LTL Model Checking of Parametric Timed Automata

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Abstract. The parameter synthesis problem for timed automata is undecidable in general even for very simple reachability properties. In this paper we introduce restrictions on parameter valuations under which the parameter synthesis problem is decidable for LTL properties. The proposed problem could be solved using an explicit enumeration of all possible parameter valuations. However, we introduce a symbolic zone-based method for synthesising bounded integer parameters of parametric timed automata with an LTL specification. Our method extends the ideas of the standard automata-based approach to LTL model checking of timed automata. Our solution employs constrained parametric difference bound matrices and a suitable notion of extrapolation.

1 Introduction

Model checking [1] is a formal verification technique applied to check for logical correctness of discrete distributed systems. While it is often used to prove the unreachability of a bad state (such as an assertion violation in a piece of code), with a proper specification formalism, such as the Linear Temporal Logic (LTL), it can also check for many interesting liveness properties of systems, such as repeated guaranteed response, eventual stability, live-lock, etc.

Timed automata have been introduced in [2] and have emerged as a useful formalism for modelling time-critical systems as found in many embedded and cyber-physical systems. The formalism is built on top of the standard finite automata enriched with a set of real-time clocks and allowing the system actions to be guarded with respect to the clock valuations. In the general case, such a timed system exhibits infinite-state semantics (the clock domains are continuous). Nevertheless, when the guards are limited to comparing clock values with integers only, there exists a bisimilar finite state representation of the original infinite-state real-time system referred to as the region abstraction. A practically efficient abstraction of the infinite-state space came with the so called zones [3]. The zone-based abstraction is much coarser and the number of zones reachable

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from the initial state is significantly smaller. This in turns allows for an efficient implementation of verification tools for timed automata, see e.g. UPPAAL [4].

Very often the correctness of a time-critical system relates to a proper timing, i.e. it does not only depend on the logical result of the computation, but also on the time at which the results are produced. To that end the designers are not only in the need of tools to verify correctness once the system is fully designed, but also in the need of tools that would help them to derive proper time parameters of individual system actions that would make the system as a whole satisfy the required specification. After all this problem of parameter synthesis is more urgent in practice than the verification as such.

The problem of the existence of a parameter valuation for a reachability property of a parametric timed automaton has been shown to be undecidable in [5] for a parametric timed automaton with as few as 3 clocks.

To obtain a decidable problem we need to restrict parameter valuations to bounded integers. When modelling a real-time system, designers can usually provide practical bounds on time parameters of individual system actions. Therefore, introducing a parameter synthesis method with such a restriction is still reasonable.

Our goal is to solve the parameter synthesis problem for linear time properties over parametric timed automata where the parameter valuation function is restricted to bounded range over integer values. As part of our goal, we propose a solution that avoids the parameter scan approach in order to provide a potentially more efficient method. To that end we introduce a finite abstraction over parametric difference bound matrices, which allows us to deploy our solution based on a zone abstraction.

An extension of the model checker Uppaal, capable of synthesising linear parameter constraints for correctness of parametric timed automata has been described in [6] together with a subclass of parametric timed automata, for which the emptiness problem is decidable.

In [7] authors show that the problem of the existence of bounded integer parameter values such that some TCTL property is satisfied is PSPACE-complete. They also give symbolic algorithms for reachability and unavoidability properties.

Contribution We show how to apply the standard automata-based approach to LTL model checking of Vardi and Wolper [8] in the context of an LTL formula, a parametric timed automaton and bounds on parameters. In particular, we show how to construct a Büchi automaton coming from the parametric system under verification using a zone-based abstraction and an extrapolation. We give the necessary proof of correctness of our construction.

2 Preliminaries and Problem Statement

In order to state our main problem formally, we need to describe the notion of a parametric timed automaton. We start by describing some basic notation.
Let $P$ be a finite set of parameters. An affine expression is an expression of the form $z_0 + z_1p_1 + \ldots + z_np_n$, where $p_1, \ldots, p_n \in P$ and $z_0, \ldots, z_n \in \mathbb{Z}$. We use $E(P)$ to denote the set of all affine expressions over $P$. A parameter valuation is a function $v : P \to \mathbb{Z}$ which assigns an integer number to each parameter. Let $lb : P \to \mathbb{Z}$ be a lower bound function and $ub : P \to \mathbb{Z}$ be an upper bound function. For an affine expression $e$, we use $e[v]$ to denote the integer value obtained by replacing each $p$ in $e$ by $v(p)$. We use $\text{max}_{lb,ub}(e)$ to denote the maximal value obtained by replacing each $p$ with a positive coefficient in $e$ by $ub(p)$ and replacing each $p$ with a negative coefficient in $e$ by $lb(p)$. We say that the parameter valuation $v$ respects $lb$ and $ub$ if for each $p \in P$ it holds that $lb(p) \leq v(p) \leq ub(p)$. We denote the set of all parameter valuations respecting $lb$ and $ub$ by $\text{Val}_{lb,ub}(P)$.

In the following, we only consider parameter valuations from $\text{Val}_{lb,ub}(P)$.

Let $X$ be a finite set of clocks. We assume the existence of a special zero clock, denoted by $x_0$, that has always the value 0. A guard is a finite conjunction of expressions of the form $x_i - x_j \sim e$ where $x_i, x_j \in X, e \in E(P)$ and $\sim \in \{\leq, <\}$. We use $G(X, P)$ to denote the set of all guards over a set of clocks $X$ and a set of parameters $P$. A plain guard is a guard containing only expressions of the form $x_i - x_j \sim e$ where $x_i, x_j \in X, e \in E(P), \sim \in \{\leq, <\}$, and $x_i = x_0$ or $x_j = x_0$. We also use $\overline{G}(X, P)$ to denote the set of all plain guards over a set of clocks $X$ and a set of parameters $P$. A clock valuation is a function $\eta : X \to \mathbb{R}_{\geq 0}$ assigning non-negative real numbers to each clock such that $\eta(x_0) = 0$. We denote the set of all clock valuations by $\text{Val}(X)$. Let $g \in \overline{G}(X, P)$ and $\eta$ a parameter valuation and $\eta$ be a clock valuation. Then $g[v, \eta]$ denotes a boolean value obtained from $g$ by replacing each parameter $p$ with $v(p)$ and each clock $x$ with $\eta(x)$. A pair $(v, \eta)$ satisfies a guard $g$, denoted by $(v, \eta) \models g$, if $g[v, \eta]$ evaluates to true. A semantics of a guard $g$, denoted by $[g]_v$, is a set of valuation pairs $(v, \eta)$ such that $(v, \eta) \models g$.

For a given parameter valuation $v$ we write $[g]_v$ for the set of clock valuations \{$(\eta \mid (v, \eta) \models g)$\}.

We define two operations on clock valuations. Let $\eta$ be a clock valuation, $d$ a non-negative real number and $R \subseteq X$ a set of clocks. We use $\eta + d$ to denote the clock valuation that adds the delay $d$ to each clock, i.e. $(\eta + d)(x) = \eta(x) + d$ for all $x \in X \setminus \{x_0\}$. We further use $\eta[R]$ to denote the clock valuation that resets clocks from the set $R$, i.e. $\eta[R](x) = 0$ if $x \in R$, $\eta[R](x) = \eta(x)$ otherwise.

