Calculation of sealing joints with elastic edge in PTC MathCAD

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Abstract. The article touches upon the issues of the development of engineering methods of calculating a thin-walled shell-plate seat for valve and flange sealing joints, followed by the formulation of tasks using modern automated calculation software. To achieve the purpose, the authors used Parametric Technology Corporation MathCAD (PTC MathCAD) with subroutines for the search for extremum of functions and the computational block 'Given - minimize'. The calculation is presented in the form of algorithms consisting of three logically interrelated parts: 1) stiffness calculation; 2) strength calculation; 3) the calculation of rational dimensions. The obtained results of the calculation of geometric dimensions are tested, which eliminates a probable error in the calculated block 'Given' of PTC MathCAD. Among the developed engineering methodologies, the article describes the following ones: a method for calculating the rational geometrical dimensions of a shell-plate valve seat, operating under shock loading at an uncertain maximum dynamic load. It ensures that the allowable stresses in a thin-walled shell-plate seat are not exceeded. Another is a method for testing the stress-strain state of a shell-plate valve seat under static loading with a force of sealing and pressure of the working medium. The last one is the method of calculating rational geometrical dimensions of the shell-plate seat of a flange connection, which minimizes the unit stiffness (minimizing sealing force), provided that the shell-plate seat strength is ensured.

1. Introduction
Sealing joints (SJ) using thin-walled elements (elastic edges), along with metal-polymer compounds, have been widely applied in valve engineering, aircraft construction, aerospace engineering, and other industries [1, 2].

The main advantage of that kind of seals is the compensation of deviations of the shape and relative position, installation and temperature deformations of the sealing surfaces as a result of the elastic deformation of the thin-walled (elastic) edge.

Figure 1 shows the diagrams of gates with a thin-walled shell (Figure 1a) and plate (Figure 1b) elements (seats) of the gate.

The widespread use of this type of SJ in industry is hampered by the lack of engineering methods for determining the optimal dimensions of a shell seat operating under shock loading (valve SJ), static loading, taking into account the effect of working medium pressure (flange SJ). There is no task of optimizing the geometric dimensions of the shell seat, as well as automating the calculations using modern software systems.
One of the ways to reduce the dynamic load acting on the shell seat is to reduce the unit stiffness of the sealing joint [3, 4]. It is necessary to note that by reducing the dynamic load acting on the shell seat, it can be made thinner, which in turn results in a decrease in the energy needed to select shape deviations, and, consequently, a decrease in the required drive force, as well as the improvement in the weight and size characteristics in general.

2. Engineering method for calculating rational geometrical dimensions (thicknesses) of the thin-walled seat under dynamic and static loading

The simplest way to reduce the unit stiffness of the SJ is to use a shell-plate seat instead of a shell seat (Figure 2) [5-11].

The computational schemes of the shell-plate seat are shown in Figure 3. The assumption is made about the application of loads on the middle surface of thin-walled elements.

In the works [12–17], the method of calculating a shell-plate seat of valve and flange sealing joints was considered in detail. The main difficulty in the analytical solution of the task was to find the optimal geometric dimensions (thicknesses) of the SJ seat, operating under shock loading with an indefinite maximum dynamic load, while not exceeding the allowable stresses.

The conditions for the compatibility of deformations are prepared in the place of the mating of the plate and the shell [18]:

\[ M_{sco}(l) = M_{sp}(r_o); \ Q_{sco}(l) = Q_{sp}(r_o); \ w(l) = \Delta r_p(r_o); \ \vartheta_{sco}(l) = \vartheta_{sp}(r_o). \]  

The first condition for the compatibility of deformations for the valve SJ is written in the following form:
\[ D \beta^2 \left[ -4A_0 K_3(\beta l) - 4A_1 K_1(\beta l) + \frac{Q}{D_3 \beta^3} K_1(\beta l) \right] = \]
\[ = D_0 \left[ C_1(1+\mu) - \frac{C_2}{r_o} (1-\mu) + \frac{Tr_o}{2D_p} + (1+\mu) \frac{Tr_o}{2D_p} \ln \frac{r_o}{R_p} \right] \]

