Classical Pendulum Feels Quantum Back-Action

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Abstract

Quantum mechanics, which has agreed with every experimental test, predicts superposition of position states even for macroscopic objects. Because of the massiveness, such states might have a key to investigate both quantum measurement problem and quantum gravity.\textsuperscript{1,2,3,4} Recently, the use of quasi-freely suspended mirror combined with a laser field was proposed to prepare such states.\textsuperscript{5,6,7} One of the key milestones for the quantum-optomechanical effects such as the generation of the entanglement state and a squeezed state of light\textsuperscript{8,9,10} is the observation of quantum back-action,\textsuperscript{11} which identifies the connection between the objects and quantumness of the light. Until now, this effect has been observed below the mesoscopic mass scale.\textsuperscript{12,13,14,15} However, it has not been observed yet in the macroscopic scale beyond Planck mass. This is partially due to a technical limitation—the radiation pressure of light will expose a free mass to instability such as anti-torsional spring effect.\textsuperscript{16} Also, there is the fundamental compromise between tolerance for the instability and sensitivity; sufficient tolerance with firm suspension makes the mass differ from free mass, and this results in increase of a thermal fluctuating force. Here we describe using a triangular optical cavity to overcome these limitations and observation of quantum back-action imposed on a suspended 5-mg mirror. The origin of quantum back-action is the momentum transfered to
the mirror by light on its reflection. For the coherent light, the photon number fluctuates according to a Poisson distribution and this fluctuation gives the radiation pressure fluctuation. The pendulum mode excited by this force fluctuation was observed. Our result paves the way to the generation of macroscopic entangled states and also has possibility to test a role of gravity\textsuperscript{5,17,18,19,6} for solving the quantum measurement problem.

Recent advances in technology have enabled experimental test of quantum mechanics at the scales close to “classical”, i.e. macroscopic, scales of our everyday life\textsuperscript{15,20,21}. However, superposition of positions of macroscopic objects beyond Planck mass has not been observed, even though quantum mechanics predicts it. This is at the heart of the so-called “quantum measurement problem”. Until now, intense theoretical and experimental works have revealed that environment such as a thermal bath plays an important role in decoherence – the loss of quantum interference\textsuperscript{22,23,24,25}. Under many-world interpretation,\textsuperscript{26} this decoherence effect will make us understand the problem without inconsistency.\textsuperscript{1} From a positivistic point of view, however, it is not the fundamental solution because linear quantum mechanics cannot destroy superposition.

Turning now to general relativity, gravity might prohibit the superposition of the massive object (i.e. gravity field) due to its non-linearity. Therefore, if we prepare a massive object isolated enough from the environment so that the superposition state is expected to be prepared from the viewpoint of the quantum mechanics, we can test what kind of a role the gravity plays in a quantum-gravity interface, e.g. gravitational collapse.\textsuperscript{17,18} For this purpose, utilization of optomechanical oscillators combined with light such as a levitated micro-sphere (e.g. satellite mission MAQRO\textsuperscript{4}) and suspended mirrors (e.g. gravitational wave detectors such as LIGO\textsuperscript{27,11,19}) have been proposed. These optomechanical systems enable us to test the effect of gravity because the oscillator is expected to be entangled with the laser field, and the resulting entanglement causes the position of the oscillator to be superposed.\textsuperscript{5,6,7} Especially the suspended mirror attracts considerable attention since the massive mirror, which enhances the effect of gravity, can enable us to precisely test the effect of gravity.\textsuperscript{10} However, at macroscopic scales, it becomes sensitive to the environmental disturbances because it is difficult to isolate the oscillator from the environment. In practice, even quantum back-action, which is the measurement-disturbance derived from Heisenberg’s uncertainty principle (HUP), has never been observed yet in macroscopic mass scales. In
an optomechanical system, quantum back-action represents a quantum fluctuation (vacuum fluctuation) of the light derived from the HUP. Therefore, observation of quantum back-action identifies the connection between the oscillator and quantumness of light, and thus it is a necessary condition for generating an entangled state. Here, we continuously measured the position of a suspended 5-mg mirror by a triangular optical cavity and succeeded in observing quantum back-action at room temperature.

Figure 1: Mechanical oscillator. The mirror was manufactured by SIGMA KOKI. It has radius of 2 mm, thickness of 0.2 mm, and mass of 5 mg. The tungsten wire of 3 μm diameter and 50 mm length is attached on the mirror with the epoxy resin. Both in the side view and front view, it appears much larger than the real size of the tungsten wire because of the overexposure of the camera. The enlarged view photographed by a stereoscopic microscope (Olympus, SZ61) shows the interface between the wire and the mirror. Scale bars, 4 mm both in side and front view, and 0.2 mm in enlarged view.

Our optomechanical system is a triangular cavity, which consist of the suspended 5-mg mirror (Fig. 1), a fixed mirror, and a mirror with actuators attached for cavity length control (Fig. 2a). The 5-mg mirror is suspended by a tungsten wire of 50 mm length with 3 μm diameter attached to the mirror with epoxy resin. The half linewidth of the cavity is $\kappa/2\pi = 1.5$ MHz and the optomechanical coupling constant is $g/2\pi = 2.8\omega_c$ Hz/m, where $\omega_c$ is the cavity resonant frequency. The triangular cavity is placed on a vibration isolation stage installed in a vacuum chamber ($10^{-3}$ Pa). We use a Nd:YAG laser source with a wavelength of 1064 nm. Two beams separated from the same source are fed into the triangular cavity in the same spatial mode but in opposite directions with the total intracavity photon number being $N_{\text{circ}} = 5.5 \times 10^9$ (i.e. 3.4 W). One of the beams is a “driving” beam
whose frequency is slightly red-detuned ($\Delta_a/\kappa \simeq -0.05$), and the other is a "spring" beam whose frequency is largely blue-detuned ($\Delta_b/\kappa = +1.3$) from the cavity resonance (Fig. 2b, c).

Figure 2: **Experimental setup.** a, The driving (red) and spring (blue) beams incident on the fixed and controlled mirrors, respectively. b, The intracavity power and detuning for each beam. The red and blue points show both laser-cavity detuning and the intracavity power. The dashed red and blue curves show the optical power as a function of cavity detuning for each beam. The driving beam dominates quantum back-action due to the higher intracavity power than the spring beam. c, The optical spring effect. The red point represents the driving beam at $\Delta_a/\kappa = -0.05$, the blue point represents the spring beam at $\Delta_b/\kappa = +1.3$, and the dotted green represents the sum of them. The dashed red and blue curves show parametric plots of the optical spring as a function of detuning for each beam and the dashed green curve is the sum of them. Inside the cyan flame, both the spring and the mechanical decay rate have positive values, and thus the mirror is stably trapped. It is impossible for single optical spring.

