Abstract

I give a pedagogical introduction to the physics of electroweak symmetry breaking and the uses of effective field theory in the context of Higgs physics. Higgs boson production and decay at the LHC and the consistency of the Higgs measurements with triviality arguments, vacuum stability, and precision electroweak measurements are discussed. Effective Lagrangian techniques are used to understand potential deviations from the Standard Model (SM) predictions. Finally, I end with a brief discussion of the future of Higgs physics.
I. INTRODUCTION

The experimental discovery of the Higgs boson [1, 2] implies that the Weinberg Salam Standard Model (SM) is a valid low energy theory at the weak scale. All current measurements are consistent with this statement and physics in the electroweak symmetry breaking (EWSB) sector beyond that predicted by the SM is highly constrained by experimental results, both at the LHC and from precision electroweak measurements. These lectures summarize the underlying theoretical framework of the SM and its experimental predictions and discuss possible high scale extensions of the theory in terms of an effective field theory.

Section II contains an introduction to the SM and Section III discusses theoretical restrictions on the EWSB sector with an aside on unitarity. Section IV presents the basics of Higgs production and decay, along with a summary of current experimental results. Pedagogical discussions of the gluon fusion production rate at leading order, low energy theorems that can be used to approximate the gluon fusion rate in Beyond the SM scenarios (BSM) and the determination of the Higgs width are also found in Section IV. Extensions of the SM in terms of an effective field theory are presented in Section V and Section VI contains some conclusions and a personal view on the future of Higgs physics. There are many excellent reviews of Higgs physics and the reader is referred to them for additional details and further references[3–9].

II. INTRODUCTION TO THE STANDARD MODEL

A. The Higgs Mechanism

We begin by discussing a simplified version of the SM with a $U(1)$ symmetry, the Abelian Higgs model, that illustrates a basic version of EWSB and the motivation for the $SU(2)_L \times U(1)_Y$ SM. To understand the problem of gauge boson masses, consider a $U(1)$ gauge theory with a single gauge field, the photon, $A_\mu$,

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad (1)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (2)$$
Local $U(1)$ gauge invariance requires that the Lagrangian be invariant under the transformation: $A_\mu(x) \to A_\mu(x) - \partial_\mu \eta(x)$ for any $\eta$ and $x$. If we add a mass term for the photon to the Lagrangian,

$$
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu,
$$

it is easy to see that the mass term violates the local gauge invariance.

The model can be extended by adding a single complex scalar field with charge $-e^1$ that couples to the photon,

$$
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 - V(\phi),
$$

where,

$$
D_\mu = \partial_\mu - ie A_\mu,
V(h) = \mu^2 |h|^2 + \lambda(|h|^2)^2.
$$

$V(\phi)$ is the most general renormalizable potential allowed by the $U(1)$ gauge invariance.

This Lagrangian is invariant under global $U(1)$ rotations, $\phi \to e^{i\theta} \phi$ and also under local gauge transformations:

$$
A_\mu(x) \to A_\mu(x) - \partial_\mu \eta(x)
\phi(x) \to e^{-ie\eta(x)} \phi(x).
$$

There are now two possibilities for the theory.\(^2\) If $\mu^2 > 0$, the potential preserves the symmetries of the Lagrangian and the state of lowest energy is that with $\phi = 0$, the vacuum state. This theory is quantum electrodynamics with a massless photon and a charged scalar field $\phi$ with mass $\mu$.

In the alternative scenario, $\mu^2 < 0$ and the potential is,

$$
V(\phi) = -|\mu^2||\phi|^2 + \lambda(|\phi|^2)^2.
$$

In this case the minimum energy state is not at $\phi = 0$, but rather at

$$
\langle \phi \rangle = \sqrt{-\frac{\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}}.
$$

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1 My conventions follow [7] and have $e > 0$ and $Q_e = -1$.

2 We assume $\lambda > 0$. If $\lambda < 0$, the potential is unbounded from below and has no state of minimum energy.
is called the vacuum expectation value (VEV) of $\phi$. The direction in which the vacuum is chosen is arbitrary, but it is conventional to choose it to lie along the direction of the real part of $\phi$. The VEV breaks the global $U(1)$ symmetry.

It is convenient to rewrite $\phi$ as

$$\phi \equiv \frac{1}{\sqrt{2}} e^{i \chi} (v + h),$$

where $\chi$ and $h$ are real fields that have no VEVs. If we substitute Eq. 10 back into the original Lagrangian, the interactions are,

$$\mathcal{L} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - evA_\mu \partial^\mu \chi + \frac{e^2 v^2}{2} A_\mu A^\mu + \frac{1}{2} \left( \partial_\mu h \partial^\mu h + 2 \mu^2 h^2 \right)$$

$$+ \frac{1}{2} \partial_\mu \chi \partial^\mu \chi + (h, \chi \text{ interactions}).$$

Eq. 11 describes a theory with a photon of mass $M_A = ev$, a physical scalar field $h$ with mass-squared $-2\mu^2 > 0$, and a massless scalar field $\chi$. The mixed $\chi - A$ propagator can be removed by making a gauge transformation,

$$A'_\mu \equiv A_\mu - \frac{1}{ev} \partial_\mu \chi.$$  (12)

After making the gauge transformation of Eq. 12, the $\chi$ field disappears from the theory. In the gauge of Eq. 12 the particle content of the theory is apparent: a massive photon and a scalar field $h$, which we call a Higgs boson. Clearly, the choice $\mu^2 < 0$ leads to very different physical consequences from $\mu^2 > 0$.

Now consider the gauge dependance of these results. The gauge choice above with the transformation $A'_\mu = A_\mu - \frac{1}{ev} \partial_\mu \chi$ is called the unitary gauge. This gauge has the advantage that the particle spectrum is obvious and there is no $\chi$ field. The unitary gauge, however, has the disadvantage that the photon propagator, $\Delta_{\mu \nu}(k)$, has bad high energy behaviour,

$$\text{Unitary gauge : } \Delta_{\mu \nu}(k) = -\frac{i}{k^2 - M_A^2} \left( g_{\mu \nu} - \frac{k^\mu k^\nu}{M_A^2} \right).$$

In the unitary gauge, scattering cross sections have contributions that grow with powers of $k^2$ (such as $k^4$, $k^6$, etc.) that cannot be removed by the conventional mass, coupling constant, and wavefunction renormalizations. More convenient gauges are the $R_\xi$ gauges that are obtained by adding the gauge fixing term to the Lagrangian[10],

$$\mathcal{L}_{GF} = -\frac{1}{2\xi} \left( \partial_\mu A^\mu + \xi ev \chi \right)^2.$$
After integration by parts, the cross term in Eq. 14 exactly cancels the mixed $\chi \partial_\mu A^\mu$ term of Eq. 11. Different choices for $\xi$ correspond to different gauges and in the limit $\xi \to \infty$, the unitary gauge is recovered.

The gauge boson propagator in $R_\xi$ gauge is given by

$$\Delta_{\mu\nu}(k) = -\frac{i}{k^2 - M_A^2} \left( g_{\mu\nu} - \frac{(1 - \xi) k_\mu k_\nu}{k^2 - \xi M_A^2} \right).$$  \hspace{1cm} (15)

In the $R_\xi$ gauges the $\chi$ field is part of the spectrum and has mass $M_\chi^2 = \xi M_A^2$. The field $\chi$ is called a Goldstone boson. Feynman gauge corresponds to the choice $\xi = 1$ and has a massive Goldstone boson, $\chi$, while Landau gauge has $\xi = 0$ and the Goldstone boson $\chi$ is massless. We see that the particle spectrum and the mass of the Goldstone boson are gauge dependent.

### B. Weinberg-Salam Model

It straightforward to obtain the Weinberg-Salam model of electroweak interactions by generalizing the results of the previous section\cite{7}. The Weinberg- Salam model is an $SU(2)_L \times U(1)_Y$ gauge theory containing three $SU(2)_L$ gauge bosons, $W^I_\mu$, $I = 1, 2, 3$, and one $U(1)_Y$ gauge boson, $B_\mu$, with kinetic energy terms,

$$L_{KE} = -\frac{1}{4} W^I_\mu W^{I\mu\nu} - \frac{1}{4} B_\mu B^{\mu\nu},$$  \hspace{1cm} (16)

where the index $I$ is summed over and,

$$W^I_\mu = \partial_\nu W^I_\mu - \partial_\mu W^I_\nu + g e^{IJK} W^J_\mu W^K_\nu,$$

$$B_\mu = \partial_\nu B_\mu - \partial_\mu B_\nu.$$  \hspace{1cm} (17)

The $SU(2)_L$ and $U(1)_Y$ coupling constants are $g$ and $g'$, respectively. Coupled to the gauge fields is a complex scalar $SU(2)$ doublet, $\Phi$,

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}.$$  \hspace{1cm} (18)

The scalar potential is given by,

$$V(\Phi) = \mu^2 |\Phi^\dagger \Phi| + \lambda \left( |\Phi^\dagger \Phi| \right)^2,$$  \hspace{1cm} (19)
where $\lambda > 0$.

Just as in the Abelian model of Section II A, the state of minimum energy for $\mu^2 < 0$ is not at $\phi^0 = 0$ and the scalar field develops a VEV$^3$. The direction of the minimum in $SU(2)_L$ space is not determined, since the potential depends only on the combination $\Phi^\dagger \Phi$ and we arbitrarily choose

$$
\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}.
$$

(20)

With this choice, the electromagnetic charge is,$^4$

$$
Q = \frac{\tau_3 + Y}{2},
$$

(21)

where we assign hypercharge $Y = 1$ to $\Phi$.

Therefore,

$$
Q \langle \Phi \rangle = 0
$$

(22)

and electromagnetism is unbroken by the scalar VEV. The VEV of Eq. 20 yields the desired symmetry breaking pattern,

$$
SU(2)_L \times U(1)_Y \to U(1)_{EM}.
$$

(23)

The scalar contribution to the Lagrangian is,

$$
\mathcal{L}_s = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi),
$$

(24)

where$^5$

$$
D_\mu = \partial_\mu + ig^2 \tau \cdot W_\mu + ig' \frac{2}{2} B_\mu Y.
$$

(25)

In unitary gauge there are no Goldstone bosons and only the physical Higgs scalar remains in the spectrum after the spontaneous symmetry breaking has occurred. In unitary gauge,

$$
\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix},
$$

(26)

$^3$ As in the Abelian model, there is no mechanism or motivation for determining the sign($\mu^2$) in the SM.

$^4$ The $\tau_I$ are the Pauli matrices with $Tr(\tau_I \tau_J) = 2\delta_{IJ}$.

$^5$ Different choices for the gauge kinetic energy and the covariant derivative depend on whether $g$ and $g'$ are chosen positive or negative. There are no physical consequences of this choice.
which gives the contribution to the gauge boson masses from the scalar kinetic energy term of Eq. 24,

\[ M^2 \sim \frac{1}{2} (0, v) \left( \frac{1}{2} g \tau \cdot W_{\mu} + \frac{1}{2} g' B_{\mu} \right)^2 \left( \begin{array}{c} 0 \\ v \end{array} \right). \]  

(27)

The physical gauge fields are two charged fields, \( W^\pm \), and two neutral gauge bosons, \( Z \) and \( \gamma \).

\[
W_{\mu}^\pm = \frac{1}{\sqrt{2}} (W_{\mu}^1 \pm i W_{\mu}^2) \\
Z_{\mu} = -g' B_{\mu} + g W_{\mu}^3 \sqrt{g^2 + g'^2} \equiv -\sin \theta_W B_{\mu} + \cos \theta_W W_{\mu}^3 \\
A_{\mu} = g B_{\mu} + g' W_{\mu}^3 \sqrt{g^2 + g'^2} \equiv \cos \theta_W B_{\mu} + \sin \theta_W W_{\mu}^3.
\]  

(28)

Eq. 28 defines a mixing angle,

\[
\sin \theta_W \equiv \frac{g'}{\sqrt{g^2 + g'^2}}.
\]  

(29)

Since the massless photon must couple with electromagnetic strength, \( e \), the coupling constants define the weak mixing angle \( \theta_W \),

\[
e = g \sin \theta_W \equiv g s_W \\
e = g' \cos \theta_W \equiv g' c_W.
\]  

(30)

The gauge bosons obtain masses from the Higgs mechanism, as demonstrated in Eq. 27:

\[
M_W^2 = \frac{1}{4} g^2 v^2, \quad M_Z^2 = \frac{1}{4} (g^2 + g'^2) v^2, \quad M_A = 0.
\]  

(31)

Just as in the case of the Abelian Higgs model, if we go to a gauge other than unitary gauge, there are Goldstone bosons in the spectrum and the scalar field can be parameterized,

\[
\Phi = \frac{1}{\sqrt{2}} e^{i \frac{\omega}{v}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}.
\]  

(32)

In the Standard Model, there are three Goldstone bosons, \( \vec{\omega} = (\omega^\pm, z) \), with masses \( M_W \) and \( M_Z \) in the Feynman gauge.