For the flange SJ, the first condition is determined from
\[ D_0 \beta^2 \left[ -4A_0 K_2(\beta l) - 4A_1 K_1(\beta l) + \frac{Q}{D_3 \beta^3} K_1(\beta l) \right] = \]
\[ = C_1 D_0 (1+\mu) - \frac{C_2 D_0}{r_o^2} (1-\mu) - \]
\[ - \frac{p_r r_o^2}{4} + \frac{Tr_o}{2} \left[ 1 + \ln \left( \frac{r_o}{R_p} \right) (1+\mu) \right]. \]

Combining the second and third compatibility condition for the valve SJ, we obtain
\[ A_0 K_0(\beta l) + A_1 K_1(\beta l) + \frac{Q}{D_3 \beta^3} K_3(\beta l) - \frac{\mu Tr_o}{Eh_o} = \frac{(R_p - r_o)}{Eh_o} (1-\mu) \times \]
\[ \times D_0 \beta^3 [ -4A_0 K_1(\beta l) - 4A_1 K_2(\beta l) + \frac{Q}{D_3 \beta^3} K_0(\beta l) ]; \]

For the flange SJ, the second and third conditions will be written in the form:
\[ A_0 K_0(\beta l) + A_1 K_1(\beta l) + \frac{Q}{D_3 \beta^3} K_3(\beta l) - \left( \frac{p_r - \frac{\mu Tr_o}{r_o}}{Eh_o} \right) \frac{r_o^2}{Eh_o} = \frac{(R_p - r_o)}{Eh_o} (1-\mu) \times \]
\[ \times D_0 \beta^3 \left[ -4A_0 K_1(\beta l) - 4A_1 K_2(\beta l) + \frac{Q}{D_3 \beta^3} K_0(\beta l) \right] \]

The fourth compatibility condition for the valve SJ:
\[ C_1 r_o + \frac{C_2}{2D_p} + \frac{Tr_o}{2D_p} \ln \frac{r_o}{R_p} = \beta [ -4A_0 K_3(\beta l) + A_1 K_0(\beta l) + \frac{Q}{D_3 \beta^3} K_2(\beta l) ]; \]
\[ C_1 r_o + \frac{C_2}{R_p} = 0 \]

For the flange SJ, the fourth compatibility condition will be determined from the expression
\[ \beta \left[ -4A_0 K_3(\beta l) + A_1 K_0(\beta l) + \frac{Q}{D_3 \beta^3} K_2(\beta l) \right] = C_1 r_o + \frac{C_2}{r_o} + \frac{Tr_o^2}{2D_p} \ln \left( \frac{r_o}{R_p} \right); \]
\[ C_1 r_o + C_2 = - \frac{p_r}{16D_p} \left[ \frac{R_p^4 - r_o^4}{R_p} - 4r_o^2 R_p \ln \left( \frac{R_p}{r_o} \right) \right]; \]
After composing the conditions of the compatibility of deformations in the place of joining plate and shell elements, the authors obtain four equations for finding four integration constants $C_1$, $C_2$, $A_1$ and $A_2$ for a valve SJ and a flange SJ.

After determining integration constants, one can calculate the deflection of the plate element for the valve and flange SJ from the expressions (10) and (11) respectively:

$$w_{sp,x} = C_2 \ln \frac{R_p}{r_o} - \frac{T_{sr}}{4D_p} (R_p^2 - r_o^2) - \frac{T_{sr}r_o^3}{4D_p} \ln \frac{r_o}{R_p} \quad (10)$$

$$w_{sp,f} = C_3 - \int g_o \, dr_p = \frac{p_p r_o^4}{64D_p} \ln \left( \frac{r_p}{r_o} \right) (p_p r_o^4 - 16C_p D_p) +$$

$$+ \frac{\ln \left( \frac{r_p}{r_o} \right)}{8D_p} \left( p_p r_o^2 r_p^2 - 2T_o r_p ^2 \right) + \frac{\ln \left( \frac{r_p}{r_o} \right)}{16D_p} \left( p_p r_o^2 + 8C_p D_p - 2T_o + 4T_o \ln \left( \frac{r_p}{r_o} \right) \right) \quad (11)$$

