Search for Physics beyond Standard Model at the Precision Frontiers

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Abstract. The article outlines the recent developments in the theoretical and computational approaches to the higher-order electroweak effects needed for the accurate interpretation of MOLLER and Belle II experimental data, and shows how new-physics particles enter at the one-loop level. By analyzing the effects of Z'-boson on the polarization asymmetry, we show how this hypothetical interaction carrier may influence the future experimental results.

1. Introduction
The recent availability of computer-algebra tools in particle physics research provides a unique opportunity to perform Next-to-Leading-Order (NLO) and Next-to-NLO (NNLO) Standard Model (SM) calculations with the high degree of precision required by MOLLER and Belle II. Here, full SM calculations are required, with no approximations at the NLO level, and must include leading order NNLO contributions, which can only be achieved with some degree of automatization. We do this for both MOLLER \((e^- + e^- \rightarrow e^- + e^-)\) and Belle II \((e^+ + e^- \rightarrow \mu^+ + \mu^-)\), and compare the results of calculations performed with different sets of renormalization conditions using on-shell renormalization. This provides a straightforward test of gauge invariance for the polarization asymmetry. A discrepancy between SM predictions and experimental measurements would signal the physics beyond the SM. Since MOLLER and Belle II are the most sensitive to the parity-violating (PV) interaction, we include U(1)' extension of SM with a mass mixing scenario, which results in the extension of the SM by the parity-violating Z' boson. Our analysis for Z' extends to NLO level, giving us a refined set of constraints on the coupling and mass. First, we start with details on the NLO and NNLO (quadratic) calculations for MOLLER and then continue with Belle II. In the second part of the paper, we provide results and analysis of the polarization asymmetry with Z' boson present at LO and NLO orders.
2. SM Predictions for Polarization Asymmetry in MOLLER and Belle II

We consider two processes, $e_k^- + e_k^+ \rightarrow e_k^- + e_k^+$, for MOLLER, and $e_k^- + e_k^+ \rightarrow \mu_k^+ + \mu_k^-$, for Belle II. For MOLLER, the most sensitive observable to PV new physics (aka $Z'$) is the polarization asymmetry:

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \simeq \frac{2\Re(M_\gamma M_Z^+ + M_\pi M_{Z'}^+ + M_Z M_{Z'}^2)_{LR}}{\sigma_L + \sigma_R}. \quad (1)$$

In Eq.1, $Z'$ will enter the numerator of the asymmetry through the interference term. For the $e^- + e^- \rightarrow e^- + e^-$ process, the asymmetry at LO order is given by the following expression:

$$A_{LR(MOLLER)}^0 = \frac{s}{m_W^2 \left( \frac{1}{1 + y^4 + (1 - y)^4} \right)} \frac{1 - 4s_W^2}{s_W^2}, \quad y = -t/s. \quad (2)$$

Here, $s = (k_1 + k_2)^2$, $t = (k_3 - k_1)^2$ and $u = (k_1 - k_4)^2$ is the usual set of Mandelstam variables, and the Weinberg mixing angle is defined as $s_W^2 \equiv \sin^2 \theta_W$. As one can see, the LO asymmetry is proportional to $1 - 4s_W^2$, which results in strong sensitivity to $s_W^2$. This provides an excellent opportunity for the precision measurements of $s_W^2$, or, accordingly, the measurement of the weak charge of electrons. Although PV asymmetry in Eq.2 is quite small, the accuracy of modern experiments exceed the accuracy of the theoretical result at LO order; the NLO order calculations have been completed by a number of authors [1, 2, 3]. We can express the perturbative expansion (up to NNLO) for differential scattering cross-section in orders of $\alpha$ as:

$$\frac{d\sigma}{d\cos \theta} = \frac{\pi^3}{2s} \left[ M_0 + M_1 + M_2 \right]^2 \simeq \frac{\pi^3}{2s} \left( \alpha^2 M'_0 M'^+_1 + \alpha^3 2\Re M'_0 M'^+_2 + \alpha^4 \left( M'_1 M'^+_1 + 2\Re M'_0 M'^+_2 \right) \right), \quad (3)$$

