Prediction on China's Energy Consumption Demand Trend Based on Bayesian Theorem

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Abstract. This paper applies SAS software for data mining and takes the energy consumption data from 2000 to 2015 as the training set and the corresponding data from 2016 to 2017 as comparative set. Due to the exponentially fluctuation of data, the author sets a nonlinear modified exponential rudimentary model, which, when combined with white noise experiment and diagrammatic figure analysis, the model's fitting effect can be inferred. The result shows that the data extraction of the model is not sufficient enough so that the model should be improved. Seen in that light, the author tries to do dimensionality reduction and differential processing to the original sequence and applies the classical time series ARIMA model for adjustment. It comes out that the fitted model should be ARIMA(4,2,1). The solved result illustrates that the information of the model has been well and sufficiently extracted. After that, the Bayesian model is constructed by taking the estimated parameters of the time series ARIMA model as priori information. The prediction effects of the three models are finally evaluated by Mean Absolute Percentage Error (MAPE) and Root-mean-square Error (RMSE). The result shows that the prediction effect of Bayesian model is somehow superior to that of single time series model. According to the prediction result of Bayesian model based on prior robust, the general demand for energy consumption in China during the 13th Five-Year Plan period tends to climb by a small margin.

1. Introduction

China, as the biggest energy consumption country in the world, encounters a continuously climbing demand of energy consumption with the boost of industrialization and urbanization. With such an increasing demand, China has reached to the main energy producer and consumer in the globe, leading to the increasingly prominent conflict between the development of both economy low-carbon and sustainable economic development pattern and reducing the dependence on energy consumption is inevitable for the future development of China. It is of great meaning for China to take the trend of future energy consumption demand for the formulation of national energy strategy and the healthy development of economy and the society.

This paper firstly uses a modified exponential model to estimate China's energy consumption. On this basis, the model is then modified and the classical time series ARIMA model is obtained, from which the parameters are solved and at the mean time the prediction is made. After setting the parameter estimation of the ARIMA model to priori information, the author gets the theoretical posteriori distribution, based on which the author figures out the Bayesian estimation of the parameters. Then the
estimation and the original data are compared. By using MAPE and RMSE two indicators, the solidity of the prediction result concluded by modified ARIMA model can be hence evaluated.

2. Literature Review and Sample Data Analysis

2.1. Sample Data Analysis
The author selects the annual data of China’s energy consumption spanning from 2000 to 2017 released by Statistical Yearbook of China. Data is shown in Table 1.

| t (Year) | X (Total Energy Consumption) | Year | X (Total Energy Consumption) |
|---------|-------------------------------|------|-------------------------------|
| 2000    | 146964                        | 2009 | 336126                        |
| 2001    | 155547                        | 2010 | 360648                        |
| 2002    | 169577                        | 2011 | 387043                        |
| 2003    | 197083                        | 2012 | 402138                        |
| 2004    | 230281                        | 2013 | 416913                        |
| 2005    | 261369                        | 2014 | 425806                        |
| 2006    | 286467                        | 2015 | 429905                        |
| 2007    | 311442                        | 2016 | 436020                        |
| 2008    | 320611                        | 2017 | 449000                        |

Note: t is a time variable that represents year; x represents the total energy consumption

2.2. Literature Review
In recent years, speaking of the analysis of stochastic volatility models, the Markov Chain method based on Bayesian estimation has been proved to be more favorable. Besides, an increasing number of researches on governmental macroeconomic forecasting and commercial economic forecasting adopt Bayesian estimation. Particularly, West delves into the Bayesian Theorem in dynamic econometric models, while Griffiths and Bewley propose a Bayesian-based logarithmic diffusion model. Tanemrua and Kasuya put forward a small-scale Bayesian-based Japanese economic forecasting model by applying MonteCarlo method and posterior information rule. Many Chinese scholars, on the other hand, start to carry out researches on statistical inference theory and method. They also try to apply the research result into real practice and have obtained certain successes. For instance, Han, Zheng, et al. put forward the posterior distribution of the AR(p) time sequence model parameters based on normal-Gamma conjugate prior distribution and the prediction distribution of the model. Zheng & Zhu use simulation software and the MCMC method to simulate the previous Bayesian prediction model. From the ARFIMA model analysis based on MCMC method proposed by Liu & Xiong, the advantages of Bayesian method in parameter estimation are well proved. By applying the research result, the prediction result of the simulation to the GDP data of China for nearly 30 years is quite satisfactory. In Performance Degradation Reliability Modeling and Evaluation Method Based on Bayesian Update and Copula Theory, Hao uses normal mean robust prior Bayesian model and concludes the Bayesian estimation to time sequence parameters by using the sequential method. The stated literatures relating to Bayesian time series model mainly focus on the estimation of model parameters. The Bayesian estimation on model parameters used not only cover the common conjugate priors but also the normal mean robust prior proposed by American statistician James O. Berger and represented by Hao huibin.

