Microwave response of an NS ring coupled to a superconducting resonator

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A long phase coherent normal (N) wire between superconductors (S) is characterized by a dense phase dependent Andreev spectrum. We probe this spectrum in a high frequency phase biased configuration, by coupling an NS ring to a multimode superconducting resonator. We detect a dc flux and frequency dependent response whose dissipative and non dissipative components are related by a simple Debye relaxation law with a characteristic time of the order of the diffusion time through the N part of the ring. The flux dependence exhibits \( h/2e \) periodic oscillations with a large harmonics content at temperatures where the Josephson current is purely sinusoidal. This is explained considering that the populations of the Andreev levels are frozen on the time-scale of the experiments.

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Most properties of a non superconducting (normal) metal connected to two superconductors (an SNS junction) can be seen as resulting from the Andreev states (AS) in the normal metal, and the occupation of those states. Andreev states are correlated electron-hole eigenstates in the normal metal, determined by boundary conditions imposed by the superconducting banks. In a long diffusive normal metal (of length \( L \) grater than the superconducting coherence length \( \xi_s \), the AS spectrum is a quasi continuum of levels with a small energy gap \( E_g \) [1, 2]. This so-called minigap depends solely on \( L \) and the diffusion constant \( D \), via the Thouless energy \( E_{th} = h/\tau_D \) where \( \tau_D = L^2/D \) is the diffusion time along the junction. \( E_g \) is fully modulated by the phase difference \( \varphi \) between the superconducting order parameters on both sides. \( E_g(\varphi) \) is maximal at \( \varphi = 0 \) with \( E_g(0) \simeq 3.1E_{Th} \) and goes linearly to zero at \( \varphi = \pi \), as was recently measured by scanning tunneling spectroscopy [3]. The phase dependent Josephson current \( I_J(\varphi) \) at equilibrium sums the contributions of each Andreev state of energy \( \epsilon_n \), via \( i_n = \frac{2e}{\hbar} \frac{\partial n}{\partial \varphi} \), the current carried by the level \( n \) of occupation factor \( p_n \).

\[
I_J(\varphi) = \sum_n p_n(\epsilon_n(\varphi))i_n(\varphi) \quad (1)
\]

As was measured recently by Hall magnetometry [6], the phase dependence of the supercurrent is non sinusoidal at low temperature, but turns sinusoidal at \( T > E_g \), with an amplitude that decreases roughly exponentially with temperature on the scale of \( E_g \).

Whereas the equilibrium properties of the proximity effect (PE) in long SNS junction are well understood theoretically and experimentally, its dynamics is a more complex and yet unresolved issue. In particular the current response of the system to a time dependent phase which affects both energy levels and their populations still needs to be determined. We expect at least the existence of two time constants, one related to the relaxation of the populations to a change of the energy levels, another one related to the dynamics of the Andreev levels, and possibly also a contribution due to the generation of quasiparticles. Which of the electron-electron, electron-phonon, diffusion or dephasing times are relevant in these processes? Experimentally, there are many ways in which to impose an out of equilibrium situation. Because of the Josephson relation linking the voltage to the time derivative of the phase, a voltage bias across the junction causes a time dependent phase, and the \( I(V) \) curve is a probe of the dynamics of the PE. Irradiating the junction with an rf field, which produces Shapiro steps in the \( I(V) \) curve, is another. Both the critical current in current biased experiments [4, 5] and the complete Josephson current phase relation [6] have been shown to be very sensitive to RF excitation at frequencies of the order of or larger than \( 1/\tau_D \). In addition, the electron-phonon scattering rate has also been shown to set the frequency scale for the hysteresis in current biased experiments [5].

In contrast to all these experiments performed in strongly non linear regimes, in this Letter we present a linear response experiment in which we measure both the non dissipative (inphase), \( \chi' \) and dissipative (out of phase), \( \chi'' \) current response of an SN ring. We measure these with imposing an oscillating phase difference, at several RF frequencies, as a function of the dc superconducting phase difference. Our results identify a current relaxation mechanism with a relaxation time of the order of the diffusion time. The temperature is greater than the minigap, so that the dc current phase relation is sinusoidal. Nevertheless, we find that the dissipative and the non dissipative components contain several harmonics. This shows on the one hand that at high frequency, the non dissipative response is not simply the flux derivative of the supercurrent, and also that dissipation is enhanced when the minigap closes.

