DEPARTURE FROM GAUSSIANITY OF THE COSMIC MICROWAVE BACKGROUND TEMPERATURE ANISOTROPIES IN THE THREE-YEAR WMAP DATA

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Received 2006 April 3; accepted 2007 April 19

ABSTRACT

We test the hypothesis that the temperature of the cosmic microwave background is consistent with a Gaussian random field defined on the celestial sphere, using the full sky debiased internal linear combination (DILC) map produced from the three-year WMAP data. We test the phases for spherical harmonic modes with $\ell \leq 10$ (which should be the cleanest) for uniformity, randomness, and correlation with phases of the foreground maps. The phases themselves are consistent with a uniform distribution, but the differences between phases (randomness) are not consistent with uniformity. For $\ell = 3$ and $\ell = 6$, the phases of the CMB maps cross-correlate with the foregrounds, suggesting the presence of residual contamination in the DILC map even on these large scales. We also use a one-dimensional Fourier representation to assemble $a_{\ell m}$ into the $\Delta T_{\ell}(\varphi)$ for each $\ell$ mode and test the positions of the resulting maxima and minima for consistency with uniformity randomness on the unit circle. The results show significant departures at the 0.5% level, with the one-dimensional peaks concentrated around $\varphi = 180^\circ$. This strongly significant alignment with the Galactic meridian, together with the cross-correlation of DILC phases with the foreground maps, strongly suggests that even the lowest spherical harmonic modes in the map are significantly contaminated with foreground radiation.

Subject headings: cosmic microwave background — cosmology: observations — methods: data analysis

1. INTRODUCTION

Since the release of the one-year Wilkinson Microwave Anisotropy Probe (WMAP) data (Bennett et al. 2003a, 2003b; Spergel et al. 2003; Hinshaw et al. 2003; Komatsu et al. 2003), great efforts have been made to detect various possible forms of non-Gaussianity in the cosmic microwave background (CMB) temperature fluctuations (Chiang et al. 2003; Coles et al. 2004; Park 2004; Eriksen et al. 2004b; Vielva et al. 2004; Copi et al. 2004; Cabella et al. 2004; Hansen et al. 2004; Mukherjee & Wang 2004; Larson & Wandelt 2004; McEwen et al. 2005; Tojeiro et al. 2006; Dineen & Coles 2006; G tanıga & Wagg 2003).

Three years of data are now available, and methods of foreground cleaning have also been improved, so the WMAP team have produced a new “debiased” version of their internal linear combination (DILC) map, which is claimed to be suitable for analysis over the full sky for spherical harmonic modes up to $\ell \leq 10$ (Spergel et al. 2007; Hinshaw et al. 2006). The statistics of these low-multipole modes provide valuable information with cosmological significance, particularly concerning statistical isotropy (or lack thereof) demonstrated by the CMB.

The three-year data analyzed by the WMAP team is claimed to be Gaussian. Non-Gaussianity, if detected, could result from primordial origin (Bartolo et al. 2004), possible foreground residues left over after cleaning (Naselsky et al. 2003, 2004, 2005b; Dineen & Coles 2004; Naselsky & Novikov 2005), and/or correlated instrument noise. One will have to be very cautious if any non-Gaussianity is detected before attributing it to a primordial origin. The method of using internal linear combination to obtain a reasonably clean CMB signal involves tuning a set of weighting coefficients in order to minimize the variance of the foregrounds. Note that the variance is minimized but not eliminated. The possibility that foreground signals remain in the DILC map to a significant extent is therefore something that should be carefully tested.

In this paper, we apply a series of stringent tests to the Gaussian hypothesis based on the behavior of the phases of the spherical harmonic modes in the data. Since these phases are highly sensitive to the morphology (Chiang 2001) of the temperature pattern, comparison between the phases of the DILC and the derived foregrounds should give a sensitive indication of the presence of contamination. Phase information can also be used to diagnose departures from statistical homogeneity over the celestial sphere.

