Estimation of precision parameters of milling machines

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Abstract. The paper considers a mathematical model that allows to evaluate the performance parameters of milling machines during the functioning of the working bodies. An example of the task of selecting equipment is given. The mathematical model is based on the mathematical apparatus of coordinate transformation. Using the proposed methodology, it is possible to solve the direct problem - to evaluate the capabilities of an existing machine for manufacturing parts with a given accuracy, or the inverse problem - to choose a machine for manufacturing parts with a given accuracy.

1. Introduction One of the indicators of the quality of the operation of machines is the accuracy of the trajectory of their movements. The development of the trajectory with a given accuracy is determined by organized programmed movements of the working bodies that ensure the functioning process, and unorganized, forming a real trajectory of movement. Unorganized movements occur under the influence of systematic and random factors, such as deviations of kinematic and dimensional chains, elastic displacements, temperature changes, wear, measurement errors, programming errors, etc. [1]. Unorganized movements are modeled by introducing component errors into the rules of movement of the coordinate systems of the links of the kinematic scheme of the machine. The accuracy of the machine plays an important role in ensuring the quality of the machined surfaces [2], therefore it is important to have a tool for preliminary assessment of the output accuracy of the machine [3].

The output accuracy of the machine is calculated in the following order [4]:

2. Building the coordinate code of a functional system

A functional system can be replaced by an equivalent scheme, as a set of solid bodies, each of which can be replaced by a coordinate system. The origin $O$ and $X, Y, Z$ axes of the coordinate system are associated with the structural or technological bases of the link.

3. Building a functional model

Instead of variables of the coordinate code, matrices of generalized movements (coordinate transformations) are substituted. Each matrix is associated with its own link in the forming system. A tool vector matrix is also added. When machining holes with an axial tool, milling grooves with end and disc mills, and round and internal grinding, the radius of the tool must be taken into account. The coordinate transformation matrices and the tool vector are multiplied in the order determined by the coordinate code.

$$r_0 = A^X \cdot A^Y \cdot A^Z \cdot A^C \cdot r_t$$ (1)
where \( r_0 \) - vector whose elements are the coordinates of the ideal surface of the workpiece; \( A^C, A^X, A^Y, A^Z \) - transformation matrices in corresponding coordinates; \( r_u \) - vector whose elements are the coordinates of the surface of the cutting tool.

Equation (1) is the basic equation of the functional model, which is a mathematical model of the shaping system and it is a description of the resulting trajectory in the form of a vector.

4. **Accounting for errors in the links of a functional system**

The purpose of this section is to find this error. The required error is the complete variation of the vector of the coordinates of the trajectory:

\[
\Delta r_0 = \delta A \cdot r_0
\]

where \( \delta A \) – variation matrix.

The variation matrix is a small displacement of the coordinate system associated with the link.

5. **Estimation of the accuracy of the generated trajectory**

To obtain estimates of the accuracy of the size, location and shape of the formed trajectory, it is necessary to build a metrological base [5].

The process of building the base surface is performed in two stages:

5.1. *The first step is to draw up the equation of the base surface.*

Due to the small deviations of the base surface from the nominal, the equation of the base surface can be written as:

\[
r_b = r_0 + \Delta r_b
\]

where \( \Delta r_b \) defined as the sum of the error vectors from position and size:

\[
\Delta r_b = e_b r_0 + d r_0
\]

where \( e_b \) – a matrix whose elements are errors (along the coordinate axes) of the location of the coordinate system associated with the base surface relative to the system in which the equation (1);

\[
d r_0 \text{ – the full differential of the radius-vector } r_0, \text{ taken for all components of the vector } q_0 \text{ of the dimensional parameters of the surface.}
\]

5.2. *The second stage of constructing the base surface is the calculation of the components of the vector } \Delta q \text{ of position and size errors.}*

The components of the vector \( \Delta q \) can be found from the matrix equation [4]:

\[
H \Delta q = d
\]

where \( H \) – matrix of order \( p \times p \) with elements:

\[
h_{ki} = \int_S f_k f_i dS
\]

