ON THE STABILITY OF SELF-GRAVITATING PROTOPLANETARY DISCS

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ABSTRACT

It has already been shown, using a local model, that accretion discs with cooling times $t_{\text{cool}} \lesssim 3\Omega^{-1}$ fragment into gravitationally bound objects, while those with cooling times $t_{\text{cool}} > 3\Omega^{-1}$ evolve into a quasi-steady state. We present results of three-dimensional simulations that test if the local result still holds globally. We find that the fragmentation boundary is close to that determined using the local model, but that fragmentation may occur for longer cooling times when the disc is more massive or when the mass is distributed in such a way as to make a particular region of the disc more susceptible to the growth of the gravitational instability. These results have significant implications for the formation of gaseous planets in protoplanetary discs and also for the redistribution of angular momentum which could be driven by the presence of relatively massive, bound objects within the disc.

1. INTRODUCTION

The discovery of the first extra-solar planet (M\textsuperscript{ayor} & Queloz, 1995) and the subsequent discovery of many (> 100) additional extra-solar planets (Marcy & Butler, 2000) has enhanced the interest in the formation of planets and the evolution of protoplanetary discs. The observation of the first transiting planet (Henry et al., 2000; Charbonneau et al., 2000), giving an estimate of the planet radius, and the relatively large masses (> 0.1$M_{\text{Jupiter}}$) of all the currently detected extra-solar planets, has led to the view that they are all giant gaseous planets. The most widely studied, and accepted, formation mechanism for gaseous planets is core accretion (Lissauer, 1993) in which planetesimals grow by direct collision to form a core which, when sufficiently massive ($m \sim 10m_{\text{Earth}}$), then accretes an envelope of gas from the disc. Models (Pollack et al., 1996), however, suggest that the timescale for planet formation via this mechanism may be longer than the lifetime of most protoplanetary accretion disc (Haisch, Lada & Lada, 2001). This has led to a renewed interest in the possibility that gas giant planets may have formed directly via a disc instability (Kuiper, 1951; Boss, 1998, 2000; Mayer et al., 2002).

A protoplanetary disc that is sufficiently massive and cool may form gravitationally bound gaseous objects via the gravitational instability. This mechanism is extremely rapid and differs from core accretion in that a rocky core is not required. The possibility that this mechanism may play a role in gaseous planet formation has been enhanced by recent models of the interiors of gaseous planets (Guillot, Gautier & Hubbard, 1997; Guillot, 1999) which suggest that Jupiter could have a relatively small core with $m_{\text{core}} < 10 m_{\text{Earth}}$. A Keplerian accretion disc with sound speed $c_s$, surface density $\Sigma$, and epicyclic frequency $\kappa$ will become gravitationally unstable if the Toomre (1964) $Q$ parameter

\begin{equation}
Q = \frac{c_s \kappa}{\pi G \Sigma}
\end{equation}

is of order unity. A gravitationally unstable disc can either fragment into one or more gravitationally bound objects, or it can evolve into a quasi-steady state in which gravitational instabilities lead to the outward transport of angular momentum. The exact outcome depends on the rate at which the disc heats up (through the dissipation of turbulence and gravitational instabilities) and the rate at which the disc cools. It has been suggested (Goldreich & Lynden-Bell, 1965) that a feedback loop may exist. When $Q$ is large the disc is stable and cooling dominates, driving the disc towards instability. When $Q$ becomes sufficiently small, heating through viscous dissipation dominates, and the disc is returned to a state of marginal stability. In this way $Q$ is maintained as a value of $\sim 1$.

It has, however, been shown using a local model (Gammie, 2001) that a quasi-stable state can only be maintained if the cooling time $t_{\text{cool}} > 3\Omega^{-1}$ where $\Omega$ is the local angular frequency. For shorter cooling times the disc fragments. This is consistent with Pickett et al. (1998, 2000) that ‘almost isothermal’
conditions are necessary for fragmentation, and defines a robust lower limit to the critical cooling time below which fragmentation occurs. It has been suggested, however, that self-gravitating discs require a strictly global treatment (Balbus & Papaloizou 1999), and while global effects are highly unlikely to stabilize a locally unstable disc, they could well allow fragmentation within discs that would be locally stable. This has led to an interest in global simulations of self-gravitating protoplanetary discs (Rice et al. 2003) and we discuss, in this paper, results of a number of these simulations.

