HIDING AND CONFINING CHARGES VIA “TUBE-LIKE” WORMHOLES

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Received
Revised

We describe two interesting effects in wormhole physics. First, we find that a genuinely charged matter source of gravity and electromagnetism may appear electrically neutral to an external observer – a phenomenon opposite to the famous Misner-Wheeler “charge without charge” effect. We show that this phenomenon takes place when coupling a bulk gravity/nonlinear-gauge-field system self-consistently to a codimension-one charged lightlike brane as a matter source. The “charge-hiding” effect occurs in a self-consistent wormhole solution of the above coupled gravity/nonlinear-gauge-field/lightlike-brane system which connects a non-compact “universe”, comprising the exterior region of Schwarzschild-(anti-)de-Sitter (or purely Schwarzschild) black hole beyond the internal (Schwarzschild) horizon, to a Levi-Civita-Bertotti-Robinson-type (“tube-like”) “universe” with two compactified dimensions via a wormhole “throat” occupied by the charged lightlike brane. In this solution the whole electric flux produced by the charged lightlike brane is expelled into the compactified Levi-Civita-Bertotti-Robinson-type “universe” and, consequently, the brane is detected as neutral by an observer in the Schwarzschild-(anti-)de-Sitter “universe”. Next, the above “charge-hiding” solution can be further generalized to a truly charge-confining wormhole solution when we couple the bulk gravity/nonlinear-gauge-field system self-consistently to two separate codimension-one charged lightlike branes with equal in magnitude but opposite charges. The latter system possesses a “two-throat” wormhole solution, where the “left-most” and the “right-most” “universes” are two identical copies of the exterior region of the neutral Schwarzschild-de-Sitter black hole beyond the Schwarzschild horizon, whereas the “middle” “universe” is of generalized Levi-Civita-Bertotti-Robinson “tube-like” form with geometry $dS_2 \times S^2$ ($dS_2$ being the two-dimensional de Sitter space). It comprises the finite-extent intermediate region of $dS_2$ between its two horizons. Both “throats” are occupied by the two oppositely charged lightlike branes and the whole electric flux produced by the latter is confined entirely within the middle finite-extent “tube-like” “universe”. A crucial ingredient is the special form of the nonlinear gauge field action, which contains both the standard Maxwell term as well as a square root of the latter. This theory was previously shown to produce a QCD-like confining dynamics in flat
space-time.

Keywords: generalized Levi-Civita-Bertotti-Robinson spaces; wormholes connecting non-compact with compactified “universes”; dynamically generated cosmological constant; wormholes via lightlike branes; QCD-like charge confinement

PACS numbers: 11.25.-w, 04.70.-s, 04.50.+h

1. Introduction

In Ref. 1 ’t Hooft has proposed a consistent quantum description of linear confinement phenomena in terms of effective nonlinear gauge field actions, where the nonlinear terms play the role of effective “infrared counterterms”. In particular, he has argued that the energy density of electrostatic field configurations should be a linear function of the electric displacement field in the infrared region (see especially Eq.(5.10) in Ref. 1), which means that an additional term of the form of square root of the standard Maxwell term should appear in the effective action. The simplest way to realize this idea in Minkowski space-time is by considering the following nonlinear effective gauge field model:

$$S = \int d^4x L(F^2), \quad L(F^2) = -\frac{1}{4}F^2 - \frac{f}{2}\sqrt{-F^2}, \quad (1)$$

with $f$ being a positive coupling constant. Since the Lagrangian $L(F^2)$ in (1) contains both the usual Maxwell term as well as a non-analytic function of $F^2$, it is thus a non-standard form of nonlinear electrodynamics. There are various reasons supporting the natural appearance of the “square-root” Maxwell term in effective gauge field actions besides ’t Hooft’s arguments in Ref. 1. Originally a purely “square-root” Lagrangian in flat space-time $-\frac{f}{2}\sqrt{F^2}$ (in “magnetic”-dominated form) was proposed by Nielsen and Olesen 8 to describe string dynamics (see also Refs. 9-11). Furthermore, it has been shown in Refs. 2-4 that the square root of the Maxwell term naturally arises (in flat space-time) as a result of spontaneous breakdown of scale symmetry of the original scale-invariant Maxwell theory with $f$ appearing as an integration constant responsible for the latter spontaneous breakdown.

As shown in Refs. 3–7 the flat space-time model (1), when coupled to charged fermions, produces a confining effective potential $V(r) = -\frac{\alpha}{r} + \beta r$ (Coulomb plus linear one) which is of the form of the well-known “Cornell” potential 12–14 in quantum chromodynamics (QCD). Also, for static field configurations the model (1) yields the following electric displacement field $\vec{D} = \vec{E} - \frac{f}{\sqrt{2}}\vec{E}/|\vec{E}|$. The pertinent energy density turns out to be (there is no contribution from the square-root term in (1)) $\frac{1}{2}E^2 = \frac{1}{2}||\vec{D}||^2 + \frac{f}{\sqrt{2}}||\vec{D}|| + \frac{1}{4}f^2$, so that it indeed contains a term linear w.r.t. $|\vec{D}|$ as argued by ‘t Hooft 1. Similar connection between $\vec{D}$ and $\vec{E}$ has been considered as an example of a “classical model of confinement” in Ref. 15 and analyzed generalizing the methods developed for the “leading logarithm model” in Ref. 16.
The gauge-field system with a square root of the Maxwell term (1) coupled to gravity (cf. Eq.(2) below) was recently studied in Ref. 17 (see the brief review in the following Section 2), where the following interesting new features of the pertinent static spherically symmetric solutions have been found:

(i) appearance of a constant vacuum radial electric field (in addition to the Coulomb one) in charged black holes within Reissner-Nordström-de-Sitter-type and/or Reissner-Nordström-anti-de-Sitter-type space-times, in particular, in electrically neutral black holes with Schwarzschild-de-Sitter and/or Schwarzschild-anti-de-Sitter geometry;

(ii) novel mechanism of dynamical generation of cosmological constant through the nonlinear gauge field dynamics due to the “square-root” Maxwell term;

(iii) appearance of a confining-type effective potential in charged test particle dynamics in the above black hole backgrounds.

Further, it is interesting to study possible new effects which can take place in the context of wormhole physics where the wormholes are generated due to the presence of nonlinear gauge fields with confining type dynamics. To this end let us recall that Misner-Wheeler “charge without charge” effect 18 stands out as one of the most interesting physical phenomena produced by wormholes. Misner and Wheeler realized that wormholes connecting two asymptotically flat space-times provide the possibility of existence of electromagnetically non-trivial solutions, where the lines of force of the electric field flow from one universe to the other without a source and giving the impression of being positively charged in one universe and negatively charged in the other universe.

For a detailed exposition of the basics of wormhole physics we refer to Visser’s book Ref. 19 (see also Refs. 20, 21) and some more recent accounts 22−26.

In a recent note 27 we found the opposite effect in wormhole physics, namely, that a genuinely charged matter source of gravity and electromagnetism may appear electrically neutral to an external observer. We showed this phenomenon to take place in the coupled gravity/nonlinear-gauge-field system (2) (without bare cosmological constant) self-consistently interacting with a charged lightlike brane as a matter source (cf. Eq.(43) below). In this case the lightlike brane, which connects as a wormhole “throat” a non-compact “universe” with a compactified “universe”, is electrically charged, however all of its flux flows into the compactified (“tube-like”) “universe” only. No Coulomb field is produced in the non-compact “universe”, therefore, the wormhole hides the charge from an external observer in the latter “universe”. This charge-hiding effect is exclusively due to the presence of the “square-root” Maxwell term in the nonlinear gauge field action.

A few remarks about the relevance of lightlike branes within the present context are in order. In our previous papers 28−36 we have provided an explicit reparametrization invariant world-volume Lagrangian formulation of lightlike p-branes (“LL-branes” for short) (a brief review is given in Section 4) and we have used them to construct various types of wormhole, regular black hole and light-
like braneworld solutions in $D = 4$ or higher-dimensional asymptotically flat or asymptotically (anti-)de Sitter bulk space-times. In particular, in Refs. 35, 36 we have shown that LL-branes can trigger a series of spontaneous compactification-decompactification transitions of space-time regions, e.g., from ordinary compactified (“tube-like”) Levi-Civita-Bertotti-Robinson space $^{37-39}$ to non-compact Reissner-Nordström or Reissner-Nordström-de-Sitter region or vice versa. Wormholes with “tube-like” structure and regular black holes with “tube-like” core have been previously obtained within different contexts in Refs. 40-48.

Here an important remark about “Einstein-Rosen bridge” wormhole is in order. The nomenclature of “Einstein-Rosen bridge” in several standard textbooks uses the Kruskal-Szekeres manifold, where the “Einstein-Rosen bridge” geometry becomes dynamical (see Ref. 49, p.839, Fig. 31.6, and Ref. 50, p.228, Fig. 5.15). The latter notion of “Einstein-Rosen bridge” is not equivalent to the original Einstein-Rosen’s construction $^{51}$, where the space-time manifold is static spherically symmetric consisting of two identical copies of the outer Schwarzschild space-time region ($r > 2m$) glued together along the horizon at $r = 2m$. Namely, the two regions in Kruskal-Szekeres space-time corresponding to the outer Schwarzschild space-time region ($r > 2m$) and labeled (I) and (III) in Ref. 49 are generally disconnected and share only a two-sphere (the angular part) as a common border ($U = 0, V = 0$ in Kruskal-Szekeres coordinates), whereas in the original Einstein-Rosen “bridge” construction $^{51}$ the boundary between the two identical copies of the outer Schwarzschild space-time region ($r > 2m$) is a three-dimensional (lightlike) hypersurface ($r = 2m$).

In Refs. 31, 34 it has been shown that the “Einstein-Rosen bridge” in its original formulation $^{51}$ naturally arises as the simplest particular case of static spherically symmetric wormhole solutions produced by lightlike branes as gravitational sources, where the two identical “universes” with Schwarzschild outer-region geometry are glued together by a lightlike brane occupying their common horizon – the wormhole “throat”. An understanding of this picture within the framework of Kruskal-Szekeres manifold was subsequently provided in Ref. 52, which involves Rindler’s elliptic identification of the two antipodal future event horizons.

Let us recall that LL-branes by themselves play an important role in modern general relativity. They are singular null (lightlike) hypersurfaces in Riemannian space-time which provide dynamical description of various physically important phenomena in cosmology and astrophysics such as: (i) impulsive lightlike signals arising in cataclysmic astrophysical events $^{53}$; (ii) the “membrane paradigm” $^{54}$ of black hole physics; (iii) the thin-wall approach to domain walls coupled to gravity $^{55-58}$. More recently, LL-branes became significant also in the context of modern non-perturbative string theory, in particular, as the so called $H$-branes describing quantum horizons (black hole and cosmological) $^{59}$, as Penrose limits of baryonic $D$-branes $^{60}$, etc (see also Refs. 61, 62).

