Benchtop Magnetic Shielding for Benchmarking Atomic Magnetometers

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Abstract—Here, a benchtop hybrid magnetic shield containing four mumetal cylinders and nine internal flexible printed circuit boards (flex-PCBs) is designed, constructed, tested, and operated. The shield is designed specifically as a test-bed for building and operating ultrasensitive quantum magnetometers. The geometry and spacing of the mumetal cylinders are optimized to maximize shielding efficiency while maintaining Johnson noise. Experimental measurements at the shield’s center show passive shielding efficiency of $(1.0 \pm 0.1) \times 10^{-6}$ for a 0.2-Hz oscillating field applied along the shield’s axis. The nine flex-PCBs generate three uniform fields, which all deviate from perfect uniformity by $\leq 5\%$ along 50\% of the inner shield axis, and five linear field gradients and one second-order gradient, which all deviate by $\leq 4\%$ from perfect linearity and curvature, respectively, over measured target regions. Together, the target field amplitudes are adjusted to minimize the remnant static field along 40\% of the inner shield axis, as mapped using an atomic magnetometer. In this region, the active null reduces the norm of the magnitudes of the three uniform fields and six gradients by factors of 19.5 and 19.8, respectively, thereby reducing the total static field from 1.68 to 0.23 nT.

Index Terms—Analytical models, coils, demagnetization, electromagnetic measurements, flexible printed circuits, Fourier transforms, magnetic shielding, magnetometers.

I. INTRODUCTION

A

N EXCEPTIONALLY low and controlled magnetic field is required to reduce noise in fundamental physics experiments [1], [2] and to benchmark ultrasensitive quantum magnetometers, including those based on NV-centers [3] and atomic vapors [4], [5], [6]. In particular, zero-field optically pumped magnetometers (OPMs) [7], [8] have diverse applications from functional neuroimaging [9], [10] to rapid diagnostics of electric batteries [11], [12], but require low static (no time variation) fields to reduce projection errors [13] and nonlinearities in sensor gain.

External magnetic fields may be attenuated by enclosing a region with passive shielding material. For low-frequency shielding, high permeability materials, such as mumetal, are used to divert magnetic flux. However, high permeability materials magnetize under applied fields, thereby limiting the shielding effect. Although this is mitigated by degaussing [14], [15], [16], some remnant magnetization usually remains. Coil systems inside passive shields are used to null offsets induced by magnetization and cancel leakage fields. These coils may be designed to account for the electromagnetic distortion induced by their coupling to passive shielding [17], [18], [19], [20], [21].

This article presents the design, construction, testing, and operation of a magnetic shield comprising nested mumetal cylinders with end caps and internal active coils. The nested cylinders are of a high permeability and their geometries and spacings are optimized to maximize their shielding effectiveness and minimize the weight of the shield, while ensuring that there is a large internal usable volume for experimentation. Entry hole positions are selected to maximize access to this usable volume without significantly diminishing the effectiveness of the passive shielding. The passive shielding performance is experimentally validated before nine active coils are constructed, housed on nested flexible printed circuit boards (flex-PCBs), within the inner mumetal shield. These coils are designed to null static offsets due to magnetization and residual external magnetic fields which pass through the passive shielding. The coupling of the active and passive components is included a priori in the design process to enhance the nulling process. The flex-PCBs are characterized in situ to validate the design procedure. Finally, the coil currents applied to the flex-PCBs are tuned to null the residual field along the inner shield’s axis as measured using a zero-field OPM.

II. PASSIVE SHIELDING

The passive shielding is constructed from four benchtop-sized nested co-axial and co-centered mumetal

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cylinders with access holes to allow optical access and cabling (Fig. 1). Following [22] and [23], the geometries of the exterior mumetal cylinders are optimized using the NGSA-II genetic algorithm [24]. This is detailed in Appendix. The optimization returns the radii and lengths of the shield of the exterior mumetal cylinders are optimized using the NGS A-II genetic algorithm [24]. This is detailed in Appendix. As the applied frequency increases, the eddy currents induced in the shield increase and tend to enhance SE A T, except for when SE T reduces between 10 and 50 Hz. The observed behavior could potentially be explained by resonant phenomena resulting from the complex permeability of the shield, which arises from the oscillating magnetization of the ferromagnetic domains and the complex susceptibility induced by eddy currents [28]. It should be noted that the strength of