\( d \) – vector of order \( p \) with elements:

\[
d_i = \int_S f_i \Delta r_n dS
\]

\( f_k, f_i \) – \( k \)-th and \( i \)-th coordinates of the vector \( f \) of normal transfer coefficients.
\[ f = G^T n \]  

(9)

\[ n - \text{unit vector of the normal to the surface } r_0, \]  

\[ \Delta r_n = \Delta r_0 n \]  

(10)

\[ \Delta r_n - \text{error measured by the normal to the surface}, \]  

\[ S - \text{the surface of integration is determined by the vector } r_0. \]

To find the values of the point coordinate errors, you must subtract the point coordinate equation from the equation, taking into account the errors (10), the point coordinate equation for an ideal surface (1).

\[ \Delta r_0(x, y, z) = r_n(x, y, z) - r_0(x, y, z) \]  

(11)

Based on this equation, you can solve the problem of determining the errors of the treated surface.

6. Determination of accuracy indicators of the machine to ensure the formation of the surface with a given accuracy

Let the results of measurements of the size and shape of finished parts are known and you need to find the best estimate of input errors from them. Formally, this problem is reduced to the problem of estimating parameters in the linear regression equation. In fact, equation (2) can be rewritten as:

\[ \Delta r_0(x, y, z) = \sum \delta q_i \cdot a_i(x, y, z) \]  

(12)

where \( \delta q_i - \text{machine errors; } a_i(x, y, z) - \text{known functions, in this case representing transfer coefficients.} \)

You need to evaluate the parameters \( \delta q_i \), if you know the result of a multiple measurement \( n \) of the value \( \Delta r_0(x, y, z) \) at least at \( m \) different points.

It is advisable to estimate \( \delta q_i \) from the conditions of minimizing the mean square deviations [4] of the observed values \( \Delta r_0(x, y, z) \) from those calculated according to equation (10), when their estimates are substituted for \( \delta q_i \). The solution of system (12) by the least squares method [4] has the form:

\[ \delta = B^{-1} \cdot A^T \cdot D^{-1} \cdot \Delta \]  

(13)

where \( \delta - \text{vector of estimated parameters } \delta q_i; \) \( B - \text{matrix of order } m \times m \) of a system of normal equations composed by the least square method:

\[ B = A^T \cdot D^{-1} \cdot A \]  

(14)

\( A - \text{structural matrix of order } n \times m \) of rank \( m \) composed of the coefficients of the system (1);

\( D - \text{a matrix of order } n \times n \) of rank \( n \), which is a covariance matrix of measurement results;

\( \Delta - \text{vector of order } n \) composed of measurement results.

Drawing up the matrix \( A \) and the vector \( \Delta \) does not cause any special difficulties. The greatest difficulties are caused by composing the covariance matrix \( D \), since its elements are usually unknown in advance. However, it is often sufficient to assume that the variances of all values \( \Delta \) are constant. Then the formula (14) is simplified:

\[ \delta = (A^T \cdot A)^{-1} \cdot A^T \cdot \Delta \]  

(15)

In this case, you only need to know the matrix \( A \), which is obtained from the equations of the variational method for calculating accuracy and measurement results.

Based on the results of the calculation, requirements were formed for the accuracy indicators of the machine for finishing milling of the stamp.
Table 1. Requirements for the accuracy of the machine for the finishing milling of the working surfaces of the stamp

| Accuracy indicators                        | Requirements for finishing milling |
|--------------------------------------------|------------------------------------|
| Radial runout of the conical spindle hole  |                                    |
| At the end                                 | 0,001                              |
| At a distance of 150 mm                    | 0,003                              |
| Spindle axial runout                       | 0,002                              |
| Repeatability of positioning               | 0,001                              |
| One-way positioning error at 120 mm length | 0,008                              |

7. Conclusions
The given system of mathematical models under the influence of errors of its links allows us to describe the deviations of the trajectory of the working body of the machine. This system allows you to evaluate the accuracy of surface treatment under specific operating conditions and purposefully correct its errors [3].

References
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