Error Performance of DPSK Satellite-to-Ground Laser Communication over Atmospheric Turbulence and Pointing Errors

Shiqi Hao¹, Xiongfeng Wan¹, Qingsong Zhao¹ and Chenlu Xu¹
¹State Key Laboratory of Pulse Power Laser Technology, National University of Defense Technology, Hefei, Anhui 230037, China
²Shiqi Hao: Liu_hsq@126.com

Abstract. In satellite-to-ground laser communication, atmospheric turbulence will seriously deteriorate the system performance. The DPSK modulation technology is usually used to control the influence of atmospheric turbulence. The Meijer G-function is used to give the closed-form expression of the average bit error rate (ABER) of the system considering the combined influence of the atmospheric turbulence and the pointing errors, where the atmospheric turbulence channel is submitted to the Gamma-Gamma distribution and the pointing errors is submitted to the Beckman distribution. Through numerical simulation, the influence of system transmitting power, normalized beam width and zenith angle on the BER performance of the system is analyzed. The simulation results show that the ABER of the system decreases with the increase of the transmitting power, the error performance of the system considered the bias errors is obviously higher, and the error performance is also different when the azimuth and elevation normalized jitter deviation are not equal. The system performance will decline rapidly with the increase of the zenith angle. The BER of the system decrease firstly and then increase with the increase of the normalized beam width, and the optimal normalized beam width can be found to achieve the best performance.

1. Introduction
Free-space optical communication has many advantages, such as higher bandwidth, lower power requirement, and smaller antenna size. It has attracted more and more attention in civil and military fields. Laser transmission in atmospheric channel will inevitably be affected by path loss and atmospheric turbulence. Because of the small beam divergence, the pointing errors between the transmitter and receiver will affect the communication performance as well. Therefore, when analysing the performance of free-space optical communication system, it is necessary to consider the path loss, atmospheric turbulence and pointing errors synthetically[1].

Farid et al[2] derived the closed expression of outage probability under the influence of atmospheric turbulence and pointing errors. Han Liqiang et al[3] analyzed the error performance and outage probability performance of free-space optical communication system under the combined influence of Gamma-Gamma atmospheric channel and pointing errors. However, the above researches adopt On-Off Keying (OOK) intensity modulation/direct detection (IM/DD). Compared with IM/DD modulation, differential phase shift keying (DPSK) modulation coherent detection has better sensitivity and makes full use of phase information[4]. Ma Xiaoping et al[5] showed that DPSK modulation technology is an effective method to overcome the influence of atmospheric turbulence in satellite-to-ground laser...
communication, but the influence of pointing errors on system performance have not been considered. The results of literature investigation show that the influence of pointing errors has not been considered before on the performance analyse of DPSK satellite-to-ground laser communication. In the current research on pointing errors, it is usually assumed that the distribution of pointing errors obeys Rayleigh distribution. It is impossible to reflect the random jitter characteristics of the satellite platform accurately. Boluda-Ruiz R et al[6] gave a generalized pointing errors model which obeys Beckman distribution. When the pointing errors obey the Beckman distribution, the research on the influence of atmospheric turbulence and pointing errors on DPSK satellite-to-ground laser communication system has not been reported.

The rest of this paper is organized as follows. Section II briefly describes the system model under consideration. Section III presents the closed-form expressions for the ABER of DPSK satellite-to-ground laser communication system by using Meijer G-function. In Section IV, numerical results as derived expressions are presented. Finally, some concluding remarks are given in section V.

2. System and channel model

2.1. System model

The schematic diagram of satellite-to-ground laser communication is shown in Fig.1. The transmitted signal is modulated by DPSK. When transmitting in atmospheric channel, the optical signal will be affected by atmospheric turbulence. In addition, the vibration of the satellite will seriously affect the pointing accuracy between the transmitter and the receiver, resulting in power loss of the received light. The received optical signal is amplified by APD and sent into the demodulation system to get the original information.

\[ y = Rhx + n \]  \hspace{1cm} (1)

where \( R \) is the detector’s responsivity factor; \( h \) is the channel state; \( x \) is the value of the transmitted data; \( n \) is the Gaussian noise.

Under the combined influence of atmospheric turbulence and pointing errors, the channel state \( h \) can be expressed as follows\[7\]

\[ h = h_l h_a h_p \]  \hspace{1cm} (2)

where \( h_l \) is the path loss; \( h_a \) is the fading due to atmospheric turbulence; \( h_p \) is the fading due to geometric spread and pointing errors.