Using spherical harmonic phases in statistical tests involves some subtleties. For one thing, they are not rotationally invariant. In other words, a different choice of $z$-axis leads to a different assignment of phases for the spherical harmonic modes of the same pattern. This can be dealt with in a number of ways (Coles et al. 2004), but in the present context we choose to fix our coordinate frame as that which makes most sense given the probable behavior of the foregrounds. Throughout this paper, we assume a Galactic coordinate system; all phase information is interpreted relative to this preferred frame. In doing this we attempt to ensure that the detection of nonuniformity or nonrandomness in the phases can be interpreted more simply in terms of Galactic foregrounds. Issues such as the claimed north-south asymmetry (Eriksen et al. 2004a) and the alignment of multipoles (Tegmark et al. 2003; de Oliveira-Costa et al. 2004; Schwarz et al. 2004; Land & Magueijo 2005), which seem to persist in the WMAP three-year data, are also measured in Galactic coordinates, so we hope to shed some additional light on these peculiarities. Owing to the visual similarity of one-year ILC and three-year DILC maps, we subject both maps to our analysis.
THE GAUSSIAN-RANDOM HYPOTHESIS AND THE CMB

The statistical characterization of CMB temperature anisotropies on a sphere can be expressed as a sum over spherical harmonics:

\[
\Delta T(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi),
\]

(1)

where the \(Y_{\ell m}(\theta, \phi)\) are spherical harmonic functions, defined in terms of the Legendre polynomials \(P_{\ell m}(\cos \theta)\) using

\[
Y_{\ell m}(\theta, \phi) = (-1)^m \sqrt{\frac{(2\ell + 1)(\ell - m)!}{4\pi(\ell + m)!}} P_{\ell m}(\cos \theta) \exp(i m \phi),
\]

(2)

and the \(a_{\ell m}\) are complex coefficients, which can be expressed as \(a_{\ell m} = |a_{\ell m}| \exp(i \phi_{\ell m})\). In standard cosmological models (i.e., those involving the simplest forms of inflation), these fluctuations constitute a realization of a statistically homogeneous and isotropic Gaussian stochastic process, or random field, defined over the celestial sphere (Bardeen et al. 1986; Bond & Efstathiou 1987). The formal definition of such a Gaussian random field requires that the real and imaginary parts of the \(a_{\ell m}\) be independent and identically distributed according to a Gaussian probability density, so that the moduli \(|a_{\ell m}|\) have a Rayleigh distribution and the phases \(\phi_{\ell m}\) are uniformly random on the interval \([0, 2\pi]\). The central limit theorem virtually guarantees that the superposition of a large number of harmonic modes will tend to a Gaussian as long as the phases are random, and this furnishes a weaker definition of Gaussianity. Moreover, statistical isotropy in general manifests itself in phase properties. Because of the importance of phases in both these definitions, we focus on their measured properties as probes of departures from Gaussianity. Many methods have been proposed to test the random phase hypothesis (Chiang et al. 2002, 2004; Dineen et al. 2005; Stannard & Coles 2005; Naselsky et al. 2004, 2005a; Chiang & Naselsky 2007).

In Figure 1, we plot on the Argand (complex) plane the \(a_{\ell m}\) of the DILC map for \(\ell \leq 10\) (amplitudes \(|a_{\ell m}|\) in units of \(\mu K\)). Because of the conjugate properties of the \(a_{\ell m}\) for a real sky signal, we plot only \(a_{\ell m}\) modes with \(m > 0\) and omit all \(m = 0\) modes. Note that 11 of the 14 phases of \(2 \leq \ell \leq 5\) are in the first two quadrants, among which 5 are clustered near 0 or \(\pi\).

![Figure 1](image1.png)

Figure 1.—The \(a_{\ell m}\) for \(\ell \leq 10\), plotted on the Argand plane. The amplitudes are in units of \(\mu K\), and only \(m > 0\) modes are displayed. **Left panel:** The plus signs, triangles, squares, and stars represent \(\ell = 2\)–5, respectively. **Right panel:** The diamonds, plus signs, triangles, squares, and stars represent \(\ell = 6\)–10, respectively. Note that 11 of the 14 phases of \(2 \leq \ell \leq 5\) are in the first two quadrants, among which 5 are clustered near 0 or \(\pi\).

### 3. TESTING THE RANDOM-PHASE HYPOTHESIS

Since throughout this paper all the tests are of circular nature, we here introduce the Kuiper statistic (KS; Kuiper 1960) to test

#### TABLE 1

| Map     | \(\phi_{\ell m}\) of \(\ell \leq 5\) (%) | \(\phi_{\ell m}\) of \(6 \leq \ell \leq 10\) (%) | All \(\phi_{\ell m}\) of \(\ell \leq 10\) (%) |
|---------|--------------------------------------|-----------------------------------------------|------------------------------------------|
| ILC     | 52.60                                | 90.91                                          | 80.62                                    |
| DILC    | 43.46                                | 98.09                                          | 74.58                                    |

