Single Epoch GPS Deformation Signals Extraction and Gross Error Detection Technique Based on Wavelet Transform

WANG Jian  GAO Jingxiang  XU Changhui

**ABSTRACT** Wavelet theory is efficient as an adequate tool for analyzing single epoch GPS deformation signal. Wavelet analysis technique on gross error detection and recovery is advanced. Criteria of wavelet function choosing and Mallat decomposition levels decision are discussed. An effective deformation signal extracting method is proposed, that is wavelet noise reduction technique considering gross error recovery, which combines wavelet multi-resolution gross error detection results. Time position recognizing of gross errors and their repairing performance are realized. In the experiment, compactly supported orthogonal wavelet with short support block is more efficient than the longer one when discerning gross errors, which can obtain more finely analyses. And the shape of discerned gross error of short support wavelet is simpler than that of the longer one. Meanwhile, the time scale is easier to identify.

**KEYWORDS** noise; single epoch GPS deformation signal; Mallat algorithm; gross error detection; gross error recovery

**CLC NUMBER** P228.42; P207

Introduction

GPS technique is widely used for deformation monitoring thanks for the high precision. Usually, there are three working modes associated with GPS deformation observation: periodical GPS deformation monitoring net, GPS monitoring array and real-time GPS deformation monitoring technique. The real-time GPS deformation monitoring technique is applicable to deformation body that owns fast deformation feature, fierce tendency change or saltation characteristics. Single epoch GPS scheme can get more credible real-time deformation information in aid of deformation control. Whereas high frequency single epoch GPS deformation signal is affected by various factors such as signal interruption, unstable troposphere, multi-path effects and strong electromagnetic field, which results in a lot of noises and some gross errors in the deformation signals. Wavelet analysis can effectively divide frequency by its bandpass filter function, moreover, it can also realize recognition through separating gross and noise in multi-resolution levels. The intrinsic characteristics of deformation signal and separation of deformation tendency can be described with multi-resolution analysis of wavelet transformation. Compared with traditional analysis methods, wavelet analysis has unique superiority.

1 Wavelet transform and multi-resolution analysis

1.1 Wavelet transform

The square-integrable function $\phi(t)$ ($t \in \mathbb{R}$),
meeting $c_\omega w^{-1} |\hat{\psi}(\omega)|^2 d\omega < + \infty$ is called a mother wavelet ($\hat{\psi}(\omega)$ denotes the Fourier transform of $\psi(t)$). The wavelet transform formula on $f(t) \in L^2(R)$ ($t \in R$) is \(^{[2]}\)

$$W_f(a,b) = \langle f(t), \psi_{a,b}(t) \rangle = \int_{-\infty}^{\infty} f(t) \overline{\psi_{a,b}(t)} dt$$

(1)

where $\psi_{a,b}(t)$ denotes the dilation by the scale factor $a$ and translation parameter $b$; $\overline{\psi_{a,b}(t)}$ is complex conjugate function of $\psi_{a,b}(t)$. $W_f(a,b)$ is translated coefficient of wavelet transform.

The transform results are decided by the wavelet function selection. The choice of wavelet function without unified criteria can be confirmed on the basis of trials considering characteristics of wavelet function. In this paper, we take "db2" wavelet as analysis function in the light of \(^{[2]}\) "dbN" wavelet functions have some excellences, such as orthogonality, compacted support block, approximate symmetry; \(^{[2]}\) "dbN" wavelet function can be applied to perform discrete wavelet transform with effectively Mallat fast decomposition and reconstruction algorithm; \(^{[2]}\) in order to satisfy the request of time precision for gross error detection, the selected “dbN” wavelet function with efficient support block length of $2N-1$ should be short; \(^{[2]}\) experimental result shows that “db2” wavelet function can recognize three types of gross errors mentioned.

1.2 Multi-resolution analysis

The principle for multi-resolution analysis (MRA) can be found in many literatures \(^{[2-4]}\). Supposing that scale function $\varphi(t) \in V_0$ is a orthonormal function. Let $\{c^{j+1}_i, j, k \in Z\}$ be the coefficients of $j+1$ decomposition level. The coefficients of $j$ decomposition level can be achieved with filter

$$c^{j+1}_i = \sum_k c^j_i h_{k-2^j}$$

(2)

$$d^j_i = \langle f^j, \psi^j_n \rangle = \sum_k c^j_i g_{k-2^j}$$

Wavelet coefficients of $j+1$ level may be reconstructed with reconstruction filter ($\tilde{h}_n, n \in Z$) and ($\tilde{g}_n, n \in Z$)

$$c^{j+1}_i = \sum_k c^j_i \tilde{h}_{k-2^j} + \sum_k d^j_i \tilde{g}_{k-2^j}$$

(3)

let $\omega$ be the highest frequency of the deformation signal, the low frequency band of $j$ level Mallat decomposition is between $0-2^{-j} \omega$ and $2^{-j} \omega-2^{-j+1} \omega$ for high frequency band. According to this dividing frequency ability, we can recognize the tendency items of the deformation by noise reduction algorithm with suitable threshold value for high frequency coefficients.

