Planetary microlensing signals from the orbital motion of the source star around the common barycentre

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ABSTRACT

With several detections, the technique of gravitational microlensing has proven useful for studying planets that orbit stars at Galactic distances, and it can even be applied to detect planets in neighbouring galaxies. So far, planet detections by microlensing have been considered to result from a change in the bending of light and the resulting magnification caused by a planet around the foreground lens star. However, in complete analogy to the annual parallax effect caused by the revolution of the Earth around the Sun, the motion of the source star around the common barycentre with an orbiting planet can also lead to observable deviations in microlensing light curves that can provide evidence for the unseen companion. We discuss this effect in some detail and study the prospects of microlensing observations for revealing planets through this alternative detection channel. Given that small distances between lens and source star are favoured, and that the effect becomes nearly independent of the source distance, planets would remain detectable even if their host star is located outside the Milky Way with a sufficiently good photometry (exceeding present-day technology) being possible. From synthetic light curves arising from a Monte Carlo simulation, we find that the chances for such detections are not overwhelming and appear practically limited to the most massive planets (at least with current observational set-ups), but they are large enough for leaving the possibility that one or the other signal has already been observed. However, it may remain undetermined whether the planet actually orbits the source star or rather the lens star, which leaves us with an ambiguity not only with respect to its location, but also to its properties.

Key words: gravitational lensing – planetary systems.

1 INTRODUCTION

Gravitational microlensing, i.e. the transient brightening of an observed star due to the bending of light caused by the gravitational field of an intervening foreground ‘lens’ star, was considered by Einstein as early as 1912, as pointed out by Renn, Sauer & Stachel (1979), but he concluded that ‘there is no great chance of observing this phenomenon’ (Einstein 1936). Only several decades of advance in technology enabled the first reported discovery of a microlensing event (Alcock et al. 1993), following the suggestion by Paczyński (1986) to use the technique as a tool for detecting compact matter in the Galactic halo.

However, microlensing provides a valuable tool for a variety of other astrophysical applications, and the most spectacular one nowadays is the detection of extrasolar planets. It was already pointed out by Liebes (1964) that the primary effect of planets as gravitational lenses would be to produce a slight perturbation of the lens action of their respective host star. The distortion of the magnification pattern of the foreground lens star by an orbiting planet, and the additional short blip or dip to the otherwise symmetric microlensing light curve was then first studied by Mao & Paczyński (1991). A super-Jupiter was the first planet detected by this technique (Bond et al. 2004), but its sensitivity even reaches below the mass of the Earth, even for ground-based observations (Bennett & Rhie 1996; Dominik et al. 2007). In fact, the detectability of planets below 10M\textsubscript{⊕} has been impressively demonstrated with the first discovery of a cool rocky/icy exoplanet (Beaulieu et al. 2006). Microlensing is already singled out amongst all ground-based current techniques aiming at the detection of extrasolar planets by the respective host stars being at Galactic distances, rather than in the solar neighbourhood, and belonging to either of two stellar populations. Significantly beyond this, even planets orbiting stars in neighbouring galaxies, such as M31, could be detected (Covone et al. 2000; Chung et al. 2006), whereas no other technique so far has been suggested that could achieve such a goal within foreseeable time.
By creating a link between received light, the gravitational field of intervening objects, and relative transverse motions between source, lens and observer, the effect of gravitational microlensing shows a substantial versatility. It is therefore not that surprising that it provides other channels for detecting planets orbiting stars other than the Sun. As suggested by Graff & Gaudi (2000), the light of close-in gas-giant planets would be detectable with large telescopes if such observations are scheduled while the planet follows its host star in exiting a caustic produced by a binary lens system, given that the light received from the planet would be far more strongly magnified than that received from its host star.

Here, we discuss a further channel for revealing the existence of extrasolar planets from the study of microlensing light curves. Rather than considering a planet around the lens star, we study the effects of the orbital motion of the source star and a planetary companion around the common barycentre. While the planet is not seen itself, the small motion of its observed host star periodically alters the line-of-sight and thereby the relative lens–source position, which results in a change of the observed source magnification. In fact, an analogous effect is caused by the revolution of the Earth around the Sun, where the line-of-sight is altered due to the motion of the observer rather than the observed object, so that it constitutes a form of parallax effect. While the annual parallax in a microlensing event was first observed by Alcock et al. (1995), the orbital motion of stellar source binaries has been studied extensively by various authors (Griest & Hu 1992; Han & Gould 1997; Paczyński 1997). Some authors refer to the latter as ‘xallarap’ effect, but in fact, this nomenclature involves a double inversion, because parallax is known as the apparent change in position of an observed object that is just the reflection of a change in position by the observer. Not surprisingly, both effects are not easy to distinguish, and a systematic analysis of 22 microlensing parallax candidate events (Poindexter et al. 2005) explicitly finds 23 per cent of them being strongly affected by ‘xallarap’. Future space missions capable of accurate astrometry such as SIM (Space Interferometry Mission) and GAIA will however allow measure the parallax along with the relative proper motion between lens and source star, allowing to obtain more accurate information about the Galaxy (e.g. Paczyński 1998; Rahvar & Ghassemi 2005).

We start our discussion in Section 2 with an introduction to the basics of gravitational microlensing and Keplerian motion, followed by a review of the annual Earth–Sun parallax based on the common formalism for orbital motion in observer, lens and source developed by Dominik (1998). The discussion of annual parallax is subsequently transferred to the case of stellar reflex motion due to an orbiting planet, where the relevant parameters are identified. In Section 3, we turn our attention to the prospects for planet detection through our proposed channel, where we first discuss the strength of the observable effect and identify the favourable scenarios. Subsequently, we discuss the input parameters and the results of a Monte Carlo simulation of synthetic light curves, thereby quantifying the planet detectability as a function of the planet mass and orbital parameters. Moreover, we study how to extract properties of the planet and its orbit from the observed light curve. A short summary and final conclusions are presented in Section 4.

2 PLANETARY ORBITS AND MICROLENSING

2.1 Microlensing events

According to the theory of general relativity, a light ray passing beside a massive object is bent, the arrival time of its photons is delayed, and a ray bundle is distorted. Allowing for several possible light rays from an observed source object to the observer, this phenomenon is commonly known as ‘gravitational lensing’. If a foreground star with mass \( M \) at distance \( D_L \) happens to be sufficiently aligned with an observed background star at distance \( D_S \), the time delay is negligible, while the angular separation between the images is of the order of microarcseconds, and therefore undetectable with current telescopes. However, contrary to extragalactic scenarios of gravitational lensing, stars in our own or neighbouring galaxies show a substantial proper motion, so that the image distortion results in an observable transient brightening as the foreground ‘lens’ star passes along the line-of-sight to the observed background source star, which is commonly referred to as a ‘microlensing event’.

Explicitly, for lens and source being separated on the sky by a position angle \( \theta_E \), where the angular Einstein radius

\[
\theta_E = \sqrt{\frac{4GM}{c^2}} (D_L^{-1} - D_S^{-1})
\]

provides the unique characteristic scale of gravitational microlensing, the combined magnification of the images reads (Einstein 1936)

\[
A(u) = \frac{u^2 + 2}{u\sqrt{u^2 + 4}},
\]

where \( u = |\theta| \).

Let us consider source and lens star being in uniform proper motion with \( \mu_s \) and \( \mu_\star \), respectively, so that the relative proper motion reads \( \mu = \mu_\star - \mu_\star = \mu(\cos \psi, \sin \psi) \). With \( u_0 \) marking the closest angular approach between lens and source star, realized at epoch \( t_0 \), the relative source–lens trajectory takes the form

\[
u_{LS}(t) = u_0 \left( -\sin \psi \cos \psi \right) + \omega_0 (t - t_0) \left( \cos \psi \sin \psi \right),
\]

where \( \omega_0 \equiv \mu/\theta_E \), so that

\[
u_{LS}(t) = \sqrt{u_0^2 + \omega_0^2 (t - t_0)^2}.
\]

In the literature, rather than \( \omega_0 \), a time-scale \( \theta_E \equiv \omega_0^{-1} \) is used more frequently. Given that \( A(u) \) is strictly decreasing with \( u \), and \( u(t) \) assumes a minimum of \( u_0 \) at epoch \( t_0 \), the magnification \( A(u(t)) \) leads to a symmetric light curve, peaking at \( t_0 \) (Paczyński 1986).

