Coverage and Throughput Analysis with a Non-Uniform Femtocell Deployment

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Abstract

In this paper, we propose a non-uniform femtocell network deployment scheme, in which the femtocell base stations (BSs) are not utilized in the region within a prescribed distance away from any macrocell BSs. Based upon a stochastic geometric framework, the downlink coverage and single user throughput are characterized with tractable results. This scheme can be implemented through two ways, namely, femtocell deactivation and smart femtocell deployment. Specifically, we define an inner region with a prescribed radius from macrocell BSs. If femtocell deactivation is implemented then femtocells BSs within this region are turned off. If instead smart femtocell deployment is used then we re-allocate these inner region femtocell BSs to new random locations outside this region. For femtocell deactivation, we find that more than 50% of femtocell BSs can be turned off to save the femtocell BS energy consumption and the operating expense, without compromising on the coverage performance. On the other hand, smart femtocell deployment can significantly improve both the coverage and the single user throughput over the uniform femtocell deployment which is commonly considered in the literature. This work demonstrates the benefits obtained from a simple non-uniform femtocell deployment, which highlights the importance of deploying femtocell BSs selectively.

I. INTRODUCTION

In recent years, the cellular communications industry has experienced an unprecedented growth in the numbers of subscribers and data traffic. This significant trend challenges cellular

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service providers’ traditional macro-only network: A much more advanced and flexible network topology is desired \[1\]. To meet this demand, the concept of heterogeneous network is proposed to most efficiently use the dimensions of space and frequency. Its network topology is composed of a diverse set of wireless technologies, macrocell base stations (BSs) and low-power access points \[2\]. Being an important part of heterogeneous networks, femtocell access points (or called femtocell BSs) are low-power devices operating in the licensed spectrum \[3\], \[4\]. By off-loading wireless traffic from the macrocells to femtocells and decreasing the distance from users to BSs, femtocells bring a multitude of benefits, including improved user experiences and more efficient spatial reuse of spectrum \[5\]. In this work, we focus on the open-access femtocells, which are operated by the cellular service providers and offer femtocell access to all the users in the networks \[3\], \[6\].

The cellular coverage performance of a heterogeneous network strongly depends on the femtocell BSs’ locations. With a constant pre-configurable transmit power, which is a mode commonly implemented in current solutions \[7\], \[8\], the femtocell coverage range is significantly reduced when it is close to a macrocell BS site \[9\], resulting in poor off-loading effects. More interestingly, when the femtocell BSs are uniformly deployed at random, increasing the density of femtocell BSs does not give any noticeable improvement in the coverage probability \[9\]–\[11\]. The main cause of this phenomenon is the increased network interference from having more femtocell BSs in satisfactory macrocell areas. Hence, one interesting question raised from the above discussion is whether or not we can improve both coverage and throughput performances by not using the femtocell BSs at undesirable locations, in other words, utilizing femtocell BSs non-uniformly.

In this work, we consider an intuitive and straightforward idea: Define the **inner region** as the union of locations within a prescribed distance from any macrocell BSs, while the union of locations outside the inner region is defined as the **outer region**. We simply avoid using femtocell BSs within the inner region. Considering the practical constraints, there are two ways to implement this non-uniform deployment scheme:

1) **Femtocell Deactivation**: If the femtocell BSs are deployed uniformly in the network, as illustrated in Fig. 1(a), the ones located within the inner region are deactivated, marked as red filled squares therein. The de facto overall femtocell BS density is decreased as compared with the uniform deployment. This method can be achieved easily in practice: A listening model to the downlink signal level from the strongest BS \[12\] can be implemented to estimate the distance and then determine the femtocell BS’s on/off
(a) Femtocell Deactivation

(b) Smart Femtocell Deployment

Fig. 1. Cellular coverage for a two-tier network with the non-uniform deployment with macrocell BSs (triangles) and femtocell BSs (squares). In the inner region (shadow areas), the deactivated femtocell BSs are marked as red filled squares in Fig. 1(a) which are reinstalled at the new locations in the outer region for the smart femtocell deployment, and marked as blue filled squares in Fig. 1(b). The remaining femtocell BSs are always active (marked as blue non-filled squares).

status. One direct benefit from the deactivation is the reduced operating expense from the decreased femtocell BS energy consumption.

2) **Smart Femtocell Deployment:** In this method we deploy the femtocell BSs in the outer region only. Compared with the above “femtocell deactivation”, this smart deployment moves the inner-region femtocell BSs to the outer region, as illustrated in Fig. 1(b), where the blue filled squares represent the newly added active BSs in the outer region.

A. Approach and Contributions

In this work, we aim to show the impact of employing the proposed non-uniform femtocell BS deployment on both the downlink coverage and throughput performance of the two-tier femtocell networks. Specifically, one of our goals is to derive the coverage probability, or equivalently the distribution of signal-to-noise ratio (SINR), based on which the throughput achievable at a randomly chosen user can further be derived.

Fortunately, modeling BSs to be randomly placed points in a plane and utilizing stochastic geometry [13], [14] to study cellular networks has been used extensively as an analytical tool with improved tractability [15]–[17]. Recent works [9], [10], [18]–[23] have shown: Compared with the practical network deployment, modeling the cellular network with BS
locations drawn from a homogeneous Poisson Point Process (PPP) is as accurate as the traditional grid models. Moreover, the stochastic geometry model can provide the randomness introduced by the femtocell deployment, and also more tractable analytical results on both the coverage and throughput performances. Based on these reasons, this stochastic geometry tool is adopted to model the locations of BSs in this work.

The main contributions of this paper are as follows:

1) A simple non-uniform femtocell deployment scheme is proposed. In this scheme, femtocell BSs are not utilized in the region within a certain distance away from any macrocell BSs. Through two ways of implementations, namely, femtocell deactivation and smart femtocell deployment, we can guarantee that most of the active femtocell BSs are located in the relatively poor macrocell coverage areas. We provide the tractable probabilistic characterization of the downlink coverage and single user throughput at a randomly located mobile user in this new scheme. To our knowledge, it is the first work to derive mathematically tractable results on a non-uniform femtocell deployment.

2) For femtocell deactivation, the same cellular coverage performance as the traditional uniform femtocell deployment can be maintained if the size of the inner region is appropriately chosen. Our numerical result demonstrates that more than 50% of femtocell BSs can be turned off to save the femtocell BS energy consumption and thus operating expense, without compromising on the coverage performance.

3) For smart femtocell deployment, we show that both the coverage and single user throughput can be significantly improved over the uniform femtocell deployment. This finding demonstrates the performance improvements achievable by implementing a simple non-uniform femtocell deployment, which highlights the significance of selectively deploying the femtocell BSs, taking their relative locations with macrocell BSs into account.

It should be noticed that Haenggi introduced a non-uniform small-cell deployment model that also incorporates dependencies between different tiers of cellular network [24]. The superposed tiers in Haenggi’s model are deployed on the edges and at the vertices of the Voronoi cells formed by the macrocell BSs, i.e., the poorest macrocell coverage locations. In contrast, our scheme regards the previously defined outer region as the poor macrocell coverage area, in which open-access femtocell BSs are utilized. Additionally, our work provides not only the new model, but also the tractable analysis on both coverage and
throughput performance.

The remainder of the paper is organized as follows: In Section II, the cellular network model and the cell association model used in this work are introduced. Section III provides the tractable result for the downlink coverage and the achievable single user throughput of the proposed non-uniform femtocell deployment scheme. Section IV presents numerical results and we conclude the paper in Section V.

II. SYSTEM MODEL

A. Two-Tier Cellular Network Model

We consider a downlink heterogeneous cellular network employing an orthogonal multiple access technique and consisting of two tiers, i.e., macrocell and femtocell tiers (or called the first and second tiers) spatially distributed as two-dimensional processes $\Phi_1$ and $\Phi_2$, with different transmit powers $P_{tx,1}$ and $P_{tx,2}$. The same transmit power holds across each tier. The macrocell tier process $\Phi_1$ is modeled as a homogeneous PPP with density $\lambda_1$. Furthermore, the collection of mobile users, located according to an independent homogeneous PPP $\Phi_{MS}$ of density $\lambda_{MS}$, is assumed in this work. We consider the process $\Phi_{MS} \cup \{0\}$ obtained by adding a user at the origin of the coordinate system, which is the typical user under consideration. This is allowed by Slivnyak’s Theorem [13], which states that the properties observed by a typical point of the PPP $\Phi_{MS}$, are the same as those observed by the origin in the process $\Phi_{MS} \cup \{0\}$.

Here we use the standard power loss propagation model with path loss exponent $\alpha > 2$ and path loss constant $L_0$ at the reference distance $r_0 = 1$ m. We assume that the typical mobile user experiences Rayleigh fading from the serving and interfering BSs. The impact of fading on the signal power follows the exponential distribution with the unitary mean value. The noise power is assumed to be additive and constant with value $\sigma^2$.

