Matter induced $\rho - \delta$ mixing : a source of dileptons

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We study the possibility of $\rho - \delta$ mixing via N-N excitations in dense nuclear matter. This mixing induces a peak in the dilepton spectra at an invariant mass equal to that of the $\delta$. We calculate the cross section for dilepton production through the mixing process and we compare its size with that of $\pi - \pi$ annihilation. In-medium masses and mixing angles are also calculated.

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Heavy ion physics has recently seen a considerable effort being devoted to the study of the properties of hadrons in a hot and/or dense nuclear medium. Those activities were stimulated in part by the suggestion that in the nuclear medium, the vector meson masses would drop from their values in free space and that this could be interpreted as a precursor phenomenon of chiral symmetry restoration [1]. Several attempts have been made to highlight and understand the in-medium behaviour of vector mesons, both in theory and experiment [1-4]. In this respect electromagnetic signals constitute valuable probes, especially lepton pairs. This owes to the fact that the leptons couple to hadrons via vector mesons and therefore hadronic processes involving $e^+e^-$ in the final channel are expected to reveal their properties in the dilepton spectra. Furthermore, the $e^+e^-$ pairs suffer minimum final state interactions and are thus likely to bring information to the detectors essentially unscathed. Several experiments have measured, or are planning to measure, the lepton pairs produced in nucleus-nucleus collisions. They have been carried out by the DLS at LBL [5], and by HELIOS [6] and CERES [2] at CERN. Two new initiatives that will focus on electromagnetic probes will be PHENIX at RHIC [7] and HADES at GSI [8]. The density-dependent characteristics of vector mesons can also be highlighted through experiments performed at TJNAF [9]. The last two projects will involve measurements performed in environments where the possible modifications from vacuum properties will mostly be density-driven. It is with those in mind that we have performed the theoretical estimates about which we report in this paper.

While several theoretical studies have sought to investigate the in-medium properties of the vector mesons (mainly the $\rho$), their possible mixing with other mesons has only started to receive attention in the context of dense baryonic matter. An exception is the case of $\rho-\omega$ [14-16,19]. This specific mixing can be omitted when dealing with symmetric nuclear matter, as we will here. The popularity of the $\rho$ meson resides in the fact that in nuclear collisions a substantial contribution to the dilepton spectra comes from $\pi-\pi$ annihilation which proceeds through $\rho$ as an intermediate state. This fact can also be stated as the dilepton spectrum sampling the in-medium vector meson spectral function [12].

We explore here the possibility of $\rho-\delta$ (or $a_0$ as listed in [13]) mixing via nucleon(n)-nucleon(n) excitations in nuclear matter. Such a mixing, in effect, is similar to the known $\omega-\sigma$ mixing [14]. This is a pure density-dependent effect and is forbidden in free space on account of Lorentz symmetry. We will show that such a mixing opens up a new channel for the dilepton productions and induces an additional peak in the $\phi$ mass region.

The interaction Lagrangian we will use can be written as

$$\mathcal{L}_{int} = g_{\sigma} \bar{\psi} \phi_{\sigma} \psi + g_{\delta} \bar{\psi} \phi_{\delta} \tau^a \psi + g_{\omega N N} \bar{\psi} \gamma_{\mu} \gamma_{\nu} \psi \omega^{\mu \nu} + g_{\rho} [\bar{\psi} \gamma_{\mu} \tau^a \psi + \frac{\kappa_{\rho}}{2m_n} \bar{\psi} \sigma_{\mu \nu} \tau^a \partial^\nu \gamma^\mu] p_{\mu}^a, \quad (1)$$

where $\psi, \phi_\sigma, \phi_\delta, \rho$ and $\omega$ correspond to nucleon, $\sigma$, $\delta$, $\rho$, and $\omega$ fields, and $\tau_a$ is a Pauli matrix. The values used for the coupling parameters are obtained from Ref. [21].