In the pioneering papers $^{55-58}$ LL-branes in the context of gravity and cosmology have been extensively studied from a phenomenological point of view, i.e., by introducing them without specifying the Lagrangian dynamics from which they
may originate\textsuperscript{a}. On the other hand, we have proposed in a series of recent papers\textsuperscript{28–36} a new class of concise reparametrization invariant world-volume Lagrangian actions (see also Section 4 below), providing a derivation from first principles of the \textit{LL-brane} dynamics. The latter feature – the explicit world-volume Lagrangian description of \textit{LL-branes} is the principal distinction of our wormhole construction via (charged) \textit{LL-branes} as sources of gravity and electromagnetism w.r.t. other non-Lagrangian “thin-shell” constructions of wormhole solutions (for the basics of the “thin-shell” cut-and-paste technique we refer to the book Ref. 19).

There are several characteristic features of \textit{LL-branes} which drastically distinguish them from ordinary Nambu-Goto branes:

(i) They describe intrinsically lightlike modes, whereas Nambu-Goto branes describe massive ones.

(ii) The tension of the \textit{LL-brane} arises as an additional dynamical degree of freedom, whereas Nambu-Goto brane tension is a given \textit{ad hoc} constant. The latter characteristic feature significantly distinguishes our \textit{LL-brane} models from the previously proposed tensionless \textit{p-branes} (for a review of the latter, see Ref. 64) which rather resemble a \textit{p}-dimensional continuous distribution of massless point-particles.

(iii) Consistency of \textit{LL-brane} dynamics in a spherically or axially symmetric gravitational background of codimension one requires the presence of a horizon which is automatically occupied by the \textit{LL-brane} (“horizon straddling” according to the terminology of Ref. 56).

(iv) When the \textit{LL-brane} moves as a test brane in spherically or axially symmetric gravitational backgrounds its dynamical tension exhibits exponential “inflation/deflation” time behavior\textsuperscript{65} – an effect similar to the “mass inflation” effect around black hole horizons\textsuperscript{66,67}.

The principal object of study in the present paper is the self-consistently coupled gravity/nonlinear-gauge-field system, containing the square root of the Maxwell term, with one or more \textit{LL-brane(s)}. We significantly extend the results of our previous note\textsuperscript{27} by constructing both more general wormhole solutions displaying a “charge-hiding” effect as well as completely new “two-throat” wormhole solution with genuinely QCD-like \textit{confining} behavior.

The plan of the present paper is as follows. In Section 2 we briefly review the Lagrangian formulation and the corresponding static spherically symmetric solutions of the coupled gravity/nonlinear-gauge-field system (2)\textsuperscript{17}, including the generation of vacuum constant-magnitude electric field as well as dynamical generation of cosmological constant.

In Section 3 we extend the results of Ref. 27 obtaining new solutions of compactified Levi-Civita-Bertotti-Robinson type in the gravity/nonlinear-gauge-field system (2) depending on the magnitude of the bare cosmological constant versus the dy-
namically generated one.

In Section 4 we briefly review the world-volume Lagrangian formulation and the basic properties of LL-brane dynamics, particularly stressing on the “horizon straddling” property (cf. Eqs.(42) below).

In Section 5 we describe the Lagrangian formulation of the self-consistently coupled bulk gravity/nonlinear-gauge-field system (2) to one or more LL-brane sources (cf. Eq.(43)) and outline a general procedure to derive wormhole solutions.

In Section 6 we construct “one-throat” wormhole solutions to the coupled gravity/nonlinear-gauge-field/LL-brane system (43) with the charged LL-brane occupying the wormhole “throat”, which connects a non-compact “universe” with Schwarzschild-(anti)-de-Sitter geometry (where the cosmological constant is partially or entirely dynamically generated) to a compactified (“tube-like”) “universe” of Levi-Civita-Bertotti-Robinson type. These wormholes exhibit the novel property of hiding electric charge from external observer in the non-compact “universe”, i.e., the whole electric flux produced by the charged LL-brane at the wormhole “throat” is expelled into the “tube-like” “universe”.

In Section 7 we construct more general “two-throat” wormhole solution to the coupled gravity/nonlinear-gauge-field/LL-brane system (43) with two separate charged LL-branes with equal in magnitude but opposite charges occupying the wormhole “throats” and connecting pairwise three different “universes”. The “left-most” and the “right-most” “universes” are two identical copies of the exterior region of the electrically neutral Schwarzschild-de-Sitter black hole beyond the Schwarzschild horizon. The “middle” “universe” is of Levi-Civita-Bertotti-Robinson “tube-like” form with geometry $dS_2 \times S^2$ ($dS_2$ being the two-dimensional de Sitter space). It comprises the finite-extent intermediate region of $dS_2$ between its two horizons. Both oppositely charged LL-branes occupying the two “throats” are producing an electric flux which turns out to be confined entirely within the middle finite-extent “tube-like” “universe”, i.e., no flux from the charged LL-branes is flowing into the non-compact outer “universes”.

2. Gravity/Nonlinear-Gauge-Field System. Spherically Symmetric Solutions

We will consider the simplest coupling to gravity of the nonlinear gauge field system with a square root of the Maxwell term (1) known to produce QCD-like confinement in flat space-time $^3$–$^7$. The relevant action is given by (we use units with Newton constant $G_N = 1$):

$$ S = \int d^4x \sqrt{-G} \left[ \frac{R(G) - 2\Lambda}{16\pi} + L(F^2) \right] , \quad L(F^2) = -\frac{1}{4} F^2 - \frac{f}{2} \sqrt{\varepsilon F^2} , \quad F^2 \equiv F_{\kappa\lambda} F_{\mu\nu} G^{\kappa\lambda} G^{\mu\nu} , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu . $$

(2)

Here $R(G)$ is the scalar curvature of the space-time metric $G_{\mu\nu}$ and $G \equiv \det |G_{\mu\nu}|$; the sign factor $\varepsilon = \pm 1$ in the square-root term in (2) corresponds to “magnetic” or “electric” dominance; $f$ is a positive coupling constant.
It is important to stress that we do not need to introduce any bare cosmological constant $\Lambda$ in (2) since the “square-root” Maxwell term dynamically generates a non-zero effective cosmological constant $\Lambda_{\text{eff}} = 2\pi f^2$. The role of the bare $\Lambda$ is just shifting the effective $\Lambda_{\text{eff}}$ (see Eqs.(9) below).

**Remark.** One could start with the non-Abelian version of the gauge field action in (2). Since we will be interested in static spherically symmetric solutions, the non-Abelian gauge theory effectively reduces to an Abelian one as pointed out in Ref. 3.

The corresponding equations of motion read:

$$R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R + \Lambda G_{\mu\nu} = 8\pi T^{(F)}_{\mu\nu},$$  

$$T^{(F)}_{\mu\nu} = \left(1 + \frac{f}{\sqrt{\varepsilon F^2}}\right) F_{\mu\kappa} F^{\nu\lambda} G^{\kappa\lambda} - \frac{1}{4} \left(F^2 + 2f\sqrt{\varepsilon F^2}\right) G_{\mu\nu},$$

and

$$\partial_{\nu} \left(\sqrt{-G} \left(1 + \frac{f}{\sqrt{\varepsilon F^2}}\right) F_{\kappa\lambda} G^{\kappa\mu} G^{\nu\lambda}\right) = 0.$$  

Here we will first consider the case of “electric dominance”, i.e., $\varepsilon = -1$ in (2).

In our preceding paper we have shown that the gravity-gauge-field system (2) with zero bare cosmological constant possesses static spherically symmetric solutions with a radial electric field containing both Coulomb and constant “vacuum” pieces:

$$F_{0r} = \frac{\varepsilon F f}{\sqrt{2}} + \frac{Q}{\sqrt{4\pi r^2}}, \quad \varepsilon_F \equiv \text{sign}(F_{0r}) = \text{sign}(Q),$$

and the space-time metric:

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2),$$

$$A(r) = 1 - \sqrt{8\pi|Q|f} - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{2\pi f^2}{3} r^2,$$

is Reissner-Nordström-de-Sitter-type with *dynamically generated* effective cosmological constant $2\pi f^2$. In the presence of the bare cosmological constant term in (2) the only effect is shifting of the effective cosmological constant, namely:

$$A(r) = 1 - \sqrt{8\pi|Q|f} - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda_{\text{eff}}}{3} r^2, \quad \Lambda_{\text{eff}} = 2\pi f^2 + \Lambda.$$  

The expression for $\Lambda_{\text{eff}}$ (9) tells us that:

- Solution (6)–(7) with (9) is Reissner-Nordström-de-Sitter-type with additional constant vacuum radial electric field even for *negative* bare cosmological constant $\Lambda < 0$ provided $|\Lambda| < 2\pi f^2$, i.e., $\Lambda_{\text{eff}} > 0$ in (9);
- Solution (6)–(7) with (9) becomes Reissner-Nordström-type with additional constant vacuum radial electric field in spite of the presence of *negative* bare cosmological constant $\Lambda < 0$ with $|\Lambda| = 2\pi f^2$, i.e., $\Lambda_{\text{eff}} = 0$ in (9);
Solution (6)–(7) with (9) is Reissner-Nordström-anti-de-Sitter-type with constant vacuum radial electric field for sufficiently large negative bare cosmological constant $\Lambda < 0$ with $|\Lambda| > 2\pi f^2$, i.e., $\Lambda_{\text{eff}} < 0$ in (9).

Notice that the “leading” constant term in the Reissner-Nordström-(anti-)de-Sitter-type metric coefficient (9) is different from 1 when $Q \neq 0$. This effect resembles the effect on gravity produced by a spherically symmetric “hedehog” configuration of a nonlinear sigma-model scalar field with $SO(3)$ symmetry (see Refs. 68, 69).

The electrically neutral case $Q = 0$ will play an important role in what follows:

$$A(r) = 1 - \frac{2m}{r} - \frac{\Lambda_{\text{eff}}}{3} r^2, \quad \Lambda_{\text{eff}} = 2\pi f^2 + \Lambda, \quad F^0_\eta = \frac{\varepsilon F f}{\sqrt{2}}. \quad \text{(10)}$$

- Solution (10) is Schwarzschild-de-Sitter black hole carrying a constant vacuum radial electric field for all $\Lambda > -2\pi f^2$, even for negative $\Lambda$ provided $|\Lambda| > 2\pi f^2$, i.e., $\Lambda_{\text{eff}} > 0$ in (10);
- Solution (10) for negative $\Lambda$ with $|\Lambda| = 2\pi f^2$ becomes asymptotically flat ordinary Schwarzschild carrying a constant vacuum radial electric field in spite of the presence of negative bare cosmological constant i.e., $\Lambda_{\text{eff}} = 0$ in (10);
- Solution (10) is Schwarzschild-anti-de-Sitter carrying a constant vacuum radial electric field for all $\Lambda < 0$ with $|\Lambda| > 2\pi f^2$, i.e., $\Lambda_{\text{eff}} < 0$ in (10).

