First principles calculations of nucleon and pion form factors: understanding the building blocks of nuclear matter from lattice QCD

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Abstract. Lattice QCD is an essential complement to the current and anticipated DOE-supported experimental program in hadronic physics. In this poster we address several key questions central to our understanding of the building blocks of nuclear matter, nucleons and pions. Firstly, we describe progress at computing the electromagnetic form factors of the nucleon, describing the distribution of charge and current, before considering the role played by the strange quarks. We then describe the study of transition form factors to the Delta resonance. Finally, we present recent work to determine the pion form factor, complementary to the current JLab experimental determination and providing insight into the approach to asymptotic freedom.

1. Introduction

Measurements from DOE-supported experiments such as those at Jefferson Laboratory, at MIT-Bates, and at BNL, are providing information of an unprecedented quality regarding the quark and gluon structure of hadrons. The only rigorous approach to solving QCD in the low-energy regime to discover the properties of hadrons is lattice QCD. Hence a vigorous program in lattice QCD is an essential complement to both guide and interpret the experimental program. Amongst the key questions which this poster will aim to address are:

- What is the distribution of charge and current within a hadron?
- What is the role of heavy flavors, and in particular strangeness, on these distributions in hadrons composed principally of light quarks?
• What are the shapes of hadrons, and do any of them have deformed intrinsic states?
• At what energy scale does a perturbative description of high-energy phenomena become valid?

These questions go to the heart of our understanding of the strong force.

The challenges and opportunities for lattice QCD encouraged the founding, in 1998, of the Lattice Hadron Physics Collaboration (LHPC), focussed on the nuclear-physics aspects of lattice QCD. The collaboration has benefited from, and contributed to, the SciDAC program in lattice QCD, both through the development of optimized software for lattice QCD, and through the provision of dedicated clusters and the special-purpose QCDOC. Furthermore, an essential part of our current program has been the use of the dynamical gauge configurations generated by the MILC collaboration[1]. This paper will describe both recent progress and near-term plans to address the questions posed above, using the resources that SciDAC has made possible.

The layout of the remainder of the paper is as follows. The next section will describe the calculation of the nucleon electromagnetic form factors. We will then review the contribution of the strange quarks to the form factors, and plans for studying such contributions in a future lattice calculation. We will then proceed to look at transition form factors to the ∆ resonance, and how these form factors provide insight into deformations of the nucleon. Finally, we will show how lattice computations can provide information about other hadrons, in particular the pion.

2. Nucleon electromagnetic form factors

At low momentum transfer \( Q^2 \), the nucleon electromagnetic form factors, \( F_1(Q^2) \) and \( F_2(Q^2) \), describe the distribution of charge and current within a nucleon. In contrast, at high \( Q^2 \), they describe the ability of a nucleon to receive high momentum while remaining in the ground state. Lattice QCD can compute these form factors from first principles.

In our current calculations, we compute the isovector form factors \( F_{1,2}^{u-d}(Q^2) \), the difference between those of the proton and neutron. The preliminary results for the calculation of the Dirac form factor are shown in Figure 1, together with the dipole expectation using the experimentally determined value of the charge radius. The dashed lines represent the dipole fits to the lattice data; the slope gives the charge radius. The right-hand plot of Figure 1 shows the charge radius against the pion mass. Using the form for the chiral extrapolation given in [2], we see that the charge radius approaches the correct value as the pion mass approaches the physical value.

One of the early successes of perturbative QCD was the understanding of how the short-range quark structure of a hadron governs the behavior of exclusive processes at large momentum transfer. However, whereas simple counting rules suggested that \( F_2 \sim F_1/Q^2 \), experimental data from JLab [3] shows that \( F_2 \) falls off much more slowly. Recently it has been shown that the next-to-leading-order calculation yields \( F_2 \sim F_1 \log^2(Q^2/\Lambda^2)/Q^2[4] \), and the agreement between this prediction and the JLab data as \( Q^2 \) approaches 6 GeV\(^2\) is striking. Lattice calculations performed with “heavy” quarks also exhibit this behavior, as illustrated in Figure 2, and an important SciDAC objective is to demonstrate that this feature persists to physical pion masses, and obtain agreement with experiment.

