Isospin dependent kaon and antikaon optical potentials in dense hadronic matter

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Abstract

Isospin effects on the optical potentials of kaons and antikaons in dense hadronic matter are investigated using a chiral SU(3) model. These effects are important for asymmetric heavy ion collision experiments. In the present work the dispersion relations are derived for kaons and antikaons, compatible with the low energy scattering data, within our model approach. The relations result from the kaonic interactions with the nucleons, vector mesons and scalar mesons in the asymmetric nuclear matter. The isospin asymmetry effects arising from the interactions with the vector-isovector $\rho$- meson as well as the scalar isovector $\delta$ mesons are considered. The density dependence of the isospin asymmetry is seen to be appreciable for the kaon and antikaon optical potentials. This can be particularly relevant for the future accelerator facility FAIR at GSI, where experiments using neutron rich beams are planned to be used in the study of compressed baryonic matter.

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I. INTRODUCTION

The study of the properties of hadrons in hot and dense matter is an important topic in the present strong interaction physics. This subject has direct implications for heavy-ion collision experiments, for the study of astrophysical compact objects (like neutron stars) as well as for the physics of the early universe. The in-medium properties of kaons have been investigated particularly because of their relevance in neutron star phenomenology as well as relativistic heavy-ion collisions. For example, in the interior of a neutron star the attractive kaon nucleon interaction might lead to kaon condensation as originally suggested by Kaplan and Nelson. The in-medium modification of kaon/antikaon properties can primarily be observed experimentally in relativistic nuclear collisions. Indeed, the experimental and theoretical studies on \( K^\pm \) production from A+A collisions at SIS energies of 1-2 A·GeV have shown that in-medium properties of kaons have been seen in the collective flow pattern of \( K^+ \) mesons as well as in the abundance and spectra of antikaons.

The theoretical research work on the topic of medium modification of hadron properties was initiated by Brown and Rho who suggested that the modifications of hadron masses should scale with the scalar quark condensate \( \langle q\bar{q} \rangle \) at finite baryon density. The first attempts to extract the antikaon-nucleus potential from the analysis of kaonic-atom data were in favor of very strong attractive potentials of the order of \(-150\) to \(-200\) MeV at normal nuclear matter density \( \rho_0 \). However, more recent self-consistent calculations based on a chiral Lagrangian or coupled-channel G-matrix theory (within meson-exchange potentials) only predicted moderate attraction with potential depths of \(-50\) to \(-80\) MeV at density \( \rho_0 \).

The problem with the antikaon potential at finite baryon density is that the antikaon-nucleon amplitude in the isospin channel \( I = 0 \) is dominated by the \( \Lambda(1405) \) resonant structure, which in free space is only \( 27 \) MeV below the \( \bar{K}N \) threshold. It is presently not clear if this physical resonance is a real excited state of a 'strange' baryon or some short lived molecular intermediate state which can be described in a coupled channel \( T \)-matrix scattering equation using a suitable meson-baryon potential. Additionally, the coupling between the \( \bar{K}N \) and \( \pi Y \) (\( Y = \Lambda, \Sigma \)) channels is essential to get the proper dynamical behavior in free space. Correspondingly, the in-medium properties of the \( \Lambda(1405) \), such as
its pole position and its width, which in turn influence strongly the antikaon-nucleus optical potential, are very sensitive to the many-body treatment of the medium effects. Previous works have shown that a self-consistent treatment of the $\bar{K}$ self energy has a strong impact on the scattering amplitudes and thus on the in-medium properties of the antikaon. Due to the complexity of this many-body problem the actual kaon and antikaon self energies (or potentials) are still a matter of debate.

The topic of isospin effects in asymmetric nuclear matter has gained interest in the recent past. The isospin effects are important in isospin asymmetric heavy ion collision experiments. Within the UrQMD model the density dependence of the symmetry potential has been studied by investigating observables like the $\pi^-/\pi^+$ ratio, the $n/p$ ratio, the $\Delta^-/\Delta^{++}$ ratio as well as the effects on the production of $K^0$ and $K^+$ and on pion flow for neutron rich heavy ion collisions. Recently, the isospin dependence of the in-medium NN cross section has also been investigated.

