Speed of sound, breaking of conformal limit and instabilities in quark-gluon plasma at finite baryon density.

Z.V Khaidukov* and Yu.A.Simonov*

+Moscow Institute of Physics and Technology, Institutskiy per. 9, 141700 Dolgoprudny, Moscow Region, Russia
*Institute for Theoretical and Experimental Physics, B. Cheremushkinskaya 25, Moscow, 117259, Russia

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Abstract

The properties of the quark-gluon plasma(QGP) in the presence of baryon chemical potential are studied using the Field Correlator Method(FCM). At low densities the QGP thermodynamics with the colormagnetic confinement and the Polyakov line interaction is in good agreement with lattice data, and the speed of sound satisfies $C_s^2 \leq \frac{1}{3}$, but in the intermediate range of densities the speed of sound displays a singular behaviour and one needs density modifications of the Polyakov line interaction to avoid instabilities. In particular for $\mu_q > 0.5$ GeV there exists a region in the $(\mu, T)$ plane where $C_s^2$ violates the conformal limit. This behaviour is strongly connected to the properties of the Polyakov loop and its density dependence.

1 Introduction

The main result of heavy ion experiments performed over the last 15 years at RHIC and then at RHIC and LHC is the discovery of a new form of matter [1–5] with its properties markedly different from the pre-RHIC era predictions-see [6–15] and references therein. Instead of the commonly assumed picture of weakly coupled Quark-Gluon Plasma(QGP) a strongly coupled liquid has emerged, subject to the law of the relativistic hydrodynamics [16–18]. The properties of the produced matter are drastically changing as it passes several stages of evolution: from formation, hydrodynamization and thermalization toward the hadron gas production. The wealth of the QCD matter phases is reflected in the QCD phase diagram drawn in the $(\mu, T)$ plane. However, the correspondence between the specific$(\mu, T)$ domains of the phase diagram and the space-time dynamics of the fireball should be considered with caution. The reason is that the phase diagram describes the limit of an infinite system in thermodynamic equilibrium.

On the theoretical side the matter created in heavy ion collisions should be described by the fundamental laws of QCD. The dynamics and thermodynamics of the Quark-Gluon Plasma (QGP) is now in the focus of numerous investigations [19]. The presence of strong interaction in QGP at zero baryon density was demonstrated in numerous lattice data [20–25], which show that the ratio of the QGP pressure to the Stefan-Boltzmann value $P_{SB}$ is around 0.8 and remains almost constant up to 1 GeV, implying a strong interaction growing with $T$.

Another striking discovery was the analysis of the temperature transition, made in the $2 + 1$ QCD lattice computations, which showed a smooth crossover in the temperature region $T = 140 \div 180$ MeV [26]. But the question of the existence of a critical point at finite baryon chemical potential is still of intense interest. [27]

1 This QCD crossover is a new phenomenon, possibly having some analogs in the material sciences and in the ionization and dissociation processes.
As a result the question about the structure of the QCD phase diagram remains open on the lattice side. This happens mostly because lattice methods are strongly restricted to a domain of small chemical potentials \((N_c=3)\) due to the "sign problem". To circumvent this difficulty in the case of \(N_c=3\) one exploits the Taylor expansion around zero chemical potential \([28,29]\), or imaginary chemical potential \([30]\). Another possibility is to use on the lattice the number of colors \(N_c=2\), where the sign problem is absent \([31,34]\).

On the theoretical side one can exploit the method, which is applicable at any chemical potential and any temperature – the Field Correlator Method (FCM), where the nonperturbative dynamics in the confinement and deconfinement regions is based on vacuum properties, described by gluonic field correlators \([35–40]\).

The strength of this method is connected with the possibility of a complete self-consistent description of both QGP plasma (also in the presence of the chemical potential) and the hadronic matter in the confinement phase \([41,49]\). The main ingredient is the vacuum average of colorelectric fields \(D^E\) and colormagnetic fields \(D^M\), which provide colorelectric confinement (CEC) with the string tension \(\sigma(T)\) and colormagnetic confinement (CMC) with the string tension \(\sigma_s(T)\). The latter, calculated from field correlators and on the lattice grows with \(T\), \(\sigma_s(T) \sim q^2(T)T^2\), and insures the strong interaction at large \(T\), mentioned above.

From the point of view of FCM the crossover phenomenon is connected with the gradual vanishing of the vacuum confining correlator \(D^E(x)\) (and the resulting string tension \(\sigma(T)\)) with the growing temperature. The same phenomenon of the "melting confinement" can be observed in the SU(3) gluodynamics \([46]\), where also the decreasing with \(T\) string tension \(\sigma(T)\), measured on the lattice \([50,53]\), explains the behaviour of pressure for \(T < T_c\), but in the case of SU(3) it cannot smoothly match the fast growing gluon pressure (in contrast to the slowly growing glueball pressure due to large glueball masses \(\gtrsim 2\) GeV). As a result, one has in SU(3) a weak first order transition, \([46]\) while in the \(n_f = 2 + 1\) QCD with low mass mesons the vanishing of \(\sigma(T)\) is complete in the course of transition.