2. NUMERICAL SIMULATIONS

2.1. Smoothed particle hydrodynamics

The three dimensional, global simulations presented here were performed using smoothed particle hydrodynamics (SPH), a Lagrangian hydrodynamics code (Benz 1990; Monaghan 1992). The central star is modelled as a point mass that may accrete gas particles if they approach within a predefined sink radius, while the gaseous disc is modelled using 250000 SPH particles. A tree is used to determine neighbours and to calculate gravitational forces between gas particles and between gas particles and point masses (Benz 1990). If in the simulation the disc starts to fragment, producing high density regions, the code can in principle continue. The high density regions do, however, slow the code down significantly. To continue simulating a fragmenting disc, high density regions that are gravitationally bound are converted into point masses (Bate, Bonnell & Price 1995) which may continue to accrete disc gas. Since the likely number of point masses is quite small, compared to the number of gas particles, the gravitational force between point masses is computed directly. An additional saving in computational time is also made by using individual particle time-steps (Bate, Bonnell & Price 1995; Navarro & White 1993). The time-steps for each particle is limited by the Courant condition and by a force condition (Monaghan 1992).

2.2. Initial conditions

We consider a system comprising a central star, modelled as a point mass with mass $M_\ast$, surrounded by a gaseous circumstellar disc with mass $M_{\text{disc}}$. We performed a number of simulations with disc masses of $M_{\text{disc}} = 0.1M_\ast$ and $M_{\text{disc}} = 0.25M_\ast$. The disc temperature is taken to have an initial radial profile of $T \propto r^{-0.5}$ (Yorke & Bodenheimer 1993) and in most of the simulations the Toomre $Q$ parameter is assumed to be initially constant and to have a value of 2. A stable accretion disc where self-gravity leads to a steady outward transport of angular momentum should have a near constant $Q$ throughout. A constant $Q$ together with Equation 1 gives a surface density profile of $\Sigma \propto r^{-7/4}$, and hydrostatic equilibrium then gives a central density profile of $\rho \propto r^{-3}$. We did, however, perform a single simulation with $M_{\text{disc}} = 0.1M_\ast$ and with $\Sigma \propto r^{-1}$. In this case, $T \propto r^{-0.5}$ gives a $Q$ that is not constant but that decreases with increasing radial distance. The temperature was chosen such that the initial minimum value of $Q$ was 1.5.

The disc is modelled using 250000 SPH particles, which are initially randomly distributed such as to give the specified density profile between inner and outer radii of $r_{\text{in}}$ and $r_{\text{out}}$. Since the calculations are scale free, we take $M_\ast = 1$, $r_{\text{in}} = 1$, and $r_{\text{out}} = 25$. If we were to assume a physical mass scale of 1 M$_\odot$ and a length scale of 1 au, the central star would have a mass of 1 M$_\odot$, the circumstellar disc would have a mass of 0.1 M$_\odot$ or 0.25 M$_\odot$ and would extend from 1 to 25 au, and 1 yr would equal 2$\pi$ code units.

2.3. Cooling

The main aim of this work is to test if the results obtained by Gammie (2001) still hold globally. Consequently we use the same approach in these simulations as was used in the local model. We use an adiabatic equation of state, with adiabatic index $\gamma = 5/3$, and allow the gas to heat due to both PdV work and dissipation. Cooling is implemented by adding a simple cooling term to the energy equation. Specifically, for a particle with internal energy per unit mass $u_i$, 

$$\frac{du_i}{dt} = -\frac{u_i}{t_{\text{cool}}}$$

where, as in Gammie (2001), $t_{\text{cool}}$ is given by $\beta \Omega^{-1}$ with the value of $\beta$ varied for each run.

Although the above cooling time is essentially chosen to compare the local model results with results using a global model, it can also be related (at least approximately) to the real physics of an accretion disc. For an optically thick disc in equilibrium, the cooling time is given by the ratio of the thermal energy per unit area to the radiative losses per unit area. It can be shown (Pringle 1981) that in such a viscous accretion disc, the cooling time is

$$t_{\text{cool}} = \frac{4}{9\gamma(\gamma - 1)} \frac{1}{\alpha \Omega}$$

where $\Omega$ is the angular frequency in the disc, and $\alpha$ is the Shakura & Sunyaev (1973) viscosity parameter.

