CP-Violating MSSM Higgs Bosons
in the Light of LEP 2

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ABSTRACT

In the MSSM, the CP parities of the neutral Higgs bosons may be mixed by radiative effects induced by explicit CP violation in the third generation of squarks. To allow for this possibility, we argue that the charged Higgs-boson mass and $\tan \beta$ should be used to parametrize the MSSM Higgs sector. We introduce a new benchmark scenario of maximal CP violation appropriate for direct searches of CP-violating MSSM Higgs bosons. We show that the bounds established by LEP 2 on the MSSM Higgs sector may be substantially relaxed at low and intermediate values of $\tan \beta$ in the presence of CP violation, and comment on possible Higgs boson signatures at LEP 2 within this framework.
The Minimal Supersymmetric extension of the Standard Model (MSSM) constitutes the minimal viable scenario of low-energy supersymmetry (SUSY), within which the problems of gauge hierarchy [1] and gauge-coupling unification [2] can be successfully addressed. Over the last 15 years, the MSSM has been the basis for many theoretical [3] and experimental Higgs-boson studies, serving as a yardstick for models beyond the Standard Model (SM). At the tree level, the Higgs sector of the MSSM contains three neutral Higgs bosons of definite CP parity: the CP-even Higgs bosons $h$ and $H$, and the CP-odd Higgs scalar $A$. Radiative corrections to the spectrum and couplings of MSSM Higgs bosons are important, leading in particular to an upper bound on the lightest neutral Higgs boson of about 130 GeV. Therefore, the presence of some Higgs boson with a mass below about 130 GeV may be considered as a ‘smoking gun’ for the MSSM. At the time of writing, searches at LEP 2 quote a lower limit of 112.3 GeV on the mass of the SM Higgs boson, excluding also a substantial part of the MSSM parameter space, particularly at low $\tan \beta$ [5].

It has been realized recently that the tree-level CP invariance of the MSSM Higgs potential may be violated sizeably beyond the Born approximation, by loop effects involving CP-violating interactions of Higgs bosons to top and bottom squarks [6,7,8,9,10,11]. In such a minimal SUSY scenario of explicit radiative CP violation, the three neutral Higgs bosons, denoted by $H_1$, $H_2$ and $H_3$ in order of increasing masses, have in general mixed CP parities. It has been found [7,9,10] that CP violation may modify drastically the tree-level couplings of the Higgs particles to fermions and to gauge bosons, thereby enabling [10] even a relatively light Higgs boson with $M_{H_1} \sim 60$ GeV to have escaped detection at LEP 2.

We re-evaluate in this paper the physics potential of LEP 2 for discovering Higgs bosons in the MSSM, in the presence of radiatively induced CP-violating effects in the Higgs sector. We emphasize that, in the presence of CP-violating mixing between the neutral Higgs bosons, it becomes necessary to parametrize the MSSM Higgs sector in terms of the charged Higgs-boson mass $M_{H^+}$, since the commonly used CP-odd Higgs boson mass is no longer associated with a physical Higgs mass eigenstate. The mass of the charged Higgs boson also controls the strength of the CP-violating effects in the lightest Higgs sector [7,10]. For large values of $M_{H^+}$, the lightest neutral Higgs acquires SM-like properties while the two heaviest neutral Higgs bosons remain in general as states of mixed CP parity.

As a framework for the discussion, we introduce a new benchmark scenario of maximal CP violation appropriate for analyzing the direct searches for CP-violating Higgs bosons.

*An excess of events consistent with a Higgs boson with SM-like couplings to gauge bosons and bottom quarks and a mass in the range $113–116$ GeV has been reported at the 2.6 $\sigma$ level [3]. As a by–product of our analysis, we propose a novel interpretation for the origin of this possible excess.
at LEP 2 and elsewhere. We then compare the MSSM Higgs discovery potential at LEP 2 for the maximal CP-violating benchmark scenario and its CP-conserving counterpart. We focus on low and intermediate values of \( \tan \beta \), i.e., \( \tan \beta \lesssim 7 \), and on low values of \( M_{H^+} \), \( M_{H^+} \lesssim 170 \) GeV, for which CP violation plays a particularly important rôle. We find that the lightest neutral Higgs bosons may be much lighter than the quoted limit, and raise the possibility that the apparent excess \([5]\) may be due to the second neutral Higgs boson.

