DIMENSIONALLY CONTINUED OPPENHEIMER-SNYDER
GRAVITATIONAL COLLAPSE II: SOLUTIONS IN ODD DIMENSIONS

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Abstract

The Lovelock gravity extends the theory of general relativity to higher dimensions in such a way that the field equations remain of second order. The theory has many constant coefficients with no a priori meaning. Nevertheless it is possible to reduce them to two, the cosmological constant and Newton’s constant. In this process one separates theories in even dimensions from theories in odd dimensions. In a previous work gravitational collapse in even dimensions was analysed. In this work attention is given to odd dimensions. It is found that black holes also emerge as the final state of gravitational collapse of a regular dust fluid.

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I. INTRODUCTION

A generalization of Einstein gravity to other dimensions while keeping the same degrees of freedom (the field equations for the metric remain of second order) is given by the Lovelock action [1]. The theory can also be considered as an extension of Einstein-Hilbert action (see e.g. [2]), in which new terms make their appearance by taking into the action the Euler densities of the spaces with dimensions lower than the space in consideration.

In a previous work [3] we have studied gravitational collapse in Lovelock gravity for a spacetime with even dimensions, thus extending the Oppenheimer-Snyder collapsing model. Following the work of [2,4], the reason for separating even from odd dimensions in the Lovelock theory comes naturally in a $D$–dimensional spacetime when one considers embedding the Lorentz group $SO(D–1,1)$ into de anti-de Sitter group $SO(D–1,2)$. The Lovelock theory then branches into two distinct classes, with Lagrangians for even dimensions and Lagrangians for odd dimensions. One also finds in this way that the number of constants, which proliferates when one goes to higher and higher dimensions, reduces drastically to two, the cosmological constant $\Lambda$ and Newton’s constant $G$.

In this work we study gravitational collapse in odd-dimensional spacetimes and show that black holes form from regular initial data consisting of a dust fluid. We follow closely the nomenclature and the division of sections made in [3]. In section II the Lovelock gravity for restricted coefficients in odd-dimensional spacetimes is presented. In section III we display the static solutions in odd dimensions found in [4]. In section IV we find some cosmological or interior matter solutions for perfect fluids. In section V we match the solutions found in section IV to the solutions of section III. In section VI we show that black holes can form through gravitational collapse in Lovelock odd-dimensional gravity. Section VI comments on the formation of naked singularities and section VII presents some conclusions. In the paper we usually do $G = c = 1$. 

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II. THE LOVELOCK THEORY

The most general action in \( \mathcal{D} \geq 3 \) spacetime dimensions that yields the same degrees of freedom of Einstein’s theory is the so called Lovelock action, given by \(^1\)\(^2\)

\[
S = \int \mathcal{L}_{\mathcal{D}} = \kappa \sum_{p=0}^{[(\mathcal{D}-1)/2]} \alpha_p \int_M \epsilon_{a_1 \cdots a_{2n}} R^{a_1 a_2} \wedge \cdots \wedge R^{a_{2p-1} a_{2p}} \wedge e^{a_{2p+1}} \wedge \cdots \wedge e^{a_{2n}} + S_m ,
\]

(2.1)

where \( R^{ab} = d\omega^{ab} + \omega^a_c \wedge \omega^{cb} \) is the curvature two-form, \( e^a \) is the local frame one-form, and \( \omega^{ab} \) is the spin connection, with \( a_i = 0, 1, \ldots, \mathcal{D} - 1 \). The symbol \([\ldots]\) over the summation symbol means one should take the integer part of \((\mathcal{D} - 1)/2\). \( S_m \) is a phenomenological action which describes the macroscopic matter sources.

In general, the constant coefficients \( \alpha_p \) are arbitrary. However, it is shown in \(^4\) that taking certain special choices one is able to get simple meaningful solutions. Following \(^3\), one first considers embedding the Lorentz group \( SO(\mathcal{D} - 1, 1) \) into de anti-de Sitter group \( SO(\mathcal{D} - 1, 2) \), and then separates into two distinct classes of Lagrangians: Lagrangians for even dimensions and Lagrangians for odd dimensions.

