Two-step Quantum Key Distribution Schemes Using Polarization and Frequency Doubly Entangled Photons

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A two-step quantum key distribution protocol using frequency and polarization doubly entangled photons is proposed. In this protocol, information is encoded by a unitary operation on each of the two doubly entangled photons and sent from Alice to Bob in two steps. State measurement device is designed. The security of the communication is analyzed.

Quantum communication, especially quantum key distribution (QKD) is an important branch of quantum information[1] which provides a secure way for creating secret keys between the communication parties called Alice and Bob. In the past few years, quantum key distribution has progressed quickly[2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15] since Bennett and Brassard presented the BB84 QKD protocol[2]. Some QKD protocols are based on the density coding using Bell states[15]. One such protocol is the two-step protocol in which the communication is realized in two steps[16, 17], it can not only be used for QKD, but also for quantum secure direct communication in which information is transmitted directly without using another classical communication of the ciphered text[14, 17, 18, 19, 20, 21]. Meanwhile experiments on quantum dense coding based on entangled photon pairs and continuous variables have also been reported in [22, 23, 24].

Entangled photon pairs have become a very important resource for quantum communication. Experimentally it is usually produced by using spontaneous parametric down conversion (SPDC) [25], in which a nonlinear crystal pumped by lasers to create two photons marked with idler (i) and signal (s) respectively under the phase match conditions: \( \omega_i = \omega_s + \omega_k \). These entangled photons are usually the polarization entangled photons. Recently, Ravaro et al have experimentally demonstrated the generation of frequency and polarization doubly entangled photons (DEPs) using AlGaAs multi-layer waveguide[26]. The high conversion efficiency of the DEP pair has attracted much attention in recent years. It is interesting to study the quantum communication protocols based on them. In this paper, we present a secure communication scheme based upon the polarization and frequency DEPs. Because the DEP pairs possess an additional degree of freedom in frequency, this protocol also has the advantage of a higher capacity of coding.

DEP and its application was first introduced by Aolita and Walborn[27]. Using nonlinear AlGaAs waveguide, Ravaro et al proposed a scheme theoretically of generating two counter propagating photons in the state:

\[
|\text{ent}\rangle = \frac{\eta_1}{\sqrt{\eta_1^2 + \eta_2^2}} |\omega_s, H\rangle |\omega_i, V\rangle + \frac{\eta_2}{\sqrt{\eta_1^2 + \eta_2^2}} |\omega_s', V\rangle |\omega_i', H\rangle, \tag{1}
\]

where \( |H\rangle (|V\rangle) \) represents the horizontal polarization of the electric field and if \( \eta_1 = \eta_2 \), the state becomes frequency and polarization maximally double entangled state.

Now we focus on the maximally entangled doubly entangled states. First lets inspect the following state

\[
\Psi_{ab}^+ = \frac{1}{\sqrt{2}} (|H, \omega_s\rangle |V, \omega_i\rangle + |V, \omega_s'\rangle |H, \omega_i'\rangle), \tag{2}
\]
as said before, \( H \) and \( V \) are polarizations of photons, \( \omega_s \), \( \omega_i \), \( \omega_s' \), \( \omega_i' \) are the frequencies of the signal and idler photons. Instead of producing entangled photons with fixed frequencies in the polarization (singly) entangled photon state, doubly entangled photon pairs have two possible frequency combinations, namely it can either state \( \omega_s \) with frequencies \( \omega_s \) and \( \omega_i \), or with frequencies \( \omega_s' \) and \( \omega_i' \). Photon \( a \) is either in the state \( |H, \omega_s\rangle \) or in \( |V, \omega_s'\rangle \), photon \( b \) is either in the state \( |H, \omega_i\rangle \) or in \( |V, \omega_i'\rangle \).

