Physics implication from a $Z_3$ symmetry of hadrons

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I show that breaking $B - L$ by one unit of this charge is suitable for neutrino mass generation through an inverse seesaw mechanism, stabilizing a dark matter candidate without supersymmetry, as well as solving the muon anomalous magnetic moment and the $W$ mass deviation via dark field contributions. The new physics is governed by the residual $Z_3$ symmetry of $B - L$ isomorphic to the center of the color group, instead of the well-studied matter parity.

I. INTRODUCTION

Of the exact��存s in physics, the conservation of baryon number minus lepton number, say $B - L$, is questionable. Of the fundamental dynamics in physics, the confinement of colors within hadrons such that only the hadronic states of types $qqq, qq^*$, and their conjugation/combination emerge causes curiosity. Such behavior of hadrons indeed obeys an exact $Z_3$ symmetry that governs constituent quarks, independent of the colors.

There is no necessary principle of the $B - L$ conservation as well as the $Z_3$ symmetry of quarks. They are direct consequences of the standard model gauge symmetry. Indeed, every interaction of the standard model separately conserves $B$ and $L$ such that $B - L$ is a conserved and anomaly-free charge and is thus quantum consistent. Whilst, the $Z_3$ symmetry of quarks is accidentally conserved by the $SU(3)_C$ color group and is never violated. Indeed, this group can be regarded as isomorphic to the center of the color group.

In contrast to electric and color charges, the excess of baryons over antibaryons of the universe suggests that the $B - L$ charge would be broken. Furthermore, the $B - L$ breaking is strongly implied by compelling neutrino mass mechanisms [11,9]. The $B - L$ charge is likely to occur in the theories of left-right symmetry [10,12] and grand unification [13]. Yet, no such traditional theories manifestly explain the existence of the accident $Z_3$ symmetry of quarks, similarly to the standard model.

I point out that such hidden features of the standard model naturally arise from a $U(1)_{B-L}$ gauge symmetry. It is noted that in a period, the matter parity—a residual symmetry of $B - L$ that transforms trivially on normal matter—has been found usefully in supersymmetry [14]. I argue in this work that there is no matter parity at all. The $Z_3$ symmetry of quarks plays the role instead, in which this $Z_3$ relates to $B - L$ as the smallest and unique residual symmetry of $B - L$ itself.

Consequently, this proposal leads to novel physical results for neutrino mass [15,16], dark matter [17,19], the muon anomalous magnetic moment [20], and the $W$ mass deviation [21], without necessity of any left-right symmetry, grand unification, or supersymmetry. Namely, the neutrino mass generation is induced by an inverse seesaw mechanism due to the breaking of $B - L$ by one unit. The dark matter stability is ensured by the residual $Z_3$ symmetry of $B - L$, i.e. the $Z_3$ symmetry of quarks, while the muon magnetic moment and the $W$ mass are contributed by the dark sector that contains the dark matter.

The rest of this work is organized as follows. In Sec. [I] I propose the model. The neutrino mass generation is examined in Sec. [II]. The residual symmetry and its resultant dark sector are investigated in Sec. [IV]. The dark matter observables are presented in Sec. [V]. The muon anomalous magnetic moment and $W$ mass deviation are supplied in Sec. [VI]. I make concluding remarks in Sec. [VII].

II. THE MODEL

The full gauge symmetry is

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L}.$$  (1)

Leptons and quarks transform under this symmetry as

$$l_{aL} = \left( \begin{array}{c} u_{aL} \\ e_{aL} \end{array} \right) \sim (1, 2, -1/2, -1),$$  (2)

$$\nu_{aR} \sim (1, 1, 0, -1),$$  (3)

$$e_{aR} \sim (1, 1, -1, -1),$$  (4)

$$q_{aL} = \left( \begin{array}{c} u_{aL} \\ d_{aL} \end{array} \right) \sim (3, 2, 1/6, 1/3),$$  (5)

$$u_{aR} \sim (3, 1, 2/3, 1/3),$$  (6)

$$d_{aR} \sim (3, 1, -1/3, 1/3),$$  (7)

where the subscript $a = 1, 2, 3$ is a family index, and the right-handed neutrinos $\nu_{aR}$ are included for $B - L$ anomaly cancelation, as usual.

