QCD Studies at the High-energy Linear $e^+e^-$ Collider

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I summarise the QCD programme at the high-energy $e^+e^-$ linear collider, as reported in the TESLA TDR and Linear Collider Physics Resource Book.

I. INTRODUCTION

Strong-interaction measurements at the Linear Collider (LC) will form an important component of the physics programme. The collider offers the possibility of testing QCD at high energy scales in the experimentally clean, theoretically tractable $e^+e^-$ environment. In addition, virtual $\gamma\gamma$ interactions will be delivered free by Nature, and a dedicated $\gamma\gamma$ collider is an additional option, allowing detailed measurements of the relatively poorly understood photon structure. The benchmark physics main topics are:

- Precise determination of the strong coupling $\alpha_s$.
- Measurement of the $Q^2$ evolution of $\alpha_s$ and constraints on the GUT scale.
- Measurement of the total $\gamma\gamma$ cross section and the photon structure function; these issues are discussed elsewhere [1].

II. PRECISE DETERMINATION OF $\alpha_s$

The current precision of individual $\alpha_s$ measurements is limited at best to several per cent [2]. Since the uncertainty on $\alpha_s$ translates directly into an uncertainty on perturbative QCD (pQCD) predictions, especially for high-order multijet processes, it would be desirable to achieve much better precision. In addition, since the weak and electromagnetic couplings are known with much greater relative precision, the error on $\alpha_s$ represents the dominant uncertainty on our ‘prediction’ of the scale for grand unification of the strong, weak and electromagnetic forces [3].

Here we will refer to the conventional yardstick of $\alpha_s$ quoted at the $Z^0$ mass scale, $\alpha_s(M_Z)$, unless explicitly stated otherwise. Several techniques for $\alpha_s(M_Z)$ determination will be available at the LC:

A. Event Shape Observables

The determination of $\alpha_s(M_Z)$ from event ‘shape’ observables that are sensitive to the 3-jet nature of the particle flow has been pursued for 2 decades and is generally well understood [4]. In this method one usually forms a differential distribution, makes corrections for detector and hadronisation effects, and fits a pQCD prediction to the data, allowing $\alpha_s(M_Z)$ to vary. Examples of such observables are the thrust, jet masses and jet rates.

The latest generation of such $\alpha_s(M_Z)$ measurements, from SLC and LEP, has shown that statistical errors below the 1% level can be obtained with samples of a few tens of thousands of hadronic events. With the current LC design luminosities of a few $\times 10^{34}$/cm$^2$/s at 500, 800 or 1000 GeV, hundreds of thousands of $e^+e^- \rightarrow q\overline{q}$ events would be produced each year, and a statistical error on $\alpha_s(M_Z)$ below the 0.5% level could be achieved.

Detector systematic errors, which relate mainly to uncertainties on the corrections made for acceptance and resolution effects and are observable-dependent, are under control in today’s detectors at the $\Delta\alpha_s(M_Z) = 1-4\%$ level [5]. If the LC detector is designed to be very hermetic, with good tracking resolution and efficiency, as well

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as good calorimetric jet energy resolution, all of which are required for the search for new physics processes, it seems reasonable to expect that the detector-related uncertainties can be beaten down to the $\Delta \alpha_s(M_Z) \simeq 1\%$ level or better.

$e^+e^- \rightarrow Z^0Z^0$, $W^+W^-$, or $t\bar{t}$ events will present significant backgrounds to $q\bar{q}$ events for QCD studies, and the selection of a highly pure $q\bar{q}$ event sample will not be quite as straightforward as at the $Z^0$ resonance. The application of kinematic cuts would cause a significant bias to the event-shape distributions, necessitating compensating corrections at the level of 25% [4]. More recent studies have shown [5] that the majority of $W^+W^-$ events can be excluded without bias by using only events produced with right-handed electron beams for the $\alpha_s(M_Z)$ analysis. Furthermore, the application of highly-efficient $b$-jet tagging can be used to reduce the $t\bar{t}$ contamination to the 1% level. After statistical subtraction of the remaining backgrounds (the $Z^0Z^0$ and $W^+W^-$ event properties have been measured accurately at SLC and LEP I/II), the residual bias on the event-shapes distributions is expected to be under control at the better than 1% level on $\alpha_s(M_Z)$.

