Generation of three-dimensional entanglement between two spatially separated atoms via shortcuts to adiabatic passage

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Abstract

We propose a scheme for generating three-dimensional entanglement between two atoms trapped in two spatially separated cavities respectively via shortcuts to adiabatic passage based on the approach of Lewis-Riesenfeld invariants in cavity quantum electronic dynamics. By combining Lewis-Riesenfeld invariants with quantum Zeno dynamics, we can generate three dimensional entanglement of the two atoms with high fidelity. The Numerical simulation results show that the scheme is robust against the decoherences caused by the photon leakage and atomic spontaneous emission.

1. Introduction

Quantum entanglement plays a significant role not only in testing quantum nonlocality, but also in a variety of quantum information tasks [1–12]. Recently, high-dimensional entanglement is becoming more and more important since they are more secure than qubit systems, especially in the aspect of quantum key distribution. Besides, it has been demonstrated that violations of local realism by two entangled high-dimensional systems are stronger than that by two-dimensional systems [13]. So a lot of works have been done theoretically and experimentally in generating high-dimensional entanglement [14–23].

In order to realize the entanglement generation or population transfer in a quantum system with time-dependent interacting field, many schemes have been put forward. Such as π pulses, composite pulses, rapid adiabatic passage(RAP), stimulated Raman adiabatic passage , and their variants [24–26]. STIRAP is widely used in time-dependent interacting field because of the robustness for variations in the experimental parameters. But it usually requires a relatively long interaction time, so that the decoherence would destroy the intended

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dynamics, and finally lead to an error result. Therefore, reducing the time of dynamics towards the perfect final outcome is necessary and perhaps the most effective method to essentially fight against the dissipation which comes from noise or losses accumulated during the operational processes. Recently, various schemes have been explored theoretically and experimentally to construct shortcuts for adiabatic passage [27–37]. Unfortunately, as far as we know, the research of constructing shortcuts to adiabatic passage for generating entanglement has not been comprehensively studied.

In this paper, we construct an effective shortcuts to adiabatic passage for generating three dimensional entanglement between two atoms trapped in two spatially separated cavities connected by a fiber based on the Lewis-Riesenfeld invariants and quantum Zeno dynamics (QZD). The time for generating three dimensional entanglement in our scheme is much shorter time than that based on adiabatic passage technique. Moreover, the strict numerical simulations demonstrate that our scheme is insensitive to the decoherence caused by spontaneous emission and photon leakage.

This paper is structured as follows: In Section 2, we give a brief description about Lewis-Riesenfeld invariants and QZD. In Section 3, we construct a shortcuts for generating three dimensional entanglement. Section 4 shows the numerical simulation results and feasibility analysis. The conclusion appears in Section 5.

2. Preliminary theory

2.1. Lewis-Riesenfeld invariants

We first give a brief description about Lewis-Riesenfeld invariants theory [38, 39]. A quantum system is governed by a time-dependent Hamiltonian $H(t)$, and the corresponding time-dependent Hermitian invariant $I(t)$ satisfies

$$i\hbar \frac{\partial I(t)}{\partial t} = [H(t), I(t)].$$

(1)

The solution of the time-dependent Schrödinger equation $i\hbar \frac{\partial \Psi(t)}{\partial t} = H(t)\Psi(t)$ can be expressed by a superposition of invariant $I(t)$ dynamical modes $|\Phi_n(t)\rangle$

$$|\Psi(t)\rangle = \sum_n C_n e^{i\alpha_n} |\Phi_n(t)\rangle,$$

(2)

where $C_n$ is time-independent amplitude, $\alpha_n$ is the Lewis-Riesenfeld phase, $|\Phi_n(t)\rangle$ is one of the orthogonal eigenvectors of the invariant $I(t)$, satisfying $I(t)|\Phi_n(t)\rangle = \lambda_n|\Phi_n(t)\rangle$, with $\lambda_n$
being real constant. And the Lewis-Riesenfeld phases are defined as

$$\alpha_n(t) = \frac{1}{\hbar} \int_0^t dt' \langle \Phi_n(t') | i\hbar \frac{\partial}{\partial t'} - H(t') | \Phi_n(t') \rangle.$$  

(3)

