Potential Sources of Gravitational Wave Emission and Laser Beam Interferometers

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ABSTRACT

The properties of potential gravitational wave sources like neutron stars, black holes and binary systems are reviewed, as well as the different contributions (stochastic and continuous) to the gravitational wave background. The detectability of these sources by the present generation of laser beam interferometers, which will be fully operational around 2002, is also considered.

Subject headings: gravitational waves, neutron stars, black holes

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1. Introduction

Gravitational waves (GW) are predicted by the General Relativity Theory (GRT) and the slow inspiral observed in the binary pulsar PSR 1913+16 (Hulse & Taylor 1975) is an indirect proof of their existence. GW are fundamentally different from electromagnetic waves. While the latter propagate in the framework of space and time, the former are waves of the spacetime itself, created by asymmetric mass motions. A direct detection of GW has not been achieved up to date.

A resonant-mass antenna is, in principle, the simplest detector of GW (Weber 1960). A suspended cylindric detector, during the passage of a gravitational wave, has its normal modes excited and monitored by transducers. Present day resonant bars (or "spheres") are fairly narrowband detectors, with bandwidths of only a few Hz around frequencies in the kHz range and sensitivities around $h \approx 10^{-19}$. Future detectors should have bandwidths of the order of 100 Hz or better and sensitivities of about $10^{-21}$.

In the early seventies emerged the idea that laser interferometers might have a better chance of detecting GW (Weiss 1971; Moss, Miller & Forward 1971). These detectors are essentially constituted by a two-arm Michelson interferometer, which measures the phase difference between a splitted laser beam having propagated along two perpendicular directions. This is the quantity that would be changed by a passing and properly oriented gravitational wave. Laser beam interferometers are wideband detectors, being sensitive to GW in the frequency range $10^{-4}$ Hz. Plans for kilometer-size interferometers have been developed in the past decades. The US project LIGO is under development at two widely separated sites (Hanford and Livingston), both localities hosting a 4 km interferometer. The 3 km French-Italian antenna VIRGO is being built in Cascina, near Pisa, and a sophisticated seismic isolation system will allow this detector to measure frequencies down to 5 Hz. GEO 600 is a 0.6 km arm interferometer in construction by a British-German collaboration, in a site near Hannover. Presently, the only operational laser interferometer is TAMA (0.3 km arms), located in Mitaka (Japan). Besides these ground based antennas, there is also a project supported by NASA and ESA to launch a large spatial interferometer (5 million km arms), constituted by three platforms. The LISA antenna will search for low-frequency (mHz) GW sources, which cannot be observed from the ground because of tectonic activity.

The best signal-to-noise (S/N) ratio that can be achieved from these detectors implies the use of "matched-filter" techniques, that require a priori knowledge of the waveform. Thus, in this context, the study of the most probable GW sources is of fundamental importance. In this work, the properties of some GW sources that have been discussed in the literature in the past years will be reviewed, as well as the expected detectability by the
major interferometers under development in the world.

2. Neutron Stars

2.1. Rotating neutron stars - pulsars

Rotating neutron stars may have a time-varying quadrupole moment and hence radiate GW, by either having a triaxial shape or a misalignment between the symmetry and the spin axes, which produces a wobble in the stellar motion (Ferrari & Ruffini 1969; Zimmerman & Szednits 1979). In the former case the GW frequency is equal to twice the rotation frequency, whereas in the latter two modes are possible: one in which the GW have the same frequency as the rotation, and another in which the GW have twice the rotation frequency. The first mode dominates by far at small wobble angles while the importance of the second increases for large values of the misalignment.

In the case of a rotating triaxial neutron star, the gravitational strain amplitude of both polarization modes are:

\[ h_+(t) = 2A(1 + \cos^2 i) \cos(2\Omega t) \]  
\[ h_\times(t) = 4A \cos i \sin(2\Omega t) \]

where \( i \) is the angle between the spin axis and the wave propagation vector, assumed to coincide with the line of sight, \( \Omega \) is the angular rotation velocity of the neutron star,

\[ A = \frac{G}{rc^4} \varepsilon I_{zz} \Omega^2 \]  
\[ \varepsilon = \frac{I_{xx} - I_{yy}}{I_{zz}} \]

with \( I_{ij} \) being the principal inertia moments of the star.

\[ h(t) = h_+(t)F_+(\theta, \phi, \psi) + h_\times(t)F_\times(\theta, \phi, \psi) \]  

where \( F_+ \) and \( F_\times \) are the beam factors of the interferometer, which are functions of the zenith distance \( \theta \), the azimuth \( \phi \) as well as of the wave polarization plane orientation \( \psi \) (see, for instance, Jaranowski et al. 1998, for details).
Detection of both gravitational polarization modes of a radio pulsar leads immediately to the value of the spin projection angle \( \theta \) and to an estimate of the ellipticity, if the distance is known by measuring the dispersion of radio signals through the interstellar plasma. Upper limits of \( \varepsilon \) have been obtained by assuming that the observed spin-down of pulsars is essentially due to the emission of GW. In this case, for "normal" pulsars one obtains \( \varepsilon \leq 10^{-3} \) whereas "recycled" millisecond pulsars seem to have equatorial deformations less than \( 10^{-8} \). Monte Carlo simulations of the galactic pulsar population by Regimbau & de Freitas Pacheco (2000a) indicate that with the planned VIRGO sensitivity (better than LIGO at lower frequencies) and integration times of the order of \( 10^7 \) s, a few detections should be possible if \( \varepsilon = 10^{-6} \).

