Negative refraction index of the mesoscopic left-handed transmission line in the thermal Fock state

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Abstract  Negative refractive index (NRI) of the quantized lossless mesoscopic left-handed transmission line (LHTL) is deduced numerically in thermal Fock state. Some specific quantum features of NRI dependent the temperature, frequency of the electromagnetic wave and photon numbers, and quantum fluctuations are shown in the lossless LHTL. The results are significant for the miniaturizing applications of LHTL and quantum circuits.

Keywords  Negative refractive index · Mesoscopic Left-handed transmission lines · Thermal Fock state

1 Introduction

The theoretical speculation of negative refractive index materials (NRM) proposed by V. Veselago[1] in 1968, in which several fundamental phenomena occurring in or in association with NRM were predicted, such as the negative Goos-Hänchen shift[2], amplification of evanescent waves[3], reversals of both Doppler shift and Cerenkov radiation[1], sub-wavelength focusing[4] and so on. Some typical approaches for NRM can be summarized as artificial structures such as metamaterials[5–7] and photonic crystals[8–10], chiral materials[11] and photonic resonant media[12,13]. Although very exciting from a physics point of view, the artificial structures seem to be of little practical interest for engineering applications because

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of these resonant structures exhibiting high loss and narrow bandwidth consequently. Due to the weaknesses of resonant-type structures, three groups almost simultaneously in June 2002 introduced a transmission line (TL) approach of N-RM: Eleftheriades et. al[15,16], Oliner[17] and Caloz et. al.[18,19]. LHTL initially the non-resonant-type one, is perhaps one of the most representative and potential candidates due to its low loss, broad operating frequency band, as well as planar configuration[20,21], which is often related with easy fabrication for N-RI applications in a suite of novel guided-wave[22,23], radiated-wave[24,25], and refracted-wave devices and structures[26–28].

![Fig. 1 (a) Equivalent circuit model for the uniform LHTL, (b) Unit cell circuit model for a uniform LHTL.](image)

In the meanwhile, with the rapid development of nanotechnology and nanoelectronics [29], the integrated circuits and components have minimized towards atomic-scale dimensions[30,31] in the last a few decades. When the scale of fabricated electric materials reached to a characteristic dimension, namely, Fermi wavelength, quantum mechanical properties of mesoscopic physics[32,33] become important while the application of classical mechanics fails. The miniaturizing applications would be undoubtedly a persistent trend for LHTL, and the quantum properties may cast some influence on NRI. So it’s significant to investigate the quantum properties of NRI in the LHTL.

Thus, from the point of against this miniaturization challenge, this paper exploits the quantum effect on NRI in the thermal Fock state of the mesoscopic LHTL. The fundamental features of LHTL in Fig.1(a) are straightforwardly derived by elementary TL theory. It consists of lossless per-unit equivalent circuit models (Fig.1.(b)) of the series-C/shunt-L prototype associated with NRI in microwave frequency band[22,34]. The per-unit-length inductance \( L_l \) (H·m) and capacitance \( C_l \) (F·m) are \( L_l = L_l \cdot \Delta_z \) and \( C_l = C_l \cdot \Delta_z \), respectively. So we have the impedances \( Z = 1/i\omega C_l \) (Ω/m) and admittances \( Y = 1/i\omega L_l \) (S/m).

According to the Fig.1(b), the complex propagation constant \( \gamma \), the propagation constant \( \beta \), the characteristic impedance \( Z_l \), the phase velocity \( v_p \), and the group velocity \( v_g \) with the the equivalent constitutive permittivity and permeabil-
ity of the unit cell equivalent circuit for LHTL are given by [34]
\[ \gamma = -i \frac{1}{\omega \sqrt{C_L L}} \sqrt{C_L}, \]  
\[ \beta = -\frac{1}{\omega \sqrt{C_L L}} < 0, \]
\[ Z_l = \sqrt{L_l C_l}, \]
\[ v_p = \frac{\omega}{\beta} = -\omega^2 \sqrt{C_L L} < 0, \]
\[ v_g = (\frac{\partial \beta}{\partial \omega})^{-1} = \omega^2 \sqrt{C_L L} > 0, \]  
\[ \mu(\omega) = -\frac{1}{\omega^2 C_L}, \epsilon(\omega) = -\frac{1}{\omega^2 L} \]  

(1)

According to Kirchhoff’s law, the classical differential equations of motion of Fig.1(b) are
\[ \frac{d^2 u(z)}{dz^2} = -\gamma^2 u(z) \]  
\[ \frac{d^2 j(z)}{dz^2} = -\gamma^2 j(z) \]  

