Relativistic spin dynamics for vector mesons

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We propose a relativistic theory for spin density matrices of vector mesons based on Kadanoff-Baym equations in the closed-time-path formalism. The theory puts the calculation of spin observables such as the spin density matrix element $\rho_{00}$ for vector mesons on a solid ground. Within the theory we formulate $\rho_{00}$ for $\phi$ mesons into a factorization form in separation of momentum and space-time variables. We argue that the main contribution to $\rho_{00}$ at lower energies should be from the $\phi$ fields that can polarize the strange quark and antiquark in the same way as electromagnetic fields. The key observation is that there is correlation inside the $\phi$ meson wave function between the $\phi$ field that polarizes the strange quark and that polarizes the strange antiquark. This is reflected by the fact that the contributions to $\rho_{00}$ are all in squares of fields which are nonvanishing even if the fields may strongly fluctuate in space-time. The fluctuation of strong force fields can be extracted from $\rho_{00}$ of quarkonium vector mesons as links to fundamental properties of quantum chromodynamics.
I. INTRODUCTION

There is an intrinsic connection between rotation and spin polarization since they are related to the conservation of total angular momentum and can be converted from one to another, as demonstrated in the Barnett effect \(1\) and the Einstein-de Haas effect \(2\) in materials. A recent example is the observation of a spin-current from the vortical motion in a liquid metal \(3\). The same effects also exist in high energy heavy-ion collisions (HIC) in which the huge orbital angular momentum (OAM) along the direction normal to the reaction plane can be partially converted to the global spin polarization of hadrons \(4–9\) (see, e.g. \(10–14\), for recent reviews). The global spin polarization of \(\Lambda\) (including \(\bar{\Lambda}\) hereafter) has been measured through their weak decays in \(\text{Au+Au}\) collisions at \(\sqrt{s_{NN}} = 7.7 - 200\) GeV \(15, 16\).

As spin-one particles, vector mesons can also be polarized in heavy ion collisions in the same way as hyperons. Normally the spin states of vector mesons are described by the spin density matrix element \(\rho_{\lambda_1\lambda_2}\) with \(\lambda_1, \lambda_2 = 0, \pm 1\) labeling spin states along the spin quantization direction. The vector mesons mainly decay through strong interaction that respects parity symmetry. So their spin polarization proportional to \(\rho_{11} - \rho_{-1,-1}\) cannot be measured through their decays. Instead, \(\rho_{00}\) can be measured through the angular distribution of its decay daughters \(15, 17, 19\). If \(\rho_{00} = 1/3\), the spin states are equally populated in the three spin states implying that the vector meson is not polarized. If \(\rho_{00} \neq 1/3\), the three spin states are not equally populated, so the spin of the vector meson is aligned either in the direction of the spin quantization or of the transverse direction perpendicular to it. In 2008, the STAR collaboration measured \(\rho_{00}\) for the vector meson \(\phi(1020)\) in \(\text{Au+Au}\) collisions at 200 GeV, but the result is consistent with 1/3 within errors due to statistics \(20\). Recently STAR has measured the \(\phi\) meson’s \(\rho_{00}\) at lower energies which shows a significant deviation from 1/3 \(21\). It can hardly be explained by conventional mechanism \(17, 22, 23\). In Ref. \(25\), some of us proposed that a large deviation of \(\rho_{00}\) from 1/3 for the \(\phi\) meson may possibly arise from the \(\phi\) field, a strong force field with vacuum quantum number in connection with pseudo-Goldstone bosons and vacuum properties of quantum chromodynamics. Such a proposal is based on a nonrelativistic quark coalescence model for the spin density matrix of vector mesons \(17, 26\).

In this paper we will present a relativistic theory for the spin density matrix of vector mesons from the Kadanoff-Baym (KB) equation \(27\) in the closed-time-path (CTP) formalism \(28, 29\) (for reviews of the KB equation and the CTP formalism, we refer the readers to Refs. \(30–33\)). Then we can derive the spin Boltzmann equation for vector mesons with their spin density matrices being expressed in terms of the matrix valued spin dependent distributions (MVSD) of the quarks and antiquarks \(34\). This puts the calculation of spin observables such as \(\rho_{00}\) for vector mesons on a solid ground.

The paper is organized as follows. In Sec. \(\text{II}\) we will give an introduction to Green functions on the CTP for vector mesons which can be expressed in MVSD. In Sec. \(\text{III}\) the KB equations for vector mesons are derived. In Sec. \(\text{IV}\) the spin density matrices for vector mesons will be formulated from the spin Boltzmann equations. In Sec. \(\text{V}\) the spin density matrices for \(\phi\) mesons will be evaluated. Discussions on the main results and conclusions are given in the final section, Sec. \(\text{VI}\).

We adopt the sign convention for the metric tensor \(g^{\mu\nu} = g_{\mu\nu} = \text{diag}(1, -1, -1, -1)\) where \(\mu, \nu = 0, 1, 2, 3\). The sign convention for the Levi-Civita symbol is \(\epsilon^{0123} = -\epsilon_{0123} = 1\). We can write the space-time coordinate as \(x = x^\mu = (x^0, \mathbf{x}) = (t, \mathbf{x})\) and \(x_\mu = (x_0, -\mathbf{x})\) with \(x_0 = x^0 = t\). The four-momentum for a particle is denoted as \(p = p^\mu = (p^0, \mathbf{p})\) or \(p_\mu = (p_0, -\mathbf{p})\), if it is on-shell we have \(p_0 = p^0 = \sqrt{p^2 + m^2} = E_p = E_\mathbf{p}\). Normally we use Greek letters to denote four-dimensional indices of four-vectors and four-tensors and Latin letters to denote their spatial components.

II. GREEN FUNCTIONS ON CTP FOR VECTOR MESONS

The massive spin-1 particle, such as the vector meson with the mass \(m_V\), can be described by the vector field \(A_V^\mu(x)\) with the classical Lagrangian density

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^V F_{\mu\nu}^V + \frac{m_V^2}{2} A_V^\mu A_V^\nu - A_V^\mu j^\mu. \tag{1}
\]
where \( j^\mu \) is the source current, \( F^{\mu\nu}_V = \partial^\mu A^\nu_V - \partial^\nu A^\mu_V \) is the field strength tensor, and \( A^\mu_V \) is assumed to be the real classical field for the charge (including flavor) neutral particle. From \( \mathcal{L} \) one can obtain the Proca equation \([35, 36]\)

\[
L^{\mu\nu}(x)A^V_\mu(x) = j^\mu(x),
\]

where the differential operator is defined as

\[
L^{\mu\nu}(x) = \left( \partial_\mu^2 + m^2_V \right) g^{\mu\nu} - \partial^\mu \partial^\nu.
\]

A constraint equation can be derived by contracting the above equation with \( \partial_\mu \) as

\[
\partial_\mu A^\mu_\nu(x) = \frac{1}{m_V} \partial_\mu j^\mu(x) = 0,
\]

if the source current is conserved \( \partial_\mu j^\mu = 0 \). The above equation means that the longitudinal component of \( A^\mu_V(x) \) is vanishing for the conserved current.

The free vector field can be quantized as

\[
A^\mu_V(x) = \sum_{\lambda=0, \pm 1} \int \frac{d^3k}{(2\pi\hbar)^3} \frac{1}{2E^V_k} \left[ e^{\mu}(\lambda, k)a^{\dagger}_V(\lambda, k)e^{-ikx/\hbar} + e^{\mu*}(\lambda, k)a^V_\mu(\lambda, k)e^{ikx/\hbar} \right],
\]

where \( E^V_k = \sqrt{k^2 + m^2_V} \) and \( \lambda \) denote the energy and the spin state in the spin quantization direction respectively, the creation and annihilation operators \( a^V_\lambda(\lambda, k) \) and \( a^{\dagger}_V(\lambda, k) \) satisfy the commutator

\[
\left[ a^V_\mu(\lambda, k), a^{\dagger}_V(\lambda', k') \right] = \delta_{\lambda\lambda'}2E_k(2\pi\hbar)^3\delta^{(3)}(k-k'),
\]

and the polarization vector \( e^{\mu}(\lambda, k) \) obeys

\[
k_\mu e^{\mu}(\lambda, k) = 0, \quad e(\lambda, k) \cdot e^{*}(\lambda', k') = -\delta_{\lambda\lambda'}, \quad \sum_{\lambda} e^{\mu}(\lambda, k)e^{*}(\lambda, k) = -\left( g^{\mu\nu} - \frac{k_\mu k_\nu}{m^2_V} \right).
\]

In above relations, the first one follows the constraint (4) and \( k^\mu = (E^V_k, k) \) denotes the on-shell four-momentum for the vector meson. By the field quantization in (5), one can check that \( A^\mu_V(x) \) is Hermitian, i.e. \( A^{\dagger}_V(\lambda, k) = A^V_\mu(x) \).

One can define the two-point Green function for the vector meson on the CTP

\[
G^\mu_V(x_1, x_2) \equiv \left\langle T_C A^\mu_V(x_1)A^{\dagger\mu}_V(x_2) \right\rangle,
\]

where \( x_1 \) and \( x_2 \) are two space-time points whose time components are defined on the CTP contour and \( T_C \) represents the time-ordering on the CTP contour. We can write \( G^\mu_V(x_1, x_2) \) in a matrix form

\[
G^\mu_V(x_1, x_2) = \begin{pmatrix} G^{++}_V(x_1, x_2) & G^{+-}_V(x_1, x_2) \\ G^{-+}_V(x_1, x_2) & G^{--}_V(x_1, x_2) \end{pmatrix} = \begin{pmatrix} G^{E}_V(x_1, x_2) & G^{<}_V(x_1, x_2) \\ G^{>}_V(x_1, x_2) & G^{F}_V(x_1, x_2) \end{pmatrix}.
\]

The \('++'\) component of \( G^\mu_V \) with both \( t_1 \) and \( t_2 \) (time components of \( x_1 \) and \( x_2 \)) on the positive time-branch is just the Feynman propagator \( G^{E}_V(x_1, x_2) \) as shown in Fig. (1a). The \('+-'\) component with \( t_1 \) on the positive time-branch while \( t_2 \) on the negative time-branch is denoted as \( G^{<}_V(x_1, x_2) \) meaning that \( t_2 \) is always later than \( t_1 \) on the CTP contour as shown in Fig. (1b). Analogously,
the relative position $y = x_1 - x_2$.

Inserting the quantized field (5) into the definition of the Wigner function (10), we obtain

$$G_{\mu\nu}^<(x_1, x_2) = 2\pi\hbar \sum_{\lambda_1, \lambda_2} \delta (p^2 - m_V^2)$$

$$\times \left\{ \theta (p^0) \epsilon_\mu (\lambda_1, p) \epsilon^*_\nu (\lambda_2, p) f_{\lambda_1 \lambda_2}(x, p) + \theta (-p^0) \epsilon^*_\mu (\lambda_1, -p) \epsilon_\nu (\lambda_2, -p) \times [\delta_{\lambda_2 \lambda_1} + f_{\lambda_2 \lambda_1}(x, -p)] \right\},$$

(11)

where the gradient expansion has been taken with spatial gradient terms being dropped, and the MVSD for the vector meson is defined as

$$f_{\lambda_1 \lambda_2}(x, p) = \int \frac{d^4 u}{2(2\pi\hbar)^3} \delta (p \cdot u) e^{-iux/h}$$

$$\times \left\langle a^+_V \left( \lambda_2, p - \frac{u}{2} \right) a_V \left( \lambda_1, p + \frac{u}{2} \right) \right\rangle.$$

(12)

One can check that $f_{\lambda_1 \lambda_2}(x, p)$ is a Hermitian matrix, i.e. $f^*_{\lambda_1 \lambda_2}(x, p) = f_{\lambda_2 \lambda_1}(x, p)$. Similarly we can define another Wigner function from $G_{\mu\nu}^>(x_1, x_2)$

$$G_{\mu\nu}^>(x, p) = \int d^4 y e^{ip' y/h} \left\langle A_\mu(x_1) A^\dagger_\nu(x_2) \right\rangle.$$

$$= 2\pi\hbar \sum_{\lambda_1, \lambda_2} \delta (p^2 - m_V^2)$$

$$\times \left\{ \theta (p^0) \epsilon_\mu (\lambda_1, p) \epsilon^*_\nu (\lambda_2, p) [\delta_{\lambda_1 \lambda_2} + f_{\lambda_1 \lambda_2}(x, p)] + \theta (-p^0) \epsilon^*_\mu (\lambda_1, -p) \epsilon_\nu (\lambda_2, -p) f_{\lambda_2 \lambda_1}(x, -p) \right\}.$$

(13)

Note that $G_{\mu\nu}^>(x, p)$ can be obtained by replacing $f_{\lambda_1 \lambda_2} \to \delta_{\lambda_1 \lambda_2} + f_{\lambda_1 \lambda_2}$ and $\delta_{\lambda_2 \lambda_1} + f_{\lambda_2 \lambda_1} \to f_{\lambda_2 \lambda_1}$ from $G_{\mu\nu}^<(x, p)$. 
III. KADANOFF-BAYM EQUATIONS FOR VECTOR MESONS

The Wigner functions for massless vector particles such as gluons and photons [24, 29, 37–40] have been studied for many years, but to our knowledge there are few works about Wigner functions for massive vector mesons in the context of spin polarization (see Ref. [41] for a recent one). In this section, we will derive the Boltzmann equation for vector mesons’ Wigner functions from two-point Green functions on the CTP. The starting point is the KB equations

\[ L_\eta(x_1) G^{\less,\eta}(x_1, x_2) = - \frac{i\hbar}{2} \int d^4 x' \left[ \Sigma^{\less,\alpha}(x_1, x') G^{\greater,\alpha\nu}(x', x_2) - \Sigma^{\greater,\alpha}(x_1, x') G^{\less,\alpha}\eta(x', x_2) \right] , \]  

and

\[ L_\nu(x_2) G^{\less,\eta}(x_1, x_2) = - \frac{i\hbar}{2} \int d^4 x' \left[ G^{\less,\alpha}(x_1, x') \Sigma^{\greater,\alpha\nu}(x', x_2) - G^{\greater,\alpha}(x_1, x') \Sigma^{\less,\alpha}\eta(x', x_2) \right] . \]  

Equations (14) and (15) are the result of the quasiparticle approximation [34]. Note that the integrations over \( x \) in Eqs. (14) and (15) are ordinary ones.

