Optimal Operation for Integrated Electricity and Natural Gas Systems Considering Dynamic Gas Linepack

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Abstract. With the increasing penetration of gas-fired turbines, the efficient and safe operation of power systems depends on the sufficient supply of natural gas. Hence, we proposed a coordinated optimal operation model of integrated electricity and natural gas systems (IENs). The gas linepack is considered by applying dynamic flow models. The non-convex Weymouth equality is converted to a second-order cone inequality constraint and a concave constraint that can be linearized by Taylor expansion. The initial operation point is obtained by optimizing the steady flow model. We proposed an algorithm based on the sequential strategy to solve this mixed-integer second order cone programming (MISOCP). The case study demonstrates that the proposed method can solve the MISOCP effectively and the operation cost can be reduced by taking full use of the gas linepack that is calculated by the dynamic gas flow model.

1. Introduction

At present, the efficient operation of integrated energy systems has caused wide public concern [1]-[2]. The energy flows through different transmission networks such as electricity and natural gas systems are mostly planned and manageddependently [3]. The core component of integrated electricity and natural gas systems (IENs) is gas-fired turbines [4]. It performs better in terms of start-up time, regulation ability, and environmental protection.

The power flow in the distribution network is described by the AC power flow model that can be solved through the Newton-Raphson method or forward and backward propagation [5]. Due to the complex topology, the DC power flow model is often adopted to calculate steady power flow in the power transmission network. The power flow simulation is very mature [6]. For natural gas networks, the gas pipelines can be divided into active pipelines and passive pipelines according to whether the pipelines are equipped with compressors [7]. In the steady-state model, the Weymouth function is used to describe the relationship between the gas flows and the pressure differences of adjacent nodes [8]. The gas flow constraints should satisfy Kirchhoff’s first and second laws. To improve the pipeline transmission capacity, the compressors are equipped and consume gas or electricity to change the node pressures [9]. Based on the steady flow model, the dynamic model takes the fluid characteristics of natural gas into consideration. As the gas can be compressed, the gas network has a certain storage capacity, which provides larger flexibility for operation.

In this paper, an optimal scheduling model of the IENs considering demand response is proposed based on the dynamic gas flow model. The detailed dynamic gas flow is considered to accurately describe the operation state of IENs. To solve the strongly non-convex model, the original problem is relaxed to a second-order cone convex optimization problem which can be solved directly, and the sequential optimization algorithm is used to ensure the strictness of the relaxation. Moreover, the
influence of the dynamic gas model is investigated in improving the operation flexibility of IENs.

2. Problem formulation

2.1. Objective function

The main objective of the presented model is to minimize the operation cost under the premise of meeting the security of IENs. The total operation cost includes startup and operation cost of coal-fired units and gas generation cost:

\[
\min \sum_i \left( \sum_{j \in CU} \left( S_{ij} + f^s_j(p_{ij}) \right) + \sum_{i \in GW} C^\text{gas}_{ij} \cdot W_{ij} \right)
\]

where \(CU\) and \(GW\) are sets of indices of coal-fired units and gas wells, respectively. \(S_{ij}\) and \(f^s_j\) are start-up cost and operation cost of coal-fired unit \(i\); \(C^\text{gas}_{ij}\) is the unit production cost of gas well \(i\); \(W_{ij}\) is the gas production of gas well \(i\) at period \(t\) and satisfies the demand of gas-fired turbines and domestic gas consumption. \(c_{0i}, c_{1i}, c_{2i}\) are cost coefficients of coal-fired unit \(i\). The binary variables \(u_{i,t}\) represents the working state of unit \(i\); \(u_{i,t} = 0\) means that the unit \(i\) is shut down at period \(t\), \(u_{i,t} = 1\) denotes that the unit \(i\) is in operation.

