Spin dynamics in relativistic ionization with highly charged ions in super-strong laser fields

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Received 23 September 2013, revised 28 January 2014
Accepted for publication 12 February 2014
Published 4 March 2014

Abstract
Spin dynamics and induced spin effects in above-threshold ionization of hydrogenlike highly charged ions in super-strong laser fields are investigated. Spin-resolved ionization rates in the tunnelling regime are calculated by employing two versions of a relativistic Coulomb-corrected strong-field approximation (SFA). An intuitive simpleman model is developed which explains the derived scaling laws for spin flip and spin asymmetry effects. The intuitive model as well as our ab initio numerical simulations support the analytical results for the spin effects obtained in the dressed SFA where the impact of the laser field on the electron spin evolution in the bound state is taken into account. In contrast, the standard SFA is shown to fail in reproducing spin effects in ionization even at a qualitative level. The anticipated spin-effects are expected to be measurable with modern laser techniques combined with an ion storage facility.

Keywords: strong-field ionization, tunneling, spin asymmetry, spin flip

(Some figures may appear in colour only in the online journal)
on the electron continuum dynamics as well as the influence of the laser field on the bound state dynamics are neglected. In particular, the latter can be of crucial relevance for the electron’s spin dynamics, decreasing the spin asymmetry dramatically as is shown in [14].

Generally, spin effects in laser fields have been in the focus of theoretical attention for a long time, in particular, for free electron motion and scattering [19–30]. In the relativistic laser–atom interaction spin effects, though small and an exemplary in the bound state is accounted for. Asymptotic scaling laws employing a relativistic Coulomb corrected dressed SFA (CC-effects are induced due to the spin motion in the strong field super-strong laser field is investigated. We show that spin ionization process of highly charged hydrogenlike ions in a dramatically, when the electron spin dynamics in the bound flip and the spin asymmetry effects at ionization is changed the conclusion that the qualitative behaviour of the spin calculated employing a relativistic Coulomb-corrected SFA (42) in [44]. The interaction Hamiltonian

\[ H = c \alpha \cdot (\hat{p} + A) + \beta c^2 + V^{(c)} - \Phi = \hat{H}_0 + \hat{H}_{\text{int}}, \]

where the vector-potential of the monochromatic laser field is chosen in the Göppert-Mayer gauge \( A^\mu \equiv (\Phi, A) = (-r \cdot E,-\hat{k}(r \cdot E)/c) \) [43, 44], with the laser field \( E \) and the unit vector in the laser propagation direction \( \hat{k} \). For the partition given by equation \( 2 \), the initial state \( \phi^{(c)}_i \) is governed by the equation

\[ i\hbar \dot{\phi}^{(c)}_i = \hat{H}_0 \phi^{(c)}_i. \]

Spin dynamics during tunnelling and motion in the continuum. The Coulomb corrected standard SFA (CC-S-SFA) is based on the partition when \( \hat{H}_0 \) corresponds to the free atomic Hamiltonian: \( \hat{H}_0^{(S)} = c \alpha \cdot \hat{p} + \beta c^2 + V^{(c)}, \) \( \hat{H}_{\text{int}}^{(S)} = c \alpha \cdot A - \Phi, \) and the spin dynamics in the bound state is completely neglected, i.e., before tunnelling from the bound state the electron spin is the same as in the initial state. In this case the spin effects are determined by the electron dynamics during tunnelling and the motion in the continuum. Because of the evident asymmetry in the spin evolution in this picture (frozen spin in the bound state and oscillating spin in the continuum) relatively large spin effects arise. Yet, it is well-known that the laser field can induce a large spin precession in the bound state [31, 32]. Moreover, the Zeeman-splitting of the bound state levels in the laser field can have an impact on the tunnelling probabilities [14], in this way inducing spin effects. Therefore, we use a CC-D-SFA [44] based on a nonstandard partition of the Hamiltonian

\[ \hat{H}_0^{(D)} = c \alpha \cdot (\hat{p} - \hat{k}(r \cdot E))/c) + \beta c^2 + V^{(c)}, \quad \hat{H}_{\text{int}}^{(D)} = r \cdot E, \]

in which \( \hat{H}_0^{(D)} \) includes the laser field, i.e., the bound state evolution in the laser field is accounted for, described by equation \( 3 \).

