Protecting qutrit-qutrit entanglement by weak measurement and reversal

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Entangled states in high dimensional systems are of great interest due to the extended possibilities they provide in quantum information processing. Recently, Sun [Phys. Rev. A 82, 052323 (2010)] and Kim [Nat. Phys. 8, 117 (2012)] pointed out that weak measurement and quantum weak measurement reversal can actively combat decoherence. We generalize their studies from qubits to qutrits under amplitude damping decoherence. We find that the qutrit-qutrit entanglement can be partially retrieved for certain initial states when only weak measurement reversals are performed. However, we can completely defeat amplitude damping decoherence for any initial states by the combination of prior weak measurements and post optimal weak measurement reversals. The experimental feasibility of our schemes is also discussed.

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I. INTRODUCTION

Quantum entanglement is not only a remarkable characteristic which distinguishes the quantum realm from the classical one, but also a key resource for quantum information and quantum computation [1]. However, in realistic quantum information processing, entanglement is inevitably affected by the interaction between the system and its environment, which leads to degradation and, in certain cases, entanglement sudden death (ESD) [2–4]. Thus, it is very important to protect entanglement from environmental noise.

Weak measurements [5] are generalizations of von Neumann measurements and are associated with a positive-operator valued measure (POVM). For weak measurements [7, 8], the information extracted from the quantum system is deliberately limited, thereby keeping the measured system’s state from randomly collapsing towards an eigenstate. Thus, it would be possible to reverse the initial state with some operations. Recently, it was pointed out that weak measurements and quantum weak measurement reversals can effectively protect the quantum states of a single qubit system from decoherence [9–11]: this idea has also been extended to protect the entanglement of two-qubit systems [12–15] from amplitude damping decoherence. Until now, probabilistic reversal with a weak measurement has already been experimentally demonstrated on a superconducting phase qubit [16], as well as on a photonic qubit [13, 17].

Most studies of weak measurements concerning the protection of entanglement are restricted to two dimensional (2D) systems. However, quantum information tasks require high dimensional bipartite entanglement. It is well known that high dimensional entangled systems such as qutrits [18–20] can offer significant advantages for the manipulation of information carriers. For instance, biphotonic qutrit-qutrit entanglement [21] enables more efficient use of communication channels [22]. Moreover, high dimensional entangled systems offer higher information-density coding and greater resilience to errors than 2D entangled systems in quantum cryptography [23]. However, practical applications of such protocols are only conceivable when the prepared high dimensional entangled states have sufficiently long coherence times for manipulation.

In this paper, we propose using weak measurements to preserve the entanglement of two initially entangled qutrits which suffer independent amplitude damping noise. Our schemes for protecting entanglement are based on the fact that weak quantum measurement can be reversed probabilistically. We specifically consider two simple schemes as shown in Fig. 1. Similar schemes have been discussed only in one or two-qubit systems [12, 13, 17], while we consider a qutrit-qutrit version in this paper. The first scheme is “amplitude damping + weak measurement reversal”. In this case, unlike the entanglement decaying exponentially to zero in amplitude damping decoherence, we show that the weak measurement reversal procedure partially recovers the entanglement under most conditions. The limitation of this scheme is that ESD still occurs in some particular situations. As an improvement on the former, the second scheme is “weak measurement + amplitude damping + weak measurement reversal”. In this case, we find the combination of prior weak measurement and post weak measurement reversal can actively combat decoherence. Moreover, it can effectively circumvent ESD. The physical mechanism of the second scheme is that a prior weak measurement intentionally moves each qutrit close to its ground state. The amplitude damping decoherence is naturally suppressed in this ‘lethargic’ state, and the entanglement is therefore preserved [24].

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This paper is organized as follows: In Section II, we introduce amplitude damping noise operators for the qutrit case, then we generalize the weak measurement and weak measurement reversal operators from qubit to qutrit. In Section III, we propose two different schemes to protect qutrit-qutrit entanglement. In Section IV, we give a brief discussion of the experimental feasibility of our schemes. Finally, we summarize our conclusions in Section V.

II. BASIC THEORY

A. amplitude damping for qutrits

The amplitude damping noise is a prototype model of a dissipative interaction between a quantum system and its environment [1]. For example, the amplitude damping noise model can be applied to describe the spontaneous emission of a photon by a two-level system into an environment of photon or phonon modes at zero (or very low) temperature in (usually) the weak Born-Markov approximation.

