NEW CONSTRAINTS ON THE VARIABLE EQUATION OF STATE PARAMETER FROM X-RAY GAS MASS FRACTIONS AND SNE Ia

J. V. CUNHA*, L. MARASSI† and R. C. SANTOS‡

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Abstract

Recent measurements are suggesting that we live in a flat Universe and that its present accelerating stage is driven by a dark energy component whose equation of state may evolve in time. Assuming two different parameterizations for the function $\omega(z)$, we constrain their free parameters from a joint analysis involving measurements from X-Ray luminosity of galaxy clusters and SNe type Ia data.

1 Introduction

In the framework of general relativity, the present accelerating stage of the Universe (as indicated by SNe type Ia observations) can be explained by assuming the existence of a substantial amount of an exotic dark energy component with negative pressure, also known as quintessence[1, 2]. The existence of this extra component filling the Universe (beyond the cold dark matter) has also been indirectly suggested by independent studies based on fluctuations of the 3K relic radiation, large scale structure, age estimates of globular clusters or old high redshift objects, as well as by the X-ray data from galaxy clusters[3, 4].

A cosmological constant ($\Lambda$), the oldest and by far the most natural candidate for dark energy, faces some theoretical difficulties. The most puzzling of them is the so-called cosmological constant problem: the present cosmological upper bound, $\Lambda_o/8\pi G \sim 10^{-47}GeV^4$, differs from natural theoretical expectations from quantum field theory, $\sim 10^{71}GeV^4$, by more than 100 orders of magnitude. Actually, such a problem has also inspired many scenarios driven

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* jvital@dfte.ufrn.br
† lucimararassi@yahoo.com
‡ rose@dfte.ufrn.br
by a $\Lambda(t)$ or a time varying decaying vacuum with constant equation of state[5]. Among the remaining candidates to dark energy, the most promising ones lead to a time dependent equation of state (EOS), usually associated to a dynamical scalar field component. Such quintessence models may also parametrically be represented by an equation of state, $\omega(z)$, as proposed by Cooray and Huterer[6], as well as the one discussed by Linder[7], and, independently, by Padmanabhan and Choudhury[8]. In principle, the time variation of the EOS parameter, $\omega(z) \equiv p/\rho$, may allow a clear distinction between a cosmological constant model and the one driven by a rolling scalar field.

In actual fact, the exploration of the expansion history of the universe using $\omega(z)$ gave origin to a heated debate with growing interest in the recent literature. The SNe type Ia test is the most promising one related to this subject. However, Maor et al.[9], and Weller and Albrecht[10], have also observed that in order to constrain the evolution of the EOS with SNe observations, it is necessary to use a tight prior on the mean matter density of the Universe. A natural way to circumvent such a problem is to consider the constraints on the density parameter from measurements of the X-Ray luminosity of galaxy clusters together in a joint analysis involving SNe Ia observations.

In this work we investigate the cosmological implications from X-ray of galaxy clusters and SNe data by considering two different classes of EOS evolving with redshift. In the first scenario (hereafter Model 1), the EOS parameter is defined by[6]

$$Model\ 1:\ \omega(z) = \omega_0 + \omega_1 z, \quad (1)$$

whereas in the second, the EOS parameter reads[7, 8]

$$Model\ 2:\ \omega(z) = \omega_0 + \frac{\omega_1 z}{1 + z} \quad (2)$$

where $\omega_0$ and $\omega_1$ are constants.

It should be noticed that the linear expression of model 1 yields a reasonable approximation for most quintessence models out to redshift of a few, and, of course, it should be exact for models where $\omega(z)$ is a constant or varies slowly. The unsuitable aspect of the first expression is that it grows with no limit at high redshifts $z > 1$ (for example, distance to the last scattering surface at $z_{lss} \simeq 1100$). In order to fix such a problem, some authors[7, 8] have proposed the second form which has the advantage of giving finite $\omega(z)$ for all $z$. In both cases, $\omega_0$ is the current value of the EOS parameter and $\omega_1$ defines its variation rate for $R$ close to the present epoch ($z = 0$).

