Premium auctions in the field

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Abstract

In a field experiment, we study the revenue-generating properties of premium auctions. In a premium auction, the runner-up obtains a premium for driving up the price paid by the winner. Previous research, both theoretical and in the lab, has shown that the relative performance of premium auctions compared to standard auction formats is context-specific. In the experiment, we compare two types of premium auctions with the standard Vickrey auction selling high-quality, limited-edition posters in an online auction. We observe that neither premium auction raises higher revenue than the Vickrey auction. Revenue dispersion in the Amsterdam auction, one of the premium auctions, is lower than that in the other two auctions.

Keywords Premium auctions · Vickrey auction · Field experiment

JEL Classification C93 · D44

1 Introduction

In his book *Auctions and Auctioneering*, marketing scholar Cassady (1967) describes a large variety of auction formats that he encountered during travels through over twenty countries across the globe. In the Netherlands, he observes the use of ‘premium auctions’, i.e., auctions where the runner-up (the highest losing bidder) obtains a premium, “called in Holland *plok or plokgeld*”, for driving up the price paid by the winner (p. 77). Van Bochove et al. (2016) report that various versions of premium...
auctions have been used since at least 1529 to sell timber, wine, spices, tea, coffee, books, art, tulips, financial securities, and real estate. Premium auctions are believed to be able to outperform standard auctions like the first-price sealed-bid auction, the Dutch auction, the English auction, and the Vickrey auction in some circumstances. The reason is that bidders compete fiercely for the premium, driving up the price paid by the winner. The additional competition may then compensate for the premium the seller has to pay to the runner-up.

In this paper, we explore the potential of premium auctions as online selling mechanisms. We do so in an online field experiment in which we compare the revenue-generating properties of two kinds of premium auctions with the Vickrey auction (i.e., the second-price sealed-bid auction). The two premium auctions that we study are variations of the Vickrey auction and only differ in that some bidders obtain a premium. In the (sealed-bid) Amsterdam auction, both the winner and the runner-up get a premium equal to 50% of the difference between the second-highest and third-highest bid. In the Fischer auction, the runner-up receives a premium equal to 5% of the price paid by the winner (i.e., the runner-up’s bid). As far as we are aware, we are the first to study the theoretical and experimental properties of the Fischer auction.

In our experiment, we sell three identical copies of a high-quality, limited-edition print using these three auction formats. We invited almost 10,000 members of a general-population panel to participate in an auction. The roughly 950 panel members that chose to enter were randomized over the three auction formats, resulting in over 300 participants per treatment. During the auction, the participants obtained no information on the number of other bidders or others’ bids. As a result, bids are arguably independent, so that we can still analyze the data in a statistically meaningful way despite the fact that we ran only one auction per treatment. More precisely, we use two methods to estimate the auctions’ revenue’s mean and variance for settings where the number of bidders equals 5–400. We use Mullin and Reiley’s (2006) recombinant estimation for low bidder numbers (up to 50) and estimates of the cumulative distributions of the bids for higher bidder numbers (between 50 and 400).

Participants also answered survey questions on their background demographics (age, gender, education, and marital status), their risk attitude (à la Dohmen et al. 2011), and their bidding strategy. Our data show that the premium auctions do not outperform the Vickrey auction in terms of average revenue. We also find that the Amsterdam auction’s revenue dispersion is lower than in the other two auctions.

Our paper adds to the experimental literature studying the relative performance of auction formats in the field. Lucking-Reiley (1999) tests revenue equivalence between first-price sealed-bid and Dutch auctions, and between English and Vickrey auctions. Häubl and Popkowski Leszczyc (2003), Reiley (2006), Katkar and Reiley (2006), Brown and Morgan (2009), Haruvy and Popkowski Leszczyc (2010) and Ostrovsky and Schwarz (2011) study the effect of reserve prices on auction revenue. Houser and Wooders (2005) and Brown and Morgan (2009) examine how auction ending rules affect auction revenue. Popkowski Leszczyc and Häubl (2010) study the profitability

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1 See Chen and Konstan (2015) for a discussion of design choices for online field experiments.
2 The Fischer auction is modeled after the auction format carrying the same name used by art dealer Simonis and Buunk. The art dealer is located in the Dutch town of Ede and chose the auction’s name to honor the Fischer family that delivered several generations of notaries in the 19th and 20th century in Ede.
of bundle auction relative to separate-component auctions. Carpenter et al. (2008) and Haruvy and Popkowski Leszczyc (2018) compare various auction formats in terms of money raised for charity.

As far as we are aware, we are the first to examine the relative revenue-generating properties of premium auctions in a field experiment. Earlier studies confirmed the intuition that premium auctions might raise more revenue than standard auctions. Milgrom (2004) shows theoretically that a premium auction may attract more entry by ‘weak’ bidders than the English auction and consequently may generate more revenue. Goeree and Offerman (2004) study ascending versions of the Amsterdam auction. They find theoretically and in the lab that two variants of the Amsterdam auction raise higher average revenue than the first-price sealed-bid auction and the English auction in a setting with strong ex ante bidder asymmetries. Hu et al. (2011a) observe in a laboratory experiment that the Amsterdam auction is less conducive to cartel formation and raises more money than the English auction and the first-price sealed-bid auction in the case of strong ex ante bidder asymmetries.

However, the received literature reports mixed results for ex ante symmetric settings. According to the celebrated revenue-equivalence theorem (Myerson 1981), expected revenue is the same in premium auctions as in standard auctions in the case of risk-neutral bidders and separating equilibria. Experimental evidence in symmetric settings or settings with weak asymmetries is in line with this finding (Goeree and Offerman 2004; Hu et al. 2011a). In theoretical work, Hu et al. (2011b) find that the Amsterdam auction generates higher [lower] revenue than the English auction in the case of risk-seeking [risk-averse] bidders. Brunner et al.’s (2014) confirm this prediction in a laboratory experiment, although bids in the Amsterdam auction are less aggressive than predicted by theory. Sufficiently risk averse sellers might still prefer the Amsterdam auction over the English auction in that the lower revenue in the Amsterdam auction is compensated by a lower revenue dispersion (Hu et al. 2018).

The setting in our field experiment is arguably a symmetric one in that anonymity assures that, from the viewpoint of the bidders, no subset of competing bidders could be identified as ‘strong’ or ‘weak.’ As a consequence, the results from our online field experiment confirm the external validity of the above results for ex ante symmetric settings. We conclude that in settings that lack strong ex ante bidder asymmetries, premium auctions are not particularly attractive for sellers. The Amsterdam auction might be an exception in that it might be an interesting format for strongly risk-averse sellers because of its relatively low revenue dispersion.

The structure of this paper is as follows. In Sect. 2, we derive the theoretical properties of the sealed-bid Amsterdam auction and the Fischer auction and compare those with the well-known properties of the Vickrey auction. In Sect. 3, we discuss our experimental design and hypotheses. Our data analysis is in Sect. 4. Section 5 contains a short conclusion. Lengthy proofs of lemmas and propositions are relegated to Appendix A.
2 Theory

In this section, we develop a theory of bidding in premium auctions. We analyze bidding in the independent private values framework. One indivisible item is sold in an auction to one out of \( n \geq 3 \) risk-neutral bidders, labeled \( i = 1, \ldots, n \). Let \( v_i \) denote bidder \( i \)'s value for the item, \( i = 1, \ldots, n \). All values are drawn independently from the interval \([v, \bar{v}]\), \( 0 \leq v < \bar{v} \), according to the same differentiable distribution function \( F \). We assume that \( F \) has no mass points and that it is strictly increasing over \([v, \bar{v}]\). Let \( f \equiv F' \) denote the corresponding density function.

