Enhancing brush tyre model accuracy through friction measurements

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ABSTRACT
Accurate tyre models are important to ensure valid and reliable simulation of vehicle behaviour. To this purpose, the Magic Formula (MF) became the de facto standard for vehicle dynamics simulations, despite requiring many empirically derived coefficients. This paper shows how the accuracy of a simple physical-based brush-type tyre model can be enhanced to simulate tyre behaviour that closely matches MF results. To do so, the real, highly complex rubber friction characteristics are incorporated into the brush model. The friction characteristics were obtained from friction measurements with a tread rubber block. The developed model is validated against experimental tyre data obtained on a flat-track test rig and the corresponding MF model. Results show that the inclusion of friction characteristics allow accurate simulation of longitudinal and lateral slip conditions over a wide range of normal loads with the simple brush model. At extreme loading scenarios, the simulation accuracy deteriorates because of the significant influence of highly nonlinear deformation behaviour of the tyre that is not accounted for in the brush model formulation.

1. Introduction
The successful development of virtual engineering capabilities of automotive manufacturers has led to a significant reduction in vehicle development times. For example, physical prototypes and the associated physical testing can be replaced by full-vehicle simulations provided that they are accurate and reliable. The quality of such simulations depends on the quality of the constituent models. With regard to tyre simulations, most tyre models currently used are derived from data obtained with indoor flat-track test rigs [1]. Once test data are collected, they are commonly parameterised with an empirical model, the Magic Formula (MF) [2].

In its most basic form, the MF is a simple equation containing nested trigonometric functions with several scaling coefficients. Over the years, the number of parameters was increased to enhance model accuracy, with the current formulation relying on over 100 coefficients. As a result, the creation of an accurate MF model requires a comprehensive
experimental dataset. To reduce the considerable MF parameterisation effort, several low-parameter models have been developed, some of which build on the understanding of tyre physics [2,3]. One of the simplest models is the brush model, which provides a good physical description of the tyre-road interaction but lacks simulation accuracy. To improve this aspect, several extended brush model formulations have been proposed. For example, the inclusion of an arbitrary pressure distribution, as opposed to an n-order polynomial commonly seen in the literature, is considered in [4], allowing for a more realistic pressure distribution (e.g. one taken from experiments) to be included. [2] proposes a brush model with a flexible carcass to enhance the accuracy of the aligning moment. Thermal effects are formulated in [5], with the inclusion of Persson’s rubber friction model and, thus, a dependence on surface roughness. In [6], the footprint geometry and lateral curvature of the tyre is considered in a 3D approach, and used to successfully predict longitudinal behaviour of a tyre on both plane and curved surfaces.

Although, the extended formulations improve simulation accuracy, they also increase model complexity. Combining several of such extensions to improve different simulation aspects can result in a highly complex model, which will again make parameterisation difficult. In addition, the computational burden will increase and the model will be less likely to be able to run in real-time simulations [5].

This paper presents a novel approach to improve the accuracy of a simple brush model, by considering the real tread-rubber/ground friction characteristics. The model formulation is deliberately kept as simple as possible to demonstrate the benefits and importance of accurately capturing the friction characteristics. Since the MF is a widely used tyre model (owing to its high utility and versatility), the brush model results are compared to the MF as a ‘benchmark’. Note that there is no intention of replacing the MF. Rather, the comparison serves to show the utility of the integration of accurate friction measurements with a very simple brush model.

Rubber friction is a physical phenomenon that is not yet fully understood, and depends on many physical parameters, including temperature, sliding velocity and surface roughness [7]. Despite excellent work developing a robust, mathematical theory of rubber friction (e.g. [7,8]), comparisons of rubber friction theory to experiments show that our understanding is not complete (e.g. [9]). Thus, to be able to simulate the real friction characteristics of a tyre, friction measurements were carried out with a tread rubber block using a purpose-built test rig under highly controllable conditions.

Based on the work described in [10], where a brush-type model was extended to include the friction measurements, here the model is further developed to predict longitudinal and lateral behaviour of the full tyre over a wider range of normal loads. The novel model results are compared to and validated against flat-track measurements obtained with the same tyre and the same surface (sandpaper) used for the friction measurements.

The paper is laid out as follows. Section 2 introduces the tyre model formulation, together with the considerations of the contact patch shape and the real friction characteristics. Section 3 describes the rubber friction experiments and results, including the creation of the friction master curve integrated into the tyre model. In Section 4, the experimental tyre data used for validation are described. Simulation results obtained with the developed model are presented and compared to the experimental tyre data and corresponding MF6.1 results in Section 5. Conclusions are drawn in Section 6.
2. Tyre model formulation

2.1. Overview of brush model

Owing to its simplicity and physical-parameter approach, the brush-type model was used for the integration of the friction measurements (see Section 4). The basic brush model is an analytical tyre model that describes the tyre as a single line of elastic bristles (or tread elements) protruding from a disk, which is assumed to deform only in the direction normal to the road plane in response to the wheel load to accommodate the contact patch. The bristles – representing the tread, belt and carcass – can move forwards, backwards and sideways across the surface, and are effectively modelled as linear springs [3].

