AN ATTEMPT AT A UNIFIED MODEL FOR THE GAMMA-RAY EMISSION OF SUPERNOVA REMNANTS

Qiang Yuan\textsuperscript{1}, Siming Liu\textsuperscript{2}, and Xiaojun Bi\textsuperscript{1}

\textsuperscript{1}Key Laboratory of Particle Astrophysics, Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
\textsuperscript{2}Key Laboratory of Dark Matter and Space Astronomy, Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China

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ABSTRACT

Shocks of supernova remnants (SNRs) are important (and perhaps the dominant) agents for the production of the Galactic cosmic rays. Recent $\gamma$-ray observations of several SNRs have made this case more compelling. However, these broadband high-energy measurements also reveal a variety of spectral shapes demanding more comprehensive modeling of emissions from SNRs. According to the locally observed fluxes of cosmic-ray protons and electrons, the electron-to-proton number ratio is known to be about 1%. Assuming such a ratio is universal for all SNRs and identical spectral shape for all kinds of accelerated particles, we propose a unified model that ascribes the distinct $\gamma$-ray spectra of different SNRs to variations of the medium density and the spectral difference between cosmic-ray electrons and protons observed from Earth to transport effects. For low-density environments, the $\gamma$-ray emission is inverse-Compton dominated. For high-density environments like systems of high-energy particles interacting with molecular clouds, the $\gamma$-ray emission is $\pi^0$-decay dominated. The model predicts a hadronic origin of $\gamma$-ray emission from very old remnants interacting mostly with molecular clouds and a leptonic origin for intermediate-age remnants whose shocks propagate in a low-density environment created by their progenitors via, e.g., strong stellar winds. These results can be regarded as evidence in support of the SNR origin of Galactic cosmic rays.

Key words: cosmic rays – gamma rays: ISM – ISM: supernova remnants – radiation mechanisms: non-thermal

Online-only material: color figures

1. INTRODUCTION

Nearly a century after the discovery of cosmic rays (CRs), their origin remains one of the biggest fundamental questions in the fields of high-energy physics and astrophysics. It is widely believed that supernova remnants (SNRs) are one of the most probable sources of Galactic CRs below the so-called knee (Bhat et al. 1985; Erlykin & Wolfendale 2001a, 2001b, 2001c, 2002a; Hillas 2005; Katz & Waxman 2008). However, there are currently no observations that can directly verify such a conjecture.

The multi-wavelength observations of SNRs, especially the high-energy $\gamma$-rays from the ground-based atmospheric Cerenkov telescope arrays and space-based telescopes, provide powerful tools for probing the particle acceleration mechanism in SNRs (Enomoto et al. 2002; Aharonian et al. 2004; Uchiyama et al. 2007; Tian et al. 2007, 2008, 2010; Neronov & Semikoz 2012). However, whether the nature of this $\gamma$-ray emission from SNRs is predominantly hadronic or leptonic is still a matter of debate (e.g., Aharonian et al. 2006a; Berezhko & Völk 2006; Butt et al. 2008; Liu et al. 2008; Plaga 2008; Morlino et al. 2009; Fang et al. 2009; Fan et al. 2010; Ellison et al. 2010; Zirakashvili & Aharonian 2010; Berezhko & Völk 2010; Yuan et al. 2011a). That is to say, SNRs are known particle accelerators, but it is not clear whether or not they dominate the observed Galactic CR flux, especially for nuclei, on Earth.

The observational $\gamma$-ray spectra of SNRs also show significant diversity. Recent Fermi observations of the young SNR RX J1713.7$-$3946 show a very hard spectrum in the GeV energy range, 1.50 $\pm$ 0.11, which implies an inverse-Compton (IC) origin of the $\gamma$-rays (Abdo et al. 2011; see Inoue et al. 2012 for an alternative explanation). On the other hand, for all of the SNRs interacting with molecular clouds (MCs), the GeV$-$TeV $\gamma$-ray spectra are generally very soft and seem to agree better with the $\pi^0$-decay model, although the model of bremsstrahlung emission from electrons cannot be ruled out (Abdo et al. 2009, 2010a, 2010b, 2010c, 2010d, 2010e; Ajello et al. 2012; Giuliani et al. 2010). It is natural to ask whether or not there is a common understanding of these $\gamma$-ray signatures of the SNRs.