For odd dimensions, \( \mathcal{D} = 2n - 1 \), one can find a construction similar to the Chern-Simons action construction in three dimensions. One starts with the Euler density in one dimension above \( \mathcal{D} \),

\[
E_{2n} = \kappa \epsilon_{A_1 \cdots A_{2n}} \hat{R}^{A_1 A_2} \wedge \cdots \wedge \hat{R}^{A_{2n-1} A_{2n}} ,
\]

(2.2)

with \( A_1, A_2 = 0, 1, \ldots 2n - 1 \) being the anti-de Sitter indices. \( \hat{R}^{AB} \) is the anti-de Sitter curvature two-form constructed with the \( SO(\mathcal{D}-1,2) \) connection \( W^{AB} \). Equation (2.2) is a local exact form, and can be written as an exterior derivative of a Lagrangian in \( 2n - 1 \) dimensions, i.e., \( E_{2n} = dL_{2n-1} \), see \(^3\). Decomposing the connection \( W^{AB} \) into the connection under \( \mathcal{D} \) rotations \( w^{ab} \) and inner translations \( e^a \), one finds the anti-de Sitter curvature \( \hat{R} \) in terms of the Lorentz curvature \( R \):
\[
\hat{R}^{ab} = R^{ab} + \frac{1}{l^2} e^a \wedge e^b ,
\]

(2.3)

where \( l \) is a scale factor which is to be related to the cosmological constant \( l^2 = -1/\Lambda \).

Using Eq. (2.3) one finds that the Lagrangian in Eq. (2.2) can be put in the form

\[
\mathcal{L}_{2n-1} = \kappa \sum_{p=0}^{n-1} \alpha_p \epsilon_{a_1 \ldots a_D} R^{a_1 a_2} \wedge \cdots \wedge R^{a_{2p-1} a_{2p}} \wedge e^{a_{2p+1}} \wedge \cdots \wedge e^{a_D}
\]

(2.4)

where the coefficients are given by

\[
\alpha_p = \frac{1}{D-2p} \left( \frac{n-1}{p} \right) l^{-D+2p} ,
\]

(2.5)

and for convenience one can choose \( \kappa \) as

\[
\kappa = \frac{D-2}{16 \pi G n} l^{D-2} .
\]

(2.6)

Given the action (2.1), the field equations are obtained by the variation with respect to the one-forms \( e^a \). Under the assumption of zero torsion, the variation with respect to the spin connection \( \omega^{ab} \) vanishes identically. Although the equations have powers in the curvatures, they remain by construction second order in the metric. The field equations are given by

\[
- \kappa \sum_{p=0}^{[(D-1)/2]} \alpha_p (D-2p) \epsilon_{a_1 \ldots a_D} R^{a_1 a_2} \wedge \cdots \wedge R^{a_{2p-1} a_{2p}} \wedge e^{a_{2p+1}} \wedge \cdots \wedge e^{a_{D-1}} = Q_{a_D} ,
\]

(2.7)

where \( Q_{a_D} \) is a \((D-1)\)-form associated with the energy momentum tensor \( T^a_b \) through the following expression

\[
Q_i = \frac{1}{(D-1)!} T^a_{i1} \epsilon_{a_1 \ldots a_D} e^{a_2} \wedge \cdots \wedge e^{a_D} .
\]

(2.8)

III. EXTERIOR VACUUM SOLUTIONS

In the vacuum all components of the energy-momentum tensor vanish, so that the field equations (2.7) are given by
\[- \kappa \sum_{p=0} \alpha_p (D - 2p) \varepsilon_{a_1 \ldots a_D} R^{a_1 a_2} \wedge \cdots \wedge R^{a_{2p-1} a_{2p}} \wedge a^{a_{2p+1}} \wedge \cdots \wedge a^{a_{D-1}} = 0. \quad (3.1)\]