As a proof of this picture one has the vanishing with \(T\) the quark condensate \([54]\) which is connected in FCM to the confinement \(\langle \bar{q}q(T)\rangle \sim \sigma^{3/2}(T)\) \([55,57]\).

Hence the FCM picture of the temperature transition is a vacuum based process, different from standard theoretical models, and this crossover does not imply the presence of a critical point.

The first study of the QGP thermodynamics at nonzero baryon density using FCM, and CMC was done in \([48]\). In general, the main interaction in QGP is provided by the colormagnetic confinement, operating both below and above transition temperature, as was observed in lattice data \([58]\), where the CMC correlators \((\text{tr} F_i(x) \phi(x, y) F_{ik}(y))\) have been measured.

It was found in \([59]\) that CMC does not support white bound states in \(q\bar{q}\) and \(gg\) systems, however it can create the screening mass \(M(T)\) of isolated quarks and gluons \([45,48,60]\), which grows with temperature, so that the ratio \(M(T)\) is constant up to the logarithmic terms.

As was shown in \([48]\), CMC mechanism produces the square root singularities in \(P(\mu, T)\) in the complex \(\mu\) plane, which are situated at the distance \(i\mu/T\) over the real axis. But the pressure still can be calculated for however large values of the chemical potential (for example in \([48]\) the pressure is calculated for \(\mu = 400\) MeV that corresponds to the baryon chemical potential \(m_B \approx 3\mu = 1200\) MeV) and it does not show any singularities. However the study at higher \(\mu\) done in the present paper, discovers new interesting phenomena.

We shall show in this paper that the behavior of QGP using CMC and the \(\mu\)-independent Polyakov line shows a good agreement with lattice data at low densities, \(\mu_q < \mu_{\text{crit}} = 400\) MeV, but at larger \(\mu\) and \(T=1.3\) \(T_c\) the sound velocity strongly violates the conformal limit and becomes a singular function of \(\mu\).

It is created by the strong NP dynamics, which gives large values to \(\frac{\partial^2 P}{\partial T^2}\) and \(\frac{\partial^2 P}{\partial T \partial \mu}\); supported by high values of \(\frac{\partial^2 L}{\partial T^2}\), where \(L = \exp(-F/T)\) is the Polyakov line value.

As will be shown below in the paper, to avoid this problem one should take into account the density modification of the Polyakov line. We shall use in what follows the speed of sound as an indicator of the instability effects in QGP.

We need to point out that the question of the possible values of the speed of sound is of interest in itself. It plays a very important role in the physics of the medium \([61]\). The question about limitations
of the speed of sound at finite density QCD is still open, because at intermediate densities the standard (perturbative) QCD theory is unable to predict the value of $C_s^2$, but in some models like ADS/CFT, the upper limit 1/3 was found \[62\,66\]. A counterexample was demonstrated by Zeldovich \[67\], and even in case of ADS/CFT correspondence there are a few counterexamples \[69\,70\]. Important restrictions could be found from the physics of neutron stars. For example, the possible maximal mass exceeding two solar masses might require the EoS with $C_s^2 > 1/3$ \[71\,72\].

Below in this paper we find domains, where the speed of sound exceeds $C_s^2 \geq 1/3$, for the used values of $L(T, \mu)$.

The paper is organized as follows. In section 2 we introduce the FCM in case of finite temperatures and densities. In section 3 we discuss the definition of the speed of sound at finite density, and make some theoretical predictions, which we confirm in section 4 using numerical results. In section 5 we discuss the behaviour of the Polyakov line at finite baryon densities, and will show that there exists a density renormalization of the Polyakov line that leads to a good behaviour of the system, and show that there exists a density renormalization of the Polyakov line which keeps the conformal limit unbroken.

The section 6 - final discussions and conclusions.

2 The Field Correlator Method

The FCM is a useful instrument to treat the physics outside the area of perturbative theory. Analysis of physics of QGP in terms of FCM made in \[41\,44\,73\,74\], has shown the important role of Polyakov loops for description of thermodynamic of QGP, while in \[45\,49\] also the CMC interaction was taken into account, providing a selfconsistent dynamical picture in a good agreement with lattice data. In the FCM the basic interaction of a quark or a gluon can be expressed via world lines affected by the vacuum fields and finally written in the form of Wilson loops and Polyakov lines. It is essential that in the deconfined phase two basic interactions define quark and gluon dynamics: the colorelectric (CE) interaction, contained in the Polyakov line $L(T)$, and the colormagnetic (CM) one in the spatial potentials via the Wilson loop integral, e.g. for the gluon pressure one has \[41,44,46\]:

$$P_{gl} = 2(N_c^2 - 1) \int_0^{\infty} \frac{ds}{s} \sum_{n=1,2} G^n(s)$$

s-proper time, and for $G^n(s)$ one can obtain:

$$G^n(s) = (Dz)_{(m)}^{(n)\alpha}\exp(-K)\hat{r}_\alpha < W^a_2(C_n) >$$

where $K = \frac{1}{T} \int_0^s d\tau (\frac{ds}{d\tau})^2$, and $W^a_2(C_n)$ is the adjoint Wilson loop defined for the gluon path $C_n$, which has both temporal $(i4)$ and spacial projections $(ij)$, and $\hat{r}_a$ is the normalized adjoint trace. When $T > T_c$ the correlation function between CE and CM fields is rather week \[41\]:

$$< E_i(x) B_k(y) \Phi(x, y) \approx 0$$
and therefore, the expression for the Wilson loops is factorized \[46\] :
\[< W^a_{\Sigma}(C_n) > = L^a_{\text{adj}}(T) < W_3 > \] (7)

with \( L^a_{\text{adj}} \approx L^a_{\text{adj}} \) for \( T \leq 1 \) GeV. One can integrate out the \( z_4 \) part of the path integral \((Dz)^{\omega}_{\text{on}} = (Dz_4)^{\omega}_{\text{on}}D^3z\), with the result
\[ G^{(n)}(s) = G_4^{(n)}(s)G_3(s), G_3^2(s) = \int (Dz_4)^{\omega}_{\text{on}}e^{-K}L^a_{\text{adj}} = \frac{1}{2\sqrt{4\pi s}}e^{-\frac{n^2}{4\pi s}}L^a_{\text{adj}} \] (8)

This factorization holds also for quarks and will be used below (changing the adjoint representation for the fundamental one).

The resulting gluon contribution is
\[ P_{\text{gl}} = \frac{N_c^2 - 1}{\sqrt{4\pi}} \int_0^\infty \frac{ds}{s^{3/2}}G_3(s) \sum_{n=0,1,2,...} e^{-\frac{n^2}{4\pi s}}L^a_{\text{adj}}, G_3(s) = \int (D^3z)xe^{-K}\sigma < \hat{\tau}_aW^a > \] (9)

To account for CMC one can introduce an approximate expression for 3d Green function \[46\] :
\[ G_3(s) = \frac{1}{(4\pi s)^{3/2}} \sqrt{\frac{(M^2_{\text{adj}})s}{\sinh(M^2_{\text{adj}})s}}, M_{\text{adj}} \approx 2M_D \] (10)

where \( M_D \) is the gluon Debye mass. It should be mentioned that the eq.(9) is in a good agreement with the lattice data \[58\].

For quarks one can write the expression of the same form as in (9), but with the quark mass term \( e^{-m^2_s} \), and the density term \( \cosh(\frac{\mu n}{T}) \)
\[ P_f = \sum_{q=u,d,s} P_qP_q = \frac{4N_c}{\sqrt{4\pi}} \int_0^\infty \frac{ds}{s^{3/2}}e^{-m^2_s}S_3(s) \sum_{n=1,2,...} (-)^{n+1}e^{-\frac{n^2}{4\pi s}}L^a_{\text{adj}}\cosh(\frac{\mu n}{T}) \] (11)
\[ S_3(s) = \frac{1}{(4\pi s)^{3/2}} \sqrt{\frac{(M^2_{\text{adj}})s}{\sinh(M^2_{\text{adj}})s}}, M^2_{\text{adj}} = \frac{9}{4}M^2_f, L_n^f = (L^a_{\text{adj}})^{4/9} \] (12)

The full pressure reads as:
\[ P_{\text{tot}} = P_f + P_{\text{gl}} \] (13)

Integrating over \( ds \) in (11) one obtains:
\[ P_f = \sum_{q=u,d,s} P_q \frac{P_q(T,\mu)}{T^4} = \frac{2N_c}{\pi^2} \sum_{n} (-)^{n+1}L^nK_2\left(\frac{\mu n}{T}\right)\frac{\tilde{M}^2}{T^2}, \] (14)

where \( \tilde{M} = \sqrt{m^2 + \frac{M^2(T)}{4}}, \quad M(T) = b\sqrt{\sigma_s(T)}, b \) is the coefficient, defined in \[46,48\].

The sum (14) can be brought to the form
\[ \frac{P_q(T,\mu)}{T^4} = f_+(T,\mu) + f_-(T,\mu), \] (15)
\[ f_\pm(T,\mu) = \frac{N_c}{3\pi^2} \int_0^\infty \frac{dz\left(z^2 + 2z\tilde{M}\right)^{3/2}}{\exp\left(z + \frac{\tilde{M}}{T} + \frac{\tilde{V}_1}{2T} + \frac{\mu}{T}\right)}, \] (16)

where we have taken into account that \( L = \exp\left(-\frac{\tilde{V}_1(\infty,T)}{2T}\right) \).
Analytical study of Eqs (14), (16) needs some efforts and was done in [48]. Two limits are simply done, one is the Stefan-Boltzmann limit at high T, and our result is some 15-20% below this limit mainly due to CMC effects, and another is the free quark limit with M tends to \( m_q \) and \( V_1 = 0 (L = 1) \) at extremely low temperatures, at this conditions the Fermi sphere is forming.