3. Theoretical Model

3.1. Modified Exponential Modeling
From the energy consumption scatter diagram, we can see that China's energy consumption presents an exponential upward trend and increases faster and faster with the moving of time t. This paper sets a nonlinear modified exponential rudimentary model: \[ X_t = at + b^t + \varepsilon_t \], in which \( \{\varepsilon_t\} \) is the
stochastic disturbance, $X_t$ represents China’s energy consumption and $t$ represents time. The author uses SAS software to solve the model and gets $\rho_{\text{ab}} = -0.9541757$, showing a high negative correlation between $a$ and $b$. As show in Figure 1, the fitting model is $X_t = 27590.4t + 1.1069^t + \varepsilon_t$, in which $(\varepsilon_t)$ is the stochastic disturbance.

| Source          | DF | Sum of Squares | Mean Square | F Value | Approx Pr > F |
|-----------------|----|----------------|-------------|---------|---------------|
| Model           | 2  | 1.907E12       | 9.635E11    | 159.36  | <.0001        |
| Error           | 18 | 9.574E10       | 5.968E9     |         |               |
| Uncorrected Total | 18 | 2.003E12       |             |         |               |

| Parameter | Estimate | Approx Std Error | Approximate 95% Confidence Limits |
|-----------|----------|------------------|-----------------------------------|
| $a$       | 27590.4  | 6528.8           | 15857.9 - 39522.9                  |
| $b$       | 1.1069   | 1370.4           | -2904.0 - 2905.2                   |

| Approximate Correlation Matrix |
|-------------------------------|
| $a$   | $b$ |
| $a$   | 1.000000 | -0.9541757   |
| $b$   | -0.9541757 | 1.000000    |

**Figure 1.** Parameter Test, Significance Test of Equation and Correlation Coefficient Matrix

**Figure 2.** China's Total Energy Consumption (1987-2014) Fitting Effect Chart

From Figure 2, we can recognize that the residual error of the modified exponential model have not passed the white noise test, indicating that the information in the model has not been extracted sufficiently. Also, since the trend of the original time series (presented by quadratic function) is non-stationary, the author tries to implement dimensionality reduction to the original sequence: $Y_t =$
in which \( u \) is the mean of the original sequence. By using SAS software, we get \( u = 317941.11 \) and the standard deviation is 103824.60.

### 3.2. Analysis to ARIMA Model

Before applying ARIMA (p, d, q) model to fit the conversed data, it is required to examine whether the first difference of the conversed data is steady or not. If not, we should continue to test the second and third difference until the sequence is steady. However, if we still cannot find a suitable difference, we should then improve the testing strategy, that is, change and use another model. Following this rule, it is discovered that the sequence is steady after trying the second difference. From SAS software, the optimal model should be ARIMA (4, 2, and 1). The prediction result based on ARIMA model is as follows.

**Table 2. 10-year Prediction Result with Adjusted ARIMA Model**

| Obs | t   | y      | x          |
|-----|-----|--------|------------|
| 1   | 19  | 382.831| 464500.8   |
| 2   | 20  | 404.264| 481370.6   |
| 3   | 21  | 425.858| 499295.7   |
| 4   | 22  | 447.492| 518190.3   |
| 5   | 23  | 469.137| 538031     |
| 6   | 24  | 490.786| 558811.5   |
| 7   | 25  | 512.434| 580529.9   |
| 8   | 26  | 534.083| 603186     |
| 9   | 27  | 555.732| 626779.3   |
| 10  | 28  | 577.381| 651310     |

### 3.3. Bayesian Model Based on Robust Prior

When estimating parameter \( \theta \) under Bayesian method, the author uses normal mean robust prior, which is firstly launched by James O. Berger. Considering the nature of data, the author sets 10 year to be a period and divide the data in the training set into 7 subsets before doing time sequence modeling analysis. Data of 2000 to 2010 is the first subset, and data of 2007 to 2017 is the last subset. Estimate that \( x^k = \left( \theta^k, \theta^k_{\beta}, k = 1, \ldots, 8 \right) \), referring to the parameter vector generated by the kth rolling of the model, and the theoretical priori variance matrix \( \tau^2 \) is equal to the first 4 covariance matrix of \( X^k \). The theoretical prior mean \( \mu \) should be the mean of the first 4 \( \tau^2 \). By now, we have got the premise for the construction of robust prior and normal conjugate prior. Next, estimate \( X^6 = X \) and apply the estimation into the above equation, and the posterior mean should be the two Bayesian estimation \( \hat{\theta}^5 \) of the 5th time sequence model parameter. Set it to be the prior \( \theta \) of parameter \( \theta \) in the next time sequence analysis and calculate, and we can get parameter \( \hat{\theta}^7 \) and the Bayesian estimation based on robust prior and conjugate prior.

