Theoretical Study of influence of MHD in an infinitely long rough Porous Journal Bearing

G K Kalavathi1,Yuvaraja B K 2, Dinesh P A3 ,Vasundhara M G 4
1 Department of Mathematics, Malnad College of Engineering, Hassan, Karnataka, India
2 Department of Mathematics, B G S Institute of Technology, Mandya, Karnataka, India
3 Department of Mathematics, M. S. Ramaiah Institute of Technology, Bangalore, Karnataka, India
4 Department of Mechanical Engineering, Malnad College of Engineering, Hassan, Karnataka, India

Abstract. Analytical study of porous rough journal bearing with magneto hydrodynamic fluid in the presence of longitudinal and transverse roughness is investigated. Surface roughness effect is studied by using Christensen stochastic theory. The generalized Reynolds equations for both types of roughness patterns are derived. Reynolds boundary conditions are used to study the characteristics of infinitely long journal bearing. Cavitation angle is obtained by using bisection method. The expressions for pressure, load carrying capacity and attitude angle are obtained. The results are shown in the graph for longitudinal roughness pattern and concluded that, load carrying capacity increases for the increasing values of Hartmann number.

Keywords Porous, Long Journal bearing, Christensen Stochastic roughness theory, Load carrying capacity.

1. Introduction

An analytical work is made on infinitely long journal bearing adopting an electrically conducting lubricant with magnetic field applied to the surfaces of bearing. M I Anwar and C M Rodkiewicz [1] investigated non uniform magnetic field effects in MHD slider bearing, a result indicate that increase of Hartmann number decreases inertia contribution but increases the load carrying capacity. The roughness effects in porous journal bearing studied by Gururajan K and Prakash [2-6] and it was found that the roughness structure increases the bearing performance. Numerical study of the MHD Reynolds equation for squeeze film lubrication between two parallel surfaces was investigated by Ramesh et al. [7] author shows that the lubricating characteristics of bearing surfaces such as pressure, load carrying capacity increased by Hartmann number. Kalavathi et al. [8-11] studied the influence of roughness on narrow, long and finite journal bearing surface with slip/no-slip and also studied the combined effect of MHD on narrow porous journal bearing surface with a heterogeneous slip/no slip and author concluded that the combined effect of roughness and MHD increases the load carrying capacity. Qiyin et al. [12] investigated the transient behavior of journal bearing with surface structure based on fluid structure interaction approach and found depending on the position of circumferential direction, surface structure increase or decrease the performance of journal bearing. Recently, Yuvaraj et al. [13] have studied the effect of MHD on narrow porous journal bearing of longitudinal roughness pattern and found that introduction of MHD increases the load carrying capacity.

In this paper, influence of MHD on long porous rough journal bearing is presented. Differential equations for two different roughness types like longitudinal and transverse types are derived and the roughness effects are studied by considering Christensen stochastic theory. Reynolds boundary conditions are used to obtain the expressions for bearing characteristics. Cavitation angle is determined by using bisection method, and expectations values are obtained by using Simpsons 1/3rd rule. Load carrying capacity is evaluated by using Gaussian Quadrature 16 point formula. In this paper the characteristic of infinitely long porous journal bearing like load carrying capacity for longitudinal roughness type is studied theoretically.
2. Analytical approach

The physical configuration of the rough porous journal bearing is as shown in the figure. Consider the journal bearing and shaft rotating with velocity $U$, eccentricity ratio $\varepsilon^*$, $H_0^*$ is the wall thickness of the porous media with a lubricant of constant viscosity.

