Ultra-Relativistic Nuclei in Crystal Channel: Coulomb Scattering, Coherence and Absorption

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Abstract

We incorporate the effect of lattice thermal vibrations into the Glauber theory description of particle- and nucleus-crystal Coulomb interactions at high-energy. The allowance for the lattice thermal vibrations is shown to produce strong absorption effect: the phase shift function of the multiple-diffraction scattering on a chain of $N$ identical atoms acquires large imaginary part and the radius of the absorption region in the impact parameter plane grows logarithmically with $N$. Consequences of this observation for the elastic and quasi-elastic Coulomb scattering are discussed. Practically interesting example of the coherent Coulomb excitation of ultra-relativistic particles and nuclei passing through the crystal is considered in detail.
1 Introduction

In this paper we develop the Glauber theory [1] description of the absorption phenomenon in coherent particle- and nucleus-crystal Coulomb interactions at high-energy.

As is well known, the multi-loop corrections generate the imaginary part of the scattering amplitude even if the tree-level amplitude is purely real. For example, the purely real Born amplitude of the high-energy Coulomb scattering in crystal acquires the imaginary part due to the multiple scattering (MS) effects. However, in the widely used static/frozen lattice approximation (SL approximation) the account of rescatterings alters only the overall real phase of the full amplitude thus producing no absorption effect. The latter is related to the creation and annihilation of excited intermediate states of crystal and as such manifests itself only beyond the SL approximation (for the analysis of elastic scattering based on the SL approximation see [2]). Indeed, the amplitude of small-angle elastic scattering on a chain of $N$ identical atoms in the impact parameter representation equals

$$1 - \langle S(b) \rangle,$$

where the scattering matrix placed between the ground states of crystal is

$$\langle S(b) \rangle = \langle \exp[i\chi(b)] \rangle$$

with the purely real phase shift function $\chi(b) = \sum_{j=1}^{N} \chi_j(b)$. In the SL approximation

$$\langle \exp[i\chi(b)] \rangle \approx \exp[i\chi(b)].$$

Therefore, only with the allowance for the lattice thermal vibrations the Coulomb phase shift function gets, in general, non-vanishing imaginary part which is interpreted as an absorption effect. The imaginary part appears only as the second order perturbation,

$$\sim \frac{i}{2} [\langle \chi^2 \rangle - \langle \chi \rangle^2].$$

But the strength of the effect is $\propto \beta^2 N$, where $\beta$ is the coupling constant. For the coherent scattering of relativistic nuclei (the electric charge $Z_1$) on the chain of $N$ atoms (the atomic number $Z_2$) in a crystal the effective coupling $\beta = 2\alpha Z_1 Z_2$ is strong and the absorption effect is strong as well. The absorption is strong for impact parameters, $b$, smaller than some characteristic value $b_a \propto \log(\beta N)$, and vanishes toward the region of larger $b$. This phenomenon provides a natural ultra-violet regulator of the theory and enables, in particular, consistent calculation of the coherent elastic scattering cross section. The latter is calculated and turns out to be equal to the one half of the total cross section. As we shall see, the absorption effect is of prime importance also for quantitative understanding of the phenomenon of the coherent Coulomb excitation of relativistic particles and nuclei passing through the crystal. A consistent description of this phenomenon is the goal of our paper.

The outline of the paper is as follows. We start with the well known example of the coherent Coulomb elastic scattering of charged particle/nucleus by a linear chain of $N$ identical atoms in a crystal target (Sec.2). We derive the scattering matrix with absorption and calculate the cross section of the coherent elastic scattering, $\sigma_{el}$ (Sec. 3) and the cross section of the incoherent excitation and break-up of the target, $\sigma_{Qel}$ (Sec. 4). We find that in the large-$N$ limit $\sigma_{el} \approx \sigma_{Qel} \approx \frac{1}{2} \sigma_{tot}$ (Sec.5). In Sec.6 we discuss the coherent Coulomb excitation of ultra-relativistic particles and nuclei passing through the crystal to the lowest order of perturbation theory. The higher order effects we consider in Sec. 7 where the cross section of the process is calculated. We finally conclude with a brief summary in Sec. 8.
2 Coherent elastic scattering and absorption.

