Quantum Dot Spin Filter in Resonant Tunneling and Kondo Regimes

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A quantum dot with spin-orbit interaction can work as an efficient spin filter if it is connected to $N \geq 3$ external leads via tunnel barriers. When an unpolarized current is injected to a quantum dot from a lead, polarized currents are ejected to other leads. A two-level quantum dot is examined as a minimal model. First, we show that the spin polarization is markedly enhanced by resonant tunneling when the level spacing in the dot is smaller than the level broadening. Next, we examine the many-body resonance induced by the Kondo effect in the Coulomb blockade regime. A large spin current is generated in the presence of the SU(4) Kondo effect when the level spacing is less than the Kondo temperature.

KEYWORDS: quantum dot, spin filter, spin-orbit interaction, Kondo effect, spin Hall effect

The generation of spin current with no magnetic field or ferromagnets is an important issue for spin-based electronics, “spintronics.”\(^1\) In this context, the spin-orbit (SO) interaction has attracted much interest. For conduction electrons in direct-gap semiconductors, an external potential $U(r)$ results in the Rashba SO interaction\(^2,3\)

$$H_{RSO} = \frac{\lambda}{\hbar} \sigma \cdot [p \times \nabla U(r)], \quad (1)$$

where $p$ is the momentum operator and $\sigma$ is the Pauli matrices indicating the electron spin $s = \sigma/2$. The coupling constant $\lambda$ is markedly enhanced by the band effect, particularly in narrow-gap semiconductors, such as InAs.\(^4,5\) The spatial inversion symmetry is broken in compound semiconductors, which gives rise to another type of SO interaction, the Dresselhaus SO interaction.\(^6\)

It is given by

$$H_{DSO} = \frac{\lambda'}{\hbar} [p_x(p_y^2 - p_z^2)\sigma_x + p_y(p_z^2 - p_x^2)\sigma_y + p_z(p_x^2 - p_y^2)\sigma_z]. \quad (2)$$

In the presence of SO interaction, the spin Hall effect (SHE) produces a spin current traverse to an electric field applied by the bias voltage. Two types of SHE have been intensively studied. One is an intrinsic SHE, which is induced by the drift motion of carriers in the SO-split band structures.\(^7-9\) The other is an extrinsic SHE caused by the spin-dependent scattering of electrons by impurities.\(^10\) Kato \textit{et al.} observed the spin accumulation at sample edges traverse to the current,\(^11\) which is ascribable to the extrinsic SHE with $U(r)$ being the screened Coulomb potential by charged impurities in eq. (1).\(^12\)

In our previous studies,\(^13,14\) we theoretically examined the extrinsic SHE in semiconductor heterostructures due to the scattering by an artificial potential created by antidots, STM tips, and others. The potential is electrically tunable. We showed that the SHE is significantly enhanced by the resonant scattering when the attractive potential is properly tuned. We proposed a three-terminal spin-filter including a single antidot.

In the present letter, we study the enhancement of the “extrinsic SHE” by resonant tunneling through a quantum dot (QD) with a strong SO interaction, e.g., InAs QD.\(^15-18\) The QD is connected to $N$ external leads via tunnel barriers. In the QD, the number of electrons can be tuned, one by one, owing to the Coulomb blockade when the electrostatic potential is changed by the gate voltage $V_g$. The current through a QD shows a peak structure as a function of $V_g$ (Coulomb oscillation). We use the term SHE in the following meaning: For $N \geq 3$, when an unpolarized current is injected to the QD from a lead, polarized currents are ejected to the other leads. In other words, the QD works as a spin filter. First, we examine the SHE around the current peaks, where the resonant tunneling takes place. We show that the spin polarization is markedly enhanced when the energy-level spacing in the QD is smaller than the level broadening due to the tunnel coupling to external leads. Next, we examine the many-body resonance induced by the Kondo effect in the Coulomb blockade regime with spin $1/2$ in the QD. We obtain a large spin current in the presence of the SU(4) Kondo effect when the level spacing is less than the Kondo temperature.

