Stability of sliding mode controlled buck converters with unmodelled dynamics of circuit elements and hall sensor

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Abstract
This paper investigates the stability problem of sliding mode controlled buck converters affected by unmodelled dynamics of circuit elements and Hall sensor. The parasitic resistors of all elements are contained in the modelling of buck converters. Different from the traditional sliding mode approach based on the nominal model, the circuit parasitic parameters are directly included in the controller design and stability analysis, which divide the regulation region located in the right half axis into four sub-ranges. It is more accurate and has no need of extra compensator. For Hall sensor, singular perturbation theory is adopted for modelling and analysing, giving a stable condition concerning its dynamic and static parameters. Finally, the influence of the two types of unmodelled dynamics on the whole closed-loop system is investigated by constructing an equivalent model, giving a more strict stability condition. Simulations and experiments are presented to illustrate the non-negligible influence of these unmodelled dynamics.

1 INTRODUCTION

The buck-type DC–DC converters are used in various applications where the required output voltage is smaller than the input voltage, for example, the computer systems and renewable energy systems [1–3]. Since buck converters are characterised as variable structure systems due to their switching operations, sliding mode (SM) control technology has been considered as a suitable non-linear control substitute over the traditional pulse width modulation (PWM) control [4] due to its major advantages such as easy implementation and strong robustness against parameter perturbations and external disturbances [5–7].

The stability is the requirement of SM-controlled buck converters in practical applications. To achieve this aim, various ways have arisen and are summarised in the recent surveys [3, 8], for example, an improved circuit topology in [9], a new SM-controlled strategy in [10] and an observer for estimating the unknown load resistors in [11]. Except for these common factors affecting the stability and control performance of buck converters mentioned in [5, 9–11], special attention should be paid to some unmodelled dynamics. These unmodelled dynamics generally originate from sensors and parasitic parameters of circuit elements such as power transistor, diode, capacitor and inductor [12–14]. They are characterised by rapidity and minor value. Taking the often-used Hall sensors ACS712, ACS706 and ACS758 as examples, the time scale of rise time is generally $10^{-6}$ s [15]. Since these unmodelled dynamics are much faster than the changes in current and voltage, it is the reason why they are often omitted in modelling and controlling.

Focused on the issue of unmodelled dynamics, some researchers have put their interest in modelling and analysing. In [16], singular perturbation theory is adopted to describe the unmodelled dynamics of actuators for a class of mechanical systems. In [17], unmodelled dynamics in SM-controlled systems are proved to be factors to degrade the control performance, or even cause the instability. By comparing the research in PI control systems [18], the switching non-linearity of SM controllers will aggravate their influence on control systems [19]. To solve this problem, many researchers design observers and compensators using various adaptive approaches such as neural network, fuzzy logic and linear matrix inequality (LMI) [20–22]. However, the isolated design of observers and controllers can only relieve the influence of unmodelled dynamics but the stable conditions have been barely investigated.

This paper introduces the unmodelled dynamics of circuit elements and Hall sensor to the design of the SM controller and stability analysis of the system. How to regulate the SM controller and what is the guaranteed stable condition if the two
types of unmodelled dynamics are considered, are two important problems to be addressed. To be specific, the contributions of this paper are concluded as follows:

(1) A novel regulation approach of the SM controller is proposed for buck converters. Differing from the traditional approach [6, 7], the parasitic resistors of all elements are directly included in the robust stability analysis of the system.

(2) A singular perturbation model of the Hall sensor is established and a stable condition concerning its dynamic and static parameters is given.

(3) By constructing an equivalent model for the whole closed-loop control system, the influence of the two types of unmodelled dynamics on the system is investigated, giving a more strict stability condition.

This paper is organised as follows. In Section 2, the SM-controlled buck converter system with unmodelled dynamics of circuit elements and Hall sensor is described. In Section 3, parasitic resistors of all elements are introduced to the robust stability analysis of SM-controlled buck converters. As a result, the regulation region located in the right half axis is divided into four sub-ranges. In Section 4, a stable condition of the Hall sensor concerning its dynamic and static parameters is given using singular perturbation theory. In its fast time scale, an equivalent model of the whole closed-loop control system is deduced at the same time. In Section 5, the influence of the two types of the unmodelled dynamics on the whole system is investigated. Finally, a more strict stability condition can be deduced. The simulations and experiments are presented in Section 6 and concluded in Section 7 to show the non-negligible influence of the unmodelled dynamics of circuit elements and Hall sensor.

### 2 SYSTEM DESCRIPTION AND MODELLING

Figure 1(a) shows the system diagram of the SM-controlled buck converter, which consists of three parts, that is, buck converter itself, SM controller and Hall sensor. Differing from the traditional design of the SM controller only based on the nominal model of the buck converter, this paper considers the non-ideality of the buck converter and Hall sensor comprehensively. Therefore, the possible unmodelled dynamics of the both are included in the modelling.

#### 2.1 Buck converter

The buck converter in Figure 1(a) consists of a DC input voltage source $E$; a diode $D$; an inductor $L$ and a capacitor $C$; a load resistor $R$ and a controlled power switch $S$; divider resistors $R_1$, $R_2$ with ratio $\beta = R_1 / (R_1 + R_2)$; $i_L$, $i_e$ and $i_R$ are the currents flowing through the inductor, capacitor and resistors, respectively; $v_e$ is the output voltage and $u$ is the control.

