Bouncing Universe with Exotic Radiation

Swastik Chowbay †, P K Suresh † and Barun Maity ‡
School of Physics, University of Hyderabad. Hyderabad 500046. India.
Department of Physics, University of Calcutta, Kolkata. India.

Abstract

Bouncing solution exists for the exotic radiation dominated universe in a non-singular bouncing model. Further, it is shown that bouncing solution exists in a universe dominated with the quintom matter-exotic radiation at their equilibrium in a non-singular bouncing model. The Hubble parameter crosses smoothly at the bouncing point for both cases.

1 Introduction

Bouncing model of the universe [1, 2] is believed to provide solutions to the singularity problem associated with the standard model of cosmology. The standard model can account most of the observed features of the universe. However, it encountered with the horizon, flatness and singularity problems and defy solutions. The singularity problem is the most challenging problem of the standard cosmology. The singularity problem is due to the classical nature of general relativity. Hence, quantum gravity probably is a viable scenario to resolve the singularity problem. Currently, there are many approaches to quantum gravity, but no conclusive model. Later, inflationary scenario was introduced into the standard cosmology to resolve some of its problems, including the singularity problem but actually it also not solves it [3, 4, 5]. Moreover, the inflation scenario itself has its own problems. Therefore, it is interesting to seek and study alternative solution(s) to avoid the singularity than only considering quantum gravity and inflation approaches to it, and bouncing model of universe is one of the alternative approaches.

In contrast to the standard model of cosmology, the bouncing scenario the universe transits from a contracting phase to an expanding phase for which the scale factor must be non-zero at the bouncing point. Hence, the present universe can be considered as the result of a previous contracted phase with a non zero minimal size to an expanding phase. Thus, it overcomes the singularity problem of the standard cosmology.

Among the different class of bouncing models non singular bouncing models received much attention (see [6, 7, 8] for review). To occur bounce, apart from the condition of non-zero scale factor, at the bouncing point, the equation of state of the responsible candidate of bounce must violate the null energy condition for a brief period of time.

---

∗e-mail: swastikchowbay@uohyd.ac.in
†e-mail: sureshpk@uohyd.ac.in
‡e-mail: barun.m003@gmail.com
around the bouncing point. It is extremely difficult for the ordinary matter or radiation to satisfy such condition, hence they are not viable candidates to occur bounce. This situation enforces to look for suitable candidate(s) for bounce. Therefore, at present, there are several candidates proposed to occur bounce in non-singular bouncing models and an interesting one is known as quintom matter [9]. Actually, the quintom model scenario is introduced to understand the nature of the equation of state of dynamical dark energy and it differs from the famous cosmological constant. It is shown that a bouncing universe is possible with the quintom matter with a phenomenological equation of state. The study of the bouncing model of the universe with quintom matter motivates to explore solutions for bouncing universe with a new proposed exotic radiation that introduced in the present work. We designate the proposed radiation as exotic radiation because its equation of state deviates from the standard radiation case. The present work is mainly aimed at to study the possibility of existence of bouncing solution in non-singular bouncing model with exotic radiation. Further, we also study a scenario in which both the quintom matter and the proposed exotic radiation coexist at their equilibrium and hence to examine the existence of bouncing solution in their coexistence form in a non-singular bouncing model.

Note that the exotic radiation and quintom matter-exotic radiation equilibrium scenarios that we consider in the present work is motivated from the study of bouncing solution with the quintom matter. Therefore the scale factor and the equation of state correspond to the exotic radiation is taken by following the similar approach as that of the quintom matter case. To know the form of scale factor in the quintom matter-exotic radiation equilibrium scenario, detailed knowledge of the physical conditions at their equilibrium are required. Since in the absence of such details and its computation is beyond the scope of the present work, the form of the scale factor and the equation of state of the equilibrium scenario case is taken as ansatz. This may be justified because it is possible to get a bouncing solution in the considered form of the scale factor and the equation of state.

