\( \eta_c' \) Hadroproduction at Next-to-Leading Order and its Relevance to \( \psi' \) Production

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We proceed to the first study of \( \eta_c' \) prompt hadroproduction at next-to-leading order in \( \alpha_s \). Based on heavy-quark-spin symmetry, which is systematically used in quarkonium-production phenomenology, we demonstrate that prompt \( \eta_c' \) can be studied at the LHC with the existing data. We emphasize its relevance to constrain \( \psi' \) production, in the same way as the first prompt \( \eta_c \) data at the LHC lately strongly impacted the phenomenology of \( J/\psi \) studies.

Introduction – Heavy-quark-spin symmetry (HQSS), whereby soft non-perturbative gluon emissions do not flip the spin of heavy quarks, is, at the heart of all phenomenological studies (see [1–3] for reviews) of quarkonium production in nonrelativistic QCD (NRQCD) [4] since more than 20 years. This symmetry sets stringent constraints between the amplitudes of different non-perturbative transitions at work in quarkonium production, encapsulated in the NRQCD Long Distance Matrix Elements (LDMEs). For the color-singlet (CS) transitions, one knows that the spin-triplet and spin-singlet states have the same spatial wave function and hence their LDME are related. Prime examples are the \( \eta_c - J/\psi \) and the \( \chi_{c0} - \chi_{c1} - \chi_{c2} \) systems for which the CS LDMEs are identical up to a mere 2\( \times \)1 factor. Another well known example, central to \( J/\psi \) and \( \psi' \) phenomenology, is that of the \( P \)-wave LDMEs for the Color-Octet (CO) transitions, \( \langle O(7p_{1/2}) \rangle \), which are also equal up to a mere 2\( \times \)1 factor.

Besides, HQSS also very strongly constrains the polarization of the produced quarkonium. Indeed, a heavy-quark pair produced at short distances in a given spin state will remain, by virtue of HQSS, in this very spin state until its hadronization in a quarkonium. Based on this, as early as in 1994, it has been predicted [5] by Cho and Wise that a high-\( P_T \) \( \psi' \) produced by gluon fragmentation should fully inherit its mother-gluon polarization and thus be transversely polarized [in the helicity frame]. The reason why current state-of-the-art NRQCD predictions [5] do not follow this simple trend is not due to an assumed violation of HQSS but to large QCD corrections in the short distance production of the pair, which is not necessarily produced in a spin \( \pm 1 \) state, even at large \( P_T \) as earlier expected based on LO arguments. As for now, all the NLO polarization predictions heavily rely on HQSS.

Moreover, HQSS lately has attracted back the attention of many, following the first experimental study of \( \eta_c \) hadroproduction at the LHC [6]. Indeed, the cross section measured by LHCb was found to be compatible with a negligible contribution of CO transitions. This, in conjunction with HQSS, in turn set extremely stringent constraints on the corresponding CO transitions at work on \( J/\psi \) production [7–9], to such an extent that only one fit [7] currently survives these constraints with a slight tension with the CDF polarization data [10] though.

This potentially casts doubts onto both the universality of the LDMEs and the validity of the factorization conjecture of NRQCD [11–13]. As such, it is of paramount importance to further explore the 2\( S \) charmonium sector to see whether similar tensions occur.

In the present Letter, through a complete NLO computation, we demonstrate that such a first study of prompt \( \eta_c' \) production is within the reach of the LHCb collaboration. We even show that it is the case for a couple of decay channels. We also emphasize how important such a measurement is to advance our understanding of \( \psi' \) production for which, contrary to the \( J/\psi \) case, no \( \epsilon \) and \( \epsilon' \) data are available to feed in NRQCD fits.

\( \eta_c' \) studies at hadron colliders : where do we stand? — Unlike the quarkonium spin-triplet vector states, which can decay into an easily detectable lepton pair, the study of the spin-singlet pseudoscalar states, such as the \( \eta_c \) and its radial excitation, the \( \eta_c' \), currently relies on hadronic decay channels. This makes their detection a real challenge at hadron colliders. As aforementioned, \( \eta_c \) hadroproduction was first studied by LHCb [6]. To do so, they used the decay channel into \( p\bar{p} \) with a branching on the order of \( 1.5 \times 10^{-5} \) [14]; that is 40 times smaller than the corresponding di-muon decay branching of the \( J/\psi \). Beside a smaller branching, such hadronic channels are much more complicated to deal with because of the high level of the combinatorial background to be suppressed by stringent requirements on the particle identification and by limiting the accessible \( P_T \) range, already at the level of the trigger system. In its study, LHCb simultaneously reported on the nonprompt and prompt yields, i.e. the \( \eta_c \) from a \( b \)-hadron decay or not. The former are significantly easier to study since the \( p\bar{p} \) pair is displaced with respect to the primary vertex from which emerges most of the particles constituting the combinatorial background.