In this paper, we illustrate that the locally observed CRs and the $\gamma$-ray emission of SNRs can be naturally understood in a unified picture. Assuming that both the protons\textsuperscript{3} and electrons are accelerated in SNRs and that the SNRs are the dominant sources of Galactic CRs, one can derive the electron-to-proton ratio at the source based on the locally observed spectra of protons and electrons. Then, the $\gamma$-ray emission of SNRs will depend primarily on the environmental gas density, and the observed diversity of $\gamma$-ray spectra of SNRs can be attributed to variations of this density.

2. ELECTRON–PROTON RATIO AT THE SOURCE

The observational flux of protons at $\sim$GeV is about two orders of magnitude higher than that of electrons, which implies that the electron–proton ratio at the source should be of the order of 1% since the energy loss of electrons is negligible in this low-energy range. This result is well known (e.g., Cohen & Ramaty 1973; Apparao & Daniel 1977; Levinson 1994; Erlykin & Wolfendale 2002b). In this section, we derive the injection parameters of the Galactic CRs at the sources by reproducing the observed spectra on Earth, considering the detailed propagation model.

After being produced at the source, charged energetic particles propagate diffusively in the random magnetic field of the Galaxy. The overall convection and reacceleration due to scattering caused by random magnetohydrodynamic waves may also change the distribution function of CR particles. Furthermore, the interactions between CRs and gas, the interstellar radiation field, and the magnetic field will lead to fragmentation and

\textsuperscript{3} The heavier nuclei that may play a similar but less important role than protons for $\gamma$-ray emission are not discussed here.
catastrophic or continuous energy loss by these particles. The transport of CRs from the source to the Earth is generally complex (Butt et al. 2008; Strong et al. 2007 for a recent review of CR propagation).

In this work, we limit our study to CR protons and electrons. We adopt the GALPROP code (Strong & Moskalenko 1998; Moskalenko & Strong 1998) version v50p to calculate the propagation of CRs. The diffusion-reacceleration frame, without convection, is assumed. The main propagation parameters are \( D_0 = 6.59 \times 10^{28} \text{cm}^2 \text{s}^{-1} \), \( \delta = 0.30 \), \( v_A = 39.2 \text{ km s}^{-1} \), and \( z_h = 3.9 \text{ kpc} \), which are derived through the fit to the B/C, \(^{10}\text{Be}/^{8}\text{Be} \), and carbon and oxygen data (Trotta et al. 2011). The spatial distribution of CR sources in cylindrical coordinates is

\[
f(R, z) \propto \left( \frac{R}{R_\odot} \right)^a \exp \left[ -\frac{\beta(R - R_\odot)}{R_\odot} \right] \exp \left( -\frac{|z|}{z_s} \right),
\]

where axis symmetry has been assumed and \( R_\odot = 8.5 \text{ kpc} \) is the distance of the solar system from the Galactic center, \( z_s \approx 0.2 \text{ kpc} \) is the scale height of the source distribution, \( a = 1.25 \), and \( b = 3.56 \). This source function is similar to the SNR spatial distribution but is altered based on the Fermi observations of diffuse Galactic \( \gamma \)-rays (Trotta et al. 2011).

The injection spectral shape as a function of momentum \( p \) (or rigidity) is assumed to be a broken power-law function

\[
q(p) \propto \begin{cases} p^{-a_1}, & p < p_{br}, \\ p^{-a_2}, & p \geq p_{br}, \end{cases}
\]

which is identical for both electrons and protons.\(^5\) The ratio of the normalization between electrons and protons, usually called \( K_{ep} \), is chosen as a free parameter. The broken power-law injection spectrum is required to fit the observed CR data (Strong et al. 2004; Trotta et al. 2011; Liu et al. 2012) as well as the \( \gamma \)-ray data (Abdo et al. 2009, 2010a, 2010c, 2010d, 2010e; Ajello et al. 2012; Neronov et al. 2012). It has been proposed that strong ion–neutral collisions near the shock front may lead to the spectral break of accelerated particles around \( \sim 10 \text{ GeV} \) (Malkov et al. 2005, 2011). Alternatively, the escape effect of particles from into the finite-size region may also lead to a break at several GeV (Ohira et al. 2011; Li & Chen 2010, 2012).

