A Model for Neutrino Masses

C. Jarlskog
Division of Mathematical Physics
LTH, Lund University
Box 118, S-22100 Lund, Sweden

Abstract
I present a model for neutrino masses based on a hypothesis proposed by Friedberg and Lee.

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1 Prelude

I am delighted to be giving a talk here in Italy on this very special day, April 15, Leonardo da Vinci’s birthday. Through my talk, I will turn to him for guidance and understanding.

2 Historical Perspective

Let me first start with a bit of history. The year 1957 was a wonderful neutrino year due to the discovery of parity violation in weak interactions. See the Nobel lectures by Lee and Yang ([1], [2]), both easily accessible on the internet. The only weak interactions studied at that time were decay processes where one or two neutrino particles were emitted. A remarkable feature of parity violation was that it was compatible with being maximal. A beautiful idea was then put forward that attributed this feature to an intrinsic property of the neutrino, its handedness. Consequently, the neutrino was automatically massless. This proposition came from no less distinguished scientists than Lee and Yang [3], Landau [4] and Salam [5], and perhaps even others. For example, Landau wrote about what he called the longitudinally polarized, or more simply longitudinal neutrino:

"In the sense of the usual scheme this would signify that the neutrino is always polarized in the direction of its motion (or in the opposite direction). The polarization of the antineutrino is correspondingly reversed. According to this model the neutrino is not a truly neutral particle".

He emphasized that "In the usual theory the neutrino mass is zero, so to say, accidentally".

Lee and Yang referred to it as the two-component neutrino theory. This theory was largely taken for granted for several decades and became a part of the weak-interactionists’ scientific heritage. A lovely, predictive theory that led to simplicity. That explains why the founders of the Standard Electroweak Model chose not to introduce right-handed neutrinos. The same goes for the Grand Unified Georgi-Glashow Model. It would have been a trivial task for the authors in question to extend their models by adding in right-handed neutrinos. One would have had what we could call Slightly Non-minimal Standard Model [6]. Nowadays, due to neutrino oscillations, we generally believe that neutrinos are massive and therefore the two-component neutrino theory is not valid. However, we do not understand why and how the neutrinos are massive.

Leonardo would tell us:

"Although nature commences with reason and ends in experience it is necessary for us to do the opposite, that is to commence with experience and from this to proceed to investigate the reason."
Thank you, Leonardo! We will keep your advice in mind. Perhaps someday we will find the reason.

3 The Friedberg-Lee Hypothesis

Recently Friedberg and Lee have, in a series of articles ([7], [8], [9]) introduced and applied a new hypothesis as I will briefly describe now.

These authors consider a model with three standard families, each containing an up-type, a down-type as well as a charged lepton and a neutrino. There are thus four "sectors", up-type, etc. For each sector, Friedberg and Lee introduce Dirac mass terms of the kind

$$L_{\text{mass}} = \overline{\psi}_j M_{jk} \psi_k$$

(1)

where, for the up-type quarks $\psi_j, j = 1, 2, 3$ denote three fields with quantum numbers of the up-type quarks and $M$ is the corresponding mass matrix. The other three sectors are treated similarly. Thus, all in all, there are four such mass terms. Friedberg and Lee require each such mass term to be invariant under a corresponding translation of the fermion fields given by

$$\psi_j \rightarrow \psi_j + \eta_j \theta$$

(2)

Here, $\theta$ is a space-time independent four-component Grassmann number, $\theta_a \theta_b + \theta_b \theta_a = 0$, $a, b$ being Dirac indices. Furthermore $\eta_j$ are three complex numbers. Note that there are four $\theta$'s, one for each sector, and each sector has its own set of $\eta$'s.

Imposing the condition (2) leads to the constraint

$$M_{jk} \eta_k = 0$$

(3)

If this condition were to be valid for arbitrary $\eta$'s (that is to be a perfect symmetry) the mass matrix would vanish identically. This is obviously too restrictive. Therefore, Friedberg and Lee require that there exist a set of $\eta$'s for which the above equation is valid. This implies that the mass matrix must have a zero eigenvalue. This is good news because in each of the three charged sectors there is one member that is much lighter than the other two: the mass of the up-quark is much smaller than that of charm and top. Similarly, the electron is much lighter than the muon and the tau lepton. To a somewhat lesser extent the same pattern also shows up in the down-type sector. Similarly, in the framework of Friedberg and Lee, the neutrinos are Dirac particles and one of them is massless.