There are three main technical features in our experiment: the extremely thin suspension wire, triangular geometry of the cavity and the use of double
optical spring. The thin wire assures that the amount of energy stored in
the pendulum is dominated by the gravitational potential over the elastic
bending energy of the wire.\footnote{29} Since the loss of the energy is only associated
with the elastic part of the stored energy, the total mechanical loss of the
pendulum is diluted with gravity by a factor $k_{\text{grav}}/k_{\text{el}}(\propto 1/r^2) = 1 \times 10^3$,
where $k_{\text{grav}}$ and $k_{\text{el}}$ are the gravitational and elastic spring constants of the
pendulum and $r$ is the radius of the wire. The reduction of the loss results in
the reduction of a thermal fluctuation force, which also drives the mechanical
motion similarly to quantum back-action, by a factor of $\sqrt{k_{\text{grav}}/k_{\text{el}}}$. The radiation pressure of the light induces a
torque on the mirror when it is rotated. In a suspended linear cavity, this torque
works as an optical anti-torsional spring and causes a mechanical instability (so-called Sidles-
Sigg instability\footnote{16}). This is especially a serious issue because the mirror is
suspended by the thin wire in order to reduce the thermal noise, which pro-
vides little mechanical torsional spring constant ($k_{t,m} \simeq 3 \times 10^{-11}$ Nm/rad) to
compete against the optical anti-torsional spring. In order to circumvent
this limitation, we use a triangular cavity, which has positive optical torsional
spring and exhibits no instability in the rotation around the suspension axis
(See supplementary information). In our setup, the optical torsional spring
constant is $k_{t,\text{opt}} = +1 \times 10^{-9}$ Nm/rad whereas it is $k_{t,\text{opt}}^{(\text{linear})} \simeq -3 \times 10^{-9}$
Nm/rad for a linear cavity with otherwise the same parameters. As a result,
we can store 100 times higher optical power in our cavity than the instability
limit for the linear cavity. Besides, compared with mechanical springs, the
optical torsional spring is free from the thermal noise penalty because the
optical field is almost in its ground state.

The third feature is the double optical spring technique\footnote{29} to increase the
resonant frequency of the pendulum without increasing the thermal noise
similarly to the optical torsional spring. By detuning a cavity from its res-
onance, one can create an optical spring effect for the position of a mirror.
However, optical spring (anti-spring) effect is always accompanied by optical
anti-damping (damping), and thus it is unstable by itself. Since our pendu-
ulum have a low mechanical decay rate ($\gamma_m/2\pi = 2.3 \times 10^{-6}$ Hz) due to the
large dilution, the anti-damping effect cannot be compensated by the
mechanical damping. In order to avoid this problem, we use a combination of
two optical springs, by injecting two laser beams, to cancel out the instability
(Fig. 2c). With this technique, we can increase the resonant frequency of
the pendulum to 130 Hz, where technical noises such as ground vibration,
Figure 3: Quantum back-action. Observed spectra of displacement fluctuation at optical power, $P_{\text{circ}} = 3.4$ W; half linewidth, $\kappa/2\pi = 1.5$ MHz; effective amplitude decay rate, $\gamma_{\text{eff}}/2\pi = 1.53 \times 10^{-2}$ Hz; effective resonant frequency, $\omega_{\text{eff}}/2\pi = 130$ Hz. Measured peak displacement spectral density (blue), estimated thermal contribution (red), estimated quantum and classical back-action contribution (cyan), the theoretical prediction (green) are shown. Measured peak is consistent with theoretical estimation in Fig. 2c both in terms of the resonant frequency and the mechanical decay rate.

The measured amplitude power spectrum of the pendulum motion of the mirror is shown in Fig. 3. The result agrees well with the sum of the estimated thermal noise (See supplementary information), the estimated classical back-action (which is generated by classical intensity fluctuation not by vacuum fluctuation) and estimated quantum back-action. Generally, the photon number fluctuation of the laser is larger than the vacuum fluctuation below MHz region. The classical back-action are also large at our measurement frequency band (80–150 Hz). In order to assure the classical back-action to be
smaller than the quantum one, the intensity fluctuation of the laser should be stabilized. The required power variations of the input beams relative to the shot noise limit have to be smaller than 2.0 dB and 26 dB for the driving and spring beams, respectively (See supplementary information). To meet this criteria, we stabilized the intensity of the driving beam with active feedback (Fig. 4a).

Figure 4: **Characterization of the light and the mechanical oscillator.** a, Observed spectra of intensity fluctuation of the input driving beam. Measured intensity fluctuation (blue), measured intensity fluctuation before the stabilization (red), and the requirement for achieving the shot noise level inside the cavity (green) are shown. b, The mechanical response of the pendulum. Measured values (blue) are shown. The peak around 130 Hz represents the pendulum resonance. The measured resonant frequency is consistent with the theoretical estimation (Fig. 2c). A low resolution of the measurement disables us from measuring the mechanical decay rate.

We also measured the optically modified dynamics of our movable mirror by intentionally adding small disturbance to the cavity length control loop in order to directly confirm the resonant frequency of the pendulum. The measured resonant frequency and mechanical decay rate of the pendulum give us precise information of the optical spring enabling us to make a consistency check on the cavity conditions (e.g. intracavity power, detuning) –namely, our estimate of the amount of the quantum back-action. As a result, we estimated the ratio of quantum back-action to thermal noise $0.33 \pm 0.03$.

In conclusion, the geometrical advantages of the triangular cavity enables for the mirror to be isolated from the thermal bath under high intracavity
power, allowing us to use the powerful double optical spring, which also enables isolation from technical noise sources. As a consequence, we were able to observe the quantum back-action imposed on a mg scale object, which is far beyond the Plank mass, at room temperature. The observation of quantum back-action is the first step toward the experimental validation of macroscopic quantum mechanics and it will certainly provide new insights into quantum mechanics and general relativity.

