Binary black holes, gravitational waves, and numerical relativity

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Abstract. The final merger of comparable mass binary black holes produces an intense burst of gravitational radiation and is one of the strongest sources for both ground-based and space-based gravitational wave detectors. Since the merger occurs in the strong-field dynamical regime of general relativity, numerical relativity simulations of the full Einstein equations in 3-D are required to calculate the resulting gravitational dynamics and waveforms. While this problem has been pursued for more than 30 years, the numerical codes have long been plagued by various instabilities and, overall, progress was incremental. Recently, however, dramatic breakthroughs have occurred, resulting in robust simulations of merging black holes. In this paper, we examine these developments and the exciting new results that are emerging.

1. Introduction

The final coalescence of a comparable mass binary black hole is a strong source of gravitational waves and proceeds in three phases: inspiral, merger, and ringdown [1]. Both the inspiral stage, when the black holes have relatively wide separations and follow quasi-circular trajectories, and the ringdown stage, during which the final merged black hole evolves towards a quiescent Kerr state, can be handled analytically. However the merger phase, during which the two black holes plunge together and merge to form a highly distorted remnant black hole, occurs in the regime of very strong, dynamical gravitational fields and can only be calculated using numerical relativity.

The merger stage will produce an intense burst of gravitational radiation with a luminosity (in gravitational waves) $\sim 10^{23}$ times the solar luminosity (in photons), briefly emitting more energy than the combined light from all the stars in the visible universe. Such bursts are expected to be among the strongest sources for the space-based LISA detector, which will observe massive binary black holes. Mergers of stellar-mass and intermediate-mass binary black holes are likely to be the strongest sources for ground-based gravitational wave detectors such as LIGO and VIRGO. Observations of the gravitational waves from the merger will enable unprecedented tests of general relativity in the dynamical, strong field regime – but only if we know the waveforms that general relativity predicts.

Merging binary black holes also have compelling applications in astrophysics. In particular, when the black holes are spinning and/or have unequal masses, the resulting emission of...
gravitational waves is asymmetric; since the gravitational waves carry momentum, the merged remnant black hole suffers a recoil kick [2]. If this kick velocity is large enough, it could eject the merged remnant from its host structure, thereby affecting the overall rate of merger events [3]. Accurate values for these kick velocities require numerical relativity simulations.

For over three decades, numerical relativists have attempted to calculate the merger of comparable mass black holes and the resulting gravitational waveforms. This has proved to be extremely difficult and, for many years, the simulation codes were plagued by a host of instabilities that caused them to crash before any sizeable fraction of a binary orbit could be evolved. However, a series of dramatic breakthroughs has recently been made, resulting in accurate and robust simulations of binary black hole mergers and the resulting gravitational waveforms.

In this paper, we examine some of these exciting developments.\footnote{Since this paper is not a full review of the subject, the reference list is representative rather than comprehensive. We have attempted to cite key papers from the major numerical relativity efforts.}

We follow the conventional practice of setting $G = 1$ and $c = 1$, which allows us to measure both time and distance in terms of mass $M$. In particular, $1M \sim (5 \times 10^{-6})(M/M_\odot)$ sec $\sim 1.5(M/M_\odot)$ km, where $M_\odot$ is the mass of the sun. We take spatial indices to have the range $i = 1, 2, 3$. Note that the simulation results scale with the masses of the black holes, and thus are equally applicable to LISA and ground-based detectors.

### Figure 1.
Spacetime is sliced into a stack of 3-D spacelike hypersurfaces labeled by time $t$.

2. The challenge of numerical relativity
In numerical relativity, a spacetime is constructed by solving the Einstein equations on a computer. In the "3+1" [4, 5] approach, 4-D spacetime is sliced into a stack of 3-D spacelike hypersurfaces labeled by time $t$, as shown in Fig. 1. The main independent variables are taken to be the 3-metric $g_{ij}$ and its first time derivative $\partial_t g_{ij}$. The Einstein equations give a set of nonlinear, partial differential equations including both constraint and evolution equations. The constraints are elliptic equations that must be satisfied on every slice; in particular, the initial data is set by solving the constraints on a slice at some initial time $t = 0$. This data is then propagated forward in time using the evolution equations. Four freely-specifiable coordinate or gauge conditions give the development of the time and spatial coordinates during the evolution. The lapse function $\alpha$ gives the lapse of proper time $\alpha \Delta t$ between neighboring slices, and the
Figure 2. The lapse function $\alpha$ and shift vector $\vec{\beta}$ provide coordinate or gauge conditions during the evolution.

