QUARK CONTRIBUTION
TO THE REGGEON-REGGEON-GLUON VERTEX IN QCD

V.Fadin†

Budker Institute for Nuclear Physics
and Novosibirsk State University, 630090 Novosibirsk, Russia

R.Fiore‡, A.Quartarolo‡

Dipartimento di Fisica, Università della Calabria
Istituto Nazionale di Fisica Nucleare, Gruppo collegato di Cosenza
Arcavacata di Rende, I-87036 Cosenza, Italy

Abstract

The quark loop contribution to the reggeon-reggeon-gluon vertex is calculated in QCD, where the reggeon is the reggeized gluon. Compared with the vertex in the Born approximation, this contribution exhibits a new spin structure as well as the gluon loop one. A remarkable but not complete cancellation between gluon and quark contributions to this new spin structure takes place for the case of three massless quark flavours.

†email address: FADIN @INP.NSK.SU
‡email address: 39022::FIORE, QUARTAROLO
            FIORE, QUARTAROLO @FIS.UNICAL.IT
1. Introduction

Semihard processes play a more and more important role at present colliders and, presumably, this role will increase in the future. Since the typical virtuality $Q^2$ for these processes is large enough to ensure a smallness of the strong coupling constant $\alpha_s(Q^2)$, perturbation theory can be applied to calculate parton distributions and cross sections. On the other hand, the energy $\sqrt{s}$ of colliding particles is sufficiently high to make the ratio $x = Q^2/s$ so small that the problem of summing up logarithmic terms $\alpha_s^n(ln(1/x))^m$ arises. In the leading logarithmic approximation (LLA), which means $n = m$, this problem was solved many years ago [1].

Unfortunately in the LLA unitarity is violated and the pomeron singularity in the $j$-plane lies on the r.h.s. of unity, yielding a strong power increase of structure functions in the small $x$ region. Therefore the evaluation of next-to-leading terms is necessary for defining the region in which the LLA can be applied, as well as for fixing the pomeron intercept.

The calculation of radiative corrections to the LLA was started by L.N. Lipatov and one of the authors (V.F.) in ref. [2]. The calculation program is based on the gluon reggeization property that was proved in the LLA and that simplified essentially the derivation of the equation for the pomeron. This equation is constructed in terms of the reggeon (reggeized gluon) trajectory

$$j(t) = 1 + \omega(t),$$

and the reggeon-reggeon-gluon (RRG) vertex. In the leading order, in the case of the SU(N) gauge group ($N = 3$ for QCD), one finds

$$\omega(t) = \omega^{(1)}(t) = \frac{g^2t}{(2\pi)^{2+\epsilon}} \frac{N}{2} \int \frac{d^{2+\epsilon}k_\perp}{k_\perp^2(q-k)^2},$$

where $g$ is the coupling constant of the gauge theory, $q$ is the momentum transfer and $t = q_\perp^2$. The integration is performed over the two-dimensional momentum
orthogonal to the initial particle momentum plane, and dimensional regularization of Feynman integrals is used:

\[
\frac{d^2k}{(2\pi)^2} \rightarrow \frac{d^{2+\varepsilon}k}{(2\pi)^{2+\varepsilon}}, \quad \varepsilon = D - 4 ,
\]

where \(D\) is the space-time dimension \((D = 4\) for the physical case).

In the LLA the only essential kinematics is the multi-Regge kinematics (see Fig. 1):

\[
s = (p_A + p_B)^2 \gg s_i \gg |t_i| , \quad i = 1, \cdots, n + 1 ,
\]

\[
s_1 \cdot s_2 \cdots s_{n+1} = s \prod_{j=1}^{n} k_{j\perp}^2 , \quad s_i = (k_{i-1} + k_i)^2 , \quad t_i = q_i^2 ,
\]

\[
q_i = q_{i-1} - k_{i-1} , \quad q_0 \equiv p_A , \quad k_0 \equiv p_A' , \quad k_{n+1} \equiv p_{B'} .
\]

Here \(p_A, p_B\) and \(p_A', p_{B'}\) are the momenta of the colliding and scattered particles respectively, \(k_1, \ldots, k_n\) are the momenta of the produced gluons and \(k_{1\perp}, \ldots, k_{n\perp}\) are their transverse components \((k_{i\perp}^2 = -k_{i\perp}^2)\). Due to the gluon reggeization, the multi-gluon production amplitude has a simple multi-Regge form in the kinematics \((4)\) \(\square\):

\[
\mathcal{A}^{A'G_1 \cdots G_n B'}_{AB} = 2s \Gamma_{i \lambda A' A}^{(0)\lambda A} = g T_{A' A}^i \delta_{\lambda A' \lambda A} ,
\]

where \(\Gamma_{i \lambda A' A}^{(0)\lambda A}\) and \(\gamma_{ij}^{G}(q_1, q_2)\) are the particle-particle-reggeon (PPR) and the RRG vertices respectively. In the LLA the helicity of each of the colliding particles is conserved and in the helicity basis the former vertices take the form

\[
\Gamma_{i \lambda A' A}^{(0)\lambda A} = g T_{A' A}^i \delta_{\lambda A' \lambda A} ,
\]

where \(\lambda_A\) is the helicity of the particle \(A\) and \(T_{A' A}^i\) stands for the matrix elements of the colour group generators in the corresponding representation (i.e. adjoint for
gluons and fundamental, $T^i = t^i = \frac{\lambda^i}{2}$, for quarks). The RRG vertex in turn is written as

$$\gamma^G_{ij}(q_1, q_2) = g\epsilon^{*d}_{G}T^d_{ij}\epsilon^{*\mu}_{G}P_{\mu}(q_1, q_2).$$

(7)