where matrix elements $M_i$ are related to $M'_i$ by $M_i = \alpha^{i+1} M'_i$. The first term corresponds to LO, the second to NLO and the third forms the NNLO contribution, which is composed of a quadratic term ($\alpha^4 M'_i M'^+_i$) and the two-loops ($\alpha^3 2\Re M'_0 M'^+_2$) contribution. The NLO contribution to LO asymmetry is rather big ($\sim 69\%$) [3], and in order to match 1% MOLLER uncertainty, we calculated a full set NNLO (quadratic) [4] and leading order NNLO (two-loops) contributions [5] (and references therein). The precision is essential, so we control it in two ways. First, we applied “on paper” on-shell calculations using the renormalization conditions of [6] and low energy approximations for $\frac{\left( t \right)_{[\text{low}]}{\pi \sqrt{s}} \ll 1$ for $\sqrt{s} \ll 30 (GeV)$. Second, we performed semi-automated [7, 8, 9, 10, 11] calculations for the full set of Feynman diagrams without any approximations and using renormalization conditions of [12]. This approach was implemented for NLO and NNLO (quadratic) contributions. The semi-automated full two-loop calculations are yet to be completed which is our next goal. Let us demonstrate how these two approaches compare to each other. First, we introduce a correction to the asymmetry, as:

$$\delta^C_A = \frac{A^C_{LR} - A_{LR}^0}{A_{LR}^0}, \quad (4)$$

where $A^C_{LR}$ stands for the NLO-corrected asymmetry. If we take $\alpha = 1/137.0359$, $m_W = 80.398 (GeV)$, $m_Z = 91.1876 (GeV)$ and kinematics relevant to MOLLER experiment ($E_{lab} = 11 (GeV)$), we can see on Fig.1 (right plot) that results obtained in both approaches differ less than 0.1%. We find that the NNLO (quadratic) contribution, Fig.1(left plot),

$$\Delta_A = \frac{A^{1\text{-loop}-Q}_{LR} - A^{1\text{-loop}}_{LR}}{A_{LR}^0}, \quad (5)$$
Figure 1. The graphs show corrections to the polarization asymmetry at the $\theta = 90^\circ$ and soft photon cut $\omega = 0.05\sqrt{s}$. On the left graph, the dashed line shows correction due to QED effects, the solid line represents weak correction based on semi-automated calculations, and the dotted line gives “on-paper” approximated calculations for the same correction. The graph on the right shows corrections for the pure quadratic (“Q”) contribution ($\Delta_A$), NLO (“1-loop”) and NLO+NNLO (“1-loop-Q”) results. The black dot on both graphs corresponds to $E_{lab} = 11 \text{(GeV)}$.

| $\theta^\circ$ | 10 | 30 | 50 | 70 | 90 | 110 | 130 | 150 | 170 |
|---------------|----|----|----|----|----|-----|-----|-----|-----|
| “on-paper”    | 0.0180 | -0.0456 | -0.0738 | -0.0935 | -0.1099 | -0.1264 | -0.1460 | -0.1743 | -0.2378 |
| SA            | 0.0179 | -0.0455 | -0.0738 | -0.0934 | -0.1099 | -0.1263 | -0.1459 | -0.1742 | -0.2372 |

Table 1. Relative NLO correction to unpolarized cross section ($\delta_{00}^C$) in the $e^+ + e^- \rightarrow \mu^+ + \mu^-$ process for the various angles in CM reference frame. Both approaches used the soft-photon approximation in the treatment of infrared divergences. Soft photon cut is $\omega = 0.05\sqrt{s}$.

is responsible for $\sim 5\%$ suppression of the total correction at $E_{lab} = 11 \text{(GeV)}$. This is a clear signal that, in the light of proposed precision experiments, the NNLO contributions are very important. Similar to the Moller process, $e^+ + e^- \rightarrow \mu^+ + \mu^-$ polarization asymmetry (first addressed in [13]), also shows strong sensitivity to the $s_W^2$:

$$A_{LR(Belle II)}^0 = -\frac{s}{4m_W^2} \frac{(y-1)^2}{2(y-1)y+1} \left( 1 - \frac{4s_W^2}{s_{W}^2} \right), \quad y = -t/s.$$  \hspace{1cm} (6)

We improve the precision by implementing the same two-way approach, for Belle II kinematics specifically, taking into account the full set of NLO electroweak corrections. For $\sqrt{s} = 10.57 \text{(GeV)}$, Tabl.1 shows our results for the NLO relative correction to the unpolarized cross section ($\delta_{00}^C = \frac{\sigma_{00}^C - \sigma_{00}^W}{\sigma_{00}^W}$) computed using the semi-automated (SA) and “on-paper” calculation methods in the on-shell renormalization. Evidently, the difference in both approaches, for the broad range of scattering angles, is negligible. Although the NLO contribution (Fig.2) for MOLLER and Belle II experiments is significant, judging by the excellent agreement between the two approaches, the theoretical uncertainty at NLO level is at a sub-percent level. However, that result does not include the NNLO contribution, needed to interpret the ultra-precision measurements.
Figure 2. LO and NLO corrected polarization asymmetries for the MOLLER (left, $E_{lab} = 11$ (GeV)) and Belle II (right, $\sqrt{s} = 10.57$ (GeV)) experiments for the $20^\circ < \theta < 160^\circ$ in the CM reference frame.