2.2. Power system constraints

\[
\begin{align*}
 u_{i,t} - u_{i,t-1} &= z_{1i,t} - z_{2i,t} \\
 z_{1i,t} + z_{2i,t} &\leq 1 \\
 p_{i,t} &\geq p_{i,t-1} \\
 p_{i,t} &\leq p^\text{min}_{i,t} \leq p_{i,t} &\leq p^\text{max}_{i,t} \\
 p_{i,t} - p_{i,t+1} &\leq r_d \cdot \Delta t \\
 p_{i,t+1} - p_{i,t} &\leq r_u \cdot \Delta t \\
 \left(T_{i,0}^{\text{on}} - T_{,i}^{\text{off}}\right) \cdot (u_{i,t-1} - u_{i,t}) &\geq 0 \\
 \left(T_{i,0}^{\text{off}} - T_{i,0}^{\text{on}}\right) \cdot (u_{i,t} - u_{i,t-1}) &\geq 0 \\
 \sum_{j \in Gen} T_{j,i}^{\text{on}} \cdot p_{i,t} + \sum_{j} T_{j,i}^{\text{line}} \cdot f_{j,i} - d_{i,t} &= 0 \\
 f_{i,j} &= B_i \cdot (\theta_{i,j} - \theta_{i,j}) \\
 -f_{i,j}^{\max} &\leq f_{i,j} \leq f_{i,j}^{\max} \\
 -\theta_{i,j}^{\max} &\leq \theta_{i,j} \leq \theta_{i,j}^{\max} \\
 \theta_{i,j}^{\text{ref}} &= 0
\end{align*}
\]

Equations (3)-(4) are state constraints and specify that the unit cannot be stated and shut down at the same time. \(z_{1i,t}\) and \(z_{2i,t}\) are both binary variables. The unit is in operation when \(z_{1i,t} = 1\), otherwise, \(z_{1i,t} = 0\) and \(z_{2i,t} = 1\). Hence, we have the start up cost of unit \(i\), \(S_{ij} = SU_i\) · \(z_{1i,t}\). (5) is capacity constraint. \(p_{i,t}^{min} / p_{i,t}^{max}\) are minimum/maximum output of unit \(i\). Equations (6)-(7) are the ramping constraints. \(r_d / r_u\) are maximum ramping down and ramping up rates, respectively. \(\Delta t\) denotes the scheduling interval. Equations (8)-(9) are the minimum start-up/shut down time constraints. \(T_{i,0}^{\text{on}} / T_{i,0}^{\text{off}}\) are the minimum up-time and minimum down-time of unit \(i\). \(T_{i,0}^{\text{on}} / T_{i,0}^{\text{off}}\) are continuous up-time and down-time until time \(t\). (10) is the nodal power balance constraint. \(f_{i,j}^{\text{on}} / f_{i,j}^{\text{off}}\) is the power flow on line \(l\); \(d_{i,t}\) is the power load at bus \(j\); \(T_{j,i}^{\text{line}} / T_{j,i}^{\text{line}}\) are correlation coefficients of generator \(i\) and line \(l\) to bus \(j\), respectively. Equations (11)-(14) are the line capacity constraints. \(\theta_{i,j}^{\text{ref}} / \theta_{i,j}^{\text{ref}}\) are voltage phases on both ends of line \(l\). \(f_{i,j}^{\text{max}}\) is the power flow limit of line \(l\); \(\theta_{i,j}^{\text{max}}\)
represents the maximum voltage phases of bus $ls$. $\theta_{\text{ref},t}$ is the voltage phase of reference bus.

2.3. Natural gas system constraints

$$f_{i,t}^{gas} = \beta_{0,i} + \beta_{1,i} \cdot p_{i,t} + \beta_{2,i} \cdot p_{i,t}^2 + G_i^{gas} \cdot z_{i,t}$$  \hspace{1cm} (15)

$$M_{mn,t} = M_{mn,t-1} + g_{mn,t} + g_{mn,t}$$  \hspace{1cm} (16)

$$p_{n,t}^{\text{min}} \leq p_{n,t} \leq p_{n,t}^{\text{max}}$$  \hspace{1cm} (17)

$$\sum_{(m,n)\in L} T_{m,n}^{\text{pipe}} g_{mn,t} + \sum_{i\in GW} T_{\text{gas},i}^{\text{pipe}} f_{i,t} + Q_{n,t}$$  \hspace{1cm} (18)

$$W_{i,t}^{\text{min}} \leq W_{i,t} \leq W_{i,t}^{\text{max}} \quad i \in GW$$  \hspace{1cm} (19)