The governing equations for MHD fluid are given by [1]

$$\frac{\partial^2 u^*}{\partial y'^2} - \left( \frac{B_0^H}{h_m^H} \right)^2 u^* = \frac{1}{\gamma^*} \frac{\partial p^H}{\partial x'^*}$$

(1)

$$\frac{\partial^2 w^*}{\partial y'^2} - \left( \frac{B_0^H}{h_m^H} \right)^2 w^* = \frac{1}{\gamma^*} \frac{\partial p^H}{\partial z'^*}$$

(2)

$$\frac{\partial p^H}{\partial y'^*} = 0.$$  

(3)

where $B_0^H = \frac{M_0 h_m^H}{\sqrt{\sigma/\gamma^*}}$ is Hartmann number, $M_0$ is applied magnetic field, $\gamma^*$ is the viscosity of the fluid, $\sigma$ is electrical conductivity of fluid, $h_m^H$ denotes the minimum film thickness. $u^*$, $v^*$ and $w^*$ are velocity components along $x^*$, $y^*$ and $z^*$ direction, $p^H$ is pressure in the porous matrix.

The equation of continuity is given by

$$\frac{\partial Q_{u^*}}{\partial x'^*} + \frac{\partial Q_{v^*}}{\partial y'^*} + \frac{\partial Q_{w^*}}{\partial z'^*} = 0$$

or

$$\frac{\partial Q_{u^*}}{\partial x'^*} + \frac{\partial Q_{v^*}}{\partial z'^*} = -\frac{\partial Q_{v^*}}{\partial y'^*}$$

(4)

where, $Q_{u^*} = \int_0^{y'^*} u^* dy'^*$

Solving Equations (1) and (2) for $u^*$ and $w^*$ we obtain

$$u^* = A_1 \cosh \left( \frac{B_0^H}{h_m^H} y^* \right) + A_2 \sinh \left( \frac{B_0^H}{h_m^H} y^* \right) - \frac{1}{\gamma^*} \left( \frac{h_m^H}{B_0^H} \right)^2 \frac{\partial p^*}{\partial x'^*}$$

(5)
\[ w^* = B_1 \cosh \left( \frac{B_{n}^{II}}{h_{m}^{II}} y^* \right) + B_2 \sinh \left( \frac{B_{n}^{II}}{h_{m}^{II}} y^* \right) - \frac{1}{\gamma} \left( \frac{h_{m}^{II}}{B_{n}^{II}} \right)^2 \partial^2 p^* \partial z^*. \]

The boundary conditions for \( u^*, v^* \) and \( w^* \) are

\[ u^*(x^*, 0, z^*) = 0, \quad v^*(x^*, 0, z^*) = -V_0^*, \quad w^*(x^*, 0, z^*) = 0. \]

\[ u^*(x^*, H^*, z^*) = U, \quad v^*(x^*, H^*, z^*) = 0, \quad w^*(x^*, H^*, z^*) = 0. \]

where \( H^* \) is the film thickness, \( V_0^* \) is velocity component in porous media, and is given by [13]

\[ V_0^* = -\frac{K}{\gamma} H_0^* \left[ \frac{\partial^2 p^{II}}{\partial x^*^2} + \frac{\partial^2 p^{II}}{\partial z^*^2} \right], \]

where \( K \) is the permeability.

Applying boundary conditions Equations (7) and (8) in to Equations (5) and (6) we obtain

\[ u^* = \frac{1}{\gamma^2} \frac{h_{m}^{II}}{B_{n}^{II}}^2 \partial^2 p^{II} \partial x^*^2 \left[ \cosh \left( \frac{B_{n}^{II}}{h_{m}^{II}} y^* \right) + \left( \frac{1 - \cosh \left( \frac{B_{n}^{II}}{h_{m}^{II}} H^* \right)}{\sinh \left( \frac{B_{n}^{II}}{h_{m}^{II}} H^* \right)} \right) \sinh \left( \frac{B_{n}^{II}}{h_{m}^{II}} y^* \right) - 1 \right] + \frac{1}{\gamma} \sinh \left( \frac{B_{n}^{II}}{h_{m}^{II}} H^* \right) \sinh \left( \frac{B_{n}^{II}}{h_{m}^{II}} H^* \right), \]