3. Beyond the Standard Model Physics with Dark Vector

The discrepancy between experimental data and SM theoretical predictions would signal new interaction carriers. We use the simple U(1)' extension of SM proposed in [14], which uses kinetic type of mixing between dark vector ($A'_\mu$) and hyper-charge ($B_\mu$) fields:

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{2} \epsilon \cos \theta_W B_{\mu\nu} A'^{\mu\nu} - \frac{1}{4} A'^{\mu\nu} A'^{\mu\nu},$$  \hspace{1cm} (7)

where fields tensors are given by $\{A'^{\mu\nu}, B_{\mu\nu}\} = \partial_{\mu} \{A'_\nu, B_\nu\} - \partial_{\nu} \{A'_\mu, B_\mu\}$, with $B_\mu = \cos \theta_W A_\mu - \sin \theta_W Z_\mu$ and $\epsilon$ is the $(B_\mu - A'_\mu)$ kinetic mixing parameter. With SM Higgs doublet plus the Higgs singlet (used for breaking the $U(1)'$ symmetry and giving mass to $A'_\mu$), a Lagrangian describing the interaction between the SM fermions and the dark vector boson $A'_\mu$ is:

$$\mathcal{L}_{\text{int}} = -e Q_f \bar{f} \gamma_\mu f \cdot (V^\mu + \epsilon A'_\mu) - \frac{e}{\sin \theta_W \cos \theta_W} \bar{f} (c_V^f \gamma_\mu + c_A^f \gamma_\mu \gamma_5) f \cdot (Z'^\mu + \epsilon Z'^A A'_\mu).$$ \hspace{1cm} (8)

Here, $Q_f$ is the charge of the fermion in units of $e$, and the $c_V^f$ and $c_A^f$ constants are the usual SM vector and axial-vector coupling strengths, respectively. As we can see from Eq.8, the dark $A'_\mu$ couples to fermions through both parity-conserving and violating terms, which is similar to the weak $Z_\mu$ coupling. This type of dark $A'_\mu$ in [15] is called the dark $Z'_\mu$ boson and is derived from an additional mass mixing term characterized by mixing parameter $\epsilon Z' = \frac{m_{Z'}}{m} \delta$. Here, $m_{Z'}$ is the the mass of the dark $Z'_\mu$ boson and $\delta$ is an arbitrary model-dependent parameter. The fact that $Z'_\mu$ is represented as a superposition of mixings between dark vector with electromagnetic and $Z$-boson fields makes it possible to include $Z'_\mu$ at the NLO level. We include the $Z'_\mu$ at NLO in order to match NLO calculations with our SM predictions. Our results are shown in the form of exclusion plots in Fig.3 for MOLLER (left) and Belle II (right).

Fig.3 explores small mixing $\epsilon$ and small $Z'_\mu$ mass scenarios for MOLLER. We show up-to-1% deviation from the SM central prediction exclusion plots for $Z'_\mu$, which is included at LO.
Figure 3. Exclusion plots for MOLLER (left) and Belle II (right). For MOLLER, we show exclusion for 1% deviation from SM prediction. Here, $Z'_\mu$ is included either at LO (Born) or up to NLO (Born+NLO) orders. Plot for Belle II shows up-to-NLO exclusion regions for 1%, 2% and 3% deviations from SM prediction. For both plots, we take $\delta^2 = 3 \cdot 10^{-5}$.

and NLO orders. Inclusion of the $Z'_\mu$ at the NLO order increases the exclusion region for $\epsilon$ for all masses of $Z'_\mu$ by about 25%. While this increase is not substantial, it could become an important factor in the determination of the $Z'_\mu$ mass and coupling if $Z'_\mu$ is discovered. According to [15, 16], if no $Z'_\mu$ is discovered, MOLLER will exclude the region where $Z'_\mu$ is used to explain the $(g - 2)_\mu$ anomaly. For Belle II (right plot of Fig.3), we concentrate on the resonance region at $\sqrt{s} = 10.57 \text{(GeV)}$, where the sensitivity of Belle II to the $Z'_\mu$ (up to NLO order) is the highest and is complimentary to the MOLLER experiment. We conclude that the inclusion of NLO and NNLO electroweak radiative corrections is essential for the search of new physics at the precision frontier, and that the computer-algebra tools are indispensable to this task.

Acknowledgments

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