B. Cell Association and Resource Allocation Model

As we described earlier, all BSs are assumed to be open access. Besides, we assume that mobile users are connected to the BS providing maximum long-term received power, which can be regarded as a widely used special case [18] of the general cell association model [9]. Specifically, the selected tier index $\omega$ can be given as

$$\omega = \arg\max_{i \in \{1,2\}} [P_{tx,i}L_0(R_i)^{-\alpha}] , \quad (1)$$
in which $R_i$ is the minimum distance from the $i$-th tier BSs to the typical mobile user at the origin, i.e., $R_i = \min_{x \in \Phi_i} \|x\|$. For simplicity, we use $P_i$ to replace the product of $P_{tx,i}$ and $L_0$, i.e., $P_i = P_{tx,i}L_0$, $i \in \{1, 2\}$, from now on.

We assume that all the BSs employ a saturated resource allocation model \cite{23}, \cite{25}, in which full buffer traffic model holds for every users; in other words, a BS always have data to transmit if there are mobile users camped on it. The available resources at each BS are assumed to be allocated evenly among all its associated mobile users, in order to simulate the Round-Robin scheduling with the most fairness.

C. Received SINR for Data Channels

Focusing on cellular data channels in this work, we assume that no interference is generated from the BSs without any user associated, since the frequency-time blocks will be left blank if they are not allocated for data transmissions in practical scenarios \cite{26}. We use \{$\Phi'_i\}_{i \in \{1, 2\}}$ to denote the process constructed by the remaining $i$-th tier BSs (i.e., the loaded BSs in the $i$-th tier) excluding the ones not associated with users (i.e., unloaded BSs).

By employing the cell association model described in Subsection II-B to choose the serving cell, the downlink received SINR at the typical user can be expressed as

$$\text{SINR} = \frac{P_\omega h(R_\omega)^{-\alpha}}{I + \sigma^2},$$

(2)

where $I = \sum_{i \in \{1, 2\}} I_i = \sum_{i \in \{1, 2\}} \sum_{x \in \Phi'_i \setminus \{x_o\}} P_i h_x \|x\|^{-\alpha}$ is the cumulative interference from all the loaded BSs (except the BS at $x_o$ from $\omega$-th tier serving for the mobile user at $o$), and $I_i$ is defined as the interference component from $i$-th tier. In our study, no intracell interference is incorporated since we assume that the orthogonal multiple access is employed among intracell users.

The coverage probability at the typical user is defined as $p_c(T) = P[\text{SINR} > T]$, i.e., the probability of a target SINR $T$ (or SINR threshold) achievable at the typical user. This coverage probability is also exactly the complementary cumulative distribution function (CCDF) of the received SINR.

D. Uniform Femtocell Deployment

Before proposing the non-uniform femtocell deployment scheme, we firstly focus on the traditional uniform femtocell deployment, where all the femtocell BSs in $\Phi_2$ stay active and are located as a homogeneous PPP with density $\lambda_2$. Following the analysis in \cite{9}, \cite{23}, the
results for this uniform femtocell deployment are briefly presented here for the purpose of comparison.

1) Mobile User Resource Sharing: Firstly, the per-tier association probabilities for the uniform femtocell deployment, i.e., the probabilities for the typical user to associate macrocell and femtocell tiers, denoted by \( Q_{1,u} \) and \( Q_{2,u} \), were derived in [9]:

\[
Q_{1,u} = \frac{\lambda_1}{\lambda_1 + \lambda_2 \left( \frac{P_2}{P_1} \right)^{2/\alpha}} \quad \text{and} \quad Q_{2,u} = \frac{\lambda_2}{\lambda_1 \left( \frac{P_1}{P_2} \right)^{2/\alpha} + \lambda_2}.
\] (3)

As proved in [23], the area \( C_{i,u} \) of the \( i \)-th tier cells can be well approximated by the Voronoi cell area formed by a homogeneous PPP with the density value \( \lambda_i / Q_{i,u} \), i.e.,

\[
C_{i,u} \approx C_0 \left( \frac{\lambda_i}{Q_{i,u}} \right),
\]

in which \( C_0(y) \) is the area of a typical Voronoi cell of a homogeneous PPP with density \( y \). We define the equivalent density \( \lambda_{i,eq,u} \) to denote the value \( \lambda_i / Q_{i,u} \), i.e.,

\[
\lambda_{1,eq,u} = \lambda_1 + \lambda_2 \left( \frac{P_2}{P_1} \right)^{2/\alpha} \quad \text{and} \quad \lambda_{2,eq,u} = \lambda_1 \left( \frac{P_1}{P_2} \right)^{2/\alpha} + \lambda_2.
\]

For the distribution of Voronoi cell area formed by a homogeneous PPP, there is no known closed form expression for its distribution [27]; however, some precise estimates can be conducted [28], [29]. By following the result in [29], the approximated probability density function (pdf) of \( i \)-th tier cell area \( C_{i,u} \) can be expressed by using gamma function

\[
f_{C_{i,u}}(x) \approx (b \lambda_{i,eq,u})^q x^{q-1} \exp(-b \lambda_{i,eq,u} x) / \Gamma(q).
\] (4)

Based on this approximation, the probability mass function (pmf) of the number of users in a randomly chosen \( i \)-th tier cell can be derived as

\[
P[N_{i,c,u} = n] = \int_0^\infty P[N_{i,c,u} = n \mid C_{i,u} = x] f_{C_{i,u}}(x) dx \approx \frac{b^q}{n!} \frac{\Gamma(n+q)}{\Gamma(q)} \cdot \frac{(\lambda_{MS})^n (\lambda_{i,eq,u})^q}{(\lambda_{MS} + b \lambda_{i,eq,u})^{n+q}}, \quad \text{for } i \in \{1, 2\},
\] (5)

where \( q = 3.61, b = 3.61 \) and \( \Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt \) is the standard gamma function.

On the other hand, given the condition that the typical user is enclosed in the cell, the pdf of \( i \)-th tier cell area \( C_{i,u} \) was derived in [22], [23], i.e.,

\[
f_{C_{i,u} \mid C_{i,u} \cap C_{i,u}}(x) = \frac{xf_{C_{i,u}}(x)}{E[C_{i,u}]},
\] (6)

which helps to obtain the distribution of the number of in-cell users sharing the resource with the typical user

\[
P[N_{i,u} = n] \approx \frac{b^q}{n!} \cdot \frac{\Gamma(n+q+1)}{\Gamma(q)} \left( \frac{\lambda_{MS}}{\lambda_{i,eq,u}} \right)^n \left( b + \frac{\lambda_{MS}}{\lambda_{i,eq,u}} \right)^{-(n+q+1)}, \quad \text{for } i \in \{1, 2\}.
\] (7)

Since the cell coverage regions are mutually disjoint and PPP \( \Phi_{MS} \) has the property of complete independence [13], the numbers of users in different cells are independent. For a
randomly chosen $i$-th tier cell, its probability to be an unloaded cell is $P[N_{i,c,u} = 0]$. Hence, the process $\Phi_i$ (i.e., the $i$-th tier loaded BSs excluding the BSs without users associated,) can be approximated by a homogeneous PPP with the density $\lambda_i = \lambda_i \cdot (1 - P[N_{i,c,u} = 0])$.

2) Coverage Probability: The coverage probability at the typical user served by the $i$-th tier can be expressed as

$$p_{c,i,u}(T) = P[\text{SINR}_{i,u} > T] = 2\pi \lambda_{i,eq,u} \int_{x>0} x \exp\left(-\frac{T x^\alpha \sigma^2}{P_i}\right) \cdot \exp\left[-\pi x^2 \left(\lambda_{i,eq,u} + \rho(T, \alpha) \lambda'_{i,eq,u}\right)\right] dx,$$

for $i \in \{1, 2\}$, (8)

where the function $\rho(x, \alpha)$ is defined as $\rho(x, \alpha) = \frac{x^{2/\alpha}}{\alpha} \int_{x^{-2/\alpha}}^{\infty} \frac{1}{1+u^{-\alpha}} du$, and $\{\lambda'_{i,eq,u}\}_{i \in \{1,2\}}$ can be defined as $\lambda'_{1,eq,u} = \lambda_{1,u} + \lambda_{2,u}(P_2/P_1)^{2/\alpha}$ and $\lambda'_{2,eq,u} = \lambda'_{1,u}(P_1/P_2)^{2/\alpha} + \lambda_{2,u}$.

3) Single User Throughput: Similar to the resource allocation model used in [23], the throughput achievable at the typical user served by the $i$-th tier can be derived as

$$\text{Pr}[R_{i,u} > \rho] = \text{Pr}\left[\frac{W}{N_{i,u} + 1} \log_2(1 + \text{SINR}_{i,u}) > \rho\right]$$

$$= \mathbb{E}_{N_{i,u}} \left[p_{c,i,u}(2^{(N_{i,u}+1)\rho/W} - 1)\right]$$

$$= \sum_{n=0}^{\infty} \text{Pr}[N_{i,u} = n] \cdot p_{c,i,u}(2^{(n+1)\rho/W} - 1), \text{ for } i \in \{1, 2\},$$

where the BS’s bandwidth $W$ are evenly allocated among all its associated users, namely, the typical user and the other $N_{i,u}$ in-cell users, as stated in the resource allocation model mentioned above.

III. NON-UNIFORM FEMTOCELL DEPLOYMENT

In this section, we analyze the proposed non-uniform femtocell deployment scheme, which aims to maintain all the active femtocell BSs deployed in the areas with unsatisfactory macrocell coverage. Based upon our analysis using the tool of the stochastic geometry cellular network model, the main results are the probabilistic characterizations of the downlink coverage and the achievable single user throughput presented in Subsections III-D and III-E respectively. The first three subsections (i.e., Subsections III-A, III-B and III-C) formally define the inner and outer regions and present important intermediate results.

