Broken R-parity, stop decays, and neutrino physics

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Abstract

We discuss the phenomenology of the lightest stop in models where R-parity is broken by bilinear superpotential terms. In this class of models we consider scenarios where the R-parity breaking two-body decay $\tilde{t}_1 \rightarrow \tau^+ b$ competes with the leading three-body decays such as $\tilde{t}_1 \rightarrow W^+ b \tilde{\chi}^0_1$. We demonstrate that the R–parity violating decay can be sizable and in some parts of the parameter space even the dominant one. Moreover we discuss the expectations for $\tilde{t}_1 \rightarrow \mu^+ b$ and $\tilde{t}_1 \rightarrow e^+ b$. The recent results from solar and atmospheric neutrinos suggest that these are as important as the $\tau^+ b$ mode. The $\tilde{t}_1 \rightarrow l^+ b$ decays are of particular interest for hadron colliders, as they may allow a full mass reconstruction of the lighter stop. Moreover these decay modes allow cross checks on the neutrino mixing angle involved in the solar neutrino puzzle complementary to those possible using neutralino decays. For the so–called small mixing angle or SMA solution $\tilde{t}_1 \rightarrow e^+ b$ should be negligible, while for the large mixing angle type solutions all $\tilde{t}_1 \rightarrow l^+ b$ decays should have comparable magnitude.
I. INTRODUCTION

The search for supersymmetry (SUSY) [1, 2] plays an important role in the experimental program of present and at future colliders, e.g. the Tevatron, LHC, or an $e^+e^-$ linear collider. Therefore many phenomenological studies have been carried out in recent years (see e.g. [3, 4, 5, 6] and references therein) focusing mainly on the minimal supersymmetric standard model (MSSM) [7]. However, neither gauge invariance nor supersymmetry require the conservation of R-parity. Indeed, there is considerable theoretical and phenomenological interest in studying possible implications of alternative scenarios [8] in which R-parity is broken [9, 10, 11, 12]. These theories are of particular interest as they lead to a pattern of neutrino masses and mixing angles [13] which can account for the observed anomalies in solar and atmospheric neutrinos [14]. In general the violation of R-parity could arise explicitly [15] as a residual effect of some larger unified theory [16], or spontaneously, through nonzero vacuum expectation values (vev’s) for scalar neutrinos [3, 14, 12]. In realistic spontaneous R-parity breaking models there is an $SU(2) \otimes U(1)$ singlet sneutrino vacuum expectation value (vev) characterizing the scale of R-parity violation [17, 18, 19, 20] which is expected to be in the order of 1 TeV.

There are two generic cases of spontaneous R-parity breaking models to consider. In the absence of any additional gauge symmetry, these models lead to the existence of a physical massless Nambu-Goldstone boson, called majoron (J) which is the lightest SUSY particle, massless and therefore stable. As in the standard case in R-parity breaking models the lightest SUSY particle (LSP) is in general a neutralino. However, it now decays mostly into visible states, therefore diluting the missing momentum signal and bringing in increased multiplicity events which arise mainly from three-body decays such as $\tilde{\chi}_1^0 \rightarrow f \bar{f} \nu$, where $f$ denotes a charged fermion [19]. If lepton number is part of the gauge symmetry and R-parity is spontaneously broken then there is an additional gauge boson which gets mass via the Higgs mechanism, and there is no physical Goldstone boson [20]. In this case R-parity violating effects relevant for collider physics are conveniently parameterized by adding bilinear terms $\epsilon_i L_i H_2$ to the MSSM superpotential and corresponding terms for
the soft SUSY breaking part of the Lagrangian. Bilinear R-parity violation may also be assumed ab initio as the fundamental theory. For example, it may be the only violation permitted by higher Abelian flavour symmetries [21].

Owing to the large top Yukawa coupling the stops have a quite different phenomenology compared to those of the first two generations of up–type squarks (see e.g. [22, 23] and references therein). The large Yukawa coupling implies a large mixing between $\tilde{t}_L$ and $\tilde{t}_R$ [24] and large couplings to the higgsino components of neutralinos and charginos. The large top quark mass also implies the existence of scenarios where all MSSM two-body decay modes of $\tilde{t}_1$ are kinematically forbidden at the tree-level (e.g. $\tilde{t}_1 \rightarrow t \tilde{\chi}_1^0, b \tilde{\chi}_j^+, t \tilde{g}$). In such case higher order decays of $\tilde{t}_1$ become relevant [25, 26, 27, 30], $\tilde{t}_1 \rightarrow c \tilde{\chi}_{1,2}^0$, $\tilde{t}_1 \rightarrow W^+ b \tilde{\chi}_1^0$, $\tilde{t}_1 \rightarrow H^+ b \tilde{\chi}_1^0$, $\tilde{t}_1 \rightarrow b \tilde{l}_1^+ \nu_l$, $\tilde{t}_1 \rightarrow b \tilde{\nu}_l l^+$, where $l$ denotes $e, \mu, \tau$. Also 4-body decays may become important if the 3-body decays are kinematically forbidden [28]. In [24, 27, 29, 30] it has been shown that in the MSSM the three-body decay modes are in general much more important than the two body decay mode. Recently it has been demonstrated that not only LSP decays but also the light stop can be a good candidate for observing R-parity violation, even if its magnitude is as small as indicated by the solutions to the present neutrino anomalies [6, 31, 32, 33]. It has been demonstrated that there exists a large parameter region where the R-parity violating decay

$$\tilde{t}_1 \rightarrow b \tau$$

is much more important than the $R_p$ conserving decays

$$\tilde{t}_1 \rightarrow c \tilde{\chi}_{1,2}^0$$

in scenarios where two-body decay modes are possible. It is therefore natural to ask if there exist scenarios where the decay $\tilde{t}_1 \rightarrow b \tau$ is as important as the three–body decays. Note that in the R-parity violating models under consideration the neutral (charged) Higgs–bosons mix with the neutral (charged) sleptons [34, 35]. These states are denoted by $S_i^0, P_j^0$, and $S_k^\pm$ for the neutral scalars, pseudoscalars and charged scalars, respectively.
Therefore in the R-parity violating case one has the following three-body decay modes:

\[
\tilde{t}_1 \rightarrow W^+ b \tilde{\chi}_1^0
\]

\[
\tilde{t}_1 \rightarrow S_k^+ b \tilde{\chi}_1^0
\]

\[
\tilde{t}_1 \rightarrow S_k^+ b \nu_3
\]

\[
\tilde{t}_1 \rightarrow b S_i^0 \tau^+
\]

\[
\tilde{t}_1 \rightarrow b P_j^0 \tau^+
\]

\[
\tilde{t}_1 \rightarrow b \bar{l}_i^+ \nu_l,
\]

\[
\tilde{t}_1 \rightarrow b \bar{l}_i^+ l^+ \quad (l = e, \mu).
\]

We will show that there exist regions in parameter space where \(\tilde{t}_1 \rightarrow b \tau^+\) is sizeable and even the most important decay mode. In particular we will consider a mass range for the light stop \(\tilde{t}_1\), where it is difficult for the LHC to discover it in the MSSM due to the large top background \[36\]. In contrast to the existing LSP decay studies in \(R_p\) models \[37, 38\] which are mainly sensitive to the atmospheric neutrino anomaly parameters, the stop decay processes considered here are very sensitive to the solar neutrino parameters and therefore gives valuable complementary information.

The paper is organized in the following way: in the next section we will introduce the model. In Sect. [II] numerical results for stop decays are presented. We first explore the extent to which the decay \(\tilde{t}_1 \rightarrow b \tau\) can be sizeable when compared with the 3-body decay modes. Moreover, we discuss the connections between the decay modes \(\tilde{t}_1 \rightarrow b l^+\) and neutrino physics, in particular we discuss a possible test of the solution to the solar neutrino puzzle. In Sect. [IV] we present our conclusions. The appendixes contain complete formulas for the total widths of the three-body decay modes as well as for the couplings.

II. THE MODEL

The supersymmetric Lagrangian is specified by the superpotential \(W\) given by

\[
W = \varepsilon_{ab} \left[ h^{ij}_U \tilde{Q}_i^a \tilde{U}_j^b \tilde{H}_2^b + h^{ij}_D \tilde{Q}_i^a \tilde{D}_j^b \tilde{H}_1^b + h^{ij}_L \tilde{L}_i^a \tilde{R}_j^b \tilde{H}_1^a - \mu \tilde{H}_1^a \tilde{H}_2^b \right] + \varepsilon_{ab} \tilde{L}_i^a \tilde{H}_2^b ,
\]
where $i,j = 1,2,3$ are generation indices, $a,b = 1,2$ are $SU(2)$ indices, and $\varepsilon$ is a completely antisymmetric $2 \times 2$ matrix, with $\varepsilon_{12} = 1$. The symbol “hat” over each letter indicates a superfield, with $\hat{Q}_i$, $\hat{L}_i$, $\hat{H}_1$, and $\hat{H}_2$ being $SU(2)$ doublets with hypercharges $1/3$, $-1$, $-1$, and $1$ respectively, and $\hat{U}$, $\hat{D}$, and $\hat{R}$ being $SU(2)$ singlets with hypercharges $-\frac{4}{3}$, $\frac{2}{3}$, and $2$ respectively. The couplings $h_U$, $h_D$ and $h_E$ are $3 \times 3$ Yukawa matrices, and $\mu$ and $\epsilon_i$ are parameters with units of mass.

Supersymmetry breaking is parameterized by the standard set of soft supersymmetry breaking terms

$$V_{soft} = M_{ij}^{Q} \hat{Q}_i^a \hat{Q}_j^a + M_{ij}^{U} \hat{U}_i \hat{U}_j + M_{ij}^{D} \hat{D}_i \hat{D}_j + M_{ij}^{L} \hat{L}_i^a \hat{L}_j^a + M_{ij}^{R} \hat{R}_i \hat{R}_j$$

$$+ m_{H_1}^2 H_1^a H_1^a + m_{H_2}^2 H_2^a H_2^a$$

$$- \left[ \frac{1}{2} m_3 \lambda_3 \lambda_3 + \frac{1}{2} M \lambda_2 \lambda_2 + \frac{1}{2} M' \lambda_1 \lambda_1 + h.c. \right]$$

$$+ \varepsilon_{ab} \left[ A_{ij}^{U} h_{ij} \hat{Q}_i \hat{D}_j H_2^b + A_{ij}^{D} h_{ij} \hat{D}_i \hat{D}_j H_1^a + A_{ij}^{E} h_{ij} \hat{L}_i \hat{L}_j H_1^a \right.$$  

$$- B_{\mu} H_1^a H_2^b + B_i \epsilon_i H_1^a H_2^b \right],$$

(1)

Note that, in the presence of soft supersymmetry breaking terms the bilinear terms proportional to the $\epsilon_i$ can not be rotated away except for the very special case $B_i = B$ and only if the scalar masses are adjusted in a special way. Such ‘fine-tuned’ assumptions at a low scale are, from our point of view, very unnatural. If realized at a high scale such as the unification or GUT-scale, then the trilinear R-parity breaking couplings introduced as a result of the rotation would re-introduce bilinear terms at the electroweak scale due to the structure of the corresponding RGEs [39]. In contrast, bilinear terms are closed under RGE evolution from the high scale to the electroweak scale [40].

In order to compare the $\tilde{t}_1 \rightarrow b \tau$ decay mode with the 3-body MSSM modes it is sufficient for us to consider a 1-generation $R_p$ model. Note however, that for the detailed connection of stop decays with neutrino physics we must consider the complete model, and we will do so in a second step when we discuss the connection with the solar neutrino mixing. Finally, notice that we also allow for R-parity-conserving Flavour Changing Neutral Currents (FCNC) effects, such as the process $\tilde{t}_1 \rightarrow c \tilde{\chi}_1^0$ involving the three generations.
of quarks in both cases.

Our 1-generation model is specified by the superpotential \[41, 42, 43, 44\]

\[
W = h_t \bar{Q}_3 \tilde{D}_3 \tilde{H}_2 + h_b \bar{Q}_3 \tilde{D}_3 \tilde{H}_1 + h_s \bar{L}_3 \tilde{R}_3 \tilde{H}_1 - \mu \tilde{H}_1 \tilde{H}_2 + \epsilon_3 \tilde{L}_3 \tilde{H}_2
\]

(2)

For simplicity, in the remaining part of this section we adopt the 1-generation model when presenting the formulas for the mass matrices which are needed in the subsequent sections. The complete formulas for the 3-generation case are given in the second paper of \[13\] and have been used whenever required. The electroweak symmetry is broken when the two Higgs doublets, \(H_1\) and \(H_2\), and left slepton doublet \(\tilde{L}_3\) acquire vacuum expectation values. We introduce the notation:

\[
H_1 = \left(\frac{1}{\sqrt{2}}[\theta_1^0 + v_1 + i\varphi_1^0]\right), \quad H_2 = \left(\frac{1}{\sqrt{2}}[\theta_2^0 + v_2 + i\varphi_2^0]\right), \quad \tilde{L}_3 = \left(\frac{1}{\sqrt{2}}[\tilde{\nu}_\tau + v_3 + i\tilde{\nu}_\tau]\right).
\]

The mass of \(W\) is given by \(m_W^2 = \frac{1}{2} g^2 v^2\), where \(v^2 = v_1^2 + v_2^2 + v_3^2 \simeq (246 \text{ GeV})^2\). We define \(\tan \beta = v_2/\sqrt{(v_1^2 + v_3^2)}\). In addition to the above MSSM parameters, our model contains three new parameters, \(\epsilon_3\), \(v_3\) and \(B_3\), of which only two are independent, because there is an additional tad-pole equation \[41\]. These may be chosen as \(\epsilon_3\) and \(v_3\).

The stop mass matrix is given by

\[
M_{\tilde{t}}^2 = \begin{bmatrix}
M_Q^2 + \frac{1}{2} v_2^2 h_t^2 + \Delta_{UL} & h_{b/2} (v_2 A_t - \mu v_1 + \epsilon_3 v_3) \\
\frac{h_{b/2}}{2} (v_2 A_t - \mu v_1 + \epsilon_3 v_3) & M_U^2 + \frac{1}{2} v_2^2 h_t^2 + \Delta_{UR}
\end{bmatrix}
\]

with \(\Delta_{UL} = \frac{1}{8} (g^2 - \frac{3}{2} g'^2) (v_1^2 - v_2^2 + v_3^2)\) and \(\Delta_{UR} = \frac{1}{6} g'^2 (v_1^2 - v_2^2 + v_3^2)\). The mass matrix for the sbottoms is given by

\[
M_{\tilde{b}}^2 = \begin{bmatrix}
M_Q^2 + \frac{1}{2} v_1^2 h_b^2 + \Delta_{DL} & h_{b/2} (v_1 A_D - \mu v_2) \\
\frac{h_{b/2}}{2} (v_1 A_D - \mu v_2) & M_D^2 + \frac{1}{2} v_1^2 h_b^2 + \Delta_{DR}
\end{bmatrix}
\]

where \(\Delta_{DL} = -\frac{1}{8} (g^2 + \frac{3}{2} g'^2) (v_1^2 - v_2^2 + v_3^2)\), \(\Delta_{DR} = -\frac{1}{12} g'^2 (v_1^2 - v_2^2 + v_3^2)\). The mass eigenstates are obtained by \((q = t, b)\):

\[
\begin{bmatrix}
\tilde{q}_1 \\
\tilde{q}_2
\end{bmatrix} = \begin{bmatrix}
\cos \theta_{\tilde{q}} & \sin \theta_{\tilde{q}} \\
-\sin \theta_{\tilde{q}} & \cos \theta_{\tilde{q}}
\end{bmatrix} \begin{bmatrix}
\tilde{q}_L \\
\tilde{q}_R
\end{bmatrix} = R^\dagger \begin{bmatrix}
\tilde{q}_L \\
\tilde{q}_R
\end{bmatrix}
\]

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\begin{align*}
\cos \theta_{\tilde{q}} &= \frac{-M_{\tilde{q}_2}^2}{\sqrt{(M_{\tilde{q}_{11}}^2 - m_{\tilde{q}_1}^2)^2 + (M_{\tilde{q}_{12}}^2)^2}}, \\
\sin \theta_{\tilde{q}} &= \frac{M_{\tilde{q}_1}^2 - m_{\tilde{q}_1}^2}{\sqrt{(M_{\tilde{q}_{11}}^2 - m_{\tilde{q}_1}^2)^2 + (M_{\tilde{q}_{12}}^2)^2}}.
\end{align*}

The bilinear term in Eq. (2) leads to a mixing between the charginos and the \( \tau \)–lepton which induces the decay \( \tilde{t}_1 \to b \tau \). The mass matrix is given by

\begin{equation*}
M_C = \begin{bmatrix}
M & \frac{1}{\sqrt{2}} g v_2 & 0 \\
\frac{1}{\sqrt{2}} g v_1 & \mu & -\frac{1}{\sqrt{2}} h \tau v_3 \\
\frac{1}{\sqrt{2}} g v_3 & -\epsilon_3 & \frac{1}{\sqrt{2}} h \tau v_1
\end{bmatrix}.
\end{equation*}

As in the MSSM, the chargino mass matrix is diagonalized by two rotation matrices \( U \) and \( V \)

\begin{equation*}
U^* M_C V^{-1} = \begin{bmatrix}
m_{\tilde{\chi}_1^\pm} & 0 & 0 \\
0 & m_{\tilde{\chi}_2^\pm} & 0 \\
0 & 0 & m_{\tilde{\chi}_3^\pm}
\end{bmatrix}.
\end{equation*}

The lightest eigenstate of this mass matrix must be the tau lepton \( \tau^\pm = \tilde{\chi}_3^\pm \) and so the mass is constrained to be \( 1.77703^{+0.30}_{-0.26} \) GeV \[45\] \[50\]

In our model, the one of the three neutrinos acquires mass at the tree level due to a mixing between the neutralino sector and one of the neutrinos \[11, 12, 46\]. The neutralino/neutrino mass matrix is

\begin{equation*}
M_N = \begin{bmatrix}
M' & 0 & -\frac{1}{2} g' v_1 & \frac{1}{2} g' v_2 & -\frac{1}{2} g' v_3 \\
0 & M & \frac{1}{2} g v_1 & -\frac{1}{2} g v_2 & \frac{1}{2} g v_3 \\
-\frac{1}{2} g' v_1 & \frac{1}{2} g v_1 & 0 & -\mu & 0 \\
\frac{1}{2} g' v_2 & -\frac{1}{2} g v_2 & -\mu & 0 & \epsilon_3 \\
-\frac{1}{2} g' v_3 & \frac{1}{2} g v_3 & 0 & \epsilon_3 & 0
\end{bmatrix}
\end{equation*}

and \( M' \) is the \( U(1) \) gaugino soft mass. This neutralino/neutrino mass matrix is diagonalized by a \( 5 \times 5 \) rotation matrix \( N' \) such that

\begin{equation*}
N'^* M_N N'^{-1} = \text{diag}(m_{\tilde{\chi}_{1_1}}, m_{\tilde{\chi}_{2}}, m_{\tilde{\chi}_{3}}, m_{\tilde{\chi}_{4}}, m_{\tilde{\chi}_{5}})
\end{equation*}

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with $m_{\nu_3} = m_{3}$. Note that in [26, 27] a different basis for the neutralinos was used.

Assuming small R-parity violating couplings the tree-level neutrino mass is approximately given by

$$m_{\nu_3} \approx - \frac{(g^2 M' + g'^2 M)\mu'^2}{4MM'\mu'^2 - 2(g^2 M' + g'^2 M)\mu'v_2v'_3 \cos \xi} v'_1^2 \sin^2 \xi$$

with

$$\sin \xi = \frac{\epsilon_3 v_1 + \mu v_3}{\mu'v'_1}, \quad \mu' = \sqrt{\mu^2 + \epsilon_3^2}, \quad v'_1 = \sqrt{v_1^2 + v_3^2}.$$  

Notice that in mSUGRA models with bilinear R–parity violation [32] $m_{\nu_3}$ is calculable through the RGE evolution and one finds in this case cancellations up to two orders of magnitude for the combination $\Lambda_3 = \epsilon_3 v_1 + \mu v_3$. In general models the smallness of $m_{\nu_3}$ requires relatively small $\epsilon_3$ values from the start as might arise, for example, in the models considered in ref. [21]. The remaining two neutrinos acquire mass radiatively. Rigorous quantitative results were given in the second paper in ref. [13]. Typically they are hierarchically lighter than the heaviest neutrino, whose mass arises at the tree-level. This way one accounts for the observed hierarchy between the solar and the atmospheric neutrino mass scales.

