A Re-analysis of Gravitino Dark Matter in the Constrained MSSM

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ABSTRACT: We re-consider the gravitino as dark matter in the framework of the Constrained MSSM. We include several recently suggested improvements on: (i) the thermal production of gravitinos, (ii) the calculation of the hadronic spectrum from NLSP decay and (iii) the BBN calculation including stau bound-state effects. In most cases we find an upper bound on the reheating temperature $T_R \lesssim a \times 10^7$ GeV from over-production of $^6$Li from bound state effects. We also find an upper limit on the stau lifetime of $3 \times 10^4$ sec, which is nearly an order of magnitude larger than the simple limit $5 \times 10^3$ sec often used to avoid the effect of bound-state catalysis. The bound on $T_R$ is relaxed to $\lesssim a \times 10^8$ GeV when we use a more conservative bound on $^6$Li/$^7$Li, in which case a new region at small stau mass at $\sim 100$ GeV and much longer lifetimes opens up. Such a low stau mass region can be easily tested at the LHC.

KEYWORDS: Supersymmetric Effective Theories, Cosmology of Theories beyond the SM, Dark Matter, Supersymmetric Standard Model.
1. Introduction

Local supersymmetry (SUSY), or supergravity, models predict the existence of a massive spin-3/2 particle, the gravitino $\tilde{G}$, whose mass in general depends on a supersymmetry breaking mechanism. Assuming standard big bang cosmology, it was shown early on that cosmological constraints require that the mass of gravitino $m_{\tilde{G}}$ be much less than 1 keV, or else heavier than some 10 TeV [1, 2]. While a primordial gravitino population can efficiently be diluted by inflation [3], subsequently the Universe can be repopulated with gravitinos via thermal production (TP) processes involving scatterings of Standard Model (SM) and SUSY particles in the hot plasma, with the number density proportional to the reheating temperature, $T_R$. If, assuming $R$-parity, the gravitino is the lightest supersymmetric particle (LSP) and stable, one can also generate gravitinos via non-thermal production (NTP) process of freeze-out and decays of the next-to-lightest superpartner (NLSP). Because of the gravitino’s exceedingly weak couplings to ordinary matter, the latter process usually takes place during or after the period of Big Bang Nucleosynthesis (BBN) and involves releasing substantial amounts of electromagnetically and/or hadronically interacting particles (the importance of the latter shown to be important in [4, 5, 6]), which could wreck havoc to successful predictions of Big Bang Nucleosynthesis (BBN). In order to avoid this, and assuming gravitinos to be a dominant component of dark matter (DM), one imposes an upper bound on the reheating temperature of $T_R < 10^{6-8}$ GeV [7, 8, 9, 10, 11, 12] (for recent updates see, e.g., [13, 5, 14, 15, 16, 6, 17]).

Considering NTP processes only, constraints on the parameter space of Minimal Supersymmetric Standard Model (MSSM) were derived in [18, 19, 20], basically eliminating
the lightest neutralino as the NLSP (unless $m_{\tilde{G}} \lesssim 1 \text{ GeV}$ [16]). In [14] some of us considered a combined impact of both thermal and nonthermal production mechanisms in the more predictive framework of the Constrained MSSM (CMSSM) [21] and derived both the maximum allowed reheating temperature and the viable regions of parameters of the model. An improved analysis using full systematic BBN calculation was next conducted in [16]. Both showed, in particular, that gravitino dark matter abundance from NTP processes alone [18, 19, 22, 20] can only agree with observations in regions of SUSY parameters where a gluino mass is very heavy, in the multi-TeV range, substantially above an approximate value derived in [23]. In other words, in order to have gravitino DM consistent with superpartner masses at or below the TeV scale, a substantial TP contribution to the total abundance is necessary. (In [16] it was also shown that the stau NLSP region consistent with the cosmological abundance of DM in gravitinos corresponds to a false (local) vacuum of the CMSSM.) Possible solutions to cosmic lithium problems were also investigated in [24]. The papers [14, 16] were next followed by similar detailed analyzes [25, 26].

More recently it was pointed out [27] that the existence of unstable (with lifetime longer than $10^3$ seconds), negatively charged electro-weak scale particles alter the predictions for lithium and other light element abundances via the formation of metastable bound states with nuclei during BBN [27, 28, 29, 30, 31]. These bound state effects were incorporated in the gravitino dark matter with charged particle NLSP [32, 33, 34, 35].

