Abstract—Despite significant economic and ecological effects, a higher level of renewable energy generation leads to increased uncertainty and variability in power injections, thus compromising grid reliability. To improve power grid security, we investigate a joint chance-constrained (CC) dc approximation of the ac optimal power flow (OPF) problem. The problem aims to find economically optimal power generation while guaranteeing that all power generation, line flows, and voltages remain within their epigraphs at the same time with a predefined probability. Unfortunately, the CC-dc-OPF problem is computationally intractable even if the distribution of renewables’ fluctuations is specified. Moreover, existing approximate solutions to the joint CC OPF problem are overly conservative and computationally demanding and, therefore, have less value for the operational practice. This article proposes an importance sampling approach for constructing an efficient and reliable scenario approximation for CC-dc-OPF with theoretical guarantees on the number of samples required, which yields better sample complexity and accuracy than current state-of-the-art methods. The algorithm efficiently reduces the number of scenarios by generating and using only a few most important, thus enabling real-time solutions for test cases with up to several hundred buses.

Index Terms—Chance-constrained (CC) optimization, optimal power flow (OPF), robust optimization, scenario approximation (SA).

I. INTRODUCTION

In 2020, electricity produced approximately 25% of greenhouse gas emissions in the US, and integration of a higher volume of renewable energy generation is seen as the primary tool to reduce the emission level [1]. In turn, a higher amount of renewable generation increases the power grid uncertainty, compromises its security, and challenges classical power grid operation and planning policies [2].

The optimal power flow (OPF) problem, which determines the economically optimal operating level of power generation under given power balance equations and security constraints, is one of the most fundamental problems in grid operation and planning [3]. Several extensions are proposed for the OPF problem for addressing the uncertainty of power generation and consumption [4]. Robust and chance-constrained power flow...
formulations are among the most popular ones. The robust OPF problem assumes bounded uncertainty and requires a solution to be feasible against any possible uncertainty realization within a given uncertainty set [5, 6, 7].

A more flexible and general chance-constrained approach requires security constraints to be satisfied with a high probability while assuming that the distribution of renewables is known in whole or in part [8, 9, 10, 11, 12, 13]. This article considers a joint chance-constrained OPF (JCC-OPF) problem, where the joint probability of at least one failure of the security constraints (line load limits, voltage stability bounds) is bounded from above by a confidence threshold.

In contrast to a single chance-constrained formulation, which imposes individual failure probability thresholds for each of the constraints, the joint chance constraint is computationally hard even for dc power balance equations, which is characterized by linear security limits, with Gaussian uncertainty [14, 15]. Several tractable convex approximations have been proposed [16, 17, 18, 19] for joint chance-constrained optimization to overcome the computational hardness of the problem. However, they often lead to conservative solutions inapplicable for operational practice.

Other approaches, such as scenario and sample average approximations [17, 20, 21, 22], consist of substituting the stochastic part with a set of deterministic inequalities based on the uncertainty realization. This approach can be distributionally robust and allow exploiting uncertainties beyond the Gaussian ones. A combination of analytical approximation and sampling [23] can further improve the accuracy of the solution. However, it may require a large number of samples. Scenario curation/modification heuristics have been designed to improve the sample complexity of JCC-OPF [24], although without formal analysis of the methodology. In work outside power grids, statistical learning has been used to approximate uncertain convex programs [25, 26, 27].

Nevertheless, the scenario approximation (SA) approach remains the most accurate algorithm for solving the joint chance-constrained dc optimal power flow (JCC-dc-OPF). At the same time, its complexity is often unacceptable for large-scale power grids [28]. To this end, the article suggests using importance sampling to reduce the complexity and improve the accuracy of the SA to chance-constrained optimal power flow (CC-OPF). The importance sampling approach generates more informative samples and results in an optimization problem with fewer constraints. Our article extends earlier results [29, 30] on importance sampling for chance-constrained optimization.

The contributions of this article are as follows.

1) We propose a novel computationally efficient approach to the JCC-dc-OPF problem. The algorithm exploits physics-informed importance sampling to refine the classical SA [20].

2) We prove the algorithm to converge to a solution of JCC-dc-OPF with a guaranteed accuracy under mild technical assumptions.

3) We demonstrate the proposed algorithm’s superior computational efficiency and accuracy relative to standard methods over multiple real and synthetic test cases.

The rest of this article is organized as follows. Section II presents the JCC-OPF problem and introduces the notation used in this article. We outline the algorithm and provide its theoretical analysis in Section III. Empirical study and comparison to state-of-the-art methods are given in Section IV. Finally, Section V concludes this article.

II. BACKGROUND AND PROBLEM SETUP

A. Notation

The dc power flow approximation remains an extremely popular yet simple model for analysis because of a linear relation between powers and phase angles in typical high-voltage power grids.

In this study, we consider a power grid given by a graph $G = (V, E)$ with a set of nodes/buses $V$ and edges/lines $E$. Assume that $n + 1$ is the number of buses and $m$ is the number of lines. Let $p$ be a vector of power injections $p = (p_F, p_R, p_S)^\top$, where $p_F$ corresponds to buses with deterministic/fixed (F) power injections, $p_R$ to buses with random (R) injections, and $p_S$ is the injection at the slack bus (S). The power system is balanced, i.e., the sum of all power injections equals zero, $\sum_{i \in V} p_i = 0$. Let $b = (b_{ij})_{i,j \in E}$ be an admittance matrix of the system, $b = B\theta$. The components $B_{ij}$ are such that $B_{ij} \neq 0$ if there is a line between nodes $i$ and $j$, the diagonal elements $B_{ii} = -\sum_{j \neq i} B_{ij}$, i.e., $B$ is a Laplacian matrix. Let $\theta$ be a vector of phase angles. Thus, $B$ is rank deficient with one eigenvalue equal to 0. Without a loss of generalization, we assume that the phase angle on the reference slack bus $\theta_S = 0$, and given that the injection $p_S$ is the negative sum of the other injections, we can analyze a reduced-order system by ignoring the reference bus $S$. This is done by removing the injection and phase angle at $S$ from $p$ and $\theta$ and removing the row and column corresponding to $S$ from $B$. This gives an invertible reduced power flow model at the nonreference buses [31, 32]. We use $\theta, p$, and $B$ to the reduced quantities and can write their inverse relation as $\theta = B^{-1}p$.

