TOPOLOGY OPTICS

Nonlinearity-induced photonic topological insulator

Lucas J. Maczewsky, Matthias Heinrich, Mark Kremer, Sergey K. Ivanov, Max Ehrhardt, Franklin Martinez, Yaroslav V. Kartashov, Vladimir V. Konotop, Luis Torner, Dieter Bauer, Alexander Szameit*

A hallmark feature of topological insulators is robust edge transport that is impervious to scattering at defects and lattice disorder. We demonstrate a topological system, using a photonic platform, in which the existence of the topological phase is brought about by optical nonlinearity. The lattice structure remains topologically trivial in the linear regime, but as the optical power is increased above a certain power threshold, the system is driven into the topologically nontrivial regime. This transition is marked by the transient emergence of a protected unidirectional transport channel along the edge of the structure. Our work studies topological properties of matter in the nonlinear regime, providing a possible route for the development of compact devices that harness topological features in an on-demand fashion.

With the discovery of topological insulators (TIs), materials science ushered in a new era of physics (1, 2). Solid-state TIs prohibit electrons from traversing the bulk while simultaneously supporting chiral surface currents that are topologically protected from scattering at defects and disorder (3). Soon after their first realizations in condensed matter systems (4, 5), topological concepts were implemented across other fields of physics, resulting in the experimental demonstrations of topological dynamics in various platforms (6–10).

Ongoing efforts have been directed toward connecting topology and nonlinearity (11), which may enable topological Mott insulators (12), interaction-induced TIs such as the Kondo insulator (13), and non-Abelian TIs (14) and may even drive the formation of topological solitons (15, 16). Only recently, bulk soliton formation within a topological bandgap was demonstrated experimentally (17). However, work on nonlinear TIs and their corresponding robust edge transport has thus far had to cope with various limitations of the respective implementations that only allowed for experiments with purely linear edge state dynamics (17–19).

We theoretically and experimentally demonstrate a nonlinearity-induced photonic TI, showing how nonlinearity can drive an initially topologically trivial system into a transient topological phase, where protected chiral edge states exist (Fig. 1A). In contrast to soliton formation in an arrangement with preexisting topological features (17), it is the action of nonlinearity itself that establishes nontrivial topology. We use lattices of coupled optical waveguides as a platform to explore nonlinear physics (20–22). In particular, we make use of a modified anomalous Floquet TI arrangement (23–25), which can exhibit unidirectional edge transport for certain parameters. Notably, the Chern number $C$ of this structure generally remains zero. Instead, the topological phase of this system is adequately described by a winding number $W$ that counts the number of topologically protected edge states (25).

In the tight-binding regime, the light dynamics in our finite $(2 + 1)$-dimensional system is modeled by the discretized Schrödinger equation (26)

$$\frac{\partial}{\partial z} a_n(z) + \sum_{m} H_{m,n}(z) a_m(z) + \gamma |a_n(z)|^2 a_n(z) = 0$$

(1)

for the field amplitudes $a_n(z)$, where $H_{m,n}(z)$ is the linear tight-binding Hamiltonian, describing the $z$-dependent coupling from site $n$ to a nearest neighbor $m$ (indicated by $(m)$ in the sum) in the two-dimensional lattice. In turn, the quantity $\gamma$ describes the strength of our Kerr-type nonlinearity. Evidently, Eq. 1 is similar to the Gross-Pitaevskii equation, with the temporal evolution being replaced by spatial dynamics in $z$ along the waveguides, and $\gamma > 0$ describing focusing or positive nonlinearity (26). We consider a $(2 + 1)$-dimensional spatially discrete system, which does not exhibit chiral edge states and remains topologically trivial for low-intensity beams. However, at sufficient excitation power, the term $\gamma |a_n(z)|^2 a_n(z)$ becomes important in Eq. 1. As we will show in our experiments, this can drive the system into a topologically nontrivial phase and promote the formation of nonlinear chiral edge states.

The underlying principle can be understood by considering two interacting waveguides that form a directional coupler (Fig. 1B). Whereas such a coupler composed of identical waveguides exhibits the characteristic sinusoidal intensity oscillation and the associated periodic full-power transfer between the waveguides, introducing a detuning between their effective refractive indices forces a certain fraction of light to remain in the initially excited guide at all times. In the linear regime, the maximum reached in the second waveguide therefore remains well below the total input power. However, when launching high-power light into the lower-index guide, a positive nonlinearity ($\gamma > 0$) can yield an intermittent phase-pulsing with the neighboring higher-index guide. Consequently, the fraction of power that can be transferred increases with the injected power (Fig. 1C). Such photonic waveguide structures, fabricated by means of femtosecond laser direct inscription ([27]; see methods [28]), serve as building blocks for our structure and indeed show this behavior when excited with intense laser pulses (Fig. 1C). For low input powers ($P_{in} = 50$ kW peak power), only about 44% of the light is transferred. We were able to increase this transfer efficiency up to 73% in the nonlinear regime at a power of $P_{NL} = 2.9$ MW. The simulated transfer dynamics illustrates how such an increased transfer ratio is achieved despite the focusing nature of the nonlinearity by judiciously tailoring the length of the interaction region (Fig. 1, D and E). This nonlinear switching is the basis for our implementation of a nonlinear photonic TI (see section 1 of [26] for details).