In light of these recent developments, we present here an updated analysis of the gravitino DM in the Constrained MSSM. Relative to the previous works [14, 16], we make improvements in the following aspects: (i) the thermal production of gravitinos including the decay allowed by thermal masses [36] as well as scattering processes; (ii) the calculation of hadronic and electromagnetic spectrum from NLSP decay including correct implementation of left-right stau mixing; (iii) the updated BBN calculation including the bound-state effects of charged massive particles.

As previously in [14, 16], we will treat $m_{\tilde{G}}$ as a free parameter and will not address the question of an underlying supergravity model and SUSY breaking mechanism. We will allow $m_{\tilde{G}}$ to vary over a wide range of values from $\mathcal{O}(\text{TeV})$ down to the $\mathcal{O}(\text{GeV})$ scale (as most natural in the CMSSM with gravity-mediated SUSY breaking), for which gravitinos (at least those produced in thermal production) would remain mostly cold DM relic, but will explore at some level also lighter gravitinos, down to 100 MeV. Less massive gravitinos could remain DM in some models of gauge mediated supersymmetry breaking [37, 38, 39, 40].

The paper is organized as follows. In section 2 we describe the improvements made in the present analysis compared to the previous ones. In section 3 we summarize the experimental and cosmological constraints, show our numerical results and derive an upper bound on $T_R$. We summarize our conclusions in section 4.
2. Improvements in the analysis

In this section we list the improvements made in this paper.

2.1 Gravitino production

We will consider gravitinos as DM relics which were produced predominantly via the TP and NTP mechanisms stated above.\(^1\)

Since the original computation of TP rates in [9] there have been a number of updates and improvements [45, 46, 47, 48, 49, 36]. In particular, a gauge invariant computation was performed in [48] (with an extension to the three SM gauge groups done in [49]), and a different technique to compute gravitino production from decays allowed by thermal masses and the effect of the top Yukawa coupling were applied in [36]. In this update, we adopt the result of [36], including the three gauge groups, which leads to about a factor of two enhancements compared to the previous calculation [47, 48]. For the renormalized gauge couplings and gaugino masses at an energy scale \( T_R \), we used a one-loop evolution by the renormalization group equation in the MSSM from GUT scale assuming the gauge coupling and gaugino mass unification,

\[
M_i(T) = \left( \frac{g_i(T)}{g_{\text{GUT}}} \right)^2 \frac{m_1}{2}.
\]  

(2.1)

The thermal production will therefore depend on the common gaugino mass \( m_{1/2} \) and the other parameters of the CMSSM: the common scalar mass \( m_0 \), \( \tan \beta \) and the trilinear soft scalar coupling \( A_0 \), as well as on the reheating temperature \( T_R \). In our analysis we use an expression for \( \Omega_{G}^{\text{TP}} h^2 \) as computed in ref. [36]

\[
\Omega_{G}^{\text{TP}} h^2 = 0.167 \left( \frac{m_{\tilde{g}}}{100 \text{ GeV}} \right) \left( \frac{T_R}{10^{10} \text{ GeV}} \right) \left( \frac{\gamma \left( T_R \right)}{T_R^6 / M_P^2} \right),
\]  

(2.2)

where the gravitino production rate \( \gamma \) is the sum of three contributions from decay, subtracted scattering and top Yukawa coupling,

\[
\gamma = \gamma_D + \gamma_{\text{sub}} + \gamma_{\text{top}},
\]  

(2.3)

and the details are given in ref. [36].

Regarding the non-thermal production of gravitinos, we proceed in the usual way. Since all the NLSP particles decay after freeze-out, the gravitino relic abundance from NTP, \( \Omega_{G}^{\text{NTP}} h^2 \), is related to \( \Omega_{\text{NLSP}} h^2 \) – the relic abundance that the NLSP would have had if it had remained stable – via a simple mass ratio

\[
\Omega_{G}^{\text{NTP}} h^2 = \frac{m_{\tilde{g}}}{m_{\text{NLSP}}} \Omega_{\text{NLSP}} h^2.
\]  

(2.4)

\(^1\)In addition, there could be other possible ways of populating the Universe with stable gravitinos, e.g. via inflaton decay or during preheating [41, 42], or from decays of moduli fields [43]. In some of these cases the gravitino production is independent of the reheating temperature and its abundance may give the measured dark matter abundance with no ensuing limit on \( T_R \). In general, such processes are, however, much more model dependent and not necessarily efficient [44], and will not be considered here.
Note that $\Omega_{\text{NTP}} \cdot h^2$ grows with the mass of the gravitino $m_{\tilde{G}}$. When we calculate the relic number density of staus, we treat correctly the mixing of left and right stau states using a supersymmetric particle spectrum calculator SuSpect [50] and employ micrOMEGAs [51] to calculate a relic density of NLSPs, which might affect the annihilation cross section when the maximal mixing happens at large $\tan \beta$, as noticed by [35].