As described in Section I there are two ways to implement the proposed non-uniform femtocell deployment scheme, i.e., femtocell deactivation and smart femtocell deployment. In femtocell deactivation, the femtocell BSs within the inner region are deactivated, while femtocell BSs are deployed in outer region only in smart femtocell deployment. From the
analysis viewpoint, the smart femtocell deployment scheme can be viewed as femtocell deactivation with an increased femtocell tier density in the outer region. Therefore, we will only focus on femtocell deactivation here and the performance analysis of smart femtocell deployment can be readily obtained by increasing the value of femtocell density.

A. Inner and Outer Regions

By implementing the femtocell deactivation scheme, all the femtocell BSs in the inner region stay inactive and other femtocell BSs in the outer region remain active. Specifically, the inner region $A_{inner}$ are defined as the union of locations where the distance from the nearest macrocell BS site is no larger than $D$, and the outer region $A_{outer}$ are defined as the union of locations where the distances from any macrocell BSs are larger than $D$, i.e.,

$$A_{inner} = \bigcup_{x \in \Phi_1} B(x, D) \quad \text{and} \quad A_{outer} = \mathbb{R}^2 \setminus A_{inner},$$

where $D$ is called the radius of inner region in this paper. By following PPP’s void probability [13], the typical user’s probabilities to be located in the inner region $A_{inner}$ and the outer region $A_{outer}$ are

$$\mathbb{P}[o \in A_{inner}] = 1 - \exp(-\pi \lambda_1 D^2) \quad \text{and} \quad \mathbb{P}[o \in A_{outer}] = \exp(-\pi \lambda_1 D^2),$$

Therefore, if the two tiers are originally distributed according to the homogeneous PPPs with the densities $\lambda_1$ and $\lambda_2$, the resultant active femtocell tier density after the proposed deployment scheme will be a function of the location, i.e.,

$$\lambda_2(x) = \begin{cases} 0 & \text{for} \ x \in A_{inner} \\ \lambda_2 & \text{for} \ x \in A_{outer}, \end{cases}$$

which makes the femtocell tier process $\Phi_2$ become a non-homogeneous PPP, and the average density for the active femtocell BSs over the whole plane become $\mathbb{E}[\lambda_2(x)] = \lambda_2 \cdot \mathbb{P}[o \in A_{outer}]$.

B. The Distribution of the Distance from Serving BS

First let us focus on the statistic characteristics of $\{X_i\}_{i \in \{1,2\}}$, the distance between the typical user and its serving BS, given the condition that the user is served by $i$-th tier. It should be noticed that $\{X_i\}_{i \in \{1,2\}}$ are random variables, because of the randomness introduced by the location of BSs. In this section, we will derive the pdf of $\{X_i\}_{i \in \{1,2\}}$ for the typical user in $A_{inner}$ and $A_{outer}$ respectively.
Lemma 1. The pdf of $X_1$, conditioned on that the typical user is in the inner region, can be approximated as

$$f_{X_1|o \in A_{\text{inner}}}(x) \approx \frac{2\pi \lambda_1 x}{[1 - \exp(-\pi \lambda_1 D^2)]} \exp(-\pi \lambda_1 x^2), \quad \text{for } x \leq D. \quad (13)$$

Proof: Since the event of $X_1 \leq x$ is the event of $R_1 \leq x$ based on the condition that the user is associated with macrocell, the cumulative distribution function (CDF) of $X_1$ for the inner region typical user can be expressed as

$$F_{X_1|o \in A_{\text{inner}}}(x) = P[X_1 \leq x | o \in A_{\text{inner}}]$$

$$= P[R_1 \leq x | \omega = 1, o \in A_{\text{inner}}]$$

$$\approx (a) P[R_1 \leq x | R_1 \leq D]$$

$$= \frac{1 - \exp(-\pi \lambda_1 x^2)}{1 - \exp(-\pi \lambda_1 D^2)}, \quad \text{for } x \leq D, \quad (14)$$

where the approximation in step $(a)$ is conducted by assuming that the inner region typical user always gets service from the macrocell tier. This assumption is reasonable from the practical implementation viewpoint: The transmit powers of macrocell tier BSs should be much larger than femtocell tier BSs, i.e., $P_1 \gg P_2$, which makes the fact that the inner region typical user is served by a femtocell BS with a small probability.

Then the pdf expression of $X_1$ in (13) can be found by differentiating the CDF, i.e.,

$$f_{X_1|o \in A_{\text{inner}}}(x) = dF_{X_1|o \in A_{\text{inner}}}(x)/dx,$$

which completes the proof. \qed

It should be noticed that the inner region users are assumed to be served by macrocell BSs only, which makes it unnecessary to derive the pdf of $X_2$ for inner region typical user, i.e., $f_{X_2|o \in A_{\text{inner}}}(x)$.

Lemma 2. The pdf of $X_1$, conditioned on that the typical user is in the outer region, can be approximated as

$$f_{X_1|o \in A_{\text{outer}}}(x) \approx \frac{2\pi \lambda_1 + \lambda_2 (\frac{P_2}{P_1})^{2/\alpha} x}{\exp \left(-\pi \lambda_1 + \lambda_2 (\frac{P_2}{P_1})^{2/\alpha} |x|^2\right)} \cdot \exp \left(-\pi \lambda_1 + \lambda_2 (\frac{P_2}{P_1})^{2/\alpha} |D|^2\right), \quad \text{for } x > D. \quad (15)$$

Proof: See Appendix A. \qed

Lemma 3. The pdf of $X_2$, conditioned on that the typical user is in the outer region, can be approximated as

$$f_{X_2|o \in A_{\text{outer}}}(x)$$
\[
\begin{align*}
M & \approx \begin{cases} 
M \cdot 2\pi \lambda_2 x \exp(-\pi \lambda_2 x^2) & \text{for } x \leq \left(\frac{P_2}{P_1}\right)^{1/\alpha} D \\
M \cdot \left[\frac{2\pi x \lambda_2}{\exp(-\pi \lambda_1 D^2)}\right] \cdot \exp\left(-\pi\left[\lambda_1\left(\frac{P_2}{P_1}\right)^{2/\alpha} + \lambda_2 x^2\right]\right) & \text{for } x > \left(\frac{P_2}{P_1}\right)^{1/\alpha} D,
\end{cases}
\end{align*}
\]

where the constant \(M\) is
\[
M = \frac{\exp(-\pi \lambda_1 D^2)}{\exp(-\pi \lambda_1 D^2) - \lambda_1 \exp\left(-\pi[\lambda_1 + \lambda_2(\frac{P_2}{P_1})^{2/\alpha}] D^2\right) / (\lambda_1 + \lambda_2(\frac{P_2}{P_1})^{2/\alpha})}.
\]

Proof: See Appendix B.

C. The Density of Loaded BSs

As stated in Section II, we assume that the interference signal at the data channel is only generated from the loaded BSs (i.e., the BSs having at least one user associated). We use the process \(\Phi_i\) to denote the \(i\)-th tier loaded BSs for the non-uniform femtocell deployment scheme, excluding the unloaded BSs without users associated. It should be noticed that \(\Phi_1\) is the process over the entire 2-D plane, while \(\Phi_2\) exists in the outer region only. We are interested in the density of \(\Phi_i\) over its deployment region, i.e., \(\lambda_i\), which will be used to estimate the interference process later on. Before obtaining this result, we will focus on the cell association probabilities firstly, i.e., the probability of the typical user served by the macrocell or femtocell tier.

Lemma 4. The probability of the typical user served by the macrocell tier, \(Q_1 = \mathbb{P}[\omega = 1]\), can be approximated by
\[
Q_1 \approx 1 - \exp(-\pi \lambda_1 D^2) + \frac{\lambda_1}{\lambda_1 + \lambda_2(\frac{P_2}{P_1})^{2/\alpha}} \cdot \exp\left(-\pi\left[\lambda_1 + \lambda_2(\frac{P_2}{P_1})^{2/\alpha}\right] D^2\right).
\]

Proof: From its definition, \(Q_1 = \mathbb{P}[\omega = 1]\) can be expanded as
\[
Q_1 = \mathbb{P}[\omega = 1, o \in A_{inner}] + \mathbb{P}[\omega = 1, o \in A_{outer}] \\
\approx \mathbb{P}[o \in A_{inner}] + \mathbb{P}[\omega = 1, o \in A_{outer}],
\]
in which the approximation is based upon the fact that the inner region typical user is associated with the femtocell tier with a small probability. The latter part of (19), i.e., the probability of accessing the macrocell tier and locating in \(A_{outer}\), is provided by
\[
\begin{align*}
\mathbb{P}[\omega = 1, o \in A_{outer}] &= \mathbb{P}[\omega = 1 | o \in A_{outer}] \cdot \mathbb{P}[o \in A_{outer}] \\
&= \mathbb{P}[\omega = 1 | R_1 = x, o \in A_{outer}] \cdot \int_{R_1|o \in A_{outer}} f_{R_1|o \in A_{outer}}(x) dx \cdot \mathbb{P}[o \in A_{outer}]
\end{align*}
\]
\[
= \int_{D}^{\infty} \mathbb{P}[R_2 > \left(\frac{P_2}{P_1}\right)^{1/\alpha} x \mid o \in A_{outer}] f_{R_1|R_2>D}(x) dx \cdot \mathbb{P}[o \in A_{outer}]
\]
\[
\approx \int_{D}^{\infty} \exp(-\pi \lambda_2 \left(\frac{P_2}{P_1}\right)^{2/\alpha} x^2) \cdot \frac{2\pi \lambda_1 r \exp(-\pi \lambda_1 r^2)}{\exp(-\pi \lambda_1 D^2)} dx \cdot \exp(-\pi \lambda_1 D^2)
\]
\[
= \frac{\lambda_1}{\lambda_1 + \lambda_2 \left(\frac{P_2}{P_1}\right)^{2/\alpha}} \cdot \exp\left( -\pi [\lambda_1 + \lambda_2 \left(\frac{P_2}{P_1}\right)^{2} D^2]\right),
\]
(20)

where we approximate the density of femtocell BSs in the vicinity of the outer region typical user as \(\lambda_2\) in step (a). By substituting (20) and the expression of \(P[o \in A_{inner}]\) in (11) into (19), we reach the result in (18), which completes the proof.