![Figure 2](image-url)

**Figure 2.** The self-energies \( \Sigma^{\less,\mu\nu}(x_1, x_2) \) and \( \Sigma^{\greater,\mu\nu}(x_1, x_2) \) of vector mesons from quark loops in the quark-meson model. Two quark propagators in the loop may have different flavors corresponding to the vector meson that is not flavor neutral.

We consider the coupling between the vector meson and the quark-antiquark in the quark-meson model [32–37]. Then at lowest order in the coupling constant, the self-energies are from quark loops as shown in Fig. 2.

\[
\Sigma^{\less,\mu\nu}(x_1, x_2) = - \text{Tr} \left[ i \Gamma^{\mu} S^{\less}(x_1, x_2) i \Gamma^{\nu} S^{\greater}(x_2, x_1) \right] ,
\]

\[
\Sigma^{\greater,\mu\nu}(x_1, x_2) = - \text{Tr} \left[ i \Gamma^{\mu} S^{\greater}(x_1, x_2) i \Gamma^{\nu} S^{\less}(x_2, x_1) \right] ,
\]

where \( S^{\greater}(x_1, x_2) \) and \( S^{\less}(x_1, x_2) \) are two-point Green functions of quarks, \( i \Gamma^{\mu} \) denotes the vertex of the vector meson and quark-antiquark, and the overall minus signs arise from quark loops. Inserting the self-energies (16) into Eq. (14) and taking a Fourier transform with respect to the difference
Since their contraction with the leading-order where we have neglected terms with Poisson brackets and those proportional to antiquarks.

The Wigner function at the leading order Taking the difference between Eq. (17) and (19), we are able to derive the Boltzmann equation for

where the Poisson bracket involves space-time and momentum gradients and is defined as

\[
\{A, B\}_{P.B.} \equiv (\partial^\mu_A)(\partial^\nu_B) - (\partial^\mu_B)(\partial^\nu_A).
\]

In the same way, we obtain from Eq. (15) another KB equation for the Wigner function

Taking the difference between Eq. (17) and (19), we are able to derive the Boltzmann equation for the Wigner function at the leading order

where we have neglected terms with Poisson brackets and those proportional to \( p_\eta \) in the left-hand-side since their contraction with the leading-order \( G^{<,\eta\nu}(x, p) \) and \( G^{<,\mu\eta}(x, p) \) is vanishing. In the next section we will rewrite the above Boltzmann equation in terms of MVSDs for vector mesons, quarks and antiquarks.
IV. SPIN DENSITY MATRIX FOR QUARK COALESCENCE AND DISSOCIATION

The two-point Green functions $S^>(x,p)$ and $S^<(x,p)$ for quarks are given by \cite{34}

$$S^<(x,p) = - (2\pi\hbar) (p_0) \delta(p^2 - m_p^2) \sum_{r,s} u(r,p) \overline{\pi}(s,p) f_{rs}^{(+)}(x,p)$$

$$- (2\pi\hbar) (p_0) \delta(p^2 - m_p^2) \sum_{r,s} v(s,-p) \overline{\pi}(r,-p) \left[ \delta_{rs} - f_{rs}^{-1}(x,-p) \right],$$

$$S^>(x,p) = (2\pi\hbar) (p_0) \delta(p^2 - m_p^2) \sum_{r,s} u(r,p) \overline{\pi}(s,p) \left[ \delta_{rs} - f_{rs}^{(+)}(x,p) \right]$$

$$+ (2\pi\hbar) (p_0) \delta(p^2 - m_p^2) \sum_{r,s} v(s,-p) \overline{\pi}(r,-p) f_{rs}^{-1}(x,-p),$$

(21)

where $f_{rs}^{(+)}$ and $f_{rs}^{-1}$ are MVSD for quarks and antiquarks respectively. We can parameterize them as

$$f_{rs}^{(+)}(x,p) = \frac{1}{2} f_q(x,p) \left[ \delta_{rs} - P^q_\mu(x,p)n_j^{(+)}(p_\mu) \tau^j_{rs} \right],$$

$$f_{rs}^{-1}(x,-p) = \frac{1}{2} f_{\overline{q}}(x,-p) \left[ \delta_{rs} - P^q_\mu(x,-p)n_j^{(-)}(p_\mu) \tau^j_{rs} \right],$$

(22)

where $f_q(x,p)$ and $f_{\overline{q}}(x,-p)$ are MVSDs for quarks and antiquarks respectively, and $P^q_\mu(x,p)$ and $P^{q\prime}_\mu(x,-p)$ are polarization four-vectors for quarks and antiquarks respectively. The spin direction four-vectors for quarks and antiquarks are given by

$$n_j^{(+)} = \left( \frac{n_j \cdot P}{m_q}, \frac{n_j \cdot p}{m_q}, \frac{n_j \cdot p}{m_q}, \frac{E^p}{m_q} \right),$$

$$n_j^{(-)} = \left( \frac{n_j \cdot P}{m_{\overline{q}}}, \frac{n_j \cdot p}{m_{\overline{q}}}, \frac{n_j \cdot p}{m_{\overline{q}}}, \frac{E^\overline{p}}{m_{\overline{q}}} \right),$$

(23)

where $n_j$ for $j = 1, 2, 3$ are three basis unit vectors that form a Cartesian coordinate system in the particle’s rest frame with $n_3$ being the spin quantization direction, and $n_j^{(+)}$ and $n_j^{(-)}$ are the Lorentz transformed four-vectors of $n_j$ for quarks and antiquarks respectively which obey the sum rules

$$n_j^{(+)} n_j^{(+)} = - \left( g^{\mu\nu} - \frac{p^\mu p^\nu}{m_q^2} \right),$$

$$n_j^{(-)} n_j^{(-)} = - \left( g^{\mu\nu} - \frac{p^\mu p^\nu}{m_{\overline{q}}^2} \right),$$

(24)

where $P^\mu = (E^p, p)$ and $P^\mu = (E^{\overline{p}}, -p)$. We note that $f_{rs}^{(+)}(x,p)$ and $f_{rs}^{-1}(x,-p)$ are actually the transpose of those MVSDs defined in Eqs. (117)-(118) of Ref. \cite{34} in spin indices. We can flip the sign of the three-momentum, $p \to -p$, in $f_{rs}^{-1}(x,-p)$ to obtain

$$f_{rs}^{-1}(x,p) = \frac{1}{2} f_{\overline{q}}(x,p) \left[ \delta_{rs} - P^q_\mu(x,p)n_j^{(-)}(p_\mu) \tau^j_{rs} \right],$$

(25)

where $n_j^{(-)}(-p)$ has the same form as $n_j^{(+)}(p)$ except the quark mass. Note that in the self-energy (16) of the vector meson that is not flavor neutral, $S^<(x,p)$ and $S^>(x,p)$ may involve different flavors of quarks and antiquarks.

Inserting $S^<(x,p)$, $S^>(x,p)$, $G^{<\mu\nu}(x,p)$, and $G^{>\mu\nu}(x,p)$ in Eqs. (21), (11) and (13) into Eq. (20),
Table I. Collision terms in the Boltzmann equation. All terms except $I_{-++}$ and $I_{+-+}$ are vanishing for on-shell quarks, antiquarks and mesons at the one-loop level of the selfenergy.

| Process | Term |
|---------|------|
| $q \to q + M$ | $I_{++-}$ |
| $q + M \to q$ | $I_{++-}$ |
| $q + \bar{q} \to M$ | $I_{++-}$ |
| $q + \bar{q} \to q + M$ | $I_{++-}$ |
| $q \to q + \bar{q}$ | $I_{++-}$ |
| $q + M \to q + \bar{q}$ | $I_{++-}$ |
| $q + \bar{q} \to q + \bar{q}$ | $I_{++-}$ |
| $q \to \bar{q} + M$ | $I_{++-}$ |
| $q + M \to \bar{q} + M$ | $I_{++-}$ |
| $q + \bar{q} \to \bar{q} + M$ | $I_{++-}$ |
| $q \to \bar{q} + \bar{q}$ | $I_{++-}$ |
| $q + M \to \bar{q} + \bar{q}$ | $I_{++-}$ |
| $q + \bar{q} \to \bar{q} + \bar{q}$ | $I_{++-}$ |

The Boltzmann equation can be put into the following form:

$$p \cdot \partial_x G^{\mu\nu}(x, p) = \frac{1}{4} \left[ p^\mu \partial_0^e G^{-\nu,\eta}(x, p) + p^\nu \partial_0^e G^{-\mu,\eta}(x, p) \right]$$

$$= \frac{1}{4(2\pi \hbar)} \int d^3p' \delta(p^2 - m_q^2) \delta \left[ (p + p')^2 - m_q^2 \right] \delta(p^2 - m_q^2)$$

$$\times \left\{ \theta(p_0') \theta(p_0 + p_0') \theta(p_0) I_{++-} + \theta(p_0') \theta(p_0 + p_0') \theta(-p_0) I_{++-} + \theta(-p_0') \theta(p_0 + p_0') \theta(p_0) I_{++-} + \theta(-p_0') \theta(-p_0 + p_0') \theta(p_0) I_{++-} \right\}.$$

(26)

The terms $I_{ijk}$, with $i, j, k = \pm$ representing the positive/negative energy, correspond to all possible processes at lowest order in the coupling constant, as shown in Table I. In Eq. (26), $I_{++-}$ and $I_{+-+}$ are absent due to incompatibility of theta functions, and $I_{-++}$ and $I_{+-+}$ contain the coalescence of quark-antiquark to the vector meson and vice versa, but $I_{++-}$ corresponds to the positive energy sector of the two-point function for the vector meson while $I_{+-+}$ corresponds to the negative energy sector. All terms except $I_{-++}$ and $I_{+-+}$ are vanishing for on-shell quarks, antiquarks and mesons at the one-loop level of the selfenergy. We distinguish $m_q$ from $m_\pi$ in Eq. (26), since the quark and antiquark may have different flavors for the vector meson that is not flavor neutral so the meson and its antiparticle are not the same particle.

In this paper, we are interested in the contribution from the coalescence and dissociation processes corresponding to $I_{-++}$. The coalescence is regarded as one of the main processes for particle production in heavy-ion collisions [43, 53]. So the spin Boltzmann equation for the vector meson’s MVSD reads

$$p \cdot \partial_x f_{\lambda_1, \lambda_2}(x, p)$$

$$= \frac{1}{16} \sum_{r_1, s_1, r_2, s_2, \lambda_1', \lambda_2'} \int \frac{d^3p'}{(2\pi \hbar)^3} \frac{1}{E_{p'} F_{p'}} 2\pi \hbar \delta \left( E_p - E_{p'} - E_p - p' \right)$$

$$\times \left\{ \delta_{\lambda_2, \lambda_2'} \epsilon^\alpha_{\lambda_1, \epsilon}(\lambda_1, p) \epsilon^\beta(\lambda_1', p') \Gamma_{\alpha \beta}(r_1, p') \Gamma_{\gamma \delta}(r_2, p) \Gamma_{\epsilon \eta}(s_2, p - p') \right\}$$

$$\times \left\{ f_{r_1, s_1}^{(+)}(x, p') f_{r_2, s_2}^{(+)}(x, p - p') \delta_{\lambda_2, \lambda_1'} + f_{\lambda_1'}(x, p) \right\}$$

$$- \left[ f_{r_1, s_1}^{(-)}(x, p') f_{r_2, s_2}^{(-)}(x, p - p') \right] \left[ \delta_{\lambda_2, \lambda_2'} - f_{\lambda_1'}(x, p) \right],$$