In the present investigation we will use a chiral SU(3) model for the description of hadrons in the medium. The nucleons – as modified in the hot hyperonic matter – have been studied previously within this model. Furthermore, the properties of vector mesons – due to their interactions with nucleons in the medium – have also been examined and have been found to have appreciable modifications due to Dirac sea polarization effects. The chiral SU(3)$_{\text{flavor}}$ model was also generalized to SU(4)$_{\text{flavor}}$ to study the mass modification of D-mesons arising from their interactions with the light hadrons in hot hadronic matter. The energies of kaons (antikaons) at zero momentum, as modified in the medium due to their interaction with nucleons, consistent with the low energy KN scattering data, were also studied within this framework. In the present work, we consider the effect of isospin asymmetry on the kaon and antikaon optical potentials in the asymmetric nuclear matter.

The outline of the paper is as follows: In section II we shall briefly review the SU(3) model used in the present investigation. Section III describes the medium modification of the $K(\bar{K})$ mesons in this effective model. In section IV, we discuss the results obtained for the optical potentials of the kaons and antikaons and the isospin-dependent effects on these optical potentials in asymmetric nuclear matter. Section V summarizes the findings of the present investigation and discusses possible extensions of the calculations.
II. THE HADRONIC CHIRAL $SU(3) \times SU(3)$ MODEL

In this section the various terms of the effective hadronic Lagrangian used

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \sum_{W=X,Y,V,A,u} \mathcal{L}_{BW} + \mathcal{L}_{VP} + \mathcal{L}_{\text{vec}} + \mathcal{L}_{0} + \mathcal{L}_{SB}$$ (1)

are discussed. Eq. (1) corresponds to a relativistic quantum field theoretical model of baryons and mesons built on a nonlinear realization of chiral symmetry and broken scale invariance (for details see [32, 33, 34]) to describe strongly interacting nuclear matter. The model was used successfully to describe nuclear matter, finite nuclei, hypernuclei and neutron stars. The Lagrangian contains the baryon octet, the spin-0 and spin-1 meson multiplets as the elementary degrees of freedom. In Eq. (1), $\mathcal{L}_{\text{kin}}$ is the kinetic energy term, $\mathcal{L}_{BW}$ contains the baryon-meson interactions in which the baryon-spin-0 meson interaction terms generate the baryon masses. $\mathcal{L}_{VP}$ describes the interactions of vector mesons with the pseudoscalar mesons (and with photons). $\mathcal{L}_{\text{vec}}$ describes the dynamical mass generation of the vector mesons via couplings to the scalar mesons and contains additionally quartic self-interactions of the vector fields. $\mathcal{L}_{0}$ contains the meson-meson interaction terms inducing the spontaneous breaking of chiral symmetry as well as a scale invariance breaking logarithmic potential. $\mathcal{L}_{SB}$ describes the explicit chiral symmetry breaking.

The kinetic energy terms are given as

$$\mathcal{L}_{\text{kin}} = i \text{Tr} \overline{B}_\mu D^\mu B + \frac{1}{2} \text{Tr} D_\mu X D^\mu X + \text{Tr}(u^\dagger Xu + Xu^\dagger X) + \frac{1}{2} \text{Tr} D_\mu Y D^\mu Y$$

$$+ \frac{1}{2} D_\mu \chi D^\mu \chi - \frac{1}{4} \text{Tr}(\tilde{V}_{\mu\nu}\tilde{V}^{\mu\nu}) - \frac{1}{4} \text{Tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{1}{4} \text{Tr}(A_{\mu\nu}A^{\mu\nu}).$$ (2)

In (2) $B$ is the baryon octet, $X$ the scalar meson multiplet, $Y$ the pseudoscalar chiral singlet, $\tilde{V}^\mu (A^\mu)$ the renormalised vector (axial vector) meson multiplet with the field strength tensor $\tilde{V}_{\mu\nu} = \partial_\mu \tilde{V}_\nu - \partial_\nu \tilde{V}_\mu$ ($A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$), $F_{\mu\nu}$ is the field strength tensor of the photon and $\chi$ is the scalar, iso-scalar dilaton (glueball) -field. In the above, $u_\mu = -\frac{i}{2}[u^\dagger \partial_\mu u - u \partial_\mu u^\dagger]$, where $u = \exp\left[\frac{i}{\sigma_0} \tau^a \lambda^a \gamma_5\right]$ is the unitary transformation operator, and the covariant derivative reads $D_\mu = \partial_\mu + [\Gamma_\mu, ]$, with $\Gamma_\mu = -\frac{i}{2}[u^\dagger \partial_\mu u + u \partial_\mu u^\dagger]$.

