Quantum interferences from cross talk in $J = 1/2 \leftrightarrow J = 1/2$ transitions

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We consider the possibility of a control field opening up multiple pathways and thereby leading to new interference and coherence effects. We illustrate the idea by considering the $J = 1/2 \leftrightarrow J = 1/2$ transition. As a result of the additional pathways, we show the possibilities of nonzero refractive index without absorption and gain without inversion. We explain these results in terms of the coherence produced by the opening of an extra pathway.

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**I. INTRODUCTION**

The usage of a coherent field to control the optical properties of a medium has led to many remarkable results such as enhanced nonlinear optical effects [1, 2], electromagnetically induced transparency (EIT) [3], lasing without inversion [4, 5, 6], ultraslow light [7, 8, 9], storage and revival of optical pulses [10] and many others [11, 12, 13, 14]. Most of these effects rely on quantum properties of a medium has led to many remarkable results such as enhanced nonlinear optical effects [1, 2], electromagnetically induced transparency (EIT) [3], lasing without inversion [4, 5, 6], ultraslow light [7, 8, 9], storage and revival of optical pulses [10] and many others [11, 12, 13, 14]. Most of these effects rely on quantum properties of a medium has led to many remarkable results such as enhanced nonlinear optical effects [1, 2], electromagnetically induced transparency (EIT) [3], lasing without inversion [4, 5, 6], ultraslow light [7, 8, 9], storage and revival of optical pulses [10] and many others [11, 12, 13, 14]. Most of these effects rely on quantum properties of a medium has led to many remarkable results such as enhanced nonlinear optical effects [1, 2], electromagnetically induced transparency (EIT) [3], lasing without inversion [4, 5, 6], ultraslow light [7, 8, 9], storage and revival of optical pulses [10] and many others [11, 12, 13, 14]. Most of these effects rely on quantum properties of a medium has led to many remarkable results such as enhanced nonlinear optical effects [1, 2], electromagnetically induced transparency (EIT) [3], lasing without inversion [4, 5, 6], ultraslow light [7, 8, 9], storage and revival of optical pulses [10] and many others [11, 12, 13, 14].

The organization of the paper is as follows. In Sec. II, we describe the atomic configuration in details. We put forward all the relevant equations and the steady state solutions. In Sec. III, we discuss the absorption and dispersion profiles of the probe field. In Sec. IV, we show how new features arise in these profiles as effects of the new coherence.

**II. MODEL CONFIGURATION**

We consider the $J = 1/2 \leftrightarrow J = 1/2$ transition in alkali atoms, as shown in Fig. 1. This kind of configuration is important for studying the effects of cross talk among different transitions [15]. Note that cross talk in a Λ-system can lead to gain in the transmission of the probe field [17]. Similar configuration with degenerate sublevels also leads to electromagnetically induced absorption while interacting with control and probe fields with the same polarization [18]. We apply a dc magnetic field to remove the degeneracy of the excited and the ground states. In general, the Zeeman separation $2B$ of the excited magnetic sublevels $m_e = \pm 1/2(\equiv |e_\pm\rangle)$ is not the same as the Zeeman separation $2B'$ of the ground levels (\equiv $|g_\mp\rangle$), due to difference in the Landé $g$-factors.
in these manifolds. For example, in $^{39}$K atom, $B' = 3B$, where $B = \mu_B g_e M / \hbar$ ($\mu_B$ is the Bohr magneton) and $g_e = 2/3$ and $g_\rho = 2$ are the Landé $g$-factors of the excited and the ground sublevels.

