I stress that spacetime is a redundant abstraction, since describing the physical content of all so-called “space-time measurements” only requires timing (by a physical/material clock) of particle detections (at a physical/material detector). It is interesting then to establish which aspects of our current theories afford us the convenient abstraction of a spacetime. I emphasize the role played by the assumed triviality of the geometry of momentum space, which makes room for an observer-independent notion of locality. This is relevant for some recent studies of the quantum-gravity problem that stumbled upon hints of a nontrivial geometry of momentum space, something which had been strikingly envisaged for quantum gravity already in 1938 by Max Born. If indeed momentum space has nontrivial geometry then the abstraction of a spacetime becomes more evidently redundant and less convenient: one may still abstract a spacetime but only allowing for the possibility of a relativity of spacetime locality. I also provide some examples of how all this could affect our attitude toward the quantum-gravity problem, including some for the program of emergent gravity and emergent spacetime and an indication of triviality of the holographic description of black holes. And in order to give an illustrative example of possible logical path for the “disappearance of spacetime” I rely on formulas inspired by the \( \kappa \)-Poincaré framework.

1. Redundancy of spacetime and primitive measurements as timed particle detections

It is easy to see [1, 2] that the most basic (“primitive”) ingredients of all of our so-called “spacetime measurements” are particle detections, possibly (but not necessarily) also recording the time of the detections, according to a local physical clock, and also recording the momentum of the particle, if the detector is, e.g., a calorimeter.

This is particularly evident when we perform “spacetime measurements” in astrophysics, such as those using observations of particles from a source to “localize” that source. But it is actually no less true for all other “spacetime measurements”, including for example those involving the use of a ruler or a tape meter.

Evidently spacetime’s popularity as a notion used in physics must be the result of its being a very convenient abstraction, and most of these notes will be about recognizing in which ways this abstraction is convenient and recognizing which aspects of our current “understanding”/formulation of the laws of physics affords us this abstraction.

1.1. Emission-detection type and wasteful redundancies

Having the objective of keeping these notes reasonably short, I shall illustrate some concepts through single simple examples of their applicability. In this subsection I work in a 1+1-dimensional spacetime and consider the case which in our presently conventional description corresponds to flat background Minkowski spacetime. The measurement of emission-detection type which I consider involves an “observer” Alice, which is an emitter, and an “observer” Bob which is a detector, and specifically the case where Alice and Bob are in relative rest at a distance \( L \) from each other, with two synchronized clocks located one at Alice and one at Bob. Alice might write the following equation to describe the “trajectory in spacetime” (the worldline) of a certain specific particle:

\[
x = ct
\]  

(1)

Of the infinitely many ”truths” codified in this equation only two facts are established experimentally: the particle is emitted at Alice at time \( t = 0 \) of Alice’s clock and the particle is detected at Bob at time \( t = x/c \) of Bob’s clock.
We do need some notion of distance between the detectors actually at work and a notion of synchronization (or specifiable lack of synchronization) of clocks at those detectors, but the spacetime picture is redundant.

This is particularly clear in relativistic theories, and plays a role in the relationship between active and passive relativistic transformations. There the redundancy is in characterizations such as “the particle was at a distance of 10 meters from Alice, where Bob is”, which evidently can be fully replaced by “the particle was at Bob”. In the case discussed above a particle emitted at Alice at \( t = 0 \) is detected at Bob at \( t = L/c \), with the relationship between Alice and Bob such that it is given by a translation transformation with translation parameter \( L \).

This is for pure translations, but the generalization that includes boosts is equally trivial.

And notice that the redundancy of spacetime becomes more manifest in cases where quantum-mechanical effects are relevant. If there is a “double slit” between Alice and Bob the redundant spacetime description must be creatively adapted, but actually nothing changes about the scientific part of the description (with the rather marginal modification that some detection probabilities which could be idealized as 100\% within classical mechanics are often unavoidably smaller than 100\% within quantum mechanics).

1.2. Bubble-chamber type and convenient redundancies

If all we had were emission-detection-type measurements the abstraction of a spacetime would have probably never been made, since in those measurements it is wastefully redundant. But many of our “spacetime measurements”, particularly in the classical regime, are not of emission-detection type but rather of a type that could be described as a sequence of measurements of localization of the same particle (or body), in which case the abstraction of a spacetime is usefully redundant. Again within the vast collection of such measurements (which of course include, e.g., those performed when we watch a game of soccer) I choose to focus on only one, which I label “bubble-chamber type”.

Of course also for a bubble-chamber setup the primitive measurements are timed particle detections, which in most bubble-chamber setups will be photons detected by our eyes (or by a photographic camera). That sequence of photons however proves to be reliably describable in terms of the abstraction of positions for certain bubbles, and in turn the collection of bubbles allows us to infer reliably the abstraction of a “trajectory” for a charged microscopic particle. Evidently here too the spacetime abstraction is redundant: we could describe a bubble-chamber measurement exclusively in terms of timed detections of many photons. But the advantages of adopting the spacetime-trajectory abstraction in such cases are undeniable.

Among these “bubble-chamber-type measurements” I should also mention the ones relevant for many studies in astrophysics: when a star bursts or explodes we detect bunches of photons (occasionally also neutrinos, and maybe one day gravitons). The discussion of the physics content of these measurements could in principle be limited to these timed particle detections, but it is evidently advantageous to recognize that in these instances the sequence of particle detections can be organized into the abstraction of a localized explosion “far away in spacetime”.

