A novel method of reconstructing semileptonic $B$ decays

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Abstract

Semileptonic decays of charged and neutral $B$ mesons play a critical role in the determination of the magnitudes of the CKM-matrix elements $V_{cb}$ and $V_{ub}$, and in the test of the lepton universality which is a basic assumption of the Standard Model. Due to the missing neutrino in the semileptonic decays, the measurements depend strongly on the tag efficiency of the $B$ meson in the recoil side which is at a few per mille to a few per cent level, and this values a limiting factor for high precision measurement and high sensitivity test mentioned above. We develop a new method of reconstructing semileptonic decays of the $B$ mesons by introducing the $B$ momentum information calculated from the $B$ decay vertex and the interaction point, as well as the kinematic information of $e^+e^- \rightarrow B\overline{B}$ at the $B$-factories such as BaBar, Belle, and Belle II. As this method does not depend on the reconstruction of the $B$ meson in the recoil side, the gain in the efficiency could be as large as two orders of magnitudes. This makes high precision measurement of CKM-matrix elements $|V_{cb}|$ and $|V_{ub}|$ and high precision test of lepton universality at $B$-factories more promising. We present the algorithms for the semileptonic decays of neutral and charged $B$ mesons separately, as they are affected by the magnetic field in the detector differently.

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I. INTRODUCTION

Semileptonic (SL) decays of charged and neutral $B$ mesons, proceed via leading-order weak interactions, play a critical role in the determination of the magnitudes of the Cabibbo-Kobayashi-Maskawa (CKM) matrix $[1]$ elements $V_{cb}$ and $V_{ub}$, which impact most studies of flavor physics and $CP$-violation in the quark sector, along with an understanding of properties of the $b$ quark bound in a meson. And it also is very important in the test of the lepton universality which is the basic assumption of the Standard Model. The leptonic part in the effective Hamiltonian and the decay matrix element factorizes from hadronic part, and QCD corrections can only occur in the $b \to q$ current $[2]$, as an important feature of SL $B$ decays.

$B$-factories collected a large $B$ meson sample to study $B$ physics. For example, there are 772 million $BB$ pairs at the Belle experiment and 471 million $BB$ pairs at the BaBar experiment $[2]$. However, in studying $B$ decays with an undetected neutrino or a missing particle (such as a neutron or $K_L$ meson), the $B$ meson in the recoil side must be fully or partially reconstructed in order to infer the information of the missing particle. This $B$-tag technique is not very efficient, because all $B$ meson ($b$ quark) weak decays are Cabibbo suppressed and there are no dominant decay modes can be used for the reconstruction.

The method of identifying signal candidates of SL $B$ decays using fully reconstructed $B$ decays in the recoil side has been employed in exclusive $\bar{B} \to X_u\ell^-\bar{\nu}_\ell$ decays (where $X_u$ denotes a light meson containing a $u$ quark, and $\ell$ denotes an electron or muon) by CLEO $[3,4]$, BaBar $[5-9]$ and Belle $[10-12]$. In such analyses, the missing energy and momentum of the whole event are used to reconstruct the neutrino from signal SL decays. In a partial reconstruction of $\bar{B} \to D^{(*)}\ell\nu$ decays as the tagging mode (SL tag), since there are two neutrinos present in the event, the kinematics cannot be fully constrained $[13]$.

Belle developed a full reconstruction tool to tag $B$ decays using a multivariable analysis based on a neural network algorithm named NeuroBayes $[14]$, in which, 1104 exclusive decay-channels were reconstructed, employing 71 neural networks. An overall efficiency of 0.28% for $B^\pm$ and of 0.18% for $B^0$ mesons is achieved, which is an improvement by roughly a factor of two comparing to the efficiency of the cut-based classical reconstruction algorithm. Recently, a new tagging method, full exclusive interpretation (FEI) $[15]$, based on machine learning, has been developed at Belle and Belle II experiments. The FEI reconstructs more than 100 explicit decay channels, leading to $O(10,000)$ distinct decay-chains. It achieves the maximum tagging efficiency to 0.76% (1.80%) for $B^+$ and 0.46% (2.04%) for $B^0$ with hadronic (SL) tag at Belle $[16]$. The $\bar{B}$ tagging efficiency is at level of $O(10^{-2})$ or even $O(10^{-3})$, which means only a very small fraction ($10^{-3} \sim 10^{-2}$) of the data sample is used in the measurement of SL decays of the $\bar{B}$ meson. Considering the advancement of FEI, it is hard to improve the tagging efficiency significantly in the further along this line. Therefore, new method should be developed to improve the efficiency of reconstructing the SL $B$ decays.

