CCDM model with spatial curvature and the breaking of “dark degeneracy”

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Abstract. Creation of Cold Dark Matter (CCDM), in the context of Einstein Field Equations, leads to a negative creation pressure, which can be used to explain the accelerated expansion of the Universe. Recently, it has been shown that the dynamics of expansion of such models can not be distinguished from the concordance ΛCDM model, even at higher orders in the evolution of density perturbations, leading at the so called “dark degeneracy”. However, depending on the form of the CDM creation rate, the inclusion of spatial curvature leads to a different behavior of CCDM when compared to ΛCDM, even at background level. With a simple form for the creation rate, namely, $\Gamma \propto \frac{H}{\gamma}$, we show that this model can be distinguished from ΛCDM, provided the Universe has some amount of spatial curvature. Observationally, however, the current limits on spatial flatness from CMB indicate that neither of the models are significantly favored against the other by current data, at least in the background level.

Keywords: dark matter theory, dark energy theory

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1 Introduction

Since 1998, the series of observations of the luminosity-redshift relation of the Supernovae Type Ia (SNe Ia) has set a new era in cosmology [1–5]. Those observations, complemented by the observations of the Cosmic Microwave Background (CMB) anisotropies, Baryon Acoustic Oscillations (BAO), Hubble parameter in different redshifts, strongly suggest that the Universe has a great order of spatial flatness and has entered in a late phase of accelerating expansion [3–15].

Inside the context of relativistic cosmology, the accelerated expansion of the Universe is usually attributed to a new dark component called dark energy which possesses as main feature the negative pressure. The most favoured candidate for dark energy is the cosmological constant, Λ. This new component not only can fit the SN Ia observations, but can also complete the matter content in order to recover the flatness of the spatial hypersection of the Universe, as predicted by inflationary theory and corroborated by CMB observations.

Nevertheless, this concordance model has some serious issues. Due to its equation of state ($p_\Lambda = -\rho_\Lambda$), the cosmological constant could rise from the vacuum energy of quantum fields. However, the theoretical estimation to the vacuum energy through quantum field theory ($\rho_{\text{vac},T\text{h}}$) is up to 122 orders of magnitude bigger than the observational density [16]. In fact, the density of cosmological constant $\Lambda$, responsible for the late acceleration of expansion of the Universe as estimated from observational data ($\rho_\Lambda$) is almost zero in natural units ($10^{-47}$ GeV$^4$) and must be fine tuned to explain quantitatively the acceleration.

One way to avoid such fine-tuning is, as usual in quintessence models, consider that $\rho_{\text{vac},T\text{h}}$ is cancelled out by some yet unknown mechanism. In this context, inside the General Relativity, the negative pressure is the key element for acceleration. This can occur naturally in thermodynamic process departing from the equilibrium, for instance, the matter creation process at expenses of gravity. This phenomenon gives rise to a term of negative pressure which should be considered at the level of Einstein Equations, as shown by Prigogine and collaborators and formulated in a manifestly covariant way by Lima, Calvão & Waga [17–19]. The inclusion of the backreaction at the level of Einstein Field Equations was determinant to the rise of a new class of cosmological models with matter creation.
Many CCDM models have been proposed in the literature, each of those have different phenomenological creation rate dependencies [18–28]. Among them, recently, a model from Lima, Jesus and Oliveira (LJO) [22] was interesting because it was shown that this model leads to the same background evolution as the concordance ΛCDM model, for any spatial curvature. Further, it was shown that even at linear density perturbation evolution, such a degeneracy persisted, leading to the so called “dark degeneracy” [26, 29].

A first alternative to break such a degeneracy was given by Jesus and Pereira [28], which have found, directly from quantum field calculations, a creation rate which depended on the dark matter mass. They have used observational data to constrain the dark matter mass and have argued that with better constraints in the future, this could be used to distinguish CCDM from ΛCDM.

In this paper we investigate a cosmological model driven only by cold dark matter (CDM) creation at expenses of gravitational field in which the rate of CDM creation evolves reciprocally to the expansion rate, and we include the possibility of nonzero spatial curvature. We assume that the created particles are described by a real scalar field and consequently the created particles are its own antiparticles. Similarly to the standard model, the scenario presented here has the same degree of freedom and is also capable of explaining the accelerating expansion. We show that, with some amount of spatial curvature, the CCDM behavior differs from ΛCDM and both can be distinguished using observational data.

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In section 2, we discuss the dynamics of the universe with the pressure due to creation. In section 3, we discuss an specific rate of dark matter creation. In section 4, we constrain the free parameters of the model. In section 5, we compare the model with other models on the literature. Finally, we summarize the main results in conclusion.

2 Cosmic dynamics on models with creation of CDM particles

We will start by considering the homogeneous and isotropic FRW line element (with $c = 1$):

$$ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right),$$  \hspace{1cm} (2.1)

where $k$ can assume values $-1$, $+1$ or $0$.

In this background, the Einstein field equations are given by

$$8\pi G (\rho_{\text{rad}} + \rho_b + \rho_{\text{dm}}) = 3 \frac{\dot{a}}{a^2} + 3 \frac{k}{a^2},$$  \hspace{1cm} (2.2)

and

$$8\pi G (p_{\text{rad}} + p_c) = -2 \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{k}{a^2},$$  \hspace{1cm} (2.3)

where $\rho_{\text{rad}}$, $\rho_b$ and $\rho_{\text{dm}}$ are the density parameters of radiation, baryons and dark matter, $\rho_{\text{rad}} = p_{\text{rad}}/3$ is the radiation pressure and $p_c$ is the creation pressure.