The item is awarded through one of the following three auction formats:

- **The Vickrey auction** Bidders independently submit a bid. The highest bidder wins the item and pays the second-highest bid.
- **The (sealed-bid) Amsterdam auction** Bidders independently submit a bid. The highest bidder wins the item and pays the second-highest bid. Both the winner and the runner-up receive a premium equal to a fraction \( \alpha \in (0, 1/2] \) of the difference between the second-highest and third-highest bid.
- **The Fischer auction** Bidders independently submit a bid. The highest bidder wins the item and pays the second-highest bid. The runner-up receives a premium equal to a fraction \( \phi \in (0, 1) \) of the second-highest bid.

The following properties of the Vickrey auction are well-known (see, e.g., Vickrey 1961).

**Lemma 1** The Vickrey auction has a unique equilibrium in weakly dominant strategies in which all bidders bid at value. In this equilibrium, the item is always awarded to the bidder with the highest value. A bidder with the lowest possible value \( v \) obtains zero expected utility.

A dynamic version of the Amsterdam auction has been studied extensively (see Goeree and Offerman 2004; Hu et al. 2011a, b, 2018; Brunner et al. 2014). The dynamic version has the same rules as the English auction with the additional feature that both the winner and the runner-up obtain a premium proportional to the difference between the second-highest and third-highest bid. In the independent private values framework, equilibrium bids do not depend on the auction history (i.e., the prices at which other bidders stepped out of the auction). Moreover, bidding below value is weakly dominated because by doing so, a bidder may lose the auction at a price below her value and she may reduce the premium that she may receive (Goeree and Offerman 2004). It is straightforward to establish the following lemma on the basis of the unique perfect Bayesian equilibrium of the dynamic version of the Amsterdam auction.

**Lemma 2** In the Amsterdam auction, bidding below value is weakly dominated. The Amsterdam auction has a unique symmetric equilibrium which is given by

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3 The independent-private-value assumption is legitimate under the condition that the poster has little resale value. We will come back to this assumption in the discussion following Result 6.

4 For all auctions, in the case of a tie at the first, second, or third highest bid, the bid order is randomized, and the outcomes are realized as if the order was strict.
In this equilibrium, the item is always awarded to the bidder with the highest value. A bidder with the lowest possible value \( v \) obtains zero expected utility.

Notice that equilibrium bids for the Vickrey auction and the Amsterdam auction are independent of the number of bidders. This result also holds true for risk-averse bidders (Hu et al. 2011b, 2018).

The following result follows immediately from Myerson’s (1981) revenue-equivalence theorem because in both the Vickrey auction and the Amsterdam auction, (1) the item is awarded to the bidder with the highest value, and (2) a bidder that has the lowest possible value obtains zero expected utility.

**Proposition 1** The Vickrey auction and Amsterdam auction yield the same expected revenue.

The following result follows immediately from Hu et al.’s (2018) study of the dynamic version of the Amsterdam auction.

**Proposition 2** The Vickrey auction’s revenue is a mean-preserving spread of that of the Amsterdam auction.

This result implies that the variance of the revenue in the Vickrey auction is higher than that in the Amsterdam auction. The intuition is as follows. First of all, bids in the Vickrey auction are more spread out than in the Amsterdam auction. In the Vickrey auction, they span over the entire value range \([v, \bar{v}]\) because all bidders bid their value, while in the Amsterdam auction, the range is \([v + \int_v^\bar{v} \left( \frac{1 - F(x)}{1 - F(v)} \right)^{1/\alpha} dx, \bar{v}]\). Second, in the Vickrey auction, revenue equals the second-highest bid while in the Amsterdam auction, it is equal to the weighted average of the third-highest and second-highest bid, which yields a lower variance even if the bids were drawn from the same distribution.

Finally, we find that expected revenue in the Fischer auction is lower than in the other two auctions, as the results below show.

**Lemma 3** The Fischer auction does not have an efficient equilibrium.

**Proposition 3** If \( F \) is log-concave, the expected revenue is strictly lower in the Fischer auction than in both the Vickrey auction and the Amsterdam auction.

A rough intuition behind Lemma 3 is the following. Suppose all other bidders bid according to the same strictly increasing bidding function, which guarantees an efficient outcome. However, a bidder having the lowest-possible value \( (v) \) has an incentive to deviate and submit a slightly higher bid, i.e., if her value was \( v + \epsilon \) rather than \( v \), where \( \epsilon > 0 \) is small. The reason is that by doing so, her probability of being second (and winning the premium) is \((n - 1)F(v + \epsilon)^{n-2} \left( 1 - F(v + \epsilon) \right)\) which
outweighs her probability of being first $F\left(v + \epsilon\right)^{n-1}$ (and potentially paying more than her value) for sufficiently small $\epsilon$. This contradicts the assumption that the bidding function constitutes an equilibrium. An alternative intuition is that bidders may have an incentive to pool with bidders having similar values. The reason is that the bidders forgo winning the premium by outbidding this set of bidders while potentially buying the object at a price above their value. Indeed, examples A.1 and A.2 in Appendix A show that for extreme sets of parameters, the Fischer auction has an equilibrium where all bidders submit the same bid.

From Myerson’s (1981) analysis of optimal auctions, it follows that in the case of a log-concave value distribution function, the revenue-maximizing auction conditional on the item always being sold is an efficient auction. So, Lemma 3 implies that the Fischer auction raises less revenue than the other two auctions because the latter two are efficient. Unfortunately, equilibria are typically not readily found for the Fischer auction so that it is often hard to predict how much the other two auction formats outperform the Fischer auction.

### 3 Experimental design and hypotheses

#### 3.1 Experimental design

The experiment was conducted in cooperation with Veylinx, a Dutch online experimental auction platform that is mainly used for marketing research. We compare three treatments in a between-subjects design: the Vickrey auction, the Amsterdam auction, and the Fischer auction. The rules of the auctions are described in the theory section above. For the Amsterdam auction, we set $\alpha = 1/2$, i.e., both the winner and the runner-up received a 50% share of the difference between the second-highest and the third-highest bid. $^5$ For the Fischer auction, we set $\phi = 1/20$, i.e., the runner-up received 5% of the own bid. $^6$ Table 1 summarizes the experimental design, including the number of bidders per treatment.

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$^5$ Brunner et al. (2014) uses the same parameter value in their laboratory experiment. This parameter yields the greatest difference in equilibrium bidding between the Vickrey auction and the Amsterdam auction cf. Lemma 2.

$^6$ This is in line with the premium auction used by art dealer Simonis and Buunk (see also footnote 2).
The experimental procedures are as follows. On May 5, 2016, we sent out an e-mail to 9763 members of a general-population panel inviting them to “participate in an auction.” In the e-mail, we did not specify either the item that was up for auction or the auction type that bidders would participate in. A potential downside of this design choice is that it does not allow us to study how the auctions differ in terms of entry. Indeed, Milgrom (2004) forcefully made the point that entry into a premium auction is relatively attractive for weak bidders that have little chance of winning in a standard auction. However, in symmetric settings, like ours, it is well known that the threshold type that is indifferent between entering and not entering is typically independent of the auction type (see, e.g., Menezes and Monteiro 2000). Of course, the auctions may differ in terms of entry empirically. Instead of studying actual entry decisions, we compare the auctions on the basis of the number of strictly positive bids. Panel members could enter the auction once by clicking on a button in the e-mail. The auction was open until noon, May 11, 2016. The 964 panel members (9.87%) that chose to enter were randomized over the three auction formats.

