A systematic phenomenological study of the $\cos 2\phi$ asymmetry in unpolarized semi-inclusive DIS

Vincenzo Barone, Alexei Prokudin, and Bo-Qiang Ma

1Di.S.T.A., Università del Piemonte Orientale “A. Avogadro”, and INFN, Gruppo Collegato di Alessandria, 15100 Alessandria, Italy
2Dipartimento di Fisica Teorica, Università di Torino, Via P. Giuria 1, I-10125 Torino, Italy
3School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

We study the $\cos 2\phi$ azimuthal asymmetry in unpolarized semi-inclusive DIS, taking into account both the perturbative contribution (gluon emission and splitting) and the non-perturbative effects arising from intrinsic transverse motion and transverse spin of quarks. In particular we explore the possibility to extract from $\langle \cos 2\phi \rangle$ some information about the Boer–Mulders function $h_1^\perp$, which represents a transverse–polarization asymmetry of quarks inside an unpolarized hadron. Predictions are presented for the HERMES, COMPASS and JLab kinematics, where $\langle \cos 2\phi \rangle$ is dominated by the kinematical higher–twist contribution, and turns to be of order of few percent. We show that a larger asymmetry in $\pi^-\pi^+$ production, compared to $\pi^+\pi^-$ production, would represent a signature of the Boer–Mulders effect.

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I. INTRODUCTION

Transverse spin and transverse momentum of quarks are by now universally recognized as two essential ingredients of the structure of hadrons. Their correlations are described by a number of $k_T$-dependent distribution functions which give rise to various observables in hard hadronic processes \[1\]. Among these distributions a special role is played by the Sivers function $f_{1T}^T(x,k_T)$ $[2,3]$, which represents an azimuthal asymmetry of unpolarized quarks inside a transversely polarized hadron, and by its chirally-odd partner $h_1^\perp(x,k_T)$, the Boer–Mulders function $[4]$, which represents a transverse–polarization asymmetry of quarks inside an unpolarized hadron. While the Sivers function is responsible of single–spin asymmetries in transversely polarized semi-inclusive DIS (SIDIS), the Boer–Mulders function generates azimuthal asymmetries in unpolarized processes. Boer $[5]$ suggested in fact that $h_1^\perp$ could explain the large $cos 2\phi$ asymmetries observed in unpolarized $\pi^-\pi^+$ production $[6,7]$. This conclusion was confirmed by more refined model calculations in $[8,9]$. An even larger effect is expected in $p\bar{p}$ Drell-Yan production $[10]$, a process to be studied by the PAX experiment at GSI-HESR $[11]$.

A $\cos 2\phi$ asymmetry also occurs in unpolarized SIDIS, where it was already investigated at high $Q^2$ by the EMC experiment $[12]$ and by ZEUS $[13]$, and is now being measured in the low–medium $Q^2$ region by many experimental collaborations (HERMES, COMPASS, JLab). In SIDIS the Boer–Mulders distribution couples to a fragmentation function, the Collins function $H_1^\perp$ $[14]$, which describes the fragmentation of transversely polarized quarks into polarized hadrons. The Boer–Mulders mechanism, however, is not the only source of a $\cos 2\phi$ asymmetry in SIDIS. Two other contributions to this asymmetry arise from non–collinear kinematics at order $k_T^2/Q^2$ (the so-called Cahn effect) $[15,16]$, and perturbative gluon radiation (i.e. order $\alpha_s$ QCD processes) $[17,18,19,20]$. It is clear that in order to get a correct understanding of the experimental results on the $\cos 2\phi$ asymmetry, and possibly to determine the Boer–Mulders function, which is one of the ultimate goals of global fits of transverse spin data, all contributing effects should be taken into account and reliably estimated (previous works $[21,22]$, including a preliminary study of ours $[23]$, considered only some of these effects). The purpose of this paper is indeed to investigate the three sources (Boer–Mulders, kinematical higher twist, order–$\alpha_s$ perturbative QCD) of the $\cos 2\phi$ asymmetry in leptoproduction, and to evaluate their contributions in the HERMES, COMPASS and JLab regimes. In particular, we will identify some signatures of the Boer–Mulders effect and the most advantageous experimental conditions to study it (for an early attempt in this direction see $24$). More generally, the phenomenological frame we will establish here is intended to be an aid in interpreting the results of the future measurements of $\langle \cos 2\phi \rangle$. To this aim, we tried to reduce the model dependence of our analysis to the minimum: thus $h_1^\perp$ is parametrized using a simple relation with the Sivers function based on the impact–parameter approach and on lattice results, and borrowing $f_{1T}^T$ from a recent fit to SIDIS data.

