Title: Day-to-Day Travel Time Perception Modeling Using an Adaptive-Network-Based Fuzzy Inference System (ANFIS)

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Abstract:
Travel time perception and learning play a central role in the modeling of day-to-day travel choice dynamics in traffic networks and have attracted the attention of many researchers, specifically for the analysis and operation of intelligent transportation systems and travel demand management scenarios. In this paper, a fuzzy learning model is proposed to capture the mechanism by which travelers update their travel time perceptions from one day to the next, taking into account their experienced travel times. In order to capture travelers’ mental representations of uncertain travel time involving imprecision and uncertainty, a combined artificial neural network and fuzzy logic (neuro-fuzzy) architecture called Adaptive-Network-based Fuzzy Inference System (ANFIS) is employed. This framework, which utilizes a set of fuzzy if-then rules, can serve as a basis for modeling the qualitative sides of travelers’ knowledge and reasoning processes. From the output of this study, the results of our laboratory-like experiment provide a good fit to the stated data of travelers’ behavior, and may reflect the fact that the neuro-fuzzy approach can be considered a promising method in learning and perception updating models. Finally, the proposed learning model is embedded in a microscopic event based simulation framework to evaluate its credibility within a day-to-day behavior of the traffic network. The results of the simulation, which converge to the equilibrium state of the test network, are finally presented, implying that the proposed perception-updating model operates properly.

Keywords: Fuzzy inference; Artificial neural networks; Day-to-day dynamics; Driver behavior; Drivers’ perception updating; Drivers’ learning.

MSC codes: 90B06 Transportation, logistics; 03E72 Fuzzy set theory; 68T37 Reasoning under uncertainty; 91A26 Rationality, learning.

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Introduction

The day-to-day modeling of commuter route choice is of considerable importance to the analysis of congestion management strategies and short-term impacts of variations in the demand or supply systems. In fact, day-to-day (or inter-periodic) modeling approaches appear to be more appropriate for capturing day-to-day traffic fluctuations and evolution processes (He et al. 2010), especially for evaluating the appropriateness of intelligent transport systems scenarios (c.f. Khademi et al. 2010) such as congestion pricing plans, pre-trip or en-route traveler information facilities, and route guidance systems.

Travel time perception and learning models play an essential role in the modeling of day-to-day commuter decisions. As a result, modeling and understanding the connection between the travelers’ perceived travel time acquired from the learning process and the time-dependent nature of traffic flows are of interest to transportation researchers and practitioners. However, one cannot neglect the presence of travel time uncertainty and its major influence on travelers’ decisions.

Travel time uncertainty is an inseparable characteristic from travelers’ cognition and perception processes. In fact, although events experienced by the traveler are precise, the human mind is always revising them in an approximate way (Dell’Orco and Teodorovic 2009). It should be mentioned that in a choice under risk, the probability distribution of the potential outcomes is known; in contrast, under uncertainty (or ambiguity), this distribution is unknown to the decision maker (de Palma et al. 2008). More often than not, for a normal traveler, route choice decisions are the decisions under uncertainty instead of risk. This is because, as stated by Zerguini et al. (2011), travelers, for the most part, do not have perfect knowledge of the objective distribution of travel time.

While a lot of mathematical theories have been developed to cover imprecision, inexactness, ambiguity, and uncertainty, the fuzzy theory (Zadeh 1965) is known as one of the most popular and successful methods in capturing the inherent uncertainties in human decisions. According to fuzzy theory (Zadeh 1965), people’s mental representation of an uncertain value like travel time is an interval rather than an exact value, and the possibility measure of the variable is defined for the interval (Fujii and Kitamura 2004). During the last decade, several studies have been performed based on fuzzy logic and fuzzy reasoning, which point out that the route choice process involves fuzziness, imprecision, vagueness, or uncertainty similar to every mental activity (Henn 2005). Modeling the traveler's decision making with fuzzy theory or possibility theory has frequently been proposed in the literature and the models were found to provide a good fit to the data of travelers’ behavior (e.g. Henn and Ottomaneelli 2006; Teodorovic 1994, 1999; Arslan and Khisty 2006).

Although many studies have incorporated fuzzy concepts into the transportation network analysis, to the best of the authors’ knowledge, no study has been reported in the literature for modeling fuzzy learning and the adaptation of travelers’ responses to the day-to-day dynamics of network parameters.

From this perspective, the aim of this paper is to suggest a novel combined artificial neural network and fuzzy logic (neuro-fuzzy) architecture called Adaptive-Network-based Fuzzy Inference System (ANFIS), which can serve as a basis for utilizing a set of fuzzy if-then rules with proper membership functions to produce the stipulated input-output pairs of the traveler behavior learning process. Based on the pioneer article (Jang 1993) of the architect of ANFIS, this method has the advantage of being able to model the qualitative sides of human knowledge and reasoning processes without employing precise quantitative analyses.

The proposed traveler’s learning model constructed in this paper by ANFIS is a fuzzy inference system capturing the imprecise aspects of reasoning that play an essential role in the human ability to make decisions in an environment of uncertainty and imprecision.
The rest of this paper is structured into four major sections. The first section provides a brief description of ANFIS. An attempt is made in this section to describe the ANFIS architecture as simply as possible so that it can be understandable for non-specialists in this field. The second section outlines the proposed learning model, which considers the travelers’ mental representations of an uncertain travel time as a triangular fuzzy number. The modal value of this triangular fuzzy number is determined by ANFIS. The next section provides a description of the laboratory-like experiment conducted in the Transportation Research Center of Iran University of Science and Technology (IUST) and among 100 students. Through this experiment, the proposed ANFIS based learning model is fine-tuned based on the stated data received from these students. Finally, in the last section, a route choice simulation on a test network is held to evaluate the quality of the learning model.

The results of our conducted laboratory-like experiment provide a good fit to the stated data of the participants and may reflect the fact that the neuro-fuzzy approach can be considered a promising method in learning and perception updating models.

**Literature Review on the Modeling of the Travelers’ Perception Updating Mechanisms**

A perception-updating model is one of the most important components of day-to-day route choice models where considerable advances have been made in this realm of modeling so far. In the last two decades, several studies have focused on the modeling of the traveler’s perception within the context of the day-to-day modeling of transportation networks. In a classification proposed by Yannaz-Tuzel and Ozbay (2009), the learning process of travelers is categorized into three classes: i) Reinforcement learning (e.g. see Nakayama et al. 2001; Arentze and Timmermans 2003; Avineri and Prashker 2003; Bogers et al. 2007), ii) Bayesian learning (c.f. Chen and Mahmassani 2004; Jha et al. 1998), and iii) Stochastic learning automata (e.g. in Ozbay et al. 2001).

Table 1 presents a brief review on some of the perception updating models. It should be mentioned that most of these models in this table do not capture the uncertainty involved in the traveler’s perception. Moreover, only few of them have attempted to calibrate their models using empirical data.

