Electromagnetic field at Finite Temperature: A first order approach

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Abstract

In this work we study the electromagnetic field at Finite Temperature via the massless DKP formalism. The constraint analysis is performed and the partition function for the theory is constructed and computed. When it is specialized to the spin 1 sector we obtain the well-known result for the thermodynamic equilibrium of the electromagnetic field.

1 Introduction

Quantum Field Theory at Finite Temperature was motivated by the increasing interest in studying the properties of matter under extreme conditions as, for example, at very high temperature or density. The pioneering works joining together the Statistical and Quantum Field Theory were developed mainly by Matsubara [1] in a non relativistic context and, the relativistic case by Fradkin [2] who, via the functional approach, studied the different methods for calculating the Thermal Green’s functions as well as the structure of the Ward identities in QED. Later other works within the Thermal Field Theory appeared [3] whose principal interest was to explore the possibility of restoring some broken symmetries that occur at zero temperature, as for example the \( SU(2) \times U(1) \) symmetry of weak interaction. The Finite Temperature gauge theories and the problems concerning to the choice of a physical gauge and its dependence was analyzed by Bernard [4], in particular, the free electromagnetic field.

On the other hand, at zero temperature, there is an alternative way to study the properties for the electromagnetic field which is known as the massless Duffin-Kemmer-Petiau (DKP) theory [5] that is not a trivial limit of the massive Duffin-Kemmer-Petiau theory (DKP) [6] that appears as an alternative formalism for the description of the spin 0 and spin 1 particles in a unified formulation. The DKP theory gives a first order linear equation and it is very similar to the Dirac one but the \( \beta^\mu \) matrices satisfy a different algebraic relation. The massive and massless case of the theory were considered in the works [7, 8, 9] where the equivalence of the DKP theory with the theories like Klein-Gordon-Fock (KGF) and Maxwell was proved in Minkowski space-time [7] and studied in curved space-time such as Riemann [8] and Riemann-Cartan [9].

At Finite Temperature the massive case is treated in the work [10] where the Bose-Einstein condensation is investigated in the spin 0 sector. The equivalence of many-photons Thermal Green’s functions of the DKP and KGF theories was also proved for the scalar sector [11] calculating the polarization operator at 1-loop order and, in [12] is shown the equivalence of many-gluons Green’s functions in the DKP and KGF Statistical Quantum Field Theories.

As above mentioned, all accomplished studies on the massless DKP theory were made at zero temperature. The aim of this work is to study the thermodynamics of the electromagnetic field by using the massless DKP theory. The paper is organized as following: In section 2, we give a brief review of the massless DKP formalism considering a theory with only one real DKP field. In section 3, the constraint analysis is performed for the DKP theory and it has been shown that the model has two first class constraints. In section 4, we calculate the partition function and, finally, we give our conclusions and commentaries.

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2 The Massless DKP Theory

The massless DKP theory is described in Minkowski space-time by the following Lagrangian density \[ \mathcal{L}_M = i \bar{\psi} \gamma^\mu \partial_\mu \psi - \bar{\psi} \gamma^\mu \beta^\mu \gamma \psi - \bar{\psi} \gamma \psi \ , \]

where \( \bar{\psi} = \psi^\dagger \eta_0 \) with \( \eta_0 = (2 \beta_0^2 - 1) \). The \( \beta^\mu \) and \( \gamma \) are singular square matrices satisfying the following algebra

\[ \beta^\mu \beta^\lambda \gamma^\nu + \beta^\nu \beta^\lambda \gamma^\mu = \gamma^\mu \gamma^\nu + \eta_0 \gamma^\mu \gamma^\nu \ , \]

\[ \beta^\mu \gamma + \gamma \beta^\mu = \beta^\mu \ , \quad \gamma^2 = \gamma \ . \]

Due to the singular character of the \( \beta^\mu \) matrices the transition of the massive case to the massless theory is non trivial and demands a different treatment. On the other hand the representations of the DKP algebra are reducible and contain the sectors of spin 0 and spin 1 in their structure.