As we know, to measure the $CP$ violation in $B$ decays, the $B$ meson decay and production vertices can be well determined at the $B$ factories. If there are at least two charged tracks in the final states of $B$ decays $[17]$, $B$ decay vertex, $\bar{V}_B=(x_B, y_B, z_B)$, can be determined by a vertex constraint fit. The interaction point (IP), $\bar{V}_{IP}=(x_{IP}, y_{IP}, z_{IP})$, which can be measured with non-$B\bar{B}$ events, can be regarded as the $B$ meson production vertex, as the lifetime of $Y(4S)$, mother of the $B$ mesons, is very short. At Belle experiment, IP is time-dependent, and is calculated every 10,000 events to take into account an observed variation of IP position during data taking.
FIG. 1: (Color online) A schematic diagram of SL decays of neutral $B$ meson $B^0_{\text{sig}} \to h\ell\nu$, $B$ decay and production vertex information is used to determine the direction of the $B$ momentum.

The $B$ meson decay and production vertices will allow us not only to calculate the lifetime of the $B$ mesons, but also to provide additional constraints, besides energy-momentum conservation constraint, in SL $B$ decays. Based on these additional constraints, we present a new method of $B$ meson reconstruction in this article, which allows to fully determine SL $B$ decays (or hadronic $B$ decays with a missing particle) without tagging the $B$ meson in the other side at an $e^+e^-$ $B$-factories running at the $\Upsilon(4S)$ energy. We present the reconstruction algorithms for SL decays of neutral $B$, positive and negative charged $B$ mesons, separately, as they are affected by the magnetic fields in the detector in different ways.

II. THE SL DECAYS OF NEUTRAL $B$ MESON

In the case of the SL decays of a neutral $B$, as shown in Fig. 1, with the magnitude of the $B$ moment $|\vec{p}_B|$ given, the momentum of the $B^0$ meson is written as a function of $B$ decay and production vertices: $\vec{p}_B = |\vec{p}_B| \cdot \vec{r}$, where momentum unit vector $\frac{\vec{r}}{|\vec{r}|}$ is determined by $B$ decay vertex and production vertex via $\vec{r} = V_B - V_{IP}$. Thus, energy-momentum conservation of the SL decays of the $B$ meson gives four constraints in Eqs. (1-4):

\[
\sqrt{m^2_B + |\vec{p}_B|^2} = \sqrt{m^2_h + |\vec{p}_h|^2} + \sqrt{m^2_{\ell} + |\vec{p}_{\ell}|^2} + \sqrt{m^2_{\nu} + |\vec{p}_{\nu}|^2}, \quad (1)
\]

\[
|\vec{p}_B| \cdot \frac{r_x}{|r|} = p_{hx} + p_{\ell x} + p_{ox}, \quad (2)
\]

\[
|\vec{p}_B| \cdot \frac{r_y}{|r|} = p_{hy} + p_{\ell y} + p_{oy}, \quad (3)
\]

\[
|\vec{p}_B| \cdot \frac{r_z}{|r|} = p_{hz} + p_{\ell z} + p_{oz}. \quad (4)
\]

Therefore this SL decay is fully determined, with known variables $\{\frac{\vec{r}}{|\vec{r}|}, \vec{p}_h, \vec{p}_{\ell}\}$ and $\{m_B, m_h, m_{\ell}, m_\nu\}$ ($m_\nu = 0$ for neutrino or nominal mass of a missing particle) and unknown $|\vec{p}_B|$ and $\vec{p}_\nu$ (four variables).
FIG. 2: (Color online) Definition of the laboratory frame at Belle detector (left panel) and Belle II detector (right panel) with different boost vector $\beta$.