Fermions can easily be included in the theory. We write the fermions in terms of their left- and right-handed projections,

\[
\psi_{L,R} = \frac{1}{2} (1 \mp \gamma_5) \psi.
\]  

(33)
From the four-Fermi theory of weak interactions\cite{7}, we know experimentally that the $W$-boson couples only to left-handed fermions and so we construct the $SU(2)_L$ doublet,

$$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}.$$  \hspace{1cm} (34)

From Eq. 21, the hypercharge of the lepton doublet must be $Y_L = -1$. In the limit where the neutrino is massless, it can have only one helicity state which is taken to be $\nu_L$. Including neutrino masses requires interactions beyond the standard construction of the Weinberg-Salam model\cite{6}. The SM is therefore constructed with no right-handed neutrinos. Further, we assume that right-handed fields do not interact with the $W$ boson, and so the right-handed electron, $e_R$, must be an $SU(2)_L$ singlet with $Y_{e_R} = -2$. Using these hypercharge assignments, the leptons can be coupled in a gauge invariant manner to the $SU(2)_L \times U(1)_Y$ gauge fields,

$$L_{\text{lepton}} = i\bar{e}_R \gamma^\mu \left( \partial_\mu + ig_2 Y_e B_\mu \right) e_R + i\bar{L}_L \gamma^\mu \left( \partial_\mu + ig_2 \tau \cdot W_\mu + ig'_2 Y_L B_\mu \right) L_L.$$ \hspace{1cm} (35)

All of the known fermions can be accommodated in the Standard Model in this fashion. The $SU(2)_L$ and $U(1)_Y$ charge assignments of the first generation of fermions are given in Table I. The quantum numbers of the 2\textsuperscript{nd} and 3\textsuperscript{rd} generation are identical to those of first generation.

A fermion mass term takes the form

$$L_{\text{mass}} = -m\bar{\psi}\psi = -m \left( \bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \right).$$ \hspace{1cm} (36)

As is obvious from Table I, the left-and right-handed fermions transform differently under $SU(2)_L$ and $U(1)_Y$ gauge transformations and so gauge invariance forbids a term like Eq. 36. The Higgs boson, however, can couple in a gauge invariant fashion to the down quarks,

$$L_d = -Y_d \bar{Q}_L \Phi d_R + h.c.,$$ \hspace{1cm} (37)

After the Higgs obtains a VEV, we have the effective coupling,

$$-Y_d \frac{1}{\sqrt{2}} (\bar{u}_L, \bar{d}_L) \begin{pmatrix} 0 \\ v + h \end{pmatrix} d_R + h.c.$$ \hspace{1cm} (38)

\hspace{1cm} \footnote{A pedagogical introduction to $\nu$ masses can be found in Ref. [11].}
which can be seen to yield a mass term for the down quark,

\[ Y_d = \frac{m_d \sqrt{2}}{v}. \]  (39)

In order to generate a mass term for the up-type quarks we use the fact that

\[ \tilde{\Phi} \equiv i\tau_2 \Phi^* = \begin{pmatrix} \phi^0 \\ -\phi^- \end{pmatrix} \]  (40)

is an \( SU(2)_L \) doublet, and write the \( SU(2)_L \) invariant coupling

\[ L_u = -Y_u \bar{Q}_L \Phi u_R + h.c. \]  (41)

which generates a mass term for the up quark. Similar couplings can be used to generate mass terms for the charged leptons. Since the neutrino has no right handed partner in the SM, it remains massless.

For the multi-family case, the Yukawa couplings, \( Y_d \) and \( Y_u \), become \( N_F \times N_F \) matrices (where \( N_F \) is the number of families). Since the fermion mass matrices and Yukawa matrices are proportional, the interactions of the Higgs boson with the fermion mass
eigenstates are flavor diagonal and the Higgs boson does not mediate flavor changing interactions. This is an important prediction of the SM.

The parameter $v$ can be found from the charged current for $\mu$ decay, $\mu \rightarrow e\nu_e\nu_\mu$, which is measured very accurately to be $G_F = 1.16638 \times 10^{-5} \text{ GeV}^{-2}$. Since the momentum carried by the $W$ boson is of order $m_\mu$ it can be neglected in comparison with $M_W$ and we make the identification,

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{1}{2v^2},$$

which gives the result

$$v = (\sqrt{2}G_F)^{-1/2} = 246 \text{ GeV}.$$ (43)

One of the most important points about the Higgs mechanism is that all of the couplings of the Higgs boson to fermions and gauge bosons are completely determined in terms of coupling constants and fermion masses. A complete set of Feynman rules can be found in Ref. [9]. The potential of Eq. 19 had two free parameters, $\mu$ and $\lambda$, which can be traded for,

$$v^2 = \frac{-\mu^2}{2\lambda},$$

$$m_h^2 = 2v^2\lambda.$$ (44)

The scalar potential is now,

$$V = \frac{m_h^2}{2}h^2 + \frac{m_h^2}{2v^2}h^3 + \frac{m_h^2}{8v^2}h^4.$$ (45)

The self-interactions of the Higgs boson are determined in terms of the Higgs mass. There are no remaining adjustable parameters and so Higgs production and decay processes can be computed unambiguously in terms of the Higgs mass.

C. Aside: Anomaly Cancellation

The requirement of gauge anomaly cancellation puts restrictions on the couplings of the fermions to vector and axial gauge bosons, denoted here by $V^\mu$ and $A^\mu$. The fermions of the Standard Model have couplings to the gauge bosons of the general form:

$$\mathcal{L} \sim g_A \bar{\psi}T^\alpha\gamma_\mu\gamma_5\psi A^{\alpha\mu} + g_\nu \bar{\psi}T^\alpha\gamma_\mu\psi V^{\alpha\mu},$$

where $T^\alpha$ is the gauge generator in the adjunct representation. These fermion-gauge
boson couplings contribute to triangle graphs (Fig. 1) that diverge at high energy,

\[ T^{abc} \sim Tr[\eta_i T^a \{ T^b, T^c \}] \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^3}, \]  

(47)

where \( \eta_i = \pm 1 \) for left- and right-handed fermions, \( \psi_{L,R} = \frac{1}{2} (1 \mp \gamma^5) \psi \). This divergence is independent of the fermion mass and depends only on the fermion couplings to the gauge bosons. Such divergences cannot exist in a physical theory. The theory can be anomaly free in a vector-like model where the left- and right-handed particles have identical couplings to gauge bosons and the contribution to Eq. 47 cancels for each pair of particles. From Table I, however, it is clear that the Standard Model is not vector-like since left- and right-handed fermions transform differently under \( SU(2)_L \). The anomaly, \( T^{abc} \), must therefore be cancelled by a judicious choice of fermion representations under the various gauge groups.

The only non-vanishing contribution to the anomaly in the Standard Model is from

\[ \Sigma Tr[Y \{ T^a, T^b \}], \]  

(48)

where \( T^a \) are the \( SU(2)_L \) generators and the sum is over all fermions in the theory. Eq. 48 vanishes for the hypercharge assignments given in Table I. Note that the anomaly cancels separately for each generation of fermions and the SM thus requires complete generations of chiral fermions.
III. THEORETICAL CONSTRAINTS

A. Bounds from Precision Measurements

The Higgs boson enters into one loop radiative corrections in the Standard Model and precision electroweak measurements test the consistency of the theory\(^7\). In the electroweak sector of the SM, there are four fundamental parameters, the $SU(2)_L \times U(1)_Y$ gauge coupling constants, $g$ and $g'$, as well as the two parameters of the Higgs potential, which are usually taken to be the vacuum expectation value of the Higgs boson, $v$, and the Higgs mass, $m_h$. Once these parameters are fixed, all other physical quantities can be derived in terms of them (and of course the fermion masses and CKM mixing parameters, along with the strong coupling constant $\alpha_s$). Equivalently, the muon decay constant, $G_\mu$, the Z-boson mass, $M_Z$, and the fine structure constant, $\alpha$, can be used as input parameters. Experimentally, the measured values for these input parameters are\(^{13, 14}\),

\[
\begin{align*}
G_\mu &= 1.16638(1) \times 10^{-5} \text{ GeV}^{-2} \\
M_Z &= 91.1876(21) \text{ GeV} \\
\alpha^{-1} &= 137.035999679(94) \\
m_h &= 125.09 \pm 0.21(\text{stat}) \pm 0.11(\text{syst}) \text{ GeV}.
\end{align*}
\]

The $W$ boson mass is thus a prediction of the theory and is defined through muon decay,

\[
\begin{align*}
M_W^2 &= \frac{\pi\alpha}{\sqrt{2}G_\mu(1 - M_W^2/M_Z^2)} \\
M_W^2 &= \frac{M_Z^2}{2} \left\{1 + \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_\mu M_Z^2}}\right\}. \quad (50)
\end{align*}
\]

At tree level, the SM prediction from Eq. 50 is,

\[
M_W(\text{tree}) = 79.829 \text{ GeV}, \quad (51)
\]

in slight disagreement with the measured value\(^{13}\),

\[
M_W(\text{experiment}) = 80.379 \pm 0.012 \text{ GeV}. \quad (52)
\]

\(^7\) An introductory review of precision measurements in the SM can be found in Ref. [12].
In order to obtain good agreement between theory and the experimental data, it is crucial to include radiative corrections. The prediction for $M_W$ can be written as[15],

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F s_W^2} \left[ 1 + \Delta r_{SM} \right],$$

where $\Delta r_{SM}$ summarizes the radiative corrections. The dependence on the top quark mass, $m_t$, is particularly significant as $\Delta r_{SM}$ depends on $m_t$ quadratically,

$$\Delta r_{SM}^t = -\frac{G_F N_c}{\sqrt{2}8\pi^2} \left( c_W^2 \right) m_t^2 + \log(m_t) \text{ terms},$$

where $N_c = 3$ is the number of colors. The dependence on $m_h$ is logarithmic,

$$\Delta r_{SM}^h \sim \frac{\alpha}{\pi s_W^2} \frac{11}{48} \log \left( \frac{m_h^2}{M_Z^2} \right) + \mathcal{O} \left( \frac{m_h^2}{M_Z^2}, v^4 \right).$$

The top quark does not decouple from the theory even at energies far above the top quark mass. This is because the top quark coupling to the Higgs boson is proportional to $m_t$.

The agreement between the radiatively corrected prediction for the $W$ mass given by Eq. 53 with the measured value is a strong test of the theory. In a similar fashion, the full set of electroweak data can be used to test the self consistency of the theory, as demonstrated in Fig. 2[16]. Similar studies have been performed by the GFITTER collaboration[17]. (The most restrictive data points are the measurements of the $Zb\bar{b}$ coupling and the $W$ boson mass.) When the experimental values of $M_W$, $m_t$, and $m_h$ are omitted, the fit is in good agreement with the directly measured values of the masses. Note that the fit excludes a large ($\sim 100'$s of GeV) value of $m_h$ and so even before the Higgs boson was discovered, we knew that if there were no new physics contributions to the predictions for electroweak quantities such as $M_W$, the Higgs boson could not be too heavy.

B. Oblique Parameters

Extensions of the SM with modified Higgs sectors are significantly restricted by the requirement of consistency with the electroweak measurements. A simple way to examine whether a theory with a complicated Higgs sector is consistent with electroweak experiments is to use the oblique parameters. Using the oblique parameters to obtain limits
FIG. 2: Experimental limits on $M_W$ and $m_t$ from precision electroweak measurements. The straight bands are the direct measurements of $M_W$ and $m_t$[16].

on BSM physics assumes that the dominant contributions resulting from the expanded theory are to the gauge boson 2-point functions[18, 19],

$$\Pi^{\mu\nu}_{XY}(p^2) = \Pi_{XY}(p^2)g^{\mu\nu} + B_{XY}(p^2)p^\mu p^\nu.$$  \hspace{1cm} (56)

with $XY = \gamma\gamma, \gamma Z, ZZ$ and $W^+W^-$. We define the $S, T$ and $U$ functions following the notation of Peskin and Takeuchi[18],

$$\alpha S = \left(\frac{4s_W^2c_W^2}{M_Z^2}\right) \left\{ \Pi_{ZZ}(M_Z^2) - \Pi_{ZZ}(0) - \Pi_{\gamma\gamma}(M_Z^2) \right\}$$

$$\alpha T = \left(\frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2} - \frac{2s_W}{c_W} \Pi_{\gamma Z}(0) \right)$$

$$\alpha U = 4s_W^2 \left\{ \frac{\Pi_{WW}(M_W^2)}{M_W^2} - \frac{\Pi_{WW}(0)}{M_W^2} - c_W^2 \left( \frac{\Pi_{ZZ}(M_Z^2) - \Pi_{ZZ}(0)}{M_Z^2} \right) \right\} - 2s_Wc_W \left( \frac{\Pi_{\gamma Z}(M_Z^2) - \Pi_{\gamma Z}(0)}{M_Z^2} \right) - s_W^2 \frac{\Pi_{\gamma\gamma}(M_Z^2)}{M_Z^2}.$$  \hspace{1cm} (57)

New physics effects are then determined by subtracting the SM contribution, e.g. $\Delta S \equiv S_{BSM} - S_{SM}$.  