The solutions of the EFE above are obtained considering an energy-momentum tensor (EMT) with the form [17, 30]:

$$T^{\mu\nu} = T_{\text{eq}}^{\mu\nu} + \Delta T^{\mu\nu},$$  \hspace{1cm} (2.4)

where $T_{\text{eq}}^{\mu\nu}$ characterizes thermodynamic equilibrium in the fluid and the creation of matter and entropy in universe are incorporated to the EFE through the correction term $\Delta T^{\mu\nu} = -p_c (g^{\mu\nu} - u^\mu u^\nu)$ [17–19, 30].
Therefore, the complete EMT (2.4) in the presence of matter creation has the explicit form:

\[
T^{\mu\nu} = (\rho_{\text{rad}} + \rho_{b} + \rho_{\text{dm}} + p_{\text{rad}} + p_{c})u^{\mu}u^{\nu} - (p_{\text{rad}} + p_{c})g^{\mu\nu},
\]

satisfying the conservation law \( T^{\mu\nu}_{\nu} = 0 \).

Assuming solely the creation of dark matter component, the densities of radiation and baryon components satisfy their respective usual conservations laws, namely:

\[
\dot{\rho}_{\text{rad}} + 4\dot{a}\rho_{\text{rad}} = 0,
\]

and

\[
\dot{\rho}_{b} + 3\dot{a}\rho_{b} = 0,
\]

where each overdot means one time derivative and we have used that \( p_{\text{rad}} = \rho_{\text{rad}}/3 \) and \( p_{b} = 0 \).

On the other hand, when the creation process is considered we should take into account a matter creation source at level of Einstein Field Equations [18, 19]:

\[
\frac{\dot{\rho}_{\text{dm}}}{\rho_{\text{dm}}} + 3\frac{\dot{a}}{a} = \Gamma,
\]

where \( \Gamma \) is the rate of dark matter creation in units of \((\text{time})^{-1}\).

As shown by [18, 19], the creation rate of cold dark matter is associated to the creation pressure \( p_{c} \) in eq. (2.3) through:

\[
p_{c} = -\frac{\rho_{\text{dm}}\Gamma}{3H},
\]

where \( H \equiv \dot{a}/a \) and we have considered an “adiabatic” creation, i.e., the case when the entropy per particle is constant. The so called “adiabatic” regime is a simplifying hypothesis in which the only source of entropy increase in the universe is the matter creation [17]. Mathematically, according to Calvão, Lima & Waga [18, 19]:

\[
\dot{\sigma} = \frac{\Psi}{nT} \left( \beta - \frac{\rho + p}{n} \right),
\]

where \( \sigma \) is the entropy per particle, \( \Psi \) is the particle creation rate, \( n \) is the particle density, \( T \) is the temperature and \( \beta \) is given from a phenomenological treatment of creation pressure:

\[
p_{c} = -\frac{\beta\Psi}{\Theta},
\]

where \( \Theta = 3H \) is the bulk expansion rate. So, in case \( \dot{\sigma} = 0 \), as we assume, we have \( \beta = \frac{\rho + p}{n} \), then creation pressure is given by \( p_{c} = -\frac{\rho + p}{n} \frac{\Theta}{3H} = -(\rho + p)\frac{\Gamma}{3H} \). On the other hand, if \( \dot{\sigma} \neq 0 \), \( \beta \) remains as an unknown parameter, which can not be constrained by thermodynamics alone, as the second law of thermodynamics demands only \( \Psi \geq -\frac{n\dot{\sigma}}{\sigma} \).

As a consequence of eq. (2.9), one can see that the dynamics of the universe is directly affected by the rate of creation of cold dark matter, \( \Gamma \). In particular, in the case \( \Gamma > 0 \) (creation of particles) we have a negative pressure creation and in the case \( \Gamma \to 0 \) we recover the well known dynamics when the universe is lately dominated by pressureless matter (baryons plus dark matter).
Creation of Cold Dark Matter (CCDM) model

The difficulties in identifying the nature of dark energy led the cosmologists to a quest for better candidates to explain the late acceleration of the Universe. In the literature, models with CDM creation has been discussed as a viable explanation to this recent phenomenon. It has been shown that under a convenient choice of the particle creation rate $\Gamma$, this scenario is able to support the observed dynamics, linear structure formation and thermodynamics features of the late Universe $[18–28]$. We argue that a natural dependence of the CDM creation rate is on the expansion rate, as already proposed in other CCDM models present on the literature. It has already been studied a linear dependence with the Hubble parameter $[21]$ and a power law dependence on Hubble parameter $[20, 31]$. It is clear to us that the expansion acceleration is a recent feature, so the CDM creation also must be recent. As the Universe evolves, the Hubble parameter decreases, and $\Gamma$ must increase. We can satisfy those conditions by assuming $\Gamma$ to be a negative power law of $H$, or, in the simplest case, $\Gamma \propto H^{-1}$. So, in this work, we consider the following creation rate:

$$
\Gamma = 3\alpha H^2 \Omega_b, \tag{3.1}
$$

where $\alpha$ is a constant free parameter of the model which drives the creation rate and the factor $3H^2_0$ has been introduced for mathematical convenience. This is also interesting because, as we shall see, with the identification $\alpha = \Omega_\Lambda$, the flat $\Lambda$CDM is a particular case of our CCDM model, when we neglect the baryon contribution.

Since we are considering only the late phase of the dynamics of the universe, we can neglect the radiation terms from now on. Thus, by combining eqs. (2.2) and (2.3), we have

$$
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho_b + \rho_{dm} + 3p_c \right). \tag{3.2}
$$

Replacing $p_c$ from eq. (2.9), we may write

$$
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho_b + \rho_{dm} \left( 1 - \frac{\Gamma}{H} \right) \right). \tag{3.3}
$$

Using that $\frac{\ddot{a}}{a} = \dot{H} + H^2$ and changing variables from time to redshift, we find

$$
\frac{dH}{dz} = \frac{H}{1+z} + \frac{H_0^2\Omega_b(1+z)^2\Gamma}{2H^2} + \frac{H^2 - H_0^2\Omega_k(1+z)^2}{2H(1+z)} \left( 1 - \frac{\Gamma}{H} \right), \tag{3.4}
$$

where we have used the solution of (2.7) to baryon density, $\rho_b = \rho_{0b}(1+z)^3$, $\Omega_b = \frac{\rho_{0b}}{\rho_0}$ is the present baryon density parameter, and $\Omega_k = -\frac{k}{H_0^2}$ is the present curvature density parameter.

