effects have the same negative sign, first is rather small, but the second one is at the edge of region of planned one per cent experimental error for MOLLER and thus will be important for future analysis of MOLLER experimental results.

**7. ACKNOWLEDGMENTS**

Many thanks to prof. A.B. Arbuzov and prof. D.Yu. Bardin for help and valuable discussions. The work of A.G.A. and S.G.B. has been supported by the Natural Science and Engineering Research Council of Canada (NSERC). Yu.M.B. acknowledges support of Heisenberg-Landau Program grant no. HLP-2015-15. V.A.Z. is grateful to the financial support of Belarus program “Convergence” (no. 20141163) and thanks JINR for hospitality in 2014–2015.

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