More recently, LHCb pioneered again with the first study of \( \eta_c' \) [15] production in exclusive \( b \) decay via the \( \eta_c' \rightarrow p\bar{p} \) decay channel. From the above argument, such a nonprompt...
η′ detection is notably simpler than that of prompt η′, which we wish to motivate in the present Letter. Yet, this first η′ production study is very encouraging and gives us reliable indications on the order of magnitude of some η′ branching. This is crucial in order to assess the feasibility of prompt η′ cross section as we wish to do here.

Indeed, with the measurements of $\langle B(B^+ → η′K^−)B(\bar{q}q → p\bar{p})\rangle = (1.58 ± 0.33 ± 0.09) × 10^{-2}$ and knowing that $B(B^+ → η′K^−) = (3.4 ± 1.8) × 10^{-4}$, $B(B^+ → J/ψ K^+) = (1.026 ± 0.031) × 10^{-3}$ and $B(J/ψ → p\bar{p}) = (2.120 ± 0.029) × 10^{-3}$ [14], one can derive that $B(η′ → p\bar{p}) = (1.0 ± 0.5) × 10^{-4}$, which we will use instead of the current PDG upper limit of $10^{-4}$.

The ratio $\mathcal{B}(η′ → p\bar{p})/\mathcal{B}(η → p\bar{p})$ is thus likely as high as 10 %, which gives us great confidence that prompt η′ studies are within the reach of the LHCb detector with data on tape or to be recorded soon. Moreover, it is reasonable to suppose that the corresponding ratios for the heavier final states $Λ\Lambda$, $Λ^∗Λ^*$, $Ξ\Xi$ would be on the same order than for the η′, if not larger, since, in general, the phase space is relatively larger for η′ compared to η. Given that $B(η → Λ\Lambda) = (1.09 ± 0.24) × 10^{-3}$ and $B(η → Ξ\Xi) = (8.9 ± 2.7) × 10^{-4}$, one can infer that the corresponding branchings for the η′ are expected to also be on the order of $10^{-4}$. As discussed in [16], the latter channels, in spite of the request to find 4(6) tracks, are also promising owing to the presence of (2-4) secondary vertices which drastically reduces the combinatorial background.

Beside the baryon-antibaryon channels, the $φφ$ decay channel is also of interest given the large $φ$ branching to charged $K$ pairs resulting in two narrow and clean peaks [16]. This channel is indeed the first via which η′ production in inclusive $b$ decay was observed by LHCb [17] and they measured $\langle B(b → η′X)B(\bar{q}q → φφ)\rangle = (4.0 ± 1.1 ± 0.4) × 10^{-2}$. Along with $B(η → φφ) = (1.75 ± 0.20) × 10^{-3}$ [14],[We take the “fit” PDG value, rather than the “average” PDG which is much more precise.] and assuming that the $2S/1S$ ratio of the partial $b$ width for the spin-singlet states (η′(ns)) is similar to that of the spin-triplet states (ψ(ns)) [and approximately accounting for the different feed-down structure], we expect $B(η′ → φφ)$ to be 6-7 times smaller, that is on the order of $2.5 × 10^{-4}$. To phrase it differently, the di-$φ$ channel is another serious alternative.

Finally, the $K^∗K^−$ channel with a branching of about 2 % [14] or the $K^+K^−π^+π^−$ one, which should be of a similar magnitude, could be used to increase the reach to large $P_T$ where the combinatorial background is getting less problematic and the statistics small. As we shall explain below, the large $P_T$ region is probably the most interesting. The di-photonic channel $\langle B(η′ → γγ)\rangle = (1.9 ± 1.2) × 10^{-4}$ [14] is a further option to be explored. Yet another method would be to look for η′ or η′ in the already recorded $J/ψ$ sample as suggested in [18]. This would allow one to bypass most of the triggering constraints. Table I summarizes the aforementioned channels and their status.