Table 2 presents summary statistics regarding the participants’ background characteristics. In each auction, we sold one copy of a framed, high-quality, limited-edition print that was available at the Amsterdam-based photo gallery YellowKorner at a retail price of €590. The bidders only competed against the subgroup of bidders that was assigned to the same auction format. We did not inform the bidders about how many others would participate in the same auction for the simple reason that this information was only available after the auctions closed. The bidders were also not shown any information about others’ bidding behavior. The resulting bids are arguably independent so that the auction formats are of the sealed-bid type. In Sect. 4.2, we elaborate how we exploit bid independence in our statistical analysis.

Upon arrival at Veylinx’ online platform, the participants browsed through several pages. The first page informed the participants that (1) they could submit only one bid, (2) they could not withdraw their bid, (3) the bid included VAT and shipment, (4) the winner paid less than their own bid, (5) the winner must pay within 48 h, (6) the auction closed at noon, May 11, 2016, and (7) the object would be shipped to the winner within 7 days after payment. On the next page, the participants were given information about the item being auctioned along with the rules of the auction. At the bottom of this page, the participants were given 6 min to enter their bid. After entering a bid, participants were asked to fill out a multiple-choice survey. The survey contained questions about (1) the participant’s education, (2) their risk attitude (à la Dohmen et al. 2011), (3) their bidding strategy, and (4) their marital status. Appendix B contains a translation of the instructions (including a picture of the print).

### 3.2 Hypotheses

We aim at testing the following hypotheses using the data from our experiment.

**Hypothesis 1** The Vickrey auction and the Amsterdam auction raise the same average revenue.
### Table 2 Summary statistics

|                        | Vickrey          | Amsterdam        | Fischer          | $p$-values $F/\chi^2$ tests on joint significance |
|------------------------|------------------|------------------|------------------|--------------------------------------------------|
|                        |                  |                  |                  |                                                  |
| **Demographics**       |                  |                  |                  |                                                  |
| Female (dummy)         | 0.514 (0.501)    | 0.489 (0.501)    | 0.506 (0.501)    | 0.8083                                           |
| Age                    | 40.9 (13.7)      | 40.6 (13.9)      | 41.1 (14.6)      | 0.8927                                           |
| Married (dummy)        | 0.397 (0.489)    | 0.403 (0.491)    | 0.421 (0.494)    | 0.7813                                           |
| University degree      | 0.662 (0.474)    | 0.636 (0.482)    | 0.664 (0.473)    | 0.7132                                           |
| Risk aversion          | 3.73 (1.27)      | 3.72 (1.26)      | 3.70 (1.32)      | 0.9502                                           |
| **Observed bids and (net) revenue** |                   |                  |                  |                                                  |
| Bid                    | €14.20 (26.52)   | €15.21 (28.80)   | €13.04 (25.87)   | 0.9066                                           |
| Highest bid            | €200.00          | €222.22          | €200.00          |                                                  |
| Second-highest bid     | €160.00          | €150.00          | €150.00          |                                                  |
| Third-highest bid      | €153.00          | €125.00          | €150.00          |                                                  |
| Strictly positive bid  | 0.539 (0.499)    | 0.521 (0.500)    | 0.529 (0.499)    | 0.8031                                           |
| (dummy)                |                  |                  |                  |                                                  |
| Bid conditional on strictly positive bid | €26.32 (31.41)  | €29.18 (34.43)   | €24.63 (31.32)   | 0.3107                                           |
| (Net) Revenue          | €160.00          | €125.00          | €142.50          |                                                  |
| **Survey responses on bids** |                   |                  |                  |                                                  |
| Bid at value (dummy)   | 0.552 (0.498)    | 0.538 (0.499)    | 0.550 (0.498)    |                                                  |
| Bid more than value    | 0.098 (0.298)    | 0.105 (0.307)    | 0.073 (0.261)    | 0.657                                            |
| (dummy)                |                  |                  |                  |                                                  |
| Bid less than value    | 0.350 (0.478)    | 0.357 (0.480)    | 0.377 (0.485)    |                                                  |
| (dummy)                |                  |                  |                  |                                                  |
| Observations           | 317              | 305              | 342              | 964                                              |

Statistics shown are averages, with SD between brackets, with the exception of ‘highest bid,’ ‘second-highest bid,’ ‘third-highest bid,’ and ‘(net) revenue,’ which are point values. The final column presents $p$-values of $F/\chi^2$ tests on jointly significant effects of the treatments on the outcome variable. For Bid the test is based on a tobit regression. For Age and Risk the tests are based on OLS regressions. For the remaining, dummy, variables the tests are based on logit regressions. A $\chi^2$ test is used to compare the distribution of the survey responses on bids. The statistics for Married and Risk aversion bid are based on 955 observations (315, 302, and 338 for the Vickrey, Amsterdam, and Fischer auctions, respectively) because nine participants failed to answer the corresponding questions in the post-auction questionnaire.

Hypothesis 1 is based on Proposition 1 that, in turn, relies on the assumption that all bidders are risk-neutral. In the case of risk-averse bidders, the expected revenue is lower in the Amsterdam auction than in the English auction (Hu et al. 2018), which is strategically equivalent to a Vickrey auction in a private-values setting. Brunner et al. (2014) find support for that prediction using lab data.

**Hypothesis 2** The average revenue of the Fischer auction is lower than that of the Vickrey auction.

**Hypothesis 3** The average revenue of the Fischer auction is lower than that of the Amsterdam auction.
Hypotheses 2 and 3 mirror Proposition 3.

**Hypothesis 4** The variance of the revenue in the Vickrey auction is greater than that of the Amsterdam auction.

Hypothesis 4 is implied by the finding in Proposition 2 that the Vickrey auction’s revenue is a mean-preserving spread of that of the Amsterdam auction. For the Fischer auction, we do not have clear hypotheses regarding its revenue variance. Example A.1 in Appendix A suggests that the variance may be as low as zero in equilibrium because all bidders submit the same bid. At the same time, it is hard to find equilibria for more general settings than the one presented in the example because the Fischer auction does not have an efficient equilibrium according to Lemma 3.

### 4 Experimental results

This section contains our experimental results. In Sect. 4.1, we present summary statistics and initial results on the basis of our raw data. In Sect. 4.2, we compare the estimates for the auctions’ revenue. In Sect. 4.3, we look in more detail at bidding behavior at the individual level.

#### 4.1 Summary statistics

Table 2 contains summary statistics of the experiment. First of all, it demonstrates that observable bidder characteristics like gender, age, marital status, education, and risk attitude were balanced across treatments. Table 2 also gives a first impression of the revenue-generating properties of the auctions. The realized revenue, net of the premium, is higher for the Vickrey auction (€160) than for both the Amsterdam auction (€125) and the Fischer auction (€142.50). The average submitted bid was the highest in the Amsterdam auction and the lowest in the Fischer auction, although the differences between the bids in the auctions are not statistically significant. This observation suggests that the Fischer auction performs relatively poorly compared to the Vickrey auction because for the Fischer auction revenue has to be discounted with the 5% premium.

The auctions also do not differ significantly in terms of the fraction of bidders submitting a strictly positive bid. This is remarkable in the sense of the attractive property of premium auctions in that even bidders without a genuine interest in the object have a reason to submit a strictly positive bid to compete for the premium. However, we do not observe more strictly positive bids in the premium auctions than in the Vickrey auction.

