We discuss the gravitino problem in contest of the Exotic see-saw mechanism for neutrinos and Leptogenesis, UV completed by intersecting D-branes Pati-Salam models. In the Exotic see-saw model, supersymmetry is broken at high scales $M_{SUSY} > 10^9$ GeV and this seems in contradiction with gravitino bounds from inflation and baryogenesis. However, if gravitino is the Lightest Stable Supersymmetric Particle, it will not decay into other SUSY particles, avoiding the gravitino problem and providing a good Cold Dark Matter. Gravitini are Super Heavy Dark Particles and they can be produced by non-adiabatic expansion during inflation. Intriguingly, from bounds on the correct abundance of dark matter, we also constrain the neutrino sector. We set a limit on the exotic instantonic coupling of $< 10^{-2} \div 10^{-3}$. This also sets important constrains on the Calabi-Yau compactifications and on the string scale. This model strongly motivates very high energy DM indirect detection of neutrini and photons of $10^{11} \div 10^{13}$ GeV: gravitini can decay on them in a cosmological time because of soft R-parity breaking effective operators.

I. INTRODUCTION

Recently, we suggested a new coherent model for leptogenesis and neutrino mass in contest of IIA superstring theory \cite{1}, inspired by a model previously suggested in Ref.\cite{2}. In particular, we discussed a simple intersecting D-branes construction reproducing a Pati-Salam model in the low energy limit. This proposal is a reincarnation of the old see-saw type I mechanism, which remains the simplest idea in order to unify neutrino mass with leptogenesis\cite{3}. But, contrary to $SO(10)$ inspired model, the main contributions to the RH neutrino mass matrix come from non-perturbative stringy effects known in literature as exotic stringy instantons\cite{4}. In IIA superstring theory, these effects can be calculated and controlled, providing a precise prediction on the neutrino mass matrix \cite{13} \cite{15}. In our model, we assume two main hypothesis: i) the non-perturbative corrections to RH mass matrix from exotic instantons dominate with respect to standard mass terms coming from Pati-Salam spontaneous symmetry breaking pattern; ii) Supersymmetry and Left-Right symmetry are assumed to be spontaneously broken at high scale, i.e. $M_{SUSY}, M_{LR} \gtrsim 10^9$ GeV. Relaxing the (i)-hypotesis and assuming non-perturbative corrections to be sub-leading, we can just obtain a similar scenario to standard $SO(10)$ one. However, this scenario is in tension with Davidson-Ibarra (DI) bound $M_{RH}^{DI} \simeq 10^9$ GeV \cite{10}. The DI mass bound is set by a correct leptogenesis mechanism, considering first generation RH neutrino decaying into leptons and Higgs. The DI bound is usually avoided assuming degeneracies among RH neutrino masses. But such an assumption is a fine-tuning. In particular, we remark that the popular $SO(10)$ PS really is in tension with DI-bound: the lightest RH neutrino is hierarchically constrained to be $M_{RH1} << 10^9$ GeV, i.e. it is not a good candidate for see-saw leptogenesis. As a consequence, in $SO(10)$ PS we necessary have to assume an accidental degeneracy among RH neutrini really not understood in their symmetry breaking pattern. On the other hand, in our scenario, exotic instantons can easily generate democratically RH neutrino masses at high scale. So that, the coincidence of masses is dynamically recovered by exotic instantonic processes, easily avoiding DI bounds. The (ii)-hypothesis is also indirectly motivated by recent results of LHC: TeV-MSSM and TeV-Left-Right scenari seem to be rule-out by recent data. On the other hand, the (ii)-assumption eliminates a lot of undesired free-parameters in our model, rendering it constrainable by low energy observables and leptogenesis consistency. For instance, supersymmetric particles are not produced by RH neutrini decays. These assumptions reduced our model to 13 free parameters mainly parametrizing our ignorance on the internal Calabi-Yau compactification. All parameters are also reduced by the rigidity of Pati-Salam symmetry, relating the CKM matrix of quarks to the Pontecorvo matrix of neutrini. However, they can be constrained by 11 low energy observables in neutrino and quark physics and the leptogenesis
consistency. From these 11 in-puts, we are able to predict a precise range for the $\theta_{13} \neq 0$ oscillation angle. The precise determination of $\theta_{13}$-angle is a hot topic of neutrino physics and it is crucially important to constrain it in future, for our understanding of baryogenesis. As we will show, we will relate measures in neutrino and quark physics with the topology and geometric shape parameters of the Calabi-Yau compactification. However the (ii)-hypothesis can lead to the gravitino problem. The gravitino problem set a bound of $m_{\tilde{G}} < 10^6$ TeV [17]. So that a generic supersymmetric scenario with $M_{SUSY} > 10^9$ GeV can lead to several problems. For instance, the gravitino could be unstable and rapidly decay into supersymmetric partners and Pati-Salam particles, essentially leading to a disastrous washing-out of our leptogenesis scenario. In this paper, we discuss the simplest way-out to this problem, saving our leptogenesis picture. It regards the gravitino stability: if the gravitino is a LSP particle in the SUSY spectrum, then we will be safe by gravitino reheating decays. Of course, for $M_{SUSY} > 10^9$ GeV, gravitino is expected to be a very heavy particle. However, it can have a mass small as $m_{\tilde{G}} \simeq 10^{-9} \times M_{SUSY}$ as for standard gravitino warm dark matter in MSSM. But it is also possible a scenario $m_{\tilde{G}} \simeq (10^{-1} \div 1) M_{SUSY}$ with $m_{\tilde{G}} < M_{SUSY} -$ particles. Of course, if stable, they can be more massive than gravitino problem bound. This offers the intriguing possibility to solve and complete our leptogenesis mechanism relating it with a candidate for Cold Dark Matter (CDM). In particular, we will discuss a scenario in which gravitini are Super Heavy Dark Matter (SHDM) particles gravitationally produced by the non-adiabatical expansion during inflation epoch. We obtain new consistency bounds from CDM abundance. In particular, we will put emphasis on the new bounds set to neutrino sector from CDM limits.