In fact, we have one additional constraint besides those listed in Eqs. (1-4). As the $B\overline{B}$ pair is produced in $\Upsilon(4S)$ decays, each $B$ meson carries an energy of half of the $\Upsilon(4S)$ mass in the center-of-mass (CM) frame. Thus, with known $\Upsilon(4S)$ energy-momentum and the direction of the $B$ momentum in the laboratory (LAB) frame, we can calculate the magnitude of the $B$ momentum, that is, $|\vec{p}_B|$ in Eqs. (1-4). We take Belle and Belle II cases as example below.

The $+z$ axis in the laboratory frame, as direction of the nominal magnetic field $\vec{B}$, are defined differently at Belle and Belle II detectors, as shown in Fig. 2. The four momenta of the $e^+e^-$ annihilation CM frame, $P_{cm}$ in Eqs. (5, 6), are determined by energy of electron and positron beams, $E_-=8$ (7.004) GeV and $E_+=3.5$ (4.002) GeV, and their cross angle $\theta=22$ (83) mrad at Belle (Belle II) experiment, therefore they both have energy $\sqrt{s} = \sqrt{2E_-E_+(1+\cos \theta)} \approx 10.58$ GeV in CM frame.

$$P_{cm} = (E_- \sin \theta, 0, E_- \cos \theta - E_+, E_- + E_+)$$

$$P_{cm} \approx (0.176, 0, 4.498, 11.5) \text{ GeV at Belle,}$$

$$P_{cm} = ((E_- + E_+) \sin \frac{\theta}{2}, 0, (E_- - E_+) \cos \frac{\theta}{2}, E_- + E_+)$$

$$P_{cm} \approx (0.457, 0, 2.999, 11.006) \text{ GeV at Belle II.}$$

The boost vector $\vec{\beta}$ of $B$ meson to mother particle $\Upsilon(4S)$ CM frame, equals the momentum $-\vec{p}_{cm}$ normalized by total energy as $\vec{\beta} = (\beta_x, \beta_y, \beta_z) = -\frac{\vec{p}_{cm}}{E_-+E_+}$. With four-momentum of the $B$ meson $P=(\vec{p}_B, E)=(p_x, p_y, p_z, E)$ in LAB frame, the boosted four-momentum $P^*=(p^*_x, p^*_y, p^*_z, E^*)$ in CM frame can be calculated with a Lorentz boost function.

$$p^*_x = p_x + \gamma_2 \cdot (\vec{\beta} \cdot \vec{p}) \cdot \beta_x + \gamma \cdot E \cdot \beta_x,$$

$$p^*_y = p_y + \gamma_2 \cdot (\vec{\beta} \cdot \vec{p}) \cdot \beta_y + \gamma \cdot E \cdot \beta_y,$$

$$p^*_z = p_z + \gamma_2 \cdot (\vec{\beta} \cdot \vec{p}) \cdot \beta_z + \gamma \cdot E \cdot \beta_z,$$

$$E^* = \gamma \cdot (E + \vec{\beta} \cdot \vec{p}).$$

Here $|\vec{\beta}|^2 = \beta_x^2 + \beta_y^2 + \beta_z^2$ and $\gamma_2=(\gamma - 1)/|\vec{\beta}|^2$, and $\gamma$ is calculated as

$$\gamma = \frac{1}{\sqrt{1 - |\vec{\beta}|^2}} = \frac{E_- + E_+}{\sqrt{2E_-E_+(1+\cos \theta)}}.$$
Thus, with these calculations and \( E^* = \sqrt{s}/2 \), Eq. (10) becomes
\[
\frac{\sqrt{s}}{2\gamma} = \sqrt{|\vec{p}_B|^2 + m_B^2} + \frac{\beta_x r_x + \beta_z r_z}{\sqrt{r_x^2 + r_y^2 + r_z^2}} \cdot |\vec{p}_B|.
\] (12)

Therefore, combining Eq. (12) with four-momentum conservation in Eqs. (1-4), the SL \( B \) decays can be fully determined even with a floated \( B \) meson mass. The fractions of signal and background components thus can be extracted by fitting the candidate \( m_B \) distribution.

