Hierarchical Quantum Network using Hybrid Entanglement

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Abstract
The advent of a new kind of entangled state known as hybrid entangled state, i.e., entanglement between different degrees of freedom, makes it possible to perform various quantum computational and communication tasks with lesser amount of resources. Here, we aim to exploit the advantage of these entangled states in communication over quantum networks. Unfortunately, the entanglement shared over the network deteriorates due to its unavoidable interaction with surroundings. Thus, an entanglement concentration protocol is proposed to obtain a maximally entangled hybrid Omega-type state from the corresponding non-maximally entangled states. The advantage of the proposed entanglement concentration protocol is that it is feasible to implement this protocol with linear optical components and present technology. The corresponding linear optical quantum circuit is provided for experimental realizations, while the success probability of the concentration protocol is also reported. Thereafter, we propose an application of maximally entangled hybrid state in the hierarchical quantum teleportation network by performing information splitting using Omega-type state, which is also the first hierarchical quantum communication scheme in the hybrid domain so far. The present hybrid entangled state has advantage in circumventing Pauli operations on the coherent state by polarization rotation of single qubit, which can be performed with lesser errors.

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1 Introduction

Quantum entanglement is an irreplaceable quantum resource that plays a very significant role in the field of quantum information processing [1]. It is an essential element for various applications in the fields of quantum computation [2] and communication [1], such as quantum teleportation [3], dense coding [4], quantum key distribution [5], quantum key agreement [6,7], quantum secret sharing [8], and quantum secure direct communication [9,10]. In the recent times, communication over quantum networks and internet is of primary interest [11–13], which often uses entanglement and teleportation. Interestingly, quantum entanglement is not sufficient for most of these applications, but maximal quantum entanglement is necessary. For example, quantum teleportation [3] and its variants [14] (and references therein) essentially require maximally entangled state (MES) to achieve the unit success probability for deterministic communication. Unavailability of such MES would lead to success probability less than unity and thus the communication becomes probabilistic [15] or the fidelity of the transmitted state not unity [16]. Similarly, security and efficiency of the secure communication schemes require MESs [17]. The practical problem associated with exploiting MES is its fragile nature due to decoherence induced by the environment. Specifically, while managing the storage, transmission, and processing, the shared entangled states (quantum channel) interact with the noisy environment which causes degradation in entanglement quality and thus the performance of the quantum communication protocols. Basically, it is impossible to avoid the decoherence which imposes a limitation in the successful execution of quantum technology as it reduces a MES into a non-MES or even a mixed states. Therefore, it is recommended to transform a non-MES into a MES before performing any quantum communication over a network. There are two important techniques in the literature to recover MES, namely entanglement concentration protocol (ECP) to recover a MES from pure non-MESs and entanglement purification protocol (EPP) for mixed non-MESs.

First such attempt to improve the quality of entanglement was performed by Bennett et al. by introducing a Schmidt decomposition-based ECP [18] and an EPP [19]. These pioneering works were followed by various ECPs [20–25] and EPPs [26–29], which were restricted to the discrete variables (DVs). Independently, an extensive effort has been put in the study of continuous variable (CV) entanglement in theory [30] (and references therein) and experiments [31,32], with applications in various quantum cryptographic [33] and information processing tasks [34,35]. This motivated CV ECPs [36,37] reported in the recent past. Several experimental realizations and applications of ECPs and EPPs play an important role in the development of quantum technology [38,39] and references therein.

Traditionally, entanglement encoded in both DV (such as polarization states of photons) and CV (superposition of coherent states with opposite phases $|\pm \alpha\rangle$ and large value of amplitude $\alpha$) has been adopted separately in the literature for various applications in DV [9,10,17] and CV [33,34] quantum communication, respec-
Both the approaches have their own pros and cons. In 2006, a new DV-CV approach has been developed [40] by combining the above two approaches to harness the advantages of discrete and continuous degrees of freedom (DOFs) together. This new DV-CV approach has been referred to as hybrid entangled states (HESs) where the entanglement is generated between different DOFs of a particle/mode pair. Hybrid entanglement-based quantum information processing tasks require fewer resources because it can be correlated within a single particle and thus outperforms the conventional method of using only a single type of DOF. In [40], the first hybrid quantum repeater using bright coherent light was proposed. Since then several researchers have made efforts independently for HES generation, such as generation of HES of light [41], HES between a single-photon polarization qubit and a coherent state [42], HES using interaction between DV and CV states [43], HES between particle-like and wave-like optical qubits induced by measurement [13]. Interestingly, many different applications have been reported based on HES, for example, near-deterministic quantum teleportation and resource-efficient quantum computation [44], hybrid long-distance entanglement distribution protocol [45], hybrid quantum information processing [46]. Further, quantum teleportation between DV and CV optical qubits has been proposed [47,48] in addition to remote preparation of CV qubits using HES [49] and more recently an efficient quantum teleportation based on DV-CV interaction mechanism [50]. Applications of HESs in remote state preparation and quantum steering [51] and a remote preparation for single-photon two-qubit HES using hyper-entanglement [52] are also proposed. Interestingly, first EPP for quantum repeaters [53], ECP assisted with single coherent state [54], ECP and EPP in spatially separated spins in nitrogen-vacancy centers [55] for HESs are also proposed. Hence, HESs have already established to be a very promising direction in advancing the quantum technology because of the combined advantages of two or more DOFs.

