The Heat and Mass Transfer Processes at the Cooling of Strong Heated Sphere in a Cold Liquid

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Abstract
Some new experimental results of continuum mechanics problems in two-phase systems are described. The processes of heat and mass transfer during cooling of strong heated sphere in the subcooled liquid are studied. Due to high level of heater temperature the stable vapor film is formed on the sphere surface. Calculation of steady-state transport processes at vapor – water interface is carried out using methods of molecular-kinetic theory. Heat transfer in vapor by thermal conductivity and natural convection in liquid are considered. Pressure balance is provided by hydrostatic pressure and non-equilibrium boundary condition. The results of the calculations are analyzed by comparison with previous data and experimental results.

1. Introduction
New peculiarities of heat and mass transfer processes in two-phase system have been found in last decade. The development of measurement equipment and appearance of high-speed video cameras provide the opportunity to fix features on vapor-liquid interface at strong non-equilibrium conditions in fast transition processes.

At the fast processes of vapor-liquid transition small effect determines dynamics of interface and heat transfer effectiveness. Measurement of microlayer properties [1] shows that the heat flux at the evaporation is over 1 MW/m², which is comparable with the critical heat flux in the general pool boiling of water. It was found that the microlayer evaporation plays a central role in the wall heat transfer. On the other hand, the rewetting heat transfer in the bubble departure process is very small compared with the microlayer evaporation. The local wall temperature beneath a bubble was measured with the MEMS sensor at a wall superheat of 8–15 K for the pool saturated isolated bubble boiling of water. The measurement results clearly indicated dynamic local heat transfer phenomena such as the microlayer evaporation, the dry-out of the microlayer, and the rewetting of the dry-out area. The wall heat transfer was evaluated by the two-dimensional transient heat conduction analysis of the heating wall using the measured temperature, and the following heat transfer characteristics were demonstrated. Bubbles are grown by the evaporation of both the microlayer and the superheated liquid layer around it in the early to middle stages of the bubble growth process. The superheated liquid layer has a significant contribution to bubble growth in the latter stage. The contribution of microlayer evaporation to bubble growth is about 50%, and does not vary much with wall superheat. The remaining heat can be supplied from the superheated liquid layer. The microlayer thickness increases with rise of distance from the nucleation site, and is, for example, 3 μm at a distance of 600 μm from the nucleation site. The microlayer tends to thickening with wall superheat (bubble growth velocity).
New interesting phenomenon was found at the investigations of Leidenfrost effect on liquid drop in different conditions. As an example, large water drops with initial mass of 1 g that evaporate at ambient pressure is investigated in [2]. The test facility allows measurement of the instantaneous mass change of the evaporating drop located on surface at constant temperature. Both thermal and video cameras register the size of a drop for further calculation of its bottom area. On the basis of the energy balance, the evaporative heat flux is calculated. A linear dependence of the evaporation rate constant on the heating surface temperature is given for drops of mass ranging from 1 g to 0.5 g. The linear dependence of the drop square diameter or its orthogonal projection on the heating surface can be assumed for only a limited period of time due to the growing relative error of calculations.

In this way the aim of this paper is to describe some effects on vapor-liquid interface at the interaction of strong heated body with cold liquid in the case of steady film boiling on the solid surface. Thus heat flux comes to interface from vapor side. Investigation of transport processes features in the vapor film is based on the methods of continuum mechanics and molecular-kinetic theory. Non-equilibrium boundary condition is used to describe the effects near the interface. Using of non-equilibrium boundary conditions gives the possibility to solve difficult problems taking into account peculiarities of the transport processes at the vapor - liquid interface.

2. Problem and mathematical description

We propose the following physical model of subcooled water steady film boiling on the sphere of radius $R_w$ immersed to the depth $h$ [3]. The temperature of this sphere is higher than critical temperature of water. Thus stable vapor film of radius $R_1$ is formed due to the high temperature of the hot sphere $T_w$. Pressure over the free surface of liquid $P_b$ as well as temperature of cold water $T_b$ are known (Fig. 1).

