Nonlinear Responses of a Cantilever Honeycomb Sandwich Plate under Aerodynamic Pressure

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Abstract. Honeycomb sandwich structures can be used on wings of airplane and the wings may be simplified to a cantilever plate. Airplanes will be affected by airflows inevitably during flight. In this paper, nonlinear vibrations of a cantilever honeycomb sandwich panel affected by aerodynamic force are investigated. The equations of the cantilever honeycomb sandwich panel under aerodynamic pressure and in-plane excitation are obtained by using Hamilton's principle and the Galerkin method. The equations are nonlinear ordinary differential equations in lateral oscillations with two degree of freedom. Nonlinear dynamics of the cantilever honeycomb sandwich panel under aerodynamic force are derived which is based on third-order piston theory. The nonlinear dynamics of the cantilever honeycomb sandwich panel under aerodynamic force and in-plane excitation are given by numerical simulations. The results show that one can adjust the frequency of the in-plane force to achieve the purpose of vibration reduction in a certain range. In the presence of in-plane forces and aerodynamic interaction, the dynamic behavior of the plate appears in a new form of vibration-intermittent flutter.

Keywords: Honeycomb plate, Cantilever, Vibration, Nonlinear response, Aerodynamic

1. Introduction

Honeycomb sandwich structure is a kind of lightweight composite laminates which was used widely in various fields, such as aerospace, ship, construction industry, home decoration and package, etc. Since the honeycomb structure has many excellent features [1], for example, high specific stiffness and high specific strength, easy to process and cheap. As a result, many research works have been studied on the mechanical properties, processing, and applications of honeycomb structures and so on. The honeycomb core has many shapes. Hexagonal honeycombs elastic properties including all nine elastic constants which are accurate given by Malek and Gibson [2]. Park et al. [3] investigated dynamic response of composite square honeycombs in dynamic compressions. The impact dynamics of circular-celled honeycombs was studied by Hu et al. [4]. The mechanical properties of a new kind of honeycombs called octagonal structure were investigated by Hedayati et al. [5].

The literatures above were focused on the mechanical properties of honeycomb cores. Impact resistance and energy absorption are salient features of honeycomb structures. Ebrahimi et al. [6] studied dynamics and failure modes of honeycomb sandwich plates subjected to shock and projectile impacts. Xing et al. [7] applied honeycomb materials to the arresting system of aircrafts for the excellent mechanical properties and higher arresting coefficient of the honeycomb materials and theoretical model for the system was given. Dynamics of aluminum honeycomb sandwich panels, including crash response and failure modes under impact velocity were studied by Wu et al. [8]. Xie et
al. [9] studied the high-speed impact deformation and the high-temperature in honeycomb sandwich panels by using experiment and finite element methods. The energy-absorbing properties of honeycomb core layer with composite tube-reinforced aluminum were given by Antali et al. [10].

Honeycomb structure is also widely used on aircraft and airplanes which are inevitably subject to airflow during flight. The following works were given to investigate the influence of airflow on composite laminated material structures. Zhang et al. [11] established dynamics models of a composite laminated cantilever plate and studied responses of the plate under aerodynamic pressures and in-plane excitations. Hao et al. [12, 13] investigated responses of functionally graded materials shell subjected to aerodynamic force and other complex loads. Meng et al. [14] indicated that chaotic motions would occur in the thin plate under subsonic airflow. Yang et al. [15] investigated dynamics of a sandwich panel in supersonic flow.

In summary, many research works on honeycomb structures are mainly investigated the mechanical properties, impact resistance and energy absorption by using finite element method or experiments. In this work, nonlinear dynamics for the cantilever honeycomb sandwich panel under aerodynamic pressure and in-plane force are given through theoretical and numerical methods. In Section 2, based on the Hamilton's principle, the Reddy's third-order shear deformation theory, the Galerkin method and third-order piston theory, the equations for the cantilever honeycomb sandwich panel are obtained. In section 3, Numerical simulations are given to obtain nonlinear response of the honeycomb panel. Finally, conclusions and acknowledge are gained.

