Off-shell effects in dilepton production from hot interacting mesons

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Abstract

The production of dielectrons in reactions involving $a_1$ mesons and pions is studied. We compare results obtained with different phenomenological Lagrangians that have been used in connection with hadronic matter and finite nuclei. We insist on the necessity for those interactions to satisfy known empirical properties of the strong interaction. Large off-shell effects in dielectron production are found and some consequences for the interpretation of heavy ion data are outlined. We also compare with results obtained using experimentally-extracted spectral functions.

1 Introduction

The field of heavy ion collisions is a very active one, straddling high energy and nuclear physics. At the upper energy limit of this flourishing area of research, the

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goal is to eventually produce and study a new state of matter in the laboratory: the quark-gluon plasma (QGP). That strongly interacting matter in conditions of extreme energy densities undergoes a phase transition is in fact a prediction of QCD [1]. To confirm whether the phase transition indeed occurs in relativistic heavy ion collisions and that the QGP is formed, one needs a clear signal as a signature for the QGP. Many approaches have been suggested to elucidate the existence of this elusive state of matter, but unfortunately no single measurement can yet be singled-out as a “smoking gun” candidate. Instead, it appears that many complementary experimental data will require simultaneous analysis [2]. One class of observables that appears especially attractive is that of electromagnetic signals. This owes to the fact that such probes essentially suffer little or no final state interactions and thus constitute reliable carriers able to report on the local conditions at their emission site. Indeed, the calculated emission rates for photons and lepton pairs have been shown to strongly depend on the local density and temperature. Those facts were established some time ago and several experiments specializing in the measurement of electromagnetic radiation are either running right now or being planned.

In this paper, we shall be concerned about lepton pairs and calculations of their production. With respect to photon measurements, they constitute an important related observable. The extra degree of freedom associated with the invariant mass makes them valuable tools in the study of annihilation reactions, for example. The emission of lepton pairs from the QGP and also from hot hadronic has been studied by many authors [3]. We will not discuss here the lepton pairs with high invariant masses ($M \geq m_{J/\psi}$), they constitute a fascinating story of their own and the literature on the subject is considerable. Rather, we shall address the issue of softer pairs and that of the reliability of calculations of their emission. Recently, a lot of attention has focused on measurements from the CERES/NA45 collaboration [4]. Several theoretical calculations have attempted to reproduce this experimental data [4]. Because of the exciting potential of such measurements to reveal new physics, extra care has to be taken in their theoretical modeling. In the very soft sector, relevant for the CERES experiment, different theoretical calculations for the fundamental processes have in fact been confronted with each other with a striking degree of agreement [4]. It has also been shown that this data supports the fact that the very soft sector of the lepton pair spectrum is relatively independent of the dynamical approach used to model the evolution of the four-volume of the colliding system. However, the disagreement between models and data has led to different interpretations [4].

Another interesting aspect of the CERES analysis is that the region of invariant mass $m_{\phi} < M < m_{J/\psi}$ also shows an “anomalous” excess of lepton pairs, when compared with estimates based on hadronic electromagnetic decays. Note that this excess is seen only in nucleus-nucleus events, the proton-nucleus results are well reproduced assuming hadronic sources [4]. Adopting the same line of reasoning as that used in most of the calculations done to interpret the low mass sector of the CERES experiment, it is natural to ask which hadronic reactions could con-
tribute significantly to the mass region in question. Some work on this aspect was performed in the past where it was found that in a hot environment of mesons, the reaction $\pi + a_1 \rightarrow e^+e^-$ should be an important source of dileptons [7, 8, 9] (from here on we shall discuss only dielectrons). Such studies concerned with the contribution of the $a_1$ to the lepton pair spectrum are based on effective chiral Lagrangians containing $\rho$, $\pi$ and $a_1$. Our purpose in this paper is the following: we review several different effective Lagrangians for the $\pi \rho a_1$ vertex, and we calculate lepton pair production from the $\pi + a_1 \rightarrow e^+e^-$ reaction, using the Vector Meson Dominance model (VMD). We shall follow a philosophy similar to that in Ref. [8], but we go further by including a survey of other known and used interactions. We shall also stress the importance of conforming to empirical measurements, and we highlight the off-shell properties. Our work is organized as follows: after introducing the different models and mentioning their origins, we compare with each other the lepton results obtained and comment on the restrictions imposed on the effective interactions by the hadronic phenomenology. We also will show results from an experimentally-extracted spectral function analysis of lepton production processes. We then conclude.