(2)  

(3)

where \( u \) and \( j \) are the position-dependent voltage and currents \( u = u(z) \) and \( j = j(z) \) along the line, respectively, where \( \gamma \) is the complex propagation constant. And the walking-wave solutions to Eq(2) and Eq(3) reach as \( j(z) = A \exp(-i\gamma z) + A^* \exp(i\gamma z), u(z) = B \exp(-i\gamma z) + B^* \exp(i\gamma z), \) in which \( A^* (B^*) \) are the conjugate complexes of \( A \) (B). We adopt the quantization method similar to Louisell [35] to achieve the current operator. In Fig.1(b) the given unit-length, i.e., \( z_0 = m\lambda \) where \( \lambda \) is the wavelength labelled typically by wavenumber \( k \) and frequency \( \omega, \) its Hamiltonian can be written as follows,
\[ H = \frac{1}{2} \int_{0}^{z_0} (L_j j^2(z) + C_l u^2(z)) dz = 2L_j A^* A z_0 \]

where
\[ A = a \sqrt{\frac{\hbar \omega}{2L_z z_0}}, A^* = a^* \sqrt{\frac{\hbar \omega}{2L_z z_0}}. \]

According to the canonical quantization principle, we can quantize the system by operators \( \hat{q} \) and \( \hat{p} \), which satisfy the commutation relation \([\hat{q}, \hat{p}] = i\hbar\). The annihilation and creation operators \( \hat{a} \) and \( \hat{a}^\dagger \) are defined by the relations
\[ \hat{a} = \frac{1}{\sqrt{2\hbar \omega}} (\omega \hat{q} + i\hat{p}), \hat{a}^\dagger = \frac{1}{\sqrt{2\hbar \omega}} (\omega \hat{q} - i\hat{p}) \]

Thus the quantum Hamiltonian of Fig.1(b) can be rewritten as \( \hat{H} = \hbar \omega \hat{a}^\dagger \hat{a} = \frac{1}{2}(\omega^2 \hat{q}^2 + \hat{p}^2) \). Thus the current in the lossless unit equivalent circuit for LHTL can be quantized as
\[ \hat{j}(z) = \sqrt{\frac{\hbar}{4\pi m L^2 C_l}} [\hat{a} \exp(\frac{i}{\omega \sqrt{C_l L}} z) + \hat{a}^\dagger \exp(-\frac{i}{\omega \sqrt{C_l L}} z)] \]

(4)
2 NRI in thermal FOCK state

The thermal noise would influence NRI when the LHTL operates at some temperature. In the following, we exploit NRI dependent thermal noise in thermal FOCK state. As for the equilibrium situation, the so-called thermo field dynamics (TFD) extends the usual quantum field theory to a finite temperature[36]. In TFD, the tilde space accompanies with the Hilbert space, and the tilde operators commute with the non-tilde operators[37]. Thus the creation and annihilation operators $\hat{a}^\dagger$, $\hat{a}$ associate with their tilde operators $\tilde{\hat{a}}^\dagger$, $\tilde{\hat{a}}$ according the rules[37]:

$$[\tilde{\hat{a}}, \tilde{\hat{a}}^\dagger] = 1,$$
$$[\hat{a}, \tilde{\hat{a}}] = [\tilde{\hat{a}}, \hat{a}^\dagger] = [\hat{a}, \hat{a}^\dagger] = 0$$

(5) (6)

The number operators in the Hilbert space and tilde space are read as $\hat{n} = \hat{a}^\dagger \hat{a}$, $\tilde{n} = \tilde{\hat{a}}^\dagger \tilde{\hat{a}}$. In the direct product space, the thermal Fock state at finite temperature $|\hat{n}\tilde{n}\rangle_T$ can be built by the thermal Bogoliubov transformation[37] through the Fock state at zero temperature $|\hat{n}\rangle \otimes |\tilde{n}\rangle = |\hat{n}\tilde{n}\rangle_T = \hat{T}(\theta)|\hat{n}\tilde{n}\rangle$, where $\hat{T}(\theta)$ is a thermal unitary operator which is defined as

$$\hat{T}(\theta) = \exp[-\beta(\hat{a}\tilde{\hat{a}} - \hat{a}^\dagger\tilde{\hat{a}}^\dagger)]$$