As can be seen from the scheme of the problem, the liquid is separated from the heater by the vapor film. The heater is cooled by liquid and heater temperature $T_w$ decreases in time. Heat comes to the interface from the vapor side. Vapor film is closed from the outside space. Interface is nearly spherical shape. Assumptions of physical model are accepted for formulation of the mathematical description. Heat mass transfer processes in two-phase system are considered as quasisteady. At this mass flux on the vapor-liquid interface is equal zero. Temperature of interface is constant on cross section of vapor film. Physical properties of a non-compressible liquid and vapor are constant. Possible fluctuations of vapor film are not considered. As the heater is cooled down the size of vapor film $R_1$ and temperature of vapor-liquid interface $T_1$ are changed in time. In this way the mathematical problem is determination of time dependence of heater temperature with considering of heat mass transfer peculiarities through vapor-liquid interface. The system of equations is simple enough.

![Figure 1. The problem.](image)

Heat flux on the vapor – liquid interface $q_1$ is determined by thermal conductivity and radiation:
\[ q_i = \frac{\bar{\lambda}''_w (T_w - T_i^1) R_w}{(R_i - R_w) R_i} + \varepsilon \sigma_0 (T_w^4 - T_i^4), \]  

(1)

where \((R_i - R_w) = \delta\) is vapor film thickness, \(\bar{\lambda}''_w\) – coefficient of average vapor thermal conductivity, \(\varepsilon\) - emissivity factor; \(\sigma_0\) - Stefan-Boltzmann constant. Due to high level of temperature difference between heater and water the radiation can reach 30% of interface heat flux.

Heat flux in liquid is transferred by natural convection:

\[ q_i = \alpha (T_i^1 - T_b), \]  

(2)

where coefficient of heat transfer \(\alpha = Nu \frac{\lambda'}{2R_i}\) is determined from Nusselt number with \(\lambda'\) – coefficient of average liquid thermal conductivity:

\[ Nu = 0.5 (Gr \cdot Pr)^{0.33}, \]  

(3)

for turbulent thermo-gravitational free convection [4] at \((Gr \cdot Pr) = \frac{g \beta (T_i^1 - T_b) (2R_i)^3}{(\mu' \rho')^2 \lambda'} > 10^6\),

where \(\beta\) – coefficient of liquid thermal expansion, \(C'\) – specific heat, \(\mu' = \nu' \rho'\) – dynamic viscosity, \(\nu'\) – kinematic viscosity, \(\rho'\) – liquid density, \(g\) – acceleration of gravity.

Pressure distribution in liquid is determined by hydrostatics:

\[ P_i' = P_b + \rho' g h, \]  

(4)

where \(P_i'\) – pressure in liquid near interface.

Let's write equation which is the relationship between the pressure, \(P''\), in vapor volume near the heater and the heat flux density, \(q_w\). The solution of steady-state Boltzmann kinetic equation for problems of vaporization-condensation in linearized statement, i.e. at \(q_w \left(P_S (T_i') \sqrt{2RT_i'}\right)^{-1} \ll 1\) gives the relationship (from paper [5]):

\[ P'' = P_S(T_i') + 0.44 \frac{q_{12}}{\sqrt{2RT_i'}}, \]  

(5)

where \(R\) is individual gas constant, \(T_i'\) is temperature of interface surface, \(P_S(T_i')\) is saturation pressure at the temperature \(T_i'\). This relationship is presented taking into account that the specific heat flux \(q_{12} = \frac{\bar{\lambda}''_w (T_w - T_i^1) R_w}{(R_i - R_w) R_i}\) is determined only by the thermal conductivity in vapor.