Let us discuss the case of a linearly polarized laser field. Spin effects in the tunnelling regime of ionization are built up in three steps: spin precession in the bound state, spin rotation during tunnelling, and spin precession during the electron motion in the continuum, see figure 1. Only the last two steps are included in the CC-S-SFA. First, we discuss the S-SFA results and the intuitive picture for the last two steps of the spin evolution. Afterwards, we discuss how the picture changes when the first step is included within the CC-D-SFA.

The CC-S-SFA predicts a spin asymmetry in ionization, i.e., the ionization probability depends on spin orientation with respect to the laser’s magnetic field

\[ A \equiv (w_{-+} - w_{+-})/w \approx 2\sqrt{2}I_0/c, \]

where \( w_{ij} \) is the ionization probability from initial state \( i \) to final state \( f, +/− \) correspond to the spin-up and -down states, \( w = (w_{++} + w_{--} + w_{+-} + w_{-+})/2 \) is the total ionization probability averaged over the initial spin states. We consider the tunnelling regime when the Keldysh parameter \( \gamma_K \) is small, \( \gamma_K \equiv (\omega/E_0)/\sqrt{2}I_0 \) and \( E_0/E_i \ll 1 \), as well as \( I_0/c^2 \ll 1 \), with the atomic field \( E_0 \equiv (2I_0)^{1/2} \), the laser field amplitude \( E_0 \). As the necessary strong field intensities are available only with infrared lasers, we use a corresponding laser frequency \( \omega = 0.05 \). The asymmetry \( 5 \) has a simple explanation. In the CC-S-SFA the laser quasi-static magnetic field acts under the tunnelling barrier, but not before tunnelling. Since the total energy \(-\hat{H}_0\) is conserved before and during tunnelling, the kinetic energy \( s_{\text{kin}} \) under the barrier is shifted due to the spin magnetic field energy \( s_M = B_H/(2c) \) for
the ionized electron along the electric field ($\Delta p_E$) that will be explained by the simpleman model below. The $I_p/2c^2$-term in equation (6) originates from the tunnelling step of ionization. The initial spin state polarized in the laser propagation direction $|\pm_s\rangle$ can be represented as a linear combination of spinors polarized in magnetic field direction $|\pm_b\rangle$. Because the tunnelling probability depends on the spin projection along the magnetic field, see equation (5), the coefficients of the superposition state will change during tunnelling, which is equivalent to a spin rotation in the B-k-plane by the angle $\Delta \theta \approx \sqrt{2I_p/c}$. The latter induces the $I_p/2c^2$-term in equation (6).

The other terms in equation (6) are due to the third step: the electron motion in the laser field after exiting the tunnelling barrier. It can be described by a semi-classical Bargmann–Michel–Telegdi equation [47], which adds a spin angular momentum $s$ to the classical electron:

$$\frac{ds}{dt} = \frac{1}{c^4} s \times (\hat{k} \times E) - \frac{1}{c(\gamma + 1)} s \times (\beta \times E),$$

(7)

where $\beta$ and $\gamma$ are the Lorentz factors. Using the classical equations of motions in a laser field and introducing the precession angle $\phi$ in the k-E-plane via $s_k = s \cos \phi$ and $s_E = s \sin \phi$, one finds that after switching off the laser field, the spin precession angle is $\Delta \phi = 2 \arctan \left( \frac{E}{\gamma E_0 \sin \eta_0} \right) = -2 \arctan \left( \frac{\xi 0}{E} \right)$, where $p_E = -c \xi 0 \sin \eta_0$ is the final electron momentum along the laser polarization direction. Averaging over $p_E$ yields at $\mu \ll 1$: $\Delta \phi^2 \approx \Delta p_E^2/(2c^2) = (3/2)\mu$, with the momentum distribution width $\Delta p_E$ [48]. After the interaction $\langle \xi k \rangle \approx 1 - \Delta \theta^2/2 - \Delta \phi^2/2$, with $\Delta \theta^2 = 2I_p/c^2$ and $\Delta \phi^2 = (3/2)\mu$, which leads to equation (6) for the spin flip probability at $\mu \ll 1$. Thus, when the electron appears in the continuum nonadiabatically in a certain laser phase, the symmetry of the spin precession in the laser field is broken resulting in an effective spin flip at the end of the laser pulse. While the sign of $\Delta \theta$ is different in the consecutive half-cycles of the laser field, the average spin after the interaction depends on $\Delta \theta^2$ and the spin flip effect is the same in both half-cycles. In strong laser fields with $\mu \gg 1$, the spin precession frequency due to free motion in the laser field is initially large $\sim \xi 0$ but vanishes quickly due to the Lorentz-boost factor in the precession frequency. This results in $\varphi \approx \pi$, i.e., in a complete spin flip $w_{+/-}/w \approx 1$, which corresponds to the first term in equation (6) at $\mu \gg 1$.