Replacing the creation rate (3.1) and changing to dimensionless variable $y \equiv \left( \frac{H}{H_0} \right)^2$, we find

$$
\frac{dy}{dz} = 3\frac{y - \alpha}{1+z} + \frac{3\alpha\Omega_b(1+z)^2}{y} - \Omega_k(1+z) \left( 1 - \frac{3\alpha}{y} \right), \tag{3.5}
$$

which can not be solved analytically, except for some special cases. For example, if the Universe is spatially flat and we can neglect baryons ($\Omega_k = \Omega_b = 0$), we can solve (3.5) to find:

$$
H(z)^2 = H_0^2 \left[ \alpha + (1 - \alpha)(1+z)^3 \right], \tag{3.6}
$$
which corresponds to the flat CCDM LJO model, or to flat ΛCDM model, with the identification $\alpha = \Omega_\Lambda$. However, by introducing baryons or curvature, this model can not recover the LJO or the ΛCDM model anymore. So, this simple, and proper, generalization can be useful to discriminate between CCDM models and ΛCDM model even in the background level. On the other hand, the LJO and ΛCDM models can only be distinguished at the perturbation levels [23] and only in the absence of the separation of dark matter components [26].

There is also one more analytical solution. If we neglect curvature ($\Omega_k = 0$), retaining baryon density ($\Omega_b \neq 0$), we find an analytical solution to (3.5),

$$\left(\frac{H}{H_0}\right)^2 = \Omega_b (1 + z)^3 \left[ 1 + W \left( \frac{(1 - \Omega_b) e^{\frac{1 + \gamma}{\gamma}} - \frac{\alpha}{\gamma\Omega_b} - 1}{\Omega_b} \right) \right],$$

(3.7)

where $W(x)$ is the principal Lambert W function, also known as product logarithm, real solution to equation $x = W(x) e^{W(x)}$.

However, if we neglect baryons only ($\Omega_b = 0$, $\Omega_k \neq 0$), or if we consider the full equation (3.5), with $\Omega_b \neq 0$ and $\Omega_k \neq 0$, we can not find an analytical solution to $H(z)$, and we have to resort to numerical methods. Nevertheless, if curvature and baryonic contributions can both be considered small ($0 < \Omega_b \ll 1$, $|\Omega_k| \ll 1$), we can find an approximation,

$$\left(\frac{H}{H_0}\right)^2 = \alpha + (1 - \alpha)(1 + z)^3 + \frac{\Omega_k}{2} [1 - (1 + z)^2] + 3\Omega_k \log(1 + z) +$$

$$+ \left( \frac{\Omega_b \alpha}{1 - \alpha} - \Omega_k \right) \log \left[ \alpha + (1 - \alpha)(1 + z)^3 \right],$$

(3.8)

which we have found by solving (3.5) with $\Omega_b = \Omega_k = 0$, replacing the solution on (3.5) and solving again, with $\Omega_b \neq 0, \Omega_k \neq 0$.

In figure 1 we show the numerical solutions of $H(z)/H_0$ for some values of the free parameters of the model, namely, $\Omega_k$ and $\alpha$. It is worthy to remark that the model has as particular cases two well known models: the Einstein-de Sitter, for $\Omega_k = 0$ and $\alpha = 0$ and the flat ΛCDM for $\Omega_k = 0$ and $\alpha \sim 0.7$ and $\Omega_b = 0$.

4 Observational constraints

In this section, we obtain constraints to the free parameters of the model, namely, $\Omega_k$ and $\alpha$. In order to do this, we considered some measurements of the Hubble parameter, $H(z)$ [32] and the Supernovae Type Ia (SN Ia) dataset of Union 2.1 [6].

4.1 $H(z)$ constraints

Hubble parameter data as function of redshift yields one of the most straightforward cosmological tests because it is inferred from astrophysical observations alone, not depending on any background cosmological models.

At the present time, the most important methods for obtaining $H(z)$ data are (i) through “cosmic chronometers”, for example, the differential age of galaxies (DAG), (ii) measurements of peaks of acoustic oscillations of baryons (BAO) and (iii) through correlation function of luminous red galaxies (LRG) [9–15]. In this work, we use the data compilation of $H(z)$ from Farooq and Ratra [32], which is, currently, the most complete compilation, with 28 measurements.
As can be seen there, the limits over $\Omega_k$ believe that such a loose constraint can be due to a underestimate of uncertainties on the Big Bang Nucleosynthesis (BBN), as shown on ref. [33].

Based on this result, along with the symmetrization process suggested by [34], we use a prior of $\Omega_k \pm \sigma_{\Omega_k} = -0.043 \pm 0.046$. We will refer to this simply as the CMB prior. The results can be seen on figure 2 (right). As can be seen there, the limits over $\alpha$ and $\Omega_k$ are quite better. We have found $\chi^2_{\min} = 16.344$, $\alpha = 0.775^{+0.059}_{-0.064} -0.011 -0.16$ and $\Omega_k = -0.041 \pm 0.069 \pm 0.11 \pm 0.16$. In order to improve these constraints, we made a combined analysis with SN Ia data.

**Figure 1.** Numerical solutions for $H/H_0$ in function of the redshift $z$, and its sensitivity to the free parameters $\alpha$ and $\Omega_k$. **Left** Evolution of $H(z)$ with $\alpha = 0.7$ and $\Omega_k = 0$ (solid curve), $\Omega_k = 0.6$ (dashed curve) and $\Omega_k = -0.5$ (dashed-dotted curve). **Right** Evolution of $H(z)$ with $\Omega_k = 0$ and $\alpha = 0.7$ (dashed-dotted curve) and $\alpha = 0.9$ (dashed curve).