The paper is outlined as follows. Next section we set up the FRW equations for both parameterizations and the angular diameter distance necessary for our analysis using the X-ray luminosity of galaxy clusters. In section 3 we obtain our main results, namely: the observational constraints to the free parameters trough a joint analysis involving X-ray luminosity of galaxy clusters and SNe type Ia data. Finally, in section 4 we summarize the main conclusions and perspectives.
2 Basic Equations

In what follows it will be assumed that the Universe is flat and its dynamics is driven by a pressureless cold dark matter (CDM) fluid plus a quintessence component. Both components are separately conserved and the EOS parameter of the quintessence component is represented by one of the parameterizations appearing in the introduction (see Eqs. (1) and (2)). By integrating the energy conservation laws for each component and combining the result with the FRW equation, it is straightforward to show that the Hubble parameter for both models can be written as:

\[ H_{\text{Model}1}^2 = H_0^2 \left[ \Omega_M (1 + z)^3 + (1 - \Omega_M) (1 + z)^{3(1+\omega_0-\omega_1)} e^{3\omega_1 z} \right], \quad (3) \]

and

\[ H_{\text{Model}2}^2 = H_0^2 \left[ \Omega_M (1 + z)^3 + (1 - \Omega_M) (1 + z)^{3(1+\omega_0+\omega_1)} e^{-3\omega_1 (z/1+z)} \right], \quad (4) \]

where the subscript “0” denotes a present day quantity and \( \Omega_M \) is the CDM density parameter.

On the other hand, the first attempts involving gas mass fraction as a cosmological test were originally performed by Pen[12] and Sasaki[13], and further fully developed by Allen et al.[4] who analyzed the X-ray observations for six relaxed lensing clusters observed with Chandra in the redshift interval \( 0.1 < z < 0.5 \). A similar analysis has also been done for conventional quintessence models having constant EOS parameter by Lima et al.[14]. These authors also discussed the case for a cosmological scenario driven by phantom energy (\( \omega < -1 \)). Further, this test was also applied in the context of a Chaplygin gas EOS[15]. More recently, a detailed analysis using an improved sample observed with Chandra (26 clusters) was performed by Allen and collaborators[17] also considering a constant EOS parameter. In such studies, it is usually assumed that the baryonic gas mass fraction in galaxy clusters provides a fair sample of the distribution of baryons in the universe. In what follows, the gas mass fraction is defined as[14, 17]

\[ f_{\text{gas}}(z_i) = \frac{b \Omega_b}{(1 + 0.19 h^{3/2}) \Omega_M} \left[ 2h \frac{D_{\text{SCDM}}^S(z_i)}{D_{\text{DE}}^A(z_i)} \right]^{1.5}, \quad (5) \]

where \( b \) is a bias factor motivated by gas dynamical simulations that takes into account the fact that the baryon fraction in clusters seems to be lower than for the universe as a whole, \( \Omega_b \) stands for the baryonic mass density parameter, and with the term \( (2h)^{3/2} \) representing the change in the Hubble parameter between the default cosmology and quintessence scenarios while the ratio \( D^S_{\text{SCDM}}(z_i)/D^A_{\text{DE}}(z_i) \) accounts for deviations in the geometry of the universe from the Einstein-de Sitter CDM model.

In order to derive the constraints from X-ray gas mass fraction in the next section we shall use the concept of angular diameter distance, \( D_A(z) \). Such a quantity is readily derived in the present context (see, for instance, Refs. [14] and [15]):

\[ D^A_{\text{DE}} = \frac{H_0^{-1}}{(1 + z)} \int_{x'}^1 \frac{dx}{x^2 H(x)}. \quad (6) \]
where $x = \frac{R(t)}{R_c} = (1 + z)^{-1}$ is a convenient integration variable. In what follows, we will consider in our statistical modelling only flat cosmological models with Gaussian priors of $h = 0.72 \pm 0.08$[16] with $b = 0.824 \pm 0.089$[17] and $\Omega_M h^2 = 0.0214 \pm 0.002$[18].

3 Observational Constraints

Let us now discuss the constraints from X-ray luminosity of galaxy clusters and SNe type Ia data. First, it is worth notice that the 26 clusters cataloged by Allen et al.[17] are all regular, relatively relaxed systems for which independent confirmation of the matter density parameter results is available from gravitational lensing studies.