(7)

the parameter $\theta$ is the thermal unitary operator relating the thermal photos $n_0$ in the thermal vacuum state: $n_0 = \sinh\theta$. The thermal photos $n_0$ and temperature $T$ are ruled by the Boltzmann distribution $n_0 = [\exp(h\omega/k_BT) - 1]^{-1}$, in which $k_B$ is the Boltzmann constant. The bosonic operators in TFD can relate each other by the thermal Bogoliubov transformation as following,

$$\hat{T}^\dagger(\theta)\hat{a}\hat{T}(\theta) = \mu\hat{a} + \tau\tilde{\hat{a}}^\dagger,$$
$$\hat{T}^\dagger(\theta)\hat{a}^\dagger\hat{T}(\theta) = \mu\hat{a}^\dagger + \tau\tilde{\hat{a}}$$

(8) (9)

where $\mu = \cosh\theta$, $\tau = \sinh\theta$. Then from Eq.(4) and Eq.(1), the quantum fluctuation of the current in the unit cell equivalent circuit is

$$\overline{(\Delta)^2} = \frac{\hbar\omega^3\sqrt{n}}{2\pi_0 Z_{ll}} [2n_0^2 + 2(n + 1)n_0 + 2n + 1]$$

(10)

where $n$ is the number of microwave field photons corresponding to the number operator $\hat{n}$ in the Hilbert space. The NRI according to the definition of the left-handed material $n_r = -\sqrt{\epsilon\mu}$ [1] can be written as follows,

$$n_r = -\frac{2\pi_0 Z_{ll}(\Delta)^2[\exp(\frac{\hbar\omega}{k_BT}) - 1][\cosh(\frac{\hbar\omega}{k_BT}) - 1]}{\hbar\omega^3[\exp(\frac{\hbar\omega}{k_BT}) - 1][\cosh(\frac{\hbar\omega}{k_BT}) - 1] + 2n[\exp(\frac{\hbar\omega}{k_BT}) - 1] + [\cosh(\frac{\hbar\omega}{k_BT}) - 1]}$$

(11)
3 Results and discussion

According Eq.(11), the expression of NRI is dependent temperature, frequency, photo numbers, and so on. However, the analytical expression of NRI in Eq.(11) corresponding to these parameters is rather cumbersome. Hence, we follow the numerical approach to get the dependence of NRI on these parameters.

Fig.2 shows NRI dependent the temperature $T$ under the different current fluctuations in one millimeter unit length of the LHTL with the characteristic impedance $Z_l = 50 \Omega/m$, the photon numbers $n = 10$ and the frequency $\omega = 2GHz$. The destructive dependence of the temperature $T$ on NRI is shown when the current fluctuation varies from 50 to 600. And NRI declines sharply in the temperature interval of $[0, 100K]$ when the current fluctuation $(\hat{\Delta}j)^2 = 50$. The phenomena demonstrates the ideal LHTL should operate at some low temperature $T$ to achieve a desired NRI. However, the quantum fluctuations of the current can improve the condition. The damping of NRI decreases with the increasing of $(\hat{\Delta}j)^2$ in the temperature interval of $[0, 100K]$, and the damping (from -1.3 to -0.8) is minimum when the quantum fluctuation of the current $(\hat{\Delta}j)^2 = 600$ in the same temperature interval.

As mentioned before[22,34], NRI of the LHTL is achieved within the microwave frequency band. In the quantum mesoscopic LHTL, some novel feature of NRI is prominent within the microwave frequency band. Fig.3 shows NRI in the microwave frequency band $[0, 3GHz]$ with $(\Delta_j)^2 = 100$ and photon numbers $n = 50$. It notes that NRI isn’t homogeneous within the microwave frequency band, and NRI decreases sharply within $[0, 1GHz]$ at an arbitrary temperature. A striking contrast between the different temperatures is obvious. The temperature of $T = 50K$ brings to the minimum damping of NRI within the frequency interval of $[0, 1GHz]$, which demonstrates the lower frequency within the microwave frequency band is constructive to NRI, as coincides with the macroscopic LHTL[38].

As the energy of the electromagnetic wave is proportional to the photon numbers $n$ in the LHTL. In the mesoscopic quantizing LHTL, the influences of the electromagnetic energy on NRI is significant feature. Fig.4 illustrates NRI de-
4 CONCLUSION

We discussed the quantum features of the NRI of the mesoscopic left-handed transmission line in the thermal Fock state. With the thermal fluctuation of current the NRI dependent the temperature, frequency of electromagnetic wave and photon numbers is discussed. The results show the lower temperature and frequency within the microwave frequency band, little photon numbers are more conducive to NRI, which is significant for the LHTL in miniaturizing applications in the coming future.
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