We can now proceed with the definition of a parametric timed automaton and its semantics.

**Definition 2.1 (PTA).** A parametric timed automaton (PTA) is a tuple $M = (L, l_0, X, P, \Delta, \text{Inv})$ where

- $L$ is a finite set of locations,
- $l_0 \in L$ is an initial location,
- $X$ is a finite set of clocks,
- $P$ is a finite set of parameters,
- $\Delta \subseteq L \times \overline{G}(X, P) \times 2^X \times L$ is a finite transition relation, and
- $\text{Inv} : L \to \overline{G}(X, P)$ is an invariant function.
We use \( q \xrightarrow{g,R} \Delta q' \) to denote \((q,g,R,q') \in \Delta \). The semantics of a PTA is given as a labelled transition system. A labelled transition system (LTS) over a set of symbols \( \Sigma \) is a triple \((S,s_0,\to)\), where \( S \) is a set of states, \( s_0 \in S \) is an initial state and \( \to \subseteq S \times \Sigma \times S \) is a transition relation. We use \( s \xrightarrow{a} s' \) to denote \((s,a,s') \in \to \).

**Definition 2.2 (PTA semantics).** Let \( M = (L,l_0,X,P,\Delta,Inv) \) be a PTA and \( v \) be a parameter valuation. The semantics of \( M \) under \( v \), denoted by \( \llbracket M \rrbracket_v \), is an LTS \((S_M,s_0,\to)\) over the set of symbols \( \{act\} \cup \mathbb{R}_{\geq 0} \), where

- \( S_M = L \times Val_{lb,ub}(X) \) is a set of all states,
- \( s_0 = (l_0,0) \), where \( 0 \) is a clock valuation with \( 0(x) = 0 \) for all \( x \), and
- the transition relation \( \to \) is specified for all \((q,\eta),(q',\eta') \in S\) such that \( \eta \) is a clock valuation as follows:
  - \((l,\eta) \xrightarrow{d} (l',\eta')\) if \( l = l' \), \( d \in \mathbb{R}_{\geq 0} \), \( \eta' = \eta + d \), and \((v,\eta') \models Inv(l')\),
  - \((l,\eta) \xrightarrow{act} (l',\eta')\) if \( \exists g,R : l \xrightarrow{g,R} \Delta l' \), \((v,\eta) \models g, \eta' = \eta[R]\), and \((v,\eta') \models Inv(l')\).

The transitions of the first kind are called delay transitions, the latter are called action transitions.

We write \( s_1 \xrightarrow{act,d} s_2 \) if there exists \( s' \in S_M \) and \( d \in \mathbb{R}_{\geq 0} \) such that \( s_1 \xrightarrow{act} s' \xrightarrow{d} s_2 \). A proper run \( \pi \) of \( \llbracket M \rrbracket_v \) is an infinite alternating sequence of delay and action transitions that begins with a delay transition \( \pi = (l_0,\eta_0) \xrightarrow{d_0} (l_0,\eta_0 + d_0) \xrightarrow{act_1} (l_1,\eta_1) \xrightarrow{d_1} \cdots \). A proper run is called a Zeno run if the sum of all its delays is finite.

For the rest of the paper, we assume that we only deal with a deadlock-free PTA, i.e. that for each considered parameter valuation \( v \) there is no state without a reachable action transition in \( \llbracket M \rrbracket_v \). We deal with Zeno runs later.

Let \( M \) be a PTA, \( L : L \to 2^{AP} \) be a labelling function that assigns a set of atomic propositions to each location of \( M \), \( v \) be a parameter valuation, and \( \varphi \) be an LTL formula. We say that \( M \) under \( v \) with \( L \) satisfies \( \varphi \), denoted by \((M,v,L) \models \varphi\) if for all proper runs \( \pi \) of \( \llbracket M \rrbracket_v \), \( \pi \) satisfies \( \varphi \) where atomic prepositions are determined by \( L \).

Unfortunately, it is known that the parameter synthesis problem for a PTA is undecidable even for very simple (reachability) properties [5]. Instead of solving the general problem, we thus focus on a more constrained version. We may now state our main problem.

**Problem 2.3 (The bounded integer parameter synthesis problem).** Given a parametric timed automaton \( M \), a labelling function \( L \), an LTL property \( \varphi \), a lower bound function \( lb \) and an upper bound function \( ub \), the problem is to compute the set of all parameter valuations \( v \) such that \((M,v,L) \models \varphi \) and \( lb(p) \leq v(p) \leq ub(p) \).
Problem 2.3 is trivially decidable using a region abstraction and parameter scan approach. Unfortunately, the size of the region-based abstraction grows exponentially with the number of clocks and the largest integer number used. As a result, the region-based abstraction is difficult to be used in practice for an analysis of more than academic toy examples, even though it has its theoretical value.

Unlike the region-based abstraction, a single state in a zone-based abstraction is no longer restricted to represent only those clock values that are between two consecutive integers. Therefore, the zone-based abstraction is much coarser and the number of zones reachable from the initial state is significantly smaller. In order to avoid the necessity of an explicit enumeration of all parameter valuations we use the zone-based abstraction together with the symbolic representation of parameter valuation sets. Our algorithmic framework which solves Problem 2.3 consists of three steps.

As the first step, we extend the standard automata-based LTL model checking of timed automata [8] to parametric timed automata. We employ this approach in the following way. From a PTA $M$ and an LTL formula $\varphi$ we produce a product parametric timed Büchi automaton (PTBA) $A$. The accepting runs of the automaton $A$ correspond to the runs of $M$ violating the formula $\varphi$ (analogously as in the case of timed automata).

As the second step, we employ a symbolic semantics of a PTBA $A$ with a suitable extrapolation. From the symbolic state space of a PTBA $A$ we finally produce a Büchi automaton $B$.

As the last step, we need to detect all parameter valuations such that there exists an accepting run in Büchi automaton $B$. This is done using our Cumulative NDFS algorithm.

Now, we proceed with the definitions of a Büchi automaton, a parametric timed Büchi automaton and its semantics.

**Definition 2.4 (BA).** A Büchi automaton (BA) is a tuple $B = (Q, q_0, \Sigma, \rightarrow, F)$, where

- $Q$ is a finite set of states,
- $q_0 \in Q$ is an initial state,
- $\Sigma$ is a finite set of symbols,
- $\rightarrow \subseteq Q \times \Sigma \times Q$ is a set of transitions, and
- $F \subseteq Q$ is the set of accepting states (acceptance condition).

An $\omega$-word $w = a_0a_1a_2\ldots \in \Sigma^\omega$ is accepting if there is an infinite sequence of states $q_0q_1q_2\ldots$ such that $q_i \xrightarrow{a_i} q_{i+1}$ for all $i \in \mathbb{N}$, and there exist infinitely many $i \in \mathbb{N}$ such that $q_i \in F$.