What are the physical mechanisms able to deform the star? Different scenarios leading to a distorted star have been discussed in the literature, as anisotropic stresses from strong magnetic fields, and tilting of the symmetry axis during the initial cooling phase when the crust solidifies. Bildsten (1998) pointed out that a neutron star in a state of accretion may develop non-axisymmetric temperature variations in the surface, which produce horizontal density patterns able to create a mass quadrupole moment of the order of \( 10^{38} \) g cm\(^{-2}\), if the elastic response of the crust is neglected. More detailed calculations by Ushomirsky et al. (2000) indicate that the inclusion of the crustal elasticity decreases by a factor 20-50 the expected mass quadrupole moment. Nevertheless, the deformation induced by the accretion process is able to balance the angular momentum gained by mass transfer to that lost by GW emission, imposing limits to the maximum rotation frequency attained by the "recycled" neutron star. Strong magnetic fields are also able to distort the star (Gunn & Ostriker 1970; Bonazzola & Gourgoulhon 1996). Recent calculations by Konno et al. (2000) including general relativity corrections permit to estimate the ellipticity by the relation

\[
\varepsilon_B \approx 4 \times 10^{-8} g B_{14}^2
\]

where \( g \) is a parameter depending on the structure of the neutron star, in particular on the adopted equation of state, and \( B_{14} \) is the magnetic field strength in units of \( 10^{14} \) Gauss. According to those authors, a typical value of the structure factor is \( g \approx 14 \). In this case, if an ellipticity of \( 10^{-6} \) is required, then from the above equation a field of \( 1.3 \times 10^{14} \) G is derived. The number of pulsars in the Galaxy with magnetic fields higher than \( 10^{14} \) may be considerable, about \( 23\% \) of the total population, if the magnetic field decay timescale is much longer than the pulsar lifetime (Regimbau & de Freitas Pacheco 2000b). However these objects can be discarded as potential GW sources since they are rapidly decelerated by magnetic dipole emission and, consequently, most of them have presently periods higher than 20 s.
2.2. Bar-mode instabilities

Rotating and self-gravitating incompressible fluids are subjected to non-axisymmetric instabilities when the ratio $\beta = T/|W|$ of the rotational energy $T$ to the gravitational energy $W$ is sufficiently large (Chandrasekhar 1969). These instabilities correspond to global nonradial toroidal modes with eigenfunctions $\propto e^{\pm im\phi}$, where $m = 2$ is the so-called "bar" mode, the fastest growing mode when rotation is very rapid. Incompressible Newtonian stars in the presence of some dissipative mechanism (viscosity or gravitational radiation reaction) become secularly unstable against bar formation when $\beta \geq 0.14$. In this case, the instability growth is essentially determined by the shortest dissipative timescale. On the other hand, when $\beta \geq 0.27$, the star becomes dynamically unstable to bar formation, and the growth of the instability is determined by the hydrodynamical timescale of the system. These instability limits have rigorously been derived for homogeneous and uniformly rotating Newtonian stars, but further relativistic numerical studies using polytropic equations of state and assuming ad-hoc rotation velocity profiles concluded that the onset of the instabilities occurs approximately at the same limits (see, for instance, Shibata et al. 2000).

Numerical simulations indicate that bar formation in dynamically unstable stars is accompanied by mass and angular momentum losses (Houser et al. 1994; Lai & Shapiro 1995), with the ejected matter forming spiral arms in the equatorial plane. The subsequent evolution is rather uncertain. Some simulations suggest that the bar shape is short lived, while other simulations predict a lifetime of many bar-rotation periods (see, for instance, New et al. 1999). The angular momentum carried out by the ejected mass and by gravitational waves reduces $\beta$ to values below the dynamical instability limit, but still above the secular stability limit. The resulting axisymmetric system may evolve into a nonaxisymmetric configuration on a secular dissipation timescale (Lai & Shapiro 1995), as we shall see later.

Estimates of the characteristic gravitational strain amplitude for the dynamical instability phase ($\beta \geq 0.27$) performed by different authors (Houser et al. 1994; Houser & CENTRELA 1996; Brown 2000; Shibata et al. 2000) seem to be in rough agreement, namely, they predict a characteristic strain amplitude $h_c \approx 3 \times 10^{-22}$ at a distance of 20 Mpc. However the characteristic frequency of the waves differs considerably from author to author, ranging from 0.49 kHz (Brown 2000) up to 4.0 kHz (Houser et al. 1994). Adopting the low frequency estimate a signal-to-noise ratio $S/N \approx 2.0$ is obtained if the planned VIRGO sensitivity curve is used (a similar result is obtained for LIGO), whereas a $S/N$ ratio one order of magnitude smaller is expected if the characteristic frequency is higher than 2-3 kHz.
Most of the numerical studies suggest that after the dynamical instability phase (if the system was initially set beyond the limit $\beta_{\text{crit}} \approx 0.27$), it recovers almost an axisymmetric shape, but with $\beta$ still above the secular instability threshold. In this case, the system may evolve away from the axisymmetric configuration, in a timescale determined by the gravitational radiation reaction, which is of the order of few seconds for $\beta$ in the range 0.20-0.25, as simulations suggest. This evolutionary path is possible if gravitation radiation reaction overcomes viscosity. Then during the evolution the fluid circulation is conserved (but not angular momentum) and the system evolves toward a Dedekind ellipsoid, whose configuration is a fixed triaxial figure with an internal fluid circulation of constant vorticity. In the opposite situation, when viscosity drives the instability, angular momentum is conserved (but not the fluid circulation) and the system evolves toward a Jacobi ellipsoid.