**Result 1** The auctions do not differ significantly in terms of the fraction of strictly positive bids.

Table 2 also gives an impression of the bidding strategies used. In all three auctions, the majority of the bidders report having bid at value, a substantial fraction of the bidders report below-value bidding, and only a small minority were shown to have bid
above value. The auctions do not differ significantly in these respects. While bidding value is consistent with equilibrium for the Vickrey auction, in the Amsterdam auction, all bidders bid more than the value in equilibrium in a symmetric setting (Hu et al. 2018), which is clearly not what we observe. At the same time, the bidding behavior in the Amsterdam auction is qualitatively in line with the observed behavior in Brunner et al.’s (2014) laboratory experiment in that they also observe many bids close to value as well as weakly-dominated bids below value. For the Fischer auction, we do not have a clear benchmark to compare bidding behavior with. Both bids above and below value might be compatible with equilibrium bidding as Examples A.1 and A.2 in Appendix A show.

**Result 2** The auctions do not differ significantly in the number of bidders reported as bidding below or above value.

Remarkably, the auctions differ substantially in terms of the bids submitted by those who overbid, as Table 3 shows. While in the Vickrey auction, those who overbid bid only €0.14 higher than those who did not (p = 0.977, t test). For the Amsterdam and Fischer auctions, those numbers are €9.10 (p = 0.091) and €26.09 (p < 0.001), respectively. Among the 31 (36) [31] bidders in the Vickrey (Amsterdam) [Fischer] auction who bid at least €50, 3 (8) [9] reported having bid more than their value. These observations suggest that overbidding in the premium auctions is partly driven by some bidders competing for the premium.

**Result 3** In the Vickrey auction, bidders who reported bidding more than their value, did not submit significantly different bids than other bidders. In the Amsterdam [Fischer] auction, those who reported having overbid submitted marginally higher [significantly higher] bids that those who did not.

### 4.2 Revenue

In this section, we present estimates for the first two moments of the auctions’ net revenues, i.e., revenues net of the premium paid by the seller. Our empirical analysis relies on the joint assumption that (1) submitted bids are independent of each other, and (2) bidders are not aware of the number of competing bidders. The first assumption is arguably a reasonable one in our setting because our participants are drawn from a large general-population panel and because they do not interact with each other on the auction platform before submitting a bid. To make sure that the second assumption is satisfied, we did not inform the bidders about how many others would participate (in
fact, we did not know this ourselves). Our theoretical analysis shows that the second
assumption may be harmless for the Vickrey and the Amsterdam auctions because
equilibrium bids do not depend on the number of competing bidders.

We use two methods to estimate the auctions’ revenue’s mean and variance for
settings where the number of bidders equals 5–400. For low bidder numbers (up to
50), we make use of Mullin and Reiley’s (2006) recombinant estimation. For higher
bidder numbers (between 50 and 400), we use estimates of the cumulative distribu-
tions of the bids. Table 4 contains the resulting estimates for the auction revenue and
its standard deviation for the three auctions based on recombinant estimation. Mullin
and Reiley’s (2006) recombinant estimation is based on averages over a large range
of group outcomes that have resulted from randomly drawn combinations from sub-
samples of the submitted bids. For our experiment, recombinant estimation is not
very useful for pools of bidders containing more than 50 bidders: the randomly drawn
group observations are too highly correlated because they are drawn from only about
300 observations per auction.

For 50 or more bidders, we rely on the following two-step procedure to estimate
the auctions’ revenue’s mean and variance. First, for each auction, we estimate the
probability distribution the bids are drawn from. More specifically, we assume that
bids for all auctions are drawn from a Weibull distribution, i.e., a distribution with cdf
\[ G(b) = \max\left\{0, 1 - e^{-\left(b/\lambda\right)^k}\right\} \]
with \( \lambda, k > 0, b \in \mathbb{R}^+ \). The parameter estimates for \( \lambda \) and \( k \) for each auction are based on OLS regressions using the 20% highest bids. The
regression model is:

\[
\log\left[-\log\left(1 - \hat{F}(b_i)\right)\right] = -k \log \lambda + k \log(b_i) + \epsilon_i
\]

where \( \hat{F} \) is the auction’s empirical distribution of the bids submitted. The regressions’
\( R^2 \) s are 0.984, 0.966, and 0.946 respectively for the Vickrey, Fischer, and Amsterdam
auctions, suggesting that the estimated bid distributions are quite accurate for all three
auctions (see also the P–P plots in Fig. 1). The second step is to use the parameter
estimates to calculate the first and second moments of the second and third highest-
order statistics to obtain the auctions’ expected revenue and its variance. The results
are in Table 5.

The estimates in Tables 4 and 5 indicate that, if anything, the Vickrey auction
raises higher revenue than both premium auctions. As a consequence, if we have
reason to reject Hypothesis 1 that states that the Vickrey and the Amsterdam are
revenue equivalent, it is in favor of the hypothesis that the Vickrey auction outperforms
the Amsterdam auction. We also find little support for Hypothesis 2: While for all
estimates, apart from \( n = 400 \) bidders, the average revenue is estimated to be higher
for the Vickrey auction than for the Fischer auction, the differences are not significant.
Similarly, we reject Hypothesis 3 in that, if anything, the data show that the Fischer

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7 Earlier applications of recombinant estimation to auction data include Mullin and Reiley (2006),
Engelbrecht-Wiggans et al. (2006), Mares and Shor (2008) and Garratt et al. (2012).
8 The reason for using the top 20% of the bids, rather than all bids, is that the auction outcomes are fixed
by the first, second, and third highest-order statistics, so that it is more important to estimate the right tail
of the distribution precisely than the complete distribution.
Table 4: Estimates for the mean and the SD of (net) auction revenue for 5, 10, 20, 30, 40, and 50 bidders

|       | Mean      | SD        | Mean      | SD        | Mean      | SD        | Mean      | SD        | Mean      | SD        |
|-------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Vickrey |           |           |           |           |           |           |           |           |           |           |
| 5     | 16.83 (1.82) | 17.74     | 33.40 (3.15) | 21.80     | 53.46 (4.15) | 25.29     | 66.55 (4.09) | 27.48     | 76.94 (3.65) | 28.86     |
| 10    | 14.02 (1.62) | 15.87     | 29.07 (2.77) | 21.58     | 48.19 (4.39) | 26.12     | 61.72 (3.69) | 28.19     | 72.42 (3.70) | 28.44     |
| 20    | 5.72 (0.96)  | 10.14     | 20.49 (2.60) | 17.85     | 44.04 (4.01) | 21.86     | 59.56 (2.77) | 20.80     | 69.52 (2.40) | 18.88     |
| Fischer |           |           |           |           |           |           |           |           |           |           |
| 5     | 16.83 (1.82) | 17.74     | 33.40 (3.15) | 21.80     | 53.46 (4.15) | 25.29     | 66.55 (4.09) | 27.48     | 76.94 (3.65) | 28.86     |
| 10    | 14.02 (1.62) | 15.87     | 29.07 (2.77) | 21.58     | 48.19 (4.39) | 26.12     | 61.72 (3.69) | 28.19     | 72.42 (3.70) | 28.44     |
| 20    | 5.72 (0.96)  | 10.14     | 20.49 (2.60) | 17.85     | 44.04 (4.01) | 21.86     | 59.56 (2.77) | 20.80     | 69.52 (2.40) | 18.88     |
| Amsterdam |           |           |           |           |           |           |           |           |           |           |
| 5     | 16.83 (1.82) | 17.74     | 33.40 (3.15) | 21.80     | 53.46 (4.15) | 25.29     | 66.55 (4.09) | 27.48     | 76.94 (3.65) | 28.86     |
| 10    | 14.02 (1.62) | 15.87     | 29.07 (2.77) | 21.58     | 48.19 (4.39) | 26.12     | 61.72 (3.69) | 28.19     | 72.42 (3.70) | 28.44     |
| 20    | 5.72 (0.96)  | 10.14     | 20.49 (2.60) | 17.85     | 44.04 (4.01) | 21.86     | 59.56 (2.77) | 20.80     | 69.52 (2.40) | 18.88     |