Shift vector $\beta^i$ provides the means to move the spatial coordinates as the evolution proceeds from one slice to the next; see Fig. 2.

In 1964, Hahn and Lindquist [6] were the first to attempt to solve the Einstein equations on a computer by evolving a head-on collision of two equal mass black holes in 2-D. Due in part to a poor choice of coordinate conditions, the evolution crashed shortly after it began. In the mid-1970s, Smarr and Eppley [7, 8, 9] pioneered the use of the the $3+1$ approach. Using improved lapse conditions to give slices that avoid crashing into singularities [10, 11], they succeeded in evolving the head-on collision in 2-D axisymmetry, and extracting some information about the resulting gravitational waves. This was a significant achievement. However, taking the next step to fully 3-D simulations of black holes proved too daunting and, during the 1980s, most of the focus in numerical relativity turned to modeling neutron stars.

In the 1990s, work on the binary black hole problem started up again, spurred on by the development of ground-based gravitational wave detectors such as LIGO. Major funding arrived in the form of a Grand Challenge grant from the National Science Foundation, resulting in the development of large 3-D codes and the ability to evolve boosted black holes [12] and grazing collisions [13, 14, 15]. However, the problem turned out to be more difficult than anticipated and the codes were plagued by instabilities that caused them to crash. By the end of that decade and into the early 2000s, work on LISA and data-taking on LIGO got underway, increasing the importance of the binary black hole problem. The presence of unstable modes in the formulations of the numerical relativity equations was recognized, and work on key areas such as gauge conditions, formalisms, boundary conditions, and the role of the constraints in evolutions was carried out. Progress in obtaining stable 3-D binary black hole evolutions was steady, but slow and incremental.

Recently, major progress in numerical relativity simulations of binary black hole mergers has been made rapidly, across a broad front. In 2004, the first complete orbit of a binary black hole was accomplished [16]. The first full simulation of a binary black hole through an orbit, plunge, merger and ringdown was carried out in 2005 [17]. In late 2005, the development of novel and highly effective gauge conditions, discovered simultaneously and independently by the numerical relativity groups at the University of Texas at Brownsville [18] and NASA’s Goddard Space Flight Center [19], led to a breakthrough in the ability to carry out accurate and stable long-term evolutions of binary black holes. These “moving puncture” techniques were rapidly and broadly adopted by the numerical relativity community, leading to stunning advances in binary black hole modeling, including simulations with unequal masses and spins [20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32].
Figure 3. Sample grid structure for the evolution of two black holes with mass ratio 1.3:1, shown in the $z = 0$ plane. Each panel shows a region centered on the origin and having extent $12.4M$ in both directions. Here we use 10 cells per block and the finest grid regions (shown in black) have resolution $h_f = 3M/160$.

3. Computational methodology

In the rest of this paper, we focus on work carried out by our numerical relativity group at NASA’s Goddard Space Flight Center. We have developed a numerical relativity code known as Hahndol based on a conformal formulation [33, 34] of the Einstein equations in which the constraints have been incorporated into the evolution equations to improve stability [35, 36, 37]. The set of evolution equations is written with first-order time derivatives and second-order spatial derivatives, and is strongly hyperbolic [38, 39]. The time integration is carried out using a 4th-order Runge-Kutta algorithm, and spatial derivatives are handled with 4th-order finite differencing stencils on a 3-D Cartesian grid [21, 25].

For successful binary black hole merger simulations, we must both resolve the black holes (with spatial scales $\sim M$) and extract the gravitational radiation (with scales $\lambda_{GW} \sim (10 - 100)M$) in the wave zone. We accomplish this using adaptive mesh refinement (AMR) to produce variable resolution on the grid, with 5th-order accurate interpolation between refinement regions and dissipation terms to minimize noise at mesh refinement boundaries. We use a 2nd-order accurate Sommerfeld condition at the outer boundary, which is kept at large enough distances $\sim 1000M$ to insure that any reflections neither impact the evolution of the sources nor the wave-extraction region.