3. Generalized Levi-Civita-Bertotti-Robinson Space-Times

Here we will look for static solutions of Levi-Civita-Bertotti-Robinson type $^{37-39}$ of the system (3)–(5), namely, with space-time geometry of the form $\mathcal{M}_2 \times S^2$, where $\mathcal{M}_2$ is some two-dimensional manifold:

$$ds^2 = -A(\eta)dt^2 + \frac{d\eta^2}{A(\eta)} + r_0^2(d\theta^2 + \sin^2 \theta d\varphi^2), \quad -\infty < \eta < \infty, \quad r_0 = \text{const}, \quad \text{(11)}$$

and being:

- either purely electric type, where the sign factor $\varepsilon = -1$ in the gauge field Lagrangian $L(F^2)$ (2):

$$F_{\mu\nu} = 0 \text{ for } \mu, \nu \neq 0, \eta, \quad F_{0\eta} = F_{0\eta}(\eta); \quad \text{(12)}$$

- or purely magnetic type, where $\varepsilon = +1$ in (2):

$$F_{\mu\nu} = 0 \text{ for } \mu, \nu \neq i, j \equiv \theta, \varphi, \quad \partial_\theta F_{ij} = \partial_\varphi F_{ij} = 0. \quad \text{(13)}$$

In the purely electric case (12) the gauge field equations of motion become:

$$\partial_\eta \left( F_{0\eta} - \frac{\varepsilon F f}{\sqrt{2}} \right) = 0, \quad \varepsilon_F \equiv \text{sign}(F_{0\eta}), \quad \text{(14)}$$

yielding a globally constant vacuum electric field:

$$F_{0\eta} = \varepsilon_F = \text{arbitrary const}. \quad \text{(15)}$$
The (mixed) components of energy-momentum tensor (4) read:

\[ T^{(F)}_{00} = T^{(F)}_{\eta\eta} = -\frac{1}{2} F^{2}_{0\eta} \] , \[ T^{(F)}_{ij} = g_{ij} \left( \frac{1}{2} F^{2}_{0\eta} - \frac{f}{\sqrt{2}} |F_{0\eta}| \right) \].

Taking into account (16), the Einstein eqs.(3) for \((ij)\), where \( R_{ij} = \frac{1}{r^{2}} g_{ij} \) because of the \( S^{2} \) factor in (11), yield:

\[ \frac{1}{r^{2}} = 4\pi c^{2} F + \Lambda \] .

(17)

The (00) Einstein eq.(3) using the expression \( R_{00} = -\frac{1}{2} \partial^{2}_{\eta} A \) (valid for metrics of the type (11), cf. Ref. 70, 71) becomes:

\[ \partial^{2}_{\eta} A = 8\pi h(|c_{F}|) , \quad h(|c_{F}|) \equiv c^{2}_{F} - \sqrt{2} f |c_{F}| - \frac{\Lambda}{4\pi} \] .

(18)

Thus, we arrive at the following three distinct types of Levi-Civita-Bertotti-Robinson solutions for gravity coupled to the non-linear gauge field system (2):

(i) \( AdS_{2} \times S^{2} \) with constant vacuum electric field \(|F_{0\eta}| = |c_{F}|\), where \( AdS_{2} \) is two-dimensional anti-de Sitter space with (using the definition of \( h(|c_{F}|) \) in (18)):

\[ A(\eta) = 4\pi h(|c_{F}|) \eta^{2} , \quad h(|c_{F}|) > 0 \] (19)

in the metric (11), \( \eta \) being the Poincare patch space-like coordinate, provided:

\[ |c_{F}| > \frac{f}{\sqrt{2}} \left( 1 + \sqrt{1 + \frac{\Lambda}{2\pi f^{2}}} \right) \quad \text{for} \quad \Lambda \geq -2\pi f^{2} \] ,

(20)

\[ |c_{F}| > \frac{1}{4\pi} |\Lambda| \quad \text{for} \quad \Lambda < 0 , \quad |\Lambda| > 2\pi f^{2} \] .

(21)

(ii) \( Rind_{2} \times S^{2} \) with constant vacuum electric field \(|F_{0\eta}| = |c_{F}|\), where \( Rind_{2} \) is the flat two-dimensional Rindler space with:

\[ A(\eta) = \eta \quad \text{for} \quad 0 < \eta < \infty \quad \text{or} \quad A(\eta) = -\eta \quad \text{for} \quad -\infty < \eta < 0 \] (22)

in the metric (11), provided:

\[ |c_{F}| = \frac{f}{\sqrt{2}} \left( 1 + \sqrt{1 + \frac{\Lambda}{2\pi f^{2}}} \right) \quad \text{for} \quad \Lambda \geq -2\pi f^{2} \] ,

(23)

(iii) \( dS_{2} \times S^{2} \) with weak constant vacuum electric field \(|F_{0\eta}| = |c_{F}|\), where \( dS_{2} \) is two-dimensional de Sitter space with:

\[ A(\eta) = 1 - K(|c_{F}|) \eta^{2} , \quad K(|c_{F}|) \equiv -4\pi h(|c_{F}|) \equiv 4\pi \left( \sqrt{2} f |c_{F}| - c^{2}_{F} + \frac{\Lambda}{4\pi} \right) > 0 \] (24)

in the metric (11), provided:

\[ |c_{F}| < \frac{f}{\sqrt{2}} \left( 1 + \sqrt{1 + \frac{\Lambda}{2\pi f^{2}}} \right) \quad \text{for} \quad \Lambda > -2\pi f^{2} \] .

(25)
When $\Lambda = 0$, for the special value $|c_F| = \frac{f}{\sqrt{2}}$ we recover the Nariai solution with $A(\eta) = 1 - 2f^2\eta^2$ and equality (up to signs) among energy density, radial and transverse pressures: $\rho = -p_r = -p_\perp = \frac{f^2}{4}$ (using standard definitions $T^{(F)}_{\mu} = \text{diag}(-\rho, p_r, p_\perp, p_\perp)$).

In all three cases above the size of the $S^2$ factor is given by (17). Solutions (22) and (24) with $\Lambda = 0$ are specifically due to the presence of the non-Maxwell square-root term (with $\varepsilon = -1$) in the gauge field Lagrangian (2).

In the purely magnetic case (13) the gauge field equations of motion (5):

$$\partial_\nu \left[ \sin \theta \left( 1 + \frac{f}{\sqrt{F^2}} \right) F^{\mu \nu} \right] = 0$$

yield magnetic monopole solution:

$$F_{ij} = Br_0^2 \sin \theta \varepsilon_{ij}, \quad B = \text{const},$$

irrespective of the presence of the “square-root” Maxwell term. The latter, however, does contribute to the energy-momentum tensor:

$$T^{(F)}_{\mu} = \sqrt{g} T^{(F)}_{\eta} = -\frac{1}{2}B^2 - f|B|, \quad T^{(F)}_{ij} = \frac{1}{2} \delta_{ij} B^2.$$

Taking into account (28), the Einstein eqs.(3) for $(ij)$ yield (cf. (17)):

$$\frac{1}{r_0^2} = 4\pi \left( B^2 + \sqrt{2}f|B| \right) + \Lambda,$$

which determines the size of the $S^2$ factor, whereas the mixed-component $(00)$ Einstein eq.(3) gives:

$$\partial_\eta^2 A = 8\pi B^2 - 2\Lambda.$$

Thus, in the purely magnetic case we recover the three types of Levi-Civita-Bertotti-Robinson solutions with constant-magnitude magnetic field:

(a) $AdS_2 \times S^2$ space-time with magnetic monopole (27) for $\Lambda < 4\pi B^2$ with $A(\eta) = 4\pi \left( B^2 - \frac{\Lambda}{4\pi} \right) \eta^2$ in the metric (11);

(b) $Rind_2 \times S^2$ space-time with magnetic monopole (27) for $\Lambda = 4\pi B^2$ with $A(\eta) = \eta$, $\eta > 0$ in the metric (11);

(c) $dS_2 \times S^2$ space-time with magnetic monopole (27) for $\Lambda > 4\pi B^2$ with $A(\eta) = 1 - 4\pi \left( \frac{4\pi}{\Lambda} - B^2 \right) \eta^2$ in the metric (11).

Here the only feature is the dependence of the size of the $S^2$-factor on the “square-root” Maxwell coupling constant $f$ (29).

Generalized Levi-Civita-Bertotti-Robinson solutions of the above type have already appeared in different contexts in Refs. 44-48 and Ref. 74 (extension to higher space-time dimensions). The main distinction in the present case is that the Levi-Civita-Bertotti-Robinson solutions are now generated due to the presence of the “square-root” Maxwell term in (2) which also produces a non-zero effective cosmological constant.
In Ref. 75 a different kind on nonlinear gauge field Lagrangian $L(F^2)$ coupled to gravity has been considered which generates locally (in the vicinity of the center of the geometry) an effective cosmological constant. However, the latter $L(F^2)$ is an analytic function of $F^2$ reducing to the ordinary Maxwell term for small $F^2$ unlike the present nonlinear Lagrangian $L(F^2)$ in (2) containing the square-root term $\sqrt{-F^2}$. This latter feature of (2) produces a globally defined dynamically generated cosmological constant $2\pi f^2$.

4. Lagrangian Formulation of Lightlike Branes. Horizon “Straddling”

In what follows we will consider gravity/gauge-field system self-consistently interacting with a lightlike $p$-brane (“LL-brane” for short) of codimension one ($D = (p + 1) + 1$); in the present Section will keep arbitrary the number of space-time dimensions). In a series of previous papers $28$–$36$ we have proposed manifestly reparametrization invariant world-volume Lagrangian formulation in several dynamically equivalent forms of $LL$-branes coupled to bulk gravity $G_{\mu\nu}$ and bulk gauge fields, in particular, electromagnetic field $A_\mu$. Here we will use our Polyakov-type formulation given by the world-volume action $35,36$:

$$S_{LL}[q] = -\frac{1}{2} \int d^{p+1}\sigma \frac{b_0^{p-1}}{T} \sqrt{-\gamma} \left[ \gamma_{ab} \bar{g}_{ab} - b_0(p-1) \right],$$

(31)

$$\bar{g}_{ab} \equiv \partial_a X^\mu G_{\mu\nu} \partial_b X^\nu - \frac{1}{T^2} (\partial_a u + qA_a)(\partial_b u + qA_b), \quad A_a \equiv \partial_a X^\mu A_\mu.$$  

(32)

Here and below the following notations are used:

- $\gamma_{ab}$ is the intrinsic Riemannian metric on the world-volume with $\gamma = \det \|\gamma_{ab}\|$; $\bar{g}_{ab}$ is the induced metric on the world-volume:

$$g_{ab} \equiv \partial_a X^\mu G_{\mu\nu}(X) \partial_b X^\nu,$$

(33)

which becomes singular on-shell (manifestation of the lightlike nature, cf. Eq.(37) below); $b_0$ is a positive constant measuring the world-volume “cosmological constant”.