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1. Aside: Restrictions on New Physics from Oblique Parameters with a Scalar Singlet

The simplest possible extension of the SM in the Higgs sector is to add a real scalar singlet, $S$, with hypercharge $Y = 0$. After imposing a $Z_2$ symmetry under which $S \rightarrow -S$, the most general scalar potential is

$$V = -\mu^2 \Phi^\dagger \Phi - m^2 S^2 + \lambda (\Phi^\dagger \Phi)^2 + \frac{a_2}{2} \Phi^\dagger \Phi S^2 + \frac{b_4}{4} S^4.$$  \hspace{1cm} (58)

After spontaneous symmetry breaking, both $\Phi$ and $S$ obtain VEVs and the mass eigenstates $h$ and $H$ are a mixture of $S$ and $\Phi$ ($s \equiv \langle S \rangle$),

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \sqrt{2}\phi_0 - v \\ S - s \end{pmatrix},$$  \hspace{1cm} (59)

with physical masses, $m_h$ and $M_H$. The singlet cannot couple directly to fermions or gauge bosons, so the only physical effect on single Higgs production is through the mixing of Eq. 59. The mixing affects the SM-like Higgs couplings to both fermions and gauge bosons in an identical fashion and all SM couplings are suppressed by the factor $\cos \alpha$. This model is particularly simple since it can be studied in terms of $M_H$ and the mixing angle $\alpha$. For $m_h, M_H >> M_W, M_Z$, the contributions to the oblique parameters are,

$$\Delta S = \frac{1}{12\pi} \sin^2 \alpha \log \left( \frac{M_H^2}{m_h^2} \right),$$

$$\Delta T = -\frac{3}{16\pi c_W^2} \sin^2 \alpha \log \left( \frac{M_H^2}{m_h^2} \right),$$

$$\Delta U = 0.$$  \hspace{1cm} (60)

and for any given value of $M_H$, an upper limit on $\sin \alpha$ can be determined[21]. Limits from the oblique parameters are an important tool in understanding what BSM models are allowed experimentally and in restricting the parameters of the models.

C. Restrictions from Unitarity

Scattering amplitudes in a weakly interacting theory are required to obey perturbative unitarity[22]. Cross sections that grow with energy will eventually violate perturbative unitarity, a simple result derived from the optical theorem. The simplest version of the unitarity requirement is,

$$| Re(a_0^0) | < \frac{1}{2},$$  \hspace{1cm} (61)
FIG. 3: Diagrams contributing to $W^+_L W^-_L \rightarrow W^+_L W^-_L$.

where $a_0^0$ is the $J = 0$ partial wave. For a $2 \rightarrow 2$ scattering process with massless particles,

$$a_0 = \frac{1}{16\pi s} \int_{-s}^{0} |A|.$$  \hspace{1cm} (62)

The interesting physics is in the longitudinal gauge boson sector, since the longitudinal polarizations are the result of the electroweak symmetry breaking. For a gauge boson $V = (W, Z)$ with momentum in the $z-$ direction,

$$p_V = (E_V, 0, 0, p),$$  \hspace{1cm} (63)

the longitudinal polarization vector is,

$$\epsilon^\mu_L = \frac{1}{M_V} (p, 0, 0, E_V) \rightarrow_{E_V \gg M_V} \frac{p^\mu_V}{M_V} + O(\frac{M_2^2}{E_V^2}).$$  \hspace{1cm} (64)

Eq. 64 makes it clear that longitudinally polarized gauge bosons can give potentially dangerous contributions to scattering processes at high energies.

The elastic scattering of longitudinally polarized gauge bosons, $W^+_L W^-_L \rightarrow W^+_L W^-_L$, illustrates this point[23]. The $s-$ and $t-$ channel diagrams containing the exchange of a $Z$ boson or $\gamma$, combined with the 4–point interaction shown in Fig. 3 give the contribution to the scattering amplitude in the limit $s \gg M_W^2, M_Z^2$,

$$\mathcal{A}(W^+_L W^-_L \rightarrow W^+_L W^-_L)_V \sim \frac{1}{v^2} \left\{ -s - t \right\}.$$  \hspace{1cm} (65)
This contribution grows with $s$. However, the $s-$ and $t-$ channel contributions from Higgs exchange shown in Fig. 4 contribute,

$$\mathcal{A}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-)_h \sim -\frac{1}{v^2} \left\{ \frac{s^2}{s - m_h^2} + \frac{t^2}{t - m_h^2} \right\}. \quad (66)$$

Combining Eqs. 65 and 66, we find the high energy limit, $M_W^2 << s$, of the $J = 0$ partial wave,

$$a_0^0(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) \equiv \frac{1}{16\pi s} \int_{-s}^{0} |A| dt + \mathcal{O} \left( \frac{s}{M_W^2} \right) = -\frac{m_h^2}{16\pi v^2} \left[ 2 + \frac{m_h^2}{s - m_h^2} - \frac{m_h^2}{s} \log \left( 1 + \frac{s}{m_h^2} \right) \right]. \quad (67)$$

This limit is well behaved in the high energy limit, due the the cancellations between the different contributions. Note that this cancellation requires the SM Higgs boson contribution with SM couplings to the gauge bosons.

In the high energy limit, $s >> m_h^2$, Eq. 67 has the limit,

$$|a_0^0(W_L^+ W_L^- \rightarrow W_L^+ W_L^-)| \rightarrow_{s >> m_h^2} \approx \frac{m_h^2}{8\pi v^2}. \quad (68)$$

Eq. 68 gives the upper bound on the SM Higgs mass from perturbative unitarity of $m_h \lesssim 870$ GeV.

Although individual contributions in Eqs. 65 and 66 grow with energy, in the SM they combine in just the right fashion to preserve perturbative unitarity. This is another strong constraint on BSM models. If the $WWZ$ coupling were altered from the SM value, the high energy cancellation of Eq. 67 would not occur and partial wave unitarity would be
violated. The Higgs boson plays a fundamental role in the theory since it cuts off the growth of the partial wave amplitudes and makes the theory obey perturbative unitarity.

D. Restrictions from Triviality

Theoretical bounds on the Higgs boson mass can be deduced on the grounds of triviality, which can be summarized as the requirement that the Higgs quartic coupling remain finite at high energy scales. If the quartic coupling becomes infinite, the theory is no longer perturbative, while if the quartic coupling goes to zero, the theory is non-interacting. The Higgs quartic coupling, $\lambda$, changes with the effective energy scale, $\Lambda$, due to the self interactions of the scalar field:

$$\frac{d\lambda}{dt} = \frac{3\lambda^2}{4\pi^2},$$

(69)

where $t \equiv \log(\Lambda^2/v^2)$. In the SM, however, there are also contributions due to gauge boson and fermion loops\(^8\). Including the top quark contribution, Eq. 70 becomes,

$$\frac{d\lambda}{dt} = \frac{3}{4\pi^2} \left\{ \lambda^2 - Y_t^2\lambda - Y_t^4 \right\},$$

(70)

where $Y_t = m_t/v$. For small $\lambda$ (small $m_h$), the $Y_t^4$ term dominates and the quartic coupling decreases with energy,

$$\lambda(\Lambda) \sim \lambda(v) - \frac{3Y_t^4}{4\pi^2} \log\left(\frac{\Lambda^2}{v^2}\right).$$

(71)

The scaling of $\lambda$ has been performed to 2-loops\[^24\], including contributions from gauge and Yukawa couplings and the result is shown in Fig. 5. The quartic coupling becomes negative at a high scale that is quite sensitive to $m_t$ and $\alpha_s$, suggesting that at this scale some new physics is required to force $\lambda$ to be positive which is need in order for the potential to be bounded from below.

IV. HIGGS PRODUCTION AND DECAY

In this section we review the SM rates for Higgs production and decay. Numerical values, including the most precisely known higher order calculations, have been tabulated by the LHC Higgs cross section working group\[^25\].

\(^8\) We neglect the gauge contributions here.
A. Higgs Decays

Expressions for the SM Higgs decay widths at leading order can be found in Ref. [9], and the QCD corrected rates, with references to the original literature, are given in Refs. [5, 6]. The QCD NLO corrected decay rates can be found using the public code, HDECAY [26].

1. $h \to f\bar{f}$

The Higgs couplings to fermions are proportional to fermion mass and the lowest order width for the Higgs decay to fermions of mass $m_f$ is,

$$
\Gamma(h \to f\bar{f}) = \frac{G_F m_f^2 N_{ci}}{4\sqrt{2}\pi} m_h \beta^3_F ,
$$

(72)

where $\beta_F \equiv \sqrt{1 - 4m_f^2/m_h^2}$ is the velocity of the final state fermions and $N_{ci} = 1(3)$ for charged leptons (fermions). The largest fermion decay channel is $h \to b\bar{b}$, which receives large QCD corrections. A significant portion of the QCD corrections can be accounted for by expressing the decay width in terms of a running quark mass, $m_f(\mu)$, evaluated at

FIG. 5: Dependence of the Higgs quartic coupling on the renormalization scale [24].

$\beta_{L} = 0.1184 \pm 0.0007$ HredL
$M_h = 125.1 \pm 0.2$ GeV HblueL
$M_t = 175.6$ GeV
$\alpha_{s}(M_Z) = 0.1163$
$\alpha_{s}(M_Z) = 0.1205$
the scale $\mu = m_h$. The QCD corrected decay width can then be approximated as\[27, 28,\]
\[
\Gamma(h \to qq) = \frac{3G_F}{4\sqrt{2}\pi} m_h^2 m_h^2 \beta_q \left( 1 + 5.67 \frac{\alpha_s(m_h^2)}{\pi} + \cdots \right),
\]
where $\alpha_s(m_h^2)$ is defined in the $\overline{MS}$ scheme with 5 flavors. In leading log QCD, the running of the $b$ quark mass is,
\[
m_b(\nu^2) = m \left[ \frac{\alpha_s(m^2)}{\alpha_s(\nu^2)} \right]^{(-12/23)} \left\{ 1 + O(\alpha_s^2) \right\},
\]
where $m_b(m^2) \equiv m$ implies that the running mass at the position of the propagator pole is equal to the location of the pole. For $m_b(m_b^2) = 4.18 \text{ GeV}$, this yields an effective value $m_b(m_h) = 125 \text{ GeV}$ \((\text{at LL} = 2.8 \text{ GeV}) \text{ (at NLL, } m_b(m_h) = 125 \text{ GeV}) \text{ (at NLL = 2.7 GeV)}\). Inserting the QCD corrected mass into the expression for the width thus leads to a suppression of the width by $\sim 0.4$. Using the running $b$ mass absorbs the large logarithms of the form $\log(m_b^2/m_h^2)$ and is important for numerical accuracy. The electroweak radiative corrections to $h \to f\bar{f}$ amount to only a few percent correction\[29,\]

2. $h \to WW, ZZ$

The Higgs boson can also decay to gauge boson pairs. At tree level, the decays $h \to WW^*$ and $h \to ZZ^*$ are possible (with one of the gauge bosons off-shell), while at one-loop the decays $h \to gg, \gamma\gamma$, and $\gamma Z$ occur.

The decay width for the off-shell decay, $h \to ZZ^* \to f_1(p_1)f_2(p_2)Z(p_3)$, is,
\[
\Gamma = \int_0^{(m_h-M_Z)^2} dq^2 \int dm^2_{23} \frac{|A|^2}{256\pi^3 m_h^3},
\]
where $m_{ij} = (p_i + p_j)^2$, $m_{12}^2 \equiv q^2$, and $m_{12}^2 + m_{23}^2 + m_{13}^2 = m_h^2 + M_Z^2$, $\lambda(m_h^2, M_Z^2, q^2) \equiv q^4 - 2q^2(m_h^2 + M_Z^2) + (m_h^2 - M_Z^2)^2$, and $m_{23}^2 |_{\text{max,min}} = \frac{1}{2} \left( m_h^2 + M_Z^2 - q^2 \pm \sqrt{\lambda} \right)$. The amplitude-squared is,
\[
| A(h \to Zf\bar{f}) |^2 = 32 (g_L^2 + g_R^2) C_Z^2 M_Z^4 \left[ \frac{2M_Z^2 q^2 - m_{13}^2 q^2 - m_h^2 M_Z^2 + m_{13}^2 M_Z^2 + m_{13}^2 m_h^2 - m_{13}^4}{(q^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^4} \right],
\]
with $g_{LL} = T_3f - Q_f s_W^2$, $g_{RF} = -Q_f s_W^2$, and $T_3 = \pm \frac{1}{2}$. We see that the amplitude is
peaked at $q^2 = M_Z^2$. Integrating over $dm_{23}^2$, 
\[
\frac{d\Gamma}{dq^2}(h \to Zf\bar{f}) = (g_L^2 + g_R^2) G_F^2 \sqrt{\lambda(m_h^2, M_Z^2, q^2)} \frac{M_Z^4}{48\pi^3 m_h^3} \left[ \left( \frac{12 M_Z^2 q^2 + \lambda(m_h^2, M_Z^2, q^2)}{(q^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \right) \right].
\]

The result for $h \to Wf\bar{f}$ can be found by making the appropriate redefinitions of the fermion - gauge boson couplings.

Performing the $q^2$ integral and summing over the final state fermions[30],
\[
\begin{align*}
\Gamma(h \to WW^*) &= \frac{3 g^4 m_h}{512\pi^3} \frac{F(M_W)}{m_h} \\
\Gamma(h \to ZZ^*) &= \frac{g^4 m_h}{2048 \cos^4 W \pi^3} \left( 7 - \frac{40}{3} s_W^2 + \frac{160}{9} s_W^4 \right) \frac{F(M_Z)}{m_h},
\end{align*}
\]

where
\[
F(x) = -\left| 1 - x^2 \right| \left( \frac{47}{2} x^2 - \frac{13}{2} + \frac{1}{x^2} \right) + 3(1 - 6x^2 + 4x^4) \ln x + \frac{3(1 - 8x^2 + 20x^4)}{\sqrt{4x^2 - 1}} \cos^{-1} \left( \frac{3x^2 - 1}{2x^3} \right).
\]

The NLO QCD and electroweak corrections to the off-shell decays, $h \to V^*V^* \to 4\text{-}fermions$, $V = (W, Z)$, are implemented in the public code, PROPHECY4f[31].

