Mechanism of thermally activated c-axis dissipation in layered High-T$_c$ superconductors at high fields

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We propose a simple model which explains experimental behavior of c-axis resistivity in layered High-T$_c$ superconductors at high fields in a limited temperature range. It is generally accepted that the in-plane dissipation at low temperatures is caused by small concentration of mobile pancake vortices. However, for high fields, the c-axis dissipation is caused by phase slips between the layers. We argue that the c-axis dissipation is caused by mobile pancake vortices in the layers, and that the in-plane dissipation is caused by phase slips in the layers. The model gives universal relation between the components of conductivity which is in good agreement with experimental data.

A well known property of layered High-T$_c$ superconductors (e.g., Bi$_2$SrCa$_2$Cu$_2$O$_x$ (BiSCCO)) is an existence of extended low-temperature tails in the temperature dependencies of the both in-plane and out-of-plane components of resistivity for magnetic field applied along the c-axis. An important experimental fact is that the thermally activated behavior for both $\rho_{ab}$ and $\rho_c$ is characterized by the same value of activation energy (see Ref. [1] and Fig. [1]). This indicates that dissipations in both directions have essentially the same origin.

The origin of the tail for the in-plane resistivity component is qualitatively understood. It is theoretically expected that at fields larger than the crossover field $B_{cT} \approx \Phi_0/(\gamma s)^2$ and temperatures larger than the decoupling temperature $T_{dec}$, the system can be treated as a set of almost independent two-dimensional (2D) vortex lattices in the layers (here $\gamma$ is the anisotropy of the London penetration depth and $s$ is the interlayer spacing). The in-plane dissipation can therefore be attributed to the thermally activated motion of point defects in the 2D lattices (weak disorder) or weakly pinned mobile pancake vortices (strong disorder) [2]. For a 2D superconductor with weak pinning a crossover between free flux flow and thermally activated motion should take place at the melting temperature of the vortex lattice $T_m \approx 0.014s\Phi_0^2/(4\pi\lambda)^2$ as observed in the thin films of $a$-Nb$_3$Ge [3] and in numerical simulations [4]. This seems not to be the case for available samples of the BiSCCO compound where crossover to the thermally activated behavior at high fields (> 1 T) takes place at temperatures 50-70 K which is significantly larger than the estimate for the melting temperature $T_m \approx 13K$ obtained using experimental values of the London penetration depth $\lambda_{ab} \approx 1800\text{A}$ and $s = 15\text{A}$. This indicates that in available BiSCCO single crystals pinning is strong enough to destroy the melting transition in the field range 1-10 T and the crossover temperature between free flux flow and thermally activated regime (50-70 K) has to be interpreted as the depinning temperature of a single pancake vortex renormalized by the intervortex interactions [5]. In this case a very natural assumption is that the lattice is strongly pinned and the in-plane dissipation is caused by the motion of mobile pancake vortices with the small concentration $n$ and diffusion constant $D$. In-plane conductivity within this model can be estimated as follows. In-plane transport current $j$ drives mobile pancakes with velocity $v = \mu s(\Phi_0/c)^j$. Here $\mu$ is the average pancake mobility, which is connected with the diffusion constant $D$ by the Einstein relation, $D = \mu T$. Pancake motion produces electric field $E = (\Phi_0/c) j n$. Therefore the conductivity in $ab$-plane $\sigma_{ab}$ in this model is given by

$$
\sigma_{ab} = \left( \frac{e}{\Phi_0} \right)^2 \frac{T}{\sin D}
$$

Both $n$ and $D$ presumably have Arrhenius-type temperature dependencies but their temperature dependence as well as detailed mechanism of in-plane motion have no influence for our further consideration.

In the region where the conductivity in the c-direction is much larger than the normal conductivity it has the Josephson origin and is caused by phase slips between neighboring layers. It was assumed [6] that these phase slips are similar to the phase slips in a single Josephson junction with the typical area $\Phi_0/B$ and therefore the Ambegaokar-Halperin formula for the resistivity of small Josephson junction can be used to describe the temperature and field dependencies of $\rho_c$. Unfortunately this physically transparent assumption does not have microscopic support.

Diffusive motion of pancake vortices at high enough temperatures leads to the time variations of the phase difference between the layers. Such pancake-induced phase slips provides a candidate mechanism for the c-axis dissipation which was not considered so far. In this Letter we obtain c-axis conductivity due to such mechanism. The key assumption of our model is that the phase slips are caused the diffusive motion of a small amount of pancake vortices independently in different layers. The model is
applicable above the decoupling temperature where the global superconducting coherence is broken. This means that the model can be used to describe dissipation in a limited temperature range (≈ 40-70 K).