The current study attempts to address the above-mentioned gaps through developing a framework that employs the fuzzy set theory to model the behavioral aspects of the human perception of travel time. In fact, by exploiting the fuzzy set theory, it is possible to capture the uncertainty involved in traveler’s perception. Furthermore, the ANFIS is employed to calibrate the parameters of the proposed model.

**Adaptive-Network-Based Fuzzy Inference System**

The adaptive-network-based fuzzy inference system (ANFIS) is a useful neural network method for the solution of function approximation problems (Buragohain and Mahanta 2008). An ANFIS yields the mapping relation among the input and output data through a hybrid learning method that determines the optimal distribution of membership functions (Chen et al. 2010).

Both fuzzy logic (Zadeh 1965) and artificial neural networks (Karayiannis and Venetsanopoulos 1993) are tools that could share a common ability to deal with uncertainties (Yager and Zadeh 1994). Both encode information in parallel and distribute architectures in a numerical framework. Therefore, it is possible to convert fuzzy logic architecture to an artificial neural network and vice versa. This makes it possible to combine the advantages of the artificial neural network and fuzzy logic. A network obtained in this way uses powerful training algorithms that artificial neural networks contain and, moreover, it leverages fuzzy logic
interpretation capabilities in terms of linguistic variables (Shoorehdeli et al. 2009). Hence, ANFIS is a combination of fuzzy logic and artificial neural networks, and holds the advantages of both in the sense that the membership functions and rules of the fuzzy systems are defined and optimized by artificial neural networks (Nguyen et al. 2003, Ch. 7).

Table 1. A Brief Summary of Traveler’s Perception Updating Models

| Author(s)               | Input variables                                                                 | Perception updating mechanism                  | Uncertainty consideration                      |
|------------------------|--------------------------------------------------------------------------------|------------------------------------------------|-----------------------------------------------|
| Horowitz (1984)        | - Experienced travel times - Weight set                                        | Weighted average of experienced travel times    | Deterministic perceived travel time           |
| Cascetta (1989)        | - Experienced travel times - Weight set                                        | Weighted average of experienced travel times    | Deterministic perceived travel time           |
| Mahmassani and Chang   | - Last experienced travel time - Schedule delay                                 | Linear combination of input variables           | Deterministic perceived travel time           |
| Iida et al. (1992)     | - Last experienced travel time - Difference between the last experienced and the perceived travel times | Linear combination of input variables           | Deterministic perceived travel time           |
| Nakayama et al. (1999) | - Minimum perceived travel times - Maximum perceived travel times - Experienced travel times - Weight set | Weighted average of experienced travel times    | Select a deterministic perceived travel time from a set of perceived travel times randomly |
| Nakayama and Kitamura  | - Last experienced travel time - Last perceived travel time                   | Convex combination of input variables           | Deterministic perceived travel time           |
| Nakayama et al. (2001) | - Average of experienced travel times - Difference between the minimum and the maximum experienced travel times | Linear combination of input variables           | Deterministic perceived travel time           |
| Oh et al. (2003)       | - Last experienced travel time - Last perceived travel time                   | Weighted average of experienced travel times    | Deterministic perceived travel time           |
| Chen and Mahmassani    | - Prior updated travel time - Variance of prior updated travel time - Sample mean of experienced travel times - Sample variance of experienced travel times | Bayesian updating model                         | Perceived mean and variance of travel time    |
| Ettema et al. (2005)   | - Experienced travel times - Weight set                                        | Weighted average of experienced travel times    | Perceived mean and variance of travel time    |
| Jotisankasa and Polak  | - Last perceived travel time - Difference between the last experienced and perceived travel times | Linear combination of input variables           | Deterministic perceived travel time           |
| Bogers et al. (2007)   | - Last experienced travel time - Last perceived travel time                   | Convex combination of input variables (i.e. belief learning) | Deterministic perceived travel time           |
| Kim and Lim (2012)     | - Experienced travel time distribution                                         | Simplifying the experienced travel time distribution | Simplified perceived travel time distribution (cognitive map) |

ANFIS is known as a promising approach to model complex phenomena. During the past two decades, several studies have employed ANFIS in the realm of transportation engineering (see Table 2).
Table 2. A Brief Summary of the ANFIS and Fuzzy-Nural Applications in Transportation Engineering

| Author(s)                               | Application                              | Number of input variables | Type of membership function | Number of rules | Size of data set | Training algorithm |
|-----------------------------------------|------------------------------------------|---------------------------|----------------------------|----------------|-----------------|--------------------|
| Dell’Orco and Ottomanelli (2012)        | Mode choice modelling                    | 8                         | N/A                        | 3              | 361             | Backpropagation    |
| Adeli and Jiang (2003)                  | Freeway work zone capacity estimation    | 18                        | Gaussian                   | N/A            | N/A             | Backpropagation algorithm |
| Pribyl and Goulias (2003)               | Analysis of the travel behavior          | 4                         | Gaussian                   | 96             | 1438            | Backpropagation algorithm |
| Hawas (2004)                            | Route choice modeling                    | 6                         | Gaussian and triangular    | N/A            | N/A             | Hybrid algorithm   |
| Andrade et al. (2006)                   | Mode choice modeling                     | 9                         | Trapezoidal                | 144            | 960             | Hybrid algorithm   |
| Murat (2006)                            | Vehicle delay modeling                   | 4                         | Gaussian                   | 8              | 85              | Hybrid algorithm   |
| Postorino and Versaci (2008)            | Mode choice modeling                     | 12                        | Gaussian                   | 16             | 500             | N/A                |
| Mucsi et al. (2011)                     | Estimating the number of vehicles in a detection zone | 3 | Trapezoidal | 27 | 14400 | N/A |
| Ottomanelli et al. (2010)               | Gap acceptance behavior of pedestrian    | 1st model: 1              | Trapezoidal and triangular | 1st model: 5   | N/A             | N/A                |
|                                      |                                          | 2nd model: 2              |                            | 2nd model: 25  | N/A             | N/A                |
| Seyedabrizhani and Shafahi (2011)       | Travel demand estimation                 | 2                         | Triangular                 | 9              | Trip generation: 156 | Modal split: 2300 | N/A    |
| Sayed et al. (2003)                     | Mode choice modeling                     | 26                        | N/A                        | 10             | 7500            | N/A                |
| Pang et al. (1999)                      | Dynamic route guidance system             | 6                         | Triangular                 | 6              | N/A             | Backpropagation    |

To illustrate the ANFIS procedures, for simplicity, suppose that the input data are \(x_1\) and \(x_2\), and the output is \(y^*\). ANFIS operates based on the fuzzy if-then rules. Fuzzy if-then rules or fuzzy conditional statements are expressions of the form “if A then B”, where A and B are labels of fuzzy sets (Zadeh 1965). For \(x_1\) and \(x_2\) as the inputs and \(y^*\) as the output, consider the problem of representing the way fuzzy control is achieved in the Takagi-Sugeno (1983) fuzzy if-then rules. That is, consider a fuzzy rule base consisting of only two rules:

Rule 1: If \(x_1\) is \(A_1\) and \(x_2\) is \(B_1\) then \(y = f_1(x)\)

Rule 2: If \(x_1\) is \(A_2\) and \(x_2\) is \(B_2\) then \(y = f_2(x)\)

where, \(A_i\) and \(B_i\) are fuzzy sets, \(x = (x_1, x_2)\) is the numerical input, and

\[
f_1(x) = m_{11} x_1 + m_{12} x_2 + m_{13} \tag{1}
\]

\[
f_2(x) = m_{21} x_1 + m_{22} x_2 + m_{23} \tag{2}
\]

where, \(m_{11}, m_{12}, m_{13}, m_{21}, m_{22}, m_{23}\) are the consequent parameters (Leondes 1999).

