Can the “standard” unitarized Regge models describe the TOTEM data?

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received 5 April 2013; accepted 29 April 2013
published online 13 May 2013

PACS 13.85.-t – Hadron-induced high- and super-high-energy interactions (energy \(> 10 \text{ GeV}\))
PACS 11.55.Jy – Regge formalism
PACS 12.40.Nn – Regge theory, duality, absorptive/optical models

Abstract – The standard Regge poles are considered as inputs for two unitarization methods: eikonal and \(U\)-matrix. It is shown that only models with three input pomerons and two input odderon can describe the high-energy data on \(pp\) and \(\bar{p}p\) elastic scattering including the new data from Tevatron and LHC. However, it seems that both the considered models (eikonal and \(U\)-matrix) require a further modification (e.g., to explore nonlinear reggeon trajectories and/or nonexponential vertex functions) for a more satisfactory description of the data at \(19.0 \text{ GeV} \leq \sqrt{s} \leq 7 \text{ TeV}\) and \(0.01 \leq |t| \leq 14.2 \text{ GeV}^2\).

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Introduction. – The first results of the TOTEM experiment on elastic proton-proton scattering at 7 TeV were published in 2011 [1]. Surprisingly none of the models (most successful at lower energies and wide intervals of \(t\)) could be able to predict correctly the \(t\)-dependence of the \(pp\) differential cross-sections at the LHC energy [2,3].

The data on elastic \(pp\) and \(\bar{p}p\) scattering at energies up to the Tevatron one were quite successfully described in three types of models. The first type deals with the intercept larger than one pomerons (e.g., Donnachie-Landschell model, see [4] and the recent papers [5,6]). However such models have a defect: they violate the Froissart bound on the total cross-sections and have to be unitarized even if they are in agreement with the data. This feature gives rise to various methods of unitarization of the input amplitudes violating the unitarity. The most known models of such type are the eikonal (see a general discussion on the eikonal method in [7]) (or quasi-eikonal [8]) and \(U\)-matrix [9] (or its generalization [10]).

The models of the third type are constructed taking into account restrictions of amplitude analyticity and unitarity from the beginning.

The TOTEM data have been analyzed in a few recent publications. Here, for the sake of shortness we do not review the obtained results. The corresponding references and discussion on various models and its agreement or disagreement with the old data and recent D0 [11] and TOTEM [2] ones are well presented in [12] and [13].

We will concentrate here on the eikonal and \(U\)-matrix models in their “standard” form with the simplest input reggeon amplitude. The name “standard” refers to employing reggeons with linear trajectories and exponential form of the vertex functions. Our aim is to check if such models are able to describe data on \(pp\) and \(\bar{p}p\) elastic scattering at high energies and in all available transferred momenta including the new data. We will also determine the minimal number of the input reggeons that contribute to amplitude and provide a reasonable agreement with the data.

“Standard” schemes of unitarization. –

Impact amplitudes. Let us define the elastic amplitude in impact parameter (two-dimensional vector \(\vec{b}\)) representation, or in short impact amplitude, as

\[ H(s, b) = \frac{1}{4s} \int \frac{d^2 k}{(2\pi)^2} e^{i \vec{k} \cdot \vec{b}} A(s, t) \]

\[ = \frac{1}{8\pi s} \int_0^\infty dk J_0(b\sqrt{-t}) A(s, t) \tag{1} \]

with the inverse transformation

\[ A(s, t) = 16\pi^2 s \int \frac{d^2 b}{(2\pi)^2} e^{-i \vec{b} \cdot \vec{b}} H(s, b) \]

\[ = 8\pi s \int_0^\infty db J_0(b\sqrt{-t}) H(s, b) \tag{2} \]
where $A(s,t)$ is an elastic amplitude and $J_0(z)$ is the Bessel function, $k$ is a two-dimensional vector and $t \approx -k^2$. One can obtain from the unitarity equation for $A(s,t)$ (see, e.g., [7]) the unitarity equation for $H(s,b)$ which in a general form can be written as

$$3mH(s,b) = |H(s,b)|^2 + G_{inel}(s,b),$$

(3)

where $G_{inel}(s,b) > 0$ takes into account inelastic intermediate states from the original unitarity condition for $A(s,t)$. Therefore,

$$3mH(s,b) > 0$$

(4)

and from the unitarity one can obtain

$$|H(s,b)| \leq 1.$$  

(5)

