Generating two dimensional ground reaction forces with a viscoelastic runner model

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Abstract

The purpose of this study was to propose a viscoelastic model which can generate two dimensional ground reaction forces in steady pace running. In our previous studies, the 3 degrees of freedom viscoelastic model was proposed to reproduce the vertical ground reaction force and rebound velocity. According to the angle of the ground reaction force calculated from the ratio of vertical and horizontal ground reaction force during contact period, the model needed the rotational control element because the rotational behavior of the model was a very complicated movement. To generate the two dimensional ground reaction force of running, the nonlinear rotational viscoelastic elements were incorporated to the model and the parameters of the model were calculated with parameter searching. The two dimensional ground reaction force was reproduced with the proposed model.

Keywords: Two dimensional ground reaction force; parameter searching; rotational viscoelastic elements

1. Introduction

To evaluate the properties of sports surfaces, friction tests and shock attenuation tests have been generally adopted to determine the horizontal and vertical characteristics, respectively [1] [2]. To assess vertical properties, these tests focus on the vertical force attenuation of sports surfaces with a dropping mass-spring model. Although the test results may be reproduced and the criteria for evaluation are very strict, this method may have an issue of ‘particularity’, because the material behavior alters when the mechanical properties of the drop mass-spring systems are changed. To avoid the difficulties, we...
proposed the method to evaluate sports surface properties by computer simulation instead of mechanical tests in previous studies. Additionally, we proposed the vertical human runner model for evaluating the cushioning of sport surfaces with runner-surface coupled system [3][4]. However these models are dealing with only vertical characteristics. Horizontal characteristics are also important for preventing the injuries and making good performances. For this reason, we developed a two dimensional shock test device and investigated two dimensional shock attenuation and impact friction compared to the static and dynamic friction test [5]. To make a progress for evaluating two dimensional shock attenuation with runner-surface model, we need to develop a runner model which can generate two dimensional ground reaction forces, too. Because of this background, the purpose of this study is to investigate a viscoelastic model which can generate two dimensional ground reaction forces in steady pace running.

### Nomenclature

| Symbol | Description |
|--------|-------------|
| $m_0, m_1, m_2$ | Mass parameters |
| $r_0, r_1, r_2$ | Displacement along with the rotational axis |
| $\theta$ | Angle of the model |
| $\theta_e$ | Equilibrium angle of the rotational elastic element $s_p$ |
| $k_0, p_0, k_2, p_2$ | Parameters of nonlinear elastic element |
| $k_1$ | Parameters of linear elastic element |
| $k_{31}...k_{35}$ | Parameters of rotational elastic element |
| $c_c, q_c$ | Parameters of nonlinear viscous element in compression phase |
| $c_r, q_r$ | Parameters of nonlinear viscous element in restore phase |
| $c_1$ | Parameters of linear viscous element |
| $c_{31}...c_{35}$ | Parameters of rotational viscous element |
| $l_0, l_1, l_2$ | Initial length of moment arm |
| $f_c$ | Ground reaction force along with the rotational axis |

### 2. Two Dimensional Ground Reaction Force in Steady Pace Running

Figure 1 shows the typical vertical-horizontal reaction force in heel-strike running (a) and that of the angle calculated from the ratio of vertical ground reaction force and horizontal reaction force (b). The vertical force is characterized by the two peaks. The first peak is known as the passive load which can not be cushioned by human movement. This is because the duration of the first peak is too short so that a human cannot make a cushioning movement. And this peak is very important for the evaluation because it is considered as the origin of the injuries. To express the two peaks in the vertical force, the model needs to have at least two masses. In previous studies [3][4], we had proposed the viscoelastic model which could generate not only the vertical ground reaction force precisely but also the rebound velocity which is almost same as the rebound velocity in the experiment.
3. Experiments

Eleven steady pace running trials ranged from 2.53 m/s to 5.68 m/s were performed by one subject. The subject ran on the force platform with right foot contact and these trials were video recorded to acquire the trajectory of the C.G. of the body in sagittal plane. Vertical and horizontal (sagittal plane) forces were acquired from the force platform and the incidental velocity and reflected velocity of the C.G. of the body were calculated by video analysis.