3. RESULTS

3.1. $M_{\text{disc}} = 0.1M_\ast$

We use a three dimensional, global model to consider how cooling times of $t_{\text{cool}} = 5\Omega^{-1}$, and $t_{\text{cool}} = 3\Omega^{-1}$ affect the gravitational stability of a protoplanetary
accretion disc with $M_{\text{disc}} = 0.1 M_*$ and with $\Sigma \propto r^{-7/4}$. This choice of surface density profile, together with $T \propto r^{-0.5}$, gives a Toomre $Q$ parameter that is initially constant throughout the disc.

Figure 1 shows the final equatorial density structure of the $t_{\text{cool}} = 5 \Omega^{-1}$ simulation. The central star (not shown) is located in the middle of the figure, and the $x$ and $y$ axes both run from $-25$ to $25$. The disc is highly structured and the instability exists at all radii. However, at no point in the disc has the density increased significantly, and no fragmentation has taken place. Figure 2 shows the Toomre $Q$ parameter for the same simulation at three different times during the simulation. At the beginning of the simulation ($t = 0$) $Q$ has an almost constant value of 2. At the end of the simulation ($t = 2932$) the value of $Q$ is almost unity between radii of 1 and 15.

Comparing $Q$ at $t = 876$ and $t = 2932$ shows that the cooling initially reduces $Q$ to a value below 1, causing the instability to grow, heating the disc and returning $Q$ to a value of order unity. The disc has clearly settled into a quasi-steady state in which the imposed cooling is balanced by heating through the dissipation of the gravitational instability.

Figure 3 shows the equatorial disc structure for a simulation with the same disc mass and surface density profile as in Figure 1 but with a cooling time of $t_{\text{cool}} = 3 \Omega^{-1}$. This simulation was only run for 504 time units and hence we show only the inner 8 radii of the simulation. The disc is again highly structured, but in this case the disc has started to fragment into gravitationally bound clumps, indicated by the bright dots in Figure 3. The density in the clumps is 4-5 orders of magnitude greater than the initial disc density, and to reach the time shown in the simulation ($t = 2932$) the disc has taken place. Figure 4 shows the Toomre $Q$ parameter at the beginning ($t = 0$), one third of the way ($t = 876$), and at the end ($t = 2932$) of the simulation in which $M_{\text{disc}} = 0.1 M_*$, $\Sigma \propto r^{-7/4}$, and $t_{\text{cool}} = 5 \Omega^{-1}$. At the end of the simulation $Q$ is of order unity between radii of 1 and 15.

3.2. $M_{\text{disc}} = 0.25 M_*$

To study how disc mass may influence the global nature of the gravitational instability we consider a disc with a mass of $M_{\text{disc}} = 0.25 M_*$. As in the previous simulations, the surface density is taken to have a radial profile of $\Sigma \propto r^{-7/4}$ which, together with $T \propto r^{-0.5}$, gives an initially constant Toomre $Q$ parameter. Although most T Tauri discs have masses considerably less than $0.25 M_*$ (Beckwith et al. 1990), there a few with such masses, and this simulation may also apply to an earlier stage of the star formation process when discs are expected to be more massive.

Figure 5 shows the final equatorial density structure of the above simulation. The disc is highly structured and the nature of the spirals, compared to the equivalent simulation with $M_{\text{disc}} = 0.1 M_*$, is consistent with the increased disc mass (Nelson et al.).
Figure 3. Equatorial density structure of the inner regions of a disc with $M_{\text{disc}} = 0.1 M_\ast$, $\Sigma \propto r^{-7/4}$, and with a cooling time of $t_{\text{cool}} = 3 \Omega^{-1}$. The disc is highly structured and is starting to fragment into high density, gravitationally bound clumps.

Figure 4. Toomre $Q$ parameter at times of $t = 192$, $t = 392$, and $t = 504$ for $M_{\text{disc}} = 0.1 M_\ast$, $\Sigma \propto r^{-7/4}$, and $t_{\text{cool}} = 3 \Omega^{-1}$.

