Component on-shell actions of supersymmetric 3-branes: I. 3-brane in $D = 6$

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Abstract

In the present and accompanying papers we explicitly construct the on-shell supersymmetric component actions for 3-branes moving in $D = 6$ and $D = 8$ within the nonlinear realizations framework. In the first paper we apply our scheme to construct the action of supersymmetric 3-brane in $D = 6$. It turns out that all ingredients entering the component action can be obtained almost algorithmically by using the nonlinear realizations approach. Within this approach, properly adapted to the construction of the on-shell component actions, we pay much attention to broken supersymmetry. Doing so, we were able to write the action in terms of purely geometric objects (vielbeins and covariant derivatives of the physical bosonic components), covariant with respect to broken supersymmetry. It turns out that all terms of the higher orders in the fermions are hidden inside these covariant derivatives and vielbeins. Moreover, the main part of the component action just mimics its bosonic cousin in which the ordinary space–time derivatives and the bosonic worldvolume are replaced by their covariant supersymmetric analogs. The Wess–Zumino term in the action, which does not exist in the bosonic case, can be also easily constructed in terms of reduced Cartan forms. Keeping the broken supersymmetry almost explicit, one may write the ansatz for the component action, fully defined up to two constant parameters. The role of the unbroken supersymmetry is just to fix these parameters.

Keywords: partial breaking of global supersymmetry, supersymmetric 3-brane, nonlinear realizations approach

1. Introduction

The usual treatment of superbranes as extended objects arising as solitons of supersymmetric field theories leads to the interpretation of the dynamics of branes fluctuations as field theories...
on the worldvolume of the superbrane. Thus, it seems to be important to have the explicit form of brane actions, preferably written in terms of covariant objects. Unfortunately, the gauge fixing procedure in the standard Green–Schwartz-type formulation is not unique. Moreover, the existing possibility to choose different worldvolume fermions makes the task of writing the full component superbrane actions with all fermions included rather complicated. The main problem is to decide which type of covariant objects has to be taken to write the actions. Clearly, even if we have at hand the proper superfield actions, after passing to the physical components the beauty of the superspace formulation will disappear, leaving us with a long tail of fermionic terms having no explicit geometric meaning.

The standard superspace description of the superbranes is based on the fact that usually the single branes preserve half of the target-space supersymmetries. Thus, it seems to be a rational choice to introduce the superfields under unbroken supersymmetry, imposing by hand the invariance of the action under broken supersymmetry. One of the methods heavily used to describe the superbranes is that of nonlinear realizations of spontaneously broken symmetries [1, 2]. In this approach, from the beginning we are dealing with the theory in the static gauge in which the worldvolume fields forming the proper supermultiplet of unbroken supersymmetry correspond to the physical modes of the branes. Within the nonlinear realizations approach one may easily construct the objects, the Cartan forms and covariant derivatives, which are covariant with respect to unbroken and broken supersymmetries [3–5]. Moreover, by imposing the proper constraints on the Cartan forms, which are similar to those used in the superembeddings approach [6], one may find the irreducibility conditions on the superfields involved and the covariant superfield equations of motion (see e.g. [7] and references therein). Nevertheless, one notices that the superfield action cannot be constructed within the nonlinear realizations approach. The reason is simple—in all known cases the superfield Lagrangian is not an invariant object. Instead, it is shifted by a total space–time derivative under supersymmetry transformations. Thus, the superfield action cannot be constructed from the Cartan forms which are the natural ingredients of this approach. Because of this, another approach in which the superfield Lagrangian is a component of the linear, with respect to both supersymmetries, supermultiplet has been elaborated [8–15].

Coming back to the nonlinear realizations method, one may wonder why the broken supersymmetry plays such a small role. Indeed, it has been well known since the pioneering papers by Volkov and Akulov [16] that the Goldstone fermions, accompanying the partial breaking of supersymmetry, cannot enter the component action in an arbitrary way. They may appear in the action either through the covariant derivatives or vielbeins, only3. Therefore, one may try to shift the attention to the broken supersymmetry while dealing with the component on-shell actions. Indeed, it was demonstrated in [19] that with a suitable choice of the parametrization of the coset, the \( \theta \) coordinates of the superspace together with the physical bosonic components do not transform under broken supersymmetry, while the physical fermions transform as the Goldstino of the Volkov–Akulov model. In such a situation, one should expect that all fermionic terms in the components can be ‘hidden’ inside the covariant derivatives and vielbeins, covariant with respect to broken supersymmetry. Therefore, one may consider the reduced coset, which does not contain the \( \theta \) coordinates at all. The corresponding reduced Cartan forms will be the building blocks for the component actions. In papers [19–22] we explicitly demonstrated that this is indeed the case and the component on-shell actions for low dimensional branes \((D < 4)\) can be written through proper covariant

3 It was explicitly demonstrated in [17, 18] that all known Goldstino models are related to the Volkov–Akulov model by non-trivial, higher nonlinear field redefinitions.
derivatives of the bosonic fields and the fermionic vielbeins. In the present paper we analyze the more interesting cases with the 3-brane moving in \( D = 6 \) and \( D = 8 \). In the case of 3-branes the worldvolume effective actions will be the actions for standard four-dimensional field theories, while the corresponding superfields will be the most interesting ones: the chiral superfields (in \( D = 6 \)) and hypermultiplet (in \( D = 8 \)).

This paper is divided into two parts. In this first part, in section 2, we will find the superfield equations of motion and the transformation properties of the coordinates and superfields within the standard method of nonlinear realizations. Then, in section 3, we will explicitly construct the component action of the 3-brane in \( D = 6 \) and check its invariance with respect to broken and unbroken supersymmetries.

The second part of the paper will be devoted to the construction of the 3-brane action in \( D = 8 \) with the hypermultiplet as the Goldstone superfield.

2. 3-brane in \( D = 6 \)

It is well known that the action of the \( N = 1 \) supersymmetric 3-brane in \( D = 6 \) is just a minimal action for the Goldstone chiral \( N = 1, d = 4 \) superfields accompanying the spontaneous breaking of \( N = 1, D = 6 \) supersymmetry down to \( N = 1, d = 4 \) one, or, in other words, the breaking of \( N = 2 \) supersymmetry to \( N = 1 \) in four dimensions [8, 10, 11, 23]. In principle, this information is enough to construct the superfield action of the 3-brane in terms of chiral \( N = 1, d = 4 \) superfields. The existence of hidden \( N = 1 \) supersymmetry completely fixes the action. However, passing to the on-shell component action almost completely destroys its very nice superfield form. Moreover, it seems there is no way to combine the long tail of fermionic terms in a more readable form, which would reflect the presence of higher symmetry in the theory. Our goal in this section is to demonstrate that one may reformulate the on-shell component action in such a way that all its ingredients will have clear geometric properties. The basic tool for this is the method of nonlinear realizations [1, 2] adopted for this task in [20]. Our procedure includes three steps: (a) construction of the superfield equations of motion, (b) checking the bosonic action, (c) construction of the full component action.

2.1. Superfield equations of motion

From the \( d = 4 \) standpoint the \( N = 1, D = 6 \) supersymmetry algebra is a two central charge extended \( N = 2 \) Poincaré superalgebra with the following basic relations

\[
\{ Q_a, \bar{Q}_\alpha \} = \{ S_a, \bar{S}_\alpha \} = 2 \sigma^\alpha \mu P_\mu, \quad \{ Q_a, S_\beta \} = 2 \epsilon_{a\beta\gamma} Z, \quad \{ \bar{Q}_\alpha, S_\beta \} = 2 \epsilon_{a\alpha\beta} \bar{Z}.
\]

(2.1)

As a reminder about its six-dimensional nature, the superalgebra (2.1) possesses the \( so(1, 5) \) automorphism algebra. Again, from the \( d = 4 \) point of view, \( so(1, 5) \) algebra contains the \( d = 4 \) Lorentz algebra generated by \( L_{\mu\nu} \) \( U(1) \) subalgebra with generator \( U \) and the generators \( K_A, \bar{K}_A \) from the coset \( SO(1, 5)/SO(1, 3) \times U(1) \). The full set of commutation relations can be found in appendix A.

Keeping the \( d = 4 \) Lorentz and \( U(1) \) symmetries linearly realized, we will choose the coset element as

\[
g = e^{i\theta^A P_A} e^{i\theta^{\bar{A}}} \bar{P}_{\bar{A}} e^{i\phi^A S_A + i\bar{\phi}^{\bar{A}}} \bar{S}_{\bar{A}} e^{i\{ \phi^{\bar{A}} + \bar{\phi}^{A} \} + i(\bar{Z} + \bar{\bar{Z}})} e^{i(\rho^A K^A + \bar{\rho}^{\bar{A}} \bar{K}_{\bar{A}})}
\]

(2.2)
Here, we associated the $N=1$, $d=4$ superspace coordinates $x^A, \theta^\alpha, \bar{\theta}^\dot{\alpha}$ with the generators $P_\alpha, Q_\mu, \bar{Q}_\dot{\mu}$ of unbroken $N=1$ supersymmetry. The remaining coset parameters are Goldstone superfields, $\psi^a(x, \theta, \bar{\theta}), \bar{\psi}^a(x, \theta, \bar{\theta}), \phi(x, \theta, \bar{\theta}), \bar{\phi}(x, \theta, \bar{\theta}), \lambda^A(x, \theta, \bar{\theta})$ and $\bar{\lambda}^\dot{A}(x, \theta, \bar{\theta})$.

