Algorithms for Shape and Trajectory Reconstruction of Obstacles in Domains with Variable Speed of Sound

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Algorithms for Shape and Trajectory Reconstruction of Obstacles in Domains with Variable Speed of Sound

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Abstract. We present new parallel algorithms for shape and trajectory reconstruction of moving obstacles using reflected rays. In contrast to tomography where the focus of the reconstruction method is to recover the velocity structure of the domain, the shape and trajectory reconstruction procedure directly finds the shape and trajectory of the obstacle. The rays are curves determined by the variable speed of sound and initial conditions and we develop ultrasonic ray models based on a system of differential equations. The method can achieve high-resolution and computational efficiency.

1. Introduction
Consider an ultrasonic wave, or signal, described by the wave equation
\[ u_{tt} - c^2(x, y, z) \Delta u = 0 \] (1)
where \( c(x, y, z) > 0 \) is the variable speed of sound in \( \Omega(t) = \Omega_0 \setminus \Omega_1(t) \) and
\[ u|_{\partial \Omega_1(t)} = 0 \]
for \( t > 0 \) and \( \Omega_1(t) \subset \Omega_0 \subset \mathbb{R}^3 \), where \( \Omega_1(t) \) is a convex moving obstacle with a smooth boundary in a bounded domain \( \Omega_0 \).

We consider an environment without caustics and look for solutions of the form
\[ u(x, y, z, t) = \sum_{j=0}^{\infty} A_j(x, y, z) e^{i\omega(W(x, y, z) - t)} \omega^j \] (2)
where the eikonal function \( W(x, y, z) = \text{const} \) defines a surface of constant phase. These solutions of the ray equation are called rays or ray solutions. Suppose that for all \( t \) we are given all integrals \( \int_{\gamma} f(l)dl = C_\gamma \) where \( \gamma \) are broken rays in \( \Omega(t) \) such that for each point \( P \in \partial \Omega_1(t) \) there is at least one broken ray \( \gamma \) reflecting at \( P \). A broken ray is a ray reflecting at the obstacle and starting and ending at the observation boundary \( \partial \Omega_0 \).

Let \( f(x) = \frac{1}{c(x, y, z)} > 0 \) in \( \Omega \). Then as we know \( C_\gamma \) correspond to signal travel times in a medium with speed of sound \( c(x, y, z) \). The shape and trajectory reconstruction problem is to find \( \partial \Omega_1(t) \) given the sets \( C_\gamma(t) \) where \( \gamma \in \Omega(t) \).
Consider $\partial\Omega_0$ as the observation boundary and signal transmitters and receivers with known locations along the boundary. Transmitters send ray signals with known initial zenith and azimuth angles at known transmission times. Receivers receive reflected ray signals and record the time when the signal was received. The combined data leads to a set of data points

$$B_k = (x_l, y_l, z_l, x_r, y_r, z_r, \phi_k, \theta_k, t_k, \xi_k)$$

where where $\phi_k$ and $\theta_k$ are the initial incident and azimuth angles of the ray from the transmitter, $x_l$, $y_l$, $z_l$ are the coordinates of the transmitter endpoint of the ray, and $x_r$, $y_r$, $z_r$ are the coordinates of the receiver endpoint of the ray, $t_k$ is the time of flight for the signal and $\xi_k$ is a frequency of the signal. We present algorithms for the shape and trajectory reconstruction problem for finite travel times and variable speed of sound derived from the equations for the Shooting Method for two-point seismic ray tracing [JG].

$$\frac{dx}{dt} = c(x, y, z)\sin \phi \cos \theta \quad (3)$$

$$\frac{dy}{dt} = c(x, y, z)\sin \phi \sin \theta \quad (4)$$

$$\frac{dz}{dt} = c(x, y, z)\cos \phi \quad (5)$$

$$\frac{\partial \phi}{\partial t} = -\cos \phi \left( \frac{\partial c}{\partial x} \cos \theta + \frac{\partial c}{\partial y} \sin \theta \right) + \frac{\partial c}{\partial z} \sin \phi \quad (6)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\sin \phi} \left( \frac{\partial c}{\partial x} \sin \theta - \frac{\partial c}{\partial y} \cos \theta \right) \quad (7)$$

This system of equations derived from the eikonal equation has wide applications in seismology and is used in algorithms for seismic ray tracing [SK]. The system of equations 3 is for a Cartesian coordinate system where $\vec{R}(t) = (x(t), y(t), z(t))$ is the ray position vector, $\phi(t)$ is the incident angle of the ray direction vector with the $z$ axis and $\theta(t)$ is the azimuth angle that the projection of the ray direction vector makes with the positive $x$ axis.

We consider that the speed of sound $c(x, y, z)$ is known inside $\Omega$. We will reconstruct the position of the reflection point $P$ given the positions of a transmitter $L$ and receiver $S$, the incident and azimuth angles of the transmitted ray with respect to a Cartesian coordinate system centered at the transmitter, and the time of flight of the signal. We know the initial position $(x(0), y(0), z(0))$ and therefore we know the velocity of the ray at $t = 0$. We know the angle and initial speed of wave propagation for the transmitted signal at time $t = 0$. Then, knowing the initial conditions, we can find numerically the signal path from the transmitter $L$ through the reflection point $P$ for a given travel time $\tau_k$. Let the travel time from the transmitter $L$ to the reflection point $P$ be $\tau_k$ and the total travel time from the transmitter $L$ to the receiver $S$ through the reflection point $P$ be $t_k$ where $t_k > \tau_k$.