So far we did not use any information of the \( \bar{B} \) decays recoiling against the \( B \) meson in signal side. In fact the above analysis can be extended to the recoil side by reconstructing another \( B \) vertex and applying the same technique by assuming there is a missing particle with unknown mass (rather than a zero mass neutrino in the signal side). As the momentum of \( \bar{B} \) meson is charged, its momentum is no more along the straight line between IP and the \( B \) decay vertex. The \( \bar{B} \) meson deflects as a circle in \( x-y \) plane, and moves uniformly as a straight line in \( z \)-direction, resulting from the magnetic field \( \vec{B} \) in the detector. In this section, the algorithms are developed for positive charged and negative charged \( B \) mesons [18] separately, considering different direction of deflection in \( x-y \) plane.

III. THE SL DECAYS OF CHARGED \( B \) MESON

When the \( B \) meson is charged, its momentum is no more along the straight line between IP and the \( B \) decay vertex. The \( B \) meson deflects as a circle in \( x-y \) plane, and moves uniformly as a straight line in \( z \)-direction, resulting from the magnetic field \( \vec{B} \) in the detector. In this section, the algorithms are developed for positive charged and negative charged \( B \) mesons [18] separately, considering different direction of deflection in \( x-y \) plane.
A. The SL decays of $B^+$ meson

For positive charged $B$ meson, it deflects on the right of the flight direction in $x$-$y$ plane with radius $r_+ = |p_{xy}|/(eB)$, as shown in Fig. 3. Different from the neutral $B$ meson case, the direction of the $B$ momentum can not be directly obtained by vector direction of two vertices. In $x$-$y$ plane, we define $\theta_0$ as the angle between the momentum of $B$ meson, $\mathbf{p}_{xy}$, and $+x$ axis; $\theta$ as the deflected angle due to magnetic field. Thus, the motion length of arc in $x$-$y$ plane is $r_+\theta$. According to the motion character in $x$-$y$ plane and in $z$-axis, the SL $B$ decay can still be determined by following equations:

\begin{align}
    f_x &: \quad p_{hx} + p_{tx} + p_{\nu x} = \sqrt{p^2_x + p^2_y \cos(\theta_0 - \theta)}, \\
    f_y &: \quad p_{hy} + p_{ty} + p_{\nu y} = \sqrt{p^2_x + p^2_y \sin(\theta_0 - \theta)}, \\
    f_z &: \quad p_{hz} + p_{tz} + p_{\nu z} = p_z, \\
    f_e &: \quad \sqrt{m^2_\rho + |\mathbf{p}_h|^2 + \sqrt{m^2_\ell + |\mathbf{p}_\ell|^2 + \sqrt{m^2_\nu + |\mathbf{p}_\nu|^2}}} = \sqrt{m^2_B + |\mathbf{p}_B|^2}, \\
    f_{r1} &: \quad r_x = \frac{2}{eB} \sqrt{p^2_x + p^2_y} \cdot \sin \frac{\theta}{2} \cos(\theta_0 - \frac{\theta}{2}), \\
    f_{r2} &: \quad r_y = \frac{2}{eB} \sqrt{p^2_x + p^2_y} \cdot \sin \frac{\theta}{2} \sin(\theta_0 - \frac{\theta}{2}), \\
    f_{r3} &: \quad r_z = v_z \cdot \frac{r_+\theta}{v_{xy}} = \frac{p_z\theta}{eB}, \\
    f_{r4} &: \quad \theta_0 = \arctan \frac{p_y}{p_x}, \\
    f_{e^*} &: \quad \frac{\sqrt{s}}{2\gamma} = \sqrt{m^2_B + |\mathbf{p}_B|^2 + (\beta_x p_x + \beta_z p_z)}. \tag{25}
\end{align}