2.2. Quantum Zeno dynamics

Quantum Zeno effect is an interesting phenomenon in quantum mechanics. Recent studies \[40–42\] show that a quantum Zeno evolution will evolve away from its initial state, but it remains in the Zeno subspace defined by the measurements \[40\] via frequently projecting onto a multidimensional subspace. This is known as QZD. We consider a system which is governed by the Hamiltonian

$$H_K = H_{\text{obs}} + K H_{\text{meas}},$$  

(4)

where $H_{\text{obs}}$ is the Hamiltonian of the investigated quantum system and the $H_{\text{meas}}$ is the interaction Hamiltonian performing the measurement. $K$ is a coupling constant, and when it satisfies $K \to \infty$, the whole system is governed by the evolution operator

$$U(t) = \exp[-it \sum_n (K \lambda_n P_n + P_n H_{\text{obs}} P_n)];$$  

(5)

where $P_n$ is one of the eigenprojections of $H_{\text{meas}}$ with eigenvalues $\lambda_n (H_{\text{meas}} = \sum_n \lambda_n P_n)$.

3. Shortcuts to adiabatic passage for generating three-dimensional entanglement of two atoms

FIG. 1: The schematic setup for generating two atoms three-dimensional entanglement. The two atoms are trapped in two spatially separated optical cavities connected by a fiber.

The schematic setup for generating three-dimensional entanglement of two atoms is shown in Fig.\[2\] We consider a cavity-fibre-cavity system, in which two atoms are trapped in the corresponding optical cavities connected by a fiber. Under the short fiber limit $(lv)/(2\pi c) \ll 1$, only the resonant mode of the fiber will interact with the cavity mode \[43\], where $l$ is the length of the fiber and $v$ is the decay rate of the cavity field into a continuum of fiber.
FIG. 2: The level configurations of atom A and B.

modes. The corresponding level structures of atoms are shown in Fig. 2. Atom A has two excited states $|e_L\rangle, |e_R\rangle$, and five ground states $|1\rangle, |R\rangle, |L\rangle, |g\rangle$ and $|0\rangle$, while atom B is a five-level system with three ground states $|R\rangle, |L\rangle$ and $|g\rangle$, two excited states $|e_L\rangle$ and $|e_R\rangle$. For atom A, the transitions $|0\rangle \leftrightarrow |e_R\rangle$ and $|1\rangle \leftrightarrow |e_L\rangle$ are driven by classical fields with the same Rabi frequency $\Omega_A(t)$. And the transitions $|R\rangle \leftrightarrow |e_R\rangle$ and $|L\rangle \leftrightarrow |e_L\rangle$ are resonantly driven by the corresponding cavity mode $a_{Aj}$ with $j$-circular polarization and the coupling strength is $g_{Aj}$ ($j = L, R$). For atom B, the transitions $|R\rangle \leftrightarrow |e_R\rangle$ and $|L\rangle \leftrightarrow |e_L\rangle$ are driven by classical fields with the same Rabi frequency $\Omega_B(t)$, and the transitions $|g\rangle \leftrightarrow |e_R\rangle$ and $|g\rangle \leftrightarrow |e_L\rangle$ are resonantly driven by the corresponding cavity mode $a_{Bj}$ with $j$-circular polarization and the coupling strength is $g_{Bj}$ ($j = L, R$). The whole Hamiltonian in the interaction picture can be written as ($\hbar = 1$):

$$H_1 = H_{a-l} + H_{a-c-f},$$

$$H_{a-l} = \Omega_A(t)(|e_L\rangle_A\langle 1| + |e_R\rangle_A\langle 0|) + \Omega_B(t)(|e_L\rangle_B\langle L| + |e_R\rangle_B\langle R|) + \text{H.c.},$$

$$H_{a-c-f} = g_{AL}a_{AL}|e_L\rangle_A\langle L| + g_{AR}a_{AR}|e_R\rangle_A\langle R| + g_{BL}a_{BL}|e_L\rangle_B\langle g| + g_{BR}a_{BR}|e_R\rangle_B\langle g| + \eta b_L(a_{AL}^\dagger + a_{BL}^\dagger) + \eta b_R(a_{AR}^\dagger + a_{BR}^\dagger) + \text{H.c.},$$