The transformation properties of the coordinates and superfields with respect to all symmetries can be found by acting from the left on the coset element $g$ (2.2) by the different elements of $N=2, d=4$ Poincaré supergroup. In particular, for the unbroken $(Q, \bar{Q})$ and broken $(S, \bar{S})$ supersymmetries we have

- **Unbroken supersymmetry:**

  \[
  \delta_Q x^A = i \left( \epsilon^a \bar{\theta}^\dot{a} + \bar{\epsilon}^\dot{a} \theta^a \right) \left( \sigma^A \right)_{a\dot{a}}, \quad \delta_Q \theta^a = \epsilon^a, \quad \delta_Q \bar{\theta}^{\dot{a}} = \bar{\epsilon}^{\dot{a}}. \quad (2.3)
  \]

- **Broken supersymmetry:**

  \[
  \delta_S x^A = i \left( \epsilon^a \bar{\psi}^\dot{a} + \bar{\epsilon}^{\dot{a}} \psi^a \right) \left( \sigma^A \right)_{a\dot{a}}, \quad \delta_S \psi^a = \epsilon^a, \quad \delta_S \bar{\psi}^{\dot{a}} = \bar{\epsilon}^{\dot{a}}, \quad \delta_S \phi = 2i \epsilon_a \theta^a, \quad \delta_S \bar{\phi} = 2i \bar{\epsilon}_{\dot{a}} \bar{\theta}^{\dot{a}}. \quad (2.4)
  \]

The local geometric properties of the system are specified by the Cartan forms. The purely technical calculations of these forms, semi-covariant derivatives and their algebra are summarized in appendix A.

The next tasks we are going to perform within the superfield approach, include

- imposing the covariant irreducibility constraints on the superfields;
- reduction of the number of independent superfields;
- finding the covariant equations of motion.

As we already demonstrated in [20], all these tasks can be solved simultaneously by imposing the following constraints on the Cartan forms

\[
\omega_Z = \bar{\omega}_Z = 0, \quad (2.5)
\]

\[
\omega_S \big| = \bar{\omega}_{\bar{S}} \big| = 0, \quad (2.6)
\]

where $|$ means the $d\theta$ and $d\bar{\theta}$-projections of the forms. These constraints are similar to superembedding conditions (see e.g. [6] and references therein).

The constraints (2.5) are purely kinematical ones, and they result in the following equations

\[
\nabla_d \varphi = 0, \quad \nabla_d \bar{\varphi} = -2i \psi_d, \quad \nabla_A \varphi = 2 \frac{\left( 2 + \lambda \bar{\lambda} \right) \lambda_A - \lambda^2 \bar{\lambda}_A}{(2 + \lambda \bar{\lambda})^2 - \lambda^2 \bar{\lambda}^2}, \quad (2.7)
\]

\[
\nabla_d \bar{\varphi} = 0, \quad \nabla_d \bar{\psi}_d = -2i \bar{\psi}_d, \quad \nabla_A \bar{\varphi} = 2 \frac{\left( 2 + \lambda \bar{\lambda} \right) \bar{\lambda}_A - \bar{\lambda}^2 \lambda_A}{(2 + \lambda \bar{\lambda})^2 - \lambda^2 \bar{\lambda}^2},
\]

where $\lambda_A$ and $\bar{\lambda}_A$ are defined in (A.9). As we can see, these equations (2.7) allow us to express the superfields $\psi_d, \bar{\psi}_d$ and $\lambda_A, \bar{\lambda}_A$ through the covariant derivatives of $\varphi$ and $\bar{\varphi}$ (this is the so called inverse Higgs phenomenon [24]). In addition, the superfields $\varphi$ and $\bar{\varphi}$ are subjected to the covariant (anti)chirality conditions. Thus, the bosonic, covariantly (anti)chiral Goldstone superfields $\varphi$ and $\bar{\varphi}$ are the only essential superfields needed for this case of partial breaking of the global supersymmetry.
The situation with the constraints (2.6) is more interesting. First, the $d\theta$ ($d\bar{\theta}$) projection of the form $\omega_S (\bar{\omega}_S)$ relates the spinor derivative of the superfield $\psi (\bar{\psi})$ and $x$-derivative of the superfield $\phi (\bar{\phi})$

$$J_{a\alpha} \equiv \nabla_{a\alpha} \rho = \partial_{a\alpha} \rho + \cdots, \quad I_{a\alpha} \equiv \nabla_{a\alpha} \bar{\psi} = \partial_{a\alpha} \bar{\psi} + \cdots, \quad (2.8)$$

where we explicitly write only the leading, linear in $\partial \rho$ and $\partial \bar{\rho}$ terms. At the same time, the $d\theta$ ($d\bar{\theta}$) projection of the form $\omega_S (\bar{\omega}_S)$ gives the equations

$$\nabla_{a\alpha} \rho = 0, \quad \nabla_{a\alpha} \bar{\psi} = 0. \quad (2.9)$$

To see that these equations are really equations of motion, note that from (2.6) and the algebra of covariant derivatives (A.14) it follows that now

$$\{ \nabla_{a\alpha}, \nabla_{b\beta} \} = 0, \quad \{ \nabla_{a\alpha}, \nabla_{b\beta} \} = 0, \quad (2.10)$$

and, therefore, from (2.7) and (2.9) we conclude that

$$\nabla^a \nabla_{a\alpha} \rho = 0, \quad \nabla^a \nabla_{a\alpha} \bar{\psi} = 0. \quad (2.11)$$

This is the covariant form of the superfield equations of motion, which is a proper covariantization of the free equations of motion.

In the next sections we will construct the on-shell component action for our 3-brane which gives the same component equations which follow from (2.11).

To close this section, note that the explicit form of the equations (2.8), which follows from the forms (2.6) is not very illuminating. It is possible to get simpler expressions for $J_{a\alpha}$ and $I_{a\alpha}$ as follows. First of all, using the equations (2.9), one may rewrite the anti-commutator $[ \nabla_{a\alpha}, \nabla_{b\beta} ] (A.14)$ as

$$\{ \nabla_{a\alpha}, \nabla_{b\beta} \} = -2i \left( \nabla_{a\alpha} + J^\beta_{a\alpha} \nabla_{\beta} \right). \quad (2.12)$$

Acting by this anti-commutator on $\rho$ and $\bar{\rho}$ and using (2.7) one may get\footnote{After passing to 4-vector notations, one should take into account, that $\nabla_{a\alpha} \rho$ is proportional to either $J_A$ or $\bar{J}_A$. Thus, the term with $\epsilon_{abcd} J_c \nabla_{d\alpha} \rho$ is zero.}

$$J_A = \left( 1 - J^{\beta} J^\beta \right) \nabla_A \rho + J^B \nabla_B \bar{\rho} J_A, \quad (2.13)$$

These equations can be easily solved for $\nabla_A \rho$, $\nabla_A \bar{\rho}$ as

$$\nabla_A \rho = \frac{J_A - J^2 J_A}{1 - J^2 J^2}, \quad \nabla_A \bar{\rho} = \frac{\bar{J}_A - \bar{J}^2 \bar{J}_A}{1 - \bar{J}^2 \bar{J}^2}, \quad (2.14)$$

where $J^2 = J^A J_A$ and $\bar{J}^2 = \bar{J}^A \bar{J}_A$. The expressions for $J_A$, $\bar{J}_A$ are more complicated to be

$$J_A = \nabla_A \rho + \frac{2 (\nabla \bar{\rho})^2 \nabla \bar{\rho}}{1 - 2 \nabla \rho \cdot \nabla \bar{\rho} + \sqrt{(1 - 2 \nabla \rho \cdot \nabla \bar{\rho})^2 - 4 (\nabla \rho)^2 (\nabla \bar{\rho})^2}}, \quad \bar{J}_A = (J_A)^\dagger. \quad (2.15)$$
3. Component action

As we already noted in the introduction, it is not clear how to construct the superfield action within the nonlinear realizations approach. For the supersymmetric 3-brane in \( D = 6 \) the corresponding superfield actions were constructed within different frameworks in \([8, 10]\). Nevertheless, the component action, being constructed from the superfield one, is very complicated. It contains a lot of fermionic terms with completely unclear geometric structure. Alternatively, as we demonstrated in \([20]\), the component actions can be constructed within the nonlinear realizations approach in such a way that the invariance with respect to broken supersymmetry becomes almost evident. The useful ingredients for this construction include the reduced Cartan forms and reduced covariant derivatives, covariant with respect to broken supersymmetry only. The basic steps of our approach are

- construction of the bosonic action,
- covariantization of the bosonic action with respect to broken supersymmetry,
- construction of the Wess–Zumino terms,
- imposing the invariance with respect to unbroken supersymmetry.

Let us perform all these steps for the supersymmetric 3-brane in \( D = 6 \).

