A photonic transistor device based on photons and phonons in a cavity electromechanical system

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Abstract

We present a scheme for photonic transistors based on photons and phonons in a cavity electromechanical system, which is composed of a superconducting microwave cavity coupled to a nanomechanical resonator. Control of the propagation of photons is achieved through the interaction of microwave field (photons) and nanomechanical vibrations (phonons). By calculating the transmission spectrum of the signal field, we show that the signal field can be efficiently attenuated or amplified, depending on the power of a second 'gating' (pump) field. This scheme may be a promising candidate for single-photon transistors and pave the way for numerous applications in telecommunication and quantum information technologies.

(Some figures may appear in colour only in the online journal)

1. Introduction

A photonic transistor is a device where the propagation of the signal photons is controlled by another 'gate' photon [1]. Over the past few decades, such a device has received a lot of interest in view of its important applications ranging from optical communication and optical quantum computers [2] to quantum information processing [3]. Schemes based on nanoscale surface plasmons [4], microtoroidal resonators [5], a single-molecule [6] and some others [7–9] have been proposed to realize photonic transistors. However, practical realization remains challenging because it necessitates large nonlinearities and low losses.

Recently, Weis et al [10] reported a novel phenomenon based on the interaction between photons and phonons in a cavity optomechanical system, which is called optomechanically induced transparency (OMIT). The transition between the absorptive and transparent regimes of the probe laser field was modulated by a second control laser field [10, 11]. In the presence of the control laser, a transparency window for the probe field appears due to the optomechanical interference effect when the beating of the two laser fields is resonant with the vibration frequency of the mechanical resonator. At the same time, electromagnetically induced transparency (EIT) and slow light with optomechanics have also been experimentally realized [12]. As a counterpart of the cavity optomechanical system, a circuit cavity electromechanical system, usually composed of a superconducting microwave cavity and a nanomechanical resonator, has been under extensive investigation in recent years [13–16]. In particular, a strong-coupling regime was reached recently [17], which paves the way for ground-state cooling of the nanomechanical resonator. Based on these achievements, and we note that they mainly consider the situation where the cavity is driven on the red sideband, in this paper, we deal with a different case where the cavity is driven on its blue sideband in a coupled superconducting microwave cavity–nanomechanical resonator system [13–17]. Theoretical study shows that the transmitted signal ('source') field can be attenuated or amplified, depending on the power of the pump ('gate') field that controls the number of photons in the cavity. Therefore, such a system can be employed to serve as a photonic transistor and may be...
realized under the existing experimental conditions \[16, 17\]. Importantly, the photonic transistor proposed here does not require large nonlinearities and is compatible with chip-scale processing.

2. Model and theory

Our cavity electromechanical system, composed of a nanomechanical resonator capacitively coupled to a superconducting microwave cavity denoted by equivalent inductance \(L\) and equivalent capacitance \(C\), is sketched in figure 1. A strong pump (‘gate’) field with frequency \(\omega_p\) and a weak signal (‘source’) field with frequency \(\omega_s\) are applied to the microwave cavity simultaneously. The beating of the two fields causes the nanomechanical resonator to vibrate, which can change the capacitance of the microwave cavity and thus its resonance frequency. The coupling capacitance can be approximated by \(C_0(x) = C_0(1 - x/d)\), where \(C_0\) represents an equilibrium capacitance \(C\) and \(x\) is the displacement of the nanomechanical resonator from its equilibrium position. Therefore, the coupled cavity has an equivalent capacitance \(C_e = C + C_0(x)\), such that the resonance frequency of the microwave cavity is \(\omega_c = 1/\sqrt{LC_e}\). In a rotating frame at the pump frequency \(\omega_p\), the system Hamiltonian reads as follows \[13, 15\]:

\[
H = \hbar \Delta_p a^\dagger a + \hbar \omega_0 b^\dagger b - \hbar \omega_c a(b^\dagger + b) + i\hbar(A_0 a^\dagger - E_s^\dagger a) + i\hbar(E_s a^\dagger e^{-i\delta t} - E_s^\dagger a e^{i\delta t}) \tag{1}
\]

