Spatial-frequency characteristics of photo matrices for colour image

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Abstract. The paper presents a spatial-frequency characteristics comparative analysis for obtaining a colour’s image several methods. The modern colour image matrix spatial-frequency characteristics photodetectors spatial-frequency characteristics are presented. Both unique and advanced matrix photodetectors of both multilayer type and standard systems with mosaic filters are being investigated. The author's matrix photodetector input signal distortion estimation, where two-layer sensors are presented, is carried out. Spatial-frequency characteristics comparative analysis showed that three-matrix colour image systems provide the best spatial resolution and the highest spatial (contrast) sensitivity than mosaic-type photodetectors.

1. Introduction
To obtain a colour image, it is necessary to create signals in several narrow spectral channels. In the modern photo and video systems, three-matrix systems, matrices with mosaic filters and matrices with three-colour pixels are used for this. Three-matrix systems contain a beam-splitting unit when passing through which the radiation flux is divided into three spectral channels coupled to the photo-matrix. In systems with mosaic filters, one photo matrix is used, which each pixel is covered with a certain colour light filter. In matrices with three-colour pixels, all three colour components are recorded in each pixel.

A universal criterion for the systems' quality based on photo matrices is the spatial-frequency characteristic, which determines a photo and video system spatial resolution and allows one to evaluate the distortion of the input signal and its spatial-frequency spectrum due to sampling. The article looks at the space-frequency characteristics' comparative analysis for the three specified ways of obtaining a colour image.

2. Three-matrix systems
Three-matrix systems are the most expensive one, as they include several full-fledged matrix photoreceivers for each main colour [1, 2]. Each spectral channel uses a matrix to sample the radiation flow distribution in the image. Let's say the image is built in the radiation receivers sensitive elements plane that make up the rectangular matrix. The matrix each element converts the falling stream into an electrical signal. There can be no signal change inside the matrix element. For simplicity, we will consider one-dimensional normalized functions of the pixel sensitivity distribution \( R(x) \) and the flow distribution \( S(x) \) along the x-axis (figure 1). Let the matrix structure have a spatial period \( T_x \), and the size of the matrix element (pixel) is equal to a. The matrix string space-frequency response (SFR) will
be the function \( R(f_x) \), which is a set of \( \delta \)-functions at frequencies divisible by the main frequency \( f_{0x} = 1/T_x \) with an envelope as a reference function equal to zero at frequencies divisible by \( 1/a \), that is

\[
R(f_x) = \frac{a}{T_x} \sum_{n=-\infty}^{\infty} \text{sinc}(anf_{0x}) \delta(f_x - nf_{0x}).
\]  

(1)

where \( \text{sinc}(anf_{0x}) = \sin(anf_{0x})/anf_{0x} \) is a point function \( n_{f_{0x}} \).

\[\text{Figure 1. Three-matrix systems spatial-frequency characteristics.}\]

Signals from matrix elements (output signal) can be represented by the functions \( R(x) \) and \( S(x) \) product. Strictly speaking, the output signal is described by more complex conversions [1], but this assumption does not affect the further outputs and can be accepted. Let us use the Fourier transform properties and these transformations graphical interpretation [3]. Then the output signal \( S_{\text{output}}(f_x) \) spectrum is determined by convolution of \( R(f_x) \) and spectrum \( S(f_x) \) of flow distribution. Taking into account the \( \delta \)-functions filtering property, we shall obtain

\[
S_{\text{output}}(f_x) = R(f_x) * S(f_x) = \frac{a}{T_x} \sum_{n=-\infty}^{\infty} \text{sinc}(anf_{0x})S(f_x - nf_{0x}) = \frac{a}{T_x} \sum_{n=-\infty}^{\infty} \text{sinc}\left(\frac{an}{T_x}\right)S(f_x - \frac{n}{T_x})
\]

(2)

Figure 1 shows the functions \( R(x), S(x) \), their spectra \( R(f_x), S(f_x) \), the product \( R(x) S(x) \) and the corresponding convolution \( R(f_x) * S(f_x) \).

