Scrutiny of entropy optimized tangent hyperbolic fluid (non-Newtonian) through perturbation and numerical methods between heated plates

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Abstract
Objective: Many methods have been used to maximize the capacity of heat transport. A constant pressure gradient or the motion of the wall can be used to increase the heat transfer rate and minimize entropy. The main goal of our investigation is to develop a mathematical model of a non-Newtonian fluid bounded within a parallel geometry. Minimization of entropy generation within the system also forms part of our objective.
Method: Perturbation theory is applied to the nonlinear complex system of equations to obtain a series solution. The regular perturbation method is used to obtain analytical solutions to the resulting dimensionless nonlinear ordinary differential equations. A numerical scheme (the shooting method) is also used to validate the series solution obtained.
Results: The flow and temperature of the fluid are accelerated as functions of the non-Newtonian parameter (via the power-law index). The pressure gradient parameter escalates the heat and volume flux fields. The energy loss due to entropy increases via the viscous heating parameter. A diminishing characteristic is predicted for the wall shear stress that occurs at the bottom plate versus the time-constant parameter. The Reynolds number suppresses the volume flux field.

Keywords
Plane Poiseuille flow, tangent hyperbolic fluid, entropy generation, perturbation method, heat transfer rate, skin friction coefficient

Date received: 19 August 2020; accepted: 13 October 2020

Handling Editor: James Baldwin

Introduction
A fluid that does not obey Newton’s law of viscosity is called a non-Newtonian fluid. Non-Newtonian fluids are more complex than Newtonian fluids because of the nonlinear relationship between their shear rates and shear stresses. Many common solutions and polymers, including ketchup, toothpaste, mud, blood, starch suspensions and paints, are examples of non-Newtonian fluids.¹⁻³ Because of the extensive number of roles of non-Newtonian fluids in the chemical, production and...
manufacturing industries, a diverse range of research has been published in the literature in this context.4–7

The tangent hyperbolic fluid is a non-Newtonian fluid that was established for use in biochemical and engineering systems. Because of its physical robustness and simplicity when used in computations, this model offers distinct benefits when compared with other non-Newtonian models. The formulation of this model is based on the kinetic principle of fluids rather than an empirical relationship. The shear thinning phenomenon in non-Newtonian fluids can be described well using the tangent hyperbolic fluid model. The magnetohydrodynamic (MHD) flow of a tangent hyperbolic fluid over a porous sheet was observed by Patil et al.8 They used the shooting method to solve the complex equations involved. They were able to capture the physical behavior of important physical quantities such as the Nusselt number, the Sherwood number and the skin friction. An analysis of the heat and flow transfer of tangent hyperbolic fluids was performed by Gaffar et al.9 They considered the flow from a sphere and provided results for the effects of the Prandtl number, the Biot number, the power-law index and the Weissenberg number on the temperature and velocity fields in detail. The combined properties of the magnetic field and viscous dissipation on the flow of a tangent hyperbolic fluid over a stretching sheet were captured by Hussain et al.10 They computed their analytical and numerical results with the aid of the shooting and homotopy analysis methods.

A porous medium or material is a surface that contains voids (pores), which can then be filled by the fluid. Porous media have a number of applications in engineering and technology, including packed absorption systems, fluid filtration, catalytic reaction systems, heat exchangers, distillation towers, ion exchange columns and transpiration cooling systems.11–15 Khan et al.16 performed a numerical study of the behavior of a Carreau-Yasuda liquid that was subject to irreversibility and activation energy effects over a stretched surface. The mass and heat transfer characteristics of microporous fluids with porous media were examined by Mohanty et al.17 The effects of the porous parameter, the Eckert and Prandtl numbers and the unsteady parameter on the flow, concentration and temperature fields were discussed in their work. A numerical investigation of the effects of magnetic fields and thermal radiation on nanofluids in a parallel porous geometry was performed by Kothandapani and Prakash.18 Prasad et al.19 developed a mathematical model to explore the heat transfer effect of a non-Newtonian fluid through a porous wall. Hosseinzadeh et al.20–22 reported the impact of hybrid nanoparticle flow over stretching sheets and cylinders and the effects of a porous medium. Rostami et al.23 selected two temperature models to perform a hydrothermal analysis of nanofluid flow through a cavity. A study of the Maxwell fluid model that considered nonlinear thermal radiation was also presented by Hosseinzadeh et al.24