The starting point of our calculation is the Kubo formula which relates $\sigma_c$ with the correlation function of the Josephson current:

$$\sigma_c = \frac{s j^2}{T} \int drdt \langle \sin \delta\phi(0,0) \sin \delta\phi(r,t) \rangle$$  \hspace{1cm} (2)

Here $j_j$ is the Josephson current, $\delta\phi = \phi_2 - \phi_1 - \frac{2\pi v}{\Phi_0} A_z$ is the Gauge invariant phase difference between neighboring layers; the phases $\phi_{1,2}(r,t)$ are mainly determined by the vortex positions $R_{1,2}$ in the layers, $\phi_{1,2}(r,t) = \sum \phi_{0}(r - R_{1,2}(t))$ where $\phi_{0}(r) = \text{atan}(y/x)$ is the phase distribution around the core of a single vortex. We calculate $\sigma_c$ in the lowest order with respect to the interplane Josephson coupling. Within this approximation one can neglect correlations between Josephson currents in different layers, which are already excluded in Eq. (2). We can also neglect interplane phase correlations in the sine-sine correlation function in Eq. (2). This means that averages like $\langle \cos \delta\phi \rangle$ assumed to be zero and

$$\langle \sin \delta\phi(0,0) \sin \delta\phi(r,t) \rangle \approx (1/2) \text{Re}[S(r,t)],$$

with $S(r,t) = \langle \exp i (\delta\phi(r,t) - \delta\phi(0,0)) \rangle$.

As follows from Eq. (3) the value of the out-of-plane conductivity is determined by how fast the phase correlations decrease in space and time. The origins of decay of the phase correlations in space and time are very different. The decay in space is determined by the randomness in the vortex arrangement while the decay in time is determined by the vortex motion. In our model the static phase difference $\delta\phi(0,0) - \delta\phi(r,0)$ is established by the almost all rigidly pinned vortices while dynamic behavior is caused by the small amount of mobile vortices. In such a situation the correlation function can be splitted into the separately averaging static and dynamic parts, $S(r,t) = S(r)S(t)$ and Eq. (3) becomes,

$$\sigma_c \approx \frac{s j^2}{2T} \int drdt S(r)S(t)$$  \hspace{1cm} (3)

with

$$S(r) = \langle \exp [i (\delta\phi(0,0) - \delta\phi(r,0))] \rangle,$$

$$S(t) = \langle \exp [i (\delta\phi(0,0) - \delta\phi(0,t))] \rangle.$$

In absence of correlations between vortex positions in neighbor layers the static part is expected to decay at distances of the order of the average intervortex spacing $a_0$. This assumption can be supported by direct calculation for the case of randomly placed pancakes, which gives

$$S(r) \approx \exp (-\pi n_v r^2 \ln(r_{max}/r))$$  \hspace{1cm} (4)

with $n_v = B/\Phi_0$. We assume that the phase at given point is not sensitive to pancake displacements at distances larger than some phase coherence distance $r_{max}$, which provides upper cutoff to the logarithmically diverging integrals. The static phase correlation function for randomly placed pancakes Eq. (4) actually has the fastest possible descend. Correlations in the pancake arrangements always increase the spatial range of $S(r)$. Decay of the dynamic part is determined by the diffusion of mobile pancakes and is given by

$$S(t) = \exp (-2\pi nD ln(r_{max}/Dt))$$  \hspace{1cm} (5)

Substituting Eqs. (1) and (3) into Eq. (3) we obtain

$$\sigma_c \approx \frac{1}{2\pi ln(r_{max}/a_0) ln(nr_{max}^2/n_v T nD)} \frac{s j^2}{j}$$  \hspace{1cm} (6)

Up to logarithmic factors and numerical constant this result immediately follows from Eq. (2) assuming that the interlayer phase difference is correlated at distances of the order of the average vortex spacing and at times of the order of the typical “phase slip” time $t_p = 1/nD$. Using the relation $j_j = \frac{2\pi v}{\Phi_0} E_j$ and Eq. (6) the last equation can be transformed to

$$\sigma_c \approx C \frac{\Phi_0 B^2 E_j^2}{BT^2} \sigma_{ab}$$  \hspace{1cm} (7)

with $C = \frac{\ln(nr_{max}^2/n_v T nD)}{\ln(r_{max}/a_0)}$. Therefore within our model the ratio of conductivities depends only upon material parameters, temperature, and field, and does not depend upon the details of the mechanism, responsible for dissipation. Eq. (7) represents the main result of our Letter.