2. Equations of motion for the panel

A cantilever honeycomb sandwich panel subjected to in-plane excitations and aerodynamic force is considered, the model and the corresponding coordinate system are given as in Figure 1. The plate is with length \(a\), width \(b\), and thickness \(h\). \(ob\) is fixed and other three sides are free. The panel is under in-plane excitations \(P = P_0 + P_\cos \Omega t\) and aerodynamic force \(\Delta P\). Make \((u, v, w)\) and \((u_0, v_0, w_0)\) be the displacements of an arbitrary point and a point in the middle plane of the panel in \(x\), \(y\), and \(z\) directions, \(\phi_x\) and \(\phi_y\) represent the mid-plane rotations of a transverse normal about the \(y\) and \(x\) axes, respectively.

![Figure 1 Model of cantilever honeycomb sandwich plate](image)

The displacement components of honeycomb sandwich panels can be represented as following based on the theory from [16]

\[
\begin{align*}
    u(x, y, z, t) &= u_0(x, y, t) + z\phi_x(x, y, t) - \frac{4}{3h^2}z^3\phi_x + \frac{\partial w_0}{\partial x}, \\
    v(x, y, z, t) &= v_0(x, y, t) + z\phi_y(x, y, t) - \frac{4}{3h^2}z^3\phi_y + \frac{\partial w_0}{\partial y},
\end{align*}
\]
density of the hexagon honeycomb core modulus, they can be computed from the formula the equivalent Poisson’s ratios.

\[
\text{equivalent elastic modulus of the honeycomb core at } \Omega_0, \text{ where } \rho_c, b_a, c_i, c, d, \text{ and } \phi_y = \frac{\partial w_0}{\partial y}, \text{ the term } \Omega_0 \text{ is the middle surface of the panel.} \]

\[
\text{The terms in Eq. (4) have the following form: }
\]

\[
\begin{bmatrix}
N_{\alphaeta} \\
M_{\alphaeta} \\
P_{\alphaeta}
\end{bmatrix} = \int_{\Omega_0} \begin{bmatrix}
\frac{h}{2} & \sigma_{\alpha\beta} \\
\frac{3}{2} & \sigma_{\alpha\beta}
\end{bmatrix} dz, \quad \begin{bmatrix}
Q_{\alpha} \\
R_{\alpha}
\end{bmatrix} = \int_{\Omega_0} \begin{bmatrix}
1 \\
2
\end{bmatrix} dz,
\]

\[
\alpha \text{ and } \beta \text{ are } x \text{ and } y, \text{ and } I_j = \int_{\Omega_0} \rho_0(z) dz, \quad (i, j = 0, 1, 2, \ldots, 6).
\]

\[
\begin{align*}
\sigma_{xx} &= Q_{11} E_x + Q_{12} E_y, \\
\sigma_{yy} &= Q_{22} E_y + Q_{22} E_y, \\
\sigma_{xy} &= Q_{66} E_y, \\
\sigma_{yx} &= Q_{44} E_y, \\
\sigma_{x} &= Q_{33} E_y, \\
\sigma_{x} &= Q_{55} E_y,
\end{align*}
\]

where \(Q_{11}, Q_{12}, Q_{22}, Q_{44}, Q_{55}, \text{ and } Q_{66} \) are elastic constants which can be written as the following

\[
Q_{11} = \frac{E_x}{1 - \nu_y \nu_y}, \\
Q_{22} = \frac{E_y}{1 - \nu_y \nu_y}, \\
Q_{22} = \frac{E_y}{1 - \nu_y \nu_y},
\]

\[
Q_{44} = G_{yx}, \\
Q_{55} = G_{yx}, \\
Q_{66} = G_{yx}.
\]

Hexagonal honeycomb core is considered in this paper, the terms \(E_x \) and \(E_y \) in equation (5) are equivalent elastic modulus of the honeycomb core at \(x \) and \(y \) directions, respectively, \(\nu_x \) and \(\nu_y \) are the equivalent Poisson’s ratios. \(G_{yx} \) and \(G_{yx} \) are transverse shear modulus. \(G_{yx} \) is in plane shear modulus, they can be computed from the formula: \(G = E/(2 + 2\nu)\). The relation between the equivalent density of the hexagon honeycomb core \(\rho_c \) and the density of the materials \(\rho_a \) is: \(\sqrt{3}\rho_c = 2d\rho_a\).
Based on the relations between strains and displacements according to von-Karman nonlinear strains-displacements, the equations of the honeycomb sandwich panel in lateral vibrations are denoted according to Eq. (2)