(27)

where $\lambda_1, \lambda_2, \lambda_1'$ and $\lambda_2'$ denote the spin states along the spin quantization direction, and $\Gamma^\alpha$ is the $q\bar{q}V$ vertex given by

$$\Gamma^\alpha \approx g_V B(p - p', p') \gamma^\alpha,$$

(28)

where $g_V$ is the coupling constant of the vector meson and quark-antiquark, and $B(p - p', p')$ denotes the Bethe-Salpeter wave function of the $\phi$ meson [54, 55] in the following parametrization form

$$B(p - p', p') = \frac{1}{1 - \exp \left\{ \left[ (E_{p - p'} - E_{p'})^2 - (p - 2p')^2 \right] / \sigma^2 \right\}}$$

$$\left[ (E_{p - p'} - E_{p'})^2 - (p - 2p')^2 \right] / \sigma^2,$$

(29)
with $\sigma \approx 0.522$ GeV being the width parameter of the wave function. The derivation of Eq. (27) is given in Appendix A. We see that there is a gain term and a loss term in Eq. (27). In heavy ion collisions, the distribution functions are normally much less than 1, $f_{\lambda_1\lambda_2}(x, p) \sim f_{\text{coal}} \sim f_{\text{diss}}(x, p) \ll 1$, so Eq. (27) can be approximated as

$$\frac{p}{E_p} \cdot \partial_x f_{\lambda_1\lambda_2}(x, p) \approx R_{\lambda_1\lambda_2}^{\text{coal}}(p) - R_{\lambda_1\lambda_2}^{\text{diss}}(p) f_{\lambda_1\lambda_2}(x, p),$$  \hspace{1cm} (30)

where $R_{\lambda_1\lambda_2}^{\text{coal}}$ and $R_{\lambda_1\lambda_2}^{\text{diss}}$ denote the coalescence and dissociation rates for the vector meson, i.e. the rates of $q + \bar{q} \rightarrow M$ and $M \rightarrow q + \bar{q}$ respectively, defined as

$$R_{\lambda_1\lambda_2}^{\text{coal}}(p) = \frac{1}{8(2\pi \hbar)^2} \sum_{r_1, s_1, r_2, s_2} \int d^3 p' \frac{1}{E_{p'} E_{p-p'}^q E_{p}^{V}}$$

$$\times \delta \left( E_p^{V} - E_{p'}^{V} - E_{p-p'}^{q} \right) \epsilon_{\alpha}(\lambda_1, p) \epsilon_{\beta}(\lambda_2, p)$$

$$\times \text{Tr} \left[ \Gamma^{\beta}(s_1, p') \pi(r_1, p') \Gamma^{\alpha} u(r_2, p - p') \pi(s_2, p - p') \right]$$

$$\times f_{r_1s_1}(x, p') f_{r_2s_2}(x, p - p').$$  \hspace{1cm} (31)

$$R_{\lambda_1\lambda_2}^{\text{diss}}(p) = -\frac{1}{12(2\pi \hbar)^2} \sum_{r_1, r_2} \int d^3 p' \frac{1}{E_{p'} E_{p-p'}^q E_{p}^{V}}$$

$$\times \delta \left( E_p^{V} - E_{p'}^{V} - E_{p-p'}^{q} \right) \left( g_{\alpha\beta} - \frac{\beta p_{\beta} p}{m_V^2} \right)$$

$$\times \text{Tr} \left[ \Gamma^{\beta}(p' \cdot \gamma - m_\gamma) \Gamma^{\alpha}(p - p' \cdot \gamma + m_\gamma) \right].$$  \hspace{1cm} (32)

Note that $R_{\lambda_1\lambda_2}^{\text{diss}}(p)$ does not depend on the MVSDs of the quark, antiquark and the vector meson, therefore it is independent of the quark polarization. Schematically the formal solution to Eq. (30) reads

$$f_{\lambda_1\lambda_2}(x, p) \sim \frac{R_{\lambda_1\lambda_2}^{\text{coal}}(p)}{R_{\lambda_1\lambda_2}^{\text{diss}}(p)} \left[ 1 - \exp \left( -R_{\lambda_1\lambda_2}^{\text{diss}}(p) \Delta t \right) \right]$$

$$\sim \begin{cases} R_{\lambda_1\lambda_2}^{\text{coal}}(p) \Delta t, & \text{for } \Delta t \ll 1/R_{\lambda_1\lambda_2}^{\text{diss}}(p) \\ R_{\lambda_1\lambda_2}^{\text{coal}}(p)/R_{\lambda_1\lambda_2}^{\text{diss}}(p), & \text{for } \Delta t \gg 1/R_{\lambda_1\lambda_2}^{\text{diss}}(p) \end{cases}$$  \hspace{1cm} (33)

if $f_{\lambda_1\lambda_2}(x, p)$ for the vector meson at the initial time is assumed to be zero, where $\Delta t$ is the formation time of the vector meson.

The spin density matrix element $\rho_{\lambda_1\lambda_2}^{V}(x, p)$ is assumed to be proportional to $f_{\lambda_1\lambda_2}(x, p)$ which is $R_{\lambda_1\lambda_2}^{\text{coal}}(p)\Delta t$ if $\Delta t \ll 1/R_{\lambda_1\lambda_2}^{\text{diss}}(p)$ or $R_{\lambda_1\lambda_2}^{\text{coal}}(p)/R_{\lambda_1\lambda_2}^{\text{diss}}(p)$ if $\Delta t \gg 1/R_{\lambda_1\lambda_2}^{\text{diss}}(p)$. In both cases, $\rho_{\lambda_1\lambda_2}^{V}(x, p)$ is proportional to $R_{\lambda_1\lambda_2}^{\text{coal}}(p)$ times a constant independent of the spin states of the vector meson. Here we assume that the coalescence of the vector meson takes place in a relatively short time, so we have

$$\rho_{\lambda_1\lambda_2}^{V}(x, p) \approx \frac{\Delta t}{8} \sum_{r_1, s_1, r_2, s_2} \int d^3 p' \frac{1}{(2\pi \hbar)^3} \frac{1}{E_{p'} E_{p-p'}^q E_{p}^{V}}$$

$$\times 2\pi \hbar \delta \left( E_p^{V} - E_{p'}^{V} - E_{p-p'}^{q} \right) \epsilon_{\alpha}(\lambda_1, p) \epsilon_{\beta}(\lambda_2, p)$$

$$\times \text{Tr} \left[ \Gamma^{\beta}(s_1, p') \pi(r_1, p') \Gamma^{\alpha} u(r_2, p - p') \pi(s_2, p - p') \right]$$

$$\times f_{r_1s_1}(x, p') f_{r_2s_2}(x, p - p').$$  \hspace{1cm} (34)

where we have changed the notation to $f_{r_1s_1}$ from $f_{r_1}(\pm)$. The spin density matrix element (34) can be put into a compact form with an explicit dependence on the polarization vector of the quark and
antiquark

\[ \rho_{\lambda_1\lambda_2}^V(x, p) = \frac{\Delta t}{32} \int \frac{d^3p'}{(2\pi \hbar)^3} \frac{1}{E_p' E_{p-p'} E_p} f_\gamma(x, p') f_\delta(x, p - p') \]

\[ \times 2\pi \hbar \delta \left( E_p' - E_\gamma - E_{p-p'} \right) \epsilon_\alpha^*(\lambda_1, p) \epsilon_\beta(\lambda_2, p) \]

\[ \times \text{Tr} \left\{ \Gamma^\beta (p' \cdot \gamma - m_\gamma) [1 + \gamma_5 \gamma \cdot P_\gamma(x, p')] \Gamma^\alpha \right\} \]

\[ \times [(p - p') \cdot \gamma + m_q] [1 + \gamma_5 \gamma \cdot P^q(x, p - p')] \],

(35)

where \( p^\mu = (E_p^q, p) \) and \( p'^\mu = (E_p'^q, p') \). The derivation of the expression inside the trace is given in Appendix B. The contraction of \( \epsilon_\alpha^*(\lambda_1, p) \) and \( \epsilon_\beta(\lambda_2, p) \) with the trace can be worked out and the result is given by Eq. (B9). The normalized \( \rho_{\lambda_1\lambda_2}^V(x, p) \) is defined as

\[ \tilde{\rho}_{\lambda_1\lambda_2}^V(x, p) = \frac{\rho_{\lambda_1\lambda_2}^V(x, p)}{\text{Tr}(\rho_V)} , \]

(36)

where \( \text{Tr}(\rho_V) \) is the trace of the spin density matrix and is evaluated using Eq. (B10) and \( \rho_{\lambda_1\lambda_2}^V(x, p) \) is evaluated using Eq. (B9).

For quarkonium vector mesons such as \( \phi \) mesons with \( m_q = m_\gamma \), \( \rho_{\lambda_1\lambda_2}^V(x, p) \) and \( \text{Tr}(\rho_V) \) can be simplified as

\[ \rho_{\lambda_1\lambda_2}^V(x, p) = \frac{\Delta t}{8 \gamma_V^2} \int \frac{d^3p'}{(2\pi \hbar)^3} \frac{1}{E_p' E_{p-p'} E_p} f_\gamma(x, p') f_\delta(x, p - p') \]

\[ \times 2\pi \hbar \delta \left( E_p' - E_\gamma - E_{p-p'} \right) B^2(p - p', p') \epsilon_\alpha^*(\lambda_1, p) \epsilon_\beta(\lambda_2, p) \]

\[ \times \left\{ (p^\alpha P_\gamma^\beta + p^\beta P_\gamma^\alpha)(p' \cdot P_q) - (p'^\alpha P_q^\beta + p'^\beta P_q^\alpha)(p \cdot P_q) \right\} \]

\[ + 2p'^\alpha p^\beta(1 - P_\gamma \cdot P_q) + g^{\alpha\beta}[p' \cdot p + (p' \cdot P_q)(p \cdot P_q)] \]

\[ + (p \cdot p') \left( P_q^\alpha P_q^\beta + P_q^\beta P_q^\alpha - g^{\alpha\beta} P_q^2 \right) \]

\[ - im_q\epsilon^{\alpha\beta\mu\nu} p_\mu (P_q^\alpha + P_q^\beta) \}, \]

(37)

\[ \text{Tr}(\rho_V) = \frac{\Delta t}{8 \gamma_V^2} \int \frac{d^3p'}{(2\pi \hbar)^3} \frac{1}{E_p' E_{p-p'} E_p} f_\gamma(x, p') f_\delta(x, p - p') \]

\[ \times 2\pi \hbar \delta \left( E_p' - E_\gamma - E_{p-p'} \right) B^2(p - p', p') \]

\[ \times [ -2m_q^2 (P_\gamma \cdot P_q) + m_\gamma^2 + 2m_q^2 ] , \]

(38)

where we have used the short-hand notation \( P_q \equiv P_q(x, p - p') \) and \( P_\gamma \equiv P_\gamma(x, p') \). Equations (37) and (38) will be used in the next section for evaluating spin density matrix elements for \( \phi \) mesons.

V. SPIN DENSITY MATRIX ELEMENTS FOR \( \phi \) MESONS

Now we consider the vector meson made of a quark and its antiquark, the so-called quarkonium. For the quarkonium vector meson such as the \( \phi \) meson, the polarization distributions in phase space in Eq. (35) are given by (17) [60, 61]

\[ P^\mu_x(x, p) = \frac{1}{4m_s} \epsilon^{\mu\nu\rho\sigma} \left( \omega_{\rho\sigma} + \frac{g_8}{E_p \gamma T_{eff}} P_\gamma^\rho \right) \right]_{\nu} [1 - f_s(x, p)] , \]

\[ P^\mu_\gamma(x, p) = \frac{1}{4m_s} \epsilon^{\mu\nu\rho\sigma} \left( \omega_{\rho\sigma} - \frac{g_8}{E_p \gamma T_{eff}} P_\gamma^\rho \right) \right]_{\nu} [1 - f_\gamma(x, p)] , \]

(39)
where \( p^\mu = (E_p^s, \mathbf{p}) \) and \( p^\mu = (E_p^\bar{s}, \mathbf{p}) \) denote the four-momenta of the strange quark \( s \) and antiquark \( \bar{s} \) respectively, with \( E_p^s = E_p^\bar{s} = \sqrt{\mathbf{p}^2 + m_s^2} \) and \( m_s = m_s \). We have assumed that \( s \) and \( \bar{s} \) are polarized by the thermal vorticity (tensor) field \( \omega_{\rho\sigma} = (1/2) (\partial_\rho (\beta u_\sigma) - \partial_\sigma (\beta u_\rho)) \) and \( \phi \) field strength tensor \( F^\phi_{\rho\sigma} = \partial_\rho A^\phi_\sigma - \partial_\sigma A^\phi_\rho \) [22], where \( u_\sigma \) is the fluid velocity, \( \beta = 1/T_{\text{eff}} \) is the inverse effective temperature, and \( A^\phi_\rho \) is the vector potential of the \( \phi \) field. Note that in some literature the definition of \( \omega_{\rho\sigma} \) may differ by a sign [14] [50] [57]. In Eq. (39), \( f_s(x, \mathbf{p}) \) and \( f_{\bar{s}}(x, \mathbf{p}) \) are unpolarized phase space distributions of \( s \) and \( \bar{s} \) respectively and given by the Fermi-Dirac distribution

\[
f_{s/\bar{s}}(x, \mathbf{p}) = \frac{1}{1 + \exp(\beta E^s_{\mathbf{p}}/\mu_s)}
\]

where \( \mu_s \) is the chemical potential for \( s \) (\( -\mu_s \) for \( \bar{s} \)). In most cases \( f_{s/\bar{s}} \) are negligible relative to 1 in \( P^\mu_{s/\bar{s}} \) in Eq. (39). The spin-field coupling in (39) can be derived from the Wigner functions for massive \( \phi \) mesons [29] and has a clear physical meaning: one contribution is from the magnetic field through the magnetic moment and the other contribution from the electric field through the spin-orbit coupling, the former is always there while the latter is only present for moving fermions. The mean field effects of vector mesons have been studied in the context of spin polarization of \( \Lambda \) hyperons [60] and different elliptic flows between hadrons of some species and their antiparticles [61] in heavy-ion collisions.