Here $(T^a)_{ij} = -if_{ij}^a$ stands for the matrix elements of the colour group generators in the adjoint representation ($f_{bc}^a$ being the group structure constants), $\epsilon^{d}_{G}$ and $\epsilon^{*\mu}_{G}$ are the colour wave function and the polarization vector of the produced gluon with momentum $k = q_1 - q_2$. The vector $P_{\mu}(q_1, q_2)$, given by

$$P^{\mu}(q_1, q_2) = -q^{*\mu}_1 - q^{*\mu}_2 + \frac{p^{\mu}_{A}p_{A} \cdot k}{p_{A} \cdot k}\left(t_1 + k_{1\perp}^2\right) - \frac{p^{\mu}_{B}p_{B} \cdot k}{p_{B} \cdot k}\left(t_2 + k_{2\perp}^2\right),$$

(8)

has the property

$$k^{\mu}P_{\mu}(q_1, q_2) = 0,$$

(9)

which makes transparent the gauge invariance of the amplitude (7).

The program of the calculation of corrections to the LLA, presented in ref. [2], includes, as a necessary step, the calculation of the radiative corrections to the PPR and RRG vertices and to the gluon trajectory. Corrections to the PPR vertices were calculated in refs. [3-5]; they lead to a new (compared with the feature found in the LLA) physical phenomenon - non conservation of the helicity for each of the colliding gluons in pure gluodynamics [3]. In QCD the situation can change: this effect indeed is absent for the case of three massless quark flavours [4].

Gluon loop corrections to the RRG vertex were calculated in ref. [3]. There a new effect also appears: the vertex, though it remains transverse, cannot be expressed only in terms of the transverse vector $P_{\mu}$ defined in eq.(8). In such a situation it becomes very interesting to calculate the quark loop corrections to the RRG vertex. This is all the more interesting because quarks do not play any role
in the LLA, therefore their role in the perturbative pomeron formation could be understood only through the corrections to the LLA.

In this paper we calculate the quark loop contribution to the RRG vertex. Section 2 is devoted to discuss the general structure of the gluon production amplitude. Section 3 deals with the calculation of the correction to the gluon production amplitude in the framework of the dispersive approach. In section 4 the quark contribution to the RRG vertex is obtained. Finally, in section 5 the spin structure of the quark correction to the RRG vertex is analyzed.

2. General structure of the gluon production amplitude

For reaching our goal, we use the dispersive approach based on the analyticity and $t$-channel unitarity, developed in refs. [3-5]. In order to obtain the corrections to the RRG vertex one needs to calculate the corrections to the gluon production amplitude and then compare this with its multi-Regge form, corresponding to the contributions of the reggeized gluons in $t_1$ and $t_2$ channels. It is necessary to remind that the simple form (5) is valid only in the LLA, where one does not make difference between $ln(s)$ and $ln(-s)$. Going beyond the LLA, from requirements of analyticity, unitarity and crossing symmetry one arrives to the following general form for the gluon production amplitude [3]:

\[
\mathcal{A}_{AB}^{A'G'B'} = \frac{s}{4} \Gamma_{A'A}(t_1) \frac{1}{t_1} \epsilon_{i_1}^d \epsilon_{i_2}^d t_{i_1} \Gamma_{B'B}(t_2) \times \left\{ \left[ \frac{s_1}{\mu^2} \right]^{\omega_1-\omega_2} + \left[ \frac{s_2}{\mu^2} \right]^{\omega_2-\omega_1} + \left[ \frac{s}{\mu^2} \right]^{\omega_2} + \left[ \frac{s}{\mu^2} \right]^{\omega_1} \right\} R_G(t_1, t_2, \vec{k}_2^\perp) + \left[ \left[ \frac{s_2}{\mu^2} \right]^{\omega_2-\omega_1} + \left[ \frac{s_2}{\mu^2} \right]^{\omega_2-\omega_1} \right] \left[ \left[ \frac{s}{\mu^2} \right]^{\omega_1} + \left[ \frac{s}{\mu^2} \right]^{\omega_1} \right] L_G(t_1, t_2, \vec{k}_2^\perp) \right\}, \quad (10)
\]
where

\[ \omega_i \equiv \omega(t_i), \quad s_1 = (p_{A'} + k)^2, \quad s_2 = (p_{B'} + k)^2, \quad \vec{k}^2_\perp = \frac{s_1 s_2}{s}. \]

The PPR vertices \( \Gamma_{PP'}^i \) depend on the polarization of particles \( P \) and \( P' \) and the squared momentum transfer \( t \). They are real for \( t < 0 \). The Born expression for these vertices is given in eq. (3); one loop corrections were calculated in refs. [3-5].

The RRG vertices \( \mathcal{R}_G \) and \( \mathcal{L}_G \) depend on the polarization of the gluon, its transverse momentum \( \vec{k}^2_\perp \) and the squared transferred momenta \( t_1, t_2 \). They are real in all channels where \( t_{1,2} < 0, \vec{k}_\perp^2 > 0 \). It is clear from eq. (10) that in each order of perturbation theory the contribution of the sum \( \mathcal{R}_G + \mathcal{L}_G \) is leading while that of the difference \( \mathcal{R}_G - \mathcal{L}_G \) is subleading. In the LLA only the sum \( \mathcal{R}_G + \mathcal{L}_G \) in the Born approximation contributes; one has

\[ \mathcal{R}_G^{(0)} + \mathcal{L}_G^{(0)} = -2g e^*_G P_\mu(q_1, q_2). \] (11)