This paper presents the buck converter working in continuous conduction mode (CCM), that is, $i_L \neq 0$. Based on Kirchhoff’s circuit law, its dynamic equations in case of switch ON ($u = 1$) and OFF ($u = 0$), respectively, as

$$
\begin{align*}
\frac{di_L}{dt} &= \frac{1}{L} (E - v_e) \\
\frac{dv_e}{dt} &= \frac{1}{C} i_L \\
\frac{di_e}{dt} &= \frac{-1}{L} v_e \\
\frac{dv_e}{dt} &= \frac{1}{C} i_e
\end{align*}
$$

(1)

In the circuit design of the buck converter, the N-channel MOSFET, Schottky barrier diode, aluminium electrolytic capacitor and planar spiral inductor are generally chosen as the elements of power switch $S$, diode $D$, capacitor $C$ and inductor $L$, respectively, shown in Table 1. This paper considers the parasitic resistors of all elements [12–14], where $R_C$, $R_1$, $R_m$ and $R_f$ are the corresponding parasitic resistors and their values are known or measured. $S_1$ and $S_2$ are ideal switches. Therefore, the circuit topology of the buck converter can be changed into Figure 1(b).

Similarly, for the two cases of Switch ON ($S_1$ is open, $S_2$ is off) and OFF ($S_1$ is off, $S_2$ is open), the dynamic equations of the circuit topology in Figure 1(b) can be deduced, respectively, as

$$
\begin{align*}
\frac{di_L}{dt} &= \frac{1}{L} [E - v_e - (R_m + R_f) i_L] \\
\frac{dv_e}{dt} &= \frac{i_e}{C} \\
\frac{di_e}{dt} &= \frac{-1}{L} [v_e + (R_f + R_L) i_L] \\
\frac{dv_e}{dt} &= \frac{i_e}{C}
\end{align*}
$$

(3)
TABLE 1 General element selection of buck converters

| Elements          | Selection       | Parasitic models |
|-------------------|-----------------|------------------|
| C                 | Aluminum electrolytic capacitor | \[ \frac{1}{C} \] |
| \( S_n \)         | N-channel MOSFET | \[ S_1, R_m \]   |
| \( L \)           | Planar spiral inductor | \[ L, R_L \]      |
| \( D \)           | Schottky barrier diode | \[ S_2, R_e, V_f \] |

By combining (3) and (4), it yields

\[
\begin{aligned}
\frac{d i_L}{dt} &= \frac{1}{L} \left[ E_{in} - v_c - (R_L + R_m u - R_F u + R_F) i_L \right], \\
\frac{d (v_c - R_c i)}{dt} &= i_c, \\
\end{aligned}
\]

(5)

where the control \( u \) is imposed on the power switch \( S_n \) to realise its direct ON/OFF switching operation as \([6, 7]\):

\[
u = -0.5 \left[ \text{sgn}(s) - 1 \right],
\]

(6)

where \( s \) is the sliding variable designed later. Equivalently there is \( u = 1 \) for switch ON and \( u = 0 \) for switch OFF. Comparing with the traditional PWM control [4], the implementation of the control \( u \) in (6) is simple. And the switching frequency is time-varying and determined by the sliding variable \( s \).

Here we define the output voltage error and its derivative as \( x_1 = \beta R_c V_{ref}, x_2 = \dot{x}_1 = \beta \dot{v}_c \), the constants \( \omega_0, \omega_1, \omega_2 \) and \( \omega_3 \) as

\[
\omega_0 = \frac{1}{LC}, \omega_1 = \frac{1}{R} + \frac{1}{R_L + R_R}, \omega_2 = R_L + R_F, \omega_3 = R_{sw} - R_F,
\]

where \( V_{ref} \) is the DC reference voltage of \( \beta R_c \).

In Figure 1(b), there are \( i_L = i_c + i_R \) and \( i_R = \omega_1 v_c \). From (5), the inductor current \( i_c \) and its derivative can be calculated, respectively, as

\[
\begin{aligned}
i_c &= \frac{C d (v_c - R_c i)}{dt} = \frac{C}{\beta} x_2 - C R_c \frac{d i_c}{dt}, \\
\frac{d i_c}{dt} &= \frac{C}{\beta} \frac{d^2 v_c}{dt^2} - C R_c \frac{d^2 i_c}{dt^2} = \frac{C}{\beta} \dot{x}_2 - C R_c \ddot{x}_2,
\end{aligned}
\]

(7)

where the terms \( \frac{d i_c}{dt} \) and \( \frac{d^2 i_c}{dt^2} \) can be easily proved to be bounded due to the charge and discharge characteristics of aluminium electrolytic capacitor [23]. Besides, since the value of \( CR_c \) is too small with \( 10^{-5} \) order of magnitude, (7) can be simplified as

\[
\begin{aligned}
i_c &= \frac{C}{\beta} x_2, \\
\frac{d i_c}{dt} &= C R_c \dot{x}_2.
\end{aligned}
\]

(8)

By combining (5) and (8), we can get

\[
\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= -\omega_0 (\omega_1 + \omega_3 C) x_2 - \omega_1 (1 + \omega_1) x_1 \\
&\quad + \omega_0 (\beta E - \omega_1 \omega_3 V_{ref}) u - \omega_0 (V_{ref} + \omega_1 \omega_2 V_{ref}) , \\
&\quad -C \omega_0 \omega_3 x_2 u - \omega_0 \omega_1 \omega_3 x_1 u
\end{aligned}
\]