\[
\frac{3}{2} A_{11} \frac{\partial^2 w_0}{\partial x^2} \left( \frac{\partial w_0}{\partial x} \right)^2 + \left( \frac{1}{2} A_{12} + A_{66} \right) \left( \frac{\partial w_0}{\partial y} \right)^2 + 2 \left( 2 A_{12} + 2 A_{66} \right) \frac{\partial^2 w_0}{\partial x \partial y} + \frac{2}{2} \frac{\partial^2 w_0}{\partial y^2} + 2 \left( A_{12} + 2 A_{66} \right) \frac{\partial^2 w_0}{\partial x \partial y} \frac{\partial w_0}{\partial y} + \frac{\partial^2 w_0}{\partial x^2} \frac{\partial w_0}{\partial y} + \frac{\partial^2 w_0}{\partial y^2} \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} + 2 \frac{\partial^2 w_0}{\partial x \partial y} \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial x} + \frac{\partial^2 w_0}{\partial x^2} \frac{\partial w_0}{\partial y} \frac{\partial w_0}{\partial y} + \frac{\partial^2 w_0}{\partial y^2} \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial x} + \frac{\partial^2 w_0}{\partial x \partial y} \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y}
\]

\[
+ \frac{3}{2} A_{12} \frac{\partial^2 w_0}{\partial y^2} \left( \frac{\partial w_0}{\partial y} \right)^2 + \left( \frac{1}{2} A_{12} + A_{66} \right) \left( \frac{\partial w_0}{\partial x} \right)^2 + \frac{2}{2} \frac{\partial^2 w_0}{\partial x \partial y} + \frac{2}{2} \frac{\partial^2 w_0}{\partial y \partial x} - H_1 c_1^2 \frac{\partial^4 w_0}{\partial x^4} - \frac{2}{2} \frac{\partial^2 w_0}{\partial x \partial y} - \frac{2}{2} \frac{\partial^2 w_0}{\partial y \partial x} - H_1 c_1^2 \frac{\partial^4 w_0}{\partial y^4}
\]

\[
+ \left( A_{55} + c_2^2 F_{55} - 2 c_2 D_{55} \right) \frac{\partial^2 w_0}{\partial x^2} + \left( A_{44} + c_2^2 F_{44} - 2 c_2 D_{44} \right) \frac{\partial^2 w_0}{\partial y^2} + \Delta p
\]

\[
- \left( 2 H_{12} c_1^2 + 4 H_{66} c_1^2 \right) \frac{\partial^4 w_0}{\partial x^2 \partial y^2} - H_2 c_1^2 \frac{\partial^4 w_0}{\partial y^2 \partial x^2} - \left( p_0 + p_i \cos(\Omega t) \right) \frac{\partial^2 w_0}{\partial y^2} = I_0 \ddot{w}_0 - c_1^2 I_0 \left( \frac{\partial^2 \ddot{w}_0}{\partial x^2} + \frac{\partial^2 \ddot{w}_0}{\partial y^2} \right),
\]

The stiffness elements of the honeycomb sandwich panel in Eq. (9) are obtained by

\[
\left( A_{ij}, B_{ij}, E_{ij}, F_{ij}, H_{ij} \right) = \int h^2 \mathcal{Q}_i^j \left( 1, z, z^2, z^3, z^4, z^5 \right) dz, \quad (i, j = 1, 2, 6),
\]

\[
\left( A_{ij}, D_{ij}, F_{ij} \right) = \int h^2 \mathcal{Q}_i^j \left( 1, z^2, z^4 \right) dz, \quad (i, j = 4, 5).
\]

Next, the aerodynamic force \( \Delta p \) in Eq. (9) will be derived. Based on the Pistons theory, it can be seen that the instantaneous pressure of the piston surface in the pipeline can be expressed as

\[
\frac{p}{p_\infty} = \left( 1 + \frac{\kappa - 1}{2} \frac{V_z}{a_\infty} \right)^{\frac{2 \kappa}{\kappa - 1}},
\]

where \( p_\infty \) is the undisturbed air pressure, \( \kappa \) is the isentropic gas coefficient, \( V_z \) is the speed of the piston, \( a_\infty \) is the speed of sound propagation in undisturbed gases.