When numerical input \(x = (x_1, x_2)\) is submitted to the ANFIS, its inference mechanism produces the output

\[
y^* = \frac{A_1(x_1)B_1(x_2)f_1(x) + A_2(x_1)B_2(x_2)f_2(x)}{A_1(x_1)B_1(x_2) + A_2(x_1)B_2(x_2)} \tag{3}
\]

The fuzzy-neural network for implementing the above inference system is depicted in Figure 1.a and the corresponding equivalent ANFIS architecture is shown in Figure 1.b. Based
on this figure, five layers are employed to outline the ANFIS inference system (Jang 1993; Leondes 1999). Each layer receives the inputs from the nodes in the previous layers and gives the layer output(s) to the next layer. The nodes in the first layer compute the membership degree of the inputs in the antecedent fuzzy sets (i.e. \( A_1, A_2, B_1, \) and \( B_2 \)). The second layer nodes \( \Pi \) execute the fuzzy AND of the antecedent part of the fuzzy rules. The normalization node \( N \) in the third layer normalizes the outputs of the previous layer. The fourth layer computes the consequent part of the fuzzy rules and, finally, the last layer calculates the output of the fuzzy system by summing up all the incoming signals. The layers of this ANFIS structure in Figure 1.a are described concisely below:

**Layer 1 (Fuzzification):** The square nodes in this layer are adaptive nodes which give the following output:

\[
(O_{11}, O_{12}, O_{13}, O_{14}) = (A_1(x_1), A_2(x_1), B_1(x_2), B_2(x_2))
\]

where, \( x_1 \) and \( x_2 \) are the inputs to the nodes; and \( A_1, A_2, B_1, \) and \( B_2 \) are the membership functions associated with the linguistic variables of the nodes. These functions are usually chosen as

\[
\Gamma_k(z) = \frac{1}{1 + ((z - c_k/a_k)^2)^b_k}
\]

or

\[
\Gamma_k(z) = \exp\left(-\left(\frac{z - c_k}{a_k}\right)^2\right)
\]

where \( k=1 \) and \( 2, \) \( z \) is \( x_1 \) or \( x_2, \) \( \Gamma_k(z) \) is then \( A_k(x_1) \) or \( B_k(x_2), \) and \( a_i, b_i, \) and \( c_i \) are the premise parameters (Buragohain and Mahanta 2008).

**Layer 2 (Fuzzy AND):** The circle nodes in Figure 1.a are named “fixed nodes”. A fixed node embodies a function to be multiplied by the input signals coming to the node. The fixed nodes in layer 2 are labeled \( \Pi \) and perform a fuzzy AND operation on the premise of the fuzzy rules that produce an output equal to the product of all the incoming signals to it, and is given by

\[
(O_{21}, O_{22}) = (A_1(x_1)B_1(x_2), A_2(x_1)B_2(x_2))
\]

**Layer 3 (Normalization):** Every node in this layer is a fixed node which is marked by a circle and labeled \( N \). Each node in this layer normalizes the outputs of the previous layer by calculating the ratio of the node’s output to the sum of the outputs of all the nodes as follows:

\[
(O_{31}, O_{32}) = \left(\frac{O_{21}}{O_{21} + O_{22}}, \frac{O_{22}}{O_{21} + O_{22}}\right)
\]

\[
= \left(\frac{A_1(x_1)B_1(x_2)}{A_1(x_1)B_1(x_2) + A_2(x_1)B_2(x_2)}, \frac{A_2(x_1)B_2(x_2)}{A_1(x_1)B_1(x_2) + A_2(x_1)B_2(x_2)}\right)
\]

**Layer 4 (Fuzzy Inference):** All nodes of this layer are adaptive nodes distinguished by squares, with node functions \( f_1 \) and \( f_2 \). It is important to note that \( f_1 \) and \( f_2 \) are fuzzy if-then rules according to the aforementioned Rules 1 and 2 with the closed-form formula provided in Equations 1 and 2. As a result, layer 4 gives

\[
(O_{41}, O_{42}) = (O_{31}f_1, O_{32}f_2)
\]

\[
= \left(\frac{A_1(x_1)B_1(x_2)(m_{11}x_1 + m_{12}x_2 + m_{13})}{A_1(x_1)B_1(x_2) + A_2(x_1)B_2(x_2)}, \frac{A_2(x_1)B_2(x_2)(m_{21}x_1 + m_{22}x_2 + m_{23})}{A_1(x_1)B_1(x_2) + A_2(x_1)B_2(x_2)}\right)
\]
Layer 5 (Defuzzification): This layer is only a fixed node, which calculates the final output as the summation of all incoming signals by

\[ y^* = O_{41} + O_{42} \]

\[ y^* = \frac{A_1(x_i)B_1(x_i)(m_{11}x_i + m_{21}x_i + m_{31}) + A_2(x_i)B_2(x_i)(m_{22}x_i + m_{32}x_i + m_{42})}{A_1(x_i)B_1(x_i) + A_2(x_i)B_2(x_i)} \]  

ANFIS employs a hybrid learning algorithm that combines the gradient method with the least squares method to update the premise parameters in Equations 5 and 6 and consequent parameters in Equations 1 and 2 (Buragohain and Mahanta 2008). In the forward phase of the learning algorithm, consequent parameters are determined through the least squares estimate. Within the backward phase, the error signals, which are the derivatives of the squared error with respect to each node output, propagate backward from the output layer to the input one. Through this backward phase the premise parameters are adjusted using the gradient descent algorithm (Jang 1993; Buragohain and Mahanta 2008). It should be noted that other optimization algorithms such as genetic algorithm (Zanaganeh et al. 2009), particle swarm optimization algorithm (Shoorehdeli et al. 2009), and ant colony (Wang et al. 2012) were applied for the calibration of the parameters of ANFIS. However, it is customary to utilize the described algorithm for the training stage as a conventional method.