(15) represents the gas consumption of gas-fired turbines. $f_{i,t}^{gas}$ is the gas consumption of gas-fired turbine $i$ at period $t$. $\beta_{0,i}/\beta_{1,i}/\beta_{2,i}$ are gas consumption coefficients; $G_i^{gas}$ is the gas consumption for turning on the gas turbine $i$. (16) can be used to calculate the gas linepack. According to the operational routine of gas networks, the directions of gas flows are predefined and will not change intra-day. $M_{mn,t}$ is the linepack of pipeline $mn$ at period $t$. $g_{mn,t}$ is gas flow from node $m$ to node $n$ at period $t$. $p_{n,t}^{\text{min}}/p_{n,t}^{\text{max}}$ are minimum and maximum pressure of node $n$. (18) is the nodal gas balance constraint. $\tau_{m,n}, \tau_{m,i}, \tau_{i}^{gas}$ are correlation coefficients of pipeline $mn$, gas well $i \in GW$, and gas-fired turbine $i \in GU$ to node $m$. $Q_{m,t}$ is the domestic gas consumption at node $m$. $L$ represents the set of pipelines. $W_{i,t}^{\text{min}}/W_{i,t}^{\text{max}}$ are minimum and maximum gas generation of gas well $i$, respectively.

The relationship between nodal pressure and gas flow can be described as partial differential equations with respect to time and space. Since the long pipeline will bring large errors to the spatial difference equation, so the virtual nodes can be introduced into the flow model to divided the pipelines into several sections. After applying finite element difference, the dynamic flow equation can be expressed as follows:

$$pr_{m,t+1} - pr_{m,t} = \Delta t \cdot C_{1,mn} \left( f_{mn,t+1} + f_{mn,t+1} \right)$$  \hspace{1cm} (20)

$$\left( f_{mn,t} \right)^2 = C_{2,mn}^2 \left( pr_{m,t}^2 - pr_{m,t}^{\text{max}} \right)$$  \hspace{1cm} (21)

where $C_{1,mn}/C_{2,mn}$ are pipeline characteristic parameters. (20) is a time-dependent difference equation of dynamic gas flow. (21) is a space-dependent difference equation of dynamic gas flow and is also called Weymouth equation.

3. Methodology

The optimization problem of IENs operation can be formulated as follows:

$$\text{Min} \quad (1)-(2)$$  \hspace{1cm} (22)

s.t. power system constraints \hspace{1cm} (3)-(14)  \hspace{1cm} (23)

natural gas system constraints \hspace{1cm} (15)-(21)  \hspace{1cm} (24)

In fact, solving problem (22)-(24) is a challenging task due to the non-convexity in Weymouth equation (21). Hence, the second order cone is often applied to relax the Weymouth function as follows:

$$\left\| f_{mn,t} \cdot C_{2,mn} \cdot \pi_{mn} \right\| \leq C_{2,mn}^2 \cdot \pi_{mn,t}^2$$  \hspace{1cm} (25)

The Weymouth equality constraint is relaxed to an inequality constraint. As a result, this method converts the feasible region of the original problem to a convex region and leads to relaxation gaps. It is an infeasible solution to the original problem if the relaxed solution locates at the relaxation gaps. Hence, we introduce concave constraints to fill the gaps.

$$\left( f_{mn,t} \right)^2 \geq C_{2,mn}^2 \cdot pr_{m,t}^2 - pr_{m,t}^{\text{max}}$$  \hspace{1cm} (26)

The second order cone programming (SOCP) is converted to convex-concave programming. (26) can be linearized by first-order Taylor expansion at the point $[\pi_{mn,t}, \pi_{mn,t}, f_{mn,t}]$.  