We assume that the SO interaction is present only in the QD and that the level spacing in the QD is comparable to the level broadening $\Gamma \sim 1$ meV, in accordance with experimental situations.\(^15-18\) The strength of SO interaction, $\Delta_{SO}$ in eq. (3), is approximately 0.2 meV.\(^16-18\) As a minimal model, we examine two levels in the QD. Note that previous theoretical papers\(^19-22\) concerned the spin-current generation in a mesoscopic region, or an open QD with no tunnel barriers, in which

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where in lead tunnel coupling, controllable by electrically tuning the tunnel barrier, \( \langle r|2 \rangle \) in the QD and hardly controllable for a given current peak. \( \{ e_\alpha \} \) and \( \Delta \) vary from peak to peak during the Coulomb oscillation. We can choose a peak with appropriate parameters for the SHE in experiments.

We assume a single channel of conduction electrons in the leads. The total Hamiltonian is
\[
H = \sum_\alpha \sum_{k,\sigma} \varepsilon_k c_{\alpha k,\sigma}^\dagger c_{\alpha k,\sigma} + H_{\text{dot}} + H_T. \tag{5}
\]

The strength of the tunnel coupling is characterized by the level broadening, \( \Gamma_\alpha = \pi \nu_\alpha (V_\alpha)^2 \), where \( \nu_\alpha \) is the density of states in lead \( \alpha \). We also introduce a matrix of \( \hat{\Gamma} = \sum_\alpha \hat{\Gamma}_\alpha \) with
\[
\hat{\Gamma}_\alpha = \Gamma_\alpha \left( \frac{(e_{\alpha,1})^2}{e_{\alpha,1} e_{\alpha,2}^*}, \frac{e_{\alpha,1} e_{\alpha,2}}{e_{\alpha,2}^*} \right). \tag{6}
\]

An unpolarized current is injected into the QD from a source lead \( \alpha = \text{S} \) and output to other leads \( \{D_n; n = 1, \ldots, (N-1)\} \). The electrochemical potential for electrons in lead S is lower than that in the other leads by \( -eV_{\text{bias}} \). The current with spin \( \sigma = \pm \) from lead \( \alpha \) to the QD is written as
\[
I_{\alpha,\sigma} = \frac{ie}{\hbar} \int d\varepsilon \text{Tr} \{ \hat{\Gamma}_\alpha [ f_\alpha(\varepsilon)(\hat{G}_\alpha^r - \hat{G}_\alpha^\text{a}) + \hat{G}_\sigma^c \} \}, \tag{7}
\]
where \( \hat{G}_\alpha^r, \hat{G}_\alpha^\text{a}, \) and \( \hat{G}_\sigma^c \) are the retarded, advanced, and lesser Green functions in the QD, respectively, in a 2 \( \times \) 2 matrix form in the pseudo-spin space.\(^{24}\) \( f_\alpha(\varepsilon) \) is the Fermi distribution function in lead \( \alpha \). In the absence of electron-electron interaction, \( H_{\text{int}} \), the conductance into lead \( D_n \) with spin \( \sigma \) is given by\(^{25}\)
\[
(3) G_{n,\sigma} = -\frac{dI_{D_n,\sigma}}{dV_{\text{bias}}}, \quad \bigg|_{V_{\text{bias}}=0} = \frac{4e^2}{h} \text{Tr} \{ \hat{G}_\sigma^\text{a}(\varepsilon_F) \hat{G}_\sigma^c(\varepsilon_F) \hat{G}_\sigma^\text{r} \}, \tag{8}
\]
where the QD Green function is
\[
\hat{G}_{\pm}(\varepsilon) = \left[ \left( \begin{array}{cc} \varepsilon - \varepsilon_d + \frac{\Delta}{2} & \mp i \frac{\Delta_{\text{SO}}}{2} \\ \mp i \frac{\Delta_{\text{SO}}}{2} & \varepsilon - \varepsilon_d - \frac{\Delta}{2} \end{array} \right) + i \hat{\Gamma} \right]^{-1} \tag{9}
\]
and \( \varepsilon_F \) is the Fermi energy.