The experiment consists in inductively coupling an NS
ring to a multimode superconducting resonator operating between 300 MHz and 6 GHZ. In the linear response regime the dc flux dependences of χ′ and χ″ are deduced from the flux induced variations of the resonator’s resonances frequency and quality factor Q. A similar technique based on single mode resonators was already used for the investigation of short Josephson junctions inbeded in superconducting rings [8, 9].

The resonator consists of two parallel superconducting Nb meander lines (2µm wide, L_R = 20 cm long, and 4µm apart, see Fig. 1) patterned on a sapphire substrate [12]. One of these lines is weakly coupled to a RF generator via a small on chip capacitance whose value is adjusted in order to preserve the high Q of the resonator(80 000 at 20 mK), the other one is grounded. The resonance conditions are L_R = nL, where the mode index n is an integer, and λ the electromagnetic wavelength. The fundamental resonance frequency (n = 1) is of the order of 360 MHz.

The resonator is enclosed in a gold plated stainless-steel box, shielding it from electromagnetic noise, and cooled down to mK temperature. The high Q factors allows us to detect extremely small variations: \( \frac{\Delta f}{f} \sim \frac{\delta Q}{Q} \sim 10^{-9} \). It therefore provides very accurate ac impedance measurements of mesoscopic objects. This technique was previously employed in contactless measurements of the response of Aharonov Bohm rings [7]. A single NS ring gives more signal than \( 10^4 \) normal rings, for two reasons:

First, the supercurrent in an NS ring is g times larger than the persistent current in a normal ring of the same size as the N wire (g is the conductance of the normal wire in \( 2e^2/h \) units). Second, the length of the S part can be adjusted to optimize the inductive coupling between the ring and the resonator.

The N part of the NS ring is a mesoscopic Au wire (99.9999% purity, 1.2 x 0.38 µm², 50 nm thick, normal state resistance \( R_N = 1/G_N = 0.52 \Omega \) corresponding to \( g = 2 \times 10^4 \)), positioned between the two meander lines by e-beam lithography, and thermally evaporated onto the sapphire substrate. We estimate the Thouless energy for this sample to be \( E_{Th} = 90 \pm 10 \text{ mK} \). The S part of the ring is constructed by connecting the Au wire to one of the resonator lines via W nanowires deposited using a focused Ga ion beam (FIB) to decompose a tungstene carbyne vapor. The superconducting transition temperature of the W wires is 4K, their critical current is 1 mA, and critical field is greater than 7 T [10]. This technique provides a good interface quality, thanks to a slight etching step prior to deposition of W. We have checked that SNS junctions fabricated using this technique exhibit supercurrents and Shapiro steps comparable to long SNS junctions made with more conventional techniques [11]. The sample is thus an ac-squid embedded in the resonator. The dc superconducting phase difference \( \varphi \) at the boundaries of the N wire is imposed by a magnetic flux \( \Phi_{dc} \) created by a magnetic field applied perpendicularly to the ring plane: \( \varphi = -2\pi \Phi_{dc}/\Phi_0 \) where \( \Phi_0 = h/2e \) is the superconducting flux quantum. In addition, an ac flux \( \delta \Phi_{ac} \cos(\omega t) \) is generated by the ac current in the resonator. At low enough frequency the current in the SNS ring should follow adiabatically the oscillating flux, and the ac response should be entirely in phase, given by the flux derivative of the Josephson current \( \partial I_J(\Phi_{dc})/\partial \Phi \), also called the inverse kinetic inductance. But at high frequency the ac current \( \delta I_{dc} \) in the ring should have both an in-phase and an out-of-phase response to the ac flux \( \delta I_{dc}(\Phi_{dc}) = \chi'(\omega, \Phi_{dc}) \delta \Phi_{ac} \cos(\omega t) + \chi''(\omega, \Phi_{dc}) \delta \Phi_{ac} \sin(\omega t) \). Here \( \chi = \chi' + i\chi'' \) is the complex susceptibility of the ring related to its complex impedance Z by \( \chi = \omega/Z \). The aim of the experiment is to determine \( \chi(\Phi_{dc}) \) of the ring at the successive resonances of the resonator by measuring the

FIG. 1: Electron micrograph of the NS ring inserted in the niobium resonator.