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**Author Contributions** N.M. had the idea. N.M. designed and performed the experiment, and analyzed the data under the supervision of Y.A. N.M. and Y.M. developed the electronic devices. N.M. and G.H. designed the optomechanical device. K.T. was a leader. N.M. and Y.M. wrote the manuscript with assistance from all other co-authors.

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A Supplementary information

A.1 Optical torsional spring effect of the triangular optical cavity

The positive torsional spring effect is the key effect in our experiment. In this section, we will theoretically and experimentally show the detail of the effect.

Figure 5: The detailed experimental setup for observing optical torsional spring effect. The laser beam (red line) were fed into the triangular cavity. An electro-optical modulator (EOM) was used to apply frequency sidebands for a Pound-Drever-Hall (PDH) method. Light was detected at various points using photodetectors (PD). The figure in the cyan flame represents the layout of the triangular cavity. The triangular cavity formed by two flat mirrors labeled M$_a$ and M$_c$ (the movable mirror) and a curved mirror labeled M$_b$. $l$ represents the distance between the curved mirror and the flat mirror, $d$ is half the distance between the two flat mirrors, $R$ is the radius of curvature of mirror M$_b$, $\theta$ is the incident angle on the curved mirror, and $\beta$ is the incident angle on the flat mirror. HWP, Half-Wave Plate; QWP, Quarter-Wave Plate; FI, Faraday Isolator.
Optomechanical model of a triangular cavity  Here, we will derive the torsional spring effect around the suspension axis (yaw) due to radiation pressure torque in a triangular cavity. We use Sigg’s result and begin by considering a two-dimensional triangular cavity formed by two flat mirrors labeled $M_a$ and $M_c$ and a curved mirror labeled $M_b$ as shown in Fig. 5. We decompose the rotations of the two flat mirrors into two basis modes: the common-mode (the same rotation direction, the same amount) and the differential-mode (the opposite rotation direction, the same amount). Any misalignment state of the two mirrors can be expressed as a linear combination of these two basis modes. In this picture, the relationship between the misalignment angle $\Delta \alpha$ of the basis modes and the change in beam position on each of the mirror $\Delta x$ is given by

$$\Delta x = lK_h \Delta \alpha$$ \hspace{1cm} (1)

with

$$K_h = \frac{1}{l(d+l-R)} \begin{pmatrix}
-\frac{2d(l-R)}{\cos \beta} & 0 & -\sqrt{2}dR \\
0 & -\frac{2(l+1-R)}{\cos \beta} & 0 \\
-\sqrt{2}dR & 0 & (d+l)R
\end{pmatrix}.$$

(2)

Here, $l$ is the distance between the curved mirror and the flat mirror, $d$ is half the distance between the two flat mirrors, $R$ is the radius of curvature of mirror $M_b$, and $\beta$ is the incident angle on the flat mirror. The torques $N_{rad}$ on the each mirror induced by the radiation pressure is given by

$$N_{rad} = \frac{2P_{circ}}{c} lTK_h$$ \hspace{1cm} (3)

with

$$T = \begin{pmatrix}
\cos \beta & 0 & 0 \\
0 & \cos \beta & 0 \\
0 & 0 & \cos \theta
\end{pmatrix}.$$

(4)

where $c$ is the speed of light in vacuum, $P_{circ}$ is the circulating power in the triangular cavity, $\theta$ is the incident angle on the curved mirror, and $\beta$ is the incident angle on the flat mirror. When the acute-angled isosceles triangular optical cavity with a positive g-factor is considered, optical spring constant is always positive because all eigenvalues of Eq. become negative. This suggests that the triangular cavity is intrinsically stable in yaw direction.
For simplicity we consider situation where only the mirror $M_a$ is movable and others are fixed. In this case, the equations of motion are given by

$$I_a \ddot{\alpha}_a = - (k_{t,\text{opt}} + k_{t,m}) \alpha_a \quad (5)$$

$$k_{t,\text{opt}} \equiv - \frac{P_{\text{circ}}}{c} l \cos \beta (K_h(1, 1) + K_h(2, 2)), \quad (6)$$

where $I_a$ is the moment of inertia about the wire axis of the mirror $M_a$, $k_{t,\text{opt}}$ is the angular spring constant of mirror $M_a$ induced by radiation pressure, and $k_{t,m}$ is the mechanical torsional spring constant of mirror $M_a$ in yaw.

From this equation, we can derive the resonant frequency of the yaw motion as

$$f_a = \frac{1}{2\pi} \sqrt{\frac{k_{t,\text{opt}} + k_{t,m}}{I_a}}. \quad (7)$$

From Eqs. (6) and (7) it is found that the angular resonant frequency is increased with the increased circulating power.

**Experimental demonstration of the torsional spring effect**

**Setup** In order to quantitatively verify the model in the previous section, we measured the angular resonant frequency of a mirror in a triangular cavity. By changing the internal power of the cavity, thus changing $k_{t,\text{opt}}$, we expect the resonant frequency to change according to Eq. (7).

A schematic of the experimental setup is shown in Fig. 5. The laser source was a monolithic non-planar Nd:YAG ring laser with 2 W continuous wave single-mode output power at 1064 nm. We used an electrooptical modulator (EOM) as a phase modulator with 15 MHz to lock the triangular cavity via a Pound-Drever-Hall (PDH) locking scheme. The triangular cavity with a length of 100 mm and a finesse of 223 was composed of two flat mirrors and a fixed curved mirror with a radius of curvature of 75 mm. One of the two flat mirrors was a half-inch fused silica mirror suspended by a tungsten wire of 30 µm diameter and 100 mm length. The suspended mirror was attached to an oxygen-free copper cylinder of 3 mm diameter and 3 mm thickness, which was damped by eddy-current using a doughnut-shaped magnet. Because of
its shape, the magnet damps only the pendulum motion without decreasing the mechanical quality factor of the yaw motion. The curved mirror was fixed and is mounted on a piezoelectric transducer (PZT), which was used as an actuator to keep the cavity in resonance with the laser. The triangular cavity and a photodetector were placed in a vacuum desiccator for acoustic shielding.