For qutrits, the situations are more complicated as there are three configurations of the 3-level system to be taken into account [25]. Here, we will focus on the so-called V-configuration. We denote the lower level as $|0\rangle$, and the two upper levels as $|1\rangle$ and $|2\rangle$, respectively. We assume that only dipole transitions between levels $|1\rangle \rightarrow |0\rangle$ and $|2\rangle \rightarrow |0\rangle$ are allowed. If the environment is in a vacuum state, the amplitude damping noise which corresponds to the spontaneous emission from the V-configuration qutrit can be represented by the following map [26]:

\[
\begin{align*}
|0\rangle_S|0\rangle_E &\rightarrow |0\rangle_S|0\rangle_E, \\
|1\rangle_S|0\rangle_E &\rightarrow \sqrt{1-d}|1\rangle_S|0\rangle_E + \sqrt{d}|0\rangle_S|1\rangle_E, \\
|2\rangle_S|0\rangle_E &\rightarrow \sqrt{1-D}|2\rangle_S|0\rangle_E + \sqrt{D}|0\rangle_S|1\rangle_E,
\end{align*}
\]

where $d, D \in [0,1]$ represents the decay rates of the upper levels $|1\rangle$ and $|2\rangle$, respectively.

B. weak measurement for qutrits

The null-result weak measurement that we consider is the POVM or partial-collapse measurement originally discussed in Refs. [7, 8]. It is different from amplitude damping in the sense that we add an ideal detector to monitor the environment function as follows: the detector clicks with a probability $p$ if there is an excitation in the environment and never clicks with a probability $1-p$ if no excitation is detected in the environment. For the qutrit case, we can construct the POVM elements as: $M_1 = \text{diag}(0,\sqrt{p}, 0)$, $M_2 = \text{diag}(0, 0, \sqrt{1-p})$ and $M_3 = \text{diag}(1,\sqrt{1-p}, \sqrt{1-q})$, where $p$ and $q$ represent the weak measurement strengths of transitions $|1\rangle \rightarrow |0\rangle$ and $|2\rangle \rightarrow |0\rangle$, respectively. The measurement operators $M_1$ and $M_2$ are identical to the normal projection measurements in which the state of the qutrit is irrevocably collapsed to the ground state and an excitation is emitted from system to environment. They are not reversible and we therefore discard the result from experiments which produced clicks, thereby removing the terms $\sqrt{p}|0\rangle_S|1\rangle_E$ and $\sqrt{q}|0\rangle_S|1\rangle_E$ from the state map. Fortunately, the measurement operator $M_3$ is a weak (or partial-collapse) measurement for the single qutrit that we are interested in this paper. We rewrite $M_3$ as

\[
\begin{align*}
|0\rangle_S|0\rangle_E &\rightarrow |0\rangle_S|0\rangle_E, \\
|1\rangle_S|0\rangle_E &\rightarrow \sqrt{1-p}|1\rangle_S|0\rangle_E, \\
|2\rangle_S|0\rangle_E &\rightarrow \sqrt{1-q}|2\rangle_S|0\rangle_E,
\end{align*}
\]

C. weak measurement reversal for qutrits

Except for von Neumann projective measurements, any weak or partial-collapse measurement could be reversed [27]. According to Ref. [27], it is easy to construct the reversed weak measurement operator of the null-result weak measurement as shown in Eq. (2). The single-qutrit reversing measurement ($M_r$) is also a non-unitary operation that can be written as

\[
M_r = \begin{pmatrix}
\sqrt{(1-p_r)(1-q_r)} & 0 & 0 \\
0 & \sqrt{1-q_r} & 0 \\
0 & 0 & \sqrt{1-p_r}
\end{pmatrix},
\]

where $p_r$ and $q_r$ are the strengths of the reversing measurements. As the matrix is non-unitary, the probability of successful reversal will always be less than unity.
### III. PROTECTION OF QUTRIT-QUTRIT ENTANGLEMENT