- $X^\mu(\sigma)$ are the $p$-brane embedding coordinates in the bulk $D$-dimensional space-time with Riemannian metric $G_{\mu\nu}(x)$ ($\mu, \nu = 0, 1, \ldots, D - 1$); ($\sigma$) $\equiv (\sigma^0 \equiv \tau, \sigma^i)$ with $i = 1, \ldots, p$; $\partial_a \equiv \frac{\partial}{\partial \sigma^a}$.

- $u$ is auxiliary world-volume scalar field defining the lightlike direction of the induced metric (see Eq.(37) below) and it is a non-propagating degree of freedom $36$.

- $T$ is dynamical (variable) brane tension (also a non-propagating degree of freedom).

- Coupling parameter $q$ is the surface charge density of the $LL$-brane.
The corresponding equations of motion w.r.t. $X^\mu$, $u$, $\gamma_{ab}$ and $T$ read accordingly (using short-hand notation (32)):

$$
\partial_a \left( T \sqrt{|g|} g^{ab} \partial_b X^\mu \right) + T \sqrt{|g|} g^{ab} \partial_a X^\lambda \partial_b X^\nu \Gamma^\mu_{\lambda \nu} + \frac{q}{77} \sqrt{|g|} g^{ab} \partial_a X^\nu (\partial_b u + q A_b) F_{\lambda \nu} G^{\nu \lambda} = 0 ,
$$

$$
\partial_a \left( \frac{1}{T} \sqrt{|g|} g^{ab} (\partial_b u + q A_b) \right) = 0 , \quad \gamma_{ab} = \frac{1}{b_0} \bar{g}_{ab} , \quad T^2 + g^{ab} (\partial_a u + q A_a)(\partial_b u + q A_b) = 0 .
$$

(34)  

(35)  

(36)

Here $\bar{g} = \det |\bar{g}_{ab}|$ and $\Gamma'^{\mu}_{\lambda \nu}$ denotes the Christoffel connection for the bulk metric $G_{\mu \nu}$.

The on-shell singularity of the induced metric $g_{ab}$ (33), i.e., the lightlike property, directly follows from Eq.(36) and the definition of $\bar{g}_{ab}$ (32):

$$
g_{ab} \left( \bar{g}^{bc} (\partial_c u + q A_c) \right) = 0 .
$$

(37)

Explicit world-volume reparametrization invariance of the $LL$-brane action (31) allows to introduce the standard synchronous gauge-fixing conditions for the intrinsic world-volume metric $\gamma_{00} = -1$ , $\gamma_{0i} = 0$ ($i = 1, \ldots , p$), which reduces Eqs.(35)–(36) to the following relations:

$$
\frac{(\partial_0 u + q A_0)^2}{T^2} = b_0 + g_{00} , \quad \partial_0 u + q A_i = (\partial_0 u + q A_0) g_{0i} (b_0 + g_{00})^{-1} ,
$$

$$
g_{00} = g^{ij} g_{0j} , \quad \partial_0 \left( \sqrt{|g|} g^{ij} \right) + \partial_0 \left( \sqrt{|g|} g^{ij} g_{0j} \right) = 0 , \quad g^{(p)} = \det |g_{ij}| ,
$$

(38)

(recall that $g_{00}, g_{0a}, g_{ij}$ are the components of the induced metric (33); $g^{ij}$ is the inverse matrix of $g_{ij}$).

In our previous papers 28–36 we have studied in some detail the consistency of $LL$-brane dynamics in static “spherically-symmetric”-type backgrounds, whose generic form reads (in what follows we will use Eddington-Finkelstein coordinates 76,77 where $dt = dv - \frac{du}{A(\eta)}$ , $F_{0\eta} = F_{v\eta}$):

$$
ds^2 = -A(\eta) dv^2 + 2dv d\eta + C(\eta) h_{ij}(\theta) d\theta^i d\theta^j , \quad F_{v\eta} = F_{v\eta}(\eta) ,
$$

(39)

all remaining components of $F_{\mu \nu}$ being zero. For the $LL$-brane we use the standard embedding ansatz:

$$
X^0 \equiv v = \tau , \quad X^1 \equiv \eta = \eta(\tau) , \quad X^i \equiv \theta^i = \sigma^i (i = 1, \ldots , p).
$$

(40)

For the class of backgrounds (39) with the embedding (40) (where the induced metric components $g_{0i} = 0$) Eqs.(38) reduce to:

$$
g_{00} = 0 , \quad \partial_0 C(\eta(\tau)) \equiv \dot{\eta} \partial_0 C \big|_{\eta = \eta(\tau)} = 0 , \quad \frac{(\partial_0 u + q A_0)^2}{T^2} = b_0 , \quad \partial_0 u = 0
$$

(41)

$$(\dot{\eta} \equiv \partial_0 \eta \equiv \partial_0 \eta(\tau))$$. Thus, in the generic case of non-trivial dependence of $C(\eta)$ on the “radial-like” coordinate $\eta$, the first two relations in (41) yield:

$$
\dot{\eta} = \frac{1}{2} A(\eta(\tau)) , \quad \dot{\eta} = 0 \rightarrow \eta(\tau) = \eta_0 = \text{const} , \quad A(\eta_0) = 0 .
$$

(42)
In other words, consistency of LL-brane dynamics requires the corresponding background (39) to possess a horizon at some $\eta = \eta_0$, which is automatically occupied by the LL-brane.

The latter property is called “horizon straddling” according to the terminology of Ref. 56. Similar “horizon straddling” has been found also for LL-branes moving in rotating axially symmetric (Kerr or Kerr-Newman) and rotating cylindrically symmetric black hole backgrounds 33,34.

5. Bulk Gravity/Nonlinear-Gauge-Field System Coupled to Lightlike Brane Sources

We consider now bulk Einstein/non-linear gauge field system (2) self-consistently coupled to $N \geq 1$ distantly separated charged codimension-one lightlike branes (in the present case $D = 4$, $p = 2$). The pertinent Lagrangian action reads:

$$S = \int d^4x \sqrt{-G} \left[ R(G) - \frac{2\Lambda}{16\pi} + L(F^2) \right] + \sum_{k=1}^{N} S_{LL}[q^{(k)}] \ , \ L(F^2) = -\frac{1}{4} F^2 - \frac{f}{2} \sqrt{-F^2} \ ,$$

(43)

where $S_{LL}[q^{(k)}]$ indicates the world-volume action of the $k$-th LL-brane of the form (31). Henceforth we will consider the case of “electric dominance” for the “square-root” Maxwell term.

The corresponding equations of motion are as follows:

$$R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R + \Lambda G_{\mu\nu} = 8\pi \left[ T_{\mu\nu}^{(F)} + \sum_{k=1}^{N} T_{\mu\nu}^{(brane-k)} \right] ,$$

(44)

$$\partial_\nu \left[ \sqrt{-G} \left( 1 - \frac{f}{\sqrt{-F^2}} \right) F_{\nu\lambda} G^{\mu\lambda} G^{\nu\lambda} \right] + \sum_{k=1}^{N} j_{(brane-k)}^{\mu} = 0 .$$

(45)

Here $T_{\mu\nu}^{(F)}$ is the same as in (4), whereas the energy-momentum tensor and the charge current density of the $k$-th LL-brane are straightforwardly derived from the pertinent LL-brane action (31):

$$T_{\mu\nu}^{(brane-k)} = -\int d^{3}\sigma \frac{\delta^{(4)}(x - X^{(k)}(\sigma))}{\sqrt{-G}} T^{(k)} \sqrt{|\bar{g}(k)| g^{ab} \partial_a X^{\mu}(k) \partial_b X^{\nu}(k)} ,$$

(46)

$$j_{(brane-k)}^{\mu} = -q^{(k)} \int d^{3}\sigma \frac{\delta^{(4)}(x - X^{(k)}(\sigma))}{\sqrt{|\bar{g}(k)| g^{ab} \partial_a X^{\mu}(k) \partial_b X^{\nu}(k) \bar{g}(k)| g^{ab} \partial_a X^{\mu}(k) \partial_b X^{\nu}(k)}} ,$$

(47)

where for each $k$-th LL-brane:

$$\bar{g}_{ab}^{(k)} \equiv \bar{g}_{ab}^{(k)} - \frac{1}{T_{(k)}^{(k)}} (\partial_a u^{(k)} + q^{(k)} A_a^{(k)}) (\partial_b u^{(k)} + q^{(k)} A_b^{(k)}) ,$$

$$g_{ab}^{(k)} = \partial_a X^{\mu}(k) G_{\mu\nu} \partial_b X^{\nu}(k) \ , \ A_a^{(k)} \equiv \partial_a X^{\mu}(k) A_\mu .$$

(48)
The LL-brane equations of motion have already been written down in (34)–(36) above.

Constructing wormhole solutions of static “spherically-symmetric”-type (39) for the coupled gravity-gauge-field-LL-brane system (43) proceeds through the following steps:

(i) Choose “vacuum” static “spherically-symmetric”-type solution s (39) of (44)–(45), i.e., without the delta-function terms due to the LL-branes, in each space-time region (separate “universe”) given by \((-\infty < \eta < \eta_{0}^{(1)})\), \((\eta_{0}^{(1)} < \eta < \eta_{0}^{(2)})\), \(\ldots\), \((\eta_{0}^{(N)} < \eta < \infty)\) with common horizon(s) at \(\eta = \eta_{0}^{(k)}\) \((k = 1, \ldots, N)\).

(ii) Each \(k\)-th LL-brane automatically locates itself on the horizon at \(\eta = \eta_{0}^{(k)}\) according to “horizon straddling” property (42) of LL-brane dynamics. It thus will play the role of a wormhole “throat” between two neighboring “universes”.

(iii) Match the discontinuities of the derivatives of the metric and the gauge field strength (39) across each horizon at \(\eta = \eta_{0}^{(k)}\) using the explicit expressions for the LL-brane stress-energy tensor and charge current density (46)–(47).