When the tyre is freely rolling, bristles enter the contact patch at the leading edge vertically and remain vertical until they leave at the trailing edge, experiencing no shear forces. Under longitudinal and/or lateral slip, a slip velocity occurs and the base point of the bristles (attached to the disk) will move at a different speed than the bristle tip that contacts the surface. As a result, the bristles are progressively displaced and build up shear forces as they move through the contact patch. The maximum bristle deflection is limited by the available friction between the tyre and the surface, typically determined by the ‘static’ friction coefficient. Once the maximum deflection value is reached, the bristles start sliding through the rest of the contact patch (see Section 2.3). There are therefore two distinct regions in the contact patch: the adhesion region, and the sliding region. In the adhesion region, the relative position of the bristle tips to their base points is a function of slip, and can be directly related to the adhesive tyre force component. In the sliding region, the generated force depends on the sliding friction coefficient in combination with the normal contact pressure. The total tyre force is a summation of the forces arising in these two regions of the contact patch, see Figure 1. At low slips – when the bristles within the contact patch are mainly adhering to the ground – the major force contributor is the adhesive tyre force, which is primarily determined by the stiffness of the tyre. At medium to high slips – when most of the bristles within the contact patch are sliding – the tyre force is mostly affected by the friction characteristics of the tread rubber.

![Figure 1. A generic force-slip curve illustrating the force contributions from the adhesion and sliding regions within the contact patch.](image-url)
Figure 2. Left: Side view of the variable-μ brush model during pure braking (with rotational velocity $\omega$), showing the adhesion and sliding regions. $\xi$ is the local co-ordinate in the contact patch, starting from the leading edge. Right: top view of variable-μ brush model during pure cornering at slip angle $\alpha$, and velocity vector, $v$. The self-aligning moment, $M_z$, (not modelled here) is shown for completeness. The maximum available stress, given by the product of the parabolic pressure distribution and the corresponding friction coefficient, is indicated for the $\mu_{\text{stick}}$, $x$, $y$ value (only valid within the adhesion region), and the $\mu_{\text{slide}}(T, v_{sl})$ value (only valid within the sliding region).

The basis for the tyre model developed here is the brush model by [3] that includes two separate friction coefficients, one for the adhesion region ($\mu_{\text{stick}}$) and one for the sliding region ($\mu_{\text{slide}}$). The model will be referred to as the ‘two-μ’ model. Qualitatively, by introducing two friction coefficients the typical peak in the force-slip curves can be replicated. However, the two-μ model fails to simulate the drop-off in force after the force peak with increasing slip [2,3], as the constant sliding friction coefficient cannot capture the complex nature of rubber friction. To overcome this shortcoming, this paper proposes to replace the discrete $\mu_{\text{slide}}$ value with real friction characteristics that are dependent on tread temperature, $T$, and sliding speed, $v_{sl}$, denoted as $\mu_{\text{slide}}(T, v_{sl})$. The resulting model will be called the ‘variable-μ’ model and is shown diagrammatically for pure longitudinal and pure lateral slips in Figure 2. Note that $\mu_{\text{stick}}$ is treated as a constant, and the tread temperature will be provided as a model input from measurement data rather than computed, as explained in Section 2.4. As an alternative to experimental temperature data, the variable-μ model could be combined with a thermal model (e.g. [11]) to align the inputs to the ones used by the MF.

Figure 3 shows the main steps for the calculation of the longitudinal and lateral forces used in the variable-μ model. Starting from the inputs (slip ratio, slip angle, normal load, wheel velocity and tread temperature), the model calculates the sliding velocity of the bristle, $v_{sl}$, in the sliding region (Section 2.3), which together with the tread temperature, $T$, provides the necessary information to determine $\mu_{\text{slide}}(T, v_{sl})$ from the friction measurement data (Section 2.4). The value of $\mu_{\text{slide}}(T, v_{sl})$ is then used in the calculation of the longitudinal and lateral forces (Section 2.5).
2.2. Contact patch considerations

The shape of the contact patch changes with a wide range of variables, including normal load, camber angle, inflation pressure and, crucially for this discussion, slip. Generally speaking, in the longitudinal case, the contact patch shape remains fairly constant (approximately rectangular) with increasing slip ratio [12–14], see Figure 4(a). During cornering, the contact patch evolves to a trapezoidal shape with increasing slip angle, see Figure 4(b) [15]. To capture this behaviour, distinctions are made between the longitudinal and lateral slip conditions. For pure braking/traction manoeuvres, the contact patch is assumed consistent across the tyre width (in terms of dimensions and contact pressure distribution, see Section 2.2.1) and, thus, can be modelled by a single row of bristles that changes in length with normal load. For cornering, the variable-μ model considers three distinct bristle rows (or ribs) – A, B and C – of different lengths that vary with slip angle and normal load to account for the trapezoidal contact patch shape, see Section 2.2.2.

2.2.1. Contact pressure distribution

The contact pressure distribution used for the variable-μ model was derived from stationary contact patch measurements obtained with the tyre investigated here. Measurements were taken on a static stiffness test rig together with a high-resolution pressure sensing mat (XSensor IX500 with 1.6 mm spatial resolution). Figure 5 shows an example of the pressure distribution of the upright tyre at a normal load of 12,150 N. At this load, the contact patch is almost square, and exhibits a reasonably even pressure distribution across its width. Pressure peaks can be observed at the left and right sides (approx. at $y = \pm 100$ mm), which stem from the share of the normal load carried by the tyre sidewalls. For the sake
of model simplicity, the localised contact pressure peaks will be ignored and a consistent pressure distribution in the lateral direction will be assumed.