* Email address: prokudin@to.infn.it
† Email address: mabq@phy.pku.edu.cn
II. THEORETICAL FRAMEWORK

The process we are interested in is unpolarized SIDIS:

\[ l(\ell) + p(P) \rightarrow l'(\ell') + h(P_h) + X(P_X). \quad (1) \]

The SIDIS cross section is expressed in terms of the invariants

\[ x = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot \ell}, \quad z = \frac{P \cdot P_h}{P \cdot q}, \quad (2) \]

where \( q = \ell - \ell' \) and \( Q^2 \equiv -q^2 \). We choose the reference frame where the virtual photon and the target proton are collinear and directed along the \( z \) axis, with the photon moving in the positive \( z \) direction (Fig. 1). We denote by \( k_T \) the transverse momentum of the quark inside the proton, and by \( P_T \) the transverse momentum of the hadron \( h \). The transverse momentum of \( h \) with respect to the direction of the fragmenting quark will be called \( p_T \). All azimuthal angles are referred to the lepton scattering plane (we call \( \phi \) the azimuthal angle of the hadron \( h \), see Fig. 1).

Taking the intrinsic motion of quarks into account, the \( \phi \)-symmetric part of the SIDIS differential cross section reads at zero-th order in \( \alpha_s \)

\[ \frac{d^5 \sigma_{\text{sym}}^{(0)}}{dx dy dz d^2P_T} = \frac{2\pi \alpha_{\text{em}}^2 s}{Q^4} \sum_a e_a^2 x [1 + (1 - y)^2] \times \int d^2k_T \int d^2p_T \delta^2(P_T - zk_T - p_T) f_1^a(x, k_T^2) D_1^a(z, p_T^2), \quad (3) \]

where \( f_1^a(x, k_T^2) \) is the unintegrated number density of quarks of flavour \( a \) and \( D_1^a(z, p_T^2) \) is the transverse-momentum dependent fragmentation function of quark \( a \) into the final hadron. The non-collinear factorization theorem for SIDIS has been proven by Ji, Ma and Yuan [25] for \( P_T \ll Q \).

Long time ago Cahn [13, 16] pointed out that the non-collinear kinematics generates a \( \cos 2\phi \) contribution to the unpolarized SIDIS cross section, which has the form

\[ \frac{d^5 \sigma_{\phi}^{(0)}}{dx dy dz d^2P_T} \bigg|_{\cos 2\phi} = \frac{8\pi \alpha_{\text{em}}^2 s}{Q^4} \sum_a e_a^2 x (1 - y) \times \int d^2k_T \int d^2p_T \delta^2(P_T - zk_T - p_T) \times \frac{2 (k_T \cdot h)^2 - k_T^2}{Q^2} f_1^a(x, k_T^2) D_1^a(z, p_T^2) \cos 2\phi, \quad (4) \]

where \( h \equiv P_T / P_T \). Notice that this contribution is of order \( k_T^2 / Q^2 \), hence it is a kinematical higher twist effect.
It is clear that, contrary to the lowest order process light-cone momentum fractions, which, in the collinear configuration with massless partons, are given by
\[ P^\gamma \] in which the quark emits a hard gluon or those initiated by gluons: \[ P^\gamma \] with large momentum, even starting from a collinear configuration. Such a contribution dominates the production of hadrons.

\[ \frac{d^5\sigma^{(0)}}{dx dy dz d^2P_T} \bigg|_{\cos 2\phi} = \frac{4\pi\alpha_s^2}{Q^4} \sum_a e_a^2 x(1 - y) \]
\[ \times \int d^2k_T \int d^2p_T \delta^2(P_T - 2k_T - p_T) \]
\[ \times \frac{2h \cdot k_T h \cdot p_T - k_T \cdot p_T}{zM_M h} h_{ij}^\perp(x, k_T^2) H_{ij}^\perp(z, p_T^2) \cos 2\phi, \]

where \( M \) is the mass of the nucleon and \( M_h \) is the mass of the produced hadron. It should be noticed that this is a leading-twist contribution, not suppressed by inverse powers of \( Q \).

In our analysis we include both the non-perturbative and the perturbative contributions. We consider now the contributions of order \( \alpha_s \), following the approach of \[ 20 \]. The relevant partonic processes, shown in Fig. 2, are those in which the quark emits a hard gluon or those initiated by gluons:
\[ \gamma^* + q \rightarrow q + g \quad \gamma^* + q \rightarrow g + q \quad \gamma^* + g \rightarrow q + \bar{q}. \]

It is clear that, contrary to the lowest order process \( \gamma^* + q \rightarrow q \), the final parton can have a large transverse momentum, even starting from a collinear configuration. Such a contribution dominates the production of hadrons with large \( P_T \) values.