The transition to a Dedekind configuration manifests in the form of strong hydrodynamic waves in the outer layers and mantle, propagating in the opposite direction of the star’s rotation. According to Lai & Shapiro (1995), the frequency of the GW is an increasing function of the angular velocity of the star. The frequency of the waves is maximum at the beginning of the transition ($\nu_{\text{max}} \sim 800$ Hz) and then it decreases monotonically. Since GW carry away the star angular momentum, the final configuration is a nonrotating triaxial ellipsoid, which no more emits GW. Thus, the wave amplitude first increases, reaches a maximum (when $\nu \sim 500$ Hz) and then decreases to zero again. Lai & Shapiro (1995) estimated that the total gravitational wave energy radiated during the transition could be as large as $4 \times 10^{-3}M\text{c}^2$. This is much larger than the expected energy ($\sim 10^{-8}M\text{c}^2$) radiated in the axisymmetric collapse and bounce preceding the neutron star formation (Zwerger & Muller 1997). Using the wave amplitude estimated by Lai & Shapiro (1995), the expected S/N ratios for sources at a distance of 20 Mpc are 4.0 and 3.0 for VIRGO and LIGO respectively.

2.3. R-modes in rotating neutron stars

The r-mode (r for rotation) is a member of the class of gravitational radiation driven instabilities (including the secular bar-mode instability) excited by the so-called CFS (Chandrasekhar-Friedman-Schutz) mechanism (see a recent review by Andersson & Kokkotas 2000). The criterion for triggering the instability is quite simple: if the pattern speed of the mode is forward-going as seen from a distant observer, but backward-going with respect to the rotation of the star, then when the disturbance radiates away the star angular momentum, the system can find a state of lower energy and angular momentum. These large scale toroidal fluid oscillations are similar to the well known geophysical Rossby
waves and, in both cases, the restoring force is the Coriolis force.

In the first tens of seconds after the formation of the neutron star, the temperature is very high ($T \approx 10^{10}$ K) and the instabilities discussed in the preceding section (bar-modes) are thought to be at work. If the temperature is still higher, the bulk viscosity is expected to suppress the CFS instability, whereas the shear viscosity plays a stabilizing role for temperatures $T \leq 10^7$ K. Thus, there is a well defined window in which the ”newly-born” neutron star is unstable. In the plane $\Omega \times T$, the instability limit region can roughly be estimated by imposing that the total energy rate of the mode is zero, or equivalently

$$\frac{1}{\tau_{gr}} + \frac{1}{\tau_b} + \frac{1}{\tau_s} = 0$$  \hfill (7)

where $\tau_{gr}$, $\tau_b$ and $\tau_s$ are respectively the gravitation radiation, bulk and shear viscosity damping timescales. For the $m=2$ mode, these timescales are approximately given by

$$\frac{1}{\tau_{gr}} = -0.303\left(\frac{\Omega^2}{\pi G \rho}\right)^3 s^{-1}$$  \hfill (8)

$$\frac{1}{\tau_b} = 5 \times 10^{-12} T_9^6 \left(\frac{\Omega^2}{\pi G \rho}\right) s^{-1}$$  \hfill (9)

and

$$\frac{1}{\tau_s} = 3.3 \times 10^{-9} T_9^{-2} s^{-1}$$  \hfill (10)

where $T_9$ is the temperature in units of $10^9$ K, $\Omega$ is the angular velocity of the star and $\rho$ its mean density. As the star cools by neutrino emission and decelerates by GW emission, it stabilizes around periods of 15-25 ms. According to this scenario, no ”newly-born” pulsar faster than this limit should be observed. However, these estimates depend not only on damping effects due to different physical mechanisms, most of which are still badly understood, but also on the role played by the crust and magnetic field.

For purposes of detection, the most important properties of the GW signal from r-modes are: a) the frequencies, which at the lowest order in the angular velocity of the star are given by

$$\omega_r = \frac{(m-1)(m+2)}{(m+1)} \Omega$$  \hfill (11)

Thus, when $m = 2$, the GW frequency is $\nu_{gr} = \frac{4}{3} \nu_{rot}$; b) the emission is connected with current multipoles instead of mass multipoles.

The amplitude of the mode is small at the beginning, but it increases in a timescale of about 100 s, until a non-linear regime is reached and when saturation effects occur. This saturation phase is the most likely to be detected and lasts about $10^4$-$10^5$ s for a crusted
star and about $10^6 - 10^7$ s for a fluid star. The expected strain amplitude in the saturation phase, calculated for a polytropic equation of state with $n=1$ and for a canonical neutron star of 1.4 $M_\odot$ is (Andersson & Kokkotas 2000)

$$h(t) \approx 9.25 \times 10^{-25} \alpha \left(\frac{20 \text{ Mpc}}{r}\right)$$

where $\alpha$ is the dimensionless radial amplitude of the mode and $r$ is the distance to the source. In order to estimate the expected S/N ratio the usual matched-filtering approach is adopted, although it must be recognized that such a technique is unlikely to be possible for this kind of signal. In this case, the S/N ratio is given by

$$\left(\frac{S}{N}\right)^2 = 4 \int_0^\infty (\frac{h_c}{h_n})^2 dln\nu$$

where $h_n = [\nu S_n(\nu)]^{1/2}$, $S_n(\nu)$ is the noise spectral density and $h_c$ is the characteristic amplitude defined by

$$h_c = h(t)[\nu^2 \left| \frac{dt}{d\nu}\right|]^{1/2}$$

This last relation is a consequence of the stationary phase approximation, meaning that the detectability of a quasi-periodic signal is improved as the square root of the number of cycles at a given frequency $\nu$. Using the planned VIRGO sensitivity curve one obtains for a source at a distance of 20 Mpc, $S/N \approx 2.2\alpha$ if no crust is present and $S/N \approx 1.5\alpha$ if a crust is already present in the star. These S/N ratios are smaller by a factor 2-3 for LIGO and almost one order of magnitude higher for the planned advanced LIGO. These estimates indicate that if the mode amplitude at saturation is nearly unity, then fast rotating newly born neutron stars could be good source candidates for the present generation of laser beam interferometers.