The total abundance of the LSPs is the sum of both thermal and non-thermal production contributions
\begin{equation}
\Omega_{\tilde{G}} h^2 = \Omega_{\text{TP}}^{\text{T}} h^2 + \Omega_{\text{NTP}}^{\text{TP}} h^2. \quad (2.5)
\end{equation}
Since it is natural to expect that the LSP makes up most of DM in the Universe, we can re-write the above as [52]
\begin{equation}
\Omega_{\tilde{G}} h^2 = \Omega_{\text{TP}}^{\text{T}} h^2 \left( T_R, m_{\tilde{G}}, m_{\tilde{G}}, m_{\text{NLSP}}, \ldots \right) + \frac{m_{\tilde{G}}}{m_{\text{NLSP}}} \Omega_{\text{NLSP}} h^2 = \Omega_{\text{DM}} \simeq 0.1. \quad (2.6)
\end{equation}

### 2.2 Hadronic and electromagnetic spectrum from NLSP decay

In order to calculate light element abundances during BBN, including the impact of massive particle decays, in our case the NLSP, we need to know the detailed decay products and their spectrum in addition to NLSP lifetime and relic number density.

The lifetime of the stau or neutralino NLSP decaying to the gravitino is dominated by two-body decays,
\begin{equation}
\Gamma_{\text{tot}} \simeq \Gamma(\tilde{\tau}_1 \to \tau \tilde{G}), \quad (2.7)
\end{equation}
\begin{equation}
\Gamma_{\text{tot}} \simeq \Gamma(\chi \to \tilde{G} \gamma) + \Gamma(\chi \to \tilde{G} Z) + \Gamma(\chi \to \tilde{G} h), \quad (2.8)
\end{equation}
where, for the stau NLSP,
\begin{equation}
\Gamma(\tilde{\tau}_1 \to \tau \tilde{G}) = \frac{1}{48 \pi} \frac{m_{\tilde{G}}^5}{M_{\text{Pl}}^2} \left( 1 - \frac{m_{\tilde{G}}^2}{m_{\tilde{G}}^2} \right)^4, \quad (2.9)
\end{equation}
and for the neutralino NLSP the dominant decay to photon and gravitino is given by [20]
\begin{equation}
\Gamma(\chi \to \gamma \tilde{G}) = \frac{|N_{11} \cos \theta_W + N_{12} \sin \theta_W|^2}{48 \pi} \frac{m_{\chi}^5}{M_{\text{Pl}}^2 m_{\chi}^2} \left( 1 - \frac{m_{\tilde{G}}^2}{m_{\chi}^2} \right)^3 \left( 1 + 3 \frac{m_{\tilde{G}}^2}{m_{\chi}^2} \right). \quad (2.10)
\end{equation}

These decay widths depend mostly on the LSP and NLSP masses, but also on $N_{ij}$, the neutralino mass mixing matrix. Note that $M_{\text{Pl}}$ denotes the reduced Planck mass. The lifetime can be sufficiently long $\tau \sim 10^2 - 10^5$ s to be interesting for BBN and the lithium anomalies.

The two-body decays also dominate the electromagnetic cascade with the decay of the tau-lepton (producing either directly leptons or mesons that decay electromagnetically before interacting strongly with the light elements). The electromagnetic branching ratio and energy injected in the cascade for the stau are given by:
\begin{equation}
B_{\text{em}}^{\tilde{\tau}} = \frac{\Gamma(\tilde{\tau}_1 \to \tau \tilde{G})}{\Gamma_{\text{tot}}} \simeq 1, \quad E_{\text{em}}^{\tilde{\tau}} \simeq 1 \left( \frac{m_{\tilde{G}}^2 - m_{\tilde{G}}^2}{2m_{\tilde{G}}} \right). \quad (2.11)
\end{equation}
The approximate fraction $1/2$ in the total tau energy is a reflection of the fact that some energy is lost into non-interacting neutrinos. Similar results are obtained for the neutralino:

$$B_{eim}^\chi \simeq 1, \quad E_{eim}^\chi = \left( \frac{m_{\chi}^2 - m_{\tilde{G}}^2}{2m_{\chi}} \right). \quad (2.12)$$