We introduce the parton variables \( x' \) and \( z' \), defined similarly to the hadronic variables \( x \) and \( z \),
\[ x' = \frac{Q^2}{2k \cdot q} = \frac{x}{\xi}, \quad z' = \frac{k \cdot k'}{k \cdot q} = \frac{z}{\zeta}, \]
where \( k \) and \( k' \) are the four-momenta of the incident and fragmenting partons, respectively. \( \xi \) and \( \zeta \) are the usual light-cone momentum fractions, which, in the collinear configuration with massless partons, are given by \( k = \xi P \) and \( P_h = \zeta k' \). We denote by \( \kappa_T \) the transverse momentum, with respect to the \( \gamma^* \) direction, of the final fragmenting parton, \( P_T = \kappa_T \).

The semi-inclusive DIS cross section, in the collinear QCD parton model, can be written in general as:
\[ \frac{d^5\sigma}{dx dy dz d^2P_T} = \sum_{i,j} \int dx' dx' d^2\kappa_T d\xi d\zeta \, \delta(x - \xi x') \, \delta(z - \zeta z') \, \delta^2(P_T - \kappa_T) \]
\[ \times f_i^Q (\xi, Q^2) \frac{d\sigma_{ij}}{dx' dy dz' d^2\kappa_T} D_1^Q (\zeta, Q^2). \]

To first order in \( \alpha_s \) the partonic cross section is given by
\[ \frac{d\sigma_{ij}}{dx' dy dz' d^2\kappa_T} = \frac{\alpha_s^2 e_i^2 e_j^2}{16\pi^2 Q^4} y L_{\mu\nu} M_{ij} \delta \left( \kappa_T^2 - \frac{z}{x'}(1 - x')(1 - z')Q^2 \right), \]

where \( ij \) denote the initial and fragmenting partons, \( ij = gg, gq, qg \). Inserting the above expression into Eq. \[ 5 \] yields, for the \( \mathcal{O}(\alpha_s) \) cross section \[ 20 \]:
\[ \frac{d^5\sigma^{(1)}}{dx dy dz d^2P_T} = \frac{\alpha_s^2 e_i^2 e_j^2}{16\pi^2 Q^4} \int_0^1 dx' P_T^2 + z_h(1 - x')Q^2 \sum_{i,j} f_i^Q (x', Q^2) \, L_{\mu\nu} M_{ij} \, D_1^Q \left( \frac{z + \frac{x' P_T^2}{z_h(1 - x')Q^2}}{Q^2} \right) \]
with $\text{[20, 26]}$

\[
L_{\mu \nu} M_{qq}^{\mu \nu} = \frac{64\pi\alpha_s}{3} Q^2 \frac{(l \cdot k)^2 + (l' \cdot k')^2 + (l \cdot k')^2 + (l \cdot k')^2}{(k \cdot k'')(k' \cdot k'')}
\]

\[
= \frac{64\pi\alpha_s}{3} Q^2 \left\{ [1 + (1 - y)^2] \left[ (1 - x')(1 - z') + \frac{1 + (x'z')^2}{(1 - x')(1 - z')} \right] + 8 x'z' (1 - y) \right. 
\]

\[
- 4 \sqrt{\frac{x'z' (1 - y)}{(1 - x')(1 - z')}} (2 - y) \left[ x'z' + (1 - x')(1 - z') \right] \cos \phi 
\]

\[
+ 4 x'z'(1 - y) \cos 2\phi \right\}, \tag{11}
\]

\[
L_{\mu \nu} M_{gg}^{\mu \nu} = \frac{64\pi\alpha_s}{3} Q^2 \frac{(l \cdot k'')^2 + (l' \cdot k')^2 + (l' \cdot k')^2 + (l \cdot k')^2}{(k \cdot k'')(k' \cdot k'')}
\]

\[
= \frac{64\pi\alpha_s}{3} Q^2 \left\{ [1 + (1 - y)^2] \left[ (1 - x')(1 - z') + \frac{1 + x'^2(1 - z')^2}{(1 - x')(1 - z')} \right] + 8 x'(1 - y)(1 - z') \right. 
\]

\[
+ 4 \sqrt{\frac{x'(1 - y)(1 - z')}{(1 - x')(1 - z')}} (2 - y) \left[ x'(1 - z') + (1 - x')z' \right] \cos \phi 
\]

\[
+ 4 x'(1 - y)(1 - z') \cos 2\phi \right\}, \tag{12}
\]

\[
L_{\mu \nu} M_{gq}^{\mu \nu} = \frac{64\pi\alpha_s}{3} Q^2 \frac{(l \cdot k'')^2 + (l' \cdot k')^2 + (l' \cdot k')^2 + (l' \cdot k')^2}{(k \cdot k'')(k' \cdot k'')}
\]

\[
= \frac{64\pi\alpha_s}{3} Q^2 \left\{ [1 + (1 - y)^2] \left[ x'^2(1 - x')^2 + \frac{z'^2 + (1 - z')^2}{z'(1 - z')} \right] + 16 x'(1 - x')(1 - y) \right. 
\]

