Topics in 3D $\mathcal{N} = 2$ AdS supergravity in superspace

Gabriele Tartaglino-Mazzucchelli$^{1,\ast}$

$^1$ Theoretical Physics, Department of Physics and Astronomy, Uppsala University
Box 516, SE-751 20 Uppsala, Sweden

Received 2012, revised 2012, accepted 2012
Published online 2012

Key words Extended Supersymmetry, AdS, Supergravity Models, Superspaces.

We review some recent results on the construction in superspace of 3D $\mathcal{N} = 2$ AdS supergravities and on the formulation of rigid supersymmetric theories in (1,1) and (2,0) AdS superspaces.

Copyright line will be provided by the publisher

1 Introduction

Recently there has been a renewed interest in 3D supersymmetric theories. On the one hand this was generated by the new insights achieved in the study of M2-brane dynamics and 3D superconformal Chern-Simons theories [3, 4, 5]. Such results triggered a large field of investigations on AdS$_4$/CFT$_3$ dualities.

On the other hand, new insights into the dynamics of 3D massive gravity theories have been achieved. The use of AdS$_3$/CFT$_2$ duality has allowed a microscopic derivation of the BTZ black hole entropy using Topological-Massive-Gravity (TMG) [6]. After that, new classes of higher-derivative, but unitary, 3D gravities, called New-Massive-Gravity (NMG) and Generalized-Massive-Gravity (GMG), were constructed [7]. These theories were extended at the non-linear level to $\mathcal{N} = 1$ supergravity [8, 9] and, in the linearized approximation, to $\mathcal{N} \geq 2$ [10]. A fully non-linear description of the supergravity extension is difficult due to the higher derivative terms. A crucial role is played by the three-dimensional AdS space which represents a maximally symmetric solution in these models, making them of interest for AdS$_3$/CFT$_2$.

The use of superspace techniques for $\mathcal{N}$-extended supergravity may give insights into the generalization of the previous results. Surprisingly this was not fully developed in the past. The $\mathcal{N} = 1$ case was studied in [11, 12]. The $\mathcal{N} \geq 2$ was sketched in [13] (more results for the $\mathcal{N} = 8$ case were given in [14]). To fill this gap, in collaboration with S. M. Kuzenko and U. Lindström, in [1] we developed the superspace description of $\mathcal{N}$-extended conformal supergravity$^1$ and, extending the superconformal results of [17], we provided formalisms to study general supergravity-matter systems with $\mathcal{N} \leq 4$.

The development of a superspace approach to study field theories in curved spaces has received a renewed attention (see [19, 20, 21, 22, 23]). For example, rigid supersymmetric sigma-models in 4D AdS have revealed new restrictions on the target space geometry and on the structure of the allowed supercurrent multiplets. New superspace techniques for rigid supersymmetric theories on curved background have clear applications if one is interested to lift off-shell theories from flat to curved backgrounds [23]. Such problems have recently arisen in studying the partition function of gauge theories on nontrivial 3D-4D, constant-curvature backgrounds (mostly spheres) when computing observables such as expectation values of Wilson loops and superconformal indices by using localization techniques [24].

The 3D constant curvature spaces present interesting features. For example, long ago, Achúcarro and Townsend discovered that 3D $\mathcal{N}$-extended anti-de Sitter (AdS) supergravity exists in several incarnations, called (p,q) supergravities [25]. The two non-negative integers $p \geq q$ are such that $\mathcal{N} = p + q$ and they classify the in-equivalent isometry supergroups $\text{OSp}(p|2;\mathbb{R}) \times \text{OSp}(q|2;\mathbb{R})$ of the AdS backgrounds.

--

$\ast$ E-mail: gabriele.tartaglino-mazzucchelli@physics.uu.se

$^1$ Independently, in [15] and [16] the superspace geometry of respectively $\mathcal{N} = 8$ and $\mathcal{N} = 16$ supergravity was studied.

