Embedded quantum dots in semiconductor nanostructures

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Abstract. In this work, we report the behavior of the tunneling current in a semiconductor nanostructure of \((Ga,Al)As/GaAs\) which takes into account the behavior of the electrons and the Rashba’s spin orbit interaction in the presence of embedded quantum dots of different geometries (lens, pyramid and ring) in voltage function, magnetic field, and the different values of the interaction spin orbit \((\pi/2, \pi/4\) and \(3\pi/4\)). The results that were obtained show, that the intensity of the current presents appreciable changes when is changed the configuration of the quantum dot as the intensities of external fields and spin polarization as well. All these internal and external effects that are studied in our model, significantly modify the transport of information of the semiconductor nanostructure, our results show that the spin effects and the quantum dot configuration contribute to the quantum memories efficiency and the spin filter devices of actual use on nanoscience and nanotechnology.

1. Introduction
Recently, several experimental and theoretical research groups have shown that the manipulation of embedded quantum dots (QDs) of InAs in low dimensionality semiconductor systems play a predominant role in the control of quantum transport [1–5]. Experimentally diverse works using QDs embedded in semiconductor quantum wires and optics microcavities that employ dual potential double barrier (DB) mechanisms together with QDs show that the quantum tunneling behavior in double-barrier-heterostructure (DBH) as a function of Rabi oscillations and excitons facilitate the manipulation of information to the extent that the QDs are embedded in a controlled manner in the different quantum prototypes [1,4,5]. These works use the formalism of strongly correlated systems and show that the entanglement states in the transport of information changes appreciably due to the presence of embedded QDs. Likewise, the results of these experimental works warn that the inclusion of optical cavities and QDs are of vital importance in the handling of information, if you have consider also the electron phonon interaction. Theoretically the study of nanodevice that contain embedded QDs plays a relevant role in the understanding and development of prototypes for nanoscience and nanotechnology [1,4,5]. For example, information control of spin through embedded QDs has facilitated the retention of quantum phase coherence allowing to examine the quantum interference in Aharonon-Bohm interferometers modulating the Fano resonance and managing the long duration molecular states in the presence of external magnetic fields [1,6]. Thus, the experimental and theoretical study of DBH involving embedded
QDs ones that allow control of quantum transport in different nanodevice is very important because these QDs allow to contrast the passage of information depending on its configuration. Currently, the behavior of the current has been analyzed in a semiconductor nanostructure where the embedded QDs model the behavior of this physical phenomenon in semiconductor nanostructures taking into account external effects such as the voltage and the influence of external magnetic fields that affects the spin-orbit process and allow to determine the information through the Rasha effect in the system [7–9]. The tuning of the current in such quantum mechanisms is possible to examine if you can manage and control both the configuration of the QDs and the phase that occurs in the Rashba coupling, which affects by the voltage variation and the magnetic field.

In this article we study quantum transport through DBH that involves QDs embedded of different structures which act as a bridge for the passage of information from one contact to another in our quantum mechanism. We work with general quantum theory of quantum transport using the tight binding model and the theory of second quantization of Keldysh for nonequilibrium in Green’s function formalism to describe the resonant states in the system taking into account that the interaction Rashba and Dresselhaus affect appreciably the current [10–12]. Here, we study the transport of electrons through behavior of the current in a DBH taking into account different configurations of QDs embedded and the phase factor that allows to control spin effects for the control of the information, using external fields.

2. Theoretical formalism

In our quantum mechanism, we carry out the study of the resonant tunneling of electrons through a QD of different geometry (lens, ring and pyramid) which is embedded in the DBH of $Ga_0.4Al_{0.6}As$. The structure is grown in the $z$ direction and is connected to two semiconductor leads in the presence of electric and magnetic fields and an antisymmetric confinement potential that facilitates the analysis of the spin orbit interaction Rashba (SO), as shown in Figure 1. Here, it is observed that the shape of the potential of confinement depends to a large extent on whether the structure of the QD is approximately equal to the reflected image of the QD contour. Inside the QD and in the DBH system there is SO interactions, while these interactions are neglected in the leads. The Hamiltonian associated with the transport of electrons through a QD-DBH in our quantum device is given by the Equation (1).

\[ H (r) = \frac{\mathbf{p}^2}{2m^*} + V (r) + \mathbf{\sigma} \cdot \mathbf{B} (r) + H_{SO} (r), \]  

\[ (1) \]
where the first term is the kinetic energy, the second term is the potential energy, the third term is the energy of Zeeman and the fourth term is the interaction of SO of Rashba, it is well known from the Dirac equation [7,10]. The term \( H_{SO} \) is reduced to the following form, expressed on the Equation (2).

\[
H_{SO} = \frac{\gamma}{2\hbar} \cdot [\alpha (\hat{\sigma} \times \mathbf{p}) + (\hat{\sigma} \times \mathbf{p}) \alpha], 
\]

