Development the assembly sequence mathematical modeling method of precision axisymmetric products based on the graph incidence matrix

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Abstract. The paper presents a method for problem solving of automated construction the assembly sequence of high-precision axially symmetric products. The purpose of this article is testing the admissibility of using a directed assembly graph, including adjacency and incidence matrices, for structuring and planning theoretical schematization of the assembly technological process. The considered method coincides with a possible mathematical implementation of the problem. The specificity of the structure modeling and the assembly process using mathematical methods has been evaluated. The graph creating technique representing the relationship between parts and the assembly process is considered. The solving the problem of assembly process automated construction is performed using algorithms and computational software based on the graphs constructing of in the matrices form. A comparison of the results achieved theoretically and computer aided in terms of designing the assembly process has been made. The achieved results make it possible in real time to adjust the product assembly sequencing, on the basis of this, it becomes possible to track parameters and control decision-making when planning assembly processes.

1. Introduction

The process of designing assembly technology involves two stages [1]. The first stage – the generation of an assembly diagram and a routing assembly technology process, containing information on the order of product elements attachment, the completeness of assembly units and assembly joint. The second stage – the generation of operations, determining the composition of the elements to be attached, types of work, tools and other parameters that form the description of assembly operations. The creative process of generation the assembly diagram and the routing assembly technology process, during which the engineer identifies the technological assembly units and determines the possible assembly order, is the most time-consuming and difficult to formalize. In this paper, is being considered the first stage of designing assembly technology.

In the designing assembly technology applies the graph theory in connection that allows you to link theoretical principles with specific computational algorithms which are easy implemented on the computer. The founder of the mathematical models application like a graph form in engineering is B. S. Mordvinov [2], which introduced the concept of the machine interconnected geometric structure and its graph. This includes, in particular, the problem of products assembly sequences. J. Yu and L. Xu
developed an approach to automatically develop an assembly sequence based on the relationship matrix method is a component-wise of the form [3]. R. Gottipolu and K. Ghosh proposed an approach to create executable assembly sequence based on the analysis of contact and mobility constraints [4]. Q. Su has developed a systematic method for analyzing the geometric constraints between assembly units, based on an interactive simulation of CAD models [5]. Alfadhlani T., Samadhi M. A., Toha I. [6] developed an automatic method to detect the collision between the Assembly units. This method is based on geometrical information obtained by pairing the assembly units. Kanthababu M., Giri R. [7] proposed an approach to generate disassembly sequence of mechanical models based on the interference matrix and connections graph. Based on the interference matrix, M. R. Bahubalendruni, B. B. Biswal has implemented the integrated methods based on the theory used to extract the criteria of the predicates, such as data communication [8, 9], the mechanical feasibility [10-12] and geometric feasibility [13] to create the optimal ASP/DSP (designing an assembly sequence / disassembly (dismantling)) [14].

2. Problem statement
The research aim is the problem of determining the joining parts sequence when assembling products based on adjacency and incidence matrices.

For definiteness, we will consider axisymmetric products as objects of assembly.

When constructing the assembly process, it should be borne in mind that the assembly process is the process of forming relationships between parts. Therefore, it is first necessary to identify the connection data, which is possible based on the use of the mathematical model of the product in the graph form, where the parts are vertices, and the bonds are edges.

If we determine the bonds forming sequence that are identified in the subassembly construction, this solves the problem of determining the product assembly sequence.

If we determine the bonds forming sequence that are identified in the assembly units construction, this solves the problem of determining the product assembly sequence.

The bond graph between the subassembly parts can be represented as an adjacency matrix (graph), and the bonds forming sequence can be represented as the incidence matrix of this graph, in which the columns sequence, in other words edges will determine the assembly sequence.

The task of research is to develop a methodology for constructing and ordering the graph incidence matrix.

3. The development theory
Methodology for constructing and ordering the graph incidence matrix:
1. Constructing the geometric model of an assembly unit (product).
2. Identification of contacts and force closure. By contact is meant the connection of parts directly touching flat or cylindrical surfaces or allowing the third part to be located between them.
3. Identification for each part the bond with other parts with the constructing the adjacency matrix.
4. Ordering details, drawing up a model of the assembly unit construction (based on the graph adjacency matrix) [15].
5. Constructing the graph with edges-bonds and the addition of force closure edges for the model of assembly unit construction.
6. Definition of the assembly sequence rules for areas limited by force closure.
7. Constructing of the initial, disordered incidence matrix from the adjacency matrix.
8. Ordering bonds in the incidence matrix.
9. Implementation of the assembly process.