The symmetry of the light curve is retained if the finite angular radius \( \theta_r \) of the source star is taken into account. With a source size parameter \( \rho_\star = \theta_r/\theta_E \) and the abbreviations

\[
n = \frac{4u \rho_\star}{(u + \rho_\star)^2}, \quad k = \sqrt{\frac{4n}{4 + (u - \rho_\star)^2}},
\]

Witt & Mao (1994) found a uniformly bright star being magnified by

\[
A(u; \rho_\star) = \frac{1}{2\pi} \left[ \frac{u + \rho_\star}{\rho_\star^2} \sqrt{4 + (u - \rho_\star)^2} \ E(k) - \frac{u - \rho_\star}{\rho_\star^2} \frac{8 + u^2 - \rho_\star^2}{\sqrt{4 + (u - \rho_\star)^2}} \ K(k) + \frac{4(u - \rho_\star)^2}{\rho_\star^2 (u + \rho_\star)} \ \frac{1 + \rho_\star^2}{\sqrt{4 + (u - \rho_\star)^2}} \ \Pi(n; k) \right]
\]

for \( u \neq \rho_\star \), where \( K(k), E(k) \) and \( \Pi(n; k) \) denote the complete elliptic integrals of first, second and third kind, respectively, whereas

\[
A(u; \rho_\star) = \frac{1}{\pi} \left[ \frac{2}{\rho_\star} + \frac{1 + \rho_\star^2}{\rho_\star^2} \left( \frac{\pi}{2} + \arcsin \frac{\rho_\star^2 - 1}{\rho_\star^2 + 1} \right) \right]
\]
for \( u = \rho_u \). The size of the source star significantly affects the observed light curve for \( u \lesssim 2 \rho_u \), while \( A(u, \rho_u) \) reveals equation (2) as \( \rho_u \to 0 \).

For \( F_S \) denoting the intrinsic flux of the magnified source star and \( F_B \) that of the background, unresolved from the observed target, the microlensing light curve is given by the received flux

\[
F(t) = F_S A[u(t); \rho_u, I] + F_B, \tag{8}
\]

where the finite-source magnification \( A[u(t); \rho_u, I] \) is completely characterized by \((t_0, \theta_0, \phi_0, \rho_u, I)\), where \( I \) stands for the brightness profile function of the source star, for which we assume \( I \equiv 1 \) (i.e., a uniformly bright source) throughout.

### 2.2 Keplerian motion

If one neglects relativistic effects, a planet and its host star of masses \( m_p \) and \( m_\ast \), respectively, both are in elliptic orbits around their common centre of mass. In fact, the motion of their separation vector \( \mathbf{r}(t) = \mathbf{r}_p(t) - \mathbf{r}_\ast(t) \) can be understood as a fixed virtual body of total mass

\[
m = m_p + m_\ast, \tag{9}
\]

being orbited by another virtual body of the reduced mass

\[
\mu = \frac{m_p m_\ast}{m}, \tag{10}
\]

and with \( \mathbf{r}_{\text{cm}}(t) \) denoting the motion of the centre of mass,

\[
\mathbf{r}_p(t) = \mathbf{r}_{\text{cm}}(t) + (m_p/m) \mathbf{r}(t), \tag{11}
\]

\[
\mathbf{r}_\ast(t) = \mathbf{r}_{\text{cm}}(t) - (m_p/m) \mathbf{r}(t).
\]

The orbit \( \mathbf{r}(t) = [x(t), y(t), 0] \) is characterized by its major semi-axis \( a \), its orbital period \( P \), its eccentricity \( e \), the orbital plane, spanned by \( x \) (along major axis) and \( y \) (along minor axis), as well as the orbital phase at a reference epoch. Kepler’s third law provides a relation between the orbital period \( P \) and the major semi-axis \( a \) by means of the total mass \( m \), where

\[
P = 2\pi \sqrt{\frac{a^3}{Gm}}. \tag{12}
\]

With positive \( x \) in the direction of periastron and positive \( y \) from periastron towards the motion of the planet, one finds

\[
x(t) = a \left[ \cos \xi(t) - e \right],
\]

\[
y(t) = a \sqrt{1 - e^2} \sin \xi(t), \tag{13}
\]

where the eccentric anomaly \( \xi(t) \in [0, 2\pi] \) is given by

\[
\xi(t) = \sin \xi(t) = \Omega(t - t_p) - [\Omega(t - t_p)], \tag{14}
\]

with \( \Omega = (2\pi)/P \), and \( t_p \) being an epoch of periastron.

For small eccentricities, an expansion in the lowest order of the eccentricity \( e \) is a fair approximation, which reads

\[
x(t) = a \rho(t) \cos \xi(t),
\]

\[
y(t) = a \rho(t) \sin \xi(t), \tag{15}
\]

with

\[
\rho(t) = 1 - e \cos[\Omega(t - t_p)] \tag{16}
\]

and

\[
\xi(t) = \Omega(t - t_p) + 2e \sin [\Omega(t - t_p)]. \tag{17}
\]

### 2.3 Earth–Sun parallax

For the Earth, \( m_\oplus \ll m_\ast \), and with the Sun at rest, \( r_p \approx r(t) \). Moreover, the orbital major semi-axis is given by \( a = 1 \) au, the orbital frequency is given by \( \Omega = (2\pi)/(1 \text{ yr}) \), and the eccentricity \( e = 0.017 \) is small. The effect of the annual parallax, i.e., the revolution of the Earth around the Sun has been discussed in some detail by Dominik (1998), and our subsequent discussion is based on the results derived in that context.

The (parallactic) shift of the position of the observer \( (\delta r)_0 \) perpendicular to the line-of-sight is equivalent to a virtual displacement of the observed source star \( (\delta r)_s \). For the corresponding shift in the dimensionless angular coordinate \( (\delta \alpha)_s = (\delta r)_s / (D_S \theta_E) \), one finds

\[
(\delta \alpha)_s = \frac{D_R - D_L}{D_R D_L} \frac{(\delta r)_0}{\theta_E} = \left( \frac{1}{D_R} - \frac{1}{D_L} \right) \frac{(\delta r)_0}{\theta_E}, \tag{18}
\]

With the relative lens–source parallax

\[
\pi_{LS} = 1 \text{ au} \left( \frac{1}{D_R} - \frac{1}{D_L} \right), \tag{19}
\]

it is customary to define a microlensing parallax \( \pi_E = \pi_{LS} / \theta_E \).

A standard coordinate system based on the Earth’s orbital plane is given by the ecliptical coordinates \( (\beta, \lambda) \), where positive \( \beta \) point towards ecliptical north, while increasing \( \lambda \) follow the apparent motion of the Sun, which is in the same sense as the actual motion of the Earth. Moreover, \( \lambda = 0 \) is defined as the position of the Sun at vernal equinox. In fact, the ecliptical latitude \( \beta \) equals the inclination of the Earth’s orbit with respect to the line-of-sight to a source star at ecliptical coordinates \( (\beta, \lambda) \).