Spin asymmetry. Now let us discuss how spin effects are modified when one accounts for laser-induced spin precession and Zeeman-splitting in the bound state. For this purpose we calculate the spin resolved ionization probabilities in the CC-D-SFA. When the initial spin is aligned along the laser’s magnetic field, no spin precession occurs neither in the bound, nor in the free electron state. However, in contrast to the CC-S-SFA, the energy of the bound state is shifted here due to Zeeman-splitting. As a consequence, the spin asymmetry in ionization, which existed in the CC-S-SFA, is significantly reduced (see also figure 2(a)):

$$A \approx 2 \left( 2I_p/c^2 \right)^{3/2}.$$  

(8)

The reduced asymmetry can be understood as follows. The electron spin interaction with the laser’s magnetic field

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**Figure 1.** The qualitatively different behaviour of spin effects within the CC-S-SFA (left column) and the CC-D-SFA (right column). The initial spin is along the laser’s magnetic field (top row). The spin is along the laser propagation direction (middle and bottom rows) with weak fields $\mu \ll 1$ (middle row), and with strong fields $\mu \gg 1$ (bottom row); red wiggled arrow, blue dotted arrow and green solid arrow indicate the initial spin, the spin after the tunnelling, and the final spin, respectively.

the two spin orientations: $\varepsilon_{\text{kin}} = -I_p \mp \varepsilon_M$. Since the kinetic energy determines the tunnelling probability, the latter becomes spin-dependent: $w_{+/-} \propto \exp \left[ -\frac{2I_p}{c^2} (I_p \pm \varepsilon_M)^{3/2} \right] \approx (1 \mp \sqrt{2I_p/c}) e^{-\frac{2I_p}{3c^2}}$, which explains the asymmetry expressed by equation (5). We stress that this asymmetry arises during one laser-half-cycle and can be observed in reduced magnitude only in a short laser pulse, e.g., in a 3-cycle cos^2-pulse the asymmetry is reduced by a factor 0.9 due to the two oppositely directed side peaks.

When the initial electron polarization is along the laser propagation direction, there is no spin asymmetry but a spin flip can occur

$$\frac{w_{+}}{w} \approx \frac{I_p}{2c^2} - \frac{3}{8} \mu \quad (\mu \ll 1); \quad \frac{w_{-}}{w} \approx -1 - \frac{I_p}{2c^2} \quad (\mu \gg 1),$$

(6)

where $\mu \equiv (E_0/E_\text{a})^{3/2} \xi 0^3 = E_0/(c \omega)$. The parameter $\mu$ is proportional to the width of the momentum distribution of
the ground state of a hydrogenlike ion with CC-D-SFA (intensity is
contributes an additional energy (∼B₀/c) which is in leading
order the same in the bound as well as in the free state. The
spin energy adds to or subtracts from the total electron energy
for the spin-up or spin-down cases, respectively, but the kinetic
energy due to the motion along the laser field that is crucial for
the tunnelling is unaffected and is the same for any spin value.
Therefore, the ionization probability does not depend on the
spin, i.e., there is no spin asymmetry in leading order by Iₚ/c².
In higher orders a small asymmetry \( A \sim (I_p/c^2)^{3/2} \) remains.
It is due to the difference in the rest-frame magnetic field
in the bound and Volkov states: \( B' \approx |B \times E| \sim |p_b|E_0/c \).
The momentum difference in the bound and the free state
during the tunnelling can be estimated with \( p_b \sim I_p/c \) [49],
which will lead to an asymmetry \( A \sim \sqrt{I_p/c^2}B' B_0 \sim (I_p/c^2)^{3/2} \).
Thus, the reduced spin asymmetry of equation (8) is due
to Zeeman splitting of the bound state energy. As a consequence,
for Ne⁹⁺, for example, there is almost no spin asymmetry in
the CC-D-SFA (\( A \approx 10^{-4} \)), while \( A \approx 0.15 \) in the CC-S-SFA.
To confirm these results, we have carried out \textit{ab initio}
numerical calculations of the ionization probability with different
initial spin states. The Dirac equation is solved on a two-dimensional
grid without any free parameter for adjustment. The ionization
rate is deduced from the bound state depletion rate. Figure 2(a)
shows that the numerical result can be well reproduced by
the CC-D-SFA.