4. Two Dimensional Runner Model

According to the angle of the ground reaction force calculated from the ratio of vertical and horizontal ground reaction force in Figure 1(b), the model should have a rotational control element because the rotational behavior of the model showed a complicated movement. Therefore, the nonlinear rotational elastic and viscous elements were incorporated in the model for controlling the rotational behavior. Figure 2 shows the two dimensional runner model proposed in this study. The model touches down and rotates at point $P$ without slip during the contact period. The masses of the model are restricted to move along the rotational axis. The equations of motion of this model are listed below:

$$m_0 \frac{d^2}{dt^2} r_0 = -k_0 \text{sign}(r_0 - r_i) \left| r_0 - r_i \right|^p_0 + D - m_0 g \sin \theta + m_0 (l_0 + r_0) \dot{\theta}^2$$  \hspace{1cm} (1)

$$m_1 \frac{d^2}{dt^2} r_1 = -k_1 (r_1 - r_2) - c_1 (r_1 - \dot{r}_2) + k_0 \text{sign}(r_0 - r_i) \left| r_0 - r_i \right|^p_0 - D - m_1 g \sin \theta + m_1 (l_1 + r_1) \dot{\theta}^2$$  \hspace{1cm} (2)

$$D = \begin{cases} -cc \text{sign}(\dot{r}_0 - \dot{r}_1) \left| \dot{r}_0 - \dot{r}_1 \right|^\psi_0 & (\dot{r}_0 - \dot{r}_1 < 0) \\ -cr \text{sign}(\dot{r}_0 - \dot{r}_1) \left| \dot{r}_0 - \dot{r}_1 \right|^\psi_0 & (\dot{r}_0 - \dot{r}_1 \geq 0) \end{cases}$$  \hspace{1cm} (3)

$$m_2 \frac{d^2}{dt^2} r_2 = k_2 (r_2 - r_1) + c_2 (\dot{r}_2 - \dot{r}_1) - k_2 \text{sign}(r_2 - r_3) \left| r_2 - r_3 \right|^p_2 - m_2 g \sin \theta + m_2 (l_2 + r_2) \dot{\theta}^2$$  \hspace{1cm} (4)

$$\left\{m_0 (l_0 + r_0)^2 + m_1 (l_1 + r_1)^2 + m_2 (l_2 + r_2)^2\right\} \frac{d^2}{dt^2} \theta = -2 \left\{m_0 (l_0 + r_0) \dot{r}_0 + m_1 (l_1 + r_1) \dot{r}_1 + m_2 (l_2 + r_2) \dot{r}_2 \right\} \dot{\theta}$$
$$- \{m_0 (l_0 + r_0) + m_1 (l_1 + r_1) + m_2 (l_2 + r_2)\} g \cos \theta - f_{k3} - f_{c3}$$  \hspace{1cm} (5)
\[ f_{i3} = \sum_{j=1}^{5} k_{3j} (\theta - \theta^*)^{i_j} \]  
(6)

\[ f_{c3} = \sum_{j=1}^{5} c_{3j} \dot{\theta}^{i_j} \]  
(7)

\[ f_r = -k_2 \text{sign}(r_2) |r_2|^p \]  
(8)

Fig. 2. Two dimensional runner model with rotational viscoelastic elements.