The polarization vector through which the $\delta$ couples to $\rho$ via the n-n loop is given by

$$\Pi_{\mu}(q_0, |q|) = 2ig_{\delta} g_{\rho} \int \frac{d^4k}{(2\pi)^4} \text{Tr}[G(k)\Gamma_{\mu}G(k+q)]. \quad (2)$$

where 2 is an isospin factor and the vertex for $\rho$-nn coupling is:

$$\Gamma_{\mu} = \gamma_{\mu} - \frac{\kappa_{\rho}}{2m_n} \sigma_{\mu \nu} q^\nu. \quad (3)$$

$G(k)$ is the in-medium nucleon propagator given by [21]

$$G(k_0, |\vec{k}|) = G_F(k) + G_D(k_0, \vec{k}) \quad (4)$$

with
\[ G_F(k) = \frac{\langle \not k + m_n^* \rangle}{k^2 - m_n^2 + i\varepsilon}, \]  
and
\[ G_D(k_0, |k|) = \langle \not k + m_n^* \rangle \frac{i\pi}{E_k^*} \delta(k_0 - E_k^*)\theta(k_0 - |k|), \]

where \( E_k^* = \sqrt{k^2 + m_n^2} \). The second term \( (G_D) \) deletes on-mass shell propagation of nucleons having momenta below the Fermi momentum \( k_F \). In Eq. \( (6) \), the subscripts \( F \) and \( D \) refer to the free and density-dependent part of the propagator. In the subsequent equations \( m_n^* \) denotes the effective nucleon mass evaluated at the mean field level \([21]\).

With the evaluation of the trace and after a little algebra Eq. \( (3) \) could be cast into a suggestive form:
\[ \Pi_\mu(q_0, |q|) = \frac{g_\mu g_k}{\pi^3} 2q^2(2m_n^* - \frac{\sqrt{q^2}}{2m_n}) \int_0^{k_F} \frac{d^3k}{E^*(k)} \frac{\delta\mu - \frac{2m_n}{q^2}(k \cdot q)}{q^2 - m_n^2 + i\varepsilon}. \]

This immediately leads to two conclusions. First, it respects the current conservation condition, viz. \( q^\mu\Pi_\mu = 0 = \Pi_\rho q^\rho \).

Secondly, there are only two components which would survive after the integration over azimuthal angle. In fact this guarantees that it is only the longitudinal component of the \( \rho \) meson which couples to the scalar meson while the transverse mode remains unaltered. Furthermore, current conservation implies that out of the two non-zero components of \( \Pi_\mu \), only one is independent. It should be noted here that the tensor interaction, as evident from Eq. \( (7) \), inhibits the mixing.

In presence of mixing the combined meson propagator might be written in a matrix form where the dressed propagator would no longer be diagonal:
\[ \mathcal{D} = \mathcal{D}^0 + \mathcal{D}^0\Pi\mathcal{D}. \]

It is to be noted that the free propagator is diagonal and has the form
\[ \mathcal{D}^0 = \begin{pmatrix} \Delta_0 & 0 \\ 0 & \Delta_0 \end{pmatrix}. \]

In Eq. \( (8) \) the noninteracting propagator for the \( \delta \) and \( \rho \) are given respectively by
\[ \Delta_0(q) = \frac{1}{q^2 - m_\delta^2 + i\varepsilon}, \]
\[ D_0^{\mu\nu}(q) = \frac{-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2} \sqrt{m_\rho^2 + i\varepsilon}}{q^2 - m_\rho^2 + i\varepsilon}. \]

The mixing is characterised by the polarization matrix which contains non-diagonal elements
\[ \Pi = \begin{pmatrix} \Pi_{\mu\nu}^\delta(q) & \Pi_\rho(q) \\ \Pi_\rho(q) & \Pi_\rho^\rho(q) \end{pmatrix}. \]

In the above expression, \( \Pi_{\mu\nu}^\delta \) and \( \Pi_{\mu\nu}^\rho \) refer to the diagonal self-energies of the \( \delta \) and \( \rho \) meson induced by the n-n polarization:
\[ \Pi_{\mu\nu}^\delta(q_0, |q|) = -2ig_{\mu\nu} \int\frac{d^4k}{(2\pi)^4} \text{Tr}[G(k)G(k + q)] \]
\[ \Pi_{\mu\nu}^\rho(q_0, |q|) = -2ig_{\rho} \int\frac{d^4k}{(2\pi)^4} \text{Tr}[G(k)\Gamma_\mu G(k + q)\Gamma_\nu]. \]