Similarly, the Higgs bosons mix with charged sleptons and the real (imaginary) parts of the sneutrino mix with the scalar (pseudoscalar) Higgs bosons. We denote the scalar bosons by $S^0_i$, the pseudo-scalar by $P^0_i$ and the charged bosons by $S^\pm_i$. The relevant formulas have all been given e.g. in [35, 37]. However, since we will take one of the pseudoscalar mass eigenvalues as input, it is worth briefly repeating here the discussion of the pseudoscalar bosons masses. The pseudo-scalar mass matrix is given by:

$$M^2_{P^0} = \begin{bmatrix}
B\mu\frac{v_1}{v_1} + \mu\epsilon_3\frac{v_3}{v_1} & B\mu & -\mu\epsilon_3 \\
B\mu\frac{v_2}{v_2} - B_3\epsilon_3\frac{v_2}{v_2} & -B_3\epsilon_3 & \\
-\mu\epsilon_3 & -B_3\epsilon_3 & \mu\epsilon_3\frac{v_3}{v_3} - B_3\epsilon_3\frac{v_3}{v_3}
\end{bmatrix}.$$  

As expected, this matrix has zero determinant, since the neutral Goldstone boson eaten by the Z is one of the corresponding states. Therefore, the masses of the two physical
states are given by the formula:
\[
m_{2,3} = \frac{1}{2} \text{Tr} M \pm \frac{1}{2} \sqrt{(\text{Tr} M)^2 - 4(M_{11} M_{22} - M_{12}^2 + M_{11} M_{33} - M_{13}^2 + M_{22} M_{33} - M_{23}^2)}.
\] (5)

Therefore we can easily take one of these masses as input and calculate \( B\mu \) from it using
\[
B\mu = \frac{-m_{P_2}^4 v_1 v_2 v_3 + B_3 \epsilon_3 \mu v_3 (v_1^2 + v_2^2 + v_3^2) + \epsilon_3 m_{P_2}^2 [\mu v_2 (v_1^2 + v_3^2)] - B_3 v_1 (v_2^2 + v_3^2)}{-m_{P_2}^2 (v_1^2 + v_2^2) v_3 + \epsilon_3 (\mu v_1 - B_3 v_2) (v_1^2 + v_2^2 + v_3^2)}.
\]

\( B_3 \) is obtained from the minimum equation for given \( \epsilon_3 \) and \( v_3 \) [35].

### III. NUMERICAL ANALYSES

In this section we present our numerical results for the branching ratios of the lighter stop \( \tilde{t}_1 \). Here we consider scenarios where all two-body decays induced at tree-level are kinematically forbidden except the \( b \ell^+ \) decays. Before going into detail it is useful to have some approximate formulas at hand [32]:
\[
\Gamma(\tilde{t}_1 \to b \tau) \approx \frac{g^2 |U_{32}|^2 h_b^2 \cos^2 \theta_{\tilde{t}} m_{\tilde{t}_i}}{16\pi} \approx \frac{g^2 |\epsilon_3|^2 h_b^2 \cos^2 \theta_{\tilde{t}} m_{\tilde{t}_i}}{16\pi |\mu|^2} \quad (6)
\]
\[
\Gamma(\tilde{t}_1 \to c\chi_1^0) \approx F h_b^4 (\delta_{m_0^2} \cos \theta_{\tilde{t}} \delta_A \sin \theta_{\tilde{t}})^2 f_L^2 m_{\tilde{t}_i} \left( 1 - \frac{m_{\chi_1^0}^2}{m_{\tilde{t}_i}^2} \right)^2,
\] (7)

where \( F = \frac{g^2}{16\pi} (\log(m_{GUT}/m_Z) K_{\text{cb}} K_{\text{tb}}/16\pi^2)^2 \sim 6 \times 10^{-7} \), \( f_L = \sqrt{2} (\tan \theta_W N_{11} + 3 N_{12})/6 \) and the parameter \( \delta_{m_0^2} \) is given by
\[
\delta_{m_0^2} = \frac{M_Q^2 + M_D^2 + m_{H_u}^2 + A_b^2}{m_{\tilde{t}_L}^2 - m_{\tilde{t}_i}^2}
\]

For the minimal SUGRA models one finds \( \delta_{m_0^2} = \mathcal{O}(1) \) which is basically independent of the initial conditions due to the \( m_0 \) dependence both in the numerator and in the denominator. Finally we have
\[
\delta_A = \frac{m_t (A_b + \frac{1}{2} A_t)}{m_{\tilde{t}_L}^2 - m_{\tilde{t}_i}^2}
\]
The complete formulas are given in [31, 32], while for the three-body decays they are given in Appendix A. They reduce to the ones given in [26, 27] for vanishing R-parity violation parameters.

We have fixed the parameters as in [27] to avoid colour breaking minima, while in the top squark sector we have used $m_{\tilde{t}_1}$, $\cos \theta_{\tilde{t}}$, $\tan \beta$, and $\mu$ as input parameters. For the sbottom sector we have fixed $M_{\tilde{Q}}$, $M_{\tilde{D}}$ and $A_b$ as input parameters whereas for the charged scalars we took $m_{P_0^c}$, $M_{\tilde{E}}$, $M_{\tilde{L}}$, and $A_{\tau}$ as input [51]. In addition we have chosen the R-parity violating parameters $\epsilon_3$ and $v_3$ in such a way that the heaviest neutrino mass is fixed with the help of Eq. (3). For simplicity, we have also assumed that the soft SUSY breaking parameters are equal for all generations.

In order to get a feeling for minimal size of branching ratios that can be measured let us first shortly discuss the expected size for the direct production of light stops at future colliders. One expects for example at the LHC a production cross section of $\sim 35$ pb for 220 GeV stop mass. Therefore, once the full luminosity has been reached, one has to expect approximately $3.5 \times 10^6$ events per year. The corresponding stop production cross section at a future $e^+e^-$ linear collider of 800 c.m.s. energy is of $O(10^{-100} fb)$ [23]. For an integrated luminosity of 500 fb$^{-1}$ per year one can expect $O(10^4)$ stop pairs per year. This implies that branching ratio as low as $10^{-3}$ can in principle be measured.

We consider first the simplest case of one generation model which, as already mentioned, is sufficient to describe the relative importance of the $\tilde{t}_1 \rightarrow \tau^+ b$ decay mode
relative to the possible 3-body decay modes

\[ \tilde{t}_1 \rightarrow W^+ b \tilde{\chi}_1^0 \]
\[ \tilde{t}_1 \rightarrow S_k^+ b \tilde{\chi}_1^0 \]
\[ \tilde{t}_1 \rightarrow S_k^+ b \nu_3 \]
\[ \tilde{t}_1 \rightarrow b S_i^0 \tau^+ , \]
\[ \tilde{t}_1 \rightarrow b P_j^0 \tau^+ \]
\[ \tilde{t}_1 \rightarrow b \tilde{\tau}_i^+ \nu_l \]
\[ \tilde{t}_1 \rightarrow b \tilde{\nu}_l t^+ \quad (l = e, \mu) . \] 

(8)

In general the important final states are those that conserve R-parity. For example, for the case of decays involving \( S_k^+ \) the most important are those in which the scalars mainly a stau. Due to the fact that the existing bounds on the MSSM sneutrinos are below 100 GeV there exists the possibility that the sneutrino has nearly the same mass as one of the Higgs boson. Similarly it could be that the charged MSSM boson has nearly the same mass as one of the staus. This implies large mixing effects even for small R-parity breaking parameters \([34, 35, 47]\). We therefore have used the complete formulas for the 3-body decay modes which are presented in the Appendix. The latter include R-parity violating decays such as \( \tilde{t}_1 \rightarrow W^+ b \nu_3 \). In addition to the above mentioned decays there is also \( \tilde{t}_1 \rightarrow b Z^0 \tau^+ \). This decay mode is kinematically suppressed compared to \( \tilde{t}_1 \rightarrow b \tau^+ \) and there is no possible enhancement due to a mixing with an R-parity conserving final state. Therefore it can be safely neglected.

In Fig. 1 we show the branching ratios for the \( \tilde{t}_1 \) as a function of \( \cos \theta_{\tilde{t}_1} \) in different scenarios. In order to calculate the partial width for the decay \( \tilde{t}_1 \rightarrow c \tilde{\chi}_1^0 \) we have taken the formula given in ref. \([25]\). According to the analysis performed in \([32]\), where the full calculation was done in the mSUGRA scenario, the result obtained with the present approximation should be taken as one upper bound. This implies also that the shown branching ratio for \( \tilde{t}_1 \rightarrow b \tau^+ \) can be viewed as a lower bound. The parameters and physical quantities used in Fig. 1 are given in Tab. I. For the case of Fig. 1(a) we have
fixed in addition the R-parity violating parameters such that $m_\nu \approx 1 \text{ eV}$. With this choice of parameters $S_1^0$, $S_3^0$, $P_3^0$, and $S_4^-$ are mainly the MSSM Higgs-bosons whereas $S_2^0$, $P_2^0$, $S_2^-$, and $S_3^-$ are mainly the MSSM sleptons of the third generation. In the plot we show the various branching ratios of the lighter stop summing up those branching ratios for the decays into sleptons that give the same final state, for example:

$$\tilde{t}_1 \to b \nu_e \tilde{e}_L^+ \to b e^+ \nu_e \bar{\chi}_1^0, \quad \tilde{t}_1 \to b e^+ \bar{\nu}_e \to b e^+ \nu_e \bar{\chi}_1^0.$$  

The branching ratios for decays into $\tilde{\mu}_L$ or $\tilde{\nu}_\mu$ are practically the same as those into $\tilde{e}_L$ or $\tilde{\nu}_e$. Note that the energy spectrum of the leptons in the final will be somewhat different depending on whether the scalar in the intermediate step is charged or neutral. This offers in principle the possibility of determining the branching ratios of the different decay chains even if the final state topology is common. Note, that states containing scalars or neutralinos will lead to additional jet and/or lepton multiplicities absent in the MSSM.

In Fig. 1(b) the slepton mass parameters are chosen such that decays into scalars are kinematically forbidden. Here we display the channels $\tilde{t}_1 \to b W^+ \bar{\chi}_1^0$, $\tilde{t}_1 \to b \tau^+$ and $\tilde{t}_1 \to c \bar{\chi}_1^0$. The remaining modes, such as $\tilde{t}_1 \to b S_1^0 \tau^+$, turn out to be completely negligible. In both cases, with and without sleptons in the final state, one can see that in general the three body mode $\tilde{t}_1 \to b W^+ \bar{\chi}_1^0$, dominates except for a somewhat narrow range of negative $\cos \theta_{\tilde{t}}$. However, the branching ratio for $\tilde{t}_1 \to b \tau^+$ is above 0.1% for most values of $|\cos \theta_{\tilde{t}}|$ implying the observability of this mode. Most importantly, note that even in the parameter ranges where the three-body decay mode is dominant, its resulting signature is rather different from that of the MSSM due to the fact the lightest neutralino decays into SM-fermions, leading to enhanced jet and/or lepton multiplicities, as discussed in detail in [37, 38]. In the remaining part of this section we assume that 3-body decays into scalars are kinematically forbidden.

In Fig. 1(c) the R-parity violating parameters are fixed in such a way that the heaviest neutrino mass is in the range suggested by the oscillation interpretation of the atmospheric neutrino anomaly [14].
FIG. 1: Branching ratios for the $\tilde{t}_1$ as a function of $\cos \theta_{\tilde{t}}$ for different scenarios. We have fixed in a) $m_{\nu_3} = 1$ eV, b) $m_{\nu_3} = 1$ eV, $M_{\tilde{E}} > 225$ GeV, $M_{\tilde{L}} > 225$ GeV, c) $m_{\nu_3} = 0.06$ eV, $M_{\tilde{E}} > 225$ GeV, $M_{\tilde{L}} > 225$ GeV, d) Branching ratios for the $\tilde{t}_1$ as a function of $\cos \theta_{\tilde{t}}$ for $\tan \beta = 3$. $m_{\nu_3} = 0.06$ eV, $M_{\tilde{E}} > 225$ GeV, $M_{\tilde{L}} > 225$ GeV. All the other inputs are given in Table I.
In Fig. 1(d) we show the same scenario as in Fig. 1(c) but for tan $\beta = 3$. The branching ratio into $b\tau$ now increases, whereas the branching ratio into $c\tilde{\chi}_0^1$ decreases. This is easily understood by inspecting Eqs. (6) and (7). Indeed for the $b\tau$ case the partial width is proportional to $h_b^2$, whereas for $c\tilde{\chi}_0^1$ it is proportional to $h_b^4$. This implies that the partial width for $\tilde{t}_1 \to c\tilde{\chi}_1^0$ grows faster with tan $\beta$ than the width for $\tilde{t}_1 \to b\tau$. This is also demonstrated in Fig. 2 where we show the tan $\beta$ dependence of the branching ratio for the decay of $\tilde{t}_1$ into $b\tau^+$ for several values of the neutrino mass. For $m_{\nu_3} = 0.06\text{ eV}$ the $B(\tilde{t}_1 \to b\tau)$ is still above 0.1% if tan $\beta$ is not too large, as favored by the explanation of the neutrino anomalies in this model [13]. As seen from the figure, the the $\tilde{t}_1 \to b\tau^+$ branching ratio is also somewhat correlated to the $\nu_3$ mass. Should one add a sterile neutrino to the model [48], then the neutrino state $\nu_3$ could in principle be heavier than assumed above, favoring $\tilde{t}_1 \to \tau^+ b$ decay mode.

Let us now turn to the general three neutrinos case. There are new features that arise in this case, as opposed to the 1-generation case considered so far. In this model the solution to the present neutrino anomalies implies that all the $\epsilon_i$ are of the same order of

| Input:          | tan $\beta = 6$ | $\mu = 500\text{ GeV}$ | $M = 250\text{ GeV}$ |
|-----------------|-----------------|--------------------------|------------------------|
| $M_D = 370\text{ GeV}$ | $M_Q = 340\text{ GeV}$ | $A_b = 150\text{ GeV}$   |
| $M_E = 210\text{ GeV}$ | $M_L = 210\text{ GeV}$ | $A_\tau = 150\text{ GeV}$|
| $m_{\tilde{t}_1} = 220\text{ GeV}$ | $\cos \theta_{\tilde{t}} = -0.8$ | $m_{P_0^3} = 300\text{ GeV}$ |

| Calculated:     | $m_{\tilde{\chi}_0^1} = 122\text{ GeV}$ | $m_{\tilde{\chi}_1^+} = 234\text{ GeV}$ | $m_{\tilde{\chi}_2^+} = 519\text{ GeV}$ |
|-----------------|-----------------|--------------------------|------------------------|
| $m_{\tilde{b}_1} = 334\text{ GeV}$ | $m_{\tilde{b}_2} = 381\text{ GeV}$ | $\cos \theta_{\tilde{b}} = 0.879$   |
| $m_{S_0^1} = 107\text{ GeV}$ | $m_{S_0^2} = 200\text{ GeV}$ | $m_{S_0^3} = 302\text{ GeV}$   |
| $m_{P_0^2} = 200\text{ GeV}$ | $m_{P_0^3} = 300\text{ GeV}$   |
| $m_{S_3^-} = 203\text{ GeV}$ | $m_{S_3^-} = 226\text{ GeV}$ | $m_{S_4^-} = 311\text{ GeV}$   |
| $m_{\tilde{\nu}_L} = 215\text{ GeV}$ | $m_{\tilde{\nu}_e} = m_{\tilde{\nu}_\mu} = 200\text{ GeV}$ |

**TABLE I:** Input parameters and resulting quantities used in Fig. 1.
FIG. 2: Branching ratios for $\tilde{t}_1$ decays for $m_{\tilde{t}_1} = 220$ GeV, $\mu = 500$ GeV, $M = 240$ GeV, and $m_\nu = 100, 1$ and 0.06 eV. The branching ratios are shown as a function of $\tan \beta$. ($\cos \theta_{\tilde{t}} = -0.8$) magnitude [13].

Two further important results of [13] are that the atmospheric neutrino angle is controlled by the ratio $(\epsilon_2 v_d + \mu v_2) / (\epsilon_3 v_d + \mu v_3)$ and that the solar mixing angle is controlled by $(\epsilon_1 / \epsilon_2)^2$. One can get approximate formulas for the decay widths $\tilde{t}_1 \to b e^+$ and $\tilde{t}_1 \to b \mu^+$ similar to Eq. (6) by replacing $\epsilon_3$ by $\epsilon_{1,2}$. This implies that (i) The decays into $b e^+$ and $b \mu^+$ are as important as the decay into $b \tau^+$. (ii) The decays $\tilde{t}_1 \to b e^+$ and $\tilde{t}_1 \to b \mu^+$ are related with the solar mixing angle. Moreover, we find that $\sum_{l=e,\mu,\tau} \Gamma(\tilde{t}_1 \to b l^+)$ in the 3-generation model is nearly equal to $\Gamma(\tilde{t}_1 \to b \tau^+)$ in the 1-generation model provided that $\sum_{i=1}^{3} \epsilon_i^2$ is identified to $\epsilon^2$ in the 1-generation model.

In Fig. 3 we show the ratio of $B(\tilde{t}_1 \to b e^+) / B(\tilde{t}_1 \to b \mu^+)$ versus $(\epsilon_1 / \epsilon_2)^2$ for different values of $\cos \theta_{\tilde{t}}$. For definiteness we have fixed the heaviest neutrino mass at the best-fit value indicated by the atmospheric neutrino anomaly. One can see that the dependence is nearly linear even for rather small $\cos \theta_{\tilde{t}}$. For $| \cos \theta_{\tilde{t}} | \lesssim 10^{-2}$ the approximation in Eq. (6) breaks down and additional pieces dependent on $\sin \theta_{\tilde{t}}$ [31, 32] become important, leading to the non-linear dependence. One sees from the figure that, as long as $\cos \theta_{\tilde{t}} \gtrsim 10^{-2}$
there is a good degree of correlation between the branching ratios into $B(\tilde{t}_1 \to be^+) \text{ and } B(\tilde{t}_1 \to b\mu^+)$ and the ratio $(\epsilon_1/\epsilon_2)^2$. Thus by measuring these branchings one will get information on the solar neutrino mixing, since $\tan^2 \theta_{\text{sol}}$ is proportional to $(\epsilon_1/\epsilon_2)^2$ \cite{13} which makes it a rather important quantity. For the so–called small mixing angle or SMA solution of the solar neutrino problem we expect $\tilde{t}_1 \to e^+ b$ to be negligible. In contrast, for the large mixing angle type solutions (LMA, LOW and QVAC, see ref. \cite{14} and references therein) we expect all $\tilde{t}_1 \to l^+ b$ decays to have comparable rates. As a result in this model one can directly test the solution to the solar neutrino problem against the lighter stop decay pattern. This is also complementary to the case of neutralino decays considered in \cite{38}. In that case the sensitivity is mainly to atmospheric mixing, as opposed to solar mixing. Testing the latter in neutralino decays at a collider experiment requires more detailed information on the complete spectrum to test the solar angle \cite{38}. In contrast we have obtained here a rather neat connection of stop decays with the solar neutrino physics.

Note, that this result is much more general than the scenarios discussed in this paper.
It is of particular importance in scenarios where only the R-parity violating decays and the decay into $\tilde{\chi}_1^0 c$ are present \cite{31, 32}. Similarly, the other ratios of the final states $b l^+$ are proportional to the square of the ratio of corresponding $\epsilon_i$ provided that $\cos \theta_\tilde{t}$ is not too small.

IV. CONCLUSIONS

We have studied the phenomenology of the lightest stop in scenarios where R-parity violating decays such as $\tilde{t}_1 \to b \tau^+$ compete with three–body decays. We have found that for $m_{\tilde{t}_1} \lesssim 250$ GeV there are regions of parameter where $\tilde{t}_1 \to b \tau^+$ is an important decay mode if not the most important one. This implies that there exists the possibility of full stop mass reconstruction from $\tau^+ \tau^- b \bar{b}$ final states, favoring the prospects for its discovery. In contrast, in the MSSM the discovery of the lightest stop might not be possible at the LHC within this mass range. This implies that it is important to take into account this new decay mode when designing the stop search strategies at a future $e^+ e^-$ Linear Collider. Spontaneously and bilinearly broken R-parity violation also imply additional leptons and/or jets in stop cascade decays. Looking at the three generation model the decays into $\tilde{t}_1 \to b l^+$ imply the possibility of probing $\epsilon_1^2 / \epsilon_2^2$ and thus the solar mixing angle. This complements information which can be obtained using neutralino decays. In the latter case the sensitivity is mainly to the atmospheric mixing, as opposed to solar mixing. In this model neutralino decays is ideal to test the atmospheric anomaly at a collider experiment, while stop decays provide neat complementary information on the solar mixing angle. Obtaining solar mixing information from neutralino decays would require more detailed knowledge on the supersymmetric spectrum, since it would be involved in the relevant loop calculations of the solar neutrino mass scale and mixing angle. By combining the two one can probe the parameters associated with both solar and atmospheric neutrino anomalies at collider experiments.
Acknowledgments

This work was supported by Spanish DGICYT under grant PB98-0693 and by the European Commission TMR network HPRN-CT-2000-00148. D. R. was supported by Colombian COLCIENCIAS fellowship, while W. P. was supported by the Spanish “Ministerio de Educación y Cultura” under the contract SB97-BU0475382.
APPENDIX

In this set of appendixes we present the formulas for the lighter stop decay widths and couplings used in the paper and omitted in previous sections.