Fig. 1. Benchtop shield consists of four nested mumetal cylinders of outer radius \( \rho_1 = 150 \) mm and length \( L_1 = 480 \) mm [red] and inner radius \( \rho_1 = 100 \) mm and length \( L_1 = 300 \) mm [blue], which enclose a co-axial and co-centered set of rolled flex-PCBs of exterior radius \( \rho_2 = 95 \) mm and length \( L_2 = 270 \) mm [yellow]. (a) Side view of the shield and end caps, (b) rolled PCB, and (c) multiple PCBs housed inside the shield with the end cap removed.
TABLE I
AXIAL AND TRANSVERSE SHIELDING EFFICIENCY, $SE_A$ AND $SE_T$, RESPECTIVELY, AT THE CENTER OF THE BENCHTOP SHIELD SUBJECT TO A SPATIALLY UNIFORM OSCILLATORY FIELD OF FREQUENCY, $f$, AND PEAK-TO-PEAK AMPLITUDE $B_0 = 505 \mu$T

| $f$ (Hz) | $SE_A \times 10^6$ | $SE_T \times 10^6$ |
|---------|-------------------|-------------------|
| 0.2     | 1.0 ± 0.1         | 20.2 ± 0.1        |
| 1       | 1.1 ± 0.1         | 23.5 ± 0.1        |
| 10      | 4.1 ± 0.1         | 39.2 ± 0.1        |
| 50      | 4.5 ± 0.1         | 19.1 ± 0.1        |

Fig. 2. (a) Axial and (b) transverse shielding efficiency, $SE_A$ and $SE_T$, respectively, measured along the $z$-axis of the inner shield cylinder using a fluxgate magnetometer [red and blue] and an OPM [black] under $f = 1$ Hz sinusoidal drive field of peak-to-peak amplitude $B_0 = 505 \mu$T inside an MSR. Dashed gray line shows the edge of the target field region and dashed black lines [b] only show the noise limit of the fluxgate magnetometer, $10 \ pT_{rms}/\sqrt{Hz}$.

Fig. 3. Residual norm($B_y, B_z$) and $B_z$ measured along the $z$-axis of the inner shield cylinder using a fluxgate magnetometer [red and blue] inside an MSR. Dashed gray line shows the edge of the target field region.

the eddy currents is greater in the case of transverse shielding, given the larger shield surface area perpendicular to the applied field. However, to establish a definitive understanding of this relationship, further investigation is required.

In addition, in Fig. 2, we show $SE_A, T$ measured in 5-mm increments along the shield’s axis measured for 1-Hz applied fields, sampled over 10 s at $f_s = 10$ kHz at each point using a Mag-13MCZ100 fluxgate magnetometer with a 24-bit Spectramag-6 DAQ (Bartington Instruments, U.K.). These measurements show good agreement with the OPM data, although the fluxgate noise floor limits the measurement of $SE_T$.

A representative residual magnetization profile along the shield’s axis inside the MSR is measured using the fluxgate magnetometer and is shown in Fig. 3. The mean absolute field norm between $z = [-L_1/2, L_1/3]$ is $(1.2 \pm 0.1) \ nT$. The static offset of the fluxgate is accounted for in these measurements by comparing the magnetic field measurements at the center of the shield with the fluxgate reading at three inverted positions, each repeated three times. As the shield has a high $SE_A, T$, the residual magnetization will dominate the field profile within the shield when compared with the transmitted field in standard conditions. However, this profile will vary between different recordings since the shield’s magnetization is determined by several factors including the background that the shield has experienced, movement of the shield relative to the background, and physical impacts.

III. FLEX-PCB COILS

Next, we consider how to null static offsets using active field coils. First, we examine a streamfunction contained on the surface of a coil cylinder of radius $\rho_c$ and length $L_c$, defined by

$$\psi(\phi', z') = -\sum_{n=1}^{N'} L_c W_{n0} \cos \left( \frac{n\pi(z' - L_c/2)}{L_c} \right)$$

$$+ \sum_{n=1}^{N'} \sum_{m=1}^{M'} \frac{L_c}{n\pi} \left( W_{nm} \cos(m\phi') + Q_{nm} \sin(m\phi') \right)$$

$$\times \sin \left( \frac{n\pi(z' - L_c/2)}{L_c} \right)$$

(1)

The streamfunction contains modes that are weighted by Fourier coefficients ($W_{n0}, W_{nm}, Q_{nm}$), of order $n \in 1, \ldots, N'$ and degree $m \in 1, \ldots, M'$. As the current is confined to the surface of the cylinder, $\nabla \cdot J(z', \phi') = 0$, we can express the azimuthal and axial components of the current density in terms of this basis. $J_{\phi}(z', \phi') = \partial \psi(\phi', z')/\partial z'$ and $J_{z}(z', \phi') = -(1/\rho_c) \partial \psi(\phi', z')/\partial \phi'$, respectively [29]. The coil patterns are generated using least-squares optimization [30], [31], [32] to find optimal values of the Fourier coefficients to generate each target field. The relationship between the magnetic field and the Fourier coefficients is encoded in [17, eqs. (37)–(39)].