Fig. 1. ANFIS Architecture.
Outline of the Learning Model

Suppose that \( ETT_i = \{ ett_{i,1}', ett_{i,2}', \ldots, ett_{i,d}', \ldots, ett_{i,n}' \} \) is the set of experienced travel times for traveler \( i \), on route \( r \), and for the duration from the first experience on route \( r \) to the last one on this route (i.e. the last time this specific route is chosen). Furthermore, consider \( PTT_i = \{ ptt_{i,1}', ptt_{i,2}', \ldots, ptt_{i,d}', \ldots, ptt_{i,n}' \} \) as the set of predicted travel time updated by the traveler \( i \) for route \( r \) from the first experience of driving on route \( r \) to the last one. Based on this explanation, it should be noted that \( d \), which varies from 1 to \( n \), is the index of experiences and is not the index of days. For example, \( ett_{3,6}' \) points to the travel time experience of traveler no. 3 for his/her travel only on route 4 and just for the 6th time that this specific route is chosen by this traveler. However, it may not correspond to the day (iteration) 6, because in some iterations, the traveler may choose other routes except for this route. Furthermore, we suppose that a traveler will update his/her perceived travel time of route \( r \) only if he/she experiences a new travel time on route \( r \); otherwise he/she does not change his/her last prediction.

It is obvious that all of the experienced travel times of a traveler do not come into the traveler’s perception updating and adaptation mechanism. That is, each individual has a memory in which relevant aspects of previous trips are stored; however, not all are retrievable (Ettema et al. 2005), owing to the fact that they may be too old. Only a limited number of travel times experienced by the traveler are retrievable, as there are limits on the number of sensations, impressions, or distinctions that can be retained in the mind briefly and grasped at once, or used as a basis for making judgments (Saaty and Ozdemir 2003).

General limitations of human capacity for processing information are very familiar in the literature of psychology (Miller 2003) and are usually categorized together as memory spans. According to one of Miller’s papers in psychology (Miller 1956), the number of objects an average human can hold in working memory is 7 ± 2. This is often referred to as Miller’s Law. Such limitations on human capacity for memory are also broadly known as “attention span”, “perceptual span”, “span of absolute judgment”, and “channel capacity” (Saaty and Ozdemir 2003).

In the realm of travel time perception modeling, Ettema and his colleagues (2004, 2005) introduced a memory strength parameter that represents the ease with which an event can be retrieved from memory; however, a fixed span has not yet been utilized. Besides, Nakayama et al. (1999) considered 3 as the number of experienced travel times that are in the working memory. Mahmassani and Chang (1986) take only the previous day into account in their myopic adjustment model of travel time prediction. Iida et al. (1992) use 1 and 3 as the spans for retrieving the data from the memory of the travelers. In their study, it was shown that the longer memory span (i.e. 3) provides sounder results.

In line with this idea, the proposed learning model in this paper considers 7 as the memory span of travelers. Thus, each traveler uses a limited set of the last 7 experiences, which is \( \{ ett_{i,n-6}', ett_{i,n-5}', \ldots, ett_{i,n}' \} \), as the set of the retrievability of the past experiences.

Our proposed method should adopt simple rules as follows:

**Rule 1.** There is no possibility (i.e. degree of membership) for a perceived travel time to be less than all the elements of the set of experienced travel time retrievable from the memory.

**Rule 2.** The perception mechanism should involve the following sub-rules:

a. An errorless perception, which is associated with the exact similarity between the last experienced travel time and the last perceived travel time, forces the traveler to predict the same travel time for the next day (iteration).
b. The greater (smaller) the difference between the last experienced travel time and the last perceived travel time, the greater (smaller) the difference between the next perceived travel time and the last perceived travel time.

c. The greater (smaller) the representativeness of the experienced travel time (i.e. the degree that the experienced travel time seems usual from the viewpoint of the traveler when he/she compares it to his/her last experienced travel times), the greater (smaller) the traveler’s inclination to incorporate this experienced travel time into his/her next perception updating step.

Rule 3. There is no possibility (i.e. degree of membership) for a perceived travel time to be greater than all the elements of the set of experienced travel times retrievable from the memory.

Based on these rules, the theoretical framework of our day-to-day learning model is designed.

As mentioned previously, travelers’ mental representation of an uncertain travel time could be an interval, and the possibility measure of travel time could be defined for this interval. Hence, it is assumed in this paper that traveler \( i \) perceives the travel time in route \( r \) for the next day \((n+1)\) as follows:

\[
P_{TT_{i_{n+1}}} = \{ \text{ptt}_{i_{n+1}}^{r_1}, \text{ptt}_{i_{n+1}}^{r_2}, \text{ptt}_{i_{n+1}}^{r_3} \} \tag{11}
\]

where, \( P_{TT_{i_{n+1}}} \) is a triangular fuzzy number (TFN); and \( \text{ptt}_{i_{n+1}}^{r_1}, \text{ptt}_{i_{n+1}}^{r_2}, \text{and} \ \text{ptt}_{i_{n+1}}^{r_3} \) are the \( t \)-coordinate of the vertices of the triangular membership function where \( \text{ptt}_{i_{n+1}}^{r_1} \leq \text{ptt}_{i_{n+1}}^{r_2} \leq \text{ptt}_{i_{n+1}}^{r_3} \) as shown in Figure 2.

According to Bojadziev (2007), the TFN \( P_{TT_{i_{n+1}}} \) is a fuzzy set with membership function \( \mu_{P_{TT_{i_{n+1}}}}(t) \) over the real number domain \( \mathbb{R} \) by:

\[
\mu_{P_{TT_{i_{n+1}}}}(t) = \begin{cases} 
\frac{t - \text{ptt}_{i_{n+1}}^{r_1}}{\text{ptt}_{i_{n+1}}^{r_2} - \text{ptt}_{i_{n+1}}^{r_1}} & \text{for } \text{ptt}_{i_{n+1}}^{r_1} \leq t \leq \text{ptt}_{i_{n+1}}^{r_2} \\
\frac{t - \text{ptt}_{i_{n+1}}^{r_3}}{\text{ptt}_{i_{n+1}}^{r_3} - \text{ptt}_{i_{n+1}}^{r_1}} & \text{for } \text{ptt}_{i_{n+1}}^{r_2} \leq t \leq \text{ptt}_{i_{n+1}}^{r_3} \\
0 & \text{otherwise,}
\end{cases} \tag{12}
\]

where, \( [\text{ptt}_{i_{n+1}}^{r_1}, \text{ptt}_{i_{n+1}}^{r_3}] \) is the supporting interval and is shown by \( \text{Supp}(P_{TT_{i_{n+1}}}) \).

![Fig.2. Triangular Fuzzy Travel Time Perceived by the Traveler.](image)
Considering 7 as the number of last experienced travel times that can be retrieved from the working memory, the proposed method calculates the vertices $ptt_{i,n+1}^{r,1}$ and $ptt_{i,n+1}^{r,3}$ as

$$ptt_{i,n+1}^{r,1} = \min \{ ett_{i,n-6}^{r}, ett_{i,n-5}^{r}, \ldots, ett_{i,n}^{r} \}$$

$$ptt_{i,n+1}^{r,3} = \max \{ ett_{i,n-6}^{r}, ett_{i,n-5}^{r}, \ldots, ett_{i,n}^{r} \}$$

where, $ett_{i,n-k}^{r}$ is the experienced travel time $n-k$ (which is $k+1$ experiences before the current travel time perception $n+1$ for the specific route $r$). It should be noted that $k$ is the index of memory span which points to the number of experienced travel times affecting the perception of the traveler. Therefore, this index is an integer ranging from 0 to 6 based on the Miller’s Law in psychology. Furthermore, Equation 13 and 14 completely follow the aforementioned Rules 1 and 3, respectively.