APPENDIX A: FORMULAS FOR THE THREE-BODY DECAY WIDTHS

1. The width $\Gamma(\tilde{t}_1 \to W^+ b \tilde{\chi}^0_i)$

$$\Gamma(\tilde{t}_1 \to W^+ b \tilde{\chi}^0_i) =$$

$$= \frac{\alpha^2}{16 \pi m_{\tilde{t}_1}^3 \sin^4 \theta_W} \int \frac{d s}{(m_b + m_{\tilde{\chi}^0_i})^2} \left( G_{\tilde{\chi}^+_i, \tilde{\chi}_i^+}^W + G_{\tilde{\chi}^+_i, \tilde{\chi}_i^0}^W + G_{\tilde{t} \tilde{t}_{\tilde{b}}}^W + G_{\tilde{t} \tilde{b}}^W + G_{\tilde{b} \tilde{b}}^W \right)$$
with

\[ G^{W}_{\tilde{\chi}^+\tilde{\chi}^+} = \sum_{j=1}^{3} \left[ \sum_{k=0}^{3} a_{ijk} s^k J^0_t (m^2_{t_1} + m^2_W + m^2_b + m^2_{\tilde{\chi}_0} - m^2_{\tilde{\chi}_j} - s, \Gamma_{\tilde{\chi}_j} m_{\tilde{\chi}_j}^+) \right. \\
+ \sum_{k=0}^{2} \frac{\left( a_{ij8} + a_{ij9} s \right) J^2_t (m^2_{t_1} + m^2_W + m^2_b + m^2_{\tilde{\chi}_0} - m^2_{\tilde{\chi}_j} - s, \Gamma_{\tilde{\chi}_j} m_{\tilde{\chi}_j}^+) \right]}{s} \\
+ \sum_{k=0}^{3} a_{44k} s^k J^0_t (m^2_{t_1} + m^2_W + m^2_b + m^2_{\tilde{\chi}_0} - m^2_{\tilde{\chi}_j} - s, \Gamma_{\tilde{\chi}_j} m_{\tilde{\chi}_j}^+) \\
\left. , m^2_{t_1} + m^2_W + m^2_b + m^2_{\tilde{\chi}_0} - m^2_{\tilde{\chi}_2} - s, \Gamma_{\tilde{\chi}_2} m_{\tilde{\chi}_2}^+ \right] \\
+ \sum_{k=0}^{2} a_{44k+5} s^k J^0_t (m^2_{t_1} + m^2_W + m^2_b + m^2_{\tilde{\chi}_0} - m^2_{\tilde{\chi}_j} - s, \Gamma_{\tilde{\chi}_j} m_{\tilde{\chi}_j}^+) \\
\left. , m^2_{t_1} + m^2_W + m^2_b + m^2_{\tilde{\chi}_0} - m^2_{\tilde{\chi}_2} - s, \Gamma_{\tilde{\chi}_2} m_{\tilde{\chi}_2}^+ \right] \\
+ \sum_{k=0}^{3} a_{45k} s^k J^0_t (m^2_{t_1} + m^2_W + m^2_b + m^2_{\tilde{\chi}_0} - m^2_{\tilde{\chi}_j} - s, \Gamma_{\tilde{\chi}_j} m_{\tilde{\chi}_j}^+) \\
\left. , m^2_{t_1} + m^2_W + m^2_b + m^2_{\tilde{\chi}_0} - m^2_{\tilde{\chi}_3} - s, \Gamma_{\tilde{\chi}_3} m_{\tilde{\chi}_3}^+ \right] \\
+ \sum_{k=0}^{2} a_{45k+5} s^k J^0_t (m^2_{t_1} + m^2_W + m^2_b + m^2_{\tilde{\chi}_0} - m^2_{\tilde{\chi}_j} - s, \Gamma_{\tilde{\chi}_j} m_{\tilde{\chi}_j}^+) \\
\left. , m^2_{t_1} + m^2_W + m^2_b + m^2_{\tilde{\chi}_0} - m^2_{\tilde{\chi}_3} - s, \Gamma_{\tilde{\chi}_3} m_{\tilde{\chi}_3}^+ \right] \\
+ \sum_{k=0}^{3} a_{46k} s^k J^0_t (m^2_{t_1} + m^2_W + m^2_b + m^2_{\tilde{\chi}_0} - m^2_{\tilde{\chi}_j} - s, \Gamma_{\tilde{\chi}_j} m_{\tilde{\chi}_j}^+) \\
\left. , m^2_{t_1} + m^2_W + m^2_b + m^2_{\tilde{\chi}_0} - m^2_{\tilde{\chi}_2} - s, \Gamma_{\tilde{\chi}_2} m_{\tilde{\chi}_2}^+ \right] \]
\[
G^W_{\chi^+t} = \sum_{j=1}^{3} \left[ \sum_{k=0}^{2} b_{ijk}s^k J^0_{it}(m^2_{t j} + m^2_W + m^2_b + m^2_{\chi^0_j} - m^2_{\chi^+_j} - s, \Gamma_{\chi^+_j} m_{\chi^+_j}, m^2_t, \Gamma_t m_t) \\
+ \sum_{k=0}^{2} b_{ij,k+l} s^k J^1_{it}(m^2_{t j} + m^2_W + m^2_b + m^2_{\chi^0_j} - m^2_{\chi^+_j} - s, \Gamma_{\chi^+_j} m_{\chi^+_j}, m^2_t, \Gamma_t m_t) \\
+ (b_{ij7} + b_{ij8}) J^2_{it}(m^2_{t j} + m^2_W + m^2_b + m^2_{\chi^0_j} - m^2_{\chi^+_j} - s, \Gamma_{\chi^+_j} m_{\chi^+_j}, m^2_t, \Gamma_t m_t) \right],
\]

\[
G^W_{\chi^+b} = -\sum_{l=0}^{3} \left[ \sum_{k=1}^{2} c_{ijkl}s^l J^0_{st}(m^2_{b_k} + \Gamma_{b_k} m_{b_k}, m^2_{t j} + m^2_W + m^2_b + m^2_{\chi^0_j} - m^2_{\chi^+_j} - s, \Gamma_{\chi^+_j} m_{\chi^+_j}) \\
+ \sum_{l=0}^{3} c_{ijkl+l} s^l J^1_{st}(m^2_{b_k} + \Gamma_{b_k} m_{b_k}, m^2_{t j} + m^2_W + m^2_b + m^2_{\chi^0_j} - m^2_{\chi^+_j} - s, \Gamma_{\chi^+_j} m_{\chi^+_j}) \right],
\]

\[
G^W_{tt} = (d_{i1} + d_{i2}s) J^0_{t}(m^2_t, \Gamma_t m_t) + (d_{i3} + d_{i4}s) J^1_{t}(m^2_t, \Gamma_t m_t) + (d_{i5} + d_{i6}s) J^2_{t}(m^2_t, \Gamma_t m_t),
\]

\[
G^W_{tb} = \sum_{k=1}^{2} \left[ (e_{ik1} + e_{ik2}s + e_{ik3}s^2) J^0_{st}(m^2_{b_k} + \Gamma_{b_k} m_{b_k}, m^2_t, \Gamma_t m_t) \\
+ (e_{ik4} + e_{ik5}s + e_{ik6}s^2) J^1_{st}(m^2_{b_k} + \Gamma_{b_k} m_{b_k}, m^2_t, \Gamma_t m_t) \right],
\]

\[
G^W_{bb} = \sqrt{\lambda(s, m^2_{t j}, m^2_W) \lambda(s, m^2_{\chi^0_j}, m^2_b)} \times \left\{ \sum_{k=1}^{2} \frac{(f_{ik1} + f_{ik2}s)}{(s - m^2_{b_k})^2 + \Gamma^2_{b_k} m^2_{b_k}} + \Re \left[ \frac{(f_{i31} + f_{i32}s)}{(s - m^2_{b_1} + i\Gamma_{b_1} m_{b_1}) (s - m^2_{b_2} - i\Gamma_{b_2} m_{b_2})} \right] \right\}.
\]
The integrals $J^{0,1,2}_{t,tt,tt}$ are:

\[
J^i_t(m_1^2, m_1 \Gamma_1) = \int_{t_{\min}}^{t_{\max}} dt \frac{t^i}{(t-m_1^2)^2 + m_1^2 \Gamma_1^2},
\]

\[
J^i_{tt}(m_1^2, m_1 \Gamma_1, m_2^2, m_2 \Gamma_2) = \Re \int_{t_{\min}}^{t_{\max}} dt \frac{t^i}{(t-m_1^2 + im_1 \Gamma_1)(t-m_2^2 - im_2 \Gamma_2)},
\]

\[
J^i_{ttt}(m_1^2, m_1 \Gamma_1, m_2^2, m_2 \Gamma_2) = \Re \frac{1}{s - m_1^2 + im_1 \Gamma_1} \int_{t_{\min}}^{t_{\max}} dt \frac{t^i}{(t-m_2^2 - im_2 \Gamma_2)}
\]

with $i = 0, 1, 2$. Their integration range is given by

\[
t_{\max} = \frac{m_2^2 + m_b^2 + m_W^2 + m_{\chi_0}^2 - s}{2} - \frac{(m_1^2 - m_W^2)(m_{\chi_0}^2 - m_b^2)}{2s} + \frac{\sqrt{\lambda(s, m_{\chi_1}^2, m_W^2)\lambda(s, m_{\chi_0}^2, m_b^2)}}{2s},
\]

where $s = (p_{t_i} - p_W)^2$ and $t = (p_{t_i} - p_t)^2$ are the usual Mandelstam variables. Note, that $-\Gamma_{\chi_j^+ m_{\chi_j^+}}$ appears in the entries of the integrals $G^{W}_{\chi^+ \chi_j}$ and $G^{W}_{\chi^+ \chi}$ because the chargino is exchanged in the $u$-channel in our convention. The coefficients are given by (no sum upon repeated index):

\[
\begin{align*}
a_{ij1} &= 6 O^L_{ij} O^R_{ij} \left( (k^i_{ij})^2 + (l^i_{ij})^2 \right) m_{\chi_j^+} m_{\chi_i} (2m_{\chi_0}^2 + m_{\chi_i}^2 + m_W^2) \\
&\quad + 2k^i_{ij} l^i_{ij} m_b m_{\chi_i} (m_b^2 + m_{\chi_0}^2 + m_{\chi_i}^2 + m_{\chi_j}^2 + m_W^2) \\
&\quad - 2k^i_{ij} l^i_{ij} (O^L_{ij})^2 + (O^R_{ij})^2 b m_{\chi_j^+} 3m_b^2 + 2m_{\chi_0}^2 + 3m_{t_i}^2 + (m_{t_i}^2 + m_b^2)^2/m_W^2 \\
&\quad - (k^i_{ij})^2 (O^R_{ij})^2 + (l^i_{ij})^2 (O^L_{ij})^2 \left[ (4m_b^2 + m_W^2)(m_{\chi_0}^2 + m_b^2) + m_{\chi_0}^4 \\
&\quad + m_{t_i}^2 (2m_W^2 + 4m_b^2 + m_{\chi_0}^2) + (m_{\chi_0}^2 + m_{\chi_i}^2 + m_{\chi_j}^2 + m_{\chi_0}^2 m_{\chi_i}^2 + m_{\chi_0}^2 m_{\chi_j}^2 + m_{\chi_i}^2 m_{\chi_j}^2)/m_W^2 \\
&\quad - (k^i_{ij})^2 (O^L_{ij})^2 + (l^i_{ij})^2 (O^R_{ij})^2 \right] m_{\chi_j}^2 \left( m_{\chi_0}^2 + m_{\chi_i}^2 + 2m_b^2 + (m_b^4 + m_{\chi_0}^2 m_{\chi_i}^2 + m_{\chi_0}^2 m_{\chi_j}^2 + m_{\chi_i}^2 m_{\chi_j}^2 + m_{\chi_0}^2 m_{\chi_i}^2 m_{\chi_j}^2)/m_W^2 \right)
\end{align*}
\]
\[ a_{ij2} = -12k^i_j\tilde{l}_{ij}O^L_{ij}O^R_{ij}m_b m_{\chi_i^0} - 6 \left( (k^i_j)^2 + (\tilde{l}_{ij})^2 \right) O^L_{ij}O^R_{ij}m_{\chi_j^+}m_{\chi_i^0} \]
\[ + 2k^i_j\tilde{l}_{ij} \left( (O^L_{ij})^2 + (O^R_{ij})^2 \right) m_b m_{\chi_j^+} (3 + 2(m_b^2 + m_{\tilde{t}_i}^2)/m_W^2) \]
\[ + \left( (k^i_j)^2(O^R_{ij})^2 + (\tilde{l}_{ij})^2(O^L_{ij})^2 \right) \left[ 6m_b^2 + 2m_{\chi_i^0}^2 + 2m_{\tilde{t}_i}^2 + m_W^2 \right] \]
\[ + (3m_b^4 + m_b^2m_{\chi_i^0}^2 + 4m_b^2m_{\tilde{t}_i}^2 + 2m_{\chi_i^0}m_{\tilde{t}_i}^2 + m_{\tilde{t}_i}^4)/m_W^2 \]
\[ + \left( (k^i_j)^2(O^L_{ij})^2 + (\tilde{l}_{ij})^2(O^R_{ij})^2 \right) m_{\chi_j^+}^2 (2 + m_b^2/m_W^2) \]

\[ a_{ij3} = -2k^i_j\tilde{l}_{ij} \left( (O^L_{ij})^2 + (O^R_{ij})^2 \right) m_b m_{\chi_i^+}/m_W^2 \]
\[ - \left( (k^i_j)^2(O^R_{ij})^2 + (\tilde{l}_{ij})^2(O^L_{ij})^2 \right) \left[ 2 + (m_{\chi_i^0}^2 + 3m_b^2 + 2m_{\tilde{t}_i}^2)/m_W^2 \right] \]

\[ a_{ij4} = \left( (k^i_j)^2(O^R_{ij})^2 + (\tilde{l}_{ij})^2(O^L_{ij})^2 \right)/m_W^2 \]

\[ a_{ij5} = -12k^i_j\tilde{l}_{ij}O^L_{ij}O^R_{ij}m_b m_{\chi_i^0} - 6 \left( (k^i_j)^2 + (\tilde{l}_{ij})^2 \right) O^L_{ij}O^R_{ij}m_{\chi_j^+}m_{\chi_i^0} \]
\[ + 2k^i_j\tilde{l}_{ij} \left( (O^L_{ij})^2 + (O^R_{ij})^2 \right) m_b m_{\chi_j^+} \left[ 3 + 2(m_b^2 + m_{\tilde{t}_i}^2)/m_W^2 \right] \]
\[ + \left( (k^i_j)^2(O^R_{ij})^2 + (\tilde{l}_{ij})^2(O^L_{ij})^2 \right) \left[ 6m_b^2 + 3m_{\chi_i^0}^2 + 2m_{\tilde{t}_i}^2 + m_W^2 \right] \]
\[ + (2m_b^4 + 2m_b^2m_{\chi_i^0}^2 + m_{\chi_i^0}m_{\tilde{t}_i}^2 + m_{\tilde{t}_i}^4)/m_W^2 \]
\[ + \left( (k^i_j)^2(O^L_{ij})^2 + (\tilde{l}_{ij})^2(O^R_{ij})^2 \right) m_{\chi_j^+}^2 \left[ 1 + (2m_b^2 + m_{\tilde{t}_i}^2)/m_W^2 \right] \]

\[ a_{ij6} = -\left\{ 4k^i_j\tilde{l}_{ij} \left( (O^L_{ij})^2 + (O^R_{ij})^2 \right) m_b m_{\chi_j^+}/m_W^2 \right\} \]
\[ + \left( (k^i_j)^2(O^R_{ij})^2 + (\tilde{l}_{ij})^2(O^L_{ij})^2 \right) \left[ 4 + (m_{\chi_i^0}^2 + 4m_b^2 + 2m_{\tilde{t}_i}^2)/m_W^2 \right] \]
\[ + \left( (k^i_j)^2(O^L_{ij})^2 + (\tilde{l}_{ij})^2(O^R_{ij})^2 \right) m_{\chi_j^+}^2/m_W^2 \}

\[ a_{ij7} = 2 \left( (k^i_j)^2(O^R_{ij})^2 + (\tilde{l}_{ij})^2(O^L_{ij})^2 \right)/m_W^2 \]

\[ a_{ij8} = -\left[ 2k^i_j\tilde{l}_{ij} \left( (O^L_{ij})^2 + (O^R_{ij})^2 \right) m_b m_{\chi_j^+}/m_W^2 \right] \]
\[ + \left( (k^i_j)^2(O^R_{ij})^2 + (\tilde{l}_{ij})^2(O^L_{ij})^2 \right) (2 + m_b^2/m_W^2) \]
\[ + \left( (k^i_j)^2(O^L_{ij})^2 + (\tilde{l}_{ij})^2(O^R_{ij})^2 \right) m_{\chi_j^+}/m_W^2 \} \]
\[ a_{ij9} = \left( (k_{ij})^2(O_{ij}^R)^2 + (l_{ij})^2(O_{ij}^L)^2 \right) / m_W^2 \]

\[ a_{4i1} = 2 \left\{ 3 \left( l_{i11}^i l_{i12}^i O_{i12}^L O_{i11}^R + k_{i11}^i k_{i12}^i O_{i11}^L O_{i12}^R \right) m_{\tilde{\chi}_1^0} m_{\tilde{\chi}_1^0} (2m_b^2 + m_{\tilde{\chi}_1^0}^2 + m_{\tilde{\tau}_1}^2) + 6 \left( l_{i11}^i l_{i12}^i O_{i12}^L O_{i11}^R + k_{i11}^i l_{i12}^i O_{i11}^L O_{i12}^R \right) m_b m_{\tilde{\chi}_1^0} (m_b^2 + m_{\tilde{\chi}_1^0}^2 + m_{\tilde{\tau}_1}^2) + 6 \left( k_{i11}^i l_{i12}^i O_{i12}^L O_{i11}^R + k_{i11}^i l_{i12}^i O_{i11}^L O_{i12}^R \right) m_b m_{\tilde{\chi}_1^0} m_{\tilde{\chi}_1^0} \right\} \]

\[ - \left( \left( l_{i12}^i l_{i11}^i O_{i11}^L O_{i12}^R + k_{i12}^i l_{i11}^i O_{i11}^R O_{i12}^L \right) m_b m_{\tilde{\chi}_1^+} \left[ 3m_b^2 + 2m_{\tilde{\chi}_1^0}^2 + 3m_{\tilde{\tau}_1}^2 \right] + (m_b^2 + m_{\tilde{\tau}_1}^2)(m_b^2 + m_{\tilde{\tau}_1}^2)/m_W^2 \right\} \]

\[ a_{4i2} = 2 \left\{ - 3 \left( l_{i11}^i l_{i12}^i O_{i12}^L O_{i11}^R + k_{i11}^i k_{i12}^i O_{i11}^L O_{i12}^R \right) m_{\tilde{\chi}_1^+} m_{\tilde{\chi}_1^0} - 6 \left( k_{i11}^i l_{i12}^i O_{i12}^R O_{i11}^L + l_{i12}^i k_{i11}^i O_{i11}^L O_{i12}^R \right) m_b m_{\tilde{\chi}_1^0} - 3 \left( k_{i11}^i k_{i12}^i O_{i11}^L O_{i12}^R + l_{i11}^i k_{i12}^i O_{i11}^L O_{i12}^R \right) m_{\tilde{\chi}_2} m_{\tilde{\chi}_1^0} + \left( l_{i12}^i l_{i11}^i O_{i11}^L O_{i12}^R + k_{i12}^i k_{i11}^i O_{i11}^R O_{i12}^L \right) m_b m_{\tilde{\chi}_1^0} \left[ 6m_b^2 + 2m_{\tilde{\chi}_1^0}^2 + 2m_{\tilde{\tau}_1}^2 + m_{\tilde{\tau}_1}^2 \right] + \left( m_{\tilde{\tau}_1}^2 + 2m_{\tilde{\tau}_1}^2 + m_{\tilde{\tau}_1}^2 \right) + \left( m_{\tilde{\tau}_1}^2 + 2m_{\tilde{\tau}_1}^2 + m_{\tilde{\tau}_1}^2 \right) + \left( m_{\tilde{\tau}_1}^2 + 2m_{\tilde{\tau}_1}^2 + m_{\tilde{\tau}_1}^2 \right) \right\} \]
\[ a_{43} = -2 \left\{ \left( t_{11}^{i} t_{12}^{i} O_{i1}^{L} O_{i2}^{L} + k_{11}^{i} k_{12}^{i} O_{i1}^{R} O_{i2}^{R} \right) \left[ 2 + (m_{X_{0}}^{2} + 3m_{b}^{2} + 2m_{t_{1}}^{2})/m_{W}^{2} \right] \\
+ \left( k_{11}^{i} t_{12}^{i} O_{j1}^{L} O_{j2}^{L} + k_{12}^{i} t_{11}^{i} O_{j1}^{R} O_{j2}^{R} \right) m_{b} m_{\tilde{X}_{j}}/m_{W}^{2} \\
+ \left( k_{12}^{i} t_{11}^{i} O_{i1}^{L} O_{i2}^{L} + k_{11}^{i} t_{12}^{i} O_{i1}^{R} O_{i2}^{R} \right) m_{b} m_{\tilde{X}_{i}}/m_{W}^{2} \right\} \]