**Definition 2.5 (PTBA).** A parametric timed Büchi automaton (PTBA) is a pair $A = (M, F)$ where

- $M = (L, l_0, X, P, \Delta, \text{Inv})$ is a PTA, and
- $F \subseteq L$ is a set of accepting locations.
Zeno runs represent non-realistic behaviours and it is desirable to ignore them in analysis. Therefore, we are interested only in non-Zeno accepting runs of a PTBA. There is a well-known transformation to the strongly non-Zeno form [9] of a PTBA, which guarantees that each accepting run is non-Zeno. For the rest, we assume that we have the strongly non-Zeno form of a PTBA, as introduced in [9].

**Definition 2.6 (PTBA semantics).** Let \( A = (M, F) \) be a PTBA and \( v \) be a parameter valuation. The semantics of \( A \) under \( v \), denoted by \([A]_v\), is defined as \([M]_v = (S_M, s_0, \to)\).

We say a state \( s = (l, \eta) \in S_M \) is accepting if \( l \in F \). A proper run \( \pi = s_0 \xrightarrow{d_0} s'_0 \xrightarrow{act} s_1 \xrightarrow{d_1} s'_1 \xrightarrow{act} \ldots \) of \([A]_v\) is accepting if there exists an infinite set of indices \( i \) such that \( s_i \) is accepting.

## 3 Symbolic Semantics

A **constraint** is an inequality of the form \( e \sim e' \) where \( e, e' \in E \) and \( \sim \in \{>, \geq, \leq, <\} \). We define \( c[v] \) as a boolean value obtained by replacing each \( p \) in \( c \) by \( v(p) \). A valuation \( v \) **satisfies** a constraint \( c \), denoted \( v \models c \), if \( c[v] \) evaluates to true. The **semantics** of a constraint \( c \), denoted \([c]\), is the set of all valuations that satisfy \( c \). A finite set of constraints \( C \) is called a **constraint set**. A valuation \( v \) **satisfies** a constrain set \( C \) if it satisfies each \( c \in C \). The **semantics** of a constraint set \( C \) is given by \([C] = \bigcap_{c \in C} [c]\). A constraint set \( C \) is **satisfiable** if \([C] \neq \emptyset \). A constraint \( c \) **covers** a constraint set \( C \), denoted \( C \models c \), exactly when \([C] \subseteq [c]\).

As in [6], we identify the relation symbol \( \leq \) with a boolean value true and \( < \) with a boolean value false. Then, we treat boolean connectives on relation symbols \( \leq, < \) as operations with boolean values. For example, \((\leq \iff <) = <\).

Now, we define a parametric difference bound matrix, a constrained parametric difference bound matrix, several operations on them, and a PTBA symbolic semantics. These definitions are introduced in detail in [6].

**Definition 3.1.** A parametric difference bound matrix (PDBM) over \( P \) and \( X \) is a set \( D \) which contains for all \( 0 \leq i, j \leq |X| \) a guard of the form \( x_i - x_j \sim_{ij} e_{ij} \) where \( x_i, x_j \in X \) and \( e_{ij} \in E(P) \cup \{\infty\} \) and \( i \neq j \). We denote by \( D_{ij} \) a guard of the form \( x_i - x_j \sim_{ij} e_{ij} \) contained in \( D \). Given a parameter valuation \( v \), the semantics of \( D \) is given by \([D]_v = \prod_{i,j} [D]_{ij}.\) A PDBM \( D \) is satisfiable with respect to \( v \) if \([D]_v \) is non-empty. If \( f \) is a guard of the form \( x_i - x_j \sim e \) with \( i \neq j \) (a proper guard), then \( D[f] \) denotes the PDBM obtained from \( D \) by replacing \( D_{ij} \) with \( f \). We denote by \( PDBMS(P, X) \) the set of all PDBM over parameters \( P \) and clocks \( X \).

**Definition 3.2.** A constrained parametric difference bound matrix (CPDBM) is a pair \((C, D)\), where \( C \) is a constraint set and \( D \) is a PDBM and for each \( 0 \leq i \leq |X| \) it holds that \( C \models e_{0i} \geq 0 \). The semantics of \((C, D)\) is given by \([C, D] = \{[v, \eta] \mid v \in [C] \land \eta \in [D]_v\}\). We call \((C, D)\) satisfiable if \([C, D] \) is non-empty. We denote by \( CPDBMS \) the set of all CPDBM. A CPDBM \((C, D)\) is in the canonical form iff for all \( i, j, k \), \( C \models e_{ij}(\sim_{ik} \land \sim_{kj})e_{ik} + e_{kj} \).
Definition 3.3 (Applying a guard). Suppose \( g \) is a simple guard of the form \( x_i - x_j \prec e \). Suppose \((C, D)\) is a constrained PDBM in the canonical form and \( D_{ij} = (e_{ij}, \prec_{ij}) \). The application of a guard \( g \) on \((C, D)\) generally results in a set of constrained PDBMs and is defined as follows:

\[
(C, D)[g] = \begin{cases} 
\{(C, D[g])\} & \text{if } C \models \neg e_{ij}(\prec_{ij} \Rightarrow \prec)e, \\
\{(C, D)\} & \text{if } C \models e_{ij}(\prec_{ij} \Rightarrow \prec)e, \\
\{(C \cup \{e_{ij}(\prec_{ij} \Rightarrow \prec)e\}, D), (C \cup \{-e_{ij}(\prec_{ij} \Rightarrow \prec)e\}, D[g])\} & \text{else}.
\end{cases}
\]

where \( D[g] \) is defined as follows:

\[
D[g]_{kl} = \begin{cases} 
(e_i, \prec) & \text{if } k = i \text{ and } l = j, \\
D_{kl} & \text{else}.
\end{cases}
\]

We can generalise this definition to conjunctions of simple guards as follows:

\[
D[g_0 \land g_1 \land \ldots \land g_k] \overset{def}{=} D[g_0][g_1] \ldots [g_k].
\]

Definition 3.4 (Resetting a clock). Suppose \( D \) is a PDBM in the canonical form. \( D \) with a reset clock \( x_r \), denoted as \( D[x_r] \), represents a PDBM \( D \) after resetting the clock \( x_r \) and is defined as follows:

\[
D[x_r]_{ij} = \begin{cases} 
D_{0j} & \text{if } i \neq j \text{ and } i = r, \\
D_{i0} & \text{if } i \neq j \text{ and } j = r, \\
D_{ij} & \text{else}.
\end{cases}
\]

We can generalise this definition to reset of a set of clocks as follows:

\[
D[x_{i_0}, x_{i_1}, \ldots, x_{i_k}] \overset{def}{=} D[x_{i_0}][x_{i_1}] \ldots [x_{i_k}].
\]

Definition 3.5 (Time successors). Suppose \( D \) is a PDBM in the canonical form. The time successor of \( D \), denoted as \( D^\uparrow \), represents a PDBM with all upper bounds on clocks removed and is defined as follows:

\[
D^\uparrow_{ij} = \begin{cases} 
(\infty, \prec) & \text{if } i \neq 0 \text{ and } j = 0, \\
D_{ij} & \text{else}.
\end{cases}
\]

It follows from the definition that the reset and time successor operations preserve the canonicity. After an application of a guard the canonical form needs to be computed.