Inserting the coefficients \(\alpha_p\) and the constant \(\kappa\) given in (2.5) and (2.6) in equation (3.1), one gets for odd dimensions \((D = 2n - 1),\)

\[(R^{a_1 a_2} + l^{-2} e^{a_1} \wedge e^{a_2}) \wedge \cdots \wedge (R^{a_{2n-3} a_{2n-2}} + l^{-2} e^{a_{2n-3}} \wedge e^{a_{2n-2}}) \varepsilon_{a_1 a_2 \ldots a_{2n-1}} = 0. \quad (3.2)\]

We consider now a static, spherical symmetric spacetime. One can write the metric in the following form,

\[ds_+^2 = -g^2(r_+) \, dt_+^2 + g^{-2}(r_+) \, dr_+^2 + r_+^2 \, d\Omega_{D-2}^2, \quad (3.3)\]

where \(t\) and \(r\) are the time and radial coordinates and \(d\Omega_{D-2}^2\) is the arc-element of a unit \((D - 2)\)-sphere. The subscript + reminds that (3.3) is to be viewed as an exterior solution.

With metric (3.3) and equations (3.1) and (3.2), Bañados, Teitelboim and Zanelli found the following exact solution for \(D = 2n - 1\) \[4\],

\[ds_+^2 = - \left[ 1 - (M + 1)^{2/(D-1)} + (r_+ / l)^2 \right] dt_+^2 + \frac{dr_+^2}{1 - (M + 1)^{2/(D-1)} + (r_+ / l)^2} + r_+^2 \, d\Omega_{D-2}^2. \quad (3.4)\]

These solutions describe black holes. We will show that they also represent the exterior vacuum solution to a collapsing (or expanding) dust cloud in Lovelock’s odd-dimensional theory, as in the even-dimensional case \[3\].

**IV. INTERIOR MATTER SOLUTIONS**

The interior spacetime is modeled by a homogeneous collapsing (or expanding) dust cloud, whose metric is described by the Friedmann-Robertson-Walker in \(D\) dimensions

\[ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 \, d\Omega_{D-2}^2 \right]. \quad (4.1)\]
The coordinates \( t \) and \( r \) are comoving coordinates (we omit throughout the subscript \(-\) to indicate an interior solution). Note that that \( k \) has dimension of \( 1/[\text{length}]^2 \). The energy-momentum tensor for a perfect fluid is given by

\[
T_{\alpha\beta} = (\rho + p) u_\alpha u_\beta + pg_{\alpha\beta}, \quad (4.2)
\]

where \( \rho \) is the energy-density, \( p \) the pressure, and \( u^\alpha \) is the \( D \)-velocity of the fluid. From (4.1)-(4.2) and Lovelock equations (2.7) we obtain

\[
- B \frac{d}{dt} \left( \frac{\dot{a}}{a} \right) + \frac{k}{a^2} = \rho + p \quad (4.3)
\]

\[
(D - 1) B \left( \frac{\dot{a}}{a} \right) \left[ - \frac{k}{a^2} + \frac{d}{dt} \left( \frac{\dot{a}}{a} \right) \right] = \dot{\rho} \quad (4.4)
\]

where

\[
B \equiv (D - 2)! \sum_p \alpha_p 2p (D - 2p) \left( \frac{\dot{a}^2 + k}{a^2} \right)^{p-1}. \quad (4.5)
\]

where the coefficients \( \alpha_p \) are given in (2.5), and \( \kappa \) is given in (2.6). Equations (4.3)-(4.4) have a first integral given by

\[
\dot{a}^2 = -k - \left( \frac{a}{l} \right)^2 + \left( \frac{a_0}{l} \right)^2 \left[ \frac{8 \pi l^2 \rho_0}{(D - 2)! (D - 2)} \right]^{2/(D-1)}, \quad (4.6)
\]

