On the quantum measurements from a reconsidered perspective

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Abstract

For theoretical approach of quantum measurements it is proposed a set of reconsidered conjectures. The proposed approach implies linear functional transformations for probability density and current but preserves the expressions for operators of observables. The measuring uncertainties appear as changes in the probabilistic estimators of observables.

Key words: quantum measurements, quantum observables, measurements uncertainties

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1 Introduction

The question of a theory of measurements is present [1,2,3] in many debates about quantum mechanics (QM) but [4] it did not exist prior to QM. The respective question arose from the discussions on the traditional interpretation of uncertainty relations (TIUR)) and it has generated a large diversity of viewpoints regarding its importance and approach. The respective diversity inserts even some extreme opinions such are:

- (i) the description of quantum measurements (QMS)is "probably the most important part of the theory" [1].
- (ii) :"the word (‘measurement’) has had such a damaging effect on the discussions that ... it should banned altogether in quantum mechanics" [5].

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As a notable aspect today one finds that many of the existing approaches of QMS (including some of the most recent ones) are TIUR-connected, because they are founded on traditional conjectures inspired someway from TIUR. In their essence the respective approaches, as well as the TIUR itself, are based on the idea that the Robertson Schrödinger uncertainty relation (RSUR) is a capital physical formula with a straightforward significance for QMS. But a minute re-examination of the facts shows that TIUR is nothing but an unjustified doctrine respectively that RSUR is a simple fluctuation formula without any significance for QMS (see the investigations progressively developed in our works [6,7,8,9,10]).

In the mentioned circumstances it becomes of real and actual interest for the theory of QMS to search new approaches which are disconnected of TIUR doctrine. Such an approach is the aim of the present paper. For our aim in the next section we present the deficiencies of the traditional conjectures regarding the QMS approaches. In section 3 we argue for a set of reconsidered conjectures regarding the QMS theory. Subsequently in section 4 we propose a new approach of QMS. The proposal is inspired from a view [11,12] about the measurement of classical (non-quantum) random observables. The proposed approach is detailed through a simple exemplification in the section 5. Some ending conclusions are given in section 6.

2 Traditional conjectures and their deficiencies

In its essence TIUR is reducible [10] to a set of main assertions for which the reference element is the RSUR

\[ \Delta A \cdot \Delta B \geq \frac{1}{2} |\langle [\hat{A}, \hat{B}] \rangle| \]  

(1)

taken as a capital formula of physics. According to the mentioned assertions \( \Delta A \) and \( \Delta B \) are considered as uncertainties in simultaneous measurements of observables A and B. The commutator \([\hat{A}, \hat{B}]\) is considered as a distinction sign between the cases of compatible and non-compatible observables (when \([\hat{A}, \hat{B}] = 0\), respectively \([\hat{A}, \hat{B}] \neq 0\)). The mentioned considerations are associated with the idea of non-null and unavoidable perturbations attributed to QMS. The asserted characteristics of RSUR (1) and the associated perturbations are considered to be specific only for QMS and without analogues in classical physics (for more details see [10]).

Inspired from the alluded considerations the TIUR-connected approaches of QMS imply someway one or more of the following traditional conjectures (TC):

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• TC.1: The descriptions of QMS must be developed as extensions/completions of TIUR. Subsequently the respective descriptions have to contain distinctive elements for the pairs of compatible respectively non-compatible observables.

• TC.2: the quantum and classical measurement must be approached with essentially distinct views because of the exclusive character of RSUR (1) and of the QMS perturbations.

• TC.3: The descriptions of QMS have to take into account the jumps caused by the mentioned perturbations on the states of the measured systems.

• TC.4: A QMS consists in a single trial (detection act) that gives as result an unique value for the measured observable. Consequently its description must be associated somehow with a collapse/reduction of the (random content of) corresponding wave function.