#### A. scheme one

We first check the efficiency of the first scheme as shown in Fig. 1(a). For simplicity, we assume two identical qutrits are initially prepared in the following state

$$|\Psi\rangle = \alpha |00\rangle + \beta |11\rangle + \gamma |22\rangle,$$

where $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$. Such a qutrit-qutrit entangled state can be experimentally prepared by utilizing the orbital angular momentum of photons [18, 20]. We assume they suffer independent but identical amplitude damping noise (i.e., $d_1 = d_2 = D_1 = D_2 = D$). Then the initial pure state inevitably evolves into a mixed state in the presence of noise.

$$\rho_d = \sum_{i=1}^{9} \varepsilon_i |\Psi\rangle \langle \Psi|_{i}^{+},$$

where $\varepsilon_i = E_j \otimes E_k$, $(j, k = 0, 1, 2)$ are the Kraus operators. In the standard product basis $\{|j, k\rangle = |3j + k + 1\rangle\}$, the non-zero elements of $\rho_d$ are:

$$\rho_{11} = |\alpha|^2 + D^2(|\beta|^2 + |\gamma|^2),$$

$$\rho_{22} = \rho_{44} = \rho_{55} = D(1 - D)|\beta|^2,$$

$$\rho_{33} = \rho_{77} = \rho_{99} = D(1 - D)|\gamma|^2,$$

$$\rho_{15} = \rho_{51} = (1 - D)|\alpha\beta^*|,$$

$$\rho_{19} = \rho_{91} = (1 - D)|\alpha\gamma^*|,$$

$$\rho_{59} = \rho_{95} = (1 - D)^2|\beta\gamma^*|.$$

After the amplitude damping decoherence, we perform quantum measurement reversal operations on each qutrit as shown in Eq. (3). The non-zero elements of the final reduced density matrix $\rho_r$ are:

$$\rho_{11} = \frac{(1 - p_r)^2}{C_1} [ |\alpha|^2 + D^2(|\beta|^2 + |\gamma|^2) ]$$

$$\rho_{22} = \rho_{44} = \rho_{55} = D(1 - D)(1 - p_r)|\beta|^2/C_1,$$

$$\rho_{33} = \rho_{77} = \rho_{99} = D(1 - D)(1 - p_r)|\gamma|^2/C_1,$$

$$\rho_{15} = \rho_{51} = (1 - D)(1 - p_r)|\alpha\beta^*|/C_1,$$

$$\rho_{19} = \rho_{91} = (1 - D)(1 - p_r)|\alpha\gamma^*|/C_1,$$

$$\rho_{59} = \rho_{95} = (1 - D)^2|\beta\gamma^*|/C_1,$$

where $C_1 = (1 - p_r)^2 |\alpha|^2 + (1 - D)^2 + 2D(1 - D)(1 - p_r) + D^2(1 - p_r)^2(|\beta|^2 + |\gamma|^2)$ is the normalization parameter.

To quantify the qutrit-qutrit entanglement change under amplitude damping noise and weak measurement reversal, we need an effective measure of mixed qutrit state entanglement since damping causes the pure states to evolve into mixed states. One usually takes the entanglement of formation [28] as such a measure, but in practice it is not known how to compute this measure for mixed states of $d \otimes d$ dimensional systems in the case when $d > 2$. A computable measure of distillable entanglement of mixed states was proposed in Ref. [29]. It is based on the trace norm of the partial transposition $\rho^T$ of the state $\rho$. From the Peres’ criterion of separability [30], it follows that if $\rho^T$ is not positive, then $\rho$ is entangled. Hence one defines the negativity of the state $\rho$ as

$$N = \frac{||\rho^T|| - 1}{2}.$$

$N$ is equal to the absolute value of the sum of negative eigenvalues of $\rho^T$ and is an entanglement monotone [29], but it cannot detect bound entangled states [31].
two particular initial states under amplitude damping decoherence and corresponding optimal reversing measurements. We have chosen the optimal reversing measurement strength to be \( p_r = D \) which gives the maximum amount of entanglement of the two-qutrit state \( \rho_r \). For \( |\Psi\rangle = 1/\sqrt{3}(|00\rangle + |11\rangle + |22\rangle) \), we note that the damped negativity \( N_d \) decays as the decoherence strength \( D \) increases, while the reversed negativity \( N_r \) approaches a finite value. The reversed negativity \( N_r \) is higher than the negativity \( N_d \) regardless of the decoherence strength parameter, as shown in Fig. 2(a). However, for \( |\Psi\rangle = \sqrt{3}/8|00\rangle + \sqrt{5}/8|11\rangle + 0|22\rangle \), we find the reversed negativity \( N_r \) is not always higher than the negativity \( N_d \). Moreover, the reversed negativity \( N_r \) suffers sudden death as well as \( N_d \), as shown in Fig. 2(b). The reason is straightforward as all operations are local, and no entanglement can be created between two independent qutrits in a separable state. The above results for qutrits are quite in accordance with those for qubits discussed in Ref. [12].