Here, this method is applied to design nine flex-PCBs to generate nine low-order magnetic field harmonics within the central half length and diameter of the inner shield cylinder. We choose to generate the full set of uniform fields and linear field gradients (see Table II) and a single quadratic field gradient with respect to the axial position, $d^2 B_z/dz^2$, to help offset the difference between $SE_A$ and $SE_T$. The PCBS are co-centered and co-axial to the shield cylinder and extend over an outer radius $\rho_c = 95 \ mm$ and length $L_c = 270 \ mm$. The streamfunctions and wire patterns which generate the $dB_y/dx$ and $B_z$ fields are presented in Fig. 4. The uniform
Table II

Benchtop Shield Contains Nine Nested Flex-PCBs Which Generate Three Order $N = 1$ Uniform Harmonics, Five $N = 2$ Linear Harmonics, and One $N = 3$ Quadratic Harmonic, With Specific Variations Along the Cartesian Unit Vectors, $(\hat{x}, \hat{y}, \hat{z})$. The Mean Field Strength, $B_0$, per Unit Current, $I$, Is Calculated by Averaging the Measured Field Along $z$ Between $z = [0, L_z/4]$, Except for the $dB_x/dx$ and $dB_y/dx$ Fields Which Are Averaged Along $x$ Between $x = [0, \rho_x/2]$. Over the Same Spatial Regions, We Also Evaluate the Maximum Deviation From the Target Field, $\Delta(B)_{\rho_x(\rho_x^N)} = B(x, y, \rho_x^N) - B_0$, as a Percentage of $B_0$. |

| $N$ | Target field | Target field harmonic | $\text{Coil efficiency, } B_0/I (\mu T/(A m^{(N-1)}))$ | $\max ([\Delta(B)/r(\rho_x^N)])$ |
|-----|--------------|-----------------------|---------------------------------|----------------------------------|
| 1   | $B_0$        | $B_0 \hat{k}$         | 68.9 ± 0.1                      | 0.21 ± 0.01                      |
|     | $B_0 \hat{y}$ | $B_0 \hat{y}$         | 68.7 ± 0.1                      | 0.22 ± 0.01                      |
|     | $B_0 \hat{x}$ | $B_0 \hat{x}$         | 74.2 ± 0.1                      | 0.44 ± 0.01                      |
| 2   | $dB_x/dz$    | $B_0 (z\hat{x} - y\hat{y})$ | 970 ± 10                        | 4 ± 2                            |
|     | $dB_y/dz$    | $B_0 (y\hat{x} + x\hat{y})$ | 940 ± 10                        | 4 ± 2                            |
|     | $dB_z/dx$    | $B_0 (x\hat{z} + 3x\hat{z})$ | 430 ± 10                        | 2 ± 1                            |
|     | $dB_x/dy$    | $B_0 (z\hat{y} + y\hat{z})$ | 440 ± 10                        | 2 ± 1                            |
|     | $dB_x/dz$    | $B_0 (-z\hat{k} - y\hat{y})$ | 1100 ± 10                       | 1 ± 1                            |
| 3   | $d^2B_z/dx^2$ | $B_0(-3z\hat{x} - 3y\hat{y} + 2x\hat{z})$ | 8800 ± 100                      | 4 ± 2                            |

Fig. 4. Uniform (a) $dB_x/dx$ coil design of radius $\rho_x = 94.8$ mm and (b) $B_z$ coil design of radius $\rho_x = 92.9$ mm, which extend azimuthally over $\phi' = [0, 2\pi]$ and axially over $z' = [-L_z/2, L_z/2]$ where $L_z = 270$ mm. Black solid and dashed linestyles show opposite current flow directions; green to white to pink color shows the value of the current flow streamfunction, $\psi(\phi', z')$, from positive to zero to negative [scale right]; and black circles show access holes.