where \( \rho_0 \) and \( a_0 \) are constants. Equations (4.3)-(4.4) have also a second integral, i.e., the solution of the Eq. (4.6) is given by (see also [3])

\[
a(t/l) = \frac{l}{r^2} \sqrt{\left\{ \left( \frac{1}{l} \right)^{D-3} \frac{8 \pi \rho_0 (a_0 r^2)^{D-1}}{(D - 2)! (D - 2)} \right\}^{2/(D-1)} - k r^2 \sin(b + t/l)}, \quad (4.7)
\]

where \( b \) is an arbitray phase which will be neglected henceforward.

The Ricci quadratic scalar and the Kretschmann scalar are given by

\[
R_{ab} R^{ab} = -(D - 1)^2 \left( \frac{\dot{a}}{a} \right)^2 + (D - 1) \left[ \frac{\dot{a}}{a} + (D - 2) \frac{\dot{a}^2 + k}{a^2} \right]^2, \quad (4.8)
\]

\[
R_{abcd} R^{abcd} = (D - 1) \left[ \left( \frac{\dot{a}}{a} \right)^2 + \left( \frac{\dot{a}^2 + k}{a^2} \right)^2 \right], \quad (4.9)
\]
respectively.

We now assume a dust fluid, \( p = 0 \). For such an equation of state we have

\[
\rho = \rho_0 \left( \frac{a_0}{a} \right)^{D-1},
\]

where \( \rho_0 \) and \( a_0 \) are the constants defined above.

Inserting Eq. (4.7) in Eq. (4.10), we obtain the evolution of the density in the dust model:

\[
\rho(t/l) = \rho_0 \left[ \frac{a_0 r_\Sigma/l}{\sqrt{\left\{ \left( \frac{1}{D-3} \right)^{D-3} \left[ \frac{8 \pi \rho_0 (a_0 r_\Sigma)^{D-1}}{(D-2)!} \right]^{2/(D-1)} - k r_\Sigma^2 \right\}}} \right]^{D-1} \sin^{-(D-1)}(t/l).
\]

We see that the density (4.11) and the curvature scalars (4.8)-(4.9) diverge at \( t/l = \pi \) which represents the formation of a singularity.

V. JUNCTION CONDITIONS

Now we match the exterior and interior spacetimes found in sections III and IV, respectively, across an interface of separation \( \Sigma \). The junctions conditions are

\[
ds^2_+|_{\Sigma} = ds^2_-|_{\Sigma} \quad (5.1)
\]

\[
K^+_{\alpha\beta}\big|_{\Sigma} = K^-_{\alpha\beta}\big|_{\Sigma} \quad (5.2)
\]

where \( K_{\alpha\beta} \) is the extrinsic curvature,

\[
K^\pm_{\alpha\beta} = -n^\pm_\epsilon \frac{\partial^2 x^\epsilon_\pm}{\partial \xi^\alpha \partial \xi^\beta} - n^\pm_\epsilon \Gamma^\gamma_\epsilon_\delta \frac{\partial x^\gamma_\pm}{\partial \xi^\alpha} \frac{\partial x^\delta_\pm}{\partial \xi^\beta} \quad (5.3)
\]

and \( n^\pm_\epsilon \) are the components of the unit normal vector to \( \Sigma \) in the coordinates \( x^\pm_\epsilon \), and \( \xi \) represents the intrinsic coordinates in \( \Sigma \). The subscripts \( \pm \) represent the quantities taken in the exterior and interior spacetimes. Both the metrics and the extrinsic curvatures in (5.1)-(5.2) are evaluated at \( \Sigma \). The metric intrinsic to \( \Sigma \) is written as
\[ ds^2_{\Sigma} = -d\tau^2 + R^2(\tau)\, d\Omega^2_{D-2}. \] (5.4)

Where \( \tau \) is the proper time on \( \Sigma \) and \( d\Omega^2_{D-2} \) denotes the line element on a \( D-2 \) dimensional sphere.