\[ w^* = \frac{1}{\gamma^2} \frac{h_{m}^{II}}{B_{n}^{II}}^2 \partial^2 p^{II} \partial z^*^2 \left[ \cosh \left( \frac{B_{n}^{II}}{h_{m}^{II}} y^* \right) + \left( \frac{1 - \cosh \left( \frac{B_{n}^{II}}{h_{m}^{II}} H^* \right)}{\sinh \left( \frac{B_{n}^{II}}{h_{m}^{II}} H^* \right)} \right) \sinh \left( \frac{B_{n}^{II}}{h_{m}^{II}} y^* \right) - 1 \right] + \frac{1}{\gamma} \sinh \left( \frac{B_{n}^{II}}{h_{m}^{II}} H^* \right) \sinh \left( \frac{B_{n}^{II}}{h_{m}^{II}} H^* \right). \]

Integrating Equations (10) and (11) across the fluid film thickness \( H^* \), substituting in to continuity Equation (4) we obtain

\[ \frac{\partial}{\partial x^*} \left\{ 2 \left( \coth \left( \frac{B_{n}^{II}}{h_{m}^{II}} H^* \right) - \cosech \left( \frac{B_{n}^{II}}{h_{m}^{II}} H^* \right) \right) \frac{\partial p^{II}}{\partial x^*} \right\} + \frac{\partial}{\partial z^*} \left\{ 2 \left( \coth \left( \frac{B_{n}^{II}}{h_{m}^{II}} H^* \right) - \cosech \left( \frac{B_{n}^{II}}{h_{m}^{II}} H^* \right) \right) \frac{\partial p^{II}}{\partial z^*} \right\} = -V_0^* \frac{B_{n}^{III}}{h_{m}^{II}} \frac{\partial}{\partial z^*} \left( \cosh \left( \frac{B_{n}^{II}}{h_{m}^{II}} H^* \right) \right) - \frac{\partial}{\partial z^*} \left( \cosech \left( \frac{B_{n}^{II}}{h_{m}^{II}} H^* \right) \right). \]

\[ \text{3. Generalized stochastic Reynolds’s equation} \]

The geometry of the lubricant film thickness is made up of two parts

\[ H^* = h_{m}^{II} + \Lambda \]

\[ H^* = \Delta \gamma^* (1 + \varepsilon^* \cos \theta^*) + \Lambda (\theta^*, z^*, \xi^*). \]

where \( h_{m}^{II} \) denotes the nominal smooth part of the film thickness, and \( \Delta \gamma^* \) is the radial clearance, \( \varepsilon^* \) is the eccentricity ratio, \( \Lambda \) represents the random part which is regarded as randomly varying quantity with zero mean, \( \theta^* \) is the angular coordinate and \( \xi^* \) is the random variable.

Taking the expected value of Equation (12) and non-dimensionalising, we obtain the modified Reynolds equation as

\[ \frac{\partial}{\partial x^*} \left\{ E[f(N^*, \tilde{H}^*) + \Theta N^*] \right\} \frac{\partial E(p^{II})}{\partial x^*} + \frac{\partial}{\partial z^*} \left\{ E[f(N^*, \tilde{H}^*) + \Theta N^*] \right\} \frac{\partial E(p^{II})}{\partial z^*} = \]

\[ \frac{\gamma U}{(\Delta \gamma^*)^2} N^* \frac{\partial}{\partial z^*} E[g(N^*, \tilde{H}^*)]. \]
where \( f(N^*,H^*) = 2[\coth(N^*H^*) - \cosech(N^*H^*)] - N^*H^*/g^3365^* \) \hspace{1cm} (15)

\[ g(N^*,H^*/g^3365^*) = -[\coth (N^*H^*) - \cosech(N^*H^*)] \hspace{1cm} (16) \]

\[ N^* = B/g^2868^*/g^2922^*/g^2922^*, h/g^2868^*/g^2922^*/g^2922 = h/g^2923^*/g^2922^*/g^2922^* \]

\[ \Delta r^* H/g^3365^* = H^* \Delta r^* \Theta = K H/g^2868^* (\Delta r^*)^3 \]