d$B_x/dx$ coil is rolled around its azimuth to form a cylinder of radius $\rho_x = 94.8$ mm [Fig. 1(b)], whereas the uniform $B_z$ coil is rolled into a cylinder of radius $\rho_z = 92.9$ mm. The coil patterns have different widths to allow them to be nested inside each other once rolled. The coil patterns are generated by contouring the streamfunction (1), at evenly spaced levels which span its full domain [33], [34], [35]. These patterns are selected according to which best emulates the continuum current [17] but is manufacturable, i.e., the individual wires are greater than 0.8 mm apart and do not intersect with the access holes. The PCBs are made of polyimide of 0.26-mm depth into which copper tracks comprising the wire patterns are printed and are connected together in series across two flex-PCB layers with vias. The unwanted magnetic fields generated by the connecting tracks are reduced by including tracks on the second PCB layer with opposite current flow. The current pattern which generates the uniform $B_z$ field is composed of current loops in series, which are constructed by soldering bridges across the PCB once it is rolled. The $dB_x/dx$ current pattern does not require solder bridges as it does not cross the edge of the PCB. The uniform $B_z$ PCB has a track width of 1.4 mm to allow 2 A of current to be passed to produce strong axial biasing ($\sim150$ $\mu$T, without heating the shield above 40 $^\circ$C from 20 $^\circ$C), whereas the remaining PCBs have track widths of 0.4 mm to allow 500 mA of current. The flex-PCBs are nested inside a nylon tube and have a radial thickness of 2.5 mm in situ, including solder bridges.

The magnetic fields generated by each flex-PCB are measured by driving sinusoidal current through each PCB sequentially at a frequency of 1 Hz for 10 s and taking the FFT of the measured field. We present the profiles generated by the $B_y$, $B_z$, $dB_z/dx$, $dB_y/dz$, $dB_z/dz$, and $d^2B_z/dx^2$ PCBs in Fig. 5, evaluated along the shield’s axis, except for the $dB_y/dx$ PCB which is evaluated radially as it is designed to generate zero field along the shield’s axis. The generated fields show close agreement to the target fields within the target region and rapidly deviate outside of it, thus minimizing the power consumption required to generate the desired field profile. We examine the deviation from perfect uniformity of the target fields generated by the uniform $B_y$ and $B_z$ PCBs in Fig. 6. The $B_y$ profile deviates more than the $B_z$ profile because of small error fields generated by the connections across the $B_z$ PCB. Notwithstanding this, the fields generated by the uniform field-generating PCBs deviate from the target only by $\leq0.5$% within the target region and compare favorably to other systems optimized in similar contexts [36]. The remaining PCBs are measured to generate fields which deviate from the target by $\leq4$%; we note that intrinsic deviations are likely
to be even smaller as gradient field measurements are highly alignment-sensitive.

IV. ACTIVE NULLING

We use a QZFM OPM to map the remnant field after degaussing at 5-mm increments along the shield’s axis by calculating $(B_x, B_y, B_z)$ values required to null the field using the onboard OPM coils using custom MATLAB code which interfaces with the OPM via NI LabVIEW. The same method as described in Section III for the fluxgate is then used to calculate the static offsets. Coil currents are calculated to null the remnant field at $N_{null} = 25$ points between $z = [-64, 56]$ mm, which extends over $40\%$ of the inner shield length, by following the methodology outlined in [37]. Each Cartesian component of the offset-corrected magnetic field is compiled into a list of measurements, $B_{\text{mes}}$, of length $(3N_{null})$. The coil currents required to null these fields may be related to the desired currents in each coil, $I_j$, using simple matrix algebra

$$B_{\text{ideal}}^j I_j = -B_{\text{mes}}^j. \quad (2)$$

where the matrix $B_{\text{ideal}}$ contains the expected magnetic field harmonics generated at each sampled coordinate by each flex-PCB used for nulling, assuming unitary current. Here, seven coils are used for nulling since the $d B_x/dx$ and $d B_y/dx$ coils generate zero field along the $z$-axis, and so this matrix is of dimension $(3N_{null} \times 7)$. The coil currents are obtained using (2) by calculating the pseudoinverse of $B_{\text{ideal}}$. The resulting coil currents, which are displayed in Table III, are at the tens of microampere level due to the low residual field within the shield. An NI-9264 voltage output module is used to generate these currents, which are then amplified using an eight-channel $\pm10$-V amplifier constructed in-house. The amplifier is experimentally tested to have a noise level $<25 \text{nV/}\sqrt{\text{Hz}}$ at $5 \text{~Hz}$. To further ensure that the coil drivers do not add significant noise, each flex-PCB is driven in series with a $47\text{-k}\Omega$ resistor.