In order to determine the effect of the orbital motion of the Earth, one needs to find its components perpendicular to the line-of-sight. While \( (x, y) \) span the orbital plane, where the positive \( x \)-axis points towards perihelion, the complementary \( z \)-coordinate points towards ecliptical north, thereby forming a right-handed three-dimensional system. We can also define a coordinate system with \( (\tilde{u}_1, \tilde{u}_2) \) in the plane perpendicular to the line-of-sight and \( \tilde{u}_3 \) pointing towards the observer, so that again a right-handed system is formed. Rather than using \( \lambda \) as the longitude, it is more straightforward to choose an angle \( \varphi \), where \( \varphi = 0 \) corresponds to the Earth being at the perihelion. In fact, \( \varphi = \lambda + \pi + \varphi_p \), where \( \varphi_p \) is the longitude of the vernal equinox as measured from the perihelion. While we can define \( \tilde{u}_1 \propto x \), \( \tilde{u}_2 \propto y \) and \( \tilde{u}_3 \propto -\tilde{z} \) for \( \beta = 0 \) and \( \varphi = 0 \), the coordinates for arbitrary \( (\beta, \varphi) \) arise from a rotation around the \( \tilde{u}_1 \)-axis by the angle \( \varphi \), and a subsequent rotation around the \( \tilde{u}_2 \)-axis by the angle \( -\beta \), so that

\[
\left( \begin{array}{c} 
\tilde{u}_1(t) \\
\tilde{u}_2(t) \\
\tilde{u}_3(t)
\end{array} \right) = \frac{\pi_E}{1 \text{ au}} \mathcal{R}(\beta, \varphi) \left( \begin{array}{c} x(t) \\
y(t) \\
\varphi(t)
\end{array} \right), \tag{20}
\]

where

\[
\mathcal{R}(\beta, \varphi) = \left( \begin{array}{cc}
-\sin \beta & \cos \beta \sin \varphi \\
-\sin \varphi & \cos \varphi \\
-\cos \beta & \sin \beta \sin \varphi
\end{array} \right). \tag{21}
\]

Since, in the small-eccentricity limit, equation (15), a rotation by \( \varphi \) is equivalent to a shift in \( \xi(t) \), one finds in general

\[
\tilde{u}_1(t) = -\pi_E \rho(t) \sin \beta \cos \xi(t),
\]

\[
\tilde{u}_2(t) = \pi_E \rho(t) \sin \xi(t), \tag{22}
\]

where \( \xi(t) = \xi(t) - \varphi \), and \( \xi(t) \) being defined by equation (17).

If we adopt the orientation angle \( \psi \) of the source trajectory, given by \( \psi_{LS} \), equation (3), as referring to the \( (\tilde{u}_1, \tilde{u}_2) \) coordinate axes,
we find for the total motion $\mathbf{u}(t) = \mathbf{u}_{LS}(t) + \mathbf{u}(t)$, so that its absolute square reads

$$[u(t)]^2 = u_0^2 + a_0^2 (t - t_0)^2 + 2 a_0 \rho(t) \sin \psi \cos \theta \sin (\chi + \varpi) [u_0 \cos \psi + a_0 \rho (t - t_0) \sin \psi] + \sin \beta \cos \Omega \sin \theta [u_0 \sin \psi - a_0 \rho (t - t_0) \cos \psi] + \pi_0^2 [\rho(t)]^2 [\sin^2 \chi(t) + \sin^2 \beta \cos^2 \Omega(t)].$$

(23)

With the Earth’s orbit defined with respect to the source at ecliptical coordinates $(\beta, \lambda)$, only the microlensing parallax $\pi_E > 0$, determining the strength of the parallax effect, and the angle $\psi$, defining the orientation of the source trajectory, are free parameters along with $(u_0, t_0, \rho_0, \varpi_0)$ that define the magnification corresponding to the ordinary light curve including finite-source effects. Since, with this parametrization, $u_0$ refers to the minimal impact of the heliocentric trajectory, $t_0$ no longer coincides with the maximum magnification.

### 2.4 Stellar reflex motion due to orbiting planet

Given that a planet and its host star orbit their common barycentre as described by equation (11), and that the motion of the observer and the source star are equivalent, it is obvious that the periodic displacement of the observed source star due to the orbiting planet and the annual parallax due to the revolution of the Earth around the Sun take exactly the same form.

In analogy to the discussion of the previous subsection, let us define a parameter

$$x_E = \frac{m_e m}{m m_{\odot} D_0 \theta_E} > 0,$$

(24)

which takes over the role held by $\pi_0$ in measuring the strength of the effect on the microlensing light curve. Let $i$ denote the inclination of the orbit with respect to a plane perpendicular to the line-of-sight, and let $\varphi$ denote a longitude in the orbital plane that decreases with the motion, where $\varphi = 0$ at periastron. The position of the source star is then given by the orbital-plane coordinates $(i, \varphi)$, and similar to before, one finds with $(\delta u_0, \delta \omega_0) = (\delta r_0)/(D_0 \theta_E)$,

$$\left(\begin{array}{c}
\delta u_1(t) \\
\delta u_2(t)
\end{array}\right) = -x_E \mathcal{R}(i, \varphi) \cdot \left(\begin{array}{c}
\sigma_1(x(t)) \\
\sigma_2(y(t))
\end{array}\right),$$

(25)

with $\mathcal{R}$ given by equation (21), where the additional sign results from considering the motion of the source star rather than its planet.

More explicitly, for small eccentricity $e$, one obtains with equation (15)

$$\mathbf{u}(t) = \mathbf{u}_{LS}(t) + \mathbf{u}(t),$$

(26)

with $(\xi(t) = \xi(t) - \varphi$ and $(\xi(t)$ being defined by equation (17). Let us choose the coordinate axes of the motion of the barycentre $\mathbf{u}_{LS}(t)$, as given by equation (3), as those of the $(\delta u_1, \delta u_2)$ coordinate system. This yields a total motion $\mathbf{u}(t) = \mathbf{u}_{LS}(t) + \mathbf{u}(t)$, whose absolute square is given by

$$[u(t)]^2 = u_0^2 + a_0^2 (t - t_0)^2 - 2 \chi_E \rho(t) \sin \psi \cos \theta \sin (\chi + \varpi) [u_0 \cos \psi + a_0 \rho (t - t_0) \sin \psi] + \sin \beta \cos \Omega \sin \theta [u_0 \sin \psi - a_0 \rho (t - t_0) \cos \psi] + \chi_E^2 [\rho(t)]^2 [\sin^2 \chi(t) + \sin^2 \beta \cos^2 \Omega(t)].$$

(27)

One indeed realizes that equations (23) and (27) have an identical form, where only $\pi_E \leftrightarrow -\chi_E$ and $\beta \leftrightarrow \varpi$. Correspondingly, $u_0$ refers to the minimal impact of the barycentre, so that again the epoch $t_0$ does in general not mark a magnification maximum.

However, in contrast to the effect of annual parallax, in addition to the strength parameter $\chi$ and the direction of source (centre-of-mass) motion, characterized by the angle $\psi$, there are a further five free parameters, namely the orbital eccentricity $e$, the orbital frequency $\Omega$, the orbital inclination $i$, the orbital longitude $\varphi$, and the time of periastron $t_\varpi$. In principle, the detection of reflex motion of the source star can be confused with the motion of the Earth around the Sun (the sign between $\pi_E$ and $\chi$ can simply be accounted for by $\psi \leftrightarrow \varphi + \pi$), and we should take extreme care in avoiding to redetect the habitable planet that we ourselves live on. It therefore needs to be ensured that at least one of the parameters determining the orbital motion is found to be incompatible with annual parallax.

For circular orbits, i.e. $e = 0$, the time of periastron $t_\varpi$ becomes arbitrary, and we can choose $t_\varpi$ as reference epoch instead, so that along with $\rho(t) \equiv 1$, one can define $\zeta(t) = \Omega(t - t_0) - \varphi$. In this case, besides $(u_0, h_0, \rho_0, \varpi)$, which describe the ordinary symmetric finite-source light curve in the absence of planets, the total number of free parameters reduces to five, namely $(\chi, \psi, \Omega, i, \varpi)$.

Another special case arises if the orbital plane is perpendicular to the line-of-sight, i.e. $i = \pm(\pi/2)$. If this happens, modifying the angles $\varphi$ or $\psi$ by the same amount has an identical effect, so that only one of them can be included in a set of independent free parameters, whereas the other is obsolete.