The contact pressure distribution along the longitudinal axis of the contact patch at $y = 33\, \text{mm}$ is shown in Figure 6. When accounting for the decrease in pressure across the tread grooves (at approx. $x = \pm 15, \pm 45$ and $\pm 75\, \text{mm}$), the average pressure across the footprint is reasonably well described by a parabolic pressure distribution. Only towards the leading and trailing edges can a significant deviation between the parabolic assumption and the static measurements be observed. Nonetheless, to keep the model formulation simple and aligned to common brush models found in the literature (e.g. [2–4]), the parabolic pressure distribution was deemed sufficiently accurate for this work.

### 2.2.2. Cornering condition

As mentioned in Section 2.2, the contact patch shape of a side slipping tyre is similar to a trapezoid, which becomes more evident with increasing slip angle. Relevant literature also demonstrates that the contact pressure distribution across the width can no longer be assumed constant. For example, finite element simulations show a contact pressure gradient across the contact patch width [15]. In addition, at very high slip angles, [15] discusses
the presence of localised 'lifting' of areas of the contact patch as a result of buckling of the tyre belts, indicating considerable changes in the local contact pressure.

As contact patch measurements of a cornering tyre were not available, indirect data were used to ascertain the contact patch characteristics of the investigated tyre. That is, the tyre temperature recordings across the tread width obtained during cornering tests on a flat-track machine (see Section 4) were analysed.

During the flat-track test, three infrared (IR) sensors are positioned across the tyre width (indicated by A, B and C) to measure the tread surface temperature. Such a temperature evolution is illustrated in Figure 7 for the outward sweep part of the test, i.e. only the part of the test starting from 0° slip angle and sweeping to the maximum/minimum slip angle (approximately ±12°). The test begins by sweeping through negative slip angles. As the tyre is steered, the temperature of the central part of the contact patch (sensor B) and the region towards the ‘outside’ of the ‘corner’ (sensor A) increases significantly, while the temperature of the area towards the ‘inside’ of the corner (sensor C) gradually reduces. Conversely, similar behaviour is seen for the positive slip angle sweep, which was performed directly after the negative slip angle sweep, while the tyre is still 'hot'. With increasing slip, the corner inside tread area (sensor A) steadily decreases in temperature, while the other parts of the contact patch first cool until about 3° slip angle and, then, significantly heat up.

For all tests, the tread centre always shows the highest temperature and reaches very similar temperatures (approx. 80°C to 85°C) at the minimum/maximum slip angle. This behaviour implies that the centre region of the contact patch is influenced by a left/right cornering manoeuvre in a consistent manner and, in turn, its characteristics (dimensions and contact pressure distribution) can be treated independently of slip angle. In contrast, the significant temperature variation between the inner and outer tread regions with positive and negative slip angles indicates considerable changes in the locally generated shear forces and, thus, local normal load and, in turn, the local contact length.

To account for the change of the contact patch characteristics, the variable-μ model divides the contact area into three regions of equal width, each simulated with a row of independent bristles (or 'ribs'), as mentioned in Section 2.2 and shown Figure 4. Effectively, the tyre can be described as three narrow tyres attached to one another. This approach is referred to as the multi-rib contact patch model. Similar approaches have been proposed in the literature (e.g. [2,16,17]). With the multi-rib model, the contact length of the inner and outer rows changes with normal load and slip angle, whereas the centre rib only varies
with normal load. For each rib, a parabolic pressure distribution as described in Section 2.2.1 will be assumed.

Assuming a direct relationship between contact area and normal load [17], the share of \( F_z \) carried by the inner and outer bristle rows is modelled as a function of slip angle. At zero slip, the three ribs share the normal load equally, i.e. the load on each is \( F_z / 3 \). The minimum load on the outer ribs is set to 100 N, to avoid local lift off. In addition, the total normal load across ribs A, B and C is kept constant, i.e. \( F_z = F_{z,A}(\alpha) + F_{z,B}(\alpha) + F_{z,C}(\alpha) \).

Thus, the normal load on each rib is given by:

\[
F_{z,i}(\alpha) = \begin{cases} 
\frac{1}{3}F_z, & \text{if } 100N \leq F_{z,i}(\alpha) \leq \frac{2}{3}F_z - 100N, \quad i = A, C \\
\frac{2}{3}F_z - 100N, & \text{if } i = B 
\end{cases}
\]

where \( i \) indicates the individual ribs, which also correspond to the temperature sensors illustrated in Figure 7.

The normal load on the outer ribs are assumed to vary linearly with slip angle, \( \alpha \), according to Equations (2) and (3):

\[
F_{z,A}(\alpha) = \frac{1}{3}F_z - jF_z\alpha \\
F_{z,C}(\alpha) = \frac{1}{3}F_z + jF_z\alpha
\]

where \( j = 1.6 \text{ N/rad} \), which was found empirically through a best-fit approach with the experimental tyre data set (see Section 4).