Figure 5. Equatorial density structure of a disc with $M_{\text{disc}} = 0.25 M_\ast$, $\Sigma \propto r^{-7/4}$, and $t_{\text{cool}} = 5 \Omega^{-1}$. There are clear signs of fragmentation with the most massive fragments being gravitationally bound.

Unlike the equivalent lower mass simulation (see Figure 4), there are a number of high density clumps present in the disc. The standard routine for checking if these clumps are bound initially found them to be unbound. This routine, however, only considers the nearest $\sim 50$ SPH neighbours. By increasing this to the nearest $\sim 250$ SPH neighbours, the densest clump was found to be just gravitationally bound. This would suggest that in more massive discs, global effects could act to make the disc more unstable, allowing gravitationally bound fragments to grow for cooling times greater than that obtained using a local model. This could have significant implications for the formation of planets, or binary companions (Adams, Ruden & Shu, 1989), reasonably early in the star formation process.

3.3. $M_{\text{disc}} = 0.1 M_\ast$, $\Sigma \propto r^{-1}$

All of the previous simulations were performed assuming $\Sigma \propto r^{-7/4}$, and $T \propto r^{-0.5}$, giving an initially constant Toomre $Q$ parameter. We have performed a single simulation with $M_{\text{disc}} = 0.1 M_\ast$, $T \propto r^{-0.5}$, and $\Sigma \propto r^{-1}$. With these parameters, the $Q$ value decreases with increasing radius. We therefore normalised our temperature such that $Q = 1.5$ at $r = 25$. Figure 6 shows the final equatorial density structure of a simulation with the above disc parameters, and with an imposed cooling time of $t_{\text{cool}} = 5 \Omega^{-1}$. The disc is again highly structured and unlike the simulation with the same disc mass and cooling time, but with the steeper surface density profile, there is clear evidence of fragmentation in the outer regions of the disc. The shallower surface density profile ($\Sigma \propto r^{-1}$) means that, compared to the steeper surface density profile ($\Sigma \propto r^{-7/4}$), there is more mass at large radii. This suggests that the global nature of the instability may depend both on the mass of the disc, and on how the mass is distributed within the disc.
Figure 6. Equatorial density structure of a disc with $M_{\text{disc}} = 0.1 M_\ast$, $\Sigma \propto r^{-1}$, and $t_{\text{cool}} = 5 \Omega^{-1}$. There are clear signs of fragmentation suggesting that the fragmentation boundary may depend both on the mass of the disc and on how the mass is distributed.

4. CONCLUSIONS

It has been shown using a local model (Gammie, 2001) that a disc will fragment for cooling times $t_{\text{cool}} \lesssim 3 \Omega^{-1}$ and will settle into a quasi-steady state for cooling times $t_{\text{cool}} > 3 \Omega^{-1}$. We present here results from three-dimensional simulations which test if the local results still holds globally (Rice et al., 2003). We impose the same cooling function as in Gammie (2001) which, although fairly simplistic, can also be justified physically (Pringle, 1981).

For a disc mass of $M_{\text{disc}} = 0.1 M_\ast$, a cooling time of $t_{\text{cool}} = 5 \Omega^{-1}$, and an initially constant $Q$, the disc settles into a quasi-steady state in which $Q$ is of order unity over a large region of the disc. Decreasing the cooling time to $t_{\text{cool}} = 3 \Omega^{-1}$ causes the disc to rapidly become unstable and produces numerous gravitationally bound clumps. For this disc mass and surface density profile ($\Sigma \propto r^{-7/4}$), the fragmentation boundary therefore seems to agree with that obtained using a local model.

Increasing the disc mass to $M_{\text{disc}} = 0.25 M_\ast$ and keeping all other parameters unchanged, however, results in fragmentation for a cooling time of $t_{\text{cool}} = 5 \Omega^{-1}$. A similar result was obtained for a simulation in which the disc mass was kept at $M_{\text{disc}} = 0.1 M_\ast$ but in which the surface density profile was changed from $\Sigma \propto r^{-7/4}$ to $\Sigma \propto r^{-1}$. These results suggest that global effects may become significant for massive disc or for discs in which the mass is distributed in such a way as to make a particular region of the disc susceptible to the growth of the gravitational instability. For T Tauri discs, which generally have masses $M_{\text{disc}} < 0.1 M_\ast$, (Beckwith et al., 1990), the fragmentation boundary is therefore likely to be close to that determined using the local model. Earlier in the star formation process, when disc may be more massive, fragmentation may occur for longer cooling times than that predicted by the local model results (Gammie, 2001).