To compute the canonical form of the given CPDBM we need to derive the tightest constraint on each clock difference. Deriving the tightest constraint on a clock difference can be seen as finding the shortest path in the graph interpretation of the CPDBM [10, 6]. The canonisation operation is usually implemented using extended Floyd-Warshall algorithm where on each relaxation a split action on the constraint set can occur. Therefore, the result of the canonisation is a set containing constrained parametric difference bound matrices in the canonical form.
Definition 3.6 (Canonicalisation). First, we define a relation $\rightarrow_{FW}$ on constrained parametric bound matrices as follows, for all $0 \leq k, i, j \leq |X| + 1$

- $(k, i, j, C_1, D_1) \rightarrow_{FW} (k, i, j + 1, C_2, D_2)$ if $(C_2, D_2) \in (C_1, D_1)[x_i - x_j (\preceq_{ik} \land \preceq_{kj}) e_{ik} + e_{kj}]$
- $(k, i, |X| + 1, C_1, D_1) \rightarrow_{FW} (k, i + 1, 0, C_2, D_2)$ if $(C_2, D_2) \in (C_1, D_1)[x_i - x_j (\preceq_{ik} \land \preceq_{kj}) e_{ik} + e_{kj}]$
- $(k, |X| + 1, 0, C_1, D_1) \rightarrow_{FW} (k + 1, 0, 0, C_2, D_2)$ if $(C_2, D_2) \in (C_1, D_1)[x_i - x_j (\preceq_{ik} \land \preceq_{kj}) e_{ik} + e_{kj}]$

The relation $\rightarrow_{FW}$ can be seen as a representation of the computation steps of the extended nondeterministic Floyd-Warshall algorithm.

Now, suppose $(C, D)$ is a CPDBM. The canonical form of $(C, D)$, denoted as $(C, D)_c$, represents a set of CPDBMs with a tightest constraint on each clock difference in $D$ and is defined as follows.

$$(C, D)_c = \{(C', D') \mid (0, 0, 0, C, D) \rightarrow_{FW} (|X| + 1, 0, 0, C', D')\}$$

Definition 3.7 (PTBA symbolic semantics). Let $A = ((L, l_0, X, P, \Delta, Inv), F)$ be a PTBA. Let $lb$ and $ub$ be a lower bound function and an upper bound function on parameters. The symbolic semantics of $A$ with respect to $lb$ and $ub$ is a transition system $(S_A, S_{init}, \Longrightarrow)$, denoted as $[A]_{lb,ub}$, where

- $S_A = L \times \{(C, D) \mid (C, D) \in CPDBMS\}$ is the set of all symbolic states,
- the set of initial states $S_0$ is defined as $\{(l_0, [C, D]) \mid (C, D) \in (\emptyset, E^\ast)[Inv(l_0)]\}$, where
  - $E$ is a PDBM with $E^{i,j}$ is $(0, \leq)$ for each $i, j$, and
  - for each $p \in P$, the constraints $p \geq lb(p)$ and $p \leq ub(p)$ are in $C$.
- There is a transition $(l, [C, D]) \Longrightarrow (l', [C', D'])$ if
  - $l \xrightarrow{g, R} l'$ and
  - $(C'', D'') \in (C, D)[g]$ and
  - $(C'_c, D'_c) \in (C'', D'')[c]$ and
  - $(C', D') \in (C''_c, D''[R]^\ast)[Inv(l')]$ and
  - $(C'_c, D'_c) \in (C''_c, D'[R]^\ast)$.

A symbolic state is represented by a tuple $(l, [C, D])$ where $l$ is a location, $(C, D)$ is a CPDBM. We say a state $S = (l, [C, D]) \in S_A$ is accepting if $l \in F$. We say $\pi = S_0 \Rightarrow S_1 \Rightarrow \ldots$ is a run of $[A]$ if $S_0 \in S_{init}$ and for each $i$, $S_i \in S_A$ and $S_{i-1} \Rightarrow S_i$. A run respects a parameter valuation $v$ if for each state $S_i = (l_i, [C_i, D_i])$ it holds that $v \in [C_i]$. A run $\pi$ is accepting if there exists an infinite set of indices $i$ such that $S_i$ is accepting.

For the rest of the paper we fix $lb$, $ub$ and we use $[A]$ to denote $[A]_{lb,ub}$. The transition system $[A]$ may be infinite. In order to obtain a finite transition system we need to apply a finite abstraction over $[A]$.

Definition 3.8 (Time-abstracting simulation). Given an LTS $(S, s_0, \rightarrow)$, a time-abstracting simulation $R$ over $S$ is a binary relation satisfying following conditions:
Lemma 3.11. Let $s_1 \overset{\alpha_{1}}{\rightarrow} s_1'$ implies the existence of $s_2 \overset{\alpha_{2}}{\rightarrow} s_2'$ such that $s_1' \vdash s_2'$, and $s_1 \vdash s_2$ and $d_1 \in \mathbb{R}^{\geq 0}$ and $s_1 \overset{d_1}{\rightarrow} s_1'$ implies the existence of $d_2 \in \mathbb{R}^{\geq 0}$ and $s_2 \overset{d_2}{\rightarrow} s_2'$ such that $s_1' \vdash s_2'$.

We define the largest simulation relation over $S (\subseteq S)$ in the following way: $s \leq s'$ if there exists a time-abstracting simulation $R$ and $(s, s') \in R$. When $S$ is clear from the context we shall only use $\leq$ instead of $\leq S$ in the following.

In the following definition, for a parameter valuation $v$, a concrete state $s_1 = (l_1, \eta)$ from $[A]_v$, and a symbolic state $S_2 = (l_2, [C, D])$ from $[A]$ we write $s_1 \in_v S_2$ if $l_1 = l_2$, $v \in C$, and $\eta \in [D]_v$.

**Definition 3.9 (PTBA abstract symbolic semantics).** Let $A = (M, F)$ be a PTBA. An abstraction over $[A] = (S_A, S_{init}, \Rightarrow)$ is a mapping $\alpha : S_A \rightarrow 2^{S_A}$ such that the following conditions hold:

- $(l', [C', D']) \in \alpha(l, [C, D])$ implies $l = l' \land [C'] \subseteq [C]$ and $[C', D] \subseteq [C', D']$.
- for each $v \in [C]$ there exists $S_1, S_2$ such that $S_2 = (l, [C', D']) \in \alpha(S_1)$ and for each $s \in_v S_2$ there exists a state $s' \in_v S_1$ satisfying $s \leq s'$.

An abstraction $\alpha$ is called finite if its image is finite. An abstraction $\alpha$ over $[A]$ induces a new transition system denoted as $[A]^\alpha = (Q_A, Q_{init}, \Rightarrow^\alpha)$ where

- $Q_A = \{ S \mid S \in \alpha(S') \text{ and } S' \in S_A \}$,
- $Q_{init} = \{ S \mid S \in \alpha(S') \text{ and } S' \in S_{init} \}$, and
- $Q \Rightarrow^\alpha Q'$ if there is $S \in S_A$ such that $Q' \in \alpha(S)$ and $Q \Rightarrow S$.