From these data, we perform a $\chi^2$-statistics, generating the $\chi^2_H$ function of free parameters:

$$
\chi^2_H = \sum_{i=1}^{28} \left( \frac{H_0 E(z_i, \alpha, \Omega_k, \Omega_b) - H_i}{\sigma_{H_i}} \right)^2,
$$

where $E(z) \equiv \frac{H(z)}{H(0)}$ and $H(z)$ is obtained by solving numerically eq. (3.5).

Throughout every analysis on this paper, we fix the baryon density parameter at the value estimated by Planck and WMAP: $\Omega_b = 0.049$ [7], a value which is in agreement with Big Bang Nucleosynthesis (BBN), as shown on ref. [33].

As the function to be fitted, $H(z) = H_0 E(z)$, is linear on the Hubble constant, $H_0$, we may analytically project over $H_0$, yielding $\tilde{\chi}^2_H$:

$$
\tilde{\chi}^2_H = C - \frac{B^2}{A},
$$

where $A \equiv \sum_{i=1}^{n} \frac{E_i^2}{\sigma_{Hi}^2}$, $B \equiv \sum_{i=1}^{n} \frac{E_i H_i}{\sigma_{Hi}^2}$, $C \equiv \sum_{i=1}^{n} \frac{H_i^2}{\sigma_{Hi}^2}$ and $E_i = \frac{H(z_i)}{H_0}$. The result of such analysis can be seen on figure 2 (left). As can be seen, the results from $H(z)$ data alone yield very loose constraints on the plane $\Omega_k-\alpha$. In fact, over the region $0 < \alpha < 1.4$ and $-1 < \Omega_k < 1$, only the 68% c.l. statistical contour could close. The minimum $\chi^2$ was $\chi^2_{\min} = 16.269$, which is too low for 28 data, yielding a $\chi^2$ per degree of freedom $\chi^2_p = 0.626$. The best fit parameters were $\alpha = 0.791^{+0.18}_{-0.085}$, $\Omega_k = 0.04^{+0.46}_{-0.44}$ in the joint analysis. We believe that such a loose constraint can be due to a underestimate of uncertainties on the $H(z)$ compilation data, which is evidenced by its low $\chi^2_p$. This issue has been addressed by [32] by combining $H(z)$ data with other constraints. While [32] uses a prior over $H_0$, we choose to use a prior over $\Omega_k$, as a prior over $H_0$ did not affect much the constraints found with $H(z)$ data only. We have considered a prior over $\Omega_k$ based on Planck and WMAP results [7].

Planck + WMAP indicate $\Omega_k = -0.037^{+0.049}_{-0.049}$, at 95% c.l., in the context of ΛCDM. Based on this result, along with the symmetrization process suggested by [34], we use a prior of $\Omega_k \pm \sigma_{\Omega_k} = -0.043 \pm 0.046$. We will refer to this simply as the CMB prior. The results can be seen on figure 2 (right). As can be seen there, the limits over $\alpha$ and $\Omega_k$ are quite better. We have found $\chi^2_{\min} = 16.344$, $\alpha = 0.775^{+0.059}_{-0.064} -0.011 -0.16$ and $\Omega_k = -0.041 \pm 0.069 \pm 0.11 \pm 0.16$. In order to improve these constraints, we made a combined analysis with SN Ia data.
4.2 Supernovae Type Ia bounds

The parameters dependent distance modulus for a supernova at the redshift $z$ can be computed through the expression

$$
\mu(z|s) = m - M = 5 \log d_L + 25,
$$

(4.3)

where $m$ and $M$ are respectively the apparent and absolute magnitudes, $s \equiv (H_0, \alpha, \Omega_k)$ is the set of the free parameters of the model and $d_L$ is the luminosity distance in unit of Megaparsecs.

Since in the general case $H(z)$ has not an analytic expression, we must define $d_L$ through a differential equation. The luminosity distance $d_L$ can be written in terms of a dimensionless comoving distance $D$ by:

$$
d_L = (1 + z) \frac{c}{H_0} D.
$$

(4.4)

The comoving distance can be related to $H(z)$, taking into account spatial curvature, by the following relation [35]:

$$
\left( \frac{H}{H_0} \right)^2 = y = \frac{\Omega_k D^2 + 1}{D'^2},
$$

(4.5)

where the prime denotes derivation with respect to redshift $z$. Inserting this relation in eq. (3.5), we obtain a differential equation for $D$:

$$
D'' = \frac{\Omega_k D D'^2}{1 + \Omega_k D^2} - \frac{3D'}{2(1 + z)} + \frac{D'^3}{2(1 + \Omega_k D^2)} \left[ \frac{3\alpha}{1 + z} + \Omega_k (1 + z) \left( 1 - \frac{3\alpha D'^2}{1 + \Omega_k D^2} \right) \right] -
$$

$$
- \frac{3\alpha \Omega_k (1 + z)^2 D'^5}{2(1 + \Omega_k D^2)^2}.
$$

(4.6)

To solve numerically this equation we have used as initial conditions $D(0) = 0$ and $D'(0) = 1$. The former condition can be derived from the relationship $d_L \approx cz/H_0$, valid for small redshifts in a FLRW universe. In order to constrain the free parameters of the model we considered the Union 2.1 SN Ia dataset from Suzuki et al. [6]. The best-fit set of parameters $s$ was estimated from a $\chi^2$ statistics with

$$
\chi^2_{\text{SN}} = \sum_{i=1}^{N} \frac{[\mu^i(z|s) - \mu^i_o(z)]^2}{\sigma^2_i},
$$

(4.7)

where $\mu^i(z|s)$ is given by (4.3), $\mu^i_o(z)$ is the corrected distance modulus for a given SNe Ia at $z_i$ being $\sigma_i$ its corresponding individual uncertainty and $N = 580$ for the Union 2.1 data compilation.