The baryon -meson interaction for a general meson field $W$ has the form

$$\mathcal{L}_{BW} = -\sqrt{2}g_8^W (\alpha_W [BOBW]_F + (1 - \alpha_W) [BBOBW]_D) - g_1^W \frac{1}{\sqrt{3}} \text{Tr}(BOB) \text{Tr} W,$$ (3)
with $[\mathcal{B}O\mathcal{B}W]^F := \text{Tr}(\mathcal{B}O\mathcal{W}B - \mathcal{B}O\mathcal{BW})$ and $[\mathcal{B}O\mathcal{B}W]^D := \text{Tr}(\mathcal{B}O\mathcal{W}B + \mathcal{B}O\mathcal{BW}) - \frac{2}{3} \text{Tr}(\mathcal{B}O\mathcal{B}) \text{Tr} \mathcal{W}$. The different terms to be considered are those for the interaction of baryons with scalar mesons ($W = X, \mathcal{O} = 1$), with vector mesons ($W = \bar{V}_\mu, \mathcal{O} = \gamma_\mu$ for the vector and $W = \bar{V}_{\mu\nu}, \mathcal{O} = \sigma^{\mu\nu}$ for the tensor interaction), with axial vector mesons ($W = \mathcal{A}_\mu, \mathcal{O} = \gamma_\mu \gamma_5$) and with pseudoscalar mesons ($W = u_\mu, \mathcal{O} = \gamma_\mu \gamma_5$), respectively.

For the current investigation the following interactions are relevant: Baryon-scalar meson interactions generate the baryon masses through coupling of the baryons to the non-strange $\sigma(\sim \langle \bar{u}u + \bar{d}d \rangle)$ and the strange $\zeta(\sim \langle \bar{s}s \rangle)$ scalar quark condensate. The parameters $g_1^S, g_8^S$ and $\alpha_S$ are adjusted to fix the baryon masses to their experimentally measured vacuum values. It should be emphasized that the nucleon mass also depends on the strange condensate $\zeta$. For the special case of ideal mixing ($\alpha_S = 1$ and $g_1^S = \sqrt{6} g_8^S$) the nucleon mass depends only on the non–strange quark condensate. In the present investigation, the general case will be used, which takes into account the baryon coupling terms to both scalar fields ($\sigma$ and $\zeta$).

In analogy to the baryon-scalar meson coupling two independent baryon-vector meson interaction terms exist corresponding to the $F$-type (antisymmetric) and $D$-type (symmetric) couplings. Here we will use the antisymmetric coupling because, following the universality principle and the vector meson dominance model, one can conclude that the symmetric coupling should be small. We realize it by setting $\alpha_V = 1$ for all fits. Additionally we decouple the strange vector field $\phi_\mu \sim \bar{s} \gamma_\mu s$ from the nucleon by setting $g_1^Y = \sqrt{6} g_8^Y$. The remaining baryon-vector meson interaction reads

$$L_{BV} = -\sqrt{2} g_8^Y \left\{ [\bar{B} \gamma_\mu B V^\mu]_F + \text{Tr}(\bar{B} \gamma_\mu B) \text{Tr} V^\mu \right\}. \quad (4)$$

The Lagrangian describing the interaction for the scalar mesons, $X$, and pseudoscalar singlet, $Y$, is given as

$$L_0 = -\frac{1}{2} k_0 \chi^2 I_2 + k_1 (I_2)^2 + k_2 I_4 + 2 k_3 \chi I_3, \quad (5)$$

with $I_2 = \text{Tr}(X + iY)^2, I_3 = \text{det}(X + iY)$ and $I_4 = \text{Tr}(X + iY)^4$. In the above, $\chi$ is the scalar color singlet glueball field. It is introduced in order to mimic the QCD trace anomaly, i.e. the non-vanishing energy-momentum tensor $\theta_\mu^\nu = (\beta_{QCD}/2g) \langle G_\mu^a G^{a,\mu\nu} \rangle$, where $G_\mu^a$ is the gluon field tensor. A scale breaking potential is introduced:

$$L_{\text{scalebreak}} = -\frac{1}{4} \chi^4 \ln \frac{\chi_0^4}{\lambda_0^4} + \frac{\delta}{3} \chi^4 \ln \frac{I_3}{\text{det}(X)_0} \quad (6)$$
which allows for the identification of the $\chi$ field width the gluon condensate $\theta^\mu_\mu = (1 - \delta)\chi^4$.
Finally the term $\mathcal{L}_\chi = -k_1\chi^4$ generates a phenomenologically consistent finite vacuum expectation value. The variation of $\chi$ in the medium is rather small \[32\]. Hence we shall use the frozen glueball approximation i.e. set $\chi$ to its vacuum value, $\chi_0$.