In recent times, the major challenge in various industrial processes has been to address energy losses. The attention of many researchers has shifted toward entropy generation in the flow of non-Newtonian fluids. Applications of entropy generation can be found in heat exchangers, pumps and electronic cooling systems. Research scientists and engineers have been working to find methods to control the waste of useful energy. For this purpose, the second law of thermodynamics is the best, most accurate and important tool for optimization of a given system. Using the work of Bejan25 as a basis, various investigations into reduction of entropy generation based on the second law of thermodynamics have been presented in the literature. Waqas et al.26 and Wang et al.27 investigated the flow of viscous fluids over variable and plate surfaces from the perspectives of two hybrid nanoparticles, i.e. graphene oxide and copper, with a Darcy–Forchheimer porous medium in terms of thermal radiation, entropy generation, and magnetohydrodynamics. Asta"{n}ina et al.28 studied the combined effects of entropy generation and natural convection on ferrofluid flow in a cavity thorough porous media. Sivaraj and Sheremet29 explored the effects of entropy generation and magnetohydrodynamic natural convection on ferrofluids. They used a uniform mesh-dependent method to solve the problem numerically. Reddy et al.30 presented an entropy generation analysis for the flow of a viscoelastic fluid over a vertical cylinder and observed that entropy generation was minimized by increasing the viscoelastic parameter. Yin et al.31 and Mukherjee et al.32 worked on the heat transport characteristics of nanomaterials in flow boiling and the thermophysical characteristics and flow boiling performance of nanomaterials in a horizontal tube, respectively. Continuous generation of entropy in the blood flow of a micropolar fluid through a channel was investigated by Asha and Deepa.33 They obtained expressions for the velocity, magnetic field, stream function, current density and entropy generation in their work.

An exclusive review of entropy generation and tangent hyperbolic fluids reveals that entropy analysis using a tangent hyperbolic fluid inside a porous heated horizontal channel has not been explored to date. Therefore, the main target of this study is to develop a mathematical analysis to reduce the energy losses due to continuous entropy generation for a tangent hyperbolic fluid inside the porous heated walls of a channel. The perturbation method is used by the authors to compute the results in a simple and easy manner. A graphical representation of the results is also presented.
follows:

we have selected the flow and the temperature as system is used here to perform the analysis, in which is shown in Figure 1. The Cartesian coordinate plates. The flow diagram for the problem to be considered is given by:

\[ \dot{S} = \left[ \tilde{\mu}_x + (\tilde{\mu}_0 + \tilde{\mu}_x) \tanh \left( \frac{\tilde{\Pi}}{\tilde{\mu}} \right) \right] \dot{A}_1. \]  

(6)

Additionally, \n\[ \tilde{\Pi} = \sqrt{\frac{1}{2} \dot{\tilde{A}_1}} \quad \]  

(7)

where \( \dot{A}_1 = tr \left( \nabla \dot{V} + (\nabla \dot{V})^T \right) \). We assume that \( \tilde{\mu}_x = 0, \tilde{\mu}_0 = \tilde{\mu} \) is the viscosity of the fluid and \( \tilde{\Pi} < 1 \) (because the tangent hyperbolic fluid shows shear thinning properties). Using these hypotheses, equation (6) becomes

\[ \dot{S} = \tilde{\mu} \left[ 1 + m \left( \tilde{\Pi} - 1 \right) \right] \dot{A}_1. \]  

(8)

Because

\[ L = \nabla \dot{V} , \]  

(9)

then for the present case, we obtain

\[ tr(\dot{A}_1)^2 = 2 \left( \frac{d\tilde{u}}{dy} \right)^2 , \]  

(10)

where \( \dot{A}_1 = L + L' \).