Therefore, inspired by these applications here we aim to propose communication over a hierarchical quantum teleportation network using Omega-type HES, which is referred to as a deterministic hybrid hierarchical quantum information splitting (HQIS). However, while network communication, Omega-type HES is expected to decohere due to the effect of environment, hence for an efficient implementation of communication over network first we have to propose an ECP for Omega-type HES which has the significance of being entangled in both DV and CV. Applications of DV Omega state have been discussed in the past [56,57] and thus CV and hybrid (DV-CV) Omega states are also expected to be useful in quantum communication tasks. Here, we have focused on quantum communication based on maximally HES, and thus it becomes essential to design an ECP for Omega-type HES to protect it from the influence of noise. To the best of our knowledge, neither an ECP nor an application in hierarchical communication for Omega-type HES has yet been proposed. A hierarchical scheme, namely hierarchical quantum information splitting [57], can be viewed as a variant of multiparty teleportation, where the receivers (agents) are graded in accordance with their power for the reconstruction of an unknown quantum state sent by a sender (boss). A couple of interesting proposals on hierarchical communication schemes have been proposed in the recent years, for example, a generalized structure of HQIS [57] using
(n + 1)—qubit states, integrated hierarchical dynamic quantum secret sharing [58] combining features of hierarchy and dynamism, a unique scheme of hierarchical joint remote state preparation [16] with joint preparation to protect the secrecy of a sensitive information, and hierarchically controlled quantum teleportation [14]. All these hierarchical schemes are shown to have wide applications in the real-world scenario ([14,16,57,58] and references therein).

The rest of the paper is organized as follows. In Sect. 2, Omega-type HES is introduced. In Sect. 3, we have described our ECP for Omega-type HES from non-MESs using a single- and two-mode coherent states with four parameters. Subsequently, in Sect. 4, the success probability for ECP has been discussed. Furthermore, a deterministic hybrid HQIS scheme using Omega-type HES has been proposed before concluding in Sect. 6.

\section{2 Omega-type hybrid entangled state}

An Omega-type hybrid MES can be written as

\[
|\Omega\rangle_{abcd} = \frac{1}{2} (|0_L\rangle|0_L\rangle|0_L\rangle|0_L\rangle + |0_L\rangle|1_L\rangle|0_L\rangle + |1_L\rangle|0_L\rangle|1_L\rangle - |1_L\rangle|1_L\rangle|1_L\rangle),
\]

(1)

where an optical logical hybrid qubit \(|0_L\rangle\) and \(|1_L\rangle\) is defined in basis \(|\mp\rangle = (|H\rangle \mp |V\rangle)/\sqrt{2}\) and coherent states \(|\pm \alpha\rangle\). Here, the subscripts \(a, b, c,\) and \(d\) represent states possessed by Alice, Bob, Charlie, and Diana, respectively. Here, a logical hybrid qubit encoded in polarization and coherent state is analogous to Ref. [44,59], and the logical state can be identified as Omega state [60,61]. Thus, Eq. (1) can be expressed as

\[
|\Omega\rangle_{abcd} = \frac{1}{2} (|+\rangle|+\rangle|+\rangle|+\rangle|\alpha\rangle|\alpha\rangle|\alpha\rangle + |+\rangle|-\rangle|-\rangle|+\rangle|\alpha\rangle|-\alpha\rangle|\alpha\rangle + |-\rangle|+\rangle|+\rangle|-\rangle|\alpha\rangle|\alpha\rangle|-\alpha\rangle|\alpha\rangle + |+\rangle|\alpha\rangle|\alpha\rangle|\alpha\rangle|-\rangle|-\rangle|-\rangle|-\rangle|\alpha\rangle|\alpha\rangle|\alpha\rangle|\alpha\rangle).
\]

(2)

As we have already discussed that due to the effect of the surroundings the hybrid MES in Eq. (2) transforms into a non-MES. Specifically, during the sharing of entanglement after its local preparation in the practical scenario, the MES becomes partially entangled. The hybrid non-MES corresponding to Eq. (2) can be written as

\[
|\Omega'\rangle_{abcd} = N_0 (\zeta |+\rangle|+\rangle|+\rangle|+\rangle|\alpha\rangle|\alpha\rangle|\alpha\rangle + \beta |+\rangle|-\rangle|-\rangle|+\rangle|\alpha\rangle|\alpha\rangle|\alpha\rangle + \gamma |+\rangle|+\rangle|+\rangle|-\rangle|\alpha\rangle|\alpha\rangle|\alpha\rangle + \delta |+\rangle|-\rangle|-\rangle|-\rangle|\alpha\rangle|\alpha\rangle|\alpha\rangle|\alpha\rangle),
\]

(3)

where the normalization constant \(N_0 = [\zeta^2 + \beta^2 + \gamma^2 + \delta^2]^{-\frac{1}{2}}\) is unity. Probability amplitudes \(\zeta, \beta, \gamma,\) and \(\delta\) are nonzero and considered real. Here, all probability amplitudes are not same because that will correspond to MES (2). We aim to obtain a hybrid
Omega-type MES (2) from Eq. (3) by performing ECP proposed in the following section.

Before we proceed further, it is imperative to discuss the preparation of quantum states in Eqs. (2)–(3). For generation of hybrid Omega-type MES, we require two single photon pairs and a hybrid entangled state of two logical qubits $|\psi_{H2}\rangle$. Specifically, this particular hybrid entangled state can be prepared using two hybrid pairs in state $|\psi_{H1}\rangle$ and a single photon pair in the Bell state $|\phi^+\rangle = \sqrt{\frac{1}{2}} |HH\rangle + |VV\rangle$. However, if all these resource states are non-maximally entangled, then the two logical qubit state $|\psi_{H2}\rangle$ will be obtained from

$$|\psi_{H2}'\rangle = |\psi_{H1}\rangle_1 \otimes |\phi^+\rangle \otimes |\psi_{H1}\rangle_2$$

$$= (a_0 |H, \alpha\rangle + a_1 |V, -\alpha\rangle) (b_0 |HH\rangle + b_1 |VV\rangle) (c_0 |H, \alpha\rangle + c_1 |V, -\alpha\rangle)$$