where $\eta$ is the coupling strength between cavity mode and the fiber mode, $b_R(L)$ is the annihilation operator for the fiber mode with $R(L)$-circular polarization, $a_{A(B)R(L)}$ is the annihilation operator for the corresponding cavity field with $R(L)$-circular polarization, and $g_{A(B)R(L)}$ is the coupling strength between the corresponding cavity mode and the trapped atom.
In order to obtain the following two atoms three-dimensional entanglement:
\[ |\Psi\rangle = \frac{1}{\sqrt{3}}(|R_A|_A R_B + |L_A|_A L_B + |g_A|_A g_B), \]
we assume atom A in the state
\[ |\Psi_A\rangle = \frac{1}{\sqrt{3}}(|1_A| + |0_A| + |g_A|), \]
while atom B in the state \(|g_B\rangle\), both the cavity modes and the fiber mode in vacuum state \(|0_{AC}|_A 0_{BC}|_B\rangle\). Then we present how to realize the evolutions of the atom state \(|1_A|g_B\rangle\) to \(-|L_A|_A L_B, |0_A|_A g_B\rangle\) to \(-|R_A|_A R_B, |g_A|_A g_B\rangle\) to \(-|g_A|_A g_B\rangle\).

For the initial state \(|0_A|g_B|0_{AC}|0_{BC}|_B\rangle\), the whole system evolves in the subspace spanned by
\[ |\phi_1\rangle = |0_A|_A g_B|0_{AC}|0_{BC}|_B\rangle, \]
\[ |\phi_2\rangle = |e_{R_A}|_A g_B|0_{AC}|0_{BC}|_B\rangle, \]
\[ |\phi_3\rangle = |R_A|_A g_B|1_{AC}|0_{BC}|_B\rangle, \]
\[ |\phi_4\rangle = |R_A|_A g_B|0_{AC}|0_{BC}|1_{B}\rangle, \]
\[ |\phi_5\rangle = |R_A|_A g_B|0_{AC}|0_{BC}|1_{B}\rangle, \]
\[ |\phi_6\rangle = |R_A|_A g_B|0_{AC}|0_{BC}|_B\rangle, \]
\[ |\phi_7\rangle = |R_A|_A R_B|0_{AC}|0_{BC}|_B\rangle. \]

Seting \(\Omega_A(t), \Omega_B(t) \ll \eta, g_{AR(L)}, g_{BR(L)}\), then both the condition \(H_{a-c-f} \gg H_{a-l}\) and the Zeno condition \(K \rightarrow \infty\) can be satisfied \((H_{a-l} \text{ and } H_{a-c-f} \text{ correspond respectively to } H_{obs} \text{ and } KH_{meas} \text{ in Eq. (4)})\). By performing the unitary transformation \(U = e^{-iH_{a-c-f}t}\) under condition \(H_{a-c-f} \gg H_{a-l}\), the Hilbert subspace can be divided into five invariant Zeno subspaces \([41, 42]\):
\[ \Gamma_{P1} = \{|\phi_1\rangle, |\phi_7\rangle, |\psi_1\rangle\}, \]
\[ \Gamma_{P2} = \{|\psi_2\rangle\}, \quad \Gamma_{P3} = \{|\psi_3\rangle\}, \]
\[ \Gamma_{P4} = \{|\psi_4\rangle\}, \quad \Gamma_{P5} = \{|\psi_5\rangle\}, \]
with the eigenvalues \(\lambda_1 = 0, \lambda_2 = -g, \lambda_3 = g, \lambda_4 = -\sqrt{g^2 + 2\eta^2} = -\varepsilon, \text{ and } \lambda_5 = g\).
\[ \sqrt{g^2 + 2\eta^2} = \varepsilon, \] where we assume \( g_{AR(L)} = g_{BR(L)} = g \) for simplicity. Here