### 3 PROSPECTS FOR PLANET DETECTION

#### 3.1 Strength of effect and favoured scenarios

For a rough assessment of the amplitude of perturbations caused by the orbital source motion, and for an identification of the favoured scenarios, as well as on dependencies on system parameters, let us first have a look at the strength parameter $\chi_E = (m_p/M) a/(D_0 \theta_E)$, introduced in equation (24). At the source distance $D_s$, the angular Einstein radius $\theta_E$ corresponds to a physical size

$$D_s \theta_E = \sqrt{\frac{4GM}{c^2}} \left(D_s - D_h\right) \frac{D_h}{D_s} = 2.8 \text{ au} \left(\frac{M}{1 M_\odot}\right)^{1/2} \left(\frac{D_s-D_h}{1 \text{kpc}}\right)^{1/2} \left(\frac{D_h}{D_s}\right)^{-1/2}.$$  

(28)

In accordance to what was found by Hamadache et al. (2006) and Sumi et al. (2006), the strongest effects are therefore expected for small $D_s - D_h$, i.e. the lens stars being close to the source stars, in contrast to deviations by annual parallax, which are the strongest for lens stars close to the observer. This means that detections on stars in the Galactic bulge are dominated by lensing events caused by bulge stars rather than disc stars. A small $D_s - D_h$ also implies $D_h/D_s \sim 1$, so that $D_h \theta_E$ practically depends on the distance difference $D_s - D_h$ only. Kepler’s third law, equation (12), allows us to eliminate the major semi-axis $a$ in favour of the orbital period $P$, so that

$$\chi_E = 6.4 \times 10^{-4} \left(\frac{m_p}{M_p}\right)^{-2/3} \left(\frac{m}{M_\odot}\right)^{1/2} \left(\frac{M_\odot}{0.3 M_\odot}\right)^{-1/2} \times \left(\frac{P}{1 \text{yr}}\right)^{2/3} \left(\frac{D_s-D_h}{1 \text{kpc}}\right)^{-1/2} \left(\frac{D_h}{D_s}\right)^{1/2}.$$  

(29)

Please note that $M$ is the mass of the lens star, whereas $m = m_\star + m_p$ is the total mass of the source system. While microlensing events prefer $M \sim 0.3 M_\odot$, solar-mass source stars are much brighter, so that those are the more reasonable target despite the fact that the mass ratio $m_p/m$ for a given planet mass is smaller than for
low-mass stars. Moreover, solar-mass stars are far more likely to host gas-giant planets than low-mass stars.

For the event time-scale \( t_E \), one finds

\[
    t_E = 17 \text{ d} \left( \frac{M}{0.3 M_\odot} \right)^{-1/2} \left( \frac{D_s - D_L}{1 \text{ kpc}} \right)^{1/2} \left( \frac{D_L}{D_s} \right)^{1/2} \times \left( \frac{D_L \mu}{160 \text{ km s}^{-1}} \right)^{-1},
\]

while for Galactic bulge–bulge lensing, \( D_L \mu \sim 160 \text{ km s}^{-1} \) and \( D_s - D_L \sim 1 \text{ kpc} \). Therefore, a characteristic value for the favourite scenario is given by \( t_E \sim 17 \text{ d} \).

### 3.2 Planetary signal

Since displacements of the source star cause larger changes to its magnification the smaller the source–lens separation (and therefore, the larger the magnification), the effects of the orbiting planet on the light curve increase with the source magnification, and the planet needs to be identified while the latter is substantial. Therefore, the characteristics of the planetary signal depend on whether the orbital period \( P \) is smaller or larger than the event time-scale \( t_E \), as illustrated in Figs 1 and 2. As by the definition of the parameters, \( u_0 \) does not refer to the closest approach between lens and source star, so that the peak magnification differs from \( A_0 \approx u_0 \) = 125, where the discrepancy is larger for the long-period case with the larger \( \chi_E \).

Given that we only altered the orbital period between the two cases shown, while leaving all other physical properties unchanged, the signal strength parameter \( \chi_E \) increases with the orbital period \( P \), according to equations (24) and (29). For the case \( P \ll t_E \), the periodic pull of the planet on its host star leads to detectable ripples on the observed light curve, while the long orbital period \( P \) deprives us of such a characteristic signature for \( P \gg t_E \). Without a good indication of \( P \), a distinction with the effect of annual parallax becomes difficult, and to the lowest order, one only observes an acceleration effect (Smith, Mao & Paczyński 2003). However, regardless of \( P/t_E \), a best-fitting ordinary light curve leaves us with a mismatch near the tip of the light curve that is not overcome by adopting a different finite-source parameter \( \rho_* \), and allows a detection if the impact parameter \( u_0 \) is sufficiently small for such a signal to be prominent enough.

### 3.3 Parameters of simulation

After having identified the basic scenarios, we carried out Monte Carlo simulations in order to study the detectability of planets orbiting Galactic bulge stars as a function of various parameters that describe the lens star, the source star and its orbiting planet. For simplicity, we have assumed circular orbits (\( e = 0 \)), so that the source magnification \( A[u(t) ; \rho_*] \) is described by the nine parameters \( (u_0, t_0, \omega, \mu, \chi_E, \psi, \Omega, i, \varphi) \).

Without loss of generality, we set \( t_0 = 0 \). Moreover, we consider planets of mass \( m_p = 1 \) or \( 10 M_{\text{jup}} \) orbiting a source star of \( m_s = 1 M_\odot \). With these choices, according to equation (29), the strength parameter \( \chi_E \) then becomes a function of the lens and source distances \( D_L \) and \( D_s \), for \( D_s - D_L \ll D_L \) essentially of \( D_s - D_L \), the orbital frequency \( \Omega = (2\pi)/P \), where \( P \) denotes the orbital period,

![Figure 1](https://academic.oup.com/mnras/article-abstract/392/3/1193/1063793)

**Figure 1.** A model light curve for which the orbital period \( P \) is smaller than the event time-scale \( t_E \). The adopted parameters are compatible with a planet of mass \( m_p = 10 M_{\text{jup}} \) orbiting a star of solar mass and solar radius at distance \( D_s = 8.5 \text{ kpc} \), whereas the lens star of mass \( M = 0.4 M_\odot \) is located at \( D_L = 8.0 \text{ kpc} \) from the observer. The upper panel shows the resulting light curve (solid) as well as a best-fitting approximation (dashed), while the lower panel displays their difference. Dotted lines refer to deviations by 2 or 0.3 per cent, respectively.

![Figure 2](https://academic.oup.com/mnras/article-abstract/392/3/1193/1063793)

**Figure 2.** A model light curve for which the orbital period \( P \) is larger than the event time-scale \( t_E \). The adopted parameters are compatible with the same scenario as chosen for Fig. 1. As before, the upper panel shows the resulting light curve (solid) together with a best-fitting ordinary light curve (dashed), while the lower panel displays their difference. Dotted lines refer to deviations by 2 or 0.3 per cent, respectively.
and the lens mass $M$. In relation to the sampling interval and the event time-scales, we generate a uniform distribution in $\log P / (1 \text{ d})$ ranging between 3 d and 1 yr. While we adopt a ‘natural’ uniform distribution of impact parameters $u_0 \in [0, 1]$, not taking into account any selection bias by the experiment, the phase angle $\varphi$, orientation angle $\psi$ and inclination angle $i$ are all naturally uniformly distributed, where $\varphi, \psi \in (0, 2\pi)$ and $i \in (0, \pi/2)$.

The event time-scale $t_E$ follows from drawing a lens distance $D_L$, source distance $D_S$, velocity $v$ and lens mass $M$ from the adopted distributions for the Galactic bulge described in Appendix A. Moreover, with assuming a source radius $R_\star = 1 R_\odot$, one obtains the source size parameter $\rho_s = [(R_\star / (v t_E))(D_L / D_S)]$.