Copyright line will be provided by the publisher
In [2], using the results of [1], we deepened the study of 3D \( \mathcal{N} = 2 \) AdS supergravities in superspace. We presented three superfield formulations for \( \mathcal{N} = 2 \) supergravity that allow for well defined cosmological terms and supersymmetric AdS solutions. We classified the consistent supercurrent multiplets in flat, (1,1) AdS and (2,0) AdS superspaces. We proved that both (1,1) and (2,0) AdS are conformally flat superspaces. Furthermore, we elaborated on rigid supersymmetric theories in (1,1) and (2,0) AdS superspaces.

In this report we review some of the results obtained in [1, 2] for the 3D \( \mathcal{N} = 2 \) case. In section 2, we review the superspace formulation of 3D \( \mathcal{N} = 2 \) conformal supergravity. Section 3 is devoted to the classification of supergravity compensators and cosmological terms that give rise to (1,1) and (2,0) AdS supergravities. In section 4 we present some results concerning the formulation of rigid supersymmetric models in (1,1) and (2,0) AdS superspaces.

## 2 3D \( \mathcal{N} = 2 \) conformal supergravity in superspace

We start by describing the superspace formulation of 3D \( \mathcal{N} = 2 \) off-shell conformal supergravity originally presented as part of the general \( \mathcal{N} \) analysis of [1] and further developed in [2].

Consider a curved 3D \( \mathcal{N} = 2 \) superspace \( M^{\beta \gamma} \) parametrized by local bosonic \((x)\) and fermionic \((\theta, \bar{\theta})\) coordinates \( z^M = (x^m, \theta^\mu, \bar{\theta}_\mu) \), where \( m = 0, 1, 2 \), \( \mu = 1, 2 \). The Grassmann variables \( \theta^\mu \) and \( \bar{\theta}_\mu \) are related to each other by complex conjugation \( \bar{\theta}_\mu = \theta^\mu \). The tangent-space group is chosen to be \( \text{SL}(2, \mathbb{R}) \times \text{U}(1)_R \) and the superspace covariant derivatives \( D_A = (D_a, D_\alpha, D^\alpha) \) have the form

\[
D_A = E_A + \Omega_A + i \Phi_A J .
\]

Here \( E_A = E_A^M(z) \partial / \partial z^M \) is the supervielbein with \( \partial_M = \partial / \partial z^M \); \( M_{bc} \) and \( \Omega_A^{bc} \) are the Lorentz generators and connection respectively (antisymmetric in \( b, c \)); \( J \) and \( \Phi_A \) are respectively the \( \text{U}(1)_R \) generator and connection. The generators of \( \text{SL}(2, \mathbb{R}) \times \text{U}(1)_R \) act on the covariant derivatives as follows:

\[
[J, D_\alpha] = D_\alpha , \quad [J, D_a] = 0 , \quad [M_{\alpha \beta}, D_\gamma] = \varepsilon_{\gamma (\alpha} D_{\beta)} , \quad [M_{ab}, D_c] = 2 \eta_{[a|c] D_b] .
\]

Here, \( M_{\alpha \beta} = (\gamma^a)_{\alpha \beta} M_a \) are the Lorentz generators with \( (\gamma^a)_{\alpha \beta} \) the symmetric and real gamma-matrices, \( \varepsilon_{\alpha \beta} \) is the antisymmetric \( \text{SL}(2, \mathbb{R}) \) invariant and \( M_a = \frac{1}{2} \varepsilon_{abc} M^{bc} \) being \( \varepsilon_{abc} (\varepsilon_{012} = -1) \) the Levi-Civita tensor (see [1, 2] for more details on our 3D notations and conventions).

The supergravity gauge group is generated by local transformations of the form

\[
\delta_K D_A = [K, D_A] , \quad \delta_K U = K U , \quad K = K^C(z) D_C + \frac{1}{2} K^{cd}(z) M_{cd} + i \tau(z) J ,
\]

with the gauge parameters obeying natural reality conditions, but otherwise arbitrary. In (2.3) we have included the transformation rule for a tensor superfield \( U(z) \), with its indices suppressed.

