Is torsion needed in theory of gravity?

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Abstract

It is known that General Relativity (GR) uses Lorentzian Manifold \((M_4; g)\) as a geometrical model of the physical space–time. \(M_4\) means here a four–dimensional differentiable manifold endowed with Lorentzian metric \(g\). The metric \(g\) satisfies Einstein equations. Since the 1970s many authors have tried to generalize this geometrical model of the physical space–time by introducing torsion and even more general metric–affine geometry. In this paper we discuss status of torsion in the theory of gravity. At first, we emphasize that up to now we have no experimental evidence for the existence of torsion in Nature. Contrary, the all experiments performed in weak gravitational field (Solar System) or in strong regime (binary pulsars) and tests of the Einstein Equivalence Principle (EEP) confirmed GR and Lorentzian manifold \((M_4; g)\) as correct geometrical model of the physical space–time. Then, we give theoretical arguments against introducing of torsion into geometrical model of the physical space–time. At last, we conclude that the general–relativistic model of the physical space–time is sufficient and it seems to be the most satisfactory.
I. INTRODUCTION

In the 70ths of the XX Century many research workers have introduced torsion into theory of gravity\textsuperscript{1}[1,2,3]. The motivation (only theoretical) was the following:

1. Analogies \textbf{GR} with the continual theory of dislocations (Theory of generalized Cosserat continuum) led to heuristic arguments for a metric space–time with torsion, i.e., to Riemann–Cartan space–time.

2. Study of spinning matter in \textbf{GR} led some people to conclusion that the canonical energy–momentum tensor of matter $cT^k_i$ is the source of the curvature and the canonical intrinsic spin density tensor $cS_{ikl} = (-)cS^{ikl}$ is the source of torsion of the underlying space–time. From these studies Einstein–Cartan–Sciama–Kibble (ECSK) theory was originated and its generalizations.

3. Trials to formulate theory of gravity as a gauge theory for Lorentz group $L$ or for Poincare’ group $P$ led to a space–time endowed with a metric– compatible connection which could have (but not necessarily) non–vanishing torsion, i.e., led to the Riemann–Cartan space–time [4–6].

The above motivation is not convincing. For example, the often used argument for torsion (It followed from 2.) based on (non-homogeneous) holonomy theorem [1]\textsuperscript{2} holds only if one uses \textit{Cartan displacements} [7]. Ordinary parallell displacements gives only Lorentz rotations (= homogeneous holonomy group) even in a Riemann-Cartan space-time [7]. Moreover, there are other geometrical interpretation of torsion, e.g., Bompiani [8] connects torsion with rotations in tangent spaces, not with translations. We also needn’t generalize \textbf{GR} in

\textsuperscript{1}We omit here older trials to introduce torsion because they have only historical meaning.

\textsuperscript{2}This theorem says that torsion gives translations and curvature gives Lorentz rotations in tangent spaces of a Riemann-Cartan manifold during displacements along loops.
order to get a gauge theory with $L$ or $P$ as a gauge group [9]. Notice especially that in
the Asthekar’s variables [10,11] the ordinary GR with Levi–Civita connection itself gives an
elementary model of a gauge theory of gravity.

Up to now, we have no experimental evidence for existence of torsion in Nature. Con-
trary, the all experiments confirmed with a very high precision ($\sim 10^{-14}$) Einstein Equiv-
alence Principle (EEP) and (with a smaller precision) General Relativity (GR) equations
[12–15]. Here by EEP we mean a formulation of this Principle given by C. W. Will [12]. In
this formulation $^3$ the EEP states:

1. Weak Equivalence Principle (WEP) is valid.

This means that the trajectory of a freely falling body (one not acted upon by such
forces as electromagnetism and too small to be affected by tidal forces) is independent
of its internal structure and composition.

2. Local Lorentz Invariance (LLI) is valid.

This means that the outcome of any local non-gravitational experiment is independent
of the velocity of the freely-falling reference frame in which it is performed

3. Local Position Invariance (LPI) is valid.

This means that the outcome of any local non-gravitational experiment is independent
of where and when in the Universe it is performed.

The only theories of gravity that can embody EEP are those that satisfy the postulates
of metric theories of gravity [12], which are:

1. Spacetime is endowed with a symmetric metric.

2. The trajectories of freely falling bodies are geodesics of that metric.

$^3$In this formulation this Principle can be experimentally tested.
3. In local freely falling reference frames the non-gravitational laws of physics are those written in the language of Special Relativity (SRT).

From the EEP it follows the universal pure metric coupling between matter and gravity. This admits GR, of course, and, at most, some of the so-called scalar–tensor theories (these, which respect EEP) [13–15] without torsion.