This paper is organized as follows: in subsections A and B we review the main aspects of Exotic see-saw from the theoretical and phenomenological side; in Section II, we discuss our main results on SHDM-gravitino production during inflation; in Section III, we show our conclusions.

A. Exotic see-saw mechanism in (un)oriented Pati-Salam quivers

Let us review the basic aspects of the exotic see-saw model proposed in Ref. [1]. The complete set-up of our model is shown in Fig.1 of Ref. [1]. In the low energy limit, our model is described by a PS gauge group $U(4) \times Sp(2)_{L} \times Sp(2)_{R}$: $U(4)$ is generated by a stacks of 4 D6-branes and their images $U(4)'$, identified under a $\Omega$-plane; $Sp(2)_{L,R}$ are supported on two stacks of two D-branes each lying on top of the $\Omega$-plane We also consider two Euclidean D2-branes (or E2-branes) on top of the $\Omega$-plane, corresponding to two Exotic O(1) Instantons. Let us call these as E2', E2''. Quarks and leptons in Left and Right fundamental representations $F_{L,R} \equiv 4_{L,R}$, are reproduced as excitations of open strings attached to the $U(4)$-stack and the Left or Right $Sp(2)_{L,R}$-stacks (respectively). Analogously, Higgses $\tilde{H} \equiv 4_{R}$ and its conjugate $H$ are introduced as extra intersections of $U(4)$-stack with $Sp(2)_{R}$. Extra color states $\Delta = (10, 1, 1)$, and their conjugates, are obtained as excitations of open strings attached to the $U(4)$-stack and its mirror image $U(4)''$-stack. $\phi_{LL} = (1, 3, 1)$ and $\phi_{RR} = (3, 1, 1)$ correspond to strings attached on the $Sp(2)_{L,R}$ with both end-points (respectively). Higgs fields $h_{LR} = (2, 2, 1)$ are massless strings attached to $Sp(2)_{L}$ and $Sp(2)_{R}$. The quiver on the left of Fig.1 automatically encodes the following super-potential terms [2]:

$$W_{Y_{uk}} = Y^{(0)}_{hLRF_{L}F_{R}} + \frac{Y^{(1)}}{M_{1}}F_{L}\phi_{LL}F_{L}\Delta$$

$$+ \frac{Y^{(2)}}{M_{2}}F_{R}\phi_{RR}F_{R}\Delta^c + \frac{Y^{(3)}}{M_{3}}h_{LR}\phi_{RR}h_{RL}\phi_{LL} + \mu h_{LR}h_{RL}$$

