Superconductivity in the t-J model

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A comparison of microscopic theories of superconductivity in the limit of strong electron correlations is presented. The results for the two-dimensional t-J model obtained within the projection technique for the Green functions in terms of the Hubbard operators and the slave-fermion representation for the RVB state are considered. It is argued that the latter approach resulting in the odd-symmetry p-wave pairing for fermions is inadequate.

I. INTRODUCTION

A mechanism of high-temperature superconductivity in cuprates is still unresolved since strong electron correlations in copper-oxygen planes prevent from applying well established methods of band structure calculations developed for conventional metals. An important role of electron correlations in cuprates was initially stressed by Anderson who proposed to consider them within the framework of the Hubbard model or the so-called t-J model which follows from the Hubbard model in the limit of strong correlations. To study these models, a lot of numerical work have been done though the obtained results are still controversial. For instance, a robust d-wave pairing was observed for the t-J model, while a long-range order was not found in the original Hubbard model.\(^5\)

In analytical approaches mostly a mean-field theory (MFA) was applied in studies of the Hubbard model or the t-J model. The resonating valence bond (RVB) state in the t-J model was proposed by Baskaran et al.\(^6\) where superconductivity was obtained as a result of spin correlations induced by the superexchange interaction. Similar results were found by Cyrot for superconducting pairing mediated by superexchange interaction. To overcome the problem of strong correlations in the t-J model and nonfermionic commutation relations for the physical electron operators a number of auxiliary field representations was proposed (see, e.g., \(^8\)–\(^32\)). However, in these methods usually a spin-charge separation is assumed for spinon and holon fields which violates rigorous restrictions imposed by nonfermionic commutation relations as the "no double occupancy" constraint which may result in unphysical conclusions.

A reliable analytical approach to deal with strong correlations in the Hubbard or the t-J model is based on the Green function methods in terms of the Hubbard operators which rigorously preserve the non-fermionic commutation relations.\(^33\) Here we may mention our results based on the Mori-type projection technique for the Green functions and the diagram technique calculations by Izyumov et al.\(^37,38\). These approaches enable one to go beyond MFA by taking into account self-energy corrections. For instance, a numerical solution of the Dyson equation in\(^36\) revealed a non Fermi-liquid behavior in the normal state at low doping and the d-wave superconductivity mediated by the exchange and spin-fluctuation pairing. In the recent paper\(^39\) a microscopical theory of superconductivity in CuO\(_2\) layer within the effective two-band p-d Hubbard model in the strong correlation limit was developed. It has been proved that the MFA for the Hubbard model results in the antiferromagnetic exchange d-wave pairing which is equivalent to pairing observed in the t-J model in MFA.

In the present paper I compare the results for the t-J model obtained within the Green function (GF) method in terms of the Hubbard operators and by applying the slave-fermion hard-boson representation for the RVB state. It will be shown that the latter approach results in an odd-symmetry p-wave pairing for the spinless fermions as in the path-integral representation which contradicts to known numerical and analytical calculations. It casts doubts on the validity of spin-charge separation approach in studying superconducting pairing in the t-J model.

In the next Section we present briefly the results of the projection technique for the GF for the t-J model. In Sec. 3 superconducting pairing within the slave-fermion representation for the Hubbard operators is considered. Results and discussions are given in Sec. 4. Concluding remarks are in Sec. 5.

II. GREEN FUNCTION METHOD

A. Dyson Equation for the t-J Model

In the present Section we consider superconducting pairing in the t-J model by applying the GF technique.\(^34\)–\(^36\). The t-J model in the standard notation\(^1,42\) reads:

\[
H_{t-J} = -t \sum_{i \neq j, \sigma} \hat{a}_{i \sigma}^\dagger \hat{a}_{j \sigma} + J \sum_{\langle ij \rangle} (\hat{S}_i \cdot \hat{S}_j - \frac{1}{4} n_i n_j),
\]

(1)
where \( \hat{a}_{i\sigma}^+ = a_{i\sigma}^\dagger(1 - n_{i-\sigma}) \) are projected operators for physical electrons, \( n_i = \sum_{\sigma} a_{i\sigma}^\dagger a_{i\sigma} \) is the number operator and \( S_\sigma^\alpha = (1/2) \sum_{i,\alpha} \hat{a}_{i\alpha}^\dagger \sigma_{\alpha,\alpha'} \hat{a}_{i\alpha'}^\dagger \) are spin-1/2 operators. Here \( t \) is effective transfer integral and \( J \) is antiferromagnetic exchange energy for a pair of nearest neighbor spins, \( (ij), \ i > j \).

To take into account on a rigorous basis the projected character of electron operators we employ the Hubbard operator technique. The Hubbard operators (HO) are defined as \( X_{i\alpha}^{\mu} = |i, \alpha\rangle \langle i, \beta| \) for three possible states \( |i, \alpha\rangle \) at a lattice site \( i \); for an empty site \( |i, 0\rangle \) and for a singly occupied site \( |i, \sigma\rangle \) by an electron with spin \( \sigma/2 \) \( (\sigma = \pm 1, \bar{\sigma} = -\sigma) \). They obey the completeness relation

\[
X_{i\alpha}^{\mu} + \sum_\sigma X_{i\sigma}^{\alpha\bar{\sigma}} = 1, \tag{2}
\]

which rigorously preserves the constraint of no double occupancy. The spin and density operators in Eq. (1) are expressed by HO as

\[
S_\sigma^\alpha = X_{i\alpha}^{\sigma\bar{\sigma}}, \quad S_i^z = \frac{1}{2} \sum_\sigma \sigma X_{i\sigma}^{\sigma\bar{\sigma}}, \quad n_i = \sum_\sigma X_{i\sigma}^{\sigma\bar{\sigma}}. \tag{3}
\]

The HO obey the following multiplication rule

\[
X_{i\alpha}^{\mu} X_{j\beta}^{\nu} = \delta_{\beta\gamma} X_{i\gamma}^{\mu\nu} \quad \text{and commutation relations}
\]

\[
[X_{i\alpha}^{\mu}, X_{j\beta}^{\nu}]_\pm = \delta_{ij} \left( \delta_{\alpha\gamma} X_{i\gamma}^{\mu\nu} \pm i \delta_{\beta\alpha} X_{i\alpha}^{\mu\nu} \right), \tag{4}
\]

where the upper sign stands for the the Fermi-like HO (as, e.g., \( \tilde{X}_{0\alpha}^{\sigma}\alpha \)) and the lower sign for the Bose-like operators (as the spin and number operators in Eq. (3)).