FIG. 2: Flux dependence of the real \( \chi' \) and imaginary susceptibility \( \chi'' \) for different temperatures and resonance frequencies (left panel, f=365MHz, right panel T=0.67K). The amplitude of the Josephson currents calculated are \( I_J(0.67K) = 18 \mu A \) and \( I_J(1K) = 6 \mu A \). The data are corrected from geometrical inductance effects, which induce corrections which are sizable at 0.67 and 0.55 K but negligible at 1K. Circles: fit with \( \chi_d(T)/(1 + i\omega\tau) \) see expressions 3 and 5 calculated from Usadel equation [17] at temperatures corresponding to \( k_{B}T = 5, 6 \) and 9E\textsubscript{Thh} giving the best agreement with experimental data. The amplitude of the theoretical curves has been rescaled by a factor 0.7. Note that this experiment only measures the flux dependence of \( \chi \), all the curves have been arbitrarily shifted along the vertical axis.
flux dependences of the resonance frequencies and quality factor which are related to $\chi(f_n)$ via:

$$\frac{\delta f_n(\Phi_{dc})}{f_n} = -\frac{1}{2} k_n \frac{M^2}{D} \left[ \chi'(1 - L_g \chi') - L_g \chi'' \right]$$

$$\frac{1}{Q_n(\Phi_{dc})} = k_n \frac{M^2 \chi''}{D} \text{ with } D = (1 - L_g \chi')^2 + L_g \chi''^2$$

(2)

$L = 0.15 \mu H$ is the total inductance of the resonator, $M \sim 5 \text{ pH}$ the mutual inductance between the ring and the two lines of the resonator and $L_g \sim 15 \text{ pH}$ the ring’s geometrical inductance. The coefficient $k_n$ accounts for the spatial dependence of the ac field amplitude at frequency $f_n$. The limit $L_g = 0$ yields the well known formulas where $\delta f/ f$ and $\delta (1/Q)$ are respectively proportional to $\chi'$ and $\chi''$. But since the geometrical inductance of the ring is finite, the internal dc flux $\Phi_{int}$ differs from the applied flux $\Phi_{ext}$ and reads $\Phi_{int} = \Phi_{ext} + L_g I \phi / \Phi_{0}$, This screening effect leads to Eq.2 and to an hysteretic behavior at low temperature when the parameter $\beta = 2 \pi L_g I \phi / \Phi_{0}$ is larger than 1. (The critical current $I_c$ is the maximum amplitude of $I \phi / \Phi_{0}$). Because of this, it is not possible to access the whole current/phase relation below 600 mK [12].

In the following we present the dependences of $\chi$ upon the internal flux through the ring. Our main findings are summarized in Fig.2 showing the flux oscillations of $\chi'$ and $\chi''$ for different temperatures and resonator modes. Each curve corresponds to an average of 30 to 100 magnetic field scans. Their periodicity corresponds, as expected, to $\Phi_0$ through the ring. The first surprise is the observation, beside the non dissipative response $\chi'(\Phi)$, of a larger dissipative response $\chi''(\Phi)$, over the entire frequency range investigated (365 MHz to 3 GHz). In addition, $\chi'(\Phi)$ and $\chi''(\Phi)$ are not harmonic functions of flux even though the dc Josephson current is sinusoidal in this temperature range. It is clear in particular that $\chi'(\Phi)$ is not equal to $\chi(J_0) = \partial I \phi / \partial \Phi$. It is also interesting to compare the flux variation amplitudes $\delta \chi'$, $\delta \chi''$ to the inverse kinetic inductance $\chi_J(T) = 2 \pi I_c(T) / \Phi_0$ and to $\chi'_{\pi} = \omega G_N$. Whereas we find that $\delta \chi'$ is smaller than $\chi_J(T)$, $\delta \chi''$ is much larger than $\chi'_{\pi}$. We now turn to the frequency dependence $\chi(\omega)$ in order to extract the relevant relaxation times. The amplitudes $\delta \chi'$ and $\delta \chi''$ are plotted in Fig.3 for the successive resonator eigen frequencies $f_n = \omega_n / 2 \pi$, at two temperatures. We find that the frequency dependence follows a simple Debye relaxation law, $\delta \chi(\omega) = \delta \chi(0) / (1 + i \omega \tau)$, with two adjustable parameters: $\delta \chi(0)$, of the order of $\chi_J$, and the relaxation time $\tau$ equal to 0.6 ± 0.2 ns, which is 7 times the diffusion time. We could not detect significant temperature dependence of this time between 0.5 and 1K.