An accepting state, a run and an accepting run are defined analogously as in the $[A]$ case. If the $\alpha$ is finite the $[A]^\alpha$ can be viewed as a Büchi automaton.

Now, we define a parametric extension of the well known $k$-extrapolation [11].

**Definition 3.10.** Let $A$ be a PTBA, $(l, [C, D])$ be a symbolic state of $[A]$ and $D_{ij} = x_i - x_j \leq e_{ij}$ for each $0 \leq i, j \leq |X|$. We define the $kp$-extrapolation $\alpha_{kp}$ in the following way: $(l, [C', D']) \in \alpha_{kp}((l, [C, D]))$ if $C' = C \land \bigwedge_{0 \leq i, j \leq |X|} e'_{ij}$ and for each $0 \leq i, j \leq |X|:

- it holds that $D'_{ij} = x_i - x_j \leq e_{ij}$ and $e'_{ij} = e_{ij} \leq M(x_j)$ or
- it holds that $D'_{ij} = x_i - x_j < \infty$ and $e'_{ij} = e_{ij} > M(x_j)$ or
- it holds that $D'_{ij} = x_i - x_j \leq e_{ij}$ and $e'_{ij} = e_{ij} \geq -M(x_j)$ or
- it holds that $D'_{ij} = x_i - x_j < -M(x_j)$ and $e'_{ij} = e_{ij} < -M(x_j)$,

where $M(x)$ is the maximum value in $\{ \max_{x_{1b,ub}}(e) \mid e \text{ is compared with } x \text{ in } A \}$.

**Lemma 3.11.** Let $A$ be a PTBA. The $kp$-extrapolation is a finite abstraction over $[A] = (S_A, S_{init}, \Rightarrow)$. 
Proof. First, we prove that the kp-extrapolation is an abstraction. It is easy to see that the kp-extrapolation satisfies the first condition \((l', [C', D']) \in \alpha((l, [C, D]))\) implies \(l = l' \land [C'] \subseteq [C] \land [C', D] \subseteq [C', D']\). The validity of the second condition follows from the following observation. For each \(v \in [C]\) and each \(\eta' \in [D']\), there exists \(\eta \in [D]_v\) such that for each clock \(x\) and each guard \(g\) the following implication holds: \(\eta'(x) \models g \implies \eta(x) \models g\).

Now, we need to show that the kp-extrapolation is finite. From the definition we have the fact that the number of locations is finite and the number of sets of bounded parameter valuations is finite. We need to show that there are only finitely many sets \([C, D]\) when the kp-extrapolation is applied. This follows from the fact that the kp-extrapolation allows values either from the finite range \(-M(x_i), M(x_i)\) or the value \(\infty\).

\[\Box\]

**Theorem 3.12.** Let \(A\) be a PTBA and \(\alpha\) be a finite abstraction. For each parameter valuation \(v\) the following holds: there exists an accepting run of \([A]_v\) if and only if there exists an accepting run respecting \(v\) of \([A]^\alpha\).

Proof idea. We can transform the proof of Theorem 1 of [12] and all corresponding lemmata into our parametric setup in a straightforward way. We refer the reader to the Appendix A.

4 Parameter Synthesis Algorithm

We recall that our main objective is to find all parameter valuations for which the parametric timed automaton satisfies its specification. In the previous sections we have described the standard automata-based method employed under a parametric setup which produces a Büchi automaton. For the rest of this section we denote for each state \(s = (l, [C, D])\) of the Büchi automaton on the input the set of valuations \([C]\) as \(s.[C]\). We say that a sequence of states \(s_1 \Rightarrow s_2 \Rightarrow \ldots \Rightarrow s_n \Rightarrow s_1\) is a cycle under the parameter valuation \(v\) if each state \(s_i\) in the sequence satisfies \(v \in s_i.[C]\). A cycle is called accepting if there exists \(0 \leq i \leq n\) such that \(s_i\) is accepting.

Contrary to the standard LTL model checking, it is not enough to check the emptiness of the produced Büchi automaton. Our objective is to check the emptiness of the produced Büchi automaton for each considered parameter valuation. We introduce the Cumulative NDFS algorithm as an extension of the well-known NDFS algorithm. Our modification is based on the set \(\text{Found}\) which accumulates all detected parametric valuations such that an accepting cycle under these valuations was found. Contrary to the NDFS algorithm, whenever Cumulative NDFS detects an accepting cycle, parameter valuations are saved to the set \(\text{Found}\) and the computation continues with a search for another accepting cycle. Note the fact that whenever we reach a state \(s'\) with \(s'.[C] \subseteq \text{Found}\) we already have found an accepting cycle under all valuations from \(s'.[C]\) and there is no need to continue with the search from \(s'\). Therefore, we are able to speed up the computation whenever we reach such a state.
Now, we mention the crucial property of monotonicity. The set of parameter valuations \( s.[C] \) can not grow along any run of the input automaton. Lemma 4.1 states this observation, which follows from the definition of successors in \([A]^{\alpha}\) and the definition of operations on CPDBMs. The clear consequence of Lemma 4.1 is the fact that each state \( s \) on a cycle has the same set \( s.[C] \).

**Lemma 4.1.** Let \( A \) be a PTBA, \( \alpha \) be an abstraction and \( s \) be a state in \([A]^{\alpha}\). For every state \( s' \) reachable from \( s \) it holds that \( s'.[C] \subseteq s.[C] \).

**Algorithm 1:** Cumulative NDFS

```plaintext
Algorithm CumulativeNDFS(G)

1. Found ← Stack ← Outer ← ∅
2. OuterDF S(sinit)
3. return Accepted ← Found

Procedure OuterDF S(s)

4. Stack ← Stack ∪ {s}
5. Outer ← Outer ∪ {s}
6. foreach s' such that s → s' do
    7. if s' ∈ Stack ∧ s'.[C] ⊈ Found then
        8. OuterDF S(s')
    9. if s ∈ Accepting ∧ s.[C] ⊈ Found then
        10. InnerDF S(s)
11. Stack ← Stack \ {s}
12. return

Procedure InnerDF S(s)

13. Inner ← Inner ∪ \{s\}
14. foreach s' such that s → s' do
    15. if s' ∈ Stack then
        16. “Cycle detected”
    17. Found ← Found ∪ s'.[C]
18. return
19. if s' ∉ Inner ∧ s'.[C] ⊈ Found then
    20. InnerDF S(s')
21. return
```

**Theorem 4.2.** Let \( A \) be a PTBA and \( \alpha \) an abstraction over \([A]\). A parameter valuation \( v \) is contained in the output of the CumulativeNDFS([A]^{\alpha}) if and only if there exists an accepting run respecting \( v \) in \([A]^{\alpha}\).