2 Effective Lagrangians and phenomenology of the $a_1$ meson

In our survey of the $\pi \rho a_1$ interaction, we in turn will consider the following Lagrangians:

1. An effective chiral Lagrangian based on the $SU(2)_L \times SU(2)_R$ linear $\sigma$ model [11].
2. A $U(2)_L \times U(2)_R$ chiral model for pseudoscalar, vector, and axial-vector mesons [11].
3. An effective chiral Lagrangian where the vector mesons are introduced as massive Yang-Mills fields of the chiral symmetry [12].
4. An effective Lagrangian previously used in connection with photon emission rates [13].
5. An effective Lagrangian used to address the issue of form factors in the Bonn potential [14].

As one can realize, many interactions claim to address the $a_1 \pi \rho$ vertex. This variety owes to the fact that there is currently no unique way of implementing the chiral symmetry in an approach uniting pseudoscalars, vectors and pseudovectors. Therefore, many versions of a given coupling can be obtained, with symmetry requirements sometimes yielding to specific practical concerns. Note that all of the
above Lagrangians (except one) have been used in the literature to calculate the lepton-producing reaction we are considering here. In such investigations, a popular practice consists of adjusting any free parameter in the above interactions to reproduce the experimental decay width $\Gamma_{a_1 \rightarrow \pi \rho}$. In cases where more than one free parameter is available, one can also fit $\Gamma_{a_1 \rightarrow \pi \gamma}$, when electromagnetic phenomena are of some relevance. However, another well-defined empirical quantity exists, regarding the decay $a_1 \rightarrow \pi \rho$. The ratio of $D$-wave to $S$-wave (eigenstates of the relative orbital angular momentum in the exit channel) amplitude has been measured and is known to be $D/S = -0.09 \pm 0.03$ \cite{13}. Theoretically, the calculation of this ratio is accomplished by expanding the decay amplitude in the helicity basis, using the standard helicity representation for the polarization vectors. On the other hand, one can expand the amplitude in spherical harmonics. Working in the rest frame of the decaying particle, in this case the $a_1$, one can then match the two representations in terms of their helicity content. It is probably fair to say that this aspect of hadronic phenomenology has not received the attention it should have in the heavy ion community by users of effective hadronic Lagrangians.

We will now proceed as advertised earlier and compare in turn the interactions described above in terms of their contribution to the process $\pi + a_1 \rightarrow e^+ e^-$. However, before we turn to this specific application, it is instructive to recall why the process in question is not determined uniquely by choosing an interaction and fitting on-shell properties. For our illustrative purpose, consider the process $a_1 \rightarrow \rho \pi$. The most general vertex for this strong decay can be written as

$$\Gamma^{\mu \nu} = ig_1 g^{\mu \nu} + ig_2 q^\mu k^\nu + ig_3 k^\mu q^\nu + ig_4 q^\mu q^\nu + ig_5 k^\mu k^\nu,$$  \hspace{1cm} (1)

where $g_i(k^2, q^2)$ is a form factor, with $k^\mu$ and $q^\nu$ being the the $a_1$ and $\rho$ four-momenta. For on-shell $a_1$ decay, three of the above terms are identically zero, owing to the transversality condition $\epsilon(a_1) \cdot k = \epsilon(\rho) \cdot q = 0$. The on-shell form factors are usually chosen such that (some of) the experimental constraints discussed above are satisfied. However in the reaction under scrutiny here, $\pi + a_1 \rightarrow \rho \rightarrow e^+ e^-$, the $\rho$ meson is not on its mass shell and the extrapolations of form factors to this region are not unique. To evaluate the importance of such effects is the purpose of this work. Note however that off shell, for our reaction, there are still only two terms that are relevant in Eq. (1). This is because the lepton electromagnetic current is conserved. More specifically, the other term of Eq. (1) that could in principle contribute to our reaction is $ig_4 q^\mu q^\nu$. However, the corresponding matrix element for electron-positron pair production, once the spin sums are carried out, will look like (the lepton masses have been set to zero, for simplicity)

$$|\overline{M}|^2 \propto q^\mu q^\nu Tr (\gamma_\mu q_1^\gamma \gamma_\nu q_2),$$  \hspace{1cm} (2)

where $q_1^\mu (q_2^\mu)$ is the four-momentum of the electron (positron). Note that now $q^\mu = q_1^\mu + q_2^\mu$. The above expression, Eq. (2), is zero.
2.1 A Chiral $SU(2)_L \times SU(2)_R$ effective Lagrangian

