No Future in Black Holes

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Abstract

The black hole information paradox has been with us for some time. We outline the nature of the paradox. We then propose a resolution based on an examination of the properties of quantum gravity under circumstances that give rise to a classical singularity. We show that the gravitational wavefunction vanishes as one gets close to the classical singularity. This results in a future boundary condition inside the black hole that allows for quantum information to be recovered in the evaporation process.

The black hole information paradox is presently one of the most puzzling problems in fundamental physics. It is widely believed that quantum mechanics should control the evolution of any physical system. At first sight, black holes appear to transcend quantum mechanics, [1].

What follows is a brief outline of a possible way to resolve the information paradox. A much more complete treatment will be published elsewhere.

In classical physics, a black hole formed by gravitational collapse will settle down to a member of the Kerr-Newman family. The black hole is completely characterised by its mass, $M$, angular momentum $J$, electric charge $Q$ and its soft gauge charges.[?] Infalling matter, once it has passed through the event horizon can no longer influence anything outside the black hole. What is more, it will inevitably arrive at the spacetime singularity in the black hole interior and disappear from the spacetime. From the point of view of external observers, the material that gave rise to the black hole has completely disappeared. The black hole will then remain as a sink for infalling material for ever.

In quantum mechanics, the situation is different because of Hawking radiation, [2, 3]. Hawking showed that black holes produce thermal radiation at a temperature inversely proportional to their mass. A black hole therefore loses mass more and more rapidly and will disappear completely on a timescale $\tau \sim M^3$. The difficulty is that the final state is just thermal Hawking radiation that appears to have a von Neumann entropy $\sim M^2$ and is independent of the nature of the incoming state. In the unitary evolution required by quantum mechanics, von Neumann entropy is constant. So either quantum mechanics breaks down or some new physics is required.

The remainder of this paper presents a concrete proposal, based on the quantization of general relativity, that allows for a possible resolution of the paradox with the black hole
obeying the usual rules of quantum mechanics as far as external observers are concerned. We will restrict the discussion to a spacetime of dimension four and only consider black holes in an asymptotically flat spacetime.

The hoop conjecture \([4]\) asserts that a black hole will form when a mass \(m\) is localised into a sufficiently small region of size \(\sim m\). Once an event horizon forms, Penrose’s theorem \([5]\) guarantees that the spacetime is singular and is usually taken to mean that there is a region of infinite curvature that is the boundary of spacetime.

The special case of spherically symmetric collapse was studied by Oppenheimer and Snyder, \([6]\). They looked at the collapse of a spherically symmetric ball of pressure-free matter. Outside the collapsing body, the metric is given precisely by the static Schwarzschild metric. The horizon forms once the ball has contracted sufficiently. The spacetime inside the ball of matter is part of a \(k = 1\) FLRW universe. The singularity forms to the future of the horizon when the density of the ball diverges. However, the singularity is not quite the same as in the familiar FLRW universe as it stretches outside the collapsing matter in a spacelike fashion.

Realistic gravitational collapse is not spherically symmetric. Weak cosmic censorship is the conjecture that any singularity is hidden from observers outside the black hole. Whilst a proof of this conjecture is lacking, the evidence for it is substantial. Unlike spherical collapse to form a Schwarzschild black hole, the Kerr-Newman solutions involve timelike singularities. It is believed that they do not form in realistic collapse. They are behind an inner horizon which is known to be perturbatively unstable. Dafermos and Luk \([7]\) have recently shown that generically singularities have spacelike, or null, components. We assume the Penrose diagram for generic collapse to be the similar to the Schwarzschild case although there are possibly null sections of the singularity. The essential point being that we expect all future-directed timelike or null lines to reach the singularity.

Hawking \([2, 3]\) showed that the temperature of the black hole is given by \(T_H = \frac{\kappa}{2\pi}\) where \(\kappa\) is the surface gravity of the black hole. Bardeen, Carter and Hawking \([8]\) proved the first law of black hole mechanics that governs infinitesimal changes in the state of a stationary black hole.

\[
dM = \frac{\kappa dA}{8\pi} + \Phi dQ + \Omega dJ
\]  
(1)

where \(M\) is the mass of the black hole, \(A\) the area of the event horizon, \(\Phi\) the electrostatic potential of the hole and \(\Omega\) its angular velocity. Since \(T_H = \frac{\kappa}{2\pi}\), we readily infer the black
hole has entropy $A/4$. It should be noted that $S \sim M^2$ is an entropy that is vastly greater than typical systems in equilibrium for which the entropy scales with mass more slowly.