First, we discuss general features of bouncing model and conditions that are required to occur bounce very briefly. Followed by a short discussion on the bouncing solution of the quintom matter. Subsequent sections are respectively for the study of bouncing for the exotic radiation and quintom matter-exotic radiation equilibrium scenario. Finally, discussion and conclusion of the present work.

2 Bouncing solution with quintom matter

Einstein’s general theory of relativity with the Friedmann-Lemaître-Robertson-Walker (FLRW) metric can provide the necessary conditions to occur bounce. The Friedmann’s equations for the FLRW metric with the perfect fluid as a source for gravity can be written by taking $c = 1$ and $8\pi G = 1$ as,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3} - \frac{k}{a^2}$$  \hspace{0.5cm} (1)

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{1}{6}(\rho + 3p)$$  \hspace{0.5cm} (2)

where $\rho$ and $p$ respectively, are the energy density and pressure for the perfect fluid. Here $k$ is the curvature parameter and $a$ is the scale factor.
The Friedmann equation can be expressed in terms of the Hubble parameter, \( H = \frac{\dot{a}}{a} \), and hence one can obtain
\[
\dot{H} = \frac{k}{a^2} - \frac{1}{2}(\rho + p)
\]
In order to occur bounce, the Hubble rate, which arises from the contracting phase with a negative value must increase, since it is positive during the expansion phase. So, it is required that \( \dot{H} > 0 \) and it means that
\[
\rho + p < 0.
\]
Thus, it implies to occur bounce, violation of the null energy condition, that is \( \rho + p > 0 \), for a brief period of bouncing around the bouncing point is essential. In other words, the existence of bouncing solution or not can be understood in terms of the violation of the null energy condition [1]. Equation (4) for a flat universe can be written as,
\[
\dot{H} = -\frac{\rho}{2}(1 + \omega) > 0,
\]
where the equation of state, \( \omega = \frac{p}{\rho} \). The condition (5) requires that the universe must transit from its past \( \omega < -1 \) to \( \omega > -1 \) to the later hot big bang phase. This transition of the equation of state must be achieved. Otherwise, when the universe enters after bounce period into the later big bang phase, the evolution of the universe continues with the same bouncing candidate with the equation of state \( \omega < -1 \). Consequently, it can lead to the big rip singularity. Hence, to achieve the bounce and the required transition for the equation of state with ordinary matter or radiation or their equilibrium is extremely difficult. Therefore, it has been proposed a candidate known as the quintom to occur bounce with appropriate equation of state. In the quintom matter scenario, the phenomenological equation of state is given by
\[
\omega(t) = -r - \frac{s}{t^2}
\]
for which the model parameters take value \( r < 1 \) and \( s > 0 \). The equation of state \( \omega \) runs from negative infinity at \( t = 0 \) to the cosmological constant boundary at \( t = \sqrt{\frac{1 - r}{s}} \). It is shown that for the quintom matter with the equation of state (6), the Friedmann equation leads to
\[
H(t) = \frac{2}{3} \frac{t}{(1 - r)t^2 + s},
\]
and the scale factor in terms of the model parameters becomes
\[
a(t) = \left( \frac{t^2 + \frac{s}{1 - r}}{1 - r} \right)^{\frac{1}{3(1 - r)}}.
\]
Note that in the absence of the model parameters the Friedmann equation and the scale factor reduce to that of the ordinary matter dominated FLRW universe case. Further, it is shown that the bouncing solution exists in the quintom matter with the phenomenological equation of state.