3.1. Bosonic action

In principle, the bosonic equations of motion can be extracted from the superfield equations (2.11). But the calculations are rather involved. Instead, one can construct the corresponding action directly, using the fact that such an action should possess invariance with respect to \( D = 6 \) Poincaré symmetry spontaneously broken to \( d = 4 \). One of the key ingredients of such a construction is the bosonic limit of the Cartan forms (A.11) which explicitly reads

\[
\begin{align*}
(\omega_P)_\text{bos}^A &= dx^B \left( \frac{1 + y/2}{1 - y/2} \right)_B^A - 2 \left( d\phi^A + d\bar{\phi}^A \right) \left( \frac{1}{1 - y/2} \right)_B^A, \\
(\omega_Z)_\text{bos}^B &= d\phi + \left( d\phi^A + d\bar{\phi}^A \right) \left( \frac{1}{1 - y/2} \right)_B^A \bar{\lambda}_B - dx^A \left( \frac{1}{1 - y/2} \right)_A^B \bar{\lambda}_B, \\
(\bar{\omega}_Z)_\text{bos}^B &= d\bar{\phi} + \left( d\phi^A + d\bar{\phi}^A \right) \left( \frac{1}{1 - y/2} \right)_B^A \lambda_B - dx^A \left( \frac{1}{1 - y/2} \right)_A^B \lambda_B.
\end{align*}
\]

(3.1)

where \( y_A^B = \lambda_A \lambda_B + \bar{\lambda}_A \bar{\lambda}_B \). Imposing now the same constraint (2.5)

\[
(\omega_Z)_\text{bos} = (\bar{\omega}_Z)_\text{bos} = 0,
\]

we will get the bosonic analog of the relations (2.7)

\[
\partial_A \phi = 2 \left( 2 + \bar{\lambda} \lambda \right) \lambda_A - \bar{\lambda}^2 \lambda_A \left( 2 + \bar{\lambda} \lambda \right)^2 - \lambda^2 \bar{\lambda}_A \left( 2 + \lambda \bar{\lambda} \right)^2 - \bar{\lambda}^2 \lambda_A \left( 2 + \lambda \bar{\lambda} \right)^2.
\]

(3.2)

Plugging these expressions in the form \((\omega_P)_\text{bos}^A\) (3.1) and using the explicit expression for the matrix \( \left( \frac{1}{1 - y/2} \right)_A^B \),
\[
\left( \frac{1}{1 - y/2} \right)_A^B = \delta_A^B + \frac{2 - \lambda \lambda}{(2 - \lambda \lambda)^2 - \lambda^2 \lambda^2} \left( \frac{\lambda A \lambda^B + \lambda \lambda A \lambda^B}{2 - \lambda \lambda} \right) + \frac{2^2 \lambda A \lambda^2 + \lambda^2 \lambda \lambda}{(2 - \lambda \lambda)^2 - \lambda^2 \lambda^2},
\]  
(3.3)

one may obtain
\[
(\omega_P)_A^B = d \epsilon^B_{aB} 
= d \epsilon^B_{aB} \left( \delta_B^A - 2 \frac{2 + \lambda \lambda}{(2 + \lambda \lambda)^2 - \lambda^2 \lambda^2} \right) 
\]  
(3.4)

Now, the unique invariant which can be constructed from the forms \((\omega_P)_A^B\) is a volume form which explicitly reads
\[
\epsilon_{ABCD} (\omega_P)_A^B \wedge (\omega_P)_B^C \wedge (\omega_P)_C^D \sim d^4 x \det(e) = d^4 x \frac{2 - \lambda \lambda}{(2 - \lambda \lambda)^2 - \lambda^2 \lambda^2}.
\]  
(3.5)

Finally, the bosonic action, being rewritten in terms of \(\partial A\phi\) and \(\partial A \bar{\phi}\) acquires the form
\[
S_{bos} = \int d^4 x \sqrt{\left(1 - 2(\partial \phi \partial \bar{\phi})^2 - 4(\partial \bar{\phi} \partial \phi)(\partial \phi \partial \bar{\phi})\right).}
\]  
(3.6)

This is the static gauge Nambu–Goto action for the 3-brane in \(D = 6\). One may explicitly check that the action (3.6) is invariant with respect to \(K_A, K_A\) transformations from the coset \(SO(1, 5)/SO(1, 3) \times U(1)\) realized as
\[
\delta \phi = 2x^A \varphi + 2\alpha x^A \varphi, \quad \delta \bar{\varphi} = \alpha x^A \bar{\phi}, \quad \delta \phi = \alpha x^A, \quad \delta \bar{\phi} = \bar{\alpha} x^A, 
\]  
(3.7)

and therefore, it is invariant with respect to the whole \(D = 6\) Poincaré group.

### 3.2. Covariantization with respect to broken supersymmetry

In contrast with the standard approach, in which the superfields with respect to unbroken \((Q, \bar{Q})\) supersymmetry play the main role and are the building blocks for the superfield actions, in the component approach we prefer to concentrate on the broken \((S, \bar{S})\) supersymmetry. Thus, the first task is to modify the bosonic action (3.6) in such a way to achieve invariance with respect to broken supersymmetry. Due to the transformation laws (2.4), the coordinates \(x^A\) and the first components of the superfields \(\varphi, \bar{\varphi}, \psi, \bar{\psi}\) transform under broken supersymmetry as follows
\[
\delta x^A = 2\bar{x}^A \varphi + 2\alpha x^A \bar{\varphi}, \quad \delta \varphi = \alpha x^A, \quad \delta \bar{\varphi} = \bar{\alpha} x^A, 
\]  
(3.8)

Thus, the volume \(d^4 x\) and the derivatives \(\partial A \varphi, \partial A \bar{\varphi}\) are not the covariant objects. To find the proper objects, let us consider the reduced coset element (2.2)
\[
\delta_{\text{red}} = e^{i x^A \varphi^a + i \bar{x}^a \bar{\varphi}}, \quad e^{i (\bar{\varphi} \varphi - \bar{\varphi} \varphi)},
\]  
(3.9)

where the fields \(\psi, \bar{\psi}, \varphi, \bar{\varphi}\) depend on the coordinates \(x^A\) only. The corresponding reduced Cartan forms (A.6) read
These forms are invariant with respect to transformations (3.8). Therefore, the covariant $x$-derivative will be

$$D_A = \left(\mathcal{E}^{-1}\right)^B_A \partial_B,$$

while the invariant volume can be constructed from the forms $(\omega_P)_{\text{red}}$. Thus, the proper covariantization of the action (3.6), having the right bosonic limit, will be

$$S_1 = \int \mathrm{d}^4x \, \det(\mathcal{E}) \sqrt{\left(1 - 2(\mathcal{D}_\mathcal{E})\right)^2 - 4(\mathcal{D}_\mathcal{E})\mathcal{D}_\mathcal{E}(\mathcal{D}_\mathcal{E})}.$$  

The action $S_1$ (3.12) reproduces the fixed kinetic terms for bosons and fermions

$$S_1 = \int \mathrm{d}^4x \left[-i(\psi^a \partial_{x^a} \psi^a + \bar{\psi}^a \partial_{x^a} \bar{\psi}^a) - 2\partial_A \phi \partial_A \phi\right].$$

This would be too strong to maintain unbroken supersymmetry. Therefore, we have to introduce one more, evidently invariant, action

$$S_2 = \alpha \int \mathrm{d}^4x \, \det(\mathcal{E}).$$

Thus, our ansatz for the invariant supersymmetric action of the 3-brane acquires the form

$$S = S_0 + S_1 + S_2 = (1 + \alpha) \int \mathrm{d}^4x - \int \mathrm{d}^4x \, \det(\mathcal{E}) \times \left[\alpha + \sqrt{1 - 2(\mathcal{D}_\mathcal{E})\mathcal{D}_\mathcal{E}(\mathcal{D}_\mathcal{E})} - 4(\mathcal{D}_\mathcal{E})\mathcal{D}_\mathcal{E}(\mathcal{D}_\mathcal{E})\right].$$

where $\alpha$ is a constant that has to be defined, and we have added the trivial invariant action

$$S_0 = \int \mathrm{d}^4x \, \partial_{x^a} \phi \partial_A \phi = 0.$$ 

### 3.3. Wess–Zumino term

The construction of the Wess–Zumino term, which is not strictly invariant but is shifted by a total derivative under broken supersymmetry (3.8), goes in a standard way [25]. First, one has to determine the close five form $\Omega_5$, which is invariant under $d = 4$ Lorentz and broken supersymmetry transformations (3.8). Moreover, in the present case this form has to disappear in the bosonic limit, because our ansatz for the action (3.15) already reproduces the proper bosonic action of the 3-brane (3.6). Such a form can be easily constructed in terms of the Cartan forms (3.10):

$$\Omega_5 = \omega_5 \wedge \omega_Z \wedge \omega_\mathcal{E}^5 \wedge \omega_\mathcal{E}^6 \wedge \omega_\mathcal{E}^7 (\sigma_{A})_{\alpha\beta} = \mathrm{d}\phi \wedge \mathrm{d}\phi \wedge \mathrm{d}\phi \wedge \mathrm{d}\phi \wedge \mathrm{d}\phi (\sigma_{A})_{\alpha\beta}.$$