So far we have discussed exclusively the linear response regime (microwave excitation powers between $10^{-15}$ and $10^{-12}$ W). Figure 4 presents non linear effects in a larger NS ring with the same N wire. They show up at greater powers corresponding to an amplitude of the ac flux of the order of $\Phi_0$ through the ring: $\delta (1/Q)(\Phi)$ features sharp peaks at odd multiples of $\Phi_0/2$, for which the minigap vanishes. The peaks widen with increasing microwave power. We attribute this extra dissipation to microwave induced Andreev pair breaking, which should occur preferentially in the range of dc flux where the minigap is the smallest. Raising temperature or frequency decreases the excitation amplitude threshold for the appearance of these peaks. The in phase response is also modified, with a striking increase of $\delta f(\Phi)$ in the whole flux range and a sign change of the flux dependence in the vicinity of $\Phi_0/2$. Non linearities also show up at lower excitation powers when temperature is increased. This behavior recalls the strong modification of the dc current/phase relation under microwave irradiation [6, 19].

We now discuss possible explanations of the linear response data. One cause of dissipation in SNS junctions is the relaxation of the population of Andreev levels which explains non equilibrium effects in voltage biased configurations such as fractional Shapiro steps [13]. This relaxation is characterized by the inelastic time $\tau_{en}$ which is the shortest between the electron-electron and the electron-phonon scattering times. It yields a frequency dependent contribution to the response function $\chi$ proportional to the sum over the whole spectrum of the square of the single level current [14].

$$\chi_{in}(\omega) = \sum_n \left[ \frac{\partial i_{en}}{\partial \phi} + i_{en}(\phi) \frac{\partial p_{en}}{\partial \phi} \frac{1}{\partial \phi} + \frac{i \omega \tau_{en}}{1 + i \omega \tau_{en}} \right]$$

$$= \frac{\partial i_{J}(\phi)}{\partial \phi} - \frac{i \omega \tau_{en}}{1 + i \omega \tau_{en}} F(\phi, T) \, F(\phi, T).$$

(3)

where $F(\phi, T) = -\sum_n \left[ \frac{\partial i_{en}}{\partial \phi} + i_{en}(\phi) \frac{\partial p_{en}}{\partial \phi} \frac{1}{\partial \phi} + \frac{i \omega \tau_{en}}{1 + i \omega \tau_{en}} \right]$ can be written in the continuous spectrum limit in terms of the spectral current $J(\epsilon)$ and the density of states $\rho(\epsilon)$ of the Andreev levels as: $F(\phi, T) = \int J(\epsilon) / [k_B T \rho(\epsilon)] \, d\epsilon$. The high frequency limit of $\chi_{in}$ is:

$$\chi_{in}(\omega) = \chi_{in}(\omega) = \lim_{\omega \rightarrow \infty} \chi_{in}(\omega) = \frac{\partial i_{J}(\phi)}{\partial \phi} - F(\phi, T)$$

(4)

which can also be written $\chi_d(\phi, T) = \sum_n \left[ \frac{\partial i_{en}}{\partial \phi} \right]$, corresponding to time independent "frozen" populations of Andreev states. We have calculated $F(\phi, T)$ [15, 17] using Usadel equations [16]. In the limit where $k_B T \gg \varepsilon_F$ it is approximatively given by $F(\phi, T) = (2 \pi G_N \varepsilon_F / e k_B T) (-\pi + \text{ mod } (\pi + \phi, 2 \pi) - \text{sgn}(\sin(\phi)) \sin^2(\phi/2)/\pi \sin(\phi))$. It is nearly $\pi$ periodic with a sharp anomaly at $\phi = \pi$ related to the closing of the minigap. This leads to an important harmonics content of $\chi(\phi)$ at finite frequency just as found experimentally (see Fig.2) and discussed below. However the experimental flux dependence of $\delta \chi''$ is similar to $\delta \chi'(\phi)$ and can not be reproduced within this model. Moreover the relaxation of the Andreev levels population is...
and $\tau$ expected to be of the order of $\tau_D$. We show in Fig. 2 that the experimental flux dependence of $\delta\chi'$ and $\delta\chi''$ at 0.55, 0.67 and 1K and 2 different frequencies are well reproduced by Eq. (5) with $\tau = 0.6ns$ as determined previously and $\chi_d(\varphi)$ calculated from expression (4). We have used a unique rescaling factor of the order of 0.7 for all the data which can be attributed to errors in the estimation of the mutual coupling between the ring and the resonator. Recent results [22] on the finite frequency response of a SINIS junction where the normal mesoscopic wire is isolated from the electrodes by an insulating barrier lead to a frequency dependent response with similar flux dependences for $\chi'$ and $\chi''$, as observed here and a characteristic time given by $\tau_D$. The extension of this work to our experimental configuration with highly transmitting N/S interfaces is in progress [17]. Finally pair breaking induced by residual magnetic impurities could also be important and needs to be considered.