The dc power flow equations, generation, and stability constraints are

$$p = B\theta \quad (1)$$

$$p_i^{\min} \leq p_i \leq p_i^{\max}, \quad i \in V \quad (2)$$

$$|\theta_i - \theta_j| \leq \bar{\theta}_{ij}, \quad (i,j) \in E. \quad (3)$$

Let $A \in \{-1, 0, 1\}^{m \times n}$ be the reduced incidence matrix, i.e., column corresponding to reference bus $S$ is removed, of grid $G$. If nodes $i$ and $j$ are connected by edge $k$, then $A_{ki} = +1$, $A_{jk} = -1$ and all other elements are equal to zero. Then, the phase angle constraints in (3) can be represented as $AB^{-1}p \leq \bar{\theta}$ and $-AB^{-1}p \leq \bar{\theta}$.

In dc OPF, we seek to minimize some cost (linear or quadratic), denoted as $\text{cost}(p)$ on power injections such that the following system of inequalities (operational constraints) are satisfied:

$$\sum_{w} \left(AB^w, -AB^w, I, -I \right)^\top p \leq \left(\bar{\theta}, \bar{\theta}, p^{\max}, -p^{\min} \right)^\top$$

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where $I$ is the $n \times n$ identity matrix. Let $J$ be the number of constraints, $J = 2m + 2n$; then, $W \in \mathbb{R}^{J \times n}$ and $b \in \mathbb{R}^{n \times 1}$. We refer to the feasibility polytope as the set $\mathcal{P}$, $\mathcal{P} = \{ p : W p \leq b \}$.

Finally, we assume that fluctuations of power injections $p$ are Gaussian, $p = x + \xi$, where $\xi$ is a zero mean Gaussian uncertainty, $\xi \sim \mathcal{N}(0, \Sigma)$ and $x$ is the deterministic part of power injections. Under this setting, we solve a JCC-OPF problem, where the dc-OPF cost is replaced by the expected value $\mathbb{E}_{\xi} \text{cost}(x, \xi)$ and the operational constraints are now satisfied probabilistically, instead of exactly. This is described in the next section.

The article notation is summarized in Nomenclature. We use lower indices for coordinates of vectors and matrices, lowercase letters for probability density functions (PDFs), and uppercase letters for cumulative distribution functions (CDFs). When it does not lead to a confusion, we use $P$ and $\mathbb{E}$ to denote probability and expectation without explicitly mentioning a distribution, respectively.

### B. Problem Setup

The JCC-OPF problem is

$$\min \mathbb{E}_{\xi \sim \mathcal{N}(0, \Sigma)} \text{cost}(x, \xi)$$

subject to

$$\mathbb{P}_{\xi \sim \mathcal{N}(0, \Sigma)}(x + \xi \in \mathcal{P}) \geq 1 - \eta$$

(4)

where $\eta$ is a preset confidence parameter with $0 < \eta < 1/2$, and $\text{cost}(x, \xi)$ is a cost function convex in $x$ for any realization of $\xi$. In other words, we assume that power flow balance equations [see (1)] are satisfied almost surely, and the probability that at least one of the security constraints [see (2) and (3)] fails is at most $\eta$.

Notice that the convexity of the cost function of Problem (4) does not retain for a low level of target confidence, i.e., $\eta < 0.5$.

### C. Scenario Approach

Over the last two decades, the scenario approach [17], [20] remains the state-of-the-art method for solving joint chance-constrained optimization. The scenario approach consists in substituting the probabilistic constraints with a large number of deterministic ones with each constraint standing for some uncertainty realization

$$\min \frac{1}{N} \sum_{t=1}^{N} \text{cost}(x, \xi^t)$$

subject to

$$p_{\text{min}} \leq x + \xi^t \leq p_{\text{max}}, \quad 1 \leq t \leq N$$

$$|\theta_i(\xi^t) - \theta_j(\xi^t)| \leq \delta_{ij}, \quad (i, j) \in \mathcal{E}, \quad 1 \leq t \leq N$$

$$x + \xi^t = B\theta(\xi^t), \quad 1 \leq t \leq N$$

(5)

where $N$ is the number of scenarios, and $\{\xi^t\}_{t=1}^{N}$ is a set of uncertainty realization. We assume below that the generation cost function $\text{cost}(x, \xi)$ does not depend on the randomness in power fluctuation (cost$(x, \xi) = \text{cost}(x)$ for any uncertainty realization $\xi$), but may depend on the uncertainty distribution (i.e., on its mean or variance). The key disadvantage of the SA (5) is the number of constraints induced by adding scenarios $\xi^t$. A classical theory, developed by Calafiore and Campi [20], suggests to include a large number of scenarios $N$. This will allow the solution of (5) to be feasible for original problem (4) with high probability $1 - \delta$. However, the number of required scenarios $N$ is given by $N \geq 2\left(\frac{\ln 1/\delta}{\eta} + d \frac{\ln 1/\eta}{\eta}\right)$, where $d$ is the number of control variables in the optimization problem. For example, with $\eta = 10^{-2}$ and $\delta = 10^{-2}$, for the IEEE 57-bus case, one would end up with $6.45 \times 10^3$ (six generators, excluding slack one) constraints, which is time- and memory-consuming to solve.