(9)

In the form of state space, (9) can be further rewritten as

\[
\dot{x} = Ax + \left[ B + \Delta B \right] u + f,
\]

(10)

where the state \( x = [x_1, x_2]^T \), the matrix \( A, B, \Delta B \) and \( f \) are

\[
A = \begin{bmatrix} 0 & 1 \\ -\omega_0 (1 + \omega_1) & -\omega_0 (\omega_2 C + \omega_1 L) \end{bmatrix},
B = \begin{bmatrix} 0 \omega_0 (\beta E - \omega_0 \omega_3 V_{ref}) \end{bmatrix},
\Delta B = \begin{bmatrix} 0 \omega_0 (\beta E - \omega_0 \omega_3 V_{ref}) \end{bmatrix},
f = \begin{bmatrix} 0 \omega_0 (1 + \omega_1) \omega_2 V_{ref} \end{bmatrix}.
\]

**Notation:** For contrastive study later, we abbreviate the model of the buck converter without circuit parasitic parameters in (1) and (2) as 'System-1', while that with circuit parasitic parameters in (9) or (10) as 'System-2'.

### 2.2 Hall sensor

Observed from Figure 1(a), the voltage signal \( x_1 \) and current signal \( x_2 \) need to be measured. In practice, \( x_1 \) can be measured using the divider resistors \( R_1 \) and \( R_2 \) as \( x_1 = R_1 v_c/(R_1 + R_2) = \beta v_c \), while \( x_2 \) is measured by Hall sensor as \( x_2 = \beta i_c/C \).

Differing from another often-used measurement approach based on by-pass small resistor, Hall sensor is more preferably based on its high accuracy and reliability.

Based on the sensor theory [23], the characteristics of the real Hall sensor are known to be composed of statics and dynamics. Therefore, this paper introduces them into the modelling in the following. And the block diagram of the whole system in Figure 1(a) can be changed as Figure 2, where \( x_2 \) and \( x_2^* \) are the input and output of the Hall sensor, respectively.

Since the Hall sensor used in buck converters are generally linear, \( b < 1 \) is a gain for describing its static linear input–output
transformation between \( x_2 \) and \( x_2^* \) as

\[
x_2^* = b \dot{x}_2
\]

where \( \dot{x}_2 \) is the fast dynamic variable corresponding to the state \( x_2 \). It should be noted that, whatever high the measurement accuracy of the sensor chip is, the static linear gain \( b \) approaches to 1 but is impossible to be 1.

For the dynamics of the Hall sensor, it is a fact that they are much faster than the changes in current and voltage [15–18]. It is also the reason why they are called unmodelled dynamics and often omitted. For buck converter system, since two types of time scales exist, singular perturbation theory is adopted to model them as [17]

\[
\dot{z} = -D \dot{z} - \frac{1}{\psi^2} z + \frac{1}{\psi^2} R x_2,
\]

where \( D \) and \( R \) are constants and \( \psi \) is a perturbation parameter. This paper chooses the rise time to act as the perturbation parameter, which value can be obtained directly from the chip manual.

**Remark 1.** By defining variables \( z_1 = z_2 = \psi \dot{z}_1 \), (12) can be rewritten as

\[
\begin{cases}
\psi \dot{z}_1 = z_2 \\
\dot{z}_2 = -D z_2 - z_1 + R x_2.
\end{cases}
\]

In the form of state space, (13) can be further changed as

\[
\psi \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -D \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ R \end{bmatrix} x_2.
\]

It means the model of the Hall sensor in (12) is equivalent to the standard form of singular perturbation systems [24].

**Remark 2.** To determine the model parameters \( b, D, R \) in (11) and (12), it is recommended to make model identification and fitting based on the test data of the sensor chip, which implementation process is illustrated in the later simulation. It is simple, accurate and close to the real performance of the real Hall sensor.

To summarise (7), (11) and (12), we can see that the possible unmodelled dynamics of the buck converter and Hall sensor are contained in the modelling simultaneously. For the former, the parasitic parameters of all elements are the main cause of unmodelled dynamics [12–14]; while for the latter, the fast dynamics are just the so-called unmodelled dynamics due to its rapidity [17].

## 3 | ROBUST STABILITY OF SLIDING MODE CONTROLLER AGAINST THE CIRCUIT PARASITIC PARAMETERS

This paper proposes a new regulation approach of the SM controller for the buck converter system in Figure 1(a). Differing from the traditional SM approach based on the nominal model in System 1, this paper adopts System 2 to introduce the circuit parasitic parameters into the controller design and stability analysis. Due to the advantages of easy implementation and wide application, this paper adopts linear SM to design a sliding surface \( s \) as

\[
s = \lambda x_1 + x_2 = \delta x,
\]

where the design parameter \( \lambda > 0 \) and the matrix \( \delta = [\lambda, 1] \). This means the regulation region of the SM controller is located in the right half axis.

Restricted by the existence condition of SM, that is, \( \lambda \delta < 0 \) [5–8], the robust stability analysis should be carried out for the two cases of switch ON and OFF. Compared with the traditional SM approach [6, 7], despite the same form of sliding surface in (15), enough information of circuit parasitic parameters can be introduced for the robust stability analysis in the following, due to the modelling of System 2 in (9).