Our topological structure is composed of optical waveguides arranged in a bipartite square lattice (Fig. 2A). The individual channels are selectively brought into evanescent contact with one another so as to form a cyclic discrete coupling pattern. At any distinct propagation step $j$, represented by a dedicated directional coupler, light from each waveguide is partially transferred with ratio $t$ to only one specific nearest neighbor. Additionally, the on-site potential (i.e., the effective refractive index) is modulated for each successive step (Fig. 2B). This Floquet driving protocol establishes periodicity in the transverse $(x,y)$ as well as the longitudinal $(z)$ direction, with (driving) periods $d$ and $Z$, respectively. Notably, the system enters a nontrivial topological phase when the transfer ratio in each step exceeds $t > 50\%$ (23). Above this topological phase transition, the winding number assumes the value of $W = 1$ (25) [see methods (28) for a detailed discussion].

Although the transfer ratio of each driving step determines the topological phase of the...
Fig. 1. Topology through nonlinearity. (A) Nonlinearity-induced photonic TI. Low-power edge excitations (red) experience diffraction, indicating a topologically trivial regime. By contrast, high-power light (yellow) produces a self-guided unidirectional edge state that travels along the perimeter of the structure. (B) Nonlinear directional detuned coupler. An intense laser pulse is launched into the lower–refractive index waveguide (smaller diameter) of a detuned coupler. A focusing Kerr nonlinearity allows the high-intensity part of the pulse to momentarily compensate for the detuning, whereby light is transferred to the higher–refractive index waveguide (larger diameter). By contrast, the lower-intensity parts exhibit linear dynamics and remain in the lower-index waveguide. (C) Measured power-dependent transfer ratio in a detuned coupler. As a guide to the eye, the regions of below- and above-50% transfer are shaded gray and magenta, respectively. Transfer ratios above 50% enable the topologically nontrivial phase of our lattice. Error bars indicate the measurement uncertainty of the individual data points. (D and E) Simulated light propagation in a detuned coupler for (D) low and (E) high intensities. The arrows schematically indicate the injected intensity (left) as well as the relative amounts of light emerging from the two output channels of the coupler (right).
Step 1

Step 2

Step 3

Step 4

Connecting the bands, when nonlinearity compensates the detuning of the on-site potential and drives the system into its topological phase.

Above the nonlinear phase transition is shown in (G) and (I), confirmed by (J) a marked increase of the bulk state occupation. In (A) to (D) and (F) to (I), the white arrows, are selectively allowed to interact, as indicated by the blue lines. The on-site potential of each waveguide (symbolized by its diameter, with larger diameter indicating greater potential) is modulated along the propagation direction. (D and E) Three-dimensional bulk band structure of the driven lattice is periodic in the quasi-momenta \( k_x \) and \( k_y \) and the quasi-energy \( e \), exhibiting a pronounced bandgap. In the presence of an edge, the band structure features (D) a trivial bandgap in the linear regime and (E) a chiral edge state (solid magenta line) connecting the bands, when nonlinearity compensates the detuning of the on-site potential and drives the system into its topological phase.

**Fig. 2. Driving protocol.**

(A) Waveguide-based implementation of the coupling sequence. The detuning is indicated by the respective waveguide diameters (lower and higher refractive index for smaller and larger diameter, respectively). (B) Visualization of the parameters of Eq. 2. The higher-index guides are characterized by a detuning \( \delta \), and selective interaction between adjacent guides is indicated by the coupling coefficient \( c \). (C) Schematic coupling sequence. The two species of the bipartite lattice (gray and black) are selectively allowed to interact, as indicated by the blue lines. The on-site potential of each waveguide (symbolized by its diameter, with larger diameter indicating greater potential) is modulated along the propagation direction. (D and E) Three-dimensional bulk band structure of the driven lattice is periodic in the quasi-momenta \( k_x \) and \( k_y \) and the quasi-energy \( e \), exhibiting a pronounced bandgap. In the presence of an edge, the band structure features (D) a trivial bandgap in the linear regime and (E) a chiral edge state (solid magenta line) connecting the bands, when nonlinearity compensates the detuning of the on-site potential and drives the system into its topological phase.

**Fig. 3. Experimental observation of nonlinearity-induced topological dynamics.** (A to D) Intensity distribution for single-site edge excitation for [(A) and (B)] one and [(C) and (D)] two driving periods. Bulk-diffractive behavior of a low-power edge excitation is shown in (A) and (C). Unidirectional edge transport at high power (3.2 MW), as indicated by magenta arrows, is shown in (B) and (D). (E) Power-dependent edge occupation after two driving periods. (F to J) Intensity distribution for single-site bulk excitation for [(F) and (G)] one driving period and [(H) and (I)] two driving periods. Bulk dynamics around the excitation site for low-power excitation are shown in (F) and (H).