On the contrary, the difference \( \mathcal{R}_G^{(0)} - \mathcal{L}_G^{(0)} \) at the same order \( g \) contributes to the amplitude (11) only as a radiative correction. It can be obtained, together with one loop corrections of order \( g^3 \) to the sum \( \mathcal{R}_G^{(1)} + \mathcal{L}_G^{(1)} \), from the gluon production amplitude, calculated with one loop accuracy assuming that the one loop corrections \( \Gamma_{PP'}^{(1)i} \) are known. In fact with such an accuracy from eq. (11) we get

\[
A_{AB}^{A'GB'} \text{(one loop)} = s \frac{1}{t_1} \epsilon^* G T_{i_1 i_2} \frac{1}{t_2} \left\{ -2g e^*_G P_\mu(q_1, q_2) \left[ \Gamma_{A'A}^{(1)i_1} \Gamma_{B'B}^{(0)i_2} + \Gamma_{A'A}^{(0)i_1} \Gamma_{B'B}^{(1)i_2} \right] + \frac{1}{4} \Gamma_{A'A}^{(0)i_1} \Gamma_{B'B}^{(0)i_2} \left( \omega_1^{(1)} + \omega_2^{(1)} \right) \ln \left( \frac{s(-s)}{\mu^4} \right) + \left( \omega_1^{(1)} - \omega_2^{(1)} \right) \ln \left( \frac{s_1(-s_1)}{s_2(-s_2)} \right) \right] \right\} .
\] (12)
We notice from eq.(12) that the difference $R_G(0) - L_G(0)$ contributes to the discontinuities of the gluon production amplitude in $s_1$, $s_2$ and $s$ channels, therefore it can be found using $s_i$-channel unitarity conditions \[1, 3\]. As for the sum $R_G(1) + L_G(1)$, it cannot be defined by means of the $s_i$-channel unitarity in the multi-Regge region. The $t$-channel approach is indeed more suitable for this purpose. This sum has been calculated for the case of pure gluodynamics \[3\]. For the real case of QCD one needs to add the quark contribution.

It is possible to calculate the corrections to the RRG vertex by considering the gluon production in various scattering processes: gluon-gluon (GG), quark-quark (QQ) and quark-gluon (QG). Of course, we should obtain the same vertex. In the approach based on $t_i$-channel unitarity relations it looks very natural. One can consider all the three processes simultaneously.

3. Corrections connected with $t_1$-channel discontinuity

Let us take into account the contribution of the amplitude discontinuity in the $t_1$-channel. The discontinuity in the $t_2$-channel could be considered analogously. The contribution is represented schematically in Fig. 2, where particles $C$ and $C'$ are quarks, as we are interested in the quark-antiquark intermediate state. In order to calculate this contribution, we need to consider both amplitudes $A_{AC}'$ and $A_{C'B}$. On one hand, we must use an exact expression for the amplitude $A_{AC}'$, as particles $C$ and $C'$ are in the intermediate state and we integrate over their momenta. On the other hand, since $s_2 = p_{B'} + k$ is fixed and large, we take the amplitude $A_{C'B}$ in the quasi-multi-Regge kinematics \[2\], which means that the gluon $G$ is produced in the fragmentation region of the quark $C'$. An expression for the part of the amplitude with the gluon quantum numbers in $t_i$ channels can
be written at once for both possible choices (quarks or gluons) of particles $B$ and $B'$. It reads

$$\mathcal{A}^{GGB'}_{C'B} = g^2 T^{ij}_{C'C} \bar{u}(p_C) \left\{ -\psi_B \frac{\psi_{C'} - \vec{k} + m_C}{(p_{C'} - \vec{k})^2 - m_C^2} \psi^*_G \\
+ \gamma^\mu \frac{\vec{q}_B \cdot \psi_{C'} + m_C^2 \psi_B}{(p_C + \vec{k})^2 - m_C^2} + \gamma^\mu \psi_B \left[ - \left( e^*_G \cdot (q_1 + q_2) \right) p_B^\mu + s_2 e^*_G \psi_B \right] + 2 \left( e^*_G \cdot p_B \right) \left( q_2^\mu - \frac{p_B^\mu t_2}{s_2} \right) \right\} u(p_B) \frac{e^d_T}{t_1} \frac{1}{t_2} \Gamma^{(0)i_2}_{B'B} \ .$$

As for the amplitude $A^{AC'}_{AC}$, it is very profitable here to decompose it into two parts [3-5] which are schematically shown in Fig. 3:

$$A^{AC'}_{AC} = A^{AC'}_{AC} (as) + A^{AC'}_{AC} (na) .$$

The first term in the r.h.s. of eq.(14) contains the contribution which is proportional to $s_A = (p_A + p_C)^2$ in the asymptotic region $s_A \gg |t_1|$, while the second one cannot include contributions increasing with $s_A$ in this region. The first term can be written in the same form for any kind (quarks or gluons) of particles $A$ and $A'$ (cf. [4, 5]):

$$A^{AC'}_{AC} (as) = 2g \Gamma^{(0)i}_{A'A} \frac{1}{t_1} T^{ij}_{C'C} \bar{u}(p_{C'}) \gamma^\mu_T \frac{1}{t_2} \Gamma^{(0)i_2}_{B'B} \ .$$

The explicit form of the second term is not quoted because we will not need it in our calculations, as it will be discussed in the next section.

According to the decomposition (14), we split the contribution represented in Fig. 2 into two terms shown in Figs 4(a) and 4(b). Let us firstly consider the contribution of Fig. 4(a). Instead of the discontinuity, we calculate the contribution to the amplitude $A^{AGB'}_{AB}$ itself, substituting the full denominator $i(p^2 - m^2 + i\varepsilon)^{-1}$ of the Feynman propagator for an intermediate particle with momentum $p$ in place of $2\pi \delta(p^2 - m^2)$ in the expression for the discontinuity. Using the expressions (13)
and (13) for the r.h.s. and l.h.s. amplitudes respectively, the contribution takes the form

\[ A^{(a)A'GB'}_{AB} = 2g^3 s \Gamma_{A'A}^{(0)i_1} \frac{1}{t_1^A} \epsilon_G^{s_1} \Gamma_{B'B}^{(0)i_2} \frac{1}{t_2^{B'}} \frac{p_A^{\mu} p_B^{\nu}}{s} \epsilon_{s_2}^{r_2} \]