**Theoretical framework** — Within NRQCD, one factorizes any differential cross section to produce a quarkonium into calculable coefficients related to the production at short distances of the heavy-quark pair in different Fock states and the LDMEs encoding the non-perturbative transitions between these states and the final-state quarkonium. Generally, the partonic quarkonium-production cross section reads

$$\sigma(Q + X) = \sum_n \hat{\sigma}(Q\bar{Q}[n] + X)(O^2(n)), \quad (1)$$

where $n$ labels the different Fock states of the $Q\bar{Q}$ pair which is produced in the partonic scattering. Correspondingly, the $\hat{\sigma}(Q\bar{Q}[n] + X)$ are the short-distance coefficients (SDC), which are perturbatively calculable using usual Feynman graphs. $(O^2(n))$ denotes the nonperturbative –but universal– NRQCD LDMEs. The hadronic cross section is then obtained by folding $\sigma(Q + X)$ with the parton distribution functions (PDFs) as usually done in collinear factorization.

The predictive power of NRQCD relies on the truncation of the sum over only a couple of Fock states $n$, namely those whose LDMEs are expected to be the least suppressed in powers of the velocity, $v$, of the heavy quarks in the rest frame of the pair. Indeed, NRQCD relies on a double expansion in both $α_s$ and $v$. As a case in point, the direct production of η′ receives contributions from four Fock states up to $O(v^5)$, i.e. $1_s^0$, $1_s^1$, $3_s^1$, $1_p^8$.

At hadron colliders, the most common analyzed observable remains the $P_T^2$ differential cross section. Explicit NLO ($α_s^2$) computations of the SDCs in the LHC kinematics show that the sole $3_s^1$ and $3_s^8$ states are relevant to predict it – unless unrealistically large LDMEs are allowed for the other transitions. In addition, we note that for $C = ± 1$ states (unlike the $C = − 1$ states like the $J/ψ$) the leading $P_T^2$ topologies already appear at $O(α_s^4)$, i.e. at NLO in $α_s$. NNLO corrections should thus be mild.

As announced, we have performed our analysis at NLO in $α_s$. To do so, we relied on the FDC framework [19, 20] which generates the Born, real-emission and virtual contributions, ensures the finiteness of their sum, performs the partonic-phase-space integration and that over the PDFs. We also note that the SDCs for η and η′ are equal up to relativistic corrections, suppressed at least as $v^3$.

We then used the LDMEs obtained from the $ψ′$ NLO fits [21–23] and, by virtue of HQSS, set $⟨O^2(3_s^8)⟩ = ⟨O^2(3_s^1)⟩$ (see Table II). For the CS channel, we take $|R(0)|^2 = 0.53$ GeV$^3$ for the radial wave function at the origin. Contrarily to the $J/ψ$ case, these fits cannot rely on $e^+e^−$ or $ep$ data. Polarization data are also more limited which explains that only three NLO fits exist in the literature.

As for the used parameters, we have set $m_t$ to 1.5 GeV and computed the cross section using the renormalization
and factorization scale values, $\mu_R$ and $\mu_F$, within the pairs $(\mu_R, \mu_F) = (1, 1; 0.5, 0.5; 2, 2; 0.5, 1; 1, 0.5; 1, 2; 1, 1)$ with $\mu_0^2 = 4 m_c^2 + P_{T}^2$. The uncertainty on $m_\psi$ is not shown as it is necessarily strongly correlated—unlike the scale uncertainties—to that in the extraction of the LDME with $\psi'$ data. We have used the default PDF sets in FDC, namely CTEQ6M (CTEQ6L1) for the NLO (LO) curves which are the ones so far used NLO LDME fits.

We note that despite this large spread of the $\langle O^{\psi'} (S_0^{[8]}) \rangle$, the 3 fits all reproduce the $\psi'$ LHC data. The variation in the size of $\langle O^{\psi'} (S_0^{[8]}) \rangle$ is compensated by that of $\langle O^{\psi'} (S_1^{[8]}) \rangle$ and $\langle O^{\psi'} (P_0^{[8]}) \rangle$—a well-known phenomenon for the $J/\psi$ which is however constrained by polarization and $ep$ data. In practice, the existing $\psi'$ data do not suffice to lift the degeneracy between these LDMEs driving the $P_T$ behavior of the $\psi'$ differential cross section. As we know, the situation is completely different for the $\eta_c$ where $\langle O^{\eta'} (S_0^{[8]}) \rangle$ is essentially the only relevant CO transition. The course of action happens for the $\eta_c$ differential cross section as we will show now.