The gauge anomaly always vanishes if including any gauge-singlet chiral fermion (or sterile fermion), such as

$$N_{aL} \sim (1, 1, 0, 0),$$  (8)

where three copies of the sterile fermion are proposed, corresponding to three families. Note that the gauge symmetry suppresses bare masses of $\nu_{R/R}$ type for right-handed neutrinos, while it allows bare masses of such type for sterile fermions, say $N_L N_L$. 

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The gauge symmetry breaking proceeds through the usual Higgs doublet,
\[ \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (1, 2, 1/2, 0), \]
and a scalar singlet,
\[ \chi \sim (1, 1, 0, 1), \]
that couples \( N_L \) to \( v_R \) through \( N_L v_R \chi \) couplings. They have vacuum expectation values (VEVs),
\[ \langle \phi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \]
\[ \langle \chi \rangle = \Lambda/\sqrt{2}, \]
such that \( \Lambda \gg v = 246 \text{ GeV} \) for consistency with the standard model.

The scheme of symmetry breaking is
\[ SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_C \otimes U(1)_Y \rightarrow SU(3)_C \]
where \( \Lambda = \Lambda. \) Here note that \( \Lambda \) breaks only
\[ \approx \begin{pmatrix} T, e \end{pmatrix}, \]
\[ \approx \begin{pmatrix} \mu, \tau \end{pmatrix}, \]
is the cube root of unity. Hence, I obtain
\[ \approx \begin{pmatrix} 1 eV \end{pmatrix}, \]
given that \( \Lambda \gg 10 \text{ TeV} \) and \( \mu \approx 1 \text{ keV}. \) Notice that the mixing of \( \nu_L \) with \( (v_R, N_L) \) is suppressed by \( m^{M-1} \ll 1 \) and is therefore neglected. The new fermions \( v_R, N_L \) obtain a Dirac mass \( \sim M \) at TeV scale.

The given \( \Lambda \) scale is suitable to the collider constraints on the \( U(1)_{B-L} \) gauge boson, called \( Z' \). Indeed, the LEPII studied processes \( e^+ e^- \rightarrow f f' \) for \( f = \mu, \tau \) contributed by \( Z' \), giving a bound \( m_{Z'}/g_{B-L} > 6 \text{ TeV} \) \[20\]. Here \( g_{B-L} \) is the \( U(1)_{B-L} \) coupling constant, and the \( Z' \) mass is \( m_{Z'} = g_{B-L}\Lambda. \) This translates to \( \Lambda > 6 \text{ TeV}. \) The LHC searched for dilepton signals through \( pp \rightarrow f f' \) contributed by \( Z' \), supplying a bound \( m_{Z'} \approx 4 \text{ TeV} \) for \( Z' \) couplings identical to those of the \( Z \) boson \[21\]. This converts to \( \Lambda \approx m_{Z'}/g \sim 6 \text{ TeV}, \) similar to the LEPII.

**IV. RESIDUAL SYMMETRY AND DARK SECTOR**

Note that \( \Lambda \) breaks only \( U(1)_{B-L} \) down to \( R \), whereas \( v \) — that breaks the electroweak symmetry—obviously conserves \( R \), as given in the breaking scheme above.

The residual symmetry \( R \) takes the form \( R = e^{i\alpha(B-L)} \) since it is a \( U(1)_{B-L} \) transformation. \( R \) conserves the vacuum \( \Lambda \) if \( R\Lambda = e^{i\alpha(1)} \Lambda = \Lambda. \) Here note that \( \Lambda \) has the charge \( B-L = 1. \) It follows that \( e^{i\alpha} = 1, \) or \( \alpha = 2\pi k, \) for \( k \) integer. Hence, I obtain
\[ R = e^{i\alpha(B-L)} = e^{i\pi k(B-L)} = [w^{3(B-L)}]^k, \]
where \( w \equiv e^{i2\pi/3} \) is the cube root of unity.

