Can collisional energy loss explain nuclear suppression factor for light hadrons?

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Abstract

We argue that in the measured \(p_T\) domain of RHIC, collisional rather than the radiative energy loss is the dominant mechanism for jet quenching. Accordingly we calculate nuclear suppression factor for light hadrons by taking only the elastic energy loss in sharp contrast with the previous calculations where only the radiative loss are considered.

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Jet quenching is one of the most promising tools to extract the initial parton density produced in high energy heavy ion collisions. This is related to the final state energy loss of the leading partons [1–3] causing depopulation of hadrons at high transverse momentum (see [4] for experimental results). The suppressions of high \(p_T\) hadrons and unbalanced back-to-back azimuthal correlations of the dijet events measured at Relativistic Heavy Ion Collider (RHIC) provide experimental evidence in support of the quenching. Based on the calculations performed by several authors [5–7] the detailed theory of ‘jet tomography’ was developed by Gyulassy et al. [3] considering only the energy loss due to induced bremsstrahlung radiation. The observed nuclear suppression of light hadrons (\(\pi, \eta\)) in \(Au + Au\) collisions at \(\sqrt{s} = 62 – 200\) AGeV at RHIC could be accounted for in these models. In all these analyses the collisional loss was ignored [8,9]. The non-photonic single electron spectrum from heavy meson decays measured by PHENIX Collaboration [10] put this assumption in
question. No realistic parameter set can explain this data using the radiative energy loss based jet tomography model which either requires violation of bulk entropy bounds or non-perturbatively large $\alpha_s$ of the theory [12], or equivalently one requires excessive transport co-efficient $\hat{q}_{\text{eff}} = 14 \text{ GeV}^2/\text{fm}$ [13].

The importance of collisional loss in the context of RHIC was first discussed by the present authors [14,15]. It is shown in ref. [14] that there exists an energy range where collisional loss is as important as or even greater than its radiative counterpart, hence cannot be neglected in any realistic model of jet quenching. Recently this is also noted in ref. [11,12,16,17]. It is similar to the passage of charged particles through material medium where the ionization loss is known to be the dominant mechanism at lower energies while at higher energies bremsstrahlung takes over. There exists a critical energy $E_c$ at which they contribute equally i.e. $(dE/dx)_{\text{rad}} = (dE/dx)_{\text{coll}}$ at $E = E_c$. For example, for an electron (proton) traversing copper target $E_c \sim 25$ MeV (1 GeV) [18]. Note that for heavier particle $E_c$ is higher. This indicates that for the heavy quark collisional loss may be more important than the radiative loss at intermediate energies. In ref. [14] we have calculated $E_c$ for light partons under RHIC conditions.

In this light, we, in the present work would like to address if the omission of collisional loss at RHIC is justified or not. We argue that, whether the collisional or radiative loss is the main mechanism is a $p_T$ dependent question. It also depends on the energies of the colliding system and expected to be different for RHIC and Large Hadron Collider (LHC). In contrast to the previous works, we, therefore, calculate nuclear suppression factor ($R_{AA}$) for pions considering only the collision energy loss. At the end we shall show that there exists a $p_T$ window where this is reasonable assumption contrary to the commonly held view that collisional loss (for light partons) can be ignored altogether.

The neutral pion production [19] (for charged hadrons see [20]) at RHIC in the $p_T$ window $\sim 1 - 13$ GeV, is found to be suppressed compared to the binary scaled $p-p$ estimation [4]. This is attributed to the final state energy loss of the partons while passing through the plasma before fragmenting into hadrons [21–23]. The energy loss in the standard
standard perturbative calculations can be incorporated by modifying the fragmentation function. This is accomplished by replacing fractional momentum $z$ carried by the hadrons with $z^* = z/(1 - \Delta z)$ in the argument of the fragmentation function, $D(z, Q^2)$, where $\Delta z = \Delta E/E$. This implementation assumes that all the partons suffer equal amount of energy loss which is questionable as argued in ref. [24,25]. We, therefore, take a different approach where the initial spectra is evolved dynamically by using Fokker Planck (FP) equation. FP equation can be derived from Boltzmann equation if the collisions are dominated by the small angle scattering involving soft momentum exchange [15,26–31]. For an expanding plasma, FP equation takes the following form:

$$\left(\frac{\partial}{\partial t} - \frac{p_\parallel}{t} \frac{\partial}{\partial p_\parallel}\right) f(p, t) = \frac{\partial}{\partial p_i} [p_i \eta f(p, t)]$$

$$+ \frac{1}{2} \frac{\partial^2}{\partial p_\parallel^2} [B_\parallel f(p, t)]$$