$$+ Y^{(5)}_{h_{LR}F_{L}\tilde{H}} + \frac{Y^{(6)}}{M_{6}}F_{R}\phi_{RR}\tilde{H}\Delta^c$$

$$+ \frac{Y^{(7)}}{M_{7}}F_{L}F_{L}F_{R}F_{R} + \frac{Y^{(8)}}{M_{8}}F_{L}F_{L}\tilde{H}\tilde{H} + \frac{Y^{(9)}}{M_{9}}F_{L}F_{L}F_{R}\tilde{H}$$

$$W_{H} = m_{\Delta}\Delta\Delta^c + \frac{1}{4M_{4}}(\Delta\Delta^c)^{2}$$

$$+ \frac{1}{2}m_{L}\phi_{LL}^{2} + \frac{1}{2}m_{R}\phi_{RR}^{2} + \frac{1}{3!}a_{L}\phi_{LL}^{3}$$

$$+ \frac{1}{3!}a_{R}\phi_{RR}^{3} + \mu' H\tilde{H} + \frac{Y^{(10)}}{M_{10}}\tilde{H}\phi_{RR}H\Delta^c + \frac{Y^{(11)}}{M_{11}}HH\Delta$$

$Y^{(\cdot\cdot)}$ are Yukawa matrices; while the mass scales $M_{\Delta}$ are considered as free parameters. In fact mass scales depend on the particular completion of our model: they can be near the string scale $M_{s}$ as well as at lower scales On the other hand, mass terms $m_{\Delta}$ and $m_{LR}$ can be generated by R-R or NS-NS 3-forms fluxes in the bulk, in a T-dual Type IIb description: $m_{\Delta} \sim \Gamma^{ijk}\langle \tilde{H}_{ijk} + iF_{ijk} \rangle$, $m_{LR} \sim \Gamma^{ijk}\langle \tilde{H}_{ijk} + iF_{ijk} \rangle$, with $H_{3}$ RR-RR and $F_{3}$ NS-NS 3-forms. The Super-potential terms [3]?? can be generated by two E2-brane instantons, with Chan-Paton group O(1)$'$. They intersect twice the $U(4)$ stack and O(1)$''$ intersects one time $Sp(2)_{R}$-stack and once the $U(4)$-stack.

We remind that fermionic moduli $\tau_{i}, \tau'_{i}, \omega'_{i}$ are massless excitations of open strings ending on $U(4) - O(1)$, $U(4) - O(1)'$, $Sp(2)_{R} - O(1)'$, respectively, where $i = 1, 4$ and $\alpha = 1, 2$ are indices of $U(4)$ and $Sp(2)_{R}$ respectively. So that, integrating Integrating over the fermionic
modulini, we exactly recover the interactions \(1\). In particular, the dynamical scales generated in \((?2)\) are \(\mathcal{M}'_0 = Y'(1)M_{SE} + S_{E'2}\) and \(\mathcal{M}''_0 = Y''(1)M_{SE} + S_{E''2}\), where \(S_{E'2}, S_{E''2}\) depend on geometric moduli, associated to 3-cycles of the CY, around which \(E', E''\) are wrapped.

The spontaneous/Stückelberg breaking pattern down to the (MS)SM (minimal supersymmetric standard model) is

\[
U(4) \times Sp(2)_L \times Sp(2)_R \rightarrow (St_{UI}) \quad SU(4) \times Sp(2)_L \times Sp(2)_R \rightarrow (R,H,h)
\]

\(St_{UI}\) for Stückelberg, \(h_{LR}\) contain the standard Higgses for the standard electroweak symmetry breaking.

The extra \(U(1)_4 \subset U(4)_c\) is anomalous in gauge theory. But anomalous abelian gauge group can be cured by a generalization of the Green-Schwarz mechanism. Usually, this mechanism necessary requires generalized Chern-Simons (GCS) terms and new massive Stückelberg vector boson \(Z^\prime\) associated to \(U(1)_4\).