Local unitary operations acting on the two particles transforms the state \( \Psi_{ab}^+ \) to other eight bases shown below

\[
\Phi_{ab}^+ = \frac{1}{\sqrt{2}} (|H, \omega_s\rangle |H, \omega_i\rangle \pm |V, \omega_s'\rangle |V, \omega_i'\rangle); \tag{3}
\]
\[
\Psi_{ab}^\pm = \frac{1}{\sqrt{2}} (|H, \omega_s\rangle |V, \omega_i\rangle \pm |V, \omega_s'\rangle |H, \omega_i'\rangle); \tag{4}
\]
\[
\Gamma_{ab}^\pm = \frac{1}{\sqrt{2}} (|V, \omega_s\rangle |H, \omega_i\rangle \pm |H, \omega_s'\rangle |V, \omega_i'\rangle); \tag{5}
\]
The correspondence between unitary operation and state is given in Table I. As listed in the table, $U_{ab} \otimes U_b$ operation transforms state $\Psi_{ab}^\pm$ to a unique state in the basis set, and Alice and Bob agree that each state corresponds to a 3-bit code word given in the table.

The two-step QKD protocol based on DEPs is described in detail as follows.

Two-step Double-QKD protocol

(Step 1): Alice, the sender, prepares a series of DEP pairs in $\Psi_{a,b}^{\pm}$. For each state, She makes a unitary operation on photon $b$. The operations are chosen from the group $\{I, \sigma_x, \sigma_z, i\sigma_y\}$. This transforms state $\Psi_{a,b}^\pm$ into one of the state in $\{\Phi_{a,b}^\pm, \Psi_{a,b}^\pm\}$. She then sends the $b$-photon sequence $\{P_1(b), P_2(b), \cdots P_n(b)\}$ to Bob.

(Step 2): Bob receives and saves the photons. Then they perform a security check, the strategy will be discussed later.

(Step 3): Alice and Bob check Eve by public comparison. If the error rate is lower than a security bound, they confirm that the first round communication is secure.

(Step 4): Alice encodes key information on the $a$ photons using $\{I, \sigma_x, \sigma_z, i\sigma_y\}$ operations. After that, she sends the $a$ photons sequence $\{P_1(a), P_2(a), \cdots P_n(a)\}$ to Bob.

(Step 5): Bob receives the photons and makes joint measurement to distinguish the eight different states.

Next we focus on the devices for state measurement. The measurement device is shown in Fig. 1. When a DEP enters Bob’s measurement device, one to the left wavelength-division multiplexing device (WDM) and the other to the right, photons with different frequencies can be distinguished. The two photons leave the port of each WDM in either the $\omega_a/\omega_b$ port or the $\omega_b/\omega_a$ port.

Then, each photon passes through a polarizing beam splitter and enters a device shown in Fig. 2 being detected by one of the two detectors at each port. Table I gives the correspondence between the ports at which detectors triggered and the states. Here there is still a two-fold degeneracy, and it is further distinguished by the two detectors at each port.

At each port, we place a wavelength converter and make a $\sigma_z$ measurement as shown in Fig. 2.

In the measurement for example, at time $t$ the detectors on port 1 and port 4 triggers, Bob knows the state are $\Psi_{a,b}^\pm$ triggers on port 3, 2 indicates $\Gamma_{a,b}^\pm$ and so on. Then passing the two photons to wavelength converter $28$. Then the state degenerates to Bell state: $\Psi_{Bell} = \frac{1}{\sqrt{2}}(|H,V\rangle \pm |V,H\rangle)$. Following a quarter wave plate Bob transforms the $H, V$ bases to $|+x\rangle = |H\rangle + |V\rangle, |−x\rangle = |H\rangle − |V\rangle$ bases. In this way, the states $\Psi_{a,b}^\pm$ can be transformed to $|+x\rangle|+x\rangle−|−x\rangle|−x\rangle$ and $|+x\rangle|−x\rangle−|−x\rangle|+x\rangle$. Bob performs a $\sigma_x \otimes \sigma_x$ measurement and distinguishes the state completely since the polarizations are either parallel or antiparallel. The other three bases can also be distinguished. In this way, the eight states can be completely distinguished.

There are two security checking strategies in our scheme, one is “decoy state” strategy and the other is wavelength convertor strategy.