Compared with the PAMELA observations of CR proton (Adriani et al. 2011b) and electron spectra (Adriani et al. 2011a), we find that \( a_1 = 1.80 \), \( a_2 = 2.52 \), and \( p_{br}c = 6 \text{ GeV} \) can trouble an acceptable fit to the data, where \( c \) is the speed of light,\(^6\) as shown in Figure 1. For energies below \( \sim 30 \text{ GeV} \), the solar modulation with a force-field approximation is employed (Gleeson & Axford 1968). To fit both the proton and electron spectra with the same injection spectrum, we need different modulation potentials, as labeled in Figure 1. This may be due to the rest mass or the sign of the charge dependence of the modulation effect (Clem et al. 1996). To reproduce the absolute fluxes, we obtain an electron-to-proton ratio \( K_{ep} \) of about 1.3%. Such a result can be reproduced in a numerical simulation (Levinson 1994). See Erlykin & Wolfendale (2002b) for more possible explanations.

\(^4\) http://galprop.stanford.edu/

\(^5\) For the sake of simplicity, we ignore the spectral evolution and potential spectral difference between high-energy electrons and protons escaping from SNRs (Erlykin & Wolfendale 2002b).

\(^6\) We do not discuss the spectral hardening of CR nuclei above \( \sim 200 \text{ GeV} \) reported by ATIC/CREAM/PAMELA (Panov et al. 2007; Ahn et al. 2010; Adriani et al. 2011b), which may imply the superposition of different source spectra (Yuan et al. 2011c) or a nearby new component of CRs (Erlykin & Wolfendale 2012; Thoudam & Hörandel 2012).

A better fit to the electron data can be obtained by increasing the magnetic field\(^7\) by a factor of two (corresponding to \( K_{ep} \approx 1.9\% \)), as shown by the red dashed line. Even with an identical spectral shape at injection, the electron spectrum is much softer than the proton one due to their energy loss in the transport processes from the source regions to Earth, which is quite different from the scenario explored by Erlykin & Wolfendale (2002b), where SNRs produce an electron distribution softer than the proton distribution.

As a comparison, the independent fit to the proton and electron data provides \( a_1 = 1.91 \), \( a_2 = 2.40 \), and \( p_{br}c = 10 \text{ GeV} \) for protons (Trotta et al. 2011), and \( a_1 = 1.50 \), \( a_2 = 2.56 \), and \( p_{br}c = 3.60 \text{ GeV} \) for electrons (Liu et al. 2012). Given the uncertainties of the propagation parameters, astrophysical inputs, solar modulation, etc., we consider these spectral fits to be consistent with each other.

3. GAMMA-RAY EMISSION OF SUPERNOVA REMNANTS

We now investigate the \( \gamma \)-ray emission of SNRs, adopting \( K_{ep} \sim 1\% \) and the spectral parameters at the source as derived in the previous section. In general, there are three major components of the \( \gamma \)-ray emission: IC and bremsstrahlung radiation by electrons and the \( \pi^0 \)-decay emission by protons. Therefore, we need further knowledge about the radiation background and the environmental gas density. For the sake of simplicity, we assume IC scattering only with the cosmic microwave background radiation in this work. The scattering with infrared and optical light may make the IC \( \gamma \)-ray spectrum broader (Porter et al. 2006), but is not expected to affect the qualitative discussion here, at least for those far away from the Galactic center. Therefore, the only parameter determining the SNR \( \gamma \)-ray spectra will be the gas density.

We classify the SNRs into three groups with low, medium, and high ambient gas densities. The lack of thermal X-ray

\(^7\) Note that the change of magnetic field may affect the synchrotron radiation (Strong et al. 2011). What we employ here is an example in order to include the uncertainties of the propagation model. The full discussion of a self-consistent propagation model is beyond the scope of the present work.
emission of RX J1713.7−3946 gives an upper limit for the gas density of 0.02−0.03 cm$^{-3}$ (Cassam-Chenaï et al. 2004; Yuan et al. 2011a). For RX J0852.0−4622, ASCA X-ray data imply a gas density $n < 0.03(d/1\text{ kpc})^{-1/2} f^{-1/2}$ cm$^{-3}$, with $d$ being the distance and $f$ being the filling factor of X-ray emitting volume (Slane et al. 2001). Although the derivation of the upper limit relies on assumptions such as the ionization equilibrium and gas temperature, we take them to be typical examples of low-density SNRs. Adopting $n = 0.01$ cm$^{-3}$, together with the CR spectra given by Equation (2) and $K_{\text{ep}} \sim 1\%$, we show the calculated $\gamma$-ray spectra of these two SNRs in Figure 2. The overall normalization of the $\gamma$-ray luminosity is determined by the observational flux of each source. To be consistent with the high-energy spectral cutoff behavior of both SNRs, we employ an exponential cutoff term with $E_i \approx 60\text{ TeV}$ of the electron spectra. The cutoff might be due to the balance of acceleration and the cooling in the vicinity of the SNR. For protons, the cutoff could be higher and is assumed not to enter the energy range discussed here. A remarkable signature of the $\gamma$-ray spectrum is that the spectrum is very hard and the luminosity of TeV emission is higher than that of the GeV emission. The $\gamma$-ray emission of these SNRs is IC dominated.