The interatomic distances in crystal, $a$, are large, compared to the Thomas-Fermi screening radius $r_0$, $a \sim 3 - 5\AA \gg r_0 = r_B Z_2^{-1/3} \sim 0.1\AA$, where $Z_2$ is the atomic number of the target atom and $r_B$ is the Bohr radius [3]. The relevant impact parameters, $b$, satisfy the condition $b \ll a$ and the amplitudes of scattering by different atomic chains parallel to a given crystallographic axis are incoherent.

The amplitude of small-angle scattering of charged particle (charge $Z_1$) by a linear chain of $N$ identical atoms in the eikonal approximation reads [1]

$$ F_{fi}(q) = \frac{ip}{2\pi} \int d^2b \exp(iqb)(\Psi_f(\{r_j\})|1 - S(b, s_1, \ldots, s_N)|\Psi_i(\{r_j\})) , \tag{1} $$

where $\Psi_i$ and $\Psi_f$ are the initial and final state wave functions of the crystal and $q$ is the 2D-vector of the momentum transfer. The incident particle momentum $p$ is assumed to be large enough to satisfy the condition of applicability of the straight paths approximation, $p/q^2 \gg aN$. The latter condition insures the coherence of interactions with different atoms.

The elastic scattering corresponds to $i = f$ and the brackets $\langle \rangle$ signify that an average is to be taken over all configurations of atoms in the ground state,

$$ \langle \Psi(\{r_j\})|1 - S(b, s_1, \ldots, s_N)|\Psi(\{r_j\})\rangle = \int d^3r_1\ldots d^3r_N|\Psi(\{r_j\})|^2 \left[ 1 - \exp(i \sum_1^N \chi(\mu|b - s_j|) ) \right] . \tag{2} $$

In (2) the total scattering phase is the sum of the phase shifts contributed by the individual atoms. The positions of the $N$ atoms which make up the target are defined by the 3D-vectors $r_j$, $j = 1, \ldots, N$. The 2D-vectors $s_j$ are the projections of these vectors on the impact parameter plane. We neglect all position correlations of the atoms and describe the ground state of crystal by the wave function $|\Psi\rangle$ such that

$$ |\Psi(\{r_j\})|^2 = \prod_{j=1}^N|\psi u_j)|^2 , \tag{3} $$

where the 3D-vectors $u_j$ are defined by $r_j = (j - 1)a + u_j$, $j = 1, \ldots, N$, $a = (0, 0, a)$ and $u_j = (s_j, z_j)$.

From eq.(2) it follows that

$$ F_{fi}(q) = F(q) = ip \int db J_0(qb) \left\{ 1 - \langle \exp[i\chi(\mu b)]\rangle^N \right\} . \tag{4} $$

Hereafter, $J_{0,1}(x)$ and $K_{0,1}(x)$ are the Bessel functions and the screened Coulomb phase shift function is

$$ \chi(\mu b) = -\beta K_0(\mu b) , \tag{5} $$

with $\beta = 2\alpha Z_1 Z_2$ and $\mu = r_0^{-1}$. After integration over longitudinal variables $\{z_j\}$ followed by the azimuthal integration the term $\langle \exp(i\chi) \rangle$ takes the form

3
\[
\langle \exp(i\chi) \rangle = \int d^2s \rho(s) \exp[i\chi(\mu|b-s|)]
\]
\[
= \exp(-\Omega^2b^2) \int_0^\infty dx \exp(-x) \times I_0(2b\sqrt{x}) \exp[-i\beta K_0(\mu\sqrt{x}/\Omega)].
\]

The 2D-vector \( s \) describes the position of the target atom in the impact parameter plane. The one-particle probability distribution \( \rho(s) \) is as follows
\[
\rho(s) = \int dz |\psi(s, z)|^2 = (\Omega^2/\pi) \exp(-\Omega^2s^2).
\]