By using the Hubbard operator representation we write the Hamiltonian of the \( t-J \) model (1) in a more general form:

\[
H_{t-J} = - \sum_{i,j,\sigma} t_{ij} X_{i\sigma}^{00} X_{j\sigma}^{00} - \mu \sum_{i,\sigma} X_{i\sigma}^{\sigma\bar{\sigma}} + \frac{1}{4} \sum_{i,j,\sigma} J_{ij} \left( X_{i\sigma}^{\sigma\sigma} X_{j\sigma}^{\sigma\sigma} - X_{i\sigma}^{\sigma\bar{\sigma}} X_{j\bar{\sigma}}^{\sigma\bar{\sigma}} \right). \tag{5}
\]

The electron hopping energy for the nearest neighbors, \( t_{ij} = t \), and the second neighbors, \( t_{ij} = t' \), on a 2D square lattice, and the exchange interaction \( J_{ij} = J \) for the nearest neighbors are considered as independent parameters if, starting from a more realistic for copper oxides three-band \( p-d \) model, we reduce it to the \( t-J \) model. In that case the parameters \( t, t' \) and \( J \) can be evaluated in terms of the original parameters of the \( p-d \) model (see, e.g., \( \tilde{X}_{0\alpha}^{\sigma}\alpha \)). We introduced also the chemical potential \( \mu \) which can be calculated from the equation for the average number of electrons

\[
n = \langle n_i \rangle = \sum_\sigma \langle X_{i\sigma}^{\sigma\bar{\sigma}} \rangle. \tag{6}
\]

To discuss the superconducting pairing within the model (5) we introduce the Nambu notation for HO:

\[
\Psi_{i\sigma} = \left( \begin{array}{c} X_{i\sigma}^{00} \\ X_{i\sigma}^{\sigma\bar{\sigma}} \end{array} \right), \quad \Psi_i^+ = \left( \begin{array}{c} X_{i\sigma}^{\sigma\sigma} \\ X_{i\bar{\sigma}}^{\sigma\bar{\sigma}} \end{array} \right),
\]

and consider the matrix GF

\[
\tilde{G}_{i\sigma}(t - t') = \langle \langle \Psi_{i\sigma}(t)|\tilde{\Psi}^+_{j\alpha}(t') \rangle \rangle = \left( \begin{array}{cc} G_{i\sigma}^{11} & G_{i\sigma}^{12} \\ G_{i\sigma}^{21} & G_{i\sigma}^{22} \end{array} \right) \tag{7}
\]

where Zubarev’s notation for the anticommutator GF is used.37

By differentiating the GF (7) over the time \( t \) we get for the Fourier component the following equation

\[
\omega \tilde{G}_{i\sigma}(\omega) = \delta_{ij} \tilde{Q}_\sigma + \langle \langle \tilde{Z}_{i\alpha} \mid \tilde{\Psi}^+_{j\alpha} \rangle \rangle, \tag{8}
\]

where \( \tilde{Z}_{i\sigma} = [\Psi_{i\sigma}, H] \), \( \tilde{Q}_\sigma = \left( \begin{array}{cc} Q_\sigma & 0 \\ 0 & \bar{Q}_\sigma \end{array} \right) \) with \( Q_\sigma = \langle \langle \tilde{X}_{i\sigma}^{00} + \tilde{X}_{i\sigma}^{\sigma\bar{\sigma}} \rangle \rangle \). Since we consider a spin-singlet state the correlation function \( Q_\sigma = Q = 1 - n/2 \) depends only on the average number of electrons (6).

Now, we project the many–particle GF in (8) on the original single–electron GF

\[
\langle \langle \tilde{Z}_{i\sigma} \mid \tilde{\Psi}^+_{j\alpha} \rangle \rangle = \sum_{\bar{t}} \tilde{E}_{\bar{t} i\sigma} \langle \langle \tilde{\Psi}_{i\sigma} \mid \tilde{\Psi}^+_{j\alpha} \rangle \rangle \tag{9}
\]

where the irreducible (irr) part of the many–particle operator \( \tilde{Z}_{i\sigma} \) is defined by the equation

\[
\langle \langle \tilde{Z}_{i\sigma}^{\text{irr}} \rangle \rangle = \langle \langle \tilde{Z}_{i\sigma} - \sum_{\bar{t}} \tilde{E}_{\bar{t} i\sigma} \tilde{\Psi}_{i\sigma} \tilde{\Psi}^+_{j\alpha} \rangle \rangle = 0,
\]

which results in the definition of the frequency matrix

\[
\tilde{E}_{i\sigma} = \langle \langle [\Psi_{i\sigma}, H], \tilde{\Psi}^+_{j\alpha} \rangle \rangle \tag{10}
\]

Now we can introduce the zero–order GF in the generalized MFA which is given by the frequency matrix (10)

\[
\tilde{G}_{i\sigma}^{00}(\omega) = Q \langle \langle \tilde{\Psi}_{i\sigma} \rangle \rangle \tilde{E}_{i\sigma}^{-1}. \tag{11}
\]

To derive the Dyson equation for the single-electron GF (7) we write down an equation of motion for the irreducible part of the GF in (9) with respect to the second time \( t' \) for the right–hand side operator \( \tilde{\Psi}^+_{j\alpha}(t') \). Then performing the same projection procedure as in Eq. (9) we obtain the Dyson equation for the GF in the form

\[
\tilde{G}_{i\sigma}^{00}(\omega) = \tilde{G}_{i\sigma}^{00}(\omega) + \sum_{k\ell} \tilde{G}_{i\sigma}^{00}(\omega) \tilde{\Sigma}_{k\ell}(\omega) \tilde{G}_{i\sigma}^{00}(\omega), \tag{12}
\]

where the self–energy operator \( \tilde{\Sigma}_{k\ell}(\omega) \) is defined by the equation

\[
\tilde{T}_{i\sigma}(\omega) = \tilde{\Sigma}_{i\alpha}(\omega) + \sum_{k\ell} \tilde{\Sigma}_{k\ell}(\omega) \tilde{G}_{i\sigma}^{00}(\omega) \tilde{T}_{j\sigma}(\omega). \tag{13}
\]

Here the scattering matrix is given by
\[
\tilde{T}_{ij\sigma}(\omega) = Q^{-1} \langle \tilde{Z}_{i\sigma}^{(\text{irr})} | \tilde{Z}_{j\sigma}^{(\text{irr})^{+}} \rangle_{\omega} Q^{-1} .
\] (14)

From Eq. (13) it follows that the self-energy operator is given by the irreducible part of the scattering matrix (14) that has no parts connected by the single zero-order GF (11):

\[
\tilde{\Sigma}_{ij\sigma}(\omega) = Q^{-1} \langle \tilde{Z}_{i\sigma}^{(\text{irr})} | \tilde{Z}_{j\sigma}^{(\text{irr})^{+}} \rangle_{\omega} Q^{-1} .
\] (15)

Equations (11), (12) and (15) give an exact representation for the single-electron GF (7). However, to solve the self-consistent system of equations one has to introduce an approximation for the many-particle GF in the self-energy matrix (15) which describes inelastic scattering of electrons on spin and charge fluctuations.

**B. Self-Consistent Equations**

Here we derive a self-consistent system of equations in MFA. To calculate the frequency matrix (10) we use the equation of motion for the HO:

\[
\left( \frac{i}{\hbar} \frac{d}{dt} + \mu \right) X_{i}^{\sigma} = -\sum_{l,\sigma'} t_{il} B_{l\sigma\sigma'} X_{l}^{\sigma'} + \frac{1}{2} \sum_{l,\sigma'} J_{il} (B_{l\sigma\sigma'} - \delta_{\sigma\sigma'}) X_{l}^{\sigma'} ,
\] (16)

where we introduced the operator

\[
B_{l\sigma\sigma'} = (X_{l}^{00} + X_{l}^{\sigma\sigma}) \delta_{\sigma\sigma'} + X_{l}^{\sigma\sigma'} \delta_{\sigma\sigma'} = (1 - \frac{1}{2} n_{l} + S_{l}^{\sigma}) \delta_{\sigma\sigma'} + S_{l}^{\sigma \sigma'} \delta_{\sigma\sigma'} .
\] (17)

The Bose-like operator (17) describes electron scattering on spin and charge fluctuations caused by the kinematic interaction (the first term in (16)) and by the exchange spin-spin interaction (the second term in (16)).