by performing a parity check measurement on the single photon of $|\psi_{H1}\rangle_1$ and the first qubit of the Bell state after application of a Hadamard operation (the second qubit of the Bell state). Here, $\sum_j A_j^2 = 1 \forall A \in \{a, b, c\}$ and Hadamard operation on polarization qubits correspond to the use of a half-wave plate at 22.5°. Conditioned on the same parity qubits in both parity check measurements, we finally perform quantum non-demolition measurement on both the qubits of the Bell state, i.e., measure both the qubits in $\{|+\rangle, |-\rangle\}$ basis. Thus, final state in the superposition of two logical qubit states after an application of Hadamard operation on both the single photon states becomes

$$|\psi_{H2}\rangle = \beta_0 |0_L\rangle |0_L\rangle + \beta_1 |0_L\rangle |1_L\rangle + \beta_2 |1_L\rangle |0_L\rangle - \beta_3 |1_L\rangle |1_L\rangle,$$  

where the probability amplitudes depend on the parameters of the initial states, such as $\beta_0 = a_0 b_0 c_0$. A close look at Eqs. (1) and (5) reveals us that we have to perform a CNOT operation with control on both these logical qubits and target on ancillae prepared in $|0_L\rangle$.

It can also be accomplished using $|\psi_{H2}\rangle$ with coherent amplitude $\sqrt{2}\alpha$ and two Bell states

$$|\psi_{H4}'\rangle = |\phi^+\rangle_1 \otimes |\psi_{H2}\rangle \otimes |\phi^+\rangle_2.$$  

After performing a Hadamard operation on one qubit each of the Bell states and both single photons in $|\psi_{H2}\rangle$, we can perform parity check measurement on one qubit of the first (second) Bell state and the first (second) single photon in $|\psi_{H2}\rangle$. Further, we condition the same parity outcomes on both these measurements and performing quantum non-demolition measurement on both the single photons of $|\psi_{H2}\rangle$. Finally, we perform a Hadamard operation on all the single photons and pass both the coherent modes through respective beamsplitters to obtain the desired state (3). Note that a beamsplitter $(BS)$ transforms the two input coherent states $|\alpha\rangle$ and $|\beta\rangle$ as
Fig. 1 (Color online) The schematic diagram of the proposed ECP for Omega-type HES. Initially, Alice, Bob, Charlie, and Diana share hybrid Omega-type non-MES. The subscripts \(a\), \(b\), \(c\), and \(d\) denote the states with Alice, Bob, Charlie, and Diana, respectively. Alice prepares a single-mode superposition of coherent state in the spatial mode \(g_1\), and Charlie (or Diana) prepares and shares a two-mode entangled coherent state in the spatial mode \(e_1\) and \(f_1\), respectively. Modes of the HES to be concentrated are mixed with the auxiliary modes at the symmetric beamsplitters \(BS_1\), \(BS_3\), and \(BS_5\). Further, to obtain the hybrid Omega-type MES, beamsplitters \(BS_2\), \(BS_4\), and \(BS_6\) are useful. Spatial modes after evolution from an optical element are labeled as \(i_j \rightarrow i_{j+1}\). Further, \(D_x\) are the photon detectors

\[
BS|\alpha\rangle|\beta\rangle \rightarrow \frac{1}{\sqrt{2}}\left(\begin{array}{c}
|\alpha + \beta\rangle \\
\frac{1}{\sqrt{2}}
\end{array}\right) \frac{1}{\sqrt{2}}\left(\begin{array}{c}
|\alpha - \beta\rangle \\
\frac{1}{\sqrt{2}}
\end{array}\right).
\]

Interestingly, generation of hybrid Omega-type MES (2) requires that all three Bell states and two hybrid pairs should be maximally entangled. If any of these entangled states is not maximally entangled, the generated quantum state will be of the form (3) and require entanglement concentration proposed here. The hybrid Omega-type MES (or non-MES) can be generated using different set of operations and input states, for instance, using parity measurement on coherent states [44] or Bell-like hybrid measurement on two three-qubit hybrid cluster states [59].

3 Entanglement concentration protocol for Omega-type hybrid entangled state

To begin with, first we propose an entanglement concentration for Omega-type HES using a single- and a two-mode superpositions of coherent states. To obtain a MES of hybrid Omega-type, we assume that Alice, Bob, Charlie, and Diana share the hybrid Omega-type non-MES as described in Eq. (3). The optical circuit of ECP for Omega-type HES is shown in Fig. 1. The subscripts \(a\), \(b\), \(c\), and \(d\) represent to the spatial modes of Alice, Bob, Charlie, and Diana, respectively. Further, Charlie (or equivalently
Diana) prepares and shares a superposition of two-mode coherent states $|\psi\rangle_{e_1 f_1}$ in the spatial mode $e_1$ and $f_1$ among Charlie and Diana, respectively,

$$|\psi\rangle_{e_1 f_1} = N_1 (\zeta |\alpha\rangle |\alpha\rangle + \beta |\alpha\rangle |\alpha\rangle + \gamma |\alpha\rangle |\alpha\rangle + \delta |\alpha\rangle |\alpha\rangle),$$

where $N_1 = \left[\zeta^2 + \beta^2 + \delta^2 + \gamma^2 + 2(\zeta \beta + \zeta \gamma + \delta \beta + \gamma \delta)e^{-2|\alpha|^2} + 2(\gamma \beta + \delta \zeta)e^{-4|\alpha|^2}\right]^{-\frac{1}{2}}$ is the normalization constant. Therefore, the combined state of the system can be expressed as

$$|\phi\rangle_{abcd e_1 f_1} = \Omega^\dagger_{abcd} \otimes |\psi\rangle_{e_1 f_1}.$$  