The major contribution of this article is a significant reduction of the requirement on the number of samples by sampling the most informative scenarios. The latter reduces the computational complexity of the SA and comes up with tight accuracy guarantees for the JCC-dc-OPF problem.

### III. ALGORITHM

#### A. Idea and Sketch

Our algorithm consists of several steps: 1) constructing an inner approximation to the feasibility set; 2) generating samples outside of this approximation; and 3) finally, solving the SA Problem (5) with the generated collection of samples.

First, using the fact that the probability of a union of events is bounded from below by the maximum of individual event probabilities, we construct a lower bound on the probability of constraint feasibility $\mathbb{P}(x + \xi \in \mathcal{P})$. The latter allows us to add a set of constraints, $x \in \mathcal{P}_{\text{out}}$, so that if $x \notin \mathcal{P}_{\text{out}}$, then $\mathbb{P}(x + \xi \notin \mathcal{P}) > \eta$. In other words, if the solution $\bar{x}$ of the SA (5) with samples from the nominal distribution $\mathcal{N}(0, \Sigma)$ satisfies $\mathbb{P}(x + \xi \in \mathcal{P}) \geq 1 - \eta$, then adding additional inequalities $x \in \mathcal{P}_{\text{out}}$ does not change $\bar{x}$. Second, using the aforementioned bound, we design a polytope $\mathcal{P}_{\text{in}}(x) = \{ p : W_{\text{in}} \xi \leq b_{\text{in}}, p = x + \xi \}$ around $x$ with $W_{\text{in}}$ and $b_{\text{in}}$ independent of $x$. We present the exact expressions for $W_{\text{in}}$ and $b_{\text{in}}$ in Theorem 3.2. Fig. 1 illustrates the idea.
Then, we show that for any sample $\xi^t$ and feasible $x$, if $p = x + \xi^t \in \mathcal{P}_{1n}(x)$, then $p$ also necessarily belongs to the constraint feasibility set $\mathcal{P}$. To this end, scenarios $\xi^t : x + \xi^t \in \mathcal{P}_{1n}$ can be eliminated from the optimization problem [see (5)] without impacting the approximation accuracy.

Finally, we sample scenarios outside of the polytope $\mathcal{P}_{1n}$ using the state-of-the-art importance sampling methods [33], [34] and solve the SA problem (5) with the collection of samples generated from importance distribution. Later, in this section, we provide rigorous proof of the algorithm’s efficiency and justify its empirical performance in Section IV.

B. Inner Approximation

Consider a probability for the power generation $p$ of being inside the feasibility polytope, $p \in \mathcal{P}$

$$
P(p \in \mathcal{P}) = P(p : Wp \leq b) = P\left(p : \sum_{i=1}^{J} \omega_i p \leq b_i\right) = 1 - P\left(p : \sum_{i=1}^{J} \omega_i p > b_i\right).
$$

Therefore, if there exists $x$ such that for some $i$, $P(\omega_i^t p > b_i) > \eta$, then $P(p \in \mathcal{P}) < 1 - \eta$. Thus, to satisfy the joint chance constraint for $p = x + \xi$, $\xi \sim \mathcal{N}(0, \Sigma)$, we need

$$
\eta \geq P(\omega_i^t p > b_i) = P(\omega_i^t x + \omega_i^t x > b_i) = P\left(\frac{\omega_i^t x}{\|\Sigma^{1/2}\omega_i\|_2} > b_i - \frac{\langle \omega_i^t x \rangle}{\|\Sigma^{1/2}\omega_i\|_2}\right)
$$

$$
= P\left(\zeta > \frac{b_i - \langle \omega_i^t x \rangle}{\|\Sigma^{1/2}\omega_i\|_2}\right) = \Phi\left(\frac{\langle \omega_i^t x \rangle - b_i}{\|\Sigma^{1/2}\omega_i\|_2}\right)
$$

(6)

where $\zeta \sim \mathcal{N}(0, 1)$ and $\Phi$ is the CDF of the standard normal distribution. Notice that the function $\Phi(\cdot)$ is convex if its argument is negative; consequently, (6) is convex if $x \in \mathcal{P}$.

A set of inequalities $\eta \geq \Phi(\langle \omega_i^t x \rangle - b_i)/\|\Sigma^{1/2}\omega_i\|_2\rangle$, $1 \leq i \leq J$, defines a polytope $\mathcal{P}_{out}$ as follows:

$$
\mathcal{P}_{out} = \{ x : \omega_i^t x \leq b_i - \Delta_i, 1 \leq i \leq J \}
$$

(7)

where $\Delta_i = \|\Sigma^{1/2}\omega_i\|_2\Phi^{-1}(1 - \eta)$. Notice that $\mathcal{P}_{out}$ gives us Problem $\mathcal{P}_{out}$. Fig. 1 illustrates the idea and the geometry of $\mathcal{P}$, $\mathcal{P}_{out}$, and $\mathcal{P}_{in}$.

Theorem 3.1: The JCC-OPF problem (4) has the same set of optimal solutions as

$$
\min_{x} \mathbb{E}_{\xi \sim \mathcal{N}(0, \Sigma)} \text{cost}(x, \xi)
$$

s.t. $P(\xi \sim \mathcal{N}(0, \Sigma))(x + \xi \in \mathcal{P}) \geq 1 - \eta, 0 < \eta \leq 1/2$,

$x \in \mathcal{P}_{out}$.

(8)

Proof: A set of additional equations $x \in \mathcal{P}_{out}$ does not affect the solution of Problem (4) because the feasibility set of the chance-constrained optimization problem is inside $\mathcal{P}_{out}$, since the probability for a scenario to be outside of the polytope $\mathcal{P}$ is lower bounded by the probability of being outside of its single face. The latter determines polytope $\mathcal{P}_{out}$; see (7) for details.