1.3. Spacetime and the ether

What is then spacetime?
I shall leave most aspects of this question to the appetites of philosophers, who have license to debate (endlessly of course) about things that “exist but are not observed”. For the science of physics one can confine the discussion of this question to two simple observations:
(i) the fact that we can add reference to a “spacetime” without adding any new item to our list of measurement procedures (still only timed particle detections) is a complete proof of the redundancy of spacetime in science;
(ii) but, while awareness of its redundancy may at some point be valuable, the abstracted notion of spacetime is in some sense tangibly useful, at least in our current theories (i.e. at least within the confines of the regimes of physics so far explored), as stressed in the previous section. And as long as this is the situation there would be no reason of course for us to change the way to do physics, using the spacetime notion.
It will be clear from these notes that it seems to me the status of spacetime in our current theories shares some aspects of the status of the ether at the beginning of the 20th century. I find evidence on the internet of Poincaré stating, in 1889, that

"Whether the ether exists or not matters little - let us leave that to the metaphysicians; what is essential for us is, that everything happens as if it existed, and that this hypothesis is found to be suitable for the explanation of phenomena. After all, have we any other reason for believing in the existence of material objects? That, too, is only a convenient hypothesis; only, it will never cease to be so, while some day, no doubt, the ether will be thrown aside as useless."

Still focusing, for no good reason, on Poincaré, and offering something else of indirect relevance to my thesis, I must highlight his rather unfortunate encounters (present in several of his works both before 1905 and even after 1905) with the notion of "true time of resting clocks in the ether", used in descriptions such as this:

"Pour que la compensation se fasse, il faut rapporter les phenomenes, non pas au temps vrai \(t\), mais a un certain temps local \(t'\) defini de la facon suivante. Je suppose que des observateurs places en differents points, reglent leurs montres a laide de signaux lumineux; quils cherchent a corriger ces signaux du temps de la transmission, mais qu'ignorant le mouvement de translation dont ils sont animes et croyant par consequent que les signaux se transmettent egalement vite dans les deux sens, ils se bornent a croiser les observations en envoyant un signal de A en B, puis un autre de B en A. Le temps local \(t'\) est le temps marque par les montres ainsi reglées. Si alors \(c\) est la vitesse de la lumiere, et \(v\) la translation de la terre que je suppose parallele a laxe des x positifs, on aura: \(t' = t - vx/c^2\)."

1.4. More on primitive measurements

I have so far used the expression "primitive measurement" assuming my readers will share an intuitive perception of what I mean by it. I am unable to go much beyond that, but in closing this section I can attempt to give more structure to the intuition.

The most primitive measurements are the ones by our "resident devices", the monolithic measurement procedures associated with smell, touch, taste, sight and sound. Confirming my thesis, these are most evidently timed particle detections at some macroscopic detectors. Timing at this stage is provided by our resident clock, which is evidently there (though I do not understand much about it) and has to do with "memory" (a very low quality but rather robust clock). No other measurement could possibly be more primitive than those of our resident devices, but since those devices have very poor resolution, it is pointless to contemplate the level of fundamental physics that codifies exclusively observations by our resident devices.

Indeed because of this issue about the resolution of our resident devices, I am including among primitive measurements also those which involve a single step of inference. For this I consider those who can at least in principle be described as measurements using devices which can be operated as sealed boxes with a digital display on the surface of the box. Among these there is evidently no spacetime-measurement device. And the two examples I do see clearly are clocks and calorimeters. One can easily imagine adopting as standard for timing a certain specific class of identical sealed boxes with digital displays. The only stage of inference involved in making use of such clocks comes in when we read the clock, a single stage of inference involving particle detections. Similarly primitive are particle detections with energy (momentum) determination by a "generalized calorimeter" (an actual calorimeter or something that can serve the same purposes of a calorimeter). Again it is not hard to imagine adopting as standard for such devices a certain specific class of identical sealed boxes with digital displays. The only stage of inference involved in making use of such "calorimeters" comes in when we read the display, again a single stage of inference involving particle detections.

It should be easily appreciated that no spacetime measurement (such as measurements of distances or lengths) can aspire to be as primitive as the particle detections by our "resident devices". And no

1Part (but not all!) of this originates from the fact that spacetime measurements are typically nonlocal. The easiest examples of this are of the type of measurements of lengths, where the nonlocality of the measurement procedures is very explicit. Other examples are more subtle, as in the case of velocity measurements: velocity is formally a "local observable" but its measurement involves necessarily a nonlocal procedure.
sealed-box setup is available for spacetime measurements: I can leave a calorimeter unattended for all of its useful time of activity, with only a human in charge of reading the digital display, and still have a sequence of measurements, whereas there is no analogous sealed-box procedure that will provide us length measurements.

Of course, some of the procedures we conventionally label energy (/momentum) measurements are not primitive: we can use the available theoretical framework to infer an energy (/momentum) measurement through analyses that rely on several layers of inference. But this is evidently irrelevant for my thesis. There is a quality that timed particle detections with momentum determination can have which spacetime measurements cannot have.

Actually it should be clear that most measurements in physics are far removed from the status of being primitive: in particular both particle detections and spacetime measurements usually are performed relying on several layers of inference. As long as we trust our inferences (as long as the assumption that the inferences are trustworthy holds up) worrying about these layers of inference falls in the “sex-of-the-angels category”. But in cases, such as quantum-gravity research, where we look for a new theory paradigm and we may fear some of these inferences to become untrustworthy, awareness of the different layers of inference can be important.