Our numerical analysis is based on our earlier study in \([10]\), in which we computed the one-loop renormalization-group (RG) improved effective potential for the MSSM Higgs sector with explicit CP violation. Using RG methods, we calculated in \([10]\) the charged and neutral Higgs-boson masses and couplings, including the two-loop leading logarithms originating from QCD effects, as well as those induced by the top- and bottom-quark Yukawa couplings \([12]\). Also, we included the leading one-loop logarithms associated with gaugino and higgsino quantum effects \([13]\). Most importantly, we implemented the potentially large two-loop non-logarithmic corrections induced by one-loop threshold effects on the top- and bottom-quark Yukawa couplings, due to the decoupling of the third-generation squarks \([14,15]\). The numerical predictions presented here are obtained by the Fortran code \textsc{cph} \([16]\), in which the aforementioned calculation of Higgs-boson masses and couplings based on the RG-improved effective potential has been implemented.

As was extensively discussed in \([7,10]\), there are two important consequences of explicit radiative CP violation in the Higgs sector.

- The first is the generation of sizeable off-diagonal scalar-pseudoscalar contributions \( M^2_{SP} \) \([6,7]\) to the general \( 3 \times 3 \) Higgs-boson mass matrix. Each of the individual CP-violating off-diagonal scalar-pseudoscalar mixing entries \( M^2_{SP} \) in the neutral MSSM mass-squared matrix contains terms scaling qualitatively as

\[
M^2_{SP} \sim \frac{m_t^4}{v^2} \frac{\text{Im}(\mu A_t)}{32\pi^2 M^2_{\text{SUSY}}} \left( 1, \frac{|A_t|^2}{M^2_{\text{SUSY}}}, \frac{|\mu|^2}{\tan \beta M^2_{\text{SUSY}}}, \frac{2\text{Re}(\mu A_t)}{M^2_{\text{SUSY}}} \right),
\]

and could be of order \( M^2_Z \). In \([4]\), \( \mu \) is the supersymmetric mixing parameter of the two Higgs superfields, \( M^2_{\text{SUSY}} \) specifies the common soft SUSY-breaking scale defined by the arithmetic average of the squared stop masses, and \( A_t \) is the soft SUSY-breaking trilinear coupling of the Higgs boson to top squarks.

- The second important consequence of CP violation is the modification of the top- and bottom-quark Yukawa couplings through CP-violating vertex effects \([4,10]\) involving gluinos and higgsinos, as well as top and bottom squarks. Although these effects enter the charged and neutral Higgs-boson masses and couplings formally at the two-loop
level, they can still modify the numerical predictions or masses and couplings in a significant way, and therefore have to be included in the analysis.

As we stressed above, the mass of the CP-odd Higgs scalar $A$ is no longer an eigenvalue in the presence of CP violation, but rather just one entry in the general $3 \times 3$ neutral-Higgs mass matrix. Therefore, in the general CP-violating case, it is no longer appropriate to parametrize the MSSM Higgs sector in terms of $M_A$, as is frequently done in the literature. The mass of the charged Higgs boson, $M_{H^+}$, instead, remains an observable physical parameter, in terms of which the parametrization of the Higgs sector becomes possible. As has been shown in [7,10], all dominant contributions to the neutral Higgs-boson mass matrix elements may be expressed as a function of $M_{H^+}$, $\tan \beta$ and the soft SUSY-breaking parameters associated with the third–generation squarks, the $\mu$ parameter and the weak and strong gaugino masses. Therefore, $M_{H^+}$ plays a very essential role in our analysis and is a preferred replacement for $M_A$ as a physical MSSM Higgs parameter. Moreover, $M_{H^+}$ is also directly related to the mass of the CP-odd Higgs boson $M_A$ in the CP-invariant limit of the theory. Therefore, $M_{H^+}$ is also an adequate physical input parameter in the simplified case in which the three neutral Higgs bosons carry definite CP parity.