The mentioned re-examination of the facts shows [10] that in reality RSUR (1) is not a capital formula of physics. Therefore one can conclude that it is unreasonable to subordinate the QMS approaches to TIUR. But such a conclusion clearly invalidates the conjecture TC.1. On the other hand through the same re-examination one finds that the RSUR (1) belongs to a general family of fluctuations formulas from both quantum and classical (non-quantum) physics. Then it results that there are not real motives for a fundamental distinction between quantum and classical situations. Evidently that such a result leaves without any base the conjecture TC.2.

The conjecture TC.3 is also TIUR-connected. Firstly it was said that the uncertainties put forward by TIUR are due to the interactions between measuring devices and measured systems. Secondly it was added that the respective interactions cause jumps in the states of measured systems. Then it was promoted the idea that, in contrast with the classical situations, in QMS the mentioned uncertainties, interactions and jumps have an unavoidable character. Subsequently it was promoted the idea that the alluded jumps must be taken into account in the descriptions of QMS. But, in spite of its genesis, the respective idea is proved to be incorrect by the following natural and indubitable remark [13]: ”it seems essential to the notion of measurement that it answers a question about the given situation existing before the measurement. Whether the measurement leaves the measured system unchanged or brings about a new and different state of that system is a second and independent question”. So we have to report an indubitable deficiency of TC.3.

The conjecture TC.4 is contradicted by the well-known character of random variables manifested by the quantum observables. Indeed, mathematically [14,15,16], in a given situation a random variable is characterized not by a unique value but by a spectrum of values. Consequently for a random observable, in a given state of the considered physical system a single trial has no significance. For a true estimation of such an observable a significant measure-
ment must consist in a statistical sampling composed by an (adequately) large number of individual trials. The mentioned features of random observables are correctly taken into account in the frame of the classical physics (e.g. in the study of the fluctuations), where to the spectrum of a random observable is attached the corresponding probability distribution. In the same frame a measurement of a random observable is not associated with a collapse/reduction of the respective probability distribution. The association can be done [11,12] through a functional transformation of the mentioned distribution. Then, because the quantum observables have random characteristics, it results directly that a QMS must be regarded as a statistical sampling (in the above noted sense). Consequently there are no reasons to represent (describe) a QMS by means of collapse/reduction of wave function. So we can conclude that the conjecture TC.4 is incorrect.

The above presented facts reveal visible deficiencies of traditional conjectures TC.1-4. The respective deficiencies have an unsurmountable character because they cannot be remedied (or avoided) by valid arguments derivable from the TIUR doctrine. Then it results that TIUR-connected approaches of QMS are attempts with wrong grounds. Due to the persistence of such attempts in the nowadays publications it result that, at least partially, the problem of QMS description is a still open question which requires new approaches founded on reconsidered conjectures and disconnected of TIUR doctrine. Such an approach we try to present in the next sections.

3 Arguments for reconsidered conjectures

A natural theory of measurements must contain elements (conjectures, reasonings) which are in an adequate correspondence with the main characteristics of the real measuring experiments. Then, for our trial, it is of first interest, to note the respective characteristics for QMS regarded in the general context of experimental physics.

In classical physics the belief [4]: "in the objective existence of material systems ... which possesses properties independently of measurements" is accepted as an axiom. The respective axiom implies the idea that a measurement aims to give information about the pre-existent state of the investigated system. We opine that the mentioned axiom and idea must be adopted also in connection with the quantum systems. Our opinion is encouraged by the following indubitable remarks expressed with regard to QMS:

- (i) "When it is said that something is 'measured' it is difficult not to think of the result are referring to some preexisting property of the object in question" [5],
respectively.

- (ii) a measurement "answer a question about the given situation existing before the measurement" [13].

In the quantum situations it is also significant the fact that, from a mathematical viewpoint, the observables are random variables and consequently their measurements must consist in statistical samplings. The essential aspects of the respective fact were pointed out in the previous section in discussions about the conjecture TC.4. Such aspects have to be integrated in both experimental and theoretical approaches of QMS.