By performing commutations in (10) we get for the normal and the anomalous parts of the frequency matrix:

\[
E_{ij\sigma}^{(1)} = -\mu \delta_{ij} + \delta_{ij} \sum_{l} \left\{ t_{il} \langle X_{l}^{\sigma0} X_{l}^{\sigma'} \rangle / Q \right. \\
+ \frac{1}{2} J_{il} (Q - 1 + \chi_{il}^{\sigma'}) / Q \left. \right\} \\
- t_{ij} (Q + \chi_{ij}^{\sigma} / Q) - \frac{1}{2} J_{ij} \langle X_{j}^{\sigma0} X_{i}^{\sigma'} \rangle / Q
\] (18)

\[
E_{ij\sigma}^{(2)} = \Delta_{ij\sigma} = \delta_{ij} \sum_{l} t_{il} \langle X_{l}^{\sigma8} X_{l}^{\sigma0} \rangle / Q \\
- \frac{1}{2} J_{ij} \langle X_{j}^{\sigma8} X_{i}^{\sigma0} \rangle / Q .
\] (19)

Here in calculation of the correlation function for the normal component of the frequency matrix:

\[
\sum_{\sigma'} \langle B_{i\sigma\sigma'} B_{j\sigma'\sigma} \rangle =
\langle (1 - \frac{1}{2} n_{i} + S_{i}^{\sigma}) (1 - \frac{1}{2} n_{j} + S_{j}^{\sigma'}) + S_{i}^{\sigma} S_{j}^{\sigma'} \rangle
= \langle (1 - \frac{1}{2} n_{i}) (1 - \frac{1}{2} n_{j}) \rangle + \langle S_{i} S_{j} \rangle = Q^{2} + \chi_{ij}^{\sigma} ,
\]

we introduce the charge- and spin-fluctuation correlation functions

\[
\chi_{ij}^{cs} = \frac{1}{4} \langle \delta n_{i} \delta n_{j} \rangle + \langle S_{i} S_{j} \rangle ,
\]

with \( \delta n_{i} = n_{i} - \langle n_{i} \rangle \). Further we neglect the charge fluctuations, \( \langle \delta n_{i} \delta n_{j} \rangle \approx 0 \), but take into account spin correlations given by spin correlation functions for the nearest (\( \chi_{1s} \)) and the next-nearest (\( \chi_{2s} \)) neighbor lattice sites:

\[
\chi_{1s} = \langle S_{i} S_{i+1} \rangle , \quad \chi_{2s} = \langle S_{i} S_{i+2} \rangle ,
\] (20)

where \( a_{1} = (\pm a_{x}, \pm a_{y}) \) and \( a_{2} = (\pm a_{x}, \pm a_{y}) \). In a paramagnetic state assumed here they depend only on the distance between the lattice sites.

In the \( k \)-representation for the GF

\[
G_{\alpha\beta}^{(0)}(k, \omega) = \sum_{j} G_{\alpha\beta}^{(0)}(j, \omega) e^{-ikj} ,
\]

we get for the zero-order GF (11):

\[
\hat{G}_{\sigma}^{(0)}(k, \omega) = Q \{ \omega \tilde{\tau}_{0} - (\varepsilon_{k} - \hat{\mu}) \tilde{\tau}_{3} - \Delta_{k}^{(0)} \tilde{\tau}_{1} \}^{-1}
= Q \frac{\omega \tilde{\tau}_{0} + (\varepsilon_{k} - \hat{\mu}) \tilde{\tau}_{3} + \Delta_{k}^{(0)} \tilde{\tau}_{1}}{Q^{2} - (\varepsilon_{k} - \hat{\mu})^{2} - |\Delta_{k}^{(0)}|^{2} .}
\] (21)

where \( \tilde{\tau}_{0} \), \( \tilde{\tau}_{1} \), \( \tilde{\tau}_{3} \) are the Pauli matrices. The quasiparticle energy \( \varepsilon_{k} \) and the renormalized chemical potential \( \hat{\mu} = \mu - \delta \mu \) in the MFA are defined by the frequency matrix (18)

\[
\varepsilon_{k} = -\nu(k) - \frac{2J}{N} \sum_{q} \gamma(k-q) N_{q} ,
\] (22)

\[
\delta \mu = \frac{1}{N} \sum_{q} t(q) N_{q} - 2J(n/2 - \chi_{1s}/Q) ,
\] (23)

where \( J(q) = 4J (\cos a_{x} q_{x} + \cos a_{y} q_{y}) \), \( \gamma(k) = (1/2)(\cos a_{x} q_{x} + \cos a_{y} q_{y}) \), \( \gamma'(k) = \cos a_{x} q_{x} \cos a_{y} q_{y} \), while the renormalized hopping integral is given by

\[
\tilde{\nu}(k) = 4t \gamma(k) Q (1 + \chi_{1s}/Q^{2}) + 4t' \gamma'(k) Q (1 + \chi_{2s}/Q^{2}) .
\] (24)

The average number of electrons in Eqs. (22), (23) in the \( k \)-representation is written in the form:

\[
n_{k,\sigma} = \langle X_{k}^{\sigma0} X_{k}^{\sigma0} \rangle = Q_{N} \chi_{k}^{\sigma} .
\]

It should be pointed out that the renormalization of the hopping parameter (24) caused by the spin correlation.
functions (20) are essential at low doping when short-range antiferromagnetic correlations are strong. For instance, for hole doping \( \delta = 1 - n \simeq 0.05 \) and \( Q = (1 + \delta)/2 \simeq 0.53 \), the correlation functions are estimated as \( \chi_{1s} \simeq -0.3, \chi_{2s} \simeq 0.2 \), which results in complete suppression of the nearest neighbors hopping, while the next-neighbor hopping is quite large:

\[
t_{\text{eff}} = t \left[ Q \right] \left[ Q + \chi_{1s}/Q^2 \right] \simeq 0,
\]

\[
t'_{\text{eff}} = t' \left[ Q \right] \left[ Q + \chi_{2s}/Q^2 \right] \simeq 0.9 t'.
\]

For large doping the antiferromagnetic correlations are suppressed and the nearest neighbor hopping prevails: \( t_{\text{eff}}/t'_{\text{eff}} \simeq t/t' \gg 1 \).

The superconducting gap \( \Delta_k^s \) in Eq. (21) is defined by the anomalous component of the the frequency matrix (19):

\[
\Delta_k^s = \frac{2}{NQ} \sum_q \left[ t(q) - \frac{1}{2} J(k - q) \right] \left( X_{-q}^{0s} X_{q}^{0s} \right), \tag{25}
\]

There are two contributions in Eq. (25) given by the \( k \)-independent kinematic interaction \( t(q) \) and the exchange interaction \( J(k - q) \). The kinematic interaction gives no contribution to the \( d \)-wave pairing in MFA, Eq. (25) (see\(^{34} \)), and we disregard it in the following equations\(^{48} \).