It is obvious from (1) that CP-violating effects on the neutral Higgs-boson mass matrix become significant when the product $\text{Im}(\mu A_t)/M^2_{\text{SUSY}}$ is large. Motivated by this observation, we introduce the following new CP-violating benchmark scenario (CPX):

$$\begin{align*}
\tilde{M}_Q &= \tilde{M}_t = \tilde{M}_b = M_{\text{SUSY}}, & \mu &= 4M_{\text{SUSY}}, \\
|A_t| &= |A_b| = 2M_{\text{SUSY}}, & \text{arg}(A_t) &= 90^\circ, \\
|m_{\tilde{g}}| &= 1 \text{ TeV}, & \text{arg}(m_{\tilde{g}}) &= 90^\circ,
\end{align*}$$

(2)

where we follow the notation of [10]. Without loss of generality, the $\mu$ parameter is considered to be real. We note that the CP-odd angles $\text{arg}(A_t)$ and $\text{arg}(m_{\tilde{g}})$ are chosen to take their maximal CP-violating values. In the following, we also discuss variants of the CPX scenario with other values of $\text{arg}(A_t)$ and $\text{arg}(m_{\tilde{g}})$, keeping the other quantities fixed at the values in (2).

Large CP-odd phases are known to lead to large contributions to the electron and neutron electric dipole moments (EDMs) [17]. The main bulk of the large EDM effects may be avoided by making the first two generation of squarks sufficiently heavy, with masses of order 1 TeV and higher [18]. However, even in this case, top and bottom squarks may give rise to observable contributions to the electron and neutron EDMs through the three-gluon operator [19], through the effective coupling of the ‘CP-odd’ components of the Higgs boson to the gauge bosons [20], and through two-loop gaugino/higgsino-mediated
EDM graphs \[21\], which may become large for large values of $\tan \beta$. However, exactly as happens at the one-loop level \[22,23\], these different EDM contributions of the third generation can also have different signs and add destructively to the electron and neutron EDMs.

For our numerical comparisons, we also consider a related CP-conserving benchmark scenario for which the stop mixing parameters are chosen in order to maximize the lightest neutral Higgs boson mass value for large values of the charged Higgs mass (MAX) \[24\]:

\[
\begin{align*}
\tilde{M}_Q = \tilde{M}_t = \tilde{M}_b = M_{\text{SUSY}}, \\
A_t = A_b = \sqrt{6} M_{\text{SUSY}}, \\
m_{\tilde{g}} = 1 \text{ TeV}, \\
\mu = m_{\tilde{B}} = m_{\tilde{W}} = 200 \text{ GeV}.
\end{align*}
\]

In this CP-conserving benchmark scenario, we take relatively small values for the $\mu$ parameter and the gaugino masses $m_{\tilde{B}}$ and $m_{\tilde{W}}$, in order to maximize the effect of the one-loop logarithmic corrections coming from chargino and neutralino interactions \[13\].

We should stress that neither the MAX nor the CPX scenario are generated in simple scenarios for SUSY breaking, such as those based on minimal supergravity or gauge mediation. For example, in supergravity models, the large values of the $A_t$ parameter, compared to the third–generation squark masses, that are needed in the MAX scenario, can only be generated by large values of this parameter at the GUT scale, an order of magnitude larger than the gaugino masses at that scale.

The large values of $|\mu|$ chosen in the CPX scenario also do not arise in minimal supergravity or gauge-mediated models. Indeed, such large values of $|\mu|$ can be consistent with electroweak symmetry breaking only if the soft SUSY-breaking parameters associated with the Higgs masses are negative and of the same order as $|\mu|$. This can easily be seen by ignoring one-loop corrections, which are inessential for this specific discussion. The soft SUSY-breaking mass parameters $m_1^2$ and $m_2^2$, which are associated with the scalar components of the Higgs-doublet superfields $\tilde{H}_1$ and $\tilde{H}_2$, are then related to the value of $\mu$ and the charged Higgs boson mass by the following relations:

\[
M_{H^+}^2 = \frac{\tan^2 \beta + 1}{\tan^2 \beta - 1} (m_1^2 - m_2^2) + M_W^2 - M_Z^2, \tag{4}
\]

\[
|\mu|^2 = \frac{m_1^2 - m_2^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{M_W^2}{2}. \tag{5}
\]

We conclude from (4) that, in order to get small values of the charged Higgs mass, the values of $m_1^2$ and $m_2^2$ must be close to each other. In addition, it follows from (5) that small values of $M_{H^+}$ are only compatible with large values of $|\mu|$ if both the parameters $m_1^2$ and $m_2^2$ are large and negative.
It is known that third-generation Yukawa-coupling effects may induce negative values of the Higgs soft SUSY-breaking parameters resulting in the breakdown of the electroweak symmetry. The large values of the trilinear coupling at the high-energy input scale which are required to generate $A_t \simeq \mathcal{O}(2M_{\text{SUSY}})$ at the weak scale, are helpful in driving $m_2^2$ to large negative values. However, for small and moderate values of $\tan \beta$, the large negative values of the soft SUSY-breaking Higgs mass parameter $m_1^2$, necessary for the realization of the CPX scheme, can only be induced if its value at the input scale is already negative and its absolute value is larger than the squark masses. These non-standard boundary conditions might be obtained, for instance, in models inspired by superstring or $M$ theory, in which SUSY breaking is induced by the vacuum expectation value of the auxiliary component of moduli fields [23].