C. Redundant Scenarios

Another practical consequence of the fact that the optimal solution of the CC-OPF problem is well separated from the boundary of polytope $\mathcal{P}$ is that some scenarios may be removed as they do not improve the accuracy of SA (5).

Let the optimal solution $\bar{x}$ of the problem (5) be feasible for the chance-constrained OPF problem. Then, by Theorem 3.1, it necessarily belongs to $\mathcal{P}_{out}$. Theorem 3.2 provides a mathematical justification of the latter.

Theorem 3.2: Any solution of the SA problem (5) that is feasible for the chance-constrained OPF problem (4) is also a solution of

$$
\min_{x} \text{cost}(x)
$$

s.t. $p_{\min} \leq x + \xi^t \leq p_{\max}$, $1 \leq t \leq N$

$$
|\theta_i(\xi^t) - \theta_j(\xi^t)| \leq \Delta_{ij}, (i, j) \in \mathcal{E}, 1 \leq t \leq N
$$

$$
x + \xi^t = B\theta(\xi^t), 1 \leq t \leq N
$$

$$
x \in \mathcal{P}_{out}.
$$

(9)

Proof: Let $\mathcal{P}_{1n}(x) = \{ p = x + \xi^t : \omega_i^t \xi \leq \Delta_i, \forall i \leq J \}$ with $\Delta_i = \|\Sigma^{1/2}\omega_i\|_2^{-1}(1 - \eta)$. Notice that

$$
\mathcal{P}_{1n}(x) = \{ p + (x - x_0) : p \in \mathcal{P}_{1n}(x_0)\}.
$$

Thus, for any $x \in \mathcal{P}_{out}$, if $\xi^t \in \mathcal{P}_{1n}(0)$, one immediately gets $p = x + \xi \in \mathcal{P}$. In other words, one can exclude scenario $\xi \in \mathcal{P}_{1n}(0)$ from Problem (5) if $p = x + \xi \in \mathcal{P}$. Removing them gives us Problem (9).

Throughout this article, using notation $\mathcal{P}_{1n}$ without specifying argument, we assume that it is $\mathcal{P}_{1n}(0)$. Fig. 1 illustrates the idea and the geometry of $\mathcal{P}$, $\mathcal{P}_{out}$, and $\mathcal{P}_{in}$.

Theorem 3.3 follows from the result of Calafiore and Campi [20] and establishes approximation properties of a solution of Problem (9). Assumption 1 is the main technical assumption used in the proof of Theorem 3.3.

Assumption 1: Assume that for all the possible uncertainty realizations $\xi^1, \ldots, \xi^n$, the optimization problem (9) is either infeasible or, if feasible, it attains a unique optimal solution.

Theorem 3.3: Let $\bar{x}_N$ be a unique solution of the scenario optimization problem (9) with $N$ independent identically distributed (i.i.d.) samples, so that none of the samples belong to $\mathcal{P}_{in}$. Moreover, assume that for any $N$, Assumption 1 is fulfilled. Then, for any $\delta \in (0, 1)$ and any $\eta \in (0, 1/2]$, $\bar{x}_N$ is also a solution for the CC-OPF Problem (4) with probability at least $1 - \delta$ if

$$
N \geq \left\lceil \frac{2(1 - \pi) \ln \frac{1}{\delta}}{\pi} + 2d + 2d(1 - \pi) \frac{\ln \frac{2(1 - \pi)}{\eta}}{\eta} \right\rceil
$$

where $d$ is a dimension of the space of controllable generators, and $\pi$ is the probability of a random scenario $\xi \sim \mathcal{N}(0, \Sigma)$ to belong to $\mathcal{P}_{in}$, and $\pi < 1$.

Proof: First, notice that discarding random scenarios $\xi \notin \mathcal{P}_{in}$ is equivalent to solving the SA problem with sampling $\xi$ from

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Our goal is to sample points in the area \( \{x : f(x) = 1\} \). Sampling from the nominal distribution \( \phi(x) \) is inefficient as one needs to sample from the distribution tail. The probability distribution \( \psi(x) \) is a better choice as the probability of getting \( f(x) = 1 \) is substantially higher when sampling from it.

a distribution \( D \), where \( \xi \sim D \leftrightarrow \xi \sim N(0, \Sigma) \) s.t. \( \xi \notin P_{\text{in}} \). From the theorem statement, \( 1 - \pi \) is the probability mass associated with samples in \( \xi \sim N(0, \Sigma) \) that are outside \( P_{\text{in}} \).

According to the result of Calafiori and Campi [20], for any probability \( \delta \in (0, 1) \) and any confidence threshold probability \( \varepsilon \), and dimension of the space of parameters \( d \), one has, for \( N_1 \),

\[
N_1 \geq \left[ \frac{2}{\varepsilon} \ln \frac{1}{\delta} + 2d + \frac{2}{\varepsilon} \ln \frac{2}{\varepsilon} \right]^{(10)}
\]

scenarios from \( D \) and the optimal solution \( \bar{x} \) of Problem (9); the probability of failure is bounded as

\[
P_D(\bar{x} \notin P) \leq \varepsilon
\]

with probability at least \( 1 - \delta \).

Notice that the bounds on the number of samples [see (10)] are strictly decreasing in \( \varepsilon \) for \( \varepsilon \in (0, 1) \). As scenarios in \( P_{\text{in}} \) do not cause failure, to get a probability of failure \( \eta \) according to \( N(0, \Sigma) \), we need the failure probability according to \( D \) to be at least \( \varepsilon = \eta/(1 - \pi) \). Thus, taking \( \varepsilon = \eta/(1 - \pi) \) and using monotonicity of (10), one gets the statement of the theorem.

Theorem 3.3 establishes the number of scenarios sufficient for the SA solution being feasible for the CC-OPF problem. This number significantly decreases if one can come up with a sufficiently restrictive inner approximation of the feasibility set that is close to the true chance-constrained set. Notice that without an inner approximation, i.e., for \( \pi = 0 \), one gets the result of [20, Th. 1].