By regarding things as above an experimental approach of QMS can be described as follows: Let be a QMS consisting of \( N \) single trials destined to measure concomitantly two observables \( A \) and \( B \). All trials operate on the same pre-existent state of the considered system (or of its identical replicas). Corresponding to the respective state for \( A \) and \( B \) the \( i \)-th trial gives the values \( \alpha_i \) respectively \( \beta_i \) \((i = 1, 2, ..., N)\). Then, according to the mathematical statistics [14,15,16], the observables \( A \) and \( B \) are evaluated through the factual (fac) estimators: mean values \( \langle A \rangle_{\text{fac}} = \langle \alpha \rangle, \langle B \rangle_{\text{fac}} = \langle \beta \rangle \), correlation \( C_{\text{fac}}(A, B) \) and standard deviations \( \Delta_{\text{fac}}A, \Delta_{\text{fac}}B \). With the notations \( \delta\alpha_i = \alpha_i - \langle \alpha \rangle \) and \( \delta\beta_i = \beta_i - \langle \beta \rangle \) the mentioned estimators are defined by the relations:

\[
\langle A \rangle_{\text{fac}} = \langle \alpha \rangle = \frac{1}{N} \sum_{i=1}^{N} \alpha_i \quad , \quad \langle B \rangle_{\text{fac}} = \langle \beta \rangle = \frac{1}{N} \sum_{i=1}^{N} \beta_i \tag{2}
\]

\[
C_{\text{fac}}(A, B) = \frac{1}{N} \sum_{i=1}^{N} \delta\alpha_i \cdot \delta\beta_i \tag{3}
\]

\[
(\Delta_{\text{fac}}A)^2 = \frac{1}{N} \sum_{i=1}^{N} (\delta\alpha_i)^2 \quad , \quad (\Delta_{\text{fac}}B)^2 = \frac{1}{N} \sum_{i=1}^{N} (\delta\beta_i)^2 \tag{4}
\]

Let us remark that in mathematical statistics when an observable \( A \) is a deterministic (non-random) variable the quantity \( \langle A \rangle_{\text{fac}} \) estimate an intrinsic characteristic of the respective observable while \( \Delta_{\text{fac}}A \) describe the error (uncertainty) with which it is evaluated \( \langle A \rangle_{\text{fac}} \). But in the case when \( A \) is a random variable both \( \langle A \rangle_{\text{fac}} \) and \( \Delta_{\text{fac}}A \) evaluate intrinsic characteristics of \( A \). In such a case the errors are specific at the same time for both \( \langle A \rangle_{\text{fac}} \) and \( \Delta_{\text{fac}}A \). The respective errors can be described by means of some uncertainty indicators \( \varepsilon[\langle A \rangle_{\text{fac}}] \) and \( \varepsilon[\Delta_{\text{fac}}A] \) defined as

\[
\varepsilon[\langle A \rangle_{\text{fac}}] = \{V[\langle A \rangle_{\text{fac}}]\}^{1/2} \quad , \quad \varepsilon[\Delta_{\text{fac}}A] = \{V[\Delta_{\text{fac}}A]\}^{1/4} \tag{5}
\]

where \( V[\eta] \) denote the variance of \( \eta \) defined in terms of \( \alpha_i \) and \( \beta_i \) by means.
Now let us observe that from the total number $N$ of values $\alpha_1, \alpha_2, \ldots, \alpha_N$ only a restricted number $n$ ($n < N$) of them are distinct. The respective distinct values, denoted by $a_1, a_2, \ldots, a_n$ constitute the spectrum of $A$. Such a value $a_i$ appears in $N_i$ trials and, consequently it is associated with the selection frequency $\nu_j = \frac{N_j}{N}$ (evidently $\sum_{j=1}^{n} \nu_j = 1$). So for $\langle A \rangle_{\text{fac}}$ and $\Delta_{\text{fac}} A$ one obtains the expressions:

$$\langle A \rangle_{\text{fac}} = \sum_{j=1}^{n} \nu_j \cdot a_j, \quad \Delta_{\text{fac}} A = \left[ \sum_{j=1}^{n} \nu_j (a_j - \langle A \rangle_{\text{fac}})^2 \right]^{1/2}$$

