Turnover Rate of Popularity Charts in Neutral Models

Tim Evans
work done with Andrea Giometto

ECCS 2012, Brussels, 7th September 2012
Turnover Rate of Popularity Charts in Neutral Models

• Neutral Models and Cultural Transmission
• Popularity Charts
• Wright-Fisher model results
• Moran model summary
The Neutral Model - as a Bipartite Network

- **$E$** individual vertices each with one edge connected to **$N$** artefact vertices
- Popularity of artefact is vertex degree **$k$**
  - $n(k)$ = degree distribution,
  - $p(k) = n(k)/N$ = degree probability distribution

[Evans 2007; Evans & Plato 2007; Evans, Plato & You 2010]
How Individuals choose in a neutral model

Individuals choose a new artefact in two ways:

- COPY the choice made by another individual (inheritance) probability \( (1-\mu) \)
- INNOVATE by choosing a new artefact at random (mutation) probability \( \mu \)

\[ \begin{array}{ccc}
1 & 0^\mu & 2 \\
\hline
\text{degree} & k & N \text{ artefacts} \\
\text{edges} & E & E \text{ individuals}
\end{array} \]

[Evans 2007; Evans & Plato 2007; Evans, Plato & You 2010]
Wright-Fisher Model and Moran Model

Many variations for update rules:

- **Wright-Fisher model**: all individuals update simultaneously.
- **Moran model**: only one individual updates at each step.
- Choices driven by simplicity and/or reality.

![Diagram of graph with nodes and edges](image)

**Notation and Definitions**:

- $N$: number of artefacts
- $E$: number of edges
- $k$: degree

[Evans 2007; Evans & Plato 2007; Evans, Plato & You 2010]
Relationship to Other Systems

• Genes not Memes [Fisher-Wright & Moran models]
• Speciation in Ecology ['Tangled Nature’, Christensen et al 2002]
• Network Rewiring [Evans & Plato, 2007]
• Statistical Physics Models [Blythe & McKane, 2007]
• Cultural Transmission [Bentley et al, 2004]
• Language Change [Baxter et al, 2006]
• Minority Game strategies [Clemson & Evans 2012]
• Opinion formation [Lambiotte et al. 2007]
• Family Names [Zanette & Manrubia, 2001]
Cultural Transmission Data

- Registrations of pedigree dogs
- Baby name registrations
- Music charts
- Archaeological pot shards

[Herzog, Bentley, Hahn 2004]
2000 (open squares); 1977 (triangles); 1962 (filled squares); 1946 (filled circles).

Frequency of registrations of each breed of pedigree dog

See Neiman (1995); Bentley, Maschner (2000, 2001); Bentley, Hahn, Shennan (2004); Bentley, Shennan (2003, 2005); Hahn, Bentley (2003); Herzog, Bentley, Hahn (2004); Bentley, Lipo, Herzog, Hahn (2007).
Old Models, New Questions

Cultural transmission context for neutral models can produce new questions

• What if can only copy from neighbour in a social network? [Evans, Plato & You, 2010; Omerod talk on Thursday]

• Measuring innovation rates from data

• How do we change innovation rate?

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Popularity Lists

- Rank artefacts by popularity (degree)
- Take list of top $y$ to form popularity list

| Rank | Artefact |
|------|----------|
| 1    | A        |
| 2    | B        |
| 3    | C        |
| 4    | D        |
| 5    | E        |
| 6    | F        |
| 7    | G        |
Turnover Rates in Popularity Lists

- Evolve system
- Update top $y$ list
- Turnover $z$ is number of new artefacts in top $y$ plus number leaving top $y$

Here $z=2$
• Neutral Models and Cultural Transmission
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Linear relationship

[BLHH = Bentley, Lipo, Herzog, & Hahn 2007]
Scaling with innovation rate $\mu$

\[
\frac{z}{2y} = \mu^{0.5}
\]

Computer Wright-Fisher Model $\mu$

[BLHH = Bentley et al 2007]
Conjecture of BLHH [Bentley, Lipo, Herzog, Hahn 2007]

\[ z = 2y \sqrt{\mu} \]

with context of Wright-Fisher Model
Testing the Hypothesis

• Update rule
  – Wright-Fisher Model or Moran Model

• Wide range of parameters
  – The value $N\mu \sim 1$ is a key scale

• Wait $\tau$ updates for equilibrium
  – We use $\tau = 4/\mu > \ln(\lambda_2)$ where $\lambda_2$ is known analytically

• Form popularity lists after $N$ individual choices

• Make $T$ measurements of $z$
  – We aim for $< 10\%$ error in $z$, using $T = 50 + \mu^{-1}$
Wright-Fisher Results – transition point