So that quiver nodes are split \(4 \rightarrow 3+1\) and \(2R \rightarrow 1+1'\) and we obtain a new effective Higgsed quiver shown in Fig.1-(b) of Ref. \([1]\). In the new effective quiver, we consider the intersections of a new exotic instanton \(E2\), intersecting once \(U(1)\) and once \(U(1)'\) - where \(U'(1)\) indicates the \(\Omega\)-image of \(U(1)\). So that, an extra non-perturbative mass matrix term is obtained:

\[
W_{E'2} = \frac{1}{2} M_{ab} N_{R}^a N_{R}^b
\]

where \(N_{R}^a\) are RH neutrini, \(a = 1,2,3\) label neutrino species. \(N_{R}\) were contained inside \(F_{R}\) as singlet components. In particular, the mass matrix has a structure \(M_{ab} = Y_{ab}^{(0)} M_{SE} \chi_{E'2}\), where \(Y_{ab}^{(0)}\) is the Yukawa matrix. The Yukawa parameterizes masses and mixings among RH neutrini. The mass matrix also depends on the particular \(E2\) intersections with ordinary D6-branes stacks. However, this sets a hierarchical bound on the Left-Right symmetry: the superpotential \(4\) can be generated only after spontaneous symmetry breaking of Pati-Salam group: \(U(4)_c \rightarrow U(3)_c\), and \(Sp(2)_R \rightarrow U'(1)\).

The electroweak symmetry breaking in our model is obtained by \((h_{LR})\) of the complex Higgs bi-doublets \(h_{LR}\). This leads to the important the tree-level mass relations

\[
m_d = m_e \quad \text{and} \quad m_u = m_D
\]

where \(V_{CKM}\) is the Cabibbo-Kobayashi-Maskawa matrix. The mass matrix has a structure

\[
M = \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}
\]

RH neutrino masses have two contributions

\[
M_R = M_P^R + M_{E'2}^R
\]

where

\[
M_P^R = \langle \phi_{RR} \rangle \langle S' \rangle / M_2
\]

This implies that light neutrino mass matrix \(m_\nu\)

\[
m_\nu \simeq - m_D (M_P^R + M_{E'2}^R)^{-1} m_D
\]

Then, the inverted see-saw formula is

\[
M_R = M_P^R + M_{E'2}^R \simeq - m_D m_{\nu}^{-1} m_D
\]

Using the matrix symmetry \(m_{D} = m_{P}^T\).

Rel.\([1]\) provides the information on the RH neutrino mass matrix \(M_{P}\), extracted using: i) LH neutrino mass matrix \(m_{\nu}\), ii) a PS-quark-lepton symmetry.

A natural situation for model is that \(E'2\) induces non-perturbative mass terms for RH neutrini of the same order:

\[
M_{E'2}^R \simeq M_{R,2}^{E'2} \simeq M_{R,3}^{E'2}
\]

where 1,2,3 are generation indexes. As a consequence, \(M_{E'2,1,2,3} \simeq 10^9 / 10^{10}\) GeV and we obtain a highly degenerate RH mass spectrum. This does not imply a highly degenerate LH mass spectrum: a large quark-lepton hierarchy is contained in Dirac matrix \(m_P\).

Let us comment that the low energy limit a IIA SUGRA plus non-perturbative non-perturbatively generated superpotentials are obtained by our IIA superstring theory model. The scalar potential in perturbative supergravity has a supersymmetric form

\[
V_P = \left( \frac{K}{M_{Pl}^2} \right) \left( (K^{-1})^{ij} D_i W D_j W^* - \frac{|W|^2}{M_{Pl}^2} \right)
\]

where the second term disappears in the global rigid susy limit \(M_{Pl} \rightarrow \infty\). The second term comes from the auxiliary fields in the gravity multiplet. The corresponding gravitino mass term is related to the Kähler and superpotentials as

\[
\mathcal{L}_{g,m} = \exp \left( \frac{K}{2 M_{Pl}^2} \right) \frac{W^*}{M_{Pl}^2} \psi_a \sigma^{ab} \psi_b
\]