In contrast to the discussion by Rahvar et al. (2003) of the observability of parallax effects towards the Large Magellanic Cloud, which used parameters specific to the EROS (Experience de la Recherche d’Objets Sombres) campaign, we adopt the simple pragmatic approach of assuming a constant photometric uncertainty for equally spaced observations over the course of the microlensing event without any loss due to bad weather. In particular, we choose (a) a sampling interval of 2 h with 2 per cent accuracy, resembling current follow-up observations, or (b) a sampling interval of 15 min with 0.3 per cent accuracy, resembling upcoming campaigns. More precisely, we demand the fractional uncertainty of the magnification to match the quoted value, or equivalently, the flux after subtraction of the background to be measured that precisely. In fact, observing campaigns need to account for such a requirement, or time is being wasted on taking data on strongly blended targets without a chance to extract meaningful results. The simple choices allow us to focus on the primary dependencies without being bound to the variety of different existing or possible set-ups, which all show different characteristics with respect to the crowding of targets, the distribution of event impact parameters for main-sequence stars, and the actually achieved photometry, where all these effects are correlated with each other.

### 3.4 Detectability in simulated events

For each of the created synthetic light curves, we obtain best-fitting model parameters by means of $\chi^2$-minimization, which corresponds to a maximum-likelihood estimate for Gaussian errors bars, for an ordinary light curve $(u_0, t_E, \omega_0, \pi_E)$ and independently the eight parameters $(u_0, t_E, \omega_0, \chi_E, \psi, \Omega, i, \varphi)$ that include the description of the motion of the source induced by a planet in a circular orbit. In both cases, we also determine effective best-fitting source and background fluxes $F_S$ and $F_B$, taking into account a potential difference between the best-fitting magnification and the true magnification arising from the simulation. We explicitly adopt a finite angular radius $\theta_E$ of the source star, but do not refit for the respective parameter $\rho_s = \theta_E / \theta_E$ in order to save computing time and to avoid parameter degeneracies if finite-source effects are not prominent.

A quantitative measure for the detectability of a planetary signal then results from a likelihood-ratio test, involving the respective $\chi^2$ minima, namely $(\chi^2_{\text{min}})^0$ and $(\chi^2_{\text{min}})^\text{planet}$. In fact, with $\Delta \chi^2_p = (\chi^2_{\text{min}})^0 - (\chi^2_{\text{min}})^\text{planet}$ following a $\chi^2$-distribution with 5 degrees of freedom, one finds a probability $P(\Delta \chi^2_p \geq 11.07) = 0.05$ for such a difference to arise. At this significance level, we therefore reject the hypothesis that an ordinary light curve explains the data whenever $\Delta \chi^2_p \geq 11.07$, and claim the detection of a signal.

A substantial fraction of such ‘detections’ however do not involve characteristic features, so that instead of the assumed effect of periodic source motion revealing the presence of an orbiting planet, these could be of different origin if found in an observed event. In order to account for false positives, we therefore adopt a second criterion. In analogy to the primary criterion, we determine the five parameters $(u_0, t_E, \omega_0, \pi_E, \psi)$ that include the annual parallax due to the Earth’s revolution, and carry out a similar likelihood-ratio test based on the criterion $\Delta \chi^2_0 = (\chi^2_{\text{min}})^0 - (\chi^2_{\text{min}})^\text{Earth} \geq 5.99$, where the adopted value results from $P(\Delta \chi^2_0 \geq 5.99) = 0.05$ for a $\chi^2$-distribution with 2 degrees of freedom. We then consider the planetary signal to be characteristic if the detection of a signal is significant ($\Delta \chi^2_p \geq 11.07$), while a signature of annual parallax is not ($\Delta \chi^2_0 < 5.99$); and the signal to be featureless otherwise. With this procedure, we do reject events with deviations that are likely to be caused by the motion of the Earth, but in an elegant way, we also get rid of all uncharacteristic deviations that are compatible, which could e.g. be due to differences in the optimal finite-source parameter or weak effects resulting from binarity of the lens or source object. At the end, we are left only with cases for which either the observed deviation is characteristic for the periodic motion of the source star, or where an asymmetric feature is closely mimicked by effects other than the annual parallax.

The relative abundances of planet detections amongst all simulated events as a function of various parameters, namely the event impact parameter $u_0$, the orbital period $P = (2\pi \Omega)/(\omega_E)$, the event time-scale $t_E = \omega_0^{-1}$, the lens–source distance $D_S - D_L$, the orbital inclination $i$ and the resulting strength parameter $\chi_E$ are shown in Figs 3 and 4 corresponding to planets of mass $m_p = 10$ or $1 M_{\text{Jup}}$, respectively, orbiting solar-mass Galactic bulge stars, and the two adopted observational capabilities with regard to the sampling frequency and achievable photometric accuracy.

For $m_p = 10 M_{\text{Jup}}$ and the less favourable 2 per cent accuracy and 2 h sampling, about 1/4 of the significant deviations are characteristic for a detection of orbital stellar motion against false positives, providing a detection efficiency of $\sim 2.5$ per cent over the adopted sample. Small impact parameters strongly support the detection, with efficiencies reaching $\sim 24$ per cent for characteristic signals arising amongst the smaller number of events with $u_0 \lesssim 0.02 (A_0 \gtrsim 50)$. While nearly all detected signals are uncharacteristic for $u_0 \gtrsim 0.1$, characteristic and uncharacteristic detections are of similar frequency for smaller $u_0$. The detection efficiency is found to increase towards larger event time-scale $t_E$ and orbital period $P$, as well as to favour smaller $D_S - D_L$ and face-on orbits $i \sim 90^\circ$ over edge-on orbits $i \sim 0^\circ$.

With a close-to-continuous sampling at 15 min intervals and 0.3 per cent photometric accuracy, planets of $10 M_{\text{Jup}}$ become hard to miss, with more than 85 per cent of them showing detectable signals, and a small $u_0$ no longer being in strong favour of a detection. As compared to the less favourable observational capabilities, more than half of the detected deviations contain characteristic features. This is a substantial improvement, pushing the detectability of planets by a factor of $\sim 20$. The much larger gain on the detectability with improved capabilities for shorter periods accounts for a decrease of the characteristic detections towards larger orbital periods (whereas the fraction of uncharacteristic detections increases). In particular, for all the shorter orbital periods $P \lesssim 10$ d, the planetary signal is characteristic, and almost 60 per cent of the planets are found. One might be puzzled by the fact that the largest encountered strength parameters $\chi_E$, the prospects for a characteristic detection decrease. While it seems a bit surprising, this behaviour results from $\chi_E$ being correlated with the orbital period $P$, and larger orbital periods mean that the light curves contain less characteristic features.

For the prospects of detecting a Jupiter-mass planet, the results of our simulations reveal mostly the same trends with the parameters
Figure 3. Efficiency of detecting a planet of mass $m_p = 10 M_{\text{Jup}}$ by means of the effect of the orbital motion of its observed host star in the Galactic bulge with $m_* = 1 M_\odot$ on the light curve arising from its light being bent by an intervening foreground Galactic bulge lens star as a function of the strength parameter $\chi_E$, defined by equation (24), the event impact parameter $u_0 \in [0, 1]$, the event time-scale $t_E$, the orbital period $P$, the lens–source distance $D_{S-L}$ and the orbital inclination $i$. For the left-hand panel, we assume observations with 2 per cent photometric accuracy regularly spaced at 2-h intervals, while the results shown in the right-hand panel correspond to increased observational capabilities of 0.3 per cent photometric accuracy and a sampling interval of 15 min. The hatched area refers to cases where the signal is characteristic enough to be distinguished from false positives.

Figure 4. Detection efficiency for a Jupiter-mass planet ($m_p = 1 M_{\text{Jup}}$) from the influence of source-star orbital motion on the microlensing light curve for two different observational capabilities, similar to Fig. 3, but considering only events with smaller impact parameters $u_0 \in [0, 0.1]$. 
as for $10 \, M_{\text{jup}}$. While for such less massive planets, one is extremely unlikely to succeed unless $u_0 \lesssim 0.03$ with a strategy of 2 h sampling at 2 per cent photometric accuracy, and a substantial rise of the detection efficiency towards smaller $u_0$ even for the more favourable observational set-up, very small $u_0 \lesssim 0.0003$ are not optimal either, given that finite-source effects wash out signals in that regime. On average, characteristic detections happen for $\sim 0.9$ per cent of all events with $u_0 < 0.1$, but for $\sim 12$ per cent of events with $u_0 \leq 0.005$. With the adopted better observational capabilities, these values rise to $\sim 30$ or $\sim 70$ per cent, respectively.