![Figure 2](image2.png)

Figure 2.—Significance levels of randomness with separation (\(\Delta \ell, \Delta m\)). The thick and thin lines are from the three-year DILC and one-year ILC maps, respectively.
the random phase hypothesis. The KS can be viewed as a variant of the standard Kolmogorov-Smirnov test, designed to cope with circular data. The standard Kolmogorov-Smirnov statistic is taken as the maximum deviation of the cumulative probability distribution being compared. For a circular function, however, one needs to take into account the maximum deviation both above and below the cumulative probability being compared. Given a set of circular variables \( X_i \) normalized in \([0, 1]\) and sorted in ascending order, where \( i = 1, \ldots, N \) (e.g., phases defined in \([0, 2\pi]\)) can be normalized by \([2\pi]^{-1}\), one can compare it with any cumulative probability functions \( P(\theta) \). For a uniformly random distribution of \( P(\theta) = 0 \) where \( \theta = 0, 1, \ldots, \), the phases seem to be concentrated in the first two quadrants. This is, however, only significant at the 43% level. Overall, therefore, the phases of the three-year DILC map for \( \ell \leq 10 \) are consistent with uniformity (\( \alpha = 0.75 \)).

We now check the randomness of the phases, which is tested by defining a set of differences, \( \Delta \phi_{\text{dilc}}(\Delta \ell, \Delta m) = \phi_{\text{dilc}}^{\Delta \ell, \Delta m} - \phi_{\text{FG}}^{\Delta \ell, \Delta m} \). To obtain the phase difference \( \Delta \phi_{\text{dilc}}(\Delta \ell, \Delta m) = (1, 0) \), for example, we take the phase differences of modes between \( (2, 1) \) and \( (3, 1) \), between \( (3, 1) \) and \( (4, 1) \), and between \( (9, 1) \) and \( (10, 1) \), then between \( (2, 2) \) and \( (3, 2) \), between \( (3, 2) \) and \( (4, 2) \), and between \( (9, 2) \) and \( (10, 2) \), then between \( (9, 9) \) and \( (10, 9) \). We then examine the collection of these phase differences with Kuiper statistics. If the phases are random with no clean modes (i.e., \( \ell \leq 10 \)), the phases of the \( m = 0 \) modes are excluded in all our tests. Table 1 shows that for the three-year DILC, the phases are consistent with a uniform distribution (\( \alpha \sim 0.98 \)) for \( 6 \leq \ell \leq 10 \), but not for \( \ell \leq 5 \) (\( \alpha \sim 0.43 \)). This is consistent with the behavior of the phases seen in Figure 1: the phases seem to be concentrated in the first two quadrants. This is, however, only significant at the 43% level. Overall, therefore, the phases of the three-year DILC map for \( \ell \leq 10 \) are consistent with uniformity (\( \alpha = 0.75 \)).

In standard frequentist fashion, we define the significance level \( \alpha \) (or \( p \)-value or \( "size" \)) for our "null" hypothesis as the probability of the measured value of \( V \) arising under the null hypothesis. For a null hypothesis (in our case the random-phase hypothesis), \( \alpha \), a measure of statistical confidence, is simply the probability that a hypothetical measurement of our test statistic would exceed the observed value if the null hypothesis were true. It is constructed to account for sampling uncertainties. In this case, \( \alpha \) can be calculated from \( \alpha = Q_{\text{Kuiper}}[V(\sqrt{N} + 0.155 + 0.24\sqrt{N})] \) and \( Q_{\text{Kuiper}}(\lambda) = 2 \sum_{j=1}^{\infty} (4j^{2}\lambda^{2} - 1)\exp(-j^{2}\lambda^{2}) \), where \( N \) is the number of data points (Kuiper 1960; Press et al. 1992).

We perform three statistical tests based on this general approach: on the uniformity of phases (i.e., consistency with a uniform distribution on the interval \([0, 2\pi]\)); on the randomness (i.e., independence) of phases by taking the difference of phases with fixed separation \( (\Delta \ell, \Delta m) \); and on the cross-correlation of each \( \ell \) between DILC and the foregrounds by \( \Delta \phi_{\text{FG}} = \phi_{\text{dilc}} - \phi_{\text{FG}} \). In each case, the resulting angles should be random: the difference between any two random angles is itself a random angle.

