Pursuit–Evasion Games with incomplete information in discrete time

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Abstract Pursuit–Evasion Games (in discrete time) are stochastic games with non-negative daily payoffs, with the final payoff being the cumulative sum of payoffs during the game. We show that such games admit a value even in the presence of incomplete information and that this value is uniform, i.e. there are $\epsilon$-optimal strategies for both players that are $\epsilon$-optimal in any long enough prefix of the game. We give an example to demonstrate that nonnegativity is essential and expand the results to Leavable Games.

Keywords Pursuit–Evasion Games · Incomplete information · Zero-sum stochastic games · Recursive games · Nonnegative payoffs

1 Introduction

Games of Pursuit and Evasion are two-player zero-sum games involving a Pursuer (P) and an Evader (E). P’s goal is to capture E, and the game consists of the space of possible locations and the allowed motions for P and E. These games are usually encountered within the domain of differential games, i.e., the location space and the allowed motions have the cardinality of the continuum and they tend to be of differentiable or at least continuous nature.

The subject of Differential Games in general, and Pursuit–Evasion Games in particular, was pioneered in the 1950s by Isaacs (1965). These games evolved from the need to solve military problems such as airfights, as opposed to classical game theory which was oriented toward solving economical problems. The basic approach was akin to differential equations techniques and optimal control, rather than standard
game theoretic tools. The underlying assumption was that of complete information, and optimal pure strategies were searched for. Conditions were given, under which a pure strategies saddle point exists (see, for example, Varaiya and Lin 1969). Usually the solution was given together with a value function, which assigned each state of the game its value. Complete information was an essential requirement in this case. For a thorough introduction to Pursuit–Evasion and Differential Games, see Basar and Olsder (1995).

A complete-information continuous-time game “intuitively” shares some relevant features with perfect-information discrete-time games. The latter are games with complete knowledge of past actions and without simultaneous actions. Indeed, if one player decides to randomly choose between two pure strategies which differ from time $t_0$ and on, his opponent will discover this “immediately” after $t_0$, thus enabling himself to respond optimally almost instantly. Assuming the payoff is continuous, the small amount of time needed to discover the strategy chosen by the opponent should affect the payoff negligibly. A well-known result of Martin (1975, 1985) implies that every perfect-information discrete-time game has $\epsilon$-optimal pure strategies (assuming a Borel payoff function) and so should, in a sense, continuous time games.

Another reason to restrict oneself to pure strategies is that unlike discrete-time games, there is no good formal framework for continuous-time games. By framework we mean a way to properly define the space of pure strategies and the measurable $\sigma$-algebra on them. There are some approaches but none is as general or complete as for discrete-time games. This kind of framework is essential when dealing with a general incomplete information setting.

This paper will therefore deal with discrete-time Pursuit–Evasion Games. We hope that our result will be applied in the future to discrete approximations of continuous-time games. Pursuit–Evasion Games in discrete time are formalized and discussed in Kumar and Shiau (1981).

Pursuit–Evasion Games are generally divided into two categories: Games of Kind and Games of Degree. Games of Kind deal with the question of capturability: whether a capture can be achieved by the Pursuer or not. In a complete-information setting this is a yes-or-no question, completely decided by the rules of the game and the starting positions. With incomplete information incorporated, we simply assign a payoff of 1 for the event of capture and payoff 0 otherwise. Games of Degree have the Pursuer try to minimize a certain payoff function such as the time needed for capture. The question of capturability is encountered here only indirectly: if the Evader have a chance of escaping capture indefinitely, the expected time of capture is infinity. The payoff, in general, can be any function, such as the minimal distance between the Evader and some target set.

What unites the two categories is that the payoff function in both is positive and cumulative. The maximizing player, be it the Pursuer or the Evader, gains his payoff and never loses anything. This is in contrast with other classes of infinitely repeated games, such as undiscounted stochastic games, where the payoff is the limit of the averages of daily payoffs.

Discrete-time stochastic games were introduced by Shapley (1953) who proved the existence of the discounted value in two-player zero-sum games with finite state and action sets. Recursive games were introduced by Everett (1957). These are stochastic