The radial stiffness of the shell element $c_2$ and the axial stiffness of the plate element $c_3$ are determined from the expressions:

$$c_2 = \frac{Q_{st}}{w(0)} = \frac{Q_{st}}{A_0 + w*}; c_3 = \frac{2T_{sr}}{w_{sp}} \quad (12)$$
The unit stiffness for the valve and flange SJ (see Figure 4) can be determined from the expressions (10) and (11), respectively:

\[
c_{sp, v} = \frac{c_1 + c_2 \tan \alpha \cdot \tan(\phi + \alpha)}{c_1 + c_2 \tan \alpha \cdot \tan(\alpha + \phi) + c_3};
\]

\[
c_{sp, f} = \frac{c_1 \tan \alpha \cdot \tan(\alpha + \phi)c_3}{c_2 \tan \alpha \cdot \tan(\alpha + \phi) + c_3},
\]

where \(c_1\) is the drive stiffness.

\[\text{Figure 4. Stiffness model of the shell-plate seat: a – of a valve; b – of a flange.}\]

It was mentioned above that the key parameter for ensuring the strength of a thin-walled shell-plate seat of the valve SJ is dynamic impact load \(F_{din}\).

Considering the above, for a thin-walled shell-plate seat, the dynamic shock load is determined from the expression

\[
F_{din} = F_w + \sqrt{\frac{F_a^2 + 2(E_t - E_f)}{c_1 + c_2 \tan \alpha \cdot \tan(\alpha + \phi) + c_3}},
\]

with the subsequent decomposition of \(F_{din}\) into components:

\[
T_{din} = -\frac{F_{din}}{2\pi r_o}; \quad Q_{din} = \frac{F_{din}}{2\pi r_o \tan(\alpha + \phi)},
\]

and performing a strength calculation.

After determining the stiffness, the equivalent stresses are determined by the fourth strength hypothesis (the form change hypothesis) [19-21].

The equivalent stresses in the shell are determined from the expressions:

\[
\sigma_{eq, min} = \sqrt{\sigma_{t, o, min}^2(x) + \sigma_{t, o, max}^2(x)} - \sigma_{z, o, min}(x)\sigma_{z, o, max}(x);
\]

\[
\sigma_{eq, max} = \sqrt{\sigma_{t, o, max}^2(x) + \sigma_{t, o, min}^2(x)} - \sigma_{z, o, max}(x)\sigma_{z, o, min}(x),
\]

where \(\sigma_{z, o, max/min}(x) = \frac{T}{h_o} \pm \frac{6M_{z, o}(x)}{h_o^2}; \quad \sigma_{t, o, max/min}(x) = \frac{T_{t, o}(x)}{h_o} \pm \frac{6M_{t, o}(x)}{h_o^2}; \quad M_{z, o}(x) = D \frac{d^2w}{dx^2}; \quad M_{t, o}(x) = \mu \cdot M_{x, o}(x).
\]

The equivalent stresses in the plate are determined from the expressions:
\[ \sigma_{eq,p\min}(r_p) = \sqrt{\sigma_{r,p\min}(r_p)^2 + \sigma_{\tau,p\min}(r_p)^2 - \sigma_{r,p\min}(r_p)\sigma_{\tau,p\min}(r_p)}; \]
\[ \sigma_{eq,p\max}(r_p) = \sqrt{\sigma_{r,p\max}(r_p)^2 + \sigma_{\tau,p\max}(r_p)^2 - \sigma_{r,p\max}(r_p)\sigma_{\tau,p\max}(r_p)}, \]

where \( \sigma_{r,p\max/min}(r_p) = \pm \frac{6M_{r,p}(r_p)}{h_p^2}; \)
\( \sigma_{\tau,p\max/min}(r_p) = \pm \frac{6M_{\tau,p}(r_p)}{h_p^2}; \)
\[ M_{r,p}(r_p) = D_P \left[ \frac{d\varphi_p(h_x,h_p,r_p)}{dr_p} + \mu \frac{\varphi_p(h_x,h_p,r_p)}{r_p} \right]; \]
\[ M_{\tau,p}(r_p) = D_P \left[ \frac{\varphi_p(r_p)}{r_p} + \mu \frac{d\varphi_p(r_p)}{dr_p} \right]. \]

