Black hole gas in TeV-gravity models

Mónica Borunda⋆ and Manuel Masip
CAFPE and Departamento de Física Teórica y del Cosmos, Universidad de Granada, E-18071 Granada, Spain
E-mail: mborunda@ugr.es, masip@ugr.es

Abstract. In a plasma at temperature close to the fundamental scale a small fraction of particles will experience transplanckian collisions that may result in microscopic black holes (BHs). We study the dynamics of a system (a black hole gas) defined by radiation at a given temperature coupled to a distribution of BHs of different mass. Our analysis includes the production of BHs in photon-photon collisions, BH evaporation, the absorption of radiation, collisions of two BHs to give a larger one, and the effects of the expansion. We find that the system may follow two different generic paths depending on the initial temperature of the plasma.

1. Introduction

The presence of extra dimensions may accommodate a fundamental scale of gravity $M_D$ much lower than the Planck mass $M_P \sim 10^{19}$ GeV [1, 2], with $M_D \approx 1$ TeV solving the so-called hierarchy problem. If that were the case, collisions of two particles at center of mass energies above $M_D$ would be dominated by gravity. In particular, one expects that at small impact parameters gravity bounds the system and the particles collapse into a microscopic BH. This process, extensively studied as a possibility for the LHC, would be also relevant in the early universe if the initial temperature $T$ was ever close to the TeV scale.

At such temperatures two particles in the high-energy tail of the distribution may experience a transplanckian ($\sqrt{s} > M_D$) collision and produce a mini-BH. If the BH is colder than the environment ($T_{BH} < T$) it will gain mass, and as it grows its temperature drops further. This would distinguish BHs from other massive particles or string excitations that may also be produced at high $T$: the heavier the BH, the colder and more stable it becomes, whereas heavier elementary particles have a shorter lifetime. BHs seem indeed a key ingredient in the dynamics of a plasma at temperatures close to the fundamental scale. Notice that the system is peculiar in the sense that the heat flows from the hot plasma to the colder BHs, but the effect is to cool the BHs.

In the usual TeV-gravity models with flat extra dimensions (ADD) a high $T$ seems inconsistent with the standard cosmology [3]. There the KK excitations of the graviton have an effective 4-dim mass proportional to $m_c \equiv L^{-1}$ and very weak couplings ($\approx \sqrt{s}/M_P$) to ordinary matter. Even if the initial configuration consists of particles only in the 4-dim brane, at $T \gg m_c$ these bulk gravitons will be abundantly produced (due to their large multiplicity) in annihilation of brane particles. If $T$ is large these (long-lived) gravitons will change the expansion rate at the time of primordial nucleosynthesis. Even if $T$ is initially as low as 1 MeV, their late decay will distort the cosmic background radiation in an unacceptable way. Obviously, a temperature...
close to $M_D$ would bring too many massive gravitons. In contrast, warped (RS) models predict KK gravitons just below $M_D$ with unsuppressed couplings, so a few resonances are enough to give an order one gravitational coupling at $M_D$. In RS models the bulk is basically empty at $T < m_c \approx 0.1M_D$.

Here we will consider *hybrid* models [4] with $n$ extra dimensions and two uncorrelated parameters: the effective compactification scale $m_c$ and the fundamental scale $M_D$. We will assume a mechanism that pushes the KK gravitons towards the 4-dim brane and increase their coupling to matter:

$$s \frac{s}{M_P^2} \rightarrow s \left(\frac{m_c}{M_D}\right)^n.$$  \hfill (1)

In this way a smaller number of KK modes will imply an order one gravitational interaction at the same scale $\sqrt{s} = M_D$. If the free parameter $m_c$ takes the value $m_c \approx M_D(M_D/M_P)^{2/n}$ we recover ADD, whereas for $m_c$ approaching $M_D$ we obtain RS. At distances below $1/m_c$ gravity would be $(4 + n)$-dim (similar to ADD) whereas at larger distances it becomes 4-dim (like in the usual RS). The KK gravitons are not produced at $T < m_c$, while their larger couplings to matter decouple them from the plasma at $T$ below their mass and make their lifetime shorter. This framework with $m_c > 1$ GeV avoids all astrophysical [5] and cosmological [3] bounds for any value of $n$.

There are two basic points that distinguish the TeV-gravity scenario from the standard 4-dim one. First, BHs are colder and live longer here than in 4-dim. Second, in these models the Hubble time $H^{-1} \approx 1/\sqrt{G_N\rho}$ at the temperature where the BHs are produced is much longer (in terms of the fundamental time scale) than in four dimensions:

$$H_{(4)}^{-1} \sim \frac{1}{M_{Pl}}, \quad H_{(4+n)}^{-1} \sim \frac{M_{Pl}}{M_D} \left(\frac{m_c}{M_D}\right)^{n/2} \frac{1}{M_D}$$  \hfill (2)

where we have assumed a bulk in thermal equilibrium with the 4-dim brane and $\rho \approx T^{4+n}/m^n_c$.

As a consequence, in a $(4 + n)$-dim universe there is plenty of time for BHs to be produced and grow, changing the dynamics of the early universe. In a 4-dim universe the temperature drops much faster, the BHs of mass near the threshold $M_P$ get then hotter than the environment and evaporate in a time $\approx 1/M_P$.