The magnetic field along the $z$-axis pre- and post-null and averaged over two runs is displayed in Fig. 7. The active null reduces the mean magnetic field from 1.68 to 0.23 nT. Refitting the field to the harmonic model, we calculate that the norms of the magnitudes of the three target uniform fields using (2) by calculating the pseudoinverse of $B_{\text{ideal}}$. The resulting field along the $z$-axis and $y$-axis after nulling, respectively (see Table III). The remaining field is dominated by contributions from higher order field harmonics at the edge of the null region. These harmonics could be alleviated by adding further target field coils to the system to null higher order gradients or using retrofitted additional coils, e.g., individually driven simple building block coils [19], for adaptive nulling of residual variations.
The magnetic noise directed along each Cartesian coordinate axis is also calculated at the shield’s center by measuring the OPM output over 300 s while sampling at $f_s = 1.2$ kHz. This is displayed in Fig. 8. Generally, the noise is limited by the OPM noise floor, 15 fT/√Hz, and so the shield-induced Johnson noise is less than this value. This is consistent with the theoretically expected Johnson noise calculated from [26, eq. (16)] of $\sim 10$ fT/√Hz. The noise floor peaks at $\sim 750$ fT/√Hz at $f = 50$ Hz due to mains electrical noise, with projections at $f = 28, 72$ Hz due to the OPM powerline.

V. CONCLUSION

In this article, we have developed a magnetic field control system using optimized arrangements of mumetal magnetic shielding in conjunction with a set of nine internal flex-PCBs. The experimental measurements show that the flex-PCBs generate target magnetic fields with $< 4\%$ error along the measured target regions which extend over $50\%$ of either the inner shield diameter or length. The flex-PCBs are demonstrated to enhance the effectiveness of the passive shields. They reduce the mean static magnetic field norm to 0.23 nT over 40% of the inner shield’s axis in a typical laboratory environment, which is a 7.3-fold improvement compared to the performance of the passive shields alone. Our results also demonstrate that the active coil nulling may be used without introducing significant magnetic noise. The magnitude of magnetic noise measured by a commercially available zero-field OPM was evaluated to be broadly at the sensor’s 15 fT/√Hz noise limit over much of the spectrum from 0 to 100 Hz.

The design, manufacturing, and experimental processes laid out in this work will be instructive to the development of magnetic shields in several application spaces outside of benchtop shielding. Larger shields designed using the same methods could be used similar to lightweight MSRs [38] for recording muscle [39] or gut activity [40]. Since these shields would require larger access holes, additional field-generating systems would be required to reduce leakage fields. Such coil systems may also supplement the existing shielding, enabling its partial removal, e.g., for weight reduction in spacecraft [41].

APPENDIX

PASSIVE SHIELDS OPTIMIZATION

Sumner [25] generated simple approximations for the static shielding effectiveness, $SE$, of $N_s$ nested layers of cylindrical passive magnetic shielding of a fixed high relative magnetic permeability, $\mu_r$, and length greater than radius, $L_i > \rho_i$, in the flux-shunting dominated limit [42]. These are

$$SE = 1 + \sum_{i=1}^{N_s} SE_i + \sum_{i=1}^{N_s-1} \sum_{j=i+1}^{N_s} SE_i SE_j F_{ij}$$

$$+ \sum_{i=1}^{N_s-2} \sum_{j=i+1}^{N_s-1} \sum_{k=j+1}^{N_s} SE_i SE_j SE_k F_{ijk} + \cdots$$

$$+ SE_{N_s} \prod_{i=1}^{N_s} SE_i F_{i(i+1)}$$

where $SE_i$ and $F_{ij}$ are replaced by different functions depending on whether the shielding efficiency axial, $SE_A$, or transverse, $SE_T$, to the shield’s axis is to be calculated. In the transverse case, they are replaced by

$$SE_{Ti} = \frac{\mu_r d_i}{2 \rho_i}$$

$$T_{ij} = 1 - \left(\frac{\rho_i}{\rho_j}\right)^2$$

whereas, in the axial case, they are replaced by

$$SE_{Ai} = 1 + \frac{\mu_r d_i}{2\rho_i} \left(\frac{K_i}{1 + \rho_i + 0.85\rho_i^2/3}\right)$$

$$A_{ij} = 1 - \left(\frac{L_j}{L_i}\right)$$

where

$$K_i = \left(1.7 - \frac{1}{\rho_i} - 1.35\left(1 + \frac{1}{4\rho_i}\right)\right)$$

$$\times \left(\ln \left(\rho_i + \sqrt{1 + \rho_i^2}\right) - 2 \left(\frac{1}{\rho_i} - 1\right)\right).$$