By substituting equation (10) into equation (7), we obtain

\[ \tilde{\Pi} = \frac{d\tilde{u}}{dy} , \]  

(11)

By substituting the values calculated above into equation (8) and subsequent simplification, the stress tensor becomes

\[ \dot{S}_{xx} = \dot{S}_{yy} = \tilde{\mu} \left[ 1 + m \left( \tilde{\Pi} - 1 \right) \right] \frac{d\tilde{u}}{dy} . \]  

(12)

**Mathematical analysis**

Consider the steady-state incompressible one-dimensional flow of a non-Newtonian fluid between the porous heated plates of a channel in which the thermal conductivity and viscosity are taken to be uniform. We assume that the two plates are set apart at a distance \( h \). Both plates are stationary and are heated to different temperatures. It is further assumed that \( -v_0 \) is the uniform suction velocity of the fluid from both plates. The flow diagram for the problem to be considered is shown in Figure 1. The Cartesian coordinate system is used here to perform the analysis, in which we have selected the flow and the temperature as follows:

\[ \dot{V} = (\tilde{u}(y), -v_0, 0) , \]  

(1)

\[ \bar{\theta} = \bar{\theta}(y) . \]  

(2)

The continuity and linear momentum equations are defined as:\(^{19}\)

\[ div \dot{V} = 0 , \]  

(3)

\[ \tilde{\rho} \frac{d\dot{V}}{dt} = div \ddot{S} + \tilde{\rho} g . \]  

(4)

The Cauchy stress tensor (\( \ddot{T} \)) of the problem to be considered is given by:

\[ \ddot{T} = -\ddot{p} \ddot{I} + \ddot{S} . \]  

(5)

The stress tensor (\( \ddot{S} \)) of the tangent hyperbolic fluid is given as:

\[ \ddot{S} = \left[ \tilde{\mu}_x + (\tilde{\mu}_0 + \tilde{\mu}_x) \tanh \left( \frac{\tilde{\Pi}}{\tilde{\mu}} \right) \right] \dot{A}_1. \]  

(6)

Linear momentum equation

In view of our assumptions above, the momentum equation for the current problem can be written as

\[ \frac{\partial \bar{p}}{\partial x} = -\tilde{\rho}v_0 \frac{\partial \tilde{u}}{\partial y} + \frac{\partial}{\partial y} \left[ \tilde{\mu} \frac{d\tilde{u}}{dy} \left\{ 1 + m \left( \tilde{\Pi} \frac{d\tilde{u}}{dy} - 1 \right) \right\} \right] . \]  

(13)

By defining the following normalized quantities, the dimensionless equation of motion can then be obtained:

\[ u = \frac{\tilde{u}}{v_0}, \quad y = \frac{\tilde{y}}{h}, \quad \bar{\theta} = \frac{\bar{\theta} - \bar{\theta}_0}{\bar{\theta}_h - \bar{\theta}_0}, \quad Re = \frac{\bar{\rho}v_0h}{\tilde{\mu}_0}, \]  

(14)

By inserting the dimensionless quantities above into equation (13) and assuming the viscosity of the fluid to
be constant, we obtain the dimensionless equation of motion as follows:

\[
\frac{\partial^2 \tilde{u}}{\partial y^2} + \left( \frac{2m}{1 - m} \frac{\partial^2 \tilde{u}}{\partial y^2} + \frac{Re}{1 - m} \frac{\partial \tilde{u}}{\partial y} + \frac{P}{1 - m} \right) = 0, \quad (15)
\]

The final form of the dimensionless momentum equation is

\[
\frac{\partial^2 \tilde{u}}{\partial y^2} + \left( \gamma_1 + \delta \gamma_2 \frac{\partial \tilde{u}}{\partial y} + \gamma_3 \right) = 0. \quad (16)
\]

\[
u = 0, \quad \text{at } y = 0, \quad (17)
\]

\[
u = 0, \quad \text{at } y = 1. \quad (18)
\]

where \(\gamma_1 = \frac{Re}{1 - m}, \gamma_2 = \frac{2m}{1 - m}, \gamma_3 = \frac{P}{1 - m}\).