D. Importance Sampling

Although a scenario optimization with scenarios that do not belong to the polytope \( P_{\text{in}} \) obeys a nice complexity bound, it requires on average \( 1/(1 - \pi) \) or more samples to generate at least one point outside of \( P_{\text{in}} \) and decreases the overall efficiency of the approach. The problem is especially challenging when dealing with rare events, i.e., the confidence level \( \eta \to 0 \).

Importance sampling is a general technique that helps to improve the efficiency of scenario generation [35]. It consists of changing the probability distribution to sample rare events with a higher probability. Fig. 2 illustrates the concept.

Unfortunately, there is no exact and time-efficient algorithm for sampling outside of a convex polytope from Gaussian distribution [15]. However, the at least one rare event (ALOE) algorithm [33], [34] proposes an elegant way for approximating the distribution of interest by sampling from a mixture of distributions.

We consider Gaussian fluctuations of power injections, \( \xi \sim N(0, \Sigma) \), with known covariance \( \Sigma \in \mathbb{R}^{n \times n} \) and aim to sample scenarios outside of \( P_{\text{in}} \) so that the probability distribution to sample from is as close as possible to the conditional Gaussian distribution \( \xi \sim N(0, \Sigma) \) s.t. \( \xi \notin P_{\text{in}} \).

The method essentially samples from a weighted mixture of conditional Gaussian distributions \( D_i \)

\[
\xi \sim D_i \iff \xi \sim N(0, \Sigma) \text{ s.t. } \omega_i^\top \xi > \Delta_i.
\]

Consider the set of inequalities \( \{\omega_i^\top \xi > \Delta_i\}_{i \leq J} \) in more detail. First, let \( \xi \sim N(0, I_n) \); then, the system is equivalent to \( \{\Sigma^{1/2} \omega_i\}^\top \xi > \Delta_i \}_{i \leq J} \).

Distribution \( D_i \) can be simulated exactly using the inverse transform method [36], [37] that admits conditional sampling \( \xi \sim N(0, \Sigma) \) s.t. \( \omega_i^\top \xi \geq \Delta_i \).

1) Sample \( z \sim N(0, I) \) and sample \( u \sim U(0, 1) \).
2) Compute \( y = \Phi^{-1}(\Phi(-\Delta_i) + u(1 - \Phi(-\Delta_i))) \).
3) Set \( \phi = \phi y + (I - \phi y^\top) z, \phi = \Sigma^{1/2} \omega_i/\|\Sigma^{1/2} \omega_i\|_2 \).
4) Set \( \xi = \Sigma^{1/2} \phi \).

The authors in [33] and [34] proposed a slightly refined version of the algorithm above that exhibits better numerical stability. We refer to the same papers for the corresponding proofs and analysis. Fig. 3 illustrates the idea: the samples can be obtained out of a certain set, e.g., feasibility polytope. In this article, \( P_{\text{in}} \) (see Fig. 1) will play the role of the set to be sample out of.

Finally, ALOE proposes to sample scenarios from a weighted mixture

\[
D = \sum_{i=1}^{J} \alpha_i D_i, \; \alpha_i \geq 0, \sum_{i=1}^{J} \alpha_i = 1,
\]
where \( \Phi \) is a CDF of the standard normal distribution. Let \( q_D(\xi) \) be a PDF of distribution \( D \); then, Theorem 3.4 established a maximal ratio of the conditional Gaussian density \( \xi \sim N(0, \Sigma) \) s.t. \( \xi \notin \mathcal{P}_{in} \) and \( q_D(\xi) \).

**Theorem 3.4:** Let \( \nu(\xi) \) and \( q_D(\xi) \) be PDFs of \( \xi \sim N(0, \Sigma) \) s.t. \( \xi \notin \mathcal{P}_{in} \) and a mixture density [see (11)], respectively. Then, for any \( \xi \notin \mathcal{P}_{in} \), we have

\[
\nu(\xi) \leq M q_D(\xi), \quad M = \frac{\sum_{i,j} \Phi\left(-\Delta_i / \|\Sigma_{i,j}^{1/2}w_i\|_2\right)}{\max_{i,j} \Phi\left(-\Delta_i / \|\Sigma_{i,j}^{1/2}w_i\|_2\right)}
\]  

where \( D \) and \( \alpha_i \) are given in (11).

**Proof:** Let \( \phi(\xi) \) be PDF of \( \xi \sim N(0, \Sigma) \). Notice that the conditional densities \( D_i \) have PDFs

\[
q_{D_i}(\xi) = \begin{cases} 
\phi(\xi)/\Phi(-\Delta_i / \|\Sigma_{i,j}^{1/2}w_i\|_2), & \text{if } \omega_i^T \xi > \Delta_i \\
0, & \text{otherwise}
\end{cases}
\]

Thus, the density of distribution \( D \) is \( \sum_{i,j} \alpha_i q_{D_i}(\xi) \). Similarly, density \( \nu(\xi) \) outside of the polytope \( \mathcal{P}_{in} \) is \( \phi(\xi)/\max_{i,j} \Phi\left(-\Delta_i / \|\Sigma_{i,j}^{1/2}w_i\|_2\right) \) for any \( \xi \notin \mathcal{P}_{in} \).

Finally, taking the ratio of \( q_D(\xi) \) and \( \nu(\xi) \) and using the value of \( \alpha_i \), we get the lemma statement.

**Theorem 3.4** implies that if a probability of a set w.r.t. measure \( q_D \) is less than or equal to \( \varepsilon \), then the probability of the same set w.r.t. measure \( p \) does not exceed \( \varepsilon/M \). The value of \( M \) has a worst case linear scaling with the number of power lines in the worst case. However, in practice, the scaling is sublinear or even constant.