Finally, the modal value of the perceived fuzzy travel time ($ptt_{i,n+1}^{r,2}$) is derived from ANFIS. In this regard, a function $y = f(x_1, x_2)$ is deemed to be achieved from ANFIS according to the following relations:

$$\left\{ \begin{array}{l}
 x_1 = \frac{ett_{i,n}^{r} - ptt_{i,n}^{r,2}}{ptt_{i,n}^{r,2}},
 x_2 = \frac{ett_{i,n}^{r} - ett^{r}}{ett^{r}}.
 y = \frac{ptt_{i,n+1}^{r,1} - ptt_{i,n}^{r,2}}{ptt_{i,n}^{r,2}}
 \end{array} \right\} : x_1, x_2, y \in \mathbb{R}$$

where $x_1, x_2$ are the inputs and $y$ is the output of the ANFIS model respectively, and $ett^{r}$ represents the mean of travel times experienced by traveler $i$ within his/her memory span.

As a result, the modal value of the perceived fuzzy travel time (Figure 2) in day $n+1$ (i.e. $ptt_{i,n+1}^{r,2}$), becomes

$$ptt_{i,n+1}^{r,2} = \max \left\{ ptt_{i,n+1}^{r,1}, \min \left\{ ptt_{i,n+1}^{r,3}, \left( y^{*} + 1 \right) \times ptt_{i,n}^{r,2} \right\} \right\}$$

where, the “max” and “min” terms in Equation 16 try to bound the modal value between the corner values of the TFN, and $y^{*}$ is the output derived from the well-trained ANFIS model:

$$y^{*} = \text{ANFIS} \left\{ \frac{ett_{i,n}^{r} - ptt_{i,n}^{r,2}}{ptt_{i,n}^{r,2}}, \frac{ett_{i,n}^{r} - ett^{r}}{ett^{r}} \right\}$$

It is worth mentioning that the fuzzy inference system of the proposed ANFIS contains four fuzzy if-then rules as follows:

**Rule 1:** If $x_1$ is small (A$_1$) and $x_2$ is small (B$_1$) then $y = f_1(x)$

**Rule 2:** If $x_1$ is small (A$_1$) and $x_2$ is large (B$_2$) then $y = f_2(x)$

**Rule 3:** If $x_1$ is large (A$_2$) and $x_2$ is small (B$_1$) then $y = f_3(x)$

**Rule 4:** If $x_1$ is large (A$_2$) and $x_2$ is large (B$_2$) then $y = f_4(x)$

Figure 3 shows this fuzzy inference system schematically.
To illustrate the proposed method, consider the traveler $i$ who has experienced a set of travel times $ETT_i' = \{20, 27, 29, 30, 30, 30, 40\}$ in his/her last 7 trips on route $r$. Moreover, the travel time of 30 minutes is anticipated based on the prediction of this traveler. According to the equations 13 and 14, the traveler considers no possibility for the perceived travel times more than 40 minutes or less than 20 minutes in his/her next travel time prediction (i.e., $\min\{ETT_i'\} = 20$, and $\max\{ETT_i'\} = 40$). Besides, based on Equation 15, the input variables $x_1$ and $x_2$ are equal to 0.33 and 0.36 respectively (i.e. $x_1 = (40 - 30)/30$, and $x_2 = (40 - 29.4)/29.4$). Since $x_1 = 0.33$ and $x_2 = 0.36$, the output of ANFIS model becomes $y^* = ANFIS(0.33, 0.36)$. $y^*$ is calculated based on the Takagi-Sugeno inference system (see Figure 3). Finally, using Equation 16 results in $\max\{20, \min\{40, (y^* + 1) \times 30\}\}$.

**Traveler’s Route Choice Based on the Proposed Learning Model**

In this study, the route choice mechanism is a simple procedure. In this procedure, the traveler selects a route between his/her origin and destination (O-D) where the perceived travel time is minimal among all the alternative routes between this O-D pair.

Perceiving fuzzy travel times for the routes as shown in Figure 2, the traveler needs to compare these routes’ fuzzy travel times. Regarding the fuzzy number comparisons, more than 35 strategies have been proposed in the literature (Khademi et al. 2013). Some studies have also provided a review of some methods for ranking fuzzy numbers or fuzzy subsets (e.g. see Bortolan and Degani 1985).
In the realm of fuzzy number comparison methods, Chen and Lu’s (2001) have proposed a method to serve the fuzzy number ordering purpose. Their method uses \( \alpha \)-cuts and performs simple arithmetic operations for comparing fuzzy numbers.

For the fuzzy perceived travel time \( PTT \), \( \alpha \)-cuts \( PTT(\alpha) = \{ t \in \mathbb{R} | \mu_{PTT}(t) \geq \alpha, \alpha \in [0,1] \} \) is a convex subsets of \( \mathbb{R} \) (Bojadziev and Bojadziev 2007, p. 14). The lower and upper limits for an \( \alpha \)-cut are represented by:

\[
\underline{m}_\alpha = \inf \left\{ t | \mu_{PTT}(t) \geq \alpha \right\}
\]

\[
\bar{m}_\alpha = \sup \left\{ t | \mu_{PTT}(t) \geq \alpha \right\}
\]

where, \( \underline{m}_\alpha \) and \( \bar{m}_\alpha \) are left and right spreads, respectively.

Comparing two fuzzy perceived travel times \( PTT_m = (m_1, m_2, m_3) \) and \( PTT_n = (n_1, n_2, n_3) \) based on Chen and Lu’s method (2001), Figure 4 illustrates the corresponding left and right spreads of these numbers at the \( \alpha \) level.

![Fig.4. Left and Right Spreads of Fuzzy Numbers PTT\(_m\) and PTT\(_n\) (Chen and Lu 2001).](image)

In order to compare the two fuzzy numbers \( PTT_m \) and \( PTT_n \), Chen and Lu’s (2001) fuzzy number comparison method defines the left and right dominance measure, \( D_{PTT_m,PTT_n} \) and \( \overline{D}_{PTT_m,PTT_n} \), of \( PTT_m \) over \( PTT_n \). \( D_{PTT_m,PTT_n} \) (\( \overline{D}_{PTT_m,PTT_n} \)) are defined as the average difference of the left (right) spreads at some \( \alpha \)-levels. While using \( k+1 \) \( \alpha \)-cuts, the left and right dominance measure are formulated as

\[
D_{PTT_m,PTT_n} = \frac{1}{k+1} \sum_{i=0}^{k} (m_i - n_i)
\]

and

\[
\overline{D}_{PTT_m,PTT_n} = \frac{1}{k+1} \sum_{i=0}^{k} (\bar{m}_i - \bar{n}_i)
\]

Finally, the total dominance of \( PTT_m \) over \( PTT_n \) with the index of optimism \( \beta \in [0,1] \) is defined as the convex combination of \( D_{PTT_m,PTT_n} \) and \( \overline{D}_{PTT_m,PTT_n} \) as

\[
D_{PTT_m,PTT_n}(\beta) = \beta \overline{D}_{PTT_m,PTT_n} + (1 - \beta) D_{PTT_m,PTT_n}
\]

This equation indicates that the total dominance is actually a comparison function. The larger the index of optimism (\( \beta \)) indicates that the right dominance is more important. The index of
optimism ($\beta$) is employed to reflect the decision maker’s degree of optimism. A more pessimistic decision maker generally takes a larger value of the index because he/she perceives longer travel time for her/his trip.

