A No-summoning theorem in Relativistic Quantum Theory

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Alice gives Bob an unknown localized physical state at some point P. At some point Q in the causal future of P, Alice will ask Bob for the state back. Bob knows this, but does not know at which point Q until the request is made. Bob can satisfy Alice’s summons, with arbitrarily short delay, for a quantum state in Galilean space-time or a classical state in Minkowski space-time. However, given an unknown quantum state in Minkowski space-time, he cannot generally fulfil her summons. This no-summoning theorem is a fundamental feature of, and intrinsic to, relativistic quantum theory. It follows from the no-signalling principle and the no-cloning theorem, but not from either alone.

I. INTRODUCTION

Although some of the fundamental properties of quantum theory – for example, the superposition principle – were understood very early, other key insights were made only later. Quantum entanglement was first described by Schrödinger [1] only in the 1930s; it was not till the 1960s that Bell showed that quantum theory violates local causality [2–5]; some other important aspects of the delicate relationship between the general quantum measurement postulates and the no-signalling principle were not completely understood until very recently [6–10]. These and other features of quantum theory have been greatly illuminated in recent decades, with the development of quantum computing, quantum cryptography and quantum information theory, which have inspired a perspective on quantum theory in terms of tasks and resources involving physical information.

Conversely, considering which quantum tasks are possible or impossible has led to significant discoveries in quantum communication (e.g. [11]) and quantum cryptography (e.g. [12–15]). One of the earliest, simplest and most celebrated results in this field was the quantum no-cloning theorem [12, 13], whose proof is mathematically trivial, but which nonetheless encapsulates a fundamental fact about quantum theory that had previously lain unremarked. The no-cloning theorem inspired several other significant results, including independent proofs of the impossibility of determining an unknown quantum state [16] and the impossibility of distinguishing between non-orthogonal states [17], the no-deletion theorem [18], the no-broadcasting theorem for mixed states [19], a general no-cloning theorem incorporating several of these results [20] and a proof that it is impossible to clone with partial ancillary information [21]. A further significant extension was the introduction of the idea of partial fidelity cloning [22], and the discovery of universal algorithms for attaining the best possible state-independent fidelities for $M \rightarrow N$ partial cloning [23–26]. The recent discovery of the principle of information causality [10] has given further interesting insight into quantum theory and its relationship with special relativity.

All of the results mentioned thus far shed interesting light on the relationship between quantum theory and special relativity, and several of them are crucial to our current understanding. However, they all describe features already evident in non-relativistic quantum mechanics. Since, as we currently understand things, relativistic quantum theory is closer to the true description of nature than quantum mechanics, there remains a compelling motivation to understand which properties and principles are intrinsic to relativistic quantum theory. The motivation is even more compelling since we understand relativistic quantum theory so poorly compared to quantum mechanics. We have an informal intuitive understanding of many features of Lorentz invariant quantum field theories with local interactions, but as yet no rigorous definition of any non-trivial relativistic quantum theory. One might hope ultimately to supply such a definition by identifying the principles that such a theory must satisfy.

This paper describes a simple new task – summoning an unknown localized physical state – which, unlike those mentioned above, distinguishes non-relativistic quantum mechanics and relativistic classical physics from relativistic quantum theory. As we show below, an unknown state can be successfully summoned in the first two theories, but not in the last.

Like the no-cloning theorem, to which it is closely related, and several of the other results mentioned above, the no-summoning theorem is relatively easy to prove once stated. It nonetheless encapsulates a significant and previously unremarked feature of relativistic quantum theory and (again like the no-cloning theorem, though more directly) leads to some significant and qualitatively new cryptographic applications.

It is also worth pointing out that, while ultimately mathematically simple, the result is not quite as transparent as naive intuition might suggest. Quantum theory allows the information in an initially localized quantum state to be delocalized, or ambiguously located, in quite counter-intuitive ways. In particular, the possibility of quantum teleportation means that, in a certain sense, an unknown initial state can be reconstructed in different
ways, from different subsystems, at different and spacelike separated locations. One cannot thus argue that a state must follow some definite timelike or null path in spacetime: indeed, it is precisely the failure of this reasoning that led recently to the breaking (27; see also 28), where this attack is generalized using earlier work of Vaidman (29) of purportedly secure protocols (30–32) for the localizability of an unknown state, to set alongside the intriguing negative results illustrated by Refs. 27–29.

II. SUMMONING A STATE: IDEALIZED SCENARIO

We initially suppose that space-time is either Galilean or Minkowski and that nature is described either by classical mechanics or quantum theory: we will distinguish these cases below. We suppose that both parties have arbitrarily efficient technology, limited only by the relevant causal structure and physical theory. In particular, to simplify the discussion initially, we suppose that Alice and Bob can independently and securely access any relevant point in space-time and instantaneously process and exchange information there. We also suppose that their preparations, information processing, communications and measurements are error-free and have unbounded precision and capacity. We will relax these idealized and somewhat unphysical assumptions later.