The black holes are represented as “punctures” [40]. On the initial slice, we write the 3-metric in the form $g_{ij} = \psi^4 \delta_{ij}$, where the conformal factor $\psi = \psi_{BL} + u$ and $i, j = 1, 2, 3$. The static,
singlar part of the conformal factor takes the form $\psiBL = 1 + \sum_{n=1}^{2} m_n/(2|\vec{r} - \vec{r}_n|)$, where the $n^{th}$ puncture has mass $m_n$ and is located at $\vec{r}_n$. The nonsingular function $u$ is found by solving one of the constraint equations. We then evolve the black holes using the coordinate conditions for moving punctures [22], which allows the black holes to move freely across the grid.

We use the PARAMESH package [41] to implement parallelization and AMR. Initially, we set up the black hole binary in a numerical domain using a box-in-box refinement structure, with grid cells of size $h_f$ in the innermost box and subsequent outer boxes with grid sizes larger by a factor of two. We begin with two or more boxes centered on each individual black hole, and then a box centered on the origin that encompasses both black holes. Subsequent boxes centered on the origin are used to give roughly a total of 11 refinement levels. As the binary evolves, the black holes move freely across the grid, changing the curvature in the surrounding region; in response, the initial grid structure changes adaptively. Paramesh works on logically Cartesian, or structured, grids and carries out the mesh refinement on grid blocks. If the curvature reaches a certain threshold (a free parameter in our code) at one point of a block, that block is bisected in each coordinate direction to produce 8 child blocks, each having half the resolution of the parent block. If all points in all the child blocks fall below the threshold, those blocks get derefined.

Figure 3 shows the grid structure at four different times during the evolution of two black holes with a mass ratio of 1.3:1. In each panel the black region represents the finest mesh, and surrounding regions are progressively coarser. In the upper left panel, the finest mesh surrounds only the smaller black hole, because this has the steeper field gradient, while the fields around the larger black hole are marginally below the refinement criterion at this time. Subsequent dynamics (by the time of the upper right panel) cause the larger black hole to also receive refinement. The grids track the black holes as they plunge together (in the lower left panel) and merge (in the lower right panel), during which the finest refinement regions also merge together.

Note that AMR is used only to resolve the sources, with the mesh becoming progressively coarser far away from the sources. The grid structure for boxes centered on the origin and having extent $\sim 40M$ or larger in each dimension remains fixed throughout the evolution. The gravitational waves that reach the outer boundary during the course of the simulation, with wavelengths of $\sim 100M$, will not be well resolved in the coarsest refinement region. However, the behavior of signals in that region, including any reflections from the outer boundary, is causally disconnected from the parts of the domain (at $R \leq 100M$) where we extract the gravitational radiation. We do not use AMR to follow the gravitational waves with fine meshes; rather we require only that the fixed mesh resolution in the region where the waves are extracted be sufficient to resolve the waves there.

4. Gravitational waves from black hole mergers
We have simulated the evolution of a nonspinning equal-mass binary black hole starting from a relatively wide separation, $\sim 1200M$ or $\sim 7$ orbits before the formation of a common event horizon [25]. Here $M$ is the total mass the system would have had when the black holes were very far apart and before radiative losses became significant. We carried out three runs using similar grid refinement structures, but at different resolutions: low ($h_f = 3M/64$), medium ($h_f = 3M/80$) and high ($h_f = M/32$). Here, $h_f$ is the grid spacing in the regions with the highest resolution in each simulation, those being the regions around each black hole.

Figure 4 shows the trajectories of the black holes in our highest resolution run; here the tracks, one shown in red (dotted line) and the other in blue (solid line), mark the paths of the punctures. Astrophysically, we expect that the black holes should spiral together on quasicircular orbits, since any initial eccentricity arising from the formation of the binary would have been radiated away due to gravitational radiation reaction earlier in the evolution. However, setting up black hole initial data on quasicircular orbits has proved to be challenging. In this model, the black holes start on nearly circular orbits, with very small eccentricity $\epsilon < 0.01$. As shown in Fig. 4,
Figure 4. The trajectories of both black holes through \( \sim 7 \) revolutions before coalescence are shown for the high resolution run.

the tracks nicely trace out a nearly circular inspiral after the first few orbits.

One way to assess the accuracy of the simulations is to examine the values of the constraints throughout the evolution. For these 3 runs, the grid structure was designed to be commensurate for all resolutions; this allows us to look at the \( L_1 \) norm of the constraints in each refinement region. We found that the Hamiltonian and momentum constraints were convergent at order 2.5 in the finest grid, where we expect errors at the punctures to dominate and the 4th-order differencing to break down. In all coarser regions, we find the Hamiltonian constraint to be 4th-order convergent, and the momentum constraint to be better than 2nd-order convergent, throughout the runs.