The reflected light was received by a photodetector and its output signal was demodulated at the modulation frequency. This signal was then low-pass filtered with a cutoff frequency of 1 Hz and fed back to the PZT actuator. The unity gain frequency of the length control servo was approximately 1 kHz. We used this signal to stabilize the cavity length and also to measure the angular (yaw) resonant frequency. The yaw motion of the suspended mirror generated the PDH signal because there was slight miscentering of the beam position on the suspended mirror. The transmitted light was also detected to measure the intracavity power. The incident light power into the cavity was varied from 60 mW to 1 W in order to measure the change in angular resonant frequency.
Figure 6: The optical torsional spring response for various power levels. a, Observed spectra of the feedback signal. The peaks correspond to the yaw resonance of the suspended mirror with the intracavity power (4 W(blue), 32 W(red), 46 W(green) and 68 W(cyan)). b, Angular resonant frequency of the mirror suspension against the intracavity power. The blue circles are the measurement data and the blue horizontal lines are the statistical errors. The solid red curve is the theoretical prediction obtained from Eq. (7) and the dashed red curve shows the systematic error.

Results and discussion  Figure 6a shows the observed spectra of the feedback signal with the intracavity power at 4 W, 32 W, 46 W, and 68 W. The peaks around 0.4 Hz are the yaw resonances. The angular resonant frequency increases with increasing circulating power. The measured angular resonant frequencies are plotted against the intracavity power in Fig. 6b. The blue circles are measured values and the horizontal lines are the statistical errors. The dashed red curves are the theoretical predictions, obtained from Eqs. (6) and (7) with \( l = 44 \text{ mm}, \ d = 10 \text{ mm}, \ \beta = 0.67 \text{ rad}, \ k_{t, \text{opt}} = 1.0 \times 10^{-9} \times P_{\text{circ}} \text{ Nm/rad}, \ I_a = 2.4 \times 10^{-8} \text{ kgm}^2, \) and \( k_{t, \text{m}} = 1.3 \times 10^{-7} \text{ Nm/rad}. \) The theoretically calculated values show good agreement with the experimental results, which suggests that Eq. (6) is suitable for modeling the torsional spring effect caused by the optical restoring force.

Until now, we only have paid attention to the yaw. Note that this is sufficient discussion in order to consider the stability of our triangular cavity. Because the suspended mirror can easily have enough mechanical positive torsional spring constants for a pitch without increasing suspension thermal noise, even though the anti-torsional spring effect occurs for the pitch. This is due to the fact that the stiffness of the pitch does not depend on the radius.
of the wire, which mainly determines the dilution factor, but depend on the radius of the mirror.
### A.2 Theoretical derivation of quantum back-action

![Diagram of cavity](image)

**Figure 7: Layout of the triangular cavity.** The cavity was made from three mirrors: the input coupler for the driving beam, with decay rate, $\kappa_{\text{in}1}$; the input coupler for the spring beam, with decay rate, $\kappa_{\text{in}2}$; the movable mirror, with decay rate, $\kappa_{\text{in}3}$; and a mirror to represent intracavity loss, with decay rate, $\kappa_{\text{in}4}$. The cavity mode is labeled $\hat{a}$ and $\hat{b}$. The extracavity field are: $\hat{A}_{\text{in}1}$, $\delta \hat{A}_{\text{in}2}$, $\delta \hat{A}_{\text{in}3}$, $\delta \hat{A}_{\text{in}4}$, $\delta \hat{B}_{\text{in}1}$, $\hat{B}_{\text{in}2}$, $\delta \hat{B}_{\text{in}3}$, and $\delta \hat{B}_{\text{in}4}$.

Here we will calculate quantum back-action in an optomechanical system. We start from Newton’s law to describe the mechanical response

$$m\ddot{x} + 2m\gamma_m \dot{x} + k_m x = F$$

(8)

where $m$ is the mass of the movable mirror (mechanical oscillator), $\omega_m$ is the mechanical resonant frequency, $\gamma_m$ is the mechanical amplitude decay rate, $k_m$ is the mechanical spring constant, and $x$ is the position for the mirror. To derive the mechanical susceptibility we Fourier transform the Eq.(8) according to the following conventions: $f(\omega) \equiv \int_{-\infty}^{\infty} dt f(t) \exp(-i\omega t)$.

$$\chi_m \equiv \frac{x}{F} = \frac{1}{m(\omega_m^2 - \omega^2 + i2\omega\gamma_m)}$$

(9)
Secondly, we calculate the response of an optomechanical system to two independent laser driving fields. The Hamiltonian describing the optomechanical coupling can be written and linearized in the form

\[ \hat{H} = \hbar \omega_c(x) \hat{a}^\dagger \hat{a} + \hbar \omega_c(x) \hat{b}^\dagger \hat{b} + \hat{H}_\kappa \]

\[ \simeq \hbar \omega_c \hat{a}^\dagger \hat{a} + \hbar \omega_c \hat{b}^\dagger \hat{b} + \hbar g \hat{a}^\dagger \hat{a} x + \hbar g \hat{b}^\dagger \hat{b} x + \hat{H}_\kappa \]  \hspace{1cm} (10)

where \( g = 2\omega_c \cos \beta / L \) is the optomechanical coupling constant, \( \omega_c \) is the cavity resonance frequency, \( \beta \) is the incident angle on the movable mirror, \( L \) is the round trip length, \( \hat{H}_\kappa \) represents the optical input and output coupling, and \( \hat{a}, \hat{b} \) is the annihilation operators (cavity modes) for two counterpropagating directions in the triangular cavity, respectively. The Heisenberg Langevin equations of motion for the cavity modes are

\[ \dot{\hat{a}} = -(\kappa + i \omega_c) \hat{a} - i g a x + \sum \sqrt{2 \kappa_i} \hat{A}_i \] \hspace{1cm} (11)

\[ \dot{\hat{b}} = -(\kappa + i \omega_c) \hat{b} - i g b x + \sum \sqrt{2 \kappa_l} \hat{B}_l \] \hspace{1cm} (12)

where the \( \kappa_{i1}, \kappa_{i2}, \kappa_{i3} \) are the cavity amplitude decay rates for the each mirror, \( \kappa_{i4} \) is the decay rate for the cavity round trip loss, \( \kappa \) is the total decay rate, and \( \hat{A}_1, \hat{B}_1 \) are the input optical fields. The equation of motion can be written in the rotating frame of reference by setting \( \hat{a} = \exp(-i \omega_a t) \hat{\tilde{a}} \) and linearized in the form:

\[ \delta \dot{\hat{a}} = -(\kappa - i \Delta_a) (\tilde{a} + \delta \tilde{a}) - i G_a \delta x + \sqrt{2 \kappa_{i1}} \delta \hat{A}_i \] \hspace{1cm} (13)