### E. SA With Importance Sampling

In this section, we present an SA for the CC-OPF with a set of scenarios generated by the ALOE algorithm [33]. A particular advantage of this approach is that every scenario is generated outside of \( \mathcal{P}_{in} \). The latter substantially improves the accuracy and efficiency of the SA. In particular, we solve the following optimization problem instead of Problem (5):

\[
\min_x \quad \text{cost}(x) \\
\text{s.t. :} \quad p_{\text{min}} \leq x + \xi^t \leq p_{\text{max}}, \quad 1 \leq t \leq N \quad (13a) \\
|\theta_i(\xi^t) - \theta_j(\xi^t)| \leq \delta_{ij}, \quad (i, j) \in \mathcal{E}, \quad 1 \leq t \leq N \quad (13b) \\
x + \xi^t = B\theta(\xi^t), \quad 1 \leq t \leq N \quad (13c) \\
x \in \mathcal{P}_{\text{out}} \quad (13d) \\
\xi^1, \xi^2, \ldots, \xi^N \sim D \quad (13e)
\]

where \( D \) is the probability distribution defined by (11).

Notice that sampling from distribution \( D \) allows us to efficiently generate scenarios outside of the polytope \( \mathcal{P}_{in} \). However, they follow distribution \( D \) instead of \( \xi \sim N(0, \Sigma) \) s.t. \( \xi \notin \mathcal{P}_{in} \).

As these distributions are close to each other, Theorem 3.5 establishes efficient complexity bounds for the SA with importance sampling.

**Theorem 3.5:** Let \( \bar{x}_N \) be a unique solution of the scenario optimization problem (13), with \( N \) i.i.d. samples following distribution \( D \). Moreover, assume that for any \( N \), Assumption 1 is fulfilled. Then, for any \( \delta \in (0, 1) \) and any \( \eta \in (0, 1/2] \), \( \bar{x}_N \) is also a solution for the CC-OPF Problem (4) with probability at least \( 1 - \delta \) if

\[
N \geq \left[ 2M \frac{(1 - \pi) \ln(\frac{1}{\delta})}{\eta} + 2d + 2dM(1 - \pi) \ln(\frac{2M(1 - \pi)}{\eta}) \right]
\]

where \( d \) is a dimension of the problem and \( \pi \) is a probability of a random scenario \( \xi \) to belong to \( \mathcal{P}_{in}, \pi < 1 \), and constant \( M \) is defined by Theorem 3.4.

**Proof:** The proof is similar to the one of Theorem 3.3. Application of theorem 3.4 allows us to upper-bound the probability of an event in measure \( D \) with respect to its probability in measure \( N(0, \Sigma) \) s.t. \( \xi \notin \mathcal{P}_{in} \). We emphasize that Theorem 3.3 is a key theoretical result demonstrating a potential reduction in the number of samples. The latter is possible if scenarios are from a distribution such that a specific subset of its domain \( \mathcal{P}_{in} \) has measure zero. However, it is not possible to sample from the latter distribution correctly. On the other hand, one can propose a mixture distribution to tackle this problem. In this case, Theorem 3.4 is a practical statement that allows one to analyze the SA complexity constructed with scenarios from a mixture distribution.

### IV. Empirical Study

We compare our importance-sampling-based approach (referred to as SA-IS) with the classical SA [20] over real and simulated test cases. Empirical results justify our theoretical results: importance-sampling-based CC-OPF required much fewer samples to achieve a highly reliable solution than classical SA.

We limit the empirical setting to considering Gaussian distributions and linear feasibility constraints only. We omit detailed comparison with other importance sampling strategies [34], [38], [39] when generating scenarios because of the article’s space limitation and for the sake of empirical study clarity. A profound discussion on the efficiency of importance samplers is given in [34].

#### A. Implementation Details

We use Python 3.9. and pandapower 2.8.0 [40] on MacBook Pro (M1 Max, 64-GB RAM). In the experiments, the computational time for each case does not exceed 5 min, which makes it applicable for the operational practice. Our code is available online on GitHub.\(^1\) When solving the optimization problem, we use CVX [41] and GLPK [42] optimization solvers.

#### B. Test Cases and Numerical Results

1) **Synthetic Example:** For elucidation of the theory, we first study the efficiency of importance-sampling-based scenarios over a one-dimensional test case

\[
\max_x \quad \text{s.t.} \quad \mathcal{P}_x(x + \xi \leq a) \geq 1 - \eta, \xi \sim N(0, 1)
\]

\(^1\)Available: https://github.com/vjugor1/IS-SA
for $0 < \eta < 1/2$ and a positive constant $a$. In this case, the chance-constrained optimization problem admits an exact solution, $x^* = a - \Phi^{-1}(1 - \eta)$.

The polytopes $\mathcal{P}_{\text{out}}$ and $\mathcal{P}_{\text{in}}$ are $\{ x : x \leq x^* \}$ and $\{ \xi : \xi \leq \Phi^{-1}(1 - \eta) \}$, respectively. To illustrate the role of an approximation $\hat{x}^*$, $x^* \in \mathcal{P}_{\text{out}} \subseteq \mathcal{P}$, we consider different polytopes $\mathcal{P}_m = \{ x : x \leq b \}$ and corresponding polytope $\mathcal{P}_b = \{ \xi : \xi \leq a - b \}$, $a - b \leq \Phi^{-1}(1 - \eta)$.

Fig. 4 illustrates that the efficiency of sampling improves (i.e., less number of samples needed) as the polytope $\mathcal{P}_{\text{out}}$ better approximates $\mathcal{P}_{\text{in}}(0)$. Indeed, the probability of a sample from the nominal distribution outside of $\mathcal{P}_b$, being also outside $\mathcal{P}_{\text{in}}$ is proportional to $\Phi(a - b)$, which becomes negligible as $a - b$ decays and leads to a high number of samples. Note that we reach the standard SA when $b = a$ and get the best possible approximation for $a - b = \Phi^{-1}(1 - \eta)$.