\[
- 4 \sqrt{\frac{x'(1 - x')(1 - y)}{z'(1 - z')}} (2 - y) (1 - 2x')(1 - 2z') \cos \phi 
\]

\[
+ 8 x'(1 - x')(1 - y) \cos 2\phi \right\}, \tag{13}
\]

where we have explicitly written the scalar products in terms of $x'$, $y$, $z'$ and $\phi$. Notice the appearance of the $\cos \phi$ and $\cos 2\phi$ terms: $\phi$ is the azimuthal angle of the fragmenting partons, which, in a collinear configuration, coincides with the azimuthal angle of the detected final hadron. Since large values of $P_T$ cannot be generated by the modest amount of intrinsic motion of quarks, we expect that Eq. (10) will dominantly describe the cross sections for the lepto-production of hadrons with $P_T$ values above 1 GeV.

The asymmetry determined experimentally is defined as

\[
\langle \cos 2\phi \rangle = \frac{\int d\sigma \cos 2\phi}{\int d\sigma}. \tag{14}
\]

The integrations are performed over the measured ranges of $x, y, z$, with a lower cutoff $P_T^{\text{cut}}$ on $P_T$, which represents the minimum value of $P_T$ of the detected charged particles. Up to order $\alpha_s$ one has

\[
\langle \cos 2\phi \rangle = \frac{\int d\sigma(0) \cos 2\phi}{\int d\sigma(0)} + \frac{\int d\sigma(1) \cos 2\phi}{\int d\sigma(1)}, \tag{15}
\]

where $\sigma(0)$ ($\sigma(1)$) is the lowest order (first order) in $\alpha_s$ cross section. In the non perturbative region ($P_T \ll 1$ GeV) one expects $\sigma(0) \gg \sigma(1)$, thus to a very good approximation we have

\[
\langle \cos 2\phi \rangle \approx \frac{\int d\sigma(0) \cos 2\phi}{\int d\sigma(0)}, \tag{16}
\]
Explicitly the numerator and the denominator are given by

\[
\int d\sigma^{(0)} \cos 2\phi = \frac{4\pi \alpha_{em}^2 s}{Q^4} \int \int \int \int \int \frac{e_a^2 x(1-y)}{A[f_1^a, D_1^a]} + \frac{1}{2} B[h_1^{+a}, H_1^{-a}] ,
\]

(17)

\[
\int d\sigma^{(0)} = \frac{2\pi \alpha_{em}^2 s}{Q^4} \int \int \int \int \int \frac{e_a^2 x[1 + (1-y)^2]}{C[f_1^a, D_1^a]},
\]

(18)

where

\[
\int \int \int \int \equiv \int_{P_T^{max}} dP_T P_T \int^{x_2} dx \int^{y_2} dy \int^{z_2} dz
\]

(19)

and \((\chi\) is the angle between \(P_T\) and \(k_T\))

\[
A[f_1^a, D_1^a] = \int d^2 k_T \int d^2 P_T \delta^2(P_T - z k_T - p_T)
\]

\[
\times \frac{2(k_T \cdot h)^2 - k_T^2}{Q^2} f_1^a(x, k_T^2) D_1^a(z, p_T^2) \cos 2\phi
\]

\[
= \int_0^\infty dk_T k_T \int_0^{2\pi} d\chi \frac{2 k_T^2 \cos^2 \chi - k_T^2}{Q^2}
\]

\[
\times f_1^a(x, k_T^2) D_1^a(z, |P_T - z k_T|^2),
\]

(20)

\[
B[h_1^{+a}, H_1^{-a}] = \int d^2 k_T \int d^2 P_T \delta^2(P_T - z k_T - p_T)
\]

\[
\times \frac{2 h \cdot k_T h \cdot P_T - k_T \cdot P_T}{z M_M h} h_1^{+a}(x, k_T^2) H_1^{-a}(z, p_T^2)
\]

\[
= \int_0^\infty dk_T k_T \int_0^{2\pi} d\chi \frac{k_T^2 + (P_T/z) k_T \cos \chi - 2 k_T^2 \cos^2 \chi}{M_M h}
\]

\[
\times h_1^{+a}(x, k_T^2) H_1^{-a}(z, |P_T - z k_T|^2),
\]

(21)

\[
C[f_1^a, D_1^a] = \int d^2 k_T \int d^2 P_T \delta^2(P_T - z k_T - p_T) f_1^a(x, k_T^2) D_1^a(z, p_T^2)
\]

\[
= \int_0^\infty dk_T k_T \int_0^{2\pi} d\chi f_1^a(x, k_T^2) D_1^a(z, |P_T - z k_T|^2).
\]

(22)

### III. PARAMETRIZATIONS OF THE BOER–MULDERS AND COLLINS FUNCTIONS

While the perturbative contribution to the \(\cos 2\phi\) asymmetry contain only standard distribution and fragmentation functions and therefore can be evaluated in a straightforward way, the non-perturbative contributions involve the Boer–Mulders distribution, at present totally unknown, and the Collins fragmentation function, for which we have some independent information coming from single-spin asymmetries in SIDIS and from \(e^+e^-\) data.