$$+ \frac{1}{2} \frac{\partial^2}{\partial p_\perp^2} [B_\perp f(p, t)], \quad (1)$$

where the second term on the left hand side arises due to expansion. Bjorken hydrodynamical model [32] has been used here for space time evolution. In Eq. (1) $\eta$ denotes drag coefficient which is related to the energy loss or the ‘stopping power’ of the plasma, $\eta = (1/E)dE/dx$. $B_\parallel, B_\perp$ denote diffusion constants along parallel and perpendicular directions of the propagating parton representing rate of longitudinal and transverse broadening (variance) i.e. $B_\parallel = d\langle(\Delta p_\parallel)^2\rangle/dt$, $B_\perp = d\langle(\Delta p_\perp)^2\rangle/dt$. These transport coefficients can be calculated from the following expressions:

$$\frac{dE}{dx} = \frac{\nu}{(2\pi)^5} \int \frac{d^3k d^3q d\omega}{2k_2k'_2p_2p'_2} \delta(\omega - v_p \cdot q) \delta(\omega - v_k \cdot q)$$

$$\langle M \rangle^2_{t \to 0} f(|k|) [1 + f(|k + q|)] \omega. \quad (2)$$

$$B_{\perp, \parallel} = \frac{\nu}{(2\pi)^5} \int \frac{d^3k d^3q d\omega}{2k_2k'_2p_2p'_2} \delta(\omega - v_p \cdot q) \delta(\omega - v_k \cdot q)$$

$$\langle M \rangle^2_{t \to 0} f(|k|) [1 + f(|k + q|)] q^2_{\perp, \parallel}. \quad (3)$$

In the above equations the small angle limit has been taken to write the arguments of the delta functions [14,15].
The matrix elements include diagrams involving exchange of massless gluons which render $\eta$ and $B_{\parallel,\perp}$ infrared divergent. Such divergences can naturally be cured by using the hard thermal loop (HTL) [33] corrected propagator for the gluons as discussed below. We work in the coulomb gauge where the gluon propagator for the transverse and longitudinal modes are denoted by $D_{00} = \Delta_{\parallel}$ and $D_{ij} = (\delta_{ij} - q^i q^j/q^2)\Delta_{\perp}$ with [34]:

$$\Delta_{\parallel}(q_0, q) = \frac{q^2}{2\omega_p^2} \left[ \frac{q_0 ln q_0 + q}{q_0 - q} - 2 \right]$$

$$\Delta_{\perp}(q_0, q) = \frac{q_0^2 - q^2}{2\omega_p^2} \left[ \frac{q_0(q_0^2 - q^2)}{2q^2} ln q_0 + q - \frac{q_0^2}{q^2} \right]$$

The HTL modified matrix element in the limit of small angle scattering takes the following form [14,15] for all the partonic processes having dominant small angle contributions like $qg \rightarrow qg, qq \rightarrow qq$ etc.:

$$|\mathcal{M}|^2 = g^4 C_R 16 (E_p E_k)^2 |\Delta_{\parallel}(q_0, q)$$

$$+ (v_p \times \hat{q}).(v_k \times \hat{q})|\Delta_{\perp}(q_0, q)|^2$$

where $C_R$ is the appropriate color factor. With the screened interaction, the drag and diffusion constants can be calculated along the line of ref. [15].

Having known the drag and diffusion, we proceed to solve the FP equation. For this purpose we require the initial parton distributions which is parametrized as [35]:

$$f(p_T, p_z, t = t_i) \equiv \frac{dN}{dp_T dy}|_{y=0} = \frac{N_0}{(1 + \frac{p_z}{p_0})^{\alpha}}$$

where $p_0$, $\alpha$ and $N_0$ are parameters. Solving the FP equation with the boundary conditions, $f(p, t) \rightarrow 0$ for $|p| \rightarrow \infty$, we are ready to evaluate the nuclear suppression factor, $R_{AA}$ defined as [36],

$$R_{AA}(p_T) = \frac{\text{“Hot QCD medium”}}{\text{“QCD vacuum”}} = \frac{\sum_a \int f_a(p', \tau_\gamma)|_{p'_{\gamma} = p_T / z} D_{a/\pi^0}(z, Q^2) dz}{\sum_a \int f_a(p', \tau_\gamma)|_{p'_{\gamma} = p_T / z} D_{a/\pi^0}(z, Q^2) dz}$$
FIG. 1. Nuclear suppression factor for pion. Experimental data are taken from PHENIX collaboration [19] for Au + Au collisions at $\sqrt{s} = 200$ GeV. Solid line indicates result from the present calculation with collisional energy loss of the partons propagating through the plasma before fragmenting into pions.

where $f(p', \tau_i)$ and $f(p', \tau_c)$ denote the parton distributions at proper time $\tau_i$ and $\tau_c$ respectively. Here $\tau_i$ is the initial time and $\tau_c$ is the time when the system cools down to the transition temperature $T_c (=190$ MeV) [37]. The result for neutral pion is shown in Fig. 1 which describes the PHENIX data [19] for Au + Au at $\sqrt{s} = 200$ GeV reasonably well.