Hadronic energy contributions come from 3-body, or from 4-body, NLSP decays:

$$\tilde{\tau}_1 \rightarrow \tau \tilde{G}Z/\gamma \rightarrow \tau \tilde{G}q\bar{q}, \quad (2.13)$$
$$\tilde{\tau}_1 \rightarrow \nu_\tau \tilde{G}W \rightarrow \nu_\tau \tilde{G}q\bar{q}, \quad (2.14)$$
$$\tilde{\tau}_1 \rightarrow \tau \tilde{G}h \rightarrow \tau \tilde{G}q\bar{q}. \quad (2.15)$$

In our previous analyzes [14, 16] we approximated these processes by a simple formula, and the process (2.14) was set to zero, since we assumed the stau to be a pure right-handed state. Here we incorporate the mixing and use the programs, CalcHEP [53] (an automatic matrix element generator) and PYTHIA [54] (a Monte-Carlo high-energy-physics event generator) to produce the spectrum of the decay products properly. With this update eq. (2.14) is not zero any more due to the left-handed component enabling this process.

Following [25] we use the $q\bar{q}$ invariant mass $m_{q\bar{q}}$ to calculate the decay width in eq. (2.13). For completeness virtual Higgs exchanges were also calculated. The decay width is given with a cut on the $q\bar{q}$ invariant mass $m_{q\bar{q}}^{\text{cut}} = 2\text{ GeV}$ below which no nucleon can be produced,

$$\Gamma(\tilde{\tau} \rightarrow \tau \tilde{G}q\bar{q}; m_{q\bar{q}}^{\text{cut}}) = \int_{m_{q\bar{q}}^{\text{cut}}}^{m_{q\bar{q}}} dm_{q\bar{q}} \frac{d\Gamma(\tilde{\tau} \rightarrow \tau \tilde{G}q\bar{q})}{dm_{q\bar{q}}}. \quad (2.16)$$

The branching ratio is given by

$$B_h(\tilde{\tau} \rightarrow \tau \tilde{G}q\bar{q}; m_{q\bar{q}}^{\text{cut}}) = \frac{\Gamma(\tilde{\tau} \rightarrow \tau \tilde{G}q\bar{q}; m_{q\bar{q}}^{\text{cut}})}{\Gamma_{\text{tot}}}. \quad (2.17)$$

The hadronic decay is dominated by an exchange of $Z/\gamma$. The $W$ processes are suppressed by one or two orders of magnitude compared to the $Z/\gamma$ because the left-handed component is comparatively small in the CMSSM. The Higgs contribution is even more negligible due to the Higgs mass in the propagator. This last contribution can be enhanced at large $\tan \beta$ which increases the Higgs boson couplings but it remains below the $W$ contribution. For the neutralino NLSP, the hadronic cascades come from 3-body decay $\chi \rightarrow \tilde{G}q\bar{q}$:

$$\chi \rightarrow \tilde{G}Z/\gamma \rightarrow \tilde{G}q\bar{q}, \quad (2.18)$$
$$\chi \rightarrow \tilde{G}h \rightarrow \tilde{G}q\bar{q}, \quad (2.19)$$

and the decay width is calculated similarly as for the stau

$$\Gamma(\chi \rightarrow \tilde{G}q\bar{q}; m_{q\bar{q}}^{\text{cut}}) = \int_{m_{q\bar{q}}^{\text{cut}}}^{m_{\chi} - m_{\tilde{G}}} dm_{q\bar{q}} \frac{d\Gamma(\chi \rightarrow \tilde{G}q\bar{q})}{dm_{q\bar{q}}}. \quad (2.20)$$
2.3 BBN calculation

As in our previous work [16], for each point in supersymmetric parameter space we perform a complete BBN calculation of light element abundances with NLSP decay-induced cascades. These calculations include all thermal nuclear reactions as well as all (nonthermal) interactions important for the developments of the electromagnetic and hadronic cascades. Details about the effects of the around 100 nonthermal interactions required to attain an estimated BBN yield accuracy of 30% may be found in [6]. The calculations are performed at \( \Omega_b h^2 = 0.02273 \), as inferred by WMAP [55].