Some relatively young SNRs have a moderate gas density, such as Cassiopeia A and Tycho. The average gas density of Cassiopeia A is estimated to be about 4.4 cm$^{-3}$ (Araya & Cui 2010). For Tycho, an upper limit $n < 0.6$ cm$^{-3}$ was derived from the absence of thermal X-ray emission from the bright outer rim of the remnant (Cassam-Chenaï et al. 2007). The density in the inner region of the remnant can be much higher. Here, we take these two SNRs as examples of the medium density sample and adopt $n = 1$ cm$^{-3}$.  The calculated spectra are shown in Figure 3. We can see that for these kinds of sources, the GeV emission is $\pi^0$-decay dominated and the TeV emission is IC dominated. The luminosities in GeV and TeV bands are comparable in this case.

Finally, we discuss the case with a high density, which is typical of the SNR–MC interacting systems. The gas density in the MCs can easily reach $10^2$−$10^3$ cm$^{-3}$. As an illustration, we adopt $n = 100$ cm$^{-3}$ in this study. The expected $\gamma$-ray spectra together with the GeV–TeV observational data of eight SNR–MC systems are shown in Figure 4. In this case, the GeV–TeV $\gamma$-ray emission is $\pi^0$-decay dominated, and the $\gamma$-ray spectrum is generally very soft.

We may need to estimate the impact of energy loss in the dense clouds of the CR proton spectrum. For $n = 100$ cm$^{-3}$ in the interstellar medium, the ionization energy loss timescale of protons is about $10^8$ yr at 1 GeV (Strong & Moskalenko 1998), and the pion-production energy-loss timescale is about $10^9$ yr.

$^8$ Although these sources clearly show complex source structure with multiple emission zones (Atoyan & Dermer 2012), in this paper we still consider the simple one-zone emission model in order to capture the dominant features.
Figure 4. Same as Figure 2, but for SNR–MC interacting systems. The gas density is adopted to be $n = 100\, \text{cm}^{-3}$. References of the observational data—W28: *Fermi* (Abdo et al. 2010a), HESS (Aharonian et al. 2008c); W41: *Fermi* (Mehault et al. 2011), HESS (Mehault et al. 2011); W49B: *Fermi* (Abdo et al. 2010c), HESS (Brun et al. 2011); W51C: *Fermi* (Abdo et al. 2009), HESS (Fissou et al. 2009), MAGIC (Carmona et al. 2011); IC 443: *Fermi* (Abdo et al. 2010e), MAGIC (Albert et al. 2007a), VERITAS (Acciari et al. 2009); CTB 37A: *Fermi* (Castro & Slane 2010), HESS (Aharonian et al. 2008b), G8.7-0.1: *Fermi* (Ajello et al. 2012), HESS (Aharonian et al. 2006b); G359.1-0.5: *Fermi* (Hui et al. 2011), HESS (Aharonian et al. 2008b).
(A color version of this figure is available in the online journal.)
The 12 SNRs studied in this work. The solid line is the model-expected result.

For most of these SNRs, the ages are estimated to be less than several tens of kyr, so we expect that for the energy range of interest in this work (GeV–TeV) the energy losses of the protons are not important. For the low-energy particles (several tens of MeV) and high-energy electrons (>GeV), the energy loss may be important and need to be considered in future modeling. As for the contribution to γ-rays from the secondary e^±, it is only important for E_γ < 10 MeV compared with the pp-induced π^0-decay component, even for very old SNRs (Fang & Zhang 2008).

In Figure 5, we show the relation between the photon index Γ and the gas density n for the SNRs studied in this work. The parameters of the SNRs are compiled in Table 1. The photon index Γ is fitted using the observational data between 1 GeV and 1 TeV, with a single power-law function. For the SNR–MC interacting system whose gas density is not well known, we assume a value of 10^2–10^3 cm^-3. A trend showing the correlation between Γ and n can be seen in Figure 5. We also show the theoretical expected result based on the unified model with the solid line. The model is consistent with the observational data.