Taking into account (39)–(42), we obtain from (46) the following expression for the energy-momentum tensor of each \(k\)-th LL-brane (here we suppress the index \((k)\)):

\[
T_{(branee)}^{\mu\nu} = S_{\mu\nu} \delta(\eta - \eta_{0})
\]

with surface energy-momentum tensor:

\[
S_{\mu\nu} = \frac{T}{b_{0}^{1/2}} \left( \partial_{\tau}X^{\mu}\partial_{\tau}X^{\nu} - b_{0}G^{ij}\partial_{i}X^{\mu}\partial_{j}X^{\nu} \right)_{v=\tau, \eta=\eta_{0}, \theta^{+}=\sigma^{+}},
\]

where \(G_{ij} = C(\eta)h_{ij}(\theta)\) (cf. (39), here \(i, j = \theta, \phi\)). For the non-zero components of (50) (with lower indices) and its trace we find:

\[
S_{\eta\eta} = \frac{T}{b_{0}^{1/2}}, \quad S_{ij} = -Tb_{0}^{1/2}G_{ij}, \quad S_{\lambda\lambda} = -2Tb_{0}^{1/2}.
\]

For the LL-brane charge current densities we get accordingly:

\[
J_{(branee-k)}^{\mu} = \delta_{0}^{(k)}q_{0}^{(k)} \sqrt{\det ||G_{ij}||} \delta(\eta - \eta_{0}^{(k)}) \cdot
\]

With the help of (49)–(52) and using again (39)–(42) the matching relations for the discontinuities at each horizon \(\eta = \eta_{0}^{(k)}\) become (cf. Refs. 35, 36):

(A) Matching relations from Einstein eqs.(44):

\[
[\partial_{\eta}A]_{\eta_{0}^{(k)}} = -16\pi T^{(k)} \sqrt{b_{0}^{(k)}} , \quad [\partial_{\eta} \ln C]_{\eta_{0}^{(k)}} = -\frac{8\pi}{\sqrt{b_{0}^{(k)}}} T^{(k)}
\]

using notation \([Y]_{\eta_{0}^{(k)}} \equiv Y \mid_{\eta_{0}^{(k)} + 0} - Y \mid_{\eta_{0}^{(k)} - 0}\) for any quantity \(Y\).

(B) Matching relations from nonlinear gauge field eqs.(45):

\[
[F_{\mu\eta}]_{\eta_{0}^{(k)}} = q_{0}^{(k)}.
\]
(C) The only non-trivial contribution of second-order \textit{LL-brane} equations of motion (34) in the case of \textit{LL-brane} coordinate embedding (40) comes from the \(X^0\)-equation of motion which yields:

\[
\partial_0 T^{(k)} + \frac{T^{(k)}}{2} \left( \langle \partial_\eta A \rangle_{\eta_0^{(k)}} + 2 b_0^{(k)} \langle \partial_\eta \ln C \rangle_{\eta_0^{(k)}} \right) - \sqrt{b_0^{(k)}} q \langle F_{\eta \eta} \rangle_{\eta_0^{(k)}} = 0 \tag{55}
\]

with notation \( \langle Y \rangle_{\eta_0} = \frac{1}{2} \left( Y \big|_{\eta \to \eta_0+0} + Y \big|_{\eta \to \eta_0-0} \right) \). In what follows we will take time-independent dynamical \textit{LL-brane} tension(s) \( (\partial_0 T^{(k)} = 0) \) because of matching static bulk space-time geometries. Let us also note that the appearance of mean values of the corresponding quantities with discontinuities across the horizons follows the resolution of the discontinuity problem given in Ref. 55 (see also Ref. 78).

The wormhole solutions presented in the next Section share the following important properties:

(a) The \textit{LL-branes} at the wormhole “throats” represent “exotic” matter with \( T \leq 0 \), i.e., negative or zero brane tension implying violation of null-energy conditions as predicted by general wormhole arguments 19 (although the latter could be remedied via quantum fluctuations).

(b) The wormhole space-times constructed via \textit{LL-branes} at their “throats” are not traversable w.r.t. the “laboratory” time of a static observer in either of the different “universes” comprising the pertinent wormhole space-time manifold since the \textit{LL-branes} sitting at the “throats” look as black hole horizons to the static observer. On the other hand, these wormholes are \textit{traversable} w.r.t. the proper time of a traveling observer.

Proper-time traversability can be easily seen by considering dynamics of test particle of mass \( m_0 \) (“traveling observer”) in a wormhole background, which is described by the reparametrization-invariant world-line action:

\[
S_{\text{particle}} = \frac{1}{2} \int d\lambda \left[ \frac{1}{e} \, (x^\mu \dot{x}_\mu) G_{\mu\nu} - em_0^2 \right]. \tag{56}
\]

Using energy \( E \) and orbital momentum \( J \) conservation and introducing the proper world-line time \( s \) \((\frac{ds}{d\lambda} = em_0)\), the “mass-shell” constraint equation (the equation w.r.t. the “einbein” \( e \)) produced by the action (56) yields:

\[
\left( \frac{d\eta}{ds} \right)^2 + \mathcal{V}_{\text{eff}}(\eta) = \frac{E^2}{m_0^2}, \quad \mathcal{V}_{\text{eff}}(\eta) \equiv A(\eta) \left( 1 + \frac{J^2}{m_0^2 C(\eta)} \right) \tag{57}
\]

where the metric coefficients \( A(\eta), C(\eta) \) are those in (39). Irrespective of the specific form of the “effective potential” in (57), a “radially” moving (with zero “impact” parameter \( J = 0 \)) traveling observer (and with sufficiently large energy \( E \)) will always cross within finite amount of proper-time through any “throat” \( (\eta = \eta_0^{(k)}) \), where \( A(\eta_0^{(k)}) = 0 \) from one “universe” to another.

6. Charge-Hiding Wormholes

First we will construct “one-throat” wormhole solutions to (43) with the charged \textit{LL-brane} occupying the wormhole “throat”, which connects a non-compact “uni-
verse” with Reissner-Nordström-(anti)-de-Sitter-type geometry (6)–(8) (where the cosmological constant is partially or entirely dynamically generated) to a compactified (“tube-like”) “universe” of (generalized) Levi-Civita-Bertotti-Robinson type (11)–(12). These wormholes possess the novel property of hiding electric charge from external observer in the non-compact “universe”, i.e., the whole electric flux produced by the charged $LL$-brane at the wormhole “throat” is pushed into the “tube-like” “universe”. As a result, the non-compact “universe” becomes electrically neutral with Schwarzschild-(anti-)de-Sitter or purely Schwarzschild geometry.

We find several types of such wormhole solutions. The first one exists when the bare cosmological constant $\Lambda > -\frac{2}{\pi f^2}$, in particular, when $\Lambda$ is absent from the very beginning, the whole effective cosmological constant being dynamically generated (9). This wormhole solution is constructed as follows:

(A-1) “Left universe” of Levi-Civita-Bertotti-Robinson (“tube-like”) type with geometry $AdS_2 \times S^2$ (19) for $\eta < 0$:

$$A(\eta) = 4\pi \left( c_F^2 - \sqrt{2f}|c_F| - \frac{\Lambda}{4\pi} \right) \eta^2, \quad C(\eta) \equiv r_0^2 = \frac{1}{4\pi c_F^2 + \Lambda} = \text{const},$$

$$|F_{\nu\eta}| = |c_F| > \sqrt{\frac{f}{\sqrt{2}}} \left( 1 + \frac{\Lambda}{2\pi f^2} \right) = \text{const};$$

(A-2) Non-compact “right universe” for $\eta > 0$ comprising the exterior region of Reissner-Nordström-anti-de-Sitter-type black hole beyond the middle (Schwarzschild-type) horizon $r_0$ (cf. (6)–(9)):

$$A(\eta) \equiv A_{RNS}(r_0 + \eta) = 1 - \sqrt{8\pi}|Q|f - \frac{2m}{r_0 + \eta} + \frac{Q^2}{(r_0 + \eta)^2} - \frac{\Lambda + 2\pi f^2}{3}(r_0 + \eta)^2,$$

$$C(\eta) = (r_0 + \eta)^2, \quad F_{\nu\eta} = F_{0\nu} = \frac{\varepsilon_F f}{\sqrt{2}} + \frac{Q}{\sqrt{4\pi}(r_0 + \eta)^2}. \quad (59)$$

Here $A(0) = A_{RNS}(r_0) = 0$ and $\partial_\eta A(0) = \partial_\eta A_{RNS}(r_0) > 0$.

The next wormhole solution, which exists for large negative bare cosmological constant $\Lambda < 0$, $|\Lambda| > 2\pi f^2$, is built by:

(B-1) The same type of “left universe” of Levi-Civita-Bertotti-Robinson type with geometry $AdS_2 \times S^2$ (19) for $\eta < 0$ as in (58):

$$A(\eta) = 4\pi \left( c_F^2 - \sqrt{2f}|c_F| + \frac{|\Lambda|}{4\pi} \right) \eta^2, \quad C(\eta) \equiv r_0^2 = \frac{1}{4\pi c_F^2 - |\Lambda|} = \text{const},$$

$$|F_{\nu\eta}| = |c_F| > \sqrt{\frac{1}{4\pi} |\Lambda|}. \quad (60)$$

(B-2) Non-compact “right universe” for $\eta > 0$ comprising the exterior region of Reissner-Nordström-anti-de-Sitter-type black hole beyond the outer (Schwarzschild-
Here again $A(r) = A_{\text{RN-AdS}}(r_0 + \eta)$ determines all parameters of the wormhole solutions $(r_0) = 0$ and $\partial_\eta A(0) = \partial_r A_{\text{RN-AdS}}(r_0) > 0$.

For the special negative value of the bare cosmological constant $\Lambda = -2\pi f^2$ we find a third wormhole solution consisting of:

(C-1) The same type of “left universe” of Levi-Civita-Bertotti-Robinson type with geometry $AdS_2 \times S^2$ (19) for $\eta < 0$ as in (58):

$$A(\eta) = 4\pi \left( |c_F| - \frac{f}{\sqrt{2}} \right)^2 \eta^2, \quad C(\eta) \equiv r_0^2 = \frac{1}{4\pi (c_F^2 - \frac{1}{2} f^2)} = \text{const},$$

$$|F_{\eta r}| = |c_F| > \frac{f}{\sqrt{2}}.$$ (62)

(C-2) Non-compact “right universe” for $\eta > 0$ comprising the exterior region of Reissner-Nordström-type black hole beyond the outer (Schwarzschild-type) horizon $r_0$:

$$A(\eta) \equiv A_{\text{RN}}(r_0 + \eta) = 1 - \sqrt{8\pi |Q| f - \frac{2m}{r_0 + \eta}} + \frac{Q^2 (r_0 + \eta)^2}{3},$$

$$C(\eta) = (r_0 + \eta)^2, \quad F_{\eta r} = F_{0 r} = \frac{\varepsilon F f}{\sqrt{2}} + \frac{Q}{\sqrt{4\pi} (r_0 + \eta)^2}.$$ (63)

Here again $A(0) = A_{\text{RN}}(r_0) = 0$ and $\partial_\eta A(0) = \partial_r A_{\text{RN}}(r_0) > 0$.