So, torsion is excluded by latest gravitational experiments (at least in vacuum if we neglect a cosmological background) which have confirmed EEP with such very high precision. In consequence, torsion can be treated only as a hypothetical field and is not needed in theory of gravity.

In Section II we give a review of the (theoretical) arguments which in our opinion additionally indicate against introducing of torsion into relativistic theory of gravity. We will conclude from these arguments that torsion rather should not be introduced into theory of gravity.

In the paper we will confine to the metric–compatible connection $\omega^i_k$. We cannot see any reasons in order to consider more general connections, for example, connexion like proposed in [16]. For a metric–compatible connection $\omega^i_k$ we have

$$Dg = 0$$

and

$$\omega^i_k = _{LC} \omega^i_k + K^i_k,$$

where $_{LC} \omega^i_k$ means Levi–Civita connection and $K^i_k$ denotes contortion.

4Torsion is excluded at least in vacuum because if we neglect a cosmological background, then the all gravitational experiment were performed in vacuum. In such a case ECSK theory can survive since this theory is identical in vacuum with GR. But we think that the arguments which we present in Section II testify against ECSK theory also.
By \( \vartheta^i \) we will denote an Lorentzian coreper; \( \Psi \) will denote a matter field and \( D \) will mean an exterior covariant derivative. For tensor fields the exterior covariant derivative \( D \) reduces to an ordinary covariant derivative \( \nabla \).

II. ARGUMENTS WHICH INDICATE AGAINST TORSION

The our main argument uses Ockham’s razor and the fact: a “wonderful”, the most simple and most symmetric Levi-Civita connection is sufficient for the all physical requirements.

The other our arguments are the following:

1. In the paper [17] the authors have showed that the geometry of free–falls and light propagation supplemented by some (very natural axioms) lead us to Riemannian geometry.

2. Torsion is topologically trivial. This means that the topology of a real manifold \( M \) and topology vector bundles over \( M \) (determined by characteristic classes) do not depend on torsion. They depend on curvature only and are determined by curvature \( LC\Omega^i_k \) of the Levi-Civita connection \( LC\omega^i_k \) [18–20]. Roughly speaking, one can continuously deform any metric–compatible connection (or even general linear connection) into Levi-Civita connections without changing topological invariants and characteristic classes. So, torsion is not relevant from the topological point of view.

3. Torsion is not relevant from the dynamical point of view too. Namely, one can reformulate every metric theory of gravitation with a metric–compatible connection \( \omega^i_k \) as a ”Levi-Civita theory”. Torsion is then treated as a matter field. An obvious example is given by ECSK theory in the so–called “combined formulation” [21].\(^5\) In general, one can prove [23] that any total Lagrangian of the type

\[
L_t = L_g(\vartheta^i, \omega^i_k) + L_m(\Psi, D\Psi)
\]

\(^5\)In this formulation ECSK theory is dynamically fully equivalent to the ordinary GR [22].
admits an unique decomposition into a pure geometric part $\tilde{L}_g(\partial^i_{,\omega^j} \omega^{ik})$ containing no torsion plus a generalized matter Lagrangian $\tilde{L}_m(\Psi_{,\omega^k} D\Psi, K^i_{,k} D K^i_{,k})$ which collects the pure matter terms and all the terms involving torsion

$$L_t = L_g + L_m = \tilde{L}_g + \tilde{L}_m.$$ (4)

$L_{C,D}$ means an exterior covariant derivative with respect to the Levi-Civita connection $\omega^j_{,k}$. From the Lagrangian

$$L_t = \tilde{L}_g + \tilde{L}_m$$ (5)

there follow the Levi-Civita equations associated with $L_t$.

So, torsion always can be treated as a matter field. This point of view has been taken e.g. in [24,25] and it is supported by transformational properties of torsion: torsion transforms like a matter field.

4. Symmetry of the energy–momentum tensor of matter. In Special Relativity (SRT) a correct energy–momentum tensor for matter (continuous medium, dust, elastic body, solids) must be symmetric [26,27]. One can always get such a tensor starting from the canonical pair $(cT^{ik}_{,c}, S^{ijkl} = (−)_{c}S^{kili})$ with properties

$$\partial_{kc}T^{ik} = 0, \quad T^{ik}_{,c} - T^{ki}_{,c} = \partial_l S^{ikkl},$$ (6)

and by use Belinfante symmetrization procedure [21,28-30]

$$sT^{ik} = T^{ik} - \frac{1}{2}\partial_j (c S^{ijk} - c S^{ikj} + S^{jki}),$$ (7)

$$S^{ijk} = c S^{ijk} - A^{jki} + A^{ikj} = 0.$$ (8)

Here

$$A^{ikj} = \frac{1}{2} (c S^{ikj} - c S^{ijk} + S^{jki}).$$ (9)
The obtained new "pair" \((sT^i, 0)\) is the most simple and the most symmetric. Note that the symmetric tensor \(sT^i = sT^i\) gives complete description of matter because the spin density tensor \(cS^{ij}k\) is entirely absorbed into \(sT^i\) during symmetrization procedure.