The spin direction four-vector for the \( \phi \) meson is given by

\[
e^\mu(\lambda, \mathbf{p}) = \left( \frac{\mathbf{p} \cdot \epsilon_\lambda}{m_\phi}, \epsilon_\lambda + \frac{\mathbf{p} \cdot \epsilon_\lambda}{m_\phi} \mathbf{p} \right),
\]

where \( E_\phi = \sqrt{m_\phi^2 + \mathbf{p}^2} \) is the energy of the \( \phi \) meson, \( \lambda = 0, \pm 1 \) denotes the spin states, and \( \epsilon_\lambda \) denotes the three-vector of the spin state (spin vector) in the \( \phi \) meson’s rest frame. In order to calculate the spin alignment along the direction of the global orbital angular momentum (the \( y \)-direction) in heavy-ion collisions, we choose the \( y \)-direction as the spin quantization direction. So the corresponding spin vectors are

\[
\epsilon_0 = (0, 1, 0),
\]

\[
\epsilon_{\pm 1} = \pm \frac{1}{\sqrt{2}} (i, 0, 1),
\]

\[
\epsilon_{-1} = \frac{1}{\sqrt{2}} (-i, 0, 1).
\]

The 00-component of the spin density matrix is what can be measured in experiments which concerns the real vector \( \epsilon_0 \) satisfying \( \epsilon_0 = \epsilon_0^\dagger \).

Substituting Eq. (39) into Eq. (35), we obtain

\[
\rho^{\phi}_{\lambda_1\lambda_2} = \rho^{\phi}_{\lambda_1\lambda_2}(0) + \rho^{\phi}_{\lambda_1\lambda_2}(\omega^1) + \rho^{\phi}_{\lambda_1\lambda_2}(F^1_\phi) + \rho^{\phi}_{\lambda_1\lambda_2}(F^2_\phi),
\]

where \( \omega^i \) and \( F^i_\phi \) with \( i = 0, 1, 2 \) denote the zeroth, first, and second order terms in the vorticity and \( \phi \) field respectively. The zeroth order term \( \rho^{\phi}_{\lambda_1\lambda_2}(0) \) represents the unpolarized contribution. In [43], we neglected mixing terms of \( \omega_{\mu\nu} \) and \( F^\phi_{\mu\nu} \), since we assume that there is no correlation between them in space-time so these terms are vanishing after taking a space-time average of \( \rho^{\phi}_{\lambda_1\lambda_2}(0) \). For \( \lambda_1 = \lambda_2 = 0 \), \( \epsilon_{\alpha}^\dagger(0, \mathbf{p}) \epsilon_{\alpha}(0, \mathbf{p}) = \epsilon_0(0, \mathbf{p}) \epsilon_0(0, \mathbf{p}) \) is symmetric in \( \alpha \) and \( \beta \), then one can verify that the first order terms \( \rho^{\phi}_{\lambda_1\lambda_2}(\omega^1) \) and \( \rho^{\phi}_{\lambda_1\lambda_2}(F^1_\phi) \) are vanishing. The zeroth order term \( \rho^{\phi}_{00}(0) \) is given by

\[
\rho^{\phi}_{00}(0) = \frac{\Delta t}{8} g^2 \int \frac{d^3p'}{(2\pi\hbar)^3} \frac{1}{E_{p'}^\phi E_p^s - E_p^s - E_{p'}^s} \rho_s(p') f_s(p-p') B^2(p-p', p') \times 2\pi\hbar \delta (E_p^\phi - E_{p'}^\phi - E_{p-p'}^s) \left\{ (p' \cdot p) - 2 [p' \cdot \epsilon(0, \mathbf{p})]^2 \right\},
\]
where we have used the second relation of Eq. (7). The second order terms $\rho_{\lambda_1\lambda_2}(\omega^2)$ and $\rho_{\lambda_1\lambda_2}(F^2_\phi)$ read
\[
\rho_{\lambda_1\lambda_2}(\omega^2) \approx -\frac{\Delta t}{32} \frac{1}{3m^2 g_0^2} \int \frac{d^3 p'}{(2\pi \hbar)^3} \frac{1}{E_{p'} E_{p-p'} E_p} B^2(p - p', p') \\
\times f_\epsilon(p') f_\mu(p - p') 2\pi \hbar \delta (E_{p'} - E_{p'})^2 \left( E_{p'} - E_{p-p'} \right) \\
\times \epsilon^\alpha_\mu_\epsilon(\lambda_1, \mu_\epsilon(p) \omega_{\beta}(x) \omega_{\gamma}(x) p_\beta (p - p') \gamma \\
\times \text{Tr} \left\{ \gamma^\beta (p' \cdot \gamma + m_\epsilon) \gamma^\alpha [(p - p') \cdot \gamma + m_\epsilon] \gamma^\sigma \right\},
\]
and
\[
\rho_{\lambda_1\lambda_2}(F^2_\phi) \approx \frac{\Delta t}{32} \frac{1}{3m^2 g_0^2} \int \frac{d^3 p'}{(2\pi \hbar)^3} \frac{1}{E_{p'} E_{p-p'} E_p} B^2(p - p', p') \\
\times f_\pi(p') f_\beta(p - p') 2\pi \hbar \delta (E_{p'})^2 \left( E_{p'} - E_{p-p'} \right) \\
\times \epsilon^\alpha_\beta(\lambda_1, \beta_\epsilon(p) \tilde{F}_{\rho \xi}(x) \tilde{F}_{\rho \xi}(x) p_\epsilon (p - p') \gamma \\
\times \text{Tr} \left\{ \gamma^\beta (p' \cdot \gamma + m_\epsilon) \gamma^\alpha [(p - p') \cdot \gamma + m_\epsilon] \gamma^\sigma \right\}.
\]
In Eqs. (45) and (46) we have used $\tilde{\omega}_{\rho \xi} = (1/2) \epsilon_{\rho \xi_\alpha_\beta} \omega_{\alpha_\beta}$, $\tilde{F}_{\rho \xi} = (1/2) \epsilon_{\rho \xi_\alpha_\beta} F_{\phi \alpha_\beta}$, and neglected $f_{s/\pi}$ relative to 1 in $P^{s/\pi}$. The tensor part of $\rho_{\phi_1\phi_2}(\omega^2)$ and $\rho_{\phi_1\phi_2}(F^2_\phi)$ that is contracted with $\epsilon^\alpha_\beta \epsilon^\rho_\sigma \tilde{\omega}_{\rho \xi} \tilde{\omega}_{\sigma \gamma}$ and $\epsilon^\alpha_\beta \epsilon^\rho_\sigma \tilde{F}_{\rho \xi} \tilde{F}_{\rho \xi}$, respectively can be evaluated as
\[
I^{\alpha_\beta\rho_\xi\sigma_\gamma} = p_\xi (p - p') \gamma \text{Tr} \left\{ \gamma^\beta (p' \cdot \gamma + m_\epsilon) \gamma^\alpha [(p - p') \cdot \gamma + m_\epsilon] \gamma^\sigma \right\} \\
= 2p_\xi p_\rho \left[ m_\phi^2 \left( g^{\beta\sigma} g^{\alpha\rho} - g^{\beta\rho} g^{\alpha\sigma} + g^{\beta\sigma} g^{\rho\alpha} \right) \\
+ 2p_\rho \left( g^{\beta\sigma} p^\alpha p^\rho - g^{\beta\rho} p^\alpha p^\sigma \right) \\
+ 2 \left( g^{\beta\rho} p^\alpha p^\rho - g^{\beta\rho} p^\alpha p^\rho - 2g^{\alpha\rho} p^\rho p^\rho \right) \\
- 2p_\rho \gamma^\rho \left[ m_\phi^2 \left( g^{\beta\sigma} g^{\alpha\rho} - g^{\beta\rho} g^{\alpha\sigma} + g^{\beta\sigma} g^{\rho\alpha} \right) \\
- 2p_\rho \left( g^{\beta\rho} p^\rho - g^{\beta\rho} p^\rho \right) - 4g^{\alpha\rho} p^\rho p^\rho \right] .
\]
With the above tensor and the quantity inside the curly brackets in (44), $\rho_{00}(0)$, $\rho_{\lambda_1\lambda_2}(\omega^2)$ and $\rho_{\lambda_1\lambda_2}(F^2_\phi)$ involve following moments of momenta
\[
\{ I_0, I^{\mu}_0, I^{\mu_\nu}_0, I^{\mu_\nu_\rho}_0, I^{\mu_\nu_\rho}_0 \} = \int \frac{d^3 p'}{(2\pi \hbar)^3} \frac{1}{E_{p'} E_{p-p'} E_p} B^2(p - p', p') \\
\times f_\epsilon(p') f_\mu(p - p') 2\pi \hbar \delta (E_{p'} - E_{p'})^2 \left( E_{p'} - E_{p-p'} \right) \\
\times \left\{ 1, p^{\mu_\epsilon}, p^{\mu_\epsilon_\nu}, p^{\mu_\epsilon_\nu_\rho}, p^{\mu_\epsilon_\nu_\rho_\sigma}, p^{\mu_\epsilon_\nu_\rho_\sigma_\tau} \right\},
\]
and
\[
\{ I^{\mu}_F, I^{\mu_\nu}_F, I^{\mu_\nu_\rho}_F, I^{\mu_\nu_\rho_\sigma}_F \} = \int \frac{d^3 p'}{(2\pi \hbar)^3} \frac{1}{E_{p'} E_{p-p'} E_p} B^2(p - p', p') \\
\times f_\pi(p') f_\beta(p - p') 2\pi \hbar \delta (E_{p'})^2 \left( E_{p'} - E_{p-p'} \right) \\
\times \left\{ 1, p^{\mu_\epsilon}, p^{\mu_\epsilon_\nu}, p^{\mu_\epsilon_\nu_\rho}, p^{\mu_\epsilon_\nu_\rho_\sigma}, p^{\mu_\epsilon_\nu_\rho_\sigma_\tau} \right\}.
\]
The tensors in (48) with the subscript ‘0’ are those in $\rho_{00}(0)$ and $\rho_{\lambda_1\lambda_2}(\omega^2)$, and the tensors in (49) with the subscript ‘F’ are those in $\rho_{\lambda_1\lambda_2}(F^2_\phi)$. The difference between Eq. (48) and Eq. (49) is in the powers of $E_{p'}$ and $E_{p-p'}$ in the denominators. Note that all above tensors with 2 or more indices are symmetric with respect to the interchange of any two indices.

Using Eqs. (17)-(49), the zeroth and second order terms of the spin density matrix in (44), (45) and (46) can be expressed in terms of moments of momenta
\[
\rho_{00}(0) = \frac{\Delta t}{16} g_0^2 m_\phi^2 \frac{1}{E_p} \left[ 1 - 4 \epsilon_\mu_0(0, p) \epsilon_\rho_0(0, p) \frac{I^{\alpha\rho}_0}{m_\phi^2 I_0} \right],
\]
From Eq. (38), the trace of the spin density matrix for the \( \phi \) meson reads

\[
\rho^{\phi}_{00}(\omega^2) = -\frac{\Delta t}{8} g^2_{\phi} \frac{1}{m_\phi^2} \epsilon_\alpha(0, \mathbf{p}) \epsilon_\beta(0, \mathbf{p}) \bar{\omega}_{\phi}(x) \bar{\omega}_{\gamma}(x) \\
\times \left[ \frac{p^\gamma m_\phi^2}{2} \left( g^{\alpha\rho} g^{\gamma\sigma} - g^{\alpha\sigma} g^{\gamma\rho} + g^{\beta\sigma} g^{\gamma\rho} \right) I_0^\xi + \frac{1}{2} \frac{p^\gamma}{I_0} \left( I_0^\xi - I_0^{\gamma\xi} \right) \right] \\
+ 2 \rho^\gamma \left( g^{\alpha\beta} I_0^{\xi\beta} - g^{\alpha\sigma} I_0^{\xi\sigma} + g^{\beta\sigma} I_0^\xi \right) \\
+ 2 \rho^\gamma \left( g^{\alpha\beta} I_F^{\xi\beta} - g^{\alpha\sigma} I_F^{\xi\sigma} + 2 g^{\beta\sigma} I_F^\xi \right) \\
- m_\phi^2 \left( g^{\alpha\beta} g^{\gamma\rho} - 2 g^{\alpha\beta} g^{\gamma\rho} + 2 g^{\alpha\beta} g^{\gamma\rho} \right) I_0^{\gamma\xi} \\
+ 2 \rho^\gamma \left( g^{\alpha\beta} I_F^{\xi\beta} + g^{\beta\sigma} I_F^{\xi\sigma} + 4 g^{\alpha\beta} I_F^\xi \right), \tag{51}
\]

\[
\rho^{\phi}_{00}(F^2_\phi) = \frac{\Delta t}{8} g^2_{\phi} \frac{1}{m_\phi^2} \epsilon_\alpha(0, \mathbf{p}) \epsilon_\beta(0, \mathbf{p}) \bar{F}_{\phi}(x) \bar{F}_{\phi}(x) \\
\times \left[ \frac{p^\gamma m_\phi^2}{2} \left( g^{\alpha\rho} g^{\gamma\sigma} - g^{\alpha\sigma} g^{\gamma\rho} + g^{\beta\sigma} g^{\gamma\rho} \right) I_F^\xi + \frac{1}{2} \frac{p^\gamma}{I_0} \left( I_F^\xi - I_0^{\gamma\xi} \right) \right] \\
+ 2 \rho^\gamma \left( g^{\alpha\beta} I_F^{\xi\beta} - g^{\alpha\sigma} I_F^{\xi\sigma} + 2 g^{\beta\sigma} I_F^\xi \right) \\
+ 2 \rho^\gamma \left( g^{\alpha\beta} I_F^{\xi\beta} + 2 g^{\beta\sigma} I_F^{\xi\sigma} + 4 g^{\alpha\beta} I_F^\xi \right), \tag{52}
\]

From Eq. (38), the trace of the spin density matrix for the \( \phi \) meson reads

\[
\text{Tr}(\rho_{\phi}) = \frac{\Delta t}{8} g^2_{\phi} (m^2_\phi + 2 m^2_\omega) \frac{1}{E_p} I_0 \\
\times \left[ 1 - \frac{\bar{\omega}_{\phi}(x) \bar{\omega}_{\gamma}(x) g^{\alpha\rho}}{I_0} \left( p^\gamma I_0^\xi - I_0^{\gamma\xi} \right) \right] \tag{53}
\]

Here we have neglected mixing terms of \( \omega_{\mu\nu} \) and \( F^\phi_{\mu\nu} \), since we assume that there is no correlation in space-time between them.