Substituting (58)--(59), (60)--(61) and (62)--(63) into the set of matching relations (53)--(55) determines all parameters of the wormhole solutions $(r_0, m, Q, b_0, T)$ in terms of $q$ (the $LL$-brane charge) and $f$ (coupling constant of the “square-root” Maxwell term in (43)):

$$Q = 0, \quad |c_F| = |q| + \frac{f}{\sqrt{2}}, \quad \text{sign}(q) = -\text{sign}(F_{\eta r}) \equiv -\text{sign}(c_F), (64)$$

$$\frac{1}{r_0^2} = 4\pi \left( |q| + \frac{f}{\sqrt{2}} \right)^2 + \Lambda, \quad m = \frac{r_0}{2} \left[ 1 - \frac{1}{3} (\Lambda + 2\pi f^2) r_0^2 \right], (65)$$

$$b_0 = \frac{1}{4} \left( q^2 + \sqrt{2} f |q| \right) \left[ \left( |q| + \frac{f}{\sqrt{2}} \right)^2 + \frac{1}{4\pi} \Lambda \right]^{-1}, \quad T^2 = \frac{1}{16\pi} \left( q^2 + \sqrt{2} f |q| \right), (66)$$

and the bare cosmological constant must be in the interval:

$$-4\pi \left( |q| + \frac{f}{\sqrt{2}} \right)^2 < \Lambda < 4\pi \left( q^2 - \frac{f^2}{2} \right),$$ (67)

in particular, $\Lambda$ could be zero.

The next wormhole solution has $\text{Rind}_2 \times S^2$ as compactified “left” universe whenever $\Lambda > -2\pi f^2$. It is built by:
(D-1) “Left universe” for $\eta < 0$ of Levi-Civita-Bertotti-Robinson (“tube-like”) type with geometry \( Rind_2 \times S^2 \) (22):

\[
A(\eta) = -\eta, \quad C(\eta) \equiv r_0^2 = \frac{1}{4\pi c_F^2 + \Lambda} = \text{const},
\]

\[
|F_{\nu\eta}| = |c_F| = \frac{f}{\sqrt{2}} \left( 1 + \sqrt{1 + \frac{\Lambda}{2\pi f^2}} \right) = \text{const}; \quad (68)
\]

(D-2) Non-compact “right universe” for $\eta > 0$ comprising the exterior region of Reissner-Nordström-de-Sitter-type black hole beyond the middle (Schwarzschild-type) horizon $r_0$ as in (59).

Again substituting (68) and (59) into the set of matching relations (53)–(55) determines all parameters of the wormhole solution (D-1)–(D-2) in complete analogy with (64)–(66):

\[
Q = 0, \quad |c_F| = |q| + \frac{f}{\sqrt{2}}, \quad \text{sign}(q) = -\text{sign}(F_{\nu\eta}) \equiv -\text{sign}(c_F), \quad (69)
\]

\[
\Lambda = 4\pi \left( q^2 - \frac{f^2}{2} \right), \quad \frac{1}{r_0^2} = 8\pi \left( q^2 + \frac{f}{\sqrt{2}} |q| \right), \quad m = \frac{r_0}{2} \left[ 1 - \frac{4\pi q^2}{3} r_0^2 \right], \quad (70)
\]

\[
b_0 = \frac{1}{4} \left[ 1 + r_0 - 4\pi q^2 r_0^2 \right], \quad T^2 = \frac{b_0}{2\pi} \left( q^2 + \frac{f}{\sqrt{2}} |q| \right). \quad (71)
\]

The result $Q = 0$ in (64) and (69) has profound consequences. Namely, the absence of Coulomb field in spite of the presence of the charged LL-brane source leads us to the following important observations:

(A) The “right-universe” in the wormhole solutions (A-1)–(A-2) (Eqs.(58)–(59)) and (D-1)–(D-2) (Eqs.(68), (59)) becomes exterior region of electrically neutral Schwarzschild-de-Sitter black hole beyond the internal (Schwarzschild-type) horizon carrying a vacuum constant radial electric field $|F_{\nu\eta}| = |F_{0r}| = \frac{f}{\sqrt{2}}$.

(B) The “right-universe” in the wormhole solution (B-1)–(B-2) (Eqs.(60)–(61)) becomes exterior region of electrically neutral Schwarzschild-anti-de-Sitter black hole beyond the sole (Schwarzschild-type) horizon carrying a vacuum constant radial electric field $|F_{\nu\eta}| = |F_{0r}| = \frac{f}{\sqrt{2}}$.

(C) The “right-universe” in the wormhole solution (C-1)–(C-2) (Eqs.(62)–(63)) becomes exterior region of the ordinary electrically neutral Schwarzschild black hole beyond the horizon carrying a vacuum constant radial electric field $|F_{\nu\eta}| = |F_{0r}| = \frac{f}{\sqrt{2}}$.

(D) According to (64) and (69) the whole flux of the electric field $|F_{0\eta}|$ with $|F_{0\eta}| = |F_{\nu\eta}| = \frac{f}{\sqrt{2}} + |q|$ produced by the LL-brane charge $q$ flows only into the compactified “left universe” of Levi-Civita-Bertotti-Robinson type (AdS$_2 \times S^2$ (19) or $Rind_2 \times S^2$ (22)). Due to the absence of electric flux in the non-compact “right universe”, an outside observer there will therefore detect the charged LL-brane as a neutral object.

A clearer explanation of above statements (A)-(D) can be given if we recall that the electric flux is defined in terms of the electric displacement field $\vec{D}$, which in the
present case is significantly different from the electric field $\vec{E}$ due to the presence of the “square-root” Maxwell term in (43):

$$\vec{D} = \left(1 - \frac{f}{\sqrt{2} |\vec{E}|}\right) \vec{E}.$$  \hspace{1cm} (72)

Indeed, in the absence of magnetic field the 0-th component of the nonlinear gauge field Eqs.(45) can be written in terms of $\vec{D}$ (72) as:

$$\vec{D} \cdot \left(\sqrt{-G} \vec{D}\right) - \sqrt{-G} J^0 = 0,$$  \hspace{1cm} (73)

where $J^\mu = \frac{1}{\sqrt{-G}} j^\mu$ is the charge vector current, so that:

$$\int_{\partial \Sigma} d\Sigma. \vec{D} = Q_{\text{total}} = \int_{\Sigma} dV J^0.$$  \hspace{1cm} (74)

Here the factors $\sqrt{-G}$ go into the definition of the corresponding volume forms (integration measures) on the three-dimensional region $\Sigma$ and its boundary $\partial \Sigma$.

Thus, Eq.(74) tells us that the electric flux from the charged $LL$-brane flowing into the non-compact “right universes”, where the constant radial vacuum electric field has magnitude $|\vec{E}| = \frac{f}{\sqrt{2}}$, is zero since $\vec{D} = 0$ there according to (72). On the other hand, inside the compactified Levi-Civita-Bertotti-Robinson-type “left universe”:

$$\vec{D} = \frac{|q|}{\sqrt{2} + |q|} \vec{E} = -q \hat{\eta},$$  \hspace{1cm} (75)

where $\hat{\eta}$ denotes the unit vector along the “radial-like” $\eta$ coordinate (here we have used relations (64) and (69)). Therefore, the whole electric flux from the charged $LL$-brane is expelled into the “tube-like” “left universe”.

The geometry of the charge-“hiding” wormholes is visualized in Figure 1.

7. Charge-Confining Wormhole

Apart from the above charge-hiding effect produced by “one-throat” wormhole connecting a non-compact “universe” to a compactified “tube-like” “universe” via $LL$-brane we find an even more interesting “two-throat” wormhole solution exhibiting QCD-like charge confinement effect. Namely, let us now consider a self-consistent coupling of the gravity/nonlinear-gauge-field system (2) with two separate oppositely charged, but otherwise identical $LL$-branes described by the action (43) and the resulting equations of motion (44)–(48) (here $N = 2$, $T^{(1)} = T^{(2)} \equiv T$, $b_0^{(1)} = b_0^{(2)} \equiv b_0$, $q^{(1)} \equiv q = -q^{(2)}$).

Using the general scheme outlined in Section 5 we construct a solution where the total “two-throat” wormhole space-time manifold is built as follows:

(E-1) “Left-most” non-compact “universe” comprising the exterior region of Reissner-Nordström-de-Sitter-type black hole beyond the middle Schwarzschild-type horizon $r_0$ for the “radial-like” $\eta$-coordinate interval (see also Eqs.(79) and
Fig. 1. Shape of $t = \text{const}$ and $\theta = \frac{\pi}{2}$ slice of charge-“hiding” wormhole geometry. The whole electric flux is expelled into the lower cylindric tube.

\begin{equation}
-\infty < \eta < -\eta_0 \equiv -\left[ 4\pi \left( \sqrt{2} |c_F| - c_F^2 \right) + \Lambda \right]^{-\frac{1}{2}},
\end{equation}
where (cf. (6)–(9)):

\[ A(\eta) = A_{RNdS}(r_0 - \eta_0 - \eta) \]

\[ = 1 - \sqrt{8\pi |Q|} f - \frac{2m}{r_0 - \eta_0 - \eta} + \frac{Q^2}{(r_0 - \eta_0 - \eta)^2} - \frac{\Lambda + 2\pi f^2}{3} (r_0 - \eta_0 - \eta)^2, \]

\[ C(\eta) = (r_0 - \eta_0 - \eta)^2, \quad F_v(\eta) = F_0(\eta) = \frac{\varepsilon F}{\sqrt{2}} + \frac{Q}{\sqrt{4\pi (r_0 - \eta_0 - \eta)^2}}. \]  

(E-2) “Middle” compactified “tube-like” “universe” of Levi-Civita-Bertotti-Robinson type with geometry \( dS \times S^2 \) comprising the finite extent (w.r.t. \( \eta \)-coordinate) region between the two horizons of \( dS_2 \) at \( \eta = \pm \eta_0 \):

\[ -\eta_0 < \eta < \eta_0 \equiv \left[ 4\pi \left( \sqrt{2} f |c_F| - c_F^2 \right) + \Lambda \right]^{-\frac{1}{2}}, \]  

where (cf. Eqs. (24)–(25)):

\[ A(\eta) = 1 - \left[ 4\pi \left( \sqrt{2} f |c_F| - c_F^2 \right) + \Lambda \right] \eta^2, \quad A(\pm \eta_0) = 0, \]

\[ C(\eta) = r_0^2 = \frac{1}{4\pi c_F^2 + \Lambda}, \quad |F_v(\eta)| = |F_v(\eta)| < \frac{f}{\sqrt{2}} \left( 1 + \sqrt{1 + \frac{2\pi f^2}{\Lambda}} \right), \]  

with \( \Lambda > -2\pi f^2 \).