The model fields transform under \( R \) with relevant values as given in Table \[3\] where the \( B-L \) charge is also supplied for convenience in reading. It is clear that \( R = 1 \) for every field corresponds to the smallest value of \( |k| = 3 \), except for the identity with \( k = 0. \) Hence, the residual symmetry \( R \) is automorphic to
\[ Z_3 = \{1, g, g^2\}, \]
where
\[ g \equiv w^{3(B-L)}, \] (19)
and \( g^3 = 1 \) for every field, as mentioned\(^1\). Obviously, the residual symmetry \( Z_3 \) is generated by \( g = w^{3(B-L)} \), called matter generator, opposite to the matter parity studied in supersymmetry.

\[ \begin{array}{|c|c|c|c|c|}
\hline
\text{Field} & \ell & q & \chi & \{N, \phi, A\} \\
\hline
B - L & -1 & 1/3 & 1 & 0 \\
R & 1 & w^k & 1 & 1 \\
\hline
\end{array} \]

TABLE I. \( B - L \) charge and \( R \) value of all fields where \( \ell \), \( q \), \( N \), and \( A \) define every lepton (including \( \nu_B \)), quark, sterile fermion, and gauge boson, respectively.

\[ \begin{array}{|c|c|c|c|}
\hline
\text{Field} & \ell & q & \chi & \{N, \phi, A\} \\
\hline
\text{g} & 1 & w & 1 & 1 \\
Z_3 & 1 & 1' & 1 & 1 \\
\hline
\end{array} \]

TABLE II. Matter generator and field representations under the residual symmetry \( Z_3 \).

\( Z_3 \) has three irreducible representations \( 1, 1' \), and \( 1'' \) corresponding to \( g = 1, w \), and \( w^2 \), respectively. The field representations under \( Z_3 \) are given in Table II. It is clear that every field transforms trivially under \( Z_3 \) with \( g = 1 \), except for quarks. Quarks are in \( 1' \) with \( g = w \), whereas antiquarks belong to \( 1'' \) with \( g = w^2 \). Hence, the hidden \( Z_3 \) symmetry of quarks in the standard model can be interpreted to be the residual symmetry of \( B - L \). In contrast to the hidden symmetry, the residual symmetry explicitly relates to \( B - L \) that would lead to dark matter with an appropriate \( B - L \) value.

That said, a dark field possesses a \( B - L \) charge such that the matter generator is nontrivial, i.e. \( g = w^{3(B-L)} \neq 1 \). Combined with \( g^3 = 1 \) that ensures the \( Z_3 \) symmetry, I obtain
\[ B - L = \left\{ \begin{array}{l}
-1/3 + k \\
-2/3 + k'
\end{array} \right\} 
= \pm 1/3, \pm 2/3, \pm 4/3, \pm 5/3, \cdots \] (20)
for \( k, k' \) integer. This identification of dark field is independent of its spin. Additionally, the signs \( \pm \) correspond to a dark field and its conjugation. Each dark field can pick up a \( B - L \) charge only differing from either of the two basic charges, say \(-1/3\) and \(-2/3\), by an integer number, because of the cyclic property of \( Z_3 \). For such reasons, it is sufficient to introduce two dark fields with respect to the two basic charges, respectively; that is, a dark (Dirac) fermion and a dark vector transform under the gauge symmetry as
\[ F \sim (1, 1, 0, -1/3), \] (21)
\[ V = \begin{pmatrix} V^0 \\ V^- \end{pmatrix} \sim (1, 2, -1/2, -2/3). \] (22)

They couple to lepton doublets, such as
\[ \mathcal{L} \supset x_a \bar{a}_L \gamma^\mu A_{\mu} + H.c., \] (23)
in order to make the model phenomenologically viable. The detailed reason of this choice comes from the muon \( g = 2 \), presented in Sec. VI. Notice that \( V \) and \( F \) transform under \( Z_3 \) as \( 1' \) and \( 1'' \), for \( g = w \) and \( w^2 \), respectively, as given in Table III.

\[ \begin{array}{|c|c|c|}
\hline
\text{Dark-field} & V & F \\
\hline
Z_3 & 1' & 1'' \\
\hline
\end{array} \]

TABLE III. Dark field identification according to \( Z_3 \).