It is encouraging that the results in Figures 2–4 are roughly consistent with the observational data, in support of our relatively simple interpretation of the complicated γ-ray spectral behaviors of SNRs. Note that here we have not tried to fit the observational data precisely because variations of the spectral and environmental parameters are expected for different sources (Ferrand & Marcorwith 2010; Yuan et al. 2011b, 2011c). With slight adjustment of these source parameters, we can easily obtain a better fit to the data (Li & Chen 2012).

4. CONCLUSION AND DISCUSSION

In this work, we propose a unified model for explaining the γ-ray emission of SNRs. In the model, by assuming that SNRs produce identical high-energy electron and proton spectral shapes and are the major sources of low-energy CRs (below the “knee”), the electron-to-proton number ratio K_{ep} at the sources is derived to be ~1%, according to a realistic CR propagation model described with the GALPROP code. With such a K_{ep} value, we calculate the expected γ-ray spectra for various SNRs with different environmental parameters. Qualitatively, the observed diversity of different SNR γ-ray spectra can naturally be understood with different gas densities. For low-density environments, the γ-ray emission is IC dominated, whereas for high-density environments the γ-ray emission is π^0-decay dominated. The model predicts that γ-ray spectra in low-density environments are generally harder than those of SNR–MC interaction systems. Since strong thermal emission is expected from shocked dense media, we expect relatively weaker thermal emission from remnants with harder γ-ray spectra than those with softer γ-ray spectra. Such a simple, self-consistent model, if further validated by observations, supports an SNR origin of low-energy CRs.

Age could also be a parameter affecting the γ-ray emission of SNRs, and it can be coupled with the density parameter. For example, considering the progenitors, the density is in the intermediate range for very young remnants, lowest for middle-age SNRs, and highest for very old remnants (Dwarkadas 2005). Of course, an improved unified model needs to consider more

Table 1
Parameters of the SNRs

| Name          | R.A.       | Decl.      | d   | Age  | n     | Γ     | References                      |
|---------------|------------|------------|-----|------|-------|-------|---------------------------------|
| RX J1713.7−3946 | 17°13′50°   | −39°45′    | 1.0 | 1.6  | < 0.03| 1.77±0.08| 1, 2                            |
| RX J0852.0−4622 | 08°52′00°   | −46°20′    | 0.75| 1.7–4.3 | < 0.03| 1.80±0.04| 3, 4                            |
| Cassiopeia A   | 23°23′26°   | +58°48′    | 3.4 | 0.32 | 4.4   | 2.19±0.04| 5–8                            |
| Tycho          | 00°25′18°   | +64°09′    | 2.5 | 3.0  | < 0.6 | 2.18±0.11| 9–12                           |
| W28            | 18°00′30°   | −23°26′    | 2.0 | 35–150 | ...   | 2.67±0.03| 13, 14                          |
| W41            | 18°34′45°   | −08°48′    | 3.9 | 4.5  | 60–200| ...   | 2.27±0.03| 15, 16                          |
| W49B           | 19°11′08′   | +09°06′    | 8.12| 1–4  | ...   | 2.73±0.05| 17, 18                          |
| W51C           | 19°23′50°   | +14°06′    | 6.0 | 30   | ...   | 2.58±0.04| 19–21                          |
| IC 443         | 06°17′00′   | +22°34′    | 0.7 | 2.0  | 3–30  | 60–240 | 2.61±0.04| 22–24                          |
| CTB 37A        | 17°14′06′   | −38°32′    | 6.3 | 9.5  | ...   | ...   | 2.48±0.07| 25–27                          |
| G8.7-0.1       | 18°05′30°   | −21°26′    | 4.8 | 6.0  | 15–28 | ...   | 2.23±0.05| 28, 29                          |
| G359.1-0.5     | 17°45′30°   | −29°57′    | 7.6 | > 10 | ...   | ...   | 2.63±0.02| 30, 31                          |

References. (1) Abdo et al. 2011; (2) Aharonian et al. 2007b; (3) Tanaka et al. 2011; (4) Aharonian et al. 2007a; (5) Abdo et al. 2010b; (6) Albert et al. 2007b; (7) Acciari et al. 2010; (8) Fesen et al. 2006; (9) Giordano et al. 2012; (10) Acciari et al. 2011; (11) Tian & Leahy 2011; (12) Cassam-Chenai et al. 2007; (13) Abdo et al. 2010a; (14) Aharonian et al. 2008c; (15) Mehault et al. 2011; (16) Leahy & Tian 2008; (17) Abdo et al. 2010c; (18) Brun et al. 2011; (19) Abdo et al. 2009; (20) Fissoni et al. 2009; (21) Carmona et al. 2011; (22) Abdo et al. 2010c; (23) Albert et al. 2007a; (24) Acciari et al. 2009; (25) Castro & Slane 2010; (26) Aharonian et al. 2008a; (27) Tian & Leahy 2012; (28) Ajello et al. 2012; (29) Aharonian et al. 2006b; (30) Hui et al. 2011; (31) Aharonian et al. 2008b.