The expression (15) has no singularities at real \( \mu \), but \( f^\pm \) may get a singularity for imaginary chemical potentials for \( \text{Im}(\mu) = \pi T \) due to vanishing of the denominator in (16) at \( z = 0 \).

So one can conclude that in the normal situation with real \( \mu \) and \( L \) \( f \) the singularity is absent, this conclusion implies that there is no critical point \( T_c (\mu) \) and the analytic structure is affected only by the complex singularities. From this point of view, it seems that our consideration could be extended without any changes to a large enough values of the chemical potential as well as in the case of finite temperatures for \( T \leq 1 \) GeV (or even higher with some modifications of the theory), but as we will show in case of finite and large \( (\mu_q \approx 600 \text{ MeV}) \) chemical potential such a trivial extension becomes inconsistent.

3 Speed of sound in a dense matter

The sound velocity is an important quantity for the description of a medium. For example it plays the crucial role in the cooling process in heavy ion collisions ([16–18]), or in physics of neutron stars (see e.g. the part 5.15 of the book [61]).

Bounds and restrictions on the speed of sound is a compelling topic [7, 62, 67, 71, 75–84]. The bound \( 0 < C_s^2 \leq 1 \) follows from thermodynamic stability and causality [84, 85]. The scale \( C_s^2 = \frac{1}{3} \) is a general property of conformal theories with the vanishing trace of the energy-momentum tensor \( \epsilon - 3P = 0 \).

In the limit of very high temperature or very large density QCD becomes nearly scale invariant and one expects that \( C_s^2 \sim \frac{1}{3} \). To describe the scale invariance breaking one can introduce two parallel quantities: a) the conformality measure \( \Delta^2_s = \frac{1}{3} - C_s^2 \) [79], and b) the interaction measure, or trace anomaly [24, 86].

In case of finite temperatures at \( \mu = 0 \) the speed of sound is defined as:

\[
C_s^2 = s = \frac{s}{\partial P/\partial T}, \quad \mu = 0.
\] (17)

At large \( T \) it tends asymptotically to \( 1/3 \) [20, 21, 24]. There are several possibilities to define the s.v at nonzero \( \mu \) [87], in this paper we will focus on the isentropic definition i.e \( s/n = \text{const} \):

\[
C_s^2 = \frac{n^2 \partial^2 P/\partial T^2 - 2sn \partial^2 P/\partial T \partial \mu + s^2 \partial^2 P/\partial \mu^2}{(\varepsilon + P)} \left( \frac{\partial^2 P/\partial T \partial \mu}{\partial^2 P/\partial \mu^2} \right)^2 = \frac{1}{\kappa_s (\varepsilon + P)},
\] (18)

where we have defined:

\[
s = \frac{\partial P}{\partial T}, \quad n = \frac{\partial P}{\partial \mu}, \quad \varepsilon + P = Ts + \mu n.
\] (19)

and the adiabatic compressibility

\[
\kappa_s = \frac{\partial^2 P/\partial T^2 - \left( \frac{\partial P}{\partial \mu} \right)^2}{n^2 \partial^2 P/\partial T^2 - 2sn \partial^2 P/\partial T \partial \mu + s^2 \partial^2 P/\partial \mu^2}
\] (20)

One can easily find that in the limit of zero \( \mu \) one has \( \partial P/\partial \mu = 0 \), \( \partial^2 P/\partial T \partial \mu = 0 \), \( \partial^2 P/\partial \mu^2 \neq 0 \), with the result:

\[
C_s^2 (\mu \to 0) = \frac{s^2}{(\varepsilon + P) \partial^2 P/\partial T^2} = \frac{\partial P/\partial T}{\partial^2 P/\partial T^2},
\] (21)

which coincides with [17].
In the opposite limit, when $\mu$ is larger than any scaleful quantity, e.g. quark masses, screening masses of quark and gluons, one can write

$$P = T^4 f(\mu/T).$$

Inserting (22) into (18), one has $\varepsilon + P = 4fT^4$ and finally one obtains the limit

$$C_s^2 = \frac{4f(4ff'' - 3f'^2)}{12f(4ff'' - 3f'^2)} = \frac{1}{3}.$$ (23)

As a next step we calculate the s.v for the pressure given in (14). One should note that the ratio $\frac{\tilde{M}}{T}$ at small quark mass $m_f$ has only logarithmic dependence in $T$, $\frac{\tilde{M}}{T} = bc^2g^2(T)$, which can be neglected at not large $T; T \sim T_c \div 4T_c$.

Considering now large $\mu/T \gg 1$, one can neglect $f_-(T, \mu)$ and writing $f_+(T, \mu) = f \left( \frac{\mu - V_1}{T} \right)$, one obtains the following form\(^2\)

$$C_s^2 = \frac{4f^2f'' - 3f'^2 - \frac{1}{4}f'^3\Delta}{3f(4ff'' - 3f'^2 - \frac{1}{4}f'^3\Delta)}, \quad \Delta = \frac{T}{2} \frac{\partial^2 V_1}{\partial T^2}. $$

(24)

The form (24) contains both of the conformal limit when $\Delta \to 0$ and $C_s^2 \to 1/3$ and the danger of strong derivations and possible singularities when $\Delta$ is large and positive (note that $4ff'' - 3f'^2$ is positive).