\[ |\psi_1\rangle = \frac{1}{\varepsilon}(\eta|\phi_2\rangle - g|\phi_4\rangle + \eta|\phi_6\rangle), \]
\[ |\psi_2\rangle = \frac{1}{2}(-|\phi_2\rangle + |\phi_3\rangle - |\phi_5\rangle + |\phi_6\rangle), \]
\[ |\psi_3\rangle = \frac{1}{2}(-|\phi_2\rangle - |\phi_3\rangle + |\phi_5\rangle + |\phi_6\rangle), \]
\[ |\psi_4\rangle = \frac{1}{2\varepsilon}(g|\phi_2\rangle - \varepsilon|\phi_3\rangle + 2\eta|\phi_4\rangle) - \varepsilon|\phi_5\rangle + g|\phi_6\rangle), \]
\[ |\psi_5\rangle = \frac{1}{2\varepsilon}(g|\phi_2\rangle + \varepsilon|\phi_3\rangle + 2\eta|\phi_4\rangle) + \varepsilon|\phi_5\rangle + g|\phi_6\rangle, \] (13)

and the corresponding projection

\[ P_i^\alpha = |\alpha\rangle \langle \alpha|, (|\alpha\rangle \in \Gamma_{\alpha}). \] (14)

Under the above condition, the system Hamiltonian can be rewritten as the following form [42]:

\[
H_{\text{total}} \simeq \sum_{i,\alpha,\beta} (\lambda_i P_i^\alpha + P_i^\alpha H_{\alpha,\beta} P_i^\beta) = -g|\psi_2\rangle \langle \psi_2| + g|\psi_3\rangle \langle \psi_3| - \varepsilon|\psi_4\rangle \langle \psi_4| + \varepsilon|\psi_5\rangle \langle \psi_5| + \frac{1}{\varepsilon}\eta(\Omega_A(t)|\psi_1\rangle \langle \phi_1| + \Omega_B(t)|\psi_1\rangle \langle \phi_7| + \text{H.c.}). 
\] (15)

When we choose the initial state \( |\phi_1\rangle = |0\rangle_A|g\rangle_B|0\rangle_{AC}|0\rangle_{BC}|0\rangle_f \), the Hamiltonian \( H_{\text{total}} \) reduces to

\[ H_{\text{eff}} = \Omega_{A1}(t)|\psi_1\rangle \langle \phi_1| + \Omega_{B1}(t)|\psi_1\rangle \langle \phi_7| + \text{H.c.}, \] (16)

where \( \Omega_{A1}(t) = \frac{1}{\varepsilon}\eta\Omega_A(t) \) and \( \Omega_{B1}(t) = \frac{1}{\varepsilon}\eta\Omega_B(t). \)

In order to construct the shortcuts for generating three-dimensional entanglement by the dynamics of invariant based inverse engineering, we need to find out the Hermitian invariant operator \( I(t) \), which satisfies \( i\hbar\frac{\partial I(t)}{\partial t} = [H_{\text{eff}}(t), I(t)] \). Since \( H_{\text{eff}}(t) \) possesses SU(2) dynamical symmetry, \( I(t) \) can be easily given by [44, 45]

\[ I(t) = \chi(\cos \nu \sin \beta|\psi_1\rangle \langle \phi_1| + \cos \nu \cos \beta|\psi_1\rangle \langle \phi_7| + i \sin \nu|\phi_7\rangle \langle \phi_1| + \text{H.c.}), \] (17)

where \( \chi \) is an arbitrary constant with units of frequency to keep \( I(t) \) with dimensions of energy, \( \nu \) and \( \beta \) are time-dependent auxiliary parameters which satisfy the equations

\[ \dot{\nu} = \Omega_{A1}(t) \cos \beta - \Omega_{B1}(t) \sin \beta, \]
\[ \dot{\beta} = \Omega \tan \nu[\Omega_{A1}(t) \cos \beta + \Omega_{B1}(t) \sin \beta]. \] (18)
Then we can derive the expressions of $\Omega_{A1}(t)$ and $\Omega_{B1}(t)$ easily as follows:

$$\Omega_{A1}(t) = (\dot{\beta} \cot \nu \sin \beta + \dot{\nu} \cos \beta),$$
$$\Omega_{B1}(t) = (\dot{\beta} \cot \nu \cos \beta - \dot{\nu} \sin \beta).$$

(19)