Now, we discuss the SHE in the vicinity of the Coulomb peaks. The electron-electron interaction is neglected in this regime. From eqs. (8) and (9), we obtain
\[
G_{n,\sigma} = \frac{e^2 4 \Gamma_S \Gamma_D n}{|D|^2} \left[ g_{n,1}^{(1)} + g_{n,\sigma}^{(2)} \right], \tag{10}
\]
\[
g_{n,1}^{(1)} = \left[ \left( \varepsilon_F - \varepsilon_d - \frac{\Delta}{2} \right) \varepsilon_{D_1,1} \varepsilon_{S_1} + (\varepsilon_F - \varepsilon_d + \frac{\Delta}{2}) \varepsilon_{D_2,2} \varepsilon_{S_2} \right]^2, \tag{11}
\]
\[
g_{n,\pm}^{(2)} = \pm \frac{\Delta_{\text{SO}}}{2} (\varepsilon_S \times \varepsilon_{D_n})_z. \tag{12}
\]
Fig. 2. Calculated results of the conductance $G_{1,\pm}^\pm$ as a function of energy level, $\varepsilon_d = (\varepsilon_1 + \varepsilon_2)/2$, in a three-terminated quantum dot. Solid (broken) lines indicate the conductance $G_{1,\pm}$ ($G_{1,-}$) for spin $\sigma = +1$ ($-1$). The level broadening by the tunnel coupling to leads S and D1 is $\Gamma_S = \Gamma_{D1} \equiv \Gamma$ ($\varepsilon_{S,1}/\varepsilon_{S,2} = 1/2, \varepsilon_{D1,1}/\varepsilon_{D1,2} = -3$), whereas that to lead D2 is (a) $\Gamma_{D2} = 0.2\Gamma$, (b) $0.5\Gamma$, (c) $\Gamma$, and (d) $2\Gamma$ ($\varepsilon_{D2,1}/\varepsilon_{D2,2} = 1$). $\Delta = \varepsilon_2 - \varepsilon_1 = 0.2\Gamma$ in the main panels and $\Delta = \Gamma$ in the insets. $\Delta_{SO} = 0.2\Gamma$.

$$+ \sum_\sigma \Gamma_\alpha(e_{Dn} \times e_\alpha)_z (e_S \times e_\alpha)_z^2,$$

$$+ \sum_\sigma [\hat{\mathcal{G}}_\alpha(\varepsilon_F)]^{-1} \Gamma_\alpha \Gamma_\beta \sum_{j=1,2} [ (e_{Dn,j} e_{S,j})^2 / (\varepsilon_j - \varepsilon_F)^2 + (\Gamma_{jj})^2 ]^2,$$

where $D$ is the determinant of $[\hat{\mathcal{G}}_\alpha(\varepsilon_F)]^{-1}$ in eq. (9), which is independent of $\sigma$, and $(a \times b)_z = a_1 b_2 - a_2 b_1$. Let us consider two simple cases. (I) When $\Delta \gg \Gamma_\sigma$ and $\Delta_{SO}$, $G_{n,\sigma}$ consists of two Lorentzian peaks as a function of $\varepsilon_d$, reflecting the resonant tunneling through one of the energy levels, $\varepsilon_{1,2} = \varepsilon_d \mp \Delta/2$:

$$G_{n,\sigma} \approx \frac{4e^2}{h} \Gamma_S \Gamma_{Dn} \sum_{j=1,2} \frac{(e_{Dn,j} e_{S,j})^2}{(\varepsilon_j - \varepsilon_F)^2 + (\Gamma_{jj})^2},$$

which two out of $e_S$, $e_{D1}$, and $e_{D2}$ are parallel to each other hereafter. The conditions for a large spin current are as follows: (i) $\Delta \approx (\text{level broadening})$, as mentioned above. Two levels in the QD should participate in the transport. (ii) The Fermi level in the leads is close to the energy levels in the QD, $\varepsilon_F \approx \varepsilon_d$ (resonant condition). (iii) The level broadening by the tunnel coupling to lead D2, $\Gamma_{D2}$, is comparable to the strength of SO interaction $\Delta_{SO}$.