The third-order piston theory is acquired by truncating the Taylor series expansion retained to the third-order items on the right side of Eq. (11)

\[
\frac{p}{p_\infty} = 1 + \kappa \frac{V_z}{a_\infty} + \frac{\kappa (\kappa + 1)}{4} \left( \frac{V_z}{a_\infty} \right)^2 + \frac{\kappa (\kappa + 1)}{12} \left( \frac{V_z}{a_\infty} \right)^3.
\]

Then, the pressure difference between the lower and upper surfaces of the wing, that is, the aerodynamic force which was derived based on the piston theory in three order with the aerodynamic correction factor, is as follows

\[
\Delta p = \frac{4 q_d r}{M_a} \left[ \frac{1}{V} \ddot{w}_0 + \frac{\partial w_0}{\partial x} + \frac{\kappa + 1}{2} \frac{r^2}{12} M_a^2 \left( \frac{1}{V} \ddot{w}_0 + \frac{\partial w_0}{\partial x} \right)^2 \right],
\]

where \( \rho_d \) is local gas density, \( q_d \) is the dynamic pressure, it has the following form

\[
q_d = \frac{1}{2} \rho_d V_z^2,
\]

\( M_a \) is the local Mach number, and the aerodynamic correction factor \( r \) is the following form
\[ r = \frac{M_a}{\sqrt{M_a^2 - 1}}. \]  

(15)

Substituting Eq. (13) to Eq. (9), it can be obtained the equation of the cantilever honeycomb sandwich plate with three order aerodynamic force and in plane force as follows

\[
\frac{3}{2} A_{11} \frac{\partial^2 w_0}{\partial x^2} \left( \frac{\partial w_0}{\partial x} \right)^2 + \left( \frac{1}{2} A_{12} + A_{66} \right) \left( \frac{\partial^2 w_0}{\partial y^2} \right)^2 + 2(A_{12} + 2A_{66}) \frac{\partial^2 w_0}{\partial x \partial y} \frac{\partial w_0}{\partial y} \\
+ \frac{3}{2} A_{22} \frac{\partial^2 w_0}{\partial y^2} \left( \frac{\partial w_0}{\partial y} \right)^2 + \left( \frac{1}{2} A_{12} + A_{66} \right) \left( \frac{\partial^2 w_0}{\partial x^2} \right)^2 - H_1 c_1 \frac{\partial^4 w_0}{\partial x^4} \\
+ \left( A_{55} + c_2^2 F_{55} - 2c_2 D_{55} \right) \frac{\partial^2 w_0}{\partial x \partial y} + \left( A_{44} + c_2^2 F_{44} - 2c_2 D_{44} \right) \frac{\partial^2 w_0}{\partial y^2} - \frac{4q_d r}{M_a} \frac{\partial w_0}{\partial x} \\
- \frac{4q_d r}{M_a} \frac{1}{V} w_0 - \frac{q_d}{3} \frac{\kappa + 1}{r^3 M_a} \left( \frac{1}{V} \frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial x} \right)^3 - 2H_{12} c_1^2 + 4H_{66} c_1^2 \frac{\partial^4 w_0}{\partial x^2 \partial y^2} \\
- H_{22} c_1 \frac{\partial^4 w_0}{\partial y^4} + (p_0 + p_1 \cos(\Omega t)) \frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 \tilde{w}_0}{\partial x^2} \right). 
\]  

(16)

The modal function which satisfied the cantilever boundary conditions is the following form

\[ w_0 = w_1(t) X_1(x) Y_1(y) + w_2(t) X_2(x) Y_2(y), \]

(17)

where

\[ X_i(x) = \sin \lambda_i x - \sinh \lambda_i x + \alpha_i (\cosh \lambda_i x - \cos \lambda_i x), \]  

(18a)

\[ Y_i(y) = \sin \mu_i y + \sin \mu_i y - \beta_i (\cosh \mu_i y + \cos \mu_i y), \]  

(18b)

\[ \cos \lambda_i a \cosh \lambda_i a + 1 = 0, \quad \alpha_i = \frac{\sin \lambda_i a + \sin \lambda_i a}{\cosh \lambda_i a + \cos \lambda_i a}, \]  