**Proof.** We refer the reader to the Appendix B.

We recall that our objective was to synthesise the set of all parameter valuations such that the given parametric timed automaton satisfies the given LTL property. In order to compute this set we employed a zone-based semantics, an extrapolation technique and the Cumulative NDFS algorithm. We have shown the way to compute all parameter valuations for which the given LTL formula is not satisfied. Now, as the last step in the solution to Problem 2.3, we need to complement the set \( Accepted \). Thus, the solution to Problem 2.3 is the complement of the set \( Accepted \), more precisely the set \( Val_{lb,ub}(X, P) \setminus Accepted \). To conclude this section, we state that Theorem 4.2 together with Theorem 3.12 imply the correctness of our solution to Problem 2.3.
5 Conclusion and Future Work

We have presented a logical and algorithmic framework for the bounded integer parameter synthesis of parametric timed automata with an LTL specification. The proposed framework allows the avoidance of the explicit enumeration of all possible parameter valuations.

In this paper we have used the parametric extension of a difference bound matrix called a constrained parametric difference bound matrix. To be able to employ a zone-based method successfully we introduced a finite abstraction called the kp-extrapolation. At the final stage of the parameter synthesis process, the cycle detection itself is performed by the introduced Cumulative NDFS algorithm which is an extension of the well-known NDFS algorithm.

As for future work we plan to introduce different finite abstractions and compare their influence on the state space size. Other area that can be investigated is the employment of different linear specification logics, e.g. Clock-Aware LTL [13] which enables the use of clock-valuation constraints as atomic propositions.

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A Proof of Theorem 3.12

In this section we transform the proof of Theorem 1 of [12] and all corresponding lemmata into our parametric setup.

For the sake of simplicity of the proof, we add labels to the transitions in $[[A]]^a$ in the following way. For each transition we use the location of a source state as the transition label. Since labels are not used in the proposed method, it is safe to do that.

For the rest, let $v$ be a parameter valuation, $A$ be a PTBA, and $\alpha$ be a finite abstraction over $[[A]]$. Then, we denote by $A | v$ a timed Büchi automaton obtained from $A$ by replacing each parameter $p$ with the value $v(p)$. We use $\equiv_N$ to denote the standard region abstraction [12] over a timed automaton $N$. In the following, we omit the proof of a lemma if the proof is an obvious modification of the original proof in [12].

We write $s_1 \overset{act_1,act_2,\ldots,act_{n+1}}{\longrightarrow}_d s_k$ if there exist $s_2, \ldots, s_{k-1}$ such that $s_1 \overset{act_1}{\longrightarrow}_d s_2, s_2 \overset{act_2}{\longrightarrow}_d s_3, \ldots$, and $s_{k-1} \overset{act_{k-1}}{\longrightarrow}_d s_k$. We write $s \overset{act_1,act_2,\ldots,act_k}{\longrightarrow}_d s'$ if $s \overset{act_1,act_2,\ldots,act_k}{\longrightarrow}_d s'$ or there exist some $s_1, s_2, \ldots, s_n(n \geq 1)$ such that $s \overset{act_1,act_2,\ldots,act_k}{\longrightarrow}_d s_1, s_1 \overset{act_1,act_2,\ldots,act_k}{\longrightarrow}_d s_2, \ldots, s_{n-1} \overset{act_1,act_2,\ldots,act_k}{\longrightarrow}_d s_n$, and $s_n \overset{act_1,act_2,\ldots,act_k}{\longrightarrow}_d s'$.

Lemma A.1. The equivalence relation $\equiv_{A|v}$ is a time-abstracting bisimulation.

Lemma A.2. Let $s_1, s_1', s_2$ be concrete states in $[[A]]_v$, $R$ be a time-abstracting simulation and $s_1 R s_1'$. If $s_1 \overset{act_1,act_2,\ldots,act_k}{\longrightarrow}_d s_2$, then there exists a concrete state $s_2'$ in $[[A]]_v$ such that $s_1' \overset{act_1,act_2,\ldots,act_k}{\longrightarrow}_d s_2'$.

Lemma A.3. Let $s_1, s_2$ be concrete states in $[[A]]_v$. If $s_1 \overset{act_1,act_2,\ldots,act_k}{\longrightarrow}_d s_2$ and $s_1 \equiv_{A|v} s_2$, then there is an infinite sequence of concrete states $s_1 s_2 \ldots$ in $[[A]]_v$ such that for each $i \geq 1$, $s_i \overset{act_1,act_2,\ldots,act_k}{\longrightarrow}_d s_{i+1}$.

Lemma A.4. Let $s', s_1, s_2$ be concrete states in $[[A]]_v$ and $S_1, S_2$ be symbolic states in $[[A]]$.

1. If $S_1 \Rightarrow S_2$ and $s' \in_v S_2$, then there exist concrete state $s$ in $[[A]]_v$ such that $s \overset{act}{\longrightarrow}_d s'$.
2. If $s_1 \overset{act}{\longrightarrow}_d s_2$ and $s_1 \in_v S_1$, then $S_1 \Rightarrow S_2$ for some symbolic state $S_2$ in $[[A]]$ with $s_2 \in_v S_2$.

Proof. See Lemma 3.16 and Lemma 3.18 of [6].

Lemma A.5. Let $s, s_1, s_2$ be concrete states in $[[A]]_v$, and $Q_1, Q_2$ be symbolic states in $[[A]]^a$.

1. If $Q_1 \Rightarrow s Q_2$ and $s \in_v Q_2$, then there exist concrete states $s_1', s_2' \in [[A]]_v$ such that $s_1' \overset{act}{\longrightarrow}_d s_2', s_1' \in_v Q_1$ and $s \leq s_2'$. 
2. If \( s_1 \xrightarrow{\gamma_d} s_2 \) and \( s_1 \in_v Q_1 \), then \( Q_1 \Rightarrow_{\alpha} Q_2 \) for some symbolic state \( Q_2 \) in \( \llbracket A \rrbracket^{\alpha} \) with \( s_2 \in_v Q_2 \).

**Proof.** 1. From \( Q_1 \Rightarrow_{\alpha} Q_2 \) we know that there exists \( S \) such that \( Q_1 \Rightarrow S \) and \( Q_2 = \alpha(S) \). For any \( s \in_v Q_2 \), since \( Q_2 = \alpha(S) \), there is \( s'_2 \in_v S \) such that \( s \preceq s'_2 \). Since \( Q_1 \Rightarrow S \) and \( s'_1 \in_v S \), by Lemma A.4, there is a \( s'_1 \in_v Q_1 \) such that \( s'_1 \xrightarrow{\gamma_d} s'_2 \).

2. By Lemma A.4 there is a \( S \) such that \( Q_1 \Rightarrow S \) with \( s_2 \in_v S \). Let \( Q_2 = \alpha(S) \).

**Lemma A.6.** Let \( s \) be a concrete state in \( \llbracket A \rrbracket_v \). \( Q_1, Q_2 \) be symbolic states in \( \llbracket A \rrbracket^{\alpha} \). If \( Q_1 \xrightarrow{\alpha_{act_1, act_2, \ldots, act_k}} Q_2 \) and \( s \in_v Q_2 \), then there exist concrete states \( s_1, s_2 \) in \( \llbracket A \rrbracket_v \) such that \( s_1 \in_v Q_1 \), \( s_1 \xrightarrow{\alpha_{act_1, act_2, \ldots, act_k}} \gamma_d s_2 \), and \( s \preceq s_2 \).