Assumptions:
1. By force closure is meant the connection of parts obtained by applying forces to them to ensure contact continuity, maintaining the position of the part relative to another part in accordance with the assembly unit construction (threaded joints, joints using locking ring, preloaded joint and others).
2. Contact will be considered a sign of parts bonds, the gap - a property of force closure.
4. Experimental results
Consider the implementation of the methodology on the example of the drive shaft [16] ‘Figure 1’ consider its characteristics, disassembly into elements.

![Figure 1](image1)

**Figure 1.** Drive subassembly, including the following parts: 1 - spline shaft, 2 - blind cap, 3 - bolt, 4 - plain washer, 5 - ring spacer, 6 - shaft seal sleeve, 7 - housing, 8, 10, 13 - locking rings, 9 and 11 - spherical ball bearing, 12 - bevel gearwheel, 14 - shaft seal ring

To carry out modeling of the “Drive Shaft” subassembly construction, it is necessary to switch to the subassembly model in the form of its geometric model, in which all parts have only flat and cylindrical surfaces ‘Figure 2’. We transform the shape of some parts, replacing conical, spherical, threaded, shaped surfaces with flat and cylindrical surfaces.

![Figure 2](image2)

**Figure 2.** A geometric model of the “Drive Shaft” subassembly construction.

1. Identification of contacts and force closure.
Mathematical modeling will be carried out on the basis of a simplified model in the graph form that takes into account the bonds in only one - axial - direction (between the end surfaces of the parts).
The details were taken as the graph vertices, and the bonds between the details as the graph edges.
By bonds is meant direct contact between parts, a certain gap or constraints on the relative position of parts in an assembly unit.

We will indicate as the bond number the numbers of the parts with which this part is associated ‘Figure 3’. If there is contact between the parts, then note the direction from the considered part to the mating one - to the right or left, depending on whether the mating part is located to the right or left, constraining the displacement of the part under consideration.

![Figure 3. Designation of directions and bonds rank between parts in the assembly unit construction model.](image)

The bond type is determined by the rank:
1st rank - direct connection (contact during assembly) - can be connected without the use of other parts - strong connection;
2nd rank - connection is possible only after preliminary assembly of other parts (there is contact) - weak connection;
3rd rank - bond indicates the relative position of parts in an assembly unit without direct contact or a ban on impossible positions.

The bond rank is indicated next to the communication number, separated from the bond number by a dot.

2. Identification for each part the bond of 1st and 2nd ranks with constructing of the adjacency matrix.

2.1. Bonds of 1st and 2nd ranks are only contacts - according to the contact graph (there will be one less than the number of parts), that is, you need to start by identifying contacts, as a result, you should get a bonds tree (contacts) between the parts ‘Figure 4’.

![Figure 4. Assembly sequence by contact graph.](image)
3. Ordering parts, drawing up a model of the assembly unit construction (based on the graph adjacency matrix) [17].

The ordering process consists in rearranging rows and corresponding columns in such a way that negative numbers are placed on top of the main diagonal, positive numbers on the bottom, while numbers equal to 1 are closer to the main diagonal, due to the removal of numbers equal to 2 from the main diagonal.

As a result, we obtain the following information: the arrangement of parts in the model of the assembly unit construction (Table 1).

![Graph-tree of bonds (contacts) and force closure between parts (solid lines - contacts, dashed - force closure).](image)

**Figure 5.** Graph-tree of bonds (contacts) and force closure between parts (solid lines - contacts, dashed - force closure).

| N  | 3  | 4  | 2  | 5  | 14 | 6  | 10 | 9  | 8  | 7  | 1  | 11 | 12 | 13 |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 3  | 0  | −1 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 4  | 1  | 0  | −2 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 2  | 0  | 2  | 0  | −2 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 5  | 0  | 0  | 2  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 14 | 0  | 0  | 0  | 0  | −2 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 6  | 0  | 0  | 0  | 2  | 0  | 0  | −2 | 0  | −2 | 0  | 0  | 0  | 0  | 0  |
| 10 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | −2 | 0  | 0  | 0  | 0  |
| 9  | 0  | 0  | 0  | 0  | 2  | 0  | 0  | 0  | 0  | −1 | 0  | 0  | 0  | 0  |
| 8  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | −1 | 0  | 0  | 0  | 0  |
| 7  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 2  | 2  | 0  | 0  | −1 | 0  | −2 |
| 11 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | −2 | 0  | 0  |
| 12 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 2  | 0  | 0  | 0  |
| 13 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 2  | 0  | 0  | 0  |

4. Constructing the graph with edges-contacts and the addition of force closure edges ‘Figure 5’.

Power contacts are also taken into account - by threads (bolts, screws) for closing circuits, locking ring.