The effective Lagrangian for the $a_1 \pi \rho$ vertex constructed by Ko and Rudaz [10] is based on the $SU(2)_L \times SU(2)_R$ linear $\sigma$ model, with the $\rho$ vector meson and the $a_1$ axial-vector meson included as phenomenological gauge fields associated with a local $SU(2)_L \times SU(2)_R$ chiral symmetry. This local chiral symmetry is then broken to global chiral symmetry.

The $a_1 \rho \pi$ vertex is generated by the following interaction Lagrangian[8, 10],

$$\mathcal{L}_{a_1 \rho \pi} = \frac{g^2 f_\pi}{Z_\pi} \left[ 2 c \vec{\pi} \cdot (\vec{\rho}_\mu \times \vec{a}_\mu) + \frac{1}{2 m_{a_1}^2} \vec{\pi} \cdot (\vec{\rho}_{\mu\nu} \times \vec{a}^{\mu\nu}) \right] + \left( \frac{1}{m_{a_1}^2} - \frac{\kappa_6 Z_\pi}{m_\rho^2} \right) \partial_\mu \vec{\pi} \cdot (\vec{\rho}^{\mu\nu} \times \vec{a}_\nu) \right]. \tag{3}$$

The $a_1(k) \to \pi(p)\rho(q)$ decay width can be obtained as

$$\Gamma(a_1 \rho \pi) = \frac{\sqrt{2}}{12\pi m_{a_1}^2} \left[ 2 f_{a_1 \rho \pi}^2 + \left( \frac{E_\rho}{m_\rho} f_{a_1 \rho \pi} + \frac{m_{a_1}}{m_\rho} g_{a_1 \rho \pi} |\vec{p}| \right)^2 \right], \tag{4}$$

where $\vec{p}$ is the momentum of the $\pi$ meson evaluated in the rest frame of the $a_1$, and

$$f_{a_1 \rho \pi} = \frac{g^2 f_\pi}{Z_\pi} \left[ 2 c + \frac{q^2}{m_{a_1}^2} \right] + \frac{\kappa_6 g^2 f_\pi}{m_\rho^2} \vec{p} \cdot q, \tag{5}$$

$$g_{a_1 \rho \pi} = -\frac{\kappa_6 g^2 f_\pi}{m_\rho^2}. \tag{6}$$

The ratio of D-wave to S-wave amplitudes for this final state is

$$\frac{D}{S} = -\frac{\sqrt{2}}{f_{a_1 \rho \pi}} \left[ f_{a_1 \rho \pi}(E_\rho - m_\rho) + g_{a_1 \rho \pi} m_{a_1} |\vec{q}|^2 \right]. \tag{7}$$

This chiral formulation admits a $a_1 \pi \gamma$ direct contact term and as such, deviates from the “traditional” Vector Meson Dominance approach. Including this, the radiative decay width one obtains for $a_1 \to \pi \gamma$ is

$$\Gamma(a_1 \pi \gamma) = \frac{1}{24} \frac{\alpha \kappa_6 g^2 f_\pi^2 (m_{a_1}^2 - m_\pi^2)^3}{m_\rho^4 m_{a_1}^3}. \tag{8}$$

Here is an opportune place for a short digression on the Vector Meson Dominance model (VMD). A recent discussion of its different representations can be found in Ref. [16]. We refer the reader to this reference, and those therein, for details. In short, the vector meson-photon vertex can either be a constant or the electromagnetic field strength tensor can couple to the rho field strength tensor to yield a $q^2$-dependent vertex, where $q$ is here the photon invariant mass [17]. In the latter case, an additional direct photon contact term is then necessary as the vector...
meson-photon mixing vanishes for real photons. The approaches described in this work invoke both versions of VMD. We have implemented the appropriate one for each case and also verified that gauge invariance was verified in the electromagnetic sector.