For the most commonly studied elements at room temperature the ratio \( \mu/\Omega \) varies in a wide range, from \( \mu/\Omega \sim 0.1 \) to \( \mu/\Omega \sim 1 \) [3]. Consider first the region of small impact parameters including \( b \simeq 1/\Omega \). For \( b \ll 1/2\Omega \) only small \( s \), such that \( \mu s \ll 1 \), contribute. One can put then \( K_0(\mu s) \simeq \log(1/\mu s) \) and integrate over \( s \),
\[
\langle \exp(i\chi) \rangle \simeq \left(\frac{\mu}{\Omega}\right)^{i\beta} \exp(-\Omega^2b^2) \times \int_0^\infty dx x^{i\beta/2} \exp(-x) I_0(2b\sqrt{x})
\]
\[
= \left(\frac{\mu}{\Omega}\right)^{i\beta} \Gamma \left(1 + \frac{i\beta}{2}\right) \Phi \left(-\frac{i\beta}{2}; 1; -\Omega^2b^2\right)
\]

In eq.(8)
\[
\Phi(a, b; z) = 1 + \frac{a}{b} z + \frac{a(a+1)}{b(b+1)} z^2 + ...
\]
is the confluent hyper-geometric function and \( \Phi(a; b; z) = \exp(z)\Phi(b-a; b; -z) \).

From eq.(8) it follows that
\[
|\langle \exp(i\chi) \rangle|_{b=0} = \left[ \frac{\pi \beta}{2 \sinh(\pi \beta/2)} \right]^{1/2},
\]
where the identity
\[
|\Gamma(i\beta/2)|^2 = \frac{2\pi}{\beta \sinh(\pi \beta/2)}
\]
has been used. In the weak coupling regime, \( \beta \ll 1 \),
\[
|\langle \exp(i\chi) \rangle|_{b=0} \simeq 1 - \frac{1}{2} (\langle \chi^2 \rangle - \langle \chi \rangle^2)
\]
and
\[2^2\text{Notice that the smallness of the ratio } r_A^2/u^2 \sim 10^{-5} - 10^{-6}, \text{ where } r_A \text{ is the nuclear radius and } u = 1/\Omega \text{ is the amplitude of lattice thermal vibrations, allows one to neglect the nuclear interactions of the projectile up to } N \sim 10^5. \text{ As we shall see, the absorption effect which we are interested in enters the game at much smaller } N.\]
\[ \langle \chi^2 \rangle - \langle \chi \rangle^2 = \frac{\pi^2 \beta^2}{24} \quad (12) \]

For \( \beta \gg 1 \)

\[ \langle \exp(i\chi) \rangle_{b=0} \simeq \sqrt{\pi\beta} \exp(-\pi\beta/4). \quad (13) \]

Therefore, at small impact parameters, \( b < 1/2\Omega \), the intensity of outgoing nuclear waves as a function of \( N \) exhibits the exponential attenuation. In terms of the unitarity cuts of the elastic scattering amplitude the imaginary part of the phase shift function comes from the cuts through the multi-photon projectile-atom blocks as shown in Fig. 1a. The account of diagrams like that of Fig. 1b which allows cuts only between projectile-atom blocks, gives the scattering matrix of the form \( \exp(iN\langle\chi\rangle) \) and affects only the overall real phase of the amplitude.

![Diagram a) and b) showing relevant multiple scattering](image)

**Figure 1:** *Example of the relevant multiple scattering diagrams to order \( \beta^3 \). The unitarity cut of the elastic amplitude 1a) which contributes to the absorption is shown by crosses. The diagram 1b) allows cuts only between the projectile-atom blocks and does not contribute to the absorption effect.*

The absorption effect becomes weaker toward the region of large impact parameters \( b \gg 1/2\Omega \),

\[ \langle \exp(i\chi) \rangle^N \simeq \langle \exp(i\chi) \rangle_{b=0}^N \left[ 1 + \frac{N\beta^2}{16} (\Omega b)^4 + \ldots \right]. \quad (14) \]

For still larger \( b, b \gg 1/2\Omega \), making use of the asymptotic form \( I_0(z) \simeq (2\pi z)^{-1/2} \exp(z) \) yields

\[ \langle \exp(i\chi) \rangle \simeq 2\Omega \int \frac{sd\omega}{\sqrt{\pi bs}} \exp[-\Omega^2(b-s)^2] \exp[i\chi(\mu s)]. \quad (15) \]