The solution of Shrödinger equation $i\hbar \partial |\Psi(t)\rangle/\partial t = H_{\text{eff}}(t) |\Psi(t)\rangle$ with respect to the instantaneous eigenstates of $I(t)$ can be written as $|\Psi(t)\rangle = \sum_{n=0,\pm} C_n e^{i \theta_n} |\Phi_n(t)\rangle$, where $\theta_n(t)$ is the Lewis-Riesenfeld phase in Eq. (3), $C_n = \langle \Phi_n(0) | \phi'_1 \rangle$, and $|\Phi_n(t)\rangle$ is the eigenstate of the invariant $I(t)$

$$|\Phi_0(t)\rangle = \cos \nu \cos \beta |\phi_1\rangle - i \sin \nu |\psi_1\rangle - \cos \nu \sin \beta |\phi_7\rangle,$$
$$|\Phi_\pm(t)\rangle = \frac{1}{\sqrt{2}} [ (\sin \nu \cos \beta \pm i \sin \beta) |\phi_1\rangle + i \cos \nu |\psi_1\rangle - (\sin \nu \sin \beta \mp i \cos \beta) |\phi_7\rangle].$$

(20)

In order to transfer the population from state $|\phi_1\rangle$ to $-|\phi'_3\rangle$, we choose the parameters as

$$\nu(t) = \epsilon, \quad \beta(t) = \frac{\pi t}{2t_f},$$

(21)

where $\epsilon$ is a time-independent small value and $t_f$ is the total pulse duration. After the precise calculation, we can easily obtain

$$\Omega_{A1}(t) = \frac{\pi}{2t_f} \cot \epsilon \sin \frac{\pi t}{2t_f},$$
$$\Omega_{B1}(t) = \frac{\pi}{2t_f} \cot \epsilon \cos \frac{\pi t}{2t_f},$$

(22)

and

$$\Omega_A(t) = \frac{\sqrt{g^2 + 2\eta^2 \pi}}{2t_f} \cot \epsilon \sin \frac{\pi t}{\eta 2t_f},$$
$$\Omega_B(t) = \frac{\sqrt{g^2 + 2\eta^2 \pi \sin^2 \epsilon}}{\eta 2t_f} \cot \epsilon \cos \frac{\pi t}{2t_f}.$$  

(23)

When $t = t_f$,

$$|\Psi(t_f)\rangle = -i \sin \epsilon \sin \theta |\phi_1\rangle + (\cos \epsilon - \sin \epsilon \cos \theta) |\psi_1\rangle$$
$$+ (\cos^2 \epsilon - \sin^2 \epsilon \cos \theta) |\phi_7\rangle,$$

(24)

where $\theta = \pi/(2 \sin \epsilon) = |\theta_\pm|$ ($\theta_\pm$ are the Lewis-Riesenfeld phases). We choose $\theta = 2N\pi (N = 1, 2, 3...)$, then $|\Psi(t_f)\rangle = -|\phi_7\rangle$. 
On the other hand, for the initial state $|\phi'_1\rangle = |1\rangle_A |g\rangle_B |0\rangle_{AC} |0\rangle_{BC} |0\rangle_f$, the whole system evolves in the subspace spanned by

\[
|\phi'_1\rangle = |1\rangle_A |g\rangle_B |0\rangle_{AC} |0\rangle_{BC} |0\rangle_f,
|\phi'_2\rangle = |e_L\rangle_A |g\rangle_B |0\rangle_{AC} |0\rangle_{BC} |0\rangle_f,
|\phi'_3\rangle = |L\rangle_A |g\rangle_B |1\rangle_{AC} |0\rangle_{BC} |0\rangle_f,
|\phi'_4\rangle = |L\rangle_A |g\rangle_B |0\rangle_{AC} |1\rangle_{BC} |0\rangle_f,
|\phi'_5\rangle = |L\rangle_A |g\rangle_B |0\rangle_{AC} |1\rangle_{BC} |0\rangle_f,
|\phi'_6\rangle = |L\rangle_A |e_L\rangle_B |0\rangle_{AC} |0\rangle_{BC} |0\rangle_f,
|\phi'_7\rangle = |L\rangle_A |L\rangle_B |0\rangle_{AC} |0\rangle_{BC} |0\rangle_f.
\]

(25)

The effective Hamiltonian in the subspace is

\[ H_{\text{eff}} = \Omega_{A1}(t) |\psi'_1\rangle \langle \phi'_1| + \Omega_{B1}(t) |\psi'_1\rangle \langle \phi'_7| + \text{H.c.}, \]

(26)

where $|\Psi'_1\rangle = \frac{1}{\sqrt{2}} (|\phi'_2\rangle - g|\phi'_4\rangle + \eta|\phi'_6\rangle)$.