In figure 2 (left), we display the space of parameters $\Omega_k - \alpha$ and the contours for $1\sigma$, $2\sigma$ and $3\sigma$ of confidence intervals. Using SN data alone, and marginalizing the nuisance parameter $h$ ($H_0 = 100 h$ km s$^{-1}$Mpc$^{-1}$), we constrain the free parameters as $\alpha = 0.788^{+0.23}_{-0.086}$, $\Omega_k = 0.04^{+0.39}_{-0.035}$ at 68% confidence level and $\chi^2 = 0.974$. We also show, on figure 2 (left), the combination SNs Ia + $H(z)$, obtained by adding $\chi^2_{\text{SN}} + \chi^2_H$ and marginalizing over $h$. As can be seen, adding $H(z)$ data alone to SN Ia data, barely changes the SN constraints. In fact, we have found, on this case, $\alpha = 0.792^{+0.11}_{-0.061}$, $\Omega_k = 0.05^{+0.25}_{-0.24}$. The main difference on this case, is that the 95% c.l. contours could close on the region considered.
summarizes the analysis results, including the analysis with the CMB prior, based on the current limit on $\Omega_k$. As one can see, the CMB prior makes a great cut restrictive, as shown in figure H full combination of SNs + $H(z) + \Omega_k$ in the joint analysis: evolution, as we have shown on our analytical solution, eq. (3.7).

### Table 1. Results of the joint analysis for the different combinations of data. Limits on the parameters correspond to 68.3%, 95.4% and 99.7% c.l. as explained on text.

| Data                     | $\alpha$                  | $\Omega_k$                  | $\chi^2_{\nu}$ |
|--------------------------|---------------------------|-----------------------------|----------------|
| $H(z)$                   | 0.791$^{+0.18}_{-0.085}$  | 0.04$^{+0.46}_{-0.40}$      | 0.626          |
| SNs                      | 0.788$^{+0.23}_{-0.086}$  | 0.04$^{+0.39}_{-0.35}$      | 0.973          |
| SNs+$H(z)$               | 0.792$^{+0.11+0.26}_{-0.061-0.093}$ | 0.05$^{+0.25+0.42}_{-0.24-0.40}$ | 0.955          |
| SNs+$H(z) + H_0$ prior   | 0.787$^{+0.11+0.25}_{-0.065-0.096}$ | 0.01$^{+0.26+0.43}_{-0.26-0.42}$ | 0.957          |
| $H(z) + H_0$ prior       | 0.786$^{+0.17+0.61}_{-0.085-0.12}$ | $-0.06^{+0.35+0.61}_{-0.35-0.57}$ | 0.649          |
| $H(z)$+CMB prior         | 0.775$^{+0.059+0.098+0.14}_{-0.064-0.11-0.17}$ | $-0.041 \pm 0.069 \pm 0.11 \pm 0.16$ | 0.629          |
| SNs+$H(z)$+CMB prior     | 0.768$^{+0.034+0.058+0.084}_{-0.031-0.051-0.069}$ | $-0.036^{+0.066+0.11+0.15}_{-0.068-0.11-0.15}$ | 0.955          |

So, in order to find the best possible constraints with the data available, we made the full combination of SNs + $H(z) + \Omega_k$. In this case, the constraints were quite restrictive, as shown in figure 2 (right). As one can see, the CMB prior makes a great cut over the Union 2.1 contours, as $\Omega_k$ is strongly constrained, in this case. We have found, on this case, in the joint analysis: $\alpha = 0.768^{+0.034+0.058+0.084}_{-0.031-0.051-0.069}$, $\Omega_k = -0.036^{+0.066+0.11+0.15}_{-0.068-0.11-0.15}$, $\chi^2_{\nu} = 0.957$. Table 1 summarizes the analysis results, including the analysis with the $H_0$ prior, based on the current limit on $H_0$ given by [36], $H_0 = 72.0 \pm 3.0$ km s$^{-1}$Mpc$^{-1}$.

5 Comparison with CCDM and $\Lambda$CDM models

In the absence of baryons and if the spatial curvature vanishes, this model coincides both with the concordance $\Lambda$CDM model and LJO model [22], where $\Gamma \propto \frac{H}{P}$. Also, this model has the same creation rate as the CCDM1 model of ref. [27]. However, it is the first time that baryons and nonzero spatial curvature are considered on this model. As baryons are separately conserved, the DM creation rate $\Gamma \propto \frac{1}{P}$ leads to a huge difference on the Universe evolution, as we have shown on our analytical solution, eq. (3.7).
It has already been shown that the LJO model can not be distinguished from ΛCDM, for any value of spatial curvature, neither at background level [22] nor at perturbation level [26]. In this manner, LJO gives rise to the so called “dark degeneracy” [26], where, through cosmological observations, one can not determine if it is the quantum vacuum energy contribution (ACDM) or the quantum vacuum dark matter creation (CCDM) which accelerates the Universe. However, in the model proposed here, if the Universe has some amount of spatial curvature or baryons, we can distinguish it from ΛCDM.

In fact, as we can see on figure 3a, the relative difference is greater than 10% on \( H(z) \), for \( z \gtrsim 1.2 \) and \( \Omega_b = 0.049 \), even for a spatially flat Universe. On figure 3b, one may see that for \( \Omega_k \neq 0 \), this difference can go up to \( \sim 60\% \) at \( z \sim 1 \) if the Universe is open enough (\( \Omega_k = 0.3 \)) or up to \( \sim 20\% \) if the Universe is closed enough (\( \Omega_k = -0.181 \)). These limits are in agreement with the CCDM constraints from SNs + \( H(z) \) alone.