5. Parameter Identification of the Runner Model

According to this model, two dimensional simulated forces were calculated as an initial-value problem of differential equations. The parameter set of the model in each trial was searched by nonlinear programming (Rosenbrock’s method) minimizing the objective function. The objective function was the summation of the Relative Standard Error (RSE) of two dimensional force and angular displacement, and Relative Error (RE) of two dimensional incident and rebound velocity. Additionally, to avoid breaking the physical constraint in the model, the length of the moment arms of the masses from contact point \( P \) are defined as follows:

\[ l_0 > l_1 > l_2 > 0 \]  
(9)

6. Results and Discussion

6.1. Two dimensional simulated force and angular displacement

Figure 3 shows the comparison between experimental data and simulated data of vertical force, horizontal force and angular displacement in trials No.1, No.5 and No.9. Although the estimated force could not represent the high frequency oscillation especially in the first half of horizontal force, the ‘passive load’ in vertical force could be represented well. The differences between the experimental and estimated data at the maximum angular displacement were observed because both the vertical and horizontal data in the experiment were close to zero.
Table 1 shows the RSE between experimental data and simulated data in vertical/horizontal force and angular displacement, and RE of incidental and reflected velocities in each trial. As shown in Figure 3, the vertical and horizontal forces and angular displacement were able to represented with this model. However, Table 1 shows that the model needs 1.7 to 2.3 times the incidental vertical velocity of the experimental value because the model has viscous elements which consume the energy. Additionally, the model needs a negative horizontal incidental velocity as an opposite direction against that of the experiment. This was indicated by the RE value of horizontal incident velocity in Table 1. In other words, simulation of all the trials needed the horizontal velocity which has same magnitude, but the opposite direction as an input. Because the angular displacement (\( \theta \)) calculated from vertical and horizontal forces start at around \( \frac{\pi}{4} \), almost above the contact point \( P \), the model did go down towards the negative horizontal direction in the compression phase, and after the maximum compression, the model moves forward to the positive running direction in the restore phase (Figure 4).

6.2. Results of parameter identification

Table 1. RSE (Relative Standard Error) between the value of experiment and that of simulation in vertical, horizontal ground reaction force and angular displacement. And RE (Relative Error) in incidental and reflected velocities.
### Trial Ground Reaction Force Angular Incident Velocity Reflected Velocity

| Trial No. | Vertical | Horizontal | Vertical | Horizontal | Vertical | Horizontal |
|-----------|----------|------------|----------|------------|----------|------------|
| 1         | 4.98     | 36.27      | 6.16     | 74.25      | 200.00   | 46.49      | 65.60      |
| 2         | 7.21     | 24.87      | 5.31     | 88.41      | 200.00   | 59.99      | 46.42      |
| 3         | 7.72     | 25.88      | 3.83     | 90.52      | 200.00   | 3.27       | 39.46      |
| 4         | 7.57     | 23.08      | 4.26     | 68.70      | 200.00   | 32.53      | 41.09      |
| 5         | 8.57     | 20.41      | 3.75     | 90.33      | 200.00   | 15.54      | 34.51      |
| 6         | 9.28     | 24.36      | 5.77     | 97.71      | 200.00   | 20.06      | 47.50      |
| 7         | 8.89     | 22.50      | 4.05     | 116.30     | 200.00   | 56.09      | 40.42      |
| 8         | 11.02    | 18.25      | 5.55     | 135.12     | 200.00   | 19.76      | 39.18      |
| 9         | 9.61     | 23.07      | 5.44     | 133.06     | 200.00   | 35.92      | 37.71      |
| 10        | 15.87    | 31.39      | 6.82     | 120.49     | 200.00   | 14.32      | 45.79      |
| 11        | 14.88    | 32.22      | 7.70     | 108.08     | 200.00   | 18.43      | 39.59      |

Fig. 4. Two dimensional (sagittal plane) trajectory of C.G. of the body in trial No.5. Whilst the running direction is left to right, the initial horizontal velocity has to have a negative direction to reproduce the two dimensional ground reaction force.

#### 7. Conclusions

A two dimensional runner model was proposed in this study. The results are summarized as follows:

- The proposed model can reproduce two dimensional forces and angular displacement with high accuracy.
- The proposed model needs the rotational viscoelastic elements because the rotational behavior is complicated.
- The proposed model needs to provide a negative incidental horizontal velocity to reproduce the forces because the model has a structural constraint.
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