We can see from (2) that the spatial frequency spectrum (spectral amplitude density) of the output signal is a band set at frequencies \( n_{f_{0x}} \), which each displays the image spectrum bands do not overlap if \( 1/T_x \geq 2f_m \), where \( f_m \) is the maximum frequency in the \( S(f_x) \) function spectrum, which corresponds to Kotelnikov’s theorem. The \( S_{\text{output}}(f_x) \) at zero frequency is proportional to the function \( R(x) \) constant component, i.e. determined by the relation \( a/T_x \). The image distortion resulting from the spectrum bands superimposition, when \( 1/T_x < 2f_m \), is called spatial frequency reduction, as well as spatial sampling noise. This noise is multiplicative because it depends not only on the parameters of the CI but also on the original image structure. A band at \( f_{0x} \) is suppressed by the greater and correspondingly smaller reduction, the smaller the gap between elements, i.e. the smaller \( T_x \) differs from \( a \).
3. Mosaic filter systems
In matrices with mosaic filters to obtain the image colour components, the matrix each pixel is covered with a certain colour light filter [4-6]. In most cases, these are red, blue and green filters. There is filters' spatial arrangement in several ways. The most common is the Bayer filter, historically the earliest. Light filters are grouped into four, with two green filters each having one red and one blue light filter. It is possible to use other filters combinations, which can improve either the sensitivity or colour rendering and the higher the sensitivity, the worse the colour rendering. However, in all cases, the coloured pixels are placed on a regular grid with a constant pitch twice as large as the matrix element pitch. The function of pixel sensitivity distribution $R(x)$ in this case has the form shown in figure 2, where signal and spectrum transformations are shown. The output signal spectrum is determined by the ratio

$$ S_{\text{output}}(f_x) = R(f_x) \ast S(f_x) = \frac{a}{2\pi x} \sum_{n=-\infty}^{\infty} \text{sinc}(anf_0x)S(f_x - n/f_0x/2) = $$

$$ = \frac{a}{2\pi x} \sum_{n=-\infty}^{\infty} \text{sinc}(\frac{an}{2\pi x})S(f_x - \frac{n}{2\pi x}), $$

where is the matrix row SFR

$$ R(f_x) = \frac{a}{2\pi x} \sum_{n=-\infty}^{\infty} \text{sinc}(anf_0x)\delta(f_x - n/f_0x). $$

Figure 2. Systems spatial-frequency characteristics with mosaic filters.

The first sideband in the output spectrum will be located at the frequency $1/2T_x$, so the matrix spatial resolution will be half as bad as using a separate matrix for each colour, because the spatial frequencies reduction does not occur in this case at $1/2T_x \geq 2f_m$, and the maximum frequency $f_m$ is half as bad. Note also that the $S$ value ($f_x$) at zero frequency in (3) will be half as low as in (2), which means that the system's sensitivity with mosaic filters is half as low as that of systems with mosaic filters.

4. Tricolor pixel systems
Such matrixes technology is offered by the American company Foveon. In the Foveon X3 matrix, all three RGB colour components are registered in each pixel [6-8]. It turned out that silicon absorbs different wavelengths light differently: the "blue" colour maximum absorption goes to a depth of 0.2$\mu$m from a silicon crystal surface, "green" - at a depth of 0.6$\mu$m, "red" - about 2$\mu$m. The matrix each pixel
is a three-layer semiconductor structure, where the p-n transitions' depth acting as diodes is selected taking into account the corresponding R, G, and B colour tones maximum absorption. On the modern technology level, it is possible to create matrices with three-colour pixels, which use colour components other than RGB, including infrared channels. In such matrices, the same spectral channel pixels may have different sizes and be arranged in groups, forming a certain periodic structure (pattern).

Let’s assume that some channel uses two sizes alternating pixels, with the matrix element pitch equal to $T_x$ and pixels alternating with the period $2T_x$, i.e. the pixel pattern has a spatial frequency $\Omega_x = 1/2T_x$ (figure 3). This pixels arrangement can be considered as a pulse-width modulation with a periodic rectangular signal frequency $f_0x = 1/T_x$.

![Figure 3. Systems with tricolour pixels spatial-frequency characteristics.](image)

The signal complex spectral composition at pulse-width modulation is determined by the expression [9], representing, in this case, the matrix SFR line

$$R(f_x) = \frac{a}{T_x} + \frac{\Delta x}{T_x} \sin \Omega_x x + \sum_{k=1}^{\infty} C_k J_0(B_k) \cos k\omega_{1x} +$$

$$+ \frac{2}{\pi} \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} j_{2n}(B_k) \cos( k\omega_{1x} \pm 2n\Omega_x) x +$$

$$+ \frac{2}{\pi} \sum_{k=1}^{\infty} D_k \sum_{n=1}^{\infty} j_{2n-1}(B_k) \sin( k\omega_{1x} \pm (2n-1)\Omega_x) x +$$

(5)

where $\Delta x$ is the maximum change in pixel size, $\omega_{1x} = 2\pi/T_x$, $B_k = k\pi\Delta x/T_x$, $C_k = 1/k \sin B_k$, $D_k = 1/k \cos B_k$, $j_{2n}(B_k)$, $j_{2n-1}(B_k)$ – values of the Bessel function of the first kind of order n and n-1 from the argument $B_k$.