With the same way as above, we can realize the transition from $|\phi'_1\rangle$ to $|\phi'_7\rangle$.

Then we make one qubit operation on atom A to make $|g\rangle_A$ become $-|g\rangle_A$ with the help of laser pulses resonant with A atomic transition $|g\rangle_A \leftrightarrow |e_R\rangle_A$ and $|R\rangle_A \leftrightarrow |e_R\rangle_A$ with the corresponding Rabi frequencies $\Omega_g(t)$ and $\Omega_R(t)$. In this step, the Hamiltonian in the interaction picture can be written as ($\hbar = 1$)

\[ H_2 = \Omega_g(t) |e_R\rangle_A \langle g| + \Omega_R(t) |e_R\rangle_A \langle R| + \text{H.c.} \]

(27)

With the same method as above, we can choose

\[
\Omega_g(t) = \frac{\pi}{2t_f} \cot \epsilon \sin \frac{\pi t}{2t_f},
\Omega_R(t) = \frac{\pi}{2t_f} \cot \epsilon \cos \frac{\pi t}{2t_f}.
\]

(28)

Here we choose $t = 2t_f$, and with the similar processes as above we can realize the transformation from $|g_A\rangle$ to $-|g_A\rangle$.

Up to now, the initial state

\[ |\Psi(0) = \frac{1}{3} (|0\rangle_A + |1\rangle_A + |g\rangle_A) |g\rangle_B |0\rangle_{AC} |0\rangle_{BC} |0\rangle_f \]

(29)
of the whole system has evolved into the state

\[ |\Psi\rangle = \frac{1}{\sqrt{3}} (|R\rangle_A|R\rangle_B + |L\rangle_A|L\rangle_B + |g\rangle_A|g\rangle_B)|0\rangle_{AC}|0\rangle_{BC}|0\rangle_f. \]  

(30)

Ignoring the global phase, the two atoms are in three-dimensional entanglement, with the cavity-modes and the fiber mode in vacuum state.

4. Numerical simulations and feasibility analysis

In the following, we present the numerical validation of the mechanism proposed for the generation of three-dimensional entanglement of the two atoms. Fig. 3 shows the time-dependence laser pulse \( \Omega_i(t)/g \) as a function of \( gt \) for a fixed value \( \epsilon = 0.25 \), and \( t_f = 15/g \). With these parameters the Zeno condition can be met well. The populations of states \( |\phi_1\rangle (|\phi'_1\rangle) \) and \( |\phi_7\rangle (|\phi'_7\rangle) \) swap perfectly when \( t = t_f \), as shown in Fig. 4(a), and the populations of states \( |R\rangle_A \) and \( |g\rangle_A \) also swap perfectly when \( t = 2t_f \) as shown in Fig. 4(b).

In addition, whether a scheme is available largely depends on the robustness to the loss and decoherence. So we consider the effects of loss and decoherence on the entanglement generation. The corresponding master equation for the whole system density matrix \( \rho(t) \) has the following form:

\[ \dot{\rho}(t) = -i[H, \rho(t)] - \sum_{j=L,R} \frac{\kappa_f}{2} [b^\dagger_j b_j \rho(t) - 2b_j \rho(t) b^\dagger_j + \rho(t) b^\dagger_j b_j]. \]
FIG. 4: (a) Time evolutions of the populations of the corresponding system states with the initial states $|\phi_1\rangle (|\phi_1'\rangle)$. (b) Time evolutions of the populations with the initial state $|g\rangle_A$. The system parameters are set to be $\epsilon = 0.25$, $g_A = g_B = g$ with $t_f = 15/g$.