(E-3) “Right-most” non-compact “universe” comprising the exterior region of Reissner-Nordström-de-Sitter-type black hole beyond the middle Schwarzschild-type horizon \( r_0 \) for the “radial-like” \( \eta \)-coordinate interval:

\[ \eta_0 < \eta < \infty \quad (\eta_0 \text{ as in (79)}), \]

i.e., a mirror image (\(-\eta \rightarrow \eta\)) of the “left-most” “universe” (77)–(78):

\[ = 1 - \sqrt{8\pi |Q|} f - \frac{2m}{r_0 + \eta - \eta_0} + \frac{Q^2}{(r_0 + \eta - \eta_0)^2} - \frac{\Lambda + 2\pi f^2}{3} (r_0 + \eta - \eta_0)^2, \]

\[ C(\eta) = (r_0 + \eta - \eta_0)^2, \quad F_v(\eta) = F_0(\eta) = \frac{\varepsilon F}{\sqrt{2}} + \frac{Q}{\sqrt{4\pi (r_0 + \eta - \eta_0)^2}}. \]  

Here \( A(\eta_0) = A_{RNdS}(r_0) = 0 \) and \( \partial_\eta A(\eta_0) = \partial_r A_{RNdS}(r_0) > 0 \).

According to the “horizon straddling” property (42) of world-volume \( LL \)-brane dynamics, each one of the two charged \( LL \)-branes (with equal world-volume parameters \( T, b_0 \) but with opposite charges \( \pm q \), cf. (31)), automatically locates itself on one of the two common horizons between “left-most” (E-1) and middle (E-2) “universes” at \( \eta = -\eta_0 \) and between middle (E-2) and “right-most” (E-3) “universes” at \( \eta = \eta_0 \), respectively.

Now, as we did in the previous Section, substituting (76)–(84) into the set of matching relations (53)–(55) determines all parameters of the wormhole solutions.
\( (r_0, \eta_0, m, Q, b_0, T) \) in terms of \(|q|\) (the magnitude of the LL-brane charges) and \(f\) (the coupling constant of the “square-root” Maxwell term in (43)):

\[
Q = 0, \quad |c_F| = |q| + \frac{f}{\sqrt{2}}, \quad \text{sign}(q) = -\text{sign}(F_{v\eta}) \equiv -\text{sign}(c_F),
\]

\[
\frac{1}{r_0^2} = 4\pi \left(|q| + \frac{f}{\sqrt{2}}\right)^2 + \Lambda, \quad m = \frac{r_0}{2} \left[1 - \frac{1}{3}(\Lambda + 2\pi f^2) r_0^2\right],
\]

\[
\eta_0 = \left[2\pi f^2 - 2q^2 + \Lambda\right]^{-\frac{1}{2}},
\]

\[
b_0 = \frac{1}{4} \left(1 - (\Lambda + 2\pi f^2) r_0^2 + 2r_0 \sqrt{\Lambda + 2\pi f^2 - 4\pi q^2}\right),
\]

\[
T^2 = \frac{b_0}{4\pi} \left(|q| + \frac{f}{\sqrt{2}}\right)^2 + \frac{1}{4\pi} \Lambda.
\]

The bare cosmological constant must be in the interval:

\[
\Lambda \leq 0, \quad |\Lambda| < 2\pi f^2 - 2q^2 \quad \rightarrow \quad |q| < \frac{f}{\sqrt{2}},
\]

in particular, \(\Lambda\) could be zero.

Again, as in the previous Section, relations (85) are of primary importance. They tell us that:

- The “left-most” (76)–(78) and “right-most” (82)–(84) non-compact “universes” become two identical copies of the electrically neutral exterior region of Schwarzschild-de-Sitter black hole beyond the Schwarzschild horizon carrying a constant vacuum radial electric field with magnitude \(|F_{v\eta}| = |F_{0r}| = \frac{f}{\sqrt{2}}\) pointing inbound towards the horizon in one of these “universes” and pointing outbound w.r.t. the horizon in the second “universe”. The corresponding electric displacement field \(\vec{D} = 0\), so there is no electric flux there (cf. Eq.(72)–(74)).

- The whole electric flux produced by the two charged LL-branes with opposite charges \(\pm q\) at the boundaries of the above non-compact “universes” is confined within the “tube-like” middle “universe” (79)–(81) where the constant electric field is \(|F_{v\eta}| = \frac{f}{\sqrt{2}} + |q|\) with associated non-zero electric displacement field \(|\vec{D}| = |q|\) (cf. Eqs.(72)–(74)).

The geometry of the charge-confining wormhole is visualized in the Figure 2.

### 8. Discussion and Conclusions

In this paper we have studied bulk gravity/nonlinear-gauge-field system self-consistently coupled to one or two charged lightlike branes as matter sources. An important feature of this system is the special form of the nonlinear gauge field sector in (43) previously known to produce QCD-like confining dynamics in flat
Hiding and Confining Charges via “Tube-like” Wormholes

Fig. 2. Shape of $t = \text{const}$ and $\theta = \frac{\pi}{2}$ slice of charge-confining wormhole geometry. The whole electric flux is confined within the middle cylindric tube.

space-time $^{3,7}$. The main objective here was to search for similar charge confining behavior in curved space-time, where the role of charged objects subject to confinement is played by charged lightlike branes.

We found that charge-confining or charge-“hiding” effects take place within wormhole solutions to the coupled gravity/nonlinear-gauge-field/lightlike-brane system (43) with the following special structure:

(i) One of the “universes” comprising the total wormhole space-time manifold must be a compactified “universe” of Levi-Civita-Bertotti-Robinson (“tube-like”)
type with geometry $\mathcal{M}_2 \times S^2$ where the two-dimensional manifold $\mathcal{M}_2$ possesses at least one horizon;

(ii) The one or two outer “universe(s)” are non-compact spherically symmetric with at least one Schwarzschild-type horizon;

(iii) The matching (gluing together) of the compactified “universe” with the (one of the two) outer non-compact “universe(s)” takes place at a common horizon of both of them, which is automatically occupied by (one of the participating) charged lightlike brane(s) (“horizon straddling” as dictated by world-volume lightlike brane dynamics);

(iv) Due to the presence of the “square-root” Maxwell term in (43) a non-zero constant vacuum electric field is generated in (any of) the outer non-compact “universe(s)”, however, the total flux is zero there because of vanishing of the pertinent electric displacement field, so that the charged lightlike brane occupying the “throat” between the non-compact and the compactified “tube-like” “universes” appears as electrically neutral to an external observer in the non-compact “universe”.

(v) In the compactified “tube-like” “universe” the charged lightlike brane(s) at its “border(s)”, where it is matched to the non-compact “universe(s)”, produce a non-zero flux entirely confined within the “tube-like” “universe”;

(vi) When only one charged lightlike brane is present, the compactified “tube-like” “universe” with geometries $\mathcal{M}_2 \times S^2$, where $\mathcal{M}_2 = AdS_2$ or $\mathcal{M}_2 = Rind_2$, has an infinite extent w.r.t. “radial-like” $\eta$-coordinate of $\mathcal{M}_2$ and it absorbs entirely the whole flux produced by the brane at the “border”. In this way it hides the charge of the brane from an outside observer in the “neighboring” non-compact “universe”.

(vii) When two oppositely charged but otherwise identical lightlike branes are present, the middle “tube-like” “universe” stretching between them has geometry $dS_2 \times S^2$ and has final extent w.r.t. “radial-like” $\eta$-coordinate of the $dS_2$ component. It absorbs entirely the whole flux produced between the branes at its “borders”, i.e., the whole flux is confined within the finite-extent “tube-like” region without flowing into any of the outside non-compact space-time regions.

It is natural to expect that in a confining theory the gauge field prefers flux-tube configurations, however, the mathematical details of how this happens might be complicated in flat space-time. On the other hand, in the present curved space-time model we obtain the following clear and simple picture:

(a) Due to the presence of lightlike brane(s) as material source(s) of gravity and gauge forces, the very special lightlike brane world-volume dynamics triggers one or more transitions between non-compact and compactified “tube-like” space-time regions in the form of special wormhole configurations with the lightlike brane(s) sitting at the “throat(s)”;

(b) Again the special lightlike brane world-volume dynamics in combination with the special properties of the additional “square-root” Maxwell term in the nonlinear gauge field action cause the whole flux generated by the charged branes to be entirely confined within the compactified “tube-like” region.
As a final remark, returning to the non-linear gauge field Eqs. (5) we see that there exists a more general vacuum solution of the latter without the assumption of staticity and spherical symmetry:

$$F^2 \equiv F_{\kappa\lambda} F_{\mu\nu} G^{\kappa\mu} G^{\lambda\nu} = -f^2 = \text{const}. \quad (90)$$

The latter automatically produces via Eq. (4) an effective positive cosmological constant:

$$T_{\mu\nu}^{(F)} = -\frac{f^2}{4} G_{\mu\nu}, \quad \text{i.e. } \Lambda_{\text{eff}} = 2\pi f^2. \quad (91)$$

This reduces the gravity/gauge-field equations of motion (3), (5) to the vacuum Einstein equations (with effective cosmological constant):

$$R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R + (\Lambda + 2\pi f^2) G_{\mu\nu} = 0 \quad (92)$$

supplemented with the constraint Eq. (90). Thus, assuming absence of magnetic field ($F_{mn} = 0$), i.e., $F^2 = 2 F_m E_m G^{mn} G_{00}$, $E_m \equiv F_{0m}$ ($m, n = 1, 2, 3$), we obtain the above described electrically neutral Schwarzschild-(anti)-de-Sitter or purely Schwarzschild solutions with a constant vacuum electric field (10), which according to (90) has constant magnitude:

$$|\vec{E}| \equiv \sqrt{-\frac{1}{2} F^2} = \frac{f}{\sqrt{2}}, \quad (93)$$

but it may point in arbitrary direction. In this vacuum with disordered constant-magnitude electric field it will not be able to pass energy to a test charged particle, which instead will undergo a kind of Brownian motion, therefore no Schwinger pair-creation mechanism will take place.