For the observed cross-sections one can derive the following expressions:

$$\sigma_t(s) = \frac{1}{s} 3mA(s,0) = 8\pi \int_0^\infty dbb \Im m H(s,b),$$

$$\sigma_{el}(s) = \frac{1}{16\pi s^2} \int_{-\infty}^{+\infty} dt |A(s,t)|^2 = 8\pi \int_0^\infty dbb |H(s,b)|^2,$$

$$\sigma_{inel}(s) = 8\pi \int_{-\infty}^{+\infty} dbb (3mH(s,b) - |H(s,b)|^2).$$

(6)

**Output amplitudes.** We consider two known methods for unitarisation of input amplitudes $h^{ap}(s,b)$ where $a = p$ or $a = \bar{p}$. Both of them can be considered as the particular cases of sum of multireggeon exchanges (see fig. 1). Just for illustration we show here only pomeron contributions, a more realistic case will be considered later on.

Assuming that the two-hadron–n-pomeron amplitude is proportional to the product of the two-hadron–pomeron vertices depending only on $n$ (it is a pole approximation for intermediate states) as shown in fig. 2, one can obtain

$$H(s,b) = \frac{1}{2i} \sum_{n=1}^{\infty} \lambda(n) \lambda_0(n) n! [2i\delta(s,b)]^n.$$  

(7)

Moreover, assuming either $\lambda(n) = \lambda^n$ or $\lambda(n) = \lambda^n n!$, where $\lambda = \lambda_0 \lambda_p = \lambda_{ap}$, we obtain from eq. (7) two well-known schemes of pomeron unitarization: eikonal [7,14, 15] or quasi-eikonal [8] and U-matrix or quasi–U-matrix models [9,10].

**Eikonal and quasi-eikonal unitarization:**

$$H^{ap}(s,b) = \frac{e^{2i\lambda_{ap} h^{ap}(s,b)} - 1}{2i\lambda_{ap}}, \quad \lambda_{ap}(n) = (\lambda_{ap})^n.$$  

(8)

If $\lambda_{ap} = 1$ then eq. (8) describes the pure eikonal unitarization.

**U-matrix and quasi–U-matrix unitarization:**

$$H^{ap}(s,b) = \frac{h^{ap}(s,b)}{1 - 2i\lambda_{ap} h^{ap}(s,b)} \lambda_{ap}(n) = (\lambda_{ap})^n n!.$$  

(9)

For $\lambda_{ap} = 1/2$ we have the pure U-matrix unitarization.

In both the cases

$$\lambda_{ap} \geq 1/2,$$  

(10)

because the full amplitude must satisfy the unitarity inequality $|H^{ab}| \leq 1$.

**Standard Regge pole input amplitudes.** For input amplitudes we consider the standard Regge pole contributions

$$a^{pp}_{\pm}(s,t) = \sum_{R_+} a_+(s,t) \pm \sum_{R_-} a_-(s,t).$$  

(11)

At high energy ($s \gg m^2_p$) the input amplitudes can be written as follows:

$$a_\pm(s,t) = \left(\frac{1}{i}\right) g_{R_+}(t)(-is)^{\alpha_{R_+}(t)},$$  

(12)

with $s = s/s_0$, $s_0 = 1$ GeV$^2$.

There are two kinds of Regge poles contributing to amplitudes. The first ones are poles with intercepts close to 1 (or larger than 1). They are crossing even pomeron (or pomerons) and crossing odd odderon (or odderons). The second kind of contributions are the so-called secondary reggeons with intercepts less than 1. They are $f, a_2, \omega, \rho$ with $\alpha(0) \sim 0.5$ and other with lower intercepts. For high-energy $pp$ ($p\bar{p}$) scattering it is sufficient to consider in the amplitudes one effective crossing even secondary reggeon ($R_+$) and one effective crossing odd secondary reggeon ($R_-$).