To evaluate the integral (15) expand \( \chi(\mu s) \) in powers of \( (s - b) \),

\[ \chi(\mu s) \simeq \chi(\mu b) + \omega(s - b). \]
If the frequency $\omega$,

$$\omega = \frac{d\chi}{db} = \mu \beta K_1(\mu b),$$

is small compared to $\Omega$,

$$\omega \ll \Omega, \quad (16)$$

$$\langle \chi^2 \rangle - \langle \chi \rangle^2 = \frac{\omega^2}{2\Omega^2}$$

and

$$\langle \exp(i\chi) \rangle \simeq \exp[i\chi - i\omega b]$$

$$\times \Omega \int_0^\infty \frac{sd{s}}{\sqrt{\pi bs}} \exp[-\Omega^2(b - s)^2] \exp[i\omega s]$$

$$\simeq \exp(i\chi) \exp[-\omega^2/4\Omega^2]. \quad (17)$$

The condition (16) is satisfied if $b \gg \beta/\Omega$. For the impact parameters from the region $\beta/\Omega \ll b \ll 1/\mu$ we may write $\omega \simeq \beta/b$ and for larger $b$ such that $b \gg \mu^{-1}$,

$$\omega \simeq \mu \beta \sqrt{\frac{\pi}{2\mu b}} \exp(-\mu b). \quad (18)$$

From the consideration presented above it follows that the absorption effect in the elastic scattering amplitude is especially strong for impact parameters

$$b \lesssim b_a = \frac{1}{2\mu} \log \frac{\pi \mu^2 \beta^2 N}{4\Omega^2}. \quad (19)$$

For $b \ll b_a$ the atomic chain acts like an opaque “black” disc. Certainly, the value of this finding differs for different observables and for different processes proceeding at different impact parameters. The only thing which is worth noticing here is the representation of the scattering matrix in the form

$$\langle S(b) \rangle \simeq \exp \left( iN\chi - \frac{N\omega^2}{4\Omega^2} \right); \quad b \gg \beta\Omega^{-1}. \quad (20)$$

The eq.(20) supplemented with the observation that

$$\langle S(b) \rangle \simeq (\pi \beta)^{N/2} \exp \left( -\frac{N\pi \beta}{4} \right); \quad b \lesssim \Omega^{-1} \quad (21)$$

simplifies all further calculations greatly.
The elastic cross section.

Integrating once by parts reduces $F(q)$ to the form convenient for evaluation of the total cross section,

$$F(q) = \frac{i p \mu N}{q} \int_0^\infty b db J_1(qb) \langle i \chi' \exp(i \chi) \rangle \langle \exp(i \chi) \rangle^{(N-1)}. \quad (22)$$

At small impact parameters, $b \ll 1/2\mu$,

$$\langle \chi' \exp(i \chi) \rangle \simeq \beta \left( \frac{\mu}{\Omega} \right)^{i \beta - 1} \exp(-\Omega^2 b^2) \times \Gamma \left( \frac{i \beta + 1}{2} \right) \Phi \left( \frac{i \beta + 1}{2}; 1; \Omega^2 b^2 \right). \quad (23)$$

Because of multiple scatterings only large impact parameters $b$ may contribute to $F(q)$ at large $N$ and small $q$. Hence,

$$F(q) \simeq \frac{i p \mu N}{q} \int_{1/\mu}^\infty b db J_1(qb) \left[ i \chi' - \omega \omega' / 2\Omega^2 \right] \times \exp(i N \chi) \exp(-N \omega^2 / 4\Omega^2). \quad (24)$$

where the explicit form of $\langle \exp(i \chi) \rangle$ at large $b$, eq.(17), has been used. For large $b$, $\omega^2 \propto \exp(-2\mu b)$ and diminishes with growing $b$ much faster than the phase shift function $\chi(\mu b)$ which is $\propto \exp(-\mu b)$. One can see that the leading contribution to the elastic scattering amplitude (24) comes from

$$b \sim \mu^{-1} \xi \gg b_a,$$

where

$$\xi = \log(\beta N)$$

and for large $N$ the second term in square brackets in (24) is small compared to the first one. Then, for

$$q \lesssim q_0 = \mu / \xi$$

and $\xi \gg 1$, the steepest descent from the saddle-point

$$b_0 = \mu^{-1} [\xi + i \pi / 2] \quad (25)$$

in eq.(24) yields

$$F(q) \simeq \frac{i p b_0}{q} J_1(qb_0). \quad (26)$$

The effect of lattice thermal vibrations at small $q$ appears to be marginal and reduces to the factor $\exp(\mu^2 / 4\Omega^2 N)$ in (26) which is irrelevant at large $N$. The amplitude $F(q)$ at $q \to 0$ coincides with the elastic scattering amplitude given by the SL approximation [2].