In a similar way, following the lines of [29, 37–39], we have made a comparison between the effective equations of state of the dark sector (\( \omega_{\text{eff}} \equiv \frac{p_{\text{dm}} + p_{\text{de}}}{\rho_{\text{dm}} + \rho_{\text{de}}} \)) for the models CCDM and ΛCDM. For CCDM, from eqs. (2.9) and (3.1) we find \( \omega_{\text{eff}} \) as:

\[
\omega_{\text{eff}},\text{CCDM} = -\alpha \left[ \frac{H_0}{H(z)} \right]^2 = -\frac{\alpha}{E(z)^2},
\]

while for ΛCDM:

\[
\omega_{\text{eff}},\text{ΛCDM} = \frac{p_\Lambda}{\rho_\Lambda + \rho_{\text{dm}}} = -\frac{\rho_\Lambda}{\rho_\Lambda + \rho_{\text{dm}}}. \tag{5.2}
\]

This comparison can be seen on figure 4, where we show how the \( \omega_{\text{eff}} \) from CCDM compares with a fiducial ΛCDM model. We choose for the fiducial ΛCDM the spatially flat best fit model from Planck + WMAP [7], with \( \Omega_\Lambda = 0.685 \pm 0.017 \). As we can see on figure 4a, the ratio of effective EOS for the spatially flat models has a small variation from unity (\( \sim 10\% \)), and has also an weak dependence with redshift. In figure 4b, however, one may see that the dependency on the curvature parameter leads to a greater variation with respect to ΛCDM, which is more accentuated for closed CCDM models, but never exceeding \( \sim 30\% \). We find, as a general tendency, that CCDM has an slightly greater effective EOS when compared to the fiducial ΛCDM model. We believe that this small deviation should not reduce the structure formation significantly.
Figure 4. Effective equation of state of the dark sector ($\omega_{\text{eff}} \equiv \frac{p_{\text{dm}} + p_{\text{de}}}{\rho_{\text{dm}} + \rho_{\text{de}}}$). For both panels, $\alpha = 0.792$ and we have used for $\Lambda$CDM the fiducial spatially flat model from Planck with parameter values: $\Omega_\Lambda = 0.685$, $\Omega_b = 0.049$. (a) Ratio between the effective equations of state in the models $\Lambda$CDM and CCDM ($\omega_{\text{eff,CDM}}/\omega_{\text{eff,CCDM}}$) for the spatially flat models ($\Omega_k = 0$). (b) The ratio between the effective equations of state for some selected values of $\Omega_k$ in the CCDM model.

We have shown that by considering this natural dependence of the creation rate $\Gamma$ with the expansion rate $H$, the direct inclusion of spatial curvature or baryons already breaks the “dark degeneracy”. The question now is: can the Universe be nonflat enough or have enough baryons in order to distinguish both models observationally? Part of the answer is on figure 2. As we have seen, SNs + $H(z)$ alone, which are observational data quite independent from cosmological models, are not enough for constraining the spatial curvature. Namely, looking at its 95% confidence limits over the curvature, we have $-0.19 < \Omega_k < 0.30$ so, the Universe can be flat, quite open ($\Omega_k \sim 0.30$) or quite closed ($\Omega_k \sim -0.19$). Spatial curvature on the border of this limit is enough for distinguishing CCDM from $\Lambda$CDM, as can be seen on figure 5.

We have plotted, on figure 5, $H(z)$ curves for some values of $\Omega_k$, in the context of CCDM and $\Lambda$CDM. In all curves, we have fixed $\alpha = \Omega_\Lambda = 0.792$, the CCDM best fit from SNs + $H(z)$, and we have used the $H_0$ value from [36], $H_0 = 72.0 \pm 3.0$ km s$^{-1}$ Mpc$^{-1}$. As we can see, if the Universe is closed, CCDM and $\Lambda$CDM are harder to be distinguished, with a small distinction only at high redshifts. If, however, the Universe is open, the distinction can be made at intermediate redshifts, and, at high redshifts, the distinction is clear even with the current set of $H(z)$ data.

However, if we take into account the prior over $\Omega_k$ given by CMB, we can not distinguish them, as we have, in this case, an strict 95% confidence limit over the curvature of $-0.104 < \Omega_k < 0.030$. This certainly is not enough to distinguish CCDM from $\Lambda$CDM. It can also be seen on figure 5, where we have used the less strict 99.7% c.l. drawn from the CMB prior, $-0.181 < \Omega_k < 0.095$. Here, one could think that CMB constraints on $\Lambda$CDM could not be used, directly, to constrain CCDM models, as the former model has not matter creation and the latter has. However, for the creation rate used here, $\Gamma \propto \frac{1}{H}$, creation is negligible on early Universe evolution, thus not changing the signatures imprinted on the last scattering surface. Thus, we conclude that the Universe is quite spatially flat, even in the context of this particular CCDM model, so we can not distinguish it from $\Lambda$CDM. The “dark degeneracy” is maintained from this analysis, at least while no more data is obtained in order to distinguish the models.
Figure 5. Comparison between $H(z)$ curves from CCDM and ΛCDM for some values of $Ω_k$. $Ω_k = -0.181$ and $Ω_k = 0.05$ correspond to 99.7% c.l. deviation over the CMB prior we have used, as explained on the text. $Ω_k = 0.3$ corresponds to a situation where ΛCDM is non-physical, while it is simply the superior limit on CCDM from SNs + $H(z)$ alone. Also shown is the $H(z)$ data used for the statistical analysis.

6 Conclusion

In this work we proposed a new cosmological model, based on the matter creation phenomena. We have shown that the proposed model is able to explain the background accelerating dynamics of the Universe, without a new dark fluid with negative pressure.

We have demonstrated that the present model is able to avoid the “dark degeneracy” simply through the presence of a baryonic content or the spatial curvature. For both cases, the model presents a distinguishable Hubble expansion from ΛCDM.

We have shown that this model can be theoretically distinguished from another CCDM model, LJO, with the inclusion of baryons and spatial curvature. It happens simply because, having different creation rates, the inclusion of baryons or spatial curvature leads to a distinction between these models.

As discussed in this work, at the background level, regarding the current data from SNe Ia and $H(z)$, CCDM and ΛCDM can lead to quite different predictions. As we have seen on section 5, SNs + $H(z)$ data alone give a 95% c.l. of $-0.19 < Ω_k < 0.30$. Thus, CCDM allows for a quite open Universe, with $Ω_k = 0.3$, while ΛCDM would lead to a non-physical situation with $Ω_m = Ω_b + Ω_{dm} < 0$.