3. Statement of the problem of calculating the rational geometric dimensions of a thin-walled seat

The above analytical method allows us to solve the following tasks of automated calculation of rational geometrical dimensions of a thin-walled shell-plate seat:

1. The dynamic calculation of the valve seat (\( E_n \) is taken into account):
   Target function: \( c_{eq}(h_x,h_p) \rightarrow \min. \)
   Limitations: \( \sigma_{eq,max}(x) \leq \sigma_{adm}; \sigma_{eq,max}(r_p) \leq \sigma_{adm} \)

2. The checking calculation of the valve seat (\( F = F_{st} \)) taking into account the action of the working medium (w*):
   Limitations: \( \sigma_{eq,max}(x) \leq \sigma_{adm}; \sigma_{eq,max}(r_p) \leq \sigma_{adm} \)

3. The design calculation of a flange connection:
   Target function: \( c_{eq}(h_x,h_p) \rightarrow \min. \)
   Limitations: \( \sigma_{eq,max}(x) \leq \sigma_{adm}; \sigma_{eq,max}(r_p) \leq \sigma_{adm} \)

The proposed technique allows setting optimization tasks using modern application software and automating the execution of operations. When setting the tasks for calculating the rational geometrical dimensions of a thin-walled shell-plate seat, for example, in PTC MathCAD, equivalent stresses must be taken as functions of thicknesses \( h_x, h_p \), coordinates \( x \) for the shell, and \( r_p \) for the plate, respectively. The values of the remaining parameters are also taken as functions from \( h_x, h_p, r_p, x \) if they are included in their expressions.

4. Solution of the set optimization problems using the mathematical software package PTC MathCAD

The solution of the set optimization problems of calculating the thin-walled shell-plate seat under the conditions of shock loading for the valve SJ is represented as an algorithm in Figures 5-7, and for a flanged SJ under the conditions of static loading and pressure of working medium in Figures 8-10.

For convenience in the description of the stages of the calculation, the represented algorithms are divided into three parts: the determination of the stiffness parameters of thin-walled seat, the determination of the strength parameters of thin-walled seat and the calculation of the rational geometric dimensions (thicknesses) of the shell and plate elements of the seat.

It should be noted that for a dynamic calculation, starting from block 6 (see Figure 6), dynamic parameters \( F_{din}(h_x,h_p), Q_{din}(h_x,h_p), T_{din}(h_x,h_p) \) are introduced. Stiffness parameters and parameters of stress-strain state (SSS) are redefined. The value of the integration constants is recalculated due to the input of dynamic parameters.
Before solving the optimization problem (choosing the rational geometrical dimensions of the seat), it is possible to build graphs of equivalent stresses, displacements, etc. to evaluate the stress-strain state of the shell-plate valve seat.

It is also possible to build equivalent stress plots for the plate and shell elements in order to further verify the results of the calculation of the subroutines of the search for extremum of functions, the operation of which is described below.

Introducing subroutines into the algorithms (blocks 12 and 14 are for a valve-type SJ, 9 and 11 are for a flange-type SJ) the search for extremum of functions was primarily due to the fact that when using a computational unit Given – minimize inside the subroutine, only the function is called but the initial approximation does not change.

As indicated in the works [22–24], the minimize and maximize functions do not basically allow the presence of variable parameters. As a result, functions inside a program module often give an incorrect result. Therefore, subroutines are used to accurately determine the initial approximations that are then
passed to the Given – minimize computational unit. In addition, to check the conditions for allowable stresses, and if they are not fulfilled, these subroutines are also used to refine the initial approximations.