(18c)

\[ \cos \mu_i b \cosh \mu_i b - 1 = 0, \quad \beta_i = \frac{\sin \mu_i b - \sin \mu_i b}{\cosh \mu_i b - \cos \mu_i b}. \]  

(18d)

Substituting Eq. (17) and (18) to Eq. (16) and using the Galerkin method, it can be obtained the following equations

\[
\begin{align*}
\tilde{w}_1 - t_1 \tilde{w}_1 - t_2 \tilde{w}_2 - t_3 \tilde{w}_3 - t_4 \tilde{w}_1^2 - t_5 \tilde{w}_2^2 - t_6 \tilde{w}_3^2 - t_7 \tilde{w}_1 \tilde{w}_2 - t_8 \tilde{w}_1 \tilde{w}_3 - t_9 \tilde{w}_2 \tilde{w}_3 \\
- t_{10} \tilde{w}_1^2 - t_{11} \tilde{w}_1 \tilde{w}_2 - t_{12} \tilde{w}_2 \tilde{w}_1 - t_{13} \tilde{w}_1 \tilde{w}_3 - t_{14} \tilde{w}_2 \tilde{w}_3 - t_{15} \tilde{w}_1^2 - t_{16} \tilde{w}_2^2 \\
- t_{17} \tilde{w}_2^2 - t_{18} \tilde{w}_1^3 - t_{19} \tilde{w}_1 \tilde{w}_2^2 - t_{20} \tilde{w}_2 \tilde{w}_1^2 - t_{21} \tilde{w}_1 \tilde{w}_2 \tilde{w}_1 - t_{22} \tilde{w}_1 \tilde{w}_2 \tilde{w}_2 \\
- t_{23} \tilde{w}_1 \tilde{w}_2 \tilde{w}_1 - t_{24} \tilde{w}_1 \tilde{w}_2 \tilde{w}_2 \left( p_0 + p_1 \cos \Omega t \right) - t_{25} \tilde{w}_1 \tilde{w}_2 \tilde{w}_1 \left( p_0 + p_1 \cos \Omega t \right) = 0,
\end{align*}
\]  

(19a)

\[
\begin{align*}
\tilde{w}_2 - s_1 \tilde{w}_1 - s_2 \tilde{w}_2 - s_3 \tilde{w}_3 - s_4 \tilde{w}_1^2 - s_5 \tilde{w}_2^2 - s_6 \tilde{w}_3^2 - s_7 \tilde{w}_1 \tilde{w}_2 - s_8 \tilde{w}_2 \tilde{w}_3 \\
- s_9 \tilde{w}_1^2 - s_{10} \tilde{w}_1 \tilde{w}_2 - s_{11} \tilde{w}_2 \tilde{w}_1 - s_{12} \tilde{w}_1 \tilde{w}_3 - s_{13} \tilde{w}_2 \tilde{w}_3 - s_{14} \tilde{w}_1^2 - s_{15} \tilde{w}_2^2 \\
- s_{16} \tilde{w}_2^2 - s_{17} \tilde{w}_1^3 - s_{18} \tilde{w}_1 \tilde{w}_2^2 - s_{19} \tilde{w}_2 \tilde{w}_1^2 - s_{20} \tilde{w}_1 \tilde{w}_2 \tilde{w}_1 - s_{21} \tilde{w}_1 \tilde{w}_2 \tilde{w}_2 - s_{22} \tilde{w}_1 \tilde{w}_2 \tilde{w}_3
\end{align*}
\]  

(19b)
Eq. (19) represents lateral vibrations of the cantilever honeycomb sandwich panel under aerodynamic force and in-plane force. \( w_i \) and \( w_z \) are vibratory amplitudes of the first two modes, \( t_i \) and \( s_j \) \((i = 1, 2, \ldots, j = 1, 2, \ldots)\) are coefficients of the system which are consisted of geometric parameters and material parameters of the plate and the aerodynamic forces, the expressions of \( t_i \) and \( s_j \) are omitted in this paper for their complexity and limit space of the paper.