The aim of the MAX and CPX benchmark scenarios, however, is to study the phenomenological consequences for Higgs searches for the most challenging values of the MSSM parameters. This allows one to study the capability of the present and near future colliders to explore the Higgs boson properties in the most generic framework.

We display in Fig. 1 contours of the masses of the two lightest neutral Higgs bosons $H_{1,2}$ in the CPX scenario, for the two values $M_{\text{SUSY}} = 0.5$, 1 TeV. As we shall see, neutral Higgs bosons as light as about 50 GeV are allowed within the CPX scenario, whereas such a light Higgs boson is not allowed in the MAX scenario. In fact, there are significant regions of parameter space in the $(M_{H^+}, \tan \beta)$ plane where the lightest neutral Higgs boson $H_1$ contains a large admixture of the CP-odd state $A$. In this case, there are important consequences for the $H_1$ couplings, as we now discuss.

At LEP 2, the main production mechanism for the neutral Higgs bosons $H_i$ and $i = 1, 2, 3$ is the Higgs-strahlung process: $e^+e^- \to Z^* \to ZH_i$ [26]. If the neutral Higgs bosons are relatively light, they may also be produced in pairs through the reaction: $e^+e^- \to Z^* \to H_iH_j$, with $i \neq j$ and $i, j = 1, 2, 3$. Therefore, the interaction Lagrangian of interest to us, which describes the effective $H_iZZ$ and $H_iH_jZ$ couplings, is given by

$$\mathcal{L}_{\text{int}} = \frac{g_w}{2\cos \theta_w} \left[ M_Z \sum_{i=1}^{3} g_{H_{iZZ}} H_i Z_\mu Z^\mu + \sum_{j>i=1}^{3} g_{H_{ijZ}} (H_i \leftrightarrow \partial_\mu H_j) Z^\mu \right], \quad (6)$$

where $\cos \theta_w \equiv M_W/M_Z$, $\leftrightarrow \partial_\mu \equiv \vec{\partial}_\mu - \vec{\partial}_\mu$, and

$$g_{H_{iZZ}} = \cos \beta O_{1i} + \sin \beta O_{2i},$$
$$g_{H_{ijZ}} = O_{3i} \left( \cos \beta O_{2j} - \sin \beta O_{1j} \right) - O_{3j} \left( \cos \beta O_{2i} - \sin \beta O_{1i} \right). \quad (7)$$

Here, $\tan \beta = v_2/v_1$ is the ratio of the vacuum expectation values of the two Higgs doublets, and $O$ is an orthogonal matrix relating the weak to the mass eigenstates of the neutral Higgs.
bosons. The effective couplings $H_i ZZ$ and $H_i H_j Z$ are related to each other through

$$g_{H_i ZZ} = \varepsilon_{ijk} g_{H_i H_j Z}.$$  \hspace{1cm} (8)

Unitarity leads to the coupling sum rule \[27\]

$$\sum_{i=1}^{3} g_{H_i ZZ}^2 = 1,$$ \hspace{1cm} (9)

which reduces the number of the independent $H_i ZZ$ and $H_i H_j Z$ couplings. Finally, there is another important sum rule involving the neutral Higgs-boson masses and their respective couplings to the $Z$ bosons:

$$\sum_{i=1}^{3} g_{H_i ZZ}^2 M_{H_i}^2 = M_{H_1}^{2, \text{max}},$$ \hspace{1cm} (10)

where $M_{H_1}^{2, \text{max}}$ is the $H_1$-boson mass in the decoupling limit, in which $M_{H^+} \gg 2M_Z$ and all other parameters, apart from the charged-Higgs mass, are assumed to be the same in the computation of both sides of (10). The mass-coupling sum rule (10) is very analogous to the one found in \[28\] for the CP-conserving case. In the decoupling limit, we have $g_{H_1 ZZ}^2 \rightarrow 1$, so the lower limit on $M_{H_1}$ reaches its maximum value.