Apart from the above couplings, \( V \) and \( F \) possess the Lagrangian terms,
\[ \mathcal{L} \supset \bar{F}(i\gamma^\mu D_\mu - m_F)F - \frac{1}{2} V^\mu V_{\mu
u} + \frac{1}{2} \mu^2 V^\mu V_{\mu} \\
+ i\kappa_1 V^\mu A^\mu V_\nu + i\kappa_2 V^\mu B^\mu V_\nu + i\kappa_3 V^\mu C^\mu V_\nu \\
+ \alpha_1 (V^\mu V_{\mu
u}) (V^\nu V_{\mu}) + \alpha_2 (V^\mu V_{\mu
u}) (V^\nu V_{\mu}) \\
+ \alpha_3 (V^\mu V_{\mu
u}) (V^\nu V_{\mu}) + \lambda_1 (\chi V_{\mu
u}) (V^\mu V_{\mu}) \\
+ \lambda_2 (\phi V_{\mu
u}) (V^\mu V_{\mu}) + \lambda_3 (\phi V^\mu V^\mu), \] (24)
where \( V_{\mu
u} = D_\mu V_\nu - D_\nu V_\mu \). The covariant derivative is \( D_\mu = \partial_\mu + igT_3 A_\mu + ig\gamma_5 B_\mu + ig_{\mu
u} (B - L) C_\mu \), where \( T_j \) \((j = 1, 2, 3)\) label the weak isospin. Additionally, \( A_\mu \) \((A_{\mu\nu}), B_\mu \((B_{\mu\nu}), \) and \( C_\mu \((C_{\mu\nu})\) stand for the gauge fields (field strengths) of \( SU(2)_L \), \( U(1)_Y \), and \( U(1)_{B-L} \), respectively. This theory preserves the \( Z_3 \) symmetry that acts on \( V, F \), in contrast to that in \([29][31]\).

It is noted that the dark vectors generally violate the unitarity condition of the \( S \)-matrix, say \( \langle V'V^\dagger |S|V'V^\dagger \rangle \), where \( V_g = \{A, B, C\} \) denotes every electroweak and \( B - L \) gauge field. Hence, the unitarity condition constrains \( \kappa_1 = g, \kappa_2 = -g\gamma_5/2, \) and \( \kappa_3 = -2g_{B-L}/3 \), where the coefficients correspond to the gauge charges of \( V \) under the symmetry in \([1]\). These conditions that match \( \kappa_{1,2,3} \) to the gauge couplings must be applied, so that the theory works well up to the current energy of colliders at TeV scale, where the standard model is still good, in agreement with \([29]\). The unitarity condition for the elements such as \( \langle V'V^\dagger |S|V'V^\dagger \rangle \) and \( \langle VV^\dagger |S|VV^\dagger \rangle \) will supply

\(^2\) Interactions such as \((\phi V^\dagger) (\phi V')\) are reducible to the given couplings and are thus suppressed.

\(^3\) Neglecting a small kinetic mixing between the two \( U(1) \) gauge fields, \( C \) is identical to \( Z' \), while \( A, B \) define \( W, Z, \gamma \).
relations between \( \alpha_{1,2,3} \). However, since the \( \alpha_{1,2,3} \) couplings are not relevant to the processes studied in this work, I will not refer to them further.

On the other hand, the interactions in \( \{23\} \) also give rise to unitarity violations like \( \langle VV\rangle \langle S[ll]\rangle \). The unitarity condition is preserved, independent of \( x_a \) couplings, by introducing either a new fermion or a new vector that appropriately couples to \( V,l,t \). However, except for this purpose/role, the extra particle would not alter our results considered below and is thus skipped.

After the symmetry breaking, the vector doublet components are separated in mass, such as

\[
\begin{align*}
m_v^2 &= m_V^2 + \lambda_1 \Lambda^2 / 2 + (\lambda_2 + \lambda_3) v^2, \\
m_{V^0}^2 &= m_V^2 + \lambda_1 \Lambda^2 / 2 + \lambda_2 v^2.
\end{align*}
\]

Here the mass splitting is proportional to the weak scale, since \( m_V^2 - m_{V^0}^2 = \lambda_3 v^2 / 2 \), which is small, compared to the \( V \) masses at \( \Lambda \) scale.