\begin{align}
\dot{\rho}(t) &= -\sum_{j=L,R} \sum_{i=A,B} \frac{\kappa_j^i}{2} [a_{ij}^\dagger a_{ij} \rho(t) - 2a_{ij} \rho(t) a_{ij}^\dagger + \rho(t) a_{ij}^\dagger a_{ij}] \\
&\quad - \sum_{j=A,B} \sum_{h=0,1,L,R} \frac{\gamma_j^h}{2} [\sigma_{e_j,e_j}^A \rho(t) - 2\sigma_{h,e_j}^A \rho(t) \sigma_{e_j,h}^A + \rho(t) \sigma_{e_j,e_j}^A] \\
&\quad - \sum_{j=A,B} \sum_{m=g,L,R} \frac{\gamma_j^m}{2} [\sigma_{e_j,e_j}^B \rho(t) - 2\sigma_{m,e_j}^B \rho(t) \sigma_{e_j,m}^B + \rho(t) \sigma_{e_j,e_j}^B],
\end{align}

where $H = H_1 + H_2$. $\kappa_j^f$ is the photon leakage rate of $j$th fiber mode, $\kappa_j^i$ is the photon leakage rate of $j$-circular polarization mode in $i$th cavity, $\gamma_j^{A(B)}$ is $j$th atomic spontaneous emission rate of cavity $A(B)$ from the excited state $|e_j\rangle$ to the corresponding ground state $|h(m)\rangle$. $\sigma_{e_j,e_j}^A = |e_j\rangle \langle e_j| (j = A, B)$, $\sigma_{e_j,h(e_j)}^A = |e_j(h)\rangle \langle h(e_j)|$ and $\sigma_{e_j,m(e_j)}^A = |e_j(m)\rangle \langle m(e_j)|, (j = A, B)$. For simplicity, we assume $\kappa_j^f = \kappa_j^i = \kappa$, $\gamma_j^{A(B)} = \gamma$. The initial condition $\rho(0) = |\Psi_0\rangle \langle \Psi_0|$. Fig. 5 shows the fidelity $F = \langle \Psi_0 | \Psi(t) \rangle$ as a function of the dimensional parameter $\gamma/g$ with different values of $\kappa$ by numerically solving the master equation (31). From Fig. 4 we can see that, the fidelity for three-dimensional entanglement is higher than 93% when $\gamma = 0.1g$ and $\kappa = g$. It shows that our scheme is robust against decoherence caused by photon leakage of cavities and fiber, and atomic spontaneous emission.

Now we give a brief analysis of the feasibility in experiment of our scheme. The ap-
FIG. 5: The effect of atomic spontaneous emission $\gamma$ on the fidelity of the three-dimensional entanglement with different values of the photon leakage rates $\kappa$ of cavities or fiber.

Propriate atomic level configuration can be obtained from the hyperfine structure of cold alkali-metal atoms [46–48]. Here we adopt the $^{133}$Cs. $5S_{1/2}$ ground level $|F = 3, m = 2\rangle (|F = 3, m = -2\rangle)$ corresponds to $|R\rangle (|L\rangle)$ and $|F = 2, m = 1\rangle (|F = 2, m = -1\rangle)$ corresponds to $|0\rangle (|1\rangle)$, respectively, while $5P_{3/2}$ excited level $|F = 3, m = 1\rangle (|F = 3, m = -1\rangle)$ corresponds to $|e_R\rangle (|e_L\rangle)$. Other hyperfine levels in the ground-state manifold can be used as $|g\rangle$ for atom A. For atom B, the states $|R\rangle, |L\rangle$ and $|g\rangle$ correspond to $|F = 2, m = -1\rangle, |F = 2, m = 1\rangle$ and $|F = 3, m = 0\rangle$ of $5S_{1/2}$ ground levels, respectively. And $|e_R\rangle (|e_L\rangle)$ corresponds to $|F = 3, m = -1\rangle (|F = 3, m = 1\rangle)$ of $5P_{3/2}$ excited level.

5. Conclusion

In conclusion, we have proposed a scheme for generating three-dimensional entanglement of two spatially separated atoms through the shortcut to adiabatic passage and QZD. We also study the influences of system parameters, such as photon leakage of cavities and fiber, and atomic spontaneous emission, on the fidelity through numerical simulation. The numerical simulation results show that our scheme is very robust against the system parameters.
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