(2)

where \( \alpha \sim \langle \Psi (z) | \left( \frac{1}{2}\hbar \right) V (z) | \Psi (z) \rangle \) is the interaction coefficient SO of Rashba [8], \( \Psi (z) \) are the bound states in the \( z \) direction, \( V (z) \) is the confinement potential, \( \hat{\sigma} (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z) \) is the matrix vector of Pauli and \( \mathbf{p} \) is the momentum operator vector. The interaction Rashba SO in Equation (2), can be divided into two terms, Equation (3), [8]:

\[
H_{SO} = H_{R1} + H_{R2}, 
\]

(3)

where \( H_{R1} = \frac{\alpha}{\hbar} \hat{\sigma}_z p_y \) y \( H_{R2} = -\frac{\alpha}{\hbar} \hat{\sigma}_y p_x \). In Equation (3), the term \( H_{R1} \) gives rise to the precession of the spin and the term \( H_{R2} \) can cause spin flip between the different energy levels.

The Equation (1) or principal Hamiltonian expressed in second quantization of the QD DBH system with two leads (\( \beta = L, R \)) semiconductors in the presence of a magnetic field applied in the \( z \) direction and considering the SO interaction of Rashba \( (H_{SO} \ con \ \alpha \neq 0) \) can be written in Anderson’s standard model, named Equation (4):

\[
H = H_{QD-DBH} + \sum_{\beta=L,R} H_{\beta} + H_T, 
\]

(4)

where \( H_{QD-DBH} \) is the Hamiltonian for the QD region embedded in the DBH, \( H_{\beta} \) (Equation (6)) is for the leads, \( H_T \) (or Equation (7)) is the coupling between the leads and QD-DBH, (known as Equation (5))

\[
H_{QD-DBH} = \sum_{n,s} (\varepsilon_n + sB_z) d_{ns}^\dagger d_{ns} + \sum_{m,n} t_{mn}^SO d_{m\downarrow}^\dagger d_{n\uparrow} + H.c 
\]

(5)

\[
H_{\beta} = \sum_{k,s} \varepsilon_{k\beta} a_{k\beta s}^\dagger a_{k\beta s} 
\]

(6)

\[
H_T = \sum_{k,s} \left[ t_{LDBH} a_{Lks}^\dagger d_s + t_{RDBH} e^{-i\frac{K_R l}{m^*} L} a_{Rks}^\dagger + H.c \right], 
\]

(7)

where \( \hat{n}_{ns} \equiv d_{ns}^\dagger d_{ns} \) is the operator number, \( s = \uparrow, \downarrow \) is the spin index, which describes the spin state with spin-up or spin-down, \( \varepsilon_n \) is the self-energy in the QD region and \( B_z \) is the magnitude of the magnetic field. The second term of Equation (5) is the \( H_{R2} \) interaction of Rashba in second quantization with intensity of \( t_{mn}^SO = 0.3 \) meV [8]. In the Equation (6), \( k_\beta \) is the quantum index for lead \( \beta = L, R \) with self-energy \( \varepsilon_{k\beta} \). In the Equation (7) the interaction of Rashba, \( H_{R1} \), gives a spin-dependent extra phase factor, -is \( -iK_R L \) with \( (K_R = \frac{m^*}{e}) \), where \( m^* \) is the effective mass and \( L \) is the dimension of our double-barrier system and \( t_{LDBH} \) (\( t_{RDBH} \)) describes the intensity of the coupling between the left lead (right) and the DBH system.

The quantum transport in the DBH device can be solved using the standard method of functions of Green proposed by Keldysh [11,12] and the equation of current of electrons with spin-up and spin-down, which involves two leads in the semiconductor nanostructure, embedded QD of different geometries and electric and magnetic fields, can be represented by the Equation (8):

\[
I = \frac{2e}{\hbar} \int \frac{d\omega}{2\pi} \text{Re} \left[ t_{LDBH} G_{LDBH}^R (\omega) + t_{RDBH} G_{RDBH}^R (\omega) \right]. 
\]

(8)
where the Green function $G^<(\omega)$ is the Fourier transform of $G^<(t)$ that allows to perform the calculations of our quantum model considering the frequency spectrum or the energies quantized of the system.

To solve $G^<$, we first calculate the retarded Green functions $G^r_s$ using the Dayson equation, referred as Equation (9).

$$G^r_s = g^r_s + g^r_s \sum_s G^r_s,$$

and the Green’s function $G^r_s$, denoted as Equation (10), is a $3 \times 3$ matrix defined as

$$G^r_s = \begin{pmatrix} G^r_{LLs} & 0 & G^r_{LQDs} \\ 0 & G^r_{RRs} & G^r_{RQDs} \\ G^r_{QDLs} & G^r_{QDRs} & G^r_{QDQDs} \end{pmatrix}. $$

