Teachers’ changes when addressing the challenges in unexpected migration to online mathematics teaching during the COVID-19 pandemic: a case study in Shanghai

Xingfeng Huang1 · Mun Yee Lai2 · Rongjin Huang3

Accepted: 5 May 2022 / Published online: 2 June 2022
© FIZ Karlsruhe 2022

Abstract
In the research reported in this paper we investigated teachers’ changes when adopting and adapting to emergency online teaching during the COVID-19 pandemic, from the perspective of the Interconnected Model of Professional Growth (IMPG). By adapting complementary accounts methodology to research into teachers’ changes when addressing the unexpected migration to online teaching, an integrated data set, including online teaching videos, teacher daily reflections, and teacher interviews from two purposefully selected teachers over two weeks of online teaching, was collected and analyzed qualitatively. Both teachers encountered different difficulties and thus had different knowledge changes displayed in different change routes. For the experienced teacher, students’ mistakes in homework and her online teaching practice triggered her knowledge changes. For the young teacher, the online video lessons, relevant resources on the Internet and students’ performance were her primary sources that triggered the changes of her knowledge for teaching. These differences between the experienced teacher and young teacher provide evidence of the complexity of teacher’s professional growth, which is related to a variety of external and internal factors. This study demonstrates how the IMPG model helps uncover teachers’ changes in such an unprecedented virtual-teaching environment. Finally, the implications of this study for teacher professional development in general are discussed.

Keywords Interconnected model of Professional Growth · Complementary accounts methodology · Teacher’s change routes · Online teaching

1 Introduction
Since early 2020, the COVID-19 pandemic has forced the physical closure of schools by pushing all schools, colleges, and universities to remodel their typical face-to-face teaching to online teaching. Migrating the physical learning environment to online platforms has become a mandatory task for almost all teachers and educators. Responses to emergencies have varied around the world. The Chinese government launched the ‘School Out but Class On’ campaign (i.e., suspending classes without stopping learning) to provide platforms and models for online learning at home, while local educational practitioners adapted their own models and support systems based on their local needs and availability for online teaching (Yao et al., 2020). Even though this unexpected re-modeling of teaching put a heavy burden on teachers, extensive teacher training programs for developing teachers’ digital competency were not available during the pandemic time. This unique situation provided an opportunity for unprecedented insight into teachers’ changes in response to changing conditions in which emergency online teaching became mandated while relevant training programs were unavailable. Researchers have called for curating what ‘people have learned or are going to learn about the current situation, for the benefit of future crises and times of stability’ (Bakker & Wagner, 2020, p. 4).
To support the shift to online teaching in Shanghai, China, the Shanghai Education Commission led expert teachers and specialists to develop a series of online video lessons (OVLs) based on the Shanghai unified curriculum. Each complete online class included two parts, namely, students watching an OVL individually, followed by an online synchronous lesson (OSL). Although some existing studies investigated the challenges in mathematics teaching and learning during the pandemic period from a macro perspective such as those dealing with cultural, social and political issues, some micro aspects such as teachers’ learning processes throughout their online teaching were largely underexplored (Chan et al., 2021). Inspired by the notion of complementary accounts methodology for classroom research (Clarke, 1997, p. 98), in this research we analyzed three data sets (i.e., recordings of OSLs, teacher daily reflections, and interviews) from the perspective of Interconnected Model of Professional Growth (IMPG) (Clarke & Hollingsworth, 2002) to investigate teachers’ changes when addressing challenges in online teaching.

2 Research background and theoretical framework

In this section, we present relevant literature and the research background. Then, the theoretical framework is described.

2.1 Studies on teaching mathematics online

Researchers have explored how to use technologies in mathematics classroom teaching (Clark-Wilson et al., 2020) and transform classrooms with the Internet (Engelbrecht et al., 2020), and have presented the benefits, strategies, and challenges. Yet, how to teach mathematics online in a hybrid model, both synchronously and asynchronously, is still a new field (Di Pietro et al., 2020; Ferdig et al., 2020). With the support of the Internet and technologies, it is possible to provide student-oriented tutoring and self-guided learning systems (Engelbrecht et al., 2020). However, various overwhelming challenges emerged in virtual environments, such as how to teach a large class with over 20 students and with limited teacher-student interactions (Goei et al., 2021; Huang et al., 2021). How to effectively adapt various platforms and applets, initially developed for commercial purposes, in online teaching is also a challenge (Joubert et al., 2020). More research-based effective strategies for conducting online teaching in large classes in a hybrid mode were needed (Clark-Wilson et al., 2020; Engelbrecht et al., 2020).

2.2 Studies on teacher professional learning

Opfer & Pedder (2011) conceptualized teacher professional learning from a complex system perspective which is related to the teacher, school, and education system. They criticized the failure of oversimplified linear models (Guskey, 1986) and appreciated the interconnected and iterative teacher professional growth model (Clarke & Hollingsworth, 2002). According to Clarke & Hollingsworth (2002), teacher change has six interpretations. Given the specification of this study, teachers’ change is regarded “as adaptation—teachers adapt their practices to changed conditions” (e.g., migrating to online teaching) and “as growth or learning—teachers are themselves learners who work in a learning community” (e.g., adapted institutionalized teaching research groups) (p. 948). Adopting the complementarity of cognitive and situative perspectives in learning (Clarke, 2001; Greeno, 1997), Clarke and Hollingsworth proposed the Interconnected Model of Professional Growth (IMPG, shown in Fig. 1). It assumes that teachers’ changes of knowledge and practices occur through interactions (i.e., reflection and enactment) among four domains. The four domains are the External Domain, which refers to external resources or stimulus, the Personal Domain, which constitutes individual teacher’s knowledge, attitudes, and beliefs, the Domain of Practice, which denotes teacher’s professional actions, and the Domain of Consequence, which comprises the inferred consequences of Domain of Practice. In IMPG, the mechanism of a cyclic learning path is mediated through the teacher’s reflection and enactment, through which change in one domain is translated into a change in another domain (Clarke & Hollingsworth, 2002) emphasized the difference between action and enactment. The enactment is deliberately