**Lemma A.7.** Let \( s \) be a concrete state in \( \llbracket A \rrbracket_v \). \( Q_1, Q_2 \) be symbolic states in \( \llbracket A \rrbracket^{\alpha} \). If \( Q \xrightarrow{\alpha_{act_1, act_2, \ldots, act_k}} \gamma_d s \in_v Q \), then for any \( n \geq 1 \), there exist concrete states \( s_1, s_2, \ldots, s_{n+1} \) in \( \llbracket A \rrbracket_v \) such that \( s_1 \in_v Q \), \( s \preceq s_{n+1} \), and for each \( i \in 1, 2, \ldots, n \), \( s_i \xrightarrow{\alpha_{act_1, act_2, \ldots, act_k}} \gamma_d s_{i+1} \).

**Lemma A.8.** Let \( s \) be a concrete state in \( \llbracket A \rrbracket_v \). \( Q_1, Q_2 \) be symbolic states in \( \llbracket A \rrbracket^{\alpha} \). If \( Q \xrightarrow{\alpha_{act_1, act_2, \ldots, act_k}} \gamma_d s \in_v Q \), then there exist concrete states \( s_1, s_2, \ldots, s_m \) in \( \llbracket A \rrbracket_v \) and \( i \in 1, 2, \ldots, m-1 \) such that \( s_1 \in_v Q \), \( s_i \equiv_{A_v} s_m \), and for each \( j \in 1, 2, \ldots, m-1 \), \( s_j \xrightarrow{\alpha_{act_1, act_2, \ldots, act_k}} \gamma_d s_{j+1} \).

**Lemma A.9.** Let \( Q = (l, [C, D]) \) be symbolic states in \( \llbracket A \rrbracket^{\alpha} \) such that \( v \in C \). If \( Q \xrightarrow{\alpha_{act_1, act_2, \ldots, act_k}} \gamma_d s \in_v Q \), then there exist concrete states \( s', s'', s''' \) in \( \llbracket A \rrbracket_v \) such that \( s' \in_v Q \), \( s' \xrightarrow{\alpha_{act_1, act_2, \ldots, act_k}} \gamma_d s'' \), \( s'' \xrightarrow{\alpha_{act_1, act_2, \ldots, act_k}} \gamma_d s''' \), and \( s''' \equiv_{A_v} s''' \).

**Lemma A.10.** Let \( s_1, s_2 \) be concrete states in \( \llbracket A \rrbracket_v \). If \( s_1 \xrightarrow{\alpha_{act_1, act_2, \ldots, act_k}} \gamma_d s_2 \) and \( s_1 \equiv_{A_v} s_2 \), then there is an infinite sequence of concrete states \( s_1 s_2 \ldots \) in \( \llbracket A \rrbracket_v \) such that for each \( i \geq 1 \), \( s_i \xrightarrow{\alpha_{act_1, act_2, \ldots, act_k}} \gamma_d s_{i+1} \).

**Lemma A.11.** Let \( Q = (l, [C, D]) \) be a symbolic state in \( \llbracket A \rrbracket^{\alpha} \) such that \( v \in C \). If \( Q \xrightarrow{\alpha_{act_1, act_2, \ldots, act_k}} \gamma_d Q \), then there is an infinite sequence of concrete states \( s_1 s_2 \ldots \) in \( \llbracket A \rrbracket_v \) such that \( s_1 \in_v Q \), and for each \( i \geq 1 \), \( s_i \xrightarrow{\alpha_{act_1, act_2, \ldots, act_k}} \gamma_d s_{i+1} \).

**Theorem (Theorem 3.12).** Let \( A = ([L, l_0, X, \Delta, Inv], F) \) be a PTBA and \( \alpha \) be a finite abstraction. For each parameter valuation \( v \) the following holds: there exists an accepting run of \( \llbracket A \rrbracket_v \) if and only if there exists an accepting run respecting \( v \) of \( \llbracket A \rrbracket^{\alpha} \).

**Proof.** The fact that the existence of an accepting run of \( \llbracket A \rrbracket_v \) implies the existence of an accepting run respecting \( v \) of \( \llbracket A \rrbracket^{\alpha} \) can be proved easily for each valuation \( v \) by induction and Lemma A.5.
Now we give the proof for the other direction. If the \([A]^\alpha = (Q_A, Q_{init}, \vDash^\alpha)\)
over \(L\) has an accepting run respecting \(v\), then there exists a \(Q = (l, \llbracket C, D \rrbracket) \in Q_A\) and \(act_0, act_1, \ldots, act_k \in L\) such that \(Q_0 \xrightarrow{act_0, act_1, \ldots, act_k} \cdots \xrightarrow{act_k} Q\) and \(Q \cap \{act_1, act_i+1, \ldots, act_k\} \neq \emptyset\) where \(Q_0\) is the initial state of \([A]^\alpha\) and \(v \in C\).

Applying Lemma A.11 to \(Q_i \xrightarrow{act_1, act_i+1, \ldots, act_k} \alpha Q_i\), we have an infinite sequence of states \(s_2', s_3', s_4' \ldots\) such that \(s_2' \in \llbracket Q, \alpha \rrbracket\) and for each \(j \geq 2\),
\(s_j' \xrightarrow{act_1, act_i+1, \ldots, act_k} \alpha s_{j+1}'\).

Applying Lemma A.6 to \(Q_0 \xrightarrow{act_0, act_1, \ldots, act_i-1} \alpha Q_i\) and \(s_2' \subseteq \llbracket Q \rrbracket\), it follows that there exist \(s', s''\) such that \(s' \in \llbracket Q_0, \alpha \rrbracket\), \(s' \xrightarrow{act_0, act_1, \ldots, act_i-1} d s''\), and \(s' \neq s''\).

By \(s' \subseteq \llbracket Q_0\) and \(Q_0 = \alpha(S_0)\), we know that there exists a \(s_1 \in \llbracket S_0\) such that \(s' \neq s_1\). From the fact that \(s'' \xrightarrow{act_0, act_1, \ldots, act_i-1} d s_2\) and \(s' \neq s_2\), we know that there exists a \(s_2\) such that \(s_1 \xrightarrow{act_0, act_1, \ldots, act_i-1} d s_2\), and \(s'' \neq s_2\). Thus we have obtained that \(s_2' \neq s_2\).

Applying Lemma A.2 to \(s_1' \subseteq s_2\) and \(s_j' \xrightarrow{act_1, act_i+1, \ldots, act_k} d s_{j+1}'\) \((j = 2, 3, \ldots)\), we can obtain an infinite sequence of states \(s_3, s_4, \ldots\) such that \(s_2 \xrightarrow{act_1, act_i+1, \ldots, act_k} s_{j+1}'\).

Furthermore, from the fact that \(s_1 \in \llbracket S_0\) it follows that there is a \(d \in \mathbb{R}_{\geq 0}\) such that \(s_0 \xrightarrow{d} s_1\) where \(s_0\) is the initial state of \([A]_v\).