Returning to the Lagrangian under scrutiny, following Refs. [10, 18] we choose the parameters $g, c, \kappa_6$, and $Z_\pi$ guided by phenomenological considerations. Two choices of those numbers are displayed in Table 1. Parameter set I was used previously to address dilepton production in the reaction we are concerned with here [8]. However, this choice produces a wrong sign for the $D/S$ ratio and its $\chi^2$ can be somewhat improved. This is achieved by parameter set II, see Table 1.

### Table 1: Two parameter sets for the model of Ref. [10], and the associated phenomenology.

|             | I     | II    | DATA               |
|-------------|-------|-------|--------------------|
| $g$         | 5.04  | 4.95  |                    |
| $c$         | -0.12 | 1.29  |                    |
| $\kappa_6$  | 1.25  | -1.9014 |                |
| $Z_\pi$     | 0.17  | 0.83  |                    |
| $\Gamma(a_1\pi\gamma)$ | 0.572 | 1.171 | 0.640 ± 0.246 MeV |
| $\Gamma(a_1\rho\pi)$  | 313.4 | 579.1 | ~ 400 MeV         |
| $D/S$       | 0.078 | -0.168 | -0.09 ± 0.03     |
| $\Gamma(\rho e^+ e^-)$ | fit  | 7.01  | 6.77 ± 0.32 KeV  |
| $\chi^2$   | 8.9   | 7.5   |                    |

2.2 A $U(2)_L \times U(2)_R$ effective chiral Lagrangian

A $U(2)_L \times U(2)_R$ chiral model for pseudoscalar, vector, and axial-vector mesons has been proposed by B. A. Li [11] which produces a successful description of meson phenomenology. In this effective chiral theory, the $a_1(k)\pi(p)\rho(q)$ coupling is described by the following interaction Lagrangian:

$$\mathcal{L}_{a_1\rho \pi} = A \bar{a}^\mu \cdot (\vec{\rho}_\mu \times \vec{\pi}) + B \bar{a}^\mu \cdot (\vec{\rho}^{\nu} \times \partial_\mu \vec{\pi}),$$

where

$$A = \frac{2}{f_\pi} \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}} \left\{ \frac{F^2}{g^2} + \frac{m_{a_1}^2}{2\pi^2 g^2} - \frac{2c}{g} (p \cdot q + p \cdot k) - \frac{3}{2\pi^2 g^2} \left(1 - \frac{2c}{g}\right) p \cdot q \right\},$$

$$B = \frac{2}{f_\pi} \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}} \left(1 - \frac{2c}{g}\right),$$

and,

$$c = \frac{f_\pi^2}{2gm_\rho^2},$$

(8)

(9)

(10)
\[ F^2 = \frac{f_\pi^2}{1 - \frac{2c}{g}} \]  

(11)

In this model, \( f_\pi = 0.186 \text{ GeV} \) and the particle masses are taken as input. Fitting \( g = 0.35 \) yields a good list of light-meson empirical properties [1] [4].

2.3 Another effective chiral Lagrangian

Yet another effective chiral Lagrangian for pseudoscalar, vector, and axial-vector mesons can be derived [12]. In this model, the \( \pi \) meson is described through the nonlinear \( \sigma \) model, and the \( \rho \) and \( a_1 \) mesons are included as massive Yang-Mills fields of the chiral symmetry. This scheme has been used to discuss photon emission [19] and dilepton production [7] from hot hadronic matter.

The lowest order interaction term for \( a_1 \rightarrow \rho \pi \) is given by a lengthy expression given in terms of the meson matrices [12]. For the physical \( a_1 \rightarrow \rho \pi \) decay, the vertex function leading to the decay width is

\[ \Gamma_{\mu\nu}(a_1 \rightarrow \rho \pi) = i(g_{a_1\rho\pi}g_{\mu\nu} - h_{a_1\rho\pi}q_{\mu}k_{\nu}), \]  

(12)

where,

\[ g_{a_1\rho\pi} = \frac{g}{\sqrt{2}} \left[ -\eta_1 q^2 + (\eta_1 - \eta_2) k \cdot q \right], \]

\[ h_{a_1\rho\pi} = \frac{g}{\sqrt{2}} (\eta_1 - \eta_2), \]  

