Spin torque and critical currents for magnetic vortex nano-oscillator in nanopillars

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Abstract. We calculated the main dynamic parameters of the spin polarized current induced magnetic vortex oscillations in nanopillars, such as the range of current density, where vortex steady oscillations exist, the oscillation frequency and orbit radius. We accounted for both the non-linear vortex frequency and non-linear vortex damping. To describe the vortex excitations by the spin polarized current we used a generalized Thiele approach to motion of the vortex core as a collective coordinate. All the calculation results are represented via the free layer sizes, saturation magnetization, and the Gilbert damping. Predictions of the developed model can be checked experimentally.

1. Introduction

Now excitations of the microwave (mw) oscillations in magnetic nanopillars, nanocontacts and tunnel junctions by spin polarized current as well as the current induced domain wall motions in nanowires are one of perspective applications of spintronics [1]. A general theoretical approach to mw generation in nanopillars/nanocontacts driven by spin-polarized current based on the universal model of a non-linear auto-oscillator was developed recently [2]. The model was applied to the case of a spin-torque oscillator (STO) excited in a uniformly magnetized free layer of nanopillar, and explains the main experimentally observed effects such as the power and frequency of the generated mw signal. However, the low generated power ~ 1 nW of such STO prevents their practical applications. Recently extremely narrow linewidth of 0.3 MHz and a relatively high generated power was detected for the magnetic vortex (strongly non-uniform state) nano-oscillators in nanopillars [3]. The considerable mw power emission from a vortex STO in the MgO-based magnetic tunnel junctions was observed [4]. It was established that the permanent perpendicular to the plane (CPP) spin polarized current $I_{c1}$ can excite magnetic vortex motion in free layer of the nanopillar if the current intensity exceeds some critical value, $I_{c1}$ [5]. Then, in the interval $I_{c1} < I < I_{c2}$ ($I_{c2}$ is a second critical current) there is mw generation at a frequency which smoothly increases with the current increasing [6]. This frequency corresponds to the vortex oscillations with a stationary orbit determined by the current value, $I$. If the current exceeds $I_{c2}$ [7,8], the vortex steady motion state is not stable anymore, presumably because the vortex reaches a critical velocity [9] and reverses its core. The vortex with opposite core polarization cannot be excited for the given current sign $I$ and the oscillations stop. Such excitation scheme is different from the current-in-plane (CIP) case, where one needs to apply a.c. CIP of about the resonance frequency to
excite the vortex motion [10]. Low value of $I_{c1}$ and high value of $I_{c2}$ make the vortex CPP nanosystem's attractive for applications as microwave devices. However, the calculations of the spin torque (ST force) gave contradictory results for the ST force and $I_{c1}$. The standard STO approach [2] is not applicable to the vortex dynamics due to a specific vortex damping term. The problem was reduced to the problem of vortex core reversal in the perpendicular to the layer plane magnetic field [7] but the vortex steady state dynamics was not accounted. Using the Thiele approach the ST force calculated in Refs. 5, 8 is in 2 times bigger than one calculated from the energy dissipation balance [6]. Non-linearity of the main governing parameters (vortex frequency and damping) and the Oersted field of current were not accounted or accounted incorrectly. The physical mechanism responsible for the critical current $I_{c2}$ has not yet been clarified and the value of $I_{c2}$ has not been calculated.