Thus, we have proved that there exists an infinite sequence of states \(s_1, s_2, \ldots\) such that \(s_0 \xrightarrow{d} s_1 \xrightarrow{act_0, act_1, \ldots, act_i-1} d s_2 \xrightarrow{act_i, act_i+1, \ldots, act_k} s_3 \xrightarrow{act_0, act_1, \ldots, act_i-1} d s_{j+1}\) \((j = 2, 3, \ldots)\). Now, by the fact that \(F \cap \{act_1, act_i+1, \ldots, act_k\} \neq \emptyset\), we know that \([A]_v\) has an infinite accepting run. \(\square\)

## B Proof of Theorem 4.2

**Lemma B.1.** If the valuation \(v\) is added to the set \(\text{Found}\) then \(v\) is returned by algorithm in the set \(\text{Accepted}\).

*Proof.* This follows from the fact that the set \(\text{Found}\) is never decreased and at the end of computation it is assigned to \(\text{Accepted}\).

**Lemma B.2.** Let \(A\) be a PTBA and \(q\) be a state in \([A]\) that does not appear on any cycle under \(v\). The OuterDFS procedure will backtrack from \(q\) only after every reachable state \(s\) such that \(v \in s.\llbracket C \rrbracket\) is already backtracked or \(s.\llbracket C \rrbracket \subseteq \text{Found}\).

*Proof.* Consider an arbitrary state \(s\) such that \(s\) is reachable from \(q\). At the time of backtracking from \(q\) there are two cases:

- Every path from \(q\) to the state \(s\) contains state \(s'\) such that \(s'.\llbracket C \rrbracket \subseteq \text{Found}\). The fact that \(s\) is reachable from \(s'\) implies \(s.\llbracket C \rrbracket \subseteq s'.\llbracket C \rrbracket\) (using Lemma 4.1). Hence, \(s.\llbracket C \rrbracket \subseteq \text{Found}\).
There exists path from $q$ to the state $s$ such that for every state $s'$ on that path holds $s'.[C] \not\subseteq \text{Found}$. In this case, the OuterDFS procedure has visited state $s$ with state $q$ on the stack. Hence, the OuterDFS procedure backtracks from the state $q$ after backtracking from $s$.

Lemma B.3. For every parameter valuation $v$, the Cumulative NDFS algorithm returns the set $\text{Accepted}$ containing valuation $v$ if and only if the given graph contains an accepting cycle $c$ under a valuation $v$.

Proof. Whenever the algorithm returns a set $\text{Accepted}$ containing $v$ there exists an accepting cycle $c$ under $v$. Such an accepting cycle can be constructed using OuterDFS and InnerDFS search stack at the time of adding the valuation $v$ to the set $\text{Found}$.

The difficult case is to show that whenever there exists an accepting cycle under $v$ in the given graph then the algorithm returns a set $\text{Accepted}$ containing $v$. Suppose an accepting cycle under a valuation $v$ exists in the given graph and the algorithm returns a set $\text{Accepted}$ such that $v \not\in \text{Accepted}$. Let $q$ be the first accepting state on a cycle under $v$ from which InnerDFS is started. There are two cases:

- There exists a path from a state $q$ to some state on the stack of OuterDFS and each state $s$ on the path is unvisited by InnerDFS and $s.[C] \not\subseteq \text{Found}$ at the time of starting InnerDFS from $q$.
- For all paths from a state $q$ to some state $p$ on the stack of OuterDFS there exists a state $s$ on the path such that $s.[C] \subseteq \text{Found}$ or $s$ is a state already visited by InnerDFS.

For the first case the algorithm will detect an accepting cycle as expected and will add the valuation $v \in q.[C]$ to the set $\text{Found}$. From Lemma B.1 we get $v \in \text{Accepted}$ and we have reached a contradiction with the assumption $v \not\in \text{Accepted}$.

For the second case, whenever the path $q \rightsquigarrow p$ contains state $s$ such that $v \in s.[C] \subseteq \text{Found}$ we reach contradiction (using Lemma B.1). Assume that for each state $s$ on path $q \rightsquigarrow p$ holds $v \not\in s.[C]$. Let $r$ be the first visited state that is reached from $q$ during InnerDFS and is on a cycle through $q$. Let $q'$ be an accepting state that started InnerDFS in which $r$ was visited for the first time. Notice the fact that InnerDFS was started from $q'$ before starting from $q$. There are two cases:

- The state $q'$ is reachable from $q$. Then there is an accepting cycle $c' = q' \rightsquigarrow r \rightsquigarrow q \rightsquigarrow q'$. If $c'$ contains state $s$ such that $v \in s.[C] \subseteq \text{Found}$ we reach a contradiction using Lemma B.1. Suppose there is no state $s$ with $v \in s.[C] \subseteq \text{Found}$ on the cycle $c'$. The cycle $c'$ was not found previously. However, this contradicts our assumption that $q$ is the first accepting state from which we missed a cycle.
The state \( q' \) is not reachable from \( q \). Notice the fact that \( v \in q'[C] \) (this follows from Lemma 4.1) and therefore every cycle containing state \( q' \) is a cycle under \( v \). If \( q' \) appears on a cycle, then an accepting cycle under \( v \) was missed before starting \( \text{InnerDFS} \) from \( q \), contrary to our assumption. If \( q' \) does not appear on a cycle then by Lemma B.2 we backtracked from \( q \) in the \( \text{OuterDFS} \) before backtracking from \( q' \) and therefore \( \text{InnerDFS} \) started from \( q \) before starting from \( q' \). We have reached a contradiction with the fact that \( \text{InnerDFS} \) started from \( q' \) before starting from \( q \).

**Lemma B.4.** The \( \text{CumulativeNDFS} \) algorithm always terminates.

**Proof.** From the fact that the number of vertices is finite we get that the size of the sets \( \text{Inner} \) and \( \text{Outer} \) is bounded. Each invocation of \( \text{InnerDFS} \) (\( \text{OuterDFS} \)) procedure increases the size of the set \( \text{Inner} \) (\( \text{Outer} \)). Hence, the \( \text{CumulativeNDFS} \) algorithm cannot proceed infinitely due to the upper bound on the size of the set \( \text{Inner} \) and \( \text{Outer} \).

**Theorem (Theorem 4.2).** Let \( A \) be a PTBA and \( \alpha \) an abstraction over \( [A] \). A parameter valuation \( v \) is contained in the output of the \( \text{CumulativeNDFS}([A]^\alpha) \) if and only if there exists an accepting run respecting \( v \) of \( [A]^\alpha \).

**Proof.** By Lemma B.4 the algorithm is guaranteed to terminate returning the set \( \text{Accepted} \). The partial correctness, the \( \Rightarrow \) case: By Lemma B.3 for each \( v \in \text{Accepted} \) there exists an accepting cycle under \( v \) and for each \( v \notin \text{Accepted} \) there is no accepting cycle under \( v \). The partial correctness, the \( \Leftarrow \) case: Analogously.