Following Boltzmann, it is presumed that $e^S$ is the density of states of the black hole. This last idea is formalised in a collection of ideas that have become known as the “central dogma” of black hole physics [9]. For external observers, the black hole behaves like any other quantum system; it has a density of states $e^S$ and its evolution is unitary. Outside the black hole, one expects the conventional ideas of general relativity to be valid; namely that spacetime can be treated by classical geometry and that fields (including gravitons) can be treated by effective field theory.

If these ideas were to hold inside a black hole we would be in trouble. Suppose we study the wave equation inside the Schwarzschild black hole, we find that its solutions typically blow up at the singularity and induce a corresponding divergence in the probability current there. Quantum information is thereby taken out of the spacetime. In the absence of cloning, information would be lost.

It is to quantum gravity that we must turn our attention. The path integral for gravity is

$$ Z \sim \int \mathcal{D}g \, e^{iI[g]} \quad (2) $$

where $I[g]$ is the Einstein action for a metric $g_{ab}$ on a manifold $M$ and boundary $\partial M$ with induced metric $\gamma_{ij}$,

$$ I[g] = \frac{1}{16\pi} \int_M R \sqrt{-g} \, d^4x \pm \frac{1}{8\pi} \int_{\partial M} K \sqrt{\gamma} \, d^3x. \quad (3) $$

$R$ is the Ricci scalar of the metric $g_{ab}$. $K$ is the trace of the second fundamental form of the boundary and the sign is chosen depending on whether the boundary is spacelike or timelike. There may be many disconnected components of the boundary. The integral is taken over all Lorentzian metrics $g_{ab}$ modulo diffeomorphisms with induced metric $\gamma_{ij}$ on the boundary.

It is not clear what, if any, precise conditions are to be imposed on the metric. If we are interested only in a semi-classical evaluation of $Z$, the path integral can be approximated using Picard-Lefshetz theory [10], in which case it seems to be necessary to deform $g_{ab}$ into the complex. The use of the path integral is limited by the failure of renormalizability of the Einstein theory. The physical meaning of $Z$ is that it gives the probability amplitude of finding a configuration with metric $\gamma_{ij}$ on the boundary, or boundaries, of $\mathcal{M}$. As an
example, $Z$ might be the transition amplitude for having a metric $\gamma^{(1)}$ on one surface and $\gamma^{(2)}$ on another, [11]. In quantum gravity, when describing a closed system, there is no way of determining whether $\gamma^{(1)}$ is to the past or future of $\gamma^{(2)}$. If however the surfaces on which $\gamma^{(i)}$ are defined stretch out to infinity, then a time can be associated with each such surface. A second possibility is that there is a single component to the boundary. $Z$ is then the probability amplitude of finding a particular geometry $\gamma$, [12]. In the case of a single boundary component, $Z$ is usually referred to as the wavefunction of the universe, $\Psi[\gamma]$. Again, there is no reference to an arrow of time for a closed system. Inside an evaporating black hole, if one has a surface close to the singularity, it does not seem to be legitimate to extend such a surface out to infinity as the interior is causally disjoint from the asymptotic region.

Canonical methods allow for another approach to determining $\Psi[\gamma]$. Take the metric and rewrite it in ADM form [13, 14] as

$$ds^2 = -N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt).$$

(4)

$N$ is referred to as the lapse and $N^i$ as the shift. $\gamma_{ij}$ is a purely spacelike metric. Using this decomposition, the Einstein action becomes

$$I[g] = \int d^3x dt \sqrt{\gamma} N \left( K_{ij} K^{ij} - K^2 + (3)R(\gamma) \right),$$

