Quasiparticles in quantum Hall effect: Smet’s fractional charges.

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It has been pointed out by Smet that there are fractional-charge values which do not fit their formula of composite fermions. We find that our formula predicts these fractional charges very well and in fact there exists a relationship between spin and the effective charge of a quasiparticle.

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1. Introduction

For some years it was thought that “flux quanta” are attached to the electron. Considerable effort was put into this “flux attached” model to explain the quantum Hall effect. An “incompressible” model was suggested but it turns out that \( a_o=1 \) is not a solution of the algebraic equations involved, \( Ba_o^2=\phi_o \). Here \( a_o=1 \) is required for incompressibility. After a lapse of twenty years, now Pan et al[1] have pointed out that quantum Hall effect occurs at some of the fractions which can not be obtained by their model. It was thought that plateaus occur at the filling factors, \( \nu=p/(2m \pm 1) \), where \( m \) and \( p \) are integers. Now, the recently reported fractions do not fit with this formula. Smet[2] has also agreed that the formula, \( \nu=p/(2m \pm 1) \) of the composite fermion model (CF) has no rigorous foundation. Similarly, composite bosons (CB) will not be feasible. The flux quanta are not so abundant, such as 10 per electron, that they can attach to electrons. Therefore, the idea of “flux quanta” attachment to the electrons should be discarded. Considerable effort has been made to examine many papers in which a claim is made that flux quanta are attached to the electrons but in all cases internal inconsistencies have been found. References to some of the papers which point out the inconsistencies are as follows.

- cond-mat/0209666 shows that CF violates classical electrodynamics;
- 0210320 shows that there are far too many parameters in CF model;
- 0211223 shows that CF effective field is incorrect;
- 0301380 shows that CF requires E, H decoupling in Maxwell equations;
- 0302009 shows that CF and CB transformation has not been carried out;
- 0302315 shows that mass of the CF is not consistent with the available space;
- 0302461 shows what quantity is measured in fractional charge experiments;
- 0303146 shows that CF lacks in Lorentz invariance;
- 0303014 shows that CF model is internally inconsistent.

Usually a Jordan-Wigner transformation transforms the spin operators into fermions. Similarly, the Holstein-Primakof transformation transforms the spin operators into magnons. Under special conditions, such as zero temperature in the case of Jordan-Wigner and low temperatures in the case of Holstein-Primakof, the solution of the transformed hamiltonian is quite close to that of the starting hamiltonian so that some
useful results can be obtained. However, in the case of “flux attachment” no transformation has been found. Therefore, there is no way except to discard the “CF” model\cite{3}. Dyakonov\cite{4} has clearly noted that CF model is not based on good theoretical principles and Farid\cite{5} has pointed out that the field formula is not correct. We are then left with no solution of the problem of quantum Hall effect. We have found the correct theory of the quantum Hall effect\cite{6}. According to this theory all of the fractional charges which do not fit in the CF model, are correctly predicted. In particular, the fractions, observed by Pan et al\cite{1} have been well predicted\cite{7}. At an earlier time, Pan et al\cite{8} thought that even feature is present in the data. However, we have shown\cite{9} that the experimental data is not consistent with the even feature. Here even feature means the quantity $2mp$ which occurs in the denominator.

In this paper, we show that the fractional charges observed by Smet\cite{2} are well predicted by our theory.

2. Smet’s observation

Motivated by the work of Pan et al\cite{1}, Smet\cite{2} searched for the fractional charges which are not found in the composite fermion model. Smet searched the region in the center of $2/5$ and $1/3$. Small steps were found at,

$\nu = \frac{4}{11}, \frac{7}{19}, \frac{10}{27}, ..., \frac{11}{29}, \frac{8}{21}, \frac{5}{13}$

in the experimental measurement of transverse resistivity. These fractions do not fit the CF model. However, if $1/2$ of these values could fit the formula, it will be sufficient and we could declare that these are two-particle states but for integer values of $m$ and $p$ the formula $p/(2mp \pm 1)$ does not fit the data. For two particle bound state, there will be the need for a binding energy and it becomes more difficult to fit the CF model with the experimental data.

Wöjs et al have suggested to use $2l + 1$ as the denominator as in our theory and add an arbitrary pseudopotential parameter until the CF agrees with the data, if not for one particle, then for any two-particle bound state. This is obviously arbitrary and not like physics because of the arbitrary parameter. Therefore, Wöjs et al have not solved the problem. There is a suggestion that the fractional charges like $4/11$, etc. are generated by a process similar to fractals. The fractal model of phase transition with fractional dimensionality does not satisfy the quantum Hall effect data. A nuclear decay type model also does not satisfy the data.