The interesting study of quintom matter motivates to explore bouncing solution for radiation of different type with a suitable equation of state. Then the behaviour of such radiation is expected to be distinct from that of the ordinary radiation. Therefore, in the present study the radiation of different type with its corresponding equation of state is termed as exotic radiation.
2.1 Bouncing solution with exotic radiation

Assume the universe in the past dominated by the exotic radiation that is at the time of occurrence of bounce. Motivated form (8), we take the scale factor for the exotic radiation in the following form

\[ a(t) = \left( t^2 + \frac{s}{1 - r} \right)^{\frac{1}{4(1 - r)}}, \]  

(9)

here again \( s \) and \( r \) are the model parameters take value \( r < 1 \) and \( s > 0 \). Thus the Hubble parameter for the exotic radiation is obtained as,

\[ H(t) = \frac{1}{2} \frac{t}{(1 - r)t^2 + s}. \]  

(10)

Hence, using eqs. (10) and (5) we get the equation of state for the exotic radiation as,

\[ \omega(t) = 1 - 4 \left[ \frac{r + s}{t^2} \right]. \]  

(11)

In order to occur the bouncing, it is required that \( \omega \) runs from negative infinity at \( t = 0 \) to the boundary at \( t = \sqrt{\frac{s}{1 - r}} \). The equation of state of the exotic radiation is found consistent with Eq.(5). For the further study, we normalize the scale factor \( a = 1 \) at the bouncing point \( t = 0 \).

We study the behaviour of the equation of state for the exotic radiation, the scale factor and the Hubble parameter and the obtained results are presented in Fig.(1). From the left top panel of Fig.(1) it can be seen that the equation of state \( \omega \) approaches to negative infinity at the bouncing point \( t = 0 \). Also, see that the equation of state of exotic radiation can lead to the contracting phase \( t < 0 \) to the subsequent expansion phase for \( t > 0 \). The right top plane of Fig. (1) shows that the scale factor for the exotic radiation is non zero at the bouncing point \( t = 0 \). From the bottom panel of Fig.(1) we can observe that the Hubble parameter running smoothly across the bouncing point \( t = 0 \). These results show that the scale factor and the Hubble parameter satisfies the required conditions to occur bounce. Thus, it is possible to get non singular bouncing universe with the proposed exotic radiation with its equation of state.

Note that the value of the model parameters, \( r < 1 \) and \( s > 0 \), holds strictly only for the bouncing case and in the absence of the model parameters the equation of state of the exotic radiation reduces to that of the standard radiation case for which bouncing solution does not exist.

2.2 Bouncing solutions for equilibrium case

In this section we consider a scenario in which both the quintom matter and the exotic radiation coexist at their equilibrium and study whether bouncing solution exist for it or not. Since the actual form of the scale factor at the equilibrium scenario is unknown, we assume the following form of scale factor, ie, an ansatz, motivated from the previous discussions, as

\[ a(t) = \left( t^2 + \frac{s}{1 - r} \right)^{\frac{1}{4(1 - r) m}}, \]  

(12)
Figure 1: Evolution of the equation of state, scale factor and the Hubble parameter for the exotic radiation case as a function cosmic time \( t \) for \( r = .6 \) and \( s = 1 \).

where \( s \) and \( r \) are once again the model parameters and respectively take value \( r < 1 \) and \( s > 0 \) for the bounce case. In the absence of the model parameters the scale factor varies as \( t^{2/m} \), where the newly introduced parameter \( m \) particularly takes only two the extreme values 3 and 4 corresponds to the quintom matter and the exotic radiation respectively. Hence it reasonable to assume that the value of \( m \) lies between 3 to 4 in the equilibrium scenario case, but its actual value depends on the physical condition of equilibrium. The physical condition of the equilibrium scenario is not known hence computation of the value of \( m \) at equilibrium is beyond the scope of the present work. However, one can estimate the value of \( m \) for the equilibrium case.