*p-value Vickrey versus Fischer*

0.247  0.139  0.302  0.466  0.383  0.577  0.381  0.567  0.385  0.456  0.244  0.338

*p-value Vickrey versus Amsterdam*

0.000 ***  0.000 ***  0.002 ***  0.089*  0.102  0.211  0.157  0.024**  0.090*  0.001 ***  0.020**  0.000 ***

*p-value Fischer versus Amsterdam*

0.000 ***  0.000 ***  0.024**  0.091*  0.485  0.165  0.639  0.012**  0.511  0.001 ***  0.254  0.000 ***


$n$ represents the number of bidders. The estimates are based on 100 representatives of each of the bids submitted resulting in 31,700, 34,200, and 30,500 random draws for the Vickrey, Fischer, and Amsterdam auctions respectively. The first [second] entry in a cell displays the estimate for average revenue [the SD (in italics)]. Standard errors are between brackets. *p*-values for two-sided *t*-tests for comparisons of means and two-sided *F*-tests for SD comparisons. Significance levels: *10%, **5%, ***1%
Premium auctions in the field

![Fig. 1 P–P plots estimated quantiles versus sample quantiles of the bid distributions](image)

**Table 5** Estimates for the mean and the SD of (net) auction revenue for 50, 100, 200, 300, and 400 bidders

|                | \( n = 50 \)   | \( n = 100 \)  | \( n = 200 \)  | \( n = 300 \)  | \( n = 400 \)  |
|----------------|----------------|----------------|----------------|----------------|----------------|
| **Mean**       |                |                |                |                |                |
| Vickrey        | 86.05          | 33.93          | 42.77          | 44.54          | 46.11          |
| (4.27)         | (4.82)         | (5.33)         | (5.61)         | (5.81)         |                |
| Fischer        | 80.25          | 35.16          | 46.00          | 49.05          | 51.19          |
| (4.26)         | (4.93)         | (5.58)         | (5.95)         | (6.21)         |                |
| Amsterdam      | 76.07          | 27.13          | 31.43          | 35.43          | 39.19          |
| (3.47)         | (4.02)         | (4.54)         | (4.82)         | (5.02)         |                |
| **SD**         |                |                |                |                |                |
| Vickrey        | 33.93          | 115.6          | 148.7          | 169.7          | 185.3          |
| (4.82)         | (8.22)         | (5.33)         | (5.61)         | (5.81)         |                |
| Fischer        | 35.16          | 110.7          | 146.0          | 168.7          | 185.8          |
| (4.93)         | (9.30)         | (5.58)         | (5.95)         | (6.21)         |                |
| Amsterdam      | 27.13          | 107.4          | 143.2          | 166.1          | 183.2          |
| (4.02)         | (4.03)         | (4.54)         | (4.82)         | (5.02)         |                |

*p*-values for two-sided *t*-tests for comparisons of means and two-sided *F*-tests for SD comparisons. Significance levels: *10%, **5%, ***1%

- The Amsterdam auction raises more money than the Amsterdam auction. Tables 4 and 5 also indicate that the variance of the revenue is substantially lower in the Amsterdam auction than in the Vickrey and Fischer auctions for almost any number of bidders used in the simulations. So, we find support for Hypothesis 4 that states that the variance of the revenue in the Vickrey auction is greater than that of the Amsterdam auction.9

9 Notice that the intuition underlying Hypothesis 4 offered in the theory section strictly speaking does not apply to our data. First of all, bids in the Vickrey auction are not more spread out than in the Amsterdam auction according to both the observed spread (€0–€222 for the Amsterdam auction and €0–€200 for the Vickrey auction) and the standard deviation of bids and bids conditional on them strictly positive reported in Table 2. Second, in the parametrization of the Amsterdam auction used in the experiment, revenue equals the third-highest bid, not the weighted average of the third-highest and second-highest bid. Still, the main explanation why we do find a lower revenue dispersion in the Amsterdam auction relative to the Vickrey auction is related to the second reason in that our parametrization puts all weight on the third-highest bid.
Result 4 summarizes the conclusions regarding Hypotheses 1–4.

**Result 4** The Vickrey and the Fischer auctions raise at least as much revenue as the Amsterdam auction. There is no significant difference among the average revenues raised in the Vickrey and the Fischer auctions. The Amsterdam auction has a significantly different (lower) revenue dispersion than either the Vickrey auction or the Fischer auction.

### 4.3 Individual bidding behavior

In this section, we zoom in on individual bidding behavior. First, we consider decisions to submit ‘serious’ bids, i.e., non-zero bids. Table 6 contains the results of a logit regression of strictly positive bids on gender, age, education, marital status, and risk attitude. Correcting for bidder characteristics does not alter the conclusion that the decision to enter a strictly positive bid is independent of auction format (see Sect. 4.1).

Men, young, low-educated, and unmarried people are significantly more likely to submit a non-zero bid. Bidders are less likely to submit a strictly positive bid the more risk averse they are, although this effect is only statistically significant for the Amsterdam auction in the regression without controls for other bidder characteristics.

**Result 5** A bidder’s decision to enter a strictly positive bid does not significantly depend on the auction format.

Table 7 includes regression results for the bids submitted. In each auction format, women submit significantly lower bids than men. Bids are decreasing in age but statistically significantly so for only the Vickrey and Fischer auctions. Bids do not depend significantly on the level of education or marital status in any of the three auctions. For all auctions, bids are decreasing in bidders’ risk attitude albeit not significantly so. The effect for the Amsterdam auction is greater than for the Vickrey auction in both regressions. This is in line with the theoretical results that bids are independent of risk attitude for the Vickrey auction and decreasing in risk aversion for the Amsterdam auction (Brunner et al. 2014).¹⁰

**Result 6** On average, risk-averse bidders tend to bid lower in all three auctions, but in none of the auctions is the effect statistically significant.

Finally, we zoom in on the bid distribution in view of our theory. Appendix C contains a table showing the frequencies of all bids submitted across auctions and auction formats.

Footnote 9 continued

It turns out that the variance of the third-highest bid in the Amsterdam auction is lower than the variance of the second-highest bid in the Vickrey auction. Similarly, the lower revenue spread in the Amsterdam auction relative to the Fischer auction is a direct consequence of the variance of the third-highest bid in the Amsterdam auction being lower than the variance of the 95% of the second-highest bid in the Fischer auction.

