Simulation Study on Circular Search and Capture Probability of Certain Torpedo

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Abstract. The influence of the distance between target and torpedo water entering point and target heading on the airdrop torpedo trajectory and acquisition probability are studied and analyzed through modeling and simulating the torpedo underwater target search and attack. In such three cases as the same target board angle different range and different target angle the same target range and different angle different target range, the trend of target acquisition probability are put forward, which can provide some theoretical references for the actual operational research.

Keywords: Torpedo, Circular Search, Capture Probability, Simulation.

1. Introduction
The typical working process of acoustic homing torpedo mainly includes launching into water, searching, capturing and attacking, re-searching and re-attacking. Each torpedo has different ballistic form in different working stages. As soon as the acoustic homing device is turned on, target search starts. The search segment ballistic forms are various, such as direct flight search, circular search, serpentine search and trapezoidal search. Among them, circular search trajectory is one of the typical search trajectories used by acoustic homing torpedoes. In order to study whether torpedo can effectively capture submarine target in the search process, the premise is to establish the search trajectory model and capture probability model. Taking an anti-submarine torpedo as an example, this paper analyzes and studies the circular ballistic search model of acoustic homing torpedo according to its basic search process and technical performance, and discusses the calculation method of its capture probability on the basis of establishing the search model.

2. Underwater Torpedo Ballistic Trajectory Model
When the helicopter is hovering, the torpedo performs a “dive + circle” search trajectory. After the torpedo entering the water dives to the preset depth, it will conduct the active homing direct navigation search trajectory. If no target is found during the direct navigation, the torpedo will enter the circular search after reaching the expected meeting point and will stay there until the target is captured or the voyage ends. All heading angles are based on true north (0°), and rotation to the right is positive; Torpedo entry coordinates \((x, y)\), water entry moment \(t\);The torpedo course \(\beta\) is within range...
When estimating torpedo trajectory, calculate from the water entry point in the ballistic trajectory, start time $t_0$ and position $(x_0, y_0)$.

### 2.1. Dive Ballistic Trajectory

Initial depth of dive trajectory $H_0$, dive angle $\alpha_0$, initial course $\beta_0$, dive speed $\nu$, search depth $H_1$, time depth is $H_1 \leq H_0$.

The torpedo's position at $t_0 + \Delta t$ is:

$$
\begin{align*}
    x_t &= x_0 + \nu \Delta t \cos \alpha_0 \sin \beta_0 \\
    y_t &= y_0 + \nu \Delta t \cos \alpha_0 \cos \beta_0
\end{align*}
$$

The torpedo's depth at $t_0 + \Delta t$ is:

$$
H_t = H_0 + \nu \Delta t \sin \alpha_0
$$

At $H_t \geq H_1$, after reaching the search depth through passing $T$, torpedo motion coordinate $x_T, y_T$ stops diving and enters into the active circular search trajectory.

### 2.2. Circular Search Trajectory

Circular search refers to the torpedo's circular motion in the XOY coordinate plane, in which the speed of the torpedo is fixed and the direction of the torpedo heading angle changes at a constant speed. The initial course of the circular trajectory is $\beta_0$, the angular velocity is $\omega$, and the linear velocity is $\nu$. Since $\omega$ is a smaller value and $\beta_0$ is a value less than $2\pi$, $\beta_0 + \omega t$ will not jump from $(0, 2\pi)$ to $(4\pi, 6\pi)$ or $(-4\pi, -2\pi)$, so $\beta_0 + \omega t$ is only considered within the case of $(-2\pi, 4\pi)$. Taking the torpedo's left-hand direction as an example, the course of the torpedo at time $T + \Delta t$ is:

$$
\begin{align*}
    \beta_i &= \beta_0 - \omega \Delta t & 0 \leq \beta_0 + \omega \Delta t \leq 2\pi \\
    \beta_i &= \beta_0 - 2\omega \Delta t & \beta_0 + \omega \Delta t \geq 2\pi \\
    \beta_i &= \beta_0 + 2\omega \Delta t & 0 \geq \beta_0 + \omega \Delta t
\end{align*}
$$

The torpedo position $(x_i, y_i)$ at the moment $T + \Delta t$ is:

$$
\begin{align*}
    x_i &= x_T + r \sin(\beta_0 - 90^\circ) + r \sin(\beta_i + 90^\circ) \\
    y_i &= y_T + r \cos(\beta_0 - 90^\circ) + r \cos(\beta_i + 90^\circ)
\end{align*}
$$

The torpedo conducts a circular search at depth $H_1$ until either the target is captured or the voyage is over.

### 3. Capturing the Target Model

In the circular search of acoustic homing torpedo, if the following two conditions are met, the torpedo can be considered to have captured the target (horizontal search is only considered here): First, the distance (range) between torpedo and the target should be less than the distance of torpedo's homing action. The other is that the position of the target relative to torpedo should be between the homing sector angles. Schematic diagram of torpedo acquisition target is shown in Figure 1.
Figure 1: Schematic diagram of torpedo acquisition target

Distance constraint:

\[ D_s = \sqrt{(X_M - X_T)^2 + (Y_M - Y_T)^2} \quad (D_s \leq R_{zd}) \]  

(5)

Orientation constraint:

\[ \beta = \cos^{-1} \left( \frac{X_M - X_T}{D_s} \right) \quad (-\gamma \leq \beta \leq \gamma) \]  

(6)

Where, \( D_s \) is the distance between the torpedo and the target (range); \( R_{zd} \) is the distance of torpedo homing action; \( \beta \) is the target board angle; \( \gamma \) is the torpedo homing sector angle. Under normal circumstances, the homing range of this type of torpedo \( R_{zd}=1000 \text{m} \), homing sector angle \( \gamma = \pi / 18 \).