To clarify the matter we shall consider the case of an arbitrary $\mu$, keeping the condition $\frac{\tilde{M}}{T} = \text{const}$. Defining $x_\pm = \pm \frac{\mu}{T} + b(T)$; $b(T) = -\frac{\tilde{M}}{T} - V_1(T)/2T$, one can write $f(x) = \int_0^\infty \frac{dz}{1 + \exp(z - x)}$.

$$f_+(T, \mu) = f_+(x_+(T)), \quad f_-(T, \mu) = f_-(x_-(T)).$$

$$f \equiv f_+(x_+) + f_-(x_-), \quad f'_+ = \frac{\partial f_+}{\partial x_+}, \text{etc.}, \quad x' = \frac{dx}{dT}.$$ (25)

The resulting expression for s.v. acquires the form

$$C_s^2 = \frac{1}{3} \left[ \frac{2f'' - \frac{3}{4}f(f'_+ - f'_-)^2 + \frac{\tilde{M}}{16}(f''_+ - f''_-)^2 + B}{1 + \frac{TV_1}{4T}} \right]. $$

(26)

where we have defined

$$A = 2T(f'_+ x'_+ + f'_- x'_-), \quad T^2(f''_+ x''_+ + f''_x x'_-),$$

$$B = (f''_+ f'^2_+ + f''_- f'^2_-) \frac{b^2}{4} + \frac{1}{2} f(f''_+ f'_+ + f''_- f'_-)(x'_+ + x'_-),$$

(27)

$$C = 4b^2 f''_+ f''_- + (2b' + b''T)(f''_+ f'_+ + f''_- f'_-) + 12b'(f''_+ f'_+ + f''_- f'_-).$$

(28)

One can again consider large $\mu$, when $f_- \ll f_+$ and then $C \simeq (2b' + b''T)f''_+ f'_+ = -\frac{V_1''}{T^2} f''_+ f'_+$, and $\frac{4C}{T^2} < 0$ in the denominator of (25). One can estimate the numerical value of $\Delta = \frac{T}{2} \frac{\partial^2 V_1}{\partial T^2}$ for the Polyakov lines in the lattice data, and e.g. for the data of Bazavov et al. [54] one finds that $\Delta(T)$ in the interval $T = (170 \div 400)$ MeV is changing approximately from 5 to 0.4 which shows a danger of $C - \Delta(T)$ to cancel in the denominator, producing high and possibly negative values of $C_s^2$, which may indicate instability.

Another interesting preliminary conclusion is that the sound velocity could exceed $1/\sqrt{3}$, both properties should be checked numerically, as it is done in the next section.

\(^2\)At this moment we are neglecting the dependence of $V_1$ on the chemical potential
4 Numerical results

We will investigate the behaviour of the speed of sound at temperatures from 160 Mev till 235 MeV. According to the predictions of the previous section one can expect some problems with the speed of sound in this interval. First of all we compare our results for the pressure at small \( \mu \) with the lattice predictions \[29\] to check the initial data before turning to larger \( \mu \). As one can see in figures (Figs.1,2,3,4,5), our results are close to the lattice data.\[4\] We also calculate the speed of sound in the range \( \mu_B = [0, 400] \) MeV , presented in Fig.6.

One should notice that the results for the pressure in \[29\] were obtained in the first order of the square of the chemical potential, however corrections from higher orders are not so important for low densities.

![Figure 1: The pressure in QGP from FCM for \( \mu_B = 0 \) MeV, in comparison with the lattice data of Borsanyi et al. \[29]\](image)

We plot in Fig.6 the changing in the speed of sound in the range \( \mu_B = [0, 400] \) MeV, where the width of the line is equal to the difference \( C_s^2(\mu_B = 400) - C_s^2(\mu_B = 0) \). One can see that at this densities there is no signal about any instabilities in the QGP.

At high densities, the speed of sound is found from the full expression of the pressure (15), (16) with inclusion of gluon contribution.

As can be seen in Fig. 7 there exists a domain where the square of the speed of sound exceeds \( \frac{1}{3} \). We treat this result with caution because, as was mentioned above, we use the fit for the Polyakov loop obtained for \( \mu = 0 \). We also find domains in Fig. 8 where the speed of sound becomes negative. Obviously, because of the continuity of the denominator of (18) as a function of \( \mu, T \) the s.v. should exceed the speed of light before it becomes negative.

These results demonstrate a big difference between the finite temperature physics and the physics of finite baryon densities. In the first case there is no problems with the system behaviour and the speed of sound has the upper limit (\( C_s^2 \leq \frac{1}{3} \)) that it never exceeds. At finite temperatures (0.16 GeV < T < 1

The speed of sound in Fig.5 a little bit different from our previous work \[89\], where another form of \( L(T) \) was used. That was done to be sure that our results are not connected to the specific form of \( V_1(\infty, T) \) in \[89\] in the confinement region. We have checked that the instability also persists for \( L(T) \) from \[89\].