The improvements with respect to ref. [16] are twofold. Firstly, the current BBN calculations take full and accurate account of catalytic effects [27, 32, 28, 29, 30, 56, 57, 58, 31, 59, 60]. Secondly, the energies of injected nucleons are treated in a much improved way. It has been recently shown that the formation of bound states between electrically charged NLSPs (here principally the stau \( \tilde{\tau} \)) and nuclei towards the end of BBN lead to drastic changes of some thermal nuclear rates [27, 30, 61, 60]. Various modifications have been proposed, but the by far most important one is the replacement of the weak quadrupole transition \( D + ^4\text{He} \rightarrow ^6\text{Li} + \gamma \) with the catalytic process \( D + (^4\text{He} - \tilde{\tau}) \rightarrow ^6\text{Li} + \tau \), with a rate \( \sim 10^7 \) times larger than the former. Since both \( (^4\text{He} - \tilde{\tau}) \) bound states as well as \(^6\text{Li} \) first survive photo-disintegration/nuclear destruction at \( \gtrsim 10^4 \) sec, \(^6\text{Li} \) overproduction rules out most of the stau-NLSP parameter space with stau decay time \( \tau > \sim 5 \times 10^3 \) sec [27, 32, 33].

The exception here are very small freeze-out NLSP stau-to-entropy ratios \( Y = n/s \lesssim 10^{-15} - 10^{-14} \) [31] as recently proposed to occur at large tan \( \beta \) [62]. Other modifications to BBN due to the bound state induced catalytic effects are the possibility of appreciable \(^9\text{Be} \) synthesis [59] which has been used to derive limits on abundances of stau NLSPs [61]. Though we fully calculate \(^9\text{Be} / \text{H} \) ratios, we refrain from using them for limits, as the catalytic rates seem currently very uncertain [60]. Finally, late-time destruction of \(^6\text{Li} \) and \(^7\text{Li} \) through reactions on \((p - \tilde{\tau}) \) bound states, as proposed in ref. [58], has proven unimportant when the newly determined catalytic rates of ref. [60] are employed.

In ref. [16] a partition of energy into hadronic three- and four-body decays, such as \( \tilde{\tau} \rightarrow \tilde{G} \tau Z \) and \( \tilde{\tau} \rightarrow \tilde{G} \tau q \bar{q} \) was approximated by \( E_Z \approx E_{q \bar{q}} \approx (m_\tau - m_\tilde{G})/3 \). In ref. [25] both processes were treated in detail and it was found that the invariant mass-squared \( m_{q \bar{q}}^2 = (P_q + P_\bar{q})^2 \) of produced \( q \bar{q} \) flux tubes is peaked around the \( Z \)-mass squared \( m_Z^2 \). It was argued that the average of \( \langle m_{q \bar{q}} \rangle \) should be used as an “effective hadronic energy” for BBN calculations rather than \( E_{q \bar{q}} \approx (m_\tau - m_\tilde{G})/3 \). In particular for heavy \( m_\tau \sim 1 \) TeV this would make a factor \( \sim 3 \) difference in hadronic effects. As described in more detail in ref. [63], the situation is actually more complicated. In fact \( \langle m_{q \bar{q}} \rangle \) is not a good estimate of the energy of the \( q \bar{q} \) flux-tube in the cosmic rest-frame. Nevertheless, for independent reasons, at early times, \( \tau \lesssim 300 \) sec taking \( \langle m_{q \bar{q}} \rangle \) is indeed a better approximation, which has been adopted in the present analysis. A completely accurate determination would require knowledge of the detailed primary nucleon energy spectrum due to the NLSP decays. This, is however, numerically currently not feasible for the large number of models to be calculated. Some detailed calculations have been performed in ref. [63] indicating a \( \sim 30\% \) uncertainty in the results.
3. Constraints and results

Unless otherwise stated, we will follow the analysis and notation of our previous papers [14, 16] to which we refer the reader for more details. Here we only summarize the main results and differences.

Mass spectra of the CMSSM are determined in terms of the usual four free parameters mentioned above: \( \tan \beta \), \( m_{1/2} \), \( m_0 \), and \( A_0 \), as well as on \( \text{sgn}(\mu) \) – the sign of the supersymmetric Higgs/higgsino mass parameter \( \mu \). The parameter \( \mu^2 \) is derived from the condition of the electroweak symmetry breaking and we take \( \mu > 0 \).

We calculate the thermal contribution \( \Omega_{\text{TP}} h^2 \) following the result of [36]. In evaluating the non-thermal part \( \Omega_{\text{NTP}} h^2 \), we first compute the number density of the NLSP after freeze-out (the neutralino or the stau) with high accuracy by numerically solving the Boltzmann equation including all (dominant and subdominant) NLSP pair annihilation and coannihilation channels. For a given value of \( m_{\tilde{G}} \), we then compute \( \Omega_{\text{NTP}} h^2 \) via eq. (2.4).