It is noteworthy that because \( F \) and \( V \) are color neutral, the lightest field of them cannot decay to colored quarks, despite the fact that both the dark field and quarks transform nontrivially under \( Z_3 \). In this case, the stability of the lightest dark field is preserved by the color charge conservation, in addition to \( Z_3 \). This stability mechanism differs from many extensions for dark matter, including supersymmetry.

V. DARK MATTER ABUNDANCE AND DETECTION

There are two candidates for dark matter, \( V^0 \) and \( F \). For the case of the vector candidate, \( V^0 \) must be the lightest field among the dark fields, namely \( m_{V^0} < m_F \) and \( m_{V^0} < m_{V^\pm} \). The last condition requires \( \lambda_3 > 0 \), which is obviously valid if this coupling results from a gauge completion. Unfortunately, this vector candidate as a complex field belongs to a weak doublet—which interacts with the usual \( Z \) boson—and is not separated in mass. The gauge interaction will induce a large scattering cross-section of \( V^0 \) with nuclei by \( t \)-channel \( Z \) exchange in direct detection, which is already ruled out by experiments analogously to the inert scalar doublet \( \{22\} \).

The model predicts the realistic dark matter to be a dark fermion, \( F \). This fermion candidate interacts with the usual particles via \( V \) and \( Z' \) portals, where \( Z' \) is the \( U(1)_{B-L} \) gauge boson. The annihilation processes of \( F \) to usual particles are described by Feynman diagrams in Fig. \( \{1\} \), where we define \( l = \{\nu_a, e_a\} \) for usual leptons and \( q = \{u_a, d_a\} \) for usual quarks.

The \( V^\pm \) mass and \( x_2 \) coupling satisfy \( |x_2|^2/4\pi m_{V^\pm}^2 \sim (800 \text{ GeV})^{-2} \) for the muon \( g - 2 \), shown below. Hence, the \( t \)-channel diagram exchanged by \( V \) largely contributes to the annihilation cross-section, unless \( m_F \) is much smaller than \( m_V \), in agreement with \( \{31\} \). I also assume \( m_F \ll m_{Z'} \), besides the condition \( m_F \ll m_V \). Therefore, the annihilation cross-section that includes both \( V,Z' \) contributions as in Figure \( \{1\} \) is approximated as

\[
\langle \sigma v \rangle \simeq \frac{m_F^2}{\pi} \left[ \sum_a |x_a|^2 \left( \frac{1}{m_{V^\pm}^2} + \frac{1}{m_{V^0}^2} \right) - \sum_a |x_a|^2 \frac{g_{B-L}^2}{6 m_{Z'}^2} \left( \frac{1}{m_{V^\pm}^2} + \frac{1}{m_{V^0}^2} \right) + \frac{37 g_{B-L}^2}{54 m_{Z'}^2} \right].
\]

Here the dark matter annihilation to top quarks is excluded, i.e. suppressed, similarly to the case of the annihilation to right-handed neutrinos, since the dark matter is radically lighter than such fields.

Because of \( m_{V^0} \approx m_{V^\pm} \) and \( m_{Z'} = g_{B-L} \Lambda \), I further estimate \( \{27\} \) to be

\[
\langle \sigma v \rangle \simeq 1 \text{ pb} \left( \frac{m_F}{6.5 \text{ GeV}} \right)^2 \left( \frac{800 \text{ GeV}}{m_{V^\pm}} \right)^4 \left[ \sum_a |x_a|^2 \right]^2 \left( \frac{1}{6 \pi} \right) \frac{1}{\Lambda^2} + \frac{37}{432 \pi^4} \frac{m_{V^\pm}^4}{\Lambda^4}.
\]