We are using below some approximation \( L_{ap}(T, \mu = 0), L(\mu, T) \approx \sqrt{L_{Baz}} \) taken from \[53\] for the Polyakov line \( L(T, \mu = 0) \), which lies between the corresponding lattice curves in \[53\] and \[88\]. The preliminary FCM data for \( L(T, \mu = 0) \) generally agrees with \( L_{ap}(T, \mu = 0) \) and is now in preparation.

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Footnotes:

1. The speed of sound in Fig.5 a little bit different from our previous work \[89\], where another form of \( L(T) \) was used. That was done to be sure that our results are not connected to the specific form of \( V_1(\infty, T) \) in \[89\] in the confinement region.

2. We are using below some approximation \( L_{ap}(T, \mu = 0), L(\mu, T) \approx \sqrt{L_{Baz}} \) taken from \[53\] for the Polyakov line \( L(T, \mu = 0) \), which lies between the corresponding lattice curves in \[53\] and \[88\]. The preliminary FCM data for \( L(T, \mu = 0) \) generally agrees with \( L_{ap}(T, \mu = 0) \) and is now in preparation.
Figure 2: The pressure in QGP from FCM for $\mu_B = 200$ MeV, in comparison with the lattice data of Borsanyi et al. [29]

Figure 3: The pressure in QGP from FCM for $\mu_B = 300$ MeV, in comparison with the lattice data of Borsanyi et al. [29]

GeV) and zero $\mu$ the FCM describes physics of QGP (see for example [89]). In the second case the system is safe in the domain of low chemical potentials and in the domain of very high chemical potentials but in the intermediate range its behaviour displays singular features, signalling the instability of the thermodynamics in the QGP phase, which might mean either the emergence of a new phase with a new
Figure 4: The pressure in QGP from FCM for $\mu_B = 400$ MeV, in comparison the lattice data of Borsanyi et al. [29]

Figure 5: The square of the speed of sound in QGP from FCM[18] for $\mu_B = 0$ MeV, in comparison with the data of Borsanyi [29] et al. and Bazavov et al. [24]

dynamics or a necessity to correct the assumed dynamics, e.g. by taking the mu-modified Polyakov lines.

In the next section we will discuss possible corrections to the Polyakov loop due to the presence of the finite chemical potential, and we will see that these corrections may change the picture of the instability.
Figure 6: The speed of sound $C_s^2$ from FCM [18] for $\mu_B = 0.400$MeV, the width of the line that is shown is equal to the change in the speed of sound in this range.

Figure 7: The square of the speed of sound in QGP from FCM. The semitransparent gray plane is at the value 1/3.

5 Polyakov lines at nonzero $\mu$

We investigate below how the nonzero baryon density can influence the magnitude of Polyakov lines in the qgp.

To this end we write the pressure for the quarks of the flavor $f$ in the np vacuum of qgp, where are present vacuum averages of the colorelectric (CE) interaction $V_1(r, T)$ and colormagnetic (CM) interaction, producing the screening mass $M(T)$, (see 48 for details).
Figure 8: The square of the speed of sound in QGP from FCM at $T = 1.25 \, T_c$.

Figure 9: The square of the speed of sound in QGP from FCM with modified Polyakov Line as a function of $T/T_c$ (left axis) and $\mu$ (right axis) in GeV. The parameter $a = 0.6$.

\[ \frac{1}{T^4} P_q^{(f)} = \frac{N_c}{4\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4} L^n(T) \cosh \frac{\mu n}{T} \phi_n(T) \]  
\[ \phi_n(T) = \frac{8m^2 M^2}{T^2} K_2 \left( \frac{M n}{T} \right), \quad M = \sqrt{m^2 + \frac{m^2(T)}{4}}, \]  

(29) and (30)
Figure 10: The square of the speed of sound in QGP from FCM with modified Polyakov Line as a function of $\mu$ in GeV. T=200 MeV. The parameter $a=0.6$.

Figure 11: The square of the speed of sound in QGP from FCM with modified Polyakov Line as a function of $\mu$ in GeV. T=200 MeV. The parameter $a=15$.

and $K_2$ is a modified Bessel function.