\[
\times \sum_f \left[ \frac{2}{t_1} \mathcal{P}_{\mu\nu}(q_1) \left( P(q_1, q_2) - 2p_A t_1 \right) \overline{\nu}_\rho(q_1, q_2) \right], \tag{16}
\]

where the summation is performed over quark flavours and the vertex vector \( P_{\mu}(q_1, q_2) \) is given by eq.(8). The tensor

\[ \mathcal{P}_{\mu\nu}(q_1) = \frac{i}{2} \int \frac{d^D p}{(2\pi)^D} \tr \left[ \gamma_\mu(p + m_f) \gamma_\nu(p + q + m_f) \right] \]

\[ = (g_{\mu\nu} q^2 - q_{\mu} q_{\nu}) \mathcal{P}(q^2) , \]

\[ \mathcal{P}(q^2) = -4 \frac{\Gamma(2 - \frac{D}{2})}{(4\pi)^{\frac{D}{2}}} \int_0^1 \frac{dx x(1 - x)}{\left( m_f^2 - q^2 x(1 - x) \right)^{2 - \frac{D}{2}}} \]

\[ (17) \]

is the well known fermion contribution to the gluon polarization operator, and the tensor

\[ \mathcal{V}_{\mu\nu\rho}(q_1, q_2) = -i \int \frac{d^D p}{(2\pi)^D} \tr \left[ \gamma_\mu(p + m_f) \gamma_\nu(p + q + m_f) \gamma_\rho(p + q_1 + m_f) \right] \]

\[ = \frac{2}{t_1} \frac{p_A^{\mu} p_B^{\nu}}{s} \]

\[ (18) \]

represents the quark contribution to the triple gluon vertex. In deriving eq.(16) we used the symmetry of the first two terms of eq.(13) under the change \( p_C \leftrightarrow p_C' \) to reduce their contribution to the term \( \mathcal{V}_{\mu\nu\rho} \) in eq.(14); on the other hand, through the usual trick for \( g^{\mu\nu} \), we approximated \( \gamma^\mu \), that appears in eq.(13), with

\[ \gamma^\mu = \gamma_\nu g^{\mu\nu} \approx \frac{2 p_A^{\mu} p_B^{\nu}}{s} . \]  \tag{19}

Subsequently, performing the Feynman parametrization and the integration over momentum \( p \) and using also the property

\[ e_C^* \cdot k = e_C^* \cdot (q_1 - q_2) = 0 \]

8
we get, in the multi-Regge asymptotic region, the following non-vanishing part of the convolution of the tensor $\frac{p_{AB}e^{\mu\nu}}{s}G^{\cdot\cdot\cdot}_{\mu\nu\rho}$ with the tensor $V^f_{\mu\nu\rho}$ shown in eq. (18):

$$\frac{p_{AB}e^{\mu\nu}}{s}G^{\cdot\cdot\cdot}_{\mu\nu\rho}(q_1, q_2) = \frac{\Gamma \left(\frac{3 - D}{2}\right)}{(4\pi)^{\frac{D}{2}}} \int_0^1 \int_0^1 dx_1 dx_2 \theta(1 - x_1 - x_2)$$

$$\times \left\{ m_f^2 + \left(\frac{2 - D}{D - 4}\right) R^f(t_1, t_2) \right\} - (e^*_G \cdot (q_1 + q_2)) (2 - x_1 - x_2)$$

$$+ 2 \frac{s_2}{s}(e^*_G \cdot p_A)(1 + x_1) - 2 \frac{s_1}{s}(e^*_G \cdot p_B)(1 + x_2)$$

$$+ (1 - x_1 - x_2) \left[ (e^*_G \cdot (q_1 + q_2)) \left( R^f(t_1, t_2) - m_f^2 + \frac{2s_1s_2}{s}x_1x_2 \right) \right.$$

$$\left. + 2 \frac{s_2}{s}(e^*_G \cdot p_A)(t_1x_1^2 + t_2x_2(1 + x_1)) \right.$$}

$$\left. - 2 \frac{s_1}{s}(e^*_G \cdot p_B)(t_2x_2^2 + t_1x_1(1 + x_2)) \right\} , \quad (20)$$

where

$$R^f(t_1, t_2) = m_f^2 - (1 - x_1 - x_2)(t_1x_1 + t_2x_2) . \quad (21)$$

Let us now turn to the contribution represented in Fig. 4(b). It is expressed in terms of the product $A^{AC'}_C(na) \cdot A^{CGB'}_{CB}$. Since the non-asymptotic part $A^{AC'}_C(na)$ does not contain terms of order $s_A$ for large values of this invariant, the essential region of integration over $p_C$ in this case is

$$s_A \sim (p_A - p_{C'})^2 \sim p_C^2 \sim p_{C'}^2 \sim q_i^2 , \quad i = 1, 2 ,$$

$$(k + p_C)^2 \sim (k - p_{C'})^2 \sim (p_{A'} + k)^2 = s_1 . \quad (22)$$

This implies that, in order to calculate the contribution of Fig. 4(b), the amplitude $A^{CGB'}_{CB}$ may be taken in the multi-Regge asymptotic region. Moreover, due to the relations

$$\frac{e^*_G \cdot p_C}{k \cdot p_C} \approx \frac{e^*_G \cdot p_A}{k \cdot p_A} , \quad \frac{k \cdot p_C}{p_B \cdot p_C} \approx \frac{k \cdot p_A}{p_B \cdot p_A} , \quad (23)$$
valid in the region \(22\), the amplitude can be written as