To see that $\Omega_5$ (3.16) is indeed a closed form, one should take into account that the exterior derivative of $(\omega_P)_{\text{red}}$ is given by the expression

$$\mathrm{d}(\omega_P)_{\text{red}} \sim (\omega_5)_{\alpha} \wedge (\omega_5)_{\beta} = \mathrm{d}\phi \wedge \mathrm{d}\phi,$$

where

$$\mathrm{d}(\omega_P)_{\text{red}} \sim (\omega_5)_{\alpha} \wedge (\omega_5)_{\beta} = \mathrm{d}\phi \wedge \mathrm{d}\phi.$$
and, therefore, $d\Omega_4 = 0$, because
\[
d\psi^\alpha \wedge d\psi_\alpha = d\bar{\psi}^\alpha \wedge d\bar{\psi}_\alpha = 0.
\]
Next, one has to write $\Omega_5$ as the exterior derivative of a 4-form $\Omega_4$:
\[
\Omega_5 = d\Omega_4 \Rightarrow \Omega_4 = d\phi \wedge \left(\psi^\alpha d\bar{\psi}^\alpha + \bar{\psi}^\alpha d\psi^\alpha\right) \wedge \alpha^A \left(\sigma_A\right)_{\alpha\alpha}.
\]
Finally, the Wess–Zumino term is given by
\[
S_{WZ} = \int \Omega_4 = \int d^4x \det(E) e^{ABCD} D_A \psi^\alpha D_B \bar{\psi}^\alpha \left(\psi^\alpha D_C \bar{\psi}^\alpha + \bar{\psi}^\alpha D_C \psi^\alpha\right) \left(\epsilon^{-1}\right)_D E \left(\sigma_E\right)_{\alpha\alpha}.
\]
Before going further, let us note that the Wess–Zumino term (3.19) could be equivalently represented as
\[
S_{WZ} = \int d^4x \det(E) e^{ABCD} D_A \psi^\alpha D_B \bar{\psi}^\alpha \left(\psi^\alpha D_C \bar{\psi}^\alpha + \bar{\psi}^\alpha D_C \psi^\alpha\right) \left(\epsilon^{-1}\right)_D E \left(\sigma_E\right)_{\alpha\alpha}.
\]
The proof is straightforward: if we substitute $\left(\epsilon^{-1}\right)_D E$, given by
\[
\left(\epsilon^{-1}\right)_D E = \delta_D E + i \left(\psi^\alpha D_D \bar{\psi}^\alpha + \bar{\psi}^\alpha D_D \psi^\alpha\right) \left(\sigma_E\right)_{\alpha\alpha}
\]
in (3.20), the difference between the expressions (3.19) and (3.20) will be
\[
\int d^4x \det(E) e^{ABCD} D_A \psi^\alpha D_B \bar{\psi}^\alpha \left(\psi^\alpha D_C \bar{\psi}^\alpha + \bar{\psi}^\alpha D_C \psi^\alpha\right) \left(\psi^\alpha D_D \bar{\psi}^\alpha + \bar{\psi}^\alpha D_D \psi^\alpha\right) = 0,
\]
which just cancels the factor $\det(E)$.

As the last step, one has to prove that the Wess–Zumino term (3.19) or (3.20) is invariant with respect to broken supersymmetry transformations (3.8). For such a proof, the action (3.20) is more suitable, because its variation contains only two pieces arising due to the shifts of $\psi^\alpha$ and $\bar{\psi}^\alpha$ explicitly presented in the integrand of $S_{WZ}$ (3.20) and in $\left(\epsilon^{-1}\right)_D E$ (3.21):
\[
\delta S_{WZ} = \delta_1 S_{WZ} + \delta_2 S_{WZ}.
\]
with
\[
\delta_1 S_{WZ} = \int d^4x \det(E) e^{ABCD} D_A \psi^\alpha D_B \bar{\psi}^\alpha \left(\epsilon D_C \bar{\psi}^\alpha + \bar{\psi}^\alpha D_C \psi^\alpha\right) \left(\epsilon^{-1}\right)_D E \left(\sigma_E\right)_{\alpha\alpha},
\]
\[
\delta_2 S_{WZ} = i \int d^4x \det(E) e^{ABCD} D_A \psi^\alpha D_B \bar{\psi}^\alpha \left(\epsilon D_C \bar{\psi}^\alpha + \bar{\psi}^\alpha D_C \psi^\alpha\right) \times \left(\epsilon D\bar{D} \bar{\psi}^\alpha + \bar{\epsilon} \bar{D} D \psi^\alpha\right).
\]
The expressions (3.26) and (3.27) can be further simplified if we note that each covariant derivative $D_A = \left(\epsilon^{-1}\right)_A B \partial_B$ contains $\left(\epsilon^{-1}\right)$. Thus, these four inverse vielbeins $\left(\epsilon^{-1}\right)$, being converted with $e^{ABCD}$, give $(\det(E))^{-1}$ which just cancels the factor $\det(E)$. Thus,
\[ \delta_1 S_{WZ} = \int d^4x \ e^{ABCD} \partial_A \phi \partial_B \bar{\psi} \left( \epsilon^a \partial_C \psi^a + \epsilon^a \partial_C \psi^a \right) (\sigma_D)_{ab}, \]  
(3.28)

\[ \delta_2 S_{WZ} = i \int d^4x \ e^{ABCD} \partial_A \phi \partial_B \bar{\psi} \left( \psi^a \partial_C \bar{\psi}^a + \bar{\psi}^a \partial_C \psi^a \right) \left( \epsilon_a \partial_D \bar{\psi}_a + \bar{\epsilon}_a \partial_D \psi_a \right). \]  
(3.29)

The integrand in (3.28) is obviously a full derivative, and therefore \( \delta_1 S_{WZ} = 0 \). Next, the integrand in (3.29) can be rewritten as

\[ e^{ABCD} \partial_A \phi \partial_B \bar{\psi} \left( \epsilon_a \psi^a \partial_C \bar{\psi}^a + \bar{\epsilon}_a \psi^a \partial_C \bar{\psi}^a \right) - \frac{1}{2} \left( \epsilon_a \psi^a \partial_D \bar{\psi}^2 + \bar{\epsilon}_a \psi^a \partial_D \bar{\psi}^2 \right). \]  
(3.30)

The first two terms in (3.30) are just zero, while the remaining two terms are full derivatives. Therefore, \( \delta_2 S_{WZ} = 0 \) and the Wess–Zumino term (3.20) is invariant with respect to broken supersymmetry, as we expected.

To conclude, let us write the full ansatz for the component action of 3-brane in \( D = 6 \)

\[ S = S_1 + S_2 + S_{WZ} = (1 + \alpha) \int d^4x - \int d^4x \ \text{det}(\mathcal{E}) \times \left[ \alpha + \sqrt{\left( 1 - 2(D\phi D\bar{\phi}) \right)^2 - 4(D\phi D\psi)(D\bar{\phi} D\bar{\psi})} \right] + \beta \int d^4x \ \text{det}(\mathcal{E}) e^{ABCD} D_A \phi D_B \bar{\phi} \left( \psi^a D_C \bar{\psi}^a + \bar{\psi}^a D_C \psi^a \right) \left( \mathcal{E}^{-1} \right)_D^E (\sigma_E)_{ab}, \]  
(3.31)

where \( \alpha \) and \( \beta \) are two constants that have to be defined by invariance with respect to unbroken supersymmetry.

Note, the purely fermionic part of the action (3.31)

\[ S_{\text{ferm}} = (1 + \alpha) \int d^4x \left[ 1 - \det(\mathcal{E}) \right] \]  
(3.32)

is just the Volkov–Akulov action for Goldstino [16] in a full agreement with the results presented in [17, 18].

Thus, after imposing broken supersymmetry, the component action (3.31) is fixed up to two constants \( \alpha \) and \( \beta \). No other terms in the action are admissible. The role of the unbroken supersymmetry is just to fix these constants.

### 3.4. Unbroken supersymmetry

In contrast with the standard superfield approach, the most technically complicated part of our approach is to maintain the unbroken supersymmetry, despite the fact that all we need is to fix two constants in the action (3.31). We have put the full proof of invariance of the action in appendix B. Nevertheless, the fixation of the constants \( \alpha \) and \( \beta \) can be achieved quite easily.

The parameter \( \alpha \) can be defined if we will consider just the kinetic terms for \( \varphi, \psi \) in the action (3.31)

\[ S_{\text{kin}} = \int d^4x \left[ 2i \left( 1 + \alpha \right) \bar{\psi}^a \partial_a \psi^a + 2 \partial^a \varphi \partial_a \bar{\psi} \right]. \]  
(3.33)

The action (3.33) has to be invariant with respect to linearized transformations (B.2):

\[ \delta_Q \partial_A \varphi = 2i \epsilon^a \partial_A \psi_a, \quad \delta_Q \partial_A \bar{\psi} = 0, \quad \delta_Q \psi_a = 0, \quad \delta_Q \bar{\psi}_a = -\epsilon^a \partial_a \bar{\psi}. \]  
(3.34)
Varying the integrand in (3.33) and integrating by parts, we will get

\[
\delta_\Omega S_{\text{kin}} = \int d^4x \left[ 2i(1 + \alpha)e^\delta\phi \partial_{\beta}^\alpha \partial_{\alpha} \psi^\alpha - 4ie^\delta\phi \partial^\lambda \partial_{\lambda} \psi^\alpha \right].
\]  

(3.35)

Therefore, we have to fix

\[
\alpha = 1.
\]

(3.36)

The fixing of the parameter \(\beta\) is more involved. The idea is to consider the part of the variation of the full action which is linear in the fermions. In this approximation, the variation of the \(S_1 + S_2\) actions (3.31) with \(\alpha = 1\) under unbroken supersymmetry can be represented as (see (B.7))

\[
\delta_\Omega (S_1 + S_2) = \int d^4x \frac{4e^\delta\phi \partial_{\lambda} \psi^\alpha}{2 - j^2 j^2} \left[ (1 - j \cdot j) \mathcal{J}_{\lambda B} \left( \sigma^{AB} \right)_{\alpha}^\beta + j \mathcal{J}_{\lambda C} \mathcal{J}_{\lambda B} \left( \sigma^{CB} \right)_{\alpha}^\beta \right].
\]  