1.3 Wavelet noise reduction

In this paper, wavelet threshold noise reduction methodology is applied to extract real deformation signal. Evaluating wavelet transform coefficients at different level for different threshold value, real signal is restored by reconstruction using quantified wavelet coefficients with the thresholds. There are two methods (soft-threshold and hard-threshold) for wavelet coefficient quantification, which has been presented in many previous literatures. Deformation signal is obtained with soft-threshold noise reduction method with Heuristic SURE threshold $\lambda^{[5-7]}$.

2 Example analysis

Strong contaminated deformation signal can be considered including stochastic noise obeying certain kind of distribution and some gross errors while deformation signal is the tendency item of non-linear motion. Simulated 1 000 epoch deformation data with strong noise can be obtained by forcing 2 mm isolated gross error at 300th epoch, 1.5 mm dispersed gross error at 400th and 403th epoch, 2 mm regional gross error from 600th to 620th epoch (Fig. 1). Deformation tendency of signal is hardly discerned in Fig. 1. Regional gross error can be found in the 600th Epoch, but the position of isolated gross error and dispersed gross error can not be decided.

With the Mallat orthogonal wavelet decomposition algorithm, we try to decompose the signal at different levels with “db2” wavelet. Through comparing gross error recognizing effects and the
can be legibly found at 5th level with relatively low time resolution; isolated gross error could be embodied at the levels of $d_2$ and $d_4$, but the time resolution of $d_3$ level is higher than that of $d_5$ level, it means that the isolated gross error is easier recognized in the high frequency band.

The frequency band of dispersed gross error mainly centralized at levels of $d_1$ and $d_2$ and can be more easily recognized at the decomposed level of $d_4$ with higher resolution; regional gross error which demonstrates low frequency feature occurs at the levels of $d_3, d_4, d_5$ with higher time precision at level of $d_4$. Thus is with higher precision at the level of $d_4$ recognizing isolated gross error, while dispersed and regional gross error can be found at the level of $d_4$ with higher time precision.

The tendency of deformation signal (Fig. 3) is discerned by wavelet soft-threshold value noise reduction with the parameters listed in Table 1. The effect of wavelet noise reduction algorithm is better than median methodology (length of filter is 7) (Fig. 4).

| Table 1 Wavelet denoising threshold value of decomposition levels |
|---|
| Levels | 1 | 2 | 3 | 4 | 5 |
| Thresholds | 3.487 | 1.427 | 0.974 | 1.136 | 1.597 |

According to the distribution detail of the decomposed deformation signal with Mallat algorithm, approximation “$a_5$” item shows tendency signal but some distortion exists at 600th epoch caused by regional gross error. Regional gross error should be further confirmed on the basis of real deformation situation.

Component at $d_1$ decomposition level mainly made up of noise and gross error is partly found at level $d_5, d_4, d_3, d_2$; three kinds of gross errors
Tendency items of signal can be separated, as well saltatorial points of gross error can be reflected and time position can also be recognized accurately in Fig. 3. When it is unable to discern the isolated or dispersed gross errors with threshold noise reduction algorithm, we can distinguish these points with Mallat decomposition algorithm. Probing to concrete situation, many kinds of gross error recovery methods can be used after confirming the discerned gross errors. In this paper, linear interpolation method is adopted to isolated gross and dispersed gross recovery while polynomial interpolation method is applied to regional gross recovery, showing satisfactory recovery results (Fig. 5).

3 Conclusions

We demonstrate a methodology for recognizing gross error from strong contaminated deformation signal with Mallat orthogonal wavelet transform and wavelet noise reduction algorithm, and some conclusions can be drawn.

1) The decomposition and reconstruction algorithm with Mallat is practicable to recognize isolated and dispersed gross error and time position in the deformation signal, but regional gross error should be confirmed and then repaired according to concrete situation in respect that it may be judged as the tendency of deformation.

2) Wavelet soft-thresholding noise reduction method is better than medium value filter method for obtaining deformation signal and gross error points when applied to strong contaminated deformation signal.

3) Though we obtain a satisfactory result through simple interpolation method, we should employ different gross error recovery principle with the concrete situation in view.

4) Orthogonal wavelet with short support block is more efficient than the longer one when discerning deformation gross errors. And the shape of discerned gross error of short support block wavelet is simpler than that of the longer one, meanwhile the time scale is easier to identify.

REFERENCES

[1] Chen Yongqi, James L (1998) Development of the methodology for single epoch GPS deformation monitoring. [J]. Journal of Wuhan Technical University of Surveying and Mapping, 23(4):324-328 (in Chinese)
[2] Ran Qiwen (2001) Theory and application of wavelet transform and mark Fourier transform [M]. Harbin: Harbin Institute of Technology, 148-152 (in Chinese)
[3] Li Yuan (2001) Wavelet analysis and non-linear wavelet estimation of time sequence change point [M]. Beijing: China Statistics Publishing House (in Chinese)
[4] Marr D (1998) Theory of vision calculation [M]. Beijing: Science Publishing House
[5] Mallat S G (1989) A theory for multi-resolution signal decomposition: the wavelet representation [J]. IEEE Trans PAMI, 11:674-693
[6] He Jun, Yu Yalun (1997) Wavelet analysis and its application to signal processing [J]. Journal of University of Science and Technology, 4(3): 49-53
[7] Zheng Zhizhen (2001) Wavelet transform and Matlab toolbox application [M]. Beijing: Earthquake Press (in Chinese)