Let us first consider the Boer–Mulders distribution. In order to estimate it, we resort to the impact–parameter approach \([27, 28, 29, 30]\), which establishes a general link between the anomalous tensor magnetic moment of quarks \(\kappa_T^q\) and the transverse deformation of quark distributions in position space. This distortion translates, in a model-dependent way, into a single-spin asymmetry for transversely polarized quarks in unpolarized hadrons, and one finally finds a correlation between \(h_1^{+q}\) and \(\kappa_T^q\) (for explicit realizations of this mechanism in spectator models and detailed discussions of its physical basis, see Ref. 31 and Ref. 32),

\[
h_1^{+q} \sim -\kappa_T^q.
\]

Since a similar connection exists between the Sivers function \(f_{1T}^{+q}\) and the anomalous magnetic moment \(\kappa_T^q\),

\[
f_{1T}^{+q} \sim -\kappa_T^q,
\]
For the Collins function we use the parametrization of [43], based on a combined analysis of SIDIS and variance with f negative (this qualitative expectation is also supported by large–u Thus the moment have been estimated by a lattice calculation in Ref. [34] and are found to be the sizes of h\textsuperscript{1}\textperp and f\textsubscript{1T} are expected to have the same sign, and in particular to be both negative (this qualitative expectation is also supported by large–N\textsubscript{c} arguments [35] and by various model calculations [36, 37, 38, 39]). Inserting the values of \(\kappa\) and \(\kappa_f\) in (23), one has

\[ h_1^{+-}(x, k_T^2) = \frac{\kappa}{\kappa^u} f_1^{+-}(x, k_T^2) \]

(23)

The contributions of u and d quarks to the anomalous magnetic moment of the proton (\(\kappa^p\)) and of the neutron (\(\kappa^n\)) can be extracted from the experimental values \(\kappa^p = 1.79\) and \(\kappa^n = -1.91\) [33] by means of \(\kappa^{p,n} = \sum_q e_q \kappa^q\) (neglecting the s quark contribution), and are given by \(\kappa^u \simeq 1.67\) and \(\kappa^d \simeq -2.03\): note the opposite sign, which explains the experimentally observed fact that \(f_1^{+u} < 0\) while \(f_1^{+d} > 0\). The flavor contributions to the anomalous tensor magnetic moment have been estimated by a lattice calculation in Ref. [34] and are found to be \(\kappa_f^u \simeq 3\), \(\kappa_f^d \simeq 1.9\). Thus, at variance with \(f_1^{+}\), we expect the u and the d components of \(h_1^{\perp}\) to have the same sign, and in particular to be both negative (this qualitative expectation is also supported by large–N\textsubscript{c} arguments [35] and by various model calculations [36, 37, 38, 39]). Inserting the values of \(\kappa\) and \(\kappa_f\) in (23), one has

\[ h_1^{+u} \simeq 1.80 f_1^{+u}, \quad h_1^{+d} = -0.94 f_1^{+d} \]

(24)

Thus the u component of \(h_1^{\perp}\) is about twice as large as the corresponding component of \(f_1^{+}\), while the d components of \(h_1^{\perp}\) and \(f_1^{+}\) have approximately the same magnitude and opposite sign. To parametrize the Boer–Mulders function we use the Ansatz [24] and get the Sivers function from a fit of single–spin asymmetry data [40]. The parametrization of Ref. [40] for \(f_1^{\perp}\) is

\[ f_1^{\perp}(x, k_T^2) = \rho_q(x) \eta(k_T) f_1^{\perp}(x, k_T^2), \]

(25)

where

\[ \rho_q(x) = A_q x^\alpha (1 - x)^\beta \left( a_q + b_q \right) \left( a_q + b_q \right) / a_q b_q, \]

\[ \eta(k_T) = \frac{2MM_0}{k_T^2 + M_0^2}. \]

(26)

(27)

Here \(A_q\), \(a_q\), \(b_q\) and \(M_0\) are free parameters, \(f_1^{\perp}(x, k_T^2)\) is the \(k_T\)-dependent unpolarized distribution function, which we assume to have a Gaussian behavior in \(k_T\):

\[ f_1^{\perp}(x, k_T^2) = e^{-k_T^2 / \langle k_T^2 \rangle} / \pi \langle k_T^2 \rangle. \]

(28)

Notice that \(f_1^{\perp}\), being a quark spin asymmetry, must satisfy a positivity bound. This bound and the valence number sum rules are automatically fulfilled by the parametrization of Ref. [40]. The average value of the intrinsic transverse momentum of quarks is taken from the SIDIS fit of Ref. [41], and is given by \(\langle k_T^2 \rangle = 0.25\) GeV\textsuperscript{2}. This value is assumed to be constant and flavor-independent. It is worth recalling that the Gaussian behavior of \(k_T\)–dependent distribution functions is supported by a recent lattice study [42], which finds a root mean squared transverse momentum very close to the one determined in Ref. [41] and used here. The fitted parameters in Eqs. (26,27) are given in Table II [40].