\[ \delta \dot{\hat{b}} = -(\kappa - i \Delta_b) (\tilde{b} + \delta \tilde{b}) - i G_b \delta x + \sqrt{2 \kappa_{i2}} \delta \hat{B}_i \] \hspace{1cm} (14)

where \( \Delta_a = \omega_a - \omega_c - G_a \bar{x}, \Delta_b = \omega_b - \omega_c - G_b \bar{x} \) are the cavity detuning, \( G_a = \bar{a} g, G_b = \bar{b} g \) are the light-enhanced optomechanical couplings for the linearized regime, \( \tilde{a}, \tilde{b} \) are the average parts for each cavity mode, \( \delta \tilde{a}, \delta \tilde{b} \) are the fluctuating parts for each cavity mode, \( \delta \hat{A}_i, \delta \hat{B}_i \) are the real valued coherent amplitudes for input lasers, and \( \delta \hat{A}_l \), for \( l = \text{in1, in2, in3, in4} \) is the vacuum fluctuation entering from each port.

The average intracavity field amplitudes are described from Eqs. (13).
and (14)

\[
\bar{a} = \frac{\sqrt{2\kappa_{in1}}}{\kappa - i\Delta_a} \tilde{A}_{in1}
\]

(15)

\[
\bar{b} = \frac{\sqrt{2\kappa_{in2}}}{\kappa - i\Delta_b} \tilde{B}_{in2}
\]

(16)

From these equations, the intracavity power is given by

\[
\bar{P}_{circ} = \frac{\hbar \omega_c |a|^2}{\tau} + \frac{\hbar \omega_c |b|^2}{\tau}
= \bar{P}_{in1,circ} + \bar{P}_{in2,circ}
= \frac{2\kappa_{in1}}{\tau (\kappa^2 + \Delta_a^2)} \bar{P}_{in1} + \frac{2\kappa_{in2}}{\tau (\kappa^2 + \Delta_b^2)} \bar{P}_{in2}
\]

(17)

where \(\tau\) is the cavity round trip time.

The fluctuation components of Eqs. (13) and (14) are similarly given by

\[
\dot{\delta a} = - (\kappa - i\Delta_a) \delta a - iG_a \delta x + \sum_l \sqrt{2\kappa_l} \delta \hat{A}_l
\]

(18)

\[
\dot{\delta b} = - (\kappa - i\Delta_b) \delta b - iG_b \delta x + \sum_l \sqrt{2\kappa_l} \delta \hat{B}_l
\]

(19)

In terms of frequency components, these can be rewritten by

\[
\delta \hat{a} = \chi_a (-iG_a \delta x + \sum_l \sqrt{2\kappa_l} \delta \hat{A}_l)
\]

(20)

\[
\delta \hat{b} = \chi_b (-iG_b \delta x + \sum_l \sqrt{2\kappa_l} \delta \hat{B}_l)
\]

(21)

where \(\chi_a = (\kappa + i(\omega - \Delta_a))^{-1}\), \(\chi_b = (\kappa + i(\omega - \Delta_b))^{-1}\) are the cavity susceptibilities for the two modes. These lead to the forces induced by the cavity
modes applying to the movable mirror, which is given by

$$F_{BA} + \delta F_{BA} + \delta \hat{F}_{BA} = -\frac{\partial \hat{H}}{\partial x}$$

$$= -\frac{(g_a + g_b)}{\omega_c} \hat{F}_{\text{circ}} + i\hbar |G_a|^2 \delta x (\chi_a(\omega) - \chi_a^*(-\omega))$$

$$- \hbar G_a \chi_a(\omega) \sum \sqrt{2\kappa_1} \delta \hat{A}_1 - \hbar G_a \chi_a^*(-\omega) \sum \sqrt{2\kappa_1} \delta \hat{A}_1^\dagger$$

$$+ i\hbar |G_b|^2 \delta x (\chi_b(\omega) - \chi_b^*(-\omega))$$

$$- \hbar G_b \chi_b(\omega) \sum \sqrt{2\kappa_1} \delta \hat{B}_1 - \hbar G_b \chi_b^*(-\omega) \sum \sqrt{2\kappa_1} \delta \hat{B}_1^\dagger$$

(22)

where $\bar{F}_{BA}$ is the average back-action force, $\delta F_{BA}$ is the dynamic back-action, which influences the dynamics of the harmonically bound mirror, and $\delta \hat{F}_{BA}$ is the quantum back-action force.

From the dynamic back-action, the optical spring effect is

$$K(\omega) = -\frac{\delta F_{BA}}{\delta x} = 2 \hbar |G_a|^2 \frac{\Delta_a}{(\kappa + i\omega)^2 + \Delta^2_a} + 2 \hbar |G_b|^2 \frac{\Delta_b}{(\kappa + i\omega)^2 + \Delta^2_b}$$

$$= \frac{8P_{\text{in1 circ}} \omega_c}{2\kappa} \frac{\Delta_a \cos^2(\beta)}{(\kappa + i\omega)^2 + \Delta^2_a} + \frac{8P_{\text{in2 circ}} \omega_c}{2\kappa} \frac{\Delta_b \cos^2(\beta)}{(\kappa + i\omega)^2 + \Delta^2_b}$$

(23)

The experiment is done under the “slowly varying” condition: $\omega \ll \sqrt{\Delta^2_a + \kappa^2}$, then the spring effect can be written by

$$K = 2 \hbar |G_a|^2 \left[ \frac{\Delta_a}{\kappa^2 + \Delta^2_a} - \frac{2i\kappa \Delta_a}{(\kappa^2 + \Delta^2_a)^2} \omega \right] + 2 \hbar |G_b|^2 \left[ \frac{\Delta_b}{\kappa^2 + \Delta^2_b} - \frac{2i\kappa \Delta_b}{(\kappa^2 + \Delta^2_b)^2} \omega \right]$$

$$\equiv K_{\text{opt}} + i\Gamma_{\text{opt}} \omega$$

(24)

This spring modifies the dynamics of the mirror as

$$\omega^2_{\text{eff}} = \omega^2_m + \frac{K_{\text{opt}}}{m}$$

$$\gamma_{\text{eff}} = \gamma_m + \frac{\Gamma_{\text{opt}}}{2m}$$

(25)

