Geometric method for researching the robot service area under the obstacles for the limitations on links movement

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Abstract. The work is devoted to solving one of the problems of robot motion virtual simulation using computer graphics. The paper offers a method for studying the shape and position of the work envelop of manipulator mechanisms in the presence of restricted work envelopes in the workspace. The method for determining the work envelope is based on the construction of allowed configuration of robot mechanisms. A parameter describing the structure and maneuverability index in different parts of the work envelope is proposed. The construction of cross sections of service areas for different positions of restricted work envelopes allows determining the areas where the output link centers of various mechanisms can not be located. The conducted research makes it possible to place the trajectories of the output link in solving motion problems, which reduce the probability of deadlocks in the automated motion synthesis along the velocity vector.

1. Introduction
A preliminary assessment of manipulator mechanism dexterity in technological process can be done by virtual modeling of manipulator movements using computer graphics [1,2]. At the same time, in direct proximity to robots performing this simulation, it is necessary to take into account auxiliary equipment (for example, tables, racks, fences, etc.). The specified environment in which the robot mechanism links can perform motions at the virtual level is usually set by various geometric objects [3-5]. These objects have a significant impact on the work envelope of robot mechanisms. The paper offers a method for studying the shape and position of the work envelope of manipulator mechanisms in the presence of obstacles in the workspace. In this case, the projected trajectories of the output link (OL) center when modeling robot motions must belong to the specified work envelope.

2. Problem Statement
Fig. 1 represents images of manipulator mechanisms and restricted areas located in the motion area of robot links. Consider the influence of geometric objects of the environment on the structure and shape of the manipulator mechanism service area presented in the figure. In figure 1a the motion of the manipulator mechanism occurs in a space limited in height. The motion of the mechanism is restricted by the horizontal plane. In figure 1b the motion of the android robot arm mechanism is restricted by a given rectangular parallelepiped (which determines the position of the rack). In figure 1ab the points \( O_i \) define the centers of the reference frames used for setting kinematic chain models [6]. The generalized coordinate \( q_1 \) of the mechanism is not taken into account when finding sections of the service area since target points reached by the gripper center determine the position of the plane to which the node points \( O_1, O_2 \) and \( O_3 \) belong when the manipulator mechanism is located on the manipulator car (Fig. 1a). In case the restricted work envelope is a horizontal plane, the service area is determined by the figure obtained by rotating its section around the axis \( z_1 \). The link lengths are set by segments \( O_1O_2 = 90cm, O_2O_3 = 70cm \) and \( O_3O_4 = 50cm \). When \( q_1 = 0 \), the spatial five-link mechanism becomes a planar four-link mechanism. The possible positions of the OL centers that define the cross sections of the work envelope are specified by the point \( O_1 \) for the planar and the point \( O_{12} \) for the spatial mechanism of the manipulators. Let us define the work envelope of the robot mechanisms shown in Fig. 1.
3. Theoretical Basis

The service area is defined by the set of the OL center positions for allowed configurations (configurations that do not cross obstacles) under the specified restrictions:

\[ q_i^{\text{min}} \leq q_i \leq q_i^{\text{max}}, \]  

where \( q_i^{\text{min}} \) and \( q_i^{\text{max}} \) are the lower and upper limits of generalized coordinate values. In examples under consideration for the planar five-link mechanism (Fig. 1a) at \( q_1 = 0 \), the values of the parameter \( i \) satisfy the conditions \( 2 \leq i \leq 4 \). The constraints \( q_2^{\text{min}} \) and \( q_2^{\text{max}} \) respectively are assumed to be equal to \( q_2^{\text{min}} (0^\circ, -120^\circ, -120^\circ) \) and \( q_2^{\text{max}} (120^\circ, 120^\circ, 120^\circ) \). Given that in the initial position of the android robot (Fig. 1b) for reaching the target point, the value of the generalized coordinate \( q_1 \) is also assumed to be equal to zero, then in order to simplify research related to the definition of the service area, the coordinate \( q_1 \) of this mechanism is also excluded in this study. Then for the six-link spatial mechanism (Fig. 1b) at \( q_1 = 0 \), the values of the parameter \( i \) satisfy the conditions \( 2 \leq i \leq 5 \), and the constraints \( q_i^{\text{min}} \) and \( q_i^{\text{max}} \) are respectively assumed to be \( q_2^{\text{min}} (0^\circ, -25^\circ, -120^\circ, -120^\circ) \) and \( q_2^{\text{max}} (120^\circ, 120^\circ, 120^\circ, 120^\circ) \). The set of the OL center positions is determined with a step \( \Delta q_i = 15^\circ \) for a planar mechanism. Accordingly, for the spatial mechanism it is with a step \( \Delta q_i = 20^\circ \).