Additional corrections must be made for the effects of the smearing of the particle momentum flow caused by hadronisation. These are traditionally evaluated using Monte Carlo models. The models have been well tuned at SLC and LEP and are widely used for evaluating systematic effects. The size of the correction factor, and hence the uncertainty, is observable dependent, but the ‘best’ observables measured at the $Z^0$ have uncertainties as low as $\Delta \alpha_s(M_Z) \simeq 1\%$. Furthermore, one expects the size of these hadronisation effects to diminish with c.m. energy at least as fast as $1/Q$. Hence 10%-level corrections at the $Z^0$ should dwindle to 1%-level corrections at $Q \geq 500$ GeV, and the associated uncertainties will be substantially below the 1% level on $\alpha_s(M_Z)$. This has been confirmed by explicit simulations using PYTHIA [4].

Currently pQCD calculations of event shapes are available complete only up to $O(\alpha_s^3)$, although resummed calculations are available for some observables [6]. One must therefore estimate the possible bias inherent in measuring $\alpha_s(M_Z)$ using the truncated QCD series. Though not universally accepted, it is customary to estimate this from the dependence of the fitted value on the QCD renormalisation scale, yielding a large and dominant uncertainty of about $\Delta \alpha_s(M_Z) \simeq \pm 6\%$ [6]. Since the missing terms are $O(\alpha_s^4)$, and since $\alpha_s(500$ GeV) is expected to be about 25% smaller than $\alpha_s(M_Z)$, one expects the uncalculated contributions to be almost a factor of two smaller at the higher energy. However, translating to the yardstick $\alpha_s(M_Z)$ yields an uncertainty of $\pm 5\%$, only slightly smaller than currently. Therefore, although a 1%-level measurement is possible experimentally, it will not be realised unless $O(\alpha_s^4)$ contributions are calculated. There is reasonable expectation that this will be achieved within the next 5 years [6].

B. The $t\bar{t}(g)$ System

The dependence of the $e^+e^- \rightarrow t\bar{t}$ production cross section, $\sigma_{t\bar{t}}$, on the top-quark mass, $m_t$, and on $\alpha_s(M_Z)$ are discussed elsewhere [4]. In order to optimise the precision on the $m_t$ measurement near threshold it is desirable to input a precise $\alpha_s(M_Z)$ measurement from elsewhere. Furthermore, the current theoretical uncertainty on $\sigma_{t\bar{t}}$ translates into $\Delta \alpha_s(M_Z) = \pm 10\%$. Hence, although extraction of $\alpha_s(M_Z)$ from $\sigma_{t\bar{t}}$ near threshold may provide a useful ‘sanity check’ of QCD in the $t\bar{t}$ system, it does not appear currently to offer the prospect of a competitive measurement. A preliminary study has also been made of the determination of $\alpha_s(M_Z)$ from $R_t \equiv \sigma_{t\bar{t}}/\sigma_{\mu+\mu-}$ above threshold. For $Q \geq 500$ GeV the uncertainty on $R_t$ due to $m_t$ is around 0.5%. The limiting precision on $R_t$ will be given by the uncertainty on the luminosity measurement. If this is as good as 0.5% [11] then $\alpha_s(M_Z)$ could be determined with an experimental precision approaching 1%, which would be extremely valuable as a complementary precision measurement from the $t\bar{t}$ system.