where \( \Theta \) is the non-dimensionalised permeability parameter

Here \( E(\cdot) \) is the expectancy operator defined by

\[ E(\cdot) = \int (\cdot) f(\cdot/d^47) d\cdot/d^47/d^2998/d^2879/d^2998 \hspace{1cm} (17) \]

where \( f(\cdot/d^47) \) is the probability distribution function for the stochastic variable. Since many of the engineering surfaces shows a roughness height distribution which is Gaussian in nature, the polynomial form of the Gaussian distribution probability density function is \[ f(\cdot/d^47) = \frac{35}{32a/d^2875/d^3435a/d^2870} \cdot \frac{1}{\sqrt{2\pi}d^47^2} \cdot e^{(-\frac{1}{2}(a^2 - \Lambda^2)^2)}, -a < \Lambda < a. \]

\[ = 0 \hspace{1cm} \text{otherwise} \hspace{1cm} (18) \]

where \( a \) is the total ridge of the random film thickness variable and the function terminates at \( a = \pm 3\sigma \) and \( \sigma \) is the standard deviation.

The film thickness of one dimensional longitudinal roughness pattern is assumed to have the form

\[ H^* = h_m^*/g^2923^*/g^2922^*/g^2922^* + (\cdot/d^47, \cdot^*\cdot) \hspace{1cm} (19) \]

\[ \frac{\partial}{\partial x^*}\left\{ E[f(N^*,H^*) + \Theta N^*^3]\right\} \frac{\partial E(p^{\|})}{\partial x^*} + \frac{\partial}{\partial x^*}\left\{ E[1/f(N^*,H^*)]\right\} \frac{\partial E(p^{\|})}{\partial x^*} = \frac{\gamma UN^*^2}{(\Delta r^*)^2} \frac{\partial}{\partial x^*} E[g(N^*,H^*)]. \hspace{1cm} (20) \]

For transverse roughness pattern Equation (22) becomes

\[ \frac{\partial}{\partial x^*}\left\{ E[f(N^*,H^*) + \Theta N^*^3]\right\} \frac{\partial E(p^{\|})}{\partial x^*} + \frac{\partial}{\partial x^*}\left\{ E[1/f(N^*,H^*)]\right\} \frac{\partial E(p^{\|})}{\partial x^*} = \frac{\gamma UN^*^2}{(\Delta r^*)^2} \frac{\partial}{\partial x^*} E[g(N^*,H^*)]. \hspace{1cm} (22) \]

4. Infinitely long porous journal bearing

It is assumed that the bearing is infinitely long in the axial direction, hence no variation of pressure in \( z^* \)-direction is found, therefore \( \frac{\partial p^{\|}}{\partial z^*} \) term can be neglected.

For longitudinal pattern Equation (20) becomes

\[ \frac{d}{d\theta^*}\left\{ E[f(N^*,H^*) + \Theta N^*^3]\right\} \frac{\partial E(p^{\|})}{\partial \theta^*} = \frac{\gamma UN^*^2 d}{(\Delta r^*)^2} d\theta^* E[g(N^*,H^*)]. \hspace{1cm} (23) \]

For transverse roughness pattern Equation (22) becomes

\[ \frac{d}{d\theta^*}\left\{ E[f(N^*,H^*) + \Theta N^*^3]\right\} \frac{\partial E(p^{\|})}{\partial \theta^*} = \frac{\gamma UN^*^2 d}{(\Delta r^*)^2} d\theta^* E[g(N^*,H^*)]. \hspace{1cm} (24) \]

where \( x^* = R^*\theta^* \), \( R^* \) is the radius of the journal
Reynolds boundary conditions are given by
\[ E(p^l) = 0 \quad \text{at} \quad \theta^* = 0. \quad (26) \]
\[ \frac{d[E(p^l)]}{d\theta^*} = 0 \quad \text{at} \quad \theta^* = \theta_c. \quad (27) \]
\[ E(p^l) = 0 \quad \text{at} \quad \theta^* = \theta_c. \quad (28) \]

where \( \theta_c \) is angular extent of the film

5. Longitudinal roughness

Integrating Equation (23) and using the boundary conditions Equations (26) and (27)