In particular I am here arguing it is important to notice that our most primitive measurements all are timed particle detections. And I feel that at some stage it may become also important to distinguish finely among the amount of inference (the number of layers of inference) needed by some specific measurement procedures: comparing two non-primitive measurements we would typically find that one is more primitive (less non-primitive) than the other. As an example of this I can compare two spacetime measurements performed on a planar capacitor: a measurement of the area of the plates and a measurement of the distance between the plates. One can measure the area of the plates from the capacity of the capacitor if the distance between the plates is determined experimentally independently. So the results of this specific type of area measurement is less primitive than the result of the distance measurement (measurement of the distance between the plates) that it uses.

And it should be also evident that the facts on which our abstraction of a macroscopic spacetime rests involve several layers of inference. The way by which in astrophysics we characterize operatively the position in spacetime of a distant burst of particles is indeed a very clear illustration of these layers of inference. And this is not of mere academic interest, since it may well be that the first evidence of the need of adopting a new theory paradigm for quantum gravity might come from anomalous properties of this sort of observations of distant bursting sources.

2. The role of absolute locality

2.1. Absolute locality and translation transformations

From the perspective I am adopting it is interesting then to ask which structural properties of our presently used theories render the abstraction of spacetime so convenient, and if we can see on the horizon any reason for these structural properties to be lost.

As recently emphasized in Refs. [1, 2], a key aspect of our current theories which plays a role in the convenience of the abstraction of a spacetime is “absolute locality”. For example when an observer \( O \) detects particles from a distant explosion, and uses those particle detections as identification of a “distant point in spacetime”, then, and this is absolute locality, all other observers agree that a point-like source exploded. The point of spacetime obtained by abstraction from this procedure of course carries different coordinates in the different reference frames of the two observers, but the laws of transformation among observers renders this all intelligible, ensuring that the two observers “leave in the same spacetime”, that they simply give different coordinates to the same objective (abstraction of) spacetime point.

It was recognized in Refs. [1, 2] that a key role in establishing this absolute locality is played by the triviality of our current description of translation transformations. Take for example a distant 3-particle interaction \( e_{\pi p \nu} \), which could be the decay of a pion of momentum \( k_\mu \) into a muon of momentum \( p_\mu \) and a neutrino of momentum \( q_\mu \). From a locality perspective this must be viewed as a coincidence of 3 events: the disappearance of the pion and the appearances of the muon and the neutrino. According to an observer \( O \) the interaction occurs at the spacetime point with coordinates \( x_\mu \), which means that at the interaction the coordinates of the three particles coincide \( x_\mu^{(1)} = x_\mu^{(2)} = x_\mu^{(3)} = x_\mu \). In our current theories, because they indeed have absolute locality, another observer \( O' \), at distance \( b_\mu \) from \( O \), will also describe the same
3-particle interaction as a coincidence of 3 events, with \( x^{\prime \mu}_{(1)} = x^{\prime \mu}_{(2)} = x^{\prime \mu}_{(3)} = x^{\prime \mu} \). This is because of the triviality of translation transformations:

\[
  x^{\prime \mu}_{(n)} = x^{\mu} + \{x^{\mu}_{(n)}, b^{\nu} P^\nu_{tot}\} = x^{\mu}_{(n)} + \{x^{\mu}_{(n)}, b^{\nu} P(n)_{\nu}\} = x^{\mu}_{(n)} + b^{\mu}
\]

Since our current description of translation transformations works this way it is inevitable that a coincidence of events for observer \( O \) will also be described as a coincidence of events by another observer, \( O' \), at rest with respect to \( O \) at distance \( b^\mu \) from \( O \). But in Planck-scale theories with deformed translation transformations we must then be prepared to loose absolute locality (more later and in Refs. [1, 2]).

### 2.2. Absolute locality and boost transformations

Before I discuss a possible path ("Born’s prophecy") for how absolute locality might be lost in the quantum-gravity realm, let me first generalize the characterization of absolute locality given in Refs. [1, 2], from the case of translation transformations there considered to the more general case of transformations involving translations and/or boosts. Also the simplicity of our present description of boost transformations is responsible for the fact that we are afforded the luxury of absolute locality.

To see this let us consider again the example of 3-particle interaction \( e_k \otimes p_\mu \otimes \bar{p}_\nu \), decay of a pion into a muon and a neutrino. As done above, for observer \( O \) the interaction occurs at the spacetime point with coordinates \( x^{\mu}_{(1)} = x^{\mu}_{(2)} = x^{\mu}_{(3)} = x^{\mu} \). Rather than taking \( O' \) as a distant observer in relative rest with respect to \( O \), I now take \( O' \) as an observer purely boosted with respect to \( O \). And of course once again, since our current theories do afford us absolute locality, also \( O' \), boosted by rapidity \( \xi \) with respect to \( O \), will describe the 3-particle interaction as a coincidence of events, with \( x^{\prime \mu}_{(1)} = x^{\prime \mu}_{(2)} = x^{\prime \mu}_{(3)} = x^{\prime \mu} \). This is because of the triviality of our boost transformations, which (focusing for brevity on the 1+1-dimensional case) take the form

\[
  x^{\prime \mu}_{(n)} = x^{\mu} + \{x^{\mu}_{(n)}, b^{\nu} P^\nu_{tot}\} = x^{\mu}_{(n)} + \{x^{\mu}_{(n)}, b^{\nu} P(n)_{\nu}\} = x^{\mu}_{(n)} + b^{\mu}
\]

As I shall stress in the following some aspects of the study of the quantum-gravity problem may suggest a path for loosing absolute locality in ways that also involve a modification of this state of affairs for boosts.