For cell association to femtocell tier, we only consider the case in which the typical user is in the outer region. This is because that the active femtocell BSs are all located in the outer region, and there is a very minimal chance for a inner region user to get associated to a femtocell BS.

**Lemma 5.** The probabilities of the outer region typical user associated with the macrocell tier and the femtocell tier, namely, \(Q_{1,outer} = \mathbb{P}[\omega = 1 \mid o \in A_{outer}]\) and \(Q_{2,outer} = \mathbb{P}[\omega = 2 \mid o \in A_{outer}]\), can be estimated by

\[
Q_{1,outer} \approx \frac{\lambda_1}{\lambda_1 + \lambda_2 \left(\frac{P_2}{P_1}\right)^{2/\alpha}} \cdot \exp\left( -\pi \lambda_2 \left(\frac{P_2}{P_1}\right)^{2} D^2\right).
\]
(21)

and

\[
Q_{2,outer} \approx 1 - \frac{\lambda_1}{\lambda_1 + \lambda_2 \left(\frac{P_2}{P_1}\right)^{2/\alpha}} \cdot \exp\left( -\pi \lambda_2 \left(\frac{P_2}{P_1}\right)^{2} D^2\right).
\]
(22)

**Proof:** This result \(Q_{2,outer} = \mathbb{P}[\omega = 2 \mid o \in A_{outer}]\) can be obtained by \(Q_{2,outer} = 1 - \mathbb{P}[\omega = 1, o \in A_{outer}] / \mathbb{P}[o \in A_{outer}]\), in which \(\mathbb{P}[\omega = 1, o \in A_{outer}]\) and \(\mathbb{P}[o \in A_{outer}]\) are provided in (20) and (11). Thus, \(Q_{1,outer}\) can be easily obtained because of \(Q_{1,outer} = 1 - Q_{2,outer}\).

**Lemma 6.** The density of \(\Phi_i'\) over the corresponding i-th tier deployment region can be approximated respectively by

\[
\lambda_i' \approx \lambda_i \left[1 - \left(\frac{\lambda_i b}{\lambda_{MS} Q_i + b \lambda_i}\right)^q\right],
\]
(23)

and

\[
\lambda_i' \approx \lambda_i \left[1 - \left(\frac{\lambda_i b}{\lambda_{MS} Q_{2,outer} + b \lambda_i}\right)^q\right].
\]
(24)

**Proof:** See Appendix C
It should be noticed that $\lambda_1'$ is the density of $\Phi_1'$ over the whole plane, and $\lambda_2'$ is the density of $\Phi_2'$ in the outer region only. This difference comes from the non-uniform femtocell deployment scheme. For simplicity, we respectively use the homogenous PPP with density $\lambda_1'$ over the entire plane and the PPP with density $\lambda_2'$ on the outer region to analyze the interference signal.

D. Coverage Probability

Now we present the result on the coverage probability $p_c(T)$, i.e., the probability that the instantaneous received SINR at the typical user’s data channel is above a target SINR threshold $T$. The coverage probability is equivalent to the CCDF of SINR. The coverage probabilities given the typical user is located in the inner region and the outer region, are presented in Theorem 1 and Theorem 2 respectively. Note that $\rho(\cdot, \cdot)$ is defined below the equation (8).

**Theorem 1.** The coverage probability for the typical user in the inner region $A_{inner}$ is approximated by

$$p_{c,A_{inner}}(T) \approx \frac{2\pi\lambda_1}{[1 - \exp(-\pi\lambda_1 D^2)]} \int_0^D \exp \left( - \frac{T \sigma^2 x^{\alpha}}{P_1} \right)$$

$$\times \exp \left( - \pi \left[ \lambda_1 + \lambda_1' \rho(T, \alpha) \right] x^2 \right) \exp \left( - \pi \lambda_2' D^2 \rho \left( \frac{P_2 T x^{\alpha}}{P_1 D^{\frac{2}{\alpha}}}, \alpha \right) \right) x \, dx. \quad (25)$$

**Proof:** See Appendix D.

**Theorem 2.** The coverage probability for the typical user in the outer region $A_{outer}$ is provided by

$$p_{c,A_{outer}}(T) = \sum_{i \in \{1, 2\}} p_{c,i,A_{outer}}(T) \cdot \mathbb{P}[\omega = i \mid o \in A_{outer}], \quad (26)$$

where

$$p_{c,1,A_{outer}}(T) \approx \frac{2\pi[\lambda_1 + \lambda_2 \left( \frac{P_2}{P_1} \right)^{2/\alpha}]}{\exp \left( - \pi [\lambda_1 + \lambda_2 \left( \frac{P_2}{P_1} \right)^{2/\alpha} D^2] \right)} \int_D^\infty \exp \left( - \frac{T \sigma^2 x^{\alpha}}{P_1} \right)$$

$$\times \exp \left( - \pi \left[ (\lambda_1 + \lambda_2 \left( \frac{P_2}{P_1} \right)^{2/\alpha}) + \rho(T, \alpha) \left( \lambda_1' + \lambda_2' \left( \frac{P_2}{P_1} \right)^{2/\alpha} \right) \right] x^2 \right) x \, dx, \quad (27)$$

and

$$p_{c,2,A_{outer}}(T) = 2\pi \lambda_2 M \cdot \int_0^{\left( \frac{P_2}{P_1} \right)^{\frac{1}{1-2/\alpha}}} x \exp \left( - \frac{T \sigma^2 x^{\alpha}}{P_2} \right) \, dx. \quad (28)$$
\[
\begin{align*}
\cdot & \exp \left( -\pi \lambda_1' D^2 \rho \left( \frac{P_1 T x^\alpha}{P_2 D^2} \right), \alpha \right) \exp \left( -\pi \left( \lambda_2 + \lambda_2' \rho(T, \alpha) \right) x^2 \right) dx \\
& + \frac{2\pi \lambda_2 M}{\exp(-\pi \lambda_1 D^2 D^2)} \int_{\alpha}^{\infty} x \exp \left( -\frac{T x^2}{2} \right) \\
& \cdot \exp \left( -\pi \left[ \left( \lambda_1 \left( \frac{P_1}{P_2} \right)^{2/\alpha} + \lambda_2' \right) + \rho(T, \alpha) \left( \lambda_1' \left( \frac{P_1}{P_2} \right)^{2/\alpha} + \lambda_2' \right) \right] x^2 \right) dx.
\end{align*}
\] (28)

Proof: See Appendix E. ■

The following Corollary 1 provides the coverage probability for a randomly chosen typical user, which can be obtained easily by expanding the coverage probability into inner and outer regions.

Corollary 1. For a randomly located typical user, the coverage probability is

\[
p_c(T) = p_{c,A_{inner}}(T) \cdot P[o \in A_{inner}] + p_{c,A_{outer}}(T) \cdot P[o \in A_{outer}],
\]

where \( P[o \in A_{inner}] \) and \( P[o \in A_{outer}] \) are given in (2). (29)

E. Single User Throughput

In this section, we derive another and the most important analytical result of this paper, the probabilistic characteristics of the achievable single user throughput for the proposed non-uniform femtocell deployment. Similar to the coverage probability analysis in Subsection III-D, we will first focus on the throughput in the inner and outer regions separately.