In the following we keep the arguments of [14] and consider the secondary crossing even and crossing odd reggeons with fixed intercepts and slopes (strictly speaking, effective trajectories may not coincide with those for $f$- and $\omega$-reggeons while at high energy their contributions to amplitudes are closed),

$$\alpha_+(0) = 0.69, \quad \alpha_+(0) = 0.47, \quad \alpha_+ = 0.84 \text{ GeV}^{-2}, \quad \alpha_-' = 0.93 \text{ GeV}^{-2}.$$  

(13)

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Fig. 3: (Colour on-line) Description of the TOTEM data in the 3P+O model [14]. This figure is taken from [14].

Fig. 4: (Colour on-line) Description of the TOTEM data in the three-pomeron and one-odderon model [14] after refitting.

One- (two-) pomeron and one-odderon model (1P(2P)+O model). –

A) One-exponent form of the vertex functions $g_i(t)$, $i = 1, 2$ and $g_0(t)$.

For eikonal form of the amplitude (9) in one- and two-pomeron models with the exponential vertex functions ($g_i(t) = g_i \exp(B_i t)$) input amplitudes have the form

$$a_i(s, t) = \eta_i g_i \exp(B_i t)(-i\tilde{s})^{\alpha_i(0)+\alpha'_{i} t},$$

(14)

where $i = P_1, P_2, O, +, -$ and $\eta_i = -1$ for pomerons and crossing even secondary reggeon, $\eta_i = i$ for odderon and crossing odd reggeon. In accordance with eq. (1) the input impact amplitudes $h_i(s, b)$ are the following:

$$h_i(s, b) = \eta_i \frac{g_i}{16\pi sr^2}(-i\tilde{s})^{\alpha_i(0)} \exp(-b^2/4r^2_i)$$

(15)

where

$$r^2_i = B_i + \alpha'_i \ln(-i\tilde{s}), \quad i = P_1, P_2, O, +, -.$$  

These models have been considered by Petrov and Prokudin [14]. They found that the P+O model as well as 2P+O one, the both with linear trajectories and with exponential form of the vertex functions $g(t)$ (for all reggeons including pomerons and odderon), failed to describe data with a sufficient quality.

Fig. 5: (Colour on-line) The solid (dashed) line shows the eikonal (U-matrix) model.

Fig. 6: (Colour on-line) Total, elastic and inelastic cross-sections. The solid (dashed) line shows the eikonal (U-matrix) model results.

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B) Two-exponents form of the vertex functions $g_{P_i}(t), i = 1, 2$ and $g_{O}(t)$. To check if a more complicated vertex function $g(t)$ can improve the data description we have considered for pomeron and odderon terms (or for some of them) a combination of two exponents with different slopes $B$. Namely, we write for $i = P_1, P_2, O$

$$g_i(t) = g_i[c_i \exp(B_{1i}t) + (1 - c_i) \exp(B_{2i}t)].$$

(17)

Changes for the impact amplitudes $h_{P_i}(s, b)$ and $h_{O}(s, b)$ quite evidently follow from eqs. (15), (17).

We do not give here the results of the fit for this modified model, we rather note that such an extension of the model in spite of some of its improvements does not lead to a satisfactory description of the TOTEM data.

Three-pomeron and one-odderon model (3P+O model). – This model has been suggested and considered by Petrov and Prokudin in [14]. The input amplitudes $h^{ab}(s, b)$ in the model are written as contribution of three pomerons, one odderon and two secondary (even and odd) reggeons

$$h_{pp}^{ab}(s, b) = h_{P_1}(s, b) + h_{P_2}(s, b) + h_{P_3}(s, b) + h_{O}(s, b) + h_{-}(s, b),$$

(18)

where input impact amplitudes are given by eq. (15). Then the eikonalized amplitudes $A_{pp}^{ab}(s, t)$ are defined by eq. (8) with $\lambda^{op} = 1$ and eq. (2). They have obtained very good description of the $pp$ and $\bar{p}p$ data for the total cross-sections $\sigma_{tot}(s)$, the ratios of the real to imaginary part of the forward elastic scattering amplitudes $\rho(s)$ at $\sqrt{s} \geq 8$ GeV and for the differential cross-sections $d\sigma(s, t)/dt$ at $\sqrt{s} \geq 23$ GeV and $0.01 \leq |t| \leq 14$ GeV$^2$ (this data set contains totally 961 points).