3. $h \to gg$

The decay of the Higgs boson to gluons only arises through fermion loops in the SM and is sensitive to new colored particles that interact with the Higgs,
\[
\Gamma(h \to gg) = \frac{G_F\alpha_s^2 m_h^3}{64\sqrt{2}\pi^3} \sum_q F_{1/2}(\tau_q) |^2,
\]

where $\tau_q \equiv 4 m_q^2 / m_h^2$ and $F_{1/2}(\tau_q)$ is defined to be,
\[
F_{1/2}(\tau_q) \equiv -2\tau_q \left[ 1 + (1 - \tau_q) f(\tau_q) \right].
\]

The function $f(\tau_q)$ is given by,
\[
f(\tau_q) = \begin{cases} 
    \sin^{-1} \left( \sqrt{1/\tau_q} \right)^2, & \text{if } \tau_q \geq 1 \\
    -\frac{1}{4} \left[ \log \left( \frac{\tau_q}{\tau_q^2} \right) - i\pi \right]^2, & \text{if } \tau_q < 1,
\end{cases}
\]
with
\[ x_\pm = 1 \pm \sqrt{1 - \tau_q}. \] (83)

In the limit in which the quark mass is much less than the Higgs boson mass,
\[ F_{1/2} \to \frac{2m_q^2}{m_h^2} \log^2 \left( \frac{m_q}{m_h} \right). \] (84)

On the other hand, for a heavy quark, \( \tau_q \to \infty \), and \( F_{1/2}(\tau_q) \) approaches a constant,
\[ F_{1/2} \to -\frac{4}{3}. \] (85)

Eqs. 84 and 85 make it clear that the top quark loop is the dominant contribution. QCD corrections to the decay \( h \to gg \) are known at NLO for a finite top quark mass and increase the rate by roughly 60%[32].

4. \( h \to \gamma\gamma \)

The decay \( h \to \gamma\gamma \) arises from fermion and \( W \) loops and is an important mode for Higgs measurements at the LHC, despite the smallness of the branching ratio. At lowest order the width is, \[ \Gamma(h \to \gamma\gamma) = \frac{\alpha^2 G_F}{128 \sqrt{2} \pi^3} m_h^3 \left| \sum_i N_{ci} Q_i^2 F_i(\tau_i) \right|^2, \] (86)
where the sum is over fermions and \( W^\pm \) bosons with \( F_{1/2}(\tau_q) \) given in Eq. 81, and
\[ F_W(\tau_W) = 2 + 3\tau_W [1 + (2 - \tau_W) f(\tau_W)], \] (87)

with \( \tau_W = 4M_W^2/m_h^2 \), \( N_{ci} = 1(3) \) for leptons (quarks), and \( Q_i \) is the electric charge in units of \( e \). In the (unphysical) limit \( \tau_W \to \infty \), \( F_W \to 7 \) and we see that the top quark and \( W \) contributions have opposite signs. The decay \( h \to \gamma\gamma \) is therefore sensitive to the sign of the top quark Yukawa coupling through the interference of the \( W \) and \( t \) loops. Similarly, the rate for \( h \to Z\gamma \) receives contributions from both fermions and the \( W \) boson. The analytic formula is given in [9] and the \( Z\gamma \) width is quite small.

The Higgs branching ratios are shown in Fig. 6 for a SM Higgs boson of arbitrary mass[25]. The width of the curves is an estimate of the theoretical uncertainties on the branching ratios. The branching ratios assume SM couplings and no new decay channels.
FIG. 6: SM Higgs Branching ratios (LHS) and total width for a SM-like Higgs boson of arbitrary mass (RHS)\cite{25}. In this figure, $H$ is the SM Higgs boson.

and include all known radiative corrections\cite{25}. Also shown in Fig. 6 is the Higgs total decay width as a function of Higgs mass. For $m_h = 125 \text{ GeV}$, the total width is very narrow, $\Gamma_h = 4 \text{ MeV}$.

B. Higgs Production in Hadronic Collisions

At the LHC, the dominant production mechanisms are gluon fusion, followed by vector boson fusion, shown in Fig. 7. The associated production mechanisms of the Higgs with vector bosons or top quarks have smaller rates, but these channels are theoretically important and are shown in Fig. 8. It is immediately apparent that gluon fusion and $t\bar{t}h$ production are sensitive to the top quark Yukawa coupling, while vector boson fusion and associated $hV$, $V = (W, Z)$, production probe the gauge-Higgs couplings.

The total rates for Higgs production in various channels are shown on the LHS of Fig. 9 for arbitrary Higgs mass at 13 $\text{ TeV}$ (LHS) and as a function of center-of-mass energy (RHS) for the physics Higgs mass. The curves include the most up-to-date theoretical calculations, and the width of the curves represents an estimate of the uncertainties. We will discuss each production channel in turn in this section\cite{33}.
FIG. 7: Contribution to Higgs boson production from (LHS) gluon fusion and (RHS) vector boson scattering. In this figure, $H$ is the SM Higgs boson.

FIG. 8: Contribution to Higgs boson production from (LHS) associated $Vh$ production and (RHS) $t\bar{t}h$ production. In this figure, $H$ is the SM Higgs boson.

1. $gg \to h$

The primary production mechanism for a Higgs boson in hadronic collisions is through the couplings to heavy fermions, $gg \to h$, which is shown on the LHS of Fig. 7. This process is dominated by the top quark loop and the loop with a bottom quark contributes roughly $-5\%$ to the SM cross section.

The lowest order (LO) amplitude for $g^{A,\mu}(p) + g^{B,\nu}(q) \to h$ from a quark of mass $m_q$ in the loop is,

$$A^{\mu\nu}(g^A g^B \to h) = -\frac{\alpha_s m_q^2}{\pi v} \delta_{AB} \left( g^{\mu\nu} \frac{m_h^2}{2} - p^\nu q^\mu \right)$$

$$\cdot \int_0^1 dx \int_0^{1-x} dy \left( \frac{1 - 4xy}{m_q^2 - m_h^2 x y} \right) \epsilon_\mu(p) \epsilon_\nu(q)$$

$$= \frac{\alpha_s}{4\pi v} \delta_{AB} \left( g^{\mu\nu} \frac{m_h^2}{2} - p^\nu q^\mu \right) \xi_1/2(\tau_q) \epsilon_\mu(p) \epsilon_\nu(q)$$

$$\to -\frac{\alpha_s}{3\pi v} \delta_{AB} \left( g^{\mu\nu} \frac{m_h^2}{2} - p^\nu q^\mu \right) \epsilon_\mu(p) \epsilon_\nu(q) \quad \text{if} \quad m_q \gg m_h. \quad (88)$$
FIG. 9: Total Higgs production cross sections\[33\]. In this figure, $H$ is the SM Higgs boson.

The partonic cross section can be found from the general resonance formula,

$$\hat{\sigma}(gg \rightarrow h) = \frac{16\pi^2}{m_h} (2J + 1) \frac{1}{64} \cdot \frac{1}{4} \cdot 2\Gamma(h \rightarrow gg)\delta(s - m_h^2),$$

where the factors of $\frac{1}{64}$ and $\frac{1}{4}$ are the color and spin averages, $J = 0$ is the Higgs spin, $s$ is the $gg$ partonic sub-energy, and the factor of 2 undoes the identical particle factor of $\frac{1}{2}$ in the decay width $\Gamma(h \rightarrow gg)$. The lowest order partonic cross section for $gg \rightarrow h$ is,

$$\hat{\sigma}(gg \rightarrow h) = \frac{\alpha_s^2}{1024\pi v^2} \sum_q F_{1/2}(\tau_q) \left| 2\Gamma(h \rightarrow gg)\delta(s - m_h^2) \right|,$$

$$\equiv \hat{\sigma}_0(gg \rightarrow h)\delta \left(1 - \frac{s}{m_h^2}\right).$$

In the heavy quark limit, the cross section is independent of the top quark mass and becomes a constant,

$$\hat{\sigma}_0(gg \rightarrow h) \sim \frac{\alpha_s^2}{512\pi v^2}.$$  

The heavy fermions do not decouple at high energy and the gluon fusion rate essentially counts the number of SM-like chiral quarks.

The Higgs boson production cross section at a hadron collider can be found by integrating the partonic cross section, $\sigma_0(pp \rightarrow h)$, with the gluon parton distribution functions, $g(x, \mu)$,

$$\sigma(pp \rightarrow h) = \hat{\sigma}_0 z \int_d^1 \frac{dx}{x} g(x, \mu) g \left( \frac{z}{x}, \mu \right),$$

where $\sigma_0$ is given in Eq. 90, $z \equiv m_h^2/S$, $\mu$ is the factorization scale and $S$ is the hadronic center of mass energy. It is particularly interesting to consider the theoretical accuracy
FIG. 10: ATLAS measurements of the gluon fusion Higgs cross section, compared to theory predictions[35] In this figure, $H$ is the SM Higgs boson mass.

at $N^3LO[34]$,

$$\sigma(pp \rightarrow h)[13 \text{ TeV}] = 48.58^{+4.6\%}_{-6.7\%}(\text{theory}) \pm 3.2\%(\text{PDF} + \alpha_s), \quad (93)$$

where the theory uncertainty arises predominantly from the scale choice and the PDF+\alpha_s uncertainty is the PDF and correlated uncertainty on \alpha_s.

The measured Higgs rate immediately rules out the possibility of a 4th generation of SM chiral fermions. Imagine that there are heavy fermions, $T$ and $B$, with identical quantum numbers as the SM top and bottom quarks. The new fermions would contribute to Higgs production from gluon fusion as on the LHS of Fig. 7. From Eq. 91, we would have,

$$\hat{\sigma}_0(gg \rightarrow h) \rightarrow \frac{\alpha_s^2}{576\pi v^2} \left[ 1 + 1 + 1 \right]^2 \rightarrow 9\hat{\sigma}_0(SM), \quad (94)$$

where the factors in the square bracket represent the contributions of the SM $t$, $T$ and $B$. This is obviously excluded by the measured rate for gluon fusion Higgs production, which is in good agreement with the SM prediction, shown in Fig. 10.

The tensor structure of Eq. 88 is exactly that required for the production of a spin-0 particle from 2-gluons with momentum, $g(k_1)$ and $g(k_2)$. Starting from a $G_{\mu\nu}G^{\mu\nu}$ term in the Lagrangian and considering only the Abelian contributions for now,

$$G_{\mu\nu}G^{\mu\nu} \rightarrow (\partial_\mu G_\nu - \partial_\nu G_\mu)(\partial^\mu G^\nu - \partial^\nu G^\mu). \quad (95)$$

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Making the replacement $\partial_\mu \to ik_\mu$,

$$G_{\mu\nu}G^{\mu\nu} \to -(k_1^\mu G^\nu_1 - k_1^\nu G^\mu_1)(k_2^\mu G^\nu_2 - k_2^\nu G^\mu_2)$$

$$= -2 \left( k_1 \cdot k_2 G_1 \cdot G_2 - k_1 \cdot G_2 k_2 \cdot G_1 \right)$$

$$= -2 k_1 \cdot k_2 G_1 G_2 \left[ g^{\mu\nu} - \frac{k_1^\mu k_2^\nu}{k_1 \cdot k_2} \right]. \quad (96)$$

Comparing Eqs. 88 and 96\(^9\) suggests that the heavy quark limit for the gluon fusion production of a Higgs boson can be obtained from the effective dimension-5 Lagrangian

$$L_{EFT} = \frac{\alpha_s}{12\pi} \frac{h}{v} G^A_{\mu\nu} G^{\mu\nu A}. \quad (97)$$

The effective Lagrangian of Eq. 97 has been used to calculate the QCD corrections to gluon fusion to NLO, NNLO, and N\(^3\)LO\[^{34}]\). The result is shown in Fig. 11. Note that there is a large correction (approximately a factor of 2) going from LO to NLO. The corrections at each order remain sizable and the dependence on the factorization scale, $\mu$, is reduced at higher order.

**Aside: Vector-like Fermions and Gluon Fusion**

The agreement of the measured rate for gluon fusion with the SM rate does not mean that all heavy fermions are excluded. A vector-like fermion is defined to have identical $SU(2)_L \times U(1)_Y$ transformation properties for both the left- and right-handed components. The simplest possibility is to add a fermionic top partner, $T$, for which both the left- and

\(^9\) The extra factor of $\frac{1}{2}$ comes from the neglected color factor, $Tr(T^AT^B) = \frac{1}{2} \delta_{AB}$. 