The model can be verified independently by measuring the frequency dependence of $\sigma_c$. The frequency dependence can be taken into account by adding the factor $\cos(\omega t)$ under the integral in Eq. (3). This gives Lorentzian frequency dependence $\sigma_c(\omega)$:

$$\frac{\sigma_c(\omega)}{\sigma_c(0)} = \frac{\omega^2 + \omega_{ps}^2}{\omega^2 + \omega_{ps}^2}$$  \hspace{1cm} (8)

The typical “phase slip” frequency $\omega_{ps} = 2\pi nD \ln(nr_{max}^2)$ can be directly related to the in-plane component of resistivity $\rho_{ab} = 1/\sigma_{ab}$ using Eq. (4), which gives $\omega_{ps} \approx 2 \times 10^6 [1/s] \cdot T[K] \cdot \rho_{ab}[\mu\Omega \cdot cm]$. The direct correlation between typical frequency in $\sigma_c(\omega)$ and $\rho_{ab}$ provides additional experimental check of the theory.

Let us discuss the limitations of our model. At low enough temperature slowing down of the pancake motion in the layers leads to increase of the interlayer correlations so that the diffusion of pancakes in different layers can not be considered independent any more. Quantitatively the influence of the Josephson coupling on the diffusive motion is characterized by value of the Josephson force acting on pancake. This force is determined by
the variation of Josephson energy under small pancake displacement and can be calculated as

$$f_{J_0}(R, t) = -E_J \int d^2r \frac{\partial \phi(r - R(t))}{\partial r_n} \sin \delta \phi(r, t)$$  \hspace{1cm} (9)

Neglecting again the interplane phase correlation we obtain the correlation function of this force

$$\langle f_{J_0}(0)f_{J_0}(R) \rangle \approx \delta_{aa'} \pi E_J^2 \frac{n_v}{n_e} \ln(r_{max}/a) S(t)$$  \hspace{1cm} (10)

As one can see from this expression the Josephson force has typical amplitude $f_{J_0} \sim aE_J$ and is correlated at times $\sim t_{ps}$ and at large distances of the order of $r_{max}$. This means that mobile pancakes within the correlation area $\sim r_{max}^2$ will drift under this force with approximately the same velocity $v \approx \mu f_{J_0}$ during the “phase slip” time $t_{ps}$. This drift motion will produce an extra phase change $\delta \phi \approx \nu r_{max} \mu f_{J_0} t_{ps} \approx r_{max} f_{J_0}/T$. One can neglect the Josephson coupling and treat pancake motion in different layers independently when this extra phase shift is small, which gives the condition $r_{max} f_{J_0} \ll T$. More precise estimate requires evaluation of the in-plane phase coherence distance $r_{max}$. This is beyond the scope of the simple model proposed in this Letter.

Several mechanisms limit applicability of this model at high temperatures. Quasistatic phase fluctuations with the amplitude $\chi = \langle (\phi^2)/2 \rangle$ lead to additional suppression of the Josephson energy, which can be described by the Debye-Waller factor $E_{J_0}^{\chi f_{J_0}} \approx E_J \exp (-\chi)$, and reduces the ratio $\sigma_c/\sigma_{ab}$ as compared to Eq. (6). Two contributions to $\chi$ come from thermal oscillations of the pinned pancakes ($\chi_1$) and from the spontaneous interplane phase fluctuations not related to pancake motion ($\chi_2$), $\chi = \chi_1 + \chi_2$. We estimate this contributions as

$$\chi_1 \approx \frac{n_v T}{\alpha_L} \ln \left( \frac{r_{max} \alpha_L}{T} \right); \quad \chi_2 = \frac{T}{4J} \ln \left( \frac{J}{E_{J_0}^{\chi f_{J_0}} / \xi^2} \right)$$

where $\alpha_L = s\Phi_0 j_c/(e\xi)$ is the Labush parameter, $j_c$ is the critical current, and $J = s\Phi_0^2/(\pi 4\pi \lambda)^2$ is the in-plane phase stiffness. Taking $j_c = 10^8 A/cm^2$, $T = 60K$, $B = 1T$, $\ln(r_{max} \alpha_L/T) = 5$ we obtain the estimate $\chi_1 \approx 0.15$. Using parameters of BISCO and taking the logarithmic factor in $\chi_2$ as 5 we obtain $\chi_2 \approx 4|K|/(\xi T)$. Therefore the spontaneous interplane phase fluctuations can be ignored at temperatures $\sim 10 K$ below $T_c$.

Another important factor, which can not be ignored at high temperatures, is the quasiparticle conductivity. This conductivity has to be added to the phase slip conductivity (6) and therefore it enlarges the conductivity ratio in comparison with prediction of Eq. (6).

To verify the prediction of the model we plot in Fig. 2 temperature dependencies of the conductivity ratio at different fields (the data 3 are the same as in Fig. 3). One can see that within the temperature range 50-70 K for fields 0.5-2 T a general trend is in agreement with Eq. (7), i.e. the ratio decreases with temperature and field. Taking the conductivity anisotropy at 60 K and 1 T ($\approx 2.7 \times 10^{-6}$) and using the relation between the Josephson coupling energy and the Ginzburg-Landau anisotropy ratio $\gamma$, $E_J = \Phi_0^2 / (\pi 4\pi \lambda)^2 s$, we estimate from Eq. (8) the value of $\gamma$ as 370 which is within the range reported in the literature (4). An abrupt increase of the ratio at low temperatures indicates probably the emergence of the global coherence in the system and the glass transition.