2. Calculation results and discussion

We present below a simple approach to calculate the ST force and the critical currents of the vortex STO in nanopillar. The approach is based on the Thiele’s formulation of the non-uniform magnetization dynamics [11] and conception of the linear spin excitations [12]. The nanopillar device consists of two ferromagnetic layers, typically FeNi or Co and a nonmagnetic metallic spacer, typically Cu, arranged in a vertical stack (figure 1). Magnetization of one layer is fixed (this layer is so called polarizer), whereas the magnetization $M(r,t)$ of the second layer of the nanopillar is free to rotate. The current of spin polarized electrons transfers some torque $\tau_s$ from the polarizer, which excites magnetization dynamics of the free layer. We start from the Landau-Lifshitz equation of motion $\dot{m} = -\gamma m \times H_{\text{eff}} + \alpha_{\text{LLG}} m \times \dot{m} + \gamma \tau_s$, where $m = M / M_s$, $M_s$ is the saturation magnetization, $\gamma > 0$ is the gyromagnetic ratio, $H_{\text{eff}}$ is the effective field, and $\alpha_{\text{LLG}}$ is the Gilbert damping. We use the ST term in the form suggested by Slonczewski [13], $\tau_s = \gamma \sigma m \times (m \times P)$, where $\sigma = \hbar \eta / (2e LM_s)$. $\eta$ is the current spin polarization ($\eta = 0.2$ for FeNi), $e$ is the electron charge, $L$ is the free layer (dot) thickness, $J$ is the current density, and $P = Pz$ is the unit vector of the polarizer magnetization $(P = \pm 1/2)$. We assume the positive vortex core polarization (direction of perpendicular to the dot plane magnetization) $\rho = +1$, $P = +1$ and define the current (flow of the positive charges) as positive $I > 0$ when it flows from the polarizer to free layer. The spin polarized current can excite a vortex motion in the free layer if only $IpP > 0$ (only the electrons bringing a magnetic moment from the polarizer to free layer opposite to the core polarization can excite a vortex motion). Except $\rho$, the magnetic vortex is described by its core position, $X = (X,Y)$, and chirality $C = \pm 1$ [14]. Let us denote the Slonczewski’s energy density of the spin polarized current as $w_s$. Then, using the Thiele approach and the ST effective field $\partial w_s / \partial m = am \times P$, the ST force acting on the vortex in the free layer can be written as

$$F_{ST}^a = -\frac{\partial}{\partial X_a} \int dV w_s = aLP \cdot \int d^2p \left( m \times \frac{\partial m}{\partial X_a} \right),$$

where $a = M_s, \sigma J_s, a = x, y, \rho = (\rho, \phi)$ is the in-plane radius vector, the derivative is taken with respect to the vortex core position $X$ assuming an ansatz $m(\rho, \phi) = m[\rho, X(\phi)]$ ($m$ dependence on the thickness coordinate $z$ is neglected). $[X]$ has sense of the amplitude of the vortex gyrotropic motion.

We use representation of $m$-components by the spherical angles $\Theta, \Phi$ (figure 1) as $m = (\sin \Theta \cos \Phi, \sin \Theta \sin \Phi, \cos \Theta)$ and find the expression for the ST force

$$F_{ST} = aL \int d^2p \sin^2 \Theta \frac{\partial \Phi}{\partial X}.$$
Figure 1. Magnetic nanopillar with the coordinate system used. The upper (free) layer is in the vortex state with the chirality $C=+1$. The polarizer (red colour) is magnetized uniformly along $Oz$ axis. The negative current $I$ (vertical arrow) flows from the free layer to the polarizer.

In the main approximation we use the decompositions $m_i(p, X) = \cos \Theta = m_i^0(\rho) + g(\rho)(X \cdot \hat{p})$, $\Phi(p, X) = \Phi_0 + m_i(\rho)[X \sin \varphi - Y \cos \varphi]$, where $m_i^0(\rho) = \rho(c^2 - \rho^2)/(c^2 + \rho^2)$, $\Phi_0 = \varphi + C\pi/2$ are the static vortex core profile and phase [14], $g(\rho) = 4pc^2\rho(1 + \rho^2)/(c^2 + \rho^2)^2$ is the excitation amplitude of the $z$-component of the vortex magnetization ($c = R_c/R$, $R_c$ is the vortex core radius, $\rho, X, Y$ are in units of $R$) and $m_0(\rho) = (1 - \rho^2)/\rho$ is the gyrotropic mode profile [12]. One can conclude from Eq. (2), by separating the core and outside-core contributions, that only the moving vortex core contributes to the ST force because the contribution of the main dot area where $\Theta = \pi/2$ is equal to zero due to vanishing integrals of the derivative $\partial X/\Theta$ over the azimuthal angle $\varphi$. This is a reason why the ST contribution is relatively small being comparable with the damping contribution.