2. One black hole

Let us first consider a single BH in an expanding universe. We take $M_D = 1$ TeV, and $m_c = 10$ GeV, with initially the brane and the bulk in thermal equilibrium at a temperature $T = T_0$. Our hybrid model at energies above $M_D$ and distances smaller than $1/m_c$ is strongly coupled and $(4 + n)$-dim, so the BH is initially a regular $(4 + n)$-dim one. If it is colder than the plasma ($T_{BH} < T_0$) it grows, and when it reaches a radius $r_H = 1/m_c$ it fills up the whole compact space. At this point the BH keeps gaining mass, but its 4-dim size does not increase because gravity is strong only up to distances of order $1/m_c$ (the inverse mass of the lightest KK graviton). Beyond this distance gravity is 4-dim, since only the massless graviton is effective. Therefore, the BH grows at constant radius and temperature $T_{BH} \approx m_c$ until its mass reaches $M \sim M_P^2/m_c$, when it becomes purely 4-dim. At $T > m_c$ the BH mass changes according to

$$\frac{dM}{dt} = \sigma_4 A_4(T^4 - T_{BH}^4) + \sigma_{4+n} A_{4+n}(T^{4+n} - T_{BH}^{4+n})$$  \hfill (3)

where the negative terms account for the evaporation into brane and bulk species and in $A_{4+n}$ (the BH area) we take a maximum radius of $1/m_c$ along the extra dimensions.
The expansion rate (we assume that the extra dimensions are frozen) of the universe is dictated by the radiation energy density in the bulk and the brane,

$$\rho_{\text{rad}} \sim \frac{\pi^2}{30} T^4 \left( g_* + g_b c_n \frac{T^n}{m_c^n} \right).$$ (4)

At $T > m_c$ the bulk energy dominates and the temperature decreases as $T \sim t^{-2/(4+n)}$ with time, whereas at $T < m_c$ all the bulk energy is transferred to the brane and the universe expands like in the standard 4-dim cosmology.

In figure 1 we show the evolution of a BH of mass $M = 100$ TeV with $M_D = 1$ TeV, $n = 1$, just photons ($g_* = 2$) and gravitons ($g_b = 5$) in the bulk at $T_0 = 100, 200$ GeV.

3. Black hole gas

Let us now simulate the evolution of the system, a black hole gas, including the different processes that may take place:

- Black hole formation in collisions of two particles.
- Collisions of two BHs into a larger one.
- Absorption and emission of radiation.
- Effect of the expansion of the universe.

We will consider an initial configuration with only radiation at $T_0$ both in the brane and the $n$ dimensional bulk. Particles with enough energy will collide to form BHs that will grow if $T_{BH} < T_0$. If $T_0$ is smaller than $M_D$ then the BHs are always non-relativistic matter, with a kinetic energy ($\approx T$) much smaller than the mass. We take $E_{BH} \sim M_{BH}$ and $v_{BH} = \sqrt{2T/M_{BH}}$.

We will assume that the BH gas is a two-component thermodynamic system formed by radiation with a time dependent temperature $T(t)$ and BHs described by a distribution $f(M, t)$. $f(M, t)$ describes the number of BHs of mass $M$ at a given time $t$ per unit mass and volume. In this way, the energy density of the universe is the addition of $\rho_{\text{rad}}$ (for brane and bulk radiation) plus $\rho_{BH}$ (the matter density in the BHs).

As the radiation cools down due to the absorption and the expansion, lighter BHs (of higher temperature) become hotter than the plasma and evaporate. This reheats the universe and provides energy to be absorbed by the larger BHs. We are interested in the evolution of the system. Does $\rho_{BH}$ ever dominate the energy density of the universe? How big the BHs grow? How much the final configuration depends on the initial temperature of the universe?
4. Results

We have identified 2 generic scenarios depending on $T_0$. Larger values of $T_0$ (above $\approx 0.2 M_D$) produce many BHs, which grow fast and dominate $\rho$. In this case the BH gas goes through four phases [6]:

- BHs of mass above a critical mass corresponding to $T_{BH} = T_0$ are produced (see Fig. 2) and absorb radiation. As their number grows collisions between two BHs become important. BHs dominate the energy density very fast ($t \approx 10^{-12} H^{-1}$), reducing the temperature of the radiation.
- As $T$ drops BH production stops. Light BHs become hotter than the plasma and evaporate. This reduces the number of BHs but the average BH mass keeps growing, since the colder (bigger) BHs keep absorbing radiation. Once a BH reaches a mass around $10^7$ GeV its radius $r \approx 1/m_c$ stops growing.
- When $T$ drops to $T_{BH} \approx m_c$, the slow absorption of radiation by the heavier BHs is compensated with the evaporation of the lighter ones, keeping $T$ basically constant. The energy density is still dominated by BHs.
- At times of order $H^{-1}$ the expansion cools the radiation. The BHs decay fast and the universe becomes radiation dominated. Likewise, the lightest KK modes also decay fast and only 4-dim photons survive below $T \sim 0.1 m_c$.

In the case with a lower initial temperature ($T_0$ below $\approx 0.1 M_D$) the basic difference is that the BHs do not dominate the energy density at any time. In this second case we distinguish three phases:

- BHs are produced at a constant rate but since their number is so small collisions between them are negligible. These BHs grow like the single BH described in Fig. 1.
- At a Hubble time the expansion cools the radiation and stops BH production. The average BH mass keeps growing, but the universe is always radiation dominated.
- Due to the expansion the temperature of the universe drops below the temperature of the BHs, which evaporate in a time scale that depends on the ratio $M_D/m_c$ (it is around $t \sim 10^{22}$ GeV$^{-1}$ for $M_D/m_c = 10^2$).

Both scenarios seem consistent with primordial nucleosynthesis. In the second case the BHs are a small contribution to $\rho$ that would not change the expansion rate of the universe. For larger values of $M_D/m_c$ one may obtain BHs whose late decay may introduce distortions in the diffuse gamma-ray background, or even BHs with a lifetime larger than the present age of the universe. These ones could account for additional dark matter or become seeds for macroscopic primordial BHs.

We think that the work presented here is a first step in the search for possible observable effects of these BHs and on the understanding of the initial conditions in the early universe.

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