Opinion Dynamics with Hopfield Neural Networks

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Abstract

In Hopfield neural networks with up to $10^8$ nodes we store two patterns through Hebb couplings. Then we start with a third random pattern which is supposed to evolve into one of the two stored patterns, simulating the cognitive process of associative memory leading to one of two possible opinions. With probability $p$ each neuron independently, instead of following the Hopfield rule, takes over the corresponding value of another network, thus simulating how different people can convince each other. A consensus is achieved for high $p$.

One of the well-studied fields in sociophysics [1] is opinion dynamics. If one can chose between only two possible opinions, the human being is reduced to a binary variable $\pm 1$, just as in an Ising magnet the spin is either up or down without consideration of the atomic structure leading to that spin. However, our cognitive processes happen in the human brain, which consists of $\sim 10^{11}$ neurons with $\sim 10^4$ connections each.

A simple brain model is the Hopfield neural network: Each neuron $i$ is a binary variable $S_i = \pm 1$ connected to all other neurons $I$ through synaptic couplings $J_{iI}$. The neuron may switch its state following the sign of the sum of all interactions:

$$S_i = \text{sign} \sum_I J_{iI}S_I.$$  (1)

Usually the cognitive process studied in this model is the associative memory: Starting from some unclear and rather random $S_i$, the above algorithm should eventually lead to one of the many previously stored patterns $\xi_i^\mu$. 

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where $\mu = 1, 2, \ldots$ counts the stored patterns. These patterns are stored by the Hebb rule

$$J_{iI} = \sum_\mu \xi_\mu^{i} \xi_\mu^{I}. \tag{2}$$

We store two random patterns $\mu = 1$ and 2 on a $L \times L$ square lattice, corresponding to two possible opinions, and start with random $S_i = \pm 1$, $i = 1, 2, \ldots, L^2$. A complete and memory saving (Penna-Oliveira trick) computer program is given in [2] and is the starting point of the new simulations here. The single $S_i$ now should not be interpreted as a single neuron but as part of our thinking leading us towards making a decision.

Our new element is the simultaneous simulation of $k$ different Hopfield networks, coupled to each other. Thus at each iteration each neuron independently deviates with probability $p$ from Eq(1) and instead takes the value of the corresponding neuron (same $i$) from a randomly selected other network. Each of the $k$ different people then can learn from the others or convince them. In this way some cognitive process is simulated, instead of simple flips of opinions $\pm 1$.

We found that for $L = 100$ (thousand samples), 1000 (hundred samples) and 10,000 (one sample) that at low $p$ the opinions end up randomly (for example agreement in half the cases if $k = 2$), at intermediate $p$ less often agreement was found, and for $p$ close to one agreement was found more often, including all thousand cases for $k = 2$, $L = 100$, $p \geq 0.97$; see Fig.1. For $L = k = 100$ no random agreement is possible and only high $p$ achieved consensus; see Fig.2. More iterations are needed at high $p$ to find agreement.

We see that for small $k$ an accidental consensus is possible for small coupling $p$. At intermediate $p$ the continuous discussions may prevent the participants to reach any of the two opinions; then even accidental agreement is impossible. For very large $p$ consensus can be reached always. If the number $k$ of people is large, then accidental consensus is impossible, and only large $p$ produce consensus.

(As in single Hopfield models, $k = 1$, the agreement with the stored pattern is sometimes but not always complete; we counted agreement if 75% of the stored pattern was recovered correctly. Sometimes, however, the system cannot decide which of the two stored patterns it should converge to, and remains blocked apart from minor fluctuations. And as in single Hopfield models, ”agreement” can also mean the convergence towards the
complementary pattern, leading for all $i$ to $S_i = +1$ where $\xi_i = -1$ and vice versa; see Fig.1b.)

If $k = 2$ (two networks A and B) and if $a$ is the fraction of $S_i$ having the correct value of a stored pattern or its complement, then we define as a criterion for “agreement” that $a \geq a_c$ where the threshold value $a_c$ was taken as $a_c = 0.75$ in our simulations. Without interactions, $p = 0$, after a few iterations we have $a = 1$ for both networks. Let us assume that network B is the opposite (complement) of network A. With probability $p \ll 1$ at each iteration, each $S_i$ of A gets a “wrong” value from network B, and thus averaged over all $i$: $a = 1 - p$. A more detailed evaluation gives $p = (1 - a)/a$ which means $p_c = 1/3$ for our numerical choice $a_c = 3/4$, in agreement with the jump observed in Fig.1a. The more accurate the required agreement is, the smaller must the probability $p$ be to lead to a random consensus, i.e. high $a_c$ require low $p_c$. (In the cases where network B is not the complement of network A, the agreements $a$ of the networks with the patterns are less vulnerable to disruption and thus the threshold value for $p$ is higher. $p = (1 - a)/a$ thus is the lowest $p$ value at which the consensus starts to be non-random.)