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right-handed components are weak singlets and color triplets\textsuperscript{10}. In this scenario, the top partner can have a Dirac mass, (which has nothing to do with electroweak symmetry breaking), and can mix with the SM top quark, \( t \). The most general Yukawa interaction for a top partner singlet is\cite{36, 37}

\[ -L_Y \sim \lambda_1 \overline{q}_L \hat{\Phi} t_R + \lambda_2 \overline{q}_L \hat{\Phi} T_R + \lambda_3 \overline{T}_L t_R + \lambda_4 \overline{T}_L T_R , \]

(corresponding to the fermion mass matrix,

\[ M^t = \begin{pmatrix} \lambda_1 \frac{v}{\sqrt{2}} & \lambda_2 \frac{v}{\sqrt{2}} \\ \lambda_3 & \lambda_4 \end{pmatrix} . \]

The mass eigenstates of charge \( \frac{2}{3} \) (\( t_1 \) and \( t_2 \), with masses \( m_{t1} \) and \( m_{t2} \)) are found through the rotations,

\[ \begin{pmatrix} t_{1,L,R} \\ t_{2,L,R} \end{pmatrix} \equiv U_{L,R} \begin{pmatrix} t_{L,R} \\ T_{L,R} \end{pmatrix} . \]

The matrices \( U_{L,R} \) are unitary and can be parameterized as

\[ U_L = \begin{pmatrix} \cos \theta_L & -\sin \theta_L \\ \sin \theta_L & \cos \theta_L \end{pmatrix} , \quad U_R = \begin{pmatrix} \cos \theta_R & -\sin \theta_R \\ \sin \theta_R & \cos \theta_R \end{pmatrix} . \]

The physical charge \( \frac{2}{3} \) particles, \( t_1 \) and \( t_2 \), are therefore mixtures of \( t \) and \( T \),

\[ t_{1(L,R)} = \cos \theta_{L,R} t_{L,R} - \sin \theta_{L,R} T_{L,R} \]
\[ t_{2(L,R)} = \sin \theta_{L,R} t_{L,R} + \cos \theta_{L,R} T_{L,R} , \]

The important point is that the couplings to the Higgs boson are changed in models with vector-like fermions,

\[ L_h \rightarrow -\frac{m_{t1}}{v} \cos^2 \theta_L \overline{t}_{1,L} t_{1,R} h - \frac{m_{t2}}{v} \sin^2 \theta_L \overline{t}_{2,L} t_{2,R} h - \frac{m_{t1}}{v} \cos \theta_L \sin \theta_L \overline{t}_{1,L} t_{2,R} h - \frac{m_{t1}}{v} \cos \theta_L \sin \theta_L \overline{t}_{2,L} t_{1,R} h + h.c. \]

In the large mass limit (\( m_{t1}, m_{t2} \gg m_h \)), the top and top partner contributions to gluon fusion yield

\[ \hat{\sigma}_0 (gg \rightarrow h) \rightarrow \frac{\alpha_s^2}{576 \pi v^2} \left[ \cos^2 \theta_L + \sin^2 \theta_L \right] + \mathcal{O} \left( \frac{m_h^2}{m_{t1}^2}, \frac{m_h^2}{m_{t2}^2} \right) \]
\[ \rightarrow \hat{\sigma}_0 (SM) . \]

\textsuperscript{10} Recall that left- and right-handed fermions contribute with opposite signs to anomalies and so the contributions cancel. Therefore, it is not required to have a full generation of vector-like fermions to cancel anomalies.
FIG. 12: General Higgs coupling to fermions. On the left-hand side is any initial or final state $X$.

This equivalence with the SM gluon fusion rate in models with heavy vector-like fermions is a general feature\cite{38, 39}. Observing the effects of top partners in single Higgs rates will be difficult, and instead models with vector-like fermions are best probed by searches for direct production of the new heavy quarks.

**Aside: Low Energy Theorems**

We have seen that both in the SM and in the vector-like top partner singlet model, the gluon fusion contribution to Higgs production takes a simple form in the heavy fermion mass limit. The idea that the Higgs gluon interactions due to heavy particles can be derived from an effective Lagrangian as in Eq. 97 gives rise to low energy theorems for Higgs-gluon couplings\cite{40–42}. Consider the Higgs coupling to a heavy fermion with mass $m$ as part of a complicated Feynman diagram as shown in Fig. 12. The sub-amplitude from the fermion-Higgs coupling can be written in the limit $p_h \to 0$ as,

$$\left(...\right) \frac{i}{k-m} \frac{-im}{v} \frac{i}{k-p_h-m} \left(...\right) \rightarrow \left(...\right) \frac{im}{v} \left(\frac{1}{k-m}\right)^2 \left(...\right)$$

$$= \left(...\right) \frac{im}{v} \frac{\partial}{m} \left(\frac{1}{k-m}\right) \left(...\right). \quad (105)$$

This observation has been formalized to a theorem,

$$\lim(p_h \to 0) A(hX) = \frac{m}{v} \frac{\partial}{\partial m} A(X), \quad (106)$$

where $A(X)$ is the amplitude for creating a state $X$. That is, adding a Higgs boson to a
diagram is equivalent to taking the derivative with respect to the heavy fermion mass\(^{11}\). It is important to note that the derivative is with respect to the unrenormalized mass. At higher orders, there are contributions from \(\partial m_R / \partial m\), where \(m_R\) is the renormalized mass.

Let us apply Eq. 106 to the gluon 2-point function,

\[
L = -\frac{1}{4g_s^2} G_{\mu\nu}^A G_{\mu\nu}^A,
\]

where we have factored all coupling constant dependence out of the gluon field strength, \(G_{\mu\nu}^A g_s \equiv G_{\mu\nu}^A\), for convenience. Applying the low energy theorem of Eq. 106,

\[
L_{EFT} = -\frac{1}{4v} \left[ m \frac{\partial}{\partial m} \frac{1}{g_s^2} \right] G_{\mu\nu}^A G_{\mu\nu}^A.
\]

The dependence of \(g_s\) on scale is given by the QCD \(\beta\) function,

\[
\beta \equiv \mu \frac{\partial g_s}{\partial \mu}.
\]

where only the heavy top quark contributes to the \(\beta\) function here,

\[
\frac{\beta}{g_s} \to \frac{\alpha_s}{6\pi}.
\]

The effective Lagrangian of Eq. 97 follows immediately.

The low energy theorem is more than just a curiosity. The effective field theory (EFT) of Eq. 97 has been used to calculate radiative corrections to Higgs production in the large \(m_t\) limit at NLO\(^{43, 44}\), NNLO\(^{45}\), and N\(^3\)LO\(^{34}\). For the NLO corrections to \(gg \rightarrow h\), the 2-loop virtual corrections in the full theory become 1-loop calculations using the EFT and so on. This greatly reduces the complexity of the problem. At NNLO, the validity of the EFT has been checked numerically in the exact (top mass dependent) theory by expanding the propagators in the large top quark mass limit, and the agreement is within a few percent\(^{46, 47}\). Practically speaking, the higher order results obtained using the EFT are typically used to rescale the LO (or NLO) kinematic distributions obtained by including the full top quark mass dependence.

The low energy theorem is particularly useful for estimating the effects of BSM physics on the gluon fusion rate\(^{41}\). Consider, for example, a model with multiple heavy fermions,

\(^{11}\) Identical reasoning holds for the coupling of a Higgs boson to gauge bosons.
\( \mathcal{F}_i \), where the interactions in the mass basis are,
\[
L \sim \Sigma_i \mathcal{F}_i \tilde{Y}_i (h + v) \mathcal{F}_i ,
\]
with the fermion masses given by \( m_i = v \tilde{Y}_i \). Then the obvious generalization of the results of the previous section is,
\[
L_{\text{EFT}} = \frac{\alpha_s}{12\pi} h \sum_i \frac{\tilde{Y}_i}{m_i} G_{\mu\nu}^A G^{\mu\nu A} .
\]
In general, however, the fermion-Higgs couplings are specified in the gauge basis, and it is quite a bit of work to obtain the results in the mass basis. This step can be eliminated using the low energy theorems. We can start from the general interactions of the fermions in the gauge basis, and diagonalize the fermion mass matrix, \( M \), using a unitary matrix, \( U \), where \( M_D = U^\dagger M U \),
\[
L \sim \Sigma_{ij} \overline{f}_i (h + v) f_j = \overline{f} U U^\dagger Y U \dagger f (h + v) ,
\]
where the diagonal mass matrix is \( M_D = v U^\dagger Y U \) and \( M = Y v \) (\( M, M_D, \tilde{Y} \) and \( Y \) are now all interpreted as matrices). The Higgs couplings to gluons are determined,
\[
R_g \equiv \Sigma_i \frac{\tilde{Y}_i}{m_i}
= \Sigma_i \left( \frac{U^\dagger Y U}{m} \right)_{ii}
= Tr(U^\dagger Y U M^{-1}_D) = Tr(Y U M^{-1}_D U^\dagger) .
\]
Using the matrix identity \( M^{-1} M = M^{-1} U M_D U^\dagger = 1 \),
\[
R_g = Tr(Y[M^{-1} U M_D U^\dagger][U M_D^{-1} U^\dagger])
= Tr(Y M^{-1})
= Tr(\frac{\partial M}{\partial v} M^{-1})
= \frac{\partial}{\partial v} \log \left( \det(M) \right)
= \frac{\partial}{\partial v} Tr \left( \log(M) \right) .
\]
The effective Lagrangian is finally given by\([39, 41, 42]\),
\[
L_{\text{EFT}} = \frac{\alpha_s}{12\pi} h \left( \frac{\partial}{\partial \log(v)} Tr(\log(M)) \right) G_{\mu\nu}^A G^{\mu\nu A} ,
\]
and there is no need to diagonalize the mass matrix.
2. $p_T$ distribution of Higgs Bosons

At LO, the Higgs boson has no $p_T$ and a transverse momentum spectrum for the Higgs is first generated by the process, $gg \to gh$, which is an NLO contribution to the gluon fusion process\[48\]. As $p_T \to 0$, the partonic cross section for Higgs plus jet production diverges as $1/p_T^2$,

$$\frac{d\hat{\sigma}}{dt}(gg \to gh) = \hat{\sigma}_0 \frac{3\alpha_s}{2\pi} \left\{ \frac{1}{p_T^2} \left[ \left( 1 - \frac{m_h^2}{s} \right)^4 + 1 + \left( \frac{m_h^2}{s} \right)^4 \right] \right\} - \frac{4}{s} \left( 1 - \frac{m_h^2}{s} \right)^2 + \frac{2p_T^2}{s},$$  \tag{117}

where $\hat{\sigma}_0$ is the LO $gg \to h$ cross section given in Eq. 90, and $s, t$ and $u$ are the partonic Mandelstam invariants. The $p_T$ spectrum for Higgs plus jet at LO is shown in Fig. 13, where the contributions from the $gg$ and $qg, qg$ initial states are shown separately. Also shown is the $m_t \to \infty$ limit of the spectrum that is derived from the effective Lagrangian of Eq. 97. The effective Lagrangian approximation fails around $p_T \sim 2m_t$. In this process, there are several distinct momentum scales ($p_T, m_h, m_t$), as opposed to gluon fusion where there is only a single scale ($m_h/m_t$) at LO. The expansion in $m_h/m_t$ for $gg \to gh$ receives corrections of $\mathcal{O}(\alpha_s^2, p_T^2)$ and for $p_T > 2m_t$, the EFT large top quark mass expansion cannot be used to obtain reliable distributions.

NLO, NNLO, and N$^3$LO radiative corrections to Higgs plus jet production have been calculated\[49–52\] using the $m_t \to \infty$ approximation. The lowest order result of Eq. 117 is then reweighted by a $K$ factor derived in the $m_t \to \infty$ limit for each kinematic bin. The effects of the higher order corrections are significant and increase the rate by a factor of around 1.8 as shown in Fig. 14. The singularity of the LO result at $p_T = 0$ is clearly visible in Fig. 14 and we note that after the inclusion of the NLO corrections, the $p_T$ spectrum no longer diverges as $p_T \to 0.$

The terms which are singular as $p_T \to 0$ can be isolated and the integrals performed explicitly. Considering only the $gg$ initial state\[53\],

$$\frac{d\sigma}{dp_T^2 dy}(pp \to gh) \bigg|_{p_T \to 0} \sim \hat{\sigma}_0 \frac{3\alpha_s}{2\pi} \left[ 6 \log \left( \frac{m_h^2}{p_T^2} \right) - \frac{20}{3} \right] g(ze^y)g(ze^{-y}) + \ldots$$  \tag{118}

where $z \equiv m_h^2/S, \quad \beta_0 = (33 - 2n_f)/6$, and $n_f = 5$ is the number of light flavors. Clearly when $p_T << m_h$, the terms containing the logarithms resulting from soft gluon emission can give a large numerical contribution. The logarithms of the form $\alpha_s^2 \log^m(m_h^2/p_T^2)$ can
FIG. 13: Lowest order $p_T$ spectrum for Higgs plus jet production from Eq. 117 and the large $m_t$ approximation of Eq. 97.

FIG. 14: QCD corrected $p_T$ spectrum for Higgs plus jet production at $\sqrt{S} = 8 \text{ TeV}$\cite{49, 50}. In this figure, $H$ is the SM Higgs boson.

be resummed\cite{53, 54} to improve the theoretical accuracy in the regime $p_T \to 0$, as can be seen in the curve labelled NLL+LO in Fig. 15. Additional logarithms can also be resummed\cite{55}, as shown in the curve labelled NNLL+NLO in Fig. 16.

3. Measuring the Higgs width with $gg \to h \to ZZ$

Gluon fusion with the subsequent Higgs decay to $ZZ \to 4$ leptons or $\gamma\gamma$ were the Higgs discovery channels. The $h \to ZZ \to 4$ lepton signals at $13 \text{ TeV}$ are shown in Fig. 17\cite{56, 57} and the Higgs resonance is clearly visible. Making a direct measurement of the
FIG. 15: QCD NLL resummed $p_T$ spectrum for Higgs plus jet production at $\sqrt{s} = 14$ TeV[53]. In this figure, $H$ is the SM Higgs boson.