Figures 2 and 3 show two typical results of the conductance $G_{1,\pm}$ as a function of $\varepsilon_d$ (Coulomb peak). In $g_1^{(1)}$, $e_{D1,1} e_{S,1}$ and $e_{D1,2} e_{S,2}$ have different (same) signs in Fig. 2 (Fig. 3): $g_1^{(1)} = 0$ has no solution (a solution) in $-\Delta/2 < \varepsilon_d - \varepsilon_F < \Delta/2$.

In Fig. 2, the conductance shows a single peak. We set $\Gamma_S = \Gamma_{D1} \equiv \Gamma$. When $\Delta = 0.2\Gamma$ (main panels), we obtain a large spin current around the current peak, which clearly indicates an enhancement of the SHE by resonant tunneling [conditions (i) and (ii)]. With increasing $\Gamma_{D2}$ from (a) $0.2\Gamma$ to (d) $2\Gamma$, the spin current increases first, takes a maximum in panel (c), and then decreases [condition (iii)]. Therefore, the SHE is tunable by changing the tunnel coupling. When $\Delta = \Gamma$ (insets), the SHE is less effective, but we still observe a spin polarization of $P = (G_{1,1} - G_{1,-})/(G_{1,1} + G_{1,-}) \approx 0.25$ at the conductance peak in panel (c).

In Fig. 3, the conductance $G_{1,\pm}$ shows a dip at $\varepsilon_d \approx \varepsilon_F$ for small $\Gamma_{D2}$. Around the dip, the spin polarization is markedly enhanced: $|P|$ is close to unity in panel (a).

Next, we examine the Kondo effect in the Coulomb blockade regime with a single electron in the QD.
The Kondo effect is not broken by the SO interaction since the time-reversal symmetry holds. For the electron-electron interaction in the QD, we assume that $H_{\text{int}} = U \sum_j n_{j,+} n_{j,-} + U' \sum_{\sigma,\sigma'} n_{1,\sigma} n_{2,\sigma'}$, where $n_{j,\sigma} = d_{j,\sigma}^\dagger d_{j,\sigma}$, with infinitely large $U$ and $U'$. The Kondo effect creates the many-body resonant state at the Fermi level, and thus condition (ii) is always satisfied. The resonant width is given by the Kondo temperature $T_K$.\(^{29}\)

When $T_K < \Delta$, the upper level in the QD is irrelevant. The spin at the lower level is screened out by the conventional SU(2) Kondo effect. When $T_K > \Delta$, on the other hand, the spin-pseudo with as well as the spin are screened by the SU(4) Kondo effect.\(^{30}\) The latter situation is required for an enhanced SHE since two levels should be relevant to the resonance [condition (i)].

The crossover between the SU(2) and SU(4) Kondo effects can be semiquantitatively described by the slave-boson mean-field theory.\(^{31}\) The theory describes the Kondo resonant state on the assumption of its presence and Fermi liquid behavior and yields the conductance at temperature $T = 0$. A boson operator $b$ is introduced to represent an empty state in the QD, $d_{j,\sigma}^\dagger = f_{j,\sigma}^\dagger b$ and $d_{j,\sigma} = b f_{j,\sigma}$, with a fermion operator $f_{j,\sigma}$ representing the pseudo-spin $j$ and spin $\sigma$. $H_{\text{int}}$ is taken into account by the constraint of $Q = \sum_{j,\sigma} f_{j,\sigma}^\dagger f_{j,\sigma} + b^\dagger b - 1 = 0$. $b$ is replaced with the mean field $(b)$, which is determined by minimizing $(H + \lambda Q)$ with the Lagrange multiplier $\lambda$.\(^{29}\)

The conductance is given by eq. (10) if $\varepsilon_d$ and $\Gamma_\alpha$ are replaced with the renormalized ones, $\varepsilon_d + \lambda (\sim \varepsilon_F)$ and $\Gamma_\alpha (b)^2 (\sim T_K)$, respectively.