3. Nonlinear dynamics of the panel

In this section, numerical simulations are given to study nonlinear dynamics of the honeycomb sandwich panel under aerodynamic and in-plane forces. The panel is made up of aluminum, its density \( \rho_s = 2.66 \times 10^3 \text{ kg} / \text{m}^3 \), Poisson’s ratio \( \nu = 0.33 \), Young’s modulus \( E_s = 72 \times 10^9 \text{ Pa} \), the length and width are \( a = 5.0 \text{ m} \), \( b = 2.0 \text{ m} \). The size of the honeycomb core is chosen as: the thickness of the core layer \( h_c = 0.01 \text{ m} \), the thickness of the honeycomb core cellular cell wall \( d = 0.0008 \text{ m} \), the length of the honeycomb core cellular cell \( l = 0.01 \text{ m} \), the isentropic gas coefficient \( \kappa = 1.4 \), the density of air \( \rho_a = 1.29 \times 10^3 \text{ g} / \text{m}^3 \), the speed of air \( V = 2040 \text{ m} / \text{s} \), the local Mach number \( M_a = 6 \).

When the in-plane force is chosen as \( P_0 = 1000 \text{ Pa} \) and \( P_1 = 1000 \text{ Pa} \), the frequency of the force \( \Omega_t = 100 \text{ Hz} \), the response of the honeycomb sandwich panel is shown in Figure 2. From Figure 2 it can be seen that the vibration of the plate is attenuated. Figure 2(a) and 2(b) are phase portraits of the displacement and speed for the first two modes \( w_i \) and \( w_z \), respectively. Figure 2(c) and 2(d) are waveforms for \( w_i \) and \( w_z \), respectively. Figure 2(e) is phase portrait for three-dimension of \( (w_i, \dot{w}_i, w_z) \).
From a large number of numerical simulations, it can be seen that when the frequency $\Omega_1$ increases, the amplitudes of the panel reduces, but it is limited that the amplitude remains essentially unchanged when the frequency increases to a certain extent. Figure 3 represents another attenuated vibration of the plate when $\Omega_1 = 400\text{Hz}$, and other parameters share the same values with that of in Figure 2.
Figure 3 When $\Omega = 400$Hz, the attenuation vibration of the system.

Change the in-plane force as $P_1 = 1500$Pa, and other coefficients are same as that in Figure 2, as shown in Figure 4. The response of the plate is not attenuated vibration any more, the amplitude is regularly increased and decreased, and the maximum and minimum amplitudes remain unchanged. We call it intermittent flutter which may be caused by the interaction between in-plane forces and aerodynamics forces.
Figure 4 When \( P = 1500 \text{Pa} \), nonlinear dynamic response of the panel

With the in-plane force \( P_1 \) increasing, the minimum amplitude is increasing too. Change the value of the in-plane force \( P_1 \), Figure 5 gives the vibration of the plate when \( P_1 = 3000 \text{Pa} \).
Figure 5 When $P_1=3000\text{Pa}$, nonlinear dynamic response of the system

Continue to change the value of the in-plane force $P_1$ and other parameters remain unchanged, Figure 6 can be obtained for $P_1=6000\text{Pa}$. It can be seen from Figure 6 that the maximum and minimum amplitudes are larger than that of in Figure 4. With the increase of the in-plane force $P_1$, the difference between the maximum and the minimum amplitudes is reduced.

Figure 6. When $P_1=6000\text{Pa}$, nonlinear dynamic response of the panel

4. Conclusions

In this work, a wing of aircraft is simplified into a cantilever panel, one can obtain the equations of cantilever honeycomb sandwich panel under aerodynamic pressure and in-plane excitations by using Hamilton principle, the Galerkin method and third-order piston theory. Nonlinear dynamics of the cantilever honeycomb sandwich panel are studied through Runge-Kutta method. The parameters of the system take the actual values. From numerical simulations, it can be concluded that the vibrations of the plate are attenuated when the in-plane force is small. The amplitudes of vibrations for the plate are regularly increased and decreased and the maximum and minimum amplitudes remain unchanged when the in-plane forces increase to a certain value. We call it intermittent flutter which it is the result
of the interaction between in-plane forces and aerodynamics forces. With the in-plane force increasing, the amplitude of the panel becomes larger and the difference between the maximum and the minimum amplitudes is reduced.

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