As the weak measurement reversals are non-unitary operations, this scheme naturally has less than unity success probability. Under the optimal reversing weak measurements (i.e., \( p_r = D \)), the corresponding success probability is:

\[
P_1 = (1 - D)^2 \left[ 1 + (|\beta|^2 + |\gamma|^2)(2D + D^2) \right].
\]

(9)

It is clear that \( P_1 \to 0 \) when \( D \to 1 \).

**B. scheme two**

As shown above, the first scheme has some limitations regarding the protection of entanglement and circumvention of ESD. In this section, we show that an improved scheme first proposed by Kim et al. [13] can completely circumvent the decoherence and protect the qutrit-qutrit entanglement even if ESD occurs. The key difference is that a prior weak measurement is applied on each qutrit before it suffers amplitude damping decoherence, as depicted in Fig. 1(b).

The whole procedure is as follows: for each qutrit, first a prior weak measurement with strength \( p \) is performed, then it goes through the amplitude damping channel, and finally a post weak measurement reversal with strength \( p_r \) is carried out. After these operations, the non-zero elements of the reduced density matrix \( \rho_{uv} \) are:

\[
\rho_{11} = \frac{[(1 - p_r)^2|\alpha|^2 + (1 - p)^2D^2(1 - p_r)^2(|\beta|^2 + |\gamma|^2)]}{C_2},
\]

\[
\rho_{15} = \rho_{51}^* = (1 - p)(1 - D)(1 - p_r)|\alpha\beta^*/C_2,
\]

\[
\rho_{19} = \rho_{91}^* = (1 - p)(1 - D)(1 - p_r)|\alpha\gamma^*/C_2,
\]

\[
\rho_{22} = \rho_{22}^* = (1 - p)^2D(1 - D)(1 - p_r)|\beta|^2/C_2,
\]

\[
\rho_{33} = \rho_{33}^* = (1 - p)(1 - D)(1 - p_r)|\alpha\beta^*/C_2,
\]

\[
\rho_{55} = (1 - p)^2(1 - D)^2|\beta|^2/C_2,
\]

\[
\rho_{59} = \rho_{95}^* = (1 - p)^2(1 - D)^2|\beta\gamma^*/C_2,
\]

\[
\rho_{99} = (1 - p)^2(1 - D)^2|\gamma|^2/C_2,
\]

where \( C_2 = (1 - p_r)^2|\alpha|^2 + (1 - p)^2(1 - D)^2 + 2D(1 - D)(1 - p_r) + D^2(1 - p_r)^2(|\beta|^2 + |\gamma|^2) \).

**Fig. 3.** (color online) For two particular initially entangled qutrit-qutrit states \( |\Psi\rangle = 1/\sqrt{3}(|00\rangle + |11\rangle + |22\rangle) \) (solid line) and \( |\Psi\rangle = \sqrt{3}/8|00\rangle + \sqrt{5}/8|11\rangle + 0|22\rangle \) (dashed line): (a) Negativity \( N_d \) as a function of decoherence strength \( D \). (b) The ratio of \( N_{uw} \) to \( N_l \) as a function of weak measurement strength \( p \) when \( D = 0.8 \) and an optimal reversing measurement is performed.

Following the methods demonstrated in Refs. [10, 13], the optimal reversing measurement strength that gives the maximum amount of entanglement for the two-qutrit state \( \rho_{uv} \) is calculated to be \( p_r = p + Dp \) where \( p = 1 - p \). We still consider the same initial states in scheme one and compare the effectiveness of these two schemes for suppressing amplitude damping noise. In Fig. 3, we show the protection of entanglement from decoherence by using a weak measurement and weak measurement reversal. As we know, the qutrit-qutrit entanglement decays monotonously with increasing \( D \) and even experiences ESD when \( |\Psi\rangle = \sqrt{3}/8|00\rangle + \sqrt{5}/8|11\rangle + 0|22\rangle \).