¹⁰ The constant in the regressions on demographics is noticeably greater for the Fischer auction than for the other two auction formats. However, this observation does not imply that the Fischer auction raises more money than the other two auctions because age and female gender turn out to have a greater negative effect in the Fischer auction than in the other two formats.
Table 6 Submitting a strictly positive bid—logit regressions

|                          | Estimate (Std. Error) | Estimate (Std. Error) |
|--------------------------|-----------------------|-----------------------|
| Constant                 | 1.64*** (0.450)       | 0.664* (0.364)       |
| Vickrey                  | −0.599 (0.507)        | −0.460 (0.505)       |
| Fischer                  | −0.440 (0.495)        | −0.435 (0.488)       |
| Gender (female = 1)      | −0.480*** (0.135)     |                       |
| Age                      | −0.011** (0.006)      |                       |
| Education (university = 1)| −0.373*** (0.146)    |                       |
| Married                  | −0.328** (0.150)      |                       |
| (Risk aversion)*Vickrey  | 0.048 (0.088)         | −0.015 (.089)        |
| (Risk aversion)*Amsterdam| −0.141 (0.095)        | −0.161* (0.092)      |
| (Risk aversion)*Fischer  | 0.016 (0.084)         | −0.014 (0.083)       |
| Number of observations   | 955                   | 955                   |
| Log pseudolikelihood     | −643                  | −658                  |
| Pseudo R²                | 0.0260                | 0.0027                |

Robust standard errors in parentheses. Significance levels: *10%; **5%; ***1%

Fig. 2 plots a histogram of the non-zero bids. Our theory is based on the independent-private-values model. The fact that the print’s shop price was communicated to the bidders begs the question whether the common-value model might be a more realistic description of our setting. In principle, the auction winners could try and resell the print and fetch a price in between the winner’s payment (less than €200 in all three auctions) and the shop value (€590). However, the data seem to offer little support for such hypothesis. First of all, the fraction of zero bids is over 40% for each auction suggesting that at least these bidders had no intention of reselling the print. Moreover, the bid frequency table in Appendix C and the histogram in Fig. 2 indicate that the bid spread is large for all formats. Only implausibly large variations between bidders in terms of beliefs about the resale value would support the common-value assumption. We also did not find any YellowKorner poster offered on marktplaats.nl, the largest online second-hand market in The Netherlands, which suggests that the resale value of the posters is low.

In the theory section, we showed that the Fischer auction does not have an efficient equilibrium. This implies that if bidders play a symmetric equilibrium, some values should pool at the same bid. Is pooling around some bids more prominent in the Fischer auction than in the other two formats? Eyeballing Fig. 2 indeed suggests that pooling at focal amounts like €5, €10, €20, €25, and €50 is at least as frequent in the Fischer auction as in the Vickrey and Amsterdam auctions. To analyze bid pooling
Table 7 Submitted bids—tobit regression, censored from below at 0

|               | Vickrey | Amsterdam | Fischer |
|---------------|---------|-----------|---------|
| **Constant**  |         |           |         |
|               | 23.8**  | 10.8      | 21.8*   |
|               | (11.4)  | (7.93)    | (13.1)  |
| **Gender (female = 1)** |         |           |         |
|               | − 14.0*** | − 11.4* | − 11.6** |
|               | (5.28) | (5.88)    | (4.72)  |
| **Age**       |         |           |         |
|               | − .133* | − .062    | − .467** |
|               | (.210)  | (.239)    | (.200)  |
| **Education (university = 1)** |         |           |         |
|               | − 6.19  | − 3.86    | − 5.27  |
|               | (5.48)  | (6.17)    | (5.05)  |
| **Married**   |         |           |         |
|               | − 5.23  | − 8.02    | − 6.80  |
|               | (5.71)  | (6.39)    | (5.59)  |
| **Risk aversion** |         |           |         |
|               | − 1.78  | − 3.22    | − 3.61  |
|               | (2.06)  | (2.00)    | (2.36)  |
| **Number of observations** |       |           |         |
|               | 315     | 315       | 302     |
| **Log likelihood** |       |           |         |
|               | − 963   | − 968     | − 912   |
|               | (338)   | (338)     | (302)   |
| **Pseudo R²** | 0.0066  | 0.0013    | 0.0046  |
|               | 0.0013  | 0.0013    | 0.0093  |
|               | 0.0003  |           |         |

Standard errors in parentheses. Significance levels: *10%; **5%; ***1%

Fig. 2 Histogram of non-zero bids

more formally, for all non-zero bids, we calculated the Hirfindahl–Hirschmann Index (HHI) of bid concentration, i.e., the sum of the squares of the fraction of bids at each amount. So, the HHI measures the extent to which bidders pool at a limited number of
Table 8 Bid concentration for the three auction formats

| Auction   | n  | HHI   | 1/n  |
|-----------|----|-------|------|
| Vickrey   | 171| 0.0568| 0.0058|
| Amsterdam | 159| 0.0685| 0.0063|
| Fischer   | 187| 0.0791| 0.0053|

\(n\) is the number of non-zero bids

bids, resulting in a number between \(1/n\) (if all \(n\) bidders bid a different amount) and 1 (if all bidders pool at the same bid). Table 8 shows that the Fischer auction’s HHI is higher than the Vickrey auction’s and the Amsterdam auction’s, even though more non-zero bids are submitted in the Fischer auction. So, indeed, the data seem to show that the Fischer has more mass concentrated around a limited number of bids than the other two auctions.

5 Conclusions

We have reported the results of an online field experiment in which we compared the revenue-generating properties of two premium auctions, the Amsterdam auction and the Fischer auction, with those of the Vickrey auction, a standard auction. We conclude that both premium auctions do not raise higher revenue than the Vickrey auction. In fact, the Amsterdam auction generates significantly lower revenue than the Vickrey auction in most scenarios with 50 or fewer bidders. This is in contrast to earlier laboratory experiments that find no significant differences between the English auction and the Amsterdam auction in terms of average revenue in symmetric settings (Goeree and Offerman 2004; Hu et al. 2011a).\(^{11}\) Revenue dispersion in the Amsterdam auction is lower than that in the Vickrey and Fischer auctions. Despite the greater incentive for bidders to submit a strictly positive bid in the premium auctions than in the Vickrey auction, the auctions do not differ in terms of fraction of strictly positive bids.

The environment that we have studied is arguably an \textit{ex ante} symmetric one in that anonymity assures that from the viewpoint of the bidders no subset of competing bidders could be identified as ‘strong’ or ‘weak.’ Our finding that the Amsterdam auction does not outperform the Vickrey auction in such setting resonates with findings in laboratory experiments and theoretical results for risk-averse bidders (Hu et al. 2011b, 2018). Our regression results indicate that risk aversion might partly drive the Amsterdam auction’s poor relative performance in that risk aversion has a relatively large downward pressure on bids in the Amsterdam auction compared to the other auctions. The differences are statistically weak, though. All in all, our results suggest that in \textit{ex ante} symmetric settings (like anonymous online auctions) there is not a strong case for using premium auctions over standard auctions. The Amsterdam auction in particular performs poorly in terms of revenue so that it is an unattractive auction.

\(^{11}\) Our result resonates with Brunner et al.’s (2014) who observe that both risk-averse and risk-seeking bidders bid less aggressively than the equilibrium bid function in the Amsterdam auction. Brunner et al. (2014) also find that revenue of the Amsterdam auction for risk-averse bidders are significantly lower than predicted revenue of the English auction.
format for risk-neutral sellers. For very risk-averse sellers, the Amsterdam auction might be attractive because of its relatively low revenue dispersion.

Of course, our conclusions may be limited to the specific parameters chosen in the experiment. Future research may highlight the effect of premium used in the Amsterdam auction (50%) and the Fischer auction (5%) on average revenue and revenue dispersion as well as the extent to which the results are robust to the specific object that was auctioned, the number of objects offered, information about the number of bidders or others’ bids, and so forth. Future research may also reveal to what extent our findings extrapolate to ex ante asymmetric field settings, e.g., settings in which a ‘strong’ bidder competes against a set of ‘weak’ bidders. Previous theoretical research and laboratory studies show the Amsterdam auction to perform better than standard auctions in such settings (Goeree and Offerman 2004; Hu et al. 2011a). Might the Fischer auction work even better than the Amsterdam auction in asymmetric field settings? It will certainly be interesting to explore this question because we find the Fischer auction to perform it at least as well in terms of revenue than the Amsterdam auction. Future research may also highlight the effect of learning. In our field experiment, many bidders tended to bid at value or below in all auctions, which is a weakly-dominated strategy for the Amsterdam auction. In the case of repeated interaction, bidders may learn that it might make sense for them to bid above value in this auction so that the auction would perform better than in our one-shot setting. Follow-up laboratory experiments may shed light on the way bidders respond to the premium offered in the Fischer auction and perhaps identify reasons why the Fischer auction raises at least much revenue as the Amsterdam auction. Finally, our finding that the auctions do not differ significantly in terms of strictly positive bids suggests that the incentive to enter the auction does not differ between auctions. Future research may test this hypothesis in an environment where the auction format is revealed to the bidders before they decide to enter.