**Energy equation**

The energy equation for the current problem can be written as:

\[
\frac{\partial^2 \tilde{\Theta}}{\partial y^2} = -\frac{1}{\tilde{K}} \left( \tilde{\rho} \tilde{C}_p \tilde{v}_0 \frac{\partial \tilde{\Theta}}{\partial y} + \text{tr}(S.L) \right). \quad (19)
\]

The heat generation (absorption), Joule heating and radiative heat flux effects are not considered in the energy equation above.

Using equations (9) and (12), we obtain

\[
\text{tr}(\mathbf{S.L}) = \tilde{\mu} \left( \frac{\partial u}{\partial y} \right)^2 \left\{ 1 + m \left( \frac{\partial u}{\partial y} - 1 \right) \right\}. \quad (20)
\]

By inserting equation (20) into equation (19), we then obtain the following dimensional form of the energy equation:

\[
\frac{\partial^2 \tilde{\Theta}}{\partial y^2} = -\frac{1}{\tilde{K}} \left( \tilde{\rho} \tilde{C}_p \tilde{v}_0 \frac{\partial \tilde{\Theta}}{\partial y} + \tilde{\mu} \left( \frac{\partial u}{\partial y} \right)^2 \left\{ 1 + m \left( \frac{\partial u}{\partial y} - 1 \right) \right\} \right). \quad (21)
\]

By using the dimensionless quantities given in equation (14) above and taking the viscosity to be a constant, we obtain the normalized form of the energy equation as follows:

\[
\frac{\partial^2 \tilde{\Theta}}{\partial y^2} = -\nu \frac{\partial \tilde{\Theta}}{\partial y} - Br \left( \frac{\partial u}{\partial y} \right)^2 \left\{ 1 + m \left( \frac{\partial u}{\partial y} - 1 \right) \right\}. \quad (22)
\]

The final form of the dimensionless energy equation with boundary conditions is defined by

\[
\frac{\partial^2 \tilde{\Theta}}{\partial y^2} + \gamma_4 \frac{\partial \tilde{\Theta}}{\partial y} + \gamma_5 \left( \frac{\partial u}{\partial y} \right)^2 \left\{ \gamma_6 + m \delta \frac{\partial u}{\partial y} \right\} = 0. \quad (23)
\]

\[
\tilde{\Theta} = 0, \quad \text{at } y = 0, \quad (24)
\]

\[
\vartheta = 1, \quad \text{at } y = 1. \quad (25)
\]

where \(\delta = \frac{\Gamma u}{h}, \gamma_4 = \frac{\rho c^2 v h}{k}, \gamma_5 = \frac{\mu^2 v}{k(\alpha_0 - \alpha_1)}, \gamma_6 = (1 - m)\).

**Perturbation method**

The governing equations (16)–(18) and (23)–(25), which represent the flow of the fluid, are nonlinear ordinary differential equations and it is not possible to obtain closed form solutions to these equations. Therefore, we are interested in finding approximate analytical solutions. For this purpose, we opted to use the regular perturbation method.\(^4\) Approximate analytical solutions are obtained for the flow and temperature distributions using the following perturbation expansions:

\[
u = \nu_0 + \varepsilon \nu_1 + o(\varepsilon^2), \quad \vartheta = \vartheta_0 + \varepsilon \vartheta_1 + O(\varepsilon^2), \quad \delta = \kappa \varepsilon. \quad (26)
\]

where \(\varepsilon\) is the perturbation parameter. By inserting the equations above into equations (23) and (24) and comparing the orders of each of the \(\varepsilon\) terms, we obtain the following systems:

\[
\varepsilon^0 : \Rightarrow \frac{\partial^2 \nu_0}{\partial y^2} + \gamma_1 \frac{\partial \nu_0}{\partial y} + \gamma_3 = 0, \quad (27)
\]

\[
\nu_0 = 0, \quad \text{at } y = 0, \quad (28)
\]

\[
\nu_0 = 0, \quad \text{at } y = 1. \quad (29)
\]

\[
\varepsilon^1 : \Rightarrow \frac{\partial^2 \nu_1}{\partial y^2} + \gamma_1 \frac{\partial \nu_1}{\partial y} + \kappa \gamma_2 \frac{\partial \nu_0}{\partial y} \frac{\partial \nu_0}{\partial y} = 0. \quad (30)
\]

\[
u_1 = 0, \quad \text{at } y = 0, \quad (31)
\]

\[
u_1 = 0, \quad \text{at } y = 1. \quad (32)
\]