However, it is clear that the qutrit-qutrit entanglement can be protected by the combined action of prior weak measurements and post weak measurement reversals. In Fig. 3(a), we note that the negativity of
\( |\Psi\rangle = 1/\sqrt{3}(|00\rangle + |11\rangle + |22\rangle) \) is 0.04 when \( D = 0.8 \), but it can be completely reversed to its initial entanglement as \( p \to 1 \) in Fig. 3(b). In Fig. 3(b), we have introduced the ratio of \( N_{wr} \) (negativity after the sequence of weak measurement, decoherence and reversing measurement) to the initial negativity \( N_i \) to highlight the entanglement recovery efficiency. To demonstrate the scheme’s ability to circumvent ESD, we choose \( D = 0.8 \), at which point ESD appears for the initial state \( |\Psi\rangle = \sqrt{3}/8|00\rangle + \sqrt{5}/8|11\rangle + 0|22\rangle \) in Fig. 3(a). We find the entanglement can be completely recovered with a certain probability by the sequence of weak measurement and weak measurement reversal, which is similar to that in Ref. [13] where a two-qubit entangled state is considered.

Similarly to the first scheme, the success probability under the optimal reversing weak measurements (i.e., \( p_r = p + D\bar{p} \)) can be written as

\[
\mathcal{P}_2 = (1 - D)^2 \bar{p}^2 \left[ 1 + (|\beta|^2 + |\gamma|^2)(2D\bar{p} + D^2\bar{p}^2) \right]. \tag{11}
\]

We observe that for \( p \to 1 \), the success probability \( \mathcal{P}_2 \to 0 \) because the prior weak measurement is reduced to an unrecoverable von Neumann projective measurement.

By comparing the two schemes, it is easy to find that the second scheme is much more efficient than the first scheme at protecting entanglement and circumventing ESD. Physically, this can be explained as follows: From Equation (2), we know that the stronger the weak measurement strength \( p \), the closer the initial qutrit is reversed towards the \( |0\rangle \) state. Once the system is in \( |0\rangle \), then it will be immune to amplitude damping decoherence. In the first scheme, no prior weak measurement is carried out before the qutrits go through the amplitude damping channel, thus the amount of reversed entanglement highly depends on the initial states and the decoherence strength \( D \). In the second scheme, prior weak measurements are performed to move the state towards the \( |00\rangle \) state, which does not experience amplitude damping decoherence. Then optimal weak measurement reversals are performed to revert the qutrits back to the initial state. Therefore, the amount of reversed entanglement is not related to the decoherence strength \( D \) but depends on the weak measurement strength \( p \). Initial entanglement can entirely be recovered for any initial state by the combined prior weak measurements and post weak measurement reversals when \( p \to 1 \).

### C. Discussions

In the above analyses, we have assumed that the two qutrits are identical and the decoherence parameters \( d \) and \( D \) are the same for states \([1]\) and \([2]\). In fact, these two schemes are universal for the most general case (i.e., \( d_1 \neq d_2 \neq D_1 \neq D_2 \)). Following the same calculation procedure as above, we plot in Fig. 4 the numerical results for the two weak measurement schemes against the amplitude damping decoherence. For the first scheme, the optimal reversing measurement strength is calculated to be \( p_{r_k} = d_k \) and \( q_{r_k} = D_k \) (\( k = 1, 2 \)). Similarly, the optimal reversing measurement strength should be \( p_{r_k} = p_k + d_k\bar{p}_k \) and \( q_{r_k} = q_k + D_k\bar{q}_k \) for the second scheme.

![Fig.4. (color online) Entanglement protection of state \( |\Psi\rangle = 1/\sqrt{3}(|00\rangle + |11\rangle + |22\rangle) \) by weak measurement and weak measurement reversal: (a) scheme one with the decoherence parameters \( d_1 = D, d_2 = 0.7D, D_1 = 0.3D \) and \( D_2 = 0.6D \), (b) scheme two with the decoherence parameters \( d_1 = 0.8, d_2 = 0.5, D_1 = 0.4 \) and \( D_2 = 0.6 \).](image)

### IV. EXPERIMENTAL FEASIBILITY

It is necessary to give a brief discussion of some key problems which are related to the experimental implementation of our procedure. Here, we restrict our discussions to the cavity QED system which we think is the best candidate for the experimental realization of our scheme.