The anomalous correlation function in Eq. (25) can be easily calculated from the anomalous part of the GF (21):

\[
\langle X_{-q}^{0s} X_{q}^{0s} \rangle = -Q \frac{\Delta_k^s}{2E_q} \tanh \frac{E_q}{2T}, \tag{26}
\]

which results in the BCS-type equation for the gap function:

\[
\Delta_k^s = \frac{1}{N} \sum_q \left( J(k - q) \right) \frac{\Delta_k^s}{2E_q} \tanh \frac{E_q}{2T}, \tag{27}
\]

where \( E_k = \left( (\varepsilon_k - \mu)^2 + |\Delta_k^s|^2 \right)^{1/2} \) is the quasiparticle energy in the superconducting state. As was proved in\(^{39} \), the retardation effects for the exchange interaction are negligible and therefore there is no restriction in integration over the energy in Eq. (27). It means that all electrons in the conduction band participate in the superconducting pairing contrary to the BCS equation for the electron-phonon model where the energy integration and pairing are restricted to a narrow energy shell of the order of the phonon energy close to the Fermi energy.

The equation (27) is identical to the results in MFA of the diagram technique\(^{37} \), while the gap equation obtained in\(^{39} \) has an additional factor \( Q = (1 - n/2) \) which is spurious. This factor appears if we apply a simple decoupling procedure in the equation of motion (16) for GF instead of the projection technique given by Eq. (10). Writing the bosonic operators in the exchange interaction as a product of two fermionic operators: \( X_i^{\sigma} = X_i^{\sigma 0} X_i^{\sigma 0} \) and performing decoupling of the fermionic operators:

\[
\langle X_i^{\sigma 0} X_i^{\sigma 0} X_j^{\sigma 0} - X_j^{\sigma 0} X_i^{\sigma 0} X_j^{\sigma 0} \rangle \omega \simeq \langle X_i^{\sigma 0} | X_j^{\sigma 0} \rangle \omega, \tag{28}
\]

we obtain the same equation (27) for the gap function but with the additional \( Q \)-factor at the right hand side. In the decoupling we miss the normalization factor for the correlation functions in the denominator of the frequency matrix, Eq. (10), which cancels out with \( Q \) in the numerator of the corresponding GF (21). So, a rigorous way to apply MFA with a proper account of projected character of HO is to use the projection technique as discussed above.

The self-energy contribution (15) in the second order of the kinematic interaction is considered in\(^{36} \) while it is omitted in\(^{37} \). As discussed in\(^{36,39} \), it mediates the spin-fluctuation pairing and results in finite life-time effects for the quasiparticle spectrum giving rise to an incoherent contribution to the single-particle density of states. Here we shall not discuss further these self-energy effects since for comparison the GF approach with the slave-fermion technique it will be sufficient to consider only MFA for the gap equation (27).

### III. SLAVE-FERMION APPROACH

#### A. Slave-fermion representation

A number of auxiliary field representations has been proposed so far (see, e.g.,\(^{8 - 32} \)). In the slave-boson model\(^{8 - 10} \) the projected electron operator is expressed as a product of the auxiliary Bose field for the charge degree of freedom (holon) and the Fermi field for the spin degree of freedom (spinon). The main problem in this approach is the so-called constraint imposed by projected character of electronic operators in the \( t-J \) model which prohibits double occupancy of any lattice site. To treat the constraint a site-dependent Lagrange multiplier is introduced. However, to solve the problem the MFA is usually applied and the Lagrange multiplier is taken to be independent on the lattice site so the local constraint is replaced by a global one with uncontrollable consequences. In the slave-fermion method the charge degree of freedom is represented by spinless fermion operators, while to describe the spin degree of freedom the Bose field (Schwinger bosons\(^{13 - 22} \) or spin operators\(^{23 - 27} \) are used. The Schwinger boson representation though being physically meaningful for the Heisenberg model\(^{11,12} \) gives poor results for the doped case: the antiferromagnetic ground state persists up to very high doping and it does not reproduce the large Fermi surface as in the slave-boson method. In the slave-fermion and spin operator representation the magnetic properties of the model are described in a more reliable way (see, e.g.,\(^{25,29,31} \)).

Below we consider the slave-fermion hard-core (CP\(^{1} \)) boson representation proposed in\(^{27} \) and later on employed in\(^{28 - 31} \) to investigate different physical proper-
ties of cuprates within the t-J model. It has some advantage since the constraint of no double occupancy can be fulfilled without introducing the Lagrange multiplier. To decouple the charge and spin degrees of freedom for physical electrons the HO in the theory is represented as a product of a spinless fermion for the charge degree of freedom (holon) and a hard-core boson for the spin degree of freedom (spinon):

\[ X_i^{\sigma} = h_i^{\uparrow} b_{i\sigma}, \quad X_i^{\sigma_0} = h_i^{\downarrow} b_{i\sigma}^{\dagger} \]

which have the following commutation relations:

\[ h_i^{\uparrow} h_j + h_j h_i^{\uparrow} = \delta_{i,j}, \quad b_{i\sigma} b_{j\sigma}^{\dagger} - b_{j\sigma}^{\dagger} b_{i\sigma} = \delta_{i,j}(1 - 2 h_i^{\uparrow} b_{i\sigma}) \]

The hard-core bosons are Pauli operators which commute on different lattice sites and anticommute on the same lattice site prohibiting double occupancy. The Pauli operators can be also represented by the spin-lowering operators for spin-1/2:

\[ b_{i\uparrow}^{\dagger} = S_i^{\uparrow}, \quad b_{i\downarrow} = S_i^{\downarrow}, \quad \text{or} \quad b_{i\uparrow} = S_i^{\uparrow}, \quad b_{i\downarrow} = S_i^{\downarrow} \]

The on-site electron local constraint

\[ \sum_\sigma X_i^{\sigma_0} X_i^{\sigma} = \sum_\sigma X_i^{\sigma_0} = h_i^{\uparrow} h_i^{\downarrow} \sum_\sigma b_{i\sigma}^{\dagger} b_{i\sigma} = h_i h_i^{\dagger} = 1 - h_i^{\dagger} h_i \leq 1 \]

is satisfied here since for the Pauli operators at any lattice site we have the equation

\[ \sum_\sigma b_{i\sigma}^{\dagger} b_{i\sigma} = S_i^{\uparrow} S_i^{\downarrow} + S_i^{\downarrow} S_i^{\uparrow} = 1, \]

and the spinless holon number \( n_i^{(h)} = h_i^{\dagger} h_i \) can be equal to 1 or 0.

However, the spin-charge separation imposed by the representation results in extra degrees of freedom: a spin 1/2 is assigned to any lattice site including an empty site, while in the HO representation, Eq. (2), we have only 3 states: an empty state and a filled state with spin ±1/2. To cure this defect one should introduce a projection operator to exclude the unphysical states of Pauli spinor representations (in 23-26). Otherwise the commutation relations for the original HO, Eq. (4), and their representation, Eq. (29), will give different results. For instance, for the physical electrons which are described by HO we have

\[ \{X_i^{0\sigma}, X_i^{0\bar{\sigma}}\} = X_i^{0\sigma} + X_i^{0\bar{\sigma}} = 1 - X_i^{\bar{\sigma}0}, \]

\[ \sum_\sigma \langle \{X_i^{0\sigma}, X_i^{0\bar{\gamma}}\} \rangle = 2 - \langle n_i \rangle = 1 + \delta \]

where the hole doping concentration \( \delta = 1 - n \). If we use the representation (29) then we can write the commutation relations as

\[ \{X_i^{0\sigma}, X_i^{0\bar{\sigma}}\} = \{h_i^{\uparrow} b_{i\sigma}, h_i^{\downarrow} b_{i\bar{\sigma}}^{\dagger}\} = h_i^{\uparrow} h_i^{\downarrow} (1 - 2 h_i^{\uparrow} h_i^{\downarrow}) b_{i\sigma} b_{i\bar{\sigma}}, \]