Let us now turn to the Collins function. We will distinguish the favored and the unfavored fragmentation functions according to the following general relations

\[ D_{\pi^+/u} = D_{\pi^+/d} = D_{\pi^-/d} = D_{\pi^-/u} = D_{\text{iso}}, \]

\[ D_{\pi^+/d} = D_{\pi^+/\bar{u}} = D_{\pi^-/u} = D_{\pi^-/\bar{d}} = D_{\pi^+/s} = D_{\pi^-/\bar{s}} = D_{\text{unf}}. \]

(29)

(30)

For the Collins function we use the parametrization of [43], based on a combined analysis of SIDIS and e\textsuperscript{+}e\textsuperscript{-} data:

\[ H_1^{\perp q}(z, p_T^2) = \rho_q^C(z) \eta_C(p_T) D_1^q(z, p_T^2), \]

(31)

| \(A_u\) | \(-0.32 \pm 0.11\) | \(A_d\) | \(1.00 \pm 0.12\) |
| \(a_u\) | \(0.29 \pm 0.35\) | \(a_d\) | \(1.16 \pm 0.47\) |
| \(b_u\) | \(0.53 \pm 3.58\) | \(b_d\) | \(3.77 \pm 2.59\) |
| \(M_0^2\) | \(0.32 \pm 0.25 \text{ GeV}^2\) |

TABLE I: Best fit values of the parameters of the Sivers function.
In these kinematical regions the cross section is dominated by the Boer-Mulders contributions to the experiment, turning to our predictions for these experiments. We let the coefficients $A_q^C$ to be flavor independent ($q = u, d$), while all the exponents $\gamma, \delta$ and the dimensional parameter $M_C$ are taken to be flavour independent. The parameterization is devised in such a way that the Collins function satisfies the positivity bound (remember that $H_{T\perp}$ is essentially a transverse momentum asymmetry). In Eq. (31) $D_1(z, p_T^2)$ is the $p_T$–dependent unpolarized fragmentation, that we take to be given by

$$D_1^q(z, p_T^2) = D_1^q(z) \frac{e^{-p_T^2/(p_T^2)}}{\pi (p_T^2)} \tag{34}$$

assuming the usual gaussian behavior in $p_T$. Again, the average value of $p_T^2$ is taken from the fit of Ref. [41] to the azimuthal dependence of the unpolarized SIDIS cross section: $\langle p_T^2 \rangle = 0.20 \text{ GeV}^2$.

Finally, we need the ordinary unpolarized distribution and fragmentation functions, $f_1(x)$ and $D_1(z)$, appearing both in the non-perturbative and in the perturbative contributions. They are taken from the GRV98 [44] and the Kretzer [45] parametrizations, respectively.

### IV. RESULTS AND PREDICTIONS FOR $\langle \cos 2\phi \rangle$

To start with, we compare our results with the available large-$Q^2$ data from the ZEUS collaboration (positron-proton collisions at 300 GeV) [13]. The integrations are performed over the following experimental ranges:

$$0.01 < x < 0.1, \quad 0.2 < y < 0.8, \quad 0.2 < z < 1.0 \ . \tag{35}$$

As shown in Fig. [3] we find quite a good agreement with the data. Notice that the average $Q^2$ value, $\langle Q^2 \rangle \approx 750 \text{ GeV}^2$, is such that the asymmetry is completely dominated by the perturbative contribution.

In order to highlight the effect of the non-perturbative contributions (Boer–Mulders and higher twist) one has to probe the kinematical region corresponding to $P_T < 1 \text{ GeV}$ and $Q^2$ of order of few GeV$^2$, where the gluon emission is quite irrelevant. Such a testing ground is investigated at the HERMES, COMPASS and JLAB facilities. We now turn to our predictions for these experiments.