The Lagrangian for the vector meson interaction is written as

$$
\mathcal{L}_{vec} = \frac{m_V^2}{2} \frac{\chi^2}{\chi_0^2} \text{Tr}(\tilde{V}_\mu \tilde{V}^\mu) + \frac{\mu}{4} \text{Tr}(\tilde{V}_{\mu\nu} \tilde{V}^{\mu\nu} \chi^2) + \frac{\lambda_V}{12} \left(\text{Tr}(\tilde{V}^\mu)\right)^2 + 2(\tilde{g}_A V^{1/2} \tilde{\chi}_\mu \tilde{V}^\mu)^2. \quad (7)
$$

The vector meson fields, $\tilde{V}_\mu$ are related to the renormalized fields by $V_\mu = Z_V^{1/2} \tilde{V}_\mu$, with $V = \omega, \rho, \phi$. The masses of $\omega, \rho$ and $\phi$ are fitted from $m_V, \mu$ and $\lambda_V$.

The explicit symmetry breaking term is given as \[32\]

$$
\mathcal{L}_{SB} = \text{Tr} A_\mu \left(u(X + iY)u + u^\dagger(X - iY)u^\dagger\right) \quad (8)
$$

with $A_\mu = 1/\sqrt{2} \text{diag}(m_\omega^2 f_\omega, m_\rho^2 f_\rho, 2m_K^2 f_K - m_\pi^2 f_\pi)$ and $m_\pi = 139\text{ MeV}, m_K = 498\text{ MeV}$.

This choice for $A_\mu$, together with the constraints $\sigma_0 = -f_\pi, \zeta_0 = -1/\sqrt{2}(2f_K - f_\pi)$ for the VEV of the scalar condensates assure that the PCAC-relations of the pion and kaon are fulfilled. With $f_\pi = 93.3\text{ MeV}$ and $f_K = 122\text{ MeV}$ we obtain $|\sigma_0| = 93.3\text{ MeV}$ and $|\zeta_0| = 106.56\text{ MeV}$.

We proceed to study the hadronic properties in the chiral SU(3) model. The Lagrangian density in the mean field approximation is given as

$$
\mathcal{L}_{BX} + \mathcal{L}_{BV} = -\sum_i \overline{\psi}_i [g_i \omega \gamma_0 \omega + g_i \phi \gamma_0 \phi + m_i^*] \psi_i \quad (9)
$$

$$
\mathcal{L}_{vec} = \frac{1}{2} m_\omega^2 \chi_0^2 \omega^2 + g_4^2 \phi^4 + \frac{1}{2} m_\rho^2 \chi_0^2 \phi^2 + g_4^2 \left(\frac{Z_\omega}{Z}\right)^2 \phi^4 \quad (10)
$$

$$
\mathcal{V}_0 = \frac{1}{2} k_0 \chi^2 (\sigma^2 + \zeta^2) - k_1 (\sigma^2 + \zeta^2)^2 - k_2 (\frac{\sigma^4}{2} + \zeta^4) - k_3 \chi \sigma^2 \zeta 
$$

$$+
 k_4 \chi^4 + \frac{1}{4} \chi^4 \ln \frac{\chi}{\chi_0} - \frac{\delta}{3} \chi^4 \ln \frac{\sigma^2 \zeta}{\sigma_0^2 \zeta_0} \quad (11)
$$

$$
\mathcal{V}_{SB} = \left(\frac{\chi}{\chi_0}\right)^2 \left[ m_\omega^2 f_\omega \sigma + (\sqrt{2} m_K^2 f_K - \frac{1}{2\sqrt{2}} m_\pi^2 f_\pi) \zeta \right], \quad (12)
$$

where $m_i^* = -g_i \sigma - g_i \zeta^2 \zeta$ is the effective mass of the baryon of type $i$ ($i = N, \Sigma, \Lambda, \Xi$). In the above, $g_4 = \sqrt{Z_\omega} \tilde{g}_4$ is the renormalized coupling for $\omega$-field. The thermodynamical potential of the grand canonical ensemble $\Omega$ per unit volume $V$ at given chemical potential $\mu$ and temperature $T$ can be written as

$$
\frac{\Omega}{V} = -\mathcal{L}_{vec} - \mathcal{L}_0 - \mathcal{L}_{SB} - \mathcal{V}_{vac} + \sum_i \frac{\gamma_i}{(2\pi)^3} \int d^3k \frac{E_i^*(k)}{E_i(k)} (f_i(k) + \tilde{f}_i(k))
$$
Here the potential at $\rho = 0$ has been subtracted in order to get a vanishing vacuum energy. In (13) $\gamma_i$ are the spin-isospin degeneracy factors. The $f_i$ and $\bar{f}_i$ are thermal distribution functions for the baryon of species $i$, given in terms of the effective single particle energy, $E^*_i$, and chemical potential, $\mu^*_i$, as

