Three-Dimensional Evacuation Model with Applicability Based on the Theory of Cellular Automata

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Abstract. With the increasing number of terror attacks in France, evacuation plans at the Louvre, which receives tens of thousands of visitors every year, has become the key to the safety of visitors. Our goal is to make a reasonable emergency evacuation plan. First, we establish a two-dimensional evacuation model, based on the model, we build a three-dimensional cellular automaton model, which takes the speed of moving up and down the stairs of the occupants, positional attraction and probability of collisions into consideration. Second, we use Pathfinder for simulation. Finally, we find that the exit 1 (secondary entrance) should be opened when the visitor density is high, while the exit 3 should be opened when the visitor density is low; and the Passage Richelieu entrance has the greatest influence on evacuation time, and emergency personnel should choose the Pyramid entrance as their emergency entrance.

1. Introduction
As one of the world's largest and most visited art museums, the Louvre attracts a diversity of visitors around the world [1]. Therefore, the number of guests in the museums varies throughout the day and year, which provides challenges in planning for regular movement within the museum. From students being killed in Toulouse to a soldier attacked outside the Louvre museum in Paris, there have been at least 12 major terror-related incidents in France since 2012 [2]. The increasing number of terror attacks in France requires a review of the emergency evacuation plans at many popular destinations [3]. Due to the endless terrorist attacks, a review of the emergency evacuation plan is essential to make sure that all occupants leave the building as quickly and safely as possible [4].

2. Model Establishment
2.1. Two-Dimensional Evacuation Model

2.1.1. The evacuation space. First, we presume the evacuation area as two-dimensional space and establish the model. Then, we separate the two-dimensional space into several grids of the same size. Each grid is a cell, and all the cells together construct the cellular space [5]. Each cell space can accommodate one person. According to the projection size of human volume, we set the cell size as 0.5m×0.5m.

Cellular space stipulates that each cellular only entrances three situations at each moment: occupied by buildings or obstacles, occupied by personnel and empty.

All the cellular vary from time to time. The cellular state at period of t+1 is related to the cellular
state at period of t and the situation of its neighboring cellular. Therefore, we can use VonNeumann or Moore models. As shown in Figure 1, the black cell is the center cell and the gray cell is its neighbor.

![VonNeumann and Moore Neighborhood Types](image)

**Figure 1.** Cellular Automata Neighbourhood Model

2.1.2. Basic assumptions about people

1. Initial position of a person: From the beginning, a person locates at one of the cells, which can be randomly generated or set based on the reality.

2. Direction of movement: Each individual can move to one of four or eight nearby cells at the next moment, and if one cell is occupied by a barrier or person, no one can enter.

3. Conflict decision: When there are more than one person competing for a cell, it is necessary to predict the entry capability. We use the competitive ability \( C \) to select the best occupier, which more competitive individual have priority access to the grid.

\[
C = \frac{A_i}{D}
\]

\[
A_i = \begin{cases} 
\sqrt{2}, & i = 1 \\
1, & i = 2 \\
\frac{\sqrt{2}}{2}, & i = 3 
\end{cases}
\]

\( A \) is the attribute of evacuation individual, \( A1 \) for young adults, \( A2 \) for old people and children, \( A3 \) for the sick and disabled. \( D \) is the direction value of personnel from the target point.

2.2. Three-Dimensional Cellular Automata Model

Based on the two-dimensional construction, we choose Moore structure of three-dimensional cellular automata. We divide Louvre into a uniform three-dimensional grid and each cell is a basic unit. Every person moves to the 26 adjacent grids or stay in place based on the probability shown in formula 1.

\[
P_{x,y,z} = N_{x,y,z} \times e^{(f_x^x \times f_y^y \times f_z^z \times C_{x,y,z})} \times (1 - f_{x,y,z}) \times O_{x,y,z} \times M_{x,y,z} \times (1 - E_{x,y,z})
\]

\( x, y, z \) is the coordinates of cells, \( P_{x,y,z} \) is probability of movement, \( f_x, y, z \) is free coefficient, \( O_x, y, z \) is obstacle coefficient, \( N_{x, y, z} \) is the normalized parameter.
\( f_{x,y,z} = \begin{cases} 0 & \text{the cell is not occupied} \\ 1 & \text{the cell is occupied} \end{cases} \)