Theorem 3. The CCDF of the throughput achieved at the typical user in the inner region \( A_{inner} \) is provided by

\[
P[R > \rho \mid o \in A_{inner}] \approx \sum_{n=0}^{\infty} P[N_1 = n] \cdot p_{c,A_{inner}} \left( 2^{(n+1)\rho/W} - 1 \right),
\]

where \( N_1 \) is the number of in-cell users sharing the resource with the macrocell typical user, and its pmf \( P[N_1 = n] \) is approximated by

\[
P[N_1 = n] \approx \frac{b^n}{n!} \cdot \frac{\Gamma(n + q + 1)}{\Gamma(q)} \cdot \left( \frac{\lambda_{MS}}{\lambda_1/Q_1} \right)^n \left( b + \frac{\lambda_{MS}}{\lambda_1/Q_1} \right)^{-(n+q+1)},
\]

in which \( Q_1 \) is given in Lemma 4. (30)

Proof: See Appendix E. ■

Theorem 4. The CCDF of the throughput achieved at the typical user in the outer region \( A_{outer} \) is provided by

\[
P[R > \rho \mid o \in A_{outer}] \approx \sum_{i=1}^{2} \sum_{n=0}^{\infty} P[N_i = n] \cdot p_{c,i,A_{outer}} \left( 2^{(n+1)\rho/W} - 1 \right) \cdot Q_{i,outer},
\]

(32)
where \{Q_{i,\text{outer}}\}_{i \in \{1,2\}} is provided in Lemma 5, \(P[N_1 = n]\) is given in (31), and \(P[N_2 = n]\) is the distribution of the number of in-cell user sharing the resource with the femtocell typical user, i.e.,

\[
P[N_2 = n] \approx \frac{b^q}{n!} \cdot \frac{\Gamma(n + q + 1)}{\Gamma(q)} \cdot \left(\frac{\lambda_{MS}}{\lambda_2/Q_{2,\text{outer}}}\right)^n \left(b + \frac{\lambda_{MS}}{\lambda_2/Q_{2,\text{outer}}}\right)^{-(n+q+1)}.
\] (33)

**Proof:** See Appendix G.

The following Corollary 2 provides the distribution of the throughput achieved for a randomly chosen typical user, which can be obtained easily by expanding the distribution expression into inner and outer regions.

**Corollary 2.** Considering resource sharing, the CCDF of the throughput achieved at the typical user can be expressed as

\[
P[R > \rho] = P[R > \rho | o \in A_{\text{inner}}] \cdot P[o \in A_{\text{inner}}] + P[R > \rho | o \in A_{\text{outer}}] \cdot P[o \in A_{\text{outer}}],
\] (34)

where \(P[o \in A_{\text{inner}}]\) and \(P[o \in A_{\text{outer}}]\) are given in (11).

### IV. Numerical Results

In this section, we present numerical results on the coverage and single user throughput for the proposed non-uniform femtocell deployment scheme. Here we assume the transmit powers of macrocell and femtocell BSs as \(P_{tx,1} = 46\) dBm and \(P_{tx,2} = 20\) dBm respectively. The macrocell tier density is \(\lambda_1 = 1\) per square km, and the mobile user density is \(\lambda_{MS} = 10\) per square km in all numerical results. The path loss constant and exponent are assumed to be \(L_0 = -34\) dB and \(\alpha = 4\). The thermal noise power is \(\sigma^2 = -104\) dBm. Monte Carlo simulations are also conducted to compare with our analysis for the purpose of model validation. The single user throughput demonstrated in our results are the rates achievable over the BS’s bandwidth of 1 Hz, i.e., \(W = 1\) Hz.

In our numerical results for femtocell deactivation, \(\lambda_2\) denotes the femtocell BS deployment density before the deactivating operation. To conduct a fair comparison with uniform deployment, the femtocell BS density of the smart femtocell deployment scheme in outer region is set to be \(\lambda_2/P[o \in A_{\text{outer}}]\), which guarantees the average density over the whole plane becomes \(\lambda_2\), the same as the uniform femtocell deployment.
Fig. 2. Coverage probability (or the CCDF of received SINR) for femtocell deactivation, $D = 500$ m and $\lambda_2/\lambda_1 = 10$.

A. Coverage Performance

Fig. 2 demonstrates the results of coverage probability (or equivalently the CCDF of received SINR) with femtocell deactivation, given the condition of $D = 500$ m, $\lambda_2/\lambda_1 = 10$. Firstly, the tractable analytical results, i.e., the approximations derived for inner and outer regions in this work, are reasonably accurate. Through combining the results of outer and inner regions by using (29), the coverage probability curve for randomly chosen users is also illustrated therein.

In Fig. 3 the coverage probability curves for different schemes are compared, in which we can conclude that the analysis on femtocell deactivation and smart femtocell deployment precisely matches the simulation result. Furthermore, we can observe two phenomena, i.e., femtocell deactivation would not hurt the coverage performance, and the non-uniform smart femtocell deployment scheme outperforms both macrocell only and two-tier uniform deployments.

Furthermore, by presenting the achievable coverage probability versus the inner region radius $D$ in Fig. 4, we can see the importance of properly dividing inner and outer regions
on the coverage performance. For an appropriately chosen $D$ value, the scheme of femtocell deactivation can achieve nearly the same coverage performance as two-tier uniform femtocell deployment, even with a significantly lower de facto femtocell density. For instance, femtocell deactivation with $D = 500$ m, which means that $54.4\%$ of femtocell BSs are deactivated, is as good as uniform deployment on coverage probability for SINR threshold $T = -5$ dB. This result is surprising since more than half of the femtocell BS energy consumption and the corresponding operating expense can be saved with the same level of coverage performance.

By compensating the femtocell density in the outer region, significant coverage improvement can be obtained for smart femtocell deployment. By taking the case of $\lambda_2/\lambda_1 = 10$ and $T = -5$ dB for example, the achievable coverage probability is around $85\%$ at $D = 600$ m for smart femtocell deployment, compared with $79\%$ for uniform two-tier deployment and $73\%$ for single macrocell tier. Similar enhancements can be observed with a different SINR threshold (i.e., $T = 10$ dB) or a different femtocell density (i.e., $\lambda_2/\lambda_1 = 5$). It should be noticed that the slight mismatches between simulation and tractable results in Fig. 4 come from the approximation used in the analysis, but the performance trend can be well
captured by the tractable results. More importantly, both schemes of femtocell deactivation and smart femtocell deployment do not incur any further network resources: we just turn off the femtocell BSs located in the inner region in femtocell deactivation scheme, or deploy femtocell BSs in the outer region only for smart femtocell deployment scheme.

B. Throughput Performance

To validate the analytical single user throughput performances for the proposed schemes, the CCDF curves for the inner and outer regions gotten from Theorem 3 and Theorem 4 are compared with the simulation counterparts in Fig. 5. It can be shown that the throughput distributions are well captured by the analytical results. By combining the inner and outer region results in Corollary 2, we compare the single user throughput performances for different deployment schemes in Fig. 6. The tractable analytical results for both femtocell deactivation and smart femtocell deployment, are reasonably accurate. For femtocell deactivation, basically it only reduces high-rate users’ performance while does not hurt low-rate users, compared with the uniform case. Since low-rate users are usually much more of a concern to the cellular service providers [30], this scheme has the desirable property of being able to significantly reduce the resource while taking care of the low-rate users. For smart femtocell deployment, it increases all users performance, especially the low-rate ones. For example, among the worst 10% users, the highest achievable rate is increased from 0.025 bps in the uniform case to 0.043 bps in the smart deployment case, which is a 72% improvement.

From the single user throughput performances versus the inner region radius $D$ demonstrated in Fig. 7 we can see the impact of the inner region radius $D$ on the single user throughput performance. For femtocell deactivation, optimally choosing $D$ can significantly reduce the resource while not hurting the low-rate users’ performance: for example, $D$ can get up to 400 m in both figures for $\rho = 0.02$ bps. For smart femtocell deployment: optimally choosing $D$ results in noticeable improvement for both low-rate and high-rate users. Our analytical results provide tools to design the value of $D$ to maximize the benefits to a target group of users of the operator’s choice. Taking $\lambda_2/\lambda_1 = 10$ for example, $D = 400$ m and $D = 500$ m can achieve near-optimal values of $\mathbb{P}[\mathcal{R} > \rho]$ for high and low rate thresholds respectively.
Fig. 4. Coverage probability over $D$ for different schemes, with the tier density ratios $\lambda_2/\lambda_1 = 10$ (upper figure) and $\lambda_2/\lambda_1 = 5$ (lower figure). The SINR thresholds are set to be $T = -5$ dB and $T = 10$ dB.
V. Conclusion

In this work, we studied the downlink coverage and throughput performance of the cellular networks with the newly proposed non-uniform femtocell deployment scheme. There are two ways to implement this scheme, i.e., femtocell deactivation and smart femtocell deployment. Using the tools from stochastic geometry, tractable results were obtained to characterize the coverage probability and single user throughput distribution. The numerical results validated the analytical expressions and approximations, and provided the following important message: By carefully choosing the parameters for the proposed femtocell deactivation scheme, more than 50% of femtocell BSs can be turned off to save the femtocell BS energy consumption and the operating expense, but achieve the same coverage performance as deploying femtocell uniformly. By compensating the femtocell density in the poor macrocell coverage regions, the smart femtocell deployment, compared with traditional uniform femtocell deployment, can obtain noticeable improvement on the coverage and the data throughput with no extra cost of network resource incurred. This interesting finding demonstrates the performance improvements achievable by implementing a simple non-uniform femtocell deployment, and

Fig. 5. Single user throughput distribution (CCDF curves) for inner and outer regions, $D = 500$ m and $\lambda_2/\lambda_1 = 10$. 
Fig. 6. Single user throughput distribution (CCDF curves) for different schemes, $D = 500$ m and $\lambda_2/\lambda_1 = 10$.

emphasizes the importance of selectively deploying the femtocell BSs by taking their relative locations with macrocell BSs into account.