Corresponding to the normal load variation, the contact lengths of the two outer ribs are assumed to also change with slip angle, while the centre rib is assumed independent of

Figure 7. Temperature variation across the tyre tread recorded with temperature sensors (A/B/C) as a function of slip angle during a pure cornering test (\( F_z = 3 \text{ kN} \)). A schematic of the tyre with sensor placement is shown on the right.
\( \alpha \), i.e.:

\[
L_i(F_z, \alpha) = \begin{cases} 
L(F_z), & i = B \\
\frac{3 F_{z,i}(\alpha)}{F_z} L(F_z), & i = A, C
\end{cases}
\] (4)

where \( L(F_z) \) is the contact length determined from the MF model used here for comparison purposes, which is given by:

\[
L(F_z) = 2 \left( q_{a1} \sqrt{\frac{F_z}{c_z r_0}} + q_{a2} \frac{F_z}{c_z r_0} \right) r_0
\] (5)

where \( r_0 \) is the free unloaded tyre radius, \( c_z \) the vertical stiffness for the nominal tyre load and inflation pressure, and \( q_{a1} \) and \( q_{a2} \) are MF parameters determined during model parameterisation [18]. For the tyre used in this study, \( c_z = 314,931 \) N/m, \( r_o = 0.3943 \) m, \( q_{a1} = 0.6015 \), \( q_{a2} = 0.8197 \).

It is acknowledged that the above assumptions for \( F_{z,i}(\alpha) \) and \( L_i(F_z, \alpha) \) contain considerable simplifications. This model approach is deliberate to keep the formulation simple and to serve as a first approximation allowing the demonstration of the benefits of the integration of friction characteristics. Furthermore, given the complexity of the contact patch geometry under lateral slip conditions, only pure longitudinal and pure lateral slip cases are examined in this paper. For combined slip cases, the contact patch geometry is more complex still, and requires further analysis.

### 2.3. Bristle sliding velocity

As discussed in Section 2.1, when a tread element enters the contact patch, it adheres to the ground and only starts to slide when the local friction force capacity is reached. The onset of sliding is a highly dynamic event that is influenced by several parameters such as the friction coefficient, the contact pressure, the velocity and the strain energy stored in the tread element [19]. However, to keep the formulation simple, similar to the TreadSim model [2], the sliding velocity is assumed to be equal to the velocity of the base point of the tread element, i.e. the slip velocity. For pure longitudinal motion, in the absence of camber, ply steer or conicity effects, the sliding velocity is given by:

\[
v_{sl}^x = |v_x \kappa|
\] (6)

where \( v_x \) is the translational velocity of the wheel hub and \( \kappa \) is the practical longitudinal slip [2]. In the lateral case, the sliding velocity is given by:

\[
v_{sl}^y = |v_x \tan \alpha|
\] (7)

Once the sliding velocity of the tread element is known, the \( \mu_{slide}(T, v_{sl}) \) value can be determined for a given tread temperature, as explained in Section 2.4.

### 2.4. Friction master curve

The real tread rubber friction characteristics are considered by incorporating the friction master curve in the variable-\( \mu \) model. Details on how the master curve is constructed are discussed in Section 3, while its implementation is presented here.
The friction master curve describes how the friction coefficient changes with sliding speed at a constant temperature, i.e. the reference temperature, $T_s$ [20]. In reality, maintaining a constant temperature at sliding speeds exceeding approximately 10 mm/s is not possible, due to local heating effects [7,20]. Thus, to allow simulation of different tyre temperatures, the friction master curve needs to be integrated with a temperature-dependent shift factor that allows replicating the friction characteristics at an arbitrary temperature, $T'$. The shift method integrated in the variable-μ model is an extension of the WLF transform [20,21,25] used to create the friction master curve (see Section 3), and is given by:

$$
\log_{10}(a_{T'}) = \frac{17.5 \times 52(T' - T_s)}{(52 + T' - T_g)(52 + T_s - T_g)}
$$

(8)

where $a_{T'}$ is the shift factor and $T_g$ is the glass transition temperature of the rubber.

Figure 8 illustrates the shift procedure, and shows how temperature can significantly affect the friction coefficient, $\mu_{\text{slide}}(T, v_{\text{sl}})$. For example, for a given sliding velocity, $v_{\text{sl},1}$, shifting the master curve to a higher temperature causes a significant reduction in the corresponding friction value. Indeed, depending on the sliding velocity, the friction coefficient can decrease, so that $\mu_{\text{slide}}(T', v_{\text{sl},1}) < \mu_{\text{slide}}(T_s, v_{\text{sl},1})$, or increase, so that $\mu_{\text{slide}}(T', v_{\text{sl},2}) > \mu_{\text{slide}}(T_s, v_{\text{sl},2})$.

Thus, temperature is an important input in the variable-μ model and, because of its importance, the tread temperature will be provided as an input to the model rather than computed to avoid introducing associated modelling inaccuracies. For the simulation of the longitudinal slip characteristics that will be compared to experiments and MF results in Section 5.1, the tread temperature recording from the centre sensor (sensor B) during
the experiments will be used. For the lateral slip characteristics (see Section 5.2), the temperature readings from the three temperature sensors (A, B and C) will be used for the force computation within the corresponding tread rib.

It should be noted that the commonly observed load-dependency of tyres is captured by the shifting of the master curve. Changes in the contact pressure lead to changes in frictional heating in the rubber, which, in turn, influences the tyre-surface friction coefficient and thus the tyre shear forces [26].