2.2. Channel model

2.2.1. Path loss
The path loss can be calculated by Lambert-Beer law. When the transmission distance is \( L \), \( h_i \) can be given by

\[
h_i(L) = \frac{P(L)}{P(0)} = \exp(-\sigma L) \tag{3}
\]

where \( P(L) \) is the optical power at the transmission distance \( z \); \( \sigma \) is the attenuation coefficient. The experimental results show that the path loss is a constant generally.

2.2.2. Atmospheric attenuation

For strong turbulence, the intensity fluctuation probability density function (PDF) is modeled as a Gamma-Gamma distribution\[^8\]:

\[
f_h(h) = \frac{2(\alpha \beta)^{\alpha+\beta/2}}{\Gamma(\alpha) \Gamma(\beta)} (h)^(\alpha+\beta) K_{\alpha-\beta}(2\sqrt{\alpha \beta h}) \tag{4}
\]

where \( \Gamma(\cdot) \) is the well-known Gamma function; \( K_{\alpha-\beta}(\cdot) \) is the \( \nu \)-th order modified Bessel function of the second kind; \( 1/\alpha \) and \( 1/\beta \) are the variances of the large and small scale eddies, respectively, and they are defined as\[^9\]:

\[
\alpha = \left\{ \exp \left[ \frac{0.49 \sigma^2_\phi}{(1 + 0.65 d^2 + 1.11 \sigma^2_\phi)^{7/5}} \right] - 1 \right\}^{-1} \tag{5}
\]

\[
\beta = \left\{ \exp \left[ \frac{0.51 \sigma^2_\phi (1 + 0.69 \sigma^2_\phi)}{1 + 0.9 d^2 + 0.62 \sigma^2_\phi^{2/3}} \right] - 1 \right\}^{-1} \tag{6}
\]

where \( \sigma^2_\phi \) is the Rytov variance, \( d = \sqrt{k a^2 / L} \) with \( k = 2\pi / \lambda \) and \( a \) being the wave number and the receive aperture size, respectively, and \( L \) being the link distance given as

\[
L = (H_s - H_g) \sec(\zeta) \tag{7}
\]

where \( H_g \) and \( H_s \) are altitudes of satellite and ground stations, respectively, and \( \zeta \) is the zenith angle.

For satellite-to-ground downlink laser communication systems, Rytov variance is defined as

\[
\sigma^2_n = 2.24 \kappa^{7/6} \sec^{11/6}(\zeta) \int_0^H C^2_n(H)(H-H_g)^{5/6} dH \tag{8}
\]

where \( C^2_n \) is the refractive index structure parameter at the altitude \( H \). The structure parameter for Huffnagel-Valley (H-V) model is given as\[^10\]:

\[
C^2_n(H) = 0.00594 \left( \frac{\nu}{27} \right) \left( H / 10^4 \right)^{10} \exp(-H/1000)
+ 2.7 \times 10^{-16} \exp(-H/1500) + C^2_n(0) \exp(-H/100) \tag{9}
\]

where \( \nu \) is the speed of wind; \( C^2_n(0) \) is the refractive index structure parameter at the ground level; \( H \) is the altitude.

2.2.3. Pointing errors

We can express the radial displacement vector at the receiver aperture plane as \( r = [r_x, r_y]^T \), where \( r_x \) and \( r_y \) denote the displacements located along the horizontal and elevation axes at the detector plane respectively, which can be described by nonzero mean Gaussian distributed RVs, i.e., \( r_x \sim N(\mu_x, \sigma^2_\phi) \), \( r_y \sim N(\mu_y, \sigma^2_\phi) \). Then the radial displacement \( r = \sqrt{r_x^2 + r_y^2} \) follows the Beckmann distribution\[^11\]:

\[
f_r(r) = \frac{r}{2\pi \sigma_r} \int_0^{2\pi} \exp\left(-\frac{(r \cos \phi - \mu_x)^2}{2\sigma^2_r} + \frac{(r \sin \phi - \mu_y)^2}{2\sigma^2_r}\right) d\phi \tag{10}
\]

It must be mentioned that finding the combined effect of the atmospheric turbulence and pointing errors might be intractable due to the fact that the closed-form solution for Beckmann distribution’s
integral in Eq. (10) is unknown. The Beckmann distribution is approximated by a modified Rayleigh distribution of parameter, which can be obtained from the third-order central moment [6]

\[ f(r) = \frac{r}{\sigma_{\text{mod}}^3} \exp\left(-\frac{r^2}{2\sigma_{\text{mod}}^3}\right) \tag{11} \]

where the \( \sigma_{\text{mod}} \) parameter is expressed as

\[ \sigma_{\text{mod}} = \left(\frac{2\mu_r^2\sigma_r^4 + 3\mu_r^4\sigma_r^6 + \sigma_r^8 + \sigma_r^{10}}{2}\right)^{1/8} \tag{12} \]