\[\text{If several copies of } F \text{ are introduced as in gauge completion theories, } F \text{ is assumed to be the lightest field of such dark fermions.} \]
It is clear that \((m_{V^\pm}/\Lambda)^2 \sim 10^{-2}|x_2|^2/4\pi\) for \(\Lambda \sim 10\) TeV. So, the contributions of the \(m_{V^\pm}/\Lambda\) terms, i.e. of the \(Z'\) boson, to the annihilation cross-section are small. The expression in brackets is dominated by the first term due to the contribution of \(V\). Taking \(\sum |x_n|^2/4\pi \sim 1\) in perturbative limit and \(m_{V^\pm} \sim 800\) GeV similar to the muon \(g-2\) below, the dark matter gets a correct abundance, i.e. \(\langle \sigma v \rangle \sim 1\) pb \([25]\), if \(m_F \sim 6.5\) GeV \(\footnote{In this work, I assume that there is no asymmetry in number density between a dark particle and a dark antiparticle.}\).

In direct detection, the dark matter \(F\) scatters with quarks confined in nucleons exchanged by \(Z'\), described by the effective Lagrangian,

\[
\mathcal{L}_{\text{eff}} \supset \frac{g_{\mathcal{B}-L}}{9m_{Z'}^2}(\bar{F}i\gamma^\mu F)(\bar{q}\gamma^\mu q).
\]  

Given that \(\Lambda = 10\) TeV, the model predicts \(\sigma_{p,n} \simeq 3.7 \times 10^{-45} \left(\frac{10\text{ TeV}}{\Lambda}\right)^4 \text{ cm}^2\). \(\footnote{In this work, I assume that there is no asymmetry in number density between a dark particle and a dark antiparticle.}\)

It is noted that monophoton events may be produced at the LEPII experiment recoiled against the missing energy carried by a pair of dark matter \(F\), governed by the effective interaction,

\[
\mathcal{L}_{\text{eff}} \supset \frac{|x_1|^2}{m_{V^\pm}^2}(\bar{e}_L\gamma^\mu F_L)(\bar{F}_L\gamma^\mu e_L),
\]

derived directly from \([23]\). With the aid of the Fierz identity, I transform this interaction to the vector and axial vector operators studied in \([32]\), such as

\[
\mathcal{L}_{\text{eff}} \supset \frac{|x_1|^2}{4m_{V^\pm}^2}(\bar{F}\gamma^\mu F)(\bar{e}\gamma^\mu e) + (AA) + (VA) + (AV),
\]

which leads to a bound

\[
m_{V^\pm} > \frac{|x_1|}{2} \times 470\text{ GeV} \sim 800\text{ GeV},
\]

according to \(|x_n|^2/4\pi \sim 0.92\), as expected. This mass limit agrees with the relic density and direction, as well as the muon \(g-2\) below.

\[
\nu \overset{\gamma}{\rightarrow} V^+ + V^-.\]

\(\gamma\)

\(V^+\)

\(V^-\)

\(\mu\)

\(\mu\)

\(\mu\)

\(\mu\)

FIG. 2. Dark field contribution to the muon \(g-2\).

VI. MUON ANOMALOUS MAGNETIC MOMENT AND W MASS DEVIATION

A. Muon \(g-2\)

Anomalous magnetic moment of muon \(a_\mu = \frac{1}{2}(g-2)_\mu\) in the standard model is now established \([30]\)

\[
a_\mu(\text{SM}) = \frac{1}{2} \times 116591810(43) \times 10^{-11}. \quad (34)
\]

The recent measurement of \(a_\mu\) provides an exciting hint for the new physics \([20]\) in which this new result combined with the old E821 result \([37]\) gives a deviation,

\[
a_\mu(\text{Exp}) = a_\mu(\text{SM}) = (251 \pm 59) \times 10^{-11}, \quad (35)
\]

at \(4.2\sigma\) from the standard model prediction. If this result is confirmed, many new physics approaches might be disfavored, since such deviation is larger than the electroweak contribution, say \(a_\mu(\text{EW}) = 153.6(1.0) \times 10^{-11}\). Indeed, the muon \(g-2\) constraint is potentially in tension with those from the electroweak precision test and current colliders. I suggest to solve this question by a contribution from the dark sector.