The dimensionless mean pressure in the integral form is obtained as
\[ p^* = \frac{E(p^l)(\Delta r)^2}{\gamma' U R^*} = N^* \int_0^{\theta_c} \frac{E[g(N^*,\bar{H}^*)] - E[\{g(N^*,\bar{H}^*)]\theta^* = \theta_c]}{E[\{g(N^*,\bar{H}^*)\theta^* + \Theta N^*\}]} d\theta^*. \quad (29) \]

where \( E[g(N^*, \bar{H}^*)] = \frac{35}{32a^2} \int_{-a}^{a} g(N^*, \bar{H}^*)(a^2 - \Lambda^2)^2 d\Lambda \)

The cavitation angle \( \theta_c \) is calculated by using the boundary condition Equation (28). Therefore, from Equation (29) we have
\[ -E[g(N^*, \bar{H}^*)]\theta^* = 0. \quad (30) \]

Above integral is evaluated to obtain \( \theta_c \) using bisection method

The average load along the line of center \( W_0^* \) is given by
\[ E(W_0^*) = E(W^*)\cos\varphi = -LR^* \int_0^{\theta_c} E(p^l)\cos\theta^* d\theta^*. \quad (31) \]

The non-dimensional form of Equation (31) is
\[ \bar{W}_0^* = \frac{E(W_0^*)(\Delta r)^2}{\gamma' U L R^*} = -\int_0^{\theta_c} p^l \cos\theta^* d\theta^*. \quad (32) \]

The average load capacity along normal to the line of centre \( \bar{W}_n^* \) is
\[ E\left(W_n^*\right) = E(W^*)\sin\varphi = LR^* \int_0^{\theta_c} E(p^l)\sin\theta^* d\theta^*. \quad (33) \]

The non-dimensional form of Equation (33) is
\[ \bar{W}_n^* = \frac{E(W_n^*)(\Delta r)^2}{\gamma' U L R^*} = \int_0^{\theta_c} p^l \sin\theta^* d\theta^*. \quad (34) \]

Integral involved in Equations (32) and (34) are evaluated using Gauss quadrature 16-point formula

The total dimensionless load carrying capacity is given by
\[ W^* = \sqrt{\bar{W}_0^* + \bar{W}_n^* / 2}. \quad (35) \]

Attitude angle \( \varphi \) is given by
\[ \varphi = \tan^{-1}\left(\frac{W \pi}{W_0}\right). \]  

(36)

6. Transverse roughness

Integrating Equation (24) with respect to \( \theta^* \) using the boundary conditions Equation (26) and Equation (27)

The mean pressure in dimensionless form is obtained as

\[ \bar{p}^* = \frac{E(p') (\Delta r)^2}{\gamma U R^*} = N^2 \int_0^{\theta^*} \frac{E [g(N^* R^*)/(N^* R^*)]}{1/[f(N^* R^*)]} + \theta N^3 d\theta^*. \]  

(37)

The angle of cavitation \( \theta_c \) is calculated by using the boundary condition Equation (28)

\[ -E \left[ \frac{g(N^* R^*)/(N^* R^*)}{1/[f(N^* R^*)]} \right] \int_0^{\theta_c} \frac{d\theta^*}{1/[f(N^* R^*)]} + \theta N^3 + \int_0^{\theta_c} \frac{E [g(N^* R^*)/(N^* R^*)]}{1/[f(N^* R^*)]} + \theta N^3 d\theta^* = 0. \]  

(38)

The non-dimensional mean load carrying capacity \( \bar{W}_0^* \) and \( \bar{W}_\pi^* \) are given by Equations (32) and (34). The total dimensionless load carrying capacity and attitude angle are given by Equations (35) and (36).