### 3. Born’s prophecy and a quantum-gravity path to relative-locality momentum spaces

Absolute locality plays a key role in rendering the abstraction of a spacetime convenient. It ensures that “we all live in the same spacetime” [2], which is the main aspect of convenience of the spacetime abstraction: if we remove absolute locality then for example a point-like distant source for observer Alice (say, a gamma-ray burst) would not be a point-like source for some observer Bob; points of Alice’s spacetime would not be points of Bob’s spacetime and vice versa; Alice and Bob would not be different observers in the same spacetime, but rather different observers, each abstracting a different spacetime. This is scary and exciting. Could we actually lose absolute locality?

My next task is to show that some aspects of the quantum-gravity problem suggest that we might loose it! And remarkably it may be connected with a speculation made by Max Born already in 1938 [5], which suggests that in order to have a consistent quantum-gravity theory one should be able to accommodate a curvature for momentum space.

I call this “Born’s prophecy” because it went largely unnoticed for several decades (with the exception of a few isolated cases such as Ref. [6]). But in recent times several different realizations of the idea of a curved momentum space have surfaced in the quantum-gravity literature [7, 8, 9, 10]. In this section I draw what I see as the most compelling path from the structure of the quantum-gravity problem to the possibility of fulfilling Born’s prophecy and loosing absolute locality.
3.1. From minimum wavelength to DSR

The structure of the quantum-gravity problem is such that we are led to expect that the Planck length (or a length scale not too different from the Planck length) should play some role in the short-distance structure of spacetime. I shall not here review the very extensive literature on this, but my readers can find in Ref. [11] a good effort of reviewing the status of the relevant debate until the mid 1990s (a more recent perspective is in Ref. [12]). The roles attributed to the Planck length include: minimum allowed value for wavelengths, minimum allowed value for length, and minimum allowed value for the uncertainty in length measurement.

Remarkably, all this was being contemplated without appreciating that there could and perhaps should be implications for Lorentz symmetry. But if any of these speculations are correct it is absolutely necessary to ask [13,14] what would be the fate of Lorentz symmetry at the Planck scale. Think for example of the statement “the Planck length is the minimum allowed value of wavelength”: this is no less hostile [13] to Lorentz symmetry than a maximum-velocity principle is hostile to Galilean-boost symmetry.

An intense scrutiny of the fate of Lorentz symmetry at the Planck scale only started with the first works [15,16,17] attracting sizable attention on the possibility of Planck-scale modifications of the dispersion relation. We now have several scenarios for accommodating notions such a Planck-length minimum wavelength and/or Planck-scale-deformed dispersion relation within a deformed-Lorentz-symmetry setup of DSR (“doubly-special relativity”) type [13,14]. Preparing for my objective of exposing the usefulness of κ-Poincaré-inspired momentum space in the development of this literature, let me consider the following on-shellness/dispersion relation (again, for simplicity, in 1+1-D spacetime)

\[
\cosh(\ell m) = \cosh(\ell p_0) - \frac{\ell^2}{2} e^{-\ell p_0} p_1^2
\]

which is invariant under the deformed boosts given by

\[
[N, p_0] = p_1, \quad [N, p_1] = \frac{e^{2\ell p_0} - 1}{2\ell} + \frac{\ell}{2} p_1^2.
\]

Notice that according to (2) the length scale |\ell| is, for negative \ell, the minimum wavelength (the inverse of the Planck length is the maximum momentum of massless particles): this can be established by studying the action of boosts on momentum space and finding [13,14] that, for negative \ell, this action on spatial momentum saturates at 1/|\ell|, and it can also be established [18] directly from the dispersion relation (when \ell is negative p_1 \rightarrow 1/|\ell| as p_0 \rightarrow \infty). Since the boosts (3) ensure that the dispersion relation (2) is observer independent, we have here a way to enforce as relativistic laws both a modified dispersion relation and a principle of minimum wavelength. In these DSR-relativistic theories the scale \ell plays a role completely analogous to the speed-of-light scale \(c\) (not visible in my formulas because of the choice of units).

3.2. From DSR to deformed momentum composition and boost-induced relative locality

It was clear from the very beginning of studies of Planck-scale-deformed relativistic kinematics [13,14] that deforming boost actions on momenta requires, in order to preserve relativistic invariance, that also the law of composition of momenta be deformed. The logical steps that lead to this conclusion are very general and elementary, since they proceed just in the same way by which, taking as starting point Galilean relativity, the introduction of an invariant speed scale (the speed-of-light scale) requires a deformation of the law of composition of velocities. Let me illustrate this general feature within the context of the setup for which I already specified the laws (2) and (3). It is evident that conservation laws based on the special-relativistic composition law, (p + p')_\mu = p_\mu + p'_\mu, such as the conservation law formally given by p_\mu + p'_\mu = 0, are not covariant with respect to the boost action (3); in particular, one finds

\[
[N[p], N[p'], p_1 + p'_1] = \left(\frac{e^{2\ell p_0} - 1}{2\ell} + \frac{\ell}{2} p_1^2\right) + \left(\frac{e^{2\ell p'_0} - 1}{2\ell} + \frac{\ell}{2} p'_1^2\right)
\]

which does not vanish enforcing on the right-hand side the conservation law itself, p_\mu + p'_\mu = 0.