5. Definition of the assembly sequence rules for areas limited by force closure.

6. Constructing of the initial, disordered incidence matrix.

6.1 The number of columns in the matrix should be 1 less than the number of parts, and the number of bonds identified for all parts is greater, because for each part, you can specify links to all other parts, i.e. \( n − 1 \) connection, thus, \( (n − 1) n / 2 \).

The table indicates all possible bonds.

6.2 When 2 parts are connected, their bonds with other parts are combined and the bonds of these two parts to the same part. Therefore, the number of columns in the matrix should not be equal to n-1, but equal to the number of identified bonds.
The bonds are entered in the incidence table (Table 2) in the order that is fixed in the adjacency matrix. Only bonds of the first and second rank are taken into account. Bonds of the third rank are needed to build a subassembly, and not to determine the assembly sequence.

**Table 2.** Adjacency matrix (disordered bonds).

|   | 1 | 2 |
|---|---|---|
| 3 |   |   |
| 4 | 1 |   |
| 2 |   | 2 |
| 5 | 2 | 2 |
| 14|   |   |
| 6 | 2 | 2 |
| 10|   | 2 |
| 9 |   |   |
| 8 |   |   |
| 7 | 2 |   |
| 1 | 1 | 2 |
| 11|   |   |
| 12|   |   |
| 13| 2 |   |

7. Ordering bonds in the incidence matrix. The next bond should cover the previous one.

The attaching parts rule must be formulated based on the fact that the graph incidence matrix will be used, which means ordering the bonds between the parts, and the bonds in the incidence matrix (Table 3).

**Table 3.** Adjacency matrix (ordered bonds)

|   |   |   |   |
|---|---|---|---|
| 3 |   | 2 | 1 |
| 4 |   | 1 |   |
| 2 | 2 | 2 |   |
| 5 |   |   |   |
| 14|   | 2 | 2 |
| 6 | 2 | 1 |   |
| 10| 2 |   |   |
| 9 |   |   |   |
| 8 |   |   |   |
| 7 |   | 2 |   |
| 1 | 1 | 2 |   |
| 11|   |   |   |
| 12|   | 2 |   |
| 13|   | 2 |   |

8. Implementation of the assembly process. Before starting the assembly, make a new table. We combine the connected bonds and reduce the matrix by one row and one column with each addition.

At the program result is the bond matrix between the parts of the assembly unit is displayed on the display screen.

With correctly prepared data, the matrix should be skew-symmetric.

Next, the program simulates the assembly unit constructions and the routing assembly technology process.

During the comparison and analysis of the differences between the theoretical and computer result of modeling the assembly unit constructions and the routing assembly technology process, a number of numerical experiments were performed.
5. Experiments discussion

As a result of the numerical experiment, the following information is displayed on the computer display screen: the order of the parts in the model of the assembly unit construction and the parts joining sequence ‘Figure 6’.

![Program result](image1)

**Figure 6.** Program result.

At the program result is a correct final model of the assembly scheme with information about the bonds was obtained ‘Figure 7’.

![Final model](image2)

**Figure 7.** Final model of the assembly scheme “Drive Shaft” with information about the bonds
6. Conclusions
The article had been analyzed the mathematical modeling features of assembly processes, a theoretical study of the construction and assembly diagrams. The adjacency and incidence graph matrices are constructed, that visually displayed the bond between the parts and the product assembly sequence. The results of theoretical studies had been supplemented by studies using a computer program for the computer aided generated the assembly process based on the matrix representation of graphs. The results had been obtained make it possible to provide automated generating model construction and automated generating the assembly technological diagram using a theoretically formed construction model, to promptly make the necessary changes to the assembly sequence, on the basis of which it becomes possible to control data and decisions when dimensional analysis of structures and when designing assembly processes.

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