2.5. Force calculation

Using the formulation of the two-μ brush model by [3], equations for the longitudinal and lateral forces can be derived for steady-state conditions. First, the shear stress, \( \tau \), built up in the bristle can be described by:

\[
\tau = G \gamma_{x,y}
\] (9)

where \( G \) is the shear modulus of the rubber and \( \gamma \) is the shear strain, which is given by the ratio of the bristle displacement and the (undeformed) bristle height. It is assumed that \( G_x = G_y = G \). Using the bristle position, \( \xi \) (measured from the leading edge, \( \xi = L/2 - x \)), the shear strain as a function of position in the contact patch reads:

\[
\gamma(\xi) = \frac{\Delta v t(\xi)}{H} = \frac{\Delta v \xi}{H |R\omega|}
\] (10)

where \( \Delta v \) is the slip speed, \( R \) is the rolling radius, \( t(\xi) = \xi / |R\omega| \) is the time taken to reach position \( \xi \) in the contact patch with respect to the leading edge, and \( H \) is the (undeformed) tread depth.

Combining Equations (9) and (10), one can write the shear stress as a function of position in the contact patch as:

\[
\tau(\xi) = \frac{G \Delta v}{H |R\omega|} \xi = \frac{G}{H} \sigma_{x,y} \xi
\] (11)

where \( \sigma_{x,y} \) is the theoretical slip [2]. In the lateral case:

\[
\sigma_y = \frac{\Delta v}{|R\omega|} = \frac{v_y}{|R\omega|} = \frac{v_y}{v_x}
\] (12)

which, for the case of pure cornering is equal to the practical slip:

\[
\sigma_y = \tan \alpha
\] (13)

In the longitudinal case:

\[
\sigma_x = \frac{(R\omega - v_x)}{|R\omega|}
\] (14)

It should be noted that \( \sigma_x \) is different to the practical slip in Equation (6). The two are related by:

\[
\sigma_x = \frac{\kappa}{1 + \kappa}
\] (15)
Thus, for the purposes of integration of flat-track data, which uses a practical slip approach, the theoretical slip is used for the force calculation, but results are plotted in terms of practical slip.

With this definition of the shear stress, one can compute the force, $F_{x,y}$, by integrating the shear stress over the contact patch area, $A$ (with $A = WL$, where $W$ is the contact patch width):

$$F_{x,y} = \int_{WL} \tau \, dA = W \int_{0}^{L} \tau \, d\xi = W \int_{0}^{L} \frac{G}{H} \sigma_{x,y} \xi \, d\xi$$  \hspace{1cm} (16)

If the friction limit is not reached within the contact patch:

$$F_{x,y} = W \int_{0}^{L} \frac{G}{H} \sigma_{x,y} \xi \, d\xi = \frac{GWL^2}{2H} \sigma_{x,y} = C_{x,y} \sigma_{x,y}$$  \hspace{1cm} (17)

where $C_{x,y}$ is the longitudinal or lateral slip stiffness, which becomes:

$$C_{x} = \frac{GWL^2}{2H} = \left. \frac{\partial F_{x}}{\partial x} \right|_{\sigma_{x}=0}$$  \hspace{1cm} (18)

$$C_{y} = \frac{GWL^2}{2H} = \left. \frac{\partial F_{y}}{\partial y} \right|_{\sigma_{y}=0}$$  \hspace{1cm} (19)

If the friction limit is reached in the contact patch, the so-called ‘breakaway point’, $\xi_{c}$ (i.e. the transition from the adhesion to sliding region of the contact patch) is found by equating the shear stress at co-ordinate $\xi_{c}$ to the maximum available frictional stress:

$$\tau(\xi_{c}) = \frac{G}{H} \sigma_{x,y} \xi_{c} = \mu_{\text{stick},x,y} p(\xi_{c})$$  \hspace{1cm} (20)

where $\mu_{\text{stick},x,y}$ is the friction coefficient in the adhesion region (cf. Section 2.1), and $p(\xi)$ is the local contact pressure. Here, the contact pressure is assumed to follow a parabolic distribution along the contact length:

$$p(\xi) = \frac{6F_{z} \xi}{WL^2} \left( 1 - \frac{\xi}{L} \right)$$  \hspace{1cm} (21)

Hence:

$$\xi_{c} = L \left( 1 - \frac{C_{x,y} \sigma_{x,y}}{3 \mu_{\text{stick},x,y} F_{z}} \right)$$  \hspace{1cm} (22)

The total tyre force is then given by two shear force contributions: $F_{x,y,\text{stick}}$, which calculates the force contribution from the leading edge up to $\xi_{c}$, and $F_{x,y,\text{slide}}$, which calculates the force contribution from $\xi_{c}$ to $L$, where the stress equals $\mu_{\text{slide}}(T, v_{sl}) \, p(\xi)$:

$$F_{x,y} = F_{x,y,\text{stick}} + F_{x,y,\text{slide}} = W \int_{0}^{\xi_{c}} \frac{G}{H} \sigma_{x,y} \xi \, d\xi + W \int_{\xi_{c}}^{L} \mu_{\text{slide}}(T, v_{sl}) \, p(\xi) \, d\xi$$  \hspace{1cm} (23)
Solving the integral (23), and considering that the sliding region extends over the entire contact patch when $\xi_c = 0$ (i.e. $|\sigma_x| > \frac{3\mu_{\text{stick},x}F_z}{C_{x,y}}$), the longitudinal force becomes:

$$F_x = \begin{cases} 
\text{sign}(\sigma_x) \left( C_x |\sigma_x| - \left( 2 - \frac{\mu_{\text{slide}}(T, v_{sl})}{\mu_{\text{stick},x}} \right) \frac{(C_x \sigma_x)^2}{3\mu_{\text{stick},x}F_z} \right) \\
\text{sign}(\sigma_x)(\mu_{\text{slide}}(T, v_{sl})F_z), \quad |\sigma_x| \leq \frac{3\mu_{\text{stick},x}F_z}{C_x}
\end{cases}$$