From Eqs. (50) - (53) we obtain the 00-component of the normalized spin density matrix for the \( \phi \) meson defined in (38)

\[
\mathcal{P}_{00}^{\phi}(x, \mathbf{p}) = c_0(\mathbf{p}) + c_\omega(x, \mathbf{p}) + c_F(x, \mathbf{p}), \tag{54}
\]

where \( c_0 \), \( c_\omega \) and \( c_F \) are given by

\[
c_0(\mathbf{p}) = \frac{m^2_\phi}{2 (m^2_\phi + 2 m^2_\omega)} \left[ 1 - 4 \epsilon_\alpha(0, \mathbf{p}) \epsilon_\beta(0, \mathbf{p}) \frac{I_0^{\alpha\beta}}{m^2_\phi I_0} \right], \tag{55}
\]
Table II. All nonvanishing moments of momenta normalized by $I_0$ are evaluated in the rest frame of the vector meson. Note that $I$ represents either $I_0$ or $I_F$. The definition for some quantities are $I_{aa} \equiv I_{11}^2 + I_{22}^2 + I_{33}^2$, $I_{aaba} \equiv I_{11}^2 + I_{22}^2 + I_{33}^2$, $I_{aaab} \equiv I_{11}^2 + I_{22}^2 + I_{33}^2$. The constant $d_0$ is defined as $d_0 \equiv 1 - 4m_\phi^2/m_\omega^2$.

$$c_\omega(x, p) = -\frac{1}{8m_\phi^2(m_\phi^2 + 2m_x^2)} \epsilon_\alpha(0, p)\epsilon_\beta(0, p)\phi(x) \omega_\gamma(x)$$

$$c_F(x, p) = \frac{1}{8m_\phi^2(m_\phi^2 + 2m_x^2)} \frac{g_\phi^2}{T_{\text{eff}}^\phi} \epsilon_\alpha(0, p)\epsilon_\beta(0, p)\phi(x) \omega_\gamma(x)$$

We see in Eqs. (55)-(57) that the momentum moments always come with the factor $1/I_0$, so they can be understood as normalized moments by $I_0$, a kind of momentum averages.

We see in Eqs. (55)-(57) that $c_0$, $c_\omega$ and $c_F$ are all Lorentz scalars, so it is convenient to evaluate them in the rest frame of the vector meson. All nonvanishing moments of momenta in Eqs. (55)-(57) that are evaluated in the rest frame of the vector meson are listed in Table III. Finally the result for $\phi_{00}$ is

$$\phi_{00}(x, p) \approx \frac{1}{3} + C_1 \left[ \frac{1}{3} \left( \omega \cdot \omega - \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^\phi} \mathbf{B}' \cdot \mathbf{B}' \right) - \left( \epsilon_0 \cdot \omega \right)^2 + \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^\phi} \left( \epsilon_0 \cdot \mathbf{B}' \right)^2 \right]$$

$$+ C_2 \left[ \frac{1}{3} \left( \epsilon' \cdot \epsilon' - \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^\phi} \mathbf{E}' \cdot \mathbf{E}' \right) - \left( \epsilon_0 \cdot \epsilon' \right)^2 + \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^\phi} \left( \epsilon_0 \cdot \mathbf{E}' \right)^2 \right].$$
where the fields with primes are in the rest frame of the vector meson, $\varepsilon$ and $\omega$ denote the electric and magnetic part of the vorticity tensor $\omega^{\mu\nu}$ respectively, $E_{\phi}$ and $B_{\phi}$ denote the electric and magnetic part of the $\phi$ field tensor $F_{\mu\nu}^{\phi}$ respectively, and $C_1$ and $C_2$ are two coefficients depending on masses of the quark and vector meson defined as

$$C_1 = \frac{8m_s^4 + 16m_s^2m_{\phi}^2 + 3m_{\phi}^4}{120m_s^2(m_{\phi}^2 + 2m_s^2)},$$

$$C_2 = \frac{8m_s^4 - 14m_s^2m_{\phi}^2 + 3m_{\phi}^4}{120m_s^2(m_{\phi}^2 + 2m_s^2)}. \quad (59)$$

The result for $\bar{\rho}_{\mu\nu}(x, p)$ in Eq. (58) is rigorous and remarkable since all contributions are in squares of the fields. This is a clear piece of evidence that there exists in the $\phi$ meson an exact correlation between the strong force field coupled to the $s$ quark and that coupled to the $\bar{s}$ quark. This feature makes $\rho_{00}$ for quarkonium vector mesons very different from that for other vector mesons carrying net charges or flavors.

One can approximate $\bar{\rho}_{00}$ by expanding $C_1$ and $C_2$ in terms of the average quark momentum inside the vector meson as

$$C_1 \approx \frac{1}{6} + \frac{1}{9}d_0 + O(d_0^2),$$

$$C_2 \approx \frac{1}{18}d_0 + O(d_0^2), \quad (60)$$

with $d_0 \equiv 1 - 4m_s^2/m_{\phi}^2$, the result is

$$\bar{\rho}_{00}(x, p) \approx \frac{1}{3} + \left( \frac{1}{6} + \frac{1}{9}d_0 \right) \left\{ \frac{1}{3} \left( \omega' \cdot \omega - \frac{4g_{\phi}^2}{m_{\phi}^2T_{\text{eff}}^2} B'_{\phi} \cdot B'_{\phi} \right) \right. $$

$$- (\varepsilon_0 \cdot \omega')^2 + \frac{4g_{\phi}^2}{m_{\phi}^2T_{\text{eff}}^2} (\varepsilon_0 \cdot B'_{\phi})^2 \right\}$$

$$+ \frac{1}{18}d_0 \left\{ \frac{1}{3} \left( \varepsilon' \cdot \varepsilon' - \frac{4g_{\phi}^2}{m_{\phi}^2T_{\text{eff}}^2} E'_{\phi} \cdot E'_{\phi} \right) \right.$$

$$- (\varepsilon_0 \cdot \varepsilon')^2 + \frac{4g_{\phi}^2}{m_{\phi}^2T_{\text{eff}}^2} (\varepsilon_0 \cdot E'_{\phi})^2 \left\} + O(d_0^2). \quad (61)$$

The above result can be compared with that in the nonrelativistic limit (see Appendix C). In order to recover the momentum dependence, one can express $\bar{\rho}_{00}$ in terms of lab-frame fields. The transformation of the fields between the lab and rest frame reads

$$B'_{\phi} = \gamma B_{\phi} - \gamma \varepsilon \times E_{\phi} + (1 - \gamma) \frac{v \cdot B_{\phi}}{v^2} v,$$

$$E'_{\phi} = \gamma E_{\phi} + \gamma \varepsilon \times B_{\phi} + (1 - \gamma) \frac{v \cdot E_{\phi}}{v^2} v,$$

$$\omega' = \gamma \omega - \gamma \varepsilon \times \varepsilon + (1 - \gamma) \frac{v \cdot \omega}{v^2} v,$$

$$\varepsilon' = \gamma \varepsilon + \gamma \varepsilon \times \omega + (1 - \gamma) \frac{v \cdot \varepsilon}{v^2} v, \quad (62)$$

where $\gamma = E_{\phi}^p/m_{\phi}$ is the Lorentz factor and $v = p/E_{\phi}^p$ is the velocity of the $\phi$ meson. Taking the $y$-direction as the spin quantization direction, $\varepsilon_0 = (0, 1, 0)$, we obtain $\bar{\rho}_{00}$ in terms of the fields in the
where the coefficients are given by

\[ I_{B,x}(p) = C_1 \left[ (E_p^\phi)^2 - \left( 1 + \frac{3p_y^2}{(m_\phi + E_p^\phi)^2} \right) p_x^2 \right] + C_2 (p_y^2 - 2p_z^2), \]

\[ I_{E,x}(p) = C_1 (p_y^2 - 2p_z^2) + C_2 \left[ (E_p^\phi)^2 - \left( 1 + \frac{3p_y^2}{(m_\phi + E_p^\phi)^2} \right) p_x^2 \right], \]

\[ I_{B,y}(p) = C_1 \left[ 6 \frac{E_p^\phi}{m_\phi + E_p^\phi} p_y^2 - 2(E_p^\phi)^2 - p_y^2 - \frac{3p_y^4}{(m_\phi + E_p^\phi)^2} \right] + C_2 (p_y^2 + p_z^2), \]

\[ I_{E,y}(p) = C_1 (p_y^2 + p_z^2) + C_2 \left[ 6 \frac{E_p^\phi}{m_\phi + E_p^\phi} p_y^2 - 2(E_p^\phi)^2 - p_y^2 - \frac{3p_y^4}{(m_\phi + E_p^\phi)^2} \right], \]

\[ I_{B,z}(p) = C_1 \left[ (E_p^\phi)^2 - \left( 1 + \frac{3p_y^2}{(m_\phi + E_p^\phi)^2} \right) p_z^2 \right] + C_2 (p_y^2 - 2p_z^2), \]

\[ I_{E,z}(p) = C_1 (p_y^2 - 2p_z^2) + C_2 \left[ (E_p^\phi)^2 - \left( 1 + \frac{3p_y^2}{(m_\phi + E_p^\phi)^2} \right) p_z^2 \right]. \]

(63)

The result in Eq. (63) is remarkable in its factorization form: the momentum functions are separated from space-time functions. This has an advantage that the momentum functions can be determined by experimental data on momentum spectra while unknown space-time functions can be extracted from data on \( \bar{\rho}^\phi_{00} \).

One can take an average of \( \bar{\rho}^\phi_{00}(x, p) \) over the local space-time volume in which the vector meson is formed as

\[
\langle \bar{\rho}^\phi_{00}(x, p) \rangle_x \approx \frac{1}{3} + \frac{1}{3} \sum_{i=1,2,3} I_{B,i}(p) \frac{1}{m_\phi} \left[ \omega_i^2 - \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2} \langle B_i^\phi \rangle^2 \right] + \frac{1}{3} \sum_{i=1,2,3} I_{E,i}(p) \frac{1}{m_\phi} \left[ \varepsilon_i^2 - \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2} \langle E_i^\phi \rangle^2 \right].
\]

(65)

These averaged field squares can play as parameters and be determined by comparing \( \langle \bar{\rho}^\phi_{00}(x, p) \rangle_x \) with the data of \( \rho^\phi_{00} \) as functions of transverse momenta. One can further take a momentum average of \( \langle \bar{\rho}^\phi_{00}(x, p) \rangle_x \) and compare with the data as functions of collision energies,

\[
\langle \bar{\rho}^\phi_{00}(x, p) \rangle_{x,p} \approx \frac{1}{3} + \frac{1}{3} \sum_{i=1,2,3} \langle I_{B,i}(p) \rangle \frac{1}{m_\phi} \left[ \omega_i^2 - \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2} \langle B_i^\phi \rangle^2 \right] + \frac{1}{3} \sum_{i=1,2,3} \langle I_{E,i}(p) \rangle \frac{1}{m_\phi} \left[ \varepsilon_i^2 - \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2} \langle E_i^\phi \rangle^2 \right],
\]

(66)

where the momentum average is defined as

\[
\langle O(p) \rangle = \frac{\int d^3p O(p) f_\phi(p)}{\int d^3p f_\phi(p)},
\]

(67)
VI. DISCUSSIONS AND CONCLUSIONS

In this section we will discuss about the main results as well as approximations or assumptions that have been made in this paper.