The “decoy state” scheme uses single photons. When the communication starts, Alice inserts single photons with frequency $\omega_i$ or $\omega_i'$ and polarization state $|H\rangle, |V\rangle, |H+V\rangle, |H−V\rangle$ randomly in the $b$ photon series as “decoy state” photons. After Bob receives the photons, Alice announces the positions of the single photons publicly. Bob chooses to measure them either in the $\sigma_i$ or the $\sigma_z$ bases. Bob then declares the bases and Alice checks which bases they match. If they happen to choose the same bases, their results are in agreement. If there are eavesdropping, there will be error. Thus after comparison, if the error rate is lower than a security bound, they confirm that the communication is secure. Otherwise they drop the data and restart the communication.

In the wavelength convertor strategy, after Bob receives $b$ photons from Alice, he chooses some photons randomly and performs a wavelength conversion. He then announces the positions of photons to Alice. Alice performs the same operation. The wavelength of $\omega_{a,i}$ or $\omega_{a,i}'$ and $\omega_{b,i}, \omega_{b,i}'$ turn to $\omega_{a0} = \min(\omega_{a,i}, \omega_{a,i}')$, $\omega_{b0} = \min(\omega_{b,i}, \omega_{b,i}')$ respectively. In this way, the DEPs degenerate to Bell states:

$$|\Phi_{Bell}^\pm\rangle = \frac{1}{\sqrt{2}}(|HH\rangle \pm |VV\rangle);$$

$$|\Psi_{Bell}^\pm\rangle = \frac{1}{\sqrt{2}}(|HV\rangle \pm |VH\rangle).$$

They both choose to measure the photons in either $\sigma_z$ or $\sigma_x$ bases, and announce the bases so as to pick up the matched particles. Finally they compare the results and check the error rate to confirm the security.

In the following we will analyze the security of the protocol under the intercept-resend (IR) attack. Eve hides herself in the communication channel and intercepts the “$V$” photons and measures them, and then prepares a series of photons according to her measurement, and resends them to Bob. Because the interception destroys the entanglement between the $a$ and $b$ photons, Alice and Bob may discover the eavesdropping in the second security checking method. Also this IR attack will introduce 25% of errors on the “decoy state” photons, so it is easily to be discovered by the first security checking method.

In conclusion, we have proposed a two-step QKD protocol using DEPs. The measurement device in distinguishing the eight states is designed. We also made an analysis on its security. Since the DEPs has a higher generation efficiency, it maybe an alternative entanglement source for communication.

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TABLE I: The correspondence between unitary operations and states

| key operation state | key operation state |
|---------------------|---------------------|
| 000 \( I \otimes I \) \( \Psi^+ \) | 100 \( \sigma_z \otimes I \) \( \Upsilon^+ \) |
| 001 \( I \otimes \sigma_z \) \( \Psi^- \) | 101 \( \sigma_z \otimes \sigma_z \) \( \Upsilon^- \) |
| 010 \( I \otimes \sigma_x \) \( \Phi^+ \) | 110 \( \sigma_x \otimes \sigma_z \) \( \Gamma^+ \) |
| 011 \( I \otimes i\sigma_y \) \( \Phi^- \) | 111 \( \sigma_z \otimes i\sigma_y \) \( \Gamma^- \) |
| 000 \( \sigma_z \otimes I \) \( \Psi^- \) | 100 \( i\sigma_y \otimes I \) \( \Upsilon^- \) |
| 001 \( \sigma_z \otimes \sigma_z \) \( \Psi^+ \) | 101 \( i\sigma_y \otimes \sigma_z \) \( \Upsilon^+ \) |
| 010 \( \sigma_x \otimes \sigma_z \) \( \Phi^+ \) | 110 \( i\sigma_y \otimes \sigma_z \) \( \Gamma^+ \) |
| 011 \( \sigma_z \otimes i\sigma_y \) \( \Phi^- \) | 111 \( i\sigma_y \otimes i\sigma_y \) \( \Gamma^- \) |

TABLE II: The correspondence between ports in Bell state measurement device and states

| Triggered Port | Corresponding states |
|----------------|----------------------|
| 1,2            | \( \Phi^\pm \)       |
| 1,4            | \( \Psi^\pm \)       |
| 3,2            | \( \Gamma^\pm \)     |
| 3,4            | \( \Upsilon^\pm \)   |

FIG. 1: Bell state measurement device.

FIG. 2: \( \hat{X} \) bases measurement device. Here QWP is the quarter wave plate and PBS+ is the \( \sigma_x \) basis polarizing beam splitter.