We display in Fig. 2 the masses $M_{H_i}$ of the two lightest neutral Higgs bosons, as well as their corresponding couplings $g_{H_i ZZ}$ to the $Z$ boson for the soft supersymmetry breaking parameters defined in the CPX scenario for several choices of $(M_{H^+}, \tan \beta)$, varying the CP-violating phases $arg(A_{t,b})$. The level-crossing phenomenon discussed in \[10\] is clearly visible in the upper panel, and is associated with a change in the strength of the couplings of the lightest and next-to-lightest Higgs bosons to the $Z$ gauge boson, as seen in the lower panel. As is apparent in the lower panel, the $g_{H_i ZZ}$ coupling is strongly suppressed for values of $M_{H^+}$ and $arg(A_{t,b})$ near those where the level crossing takes place.

On the other hand, the branching ratios for $H_{1,2} \rightarrow \bar{b}b$ decay are not in general greatly modified in comparison with the CP-conserving MAX scenario, as can be seen in Fig. 3. This property is expected to be valid for low values of $\tan \beta$, such as the ones considered here. For larger values of $\tan \beta$, scalar-pseudoscalar mixing effects induced by stop and sbottom quantum effects become relatively less significant\[1\] and CP-violating vertex effects on Higgs-boson decays become more important \[10,30\]. Consistent with Fig. 3, in what follows we assume that the branching ratios for the decays of the $H_1$ and $H_2$ into $\bar{b}b$ are $\simeq 1$, and pursue the implications of the differences in production cross sections for $e^+e^- \rightarrow Z + H_{1,2}$ and $e^+e^- \rightarrow H_1 H_2$ between the MAX and CPX benchmarks.\[1\]

\[1\] CP-violating chargino effects were recently computed in \[29\], and found to be of relevance only for large values of $\tan \beta \gtrsim 30$. They are comparable with the small squark effects of the third generation in this large-$\tan \beta$ regime.
In Fig. 4 we compare the 95% confidence-level exclusion limits [31,32] on the neutral Higgs bosons in the \((M_{H^+}, \tan \beta)\) plane for the two scenarios CPX and MAX, for the choices \(M_{\text{SUSY}} = 0.5\) and 1 TeV in the upper and lower panels, respectively. The solid lines in Fig. 4 refer to the CP-violating CPX benchmark scenario (2), whilst the dashed ones are for the CP-conserving MAX benchmark scenario (3). In order to obtain the limits shown, we have rescaled the quoted SM Higgs mass limits [32] to take account of the fact that no SM-like Higgs boson has yet been observed with a mass up to about 112.3 GeV at the 95% confidence level (CL) [32]. The areas lying to the left of the lines are excluded. We concentrate on small values of the charged-Higgs-boson mass, for which the effect of CP violation is maximized.

As we mentioned above, for large values of the charged-Higgs-boson mass, CP-violation effects decouple from the lightest neutral Higgs boson, whose couplings to fermions and gauge bosons resemble those of the SM Higgs boson. Therefore, for large values of the charged-Higgs-boson mass and for fixed values of the third-generation squark-mass parameters, the MAX scenario leads to the most conservative limits on \(\tan \beta\). For smaller values of the charged Higgs mass, instead, CP-violating effects can considerably weaken the quoted LEP Higgs bound. Indeed, we see in Fig. 4 that, for any given value of \(\tan \beta\) and \(M_{\text{SUSY}}\), lower values of \(M_{H^+}\) are allowed in the CPX scenario. Comparing with Fig. 1, we see that, due the effects of CP violation the lightest neutral Higgs boson \(H_1\) could be as light as about 50 GeV and remain undetected at LEP.

We also observe in Figs 1 and 2 another interesting feature, namely that \(M_{H_2}\) lies between 110 and 120 GeV over much of the parameter region where \(M_{H_1} < 100\) GeV [4]. In this region, the lightest neutral Higgs boson may remain unobserved due to a strong suppression of its coupling to the \(Z\) gauge boson. The second-lightest neutral Higgs-boson, instead, has couplings to gauge bosons and to bottom quarks that are similar to those in the SM, and its mass may be consistent with the apparent excess of events reported recently at LEP 2 [5]. Therefore, it is conceivable that LEP has evidence for the second-lightest Higgs boson, not the lightest [4], even though this may not be the most obvious interpretation.