It should be noted here that the $R_{AA}(p_T)$ with collisional loss has a tendency to increase for higher $p_T$, indicating less importance of collisional loss at this domain, where the radiative loss may become important. Therefore, a detailed calculation with both collisional and radiative loss may be useful to delineate the importance of individual mechanism.
FIG. 2. Excitation function of the nuclear modification factor for neutral pions in central A+A reactions at a fixed $p_T = 4$ GeV where only the elastic energy loss is considered. Experimental data are taken from [36].

To stress our point further we also analyse the excitation function of the nuclear suppression factor. Results are shown in Fig. 2. It is clear that $R_{AA}(p_T)$ at $p_T = 4$ GeV for various beam energies are found to be well described. The values of parton (quarks, anti-quarks and gluons) densities ($n_{g+q+\bar{q}}$) of the QCD medium which describe the data for various beam energies are shown in table I.

To pin down the relative importance of $2 \leftrightarrow 2 \rightarrow 3$ processes, we determine the average energy of the parton which contribute to the measured $p_T$ window of the hadrons. To this end, the average fractional momentum ($\langle z \rangle$) of the fragmenting partons carried by the pion is calculated using relevant parton distribution and fragmentation functions. For the former, we use CTEQ [39] including shadowing via EKS98 parametrization [40], while for the fragmentation function KKP parametrization is used [41]. The average energy of the parton, $\langle E_{\text{parton}} \rangle$ is obtained by using the relation $\langle E_{\text{parton}} \rangle = p_T^2/ \langle z \rangle$ for $y_\pi = 0$. The results are shown in Fig. 3. Our results are consistent with that of ref. [36] which quotes $\langle z \rangle = p_{\text{hadron}}/p_{\text{parton}} \simeq 0.5 - 0.7$ for $p_{\text{hadron}} \geq 4$ GeV at RHIC energies.
TABLE I. The extracted initial parton densities and nuclear modification factors at various beam energies.

| $\sqrt{s_{NN}}$ | $n_{g+q+\bar{q}}$ | $R_{AA}$ |
|-----------------|----------------|--------|
| 17.3            | 4              | 0.97   |
| 62.4            | 15             | 0.59   |
| 130             | 27             | 0.39   |
| 200             | 49             | 0.27   |
It might be recalled that at RHIC energies the nuclear modification factor $R_{AA}(p_T)$ has been measured in the pion transverse momenta range $p_T \sim 1 - 13$ GeV. Assuming that these pions are originated from the fragmenting partons, we ask the question, what is the average parton energy required to produce these pions? From Fig. 3, it is clear that the maximum average parton energy required is about 26 GeV here.

Now the next question is, what is the dominant energy loss mechanism for partons with energy $\sim 26$ GeV or less? We might compare this value with the determined $E_c^1$ given in refs. [14] (can also be read out from [16]) and note that at these energies collisional loss cannot be neglected. For lower beam energy, 62.4 (130) AGeV the value of maximum average parton energy required to produce a 13 GeV pion is 16 (22) GeV, where the collisional loss will definitely be more important. It is worthwhile to mention here that this estimation of $E_c$ has some uncertainty as it depends on the length of the plasma, initial temperature, mean free path, dynamical screening mass etc. Those would affect both the mechanisms (i.e. radiative and collisional) of energy loss. Our chosen parameter set is consistent with that of ref. [7,42] used to study the radiative energy loss.

In conclusion, our investigations clearly suggest that in the measured $p_T$ range of light

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<sup>1</sup>Note that $E_c$ is defined to be the energy below which elastic loss dominates [14].
hadrons at RHIC collisional, rather than the radiative, is the dominant mechanism of jet quenching. This is in sharp contrast to all the previous analyses. The determination of the critical energy ($E_c$), however, might change depending upon the detailed model of ‘jet quenching’. Inclusion of three body elastic channels for heavy quark energy loss, which are considered in ref. [43], if applied for light flavours, might even increase $E_c$, making our point stronger. $E_c$ will also increase if there exist partonic bound state in the plasma due to ionization loss [44]. It should be mentioned that for the collisional energy loss we have not included finite size effect which however is shown to be small [16]. In light of these new findings the theory of jet tomography is expected to change considerably.

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