From the Lagrangian \[ \text{(1)} \] we obtain the equation of motion for the massless DKP field

\[ i \beta^\mu \partial_\mu \psi - \gamma \psi = 0 \ . \]

It can be shown that the Lagrangian \[ \text{(1)} \] and the massless DKP equation remain invariants under the following gauge transformation

\[ \psi \rightarrow \psi' = \psi + (1 - \gamma) \Phi \ , \]

iff the field \( \Phi \) satisfies the condition

\[ i \beta^\mu \partial_\mu (1 - \gamma) \Phi = 0 \ . \]

When the fields under consideration are no charged we have a real DKP field, in such situation the Lagrangian \[ \text{(1)} \] takes the following form

\[ \mathcal{L} = \frac{i}{2} \psi^{T} (\eta_0 \gamma \beta^\mu) \partial_\mu \psi - \frac{i}{2} \partial_\mu \psi^{T} (\eta_0 \beta^\mu \gamma) \psi - \frac{1}{2} \psi^{T} (\eta_0 \gamma) \psi \]

3 Constraint Analysis

We proceed the study of the constraint analysis to the real massless DKP theory from the Lagrangian \[ \text{(7)} \] which is written as

\[ \mathcal{L} = \frac{i}{2} \psi^a (\eta_0 \gamma \beta^a) \partial_\mu \psi^b - \frac{i}{2} \partial_\mu \psi^a (\eta_0 \beta^a \gamma) \psi^b - \frac{1}{2} \psi^a (\eta_0 \gamma) \psi^b \ , \]

as usual we define the canonical momentum \( \pi_a \) as

\[ \pi_a = \frac{\delta \mathcal{L}}{\delta \dot{\psi}^a} = -i (\beta^a \gamma)_{ab} \psi^b \ , \]

from which a set of primary constraints appear \( \theta \)

\[ \theta_a = \pi_a + i (\beta^a \gamma)_{ab} \psi^b \ , \]

because the two different representations for the \( \beta^\mu \) matrices we have for the spin 0 sector that \( a = 0, 1, 2, 3, 9 \) and for spin 1 sector that \( a = 0, 1, 2, ..., 9 \).

The canonical Hamiltonian density \( \mathcal{H}_C \) that follows from the Lagrangian \[ \text{(8)} \] is given by

\[ \mathcal{H}_C = \frac{i}{2} \partial_\kappa \psi^a (\eta_0 \beta^k \gamma)_{ab} \psi^b - \frac{i}{2} \psi^a (\eta_0 \gamma \beta^k)_{ab} \partial_\kappa \psi^b + \frac{1}{2} \psi^a (\eta_0 \gamma)_{ab} \psi^b \]

and considering the set of constraints \[ \text{(10)} \] we have the primary Hamiltonian density \( \mathcal{H}_P \) as

\[ \mathcal{H}_P = \mathcal{H}_C + \lambda^a \theta_a \ , \]

where \( \lambda^a \) are the Lagrange multiplier.

The Poisson bracket (PB) for the primary constraints results in

\[ \{ \theta_a (x) , \theta_b (y) \} = i (\beta^a)_{ab} \delta (x - y) \ . \]

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1. When the DKP field is real, the \( \beta^\mu \) matrices must be satisfied \( \beta^a_0 = -\beta_0 \), \( \beta^a_k = \beta_k \). The representation of the \( \beta^\mu \) matrices for the spin 1 sector is given in the appendix A, where we also included the \( \gamma \) matrix.
To investigate the possibility of obtaining more constraints in the theory we apply the preservation in time of $\theta_a$, i.e.

$$\hat{\theta}_a(x) = \{\theta_a(x), H_P\}, \quad (14)$$

where $H_P = \int d^3x \mathcal{H}_P$ is the primary Hamiltonian. Thus, the stability condition provides

$$\hat{\theta}_a = i \left( \eta^0 \beta^k \right)_{ab} \partial_k \psi^b - \left( \eta^0 \gamma \right)_{ab} \psi^b + i \beta^0 \lambda^b \approx 0 \quad , \quad (15)$$

being $\beta^\mu$ singular matrices we conclude that not all $\lambda$ coefficients can be obtained from the relation and more constraints appear. These new constraints are selected by means of the projector $M = (1 - \beta^0_\mu)$, thus, we obtain

$$\theta^{(2)}_a = (M \beta^k)_{ac} i \partial_k \psi^c - (M \gamma)_{ac} \psi^c \quad (16)$$

that are a set of secondary constrains. When the preservation in time of these secondary constraints is imposed no more constraints appear in the theory.