Consider two agencies, Alice and Bob, who agree in advance on some space-time point \( P \) that they can access independently. Idealizing, we suppose they are independently able to access every point in the causal future of \( P \), and that each is able to keep information everywhere secure from the other party unless and until they choose to disclose it.\(^1\) At the point \( P \), Alice gives Bob a localized physical system \( S \) (which in the ideal case we treat as pointlike, and which we will assume has zero internal Hamiltonian and can, if Bob wishes, easily be kept isolated with no interaction with the environment) whose state is known to her but not known in advance to Bob. For definiteness, let us say that the state is drawn randomly from some probability distribution \( \mu \) known to them both. Until he receives \( S \), Bob has no more information about \( S \) than is implied by \( \mu \). Once he receives \( S \), he has the additional information carried by \( S \) (but nothing further).

It is agreed that at some point \( Q \) in the causal future of \( P \), to be decided by Alice but not known in advance by Bob, Alice will ask Bob to return \( S \) in its original state. Bob is required to do this in a way that identifies the returned system \( S \), by specifying the relevant classical or quantum degrees of freedom.\(^2\)

Again, for definiteness, let us say that Alice chooses \( Q \) randomly from some probability distribution \( \mu' \) known to both parties. At any point \( Q' \) that does not lie in the causal future of the chosen \( Q \), Bob has no more information about the choice of \( Q \) than is implied by \( \mu' \) and the fact that \( Q \) is not in the causal past of \( Q' \). We say that Alice summons the state from Bob at point \( Q \), and that Bob complies with the summons if he returns an identified verifiable copy of the system \( S \) to Alice at \( Q \). Here a verifiable copy is defined operationally, as a state that Alice has zero probability of distinguishing from \( S \) by any physical means.\(^3\)

III. SUMMONING IN CLASSICAL THEORIES AND IN NON-RELATIVISTIC QUANTUM MECHANICS

It is easy to see that, in our idealized scenario, Bob can satisfy Alice’s request in classical mechanics, in either Galilean or Minkowski space-time. Once he receives the state at \( P \), he immediately measures it to infinite precision, and broadcasts the result to all points in the causal future of \( P \). When he receives a summons at \( Q \), he instantaneously receives and processes this broadcast, making a perfect copy of \( S \), which he returns to Alice.

The same is true of quantum mechanics in Galilean space-time. For example, if the point \( P \) has coordinates \((x_P,t_P)\) in some inertial frame, Bob can simply hold the state \( S \) at position \( x_P \) until he receives a summons at \( Q = (x_Q,t_Q) \). He then sends an instantaneous signal from \( Q \) to \((x_P,t_Q)\), on receipt of which he instantaneously sends the state \( S \) to \( Q \) and returns it to Alice.

IV. NO SUMMONING IN RELATIVISTIC QUANTUM THEORY

GLENDEWOUR I can call spirits from the vasty deep.

HOTSPUR Why, so can I, or so can any man: But will they come when you do call for them?\(^4\)

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\(^1\) In a more realistic model, which deviates from our idealized scenario but can still illustrate all the key features of our discussion, Alice and Bob could both be large independent collaborative groups of people, with each group having their own independent secure network of quantum devices distributed throughout a region.

\(^2\) He may not give Alice some larger system – for instance, a collection of all possible states – and then argue that he has satisfied the summons because he has indeed supplied \( S \), albeit in the form of an unspecified subsystem.

\(^3\) We discuss the case of probabilistic discrimination, which is relevant when we consider quantum theory, below.

\(^4\) As Hotspur notes, strictly speaking it would be more precise to
To fix a definite example, take the probability distribution $\mu$ to be the uniform distribution over pure quantum states in some dimension $d \geq 2$, and suppose that at point $P = (x_P, t_P)$ Alice gives Bob an unknown pure state $\rho$ drawn from this distribution. Suppose that the point $Q$ is drawn from a probability distribution $\mu'$ uniform over the set $X$ of points that have time coordinate $t_Q = t_P + t$ in some given inertial frame, for some fixed $t > 0$, and that are lightlike separated from $P$; i.e. $X$ is a spatial sphere with centre $x_P$ and radius $t$, at time $t_Q$. Each of these points is spacelike separated from all of the others, so Bob learns no extra information about the choice of $Q$ at any point in the causal past of any possible $Q$.