1. If \( s < 0 \), there is \( u = 1 \) got from (6). And \( \dot{s} > 0 \) should be held. Therefore, by combining (9) and (15), it yields

\[
\begin{align*}
l_1 &= \left[ \lambda - (R_{so} + R_{f}) \omega_0 C - \omega_0 \omega_1 L \right] x_2 \\
&\quad - \left[ \omega_0 + (R_{so} + R_{f}) \omega_1 \right] x_1 \\
&\quad + \omega_1 \left[ \delta E - V_{ref} - (R_{so} + R_{f}) \omega_1 V_{ref} \right] > 0
\end{align*}
\]

2. If \( s > 0 \), there is \( u = 0 \). To make \( \dot{s} < 0 \), the following inequation can be similarly obtained as

\[
\begin{align*}
l_2 &= \left[ \lambda \omega_0 (R_{so} + R_{f}) C - \omega_0 \omega_1 L \right] x_2 \\
&\quad - \omega_0 \left[ 1 + \omega_1 (R_{so} + R_{f}) \right] x_1 \\
&\quad - \omega_1 \left[ V_{ref} + (R_{so} + R_{f}) \omega_1 V_{ref} \right] < 0
\end{align*}
\]

From (16) and (17), the slope of the line \( l_1 \) and \( l_2 \) can be calculated, respectively, as

\[
\begin{align*}
m_1 &= \frac{\omega_0 + \omega_1 \omega_1 (R_{so} + R_{f})}{\lambda - (R_{so} + R_{f}) \omega_0 C - \omega_0 \omega_1} \\
m_2 &= \frac{\omega_0 + \omega_1 \omega_1 (R_{so} + R_{f})}{\lambda - (R_{so} + R_{f}) \omega_0 C - \omega_0 \omega_1}
\end{align*}
\]

Two critical values of the SM parameter \( \lambda \) can be obtained from the denominator of the slope \( m_1 \) and \( m_2 \) as

\[
\lambda^{*} = \frac{(R_{so} + R_{f}) \omega_0 C + \omega_0 \omega_1}{L} \left( \frac{1}{R} + \frac{1}{R_1 + R_2} \right)
\]

### Block diagram of SM-controlled buck converter

**FIGURE 2** Block diagram of SM-controlled buck converter
\[ \lambda^* - 2 = C \omega_0 (R_f + R_L) + \omega_0 \omega_1 L = \frac{R_f + R_L}{L} + \frac{1}{C} \left( \frac{1}{R} + \frac{1}{R_1 + R_2} \right). \] (21)

By contrast, the nominal model or System 1 in (1) and (2) is generally adopted for the traditional design of SM controller [6, 7]. It is also based on the existence condition of SM, that is, \( \dot{s} > 0 \). Similarly, for the two cases of switch ON and OFF, the guaranteed stability conditions can be deduced, respectively, as

\[
l'_1 = (\lambda - L \omega_0 \omega_1) x_2 - \omega_0 x_1 + \omega_0 (\beta E - V_{\text{ref}}) > 0, \quad \text{for } s < 0, n = 1
\]

(22)

\[
l'_2 = (\lambda - \omega_0 \omega_1 L) x_2 - \omega_0 x_1 - \omega_0 V_{\text{ref}} < 0, \quad \text{for } s > 0, n = 0.
\] (23)

The slopes of the lines \( l'_1 \) and \( l'_2 \) are similar to

\[
n'_1 = n'_2 = \frac{\omega_0}{\lambda - \omega_0 \omega_1 L},
\] (24)

which gives the reason that only one critical value of SM parameter \( \lambda \) can be obtained as

\[
\lambda_0 = \frac{1}{\omega_0 \omega_1} \left( \frac{1}{R} + \frac{1}{R_1 + R_2} \right).
\] (25)

By combining (15) and (25), the traditional SM approach can only provide two sub-ranges for the controller regulation, that is, \( \lambda > \lambda_0 \) and \( 0 < \lambda < \lambda_0 \).

To investigate the influence of circuit parasitic parameters on the stability and controller regulation, we combine the three critical values in (20), (21) and (25) and divide the regulation region located in the right half axis into four sub-ranges, that is, \( \lambda > \max(\lambda_1^*, \lambda_2^*) \), \( 0 < \lambda < \lambda_0 \), \( \min(\lambda_1^*, \lambda_2^*) < \lambda < \max(\lambda_1^*, \lambda_2^*) \) and \( \lambda_0 < \lambda < \min(\lambda_1^*, \lambda_2^*) \). In the following, we will illustrate and compare the stability regions of Systems 1 and 2 in the four sub-ranges.

Case 1 (\( \lambda > \max(\lambda_1^*, \lambda_2^*) \)): From (16)–(19) and (22)–(24), the stability regions of Systems 1 and 2 can be illustrated in Figure 3, where the individual stability region is mutually intersected by the sliding line \( s = 0 \) and the defined line \( l_i \) (or \( l'_i, l'_i' \)), respectively. According to the two cases of Switch ON and OFF, the stability regions of System-1 are composed of Regions I and II, where the conditions \( s < 0, l'_i > 0 \) and \( s > 0, n_i < 0 \) and \( n = 0 \) are satisfied, respectively. Similarly, the stability regions of System-2 are composed of Region III (\( s < 0, l'_i > 0 \) and \( s > 0, n_i < 0 \)) and Region IV (\( s > 0, l'_i < 0 \) and \( n = 0 \)). From (18), (19) and (24), since the slope \( m_i = m'_i > 0 \), there is \( m_i < 0 \) and \( m'_i < 0 \). Therefore, the stability regions of Systems 1 and 2 can be illustrated in Figure 4(a) and (b), respectively, due to the different symbols of \( m_i \) and \( m'_i \). Since Region III is larger than Region I, and Region IV is larger than Region II, the same conclusion can be obtained as Case 1.