Figure 5. Photon index Γ (between 1 GeV and 1 TeV) vs. the gas density n of the 12 SNRs studied in this work. The solid line is the model-expected result. (A color version of this figure is available in the online journal.)
factors, including the effect of multiple emission zones as observed in some sources (Atoyan & Dermer 2012), which will affect the detailed spectral fit. In general, for $K_{ep} \sim 1\%$, the hadronic component will always dominate the bremsstrahlung component, which is distinguishable from the model predicting that the bremsstrahlung component may dominate the $\gamma$-ray emission (Atoyan & Dermer 2012), and can be tested by observing $\gamma$-rays in a lower energy range ($< 100$ MeV).

When calculating the $\gamma$-ray emission of the SNRs, we employ particle spectra, which are the same as the injection spectra, giving rise to locally observed ones. However, we should keep in mind that the CR spectra accelerated by the source at a specific epoch may be different from that injected into the Milky Way, which should be the time-integrated spectrum (Caprioli et al. 2010). More detailed modeling may need to take into account the evolution history of the SNRs (e.g., Sturmer et al. 1997; Reynolds 2008; Lee et al. 2008; Fang & Zhang 2008; Finke & Dermer 2012; Yan et al. 2012). Furthermore, the electron-to-proton ratio is assumed to be a constant in this work, independent of the source. Such an assumption can break the degeneracy between medium density $n$ and $K_{ep}$. This is a strong assumption, but is not in conflict with observations. Detailed study of individual sources is needed to verify this assumption.

The magnetic field of the shocked emission region is an important parameter but does not appear explicitly in our model. However, for the multi-wavelength modeling including the synchrotron emission component, the average magnetic field might be determined well via a detailed spectral fit (Fan et al. 2010). Indeed, with the increase of the magnetic field, the IC component will be suppressed for a given synchrotron flux due to the decrease of the number of energetic electrons. The gas density (or the number of CR protons and therefore $1/\epsilon_{IC}$) needs to be increased in this case to account for the observed $\gamma$-ray flux. The $\gamma$-ray emission is likely dominated by pionic emission, in this case due to higher gas density or proton flux, and it will be difficult to detect the IC component. Another effect of the high magnetic field is the cooling of accelerated electrons via synchrotron emission. Thus, the electron spectrum should be in contradiction with our assumption of a unified spectral shape. Except for this particular case, our study will remain valid as long as this strong magnetic field does not dominate the overall particle acceleration in SNRs. In cases where the IC dominates the $\gamma$-ray emission, the magnetic field can be directly derived through the radio-to-$X$-ray emission of the SNRs. The magnetic field is usually weak ($\sim 1\mu$G) with an energy density of only a factor of a few times higher than that of the background photons, according to the synchrotron X-ray-to-IC $\gamma$-ray luminosity ratio (Liu et al. 2008). Lagage & Cesarsky (1983) showed that with such a weak field, SNRs can barely accelerate charged particles up to $10^{15}$ eV. Although the discussion in this work may still hold ($E \lesssim 100$ TeV) even in the case of a weak magnetic field, for CRs to be accelerated up to the knee via the diffusive shock acceleration, magnetic fields need to be amplified further by some mechanisms (e.g., Bell 2004; Guo et al. 2012). Inhomogeneity of the magnetic field and multi-zone acceleration scenario (Atoyan & Dermer 2012) may alleviate challenges to the weak field case. Better determination of the magnetic field, together with the gas density parameter, will provide crucial tests of the assumptions made in this work.

Finally, the radio observations of SNRs indicate that the electron spectrum is about $E^{-2}$ with a remarkable dispersion (Clark & Caswell 1976; Bogdan et al. 1985; Schlickeiser 2002), which is not exactly the same as that inferred in Section 2. We also expect dispersion of other parameters characterizing the energetic particle distribution, which may be used to improve the fit to the observed $\gamma$-ray spectra. The present work gives a zero-order approximation to the problem of the SNR $\gamma$-ray emission and the origin of CRs. Some common features of the $\gamma$-ray emission of different populations of SNRs are revealed. Further works about the details may be helpful for refining the present model.

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YUAN, LIU, & BI