(26)

which indicates the positive (negative) rigidity is always accompanied by a negative (positive) damping. In either case, the system is unstable if we
use the single optical spring. To stabilize the system, one can use a feedback control, however, it is difficult to control if we use the tiny oscillator. An appropriate alternative is to implement the idea of the double optical spring by inputting two lasers to the cavity at different frequencies. One laser with a small detuning provides a large positive damping while another higher input-power beam with a large detuning provides a strong restoring force. The resulting system is self-stabilized with both positive rigidity and positive damping, as shown in Fig 2. In addition, unlike mechanical springs, the optical spring effect does not change the thermal excitation spectrum of the mirror since the optical field is almost in its ground state (in our case, infrared optical field has an effective temperature of 15,000 K). We can measure quantum back-action force fluctuation as displacement fluctuation via the effective susceptibility $\chi_{\text{eff}}$. The double-sided force spectrum $S^{(2)}_{FF,q}$ is written by

$$S^{(2)}_{FF,q} = \langle \delta \hat{F}_{BA}(-\omega) \delta \hat{F}_{BA}(\omega) \rangle$$

$$= 2\hbar^2 \kappa |G_a|^2 |\chi_a(-\omega)|^2 + 2\hbar^2 \kappa |G_b|^2 |\chi_b(-\omega)|^2$$

$$= 2N_{\text{in1,circ}} \frac{\hbar^2 g^2}{\kappa} \left( 1 + \left( \frac{\omega + \Delta_a}{\kappa} \right)^2 \right)^{-1} + 2N_{\text{in2,circ}} \frac{\hbar^2 g^2}{\kappa} \left( 1 + \left( \frac{\omega + \Delta_b}{\kappa} \right)^2 \right)^{-1}$$

Therefore, quantum back-action is given by $|\chi_{\text{eff}}|^2 S^{(2)}_{FF,q}$. In practice, laser has classical intensity fluctuation generating the “classical” back-action force. This effect is given by

$$S^{(2)}_{FF,c} = 2(B_{\text{in1}} - 1) \hbar^2 \kappa_{\text{in1}} |G_a|^2 \left( |\chi_a(\omega)|^2 + |\chi_a(-\omega)|^2 \right)$$

$$+ 2(B_{\text{in2}} - 1) \hbar^2 \kappa_{\text{in2}} |G_b|^2 \left( |\chi_b(\omega)|^2 + |\chi_b(-\omega)|^2 \right)$$

where the $B_{\text{in1}}, B_{\text{in2}}$ are the relative shot noise levels for each beam. In slowly varying regime, the ratio of classical back-action to quantum back-action for each beam is $S^{(2)}_{FF,c,j} / S^{(2)}_{FF,q,j} \propto \kappa_{j} / \kappa$, for $j = \text{in1, in2}$ and enables us to estimate the requirement for the intensity stability of the input beam.
A.3 Main experiment for observing of quantum back-action

A.3.1 Setup

Figure 8: The detailed experimental setup for observing quantum back-action. The driving beam (red line) and spring beam (blue line) were fed into the triangular cavity in the same spatial mode but in different directions. $\beta$ represents the incident angle on the movable mirror. Acousto-optical modulators (AOM) were used to shift the laser frequency. An electro-optical modulator (EOM) was used to apply frequency sidebands for the PDH method. Light was detected at various points using photodetectors (PD). HWP, Half-Wave Plate; QWP, Quarter-Wave Plate; FI, Faraday Isolator.

The detailed experimental setup is shown in Fig. 8. Our optical cavity was a triangular cavity constructed from one movable mirror (mass: 5 mg), one half-inch fixed mirror, and one half-inch suspended mirror with a coil-magnet actuator attached on its aluminum mirror holder (mass: 97 g, radius of curvature: 200 mm). The actuator was used for the cavity length control.
The movable mirror was suspended by the wire from a picomotor actuated stage for yaw alignment. The fixed mirror and the controlled mirror had picomotors for both pitch and yaw alignment. Also, there were two picomotor actuated folding mirrors for aligning each incident beam. These adjustment mechanism allow us to align the optics inside the vacuum chamber remotely.

The shape of the optical path of the cavity was an isosceles triangle and it had round-trip length of $L = 90$ mm. The incident angle to the movable mirror was $\beta = 0.64$ rad. The finesse of the cavity was $1.10 \times 10^3$ and each mirror had the transmittance of $T_{\text{in}1} = 4.8 \times 10^{-3}$, $T_{\text{in}2} \simeq 1 \times 10^{-4}$, and $T_{\text{in}3} < 8 \times 10^{-4}$. The Nd:YAG laser source at operational wavelength 1064 nm was used and the whole input optics were covered by a sound-proof acryl box. The two beams split from the same laser source was injected to the cavity in counter-propagating directions. One of the two beams was injected from the fixed mirror in a clockwise direction (spring beam, illustrated as blue line in Fig. 7 and 8), and the other one was injected from the controlled mirror in a counterclockwise direction (driving beam, illustrated as red line in Fig. 7 and 8). The incident beam power were $P_{\text{in}1} = 5$ mW and $P_{\text{in}2} = 0.1$ W, and the intra-cavity power were $P_{\text{in}1,\text{circ}} = 2.9$ W and $P_{\text{in}2,\text{circ}} = 0.5$ W, respectively.

Since the intra-cavity power of the driving beam was larger, this beam was the major source of the quantum back-action.

The driving beam was frequency shifted by 80 MHz with an AOM and phase modulated in 15 MHz with an EOM. A portion of the driving beam was picked-off and detected by a photodetector with a high quantum efficiency (Perkin Elmer, C30632, InGaAs photodiode). In order to stabilize the intensity of the laser beam sufficiently, we have to take into account of the vacuum fluctuation $\delta \hat{a}_1$ and $\delta \hat{a}_2$ unavoidably injected from the open ports of the BS1 and B2, respectively. Because the vacuum fluctuation $\delta \hat{a}_1$ has the anti-correlation between a in-loop (PD1) and a out-of-loop (PD2), a correlation between the laser intensity fluctuation and the vacuum $\delta \hat{a}_1$ will be generated in the out-of-loop after the stabilization. This results in increasing the noise level in out-of-loop, which is so-called “noise penalty”\[1\]. In addition, a possible minimum relative to shot noise level in out-of-loop is also limited by the uncorrelated vacuum $\delta \hat{a}_2$. More concretely, in our case, required relative shot noise levels of input beams were smaller than 2.0 dB and 26 dB for the driving and spring beams, respectively from Eqs. (27) and (28). Thus, the power injected to the photodetector (PD1; in-loop) used for the intensity stabilization was two times larger than the power injected to the photodetector (PD2; out-of-loop) for monitoring the intensity fluctuation. This power

\[1\]
balance allows intensity stabilization of the beam at relative shot noise level of 1.7 dB. Besides, there were picomotors actuated mirrors before these two PDs to adjust the position of the beam spot on the detectors to find the spot position where the effect of the beam jitter is minimized. Moreover, incident angle to these two PDs were adjusted to the Brewster angle in order to minimize the effect of the back scattering.