APPENDIX

A. Proof of Lemma 2

Since the event of $X_1 \leq x$ is the event of $R_1 \leq x$ based on the condition that the user is associated with macrocell, the CDF of $X_1$ for the outer region typical user is

$$F_{X_1|o\in A_{outer}}(x) = \mathbb{P}[X_1 \leq x \mid o \in A_{outer}]$$

$$= \mathbb{P}[R_1 \leq x \mid \omega = 1, o \in A_{outer}]$$

$$= \mathbb{P}[R_1 \leq x \mid R_1 < \left(\frac{P_1}{P_2}\right)^{1/\alpha} R_2, R_1 > D]$$

$$= \frac{\mathbb{P}[R_1 \leq x, R_1 < \left(\frac{P_1}{P_2}\right)^{1/\alpha} R_2 \mid R_1 > D] \cdot \mathbb{P}[R_1 > D]}{\mathbb{P}[R_1 < \left(\frac{P_1}{P_2}\right)^{1/\alpha} R_2, R_1 > D]}, \quad \text{for } x > D, \quad (35)$$

in which step (a) follows Bayes’ theorem. The denominator of (35) can be derived as

$$\mathbb{P}[R_1 < \left(\frac{P_1}{P_2}\right)^{1/\alpha} R_2, R_1 > D]$$
Fig. 7. $P[R > \rho]$ over $D$ for different schemes, with the tier density ratios $\lambda_2/\lambda_1 = 10$ (upper figure) and $\lambda_2/\lambda_1 = 5$ (lower figure). The rate thresholds are set to be $\rho = 0.02$ bps and $\rho = 1$ bps.
\[ \int_{D_1} \cdots \int_{D_2} \mathbb{P}[D < R_1 < (\frac{P_1}{P_2})^{1/\alpha} R_2] f_{R_1}(r) dr \]

\[ \approx \int_{D_1} \cdots \int_{D_2} \left[ \exp(-\pi \lambda_1 D^2) - \exp(-\pi \lambda_1 (\frac{P_1}{P_2})^{2/\alpha} R_2) \right] 2\pi \lambda_2 r \exp(-\pi \lambda_2 r^2) dr \]

\[ = \frac{\lambda_1}{\lambda_1 + \lambda_2 (\frac{P_2}{P_1})^{2/\alpha}} \exp\left( -\pi \left[ \lambda_1 + \lambda_2 (\frac{P_2}{P_1})^{2/\alpha} \right] D^2 \right), \quad (36) \]

where we approximate the density of femtocell BSs in the vicinity of the outer region typical user as \( \lambda_2 \) in step (b). This approximation is accurate as long as the typical user is not located close to the boundary between the inner and outer regions. Based upon the same approximation, the former part of (35)’s numerator is derived as

\[ \mathbb{P}[R_1 \leq x, R_1 < (\frac{P_1}{P_2})^{1/\alpha} R_2 \mid R_1 > D] \]

\[ = \int_D \mathbb{P}[R_2 > (\frac{P_2}{P_1})^{1/\alpha} R_1 \mid R_1 > D] f_{R_1}(r) dr \]

\[ \approx \int_D \exp(-\pi \lambda_2 (\frac{P_2}{P_1})^{2/\alpha} D^2) \cdot \frac{2\pi \lambda_1 r \exp(-\pi \lambda_1 r^2)}{\exp(-\pi \lambda_1 D^2)} dr \]

\[ = \frac{\lambda_1}{\lambda_1 + \lambda_2 (\frac{P_2}{P_1})^{2/\alpha}} \cdot \frac{1}{\exp(-\pi \lambda_1 D^2)} \cdot \left[ \exp\left( -\pi \left[ \lambda_1 + \lambda_2 (\frac{P_2}{P_1})^{2/\alpha} \right] D^2 \right) \right] \]

\[ - \exp\left( -\pi \left[ \lambda_1 + \lambda_2 (\frac{P_2}{P_1})^{2/\alpha} \right] x^2 \right) \right]. \quad (37) \]

By substituting (36) and (37) into (35) and differentiating the resultant CDF, we can reach \( X_1 \)'s pdf in (15), which completes the proof.

**B. Proof of Lemma 3**

Since the event of \( X_2 \leq x \) is the event of \( R_2 \leq x \) based on the condition that the user is associated with femtocell, the CDF of \( X_2 \) for the outer region typical user can be derived as

\[ F_{X_2 \mid o \in A_{outer}}(x) = \mathbb{P}[X_2 \leq x \mid o \in A_{outer}] \]

\[ = \mathbb{P}[R_2 \leq x \mid \omega = 2, o \in A_{outer}] \]

\[ = \mathbb{P}[R_2 \leq x \mid R_2 < (\frac{P_2}{P_1})^{1/\alpha} R_1, R_1 > D] \]

\[ \approx \frac{\mathbb{P}[R_2 \leq x, R_2 < (\frac{P_2}{P_1})^{1/\alpha} R_1 \mid R_1 > D] \cdot \mathbb{P}[R_1 > D]}{\mathbb{P}[R_2 < (\frac{P_2}{P_1})^{1/\alpha} R_1, R_1 > D]}, \quad \text{for } x > D, \quad (38) \]

where step (a) follows Bayes’ theorem. The former part of (38)’s numerator is expressed as

\[ \mathbb{P}[R_2 \leq x, R_2 < (\frac{P_2}{P_1})^{1/\alpha} R_1 \mid R_1 > D] \]
PPP with certain density values to approximate the area of the macrocells and femtocells.

C. Proof of Lemma 6

in which step 

in which the integration under the condition of $x > (P_2/P_1)^{1/\alpha} D$ can be derived as

$$
\int_D^\infty \mathbb{P}[R_2 \leq x, R_2 < (P_2/P_1)^{1/\alpha} r] f_{R_1|R_1>D}(r) dr
= \int_{(P_2/P_1)^{1/\alpha} x}^\infty \mathbb{P}[R_2 < x] \cdot 2\pi \lambda_1 r \exp(-\pi \lambda_1 r^2) \exp(-\pi \lambda_1 D^2) dr 
+ \int_{(P_2/P_1)^{1/\alpha} x}^\infty \mathbb{P}[R_2 < (P_2/P_1)^{1/\alpha} r] \cdot 2\pi \lambda_1 r \exp(-\pi \lambda_1 r^2) \exp(-\pi \lambda_1 D^2) dr 
\approx \int_{(P_2/P_1)^{1/\alpha} x}^\infty \left[1 - \exp(-\pi \lambda_2 x^2)\right] \cdot 2\pi \lambda_1 r \exp(-\pi \lambda_1 r^2) \exp(-\pi \lambda_1 D^2) dr 
+ \int_{(P_2/P_1)^{1/\alpha} x}^\infty \left[1 - \exp(-\pi \lambda_2 (P_2/P_1)^{2/\alpha} r^2)\right] \cdot 2\pi \lambda_1 r \exp(-\pi \lambda_1 r^2) \exp(-\pi \lambda_1 D^2) dr 
= 1 - \frac{\lambda_1}{\exp(-\pi \lambda_1 D^2)} \cdot \frac{\lambda_1}{\lambda_1 + \lambda_2 (P_2/P_1)^{2/\alpha}} \exp\left(-\pi \left[\lambda_1 + \lambda_2 (P_2/P_1)^{2/\alpha}\right] D^2\right) 
- \frac{1}{\exp(-\pi \lambda_1 D^2)} \cdot \frac{\lambda_2 (P_2/P_1)^{2/\alpha}}{\lambda_1 + \lambda_2 (P_2/P_1)^{2/\alpha}} \exp\left(-\pi \left[\lambda_1 + \lambda_2 (P_2/P_1)^{2/\alpha}\right] D^2\right),
$$

in which step (b) is approximated by assuming that the density of femtocell BSs in the vicinity of the outer region typical user is $\lambda_2$, similar to the proof in Appendix A. The same approximation will help us to derive the denominator of (38), i.e.,

$$
\mathbb{P}[R_2 < (P_2/P_1)^{1/\alpha} R_1, R_1 > D]
= \int_D^\infty \mathbb{P}[R_2 < (P_2/P_1)^{1/\alpha} r] \cdot 2\pi \lambda_1 r \exp(-\pi \lambda_1 r^2) dr 
\approx \int_D^\infty \left[1 - \exp(-\pi \lambda_2 (P_2/P_1)^{2/\alpha} r^2)\right] \cdot 2\pi \lambda_1 r \exp(-\pi \lambda_1 r^2) dr 
= \exp(-\pi \lambda_1 D^2) \cdot \frac{\lambda_1}{\lambda_1 + \lambda_2 (P_2/P_1)^{2/\alpha}} \exp\left(-\pi \left[\lambda_1 + \lambda_2 (P_2/P_1)^{2/\alpha}\right] D^2\right). (41)
$$

By substituting (41), (39) and (40) into (38) and differentiating the resultant CDF, $X_2$’s pdf in (16) can be obtained and the proof is completed.