Case 2 (\( \min(\lambda_1^*, \lambda_2^*) < \lambda < \max(\lambda_1^*, \lambda_2^*) \)): From (18), (19) and (24), it has \( m'_i = m'_i > 0 \). Based on the two cases of \( R_{\text{out}} < R_F \) and \( R_{\text{out}} > R_F \), the stability regions of Systems 1 and 2 can be illustrated in Figure 4(a) and (b), respectively.

Case 3 (\( \lambda_0 < \lambda < \min(\lambda_1^*, \lambda_2^*) \)): From (18), (19) and (24), since \( m'_i = m'_i > 0 \), there is \( m_i < 0 \) and \( m'_i < 0 \). Therefore, the stability regions of Systems 1 and 2 can be illustrated in Figure 5. Still, Region III is larger than Region I, and Region IV is larger than Region II.
Stable condition of the Hall sensor

Differing from the traditional approach based on the stability analysis of System 1 [6, 7], the regulation region of SM parameter $\lambda$ is extended from two sub-ranges with $\lambda > \lambda_0$ and $0 < \lambda < \lambda_0$ into four sub-ranges. The difference of the two approaches can be illustrated in Figure 7. Besides due to the inherent robustness of SM, no extra compensator is needed to overcome the stability influence of circuit parasitic parameters.

4.1 Stable condition of the Hall sensor

Since the sliding variable $s$ in (15) is the linear combination of the states $x_1$ and $x_2$, here we further simplify Figure 2 into Figure 8, where $s$ and $\dot{s}$ are the input and output of the Hall sensor, respectively. Correspondingly, the SM control law $u$ in (6) and the output of the Hall sensor in (11) can be expressed, respectively, as

$$\dot{s} = b \zeta$$  

$$u = -\frac{1}{2} \left[ \text{sgn} (\dot{s}) - 1 \right].$$  

The fast dynamics of the Hall sensor in (12) are rewritten as

$$\dot{\psi} = -\psi \frac{D_2}{\sqrt{I_c}} \zeta + R x_2.$$  

If the Hall sensor can still work stably and the variable $s$ and its derivative $\dot{s}$ are bounded, then from (27) and (29), in a sufficiently small time $\Delta t (\Delta t \to 0, \psi \to 0)$, we can get

$$\zeta = R s + \varepsilon_1 (\psi)$$  

$$\dot{s} = b \zeta = h R s + h \varepsilon_1 (\psi),$$  

where $\varepsilon_1$ tends to zero as $\psi \to 0.$
From (11), since the static linear gain \( b < 1 \) and approximates to one, the stable condition of the Hall sensor concerning its dynamic and static parameters can be deduced as

\[
h_R \cong 1.
\]  
(32)

By substituting (32) into (31), the stable output \( e^* \cong e + b e_1(\psi) \) of the Hall sensor can be guaranteed.

**Remark 3.** By combining the above Remark 2, the significance of (32) lies on the choice and regulation of the real Hall sensor in practical applications.

### 4.2 Equivalent model transformation

Observed from Figure 8, the insertion of the Hall sensor between the SM controller and buck converter hinders the stability analysis of the whole system. Therefore, the equivalent model transformation is carried out as follows.

Based on the singular perturbation theory [17, 24–25] since the time scale of the current and voltage of buck converter is much smaller than that of the fast dynamics of the Hall sensor, states \( x_1 \) and \( x_2 \) can be 'frozen' as constants. For the disturbance term \( \Delta B(t) \) in (10), since it is composed of the states \( x_1 \) and \( x_2 \), it can also be 'frozen' as a constant.

Here we assume that the initial value of state \( x(0) = [x_1(0), x_2(0)]^T = [0, 0]^T \). Based on the singular perturbation theory, the Laplace transform of System 2 in (10) can be made in the fast time scale of the Hall sensor as

\[
X(s) = (sI - A)^{-1}(B + \Delta B)U(s) + \frac{(sI - A)^{-1}f}{s} \epsilon_1(\psi)\left[ \begin{array}{c} 0 \\ 1 \end{array} \right].
\]  
(33)

Similarly, the Laplace transforms of the sliding variable \( \Sigma \) in (15), the input and output of the Hall sensor in (30) and (31) are given, respectively, as

\[
\Sigma = \delta X(s) = R \Sigma + \frac{e_1(\psi)}{s},
\]  
(34)

\[
\chi = R \Sigma + \frac{e_1(\psi)}{s},
\]  
(35)

\[
\Sigma^* = h \chi(s).
\]  
(36)