These expressions allow to point out the idea of spectrum preservation often assumed in experimental practice. According to the respective idea in measurements of a random quantity $A$ the changes of experimental performances aim ameliorative transformations for the frequencies $\nu_j$ but assume the preservation of the values $a_j$ which define the spectrum of $A$.

Based on the above considerations we think that a natural approach of QMS can be founded on the following reconsidered conjectures (RC):

- **RC.1**: The purpose of a QMS is to give information about the pre-existent state of the investigated system.
- **RC.2**: Because QMS view the systems studied in QM, their theoretical descriptions must be done in terms of quantum operators and wave functions.
- **RC.3**: A QMS consists in a statistical sampling which preserves the spectra of observables but can modify the corresponding probabilities. That is why theoretically a QMS must be described as an operation which preserves the mathematical expressions of operators but which is associated with functional transformations for the wave functions (or for related probabilistic quantities). So the description of QMS is dissociated of any idea about the collapse (reduction) of wave function.
- **RC.4**: Since the simple usual QM refers only to the intrinsic properties of quantum systems the transformations mentioned in **RC.3** must contain some extra-QM elements regarding the measuring devices and procedures. Then the description of QMS appears not as a part of QM theory but as a distinct and independent task comparatively with the objectives of usual QM.

### 4 An approach from the reconsidered perspective

Now let us develop a theoretical model for description of a QMS regarded as an operation with the characteristics announced in **RC.1-3**.
To the respective characteristics we add firstly the observation that the wave functions incorporate information (of probabilistic nature) about the measured system. That is why a QMS can be regarded as a process of information transmission, from the respective system to the recorder of the measuring device. The input ($in$) information, described by a wave function $\psi_{in}$, regards the measured system considered as information source. The output ($out$) information, described by a wave function $\psi_{out}$, refers to the data received on the recorder taken as information receiver. So the measuring device plays the role of a channel for information transmission. Then the errors (uncertainties) induced by measurement appears as alteration of the transmitted information.

As a matter of fact a QMS description of the mentioned kind can be depicted as follows. Let be a spinless microparticle (quantum system) whose own characteristics of orbital type are described by the intrinsic ($in$) wave function $\psi_{in}(\vec{r})$, regarded as solution of the corresponding Schrödinger equation. The observables of interest (such are coordinates, the components of linear and angular momenta or energy) will be designed by $A_k (k = 1, 2, ..., s)$. They are described by the corresponding usual QM operators $\hat{A}_k$ regarded as generalized random variables. In spirit of the discussions from section 3 we suggest that within the theoretical description the QMS operation must leave unchanged the spectra (and consequently the mathematical expressions) of operators $\hat{A}_k$. The same operation has to transform the quantum probabilities from $in$-reading into $out$-reading. The mentioned probabilities are associated with the densities $\rho_z$ and currents $\vec{J}_z (z = in, out)$ defined by:

$$\rho_z = |\psi_z|^2, \quad \vec{J}_z = \frac{\hbar}{m}|\psi_z|^2 \cdot \nabla \phi_z$$

(7)

Here $|\psi_z|$ and $\phi_z$ denote the modulus respectively the argument of $\psi_z$ (i.e. $\psi_z = |\psi_z| \cdot \exp(i\phi_z)$) and $m$ represents the particle mass.