Suppression of bulk diffraction above the nonlinear phase transition is shown in (G) and (I), confirmed by (J) a marked increase of the bulk state occupation. In (A) to (D) and (F) to (I), the white dashed circles indicate the excited waveguides. In (E) and (J), error bars indicate the measurement uncertainty of the individual data points.

Floquet periods fabricated in samples up to 15 cm in length. The required linear detuning sequence was implemented by choosing appropriate writing speeds [see methods (28)]. In each coupling step, the interaction region was tailored such that an identical effective linear coupling was achieved [for details, see section 1 of (28)]. When exciting an edge waveguide with low-power light at \( P_{\text{lin}} = 50 \text{ kW} \) (i.e., \( t < 50\% \)), diffraction into the bulk of the lattice and especially in all surrounding edge sites occurs, indicating the topologically trivial phase (see Fig. 3, A and C). By contrast, as the input peak power is increased up to \( P_{\text{NL}} = 3.5 \text{ MW} \) (i.e., \( t > 50\% \)), bulk diffraction is suppressed and the chiral edge state forms (Fig. 3, B and D). The observed edge state occupation for two driving periods, determined as the ratio \( L_{\text{edge}}/L_{\text{total}} \) of the intensity of the five involved edge lattice sites (indicated by the magenta outline in Fig. 3D) over the total intensity in the lattice, clearly demonstrates the transition into the topological phase at the threshold power of 0.8 MW (Fig. 3E; see also section 4 of (28) and fig. S8). By contrast, a bulk excitation (Fig. 3, D to J) contracts toward its initially excited site at higher launched...
powers (Fig. 3, G and I), resulting in a marked increase of the bulk state occupation, calculated as the ratio $I_{\text{bulk}}/I_{\text{total}}$ of the intensity at the linear bulk state lattice sites outlined by a rounded square in Fig. 3I over the total intensity. Crucially, this contraction does not arise from a nonlinear cancellation of coupling (J7), as would occur in the case of soliton formation, but is instead a signature of bulk states that arise in the nonlinearity-induced topological phase of the lattice and localize light similar to the behavior known from conventional linear TIs.

Similar to their counterparts in conventional linear TIs, our nonlinear edge states are topologically protected against scattering, as evidenced by their scatter-free unidirectional propagation around a corner of the lattice (see Fig. 4). Despite the transient nature of the underlying topological phase, the edge light transport is robust and could persist even in the presence of imposed coupling disorder and artificial defects—as long as the power of the propagating beam is sufficient to induce the required amount of effective coupling. Therefore, nonlinearly induced topologically protected edge state transport can also be terminated at will by introducing losses so as to reduce the intensity below the phase transition threshold. Alternatively, this reverse transition can also be brought about at a predetermined propagation distance by making use of the gradual decrease of peak intensity owing to propagation losses or dispersive pulse broadening. Notably, the ability to terminate topological protection at will, and at a location encoded by the initially injected power or pulse prechirp, addresses one of the main challenges hindering the application of topological systems, namely, how to efficiently extract signals at the destination of their topologically protected journey.

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**ACKNOWLEDGMENTS**

We thank C. Otto for preparing the high-quality fused silica samples used in this work. Funding: A.S. acknowledges funding from the Deutsche Forschungsgemeinschaft (grants SCHE 612/6-1, BL 574/13-1, BL 574/15-1, and BL 574/20-1) and the Alfred Krupp von Bohlen und Halbach Foundation. V.V.K. acknowledges support from the Portuguese Foundation for Science and Technology (FCT) under contract no. UIDB/00638/2020. V.Y.K. and L.T. acknowledge support from the Government of Spain (Severo Ochoa CEX2019-000910-S), Fundación Cellex, Fundación Mir-Pug, and Generalitat de Catalunya (CERCA). Y.V.K. and V.K. acknowledge funding of this study by RFBR and DFG according to research project no. 18-512-12080. Author contributions: L.J.M., M.H., and A.S. developed the driving protocol and its experimental implementation. L.J.M. carried out the experiments. M.E. and F.M. provided technical support and assisted in the sample fabrication. V.V.K., S.K.I., and L.T. formulated the numerical model and conducted the continuous simulations. D.B., M.K., L.J.M., and V.V.K. formulated the linearized Hamiltonian model. A.S. supervised the project. All authors discussed the results and co-wrote the paper. Competing interests: The authors declare no competing interests.

**SUPPLEMENTARY MATERIALS**

science.sciencemag.org/content/370/6517/701/suppl/DC1

Materials and Methods

Supplementary Text

Figs. S1 to S8

References (SI–35)

7 June 2020; accepted 17 September 2020
10.1126/science.abd2033