Bob may instantaneously carry out any quantum operations on $\rho$, together with an ancilla, at $P$, and then send outputs at light speed to his representatives at any or all of the possible points $Q$.\(^5\) Bob chooses and follows some strategy, with the result that at each possible $Q$ he has some (not necessarily pure) state $\rho_Q$ which he will hand over to Alice in response to the summons, if it arrives at $Q$.\(^6\)

Fix now on two possible summoning points, $Q_0$ and $Q_1$. If the summons arrives at $Q_1$, then Bob successfully complies only if $\rho = \rho_{Q_1}$. However, the no-signalling principle means that the state $\rho_{Q_1}$ must be independent of whether or not Alice chooses to summon at $Q_0$, and vice versa. So Bob can successfully comply with both possible summonses only if $\rho_{Q_0} = \rho_{Q_1} = \rho$. But this would imply that, on the spacelike surface defined by time coordinate $t_Q$, Bob has made two perfect copies of $\rho$, which contradicts the no-cloning theorem. Summoning an unknown state is thus not generally possible in relativistic quantum theory.

### A. The non-ideal case: finite processing and response time

At this point, it is natural to ask whether the no-summoning theorem could be an artefact of some of the idealizations in our model, and so to consider more realistic models. We start by considering ways of relaxing the assumptions that Bob should be able to carry out information processing instantaneously, communicate at light speed, or respond instantaneously to a summons.

One way of doing this is to allow finite time margins of error, in a given fixed inertial frame, for each of these processes. In the concrete example considered above, we can allow Bob some time for information processing after receiving $\rho$ at $P$, and also some margin for slower than light communication, by taking the set of possible $Q$ to be the set of points $X'$ with time coordinate $t_Q = t_P + t$ (as before) and some fixed timelike separation $t'$ from $P$, where $0 < t' < t$, i.e., the set of points in $X'$ form a spatial sphere with centre $x_P$ and radius $r = (t^2 - (t')^2)^{1/2}$, at time $t_Q$. We can then allow Bob a further finite time $\delta$ to process information and respond to the summons after it is received at $Q = (x_Q', t_Q)$, so that Bob is now required to return the state at the spacetime point $Q^\delta = (x_Q', t_Q + \delta)$. We require $\delta \ll r$, so that for most pairs $(Q_0, Q_1)$ of points with $Q_i \in X'$ the causal pasts of $Q_0^\delta$ and $Q_1^\delta$ do not intersect at time $t_Q$.

Now let $(Q_0, Q_1)$ be a pair of summoning points with this last property — for example, a pair of antipodes on the sphere defining $X'$. If Bob receives a summons at $Q_0$, he can successfully comply only if he can produce the state $\rho$ by quantum operations on the distributed state $\phi_0$ available to him in the spatial region lying in the causal past of $Q_0^\delta$ at time $t_Q$.\(^7\) Similarly, if he receives a summons at $Q_1$, he can successfully comply only if he can produce the state $\rho$ by quantum operations on the distributed state $\phi_1$ available to him in the spatial region lying in the causal past of $Q_1^\delta$ at time $t_Q$. But the states $\phi_0$ and $\phi_1$ belong to disjoint factors of the space of states in Bob's control at time $t_Q$, since they are localized in disjoint regions. Hence our previous argument applies. The state $\phi_0$ is independent of whether Alice chooses to summon at $Q_1$, and vice versa, by the no-signalling principle. Hence Bob is in a position to comply with a summons at $Q_0$ or (alternatively) a summons at $Q_1$ only if he can separately create copies of $\rho$ from both $\phi_0$ and $\phi_1$, which again violates the no-cloning theorem.

We have expressed Bob’s allowed response time in a fixed frame here, as it makes his constraints easy to visualize and understand. The frame choice is defined by the set of allowed summoning points. This breaking of Lorentz invariance is necessary in any non-trivial practical example — in practice the set of summoning points will always be compact and so not Lorentz invariant, and the parties’ devices will define preferred sets of coordinates. Abstractly, though, it is interesting to consider Lorentz invariant versions of summoning incorporating time de-
lays. One way of doing this is to define the set of allowed summoning points to be all points \( Q \) in the future light cone of \( P \) such that \( \tau(P, Q) = t' \), the set of allowed response points to be points \( R \) in the future light cone such that \( \tau(P, R) = t' + \delta \), and require that \( R \) is determined, once \( Q \) is fixed, by the constraint that \( P, Q \) and \( R \) are colinear. Here we require \( \delta \ll t' \).

B. Relation to standard mistrustful cryptography

Note that relaxing our idealized assumptions allows us to fit the task of summoning into the standard cryptographic model \[34\] for mistrustful parties in Minkowski space-time. The non-idealized case allows us to drop the assumption that Alice and Bob each have independent secure access to every space-time point. Instead, we can suppose that Alice and Bob control suitably configured disjoint regions of space-time, their “laboratories”. Each trusts the security of their laboratory and all devices contained within it, but need not trust anything outside their laboratory.