### 2.4. Oscillating neutron stars

Neutron stars have a large number of families of distinct pulsating modes (see Kokkotas 1997 for a review). Nonradial oscillating modes as an emission mechanism of GW were already discussed in the late sixties (Thorne & Campolattaro 1967; Thorne 1969). These early calculations were concentrated mainly in the so-called $f$ (fundamental) mode, since this is the mode through which most of the mechanical energy of the star is radiated away. All the other possible fluid modes as $g$ (gravity), $p$ (pressure), $s$ (shear), $t$ (toroidal) and $i$ (interface) can be calculated with sufficient accuracy using Newtonian dynamics, since they don’t emit significant amounts of gravitational radiation.
The eigenfunctions of the f-modes have no nodes inside the star, reaching a maximum at the surface of the star. Lindblom & Detweiler (1983) calculated quadrupole f-modes for a series of neutron stars characterized by different equations of state (see also Kokkotas & Schutz 1992). Frequencies are in the range 1.5-4.0 kHz and the damping timescales are of the order of few tenths of second. In a first approximation, f-mode frequencies and damping timescales can be estimated from the formulae

\[ \nu_{kHz} = 46.13M^{0.255}R^{-1.32} \]  

and

\[ \tau_{sec} = 5.164 \times 10^{-3}R^{1.60}M^{-0.813} \]  

where M and R are in solar units and km respectively.

As mentioned above, other oscillation modes are present in a real star. Pressure is the restoring force for p-modes and frequencies associated to these modes depend on the travel time for acoustic waves to cross the star. These frequencies are higher than 5 kHz and the oscillations are damped in timescales longer than those of f-modes. The g-modes, restored by gravity, depend critically on the internal composition and temperature profile, having frequencies typically of the order of few hundred Hz. The interplay of all these modes in a neutron star is quite complex, since these objects have a solid crust and a central fluid interior. The modes f, p and g belong to the class of polar modes, but if the shear modulus in the crust is non-zero, axial modes should exist as well as families (i-modes) associated with the interface between distinct phases of the matter inside neutron stars (McDermott et al. 1988). Besides these "Newtonian-modes", there is a class of modes uniquely associated with perturbations of the spacetime itself, the so-called w-mode (Kojima 1988; Kokkotas & Schultz 1992). These gravitational wave modes arise because of the trapping of GW by the spacetime curvature generated by the background density. The w-modes exist for both polar and axial perturbations since they do not depend on perturbations of a fluid. These modes have high frequencies (above 7 kHz) and are damped in timescales shorter than one millisecond. They are probably the natural way to recover any initial deformation of the spacetime, as those expected to occur during the gravitational collapse, leading to black hole formation.

One of the problems concerning the gravitational wave emission from oscillating neutron stars is the absence of a convincing mechanism able to excite those modes. Once the solid crust is formed, stresses will exist by the presence of a strong magnetic field or/and rotation. These stresses induce tectonic activity and the stored elastic energy may be released as a consequence of quakes. The elastic energy is converted into shear waves that excite nonradial oscillation modes damped by GW. However, the maximum elastic
energy that can be channeled into oscillating modes is likely to be about $10^{45}$ erg, restricting considerably the detection of gravitational wave sources excited by tectonic activity.

The maximum distance that a given source can be detected by a gravitational wave antenna can be estimated by the following considerations. After filtering the signal, the expected S/N ratio is given by the equation

$$\frac{(S/N)^2}{2} = 4 \int_0^\infty \frac{\langle \tilde{h}(\nu) \rangle^2}{S_n(\nu)} d\nu$$

(17)

where $\tilde{h}(\nu)$ is the Fourier transform of the signal, here a sinusoidal damped oscillation of amplitude $h_0$, and $S_n(\nu)$ is the noise power spectrum of the detector. Performing the required calculations, the S/N ratio can be written as

$$\frac{(S/N)^2}{2} = 4 \frac{h_0^2}{\tau_f} \frac{Q^2}{S_n(\nu_0)}$$

(18)

where $\tau_f$ is the damping time, $Q = \pi \nu_0 \tau_f$ is the quality factor of the oscillation and $\nu_0$ is the frequency of the considered mode. In the above equation, the angle averaged beam factors of the detector were already included. The amplitude of the signal is related with the total released energy $E$ by the equation

$$h_0 = \frac{2 \pi \nu_0}{c} \left( \frac{GE}{\tau_f c^3} \right)^{1/2}$$

(19)

where $r$ is again the distance to the source. From these equations, once the energy and the S/N ratio are fixed, the maximum distance to a given source that a gravitational antenna can probe can be estimated. Using the planned sensitivities of VIRGO and LIGO as well as the oscillation properties of neutron star models calculated by Lindblom & Detweiler (1983), it is clear that sources excited by starquakes cannot be detected beyond distances of about $2.5$ kpc (de Freitas Pacheco 1998).

Another possible excitation mechanism of nonradial oscillations is the micro-collapse suffered by a neutron star when a phase transition occurs in the core, caused by a softening of the equation of state. There are presently several arguments in favor of a stiff equation of state for the nuclear matter:

Firstly, high frequency quasi-periodic oscillations (QPOs) have been found in the X-ray emission originated from the accretion disk around neutron stars associated to low-mass binaries (see van der Klis 2000, for a review). These oscillations are observed in the range $300$-$1300$ Hz, and are often splitted into pairs with a nearly constant separation of $250$-$350$ Hz. A possible interpretation of such a modulation in the X-ray emission is the presence of instabilities ("hot spots") near the "last stable" orbit ($r = \frac{6GM}{c^2}$) and modulated by the
local orbital frequency. In this case, the mass of the neutron star is directly related with the orbital frequency by

\[
M = \frac{1}{(216)^{1/2}} \frac{c^3}{G \Omega_{\text{orb}}} \approx 2.2 \nu_{\text{Hz}}^{-1} M_\odot
\]  

(20)

If this interpretation is correct, neutron stars with masses up to 2.0 M_\odot are present in those low-mass X-ray binaries as a consequence of the accretion process. Moreover, the analysis of the orbital motion of some massive X-ray binaries seems also to suggest the presence of massive neutron stars (for instance, 1.8 M_\odot for 4U1700-37 and 1.75 M_\odot for Vela X-1). These high mass values favor stiff equations of state (Akmal et al. 1998). Secondly, post-glitch recovery analyzes of isolated pulsars are not consistent with soft equations of state (Link et al. 1992; Alpar et al. 1993).