\[2\]In these notes, for brevity, I discuss processes handling all particles as incoming. For example, momentum conservation in propagation, when the composition of momentum is trivial, is written as p + p' = 0 with, say, p = \(\text{incoming}\) and p' = \(\text{outgoing}\). An example with deformed law of composition of momenta p \(\oplus k\), would be momentum conservation in propagation written as p \(\oplus p' = 0\) with, say, p = \(\text{incoming}\) and p' = \(\text{outgoing}\) (where \(\oplus\) denotes the “antipode” such that p \(\oplus [\ominus p] = 0\).
But relativistic covariance of the conservation laws can be reinstated by introducing suitably nontrivial [13] composition of momenta (and possibly suitably nontrivial composition of boosts, see later) such that

\[
[N[p] \oplus L N[p'], \langle p \oplus L p' \rangle_\mu] \big|_{\langle p \oplus L p' \rangle_\mu = 0} = 0
\]

The list of setups that satisfy this criterion is at this point very large. I’ll discuss one explicitly later in this section, and other examples are provided in Refs. [13, 19, 20, 21, 22, 23] and references therein.

DSR-deformed Lorentz transformations also have another consequence, no less dramatic than the deformation of the law of composition of momenta, and particularly relevant for the subject of these notes: relative locality (see Fig. 1).

Figure 1: I here illustrate a feature of relative spacetime locality that follows from certain fashionable DSR-deformations of Lorentz boosts, as shown in Ref. [22]. The left panel shows an idealized case in which Alice’s description has a coincidence of events, which she observes (in the origin of her reference frame), and two distant coincidences of events, which she infers. The right panel is for the description by Bob, purely boosted with respect to Alice, of the same idealized situation: also Bob observes the coincidence of nearby events, but for Bob the distant events are not exactly coincident. Red lines are for low-energy worldlines, unaffected by the deformation, while blue lines are for particles of energy high enough to be tangibly affected (within the resolution available experimentally) by the deformation.

As first noticed in Ref. [24], typically DSR-deformed boosts affect not the description of coincidences, but the description of distant events. For simplicity take the case of an observer Alice who is a local witness of a pion decay (a pion decay occurs in the origin of her coordinate system) and also infers an ordinary coincidence of events characterizing a distant pion decay (detects the decay products of a pion decay which occurred far from her origin, but from the spatial-momentum components and timing of those detections Alice infers a distant coincidence between the disappearance of the pion and the appearance of, say, the muon and the neutrino). It follows from the structure of typical DSR-deformed boosts that an observer Bob purely boosted with respect to Alice will still necessarily describe as “local”, i.e. as a coincidence of events, the pion decay in the origin of Alice (which is also the origin of Bob, since they are related by a pure boost), but instead in such a situation if Alice inferred a coincidence of events for the distant pion decay then Bob will necessarily infer such a distant pion decay in terms of events which are not coincident.

\footnotetext{3}{In retrospect, the relativity of locality of distant coincidences of events first described in Ref. [24] can be viewed as the answer to some puzzles for absolute locality of distant events which had been noticed in Refs. [25, 26, 27, 28, 29, 30]. A useful complementary way to characterize the presence of relativity of spacetime locality is obtained, following Ref. [31], by establishing the finiteness of the range of distances and boosts from a given observer within which (for given value of the relevant experimental resolution) one could still effectively rely on absolute locality.}
3.3. From deformed momentum composition and relative spacetime locality to curved momentum space

The two features I reviewed in the previous subsection, deformed laws of composition of momentum and loss of absoluteness of distant coincidences of events, must be clearly connected, but how? They must be connected because they are both linked to translation transformations: a distant event for observer $O$ is a nearby event for an observer $O'$ obtained from $O$ by a suitable translation transformation; and momenta are charges associated with translational symmetries. Essentially what was clearly missing was a satisfactory understanding of some deformed notion of translation transformations which would be compatible with the assumed deformations of Lorentz transformations. A satisfactory way to address this challenge was offered in Refs. [1, 2], within the novel “relative-locality framework” centered on the geometry of momentum space.

For what concerns translations, roughly speaking (detailed descriptions of these issues for translations are in Refs. [1,2,32,33]), the relative locality framework uses the modified conservation laws themselves as generators of the translation transformations. And it turns out [1,2,32,33], as here illustrated in Fig. 2, that these translations by themselves (even pure relative-locality-framework translations, without any boosting) produce relative locality: taken a pion-decay process described of course as a coincidence of events by the nearby observer Alice this pion decay will be in general inferred in terms of non-coincident events by a distant observer Bob (even if Bob is at rest with respect to Alice).

![Figure 2](image)

Figure 2: I here illustrate a feature of relative spacetime locality that can arise for suitable deformations of translation transformations, as shown in Refs. [1,2]. I am imagining something like an atom being deexcited at Alice (near the origin of Alice’s reference frame) thereby exchanging a photon to a distant atom, which is then excited, at Bob (near the origin of Bob’s reference frame), with Alice and Bob two distant observers in relative rest. The left panel shows Alice’s description of the coincidence of events she observes for the first process and the lack of coincidence of events by which she would infer the second process. The right panel shows Bob’s description of the coincidence of events he observes for the second process and the lack of coincidence of events by which he would infer the first process.