In order to determine the cosmological parameters $\omega_o$ and $\omega_1$ we use a $\chi^2$ minimization for the range of $\omega_0$ and $\omega_1$ spanning the interval $[-2.3,-0.4]$ and $[-4.6,5]$, respectively, in steps of 0.025. The 68.3%, 90.0% and 95.4% confidence levels are defined by the conventional two-parameters $\chi^2$ levels 2.30, 4.61 and 6.17, respectively. It is very important to note that we do not consider any prior in $\Omega_M$, as usually required by the SNe Ia test.

In addition to our gas mass fraction analysis we consider the SNe Ia measurements as given by Riess et al.[2]. The best fit of the model of Eq.(3) is $\chi^2 = 202.06$, $\omega_o = -1.25$, $\omega_1 = 1.3$ and $\Omega_M = 0.26$. For the Model 2, the best fit is $\chi^2 = 202.02$, $\omega_o = -1.4$, $\omega_1 = 2.57$ and $\Omega_M = 0.26$.

We now present our joint analysis (X-Ray luminosity from galaxy clusters and SNe Ia data). In the first EOS parameter we find at 2σ of likelihood that $-1.78 \leq \omega_o \leq -0.82$ and $-1.2 \leq \omega_1 \leq 2.7$. For the other model we get $-2.0 \leq \omega_o \leq -0.8$ and $-2.0 \leq \omega_1 \leq 5.5$ with 2σ. In Fig. 1, we show contours of constant likelihood in the $\omega_o$-$\omega_1$ plane. Note that the allowed range for both $\omega_o$ and $\omega_1$ is reasonably large thereby showing the impossibility of placing restrictive limits on these quintessence scenarios. However, these limits are better than those obtained by a simple SNe Ia analysis since in this case, the uncertainties on both parameters are strongly correlated when one marginalizes over $\Omega_M$.

At this point, it is interesting to compare our results with other recent determinations of $\omega_o$ and $\omega_1$ derived from independent methods. For example, the age constraints recently derived by Jain and Dev[19] are $\omega_o \leq -0.31$ and $\omega_1 \leq 0.96$ for the first model, and $\omega_0 \leq -0.31$ and $\omega_1 \leq 3.29$ for the second one. Riess et al.[2] found $\omega_o = -1.31^{+0.22}_{-0.28}$ (1σ) and $\omega_1 = 1.48^{+0.81}_{-0.90}$ (1σ) with the uncertainties in both parameters strongly correlated. In the article of Padmanabhan and Choudhury we must to use a constant $\Omega_M$ in order to analyze the $\omega_o - \omega_1$ plane. It should be stressed that the EOS corresponding to the cosmological constant is within the 1σ contour for $0.21 < \Omega_M < 0.41$, and models with $\omega_o > -1/3$ are ruled out at a high significance level for $\Omega_M < 0.4$ (we must to have very high negative values of $\omega_1$ in this case, and despite the high uncertainties in $\omega_1$ present in this data set, we know that it cannot vary but a few from $\omega_o$); this supernova observation analysis clearly indicates that the data are not sensitive to $\omega_1$ as compared to $\omega_o$. 
Figure 1: Marginalized constraints on plane $\omega_0$ and $\omega_1$ from joint analysis of the Chandra $f_{\text{gas}}(z)$ and SNe Ia data shown above for models 1 (left panel) and 2 (right panel). The solid lines mark the 1, 2 and 3 $\sigma$ confidence limits. See text for more details.

4 Conclusion and Perspectives

Nowadays, the signature of a dark energy or quintessence component is the most impressive observational fact of cosmology. As remarked elsewhere, we are living at a special moment where the emergence of new kind of “standard cosmology” seems to be inevitable. In the last few years, a growing attention has been paid for models with a time varying EOS parameter $\omega(z)$. With basis on this sort of cosmological scenario, we have discussed here two simple possible parameterizations of the EOS obeyed by quintessence models as recently presented in the literature. Our results suggest that it is worthwhile to use the estimates of the gas mass fraction from galaxy clusters in joint analysis with the SNe Ia data since the derived constraints for $\Omega_M$ (and other quantities) do not require any prior to this parameter. More important, we have also obtained constraints for $w_o$ and $w_1$ which have not been obtained before without a prior in $\Omega_M$.

The parameterizations seems to be more efficient to explain these data set, once they get a lower $\chi^2$; however they also have an additional parameter, and that is worthy of a study with more specific statistical criteria (Akaike or Bayesian information criteria, for example). A more detailed analysis of this kind will be investigated in the near future.

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