Fig. 1 The Interconnected Model of Professional Growth (adopted from Clarke & Hollingsworth 2002)
chosen to stress ‘a belief or a pedagogical model into action from simply acting, on the ground that acting occurs in the domain of practice, and each action represents the performance of something a teacher knows, believes or has experienced’ (p.951). Within the change environment, a teacher’s change (i.e., adaptation to new teaching practices and professional learning) occurs through interactions between different domains along with his/her reflective and/or enactive processes.

This model has been widely used to investigate teachers’ professional learning (e.g., Justi & Driel 2006; Hilton et al., 2015; Witterholt et al., 2012; Widjaja et al., 2015; Wilkie 2019). A systematic review of studies on mathematics teachers’ learning based on the IMPG framework (Goldsmith et al., 2014) revealed that teachers’ learning occurs in small, incremental, and iterative steps, demonstrating the usefulness of this framework in capturing teacher learning. However, most of the existing research reported how IMPG was used to investigate teachers’ knowledge changes after engaging in face-to-face professional development programs or in collaborative situations (e.g., Justi & Driel 2006; Wilkie, 2019). The IMPG was employed as an analytical framework in this study because the findings of some previous studies (e.g., Hilton et al., 2015; Wilkie, 2019) revealed that IMPG was effective in identifying both the knowledge changes and different learning paths that may occur in individual teachers.

2.3 Teacher knowledge for teaching decimals online

Within the study, the mathematical focus of the lessons was decimals. Therefore, a brief summary of teacher knowledge for teaching decimals online follows.

Decimals are an extension of base-10 whole numbers and are constructed on a concept of units (Strother et al., 2016). The essential concept of unit defines decimal place value that any decimal place value is created by equally partitioning “the previous place value units into ten equivalent parts” (Strother et al., 2016; p. 133). This fundamental knowledge of quantities forms the basis of decimal notation such that the place value on the immediate right of ‘one’ denotes tenths (e.g., 0.1), on the immediate right of tenths denotes hundredths (e.g., 0.01), and so on. Using the framework of MKT (Ball et al., 2008), The conceptual knowledge and reasoning for justifying the mathematical procedures and their legitimacy are identified as Specialized Content Knowledge (SCK). It is a set of knowledge limited to classroom teaching and practices. An example of SCK includes “knowing why the decimal point shifts to the right when multiplying with ten or why annexing a zero does not change the decimals” (Takker & Subramaniam, 2019, p. 260). Horizontal Content Knowledge (HCK) refers to a set of knowledge for connecting different mathematics topics, such as making connections between integers, fractions and decimals. Identifying student’s mistakes and understanding their difficulties are parts of Knowledge of Content and Students (KCS). It is a set of knowledge combining knowledge about students and mathematics. For instance, adding a zero to make the length of two decimals congruent for comparing their size is a common strategy among students who have not fully mastered the concept of decimal place value. Knowledge of Content and Teaching (KCT) combines knowing about teaching and mathematics. Selecting appropriate teaching materials, sequencing examples, and explaining mathematically with adequate representations, are part of KCT. For instance, using the part-whole concept and different representations can support students to understand unit conversion (e.g., 10 centimeters = 10/100 meters = 0.1 m).

In addition to the MKT framework, in the study we also adopted one concept from the framework of Technology, Pedagogy, and Content Knowledge (TPACK)—Technological Pedagogical Knowledge (TPK) (Mishra & Koehler, 2006), because the online teaching included the use of videos and a platform. TPK refers to the knowledge about how technology supports effective teaching that enhances students’ learning, without considering the subject matter. Although the TPACK framework encompasses seven knowledge categories, only TPK was adopted because the technologies used in online teaching are mainly presenting/sharing/communicating technologies (rather than mathematical action technologies) (Dick & Hollebrands, 2011).

2.4 Research questions

In this study we aimed to answer the following two research questions, using the lens of IMPG:

1. How did the teachers develop their knowledge through addressing the challenges in online teaching?
2. How did the interactions among different domains influence the changes in teachers’ knowledge?

3 Methods

In the Learner’s Perspective Study, complementary accounts methodology was developed to understand how learning occurred in a complex social setting of classroom (Clarke, 1997). This methodology is distinguished from other approaches to classroom research in terms of the construction of ‘integrated data sets’ which include the reflective voice of different participants, and an analytical approach that utilises a research team with complementary but diverse expertise to conduct a multi-faceted analysis of a common body of classroom data (Clarke, 1997, 2001). According to
Although there are differences in classifying and defining teacher professional developmental stages across different cultural contexts and social systems (Li, et al., 2011), the experienced teacher and young teacher are generally considered as falling at opposite ends of the teacher professional development spectrum. The reason for investigating one experienced teacher and one young teacher in this study was to examine and make explicit the tacit understanding of the teaching knowledge changes of both teachers (Chi, 2011).

3.3 Defining the components of the four domains in this study

A detailed definition of the meaning of each of the domains in the context of this study is essential to facilitate the analysis of the data (see Fig. 1). The domains are as follows:

- **External domain**: This domain includes the recorded lectures (e.g., OVLs), textbooks, lesson plans, and support and materials provided by TRG. They serve as the agency for the instructional design and teaching of the OSLs.
- **Domain of practice**: This refers to “teachers’ classroom practice, in particular the implementation of the improved teaching design and teachers’ behavior” (Witterholt et al., 2012, p. 665), which was designed to facilitate students’ learning, expression, and negotiation of ideas during the OSLs.
- **Personal domain**: This aspect focuses on the teachers’ knowledge, which refers to the different components of MKT and TPK as discussed in Sect. 2.3.
- **Domain of consequence**: In this study, the salient outcomes are interactions between teacher and students or interactions among students, and students’ performance in homework.