To reduce the unknown variables, a set of simplified equations are obtained as Eqs. (26-32)
with the unknown momenta of \( B \) meson and \( \nu \), and the angle \( \theta \):

\[
\begin{align*}
\nu_x : & \quad p_{\nu x} = \frac{eB}{2}(r_x \cot \frac{\theta}{2} + r_y) - (p_{hx} + p_{\nu x}), \\
\nu_y : & \quad p_{\nu y} = \frac{eB}{2}(-r_x + r_y \cot \frac{\theta}{2}) - (p_{hy} + p_{\nu y}), \\
\nu_z : & \quad p_{\nu z} = eB \frac{r_z}{\theta} - (p_{hz} + p_{\nu z}), \\
B_x : & \quad p_x = \frac{eB}{2}(r_x \cot \frac{\theta}{2} - r_y), \\
B_y : & \quad p_y = \frac{eB}{2}(r_x + r_y \cot \frac{\theta}{2}), \\
B_z : & \quad p_z = eB \frac{r_z}{\theta}, \\
\varepsilon : & \quad \sqrt{m_h^2 + |\vec{p}_h|^2} + \sqrt{m_{\ell}^2 + |\vec{p}_\ell|^2} + \sqrt{m_{\nu}^2 + |\vec{p}_\nu|^2} = \frac{\sqrt{s}}{2\gamma} - (\beta_x p_x + \beta_z p_z). \tag{32}
\end{align*}
\]

Then an equation with only one variable \( \theta \) can be easily obtained by Eq. (32) with other Eqs. (26-31). With solved \( \theta \) and the nine equations, all the unknown variables can be determined. Thus, SL \( B^+ \) decays can be fully determined, including the momentum of the undetectable neutrino.

**B. The SL decays of \( B^- \) meson**

For negative charged \( B \) meson, it deflects on the left of the flight direction in \( x-y \) plane with radius \( r_- = |p_{xy}|/(eB) \), as shown in Fig. 4. Similar to the positive charged \( B \) meson case, following equations constrain the SL \( B^- \) decays:

\[
\begin{align*}
\nu_x : & \quad p_{hx} + p_{\ell x} + p_{\nu x} = \sqrt{p_x^2 + p_y^2} \cos(\theta_0 + \theta), \\
\nu_y : & \quad p_{hy} + p_{\ell y} + p_{\nu y} = \sqrt{p_x^2 + p_y^2} \sin(\theta_0 + \theta), \\
\nu_z : & \quad p_{hz} + p_{\ell z} + p_{\nu z} = p_z, \\
\varepsilon : & \quad \sqrt{m_h^2 + |\vec{p}_h|^2} + \sqrt{m_{\ell}^2 + |\vec{p}_\ell|^2} + \sqrt{m_{\nu}^2 + |\vec{p}_\nu|^2} = \sqrt{m_B^2 + |\vec{p}_B|^2}, \\
r_1 : & \quad r_x = \frac{2}{eB} \sqrt{p_x^2 + p_y^2} \cdot \sin \frac{\theta}{2} \cos(\theta_0 + \frac{\theta}{2}), \\
r_2 : & \quad r_y = \frac{2}{eB} \sqrt{p_x^2 + p_y^2} \cdot \sin \frac{\theta}{2} \sin(\theta_0 + \frac{\theta}{2}), \\
r_3 : & \quad r_z = v_z \cdot \frac{r_- \theta}{v_{xy}} = p_z \theta \frac{eB}{p_x}, \\
\theta_0 : & \quad \theta_0 = \arctan \frac{p_y}{p_x}, \\
\varepsilon : & \quad \frac{\sqrt{s}}{2\gamma} = \sqrt{m_B^2 + |\vec{p}_B|^2} + (\beta_x p_x + \beta_z p_z). \tag{41}
\end{align*}
\]
FIG. 4: (Color online) A schematic diagram of SL decays of $B^-$ meson with vertices information.