In SUGRA the goldstino is eaten by the gravitino becoming massive. So that, in the limit of \(M_{Pl}^2 \rightarrow \infty\),
the gravitino is massless, as also explicitly obtained performing such a limit in \([11]\). In flat space-time vacua, it rigid SUSY is unbroken the gravitino mass must be zero, related to the condition \(V_F = 0 \rightarrow F = 0\). On the other hand, in AdS vacua, the F-term can be null even allowing a non-zero scalar potential and consequently a massive gravitino. In fact the supersymmetric algebra in AdS is different by SuperPoincaré group algebra. Supersymmetry in AdS relates superpartners without mass degeneracy. However, Rel. \([10]-[11]\) can be affected by non-perturbative stringy effects not present at all in standard supergravity. For instance, the scalar potential and the perturbative vacua can be lifted by R-R and NS-NS supersymmetric fluxes in the Calabi-Yau compactification, consequently lifting the gravitino mass. An example is provided in Ref. \([18]\), where in a T-dual IIB context supersymmetric fluxes are introduced in order to stabilize Calabi-Yau moduli. On the other hand, also the presence of non-supersymmetric R-R and NS-NS three-forms fluxes in the bulk can generate an extra soft-susy mass term for the gravitino, as \(m_{3/2}^{NP} \psi \sigma^{ab} \bar{\psi}_b\), where \(m_{3/2}^{NP} \sim (\tau H + iF)\) (flux indices contractions omitted). In our scenario, the gravitino has to be the Lightest Supersymmetric Particles. So that we assume that non-perturbative fluxes will not be higher than the supersymmetric scale and superpartners masses.

**B. Region of parameters and main results**

Under model generalities discussed above, In Ref. \([1]\) we consider the following assumptions:

I) Supersymmetry and Left-Right symmetry are spontaneously broken at high scales: \(M_{SUSY}, M_{LR} > M_R \approx 10^9\) GeV.

II) Dominance of non-perturbative contributions from exotic instantons to perturbative terms: \(M_R^p << M_R^{NP}\).

Under these assumptions, we counted the number of free-parameters as 13. The (II) condition is exactly the definition of exotic see-saw mechanism: a see-saw mechanism completely dominated by exotic instantonic contributions.

In this framework, acquiring 11 low energy inputs from neutrino and quark physics, we demonstrated a successful leptogenesis for \(M_1 \approx 3.5 \times 10^6\) GeV, \(M_2 \approx M_3 \approx 8.7 \times 10^9\) GeV and \(\theta_{13} \approx 8.5^\circ\), while a precise dependence of the \(m_1\) mass eigenvalues with the PMNS phase \(\delta\) was shown in Fig.2 of Ref.\([1]\).

**II. GRAVITINI AS SHDM**

Assuming gravitini as Super Heavy LSP, we discuss bounds from CDM abundance. In particular, let us consider a scenario in which gravitini are so heavy to be produced not by thermal relic mechanisms but by the non-thermal non-adiabatic expansion during inflation.

This production mechanism is similar to every productions of pairs in an external background field. For example, an external electric field can promote a virtual electron-positron pair to a become real pair, as well as in Bekenstein-Hawking mechanism the external gravitational field can source B.H. pair to become a real pair nearby the black hole horizon. In our case, gravitini can be produced by the tremendous expansion rate during inflation of the FRW metric

\[
ds^2 = a^2(\eta)(d\eta^2 - dx^2)\]

For a massive heavy scalar field the action in FRW is easier than a spin 3/2 field:

\[
S = \int dt \int d^3x a^3 \left( \dot{\Phi}^2 - \frac{\nabla \Phi^2}{a^2} - M^2_\Phi \Phi^2 - \zeta R \Phi^2 \right)
\]

where \(R\) is the Ricci scalar. Then, change the to time variable to the to conformal time variable and we can perform a standard mode expansion of a QFT in curved space-time,

\[
\Phi(x) = \int \frac{d^3k}{(2\pi)^{3/2}} a_k \left[ e^{ik \cdot x} + a_k^\dag e^{-ik \cdot x} \right]
\]

where the coefficients of creation/destruction depend on the conformal time variable. This case was analyzed in Ref. \([19]\). However, our case is formally different and not still discussed in literature.