3.4 Data sources

This study involved researchers from diverse but complementary research expertise, in order to conduct a multi-faceted analysis of the integrated data set (i.e., OSLs, teacher daily reflections, and teacher interviews) collected from the two teachers over two weeks. The research expertise of the first author is in mathematics curriculum, the second author in student learning and the third author in teacher teaching. The comprehensive expertise of the research team enabled us to examine the data from the perspective of IMPG in a holistic and in-depth way.

3.4.1 Online synchronous lessons

A total of 22 OSLs (11 from the experienced teacher and 11 from the young teacher) were recorded through the online teaching platform, focusing on the teachers’ activities during
the lesson. Each OSL lasted for about 20 min. Everything the teachers said was subsequently transcribed. These transcriptions had the aim of examining teachers’ enactment. Schön (1987) emphasized that an epistemology of teaching professional practice should be grounded in teachers’ classroom teaching practice. Observation of a teacher’s teaching practice is vital, for other teachers and education researchers to access his/her knowledge of ‘knowing-how’ (to teach well) because this is particularly apparent in action; as such Schön named it knowing-in-action (Russell, 1993). Schön asserted that teacher knowledge in their hearts may not be easy to express in words but is prominently manifested in their enactment.

3.4.2 Teachers’ daily reflections

Teachers’ daily post-lesson reflections were collected. The daily reflections had the purpose of revealing the rationale of their teaching practices, their perception of successful and unsuccessful teaching practices, major changes in teaching practices, and the reasons for the changes in the OSLs over the two weeks of the study. Schön (1987) encouraged teachers to engage in reflection-in-action, in which teachers reviewed their teaching practice in action and evaluated students’ learning outcomes. The process of reflection-in-action involves teachers with active interpretation and development of knowledge in terms of personally and professionally relevant frames of thinking, which may lead to reframing their experience and new ways of seeing the situation (Russell, 1993).

3.4.3 Teacher interviews

After observing the teachers’ online teaching for two weeks, a semi-structured interview with each teacher was conducted for about 45 min. The teachers were expected to provide holistic reflections on their online teaching experience during the two weeks. The interview had the purpose of understanding (1) the difficulties in online teaching they encountered, (2) how they tried to understand their students’ learning difficulties, and (3) the general and specific knowledge for teaching that the teachers learnt.

3.5 Data analysis

The data analysis included the following four phases:

Phase 1: Analysis of lesson observations. The recordings of all the OSLs were transcribed verbatim for data analysis. The first and second researchers of the study watched all the OSLs along with the transcribed texts to identify the teaching moments that reflected changes in the teachers’ teaching practices (i.e., Enactment).

Phase 2: An in-depth interpretation of the teachers’ post-lesson daily reflections was conducted by the first and second researchers separately, to understand how the teachers evaluated their teaching per se, how they thought their teaching influenced students’ learning, and what improvement would be implemented in the subsequent lessons (i.e., Reflection). These data provided evidence of the identified changes of teaching practices that were observed in Phase 1, and were coded into the five types of knowledge mentioned in Sect. 2.3 (SCK, HCK, KCS, KCT and TPK). For any inconsistencies between the two researchers, the third researcher read closely the corresponding transcript texts of lessons and daily reflections, to make changes where appropriate. The identified changes in the teachers’ knowledge are reported in the result section and received consent from all the three authors.

Phase 3: Triangulation of interview data. Data from the interviews were used to confirm and support the changes in teachers’ practices and knowledge identified in the first two phases. When the interview data revealed any inconsistency with the identified changes, the researchers revisited the recordings of the OSLs and daily reflections, to sort out the reasons for the inconsistency and to adjust the findings when necessary.

Phase 4: Building the teachers’ change paths and routes. Based on the identified changes in teacher knowledge in the first three phases (see a teacher’s sample in the 6., and another one, as supplementary material, due to the limited space in this paper), we sought the interactions (enactment and reflection) among different domains, which resulted in the changes in practice and knowledge in the Personal Domain and the Domain of Practice. The OSLs, along with evidence from daily reflections, were then used to generate the paths (enactment or reflection) to represent the change sequences among domains. For each teacher, the change routes between the four domains were identified according to the paths as shown in Figs. 2 and 3, which are explained and evidenced in the Sect. 4.

4 Findings

The learning routes of the experienced teacher and young teacher were very different. The following sections describe their changes in practice and knowledge and the related learning routes in detail.

4.1 The experienced teacher

The connections among the four domains of the experienced teacher are illustrated in Fig. 2. The solid line represents the change through Enactment, and the dotted line represents
3.1  Route 1 of change in KCS through using student homework to promote students’ understanding: path 1 → path 4 → path 3

As recommended by the Shanghai Education Commission, the teaching model during the pandemic was that students watched an OVL independently at home, then the class teacher interacted with the students online, and finally the students completed and submitted their homework to their class teacher. Before the OSL2, she enacted the recommended model but found it was ineffective because she was not able to instantly address her students’ learning difficulties. She decided to change the teaching model by requiring her students to complete and submit their homework right after watching the OVL (Enactment path 1). She then quickly marked the homework and adjusted her online teaching based on the students’ mistakes to rectify their misconceptions (Reflection path 4 and Enactment path 3). The teacher found this teaching strategy very helpful and made the following comment.