It should be mentioned that, unlike mixing, Eqs. \( (13) \) and \( (14) \) also involve a free part stemming from the \( G_F(k)G_F(k) \) combination which is divergent. This therefore needs to be regularized. The regularization condition we employ is \( \partial^\nu\Pi^F(q^2)/\partial(q^2)^n|_{m_n^* m_n^* m_n^* m_n^* \rightarrow m_\rho^2 m_\rho^2 m_\rho^2 m_\rho^2} = 0 \) \( (n = 0, 1, 2, \ldots , \infty) \) \([17]\). For \( \rho \) the results may be found in Ref. \([17]\) which we do not present here and for \( \delta \) the free part of the self-energy is given by:
\[ \Pi_{\mu\nu}^\delta(q^2) = \frac{3g_{\mu\nu}^2}{2\pi^2} [3(m_n^* - m_n^2) - 4(m_n^* - m_n)m_n - (m_n^* - m_n^2)^2] \int_0^1 dx \ln \left( \frac{m_n^2 - x(1 - x)q^2}{m_n^2} \right) \]
\[ - \int_0^1 dx \left( m_n^2 - x(1 - x)q^2 \right) \ln \left( \frac{m_n^2 - x(1 - x)q^2}{m_n^2 - x(1 - x)q^2} \right). \]
It might be worthwhile to say here that the $\rho$ meson, being a vector, the collective oscillations set by its propagation through matter would have longitudinal (L) and transverse (T) components depending upon whether its spin is aligned along or perpendicular to the direction of propagation. Accordingly, with a special choice of z-axis along the direction of the momenta ($\vec{q}$), one can define the longitudinal and transverse polarization as $\Pi_L = -\Pi_{00} + \Pi_{33}$ and $\Pi_T = \Pi_{11} = \Pi_{22}$ respectively \cite{14}. To determine the collective modes, one defines the dielectric function as \cite{14}:

$$\epsilon(q_0, |\vec{q}|) = \det(1 - D^0\Pi) = \epsilon_T^2 \times \epsilon_{mix},$$

where $\epsilon_T$ corresponds to two identical transverse (T) modes and $\epsilon_{mix}$ correspond to the longitudinal mode with the mixing. The latter, of course, also characterizes the mode relevant for the $\delta$ meson propagation.

$$\epsilon_T = 1 - d_0\Pi_T, \quad d_0 = \frac{1}{q^2 - m_{\delta}^2 + i\epsilon}$$

$$\epsilon_{mix} = (1 - d_0\Pi_L)(1 - \Delta_0\Pi_s) - \frac{q^2}{|\vec{q}|^2}\Delta_0 d_0 (\Pi_0)^2$$

The zeros of the dielectric functions characterize the dispersion relation for the meson propagation. Fig. 1 shows the relevant dispersion curves with and without mixing at density $\rho=2.5\rho_0$. As only the L mode mixes with the scalar mode, we do not consider the T mode. The latter in fact is the same as presented in Ref. \cite{17} for $\rho$ meson. The effect of mixing on the pole masses, as evident from Fig. 1, are found to be very small. However, the mixing could be large when the mesons involved go off-shell. It should be noted that the modes with mixing move away from each other compared to what one obtains without mixing. This can be understood in terms of “level-level” repulsion driven by the off-diagonal terms of the dressed propagator \cite{16}.

![FIG. 1. The dispersion curve for $\rho$ and $\delta$ meson with and without mixing at $\rho=2.5\rho_0$.](image)

Fig. 2 shows the dependence of the invariant masses ($M_i = \sqrt{q_{0(i)}^2 - |\vec{q}_i|^2}$) ($i = \rho, \delta$) on nuclear densities where $q_0$’s are determined from the zeros of the dielectric function, Eq. (17). It is evident that the difference of the invariant masses first decreases with density reaching a minima and then again starts increasing. This behavior arises from the non-monotonic density dependence of the polarization functions.
To calculate the mixing angle, one diagonalizes the mass matrix \[17\] with the mixing and obtains

\[ \theta_{\text{mix}} = \frac{1}{2} \arctan\left( \frac{2\Pi_{\rho\delta}^0}{m^2_{\delta} - m^2_{\rho} - \Pi_L^0 + \Pi_L^0} \right) \]  

In Eq. \[18\] \( \Pi_{\text{mix}}^0 = M_i/|\vec{q}|\Pi_0 \) which increases with density. \( \Pi_0 \) is the zero component of Eq. \[6\]. Eq. \[18\] clearly shows that the mixing angle depends not only on the mixing amplitude (\( \Pi_0 \)) but also on the “energy denominator”. The latter, as seen in Fig. 2, first decreases as a function of density then again shows an increase characterizing the density dependence of the mixing angle as presented in Fig. 3. The mixing angle in Fig. 3 corresponds to \( |\vec{q}|=0.3 \) GeV/c. The momentum dependence for a density 2.5 times higher than the normal nuclear matter density is shown in the right panel of the same figure. This shows that for momenta beyond \( |\vec{q}| \approx 0.2 \) GeV/c the mixing is quite appreciable which affects the dilepton yield substantially as shown later. It should also be noted that the mixing angle vanishes at \( |\vec{q}| = 0 \) or at \( \rho = 0 \), as it should.
The $\rho-\delta$ mixing opens a new channel \textit{viz} $\pi + \eta \rightarrow e^+ + e^-$ in dense nuclear matter through n-n excitations. The Feynman diagram of the process is depicted in Fig. 4.