**Initial state preparation.** The V-type qutrit-qutrit entanglement of Eq. (4) could be generated by sending a pair of momentum and polarization-entangled photons to two spatially separated cavities in which a V-type atom is trapped [32]. For the atomic level structure, we can
take $^{87}\text{Rb}$ as our choice. The state $|0\rangle$ corresponds to $|F = 1, m_F = 0\rangle$ of $5^2S_{1/2}$, while the states $|1\rangle$ and $|2\rangle$ correspond to the degenerate levels $|F = 1, m_F = 1\rangle$ and $|F = 1, m_F = -1\rangle$ of $5^2P_{1/2}$, respectively. The transitions $|1\rangle \rightarrow |0\rangle$ and $|2\rangle \rightarrow |0\rangle$ emit right-circularly and left-circularly polarized photons, so we can distinguish the parameters $p$ and $q$ during the weak measurement.

**Amplitude damping decoherence.** In a cavity QED system, the amplitude damping decoherence is the natural spontaneous emission of a photon from the excited state of an atom to its ground state. The dynamic map of Eq. (1) describes a dissipative interaction between a V-type qutrit and its vacuum environment [26].

**Weak measurement.** As shown in Sect. II, we note that the only difference between the AD decoherence map Eq. (1) and weak measurement map Eq. (2) is the inclusion of the $\sqrt{p}\langle 0|S|1\rangle_E$ and $\sqrt{q}\langle 0|S|1\rangle_E$ terms. In this sense, we can add an ideal single-photon detector to monitor the cavity. Whenever there is a detector click, we discard the result. This postselection removes the $\sqrt{p}\langle 0|S|1\rangle_E$ and $\sqrt{q}\langle 0|S|1\rangle_E$ terms and hence a null-result weak measurement is implemented.

**Weak measurement reversal.** To reverse the effect of the weak measurement ($M_3$), we only need to apply the inverse of $M_3$

$$M_3^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{1-p}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{1-q}} \end{pmatrix} \text{ (12)}$$

since $M_3^{-1}$ can be re-written as

$$M_3^{-1} = \frac{1}{\sqrt{(1-p)(1-q)}} FM_3 FM_3 F$$

$$= \frac{1}{\sqrt{(1-p)(1-q)}} M_F, \text{ (13)}$$

where $F$ is the trit-flip operation

$$F = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \text{ (14)}$$

Thus, the weak measurement reversal procedure of Eq. (3) can be constructed by the following five sequential operations on each system qutrit: trit-flip ($F$), weak measurement ($M_3$), trit-flip ($F$), another weak measurement ($M_3$), and trit-flip ($F$). The trit-flip operation $F$ can be realized by a $\pi$ pulse applied on the transition $|1\rangle \leftrightarrow |2\rangle$ and followed by another $\pi$ pulse to interchange the populations between $|0\rangle$ and $|1\rangle$. (i.e., by the series of two $\pi$ pulses $\pi^{(1)} \leftrightarrow \pi^{(2)} \pi^{(0)} \leftrightarrow \pi^{(1)}$) [33].

### V. CONCLUSIONS

In conclusion, we have demonstrated that weak measurement reversal can indeed be useful for combating amplitude damping decoherence and recovering the entanglement of two qutrits. In particular, we have examined two simple schemes: one is “amplitude damping + weak measurement reversal” and the other is “weak measurement + amplitude damping + weak measurement reversal”. We have shown that the first scheme can partially recover qutrit–qutrit entanglement for certain initial states, but it has some limitations with respect to entanglement protection efficiency and ESD circumvention. For the second scheme, in which prior weak measurements and post weak measurement reversals are carried out sequentially, the amplitude damping decoherence can be completely suppressed for any initial states even if ESD occurs. Even though the method is risky (i.e., a stronger procedure is required for a longer preservation, which decreases the probability of success), this procedure for entanglement preservation is useful in entanglement distillation protocols and some quantum communication tasks.

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