If one imposes conventional constraints [13] and solves the the Bianchi identities [1], the covariant derivatives algebra turn out to obey the (anti)commutation relations \((D_{\alpha \beta} = (\gamma^a)_{\alpha \beta} D_a)\)

\[
(D_\alpha, D_\beta) = -4 \bar{R} M_{\alpha \beta} , \quad (D_\alpha, \bar{D}_\beta) = 4 R M_{\alpha \beta} ,
\]

\[
(D_\alpha, \bar{D}_\beta) = -2 i \bar{D}_\alpha - 2 C_{\alpha \beta} J - i \varepsilon_{\alpha \beta} S J + i S M_{\alpha \beta} - 2 \varepsilon_{\alpha \beta} \bar{C}^{\delta} M_{\gamma \delta} , \quad
\]

\[
[D_{\alpha \beta}, D_\gamma] = -i \varepsilon_{\gamma (\alpha} \bar{C}_{\beta) \delta} D^\delta + i C_{\gamma (\alpha} D_{\beta)} - \frac{1}{2} \varepsilon_{\gamma (\alpha} S D_{\beta)} - 2 i \varepsilon_{\gamma (\alpha} \bar{R} \bar{D}_{\beta)} + \text{curvature terms} .
\]

The algebra is parametrized by three dimension-1 torsion superfields: a real scalar \( S \), a complex scalar \( R \) and its conjugate \( \bar{R} \), and a real vector \( C_a \) \((C_{\alpha \beta} := (\gamma^a)_{\alpha \beta} C_a)\). The superfields \( S \) and \( C_a \) are neutral under the \( \text{U}(1)_R \) group, while the \( \text{U}(1)_R \) charge of \( R \) is \(-2\), \( \bar{R} J R = -2 R \) and \( J \bar{R} = 2 \bar{R} \). The torsion superfields obey differential constraints implied by the Bianchi identities. At dimension-3/2 these are

\[
\bar{D}_\alpha R = 0 , \quad D_\alpha C_{\beta \gamma} = i C_{\alpha \beta \gamma} + \frac{1}{3} \varepsilon_{\alpha \beta \gamma} \left( i \bar{D}_{\alpha} \bar{R} - D_\gamma S \right) ,
\]

---

\(2 \) Note that the (anti)symmetrization of \( n \) indices is defined to include a factor of \( (n!)^{-1} \).
together with their complex conjugates. These imply at dimension-2 the following descendant equation
\[(\mathcal{D}^2 - 4\mathcal{R})\mathcal{S} = (\bar{\mathcal{D}}^2 - 4\mathcal{R})\bar{\mathcal{S}} = 0, \quad \mathcal{D}^2 := \mathcal{D}^\gamma \mathcal{D}_\gamma, \quad \bar{\mathcal{D}}^2 = \bar{\mathcal{D}}_\gamma \mathcal{D}^\gamma.\] (2.6)

The constraints tell us that \(R\) and \(\mathcal{S}\) are respectively chiral and real linear superfields. It is not surprising that the 3D \(\mathcal{N} = 2\) geometry has the resemblance of a dimensionally reduced version of 4D \(\mathcal{N} = 1\) conformal supergravity in superspace (for reviews on 4D \(\mathcal{N} = 1\) supergravity see [26, 27]).

The fact that the supergeometry introduced corresponds to 3D \(\mathcal{N} = 2\) conformal supergravity, relies on the fact that the algebra (2.4a)–(2.4c) and the Bianchi identities are invariant under super-Weyl transformations of the covariant derivatives\(^3\)
\[\delta_\sigma \mathcal{D}_\alpha = \frac{1}{2} \sigma \mathcal{D}_\alpha + (\mathcal{D}^\gamma \sigma) \mathcal{M}_{\gamma\alpha} - (\mathcal{D}_\alpha \sigma) \mathcal{J},\] (2.7a)
where the scalar superfield \(\sigma\) is real and unconstrained. The dimension-1 torsion components transform as
\[\delta_\sigma R = \sigma R + \frac{1}{4} (\mathcal{D}^2 \sigma), \quad \delta_\sigma \mathcal{S} = \sigma \mathcal{S} + \mathcal{J}(\mathcal{D}^2 \sigma), \quad \delta_\sigma \mathcal{C}_a = \sigma \mathcal{C}_a + \frac{1}{8} (\gamma_\alpha) \gamma^\beta (\mathcal{D}_\beta \mathcal{D}^2 \sigma).\] (2.8)