(3.37)

where \(j_A, j_A\) are defined in terms of \(\partial_{\lambda} \phi, \partial_{\lambda} \tilde{\phi}\) by the same formulas (2.15) with \(V_A \phi, V_A \tilde{\phi}\) replaced by the \(\partial_{\lambda} \phi, \partial_{\lambda} \tilde{\phi}\). To get the variation of the Wess–Zumino term in this approximation one has to vary only fermions in \(S_{WZ}\) (3.31). The corresponding variation reads

\[
\delta_\Omega S_{WZ} = -2\beta \int d^4x \frac{4e^\delta\phi \partial_{\lambda} \psi^\alpha}{2 - j^2 j^2} \left[ -j \cdot j \mathcal{J}_{\lambda B} \left( \sigma^{AB} \right)_{\alpha}^\beta + j^2 \mathcal{J}_{\lambda B} \left( \sigma^{AB} \right)_{\alpha}^\beta + j \mathcal{J}_{\lambda C} \mathcal{J}_{\lambda B} \left( \sigma^{CB} \right)_{\alpha}^\beta \right].
\]  

(3.38)

If we choose now \(\beta = 2\), then the variation of the full action acquires the form

\[
\delta_\Omega S = \int d^4x \frac{4e^\delta\phi \partial_{\lambda} \psi^\alpha}{2 - j^2 j^2} \left( \mathcal{J}_{\lambda B} - j^2 \mathcal{J}_{\lambda B} \right) \left( \sigma^{AB} \right)_{\alpha}^\beta = 4 \int d^4x e^\delta\phi \partial_{\lambda} \psi^\alpha \partial_{\lambda} \tilde{\phi} \left( \sigma^{AB} \right)_{\alpha}^\beta.
\]  

(3.39)

Thus, the integrand in (3.39) is a full derivative and, therefore, the action (3.31) is invariant in this approximation. The careful analysis of the terms with higher order in the fermions presented in appendix B shows that they also cancel out. To conclude, let us write the full component action of 3-brane in \(D = 6\)

\[
S = 2 \int d^4x - \int d^4x \det(\mathcal{E}) \left[ 1 + \sqrt{ \left( 1 - 2(\mathcal{D}_\phi \mathcal{D}_\phi) \right)^2 - 4(\mathcal{D}_\phi \mathcal{D}_\phi)(\mathcal{D}_\phi \mathcal{D}_\phi) } \right]
\]  

\[
+ 2 \int d^4x \det(\mathcal{E}) e^{ABCD} \mathcal{D}_{\lambda} \phi \mathcal{D}_{\lambda} \tilde{\phi} \left( \psi^\alpha D_{\lambda} \psi^\alpha + \tilde{\psi}^\alpha D_{\lambda} \tilde{\psi}^\alpha \right) \left( \mathcal{E}^{-1} \right)^{E}_{D} \left( \sigma_{E} \right)_{\alpha}^\beta.
\]  

(3.40)

This action is invariant with respect to both, broken and unbroken supersymmetries and, therefore, it is the supersymmetric 3-brane action in \(D = 6\).

4. Conclusion

In this first part of our paper we have constructed the on-shell component action for \(N = 1\), \(D = 6\) supersymmetric 3-brane within the nonlinear realizations approach. We treated the worldvolume action of this 3-brane as the action of four-dimensional field theory which realized the partial breaking of \(N = 2, d = 4\) global supersymmetry down to the \(N = 1, d = 4\) one. The \(N = 2, d = 4\) Poincaré superalgebra we considered contains two central charges, which are just two translation generators from the \(D = 6\) point of view. These two translations, as well as half of the \(N = 2, d = 4\) supersymmetry, are supposed to be spontaneously broken. As the result of this spontaneous breakdown we have one complex bosonic and one fermionic Goldstone superfield in the theory. The first components of these superfields constitute the
physical fields of the corresponding 3-brane. Within the superspace part of our consideration we have constructed the covariant irreducibility constraints on the bosonic superfields and, in addition, we imposed the constraints which expressed the fermionic Goldstone superfields in terms of the bosonic ones. By using an analog of superembedding constraints we were able to find the covariant superfield equations of motion, which lead to the proper bosonic equations of motion. This part is not new and has a close relation with paper [23] where similar results were obtained. In the main part of this paper (section 3) we have constructed the component action of this 3-brane. By shifting attention to the broken supersymmetry and considering the reduced coset, we introduced the covariant derivatives and the fermionic vielbeins (covariant with respect to broken supersymmetry), which are the building blocks of the action. It turns out that all terms of higher orders in the fermions are hidden inside our covariant derivatives and vielbeins. Moreover, the main part of the component action just mimics its bosonic cousin in which the ordinary space–time derivatives and the bosonic worldvolume are replaced by their covariant supersymmetric analogs. It is funny that the Wess–Zumino term in the action, which does not exist in the bosonic case, can also be easily constructed in terms of reduced Cartan forms. Keeping the broken supersymmetry almost explicit, one may write the ansatz for the component action fully defined up to two constant parameters. The role of the unbroken supersymmetry is to fix these parameters. Of course, the component action we have explicitly constructed in this paper can in principle be obtained from the superfield action of the papers [8, 10]. Nevertheless, using the introduced covariant derivatives and fermionic vielbeins makes the action quite simple. Moreover, with our action one may, for example, perform the duality transformations, considered in [11], in full generality with all fermionic terms taken into account.

In the second part of this paper we will consider the supersymmetric 3-brane in $D = 8$. From the $d = 4$ point of view such a 3-brane corresponds to partial breaking of $N = 4, d = 4$ supersymmetry down to the $N = 2, d = 4$ one. The corresponding Goldstone superfield, accompanying this breakdown of supersymmetry has to be a $N = 2, d = 4$ hypermultiplet, which makes such a system quite interesting.

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Appendix A. Superalgebra, coset space, transformations and Cartan forms

In this appendix we collected some formulas describing the nonlinear realization of $N = 1, D = 6$ Poincaré group in its coset over its $N = 1, d = 4$ subgroup.

In $d = 4$ notation the $N = 1, D = 6$ Poincaré superalgebra is a two central charge extended $N = 2$ super-Poincaré algebra containing the following set of generators:

$$N = 2, \ d = 4 \ \text{SUSY} \propto \{ P_a, Q_a, \overline{Q}_a, S_a, \overline{S}_a, Z, \overline{Z}, L_{AB}, K_A, \overline{K}_A, U \}.$$  \hspace{1cm} (A.1)

Here, $P_a, Z$ and $\overline{Z}$ are $D = 6$ translation generators, $Q_a, \overline{Q}_a$ and $S_a, \overline{S}_a$ are the generators of super-translations, the generators $L_{AB}$ form $d = 4$ Lorentz algebra $so(1,3)$, the generators $K_A$ and $\overline{K}_A$ belong to the coset $SO(1,5)/SO(1,3) \times U(1)$, while $U$ span $u(1)$. The commutation relations of $D = 6$ Poincaré algebra in this basis read
\[ [L_{AB}, L_{CD}] = i\left( -\eta_{AC}L_{BD} + \eta_{BC}L_{AD} - \eta_{BD}L_{AC} + \eta_{AD}L_{BC} \right), \quad [L_{AB}, P_C] = -i\eta_{AC}P_B + i\eta_{BC}P_A; \]

\[ [L_{AB}, K_c] = -i\eta_{AC}K_B + i\eta_{BC}K_A, \quad [L_{AB}, \bar{K}_c] = -i\eta_{AC}\bar{K}_B + i\eta_{BC}\bar{K}_A, \]

\[ [U, K_A] = K_A, \quad [U, \bar{K}_A] = -\bar{K}_A, \quad [U, Z] = Z, \quad [U, \bar{Z}] = -Z; \]

\[ [K_A, Z] = [\bar{K}_A, \bar{Z}] = -2iP_A, \quad [K_A, P_B] = -i\eta_{AB}Z, \quad [\bar{K}_A, P_B] = -i\eta_{AB}\bar{Z}; \]

\[ [K_A, \bar{K}_B] = 2iL_{AB} - 2i\eta_{AB}U. \] (A.2)

Here, \( \eta = \text{diag}(1, -1, -1, -1). \)

The four supercharges \( Q_a, \bar{Q}_a, S_a, \bar{S}_a \) obey the following (anti)commutation relations:

\[ \{ Q_a, \bar{Q}_a \} = \{ S_a, \bar{S}_a \} = 2\left( \sigma^A \right)_{ab} P_a, \quad \{ Q_a, S_b \} = 2\epsilon_{ab}Z, \quad \{ \bar{Q}_a, \bar{S}_b \} = 2\epsilon_{ab}\bar{Z}; \]

\[ [L_{AB}, Q_a] = -\frac{1}{2}(\sigma_{AB})_a^\beta \bar{Q}_\beta, \quad [L_{AB}, \bar{Q}_a] = \frac{1}{2}\bar{Q}_\beta(\bar{\sigma}_{AB})^\beta_a, \]

\[ [L_{AB}, S_a] = -\frac{1}{2}(\sigma_{AB})_a^\beta S_\beta, \quad [L_{AB}, \bar{S}_a] = \frac{1}{2}\bar{S}_\beta(\bar{\sigma}_{AB})^\beta_a; \]

\[ [K_A, Q_a] = i(\sigma_{AA})_{ab} S_b, \quad [K_A, S_b] = -i(\sigma_{AA})_{ab}\bar{Q}_b, \]

\[ [K_A, \bar{Q}_a] = i(\sigma_{AA})_{ab} S_b, \quad [K_A, \bar{S}_b] = -i(\sigma_{AA})_{ab}Q_a; \]

\[ [U, Q_a] = \frac{1}{2}Q_a, \quad [U, S_a] = \frac{1}{2}S_a, \quad [U, \bar{Q}_a] = -\frac{1}{2}\bar{Q}_a, \quad [U, \bar{S}_b] = -\frac{1}{2}\bar{S}_b. \] (A.3)