For quiescent T Tauri discs, the Shakura & Sunyaev (1973) $\alpha$ is conventionally estimated to be of the order of $10^{-2}$ or smaller (Hartmann et al., 1998; Bell & Lin, 1994). The simple estimate quoted earlier (see Eq. 14) would suggest that such discs are comfortably stable. Boss (2002), however, has shown using an approximate treatment of disc heating and cooling, that there may be periods when the cooling time is comparable to the orbital period. Our results suggest that if the disc is fairly massive, such short cooling times may open a window of opportunity for the formation of substellar objects, probably in the form of a multiple system (Armitage & Hansen, 1999). This has important implications for the formation of giant gaseous planets in protoplanetary discs and may also play a role in the rapid transfer of angular momentum (Larson, 2002).

ACKNOWLEDGMENTS

The simulations reported in this paper made use of the UK Astrophysical Fluids Facility (UKAFF). WKMR acknowledges support from a PPARC standard grant.

REFERENCES

Adams F.C., Ruden S.P., Shu F.H., 1989, ApJ, 347, 989
Armitage P.J., Hansen B.M.S., 1999, Nat, 467, 633
Balbus S.A., Papaloizou J.C.B., 1999, ApJ, 521, 650
Bate M.R., Bonnell I.A., Price N.M., 1995, MNRAS, 277, 362
Beckwith S.V.W., Sargent A.I., Chini R.S., Guesten R., 1990, AJ, 99, 924
Bell K.R., Lin D.N.C., 1994, ApJ, 427, 987
Benz W., 1990, in Buchler J.R. ed., The numerical modelling of nonlinear stellar pulsations, Kluwer, Dordrecht, p. 269
Boss A.P., 1998, Nat., 393, 141
Boss A.P., 2000, ApJ, 536, L101
Boss A.P., 2002, E&PSL, 202, 513
Charbonneau D., Brown T.M., Latham D.W., Mayor M., 2000, apJ, 529, L45
Gammie C.F., 2001, ApJ, 553, 174.
Goldreich P., Lynden-Bell D., 1965, MNRAS, 130, 125
Guillot T., Gautier D., Hubbard W.B., 1997, Icarus, 130, 534.
Guillot T., 1999, Planet. Space Sci., 47, 1183.
Haisch K.E., Lada E.A., Lada C.J., 2001, ApJ, 553, L153
Hartmann L., Calvet N., Gullbring E., D’Alessio P., 1998, ApJ, 495, 385.

Henry G.W., Marcy G.W., Butler R.P., Vogt S.S., 2000, ApJ, 529, L41

Kuiper G.P., 1951, in Hynek J.A., ed., Proceedings of a topical symposium, commemorating the 50th anniversary of the Yerkes Observatory and half a century of progress in astrophysics, McGraw-Hill, New York, p.357

Larson R.B., 2002, MNRAS, 332, 155.

Lissauer J.J., 1993, ARA&A, 31, 129

Marcy G.W., Butler R.P., 2000, PASP, 112, 137.

Mayer L., Quinn T., Wadsley J., Stadel J., 2002, Science, 298, 1756

Mayor M., Queloz D., 1995, Nat., 378, 335

Monaghan J.J., 1992, ARA&A 30, 543

Navarro J.F., White S.D.M., 1993, MNRAS, 265, 271

Nelson A.F., Benz W., Adams F.C., Arnett D., 1998, ApJ, 502, 342

Pickett B.K., Cassen P., Durisen R.H., Link R., 1998, ApJ, 504, 468

Pickett B.K., Durisen R.H., Cassen P., Mejia A.C., 2000, ApJ, 540, L95

Pollack J.B., et al., 1996, Icarus, 124, 62

Pringle J.E., 1981, ARA&A, 19, 137

Rice W.K.M., Armitage P.J., Bate M.R., Bonnell I.A., 2003, MNRAS, 339, 1025

Shakura N.I., Sunyaev R.A., 1973, A&A, 24, 337

Toomre A., 1964, ApJ, 139, 1217

Yorke H.W., Bodenheimer P., 1999, ApJ, 525, 330.