The cross-section for this process might be expressed in terms of the mixing amplitude ($\Pi_0$)

$$
\sigma_{\pi\eta\rightarrow e^+e^-} = \frac{4\pi\alpha^2}{3q^2M} \frac{g^2_{\delta\eta}}{g^2_{\rho}} \frac{m^4_{\rho}}{(M^2 - m^2_{\rho})^2 + m^4_{\rho} \Gamma^2(\rho) (M^2 - m^2_{\delta})^2 + m^4_{\delta} \Gamma^2(\delta)} \frac{1}{\sqrt{M^2 - 4m^2_{\pi}}} |\Pi_0|^2. \tag{19}
$$

For the decay widths we consider the invariant mass dependence as presented below:

$$
\Gamma_{\rho}(M) = \frac{g^2_{\rho\pi\pi}}{6\pi} \frac{(M^2 - m^2_{\pi})^2}{M^2}. \tag{20}
$$

and

$$
\Gamma_{\delta}(M) = \frac{g^2_{\delta\pi\eta}}{16\pi} \sqrt{(M^2 - (m_{\pi} + m_{\eta})^2)(M^2 - (m_{\pi} - m_{\eta})^2)} \frac{1}{M^3}. \tag{21}
$$

To describe the $\pi\delta\eta$ vertex we use

$$
L_{\delta\pi\eta} = f_{\delta\pi\eta} \frac{m^2_{\delta} - m^2_{\eta}}{m_{\pi}} \phi_{\eta} \phi_{\pi} \cdot \phi_{\delta}, \tag{22}
$$

where for later convenience we define $g_{\pi\delta\eta} = f_{\delta\pi\eta}(m^2_{\delta} - m^2_{\eta})/m_{\pi}$. 

![Dilepton spectrum induced by $\pi + \pi \rightarrow e^+ + e^-$ and $\pi + \eta \rightarrow e^+ + e^-$ considering matter induced $\rho - \delta$ mixing](image)
Of course, there is an uncertainty involved with the coupling parameter $f_{\delta\pi\eta}$ as discussed in Refs. [22,13]. This arises from the fact that $\delta$ (or $a_0$) lies close to the opening of the $K\bar{K}$ channel leading to a cusp-like behavior in the resonant amplitude, therefore a naive Breit-Wigner form for the decay width is inadequate. Furthermore, as mentioned before, there is also uncertainty involved with the $\delta$-$\pi\eta$ coupling which renders the precise extraction of $\delta$-$\pi\eta$ coupling even more difficult [22]. We take a value for $f_{\delta\eta\pi}=0.44$ from Ref. [22] which gives $\Gamma_{\delta\to\pi\eta}(m_{\delta})=59$ MeV, while the experimental vacuum width of $\delta$ is between 50 $-$ 100 MeV [13].

One can notice in Fig. 5 that the process, $\pi + \eta \to e^+ + e^-$, at densities higher than $\rho_0$, not only enhances the overall production of lepton pairs but also induces an additional peak near the $\phi$ mass region. The contribution at the $\delta$ mass is comparable to that of $\pi + \pi \to e^+ + e^-$ near the $\rho$ peak, for densities higher than $\rho_0$. Fig. 5 also shows that as the density goes even higher the dilepton yield arising out of the mixing also increases further. The cross-section increases with increasing momenta of the mesons in keeping with the mixing angle as shown in Fig. 3.

We have highlighted the possibility of $\rho$-$\delta$ mixing in dense nuclear matter. We observe the appearance of an additional peak at a dilepton invariant mass that corresponds to that of the $\delta$. With sufficient experimental resolution, this effect could be observable. Probably not as an individual peak, because of the $\delta$'s vacuum width which is already not small, but more realistically as a shoulder in the $\phi$ spectrum. This feature is then exclusively density-dependent. Our aim here was to establish the existence of the signal. Our calculation can, and will be improved upon: further studies are in progress to assess finite temperature effects and to self-consistently incorporate the necessary many-body machinery. For example, the characteristics of the $\rho$ can be modified in the nuclear medium [23] and the in-medium behaviour of the $\delta$ needs to be addressed. We have verified that the inclusion of hadronic form factors does not change the conclusions we reach in this work. Detailed results will be presented elsewhere.

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