(5)

where $(3)R(\gamma)$ is the Ricci scalar of the three-metric $\gamma$ and $K_{ij}$ is the second fundamental form of the $t = \text{const}$ surfaces. Explicitly

$$K_{ij} = \frac{1}{2N}(D_i N_j + D_j N_i - \dot{\gamma}_{ij}),$$

(6)

where $D_i$ is the covariant derivative based on the 3-metric $\gamma_{ij}$ and a dot denotes the time derivative. Using Hamiltonian techniques, one finds that in the Gaussian gauge $N = 1, N^i = 0$ the system is described entirely in terms of two constraints: the diffeomorphism constraint

$$\chi^j \equiv D_i \pi^{ij} = 0$$

(7)

and the Hamiltonian constraint

$$\mathcal{H} \equiv \gamma^{-1/2} \left( \pi^{ij} \pi_{ij} - \frac{1}{2} \pi^i \pi^j - (3)R(\gamma) \right) = 0.$$  

(8)

$\pi^{ij}$ is the momentum conjugate to $\gamma_{ij}$ and can be written as

$$\pi^{ij} = -\gamma^{1/2}(K^{ij} - \dot{\gamma}^{ij}K).$$

(9)
One quantizes the system by replacing \( \pi^{ij} \) by \(-i\delta/\delta \gamma^{ij}\). The Hamiltonian constraint then becomes the Wheeler-DeWitt equation \[15, 16\]

\[
\left( G_{ijkl} \frac{\delta}{\delta \gamma^{ij}} \frac{\delta}{\delta \gamma^{kl}} - \gamma^{1/2(3)} R \right) \Psi[\gamma] = 0. 
\]

(10)

\( G_{ijkl} \) is the DeWitt co-metric

\[
G_{ijkl} = \frac{1}{2} \gamma^{-1/2} (\gamma_{ik} \gamma_{jl} + \gamma_{il} \gamma_{jk} - \gamma_{ij} \gamma_{kl}).
\]

(11)

\( \Psi[\gamma] \) here is the same object as that defined by the path integral and is a functional on superspace, the space of all spatial metrics modulo diffeomorphisms. \[? \]

When one studies systems that are far from equilibrium, it is convenient to suppose the initial state is specified by a density matrix \( \rho^{(i)} \) rather than a pure state with some specific geometry \( \gamma \). The usual path integral is then generalised by a method of Schwinger \[17\], Keldysh \[18\], and Kadanoff and Baym \[19\]. The path integral defines the probability \( P[\gamma^{(f)}, \rho_i] \) of finding a final state geometry \( \gamma^{(f)} \) given the initial density matrix \( \rho^{(i)} \).

\[
P[\gamma^{(f)}, \rho_i] = \mathcal{N} \text{ Tr} \int \mathcal{D}g^{(+)} \mathcal{D}g^{(-)} e^{i I[\gamma^{(+)}]} \rho^{(i)} e^{-i I[\gamma^{(-)}]}.
\]

(12)

\( \rho^{(i)} = \rho^{(i)}_{\gamma^{(i)}} \gamma \langle \gamma' | \) where \( \gamma \) and \( \gamma' \) refer to some geometries. The forward part of the path integral is over spacetime metrics \( g^{(+)} \) with boundary \( \gamma^{(f)} \) in the future and \( \gamma \) in the past. The time-reversed path integral is over \( g^{(-)} \) with \( \gamma^{(f)} \) in the future and \( \gamma' \) in the past. \( \mathcal{N} \) is a normalization constant.

Aharonov, Bergmann and Lebowitz \[20\] observed that since the laws of physics are invariant under time reversal, it might be possible under certain circumstances to impose the condition that there is particular density matrix \( \rho_f \) in the future. In the case of gravity, where distinguishing between the past and future is not straightforward, the impetus to do so is greater. They suggested the modification

\[
P[\rho_f, \rho_i] = \mathcal{N} \text{ Tr} \int \mathcal{D}g^{(+)} \mathcal{D}g^{(-)} \rho^{(f)} e^{i I[\gamma^{(+)}]} \rho^{(i)} e^{-i I[\gamma^{(-)}]}.
\]

(13)

where the two path integrals are over spacetime metrics that have spatial metrics reproducing the density matrices \( \rho^{(i)} \) and \( \rho^{(f)} \) in the past and future respectively. Setting \( \rho^{(f)} \) to the identity reproduces the usual formulation of quantum mechanics and has been termed by Gell-Mann and Hartle \[21\] as the principle of indifference. Specifying a non-trivial \( \rho_f \) is termed post-selection.
Some time ago, Horowitz and Maldacena [22] suggested this type of boundary condition in the future might be able to resolve the information paradox. In fact, it seems that in order to prevent information leaking out of the spacetime through the singularity, setting a boundary condition there is a necessity. Setting boundary conditions in the future risks unusual behavior such as apparent violations of no-cloning, unitarity and causality, [23].