We first test the uniformity of the phases for two groups, \( (2 \leq \ell \leq 5) \) and \( (6 \leq \ell \leq 10) \), and for all phases claimed to describe...
association, the phase difference defined this way should distribute uniformly in $[0, 2\pi]$. In Figure 2, we show the significance levels for randomness between phases with different $\ell$ and $m$. There are three separations that show significant departures from uniformity: $(\ell, m) = (0, 0.047, 0.0421, 0.0053$ for $(\ell, m) = (0, 1), (0, 3), and (1, 2)$, respectively. This corresponds to a significant coupling of the phases. It is complicated to understand coupling across both $\ell$ and $m$ for $(\ell, m) = (1, 2)$, because of the lack of rotational invariance described earlier. Nevertheless, because the first two examples involve coupling between azimuthal numbers $m$ within each $\ell$, we plot in Figure 3 the sequences of phase difference $\Delta\phi^{\text{dilc}}(0, 1)$ and $\Delta\phi^{\text{dilc}}(0, 3)$ on a unit circle [or $\exp(i\Delta\phi^{\text{dilc}})$ on an Argand plane]. For both distributions, one can see the deficits around $\Delta\phi = 0$ that cause the significance levels for the cases to appear below 5%. We also include $(0, 2)$ in Figure 3 for comparison, which has the apparent tendency of phase differences to avoid $\pi$, although this is not significant at the 5% level.

We would like to emphasize that the statistics derived from these 35 separations shown in Figure 2 should not be treated as a statistical ensemble. For example, a primordial cosmological magnetic field induces and supports vorticity or Alfvén waves, which induce anisotropies in the CMB, with correlation between $a_{\ell+1, m}$ and $a_{\ell-1, m}$, i.e., $\Delta\ell = 2$ (Durrer et al. 1998; Chen et al. 2004). Therefore, examining $\Delta\ell = 2$ correlation alone is qualified as an independent method. Another example is that symmetric signals defined on the Galactic coordinate system with respect to the meridian ($\varphi = 0$) on a sphere induce correlations between $\Delta\ell = 4$ (Naselsky & Novikov 2005; Naselsky et al. 2006). Therefore, each of the $(\ell, m)$ should be treated as a separate non-Gaussianity test, such as bispectrum, trispectrum, etc.

4. CROSS-CORRELATION BETWEEN THE DILC AND FOREGROUND MAPS

Since the DILC map is obtained from an internal combination of the frequency maps, some foreground residues might be left unsubtracted. Based on the assumption that the CMB signal should not correlate with the foregrounds and that the characteristics of phases reflect the morphology of the CMB anisotropy pattern, we also test the cross-correlation of phases for the DILC and the...
*WMAP* three-year foreground maps at K, Ka, Q, V, and W channels. The foreground maps we test are the sum of the synchrotron, free-free, and dust maps derived from the maximum entropy method (Hinshaw et al. 2006). We take the phase difference \( \Delta \phi_{\ell m} = \phi_{\ell m}^{\text{dilc}} - \phi_{\ell m}^{\text{FG}} \) for each \( \ell \), assuming that \( \Delta \phi_{\ell m} \) at each \( \ell \) should be uniformly distributed. In Figure 4, one can see that for \( \ell = 3 \) and 6, the DILC phases correlate with the foregrounds with significance around 10%. One particular point about the quadrupole is easily seen from Figure 1: two of the three quadrupole phases are near 0 and \( \pi \). Note also that, for \( \ell = 6 \), in Figure 3 (red diamonds), the phase differences for \( \Delta m = 1 \) are strongly clustered.

5. EXTREMUM STATISTICS

The \( \Delta m \) phase coupling in each \( \ell \) leads to a departure from Gaussianity in the resulting signal. Following Chiang & Naselsky (2007), we represent the effect of phase coupling by assembling the \( a_{\ell m} \) (now only single variable \( m \)) with an inverse Fourier transform:

\[
\Delta T_\ell (\varphi) = \sum_m a_{\ell m}^{\text{dilc}} \exp(i m \varphi),
\]

where the negative \( m \) in the sum are replaced with \( a_{\ell m}^{\text{dilc}} \) to ensure that \( \Delta T_\ell \) is real. The morphology of the signal obtained by this method is similar to the signal \( \Delta T_\ell (\varphi) = \sum_{m=-\ell}^\ell a_{\ell m} Y_{\ell m}(\theta, \varphi) \) obtained for each \( \ell \) by summing over all \( \theta \) onto the \( \varphi \)-axis on the map. One should note the following subtleties of such a comparison: the map can be written

\[
\Delta T_\ell = \sqrt{\frac{2\ell+1}{4\pi}} \sum_m a_{\ell m} P_\ell (\cos \theta) + 2 \sum_{m=1}^{\ell} (-1)^m \frac{(\ell - m)!}{(\ell + m)!} |a_{\ell m}| \cos(m \varphi + \phi_{\ell m}) P_\ell (\cos \theta),
\]