The alluded association is connected with the facts that [17] the set $\rho_z$ and $\vec{J}_z$ "encodes the probability distributions of quantum mechanics" and it "is in principle measurable by virtue of its effects on other systems". To be added here the possibility [18] for taking in QM as primary entity the set $\rho_{in} - \vec{J}_{in}$ but not the wave function $\psi_{in}$ (i.e. the possibility to start the QM considerations with the continuity equation for $\rho_{in} - \vec{J}_{in}$ and subsequently to derive the Schrödinger equation for $\psi_{in}$).

The above noted observations suggest that the transformations $\psi_{in} \rightarrow \psi_{out}$ to be described in terms of $\rho_z$ and $\vec{J}_z (z = in, out)$. But $\rho_z$ and $\vec{J}_z$ refer to the position respectively motion kinds of probabilities. Experimentally the two kinds are regarded as measurable by distinct devices and procedures. Then the aimed description of QMS has to combine the distinct functional transformation of $\rho_{in}$ in $\rho_{out}$ respectively of $\vec{J}_{in}$ in $\vec{J}_{out}$.
Similarly with the classical situations [11,12], for which are implied measuring devices with linear and stationary characteristics, the alluded functional transformations can be written as:

\[ \rho_{\text{out}}(\vec{r}) = \int \Gamma(\vec{r}, \vec{r}') \rho_{\text{in}}(\vec{r}') \, d^3\vec{r}' \]  

(8)

\[ J_{\text{out}; k}(\vec{r}) = \sum_{l=1}^{3} \int \Lambda_{kl}(\vec{r}, \vec{r}') J_{\text{in}; l}(\vec{r}') \, d^3\vec{r}' \]  

(9)

\( J_{z; k} \) with \( z = \text{in, out} \) and \( k = 1, 2, 3 = X, Y, Z \) denote the cartesian components of \( J_z \). Due to the probabilistic correspondence between \( \rho_{\text{in}} \) and \( \rho_{\text{out}} \) respectively between \( J_{\text{in}} \) and \( J_{\text{out}} \) the kernels \( \Gamma(\vec{r}, \vec{r}') \) and \( \Lambda_{kl}(\vec{r}, \vec{r}') \) satisfy the conditions:

\[ \int \Gamma(\vec{r}, \vec{r}') \, d^3\vec{r} = \int \Gamma(\vec{r}, \vec{r}') \, d^3\vec{r}' = 1 \]  

(10)

\[ \sum_{k=1}^{3} \int \Lambda_{kl}(\vec{r}, \vec{r}') \, d^3\vec{r} = \sum_{l=1}^{3} \int \Lambda_{kl}(\vec{r}, \vec{r}') \, d^3\vec{r}' = 1 \]  

(11)

The kernels \( \Gamma \) and \( \Lambda_{kl} \) describe the transformations induced by QMS in the information regarding the measured system (microparticle). Therefore they must incorporate some extra-QM elements regarding the characteristics of the measuring devices and procedures. The respective elements do not belong to the usual QM description for the intrinsic properties of the measured system.

The above considerations facilitate an evaluation of the effects induced by QMS on the probabilistic estimators of orbital observables specific to the measured system. Such observables are described by the operators \( \hat{A}_k \) whose expressions depend on \( \vec{r} \) and \( \nabla \). According with RC.3 the mentioned expressions are supposed to remain invariant under the transformations which describe QMS. So one can say that in the situations associated with the wave functions \( \psi_z \) \((z = \text{in, out})\) two observables \( A \) and \( B \) are described by the following probabilistic estimators: mean values \( \langle A \rangle_z \) and \( \langle B \rangle_z \), correlation \( C_z(A, B) \) and standard deviations \( \Delta_z A \) and \( \Delta_z B \). With the usual notation \((f, g) = \int f^* g \, d^3\vec{r}\) for the product of two functions \( f \) and \( g \), the mentioned estimators are defined by the relations:

\[ \langle A \rangle_z = (\psi_z, \hat{A} \psi_z) \quad , \quad \langle B \rangle_z = (\psi_z, \hat{B} \psi_z) \]  