In relation to Born’s “prophecy” it is noteworthy that all these properties of the relative-locality framework can be described in terms of the geometry of momentum space. A metric in momentum space $ds^2 = g^{\mu\nu}(p)dp_\mu dp_\nu$ is used for writing the energy-momentum on-shell relation,

$$m^2 = D^2(p),$$

where $D(p)$ is the distance of the point $p_\mu$ from the origin $p_\mu = 0$. And a non-trivial affine connection is
needed for writing the law of composition of momenta\footnote{The perspective on modifications of the on-shell relation describing them in terms of an energy-dependent metric already surfaced in the DSR literature before the advent of the relative-locality framework (see, e.g., Refs. \cite{34,35}). But only a metric (without a specification of affine connection) does not make a geometry. The key conceptual ingredient of the relative-locality-framework proposal is the description of deformed laws of composition of momenta in terms of an affine connection on momentum space.}

\[
\langle p \oplus \ell \, q \rangle_\mu \simeq p_\mu + q_\mu - \ell_\mu^{\alpha \beta} p_\alpha q_\beta
\]

where on the right-hand side the momenta are assumed to be small with respect to $1/\ell$ and $\Gamma_\mu^{\alpha \beta}$ are the connection coefficients evaluated at $p_\mu = 0$.

Another key achievement of the recent work on the relative-locality curved-momentum-space framework is the ability to describe consistently and systematically interacting theories\footnote{I focus here, to keep things short, on two-particle events, which essentially describes the propagator and also describes for example a transition from a $K_0$ to a $\bar{K}_0$. The generalization to the more interesting case of events with more than two particles is in Refs. \cite{22,23}.} with deformed Lorentz symmetry, whereas earlier work with deformed Lorentz symmetry was mainly confined to free theories. And this relative-locality curved-momentum-space framework is not exclusively for theories with deformed Lorentz symmetry: on the contrary it handles on the same footing and with equal descriptive power both cases with geometry of momentum space compatible with the introduction of DSR-deformations and cases with geometry of momentum space that necessarily requires\footnote{\cite{22}} the introduction of a preferred class of observers.

### 3.4. From curved momentum space to more boost-induced relative locality

I have one more step in this long logical chain from aspects of the quantum-gravity problem through deformed boost transformations to curved momentum space. The features of distant relative locality uncovered by DSR-deformed boosts in Ref. \cite{24} were seen studying free particles. Now that we have available the description of interacting theories given by the relative-locality curved-momentum-space framework we can ask which additional features of relative locality are produced by deformed boosts in an interacting theory. This is the issue I studied in the recent Refs. \cite{22,23}. An intriguing case is the one where the momentum-composition law is

\[
(p \oplus \ell \, p')_1 = p_1 + e^\ell p_1', \quad (p \oplus \ell \, p')_0 = p_0 + p_0'.
\]

(5)

which within the relative-locality framework is viewed as a case with torsionful momentum space.

A relativistic description of interactions governed by the noncommutative composition law (5) requires another conceptual leap in the logical chain I am describing in this section. In our current theories (with linear momentum-composition law) we can simply impose that the boost of a two-particle\footnote{I focus here, to keep things short, on two-particle events, which essentially describes the propagator and also describes for example a transition from a $K_0$ to a $\bar{K}_0$. The generalization to the more interesting case of events with more than two particles is in Refs. \cite{22,23}.} event $e_{k \oplus p}$ would be governed by $N_{k \oplus p} = N_{[p]} + N_{[k]}$, i.e. we assume implicitly that the boost of such a two-valent interaction can be decomposed into two pieces, each given in terms of a boost acting exclusively on a certain momentum in the interaction. This is our standard concept of “total boost” generator obtained by combining trivially the boost generators acting on each individual particle. With noncommutative law of composition of momentum this is in general no longer possible. In particular, for the composition law (5) one easily verifies that

\[
[N_{[p]} + N_{[p']}, (p \oplus \ell \, p')_\mu]_{(p \oplus \ell \, p')_\mu = 0} \neq 0.
\]

So if a relativistic description is at all available it will have to be in terms of a deformed law of composition of boosts.

Looking for a suitable boost-composition law of this sort one quickly realizes that, since the only deformation scale here available is an inverse-momentum scale, the boost-composition law would have to involve
momentum. It is then not difficult to establish what does work: adopting the following boost-composition law

\[ N(p_0 p') = N[p] + e^{\ell p_0} N[p'] \]

(6)

one does find that \([N(p_0 p'), (p + \ell p')_\mu]_{(p_0 p')_\mu = 0} = 0\). This is easily verified:

\[ [N[p] + e^{\ell p_0} N[p'], p_0 + p'_0] = p_1 + e^{\ell p_0} p'_1 = 0 \]

(7)

\[ [N[p] + e^{\ell p_0} N[p'], p_1 + e^{\ell p_0} p'_1] = \frac{\ell}{2} \left( p_1 + e^{\ell p_0} p'_1 \right)^2 + \left( e^{2\ell(p_0 + p'_0)} - 1 \right) = 0 \]

(8)

where for both these results I of course enforced on the right-hand side the conservation law \( p + \ell p' = 0 \) itself.

4. A key example: \( \kappa \)-Poincaré-inspired momentum space

In the previous section I discuss some new scenarios for relativistic kinematics. We should expect that a full empowerment of research on these new (DSR-deformed) scenarios for relativistic kinematics will also need the counterpart of a corresponding description given in terms of symmetry algebras. This balance is rather visible in special relativity, whose full understanding requires combining the Poincaré symmetry algebra and Einstein kinematics. And we have one clear example (or at least a clear candidate example) of notion of symmetry algebra which can accommodate all of the features of relativistic kinematics highlighted in the previous section. This is the \( \kappa \)-Poincaré Hopf-algebra framework (see, e.g., Refs. [36, 37, 38, 39, 40]).