If we thought that the approach to singularity was smooth, as seems to be the case for Oppenheimer-Snyder collapse, one would be wrong. Belinsky, Khalatnikov and E.M.Lifshitz (BKL) [24] showed that collapse is generally chaotic. We illustrate this for the case of pure gravity but the extension to include other fields is straightforward. Classically, as we approach a spacelike singularity, space starts to break up into individual regions that do not interact with each other. Each of these regions behaves a bit like a Kasner spacetime. The Kasner spacetime has metric

$$ds^2 = -dt^2 + a^2(t)dx^2 + b^2(t)dy^2 + c^2(t)dz^2$$  \hspace{1cm} (14)$$

with $a(t) = (t_0 - t)^{p_1}, b(t) = (t_0 - t)^{p_2}, c(t) = (t_0 - t)^{p_3}$ and $t \leq t_0$. The singularity is reached at $t = t_0$. The exponents $p_i$ obey $p_1 + p_2 + p_3 = p_1^2 + p_2^2 + p_3^2 = 1$ so that one of the $p_i$ is negative and two are positive. Two dimensions of space are contracting as one moves towards the singularity and one is expanding. Any object approaching the singularity will therefore undergo spaghettification. Note that the volume of space is decreasing as one gets close to the singularity. BKL showed that a region undergoes Kasner behavior for a period but is then interrupted by two different types of process. The first is that the Kasner exponents are not fixed but experience a rapid change at certain intervals. The second is that the principal directions of expansion and contraction $x, y$ and $z$ undergo rotation from time to time. The complicated behavior found by BKL can be succinctly summarised in a way first found by Damour, Henneaux and Nicolai, [25]. The Einstein equations amount to null geodesic motion in a space with metric

$$d\sigma^2 = -d\rho^2 + \rho^2 \left( \frac{du^2 + dv^2}{v^2} \right).$$ \hspace{1cm} (15)$$
The $z = u + iv$ plane is a space of constant negative curvature. Motion is however restricted to the domain $F$ in the $z$ plane bounded by the unit circle centered on the origin and the lines $u = 0, v > 1$ and $u = 1/2, v > \sqrt{3}/2$. This is precisely half of the more familiar fundamental region for $SL(2, \mathbb{Z})$. When a null geodesic meets the boundary of $F$, it bounces off it by
specular reflection. Approach to the singularity corresponds to $\rho \to \infty$. Each point in this space corresponds to some three-geometry and the null geodesic sweeps out a curve that represents the time evolution of this geometry. In this picture, the Hamiltonian constraint takes the form

$$\mathcal{H} = \frac{1}{2} \left( -\pi^2 + \frac{v^2}{\rho^2} (\pi^2 + \pi^2) \right).$$

Here $\pi_\rho, \pi_u$ and $\pi_v$ are the momenta conjugate to $\rho, u$ and $v$. Treating $\mathcal{H}$ as the Hamiltonian, together with the constraint $\mathcal{H} = 0$ and the reflective conditions at the boundary of $F$, reproduces the solution to the Einstein equations close to the singularity.

This system is quantized by replacing $\pi_\rho, \pi_u$ and $\pi_v$ by $-i\partial/\partial \rho, -i\partial/\partial u$ and $-i\partial/\partial v$ respectively. The Hamiltonian constraint then turns into the Wheeler-DeWitt equation on a minisuperspace. There is an operator ordering ambiguity in carrying this out, so we have adopted an ordering such that the Wheeler-DeWitt equation is the Laplacian for the metric in (15),

$$\left( -\rho^2 \frac{\partial^2}{\partial \rho^2} - 2\rho \frac{\partial}{\partial \rho} - \Delta_F \right) \Psi = 0$$

(17)

$\Delta_F$ is the Laplacian on the domain $F$ and is given by

$$\Delta_F = -v^2 \left( \frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2} \right).$$

(18)

Since the walls are classically reflective, the boundary condition on $\Psi$ is that it vanishes on the boundary of $F$. The eigenfunctions $f_n$ of $\Delta_F$ obey

$$\Delta_F f_n = s_n (1 - s_n) f_n.$$  

(19)