The travel times are compared based on the rules:

\begin{align}
(1) & \text{ if } D_{\text{PTT}_m, \text{PTT}_n} (\beta) > 0 \text{ then } \text{PTT}_m > \text{PTT}_n \\
(2) & \text{ if } D_{\text{PTT}_m, \text{PTT}_n} (\beta) = 0 \text{ then } \text{PTT}_m = \text{PTT}_n \\
(3) & \text{ if } D_{\text{PTT}_m, \text{PTT}_n} (\beta) < 0 \text{ then } \text{PTT}_m < \text{PTT}_n
\end{align}

Finally, the route with the least predicted travel time is finally selected by the traveler.

It is worth to mention that if one aims to use the random utility theory to model the route choice behavior, careful attention must be paid that the random utility theory deals with the choice problems from a different point of view. For example, consider a traveler faces a choice between two routes, $m$ and $n$. For notational simplicity, we omit the subscripts denoting the day and the traveler. The traveler would perceive a utility for each route on the next day (i.e. $u_n$ and $u_m$). In the random utility context, it is supposed that these utilities ($u_n$ and $u_m$) are known to the traveler but not by the researcher (Train 2009). Let us consider that the researcher does not observe the traveler’s utility and suppose that only the travel time of the routes ($t_n$ and $t_m$) is a factor influencing the traveler’s route choice for the next day. Yet, other aspects of utility that the researcher cannot observe may exist. These aspects of utility are not known to the analyst with certainty and are therefore treated by the analyst as random variables (Ben-Akiva and Lerman 1985). So, $t_n \neq u_n$ and $t_m \neq u_m$, and utilities are written as $u_m = t_m + \varepsilon_m$ and $u_n = t_n + \varepsilon_n$, where $\varepsilon_m$ and $\varepsilon_n$ are stochastic terms to cover the factors included in the utility but are not observed by the researcher (Train 2009). The traveler is always assumed to select the alternative with the highest utility (Ben-Akiva and Lerman 1985). According to Prashker and Bekhor (2004), individual perception errors, measurement errors, and specification errors are also captured by these stochastic terms.

However, in the route choice model proposed in this paper, the uncertainty existing in traveler’s perception about the travel time is captured by fuzzy travel time instead of the stochastic terms in random utility models.

**Laboratory Study**

**Modeling the Travel Time Updating Mechanism**

In order to set up the learning model by ANFIS, the MATLAB software (version 7.10.0) fuzzy logic toolbox (Fuzzy Logic Toolbox User’s Guide) was used. The experimental data for training and testing ANFIS model was collected from civil engineering students of IUST. There were no restrictions on any other socioeconomic attributes in the selection of the respondents. Some characteristics of the respondents are as follows:

- 65 % male and 35 % female
- 76 % undergraduate and 24 % graduate students
- Between 18 and 34 years old

An interactive internet-based survey was designed to provide ANFIS training and testing data. In order to avoid biased and incomplete data set that might have led to erroneous conclusions, the survey used four different patterns of travel time, normally distributed with pre-defined means and variances. Table 3 illustrates the means and variances of the day-to-day sets of travel times used in the four stages of the internet-based survey. The four different patterns corresponding to the four different stages are intended to present different situations from the most reliable situation (less variable pattern of travel time when the variance is the
lowest and the mean of travel time is the highest) in stage one to the most unreliable situation (the highest variance of travel time with the lowest mean) in stage four.

| Stages of survey | Mean of travel times (in minute) | Variance of travel times |
|------------------|---------------------------------|--------------------------|
| Stage 1          | $\bar{x} = 33.3$                | $s^2 = 6.7$              |
| Stage 2          | $\bar{x} = 31.5$                | $s^2 = 19.7$             |
| Stage 3          | $\bar{x} = 30.4$                | $s^2 = 70.5$             |
| Stage 4          | $\bar{x} = 26.6$                | $s^2 = 165.8$            |

Each respondent was given one of the patterns randomly, and the respondent had to make consecutive predictions for 45 iterations. That is, a respondent had to predict the next-day travel time of his or her trip in each iteration. Afterward, the real travel time was presented to him/her and then, he/she was asked to again predict the travel time of the next iteration (day). The basic question in each iteration was: “Most possibly, how long will your trip take?” where the term “Most possibly” points to the modal value of the perceived fuzzy travel time (see Equation 16) which was used for ANFIS training according to Equation 17.

It should be noted that there was no evidence of survey fatigue on the part of the participants after completion of the survey. The data set obtained from the internet-based survey was divided randomly into a training data set made of 80% of the original set and a test data set made of 20% of it. The training data set was used to determine the premise and consequent parameters of the ANFIS model, whereas the test data set was used for the model validation.

The parameters and the structure of the ANFIS used in this study are shown in Table 4.a. Let us suppose that the ANFIS model has $p$ parameters corresponding to each membership function (associated with the linguistic variables), $r$ rules, $v$ input variables, and one output variable. Then, according to Figure 1 or 3, the number of premise parameters is $v \times r \times p$, the number of consequent parameters is $r \times (v+1)$, and the total number of parameters within the ANFIS structure is equal to $r \times (vp + v + 1)$. From Table 4.a, $r=4$, $v=2$, and $p=4$; hence, the total number of parameters in the proposed ANFIS structure is 44.

In order to determine the error between the predicted and the observed data, the root mean square error (RMSE) is employed as follows:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y - y^*)^2}$$

(23)

where, $y^*$ is the output of the ANFIS and $y$ is the experimental values calculated based on Equation 15.

Table 4.b shows the error measure RMSE for the training and test data sets. The RMSE values for the training and test data sets become 5.74% and 5.94%, respectively, which indicates a good fit.
Table 4. ANFIS Parameters and Results

| a. ANFIS architecture and the training parameters |
|-----------------------------------------------|
| Number of layers | 5 |
| Number of rules (r) | 4 |
| Number of input variables (v) | 2 |
| Number of output variables | 1 |
| Membership function | gauss2mf* |
| Number of parameters in each membership function (p) | 4 |
| Learning rules | hybrid** |
| Size of data set | 2035 |
| Number of epochs | 100 |

b. RMSE Values for the training data set and the test data set

| RMSE- ANFIS training | 0.0574 |
|-----------------------|-------|
| RMSE- ANFIS test      | 0.0594 |

* gauss2mf is the combination of two Gaussian functions. The first one determines the shape of the left side of the function and the second one determines the shape of the right side (see Fuzzy Logic Toolbox User’s Guide).