In the Equation (9), $g^r_s$ is the Green’s function of the system without coupling between the leads and DBH (i.e., when $t_{LDBH} = t_{RDBH} = 0$). This is represented in Equation (11) and it is obtained exactly as

$$g^r_s = \begin{pmatrix} -i\pi\rho & 0 & 0 \\ 0 & -i\pi\rho & 0 \\ 0 & 0 & g^r_{QDQDs}(\omega) \end{pmatrix},$$

where $g^r_{QDQDs} = (\omega - \varepsilon_{QD})^{-1}$ is the green function of the QD and $\rho$ is the density of the states of the leads. The self-energy $\sum_s(\omega)$, designated as Equation (12), in the Equation (9) is

$$\sum_s = \begin{pmatrix} 0 & 0 & t_{LQD} \\ 0 & 0 & \tilde{t}_{RQD} \\ t_{LQD}^* & \tilde{t}_{RQD}^* & 0 \end{pmatrix},$$

where $\tilde{t}_{RQDs} = \tilde{t}_{RDe}^{-isK_Q}$. Using Equation (11) and Equation (12), $G^r_s$ can easily be obtained by solving Equation (9) as $G^r_s = (g^r_s - \sum_s)^{-1}.

After solving $G^r_s(\omega)$, the Keldysh Green’s function $G^<_s(\omega)$ is obtained from the standard Keldysh equation, and it is numbered as Equation (13):

$$G^<_s = \left(1 + G^r_s \sum_s \right) g^<_s \left(1 + \sum_s G^a_s \right) + G^r_s \sum_s G^a_s,$$

where $G^r(a)$ are the retarded (advanced) Green functions.

3. Results and conclusions

We report the behavior of the tunneling current in a nanostructure semiconductor of ($Ga,Al$)As/GaAs which contains both quantum electron transport as the orbital interaction of Rashba spin in the presence of embedded InAs quantum dots of different geometries (lens, pyramid and ring) as a function of voltage, magnetic field, and different spin orbit interaction values. In the first instance we report the behavior of the current as a function of the voltage taking into account different geometries, and fixed values of magnetic fields (B = 0.5 mT) considering the effect of spin up and spin down, which provides information about the Zeman effect and in turn a degeneration of the energy levels (Figure 2(a) to Figure 2(f)). This results show that the intensity of the current presents appreciable changes when the configuration of the QDs change as the system phase. Here, there are three cases: for voltages less than or equal to 0.1 V, for voltages between 0.1 V and 0.2 V and for voltages greater than 0.2 V.
Figure 2. These figures exhibit current as a function of the applied voltage, the magnetic field of 0.5 mT, the spin up and down polarization and the phase: (a) and (b) \( K_R L = \pi/4 \), (c) and (d) \( K_R L = \pi/2 \), (e) and (f) \( K_R L = 3\pi/4 \), for different configuration of the QDs lens (black curve), ring (blue curve) and pyramid (red curve).

For the first case and spin up (Figure 2(b), Figure 2(d) and Figure 2(f)) we observe that they appear the first bound states and therefore the first current values in the nanostructure when using pyramid type QDs being their values, increasingly larger as the value of the phase
increases from $\pi/4$ to $3\pi/4$. For this first voltage range also it is observed that another picks of lower intensity appear for the ring type configuration and finally with an even lower intensity of a lens. At the second interval reveals a clear domain of lens-like configuration in the different values of the phase that we use here; the value of the ring being the weakest. Likewise, for the third case and phase of $3\pi/4$ it is observed that the dominant states are lens and ring respectively generating different values of current and passage of information in the system. Punctually in this phase value is seen a clear domain of the tunnel effect in the system by the type QDs lens, which is interpreted as a domain due to the high symmetry of the QDs. This allows us to conclude that for spin up and small voltage values the transport domain of information the system occurs through pyramid type QDs or less symmetric QDs. For voltages greater than 0.2 V clearly dominate the QDs with greater symmetry (lens and ring, respectively). In the case of spin down (Figure 2(a), Figure 2(c) and Figure 2(e)), in the system it is observed that for the first case (voltages less than or equal to 0.1 V) the dominant configuration is that of the lens because here the greater current intensities. However, in a timely manner the first states appear confined for the case of the ring when the phase changes to $\pi/2$ but the values are very small. In the range between 0.1 V and 0.2 V a clear domain of lens type configuration is observed followed by ring; and for voltages greater than 0.2 V a clear control of the pyramid type configuration is observed.

4. Conclusions
The previous results allow us to conclude that morphologies with greater symmetry control in small voltage values, but when the voltage is greater than 0.2 V the transmission record of information is governed by pyramidal configuration. Then in our quantum device of study, there is a change of domain in the transmission of information when presented an inversion of the spin upwards to spin downwards generating a transmission change of information of QDs of greater symmetry to asymmetric (pyramid type). We also observe that there is a greater electron flow of information when you have spin polarization up (Figure 2(b), Figure 2(d) and Figure 2(f)) and the phase is between $\pi/4$ and $\pi/2$ (Figure 2(b) and Figure 2(d)). Likewise, when the phase takes the value of $3\pi/4$ the information is better appreciated when the system is polarized spin down (Figure 2(e)).

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