\[ \sum_\sigma \{h_i^{\uparrow} b_{i\sigma}, h_i^{\downarrow} b_{i\bar{\sigma}}^{\dagger}\} = 1, \]

where we have used Eq. (31). For the average value in Eq. (33) we get, respectively,

\[ \sum_\sigma \langle h_i^{\uparrow} b_{i\sigma}, h_i^{\downarrow} b_{i\bar{\sigma}}^{\dagger}\rangle = 1, \]

which contradicts to the rigorous result for HO, Eq. (32). For the average number of electrons in the representation (29) by using the definition (6) we can write

\[ n = \langle n_i \rangle = \sum_\sigma \langle X_i^{0\sigma} X_i^{0\bar{\sigma}}\rangle = \sum_\sigma \langle h_i b_{i\sigma}^{\dagger} h_i^{\dagger} b_{i\bar{\sigma}}\rangle = \langle h_i h_i^{\dagger} \rangle = 1 - \delta, \]

which coincides with the definition (6) if we take for the hole number operator the following definition: \( X_i^{00} = h_i^{\dagger} h_i \). However, this definition is not unique. For instance we can write: \( X_i^{00} = X_i^{0\sigma} X_i^{0\bar{\sigma}} = X_i^{0\sigma} X_i^{0\bar{\bar{\sigma}}}, \) which results in the equation: \( X_i^{00} = (1/2) h_i^{\dagger} h_i \) if we use the representation (29) and the condition (31). So the double counting of empty sites results in controversial equation for an average number of electrons which is therefore valid only with accuracy of \( \pm \delta \).

### B. Mean-field approximation

Let us consider the resonating valence bond (RVB) state in the original Hamiltonian (5) as proposed by Baskaran et al. for this we should write the Bose-like spin operators in the exchange energy in \( H_{t-J} \) (5) as a product of two single-particle Fermi-like operators: \( X_i^{\sigma} = X_i^{\sigma_0} X_i^{\bar{\sigma}} \) and introduce the singlet operators

\[ b_{ij}^{\dagger} = b_{ij} = \frac{1}{\sqrt{2}} (X_i^{\sigma} X_j^{\bar{\sigma}} - X_i^{\bar{\sigma}} X_j^{\sigma}) \]

Then using MFA for the singlet operators in the exchange interaction of the \( t-J \) model we get the RVB effective Hamiltonian:

\[ H_J = \frac{1}{2} \sum_{i \neq j} J_{ij} (X_i^{\sigma} X_j^{\bar{\sigma}} - X_i^{\bar{\sigma}} X_j^{\sigma}) = \frac{1}{2} \sum_{i \neq j} J_{ij} b_{ij}^{\dagger} b_{ij} \]

\[ \approx - \frac{1}{2} \sum_{i \neq j} J_{ij} (B_{ij}^{\dagger} b_{ij} + b_{ij}^{\dagger} B_{ij} - |B_{ij}|^2), \]

where we introduced the RVB order parameter:

\[ B_{ij}^{\dagger} \equiv B_{ij} = \langle b_{ij} \rangle = \frac{1}{\sqrt{2}} (X_i^{\sigma} X_j^{\bar{\sigma}} - X_i^{\bar{\sigma}} X_j^{\sigma}). \]

Here we should point out that in MFA (37) a decoupling of the Hubbard operators on the same lattice site is used:
\[ X_i^+ X_j^- = X_i^{+0} X_j^{0-} X_j^{0+} X_i^{+0}, \]
\[ \Rightarrow \langle X_i^{+0} X_j^{0-} \rangle X_j^{0+} X_i^{+0}, \]

which is not unique and results in uncontrollable approximation since the Hubbard operators obey the multiplication rule: \( X_i^{a\gamma} X_j^{b\gamma} = X_i^{a\gamma} X_j^{b\gamma} \), and any intermediate state \( \beta \) can be used in the decoupling.

To obtain a self-consistent equation for the order parameter we assume a spin-charge separation as it is usually done in the slave-particle methods by applying a decoupling of spinon and holon degrees of freedom introduced in Eq. (29):

\[ B_{ij} = \frac{1}{\sqrt{2}} (h_i b_{i\uparrow} h_j b_{j\uparrow} - h_i b_{i\downarrow} h_j b_{j\uparrow}) \]
\[ \simeq \langle h_i h_j \rangle \frac{1}{\sqrt{2}} (b_{i\uparrow} b_{j\uparrow} - b_{i\downarrow} b_{j\uparrow}) \equiv F_{ij} \varphi_{ij}, \]

\[ b_{ij} = \frac{1}{\sqrt{2}} (h_i b_{i\uparrow} h_j b_{j\uparrow} - h_i b_{i\downarrow} h_j b_{j\uparrow}) \simeq h_i h_j \varphi_{ij}. \]

Within these approximations we obtain the effective Hamiltonian for holons

\[ H_h \simeq -\sum_{i \neq j} \tilde{t}_{ij} h_i h_j^+ - \mu \sum_i h_i h_i^+ \]
\[ -\frac{1}{2} \sum_{i \neq j} \tilde{J}_{ij} (\langle h_j h_i \rangle h_i^+ h_j^+ + h_j h_i (h_i h_i^+)), \]

where the effective hopping parameter and the exchange energy are given by

\[ \tilde{t}_{ij} = t_{ij} (b_{i\uparrow} b_{j\uparrow} + b_{i\downarrow} b_{j\downarrow}) = t_{ij} \langle S_i^+ S_j^- + S_i^- S_j^+ \rangle \]
\[ \tilde{J}_{ij} = J_{ij} |\varphi_{ij}|^2 = J_{ij} \frac{1}{2} |(b_{i\uparrow} b_{j\uparrow} - b_{i\downarrow} b_{j\downarrow})|^2. \]

To obtain a phase diagram for the RVB order parameter \( B_{ij} = F_{ij} \varphi_{ij} \) as a function of temperature \( T \) and hole doping \( \delta \) one should solve a self-consistent system of equations for both order parameters, holon \( F_{ij} \) and spinon \( \varphi_{ij} \) ones. Here we consider only equations for the holon order parameter by suggesting that there exists a region of \( (T, \delta) \) where the spinon order parameter is nonzero.

Introducing \( k \)-vector representation for the correlation functions:

\[ F_{ij} = \langle h_i h_j \rangle = \frac{1}{N} \sum_k \exp(i \mathbf{k}(i-j)) F(k) \]
\[ = \frac{i}{N} \sum_k \sin(k(i-j)) \langle h_k h_{-k} \rangle, \]
\[ \varphi_{ij} = \frac{1}{\sqrt{2}} (b_{i\uparrow} b_{j\downarrow} - b_{i\downarrow} b_{j\uparrow}) = \frac{1}{N} \sum_k \exp(i \mathbf{k}(i-j)) \varphi(k) \]
\[ = \frac{2i}{N} \sum_k \sin(k(i-j)) \frac{1}{\sqrt{2}} \langle h_k b_{-k} \rangle, \]

the Hamiltonian (42) can be written in \( k \)-space as

\[ H_h = \sum_k (\varepsilon(k) - \mu_h) h_{k\uparrow} h_{k\downarrow} \]
\[ -\frac{1}{2} \sum_k \{ \Delta(k) h_{k\uparrow} h_{k\downarrow}^+ + \Delta^+(k) h_{-k\downarrow} h_{k\uparrow} \}, \]

where the chemical potential for holons \( \mu_h = -\mu \) and the holon spectrum \( \varepsilon(k) \) according to Eq. (43) is written in the form (cf. Eq. (24)):

\[ \varepsilon(k) = \tilde{t}(k) = 4t \gamma(k) 2(S_i^+ S_i^- + S_i^- S_i^+) \]
\[ + 4t' \gamma'(k) 2(S_i^+ S_i^- + S_i^- S_i^+), \]
\[ \Delta(k) = -\Delta(-k) = \frac{1}{N} \sum_{\mathbf{q}} \tilde{J}(k - \mathbf{q}) \langle h_{\mathbf{q}} h_{-\mathbf{q}} \rangle. \]

In Eqs. (47), (48) we used notation \( \varepsilon(k), \Delta(k) \) for the holon spectrum and the holon gap to distinguish them from that ones of physical electrons in Sec. 2.2.