The Hubble parameter corresponds to the scale factor for the equilibrium scenario is obtained as,

\[
H(t) = \frac{2}{m} \frac{t}{(1-r)t^2 + s}.
\]

Therefore, by using eqs. (13) and (5), we obtain the equation of state for the quintom-exotic radiation equilibrium case as,

\[
\omega = -1 + \frac{m}{3} \left[ 1 - r - \frac{s}{t^2} \right].
\]

In order to occur bounce it is required that \( \omega \) runs from negative infinity at \( t = 0 \) to the cosmological constant boundary at \( t = \sqrt{\frac{s}{r}} \). Thus next, we need to study the behaviour of the equation of state, the scale factor and the Hubble parameter for the coexistence case of the quintom matter and the exotic radiation at their equilibrium. For
that the value of \( m \) is unavoidable, thus the value of the parameter for the equilibrium case can be estimated as follows.

To estimate the value of \( m \) for the equilibrium case, consider the Hubble parameter

\[
\frac{\dot{a}}{a} = \frac{2}{m (1 - r)t^2 + s}.
\]

(15)

Upon differentiating both sides of (13) with respect to \( t \) we get,

\[
\frac{\ddot{a}}{a} - \left( \frac{\dot{a}}{a} \right)^2 = \frac{2}{m (1 - r)t^2 + s} - \frac{4t^2}{m (1 - r)(t^2 + \frac{s}{1 - r})^2}.
\]

(16)

Therefore, by using (15) in (16), we obtain

\[
\frac{\ddot{a}}{a} = \frac{2}{m (1 - r)t^2 + s} + \frac{4t^2}{m^2 (1 - r)^2} \left[ \frac{1}{m^2 (1 - r)^2} - \frac{1}{m(1 - r)} \right].
\]

(17)

Hence, by using the condition \( \dot{H} > 0 \) at the bouncing point that is at \( t = 0 \), we get

\[
\frac{\ddot{a}}{a} = \frac{2}{ms} > 0.
\]

(18)

Since \( s > 0 \), the condition (18) implies that \( m \) must be positive. Thus, by interpolating \( \frac{\dot{a}}{a} \) for \( s = 1 \) with values of \( m \) bounded between the quintom matter and the exotic-radiation, we obtained the value of \( m \) for the equilibrium case as, \( m \approx 3.42 \). One can repeats the estimation of \( m \) at the equilibrium case for the different allowed values of \( s \) and it can be observed that its value remains invariable. Hence, the value of \( m \) at the equilibrium case is independent of the model parameter as it should be.

With the aforementioned estimated value of \( m \), we study the scale factor, the Hubble parameter and equation of state by normalizing the scale factor \( a = 1 \) at the bouncing point \( t = 0 \) for the quintom matter and the exotic radiation equilibrium case. The obtained results are presented in Fig. (2). From the left top panel of Fig. (2) it can be seen that the equation of state of the quintom matter exotic radiation equilibrium approaches to negative infinity at the bouncing point \( t = 0 \). The right top panel of Fig. (2) shows that the scale factor \( a \) for the exotic radiation is non zero at the bouncing point \( t = 0 \). From the bottom panel of Fig. (2) we can observe that the Hubble parameter running smoothly across the bouncing point \( t = 0 \) for the quintom matter-exotic radiation equilibrium case. These results show that the equation of the state and the Hubble parameter satisfy the needed condition to occur bounce. Thus, it can be concluded that the coexistence of quintom matter with exotic ration at equilibrium can lead to the evolution of the universe from the contracting phase \( t < 0 \) to the subsequent expansion phase for \( t > 0 \) with a bouncing point at \( t = 0 \) for the non singular class of bouncing model.