Now we calculate the PB for all primary and secondary constraints

$$\{\theta_a(x), \theta^{(2)}_b(y)\} = \left[(i \beta_k \partial^k + \gamma) M\right]_{ab} \delta(x - y),$$

$$\{\theta^{(2)}_b(x), \theta_a(y)\} = \left[M(i \beta_k \partial^k - \gamma)\right]_{ab} \delta(x - y), \quad (17)$$

$$\{\theta^{(2)}_a(x), \theta^{(2)}_b(y)\} = 0 \quad .$$

We can write all set of constraints as $\zeta_a = \{\theta_a, \theta^{(2)}_a\}$ such that its matrix is

$$\{\zeta_a(x), \zeta_b(y)\} = \begin{bmatrix}
i \beta^0 & (i \beta_k \partial^k + \gamma) M \\
M(i \beta_k \partial^k - \gamma) & 0
\end{bmatrix} \delta(x - y) \quad (18)$$

and the determinant of $\{\zeta_a, \zeta_b\}$ is zero. But the constraint structure depends of the chosen representation for the DKP algebra as we will see go on.

### 3.1 Spin 1 sector

For this sector the representation of the $\beta^\mu$ matrices is $10 \times 10$ and the spin 1 DKP field is a column matrix with ten real components

$$\psi = (\psi^0, \psi^1, \psi^2, \psi^3, \psi^4, \psi^5, \psi^6, \psi^7, \psi^8, \psi^9)^T \quad . \quad (19)$$

Consequently, in this sector, from (10) we observe that there are ten primary constraints and write explicitly

$$\theta_0 = \pi_0 \quad , \quad (20)$$

$$\theta_1 = \pi_1 + \psi^7 \quad , \quad \theta_2 = \pi_2 + \psi^5 \quad , \quad \theta_3 = \pi_3 + \psi^9 \quad , \quad (21)$$

$$\theta_n = \pi_n \quad , \quad n = 4, 5, 6, 7, 8, 9 \quad . \quad (22)$$

And from (16) we obtain four secondary constraints given by

$$\theta^{(2)}_0 = -\partial_1 \psi^7 - \partial_2 \psi^8 - \partial_3 \psi^9 \quad , \quad (23)$$

$$\theta^{(2)}_4 = \partial_3 \psi^2 - \partial_2 \psi^3 - \psi^4 \quad , \quad \theta^{(2)}_5 = \partial_1 \psi^3 - \partial_3 \psi^1 - \psi^5 \quad , \quad \theta^{(2)}_6 = \partial_2 \psi^1 - \partial_1 \psi^2 - \psi^6 \quad . \quad (24)$$

To classify these constraints as first and second class we perform the calculation of the PB between all these primary and secondary constraints such as it is shown by the matrix $\{\zeta_a(x), \zeta_b(y)\}$ in (15).

In our case, the rank of the matrix $\{\zeta_a, \zeta_b\}$ is 12 which is the number of second class constraints and, for the spin 0, the representation for the $\beta^\mu$ matrices is $5 \times 5$. The rank of the constraint matrix $\{\zeta_a, \zeta_b\}$ is 8, the null space has two trivial constraints which are irrelevant, and the set of constraints is second class.
the dimension of the null space is 2 that gives the number of the first class constraints. The null space is formed by the constraint $\theta_0 = \pi_0$ and by the linear combination of second class constraints $\partial_1 \theta_1 + \partial_2 \theta_2 + \partial_3 \theta_3 + \theta_0^{(2)}$ that defines another first class constraint $G = \partial_k \pi_k$. Thus, we obtain two first class constraints

$$\theta_0 = \pi_0 , \quad G = \partial_k \pi_k , \quad k = 1, 2, 3 \tag{25}$$

and twelve second class constraints given by the equations (21), (22) and (24).

The projectors of the spin 1 sector are defined as

$$R^\mu = (\beta^1)^2 (\beta^2)^2 (\beta^3)^2 \left[ \beta^\mu \beta^0 - \eta^\mu^0 \right]$$

$$R^\mu \nu = R^\mu \beta^\nu , \quad \mu, \nu = 0, 1, 2, 3. \tag{26}$$

such that the field $\psi^\mu = R^\mu \psi$ is a Lorentz vector and $\psi^\mu \nu = R^\mu \nu \psi$ is an antisymmetric second-rank Lorentz tensor; when multiplied with the $\gamma$ matrix we also get $R^\mu \gamma \psi = 0$ and $R^\mu \nu \gamma \psi = R^\mu \nu \psi$.

