Parallel Newton-Chebyshev Polynomial Preconditioners for the Conjugate Gradient method

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Discretization of PDEs modeling different processes and constrained/unconstrained optimization problems often require the repeated solution of large and sparse linear systems $Ax = b$. The size of these systems can be of order $10^6 \div 10^9$ and this calls for the use of iterative methods, equipped with ad-hoc preconditioners as accelerators running on a parallel computing environment. In most cases, the huge size of the matrices involved prevents their complete storage. In these instances only the application of the matrix to a vector is available as a routine (matrix-free regime). Differently from direct factorization methods, iterative methods do not need the explicit knowledge of the coefficient matrix. The issue is the construction of a preconditioner which also work in a matrix-free regime. Polynomial preconditioners, i.e. preconditioners that can be expressed as $P_k(A)$, are very attractive for several reasons i.e. their construction is only theoretical, namely only the coefficients of the polynomial are to be computed with negligible computational cost, the application of $P_k(A)$ requires a number, $k$, of matrix-vector products so that they can be implemented in a matrix-free regime, and the eigenvectors of the preconditioned matrix are the same as those of $A$.

We consider polynomial preconditioners to accelerate the Conjugate Gradient method in the solution of large symmetric positive definite linear systems in massively parallel environments. We put in connection a specialized Newton method to solve the matrix equation $X^{-1} = A$ [1] and the Chebyshev polynomials for preconditioning. We propose a simple strategy to avoid clustering of the extremal eigenvalues in order to speed-up convergence. Numerical results on very large linear systems (up to 8 billion unknowns) in a parallel environment show the efficiency of the proposed class of preconditioners.
References

[1] L. Bergamaschi and A. Martínez, Parallel Newton-Chebyshev polynomial preconditioners for the Conjugate Gradient method, Computational and Mathematical Methods, (2021). to appear.