The first term is the energy of the microwave cavity, where \(a^\dagger (a)\) is the creation (annihilation) operator of the microwave cavity and \(\Delta_p = \omega_c - \omega_p\) is the cavity–pump field detuning. The second term gives the energy of the nanomechanical resonator (annihilation) operator \(b^\dagger (b)\), resonance frequency \(\omega_0\) and effective mass \(m\). The third term corresponds to the capacitive coupling between the microwave cavity and the nanomechanical resonator, where \(\lambda = g\delta{x}_{cp}\) is the coupling strength between the cavity and the resonator, \(g = \lambda_{0}\sqrt{\frac{m}{m_{nm}}}\) is the effect of the displacement \(x = (b^\dagger + b)\delta{x}_{cp}\) on the perturbed cavity resonance frequency and \(\delta{x}_{cp} = \sqrt{\frac{2}{m_{nm}}}\) is the zero-point motion of the nanomechanical resonator. The last two terms represent the interaction between the cavity field and the two rf fields with frequency \(\omega_p\) and \(\omega_s\). \(\delta = \omega_s - \omega_p\) is the signal–pump detuning. \(E_p\) and \(E_s\) are, respectively, the amplitudes of the pump field and the signal field, and they are defined by \(|E_p| = \sqrt{2P_p/\hbar\omega_p}\) and \(|E_s| = \sqrt{2P_s/\hbar\omega_s}\), where \(P_p\) is the pump power, \(P_s\) is the power of the signal field and \(\kappa\) is the decay rate of the cavity.

Applying the Heisenberg equations of motion for operators \(\hat{a}\) and \(\hat{Q}\), and introducing the corresponding damping and noise terms \[18, 19\], we derive the quantum Langevin equations as follows:

\[
\dot{a} = -(\delta + i\lambda) a + i\lambda Q + E_p e^{-i\omega t} + \sqrt{2\kappa} a_n, \tag{2}
\]

\[
\dot{Q} + \gamma_a Q + \omega_s^2 Q = 2\kappa\omega_s a + \xi, \tag{3}
\]

where we have set \(Q = b^\dagger + b\) as the resonator amplitude. The cavity mode decays at the rate \(\kappa\) and is affected by the input vacuum noise operator \(a_n\) with zero mean value, which obeys the correlation function in the time domain,

\[
\langle \delta a_n(t) \delta a_n(t') \rangle = \delta(t - t'), \tag{4}
\]

\[
\langle \delta a_n(t) \delta a_n(t') \rangle = \langle \delta a_n^\dagger(t) \delta a_n(t') \rangle = 0. \tag{5}
\]

The mechanical mode is affected by a viscous force with damping rate \(\gamma_a\) and by a Brownian stochastic force with zero mean value \(\xi\) that has the following correlation function \[20\]:

\[
\langle \xi(t) \xi(t') \rangle = \frac{\gamma_n}{\omega_n} \int \frac{d\omega}{2\pi} |\omega| e^{-i\omega(t-t')} \left[ 1 + \coth \left( \frac{\hbar\omega}{2k_B T} \right) \right], \tag{6}
\]

where \(k_B\) is the Boltzmann constant and \(T\) is the temperature of the reservoir of the mechanical resonator. Following standard methods from quantum optics, we derive the steady-state solution to equations (2) and (3) by setting all the time derivatives to zero. They are given by

\[
a_s = \frac{E_p}{\kappa + i(\Delta_p - \lambda Q_s)}, \quad Q_s = \frac{2\lambda |a_s|^2}{\omega_n}. \tag{7}
\]

To go beyond weak coupling, we can always rewrite each Heisenberg operator as the sum of its steady-state mean value and a small fluctuation with zero mean value,

\[
a = a_s + \delta a, \quad Q = Q_s + \delta Q. \tag{8}
\]