The Lagrangian (1) is for real vector fields since we are concerned about the charge or flavor neutral particles such as quarkonia made of a quark and its antiquark. To describe those particles that carry net charge or flavor, we have to consider complex vector fields. The generalization of the formalism to complex vector fields is straightforward.

The vector fields that polarize $s$ and $\overline{s}$ are assumed to be the $\phi$ fields, the effective (color singlet) modes of the strong force that carry vacuum quantum number. As an input to the general formula (35) we assume that $P^\mu_\sigma$ and $P^\mu_\pi$ have the linear form in Eq. (39) in the vorticity and $\phi$ fields. The coupling between the spin and fluid velocity field is assumed to be through the vorticity. One can also introduce other coupling forms such as spin-shear couplings [63–66]. The spin coupling to the $\phi$ field is assumed to have a covariant form $\sim e^{\mu\nu\alpha\beta} F^\alpha_{\gamma\delta} P^\nu_{\gamma\sigma}$. This is one of our main assumptions. Of course one can use other forms of spin-field couplings or add more terms to Eq. (39). An alternative choice is to use the coupling of the spin and gluon field as in the nonrelativistic quantum chromodynamics (NRQCD) [67, 68]. But the Hamiltonian of NRQCD is not covariant at all and may be different from the covariant formalism to complex vector fields.

In evaluating the integrals in momentum moments in the rest frame of the vector meson, we assume a simple form for the fluid four-velocity $u^\mu = (1, 0)$ so that the quark and antiquark distributions depend only on energies. Then we obtain the simple form of $\rho^\phi_{00}(x, p)$ in Eq. (58) with $C_1$ and $C_2$ depending only on masses as shown in (60). In general the fluid four-velocity has also a spatial component or three-velocity, in this case $\rho^\phi_{00}(x, p)$ should have much more complicated form than Eq. (58) where the coefficients also depend on the three-velocity of the fluid in a more sophisticated way.

According to the chiral quark model in Ref. [42] the local averaged field squares $\langle (B^\rho)^2 \rangle$ and $\langle (E^\rho)^2 \rangle$ are related to the fields of pseudo-Goldstone bosons. They are also related to gluon fluctuation of instantons [69, 70] according to the quark model based on instanton vacuum [71]. If quarks and antiquarks are polarized by gluon fields, the local averaged field squares are related to the gluon condensate which contributes to the trace anomaly of the energy momentum tensor. Therefore the local averaged field squares are in connection with fundamental properties of the QCD vacuum which play an important role in hadron structures [72, 73].

In summary, a relativistic theory for the spin density matrix of vector mesons is constructed based on Kadanoff-Baym (KB) equations from which the spin Boltzmann equations are derived. With the spin Boltzmann equations we formulate the spin density matrix element $\rho_{00}$ for $\phi$ mesons. The dominant contributions to $\rho^\phi_{00}$ at lower energies are assumed to come from the $\phi$ field, a kind of the strong force field that can polarize the strange quark and antiquark in the same way as the electromagnetic field. The key observation is that there is correlation inside the $\phi$ meson wave function between the $\phi$ field
that polarizes the strange quark and that polarizes the strange antiquark. This is reflected by the
fact that the contributions to $\rho_{00}$ are all in squares of the fields which are nonvanishing even if the
fields may strongly fluctuate. Then the fluctuations of strong force fields can be extracted from $\rho_{00}$ of
quarkonium vector mesons as links to fundamental properties of QCD.

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Appendix A: Collision terms for coalescence and dissoication

In this appendix, we will derive the collision term for the coalescence process of the quark and
antiquark into the vector meson corresponding to $I_{-+}$ in Eq. (26).

The explicit form of $I_{-+}$ is

$$I_{-+} = \sum_{\{\epsilon^\alpha (\lambda_1', p) \epsilon^{\nu*} (\lambda_2', p) \times} \delta \left( E_0 - E_{p'} - E_{p' - p'} \right) \delta (p^0 - E_p')$$

$$\times \left\{ \left[ f_{r_{1} s_{1}}^{(-)} (x, p') \right] \left[ f_{r_{2} s_{2}}^{(+)} (x, p + p') \right] f_{\lambda_1' \lambda_2'} (x, p) \right\}. \tag{A1}$$

The corresponding collision term reads

$$C^{\mu\nu}_{\text{coalescence}} = \frac{1}{4(2\pi)^3} \sum_{r_{1},s_{1},r_{2},s_{2},\lambda_{1}',\lambda_{2}'} \int d^{3}p' \frac{1}{E_p' E_{p' + p} V} \delta \left( E_0 - E_{p'} - E_{p' - p'} \right) \delta (p^0 - E_p')$$

$$\times \left\{ \left[ f_{r_{1} s_{1}}^{(-)} (x, p') \right] \left[ f_{r_{2} s_{2}}^{(+)} (x, p + p') \right] f_{\lambda_1' \lambda_2'} (x, p) \right\}. \tag{A2}$$

where we have changed the sign of the antiquark’s three-momentum as $p' \to -p'$ in the integral, used
Eq. (7) and the relation

$$\delta (p^2 - m^2_q) \delta \left( p + p' \right) \delta \left( p^2 - m^2_V \right) \theta (-p_0') \theta (p_0 + p_0') \theta (p_0)$$

$$= \frac{1}{8E_p' E_{p' + p} V} \delta (p_0 + E_{p'}) \delta \left( E_0 + E_{p' + p} \right) \delta (p_0 - E_p')$$

$$= \frac{1}{8E_p' E_{p' + p} V} \delta (p_0 + E_{p'}) \delta \left( E_0 - E_{p'} - E_{p' + p} \right) \delta (p_0 - E_p'). \tag{A3}$$
From [11], the particle sector of \( p \cdot \partial_x G^{<,\mu
u}(x,p) \) in the left-hand-side of Eq. (26) becomes

\[
p \cdot \partial_x G^{<,\mu
u}(x,p) = 2\pi \hbar \frac{1}{2E_p^\mu} \delta(p_0 - E_p^\nu)
\times \sum_{\lambda_1',\lambda_2'} \epsilon^{\mu*}(\lambda_1',p) \epsilon^{\nu*}(\lambda_2',p) p \cdot \partial_x f_{\lambda_1',\lambda_2'}(x,p).
\] (A4)

Using Eqs. (A2) and (A4) into Eq. (26), taking a contraction of the resulting equation with \( \epsilon^{\mu*}(\lambda_1, p) \) and \( \epsilon^{\nu*}(\lambda_2, p) \), and using the first identity in (7), we obtain

\[
p \cdot \partial_x f_{\lambda_1,\lambda_2}(x,p)
= \frac{1}{16} \sum_{r_1,s_1,r_2,s_2,\lambda_1',\lambda_2'} \int \frac{d^3p'}{(2\pi\hbar)^3} \frac{1}{E_{p'}^\mu E_{p'}^\nu} \frac{1}{2\pi\hbar} \delta \left( E_p^\mu - E_{p'}^\mu - E_{p'}^\nu - E_{p'}^\nu \right)
\times \left\{ \delta_{\lambda_1,\lambda_1'} \epsilon^{\mu*}(\lambda_1, p) \epsilon^{\nu*}(\lambda_1', p) \text{Tr} \left[ \Gamma_{\alpha\nu}(s_1, p') \pi(r_1, p') \Gamma^\mu u(r_2, p - p') \pi(s_2, p - p') \right] + \delta_{\lambda_1,\lambda_2} \epsilon^{\nu*}(\lambda_2, p) \epsilon^{\mu*}(\lambda_2', p) \text{Tr} \left[ \Gamma^\nu u(s_1, p') \pi(r_1, p') \Gamma^\mu u(r_2, p - p') \pi(s_2, p - p') \right] \right\}
\times \left\{ f_{r_1,s_1}^{(+)}(x, p, p') f_{r_2,s_2}^{(+)}(x, p - p') f_{\lambda_1',\lambda_2'}(x,p) \right\}
\times \left\{ f_{r_1,s_1}^{(-)}(x, p, p') f_{r_2,s_2}^{(-)}(x, p - p') f_{\lambda_1,\lambda_2}(x,p) \right\}.
\] (A5)

which reproduces Eq. (27). Note that the terms proportional to \( p^\mu \) and \( p^\nu \) in the left-hand-side of Eq. (26) do not contribute since their contraction with \( \epsilon^{\mu*}(\lambda_1, p) \) and \( \epsilon^{\nu*}(\lambda_2, p) \) is vanishing.

We consider the coalescence process in heavy ion collisions in which the MVSDs of quarks, antiquarks and vector mesons are assumed to be much smaller than unity. So the term with \( \delta_{\lambda_1,\lambda_2} \) dominates the gain term which can be simplified as

\[
gain \approx \frac{1}{8(2\pi\hbar)^2} \sum_{r_1,s_1,r_2,s_2} \int \frac{d^3p'}{(2\pi\hbar)^3} \frac{1}{E_{p'}^\mu E_{p'}^\nu} \delta \left( E_p^\mu - E_{p'}^\mu - E_{p'}^\nu - E_{p'}^\nu \right)
\times \epsilon^{\mu*}(\lambda_1, p) \epsilon^{\nu*}(\lambda_2, p) \text{Tr} \left[ \Gamma^\nu u(s_1, p') \pi(r_1, p') \Gamma^\mu u(r_2, p - p') \pi(s_2, p - p') \right]
\times f_{r_1,s_1}^{(-)}(x, p, p') f_{r_2,s_2}^{(+)}(x, p - p') f_{\lambda_1,\lambda_2}(x,p).
\] (A6)
which gives Eq. (31). The loss term can be simplified as

\[
\text{loss} \approx -\frac{1}{16(2\pi\hbar)^2} \sum_{r_1,s_1,r_2,s_2} \int d^3p' \frac{1}{E_{p'}} \frac{1}{E_{p'-p'}} \delta \left( E_{p'} - E_{p'}^\pi - E_{p-p'}^q \right) 
\times \left\{ \delta_{\lambda_1'\lambda_2'} (\lambda_1, p) \epsilon^\alpha (\lambda_1', p) \epsilon^\alpha (\lambda_1', p) \text{Tr} [\Gamma_\alpha (p' \cdot \gamma - m_\pi) \Gamma^\mu ((p - p') \cdot \gamma + m_q)] 
\right.
\]

\[
+ \sum_{\lambda_2'} f_{\lambda_1'\lambda_2'} (x, p) \epsilon^\alpha (\lambda_1', p) \epsilon^\alpha (\lambda_1', p) \text{Tr} [\Gamma^\mu ((p - p') \cdot \gamma + m_q)] \right\}
\]

\[
= -\frac{1}{16(2\pi\hbar)^2} \int d^3p' \frac{1}{E_{p'}} \frac{1}{E_{p'-p'}} \delta \left( E_{p'} - E_{p'}^\pi - E_{p-p'}^q \right) 
\times \left\{ \sum_{\lambda_1'} f_{\lambda_1'\lambda_2'} (x, p) \epsilon^\alpha (\lambda_1', p) \epsilon^\alpha (\lambda_1', p) \text{Tr} [\Gamma_\alpha (p' \cdot \gamma - m_\pi) \Gamma^\mu ((p - p') \cdot \gamma + m_q)] 
\right.
\]

\[
+ \sum_{\lambda_2'} f_{\lambda_1'\lambda_2'} (x, p) \epsilon^\alpha (\lambda_1', p) \epsilon^\alpha (\lambda_1', p) \text{Tr} [\Gamma^\mu ((p - p') \cdot \gamma + m_q)] \right\}
\]

\[
= -\frac{1}{16(2\pi\hbar)^2} \int d^3p' \frac{1}{E_{p'}} \frac{1}{E_{p'-p'}} \delta \left( E_{p'} - E_{p'}^\pi - E_{p-p'}^q \right) 
\times \left\{ \sum_{\lambda_1'} f_{\lambda_1'\lambda_2'} (x, p) \epsilon^\alpha (\lambda_1', p) \epsilon^\alpha (\lambda_1', p) + \sum_{\lambda_2'} f_{\lambda_1'\lambda_2'} (x, p) \epsilon^\alpha (\lambda_1', p) \epsilon^\alpha (\lambda_1', p) \right\}
\times \text{Tr} \{\Gamma^\alpha (p' \cdot \gamma - m_\pi) \Gamma^\mu ((p - p') \cdot \gamma + m_q) \},
\]  

(A7)

where we have neglected $f^{(s)}_{r_1s_1}$ and $f^{(s)}_{r_2s_2}$ relative to $\delta_{r_1s_1}$ and $\delta_{r_2s_2}$ respectively. After completing the integral over $p'$ in the vector meson’s rest frame, one can prove

\[
\epsilon^\alpha (\lambda_1', p) \epsilon^\alpha (\lambda_1', p) \int d^3p' \frac{1}{4E_{p'} E_{p'-p'}} \delta \left( E_{p'} - E_{p'}^\pi - E_{p-p'}^q \right) 
\times \text{Tr} \{\Gamma^\alpha (p' \cdot \gamma - m_\pi) \Gamma^\mu ((p - p') \cdot \gamma + m_q) \} \propto \delta_{\lambda_1'\lambda_1'},
\]  

(A8)

so we can replace

\[
\sum_{\lambda_1'} f_{\lambda_1'\lambda_2'} (x, p) \epsilon^\alpha (\lambda_1', p) \epsilon^\alpha (\lambda_1', p) + \sum_{\lambda_2'} f_{\lambda_1'\lambda_2'} (x, p) \epsilon^\alpha (\lambda_1', p) \epsilon^\alpha (\lambda_1', p)
\]

\[
\rightarrow \frac{2}{3} f_{\lambda_1'\lambda_2'} (x, p) \left( g_{\mu\alpha} - \frac{p_\mu p_\alpha}{m_V^2} \right),
\]  

(A9)

then the loss term becomes

\[
\text{loss} \approx \frac{1}{12(2\pi\hbar)^2} f_{\lambda_1'\lambda_2'} (x, p) \left( g_{\mu\alpha} - \frac{p_\mu p_\alpha}{m_V^2} \right) \int d^3p' \frac{1}{E_{p'}^\pi E_{p'-p'}} \delta \left( E_{p'} - E_{p'}^\pi - E_{p-p'}^q \right) 
\times \text{Tr} \{\Gamma^\alpha (p' \cdot \gamma - m_\pi) \Gamma^\mu ((p - p') \cdot \gamma + m_q) \}.
\]  

(A10)

This gives Eq. (32).