In the previous section I have set the stage for making this point by choosing accordingly my illustrative examples of the types of laws of relativistic kinematics that could be adopted as a DSR scenario with a relative-locality momentum-space description. Essentially I now must only show the possible connection between relativistic kinematics and Einstein kinematics. And we have one clear example (or at least a clear candidate example) of \( \kappa \)-Poincaré generators of boost and momenta in that basis (again specializing to the 1+1-D case):

\[ [\mathcal{N}, \mathcal{P}_0] = \mathcal{P}_1 \quad [\mathcal{N}, \mathcal{P}_1] = \frac{e^{2\ell p_0} - 1}{2\ell} + \frac{\ell}{2} \mathcal{P}_1^2. \]

(9)

from which one finds the following deformed mass Casimir:

\[ \cosh(\ell \mathcal{P}_0) - \frac{\ell^2}{2} e^{-\ell p_0} \mathcal{P}_1^2 \]

(10)

In relation to [5, 6] a connection is visible with the co-algebra (co-products) structure of the \( \kappa \)-Poincaré Hopf algebra in the basis of Ref. [38], which read

\[ \Delta(\mathcal{P}_0) = \mathcal{P}_0 \otimes 1 + 1 \otimes \mathcal{P}_0 \quad \Delta(\mathcal{P}_1) = \mathcal{P}_1 \otimes 1 + e^{\ell p_0} \otimes \mathcal{P}_1 \]

(11)

\[ \Delta(\mathcal{N}) = \mathcal{N} \otimes 1 + e^{\ell p_0} \otimes \mathcal{N} \]

(12)

Starting from these connections here highlighted I think it is legitimate to expect that the \( \kappa \)-Poincaré Hopf algebra will continue to be a precious “theoretical laboratory” for the future development of the quantum-gravity research line that I described schematically in the preceding section. It has been so far the most valuable “theoretical laboratory” of this sort (see Fig. 3). We now are in better position for seeing other similarly useful examples of novel symmetry-algebra structures potentially relevant for the development of Planck-scale-deformed kinematics. I conjecture for example that something similar to (but ultimately different from) the \( \kappa \)-Poincaré Hopf algebra will be needed in order to fully investigate some of most recent ideas that emerged on the deformed-kinematics side, such as the notion of “dual-gravity lensing” [32, 41].
5. Implications for quantum gravity

Relative locality is a spacetime feature which we can (and must) consider once we appreciate that spacetime is redundant. It is a way by which observers can still abstract a rather conventional spacetime picture, but since those pictures are only collections of inferences about a redundant structure, the spacetime abstraction of one observer does not need to match the spacetime abstraction of another observer. Spacetime points (coincident events) for one observer do not need to be spacetime points for another observer.

I have discussed a path toward the adoption of relative locality which is based on a perspective on the quantum-gravity problem. This is a path going through deformed Lorentz symmetry and deformed relativistic kinematics. Evidently if there is a deformation of relativistic kinematics it will have very significant implications on the nature of the quantum-gravity realm.

In this closing section I want to contemplate other sorts of implications for quantum-gravity research that could come from the realization of the redundancy of spacetime. I stress again that this is the main objective of these notes: the question of whether spacetime “exists” or “does not exist”, and even to a large extent the issues of the redundancy of spacetime, falls well beyond the boundary of science, so it is of
no interest to me, but an appreciation of the redundancy of spacetime in physics (in the sense discussed in the previous sections) may be relevant for building intuition for how future theories may be structured. Relativity of spacetime locality is an example of this, but other even wilder pictures should in principle be considered once we appreciate the redundancy of spacetime.

5.1. Implications for early-Universe cosmology

One of the limitations of the way humans learn physics is that our condition usually puts us in the position of going “upstream”. A good example is the program of “quantization of theories”: from Nature’s perspective theories start off being quantum theories and happen to be amenable to description in terms of classical mechanics only in some peculiar limiting cases, but our condition is such that we experience more easily those limiting cases rather than the full quantum manifestation of the laws. So we started with, e.g., Maxwell’s theory and then we learned about QED, a “quantization” of Maxwell’s theory. The logical path would have been much clearer (or at least clear much sooner) if we had the luxury of going downstream, first experiencing QED and then discovering Maxwell’s theory as a special limit of QED.

In light of the observations here reported we must infer that our “scientific relationship” with spacetime is another example of going upstream. Spacetime is a feature (an abstraction characterizing) our primitive measurements which are timelike particle detections. I do not see any way to introduce spacetime operatively without clocks and detectors. And yet it is standard to develop theories by introducing the spacetime picture as first ingredient, than introducing a long list of formal properties of fields (or particles) in that spacetime, and only in the end (for those brave enough to get that far) we worry about actually having detectors and clocks in our theory.

This works. It worked so far. It works in the same sense (and for the same reasons) of the success of the program of “quantization of theories”. But this luxury of going upstream might be lost at some point.

An example is early-Universe cosmology: our current models, even for sub-Planckian times, assume a spacetime picture. In light of the line of reasoning I have here advocated I would be very curious to see proposals of a description of the early Universe which initially (for a finite “duration”) has no spacetime. At Planckian energies, and without any macroscopic measuring device (detectors, clocks) to speak of, we might have already processes and yet not being provided the luxury of abstracting a spacetime, not even a spacetime affected by relative locality.