(24)

Similarly, the lateral force is given by Equation (25), noting that $C_y$ and $F_z$ are functions of slip angle, $\alpha$, for the multi-rib model (see Section 2.2.2):

$$F_y = \begin{cases} 
-\text{sign}(\tan \alpha) \left( C_y(\alpha) \tan \alpha - \left( 2 - \frac{\mu_{\text{slide}}(T, v_{sl})}{\mu_{\text{stick},y}} \right) \frac{(C_y(\alpha) \tan \alpha)^2}{3\mu_{\text{stick},y}F_z(\alpha)} \right) \\
-\text{sign}(\tan \alpha)(\mu_{\text{slide}}(T, v_{sl})F_z(\alpha)), \quad |\tan \alpha| > \frac{3\mu_{\text{stick},y}F_z(\alpha)}{C_y(\alpha)}
\end{cases}$$

(25)

As rubber has a complex shear modulus [22], instead of calculating $C_x$ and $C_y$ from (18) and (19), the relevant stiffness values were directly derived from the experimental flat-track data, which will be denoted as $C_{x,y,\text{exp}}$. $C_{x,\text{exp}}$ was found from the force gradient over a $\pm 1\%$ longitudinal slip range and $C_{y,\text{exp}}$ over a $\pm 1^\circ$ slip angle, respectively.

Equation (19) also shows that $C_y$ depends on the length and width of the contact patch. Hence, for the multi-rib model (cf. Section 2.2.2), the lateral slip stiffness is computed for each rib. Since each rib has equal width, but a varying contact length $L_i(F_z, \alpha)$, the lateral slip stiffness per rib reads:

$$C_{y,i}(F_z, \alpha) = \begin{cases} 
\frac{1}{3}C_{y,\text{exp}}(F_z) \left( \frac{L_i(F_z, \alpha)}{L(F_z)} \right)^2 & i = B \\
\frac{1}{3}C_{y,\text{exp}}(F_z) & i = A, C
\end{cases}$$

(26)

where $L_i(F_z, \alpha)$ is given by Equation (4) and $L(F_z)$ is obtained from Equation (5).

3. Friction measurements

3.1. Rubber friction testing

A state-of-the-art, purpose-built test rig (Figure 9) was used to measure the rubber friction characteristics. The rig consists of a linear actuator that moves a surface at a user-defined velocity within a climate chamber. A tread rubber sample, fixed in a sample holder, is pressed against the test surface by weights while the frictional, normal and lateral forces are measured. Similar to the fundamental work by Grosch [20], measurements were carried out at sliding velocities $\leq 10 \text{ mm/s}$ to avoid frictional heating [7,20]. By repeating friction experiments at different temperatures, the measurements allow for the construction of a
friction master curve as explained in Section 3.2. Measurements were carried out on the same sandpaper surface as installed on the flat-track test rig that was used to obtain the real tyre force slip characteristics (see Section 4) for the assessment of the accuracy of the variable-μ model.

The rubber sample was cut from a new tyre (Figure 10). To ensure consistent and repeatable results, a robust sample preparation and run-in procedure was carried out before friction data were recorded. This consisted of wearing-in the sample by, firstly, hand-sanding to remove the outer layer and, secondly, running it at a constant velocity in the rig for several hours until a stable friction coefficient was reached. A more detailed account of the sample preparation is given in [10]. After a sample was run-in, test runs began. A test run is defined as the sample traversing a 70 mm section of the sandpaper surface at 14 different velocities, ranging from 0.03 to 10 mm/s, at one temperature. For each velocity in a test run, the sample traversed the 70 mm section back and forth twice, and the friction coefficient measurement was taken as the average value across this distance.

Test runs were carried out at 16 different temperatures ranging from $-40^\circ$C to $+50^\circ$C. During a test run, the sliding velocity was changed from the lowest to the highest one to minimise the effects of sample wear. For example, at cold temperatures in particular, wear was greater for higher sliding velocities, evident by the observable debris on the surface after the test, see Figure 10. The surface and the sample were cleaned after each test run. In addition, testing was peppered with ‘calibration runs’, where the sample was slid at the run-in speed to monitor consistency and repeatability of the measurements. Tests were also repeated over a period of months to check and ensure consistency and repeatability.

### 3.2. Friction measurement results

Following the test procedure outlined in Section 3.1, a series of friction curves over a narrow velocity range (between 0.03 and 10 mm/s) and at different constant temperatures was obtained, see Figure 11. As the very low sliding velocities, $v$, of the friction tests are not representative of real tyre contact patch sliding speeds, the individual friction curves were merged to form the friction master curve. Similar to the fundamental work by Grosch [20], the Williams-Landel-Ferry (WLF) transform was used.
Figure 10. Examples of surface (left) and sample (right) conditions after a test run; top: wear particles noticeable after low temperature (−15°C) and high speed (10 mm/s) test; bottom: clean surfaces after a higher temperature (22°C) test.

Figure 11. Measured friction-velocity curves for temperatures ranging between −40°C and +50°C.