$$f_i(k) = \frac{1}{e^{\beta (E^*_i(k) - \mu^*_i)} + 1}, \quad \bar{f}_i(k) = \frac{1}{e^{\beta (E^*_i(k) + \mu^*_i)} + 1},$$

with $E^*_i(k) = \sqrt{k_i^2 + m_i^2}$ and $\mu^*_i = \mu_i - g_i \omega$. The mesonic field equations are determined by minimizing the thermodynamical potential $[33, 34]$. They depend on the scalar and vector densities for the baryons at finite temperature

$$\rho^s_i = \gamma_i \int \frac{d^3k}{(2\pi)^3} \frac{m^*_i}{E^*_i} \left( f_i(k) + \bar{f}_i(k) \right) ; \quad \rho_i = \gamma_i \int \frac{d^3k}{(2\pi)^3} \left( f_i(k) - \bar{f}_i(k) \right).$$

The energy density and the pressure are given as, $\epsilon = \Omega/V + \mu_i \rho_i + TS$ and $p = -\Omega/V$.

### III. KAON (ANTIKAON) INTERACTIONS IN THE CHIRAL SU(3) MODEL

In this section, we derive the dispersion relations for the $K$($\bar{K}$) and calculate their optical potentials in the asymmetric nuclear matter. The medium modified energies of the kaons and antikaons arise from their interactions with the nucleons, vector mesons and scalar mesons within the chiral SU(3) model.

In this model the interactions with the scalar fields (non-strange, $\sigma$ and strange, $\zeta$), scalar–isovector field $\delta$ as well as a vectorial interaction and the vector meson ($\omega$ and $\rho$) - exchange terms modify the energies for $K$($\bar{K}$) mesons in the medium. In the following, we shall derive the dispersion relations for the kaons and antikaons, including the effects from isospin asymmetry originating from both the vector-isovector $\rho$-field as well as the scalar-isovector $\delta$ field.

The scalar meson multiplet has the expectation value $\langle X \rangle = \text{diag}((\sigma + \delta)/\sqrt{2},(\sigma - \delta)/\sqrt{2},\zeta)$, with $\sigma$ and $\zeta$ corresponding to the non-strange and strange scalar condensates, and $\delta$ is the third isospin component of the scalar-isovector field, $\vec{\delta}$. The pseudoscalar meson
field $P$ can be written as,

$$
P = \begin{pmatrix}
\frac{\pi^0}{\sqrt{2}} & \pi^+ & \frac{2K^+}{1+w} \\
\pi^- & -\frac{\pi^0}{\sqrt{2}} & \frac{2K^0}{1+w} \\
\frac{2K^-}{1+w} & \frac{2K^0}{1+w} & 0
\end{pmatrix},
$$

(15)

where $w = \sqrt{2}\zeta/\sigma$ and we have written down the terms that are relevant for the present investigation. From PCAC one gets the decay constants for the pseudoscalar mesons as $f_\pi = -\sigma$ and $f_K = -(\sigma + \sqrt{2}\zeta)/2$. The vector meson interaction with the pseudoscalar mesons, which modifies the energies of the $K$ ($m\bar{m}K$ mesons, is given as

$$
\mathcal{L}_{VP} = -\frac{m_V^2}{2g_V} \text{Tr}(\Gamma_\mu V^\mu) + \text{h.c.}
$$

(16)

The vector meson multiplet is given as $V = \text{diag}((\omega + \rho_0)/\sqrt{2}, (\omega - \rho_0)/\sqrt{2}, \phi)$. The non-diagonal components in the multiplet, which are not relevant in the present investigation,
FIG. 2: The energies of the kaons, $K^+$ and $K^0$, at zero momentum and for $\rho_B=0.16$ fm$^{-3}$, are plotted as functions of the isospin asymmetry parameter, $t$ in (a) and (b). The medium modifications to the energies are also shown for the situations when either the isospin asymmetric contribution from the $\rho$-meson or $\delta$ meson, or, both, are not taken into account. The solid line shows the total contribution.
FIG. 3: The energies of the antikaons, $K^-$ and $K^0$, at zero momentum and for $\rho_B=0.16$ fm$^{-3}$ are plotted as functions of the isospin asymmetry parameter, $t$ in (a) and (b). The medium modifications to the energies are also shown for the situations when the isospin asymmetric contribution from the $\rho$-meson or $\delta$ meson, or, both, are not taken into account. The solid line shows the total contribution. The symmetry energy in MeV plotted as a function of the baryon density, $\rho_B$ (in fm$^{-3}$).
are omitted. With the interaction (16), the coupling of the $K$-meson to the $\omega$-meson is related to the pion-rho coupling as $g_{\omega K}/g_{\rho \pi \pi} = f_\pi^2/(2f_K^2)$.