Acknowledgments

E.N. and S.P. are supported by Bulgarian NSF grant DO 02-257. Also, all of us acknowledge support of our collaboration through the exchange agreement between the Ben-Gurion University and the Bulgarian Academy of Sciences. We are grateful to Stoycho Yazadjiev for constructive discussions and Doug Singleton for correspondence. Thanks are also due to the referee for useful remarks.

References

1. G. ’t Hooft, *Nucl. Phys. B (Proc. Suppl.)* 121, 333-340 (2003) (arxiv:0208054[hep-th]).
2. E. Guendelman, *Int. J. Mod. Phys.* A19, 3255 (2004) (arxiv:0306162[hep-th]).
3. P. Gaete and E. Guendelman, *Phys. Lett.* B640B, 201-204 (2006) (arxiv:0607113[hep-th]).
4. P. Gaete, E. Guendelman and E. Spalluci, *Phys. Lett.* B649B, 217 (2007) (arxiv:0702067[hep-th]).
5. E. Guendelman, *Mod. Phys. Lett.* A22, 1209-1215 (2007) (arxiv:0703139[hep-th]).
6. I. Korover and E. Guendelman, *Int. J. Mod. Phys.* A24, 1443-1456 (2009).
7. E. Guendelman, *Int. J. Mod. Phys.* A25, 4195-4220 (2010) (arxiv:1005.1421[hep-th]).
8. H. Nielsen and P. Olesen, *Nucl. Phys.* B57, 367-380 (1973).
9. A. Aurilia, A. Smailagic and E. Spallucci, *Phys. Rev.* D47, 2536 (1993).
10. N. Amer and E. Guendelman, *Int. J. Mod. Phys.* A15, 4407 (2000).
11. E. Guendelman, *Int. J. Mod. Phys.* A24, 1443-1456 (2009).
12. E. Guendelman, *Int. J. Mod. Phys.* A25, 4195-4220 (2010) (arxiv:1005.1421[hep-th]).
13. H. Nielsen and P. Olesen, *Nucl. Phys.* B57, 367-380 (1973).
14. A. Aurilia, A. Smailagic and E. Spallucci, *Phys. Rev.* D47, 2536 (2010) (arxiv:1005.1421[hep-th]).
15. H. Lehmann and T.T. Wu, *Nucl. Phys.* B237, 205 (1984).
16. S.L. Adler and T. Piran, *Phys. Rev.* D47, 2536 (1993).
17. E. Guendelman, A. Kaganovich, E. Nissimov and S. Pacheva, *Phys. Lett.* 704B, 230 (2011) (arxiv:1108.0160[hep-th]).
18. C. Misner and J.A. Wheeler, *Ann. of Phys.* 324, 2-15 (2009) (arxiv:0804.1575[hep-ph]).
19. H. Lehmann and T.T. Wu, *Nucl. Phys.* B237, 205 (1984).
20. M. Visser, *“Lorentzian Wormholes. From Einstein to Hawking”* (Springer, Berlin, 1996).
21. D. Hochberg and M. Visser, *Phys. Rev.* D56, 4745 (1997) (arxiv:9704082[gr-qc]).
22. J. Lemos, F. Lobo and S. de Oliveira, *Phys. Rev.* D68, 064004 (2003) (arxiv:0302049[gr-qc]).
23. S. Sushkov, *Phys. Rev.* D71, 043520 (2005) (arxiv:0502084[gr-qc]).
24. F. Lobo, *arxiv:0710.4474[gr-qc].
25. K. Bronnikov and J. Lemos, *Phys. Rev.* D79, 104019 (2009) (arxiv:0902.2360[gr-qc]).
26. K. Bronnikov, M. Skvortsova and A. Starobinsky, *Grav. Cosmol.* 16, 216 (2010) (arxiv:1005.3262[gr-qc]).
27. E. Guendelman, A. Kaganovich, E. Nissimov and S. Pacheva, *Phys. Rev.* D72, 0806011 (2005) (hep-th/0507193).
28. E. Guendelman, A. Kaganovich, E. Nissimov and S. Pacheva, *Fortschr. der Physik* 55, 579 (2007) (hep-th/0612091).
29. E. Guendelman, A. Kaganovich, E. Nissimov and S. Pacheva, *Fortschr. der Phys.* 57, 566 (2009) (arxiv:0901.4443[hep-th]).
30. E. Guendelman, A. Kaganovich, E. Nissimov and S. Pacheva, *Phys. Lett.* 681B, 457 (2009) (arxiv:0904.3198[hep-th]).
31. E. Guendelman, A. Kaganovich, E. Nissimov and S. Pacheva, *Int. J. Mod. Phys.* A25, 1571-1596 (2010) (arxiv:0908.4115[hep-th]).
32. E. Guendelman, A. Kaganovich, E. Nissimov and S. Pacheva, *Phys. Lett.* 673B, 288-292 (2009) (arxiv:0811.2882[hep-th]).
33. E. Guendelman, A. Kaganovich, E. Nissimov and S. Pacheva, *Int. J. Mod. Phys.* A25, 1405 (2010) (arxiv:0904.0401[hep-th]).
34. E. Guendelman, A. Kaganovich, E. Nissimov and S. Pacheva, *Gen. Rel. Grav.* 43, 1487-1513 (2011) (arxiv:1007.4893[hep-th]).
35. E. Guendelman, A. Kaganovich, E. Nissimov and S. Pacheva, in “Sixth Meeting in Modern Mathematical Physics”, B. Dragovic and Z. Rakic (eds.), Belgrade Inst. Phys. Press, 2011 (arxiv:1007.4893[hep-th]).
36. T. Levi-Civita, *Rend. R. Acad. Naz. Lincei*, 26, 519 (1917).
37. B. Bertotti, *Phys. Rev.* D116, 1331 (1959).
39. I. Robinson, Bull. Akad. Pol., 7, 351 (1959).
40. E. Guendelman, Gen. Rel. Grav. 23, 1415 (1991).
41. V. Dzhunushaliev and D. Singleton, Class. Quantum Grav. 16, 973 (1999) (arxiv:gr-qc/9805104).
42. V. Dzhunushaliev and D. Singleton, Phys. Rev. D59, 064018 (1999) (arxiv:gr-qc/9807086).
43. V. Dzhunushaliev, U. Kasper and D. Singleton, Phys. Lett. 479B, 249 (2000) (arxiv:gr-qc/9910092).
44. J. Matyjasek, O. Zaslavsky, Phys. Rev. D64, 044005 (2001) (arxiv:gr-qc/0006014).
45. V. Dzhunushaliev, Gen. Rel. Grav. bf 35, 1481 (2003) (arxiv:gr-qc/0301046).
46. O. Zaslavskii, Phys. Rev. D70, 104017 (2004) (arxiv:gr-qc/0410101).
47. O. Zaslavskii, Phys. Lett. 634B, 111 (2006) (arxiv:gr-qc/0601017).
48. O. Zaslavskii, Phys. Rev. D80, 064034 (2009) (arxiv:0909.2270[gr-qc]).
49. Ch. Misner, K. Thorne and J.A. Wheeler, “Gravitation” (W.H. Freeman and Co., San Francisco, 1973).
50. S. Carroll, “Spacetime and Geometry. An Introduction to General Relativity” (Addison Wesley, San Francisco, 2003).
51. A. Einstein and N. Rosen, Phys. Rev. 43, 73 (1935).
52. N. Poplawski, Phys. Lett. 189B, 110 (2010) (arxiv:0902.1994[gr-qc]).
53. C. Barrabés and P. Hogan, “Singular Null-Hypersurfaces in General Relativity” (World Scientific, Singapore, 2004).
54. K. Thorne, R. Price and D. Macdonald (Eds.), “Black Holes: The Membrane Paradigm” (Yale Univ. Press, New Haven, CT, 1986).
55. W. Israel, Nuovo Cim. B44, 1 (1966); erratum, Nuovo Cim. B48, 463 (1967).
56. C. Barrabés and W. Israel, Phys. Rev. D43, 1129 (1991).
57. T. Dray and G. ’t Hooft, Class. Quantum Grav. 3, 825 (1986).
58. V. Berezin, A. Kuzmin and I. Tkachev, Phys. Rev. D36, 2919 (1987).
59. I. Kogan and N. Reis, Int. J. Mod. Phys. A16, 4567 (2001) (arxiv:hep-th/0107163).
60. D. Mateos and S. Ng, JHEP 0208, 005 (2002) (arxiv:hep-th/0205291).
61. D. Mateos, T. Mateos and P.K. Townsend, JHEP 0312, 017 (2003) (arxiv:hep-th/0309114).
62. A. Bredthauer, U. Lindström, J. Persson and L. Wulff, JHEP 0402, 051 (2004) (arxiv:hep-th/0401159).
63. C. Barrabés and W. Israel, Phys. Rev. D71, 064008 (2005) (arxiv:gr-qc/0502108).
64. U. Lindström and H. Svendsen, Int. J. Mod. Phys. A16, 1347 (2001) (arxiv:hep-th/0007101).
65. E. Guendelman, A. Kaganovich, E. Nissimov and S. Pacheva, Centr. Europ. Journ. Phys. 7, 668 (2009) (arxiv:0711.2877[hep-th]).
66. W. Israel and E. Poisson, Phys. Rev. Lett. 63, 1663 (1989).
67. W. Israel and E. Poisson, Phys. Rev. D41, 1796 (1990).
68. E. Guendelman and A. Rabinowitz, Phys. Rev. D44, 3152 (1991).
69. M. Barriola and A Vilenkin, Phys. Rev. Lett. 63, 341 (1989).
70. E. Guendelman and A. Rabinowitz, Gen. Rel. Grav. 28, 117 (1996).
71. T. Jacobson, Class. Quantum Grav. 24, 5717 (2007) (arxiv:0707.3272[gr-qc]).
72. H. Nariai, Sci. Rep. Tohoku Univ. 34, 160 (1950).
73. H. Nariai and Y. Ueno, Progr. Theor. Phys. 24, 1149 (1960).
74. V. Cardoso, O. Dias and J. Lemos, Phys. Rev. D70, 024002 (2004) (arxiv:hep-th/0401192).
75. K. Bronnikov, Phys. Rev. D63, 044005 (2001) (arxiv:gr-qc/0006014).
76. A. Eddington, Nature, 113, 192 (1924).
28  E. Guendelman, A. Kaganovich, E. Nissimov and S. Pacheva

77.  D. Finkelstein, Phys. Rev. D110, 965 (1958).
78.  S. Blau, E. Guendelman and A. Guth, Phys. Rev. D35, 1747 (1987).