Here $L(T)$ is expressed via $V_1(r,T)$ as for the isolated quark line, i.e. distant from other quark or antiquark lines, $V_1(r,T) \to V_1(\infty, T)$,

$$L(T) = \exp \left( - \frac{V_1(\infty, T)}{2T} \right).$$

One should have in mind, that the actual origin of the Polyakov line in the pressure is the vacuum average of the phase factor $\Lambda(C) = \exp(ig \int_c A_\mu dz_\mu)$ along the path of the quark. Indeed, using the
The Fock-Schwinger representation for the free energy of an isolated quark one has \([41,46]\), neglecting CM interaction

\[
\frac{1}{T}F_0^q = -\frac{1}{2}Sp\int_0^\infty \xi(s)\frac{ds}{s}e^{-sm_0^2+\frac{1}{2}(D^2_\mu-gF_{\mu\nu}\sigma_{\mu\nu})} \tag{32}
\]

and neglecting the spin-dependent term \(\sigma_{\mu\nu}F_{\mu\nu}\) one has

\[
\frac{1}{T}F_0^q = -\frac{1}{2}\int_0^\infty \xi(s)\frac{ds}{s}d^4x \sum_n (Dz)^w_{xx}e^{-K-sm_0^2}\Lambda(C_n)), \tag{33}
\]

where the kinematic factor \(K\) and the winding path measure \((Dz)^w_{xx}\) are defined in \([41,46]\).

As a result one obtains the vacuum averaged factor with a still undefined contour \(C\)

\[
\bar{\Lambda}(T) = \frac{1}{N_c}\langle tr\Lambda(C)\rangle. \tag{34}
\]

At this point one can introduce two important simplifications, which lead to the final result \([31]\).

a) \(\Lambda(C_n)\) is a phase factor along the quark trajectory \(C_n\) from the point \(x_4 = 0\) to the gauge equivalent point \(x_4 = n/T\). Introducing the colormagnetic confinement (CMC) acting inside the closed loop, one can close the trajectory \(C_n\) by a straight line from \(x_4 = n/T\) to \(x_4 = 0\), and writing the same straight line in the opposite direction. In this way after vacuum averaging one obtains

\[
\bar{\Lambda}(T) = L_n(T)\exp(-\sigma_s \cdot \Sigma_2), \tag{35}
\]

where \(\Sigma_2\) is the area in the 3d projection of the closed loop \(C_n\), which produces finally the screening mass \(M(T)\) in \([35]\), and \(L_n(T)\) is the vacuum average of the integral along \(x_4\) axis

\[
L_n(T) = \langle P \exp(i\int_0^{n/T} A_1 dz_4) \rangle \simeq \exp\left(-\frac{n}{2T}\frac{\partial V_1(\infty,T)}{\partial T}\right). \tag{36}
\]

To calculate \(L(T)\) in the dense QGP one can use the property of the Polyakov lines studied in \([73,74]\), namely that the static quark free energy \(F_Q(T) = \frac{V_1(\infty,T)}{2}\) is connected to the static quark-antiquark free energy \(F_{QQ}(r \to \infty, T) = 2F_Q(T)\), or in our notations the identity \(V_1(r \to \infty, T) = 2\frac{V_1(\infty,T)}{2}\).

In this way one can define the correlator of Polyakov lines

\[
\langle \tilde{r}L_x \tilde{r}L_y \rangle = P(x - y), \quad \tilde{r} = \frac{1}{N_c}tr, \tag{37}
\]

which can be written as \([73]\)

\[
P(x - y) = \frac{1}{N_c^2} \exp\left(-\frac{V_1(r,T)}{T}\right) + \frac{N_c^2 - \frac{1}{N_c^2}}{N_c^2} \exp\left(-\frac{9}{8}V_1(\infty) - \frac{1}{8}V_1(r)/T\right), \tag{38}
\]

where the proper renormalization is to be done, as in \([73]\).

Usually one exploits, as was mentioned above, \(F_{QQ}(T) = V_1(\infty,T)\) from \(P(\infty)\) to define Polyakov line, but it is clear, that for high density the smaller distances \(|x - y| \equiv R\) between \(Q\) and \(\bar{Q}\) should effectively enter the corresponding interaction \(V_1(R,T)\), so that \(L(T) \to L(R,T)\).

One can associate \(R\) at large \(\mu\) with the quark density \(n(\mu) = \frac{\partial P}{\partial \mu}\) as follows \(R = R_0 \left(\frac{n(\mu)}{n(0)}\right)^{-1/3}\), where \(R_0\) is an average \(q\bar{q}\) distance at \(\mu = 0\). One can realize that \(n(\mu) > n(0)\) and hence \(R(\mu) < R(0)\). Since \(V_1(R,T)\) satisfies the relation \(\frac{\partial V_1(R,T)}{\partial R} > 0\) (see \([73]\) for details of behaviour of \(V_1(R,T)\)), one obtains the relation

\[
V_1(R(\mu),T) < V_1(R(0),T), L(T,\mu) > L(T,0). \tag{39}
\]

This behaviour is supported by lattice data for \(L(T,\mu)\) in the case of SU(2) \([81,90]\). To check this relation and to understand the role of this effect on the sound velocity we have calculated \(C_s^2(\mu,T)\),
assuming \( V_1(\infty, T, \mu) = \frac{V_1(\infty, T)}{\sqrt{1 + \alpha \mu^2}} \), with \( a > 0 \). We investigated properties of the speed of sound for different values of parameter \( a \), and obtained following results:

The instability exists for \( a \lesssim 0.5 \)

At larger values the speed of sound never exceeds the speed of light but it could break the conformal limit as it is shown in Fig.10 and tends to \( \sqrt{1/3} \) from above.