**Spin flip.** The role of the spin precession in the bound
state on the spin flip effect at ionization is investigated with
the CC-D-SFA. Spin flip happens when the initial polarization
is along the laser propagation direction. When solving equation
(3) for the electron bound state, we restrict the expansion basis
of the time-dependent bound-state wave function only over
the magnetic quantum number \( m_j = \pm 1/2 \), representing it
as a linear combination of the states \( \phi_{\pm \pm}^{(c)}(t) = \frac{1}{\sqrt{2}}[\phi_{B,\pm}^{(c)}(t) \mp i\phi_{B,\mp}^{(c)}(t)] \) [44], where \( \phi_{B,\pm}^{(c)}(t) = \phi_{B,\pm}^{(c)}(0) \exp \left[ \pm i\frac{\delta_{B}(t)}{2}(1 \mp 2\eta) \right] \)
and \( \phi_{B,\pm}(t) \) are the dressed and initial wave functions of
the ground state of a hydrogenlike ion with \( m_j = \pm 1/2 \),
respectively (quantization axis along the laser magnetic field
direction), \( \tilde{A}(\eta) = -\int_{-\infty}^{\infty} E(\eta')d\eta' \). Using these states in
equation (1) leads to the ionization probabilities shown in
figure 2(b). The behaviour of the spin flip probability is
qualitatively different than the CC-S-SFA result. In weak laser
fields \( \mu \ll 1 \), almost no spin flip occurs:
\[
w_{-+}/w \approx O((I_p/c^2)²) \pm O((I_p/c^2)²\mu). \tag{9}
\]
This is again in contrast to the large spin flip prediction of
the CC-S-SFA, see equation (6). In strong fields \( \mu \gg 1 \), the probabilities for the spin-up and -down states are almost equal
in the final state:
\[
w_{-+}/w \approx 1/2 + O(1/\mu), \tag{10}
\]
whereas the CC-S-SFA predicts almost unity spin flip probability.
Thus, the spin effects are governed by the parameter \( \mu = \delta_{B}/E_0 \) and can be rather large at \( \mu \gtrsim 1 \), in contrast to the relativistic corrections to the spin-averaged total
probability of tunnelling ionization which are proportional to
\( I_p/c^2 \) and small [15]. Note that the remarkably fast increase of
the spin flip probability within a short intensity interval around
\( \mu \approx 1 \) even offers a means for measurement of super-high laser
intensities.

The spin flip behaviour is explained as follows. In the bound state a precession of the electron spin occurs with a
frequency \( \Omega_{b} \approx \omega_0(1 - 2\eta/3c²) \sim \omega_0 \) (cf [31]). For
the electron in the continuum, the spin precession frequency is
\( \Omega(\eta) \sim \omega_0 \) in weak fields \( \mu \ll 1 \), which coincides with the
spin dynamics in the bound state. There is no asymmetry in
the spin dynamics and the spin state does not change when
adiabatically switching on and off the laser field. Conversely,
when the laser field is strong \( \mu \gg 1 \), the bound and free
spin dynamics are different. The electron spin in the bound
state still precesses rapidly with \( \Omega_{b} \). During the dominant
ionization phase \( \delta_{\eta} \), the spin has many different orientations
because of \( \Omega_b \delta_{\eta}/\omega \gg 1 \), with \( \delta_{\eta} \sim \sqrt{E_0/E_c} \), and on average
spin-up and -down orientations are equally probable, i.e., the,
spin flip probability is approximately equal to 1/2. During the
continuum motion in the laser pulse, a total spin flip occurs
\( \psi \approx \pi \) at \( \mu \gg 1 \). However, the probability ratio of the spin-up
and -down states remain unchanged.