Figure 4 shows $G_{1,\pm}$ as a function of $\varepsilon_d$ in the three-terminal system. The parameters are the same as those in the main panels in Fig. 2. In the two-terminal situation (curve $a$; $G_{1,+} = G_{1,-}$), the conductance increases with decreasing $\varepsilon_d$ and saturates, indicating the charge fluctuation regime and Kondo regime, respectively. With three leads (curves $b$–$e$), we observe a spin current around the beginning of the Kondo regime. $P \approx 0.5$ in the case of curve $c$. As $\varepsilon_d$ is decreased further, $T_K$ decreases and becomes smaller than $\Delta$, which weakens the SHE. We obtain similar results using the parameters in Fig. 3.

In summary, we have examined the SHE in a multi-terminated QD with SO interaction. The spin polarization in the output currents is markedly enhanced by resonant tunneling if the level spacing in the QD is smaller than the level broadening. The spin current is also enlarged by the many-body resonances due to the SU(4) Kondo effect. The SHE is electrically tunable by changing the tunnel coupling to the leads.

Hamaya et al. fabricated InAs QDs connected to ferromagnets.\(^{32}\) If a ferromagnet is used as a source lead in our model, spin filtering is electrically detected through an “inverse SHE.” The current to lead D1 is proportional to $(1 + p \cos \theta)G_{D_{1,+}} + (1 - p \cos \theta)G_{D_{1,-}}$, where $p$ is the polarization in the ferromagnet and $\theta$ is the angle between the magnetization and $\mathbf{h}_{\text{SO}}$.

The SHE in QDs is useful for the fundamental research as well as for the application to an efficient spin filter. The SHE enhanced by resonant scattering or Kondo resonance was examined for metallic systems with magnetic impurities.\(^{33-35}\) In semiconductor QDs, we can observe the SHE due to the scattering by a single “impurity” with the tuning of various conditions.