We apply a weak $\hat{x}$ polarized field $\vec{E}_p = \hat{x} \vec{E}_p e^{-i\omega t} + c.c.$ to probe the properties of the atom, where $\omega$ is the angular frequency of the field. The $\sigma_{\pm}$ components of this probe field interact with the $|e_\pm\rangle \leftrightarrow |g_\pm\rangle$ transitions. The Rabi frequencies are given by $2g_\pm = 2(\vec{d}_{e_g} \cdot \hat{x} \vec{E}_p) / \hbar$, where $\vec{d}_{ij}$ is the electric dipole moment between the levels $|i\rangle$ and $|j\rangle$. We also apply a strong $\pi$-polarized control field

$$\vec{E}_c = \vec{E}_c e^{-i\omega_c t} + c.c.,$$

which interacts with the $|e_\pm\rangle \leftrightarrow |g_\pm\rangle$ transitions. We assume that the corresponding Rabi frequencies $2\vec{d}_{e_g} \cdot \vec{E}_c / \hbar$ are equal to $2G$. We emphasize that the system of Fig. 1 can be visualized as two $\Lambda$ systems (as shown in Fig. 2), which talk to each other, as the same control field $G$ drives both the transitions $|e_\pm\rangle \leftrightarrow |g_\pm\rangle$.

The interaction Hamiltonian of this system in dipole approximation is

$$\frac{\hbar}{\hbar} = \left[ \langle \omega_e g_- | e_- \rangle \langle e_- | + \omega_{e_+ g_-} | e_+ \rangle \langle e_+ | + \omega_{g_- g_+} | g_+ \rangle \langle g_+ | \right]$$

$$- \left( \langle \tilde{d}_{e_+ g_-} | e_+ \rangle \langle g_+ | + \langle \tilde{d}_{e_+ g_-} | e_- \rangle \langle g_- | + h.c. \right) \vec{E}_p$$

$$- \left( \langle \tilde{d}_{e_+ g_-} | e_+ \rangle \langle g_+ | + \langle \tilde{d}_{e_+ g_-} | e_- \rangle \langle g_- | + h.c. \right) \vec{E}_c \right].$$

Here the zero of energy is defined at the level $|g_-\rangle$ and $i\hbar\omega_{\alpha\beta}$ is the energy difference between the levels $|\alpha\rangle$ and $|\beta\rangle$.

We consider the natural decay terms in our analysis and hence invoke the density matrix formalism to find the following equations for different density matrix elements:

$$\dot{\rho}_{e_+ e_-} = -i(\Delta - 2B') \rho_{e_+ e_-} + i \left[ g_+ e^{i\omega_{\alpha\beta} t} \rho_{g_+ g_-} - \rho_{e_+ e_-} + G \rho_{g_+ g_-} - G \rho_{e_+ e_-} \right],$$

$$\dot{\rho}_{e_- e_+} = -i(\Delta - 2B) \rho_{e_- e_+} + i \left[ g_+ e^{-i\omega_{\alpha\beta} t} \rho_{g_+ g_-} - \rho_{e_- e_+} + G \rho_{g_+ g_-} - G \rho_{e_- e_+} \right],$$

$$\dot{\rho}_{e_+ g_-} = -i(\Delta + 2B) \rho_{e_+ g_-} + i \left[ G \rho_{e_+ e_-} - G \rho_{e_+ g_-} + g_+ e^{-i\omega_{\alpha\beta} t} \rho_{g_+ g_-} - g_+ e^{-i\omega_{\alpha\beta} t} \rho_{g_+ g_-} \right],$$

$$\dot{\rho}_{e_- g_+} = -(2 \rho_{e_- e_+}) \rho_{e_- g_+} + i \left[ G \rho_{e_+ e_-} - G \rho_{e_- g_+} + g_+ e^{-i\omega_{\alpha\beta} t} \rho_{g_+ g_-} - g_+ e^{-i\omega_{\alpha\beta} t} \rho_{g_+ g_-} \right],$$

$$\dot{\rho}_{e_+ e_+} = -(2 \rho_{e_- e_+}) \rho_{e_+ e_+} - i \left[ G^* \rho_{e_+ e_-} - G^* \rho_{e_+ e_+} + g_+ e^{i\omega_{\alpha\beta} t} \rho_{g_+ g_-} - g_+ e^{i\omega_{\alpha\beta} t} \rho_{g_+ g_-} \right],$$