To represent the models of kinematic chains, we use the method from the work [6]. This method involves using codes that define coordinate system transformations associated with links of mechanisms. The models of the two manipulator mechanisms shown in figure 1 are set by elements of the arrays given in tables 1 and 2.

### Table 1. Values of list items of the four-link planar manipulator mechanism

| Arrays | Conversion number of coordinates system |
|--------|----------------------------------------|
|        | 2                                       |
|        | 3                                       |
|        | 4                                       |
|        | 5                                       |
| \( q_i^0 \) | \( q_2 = 45^0 \) | \( q_3 = -45^0 \) | \( q_4 = -45^0 \) |
| \( l_i, \text{ cm} \) | 90 | 70 | 50 |
| \( l_m \) | 0 | 0 | 0 |
| \( n_{bow} \) | 2 | 2 | 2 |
These arrays set the values of the generalized coordinates $q_i$, the lengths of mechanism links $l_i$, the displacement along the axes of the coordinate systems $l_{cm}$ that are fixed to the links of the mechanism, and the coordinate system transformations codes $n_{kod}$ [6]. The code values ($1 \leq n_{kod} \leq 3$) specify coordinate system transformations, respectively, when the system is rotated around the axes $O_kx_k$, $O_ky_k$ and $O_zy_k$ ($4 \leq n_{kod} \leq 6$) of translational motions along the specified axes using generalized coordinates. The code values ($7 \leq n_{kod} \leq 12$) correspond to the same transformations, but without using generalized coordinates. As is clear from the tables, three and twelve coordinate systems are used to set the planar and spatial mechanisms, accordingly.

Table 2. Values of list items of the six-link spatial mechanism of the android robot arm

| Arrays   | Conversion number of coordinates system |
|----------|-----------------------------------------|
| $q_i$    | $q_1 = 25^0$  | 0  | 0  | 0  | $q_2 = 20^0$  | 0  |
| $l_i$, cm| 0            | $l_1 = 300$ | $l_2 = -120$ | $a = 80^0$ | 0  | $l_3 = 100$  |
| $l_{cm}$ | 0            | 0  | 0  | 0  | 0  | 0  |
| $n_{kod}$| 3            | 12 | 11 | 7  | 3  | 12 |
| $q_i$    | $q_3 = 65^0$  | 0  | 0  | $q_4 = 25^0$  | 0  | $q_5 = 65^0$  | 0  |
| $l_i$, cm| 0            | $l_4 = 80$ | 0  | $l_5 = 150$ | 0  | $l_6 = 250$  |
| $l_{cm}$ | 0            | 0  | 0  | 0  | 0  | 0  |
| $n_{kod}$| 2            | 12 | 3  | 12 | 2  | 12 |

Fig. 2. represents multiple configurations for manipulator mechanisms that specify allowed configurations that do not cross restricted work envelopes and meet the conditions (1). These configurations define the output link centers, which together define the work envelope.
4. Experimental Results