7. Theoretical case study

In this paper, the bearing characteristics are the functions of permeability parameter \( \Theta = \frac{K H_0^*}{(\Delta r)^3} \), eccentricity ratio, Hartmann number \( N^* = \frac{B_0 H}{l_h t} \) and the roughness height parameter \( \chi = \frac{h^*_R}{\Delta r} \). It may be noted that definite integrals involved in various parts of analysis are evaluated numerically by using Simpson’s 1/3rd rule and Gaussian quadrature formulae. Cavitation angle \( \theta_c \) is obtained by using Bisection method. The results are compared with some of Gururajan and Prakash [12], as \( N^* \rightarrow 0 \) the results of non-magnetic fluid are recovered.

8. Angle of cavitation

Figure 2, explains the angular extent of the film \( \theta_c \) for different values of eccentricity ratio. The roughness parameter is fixed at \( \chi = 0.2 \). It is observed that longitudinal roughness decreases cavitation angle, \( \theta_c \) as compared with smooth. The increasing value of \( \Theta \) increases the \( \theta_c \).

![Figure 2](image_url)

**Figure 2** Angle of cavitation \( \theta_c \) versus permeability parameter \( \Theta \) for various values of \( \varepsilon^* \), \( \chi = 0.2 \) and \( N^* = 1 \).
9. Load Carrying Capacity:
The Variation of load carrying capacity as a function of eccentricity ratio ($\epsilon^*$) for different values of Hartmann number $N^*$ by keeping $\chi = 0.49$ and $\Theta = 0.01$ is shown in the figure 3. The Hartmann number and eccentricity ratio ($\epsilon^*$) increases the load carrying capacities compared to the smooth case. Figure 4 shows that the variation of load capacity with respect to the $N^*$ for different values of eccentricity ratio, with fixed value of roughness and permeability parameters. It is seen that the load increases slowly with increasing value of $N^*$ from 0 to 3, and after that it decreases. The variation of dimensionless load carrying capacity versus the roughness parameter $\chi$ for the various values of the $N^*$ is shown in figure 5. It was observed that load carrying capacity increases with increasing $N^*$ and also it decreases for increasing the value of $\epsilon^*$. Interpretation of the variation of load carrying capacity as a function of $\Theta$ by fixing the other parameters constant is shown in figure 6. The Hartmann number is to increases the load capacity. It is observed that load capacity decreases with increasing value of $\Theta$, this is because permeable surface becomes the main path to pass a fluid through a porous region, this decreases the load carrying capacity and is significantly high for the particular value $\Theta = 0.01$.

10. Attitude angle:
Attitude angle $\varphi$ for various values of $\epsilon^*$ and $\Theta$ is shown in the table 1 by considering $\chi = 0.2$ and $N^* = 3$. It is observed that attitude angle varies insignificantly with increase of $\Theta$ and $\epsilon^*$.

Table 1: Attitude Angle

| $\epsilon^*$ | $\Theta = 0.001$ | $\Theta = 0.01$ | $\Theta = 0.1$ | $\Theta = 1.0$ |
|--------------|-----------------|----------------|----------------|----------------|
| 0.100000     | 68.935165       | 68.762904      | 70.937906      | 69.956387      |
| 0.200000     | 53.571599       | 54.075043      | 53.574444      | 53.163393      |
| 0.300000     | 38.355185       | 40.095019      | 36.373348      | 36.840805      |
| 0.400000     | 22.921001       | 23.455840      | 23.214575      | 23.855422      |
| 0.500000     | 78.896767       | 33.779022      | 13.987312      | 14.431891      |
| 0.600000     | 23.495498       | 18.601278      | 7.306093       | 7.558808       |
| 0.700000     | 15.763951       | 10.265764      | 1.944821       | 2.317761       |
| 0.800000     | 19.718913       | 8.353334       | 10.096708      | -1.962381      |
Figure 3 Load carrying capacity versus $\varepsilon$ for various values of Hartmann Number $N^*$ with $c = 0.49$ and $Q = 0.01$

Figure 4 Load carrying capacity versus Hartmann Number $N^*$ for Various values of Eccentricity $\varepsilon^*$ with $\chi = 0.49$ and $\Theta = 0.01$