(12)

\[ C_z(A, B) = (\delta_z \hat{A} \psi_z, \delta_z \hat{B} \psi_z), \quad \delta_z \hat{A} = \hat{A} - \langle A \rangle_z \]  

(13)
\[ \Delta_z X = \sqrt{\mathcal{C}_z(X,X)} , \quad X = A, B \] (14)

Note that the estimators (12) - (14) can be calculated by utilization of the basic probability elements \( \varrho_z \) and \( \vec{J}_z \). So if \( \hat{A} \) does not depend on \( \nabla \) (i.e. \( \hat{A} = A(\vec{r}) \)) in evaluating the scalar products from (12) - (14) one can use the evident equality \( \psi_z^* \hat{A} \psi_z = A(\vec{r})\rho_z \). When \( \hat{A} \) depends on \( \nabla \) (i.e \( \hat{A} = A(\nabla) \)) in the same products can be appealed the substitutions

\[ \psi_z^* \nabla \psi_z = \frac{1}{2} \nabla \rho_z + \frac{im}{\hbar} \vec{J}_z \] (15)

\[ \psi_z^* \nabla^2 \psi_z = \rho_z^{1/2} \nabla^2 \rho_z^{1/2} + \frac{im}{\hbar} \nabla \vec{J}_z - \frac{m^2 \vec{J}_z^2}{\hbar^2 \rho_z} \] (16)

For the evaluation of the estimators (12), (13) and (14) the above mentioned utilization seems to allow the avoidance the implications of [17] "a possible nonuniqueness of current" (i.e. of the set \( \varrho_z \) and \( \vec{J}_z \)". Within the above approach, for two observables \( A \) and \( B \), the measuring errors can be evaluated through the following predicted (prd) uncertainty indicators:

\[ \varepsilon_{\text{prd}}(\langle A \rangle) = |\langle A \rangle_{\text{out}} - \langle A \rangle_{\text{in}}| \] (17)

\[ \varepsilon_{\text{prd}}(\mathcal{C}(A,B)) = |\mathcal{C}_{\text{out}}(A,B) - \mathcal{C}_{\text{in}}(A,B)| \] (18)

\[ \varepsilon_{\text{prd}}(\Delta A) = |\Delta_{\text{out}} A - \Delta_{\text{in}} A| \] (19)

These indicators incorporate errors regarding the whole spectrum of the observables \( A \) and \( B \) considered as random variables.

Now note that the \( z = \text{out} \) version of probabilistic indicators (12)-(14) as well as uncertainties indicators (15)-(17) have a theoretical significance. The adequacy of such estimators and indicators must be tested by comparing them with their factual (fac) correspondents (2)-(4) respectively (5). If the test is affirmative both theoretical descriptions, of QM intrinsic properties of system and of QMS, can be accepted as adequate. But if test gives an invalidation, at least one of the mentioned description must be regarded as inadequate.
5 A simple exemplification

For an exemplification of the above considerations let us refer to a microparticle in a one-dimensional motion along the x-axis. We take $\psi_{in}(x) = |\psi_{in}(x)| \cdot \exp \{i\phi_{in}\}$ with

$$|\psi_{in}(x)| = (\sigma \sqrt{2\pi})^{-1/4} \cdot \exp \left\{ -\frac{(x - x_0)^2}{4\sigma^2} \right\}, \quad \phi_{in}(x) = k \cdot x \quad (20)$$

Correspondingly we have

$$\rho_{in}(x) = |\psi_{in}(x)|^2, \quad J_{in}(x) = \frac{\hbar k}{m} |\psi_{in}(x)|^2 \quad (21)$$

So the intrinsic characteristics of the microparticle are described by the parameters $x_0$, $\sigma$ and $k$.