We define the coset element as follows:

\[ g = e^{i\omega^a\xi_a e^\theta_0 Q_a + \bar{\theta}_0 \bar{Q}_a e^{\psi^a S_a + \bar{\psi}^a \bar{S}_a} e^{i(\phi z + \bar{\phi} z \bar{Z})} e^{i(\lambda^A K_A + \bar{\lambda}^A \bar{K}_A)}. \] (A.4)

Here, \( \{ x^A, \theta^a, \bar{\theta}^\alpha \} \) are \( N = 1, d = 4 \) superspace coordinates, while the remaining coset parameters are Goldstone superfields. The local geometric properties of the system are specified by the Cartan forms:

\[ g^{-1}dg = i(\omega_P)^a P_a + i\omega_Z Z + i\omega_{\bar{Z}} \bar{Z} + (\omega_Q)^a Q_a + (\omega_{\bar{Q}})^a \bar{Q}_a + (\omega_S)^a S_a + (\omega_{\bar{S}})^a \bar{S}_a \]

\[ + i(\omega_K)^a K_A + i(\omega_{\bar{K}})^a \bar{K}_A + i(\omega_L)^{AB} L_{AB} + i(\omega_U)U. \] (A.5)

In what follows, we will need the explicit expressions of the following forms:

\[ (\omega_P)^a = \Delta x^A \left( \frac{\cosh \sqrt{2Y}}{\sqrt{Y}} \right)^A_B - 2\left( \Delta \psi^A + \Delta \bar{\psi}^\beta \right) \left( \frac{\sinh \sqrt{2Y}}{\sqrt{2Y}} \right)^A_B, \]

\[ \omega_Z = \Delta x^A + \left( \Delta \psi^A + \Delta \bar{\psi}^\beta \right) \left( \frac{\cosh \sqrt{2Y} - 1}{Y} \right) \Delta x^A \left( \frac{\sinh \sqrt{2Y}}{\sqrt{2Y}} \right)^A_B, \]

\[ (\omega_Q)^a = d\theta^\beta \left( \cosh \sqrt{\bar{W}} \right)^a_\beta + d\bar{\psi}^\alpha \left( \frac{\sinh \sqrt{\bar{W}}}{\sqrt{\bar{W}}} \right)^\alpha_\beta A_\beta^\alpha, \]

\[ (\omega_S)^a = d\bar{\psi}^\beta \left( \cosh \sqrt{\bar{W}} \right)^a_\beta - d\theta^\alpha \left( \frac{\sinh \sqrt{\bar{W}}}{\sqrt{\bar{W}}} \right)^\beta_\alpha A_\beta^\alpha. \] (A.6)
where

\[ \triangle x^A = dx^A - i \left( \theta^a d\bar{\theta}^a + \bar{\theta}^a d\theta^a + \psi^a d\bar{\psi}^a + \bar{\psi}^a d\psi^a \right) \left( \sigma^A \right)_{\alpha a}, \]

\[ \triangle \varphi = d\varphi - 2i\psi_a d\bar{\theta}^a, \quad \triangle \bar{\varphi} = d\bar{\varphi} - 2i\bar{\psi}_a d\theta^a. \]  

(A.7)

The matrix-valued functions \( Y^B_A, W^a_B, \overline{W}^a_B \) are defined as follows

\[ Y^B_A = \Lambda_A \overline{\Lambda}^B + \overline{\Lambda}_A \Lambda^B, \quad W^a_B = \Lambda^a \overline{\Lambda}_B, \quad \overline{W}^a_B = \overline{\Lambda}^a \Lambda_B. \]  

(A.8)

For the bosonic forms \((\omega_F)^A, \omega_Z, \bar{\omega}_Z\) we find useful to perform the changing of the variables

\[ \lambda^A = \left( \tanh \left( \frac{y}{2} \right) \right)^A, \quad \bar{\lambda}^A = \left( \tanh \left( \frac{y}{2} \right) \right)^A \overline{\Lambda}^B. \]  

(A.9)

This is just a stereographic projection. Introducing now the new matrix \( Y^B_A = \lambda_A \bar{\lambda}^B + \bar{\lambda}_A \lambda^B \), and noting that

\[ Y^B_A = 2 \left( \text{arctanh}^2 \left( \frac{y}{2} \right) \right)^B_A, \]  

(A.10)

one may rewrite the bosonic forms in (A.6) as

\[ (\omega_F)^A = \triangle x^B \left( \frac{1 + y/2}{1 - y/2} \right)^A_B - 2 \left( \triangle \varphi \lambda^B + \triangle \bar{\varphi} \bar{\lambda}^B \right) \left( \frac{1}{1 - y/2} \right)_B^A, \]

\[ \omega_Z = \triangle \varphi + \left( \triangle \varphi \lambda^A + \triangle \bar{\varphi} \bar{\lambda}^A \right) \left( \frac{1}{1 - y/2} \right)_A^B \lambda_B^A, \]

\[ \bar{\omega}_Z = \triangle \bar{\varphi} + \left( \triangle \varphi \lambda^A + \triangle \bar{\varphi} \bar{\lambda}^A \right) \left( \frac{1}{1 - y/2} \right)_A^B \bar{\lambda}^A_B. \]  

(A.11)

Keeping in mind, that the quantities \( \triangle x^A, d\theta^a \) and \( d\bar{\theta}^a \) are invariant with respect to both supersymmetries (2.3), (2.4), one may define the semi-covariant derivatives \( \nabla_A, \nabla_a, \overline{\nabla}_a \) as

\[ dF = \left( dx^A \frac{\partial}{\partial x^A} + d\theta^a \frac{\partial}{\partial \theta^a} + d\bar{\theta}^a \frac{\partial}{\partial \bar{\theta}^a} \right) F = \left( \triangle x^A \nabla_A + d\theta^a \nabla_a + d\bar{\theta}^a \overline{\nabla}_a \right) F, \]  

(A.12)

and, therefore,

\[ \nabla_A = \left( E^{-1} \right)^B_A \partial_B, \quad E_A^B = \delta_A^B - i \left( \psi^a \partial_A \bar{\psi}^a + \bar{\psi}^a \partial_A \psi^a \right) \left( \sigma^B \right)_{\alpha a}, \]

\[ \nabla_{\bar{\beta}} = \overline{D}_{\bar{\beta}} - i \left( \psi^a D_{\bar{\beta}} \bar{\psi}^a + \bar{\psi}^a D_{\bar{\beta}} \psi^a \right) \left( \sigma^B \right)_{\alpha a}, \]

\[ \nabla_B = D_B - i \left( \psi^a \nabla_B \bar{\psi}^a + \bar{\psi}^a \nabla_B \psi^a \right) \left( \sigma^B \right)_{\alpha a}, \]

\[ \nabla_{\bar{\beta}} = \overline{D}_{\bar{\beta}} - i \left( \psi^a \overline{D}_{\bar{\beta}} \bar{\psi}^a + \bar{\psi}^a \overline{D}_{\bar{\beta}} \psi^a \right) \left( \sigma^B \right)_{\alpha a}, \]

(A.13)
These derivatives satisfy the following (anti)commutation relations
\[
\{ V_\alpha, V_\beta \} = -2i \left( V_\alpha \psi^\dagger V_\beta \psi - V_\beta \psi^\dagger V_\alpha \psi \right) \left( \sigma^C \right)_{\gamma\gamma} V_\gamma,
\]
\[
\{ V_\alpha, V_\beta \} = 2i \left( V_\alpha \psi^\dagger V_\beta \psi + V_\beta \psi^\dagger V_\alpha \psi \right) \left( \sigma^C \right)_{\gamma\gamma} V_\gamma.
\]
\[
\{ V_\alpha, V_\beta \} = -2i \left( \delta^\alpha_\gamma \delta^\beta_\gamma + V_\alpha \psi^\dagger V_\beta \psi + V_\beta \psi^\dagger V_\alpha \psi \right) \left( \sigma^C \right)_{\gamma\gamma} V_\gamma.
\]
\[
\{ V_\alpha, V_\beta \} = -2i \left( V_\alpha \psi^\dagger V_\beta \psi + V_\beta \psi^\dagger V_\alpha \psi \right) \left( \sigma^C \right)_{\gamma\gamma} V_\gamma.
\]
(A.14)

Here, $D_\alpha, \overline{D}_\alpha$ are flat covariant derivatives obeying the relations
\[
\{ D_\alpha, D_\beta \} = -2i \left( \sigma^A \right)_{\alpha\alpha} \partial_\alpha, \quad \{ \overline{D}_\alpha, D_\beta \} = \{ \overline{D}_\alpha, \overline{D}_\beta \} = 0. \quad (A.15)
\]

Finally, we will define the transition to the vectors as
\[
V^A = \frac{1}{2} V^{\alpha\alpha} \left( \sigma^A \right)_{\alpha\alpha}.
\]
(A.16)

We used the following definitions of the $\sigma$-matrices:
\[
\left( \sigma^A \right)_{\alpha\beta} = (1, \overline{\sigma}), \quad \left( \overline{\sigma}^A \right)_{\alpha\beta} = c_{\alpha\beta} c_{\beta\alpha} \left( \sigma^A \right)_{\beta\alpha} = (1, -\sigma).
\]
(A.17)

(\overline{\sigma} is the ordinary set of three-dimensional Pauli matrices). Indices of these matrices and spinors are raised and lowered by $\epsilon_{\alpha\beta}, \epsilon_{\alpha\beta}$ with properties
\[
\epsilon_{\alpha\beta} \epsilon_{\beta\gamma} = \delta^\alpha_\gamma, \quad \epsilon_{\alpha\beta} \epsilon_{\beta\gamma} = \delta^\alpha_\gamma, \quad \epsilon_{12} = \epsilon_{12} = 1.
\]
(A.18)