**Experimental observability.** Although, our calculations
show that the spin asymmetry is rather small and difficult
to observe, the spin flip probability can have rather large
values in the strong fields. For typical experimental parameters,
e.g., ionization of hydrogenlike Ne⁹⁺ (Fe²⁵⁺) ions in a

**Figure 2.** (a) Spin asymmetry parameter \( A \) versus nuclear charge \( Z \), when the electron initial spin is along the laser magnetic field; (b) spin flip probability when the initial spin is along the laser propagation direction; (black, solid) with CC-D-SFA, (red, dashed) with CC-S-SFA, \textit{ab initio} numerical calculations are shown with circles; in the quasistatic tunnelling regime with \( \omega = 0.05 \) and \( E_0/E_c = 1/25 \) when the laser intensity is \( I = 5.6 \times 10^{13} \times Z^0 \) W cm⁻².
strong infrared laser field with an intensity of 8.5 × 10^{19} (I = 1.7 × 10^{22}) W cm^{-2}, the CC-D-SFA predicts a relative spin flip probability of about 0.1 (0.4). Such measurements require an initially polarized target of ions and detection of the photoelectron polarization via Mott polarimetry [50, 51]. The electron spin flip can also be revealed via measurement of ion parameters, relating the angular momentum change of the ion during ionization to the electron spin change. The spin flip will be indicated by a non-vanishing signal for the difference in the ion angular distribution with application of left versus right circularly polarized laser fields.

**Conclusion.** We have shown that the bound electron dynamics in a laser field plays an essential role for the development of a spin-flip effect during laser induced tunnelling ionization. This effect is beyond the common strong-field approximation theory and can be confirmed in a challenging experiment employing the relativistic regime of tunnelling with strong laser fields and highly charged ions. The effect indicates that, even if an electron is very tightly bound by the strong Coulomb field in a highly charged ion, it may still be crucially affected by a laser field of relatively moderate intensity.

**Acknowledgment**

We acknowledge valuable discussions with C H Keitel.

**References**

[1] Di Piazza A, Müller C, Hatsagortsyan K Z and Keitel C H 2012 Rev. Mod. Phys. 84 1177

[2] Moore C I, Ting A, McNaught S J, Qiu J, Burris H R and Sprangle P 1999 Phys. Rev. Lett. 82 1688

[3] Chowdhury E A, Barty C P J and Walker B C 2001 Phys. Rev. A 63 042712

[4] Dammash M, Dörr M, Eichmann U, Lenz E and Sandner W 2001 Phys. Rev. A 64 061402

[5] Yamakawa K, Akahane Y, Fukuda Y, Aoyama Y, Inoue N and Ueda H 2002 Phys. Rev. A 65 063402

[6] Gubbini E 2005 J. Phys. B: At. Mol. Opt. Phys. 38 L87

[7] DiChiara A D, Ghebregziabher I, Sauer R, Waesche J, Palaniyappan S, Wen B L and Walker B C 2008 Phys. Rev. Lett. 101 173002

[8] Palaniyappan S, Mitchell R, Sauer R, Ghebregziabher I, White S L, Decamp M F and Walker B C 2008 Phys. Rev. Lett. 100 183001

[9] DiChiara A D et al 2010 Phys. Rev. A 81 043417

[10] Reiss H R 1990 Phys. Rev. A 42 1476.

[11] Reiss H R 1990 J. Opt. Soc. Am. B 7 574.