From (33)-(36), the open-loop flowchart of Figure 8 can be shown in Figure 9(a). Here we further change it by three steps of transformation, that is, the position of the constant \( R \) is first adjusted to Figure 9(b), then the constant matrix gain \( \delta \in \mathbb{R}^{1 \times 2} \) is adjusted to Figure 9(c), and finally \((sI - A)^{-1}(B + \Delta B)\) and the constant \( b \) are adjusted to Figure 9(d) by defining a constant \( \epsilon'_1(\psi) \) as

\[
(sI - A)^{-1}(B + \Delta B) \frac{\epsilon'_1(\psi)}{s} = \frac{e_1(\psi)}{s} \left[ \begin{array}{c} 0 \\ 1 \end{array} \right],
\]  
(37)

or equivalently, it has

\[
\frac{e'_1(\psi)}{s} = (B^+ + \Delta B^+) \frac{e_1(\psi)}{s} \left[ \begin{array}{c} 0 \\ 1 \end{array} \right],
\]  
(38)

where \( B^+ = B^T(BB^T)^{-1} \) and \( \Delta B^+ = \Delta B^T(\Delta B\Delta B^T)^{-1} \) are denoted as the pseudo-inverse matrix of the 'frozen' \( B \) and \( \Delta B \) in the fast time scale of the Hall sensor, respectively.

For Figure 9(d), we continue to make the inverse Laplace transform for it. Therefore, Figure 10 can be got and is equivalent to Figure 8, where the variable \( w \) is the new output of the Hall sensor and \( W(s) \) is the Laplace transform of \( w \).

By comparing with Figure 8, attention should be paid to the position of the Hall sensor in Figure 10. It is impossible for the practical application of the Hall sensor in buck converters. In other words, Figure 10 is only the ‘mathematical’ transformation model of Figure 8 in the fast time scale of the Hall sensor. Therefore by combining (10), the buck converter in Figure 10 can be described as

\[
\dot{x} = Ax + [B + \Delta B(t)]w + hRf.
\]  
(39)
If we substitute the guaranteed stable condition of the Hall sensor in (32), (39) can be changed as
\[
\dot{x} = Ax + [B + \Delta B(i)]u + f.
\] (40)

Correspondingly from (28), (29) and (31), the SM controller and the Hall sensor located in Figure 10 can be described, respectively, as
\[
u = -\frac{1}{2} \left[ \text{sgn}(s) - 1 \right]
\]
\[
\begin{cases} 
\dot{z}_1 = z_2 \\
\dot{z}_2 = -\psi Dz_2 - z_1 + Ru \\
w = h\bar{z} = h\psi_2'(\psi)
\end{cases}
\] (42)

where \( \psi_2' \) tends to be zero as \( \psi \to 0 \).

**Remark 4.** The purpose of the equivalent model transformations in Figure 9(a)–(d) is just to prove that Figure 10 is equivalent to Figure 8 only in the fast time scale of the Hall sensor. Since the Laplace transform and inverse Laplace transform are made simultaneously, we assume the initial values of states \( x_1, x_2 \) and \( z \) are all zeros in (33)–(36) just to facilitate the analysis.

**Remark 5.** Since Figure 10 and (40)–(43) are only mathematically equivalent to the real SM-controlled buck converter system, they have no practical significance for the controller design due to their incompatible fast time scale with the real buck converter in (10).

### 5 Stability Analysis of the Whole Closed-Loop System

By combining the isolated regulation approach of the SM controller in Table 2 and the stable condition of the Hall sensor in (32), it is necessary to further investigate the stability influence of the above two types of unmodelled dynamics on the whole system.

Here we construct a Lyapunov function \( V' = 0.5z^2 \). From (40), we differentiate \( V' \) to the time yielding
\[
\dot{V'} = s' V' = s' \left( \delta Ax + \delta Bu + \delta f + \delta \Delta Bu \right)
\]
\[
= s' \left[ -\omega_0(1 + \omega_1\omega_2)x_1 + \lambda x_2 - \omega_0(\omega_2 C + \omega_1 L)x_2 + \omega_0\beta E w - \omega_0\omega_1\omega_2 V_{ref} - \omega_0(1 + \omega_1\omega_2) V_{ref} 
\right.
\]
\[
\left. - C\omega_0\omega_2 x_2 w - \omega_0\omega_1\omega_3 x_1 w \right].
\] (44)

To keep the stability of the whole closed-loop system, \( \dot{V'} < 0 \) or \( s' < 0 \) should still be held. According to switch ON and OFF, two cases need to be discussed in the following.

1. If \( s < 0, s' > 0 \) should be satisfied with
\[
\begin{align*}
&\lambda x_2 - \omega_0(1 + \omega_1\omega_2)x_1 - \omega_0(\omega_2 C + \omega_1 L)x_2 + \omega_0\beta E w - \omega_0\omega_1\omega_2 V_{ref} - \omega_0(1 + \omega_1\omega_2) V_{ref} - C\omega_0\omega_2 x_2 w - \omega_0\omega_1\omega_3 x_1 w < 0.
\end{align*}
\] (45)

Since \( u = 1 \) for switch ON and \( w \equiv u \) from (43), (45) can be further simplified as
\[
[\lambda - \omega_0(\omega_2 C + \omega_3 C + \omega_1 L)] x_2 - \omega_0(1 + \omega_1\omega_2 + \omega_1\omega_3) (x_1 + V_{ref}) + \omega_0(\omega_2 C + \omega_3 C + \omega_1 L) x_2 + \omega_0\beta E > 0.
\] (46)