(13)

with

\[ \eta_1 = \left( \frac{1 - \sigma}{1 + \sigma} \right)^{1/2} \frac{gF_\pi}{2m_\rho^2} + \frac{4\xi Z^2}{F_\pi \sqrt{1 + \sigma}}, \]

\[ \eta_2 = \left( \frac{1 + \sigma}{1 - \sigma} \right)^{1/2} \frac{gF_\pi}{2m_\rho^2} - \frac{4\sigma}{gF_\pi \sqrt{1 - \sigma^2}}, \]

and

\[ Z^2 = 1 - \frac{g^2 F_\pi^2}{4m_\rho^2}. \]  

(14)

Here, \( F_\pi \approx 135 \text{ MeV} \). With the two sets of parameters which give the \( a_1 \rightarrow \rho \pi \) decay width \( \Gamma(a_1 \rightarrow \rho \pi) \approx 400 \text{ MeV} \) [19], we further obtained the \( D/S \) ratio for this effective Lagrangian:

set I : \( g = 10.3063, \quad \sigma = 0.3405, \quad \xi = 0.4473, \quad D/S = 0.357 \)

set II : \( g = 6.4483, \quad \sigma = -0.2913, \quad \xi = 0.0585, \quad D/S = -0.099 \).  

(15)

In this work, we shall use parameter set II. The electromagnetic interaction is introduced by imposing the \( U(1)_{em} \) gauge symmetry on the effective chiral Lagrangian.

\(^{1}\)In Eq.(62) of ref. [11], we argue that a factor of \( \sqrt{2} \) is missing. For \( m_{a_1} = 1230 \text{ MeV} \), we then get \( D/S = -0.0895 \). The experimental value is \(-0.09 \pm 0.03 \pm 0.01\).
2.4 Two more simple effective Lagrangians for $a_1\rho\pi$ interactions

Xiong, Shuryak and Brown [13] have defined an effective Lagrangian for $a_1\rho\pi$ interactions, in order to calculate photon production from hot hadronic matter. This Lagrangian is

$$\mathcal{L}_{a_1\rho\pi} = G_\rho a^{\mu}(p \cdot q g_{\mu\nu} - q_\mu p_\nu)\rho^\nu\pi.$$  \hspace{1cm} (16)

The coupling constant $G_\rho$ is determined by fitting the $a_1 \to \rho\pi$ decay width. Unfortunately, we cannot reproduce the numerical results of Ref. [13]. Using $m_{a_1} = 1230$ MeV and fitting the total decay width, we get $G_\rho = 11.425$ GeV$^{-1}$. Using the Vector Dominance Model we obtain the coupling constant of the $a_1\pi\gamma$ interaction as $G_\gamma = 0.573$ GeV$^{-1}$. This then implies a value of $\Gamma_{a_1 \to \pi\gamma} = 1.94$ MeV, which is somewhat larger than the experimental measurement of $0.640 \pm 0.246$ MeV. This effective Lagrangian also predicts $D/S = 0.185$.

| Source: | I | II | [12] | [14] | [13] | DATA |
|---------|---|----|------|------|------|------|
| $\Gamma(a_1\rho\pi)$ | 313.4 | 579.1 | fit | 331.7 | fit | fit | $\sim 400$ MeV |
| $\Gamma(a_1\pi\gamma)$ | 0.572 | 1.171 | 0.067 | 0.331 | 1.940 | 0.312 | 0.640 $\pm$ 0.246 MeV |
| $D/S$ | 0.078 | $-0.168$ | $-0.099$ | $-0.161$ | 0.185 | 0.045 | $-0.09 \pm 0.03$ |
| $\chi^2$ | 11.8 | 9.6 | 1.8 | 3.1 | 37.3 | 7.3 |

Table 2: A comparison of hadronic properties for the interactions discussed in this work. Note that the $\chi^2$'s appearing in this Table were calculated using the experimental measurements that appear on it only. When different parameter sets are involved for a given interaction, the distinction is explained in the text.

The other simple effective Lagrangian which considers the $a_1\rho\pi$ interaction was used to formulate a meson-exchange model for $\pi\rho$ scattering by Janssen, Holinde and Speth [14]. It is

$$\mathcal{L}_{a_1\rho\pi} = g_{a_1}(\partial_\mu \vec{p}_\nu - \partial_\nu \vec{p}_\mu) \cdot [\vec{\pi} \times (\partial^\mu \vec{a}^\nu - \partial^\nu \vec{a}^\mu)].$$  \hspace{1cm} (17)

For $m_{a_1} = 1230$ MeV, the coupling constant $g_{a_1} = 2.285$ GeV$^{-1}$ follows from the usual fit of the $a_1$ decay width. The $D/S$ ratio here is $D/S = 0.045$.