4. Simulation Results and Analysis

When it is necessary to adopt simulation method to make statistics on the acquisition probability of torpedoes under different attack and defense situations, a large number of analytical calculations are carried out on the computer software for implementing simulation, obtaining statistical results, and drawing a conclusion.

The simulation process of simulation method is shown in Figure 2. After the initialization of MATLAB software and the simulation calculation, it can be obtained as follows:

After the initialization of MATLAB software and the simulation calculation, it can be obtained as follows:

The acquisition probability of torpedoes at different range at the same target board angle is shown in FIG. 3: X-coordinate is the torpedo firing range and y-coordinate is the acquisition probability P. The overall curve shows a downward trend. When the torpedo range is at 600m, the torpedo's acquisition probability of the target is as high as 100%. With the continuous increase of the range, the torpedo acquisition probability gradually drops to 0.
input basic parameters

calculate mean square error

firing range: $D_s(i) = 0:1:2000$
For $i=1:2000$
target board angle: $Q_m(l) = 0:0.1:2\pi$
For $l=1:63$

generate random entry points: $(X_0, Y_0)$

$n=2000$
For $j=1:2000$

determine the initial position of the target

Simulate the dive section

whether the set depth has been reached

simulate the circular searching segment

$D<r && -r<\beta <\gamma$

capture times: $M=M+1$

capture times: $M=M$

capture times: $P=M/N$

whether the simulation times are satisfied?

whether the voyage is over?

whether the simulation time has been completed?

plot the capture probability curve

end

start

Figure 2 Simulation flow of simulation method
Figure 3 Relation curves between different firing distances and acquisition probability

The probability of torpedo acquisition under the same range and different target board angle is shown in FIG. 4. The x-coordinate is the target board angle, and the y-coordinate is the acquisition probability P. The overall curve shows a downward trend, and the torpedo's capture probability will be up to 100% when the target board angle is at $-\pi/2$. At the target heading $-\pi/2 \sim -\pi/3$, torpedo acquisition probability of the target decreased gradually from 100% to 87%, and at the target heading $-\pi/3 \sim \pi/6$, torpedo acquisition probability of the target decreased rapidly from 87% to 45%. The faster the target is away from the center of the torpedo circular search trajectory, the less likely the torpedo is to be captured.

Figure 4 Relation curves between different target board angles and acquisition probability

The torpedo acquisition probability varies between different firing range and different target board angle. When the target board range in 0 ~ 2000 m, and the target board angle between 0 ~ 2π, the acquisition probability trend along with the change of target range and target board angle are shown in figure 5 in which the red areas show that torpedo acquisition probability is greater than 80%, dark red areas represent torpedo acquisition probability is greater than 90%, and the blue areas mean torpedo acquisition probability is less than 50%; When the firing range is less than 400m and the target board angle is less than 1.2rad or greater than 5.8rad, the torpedo acquisition probability is high, therefore, when torpedoes are projected, this area should be selected as much as possible. When the range is at 800 ~ 1000m, the probability of torpedo acquisition is low, therefore torpedo projection should avoid this area.
5. Conclusion
Taking an anti-submarine torpedo as an example, the circular ballistic search model of an acoustic homing torpedo is established and analyzed by means of modeling and simulation, based on which, the research method of the probability of the circular ballistic target acquisition are discussed and the influence of the change of torpedo range and target board Angle on target acquisition probability is studied. At last the results are compared and analyzed, which provides valuable reference for practical application of torpedo.

References
[1] Lin Zongxiang, Zhou Ming, Chu Lei, Simulation of acquisition probability of aerial hovering torpedo [J]. Command control and simulation, 2008,30 (5) : 75-77.
[2] Li Changjun, Yu Xueyong. Calculation of anti-submarine hit probability of air-dropped torpedo by simulation method [J]. Firepower and command control. 2008,3 (3): 131-134.
[3] Wang Shunjie, Zhu Weiliang, LI Bin. Research on wire-guided + wake-guided Torpedo Attack Based on past azimuth-guided method. Ordnance Automation,2014, 33 (11): 11-13.
[4] Li Guofu. Research on distance Estimation method of torpedo homing effect. [Master's thesis]. Northwestern Polytechnical University Library.2007.
[5] Li Gangqiang, Huang Wenbin, Overview of wire-guided torpedo guidance methods. Torpedo Technology,2003,11(2):38-42.
[6] Chu Lei, Lou Xiaoping, Li Shiling,.Simulation analysis of influencing factors of Raythe on homing distance. Journal of missile and guidance 2010,30(1): 68-71.
[7] Sun Jian. Mathematical Simulation Research and Implementation of Multi-mode Guidance System. Northwestern Polytechnical University Library.2007.
[8] Li Weibo, Fu diaoping, Liu Bin. (in Chinese) Simulation of line-guided torpedo attack feasible region of current azimuth guidance method. Computer Simulation.2012,29(5):28-32