As shown before, appropriate dark fields may be identified independent of their spins. Additionally, the model is relevant if two dark fields are sufficiently presented, coupled to lepton doublets. It turns out that there are only two types of such couplings, either \(\bar{l}_L\gamma F_L\) for the case of a proposed dark vector or \(\bar{l}_L\gamma F_R\) for the case of a proposed dark scalar, accompanied with the dark fermion. The latter case with a dark scalar contributes insignificantly and negatively valued to the muon \(g-2\), analogous to the minimal scotogenic setup (cf. \([38]\)). Hence, the former case with a dark vector may be significant. Additionally, the dark vector would be a weak doublet, while the dark fermion is a weak singlet, as proposed in the previous section. Otherwise, a vector singlet and a fermion doublet coupled to usual leptons yield an unsuitable negative contribution to the muon \(g-2\) similar to the above scalar case \([39]\), which would be suppressed.

\[\text{FIG. 2. Dark field contribution to the muon } g-2.\]
in Fig. 2. Assuming \( m_\mu \ll m_F, m_{V^\pm} \), I obtain
\[
\Delta a_\mu = \frac{|x_2|^2}{8\pi^2} \frac{m_\mu^2}{m_{V^\pm}^2} 
\times \int_0^1 dt \frac{t(1+t)(m_{V^\pm}^2 + (1-t)(1-\frac{1}{4})m_F^2)}{m_{V^\pm}^2 + (1-t)m_F^2}. \tag{36}
\]
The integral is of the order of 1, thus
\[
\Delta a_\mu \sim 2.5 \times 10^{-9} \left( \frac{|x_2|^2}{4\pi} \right) \left( \frac{800 \text{ GeV}}{m_{V^\pm}} \right)^2. \tag{37}
\]
Compared to the muon \( g-2 \) deviation in \( \text{[35]} \), it gives
\[
m_{V^\pm} \sim 800 \sqrt{\frac{|x_2|^2}{4\pi}} \text{ GeV}. \tag{38}
\]
This prediction agrees with the dark matter constraint given above. The charged vector gains \( m_{V^\pm} \sim 800 \text{ GeV} \) for \( |x_2|^2/4\pi \sim 1 \). It is noted that \( x_2 \) couples \( F \) to the muon doublet of interest.

**B. \( \Delta m_W \)**

The renormalized masses of \( W, Z \) bosons in the on-shell scheme are related through
\[
m_W^2 \left( 1 - \frac{m_W^2}{m_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_F} (1 + \Delta r), \tag{39}
\]
where \( \Delta r = (\Delta r)^{\text{SM}} + (\Delta r)^{\text{NP}} \) represents quantum corrections that come from the standard model and the new physics, respectively. Given that the new physics arises as oblique contributions, one obtains
\[
(\Delta r)^{\text{NP}} = -\langle \xi_V^2/s_W^2 \rangle \Delta \rho, \tag{40}
\]
where \( \Delta \rho = \alpha(m_Z)T \) is the \( \rho \)-parameter deviation from the standard model value related via the \( T \) parameter.

The \( W \) mass in the standard model is extracted, based upon the precisely measured parameters, say \( G_F, \alpha, m_Z \), as well as the top quark and Higgs boson masses and the strong coupling that radiatively contribute to \( (\Delta r)^{\text{SM}} \). That said, the standard model predicts
\[
m_W^{\text{SM}} = 80.357 \pm 0.006 \text{ GeV}, \tag{41}
\]
with respect to \( (\Delta r)^{\text{SM}} \approx 0.038 \) \( \text{[40]} \).

The CDF II collaboration has recently reported a novel result of the \( W \) boson mass,
\[
m_W = 80.4335 \pm 0.0094 \text{ GeV}, \tag{42}
\]
which differs from the standard model prediction given above at 4\( \sigma \) confidence level \( \text{[21]} \). This high precision measurement of the \( W \) mass reveals an exciting hint for the new physics, modifying the standard model value by \( (\Delta r)^{\text{NP}} \approx -0.00489 \). With \( \alpha(m_Z) = 1/128 \) and \( s_W^2 = 0.231 \), it gives rise to \( T \approx 0.188 \).