A comparative summary of the different on-shell properties and predictions is shown in Table 2.

3 Results for dilepton production from hot hadronic matter
3.1 Effective Lagrangian approach

In this section we shall compute rates of lepton pair emission with the effective interactions introduced earlier. In this study we rely on relativistic kinetic theory to provide an idealized dynamical framework. Because we are mainly interested in comparisons between different models, this line of reasoning is entirely adequate. We also set any chemical potential to zero, for simplicity. The temperature chosen for our baseline study is \( T = 150 \text{ MeV} \).

In general, the dilepton production rate from the annihilation of two particles \( a(p_1) \) and \( b(p_2) \) can be written as

\[
\frac{dN}{d^4xdM^2} = \mathcal{N} \int ds \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} f_a(p_1) f_b(p_2) \frac{d\sigma(ab \rightarrow l^+l^-)}{dM^2} v_{\text{rel}} \delta \left(s - (p_1 + p_2)^2\right),
\]

where \( \mathcal{N} \) is an overall degeneracy factor, \( f(p) \) is the distribution function of the incoming particles at temperature \( T \), \( v_{\text{rel}} \) is the relative velocity of the two particles,

\[
v_{\text{rel}} = \sqrt{(p_1 \cdot p_2)^2 - m_a^2 m_b^2 \over E_a E_b},
\]

and \( \sigma \) is the dilepton production cross section for the reaction \( a + b \rightarrow l^+ + l^- \). Using Boltzmann distributions, one performs the phase space integrations and obtains

\[
\frac{dN}{d^4xdM^2} = \mathcal{N} \frac{T}{32\pi^4M} K_1(M/T) \lambda(s, m_a^2, m_b^2) \sigma(M),
\]

where \( K_1 \) is a modified Bessel function, and \( \lambda \) is the kinematic triangle function

\[
\lambda(x, y, z) = x^2 - 2x(y + z) + (y - z)^2.
\]

The cross section \( \sigma(M) \) depends only on the square of the invariant mass of the dilepton \( s = (p_1 + p_2)^2 = M^2 \) and is easily related to the square of a spin-averaged scattering amplitude. This in turn can be written as

\[
|\overrightarrow{M}|^2 = 4 \left( \frac{4\pi\alpha}{s} \right)^2 L_{\mu\nu}^\ast H^{\mu\nu},
\]

where, \( \alpha \) is the fine structure constant, and \( L_{\mu\nu}^\ast \) is a leptonic tensor given by

\[
L_{\mu\nu} = q_1^\mu q_2^\nu + q_2^\mu q_1^\nu - g^{\mu\nu} q_1 \cdot q_2,
\]

and \( H^{\mu\nu} \) is a hadronic tensor for the process. Note that \( (q_1 + q_2)^2 = s = M^2 \). The hadronic tensor is calculated from the meson vertex function of the appropriate Feynman diagram with the relevant Lagrangian.

We plot in Fig. [1] the rates for dielectron production from a gas of hot mesons, at a temperature of \( T = 150 \text{ MeV} \). We concentrate on \( \pi a_1 \) reactions. All of the different interactions enumerated previously have been considered and their contribution
appears here. A striking feature is that the rates calculated with those Lagrangians span two-and-a-half orders of magnitude. Recall that all of them (except one) have been used in the literature to perform dilepton calculations very similar to the one done here. The features illustrated in Fig. 1 are essentially temperature-independent. We have verified this by also performing calculations at a temperature of $T = 100$ MeV. We conclude from this that off-shell effects are indeed quite important. Considering the curves labeled (d1) and (d2), one can verify by looking again at Table 2 that a modest change in the $\chi^2$ calculated on shell can result in an important variation in off-shell behaviour. This tells us that although the on-shell $\chi^2$ can be used as a goodness-of-fit parameter, it is far from being enough to specify unambiguously which interaction to use in situations like the one at hand. One has to proceed with caution.

We now turn to an approach which should have the off-shell effects built-in, in order to help us deciding upon an interaction to use.