Therefore,
\[
(\sigma_A)_\alpha (\sigma_B)^\alpha = \text{Tr} (\sigma_A \sigma_B) = 2\eta_{AB}, \quad \left( \sigma^A \right)_{\alpha\alpha} (\sigma^A)^{\beta\beta} = 2 \delta^\beta_\alpha \delta^\alpha_\beta, \quad (A.19)
\]
and
\[
\left( \sigma^A \right)_{\alpha\beta} (\sigma^B)^{\alpha\beta} = \eta^{AB} \delta^\beta_\alpha - i (\sigma^B)^{\alpha\beta}, \quad \left( \sigma^A \right)^{\beta\alpha} (\sigma^B)_{\alpha\beta} = \eta^{AB} \delta^\beta_\alpha - i (\sigma^B)^{\alpha\beta},
\]
\[
(\sigma_A \sigma_B \sigma_C)_{\alpha\beta} = \eta_{AB} (\sigma_C)_{\alpha\beta} + \eta_{BC} (\sigma_A)_{\alpha\beta} - \eta_{AC} (\sigma_B)_{\alpha\beta} + i \epsilon_{ABCD} (\sigma_A)_{\alpha\beta}, \quad (A.20)
\]

where two-indices matrices are defined as
\[
\left( \sigma^{AB} \right)^{\beta}_{\alpha} = \frac{1}{2} \left[ \left( \sigma^A \right)_{\alpha\alpha} \left( \sigma^B \right)^{\beta\beta} - \left( \sigma^B \right)_{\alpha\alpha} \left( \sigma^A \right)^{\beta\beta} \right],
\]
\[
\left( \overline{\sigma}^{AB} \right)^{\beta}_{\alpha} = \frac{1}{2} \left[ \left( \overline{\sigma}^A \right)_{\alpha\alpha} \left( \sigma^B \right)^{\beta\beta} - \left( \overline{\sigma}^B \right)_{\alpha\alpha} \left( \sigma^A \right)^{\beta\beta} \right].
\]
(A.21)

Note, that the matrices $\sigma^{AB}$ ($\overline{\sigma}^{AB}$) are self-dual (anti-self-dual), respectively
\[
\left( \sigma^{AB} \right)^{\beta}_{\alpha} = -\frac{1}{2} \epsilon^{ABCD} (\sigma_{CD})_{\alpha\beta}, \quad \left( \overline{\sigma}^{AB} \right)^{\beta}_{\alpha} = \frac{1}{2} \epsilon^{ABCD} (\overline{\sigma}_{CD})_{\alpha\beta}.
\]
(A.22)
Finally, we define the $d = 4$ volume form in a standard manner as
\[
d^4 x \equiv \epsilon_{ABCD} dx^A \wedge dx^B \wedge dx^C \wedge dx^D
\]
\[
\Rightarrow \quad dx^A \wedge dx^B \wedge dx^C \wedge dx^D = -\frac{1}{24} \epsilon^{ABCD} d^4 x,
\]
with
\[
\epsilon_{0123} = -\epsilon^{0123} = 1.
\]

Appendix B. Invariance of the action with respect to unbroken supersymmetry

In this appendix we will prove the invariance of the 3-brane action (3.40) under unbroken supersymmetry.

Let us start from the variation of the $S_2$ (3.31)
\[
S_2 = -\int d^4 x \det(\epsilon) \left[ 1 + \sqrt{\left( 1 - 2(\bar{D}\phi D\phi) \right)^2 - 4(\bar{D}\phi D\phi)(\bar{D}\phi D\phi)} \right]
\]
\[
\equiv -\int d^4 x \det(\epsilon) F.
\]

Keeping in the mind that under unbroken $(Q)$ supersymmetry the covariant derivatives $V_A$ (A.13) are invariant, one may find the transformation properties of all ingredients in the action (B.1)
\[
\delta_Q D_A \psi = 2i\epsilon^\alpha D_A \psi^\alpha - 2i\epsilon^\beta D_A \psi^\beta \tilde{T}_\beta D_{\alpha\beta} \varphi - 2iH^C \partial_c D_A \varphi,
\]
\[
\delta_Q D_A \bar{\psi} = -2i\epsilon^\beta D_A \psi^\beta \tilde{T}_\beta D_{\alpha\beta} \bar{\psi} - 2iH^C \partial_c D_A \bar{\psi},
\]
\[
\delta_Q \psi^\alpha_a = -2iH^A \partial_{\alpha \beta} \psi^\beta_a, \quad \delta_Q \bar{\psi}_{\alpha a} = -\epsilon^\alpha \bar{T}_{\alpha a} - 2iH^A \partial_{\alpha \beta} \bar{\psi}_{\beta a},
\]
\[
\delta_Q D_A \psi^\alpha_a = 2i\epsilon^\beta D_A \psi^\beta \tilde{T}_\beta D_{\gamma\beta} \psi^\gamma_a - 2iH^C \partial_c D_A \psi^\alpha_a,
\]
\[
\delta_Q D_A \bar{\psi}_{\alpha a} = -\epsilon^\alpha D_A \bar{T}_{\alpha a} - 2i\epsilon^\beta D_A \psi^\beta \tilde{T}_\beta D_{\gamma\beta} \bar{\psi}_{\gamma a} - 2iH^C \partial_c D_A \bar{\psi}_{\alpha a},
\]
where
\[
H^a \equiv \epsilon^\alpha \psi^\beta \tilde{T}_\beta \psi^\alpha.
\]

As a consequence of (B.2) we will have
\[
\delta_Q \det(\epsilon) = 2i\epsilon^\beta D_{\alpha \beta} \psi^\alpha \bar{T}_\beta \psi^\beta \det(\epsilon) - 2i\partial_A \left( H^a \det(\epsilon) \right).
\]

Representing the variation of the integrand of (B.1) as
\[
\delta_Q L_2 = \delta_Q \det(\epsilon) F + \det(\epsilon) \delta_Q F
\]
\[
\equiv \delta_Q \det(\epsilon) F + \det(\epsilon) \left[ \frac{\partial F}{\partial D_A \varphi} \delta_Q D_A \varphi + \frac{\partial F}{\partial D_A \bar{\psi}} \delta_Q D_A \bar{\psi} \right]
\]
we conclude that $H$-dependent terms convert into the full derivative $-2i\partial_A \left( H^a \det(\epsilon) F \right)$. To represent the rest terms in the variation (B.5) in a more readable form it is useful to pass to variables $J_A$, $\bar{T}_A$ (2.15), in terms of which we have
\[
\mathcal{L} = \frac{1}{2} J - \mathcal{T}, \quad \frac{\partial \mathcal{L}}{\partial D_A \varphi} = -\frac{2}{1 - J^2 \mathcal{T}^2} \left( \mathcal{T}_A + J^2 \mathcal{T}_A \right),
\]

\[
\frac{\partial \mathcal{L}}{\partial D_A \bar{\varphi}} = -\frac{2}{1 - J^2 \mathcal{T}^2} \left( \mathcal{T}_A + J^2 \mathcal{T}_A \right).
\]

Combining all the terms in the variation \( \delta_Q S_2 \) we will get the following expression for it

\[
\delta_Q(S_2) = \int d^4x \text{det}(\mathcal{E}) \frac{4e_B D_A \psi^a}{1 - J^2 \mathcal{T}^2} \left[ (1 - J \cdot \mathcal{T}) \mathcal{T}_B \left( \sigma^{AB} \right)_a^\beta + \mathcal{T}_A \mathcal{T}_B \left( \sigma^{BC} \right)_a^\beta \right]
\]

(B.6)

The last step is to use the self-duality property of \( \sigma^{AB} \) (A.22) and the identity

\[
\epsilon^{ABCD}X^E - \epsilon^{EBCD}X^A - \epsilon^{AECB}X^R - \epsilon^{ABCE}X^D = 0,
\]

(B.8)

which is valid for an arbitrary vector \( X^A \), to represent the second term in the square brackets in (B.7) as

\[
\mathcal{T}_B \mathcal{T}_C \left( \sigma^{BC} \right)_a^\beta = \left[ (J \cdot \mathcal{T}) \mathcal{T}_B - J^2 \mathcal{T}_B \right] \left( \sigma^{AB} \right)_a^\beta - i\epsilon^{ABCE} J_B \mathcal{T}_C \mathcal{T}_D\left( \sigma_{DE} \right)_a^\beta.
\]

(B.9)

Plugging (B.9) back into (B.7) and using (2.14) we will finally get

\[
\delta_Q S_2 = \int d^4x \text{det}(\mathcal{E}) \frac{4e_B D_A \psi^a}{1 - J^2 \mathcal{T}^2} \left[ (J \cdot \mathcal{T}) \mathcal{T}_B - J^2 \mathcal{T}_B \right] \left( \sigma^{AB} \right)_a^\beta - i\epsilon^{ABCE} J_B \mathcal{T}_C \mathcal{T}_D\left( \sigma_{DE} \right)_a^\beta.
\]

(B.10)

To find the variation of Wess–Zumino term it is preferable to use it in the form (3.31) (with parameter \( \beta = 2 \))

\[
S_{WZ} = 2 \int d^4x \text{det}(\mathcal{E}) \epsilon^{ABCD} D_A \varphi D_B \varphi \left( \psi^a D_C \psi^a + \psi^a D_C \psi^a \right) \left( \mathcal{E}^{-1} \right)_D^E \left( \sigma_E \right)_{\alpha \alpha}
\]

\[
= 2 \int d^4x \epsilon^{ABCD} D_A \varphi \partial_B \varphi \left( \psi^a \partial_C \psi^a + \psi^a \partial_C \psi^a \right) \left( \sigma_D \right)_{\alpha \alpha}
\]

\[
= 4 \int d^4x \epsilon^{ABCD} D_A \varphi \partial_B \varphi \psi^a \partial_C \psi^a \left( \sigma_D \right)_{\alpha \alpha}.
\]