It is interesting that one can easily generalize the above symmetrization procedure onto a general metric manifold \((M_4, g)\) [9,21] by using Levi-Civita connection associated with the metric \(g\). The generalized symmetrization procedure has the same form as above with replacing \(\eta_{ik} \rightarrow g_{ik}, \ \partial_i \rightarrow _{LC} \nabla_i\).

So, one can always get in a metric manifold \((M_4, g)\) a symmetric energy–momentum tensor \(sT^i = sT^i\) for matter (then, of course, corresponding \(S^{ij}k = 0\)). Observe that the symmetric tensor \(sT^i\), like as in SRT, consists of \(cT^i\) and \(cS^{ikl}\).

The symmetric energy–momentum tensor for matter is uniquely determined by the matter equations of motion [31]. This fact is very important for the uniqueness of the gravitational field equations. Moreover, the symmetric energy–momentum tensor is covariantly conserved.

L. Rosenfeld has proved [32] that

\[
sT^i = \frac{\delta L_m}{\delta g_{ik}},
\]

where \(L_m = L_m(\Psi, _{LC}D\Psi)\) is a covariant Lagrangian density for matter.

The tensor \(sT^i\) given by (10) is the source in the Einstein equations

\[
G_{ik} = \chi sT_{ik},
\]

where \(\chi = \frac{8\pi G}{c^4}\).

Note that these equations geometrize the both canonical quantities \(cT^i\) and \(cS^{ikl} = (-)cS^{kil}\) in a some equivalent way because the tensor \(sT^i\) is built from these two canonical tensors.
So, it is the most natural and most simple to postulate that, in general, the correct energy–momentum tensor for matter is the symmetric tensor $T^{ik}$. This leads us to a pure metric, torsion-free, theory of gravity which has the field equations of the form

$$\frac{\delta L_g}{\delta g_{ik}} = \frac{\delta L_m}{\delta g_{ik}}.$$  \hspace{1cm} (12)

Then, if we take into account the universality of the Einstein equations [33–38], we will end up with General Relativity equations (possible with $\Lambda \neq 0$) which will have a sophisticated, symmetric energy-momentum tensor as a source.

5. A gravitational theory with torsion violates EEP which has so very good experimental evidence (up to $10^{-14}$).

6. In a space-time with torsion a tangent space $T(P)$ cannot be identified with Minkowskian spacetime because there do not exist holonomic coordinates in which $g_{ik}(P) = \eta_{ik}$, $\Gamma^i_{kl}(P) = 0$. So, a gravitational theory with torsion is not a covering theory for SRT. We also lose Fermi coordinates [7,39] in Riemann-Cartan space-time.

7. A low-energetic superstrings gravity needn’t torsion. It uses only metric $g$ and scalar fields and can always be formulated as Einstein theory (in “Einstein frame”) without torsion.

8. A connection having torsion can be determined neither by its own autoparallels (paths) nor by geodesics [7]. So, one cannot determine a connection which has torsion by observation of the test particles (which move along geodesics or autoparallels).

Torsion leads to ambiguities:

1. *Minimal Coupling Principle (MCP) ≠ Minimal Action Principle (MAP)* in a space-time with torsion [29].

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6Fermi coordinates realize in GR a local (freely-falling) inertial frame in which SRT is valid.
MCP can be formulated as follows. In a SRT field equations obtained from the SRT Lagrangian density $L = L(\Psi, \partial_i \Psi)$ we replace $\partial_i \rightarrow \nabla_i$, $\eta_{ik} \rightarrow g_{ik}$ and get covariant field equations in $(M_4, g)$.

By MAP we mean an application of the Minimal Action Principle (Hamiltonian Principle) to the covariant action integral $S = \int_\Omega L(\Psi, \nabla_i \Psi) d^4\Omega$, where $L(\Psi, \nabla_i \Psi)$ is a covariant Lagrangian density obtained from the SRT Lagrangian density $L(\Psi, \partial_i \Psi)$ by MCP.

It is natural to expect that the field equations in $(M_4, g)$ obtained by using MCP on SRT equations should coincide with the Euler-Lagrange equations obtained from $L(\Psi, \nabla_i \Psi)$ by MAP. This holds in GR but not in the framework of the Riemann-Cartan geometry. So, we have there an ambiguity in the field equations.

2. In the framework of the ECSK theory of gravity we have four energy-momentum tensors for matter: Hilbert, canonical, combined, formal [21]. Which one is correct?