$$\dot{\rho}_{e_+ e_-} = -(2 \rho_{e_- e_+}) \rho_{e_+ e_-} - i \left[ G^* \rho_{e_+ e_-} - G^* \rho_{e_+ e_+} + g_+ e^{i\omega_{\alpha\beta} t} \rho_{g_+ g_-} - g_+ e^{i\omega_{\alpha\beta} t} \rho_{g_+ g_-} \right],$$

$$\dot{\rho}_{e_- g_+} = \gamma_{e_-, e_+} \rho_{e_- e_+} + \gamma_{e_-, g_+} \rho_{e_+ g_+} + i \left[ g_+ e^{i\omega_{\alpha\beta} t} \rho_{g_+ g_-} + G \rho_{g_+ g_-} - h.c. \right],$$

$$\dot{\rho}_{e_+ g_-} = \gamma_{e_+, e_-} \rho_{e_+ e_-} + \gamma_{e_+, g_-} \rho_{e_+ g_-} + i \left[ g_+ e^{-i\omega_{\alpha\beta} t} \rho_{g_+ g_-} + G \rho_{g_+ g_-} - h.c. \right],$$

where $\Delta = \omega_2 - \omega_{e_+ g_-}$ is the detuning of the control field from the transition $|e_+\rangle \leftrightarrow |g_+\rangle$, $\delta = \omega_1 - \omega_{e_+ g_-}$ is the detuning of the $\sigma_-$ component of the probe field from the transition $|e_+\rangle \leftrightarrow |g_-\rangle$, $\omega_{12} = \omega_1 - \omega_2 = \delta - \Delta - 2B'$ is the difference between frequencies of the probe and control fields, $\gamma_{\alpha\beta}$ is the spontaneous emission rate from the level $|\beta\rangle$ to $|\alpha\rangle$, $\Gamma_{\alpha\beta} = \frac{1}{2} \sum_k (\gamma_{\alpha k} + \gamma_{k\beta})$ is the dephasing rate of the coherence between the levels $|\alpha\rangle$ and $|\beta\rangle$.

Here onwards, we assume that $\gamma_{e_-, e_+} = \gamma_{e_-, g_+} = \gamma_{e_+, e_-} = \gamma_2$ and $\gamma_{e_+, g_-} = \gamma_1$ without loss of generality, so that $\Gamma_{e_+, e_+} = \Gamma_{e_+, g_-} = \Gamma = (\gamma_1 + \gamma_2)/2$, $\Gamma_{e_-, e_-} = \Gamma_{e_-, g_+} = \gamma_1 + \gamma_2$, and $\Gamma_{e_-, g_-} = 0$. To obtain the above equations, we have applied the rotating wave approximation to neglect the highly oscillating terms. The transformed matrix elements are given by $\tilde{\rho}_{e_\pm g_\pm} = \rho_{e_\pm g_\pm} e^{i\omega t}$ and $\tilde{\rho}_{e\pm e\mp} = \rho_{e\pm e\mp} e^{i\omega t}$, whereas the other elements remain unchanged. Before solving Eqs. (3), let us first analyze two different cases.

Case I: We consider Fig. 2(b). If the probe field $\sigma_-$ is absent, all the populations from the level $|g_+\rangle$ would be optically pumped to the state $|g_-\rangle$ by the action of the control field $G$. Thus, the level $|g_-\rangle$ would be the steady state. When the probe field is on, population transfer from the level $|g_-\rangle$ to $|g_+\rangle$ would occur via the following pathway: absorption from the $\sigma_-$ component followed by the emission in the control field in the $|e_+\rangle \rightarrow |g_+\rangle$ transition. The relevant susceptibility of the probe field would be the same as that in case of a $\Lambda$ system. We provide the corresponding expression at the end of this section.