The $f_n$ are the odd Maass waveforms of $SL(2, \mathbb{Z})$ with $s_n = \frac{1}{2} \pm it_n, t_n$ real and come in complex conjugate pairs. The spectrum of $\Delta_F$ has two distinct components; a discrete set of eigenfunctions that are the odd cusp forms of $SL(2, \mathbb{Z})$ and a continuum of the odd non-holomorphic Eisenstein series (NHES), \[26\]. The cusp forms are square integrable in $F$ whereas the NHES are not. Despite this, square integrable solutions of (19) can be written as a linear combination of the $f_n$, \[26\]. Each $f_n$ provides a solution of (17), $\psi_n = \rho^{-s_n} f_n$. Hence a general solution of (17) is a linear combination $\psi_n$. The wavefunctions are not square integrable in minisuperspace even if they are square integrable in $F$. Such wavefunctions have appeared in the literature before in a description of the initial cosmological singularity \[27\]. The natural inner product on functions in superspace is an analog of the Klein-Gordon
On minisuperspace, the inner product of two wavefunctions is

\[
(\Psi_a, \Psi_b) = i \int_\Sigma \left( \Psi_a^* \frac{\partial \Psi_b}{\partial \rho} - \Psi_b^* \frac{\partial \Psi_a}{\partial \rho} \right) \frac{\rho^2}{v^2} \, du \, dv
\]

with \(\Sigma\) being a “spacelike” surface of \(\rho = \text{constant}\) in the minisuperspace. By virtue of the Wheeler-DeWitt equation, this is independent of \(\rho\). Using this norm, any wavefunction that is integrable in \(F\) descends to one with finite norm.

As one approaches the singularity where \(\rho \to \infty\), the wavefunction behaves like \(\rho^{-1/2}\). The wavefunction therefore vanishes at the singularity, a condition that was proposed by DeWitt, as necessary for singularities to make sense quantum mechanically. \((\Psi, \Psi)\) is in some sense the probability flux for gravitational information flowing through \(\Sigma\). For generic \(\Psi_a\), \((\Psi_a, \Psi_b)\) does not vanish as \(\rho \to \infty\) even though \(\Psi \sim \rho^{-1/2}\). However, one can choose perfectly reflecting boundary conditions at the singularity, the information falling into the singularity is perfectly reflected. Under these circumstances \(\Psi\) is real and \((\Psi, \Psi) = 0\). Again this possibility was anticipated by DeWitt as a mechanism for singularity avoidance.

This is a boundary condition that specifies \(\rho_f = 0\). Under these circumstances, information is not lost from the spacetime and opens the possibility of rescuing unitary time evolution for black holes. As spatial geometries approach the singularity one concludes that the probability of finding them is going to zero because of the perfectly reflecting boundary conditions there. The approximation used will be accurate only for geometries close to being singular because it is only there that the BKL walls become exactly reflective.

We have seen how to set a future boundary condition for the singularity, namely that \(\rho_f = 0\). Outside the black hole, we assume that the principle of indifference holds. Such behavior gives a way of resolving the information paradox.

The singularity is required to reflect anything incident on it. One might be able to think of particles getting close to the singularity as being annihilated by their antiparticles which travel backwards in time and once they get outside the horizon scatter again and become outgoing Hawking radiation. In some sense what happens close to the singularity is the time reverse of the Hawking pair creation process in which particle-antiparticle pairs are created outside the horizon. The interior of the black hole is therefore a strange place where ones classical notions of causality and unitarity are violated. This does not matter as long as outside the black hole such pathologies do not bother us. Lloyd has found that classical information completely escapes from the black hole and that quantum information mostly
does. He estimated that at most half a qubit of information would be lost as the black
hole disappears. Subsequently Lloyd and Preskill showed that it was likely that both
causality violation and unitarity could occur outside the black hole, but in practice it was
most probably unobservable. We therefore conjecture that there is a quantum cosmic censor
that forbids strange behavior outside the black hole. A slightly weaker form of this conjecture
would be one that forbids the *observation* of strange behavior outside the black hole. One
should also note that the conditions under which the firewall paradox was derived, do not
hold for systems with post-selection, \[30\]. We should also note an intriguing connection with
number theory based on the nature of the wavefunctions in $F$.  

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