It is necessary to note here that our data set differs from those used in [14,15]. Our data set has been proposed by Cudell, Lengyel and Martynov (CLM) [16], who built a coherent set of all existing data for $4 \leq \sqrt{s} \leq 1800$ GeV and $0 \leq |t| \leq 15$ GeV$^2$ [16] which can be found in the HEP DATA system [17,18]. CLM have performed a detailed study of the systematic errors and gathered in a common format more than 260 subsets of data from more than 80 experimental papers. We suggest to use the CLM data set as a standard dataset. The latest (with some corrections) updated version of it including TOTEM [1–3] and D0 [11] data is available online [19]. The set used for the given analysis contains 1723 points in the region described above. There are only 31 points from the 3 groups, 8 points at $\sqrt{s} = 26.946$ GeV, 11 points at 30.7 GeV and 12 points at 53.018 GeV for $d\sigma_{pp}/dt$ were excluded from all the data presented in [18] because these groups are strongly deviated from the rest of the data points and can slightly distort the fit.

As was shown in paper [1] where the first data at the LHC energy of 7 TeV was presented, the model [14] does not describe the TOTEM data. This statement is confirmed in the recent paper [15] by the authors of the model (we reproduce fig. 4 of their paper in fig. 3). We have performed the refit of the model with our data set including the TOTEM data and found that the resulting
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Fig. 9: (Colour on-line) Differential elastic \( \bar{p}p \) cross-sections. The solid (dashed) line shows the eikonal (\( U \)-matrix) model results.

Fig. 10: (Colour on-line) Description of the TOTEM data and prediction for higher \(|t|\).

Theoretical curve for \( d\sigma/dt \) at 7 TeV is still out of the data at \(|t| > 0.4 \text{ GeV}^2\) as shown in fig. 4. Thus we can conclude that the 3P+O model should be modified either adding new terms or using nonlinear pomeron (odderon) trajectories and more complicated nonexponential vertex functions. In what follows we consider the first possibility, namely we would like to check if adding the second odderon term can help to improve the agreement of the model with the new data.

The big difference for \( \rho_{pp} \) and \( \rho_{p\bar{p}} \) (shown in fig. 7) disappeared at very high energy. In the eikonal model the

Three-pomeron and two-odderon model 3P+2O model. – In this section we consider the minimal extension of the 3P + O model (18) adding the second odderon term:

\[
\begin{align*}
\rho_{PP}^{\alpha}(s, b) &= h_p(s, b) + h_l(s, b) + \left( h_O(s, b) + h_{-}(s, b) \right), \\
\rho_{Pp}^{\alpha}(s, b) &= \sum_{i=1}^{3} h_{P_i}(s, b), \\
\rho_{O}(s, b) &= \sum_{i=1}^{2} h_{O_i}(s, b),
\end{align*}
\]

(19)

where each input term is still in a “standard” form (12), (14) with linear trajectory \( \alpha = \alpha_i(0) + \alpha'_i t \) and exponential vertices \( g_i(t) = g_i \exp(B_i t) \). Later, we would like to check other possibilities to improve the eikonal and \( U \)-matrix models (nonlinear trajectories, nonexponential vertices and so on).

We considered both the methods of unitarization, the eikonal one, eq. (8), with \( \lambda^{np} = 1 \) and the \( U \)-matrix one, eq. (9), with \( \lambda^{np} = 1/2 \). The fit has been performed with the data in the region,

\[
\begin{align*}
\text{for } \sigma_{tot}(s) \text{ and } \rho(s) \text{ at } \sqrt{s} \geq 5 \text{ GeV}, \\
\text{for } d\sigma(s, t)/dt \text{ at } \sqrt{s} \geq 19.0 \text{ GeV}, \\
&\text{0.01 } |t| \leq 14.2 \text{ GeV}^2.
\end{align*}
\]

(20)

The resulting description of the data is shown in figs. 5–11. Parameters of the models are given in table 1.

Let us briefly comment the results.

The addition of the second odderon term leads to a certain improvement of the data description.

The agreement with the TOTEM data is essentially better but some defects appeared at lower energies, especially at large \(|t|\) and for \( pp \) differential cross-sections (see figs. 8 and 9).