Case II: We consider Fig. 4. If both the $\sigma_\pm$ components are absent, then in the steady state, the population gets distributed in four levels depending upon the detunings and the Rabi frequencies of the control fields. In this case, when both the $\sigma_\pm$ components are switched on, an additional pathway for population transfer from the level $|g_-\rangle$ to $|g_+\rangle$ would arise, in addition to the one described in the Case I. This pathway can be described as follows: absorption from the control field in $|g_-\rangle \rightarrow |e_-\rangle$ transition followed by the emission in the $\sigma_+$ component. In-
terference of these two pathways (i.e., cross talk between two Λ systems in Fig. 2) leads to new coherence in the four-level system of Fig. 1 which would not arise in a Λ-system (Case I). Later we show that all the new features described in this paper can be attributed to this coherence. Note that coherences have been recognized as the major source of newer effects in multilevel systems.

For the Case II, the steady state solutions of the equations (4) can be found by expanding the density matrix elements in terms of the harmonics of \( \omega_{12} \) as

\[
\rho_{\alpha\beta} = \rho^{(0)}_{\alpha\beta} + g_\beta e^{-i\omega_{12}t} \rho^{(-1)}_{\alpha\beta} + g_\alpha^* e^{i\omega_{12}t} \rho^{(-1)*}_{\alpha\beta} + g_\beta e^{-i\omega_{12}t}\rho^{(+1)}_{\alpha\beta} + g_\alpha^* e^{i\omega_{12}t}\rho^{(+1)*}_{\alpha\beta}.
\]

Thus, we obtain a set of algebraic equations for \( \rho^{(0)}_{\alpha\beta} \)’s. Solving them, we find the following zeroth order population terms (i.e., when both the \( \sigma_\pm \) components are absent):

\[
\begin{align*}
\tilde{\rho}_{e_+e_+}^{(0)} &= \frac{xy}{Q}, \\
\tilde{\rho}_{e_-e_-}^{(0)} &= \frac{y}{Q}(\gamma_1 + \gamma_2 + x), \\
\tilde{\rho}_{e_+e_-}^{(0)} &= \frac{x}{Q}(\gamma_1 + \gamma_2 + y),
\end{align*}
\]

where

\[
x = \frac{2|G|^2\Gamma}{|d|^2}; \quad y = \frac{2|G|^2\Gamma}{|c|^2},
\]

\[
Q = (x + y)(\gamma_1 + \gamma_2) + 4xy,
\]

\[
c = i\Delta + \Gamma,
\]

\[
d = -i(-\Delta - 2B + 2B') + \Gamma.
\]

Clearly, the population is distributed among the levels \( |e_\pm\rangle \) and \( |g_\pm\rangle \). This is due to optical pumping as we have discussed earlier. The relevant zeroth order coherence terms turn out to be

\[
\begin{align*}
\tilde{\rho}_{g_+e_+}^{(0)} &= -i\frac{G^*}{cQ}(\gamma_1 + \gamma_2), \quad (7a) \\
\tilde{\rho}_{g_-e_-}^{(0)} &= -i\frac{G^*}{dQ}(\gamma_1 + \gamma_2), \quad (7b)
\end{align*}
\]

which vanish in absence of any control field (i.e., for \( G = 0 \)).

The susceptibilities of the \( \sigma_\mp \) components of the probe field can be obtained from first-order solutions which we write as

\[
\begin{align*}
\tilde{\rho}_{e_+e_-}^{(-1)} &= \frac{1}{M_1} \left[G_{a_+} p_+ + G_{b_+} p_- + i\{a_+ b_+ p_+ + |G|^2(a_+ + b_+)\}(\tilde{\rho}_{g_+e_+}^{(0)} - \tilde{\rho}_{g_-e_-}^{(0)})\right], \\
\tilde{\rho}_{e_-e_+}^{(+1)} &= \frac{1}{M_2} \left[G_{a_-} q_+ - G_{b_-} q_- + i\{a_- b_- q_+ + |G|^2(a_- + b_-)\}(\tilde{\rho}_{g_+e_+}^{(0)} - \tilde{\rho}_{g_-e_-}^{(0)})\right],
\end{align*}
\]

where

\[
M_1 = a_+ b_+ p_+ + |G|^2(a_+ + b_+),
\]

\[
M_2 = a_- b_- p_- - |G|^2(a_- + b_-),
\]