If the errors induced by QMS are small in (8)-(9) we can operate with the kernels of Gaussian forms given by:

$$\Gamma(x, x') = (\gamma \sqrt{2\pi})^{-1} \cdot \exp \left\{ -\frac{(x - x')^2}{2\gamma^2} \right\} \quad (22)$$

$$\Lambda(x, x') = (\lambda \sqrt{2\pi})^{-1} \cdot \exp \left\{ -\frac{(x - x')^2}{2\lambda^2} \right\} \quad (23)$$

where $\gamma$ and $\lambda$ describe the characteristics of the measuring devices. Then for $\rho_{out}$ and $J_{out}$ one finds

$$\rho_{out}(x) = \left[ 2\pi \left( \sigma^2 + \gamma^2 \right) \right]^{-1/2} \cdot \exp \left\{ -\frac{(x - x_0)^2}{2(\sigma^2 + \gamma^2)} \right\} \quad (24)$$

$$J_{out}(x) = \hbar k \left[ 2\pi m^2 \left( \sigma^2 + \lambda^2 \right) \right]^{-1/2} \cdot \exp \left\{ -\frac{(x - x_0)^2}{2(\sigma^2 + \lambda^2)} \right\} \quad (25)$$

One can see that in the case when both $\gamma \to 0$ and $\lambda \to 0$ the kernels $\Gamma(x)$ and $\Lambda(x)$ degenerate into the Dirac’s function $\delta(x - x')$. Then $\rho_{out} \to \rho_{in}$ and $J_{out} \to J_{in}$. Such a case corresponds to an ideal measurement. Alternatively the cases when $\gamma \neq 0$ and/or $\lambda \neq 0$ are associated with non-ideal measurements.

As observables of interest we take the coordinate $x$ and momentum $p$ described by the operators $\hat{x} = x \cdot$ and $\hat{p} = -i\hbar \frac{\partial}{\partial x}$. Then, according to the scheme presented in the previous section, one obtains

$$\langle x \rangle_{in} = \langle x \rangle_{out} = x_0 \quad , \quad \langle p \rangle_{in} = \langle p \rangle_{out} = \hbar \cdot k \quad (26)$$
\( C_{in}(x,p) = C_{out}(x,p) = \frac{i\hbar}{2} \) \tag{27}

\[
\Delta_{in}x = \sigma \quad , \quad \Delta_{out}x = \sqrt{\sigma^2 + \gamma^2}
\] \tag{28}

\[
\Delta_{in}p = \frac{\hbar}{2\sigma}
\] \tag{29}

\[
\Delta_{out}p = \hbar \frac{k^2(\sigma^2 + \gamma^2)}{\sqrt{\sigma^4 - \gamma^4 + 2\gamma^2(\sigma^2 + \lambda^2) - k^2 + \frac{1}{4(\sigma^2 + \gamma^2)}}}
\] \tag{30}

Subsequently for the corresponding uncertainty indicators of predicted \((prd)\) type defined in (17),(18) and (19) one finds

\[
\varepsilon_{prd}(\langle x \rangle) = 0, \quad \varepsilon_{prd}(\langle p \rangle) = 0, \quad \varepsilon_{prd}(\langle C(x,p) \rangle) = 0 \tag{31}
\]

\[
\varepsilon_{prd}(\Delta x) = \sqrt{\sigma^2 + \gamma^2} - \sigma \tag{32}
\]

\[
\varepsilon_{prd}(\Delta p) = \Delta_{out}p - \Delta_{in}p \neq 0 \tag{33}
\]

These relations show that in the considered model the estimators \(\langle x \rangle, \langle p \rangle\) and \(C(x,p)\) have not predicted uncertainties. But within the same model the estimators \(\Delta x\) and \(\Delta p\) are characterized by non-null uncertainties.

For an evaluation of the interdependencies between the uncertainties of \(x\) and \(p\) from (31)-(33) one obtains

\[
\varepsilon(\langle x \rangle) \cdot \varepsilon(\langle p \rangle) = 0 \tag{34}
\]

\[
\varepsilon(\Delta x) \cdot \varepsilon(\Delta p) = \hbar \cdot W \tag{35}
\]

Here \(W\) is a real, non-negative and dimensionless quantity which can be evaluated by means of the relations (33),(28), (29) and (30).