\[ 
\]
The production of the gas wells can be obtained by optimizing (22)-(24) without gas flow constraints. Then, calculate steady-state gas flow based on the Newton method and take the results as initial points \( \pi_{n,t}^0, \pi_{n,t}^0, f_{n,t}^0 \). As the dynamic gas model has time coupling constraints, it is necessary to specify the reasonable initial operation state of gas systems. The initial state can be obtained by replacing (20) with (28). It means that the gas linepack is not considered.

\[
f_{mn,t+1} + f_{mn,t+1} = 0
\]

It should be noted that the convergence of the sequential algorithm cannot be guaranteed mathematically because the problem (22)-(24) is a mixed-integer second order cone programming (MISOCP). However, given the relatively small number of integer variables, the integer variables are no longer changed after several initial iterations. Then, we fix these integer variables and convert the MISOCP to standard SOCP to utilize the convergence of convex optimization.

The proposed algorithm has two parts and is summarized in Figure 1.

**Figure 1.** Algorithm flow chart.

### 4. Case Study

Figure 2 shows the network topology of test IENs consisting of a 6-bus power system and a 6-node natural gas system. G4 is a coal-fired unit and G1-G3 are gas-fired units. The parameters of gas-fired turbines are listed in Table 1.

To show the impact of the modeling method on optimization results, we compare three different cases. Case 1: Steady modeling of the gas network; Case 2: Dynamic modeling of gas network without a virtual gas node; Case 3: Dynamic modeling of gas network with virtual gas node.
Figure 2. Network topology of IENs.

Table 1. Parameters of Gas-fired Turbines

| Unit | Max output | Min output | $\beta_0$ | $\beta_1$ | $\beta_2$ | Start-up cost | Ramp rate |
|------|------------|------------|-----------|-----------|-----------|---------------|-----------|
| 1    | 100        | 30         | 0.0004    | 33.5      | 177       | 500           | 50        |
| 2    | 80         | 10         | 0.001     | 42.6      | 130       | 500           | 40        |
| 3    | 80         | 10         | 0.005     | 37.7      | 137       | 500           | 40        |

Figure 3. Results of unit commitment.

Figure 3 shows the operation plan of generating units obtained by different modeling methods. During peak load periods, all four units are supposed to be started up to satisfy the power demand. During other periods when the power load is low, the units with the worse economy are shut down to decrease the operation cost. Compared with these three cases, no matter in which periods, the number of units start up is the largest if the steady modeling method is applied. This is because the IENs do not have enough adjustment ability in the steady flow model. The linepack can be used in the dynamic model so the unit 4 with higher cost can be shut down during periods 2-5 and 20-21. In Case 3, the flow dynamic characteristic is described in more detail than that in Case 2 with the help of virtual nodes. By making full use of the buffer capacity of gas linepack, the coal-fired units can be shut down at periods 1 and 6 in Case 3.

Table 2. Performance comparison among different methods

|         | Total cost ($) | Gas cost ($) | Coal cost ($) | Computation time |
|---------|----------------|--------------|---------------|------------------|
| Case 1  | 230,603        | 192,631      | 37,966        | 10.09            |
Table 2 compares the operation cost and calculation time. It can be observed that the operation costs in Case 2 and Case 3 considering the gas linepack are much lower than that in Case 1. In Case 2, the gas cost is higher than that in Case 1, because the economical gas-fired units take more power load that is supplied by coal-fired units in Case 1. Though the computation time is the longest after introducing virtual nodes, the gas linepack can be fully used to enhance the flexibility of IENs. The number of virtual nodes can be adjusted according to actual demands.

5. Conclusion
A non-convex optimal operation model of integrated electricity and natural gas system is established considering dynamic gas flow. For the convenience of calculation, the second-order cone relaxation is applied to relax the Weymouth equation and the concave constraints are also added to fill the relaxation gaps. A modified sequential optimization algorithm is proposed to ensure the strictness of relaxation. The case study demonstrates that the proposed model is effective in improving the operation flexibility of IENs and the proposed algorithm is feasible to solve this non-convex programming. The linepack can be used rationally to reduce the total operation cost.

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