If $q \gtrsim q_0$ the stationary phase approximation gives the elastic scattering amplitude of the form
\[ F(q) \simeq \frac{-ip\sqrt{\eta}}{\mu q} \exp \left( -\frac{iq\eta}{\mu} \right) \exp \left( -\frac{q^2}{4\Omega^2 N} \right) \]

where \( \eta = \log(\mu \beta N/q) \gg 1 \). The account of the lattice thermal vibrations insures the convergence of the integral for the coherent elastic scattering cross section,

\[ \sigma_{el} = \frac{\pi}{p^2} \int dq |F(q)|^2 \simeq \frac{\pi \xi^2}{\mu^2} \int_0^{q_0} \frac{dq^2}{q^2} J_1^2 \left( \frac{q \xi}{\mu} \right) \]

\[ + \frac{\mu^2}{p^2} \int_{q_0}^{\infty} dq^2 \log \left( \frac{\mu \beta N}{q} \right) \exp \left( -\frac{q^2}{2\Omega^2 N} \right), \]

which for \( \xi \gg 1 \) is simply

\[ \sigma_{el} \approx \frac{\pi}{\mu^2} \xi^2. \]

4 The quasi-elastic cross section.

In this paper we focus on the coherent nucleus-atom interactions. The incoherent process of ionization of the target atom is suppressed by the factor \( \sim Z^{-2} \). Then, the inelastic process which via unitarity gives rise to the attenuation of elastic amplitude is the process of the quasi-elastic scattering (Fig.1a). Its cross section is the quantity [1]

\[ p^2 \frac{d\sigma_{Qel}}{d^2 q} = \sum_f |F_{fi}(q)|^2 - |F_{ii}(q)|^2 \]

where the sum extends over all final states of crystal, in which no particle production takes place. The closure relation then yields

\[ \frac{d\sigma_{Qel}}{d^2 q} = \frac{1}{4\pi^2} \int d^2 b d^2 b' \exp[iq(b - b')] \]

\[ \times \left\{ \langle \exp[i\chi(\mu b) - i\chi^*(\mu b')] \rangle^N \right. \]

\[ - \left. \langle \exp[i\chi(\mu b)] \rangle^N \langle \exp[-i\chi^*(\mu b')] \rangle^N \right\} \]

and

\[ \sigma_{Qel} = \int d^2 b \left\{ 1 - |\langle \exp[i\chi(\mu b)] \rangle |^{2N} \right\}. \]

In the SL approximation \( |\langle \exp[i\chi(\mu b)] \rangle | = 1 \) and \( \sigma_{Qel} = 0 \). From (20) and the discussion of the absorption radius, \( b_a \), presented above it follows that for \( \xi \gg 1 \)

\[ 1 - |\langle \exp[i\chi(\mu b)] \rangle |^{2N} \approx \theta(2b_a - b) \]

and

\[ \sigma_{Qel} \approx \pi (2b_a)^2 \approx \frac{\pi}{\mu^2} \xi^2 \]
5 The total cross section.

From eq.(26) by means of the optical theorem we find the total cross section

$$\sigma_{tot} = \frac{4\pi}{p} \text{Im} F(0) \approx \frac{2\pi}{\mu^2} \xi^2. \quad (35)$$

Thus, we conclude that at high energy and in the large-$N$ limit

$$\sigma_{el} \approx \sigma_{Qel} \approx \frac{1}{2} \sigma_{tot}. \quad (36)$$

6 Coulomb excitation of ultra-relativistic particles and nuclei in crystal channel. The excitation cross section to the lowest order. The Born approximation.