Fig. 3 abcd represents the positions for the calculation points $O_i$ and the service zone $\Lambda_\omega$ in the absence and presence of the restricted zone $P$ (Fig. 1a). In Fig. 3 only points $O_i$ are constructed, their configurations not crossing the restricted area $P_i$ and are permitted (Fig. 1a). In Fig. 3a the form of the service zone depends significantly on the parameter $z_{op}$ that sets the position of the restricted zone $P$ (Fig. 3abc). Fig. 4. shows two sections of the work envelope for the spatial mechanism of the android robot arm. In this case, the position of the restricted work envelope is set by the parameter $z_{op} = 400 \text{ mm}$, and the cross-section positions are $y_{c1}^{\min} = 0 \text{ cm}$, $y_{c1}^{\max} = 40 \text{ cm}$, $y_{c2}^{\min} = 100 \text{ cm}$, $y_{c2}^{\max} = 150 \text{ cm}$ (Fig. 2.). The following parameters characterize the service area of manipulator mechanisms taking into account the position of the restricted zones $P$: the area $S_{\phi\theta}$ of the work envelope $\Lambda_\omega$; the number of points $N_{kol}$ belonging to the area $\Lambda_\omega$ specifying points $O_i$ and allowed configurations (points are set in grid nodes with the accepted step $\Delta q$); the number of OL centers of allowed configurations $N_{kol}^\phi$ per unit area (fragment of the service area). This parameter is calculated by the formula:

$$\mu = \frac{N_{kol}^\phi}{S_{\phi\theta}^\theta},$$

where parameters $N_{kol}^\phi$ and $S_{\phi\theta}^\theta$ characterize a separate area fragment 1x1cm of the workspace $\Lambda_\omega$.

Figures 3a-d show the lines of the level $\mu_k$ that define the fragments of the workspace of the manipulator mechanism that correspond to different values of the parameter $\mu$. When planning the trajectory of the OL center motion in the fragments of the manipulator workspace, where the value of $\mu$ is the highest, the probability of a deadlock in the motion synthesis along the velocity vector is significantly reduced. In figure 3 it is possible to make the analysis of the most optimal values of the parameter $l$ (the parameter is not shown in fig.) defining the target point which moves from the point $O_i$ of the manipulator car, the target point being at a height of $h = 25 \text{ cm}$.

The modeling results of the planar manipulator motion along the velocity vector are presented in Fig. 5 [7]. Motion modeling is performed in AutoCAD development system using the facilities of the language AutoLISP.
Figure 3. The analysis of the parameter $N_{kol}$ values in different locations of the work envelope for the planar four-link manipulator mechanism: а is $z_{op} = 50$ cm, $N_{kol} = 2153$; b is $z_{op} = 80$ cm, $N_{kol} = 6935$ points; c is $z_{op} = 150$ cm, $N_{kol} = 21878$; d is without obstacles, $N_{kol} = 28212$

Figure 4. Analysis of the parameter $N_{kol}$ values in various locations in the work envelope of the android robot arm mechanism (grid step is 10°): a is $N_{kol} = 4590$, workspace section at $0cm < y_{c1} < 40cm$; b is $N_{kol} = 4877$, workspace section at $100cm < y_{c2} < 150cm$
5. Results Discussion
Analysis of the level lines $\mu_k$ proves that when gripping manipulated objects specified by the position of the target point at a height of 25 cm for the parameter values $z_{op} = 50$ cm, $z_{op} = 80$ cm and $z_{op} = 150$ cm, the point $O_1$ should have a rational distance $l$ to the target point being respectively equal to $190 \geq l \geq 210$, $150 \geq l \geq 180$ and $140 \geq l \geq 160$. This is due the maximum value of the parameter $\mu > 5$ in the specified ranges. Therefore, at these distances in the vicinity of the target point, maximum maneuverability is provided, which allows for the synthesis of the OL center motion along the velocity vector [7].

Summary and Conclusion
Based on the conducted research related to defining the service area for various robot mechanisms, it is possible to determine the most optimal distance of the manipulator base to the target point to be reached. These distances ensure maximum maneuverability of the mechanism in the vicinity of the target point. The results of the research can be used for virtual simulation of robot motions that perform various technological processes using computer graphics.

References
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