C. A High-luminosity Run at the $Z^0$ Resonance

A Giga $Z^0$ sample offers two additional options for $\alpha_s(M_Z)$ determination via measurements of the inclusive ratios $\Gamma_{Z \rightarrow \mu\mu}^{had}/\Gamma_{Z \rightarrow \mu\mu}^{lept}$ and $\Gamma_{t\bar{t}}^{had}/\Gamma_{t\bar{t}}^{lept}$. Both are indirectly proportional to $\alpha_s$, and hence require a very large event sample for a precise measurement. For example, the current LEP data sample of 16M $Z^0$ yields an error of 2.5% on $\alpha_s(M_Z)$ from $\Gamma_{Z \rightarrow \mu\mu}^{had}/\Gamma_{Z \rightarrow \mu\mu}^{lept}$. The statistical error could, naïvely, be pushed to below the $\Delta \alpha_s(M_Z) = 0.4\%$ level, but systematic errors arising from the hadronic and leptonic event selection will probably limit the precision to 0.8% [10]. This would be a very precise, reliable measurement. In the case of $\Gamma_{t\bar{t}}^{had}/\Gamma_{t\bar{t}}^{lept}$ the experimental precision from LEP and CLEO is already at the 1% level on $\alpha_s(M_Z)$. However, there has been considerable debate about the size of the theoretical uncertainties, with estimates as large as 5% [12]. If this situation is clarified, and the theoretical uncertainty is small, $\Gamma_{t\bar{t}}^{had}/\Gamma_{t\bar{t}}^{lept}$ may offer a further 1%-level $\alpha_s(M_Z)$ measurement.
In the preceding sections we discussed the expected attainable precision on the yardstick $\alpha_s(M_Z)$. Translation of the measurements of $\alpha_s(Q)$ ($Q \neq M_Z$) to $\alpha_s(M_Z)$ requires the assumption that the ‘running’ of the coupling is determined by the QCD $\beta$ function. However, since the logarithmic decrease of $\alpha_s$ with $Q$ is an essential component of QCD, reflecting the underlying non-Abelian dynamics, it is vital also to test this $Q$-dependence explicitly. Such a test would be particularly interesting if new coloured particles were discovered, since deviations from QCD running would be expected at energies above the threshold for pair-production of the new particles. Furthermore, extrapolation of $\alpha_s$ to very high energies of the order of $10^{15}$ GeV can be combined with corresponding extrapolations of the dimensionless weak and electromagnetic couplings in order to constrain the coupling-unification, or GUT, scale. Hence it would be desirable to measure $\alpha_s$ in the same detector, with the same technique, and by applying the same treatment to the data at a series of different energies $Q$, so as to maximise the lever-arm for constraining the running.

Simulated measurements of $\alpha_s(Q)$ at $Q = 91$, 500 and 800 GeV are shown in Fig. 1 together with existing measurements which span the range $20 \leq Q \leq 200$ GeV. The highest-energy measurements are currently provided by LEP II. The point at $Q = 91$ GeV is based on the $\Gamma_{\text{had}}/\Gamma_{\text{lept}}$ technique, and those at 500 and 800 GeV are based on the event shapes technique. The last two include the current theoretical uncertainty, which yields a total error on each point equivalent to $\Delta\alpha_s(M_Z) = 4\%$. It is clear that the LC data would add significantly to the lever-arm in $Q$, and would allow a substantially improved extrapolation to the GUT scale.

IV. FURTHER IMPORTANT TOPICS

Limited space allows only a brief mention of several other important topics:

- Hard gluon radiation in $t\bar{t}$ events would allow several tests of the strong dynamics of the top quark: test of the flavour-independence of strong interactions; limits on anomalous chromo-electric and/or chromo-magnetic dipole moments; determination of the running $m_t$. 

• **Soft gluon radiation in \(\bar{t}t\) events** is expected to be strongly regulated by the large mass and width of the top quark. Precise measurements of gluon radiation patterns in \(\bar{t}g\) events would provide additional constraints on the top decay width \([16]\).

• **Polarised electron (and positron) beams** can be exploited to test symmetries using multi-jet final states. For polarized \(e^+e^-\) annihilation to three hadronic jets one can define \(S_e \cdot (k_1 \times k_2)\), which correlates the electron-beam polarization vector \(S_e\) with the normal to the three-jet plane defined by \(k_1\) and \(k_2\), the momenta of the two quark jets. If the jets are ordered by momentum (flavour) the triple-product is CP even (odd) and T odd. Standard Model T-odd contributions of this form are expected \([17]\) to be immeasurably small, and limits have been set for the \(bg\) system \([18]\). At the LC these observables will provide an additional search-ground for anomalous effects in the \(t\bar{t}\) system.

• **The difference between the particle multiplicity in heavy- (b, c) and light-quark events** is predicted \([19]\) to be independent of c.m. energy. Precise measurements have been made at the \(Z^0\), but measurements at other energies are statistically limited in precision, rendering a limited test of this important prediction. High-precision measurements at the LC would add the lever-arm for a powerful test.

• **Colour reconnection and Bose-Einstein correlations** are important to study precisely since they may affect the precision with which the masses of heavy particles, such as the \(W^\pm\) and top-quark, can be reconstructed kinematically via their multijet decays \([20]\).

• **Hadronisation studies and renormalon physics** can be explored via measurements of event-shape observables over a range of \(Q\) values.

V. SUMMARY

There is a rich programme of QCD studies at the Linear Collider. Precision measurements of the strong coupling and of the strong dynamics of the \(\bar{t}t\) system will complement inclusive measurements that will be made at the LHC.

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