\[
\mathcal{A}_{C'B}^{GB'} = 2gT_{(C'B)}^{v_i}(p_C)\bar{u}(p_{C'})u(p_C')\frac{1}{t_1}\gamma_{i_1i_2}^G(q_1, q_2)\frac{1}{t_2}\Gamma_{B'B}^{(0)i_2}, \tag{24}
\]

where the vertex \(\gamma_{i_1i_2}^G(q_1, q_2)\) is given by eqs. (7) and (8). It means that the amplitude (24) differs from the asymptotic part \(\mathcal{A}_{C'B}^{GB'}(as)\) of the elastic scattering amplitude only for factors which do not depend on the momenta \(p_C\) and \(p_{C'}\) of the intermediate particles. Consequently, the contribution of Fig. 4(b) can be calculated in the same way as the corresponding contribution to the elastic scattering amplitude \(\mathcal{A}_{AB}^{A'B'}\) coming from the product \(\mathcal{A}_{AC}^{A'C'}(na) \cdot \mathcal{A}_{C'B}^{GB'}(as)\) (see refs. [4, 5]). As a matter of fact, it is not necessary to calculate this contribution at all, since it is totally absorbed by the corrections to the PPR vertices \(\Gamma_{AB}^{(1)i}\). Let us stress that one-loop corrections to the gluon production amplitude include the corrections to these vertices, as one may observe in eq.(12).

4. Quark contribution to the reggeon-reggeon-gluon vertex

In order to obtain the corrections to the RRG vertex one could calculate the corrections to the gluon production amplitude and subtract those ones coming from the PPR vertices. However, it is more preferable to identify and subtract these last corrections without calculating them. This indeed can be easily done by comparing the corrections to the elastic scattering amplitude with those ones to the gluon production amplitude. In the case of the elastic scattering the corrections connected with the quark-antiquark \(t\)-channel intermediate state are expressed in terms of the corrections to the PPR vertices \(\Gamma_{A'\bar{A}}^{(1)i}\) and \(\Gamma_{B'B}^{(1)i}\). They are found by using the \(t\)-channel unitarity and by performing the decomposition (14) for \(\mathcal{A}_{AC}^{A'C'}\) and the analogous one for \(\mathcal{A}_{C'B}^{GB'}\). The contribution coming from \(\mathcal{A}_{AC}^{A'C'}(na) \cdot \mathcal{A}_{C'B}^{GB'}(as)\) represents a part of the corrections to the amplitude connected with a piece of \(\Gamma_{A'\bar{A}}^{(1)i}\). In
the case of the gluon production, because of the factorization property of $A_{C'B'}^C(a)$ (cf. eq.(15)) and the analogous property of $A_{C'B'}^C(a)$. Therefore we may exclude this piece from the corrections to the gluon production amplitude and thus we do not need to calculate it. After that, we have to consider only the piece of $\Gamma^{(1)i}_{A'HA}$ defined by $A_{A'C'B'}(a) \cdot A_{C'B'}(a)$ and subtract from the corrections to the gluon production amplitude only the part connected with this piece. We know from refs. [4, 5] that

$$\Gamma^{(1)i}_{A'HA}(as \cdot as) = g^2 \Gamma^{(0)i}_{A'HA} \frac{1}{t_1} \frac{P_{aA}^{\mu} P_{aB}^{\nu}}{s} \sum_f \mathcal{P}_{\mu\nu}^f(q_1).$$

(25)

Using eq.(25) and comparing eqs.(12) and (16) we conclude that, in order to avoid taking into consideration the corrections to $\Gamma^{(1)i}_{A'HA}$, we only need to divide by a factor 2 the term with $P_{\mu}(q_1, q_2)$ in eq.(16).

Consequently, the part of the corrections to the the RRG vertex connected with the quark-antiquark intermediate state in the $t_1$-channel is given by

$$(\mathcal{R}^{(1)}_G + \mathcal{L}^{(1)}_G)_{t_1} =$$

$$-2g^3 \frac{P_{A}^{\mu} P_{B}^{\nu}}{s} \epsilon_{G}^{\rho} \sum_f \left[ \frac{1}{t_1} \mathcal{P}_{\mu\nu}^f(q_1) \left( P(q_1, q_2) - 4p_A \frac{t_1}{s_f} \right) + \mathcal{V}_{\mu\nu\rho}^f(q_1, q_2) \right],$$

(26)

where $P_{\mu}(q_1, q_2)$, $\mathcal{P}_{\mu\nu}^f(q_1)$ and $\mathcal{V}_{\mu\nu\rho}^f(q_1, q_2)$ are given by eqs.(8), (17) and (18) respectively.

The sum in eq.(26) has a correct discontinuity in the $t_1$-channel. It has also a $t_2$-channel discontinuity, but this is correct only for terms which have both $t_1$ and $t_2$-channel discontinuities. An expression for the sum $\mathcal{R}^{(1)}_G + \mathcal{L}^{(1)}_G$ with correct discontinuities in both channels can be easily yielded by symmetry considerations. Let us notice that, due to the colour structure of the amplitude (10) and the momentum
flow (see Fig. 1), the sum $\mathcal{R}_G^{(1)} + \mathcal{L}_G^{(1)}$ must change sign under the substitution

$$q_1 \leftrightarrow -q_2, \quad p_A \leftrightarrow p_B,$$

as the vector $P_{\mu}(q_1, q_2)$ does. The function $\frac{p_A^{\mu} p_B^{\nu}}{2} e^{*}_G \mathcal{V}^{f}_{\mu\rho}(q_1, q_2)$, given in eq.(20), has such a property, so we obtain correct discontinuities in both $t_i$ channels if we add the term

$$\frac{1}{t_2} \mathcal{P}^f_{\mu\nu}(q_2) \left( P(q_1, q_2) + 4p_B t_2 \right)_{\rho}$$

into the square brackets of eq.(26).