It can be proved that, by using super-Weyl transformations, many components of the supergravity multiplet embedded in the geometry are gauged away. The remaining component fields are the vielbein \(e_\alpha^m\), the gravitini \(\Psi_\alpha^\mu\) and the U(1)\(_R\) connection \(A_\alpha\) with no auxiliary fields and no Weyl tensor. These are exactly the field components of the 3D \(\mathcal{N} = 2\) Weyl multiplet of conformal supergravity (see for example [28]).

### 3 3D \(\mathcal{N} = 2\) AdS supergravities

We have reviewed the geometric description of 3D \(\mathcal{N} = 2\) conformal supergravity in superspace. To generate the different 3D \(\mathcal{N} = 2\) AdS supergravities we first need to study the classes of scalar multiplets that can be used as conformal compensators. Depending on the compensator chosen, the different cosmological terms give rise to the two inequivalent 3D \(\mathcal{N} = 2\) AdS superspaces as solution of the supergravity equation of motions. Below we illustrate these steps.

As pointed out above, 3D \(\mathcal{N} = 2\) conformal supergravity is analogue to the 4D \(\mathcal{N} = 1\) one. In fact, there are three different natural types of scalar multiplets that can be used as conformal compensators to generate Poincaré supergravity. As in 4D [26, 27], these are:

(i) a complex chiral superfield \(\Phi\) satisfying
\[\mathcal{D}_\alpha \Phi = 0, \quad \delta_\sigma \Phi = \frac{1}{2} \sigma \Phi, \quad \mathcal{J} \Phi = -\frac{1}{2} \Phi;\] (3.9)

(ii) a real linear superfield \(G\) such that
\[(\mathcal{D}^2 - 4\mathcal{R})G = 0, \quad \delta_\sigma G = \sigma G, \quad \mathcal{J} G = 0;\] (3.10)

(iii) a complex linear superfield \(\Sigma\) that obeys the conditions
\[(\mathcal{D}^2 - 4\mathcal{R})\Sigma = 0, \quad \delta_\sigma \Sigma = w \sigma \Sigma, \quad \mathcal{J} \Sigma = (1 - w) \Sigma.\] (3.11)

The constant parameter \(w\) is real and constrained to be different from zero and one, \(w \neq 0, 1\), to ensure that \(\Sigma\) has nontrivial super-Weyl and U(1)\(_R\) transformations. The three choices of compensators respectively provide the 3D analogous of 4D \(\mathcal{N} = 1\): (i) old-minimal, (ii) new-minimal and (iii) non-minimal supergravities (look at [27] for a detailed list of references on the 4D \(\mathcal{N} = 1\) supergravities).

To describe 3D AdS supergravity one has to introduce cosmological terms that depend on the choice of the compensators. In the case (i), the supergravity action is\(^4\)
\[S_{\text{AdS}}^{(1,1)} = -4 \int d^3 x d^4 \theta E \Phi \Phi^4 + \mu \int d^3 x d^2 \theta \mathcal{E} \Phi^4 + \bar{\mu} \int d^3 x d^2 \bar{\theta} \bar{\mathcal{E}} \bar{\Phi}^4,\] (3.12)

\(^3\) We omit the transformations \(\delta \mathcal{D}_\alpha\) which are induced by the one of the spinor covariant derivatives \(\delta_\sigma \mathcal{D}_\alpha\) [1].