and

\[
\begin{align*}
a_\pm &= -i\omega_{12} \pm 2iB + \Gamma_{e_\pm e_\mp}, \\
b_\pm &= -i\omega_{12} \pm 2iB' + \Gamma_{g_\pm g_\mp}, \\
p_\pm &= -i\omega_{12} \pm i(\Delta + 2B) + \Gamma, \\
q_\pm &= -i\omega_{12} \pm i(-\Delta + 2B') + \Gamma.
\end{align*}
\]
Note that the difference between the frequencies of the probe and control fields is given by \( \omega_{12} = \delta - \Delta + 2B' \).

The above susceptibility is to be compared with the one in the absence of cross-talk (Case I). For the Case I, we have

\[
\hat{\rho}_{e+g-}^{(0)} = \frac{i\hat{b}_+}{\beta_+g_+ + |G|^2}(\hat{\rho}_{g-g-}^{(0)} - \hat{\rho}_{e+e+}^{(0)})
\]

\[
= \frac{-i\{i(\delta - \Delta) - \Gamma_{g+g-}\}}{\{i(\delta - \Delta) - \Gamma_{g+g-}\}(i\delta - \Gamma) + |G|^2}
\]

where we have used the fact that in steady state, \( \hat{\rho}_{g-g-}^{(0)} = 1 \) and the populations in the other levels vanish. In this case, the zeroth-order coherence between the levels \( |e_+\rangle, |g_+\rangle \) also vanishes. From Eq. (10b), we find that the real part of the susceptibility vanishes at the detunings \( \delta = \Delta + \Delta', \), satisfying the following equation:

\[
\Delta^3 + \Delta^2 \Lambda + \Lambda^2 \delta + 2\Gamma g_{+g-} + |G|^2 \Lambda + \Delta \Lambda' = 0 \ .
\]

Three different solutions of the above equation for \( \Delta' \) corresponding to vanishing real part of \( \chi_- \), can be obtained from the Cardano’s formula \([20]\), given by

\[
\Delta_- = -\frac{\Lambda}{3} + A_+ \ , \Delta_{-2,3} = -\frac{\Lambda}{3} - \frac{1}{2} A_+ \pm \frac{i}{2} \sqrt{3} A_- \ ,
\]

where

\[
A_+ = \left[ R + \sqrt{Q^3 + R^2} \right]^{1/3} + \left[ R - \sqrt{Q^3 + R^2} \right]^{1/3} \ , \quad Q = \frac{1}{9}(3\Delta_1 - \Delta^2) \ , \quad R = \frac{1}{54}(9\Delta_1 - 27a_0 - 2\Delta^3) \ ,
\]

\[
a_0 = \Delta' \Lambda g_{+g-} \ , \ a_1 = \Delta \Lambda' g_{+g-} + 2\Gamma g_{+g-} + |G|^2 \ .
\]

III. ABSORPTION AND DISPERSION PROFILES

We first recall the features of the \( \Lambda \) system of Fig. 2(b). The usual line-shapes can be obtained for a resonant control field, i.e., by putting \( \Delta = 0 \) in the susceptibility given by Eq. (10a). Then the absorption and dispersion profiles would be symmetric around \( \delta = 0 \). However, for non-zero \( \Delta \), the line-shapes depend upon the values of \( \Delta \). We show the dispersion and absorption profiles of the \( \sigma_- \) component in Figs. 3(a) and 3(b) for a fixed detuning \( \Delta = B' - B \) of the control field. Clearly, the real and imaginary parts of the susceptibility \( \chi_- \) [\( \equiv \langle |\Delta e_{+g-}|^2/\hbar \gamma \rangle \hat{\rho}_{e+g-}^{(-1)} \), \( N \) being the number density of the atomic medium] vanish at the two-photon resonance \( \delta = \Delta \), which occurs at \( \delta = \Delta = 4\gamma \) for \( B = 2\gamma \). These are the usual features of a \( \Lambda \) system at two-photon resonance.