Contrary to the massless case, where correct analytical properties together with the unitarity requirement in $t_i$ channels determine the amplitude in an unambiguous way [3], in the massive quark case we can add an expression which is equal to the Born amplitude with some constant coefficient [5]. If we want to use a customary renormalization scheme, this coefficient should be determined by Feynman diagrams. This means that we need to add the contribution of Feynman diagrams with a quark loop inserted in the produced gluon line. This contribution is equal to the Born amplitude with the coefficient

$$\frac{g^2}{2} \sum_f \mathcal{P}^f(0) = -\frac{g^2}{3} \frac{\Gamma \left( \frac{2 - D}{2} \right)}{(4\pi)^{D/2}} \sum_f (m_f^2)^{D/2 - 2}. \quad (29)$$

The last equality can be easily derived from the second of eqs.(17).

Finally, we find that the quark contribution to the one loop corrections to the RRG vertex is given by

$$\mathcal{R}_G^{(1)} + \mathcal{L}_G^{(1)} = -g^3 e^{*}_G \sum_f \left[ P_\rho(q_1, q_2) \left( \mathcal{P}^f(t_1) + \mathcal{P}^f(t_2) + \mathcal{P}^f(0) \right) ight.$$ 

$$-4 \frac{p_A^{\mu} t_1}{s_1} \mathcal{P}^f(t_1) + 4 \frac{p_B^{\mu} t_2}{s_2} \mathcal{P}^f(t_2) + 2 \frac{p_A^{\mu} p_B^{\nu}}{s} \mathcal{V}^{f}_{\mu\rho}(q_1, q_2) \right]. \quad (30)$$
By integrating eq.(20) over one of the two \( x \) variables keeping fixed their sum, the quantity \( \mathcal{R}_G^{(1)} + \mathcal{L}_G^{(1)} \) can be written in a manifestly gauge invariant form:

\[
\mathcal{R}_G^{(1)} + \mathcal{L}_G^{(1)} = e_G^* \sum_f \left[ P^\rho(q_1, q_2)(a_1^f + a_2^f) + \left( \frac{p_A}{s_1} - \frac{p_B}{s_2} \right)^\rho \left( -(t_1 + t_2 + 2\vec{k}_\perp^2) a_2^f + a_3^f \right) \right].
\]

The coefficients \( a_i \) in eq.(31) depend on \( t_i \) and \( \vec{k}_\perp^2 \) and read

\[
a_1^f = g^3 \left[ \frac{t_1 + t_2}{t_1 - t_2} \left( \mathcal{P}^f(t_1) - \mathcal{P}^f(t_2) \right) - \mathcal{P}^f(0) \right],
\]

\[
a_2^f = \frac{8g^3}{D - 2} \frac{\Gamma \left( \frac{3 - D}{2} \right)}{(4\pi)^{\frac{D}{2}}} \frac{\vec{k}_\perp^2}{(t_1 - t_2)^3} \times \int_0^1 \frac{dz}{z^2} \left[ \frac{2R_1 R_2}{D - 4} \left( R_{\frac{D}{2} - 2}^1 - R_{\frac{D}{2} - 2}^2 \right) - \frac{2}{D} \left( R_{\frac{D}{2} - 1}^1 - R_{\frac{D}{2} - 1}^2 \right) \right],
\]

\[
a_3^f = g^3 \frac{\Gamma \left( \frac{2 - D}{2} \right)}{(4\pi)^{\frac{D}{2}}} \frac{4}{t_1 - t_2} \int_0^1 \frac{dz}{z} \left[ 4t_1 t_2 z(1 - z) \left( R_{\frac{D}{2} - 2}^1 - R_{\frac{D}{2} - 2}^2 \right) + \frac{2\vec{k}_\perp^2}{D - 2} \left( 3 - \frac{1}{2z(1 - z)} \right) \left( R_{\frac{D}{2} - 1}^1 - R_{\frac{D}{2} - 1}^2 \right) \right],
\]

where

\[
R_i = m_f^2 - z(1 - z)t_i.
\]

Let us notice that the coefficients \( a_2^f \) and \( a_3^f \) do not contain ultraviolet (as well as infrared) singularities. Only the term \( \mathcal{P}^f(0) \) has such singularities. Evidently, poles at \( t_1 = t_2 \) in formulae (32) for \( a_i^f \) are fictitious. One can verify that they cancel and the coefficients have only logarithmic singularities in \( t_1 \) and \( t_2 \).

In the massless case the integrals in eqs.(32) can be calculated for arbitrary \( D \) yielding

\[
a_1^f \big|_{m_f=0} = -4g^3 \frac{\Gamma \left( \frac{2 - D}{2} \right)}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma^2 \left( \frac{D}{2} \right)}{\Gamma(D)} \frac{t_1 + t_2}{t_1 - t_2} \left( (-t_1)_{\frac{D}{2} - 2} - (-t_2)_{\frac{D}{2} - 2} \right),
\]

\[
\]
\[
a_{2}^{f}|_{m_f=0} = -4g^{3}\frac{\Gamma\left(2 - \frac{D}{2}\right)\Gamma^{2}\left(\frac{D}{2} - 1\right)}{(4\pi)^{\frac{D}{2}}\Gamma(D)} \frac{\vec{k}_{\perp}^{2}}{(t_{1} - t_{2})^{3}} \\
\times \left[ \frac{D}{2}t_{1}t_{2} \left( (-t_{1})^{\frac{D}{2} - 2} - (-t_{2})^{\frac{D}{2} - 2} \right) + (2 - \frac{D}{2}) \left( (-t_{1})^{\frac{D}{2} - 1} - (-t_{2})^{\frac{D}{2} - 1} \right) \right],
\]

\[
a_{3}^{f}|_{m_f=0} = 4g^{3}\frac{\Gamma\left(2 - \frac{D}{2}\right)\Gamma^{2}\left(\frac{D}{2} - 1\right)}{(4\pi)^{\frac{D}{2}}\Gamma(D)} \\
\times \frac{1}{t_{1} - t_{2}} \left[ (D - 2)^{2}t_{1}t_{2} \left( (-t_{1})^{\frac{D}{2} - 2} - (-t_{2})^{\frac{D}{2} - 2} \right) \right. \\
\left. - \left( \frac{2 - D}{2} \right) \vec{k}_{\perp}^{2} \left( (-t_{1})^{\frac{D}{2} - 1} - (-t_{2})^{\frac{D}{2} - 1} \right) \right].
\] (34)