\( O_{x,y,z} = \begin{cases} 1 & \text{the cell is a barrier} \\ 0 & \text{the cell is not a barrier} \end{cases} \)

\[
N_{x,y,z} = \frac{1}{\sum_{x',y',z'} e^{(P_{x',y',z'} + I_{x',y',z'})} \times (1 - f_{x',y',z'}) \times O_{x',y',z'} \times M_{x',y',z'} \times (1 - E_{x',y',z'})}
\]

\[
\begin{cases}
I_c + I_p = 1 \\
I_c \geq 0 \\
I_p \geq 0
\end{cases}
\]

\( P_x, y, z \) and \( I_p \) respectively stand for the position attraction of grid points and its influence coefficient. \( C_x, y, z \) and \( I_c \) respectively stand for the possibility of collision around grid point and its influence coefficient. The higher the collision coefficient \( I_c \) is, the more likely visitors will adopt avoidance strategies to reduce the occurrence of collisions. The lower the position attraction coefficient \( I_p \) is, the more likely people will take the shortest path for evacuation.

Compared with two-dimensional cellular automata, we define the parameter \( E_{x, y, z} \) as whether the cell entrances. The cell \((1, 1, 1)\) does not entrance, so \( E_{1, 1, 1} = 1 \). The parameter \( M_{x, y, z} \) is the movement income of visitors walking up and down stairs and on the same floor. Normally, people tend to walk on the same level, which can save more energy. Then, they choose to go down the stairs, and at last they go up the stairs. As a result, we set the parameter \( M_{x, y, z} \)

\[
\text{velocity} = \begin{cases} 
1.33 & (\text{dis tan ce} = 0.5) \\
1.33\sqrt{2} & (\text{dis tan ce} = 0.4\sqrt{2}) \\
1.33\sqrt{3} & (\text{dis tan ce} = 0.4\sqrt{3})
\end{cases}
\]

\textbf{Figure 2. Three-Dimensional Cellular Automata Moore structure}
Meanwhile, the moving speed of pedestrian walking up and down stairs is different from that of the same flat floor in the three-dimensional space so it is necessary for us to take the change of velocity into account. We set the size of the cell as 0.5×0.5×0.5. The pedestrian moves a cell distance to an adjacent position in the period of a step, which we set as 0.3s. However, the velocity of pedestrians is different because of the varied distance between the adjacent cells.

As shown in Figure 4, we get the following velocity equation:

\[
velocity = \begin{cases} 
1.33 & (\text{dis} \ tan \ ce = 0.5) \\
1.33\sqrt{2} & (\text{dis} \ tan \ ce = 0.4\sqrt{2}) \\
1.33\sqrt{3} & (\text{dis} \ tan \ ce = 0.4\sqrt{3}) 
\end{cases}
\]

According to the following rules, we determine the relationship between each cell and the attraction of the entrance.

The position attraction \(P_{x, y, z}\) of the surrounding walls and the obstacles is 500, which indicates that pedestrians cannot get through.

1. The positional attraction \(P_{x, y, z}\) at the entrance is 0.
2. If the current cell and the adjacent cell are in the same layer, the position attraction of the front, back, left and right adjacent cells will plus 1, and the position attraction of the diagonal adjacent cells will plus \(\sqrt{2}\).
3. If the current cell and the adjacent cell are in the adjacent layer, the position attraction of the front, back, left and right adjacent cells will plus \(2\), and the position attraction of the diagonal adjacent cells will plus \(\sqrt{3}\).
4. Diagonal movement is not allowed at the entrance.
We consider that if there is an emergency that requires security or fire personnel to enter the museum for evacuation, these personnel will occupy an entrance, which will have a position attraction of 1000 for pedestrians and cannot be crossed.

Figure 5. Schematic Diagram of Adjacent Location Distribution of Cells

Figure 6. Schematic Diagram of Position Attraction

The probability of collision:

It is assumed that pedestrian A is located at a cell \((x, y, z)\). The possibility of collision comes from the region of the cube centered at a distance of 1. When other pedestrian B enters the cell, it can be considered that there enterances possibility of collision between pedestrian A and B [6].