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‡ The irregularities of the lines originate from our approximate reading of the experimental limits [32]. Our intention here is to establish the differences between the CPX and MAX benchmark scenarios. We leave more complete studies to the LEP Collaborations and the LEP Higgs Working Group.

§ However, we respect the direct experimental limits for \(M_{H^+}\) established at LEP 2 and the Tevatron [33].

¶ At present, the theoretical uncertainties in the mass calculation are of about 3 GeV [14,15] and, moreover, there are uncertainties due to the value of \(m_t\), which may lie in the range 170 to 180 GeV. The top panel of Fig 1 illustrates the corresponding uncertainty in the \((M_{H^+}, \tan \beta)\) plane.

∥ A second-lightest Higgs boson with SM-like couplings to gauge bosons and fermions can also appear in the absence of CP violation [28]. However, in this case, it tends to occur for larger values of \(\tan \beta\), for
On a more speculative note, we observe that the combination of the data of the four LEP experiments has not only led to an apparent excess around 115 GeV, but may also not be able to exclude an additional excess of events around 95 GeV at a level corresponding to a Higgs cross section about a tenth of the SM cross section \[34\]. As is shown in Fig. 2, both ‘excesses’ could be reproduced simultaneously in the CP-violating scenario, for \(M_{H^+} \sim 160\) GeV, \(\tan \beta \sim 4\) and \(\arg(A_t) \sim 85\) degrees.

In conclusion: we have demonstrated how radiative corrections in the MSSM with explicit CP violation can enlarge the parameter space allowed for the MSSM Higgs sector at low values of \(\tan \beta\) and the charged Higgs-boson mass \(M_{H^+}\), i.e., for \(\tan \beta \lesssim 7\) and \(M_{H^+} \lesssim 170\) GeV. In developing this analysis, we have introduced a new CP-violating benchmark scenario for Higgs mixing, which should be useful for testing the CP-violating scenario at LEP2 and other colliders. We have also emphasized that the optimal parametrization of MSSM Higgs bosons is in terms of \(M_{H^\pm}\) and \(\tan \beta\). Finally, as a by-product, we have given a novel interpretation of the possible excess of events recently reported at LEP 2 \[5\]. If this excess were to be confirmed by further LEP running, data from the Tevatron collider and from the LHC would be necessary in order to discriminate this possibility from more standard explanations.

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  http://alephwww.cern.ch/
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  J.-J. Blaising, for the L3 Collaboration,
  http://l3www.cern.ch/analysis/latestresults.html
  C. Rembser, for the OPAL Collaboration,
  http://opal.web.cern.ch/Opal/PPwelcome.html
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Figure 1: Contours of the lightest neutral Higgs boson mass $M_{H_1}$ in the $(M_{H^+}, \tan \beta)$ plane, for the CP-violating benchmark scenario (3) CPX and values of $M_{\text{SUSY}} = 0.5$ and 1 TeV. Contours of $M_{H_1} = 113$ GeV are also indicated by dash-dotted lines. The three such lines in the upper panel correspond (from right to left) to $m_t = 170, 175, 180$ GeV: all the other contours in this and subsequent figures are drawn for our default choice $m_t = 175$ GeV.
Figure 2: Predicted values of (a) $M_{H_1}$ and $M_{H_2}$ and (b) $g_{H_1 ZZ}^2$, $g_{H_2 ZZ}^2$, $g_{H_3 ZZ}^2$ as functions of $\arg (A_t) = \arg (A_b)$, in the CPX scenario for $M_{\text{SUSY}} = 0.5$ TeV and for the following choices of $(M_{H^+}, \tan \beta)$: (160 GeV, 4) (solid lines), (150 GeV, 5) (dashed lines) and (140 GeV, 6) (dotted lines).
Figure 3: Numerical estimates of the squared $H_1 b\bar{b}$ and $H_2 b\bar{b}$ couplings, normalized to their SM values, as functions of $\tan\beta$, in the CP-violating benchmark scenario (a) CPX (top panel) and the maximal stop-mixing CP-conserving scenario (b) MAX (bottom panel).
Figure 4: Approximate 95 % C.L. exclusion plots in the \((M_{H^+}, \tan \beta)\) plane, for the CP-violating benchmark scenario (a) CPX (solid lines) and the maximal stop-mixing CP-conserving scenario (b) MAX (dashed lines), for the two indicated sets of soft SUSY-breaking parameters.