The normal and anomalous correlation functions can be easily calculated for the BCS-type Hamiltonian (46):

\[ \langle h_{\mathbf{q}}^+ h_{\mathbf{q}} \rangle = \frac{1}{2} \left( 1 - \frac{\varepsilon(\mathbf{q}) - \mu_h}{E(\mathbf{q})} \tanh \frac{E(\mathbf{q})}{2T} \right), \]
\[ F(\mathbf{q}) = \langle h_{\mathbf{q}} h_{-\mathbf{q}} \rangle = \frac{\Delta(\mathbf{q})}{E(\mathbf{q})} \tanh \frac{E(\mathbf{q})}{2T}, \]

where the quasiparticle spectrum \( E(\mathbf{k}) = [\varepsilon(\mathbf{k}) - \mu_h]^2 + |\Delta(\mathbf{k})|^2 |^{1/2} \). From these equations follow the self-consistent equations for the gap function and the average number of holes \( \delta \) which defines the holon chemical potential \( \mu_h \):

\[ \Delta(k) = \frac{1}{N} \sum_{\mathbf{q}} \tilde{J}(k - \mathbf{q}) \Delta(\mathbf{q}) \tanh \frac{E(\mathbf{q})}{2T}, \]
\[ \delta = \frac{1}{N} \sum_{\mathbf{q}} \langle h_{\mathbf{q}}^+ h_{\mathbf{q}} \rangle \]
\[ = \frac{1}{N} \sum_{\mathbf{q}} \frac{1}{2} \left( 1 - \frac{\varepsilon(\mathbf{q}) - \mu_h}{E(\mathbf{q})} \right) \tanh \frac{E(\mathbf{q})}{2T}. \]

These equations are identical to the results obtained in\(^{32}\) where the path-integral representation for the \( t-J \) model was applied and an effective BCS-type Hamiltonian analogous to Eq. (46) was derived. Starting from the MFA for RVB order parameter as in Eq. (37), the authors introduced MFA for the spinon auxiliary field that resulted in the spin-charge separation as in our Eq. (41). Therefore, we have proved that the results of the path-integral representation for the \( t-J \) employed in Ref.\(^{32}\) are equivalent to the MFA for the slave fermion - hard-core boson approach considered in this section.
C. Holon Green functions

To avoid uncontrollable approximation caused by the decoupling of the Hubbard operators on the same lattice site in Eq. (39) used in MFA for RVB state in the Hamiltonian (37), in the present section we consider projection technique for the holon GF for the spinon-holon model. By using the spinless fermion hard-core boson representation (29) we write the Hamiltonian of the t-J model (5) as follows:

\[ H_{t-J} = - \sum_{i \neq j} t_{ij} h_i h_j^+ (S_i^+ S_j^- + S_i^- S_j^+) \]

\[ - \mu \sum_i h_i h_i^+ - \frac{1}{4} \sum_{i \neq j} J_{ij} h_i h_j^+ h_i^+ h_j \]

\[ \times (S_i^+ S_j^- - S_i^- S_j^+)(S_i^+ S_j^- - S_i^- S_j^+). \]  

(53)

To obtain equation for the holon GF we apply the projection technique described in Sec. 2.1. By introducing the matrix GF for holon operators:

\[ \tilde{\chi}^{(h)}(t - t') \]

we can obtain equation of motion for the GF as discussed in Sec. 2.1 (see Eqs. (8)–(11)). For the zero-order GF in MFA we get the following result:

\[ \tilde{\chi}^{(h,0)}(k, \omega) = \frac{\omega \tau_0 + (\hat{\varepsilon}(k) - \mu_h) \tau_3 + \hat{\Delta}(k) \tau_3}{\omega^2 + (\hat{\varepsilon}(k) - \mu_h)^2 - |\hat{\Delta}(k)|^2}, \]  

(54)

where the holon dispersion and the gap function are given by the equations:

\[ \varepsilon(k) = \tilde{\varepsilon}(k) - \frac{1}{N} \sum_q \{ \tilde{J}^+(k - q) - \tilde{J}^-(0) \} \langle b^+_q h_q \rangle, \]  

\[ \hat{\Delta}(k) = \frac{1}{N} \sum_q \tilde{J}^-(k - q) \langle h_q h_{-q} \rangle. \]  

(55)

(56)

In comparison with MFA for RVB state considered previously, in the GF approach we obtain additional renormalization for the holon dispersion, Eq. (47), while the gap equation has the same form, Eq. (48), but with a different renormalized exchange interaction:

\[ \tilde{J}^+_q = J_{ij} \frac{1}{2} \langle | S_i^+ S_j^- - S_i^- S_j^+ |^2 \rangle \]

\[ = J_{ij} \langle | S_i^+ S_j^- |^2 \rangle. \]  

(57)

By using equation of motion method instead of MFA for RVB state we managed to take into account spinon correlations in the effective holon interaction (57) and obtained a simple formula for it by using the identities: \( S_i^+ S_i^+ = 0, S_i^- S_i^- = (1/2) - S_i^+ \) in the last equation. From Eq. (57) we get the following estimation for the effective exchange energy for the nearest neighbors: \( J' \approx (0.5 \div 0.25) J \) if we assume the AFM Neel phase: \( \langle S_i^+ S_j^- \rangle = -1/4 \) or completely neglect AFM correlations: \( \langle S_i^+ S_j^- \rangle = 0 \), respectively.

IV. RESULTS AND DISCUSSION

A. Spectral density

Spectral density for physical electron excitations in the lower Hubbard subband within the t-J model which is defined by the equation

\[ A_\sigma(k, \omega) = -\frac{1}{\pi} \text{Im} \langle \langle X^0_\sigma | X^{0\sigma}_k \rangle \rangle_{\omega+i\epsilon}, \]  

(58)

satisfies the sum rule:

\[ \sum_{\sigma} \int_{-\infty}^{+\infty} d\omega A_\sigma(k, \omega) = \sum_{\sigma} \langle \langle X^{0\sigma}_\sigma | X^{0\sigma}_0 \rangle \rangle \]

\[ = 2Q = 1 + \delta. \]  

(59)

In the MFA for the GF in terms of the HO, Eq. (21), we get:

\[ A_\sigma(k, \omega) = Q u_k \delta(\omega - E_k) + Q v_k \delta(\omega + E_k), \]  

(60)

where

\[ u_k = \frac{1}{2} \left( 1 + \frac{\varepsilon_k - \tilde{\mu}}{E_k} \right), \]  

\[ v_k = \frac{1}{2} \left( 1 - \frac{\varepsilon_k - \tilde{\mu}}{E_k} \right). \]  

(61)

In MFA the spectral density (60) satisfies the sum rule (59) but have no incoherent background which appears if one takes into account the self-energy corrections as shown in Ref. 36.