For a gravitino, the main dynamical aspects provoked by expansions are the same, but the action has a Rarita-Schwinger fashion \([20]\):

\[
S = \int d^4x e \bar{\psi}_\mu R^\mu[\psi]
\]

where \(R\) is the Rarita-Schwinger operator:

\[
R^\mu[\psi] = i \gamma^\mu \psi_\rho D_\rho \psi_\nu + m \gamma^\mu \psi_\nu
\]

with the covariant derivative

\[
D_\mu \psi_\nu = \partial_\mu \psi_\nu + \frac{1}{4} \omega^{abc}_\mu \Gamma^{ab\gamma}_\mu \psi_\nu
\]

where \(\gamma^{\mu_1 \ldots \mu_n} = \gamma^{[\mu_1} ... \gamma^{\mu_n]}\) and \(e = \text{det} e^{ab}_\mu\) is the determinant of the inverse vielbein \(e_{\mu}^a\).

We can assume a torsion-free background metric so that \(\Gamma^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\nu\mu}\). Let us note that the mass is given by the Kähler and the superpotential as \(m = eK/3W/M_{Pl}^2\) and it can depend on the space-time expansion. The related equation of motion is

\[
(iD - m) \psi_\mu - (iD + \frac{m}{2} \gamma_\mu) \gamma^\rho \psi_\rho = 0
\]

Now, in FRW, we have

\[
e_\mu = a(\eta) e_\mu^a, \quad m = m(\eta), \quad \omega_{\mu ab} = 2aa^{-1} e_\mu[a e_b]^0
\]
obtaining the EoM
\[
i\gamma^{mn}\partial_m\psi_n = -(m + \frac{a'}{a} \gamma^0)\gamma^m\partial_m
\]
So that, we expand the gravitino field as
\[
\psi(x) = \int \frac{d^3p}{(2\pi)^3 2\rho_0} \sum_\lambda \{e^{ik\cdot x} c_\mu(\eta, \lambda) a_k(\lambda) + e^{-ik\cdot x} c_\mu^C(\eta, \lambda) a_k^G(\lambda)\}
\]
and note that the two coefficients are dependent in order to guarantee Majorana fermion condition \(\psi_C = \psi\).

At this point, we can perform a Bogoliubov transformation
\[
c_{\mu k}^\eta = \alpha_k h_k^\eta(\eta) + \beta_k C_k^\eta(\eta)
\]
so that we can calculate the energy density of gravitino produced:
\[
\rho_\phi(\eta) = m n_\phi(\eta) = m H_e^2 \left( \frac{1}{a(\eta)} \right) I
\]
\[
I = \int \frac{dk}{2\pi^2} k^2 |\beta_k|^2
\]
in which we consider Bogoliubov transformations from the Cauchy surface foliated by \(\eta = \eta_0\) into another Cauchy surface in a cosmological time \(\eta > \eta_0\), assuming conditions \(\dot{a}/a^2 << 1\), and we tacitly normalized \(k \rightarrow k/a H_e, \eta \rightarrow \eta_0 h_e, a \rightarrow a/a_e\), where \(e\)-label indicates variables of the starting oscillating inflaton epoch.