Inserting this equation into the Langevin equations (equations (2) and (3) and assuming \(|a_s| \gg 1\), one can safely neglect the nonlinear terms \(\delta a\delta a\) and \(\delta a\delta Q\) and get the linearized Langevin equations \[10, 18, 21\]:

\[
\delta \dot{a} = -(\kappa + i\Delta_p) \delta a + i\kappa Q_s \delta a + i\lambda a_s \delta Q + E_p e^{-i\omega t} + \sqrt{2\kappa} a_n, \tag{9}
\]

\[
\delta \dot{Q} + \gamma_a \delta a + \omega_s^2 \delta Q = 2\kappa\omega_s a_s (\delta a + \delta a^\dagger) + \xi. \tag{10}
\]

Such linearized Langevin equations have been used for investigating optomechanically induced transparency \[10\], parametric normal-mode splitting \[18\] and sideband cooling of mechanical motion \[21\] in optomechanical systems, where strong coupling is required. In the circuit cavity electromechanical system we study here, a strong-coupling regime has been achieved recently, where the cooperativity \(C \approx 2000000\) \[17\], larger than those previously achieved.
in optomechanical systems \([10, 22]\). Here, \(C = 4G^2/\gamma_0 \kappa\) \((G = \lambda/\sqrt{\pi} \kappa\) is the effective coupling strength) is an equivalent opto- or electromechanical cooperativity parameter. In this situation, the eigenmodes of the driven system are hybrids of the original mechanical mode and the cavity mode. The coupled system shows the normal-mode splitting, a phenomenon well known to both classical and quantum physics. The theoretical transmission spectrum obtained through the linearized Langevin equations is in good agreement with the experimental result \([17]\). In the following, since the drives are weak, but classical coherent fields, we will identify all operators with their expectation values, and drop the quantum and thermal noise terms \([10]\). Then the linearized Langevin equations can be written as

\[
\langle \dot{\delta a} \rangle = -(\kappa + i\Delta_p)\langle \delta a \rangle + i\lambda Q_t \langle \delta a \rangle + i\delta \alpha_s \langle \delta Q \rangle + E_s e^{-i\theta t},
\]

\[
\langle \dot{\delta Q} \rangle + \gamma_s \langle \delta Q \rangle + \alpha_s^2 \delta \langle \delta Q \rangle = 2\omega_0 \lambda \alpha_s \langle \delta a \rangle \langle \delta a^\dagger \rangle.
\]

In order to solve equations (11) and (12), we make the ansatz \([23, 24]\)

\[
\langle \delta a \rangle = a_s e^{-i\theta t} + a_s e^{i\theta t}, \quad \langle \delta Q \rangle = Q_s e^{-i\theta t} + Q_s e^{i\theta t}.
\]

Upon substituting the above ansatz into equations (11) and (12), we derive the following solution

\[
\alpha_s = \frac{i(\delta + \Delta_p) - (\kappa + \theta)}{(\theta - i(\Delta_p)^2 + \beta) E_s},
\]

where \(\eta = \frac{\alpha_s^2}{\sqrt{\alpha_s^2 - \omega_0^2 \gamma_s^2}}, \beta = \alpha_s^2 \eta^2 \omega_0^2 \gamma_s^2, \theta = \omega_0 \omega_s \eta_s (\eta + 1)\) and \(n_p = |a_s|^2\). Here \(n_p\) is approximately equal to the number of pump photons in the cavity, is determined by the following equation:

\[
n_p \left[ \kappa^2 + (\Delta_p - \omega_0 \alpha n_p) \right] = |E_p|^2.
\]

This form of cubic equation is characteristic of optical multistability \([25, 26]\).