FIG. 16: NNLL QCD resummed $p_T$ spectrum for Higgs plus jet production[55]. In this figure, $H$ is the SM Higgs boson.

Higgs width by fitting a Breit-Wigner function to the resonance shape is not possible since the detector resolution is $\mathcal{O}(1-2)$ GeV, much larger than the Higgs width, $\Gamma_H \sim 4$ MeV.

A clever idea uses the properties of the longitudinal $Z$ polarizations[58, 59]. Consider
FIG. 17: $h \rightarrow ZZ \rightarrow 4\ell$ signal at 13 TeV[56, 57].

FIG. 18: Contributions to $gg \rightarrow ZZ \rightarrow 4\ell$. The dominant contributions to the triangle and box diagrams are from the top quark.

the process $gg \rightarrow ZZ \rightarrow 4\ell$ shown in Fig. 18. The Higgs contribution is shown on the LHS of Fig. 18 and the partonic cross section from the Higgs contribution alone is generically given by,

$$\hat{\sigma}(gg \rightarrow h \rightarrow ZZ) \sim \int ds \frac{|A(gg \rightarrow h)|^2 |A(h \rightarrow ZZ)|^2}{(s - m_h^2)^2 + \Gamma_h^2 m_h^2}.$$  \hspace{1cm} (119)

We allow the effective $gg \rightarrow h$ and $h \rightarrow ZZ$ couplings to be scaled from the SM values by arbitrary factors $\kappa_g(s)$ and $\kappa_Z(s)$, where we explicitly note that the $\kappa$ factors can in principle depend on scale,

$$|A(gg \rightarrow h)|^2 |A(h \rightarrow ZZ)|^2 \sim \kappa_g^2(s) \kappa_Z^2(s) |\epsilon_{Z1} \cdot \epsilon_{Z2}|^2,$$  \hspace{1cm} (120)

where $\epsilon_{Zi}$ are the Z polarization vectors.

The interesting observation is that Eq. 119 behaves very differently above the Higgs resonance and near the resonance. Above the resonance, $s >> m_h^2$, Eq. 119 becomes,

$$\hat{\sigma}(gg \rightarrow h \rightarrow ZZ)_{\text{above}} \sim \int ds \frac{\kappa_g^2(s) \kappa_Z^2(s)}{s^2} |\epsilon_{Z1} \cdot \epsilon_{Z2}|^2.$$  \hspace{1cm} (121)
For transverse polarizations, nothing particularly interesting happens, but because of the electroweak symmetry breaking the longitudinally polarized $Z$ bosons have a novel feature. Defining the momenta of the outgoing $Z$ bosons as $p_{Z1}$ and $p_{Z2}$ and remembering that the longitudinal polarization is approximately given by,

$$\epsilon^\mu_L(p_{Z}) \sim \frac{p^\mu_{Z}}{M_Z} + O\left(\frac{M_Z^2}{s}\right),$$

we observe that $\epsilon_L \cdot \epsilon_L \sim \frac{p_{Z1} \cdot p_{Z2}}{M_Z^2}$. Eq. 121 has the approximate form for $s >> m_h^2$,

$$\hat{\sigma}(gg \rightarrow h \rightarrow Z_LZ_L)^{\text{above}} \sim \int ds \frac{\kappa_y^2(s)\kappa_Z^2(s)}{M_Z^2}. \quad \text{(123)}$$

We note that Eq. 123 exhibits no dependence on the Higgs width.

Near the Higgs resonance, we can use the narrow width approximation, which amounts to the replacement,

$$\frac{1}{(s-m_h^2)^2 + (m_h\Gamma_h)^2} \rightarrow \frac{\pi}{m_h\Gamma_h} \delta(s-m_h^2)$$

and Eq. 119 is approximately,

$$\hat{\sigma}(gg \rightarrow h \rightarrow ZZ)^{\text{on}} \sim \frac{\kappa_y^2(m_h^2)\kappa_Z^2(m_h^2)}{m_h\Gamma_h}. \quad \text{(125)}$$

The idea is that by measuring the $gg \rightarrow h \rightarrow ZZ$ rate above and on the resonance, information can be extracted about the Higgs width. Assuming the $\kappa$ factors do not depend on scale,

$$\Gamma_h \sim \frac{\hat{\sigma}^{\text{above}}}{\hat{\sigma}^{\text{on}}}. \quad \text{(126)}$$

At 8 TeV, approximately 15% of the cross section has $m_{4l} > 140$ GeV, so this is a promising idea. If the $\kappa$ factors have an energy dependence, they do not cancel in Eq. 126 and the interpretation of the measurement becomes more complicated.

Of course, a real calculation needs to include both the diagrams of Fig. 18, along with the interference, and this has been done by several groups with results shown in Fig. 19. The importance of including the interference terms is apparent, but the long tail at high $m_{4l}$ (shown in red) is clear. ATLAS and CMS have used this technique to place limits on the Higgs width[60, 61],

$$\Gamma_h \lesssim (4 - 5)\Gamma_{h}^{SM}. \quad \text{(127)}$$
FIG. 19: Contributions to $gg \rightarrow ZZ \rightarrow 4l$ at 8 TeV. The Higgs contributions are shown in red, while the total rate from gluon fusion including interference is given in magenta[62].

There are some big assumptions in this extraction of the Higgs width, the most obvious of which is the assumption that the $\kappa$ factors are the same on and off the Higgs resonance peak. This is clearly a false assumption, since in a quantum field theory all couplings run. If there are anomalous $hZZ$ (or $hgg$) couplings, than the running could be changed significantly[63, 64]. For example, a contribution to the EFT of the form,

$$L \sim \frac{c_Z}{\Lambda^2} \frac{h}{v} Z_{\mu\nu}^2 Z_{\mu\nu}^2$$

would give contributions of $O\left(\frac{s}{\Lambda^2}\right)$ and would cause $m_4l$ to grow above the peak, and would invalidate the extraction of $\Gamma_h$. Additional colored particles in the $ggh$ loop would also change the interpretation of the $gg \rightarrow ZZ \rightarrow 4\text{ lepton}$ result as a measurement of the Higgs width[65].

It is worth noting that an $e^+e^-$ collider with an energy of $\sqrt{s} = 500\, GeV$ can make a 5% measurement of $\Gamma_h$ with an integrated luminosity of 500 GeV[66]. First the measurement of $e^+e^- \rightarrow Zh$ is made by tagging the $Zh$ events where the recoil mass is consistent with a Higgs boson. This is done using conservation of momenta and determines $\sigma(Zh)$. Next we can measure the $h \rightarrow ZZ$ rate to determine $BR(h \rightarrow ZZ)$. The Higgs width is then determined in a model independent fashion,

$$\Gamma_h = \Gamma(h \rightarrow ZZ)BR(h \rightarrow ZZ)$$

$$\sim \frac{\sigma(Zh)}{BR(h \rightarrow ZZ)}.$$

(129)
4. Vector Boson Scattering

The vector boson scattering (VBS) process is shown on the RHS of Fig. 7. It can be thought of as 2 incoming quarks each radiating a $W$ or $Z$ boson, which then form a Higgs. Vector boson fusion also offers the opportunity to observe the $2 \rightarrow 2$ scattering process, $VV \rightarrow VV$, ($V = Z, W$), which is extremely sensitive to new physics in the electroweak sector. The $VV \rightarrow VV$ sub-process plays a special role in Higgs physics since the Higgs exchange contributions unitarize the scattering amplitude, as discussed in Sec. III C.

VBS production of a Higgs occurs through the purely electroweak process $q\bar{q}' \rightarrow q\bar{q}'h$ which has a distinctive experimental signature and vanishes in the limit $v = 0$. The outgoing jets are peaked in the forward and backward regions and can be used to tag the VBF event. This can easily be seen by considering the top leg of the RHS of Fig. 7:

$$q(p) \rightarrow q'(p')V(k).$$

(130)

In the lab frame,

$$p \equiv E(1, 0, 0, 1)$$

$$p' \equiv E'(1, 0, \sin \theta, \cos \theta).$$

(131)

The integral over the final state phase space for the VBS scattering cross section has a generic contribution,

$$\sigma \sim \int \frac{\text{Phase Space}}{[(p - p')^2 - M_V^2]^2} \sim \int \frac{\theta d\theta}{[2EE'(1 - \cos \theta) - M_V^2]^2} \sim \int \frac{\theta d\theta}{[\theta^2 - M_V^2 / EE']^2}$$

(132)

which is enhanced in the $\theta \rightarrow 0$ region for $E, E' >> M_V^2$. In addition, these forward tagging jets have a large invariant mass and small $p_T$. Typical cuts on the jets are,

$$p_T > 20 \text{ GeV, } |y_j| < 5, |y_{j_1} - y_{j_2}| > 3, M_{jj} > 130 \text{ GeV}.$$

(133)

The decay products from the intermediate $VV$ scattering are mostly contained in the central rapidity region. These characteristics can be used to separate VBS scattering from QCD gluon initiated events and the non-VBS contributions can be suppressed to $\sim 1 - 2\%$[67]. The ability to separate the Higgs signal into gluon initiated events and VBF events is crucial for the extraction of Higgs coupling constants.
5. Associated Production

At the LHC the process $q\bar{q} \rightarrow Vh$ offers the hope of being able to tag the Higgs boson by the $V$ boson decay products$[68]$, although as shown in Fig. 9 the rate is significantly smaller than the dominant $gg \rightarrow h$ production mechanism. The cross section for $Wh$ production is,

$$\hat{\sigma}(q_i\bar{q}_j \rightarrow W^\pm h) = \frac{G_F^2 M_W^6 |V_{ij}|^2}{6\pi s^2 (1 - M_W^2/s)^2} \lambda_{Wh}^{1/2} \left[ 1 + \frac{s \lambda_{Wh}}{12 M_W^2} \right],$$

(134)

where $\lambda_{Wh} = 1 - 2(M_W^2 + m_h^2)/s + (M_W^2 - m_h^2)^2/s^2$ and $V_{ij}$ is the CKM angle associated with the $q_i\bar{q}_jW$ vertex. The rate for $Zh$ is about a factor of 3 smaller than that for $Wh$ and analytic results can be found in Ref. [5]. The NNLO QCD and NLO electroweak corrections are known, so there is relatively little uncertainty on the prediction$[69, 70]$. The $Vh$ associated channel has recently been used to observe the decay $h \rightarrow bb^\pm$[71, 72], using the jet substructure techniques first proposed in Ref. [73]. The idea is that by going to high transverse momentum for the Higgs, the backgrounds can be significantly reduced. Jet substructure techniques are discussed in the lectures of Schwartz at this school[74].

6. $t\bar{t}h$ Production

The top quark Yukawa coupling, $Y_t$, can be directly measured in the $t\bar{t}h$ process shown on the RHS of Fig. 8. Recall that the gluon fusion production of the Higgs is also proportional to the top quark Yukawa, but in addition it can receive enhanced contributions from the bottom quark Yukawa interactions in some BSM scenarios, along with contributions from new colored scalars. The NLO QCD$[75–78]$ and electroweak corrections$[79, 80]$ for $t\bar{t}h$ production are known and contribute to very precise predictions$[33]$:

$$\sqrt{S} = 8 \text{ TeV} \quad \sigma_{t\bar{t}h} = .133 \, pb^{+4\%}_{-9\%}(scale) \pm 4.3\% (PDF + \alpha_s)$$

$$\sqrt{S} = 13 \text{ TeV} \quad \sigma_{t\bar{t}h} = .507 \, pb^{+5.8\%}_{-9.2\%}(scale) \pm 3.6\% (PDF + \alpha_s).$$

(135)

Although numerically small, electroweak corrections spoil the direct proportionality of the lowest order cross section to $Y_t^2$.

This process has large backgrounds from $t\bar{t}b\bar{b}$ and $t\bar{t}jj$. In order to suppress the backgrounds, many $t\bar{t}h$ searches are done in the boosted regime, where the electroweak Sudakov
FIG. 20: Contributions to $gg \rightarrow hh$ in the SM. The dominant contribution to the triangle and box diagrams are from the top quark. In this figure, $H$ is the SM Higgs boson.

logarithms become relevant. A definitive measurement of this channel has not yet been made, and will be one of the important milestones of the coming LHC run.

The associated production of $b\bar{b}h$ is not relevant in the SM, but can be important in models with enhanced $b$ Yukawa couplings.

7. Double Higgs Production

Finally, we need to measure the parameters of the Higgs potential, Eq. 45, to determine if electroweak symmetry breaking really proceeds as in the SM. In the SM, the Higgs potential from Eq. 5 is,

$$V = \frac{m_h^2}{2} h^2 + \lambda_3 h^3 + \lambda_4 h^4,$$

(136)

where $\lambda_3^{SM} = m_h^2/(2v)$ and $\lambda_4^{SM} = h^2/(8v^2)$. It is apparent that the Higgs self- couplings are weak,

$$\lambda_3^{SM} = .13v, \quad \lambda_4^{SM} = .03.$$

(137)

The only way to directly probe the $h^3$ coupling is by double Higgs production and the dominant production mechanism is gluon fusion as shown in Fig. 23. The result is sensitive to new colored particles running in the loops, along with modifications to the Higgs tri-linear self-coupling and the top quark Yukawa coupling (Eqs. 136 and 41).