From symmetry, we can imagine that the signal received at the receiver $S$ is transmitted from the receiver $S$ and that it arrives at the reflection point $P$ in time $t = t_k - \tau_k$ because its travel time is $t_k - \tau_k$. The ray path from $S$ to $P$ is again described by the above system of equations for seismic ray tracing, however we do not know the initial angles for the signal from the receiver $S$.

In order to reconstruct the intersection point $P_k$ of rays starting from the transmitter and receiver for a given data point

$$B_k = (x_l, y_l, z_l, x_r, y_r, z_r, \phi_k, \theta_k, t_k, \xi_k)$$

containing the measured values for the signal, we step through a discrete set of values for $\tau_k$. Starting to trace a ray path from $L$, at each time step, each new point on the path is a candidate
reflection point $P$. At this stage, this is basic initial value ray tracing. Then we apply the two point seismic ray tracing shooting method to see if we can reach from receiver $S$ candidate point $P$ for the remainder of our time budget. If we can then we have found the reflection point $P$ for this data point. Otherwise, we continue with the next time step of the initial value ray tracing and check the next candidate point $P$. We repeat this until we find a reflection point $P$ or exhaust our time budget $t_k$. For example, let

$$
\tau_{ks} = \frac{t_k n_s}{N_r}
$$

where $N_r$ is an integer that specifies the time-step resolution and $n_s$ is an integer that specifies the number of the time step and such that $0 \leq n_s < N_r$. Each time $\tau_{ks}$ corresponds to a unique point $P_{ks}$ on the ray path starting from the transmitter $L$ such that $P_{ks}$ can be reached from $L$ in time $\tau_{ks}$. Next, for each time step corresponding to time $\tau_{ks}$ to reach candidate $P_{ks}$ from $L$, we step through the range of initial angles for the signal starting from the receiver $S$ and check whether the curve from $S$ will intersect $P_{ks}$ in time $t_k - \tau_{ks}$. Alternatively, we can step through a range of reflection angles at the point $P_{ks}$ that is found for the given $\tau_{ks}$ and check whether the reflected signal will intersect $S$ in time $t_k - \tau_{ks}$.

2. Reconstruction algorithms

The input to the following algorithm is the speed of sound $c(x)$ for the domain $\Omega$ and a set of data points or ray coordinates corresponding to broken rays. The output is a unique set of points in $\mathbb{R}^3$. The points from the output are the reflection points reconstructed from the input data.

Require: Set of broken ray data points $B_k = (x_l, y_l, z_l, x_r, y_r, z_r, \phi_k, \theta_k, t_k, \xi_k)$

Require: Speed of sound $c(x)$ for domain $\Omega$

{Algorithm for Shape and Trajectory Reconstruction of Moving Obstacles}

{Estimated time complexity is $O(T^2 A)$ where $T$ is the number of discretization points for the time of flight, and $A$ is the number of discretization points for the angle space}

for all data points $B_k$ do

$h_k = \frac{t_k}{N_r}$

$L = (X_0, Y_0, Z_0) = (x_l, y_l, z_l)$ set this initial position to be position of transmitter

$S = (aX_0, aY_0, aZ_0) = (x_r, y_r, z_r)$ set this initial position to be position of receiver

$\Phi_0 = \phi_k$

$\Theta_0 = \theta_k$

$T_0 = 0$

$aT_0 = 0$

for $s = 0 \rightarrow N_r - 1$ do

{Compute the next point on the ray from the transmitter by fourth order Runge-Kutta step and the ray tracing system 3}

$$X_{s+1} = RK4X(h_k, T_s, X_s, Y_s, Z_s, \Phi_s, \Theta_s)$$

$$Y_{s+1} = RK4Y(h_k, T_s, X_s, Y_s, Z_s, \Phi_s, \Theta_s)$$

$$Z_{s+1} = RK4Z(h_k, T_s, X_s, Y_s, Z_s, \Phi_s, \Theta_s)$$

$$\Phi_{s+1} = RK4\phi(h_k, T_s, X_s, Y_s, Z_s, \Phi_s, \Theta_s)$$

$$\Theta_{s+1} = RK4\Theta(h_k, T_s, X_s, Y_s, Z_s, \Phi_s, \Theta_s)$$

$$T_{s+1} = T_s + h_k$$

$P_{s+1} = (X_{s+1}, Y_{s+1}, Z_{s+1})$ point on solution of ray tracing equations with initial values for transmitter that is at time $T_{s+1}$ away from the transmitter $L$

if $!(P_{s+1} \in \Omega_0)$ then

There must be a measurement error. Continue with next data point $B_k$

end if
When the above algorithm is run on a set of points \( \{P_i\} \), the computation for tracing the rays from the receiver can be parallelized in multiple threads. This key for efficient real-time processing, performance and computational efficiency. For example, these algorithms are very suitable for parallelization and their parallelized implementation is methods can be used as well. The above algorithm leads to a class of new algorithms when data structures are used to store the rays for faster processing. For example, when rays from the receiver in its set of directions are stored and looked up in a data structure such as an array, the performance of the algorithm can be improved to \( O(NT^2) \) but \( T \) is the number of discretization points for the time of flight, and A is the number of discretization points for the angle space. These algorithms are very suitable for parallelization and their parallelized implementation is key for efficient real-time processing, performance and computational efficiency. For example, the computation for tracing the rays from the receiver can be parallelized in multiple threads. When the above algorithm is run on a set of points \( \{B_k\} \) from one sampling time interval \( T_k \), the algorithm reconstructs the shape of the obstacle during this sampling interval. In order to reconstruct the trajectory of the obstacle, the algorithm is run on the data points for each of the sampling intervals. The method can achieve high resolution because it can process a very large number of distinct points on the obstacle’s boundary.

3. Acknowledgements
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4. References
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