After freeze-out from the thermal plasma at \( t \sim 10^{-12} \) sec, the NLSPs decay into gravitinos at late times which strongly depend on the NLSP composition and mass, on \( m_{\tilde{G}} \) and on the final states of the NLSP decay [22, 20]. The exact value of the NLSP lifetime in the CMSSM further depends on a possible relation between \( m_{\tilde{G}} \) and \( m_{1/2} \) and/or \( m_0 \) but in the parameter space allowed by other constraints it can vary from \( \gtrsim 10^8 \) sec at smaller \( m_{\text{NLSP}} \) down to \( 10^2 \) sec, or even less, for \( m_{1/2} \) and/or \( m_0 \) in the TeV range or for small gravitino mass \( m_{\tilde{G}} \).

When the NLSPs decay, the high energy electromagnetic and hadronic particles are produced and may interact with background light nuclei and change the abundances. Using the output from low energy spectrum and interactions, we calculate the spectrum of decay products of NLSP and implement into BBN calculation to predict the primordial light element abundances. Then we require that the light element abundances after decay of NLSP are within the bounds of those inferred from the observational data.

3.1 Collider and cosmological constraints

We apply the same experimental bounds as in [14, 16]: (i) the lightest chargino mass \( m_{\chi_1^\pm} > 104 \text{ GeV} \); (ii) the lightest Higgs mass \( m_h > 114.4 \text{ GeV} \); (iii) \( \text{BR}(B \to X_s \gamma) = (3.55 \pm 0.68) \times 10^{-4} \) and (iv) the stau mass bound \( m_{\tilde{\tau}} > 87 \text{ GeV} \). In this analysis we also use the top quark mass \( m_t = 172.7 \text{ GeV} \) [64]. However slight change of these experimental limit does not modify the final results, since the BBN constraints (especially from \( ^6\text{Li}/^7\text{Li} \)) are more severe.

For the observational constraints on the primordial light element abundances, we use [6]

\[
1.2 \times 10^{-5} < \frac{\text{D/H}}{Y_p} < 5.3 \times 10^{-5} \\
8.5 \times 10^{-11} < \frac{^2\text{He}/\text{D}}{Y_p} < 1.52 \\
^6\text{Li}/^7\text{Li} < 0.1 (0.66).
\]
Figure 1: The plane $(m_{1/2}, m_0)$ for $\tan \beta = 10$, $m_\tilde{\chi} = m_0$ (left window) and $\tan \beta = 50$, $m_\tilde{\chi} = 0.2 m_0$ (right window) and for $A_0 = 0$, $\mu > 0$. The light brown regions labeled “LEP $\chi^+$” and “LEP Higgs” are excluded by unsuccessful chargino and Higgs searches at LEP, respectively. In the right window the darker brown region labeled “Tachyonic” because of some sfermion masses becoming tachyonic, and is also excluded. In the rest of the grey region (above the dashed line) the stau mass bound $m_{\tilde{\tau}_1} > 87$ GeV is violated. In the region “No EWSB” the conditions of EWSB are not satisfied. The dotted diagonal line marks the boundary between the neutralino ($\chi$) and the stau ($\tilde{\tau}$) NLSP. The regions excluded by the various BBN constraints are denoted in violet. A solid magenta curve labeled “CMB” delineates the region (on the side of label) which is inconsistent with the CMB spectrum. In both windows, the dark green bands labeled “$T_R = 10^7$ GeV” and “10$^8$” correspond to the total relic abundance of the gravitino (from the sum of thermal and non-thermal production), for a given (denoted) reheating temperature, lying in the favored range. In the light green regions (marked “NTP”) the same is the case for the relic abundance from NTP process alone. The blue dashed line denotes the relaxed boundary of the BBN constrains when we use the conservative limit $^6\text{Li}/^7\text{Li} < 0.66$.

For $^6\text{Li}/^7\text{Li}$ we will also use the conservative upper limit which is given in the bracket. It allows for the possibility of stellar $^6\text{Li}$ (and $^7\text{Li}$) depletion. Since $^6\text{Li}$ is more fragile than $^7\text{Li}$, post-BBN lithium processing may conceivably reduce the $^6\text{Li}/^7\text{Li}$ ratio. In the first two figures below, the regions excluded by BBN constraints will be shaded violet and marked “BBN”. We also show the BBN constraints with the conservative bound on $^6\text{Li}/^7\text{Li}$ with blue dashed lines.

As regards the total gravitino relic abundance $\Omega_\tilde{G} h^2$, we apply the 3σ range derived from WMAP 5 year data [65]

$$0.091 < \Omega_\tilde{G} h^2 < 0.128,$$

which in the figures below will be marked as green bands and labeled “$\Omega_\tilde{G} h^2$”.
As previously in [16], we also include the bound on the possible distortion in the nearly perfect black-body shape of the CMB spectrum [66] by the injection of energetic photons into the plasma. However we note that this constraint (delineated with magenta line with a label “CMB” over it) seems generally less important than that due to the BBN [67, 68].