**Appendix B: Collision kernel**

The spin density matrix element for vector mesons is given by Eq. (35). In this appendix we evaluate the collision kernel in Eq. (35)

\[
I_{\lambda_1,\lambda_2} (p, p') = I^{\alpha\beta} (p, p') \epsilon^\alpha (\lambda_1, p) \epsilon^\beta (\lambda_2, p),
\]  

(B1)
where $I^{\alpha\beta}(p, p')$ is defined as
\[
I^{\alpha\beta}(p, p') \equiv \text{Tr} \left[ \Gamma^\beta v(r_1, p') \pi(r_1, p') \Gamma^\alpha u(r_2, p - p') \pi(s_2, p - p') \right] 
\times f_{r_1s_1}^{(q)}(x, p') f_{r_2s_2}^{(q)}(x, p - p'). \tag{B2}
\]
Now we use following formula to simplify $I^{\alpha\beta}$. For quark spinors of particles and antiparticles we have
\[
u(r, p)\pi(s, p) = \frac{1}{2} (m_\gamma + \gamma^\mu p_\mu) \delta_{rs} + \frac{1}{2} m_\gamma \gamma^\mu n_\mu (n_{sr}, p, m_q) - \frac{1}{4} \epsilon_{\mu\nu\alpha\beta} \sigma^{\mu\nu} p^\alpha n^\beta (n_{sr}, p, m_q),
\]
\[
u(r, p)\pi(s, p) = \frac{1}{2} (m_\gamma + \gamma^\mu p_\mu) \delta_{rs} - \frac{1}{2} m_\gamma \gamma^\mu n^\mu (n^*_{sr}, p, m_\gamma) - \frac{1}{4} \epsilon_{\mu\nu\alpha\beta} \sigma^{\mu\nu} p_\alpha n_\beta (n^*_{sr}, p, m_\gamma),
\]
\[
u(r, p)\pi(s, p) = -m_\gamma \gamma^\mu n_\mu (n_{jr}, p, m_\gamma) - \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} \sigma^{\mu\nu} p_\alpha n_\beta (n_{jr}, p, m_\gamma), \tag{B3}
\]
where we have used $n^*_{sr} = n_{rs} = n_{(ri)_s}$. Inserting Eq. (B2) into Eq. (B2) gives
\[
I^{\alpha\beta}(p, p') = \text{Tr} \left[ \Gamma^\beta v(s_1, p') \pi(r_1, p') \Gamma^\alpha u(r_2, p - p') \pi(s_2, p - p') \right] 
\times \frac{1}{2} f_\pi(x, p') \left[ \delta_{r_1s_1} - P_\pi^7(x, p') n_{j}^{(-\mu)} (-p') \tau_{r_1s_1}^{j} \right] 
\times \frac{1}{2} f_\pi(x, p - p') \left[ \delta_{r_2s_2} - P_\pi^7(x, p - p') n_{j}^{(+\mu)} (p - p') \tau_{r_2s_2}^{j} \right] 
= \frac{1}{4} f_\pi(x, p') f_\pi(x, p - p') \times \text{Tr} \left[ \Gamma^\beta (p' \cdot \gamma - m_\gamma) \left[ 1 + \gamma_5 \gamma \cdot P_\pi^7(x, p') \right] \Gamma^\alpha \times \left[ (p - p') \cdot \gamma + m_\gamma \right] \left[ 1 + \gamma_5 \gamma \cdot P_\pi^7(x, p - p') \right] \right], \tag{B4}
\]
where we have used
\[
v(s_1, p')\pi(r_1, p') \left[ \delta_{r_1s_1} - P_\pi^7(x, p') n_{j}^{(-\mu)} (-p') \tau_{r_1s_1}^{j} \right] 
= (p' \cdot \gamma - m_\gamma) \left[ 1 + \gamma_5 \gamma \cdot P_\pi^7(x, p') \right], \tag{B5}
\]
and
\[
u(r_2, p - p')\pi(s_2, p - p') \times \left[ \delta_{r_2s_2} - P_\pi^7(x, p - p') n_{j}^{(+\mu)} (p - p') \tau_{r_2s_2}^{j} \right] 
= \left[ (p - p') \cdot \gamma + m_\gamma \right] \left[ 1 + \gamma_5 \gamma \cdot P_\pi^7(x, p - p') \right]. \tag{B6}
\]
In deriving (B5) and (B6) we have used
\[
\frac{1}{2} \epsilon^{\alpha\beta\mu\nu} \sigma_{\mu\nu} = \gamma_5 \gamma^\alpha \gamma^\beta - g^\alpha\beta \gamma_5,
\]
\[
n_{j}^{(-\mu)} (-p') = n_{j}^{(\mu)} (n_{jr}, p', m_\gamma),
\]
\[
p^\mu P_\pi^{7}(x, p') = (p - p')^\mu P_\pi^{7}(x, p - p') = 0. \tag{B7}
\]
Inserting (B4) into (B1) we obtain
\[
I_{\lambda_1\lambda_2} = \epsilon_\alpha(\lambda_1, p) \epsilon_\beta(\lambda_2, p) \times \text{Tr} \left[ \Gamma^\beta (p' \cdot \gamma - m_\gamma) \left[ 1 + \gamma_5 \gamma \cdot P_\pi^7(x, p') \right] \Gamma^\alpha \times \left[ (p - p') \cdot \gamma + m_\gamma \right] \left[ 1 + \gamma_5 \gamma \cdot P_\pi^7(x, p - p') \right] \right]. \tag{B8}
\]
From Eq. \(B8\) one arrives at Eq. 35. Using \(28\), the trace in \(B8\) can be worked out and the result of \(I_{\lambda_1\lambda_2}\) is

\[
I_{\lambda_1\lambda_2} = -4g_\lambda^2 B^2 (p - p', p') \epsilon_\alpha(\lambda_1) \epsilon_\beta(\lambda_2) \times \left\{ \left( p'^\alpha P_{\eta}^\beta + p'^\beta P_{\eta}^\alpha \right) (p' - p_q) - \left( p'^\alpha P_{\eta}^\beta + p'^\beta P_{\eta}^\alpha \right) (p \cdot P_{\eta}) \right. \\
+ 2p'^\alpha p'^\beta (1 - P_{\eta} \cdot P_q) + g^{\alpha}\beta \left[ p' \cdot p + (p' \cdot P_q)(p \cdot P_{\eta}) \right] \\
+ \left[ (m_q - m_\eta)m_q + p \cdot p' \right] \left( P_{\eta}^\alpha P_{\eta}^\beta + P_{\eta}^\beta P_{\eta}^\alpha - g^{\alpha}\beta P_{\eta} \cdot P_q \right) \\
- \left( m_q - m_\eta \right)m_\eta g^{\alpha}\beta - i \left( m_q - m_\eta \right) \epsilon^{\alpha\beta\mu\nu} P_{\mu}^\alpha (P_{\nu}^\beta + P_{\eta}^\beta) \\
- \left. 2m_\eta^2 + 6m_qm_\eta - m_q^2 - m_\eta^2 - \frac{1}{m_\eta^2} \left( m_\eta^2 - m_q^2 \right)^2 \right\}. \tag{B9}
\]

where we have used shorthand notations \(\epsilon(\lambda) \equiv \epsilon(\lambda, p), P_q \equiv P_q(x, p - p'), \) and \(P_\eta \equiv P_\eta(x, p').\) We can take the sum of \(I_{\lambda\lambda}\) over \(\lambda\) as

\[
\sum_\lambda I_{\lambda\lambda} = 2g_\lambda^2 B^2 (p - p', p') \\
\times \left\{ \left[ -(m_q + m_\eta)^2 + \frac{1}{m_\eta^2} \left( m_\eta^2 - m_q^2 \right)^2 \right] (P_{\eta} \cdot P_q) \\
+ \frac{2}{m_\eta^2} (m_q - m_\eta)^2 (p \cdot P_q)(p \cdot P_\eta) \\
+ 2m_\eta^2 + 6m_qm_\eta - m_q^2 - m_\eta^2 - \frac{1}{m_\eta^2} \left( m_\eta^2 - m_q^2 \right)^2 \right\}. \tag{B10}
\]

From Eq. \(B10\) one can obtain \(\text{Tr}(\rho_V)\) in Eq. \(36.\)

**Appendix C: Spin density matrix in nonrelativistic limit**

We consider \(m_q = m_\eta\) and assume \(m_\eta \approx 2m_q.\) In non-relativistic limit, we can approximate

\[
p^\mu \approx (m_\eta, 0), \\
p'^\mu \approx (m_\eta, 0) \approx (m_q, 0), \\
p'^\mu - p'^\mu \approx (m_\eta - m_q, 0) \approx (m_q, 0), \\
P_{\eta}^\mu(x, p') \approx (0, P_{\eta}(x, p')), \\
P_{\eta}^\mu(x, p - p') \approx (0, P_{\eta}(x, p - p')), \\
\epsilon^\mu(\lambda_1) \approx 0, \epsilon(\lambda_1), \\
\epsilon^\mu(\lambda_2) \approx 0, \epsilon(\lambda_2), \tag{C1}
\]

which leads to

\[
p' \cdot \epsilon^*(\lambda_1) \approx 0, \\
p' \cdot \epsilon(\lambda_2) \approx 0, \\
p' \cdot p \approx m_\eta m_q, \\
\epsilon^*(\lambda_1) \cdot \epsilon(\lambda_2) \approx -\epsilon^*(\lambda_1) \cdot \epsilon(\lambda_2). \tag{C2}
\]

In Eqs. \(C1\) and \(C2\) we have used the shorthand notation \(\epsilon(\lambda) \equiv \epsilon(\lambda, p).\) Using \(C1\) and \(C2,\)

\[
I_{\lambda_1\lambda_2} = 4g_\lambda^2 m_\eta m_q \left\{ \epsilon^*(\lambda_1) \cdot \epsilon(\lambda_2) (1 + P_{\eta} \cdot P_q) \\
- [P_{\eta} \cdot \epsilon^*(\lambda_1)][P_q \cdot \epsilon(\lambda_2)] - [P_q \cdot \epsilon^*(\lambda_1)][P_{\eta} \cdot \epsilon(\lambda_2)] \\
- i[\epsilon^*(\lambda_1) \times \epsilon(\lambda_2)] \cdot (P_q + P_{\eta}) \right\}. \tag{C3}
\]
One can verify \( I_{\lambda_3\lambda_1} = I_{\lambda_1\lambda_2} \).

From (35), the spin density matrix for the vector meson in the non-relativistic limit is given by

\[
\rho^V_{\lambda_1\lambda_2}(x, p) = \frac{\Delta t}{8} g^2_{V} m_V m_q \int \frac{d^3p'}{(2\pi\hbar)^3} \frac{1}{E^q_{p'} E^q_{p-p'} E^V_p} \\
\times f_\pi(x, p') f_q(x, p - p') 2\pi\hbar \delta \left( E^V_p - E^q_{p'} - E^q_{p-p'} \right) \\
\times \{ \epsilon^*(\lambda_1) \cdot \epsilon(\lambda_2) \} [1 + P_q(x, p - p') \cdot P_\pi(x, p')] \\
- [P_q(x, p - p') \cdot \epsilon(\lambda_2)] [P_\pi(x, p') \cdot \epsilon^*(\lambda_1)] \\
- [P_q(x, p - p') \cdot \epsilon^*(\lambda_1)] [P_\pi(x, p') \cdot \epsilon(\lambda_2)] \\
- i [\epsilon^*(\lambda_1) \times \epsilon(\lambda_2)] \cdot [P_q(x, p - p') + P_\pi(x, p')] \}.
\]  

(C4)

We can simplify the above formula by using the shorthand notation

\[
Dp' \equiv \frac{\Delta t}{8} g^2_{V} m_V m_q \int \frac{d^3p'}{(2\pi\hbar)^3} \frac{1}{E^q_{p'} E^q_{p-p'} E^V_p} \\
\times f_\pi(x, p') f_q(x, p - p') 2\pi\hbar \delta \left( E^V_p - E^q_{p'} - E^q_{p-p'} \right) .
\]  

(C5)

We can put \( \rho^V_{\lambda_1\lambda_2} \) into a matrix form

\[
\rho^V = \begin{pmatrix}
\rho_{11} & \rho_{10} & \rho_{1,-1} \\
\rho^*_{10} & \rho_{00} & \rho_{0,-1} \\
\rho^*_{1,-1} & \rho^*_{0,-1} & \rho_{-1,-1}
\end{pmatrix}.
\]  

(C6)

Note that \( \rho^V \) is a Hermitian matrix and we have suppressed the index 'V' in all elements.