In the present model, the deviation of the measured \( W \) mass from the standard model expectation comes from a positive contribution of the non-degenerate vector doublet to the \( T \)-parameter, evaluated by
\[
T = \frac{3}{16\pi^2 \alpha(m_Z)v^2} \times \left[ m_{V^\pm}^2 + m_{V^0}^2 - \frac{2m_{V^\pm}^2 m_{V^0}^2}{m_{V^\pm}^2 - m_{V^0}^2} \ln \frac{m_{V^\pm}^2}{m_{V^0}^2} \right], \tag{43}
\]
where the coefficient 3 comes from three physical degrees of freedom of massive vectors\( \text{[41]} \). I have included the contributions of \( V \) to \( W, Z \) self-energies arising from both gauge interactions of \( V \) and \( \kappa_{1,2,3} \) couplings furnished by the unitarity constraint. The computation in \( \text{[41]} \) for \( T \) in 't Hooft-Feynman gauge coincides with the above result in the unitary gauge. Notice that gauge dependence similar to the standard model \( W, Z, \gamma \) contributions to \( T \) does not arise, since \( V \) is not a gauge field \( \text{[42]} \).

Because the mass splitting is small, i.e. \( m_{V^\pm} - m_{V^0} = \lambda_3^2 \ll m_{V^0} \), I further approximate
\[
T \approx \frac{3}{16\pi^2 \alpha(m_Z)v^2} \left( \frac{\lambda_3 v^2/2}{m_{V^0}^2} \right)^2 \approx 0.188 \frac{\lambda_3^2}{\pi} \left( \frac{783 \text{ GeV}}{m_{V^0}} \right)^2. \tag{44}
\]
This coincides with the measured value of the \( W \) mass, i.e. \( T \approx 0.188 \), given that
\[
m_{V^0} \approx 783 \sqrt{\frac{\lambda_3^2}{\pi}} \text{ GeV}. \tag{45}
\]
This mass is comparable to that of the dark charged vector, if \( \lambda_3 \) is similar in size to \( x_2 \).

**VII. CONCLUDING REMARKS**

I have investigated a \( Z_3 \) symmetry of matter, determined by the generator \( g = w^{3(\delta-1)} \), which acts on quarks and governs the inverse seesaw mechanism responsible for neutrino masses. Especially, this symmetry reveals two dark fields—a fermion singlet \( F \) and a vector doublet \( V \)—as necessary elements that solve the questions of dark matter, muon anomalous magnetic moment, and \( W \) mass deviation.

I have shown that the components of \( V \) gains a mass around 800 GeV, whereas \( F \) obtains a mass at 6.5 GeV. Additionally, the couplings of \( V \) to leptons and Higgs boson are near the perturbative limit, say \( |x_2|^2/4\pi \sim 1 \)

6 \( S, U \)-parameters are also modified by the \( V \) doublet but the results are respectively suppressed by \( m_W/m_V \) and \( (m_W/m_V)^2 \) in comparison to \( T \). Hence, such kinds of contribution to the \( W \) mass deviation are not significant.
$|x_1|^2/4\pi \lesssim 0.92$, and $\lambda_5^2/\pi \sim 1$. Furthermore, given that the couplings of $V$ are perturbative, Ref. [29] shows that the dark vectors with masses $m_{V^\pm, V^0} \sim 800$ GeV would satisfy all the high energy collider bounds. So, I would not account for such constraints here.

Such $V$ doublet is hinted from non-Abelian extensions of electroweak symmetry that contain a higher weak isospin $SU(3)_L$ as a sub-/residual group [33], where three copies of $F$ according to three families of leptons are warranted [44–49]. However, the vector doublet with charge $B - L = -2/3$ necessarily coupled to usual leptons may only be understood in the 3-3-1-1 setup [50, 51] or flipped trinifcation [52 53]. While a gauge completion makes more constraint on $V, F$, this work presents an effective theory, by contrast, in which the behaviors of $V, F$ couplings and masses are phenomenologically determined. It is noteworthy that the more fundamental theory would explain the strong couplings of $V$ where either a Landau pole or a technicolor may be predicted at TeV scale.

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