3.2 Numerical results

In figs. 1 and 2 we give the updated results with the constraints and the relic abundance of the gravitino in the same format as used in [16] to facilitate the comparison. Generally, the regions in white are left allowed after applying collider and BBN constraints. Green bands denote ranges of parameters where the total gravitino abundance reproduces the DM abundance (3.1), while in the light green bands (marked “NTP”) only the NTP part of the gravitino abundance agrees with that range.

In fig. 1 we present two cases with gravitino mass related to the soft scalar mass $m_0$. In the left window we take $\tan \beta = 10$ and $m_{\tilde{G}} = m_0$ and in the right window $\tan \beta = 50$ and $m_{\tilde{G}} = 0.2m_0$. In fig. 2 we present two cases with a fixed $m_{\tilde{G}}$. In the left window we fix $\tan \beta = 10$ and $m_{\tilde{G}} = 10\text{ GeV}$ while in the right one $\tan \beta = 50$ and $m_{\tilde{G}} = 100\text{ GeV}$.

We can see that, as before, the whole neutralino NLSP region (above the black dotted diagonal line) is again ruled out, as well as a part of the stau NLSP region corresponding to smaller $m_{1/2}$ (and therefore smaller $m_{\tilde{\tau}}$ and hence larger stau lifetimes). By comparing with the corresponding figures in [16], we can see that including the bound state effects causes the allowed region of stau NLSP consistent with DM abundance (green band) to be more constrained, and also strengthens an upper bound on $T_R$ to

$T_R < a \text{ few } \times 10^7 \text{ GeV}.$

(3.2)
(For comparison, without the bound state effect, the bound was a few times $10^8$ GeV [16].) This bound is consistent with the result of ref. [33, 34] where the bound state effect was mimicked by making a simple and approximate constraint on the lifetime of the stau, $\tau < 5 \times 10^3$ sec. However, as we shall now show, in a number of cases the stau lifetime that can be consistent with avoiding strong bound-state catalytic effect can be almost an order of magnitude larger. Furthermore, by applying more conservative bounds on $^6\text{Li}/^7\text{Li}$, the upper bound (3.2) can in some cases be violated by up to an order of magnitude.

In order to examine the upper bound on $T_R$ more closely, in fig. 3 we plot $T_R$ vs. $m_{\tilde{g}}$ for all the points in our scans which satisfy the constraints from BBN and collider experiment and for $\tan \beta = 10$ (left) and $\tan \beta = 50$ (right). In these plots we use several choices of the gravitino mass: linear in the scalar mass ($0.2 m_0, 0.4 m_0$), linear in the gaugino mass ($0.2 m_{1/2}, 0.4 m_{1/2}$) or constant (24 values between 0.1 GeV and 100 GeV). For each point, $T_R$ is calculated using eqs. (2.2) and (2.6) for a given gravitino mass and for the mass spectra and the yield of NLSP at each point of parameter space, imposing $\Omega_{\tilde{g}} h^2 = 0.11$. Points marked by green dots survive all the constraints from experiments and cosmology while those marked by grey dots are allowed by experiments but disallowed by BBN constraints from light element abundances. Again, we can see that, for both choices of $\tan \beta$, we find the upper bound (3.2). This result does not change if one alters collider constraints since the bound is set by the $^6\text{Li}$ abundance.

However, when we use the conservative bound on $^6\text{Li}/^7\text{Li} < 0.66$ then we find new isolated, albeit rather small allowed regions (marked by blue dots), where the reheating temperature can go up to $3 \times 10^8$ GeV. (They correspond to vertical blue dashed region around $m_{1/2} \sim 1$ TeV and small $m_0$ in the right window in figs. 1 and 2). Clearly, those points evade the simple bound $\tau < 5 \times 10^3$ sec (marked with a red dashed line). On the other hand, this region corresponds to a rather low stau mass around 100 GeV (but still above the LEP limit). This can be seen in fig. 4 where we plot stau lifetime for the data points shown in fig. 3. We can see that the (green) points allowed by $^6\text{Li}/^7\text{Li} < 0.1$ are located below the stau lifetime of $5 \times 10^3$ sec for $\tan \beta = 10$ and $3 \times 10^4$ sec for $\tan \beta = 50$. It is also clear that, at smaller $\tan \beta$ (left panel) and larger $m_{\tilde{\tau}}$ the bound $\tau < 5 \times 10^3$ sec can actually be too weak. On the other hand, assuming the conservative bound $^6\text{Li}/^7\text{Li} < 0.66$ (blue points), at stau mass around 100 GeV, for $\tan \beta = 50$ the lifetime can exceed $10^4$ sec even by a few orders of magnitude. Such low stau mass will be accessible to the LHC, thus allowing one to scrutinize such cases fairly easily.