Here, we will use (4) and (6) with $\mu_r = 40000$ as objective functions for a genetic algorithm. An additional objective function is the total volume of shielding material in the cylindrical shield wall and circular end caps of the shielding layers

$$V = \sum_{i=1}^{N_s} 2\pi d_i \rho_i^2 + \pi (2d_i \rho_i - d_i^2)(L_i - 2d_i).$$

In the optimization system, as detailed in the main text, we fix the geometry of the inner shield cylinder and the thicknesses of all the shielding layers. The fixed values imposed in the optimization are therefore

\[\begin{aligned}
\rho_1 &= 100\text{ mm} \\
L_1 &= 300\text{ mm} \\
d_1 &= 0.5\text{ mm} \\
d_i &= 1.5\text{ mm}, \text{ for } i \in [2, 3, 4].
\end{aligned}\]

To ensure that the shielding layers have sufficient clearance for manufacture and so the shield fits on a laboratory benchtop,
the boundaries of the search domain are constrained to
\[
\begin{aligned}
\rho_{i+1} - \rho_i &> 5 \text{ mm}, \quad \text{for } i \in [1, 2, 3] \\
L_{i+1} - L_i &> 5 \text{ mm}, \quad \text{for } i \in [1, 2, 3] \\
\rho_4 &\leq 150 \text{ mm} \\
L_4 &\leq 600 \text{ mm}.
\end{aligned}
\]

The objectives of the optimization procedure are therefore
\[
\begin{aligned}
\text{obj.} = &\begin{cases} 
\max SE_T(\rho_2, \rho_3, \rho_4, L_2, L_3, L_4) \\
\max SE_A(\rho_2, \rho_3, \rho_4, L_2, L_3, L_4) \\
\min V(\rho_2, \rho_3, \rho_4, L_2, L_3, L_4)
\end{cases},
\end{aligned}
\]

The NSGA-II genetic algorithm is used to find solutions for the objective functions laid out in (12), subject to the search domain constraints outlined in (11). The NSGA-II control parameters used match those in [19], except that the population size is set to 250. Convergence is achieved after 112 generations of the algorithm, requiring 28,000 evaluations of each objective function. This takes 42.6 s on a MacBook Pro16,1 which is equipped with a 6-Core Intel Core i7 2.6-GHz processor and 16 GB of DDR4 RAM. The output Pareto front is presented in Fig. 9. The solution with the highest axial shielding efficiency is selected, where the axial and transverse shielding efficiencies are \(SE_A = 1.1 \times 10^6\) and \(SE_T = 19 \times 10^7\), respectively, for fixed \(\mu_r = 40,000\). These values deviate from the lowest frequency (0.2 Hz) experimentally measured values in Table 1 by 10\%, although they are highly sensitive to the value of initial relative permeability selected in the optimization. Thus, discrepancies are anticipated as the analytic formulae are approximations in the static limit and assume fixed permeability of each layer. Conversely, in the experimental measurements, the inner layers of mumetal will experience a reduced field magnitude, changing their effective permeability [43].

**DECLARATIONS**

Peter J. Hobson, Michael Packer, Dominic Sims, Matthew J. Brookes, Richard Bowtell, and Mark Fromhold have a worldwide patent (WO/2021/053356) which includes the coil design technique applied in this work. Ben Styles, Prashant Patel, James Chalmers, and David Woolger are employees of Magnetic Shields Ltd., U.K., who sell benchtop shields commercially. Sintija Raudonyte, Darragh Holmes, and Robert Harrison are ex-employees of MSL. Niall Holmes, David Woolger, Matthew J. Brookes, and Richard Bowtell hold founding equity in Cerca Magnetics Ltd., U.K., who commercialize OPM technology. Chris Morley, Alister Davis, and Thomas X. Smith declare no competing interests.

Magnetic Shields Limited have made the benchtop shield available for purchase (see [44]).

This work has been submitted to the IEEE for possible publication. Copyright may be transferred without notice, after which this version may no longer be accessible. All supporting data may be made available on request.

**REFERENCES**

[1] J. Xu, Q. You, Z. Li, B. Han, Z. Zhang, and Q. He, “The elimination of the effect of the external field in the Joule balance,” IEEE Trans. Instrum. Meas., vol. 69, no. 4, pp. 1745–1752, Apr. 2020.

[2] S. Afach et al., “Dynamic stabilization of the magnetic field surrounding the neutron electric dipole moment spectrometer at the Paul Scherrer institute,” J. Appl. Phys., vol. 116, no. 8, Aug. 2014, Art. no. 084510.