Given that the same value of the strength parameter $\chi_E$ for a larger planet mass $m_p$ implies a smaller orbital period $P$, which eases the detection of a planetary signal, the detection efficiency for more massive planets at same $\chi_E$ should be larger. However, the smaller detection efficiency for some of the ranges shown in the figures is a result of averaging over events with $u_0 \in [0, 0.1]$ for $m_p = 1 \, M_{\text{jup}}$, whereas an average over $u_0 \in [0, 1]$ has been taken for $m_p = 10 \, M_{\text{jup}}$.

Rather than the planet detection efficiencies for certain parameter ranges, Figs 5 and 6 show the distribution of the detections with these parameters, i.e. in contrast to before, the distribution of the respective parameter ranges among the simulated events has been considered (which makes no difference for uniformly distributed quantities, which have been omitted). In all four considered cases (two planet masses and two sets of observational capabilities), the fact that microlensing events preferentially arise with time-scales $5 \leq t_E \leq 80$ d leaves such small $t_E$ with the largest number of planet detections, despite the fact the detection efficiency increases with $t_E$. With 2 h sampling and 2 per cent accuracy, there are different preferences for the impact parameter $u_0$, depending on the mass

![Distribution of detections](#)

**Figure 5.** Distribution with the signal strength parameter $\chi_E$, the event impact parameter $u_0 \in [0, 1]$, the event time-scale $t_E$ or the lens–source distance $D_S - D_L$ of the detection of a planet of mass $m_p = 10 \, M_{\text{jup}}$ orbiting a Galactic bulge star with $m_\star = 1 \, M_{\odot}$ from the change in the microlensing light curve resulting from the periodic displacement of the observed star caused by the gravitational pull of the orbiting planet. As for Fig. 3, the two panels refer to different observational capabilities, namely 2 h sampling with 2 per cent accuracy or 15 min sampling with 0.3 per cent accuracy, and the hatched area marks the characteristic detections. The total area of the bins gives the average detection efficiency. For the uniformly distributed quantities $\text{lg}[P/(1 \, \text{yr})]$ and $i$, the distribution of the detections is proportional to the detection efficiencies plotted in Fig. 3.

![Distribution of detections](#)

**Figure 6.** Distribution of the planet detections with several parameters, similar to Fig. 5, but now for a planet mass of $m_p = 1 \, M_{\text{jup}}$ and the impact parameter in the range $u_0 \in [0, 0.1]$. 

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of the planet. While for \(m_p = 10 M_{\text{Jup}}\), the bulk of detections is expected from 0.01 \(\lesssim u_0 \lesssim 0.3\), the difficulty of revealing a signal for larger \(u_0\) in the case of for \(m_p = 1 M_{\text{Jup}}\) moves the preferred range to 0.0003 \(\lesssim u_0 \lesssim 0.03\). For both planet masses, the better observational capabilities of 15 min sampling and 0.3 per cent photometric accuracy lead to the larger number of events with larger \(u_0\) providing the larger number of detections.

### 3.5 The challenge of detecting extragalactic planets

No other technique but gravitational microlensing has so far been suggested to achieve the detection of planets orbiting stars in neighboring galaxies, such as M31, within foreseeable time (Covone et al. 2000; Chung et al. 2006). Given that the signal strength \(\chi_E\) related to the periodic reflex motion of the source star due to an orbiting planet lacks of practical dependence on the distance \(D_S\), one might hope that this planet detection channel could also be a viable alternative for extragalactic planets.

As compared to the Galactic bulge, there are however some severe difficulties that make any such attempts a quite demanding challenge. Most importantly, one faces the problem that the target stars are not resolved, so that the effectively achieved photometric accuracy is quite limited. Unless major technology leaps are made, one is therefore restricted to very strong signals, which are unlikely to occur. Catching main-sequence source stars rather than giants will only be possible during phases of extreme magnification, limiting the number of suitable events further. Beyond that, detections on M31 targets are disfavoured due to the larger typical \(D_B - D_L \sim 10\) kpc as compared to the Galactic bulge, where \(D_B - D_L \sim 1\) kpc.

### 3.6 Does the planet orbit the source star?

So far, we have assumed by construction that the periodic alteration of the relative position between lens and source star as seen from the Earth is the result of a planet orbiting the observed source star, and only the revolution of the Earth itself has been considered as an alternative. However, as explicitly pointed out to us by Bozza (private communication), a similar effect might also arise from a planet orbiting the foreground lens star rather than the observed background source star.

We are quite familiar with planets orbiting the foreground lens star revealing their presence due to them affecting the gravitational bending of light received from the source star (Mao & Paczyński 1991). Notably, the arising characteristic signal represents a snapshot of the planet at its current angular separation from its host star, and does not depend on its orbital period. Moreover, the chances of detecting a planetary signal rely on a resonance of the angular Einstein radius \(\theta_E\) with the angular separation of the planet \(\theta_p = d \theta_E\), whereas the planet is likely to escape detection for either \(d \ll 1\) or \(d \gg 1\).

It is obvious that similar to a source star of mass \(M\), a lens star of mass \(M\) also exhibits a periodic shift due to a planet orbiting at a semimajor axis \(a\), whose strength in full analogy to the discussion of Section 2.4 is given by

\[
\lambda_E = \frac{m_p}{M_{\text{Jup}}} \frac{a}{D_B \theta_E}. \tag{31}
\]

Inserting characteristic values yields

\[
\lambda_E = 1.4 \times 10^{-3} \frac{m_p}{M_{\text{Jup}}} \left( \frac{M}{0.5 M_{\odot}} \right)^{-7/6} \times \left( \frac{P}{1 \text{ yr}} \right)^{2/3} \left( \frac{D_S - D_L}{1 \text{ kpc}} \right)^{-1/2} \left( \frac{D_B}{D_S} \right)^{-1/2}, \tag{32}
\]

and by comparing this expression with the corresponding equation (29), one sees that the detection as compared to planets orbiting the source star is not only facilitated by the smaller typical stellar mass, but lens distances not only close to the source star, but also close to the observer provide favourable configurations.

Assuming circular orbits, a typical orbital radius is given by \(a \sim 1.36 d D_L \theta_E\) (Dominik 2006), so that for \(d = 1\), characteristic orbital periods are of the order of

\[
P_0 = 5.6 \text{ yr} \left( \frac{D_B - D_S}{1 \text{ kpc}} \right)^{3/4} \left( \frac{D_L}{D_S} \right)^{3/4} \left( \frac{M}{0.3 M_{\odot}} \right)^{1/4}, \tag{33}
\]

which usually exceed those for which we expect to detect signatures of the period reflex motion. Given that \(P_0 \propto d^{1/2}\), we find in particular that orbital periods \(P_0 \lesssim 50\) d roughly correspond to separation parameters \(d \lesssim 0.08\), so that substantial deviations due to gravitational bending of light are not likely to occur. In fact, such can always be checked for by means of constructing model light curves that take this effect into account.

However, as a consequence, we are facing the situation that even in the case that one identifies a characteristic signal that cannot arise from the revolution of the Earth, it remains possible that one ends up with two competing interpretations putting the planet in orbit around the source star or the lens star, respectively.

### 3.7 Properties of the planet and its orbit

For the ‘usual’ channel of planet detection by microlensing, the properties of the planet and its orbit affect the light curve only by means of two dimensionless parameters, which can be taken as the planet-to-star mass ratio \(q\) and the separation parameter \(d\), where \(d \theta_E\) is the instantaneous angular separation of the planet from its host star. With the technique being most sensitive to planets around \(d \sim 1\), and planetary signals lasting between hours and days, these are substantially smaller than the orbital period \(P\), and therefore the only information about the planetary orbit is provided by the separation parameter \(d\), whereas neither the orbital eccentricity \(e\) nor the orbital inclination \(i\) can be inferred from the snapshot.