The output field can be obtained by employing the standard input–output theory \([27]\),

\[
|a_{out}(t)|^2 = |a_p(t) - \sqrt{2\kappa} a_s(t)|^2,
\]

where \(a_{out}(t)\) is the output field operator; we have

\[
|a_{out}(t)|^2 = (E_p - \sqrt{2\kappa} a_s) e^{-i\theta t} + (E_s - \sqrt{2\kappa} a_s) e^{-i(\omega_0 + \theta) t} - \sqrt{2\kappa} a_{-s} e^{i(\omega_0 - \theta) t}.
\]

The transmission of the signal field, defined by the ratio of the output and input field amplitudes at the signal frequency, is then given by

\[
t_p = \frac{E_s - \sqrt{2\kappa} a_s}{E_s} = 1 - \sqrt{2\kappa} a_s / E_s.
\]

3. Numerical results and discussion

In what follows, we choose a realistic cavity electromechanical system used to calculate the transmission spectrum of the signal field. The parameters used in the numerical simulation are as follows \([16]\): \(\omega_0 = 2\pi \times 7.5\) GHz, \(\omega_s = 2\pi \times 6.3\) MHz, \(\kappa = 2\pi \times 600\) kHz, \(\lambda = 250\) Hz, \(\Delta_p = 10^6\), where \(Q_n\) is the quality factor of the nanomechanical resonator, and the damping rate \(\gamma_s\) is given by \(\omega_0 / Q_n\). We can see that \(\omega_0 > \kappa\), therefore the system operates in the resolved-sideband regime and also termed the good-cavity limit. When the cavity is driven on its red sideband, i.e. \(\Delta_p = \omega_0\), the analogy of EIT could appear, which has been extensively discussed \([10–12, 17]\). If we choose \(\Delta_p = \omega_0\), we can also obtain a similar EIT effect, as shown in figure 2(a). When the pump power increases from zero, the transmission of the signal field at signal–cavity detuning \(\Delta_s = 0\) increases to unity gradually, and then the transparency window is broadened by further increasing the pump power. Our result is in good agreement with that in \([17]\). The physical origin of the EIT effect can be understood as follows. The simultaneous presence of the pump and signal fields induces a radiation pressure force at the beat frequency \(\delta = \omega_s - \omega_p\). When this driving force is close to the mechanical resonance frequency \(\omega_p\), the vibrational mode is excited, giving rise to Stokes and anti-Stokes scattering of photons from the strong pump field. If the cavity is driven on its red sideband, the Stokes scattering at the frequency \(\omega_p - \omega_s\) is strongly suppressed and only the anti-Stokes scattering at the frequency \(\omega_p + \omega_s\) builds up within the cavity. However, the anti-Stokes field is degenerate with the near-resonant signal field sent to the cavity. Destructive interference between these two fields can suppress the build-up of an intracavity signal field \([17]\). Here, we mainly consider the situation where the cavity is driven on its blue sideband, i.e. \(\Delta_p = -\omega_0\). Under blue-detuning pumping, the effective interaction Hamiltonian for the cavity field and the mechanical phonon mode becomes one of the parametric amplifications, \(H_{int} = \hbar G(a b^* + a b)\), where \(G = \lambda/\sqrt{\pi} \kappa\) is the effective coupling strength. Figure 2(b) displays a series of transmission spectra of the signal field as a function of the signal–cavity detuning \(\Delta_s = \omega_s - \omega_p\) for various pump powers. When the pump field is off, i.e. \(P_p = 0\), the transmission spectrum of the signal field shows the usual Lorentzian line shape of the bare cavity. However, as the pump power is raised (\(P_p = 0.3\) and 0.5 pW), we can see that the transmission is attenuated around the signal-cavity detuning \(\Delta_s = 0\) compared with the situation where the pump field is on, a result of the increased feeding of photons into the cavity. If the pump power is increased further, the system switches from electromagnetically induced absorption (EIA) \([28]\) to parametric amplification (PA) \([29]\), leading to signal amplification (\(P_p = 0.6, 0.8\) and 0.9 pW). When the pump power equals 0.9 pW, the transmitted signal field can be amplified significantly. Similar to figure 2(a), at the low pump power, the normal-mode splitting is not apparent even if the coupled system has entered the strong-coupling regime. When the pump power is large enough (\(P_p = 10\) nW), normal-mode splitting can be easily observed. In the strong-coupling regime, the eigenmodes of the coupled system are hybrids of the original cavity modes characterized by \(n_p\) cavity photons and mechanical modes represented by \(n_m\) mechanical quanta. Figure 2(c) is the level diagram of the driven, coupled system. When the cavity is driven on its red sideband, corresponding to the transition between \(|n_p, n_m + 1\rangle\) and \(|n_p + 1, n_m\rangle\), the system exhibits the mechanical analogue of EIT. However, the blue-detuned pump field induces a transition between \(|n_p, n_m\rangle\) and \(|n_p + 1, n_m + 1\rangle\), which can efficiently amplify the signal field at the cavity resonance. Figure 2(b) demonstrates that such
Figure 2. The normalized magnitude of the cavity transmission $|t|^2$ as a function of signal–cavity detuning $\Delta_s = \omega_s - \omega_c$ for various pump powers with (a) $\Delta_p = \omega_n$ and (b) $\Delta_p = -\omega_n$, respectively. (c) Level diagram of the coupled system under red-detuned pumping ($\Delta_p = \omega_n$) and blue-detuned pumping ($\Delta_p = -\omega_n$), respectively. The signal field probes the transition in which the mechanical occupation is unchanged. Other parameters used are $\omega_c = 2\pi \times 7.5$ GHz, $\kappa = 2\pi \times 600$ kHz, $\lambda = 250$ Hz, $\gamma_n = 40$ Hz and $\omega_n = 2\pi \times 6.3$ MHz.