when considering detector aperture size, beam width, and jitter variance, the PDF of instantaneous channel gain can be expressed as

\[ f_{h_p}(h_p) = \frac{\varphi_{\text{mod}}}{(A_0G)^{\lambda h_p}} \cdot h_p^{\varphi_{\text{mod}} - 1}, \quad 0 \leq h_p \leq A_0G \tag{13} \]

where \( \varphi_{\text{mod}} = \omega_{\text{eq}} / \sigma_{\text{mod}} \), \( \omega_{\text{eq}} = \sqrt{\pi} \text{erf}(v) \) \( 2v \exp(-v^2) \); \( v = \sqrt{\pi a} / \sqrt{2\omega_{\text{eq}}} \); \( A_0 = (\text{erf}(v))^2 \); \( a \) is the radius of receiver antenna; \( \omega_{\text{eq}} \) is the equivalent beam width; \( \omega_{\text{eq}} \) is the beam waist at distance \( z \); \( Q \) is the Correction parameter, which can be expressed as

\[ Q = \exp\left\{ \frac{1}{\varphi_{\text{mod}}^2} \cdot \left( \frac{2(\mu_r^2 + \mu_r^4 + \sigma_r^8) - \omega_{\text{eq}}^2}{\omega_{\text{eq}}^2} \right) \right\} \tag{14} \]

2.2.4. Channel model under the combined effects

Using the PDFs of atmospheric turbulence and pointing errors, the PDF of \( h \) is given as

\[ f_h(h) = \int_{h} f_{h|\lambda}(h|h) \cdot f_{\lambda}(h) dh \tag{15} \]

where \( f_{h|\lambda}(h|h) \) is the conditional probability given \( h \) state and is expressed by

\[ f_{h|\lambda}(h|h) = \frac{\varphi_{\text{mod}}^\lambda}{(A_0Q)^{\lambda h}} \cdot h^\lambda \exp(-\lambda h), \quad 0 \leq h \leq A_0Qh \tag{16} \]

By substituting (4) and (16) in (15), \( f_h(h) \) is given by

\[ f_h(h) = \int_{0}^{h} \frac{\varphi_{\text{mod}}^\lambda(\alpha\beta)^{2\lambda/2}}{(A_0Qh)^{\lambda h}} \cdot h^{\lambda h - 1} \exp(-\lambda h) dh \tag{17} \]

The above integral is very complicated. In order to derive a closed-form solution, we express the \( K_v(.) \) in terms of the Meijer’s G-function, defined as [12]

\[ K_v(\lambda) = \frac{1}{2} G_{0,2}^{2,0} \left[ \frac{x^2}{4} \right]_{\lambda h - 2 / \lambda h - 2}^{\lambda h - 1 / \lambda h - 1} \tag{18} \]

Then

\[ f_h(h) = \frac{\alpha\beta\varphi_{\text{mod}}^\lambda}{A_0Qh^\lambda} G_{0,2}^{2,0} \left[ \frac{\alpha\beta}{A_0Qh} \right] \left[ \frac{\varphi_{\text{mod}}^\lambda - 1}{\alpha - 1, \beta - 1} \right] \tag{19} \]

3. Error performance

In the satellite-to-underground laser communication, because of the long transmission distance and the influence of atmospheric turbulence and pointing errors, the APD detector is usually used to amplify the signal at the receiver. In this case, the instantaneous signal-to-noise ratio of the output signal can be expressed as [13]

\[ \gamma(h) = \frac{(\eta T P h / hc + K_s) + I_g T}{2(\eta T P h / hc + K_s) + I_g T} \]  

\[ = \frac{(\eta T P h / hc + K_s) + I_g T}{2(\eta T P h / hc + K_s) + I_g T} \]  

\[ = \frac{(\eta T P h / hc + K_s) + I_g T}{2(\eta T P h / hc + K_s) + I_g T} \]

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where $P_t$ is the average transmitting power; $G$ is the photomultiplier gain factor; $F$ is the additional noise factor; $\hbar$ is the Planck constant; $q$ is the electron charge; $\eta$ is quantum efficiency; $c$ is the frequency of the signal light; $T_s$ is the optical pulse time; $K_b$ is the photon count of the background light; $I_{dc}$ is the dark current, $T$ is the temperature; $\sigma^2$ is the thermal noise; If the dark current, thermal noise and the photon count of the background light are neglected, the instantaneous signal-to-noise ratio can be expressed as

$$\gamma(h) = \frac{\eta T_s P_t}{2\hbar c F}$$  \hspace{1cm} (21)

For a 2DPSK scheme, the BER is obtained as

$$P(e \mid h) = \frac{1}{2} \text{erfc} (\gamma_e h)$$  \hspace{1cm} (22)

where $\gamma_e = \frac{\eta T_s P_t}{2\hbar c F}$.