\[ a_{44} = 2 \left( t_{11}^{i} t_{12}^{i} O_{i1}^{L} O_{i2}^{L} + k_{11}^{i} k_{12}^{i} O_{i1}^{R} O_{i2}^{R} / m_{W}^{2} \right) \]

\[ a_{45} = 2 \left\{ -3 \left( t_{11}^{i} t_{12}^{i} O_{i1}^{L} O_{i2}^{L} + k_{11}^{i} k_{12}^{i} O_{i1}^{R} O_{i2}^{R} \right) m_{\tilde{X}_{j}} m_{\tilde{X}_{j}}^{0} \\
-6 \left( k_{11}^{i} t_{12}^{i} O_{i1}^{L} O_{i2}^{L} + k_{12}^{i} t_{11}^{i} O_{i1}^{R} O_{i2}^{R} \right) m_{b} m_{\tilde{X}_{j}}^{0} \\
-3 \left( k_{11}^{i} k_{12}^{i} O_{i1}^{L} O_{i2}^{L} + t_{11}^{i} t_{12}^{i} O_{i1}^{R} O_{i2}^{R} \right) m_{\tilde{X}_{j}} m_{\tilde{X}_{j}}^{0} \\
+ \left( k_{11}^{i} t_{12}^{i} O_{i1}^{L} O_{i2}^{L} + k_{11}^{i} k_{12}^{i} O_{i1}^{R} O_{i2}^{R} \right) \left[ 6m_{b}^{2} + 3m_{\tilde{X}_{j}}^{2} + 2m_{t_{1}}^{2} + 2m_{W}^{2} + (2m_{b}^{4} + m_{t_{1}}^{2}(m_{\tilde{X}_{j}}^{2} + 2m_{b}^{2}))/m_{W}^{2} \right] \\
+ \left( k_{11}^{i} k_{12}^{i} O_{i1}^{L} O_{i2}^{L} + k_{12}^{i} k_{11}^{i} O_{i1}^{R} O_{i2}^{R} \right) \left[ 3 + 2(m_{b}^{2} + m_{t_{1}}^{2})/m_{W}^{2} \right] \\
+ \left( k_{11}^{i} t_{12}^{i} O_{i1}^{L} O_{i2}^{L} + k_{11}^{i} t_{12}^{i} O_{i1}^{R} O_{i2}^{R} \right) \left[ 3 + 2(m_{b}^{2} + m_{t_{1}}^{2})/m_{W}^{2} \right] \\
+ \left( k_{11}^{i} k_{12}^{i} O_{i1}^{L} O_{i2}^{L} + t_{11}^{i} t_{12}^{i} O_{i1}^{R} O_{i2}^{R} \right) \left[ 1 + (2m_{b}^{2} + m_{t_{1}}^{2})/m_{W}^{2} \right] \right\} \]

\[ a_{46} = -2 \left\{ \left( t_{11}^{i} t_{12}^{i} O_{i1}^{L} O_{i2}^{L} + k_{11}^{i} k_{12}^{i} O_{i1}^{R} O_{i2}^{R} \right) \left[ 4 + (m_{\tilde{X}_{j}}^{2} + 4m_{b}^{2} + 2m_{t_{1}}^{2})/m_{W}^{2} \right] \\
+ 2 \left( k_{11}^{i} t_{12}^{i} O_{i1}^{L} O_{i2}^{L} + k_{12}^{i} t_{11}^{i} O_{i1}^{R} O_{i2}^{R} \right) m_{b} m_{\tilde{X}_{j}}^{+}/m_{W}^{2} \\
+ 2 \left( t_{12}^{i} t_{11}^{i} O_{i1}^{L} O_{i2}^{L} + k_{11}^{i} t_{12}^{i} O_{i1}^{R} O_{i2}^{R} \right) m_{b} m_{\tilde{X}_{j}}^{+}/m_{W}^{2} \\
+ \left( k_{11}^{i} k_{12}^{i} O_{i1}^{L} O_{i2}^{L} + t_{11}^{i} t_{12}^{i} O_{i1}^{R} O_{i2}^{R} \right) m_{\tilde{X}_{j}}^{+} m_{\tilde{X}_{j}}^{+}/m_{W}^{2} \right\} \]

\[ a_{47} = 4 \left( t_{11}^{i} t_{12}^{i} O_{i1}^{L} O_{i2}^{L} + k_{11}^{i} k_{12}^{i} O_{i1}^{R} O_{i2}^{R} / m_{W}^{2} \right) \]

\[ a_{48} = -2 \left[ \left( t_{11}^{i} t_{12}^{i} O_{i1}^{L} O_{i2}^{L} + k_{11}^{i} k_{12}^{i} O_{i1}^{R} O_{i2}^{R} \right) (2 + m_{b}^{2}/m_{W}^{2}) \\
+ \left( k_{11}^{i} t_{12}^{i} O_{i1}^{L} O_{i2}^{L} + k_{12}^{i} t_{11}^{i} O_{i1}^{R} O_{i2}^{R} \right) m_{b} m_{\tilde{X}_{j}}^{+}/m_{W}^{2} \\
+ \left( k_{12}^{i} t_{11}^{i} O_{i1}^{L} O_{i2}^{L} + k_{11}^{i} t_{12}^{i} O_{i1}^{R} O_{i2}^{R} \right) m_{b} m_{\tilde{X}_{j}}^{+}/m_{W}^{2} \\
+ \left( k_{11}^{i} k_{12}^{i} O_{i1}^{L} O_{i2}^{L} + t_{11}^{i} t_{12}^{i} O_{i1}^{R} O_{i2}^{R} \right) m_{\tilde{X}_{j}}^{+} m_{\tilde{X}_{j}}^{+}/m_{W}^{2} \right] \]
\[ a_{i49} = 2 \left( \tilde{l}_{11} \tilde{l}_{12} O_{11}^L O_{12}^L + k_{11}^i k_{12}^i O_{11}^R O_{12}^R \right) / m_W^2 \]

The coefficients \( a_{i5l} \) are obtained from \( a_{i4l} \) by replacing: \( \tilde{l}_{12} \to \tilde{l}_{13} \), \( k_{12}^i \to k_{13}^i \), \( O_{12}^L \to O_{13}^L \), \( O_{12}^R \to O_{13}^R \), \( m_{\tilde{\chi}_2} \to m_{\tilde{\chi}_3} \) and the coefficients \( a_{i6l} \) are obtained from \( a_{i4l} \) by replacing: \( \tilde{l}_{11} \to \tilde{l}_{13} \), \( k_{11}^i \to k_{13}^i \), \( O_{11}^L \to O_{13}^L \), \( O_{11}^R \to O_{13}^R \), \( m_{\tilde{\chi}_1} \to m_{\tilde{\chi}_3} \)

\[ b_{i12} = \sqrt{2} \left\{ a_{i1}^i \tilde{l}_{1j} O_{1j}^L \left[ (2m_{\tilde{\chi}_i}^2 - m_{\tilde{\chi}_i}^2)(m_{W}^2 + m_{W}^2) \right] + 3b_{i1}^i \tilde{l}_{1j} O_{1j}^R m_b \right( m_{\tilde{\chi}_i}^2 - m_{\tilde{\chi}_i}^2 \right) \] 
\[ + 3b_{i1}^i \tilde{l}_{1j} O_{1j}^L \left( 2m_{b}^2 + m_{\tilde{\chi}_i}^2 + m_{W}^2 \right) + 6b_{i1}^i \tilde{l}_{1j} O_{1j}^R m_{\tilde{\chi}_i} m_b m_{\tilde{\chi}_i} \] 
\[ + a_{i1}^i \tilde{l}_{1j} O_{1j}^R m_{\tilde{\chi}_i} m_{\tilde{\chi}_i} \left[ 2m_{W}^2 - m_{b}^2 (1 + m_{b}^2 / m_{W}^2) \right] \] 
\[ + a_{i1}^i \tilde{l}_{1j} O_{1j}^R m_{\tilde{\chi}_i} m_{\tilde{\chi}_i} \left[ 2m_{W}^2 + m_{b}^2 (1 + m_{b}^2 / m_{W}^2) - m_{\tilde{\chi}_i}^2 \right] \] 
\[ - b_{i1}^i \tilde{l}_{1j} O_{1j}^R m_{\tilde{\chi}_i} m_{l} \left( m_{\tilde{\chi}_i}^2 + m_{\tilde{\chi}_i}^2 \right) (3 + (m_{b}^2 + m_{\tilde{\chi}_i}^2) / m_{W}^2) \] 
\[ - b_{i1}^i \tilde{l}_{1j} O_{1j}^R m_{\tilde{\chi}_i} m_{l} \left( m_{\tilde{\chi}_i}^2 + m_{\tilde{\chi}_i}^2 \right) (3 + (m_{b}^2 + m_{\tilde{\chi}_i}^2) / m_{W}^2) \] 

\[ b_{i3} = - \sqrt{2} b_{i1}^i \tilde{l}_{1j} O_{1j}^R m_b m_l / m_{W}^2 \]

\[ b_{i4} = \sqrt{2} \left\{ a_{i1}^i \tilde{l}_{1j} O_{1j}^L \left[ 4m_{b}^2 + 2m_{W}^2 + m_{\tilde{\chi}_i}^2 m_{l}^2 / m_{W}^2 \right] + 3a_{i1}^i \tilde{l}_{1j} O_{1j}^R m_b m_{\tilde{\chi}_i} \right. \] 
\[ - 3b_{i1}^i \tilde{l}_{1j} O_{1j}^L m_{\tilde{\chi}_i} m_{l} - a_{i1}^i \tilde{l}_{1j} O_{1j}^R m_{\tilde{\chi}_i} m_{\tilde{\chi}_i} \left( 1 - 2m_{b}^2 / m_{W}^2 \right) \] 
\[ + a_{i1}^i \tilde{l}_{1j} O_{1j}^L m_b m_2 \left( 1 + 2m_{b}^2 / m_{W}^2 + m_{l}^2 / m_{W}^2 \right) \] 
\[ + b_{i1}^i \tilde{l}_{1j} O_{1j}^R m_{\tilde{\chi}_i} m_{l} \left( 1 + 2m_{b}^2 / m_{W}^2 + m_{l}^2 / m_{W}^2 \right) \] 
\[ + b_{i1}^i \tilde{l}_{1j} O_{1j}^R m_{\tilde{\chi}_i} m_{l} \left[ 3 + 2(m_{b}^2 + m_{\tilde{\chi}_i}^2) / m_{W}^2 \right] \} \] 

\[ b_{i5} = - \sqrt{2} \left\{ a_{i1}^i \tilde{l}_{1j} O_{1j}^L \left[ 3 + m_{b}^2 / m_{W}^2 + (m_{\tilde{\chi}_i}^2 + m_{\tilde{\chi}_i}^2) / m_{W}^2 \right] + a_{i1}^i \tilde{l}_{1j} O_{1j}^R m_b m_{\tilde{\chi}_i} \right. \] 
\[ + b_{i1}^i \tilde{l}_{1j} O_{1j}^R m_{\tilde{\chi}_i} m_l / m_{W}^2 + 2b_{i1}^i \tilde{l}_{1j} O_{1j}^R m_b m_l / m_{W}^2 \} \]
\[ b_{ij6} = \sqrt{2} a_{ii}^L l_{ij}^L O_{ij}^L/m_{W}^2 \]

\[ b_{ij7} = -\sqrt{2} \left[ 2 a_{ii}^L l_{ij}^L O_{ij}^L + a_{ii}^L k_{ij}^L O_{ij}^R m_{\chi_i^0} m_{\chi_j^0}/m_{W}^2 + a_{ii}^L k_{ij}^L O_{ij}^R m_{\chi_i^0} m_{\chi_j^0}/m_{W}^2 + b_{ii}^L l_{ij}^L O_{ij}^R m_{\chi_i^0} m_{t_2}/m_{W}^2 \right] \]

\[ b_{ij8} = \sqrt{2} a_{ii}^L l_{ij}^L O_{ij}^L/m_{W}^2 \]

\[ c_{ijk1} = 2 A_{i \bar{b} k}^W \left\{ b_{ii}^L l_{ij}^L O_{ij}^L m_{b} m_{\chi_i^0} \left[ m_{b}^2 - m_{t_1}^2 - 2m_{\chi_i^0}^2 + m_{t_1}^2 (m_{b}^2/m_{W}^2 + m_{t_1}^2/m_{W}^2) \right] \\
+ a_{ii}^L k_{ij}^L O_{ij}^L \left[ m_{b}^2 m_{W}^2 + m_{\chi_i^0}^2 m_{t_1}^2 (m_{t_1}/m_{W}^2 - 1) + m_{b}^2 m_{t_1}^2 (m_{b}^2/m_{W}^2 - 2) + m_{b}^2 - 2m_{b}^2 m_{\chi_i^0}^2 + m_{t_1}^2 m_{t_1}^2/m_{W}^2 \right] \\
+ b_{ii}^L k_{ij}^L O_{ij}^R m_{\chi_i^0} m_{\chi_j^0} (m_{b}^2 - m_{W}^2 - 2m_{\chi_i^0}^2 + m_{t_1}^2 (1 + m_{b}^2/m_{W}^2)) \\
+ a_{ii}^L k_{ij}^L O_{ij}^L m_{b} m_{\chi_j^0}^+ \left[ m_{b}^2 - 2m_{\chi_j^0}^2 + m_{t_1}^2 (m_{b}^2/m_{W}^2 + m_{t_1}^2/m_{W}^2 - 1) \right] \right\} \]

\[ c_{ijk2} = -2 A_{i \bar{b} k}^W \left\{ b_{ii}^L l_{ij}^L O_{ij}^L m_{b} m_{\chi_i^0} (1 + m_{b}^2/m_{W}^2 + 2m_{t_1}^2/m_{W}^2) \\
+ a_{ii}^L k_{ij}^L O_{ij}^L \left[ m_{b}^2 (1 + 2m_{t_1}^2/m_{W}^2) + m_{t_1}^2 m_{t_1}^2/m_{W}^2 + m_{b}^2 \\
+ m_{b}^2 (2 + 2m_{b}^2/m_{W}^2 + 3m_{t_1}^2/m_{W}^2) \right] + b_{ii}^L k_{ij}^L O_{ij}^R m_{\chi_i^0} m_{\chi_j^0} (m_{b}^2/m_{W}^2 - 1) \\
+ a_{ii}^L k_{ij}^L O_{ij}^L m_{b} m_{\chi_j^0}^+ (1 + m_{b}^2/m_{W}^2 + 2m_{t_1}^2/m_{W}^2) \right\} \]

\[ c_{ijk3} = 2 A_{i \bar{b} k}^W \left[ b_{ii}^L l_{ij}^L O_{ij}^L m_{b} m_{\chi_i^0}^+ /m_{W}^2 + a_{ii}^L l_{ij}^L O_{ij}^L (2 + 2m_{b}^2/m_{W}^2 + m_{\chi_i^0}^2/m_{W}^2 + 2m_{t_1}^2/m_{W}^2) \\
+ a_{ii}^L k_{ij}^L O_{ij}^L m_{b} m_{\chi_j^0}^+ /m_{W}^2 \right] \]

\[ c_{ijk4} = -2 A_{i \bar{b} k}^W a_{ii}^L l_{ij}^L O_{ij}^L /m_{W}^2 \]

\[ c_{ijk5} = 2 A_{i \bar{b} k}^W \left[ b_{ii}^L k_{ij}^L O_{ij}^L m_{\chi_j^0} m_{\chi_i^0}^+ + a_{ii}^L k_{ij}^L O_{ij}^L m_{b} m_{\chi_j^0}^+ + b_{ii}^L l_{ij}^L O_{ij}^L m_{b} m_{\chi_i^0}^+ \\
+ a_{ii}^L k_{ij}^L O_{ij}^L m_{b}^2 \right] \left( 1 - m_{t_1}^2/m_{W}^2 \right) \]

\[ c_{ijk6} = 2 A_{i \bar{b} k}^W \left[ b_{ii}^L k_{ij}^L O_{ij}^L m_{b} m_{\chi_i^0}^+ /m_{W}^2 + a_{ii}^L l_{ij}^L O_{ij}^L (1 + m_{b}^2/m_{W}^2 + m_{t_1}^2/m_{W}^2) \\
+ b_{ii}^L k_{ij}^L O_{ij}^L m_{\chi_j^0} m_{\chi_i^0}^+ /m_{W}^2 + a_{ii}^L k_{ij}^L O_{ij}^L m_{b} m_{\chi_j^0}^+ /m_{W}^2 \right] \]

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\[ c_{ijk7} = c_{ijk4} \]

\[ d_{i1} = \frac{1}{2} \left\{ (a_{i1}^i)^2 m_{\chi_i^0}^2 - m_{i1}^2 \left[ 2m_{W}^2 - m_{b1}^2(1 + m_{i1}^2/m_{W}^2) \right] - (b_{i1}^i)^2 m_{i1}^2 \left[ m_{\chi_i^0}^2 + m_{i1}^2 + m_{b1}^2(2 + m_{i1}^2/m_{W}^2 + m_{i1}^2/m_{W}^2) \right] + 2a_{i1}^i b_{i1}^i m_{\chi_i^0} m_{t1} \left[ 2m_{W}^2 - m_{b1}^2(1 + m_{i1}^2/m_{W}^2) \right] \right\} \]

\[ d_{i2} = \frac{1}{2} (b_{i1}^i)^2 m_{i1}^2 (2 + m_{i1}^2/m_{W}^2) \]

\[ d_{i3} = \frac{1}{2} \left\{ (a_{i1}^i)^2 \left[ m_{b1}^2 + 2m_{i1}^2 + 2m_{W}^2 + (2m_{\chi_i^0}^2 - m_{i1}^2)m_{b1}^2/m_{W}^2 \right] + (b_{i1}^i)^2 m_{i1}^2 \left[ 1 + (2m_{b1}^2 + m_{i1}^2)/m_{W}^2 \right] - 2a_{i1}^i b_{i1}^i m_{\chi_i^0} m_{t1}(1 - 2m_{i1}^2/m_{W}^2) \right\} \]

\[ d_{i4} = -\frac{1}{2} \left\{ a_{i1}^i)^2 (2 + m_{b1}^2/m_{W}^2) - (b_{i1}^i)^2 m_{i1}^2/m_{W}^2 \right\} \]

\[ d_{i5} = -\frac{1}{2} \left\{ (a_{i1}^i)^2 (2 + m_{\chi_i^0}/m_{W}^2) + 2a_{i1}^i b_{i1}^i m_{\chi_i^0} m_{t1}/m_{W}^2 + (b_{i1}^i)^2 m_{i1}^2/m_{W}^2 \right\} \]

\[ d_{i6} = \frac{1}{2} (a_{i1}^i)^2 / m_{W}^2 \]

\[ e_{i1} = \sqrt{2} A_{i1b_k}^W \left\{ a_{k1}^b a_{i1}^i \left[ m_{b1}^2 (m_{\chi_i^0}^2 + m_{i1}^2) + 2m_{\chi_i^0}^2 m_{i1}^2 - m_{\chi_i^0}^2 - m_{W}^2 \right] + (m_{\chi_i^0}^2 - m_{i1}^2) m_{b1}^2 m_{i1}^2 / m_{W}^2 \right\} \]

\[ e_{i2} = \sqrt{2} A_{i1b_k}^W \left[ a_{k1}^b a_{i1}^i (m_{\chi_i^0}^2 - m_{i1}^2) (2 - m_{b1}^2/m_{W}^2) + (a_{k1}^b b_{i1}^i m_{i1}^2 m_{t1} + b_{k1}^b a_{i1}^i m_{b1} m_{\chi_i^0}^2) (1 - m_{b1}^2/m_{W}^2) - b_{k1}^b b_{i1}^i m_{b1} m_{t1}(1 + m_{b1}^2/m_{W}^2 + 2m_{i1}^2/m_{W}^2) \right] \]

\[ e_{i3} = \sqrt{2} A_{i1b_k}^W \left[ b_{k1}^b b_{i1}^i m_{b1} m_{t1} / m_{W}^2 \right] \]

\[ e_{i4} = \sqrt{2} \left[ a_{k1}^b a_{i1}^i m_{\chi_i^0}^2 + a_{k1}^b b_{i1}^i m_{i1}^2 m_{t1} + b_{k1}^b a_{i1}^i m_{b1} m_{\chi_i^0}^2 + b_{k1}^b b_{i1}^i m_{b1} m_{t1} \left( 1 - m_{i1}^2/m_{W}^2 \right) \right] \]