Zhu: After reviewing my students’ homework [before the OSL], I was able to understand more about my students’ learning difficulties that I did not encounter before. With this, I knew how to adjust my online teaching [OSL] plan. With the adjustment made to the learning content in the online teaching [OSL], the overall performance of student learning met my expectations eventually. Using my model of teaching, I learned that understanding students’ learning outcomes is a key to effective teaching especially when they learn from home. (Daily reflection 2)

The teacher’s response in the interview was further evidence that reviewing students' homework prior to the OSLs enabled her to better understand students’ misconceptions and learning difficulties, which helped adjust the teaching goals and foster student interactions during the OSL. It also helped her to make a better connection between OVL and OSL. Ms. Zhu mentioned that the biggest problem in learning through watching the OVLs (compared to in a normal classroom) was that students could not physically participate in collective interactions, formative assessment, and manipulating materials, all of which could enhance their concept construction. As a result, the mistakes that appeared in students’ homework that rarely occurred in the past inspired her to have a new understanding of students’ difficulties in learning decimals (change in KCS). For example, after watching the OVL, most students still did not understand why $0.6 = 0.60$ and thus committed many mistakes in their homework. When she was reviewing the homework, she found out that students’ difficulty was due to their struggle to understand the place value: the students did not understand why 60 groups of 0.01 is equal to 6 groups of 0.1.

Overall, the student homework became a useful resource for her to facilitate online learning.

Zhu: I can better understand my students’ learning difficulties through reviewing their homework prior to the lesson. It helps me clear up my teaching goal. I can use their mistakes identified in the homework to facilitate student discussion. …For online teaching, three things I must consider are understanding students’ difficulties, clarifying teaching goals, and encouraging students’ interactions. (Interview)
4.1.2 Route 2 change in KCT through revisiting the OVLs to support student further understanding: path 1 → path 2 → path 4 → path 3

In the OSL 2, Ms. Zhu re-used some slides from the OVL to further explain some important mathematical concepts. However, in the OSL 4 when she highlighted the examples that had been already taught in the OVL 4, some of her students did not have any idea, and some of them even did not notice these examples. In her reflection, Ms. Zhu expressed her disappointment and mentioned that she would revisit the important concepts and examples in every subsequent OSL.

Zhu: I am really disappointed. These are important examples that have been explained clearly in the video [OVL]. Why didn’t the students notice at all? There were so many mistakes in their homework. I will discuss the important concepts in every lesson [OSL].
(Daily reflection 4)

She then, from the OSL 4, adjusted her teaching strategy by requesting her students to summarize the significant mathematical content that was delivered in the OVLs at the beginning of the OSLs. It was followed by discussing students’ common mistakes that were identified in their homework (Enactment path 1 and path 2). However, this was not effective in engaging those students who had watched the OVLs and who understood the concepts. She then gave her students a new question that was built on the examples and knowledge learnt from the OVLs (change in KCT) to promote further understanding (Enactment path 4 and Enactment path 3). For example, in the OVL 6, the students learnt about the movements of the decimal point when 0.001, 0.01 and 0.1 were multiplied or divided by 10, 100 and so on. A calculator was then used to verify the movements of the decimal point when 0.111 was multiplied or divided by 10 and 100. In the OSL 6, Ms. Zhu first summarized this concept and then asked the students to use the knowledge of place value and distributive law to think about other ways to explain for the decimal point movement when 0.111 was multiplied by 10: \[0.111 \times 10 = (0.1 + 0.01 + 0.001) \times 10 = 1 + 0.1 + 0.01 = 1.11\]. This change of teaching strategy revealed the change in her KCT, which enabled her to develop new but related questions to check as well as to promote students’ understanding of some important concepts. The interview further supports this observation.

Zhu: When using the content from the videos [OVLs], I wanted to encourage students to think deeply. With the examples used in the videos [OVLs], I learned about how to design new but related questions for online teaching. Also, I had to rethink the design of the videos [OVLs], the textbook and my students’ homework to design online teaching.

4.1.3 Route 3 of change in TPK through using platform interactive functions: path 1 → path 5 → path 6 → path 3

Ms. Zhu found that the limitations of the online teaching platform impeded her teaching. First, the online teaching platform did not allow her to see all the students: their facial expressions and body gestures that might reflect their understanding, were invisible. Second, if students wanted to answer the teacher’s questions, they had to press a hands-up button to indicate this, then the teacher switched on the microphone for them to speak. However, the teacher could see only up to six students, and only one student could speak at one time on this platform; thus class interactions were greatly reduced. Third, if the teacher wanted a particular student to speak, she had to search for this student from a long name list displayed on her screen. To save time, the teacher chose only the students who put their hands up in the first few OSLs. However, most of the time, those students who put their hands up were the high achievers. She then realized that only presenting students’ mistakes to promote class discussion was insufficient. To better understand how the low achievers interpreted the concepts, she also needed to listen to their explanations (Enactment path 1). As the class had 45 students, it was not easy to search for her targeted students from such a long name list in a short period of time on the screen. This adversely impacted her teaching pace and flow. She mentioned this issue in her OSL 4 daily reflection (Reflection path 5).

Zhu: Since the online platform does not allow me to see all my students at one time, I need to remind them to stay online and to participate actively throughout the lesson. At present, I use a hands-up approach to invite students to talk about their understanding. But I always miss a few students who truly have difficulties with their homework. … It would save some time if I could identify those students who are having difficulties and quickly switch on their microphones for them.
(Daily reflection 4)

To address this issue, she explored how to use different functions on the platform to support her online teaching (Reflection path 6 and Enactment path 3). In the OSL 4 she changed her teaching practice. Before the OSL started, she turned on the microphones for her target students in advance. When the students were called, they could immediately give responses (change in TPK). However, in the OSL 5, some students used the chat function to privately communicate with other students when the teacher was teaching. To improve students’ online engagement and seek their attention, she occasionally asked students some simple ‘yes or no’ questions and provided their answers using the hands-up button (change in TPK). However, Ms. Zhu mentioned
in her OSL 8 reflection that though she had tried to engage her students, their performance was still not satisfactory, especially for the students who struggled to learn.