Consider now the process of the coherent Coulomb excitation of ultra-relativistic particles and nuclei passing through the crystal. This way of the experimental study of rare processes has been proposed in [4, 5, 6, 7, 8, 9, 10].

The ultra-relativistic projectile-nucleus (the mass number $A$, the charge $Z_1$ and the four-momentum $p$) moving along a crystal axis undergoes a correlated series of soft collisions which give rise to diagonal ($A \to A$, $A^* \to A^*$) and off-diagonal ($A \to A^*$, $A^* \to A$) transitions.

In [9, 4, 5] it has been proposed to study the electric dipole transition in $^{19}\text{F}$, the excitation of the state $|J^\pi = 1/2^-\rangle$ from the ground state $|1/2^+\rangle$. The phenomenological matrix element of the transition $1/2^+ \to 1/2^-$ is [14]

$$\mathcal{M} = \frac{1}{2} d \bar{u}(p') \gamma_5 (\hat{q} \vec{\varepsilon} - \varepsilon \hat{q}) u(p), \quad (37)$$

where both $u(p')$ and $u(p)$ are bispinors of initial and final states of the projectile, $d$ is the transition dipole moment and $\varepsilon$ is the photon polarization vector. The transverse and longitudinal components the 4-vector $p - p'$ are denoted by $q$ and $\kappa$, respectively. In what follows $q = |q|$. The only phenomenological parameter of the problem is the dipole moment $d$. The measured life-time of the 110 KeV level $^{19}F(1/2^-)$ is $\tau = (0.853 \pm 0.010) \times 10^{-9}$ sec [13] and the dipole moment of the $1/2^+ \to 1/2^-$ transition, determined from the width of the level $^{19}F(1/2^-)$ is $d \approx 5 \times 10^{-8}$ KeV$^{-1}$ [14]. Then, first, because of large value of $\tau$ the decay of excited state inside the target crystal can be safely neglected and, second, due to the smallness of $d$, the excitation amplitude is much smaller than the elastic Coulomb amplitude for all $q$ up to $q \sim \sqrt{4\pi\alpha Z_1/d}$ and can be considered as a perturbation. Thus, the multi-channel problem reduces to the one-channel one.

The high-energy helicity-flip Born amplitude of the transition $1/2^+ \to 1/2^-$ in collision of the projectile-nucleus with $N$ bound atoms in crystal reads

$$F_{ex}^B(q) = S(\kappa) \frac{p}{2\pi} \frac{g(\sigma q)}{q^2 + \lambda^2} \exp \left( -\frac{q^2}{4\Omega^2} \right), \quad (38)$$

where $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ is the Pauli spin vector, $\{\sigma_i, \sigma_j\} = 2\delta_{ij}$ and the amplitude we are constructed is to be regarded as an operator which transforms the initial helicity state of
the projectile into its final state. In the denominator of eq. (38) \( \lambda^2 = \mu^2 + \kappa^2 \). In the Glauber approximation the longitudinal momentum transfer which determines the coherency length, \( l_c \sim \kappa^{-1} \), reads \[11\]

\[ \kappa = \frac{M \Delta E}{p}, \tag{39} \]

where \( M \) is the mass of projectile and \( \Delta E \) is the excitation energy \(^3\).

The structure factor of crystal, \( S(\kappa) \), to the first order in \( g \) is

\[ S(\kappa) = \exp \left[ -\frac{\kappa^2}{4 \Omega^2} \right] \frac{\sin(\kappa Na/2)}{\sin(\kappa a/2)}. \tag{40} \]

If the projectile momentum satisfies the resonance condition \[4, 5, 9, 7\]

\[ \frac{M \Delta E}{p} = \frac{2\pi n}{a}, \quad n = 0, 1, 2... \tag{41} \]