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\[ e_{ik5} = \sqrt{2} A^W_{i_1b_k} \left[ a^b_{k_1} a^{i_1} (1 + m^2_{\tilde{\chi}_1} / m^2_W + m^2_{\tilde{\chi}_i} / m^2_W) + a^b_{k_1} b^{i_1} m_{\tilde{\chi}_1} m_{\tilde{\chi}_i} / m^2_W \\
+ b^b_{k_1} a^{i_1} m_b m_{\tilde{\chi}_i} / m^2_W + b^b_{k_1} b^{i_1} m_b m_{\tilde{\chi}_i} / m^2_W \right] \]

\[ e_{ik6} = -\sqrt{2} A^W_{i_1b_k} a^{i_1} \]

\[ f_{ik1} = -\sqrt{2} (A^W_{i_1b_k})^2 \left[ (a^b_{k_1})^2 + (b^b_{k_1})^2 \left( m^2_b + m^2_{\tilde{\chi}_i} \right) + 4 a^b_{k_1} b^b_{k_1} m_b m_{\tilde{\chi}_i} \right], \]

\[ f_{ik2} = \sqrt{2} (A^W_{i_1b_k})^2 \left( (a^b_{k_1})^2 + (b^b_{k_1})^2 \right), \]

\[ f_{i31} = -4 A^W_{i_1b_1} A^W_{i_1b_2} \left[ \left( a^b_{1_1} a^{i_2} + b^{b_1} b^{i_2} \right) \left( m^2_b + m^2_{\tilde{\chi}_i} \right) + 2 \left( a^b_{1_1} b^{i_2} + b^{b_1} a^{i_2} \right) m_b m_{\tilde{\chi}_i} \right], \]

\[ f_{i31} = 4 A^W_{i_1b_1} A^W_{i_1b_2} \left( a^b_{1_1} a^{i_2} + b^{b_1} b^{i_2} \right). \]

\[ 2. \text{ The width } \Gamma(\tilde{t}_1 \to S^+_k b_{\tilde{\chi}_i}^0) \]

The decay width is given by

\[ \Gamma(\tilde{t}_1 \to S^+_k b_{\tilde{\chi}_i}^0) = \frac{\alpha^2}{16 \pi m^3_{t_1} \sin^3 \theta_W} \int d s \left( G_{\tilde{\chi}^+_1 + \tilde{\chi}^+_i} + G_{\tilde{\chi}^+_1 + \tilde{\chi}^+_i} + G_{\tilde{t} + \tilde{b}} + G_{\tilde{t} + \tilde{b}} \right) \]

\[ \frac{(m_{t_1} - m_{S^+_k})^2}{(m_b + m_{\tilde{\chi}_i})^2} \]
\[
G_{\tilde{\chi}^+\tilde{\chi}^+} = \sum_{j=1}^{3} \left[ (a_{ijk1} + a_{ijk2}s) J^0_{t} \left( m_{t_1}^2 + m_{s_{k_1}}^2 + m_{b}^2 + m_{\chi_{j}^0}^2 - m_{\tilde{\chi}_{j}^+}^2 - s, \Gamma_{\tilde{\chi}_{j}^+} m_{\tilde{\chi}_{j}^+} \right) \\
+ (a_{ijk3} + a_{ijk}s) J^1_{t} \left( m_{t_1}^2 + m_{s_{k_1}}^2 + m_{b}^2 + m_{\chi_{j}^0}^2 - m_{\tilde{\chi}_{j}^+}^2 - s, \Gamma_{\tilde{\chi}_{j}^+} m_{\tilde{\chi}_{j}^+} \right) \\
+ a_{ijk4} J^2_{t} \left( m_{t_1}^2 + m_{s_{k_1}}^2 + m_{b}^2 + m_{\chi_{j}^0}^2 - m_{\tilde{\chi}_{j}^+}^2 - s, \Gamma_{\tilde{\chi}_{j}^+} m_{\tilde{\chi}_{j}^+} \right) \right] \\
+ (a_{4k1} + a_{4k2}s) J^0_{u} \left( m_{t_1}^2 + m_{s_{k_1}}^2 + m_{b}^2 + m_{\chi_{j}^0}^2 - m_{\tilde{\chi}_{j}^+}^2 - s, \Gamma_{\tilde{\chi}_{j}^+} m_{\tilde{\chi}_{j}^+} \right) \\
+ (a_{4k3} + a_{4k4}s) J^1_{u} \left( m_{t_1}^2 + m_{s_{k_1}}^2 + m_{b}^2 + m_{\chi_{j}^0}^2 - m_{\tilde{\chi}_{j}^+}^2 - s, \Gamma_{\tilde{\chi}_{j}^+} m_{\tilde{\chi}_{j}^+} \right) \\
+ (a_{5k1} + a_{5k2}s) J^0_{u} \left( m_{t_1}^2 + m_{s_{k_1}}^2 + m_{b}^2 + m_{\chi_{j}^0}^2 - m_{\tilde{\chi}_{j}^+}^2 - s, \Gamma_{\tilde{\chi}_{j}^+} m_{\tilde{\chi}_{j}^+} \right) \\
+ (a_{5k3} + a_{5k4}s) J^1_{u} \left( m_{t_1}^2 + m_{s_{k_1}}^2 + m_{b}^2 + m_{\chi_{j}^0}^2 - m_{\tilde{\chi}_{j}^+}^2 - s, \Gamma_{\tilde{\chi}_{j}^+} m_{\tilde{\chi}_{j}^+} \right) \\
+ (a_{6k1} + a_{6k2}s) J^0_{u} \left( m_{t_1}^2 + m_{s_{k_1}}^2 + m_{b}^2 + m_{\chi_{j}^0}^2 - m_{\tilde{\chi}_{j}^+}^2 - s, \Gamma_{\tilde{\chi}_{j}^+} m_{\tilde{\chi}_{j}^+} \right) \\
+ (a_{6k3} + a_{6k4}s) J^1_{u} \left( m_{t_1}^2 + m_{s_{k_1}}^2 + m_{b}^2 + m_{\chi_{j}^0}^2 - m_{\tilde{\chi}_{j}^+}^2 - s, \Gamma_{\tilde{\chi}_{j}^+} m_{\tilde{\chi}_{j}^+} \right) \\
+ (a_{6k4} J^2_{u} \left( m_{t_1}^2 + m_{s_{k_1}}^2 + m_{b}^2 + m_{\chi_{j}^0}^2 - m_{\tilde{\chi}_{j}^+}^2 - s, \Gamma_{\tilde{\chi}_{j}^+} m_{\tilde{\chi}_{j}^+} \right) \\
, m_{t_1}^2 + m_{s_{k_1}}^2 + m_{b}^2 + m_{\chi_{j}^0}^2 - m_{\tilde{\chi}_{j}^+}^2 - s, \Gamma_{\tilde{\chi}_{j}^+} m_{\tilde{\chi}_{j}^+} \right) 
\right),
\]
\[ G_{\tilde{\chi}^+ t} = \sum_{j=1}^{3} \left[ (b_{ijk1} + b_{ijk2}s)J^0_{tt}(m_{t_1}^2 + m_{t_2}^2 + m_{b}^2 + m_{\tilde{\chi}_i}^2 - m_{\tilde{\chi}_j}^2 - s, -\Gamma_{\tilde{\chi}_j}m_{\tilde{\chi}_j} + m_{t_1}^2, \Gamma_{t_1}m_{t_1}) 
+ (b_{ijk3} + b_{ijk4}s)J^1_{tt}(m_{t_1}^2 + m_{S_k}^2 + m_{b}^2 + m_{\tilde{\chi}_i}^2 - m_{\tilde{\chi}_j}^2 - s, -\Gamma_{\tilde{\chi}_j}m_{\tilde{\chi}_j}, m_{t_2}^2, \Gamma_{t_2}m_{t_2}) 
+ b_{ijk4}J^2_{tt}(m_{t_1}^2 + m_{S_k}^2 + m_{b}^2 + m_{\tilde{\chi}_i}^2 - m_{\tilde{\chi}_j}^2 - s, -\Gamma_{\tilde{\chi}_j}m_{\tilde{\chi}_j}, m_{t_2}^2, \Gamma_{t_2}m_{t_2}) \right], \]

\[ G_{\tilde{\chi}^+ \tilde{b}} = \sum_{l=1}^{2} \left[ (c_{ijkl1} + c_{ijkl2}s) \times \left. J^0_{st}(m_{b_1}^2, \Gamma_{b_1}m_{b_1}, m_{t}^2 + m_{S_k}^2 + m_{b}^2 + m_{\tilde{\chi}_i}^2 - m_{\tilde{\chi}_j}^2 - s, -\Gamma_{\tilde{\chi}_j}m_{\tilde{\chi}_j}) \right. \right] \]

\[ G_{tt} = (d_{ik1} + d_{ik2}s)J^0_{tt}(m_{t_1}^2, \Gamma_{t_1}m_{t_1}) + (d_{ik3} + d_{ik4}s)J^1_{tt}(m_{t_1}^2, \Gamma_{t_2}m_{t_2}) + d_{ik4}J^2_{tt}(m_{t_1}^2, \Gamma_{t_2}m_{t_2}), \]

\[ G_{t\tilde{b}} = \sum_{l=1}^{2} \left[ (e_{ikl1} + e_{ikl2}s)J^0_{st}(m_{b_1}^2, \Gamma_{b_1}m_{b_1}, m_{t}^2, \Gamma_{t_1}m_{t_1}) + e_{ikl3}J^1_{st}(m_{b_1}^2, \Gamma_{b_1}m_{b_1}, m_{t_2}^2, \Gamma_{t_2}m_{t_2}) \right], \]

\[ G_{\tilde{b}\tilde{b}} = \sqrt{\lambda(s, m_{t_1}^2, m_{S_k}^2)}\lambda(s, m_{\tilde{\chi}_i}^2, m_{b}^2) \times \left\{ \sum_{l=1}^{2} \left[ \frac{(f_{ikl1} + f_{ikl2}s)}{(s - m_{b_1}^2 + \Gamma_{b_1}m_{b_1})} + \text{Re} \left[ \frac{(f_{ikl3} + f_{ikl4}s)}{(s - m_{b_2}^2 + \Gamma_{b_2}m_{b_2})} \right] \right] \right\}. \]

Their integration range is given by

\[ t_{\text{max}} - t_{\text{min}} = \frac{m_{t_1}^2 + m_{b}^2 + m_{S_k}^2 + m_{\tilde{\chi}_i}^2 - s}{2} - \frac{(m_{t_1}^2 - m_{S_k}^2)(m_{\tilde{\chi}_i}^2 - m_{b}^2)}{2s} \left( \pm \sqrt{\lambda(s, m_{t_1}^2, m_{S_k}^2)}\lambda(s, m_{\tilde{\chi}_i}^2, m_{b}^2) \right)/2s, \]

where \( s = (p_{t_1} - p_{S_k}^+)^2 \) and \( t = (p_{t_1} - p_t)^2 \) are the usual Mandelstam variables. Note, that \(-\Gamma_{\tilde{\chi}_j}m_{\tilde{\chi}_j}^+\) appears in the entries of the integrals \( G_{\tilde{\chi}^+ \tilde{b}_j} \) and \( G_{\tilde{\chi}^+ t} \) because the chargino is
exchanged in the $u$-channel in our convention. The coefficients are given by (no sum upon repeated index):

$$a_{ijk1} = -4 \kappa_{ijk}^i \kappa_{ijk}^j Q_{ijkl} \lbrack \kappa_{ijkl}^i \kappa_{ijkl}^j m_b m_{\chi_i^0} \left( m_b^2 + m_{\chi_i^0}^2 + m_{\chi_i^+}^2 + m_{\chi_i^{-}}^2 \right) \\
-2 Q_{ijkl}^R \left( (k_{ij}^i)^2 + (l_{ij}^i)^2 \right) m_{\chi_i^0} m_{\chi_j^+} \left( 2 m_b^2 + m_{\chi_i^0}^2 + m_{\chi_j^+}^2 \right) \\
-2 k_{ij}^i \kappa_{ijkl}^j \left( (Q_{ij}^L)^2 + (Q_{ijkl}^R)^2 \right) m_b m_{\chi_j^+} \left( m_b^2 + 2 m_{\chi_i^0}^2 + m_{\chi_j^+}^2 \right) \\
- \left( (k_{ij}^i)^2 (Q_{ijkl}^L)^2 + (l_{ij}^i)^2 (Q_{ijkl}^L)^2 \right) \\
\times \left[ \left( m_b^2 + m_{\chi_i^0}^2 \right)^2 + \left( m_b^2 + m_{\chi_i^+}^2 \right) \left( m_{\chi_i^0}^2 + m_{\chi_i^+}^2 \right) \right] \\
- \left( (k_{ij}^i)^2 (Q_{ijkl}^L)^2 + (l_{ij}^i)^2 (Q_{ijkl}^L)^2 \right) m_{\chi_j^+}^2 \left( m_b^2 + m_{\chi_j^+}^2 \right),$$

$$a_{ijk2} = 4 \kappa_{ijk}^i \kappa_{ijk}^j Q_{ijkl}^L \lbrack \kappa_{ijkl}^i \kappa_{ijkl}^j m_b m_{\chi_i^0} + \left( (k_{ij}^i)^2 (Q_{ijkl}^L)^2 + (l_{ij}^i)^2 (Q_{ijkl}^L)^2 \right) \left( m_b^2 + m_{\chi_i^0}^2 \right) \\
+ 2 k_{ij}^i \kappa_{ijkl}^j \left( (Q_{ijkl}^L)^2 + (Q_{ijkl}^R)^2 \right) m_b m_{\chi_j^+} + 2 Q_{ijkl}^R \left( (k_{ij}^i)^2 + (l_{ij}^i)^2 \right) m_{\chi_i^0} m_{\chi_j^+} \\
+ \left( (k_{ij}^i)^2 (Q_{ijkl}^L)^2 + (l_{ij}^i)^2 (Q_{ijkl}^L)^2 \right) m_{\chi_j^+}^2 \left( m_b^2 + m_{\chi_j^+}^2 \right),$$

$$a_{ijk3} = 4 \kappa_{ijk}^i \kappa_{ijk}^j Q_{ijkl}^L \lbrack \kappa_{ijkl}^i \kappa_{ijkl}^j m_b m_{\chi_i^0} + 2 Q_{ijkl}^L \lbrack \kappa_{ijkl}^i \kappa_{ijkl}^j m_b m_{\chi_j^+} + \left( (k_{ij}^i)^2 + (l_{ij}^i)^2 \right) m_{\chi_i^0} m_{\chi_j^+} \\
+ \left( (k_{ij}^i)^2 (Q_{ijkl}^L)^2 + (l_{ij}^i)^2 (Q_{ijkl}^L)^2 \right) \left( m_b^2 + 2 m_{\chi_i^0}^2 + m_{\chi_j^+}^2 + m_{\chi_i^{-}}^2 \right) \\
+ 2 k_{ij}^i \kappa_{ijkl}^j \left( (Q_{ijkl}^L)^2 + (Q_{ijkl}^R)^2 \right) m_b m_{\chi_j^+},$$
\[ a_{ijk4} = -(k_{ij}^b)^2(Q_{i j k}^L)^2 - (k_{ij}^b)^2(Q_{i j k}^R)^2, \]
\[ a_{ijk1} = -2 \left( t_{11}^{ij} \bar{t}_{12}^{ij} Q_{i j k}^L Q_{i j k}^R + k_{11}^{ij} k_{12}^{ij} Q_{i j i i k}^L Q_{i j i i k}^R \right) m_{\chi_{0}^i} m_{\chi_{1}^i} \left( m_{\chi_{0}^i}^2 + m_{\chi_{1}^i}^2 + 2 m_{\tilde{b}}^2 \right) \]
\[ - 4 \left( k_{11}^{ij} k_{11}^{ij} Q_{i j k}^L Q_{i j k}^R + k_{12}^{ij} k_{12}^{ij} Q_{i j i i k}^L Q_{i j i i k}^R \right) m_b m_{\chi_{0}^i} \left( m_{\chi_{0}^i}^2 + m_{\tilde{t}_1}^2 + m_{\tilde{b}}^2 + m_{\tilde{t}_1}^2 \right) \]
\[ - 4 \left( k_{11}^{ij} k_{12}^{ij} Q_{i j k}^L Q_{i j k}^R + k_{11}^{ij} k_{12}^{ij} Q_{i j i i k}^L Q_{i j i i k}^R \right) m_b m_{\chi_{0}^i} m_{\chi_{1}^i} m_{\chi_{2}^i} \]
\[ - 2 \left( k_{11}^{ij} k_{11}^{ij} Q_{i j k}^L Q_{i j k}^R + k_{11}^{ij} k_{12}^{ij} Q_{i j i i k}^L Q_{i j i i k}^R \right) m_{\chi_{1}^i} m_{\tilde{\chi}_{1}^i} \left( m_{\chi_{0}^i}^2 + m_{\tilde{t}_1}^2 + 2 m_{\tilde{t}_1}^2 \right) \]
\[ - 2 \left( k_{11}^{ij} k_{11}^{ij} Q_{i j k}^L Q_{i j k}^R + k_{11}^{ij} k_{12}^{ij} Q_{i j i i k}^L Q_{i j i i k}^R \right) m_{\chi_{2}^i} m_{\tilde{\chi}_{2}^i} \left( m_{\chi_{0}^i}^2 + m_{\tilde{t}_1}^2 + 2 m_{\tilde{t}_1}^2 \right) \]
\[ \times \left[ \left( m_{\tilde{b}}^2 + m_{\tilde{t}_1}^2 + m_{\tilde{t}_1}^2 \right) \left( m_{\chi_{0}^i}^2 + m_{\chi_{1}^i}^2 \right) \right] \]
\[ - 2 \left( k_{11}^{ij} k_{12}^{ij} Q_{i j k}^L Q_{i j k}^R + k_{11}^{ij} k_{12}^{ij} Q_{i j i i k}^L Q_{i j i i k}^R \right) m_{\chi_{1}^i} m_{\tilde{\chi}_{1}^i} \left( m_{\tilde{b}}^2 + m_{\tilde{t}_1}^2 + 2 m_{\tilde{t}_1}^2 \right) \]
\[ - 2 \left( k_{11}^{ij} k_{12}^{ij} Q_{i j k}^L Q_{i j k}^R + k_{11}^{ij} k_{12}^{ij} Q_{i j i i k}^L Q_{i j i i k}^R \right) m_{\chi_{2}^i} m_{\tilde{\chi}_{2}^i} \left( m_{\tilde{b}}^2 + m_{\tilde{t}_1}^2 + 2 m_{\tilde{t}_1}^2 \right) \]
\[ a_{ijk2} = 2 \left( t_{11}^{ij} t_{12}^{ij} Q_{i j k}^L Q_{i j k}^R + k_{11}^{ij} k_{12}^{ij} Q_{i j i i k}^L Q_{i j i i k}^R \right) m_{\chi_{0}^i} m_{\chi_{1}^i} \]
\[ + 4 \left( k_{11}^{ij} t_{12}^{ij} Q_{i j k}^L Q_{i j k}^R + k_{11}^{ij} k_{12}^{ij} Q_{i j i i k}^L Q_{i j i i k}^R \right) m_b m_{\chi_{0}^i} \]
\[ + 2 \left( k_{11}^{ij} k_{11}^{ij} Q_{i j k}^L Q_{i j k}^R + t_{11}^{ij} t_{11}^{ij} Q_{i j i i k}^L Q_{i j i i k}^R \right) m_{\chi_{1}^i} m_{\chi_{2}^i} \]
\[ + 2 \left( t_{11}^{ij} t_{12}^{ij} Q_{i j k}^L Q_{i j k}^R + k_{11}^{ij} k_{12}^{ij} Q_{i j i i k}^L Q_{i j i i k}^R \right) \left( m_{\tilde{b}}^2 + m_{\tilde{t}_1}^2 \right) \]
\[ + 2 \left( k_{11}^{ij} k_{12}^{ij} Q_{i j k}^L Q_{i j k}^R + k_{11}^{ij} k_{12}^{ij} Q_{i j i i k}^L Q_{i j i i k}^R \right) m_b m_{\chi_{1}^i} \]
\[ + 2 \left( k_{12}^{ij} t_{11}^{ij} Q_{i j k}^L Q_{i j k}^R + k_{11}^{ij} k_{12}^{ij} Q_{i j i i k}^L Q_{i j i i k}^R \right) m_{\chi_{2}^i} m_{\tilde{\chi}_{2}^i} \]
\[ + 2 \left( k_{11}^{ij} k_{12}^{ij} Q_{i j k}^L Q_{i j k}^R + t_{11}^{ij} t_{12}^{ij} Q_{i j i i k}^L Q_{i j i i k}^R \right) m_{\tilde{\chi}_{1}^i} m_{\chi_{2}^i} \],
\[ a_{i4k3} = 2 \left( i_{11}^i i_{12}^j Q_{1;ik}^L Q_{1;ik}^R + k_{11}^i k_{12}^j Q_{1;ik}^L Q_{1;ik}^R \right) m_{i1}^0 m_{i1}^0 + 4 \left( k_{11}^i i_{12}^j Q_{1;ik}^L Q_{1;ik}^R + k_{12}^i i_{11}^j Q_{1;ik}^L Q_{1;ik}^R \right) m_{i2}^0 m_{i2}^0 + 2 \left( k_{11}^i k_{12}^j Q_{1;ik}^L Q_{1;ik}^R + i_{11}^i i_{12}^j Q_{1;ik}^L Q_{1;ik}^R \right) m_{i3}^0 m_{i3}^0 + 2 \left( i_{11}^i i_{12}^j Q_{1;ik}^L Q_{1;ik}^R + k_{11}^i k_{12}^j Q_{1;ik}^L Q_{1;ik}^R \right) \left( 2 m_{i1}^0 + 2 m_{i1}^0 + m_{i1}^0 + m_{i1}^0 \right) m_{i3}^0 m_{i3}^0 + 2 \left( k_{11}^i i_{12}^j Q_{1;ik}^L Q_{1;ik}^R + k_{12}^i i_{11}^j Q_{1;ik}^L Q_{1;ik}^R \right) m_{i3}^0 m_{i3}^0 + 2 \left( k_{12}^i i_{11}^j Q_{1;ik}^L Q_{1;ik}^R + k_{11}^i k_{12}^j Q_{1;ik}^L Q_{1;ik}^R \right) m_{i3}^0 m_{i3}^0 \]