**Results and discussion** — Fig. 1 shows our (N)LO cross sections for the direct $\eta_c'$ yield at $\sqrt{s} = 13$ TeV in the rapidity acceptance of the LHCB detector. On purpose, we have not combined the CS and the CO contributions since the latter is essentially unconstrained. The displayed CS curve can be seen as a lower value [24] fit and the CO one—with the chosen LDME value, $\langle O^{\eta'} (S_0^{[8]}) \rangle = 0.0382$ GeV$^3$—is a likely upper limit. In addition to the size of the cross section, on which we elaborate further below, we note that of the impact of the NLO corrections differs in each case. In the CO case, one observes a classical reduction of the theoretical uncertainties along the lines of a good convergence of the perturbative series. In practice the $K$ factor happens to be slightly below unity at large $P_T$. In the CS case, the main effect is the expected appearance of new leading $P_T$ topologies which results in a $K$ factor increasing with $P_T$ (see [18, 25–28] for analogous cases). Since the leading-$P_T$ CS contributions eventually consist in $\alpha_s^3$. Born topologies, the $\mu_R$ sensitivity also naturally increases in the absence of the corresponding virtual corrections to these topologies which only appear at $\alpha_s^2$.

In addition, we note that the large difference between the CS and CO curves reinforce our confidence in the discriminating power of forthcoming $\eta_c'$ data, even with a moderate precision, especially above $P_T = 10$ GeV where the $S_1^{[8]}$ channel should dominate.

In fact, data already on tape or to be recorded very soon most probably offer enough precision in this region. Fig. 2 shows the projected statistical uncertainties expected with the central value of theoretical range for each LDME fit. It has been derived with a detection efficiency of 2% [29] in the $p\bar{p}$ decay channel and with a luminosity—already collected at $\sqrt{s} = 13$ TeV—of 1.5 fb$^{-1}$. Clearly, the $\eta_c'$ cross section should be measurable up to $P_T = 20$ GeV with a decent precision, and significantly further if other channels can be used as already done for the nonprompt sample.

Going further, one sees that the expected cross sections significantly differ depending on the $\psi'$ fit used, compare e.g. Fig. 2b and Fig. 2c. It is clear that forthcoming $\eta_c'$ data will significantly narrow down the allowed range for $\langle O^{\psi'} (S_0^{[8]}) \rangle$, and thus of $\langle O^{\psi'} (S_1^{[8]}) \rangle$. In fact, if the measured cross section coincides with that predicted based on the Bodwin et al. $\psi'$ fit, it would virtually exclude the Gong et al. $\psi'$ fit and reduce by an order of magnitude the range of the Shao et al. $\psi'$ fit. Conversely, since the CS cross section [with a vanishing CO LDME] nearly coincides with the central value of the Gong et al. band, one also concludes that a cross section compatible with the CS contribution alone—as for the $\eta_c$ [6]—would have the statistical power to exclude the Bodwin et al. fit as well as to significantly reduce the range of the Shao et al. fit. To phrase it differently, a measurement compatible with the CS contribution alone directly sets an upper limit on $\langle O^{\psi'} (S_1^{[8]}) \rangle$.

**Conclusions.**— We have performed the first—and complete—one-loop analysis of $\eta_c'$ hadroproduction. We have focused on the LHC case, in particular on the potential of the LHCB experiment to measure its cross section in a $P_T$ range which would severely constrain—by virtue of HQSS—

| $\langle O^{\psi'} (S_1^{[8]}) \rangle$ | $[0.3, 82]$ | $[-0.881, 0.857]$ | $[2.35, 3.93]$ |
|---|---|---|---|

**TABLE II.** Allowed range of the LDME $\langle O^{\psi'} (S_1^{[8]}) \rangle$ from 3 existing NLO fits of $\langle O^{\psi'} (S_0^{[8]}) \rangle$.
ψ’ theoretical predictions.

After discussing the usable η’ decay channels, we have demonstrated that such studies are well within the reach of LHCb already with data on tape with a channel which they already studied for the nonprompt yield. Given the smaller ψ’ data set compared to the J/ψ ones and the quasi absence of feed down effects, we expect that the impact of such a measurement will surpass that of the first LHCb study of ηc production at 7 and 8 TeV and to impact the field at large.

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