(B.11)

The variations of the ingredients in (B.11) read

\[
\delta_Q \partial_A \varphi = -\partial_A (2i \epsilon^a \psi_a + 2iH^B \partial_B \varphi), \quad \delta_Q \partial_A \bar{\varphi} = -2i \partial_A \left( H^B \partial_B \bar{\varphi} \right).
\]

\[
\delta_Q \psi_a = -2i H^A \partial_A \psi_a, \quad \delta_Q \partial_A \bar{\psi}_a = -\partial_A \left( \epsilon^a \mathcal{T}_a + 2iH^B \partial_B \bar{\psi}_a \right).
\]

(B.12)

To simplify consideration, we will split the variation of \( S_{WZ} \) into two parts

\[
\delta_Q S_{WZ} = \delta_H^H S_{WZ} + \delta_H S_{WZ}.
\]

(B.13)

where \( \delta_H^H S_{WZ} \) contains \( H \)-dependent terms in (B.12), while \( \delta_H S_{WZ} \) includes \( H \)-independent variations in (B.12).
The variation $\delta^H_{SQ} S_{WZ}$ reads

\[
\delta^H_{SQ} S_{WZ} = 8i \int d^4x e^{ABCD} \left[ H^F \partial_F \phi \partial_B \bar{\psi} \partial_A \psi^\alpha \partial_C \bar{\psi}^\beta + H^F \partial_A \phi \partial_B \bar{\psi} \partial_F \psi^\alpha \partial_C \bar{\psi}^\beta + H^F \partial_A \phi \partial_B \bar{\psi} \partial_C \psi^\alpha \partial_F \bar{\psi}^\beta + H^F \partial_F \phi \partial_B \bar{\psi} \partial_C \psi^\alpha \partial_A \bar{\psi}^\beta \right] (\sigma_D)_{\alpha \alpha}
\]

\[
= 8i \int d^4x e^{ABCD} \partial_A \phi \partial_B \bar{\psi} \partial_C \psi^\alpha \partial_D \bar{\psi}^\beta H_{\alpha \alpha}, \quad (\text{B.14})
\]

where on the last step we have used the identity (B.8) for the first term in the square brackets.

The $H$-independent variation of $S_{WZ}$ has the form

\[
\tilde{\delta}^H_{SQ} S_{WZ} = 4 \int d^4x e^{ABCD} \left[ 2e^\beta \partial_A \psi^\beta \partial_B \psi^\alpha \partial_C \bar{\psi}^\beta + \partial_A \psi^\alpha \partial_B \psi^\beta \partial_C \bar{\psi}^\beta \partial_D \bar{\psi}^\beta \right] (\sigma_D)_{\alpha \alpha}
\]

\[
= -8i \int d^4x e^{ABCD} e^{\beta \gamma} \partial_B \psi^\beta \partial_C \bar{\psi}^\beta \partial_D \bar{\psi}^\beta (\sigma_D)_{\alpha \alpha}
\]

\[
+ 4 \int d^4x e^{ABCD} \partial_A \phi \partial_B \bar{\psi} \partial_C \psi^\alpha \partial_D \bar{\psi}^\beta \tilde{T}_D
\]

\[
- 4i \int d^4x e^{ABCD} \partial_A \phi \partial_B \bar{\psi} \partial_C \psi^\alpha \partial_D \bar{\psi}^\beta (\sigma_{DF})_{\alpha \alpha}. \quad (\text{B.15})
\]

The term (B.16) may be rewritten as

\[
4 \int d^4x \varepsilon^{ABCD} \partial_A \phi \partial_B \bar{\psi} \partial_C \psi^\alpha \partial_D \bar{\psi}^\beta \tilde{T}_D = 4i \int d^4x \varepsilon^{ABCD} \partial_A \phi \partial_B \bar{\psi} \partial_C \psi^\alpha \partial_D \bar{\psi}^\beta D_A \psi^\beta D_B \bar{\psi}^\beta\psi^\alpha
\]

\[
\times \left( \varepsilon^{-1} \right)_D^{\gamma} \psi^\gamma \bar{\psi}^\gamma \partial_D \bar{\psi}^\beta + \psi^\gamma \bar{\psi}^\gamma \partial_D \bar{\psi}^\beta \right) \tilde{T}_D. \quad (\text{B.16})
\]

On the last step in (B.18) we have used the equality

\[
4i \int d^4x \varepsilon^{ABCD} \partial_A \phi \partial_B \bar{\psi} \partial_C \psi^\alpha \partial_D \bar{\psi}^\beta D_A \psi^\beta D_B \bar{\psi}^\beta\psi^\alpha D_C \psi^\beta D_D \bar{\psi}^\beta = 0
\]

which is valid in virtue of (2.15).

Playing the same game with the term (B.17) we will get

\[
\int d^4x e^{ABCD} \partial_A \phi \partial_B \bar{\psi} \partial_C \psi^\alpha \partial_D \bar{\psi}^\beta \tilde{T}_D = \int d^4x \varepsilon^{ABCD} \partial_A \phi \partial_B \bar{\psi} \partial_C \psi^\alpha \partial_D \bar{\psi}^\beta \psi^\gamma \bar{\psi}^\gamma \partial_D \bar{\psi}^\beta \tilde{T}_D.
\]

\[
\times \left[ (\sigma_{DF})_{\alpha \alpha} \tilde{T}_D + \psi^\alpha \bar{\psi}^\alpha \partial_D \bar{\psi}^\beta \tilde{T}_D + \psi^\beta \bar{\psi}^\beta \partial_D \bar{\psi}^\alpha \tilde{T}_D + \bar{\psi}^\alpha \partial_D \bar{\psi}^\beta \tilde{T}_D \right]. \quad (\text{B.19})
\]

Combining now (B.15), (B.18) and (B.19) we will finally get

\[
\delta^H_{SQ} S_{WZ} = -8i \int d^4x e^{ABCD} e^{\beta \gamma} \partial_B \psi^\beta \partial_C \bar{\psi}^\beta \partial_D \bar{\psi}^\beta (\sigma_D)_{\alpha \alpha}
\]

\[
+ 4i \int d^4x \varepsilon^{ABCD} \partial_A \phi \partial_B \bar{\psi} \partial_C \psi^\alpha \partial_D \bar{\psi}^\beta \tilde{T}_D (\sigma_{DF})_{\alpha \alpha}. \quad (\text{B.20})
\]
Therefore, the full variation of our action (3.40) reads

\[
\delta_Q S = \delta_Q S_2 + \delta_Q S_{WZ} = -8i \int d^4xe^{ABCD} \epsilon_\beta \partial A \psi^\beta \partial B \bar{\psi} \bar{\psi}^a \partial C \bar{\psi}^a (\sigma_D)_{a a}
\]

\[
+ 4 \int d^4x \det(E) \epsilon_\beta E_D \psi^a \psi^a D_B \bar{\psi}^a (\sigma^{AB})_a^\beta.
\]

(B.21)

To demonstrate that \(\delta_Q S\) (B.21) is zero, let us start by rewriting the first term in (B.21) as follows

\[
-8i \int d^4xe^{ABCD} \epsilon_\beta \partial A \psi^\beta \partial B \bar{\psi} \bar{\psi}^a \partial C \bar{\psi}^a (\sigma_D)_{a a} = 4 \int d^4xe^{ABCD} \epsilon_\beta \partial A \psi^\beta \partial B \bar{\psi} \bar{\psi} E_{C|D}.
\]

(B.22)

where

\[
E_{C|D} \equiv E_F^{c} \eta_{EF}.
\]

(B.23)

Using the self-duality property of \((\sigma_{AB})\) (A.22) and the definition of the covariant derivatives \(D_A\) (3.11), the second term in (B.21) can be rewritten as

\[
-2i \int d^4x e_a e^{ABCD} \partial A \psi^\beta \partial B \bar{\psi} E_C^E \ E_D^F (\sigma_{EF})_\alpha^\beta.
\]

(B.24)

Substituting here the explicit expression for \(E_A^\beta\) (3.10) and having used the following property of \(\sigma\)-matrices

\[
(\sigma^A)_{a a} (\sigma_A^B)^{b c} = -i \delta_a^b (\sigma_b^c)^{a c} - i \delta_a^c (\sigma_b^c)^{a b},
\]

we will receive for (B.24)

\[
-4i \int d^4x e_a e^{ABCD} \partial A \psi^\beta \partial B \bar{\psi} (\bar{\psi} \partial C \bar{\psi} + \bar{\psi} \partial C \bar{\psi}) \left[ \delta_c^\beta (\sigma_D)_{a a} + \delta_a^c (\sigma_D)_{a a} \right].
\]

(B.26)

After integrating by parts with respect to the partial derivative \(\partial_B\), we finally get for the second term in (B.21)

\[
4 \int d^4x e_a e^{ABCD} \partial A \psi^\beta \partial B \bar{\psi} E_{C|D}.
\]

(B.27)

Combining now (B.22) and (B.27), we will get

\[
\delta_Q S = 4 \int d^4xe^{ABCD} \epsilon_\beta \partial A \psi^\beta \partial B \bar{\psi} E_{C|D} = 0.
\]

(B.28)

Thus, our action (3.40) is indeed invariant with respect to broken and unbroken supersymmetries.

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