We set the probability that pedestrian A and pedestrian B overlap in longitudinal, lateral and vertical directions as \(P_u\) \((u=x, y, z)\). When there is overlap in a certain direction, in other words, pedestrian A and B select the direction at the same time, and the overlap probability is the result of the movement probability of pedestrian A and B in this direction [7].

\[
p_z = p_{x,y,z}(A) \times p_{x,y,z}(B) = \frac{\sqrt{6}}{6} p_{x,y,z}(A) \times p_{x,y,z}(B)
\]

And \(p_x, y, z (A, u)\) and \(p_x, y, z (B, u)\) are the moving probability components in the direction respectively. When the pedestrian steps up and down, it will produce a component in the vertical direction [8]. We assume that both pedestrian A and B are moving towards position \((x, y, z)\) at the same time, we can get
The probability of overlap is

\[ p_{x,y,z}(A,x) = \frac{\sqrt{3}}{3} p_{x,y,z}(A), \quad p_{x,y,z}(A,y) = \frac{\sqrt{3}}{3} p_{x,y,z}(A) \]

\[ p_{x,y,z}(A,z) = \frac{\sqrt{3}}{3} p_{x,y,z}(A), \quad p_{x,y,z}(B,x) = 0 \]

\[ p_{x,y,z}(B,y) = \frac{\sqrt{2}}{2} p_{x,y,z}(B), \quad p_{x,y,z}(B,z) = \frac{\sqrt{2}}{2} p_{x,y,z}(B) \]

The probability of overlap is

\[ p_x = p_{x,y,z}(A,x) \times p_{x,y,z}(B,x) = 0, \]

\[ p_y = p_{x,y,z}(A,y) \times p_{x,y,z}(B,y) = \frac{\sqrt{6}}{6} p_{x,y,z}(A) \times p_{x,y,z}(B), \]

\[ p_z = p_{x,y,z}(A,z) \times p_{x,y,z}(B,z) = \frac{\sqrt{6}}{6} p_{x,y,z}(A) \times p_{x,y,z}(B), \]

Meanwhile, the collision between pedestrian A and pedestrian B is not only related to the spatial position, but also related to the passing time [9]. Therefore, we introduce the time factor into the collision probability. We assume that the relative velocities of pedestrian A and B in the longitudinal, lateral and vertical directions are respectively \( v_x, v_y, v_z \). The average time through the cube is

\[ t_u = \frac{d_u}{v_u}, u = x, y, z \]

The collision probability of pedestrian A and B overlapping in a certain direction is

\[ C_u = t_u \times P_{x,y,z}(A,u) \times P_{x,y,z}(B,u) \]

The collision probability of pedestrian A and B is equal to the sum of the collision probabilities in the three directions.

\[ C_{x,y,z} = \sum_u C_u = \sum_u t_u \times P_{x,y,z}(A,u) \times P_{x,y,z}(B,u) \]

In the evacuation process from time \( t \) to \( t+1 \), pedestrians obey the following evolution rules:

1. At time \( t \), pedestrians will take the maximum probability of movement as the target position at time \( t+1 \). If there are movements with the same multiple maximal probability, pedestrians will select one of them randomly with equal probability.

2. When many pedestrians are competing for the same target position at the same time, the system randomly selects one of them to move to the target position with equal probability at time \( t+1 \), and the rest of them remain motionless [10].

3. If two pedestrians simultaneously take the position occupied by the other side as the target position for the next step, the two pedestrians will exchange positions. Otherwise, they will remain motionless.

4. When the pedestrian moves to an entrance, it will be evacuated from the system at time \( t+1 \).

5. When all pedestrians in the system are evacuated, the simulation ends.

3. Model Solution

Under the simulation of the three-dimensional cell automata, we can get the image shown in Figure 7 and Figure 8.