### 3.4. Bayesian Model under Normal-Wishart Prior Estimation

From equation:
\[ \hat{\beta}_i^* = \int \beta_i \, p(\beta_i | y_i, x_i) \, d\beta_i = \int \frac{(2\pi)^{-\frac{n}{2}} \exp \left( -\frac{1}{2} \sum_{i=1}^{n} \frac{(y_i - \beta_i x_i)^2}{\sigma_i^2} \right)}{A} \, \left( \frac{1}{\sigma_i^2} \right) \, d\beta_i \]

\[ = \int \frac{1}{\sigma_i^2} \, \exp \left( -\frac{1}{2} \sum_{i=1}^{n} \frac{(y_i - \beta_i x_i)^2}{\sigma_i^2} \right) \, d\beta_i \]

\[ = \exp \left( \frac{1}{\sigma_i^2} \sum_{i=1}^{n} \frac{(y_i - \beta_i x_i)^2}{\sigma_i^2} \right) \]  

we know \( \hat{\beta}_i^* = \frac{\beta_i}{\sum_{i=1}^{n} \frac{(y_i - \beta_i x_i)^2}{\sigma_i^2}} \), the Bayesian estimation \( \hat{\beta}_i^* \) should be the expected number of \( \frac{\beta_i}{\sum_{i=1}^{n} \frac{(y_i - \beta_i x_i)^2}{\sigma_i^2}} \) divided by the expected number of \( \frac{1}{\sum_{i=1}^{n} \frac{(y_i - \beta_i x_i)^2}{\sigma_i^2}} \). Hence, we need random sampling method to do estimation and calculate the expected numbers. The method is as follows:

Firstly, select 1000 random numbers that obey \( \mathcal{N}(\beta_i, \Sigma_i) \), in which the estimation of \( \beta_i \), that is, \( \hat{\beta}_i \), is the parameter estimation vector calculated from the last modeling. Besides, the estimation of \( \Sigma_i \) is \( \hat{\Sigma}_i \), the covariance matrix of \( \beta_i^{\text{last}}, \beta_i^{\text{last}+1}, \ldots, \beta_i^{\text{last}-1} \).

Secondly, calculate 1000 groups of \( \frac{\beta_i}{\sum_{i=1}^{n} \frac{(y_i - \beta_i x_i)^2}{\sigma_i^2}} \) and \( \frac{1}{\sum_{i=1}^{n} \frac{(y_i - \beta_i x_i)^2}{\sigma_i^2}} \).

Thirdly, set the mean of 1000 groups of results into the prediction of two estimations, and apply the estimations into the equation and we can get \( \hat{\beta}_i^* \).

3.5. Model Comparison

After comparing the prediction result carried out by different prediction models with the same group of data by using MAPE and RMSE two indicators, the author selects an optimal prediction model. It is defined that RMSE = \( \left( \frac{1}{N} \sum_{i=1}^{N} (\hat{y}_i - y_i)^2 \right)^{\frac{1}{2}} \) and MAPE = \( \frac{1}{N} \sum_{i=1}^{N} \left| \frac{\hat{y}_i - y_i}{y_i} \right| \times 100\% \), in which \( N \) represents sample size, \( (y_1, y_2, \ldots, y_n) \) represents the historical observed value, and \( (\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_n) \) represents the prediction value. From the equations, smaller values of RMSE and MAPE stand for more favorable model fitting and better predictive effect.

**Table 3.** Comparison on the Predictive Effect among Different Models

| Criteria          | ARIMA (4, 2, 1) | Bayesian Model Based on Robust Prior | Bayesian Model Under Normal—Wishart Prior Estimation |
|-------------------|----------------|-------------------------------------|------------------------------------------------------|
| MAPE              | 12.10%         | 6.38%                               | 13.05%                                               |
| RMSE              | 4857.66        | 2284.35                             | 5051.67                                              |

From Table 3, it can be observed that the predictive effect of Bayesian model is superior to that of single time sequence mode to some extent.

4. Conclusion

This paper firstly constructs a modified exponential model to China’s energy consumption sequence and then establishes a classical time sequence ARIMA to amend the model. The author sets the parameter estimation of the model into prior information, applies robust prior distribution and normal—Wishart...
prior estimation to figure out the Bayesian estimation of the parameter. In this way, the author tries to predict China’s energy consumption. When using Bayesian model, the result got is more reliable and reasonable than that got from ARIMA model because of the sufficient extraction of model and sample information and the integration of the prior information of unknown parameters. It is concluded from the result that the predictive effect of Bayesian model that based on robust prior assumption can reduce prediction error when compared with traditional time sequence model. However, it should be noted that the prediction effect of the Bayesian Model under Normal-Wishart Prior estimation is not adequate enough, explaining that Bayesian Model is reasonable not somehow inadequate in parameter estimations.

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The Bayesian estimation parameter method used only considers the mean value. It is also necessary to consider the case where the covariance matrix of the initial sample is a unit covariance matrix, and obtain a Bayesian estimate to implement the prediction.

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