The coefficients \( a_{i5kl} \) are obtained from \( a_{i4kl} \) by replacing: \( i_{12}^i \rightarrow i_{13}^i \) \( k_{12}^i \rightarrow k_{13}^i \) \( Q_{1;ik}^L \rightarrow Q_{1;ik}^L \) \( Q_{1;ik}^R \rightarrow Q_{1;ik}^R \) \( m_{i1}^0 \rightarrow m_{i1}^0 \) \( m_{i1}^0 \rightarrow m_{i1}^0 \) and the coefficients \( a_{i6kl} \) are obtained from \( a_{i4kl} \) by replacing: \( a_{i4kl} \rightarrow a_{i6kl} \) \( i_{11}^i \rightarrow i_{11}^i \) \( k_{11}^i \rightarrow k_{11}^i \) \( Q_{1;ik}^L \rightarrow Q_{1;ik}^L \) \( Q_{1;ik}^R \rightarrow Q_{1;ik}^R \) \( m_{i1}^0 \rightarrow m_{i1}^0 \) \( m_{i1}^0 \rightarrow m_{i1}^0 \)

\[ b_{ijkl} = \frac{2\sqrt{2}}{g} \left[ b_{ij}^i k_{ij}^j Q_{ij;kl}^L m_{ij}^0 + b_{ij}^i k_{ij}^j Q_{ij;kl}^L m_{ij}^0 \left( m_{ij}^0 - m_{ij}^0 \right) \frac{R_{kl}^{S^\pm}}{v_2} + \left( m_{ij}^0 + m_{ij}^0 + 2 m_{ij}^0 \right) \frac{R_{kl}^{S^\pm}}{v_2} \right] \]

\[ + a_{ij}^i k_{ij}^j Q_{ij;kl}^L m_{ij}^0 \]

\[ + a_{ij}^i k_{ij}^j Q_{ij;kl}^L m_{ij}^0 \]
\[ b_{ijk} = \frac{2\sqrt{2}}{g} \left\{ b_{i1} k_{i} Q^{L}_{ijk}' m_{\chi_{j}} m_{b} - a_{i1} k_{i} Q^{R}_{ijk}' m_{b} m_{b} + b_{i1} k_{i} Q^{L}_{ijk}' m_{\chi_{j}} m_{b} + a_{i1} k_{i} Q^{R}_{ijk}' m_{b} m_{b} \right\} \]

\[ b_{ijk} = -\frac{2\sqrt{2}}{g} \left\{ b_{i1} k_{i} Q^{L}_{ijk}' m_{\chi_{j}} m_{b} m_{b} m_{b} + b_{i1} k_{i} Q^{L}_{ijk}' m_{\chi_{j}} m_{b} m_{b} - b_{i1} k_{i} Q^{R}_{ijk}' m_{\chi_{j}} m_{b} m_{b} + b_{i1} k_{i} Q^{R}_{ijk}' m_{\chi_{j}} m_{b} m_{b} \right\} \]

\[ b_{ijk} = \frac{2\sqrt{2}}{g} \left( b_{i1} k_{i} Q^{L}_{ijk}' m_{b} m_{b} + a_{i1} k_{i} Q^{R}_{ijk}' m_{b} m_{b} \right) \]

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\[ c_{ijkl1} = 2 C^{S_+}_{i_1 b_1} \left[ \left( b_{i_1 j_1}^i Q_{i_1 j}^{L} + a_{i_1 j_1}^i Q_{i_1 j}^{R} \right) m_b \left( m_b^2 + m_{\chi_1}^2 + 2 m_{\chi_0}^2 \right) \right. \\
+ \left( a_{i_1 j_1}^i Q_{i_1 j}^{L} + b_{i_1 j_1}^i Q_{i_1 j}^{R} \right) m_{\chi_0} \left( m_{\chi_0}^2 + m_{S_k}^2 + 2 m_b^2 \right) \right. \\
+ \left( b_{i_1 j_1}^i Q_{i_1 j}^{L} + a_{i_1 j_1}^i Q_{i_1 j}^{R} \right) m_{\chi_1} \left( m_{\chi_1}^2 + m_{\chi_0}^2 \right) \right. \\
+ \left. \left( b_{i_1 j_1}^i Q_{i_1 j}^{L} + b_{i_1 j_1}^i Q_{i_1 j}^{R} \right) 2 m_b m_{\chi_0} m_{\chi_1}^+ \right], \\
\]

\[ c_{ijkl2} = -2 C^{S_+}_{i_1 b_1} \left[ \left( b_{i_1 j_1}^i Q_{i_1 j}^{L} + a_{i_1 j_1}^i Q_{i_1 j}^{R} \right) m_b + \left( a_{i_1 j_1}^i Q_{i_1 j}^{L} + b_{i_1 j_1}^i Q_{i_1 j}^{R} \right) m_{\chi_0} \right. \\
+ \left( b_{i_1 j_1}^i Q_{i_1 j}^{L} + a_{i_1 j_1}^i Q_{i_1 j}^{R} \right) m_{\chi_1} \right], \\
\]

\[ c_{ijkl3} = -2 C^{S_+}_{i_1 b_1} \left[ \left( b_{i_1 j_1}^i Q_{i_1 j}^{L} + a_{i_1 j_1}^i Q_{i_1 j}^{R} \right) m_b + \left( a_{i_1 j_1}^i Q_{i_1 j}^{L} + b_{i_1 j_1}^i Q_{i_1 j}^{R} \right) m_{\chi_0} \right. \\
+ \left( b_{i_1 j_1}^i Q_{i_1 j}^{L} + a_{i_1 j_1}^i Q_{i_1 j}^{R} \right) m_{\chi_1} \right], \\
\]

\[ d_{ik1} = \frac{2}{g^2} \left\{ - \left[ (a_{1i}^i)^2 m_b^2 \left( \frac{R_{k_2}^{S_+}}{v_2} \right)^2 + (b_{1i}^i)^2 m_b^2 \left( \frac{R_{k_1}^{S_+}}{v_1} \right)^2 \right] m_b^2 \left( m_b^2 + m_{\chi_0}^2 \right) \right. \\
+ \left[ (a_{1i}^i)^2 m_b^2 \left( \frac{R_{k_1}^{S_+}}{v_1} \right)^2 + (b_{1i}^i)^2 m_t^2 \left( \frac{R_{k_2}^{S_+}}{v_1} \right)^2 \right] \left( m_{\chi_0}^2 - m_t^2 \right) \left( m_{S_k}^2 - m_b^2 \right) \right. \\
+ \left. 2 a_{1i}^i b_{1i}^i m_t m_{\chi_0} \left[ m_{S_k}^2 - m_b^2 \right] \left( m_b^2 \left( \frac{R_{k_2}^{S_+}}{v_2} \right)^2 + m_t^2 \left( \frac{R_{k_1}^{S_+}}{v_1} \right)^2 \right) - 2 m_b^2 m_t^2 \right. \\
+ \left. 2 \left( (a_{1i}^i)^2 + (b_{1i}^i)^2 \right) m_b^2 m_t^2 \left( m_t^2 - m_{\chi_0}^2 \right) \right\}, \\
\]

\[ d_{ik2} = \frac{2 m_t^2}{g^2} \left[ (a_{1i}^i)^2 m_t^2 \left( \frac{R_{k_2}^{S_+}}{v_2} \right)^2 + (b_{1i}^i)^2 m_b^2 \left( \frac{R_{k_1}^{S_+}}{v_1} \right)^2 \right], \]
\[ d_{ik3} = \frac{-2}{g^2} \left\{ 2 \left( (a_{i1})^2 + (b_{i1})^2 \right) m_b^2 m_i^2 - \left[ (a_{i1})^2 m_b^2 \left( \frac{R_{k1}^{S\pm}}{v_1} \right)^2 + (b_{i1})^2 m_i^2 \left( \frac{R_{k2}^{S\pm}}{v_2} \right)^2 \right] \right\} , \]

\[ d_{ik4} = -\frac{2}{g^2} \left\{ (a_{i1})^2 m_b^2 \left( \frac{R_{k1}^{S\pm}}{v_1} \right)^2 + (b_{i1})^2 m_i^2 \left( \frac{R_{k2}^{S\pm}}{v_2} \right)^2 \right\} , \]

\[ e_{ik1} = -\frac{2\sqrt{2}}{g} C_{ti1b}^{S\pm} \left\{ a_{i1} b_{i1} m_t m_{\chi_i} \left[ \left( m_b^2 - m_i^2 \right) - \frac{R_{k2}^{S\pm}}{v_2} - 2 m_b^2 \frac{R_{k1}^{S\pm}}{v_1} \right] + b_{ki} a_{i1} m_b m_t \left[ \left( m_t^2 + m_{\chi_i}^2 \right) - \frac{R_{k2}^{S\pm}}{v_2} + m_b^2 \left( m_{\chi_i}^2 - m_i^2 \right) \frac{R_{k1}^{S\pm}}{v_1} \right] \right\} , \]

\[ e_{ik2} = -\frac{2\sqrt{2}}{g} C_{ti1b}^{S\pm} \left( a_{i1} b_{i1} m_t m_{\chi_i} \frac{R_{k2}^{S\pm}}{v_2} + b_{ki} b_{i1} m_b m_t \frac{R_{k1}^{S\pm}}{v_1} \right) , \]

\[ e_{ik3} = \frac{2\sqrt{2}}{g} C_{ti1b}^{S\pm} \left( a_{i1} b_{i1} m_t m_{\chi_i} \frac{R_{k2}^{S\pm}}{v_2} + b_{ki} b_{i1} m_b m_t \frac{R_{k1}^{S\pm}}{v_1} \right) + a_{i1} b_{i1} m_b^2 \frac{R_{k1}^{S\pm}}{v_1} + b_{ki} a_{i1} m_b m_{\chi_i} \frac{R_{k1}^{S\pm}}{v_1} , \]

\[ f_{ik1} = -(C_{ti1b}^{S\pm})^2 \left[ \left( (a_{i1})^2 + (b_{i1})^2 \right) \left( m_b^2 + m_{\chi_i}^2 \right) + 4 a_{i1} b_{ki} m_b m_{\chi_i} \right] , \]

\[ f_{ik2} = (C_{ti1b}^{S\pm})^2 \left( (a_{i1})^2 + (b_{i1})^2 \right) , \]

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\[ f_{ik31} = -2 C^S_{t_i b_1} C^S_{t_i b_2} \left[ \left( a_{1i} b_{2i} + b_{1i} b_{2i} \right) m_b^2 + m_{S_0}^2 \right] + 2 \left( a_{1i} b_{2i} + b_{1i} b_{2i} \right) m_b m_{\tilde{S}_i} \right], \]
\[ f_{ik32} = 2 C^S_{t_i b_1} C^S_{t_i b_2} \left( a_{1i} b_{2i} + b_{1i} b_{2i} \right). \]

3. The width \( \Gamma(\tilde{t}_1 \rightarrow S_k^0 b \tilde{\chi}_i^+) \)

The decay width is given by
\[
\Gamma(\tilde{t}_1 \rightarrow S_k^0 b \tilde{\chi}_i^+) = \frac{\alpha^2}{16 \pi m_{\tilde{t}_1}^3 \sin^4 \theta_W} \int ds \left( G_{\tilde{\chi}_i^+ \tilde{\chi}_i^+} + G_{\tilde{\chi}_i^+ \tilde{\chi}_i^+} + G_{\tilde{\chi}_i^+ \tilde{\chi}_i^+} + G_{\tilde{\chi}_i^+ \tilde{\chi}_i^+} \right)
\]
\[
G^S_{\tilde{\chi}^+ \chi^+} = \sum_{j=1}^{3} \left[ (a_{ijk1} + a_{ijk2}s)J^0_{b_1}(m_{t_1}^2 + m_{s_0}^2 + m_b^2 + m_{\tilde{\chi}_i^+}^2 - m_{\tilde{\chi}_j^+}^2 - s, \Gamma_{\tilde{\chi}_j^+} m_{\tilde{\chi}_i^+}) \\
+ (a_{ijk3} + a_{ijk s})J^1_{b_1}(m_{t_1}^2 + m_{s_0}^2 + m_b^2 + m_{\tilde{\chi}_i^+}^2 - m_{\tilde{\chi}_j^+}^2 - s, \Gamma_{\tilde{\chi}_j^+} m_{\tilde{\chi}_i^+}) \\
+ a_{ijk4} J^2_{b_1}(m_{s_0}^2 + m_{\tilde{\chi}_j^+}^2 - m_{\tilde{\chi}_k^+}^2 - s, \Gamma_{\tilde{\chi}_k^+} m_{\tilde{\chi}_j^+}) \right] \\
+ (a_{ijk1} + a_{ijk2}s)J^0_{b_2}(m_{t_1}^2 + m_{s_0}^2 + m_b^2 + m_{\tilde{\chi}_i^+}^2 - m_{\tilde{\chi}_j^+}^2 - s, \Gamma_{\tilde{\chi}_j^+} m_{\tilde{\chi}_i^+}) \\
+ (a_{ijk3} + a_{ijk s})J^1_{b_2}(m_{t_1}^2 + m_{s_0}^2 + m_b^2 + m_{\tilde{\chi}_i^+}^2 - m_{\tilde{\chi}_j^+}^2 - s, \Gamma_{\tilde{\chi}_j^+} m_{\tilde{\chi}_i^+}) \\
+ a_{ijk4} J^2_{b_2}(m_{s_0}^2 + m_{\tilde{\chi}_j^+}^2 - m_{\tilde{\chi}_k^+}^2 - s, \Gamma_{\tilde{\chi}_k^+} m_{\tilde{\chi}_j^+}) \\
+ (a_{ijk1} + a_{ijk2}s)J^0_{b_3}(m_{t_1}^2 + m_{s_0}^2 + m_b^2 + m_{\tilde{\chi}_i^+}^2 - m_{\tilde{\chi}_j^+}^2 - s, \Gamma_{\tilde{\chi}_j^+} m_{\tilde{\chi}_i^+}) \\
+ (a_{ijk3} + a_{ijk s})J^1_{b_3}(m_{t_1}^2 + m_{s_0}^2 + m_b^2 + m_{\tilde{\chi}_i^+}^2 - m_{\tilde{\chi}_j^+}^2 - s, \Gamma_{\tilde{\chi}_j^+} m_{\tilde{\chi}_i^+}) \\
+ a_{ijk4} J^2_{b_3}(m_{s_0}^2 + m_{\tilde{\chi}_j^+}^2 - m_{\tilde{\chi}_k^+}^2 - s, \Gamma_{\tilde{\chi}_k^+} m_{\tilde{\chi}_j^+}) \right],
\]
\[ G_{\chi+ t}^{S_0} = \sum_{j=1}^{3} \left[(b_{ijk1} + b_{ijk2}) J_{b}^{0}(m_{1}^{2}, m_{1}^{2}) + \frac{m_{1}^{2} + m_{2}^{2} + m_{2}^{2} - m_{3}^{2} - \chi_{+}^{m} - m_{4}^{2} - \chi_{+}^{m} + \delta - \chi_{+}^{m} + m_{5}^{2} \Gamma_{b} m_{b})}{s} \right] + \frac{m_{1}^{2} + m_{2}^{2} + m_{2}^{2} - m_{3}^{2} - \chi_{+}^{m} - m_{4}^{2} - \chi_{+}^{m} + \delta - \chi_{+}^{m} + m_{5}^{2} \Gamma_{b} m_{b})}{s}, \]

\[ G_{\chi+ i}^{S_0} = \sum_{l=1}^{2} \left[(c_{ijkl1} + c_{ijkl2}) s) \right] * J_{st}^{0}(m_{1}^{2}, m_{1}^{2}, m_{2}^{2}, m_{3}^{2}, m_{4}^{2}, m_{5}^{2} - \chi_{+}^{m} + m_{6}^{2} \Gamma_{b} m_{b}) \right] + \frac{m_{1}^{2} + m_{2}^{2} + m_{2}^{2} - m_{3}^{2} - \chi_{+}^{m} - m_{4}^{2} - \chi_{+}^{m} + \delta - \chi_{+}^{m} + m_{5}^{2} \Gamma_{b} m_{b})}{s}, \]

\[ G_{tt}^{S_0} = (d_{ik1} + d_{ik2}) J_{b}^{0}(m_{1}^{2}, m_{1}^{2}, m_{2}^{2}, m_{3}^{2}, m_{4}^{2}, m_{5}^{2} - \chi_{+}^{m} + m_{6}^{2} \Gamma_{b} m_{b}) \right] + \frac{m_{1}^{2} + m_{2}^{2} + m_{2}^{2} - m_{3}^{2} - \chi_{+}^{m} - m_{4}^{2} - \chi_{+}^{m} + \delta - \chi_{+}^{m} + m_{5}^{2} \Gamma_{b} m_{b})}{s}, \]

\[ G_{tt}^{S_0} = \frac{\lambda(s, m_{1}^{2}, m_{2}^{2}) \lambda(s, m_{1}^{2}, m_{2}^{2})}{s} \]