Decision-making committees are often dominated by a smaller core of interacting people. We simulate this “old-boy network” by assuming that only the core members interact with each other in the above way, while the remaining people do not interact with anybody and thus arrive at random decisions. A victory for the core is defined as a case where the majority vote of all agrees with the majority vote of only the core. Fig.3 shows random victories at low $p$ and nearly complete victories for very high $p$. But at intermediate $p$ again many core members only discuss instead of arriving at a decision, and often the core does not cast a single vote. (It does not matter if we give the core 100 or 1000 iterations for deliberations.)

This feasibility study is a neural network generalisation of the voter model, where everybody can take over the opinion of a randomly selected neighbour [3], somewhat similar to the Axelrod model [3]. Another application could be the Sznajd model of convincing [4,2].

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References

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program neuropin
  c  ixi(i,m,j): neuron i=1...L*L;pattern mu=1,2;network j=1...k.
  parameter(L=100 ,n=L*L,k= 4)
  dimension ixi(n,2,k),is(n,k),m(2,k),ifixed(k)
  logical same,samp
  byte ixi,is,one
  data iseed/1/,nrun/100/,max/1000/,one/1/
  fact=0.5/2147483647
  ibm=2*iseed-1
  print *, L,k,iseed,nrun,max
  do 10 ipr= 1,88,1
    p=0.01*ipr
    ip=(2*p-1)*2147483648.0d0
    icnt=0
    do 2 mu=1,2
      do 2 i=1,n
        ixi(i,mu,1)=-one
        ibm=ibm*16807
      2   if(ibm.gt.0) ixi(i,mu,1)=one
      do 1 j=2,k
        do 1 mu=1,2
          do 1 i=1,n
            ixi(i,mu,j)=ixi(i,mu,1)
          1 icount=0
          do 8 irun=1,nrun
            call flush(6)
do 3 j=1,k
do 3 i=1,n
    ibm=ibm*16807
    is(i,j)=one
3    if(ibm.lt.0) is(i,j)=-one
    if(L.eq.38) print 100, (ixi(i,1,1),i=1,n)
c initialization of 2 fixed patterns + 1 variable pattern
    do 4 itime=1,max
    do 5 j=1,k
    do 5 mu=1,2
        m(mu,j)=0
        do 5 i=1,n
            5 m(mu,j)=m(mu,j)+is(i,j)*ixi(i,mu,j)
    do 6 j=1,k
    ifixed(j)=0
    do 6 i=1,n
        isold=is(i,j)
        ifield=ixi(i,1,j)*m(1,j)+ixi(i,2,j)*m(2,j)
        is(i,j)=one
        if(ifield.lt.0) is(i,j)=-one
        ibm=ibm*16807
    if(ibm.gt.ip) goto 6
    9 ibm=ibm*65539
    jj=1+(ibm*fact+0.5)*k
    if(jj.le.0.or.jj.gt.k.or.jj.eq.j) goto 9
    is(i,j)=is(i,jj)
6    ifixed(j)=ifixed(j)+isold*is(i,j)
same=.true.
samp=.true.
    do 12 j=1,k
        same=same.and.(ifixed(j).eq.n)
    12 samp=samp.and.((iabs(m(1,j)).eq.n.or.iabs(m(2,j)).eq.n)
        if(L.gt.5000) print *, irun,itime,m,ifixed,icnt,same,samp
        if(same.and.samp) icount=icount+1
        if(same.and.samp) goto 13
    4 continue
    if(L.eq.38) print 100, ((is(i,j),i=1,n),j=1,k)
13 samp=.true.
    do 11 j=2,k
    do 11 jj=1,j-1
same=.false.
do 7 mu=1,2
   x1=iabs(m(mu,j))
   x2=iabs(m(mu,jj))
7    same=(x1+x2.gt.n.and.abs(1.00-x1/x2).lt.0.1).or.same
    print *, j, jj, samp, same
   samp=samp.and.same
   if(samp) icnt=icnt+1
   print *, irun, itime, icnt, samp
   call flush(6)
8    continue
10   print *, p, icnt, icount
100  format(1x,38i2)
end
Figure 1: Top: Fraction of samples leading to agreement for $k = 2$ (+,x) and 4 (*, squares) with linear pattern dimension $L = 100$ (+,*) and 1000 (x, squares). 1000 iterations were used. Bottom: Neural dynamics of two people with $p = 0.97$, $L = 10,000$ ($10^8$ neurons). We show the overlap $\sum_i S_i \xi_i^2$ with the finally winning second opinion, which reaches $-95$ million after 100 iterations.
Figure 2: Top: Number of samples (from 100) leading to agreement for $L = k = 100$, $t = 1000$. Centre: Average number of iterations needed for consensus of top part, ignoring the samples where no consensus was reached within 1000 iterations. Bottom: As top, but for hundred times more neurons.
Figure 3: Influence of an interacting core of 3 or 21 members on the majority of all $k = 15$ or 101 voters, respectively, summing over 1000 samples. The top shows victories, defined as samples with the overall majority agreeing with the core majority; the lower case shows the summed number of core members who were unable to converge on a pattern. At least 75% agreement means convergence.