Masses of neutron stars derived from NS-NS binaries cluster around 1.4 M_\odot. Neutron star models of 1.4 M_\odot built with stiff and moderately stiff equations of state suggest central densities of about (4-5)n_0 (de Freitas Pacheco et al. 1993; Akmal et al. 1998), where n_0 = 0.16 fm^{-3} is the nuclear saturation density. These densities are lower than the critical density for kaon condensation, if kaon-nucleon and nucleon-nucleon correlations are taken into account (Pandharipande et al. 1995). In this case, according to those calculations, the kaon condensation should only occur for densities around (6-7)n_0. The appearance of a kaon condensate will soften the equation of state reducing the maximum stable mass, yielding the star unstable (however, there are no evidences in favor of the presence of black holes in those low-mass X-ray binary systems!). It is worth mentioning that the deconfinement of cold matter is expected to occur around (7-9)n_0 (de Freitas Pacheco et al. 1993). If the neutron star accretes mass, the central density increases and may eventually to overcome the critical density necessary to occur a phase transition. Further increase of the neutron star mass leads the star into a metastable situation, until the occurrence of a structural transition of the whole star into a new configuration of minimum energy. Consider, for instance, the appearance of quark matter in the core. Since the energy density of the quark matter is higher than that of the hadron matter, the star must to contract (in a dynamical timescale) in order to extract gravitational energy, which will provide the "latent" heat of the phase transition. In the "metastable" hadron state, a 1.81 M_\odot neutron star has a central density n_c = 7.6n_0 and a radius R = 9.9127 km, while after the micro-collapse the radius is reduced by 5.2 m and a quark core matter of 0.44 km radius is formed. The event releases a total gravitational energy of about 4.7 \times 10^{50} \text{erg}, from which 85% go to the phase transition and, about 7 \times 10^{49} \text{erg} are converted into heat and/or into mechanical energy (de Freitas Pacheco 1999). In the ideal case, if the latter amount of energy is channeled into the quadrupole f-mode (the coupling with rotation favors the excitation of nonradial modes), the emitted GW from the considered neutron star model will have a frequency of
2.6 kHz and the corresponding damping timescale will be \( \tau_f = 0.124 \text{ s} \). Using equations (18) and (19), the maximum distance that these signals can be detected by the present generation of interferometers is about 85 kpc, which includes all sources in the Milky Way and in the Magellanic Clouds.

3. Gravitational Waves from Black Holes

Black holes are expected to exist in our universe with masses ranging from a few solar masses up to \( 10^{10} \text{ M}_\odot \). Stellar mass black holes may be formed in the core collapse of massive stars, by accretion either of a massive neutron star or a small hole, by merging of two neutron stars at the end-point of their inspiral in a binary system, whereas very massive holes, probably present in a considerable number of galactic nuclei, can be formed by different routes.

Recent 2D-hydrodynamic simulations of core collapse (Fryer 1999) for a large range of progenitor masses, indicate that partial fallback of the envelope drives the compact core to collapse into a black hole. This occurs for progenitors with masses \( M > 20 \text{ M}_\odot \) while progenitors more massive than \( 40 \text{ M}_\odot \) form black holes directly. In spite all the uncertainties present in those simulations, they permit nevertheless an estimate of the mass distribution and of the formation rate of these objects.

Newly formed black holes are likely to be quite "deformed". They need to radiate away energy in order to settle down into a quiescent and axisymmetric state characterized only by their mass \( M \) and angular momentum \( J \) (a Kerr black hole). Detailed calculations suggest that black hole oscillations are easy to trigger and that quadrupole modes dominate the emission (Stark & Piran 1985a, 1985b). The waveform of the initial burst depends on the details of the collapse, but the late-time behavior (the "ring-down" phase) has a well established damped oscillatory form, which is essentially a function of \( M \) and \( J \) (see, for instance, Echeverria 1989; Finn 1992). For the mode \( l = m = 2 \), using the results of Echeverria (1989), the frequency and the damping timescale are respectively estimated from

\[
\nu_{gw} \approx 12 \frac{M_\odot}{M_{bh}} F(a) \text{ kHz}
\]

and

\[
\tau_{gw} \approx 5.55 \times 10^{-5} \frac{M_{bh}}{M_\odot} K(a) \text{ s}
\]

where \( a = \frac{Jc}{GM^2} \) and useful fitting formulae for the functions \( F(a) \), \( K(a) \) are

\[
F(a) \approx \frac{100}{37} - \frac{63}{37} (1 - a)^{0.3} \quad \text{and} \quad K(a) \approx (1 - a)^{-0.45}
\]
The amplitude of the signal can be written as

\[ h_0 = \frac{2.3 \times 10^{-21}}{D_{\text{Mpc}}\sqrt{F(a)Q}} \left( \frac{\varepsilon}{10^{-4}} \right)^{1/2} \left( \frac{M_{\text{bh}}}{M_{\odot}} \right) \]  

(24)

where \( Q \) is again the quality factor of the oscillation, \( D_{\text{Mpc}} \) is the distance to the source in megaparsec and the radiation efficiency was defined such as \( E = \varepsilon M_{\text{bh}} c^2 \) be the total gravitational wave energy emitted by the source. Equations (24) and (18) permit an estimate of the maximum distance that a given event could be detected, once the S/N ratio, the radiation efficiency, the mass and the angular momentum of the hole are fixed. Here, estimates were performed for a typical black hole mass of \( 9.0 M_{\odot} \), a radiation efficiency \( \varepsilon = 10^{-4} \) and a S/N ratio equal to two. The results are given in table 1 for two values of the angular momentum parameter \( a \).