Using the projectors $R_\mu$ and $R_\mu \nu$ and, from the relation (3) we conclude that only the vector components of the DKP field are transformed as it is shown to follow

$$\psi_\mu' = \psi_\mu + \Phi_\mu ,$$

$$\psi_n' = \psi_n , \quad n = 4, 5, 6, 7, 8, 9 \tag{27}$$

and from (20) we get $R_\mu \Phi = \Phi_\mu = \pm \partial_\mu \Lambda$, being $\Lambda$ an arbitrary scalar function, thus, we can conclude that the theory under consideration is a local $U(1)$ gauge field theory. Then, we impose the following gauge fixing conditions

$$\Omega_1 = \partial_k \psi^k , \quad \Omega_2 = \psi^0 , \tag{28}$$

such that the set $\chi_{A'} = \{ \theta_0, G, \Omega_1, \Omega_2 \}$ is second class, thus, the PB matrix of the set is given by

$$D_{A'B'} = \{ \chi_{A'}(x), \chi_{A'}(y) \} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -\triangle x & 0 \\ 0 & \triangle x & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \delta(x - y) \tag{29}$$

where $\triangle = (\partial_k)^2 = (\partial_1)^2 + (\partial_2)^2 + (\partial_3)^2$, computing the functional determinant we get

$$\det D_{A'B'} = |\det \triangle|^2 . \tag{30}$$

## 4 The Partition Function

Now we study the thermodynamic equilibrium of the electromagnetic field using the DKP formalism. Such as we see the constraint analysis of the theory gives for the spin 1 sector two first class constraints characterizing a local $U(1)$ gauge field theory. Then, we write the partition function for the massless real DKP field using the Hamiltonian formalism

$$Z = \int_{\text{periodic}} D\psi D\pi \delta(\Theta_A) \delta(\chi_{A'}) (\det C_{AB})^{1/2} (\det D_{A'B'})^{1/2} \exp \left\{ \int d^d x \left( i \pi_\alpha \partial_\alpha \psi^\alpha - \mathcal{H}_C \right) \right\} , \tag{31}$$

where $\mathcal{H}_C$ is given by (11) and

$$D\psi = D\psi^\alpha , \quad D\pi = D\pi_\alpha , \quad a = 0, 1, 2, ..., 9. \tag{32}$$

The fields $\psi$ are restricted by the periodicity condition

$$\psi(0, x) = \psi(\beta, x) , \tag{33}$$

where $\Theta_A = \{ \theta_a, \theta_b^{(2)} \}$ is the set of second class constraints given by the equations (21), (22) and (24); the set $\chi_{A'} = \{ \theta_0, G, \Omega_1, \Omega_2 \}$ is given by the set of first class constraints (25) and its respective gauge fixing conditions (28). The matrix $C_{AB} = \{ \Theta_A, \Theta_B \}$ can be obtained from (18) and its determinant is $\det C_{AB} = 1$. The matrix $D_{A'B'} = \{ \chi_{A'}, \chi_{B'} \}$ and its determinant $\det D_{A'B'}$ are given by the equations (29) and (30), respectively.
But, it is interesting to perform the calculation of the partition function in a manifest covariant way. Thus, it is possible to show that the equation (31) becomes

\[
Z = N(\beta) \int \mathcal{D}\psi \left| \frac{1}{\sqrt{\xi}} \partial_A \psi^A - f \right| \prod_{\beta} \left\{ \int d^4x \left[ \frac{1}{2} \psi^T \eta^0 (i\beta^A \partial_A - \gamma) \psi \right] \right\} ,
\]

(34)

where \( F[\psi^A] \) is an arbitrary gauge fixing condition. Here we consider

\[
F[\psi^A] = 1 \frac{1}{\sqrt{\xi}} \partial_A \psi^A - f , \quad F^g[\psi^A] = F[\psi^A] - \frac{1}{\sqrt{\xi}} \bigtriangledown \Lambda .
\]

(35)

with the gauge transformation \( \psi^A \to \psi^A - \beta^A \Lambda \), and \( f \) is an arbitrary scalar function.