3. Role of strange quarks

The search for evidence of the strange-quark contribution to the structure of the nucleon has been a goal of SAMPLE at MIT-Bates, of HAPPEX and G-Zero at Jefferson Laboratory, and of the A4 experiment at Mainz in Germany. Recently, the G-Zero collaboration has presented evidence that the strange quarks do indeed have a non-zero contribution both to the charge and current distributions in the proton[6], as illustrated in Figure 3. A calculation of the strangeness magnetic moment of the nucleon has been performed by employing an amalgam of lattice QCD
Figure 1. The left-hand plot shows preliminary results for the lattice computation of the isovector form factor $F_1(Q^2)$ at three values of the lattice pion mass; the dashed lines are dipole fits to the data. The solid line is the dipole form using the corresponding experimentally measured charge radius. The right-hand plot shows the preliminary determination of the charge radius, together with a chiral fit to the data[2]; the experimental value is shown as the burst.

Figure 2. The electromagnetic form factor ratio $\frac{Q^2 F_2(Q^2)}{\log^2(Q^2/\Lambda^2) F_1(Q^2)}$, plotted with $\Lambda = 300$ MeV. The data were obtained on approximately 200 SESAM full QCD configurations[5], and the circles and squares denote quark masses corresponding to pion masses of 897 and 744 MeV respectively.

calculations, the constraints of charge symmetry, and phenomenological input[7]; \textit{ab initio} lattice QCD computations have been in the quenched approximation[8, 9]. Computations at light quark masses in full QCD are an approved part of the LHPC program, using the JLab QCD clusters, and at BNL.

4. $\gamma N \rightarrow \Delta$ transition form factors
Electric and Coulomb quadrupole transition form factors, $G_E$ and $G_C$, are signatures of deformation in the nucleon or delta. More generally, the study of the transition form factors to a resonance can yield information about the production mechanism, and hence guide experiment. Recent quenched QCD calculations have yielded non-zero form factors $G_E$ and $G_C$, in qualitative agreement with experiments at JLab and Bates[10], as shown in Figure 4. The LHPC is now performing full QCD calculations with light quark masses.
Figure 3. The world data for strange-quark contribution to the distribution of charge and current in the nucleon[6]; the ovals represent one- and two-standard-deviation contours respectively.

Figure 4. The left- and right-hand plots show lattice calculations of the ratios \( R_{EM} \equiv -G_E/G_M \) and \( R_{SM} \equiv G_C/G_M \), respectively, obtained in the quenched approximation to QCD[10]. The experimental points are from data taken at JLab and at MIT-Bates.

5. Pion form factor
The electromagnetic form factor of the pion, the lightest and simplest hadron, is often considered a good observable for studying the onset, with increasing energy, of the perturbative QCD regime for exclusive processes. Furthermore, its asymptotic normalization can be determined from pion decay. A summary of the experimental measurements of the pion form factor is shown in Figure 5, together with projected range of data from Jefferson Laboratory at 6 GeV and at 12 GeV; a word of caution should be noted in that, except for the data at the very lowest \( Q^2 \), the data is obtained from quasi-elastic scattering from a virtual pion in the proton, necessitating an extrapolation to the expected scattering for on-shell pions. Shown as the right-hand plot in Figure 5 is the full QCD lattice calculation, at two values of the lattice pion mass[11]. The
Figure 5. The left-hand plot shows a summary of experimental data for the pion form factor; the shaded areas are anticipated future results, and the lines correspond to various theoretical expectations. The right-hand plot shows the lattice determination at two values of the lattice pion mass; the shaded bands correspond to the uncertainties on the dipole fits to the lattice data shaded bands correspond to vector-meson-dominance (VMD) fits to the lattice data. The VMD form provides a faithful description of the lattice data for the range over which it has been calculated, with a pole mass in accord with the mass of the lightest vector meson. That the pion form factor is still far from the asymptotic pQCD prediction may be largely ameliorated by an improved choice of strong-interaction scale[12].

6. Conclusions
In this paper, we have demonstrated the utility of lattice QCD in studying the electromagnetic properties of hadrons, and hence gleaning important information about the distribution of charge and current. Future resources will enable us both to refine our calculations, principally by decreasing the pion mass, and extend the range of quantities accessible.

Acknowledgments
This work was supported in part by DOE contract DE-AC05-84ER40150 under which the Southeastern Universities Research Association (SURA) operates the Thomas Jefferson National Accelerator Facility. We are grateful to the MILC collaboration for access to their dynamical gauge configurations. We thank Tony Thomas and Ross Young for their helpful suggestions.

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