Here and in what follows, we label the spatial modes on which we perform operations as $a_1$, $c_1$ and $d_1$ while $b$ mode is unchanged as we do not perform any operation on this mode. Further, all the operations are only performed on the CV DOF and thus polarization remains unchanged. Thus, we can write

$$|\phi\rangle_{a_1 b_1 c_1 d_1 e_1 f_1} = N_1 \left( \zeta^2 |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle + \beta |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle + \gamma |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle + \delta |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle \right)\cdot$$

First of all, Charlie (Diana) passes modes $c_1$ and $e_1$ ($d_1$ and $f_1$) through 50:50 (symmetric or balanced) beamsplitter $BS_5$, ($BS_3$). Using Eq. (7), the state post-beamsplitter can be written as

$$|\phi\rangle_{a_1 b_2 c_2 d_2 e_2 f_2} = N_1 \left( \zeta^2 |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle + \beta |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle + \gamma |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle + \delta |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle \right).$$
Postselecting the condition that there is no photon in modes $e_2$ and $f_2$, the state in these cases can be written as

$$\langle \phi \rangle_{a_1bc2_{d2}} = N_1 \left( \zeta \beta |+\rangle |−\rangle |−\rangle |+\rangle |α\rangle |−α\rangle \right) \left| \sqrt{2\alpha} \right\rangle.$$  \hspace{1cm} (12)

Furthermore, Charlie and Diana let modes $c_2$ and $d_2$ pass through 50:50 beamsplitters $BS_6$ and $BS_4$, respectively, and using Eq. (7) the state becomes

$$\langle \phi \rangle_{a_1bc_3c_4d_3d_4} = N_1 \left( \zeta \beta |+\rangle |−\rangle |−\rangle |+\rangle |α\rangle |−α\rangle |−α\rangle |α\rangle |α\rangle \right)$$

$$+ β\zeta |+\rangle |+\rangle |+\rangle |+\rangle |α\rangle |α\rangle |α\rangle |α\rangle |α\rangle$$

$$− δγ |−\rangle |−\rangle |−\rangle |−\rangle |α\rangle |−α\rangle |α\rangle |α\rangle |α\rangle$$

$$+ δγ |−\rangle |+\rangle |+\rangle |+\rangle |−\rangle |−α\rangle |α\rangle |α\rangle |α\rangle |α\rangle.$$ \hspace{1cm} (13)

Modes $c_4$ and $d_4$ are redundant after this step, but a measurement here would collapse the superposition. Thus, by performing a photon number measurement on these two modes Charlie and Diana cannot distinguish $|±α\rangle$ as they have same photon number distribution. Afterward, the state can be written as

$$\langle \phi \rangle_{a_1bc_3d_3} = N_1 \left( \zeta \beta |+\rangle |−\rangle |−\rangle |+\rangle |α\rangle |−α\rangle |−α\rangle |α\rangle |α\rangle \right)$$

$$+ β\zeta |+\rangle |+\rangle |+\rangle |+\rangle |α\rangle |α\rangle |α\rangle |α\rangle |α\rangle$$

$$− δγ |−\rangle |−\rangle |−\rangle |−\rangle |α\rangle |−α\rangle |α\rangle |α\rangle |α\rangle$$

$$+ δγ |−\rangle |+\rangle |+\rangle |+\rangle |−\rangle |−α\rangle |α\rangle |α\rangle |α\rangle |α\rangle.$$ \hspace{1cm} (14)

Subsequently, Alice prepares an single-mode ancilla in superposition of coherent states in spatial mode $g_1$ as

$$\langle \phi \rangle_{g_1} = N_2 \left( \zeta \beta |−α\rangle + δγ |α\rangle \right),$$ \hspace{1cm} (15)

where the normalization constant $N_2 = \left[ \zeta^2 β^2 + δ^2 γ^2 + 2ζβδγe^{−2|α|^2} \right]^{−\frac{1}{2}}$. Thus, the combined state of the system can be expressed as

$$\langle \phi \rangle_{a_1bc_3d_3g_1} = \langle \phi \rangle_{a_1bc_3d_3} \otimes \langle \phi \rangle_{g_1}$$

$$= N_1N_2 \left( \zeta^2 β^2 |+\rangle |−\rangle |+\rangle |+\rangle |α\rangle |−α\rangle |−α\rangle |α\rangle |α\rangle \right)$$

$$+ \zeta β|+\rangle |−\rangle |+\rangle |+\rangle |α\rangle |−α\rangle |−α\rangle |α\rangle |α\rangle$$

$$+ \zeta ζ |+\rangle |−\rangle |+\rangle |+\rangle |−\rangle |−α\rangle |−α\rangle |α\rangle |α\rangle$$

$$+ \zeta βδγ |+\rangle |−\rangle |+\rangle |+\rangle |−\rangle |−α\rangle |−α\rangle |α\rangle |α\rangle$$

$$+ δ^2 γ^2 |+\rangle |+\rangle |+\rangle |+\rangle |−\rangle |−α\rangle |−α\rangle |α\rangle |α\rangle.$$ \hspace{1cm} (16)
Alice further passes modes $a_1$ and $g_1$ through 50:50 beamsplitter $B_{S_1}$. Now, using Eq. (7) the post-beamsplitter state can be expressed as

$$|\phi\rangle_{a_2b_3c_3d_3g_2} = N_1 N_2 \left( \zeta^2 \beta^2 |+\rangle |+\rangle |+\rangle |0\rangle |+\rangle |+\rangle |+\rangle \right) \sqrt{2|\alpha\rangle}$$

$$+ \zeta^2 \beta^2 |+\rangle |+\rangle |+\rangle |+\rangle |0\rangle |\alpha\rangle |\alpha\rangle \left( \sqrt{2|\alpha\rangle} \right)$$

$$+ \zeta \beta \delta \gamma |+\rangle |+\rangle |+\rangle \sqrt{2|\alpha\rangle} |-\alpha\rangle |-\alpha\rangle |0\rangle$$