The sufficiently stabilized beam was injected into the cavity, and the cavity reflected beam was detected with fast responding PD (PD3; HAMA-MATSU, G10899-01K, InGaAs photodiode). The output of this PD was demodulated with 15 MHz RF signal to obtain cavity length signal. We used this signal for the cavity length control and also to extract quantum back-action signal. The one of the transmitted beams from the cavity was monitored by CCD or PD4 (flipped by a flipped mirror), and the other one was eventually rejected at the Faraday isolator.

The frequency of the spring beam was shifted by 82 MHz with an AOM in the double pass configuration, before injecting into the cavity. The cavity reflected beam was monitored with PD5. One of the transmitted beams was monitored by PD6 and the other one was monitored by PD7 and CCD.
A.3.2 Optical Characterization

Figure 9: Cavity scan. The optical characterization of our devices was done by sweeping the laser frequency across the optical resonance while detecting the transmitted light in a photo-detector. The blue circles are measured values and the horizontal blue lines are the statistical errors, and the red line is the fitting line. From this measurement, the total decay rate, $\kappa/2\pi = (1.52 \pm 0.03) \times 10^6$ Hz (i.e. finesse, $F = (1.10 \pm 0.03) \times 10^3$), was estimated.

From Eq. (27), optical parameters $\kappa, \kappa_{\text{in}1}$, and $\kappa_{\text{in}2}$ are necessary for estimating the power spectrum of quantum back-action. Here, we will show the estimation of these parameters.

The experimental setup for the estimation is shown in Fig. 8. We measured the transmittance of the triangular cavity using the spring beam (blue line in Fig. 8), which was not used for the control of the cavity length. The shifted frequency of spring beam was changed within the range from 77 MHz to 83 MHz by AOM, while the driving (control) beam was shifted at 80 MHz, thus the transmittance of the cavity for the broadband frequency could be measured using PD6 and PD7. Figure 9 shows the result. From this measurement, the total cavity decay rate $\kappa$ and the combination of $\kappa_{\text{in}2}(\kappa_{\text{in}1} + \kappa_{\text{in}3} + \kappa_{\text{in}4})$ could be estimated. In addition to this information, the ratio of the decay rate of the fixed mirror to of the controlled one (indexed in1 and in2, respectively), which was measured from the ratio of tuned-transmittance of driving beam to one of spring beam, was used to separate them. As a result, $\kappa/2\pi = (1.52 \pm 0.03) \times 10^6$ Hz (i.e. finesse,
\( \mathcal{F} = (1.10 \pm 0.03) \times 10^3 \), \( \kappa_{in1}/2\pi = 1.3 \times 10^6 \) Hz, and \( \kappa_{in2}/2\pi \simeq 3.2 \times 10^4 \) Hz were estimated.
A.3.3 Mechanical Characterization

Figure 10: **Ringdown measurement.** Measured damped oscillation for pendulum and yawing motion are shown, respectively. The blue solid curve is the measured value and the red line is the fitting curve.  

- **a**, Decay of the pendulum motion. The mechanical Q-value of the pendulum, $Q_{\text{pend}} = 4.7 \times 10^5$, was estimated.  
- **b**, Decay of the rotational motion. The mechanical Q-value of the yaw, $Q_{\text{yaw}} = 3.8 \times 10^3$, was estimated.

Here we will present details of the Q-value measurement of the mechanical oscillator. Mechanical Q-value can be written as $Q_{m} = \frac{\omega_{m}}{2\gamma_{m}}$, using $\omega_{m}$ and $\gamma_{m}$ in Eq. (8). Also, from the fluctuation-dissipation theorem, pendulum thermal noise can be written as

$$S_{\text{FF,th}}^{(2)} = 4k_{B}T\gamma_{m}m$$  

(29)

Thus, by measuring the Q-value of the pendulum, thermal noise level can be estimated. Here, $T$ is the temperature of the pendulum and $k_{B}$ is the Boltzmann constant. From Eq. (29), it follows that the thermal noise level is proportional to the pendulum loss $\gamma_{m}$, and so the displacement sensitivity can be improved by trapping the pendulum with gravitational potential and diluting the loss. Our oscillator was also trapped by the optical fields. This stabilizes the system without changing the thermal noise level since the force fluctuation caused by the thermal fluctuation does not change with optical springs. Our oscillator is effectively cooled down to

$$T_{\text{eff}} = T\frac{\omega_{m}Q_{\text{eff}}}{\omega_{\text{eff}}Q_{m}}$$  

(30)
It is worth pointing out that this effective cooling reduces the thermal noise at the resonant frequency, but it does not change the signal-to-noise ratio (SNR) with respect to the quantum back-action. This is because the reduction of the thermal noise at the resonant frequency is caused by the change in the susceptibility, not the reduction in the force fluctuation. The change in the susceptibility also reduces the quantum back-action and therefore SNR stays the same.

In our experiment, we used a thin wire to suspend a mirror to dilute the mechanical loss by the gravitational potential and increase the pendulum Q-value. Generally, a thin wire has a low material Q-value because the loss of a material comes mainly from the surface loss. Ultra thin wires have been used for discharging test masses for inertia sensors. There are some literatures reporting the measurement of the Q-value for golden thin wires and one example reports the Q-value of 10 $\mu$m diameter golden wire at $Q=270$.\cite{liu2007}

However, a Q-value of the ultra thin tungsten wire, as far as we know, has not been reported yet.

Our experimental setup for the Q-value measurement was almost the same as the one shown in Fig. 8 except for the fixed mirror was removed. The incident beam was aligned such that the beam hits the movable mirror at its edge and the mirror blocks the portion of the beam before the beam was detected by PD4. The amplitude of the resonant motion can be obtained by demodulating the output of PD4 with the resonant frequencies of the pendulum mode and the torsional mode because a small oscillation of the mirror creates amplitude modulation of the beam. The Q-values were measured by exciting the mirror motion and measuring the decay time of each mode.