The resulting curve is shown in Fig.9 and one can see, that \( C_s^2 \) approaches the value \( C_s^2 \approx 0.55 \) at \( T/T_C = 1.3 \) and \( \mu_q = 0.9 \) GeV. It is rather curious that there are values for the parameter, for which the square of the speed of sound does not exceed \( 1/3 \) Fig.11 and tends to \( 1/3 \) from below, in accordance with [62–66]. Thus one can say that FCM could describe both cases with and without exceeding of the conformal limit. And which one of them is realized depend upon details of the microscopic theory, which should predict the explicit behavior of the Polyakov line \( L(T, \mu) \) as a function of \( \mu \).

6 Conclusions and discussions.

The present paper is devoted to the effects of baryon chemical potential \( \mu \) in the dynamics of QGP.

It is an extension of the study of QCD thermodynamics at zero \( \mu \) in terms of the sound velocity made in [89], and is in the line of the series of papers [41–49] where the QCD thermodynamics is worked out on the basis of FCM. In particular, it was shown in [48], that nonzero \( \mu \) do not present any difficulty for the method up to \( \mu_q = 0.4 \) GeV.

In the present paper this topic was addressed from the point of view of sound velocity and in the temperature range which corresponds to QGP (at least for small \( \mu \)). This analysis with the help of the sound velocity allows to find instability regions independently of the visible \( P(T) \) behavior, since s.v. is sensitive to the second derivatives of \( P(T) \).

As it is seen in Figs. [14, 23] which scan the region \( \mu_B = [0, 400] \) MeV, there is a good agreement between the FCM results for the pressure \( P(T) \) and the lattice data [28]. In this region of \( \mu_B \) the sound velocity is rather insensitive to \( \mu_B \), as it is demonstrated in Fig.4, and \( C_s^2 \) is fully in the conformal domain.

However \( \mu_B = 400 \) MeV corresponds to \( \mu_q \approx 130 \) MeV and the main analysis of the present paper refers to the region \( \mu_q > 500 \) MeV. In this region without inclusion of the chemical potential in the Polyakov line the instability domains are discovered in Figs.8.

One can see in Fig.7 a spectacular area in the (\( \mu_q, T \)) plane \( \mu_q \geq 0.5 \) GeV and \( T/T_c = 1.1 \div 1.4 \), where \( C_s^2 \) exceeds the \( 1/3 \) limit and approaches the value 0.8. Even more dangerous region is shown in Fig.8 for \( \mu_q \geq 0.6 \) GeV, where \( C_s^2 \) may exceed unity and even become negative, which means the instability region, not subject to the chosen dynamics. This results are obtained with the Polyakov loop \( L(T) \) independent of \( \mu \). So one must expand a little on the importance of the \( L(T, \mu) \) dynamics. In FCM the Polyakov loop enters self-consistently from the very beginning, since it belongs to the quark and gluon propagators in the formalism.

The importance of \( L(T) \) was numerically proved in the first stage of the FCM formalism in [41,44], where it was the main dynamical effect (without CMC).

In the present paper we have chosen in Figs.1-7 the Polyakov line values \( L(T, \mu = 0) = L_{ap}(T) \), which lie between the lattice data for \( L(T, 0) \) obtained in [54] and [88], and is very close to the preliminary values of \( L(T, 0) \) obtained within the FCM formalism (to be published).

As was discussed in section 5, the introduction of \( \mu \) in \( L(T) \) leads to the decrease of \( V_1(T) \) and the increasing of \( L(T, \mu) \), and as it was discussed above in section 3, this leads to smaller values of \( \Delta \) in Eq. (24), and \( C \) in Eq. (25), i.e. the terms which effectively lead to instabilities of \( C_s^2 \). To prove this dependence and to investigate characteristics of the instability (i.e. the effect of the singular behaviour of the sound velocity) we included the chemical potential in the Polyakov line in the following way \( V = \frac{V_1(T)}{\sqrt{1 + \alpha \mu^2}} \), and observed that the instability survives until parameter \( a \) exceeds critical value (in our case \( a \approx 0.55 \)). With increasing of \( a \)-parameter a dangerous region is shifted to larger \( \mu \), and eventually the instability vanishes.

When this happens the speed of sound becomes well behaved quantity but it could still exceed the speed of light, but for \( a > 0.58 \) the last problem is absent.
There is a range for \(a\)-parameter where s.v breaks the conformal limit. Thats clearly seen on FiG.9 and FiG.8. In FiG.8 the speed of sound as a function of chemical potential was shown. As one can see, in the limit of large potentials the square of s.v tends to 1/3 from above, as predicted in [23]. With further increasing of the \(a\)-parameter, breaking of conformal limit becomes less significant and eventually it disappears FiG.11.

Summarizing one can say that the dynamics of QGP is well described by the FCM formalism, which agrees with lattice data for \(\mu = 0\) and predicts a reasonable extension to nonzero \(\mu\), being however parameter dependent.

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