The amplitude for $g^{A,\mu}(p_1)g^{B,\nu}(p_2) \rightarrow h(p_3)h(p_4)$ is

$$A_{AB}^{\mu\nu} = \frac{\alpha_s}{8\pi v^2} \delta_{AB} \left[ P_0^{\mu\nu} (p_1, p_2) \tilde{F}_1(s, t, u, m_t^2) + P_2^{\mu\nu} (p_1, p_2, p_3) \tilde{F}_2(s, t, u, m_t^2) \right],$$

(138)

where $P_0$ and $P_2$ are the orthogonal projectors onto the spin-0 and spin-2 states respec-
tively,
\[ P_0^{\mu\nu}(p_1, p_2) = g^{\mu\nu} - \frac{p_1^\mu p_2^\nu}{p_1 \cdot p_2}, \]
\[ P_2^{\mu\nu}(p_1, p_2, p_3) = g^{\mu\nu} + \frac{2}{s p_T^2} \left( m_h^2 p_1^\mu p_2^\nu - 2 p_1 \cdot p_3 p_2^\mu p_3^\nu - 2 p_2 \cdot p_3 p_1^\mu p_3^\nu + s p_3^\mu p_3^\nu \right), \]
(139)
\[ s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2, \quad u = (p_2 - p_3)^2, \]
and \( p_T \) is the transverse momentum of the Higgs boson,
\[ p_T^2 = \frac{ut - m_h^4}{s}. \]
(140)
The functions \( \hat{F}_1 \) and \( \hat{F}_2 \) are known analytically \cite{81, 82}.

In the SM, the largest contributions come from top quark loops and in the limit \( m_t^2 \gg s \), the leading terms are,
\[ \hat{F}_1(s, t, u, m_t^2) = \hat{F}_1^{\text{tri}}(s, t, u, m_t^2) + \hat{F}_1^{\text{box}}(s, t, u, m_t^2) \]
\[ \hat{F}_1^{\text{tri}}(s, t, u, m_t^2) = -4 \frac{m_h^2}{s - m_h^2} s \left( \frac{\lambda_3}{\lambda_3^{\text{SM}}} \right) \]
\[ \hat{F}_1^{\text{box}}(s, t, u, m_t^2) = -4 \frac{s}{3} \]
\[ \hat{F}_2(s, t, u, m_t^2) = -11 \frac{p_T^2}{45 s} \]
(141)
where we have allowed an arbitrary rescaling of the Higgs tri-linear coupling. It is important to remember that in the SM, there is no freedom to rescale \( \lambda_3 \), making this a BSM effect.

We see that the amplitude vanishes at threshold in the SM in the large \( m_t \) limit, reducing the sensitivity to \( \lambda_3 \). The expansion in powers of \( 1/m_t \) poorly reproduces kinematic distributions, due to the presence of contributions proportional to \( s/m_t^2 \), as is obvious in Fig. 21\cite{39, 83}.

The large \( m_t \) limit has been used to compute QCD corrections to NLO \cite{84} and NNLO\cite{85}. In this approach, a K factor is computed:
\[ K = \frac{d\sigma_{\text{NNLO}}}{d\sigma_{\text{LO}}}, \]
(142)
where the distributions in Eq. 142 are computed in the \( m_t \to \infty \) limit and are then used to rescale the lowest order distributions computed with finite \( m_t \)\footnote{This is termed the B.i. NLO HEFT in Fig. 22.}. The exact
FIG. 21: LO transverse momentum distribution for double Higgs production in the SM, compared with the large $m_t$ limit, along with the first correction of $\mathcal{O}(s/m_t^2)$.

FIG. 22: Transverse momentum distribution for double Higgs production in the SM, including various approximations for the QCD corrections. The curve labelled NLO includes all finite $m_t$ effects\cite{91}.

NLO result for double Higgs production including all top mass effects is now known and can be used to obtain distributions\cite{90, 91}. The effects of including the top quark mass exactly at NLO are significant and reduce the total cross section by $\sim 14\%$ at 14 TeV from the B.i. NLO HEFT limit. Including the top quark mass effects also has significant effects on distributions, as demonstrated in Fig. 22.

The dependence of $hh$ production on $\lambda_3$ from various production mechanisms is shown...
in Fig. 23[92] as a function of \( \delta_3 \equiv \frac{\lambda_3}{\lambda_{3SM}} \).\textsuperscript{13}

The best current limits from the 8 TeV data on double Higgs production are,

\[
\frac{\sigma(pp \rightarrow hh)}{\sigma(pp \rightarrow hh) |_{SM}} < 29 \quad \text{ATLAS},
\]

\[
\frac{\sigma(pp \rightarrow hh)}{\sigma(pp \rightarrow hh) |_{SM}} < 19 \quad \text{CMS},
\]  \hspace{1cm} (143)

which still leaves a way to go before we get to an interesting regime. The ATLAS limit is
from the \( b\bar{b}bb \) final state\textsuperscript{93}, while the CMS limit is from the \( b\bar{b}\gamma\gamma \) final state\textsuperscript{94}. ATLAS
estimates that a luminosity of 3 ab\(^{-1}\) will be sensitive to \( \delta_3 > 8.7 \) and \( \delta_3 < -1.3 \)\textsuperscript{95}. This
is clearly not the precision measurement we desire and the need to measure the Higgs
tri-linear coupling is one of the major motivations for a 100 TeV collider.

The fact that the SM rate for double Higgs production is quite small makes it an
ideal place to search for new physics. Many models (singlet, 2HDM, MSSM, NMSSM,
etc)\textsuperscript{96–100} contain heavy neutral scalars that can decay into 2 SM Higgs bosons with a
significant (\( \sim 30\% \)) branching ratio. In these models, there is an \( s-\) channel resonance
from the heavy Higgs particle, and there will be interference between this new scalar and
the SM Higgs giving the classic dip structure shown in Fig. 24 for the example of the
singlet model. Limits on resonant decays in the generic BSM process, \( gg \rightarrow X \rightarrow hh \) for
various final states are shown in Fig. 25, where for heavy resonances, the most important
search channel is the 4\( b \) final state.

\textsuperscript{13} The curve labelled EFT loop-improved is identical to the B.i. NLO HEFT approximation.
FIG. 24: Double Higgs production in the $Z_2$ symmetric singlet model with a heavy neutral scalar of mass $M_H = 300$ GeV\cite{96}.

FIG. 25: Experimental limits from the LHC on $hh$ production in a BSM theory containing an $s-$ channel scalar resonance with mass $M_X$\cite{33}.

It has been proposed that indirect limits on $\lambda_3$ may be extracted from the dependence of electroweak radiative corrections to single Higgs production on the Higgs tri-linear coupling. This coupling enters the rate for $gg \to h$ at 2− loops and contributes to the $t\bar{t}h$, $Vh$, and VBS processes at 1− loop. Of course $\lambda_3$ is not a free parameter in the SM, and some care must be taken with the renormalization prescription. Ref. \cite{101} obtains the allowed 2σ region from a fit to single Higgs production,

$$-9.4 < \delta_3 < 16.$$ \hspace{1cm} (144)

Similar allowed regions are obtained in Refs. \cite{102}–\cite{104}. The allowed parameter space from current fits to single Higgs production are not significantly different from the ex-
pected limits on $\lambda_3$ with $3ab^{-1}$ at the LHC.

V. EFFECTIVE FIELD THEORY AND THE HIGGS BOSON

A. Higgs Boson Coupling measurements

The production of the Higgs boson in Run-I at the LHC produced results which basically agree with the SM predictions at the $10-20\%$ level[105]. Preliminary Higgs coupling results at $13\,TeV$ [72, 106–110], are also in reasonable agreement with expectations. The rates are as predicted, and there are no non-SM like light (EW scale) particles observed.

What we need is a way to quantify small deviations from the SM predictions. The simplest way is to introduce an arbitrary scaling into the SM interactions,

$$L_{\kappa} = \sum_f \kappa_f \frac{m_f}{v} \bar{f} h + \kappa_W g M_W W^+ h + \kappa_Z g \frac{M_Z}{c_W} Z^\mu Z^\nu h. \tag{145}$$

In the SM, all $\kappa$ parameters are 1, so a deviation would indicate some physics not contained in the SM. Of course, Eq. 145 is not $SU(2)_L \times U(1)_Y$ gauge invariant, but it serves as a starting point for study.

For a given production and decay channel, $i \rightarrow h \rightarrow j$,

$$\kappa_i^2 = \frac{\sigma(i \rightarrow h)}{\sigma(i \rightarrow h)_{SM}}$$
$$\kappa_j^2 = \frac{\Gamma(h \rightarrow j)}{\Gamma(h \rightarrow j)_{SM}}. \tag{146}$$

The $\kappa$ formalism also rescales the total width,

$$\kappa_h \equiv \frac{\Gamma_h}{\Gamma_h^{SM}}$$
$$\Gamma_h = \sum_X \kappa_X^2 \Gamma(h \rightarrow XX) + \Gamma(h \rightarrow \text{invisible}), \tag{147}$$

where $\Gamma(h \rightarrow \text{invisible})$ is any unobserved decay. This approach assumes that there are no new light resonances, no new tensor structures in the Higgs interactions beyond those of the SM, that the narrow width approximation for Higgs decays is valid, and is based on rescaling total rates (that is, no new dynamics is included).

A combined CMS/ATLAS fit is shown in Fig. 26. This particular fit does not allow for new physics in the $gg \rightarrow h$ and $h \rightarrow \gamma\gamma$ channels, but instead parameterizes the effective
couplings in terms of the SM interactions of the Higgs with the top and bottom ($\kappa_g$) and
with the $W$ and top ($\kappa_\gamma$) as,
\[
\kappa_g^2 \sim 1.06\kappa_t^2 + 0.01\kappa_b^2 - 0.07\kappa_t\kappa_b
\]
\[
\kappa_\gamma^2 \sim 1.59\kappa_W^2 + 0.07\kappa_t^2 - 0.66\kappa_W\kappa_t.
\] (148)

Similar results are shown in Fig. 27, and again the results are in general agreement with
the SM predictions. With the addition of 13 $TeV$ data, the Higgs couplings should become
even more constrained. In particular, the $tth$ and $bbh$ coupling measurements have been
significantly updated from Fig. 27.

ATLAS and CMS have various types of fits. In some fits, they separate Higgs bosons
from different production and decay channels. Other fits allow for unobserved decay
channels, or new contributions to gluon fusion or the decay to $\gamma\gamma$. None of the fits show
any significant deviation from the SM predictions.

Finally, a fit to all Higgs production and decay channels yields the combined AT-
LAS/CMS result[105],
\[
\mu \equiv \frac{\sigma_h}{\sigma_h(SM)} = 1.0 \pm 0.07(stat) \pm 0.04(syst) \pm 0.03(theory).
\] (149)

From Eq. 149, it is clear that the accuracy of the theoretical predictions will soon be the
limiting factor in the interpretation of Higgs measurements.

To improve on the fits to total rates, we need to construct an effective field theory,
which is the topic of the next section.

B. Effective Field Theory Basics

The effective field theory (EFT) Lagrangian we use assumes that there are no new light
degrees of freedom and is constructed by writing an $SU(2)_L \times U(1)_Y$ invariant Lagrangian
as an expansion in powers of $v/\Lambda$, where $\Lambda$ is some high scale where we envision that there
is a UV complete theory[113, 114],
\[
L_{EFT} = L_{SM} + \sum_i \frac{c_5^i}{\Lambda} O_i^5 + \sum_i \frac{c_6^i}{\Lambda^2} O_i^6 + ....
\] (150)

and $O_i^n$ is a dimension-$n$ operator constructed from SM fields. The EFT allows for a sys-
tematic study of BSM physics effects in a gauge invariant fashion and radiative corrections
can be implemented order by order in $\frac{v}{\Lambda}$.
The only possible dimension-5 operator violates lepton number conservation and is typically neglected in studies of Higgs physics. There are many possible bases for constructing the dimension-6 operators, of which the most well-known are the Warsaw[115], HISZ[116], and SILH[117] bases. By using the equations of motion, there is a mapping from one basis to the next[118, 119]. Note that the HISZ basis does not contain fermion interactions.

There are several approaches to using the dimension-6 truncation of the EFT of Eq.
One could calculate an amplitude to $\mathcal{O}\left(\frac{v^2}{\Lambda^2}\right)$,

$$A \sim A_{SM} + \frac{A_{EFT}^6}{\Lambda^2}.$$  

(S151)

Squaring the amplitude,

$$|A|^2 \sim |A_{SM} + \frac{A_{EFT}^6}{\Lambda^2}|^2,$$  

(S152)

we obtain results that are guaranteed to be positive-definite. The problem is that Eq. 152 contains terms $\sim (\frac{A_{EFT}^6}{\Lambda^2})^2$ that are of the same order in $v^2/\Lambda^2$ as the neglected dimension-8 terms. The expansion only makes sense if

$$|A_{EFT}^6|^2 << |A_{SM}^* A_{EFT}^8|,$$  

(S153)

which can be arranged in some BSM models[120].