Perhaps Leonardo would say:

"Life is pretty simple: You do some stuff. Most fail. Some work. You do more of what works. If it works big, others quickly copy it."
Thank you Leonardo! I think that the Friedberg-Lee idea works big. It deserves to be copied. I’ll wander down their path hoping to find an exciting new vista.

4 Choices within Slightly Non-minimal Standard Model

The Standard Model is a chiral theory. Thus the fermion mass terms are of the form

\[ L_{\text{mass}}^{SM} = \bar{\psi}_{jL} M_{jk}^{\text{Dir}} \psi_{kR} + h.c. \]  \hspace{1cm} (4)

for the quarks and charged leptons. Here Dir stands for Dirac. In Slightly Non-minimal Standard Model, i.e., with right-handed neutrinos, we generally expect to have, in addition to Dirac mass terms, also Majorana mass terms for the right-handed neutrinos because there is no symmetry that forbids such terms.

Consider the fermionic translation in (2). In a chiral theory, there are several possibilities for introducing it because the left-handed and the right-handed fermions are independent of each other. Therefore, one may choose to introduce the Friedberg-Lee condition for:

1. only the left-handed fermions
2. only the right-handed fermions
3. both chiralities.

One could also choose some combinations of the above alternatives. Let us consider not only the mass terms but the entire Lagrangian of this Slightly Non-minimal Standard Model. Option 1, applied to the entire Lagrangian, would kill all interactions of the corresponding fermions with the W-bosons. In option 2, the interactions with W-bosons are not affected but the interactions of right-handed fermions with the B-boson will be forbidden. This is a less severe restriction than we had in the former case. The right-handed neutrino sector is very special because right-handed neutrinos don’t interact with the gauge bosons. Furthermore, the kinetic terms of fermions change, under the above translation, by a total derivative whereby the resulting action is invariant. This means that all the terms in the Lagrangian, except the interactions with the Higgses and Majorana mass terms, are automatically invariant under the Friedberg-Lee translation of the right-handed neutrino fields. In other words, the right-handed neutrinos constitute a basis for maximal symmetry. This was the motivation for the work in [10] that I would like to discuss now.

Leonardo, what would you say about this? I hear you say

“Human subtlety will never devise an invention more beautiful, more simple
or more direct than does nature because in her inventions nothing is lacking, and nothing is superfluous.”

Yes, Leonardo. You are absolutely right. Please forgive me for pursuing an idea that is by no means more beautiful or more direct than does nature. It may be wrong but at least it is simple.

5 The \(\nu_R\)-sector

Following the discussion above, we assume that the Lagrangian is invariant under the translation

\[ \nu_{Rj} \rightarrow \nu_{Rj} + \eta_j \theta \]  

Again \(\eta_j, j = 1 - 3\) are complex numbers, and \(\theta\) is a space-time independent Grassmann number. As noted before, this translation is a symmetry of the Standard Model excluding the interactions of the neutrinos with the Higgs fields (which generate the Dirac mass terms) and the Majorana mass terms. Therefore, requiring the above translations to be a symmetry (i.e., to be valid for arbitrary \(\eta\)) of the entire action immediately would yield that the neutrino masses are all zero. This result is interesting because it could perhaps be the reason why the neutrino masses are small - they start off by being zero in the symmetry limit! One could imagine that there is a scenario beyond the Standard Model in which the above symmetry holds and somehow in the broken version the \(\eta\)'s get fixed.

The neutrino mass matrix, including Majorana terms, is given by

\[ L_{\text{mass}}^{(\nu)} = -\frac{1}{2}(\bar{\nu}_L, \nu_R^C) \mathcal{M} \begin{pmatrix} \nu_L^C \\ \nu_R \end{pmatrix} + \text{h.c.} \]  

where

\[ \nu_X = \begin{pmatrix} \nu_{1X} \\ \nu_{2X} \\ \nu_{3X} \end{pmatrix} \]  

\(X = L, R\). Here \(\mathcal{M}\) is the six-by-six neutrino mass matrix given by

\[ \mathcal{M} = \begin{pmatrix} 0 & A \\ A^T & M \end{pmatrix} \]  

\(\mathcal{M}\) is a symmetric matrix; \(A\) and \(M\) being respectively the three-by-three Dirac and Majorana mass matrices and \(T\) stands for transpose.