After that, it is crucially important to constrain the region of parameters from CDM abundance. The relation among gravitini energy density and radiation is
\[
\frac{\rho_G(t_0)}{\rho_R(t_0)} = \frac{\rho_G(t_{Re})}{\rho_R(t_{Re})} \left( \frac{T_R}{T_0} \right)
\]
where \(t_0\) is the today day, while the ratio \(\rho_G(t_{Re})/\rho_R(t_{Re})\) is determined after the reheating epoch, while gravitini were produced during \(t_c > t_{Rh}\) epoch, where inflaton starts to oscillate and decays into SM particles. During the radiation dominated epoch, the ratio \(\rho_G(t_{Re})/\rho_R(t_{Re})\) can be estimated as
\[
\frac{\rho_G(t_{Rh})}{\rho_R(t_{Rh})} \approx \frac{8\pi}{3} \left( \frac{\rho_G(t_0)}{M_P^2 H^2(t_0)} \right)
\]
where \(H_0 \approx 100\) km s\(^{-1}\) Mpc\(^{-1}\). Then, the natural hierarchies of Hubble scales and densities with inflaton parameters are \(H^2(t_c) \sim m_\phi^2\) and \(\rho(t_c) \sim m_\phi^2 M_P^2\), so that
\[
\Omega_G h^2 \approx 10^{17} \left( \frac{T_{Rh}}{10^9 \text{GeV}} \right) \left( \frac{\rho_G(t_c)}{\rho_c(t_c)} \right)
\]
where \(\rho_{\text{c}} = 3H(t_c)^2 M_P^2/8\pi\) is the critical energy density during \(t_c\). Eq. \((26)\) can also be rewritten as
\[
\Omega_G h^2 \approx \Omega_R h^2 \left( \frac{T_{Rh}}{T_0} \right) \frac{8\pi}{3} \left( \frac{M_G}{M_P} \right) \frac{n_G}{M_P H^2(t_c)}
\]
But the limit on the inflaton mass is \(m_\phi \simeq 10^{13}\) GeV or so, while the limit on \(T_{Rh}\) for a successful reheating is \(T_{Rh}/T_0 \approx 4.2 \times 10^4\), for \(\Omega_G h^2 \lesssim 1\) In this range of parameters, we are able to constrain the Gravitino mass. How heavy might it be? We can set the limit
\[
m_G \approx (10^{-2} \div 2) \times H \approx 10^{11} \div 10^{13}\text{ GeV}.
\]
This sets a limit on the SUSY symmetry breaking scale of
\[
M_{SUSY} > m_G \approx 10^{11} \div 10^{13}\text{ GeV},
\]
that is well compatible with our D-brane leptogenesis bound \(M_{SUSY} > 10^9\) GeV. More precisely, not only the supersymmetric scale has to satisfy this hierarchy with the gravitino mass, but we cannot allow any superparticles to be smaller than the gravitino, otherwise gravitini will suddenly decay on them. Let us not that this also set a bound on the string scale and on the size of the 3-cycles wrapped by the E2-brane responsible for RH neutrino masses. In particular, stability of exotic instanton calculations are trustable if \(M_s \geq M_{SUSY} \approx 10^{11} \div 10^{13}\) GeV. The order of neutrino masses are \(M_{1,2,3,RH} \approx e^{-S_{22}(\rho_i)}/M_S \approx 10^9\) GeV, where \(e^{-S_{22}(\rho_i)}\) is the non-perturbative coupling constant associated to the exotic instanton. In particular, it depends on geometric moduli fields \(\rho_i\), parametrizing the size of the 3-cycles wrapped by the E2-brane in the internal Calabi-Yau compactification CY\(_3\). So that, this imposes an interesting bound on the E2-brane: \(e^{-S_{22}} \approx M_{RH,1,2,3}/M_S < 10^{-2} \div 10^{-3}\).

### III. CONCLUSIONS AND DISCUSSIONS

In this paper, we discussed the gravitino problem in context of a see-saw model for neutrini and leptogenesis strongly motivated by IIA open superstring theory. In particular, we suggest an effective Pati-Salam (un-)oriented quiver field theory, UV completed by an intersecting D-brane superstring theory. The quiver theory encodes informations on D-branes and its consistency with respect to gauge and stringy anomalies or tadpoles. This model assumes supersymmetry to be spontaneously broken at very high scale: \(M_{SUSY} > 10^9\) GeV. We individuated a possible problem on this assumption: if the gravitino is heavy and unstable, it will decay inside the thermal bath and our leptogenesis scenario will not be trustable! Essentially gravitini decays will jeopardize the leptogenesis picture, badly washing-out the lepton-antilepton asymmetry generated by RH neutrini decays. So that, we moved ourself to try a way-out on this problem in contest of our model. We not only demonstrated that a way-out is possible, but we also provide a related candidate for cold dark matter. In particular, we noticed that if the gravitino is the Lightest Stable Particle of the Superworlds, then the gravitino bound is automatically avoided: gravitini simply cannot decay into more massive particles.
superparticles. On the other hand, gravitini are expected to be very massive and they can provide a good candidate for Super Heavy Dark Matter or WIMPZILLA. In particular, we re-discussed the gravitational production of massive gravitini during the inflation due to the tremendous expansion rate. We show that gravitino mass has to be around the inflaton mass or so in order to guarantee a correct abundance of CDM: $10^{13} \div 10^{15}$ GeV. This also sets a limit on the instantonic coupling of $e^{-3 S_{\text{ex}}/\Lambda} < 10^{-2} \div 10^{-3}$. So that, we tried a relation of the mechanism of DM production with parameters of neutrino and quark physics. So, we conclude that our model naturally provides a coherent picture of baryogenesis and dark matter genesis: the parameters of the two genesis mechanisms are rigidly related each others. Our model inspired by first principles of superstring theory naturally reproduces and relates ordinary and dark matter. On the other hand, we can relax the hypothesis that all cold dark matter is composed of superheavy gravitini. In this case, the bound on the gravitino mass set by the non-adiabatic genesis mechanism can be relaxed as well as constrains on the instantonic coupling. Let us also comment about possible implications in phenomenology. Clearly, such a candidate is so heavy and rarely distributed in the galactic halo that cannot be seen by direct detection experiments. Such a candidate could be seen by high energy DM indirect detection. It is also possible that gravitini are destabilized by trilinear R-parity breaking terms like $\frac{y_{\alpha}}{\sqrt{2}} H_{\alpha} L^2$ so that it can decay as $G \to \gamma \nu$, with a rate $\frac{\cos^2 \theta_W m_{\nu} m_{\chi}^2}{32\pi m_{\chi}} \left( 1 - \frac{m_{\nu}^2}{m_{\chi}^2} \right)^3 \left( 1 + \frac{m_{\nu}^2}{3m_{\chi}^2} \right) \Gamma_G \to \gamma \nu$