The integrals \( J_{t,tt,sl}^{0,1,2} \) are:

\[ J_{i}^{1}(m_{1}^{2}, m_{1}^{2}, m_{1}^{2}) = \int_{t_{\min}}^{t_{\max}} dt \frac{t^{i}}{(t - m_{1}^{2})^{2} + m_{1}^{2} \Gamma_{1}} ; \]

\[ J_{b}^{0}(m_{1}^{2}, m_{1}^{2}, m_{2}^{2}, m_{2}^{2}, m_{2}^{2}) = \text{Re} \int_{t_{\min}}^{t_{\max}} dt \frac{t^{i}}{(t - m_{1}^{2} + im_{1} \Gamma_{1})(t - m_{2}^{2} - im_{2} \Gamma_{2})} ; \]

\[ J_{sl}^{1}(m_{1}^{2}, m_{1}^{2}, m_{2}^{2}, m_{2}^{2}, m_{2}^{2}) = \text{Re} \int_{t_{\min}}^{t_{\max}} dt \frac{1}{s - m_{1}^{2} + im_{1} \Gamma_{1}} \int_{t_{\min}}^{t_{\max}} dt \frac{t^{i}}{(t - m_{2}^{2} - im_{2} \Gamma_{2})} ; \]
with \( i = 0, 1, 2 \). Their integration range is given by

\[
\begin{split}
t_{\text{max}}^{\text{min}} &= \frac{m_{l_1}^2 + m_{l_2}^2 + m_{s_0}^2}{2} \sqrt{\frac{(m_{l_1}^2 - m_{s_0}^2)(m_{l_2}^2 - m_{s_0}^2)}{2s}} - \frac{(m_{l_1}^2 - m_{s_0}^2)(m_{l_2}^2 - m_{s_0}^2)}{2s} \sqrt{\frac{\lambda(s, m_{l_1}^2, m_{s_0}^2)\lambda(s, m_{l_2}^2, m_{s_0}^2)}{2s}},
\end{split}
\]

where \( s = (p_{l_1} - p_{l_2})^2 \) and \( t = (p_{l_1} - p_{l_3})^2 \) are the usual Mandelstam variables. Note, that \(-\Gamma_{\tilde{\chi}^+_j} m_{\tilde{\chi}^+_j}\) appears in the entries of the integrals \( G_{\tilde{\chi}^+_j}^{s_0} \) and \( G_{\tilde{\chi}^+_j}^{s_0} \), because the chargino is exchanged in the \( u \)-channel in our convention. The coefficients are given by (no sum upon repeated index):

\[
a_{ijk1} = -4k_{ij}^1 t_{ij}^1 l_{ijk}^0 k_{ijk}^0 m_b m_{\chi^+_i} \left( m_b^2 + m_{\chi^+_j}^2 + m_{\chi^+_i}^2 + m_{l_1}^2 + m_{s_0}^2 \right)
- 2t_{ij}^1 k_{ijk}^0 \left( (k_{ij}^1)^2 + (l_{ij}^1)^2 \right) m_{\chi^+_i} m_{\chi^+_j} \left( 2m_b^2 + m_{\chi^+_i}^2 + m_{s_0}^2 \right)
- 2k_{ij}^1 l_{ij}^1 \left( (l_{ijk}^0)^2 + (k_{ijk}^0)^2 \right) m_b m_{\chi^+_j} \left( m_b^2 + 2m_{\chi^+_i}^2 + m_{l_1}^2 \right)
- \left( (k_{ij}^1)^2 (k_{ijk}^0)^2 + (l_{ij}^1)^2 (l_{ijk}^0)^2 \right)
\times \left[ \left( m_b^2 + m_{\chi^+_i}^2 \right)^2 + \left( m_b^2 + m_{s_0}^2 \right)^2 + \left( m_{\chi^+_i}^2 + m_{l_1}^2 \right)^2 \right]
- \left( (k_{ij}^1)^2 (l_{ijk}^0)^2 + (l_{ij}^1)^2 (k_{ijk}^0)^2 \right) m_{\chi^+_i}^2 \left( m_b^2 + m_{\chi^+_i}^2 \right),
\]

\[
a_{ijk2} = 4k_{ij}^1 t_{ij}^1 l_{ijk}^0 k_{ijk}^0 m_b m_{\chi^+_i} + \left( (k_{ij}^1)^2 (k_{ijk}^0)^2 + (l_{ij}^1)^2 (l_{ijk}^0)^2 \right) \left( m_b^2 + m_{\chi^+_i}^2 \right)
+ 2k_{ij}^1 l_{ij}^1 \left( (l_{ijk}^0)^2 + (k_{ijk}^0)^2 \right) m_b m_{\chi^+_j} + 2l_{ijk}^0 k_{ijk}^0 \left( (k_{ij}^1)^2 + (l_{ij}^1)^2 \right) m_{\chi^+_i} m_{\chi^+_j}
+ \left( (k_{ij}^1)^2 (l_{ijk}^0)^2 + (l_{ij}^1)^2 (k_{ijk}^0)^2 \right) m_{\chi^+_i}^2 \left( m_b^2 + m_{\chi^+_i}^2 \right),
\]

\[
a_{ijk3} = 4k_{ij}^1 t_{ij}^1 l_{ijk}^0 k_{ijk}^0 m_b m_{\chi^+_i} + 2l_{ijk}^0 k_{ijk}^0 \left( (k_{ij}^1)^2 + (l_{ij}^1)^2 \right) m_{\chi^+_i} m_{\chi^+_j}
+ \left( (k_{ij}^1)^2 (k_{ijk}^0)^2 + (l_{ij}^1)^2 (l_{ijk}^0)^2 \right) \left( 2m_b^2 + 2m_{\chi^+_i}^2 + m_{s_0}^2 + m_{l_1}^2 \right)
+ 2k_{ij}^1 l_{ij}^1 \left( (l_{ijk}^0)^2 + (k_{ijk}^0)^2 \right) m_b m_{\chi^+_j},
\]

\[
a_{ijk4} = -(k_{ij}^1)^2 (k_{ijk}^0)^2 - (l_{ij}^1)^2 (l_{ijk}^0)^2,
\]

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\[ a_{i4k1} = -2 \left( l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i + l_{t1}^i k_{t2}^i k_{t1}^i k_{t2}^i \right) m_{\chi_1^+} m_{\chi_1^+} \left( m_{\chi_1^+}^2 + m_{\chi_1^+}^2 + 2 m_b^2 \right) \\
-4 \left( l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i + l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i \right) m_b m_{\chi_1^+} \left( m_{\chi_1^+}^2 + m_{\chi_1^+}^2 + m_b^2 + m_{t_1}^2 \right) \\
-4 \left( l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i + l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i \right) m_b m_{\chi_1^+} m_{\chi_1^+} m_{\chi_2^+} \\
-2 \left( l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i + l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i \right) m_{\chi_1^+} m_{\chi_2^+} \left( m_{\chi_1^+}^2 + m_{\chi_1^+}^2 + 2 m_b^2 \right) \\
-2 \left( l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i + l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i \right) \times \left[ \left( m_b^2 + m_{\chi_1^+}^2 \right)^2 + \left( m_{\chi_1^+}^2 + m_{\chi_1^+}^2 \right) \left( m_b^2 + m_{\chi_1^+}^2 \right) \right] \\
-2 \left( l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i + l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i \right) m_b m_{\chi_1^+} \left( m_{\chi_1^+}^2 + m_{\chi_1^+}^2 + 2 m_b^2 \right) \\
-2 \left( l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i + l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i \right) m_b m_{\chi_1^+} \left( m_{\chi_1^+}^2 + m_{\chi_1^+}^2 + 2 m_b^2 \right) \\
-2 \left( l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i + l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i \right) m_{\chi_1^+} m_{\chi_2^+} \left( m_{\chi_1^+}^2 + m_{\chi_1^+}^2 \right) , \]

\[ a_{i4k2} = 2 \left( l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i + l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i \right) m_{\chi_1^+} m_{\chi_1^+} + 4 \left( l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i + l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i \right) m_b m_{\chi_1^+} + 2 \left( l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i + l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i \right) m_{\chi_1^+} m_{\chi_2^+} + 2 \left( l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i + l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i \right) m_{\chi_1^+} m_{\chi_2^+} + 2 \left( l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i + l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i \right) m_{\chi_1^+} m_{\chi_2^+} , \]

\[ a_{i4k3} = 2 \left( l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i + l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i \right) m_{\chi_1^+} m_{\chi_1^+} + 4 \left( l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i + l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i \right) m_b m_{\chi_1^+} + 2 \left( l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i + l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i \right) m_{\chi_1^+} m_{\chi_2^+} + 2 \left( l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i + l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i \right) m_{\chi_1^+} m_{\chi_2^+} + 2 \left( l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i + l_{t1}^i l_{t2}^i k_{t1}^i k_{t2}^i \right) m_{\chi_1^+} m_{\chi_2^+} , \]

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\[ a_{i4k4} = -2 \left( \tilde{t}_{11}^i t_{12} k_{11k} k_{12k} + \tilde{t}_{11}^i t_{12k} k_{11k}^i \right), \]

The coefficients \( a_{i5kl} \) are obtained from \( a_{i4kl} \) by replacing: \( \tilde{l}_{12} \rightarrow \tilde{l}_{13} \) \( \tilde{k}_{12} \rightarrow \tilde{k}_{13} \) \( k_{ij2k} \rightarrow k_{ij3k} \) \( l_{i2k} \rightarrow l_{i3k} \) \( m_{\chi_2^+} \rightarrow m_{\chi_3^+} \) and the coefficients \( a_{i6kl} \) are obtained from \( a_{i4kl} \) by replacing: \( a_{i4kl} \rightarrow a_{i6kl} \):

\[ b_{ijk1} = 2 \left\{ k_{i1} k_{1j} l_{ijk}^{S_0} m_{\chi_1^+} m_{\chi_1^+} \left[ \left( m_{t_1}^2 - m_{\chi_1^+}^2 \right) l_{b_0}^{S_0} - \left( m_{b}^2 + m_{\chi_1^+}^2 \right) k_{b_0}^{S_0} \right] ight. \]

\[ + k_{i1} k_{1j} l_{ijk}^{S_0} m_{b} \left( m_{t_1}^2 - m_{\chi_1^+}^2 \right) l_{b_0}^{S_0} - m_{b} \left( m_{b}^2 + m_{t_1}^2 + 2 m_{\chi_1^+}^2 \right) k_{b_0}^{S_0} \]

\[ + k_{i1} k_{1j} l_{ijk}^{S_0} m_{\chi_1^+} \left[ \left( m_{t_1}^2 + m_{\chi_1^+}^2 \right) l_{b_0}^{S_0} - \left( m_{b}^2 + 2 m_{t_1}^2 + m_{\chi_1^+}^2 \right) k_{b_0}^{S_0} \right] \]

\[ - l_{i1} k_{1j} k_{ijk}^{S_0} m_{b} \left( m_{t_1}^2 - m_{\chi_1^+}^2 \right) l_{b_0}^{S_0} - m_{b} \left( m_{b}^2 + m_{t_1}^2 + 2 m_{\chi_1^+}^2 \right) k_{b_0}^{S_0} \right\}, \]

\[ b_{ijk2} = 2 \left\{ k_{i1} k_{1j} l_{ijk}^{S_0} m_{\chi_1^+} m_{\chi_1^+} + k_{i1} k_{1j} l_{ijk}^{S_0} m_{b} k_{b_0}^{S_0} \right. \]

\[ + l_{i1} k_{1j} k_{ijk}^{S_0} m_{\chi_1^+} l_{b_0}^{S_0} + \left. k_{i1} k_{1j} k_{ijk}^{S_0} m_{\chi_1^+} m_{\chi_1^+} \right\}, \]

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\[ b_{ijk} = -2 \left\{ k_{i} k_{j} k_{l} l^{S_{0}} L_{ijk} m_{\bar{\chi}_{i}} m_{\bar{\chi}_{j}} m_{\bar{\chi}_{l}} - l_{l} l_{i} l_{j} l^{S_{0}} L_{ijk} m_{\bar{\chi}_{i}} m_{\bar{\chi}_{j}} m_{\bar{\chi}_{l}} l^{S_{0}} \right\} \\
+ k_{i} k_{l} l^{S_{0}} L_{ijk} m_{b} \left[ \left( m_{\bar{\chi}_{i}}^{2} + m_{\bar{\chi}_{l}}^{2} + m_{\bar{\chi}_{j}}^{2} \right) l^{S_{0}} - m_{\bar{\chi}_{i}}^{2} k^{S_{0}} \right] \\
+ k_{i} l^{S_{0}} L_{ijk} m_{\bar{\chi}_{l}}^{2} m_{\bar{\chi}_{l}}^{2} m_{\bar{\chi}_{j}}^{2} m_{\bar{\chi}_{l}}^{2} k^{S_{0}} \\
+ l^{S_{0}} L_{ijk} m_{\bar{\chi}_{i}}^{2} m_{\bar{\chi}_{j}}^{2} m_{\bar{\chi}_{l}}^{2} l^{S_{0}} + l^{S_{0}} L_{ijk} m_{\bar{\chi}_{i}}^{2} m_{\bar{\chi}_{j}}^{2} m_{\bar{\chi}_{l}}^{2} k^{S_{0}} \\
+ l^{S_{0}} L_{ijk} m_{b} \left[ \left( m_{\bar{\chi}_{i}}^{2} + m_{\bar{\chi}_{l}}^{2} + m_{\bar{\chi}_{j}}^{2} \right) k^{S_{0}} - m_{\bar{\chi}_{i}}^{2} l^{S_{0}} \right] \right\} , \\
\]

\[ b_{ijk} = 2 \left( k_{i} k_{j} k_{l} l^{S_{0}} L_{ijk} m_{b} l^{S_{0}} + l_{l} l_{i} l_{j} l^{S_{0}} L_{ijk} m_{b} k^{S_{0}} \right) , \]

\[ c_{ijkl} = 2 C_{i}^{S_{0}} \left[ \left( k_{i} i_{j} k_{l} l^{S_{0}} L_{ijk} + a_{i} i_{j} l^{S_{0}} L_{ijk} \right) m_{b} \left( m_{\bar{\chi}_{i}}^{2} + m_{\bar{\chi}_{l}}^{2} + 2 m_{\bar{\chi}_{l}}^{2} \right) \\
+ \left( l_{l} i_{j} l^{S_{0}} L_{ijk} + k_{l} i_{j} l^{S_{0}} L_{ijk} \right) m_{\bar{\chi}_{l}}^{2} \left( m_{\bar{\chi}_{i}}^{2} + m_{\bar{\chi}_{l}}^{2} + 2 m_{\bar{\chi}_{l}}^{2} \right) \\
+ \left( k_{l} i_{j} k_{l} l^{S_{0}} L_{ijk} + l_{l} i_{j} k_{l} l^{S_{0}} L_{ijk} \right) m_{\bar{\chi}_{l}}^{2} \left( m_{\bar{\chi}_{i}}^{2} + m_{\bar{\chi}_{l}}^{2} + 2 m_{\bar{\chi}_{l}}^{2} \right) \\
+ \left( l_{l} i_{j} l^{S_{0}} L_{ijk} + k_{l} i_{j} l^{S_{0}} L_{ijk} \right) 2 m_{b} k_{\bar{\chi}_{l}}^{2} m_{\bar{\chi}_{l}}^{2} \right] , \]

\[ c_{ijkl} = -2 C_{i}^{S_{0}} \left[ \left( k_{i} i_{j} k_{l} l^{S_{0}} L_{ijk} + k_{l} i_{j} l^{S_{0}} L_{ijk} \right) m_{b} \left( m_{\bar{\chi}_{i}}^{2} + m_{\bar{\chi}_{l}}^{2} + 2 m_{\bar{\chi}_{l}}^{2} \right) \\
+ \left( k_{l} i_{j} k_{l} l^{S_{0}} L_{ijk} + l_{l} i_{j} k_{l} l^{S_{0}} L_{ijk} \right) m_{\bar{\chi}_{l}}^{2} \left( m_{\bar{\chi}_{i}}^{2} + m_{\bar{\chi}_{l}}^{2} + 2 m_{\bar{\chi}_{l}}^{2} \right) \\
+ \left( k_{l} i_{j} l^{S_{0}} L_{ijk} + l_{l} i_{j} k^{S_{0}} \right) m_{\bar{\chi}_{l}}^{2} \left( m_{\bar{\chi}_{i}}^{2} + m_{\bar{\chi}_{l}}^{2} + 2 m_{\bar{\chi}_{l}}^{2} \right) \\
+ \left( l_{l} i_{j} l^{S_{0}} L_{ijk} + k_{l} i_{j} l^{S_{0}} L_{ijk} \right) 2 m_{b} m_{\bar{\chi}_{l}}^{2} m_{\bar{\chi}_{l}}^{2} \right] , \]

\[ c_{ijkl} = -2 C_{k}^{S_{0}} \left[ \left( k_{i} i_{j} k_{l} l^{S_{0}} L_{ijk} + k_{l} i_{j} l^{S_{0}} L_{ijk} \right) m_{b} \left( m_{\bar{\chi}_{i}}^{2} + m_{\bar{\chi}_{l}}^{2} + 2 m_{\bar{\chi}_{l}}^{2} \right) \\
+ \left( k_{l} i_{j} k_{l} l^{S_{0}} L_{ijk} + l_{l} i_{j} k^{S_{0}} \right) m_{\bar{\chi}_{l}}^{2} \left( m_{\bar{\chi}_{i}}^{2} + m_{\bar{\chi}_{l}}^{2} + 2 m_{\bar{\chi}_{l}}^{2} \right) \\
+ \left( k_{l} i_{j} l^{S_{0}} L_{ijk} + l_{l} i_{j} k^{S_{0}} \right) m_{\bar{\chi}_{l}}^{2} \left( m_{\bar{\chi}_{i}}^{2} + m_{\bar{\chi}_{l}}^{2} + 2 m_{\bar{\chi}_{l}}^{2} \right) \\
+ \left( k_{l} i_{j} l^{S_{0}} L_{ijk} + k_{l} i_{j} l^{S_{0}} L_{ijk} \right) 2 m_{b} m_{\bar{\chi}_{l}}^{2} m_{\bar{\chi}_{l}}^{2} \right] , \]
\[
d_{ik1} = \left\{ - \left[ (l_{1i}^{\tilde{t}})^2 m_b^2 \left( t_{bb}^{S_0} \right)^2 + (k_{1i}^{\tilde{t}})^2 m_b^2 \left( k_{bb}^{S_0} \right)^2 \right] m_b^2 \left( m_b^2 + m_{\chi_i^+}^2 \right) \\
+ \left[ (l_{1i}^{\tilde{t}})^2 m_b^2 \left( k_{bb}^{S_0} \right)^2 + (k_{1i}^{\tilde{t}})^2 m_b^2 \left( t_{bb}^{S_0} \right)^2 \right] \left( m_{\chi_i^+}^2 - m_{t_i}^2 \right) \left( m_{S_k}^2 - m_b^2 \right) \\
+ 2 l_{1i}^{\tilde{t}} k_{1i}^{\tilde{t}} m_b m_{\chi_i^+} \left[ (m_{S_k}^2 - m_b^2) \left( m_b^2 \left( k_{bb}^{S_0} \right)^2 + m_b^2 \left( t_{bb}^{S_0} \right)^2 \right) - 2 m_b^2 m_b^2 \right] \\
+ 2 \left( (l_{1i}^{\tilde{t}})^2 + (k_{1i}^{\tilde{t}})^2 \right) m_b^2 m_b^2 \left( m_{t_i}^2 - m_{\chi_i^+}^2 \right) \right\}, \\
\]

\[
d_{ik2} = m_b^2 \left[ (l_{1i}^{\tilde{t}})^2 m_b^2 \left( t_{bb}^{S_0} \right)^2 + (k_{1i}^{\tilde{t}})^2 m_b^2 \left( k_{bb}^{S_0} \right)^2 \right], \\
d_{ik3} = - \left\{ 2 \left( (l_{1i}^{\tilde{t}})^2 + (k_{1i}^{\tilde{t}})^2 \right) m_b^2 m_b^2 + 2 l_{1i}^{\tilde{t}} k_{1i}^{\tilde{t}} m_b m_{\chi_i^+} \left[ m_b^2 \left( 2 + \left( k_{bb}^{S_0} \right)^2 \right) + m_b^2 \left( t_{bb}^{S_0} \right)^2 \right] \\
- \left[ (l_{1i}^{\tilde{t}})^2 m_b^2 \left( k_{bb}^{S_0} \right)^2 + (k_{1i}^{\tilde{t}})^2 m_b^2 \left( t_{bb}^{S_0} \right)^2 \right] \left( m_{S_k}^2 + m_{t_i}^2 \right) \right\}, \\
d_{ik4} = - \left\{ (l_{1i}^{\tilde{t}})^2 m_b^2 \left( k_{bb}^{S_0} \right)^2 + (k_{1i}^{\tilde{t}})^2 m_b^2 \left( t_{bb}^{S_0} \right)^2 \right\}, \\
\]

\[
e_{ik1} = -2 C^{S_0}_{t_{1i}t_i} \left\{ l_{1i}^{\tilde{t}} k_{1i}^{\tilde{t}} m_b m_{\chi_i^+} \left[ \left( m_{S_k}^2 - m_b^2 \right) l_{bb}^{S_0} - 2 m_b^2 k_{bb}^{S_0} \right] \\
+ k_{1i}^{\tilde{t}} k_{1i}^{\tilde{t}} m_b^2 \left[ \left( m_{t_i}^2 - m_{\chi_i^+}^2 \right) l_{bb}^{S_0} - \left( m_b^2 + m_{\chi_i^+}^2 \right) k_{bb}^{S_0} \right] \\
- \tilde{t}_{1i}^{\tilde{t}} l_{1i}^{\tilde{t}} \left[ m_b^2 \left( m_b^2 + m_{\chi_i^+}^2 \right) l_{bb}^{S_0} + m_b \left( m_{\chi_i^+}^2 - m_{t_i}^2 \right) k_{bb}^{S_0} \right] \\
- k_{1i}^{\tilde{t}} l_{1i}^{\tilde{t}} m_b m_{\chi_i^+} \left[ 2 m_b^2 l_{bb}^{S_0} + \left( m_b^2 - m_{S_k}^2 \right) k_{bb}^{S_0} \right] \right\}, \\
\]

\[
e_{ik2} = -2 C^{S_0}_{t_{1i}t_i} \left( l_{1i}^{\tilde{t}} l_{1i}^{\tilde{t}} m_b^2 l_{bb}^{S_0} + k_{1i}^{\tilde{t}} k_{1i}^{\tilde{t}} m_b^2 k_{bb}^{S_0} \right), \\
e_{ik3} = 2 C^{S_0}_{t_{1i}t_i} \left( l_{1i}^{\tilde{t}} k_{1i}^{\tilde{t}} m_b m_{\chi_i^+} l_{bb}^{S_0} + k_{1i}^{\tilde{t}} l_{1i}^{\tilde{t}} m_b^2 l_{bb}^{S_0} \\
+ \tilde{t}_{1i}^{\tilde{t}} l_{1i}^{\tilde{t}} m_b^2 k_{bb}^{S_0} + k_{1i}^{\tilde{t}} l_{1i}^{\tilde{t}} m_b m_{\chi_i^+} k_{bb}^{S_0} \right), \\
\]

45
\[
\begin{align*}
    f_{ikl1} &= -(C^{S_0}_{i t_1 t_2})^2 \left[ \left( \left( l_{i t_1}^f \right)^2 + \left( k_{k t_2}^f \right)^2 \right) \left( m_b^2 + m_{\chi_i^+}^2 \right) + 4 l_{i t_1}^f k_{k t_2}^f m_b m_{\chi_i^+} \right], \\
    f_{ikl2} &= (C^{S_0}_{i t_1 t_2})^2 \left( \left( l_{i t_1}^f \right)^2 + \left( k_{k t_2}^f \right)^2 \right), \\
    f_{ikl31} &= -2 C^{S_0}_{i t_1 t_2} \left[ \left( \left( l_{i t_1}^f \right)^2 + k_{k t_2}^f \right) \left( m_b^2 + m_{\chi_i}^2 \right) + 2 \left( l_{i t_1}^f k_{k t_2}^f \right) m_b m_{\chi_i^+} \right], \\
    f_{ikl32} &= 2 C^{S_0}_{i t_1 t_2} \left( l_{i t_1}^f k_{k t_2}^f \right)
\end{align*}
\]

APPENDIX B: COUPLINGS

Here we give the couplings that were used in sec. [A].