In the spinon-holon representation (29) the spectral density, as follows from Eq. (34), does not obey the sum rule (59):

\[ \sum_{\sigma} \int_{-\infty}^{+\infty} d\omega A^{(sh)}_\sigma(k, \omega) = \sum_{\sigma} \int_{-\infty}^{+\infty} d\omega \{ -\frac{1}{\pi} \text{Im} \langle \langle h^+_i b_{i\sigma} | h^+_i b^+_{i\sigma} \rangle \rangle_{\omega+i\epsilon} \} \]

\[ = \sum_{\sigma} \langle \langle h^+_i b_{i\sigma} | h^+_i b^+_{i\sigma} \rangle \rangle = 1. \]  

(62)

MFA for the spinon-holon GF in Eq. (62) results in the spin-charge separation which defines the spectral density (62) as a convolution of the anticommutator holon and the commutator spinon GF38:

\[ A^{(sh)}(k, \omega) = \frac{1}{N} \sum_q \frac{1}{2\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{d\omega_1 d\omega_2}{\omega - \omega_1 - \omega_2} \]

\[ \times [ \tanh(\omega_1/2T) + \coth(\omega_2/2T) ] \]

\[ \times \text{Im} \langle \langle h^+_q | h^+_q \rangle \rangle_{\omega_1} \text{Im} \langle \langle S^\sigma_{k-q} | S^\sigma_{k-q} \rangle \rangle_{\omega_2}. \]  

(63)

In this representation the spectral density of physical electron excitations is given by a superposition of the
satisfy the condition (66) for the RVB order parameter. The condition (65) for the RVB order parameter reads

\[ B_{ij} = \frac{1}{N} \sum_{k} B(k) = \frac{1}{N} \sum_{k,q} \varphi(k - q) F^+(q) = 0, \]  

which also imposes a constraint on the symmetry of the spinon and holon order parameters. Since the symmetry of the holon pairing order parameter, Eq. (44), for spinless fermions is odd, imposed by their anticommutation relations, it results in the odd symmetry of the gap function \( \Delta(k) \) in Eq. (48). For a tetragonal lattice the symmetry is given by the odd two-dimensional irreducible representation \( E_u \) which can be modelled by the function:

\[ \Delta(k) \propto \eta^\pm_{ij}(k) = (\sin k_x \pm \sin k_y) \text{ as in the } p\text{-wave triplet pairing}. \]

The RVB order parameter \( B_{ij} = \langle b_{ij} \rangle = \varphi_{ij} F^+_{ij} \) as a product of two antisymmetric order parameters with \( E_u \) symmetry has either \( A \) (s-wave) or \( B_{ij} \) (d-wave) symmetry. Namely, if we adopt only the nearest neighbor pairing for both the order parameters:

\[ \varphi_{ij} = \varphi_{i,\mu+a_x} \{ \langle \delta_{,i+a_x} - \delta_{,i-a_x} \rangle \pm \langle \delta_{,i+a_y} - \delta_{,i-a_y} \rangle \} F^+ + F^+_{ij} = \varphi_{i,\mu+a_x} \{ \langle \delta_{,i+a_x} - \delta_{,i-a_x} \rangle \pm \langle \delta_{,i+a_y} - \delta_{,i-a_y} \rangle \}, \]

then for the Fourier transform of the RVB order parameter we get

\[ B(k) = \sum_{\mathbf{q}} \varphi(k - q) F^+(\mathbf{q}) = 2b \sum_{\mathbf{q}} \{\sin(k_x - q_x) \pm \sin(k_y - q_y)\} \{\sin q_x \pm \sin q_y\} = -b(\cos k_x \pm \cos k_y) \]

where \( b = \varphi_{i,\mu+a_x} F^+_{i,\mu+a_x} \) and the sign (+) in the last line corresponds to the same (different) signs in the first line. The solution (68) satisfies the condition (67) for both the s- and d-wave symmetry:

\[ \sum_k B(k) \propto \sum_k (\cos k_x \pm \cos k_y) = 0. \]

However, the explicit solution for the holon order parameter (50) shows its more complicated \( \mathbf{k} \)-dependence which proves that the frequently used model for the nearest neighbor pairing is inadequate. For the solution (50) the condition (67) for the RVB order parameter reads:

\[ \sum_{\mathbf{k},\mathbf{q}} \varphi(k - q) F^+(\mathbf{q}) = \]

\[ \sum_{\mathbf{k},\mathbf{q}} \varphi(k - q) \frac{\Delta^+(\mathbf{q})}{2E(\mathbf{q})} \tanh \frac{E(\mathbf{q})}{2T} = 0, \]

which also imposes a requirement on the symmetry of the singlet (RVB) order parameter which should have only the d-wave symmetry, \( B(k_x, k_y) = -B(k_y, k_x) \), to satisfy this condition.

It is interesting to compare results for the gap equation derived in MFA for the t-J model and that one for

B. Gap symmetry

For models with strong electron correlations the s-wave component of superconducting gap must be strongly suppressed due to on-site Coulomb correlations. For the t-J model it follows from the constraint of no double occupancy on a single site given by the identity:

\[ \langle \hat{c}_{i,\sigma} \hat{c}_{i,\sigma} \rangle = \frac{1}{N} \sum_{\mathbf{k}} \langle \hat{c}_{\mathbf{k},\sigma} \hat{c}_{\mathbf{k},-\sigma} \rangle = 0, \]

for the physical electron operators \( \hat{c}_{i,\sigma} = c_{i,\sigma}(1 - n_{i,\sigma}) \).

Since the anomalous correlation function \( \langle \hat{c}_{\mathbf{k},\sigma} \hat{c}_{\mathbf{k},-\sigma} \rangle \) is proportional to the gap function \( \Delta(k) \), Eq. (65) imposes a certain constraint on the symmetry of the gap function. In particular, for the solution (26) in MFA Eq. (65) reads

\[ \langle X_i^{0\sigma} X_i^{0\sigma} \rangle = \frac{1}{N} \sum_{\mathbf{q}} \langle X_{\mathbf{q}}^{0\sigma} X_{\mathbf{q}}^{0\sigma} \rangle = \]

\[ = \frac{Q}{N} \sum_{\mathbf{q}} \frac{\Delta^0_{\mathbf{q}}}{2E_{\mathbf{q}}} \tanh \frac{E_{\mathbf{q}}}{2T} = 0. \]

For a tetragonal lattice the Fermi surface (FS) is invariant under the \( C_4 \)-axis rotation in the \( \mathbf{k} \)-space and therefore to satisfy the condition (66) for \( E_{\mathbf{q}} > 0 \) the gap function \( \Delta^0_{\mathbf{q}} \) should change its sign along the FS. It means that the symmetric s-wave solution, \( \Delta_s(k_x, k_y) \propto (\cos q_x + \cos q_y) \), does not fit Eq. (66), while the d-wave solution with \( B_{1g} \) or \( B_{2g} \) symmetry, \( \Delta_d(k_x, k_y) = -\Delta_d(k_y, k_x) \), satisfy the condition (66). In general, Eq. (66) should be considered as a constraint on the symmetry of the gap function in the superconducting phase and solutions which violate this constraint should be disregarded.
the Hubbard model. By applying the projection technique for the GF for the Hubbard model one can get the following equation for the gap function (see, e.g., 39):

\[ \Delta_{ij\sigma} \propto \langle \chi_{ij}^{\nu N} \rangle = \langle c_{i\downarrow} c_{i\uparrow}^\dagger N_i \rangle, \]  