1. If \( s < 0 \), similarly it has \( w = 0 \) and \( s' < 0 \), that is,
\[
\begin{align*}
&\omega_0(1 + \omega_1\omega_2) (x_1 + V_{ref}) + \omega_0(\omega_2 C + \omega_1 L) > \lambda x_2 \\
&> \omega_0(1 + \omega_1\omega_2 + \omega_1\omega_3) (x_1 + V_{ref}) \\
&+ \omega_0(\omega_2 C + \omega_3 C + \omega_1 L) x_2 - \omega_0\beta E.
\end{align*}
\] (47)

By combining (46) and (47), if the whole closed-loop system can be still stable around the equilibrium point \( (x_1 \cong 0, x_2 \cong 0) \), a more strict stability condition can be achieved as
\[
(1 + \omega_1\omega_2 + \omega_1\omega_3) V_{ref} - \beta E < 0.
\] (48)

Or in other words, the influence of the two types of unmodelled dynamics on the system stability can be concluded as the constraint of output voltage for buck converter as
\[
0 < V_{ref} < \frac{\beta E}{1 + \omega_1\omega_2 + \omega_1\omega_3}.
\] (49)

### 6 Simulations and Experiments

To validate the proposed regulation approach of the SM controller in Table 2 and to show the stability influence of the two types of unmodelled dynamics, the parameters of the buck converter in Figure 1(b) are designed and listed in Table 3, which

| Description                  | Parameter | Value  |
|------------------------------|-----------|--------|
| Input voltage                | \( E \)   | 18 V   |
| Inductor                     | \( L \)   | 1000 \( \mu \)H |
| Capacitor                    | \( C \)   | 3200 \( \mu \)F |
| Load resistor                | \( R \)   | 10 \( \Omega \) |
| Reference output voltage     | \( V_{ref}/\beta \) | 9 V |
| Divider resistor             | \( R_1 \) | 1 k\( \Omega \) |
| Variable divider resistor    | \( R_2 \) | [1 k\( \Omega \), 9 k\( \Omega \)] |
can guarantee the buck converter to work in CCM [26]. And the parasitic resistors in Table 1 are listed as $R_C = 20 \text{ m}\Omega$, $R_L = 100 \text{ m}\Omega$, $R_m = 75 \text{ m}\Omega$, $R_F = 25 \text{ m}\Omega$. The variable divider resistor $R_2$ is fixed as $3 \text{ k}\Omega$ so that the ratio $\beta = R_1/(R_1 + R_2) = 0.25$. From (49), the range of reference output voltage is decreased from the expected (0 V, 18 V) to the real (0 V, 17.6 V).

6.1 | Simulation results

This paper uses a linear Hall sensor ACS712 in buck converter system due to its adjustable rising time for testing the influence of unmodelled dynamics in (12).

6.1.1 | Model identification of Hall sensor ACS712

In Figure 11(a), it gives the test data of ACS712 got from its chip manual [15]. Using the software of GetData and MATLAB Identification Toolbox, its transfer function can be obtained as

$$y = \frac{2.235e^{11}}{s^2 + 6.681e^5 s + 2.257e^{11}}. \tag{50}$$

And the step response of (50) is illustrated in Figure 11(b) to evaluate the model identification. The static linear gain $b$ in (11) can be obtained as 0.99897. Accordingly from (12), the constant $R = 0.9999$ can be calculated. Since $Rb = 0.9989$, the stable condition of the Hall sensor in (32) is satisfied.

6.1.2 | Performance influence of the Hall sensor

In order to test the influence of the Hall sensor on the system, here we ignore the circuit parasitic parameters and take System 1 as the simulation model. The SM controller parameter in (15) is chosen as $\lambda = 100$ and the rise time of the Hall sensor ACS712 is chosen as $\psi = 6.647 \mu s$.

The simulation results are given in Figure 12(a)–(d). It can be seen that, if the unmodelled dynamics of the Hall sensor are not considered, the sliding variable $s$ in Figure 12(a) is smooth and has no overshoot. On the contrary, if the Hall sensor is included in the system, the overshoots and oscillations of the input variable $s$ and output variable $s^*$ appear simultaneously. For the identified model in (50), since its order is 2, the oscillation frequency and damping ratio are calculated as 0.106 Hz and $1.2774 \times 10^{-11} < 1$, respectively. In other words, (50) is a second-order under-damped system with the oscillation period $2\pi/0.106 = 59.28$ s. Therefore, there is one and only one big waveform with peak value $-748$ for variables $s$ and $s^*$ at the beginning, due to the characteristics of the under-damped system. From the control $u$ in Figure 12(b), the switching frequency of the power switch is reduced from 12.50 to 8.33 kHz affected by the Hall sensor. Figure 12(c) and (d) shows the output voltage and inductor current, respectively. It is obvious that the both performance is deteriorated, which is also due to the inclusion of the Hall sensor.

To further test the influence of the rise time $\psi$ (perturbation parameter) of Hall sensor ACS712 in (12), we choose four values of 6.647, 32.09, 68.15 and 88.18 $\mu s$. The contrastive simulations can be seen in Figure 13(a) and (b). Although the time scale of the rise time $\psi$ is very small, the oscillations of the output...
FIGURE 14  Output voltage of SM controller with different $\lambda$: (a) $\lambda = 31.25$, (b) $\lambda = 100$, (c) $\lambda = 200$ and (d) $\lambda = 1000$

voltage $v_c$ and inductor current $i_L$ get bigger as the increase of $\psi$. Besides the phenomenon of phase shift appears.