\(^4\) Here \(E\) is the full superspace density with \(E^{-1} = \text{Ber}(E_A M)\) and \(\mathcal{E}\) is the chiral density.
where $\mu$ is a complex constant parameter that plays the role of the cosmological constant. One can prove that the following equations hold on-shell

$$C_a = S = 0, \quad R = \mu.$$  \hspace{1cm} (3.13)

Denoting with $\nabla_A = (\nabla_a, \nabla_\alpha, \nabla_\bar{\alpha})$ the on-shell covariant derivatives, their algebra turn out to be

$$\{\nabla_\alpha, \nabla_\beta\} = -4\bar{\mu}M_{\alpha\beta}, \quad \{\nabla_\alpha, \nabla_\bar{\beta}\} = -2i\nabla_\alpha\bar{\beta}, \quad \{\nabla_\alpha, \nabla_\gamma\} = -2i\varepsilon_\gamma(\alpha \nabla_\beta), \quad \{\nabla_a, \nabla_b\} = -4\bar{\mu}M_{ab}.$$  \hspace{1cm} (3.14a)\hspace{1cm} (3.14b)

According to the classification of [25], the latter describes the 3D $(1,1)$ AdS superspace and the $(3.12)$ theory describes AdS supergravity. Note that on-shell the $U(1)_R$ generator disappear from the algebra.

The previous theory is the analogue of 4D $\mathcal{N} = 1$ old-minimal supergravity with a cosmological constant term [27]. Recently it was proved that an equivalent description can be achieved in non-minimal supergravity [29]. The same is true in 3D [2]. The basic idea is to notice that, given a general scalar complex superfield such that $\delta \Sigma = \bar{\mu}M_{\alpha\beta}$, recently it was proved that an equivalent description can be achieved in non-minimal supergravity. Note that on-shell the $U(1)_R$ generator disappear from the algebra.

A dual formulation of the 3D, $(1,1)$ AdS theory (3.12) is described by the first-order action [2]

$$S_{\text{AdS}}^{(1,1)} = -2 \int d^3 x d^4 \theta E(\bar{\Gamma}\Gamma)^{-1/2}.$$  \hspace{1cm} (3.16)

In fact, the equations of motion arising from (3.16) are exactly (3.13). The on-shell algebra is then the one of $(1,1)$ AdS superspace (3.14a)–(3.14b).

We are left with the case (ii) of a real linear compensator. In 4D it is known that no cosmological constant term is allowed in new minimal supergravity. However, in 3D the situation is different and more interesting. Note that the constraints (3.10) describe a 3D $\mathcal{N} = 2$ Abelian vector multiplet. Instead of the field strength $\mathcal{G}$, we can be use the real unconstrained prepotential superfield $\mathcal{G}$ such that: $\mathcal{G} = i\mathcal{D}^a\mathcal{D}_a\mathcal{G}$, $\delta_\sigma \mathcal{G} = J\mathcal{G} = 0$. This is defined up to gauge transformations $\delta \mathcal{G} = (\lambda + \bar{\lambda})$ generated by a chiral superfield $\lambda$ such that: $\mathcal{D}_\alpha \lambda = 0$ and $\delta_\sigma \lambda = J\lambda = 0$. The super-Weyl invariant, AdS supergravity action for a vector multiplet compensator has the following form [2] ($\rho$ is a real coupling constant)

$$S_{\text{AdS}}^{(2,0)} = \int d^3 x d^4 \theta E(\bar{\Gamma}\Gamma)^{-1/2}.$$  \hspace{1cm} (3.17)

Here $L_{\text{IT}}$ is a supergravity extension the improved tensor multiplet Lagrangian [32, 33] and is the 3D analogue of the 4D $\mathcal{N} = 1$ new-minimal supergravity Lagrangian [26, 27]. The Chern-Simons Lagrangian $L_{\text{CS}}$ is the 3D novelty and it represents a cosmological term. The equations of motion of (3.17) are

$$C_a = R = 0, \quad S = \rho.$$  \hspace{1cm} (3.18)

Denoting with $\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha, \mathcal{D}_{\bar{\alpha}})$ the on-shell covariant derivatives, their algebra is given by