[3] F. M. Stü rner et al., “Integrated and portable magnetometer based on nitrogen-vacancy ensembles in diamond,” Adv. Quantum Technol., vol. 4, no. 4, Apr. 2021, Art. no. 2000111.

[4] F. Bertrand et al., “A 4He vector zero-field optically pumped magnetometer operated in the earth-field,” Rev. Sci. Instrum., vol. 92, no. 10, Oct. 2021, Art. no. 105005.

[5] S. P. Krzyzewski, A. R. Perry, V. Gerginov, and S. Knappe, “Characterization of noise sources in a microfabricated single-beam zero-field optically-pumped magnetometer,” J. Appl. Phys., vol. 126, no. 4, Jul. 2019, Art. no. 044504.

[6] P. Bevington, R. Gartman, W. Chalupczak, C. Deans, L. Marmugi, and F. Renzoni, “Non-destructive structural imaging of steelwork with atomic magnetometers,” Appl. Phys. Lett., vol. 113, no. 6, Aug. 2018, Art. no. 063503.

[7] I. Kominis, T. Kornack, J. Allred, and M. Romalis, “A subfemtotesla multichannel atomic magnetometer,” Nature, vol. 422, pp. 596–599, May 2003.

[8] V. K. Shah and R. T. Wakai, “A compact, high performance atomic magnetometer for biomedical applications,” Phys. Med. Biol., vol. 58, no. 22, pp. 8153–8161, Nov. 2013.

[9] R. M. Hill et al., “Using OPM-MEG in contrasting magnetic environments,” Neuroimage, vol. 253, Jun. 2022, Art. no. 119084.

[10] E. Boto et al., “Triaxial detection of the neuromagnetic field using optically-pumped magnetometry: Feasibility and application in children,” Neuroimage, vol. 252, May 2022, Art. no. 119027.

[11] Y. Hu et al., “Rapid online solid-state battery diagnostics with optically pumped magnetometers,” Appl. Sci., vol. 10, no. 21, p. 7864, Nov. 2020.

[12] Y. Hu et al., “Sensitive magnetometry reveals inhomogeneities in charge storage and weak transient internal currents in Li-ion cells,” Proc. Nat. Acad. Sci. USA, vol. 117, no. 20, pp. 10667–10672, May 2020.

[13] A. Borna et al., “Cross-axis projection error in optically pumped magnetometers and its implication for magnetoencephalography systems,” Neuroimage, vol. 247, Feb. 2022, Art. no. 118818.

[14] Y. Fu, Z. Wang, L. Xing, W. Fan, J. Ruan, and H. Pang, “Suppression of nonuniform magnetic fields in magnetic shielding system for SERF co-magnetometer,” IEEE Trans. Instrum. Meas., vol. 71, pp. 1–8, 2022.

[15] I. Altarev et al., “Minimizing magnetic fields for precision experiments,” J. Appl. Phys., vol. 117, no. 23, Jun. 2015, Art. no. 233903.

[16] Z. Sun, M. Reisner, P. Fierlinger, A. Schnabel, S. Stuiber, and L. Li, “Dynamic modeling of the behavior of permalloy for magnetic shielding,” J. Appl. Phys., vol. 116, no. 8, Aug. 2014, Art. no. 084510.

[17] M. Packer et al., “Planar coil optimization in a magnetically shielded cylinder,” Phys. Rev. Appl., vol. 15, no. 6, Jun. 2021, Art. no. 064006.
[19] M. Packer et al., “Magnetic field design in a cylindrical high-permeability shield: The combination of simple building blocks and a genetic algorithm,” J. Appl. Phys., vol. 131, no. 9, Mar. 2022, Art. no. 093902.

[20] F. Zhao, X. Zhou, W. Zhou, X. Zhang, K. Wang, and W. Wang, “Research on the design of axial uniform coils for residual field compensation in magnetically shielded cylinder,” IEEE Trans. Instrum. Meas., vol. 71, pp. 1–9, 2022.

[21] H. Pang et al., “Design of highly uniform field coils based on the magnetic field coupling model and improved PSO algorithm in atomic sensors,” IEEE Trans. Instrum. Meas., vol. 71, pp. 1–11, 2022.

[22] P. J. Hobson et al., “Optimised hybrid shielding and magnetic field control for emerging quantum technologies,” Proc. SPIE, vol. 11881, pp. 121–113–123, Mar. 2021.

[23] J. Li, Z. Wang, and W. Quan, “Multi-objective optimization of multilayer passive magnetic shield based on genetic algorithm,” AIP Adv., vol. 9, no. 12, Dec. 2019, Art. no. 125210.