Using the junction condition (5.1), metric (5.4) and the exterior metric (3.4) we obtain

\[ r_+]_{\Sigma} = R(\tau), \] (5.5)

and

\[ \left[ 1 - (M + 1)^{2/(D-1)} + (r_+/l)^2 \right] \dot{r}_+^2 - \left[ 1 - (M + 1)^{2/(D-1)} + (r_+/l)^2 \right]^{-1} \dot{R}^2 = 1, \] (5.6)

where \( \cdot \equiv d/d\tau \), and both equations are evaluated at \( \Sigma \). From now on, we will usually omit the subscript \( \Sigma \) to denote evaluation at the interface. Using (5.5) in (5.6) we find

\[ \frac{d\tau}{d\tau} = \frac{\sqrt{\left[ 1 - (M + 1)^{2/(D-1)} + (R/l)^2 \right] + \dot{R}^2}}{\left[ 1 - (M + 1)^{2/(D-1)} + (R/l)^2 \right]}. \] (5.7)

The unit normal to \( \Sigma \) in the exterior spacetime is

\[ n^+_e = \left( -\frac{d\tau}{d\tau}, \frac{dt}{d\tau}, 0, \ldots, 0 \right). \] (5.8)

From (5.3) we then get

\[ K^{+}_{\theta\theta} = R \sqrt{\left[ 1 - (M + 1)^{2/(D-1)} + \left( \frac{R}{l} \right)^2 \right] + \dot{R}^2}. \] (5.9)

In what follows the other components of \( K^{+}_{ab} \) are not needed.

The unit normal to \( \Sigma \) in the interior spacetime is

\[ n^-_e = \left( 0, \frac{a}{\sqrt{1 - k r_+^2}}, 0, \ldots, 0 \right). \] (5.10)

and from (5.3) we have

\[ K^{-}_{\theta\theta} = R(\tau) \sqrt{1 - k r^2_{\Sigma}}. \] (5.11)
Using the junction condition \( \Sigma = R(\tau) \) for the interior spacetime yields \( ar_\Sigma = R(\tau) \). From the condition \( K_{\theta\theta}^+ = K_{\theta\theta}^- \), (5.9) and (5.11) we obtain

\[
\dot{R}^2 + \left( \frac{R}{l} \right)^2 + k r^2_\Sigma = (M + 1)^{2/(D-1)}, \tag{5.12}
\]

Multiplying equation (4.6) by \( r^2_\Sigma \) we get

\[
\dot{R}^2 + \left( \frac{R}{l} \right)^2 + k r^2_\Sigma = \left( \frac{R_0}{l} \right)^2 \left[ \frac{8 \pi l^2 \rho_0}{(D - 2)! (D - 2)} \right]^{2/(D-1)}. \tag{5.13}
\]

Comparing equation (5.12) and (5.13) we have

\[
M = \left( \frac{1}{7} \right)^{D-3} \left[ \frac{8 \pi \rho_0 (a_0 r_\Sigma)^{D-1}}{(D - 2)! (D - 2)} \right] - 1, \tag{5.14}
\]

which is the mass of the cloud expressed in terms of the constants given in the problem.

**VI. BLACK HOLE FORMATION**

In order to study black hole formation in this theory we work with the solution found in (4.7). The interior and exterior metrics are given in (4.1) and in (3.4) respectively, and as we have shown in section V, it is possible to make a smooth junction between both spacetimes. To be complete we treat the cases \( D \geq 3 \). The case \( D = 3 \) reduces to the collapse studied in [7].