For a given spin quantization direction \( n_3 \), we can construct \( \epsilon(\lambda) \) as follows

\[
\epsilon(0) = n_3, \\
\epsilon(1) = -\frac{1}{\sqrt{2}} (n_1 + i n_2), \\
\epsilon(-1) = \frac{1}{\sqrt{2}} (n_1 - i n_2),
\]  

(C7)

where \( n_1 \), \( n_2 \) and \( n_3 \) form orthogonal basis vectors in the rest frame of the vector meson. From (C4) we obtain

\[
\rho_{11} = \int Dp' \left( 1 + n_3 \cdot P_q \right) \left( 1 + n_3 \cdot P_\pi \right),
\]

\[
\rho_{10} = \frac{1}{\sqrt{2}} \int Dp' \left\{ (n_1 - i n_2) \cdot P_q \left( 1 + n_3 \cdot P_\pi \right) \\
+ (n_1 - i n_2) \cdot P_\pi \left( 1 + n_3 \cdot P_q \right) \right\},
\]

\[
\rho_{1,-1} = \int Dp' \left[ (n_1 - i n_2) \cdot P_q \right] \left[ (n_1 - i n_2) \cdot P_\pi \right],
\]

\[
\rho_{00} = \int Dp' \left[ 1 + P_q \cdot P_\pi - 2 (n_3 \cdot P_q)(n_3 \cdot P_\pi) \right],
\]

\[
\rho_{-1,0} = -\frac{1}{\sqrt{2}} \int Dp' \left\{ (n_1 + i n_2) \cdot P_q \left( 1 - n_3 \cdot P_\pi \right) \\
+ (n_1 + i n_2) \cdot P_\pi \left( 1 - n_3 \cdot P_q \right) \right\},
\]

\[
\rho_{-1,-1} = \int Dp' \left( 1 - n_3 \cdot P_q \right) \left( 1 - n_3 \cdot P_\pi \right),
\]  

(C8)
where we have used shorthand notations $P_q \equiv P_q(x, p - p')$ and $P_\tau \equiv P_\tau(x, p')$. The 00-element of the normalized density matrix is given by

$$\bar{\rho}_{00} = \frac{\rho_{00}}{\rho_{11} + \rho_{00} + \rho_{-1,-1}}$$

$$= \frac{\int Dp' \left[ 1 + P_q \cdot P_\tau - 2 (n_3 \cdot P_q) (n_3 \cdot P_\tau) \right]}{\int Dp' (3 + P_q \cdot P_\tau)} ,$$

(C9)

where $N \equiv \int Dp'$ is the normalization constant. If the magnitude of the polarization is much smaller than 1, we can make a Taylor expansion in it and obtain

$$\bar{\rho}_{00}(x, p) \approx \frac{1}{3} + \frac{2}{9N} \int Dp' \left[ (n_1 \cdot P_q) (n_1 \cdot P_\tau) + (n_2 \cdot P_q) (n_2 \cdot P_\tau) \right. \left. - 2 (n_3 \cdot P_q) (n_3 \cdot P_\tau) \right].$$

(C10)

If we assume $P_q$ and $P_\tau$ are only in the direction of $n_3$, i.e.

$$n_x \cdot P_q = n_y \cdot P_q = n_x \cdot P_\tau = n_y \cdot P_\tau = 0,$$

(C11)

we obtain

$$\bar{\rho}_{00}(x,p) \approx \frac{1}{3} - \frac{4}{9N} \int Dp' \left[ n_3 \cdot P_q(x,p - p') \right] \left[ n_3 \cdot P_\tau(x,p') \right].$$

(C12)

which recovers the similar form to the previous result but expressed in terms of the weighted integral $Dp'$ in (C5).

[1] S. Barnett, Rev. Mod. Phys. 7, 129 (1935).
[2] A. Einstein and W. de Haas, Deutsche Physikalische Gesellschaft, Verhandlungen 17, 152 (1915).
[3] R. Takahashi, M. Matsu, M. Ono, K. Harri, H. Chudo, S. Okayasu, J. Ieda, S. Takahashi, S. Maekawa, and E. Saitoh, Nat. Phys. 12, 52 (2016).
[4] Z.-T. Liang and X.-N. Wang, Phys. Rev. Lett. 94, 102301 (2005), [Erratum: Phys. Rev. Lett. 96,039901(2006)], nucl-th/0410079.
[5] Z.-T. Liang and X.-N. Wang, Phys. Lett. B629, 20 (2005), nucl-th/0411101.
[6] S. A. Voloshin (2004), nucl-th/0410089.
[7] B. Betz, M. Gyulassy, and G. Torrieri, Phys. Rev. C76, 044901 (2007), 0708.0035.
[8] F. Becattini, F. Piccinini, and J. Rizzo, Phys. Rev. C77, 024906 (2008), 0711.1253.
[9] J.-H. Gao, S.-W. Chen, W.-t. Deng, Z.-T. Liang, Q. Wang, and X.-N. Wang, Phys. Rev. C77, 044902 (2008), 0710.2943.
[10] Q. Wang, Nucl. Phys. A967, 225 (2017), 1704.04022.
[11] W. Florkowski, A. Kumar, and R. Ryblewski, Prog. Part. Nucl. Phys. 108, 103709 (2019), 1811.04409.
[12] F. Becattini and M. A. Lisa, Ann. Rev. Nucl. Part. Sci. 70, 395 (2020), 2003.03640.
[13] J.-H. Gao, Z.-T. Liang, Q. Wang, and X.-N. Wang, Lect. Notes Phys. 987, 195 (2021), 2009.04803.
[14] X.-G. Huang, J. Liao, Q. Wang, and X.-L. Xia (2020), 2010.08937.
[15] L. Adamczyk et al. (STAR), Nature 548, 62 (2017), 1701.06657.
[16] J. Adam et al. (STAR), Phys. Rev. C98, 014910 (2018), 1805.04400.
[17] Y.-G. Yang, R.-H. Fang, Q. Wang, and X.-N. Wang, Phys. Rev. C97, 034917 (2018), 1711.06008.
[18] A. H. Tang, B. Tu, and C. S. Zhou, Phys. Rev. C98, 044907 (2018), 1803.05777.
[19] K. J. Gonçalves and G. Torrieri, Phys. Rev. C105, 034913 (2022), 2104.12941.
[20] B. I. Abelev et al. (STAR), Phys. Rev. C77, 061902 (2008), 0710.2943.
[21] M. Abdallah et al. (STAR) (2022), 2204.02302.
[22] X.-L. Xia, H. Li, X.-G. Huang, and H. Zhong Huang, Phys. Lett. B 817, 136325 (2021), 2010.01474.
[23] J.-H. Gao, Phys. Rev. D 104, 076016 (2021), 2105.08293.
[24] B. Müller and D.-L. Yang, Phys. Rev. D 105, L011901 (2022), 2110.15630.
[25] X.-L. Sheng, L. Oliva, and Q. Wang, Phys. Rev. D 101, 096005 (2020), 1910.16364.
[26] X.-L. Sheng, Q. Wang, and X.-N. Wang, Phys. Rev. D 102, 056013 (2020), 2007.05106.
[27] L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, New York, 1962).

[28] P. C. Martin and J. S. Schwinger, Phys. Rev. **115**, 1342 (1959), [427(1959)].

[29] L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964), [Sov. Phys. JETP20,1018(1965)].

[30] K.-c. Chou, Z.-b. Su, B.-l. Hao, and L. Yu, Phys. Rept. **118**, 1 (1985).

[31] J.-P. Blaizot and E. Iancu, Phys. Rept. **359**, 355 (2002), hep-ph/0110113.

[32] J. Berges, AIP Conf. Proc. **739**, 3 (2004), hep-ph/0409233.

[33] W. Cassing, Eur. Phys. J. ST **168**, 3 (2009), 0808.0715.

[34] X.-L. Sheng, N. Weickgenannt, E. Speranza, D. H. Rischke, and Q. Wang, Phys. Rev. D **104**, 016029 (2021), 2103.10636.

[35] A. Proca, Journal de Physique et le Radium **7**, 347 (1936).

[36] C. Itzykson and J. B. Zuber, *Quantum Field Theory*, International Series In Pure and Applied Physics (McGraw-Hill, New York, 1980), ISBN 978-0-486-44568-7.

[37] H. T. Elze, M. Gyulassy, and D. Vasak, Phys. Lett. B **177**, 402 (1986).

[38] Q. Wang, K. Redlich, H. Stoecker, and W. Greiner, Phys. Rev. Lett. **88**, 132303 (2002), nucl-th/0111040.

[39] K. Hattori, Y. Hidaka, N. Yamamoto, and D.-L. Yang, JHEP **02**, 001 (2021), 2010.13368.

[40] N. Weickgenannt, D. Wagner, and E. Speranza (2022), 2204.01797.

[41] A. Manohar and H. Georgi, Nucl. Phys. B **234**, 189 (1984).

[42] F. Fernandez, A. Valcarce, U. Straub, and A. Faessler, J. Phys. G **19**, 2013 (1993).

[43] Z.-p. Li, J.-x. Ye, and M.-h. Lu, Phys. Rev. C **56**, 1099 (1997), nucl-th/9706010.

[44] Q. Zhao, Z.-p. Li, and C. Bennhold, Phys. Rev. C **58**, 2393 (1998), nucl-th/9806100.

[45] A. Zacchi, R. Stiele, and J. Schaffner-Bielich, Phys. Rev. D **92**, 045022 (2015), 1506.01868.

[46] A. Zacchi, L. Tolos, and J. Schaffner-Bielich, Phys. Rev. D **95**, 103008 (2017), 1612.06167.

[47] V. Greco, C. M. Ko, and P. Levai, Phys. Rev. Lett. **90**, 202302 (2003), nucl-th/0301093.

[48] V. Greco, C. M. Ko, and P. Levai, Phys. Rev. C **68**, 034904 (2003), nucl-th/0305024.

[49] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. B **595**, 202 (2004), nucl-th/0312100.

[50] W. Zhao, C. M. Ko, Y.-X. Liu, G.-Y. Qin, and H. Song, Phys. Rev. Lett. **125**, 072301 (2020), 1911.00826.

[51] Y.-Z. Xu, D. Binosi, Z.-F. Cui, B.-L. Li, C. D. Roberts, S.-S. Xu, and H. S. Zong, Phys. Rev. D **100**, 114038 (2019), 1911.05199.

[52] F. Becattini, V. Chandra, L. Del Zanna, and E. Grossi, Annals Phys. **338**, 32 (2013), 1303.3431.

[53] F. Becattini, I. Karpenko, M. Lisa, I. Upsal, and S. Voloshin, Phys. Rev. C **95**, 054902 (2017), 1610.02506.

[54] R.-h. Fang, L.-g. Pang, Q. Wang, and X.-n. Wang, Phys. Rev. C **94**, 024904 (2016), 1604.04036.

[55] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. Lett. **90**, 202303 (2003), nucl-th/0301087.

[56] V. Greco, C. M. Ko, and P. Levai, Phys. Rev. C **68**, 034904 (2003), nucl-th/0305024.

[57] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. C **68**, 034902 (2003), nucl-th/0306027.

[58] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. C **68**, 034902 (2003), nucl-th/0306027.

[59] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. C **68**, 034902 (2003), nucl-th/0306027.

[60] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. C **68**, 034902 (2003), nucl-th/0306027.

[61] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. C **68**, 034902 (2003), nucl-th/0306027.

[62] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. C **68**, 034902 (2003), nucl-th/0306027.

[63] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. C **68**, 034902 (2003), nucl-th/0306027.

[64] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. C **68**, 034902 (2003), nucl-th/0306027.

[65] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. C **68**, 034902 (2003), nucl-th/0306027.

[66] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. C **68**, 034902 (2003), nucl-th/0306027.

[67] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. C **68**, 034902 (2003), nucl-th/0306027.

[68] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. C **68**, 034902 (2003), nucl-th/0306027.

[69] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. C **68**, 034902 (2003), nucl-th/0306027.

[70] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. C **68**, 034902 (2003), nucl-th/0306027.

[71] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. C **68**, 034902 (2003), nucl-th/0306027.

[72] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. C **68**, 034902 (2003), nucl-th/0306027.

[73] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. C **68**, 034902 (2003), nucl-th/0306027.

[74] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. C **68**, 034902 (2003), nucl-th/0306027.

[75] V. Greco, C. M. Ko, and R. Rapp, Phys. Rev. C **68**, 034902 (2003), nucl-th/0306027.