Requiring that the translation \(\nu_{Rj} \rightarrow \nu_{Rj} + \eta_j \theta\) leave the Lagrangian invariant yields

\[ A_{jk} \eta_k = 0, \quad M_{jk} \eta_k = 0 \]  

We may now redefine the right-handed neutrino fields by

\[ \nu_R \rightarrow U \nu_R \]
where $U$ is a three-by-three unitary matrix. Under this transformation the Lagrangian retains its form. All that happens is a redefinition of the mass matrices

$$ M \rightarrow M' = U^* M U^\dagger, \quad A \rightarrow A' = A U^\dagger $$

(11)

The Friedberg-Lee condition now reads

$$ A'_{jk} \eta'_k = 0, \quad M'_{jk} \eta'_k = 0 $$

(12)

where $\eta' = U \eta$. We may choose $U$ such that $M'$ is diagonal. $M'$ has a zero eigenvalue. Therefore, by permutation of the right-handed neutrino fields (which leaves the rest of Lagrangian invariant) we may write $M'$ in the form

$$ M' = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} $$

(13)

Assuming that $M_1$ and $M_2$ are nonzero, we find that the vector $\eta'$ is proportional to $(0, 0, 1)$. This in turn implies that the third column in matrix $A'$ is zero. Therefore it is of the form

$$ A' = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & 0 \end{pmatrix} $$

(14)

Thus a remarkable consequence of the Friedberg-Lee condition is that one of the three right-handed neutrinos simply decouples, in the sense that it has only gravitational interactions due to its kinetic energy term. We end up with three left-handed neutrinos and just two right-handed neutrinos. The mass matrix is then of the form

$$ M = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & 0 \end{pmatrix} $$

(15)

To sum up, in this model there are two massless neutrinos one of them being a non-interacting massless right-handed neutrino and the other one a massless interacting neutrino. A natural scenario would be to have small values for magnitudes of $M_1$ and $M_2$ because in the symmetry limit these are zero. In the limit $M_1, M_2 \rightarrow 0$ one obtains three Dirac neutrinos, one of them being massless. The model may or may not violate CP symmetry depending on how the charged lepton mass matrix looks like.

However, since we don’t really know what is natural or not, we should also keep in mind that the see-saw mechanism is also allowed. It is obtained by taking the magnitudes of $M_1$ and $M_2$ to be very large. Then there will be two
very heavy neutrinos and three light ones, one of them being massless. The phenomenology of see-saw models with two right-handed neutrinos has been the subject of several studies in the literature (see, for example [11] and [12] and references cited therein). See also Micha Shaposhnikov’s contribution to this meeting [13].

I hear Leonardo declaring.

"The noblest pleasure is the joy of understanding."

6 Lorentz invariance

I would like to mention that fermionic translations are far more subtle than their bosonic counterparts, such as \( \phi \rightarrow \phi + v \), or \( A_\mu \rightarrow A_\mu + \partial_\mu \Lambda \). For example, under Lorentz transformations \( \theta \) must transform as a fermionic field, i.e., the transformation \( \nu_R \rightarrow S \nu_R \) implies \( \theta \rightarrow S \theta \), where \( S \) is the appropriate transformation matrix. A nonzero vacuum expectation value of a fermionic operator will break Lorentz invariance. One could imagine that the Friedberg-Lee translation is actually driven by the requirement of Lorentz invariance, such as in a brane world picture, where Lorentz invariance is violated in the bulk but restored on the brane.

I am not sure that Leonardo would agree with this. Perhaps he would say:

"Blinding ignorance does mislead us. O! Wretched mortals, open your eyes!"

7 Acknowledgements

My most sincere thanks go to Milla Baldo-Ceolin, for inviting me to this meeting and asking me to speak. For me she is a role model, by her kindness and dedication to our field. I would like also to thank you, Leonardo, for your fantastic multidisciplinary contributions and for being born on 15 April. It has been great to get to know you better.

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