The first term in (5) is a constant component, the second is a harmonic at the frequency $\Omega_x$ with the amplitude $\Delta x/T_x$. Also, the spectrum contains harmonics with frequencies that are multiples of $f_{0x} = 1/T_x$ with upper and lower sidebands with frequencies $k\omega_{1x} \pm 2n\Omega_x$ and $k\omega_{1x} \pm (2n-1)\Omega_x$. The harmonic with the frequency $\Omega_x$ and these sidebands' harmonics may be in the spatial frequency reduction region. The greatest influence is exerted by the harmonic at the frequency $\Omega_x$ as the lowest frequency and with the largest amplitude compared to other side harmonics.

In this case, the frequency $\Omega_x$ is the matrix elements half the frequency, which, accordingly, reduces the matrix spatial resolution by half.
In matrices with three-colour pixels, the same colour pixels can form a periodic structure with elements arranged in pairs in a staggered order. In a row, such pairs will alternate with a two elements interval, that is, with a $4\, T_x$ period. It can be shown that a rectangular pulses $R_1(x)$ pair spectral density (figure 4) is defined as [1]:

$$R_1(f_x) = a \text{sinc}(af_x) \cos(2\pi f_x T_x)$$

Rectangular pulses $R(x)$ pairs alternating with the period $4T_x$ are described by the convolution $R_1(x)$ with the lattice function $N(x)$, and this convolution $R(f_x)$ spectrum is the corresponding spectra product (figure 4):

$$R(f_x) = R_1(f_x)\, N(f_x) = a \text{sinc}(af_x) \cos(2\pi f_x T_x) \times N(f_x) =\frac{a}{4T_x} \text{sinc}(af_x) \cos(2\pi f_x T_x) \times \sum_{n=-\infty}^{\infty} \delta(f_x - \frac{n}{2T_x})$$

(6)

where $N(f_x) = \frac{1}{4T_x} \sum_{n=-\infty}^{\infty} \delta(f_x - n/4T_x)$ – a comb function with a $1/4T_x$ frequency period, which is the comb function Fourier image

$$N(x) = \sum_{n=-\infty}^{\infty} \delta(x - 4nT_x).$$

Figure 4. Systems space-frequency response with three-colour pixels with convolution $R_1(x)$ with a lattice function $N(x)$.

The matrix row product (6) is the SFR If we take into account that the function $\cos(2\pi f_x T_x)$ takes the values either 0 or 1 at the frequencies $f_x = n/4T_x$, and therefore there are no odd harmonics in $R(f_x)$ they coincide with the zeros of the function $\cos(2\pi f_x T_x)$, then (6) takes the form
\[ R(f_x) = \frac{a}{4T_x} \sum_{n=-\infty}^{\infty} \text{sinc} \left( \frac{an}{2T_x} \right) \delta(f_x - \frac{n}{2T_x}) \] (7)

that is, the matrix row SFR is a \( \delta \)-functions set located at frequencies that are multiples of \( 1/2T_x \), inscribed in the envelope a \( \text{sinc}(af_x) \). The output signal spectrum is a convolution

\[ S_{\text{output}}(f_x) = R(f_x) * S(f_x) = \frac{a}{4T_x} \sum_{n=-\infty}^{\infty} \text{sinc} \left( \frac{an}{2T_x} \right) S \left( f_x - \frac{n}{2T_x} \right), \] (8)

It can be seen from (4) and (8) that the matrix row considered SFR is similar to the matrices SFR structure with mosaic filters and differs only by a constant multiplier before the sum.

5. Results
Thus, a colour photo matrices spatial-frequency characteristics comparative analysis has shown that three-matrix colour image systems provide the best spatial resolution and the highest spatial (contrast) sensitivity. The maximum spatial frequency allowed by a three-matrix system is defined as \( f_m = 1/2T_x \). In systems with mosaic filters, the spatial resolution is twice as bad, the maximum allowed frequency is \( f_m = 1/4T_x \), and the sensitivity is half as high as when using a separate matrix for each colour. In systems with three-colour matrix pixels, it is fundamentally possible to provide spatial resolution at the systems level with mosaic filters in combination with increased spatial (contrast) sensitivity of one of the selected layers.

6. Conclusion
The paper presents the photodetectors current matrix assessment as a standard system multilayer type or with mosaic filters.

Such issues are the Russian Federation STG strategy problems in the transition to advanced digital, intelligent production technologies, robotic systems, construction new materials and methods, systems creation for processing data large amounts, machine learning and artificial intelligence. As well as to the information and telecommunications systems priority direction.

The spatial frequency response, which determines the photo and video system spatial resolution and allows us to evaluate the distortion of the input signal and its spatial frequency spectrum due to sampling, allows us to qualitatively and quantitatively evaluate the modern photo matrices capabilities. This assessment is necessary when choosing the matrix photodetector type for scientific research and space and aerial photography.

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