The equation for temperature difference on the vapor-liquid interface is written in paper [5] as:

\[ T_i^1 = T_i' + 1.05 \frac{q_{12} T_i'}{P_S(T_i') \sqrt{2RT_i'}}, \]  

(6)

The last one equation is Laplace equation:

\[ P'' - P_i' = \frac{2\sigma}{R_i'}, \]  

(7)

where \(\sigma\) – surface tension.
The system of equations (1)-(7) can be solved for prescribed temperature of heater $T_w$. After this the balance correlation for heater temperature can be used to find time dependence $T_w(\tau)$:

$$\frac{dT_w}{d\tau} = -\frac{3q_w}{R_w\rho_wC_w},$$

where $q_w = \frac{\lambda^*(T_w - T_i^*)R_i}{(R_i - R_w)R_w} + \varepsilon\sigma(T_w^4 - T_i^4)$. At this system of equations (1)-(7) is transformed into the following form:

$$P_3(T_i') + 0.44\frac{\lambda^*(T_w - T_i^*)R_w}{R_i(R_i - R_w)\sqrt{2RT_i^3}} - (P_i + \rho'gh) = \frac{2\sigma}{R_i},$$

$$\frac{\lambda^*(T_w - T_i^*)R_w}{R_i(R_i - R_w)\sqrt{2RT_i^3}} = \frac{\lambda^*(T_w - T_i^*)R_w}{R_i(R_i - R_w)\sqrt{2RT_i^3}} + \varepsilon\sigma(T_w^4 - T_i^4),$$

$$T_i^* = T_i' + 1.05\frac{\lambda^*(T_w - T_i^*)R_w}{P_3(T_i')R_i(R_i - R_w)\sqrt{2RT_i^3}}.$$

Equation (9) is mechanical balance of the system heater – vapor film – liquid, equation (10) – thermal balance and (11) – correlations between temperatures on and near the vapor-liquid interface. The system of equations was solved numerically by standard procedures of MathCad.

3. The calculation results and discussion

For the cases of three temperatures of subcooled water $T_b$ the time dependences of heater temperature with radius of 30 mm were obtained. These dependencies are presented in Fig. 2. Experimental data [6] for steel ball are depicted also in this figure for comparison. Corresponding time dependencies of stationary film radius $R_i$ are presented in the Fig. 3. This is not a solution of dynamic equation because the problem is quasi-steady. As we can see the rate of cooling is higher for lower temperature of liquid, but experimental rate even for bath temperature $T_b=65^\circ C$ is much more than calculation value. It can be suppose that heat transfer in liquid due to natural convection is more efficient and exponent in equation (3) should be evaluated as 0.36. The thickness of vapor film $\delta=R_i-R_w$ is larger for higher temperature of liquid $T_w$. With time this value $\delta$ is decreases due to diminishing of heat flux $q_1$ from heater. But if the time dependence of heater temperature $T_w$ is linear, corresponding vapor film thickness $\delta$ dependence is non-linear.

![Figure 2](image.png)

**Figure 2.** Time dependences of heater temperature $T_w$ at different water temperature $T_b$:
1 – $T_b=30^\circ C$, 2 – $T_b=50^\circ C$, 3 – $T_b=65^\circ C$, 4 – experimental result at $T_b=65^\circ C$ [6].
Figure 3. Time dependences of vapor film radius $R_{1}$ at different water temperature $T_{b}$:
1 – $T_{b}$=30°C, 2 – $T_{b}$=50°C, 3 – $T_{b}$=65°C

Figure 4. Time dependences of heater temperature $T_{w}$ (1), experimental curve (2) and vapor film thickness $\delta$ (3)

Comparison between calculation and experimental results for nickel heater of radius $R_{w}$=22.5 mm and water temperature $T_{b}$=75°C is presented on Fig. 4. As for curves on Fig. 2 the rate of cooling in experiment is larger than corresponding rate in calculations at this initial data.

4. Conclusion
The analysis of cooling of strong heated sphere in cold water was carried out with taking into account of non-equilibrium processes at the vapor-liquid interface. Stable vapor film was described by system of equations. Heat transfer in vapor and in liquid was analyzed. Pressure balance was considered. Thus, the numerical solution was obtained. At this time dependences of heater temperature and corresponding vapor film thickness and vapor-liquid interface temperature were obtained for different water temperature. The main result is that rate of cooling in experiment is larger than calculative one. Thus, it can be concluded that heat transfer in liquid is more efficient that described by usual natural convection.

Acknowledgments
This work was supported by Russian Foundation of Basic Research (project No. 15-08-06145).
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