(70)

where we have used the identity for the Hubbard operators, \( \chi_{ij}^{\nu N} = \chi_{ij}^{\nu N} \chi_{ij}^{\downarrow} = c_{i\downarrow} c_{i\uparrow}^\dagger \). From Eq. (70) follows that the pairing occurs on one lattice site but in the different Hubbard subbands. By using equation of motion for the GF \( \langle \chi_{ij}^{\nu N} (t) | N_j (t') \rangle \) the anomalous correlation function \( \chi_{ij}^{\nu N} \) can be calculated without any decoupling that results in the same gap equation as in the MFA for the \( t\)-\( J \) model 39: \( \Delta_{ij\sigma} = J_{ij} \langle \chi_{ij}^{\nu N} X_{ij}^{\sigma} \rangle / Q \) where we have used the notation of the present paper. Therefore the symmetry constraint considered above is also applicable for the gap solutions obtained in the Hubbard model.

C. Superconducting \( T_c \)

In the present section we compare the pairing temperature \( T_c \) defined by the gap equation for physical electrons (27) and that one for holons, Eq. (51). We study \( T_c \) as a function of corresponding chemical potential \( \nu = \mu / W \) for a given value of the pairing interaction \( \lambda = J / W \) where all energies are measured in units of the renormalized half bandwidth for pairing fermions \( W = 4 \varepsilon \):

\[ e(k) = \frac{\varepsilon(k)}{W} = \gamma(k) + \tau \gamma'(k), \quad d(k) = \frac{\Delta(k)}{W}, \]  

(71)

where \( \tau = t_{eff} / t_{eff} \). The gap Eqs. (27), (51) (or (56)) can be written in the form

\[ d(k) = \frac{4\lambda}{N} \sum_q \frac{\gamma(k - q) d(q)}{2(e(q) - \nu)^2 + d^2(q)} \]

\[ \times \tanh \left( \frac{(e(q) - \nu)^2 + d^2(q)}{2T} \right), \]  

(72)

Let us consider \( T_c(\nu) \) for different pairing symmetry, \( d(k) = d_\alpha \eta_\alpha(k) \):

- \( s \)-wave: \( \eta_s(k) = \cos k_x + \cos k_y \),
- \( d \)-wave: \( \eta_d(k) = \cos k_x - \cos k_y \),
- \( p \)-wave: \( \eta_p(k) = \sin k_x \pm \sin k_y \).  

(73)

By integrating Eq. (72) with the corresponding symmetry parameter \( \eta_\alpha(k) \) we obtain Eq. (72) for \( T = T_c \) in the same form for any symmetry:

\[ \frac{1}{\lambda} = \frac{1}{N} \sum_k (\eta_\alpha(k))^2 \frac{1}{2(e(k) - \nu)} \tanh \frac{e(k) - \nu}{2T_c^{(\alpha)}} \]

\[ = \frac{1}{2} \int_{-1}^{+1} \frac{de}{e - \nu} N_\alpha(e) \tanh \frac{e - \nu}{2T_c^{(\alpha)}}, \]  

(74)

if we introduce an effective density of state (DOS) for the corresponding symmetry, \( \alpha = s, d, p \):

![FIG. 1. Effective density of states, Eq. (75), for \( s \) (solid line), \( d \) (dot-dashed line) and \( p \) (dashed line) symmetry.]

\[ N_\alpha(e) = \frac{1}{N} \sum_k (\eta_\alpha(k))^2 \delta(e - e(k)), \]

(75)

which is normalized

\[ \int_{-1}^{+1} de N_\alpha(e) = 1, \quad \text{since} \quad \frac{1}{N} \sum_k (\eta_\alpha(k))^2 = 1. \]

The results of calculation of the effective DOS, Eq. (75), for different symmetry \( \alpha = s, d, p \) is presented on Fig. 1 for \( t' = 0 \). From this dependence it is easy to draw a conclusion that the \( T_c(\nu) \) function will follow the dependence \( N_\alpha(\nu) \) for corresponding symmetry since the effective coupling constant \( V \simeq \lambda N_\alpha(\nu) \) reaches its maximum value at the maximum value of \( N_\alpha(\nu) \). In the logarithmic approximation a solution for \( T_c(\mu) \), Eq. (74), can be written in the conventional BCS form:

\[ T_c \simeq \mu(W - \mu) \exp(-1/V), \quad V \approx J N_\mu, \]  

(76)

but with the prefactor proportional to the Fermi energy, \( \mu = E_F \) which can result in high-\( T_c \). The highest \( T_c(\nu) \) appears for the \( d \)-wave pairing and the lowest \( T_c(\nu) \) for the \( p \)-wave pairing since in the former case the van Hove singularity gives a strong contribution for DOS \( (|\eta_d(k = (\pm \pi, 0))|^2 = 4) \), while in the latter case it is completely suppressed \( (|\eta_p(k = (\pm \pi, 0))| = 0) \). This estimation can be checked by a direct numerical solution of Eq. (74) (see, e.g., 35,36 and 49 for the \( p \)-wave symmetry).

V. CONCLUSION

In the present paper we consider superconducting pairing mediated by the exchange interaction which is generic
for system with strong electron correlations as cuprates. The mechanism of the exchange pairing is the lowering of kinetic energy of electron pairs due to their coherent hopping between different Hubbard subbands. Since the exciton energy of this hopping is much larger than the Fermi energy the retardation effects in the exchange interaction are negligible, which results in pairing of all electrons (holes) in the conduction band and a high-$T_c$ proportional to the Fermi energy, Eq. (76).

To obtain estimation for the superconducting $T_c$ it is tempting to use a mean-field approximation for the exchange interaction within the $t$-$J$ model. However, meaningful physical results can be obtained only if one takes into account strong electron correlations on a rigorous basis which is provided by the Hubbard operator technique. Any auxiliary field representations applied within MFA inevitably violates rigorous commutation relations for HO which may result in unphysical conclusions. In the present paper we have proved this by comparing the results for superconducting gap equation derived within the slave fermion - hard-core boson representation, Eq. (29), and HO technique for the Green functions (Sec. 2.1). In the former method the projected character of physical electron operators is neglected that results in double counting of empty states and violation of the sum rule, Eq. (62). The spin-charge separation which occurs in MFA, Eqs. (40), (41), results in separate equations for two order parameters for spinons and holons instead of one equation for physical electrons as in the GF method. The gap equation (51) for spinless fermions has only antisymmetric solutions which results in the $p$-wave gap for the quasiparticle excitations in superconducting state never observed in ARPES experiments in cuprates and much lower $T_c$ then for the $d$-wave pairing given by Eq. (27) for physical electrons.

The obtained results within MFA for the spinless fermion – hard core bosons in Sec. 2 appears to be identical to the path-integral representation for the $t$-$J$ model employed in Ref. 52. Therefore the latter results have the same flaws as discussed above which casts doubts on approaches based on the idea of spin-charge separation. Violation of the local constraint in the two-band Hubbard model may lead also to unphysical results, as shown in Ref. 50.

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