The mass of the planet \(M_p = q M\) is related to the mass of the lens star \(M\), which does not follow directly from the light curve, but in general needs to be inferred probabilistically from the event time-scale \(t_e = \theta_E/\mu\), with the angular Einstein radius, given by equation (1), being a function of the lens mass as well as of the lens and source distances \(D_L\) and \(D_S\). As discussed in detail by Dominik (2006), this requires the adoption of a kinematic model of the Galaxy as well as of mass functions of the underlying stellar populations that make up the lens stars. Light curves that involve planets with small masses, however, are likely to be influenced by the finite angular size \(\theta_E\) of the source star. With the possibility to determine \(\theta_E\) from stellar typing based on its magnitude and colour, and better with a spectrum, the extractable time-scale \(t_e = \theta_E/\mu\), in which the source moves by its own angular radius with respect to the lens, allows to infer the proper motion \(\mu\) and thereby with the event time-scale \(t_e = \theta_E/\mu\) of the angular Einstein radius \(\theta_E\). With a reliable estimate for the source distance \(D_S\), the mass of the lens star \(M\) thereby only becomes a function of the lens distance \(D_L\), reducing the uncertainty substantially. However, the mass of the lens star \(M\), and thereby the mass of the planet \(M_p = q M\) only follows from the observed light curve, if moreover the microlensing parallax \(\pi_L = \pi_{LE}/\theta_E\) can be determined. As for the lens mass \(M\), only a probability density can in general be obtained for the instantaneous physical projected separation \(d = D_L \theta_E\), while a stochastic distribution for the orbital
semimajor axis $a$ further follows with an orbital deprojection and assumption of a distribution of orbital eccentricities $e$. Similarly, probabilistic estimates for the orbital period $P = 2\pi[a^3/(GM)]^{1/2}$ can be derived.

In contrast to the just two additional parameters ($d, q$) as compared to an ordinary microlensing light curve, our alternative channel of detecting planets from the orbital motion of observed microlensing source stars around the common barycentre involves seven parameters ($x_E, \psi, \Omega, i, \varphi, \varepsilon, t_p$). For studies of the planet population, the values of the inclination $i$, the phase angle $\varphi$ and the time of periastron $t_p$ are of little use. Moreover, while for the annual parallax, the direction angle of the source trajectory $\psi$ with respect to the ecliptic plane, thereby well defined in space, carries useful information, the direction angle $\psi$ with respect to the orbital plane of the planet does not. In sharp contrast, a direct determination of the orbital period $P = (2\pi)/\Omega$ and the orbital eccentricity $e$ are valuable.

Since spectral typing not only provides an estimate of the angular source size $\theta_1$, but also of the stellar mass $m_\star$, the orbital semimajor axis $a = Gm_\star P^2/(4\pi^2)$ is determined along with the orbital period $P$, given that $m \approx m_\star$. The mass of the planet therefore follows as $m_p = (x_E/\theta_E)/a$ $m_\star$, where the main indeterminacy results from the unknown $\theta_E$ with in general only $t_E = \theta_E/\mu$ being extractable. However, with finite-source effects being observed, $\theta_E$ is measured, and thereby the planet mass $m_p$ will result. Given the fact that we do not require any knowledge about the lens distance $D_L$, a measurement of the microlens parallax $\pi_E = \pi_{LS}/\theta_E$ is not helpful in this case. Otherwise, as for the ‘standard’ channel, one only finds a probability density for $m_p$, but in contrast to that case, one obtains measurements for the orbital semimajor axis $a$, the orbital period $P$ and the orbital eccentricity $e$.

While the parameters ($x_E, \psi, \Omega, i, \varphi, \varepsilon, t_p$) are extractable in principle, the power for determining the properties of the planet and its orbit is limited by severe degeneracies that occur in several (not unlikely) cases. For example, a proper measurement of the orbital period $P$ is only possible for $P \lesssim 3t_E$, whereas for wider orbits, the effect on the microlensing light curve is mainly described by the acceleration of the source trajectory in the vicinity of the epoch at which the ordinary, unperturbed light curve reaches its peak (Smith et al. 2003). Moreover, for $t_E \lesssim P \lesssim 3t_E$, the orbital period $P$ appears to be strongly degenerate with the orbital inclination $i$ and cannot be properly disentangled.

Finally, with a large variety of effects causing small deviations, one needs to take care to investigate all possible alternatives such as lens or source binarity, and in particular the annual Earth–Sun parallax. As already pointed out in Section 2.4, the latter causes an identical signature, with the only difference that the parameter space is restricted to the fixed values that define the Earth’s orbit.

4 SUMMARY AND FINAL CONCLUSIONS

We find that an alternative channel for the detection of extrasolar planets by microlensing is provided by the orbital motion of the source star around the common barycentre with an unseen planetary companion, as opposed to the standard channel where a planet orbiting the lens stars alters the bending angle and thereby the observed magnification. We derived a formalism in exact analogy to the treatment of the annual parallax that results from the revolution of the Earth around the Sun, mainly following Dominik (1998), which however involves seven additional free parameters as compared to an ordinary microlensing light curve, which reduce to five for circular orbits, whereas there are just two for the annual parallax due to the known properties of the Earth’s orbit.

Constituting an exchange of the roles of source star and observer as compared to parallax effects, which are most prominent for lens stars close to the observer, the strongest signals arise for lens stars close to the source star. In this limit, the signal strength practically depends on the difference $D_L - D_\star$ of the source and lens distances only, rather than on the individual values, so that planetary signals on observed Galactic bulge stars will show up predominantly in events that involve lens stars in the Galactic bulge rather than the Galactic disc. Moreover, the signal strength persists for source stars in neighbouring galaxies, such as M31, but achieving a sufficient photometric accuracy (on an unresolved) target for being able to claim a detection is extremely challenging.

Other than for the standard microlensing channel, which is strongly biased in favour of K- and M dwarfs due to their larger abundance, more massive stars are the most prominent targets due to their greater brightness, which results in favourable prospects for studying gas-giant planets, which are known to be extremely rare around low-mass stars.

Unless the orbital period can be identified from the observed light curve, there are a variety of alternative explanations for the nature of the event that are compatible with the acquired data. Rather featureless deviations might also result from finite-source effects, the revolution of the Earth around the Sun, as well as lens or source binarity. Therefore, a proper characterization can only be expected if the orbital period $P$ does not substantially exceed the event timescale $t_E$, which works against the fact that signals increase with $P$. Apart from this, planets in closer orbits, in particular with $P \lesssim 50$ d, could be as well orbiting the lens star rather than the source star, given that their effect on the bending of light would be expected to be negligible.

A rough estimate of the signal strength showed that the observability of signals with current experimental set-ups is practically limited to massive gas giants, and a Monte Carlo simulation of a survey with 2 h sampling and 2 per cent photometric uncertainties revealed that, for the Galactic bulge, the detection of Jupiter-mass planets will be dominated by events with small impact parameters $u_0 \lesssim 0.03$, whereas there is a substantial chance to detect planets of mass $m_p \sim 10 M_{\text{Jup}}$ for the larger sample of events with $u_0 \lesssim 0.1$. Microlensing observations with improved capabilities, namely 15 min sampling with 0.3 per cent accuracy, would substantially increase the prospects, where the detection efficiency for Jupiter-mass planets in events with $u_0 < 0.1$ being boosted from 0.9 to 30 per cent, whereas the prospect for detecting planets that are 10 times more massive amongst all events with $u_0 < 1$ is pushed from 2.5 to 50 per cent. With the much higher detection efficiencies for moderate impact parameters, upper decades of $u_0$ (which carry more events) would provide a larger fraction of the detections than lower decades.