The scalar meson exchange interaction term is determined from the explicit symmetry breaking term by equation (8), where $A_p = 1/\sqrt{2} \text{ diag } (m_\pi^2 f_\pi, m_\pi^2 f_\pi, 2m_K^2 f_K - m_\pi^2 f_\pi)$.

The interaction Lagrangian modifying the energies of the $K$($\bar{K}$)-mesons can be written as

$$\mathcal{L}_{KN} = -\frac{i}{8f_K^2} \left[ 3(\bar{N}\gamma^\mu N)(\bar{K}(\partial_\mu K) - (\partial_\mu \bar{K})K) + (\bar{N}\gamma^\mu \tau^a N)(\bar{K}\tau^a(\partial_\mu K) - (\partial_\mu \bar{K})\tau^a K) \right]$$

$$+ \frac{m_\pi^2}{2f_K^2} \left[ (\sigma + \sqrt{2}\zeta)(\bar{K}K) + \delta^a(\bar{K}\tau^a K) \right]$$

$$- ig_{\omega K} \left[ (\bar{K}(\partial_\mu K) - (\partial_\mu \bar{K})K)\omega^\mu + (\bar{K}\tau^a(\partial_\mu K) - (\partial_\mu \bar{K})\tau^a K)\rho^\mu a \right]$$

$$- \frac{1}{f_K^2} \left[ (\sigma + \sqrt{2}\zeta)(\partial_\mu \bar{K})(\partial^\mu K) + (\partial_\mu \bar{K})\tau^a(\partial^\mu K)\delta^a \right]$$

$$+ \frac{d_1}{2f_K^2} (\bar{N} N)(\partial_\mu K)(\partial^\mu K). \quad (17)$$

In the above, $K$ and $\bar{K}$ are the kaon and antikaon doublets. In (17) the first line is the vectorial interaction term obtained from the first term in (2) (Weinberg-Tomozawa term). The second term, which gives an attractive interaction for the $K$-mesons, is obtained from the explicit symmetry breaking term (8). The third term, arising from equation (16), refers to the interaction in terms of the $\omega$-meson and $\rho$-meson exchanges. The fourth term arises within the present chiral model from the kinetic term of the pseudoscalar mesons given by the third term in equation (2), when the scalar fields in one of the meson multiplets, $X$, are replaced by their vacuum expectation values. The fifth term in (17) for the KN interactions arises from the term

$$\mathcal{L}^{BM} = d_1 Tr(u_\mu u^\mu \bar{B} B), \quad (18)$$

in the SU(3) chiral model [37]. The last two terms in (17) represent the range term in the chiral model. The Fourier transformation of the equation of motion for kaons (antikaons) leads to the dispersion relations,

$$-\omega^2 + \vec{k}^2 + m_K^2 - \Pi_K(\omega, |\vec{k}|, \rho) = 0,$$

where $\Pi_K$ denotes the kaon (antikaon) self energy in the medium.

Explicitly, the self energy $\Pi(\omega, |\vec{k}|)$ for the kaon doublet arising from the interaction (17) is given as
FIG. 4: The energies of the kaons, relative to the t=0 values, are plotted as functions of the momentum for t=0.5 and 1. The subplots (a), (c) and (e) refer to $K^+$ and, (b), (d) and (f) refer to $K^0$ at different densities.
FIG. 5: The energies of the antikaons, relative to the $t=0$ values, are plotted as functions of the momentum for $t=0.5$ and 1. The subplots (a), (c) and (e) refer to $K^-$ and, (b), (d) and (f) refer to $\bar{K}^0$ at different densities.
\[ \Pi(\omega, |\vec{k}|) = -\frac{3}{4f_{K}^{2}}(\rho_{p} + \rho_{n})\omega + \frac{m_{K}^{2}}{2f_{K}}(\sigma' + \sqrt{2}\zeta' \pm \delta') - 2g_{\omega K}\omega(\omega_0 \pm \rho_{0}) \]
\[ + \left[ -\frac{1}{f_{K}}(\sigma' + \sqrt{2}\zeta' \pm \delta') + \frac{d_{1}}{2f_{K}}(\rho_{s}^{p} + \rho_{s}^{n}) \right](\omega^{2} - \vec{k}^{2}), \quad (19) \]