Considering the combined effects of atmospheric turbulence and pointing errors, the average BER of the system can be expressed as

$$P(e) = \int_0^\infty P(e \mid h)f_h(h)dh$$  \hspace{1cm} (23)

According to [14], it is known as

$$\int_0^\infty \text{erfc}(ax)G_{\alpha,\beta}^D \begin{bmatrix} b \\ h \\ \epsilon \\ \epsilon \end{bmatrix} dx = \frac{2^{1/2} - 1}{\pi^{1/2}} G_{\alpha,\beta}^D \begin{bmatrix} b \\ h \\ \epsilon \\ \epsilon \end{bmatrix}$$  \hspace{1cm} (24)

By submitting (19) and (22) in (23), the average BER is given as

$$P(e) = \frac{\phi_{\text{max}}^2 2^{\alpha+\beta-1}}{\pi^{1/2} \Gamma(\alpha) \Gamma(\beta)} \frac{\alpha \beta}{4 A_G \gamma(h)} \left[ \frac{1}{2}, 1, 1+\frac{\phi_{\text{max}}^2}{2} \right] \left[ \frac{1}{2}, 1, 1+\frac{\phi_{\text{max}}^2}{2} \right]$$  \hspace{1cm} (25)

4. Numerical results

The simulation parameters are set as shown in Table 1.

Table 1. Satellite-to-ground laser communication parameters.

| Parameter                  | Symbol | Value      |
|----------------------------|--------|------------|
| Satellite altitude         | $H_s$  | 700km      |
| Ground station altitude    | $H_g$  | 100m       |
| Zenith angle               | $\varsigma$ | 60°     |
| Wavelength                 | $\lambda$ | 850nm     |
| Normalized beam width      | $\omega/a$ | 20       |
| Quantum efficiency         | $\eta$  | 0.75       |
| Additional noise factor    | $F$    | $G^{0.5}$  |
| photomultiplier gain factor| $G$    | 100        |
| Radius of receiver antenna | $a$    | 0.2m       |
Optical pulse time | $T_s$ | 10ns  
Average transmitting power | $P_t$ | 1W  
Path loss | $h_t$ | 0.1  
Wind speed | $v$ | 21 m/s  
Structure parameter | $c_j(0)$ | 1.7×10$^{-31}$m$^{-3}$

**Fig. 2.** ABER versus transmitting power

Fig. 2 shows that the ABER decreases stepwise with the increase of transmitting power under the condition that normalized bias errors and normalized jitter standard deviation in azimuth and pitch directions are different. And the BER considering the bias errors is obviously higher. In addition, when the normalized jitter standard deviation in azimuth and pitch directions is different, the system performance varies greatly, which shows that the distribution of pointing errors have a great impact on the system error performance. It further explains the necessity of accurately describing the distribution of pointing errors.

**Fig. 3.** ABER versus transmitting power at different zenith angle

Fig. 3 shows that the system performance decline rapidly with the zenith angle increasing. When the zenith angle is large, the signal light transmission distance in the atmospheric channel is longer. At this time, influenced by path loss and atmospheric turbulence, the BER will rise rapidly.
Fig. 4 shows that the ABER decreases firstly and then increases with the increase of normalized beam width. There is an optimal normalized beam width value to optimize the system performance. When the normalized jitter standard deviation in azimuth and pitch directions is different, the optimum normalized beam width will change accordingly. Taking the numerical results in Figure 4 as an example, the normalized beam width of the average BER of the system is lowest, when $\omega_z / a \approx 17$.

5. Conclusions
The closed-form expression of BER for DPSK satellite-to-ground laser communication under the combined effect of path loss, atmospheric turbulence and pointing errors is given by using Meijer G-function. The results of numerical simulation show that the BER decreases stepwise with the increase of transmitting power, and the BER considering the bias error is significantly higher. When the normalized jitter standard deviations in azimuth and pitch directions are different, the BER also varies greatly. With the increase of zenith angle, the system performance decline rapidly. And the BER decreases firstly and then increases with the increase of normalized beam width. There exists an optimal normalized beam width value to optimize the system performance. Taking the simulation results as an example, the average BER of the system is lowest when $\omega_z / a = 17$. The research in this paper has certain theoretical guiding significance in the design of DPSK satellite-to-ground laser communication system.

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