For convenience we rewrite Eqs. (4.6)-(4.9) and (4.11) in terms of the mass \( M \). We have thus

\[
a(t/l) = \frac{l}{r_\Sigma} \sqrt{(M + 1)^{2/(D-1)} - k r^2_\Sigma} \sin(t/l), \tag{6.1}
\]

for the scale factor,

\[
\rho(t/l) = \rho_0 \left[ \frac{a_0 r_\Sigma/l}{\sqrt{(M + 1)^{2/(D-1)} - k r^2_\Sigma}} \right]^{D-1} \sin^{-(D-1)}(t/l), \tag{6.2}
\]

for the density and
\[ R_{ab} R^{ab} = -\frac{(D - 1)^2}{l^4} + \frac{(D - 1)}{l^4} \left\{ -1 + (D - 2) \left[ \frac{(M + 1)^{2/(D-1)} - k r_\Sigma^2}{(M + 1)^{2/(D-1)} - k r_\Sigma^2} \cos^2(t/l) + k r_\Sigma^2 \sin^2(t/l) \right] \right\}^2 \], \quad (6.3)

and

\[ R_{abcd} R^{abcd} = \frac{(D - 1)}{l^4} \left\{ 1 + \left[ \frac{(M + 1)^{2/(D-1)} - k r_\Sigma^2}{(M + 1)^{2/(D-1)} - k r_\Sigma^2} \cos^2(t/l) + k r_\Sigma^2 \sin^2(t/l) \right] \right\}^2 \], \quad (6.4)

for the quadratic Ricci and Kretschmann scalars respectively. In this work we restrict the values of the quantity \( k r_\Sigma^2 \), assuming \( k r_\Sigma^2 = 0, \pm 1/2 \). These values have no special meaning, although for \( k r_\Sigma^2 \) positive and large enough there is no solution at all. Note also that the expression (5.14) for the mass is independent of the value chosen for \( k r_\Sigma^2 \).

Gravitational collapse occurs for \( \pi/2 \leq t/l \leq \pi \). The time \( t/l = \pi/2 \) marks the onset of collapse. At this moment there are no singularities in spacetime, as the curvature scalars (6.3)-(6.4) and the density (6.2) indicate. In fact, the singularity appears only at \( t/l = \pi \), where all these quantities blow up.

To know whether a black hole has formed or not, one has to search for the appearance of an apparent horizon and an event horizon. The apparent horizon is defined to be the boundary of the region of trapped two-spheres in spacetime. To find this boundary on the interior spacetime one looks for two spheres \( Y \equiv a(t) = \text{constant\ whose\ outward\ normals} \) are null, i.e., \( \nabla Y \cdot \nabla Y = 0 \). Using metric (4.1) this yields,

\[ \frac{da(t)}{dt} = -\sqrt{1 - k r_\Sigma^2} \frac{r}{r} \] \quad (6.5)

Using (6.1) in (6.5) gives the evolution of the apparent horizon in comoving coordinates,

\[ \sqrt{\frac{(M + 1)^{2/(D-1)} - k r_\Sigma^2}{1 - k r_\Sigma^2 (r/r_\Sigma)^2}} \left( \frac{r}{r_\Sigma} \right) = -\frac{1}{\cos(t/l)} \] \quad (6.6)

Now, the apparent horizon first forms at the surface \( r_\Sigma \). Then, for \( r = r_\Sigma \), equation (6.6) gives the time \( t/l \) at which the apparent horizon first forms. On the other hand, one should
also be able to find the formation time of the apparent horizon on the surface $\Sigma$ through an equation on $\Sigma$, equation (5.12). Indeed, at the junction one has $R = a(t) r_{\Sigma}$. Then from junction condition (5.12) and equation (6.5) we have that the apparent horizon first forms when

$$\frac{R_{AH}}{l} = \sqrt{(M + 1)^{2/(D-1)} - 1}.$$  \hspace{1cm} (6.7)

Now, using (6.1), the time of formation of the apparent horizon can be found through the equation

$$\frac{R_{AH}}{l} = a(t_{AH}) \frac{r_{\Sigma}}{l} = \sqrt{(M + 1)^{2/(D-1)} - k r_{\Sigma}^2 \sin \left( \frac{t_{AH}}{l} \right)}.$$  \hspace{1cm} (6.8)

Given a dimension $D$ and an $M$ one can obtain $R_{AH}$ through equation (6.7). Then equation (6.8) gives implicitly $t_{AH}$, the time of the formation of the apparent horizon on the surface $\Sigma$ for a given $k$. For instance, for $D = 3$, $M = 0.25$ and $k r_{\Sigma}^2 = 0$ we find $t_{AH}/l = 2.68$. Putting this value back in equation (6.6) we verify that everything checks.