$$+ \zeta \beta \delta \gamma |+\rangle |+\rangle |+\rangle |+\rangle \sqrt{2|\alpha\rangle} |\alpha\rangle |\alpha\rangle |0\rangle$$

$$- \zeta \beta \delta \gamma |+\rangle |+\rangle |+\rangle \sqrt{2|\alpha\rangle} |\alpha\rangle |\alpha\rangle |0\rangle$$

$$+ \delta^2 \gamma^2 |+\rangle |+\rangle |+\rangle |+\rangle |0\rangle |\alpha\rangle |\alpha\rangle \left( -\sqrt{2|\alpha\rangle} \right)$$

$$- \delta^2 \gamma^2 |+\rangle |+\rangle |+\rangle |+\rangle |+\rangle |\alpha\rangle |\alpha\rangle \left( -\sqrt{2|\alpha\rangle} \right)$$

(17)

She further postselects cases with no photon in mode $g_2$, and that leads to a reduced state which can be written as

$$|\phi\rangle_{a_2b_3c_3d_3} = N_1 N_2 \zeta \beta \delta \gamma \left( |+\rangle |+\rangle |+\rangle |+\rangle \sqrt{2|\alpha\rangle} |\alpha\rangle \right)$$

$$+ |+\rangle |+\rangle |+\rangle |\alpha\rangle |\alpha\rangle \left( \sqrt{2|\alpha\rangle} \right)$$

$$+ |+\rangle |+\rangle |+\rangle |+\rangle \sqrt{2|\alpha\rangle} |\alpha\rangle |\alpha\rangle$$

$$- |+\rangle |+\rangle |+\rangle |+\rangle \sqrt{2|\alpha\rangle} |\alpha\rangle |\alpha\rangle$$

$$+ |+\rangle |+\rangle |+\rangle |+\rangle \sqrt{2|\alpha\rangle} |\alpha\rangle |\alpha\rangle$$

$$- |+\rangle |+\rangle |+\rangle |+\rangle \sqrt{2|\alpha\rangle} |\alpha\rangle |\alpha\rangle$$

(18)

Notice that the amplitude of coherent state in mode $a_2$ in Eq. (18) is amplified by a factor of $\sqrt{2}$ (cf. Eq. (2)), which may have advantages in long-range quantum communication. However, for the sake of accomplishment of ECP task here Alice passes mode $a_2$ through a 50:50 beamsplitter $B_{S_2}$ in order to concentrate the desired MES (2). Similar beamsplitter operations were used on modes $c_2$ and $b_2$ by using $B_{S_6}$ and $B_{S_4}$ in Eq. (12). Therefore, after the beamsplitter operation on $a_2$ mode using Eq. (7), the evolved state can be expressed as

$$|\phi\rangle_{a_3a_4b_3c_3d_3} = N_1 N_2 \zeta \beta \delta \gamma \left( |+\rangle |+\rangle |+\rangle |+\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle \right)$$

$$+ |+\rangle |+\rangle |+\rangle |+\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle$$

$$+ |+\rangle |+\rangle |+\rangle |+\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle$$

$$- |+\rangle |+\rangle |+\rangle |+\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle$$

$$- |+\rangle |+\rangle |+\rangle |+\rangle |\alpha\rangle |\alpha\rangle |\alpha\rangle$$

(19)

At the last step, by performing the photon number measurement of the coherent state in $a_4$ mode, which keeps $|\pm \alpha\rangle$ indistinguishable, they finally obtain the hybrid Omega-type MES

$$|\phi\rangle_{a_3b_3c_3d_3} = |\Omega\rangle_{abcd}$$

(20)

which is the desired concentrated hybrid Omega-type MES as shown in Eq. (2).
Fig. 2 (Color online) Variation of success probability ($P$) with (a) $\theta_1$, (b) $\theta_2$, and (c) $\theta_3$ considering the rest of the state parameters $\theta_1 = \frac{\pi}{4}$, $\theta_2 = \frac{\pi}{4}$, and $\theta_3 = \frac{3\pi}{8}$, wherever needed. We have chosen the amplitude of coherent state $\alpha = 0.5$ (smooth blue line), $\alpha = 1$ (magenta dashed line), and $\alpha = 2$ (red dot-dashed line).

Note that we have only used optical elements required for ECP of entangled coherent state ([37] and references therein). This is the advantage of hybrid ECPs that using solely one DOF and corresponding technology the desired state can be concentrated. Therefore, the present state can be concentrated using polarization DOF equivalently using a polarization beamsplitter, half-waveplates, and the similar set of auxiliary single- and two-qubit polarization states. Interestingly, the success probability with polarization qubit-based ECP is similar to that obtained in the present case (discussed in detail in the next section).

There is nonzero probability of photon losses from both polarization and continuous variable part of the hybrid Omega-type MES during transmission and storage (see [44,59] for detailed discussion). Further, imperfect sources, detectors, or absorption in the rest of the optical elements used for various operations may also cause losses. The amplitude of coherent state reduces with photon loss rate parameter $\eta$ as $\alpha' = \alpha \sqrt{1 - \eta}$. Loss of single photons with polarization states will leave the state in continuous variable only. In general, loss of photons leads to mixedness of the reduced state, and in that case, we require entanglement purification scheme for the distillation of entanglement. This scenario will be discussed in detail in the future.

In general, we observed that the maxima (minima) of success probability are obtained for state parameters $\theta_j = \left(\frac{2n+1}{4}\pi\right) \left(\theta_j = \frac{n\pi}{2}\right)$ (cf. Fig. 3). In Fig. 3, a symmetric variation of $P$ for $\alpha = 1$ is illustrated. This shows that for a particular state (with known amplitude of coherent state) the success probability depends on the state parameters of non-MES.