C. Proof of Lemma 6

Similar to the method used in [23], we use the Voronoi cell area formed by a homogeneous PPP with certain density values to approximate the area of the macrocells and femtocells.
The area of the macrocell tier cells can be approximated by
\[ C_1 \approx C_0 \left( \frac{\lambda_1}{Q_1} \right). \] (42)

On the other hand, the femtocell-deployed region \( A_{\text{outer}} \) is the area where femtocell BSs are deployed with the density \( \lambda_2 \). Hence, the area of cells formed by the femtocell tier can be similarly approximated by
\[ C_2 \approx C_0 \left( \frac{\lambda_2}{Q_{2,\text{outer}}} \right). \] (43)

Similar to the analysis in Section II, the probabilities of \( i \)-th tier cells with no user associated are \( P[N_{i,c} = 0] \), in which \( N_{i,c} \) is the number of users in a randomly chosen \( i \)-th tier cell,
\[ P[N_{1,c} = n] \approx \frac{b^n}{n!} \cdot \frac{\Gamma(n + q)}{\Gamma(q)} \cdot \frac{(\lambda_{MS})^n(\lambda_1/Q_1)^q}{(\lambda_{MS} + b\lambda_1/Q_1)^{n+q}}, \] (44)
and
\[ P[N_{2,c} = n] \approx \frac{b^n}{n!} \cdot \frac{\Gamma(n + q)}{\Gamma(q)} \cdot \frac{(\lambda_{MS})^n(\lambda_2/Q_{2,\text{outer}})^q}{(\lambda_{MS} + b\lambda_2/Q_{2,\text{outer}})^{n+q}}. \] (45)

Then the average density can be obtained by
\[ \lambda'_1 = \lambda_i \cdot (1 - P[N_{i,c} = 0]), \] which completes the proof.

D. Proof of Theorem 1

By assuming that the typical user located in the inner region \( A_{\text{inner}} \) always gets service from macrocell BSs, its coverage probability can be derived as
\[ p_{c,A_{\text{inner}}} (T) = \mathbb{P}[\text{SINR} > T \mid o \in A_{\text{inner}}] \]
\[ = \int_0^D \mathbb{P}[\text{SINR} > T \mid X_1 = x, o \in A_{\text{inner}}] f_{X_1|o\in A_{\text{inner}}}(x) dx \]
\[ \approx \int_0^D \exp \left( -\frac{T \sigma^2 x^\alpha}{P_1} \right) \prod_{i=1}^2 \mathcal{L}_{I_i|X_1=x,o\in A_{\text{inner}}}(\frac{T x^\alpha}{P_1}) f_{X_1|o\in A_{\text{inner}}}(x) dx, \] (46)

where step (a) comes from the Rayleigh fading assumption, and \( \mathcal{L}_{I_i|X_1=x,o\in A_{\text{inner}}}(\cdot) \) is the Laplace transform of random variable \( I_i \) given the condition that the typical user \( x \) away from the macrocell serving BS is located in the inner region \( A_{\text{inner}} \). Here we assume the femtocell interference comes from the whole region out of the area \( B(o, D) \), which is an optimistic estimation (proved to be accurate by the numerical results). Then, we can have
\[ \prod_{i=1}^2 \mathcal{L}_{I_i|X_1=x,o\in A_{\text{inner}}}(\frac{T x^\alpha}{P_1}) \approx \exp \left( -\pi \lambda'_1 \rho(T, \alpha)x^2 \right) \exp \left( -\pi \lambda'_2 \rho(D, \alpha) \right). \] (47)

By substituting (47) and (13) into (46), we can obtain (25) and complete the proof.
E. Proof of Theorem 2

For the outer region typical user served by the macrocell tier, its coverage probability can be expressed as

\[
p_{c,1,A_{outer}}(T) = \mathbb{P}[\text{SINR}_1 > T \mid o \in A_{outer}] \\
= \int_D \mathbb{P}[\text{SINR}_1 > T \mid X_1 = x, o \in A_{outer}] f_{X_1|o\in A_{outer}}(x) dx \\
\overset{(a)}{=} \int_D \exp \left( - \frac{T \sigma^2 x^\alpha}{P_1} \right) \prod_{i=1}^{2} \mathcal{L}_{I_i|X_1 = x, o \in A_{outer}} \left( \frac{T x^\alpha}{P_1} \right) f_{X_1|o\in A_{outer}}(x) dx,
\]

(48)

where step (a) still follows from the Rayleigh fading assumption, and \( \mathcal{L}_{I_i|X_1 = x, o \in A_{outer}}(\cdot) \) is the Laplace transform of random variable \( I_i \) given the condition that the typical user \( x \) away from the macrocell serving BS is located in the outer region \( A_{outer} \). Assuming the interference from femtocell BSs comes from the whole plane, we can approximate this Laplace transform as

\[
\mathcal{L}_{I_i|X_1 = x, o \in A_{outer}} \left( \frac{T x^\alpha}{P_1} \right) \approx \exp \left( - \pi x^2 \rho(T, \alpha) \mathbb{E}[\lambda_i'] \left( \frac{P_i}{P_1} \right)^{2/\alpha} \right). \quad (49)
\]

It is a pessimistic assumption since the original femtocell interference from inner area is eliminated due to this non-uniform deployment scheme, and the numerical results in Section IV show that it is still a reasonably accurate approximation.

By substituting (15) and (49) into (48), we can have

\[
p_{c,1,A_{outer}}(T) \approx \int_D \exp \left( - \frac{T \sigma^2 x^\alpha}{P_1} \right) \exp \left( - \pi x^2 \rho(T, \alpha) \mathbb{E}[\lambda_1'] \left( \frac{P_2}{P_1} \right)^{2/\alpha} \right) \cdot \exp \left( - \pi \left[ \lambda_1 \lambda_2 \left( \frac{P_2}{P_1} \right)^{2/\alpha} \right] x^2 \right) \cdot \exp \left( - \pi \left[ \lambda_1 \lambda_2 \left( \frac{P_2}{P_1} \right)^{2/\alpha} \right] D^2 \right) dx
\]

\[
= \frac{2\pi [\lambda_1 + \lambda_2 \left( \frac{P_2}{P_1} \right)^{2/\alpha}] \exp \left( - \frac{T \sigma^2 x^\alpha}{P_1} \right) \int_D \exp \left( - \pi \left[ \lambda_1 \lambda_2 \left( \frac{P_2}{P_1} \right)^{2/\alpha} \right] x^2 \right) dx}{\exp \left( - \pi \left[ \lambda_1 \lambda_2 \left( \frac{P_2}{P_1} \right)^{2/\alpha} \right] D^2 \right)}
\]

\[
\cdot \exp \left( - \pi \left[ \lambda_1 \lambda_2 \left( \frac{P_2}{P_1} \right)^{2/\alpha} \right] + \rho(T, \alpha) \mathbb{E}[\lambda_i'] \left( \frac{P_i}{P_1} \right)^{2/\alpha} \right) \int_D \exp \left( - \pi \left[ \lambda_1 \lambda_2 \left( \frac{P_2}{P_1} \right)^{2/\alpha} \right] D^2 \right) \cdot \exp \left( - \pi \left[ \lambda_1 \lambda_2 \left( \frac{P_2}{P_1} \right)^{2/\alpha} \right] x^2 \right) dx.
\]

\[
(50)
\]

If the outer region typical user is served by the femtocell tier, its coverage probability can be given as

\[
p_{c,2,A_{outer}}(T) = \mathbb{P}[\text{SINR}_2 > T \mid o \in A_{outer}] \\
= \int_0^\infty \mathbb{P}[\text{SINR}_2 > T \mid X_2 = x, o \in A_{outer}] f_{X_2|o\in A_{outer}}(x) dx
\]
\[
(a) \int_{0}^{\infty} \exp \left( - \frac{T \sigma^2 x^\alpha}{P_2} \right) \prod_{i=1}^{2} L_{I_1|x=x, o \in A_{outer}} \left( \frac{T x^\alpha}{P_2} \right) f_{X_2|o \in A_{outer}}(x) \, dx,
\]

where step (a) follows from the Rayleigh fading assumption. The Laplace transform of random variable \( I_2 \) given the condition that the typical user \( x \) away from the femtocell serving BS is located in the outer region \( A_{outer} \), i.e., \( L_{I_2|x=x, o \in A_{outer}}(T x^\alpha / P_2) \), is

\[
L_{I_2|x=x, o \in A_{outer}} \left( \frac{T x^\alpha}{P_2} \right) \approx \exp \left( - \pi \lambda'_2 \rho(x, o)^2 \right),
\]

and the Laplace transform of random variable \( I_1 \) given that condition can be derived as

\[
L_{I_1|x=x, o \in A_{outer}} \left( \frac{T x^\alpha}{P_2} \right) = \begin{cases} 
\exp \left( - 2 \pi \lambda_1 \int_{0}^{\infty} \left( 1 - \frac{1}{1+x^\alpha(T y)^\alpha} \right) y \, dy \right) & \text{for } x \leq \left( \frac{P_2}{P_1} \right)^{1/\alpha} D \\
\exp \left( - 2 \pi \lambda_1 \int_{\left( \frac{P_2}{P_1} \right)^{1/\alpha} D}^{\infty} \left( 1 - \frac{1}{1+x^\alpha(T y)^\alpha} \right) y \, dy \right) & \text{for } x > \left( \frac{P_2}{P_1} \right)^{1/\alpha} D 
\end{cases}
\]

Then we can substitute (52), (53) and (16) into (51), and obtain the expression of the coverage probability in (28).

By combining the results in (27) and (28), we can reach the coverage performance for a randomly chosen outer region typical user, provided by (26). Till here, we complete the proof.