The event horizon, being a null spherical surface, is determined through the null outgoing lines of the metric (4.1), i.e.,

$$\frac{dt}{dr} = \frac{a(t)}{\sqrt{1 - k r^2}}.$$  \hspace{1cm} (6.9)

Equation (6.9) can be put in the following integral form,

$$\sqrt{(M + 1)^{2/(D-1)} - k r_{\Sigma}^2} \int_{r_0/r_{\Sigma}}^{r_1/r_{\Sigma}} \frac{d(r/r_{\Sigma})}{\sqrt{1 - k r_{\Sigma}^2 (r/r_{\Sigma})^2}} = \ln \left[ \frac{\tan(x_1)}{\tan(x_0)} \right],$$  \hspace{1cm} (6.10)

where $x \equiv (1/2)t/l$. Now, the time $x_1$ is to be precisely equal to the formation time of the apparent horizon, since one expects that in vacuum both horizons coincide [8]. One has then to integrate (6.10) to find the time $x_0$ at which the event horizon first forms, at $r = 0$. For instance, $D = 3$, $M = 0.25$ and $k r_{\Sigma}^2 = -1/2$ we obtain $t_0/l = 1.96$. A plot in comoving coordinates ($t/l, r/r_{\Sigma}$) shows the evolution of the apparent and event horizons in
Fig. 1. There we repeat the numerical calculations for the same value of \(D\) and \(M\) but with \(k r_+^2 = 0\) and \(k r_+^2 = 1/2\), as is shown in lines (b) and (c). In Fig. 2 we show the formation of the apparent and event horizons for \(D = 25\), and \(M = 0.25\) and \(k r_+^2 = 0\). Intermediate \(D\) dimensions have similar behavior. Making a matching to the vacuum exterior spacetime one finds the usual Penrose diagram for gravitational collapse and formation of a black hole in an anti-de Sitter background, see Fig. 3.

To study what happens to external observers we note that a light signal emitted from the surface \(r_+\) at the exterior time \(t_+\) obeys the null condition

\[
\frac{dr_+}{dt_+} = 1 - (M + 1)^{2/(D-1)} + \left(\frac{r_+}{l}\right)^2,
\]

(see Eq. (3.4)). This light ray arrives at a point \(r_+\) at time \(t_+\) given by

\[
\frac{t_+}{l} = t_+^{\Sigma/l} + \frac{1}{2(M + 1)^{2/(D-1)} - 2} \ln \left[\frac{(r_+/l) - [2(M + 1)^{2/(D-1)} - 2]}{(r_+/l) + [2(M + 1)^{2/(D-1)} - 2]}\right]^{r_+/l}_{r_+^{\Sigma/l}}.
\]

(6.12)

Thus \(t_+/l \to \infty\) when \(r_+^{\Sigma/l} \to \sqrt{(M + 1)^{2/(D-1)} - 1}\), so the collapse to the event horizon appears to take an infinite amount of time to an exterior observer, and the collapse to \(r_+ = 0\) is unobservable from the outside. Also, the redshift from the dust edge is given by

\[
z = \frac{dt_+}{dt} - 1 = \frac{1}{\sqrt{1 - k r_+^2} + \dot{R}} - 1.
\]

(6.13)

When the dust edge crosses the event horizon we have \(\dot{R} = -\sqrt{1 - k r_+^2}\), so \(z \to \infty\). Thus the collapsing dust will fade from sight, as the redshift of the light from its surface diverges.