In Fig. [4] we plot $\langle \cos 2\phi \rangle$ for $\pi^+$ and $\pi^-$ production at HERMES, as a function of one variable at a time, $x$, $z$ and $P_T$, the integration over the unobserved variables has been performed over the measured ranges of the HERMES experiment,

$$Q^2 > 1 \text{ GeV}^2, \quad W^2 > 10 \text{ GeV}^2, \quad P_T > 0.05 \text{ GeV}$$

$$0.023 < x < 0.4, \quad 0.2 < z < 0.7, \quad 0.1 < y < 0.85$$

$$2 < E_h < 15 \text{ GeV} \ . \tag{36}$$

In these kinematical regions the cross section is dominated by $\sigma^{(0)}$. An interesting feature of the asymmetry is that the Boer-Mulders contributions to $\pi^+$ and $\pi^-$ production are opposite in sign. In fact, we have

$$\langle \cos 2\phi \rangle_{\pi^+_{BM}} \sim e_u^2 h_1^{u}(x) H_1^{u\text{fav}}(z) + e_d^2 h_1^{d}(x) H_1^{d\text{fav}}(z) \ ,$$

$$\langle \cos 2\phi \rangle_{\pi^-_{BM}} \sim e_u^2 h_1^{u}(x) H_1^{u\text{unf}}(z) + e_d^2 h_1^{d}(x) H_1^{d\text{unf}}(z) \ , \tag{37}$$

| Collins fragmentation function | $A_{fav}^C = 0.41 \pm 0.91$ | $A_{unf}^C = -1.00 \pm 0.96$ |
|-----------------------------|---------------------------|---------------------------|
| $\gamma$                    | $1.04 \pm 0.38$          | $0.13 \pm 0.25$          |
| $\langle p_T^2 \rangle$    | $0.2 \text{ GeV}^2$      | $0.71 \pm 0.65 \text{ GeV}^2$ |

### Table II: Best values of the favored and unfavored Collins fragmentation functions.
FIG. 3: Our prediction for $\langle \cos 2\phi \rangle$ in charged pion production at ZEUS, compared with the data. The asymmetry is completely dominated by the perturbative contribution.

and, as far as $H_1^{\perp unf}(z) \simeq -H_1^{\perp fav}(z)$ [43], one gets different signs for the Boer-Mulders effect for positive and negative pions. Another important finding is the quantitative relevance of the higher-twist (Cahn) component of $\langle \cos 2\phi \rangle$. This contribution, which is positive, is the same for $\pi^+$ and $\pi^-$, if the $k_T$–dependence of the quark distributions is flavor–independent. Thus the asymmetry resulting from the combination of the Boer–Mulders and Cahn contributions turns out to be larger for $\pi^-$ than for $\pi^+$. We conclude that a difference between $\langle \cos 2\phi \rangle_{\pi^+}$ and $\langle \cos 2\phi \rangle_{\pi^-}$ is a clear signature of the Boer–Mulders effect. This is a definite prediction, to be checked experimentally. Moreover, the bigger in magnitude $h_1^D$, the more pronounced is the difference between the $\pi^-$ and the $\pi^+$ asymmetry. To illustrate this, we show in Fig. 5 our predictions for $\langle \cos 2\phi \rangle$ with three different choices of $h_1^D$: one corresponding to the Ansatz [24], the other two corresponding to a smaller and to a larger (in magnitude) $h_1^D$.

Fig. 6 shows our predictions for $\langle \cos 2\phi \rangle_h$ at COMPASS with a deuteron target. The experimental cuts are

$$Q^2 \geq 1 \text{ GeV}^2, \quad W^2 \geq 25 \text{ GeV}^2, \quad 0.2 \leq z \leq 1, \quad 0.1 \leq y \leq 0.9, \quad E_h \leq 15 \text{ GeV}. \quad (38)$$

We neglect nuclear corrections and use isospin symmetry to relate the distribution functions of the neutron to those of the proton. For the Boer–Mulders contribution we have

$$\langle \cos 2\phi \rangle_{BM}^{\pi^+D} \sim (h_1^{+u}(x) + h_1^{+d}(x)) (e_u^2 H_1^{+fav}(z) + e_d^2 H_1^{+unf}(z)), $$

$$\langle \cos 2\phi \rangle_{BM}^{\pi^0D} \sim (h_1^{+u}(x) + h_1^{+d}(x)) (e_u^2 H_1^{unf}(z) + e_d^2 H_1^{fav}(z)). \quad (39)$$

Since $h_1^{+u}$ has the same sign as $h_1^{+d}$ the deuteron target tends to exalt the Boer–Mulders effect. The opposite happens for the Sivers effect in transversely polarized SIDIS, which is suppressed for a deuteron target since $f_{1T}^{+u}$ and $f_{1T}^{+d}$, having different sign, partly cancel each other. Notice also that the perturbative contribution to the asymmetry at COMPASS becomes non negligible for $P_T > 1$ GeV.

For completeness, in Fig. 5 we present our predictions for the COMPASS experiment operating with a proton target.