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**Appendix A: Proofs and examples**

**Proof of Lemma 2** Goeree and Offerman (2004) and Hu et al. (2011b) study a dynamic version of the Amsterdam auction. In this auction, the auctioneer raises the price. Bidders can signal at any price that they wish to leave the auction. All bidders are informed of other bidders leaving the auction at any price. The auction stops at the price at which only one bidder remains. This bidder wins the auctioned item and pays the price at which the auction stops. Both the winner and the runner-up receive a
premium equal to \( \alpha \) times the difference between the final price and the price where the third-last bidder left the auction. Goeree and Offerman (2004) find that bidding below value is weakly dominated. Hu et al. (2011b) show that this auction has a unique perfect Bayesian equilibrium in which a bidder with value \( v \) leaves the auction at \( b(v) \), independent of the observed history, i.e., the prices at which the other bidders left the auction. As a consequence, \( b(v) \) is also the unique symmetric equilibrium of the sealed-bid Amsterdam auction. Because \( b \) is a strictly increasing function on \([v, \bar{v}]\), the item is always awarded to the bidder with the highest value. Moreover, a bidder value \( v \) never obtains the item or the premium so her expected utility equals zero in equilibrium.

**Proof of Lemma 3** Suppose the Fischer auction had an efficient equilibrium. Then all bidders bid according to the same, strictly increasing, bidding function. (Suppose two bidders bid according to a different equilibrium bidding function. Let \( b_i \) denote bidder \( i \)’s equilibrium bidding function, \( i = 1, \ldots, n \). Suppose \( b_j(v) < b_k(v) \) for so \( v \in (v, \bar{v}) \) and bidder pair \( j, k \). Efficiency implies that in equilibrium, zero mass of bids is submitted in the interval \( (b_j(v), b_k(v)) \). But then bidder \( j \) is better off bidding \( b_k(v) \) as that increases the premium that she receives with non-zero probability while not affecting the winning probability nor payment upon winning. A contradiction.) We call this function \( B \). Let \( F[1] \) denote the distribution of the highest value among \( n - 1 \) independent draws from \( F \) and \( f[1] \) the corresponding density function. Provided that all other bidders stick to the equilibrium bidding strategy, a bidder that has value \( v \) solves

\[
\max_w \varphi B(w) P^{[2]}(w) + \int_v^w (v - B(x)) dF[1](x)
\]

where \( P^{[2]}(w) \) represents the probability that the bidder will submit the second-highest bid, i.e., the probability that exactly one bidder has a value greater than \( w \). Observe that for all \( v \in [v, \bar{v}] \)

\[
P^{[2]}(v) = (n - 1) F(v)^{n-2} (1 - F(v)) = \frac{f^{[1]}(v)}{f(v)} (1 - F(v)).
\]

Let \( p^{[2]}(v) \equiv P^{[2]}(v) \). for \( v \in [v, \bar{v}] \). The equilibrium first-order condition is given by

\[
\varphi B'(v) P^{[2]}(v) + \varphi B(v) p^{[2]}(v) + (v - B(v)) f^{[1]}(v) = 0
\]
or equivalently,

\[
\phi B'(v) P^2(v) (1 - F(v))^\frac{1}{\phi} + \phi B(v) P^2(v) (1 - F(v))^\frac{1}{\phi} - \frac{B(v) P^2(v) f(v)}{(1 - F(v))^{1 - \frac{1}{\phi}}}
\]

\[
= -\frac{v P^2(v) f(v)}{(1 - F(v))^{1 - \frac{1}{\phi}}}
\]

The resulting differential equation is solved by

\[
\phi B(v) P^2(v) (1 - F(v))^\frac{1}{\phi} = \int_{v_0}^{v} \frac{x P^2(x) f(x)}{(1 - F(x))^{1 - \frac{1}{\phi}}} dx + C
\]

where \(C\) is a constant. \(C = 0\) because at the boundary \(v = \bar{v}\), it holds true that \(P^2(\bar{v}) = 1 - F(\bar{v}) = \lim_{v \uparrow \bar{v}} \int_{v}^{\bar{v}} \frac{x P^2(x) f(x)}{(1 - F(x))^{1 - \frac{1}{\phi}}} dx = 0\). Now, an inconsistency arises because at \(v = v\), the left-hand side is zero (since \(P^2(v) = 0\) for \(n \geq 3\)) while the right-hand side is strictly positive. Therefore, the assumption that \(B\) is a strictly increasing function must be violated. As a consequence, the Fischer auction does not have an efficient equilibrium.

**Proof of Proposition 3** Myerson (1981) shows that in the case of risk-neutral bidders, a mechanism’s expected revenue \(R\) is given by

\[
R = \sum_{i=1}^{n} \left( v_i - \frac{1 - F(v_i)}{f(v_i)} \right) p_i(v_1, \ldots, v_n) - \sum_{i=1}^{n} U_i
\]

where \(p_i(v_1, \ldots, v_n)\) denotes bidder \(i\)’s probability of winning the auctioned item in equilibrium conditional on the bidders’ values \(v_1, \ldots, v_n\). \(U_i\) represents bidder \(i\)’s expected utility in equilibrium when she has the lowest possible value \(v\). Because \(F\) is log-concave, \(v_i - \frac{1 - F(v_i)}{f(v_i)}\) is strictly increasing in \(v_i\). As a consequence, under the condition that the item is always awarded, it is optimal to award it to the bidder with the highest ‘marginal value’ \(v_i - \frac{1 - F(v_i)}{f(v_i)}\) and letting the bidder with the lowest possible value obtain zero expected utility. Expected revenue is lower in the Fischer auction than in both the Vickrey auction and the Amsterdam auction because according to the auction rules and Lemmas 1–3, (1) the item is always awarded in all three auctions, (2) the latter two auctions are efficient in contrast to the Fischer auction, and (3) bidders that have the lowest possible value obtain zero expected utility in the latter two auctions.

**Example A.1** Suppose \((n - 1)\bar{v} \leq (n - 1 + \phi)v\). Then the Fischer auction has an equilibrium in which all bidders bid \(B = v\) independently of their value. To see that this is an equilibrium, first note that none of the bidders have a reason to bid lower than \(v\): by bidding \(v\) they obtain positive utility because they have a strictly positive probability
both of winning the item and paying a price lower than their value and of obtaining the premium amounting to $\varphi v$. Second, bidders do not have a reason to bid higher than $v$. Suppose they did, then their payoff is less than their expected payoff when bidding $v$:

$$v_i - v = \frac{v_i - v}{n} + \frac{\varphi v}{n} + \frac{(n-1)v_i}{n} - \frac{n-1 + \varphi}{n}v$$

$$\leq \frac{v_i - v}{n} + \frac{\varphi v}{n} + \frac{(n-1)v}{n} - \frac{(n-1 + \varphi)v}{n} \leq \frac{v_i - v}{n} + \frac{\varphi v}{n}.$$

The first [second] inequality is implied by $v_i \leq \bar{v}$ [the assumption that $(n-1)\bar{v} \leq (n-1 + \varphi)\bar{v}$]. The auction’s revenue equals $(1 - \varphi)v$ which is more than a factor $(1 - \varphi)$ less than the expected revenue in the Vickrey and Amsterdam auctions (i.e., the expected value of the second highest value).