The solution to equation (27) can be obtained easily and by making some simplifications, we then obtain

\[
\nu_0 = \alpha_0 \nu_1 + \alpha_1 (1 - e^{-\gamma_1}), \quad (33)
\]

By inserting equation (33) into equation (30), we obtain the solution to the correction term in the following form:

\[
\nu_1 = \kappa (\alpha_2 + (\alpha_3 + \alpha_4) e^{-\gamma_1} + \alpha_5 e^{-2\gamma_1}). \quad (34)
\]

To find the final expression for the velocity profile, we substitute equations (33) and (34) into equation (26). We then obtain the following final form for the velocity distribution:

\[
\nu = (\alpha_0 \nu_1 + \alpha_1 (1 - e^{-\gamma_1})) + \delta (\alpha_2 + (\alpha_3 + \alpha_4) e^{-\gamma_1} + \alpha_5 e^{-2\gamma_1}). \quad (35)
\]

Next, to determine the analytical solution to the temperature equation, we insert equation (26) into
By inserting the value of the temperature distribution: 

\[ \theta(y) = \frac{1}{2} \rho \beta_0 \gamma (1 - e^{-\gamma y}) + \delta (a_2 + (a_3 + a_4) e^{-\gamma y} + a_5 e^{-2\gamma y}) \]

(46)

Then, by integrating from zero to one, we obtain the following analytical expression for the volumetric flow rate:

\[ Q = \frac{1}{2} \rho \beta_0 \gamma \left[ \frac{\mu}{\rho \beta_0 \gamma} \left( 1 + m \left( \frac{\mu}{\rho \beta_0 \gamma} - 1 \right) \right) \right], \]

(47)

The equation above is the volumetric flow rate required for our current exploration.

### Wall shear rate

The formula for the wall shear rate can be written as

\[ \frac{1}{2} \rho \nu_0^2 \gamma C_F = S_{\text{wall}}, \]

(48)

By inserting the value of \( S_{\text{wall}} \) from equation (12) into equation (48), we then obtain:

\[ C_F = \frac{2}{\rho \nu_0^2 \gamma} \left[ \frac{\mu}{\rho \beta_0 \gamma} \left( 1 + m \left( \frac{\mu}{\rho \beta_0 \gamma} - 1 \right) \right) \right], \]

(49)

The equation above in normalized form is given by:

\[ C_F = \frac{2}{Re} \left( \gamma_6 \frac{d\theta}{dy} + m\delta \left( \frac{\partial \theta}{\partial y} \right)^2 \right). \]

(50)

The wall shear rate at the lower wall is denoted by \( C_{F,\text{lower wall}} \) and is given by:

\[ C_{F,\text{lower wall}} = C_F \big|_{y=0} \]

\[ = \frac{2}{Re} \left( \gamma_6 \frac{d\theta}{dy} + m\delta (a_0 + a_1 \gamma_1 + \delta (a_4 - a_3 \gamma_1 - 2a_5 \gamma_1)) \right) \]

(51)

The wall shear rate at the upper wall is denoted by \( C_{F,\text{upper wall}} \) and is given by:

\[ C_{F,\text{upper wall}} = C_F \big|_{y=1} \]

\[ = \frac{2}{Re} \left( \gamma_6 \frac{d\theta}{dy} + m\delta (a_0 + a_1 \gamma_1 e^{-\gamma y} + \delta (a_4 e^{-\gamma y} + (a_3 + a_4) e^{-2\gamma y} + a_5 e^{-2\gamma y})) \right) \]

(52)
The heat flux quantity is given in normalized form by

$$Z = \frac{hA}{K},$$

Equations (51) and (52) are thus the expressions required for the skin friction coefficients at the lower and upper walls of the channel, respectively.