Thus, to reduce unknown variables, a combination of equations is simplified as follows:

$$f_{\nu x} : \quad p_{\nu x} = \frac{eB}{2} \left( r_x \cot \frac{\theta}{2} - r_y \right) - (p_{hx} + p_{\ell x}),$$  \hspace{1cm} (42)

$$f_{\nu y} : \quad p_{\nu y} = \frac{eB}{2} \left( r_x + r_y \cot \frac{\theta}{2} \right) - (p_{hy} + p_{\ell y}),$$ \hspace{1cm} (43)

$$f_{\nu z} : \quad p_{\nu z} = eB \frac{r_z}{\theta} - (p_{hz} + p_{\ell z}),$$ \hspace{1cm} (44)

$$f_{B x} : \quad p_x = \frac{eB}{2} \left( r_x \cot \frac{\theta}{2} + r_y \right),$$ \hspace{1cm} (45)

$$f_{B y} : \quad p_y = \frac{eB}{2} \left( -r_x + r_y \cot \frac{\theta}{2} \right),$$ \hspace{1cm} (46)

$$f_{B z} : \quad p_z = eB \frac{r_z}{\theta},$$ \hspace{1cm} (47)

$$f_e : \quad \sqrt{m_h^2 + |\vec{p}_h|^2} + \sqrt{m_{\ell}^2 + |\vec{p}_{\ell}|^2} + \sqrt{m_{\nu}^2 + |\vec{p}_{\nu}|^2} = \frac{\sqrt{s}}{2\gamma} - (\beta_x p_x + \beta_z p_z).$$ \hspace{1cm} (48)

Through solving an equation with one variable $\theta$ which can be obtained from Eq. (48) by using Eqs. (42-47), SL $B^-$ decays can be fully determined, similar to the $B^+$ case.

IV. CONCLUSION AND DISCUSSION

We present a novel method to reconstruct SL $B$ decays with a missing neutrino or hadronic $B$ decays with a missing particle without tagging the $B$ meson in the recoiling side. Without suffering from the very low tagging-efficiency (a few per mille to a few per cent), our method is very promising to achieve a much larger utilization of $B$ decay sample for studying the SL $B$ decays.
This method depends strongly on the precision of the reconstruction of the $IP$ and the $B$ decay vertex. Typical resolution achieved at Belle experiment is $\sigma_x \sim 100 \mu m$, $\sigma_y \sim 1.9 \mu m$ and $\sigma_z \sim 3.6 \text{ mm}$ for $IP$ uncertainty and $\sim 100 \mu m$ for vertex reconstruction both along the beam direction and in transverse plane [2]. The resolution at Belle II is much improved with nano-beam scheme for the $IP$ and better inner detectors for secondary vertex reconstruction. The design resolution of $IP$ is $\sigma_x \sim 10 \mu m$, $\sigma_y \sim 50 \text{ nm}$ and $\sigma_z \sim 0.15 \text{ mm}$ and excellent vertex resolution is $\sim 50 \mu m$ at Belle II [19].

Compared with the conventional way of studying SL $B$ decays, the background level could be higher in the method presented here, so some other techniques may need to be developed to further suppress the background. An educated guess is even these background suppressions further reduce the efficiency by an order of magnitude, the efficiency of the new method is still higher than the tag method by at least an order of magnitude. Of course, the factors are mode dependent, detailed results can be obtained with Monte Carlo study or with the real data at Belle and BaBar experiments.

As we have mentioned, this new method can be used for the SL $B^0$ decays, such as $B^0 \to \pi^- \ell^+ \nu_\ell$ with an undetectable neutrino for improved measurement of form factor and $|V_{ub}|$ [20]; hadronic decays such as $B^0 \to D^- \bar{p}\bar{n}$, which has not been observed yet but a similar decay $B^0 \to D^{*-} \bar{p}\bar{n}$ has been observed [21], with a missing anti-neutron for searching of exited charmed baryon $\Lambda^*_c$, pentaquark $\Theta_c[ccuudd]$, or hadronic structures in $D^-p$ system; SL decays of charged $B$ such as $B^+ \to p\bar{p}\ell^+ \nu_\ell$ to target for the first observation, which is now only an evidence from Belle [22].

Besides $B$ decays, this method can also be used in analyzing SL $D$ decays or $\tau$ decays at BaBar, Belle, and Belle II experiments where large momentum $D$ and $\tau$ may have even longer decay length thus better momentum resolution. This method also allows a study of the semileptonic decays of hyperons ($\Lambda$, $\Sigma$, $\Xi$, and so on) pair produced copiously in $\tau$-charm factories such as BESIII experiment [23], and super $\tau$-charm factories under discussion [24, 25].

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