The reason why the points corresponding to stau NLSP with such a long lifetime are allowed is that, as the stau mass decreases, its yield $Y$ also decreases and can drop below $10^{-14}$. In this case the BBN constraints from the bound state effects can be avoided even with stau lifetime as long as $3 \times 10^4$ sec (green points).

To see this, we present the left window of fig. 5. Assuming a standard (more conservative) limit on $^6\text{Li}/^7\text{Li}$ we find green (blue) points corresponding to the lifetime of up to $3 \times 10^4$ sec ($2 \times 10^7$ sec) and the stau yield of less than $10^{-14}$. For lifetimes longer than $10^7$ sec the constraint due to violating the upper limit on $^3\text{He}/D$ due to $^4\text{He}$ photo-disintegration becomes severe [31]. On the other hand, for much longer lifetimes, between $10^{10-11}$ sec, we again find some isolated allowed regions of (blue) points. They become allowed because
Figure 3: The $T_R$ vs. $m_G$ plane for $\tan \beta = 10$ (left) and $\tan \beta = 50$ (right). Data points which survived all BBN and experimental constraints are shown with green dots, while grey dots are disallowed by the BBN while allowed by collider constraints. Blue points are added to the green when we use a more conservative limit $^6\text{Li}/^7\text{Li} < 0.66$. $T_R$ is determined so that the total (thermal and non-thermal) production of gravitinos satisfies the WMAP value; here we used $\Omega_G^{-1}h^2 = \Omega_G^T h^2 + \Omega_G^{NTP} h^2 = 0.11$. We also show the lifetime contour of stau, $\tau = 5 \times 10^3 \text{sec}$, which was obtained using the specific relation between stau mass and gaugino masses as well as the approximate relic density of the $\tilde{\tau}$ in the CMSSM.

the mass difference between the gravitino LSP and the stau NLSP is very small there and the electromagnetic energy released is not enough to be a potential problem for BBN.

Finally, in the right window of fig. 5 we show $T_R$ vs. stau lifetime for all the cases we have considered, and which summarizes many of our points made above. In particular, with the ranges of primordial light element abundances adopted in this analysis (green points), the upper limit on $T_R$ scales with the stau lifetime $\tau$ but does not exceed the limit (3.2). Allowing the more conservative limit on $^6\text{Li}/^7\text{Li}$ pushes it up to nearly $10^8 \text{GeV}$, with some isolated points tolerating $T_R$ as high as $3 \times 10^8 \text{GeV}$, as stated above.

4. Summary

We have re-analyzed the gravitino as dark matter in the Universe in the framework of the CMSSM, taking into account of a number of recent improvements in the calculation. The improvements concern the thermal production of gravitinos, including the decay allowed by thermal masses as well as scatterings, the computation of the hadronic spectrum from NLSP decay including the correct implementation of left-right stau mixing, as well as an updated BBN calculation fully treating the effects of bound state effects between charged massive particles and nuclei. We found that the over-abundance of $^6\text{Li}$ from bound state effects puts more severe constraints on allowed ranges of CMSSM parameters than before
and puts an upper bound on $T_R$ at a few times $10^7$ GeV and the stau lifetime less than around $3 \times 10^4$ sec.

We also analyzed the impact of applying a more conservative upper limit on $^6\text{Li}/^7\text{Li}$. In this case a new region at small stau mass opens up because of the reduced relic abundance of the stau, even though the lifetime of the stau is much longer than $10^4$ sec, and can reach
up to $5 \times 10^7$ sec, in some cases even between $10^{10}$ and $10^{11}$ sec. In these latter cases, which are easily testable at the LHC, $T_R$ can exceed $10^8$ GeV.

Acknowledgments

K.-Y.C. is supported by the Ministerio de Educacion y Ciencia of Spain under Proyecto Nacional FPA2006-05423 and by the Comunidad de Madrid under Proyecto HEPHACOS, Ayudas de I+D S-0505/ESP-0346. K.-Y.C. and L.R. would like to thank the European Network of Theoretical Astroparticle Physics ILIAS/ENTApP under contract number RII3-CT-2004-506222 for financial support.

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