In the interview, we noticed that Ms. Zhu had established a deep understanding of teaching decimals through working collaboratively with other teachers on developing the OVLs. She appreciated the learning opportunity and made the following comment:

Zhu: The process of developing the videos [OVLs] was a very significant learning opportunity for me. I worked with specialists [from the Education Commission] and colleagues [at her school] to discuss and revise the teaching points, to choose the representations of concepts, and to think about the possible student learning difficulties. Although the process was very tough, I gained a lot, including understanding the teaching content, concepts and pedagogies.

4.2 The young teacher

Ms. Gu used the same set of the OVLs as Ms. Zhu. But unlike Ms. Zhu, Ms. Gu met her students in the OSLs immediately after her students had watched the OVLs. She then assigned student homework at the conclusion of the OSLs. Throughout the two weeks of Ms. Gu’s online teaching, two routes of changes in KCT, SCK and HCK were identified, as illustrated in Fig. 3.

4.2.1 Route 1 of change in KCT through using multiple representations: path 1 → path 5

In Ms. Gu’s reflection, she mentioned several times that the design and content of the OVLs were very helpful and thus she gained a lot of knowledge for teaching from the OVLs (Reflection path 1).

Gu: The teacher in the video lessons used various ways to help students understand the same mathematical concept and such teaching strategies have inspired my teaching. (Daily reflection 2).

She adopted the materials from the OVL and slightly modified them by incorporating different ideas in the lesson plans that were shared among her colleagues (Enactment path 5). For example, in the OVL 1, she found that the visual models including the grids and number line were effective in developing students’ understanding of the properties of decimals such as 0.3 = 0.30. Building on this idea, she believed that the process for converting units—0.30 m = 30/100 meters = 30 centimeters, 0.3 m = 3/10 meters = 3 decimeters = 30 centimeters—could also facilitate students’ understanding. She thus included this in her OSL.

However, when she was not satisfied with the representations used in the OVLs, she searched relevant materials on the Internet, and integrated them into her online teaching. In the OVL 6, she did not think that the number line model was effective for explaining the movement of the decimal point when multiplying or dividing a decimal by 10, 100, and so on. Borrowing from the modelling of whole numbers, she used the base-10 blocks, in which a big cube represents 1, a flat represents 0.1 (one-tenth), a rod represents 0.01 (one-hundredth) and a small cube represents 0.001 (one-thousandth) to support her students’ learning.

Ms. Gu’s lesson design and teaching practice in the OSLs were greatly influenced by the multiple representations introduced in the OVLs and the resources shared between her colleagues, which enhanced her KCT. The evidence from the interview also supported her change in KCT. In the interview, Ms. Gu mentioned how the multiple representations used in OVLs promoted her new understanding of multiple representations, which enabled her to provide students with different ways to understand mathematics. This also inspired her to explore the use of concrete materials to support her students’ understanding of mathematical concepts. She made the following comment, which indicated the change in KCT, in the interview.

Gu: I am now trying to design a scale that can be stretched to help students understand the relationship between millimeters, centimeters, and decimeters. In the textbook and videos [OVLs], the representations of these relationships are pictorial or symbolic, which may be difficult to support students’ concept development. I, thus, tried to use some concrete materials as a kind of representation to develop students’ conceptual understanding.

4.2.2 Route 2 of change in SCK and HCK through promoting conceptual understanding: path 2 → path 3 → path 4 → path 5

Although Ms. Gu found the OVLs very useful, there were still some discussions on the use of OVLs, lesson plans, and resources among her colleagues on social media, Ms. Gu mentioned in her reflections that the discussions were superficial and did not resolve the issues that she encountered in the actual implementation. For instance, the teaching strategy and resources on converting units introduced in the OVL 9 (e.g., 1 L = 1000 milliliters, and 0.98×1000 = 980, so 0.98 L = 980 milliliters) impressed her greatly. She then implemented these in her OSL and requested her students to follow the steps (Enactment path 2). However, when the students were asked to convert between large and small units directly without going through the steps, they encountered
some difficulties (Reflection path 3). She made the following comment (Reflection path 4).

Gu: I’m so worried that some of my low-achieving students are not able to understand unit conversion. So, I attempted to simplify the procedure. .... However, my colleagues had different approaches. Some of them did not emphasize the steps and others required students to follow the steps strictly. I prefer simplifying the procedure by connecting the movement of the decimal point to multiplying/dividing 10, 100, 1000 and so on. (Daily reflection 9)

So, she began to implement her own strategy (Enactment path 5). She created a rule—move the decimal point $n$ place to the right (left) when multiplying (dividing) the decimals by $10^n$. With this rule, she easily converted 0.76 m to 76 cm by moving the decimal point two places to the right because one meter had 100 centimeters. Thus, to convert 0.76 m to centimeters, she had to multiply 0.76 m by 100. She assumed that her students could apply this rule with ease. However, her students’ homework displayed many mistakes (Reflection path 3) and she made the following comment.