The gravitational radiation produced by the system can be calculated using the complex Weyl tensor component \( \psi_4 \). The gravitational wave strain is the physical observable that will be measured by the detector and is related to \( \psi_4 \) by \( -\ddot{h}_+ + i\ddot{h}_\times = 2\psi_4 \); here, \( h_+ \) and \( h_\times \) are the two polarization states of the gravitational wave. Figure 5 shows the distribution in space of \( \text{Re}(\psi_4) \), corresponding to the + polarization, at two times: shortly before the black holes merge (left) and shortly after the time of merger (right). The colors denote the amplitude of the waves, increasing from red through orange and into yellow. We extract waveforms from the simulation on coordinate spheres of different radii \( R_{\text{ext}}/M \) on which we measure spin-weighted spherical harmonic components of \( \psi_4 \) using a 2nd-order algorithm [42, 43]. Figure 6 shows the \( l = 2, m = 2 \) component of the strain extracted at \( R_{\text{ext}} = 40M \) and observed on the equatorial plane, where only the \( h_+ \) component contributes to the measured strain.

5. Mergers of spinning black holes

Rotating black holes are specified by their mass \( M \) and spin parameter \( a = |\vec{S}|/M \), where \( \vec{S} \) is the spin angular momentum. The dimensionless spin parameter is then \( 0 \leq a/M \leq 1 \), where \( a = 0 \) is a nonrotating Schwarzschild black hole and \( a = 1 \) is an extremal Kerr hole. Since astrophysical black holes are expected to be rotating, it is important to study the dynamics and waveforms of black hole binaries with spin.

Figure 7 shows the evolution of a black hole binary with equal masses and spins \( a \sim 0.9M \).
Figure 5. Contours of gravitational radiation for the merger of equal mass binary black holes. The radiation amplitude is denoted by the colors, increasing from red through orange and into yellow. (left) just before the black holes merge (right) shortly after the merger.

Figure 6. Gravitational waveform from the merger of equal-mass Schwarzschild black holes. The black holes start out with their spins lying in the equatorial plane, oppositely directed along the $y$--axis and normal to their orbital trajectories. Here the directions of the spins were deduced by computing a certain curvature-related quantity on the surface of each black hole horizon, and assuming that it should behave as that of a Kerr black hole, where curvature depends on spin in a well-understood way. As the system evolves, the spins precess. The black holes merge to form a final hole with $a/M \sim 0.67$ and spin axis along the $+z$--axis, moving with a velocity $v_{\text{kick}} \sim 1500\text{km/s}$ in the $+z$ direction.

6. Outlook
A remarkable series of breakthrough has occurred recently, enabling robust modeling of comparable mass binary black hole mergers using numerical relativity. We now have a broad
Figure 7. The evolution of an equal-mass binary black hole with spins \( a/M \sim 0.9 \). The spin vectors have equal magnitudes and start out in the equatorial plane, oppositely directed and normal to the orbits. The spins precess as the system evolves. The final black hole has spin \( a/M = 0.67 \) and is moving with velocity \( v_{\text{kick}} \sim 1500\text{km/s} \) in the +z direction.

Consensus (a) on the overall shape of the waveform resulting from the merger of two equal mass Schwarzschild black holes (cf. Fig. 6), (b) that this merger produces a final remnant black hole with spin \( a \sim 0.7M \), and (c) that the amount of energy radiated in the form of gravitational waves, starting with the final few orbits and proceeding through the plunge, merger and ringdown, is \( \sim 0.04M \); cf. [44]. Some mergers of binary black holes with unequal masses and with spins have been simulated, and the resulting recoil kicks have been calculated; certain configurations, notably those with spins originally in the equatorial plane, have been shown to produce very large kicks, \( \sim 2000\text{km/s} \) or even larger [30, 32, 28]. Comparisons of the numerical relativity waveforms and those calculated for the late inspiral regime using analytic post-Newtonian methods have begun [45, 46, 47], and applications to gravitational wave data analysis are underway [48, 49].

Important and exciting work remains to be done, including the exploration of a large parameter space and the achievement of greater numerical precision. In addition, new effects might arise when matter is incorporated into the simulations. We are privileged to be in a true “golden era” of scientific discovery!

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