4. Theory

Our theory is explained in a recent book\cite{11}. Needless to say that this theory agrees with the quantum Hall effect data. We find that Laughlin has not resolved whether the area becomes fractional or the charge. Since the magnetic area is $a_0^2 = \phi_0/B$, it is important to know that the quantity which enters in the Laughlin’s paper is the product $eB$ and not $e$ alone. Schrieffer has used the Laughlin’s work as if it is $e$. There is no serious problem created by this type of treatment as long as we keep track of the product $eB$. We have explained our theory in comparison with Laughlin and Schrieffer in ref.\cite{12}. Ref.\cite{7} shows the interpretation of the data of Pan et al\cite{1}. At this stage our attention was drawn to a paper by Mani and von Klitzing\cite{13} which has 146 different fractions.
Surprisingly, we are able to understand all of these 146 values [14,15]. Pan et al[1] are concerned with 4/11, 5/13, 6/17, 4/13, 5/17 and 7/11. Let us add these 6 values to our 146 values so that it may be thought that 152 values come out to be correct from our theory. To this we add another 6 values from Smet[2] so that we have 158 values. The interpretation of Smet’s values is given below. In our theory, the effective charge is given by the formula,

\[ \frac{e_{\text{eff}}}{e} = \frac{l + \frac{1}{2} \pm s}{2l + 1} \]  

(2)

which gives 4/11 for \( l=5 \) and \( s=-3/2 \), 7/19 comes for \( l=9 \), \( s=-5/2 \); 10/27 comes for \( l=13 \), \( s=-7/2 \); 11/29 comes for \( l=14 \), \( s=-7/2 \); 8/21 has \( l=10 \), \( s=-5/2 \); 5/13 has \( l=6 \) and \( s=-3/2 \). Thus 4/11 belongs to a cluster of 3 electrons with \( l=5 \) and all polarized with negative sign for the spin. The fraction 7/19 has five electrons in \( l=9 \) with all spin polarized with negative sign, etc. Thus all of Smet’s experimentally measured values fit well in our formula showing that there are clusters of electrons with a small number of electrons in each cluster. For \( l=7 \), \( s=-3/2 \), \( \nu_+ =6/15 \); for \( l=7 \), \( s=-1/2 \), \( \nu_- =7/15 \); for \( l=8 \), \( s=-3/2 \), \( \nu_- =7/17 \), etc are predicted but not noted by Smet from experimental work. Thus we learn that there is a linear relationship between charge and spin. For \( l=0 \), the charge \( \nu_\pm \) is related to spin, \( s \) as,

\[ \nu_\pm = \frac{1}{2} \pm s. \]  

(3)

We can tabulate this expression as in Table 1 below.

| S.No. | \( s \) | \( \nu_+ \) | \( \nu_- \) |
|-------|-------|-------|-------|
| 1     | 1/2   | 1     | 0     |
| 2     | 3/2   | 2     | -1    |
| 3     | 5/2   | 3     | -2    |
| 4     | 7/2   | 4     | -3    |

This means that in the case of spherically symmetric states, half integer spin gives rise to integer charge with pairwise production of quasiparticles such that \( \nu_+ + \nu_- =1 \) but there is spin polarization. The zero charge for \( s=1/2 \) means that there is a charge-density wave which is linked to distortions in the solid. Integer charge is seen in electron clusters. For \( l=0 \), one electron gives rise to quasiparticles of zero charge and also equal to that of one electron. Three electrons must occupy three sites and then the quasiparticle charge may be 2 for one quasiparticle and -1 for the other, etc. Why should three electrons give a charge of 2? Actually, it is quite simple to understand. If all the three electrons are at one point, then it is justified that charge should be 3, otherwise, charge must be transported to one point before it is added. Fractional charge is developed due to this transport.

3. Conclusions.

It is clear that our formula predicts all of Smet’s values correctly. We have explained[16] that composite fermion(CF) model is internally inconsistent and the transformation from electrons to composite fermions has not been done. In any case, the
composite fermion series $p/(2mp \pm 1)$ is inconsistent with the experimental data of Smet. Our theory [11, 12, 14, 15] is based on quantum mechanics and works very well on all of the experimentally observed values of the fractional charges. We have discovered a new spin-charge relationship.

4. References

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Note: Ref. 11 is available from: Nova Science Publishers, Inc., 400 Oser Avenue, Suite 1600, Hauppauge, N. Y. 11788-3619, Tel. (631)-231-7269, Fax: (631)-231-8175, ISBN 1-59033-419-1 US$69. E-mail: novascience@Earthlink.net