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = 0, \quad \{\mathcal{D}_\alpha, \mathcal{D}_\bar{\beta}\} = -2i\mathcal{D}_\alpha\bar{\beta} - i\rho e_{\alpha\beta}\bar{\mathcal{J}} + i\rho M_{\alpha\beta}, \quad \{\mathcal{D}_\alpha, \mathcal{D}_\gamma\} = -\frac{i}{2}\rho e_{\gamma(\alpha \mathcal{D}_\beta)}, \quad \{\mathcal{D}_a, \mathcal{D}_b\} = -\frac{i}{2}\rho^2 M_{ab}.$$  \hspace{1cm} (3.19a)\hspace{1cm} (3.19b)

---

\textsuperscript{5} In global 4D $\mathcal{N} = 1$ supersymmetry, constraints of this form were introduced by Deo and Gates [30]. In the context of 4D supergravity, such constraints have recently been used in [31] and then in [29].
According to the classification of [25], the latter describes the 3D (2,0) AdS superspace and (3.17) describes an AdS supergravity theory. This is inequivalent to the (1,1) supergravity described by the chiral and non-minimal compensators. Note that in the (2,0) case the U(1) generator remains part of the algebra. The (1,1) and (2,0) AdS superspaces are characterized by quite different geometrical features that affect the matter systems that can be consistently formulated in these geometries. In what follows, we turn our attention to describing some matter systems in the two 3D AdS superspaces and to pointing out some of their differences.

4 Matter couplings in AdS superspaces

We turn to considering rigid supersymmetric field theories in (1,1) AdS superspace. These are theories invariant under the isometry transformations of the (1,1) AdS geometry. The isometries are generated by Killing vector fields, \( \Lambda = \lambda^a \nabla_a + \lambda^\alpha \nabla_\alpha + \lambda_\alpha \nabla^\alpha \), which, combined with an appropriate Lorentz transformation, leave invariant the covariant derivatives: \( [\Lambda + \frac{i}{2} \omega_{ab} \mathcal{M}_{ab}, \nabla_C] = 0 \) [2]. It can be shown that the (1,1) AdS Killing vector fields generate the supergroup \( \text{OSp}(1|2; \mathbb{R}) \times \text{OSp}(1|2; \mathbb{R}) \).

Matter couplings in (1,1) AdS superspace are very similar to those in 4D \( \mathcal{N} = 1 \) AdS [22, 23, 21], and they are more restrictive than their flat counterparts. As a nontrivial example, here we consider the most general supersymmetric nonlinear \( \sigma \)-model in (1,1) AdS superspace described by the action

\[
S = \int d^3 x d^4 \theta E \mathcal{K}(\phi^I, \bar{\phi}^\bar{J}) .
\] (4.1)

Here \( \phi^I \) are chiral superfields, \( \bar{\nabla}_\alpha \phi^I = 0 \), and at the same time local complex coordinates of a complex manifold \( \mathcal{M} \). The action is invariant under (1,1) AdS isometry transformations \( \delta \phi^I = \Lambda \phi^I \).

Unlike in the Minkowski case, the action does not possess Kähler invariance. This relies on the relation

\[
\int d^3 x d^4 \theta E F(\phi) = \int d^3 x d^2 \theta \mathcal{E} \mu F(\phi) \neq 0 ,
\] (4.2)

which relates every chiral integral of a holomorphic function to a full superspace integral. It turns out that, because of (4.2), the Lagrangian \( \mathcal{K} \) in (4.1) should be a globally defined function on the Kähler target space \( \mathcal{M} \). This implies that the Kähler two-form, \( \Omega = 2i \, \mathcal{J}(\phi^I, \bar{\phi}^\bar{J}) \), associated with the Kähler metric \( g_{I\bar{J}} := \partial_I \partial_{\bar{J}} \mathcal{K} \), is exact and hence \( \mathcal{M} \) is necessarily non-compact exactly as the 4D AdS case [22, 23, 21]. The \( \sigma \)-model couplings in (1,1) AdS are more restrictive than in the 3D Minkowski case.