6.1.3 Performance influence of SM parameter $\lambda$

In order to validate the proposed regulation approach of SM controller, we calculate $\lambda^* = 215.46$, $\lambda^* = 162.83$, $\lambda_0 = 31.25$ from (20), (21) and (25). Therefore, the four sub-regions in Table 2 can be determined as $\lambda > 215.46$, $\lambda \in [162.83, 215.46]$, $\lambda \in [31.25, 162.83]$ and $\lambda \in [0, 31.25]$. Here we choose $\lambda = 31.25$, $100$, $200$ and $1000$ in the individual sub-region. Special attention should be paid to $\lambda = \lambda_0 = 31.25$, which is the recommended value of the traditional approach in [6, 7]. Taking the output voltage for comparison, the simulations are given in Figure 14(a)–(d).

In Figure 14(a), we can see that despite the traditional approach can provide better performance for System 1, but the SM parameter $\lambda = \lambda_0 = 31.25$ is not suitable for System 2 with 0.3 V steady error, which is caused by the circuit parasitic parameters. With the increase in $\lambda$ from 31.25 to 100 in Figure 14(b), the output voltage of System 1 is deteriorated with 0.02 V steady error. While for System 2, regardless of 0.022 V static error, the performance of the output voltage is improved. With the continuous increase of $\lambda$ in Figure 14(c) and (d), the problem of static error can be overcome at the price of overshoots both in Systems 1 and 2 due to an excessive $\lambda$.

Based on the above comparative simulations, we can summarise the influence of the unmodelled dynamics of circuit elements and Hall sensor as the following two points.

(1) Observed from Figures 12 and 13, the influence of Hall sensor should not be ignored in modelling and controlling.

(2) Observed from Figure 14, the new regulation approach proposed in Table 2 is more effective than the traditional approach in [6, 7], due to the inclusion of circuit parasitic parameters in controller design. It is more accurate and has no need of extra compensator.

6.2 Experiment results

In Figure 15(a), the experimental platform of the buck converter control system based on dSPACE is given, where the sampling time is 0.5 ms and the switching frequency is 1 kHz; while in Figure 15(b), the diagram of the hardware realisation of the buck converter with the chosen Hall sensor ACS712 is illustrated, where the external filter capacitor $c_F$ determines the rising time of the Hall sensor $\psi$ in (12).

In the following, we want to further test the proposed SM regulation approach and the influence of the Hall sensor by experiment. And two cases are considered, that is,

1. we fix the rising time $\psi = 6.647 \mu s$ and vary the SM parameter $\lambda$ with 31.25, 80, 200 and 400 in the four sub-regions of Table 2 to test the influence of the Hall sensor;
2. we fix the design parameter $\lambda = 200$ and vary the rising time $\psi$ with 6.647, 32.09, 68.15 and 88.18 $\mu$s to test the influence of circuit parasitic parameters of controller regulation.

The comparative experimental results of both cases are given in Figure 16, and Tables 4 and 5.

By combining Figure 16(a) and Table 4, we can see that the design parameter $\lambda = \lambda_0 = 31.25$ recommended by the traditional approach in [6, 7] still cannot provide a better performance as the simulations in Figure 14. By contrast, if we adopt the new regulation approach in Table 2, it is easier to get an appropriate $\lambda = 200$ within the four more accurate sub-regions of controller regulation. Besides by comparing Figure 14 with Figure 16, $\lambda = 200$ got by simulation can still guarantee the best performance in the experiment, which is greatly attributed to the inclusion of circuit parasitic parameters at the stage of controller design.

By combining Figure 16(b) and Table 5, we can see that the steady error of output voltage will increase from 0.102 to 1.099 V, as the rising time $\psi$ increasing from 6.647 to 88.18 $\mu$s. Especially it should be noted, if $\psi = 68.15 \mu$s, the relative steady error of output voltage is 6.83%. It is far from the accepted index 2% for buck converters, even a good choice of $\lambda = 200$ is adopted by comparing Figure 16(a) and (b), which proves that the influence of the Hall sensor should also not be ignored.

7 | CONCLUSION

This paper proposes a new regulation approach of the SM controller for the direct ON/OFF control of buck converters with circuit parasitic parameters and the Hall sensor. On the one side, differing from the traditional approach, the parasitic parameters of all elements are directly introduced into the design of the SM controller. No extra compensator is needed to overcome their influence on the system. And the regulation region located in the right half axis is divided into four sub-regions, which will bring benefits of high effectiveness and high accuracy in the process of controller regulation and practical applications. On the other side, a condition for guaranteeing the stable output of the Hall sensor is proposed. Finally, the two types of unmodelled dynamics are investigated in the whole closed-loop system in the fast time scale of the Hall sensor. And their non-negligible influence on the system can be concluded as the constrained range of the output voltage. Simulations and experiments based on Hall sensor ACS712 and dSPACE validate the effectiveness of this paper.

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How to cite this article: Wang Y, Niu Z, Yang M, Sun I, Zhang W. Stability of sliding mode controlled buck converters with unmodelled dynamics of circuit elements and hall sensor. IET Power Electron. 2021;14:602–613. https://doi.org/10.1049/pe12.12043