If in (20) we restrict to the values \(x = 0, k = 0\) and \(\sigma = \sqrt{\frac{\hbar}{2m\omega}}\) our system is just a linear oscillator situated in its ground state \((m = \text{mass} \text{ and } \omega = \text{frequency})\).
pulsation). For an observable of interest we refer to the energy $E$ described by the Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{m\omega^2}{2} x^2$$

(36)

Then for the probabilistic estimators of energy one finds

$$\langle \hat{H} \rangle_{in} = \frac{\hbar \omega}{2}, \quad \Delta_{in} H = 0$$

(37)

$$\langle \hat{H} \rangle_{out} = \frac{\omega [\hbar^2 + (\hbar + 2m\omega\gamma^2)^2]}{4 (\hbar + 2m\omega\gamma^2)}$$

(38)

$$\Delta_{out} H = \frac{\sqrt{2m\omega^2\gamma^2} (\hbar + m\omega\gamma^2)}{\hbar + 2m\omega\gamma^2}$$

(39)

The corresponding predicted uncertainty indicators are

$$\varepsilon_{prd} (\langle \hat{H} \rangle) = \langle \hat{H} \rangle_{out} - \langle \hat{H} \rangle_{in} \neq 0$$

(40)

$$\varepsilon_{prd} (\Delta H) = \Delta_{out} H - \Delta_{in} H \neq 0$$

(41)

6 Conclusions

The problem of QMS description persists in our days as an open and disputed question while many of its approaches are founded on traditional conjectures inspired from TIHR. But indubitable facts show [10] that TIUR is an unjustified doctrine. Consequently in this paper we find that the mentioned conjectures imply unsurmountable deficiencies. The respective finding motivates our search for a possible new approach of QMS, based on reconsidered conjectures. We propose a set of four such conjectures and develop an adequate approach of QMS.

Our approach is motivated by some considerations about the characteristics of real QMS. So we regard a QMS as a statistical sampling (i.e. as a set of a large number of trials) which preserves the spectra of the observables but can modify the probabilities associated with the values from the respective spectra. Consequently in the proposed approach the QM operators of observables are preserved. Concomitantly the probability density and current
(directly connected with the wave function) are subjected to linear functional transformations, from input (in) into output (out) expressions. The respective transformations imply the characteristics of measuring devices. The quantum observables are evaluated through probabilistic estimators such are: mean values, correlations and standard deviations. Within the proposed approach the mentioned estimators are characterized by both in and out values. Then for each estimator the difference between the respective value gives a quantitative evaluation of the measuring uncertainty. A concrete exemplification of the above ideas is given in section 5.

Our approach of QMS is quite different from the approaches founded conjectures inspired from TIUR. The difference is evidenced on the one hand by the opinion that a QMS must be regarded as a statistical sampling but not as a single trial. Consequently we can avoid the controversial idea of wave function collapse (reduction). On the other hand the alluded difference is pointed out by the presumption that the description of QMS must be regarded not as a part of QM theory but as a distinct and independent task comparatively with the objectives of usual QM. Note that the opinions promoted here and in [6,7,8,9,10] in connection with QMS respectively with RSUR are mainly consonant with usual QM. Therefore they must not be tested "against quantum mechanics" as it is suggested [3] for the theories which promote the idea of wave function reduction.

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List of abbreviations

\begin{itemize}
  \item \textit{in} = input
  \item \textit{fac} = factual
  \item \textit{out} = output
  \item \textit{prd} = predicted
  \item QM = quantum mechanics
  \item QMS = quantum measurement(s)
  \item RC = reconsidered conjecture
  \item RSUR = Robertson Schrödinger uncertainty relation
\end{itemize}
TC = traditional conjecture
TIUR = traditional interpretation of uncertainty relations

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