**Heat flux**

The heat flux is another important physical quantity and is given by the following formula:

$$Z = \frac{hA}{K},$$

$$q_s = K \frac{\partial \theta}{\partial y},$$

By inserting the values $A = \bar{h}$, $h = - \frac{\partial u}{\partial y}$, $\bar{q}_s = \bar{K} \frac{\partial \theta}{\partial y}$, into equation (53), we obtain

$$Z = - \frac{\bar{h} \bar{q}_s}{\theta_s - \theta_0} \frac{\partial \theta}{\partial y},$$

The heat flux quantity is given in normalized form by

$$Z = - \frac{\partial \theta}{\partial y}.$$}

The heat flux at the lower wall is denoted by $Z_{\text{Lower wall}} = - \frac{\partial \theta}{\partial y}$ at $y = 0$, where

$$Z_{\text{Lower wall}} = \beta_1 - \beta_3 \gamma_1 - 2 \beta_4 \gamma_1 - \beta_2 \gamma_4 + \delta \left( \beta_7 + \beta_9 + \beta_{11} \right)$$

$$- \beta_8 \gamma_1 - 2 \beta_{10} \gamma_1 - 3 \beta_{12} \gamma_1 - \beta_6 \gamma_4),$$

The heat flux at the upper wall is denoted by $Z_{\text{Upper wall}} = - \frac{\partial \theta}{\partial y}$ at $y = 1$. Here,

$$Z_{\text{Upper wall}} = \beta_1 - \beta_3 \gamma_1 e^{-\gamma_1} - 2 \beta_4 \gamma_1 e^{-2\gamma_1} - \beta_2 \gamma_4 e^{-\gamma_4} + \delta \left( \beta_1 - \beta_3 e^{-\gamma_1} + 2 \beta_{11} e^{-2\gamma_1} - \beta_8 \gamma_1 e^{-\gamma_1} - \beta_{12} \gamma_1 e^{-3\gamma_1} - \beta_6\gamma_4 e^{-\gamma_4} \right).$$

Equations (57) and (58) are the final expressions for the heat transfer rates at the lower and upper walls, respectively.

**Entropy analysis**

The dimensionless expressions for the total entropy generation ($E_{gr}$) and the Bejan number ($B_E$) are given by:

$$E_{gr} = \left( \frac{\partial \theta}{\partial y} \right)^2 + \gamma_b \left( \frac{\partial u}{\partial y} \right)^2 \left\{ \mu \left( \gamma_b + m \delta \frac{\partial \theta}{\partial y} \right) \right\},$$

$$B_E = \frac{\gamma_b \left( \frac{\partial u}{\partial y} \right)^2}{\left( \frac{\partial \theta}{\partial y} \right)^2},$$

**Numerical solution**

To validate the perturbation solution for the current problem, the numerical solution with the absolute error is also calculated (as given in Table 1). The numerical and analytical solutions to the current problem are clearly in good agreement with each other. The numerical solution is achieved here using the shooting method, which is based on the fourth-order Runge–Kutta method. The missing conditions are determined via use of the secant iterative scheme. Our system is based on nonlinear ordinary differential equations and the shooting method is thus selected because of its excellent convergence. A detailed discussion of the shooting method is available in the literature.

**Comparison with previous study**

We have matched our calculation results with the results reported by Gupta and Massoudi for the uniform viscosity model case. Gupta and Massoudi discussed a generalized second grade fluid between two heated walls for the cases of Newtonian ($m = 0$) and non-Newtonian ($m \neq 0$) fluids based on consideration of both constant and variable viscosity models. For comparison purposes, we have presented the values of the velocity and temperature gradients versus the variations in the pressure gradient and the viscous dissipation parameters for the limiting case ($\gamma_1 = m = \delta = 0$) in Table 2; note that our solutions and the solutions of Gupta and Massoudi are well matched with each other.
Physical interpretation of results

This section includes a discussion of the physical behavior of the emerging parameters, i.e. the power-law index \( m \), the Reynolds number \( Re \), the time-constant parameter \( d \), the pressure gradient parameter \( P \), the Peclet number \( \gamma_4 \), the viscous heating parameter \( \gamma_5 \) and the temperature difference parameter \( \Omega \), in terms of their effects on the flow and temperature distributions, the volume flux, the heat flux, the skin friction coefficient, entropy generation and the Bejan number profiles (see Figures 2–10).