**F. Proof of Theorem 3**

The CCDF of the throughput achieved at the inner region typical user, \( P[R > \rho | o \in A_{inner}] \), can be obtained by assuming that all inner region users are served by the macrocell tier, i.e.,

\[
P[R > \rho | o \in A_{inner}] \approx P[R > \rho | \omega = 1, o \in A_{inner}] \\
= P \left[ \frac{W}{N_1 + 1} \log_2(1 + \text{SINR}) > \rho | \omega = 1, o \in A_{inner} \right] \\
= \mathbb{E}_{N_1} \left[ p_{c,A_{inner}} (2(N_1+1)\rho/W - 1) \right] \\
= \sum_{n=0}^{\infty} P[N_1 = n] \cdot p_{c,A_{inner}} (2^{(n+1)\rho/W} - 1),
\]

where \( N_1 \) is the number of in-cell macrocell MSs sharing the resource with the typical user. As indicated in Appendix [C] the area of the macrocell tier tier cells can be approximated by \( C_1 \approx C_0 \left( \frac{\lambda_1}{\lambda_2} \right) \), which helps us to reach the pmf of \( N_1 \) in (31).
G. Proof of Theorem 4

The CCDF of the throughput achieved at the outer region typical user, $P[R > \rho \mid o \in A_{outer}]$, can be provided by

$$P[R > \rho \mid o \in A_{outer}] = \sum_{i \in \{1,2\}} P[R > \rho \mid \omega = i, o \in A_{outer}] \cdot P[\omega = i \mid o \in A_{outer}]$$

$$= \sum_{i \in \{1,2\}} \sum_{n=0}^{\infty} P[N_i = n] \cdot p_{c,i,A_{outer}} \left(2^{(n+1)\rho/W} - 1\right) \cdot P[\omega = i \mid o \in A_{outer}].$$

(55)

where $N_2$ is the number of in-cell femtocell MSs sharing the resource with the typical user. As indicated in Appendix C, the area of the femtocell tier tier cells can be approximated by $C_2 \approx C_0 \left(\frac{\lambda_2}{Q_{2,outer}}\right)$, which helps us to reach the pmf of $N_2$ in (33).

REFERENCES

[1] “LTE Advanced: Heterogeneous networks,” Qualcomm Inc. White Paper, Jan. 2011.

[2] J. G. Andrews, “Seven ways that HetNets are a cellular paradigm shift,” IEEE Commun. Mag., vol. 51, no. 3, pp. 136–144, 2013.

[3] S. R. Saunders, S. Carlaw, A. Giustina, R. R. Bhat, V. S. Rao, and R. Siegberg, Femtocells: Opportunities and Challenges for Business and Technology, 1st ed. Chichester, U.K.: John Wiley & Sons Ltd., 2009.

[4] C. Chandrasekhar, J. G. Andrews, and A. Gatherer, “Femtocell networks: a survey,” IEEE Commun. Mag., vol. 46, no. 9, pp. 59–67, Sep. 2008.

[5] J. G. Andrews, H. Claussen, M. Dohler, S. Rangan, and M. C. Reed, “Femtocells: Past, present, and future,” IEEE J. Select. Areas Commun., vol. 30, no. 3, pp. 497–508, Apr. 2012.

[6] P. Xia, V. Chandrasekhar, and J. G. Andrews, “Open vs. closed access femtocells in the uplink,” IEEE Trans. Wireless Commun., vol. 9, no. 12, pp. 3798–3809, Dec. 2010.

[7] V. Chandrasekhar, M. Kountouris, and J. G. Andrews, “Coverage in multi-antenna two-tier networks,” IEEE Trans. Wireless Commun., vol. 8, no. 10, pp. 5314–5327, Oct. 2009.

[8] J. Weitzen and T. Grosch, “Comparing coverage quality for femtocell and macrocell broadband data services,” IEEE Commun. Mag., vol. 48, no. 1, pp. 40–44, Jan. 2010.

[9] H. Jo, Y. Sang, P. Xia, and J. G. Andrews, “Heterogeneous cellular networks with flexible cell association: A comprehensive downlink SINR analysis,” IEEE Trans. Wireless Commun., vol. PP, no. 99, pp. 1–12, Aug. 2012.

[10] H. S. Dhillon, R. K. Ganti, F. Baccelli, and J. G. Andrews, “Modeling and analysis of K-tier downlink heterogeneous cellular networks,” IEEE J. Select. Areas Commun., vol. 30, no. 3, pp. 550–560, Apr. 2012.

[11] H. Wang and M. C. Reed, “Tractable model for heterogeneous cellular networks with directional antennas,” in Proc. 2012 Australian Commun. Theory Workshop (AusCTW’12), Wellington, New Zealand, Jan./Feb. 2012, pp. 61–65.
[12] M. Yavuz, F. Meshkati, S. Nanda, A. Pokhriyal, N. Johnson, B. Raghothaman, and A. Richardson, “Interference management and performance analysis of UMTS/HSPA+ femtocells,” IEEE Commun. Mag., vol. 47, no. 9, pp. 102–109, Sep. 2009.

[13] D. Stoyan, W. S. Kendall, and J. Mecke, Stochastic Geometry and its Applications, 2nd ed. New York, NY: John Wiley & Sons Ltd., 1995.

[14] F. Baccelli and B. Blaszczyszyn, Stochastic Geometry and Wireless Networks, Volume I: Theory, 1st ed. Hanover, MA: Now Publishers Inc., 2009.

[15] T. X. Brown, “Cellular performance bounds via shotgun cellular systems,” IEEE J. Select. Areas Commun., vol. 18, no. 11, pp. 2443–2455, Nov. 2000.

[16] X. Yang and A. P. Petropulu, “Co-channel interference modeling and analysis in a Poisson field of interferers in wireless communications,” IEEE Trans. Signal Processing, vol. 51, no. 1, pp. 64–76, Jan. 2003.

[17] M. Haenggi, “A geometric interpretation of fading in wireless networks: Theory and applications,” IEEE Trans. Inform. Theory, vol. 54, no. 12, pp. 5500–5510, Dec. 2008.

[18] J. G. Andrews, F. Baccelli, and R. K. Ganti, “A tractable approach to coverage and rate in cellular networks,” IEEE Trans. Commun., vol. 59, no. 11, pp. 3122–3134, Nov. 2011.

[19] T. D. Novlan, R. K. Ganti, A. Ghosh, and J. G. Andrews, “Analytical evaluation of fractional frequency reuse for OFDMA cellular networks,” IEEE Trans. Wireless Commun., vol. 10, no. 12, pp. 4294–4305, Dec. 2011.

[20] W. C. Cheung, T. Q. Quek, and M. Kountouris, “Throughput optimization, spectrum allocation, and access control in two-tier femtocell networks,” IEEE J. Select. Areas Commun., vol. 30, no. 3, pp. 561–574, Apr. 2012.

[21] C. S. Chen, V. M. Nguyen, and L. Thomas, “On small cell network deployment: A comparative study of random and grid topologies,” in Proc. IEEE 76th Vehic. Tech. Conf. (VTC’12-Fall), Québec City, Canada, Sep. 2012, pp. 1–5.

[22] S. M. Yu and S.-L. Kim. Downlink capacity and base station density in cellular networks. [Online]. Available: http://arxiv.org/abs/1109.2992

[23] S. Singh, H. S. Dhillon, and J. G. Andrews. Offloading in heterogeneous networks: Modeling, analysis and design insights. [Online]. Available: [http://arxiv.org/abs/1208.1977](http://arxiv.org/abs/1208.1977)

[24] M. Haenggi. A versatile dependent model for heterogeneous cellular networks. [Online]. Available: [http://arxiv.org/abs/1305.3947](http://arxiv.org/abs/1305.3947)

[25] J. Niu, D. Lee, X. Ren, G. Y. Li, and T. Su, “Scheduling exploiting frequency and multi-user diversity in LTE downlink systems,” IEEE Trans. Wireless Commun., vol. 12, no. 4, pp. 1843–1849, 2013.

[26] E. Dahlman, S. Parkvall, J. Skold, and P. Beming, 3G Evolution: HSPA and LTE for Mobile Broadband, 2nd ed. Burlington, MA: Academic Press, 2008.

[27] A. Okabe, B. Boots, and K. Sugihara, Spatial Tessellations: Concepts and Applications of Voronoi Diagrams, 1st ed. West Sussex, England: John Wiley & Sons Ltd., 1992.

[28] A. L. Hinde and R. E. Miles, “Monte Carlo estimates of the distributions of the random polygons of the Voronoi tessellation with respect to a Poisson process,” Journal of Statistical Computation and Simulation, vol. 10, no. 3-4, pp. 205–223, 1980.

[29] D. Weaire, J. P. Kermode, and J. Weichert, “On the distribution of cell areas in a Voronoi network,” Philosophical Magazine Part B, vol. 53, no. 5, pp. L101–L105, 1986.

[30] M. Efthymiou, A. Mackay, A. Dow, and M. Flanagan, “Spatial optimisation: How subscribers can help you optimize your CDMA network,” in Proc. 12th Int’l Telecom. Network Strategy and Planning Symp. (Networks’06), New Delhi, India, Nov. 2006, pp. 1–11.