Figure 7. Algorithm for determining the strength parameters of the shell-plate seat.

Figure 8. Algorithm for determining the stiffness parameters of the shell-plate seat.
Figure 9. Algorithm for determining the strength parameters of the shell-plate seat

Figure 10. Algorithm for determining the strength parameters of the shell-plate seat
Figure 11 shows the subroutines for finding the extremum of the function for the shell and the plate. The subroutines are executed in accordance with the instructions provided in [20–22], according to which, first, the subroutines define the values of the arguments $R$, $X$ and the values of the functions $\sigma(R)$, $\sigma(X)$ for the plate and the shell respectively. Then within the cycles (while $R<r_k$ and while $X<x_k$) for N argument values, the values of the functions $\sigma(R)$ and $\sigma(X)$ are determined. Each value of the function is compared with the previous one and, if it is greater than the previous one, is recorded as the maximum $A_1$, also $R_1$ and $X_1$ are written as the values of the arguments. At the end of the calculations (closing the cycle while), the last values $A_1$, $R_1$ and $X_1$ are derived from the subroutines using concatenated data sets.

$$\sigma_{eq,p}(\sigma, r_n, r_k, N) := \begin{cases} R \leftarrow r_n \\ A1 \leftarrow \sigma(R) \\ \text{while } R \leq r_k & \begin{cases} R \leftarrow R + \frac{r_k - r_n}{N} \\ A2 \leftarrow \sigma(R) \\ \text{if } A2 > A1 & \begin{cases} A1 \leftarrow A2 \\ R1 \leftarrow R \end{cases} \\ X \leftarrow x_n \\ A1 \leftarrow \sigma(X) \\ \text{while } X \leq x_k & \begin{cases} X \leftarrow X + \frac{x_k - x_n}{N} \\ A2 \leftarrow \sigma(X) \\ \text{if } A2 > A1 & \begin{cases} A1 \leftarrow A2 \\ X1 \leftarrow X \end{cases} \end{cases} \end{cases} \end{cases}$$

**Figure 11.** Algorithm for determining the strength parameters of the shell-plate seat

The output value of radius $r_{max,p}$ and coordinates $x_{max,o}$ passed as the initial approximations in the computational unit *Given – minimize*, which performed optimization geometrical parameters (thicknesses) of a shell-plate seat with specified limitations on allowable stresses.

The results of the calculation of the thicknesses $h_o, h_p$ after the computational unit *Given - minimize* are tested for allowable stresses. If these conditions are not fulfilled, the values of radius $r_{max,p}$ and coordinates $x_{max,o}$ are again analyzed by the subroutines and transmitted to the computational unit *Given – minimize*.

After performing the calculation and obtaining the results of rational thicknesses for the seat, it is necessary to check the initial approximation, since the conditions for allowable stresses might not be satisfied. This is because the computational unit *Given* (in this calculation, *Given – minimize*) is limited by initial approximations and fulfills the specified limits on the allowable stresses only in them.

5. Conclusion

The use of automated calculation systems can significantly reduce the time to solve the problem. The programming functions built into the system make it possible to create simple program modules necessary for repeated calculations, for example, to find the coordinate $x_{max,o}$ and the radius $r_{max,p}$, used in further calculations as the initial approximation for the *Given – minimize* computational unit.

The built-in *Given* block allows solving problems of exploring functions for an extremum (*Given – minimize, – maximize, – minerr*, etc.), however, it is limited by initial approximations, which in most cases leads to incorrect results. The main advantage of the *Given* block, in this case, is the ability to calculate rational geometric parameters of the seat (thickness) by solving an optimization problem with constraints on allowable stresses.

The presented example of solving the problem of optimizing the geometric dimensions of a thin-walled seat under dynamic and static loading in PTC MathCAD, its automation using the built-in *Given*
block can also be supplemented with data obtained from modeling in various systems (MSC.vN4W, APM WinMachine, etc.).

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