If one is able to properly measure the orbital period $P$ from the microlensing light curve, which requires $P \lesssim t_E$, the orbital semimajor axis $a$ can also be determined provided that typing of the source star yields its mass $m_\star$. Along with the orbital eccentricity $e$, substantially more information can be extracted as compared to the standard channel that lacks of vital constraints on the planetary orbit. The planet mass $m_p$, however, still depends on the angular Einstein radius $\theta_E$, which is not known in general, leaving the need to adopt a kinematic model for the lens and source populations, as well as a mass function for the lens stars, as discussed by Dominik (2006). This can only be overcome by additional measurements of either
θ_χ or the lens–source proper motion \mu, e.g. from the assessment of finite-source effects involving the time-scale \tau_* = \theta_χ/\mu, where the angular size of the source star \theta_χ follows from typing alongside the mass \mu_s.

Current microlensing observing campaigns are not expected to provide many such detections, but with a detection probability of a few per cent for \mu_s = 10 M_☉, and an estimated abundance also of a few per cent (Udry & Santos 2007), a corresponding signal may already be present among the several thousand events that have been monitored so far (with 700–1000 new ones currently being discovered every year). However, finding it may require a careful extensive data-mining effort, given that deviations of this kind are easily missed or misidentified.

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APPENDIX A: MODELS OF THE GALACTIC BULGE POPULATION

Given that the number of source stars in the observed field is proportional to \rho_\text{S}(D_l) D_l^2, while the event rate for a given source star is proportional to \rho_\text{S}(D_l) D_l \theta_\chi \rho_\text{L}, where \rho_\text{S} and \rho_\text{L} denote the volume mass densities of the source and lens stars, respectively, a joint probability density for the source and lens distances is proportional to the differential total event rate:

\[ \frac{d\Gamma}{dD_\text{L} \, dD_\text{S}} \propto \rho_\text{S}(D_\text{L}) \rho_\text{S}(D_\text{S}) \]

\[ \times D_\text{S}^2 \sqrt{D_\text{L} \left(1 - \frac{D_\text{L}}{D_\text{S}}\right)} \Theta(D_\text{S} - D_\text{L}) \Theta(D_\text{L}). \]  \hspace{1cm} (A1)

with \Theta(x) denoting the step function. This, however, does not account for the fact that more distant source stars appear fainter on average and that their light is more likely to be affected by extinction. Integration of equation (A1) over the source distance \(D_\text{S}\) leads to the probability density of the lens distance \(D_\text{L} > 0\) being proportional to

\[ \frac{d\Gamma}{dD_\text{L}} \propto \rho_\text{L}(D_\text{L}) \sqrt{D_\text{L}} \int_{D_\text{L}}^\infty \rho_\text{S}(D_\text{S}) D_\text{S}^2 \sqrt{1 - \frac{D_\text{L}}{D_\text{S}}} dD_\text{S}. \]  \hspace{1cm} (A2)

Alternatively, the joint probability density as given by equation (A1) can be transformed to refer to the source–lens distance \(D_{\text{LS}} = D_\text{S} - D_\text{L}\) rather than the lens distance \(D_\text{L}\), yielding

\[ \frac{d\Gamma}{dD_{\text{LS}} \, dD_\text{S}} \propto \rho_\text{L}(D_\text{S} - D_{\text{LS}}) \rho_\text{S}(D_\text{S}) \]

\[ \times D_\text{S}^2 \sqrt{D_{\text{LS}} \left(1 - \frac{D_{\text{LS}}}{D_\text{S}}\right)} \Theta(D_{\text{LS}}) \Theta(D_\text{S} - D_{\text{LS}}). \]  \hspace{1cm} (A3)

Therefore, one finds the respective probability density of \(D_{\text{LS}}\) as

\[ \frac{d\Gamma}{dD_{\text{LS}}} \propto \sqrt{D_{\text{LS}}} \int_{D_{\text{LS}}}^{\infty} \rho_\text{L}(D_\text{S} - D_{\text{LS}}) \rho_\text{S}(D_\text{S}) \]

\[ \times D_\text{S}^2 \sqrt{1 - \frac{D_{\text{LS}}}{D_\text{S}}} dD_\text{S}. \]  \hspace{1cm} (A4)

Amongst the various laws suggested for the mass density of the Galactic bulge, we adopt a triaxial barred shape of the form

\[ \rho_{\text{bulge}} = \frac{M_{\text{bulge}}}{6.57 \pi x_0 s_0 s_0} \exp(-r^2/2), \]  \hspace{1cm} (A5)

where

\[ r^4 = \left(\frac{x}{s_0}\right)^2 + \left(\frac{z}{s_0}\right)^2 + \left(\frac{\bar{y}}{s_0}\right)^4, \]  \hspace{1cm} (A6)

which is a favourite for matching COBE data (Dwek et al. 1995) with \(s_0 = 1.49\) kpc, \(s_0 = 0.58\) kpc and \(s_0 = 0.40\) kpc and \(M_{\text{bulge}} = 1.7 \times 10^{10} M_\odot\). While \(\bar{y}\) measures the distance towards Galactic north, the axes referring to the \(x\) and \(y\) coordinates are tilted by an angle \(\theta \sim 20^\circ\) with respect to the direction towards the Galactic Centre and the direction of the local circular motion, respectively.

We neglect the variation with the sky coordinates of the source star, and adopt a typical average Galactic longitude \(l = 0^\circ\) and latitude \(b = -3^\circ\), so that

\[ x = (D_L \cos b - R_{\text{GC}}) \cos \theta, \]

\[ y = -(D_L \cos b - R_{\text{GC}}) \sin \theta, \]

\[ z = D_L \sin b. \]  \hspace{1cm} (A7)

where \(R_{\text{GC}} \sim 7.62\) kpc denotes the distance to the Galactic Centre.

Given that the bulge self-lensing is dominant, \(\rho_{\text{bulge}}\) describes both the lens and the source stars. The joint probability density of the lens distance \(D_\text{L}\) and source distance \(D_\text{S}\) according to equation (A1), as well as the probability density of the lens–source distance \(D_{\text{LS}} = D_\text{S} - D_\text{L}\), according to equation (A4), respectively, that result from this choice, are displayed in Figs A1 and A2.

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Figure A1. Joint probability density $\Gamma^{-1} [d^2 \Gamma / (dD_L dD_S)](D_L, D_S)$, indicated by equidistant contours between zero and the maximum, of the lens distance $D_L$ and the source distance $D_S$ for self-lensing within the Galactic bulge in the direction of $l = 0^\circ$ and $b = -3^\circ$.

Figure A2. Probability density of the lens–source distance $\Gamma^{-1} [d^2 \Gamma / (dD_L dD_S)](D_L, D_S)$ for bulge–bulge lensing, where the source stars have been located towards $l = 0^\circ$ and $b = -3^\circ$ and the bulge mass density is given by equation (A5).

For the velocities of lens and source stars, we adopt a two-dimensional Maxwell–Boltzmann distribution in the absolute transversal velocity $v$, where the probability to find $v$ in the interval $(v, v + dv)$ is given by

$$\Phi_v(v) \, dv = \frac{v}{2\sigma^2} \exp \left( -\frac{v^2}{4\sigma^2} \right) \, dv.$$  \hspace{1cm} (A8)

While we adopted $\sigma = 100 \, \text{km} \, \text{s}^{-1}$, we neglected its effective variation with $D_L$ and $D_S$, as well as the motion of the Sun, which are fair approximations for $D_L / D_S \sim 1$, marking the region dominating the event count.

For the mass of Galactic bulge stars, a probability density in $\log(M/M_\odot)$ is given by the normalized mass function (e.g. Chabrier 2003)

$$\Psi_{\log}(\log(M/M_\odot)) = \begin{cases} 
1.292 \exp \left\{ -0.5 \left[ \log(M/M_\odot) + 0.658 \right]^2 \right\} \\
0.2546 (M/M_\odot)^{-1.3} \\
0 
\end{cases} \quad \text{for} \quad -2.0 \leq \log(M/M_\odot) \leq -0.155,$$

$$\quad \text{for} \quad -0.155 < \log(M/M_\odot) \leq 1.8,$$

$$\quad \text{for} \quad \log(M/M_\odot) < -2.0 \text{ or } \log(M/M_\odot) > 1.8,$$

which is shown in Fig. A3.

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