Finally, JLab collects data in the collisions of 6 and 12 GeV electrons from proton and neutron targets. The experimental cuts for JLab operating with a proton target and a 6 GeV beam are the following

$$Q^2 \geq 1 \text{ GeV}^2, \quad W^2 \geq 4 \text{ GeV}^2, \quad 0.02 \leq P_T \leq 1 \text{ GeV}.$$
Our results for $\langle \cos 2\phi_h \rangle$ are shown in Figs. 8, 9, 10, and 11. Notice that unlike the predictions for HERMES and COMPASS, where the dependence on $P_T^{cut}$ is presented, for JLab we show the dependence on $P_T$ (thus $\langle \cos 2\phi \rangle$ vanishes when $P_T \to 0$). As one can see, the JLab measurements are insensitive to the perturbative QCD corrections and completely dominated by $O(\alpha_s^n)$ effects. Again, due to the Boer–Mulders contribution, the $\pi^-$ asymmetry is larger than the $\pi^+$ asymmetry. In particular, in the case of a neutron target, the Boer–Mulders effect and the Cahn higher twist effect combine to yield a vanishing $\pi^+$ asymmetry and a 4-5 % $\pi^-$ asymmetry.

Our prediction of a larger $\pi^-$ asymmetry as a signature of the Boer–Mulders effect is based on the assumption of a flavor–independent $k_T$–distribution. In order to check the robustness of this result, we varied the width of the Gaussian distribution for $d$ quarks (the $u$ distribution is well constrained by SIDIS data [41]), allowing it to be 50 % larger or smaller than the $u$ width, which is fixed to the value we used above, $\langle k_T^2 \rangle = 0.25 \text{ GeV}^2$. As one can see in Fig. 12 even such a large difference between the $u$ and $d$ widths does not modify much the results.
FIG. 5: Our prediction for the $\cos 2\phi$ asymmetry at HERMES, with three different assumptions for $h_1^\perp u$. The solid line corresponds to the Ansatz adopted here.

V. CONCLUSIONS

The Boer–Mulders function $h_1^\perp$, one of the distributions describing the transverse spin and transverse momentum structure of the nucleon, is so far completely unknown. Its main effect is an azimuthal $\cos 2\phi$ asymmetry in unpolarized SIDIS, an observable under an intense scrutiny by many ongoing and planned experiments. In this paper we presented some predictions for this asymmetry, taking all perturbative and non perturbative contributions into account. We found that $\langle \cos 2\phi \rangle$ is generally of order of few percent, and in most cases is dominated, in the moderate $Q^2$ region, by a kinematical higher–twist effect arising from the intrinsic transverse motion of quarks, while the perturbative component is negligible. Concerning the Boer–Mulders mechanism, we showed that it is possible to learn about it by comparing $\pi^+$ and $\pi^−$ production data, since we predict that the contribution related to $h_1^\perp$ should be positive for $\pi^−$ and negative for $\pi^+$, which results in a $\pi^−$ asymmetry larger than the $\pi^+$ asymmetry.

What emerges from our analysis is also the complementarity of the various experiments (HERMES, COMPASS, JLab). Taken altogether, the planned measurements of $\langle \cos 2\phi \rangle$, with their variety of kinematical regimes and targets, will represent a very important piece of information on transverse spin and transverse momentum effects in the nucleon.

VI. ACKNOWLEDGEMENT

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FIG. 6: Our prediction for the $\cos 2\phi$ asymmetry at COMPASS, with a deuteron target. The line labels are the same as in Fig. 4.

FIG. 7: Our prediction for the $\cos 2\phi$ asymmetry at COMPASS, with a proton target. The line labels are the same as in Fig. 4.

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FIG. 8: Our prediction for $\langle \cos 2\phi \rangle$ at JLab, with an incident beam energy of 6 GeV operating and a proton target. The line labels are the same as in Fig. [1]
FIG. 9: Our prediction for $\langle \cos 2\phi \rangle$ at JLab, with an incident beam energy of 12 GeV operating and a proton target. The line labels are the same as in Fig. 4.

FIG. 10: Our prediction for $\langle \cos 2\phi \rangle$ at JLab, with an incident beam energy of 6 GeV and a neutron target. The line labels are the same as in Fig. 4.
FIG. 11: Our prediction for $\langle \cos 2\phi \rangle$ asymmetry at JLab, with an incident beam energy of 12 GeV and a neutron target. The line labels are the same as in Fig. 4.

FIG. 12: Our prediction for $\langle \cos 2\phi \rangle$ asymmetry at JLab, with an incident beam energy of 6 GeV and a proton target using different $k_T$ widths for $u$ and $d$ quarks. The bands correspond to a $\pm 50\%$ variation of $\langle k_T^2 \rangle_d$ with respect to $\langle k_T^2 \rangle_u$, for which we take the value 0.25 GeV$^2$ (the upper end corresponds to $\langle k_T^2 \rangle_d = 0.375$ GeV$^2$, the lower end to $\langle k_T^2 \rangle_d = 0.125$ GeV$^2$).