\( S(\kappa) \sim N. \) Then, to the first order in \( g \) and to the zero order in \( \beta \) (Born approximation) the cross section of the coherent excitation of the projectile in scattering on a chain of \( N \) atoms in crystal is

\[ \sigma_{ex}^B = \frac{\pi}{p^2} \int dq^2 |F_{ex}^B(q)|^2 \]

\[ \sim \frac{g^2 N^2}{4\pi} \left[ \log \left( 1 + \frac{2\Omega^2}{\lambda^2} \right) - \frac{2\Omega^2}{\lambda^2 + 2\Omega^2} \right], \tag{42} \]

where \( g = \sqrt{4\pi \alpha d Z} \). The central idea of \[4, 5, 6, 7, 9, 10\] based on the Born approximation is that the transition rate can be enhanced substantially due to coherency of interactions which is assumed to sustain over the large distance scale. The law \( \sigma_{ex} \propto N^2 \) is expected to hold true up to the crystal thicknesses \( N = L/a \sim 10^5 - 10^6 \) in tungsten target. In \[10\] the Born approximation for the coherent excitation of \( \Sigma^+ \) in high-energy proton-crystal interactions \( p\gamma \rightarrow \Sigma^+ \) has been assumed to be valid up to \( N \sim 10^8 \). However, the account of the initial and final state Coulomb interactions dramatically changes the dependence of \( \sigma_{ex} \) on \( N \). For instance, at \( N = 2 \) the excitation amplitude is of the following form

\[ F_{ex}^{(2)}(q) = \frac{p}{\pi} \int d^2 b \exp(iqb) \langle f_{ex}^B \exp(i\chi) \rangle \langle \exp(i\chi) \rangle \tag{43} \]

The first one of two bracketed factors in eq. (43) corresponds to the nuclear excitation amplitude in scattering on a bound atom. It differs from the excitation amplitude of the Born approximation, \( f_{ex}^B(b) \), by the multiplicative phase factor which is due to the initial and final state multiple Coulomb scattering. At small impact parameters, \( b \ll 1/2\Omega \),

\[ \langle f_{ex}^B \exp(i\chi) \rangle \approx S(\kappa) \frac{g}{2\pi b} (\sigma_{mb}) \sinh \left( \frac{1}{2} \Omega^2 b^2 \right) \exp \left( -\frac{1}{2} \Omega^2 b^2 \right). \tag{44} \]

For large \( b \)

\(^3\)The Fresnel corrections to the eikonal approximation which are neglected here become important at large \( N \) or at large \( q \) diminish the coherency length and bring about an additional suppression of coherent processes \[12\].
\[ \langle f^B_{\text{ex}} \exp(i\chi) \rangle = S(\kappa) \frac{g}{4\pi} \int d^2 s \rho(s) (\sigma(n_b - n_s)) \times \lambda K_1(\lambda|n_b - n_s|) \exp[i\chi(\mu|n_b - n_s|)] \]
\[ \simeq S(\kappa) \frac{g}{4\pi} (\sigma n_b) \lambda K_1(\lambda b) \exp(i\chi) \exp \left( -\frac{\omega^2}{4\Omega^2} \right), \]  
(45)

where \( n_b = b/|b|, n_s = s/|s| \) and \( b \gtrsim \mu^{-1} \). As far as for small \( b \)
\[ |\langle f^B_{\text{ex}} \exp(i\chi) \rangle|^2 \propto \Omega^2 b^2 \]
and for large impact parameters, \( b \gtrsim 1/\mu \),
\[ bK_1^2(\mu b) \propto \exp(-2\mu b) \]
the cross section
\[ \sigma_{\text{ex}}^{(2)} = \int d^2 b |\langle f^B_{\text{ex}} \exp(i\chi) \rangle|^2 |\langle \exp(i\chi) \rangle|^2 \]
\[ \simeq 4\sigma_{\text{ex}}^{(1)} \left( 1 - \frac{\omega^2}{2\Omega^2} \right) \]  
(46)

is dominated by \( b \sim 1/2\mu \). For the diamond crystal \( \mu/\Omega \simeq 0.16 \) [3]. Hence, \( \omega^2/2\Omega^2 \simeq 2\beta^2\mu^2/\Omega^2 \sim 1/20 \). This estimate shows that even for the diamond crystal target the Born approximation is irrelevant already at \( N \gtrsim 10 \).