Inspection of table 1 shows that the detection of slow rotating holes are more favorable, in spite of fast rotating holes have a higher oscillation quality factor. The reason is that the fundamental quadrupole frequency increases for higher angular momentum values and the sensitivity of most interferometers decreases for \( \nu > 1 \) kHz. A serious handicap to detect these events is that the signal is completely damped out after only a few cycles and could easily be confounded with transient disturbances produced, for instance, by the suspension of the mirrors. Moreover, the expected maximum distances for the present planned sensitivity of VIRGO and LIGO imply quite small detection rates and, only for the advanced LIGO a high event rate should be expected, namely, about one event/month under the assumption that all stars with \( M \geq 40 M_{\odot} \) produce black holes.

4. Binary Systems

The gravitational wave emission resulting from the merger of two compact stars (NS-NS, NS-BH, BH-BH) is a very attractive possibility due to the huge energy power implied in the process. This mechanism was already discussed in the late seventies by Clark.

### Table 1: Maximum Probed Distances (Mpc)

| \( a \) | \( Q \) | \( \nu_{gw} \) (kHz) | VIRGO | LIGO | LIGO-ad |
|------|------|-----------------|-------|------|---------|
| 0.2  | 2.57 | 1.48            | 1.25  | 0.43 | 83      |
| 0.9  | 10.9 | 2.46            | 0.44  | 0.16 | 9       |
et al. (1979), who suggested that gravitational antennas could be able to probe the universe up to distances of about 200 Mpc. Here, the case of a binary system constituted by two neutron stars (NS-NS system) will be discussed in more detail.

In the late inspiral phase, which precedes the final coalescence, relativistic effects or post-Newtonian (PN) corrections are necessary in order to describe the waveform and the amplitude of the signal (Damour & Deruelle 1981; Blanchet & Sathyaprakash 1994, 1995). The main features of the GW emission during the inspiral motion are an increasing radiated power and a wave frequency equal to twice the orbital frequency. The ”last” stable orbit imposes a limit to the maximum wave frequency (see eq. (20), with M now being the total mass of the system). For a binary system constituted by two ”canonical” neutron stars (1.4 M_☉) such a limit frequency is about 1.6 kHz. As a consequence, during the epoch when the GW emission is in the bandwidth where the sensitivity of VIRGO or LIGO is the greatest, the motion is still well described by the Newtonian picture, whereas PN corrections become important at frequencies where the sensitivity of those detectors is already considerably reduced.

After averaging both polarization components with respect to the inclination angle of the orbital plane, the Fourier transform of the signal is

$$|\vec{h}(\nu)|^2 = <|\vec{h}_+ (\nu)|^2 + |\vec{h}_\times (\nu)|^2> = \frac{1}{12} \left(\frac{G^5 M^2}{\pi^4 c^9}\right)^{1/3} \frac{\mu}{r^2 \nu^{7/3}}$$

(25)

where M is the total mass of the system and μ is the reduced mass. Combining eqs.(17) and (25), the maximum distance probed by the detector can be estimated by assuming M = 2.8 M_☉ and S/N = 2. For VIRGO, r_max = 46 Mpc and for LIGO, r_max = 36 Mpc, if the planned sensitivity of these detectors is used.

The maximum distance permits to evaluate the expected event rate if the coalescence rate is known. Regimbau (private communication) has recently reviewed the formation rate of NS-NS systems. She assumed an initial binary system constituted by progenitors with masses greater than 9 M_☉. The more massive evolves faster, explodes and produces a neutron star. Then, as the less massive star evolves it loses mass, affecting the rotation period of the first formed pulsar. The evolving star becomes a ”He-star” and explodes probably as a type Ib supernovae. The system is supposed to remain bounded after both explosions. If evolution of the newly formed pulsar is not affected by external torques other than the canonical magnetic dipole, then its rotation period will have a secular increase similar to that observed for ”isolated” pulsars. Table 2 summarizes the present census of NS-NS systems. The first column identifies the pulsar and the others give respectively the rotation period in ms, the logarithm of the period derivative, the orbital period in days, the orbital eccentricity and the total mass of the system. Notice that for B1820-11 only
a lower limit for the total mass is known since the rate of the relativistic advance of the longitude of the periastron is poorly determined. All the systems, excepting PSR B1820-11, seem to contain recycled pulsars, indicating that the first formed neutron star is the one being observed. On the other hand, in the case of B1820-11, the period derivative, the magnetic field and the indicative age favor the interpretation of a young and non-recycled pulsar. However, the nature of the companion is not yet well established. Lyne & McKenna (1989) suggested that the system is indeed constituted by two neutron stars, but alternative possibilities like a main-sequence star (Phinney & Verbunt 1991) or a white dwarf (Portegies Zwart & Yungelson 1999) have also been discussed in the literature. According to the simulations by Regimbau, in order to observe one system like B1820-11, the present formation rate \( R_b(T) \) of NS-NS binaries must be equal to \( 1.2 \times 10^{-5} \text{ yr}^{-1} \), consistent with other recent estimates (Kalogera & Lorimer 2000).