**hybrid: Least square estimation & gradient descent algorithm.

Figure 5 shows a snapshot of the ANFIS editor GUI of the Fuzzy Logic Toolbox of the MATLAB software (version 7.10.0). It illustrates the final membership functions associated with linguistic variables after training the ANFIS model.

![Fig.5. Bell-Shape Gaussian Membership Functions After Training.](image)

Figure 6.a depicts the surface $y^* = ANFIS(x_1, x_2)$ obtained from the ANFIS learning model. Moreover, Figure 6.b illustrates two cross-sections of the surface perpendicular to the $x_2$-axis at $x_2=0$ and $x_2=1.6$. In other words, two curved lines are shown in Figure 6.b which are the intersections of the ANFIS surface with the planes $x_2=0$ and $x_2=1.6$. It is obvious that the results of Figure 6.b are consistent with Rule 2.b. In fact, the greater difference between the last experienced travel time and the last perceived travel time ($x_1$) results in the greater difference between the next perceived travel time and the last perceived travel time ($y^*$). For example, consider a route with the mean travel time of 47.5 minutes at a particular time of day. On a given day, a traveler predicts 40 minutes as the travel time for his/her next experience on this route. Afterwards, the traveler experiences the actual travel time of 45 minutes. Based on Equation 15, the input variables $x_1$ and $x_2$ equal to 0.125 and 0.052 respectively (i.e. $x_1 = (45 - 40)/40$, and $x_2 = \lceil (45 - 47.5)/47.5 \rceil$). Therefore, the output of the trained ANFIS
model ($y^*$) is equal to 0.11 based on Figure 6.a. By contrast, if the traveler predicts 40 minutes as the next day travel time and he/she experiences 50 minutes as the actual travel time, then the input variables $x_1'$ and $x_2'$ are 0.25 and 0.052 respectively (i.e. $x_1'=(50-40)/40$ and $x_2'=|50-47.5|/47.5$). As a result, the output of the trained ANFIS model is equal to 0.21, which is greater than 0.11 (i.e. $x_1' = 0.25 \succ x_1 = 0.125$ then $y^* = 0.21 \succ y^* = 0.11$).

In Figure 6.b, the part of each curved line before $x_1=0$ denotes the perception of traveler of the early arrival penalty (when $ett_{i,n} < ppt_{i,n}^2$) while, the part of each curved line after the point $x_1=0$ implies the perception of the late arrival penalty (when $ett_{i,n} > ppt_{i,n}^2$). It can be observed that the slope of these curved lines for the early arrival span is smaller than the slope for the late arrival span. This finding corresponds to the fact that the travelers are more sensitive to their late arrivals (losses) than their early arrivals (gains) which is known as a well-known phenomenon in transportation economics (Zerguini et al. 2011).

Figure 6.c also illustrates two cross-sections of the surface in Figure 6.a perpendicular to the $x_1$-axis at $x_1=-0.6$ and $x_1=1.8$ (i.e. the intersections of the ANFIS surface with the planes $x_1=-0.6$ and $x_1=1.8$). The results of this figure correspond closely to the concept of the representativeness factor explained in Rule 2.c. In fact, travelers tend to consider the non-representative experienced travel times less in their perception updating process. As an example of this rule, consider a route with the mean travel time of 47.5 minutes at a particular time of day. Suppose a traveler predicts the travel time of 40 minutes for his/her next experience. Subsequently, the traveler experiences the travel time of 45 minutes. The input variables $x_1$ and $x_2$ are equal to 0.125 and 0.052 respectively. Consequently, the output of the ANFIS model is $y^*=0.11$. Following the same approach mentioned earlier, if the mean travel time of the route is about 30 minutes, then the input variables $x_1'$ and $x_2'$ are 0.125 and 0.5 respectively (i.e. $x_1'=(45-40)/40$, and $x_2'=|(45-30)/30|$). As a result, the output of the trained ANFIS model is equal to 0.095, which is less than 0.11. In fact, the traveler has experienced a non-representative travel time in the second case. Therefore, he/she weights a lower value to this non-representative experience for the perception updating process (i.e. $x_1' = 0.5 \succ x_2' = 0.052$ then $y^*=0.095 \prec y^* = 0.11$).

In order to highlight the pros and cons in using the proposed ANFIS model, a linear regression model (with the same dependent and independent variables as those exist in the ANFIS model) was developed using the SPSS Statistics software version 13.

This linear regression model has the formula $y^* = 0.614x_1 - 0.04x_2 + 0.0058$ where $x_1 = \left(ett_{1,n} - ppt_{1,n}^2\right)/ppt_{1,n}^2$ and $x_2 = \left|ett_{1,n} - ett'\right|/ett'$ and it has the surface shown in Figure 6.d. Using equation 23, the RMSE values of the regression model for the training (i.e. calibration) and test (i.e. validation) data sets are 0.0634 and 0.0852, respectively, which are higher than the RMSE values in Table 4.b derived from the ANFIS model.

Figure 6.e shows two cross-sections of the surface of Figure 6.d at $x_2=0$ and $x_2=1.6$. In addition, Figure 6.f is the intersections of this surface with the planes $x_1=-0.6$ and $x_1=1.8$.

All the cross-sections of the ANFIS surface along the $x_1$-axis are strictly increasing (see Figure 6.b), whereas depending on the value of $x_1$, the cross-sections of the surface along the $x_2$-axis can be increasing or decreasing. That is, the surface curves up and down along $x_2$ depending on the values of $x_1$.

As can be seen from Figure 6.a, the ability of the ANFIS to model nonlinear functions makes it possible to capture the increasing nature of the dependent variable ($y$) with respect to $x_2$ when $x_1$ is low and, on the contrary, it can capture the decreasing nature of the dependent variable ($y$) with respect to $x_2$ when $x_1$ is high enough. Such interesting
behavior is in accordance with the concept of the representativeness factor explained by Rule 2.c. As the multivariate linear regression does not have the capability to fully capture such non-linear behavior, the coefficient of $x_2$ in the fitted regression is small (see Figure 6.f where the slopes of the lines are low).

Fig.6. Relationship between the Input and Output Variables.

**Experimental Set-Up of the Day-to-Day Route Choice Decisions**

The proposed learning model is embedded into a microscopic simulation framework in order to evaluate its credibility for the day-to-day modeling of transportation networks.

The test network is relatively similar to the network presented in (Chen and Mahmassani 2004; He et al. 2010). However, some differences can be found in the type of congestion function when compared to (Chen and Mahmassani 2004), and the number of O-D pairs when compared to (He et al. 2010). The proposed test network contains 9 nodes, 12 links, and 7 O-D pairs. Figure 7 shows the test network and its characteristics. It should be mentioned that some of the O-D pairs in this network are connected by two or several routes (e.g. O-D pairs A-I, A-E, and E-I), and the others are connected by only one route as mentioned in the bottom of Figure 7.