Gu: … I … focused on moving the decimal point. … I did not realize that the students do not know the size of different units and the ratios between different units of the same kind. Thus, they do not know when to multiply or divide and by how many. My understanding of students’ learning difficulties is insufficient. … The lesson that struck me was that just getting students to follow procedures doesn’t work when they do not understand. So once again I realized the importance of conceptual understanding. (Daily reflection 10)

She then adjusted her teaching by using the method as shown in the OVL (Reflection path 4 and Enactment path 5) though the steps were still procedure in nature (change in KCT). Driven by the desire for helping students gain conceptual understanding of the unit conversion, she did some study and deepened her understanding as explained in the interview.

Gu: … I found a new way from a textbook. For example, how to convert 0.98 liters to milliliters? Because 1 liter = 1000 milliliters, we know 0.1 liter = 100 milliliters, and 0.01 liter = 10 milliliters. Then 0.9 liters = 900 milliliters, and 0.08 liters = 80 milliliters. Because 0.98 liters = 0.9 liters + 0.08 liters, we can get 0.98 liters = 900 milliliters + 80 milliliters = 980 milliliters. …Based on a basic relationship between units, more quantitative relationships are developed. This is very similar to the place value system. Although the conversion process is a bit slow, the process can promote students’ understanding of unit conversion rather than just quickly applying the procedure.

This interview response indicated that based on the online teaching, reflection and study, Ms. Gu also developed a better conceptual understanding of unit conversion (SCK), and was able to make connections to the place value system (HCK).

### 5 Discussion and conclusions

In this section, we answer the research questions, discuss the contribution and implications, and acknowledge the limitations.

#### 5.1 The teachers’ knowledge changes

The experienced teacher developed her personal online teaching mode different from the officially recommended teaching mode in order to better understand students’ leaning effects after watching the OVLs at home. Thus, the experienced teacher was attentive to making connection between OVLs and OSLs, and to the effective use of students’ mistakes identified in homework as part of her scaffolding to rectify students’ misconceptions. The limitations of the online platform also enabled the experienced teacher to think differently about how to make use of students’ mistakes to promote students’ learning, and eventually to change her KCT, KCS and TPK. This finding could be partly due to the teacher’s knowledge that she had developed when participating in the production of the OVLs. The developed knowledge empowered her to be able to diagnose quickly students’ various mistakes and to provide timely support in her OSLs.

However, the young teacher’s changes in KCT, SCK and HCK were mainly triggered by the OVLs. When she found mistakes in students’ homework, she was not able to figure out the underlying reasons for students’ committing the mistakes. As she could not receive adequate collegial support from her TRG, she attempted to solve her problems drawn from her reflections and searching for relevant resources on the internet. So, the young teacher’s focus was on researching alternative representations and materials to help students understand the mathematical concepts.

The difference in the knowledge change between the experienced and young teacher is aligned with Coenders & Terlouw (2015)’s finding that a group of teachers who developed a set of reform-oriented teaching materials demonstrated stronger gain in overall MKT than a control group of teachers who used the materials to teach only. Thus, for young teachers, external resources such as textbooks, existing teaching materials, and online resources are significant in enhancing their overall MKT. A strong teacher
professional community with qualified facilitators is also beneficial to young teachers. For experienced teachers, the experiences in collectively developing the OVLs could foster their knowledge development. It might be helpful if experienced teachers served as facilitators of an online TRG that includes young teachers and other colleagues.

5.2 Complexity of the factors impacting on teacher professional learning

In the experienced teacher’s knowledge change routes, each route included a two-way interaction of enactment and reflection between the Personal Domain and the Domain of Practice. But for the younger teacher, each route missed the reflection from the Personal Domain to the Domain of Practice. This is a significant difference between the two teachers’ routes in this study. Schön (1987) emphasizes teachers’ reflection through which teachers review their teaching practices to reconstruct their experiences and new ways of looking at situations (Russell, 1993). In this study, it appeared that when the young teacher made any changes in her teaching practices, she did not actively engage herself in reviewing her teaching to sort out the possible ‘gaps’, but researched alternative materials/representations and took them as her ‘new teaching strategies’. In the context of online teaching in the future, the senior management of schools could consider organizing a virtual TRG as a teacher learning community, in which experienced teachers are encouraged to share with young teachers their experiences and reflections on teaching practices. This is an important self-learning process for young teachers to be able to engage themselves in reflection before seeking for solutions. Being able to review their own deficiencies is a significant step in identifying an effective solution.

Regarding the interactions between different domains that trigger changes in teacher knowledge (Personal Domain), there is a major difference between the experienced teacher and young teacher. For the experienced teacher, students’ mistakes in homework (Domain of Consequence) and the deficiency of her online teaching practice (Domain of Practice) triggered her knowledge changes. For the young teacher, the OVLs, relevant resources on the Internet (External Domain), and students’ under-performance (Domain of Consequence) were her primary sources that triggered her knowledge changes. This difference between the experienced teacher and young teacher provides evidence of the complexity of teacher professional learning that is related to a variety of external and internal factors, such as work environment, available resources, personal experience, knowledge and beliefs (Wilkie, 2019; Witterholt et al., 2012). Although the results revealed in this study could not be generalized, the comparison could provide insight into ways for helping teachers address different challenges (Bakker & Wagner, 2020). Teacher professional development programs should be organized to accommodate the different needs of experienced teachers and young teachers.

5.3 Implications for using IMPG model

This study demonstrates that the IMPG could be used to explore teacher professional learning in an unprecedented work environment (e.g., sudden school lockdown, migrating to online teaching without needed preparation). Methodologically, given the difficulties in collecting relevant data during the pandemic lockdown, the complementary accounts methodology (Clarke, 1997; Clarke et al., 2006) (at the level of data) inspired the research team to collect an ‘integrated data set’—videos, daily reflections and interviews—from the two teachers over two weeks. The uniqueness of this study was the holistic and multi-faceted analysis of data using the lens of IMPG through (i) the complementary data set, which is different from the previous IMPG studies that typically used interview and assessment data only, and (ii) the diverse but complementary research expertise of the research team. This study revealed that the teachers’ knowledge changed in the novel environment in a detailed and rich way.