It is worth noticing that these coefficients have not any singularity at \( D = 4 \). However, the coefficient \( a_{1}^{f} \) must have an ultraviolet singularity in order to cancel the corresponding one in the sum \( \mathcal{R}_{\perp} + \mathcal{L}_{\perp} \) when this quantity is expressed in terms of the renormalized coupling constant. If \( g_{\mu} \) is such a constant in the \( \overline{\text{MS}} \) scheme at the renormalization point \( \mu \), then one has

\[
g = g_{\mu}\mu^{2-D} \left\{ 1 + \left( \frac{11}{3}N - \frac{2}{3}n_{f} \right) \frac{g_{\mu}^{2}}{(4\pi)^{2}} \left[ \frac{1}{D - 4} - \frac{1}{2}(\ln(4\pi) - \gamma) \right] + \cdots \right\},
\] (35)

where \( \gamma \) is the Euler constant and the term with the quark flavour number \( n_{f} \) yields the “corresponding” quark induced singularity. The gluon induced singularity, originating from the term with \( N \), is absorbed by the gluon contribution to the RRG vertex [3]. At first sight the absence of the singularity in eq.(34) could disturb, but one should realize that the term with the ultraviolet singularity is cancelled there by the one with the infrared singularity [7] which arises in \( \mathcal{P}^{f}(0) \) at \( m_{f} = 0 \).

In the massive quark case for arbitrary \( D \) the integrals in eq.(32) cannot be expressed in terms of elementary functions but it is possible for \( D = 4 \). After performing the charge renormalization (33), taking into account that the gluon induced terms in eq.(34) are absorbed by the gluon contribution to the RRG vertex,
the sum \( R_G^{(1)} + L_G^{(1)} \) of order \( g_\mu^3 \) to which we arrive is given by eq.(31) with the following coefficients:

\[
a_{1R}|_{D=4} = \frac{2}{3} \frac{g_\mu^3}{(4\pi)^2} \left\{ \frac{4m_f^2(t_1 + t_2)}{t_1t_2} + \frac{t_1 + t_2}{t_1 - t_2} \left[ \left( 2 + \frac{4m_f^2}{t_1} \right) \frac{L_1}{\beta_1} \right. \right. \\
- \left. \left. \left( 2 + \frac{4m_f^2}{t_2} \right) \frac{L_2}{\beta_2} \right] - \ln \left( \frac{m_f^2}{\mu^2} \right) \right\},
\]

\[
a_{2}|_{D=4} = 4 \frac{g_\mu^3}{(4\pi)^2} \left\{ \frac{\vec{k}_2^2}{(t_1 - t_2)^3} \left[ -2m_f^2(t_1 + t_2)(L_1^2 - L_2^2) + \left( 4m_f^2(t_1 + t_2) \\
+ \frac{2}{3}t_1t_2 \right) \left( \frac{L_1}{\beta_1} - \frac{L_2}{\beta_2} \right) - \frac{8m_f^2}{3} \left( \frac{t_2L_1}{\beta_1} - \frac{t_1L_2}{\beta_2} \right) - \frac{(t_1-t_2)}{6} (t_1 + t_2 + 16m_f^2) \right] \right\},
\]

\[
a_{3}|_{D=4} = 4 \frac{g_\mu^3}{(4\pi)^2} \left\{ \frac{\vec{k}_2^2}{t_1 - t_2} \left[ 2m_f^2(L_1^2 - L_2^2) - 4m_f^2 \left( \frac{L_1}{\beta_1} - \frac{L_2}{\beta_2} \right) + \frac{t_1 - t_2}{6} \right] \right\},
\]

where the index \( R \) in \( a_1^f \) denotes that the renormalization was performed and

\[
\beta = \sqrt{-\frac{t}{4m_f^2 - t}}, \quad L = \frac{1}{2} \ln \left( \frac{1 + \beta}{1 - \beta} \right) = \ln \left( \sqrt{1 - \frac{t}{4m_f^2}} + \sqrt{\frac{t}{4m_f^2}} \right).
\]

Finally, let us consider the massless quark case in the physical space-time dimension \( D = 4 \). We may approach this case moving either from eq.(36) or from eq.(34) taking into account the charge renormalization (35) in the Born term (11). These two ways are characterized by different regularizations of the infrared singularity which arises in \( \mathcal{P}^f(0) \) at \( m_f = 0 \). Following the second approach we get

\[
\mu^{D-2} \left. \left( R_G^{(1)} + L_G^{(1)} \right) \right|_{m_f=0} = \frac{4}{(4\pi)^2} n_f \left\{ \frac{1}{3} e_G^2 \rho(q_1, q_2) \left[ \frac{1}{D-4} - \frac{1}{2} (\ln(4\pi) - \gamma) + \left( \frac{t_1 + t_2}{2(t_1 - t_2)} \right) \right] \right\},
\]

15
\[ + \frac{\vec{k}^2 t_1 t_2}{(t_1 - t_2)^3} \ln \left( \frac{t_1}{t_2} \right) - \frac{\vec{k}^2 (t_1 + t_2)}{2(t_1 - t_2)^2} + e_G^* \left( \frac{p_A}{s_1} - \frac{p_B}{s_2} \right) \rho \times \left[ \frac{t_1 t_2}{3(t_1 - t_2)} \left( -2 - \frac{\vec{k}^2 (t_1 + t_2 + 2\vec{k}^2 \perp)}{(t_1 - t_2)^2} \right) \ln \left( \frac{t_1}{t_2} \right) \right. \]
\[ \left. + \frac{\vec{k}^2 \perp}{6} \left( 1 + \frac{(t_1 + t_2)(t_1 + t_2 + 2\vec{k}^2 \perp)}{(t_1 - t_2)^2} \right) \right\}. \tag{38} \]

If the infrared singularity was regularized by mass \( m \), in eq. (38) we would have \( \ln \left( \frac{m}{\mu} \right) \) instead of \( \frac{1}{D - 4} - \frac{i}{2} (\ln(4\pi) - \gamma) \).