induced by an interaction term

$$L_{\text{int}} = -\frac{i}{8M_{\text{Pl}}} \bar{\chi}_\nu [\gamma^\mu, \gamma^\rho] \gamma^\mu \lambda F_{\nu\rho}$$

where $\lambda$ is the photino and $\chi$ is the bino-dominated lightest neutralino eigenstate. In other words, the photino and the higgsino are rotated into neutralino so that and effective neutralino-neutrino mixing mediates the gravitino decays into a neutrino and a photon. But, assuming $m_{\chi} \simeq 10^{13}$ GeV and $m_{\tilde{G}} \simeq 10^{11}$ GeV, we obtain $\Gamma \simeq 10^{-24} \times 10^{-16} \times 10^{11}$ GeV $\simeq 10^{-20}$ eV corresponding to $\tau \simeq 10^5$ s that is clearly a too fast rate for a good dark matter candidate. At least, considering a very heavy $\chi$-particle, lets say $10^{15}$ GeV, we can enhance $\tau \simeq 1$ yr or so. But these R-parity violating perturbative terms are expected to be suppressed by non-perturbative stringy effects, as discussed in section II. So that, the gravitino decay can be only induced by higher order effective operator like $\frac{S_n}{h_{\alpha}} L^2$ suppressing the rate of at least a factor $10^{-13}$ for $m_{\chi} \simeq 10^{13}$ GeV in order to obtain a cosmological lifetime $1 \div 10$ Gyrs, where $\Lambda$ can be for example the non-perturbative scale of a R-R or NS-NS flux in the compactification. For example for supposing $n = 1$, the rate will be suppressed as $(\langle S / \Lambda \rangle)^2 \simeq 10^{-13}$. Several different operators involving other heavier Higgs in Pati-Salam could be generated by non-perturbative stringy effects as understood. These terms can be generated in our model by expectation values of non-perturbative R-R or NS-NS fluxes. To find a $10^{11} \div 10^{13}$ GeV neutrino or photon with two-body decay peaks would provide a testbed in favor of our suggestion. Experiments like IceCube and ANTARES would search for such a very rare decays. Of course, other possible mechanism of DM genesis can be considered. For example, in [22–24], several different DM genesis mechanisms from Bekenstein-Hawking evaporation of primordial black holes were suggested. A complete study of these mechanisms in contest of our model deserves further investigations beyond the purposes of this letter.

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3 This assumption can be strongly motivated by unifying picture of dark matter and dark energy by hidden gauge sectors recently suggested in Refs. [25, 26] as well as by DAMA dark matter direct detection result recently analyzed in a fully detailed analysis in Ref. [27], in the framework of asymmetric Mirror dark matter.

4 We mention that recently we suggested a possible way-out to the information paradox related to Bekenstein-Hawking radiation, relating it to quantum chaotization of infalling information [28–31]. On the other hand, we found in contest of $f(R)$-gravity that primordial Nariai black holes have an antievaporation instability, turning-off Bekenstein-Hawking radiation. So that in these models, a DM production from an evaporation of primordial BH seems to be not possible [31].
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