a circuit cavity electromechanical system can indeed act as a photonic transistor, where the pump (‘gate’) field regulates the flow of the signal (‘source’) field by controlling the number of photons in the cavity. We mainly use the narrow region around signal–cavity detuning $\Delta_s \approx 0$ to attenuate or amplify the signal field. The physical origin of this phenomenon is from the radiation pressure force oscillating at the beat frequency between the pump field and the signal field, which induces the vibration of the nanomechanical resonator. When the beat frequency is resonant with the mechanical resonance frequency $\omega_n$, the frequency of the pump field $\omega_p$ is downshifted to the Stokes frequency $\omega_p - \omega_n$, which is degenerate with the signal field. Constructive interference between the Stokes field and the signal field amplifies the weak signal field. A similar amplification of a signal due to radiation pressure back-action in a detuned cavity optomechanical system was recently demonstrated by Verlot et al [30]. Note that the phenomenon of parametric oscillation instability can occur at some pump power threshold when the Stokes field coincides with the cavity resonance, which has been predicted by Braginsky et al [31] and demonstrated for the first time at Caltech [32].

To better demonstrate the transistor action of the system, we plot the transmission of the signal field as a function of the signal–cavity detuning in figure 3(a) for $P_p = 0.8$ pW, 0.9 pW and 1.0 pW, respectively. It is clearly seen that transmission is considerably enhanced around the signal–cavity detuning $\Delta_s = 0$ at the higher pump power. Figure 3(b) summarizes the transistor characteristic curve by plotting the gain of the transmitted signal field as a function of the pump power when the two-photon detuning $\delta = \omega_n$. The transmitted signal field can be amplified when the pump power increases above a critical value. Recently, similar results have been obtained experimentally [33]. Therefore, the transmitted signal field can be attenuated or amplified in this coupled system under the control of the strong pump field when the cavity–pump detuning $\Delta_p = -\omega_n$.

4. Conclusion

In conclusion, we have demonstrated that the coupled nanomechanical resonator–microwave cavity system can serve...
Figure 3. (a) Amplification of the signal field around the region $\Delta_s = 0$ for three different pump powers with $\Delta_p = -\omega_n$. (b) The photonic transistor characteristic curve by plotting the gain of the transmitted signal field with respect to the pump power when the signal field is resonant with the cavity resonance frequency.

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