### 7 The multiple scattering effects and absorption in the coherent Coulomb excitation processes

The transition amplitude on a chain of \( N \) identical atoms including all the multi-photon t-channel exchanges reads
\[ F_{\text{ex}}(q) = \frac{p}{\pi} \int d^2 b \exp(iqb) \langle f^B_{\text{ex}} \exp(i\chi) \rangle \langle \exp(i\chi) \rangle^{N-1} \]  
(47)

Because of both the multiple scattering effect and absorption only large impact parameters, \( b \gg \mu^{-1} \), may contribute to \( F_{\text{ex}}(q) \). Then, the evaluation of \( F_{\text{ex}}(q) \) reads
\[ F_{\text{ex}}(q) \approx \frac{2p}{\pi} S(\kappa) (\sigma n_q) \int_{1/\mu}^{\infty} b db J_1(qb) \]
\[ \times \lambda K_1(\lambda b) \exp(iN\chi) \exp(-N\omega^2/4\Omega^2), \]  
(48)

where \( n_q = q/|q| \). The contribution of the domain \( q \lesssim q_0 = \mu/\xi \) to the excitation cross section can be neglected as far as \( F_{\text{ex}} \propto q \) in this region. If \( q \gg q_0 \) and \( \xi \gg 1 \), the stationary phase approximation gives the coherent excitation amplitude of the form
\[ F_{\text{ex}}(q) \approx \frac{ipg(\sigma n_q)}{2\pi\beta} S(\kappa) \frac{\lambda}{N \mu} \sqrt{\eta} \exp(-\delta\eta) \]
\[ \times \exp \left( -\frac{iq\eta}{\mu} \right) \exp \left( -\frac{q^2}{4\Omega^2 N} \right) . \]  
(49)

11
We see that the helicity-flip dynamics removes the factor $1/q$ from the elastic amplitude (27) thus making the UV-regularization of the excitation cross section indispensable. The latter is evaluated as,

$$
\sigma_{ex} = \frac{\pi}{p^2} \int dq^2 |F_{ex}(q)|^2 \\
\sim \frac{g^2 N^{1-\delta}}{8\pi} C \log \left( \frac{N}{\delta \gamma} \right),
$$

(50)

where $C = \gamma^2 \Delta^2 \Gamma(\Delta)$, $\gamma = 2\Omega^2/\beta^2 \mu^2$, $\Delta = \lambda/\mu$ and $\delta = \Delta - 1 \sim \kappa^2/2 \mu^2 \ll 1$. In (50) we put simply $S(\kappa) = N$. Thus, the account of multiple scatterings and absorption turns the Born approximation cross section $\sigma_{ex} \propto N^2$ into $\sigma_{ex} \propto N^{1-\delta} \log N$. In the limit of $p \to \infty$ and $\delta \to 0$,

$$
\sigma_{ex} \sim \frac{g^2 N}{8\pi} \gamma \log \left( \frac{N}{\gamma} \right)
$$

(51)

The dependence of $\sigma_{ex}$ on $N$ differs from that of the fully unitarized elastic cross section, $\sigma_{el} \propto \log^2 N$. The reason is that in $\sigma_{ex}$ we sum the eikonal diagrams to all orders in $\beta$ but only to the first order in $g$. Such a procedure of unitarization is, of course, incomplete, but this is of no importance for practical purposes since the smallness of $d^2 \Omega^2$ makes the next to leading order terms negligibly small up to $N \sim \alpha Z^2_1/\delta \Omega^2 d^2 \sim 10^{12}$.

8 Summary

The main goal we pursued in this paper is a consistent description of the coherent Coulomb excitation of ultra-relativistic particles and nuclei passing through the aligned crystal. We started with the discussion of the elastic scattering and found that the account of the lattice thermal vibrations within the Glauber multiple scattering theory gives rise to the strong absorption effect. The radius of the absorption region in the impact parameter space appeared to grow logarithmically with growing crystal thickness. We derive convenient representation for the scattering matrix with absorption and calculate the coherent elastic and the incoherent quasi-elastic cross sections. The suppression of scattering amplitudes in the absorption region is shown to serve as a natural UV regulator and enables consistent calculation of the cross section of the coherent nuclear excitation, $\sigma_{ex}$. The dependence of $\sigma_{ex}$ on the crystal thickness is found. The multiple scattering effects are shown to become numerically important already at $N \gtrsim 1$ thus leaving no room for the Born approximation widely used in early analyses of the problem.

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