To verify quantitatively the validity of Eq. (7) we need to know the temperature dependence of the Josephson coupling energy. This dependence is determined by the temperature dependencies of the tunneling integral $t$ and superconducting order parameter $\Psi$, $E_J \propto |\Psi|^2$. Usually the first dependence is neglected. However, as recently was proposed by Bulaevskii et al (15), the semiconducting-temperature dependence of the normal-state c-axis conductivity $\sigma_{cn}$ at low temperatures can be ascribed to the temperature dependence of $t$. We therefore assume that $t$ and $\sigma_{cn}$ have the same temperature dependence.

The temperature dependence of $\sigma_{cn}$ have been recently measured by Cho et al (3) for the temperatures down to $\approx 55 K$ using high magnetic fields (up to 18 T). This dependence in the interval 55-90 K can be very well fitted by the exponential formula $\sigma_{cn} \propto \exp(T/29[K])$. Therefore to verify Eq. (7) we try to fit the conductivity ratios by the formula

$$\frac{\sigma_c}{\sigma_{ab}}(B,T) = A(B)\exp(2T/29)(1 - T/T_c)^2T^{-2}$$  \hspace{1cm} (11)

with $A(B)$ being the fitting parameter. The factor $1 - T/T_c$ accounts for temperature dependence of $|\Psi|^2$. As one can see this formula gives very good fit to the data in the temperature range 45-70 K. However the field dependence of the fitting parameter $A(B)$ is weaker than predicted by Eq. (7), $A(B) \propto B^{-0.52}$. The most probable origin of this discrepancy is that increasing of the field reduces randomness in vortex arrangement within the layers and, as a consequence, the static phase correlations decay in space slower than is predicted by Eq. (7).

In conclusion, we propose simple model, which provides the mechanism for thermally activated dissipation along the c-axis in the vortex state of disordered layered superconductors. The model gives the universal relation between the conductivity components.

I would like to thank A.I.Larkin for useful discussion. This work was supported by the National Science Foundation Office of the Science and Technology Center under contract No. DMR-91-20000. and by the U. S. Department of Energy, BES-Materials Sciences, under contract No. W-31-109-ENG-38.
We should emphasize that this statement is related to large fields where the melting temperature is close to the melting point of a single layer. As field decreases the melting temperature increases due to 3D effects and at some point becomes larger than the depinning temperature. This means that the melting transition is expected to be restored at small fields (< 1 kOe). Ordered vortex lattice at small fields indeed was observed by decorations (R.N.Kleiman Phys.Rev.Lett. 62, 2331 (1989)), neutron diffraction (R.Cubitt et al Nature(London) 365, 407 (1993)), and Lorentz Microscopy (Harada et al Phys.Rev.Lett. 71, 3371 (1993)).

Note that Eqs. (4) and (5) give \( \langle \exp i\delta \phi \rangle = 0 \), which is valid only if the interlayer coupling is completely neglected. If small interlayer coupling is included than the correlation function \( S(r,t) \) does not vanish at large distances and times but saturates at \( S(\infty, \infty) = \langle \cos \delta \phi \rangle^2 \ll 1 \). This large-scale behavior does not influence much the integral in Eq. (2) and result (5), because in higher order with respect to interlayer coupling one has to keep term \( \langle \exp i[\delta \phi(0,0) + \delta \phi(r,t)] \rangle \), which was skipped in the transformation from Eq.(2) to Eq.(3). This term being added in Eq.(3) exactly cancels \( S(\infty, \infty) \) at large scales and the main contribution still comes from small distances and times where approximations (4) and (5) are applicable.

FIGURE CAPTIONS

FIG. 1. Temperature dependencies of the resistivity components \( \rho_{ab} \) and \( \rho_c \) for BiSCCO for fields 0.5,1,2, and 5 T. Data are taken from Ref. [2]. Note that in spite of huge difference in absolute values, \( \rho_{ab} \) and \( \rho_c \) have almost identical temperature dependencies.

FIG. 2. Temperature dependencies of the conductivity ratio for the data from Ref. [2]. Solid lines give the fits by the formula (6), which were made to verify the prediction of Eq.(7). Table gives the values of parameter \( A \) at different fields obtained from the fits.
| $B$ (T) | $A$ (K$^{-2}$) |
|---------|----------------|
| 0.5     | 0.00234        |
| 1       | 0.00165        |
| 2       | 0.00107        |
| 5       | 0.00067        |

Graph showing the relationship between $\frac{\sigma_c}{\sigma_{ab}}$ and $T$ (K) for different magnetic fields ($B$ (T)).