4 Success probability

In the previous section, we have obtained hybrid Omega-type MES from the corresponding non-MESs with the success probability $P = 4 \left(N_1 N_2 \zeta \beta \delta \gamma\right)^2$, which can be easily calculated from Eq. (19). Interestingly, the same success probability for hybrid Cluster-type MES was recently reported [54], which was of the order of that obtained for CV Cluster state [37]. It is worth stressing here using polarization DOF for concentration of the present Omega-type HES analogously, the success probability is independent of $\alpha$ as the auxiliary state (8) will have $N_1 = 1$. We have further discussed the variation of the success probability ($P$) with state parameters ($\zeta$, $\beta$, $\delta$, and $\gamma$) in...
Fig. 3 (Color online) The contour plot shows the variation of success probability ($P$) with two state parameters $\theta_j$. In all cases, we assumed $\theta_1 = \frac{\pi}{4}$, $\theta_2 = \frac{\pi}{4}$, and $\theta_3 = \frac{3\pi}{8}$, wherever needed, and the amplitude of coherent state $\alpha = 1$.

Figs. 2 and 3. Exploiting the normalization condition $\zeta^2 + \beta^2 + \delta^2 + \gamma^2 = 1$ for the real probability amplitudes, we can parameterize them as $\zeta = \cos \theta_3$, $\beta = \sin \theta_3 \cos \theta_2$, $\delta = \sin \theta_3 \sin \theta_2 \cos \theta_1$, and $\gamma = \sin \theta_3 \sin \theta_2 \sin \theta_1$. In Fig. 2, the maximum possible value of $P$ with $\theta_j$ is obtained for $\alpha = 0.5$ and further decreases with increasing value of $\alpha$. The nature of the plot in Fig. 2b, c is not as symmetric around $\theta_j = \frac{(2n+1)\pi}{4}$ as in Fig. 2a. Clearly, Fig. 2 shows that the dependence of $P$ on state parameters $\theta_1$, $\theta_2$, and $\theta_3$ depends on the value of $\alpha$. In brief, the symmetry and periodicity observed for $\alpha \geq 1$ disappears for $\alpha < 1$.

5 Application of Omega-type hybrid entangled state: Hierarchical quantum information splitting of hybrid entanglement

In a hierarchical communication network of four participants, a boss Alice (sender) distributes her quantum state among her agents (receivers) Bob, Charlie, and Diana in such a way that receivers are arranged in a hierarchy. Specifically, Diana (high power agent) can reconstruct Alice’s state with the cooperation of fewer agents (either Bob or Charlie) after Alice’s announcement; whereas Bob (Charlie), a low power agent, can reconstruct Alice’s state with the cooperation of Diana as well as Charlie (Bob). Consequently, hierarchy exists among the powers of agents (receivers) to reconstruct the quantum information split between all the receivers. A schematic diagram of the hierarchical communication network is shown in Fig. 4. Suppose Alice wishes to teleport (share) among her agents an unknown logical single qubit $|\psi\rangle_{in}$ of the form

$$|\psi\rangle_{in} = (\lambda |0_L\rangle + \eta |1_L\rangle)_{A_0},$$

where $\lambda$ and $\eta$ are complex numbers with $|\lambda|^2 + |\eta|^2 = 1$, and the subscript $L$ stands for the logical qubit $\{ |0_L\rangle = |+\rangle |\alpha\rangle ; |1_L\rangle = |-\rangle |-\alpha\rangle \}$. Notice that state (21) is CV-DV hybrid entangled state. Alice chooses a hybrid Omega-type MES (1) as quantum channel.
Fig. 4 (Color online) A schematic diagram for the proposed hierarchical communication network using hybrid entangled state. The solid (black) line corresponds to the hybrid entanglement shared among all the parties while arrows represent the classical communication required by each receiver to accomplish the task. Specifically, the dashed (black) arrow shows the information Alice has to broadcast to all the receivers. The dashed (purple) and double-dashed (purple) lines correspond to the classical communication required by Bob and Charlie, respectively. Diana requires information either from Bob [shown by double (brown) line] or Charlie [shown by dashed (brown) line].

\[
\ket{\Omega}_{ch} = \frac{1}{2} \left[ \ket{0_L} \ket{0_L} \ket{0_L} \ket{0_L} + \ket{0_L} \ket{1_L} \ket{1_L} \ket{0_L} + \ket{1_L} \ket{0_L} \ket{0_L} \ket{1_L} - \ket{1_L} \ket{1_L} \ket{1_L} \ket{1_L} \right]_{ABCD}
\]

and shares the logical qubits \( B, C, \) and \( D \) with her agents Bob, Charlie, and Diana, respectively, keeping the logical qubit \( A \) with herself. We know that hybrid entangled logical qubits undergo decoherence during transmission transforming HES into non-MES, and thus ECP discussed in the previous section is used by all the parties to extract MES. The quantum channel can also be expressed as

\[
\ket{\Omega}_{ch} = \frac{1}{\sqrt{2}} \left[ \ket{0_L}_A \ket{\psi_{0_L}}_{BCD} + \ket{1_L}_A \ket{\psi_{1_L}}_{BCD} \right],
\]

where the combined states of Bob, Charlie and Diana are

\[
\ket{\psi_{0_L}}_{BCD} = \frac{1}{\sqrt{2}} \left[ \ket{0_L} \ket{0_L} \ket{0_L} + \ket{1_L} \ket{1_L} \ket{0_L} \right]_{BCD}
\]

and

\[
\ket{\psi_{1_L}}_{BCD} = \frac{1}{\sqrt{2}} \left[ \ket{0_L} \ket{0_L} \ket{1_L} - \ket{1_L} \ket{1_L} \ket{1_L} \right]_{BCD}.
\]