where the \pm signs refer to the \(K^{+}\) and \(K^{0}\) respectively. In the above, \(\sigma' (=\sigma - \sigma_{0})\), \(\zeta' (=\zeta - \zeta_{0})\) and \(\delta' (=\delta - \delta_{0})\), are the fluctuations of the scalar-isoscalar fields \(\sigma\) and \(\zeta\), and the third component of the scalar-isovector field, \(\delta\), from their vacuum expectation values. The vacuum expectation value of \(\delta\) is zero \((\delta_{0}=0)\), since a nonzero value for it will break the isospin symmetry of the vacuum (the small isospin breaking effect arising from the mass and charge difference of the up and down quarks, respectively, has been neglected here). \(\rho_{p}\) and \(\rho_{n}\) are the number densities for the proton and the neutron, \(\rho_{s}^{p}\) and \(\rho_{s}^{n}\) are their corresponding scalar densities.

Similarly, for the antikaon doublet, the self-energy is calculated as
\[ \Pi(\omega, |\vec{k}|) = \frac{3}{4f_{K}^{2}}(\rho_{p} + \rho_{n})\omega + \frac{m_{K}^{2}}{2f_{K}}(\sigma' + \sqrt{2}\zeta' \pm \delta') + 2g_{\omega K}\omega(\omega_0 \pm \rho_{0}) \]
\[ + \left[ -\frac{1}{f_{K}}(\sigma' + \sqrt{2}\zeta' \pm \delta') + \frac{d_{1}}{2f_{K}}(\rho_{s}^{p} + \rho_{s}^{n}) \right](\omega^{2} - \vec{k}^{2}), \quad (20) \]

where the \pm signs refer to the \(K^{-}\) and \(\bar{K}^{0}\) respectively.

After solving the above dispersion relations for the kaons and antikaons, their optical potentials can be calculated from
\[ U(\omega, k) = \omega(k) - \sqrt{k^{2} + m_{K}^{2}}, \quad (21) \]
where \(m_{K}\) is the vacuum mass for the kaon (antikaon).

The parameter \(d_{1}\) is calculated from the the empirical value of the isospin averaged KN scattering length [36, 40, 41] taken to be
\[ \bar{a}_{KN} \approx -0.255 \text{ fm}. \quad (22) \]

**IV. RESULTS AND DISCUSSIONS**

The present calculation uses the model parameters from [32]. The values, \(g_{\sigma N} = 10.623\), and \(g_{\zeta N} = -0.4894\) are determined by fitting vacuum baryon masses. The other
FIG. 6: The optical potentials of the kaons are plotted as functions of the momentum for various values of the isospin asymmetry parameter $t$. The subplots (a), (c) and (e) refer to $K^+$ and, (b), (d) and (f) refer to $K^0$ at different densities.
FIG. 7: The optical potentials of the antikaons are plotted as functions of the momentum for various values of the isospin asymmetry parameter, $t$. The subplots (a),(c) and (e) refer to $K^-$ and, (b), (d) and (f) refer to $\bar{K}^0$ at different densities.
parameters as fitted to the nuclear matter saturation properties in the mean field approximation are: \( g_{\omega N} = 13.606 \), \( g_4 = 61.466 \), \( m_\zeta = 1038.5 \text{ MeV} \), \( m_\sigma = 474.3 \text{ MeV} \). The coefficient \( d_1 \), calculated from the empirical value of the isospin averaged scattering length \( (22) \), is \( 5.196/m_K \). Using these parameters, the symmetry energy defined as

\[
a_4 = \frac{1}{2} \frac{d^2 E}{dt^2} |_{t=0} \tag{23}
\]

with the asymmetry parameter \( t = (\rho_n - \rho_p)/\rho_B \), has a value of \( a_4 = 28.4 \text{ MeV} \) at saturation nuclear matter density of \( \rho_0 = 0.15 \text{ fm}^{-3} \). Figure 1 shows the density dependence of the symmetry energy, which increases with density similar to previous calculations \([42]\).

The kaon and antikaon properties were studied in the isospin symmetric hadronic matter within the chiral SU(3) model in ref. \([37]\). The contributions from the vector interaction as well as the vector meson \( \omega \)-exchange terms lead to a drop for the antikaons energy, whereas they are repulsive for the kaons. The scalar meson exchange term arising from the scalar-isoscalar fields (\( \sigma \) and \( \zeta \)) is attractive for both \( K \) and \( \bar{K} \). The first term of the range term of eq. \((17)\) is repulsive whereas the second term has an attractive contribution for the isospin symmetric matter \([37]\) for both kaons and antikaons.