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1) I. Žutić, J. Fabian, and S. Das Sarma: Rev. Mod. Phys. 76 (2004) 323.
2) E. I. Rashba: Fiz. Tverd. Tela (Leningrad) 2 (1960) 1224 [in Russian].
3) Yu. A. Bychkov and E. I. Rashba: J. Phys. C 17 (1984) 6039.
4) R. Winkler: *Spin-Orbit Coupling Effects in Two-Dimensional Electron and Hole Systems* (Springer, Heidelberg, 2003).
5) J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki: Phys. Rev. Lett. 78 (1997) 1335.
6) G. Dresselhaus: Phys. Rev. 100 (1955) 580.
7) S. Murakami, N. Nagaosa, and S. C. Zhang: Science 301 (2003) 1348.
8) J. Wunderlich, B. Kaestner, J. Sinova, and T. Jungwirth: Phys. Rev. Lett. 94 (2005) 47204.
9) J. Sinova, D. Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth, and A. H. MacDonald: Phys. Rev. Lett. 92 (2004) 126603.
10) M. I. Dyakonov and V. I. Perel: Phys. Lett. 35A (1971) 459.
11) Y. K. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom: Science 306 (2004) 1910.
12) H. Engel, B. I. Halperin, and E. I. Rashba: Phys. Rev. Lett. 95 (2005) 166605.
13) M. Eto and T. Yokoyama: J. Phys. Soc. Jpn. 78 (2009) 073710.
14) T. Yokoyama and M. Eto: Phys. Rev. B 80 (2009) 125311.
15) Y. Igarashi, M. Jung, M. Yamamoto, A. Oiwa, T. Machida, K. Hirakawa, and S. Tarucha: Phys. Rev. B 76 (2007) 081303(R).
16) C. Fasth, A. Fuhrer, L. Samuelson, V. N. Golovach, and D. Loss: Phys. Rev. Lett. 98 (2007) 266801.
17) A. Pfund, I. Shorubalko, K. Ensslin, and R. Leturcq: Phys. Rev. B 76 (2007) 161308(R).
18) S. Takahashi, R. S. Deacon, K. Yoshida, A. Oiwa, K. Shibata, K. Hirakawa, Y. Tokura, and S. Tarucha: Phys. Rev. Lett. 104 (2010) 246801.
19) A. A. Kiselev and K. W. Kim: Phys. Rev. B 71 (2005) 153315.
20) J. H. Bardarson, I. Adagideli, and Ph. Jacquod: Phys. Rev. Lett. 98 (2007) 196601.
21) J. J. Krich: Phys. Rev. B 80 (2009) 245313.
22) J. J. Krich and B. I. Halperin: Phys. Rev. B 78 (2008) 035338.
23) For $H_{DSO}$, $h_{SO}$ is replaced with $h_{DSO}$, where $ih_{DSO} \sigma / 2 = (\lambda'/\hbar)|p_x(p_y - p_z)|$, etc. In InAs QDs, $H_{RSO}$ and $H_{DSO}$ may coexist. Then $h_{SO}$ is changed to $h_{SO} + h_{DSO}$.
24) Y. Meir and N. S. Wingreen: Phys. Rev. Lett. 68 (1992) 2512.
25) For noninteracting electrons, $\hat{G}_\sigma - \hat{G}_\sigma = -2i\hat{G}_\sigma \hat{\Gamma} \hat{G}_\sigma$ and $\tilde{G}_\sigma = 2\tilde{G}_\sigma (\sum_{\alpha} \hat{\Gamma}_\alpha f_{\alpha}) \hat{G}_\sigma$. The substitution of these relations into eq. (7) yields $I_{Dn,\sigma} = (4e/h) \int d\epsilon[f_D(\epsilon) - f_S(\epsilon)]Tr(\hat{G}_\sigma \hat{\Gamma}_{Dn} \hat{G}_\sigma \hat{\Gamma}_S)$, where $f_{Dn}(\epsilon) \equiv f_D(\epsilon)$.
26) In the presence of more than one channel in a lead, a spin current can be generated in two-terminal systems, e.g., see, M. Eto, T. Hayashi, and Y. Kurotani: J. Phys. Soc. Jpn. 74 (2005) 1934.
27) F. Zhai and H. Q. Xu: Phys. Rev. Lett. 94 (2005) 246601.
28) The conductance dip is caused by the destructive interference between propagating waves through two orbitals in the QD. In the two-terminal system without SO interaction, the conductance $G_{1,\sigma} = g_1(1)$ completely vanishes at the dip, where the “phase lapse” of the transmission phase takes place. See ref. 36 and related references cited therein.
29) A. C. Hewson: The Kondo Problem to Heavy Fermions (Cambridge University Press, Cambridge, 1993).
30) We have two sets of Kramers’ degenerate levels in the QD and $N$ channels in the leads. By the unitary transformation for the $N$ channels, we obtain two channels connected to the QD and ($N-2$) channels disconnected from the QD. This is the situation of the full-screening Kondo effect. When $\Delta < T_K$, the SU(4) Kondo effect is realized even when the strengths of the tunnel coupling are not identical between the two levels in the QD. This is because the fixed point is marginal (and thus not unstable) in the Kondo scaling.
31) J. S. Lim, M.-S. Choi, M. Y. Choi, R. López, and R. Aguado: Phys. Rev. B 74 (2006) 205119.
32) K. Hamaya, M. Kitabatake, K. Shibata, M. Jung, M. Kawamura, K. Hirakawa, T. Machida, T. Taniyama, S. Ishida, and Y. Arakawa: Appl. Phys. Lett. 91 (2007) 022107.
33) A. L. Fert and O. Jaoul: Phys. Rev. Lett. 28 (1972) 303.
34) A. Fert, A. Friederich, and A. Hamzic: J. Magn. Magn. Mater. 24 (1981) 231.
35) G. Y. Guo, S. Maekawa, and N. Nagaosa: Phys. Rev. Lett. 102 (2009) 036401.
36) C. Karrasch, T. Hecht, A. Weichselbaum, Y. Oreg, J. von Delft, and V. Meden: Phys. Rev. Lett. 98 (2007) 186802.
37) P. Nozières and A. Blandin: J. Phys. (Paris) 41 (1980) 193.
38) M. Eto: J. Phys. Soc. Jpn. 74 (2005) 95.