The integration in Eq. (2) yields the ST force $F_{ST} = \pi a L(\hat{z} \times X)$, which is in 2 times smaller than in Refs. 5, 8 due to incorrect using of the Thiele’s approach there. This force contributes to the Thiele’s equation of motion $G \times \dot{X} = -\partial X W + \dot{D}X + F_{ST}$, where $G = 2\pi a L M_s/\gamma$ is the gyrovector, $\dot{D}$ is the damping tensor. The vortex energy $W(X)$ and restoring force $F_R = -\partial X W$ can be calculated from an appropriate model of the shifted vortex [14] (the force balance is shown in figure 2). For a circular steady state vortex core motion the $X = \omega \times X$ relation holds, which allows calculating $J_{cl}$. To calculate the vortex steady orbit radius $R_e = |X|$ we need to account non-linear on $X_a$ terms in the vortex damping and frequency (the account only non-linear frequency as in Ref. 5 is not sufficient). The gyrovector also depends on $X$, but this dependence is essential only for the vortex core $p$ reversal, when $G$ changes its sign. The most important non-linearity comes from the damping tensor defined as

$$D_{\alpha\beta}(X) = -\alpha_{LLG} \frac{M_s}{\gamma} \int dV \frac{\partial m}{\partial X_{\alpha}} \cdot \frac{\partial m}{\partial X_{\beta}},$$