[24] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, “A fast and elitist multiobjective genetic algorithm: NSGA-II,” IEEE Trans. Evol. Comput., vol. 6, no. 2, pp. 182–197, Apr. 2002.

[25] T. J. Sumner, J. M. Pendlebury, and K. F. Smith, “Convectional magnetic shielding,” J. Phys. D: Appl. Phys., vol. 20, no. 9, pp. 1095–1101, Sep. 1987.

[26] S.-K. Lee and M. V. Romalis, “Calculation of magnetic field noise from high-permeability magnetic shields and conducting objects with simple geometry,” J. Appl. Phys., vol. 103, no. 8, Apr. 2008, Art. no. 084904.

[27] F. Thiel, A. Schnabel, S. Knappe-Grüneberg, D. Stollfuß, and M. Burghoff, “Demagnetization of magnetically shielded rooms,” Rev. Sci. Instrum., vol. 78, no. 3, Mar. 2007, Art. no. 035106.

[28] D.-X. Chen and C. Gu, “AC susceptibilities of conducting cylinders and their applications in electromagnetic measurements,” IEEE Trans. Magn., vol. 41, no. 9, pp. 2436–2446, Sep. 2005.

[29] J. D. Jackson, Classical Electrodynamics, 3rd ed. New York, NY, USA: Wiley, 1999.

[30] P. T. Boggs and J. W. Tolle, “Sequential quadratic programming,” Acta Numer., vol. 4, pp. 1–81, Jan. 1995.

[31] C. A. Floudas and V. Visweswaran, “Quadratic optimization,” in Handbook of Global Optimization. Cham, Switzerland: Springer, 1995, pp. 217–269.

[32] J. W. Carlson, K. A. Derby, K. C. Hawryszko, and M. Weideman, “Design and evaluation of shielded gradient coils,” Magn. Reson. Med., vol. 26, no. 2, pp. 191–206, Aug. 1992.

[33] L. K. Forbes and S. Crozier, “A novel target-field method for finite-length magnetic resonance shim coils: I. Zonal shims,” J. Phys. D: Appl. Phys., vol. 34, no. 24, pp. 3447–3455, Dec. 2001.

[34] L. K. Forbes and S. Crozier, “A novel target-field method for finite-length magnetic resonance shim coils: II. Tesseroidal shims,” J. Phys. D: Appl. Phys., vol. 35, no. 9, pp. 839–849, May 2002.

[35] L. K. Forbes and S. Crozier, “A novel target-field method for magnetic resonance shim coils: III. Shielded zonal and tesseroidal shims,” J. Phys. D: Appl. Phys., vol. 36, no. 2, pp. 68–80, Jan. 2003.

[36] P. J. Hobson et al., “Bespoke magnetic field design for a magnetically shielded cold atom interferometer,” Sci. Rep., vol. 12, no. 1, pp. 1–12, Jun. 2022.

[37] M. Rea et al., “Precision magnetic field modelling and control for wearable magnetoencephalography,” NeuroImage, vol. 241, Nov. 2021, Art. no. 118401.

[38] N. Holmes et al., “A lightweight magnetically shielded room with active shielding,” Sci. Rep., vol. 12, no. 1, p. 8, Aug. 2022.

[39] P. J. Broser, T. Middelmann, D. Sometti, and C. Braun, “Optically pumped magnetometers disclose magnetic field components of the muscular action potential,” J. Electromyogr. Kinesiol., vol. 56, Feb. 2021, Art. no. 102490.

[40] C. E. Eichler, L. K. Cheng, N. Paskaranandavadivel, P. Du, L. A. Bradshaw, and R. Avci, “Effects of magnetogastroscopy sensor configurations in tracking slow wave propagation,” Comput. Biol. Med., vol. 129, Feb. 2021, Art. no. 104169.

[41] A. Junge and F. Marliani, “Prediction of DC magnetic fields for magnetic cleanliness on spacecraft,” in Proc. IEEE Int. Symp. Electromagn. Compat., Aug. 2011, pp. 834–839.

[42] A. R. S. Celozzi and G. Lovat, Appendix B: Magnetic Shielding. Hoboken, NJ, USA: Wiley, 2008.

[43] T. Andalib et al., “Sensitivity of fields generated within magnetically shielded volumes to changes in magnetic permeability,” Nucl. Instrum. Methods Phys. Res. A, Accel. Spectrom. Detect. Assoc. Equip., vol. 867, pp. 139–147, Sep. 2017.

[44] Magnetic Shields Limited. GA4 Hybrid Shield. [Online]. Available: http://www.magneticshields.co.uk/product-munetal-shields

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