The big difference for \( \rho_{pp} \) and \( \rho_{p\bar{p}} \) (shown in fig. 7) disappeared at very high energy. In the eikonal model the
Table 1: Parameters of the eikonal and $U$-matrix models obtained by fitting to the data. Parameters $\alpha_i$ and $B_i$ are given in GeV$^{-2}$, other parameters are dimensionless. Errors are taken from the MINUIT output. $\chi^2$/dof $= \chi^2/(N_p - N_{par})$, where $N_p$ is the number of experimental points in the fit, $N_{par}$ is the number of free parameters in the model.

| Param. | Eikonal model | $U$-matrix model |
|--------|---------------|------------------|
| $\alpha_{p_1}$ | 1.2209 | 0.0012 |
| $\alpha_{p_2}$ | 0.1171 | 0.0007 |
| $g_{p_1}$ | 1.3249 | 0.0196 |
| $B_{p_1}$ | 0.3977 | 0.0055 |
| $\alpha_{p_2}$ | 1.1675 | 0.0006 |
| $g_{p_2}$ | 0.3036 | 0.0014 |
| $B_{p_2}$ | 8.753 | 0.064 |
| $\alpha_{p_3}$ | 0.4125 | 0.0089 |
| $g_{p_3}$ | 0.1073 | 0.0005 |
| $B_{p_3}$ | 0.5912 | 0.0024 |
| $\alpha_{g_1}$ | 0.1318 | 0.0198 |
| $g_{B_1}$ | 1.2046 | 0.0169 |
| $\alpha_{B_1}$ | 1.220 | 0.0004 |
| $\alpha_{B_2}$ | 0.0723 | 0.7E-05 |
| $g_{B_2}$ | 27.133 | 0.007 |
| $B_{B_2}$ | 1.2996 | 0.0002 |
| $\alpha_{p_2}$ | 0.07222 | 0.5E-05 |
| $g_{p_2}$ | 27.171 | 0.0067 |
| $B_{p_2}$ | 0.01374 | 0.00003 |
| $\alpha_{p_3}$ | 0.69 | fixed |
| $g_{p_3}$ | 2.2155 | 1.18 |
| $B_{p_3}$ | 5.460 | 0.175 |
| $\alpha_{g_1}$ | 0.47 | fixed |
| $g_{B_1}$ | 0.93 | fixed |
| $B_{B_1}$ | 149.17 | 4.08 |
| $\chi^2$/dof | 1.673 | 1.973 |

Table 2: Cross-sections and ratio of the real to imaginary part of the forward scattering $pp$ amplitude in the eikonal model $3P + 2O$ defined by eqs. (8), (19).

\[
\sqrt{s} \ (\text{TeV}) \quad \sigma_{el} \ (\text{mb}) \quad \sigma_{sl} \ (\text{mb}) \quad \sigma_{sl/\text{inst}} \ (\text{mb}) \quad \rho \\
\hline
7 \quad 98.97 \quad 25.53 \quad 73.45 \quad 0.153 \quad \\
8 \quad 101.5 \quad 26.48 \quad 75.02 \quad 0.152 \quad \\
13 \quad 111.14 \quad 30.18 \quad 80.96 \quad 0.150 \quad \\
14 \quad 112.68 \quad 30.78 \quad 81.9 \quad 0.150 \quad \\
\hline
\]

difference is negligible at $\sqrt{s} \gtrsim 10^4$ GeV while in the $U$-matrix model it is at $\sqrt{s} \gtrsim 10^5$ GeV. Thus an asymptotic regime comes quite late in energy.

In table 2 the calculated values of the total, elastic and inelastic cross-section are given for the LHC energies. One can see a very good agreement with the results of TOTEM [3], ATLAS [20], CMS [21] and ALICE [22].

More problems are for the $U$-matrix model rather than for the eikonal one, however both models do not strongly contradict the TOTEM data.

**Conclusion.** – Answering to the question put in the title we would like to say that in our opinion the obtained results demonstrate an ability of the unitarized eikonal and $U$-matrix $3P + 2O$ models to describe the data up to LHC energies, however, it seems that for further improvement nonlinear and/or nonexponential (for vertices) effects are very important. Exploring another possibility, namely, a construction of the model that does not explicitly violate unitarity (at least the unitarity bounds on the cross-sections) just from the beginning of its construction is in progress, results will be presented later.

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