or $D_{\alpha\beta} = -\alpha_{LLG} (M, L, \gamma) \int d^2 p [\partial_{\alpha} \Theta \partial_{\beta} \Theta + \sin^2 \Theta \partial_{\alpha} \Theta \partial_{\beta} \Phi \Phi]$. Accounting $D_{\alpha\beta} = D\delta_{\alpha\beta}$ and introducing dimensionless damping parameter $d = -D/|G| > 0$ [15] we can write the
equation for a steady state vortex motion with the orbit radius \( R_x = |X| \):
\[
d(s)G|\omega_0(s)|sR = F_{ST}^v
\]
from which \( s = R_x / R \) and the critical currents \( J_{c1}, J_{c2} \) can be found. In the second order non-linear approximation
\[
d(s) = d_0 + d_1 s^2, \quad \omega_0(s) = \omega_0 + \omega_1 s^2 \quad \text{and} \quad F_{ST}^v = \pi a L R s,
\]
where \( \omega_0 \) is the vortex precession frequency, \( \omega_0 > 0 \) is a function of the dot aspect ratio \( \beta = L/R \) calculated from the vortex energy decomposition \( W(s) \) in series of \( s = |X| / R \) within the pole free model of the shifted vortex [13]. It can be shown that \( \omega_0(\beta) = \frac{20}{9} \pi M \beta [1 - 4 \beta / 3] \) and \( \omega_2(\beta) = 4 \omega_0(\beta) \) for quite wide range of \( \beta \approx 0.01-0.2 \) of practical interest, whereas considerably larger non-linearity \( \omega_0(\beta) / \omega_0(\beta) = 42.8 \) was calculated in Ref. 5 due to incorrect account of the magnetostatic energy. We use the pole free model of the shifted vortex \( m[p, X(t)] \), where the dynamic magnetization satisfies the strong pinning boundary condition at the dot circumference [16] \( \rho = R \). The damping parameters are
\[
d_0 = \alpha_{\text{LLG}} \left( 5 / 8 + \ln(R / R_s) / 2 \right), \quad d_1 = \alpha_{\text{LLG}} \left( R^2 / R_s^2 - 8 / 3 \right) / 4.
\]
We need also to account for the Oersted field of the current, which leads to contribution to the vortex frequency proportional to the current density \( \omega_c = \omega_0(\beta, J) + \omega_1 J \), where \( \omega_1 = (8 \pi / 15) \beta R / c \xi C \), \( \xi = 1 + 15 \ln(2 - 1 / 2) \), \( / 8R \) is the correction to \( \omega_c \) of Ref. 6 (typically \( \sim 10\% \)) for the finite core radius \( R_c << R \). In the linear approximation we get the equality of the damping force and the ST force (negative damping) as the condition to find the value of \( J_{c1} \), with the contribution of the Oersted field of the current accounted. The values of \( F_{ST}, J_{c1} \) coincide with calculation of Ref. 6 conducted by the method of the damping energy balance and differ by the multiplayer 2 from Refs. 5, 8. The first critical current is
\[
J_{c1} = d_0 \omega_0 / (\gamma \sigma / 2 - d_0 \omega_0)
\]
and the steady state vortex orbit radius is
\[
s(J) = \lambda \sqrt{J / J_{c1} - 1}, \quad J > J_{c1}, \quad \lambda^2 = \frac{1}{2} \left[ \frac{\gamma \sigma J_{c1}}{d_1 \omega_0 (J_{c1}) + d_0 \omega_0} \right]. \quad (4)
\]
In this approximation the vortex orbit radius \( s(J) \) increases as square root of the current overcriticality \( (J - J_{c1}) / J_{c1} \) (for the typical parameters and \( R=80-120 \text{ nm} \) we get \( \lambda = 0.25-0.30 \)) and the vortex gyrotropic frequency \( \omega_0 = \omega_0(\beta) + \omega_1 J + \lambda^2 (J / J_{c1} - 1) \omega_0 \) increases linearly with \( J \) increasing. The vortex steady orbit can exist until the moving vortex core crosses the dot border \( s=1 \) or its velocity \( |X| \) reaches the critical velocity \( v_c \) of the core polarization reversal defined in Ref. 9. The vortex with opposite core polarization cannot be excited for the given current sign \( I \) and the oscillations stop. The later condition allows to write equation for the second critical current \( J_{c2} \) as
\[
\omega_0(J)s(J)R = \nu_c. \quad \text{Substituting to this expression the equations for} \quad \omega_0(J) \quad \text{and} \quad s(J) \quad \text{derived above we get a cubic equation for} \quad J_{c2} \quad \text{in the form} \quad \eta \sigma J_{c1} / 2 d_0 + (\omega_1 J_{c1} + \omega_1 \lambda^2) x^{1 / 2} = \nu_c / \lambda R, \quad (x = (J_{c2} / J_{c1} - 1). \quad \text{This equation has one positive root} \quad \beta \text{ and the value of} \quad J_{c2} \quad \text{can be easily calculated (see figure 3).} \quad \text{The former condition} \quad (s=1) \quad \text{yields the second critical current} \quad J_{c2}' = (1 + 1 / \lambda^2) J_{c1}'. \quad \text{More detailed numerical analysis shows that both the mechanisms of the high current instability of the vortex motion are possible depending on the dot sizes} \quad L, \quad R, \quad \text{and the second critical current is the lower value of the calculated currents} \quad J_{c2}, \quad J_{c2}'. \quad \text{The moving vortex core reversal inside the dot occurs for large enough} \quad R > 100 \text{ nm} \quad \text{and} \quad L, \quad \text{whereas the vortex core is expelled from the dot at} \quad J = J_{c2}' \quad \text{for small} \quad R \leq 100 \text{ nm} \quad \text{of practical interest for the vortex STO applications in nanopillars. For the typical free layer sizes} \quad L = 10 \text{ nm}, \quad R = 120 \text{ nm} \quad \text{and} \quad C = 1, \quad \text{the critical currents calculated by the above listed equations are} \quad J_{c1} = 6.3 \times 10^6 \text{ A/cm}^2 (I_{c1} = 2.9 \text{ mA}), \quad J_{c2} = 1.13 \times 10^7 \text{ A/cm}^2 (I_{c2} = 51 \text{ mA}), \quad \text{and for} \quad L = 5 \text{ nm}, \quad R = 100 \text{ nm} \quad \text{we get} \quad J_{c1} = 1.8 \times 10^6 \text{ A/cm}^2 (I_{c1} = 0.56 \text{ mA}), \quad J_{c2}' = 2.7 \times 10^7 \text{ A/cm}^2 (I_{c2}' = 8.4 \text{ mA}).
3. Conclusions
We calculated the main physical parameters of the spin polarized CPP current induced magnetic vortex oscillations in nanopillars, such as the critical currents $J_{c1}$, $J_{c2}$, the vortex steady state oscillations frequency and orbit radius. All the results are represented via the free layer sizes ($L$, $R$), saturation magnetization, Gilbert damping and the degree of spin polarization of the current. These parameters can be obtained from independent experiments. The generalized Thiele approach to the dynamics of non-uniform magnetization is applicable to the problem of the vortex STO excitations by the CPP spin polarized current. The spin transfer torque force is related to the vortex core only.

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