Transition at 
$N\mu = 0.15 \, y$
Wright-Fisher Results – Low Innovation

Low Innovation ($N\mu < 0.15\,\text{y}$) find

$z = 2\,N\mu$
### Wright-Fisher Results – Low Innovation

List too long, \( y > y^* = 7N\mu \), for low innovation where too few artefacts chosen \( z = 2N\mu \)

| Rank | Artefact |
|------|----------|
| 1    | B        |
| 2    | R        |
| 3    | T        |
| 4    | -        |
| 5    | -        |
| 6    | -        |
| 7    | -        |
Wright-Fisher Results – High Innovation

For high innovation, the BLHH formula is

\[ z = 2y\mu^{0.5} \]

roughly right.

\[ N_\mu > 0.15y \]
Wright-Fisher Results – High Innovation

For $180 < N < 3993$ we tried to fit

$$z = A \mu^a y^b N^c$$

and found [BLHH values]

$A = 1.38(2) \quad [2.0]$  
$a = 0.550(2) \quad [0.5]$  
$b = 0.860(1) \quad [1.0]$  
$c = 0.130(2) \quad [0.0]$  

Weak $N$ dependence

$N \mu > 0.15 \ y$
Excellent data collapse

\[ A\mu^a y^b N^c \]

Wright-Fisher – High Innovation Data Collapse

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Wright-Fisher – Large Populations

• So far $N \leq 4000$ as in BLHH
• Real data sets are much larger
  – One million births per year for baby names
  – Six hundred thousand dog breed registrations per year
  – Seven thousand new CD issued by major labels per year

So we extended to $N=100K, 120K, 144K$ for $y=200,400$ and $\mu=0.0012$ and 0.0024
Fisher-Wright – large populations

Now for $N = 100K$ to $144K$ again fit to

$$z = A \mu^a y^b N^c$$

and found large $N$ small $N$ [BLHH values]

\[
\begin{align*}
A &= 1.79(2) \quad 1.38(2) \quad [2.0] \\
a &= 0.558(1) \quad 0.550(2) \quad [0.5] \\
b &= 0.879(1) \quad 0.860(1) \quad [1.0] \\
c &= 0.091(1) \quad 0.130(2) \quad [0.0]
\end{align*}
\]
Fisher-Wright – Conclusions

• Simple BJHH formula excluded statistically
• Better formulae provided
• Still suggestion of weak dependence on population size \( N \)

• However fluctuations may mean differences difficult to detect in actual data

\[
z = A\mu^a y^b N^c
\]

\[
\begin{align*}
A &= 1.79(2) & 1.38(2) & [2.0] \\
a &= 0.558(1) & 0.550(2) & [0.5] \\
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\end{align*}
\]
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Moran Model

• Found similar transition but now at $y^* = N\mu^{3/2}$ (for FW $y^* = N\mu / 0.15$)

• Found same low innovation behaviour

• Could not get simple power law fits to work

• Formula of Erikson et al 2010 works better but assumptions used not clearly satisfied (is 5.83 infinity?)
THANKS

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For papers, talks and other material search for Tim Evans Networks or look at netplexity.org
Primary sources for this talk:-

Evans, T.S. & Giometto, A., “Turnover Rate of Popularity Charts in Neutral Models”, 2011 [arxiv:1105.4044]

Bentley, R. A.; Lipo, C. P.; Herzog, H. A. & Hahn, M. W., “Regular rates of popular culture change reflect random copying”, Evolution and Human Behavior, 2007, 28, 151-158.

Eriksson, K.; Jansson, F. & Sjöstrand, J., “Bentley's conjecture on popularity toplist turnover under random copying”, The Ramanujan Journal, 2010, 23, 371-396-396

Other work in this talk by TSE and on this topic:-

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Evans T.S. & Plato A.D.K. , “Exact Solution for the Time Evolution of Network Rewiring Models” Phys. Rev. E 75 (2007) 056101 [cond-mat/0612214] (includes review of applications)

Evans, T.S., Plato, A.D.K. & You, T., “Are Copying and Innovation Enough?”, Progress in Industrial Mathematics at ECMI 2008, Fitt, A.; Norbury, J.; Ockendon, H. & Wilson, E. (Eds.) , Springer-Verlag, 2010, 15, 825-831

For material search for Tim Evans Networks or look at netplexity.org
Other papers mentioned in this talk:

Christensen, K.; Di Collobiano, S. A.; Hall, M. & Jensen, H. J. Tangled Nature: A Model of Evolutionary Ecology *Journal of Theoretical Biology*, 2002, 216, 73-84

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*Physica A, Imperial College London*, 2012, 391, 1434-1444  [arXiv:1106.0296]

Blythe, R. A. & McKane, A. J. Stochastic models of evolution in genetics, ecology and linguistics *Journal of Statistical Mechanics: Theory and Experiment*, 2007, 2007, P07018-P07018

D.Zanette and S.Manrubia, *Vertical transmission of culture and the distribution of family names*, Physica A 295 1 (2001).