The coalescence rate can now be calculated following the procedure by de Freitas Pacheco (1997). The first step is to compute the fraction \( \gamma_b \) of formed stars giving rise to NS-NS systems. The formation rate of NS-NS systems can be written as

\[
R_b(t) = \gamma_b k M_g(t) \int_{m_1}^{m_2} \xi(m) dm
\]

where \( k M_g(t) \) is the star formation rate (\( k \) in \( \text{yr}^{-1} \) is the star formation efficiency and \( M_g(t) \) is the gas mass in solar masses in the Galaxy), \( \xi(m) \) is the initial mass function and \( m_1 = 0.1M_\odot \), \( m_2 = 80M_\odot \) are respectively the minimum and the maximum stellar masses. Adopting a galactic age \( T=15 \text{ Gyr} \) and using the NS-NS formation rate estimated above, from eq. (26) one obtains for the fraction of formed NS-NS binaries \( \gamma_b = 2.6 \times 10^{-6} \). Then, assuming \( \gamma_b \) constant in time, the second step consists to compute the coalescence rate \( \varpi \)

| PSR       | P (ms) | \( \log(\dot{P}) \) | \( P_{\text{orb}} \) (days) | e   | \( M/M_\odot \) |
|-----------|--------|---------------------|-----------------------------|-----|--------------|
| B1913+16  | 59.03  | -17.10              | 0.323                       | 0.617 | 2.83         |
| J1518+49  | 40.94  | -19.40              | 8.634                       | 0.248 | 2.62         |
| B1534+12  | 37.90  | -17.60              | 0.420                       | 0.274 | 2.68         |
| B2127+11C | 30.53  | -17.30              | 0.335                       | 0.681 | 2.71         |
| J1811-17  | 104.18 | -17.74              | 18.779                      | 0.828 | 2.60         |
| B1820-11  | 279.83 | -14.86              | 357.76                      | 0.795 | >0.54        |
from the equation (de Freitas Pacheco 1997)
\[ \varpi(t) = \int_{t_0}^{t} R_b(t - t')\phi(t')dt' \] (27)

where \( \phi(\tau) = \frac{B}{\tau} \) is the probability for a NS-NS system to merge in a timescale \( \tau \) and \( t_0 \) is the minimum time required for a given pair to coalesce. Solving the above integral, the resulting coalescence rate in the Galaxy at present is \( \varpi(T) = 3.0 \times 10^{-5} \text{ yr}^{-1} \). This procedure can now be applied to an elliptical galaxy, assuming the same value of \( \gamma_b \), but a higher efficiency for the star formation rate. Then, a "cosmic" mean coalescence rate can be estimated by using a weighted contribution of 20% of ellipticals and 80% of spiral galaxies. These calculations give \( \varpi = 7.0 \times 10^{-7} \text{ Mpc}^{-3} \text{yr}^{-1} \), from which an event rate of one merging every 3.5 yr is expected for VIRGO and one every 7.3 yr for LIGO. Clearly, reasonable event rates will only be obtained with the "enhanced" versions of these detectors. However, it is worth mentioning that already a gain of a factor of two in the sensitivity implies an increase of one order of magnitude in the probed volume and the consequences of such an achievement may drastically change the situation. It seems to be rather well established now that at a distance of about 60 Mpc, in the direction of Centaurus, there is a huge concentration of galaxies dubbed the Great Attractor (Burstein 1990). The estimated mass of this supercluster is about \( 5 \times 10^{16} \text{ M}_\odot \) (but see Staveley-Smith et al. 2000), implying a coalescence rate of 11 events/yr from the Great Attractor alone. However, we should have in mind the uncertainties still present in all these estimates, once they are based on rather poor statistics of NS-NS pairs.

5. The Stochastic and the Cosmological Background

The stochastic background is the GW emission arising from a very large number of unresolved and uncorrelated sources. A quantity which is often used to measure whether the collective effect of bursts generates a continuous background or not, is the so-called duty cycle defined by
\[ D(z) = \int_{0}^{z} \tau_s(1 + z')dR_s(z') \] (28)

where \( \tau_s \) is the average duration of a single burst at emission, \( R_s(z) \) is event rate at redshift \( z \) and the factor \( (1+z) \) takes into account the time dilation. If \( D<<1 \), the background cannot be considered as a continuous one, but should rather be seen as a shot noise process. Supernovae (Blair et al.1997) and distorted stellar black-holes (Ferrari et al. 1999a; de Araújo et al. 2000) are examples of sources which may produce a shot noise like background. On the opposite situation, when \( D>>1 \) a truly continuous background will be produced, as
that due to the r-mode emission from neutron stars (Owen et al. 1998; Ferrari et al. 1999b) or from processes that took place very shortly after the big-bang.

In order to compare the importance of the GW background with other forms of energy which dominate the expansion of the universe, it is usual to define an equivalent energy density parameter \( \Omega_{gw} \) associated to GW by the relation

\[
\Omega_{gw}(\nu) = \frac{1}{\rho_c} \frac{d\rho_{gw}}{d\ln \nu}
\]

(29)

where \( \rho_c = \frac{3c^3H_0^2}{8\pi G} \) is the critical density and \( \rho_{gw} \) is the energy density under the form of gravitational waves.

Primordial nucleosynthesis and millisecond pulsars impose relevant constraints on the closure density due to GW. The outcome from primordial nucleosynthesis depends on the balance between the nuclear reaction rates and the expansion of the universe. Thus, in order not to spoil the predict abundances of \(^2\text{H}, ^3\text{He}, ^4\text{He} \) and \(^7\text{Li} \), which are in agreement with present data, the energy density under the form of GW cannot exceed a certain limit. This bound is usually established in terms of an effective number of neutrino species \( N_\nu \) and is given by (Kolb & Turner 1990)

\[
\int \Omega_{gw}(\nu)d\ln \nu \leq 1.3 \times 10^{-5}(N_\nu - 3)
\]

(30)

if the Hubble expansion parameter \( H_0 \) is taken to be equal to 65 km/s/Mpc. Primordial nucleosynthesis constrains the number of massless neutrinos to be \( N_\nu \leq 3.2 \), restricting the background spectrum of GW over a wide range of frequencies to have an equivalent amplitude \( \Omega_{gw} < 10^{-6} \). Notice that this is not a very restrictive constraint.