Flow and temperature distribution

Figure 2 portrays the trends in the flow and temperature of the fluid using various values of the power-law index parameter. The figure shows that enlarging the value of the power-law index parameter causes both the flow and the temperature of the fluid to increase. This occurs because, for increasing values of \( m \), the nature of the fluid changes from shear thickening to shear thinning. Increasing trends in the temperature field with increments in \( \gamma_4 \) and \( \gamma_5 \) are illustrated in Figure 3. The Peclet number is a measure of the heat transfer ratio caused by the motion of the fluid and thermal conduction, which means that an increment in the Peclet number causes increased heat dissipation within the fluid; this then causes a temperature increase in the fluid. Additionally, an increase in the viscous heating parameter raises the kinetic energy of the fluid and thus further elevation in the temperature field is observed.

Table 2. Comparison with the results of Gupta and Massoudi\(^ {48} \) when \( m = \delta = \gamma_1 = 0 \).

| \( \gamma_3 \) | \( \gamma_5 \) | Gupta and Massoudi\(^ {48} \) | Present method |
|---------|---------|----------------|---------------|
|         |         | \( u'(0) \) | \( u'(1) \) | \( \vartheta'(0) \) | \( \vartheta'(1) \) |
| 2       | 0       | 0.9990      | -0.9980    | 1.0000  | 1.0000  |
|         | 2       | 0.9990      | -0.9980    | 1.3343  | 0.6687  |
| 3       | 0       | 1.4985      | -1.4700    | 1.0000  | 1.0000  |
|         | 2       | 1.4985      | -1.4700    | 1.7493  | 0.2545  |

Figure 2. Variations in flow and temperature for various values of power law index \( m \) along channel height \( y \).

Figure 3. Variations in temperature for various values of Peclet number \( \gamma_4 \) along channel height \( y \).
Volume flux

The Reynolds number and the pressure gradient parameter have opposite effects on the volume flux field (see Figure 4). When the value of the Reynolds number increases, it causes flow instability, with the flow even becoming turbulent at larger values; this instability reduces the volume flux rate. In contrast, an increase in the pressure gradient parameter is shown to enhance the volume flux field. It is obvious that when the value of $P$ increases, the fluid flow then covers additional volume within the channel and thus accelerates the volume flux field.
Skin friction coefficient

Figure 5 illustrates the effect of the time-constant parameter on the skin friction field. It is observed that increases in the time-constant parameter value lead to strong reductions in the $C_{f,\text{wall}}$ field near the right limit of the channel. This behavior is physically correct because increasing $\delta$ also increases the friction of the particles in the fluid, which slows the skin friction coefficient rate.

Heat flux

Increases in the values of $(P)$, also accelerate the heat flux rate ($Z_{\text{Upper wall}}$) of the channel (see Figure 6). The reason for this behavior is that the pressure gradient parameter has direct relationships with the flow and the heat transfer within the fluid. Therefore, an increase in the pressure gradient parameter causes an increase in the heat transfer rate, which then accelerates the heat flux profile.