5. Conclusion

Let us discuss the results we obtained. We already mentioned that poles at \( t_1 = t_2 \) in eqs. (32), (34), (36) and (38) are fictitious and the vertex has only logarithmic singularities in \( t_1 \) and \( t_2 \). As for the dependence on \( \vec{k}^2 \perp \), it is only polynomial for the quark contribution considered here. There are some a priori restrictions on the coefficients with which the spin structures \( e^*_G P_\mu(q_1, q_2) \) and \( e^*_G \left( \frac{p_A}{s_1} - \frac{p_B}{s_2} \right) \) enter the vertex. Such restrictions are connected with the structure of QCD in the infrared region, where only logarithmic singularities are permitted. One can easily verify from eq. (8) that, when \( |\vec{q}_1 \perp| \) or \( |\vec{q}_2 \perp| \) tends to zero, the vector \( P_\mu(q_1, q_2) \) becomes proportional to \( k_\mu \), which in turn means that \( e^*_G P_\mu(q_1, q_2) \) tends to zero. This last result can be proved summing up over the physical polarization states:

\[ \sum_{\lambda_G = 1, 2} |e^*_G P_\mu(q_1, q_2)|^2 = - (P(q_1, q_2))^2 = \frac{4q_1^2 \cdot q_2^2}{\vec{k}^2 \perp}. \tag{39} \]

This property of \( P_\mu(q_1, q_2) \) guarantees a correct infrared behaviour of the vertex and determines, in the Born approximation, the spin structure of the RRG vertex (7) in a unique way. The matter is that the other spin structure does not vanish at
\[ \vec{q}_1 \perp \text{ or } \vec{q}_2 \perp \text{ equal to zero:} \]
\[ \sum_{\lambda_G=1,2} |\epsilon^{*\mu}_G \left( \frac{p_A}{s_1} - \frac{p_B}{s_2} \right)_\mu |^2 = \frac{1}{k^2_\perp}. \quad (40) \]

It means that this spin structure can enter the vertex only with a coefficient which turns to zero at \( \vec{q}_1 \perp \) or \( \vec{q}_2 \perp \) equal to zero. In the Born approximation a coefficient with such properties cannot be found, as it could only exhibit \( t_1 \) and \( t_2 \) singularities at infinity. This spin structure may instead appear in the corrections as it really does, as well as in the gluon contribution to the RRG vertex [3]. On the contrary, for a coefficient of \( e^{*\mu}_G P_\mu \) we only require the absence of power singularities at small \( \vec{q}_1 \perp \) or \( \vec{q}_2 \perp \). One might check that these requirements are fulfilled by inspecting eqs. (31), (32), (34), (36) and (38).

Comparing the coefficient of the new spin structure in eq. (38) with the corresponding one in the gluon contribution (see eq. (86) in the second paper of ref. [3]), one observes a striking similarity of both coefficients. Furthermore, for \( n_f = N \) we find almost full cancellation of the two contributions to the new spin structure, apart from one term with the coefficient \(-\frac{2}{3}n_f\) for the quark case and \( \frac{11}{3}N \) for the gluon case. Unfortunately, we have not an explanation for it.

Acknowledgement: One of us (V.F.) thanks the Dipartimento di Fisica dell’Università della Calabria and the Istituto Nazionale di Fisica Nucleare - Gruppo collegato di Cosenza for their warm hospitality while this work was done.
References

[1] V.S. Fadin, E.A. Kuraev and L.N. Lipatov, Phys. Lett. B60 (1975) 50;
    E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Sov. Phys. JEPT 44 (1976) 443;
    Sov. Phys. JEPT 45 (1977) 199.

[2] L.N. Lipatov and V.S. Fadin, ZHETF Pis’ma 49 (1989) 311; L.N. Lipatov and
    V.S. Fadin, Yad. Fiz. 50 (1989) 1141.

[3] V.S. Fadin and L.N. Lipatov, Nucl. Phys. B (Proc. Suppl.) 29A (1992) 93;
    V.S. Fadin and L.N. Lipatov, Nucl. Phys. B406 (1993) 259.

[4] V.S. Fadin and R. Fiore, Phys. Lett. B294 (1992) 286.

[5] V.S. Fadin, R. Fiore and A. Quartarolo, Phys. Rev. D (1994) in press.

[6] J. Bartels, Nucl. Phys. B175 (1980) 365.

[7] G. ’t Hooft and M. Veltman, Nucl. Phys. B44 (1972) 189.
Figure Captions

Fig. 1: Diagram for the multiple gluon production.

Fig. 2: $t_1$-channel discontinuity of the gluon production amplitude.

Fig. 3: Decomposition of the elastic scattering amplitude in its asymptotic and non asymptotic parts.

Fig. 4(a): Contribution to $t_1$-channel discontinuity from the product of the asymptotic part of the elastic amplitude with the gluon production amplitude.

Fig. 4(b): Contribution to $t_1$-channel discontinuity from the product of the non-asymptotic part of the elastic amplitude with the gluon production amplitude.
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-th/9405127v1