A fully disaggregate method of discrete event-based simulation that employs the bottleneck model as the travel time function (Arnott et al. 1993) is developed. The route choice procedure has the following characteristics in this simulation:
- All the travelers are assumed to depart from their origin together. That is, there is no departure time choice model; or in other words, the O-D pairs are only for one departure time slot.
- The travelers confront the bottleneck model when they enter the network links. A bottleneck has a fixed capacity, and if the number of drivers arriving at the bottleneck exceeds this capacity, a queue forms. At most $s$ cars can pass per unit time from a bottleneck. The queue discipline is first-in-first-out (FIFO). Thus, the experienced travel time in link $l$, for traveler $i$, and in day $n$ becomes

$$ett^l_{i,n} = t^l_0 + qt^l_{i,n}$$

where, $t^l_0$ is the fixed travel time in link $l$, and $qt^l_{i,n}$ is the variable travel time incurred by the traveler because of the waiting time in the queue. Moreover, we have: $qt^l_{i,n} = D/s$, in which $D$ is the queue length and $s$ is the link capacity. It is obvious that in a no-congestion case, $qt^l_{i,n} = 0$, the outflow from the link is equal to the arrival rate. With congestion, $D>0$, the outflow is equal to the capacity. The parameter $\zeta$ in Figure 7 is a capacity scalar parameter that we change it from 1 to 0.00125 in this paper to trace the state of the network from the uncongested situation to the dense one.
- The traveler’s perception and updating mechanism is not at an aggregate level using a common pool of information which is accessed by all travelers as proposed by de Palma and Marchal (2002); in contrast, each traveler experiences the travel time in an individual process.
- 40 day-to-day iterations (days) are considered for this simulation.
- The index of optimism ($\beta$) in Equation 21 is assumed to be the constant value of 0.5.
- Four different simulations are presented in this paper according to the values 0.00125, 0.0125, 0.1, and 1 for the capacity scalar parameter ($\zeta$).
- We force the travelers to take a specific route at the initial time of the simulation when they have not experienced any travel time yet. This initial pre-allocation is fixed for all the four above-mentioned simulations (i.e. four simulations with $\zeta = 0.00125, 0.0125, 0.1, \text{and } 1$).
- This initial pre-allocation is devised in a manner that the travelers’ route choices in the first iteration corresponds to their route choices derived from the static equilibrium solution in congested networks when $\zeta=0.00125$. In other words, we force the travelers to choose those routes in the first iteration to make the state of the network similar to the result of the static equilibrium when the network has the capacity scalar of $\zeta=0.00125$. Through this tactic, we want to check whether our dynamic day-to-day model for the congested network converges to the equilibrium state or not.
- Because the pre-allocation is fixed for all the four above-mentioned simulations (i.e. $\zeta = 0.00125, 0.0125, 0.1, \text{and } 1$), and due to the fact that the pre-allocation was performed based on the static equilibrium state of the congested network, it could be deduced that this pre-allocation does not accord with the uncongested equilibrium state.
This day-to-day route choice process in this paper presents a deterministic discrete-event simulation model (Rubinstein and Kroese 2011). This simulation model is deterministic as all the mathematical and logical relationships between network elements (the bottleneck model and the FIFO discipline) are fixed in advance. It is also a discrete simulation model, since the state of the network changes instantaneously at discrete points in time. The discrete-event simulation is the standard structure for the simulation of a class of models in which the system state requires to be monitored only at certain epochs (event times). Between these epochs, the system state either stays the same or changes in a predictable fashion (Rubinstein and Kroese 2011).

The simulation has been performed based on the constructed learning model described in the previous section. The simulation code was written in MATLAB (version 7.10.0). The process of the simulation is shown in Figure 8 composed of a series of day-to-day learning, decision-making, and experiencing process.
The results of the simulations are shown in Figure 9. The following points are noteworthy to mention from Figure 9:

- The first column of this figure shows the travel time of each traveler at the last iteration (iteration no. 40). At the beginning of the simulation, all travelers who have the same route are considered as a platoon. As we use the disaggregate traffic representation for the proposed discrete-event simulation model, the platoon forms a queue and the position of each traveler in the queue is specific. The people in each platoon (queue) start their trip together, but they may be separated from each other after passing through the first bottleneck, and travelers from other platoons (queues) may be placed between them depending on the level of congestion of the network. It is clear that in the uncongested network (ζ=1), all the travelers of each O-D pass the links in platoons, come to and out the bottleneck together, and arrive to their destination nearly within the same period. It is shown clearly by the discrete segments shown in the bottom-left part of Figure 9. However, in a fully congested network (ζ=0.00125), we can see that the travelers get stuck in the queues formed from different O-D pairs (see the top-left part of Figure 9).

- The second column of this figure points to the ratio of the number of travelers who swap their routes to the total number of travelers. It is a measure to test when the equilibrium state is achieved. On the uncongested network (ζ=1), the equilibrium state is achieved after 7 days; the equilibrium state of the congested network (ζ=0.00125) is reached after 20 days.
• The figures in the third column show the average travel time of all the travelers in each iteration. As was mentioned above, the initial pre-allocation is devised in a manner that the travelers' route choices in the first iteration corresponds to their route choices derived from the static equilibrium solution in the congested network (\(\zeta=0.00125\)). The simulation model clearly shows that the dynamic equilibrium in the last day and static equilibrium in the first day are the same. However, The observed convergence in Figure 9, may depend on the assumptions of disaggregate and non-shared memory. As a result, the other case of aggregate shared memory and any intermediate behavior might lead to different results. However, understanding how the level of aggregation could change the results remains open for further investigation.

• The average experienced travel time fluctuations decrease gradually when people learn and update their perceived travel times.

![Fig.9. Results of the Day-to-Day Route Choice Simulations.](image-url)
Conclusion

Modeling the day-to-day travel choice dynamics of a traffic network requires a reasonably well-fitted model of travel time perception and learning of travelers. It becomes even more important when travel demand management policies are aimed at alleviating the traffic jam in the network.

The focus of this paper was to develop a day-to-day learning model considering imprecision, vagueness, and uncertainty of the traveler’s perception. In this paper, a fuzzy learning model was proposed to capture the mechanism by which travelers update their travel time perceptions from one day to the next taking into account their experienced travel times. A neuro-fuzzy architecture, Adaptive-Network-based Fuzzy Inference System (ANFIS), was used for modeling the travelers’ reasoning processes.

In our study, an interactive internet-based survey was designed to provide ANFIS training data. The results of this laboratory-like experiment provide a good fit to the stated data of travelers’ behavior. It spotlights the fact that the neuro-fuzzy approach is a promising method in learning and perception updating models.

Last but not least, the fine-tuned learning model was utilized for a microscopic simulation of a test network. The results of this simulation verify that the learning model works properly and can lead the network to the equilibrium state.

Acknowledgment

The authors would like to convey their deep appreciation to the anonymous referees for their valuable comments that considerably improved the overall quality of this paper.

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