[12] Popov V, Mur V and Karnakov B 1997 JETP Lett. 66 229

[13] Mur V, Karnakov B and Popov V 1998 JETP Lett. 67 833

[14] Popov V S, Karnakov B M and Mur V D 2004 Zh. Eksp. Teor. Fiz. 79 320

[15] Milosevic N, Krainov V P and Brabec T 2002 Phys. Rev. Lett. 89 193001

[16] Milosevic N, Krainov V P and Brabec T 2002 J. Phys. B: At. Mol. Opt. Phys. 35 3515

[17] Becker W, Grasbon F, Kopold R, Milosšević D, Paulus G G and Walther H 2002 Adv. Arom. Mol. Opt. Phys. 48 35

[18] Faisal F H M and Bhattacharya S 2004 Phys. Rev. Lett. 93 053002

[19] Bunkin F, Kazakov A and Fedorov M 1972 Usp. Fiz. Nauk 100 173002

[20] Szymanowski C, Véniard V, Thérèse R, Maquet A and Keitel C H 1997 Phys. Rev. A 56 3846

[21] Walser M W, Szymanowski C and Keitel C H 1999 Europhys. Lett. 48 533

[22] Walser M W and Keitel C H 2000 J. Phys. B: At. Mol. Opt. Phys. 33 L221

[23] Panek P, Kamiński J Z and Ehlotzky F 2002 Phys. Rev. A 65 033408

[24] Panek P, Kamiński J Z and Ehlotzky F 2004 Phys. Rev. A 69 013404

[25] Ivanov D Y, Kotkin G L and Serbo V G 2005 Eur. Phys. J. C 40 27

[26] Ahrens S, Bäke H, Keitel C H and Müller C 2012 Phys. Rev. Lett. 109 043601

[27] Di Piazza A, Milstein A I and Müller C 2010 Phys. Rev. A 82 062110

[28] Müller T O and Müller C 2011 Phys. Lett. B 696 201

[29] Müller T O and Müller C 2012 Phys. Rev. A 86 022109

[30] Skoromnik O D, Feranchuk I D and Keitel C H 2013 Phys. Rev. A 87 052107

[31] Hu S X and Keitel C H 1999 Phys. Rev. Lett. 83 4709

[32] Hu S X and Keitel C H 2001 Phys. Rev. A 63 053402

[33] Walser M W and Keitel C H 2001 Opt. Commun. 199 447

[34] Walser M W, Urbach D J, Hatsagortsyan K Z, Hu S X and Keitel C H 2002 Phys. Rev. A 65 043410

[35] Bhattacharyya S, Mukherjee M, Chakrabarti J and Faisal F H M 2007 J. Phys.: Conf. Ser. 80 012029

[36] Bhattacharyya S, Mazumder M, Chakrabarti J and Faisal F H M 2011 Phys. Rev. A 83 043407

[37] Barth I and Smirnova O 2013 Phys. Rev. A 88 013401

[38] Gersten J I and Mittleman M H 1975 Phys. Rev. A 12 1840

[39] Avetisissian H K, Markossian A G, Mkrtchian G F and Movsissian S V 1997 Phys. Rev. A 56 4905

[40] Krainov V P 1997 J. Opt. Soc. Am. B 14 425

[41] Avetisissian H K, Hatsagortsyan K Z, Markossian A G and Movsissian S V 1999 Phys. Rev. A 59 549

[42] Smirnova O, Spanner M and Ivanov M 2008 Phys. Rev. A 77 033407

[43] Klaiber M, Yakaboylu E and Hatsagortsyan K Z 2013 Phys. Rev. A 87 023417

[44] Klaiber M, Yakaboylu E and Hatsagortsyan K Z 2013 Phys. Rev. A 87 023418

[45] Faisal F H M 2007 J. Phys. B: At. Mol. Opt. Phys. 40 F145

[46] Faisal F H M 2007 Phys. Rev. Lett. 109 033412

[47] Bargmann V, Michel L and Telegdi V L 1959 Phys. Rev. Lett. 8 2435

[48] Popov V S 2004 Phys.—Usp. 47 475

[49] Klaiber M, Yakaboylu E, Bäke H, Hatsagortsyan K Z and Keitel C H 2013 Phys. Rev. Lett. 110 153004

[50] Kessler J 1985 *Polarized Electrons. Series on Atoms and Plasmas* (New York: Springer).

[51] Tioukine V, Aulenbacher K and Riehn E 2011 Rev. Sci.Instrum. 82 033303