Next we analyze the four-level system of Fig. 2(a). We show the dispersion and absorption profiles of the \( \sigma_- \) component in Figs. 3 for \( \Delta = B' - B \[21\]. At two-photon resonance (i.e., at \( \delta = \Delta \)), the real part of the susceptibility \( \chi_- \) is non-zero and negative in contrast to the case of a \( \Lambda \) system. On the other hand, at two-photon resonance \( \delta = \Delta \), the medium continues to remain transparent as in case of a \( \Lambda \)-system. Further, at certain region of the detuning \( \delta (> \Delta) \) of the probe field, the imaginary part of \( \chi_- \) becomes negative, leading to the gain in the \( \sigma_- \) component. For the case of a \( \Lambda \)-system, there would be no possibility of gain in the medium [solid line in Fig. 3(b)] for any \( \delta \). In the next section, we analyze these new features in terms of the new coherence that we discussed in the Sec. II.

We should mention here that two-photon gain in hot alkali vapor has been reported in recent experiments \([22]\). The gain involves absorption of two photons from the detuned \( \sigma_- \) polarized control field and emission of two photons in \( \pi \)-polarized probe field at Raman resonance. Detailed theoretical analysis \([23]\) has shown that these are due to quantum interference of several excitation pathways, originating from hyperfine structures. Because in our model, gain arises due to interference of two different pathways, as described Sec. II, we should emphasize the
The main difference between the model in [22] and our model. We consider a different set of polarizations and frequency of the electric fields interacting with the non-degenerate electronic levels, contrary to [22], where degenerate hyperfine levels have been considered. In our model, the gain, associated with emission of a single photon in \( g_\sigma \) component, is essentially a two-photon process, and thus much larger in effect compared to that arising due to four-photon process described in [22, 22]. In addition, gain occurs when the probe field is not at Raman resonance with the control field.

IV. DISCUSSIONS

A. Origin of non-zero susceptibility

We start with the expression \[ \chi^{(1)}_{\sigma_+ g_+} \] for \( \tilde{\chi}_{e_+ g_-}^{(-1)} \) in the \( \Lambda \) system. Clearly, the susceptibility of the probe field arises only by the population difference between the relevant levels \( |e_+\rangle \) and \( |g_-\rangle \), as the zeroth-order coherence between the levels \( |g_+\rangle \) and \( |e_+\rangle \) is zero in this case. In addition, at two-photon resonance \( (\delta = \Delta) \), \( b_\perp = 0 \) and the susceptibility vanishes. But, in case of the four-level system of Fig. 4 the coherence \( \tilde{\rho}_{g_+ e_+}^{(0)} \) also contributes to the susceptibility \( \chi_- \) [see Eq. (5)], while contribution from \( \tilde{\rho}_{g_+ e_-}^{(0)} \) to \( \chi_- \) vanishes at two-photon resonance \( (\delta = \Delta) \) as \( b_\perp = 0 \). Then, using Eqs. (5) and (8a), we can write for \( \delta = \Delta = B' - B \)

\[ \tilde{\rho}_{e_+ g_-}^{(-1)} = \frac{ix(\gamma_1 + \gamma_2)}{2q + Q} \left( 1 - \frac{q_+}{c} \right), \tag{14} \]

which is non-zero as \( B' \neq B \). Thus nonzero susceptibility of the system of Fig. 4 manifests itself as an effect of the zeroth order coherence in the \( |e_+\rangle \leftrightarrow |g_+\rangle \) transition. Further, it is essentially associated with no absorption, as Eq. (15) has no imaginary part. We should mention here that the susceptibility becomes negative due to larger Landé \( g \) factor of the ground state manifold (i.e., \( B' > B \)). Clearly, in absence of the magnetic field, the medium becomes isotropic, and no special features can be found in the dispersion profile.