It is worthwhile to note that (34) is exactly the Faddeev-Popov technique [14] used to quantize a local gauge theory. Consequently, the equation (34) can be expressed as being

\[
Z = N(\beta) \int \mathcal{D}\psi \det \left| \frac{1}{\sqrt{\xi}} \right| \prod_{\beta} \left\{ \int d^4x \left[ \frac{1}{2} \psi^T \eta^0 (i\beta^A \partial_A - \gamma) \psi - \frac{1}{2\xi} (\partial_A \psi^A)^2 \right] \right\} .
\]

(36)

where the index \( A = \tau, 1, 2, 3 \) and \( \beta^A \partial_A = i\beta^A \partial_\tau + \beta^k \partial_k \). We remark that at zero temperature the \( \det(\bigtriangledown) \) is a constant that can be ignored, however, at Finite Temperature it turns out a very important temperature dependent term. Using the projectors \( R^A \) we rewrite the gauge fixing term in matrix form such that the partition function reads as

\[
Z = N(\beta) \det \left| \frac{1}{\sqrt{\xi}} \right| Z' ,
\]

(37)

where

\[
Z' = \int \mathcal{D}\psi \exp \left\{ \int d^4x \left[ \frac{1}{2} \psi^T \eta^0 \left( i\beta^A \partial_A - \gamma + \frac{1}{\xi} \eta^0 (R^A)^T R^B \partial_A \partial_B \right) \psi \right] \right\} ,
\]

(38)

and \( R^A \partial_A = iR^0 \partial_\tau + R^k \partial_k \).

To perform the calculation of the functional integral we use the Fourier series of the DKP field

\[
\psi(\tau, x) = \frac{1}{\beta} \sum_n \int \frac{d^3p}{(2\pi)^3} \tilde{\psi}(\omega_n, p) e^{i(px + \omega_n \tau)} ,
\]

(39)

with \( \omega_n = 2\pi n/\beta \) and the periodicity conditions (33) for the DKP field are imposed.

Then, substituting (39) in (38) we get

\[
Z' = \prod_n \det \left[ i\omega_n \beta^0 + p_k \beta^k + \gamma + \frac{1}{\xi} \eta^0 \left( i\omega_n R^0 + p_k R^k \right)^T \left( i\omega_n R^0 + p_k R^k \right) \right]^{-1/2}
\]

\[
= \prod_n \left[ \frac{\omega_n^2 + \omega_p^2}{\xi} \right]^{-1/2} = \prod_n \left( \omega_n^2 + \omega_p^2 \right)^{-2} \sqrt{\xi} ,
\]

(40)

where \( \omega_p = |p| \). The determinant in (37) is

\[
\det \left| \frac{1}{\sqrt{\xi}} \bigtriangledown \right| = \prod_n \frac{\omega_n^2 + \omega_p^2}{\sqrt{\xi}} .
\]

(41)

Finally, the partition function reads

\[
Z = N(\beta) \prod_n \left( \omega_n^2 + \omega_p^2 \right)^{-1} ,
\]

(42)

as we are interesting in \( \ln Z \), then, from the last expression we obtain

\[
\ln Z = \ln N(\beta) + 2V \sum_n \int \frac{d^3p}{(2\pi)^3} \ln \beta - V \sum_n \int \frac{d^3p}{(2\pi)^3} \ln \beta^2 \left( \omega_n^2 + \omega_p^2 \right) .
\]

(43)
The value for the $N(\beta)$ is selected in a manner that it cancels the divergent term, i.e. we choose

$$\ln N(\beta) = -2V \sum_n \int \frac{d^3p}{(2\pi)^3} \ln \beta ,$$

(44)

next, we perform the sum (see, for example, [4, 15]) and get the following expression for the partition function

$$\ln Z = -2V \int \frac{d^3p}{(2\pi)^3} \left[ \frac{\beta \omega_p}{2} + \ln \left(1 - e^{-\beta \omega_p}\right) \right] ,$$

(45)

which describe a massless bosonic field with two polarization states that is the characteristic behavior of the electromagnetic field in thermodynamical equilibrium.

5 Conclusions

In this work we study the massless DKP theory at Finite Temperature, and it is shown the constraint structure of the model leads to conclude that it is a local $U(1)$ gauge theory in its spin 1 sector. Such analysis allow to construct the correct partition function using the Hamiltonian procedure. Also, we show that it is possible to arrive to the covariant expression which is exactly the covariant quantization of a gauge theory using the Faddeev-Popov approach. Consequently, the partition function of the spin 1 sector gives the partition function of a zero-mass Bose gas with two polarization states, i.e. the electromagnetic field modes in thermodynamical equilibrium.

The perspectives to be followed are to study the Finite Temperature properties of the massless DKP field in curved space-time and, consequently, analyze the curvature effects in the thermodynamics of the electromagnetic field via the DKP formalism. Advances in this directions will be reported elsewhere.

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A Spin 1 representation for the massless DKP algebra

\[ \gamma = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \]

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