We can take Bob’s laboratory to be a connected region of space-time that includes, near its boundary, \( P \) and all allowed summoning points \( Q_i \), and includes line segments joining \( P \) to each \( Q_i \); this allows Bob to receive a state at \( P \) and transmit it securely to any \( Q_i \). We can take Alice’s laboratory to be a disjoint connected region of space-time that includes a point \( P' \) in the near causal past of \( P \), from which she sends the unknown state to \( P' \), points \( Q'_i \) in the near casual past of each possible summoning point \( Q_i \), from which she sends a summoning request to the relevant \( Q_i \), and points \( Q''_i \) in the near causal future of each summoning point \( Q_i \), to which Bob is supposed to send the summoned state if the summons arrives at \( Q_i \). This allows Alice to generate the unknown state securely, transmit it to \( P \), generate a summoning request securely (so that Bob cannot predict it in advance), transmit it to \( Q_i \), receive the summoned state at \( Q''_i \), and test it securely.

As is standard in mistrustful cryptographic scenarios, we assume that Alice and Bob are the only relevant parties – no one else is trying to interfere with their communications – and that they have classical and quantum channels (which in principle can be made arbitrarily close to error-free) allowing them to send classical and quantum signals between the relevant points.

C. No approximately successful summoning

Another idealization in the model above is that we take the state \( \rho \) to be localized at a point. We could, instead, consider the state to be localized in a finite region – i.e. for its wave function to be zero outside the region. This case can easily be handled as above, with a few more epsilonics. However, this is still an idealization: physical states are only approximately localized. So, if Bob and Alice treat the system as perfectly localized in a finite region, they will introduce some probability of error in the verification step. A realistic model would also allow for errors in Alice’s preparation and measurement. Independently of these points, even in an ideal model, it is also interesting to ask whether Bob can generally respond to summonses in such a way that he will pass Alice’s verification test with a probability \( p \) that can be made arbitrarily close to 1.

We are thus motivated to consider the possibility of approximately successful summoning. We say that Bob can guarantee \( p \)-compliance with the summons if he has a strategy which, for each possible summoning point \( Q \), allows him to generate a state \( \rho_Q \) that he can return to Alice at (or, in the non-ideal case, appropriately near) \( Q \), such that \( \text{Tr}(\rho_Q \rho) \geq p \) for all \( Q \). We can show that this is also not generally possible, for \( p \) arbitrarily close to 1, by generalizing the arguments above, using the bounds \[23\] \[24\] \[35\] \[39\] on the fidelity of \( 1 \rightarrow 2 \) quantum cloning for qudits.\(^8\)

V. DISCUSSION

Summoning is most simply and naturally defined in the idealized case, but also has simple natural extensions to realistic models of information processing and to the probabilistic case. It may be the first significant example of a simple information theoretic task that essentially distinguishes relativistic quantum theory from non-relativistic quantum theory and from relativistic classical physics. Its impossibility has significant implications for quantum computing and quantum cryptography, which deserve further exploration.

We have framed the task in terms of two parties, Alice, who creates and knows the physical state, and Bob, who knows nothing about the state until he receives it, and then only the information conveyed by the state itself. One could, alternatively, suppose that the state is not known to either Alice or Bob, but obtained by Alice from a third party or from nature. In this case, Alice cannot verify whether or not Bob (approximately) complies with the summons, and so we cannot define the task operationally. However, we can still define success depending on whether the fidelity of the returned state to the original is (close to) one. In another version of this picture, one could instead think of nature as one of the parties: for example, Bob could find a state in nature, and be required to produce it at a summoning point that depends on some natural event which he cannot predict in advance.

We have given examples to show that summoning is not generally possible in relativistic quantum theory. In essence, this follows from the fact that summoning is

\[^8\] See Ref. \[44\] for a quantitative discussion.
not possible even in the simple case in which Bob knows that Alice will summon at one of two spacelike separated points, or, in the non-ideal case, within one of two spacelike separated regions. Of course, one can construct examples in which summoning is possible: for example, if all the possible summoning points lie on a timelike curve in the causal future of \( P \). It is not hard (if perhaps not always very illuminating) to list conditions that ensure that summoning is impossible in a particular example, for either the ideal or non-ideal cases.

Summoning is closely related to cloning, and we can extend the parallel to define other related impossible tasks in relativistic quantum theory, such as summoning an unknown member of a set of non-orthogonal states, summoning an unknown state given partial ancillary information, or summoning \( N > M \) returned copies of \( M \) states (where the states may have been supplied at different spacetime points, and the summonses may arrive at different spacetime points). One can also consider the probabilistic versions of these tasks. The arguments above generalize straight away to these cases, using the exist-

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