We begin by considering a simple EFT with just 2 non-SM terms,

$$L \sim L_{SM} + \frac{\alpha_s}{4\pi} \frac{c_g}{\Lambda^2} (\Phi^\dagger \Phi) G^A_{\mu\nu} G^{\mu\nu A} + \left( \frac{c_t Y_t}{\Lambda^2} \bar{q}_L \Phi q_R (\Phi^\dagger \Phi) + h.c. \right).$$  

(S154)

After spontaneous symmetry breaking, the top mass is shifted,

$$m_t = \frac{Y_t v}{\sqrt{2}} \left( 1 - \frac{v^2 c_t}{2\Lambda^2} \right).$$  

(S155)

The Higgs coupling to the top quark is no longer proportional to $m_t$ and Eq. 154 becomes

$$L \rightarrow \frac{\alpha_s}{4\pi} \frac{c_g}{\Lambda^2} h G^A_{\mu\nu} G^{\mu\nu A} - m_t \bar{t} t \left[ 1 + \frac{h}{v} \left( 1 - \frac{v^2 c_t}{2\Lambda^2} \right) \right] + ...$$  

(S156)

When flavor indices are included in the fermion interactions, Eq. 154 can generate flavor violation in the Higgs sector[121].

Both $c_g$ and $c_t$ contribute to $gg \rightarrow h$,

$$\sigma(gg \rightarrow h) = \sigma(gg \rightarrow h)_{SM} \left( 1 + 2 \frac{v^2}{\Lambda^2} (3c_g - c_t) \right) + \mathcal{O}\left( \frac{m_h^2}{m_t^2}, \frac{v^4}{\Lambda^2} \right),$$  

(S157)

and so gluon fusion cannot distinguish between $c_g$ and $c_t$[113, 122–126]. The $t\bar{t}h$ process is independent of $c_g$ at leading order and can be used to obtain a measurement of $c_t$. Once

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\textit{Caveat emptor}: Practically every EFT paper uses different normalization and sign conventions for the EFT operators. The only way to check results like Eq. 157 is to start from the definition of the operators in the Lagrangian.
radiative corrections (both QCD and electroweak) are included, however, the situation becomes murkier and the $t\bar{t}h$ rate is no longer directly proportional to $c_t$.

At dimension-6, the unique operator contributing to gluon fusion of the Higgs is

$$O_1 = G^A_{\mu\nu} G^{\mu\nu, A} \Phi^\dagger \Phi,$$  \hspace{1cm} (158)

generating the effective Lagrangian of Eq. 154 and discussed in Sec. IV B.1. The Higgs gluon effective interactions can be further altered at dimension-8 by the inclusion of the operators,[115, 127–131],

$$O_2 = D_\sigma G^A_{\mu\nu} D^\sigma G^{A,\mu\nu} h$$

$$O_3 = f_{ABC} G^A_{\nu} G^{B,\nu} G^{C,\sigma} h$$

$$O_4 = g_s^2 h \Sigma_{i,j=1}^{\text{jet}} \bar{\psi}_i \gamma_\mu T^A \psi_j \bar{\psi}_j \gamma^\mu T^A \psi_j$$

$$O_5 = g_s h \Sigma_{i=1}^{\text{jet}} G^A_{\mu\nu} D_\mu \bar{\psi}_i \gamma_\nu T^A \psi_i. \hspace{1cm} (159)$$

The operator, $O_3$, of Eq. 159 not only affects Higgs interactions, but also changes the kinematics of dijet production[132]. The Higgs $p_T$ spectrum discussed in Sect. IV B.2 can be significantly affected by the presence of the dimension-8 operators[127, 128, 130, 131].

In Fig. 28, we show the effects on the Higgs $p_T$ spectrum for the lowest order rate for $gg \rightarrow gh$ with a cut implemented on the jet energy of $p_T \text{cut}$. For $p_T \gtrsim 300 \text{ GeV}$, the effects of the higher dimension operators can be numerically relevant. This plot illustrates an important point about the EFT expansion. For $2 \rightarrow 2$ processes, there are contributions of $O\left(\frac{\Lambda^2}{\Lambda^2}\right)$, so care needs to be taken to stay in the region of validity of the expansion\(^\text{16}\).

We turn now to a discussion of the effects of dimension-6 operators in the electroweak sector. As an example, we consider the SILH basis relevant for gauge-Higgs

\(^{15}\) $\kappa_t$ and $\kappa_g$ are defined in Eq. 145 and $\kappa_5$ is the scaling relative to the contribution of a 500 GeV scalar as discussed in Ref. [127].

\(^{16}\) This failure of the EFT also occurs in the $m_t \rightarrow \infty$ limit of the $gg \rightarrow hh$ process discussed in Sec. IV B.7.
FIG. 28: Effects of the dimension-8 operators of Eq. 159 on the $p_T$ spectrum of $gg \rightarrow gh$[127].

interactions[117],

$$L_{SILH} = \frac{c_H}{2\Lambda^2} \left( \partial^\mu \Phi \right)^2 + \frac{c_T}{2\Lambda^2} \left( \Phi \langle \overleftrightarrow{D^\mu \Phi} \rangle \right)^2 + \left( \frac{c_f y_f}{\Lambda^2} \Phi \right)^2 \mathcal{F}_L \Phi f_R + h.c. \right) - \frac{c_6}{\Lambda^2} | \phi |^6$$

$$+ \frac{i g c_W}{2\Lambda^2} \left( \phi \sigma^I \overleftrightarrow{D^\mu \Phi} \right) \left( D^\nu W^I_{\mu \nu} \right) + \frac{i g' c_B}{2\Lambda^2} \left( \phi \overleftrightarrow{D^\mu \Phi} \right) \left( D^\nu B_{\mu \nu} \right)$$

$$+ \frac{i g c_{HW}}{16\pi^2\Lambda^2} \left( D^\mu \Phi \right)^I \sigma^I \left( D^\nu \Phi \right) W_{\mu \nu}^I + \frac{i g' c_{HB}}{16\pi^2\Lambda^2} \left( D^\mu \Phi \right)^I \left( D^\nu \Phi \right) B_{\mu \nu}$$

$$+ \frac{c_7 g' g s}{16\pi^2\Lambda^2} | \Phi |^2 B_{\mu \nu} B^{\mu \nu} + \frac{c_9 g s}{16\pi^2\Lambda^2} | \Phi |^2 G^{A,\mu \nu} G^{A,\mu \nu}. \tag{160}$$

Note that the normalization of the operators is arbitrary and merely reflects a prejudice about the origins of the new physics, $I = 1, 2, 3$ are SU(2) indices and we have not written terms involving only fermions, or terms that do not contain a Higgs field. Many of the operators of Eq. 160 introduce momentum dependence into the Higgs couplings to SM fermions and so the kinematic distributions of the Higgs will be affected.

We briefly discuss some of the phenomenological effects of Eq. 160. Three of the coefficients are strongly limited by precision electroweak measurements as parameterized
by the oblique parameters,

\[ \Delta T = \frac{v^2}{\Lambda^2} c_T \]
\[ \Delta S = \frac{M_W^2}{\Lambda^2} (c_W + c_B). \]  

Using the fit from Ref. [16], \( |c_T| \lesssim O(0.03) \) and \( |c_W + c_B| \lesssim O(1) \) for \( \Lambda \sim 1 \text{ TeV} \).

The coefficient \( c_H \) modifies the Higgs boson kinetic energy. The physical Higgs field needs to be rescaled,

\[ h \rightarrow h \left( 1 - \frac{c_H v^2}{2 \Lambda^2} \right), \]  

in order to have canonically normalized kinetic energy. This shift introduces a dependence on \( c_H \) into all of the Higgs decay widths. The tree level Higgs decay widths to \( \mathcal{O}(v^2/\Lambda^2) \) in the SILH formalism are,

\[ \frac{\Gamma(h \rightarrow W W^*)}{\Gamma(h \rightarrow W W^*)_{|SM}} = 1 - \frac{v^2}{\Lambda^2} \left[ c_H - g^2 \left( c_W + \frac{c_{HW}}{16 \pi^2} \right) \right] \]
\[ \frac{\Gamma(h \rightarrow Z Z^*)}{\Gamma(h \rightarrow Z Z^*)_{|SM}} = 1 - \frac{v^2}{\Lambda^2} \left[ c_H - g^2 \left( c_W + \tan^2 \theta_W c_B + \frac{c_{HW} + \tan^2 \theta_2 c_{HB}}{16 \pi^2} \right) \right] \]
\[ \frac{\Gamma(h \rightarrow f \bar{f})}{\Gamma(h \rightarrow f \bar{f})_{|SM}} = 1 - \frac{v^2}{\Lambda^2} (c_H + 2 c_f). \]  

The loop processes, \( gg \rightarrow h \) and \( h \rightarrow \gamma \gamma \), also receive corrections from the EFT operators. The expressions for Higgs decays in the SILH Lagrangian have been implemented into an update of the HDECAY program, EDECAY[133]. In the Warsaw basis, they can be obtained using the SMEFTsim code[134]. Fits to the EFT coefficients can be performed using total Higgs rates (as is done in the \( \kappa \) formalism) or including information from distributions[102, 135]. The kinematic information provides a significant improvement to the fits from using only the total rates.

Some of the operators of Eq. 160 not only affect Higgs production, but they also change the \( WWZ \) and \( W W \gamma \) vertices. Assuming CP conservation, the most general Lorentz invariant 3–gauge boson couplings can be written as [136, 137]

\[ L_V = -i g_{WWV} \left[ g_1^V \left( W_{\mu \nu}^+ W^{-\mu \nu} - W_{\mu \nu}^- W^{+ \mu \nu} \right) + \kappa^V W_{\mu}^+ W_{\nu}^- V^{\mu \nu} \right. \]
\[ \left. + \frac{\lambda^V}{M_W^2} W_{\rho \mu}^+ W_{\nu}^- V^{\mu \rho} \right], \]  

51
where $V = (Z, \gamma)$, $g_{WW}^{\gamma} = e$, and $g_{WW}^{Z} = gc_{W}$. In the SM, $g_{1}^{Z} = g_{1}^{\gamma} = \kappa^{Z} = \kappa^{\gamma} = 1$,
$\chi^{Z} = \chi^{\gamma} = 0$ and $SU(2)$ gauge invariance implies,

$$
\chi^{\gamma} = \chi^{Z},
$$

$$
g_{1}^{Z} = \kappa^{Z} + \frac{s_{W}^{2}}{c_{W}}(\kappa^{\gamma} - 1).
$$

(165)

The fields in Eq. 164 are the canonically normalized mass eigenstate fields. These coefficients can be mapped to EFT coefficients in a straightforward manner and a subset of the dimension-6 coefficients contribute both to gauge boson pair production and Higgs production[135, 138, 139].

A consistent fit must include not only Higgs data, but also fits to anomalous gauge couplings. In Fig. 29, we show fits to 3 of the EFT couplings that contribute to both $W^{+}W^{-}$ and Higgs production, including only LEP data on $W^{+}W^{-}$ pair production, only LHC data on $W^{+}W^{-}$ and Higgs production, and the resulting fit combining the two. The LHC results have now surpassed the LEP results in terms of precision[135]. This figure includes the full set of dimension-6 squared contributions. In terms of the parameters of Eq. 164,

$$
f_{W} = \frac{2\Lambda^{2}}{M_{Z}^{2}}(g_{1}^{Z} - 1),
$$

$$
f_{B} = \frac{2\Lambda^{2}}{M_{W}^{2}}[(\kappa_{\gamma} - 1) - c_{W}^{2}(g_{1}^{Z} - 1)],
$$

$$
f_{WWW} = \frac{4\Lambda^{2}}{3g_{W}^{2}M_{W}^{2}}\chi^{\gamma}.
$$

(166)

Global fits to EFT coefficients in the SILH basis can be found in Ref. [102, 140] and in the Warsaw basis in Ref. [139]. Many of the EFT coefficients are only weakly constrained. These results illustrate, however, that fits performed to only a single operator typically significantly overestimate the sensitivity. As of this writing, the experimental collaborations have not performed such global EFT fits.

Finally, it is interesting to ask what the target precision is for measuring EFT coefficients. In any given UV complete model, these coefficients can be calculated, and the scale $\Lambda$ will be of the same order of magnitude as the mass of the new particles. This suggests that as direct searches for new particles get more and more precise, it is necessary to measure the EFT coefficients more and more precisely. In a specific UV complete
model, not all coefficients will be generated, and the pattern of non-zero coefficients will be a guide to the underlying model. The EFT coefficients for numerous models with heavy scalars[141–145] and heavy vector-like quarks[146, 147] are known and suggest that measurements of $O(2−3\%)$ will be necessary to probe models with new particles at the $2−3\ TeV$ scale.

VI. OUTLOOK

The discovery of a SM-like Higgs boson opened a new era in particle physics. We do not yet know if we have discovered a Higgs boson or the Higgs boson. To make this determination, the measurements of Higgs interactions need to be improved to the few % level and the Higgs self-interactions need to be observed. These precision measurements will begin during the high luminosity run of the LHC, but will require a future high energy hadron collider or $e^+e^−$ collider to reach the desired accuracy. A limiting factor will be the precision of theoretical predictions–predictions accurate at the few % level will require

FIG. 29: Fits to LEP data, LHC data and the combination of both[135].
a dedicated effort in the coming years and improvement of our knowledge of PDFs. I have not discussed models with extra scalar particles other than the singlet model. One of the most important efforts of the Higgs program in the next few years will be the search for additional Higgs-like particles. The observation of another scalar would be the cleanest possible indication of new BSM physics in the scalar sector.

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