5.4 Limitation

In this study we compared the learning routes of two teachers with very different experiences and backgrounds. Their distinctive learning routes for teaching decimals were identified. However, those teachers who were experienced in teaching, but did not participate in any curriculum and teaching resource development, were not included in this study. Due to the sudden onset of the pandemic and the various social constraints during the pandemic, no data was collected prior to the mandatory online teaching, in order to understand how much change actually occurred in teaching practice. Similarly, no data were collected after teachers returned to normal classroom teaching, in order to see how the developed knowledge was translated into normal classroom teaching.
Appendix

Ms. Zhu’s knowledge change during the two-week online teaching in the pandemic.

| Category of teacher knowledge | Changes of knowledge are related to | Change sequence across the OSLs observed by researchers | Reflection written by teachers in daily reflections | Evidence to support the changes from the interviews |
|-------------------------------|-----------------------------------|--------------------------------------------------------|---------------------------------------------------|--------------------------------------------------|
| Change in KCS through using student homework to promote students’ understanding | Teachers used the students’ homework as a key resource to promote students’ understanding | OSL2: The teacher interacted with the students online based on their mistakes that occurred in homework. (She adjusted the order of components of the OVL as ‘students watching the OVLs at home, then students completing their work at home, and finally teacher interacting with students online’). | Daily reflection 2: After reviewing my students’ homework, I was able to learn the learning difficulties that I unanticipated. With this, I know how to adjust my teaching plan for online teaching. | I followed the recommendation by the Education Commission at the beginning, but I found that I didn’t know what to teach online. The online videos are only twenty minutes long, but have all the content included to be taught. I could not just repeat those contents online. However, I just asked students what they are doing online and then gave them some questions to answer. |
|                               |                                   | OSL3: The teacher used students’ mistakes related to 0.6 = 0.60 as a resource to elicit students’ interactions online | Daily reflection 3: I find students’ mistakes related to 0.6 = 0.60 very interesting. I didn’t notice before. Actually, they have difficulties in understanding place value. | However, I feel that the teaching is not relevant. In addition, the students are learning at home unlike at school, I have no idea how much they have learned through the videos. |
|                               |                                   | OSL6: The teacher used the mistakes on the movement of the decimal point to elicit students’ discussion | Daily reflection 6: The students have so many mistakes related to the movement of the decimal point. I am very surprised by this, but I learned a lot. | Therefore, in order to understand the students’ learning and thus provide targeted online teaching, I changed the order by having students complete the homework first, and then I understood the students’ difficulties through the homework, so the teaching purpose was very clear. |
|                               |                                   | OSL10: The teacher used the mistakes on the unit conversion to enact her online teaching | Daily reflection 10: Why is unit conversion so difficult for my students? Maybe the method used in the video is so procedural that they could not really understand. | I used the students’ problems at homework as resources to facilitate them to discuss online. |

But I think there are three things I must consider: understanding students’ difficulties, clarifying teaching goals, and encouraging student-student interactions.
| Category of teacher knowledge | Changes of knowledge are related to | Change sequence across the OSLs observed by researchers | Reflection written by teachers in daily reflections | Evidence to support the changes from the interviews |
|--------------------------------|------------------------------------|-------------------------------------------------------|-------------------------------------------------|----------------------------------------------------------|
| Change in KCT through revisiting the OVLs to support student further understanding | The teacher used the contents in the OVLs to connect students’ homework with her online interactions | OSL2: Although the teacher always selected the key points and examples from the videos as resources for online interactions, she found that the students seemed unfamiliar with the content of the videos. She began to remind the students to watch the videos carefully and used some slides from the video to explain the key concepts again. OSL4: The teacher began to change her teaching strategy. At the beginning of the online interaction, she first asked the students to recall and summarize what they learned in the video or what impressed them, and then ask further questions on the key points. OSL6: The teacher first used the examples from the video and provided opportunities for the students to briefly review (three minutes) the property of the decimal point movement. The teacher then elicited students to explore this property further. OSL8: The teacher spent a long time (15 min) reviewing the use of inverse operations in the video to check the relation between before and after the movement of the decimal point, and then discussed some questions related to the topic. OSL10 & 11: The online interactions were structured by the teacher as a cycle of ‘review of important concepts in the video – further new questions - discussion of typical issues occurred on the students’ homework’ | Daily reflection 4: I am really disappointed. These are important examples that have been explained clearly in the videos. Why didn’t they notice at all? There were so many mistakes in their homework. I need to discuss the important concepts online again. Daily reflection 6: The students did not learn very well by watching the video at home. In the video, it is not possible to explain the nature of decimal point movement thoroughly because I cannot go too deeply as I have to meet students in different schools. So, in online interaction, I have to explore this property again with my students. Daily reflection 8: I now find it essential to review and address the important contents in the videos online | I learned how to design new questions based on the content of the videos, because I had to rethink the design of the videos, study the textbook and the students’ homework. When using the content from the videos, I wanted to encourage students to think deeply. With the examples used in the videos, I learned about how to design new but related questions for online teaching |
Supplementary Information The online version contains supplementary material available at https://doi.org/10.1007/s11858-022-01378-y.

Funding Shanghai Mathematics Education under the Condition of Online Interaction (2019–2028) supported by the Teacher Education Center under the Auspices of UNESCO.

References

Bakker, A., & Wagner, D. (2020). Pandemic: Lessons for today and tomorrow? Educational Studies in Mathematics, 104, 1–4.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? Journal of Teacher Education, 59(5), 389–407.