In conclusion we have found that the high frequency current response at GHz frequencies of NS rings is dramatically different from the flux derivative of the Josephson current. The harmonics content of the dc flux dependence is well understood as a result of the freezing of the populations of the Andreev states which cannot follow the time dependent flux. The physical origin of the nanosecond time scale responsible for the observed dissipative response and the associated high frequency decrease of the in phase response seems to be related to the diffusion time through the normal part of the ring.

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![Figure 3: Frequency dependence of $\delta\chi'$ and $\delta\chi''$ for two different temperatures. Continuous lines: fits according to Debye relaxation laws with $\tau = 0.6 ns$. Inset: frequency dependence of the ratio $\delta\chi''/\delta\chi'$ for several temperatures. The linear dependence in $2\pi f$ confirms the validity of the Debye relaxation fits. Circles: $T=1K$, crosses: $T=0.67K$.](image)

![Figure 4: Non linear regime. Flux dependent dissipative and non-dissipative responses for different excitation microwave powers at $T = 0.7K$. This data was taken for the same N Au wire inbedded in a larger ring than the previous measurements.](image)

1. T. T. Heikkilä, J. Sarkka, and F. K. Wilhelm Phys. Rev. B 66, 184513 (2002).
2. F. Zhou et al., J. Low Temp. Phys. 110, 841 (1998).
3. H. le Sueur et al., Phys. Rev. Lett. 100, 197002 (2008).
4. J.M. Warlaumont et al., Phys. Rev. Lett. 43, 169 (1979).
5. F. Chiodi, M. Aprili, and B. Reulet, Phys. Rev. Lett. 103, 177002 (2009).
6. M. Fuechsle et al., Phys. Rev. Lett. 102, 127001 (2009).
7. R. Deblock, et al Phys. Rev. Lett. 89, 206803 (2002), Phys. Rev. B 65, 075301 (2002).
8. R. Rifkin and B. S. Deaver Phys. Rev. B 13, 3894 (1976).
9. A. A. Golubov, M. Yu. Kupriyanov, and E. Ilichev Rev. Mod. Phys. 76, 411 (2004),Mark Ebel et al Phys. Rev. B71, 052506 (2005).
10. A.Yu. Kasumov, et al Phys. Rev. B72, 033414 (2005).
11. F.Chiosti et al unpublished
12. We have checked that the condition $\beta = 1$ is consistent with the calculated geometrical ring inductance, and the critical current at this temperature. We have determined $I_c(T)$ from the temperature dependence of amplitude of the hysteresis on the flux axis. The Thouless energy was then determined from the expression of $I_c(T)$ calculated from P. Dubos et al., Phys. Rev. B 63, 064502 (2001),
with the ratio $\Delta/E_{Th} = 84$ where $\Delta$ is the gap of the superconducting electrodes.

[13] K.W. Lehnert et al. Phys. Rev. Lett. 82, 1265 (1999); N. Argaman, Superlattices Microstrct; 25, 861 (1999).

[14] Note the similarity between this relaxation mechanism and the persistent current relaxation mechanism discussed in Aharonov Bohm mesoscopic rings discussed in: N. Trivedi and D.A. Browne, Phys. Rev. B 38(14), 9581 (1988); B. Reulet and H. Bouchiat, Phys. Rev. B 50(4), 2259 (1994); A. Kamenev, B. Reulet, H. Bouchiat and Y. Gefen, Europhys. Lett. 28 (6), 391 (1994).

[15] S.V. Lempitsky Sov. Phys. Jetp 57, 910(1983).

[16] K. D. Usadel, Phys. Rev. Lett. 25, 507 (1970).

[17] P. Virtanen, T.T. Heikkilä (to be published).

[18] W.J. Scocpol and M. Tinkham Rep. Prog. Phys. 39 1049, (1975).

[19] Pauli Virtanen et al arxiv.1001.5149

[20] Y. Blanter Phys. Rev. B54 12807, (1997).

[21] C. Texier and G. Montambaux Phys. Rev. B72, 115327 (2005).

[22] K. Tikhonov, M.S. diploma thesis (in Russian): http://qmeso.itp.ac.ru/Publications/Manuscripts/tikhonov_diplom_2009.pdf; K. Tikhonov and M. Feigel’man (to be published).