In Fig. 4(a), we show how the real part of \( \chi_- \) varies with the detuning \( \Delta \) of the control field at two-photon resonance \( \delta = \Delta \). Clearly, even for resonant control field (i.e., \( \delta = 0 \)), the susceptibility is negative. Moreover, the real part of \( \chi_- \) becomes zero at two-photon resonance for certain value of \( \delta \). Putting \( b_\perp = 0 \) in Eq. (8), one can calculate the corresponding value

\[ \delta_0 = \frac{2(B' - B)^2 + \Gamma^2}{B' - B}. \tag{16} \]

For the present parameters, we find that \( \delta_0 = 10.25\gamma \), which is shown in Fig. 4(a). In fact, for \( \delta = \Delta = \delta_0 \), the contributions of the population difference term and the coherence term to \( \text{Re}(\chi_-) \) cancel each other and the susceptibility of the probe field vanishes. In Fig. 4(b), we show that the control field can be a good control.
parameter for the susceptibility. The susceptibility remains negative for the entire range of $G$ at two-photon resonance. This is unlike the case of a $\Lambda$ system [see Fig. 2(b)] for which the real part of $\chi_-$ would remain zero at two-photon resonance irrespective of the value of Rabi frequency $G$ of the control field.

B. Origin of gain

It is well understood that, the absorption spectrum of the probe field in the $\Lambda$ system of Fig. 2(b) shows Autler-Townes doublet. Moreover, there does not arise any gain in the medium. At two-photon resonance $\delta = \Delta$, the absorption becomes zero. However, the case of the four-level system of Fig. 1 is different. We already have noted that the susceptibility $\chi_-$ of the $\sigma_-$ component is contributed by two terms: $\tilde{\rho}^{(0)}_{g_+e_+}$ and $\tilde{\rho}^{(0)}_{g_-e_-} - \tilde{\rho}^{(0)}_{e_+e_-}$. We show the individual contribution of these two terms in the absorption spectra in Fig. 5. From this figure one can see that at certain region of frequency ($\delta > B' - B$), the negative contribution of $\tilde{\rho}^{(0)}_{g_+e_+}$ is larger in magnitude than the positive contribution of $\tilde{\rho}^{(0)}_{g_-e_-} - \tilde{\rho}^{(0)}_{e_+e_-}$. Thus, gain arises in the medium. In our model, this novel feature can be attributed to the control field $G$ which gives rise to the non-zero coherence $\tilde{\rho}^{(0)}_{g_+e_+}$. Further, from the expressions of $\tilde{\rho}^{(0)}_{g_\pm e_\pm}$ and $\tilde{\rho}^{(0)}_{e_\pm e_\mp}$ [Eqs. 5], one sees that the zeroth-order populations in both the $|g_\pm\rangle$ levels are larger than those in the levels $|e_\pm\rangle$ due to the presence of non-zero decay terms $\gamma_1$ and $\gamma_2$. Thus there is no population inversion in bare basis and we have gain without inversion. Note that at two-photon resonance ($\delta = \Delta$), the contributions from the terms $\tilde{\rho}^{(0)}_{g_+e_+}$ and $\tilde{\rho}^{(0)}_{g_-e_-} - \tilde{\rho}^{(0)}_{e_+e_-}$ to the absorption profile cancel each other, leading to transparency. We should further mention here that the contribution of $\tilde{\rho}^{(0)}_{g_-e_-}$ to the gain is negligible for all $\delta$.

V. CONCLUSIONS

In conclusion, we have shown how a control field can give rise to new coherence effects in a specific four-level system. The control field leads to multiple pathways, interference between which leads to the effects like gain without inversion and non-zero susceptibility associated with zero absorption. We have explained these results in terms of the new coherence arising due to this interference.

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[21] The similar profiles of $\sigma_\pm$ component could also be considered. However in terms of physical origin, the profiles of $\sigma_\pm$ components are complementary to each other. So it suffices to consider the either component.
[22] H. P. Concannon, W. J. Brown, J. R. Gardner, D. J. Gauthier, Phys. Rev. A 56, (1997) 1519 ; O. Pfister, W. J. Brown, M. D. Stenner, D. J. Gauthier, Phys. Rev. A 60 (1999) 4249.
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