As shown in Figure 7, we can get conclusion that when the visitor density is constant, the evacuation time is negatively correlated with the width of the entrance within a certain range. The most suitable width of the entrance is 6m. In other words, the evacuation time drops down slowly after the width of the door exceeds 6m. When the width of the entrance is constant, the evacuation time is
positively correlated with the visitor density.

Figure 7. The Relationship between Evacuation Time and Entrance Width under Different Initial Visitor Density

As shown in Figure 8, we can deduce that most people choose Passage Richelieu Entrances which play an important role in the evacuation process. In addition, there are only few visitors’ escape from the Portes Des Lions Entrances.

Figure 8. the Relationship between Flow Rate and Time

4. Simulation
Based on the Google satellite map, we measure the size of the Louvre and use Pathfinder to establish the Louvre building model for simulation, and we divide our simulation results into three parts.

4.1. Diversity of Visitors
We first carried out the simulation with a visitor density of 50 square meters per person (94% for young people, 1% for disabled people, and 5% for old people). We set the main entrance width as 3m. The average speed for young people is 4 m/s, the average speed for disabled people is 1m/s, and the average speed for old people is 2 m/s. Therefore, we analyze the function image of specific evacuation situation and time. The image is shown in Figure 9.
Figure 9. The Relationship between the Remained Visitors and Time

We clearly find that the unreasonable factors affecting evacuation time are caused by the elderly and the disabled, so the evacuation of the elderly and the disabled has a great impact on the final evacuation time.

4.2. The Condition of the Entrances

In order to discuss how to reasonably use the secondary entrances other than the primary ones, we set four secondary entrances based on relevant information to discuss this issue. The specific locations of these four entrances are shown in Figure 10 below.

Figure 10. Four Exit

There are five situations in all. One is the normal situation (the three main entrances are open), the other four is that one of the four exits is open. Under different visitor densities, we carry out many simulation analyses on various situations, and obtained the relationship between various situations and evacuation time.
As shown in Figure 11, we can find that the opening of Entrance 1 is optimal when the visitor density is high. When visitor density is low, opening Entrance 1 or Entrance 3 can reduce evacuation time. As a result, the condition of the secondary entrances depends on the specific situation.

4.3. The Condition of the Emergency Entrance and the Position of the Bottlenecks
Considering the emergency personnel entering the building, here we discuss the main entrances and entrances. When an entrance is used as an entrance for emergency personnel, we regarded it as closed (visitors in the building cannot enter or leave through the entrance). The simulation results are as follows:

We can find that the evacuation time is the shortest when the Pyramid entrance is closed, while the evacuation time is the longest when the Passage Richelieu entrance is closed. Therefore, the entrance of Passage Richelieu entrance has the greatest influence on evacuation time and emergency personnel should choose pyramid entrance as emergency entrance.

![Figure 11. The Relationship between Evacuation Time and Entrance Width](image1)

As shown in Figure 11, we can find that the opening of Entrance 1 is optimal when the visitor density is high. When visitor density is low, opening Entrance 1 or Entrance 3 can reduce evacuation time. As a result, the condition of the secondary entrances depends on the specific situation.

![Figure 12. The Location of Stairways](image2)
We run many simulations to find out where the important bottlenecks are. In order to describe the location of bottlenecks better, we mark the exact location of stairways shown in Figure 12. (The first number indicates the floor followed by a number indicating the ordinal number)

We use density distribution images to select bottlenecks. When there is a bottleneck at a certain location, the density distribution image is shown in red. After observation, we found that the bottleneck entrances in the following positions: Stair- 2_2, Stair-2_3, Stair-2_4, Stair-2_5, Stair-2_6, Stair-2_27, Stair-1_1, Stair-1_2, Stair- 1_3, Stair-1_4, Stair-1_5, Stair-1_14, Stair-1_24, Stair-1_26, Stair-1_27, Stair0_4, Stair0_24, Stair0_26, Stair0_27, Stair1_4, Stair1_31, the Passage Richelieu entrance. The most crowded positions are the Passage Richelieu entrance and Stair-1_14. We list images of the two main bottlenecks and some other bottlenecks.

Figure 13. The Passage Richelieu Entrance

Figure 14. Stair-1_14

Figure 15. Other Bottlenecks

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