Our approach crucially relies on a nonconservative/tight approximation of the joint chance-constrained feasible set and is crucial for the success of the importance sampling approach, as discussed next for power grid test cases.

2) Power Grid Test Cases: We address the scenario-based CC-OPF problem under Gaussian fluctuations in four different test cases (IEEE 30-, IEEE 57-, IEEE 118-, and IEEE 300-bus systems with 30, 57, 118, and 300 buses/nodes, respectively). For all the considered cases, we assume that the power generation and consumption level fluctuate with the standard deviation of 0.07 of its nominal value.

To demonstrate the practical benefits of importance sampling (SA-IS) versus standard SA, we compare their respective number of samples $N$ needed to solve CC-OPF for different test cases, given prescribed $\eta$ and $\delta$. As stated in theorems in Section III, $1 - \eta$ is the required confidence threshold for constraint feasibility (of joint chance constraint) by a solution, while $1 - \delta$ is the required reliability of the SA’s solution obtained.

We consider the setting of fixed confidence threshold $1 - \eta$ and $N$, the number of samples used in CC-OPF (with SA-IS or SA), and determine their effect on empirical reliability $1 - \hat{\delta}$ using Algorithm 1. In Algorithm 1, we independently form $L = 100$ different SA problems, each with $N$ scenarios constructed using SA or SA-IS. We, thus, obtain $L$ different solutions $(x_{N}^l, l = 1, \ldots, L$. Using separately generated $10^4$ Monte Carlo samples of uncertainty for each $(x_{N}^l)$, we estimate each solution’s probability of constraint satisfaction $(\hat{P}_N)$.

\begin{algorithm}[h]
\begin{algorithmic}
\Require $L$—number of trials, $\text{dc-OPF}$ problem parameters, $\eta$—confidence level, $N_0$—initial size of scenario approximation, $N_{\max}$—maximum size of scenario approximation
\State $N \leftarrow N_0$
\State $\delta$—storage for $\hat{\delta}_N$
\While{$N \leq N_{\max}$}
\State $C_N \leftarrow 0$—feasibility counter
\State $l \leftarrow 1$
\While{$l \leq L$}
\State Obtain $(x_{N}^l)$—scenario approximation with $N$ samples (using SA-IS or SA)
\State Estimate constraint satisfaction probability $(\hat{P}_N)$ using Monte Carlo samples.
\If{$(\hat{P}_N) \geq 1 - \eta$} $C_N \leftarrow C_N + 1$
\EndIf
\State $n \leftarrow n + N_{\max}/10$
\EndWhile
\State \Return $\hat{\delta}_N$
\end{algorithmic}
\end{algorithm}

The estimated reliability $1 - \hat{\delta}$ is then given by the fraction of $L$ $(x_{N}^l)$ solutions with $(\hat{P}_N) \geq 1 - \eta$.

Table I summarizes the number of samples needed using SA and SA-IS to ensure an empirical reliability of 0.99, for two confidence thresholds $1 - \eta$ (0.95 and 0.99). Our experiments show that SA-IS requires much fewer samples to provide a reliable CC-OPF feasible solution while maintaining the same cost for each test case compared to CC-OPF with SA. The improvement in the number of scenarios is bigger for the higher confidence threshold value ($1 - \eta = 0.99$).

We illustrate the dependence between empirical reliability $1 - \hat{\delta}$ and $N$ over a range of values for the IEEE 118-bus system in Fig. 5(c). Here, we keep a confidence threshold of joint chance constraint feasibility $1 - \eta = 0.99$. In addition to SA-IS and SA, we also consider an intermediate setting—scenario approximation with polygon-set (SA-O), as described in (9).

Compared to SA, SA-O includes the inner approximation constraints $x \in \mathcal{P}_{\text{out}}$. However, unlike SA-IS, SA-O does not involve importance-sampling-based samples. It is clear from Fig. 5(c) that at all values of $N$, SA-IS’s reliability $1 - \hat{\delta}$ is much higher than that of SA or SA-O. In fact, for $3500 \leq N \leq 4000$, the SA-IS is around ten times more reliable than SA. As a result, the solution of SA-IS also becomes conservative faster (highlighted by the decrease in slope at higher reliability). On the other hand, SA and SA-O have almost similar reliability, which follows from Theorem 3.2.

Finally, for the same $L$ optimization instances used for Fig. 5(c), we present boxplots for the spread of $(\hat{P}_N)$ for different $N$, in Fig. 5(d). Here, $(\hat{P}_N)$ is the probability of constraint satisfaction, empirically computed using $10^4$ Monte Carlo samples of uncertainty. It is clear from the boxplot that the reliability of solution $1 - \hat{\delta}$ is higher for SA-IS compared to SA for all values of $N$, with SA-IS being much higher than that of SA or SA-O. In fact, for $3500 \leq N \leq 4000$, the SA-IS is around ten times more reliable than SA. As a result, the solution of SA-IS also becomes conservative faster (highlighted by the decrease in slope at higher reliability). On the other hand, SA and SA-O have almost similar reliability, which follows from Theorem 3.2.