The WLF provides a temperature-dependent transformation factor, $a_T$, that can be applied to each friction curve by plotting the measured $\mu$ as a function of $(a_T \cdot v)$, rather than $v$. As a result, the individual curves are shifted over a wide range of sliding velocities forming a single ‘master curve’, but at a specific reference temperature, $T_s$. The WLF is
given by:

$$\log_{10}a_T = \frac{-8.86(T - T_s)}{101.5 + (T - T_s)}$$  \hspace{1cm} (27)$$

where $T$ is the temperature of the curve which is being shifted. The reference temperature is set at 50°C above the glass transition temperature, $T_g$, of the rubber compound [20]. $T_g$ was determined through DMA testing using three tread compound samples. Similar to the literature (e.g. [22–24]), here, $T_g$ was taken as the temperature corresponding to the peak in the experimentally determined tan δ curve. By averaging the results from all samples, $T_g$ was found to be equal to $-30°C$. Thus, $T_s = 20°C$.

Figure 12 shows the individual friction measurements shifted to the reference temperature yielding a very wide sliding velocity range, plotted as log($a_T \cdot v$). The master curve was approximated from the measurements by fitting a spline, which is shown by the solid red line. The spline was integrated into the variable-μ brush model as explained in Section 2.4.

4. Experimental tyre data

To assess the accuracy of the developed variable-μ model, simulation results are compared to experimental tyre data obtained on an indoor flat-track test rig at Calspan (USA), and the corresponding Magic Formula model. In this paper, only pure slip condition tests with an upright tyre at one inflation pressure are presented. Due to the nature of the flat-track procedure, a different number of normal loads were tested for longitudinal and lateral slip tests (five and three, respectively).

For both longitudinal and lateral tests, the input to the flat-track machine for the $F_z$ value was kept constant, while the slip ratio or slip angle was changed. Tests were conducted at
a constant velocity $v_x = 16.7$ m/s, on a 120-grit sandpaper belt – the same sandpaper that was used for friction testing described in Section 3.

Flat-track data were post-processed to ensure reliable comparison with the variable-$\mu$ brush model. Post-processing consisted of cropping data so that only the ‘outbound’ sweeps were retained. That is, sweeps from 0 slip to maximum/minimum slip were retained; data of the ‘return’ sweep from the slip extrema back to 0 were discarded. The primary reason for this was to ensure the validity of the WLF transform, which is only applicable for temperatures of up to $100^\circ C$ above $T_g$ [20], i.e. approx. $70^\circ C$. Since the tyre undergoes significant energy dissipation at high slips, the tyre was often heated to temperatures outside of this range at the maximum/minimum slip, and did not cool down until it had passed through 0 slip, before undergoing the remainder of the test. An example of the cropping process of the dataset is shown in Figure 13.

Variable-$\mu$ model and MF results (see Section 5) were compared to the flat-track data, using the root mean squared error (RMSE):

$$\text{RMSE} = \sqrt{\frac{\sum_{k=1}^{N} (m_k - f_k)^2}{N}}$$

where $k$ is a data point, $N$ is the maximum number of data points, $m$ is the model output and $f$ is the flat-track measurement.

5. Results

5.1. Longitudinal slip characteristics

The normal loads for which flat-track data were extracted are shown in Table 1, alongside the values of $C_{x,\text{exp}}$ and $\mu_{\text{stick},x}$ used in the variable-$\mu$ model. $C_{x,\text{exp}}$ was calculated from
Table 1. Longitudinal model parameters

| $F_z$ (N) | $C_{x,exp}$ (N) | $\mu_{stick,x}$ | $\omega$ |
|-----------|-----------------|-----------------|----------|
| 3400      | 119,510          | 1.6             |          |
| 6865      | 262,224          | 1.5             |          |
| 8573      | 338,336          | 1.4             |          |
| 10,299    | 452,809          | 1.4             |          |
| 13,757    | 599,729          | 1.0             |          |

Figure 14. $F_z = 3400$ N variable-$\mu$ model results, split into adhesive and sliding region force contributions ($F_{x,stick}$ and $F_{x,slide}$, respectively), alongside the experiments (flat-track data), MF model, and two-$\mu$ model ($\mu_{stick,x}$ of Table 1 and $\mu_{slide} = 1.25$).

The force-slip gradient of the flat-track data (see Section 2.5), while $\mu_{stick,x}$ was empirically obtained by fitting the model results to the experimental tyre data.

To visualise the summation from the adhesive and sliding regions of the contact patch, the force contribution from each region is shown separately (and the two summed together) in Figure 14, together with the flat-track data, two-$\mu$ model (with $\mu_{stick,x}$ of Table 1 and $\mu_{slide} = 1.25$), and the corresponding MF6.1 model results. The variable-$\mu$ model predicts tyre behaviour well, capturing the peak force and drop-off accurately, unlike the two-$\mu$ model as discussed in Section 2.1. For negative slips, it can be argued that the variable-$\mu$ model predicts tyre behaviour better than the MF. For example, utilising Equation (28), the RMSE for $\kappa < 0$ was 225 N for the variable-$\mu$ model, compared with 270 N for the MF. After the peak force for $\kappa < 0$, the RMSE was 237 N for the variable-$\mu$ model compared with 275 N for the MF. Figure 14 also illustrates how the relative contributions from the two contact patch regions affect the overall force characteristics: the size of the adhesion region affect the slip stiffness, while the frictional characteristics of tyre-road interaction affect the peak and drop-off forces. Interestingly, the slight asymmetry around the peak force seen in the flat-track data with positive and negative slip ratios – primarily a temperature effect – can be captured through the integration of the measured friction characteristics.
Figure 15. Longitudinal slip characteristics at the normal loads shown in Table 1 obtained with the MF model (blue line) and the variable-\(\mu\) model, which are coloured by the tread temperature (sensor B, cf. Figure 7). Experimental flat-track data are given by the black dotted lines.