The integration of the associated Legendre function is zero for odd \( \ell + m \). When one compares the one-dimensional curve with the composite map for odd \( \ell \), only the odd \( m \) contribute to the integration, while the \( (-1)^m \) reverses the signal as seen from the simple inverse Fourier transform \( \sum_m |a_{\ell m}| \cos (m \varphi + \phi_{\ell m}) \).

Such a one-dimensional construction obviously loses information in one of the available dimensions, but despite the fact that the \( a_{\ell m} \) are spherical harmonic coefficients, the statistics registered in the one-dimensional complex \( a_{\ell m} \) should still manifest themselves in the one-dimensional \( \Delta T \) curves we create. In Figure 5, we plot \( \Delta T_\ell (\varphi) \) summing from the DILC \( a_{\ell m} \) (left panel), and from the whitened DILC: \( a_{\ell m} / |a_{\ell m}| \) (right panel). If the signal is Gaussian, the locations of the highest and lowest peaks should be randomly distributed in \( \varphi \) between \(-180^\circ\) and \(180^\circ\). We plot in Figure 6 the distribution of these extremal locations in a unit circle.

In Table 2, we list the significance of the distribution of the extremal locations. Not only do they show low significance levels of random distribution in \( \varphi \), but the peak locations also cluster in \( \varphi = 180^\circ \). In Figure 4, \( \ell = 3 \) and 6 show significant cross-correlation with the foregrounds. We therefore test the peak distribution by excluding the four extrema belonging to these two modes. Although the \( \alpha \)-values increase slightly, the results are still significant at a level below 5%.

6. CONCLUSION

In this paper, we test the Gaussian random hypothesis of the full-sky map of the CMB temperature anisotropies. The behavior of the three-year DILC does not differ strongly from that of the one-year ILC version: all the famous peculiarities still exist, as mentioned in Spergel et al. (2007). We have found in this paper that this also extends to the behavior of the spherical harmonic phases. In particular, we find that phase differences (which should be uniformly distributed) tend to avoid the region of the complex plane close to \( \Delta \varphi = 0 \). We also find that the phases for \( \ell = 3 \) and \( \ell = 6 \) are significantly correlated with those of the foreground maps. We also test the real space alignment of the resulting features using the temperature extrema resulting from a Fourier summation for each \( \ell \). The resulting peaks are indeed concentrated opposite the center of our Galaxy, i.e., \( \ell = 180^\circ \) in Galactic coordinates, with respect to which the phases are themselves defined.

On the basis of these results, we reject the null hypothesis that the modes with \( \ell \leq 10 \) are a realization of a statistically homogeneous Gaussian random field at a significance level better than 5%. We also infer that the origin of the observed departures is consistent with being some form of Galactic foreground.

Of course, it is possible that the apparent alignment between the CMB temperature pattern and galactic foreground morphology is simply fortuitous. It could be that large-scale anisotropies in both line up accidentally. If this is the case, then we just happen to live at a place in the universe where our past light cone presents us with this coincidence, and we will just have to cope with it; all future diagnostics of foreground contamination will have to incorporate this alignment as conditioning information. We believe, however, that it is important to take such coincidences very seriously, until we know for certain that that is all they are.

We acknowledge the use of the NASA Legacy Archive for extracting the *WMAP* data. We also acknowledge the use of the HEALPix\(^4\) package (Gorski et al. 1999) to produce \( a_{\ell m} \) from the *WMAP* data and the use of the GLESP\(^5\) package (Doroshkevich et al. 2003).

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### Table 2

**Significance Levels of the Uniformity of the Distribution of the Extrema Locations in \( T_\ell \)**

| Parameter                  | Three-Year DILC (%) | Whitened Three-Year DILC (%) |
|----------------------------|---------------------|------------------------------|
| All peaks \( \ell \leq 10 \)................. | 0.450               | 0.552                        |
| Excluding peaks of \( \ell = 3, 6 \)........ | 2.759               | 4.218                        |

4 See http://healpix.jpl.nasa.gov/.
5 See http://www.glesp.nbi.dk.

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