The combined state of the system can be written as

\[
\ket{\psi}_{in} \otimes \ket{\Omega}_{ch} = (\lambda \ket{0_L} + \eta \ket{1_L}) A_0 \otimes \frac{1}{\sqrt{2}} \left[ \ket{0_L}_A \ket{\psi_{0_L}}_{BCD} + \ket{1_L}_A \ket{\psi_{1_L}}_{BCD} \right].
\]
Further, we define Bell-type state in the logical qubits as

\[
\begin{align*}
|\phi^+\rangle &= \frac{1}{\sqrt{2}} (|0\rangle|0\rangle + |1\rangle|1\rangle), \\
|\phi^-\rangle &= \frac{1}{\sqrt{2}} (|0\rangle|1\rangle - |1\rangle|0\rangle), \\
|\psi^+\rangle &= \frac{1}{\sqrt{2}} (|0\rangle|0\rangle + |1\rangle|1\rangle), \\
|\psi^-\rangle &= \frac{1}{\sqrt{2}} (|0\rangle|1\rangle - |1\rangle|0\rangle).
\end{align*}
\]  

(27)

Here, the Bell states in polarization DOF are denoted as  
\[|\phi^\pm\rangle = \frac{|0\rangle|0\rangle \pm |1\rangle|1\rangle}{\sqrt{2}} \] and  
\[|\psi^\pm\rangle = \frac{|0\rangle|1\rangle \pm |1\rangle|0\rangle}{\sqrt{2}}.\]  

Also, the quasi-Bell states in coherent state DOF are  
\[|\phi_\pm\rangle = N_\pm (|\alpha\rangle |\alpha\rangle \pm |-\alpha\rangle |-\alpha\rangle) \] and  
\[|\psi_\pm\rangle = N_\pm (|\alpha\rangle |-\alpha\rangle \pm |\alpha\rangle |-\alpha\rangle).\]  

With normalization constant for quasi-Bell state  
\[N_\pm = \frac{1}{\sqrt{2(1+\exp[-4|\alpha|^2])}}.\]  

This gives us a Bell-type basis in HES, and thus we can decompose Alice’s qubits  
\[A_0A \] in Eq. (26) into the above logical Bell states (27) as follows

\[
|\psi\rangle_{in} \otimes |\Omega\rangle_{ch} = \frac{1}{2} \left[ |\phi^+\rangle_{A_0A} (\lambda |\psi_0\rangle_{BCD} + \eta |\psi_1\rangle_{BCD}) + |\phi^-\rangle_{A_0A} (\lambda |\psi_0\rangle_{BCD} - \eta |\psi_1\rangle_{BCD}) + |\psi^+\rangle_{A_0A} (\lambda |\psi_1\rangle_{BCD} + \eta |\psi_0\rangle_{BCD}) + |\psi^-\rangle_{A_0A} (\lambda |\psi_1\rangle_{BCD} - \eta |\psi_0\rangle_{BCD}) \right].
\]  

(28)

This shows that after Alice performs a logical Bell (basis) measurement on her qubits  
\[A_0A \] and broadcasts her outcome the reduced state is shared among Bob, Charlie and Diana. Alice’s measurement outcomes and the reduced tripartite state sharing among the receivers is summarized in Table 1.

| Alice’s logical Bell measurement outcome | Combined state of all three agents |
|-----------------------------------------|----------------------------------|
|  \( |\phi^\pm\rangle_{A_0A} \)         |  \( (\lambda |\psi_0\rangle \pm \eta |\psi_1\rangle)_{BCD} \) |
|  \( |\psi^\pm\rangle_{A_0A} \)         |  \( (\lambda |\psi_1\rangle \pm \eta |\psi_0\rangle)_{BCD} \) |

For instance, if Alice’s measurement outcome is  
\[|\phi^+\rangle_{A_0A} \] then the corresponding receivers’ state collapses to  
\[ (\lambda |\psi_0\rangle \pm \eta |\psi_1\rangle)_{BCD}, \] which can further be expanded using the values of  
\[ |\psi_0\rangle_{BCD} \] and  
\[ |\psi_1\rangle_{BCD} \] from Eqs. (24–25) as follows

\[
|\psi\rangle = \frac{1}{\sqrt{2}} \left[ (\lambda (|0\rangle_0 |0\rangle_1 + |1\rangle_0 |1\rangle_1) |0\rangle_{L} + \eta (|0\rangle_0 |0\rangle_1 |1\rangle_1) |0\rangle_{L} - |1\rangle_0 |1\rangle_1 |1\rangle_1) \right]_{BCD}.
\]  

(29)
Table 2  Bob’s and Charlie’s measurement outcomes, Diana’s collapsed state, and the unitary operations to be applied by Diana to recover the unknown state sent by Alice

| Measurement outcome of Bob and Charlie | Diana’s state before operation | Diana’s operation |
|----------------------------------------|-------------------------------|-------------------|
| $|0\rangle_B \ 0\rangle_C$                | $(\lambda |0\rangle + \eta |1\rangle)_D$ | $I$               |
| $|1\rangle_B \ 1\rangle_C$                | $(\lambda |0\rangle - \eta |1\rangle)_D$ | $Z$               |

Notice that Alice’s quantum information is split among the agents, and now $|\psi\rangle_{in}$ has to be hierarchically recovered by agents. In what follows, we consider two cases, i.e., when the higher and lower power agents recover the quantum information.