Entropy generation

Figures 7 and 8 illustrate the effects of the parameters involved on the entropy generation profile. Increasing the value of $(m)$ reduces the viscosity of the fluid, which causes accelerations in the velocity and the shear strain rate within the fluid. Therefore, an increment in entropy generation is observed (as shown in Figure 7(a)). The influence of the time-constant parameter $(\delta)$ on entropy generation is shown in Figure 7(b). It is noted that the entropy generation accelerates near the left boundary of the channel, but it then declines drastically beyond the mid-point of the channel with increasing values of $(\delta)$. This behavioral fluctuation is due to fluid friction irreversibility, which increases with larger values of the time-constant parameter. Figure 8(a) and (b) show opposite trends for entropy generation versus $(\gamma_5)$ and $(\Omega)$. For increasing values of $(\gamma_5)$, increments in entropy generation are observed near the left and right boundaries of the channel. This is physically true because the viscous heating parameter is actually a heat source parameter that enhances the heat energy in the fluid particles. In this situation, the heat transfer increases and this causes the entropy generation rate to accelerate. At the middle of the channel, however, the heat transfer rate remains low because of the temperature gradients. In contrast, inverse trends in entropy generation are noted with respect to variations in $(\Omega)$.
Therefore, to reduce energy losses, it is essential that
the values of all parameters involved are measured.

**Bejan number**

The effects of the pertinent parameters on the Bejan number are illustrated in Figures 9 and 10. It is shown that increases in the power-law index parameter cause the Bejan number to decrease and that an extreme elevation in the profile is observed at the center of the channel (see Figure 9(a)). It is also observed that increasing magnitudes of \( P \) and \( \gamma_s \) cause the Bejan number to decrease (as shown in Figures 9(b) and 10(a), respectively). This occurs because an increase in the value of the viscous heating parameter reduces the thermal conductivity, which causes the Bejan number to decline. However, the temperature difference parameter \( \Omega \) shows the inverse effect on the Bejan number. Figure 10(b) demonstrates that when the value of \( \Omega \) increases, the Bejan number is also enhanced. This is also true because increased \( \Omega \) enhances the heat coefficient rate, which increases the Bejan number. Therefore, the expected outcomes for these results coincide with our experimental outcomes.

**Concluding remarks**

This investigation describes a heat irreversibility analysis for a tangent hyperbolic fluid between heated plates. The main conclusions drawn from the current investigation are summarized as follows:

- The power-law index parameter enhances the flow and temperature of the fluid.
- The pressure gradient parameter enhances the volume and the heat flux but reduces the Bejan number.
- The wall shear rate at the bottom plate diminishes as the time-constant parameter increases.
- Entropy generation is enhanced via increases in the power-law index parameter, the viscous heating parameter and the time-constant parameter.
- Increases in both the power law index parameter and the pressure gradient parameter reduce the Bejan number.
- In the case of the Bejan number, the maximum height of the number profile is observed at the center of the channel.

**Declaration of conflicting interests**

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

**Funding**

The author(s) received no financial support for the research, authorship, and/or publication of this article.

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### Appendix

#### Nomenclature

| Symbol | Definition |
|--------|------------|
| $\bar{h}$ | Distance between plates |
| $\theta_0$ | Temperature at lower plate |
| $A_1$ | Second invariant tensor |
| $\theta_1$ | Temperature at upper plate |
| $\bar{\theta}$ | Dimensional form of temperature |
| $\bar{V}$ | Velocity vector |
| $\bar{t}$ | Dimensional time constant |
| $\bar{S}$ | Cauchy stress tensor |
| $\bar{r}$ | Time |
| $P$ | Dimensionless pressure gradient parameter |
| $\gamma_S$ | Viscous heating parameter |
| $u$ | Dimensionless velocity of fluid |
| $Re$ | Reynolds number |
| $C_F$ | Skin friction coefficient |
| $E_{eg}$ | Total entropy generation |
| $-\bar{v}_0$ | Suction velocity of fluid |
| $\bar{\rho}$ | Density of fluid |
| $\bar{g}$ | Gravitational acceleration |
| $\bar{K}$ | Thermal conductivity |
| $\bar{u}$ | Dimensional form of velocity component |
| $\bar{m}$ | Dimensional power-law index parameter |
| $\delta$ | Dimensionless time constant parameter |
| $\mu$ | Viscosity of the fluid |
| $\epsilon$ | Perturbation parameter |
| $\gamma_A$ | Péclet number in dimensionless form |
| $Q$ | Volume flux |
| $\delta$ | Dimensionless form of temperature |
| $\Omega$ | Temperature difference parameter |
| $Z$ | Heat transfer rate |
| $B_E$ | Bejan number |