5.2. Implications for “emergent gravity”/“emergent spacetime”

For reasons similar to the ones I just stressed in relation to the early Universe, awareness of the redundancy of spacetime may be valuable for the fashionable idea of “emergent spacetime” and “emergent gravity”. The point here is in a subtle change of intuition: with “emergent spacetime” one is prepared to describe spacetime as not being “fundamental”, typically being a structure that takes the shape we are familiar with only for low-energy probes and instead has a different structure “fundamentally”. The line of reasoning I have advocated is such that spacetime might “emerge” in a rather different sense. It may well be that the abstraction of a spacetime admits a deeper level of abstraction, but this is not the key “emergence issue” we would worry about from a spacetime-redundancy perspective: we would want to know how and when (under which conditions) we are afforded the abstraction of a spacetime, and what replaces reliance on that abstraction in regimes where it does not apply. And is it not the case that we should only expect a spacetime picture to arise/emanate when actual physical/material clocks and detectors can be properly contemplated?

5.3. Triviality of a holographic description of black holes

The indirect evidence we have concerning the possibility to describe holographically black holes appears to be puzzling if one conceptualizes the physics of black holes to be “contained” in the 4D spacetime region with boundary the black-hole horizon. According to the perspective I here advocated one should conclude that describing the physics of black holes as contained in the 4D spacetime region with boundary the black-hole horizon is very naive. In the situation we colloquially describe as being in presence of a black hole we are ultimately describing emission/detection correlations among a collection of emitter/detectors at (or all around) the horizon of the black hole. The idea of “Hawking radiation” for example pertains to the cause/effect correlations that could be found when one of these emitter/detectors emits a particle (e.g. “toward the black hole”) and when one of these emitter/detectors detects a particle (a particle of Hawking radiation in the most interesting cases). The presence of an horizon should be more carefully described not in terms of the structure of the abstracted spacetime but rather in terms of the network of emitter/detectors.
that can be setup. Ultimately horizons are not to be described in terms of properties of the (abstracted) spacetime, but rather as limitations to signal exchange among (real, physical) emitter/detectors. There is no use for emitter/detectors to be contemplated inside the black hole: there is an (at least one-way) horizon between emitter/detectors inside the black hole and emitter/detectors outside the black hole.

From all this we deduce that the description of black holes is necessarily holographic, since it pertains to a distribution of emitter/detectors which is 2+1-dimensional. As a matter of fact even for classical black holes, where formally it seems we can introduce a non-holographic description of black holes, by assigning fields at each point of spacetime, including the interior of the black hole, the description of black holes is ultimately holographic: the values of the fields inside the black hole are not observable. Any theory which describes classical gravity exactly like Einstein gravity for the outside of black holes but gives a different description of the inside of black holes is to be described as exactly equivalent scientifically to Einstein gravity, since no measurement result could describe the difference between these two theories. It is rather intriguing that we have some evidence of the fact that the quantum description of black holes may be explicitly holographic (rather than being implicitly holographic like its classical-limit counterpart). But it should be acknowledged that the physics of black holes is inevitably holographic: the formalization may or may not be explicitly holographic but the physics is indeed inevitably holographic.

5.4. New paths to quantum gravity

An example of new path toward a solution of the quantum-gravity problem that can be inspired by awareness of the redundancy of spacetime can take as starting point the novel relative-locality curved-momentum-space framework.

In proposing this picture, Freidel, Kowalski-Glikman, Smolin and I adopted the standpoint [1, 2] that theories are formulated on momentum space, which works well with the redundancy of spacetime. I here just want to stress that in the relative-locality curved-momentum-space framework Newton constant, $G_N$, may be redundant. This is simply because one needs [1, 2] the Planck scale as scale of geometry of momentum space, and geometry of momentum space is primitive in the relative-locality framework. So rather than viewing the Planck scale as derived from Newton constant (plus Planck constant “$\hbar$” and speed-of-light scale “$c$”), within the relative-locality framework it is more natural to view Newton constant as derived from the Planck scale:

$$G_N = \frac{\hbar^2 c^5}{E_P^2}$$

Combining awareness of the redundancy of spacetime with the perspective of the relative-locality curved-momentum-space framework, one can envisage profoundly new ways for describing “spacetime physics”. As an example of what this might lead to, I observe in Fig. 4 that one could contemplate a new version of the Bronstein cube.
Figure 4: If the Planck scale is primitive (together with $\hbar$ and $c$) while Newton constant is derived, the Bronstein cube would have to be redrawn as here shown. I am assuming that the relative-locality regime is described by the relative-locality framework or something similar to it. This may also suggest that that the theory encompassing all these regimes ("quantum gravity") could be obtained as a quantum theory on the relative-locality momentum space.

From this perspective I am particularly intrigued by the fact that we are used to thinking that spacetime curvature produces noncommutativity of the momenta/translation generators (readers unfamiliar with this may think in particular of de Sitter spacetime and its symmetry algebra). So one could perhaps attempt to codify the geometry of the (inferred/redundant) spacetime within rules of noncommutativity of momenta at a given observer/detector. And something like Einstein’s equations for the dynamics of geometry of spacetime could perhaps also be written in terms of laws for the noncommutativity of momenta for different observers/detectors.

And it could be something even wilder than this. Gravity could be a purely quantum-mechanical effect: on the basis of $G_N = \hbar^2 c^5 / E_P^2$ one could be tempted of viewing Newton constant as a feature of the structure at order $\hbar^2$ of quantum theories on a curved momentum space.

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References

[1] G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman and L. Smolin, arXiv:1101.0931, Phys. Rev. D84 (2011) 084010