### 3 Discussion and conclusion

The standard model of cosmology based on the general relativity and cosmological principle is a very successful model to account for several observed features of the universe. However, it faces major problems like flatness, horizon and singularity etc. To resolve some of these problems a scenario called inflation is introduced into it, but
the singularity problem remained as a challenge. Meanwhile, a different kind of scenario known as bouncing model is proposed to deal with the singularity problem. According to the bouncing model of cosmology the universe had a contracting phase, then followed by a bounce without the initial singularity. But in order to occur bounce, violation of the null energy condition of the responsible bouncing candidate is essential. Since the ordinary matter and the radiation cannot satisfy that condition, bouncing solution with them is difficult to achieve. Hence, to get bouncing several candidates are proposed in nonsingular bouncing models. An interesting candidate called the quintom matter is proposed with a phenomenological equation of state with hope that it can account for the dynamical dark energy. It is shown that bouncing is possible with the quintom matter in a nonsingular class of bouncing model. This study motives to introduce a nonstandard form of radiation with a suitable equation of state, termed as the exotic radiation, and explored whether the bouncing solution exist for it or not in nonsingular bouncing model.

The main aim of the present work is to study existence of bouncing solution in a nonsingular class of bouncing cosmology with the exotic radiation. The equation of state corresponds to the exotic radiation is taken with two model parameters as in the case of the quintom matter. It is found that with the equation of state of the exotic radiation can violate the null energy condition. Also, a study of the associated scale factor, the Hubble parameter and the equation of state found satisfy with the required condition to occur bounce. From these results, it may be concluded that exotic radiation with the equation of state leads to bounce in the nonsingular cosmological model. Further,
in the absence of the model parameters the solution reduces to that of the standard radiation case for which bouncing solution does not exist.

Another scenario that considered in the present work is the coexistence of the quintom matter and the exotic radiation at their equilibrium and hence examined whether it can lead to bouncing solution or not in nonsingular class of bouncing cosmology. Since the actual form of the scale factor is not known a very general form of it is assumed with an additional parameter $m$. Consequently, the corresponding Hubble parameter and hence its equation state are obtained without the actual value of $m$. This due to the fact that to get the actual value of the parameter $m$, detailed knowledge of the physical condition at equilibrium of the quintom matter with the exotic radiation must be known. Since absence of such details, which is beyond the consideration of the present study, however, an estimate of $m$ is obtained as $\approx 3.42$ and is independent of the model parameters. Hence, existence of bouncing solution for the equilibrium case is examined with the obtained value of $m$ by studying the associated scale factor, the Hubble parameter and the equation of state. The obtained results show that the coexistence of the quintom matter with the exotic radiation in equilibrium satisfy the required condition to occur bounce. Therefore, it may be concluded that bouncing solution exists for the co-existing phase of the quintom matter with the exotic radiation at their equilibrium.

In the present study it is shown that bouncing solution exists for the exotic radiation case as well as the quintom matter and exotic radiation at their equilibrium scenario with their corresponding equation of state. The viability of bouncing cosmology with the exotic radiation and the quintom matter-exotic radiation equilibrium may be interesting and hopefully can be scrutinised with appropriate observations in the future.

References

[1] D. Battefeld, and P. Peter, *Phys.Rept.* **571** (2015) 1.

[2] Yi-Fu Cai, T. Qiu, Yun-Song Piao, and M. Li, X.Zhang, *JHEP* **2007** (2007) 10.

[3] A. A. Starobinsky, *Phys. Lett. B* **91** (1980) 99.

[4] A. H. Guth, *Phys. Rev. D* **23** (1981) 347.

[5] A. D. Linde, *Phys. Lett. B* **129** (1983) 177.

[6] M. Novello and S. E. P. Bergliaffa, *Phys.Rept.* **463** (2008) 127.

[7] Yi-Fu Cai,and D. A. Easson and R. Brandenberger, *Phys. Rev. D* **80** (2009) 023511.

[8] Jun-Qing Xia, Yi-Fu Cai, Hong Li,and X. Zhang *Phys. Rev. Lett.* **112** (2014) 251301.

[9] B. Feng, X. Wang and X. Zhang, *Phys. Lett.* **B607** (2005) 35.

[10] G.-B. Zhao, J.-Q. Xia, H. Li, C. Tao, J. M. Virey, Z.-H. Zhu and X. Zhang, *Phys. Lett.* **B648** (2007) 8.