What is the situation in the (2,0) AdS superspace? The Killing vector fields, \( \tau = \tau^a \mathbf{D}_a + \tau^\alpha \mathbf{D}_\alpha + \tau_\alpha \mathbf{D}^\alpha \), generating the isometries of (2,0) AdS superspace, obey the equation

\[
[\tau + it J + \frac{\tau}{2} t^{bc} \mathcal{M}_{bc}, \mathbf{D}_A] = 0
\]

where \( t \) and \( t^{bc} \) are constrained U(1) \( _R \) and Lorentz parameters [2]. The (2,0) AdS Killing vector fields prove to generate the supergroup \( \text{OSp}(2|2; \mathbb{R}) \times \text{Sp}(2, \mathbb{R}) \). Note that, due to the U(1) subgroup, matter couplings in (2,0) AdS superspace differ from those in the (1,1) case. In fact, only \( R \)-invariant actions can be consistently defined in (2,0) AdS superspace. Moreover, since in (2,0) superspace the chiral curvature is zero, \( R = 0 \), chiral integrals cannot be rewritten as a full superspace integral in contrast with the (1,1) case spelled out by eq. (4.2). This indicates that holomorphicity may still be an important ingredient in studying the dynamics in (2,0) AdS similarly to the flat case. As an example, consider the action

\[
S = \int d^3 x d^4 \theta E K(\phi^I, \bar{\phi}^\bar{J}) + \left\{ \int d^3 x d^2 \theta \mathcal{E} W(\phi^I) + \text{c.c.} \right\} .
\] (4.3)

Here \( \phi^I \) are chiral superfields, \( \mathbf{D}_a \phi^I = 0 \), with U(1) \( _R \) charges \( r_I \): \( J \phi^I = -r_I \phi^I \) (no sum over \( I \)). For \( R \)-invariance, the Kähler potential \( K(\phi, \bar{\phi}) \) and the superpotential \( W(\phi) \) should obey:

\[
\sum_I r_I \phi^I K_I = \sum_I r_I \phi^I K_I , \quad \sum_I r_I \phi^I W_I = 2W .
\] (4.4)
The action is invariant under the isometry transformations $\delta \phi^I = (\tau + i t^I) \phi^I$. An important class of $\sigma$-models in (2,0) AdS superspace is specified by the conditions $r_I = 0$ and $W(\varphi) = 0$. In this case no restrictions on the Kähler target space occur and, unlike the (1,1) case, compact target spaces are allowed.

Let us conclude by describing a system of self-interacting Abelian vector multiplets described by real linear field strengths $F^i$, with $i = 1, \ldots, n$. A general gauge invariant action in (2,0) AdS is

$$S = \int d^3 x d^4 \theta \left\{ L(F^i) + \frac{1}{2} m_{ij} F^i F^j + \xi_i F^i \right\}, \quad (4.5)$$

with $m_{ij} = m_{ji} = (m_{ij})^*$ and $\xi_i$ being Chern-Simons and Fayet-Iliopoulos coupling constants respectively. Here $F^i$ is the gauge prepotential for $F^i$ and $L$ is an arbitrary real function of $F^i$. The scalar superfields $F^i$ and $\bar{F}^i$ have isometry transformations $\delta F^i = \tau F^i$, $\delta \bar{F}^i = \tau \bar{F}^i$.

It is interesting to note that the very same action (4.5) is well defined in (1,1) AdS only if the Fayet-Iliopoulos term is not present, $\xi_i \equiv 0$. In fact, these are gauge invariant only in (2,0) AdS where $R = 0$.

Acknowledgements We thank the organizers of the XVII European Workshop on String Theory 2011 for the opportunity to report on these results. We are grateful to S. M. Kuzenko and U. Lindström for collaborations and discussions. This work is supported by the European Commission, Marie Curie IEF under contract No. PIEF-GA-2009-236454.