Figure 5 Load carrying capacity versus $\chi$ for various values of Hartman number $N^*$ with $\varepsilon^* = 0.5$ and $\Theta = 0.01$

Figure 6 Load carrying capacity versus $\Theta$ for various values of Hartmann number $N^*$ with $\chi = 0.5$ and $\varepsilon^* = 0.49$
11. Conclusion

In this paper, using Christensen stochastic method, a theory was developed to study the influence of MHD fluids in a hydrodynamically lubricated rough porous journal bearing. As an illustration, the case of an infinitely long journal bearing for longitudinal roughness pattern is analyzed. It is observed that the effect of magnetic field is to increase the load carrying capacity. Load carrying capacity is highly significant for the values of $N^* = 3$, $\chi$ between 0.1 to 0.2 and $\Theta = 0.01$. Finally, it is concluded that addition of MHD considerably influences the porous bearing performance.

References

[1] Anwar M I, and C. M Rodkiewicz C M, 1972 Non uniform Magnetic Field Effects in MHD Slider Bearing Journal of Lubrication Tech. ASME, 94 101-105.
[2] Gururajan K, and Prakash J, 1999 Surface roughness effects in infinitely long porous journal bearing Journal Bearing. Tribology Transactions, 121 139-147.
[3] Prakash J, and Gururajan K, 1999 Effect of Velocity Slip in an Infinitely Long Rough Porous Journal Bearing, Tribology Transactions, 42 661-667.
[4] Gururajan K, and Prakash J, 2000 Effect of surface roughness in a narrow porous journal bearing, ASME Journal of Tribology, 122 472-475.
[5] Gururajan K, and Prakash J, 2002 Roughness effects in a narrow porous journal bearing with arbitrary porous wall thickness, International Journal of Mechanical Sciences, 44 1003-1016.
[6] Gururajan K, and Prakash J, 2003 Effect of velocity slip on a narrow porous journal bearing In proceedings of institution of mechanical Engineers, Part J: Journal of Eng. Tribology, 217 59-70.
[7] Ramesh B Kudenetti, and Basti D P, 2012 Numerical solution of the MHD Reynolds equation for squeeze film lubrication between two parallel surfaces Applied Mathematics and Computation, 218 9372-9382.
[8] Kalavathi G K, Gururajan K, Dinesh P A, Gurubasavaraj G, 2014 Effect of surface roughness in a Narrow porous journal bearing with a heterogeneous slip/no-slip surface, International journal of Scientific and Innovative Mathematical Research, 2 944-959.
[9] Kalavathi G K, Dinesh P A, Gururajan K, 2016 Influence of Roughness on Porous Finite Journal Bearing with Heterogeneous slip / no-slip surface, Tribology International, 102 74-181.
[10] Kalavathi G K, Dinesh P A, Gururajan K, 2016 Numerical Study of Effect of Roughness on Porous Long Journal Bearing with Heterogeneous Surface, Journal of Nigerian Mathematical Society. 35 468 – 487.
[11] Kalavathi G K, Dinesh P A, Gururajan K, 2017 Numerical Study of Magnetic Field on Rough Porous Narrow Journal Bearing with Heterogeneous Surface, Materialstoday: Proceedings 4 10539–10543.
[12] Qiyin Lin, Qingkang Bao, Kejia Li, M M, Khonsari, Hong Zhao, 2018 An investigation into the transient behavior of journal bearing with surface structure based on fluid- structure interaction approach”, Tribology international, 118 246-255.
[13] Yuvaraja B K, and Kalavathi G K, Dinesh P A, Gururajan K, 2018 Effects of Magneto Hydrodynamic Fluids in a narrow Porous rough Journal Bearing, IJISM, 6 7-15.
[14] Prakash J, and Vij S K, 1973 Load capacity and time height relations for squeeze films between porous plates, Wear, 24 309-12.
[15] Christensen H 1969 Stochastic model for hydrodynamic lubrication of rough surfaces, Part I. Proceedings of the Institution of Mechanical Engineers, 184 1013-1026.