**Case 1:** If the agents decide that Diana (high power agent) will recover the quantum state sent by Alice, then we can decompose Eq. (29) as

$$|\psi'\rangle = \frac{1}{\sqrt{2}} [ |0\rangle_B |0\rangle_C (\lambda |0\rangle + \eta |1\rangle)_D + |1\rangle_B |1\rangle_C (\lambda |0\rangle - \eta |1\rangle)_D].$$

(30)

It is to be noted that both Bob and Charlie measure in the logical qubit basis $\{|0\rangle_L, |1\rangle_L\}$ and would always have the same measurement outcomes as shown in Column 1 of Table 2, hence the communication from one of them in addition to Alice’s announcement would be enough to enable Diana to reconstruct the unknown state sent by Alice. The significance of using this logical Omega-type HES lies in the fact that $Z$ gate to be applied by Diana can be applied using waveplate on single-qubit polarization states. Note that $Z$ gate applied on coherent state gives errors due to non-orthogonality of $|\pm\alpha\rangle$ [44], and thus single qubit in HES provides that advantage.

**Case 2:** If the agents decide that Bob (low power agent) will recover the quantum state sent by Alice, then we can decompose Eq. (29) as

$$|\psi'\rangle = \frac{1}{2} \left[ |\phi^{+}_{L}\rangle_{CD} (\lambda |0\rangle - \eta |1\rangle)_B + |\phi^{-}_{L}\rangle_{CD} (\lambda |0\rangle + \eta |1\rangle)_B + |\psi^{+}_{L}\rangle_{CD} (\eta |0\rangle + \lambda |1\rangle)_B + |\psi^{-}_{L}\rangle_{CD} (\lambda |1\rangle + \eta |0\rangle)_B \right].$$

(31)

Here, Charlie and Diana are required to perform a joint logical Bell (basis) measurement $\{|\phi^{\pm}_{L}\rangle, |\psi^{\pm}_{L}\rangle\}$ and it is therefore necessary for Bob seeking the cooperation of both of them and Alice to reconstruct the unknown state sent by Alice (summarized in Table 3). Notice that Bob’s ignorance is 2 bits, and Bob’s state is a mixed state $I^L_2$ in logical qubits after tracing out Charlie’s and Diana’s logical qubits. Bob (Charlie) too can apply Pauli operations on single-qubit polarization states to reconstruct the state instead of operations on coherent states.

It is worth mentioning here that if Charlie decides to reconstruct the state, this case would be same as Case 2. Similarly, for the rest of the three cases of Alice’s measurement outcomes $|\phi^{-}_{L}\rangle_{A0A}$ and $|\psi^{\pm}_{L}\rangle_{A0A}$ as shown in Table 1, the whole scheme
for both high and low power agents will be followed exactly the same way as we have shown for $|\phi^+_A\rangle$.

Notice that while recovering the initial state after Alice’s measurement outcome, Bob/Charlie (Diana) require(s) the help from both Diana and Charlie/Bob (either Bob or Charlie), and clearly, Diana has higher power than Bob (or Charlie) to recover Alice’s state. Thus, a hierarchy exists among the agents in such a way that Bob requires more information than that by Diana for the reconstruction of the unknown quantum state sent by Alice. This is why this scheme is referred to as HQIS. Similar HQIS of HES can be proposed using the Cluster-type HES [54]. Further, by modifying our hybrid HQIS scheme proposed here, we aim to investigate the hybrid hierarchical quantum secret sharing scheme in our future work.

Before concluding the work it is imperative to stress that the present HES cannot be used for HQIS of an arbitrary two-qubit state as according to Nielsen’s criterion [62] the present state (Omega-type HES) cannot be transformed to the product of two Omega states (hyperentangled state in the present case) using local operation and classical communication [63]. Therefore, we restricted ourselves to the application of hybrid Omega-type MES in HQIS of HES.

### 6 Conclusion

As emphasized in the introduction, the success of a quantum communication or computation task, and quantum technology in general, MES is an essential quantum resource. First of all, it is difficult to distribute a MES between distant parties and even if we succeed in achieving that, it would be hard to maintain entanglement for a longer period of time. Thus, a MES would be transformed into a non-MES due to its interaction with the surroundings before its application. Therefore, it becomes necessary to extract the MES from the non-MESs using an ECP before performing an application using such quantum resources. Essentially, ECPs/EPPs are the initial steps for performing entangled state-based long-distance quantum communication and secure computation.
Here, we have chosen a hybrid Omega-type MES inspired by the possibility of its application in quantum communication over network. For the sake of completeness of the present work and to achieve the unit success probability, first we have proposed ECP of the HES using a single- and two-mode superposition of coherent states. Interestingly, the proposed ECP can be performed with linear optics and available technology only. The proposed scheme not only provides ECP but a possibility of amplification of the transmitted modes, which may have advantages in some application, for instance, to counter photon loss in long-distance quantum communication. Even when this amplification is not required the desired state can be obtained by performing a photon number measurement on one of the modes after beamsplitter as it would not disturb the entanglement. We further obtained the success probability $P$ for the ECP and found that it depends not only on the amplitude of coherent state ($\alpha$) but the coefficients of the non-MES.

Further, to accomplish our aim, we have discussed an application of the obtained hybrid Omega-type MES in quantum information splitting over a hierarchical network. Specifically, we have performed HQIS of a HES using the present scheme, which is the first such hierarchical communication scheme. The advantage of the present HES is visible here as the receiver can recover the shared quantum information by performing single-qubit polarization rotation, which can be implemented with less errors than the corresponding operations on coherent state. The present HQIS scheme can also be viewed as an application of hybrid Omega-type MES in a hierarchical controlled quantum teleportation network in view of some of the recent schemes along the same line [14]. Similarly, the proposed ECP and the concerned state can also be used in HES-based two- and multi-party networks for secure quantum communication and computation [56,64,65]. Thus, hybrid entanglement is a powerful resource for such communication tasks, and ECPs for such states are the demand of time as we are generally going to implement communication tasks over quantum networks. We conclude this work focused on the first hybrid quantum information splitting over hierarchical network using Omega-type HES and an ECP required to accomplish the task efficiently, with a hope that it can be realized experimentally and would be significant in long-range quantum communication and distributed quantum computing in the near future.

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