Nevertheless, when a CMB prior over the curvature is considered, the CCDM and ΛCDM models can not be distinguished from each other. As we have shown on section 5, while the inclusion of a CMB prior improves the constraints on the free parameters, it also tends to prefer a spatially flat Universe, thus reducing the contrast between the ΛCDM and CCDM theoretical predictions. In fact, the 95% c.l. constraint over the curvature was $-0.104 < Ω_k < 0.030$, when including the CMB prior.

During the writing of this paper, Sarkar et al. [40] have made an analysis where they put limits on epoch of DM formation. By summarizing, they conclude that for models endowed
with CDM creation, not all the observed CDM can be created too late on the Universe evolution. However, their analysis can not be applied to our model, as they study the case where CDM is created at radiation expenses, thus constraining the CDM created with $N_{\text{eff}}$ from CMB data. In our case, CDM comes from gravitational particle production, thus being more in line with the work by Lima and Baranov [30]. That is, our model does not rely on any dark radiation in order to CDM creation take part on Universe evolution, as assumed by ref. [40]. Even though gravitational particle production can not be ruled out by their analysis, we perform here a small proof of early dark matter existence in our CCDM model.

Although we can not access in our model the CDM creation at early times, as we neglected the radiation contribution, we can make a rough estimate of the early CDM in our model by taking the approximation $\Omega_b \approx \Omega_k \approx 0$ (which corresponds to LJO). In this case, we have for DM density: $\rho_{\text{dm}} = \rho_{\text{dm}0} \left[ \alpha + (1 - \alpha)(1 + z)^3 \right]$. Estimating the dark matter mass inside a comoving volume by $M \propto \rho_{\text{dm}} a^3$, we have

$$M = M_0 \left[ \alpha (1 + z)^{-3} + 1 - \alpha \right],$$

where $M_0$ would be the dark matter mass today. Thus it is clear that, even at higher redshifts, a quantity of CDM is already present, as we can see from the limit $M(z \rightarrow \infty) = M_0(1 - \alpha)$.

Of course, we can not take such a limit, as for much high redshifts radiation contribution becomes non-negligible, so it changes $H(z)$, thus affecting $\Gamma$ and $\rho_{\text{dm}}(z)$. However, one can fix an initial mass $M_i = M(z_i)$ for some high $z_i \gtrsim 100$, so $M(z_i) = \alpha(1 + z_i)^{-3} + 1 - \alpha \approx 1 - \alpha$. So, we can see that for such redshift the creation is negligible and CDM mass is dominated by dark matter created during the inflation.

Further investigations including perturbation analysis should be made, in order to realize if more significant distinctions between CCDM and $\Lambda$CDM models can be found at higher orders of density perturbations evolution.

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References

[1] Supernova Search Team collaboration, A.G. Riess et al., Observational evidence from supernovae for an accelerating universe and a cosmological constant, *Astron. J.* 116 (1998) 1009 [astro-ph/9805201] [SPIRE].

[2] Supernova Cosmology Project collaboration, S. Perlmutter et al., Measurements of Omega and Lambda from 42 high redshift supernovae, *Astrophys. J.* 517 (1999) 565 [astro-ph/9812133] [SPIRE].

[3] SNLS collaboration, P. Astier et al., The Supernova legacy survey: Measurement of $\Omega_m$, $\Omega_\Lambda$ and $W$ from the first year data set, *Astron. Astrophys.* 447 (2006) 31 [astro-ph/0510447] [SPIRE].

[4] A.G. Riess et al., New Hubble Space Telescope Discoveries of Type Ia Supernovae at $z \geq 1$: Narrowing Constraints on the Early Behavior of Dark Energy, *Astrophys. J.* 659 (2007) 98 [astro-ph/0611572] [SPIRE].
[5] Supernova Cosmology Project collaboration, M. Kowalski et al., Improved Cosmological Constraints from New, Old and Combined Supernova Datasets, *Astrophys. J.* 686 (2008) 749 [arXiv:0804.4142] [nSPIRE].

[6] N. Suzuki et al., The Hubble Space Telescope Cluster Supernova Survey: V. Improving the Dark Energy Constraints Above $z > 1$ and Building an Early-Type-Hosted Supernova Sample, *Astrophys. J.* 746 (2012) 85 [arXiv:1105.3470] [nSPIRE].

[7] Planck collaboration, P.A.R. Ade et al., Planck 2013 results. XVI. Cosmological parameters, *Astron. Astrophys.* 571 (2014) A16 [arXiv:1303.5076] [nSPIRE].

[8] SDSS collaboration, D.J. Eisenstein et al., Detection of the baryon acoustic peak in the large-scale correlation function of SDSS luminous red galaxies, *Astrophys. J.* 633 (2005) 560 [astro-ph/0501171] [nSPIRE].

[9] J. Simon, L. Verde and R. Jimenez, Constraints on the redshift dependence of the dark energy potential, *Phys. Rev.* D 71 (2005) 123001 [astro-ph/0412269] [nSPIRE].

[10] D. Stern, R. Jimenez, L. Verde, M. Kamionkowski and S.A. Stanford, Cosmic Chronometers: Constraining the Equation of State of Dark Energy. I: $H(z)$ Measurements, *JCAP* 02 (2010) 008 [arXiv:0907.3149] [nSPIRE].

[11] C. Blake et al., The WiggleZ Dark Energy Survey: Joint measurements of the expansion and growth history at $z < 1$, *Mon. Not. Roy. Astron. Soc.* 425 (2012) 405 [arXiv:1204.3674] [nSPIRE].

[12] M. Moresco et al., Improved constraints on the expansion rate of the Universe up to $z \approx 1.1$ from the spectroscopic evolution of cosmic chronometers, *JCAP* 08 (2012) 006 [arXiv:1201.3609] [nSPIRE].

[13] C. Zhang, H. Zhang, S. Yuan, T.-J. Zhang and Y.-C. Sun, Four new observational $H(z)$ data from luminous red galaxies in the Sloan Digital Sky Survey data release seven, *Res. Astron. Astrophys.* 14 (2014) 1221 [arXiv:1207.4541] [nSPIRE].