The longitudinal slip characteristics at five different normal loads are collated in Figure 15. In general, compared to the experimental data, the variable-\(\mu\) model describes the longitudinal force against slip ratio behaviour well for a wide range of normal loads and even at elevated temperatures beyond the commonly assumed validity of the WLF transform. Also, for many slip conditions, the variable-\(\mu\) model matches the tyre data better than the MF model. For example, the RMSE value for \(\kappa > 0.1\) (corresponding to the peak force) at 7 kN was 116 N for the variable-\(\mu\) model, versus 214 N for the MF. However, at the highest investigated normal load (13,757 N), the variable-\(\mu\) model displays a significant overshoot around the peak force. This model discrepancy is due to the extreme test condition with a normal load that exceeds the rated load of the tested tyre (1060 kg). As a consequence, during the test, large deformations occurred within the tyre structure, yielding geometric nonlinearities that cannot be captured by the simple contact patch formulation of the variable-\(\mu\) model.

5.2. Lateral slip characteristics

As discussed in Section 2.2.2, the multi-rib contact patch model was used for the lateral force calculation. The corresponding model parameters in terms of \(C_y, \text{exp}\) and \(\mu_{\text{stick}, y}\) are shown in Table 2. To illustrate the multi-rib approach, Figure 16 shows the force contribution of each rib in the calculation of the total force together with experimental results. The discontinuity of the experimental data for small slip angles is a consequence of the post-processing outlined in Section 4. Similar to the longitudinal slip characteristics, the variable-\(\mu\) model matches the lateral force experiments well, especially for the negative slip sweep. For the positive slip sweep, which was carried out directly after the positive slip angle sweep, small differences can be observed for slip angles approx. > 6°. The difference
can be primarily ascribed to a combination of elevated temperatures during the test and tyre structure nonlinearities, i.e. ply-steer and conicity [2].

Figure 17 compares the simulation results at three different normal loads with the experimental data and the MF model. Generally, the variable-μ model shows a satisfactory correlation at the different loads. It can match the cornering stiffness well for all loads and better than the MF model, but shows differences in the simulated lateral force for slip angles greater than about 5° with increasing normal load. Again, the model mismatch is more evident for the positive slip angles than for negative slip angles.

Similar to the longitudinal slip characteristics, the variable-μ model shows a significant overshoot compared to the experiments and the MF results at the highest normal load (9 kN), resulting in an RMSE value of 947 N for the variable-μ model vs 225 N for the MF for $\alpha \leq -6^\circ$. This behaviour can be mostly ascribed to nonlinear tyre deformation behaviour which is more complex compared to the longitudinal manoeuvre because of the off-axis resultant force vector. That is, during cornering, the contact patch will be deformed laterally (twisted) and longitudinally (stretched). Also, the model discrepancy demonstrates that the tyre force nonlinearities at high slip are not only due to friction influences, but also due to carcass/tyre structure effects, which are not accounted for in the contact patch model of the variable-μ model. These effects – in conjunction with the limited validity of the WLF transform (cf. Section 4) – also account for the uptick in predicted lateral forces at the highest slip angles at the 6 and 9 kN loads.
Figure 17. Lateral slip characteristics at three different loads, 2968, 6057, and 9094 N. MF model results are shown in solid blue lines, experiments in black, and the variable-μ model results are coloured according to the average temperature of the tread ribs.

In sum, the close match of the results demonstrates the benefit of the inclusion of the real friction characteristics and the observations illustrate the benefits from employing a physical modelling approach.

6. Conclusions

In this paper, friction measurements with a tread rubber block slid against a sandpaper surface were performed under controlled conditions and integrated with a simple, low-parameter brush model, producing the so-called ‘variable-μ model’. Comparison of the model results with experimental slip characteristics obtained on the same sandpaper surface and the corresponding MF model allow the following conclusions:

- Despite the very simple, low-parameter brush model approach used here, the integration of the real friction characteristics allows very good replication of the longitudinal tyre behaviour over a wide range of operating conditions in terms of normal load and slip ratio. For lateral slip characteristics, good agreement between simulations and experiments was observed for low to medium vertical loads.
- With very high normal loads, the variable-μ model overpredicted the experiments due to nonlinear effects arising from large tyre deformations that are not captured in the simple contact patch formulation adopted here.
- Compared to the MF model, requiring a total of 42 parameters to simulate pure longitudinal and pure lateral slip conditions [18], the variable-μ model contains considerably fewer parameters – 12 parameters or 25 if modelling of the temperature according to [11] would be considered. Nonetheless, the variable-μ model can replicate very similar behaviour for the longitudinal force characteristics and reasonable behaviour for lateral
slip behaviour. While not seeking to replace the MF, the proposed approach shows the value and importance in capturing accurate friction characteristics for tyre modelling.

- Under certain operating conditions the variable-μ model can provide a better match with the experiments than the MF model; for example, this includes small slip angle and large slip ratio conditions.

In addition, the information associated with the physical modelling approach (e.g. breakdown of the contact patch contributions) was shown to provide valuable insights into tyre behaviour, which is not available using the empirical MF.

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