Our pendulum had a very high Q-value since the loss was diluted by gravitational potential by a factor of 1000. Therefore, the pendulum mode had a very long decay time and the measurement of the Q-value without excitations was difficult. In order to prevent the overestimation of the Q-value (i.e. thermal noise), we also measured Q-value of the yaw mode, which had less decay time, to have estimated the maximum Q-value of the pendulum. In addition to the less decay time, the yaw mode has tolerance to the mechanical loss of the clamping mechanism such as the epoxy due to its mode function, which represents the mechanical displacement patterns associated with mechanical motion. Therefore we can estimate the Q-value of the yaw mode as the intrinsic mechanical Q-value of the wire. As a result, the measured pendulum Q-value should be smaller than the maximum Q-value of the pendulum estimated from $Q_{m, \text{max}} = Q_{t, m} \times 1.0 \times 10^3$.\cite{liu2007}
The result of the ring down measurement is shown in Fig. 10. The measured Q-value for the pendulum mode was $Q_m = 4.7 \times 10^5$ (the resonant frequency was $\omega/2\pi = 2.2$ Hz). There was an amplitude modulation at a period of approximately 30 min, which might be from the temperature change of the vacuum chamber. However, the error caused by this amplitude modulation was negligible. The measured Q-value for the torsion mode was $Q_{t,m} = 3.8 \times 10^3$ (the resonant frequency was $\omega_{yaw}/2\pi = 0.23$Hz). This insists the upper limit of the Q-value for the pendulum mode to be $Q_{m,max} = 3.8 \times 10^6$, which is consistent with the Q-value from the direct measurement above. From the $Q_m = 4.7 \times 10^5$, we estimated the suspension thermal noise level as in Fig. 2 (red).
A.3.4 Optomechanical characterization

Here we will present the characterization of the optical spring of our optomechanical system. We measured the effect of the restoring force from the optical spring to estimate the optomechanical coupling constant $g$ of the cavity using Eq. (24). Together with the measured value of $\kappa, \kappa_{\text{in1}}$, and $\kappa_{\text{in2}}$ from the optical characterization, the amplitude of the quantum back-action can be estimated.

The optical restoring force was measured using the same setup as in Fig. 8, at higher pressure ($1 \times 10^5$ Pa) than the main quantum back-action measurement. The driving beam (shown as the red line in Fig. 8) at 2 mW was injected to the cavity and the cavity length was controlled using the same light (PD3). Under these conditions, the decay rate from the gas damping was $\gamma_{\text{gas}}/2\pi \simeq +1$ Hz and the minimum decay rate from the optical spring was $\Gamma_{\text{opt, min}}/2m \cdot 2\pi \simeq -0.03$ Hz. Thus, the cavity stay sufficiently stable without using the double optical spring technique. After closing the cavity length control loop, a small electrical signal was injected into the loop to
measure the resonant frequency of the movable mirror.

Our optomechanical system can be modeled by the following coupled oscillator without the damping term:

\[
\begin{align*}
M \ddot{x}_c &= -(k_c + k_{\text{opt}}) x_c + k_{\text{opt}} x_m + F_c \\
m \ddot{x}_m &= k_{\text{opt}} (x_c - x_m) - k_m x_m
\end{align*}
\]  

(31)  

(32)

Here, \( k_c \) is the mechanical spring constant of the controlled mirror, \( F_c \) is the feedback force acting on the controlled mirror, \( k_m \) is the mechanical spring constant of the movable mirror, \( x_c \) and \( x_m \) are the displacements of the controlled mirror and the movable mirror, and \( M \) and \( m \) are the masses of the controlled mirror and the movable mirror, respectively.

In our setup, the controlled mirror was heavier (\( M = 97 \text{ g} \)) and had higher mechanical spring constant (\( k_c \approx 32 \text{ N/m} \)). The movable mirror was lighter (\( m = 5 \text{ mg} \)) and had lower mechanical spring constant (\( k_m \approx 1 \times 10^{-3} \text{ N/m} \)). The optical spring connected the controlled mirror and the movable mirror with the spring in between (\( k_{\text{opt}} \approx 10 \text{ N/m} \)). Thus, the cavity length change caused by the force on the controlled mirror can be written as

\[
\frac{x_c(\omega) - x_m(\omega)}{F_c(\omega)} \approx \frac{1}{M \omega_{\text{eff}}^2} \frac{1}{\omega^2}
\]

(33)

in the frequency domain, where \( \omega_{\text{eff}} \) is the effective resonant frequency of the movable mirror. This means that the resonant frequency of the movable mirror can be measured by exciting the controlled mirror and measuring the cavity length change.

Figure 11 shows the change of the resonant frequency of the movable mirror under different detuning of the cavity. The measured dependence of the resonant frequency to the cavity detuning was fitted by least squares and the estimated value \( g/2\pi = (2.8 \pm 0.1) \omega_c \text{ Hz/m} \) was obtained.
A.3.5 Calibration

Figure 12: **Measurement of the efficiency of the coil-magnet actuator.** a, Experimental setup. The laser beam was fed into the Michelson interferometer (MI). Light was detected using a photodetector (PD), and the MI was locked at the mid-fringe point. b, Measured open loop transfer functions of displacement control are shown as blue points, while the red solid lines are the fitting curves. From this measurement, the actuation efficiency, \((1.4 \pm 0.1) \times 10^{-4} \text{ N/V}\), was estimated.

In order to calibrate the displacement noise spectrum from the voltage signal, we firstly measured the actuator efficiency and determined the voltage-to-force conversion factor. Here we will present the estimation of the factor using Michelson interferometer.

Fig. 12 shows the configuration to measure the actuation efficiencies. We locked the Michelson interferometer using a PD, an appropriate servo circuit and the same coil-magnet actuator as the main measurement for observing quantum back-action. From this measurement, we could estimate the voltage-to-force conversion factor of the actuator because the others composing the loop such as the response of MI, the pendulum, PD, and servo filter had been measured by other experiments. As a result, we experimentally determined the actuation efficiency \((1.4 \pm 0.1) \times 10^{-4} \text{ N/V}\).

Secondly, we determined the force-to-voltage conversion factor from Eq. (33). As a result, the voltage-to-displacement conversion factor at \(\omega \ll \omega_m\) was estimated \((8.5 \pm 0.6) \times 10^{-11} \text{ m/V}\).
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