[14] N.G. Busca et al., Baryon Acoustic Oscillations in the Ly-$\alpha$ forest of BOSS quasars, *Astron. Astrophys.* 552 (2013) A96 [arXiv:1211.2616] [nSPIRE].

[15] C.-H. Chuang and Y. Wang, Modeling the Anisotropic Two-Point Galaxy Correlation Function on Small Scales and Improved Measurements of $H(z)$, $D_A(z)$ and $\beta(z)$ from the Sloan Digital Sky Survey DR7 Luminous Red Galaxies, *Mon. Not. Roy. Astron. Soc.* 435 (2013) 255 [arXiv:1209.0210] [nSPIRE].

[16] S. Weinberg, The Cosmological Constant Problem, *Rev. Mod. Phys.* 61 (1989) 1 [nSPIRE].

[17] I. Prigogine, J. Geheniau, E. Gunzig and P. Nardone, Thermodynamics and cosmology, *Gen. Rel. Grav.* 21 (1989) 767 [nSPIRE].

[18] J.A.S. Lima, M.O. Calvao and I. Waga, Cosmology, Thermodynamics and Matter Creation, *arXiv:0708.3397* [nSPIRE].

[19] M.O. Calvao, J.A.S. Lima and I. Waga, On the thermodynamics of matter creation in cosmology, *Phys. Lett. A* 162 (1992) 223 [nSPIRE].

[20] M.P. Freaza, R.S. de Souza and I. Waga, Cosmic acceleration and matter creation, *Phys. Rev. D* 66 (2002) 103502 [nSPIRE].

[21] J.A.S. Lima, F.E. Silva and R.C. Santos, Accelerating Cold Dark Matter Cosmology ($\Omega_\Lambda \equiv 0$), *Class. Quant. Grav.* 25 (2008) 205006 [arXiv:0807.3379] [nSPIRE].

[22] J.A.S. Lima, J.F. Jesus and F.A. Oliveira, CDM Accelerating Cosmology as an Alternative to $\Lambda$CDM model, *JCAP* 11 (2010) 027 [arXiv:0911.5727] [nSPIRE].

[23] J.F. Jesus, F.A. Oliveira, S. Basilakos and J.A.S. Lima, Newtonian Perturbations on Models with Matter Creation, *Phys. Rev. D* 84 (2011) 063511 [arXiv:1105.1027] [nSPIRE].
[24] J.A.S. Lima and S. Basilakos, From de Sitter to de Sitter: A New Cosmic Scenario without Dark Energy, arXiv:1106.1938 [nSPIRE].
[25] J.A.S. Lima, S. Basilakos and F.E.M. Costa, New Cosmic Accelerating Scenario without Dark Energy, Phys. Rev. D 86 (2012) 103534 [arXiv:1205.0868] [nSPIRE].
[26] R.O. Ramos, M.V.d. Santos and I. Waga, Matter creation and cosmic acceleration, Phys. Rev. D 89 (2014) 083524 [arXiv:1404.2604] [nSPIRE].
[27] L.L. Graef, F.E.M. Costa and J.A.S. Lima, On the equivalence of $\Lambda(t)$ and gravitationally induced particle production cosmologies, Phys. Lett. B 728 (2014) 400 [arXiv:1303.2075] [nSPIRE].
[28] J.F. Jesus and S.H. Pereira, CCDM model from quantum particle creation: constraints on dark matter mass, JCAP 07 (2014) 040 [arXiv:1403.3679] [nSPIRE].
[29] M. Kunz, The dark degeneracy: On the number and nature of dark components, Phys. Rev. D 80 (2009) 123001 [astro-ph/0702615] [nSPIRE].
[30] J.A.S. Lima and I. Baranov, Gravitationally Induced Particle Production: Thermodynamics and Kinetic Theory, Phys. Rev. D 90 (2014) 043515 [arXiv:1411.6589] [nSPIRE].
[31] J.A.S. Lima, L.L. Graef, D. Pavon and S. Basilakos, Cosmic acceleration without dark energy: Background tests and thermodynamic analysis, JCAP 10 (2014) 042 [arXiv:1406.5538] [nSPIRE].
[32] O. Farooq and B. Ratra, Hubble parameter measurement constraints on the cosmological deceleration-acceleration transition redshift, Astrophys. J. 766 (2013) L7 [arXiv:1301.5243] [nSPIRE].
[33] Particle Data Group collaboration, K.A. Olive et al., Review of Particle Physics, Chin. Phys. C 38 (2014) 090001 [nSPIRE].
[34] G. D’Agostini, Asymmetric uncertainties: Sources, treatment and potential dangers, physics/0403086 [nSPIRE].
[35] C. Clarkson, B. Bassett and T. H.-C. Lu, A general test of the Copernican Principle, Phys. Rev. Lett. 101 (2008) 011301 [arXiv:0712.3457] [nSPIRE].
[36] E.M.L. Humphreys, M.J. Reid, J.M. Moran, L.J. Greenhill and A.L. Argon, Toward a New Geometric Distance to the Active Galaxy NGC 4258. III. Final Results and the Hubble Constant, Astrophys. J. 775 (2013) 13 [arXiv:1307.6031] [nSPIRE].
[37] C. Clarkson, M. Cortes and B.A. Bassett, Dynamical Dark Energy or Simply Cosmic Curvature?, JCAP 08 (2007) 011 [astro-ph/0702670] [nSPIRE].
[38] M. Seikel, C. Clarkson and M. Smith, Reconstruction of dark energy and expansion dynamics using Gaussian processes, JCAP 06 (2012) 036 [arXiv:1204.2832] [nSPIRE].
[39] V.C. Busti and C. Clarkson, Dodging the dark matter degeneracy while determining the dynamics of dark energy, arXiv:1505.01821 [nSPIRE].
[40] A. Sarkar, S. Das and S.K. Sethi, How Late can the Dark Matter form in our universe?, JCAP 03 (2015) 004 [arXiv:1410.7129] [nSPIRE].