**VII. NAKED SINGULARITIES**

To study the presence of naked singularities, i.e., singularities not hidden by an event horizon we analyse equations (3.4), (6.1)-(6.4) and (5.14). Naked singularities appear only when \(M < 0\). Although solutions with negative mass are usually considered unphysical,
they will be studied here because these generalize the three-dimensional solutions found in [4,5]. In the model adopted here it is useful to separate two distinct classes:

i) If \( l \) remains finite (in which case \( \Lambda \neq 0 \)), for any \( D \geq 3 \) the curvature scalars (6.3)-(6.4) will blow up when \( t/l = \pi \), indicating the formation of a curvature naked singularity.

ii) If we take the limit \( l \to \infty \) (in which case \( \Lambda = 0 \)) we see from the exterior metric (3.4) that the event horizon is no longer present, and the collapse will form a naked singularity.

Taking the limit on Eqs. (6.3)-(6.4) we have

\[
R_{ab} R^{ab} = \left( \frac{D-1}{t^4} \right) \left[ \frac{(M+1)^{2/(D-1)}}{(M+1)^{2/(D-1)} - k r_0^2} \right]^2,
\]

(7.1)

\[
R_{abcd} R^{abcd} = \frac{D-1}{t^4} \frac{(M+1)^{2/(D-1)}}{(M+1)^{2/(D-1)} - k r_0^2}.
\]

(7.2)

For any \( D > 3 \) both (7.1)-(7.2) will vanish because from Eq. (5.14) \( M = -1 + \mathcal{O}(l^{-D+3}) \), so in the limit we have \( M = -1 \). Also, from Eq. (6.1) we have in the limit, \( a(t) = \sqrt{-k t} \), so that the only possible solution is when \( k r_0^2 < 0 \). Note also that \( M = -1 \) implies that the exterior metric (3.4) is a Minkowski one, although the interior density (6.2) is non-zero everywhere in the dust cloud. So at \( t/l = \pi \) we will have \( \rho \to \infty \) in a flat Minkowski spacetime. This is analogous to a Newtonian singularity.

For \( D = 3 \) we have \( M = 8 \pi \rho_0 (a_0 r_0)^2 - 1 \) and \( a(t) = \sqrt{8 \pi \rho_0 a_0^2 - k t} \), so that Eqs. (7.1)-(7.2) will be finite but non-zero and the collapse will form a naked conical singularity [10,11].

**VIII. CONCLUSIONS**

We have analysed gravitational collapse in Lovelock gravity for odd-dimensional spacetimes. We have showed that gravitational collapse of a regular initial non-rotating dust cloud proceeds, to form event and apparent horizons, and terminates at a spacelike curvature singularity.
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Figure Captions

Figure 1. Gravitational collapse in $D = 3$ dimensions in an asymptotically anti-de Sitter spacetime. The interior dust cloud in comoving coordinates $(t/l, r/r_\Sigma)$ fills the whole diagram. The left side represents the center of the cloud $r/r_\Sigma = 0$, the right side the surface of the cloud $r/r_\Sigma = 1$. The evolution of the event horizon (dashed line) and apparent horizon (full line) are drawn. The singularity occurs at $t/l = 0$. It was used $M = 0.25$. The three different cases are (a) $k r_\Sigma^2 = -1/2$, (b) $k r_\Sigma^2 = 0$, and (c) $k r_\Sigma^2 = 1/2$.

Figure 2. Dimensionally continued Oppenheimer-Snyder collapse in $D = 25$ dimensions in an asymptotically anti-de Sitter spacetime. It was used $M = 0.25$ and $k r_\Sigma^2 = 0$. See subtitle of figure 1 for more detailed explanation.

Figure 3. Penrose diagram for the collapse of a dust cloud in an asymptotically anti-de Sitter spacetime. Each point in the diagram represents a $D - 2$ sphere. (eh=event horizon, ah=apparent horizon).
$D = 3$
$D = 25$