References

[1] S. M. Kuzenko, U. Lindström and G. Tartaglino-Mazzucchelli, JHEP 1103, 120 (2011).
[2] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, JHEP 1112, 052 (2011) [arXiv:1109.0496 [hep-th]].
[3] J. Bagger and N. Lambert, Phys. Rev. D 75, 045020 (2007); Phys. Rev. D 77, 065008 (2008); JHEP 0802, 105 (2008).
[4] A. Gustavsson, Nucl. Phys. B 811, 66 (2009).
[5] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, JHEP 0810, 091 (2008).
[6] W. Li, W. Song and A. Strominger, JHEP 0804, 082 (2008).
[7] E. A. Bergshoeff, O. Hohm and P. K. Townsend, Phys. Rev. Lett. 102, 201301 (2009).
[8] R. Andringa, E. A. Bergshoeff, M. de Roo, O. Hohm, E. Sezgin and P. K. Townsend, Class. Quant. Grav. 27, 025010 (2010).
[9] E. A. Bergshoeff, O. Hohm, J. Rosseel, E. Sezgin and P. K. Townsend, Class. Quant. Grav. 28, 015002 (2011).
[10] E. A. Bergshoeff, O. Hohm, J. Rosseel and P. K. Townsend, Class. Quant. Grav. 27, 235012 (2010).
[11] P. S. Howe and R. W. Tucker, J. Phys. A 10, L155 (1977); J. Math. Phys. 19, 869 (1978); J. Math. Phys. 19, 981 (1978).
[12] M. Brown and S. J. Gates Jr., Annals Phys. 122, 443 (1979).
[13] P. S. Howe, J. M. Izquierdo, G. Papadopoulos and P. K. Townsend, Nucl. Phys. B 467, 183 (1996).
[14] P. S. Howe and E. Sezgin, Class. Quant. Grav. 22, 2167 (2005).
[15] M. Cederwall, U. Gran and B. E. W. Nilsson, JHEP 1109, 011 (2011).
[16] J. Greitz and P. S. Howe, JHEP 1107, 071 (2011).
[17] S. M. Kuzenko, J.-H. Park, G. Tartaglino-Mazzucchelli and R. Unge, JHEP 1101, 146 (2011).
[18] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, Nucl. Phys. B 785, 34 (2007); JHEP 0806, 097 (2008); JHEP 0810, 001 (2008).
[19] D. Butter and S. M. Kuzenko, JHEP 1107, 081 (2011)
[20] J. Bagger and C. Xiong, JHEP 1107, 119 (2011).
[21] D. Butter and S. M. Kuzenko, Phys. Lett. B 703, 620 (2011); JHEP 1111, 080 (2011).
[22] A. Adams, H. Jockers, V. Kumar and J. M. Lapan, JHEP 1112, 042 (2011).
[23] G. Festuccia and N. Seiberg, JHEP 1106, 114 (2011).
[24] V. Pestun, [arXiv:0712.2824 [hep-th]]; [arXiv:0906.0638 [hep-th]]; D. L. Jafferis, arXiv:1012.3210 [hep-th].
[25] A. Achúcarro and P. K. Townsend, Phys. Lett. B 180, 89 (1986).
[26] S. J. Gates Jr., M. T. Grisaru, M. Roček and W. Siegel, Superspace, or One Thousand and One Lessons in Supersymmetry, Benjamin/Cummings (Reading, MA), 1983, hep-th/0108200.
[27] I. L. Buchbinder and S. M. Kuzenko, Ideas and Methods of Supersymmetry and Supergravity, Or a Walk Through Superspace, IOP, Bristol, 1998.
[28] M. Roček and P. van Nieuwenhuizen, Class. Quant. Grav. 3, 43 (1986).
[29] D. Butter and S. M. Kuzenko, Nucl. Phys. B 854, 1 (2012).
[30] B. B. Deo and S. J. Gates Jr., Nucl. Phys. B 254, 187 (1985).
[31] S. M. Kuzenko and S. J. Tyler, JHEP 1104, 057 (2011) [arXiv:1102.3042 [hep-th]].
[32] N. J. Hitchin, A. Karlhede, U. Lindström and M. Roček, Commun. Math. Phys. 108, 535 (1987).
[33] B. de Wit and M. Roček, Phys. Lett. B 109, 439 (1982).