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TABLE I
NUMBER OF SA-IS AND SA SAMPLES REQUIRED TO REACH RELIABILITY LEVEL 1 − \( \hat{\delta} \) = 0.99 IN CC-OPF WITH CONFIDENCE THRESHOLD 1 − \( \eta \)

| Case   | \( \eta \) | SA No | SA Cost     | SA \( (\hat{P}_N) \)  | IS-SA No | IS-SA Cost     | IS-SA \( (\hat{P}_N) \)  | DC-OPF Cost |
|--------|-----------|-------|-------------|-------------------------|-----------|----------------|-------------------------|-------------|
| grid30 | 0.05      | 160   | 5.89e+03    | 9.82e-01±7.09e-03       | 60        | 5.87e+03       | 9.80e-01±8.86e-03       | 5.67e+03    |
| grid57 | 0.05      | 210   | 2.52e+04    | 9.78e-01±8.95e-03       | 160       | 2.52e+04       | 9.89e-01±7.71e-03       | 2.50e+04    |
| grid118| 0.05      | 1300  | 8.72e+04    | 9.68e-01±4.18e-03       | 1050      | 8.72e+04       | 9.68e-01±4.16e-03       | 8.48e+04    |
| grid300| 0.05      | 1550  | 4.72e+05    | 9.63e-01±4.36e-03       | 1250      | 4.72e+05       | 9.62e-01±3.97e-03       | 4.71e+05    |
| grid30 | 0.01      | 800   | 5.94e+03    | 9.96e-01±1.83e-03       | 300       | 5.96e+03       | 9.99e-01±6.57e-04       | 5.67e+03    |
| grid57 | 0.01      | 1300  | 2.52e+04    | 9.96e-01±1.55e-03       | 300       | 2.53e+04       | 9.97e-01±1.88e-03       | 2.50e+04    |
| grid118| 0.01      | 6000  | 8.74e+04    | 9.93e-01±1.16e-03       | 3600      | 8.74e+04       | 9.94e-01±9.11e-04       | 8.48e+04    |
| grid300| 0.01      | 9000  | 4.72e+05    | 9.93e-01±8.42e-04       | 4500      | 4.72e+05       | 9.92e-01±9.53e-04       | 4.71e+05    |

The number of scenarios required for target reliability were obtained empirically, iterating over a predefined grid for \( N \) with the step of 10. The confidence threshold 1 − \( \eta \) is estimated using Monte Carlo samples (out of sample) and empirical reliability is computed by averaging over \( L = 100 \) independent CC-OPF problem instances, as stated in algorithm 1. The costs of solutions obtained are depicted alongside with the corresponding costs of deterministic DC-OPF solution. It is clear that SA-IS requires much less samples compared to SA, while maintaining the same reliability of the solution.

![Empirical reliability](image)

Fig. 5. Empirical reliability (1 − \( \hat{\delta} \)) and the spread of constraint feasibility (\( \hat{\hat{P}}_N \)) for CC-OPF (1 − \( \eta \) = 0.99) in the IEEE 300-bus (IEEE 118-bus, respectively) system. The three cases correspond to samples in CC-OPF being drawn by SA, SA-IS, and SA-O. The empirical estimates are computed with \( L = 100 \) optimization instances (for 1 − \( \hat{\delta} \)), and \( N_{MC} = 10^4 \) Monte Carlo samples for each instance to determine constraint validation (for boxplot of (\( \hat{\hat{P}}_N \))), as described in Algorithm 1. Colored boxes stands for the 25–75% interquantile range (IQR). Diamonds shows samples outside of the ±1.5 \( \times \) IQR. Note that both in reliability and the IQR of constraint feasibility, SA-IS requires much less number of samples compared to SA or SA-O. (a) Empirical reliability (1 − \( \hat{\delta} \)) versus number of samples in CC-OPF (\( N \)) for the IEEE 300-bus system. (b) Spread of probability of constraint feasibility (\( \hat{\hat{P}}_N \)) versus number of samples in CC-OPF (\( N \)) for the IEEE 300-bus system. (c) Empirical reliability (1 − \( \hat{\delta} \)) versus number of samples in CC-OPF (\( N \)) for the IEEE 118-bus system. (d) Spread of probability of constraint feasibility (\( \hat{\hat{P}}_N \)) versus number of samples in CC-OPF (\( N \)) for the IEEE 118-bus system.

Carlo separately generated samples of Algorithm 1 [see (9)]. Note that SA-IS reaches a higher reliability level (1 − \( \hat{\delta} \)) at a fewer number of samples \( N \), observed when almost all of the box is above the 1 − \( \eta \) threshold. The boxplots also indicate that the variance in the obtained solution’s chance-constrained feasibility reduces faster for SA-IS, noted by thinner boxplots and lack of outliers, compared to SA and SA-O.

Finally, we address the question if the total computational time for SA-IS is better than for classical SA. First, we address the amount of time required to generate samples for classical Monte Carlo SA, and those for IS-SA using the importance sampling technique. Next, we analyze how too much time totally is required to obtain 1 − \( \hat{\delta} \) = 0.99-reliable solution. For such an experiment, we consider generating samples for the IEEE 30-bus power system.

In order to compare preprocessing step, i.e., sampling generation step, we generate \( N = 50, 150, 250, 350, \) and 450 samples with ALOE, and the same amount of samples from multivariate normal distribution. For each \( N \), we repeat generation 15 times to obtain statistics on execution time. Similarly, with the same strategy, we generate the samples for each \( N \) and solve the SA and collect execution time, repeating for 15 times. The

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execution time of the preprocessing step comparison is visualized in Fig. 6(a). From here, one can observe that ALOE is a more time consuming procedure. However, one can observe that the time required to solve SAs together with preprocessing does not differ significantly for IS-SA and SA and [see Fig. 6(b)]. The latter figure also shows that the computational time required to obtain the same reliability level is five times less for SA-IS (the number of samples is taken from Table I. IEEE 30-bus case, \( \eta = 0.05 \)).

V. CONCLUSION

In this article, we investigated the SA for the CC-OPF. We showed that the importance sampling technique used for scenario generation leads to a better accuracy. Moreover, the numerical complexity is much lower in theory and practice for stochastic OPF.

The theoretical study indicates benefit from using violative samples. The results are presented and proven alongside with numerical experiments that indicate significant reduction of sample size in SA required to reach a high reliability level. Finally, the approach can be extended to automated real-time control of bulk power systems.

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