### 3.2 Using experimentally-constrained spectral functions

Above, we have found large differences in lepton producing rates using different hadronic Lagrangians to model the hard vertices. This result is worrisome, as many of those interactions have been used in the past for dilepton calculations, and there is no doubt that they will be used again in the future. We then turn to an alternative approach, with the hope that this will assist us in the selection of an appropriate
theory. In the context of heavy ion reactions, it has been argued recently that lepton pair production cross sections could in fact be determined from the spectral functions extracted from the inverse process: $e^+ + e^- \rightarrow \text{hadrons}$ \cite{21}. We refer the reader to the appropriate reference for the complete details. It will suffice here to state that the rate for the emission of dilepton pairs of invariant mass $M$, at temperature $T$ in a given reaction is given by

$$\frac{dR}{dM^2} = \frac{4\alpha^2}{2\pi} MTK_1(M/T) \rho_{em}(M), \quad (24)$$

where $\alpha$ is the usual fine structure constant, $K_1(x)$ is a modified Bessel function, and $\rho_{em}(M)$ is a zero-temperature spectral function. This quantity is extracted from $e^+ e^-$ annihilation data through

$$\rho_{em}(s) = \frac{s \sigma(e^+ e^- \rightarrow \text{hadrons})}{16\pi^3\alpha^2}. \quad (25)$$

We then follow the same approach as that of a current algebra calculation \cite{22} to extract the $\pi a_1$ contribution from $e^+ e^- \rightarrow 4\pi^{\pm}$ data. One can then obtain $\sigma(e^+ e^- \rightarrow \pi a_1)$, which can then be used in Eq. (25) to extract the spectral function. Note that this procedure is straightforward and relatively free from ambiguity, numerical or otherwise. It is tantamount to a direct evaluation of the dilepton cross sections by the same theoretical methods.

We plot on Fig. 2 the rates previously shown on Fig. 1, accompanied by the rate evaluated with the method outlined above. Note that this latter calculation does
not run over the entire invariant mass range covered in the plot, as we have chosen not to stretch the validity of the theoretical approach which was deemed optimal in the invariant mass range covered by the double line [22]. This coverage is sufficient for us to make our point. Considering that the spectral function is constrained by data, we use this analysis as an extra tool to discriminate between the various interactions considered in this work. We observe that the spectral function results are overshot by the interaction from Ref. [11]. They are underpredicted by the rates related to the Lagrangians in Refs. [13] and [14], and by rates obtained with parameter set II of the interaction from Ref. [10]. The interactions from Refs. [12] and [10] (parameter set I) yield results in good agreement with the spectral function determination. Even though both Lagrangians seem to exhibit a satisfactory off-shell behaviour, going back to Table 2 one observes that the one from Ref. [12] also produces an excellent $\chi^2$ for on-shell hadronic properties. It appears that, with the parameters used in this study and considering both on-shell and off-shell behaviours, this interaction achieves the compromise we were seeking. It is also a satisfying one from a theoretical point of view [23]. Note in closing that the rates above were determined in the narrow-width approximation for the $a_1$. While at first sight this limit seems unreasonable, it has been shown to affect little the magnitude of the thermal rates [24]. Its main effect is to soften the threshold of this specific reaction.

4 Conclusion

We have calculated dilepton emission from $\pi a_1$ reactions at finite temperature using several different Lagrangians found in the literature. We have found widely different results. One might argue that some of those interactions were derived for entirely different purposes, therefore comparing them on the basis of dilepton emission seems vaguely inappropriate. One should however remember that all of the Lagrangians (except that of Ref. [11]) have been used previously in such exercises. Up to now, a critical comparison of their results was lacking. With the help of an experimentally-constrained spectral function, combined with a quantitative analysis of on-shell properties, we were able to select an adequate interaction. It is clear that a companion study to this one will consider the rates for photon emission. This work is in progress and will be reported on later. There are however indications that for photons, the differences arising from the use of different Lagrangians will be less severe [8]. This probably owes to the fact that the mass shell condition for real photons has a restraining effect.

It should be clear that our goal was not an exhaustive survey of the parameter space relevant to each of the models we have discussed. We viewed these interactions as representative of what is currently on the market. One could in principle devise a new completely phenomenological interaction. Our study highlights the important issue of what needs to be done in order to give credibility to results obtained in an environment as potentially complex as that of high energy heavy ion collisions.
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