Chan, M. C. E., Sabena, C., & Wagner, D. (2021). Mathematics education in a time of crisis—a viral pandemic. Educational Studies Mathematics, 108, 1–13.

Chi, M. T. H. (2011). Theoretical perspectives, methodological approaches, and trends in the study of expertise. In Y. Li, & G. Kaiser (Eds.), Expertise in mathematics education: An international perspective (pp. 17–39). New York: Springer.

Clarke, D. (1997). Studying the classroom negotiation of meaning: Complementary accounts methodology. Journal for Research in Mathematics Education, 9, 98–111.

Clarke, D. (Ed.). (2001). Perspectives on practice and meaning in mathematics and science classrooms. Dordrecht: Kluwer Academic Publishers.

Clarke, D., Emanuelessson, J., Jablonka, E., & Mok, I. (2006). The learner’s perspective study and international comparisons of classroom practice. In D. Clarke, J. Emanuelesson, E. Jablonka, & I. Mok (Eds.), Making connections: Comparing mathematics classrooms around the world (pp. 1–22). Netherlands: Sense publishers.

Clarke, D., & Hollingsworth, H. (2002). Elaborating a model of teacher professional growth. Teaching and teacher education, 18(8), 947–967.

Clark-Wilson, A., Robotti, O., & Thoma, M. (2020). Teaching with digital technology. ZDM – Mathematics Education, 52, 1223–1242.

Coenders, F., & Terlouw, C. (2015). A model for in-service teacher learning in the context of an innovation. Journal Science Teacher Education, 26, 451–470.

Dick, T. P., & Hollebrands, K. F. (2011). Focus in high school mathematics: Technology to support reasoning and sense making. National Council of Teachers of Mathematics.

Di Pietro, G., Biagi, F., Costa, P., Karpinski, Z., & Mazza, J. (2020). The likely impact of COVID-19 on education: Reflections based...
Engelbrecht, J., Borba, M. C., Llinares, S., & Kaiser, G. (2020). Will 2020 be remembered as the year in which education was changed? ZDM – Mathematics Education, 52, 821–824

Ferdig, R. E., Baumgartner, E., Hartshorne, R., Kaplan-Rakowski, R., & Mouza, C. (2020). Teaching, technology, and teacher education during the COVID-19 pandemic: Stories from the field. Association for the Advancement of Computing in Education (AACE)

Goei, S. L., van Joolingen, W. R., Goettsch, F., Khaled, A., Coenen, T., In ’t Veld, de Vries, S. G. J. G., S., & Schipper, T. M. (2021). Online lesson study: Virtual teaming in a new normal. International Journal for Lesson and Learning Studies, 10(2), 217–229

Goldsmith, L. T., Doerr, H. M., & Lewis, C. C. (2014). Mathematics teachers’ learning: A conceptual framework and synthesis of research. Journal of Mathematics Teacher Education, 17, 5–36

Greeno, J. G. (1997). On claims that answer the wrong questions. Educational Researcher, 26(1), 5–17

Guskey, T. R. (1986). Staff development and the process of teacher change. Educational Researcher, 15(5), 5–12

Hilton, A., Hilton, G., Dole, S., & Goos, M. (2015). School leaders as participants in teachers’ professional development: The impact on teachers’ and school leaders’ professional growth. Australian Journal of Teacher Education, 40(12), 104–125

Huang, X. F., Lai, M. Y., & Huang, R. (2021). Teachers’ learning through an online lesson study: An analysis from the expansive learning perspective. International Journal for Lesson and Learning Studies, 10(2), 202–216

Joubert, J., Callaghan, R., & Engelbrecht, J. (2020). Lesson study in a blended approach to support isolated teachers in teaching with technology. ZDM – Mathematics Education, 52, 907–925

Justi, R., & Van Driel, J. (2006). The use of the interconnected model of teacher professional growth for understanding the development of science teachers’ knowledge on models and modelling. Teaching and Teacher Education, 22(4), 437–450

Li, Y., & Kaiser, G. (2011). Expertise in mathematics instruction (pp. 3–5). Boston, MA: Springer

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. Teachers College Record, 108(6), 1017–1054

Opfer, V. D., & Pedder, D. (2011). Conceptualizing teacher professional learning. Review of Education Research, 81(3), 376–407

Russell, T. (1993). Reflection-in-action and the development of professional expertise. Teacher Education Quarterly, 20(1), 51–62

Schön, D. (1987). Educating the reflective practitioner. San Francisco, CA: Jossey-Bass

Strother, S., Brendefur, J. L., Thiede, K., & Appleton, S. (2016). Five key ideas to teach fractions and decimals with understanding. Advances in Social Sciences Research Journal, 3(2), 132–137

Takker, S., & Subramaniam, K. (2019). Knowledge demands in teaching decimal numbers. Journal of Mathematics Teacher Education, 22(3), 257–280

Widjaja, W., Vale, C., Groves, S., & Doig, B. (2017). Teachers’ professional growth through engagement with lesson study. Journal of Mathematics Teacher Education, 20(4), 357–383

Wilkie, K. J. (2019). The challenge of changing teaching: investigating the interplay of external and internal influences during professional learning with secondary mathematics teachers. Journal of Mathematics Teacher Education, 22(1), 95–124

Witterholt, M., Goedhart, M., Suhre, C., & van Streun, A. (2012). The interconnected model of professional growth as a means to assess the development of a mathematics teacher. Teaching and Teacher Education, 28(5), 661–674

Yang, Y. (2009). How a Chinese teacher improved classroom teaching in Teaching Research Group: a case study on Pythagoras theorem teaching in Shanghai. ZDM – Mathematics Education, 41, 279–296

Publisher’s note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.