The contributions due to the scalar-isovector, \( \delta \)-field as well as the vector-isovector \( \rho \)-meson, introduce isotopic asymmetry in the \( K \) and \( \bar{K} \)-energies. For \( \rho_n > \rho_p \), in the kaon sector, \( K^+ (K^0) \) has negative (positive) contributions from both \( \delta \) and \( \rho \) mesons. The \( \delta \) contribution from the scalar exchange term is positive (negative) for \( K^+ (K^0) \), whereas that arising from the range term has the opposite sign and dominates over the former contribution. The contribution from the \( \rho \) to the kaon (antikaon) masses at \( \rho_B = 0.16 \text{ fm}^{-3} \) is large as compared to that of the \( \delta \) contribution, as can be seen from the figures 2 and 3. When we do not account for the isospin asymmetry effects arising due to the \( \rho \) and \( \delta \) fields, then the masses of kaons and antikaons stay almost constant. The small fluctuations are reflections of the deviation of the scalar density occurring in the last term of equation \((19)\) for the kaon (antikaon) self energy from the baryon density, \( \rho_B \).

In figure 4, the energies of the \( K^+ \) and \( K^0 \), relative to the \( t=0 \) values, are plotted for different values of the isospin asymmetry parameter, \( t \), at various densities. For \( \rho_B = 0.16 \text{ fm}^{-3} \), the energy of \( K^+ \) is seen to drop by about 15 MeV at zero momentum when \( t \) changes from 0 to 1. On the other hand, the \( K^0 \) energy is seen to increase by a similar amount for \( t=1 \), from the isospin symmetric case of \( t=0 \). The energies of the kaons are also plotted for
densities $\rho_B = 0.31 \text{ fm}^{-3}$ and $\rho_B = 0.61 \text{ fm}^{-3}$ in the same figure. For $K^+$, the t-dependence of the energy is seen to be less sensitive at higher densities, whereas the energy of $K^0$ is seen to have a larger drop from the $t=0$ case, as we increase the density. The reason for this opposite behavior for the $K^+$ and $K^0$ on the isospin asymmetry comes from the relative vector-meson contributions. For $K^+$, the isospin asymmetric effect to the energy arising from the $\rho$ meson (which dominates over that from the $\delta$ meson) is opposite in sign to that of the $\omega$ meson, whereas for $K^0$, it has the same sign as that of the $\omega$ meson.

For the antikaons, the $K^-(\bar{K}^0)$ energy is seen to increase (drop) with $t$, as seen in figure 5. The sensitivity of the isospin asymmetry dependence of the energies is seen to be larger for $K^-$ with density, whereas it becomes smaller for $\bar{K}^0$ at high densities. However, $\bar{K}^0$ shows an appreciable drop as we increase the momentum, whereas the value for $K^-$ is not as sensitive to momentum change as $\bar{K}^0$.

The qualitative behavior of the isospin asymmetry dependencies of the energies of the kaons and antikaons are also reflected in their optical potentials plotted in figure 6 for the kaons, and in figure 7, for the antikaons, at selected densities. The different behavior of the $K^+$ and $K^0$, as well as for the $K^-$ and $\bar{K}^0$ optical potentials in the dense asymmetric nuclear matter should be seen in their production as well as propagation in isospin asymmetric heavy ion collisions. The effects of the isospin asymmetric optical potentials could thus be observed in nuclear collisions at the CBM experiment at the proposed project FAIR at GSI, where experiments with neutron rich beams are planned to be adopted.

V. SUMMARY

To summarize, we have investigated, within a chiral SU(3) model, the density dependence of the $K, \bar{K}$-meson optical potentials in asymmetric nuclear matter, arising from the interactions with nucleons and scalar and vector mesons. The properties of the light hadrons – as studied in a SU(3) chiral model – modify the $K(\bar{K})$-meson properties in the hadronic medium. The model with parameters fixed from the properties of hadron masses, nuclei and KN scattering data, takes into account all terms up to the next to leading order arising in chiral perturbative expansion for the interactions of $K(\bar{K})$-mesons with baryons. One can observe a significant density dependence of the isospin asymmetry on the optical potentials of the kaons and antikaons. The results can be used in heavy-ion simulations that include
mean fields for the propagation of mesons \[37\]. The different potentials of kaons and antikaons can be particularly relevant for neutron-rich beams in the CBM experiment at the future facility FAIR at GSI, Germany, as well as at the experiments at the planned Rare Isotope Accelerator (RIA) laboratory, USA.

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