Millisecond pulsars are very precise and stable clocks. The regularity of the pulses can be described in terms of timing residuals, which represent the errors in predicting the arrival time of these pulses. GW passing between the pulsar and the observer cause a fluctuation in the arrival time proportional to the amplitude of the perturbing waves. Eight years of monitoring of PSR B1855+09 give an upper limit at \( \nu \sim 10^{-8} \) Hz of \( \Omega_{gw} < 10^{-8} \).

It is interesting to compare these upper bounds with the expected sensitivity of laser beam interferometers. At lower frequencies, \( \nu \sim 1 \) mHz, LISA will be able to detect after one year integration, a GW background corresponding to a closure density \( \Omega_{gw} \sim 10^{-12} \). The present generation of ground based interferometers like VIRGO and LIGO, after one year integration and using signal correlation techniques, may detect a GW background equivalent to \( \Omega_{gw} \sim 10^{-5} \), while the next generation will be able to detect at frequencies in the range 0.1 - 1.0 kHz, amplitudes several orders of magnitude smaller, namely, \( \Omega_{gw} \sim 10^{-10} \) (Maggiore 2000).
The GW emission from core collapse (see section 2.2) is likely to be of the order of \(10^{-8} \text{ Me}^2\). Thus, the expected contribution to the stochastic background will be about \(\Omega_{gw} \leq 10^{-15}\), an extremely small value in comparison with the expected amplitude from other sources. The GW stochastic background due to black hole formation after the core collapse of a massive star was estimated by Ferrari et al. (1999a) and de Araújo et al. (2000). Both studies conclude that the spectral energy distribution attains a maximum around 1 kHz with an amplitude corresponding to a closure density of about \(\Omega_{gw} \approx 10^{-10}\). This result indicates that such an emission will not be detected by the present generation of laser beam interferometers but it is within the capabilities of the advanced versions. The GW background resulting from the r-mode instability in young and rapidly rotating neutron stars may be a more promising possibility. According to the computations by Ferrari et al. (1999b), the closure density has a maximum amplitude plateau of \(\Omega_{gw} \sim 7 \times 10^{-8}\) in the frequency range 0.5 - 1.7 Hz, which is about two orders of magnitude above the expected detection threshold for the advanced interferometers.

Different physical processes which could have taken place in the early universe and which produce a continuous GW background, have been discussed in the literature. The first mechanism consists on GW generated by quantum perturbations in a inflationary scenario. As the universe cools, it passes through a phase dominated by the vacuum energy, where the scale factor increases exponentially, followed by a rapid transition to a radiation dominated phase. GW quanta are produced by quantum fluctuations during such a transition and the resulting closure density is (Allen 1997)

\[
\Omega_{gw} = \frac{16hG^2}{9c^7}\rho_v
\]

where \(\rho_v\) is the energy density of the vacuum in the inflationary phase. The spectrum is flat at frequencies seen by LISA or ground based interferometers like VIRGO and LIGO, and the expected amplitude is of the order of \(\Omega_{gw} \sim 10^{-13}\). This value is well below the expected detection limit of LISA and ground based advanced interferometers. Another scenario was developed by Vilenkin (1985) (see also Vilenkin & Shelard 1994), who considered the GW background produced by a network of cosmic strings formed during a symmetry breaking phase transition. This scenario offers a situation more favorable for a future detection. The spectrum is almost flat in the frequency range \(10^{-9}-10^9\) Hz with an amplitude \(\Omega_{gw} \sim 10^{-8}\). The existence of this possible background component can be confirmed or not by a continuous monitoring of the timing residuals of millisecond pulsars, by future space observations as well as by the next generation of laser interferometers. More recent studies in this domain (Damour & Vilenkin 2000) seem to indicate that the stochastic background produced by strings is non-Gaussian, including occasional bursts emanating from cusps, that stands above the mean amplitude. According to those authors,
the GW bursts might be accompanied by gamma-ray bursts, what could be a decisive signature of such a mechanism.

A more fascinating possibility is related with theories which describe our world as a (mem)brane embedded in a higher dimensional space, having the purpose of solving the huge energy gap that separates the electroweak scale from the Planck scale, the so-called hierarchy problem (Randall & Sundrum 1999). In this scenario, GW are produced by coherently excited radions (geometrical degree of freedom controlling the size or curvature of the extra dimensions) and Nambu-Goldstone modes (Hogan 2000). According to estimates by Hogan (2000), the typical GW frequency is $\nu \sim 10^{-4}$ Hz and the amplitude corresponds to a closure density of about $\Omega_{gw} \sim 8 \times 10^{-5}$. Notice that this amplitude is higher than the bound imposed by primordial nucleosynthesis.

6. Concluding Remarks

The direct detection of GW will constitute an extraordinary scientific accomplishment, giving further support to the General Relativity Theory, opening a new window to explore the Universe.

GW from oscillating neutron stars excited by core phase transitions, from unstable newly born proto-neutron stars (r-modes) or from the last phases of the coalescence of a NS-NS binary, will give an important insight on the properties of the nuclear matter in these objects. Moreover, the eventual detection of GW from the ”ringdown” phase of a newly formed black hole will permit to probe directly their properties like mass and angular momentum. The detection rate of these events, if estimated by the future advanced interferometers, will impose severe constraints on the last evolutionary phases of massive stars.

The Universe is highly transparent to GW and the detection of a background will probe the very early phases of the big-bang, when radiation and matter are still strongly coupled. Modern unified theories can also be tested as well as the spacetime structure of our world.

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