**Example A.2** If $v \geq (1 - \varphi)\bar{v}$, the Fischer auction has an equilibrium in which all bidders bid $B = \bar{v}$ independently of their value. Notice that for a bidder, deviating to a higher bid is not interesting because then they are guaranteed to win the auction and pay more than their value for the item without getting the premium. For bidder $i$, bids below $\bar{v}$ yield a payoff of zero while the expected payoff when bidding $\bar{v}$ equals

$$\frac{1}{n}(v_i - \bar{v}) + \frac{\varphi \bar{v}}{n} \geq \frac{1}{n}(v - \bar{v}) + \frac{\varphi \bar{v}}{n} = \frac{v}{n} - \frac{(1 - \varphi)\bar{v}}{n} \geq 0.$$

The first [second] inequality follows from $v_i \geq v$ [the assumption that $v \geq (1 - \varphi)\bar{v}$]. So, deviating to a lower bid is not interesting either. The auction’s revenue equals $(1 - \varphi)\bar{v} \leq v$ which is strictly lower than the expected revenue in the Vickrey and Amsterdam auctions (i.e., the expected value of the second-highest value).

**Appendix B: Instructions**

In this appendix, we present an English translation of the instructions on the Veylinx website for each of the three auctions. The instructions were presented to participants when they entered the website. Between treatments, the instructions only differed as far as the description of the auction rules is concerned. [We indicate the relevant auction format between square brackets.]
Screen 1

1 One bid
You can only submit a single bid.

Legally binding
It is not possible to withdraw your bid.

Everything included
Your bid includes VAT and shipping.

Payment
The winner pays less than his own bid. The winner must pay within 48 hours.

Closing
This auction closes on Wednesday, May 11, at 12 noon.

Shipment
The object will be shipped within a week once the payment is received.

Screen 2

Framed print, shop price €590

Description:
- Exclusive print, limited edition of 500 copies
- Including black wooden frame, 60 cm x 90 cm
- Acrylic glass premium sheet

Auction rules [Vickrey]:
- The highest bidder wins the framed print
- The winner does not pay his own bid but the second-highest bid

Auction rules [Amsterdam]:
- The highest bidder wins the framed print
- The winner does not pay his own bid but the second-highest bid
- Both the winner and the runner-up receive a bonus
- Both get 50% of the difference between the second-highest and the third-highest bid

Auction rules [Pischer]:
- The highest bidder wins the framed print
- The winner does not pay his own bid but the second-highest bid
- The runner-up receives a bonus equal to 5% of his or her bid

05:53
Premium auctions in the field

Appendix C

See Table 9.

| Table 9 | Frequencies of bids submitted across auctions |
|---------|-----------------------------------------------|
|         | Vickrey | Amsterdam | Fischer | Vickrey | Amsterdam | Fischer |
| €222.22 | 0       | 1         | 0       | €25.05  | 0         | 0       | 1       |
| €200.00 | 1       | 0         | 1       | €25.00  | 18        | 15      | 22      |
| €160.00 | 1       | 0         | 0       | €24.00  | 1         | 0       | 1       |
| €153.00 | 1       | 0         | 0       | €23.23  | 0         | 1       | 0       |
| €150.00 | 0       | 1         | 3       | €21.30  | 1         | 0       | 0       |
| €126.00 | 1       | 0         | 0       | €20.05  | 0         | 0       | 1       |
| €125.00 | 1       | 2         | 0       | €20.00  | 13        | 5       | 14      |
| €121.00 | 0       | 1         | 0       | €19.00  | 1         | 0       | 1       |
| €105.00 | 0       | 0         | 1       | €18.99  | 1         | 0       | 0       |
| €101.00 | 0       | 0         | 1       | €16.00  | 1         | 0       | 0       |
| €100.00 | 3       | 4         | 3       | €15.06  | 1         | 0       | 0       |
Table 9 continued

| Vickrey | Amsterdam | Fischer | Vickrey | Amsterdam | Fischer |
|---------|-----------|---------|---------|-----------|---------|
| €99.00  | 0         | 0       | 1       | €15.01    | 1       | 0       | 0       |
| €90.00  | 0         | 1       | 0       | €15.00    | 7       | 13      | 9       |
| €89.00  | 1         | 0       | 1       | €14.00    | 0       | 0       | 1       |
| €88.00  | 1         | 0       | 0       | €13.65    | 1       | 0       | 0       |
| €85.00  | 0         | 1       | 0       | €12.57    | 1       | 0       | 0       |
| €80.00  | 0         | 4       | 1       | €12.50    | 1       | 1       | 0       |
| €79.00  | 0         | 0       | 1       | €12.00    | 2       | 1       | 1       |
| €75.00  | 3         | 6       | 1       | €11.11    | 1       | 1       | 0       |
| €74.00  | 0         | 1       | 0       | €11.00    | 0       | 2       | 0       |
| €70.00  | 1         | 1       | 0       | €10.98    | 0       | 1       | 0       |
| €66.00  | 0         | 0       | 1       | €10.01    | 0       | 1       | 0       |
| €65.00  | 1         | 2       | 1       | €10.00    | 21      | 25      | 28      |
| €63.00  | 0         | 0       | 1       | €9.95     | 1       | 0       | 2       |
| €60.00  | 1         | 1       | 1       | €9.00     | 1       | 1       | 1       |
| €59.00  | 2         | 1       | 0       | €8.05     | 1       | 0       | 0       |
| €56.00  | 1         | 0       | 0       | €7.50     | 1       | 0       | 1       |
| €55.55  | 1         | 0       | 1       | €7.00     | 1       | 1       | 0       |
| €55.00  | 1         | 2       | 1       | €6.85     | 0       | 1       | 0       |
| €53.00  | 0         | 1       | 0       | €6.24     | 0       | 1       | 0       |
| €52.00  | 1         | 0       | 0       | €6.00     | 0       | 1       | 1       |
| €51.00  | 3         | 0       | 0       | €5.98     | 1       | 0       | 0       |
| €50.00  | 6         | 6       | 11      | €5.90     | 0       | 0       | 1       |
| €49.00  | 0         | 1       | 1       | €5.05     | 1       | 0       | 0       |
| €46.00  | 0         | 1       | 0       | €5.01     | 1       | 0       | 0       |
| €45.45  | 0         | 1       | 0       | €5.00     | 14      | 12      | 23      |
| €45.00  | 2         | 1       | 0       | €4.00     | 1       | 0       | 1       |
| €43.00  | 1         | 0       | 0       | €3.00     | 2       | 1       | 3       |
| €40.00  | 5         | 4       | 3       | €2.50     | 1       | 0       | 1       |
| €37.00  | 1         | 0       | 0       | €2.00     | 4       | 6       | 9       |
| €35.95  | 1         | 0       | 0       | €1.95     | 1       | 0       | 0       |
| €35.00  | 2         | 4       | 3       | €1.00     | 17      | 18      | 20      |
| €30.00  | 5         | 2       | 3       | €0.50     | 0       | 1       | 1       |
| €29.00  | 1         | 0       | 0       | €0.01     | 2       | 0       | 2       |
| €28.00  | 0         | 0       | 1       | €–        | 146     | 146     | 155     |

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