The \( \tilde{q}_i q' - \tilde{\chi}_j^\pm \) couplings read then

\[
\begin{align*}
    l_{ij}^q &= R_{in}^{i} \mathcal{O}_{jn}^{q}, \quad k_{ij}^{q'} &= R_{i1}^{q} \mathcal{O}_{j2}^{q'}
\end{align*}
\]

with

\[
\begin{align*}
    \mathcal{O}_j^q &= \begin{pmatrix}
        -V_{j1} \\
        h_t V_{j2}
    \end{pmatrix}, \quad \mathcal{O}_j^{q'} = \begin{pmatrix}
        -U_{j1} \\
        h_u U_{j2}
    \end{pmatrix}
\end{align*}
\]

where \( U_{ij} \) and \( V_{ij} \) are the mixing matrices of the charginos [33].

For the couplings \( \tilde{\chi}_j^\pm - S^0/P^0 - \tilde{\chi}_j^\pm \) we have

\[
\begin{align*}
    l_{ijk}^{S_0} &= \left[ -R_{k3}^{S_0} V_{j3} U_{i3} - R_{k2}^{S_0} V_{j2} U_{i1} - R_{k1}^{S_0} V_{j1} U_{i2} + \hat{h}_t \left( R_{k3}^{S_0} V_{j3} U_{i2} - R_{k1}^{S_0} V_{j1} U_{i3} \right) \right] \\
    k_{ijk}^{S_0} &= \left[ -R_{k3}^{S_0} U_{j3} V_{i1} - R_{k2}^{S_0} U_{j2} V_{i1} - R_{k1}^{S_0} U_{j1} V_{i2} + \hat{h}_t \left( R_{k3}^{S_0} U_{j3} V_{i2} - R_{k1}^{S_0} U_{j1} V_{i3} \right) \right] \\
    l_{ijk}^{P_0} &= -i \left[ -R_{k3}^{P_0} V_{j3} U_{i3} - R_{k2}^{P_0} V_{j2} U_{i1} - R_{k1}^{P_0} V_{j1} U_{i2} - \hat{h}_t \left( R_{k3}^{P_0} V_{j3} U_{i2} - R_{k1}^{P_0} V_{j1} U_{i3} \right) \right] \\
    k_{ijk}^{P_0} &= -i \left[ -R_{k3}^{P_0} U_{j3} V_{i1} - R_{k2}^{P_0} U_{j2} V_{i1} - R_{k1}^{P_0} U_{j1} V_{i2} - \hat{h}_t \left( R_{k3}^{P_0} U_{j3} V_{i2} - R_{k1}^{P_0} U_{j1} V_{i3} \right) \right]
\end{align*}
\]

The \( \tilde{q}_i q - \tilde{\chi}_j^0 \) couplings are given by

\[
\begin{align*}
    a_{ik}^{q} &= R_{in}^{i} \mathcal{A}_{kn}^{f}, \quad b_{ik}^{q} &= R_{in}^{i} \mathcal{B}_{kn}^{f}
\end{align*}
\]
\[ A_k^f = \begin{pmatrix} f_{Lk}^f \\ f_{Rk}^f \end{pmatrix}, \quad B_k^f = \begin{pmatrix} h_{Lk}^f \\ f_{Rk}^f \end{pmatrix}, \]

and

\[
\begin{align*}
   h_{Lk}^t &= -h_t N_{k4} \\
   f_{Lk}^t &= -\frac{\sqrt{2}}{2} \left[ N_{k2} + \frac{1}{3} \frac{g'}{g} N_{k1} \right] \\
   h_{Rk}^t &= -h_t N_{k4} \\
   f_{Rk}^t &= -\frac{2\sqrt{2}}{3} N_{k1} \\
   h_{Lk}^b &= -h_b N_{k3} \\
   f_{Lk}^b &= \frac{\sqrt{2}}{2} \left( N_{k2} - \frac{1}{3} \frac{g'}{g} N_{k2} \right) \\
   h_{Rk}^b &= -h_b N_{k4} \\
   f_{Rk}^b &= \frac{\sqrt{2}}{3} N_{k1}
\end{align*}
\]

where \( N_{ij} \) is the mixing matrix of the neutralinos. The couplings \( \tilde{t}_i \tilde{b}_j W^+ \) read

\[
A_{\tilde{t}_i \tilde{b}_j}^W = \left( A_{\tilde{b}_i \tilde{t}_j}^W \right)^T = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos \theta_b \cos \theta_t & -\sin \theta_b \cos \theta_t \\ -\cos \theta_b \sin \theta_t & \sin \theta_b \sin \theta_t \end{pmatrix}.
\]

The couplings \( \tilde{t}_i \tilde{b}_j S_k^+ \) are given by

\[
C_{\tilde{t}_i \tilde{b}_j}^{S_k^+} = \left( C_{\tilde{b}_i \tilde{t}_j}^{S_k^+} \right)^T = \frac{1}{\sqrt{2}} R^\dagger \begin{pmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} \end{pmatrix} \left( R^b \right)^\dagger.
\]

where

\[
\begin{align*}
   \mathcal{A}_{11} &= v_1 h_b^2 R_{k1}^{S_k^+} + v_2 h_t^2 R_{k2}^{S_k^+} - \frac{1}{2} g^2 \sum_{j=1}^3 v_j R_{kj}^{S_k^+} \\
   \mathcal{A}_{12} &= \sqrt{2} h_b \left( A_b R_{k1}^{S_k^+} + \mu R_{k2}^{S_k^+} - v_3 h_t R_{k1}^{S_k^+} \right) \\
   \mathcal{A}_{21} &= \sqrt{2} h_t \left( A_t R_{k2}^{S_k^+} + \mu R_{k1}^{S_k^+} - \epsilon_3 R_{k2}^{S_k^+} \right) \\
   \mathcal{A}_{22} &= h_b h_t \left( v_2 R_{k1}^{S_k^+} + v_1 R_{k2}^{S_k^+} \right)
\end{align*}
\]
The couplings $\bar{t}bS^+$ are given by

$$C^S_{tb} = \tilde{t}(h_t R^{S^+}_{R2} P_L + h_t R^{S^+}_{R1} P_R)$$

The $W^+\tilde{\chi}_j^0 - \tilde{\chi}_j^0$ couplings read: The $W^+\tilde{\chi}_j^0 - \tilde{\chi}_j^0$ couplings read:

$$O_{L}^{kj} = -\frac{V_{j2}}{\sqrt{2}} N_{k4} + V_{j1} N_{k2}$$
$$O_{R}^{kj} = \frac{U_{j2}}{\sqrt{2}} N_{k3} + U_{j1} N_{j2}$$

The $S^+\tilde{\chi}_j^0 - \tilde{\chi}_j^0$ couplings are given by:

$$Q_{L}^{ijk} = -g \left[ R^{S^+}_{R2} \left( V_{j1} N_{j4} + V_{j2} \frac{g'}{g} N_{i1} + N_{i2} \right) + \hat{h}_\tau \left( R^{S^+}_{R1} V_{j3} N_{i5} + R^{S^+}_{R3} V_{j3} N_{i3} \right) \right]$$

$$+ \sqrt{2} N_{i1} V_{j3} R^{S^+}_{R4}$$

$$Q_{R}^{ikj} = -g \left\{ R^{S^+}_{R1} \left[ U_{j1} N_{i3} - \frac{U_{j2}}{\sqrt{2}} \left( \frac{g'}{g} N_{i1} + N_{i2} \right) \right] - \hat{h}_\tau \left( R^{S^+}_{R3} U_{j2} N_{i5} + R^{S^+}_{R3} U_{j3} N_{i3} \right) \right\}$$

$$- \frac{1}{\sqrt{2}} R^{S^+}_{R3} U_{j3} \left( \frac{g'}{g} N_{i1} + N_{i2} \right)$$

[1] H. P. Nilles, “Supersymmetry, Supergravity And Particle Physics,” Phys. Rept. 110 (1984) 1.
[2] H. E. Haber and G. L. Kane, Phys. Rept. 117 (1985) 75.
[3] Proc. of the Workshop on Physics at LEP2, CERN 96-01, Vol. I, edited by G. Altarelli, T. Sjöstrand, and F. Zwirner; J. Amundson et al., Proceedings of the 1996 DPF/DPB Summer Study on High-Energy Physics, Snowmass, Colorado, 1996, edited by D.G. Cassel, L. Trindle Gennari, R.H. Siemann, p. 655; A. Bartl et al., ibid., p. 693; S. Mrenna et al., ibid., p. 681.
[4] M. Carena, R. L. Culbertson, S. Eno, H. J. Frisch and S. Mrenna, hep-ex/9802006.
[5] E. Accomando et al. [ECFA/DESY LC Physics Working Group Collaboration], Phys. Rept. 299 (1998) 1 [hep-ph/9705442].

[6] B. Allanach et al., “Searching for R-parity violation at Run-II of the Tevatron,” [hep-ph/9906224].

[7] X. Tata, [hep-ph/9510287]. Talk given at Theoretical Advanced Study Institute in Elementary Particle Physics (TASI 95): QCD and Beyond, Boulder, CO. In ”Boulder 1995, QCD and beyond” 163-219.

[8] For a recent review see J. W. F. Valle, “Super-gravity unification with bilinear R-parity violation,” [hep-ph/9808292]. Proceedings of PASCOS98, ed. P. Nath, W. Scientific; J. W. F. Valle, “Physics beyond the standard model,” [hep-ph/9603307], lectures given at the VIII Jorge Andre Swieca Summer School (Rio de Janeiro, February 1995) and at V Taller Latinoamericano de Fenomenología de las Interacciones Fundamentales (Puebla, Mexico, October 1995)

[9] C. S. Aulakh and R. N. Mohapatra, Phys. Lett. B119 (1982) 136.

[10] L. J. Hall and M. Suzuki, Nucl. Phys. B231 (1984) 419.

[11] G. G. Ross and J. W. F. Valle, Phys. Lett. B151 (1985) 375; J. Ellis, G. Gelmini, C. Jarlskog, G. G.Ross and J. W. F. Valle, Phys. Lett. B150 (1985) 142.

[12] A. Santamaria and J. W. F. Valle, Phys. Lett. B195 (1987) 423; Phys. Rev. D39 (1989) 1780; Phys. Rev. Lett. 60 (1988) 397.

[13] J. C. Romao, M. A. Diaz, M. Hirsch, W. Porod and J. W. F. Valle, Phys. Rev. D61 (2000) 071703 [hep-ph/9907499]; M. Hirsch, M. A. Diaz, W. Porod, J. C. Romao and J. W. F. Valle, Phys. Rev. D 62 (2000) 113008 [hep-ph/0004115] and references therein.

[14] M. C. Gonzalez-Garcia, M. Maltoni, C. Pena-Garay and J. W. F. Valle, Phys. Rev. D 63 (2001) 033005 [hep-ph/0009350] and references therein.

[15] S. Dimopoulos and L. J. Hall, Phys. Lett. B207 (1988) 210; E. Ma and D. Ng, Phys. Rev. D41 (1990) 1005; V. Barger, G. F. Giudice and T. Han, Phys. Rev. D40 (1989) 2987; T. Banks, Y. Grossman, E. Nardi and Y. Nir, Phys. Rev. D52 (1995) 5319 [hep-ph/9505248]; M. Nowakowski and A. Pilaftsis, Nucl. Phys. B461 (1996) 19 [hep-
for recent analyses of spontaneous R-parity violation at LEP II see P. Abreu et al. [DELPHI Collaboration], Phys. Lett. B 502 (2001) 24. and F. de Campos, O. J. Eboli, M. A. Garcia-Jareno and J. W. F. Valle, Nucl. Phys. B 546 (1999) 33 [hep-ph/9710545].

[17] A. Masiero and J. W. F. Valle, Phys. Lett. B251 (1990) 273; J. C. Romao, C. A. Santos and J. W. F. Valle, Phys. Lett. B288 (1992) 311; J. C. Romao, A. Ioannisian and J. W. F. Valle, Phys. Rev. D55 (1997) 427 [hep-ph/9607401].

[18] G. F. Giudice, A. Masiero, M. Pietroni and A. Riotto, Nucl. Phys. B396 (1993) 243 [hep-ph/9209296]; M. Shiraishi, I. Umemura and K. Yamamoto, Phys. Lett. B313 (1993) 89; I. Umemura and K. Yamamoto, Nucl. Phys. B423 (1994) 405.

[19] P. Nogueira, J. C. Romao and J. W. F. Valle, Phys. Lett. B 251 (1990) 142. R. Barbieri, D. E. Brahm, L. J. Hall and S. D. Hsu, Phys. Lett. B 238 (1990) 86. J. Romão, J. Rosiek and J. W. F. F. Valle, Phys. Lett. B 351 (1995) 497 [hep-ph/9502211].

[20] J. W. F. Valle, Phys. Lett. B 196 (1987) 157. M. C. Gonzalez-Garcia and J. W. F. Valle, Nucl. Phys. B 355 (1991) 330. K. Huitu and J. Maalampi, Phys. Lett. B 344 (1995) 217 [hep-ph/9410342].

[21] J. M. Mira, E. Nardi, D. A. Restrepo and J. W. F. Valle, Phys. Lett. B 492 (2000) 81 [hep-ph/0007266].

[22] A. Bartl, W. Majerotto and W. Porod, Z. Phys. C 64 (1994) 499; erratum ibid. C 68, 518 (1995);

[23] A. Bartl, H. Eberl, S. Kraml, W. Majerotto and W. Porod, Z. Phys. C 73 (1997) 469 [hep-ph/9603410]. A. Bartl, H. Eberl, S. Kraml, W. Majerotto, W. Porod and A. Sopczak, Z. Phys. C 76 (1997) 549 [hep-ph/9701336].

[24] J. Ellis and S. Rudaz, Phys. Lett. B 128 (1983) 248; G. Altarelli and R. Ruckl, Phys. Lett. B 144 (1984) 126; I. I. Bigi and S. Rudaz, Phys. Lett. B 153 (1985) 335.

[25] K. Hikasa and M. Kobayashi, Phys. Rev. D 36 (1987) 724.
[26] W. Porod and T. Wohrmann, Phys. Rev. D55 (1997) 2907 [hep-ph/9608472].

[27] W. Porod, Phys. Rev. D59 (1999) 095009 [hep-ph/9812230].

[28] C. Boehm, A. Djouadi and Y. Mambrini, Phys. Rev. D 61 (2000) 095006 [hep-ph/9907428].

[29] A. Bartl, H. Eberl, S. Kraml, W. Majerotto and W. Porod, [hep-ph/0002115].

[30] A. Djouadi and Y. Mambrini, [hep-ph/0011364].

[31] A. Bartl, W. Porod, M. A. Garcia-Jareno, M. B. Magro, J. W. F. Valle and W. Majerotto, Phys. Lett. B 384 (1996) 151 [hep-ph/9606256].

[32] M. A. Diaz, D. A. Restrepo and J. W. F. Valle, Nucl. Phys. B 583 (2000) 182 [hep-ph/9908286].

[33] A. Datta and B. Mukhopadhyaya, Phys. Rev. Lett. 85 (2000) 248, [hep-ph/0003174].

[34] F. de Campos, M. A. Garcia-Jareno, A. S. Joshipura, J. Rosiek and J. W. F. Valle, Nucl. Phys. B 451 (1995) 3 [hep-ph/9502237].

[35] A. Akeroyd, M. A. Diaz, J. Ferrandis, M. A. Garcia-Jareno and J. W. F. Valle, Nucl. Phys. B529 (1998) 3 [hep-ph/9707395].

[36] U. Dydak, Search for the supersymmetric Scalar top Quark with the CMS Detector at the LHC, Diploma Thesis, University of Vienna, 1996; F. Gianotti, ATLAS Internal Note PHYS-No-110 (1997); L. Poggiali, G. Polesello, E. Richter-Was, and J. Soderqvist, ATL-PHYS-97-111.

[37] A. Bartl, W. Porod, D. Restrepo, J. Romao and J. W. F. Valle, Nucl. Phys. B600 (2001) 39 [hep-ph/0007157].

[38] W. Porod, M. Hirsch, J. Romao and J. W. F. Valle, [hep-ph/0011248], to appear in Phys. Rev. D.

[39] B. de Carlos and P. L. White, Phys. Rev. D 54 (1996) 3427; [hep-ph/9602381]. B. C. Allanach, A. Dedes and H. K. Dreiner, Phys. Rev. D 60 (1999) 056002 [hep-ph/9902251].

[40] M. A. Diaz, J. Ferrandis, J. C. Romao and J. W. F. Valle, Nucl. Phys. B 590 (2000) 3 [hep-ph/9906343].

[41] M. A. Diaz, J. C. Romao and J. W. F. Valle, Nucl. Phys. B 524 (1998) 23 [hep-ph/9706313]. J. W. F. Valle, [hep-ph/9808292], and references therein.
M. A. Diaz, J. Ferrandis, J. C. Romao and J. W. F. Valle, Phys. Lett. B 453 (1999) 263 [hep-ph/9801391]. M. A. Diaz, E. Torrente-Lujan and J. W. F. Valle, Nucl. Phys. B 551 (1999) 78 [hep-ph/9808412]. L. Navarro, W. Porod and J. W. F. Valle, Phys. Lett. B 459 (1999) 615 [hep-ph/9903473].

S. Roy and B. Mukhopadhyaya, Phys. Rev. D 55 (1997) 7020 [hep-ph/9612447]; Phys. Rev. D 60 (1999) 115012 [hep-ph/9903418]; A. Datta, B. Mukhopadhyaya and S. Roy, Phys. Rev. D 61 (2000) 055006 [hep-ph/9905549]; T. Feng, Commun. Theor. Phys. 33 (2000) 421 [hep-ph/9806505]; T. Feng, [hep-ph/9808379]. C. Chang and T. Feng, Eur. Phys. J. C 12 (2000) 137 [hep-ph/9901260].

J. Ferrandis, Phys. Rev. D 60 (1999) 095012 [hep-ph/9810371]; D. E. Groom et al. [Particle Data Group], Eur. Phys. J. C 15 (2000) 1.

J. C. Romao and J. W. F. Valle, Nucl. Phys. B 381 (1992) 87

W. Porod, D. Restrepo and J. W. F. Valle, [hep-ph/0001033].

M. Hirsch and J. W. F. Valle, Phys. Lett. B 495 (2000) 121 [hep-ph/0009066].

Note that some of the neutralino decay modes such as $\tilde{\chi}_1^0 \rightarrow 3\nu$ and $\tilde{\chi}_1^0 \rightarrow \nu J$ are experimentally “invisible” and maintain the missing momentum signal unchanged. This last decay conserves R-parity since the majoron has a large R-odd singlet sneutrino component.

Strictly speaking one must take into account that the tau Yukawa coupling becomes a function of the parameters in the mass matrix, as given in [35]. In practice the corrections are negligible for the case of interest.

$m_{\tau^2}$ plays here the same role as $m_{A^0}$ in the MSSM case [2].