3. Let us consider now normal coordinates $NC(P)$ [7,40] which are so very important in GR (See, eg., [41-43]). In the framework of the Riemann-Cartan geometry we have two $NC(P)$: normal coordinates for the Levi-Civita part of the Riemann-Cartan connection $NC(LC_\omega, P)$ and normal coordinates for the symmetric part of the full connection $NC(s_\omega, P)$ [44,45]. Which one has a greater physical meaning?

The above ambiguity of the normal coordinates leads us to ambiguities in superenergy and supermomentum tensors [44,45]. Moreover, the obtained expressions are too complicated for practical using. In fact, we lose here a possibility of effective using of the normal coordinates.

7 Axial torsion removes this ambiguity. By $(M_4, g)$ we mean here a general metric manifold; not necessarily Riemannian.

8 Axial torsion removes this ambiguity.
4. In the framework of the Riemann-Cartan geometry [7]

\[ R_{(ik)lm} = R_{ik(lm)} = 0, \] (13)

but

\[ R_{iklm} \neq R_{lmik}. \] (14)

The last asymmetry leads to an ambiguity in construction of the so-called “Maxwellian superenergy tensor” for the field \( R_{iklm} \) [46]. This tensor is uniquely constructed in GR owing to the symmetry \( R_{iklm} = R_{lmik} \) and it is proportional to the Bel-Robinson tensor [47,48]. In the framework of the Riemann-Cartan geometry the obtained result depends on which antisymmetric pair of the \( R_{iklm} \), first or second, is used in construction.

5. In a Riemann-Cartan space-time we have geodesics and autoparallels (paths). Hamiltonian Principle demands geodesics as trajectories for the test particles. Then, what about physical meaning of the autoparallels? \( ^9 \)

6. In a space-time with torsion we have in fact two kinds of parallell displacement defined by

\[ dv^k = (-)\Gamma^k_{ij} v^j dx^i \] (15)

and

\[ dv^k = (-)\Gamma^k_{ij} v^j dx^i. \] (16)

There follow from that two kinds of absolute (and covariant) differentials

\[ \nabla_i v^k = \partial_i v^k + \Gamma^k_{il} v^l \] (17)

\( ^9 \)Axial torsion removes this problem.
\[ \nabla_i v^k = \partial_i v^k + \Gamma^k_{li} v^l. \]  

(18)

Which one of the two above possibilities could be eventually realized in Nature?

In practice, one must consequently use one of the two above possibilities (or conventions) during any calculations in order to avoid mistakes. For example, in Hehl’s papers

\[ \nabla_i \sqrt{|g|} = 0 \]  

(19)

in the Riemann-Cartan space-time (and it is, of course, correct result). But in the paper [49] you can find

\[ \nabla_i \sqrt{|g|} \neq 0. \]  

(20)

The last result is, of course, uncorrect and it is a consequence of mixing of the two above covariant differentiation.

The other source of the computational mistakes connected with torsion is the following: different Authors use definitions of torsion which differ by sign and by factor 1/2.

### III. CONCLUDING REMARKS

As we have seen, the GR model of the space-time has very good experimental confirmation. On the other hand, torsion has no experimental evidence (at least in vacuum) and it is not needed in the theory of gravity. Moreover, introducing of torsion into geometric structure of the space-time leads us to many problems (apart from calculational, of course). Some of these problems removes an axial torsion \( A_i = \frac{1}{6} \eta_{abc} Q^{abc}, \ Q^{[abc]} = Q^{abc}. \) So, it would be reasonable to confine themselves to the axial torsion only (If one still want to keep on torsion). This is also supported by an important fact that the matter fields (= Dirac’s particles) are coupled only to the axial part of torsion in the Riemann-Cartan space-time. However, if we confine to the axial torsion, then (if you remember
dynamical triviality of torsion and universality of the Einstein equations) we effectively will end up with $\text{GR} + \text{an additional pseudovector field } A_i$ (or with an additional pseudoscalar field $\varphi$ if the field $A_i$ is potential, i.e., if $A_i = \partial_i \varphi$) [29]. But $\text{GR}$ with an additional dynamical pseudovector field $A_i$ yields local gravitational physics which may have both location and velocity-dependent effects [13] unobserved up to now. So, we will finish with the conclusion that the geometric model of the space-time given by ordinary $\text{GR}$ with “wonderful” Levi-Civita connection seems to be the most satisfactory.  

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\footnote{During Symposium LISA 3 (Golm, July 10-14, 2000) I have discussed with many Participants of the Symposium about geometric model of the space-time. They all agreed with this conclusion. See also conclusion about correct theory of gravity and space-time structure given recently in [50].}
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