Comment on

“Single-inclusive jet production in electron-nucleon collisions through next-to-next-to-leading order in perturbative QCD”

[Phys. Lett. B763, 52–59 (2016)]

Geoffrey T. Bodwin$^{1}$ and Eric Braaten$^{2}$

$^1$High Energy Physics Division, Argonne National Laboratory, Argonne, IL 60439, USA

$^2$Department of Physics, The Ohio State University, Columbus, Ohio 43210, USA

(Dated: September 19, 2018)

Abstract

In the cross section for single-inclusive jet production in electron-nucleon collisions, the distribution of a quark in an electron appears at next-to-next-to-leading order. The numerical calculations in Ref. [1] were carried out using a perturbative approximation for the distribution of a quark in an electron. We point out that that distribution receives nonperturbative QCD contributions that invalidate the perturbative approximation. Those nonperturbative effects enter into cross sections for hard-scattering processes through resolved-electron contributions and can be taken into account by determining the distribution of a quark in an electron phenomenologically.

$^1$gtb@anl.gov

$^2$braaten@mps.ohio-state.edu
In Ref. [1], the cross section for single-jet inclusive production in lepton-nucleon collisions is computed through next-to-next-to-leading order in perturbative quantum chromodynamics (QCD). That computation advances significantly the potential for precision comparisons between theory and experiment for this process. The cross section contains a contribution that is proportional to the distribution of a quark in a lepton, namely, \( f_{q/l}(\xi, \mu^2) \), where \( \xi \) is the light-cone momentum fraction of the quark and \( \mu \) is the renormalization scale. Such a contribution could be termed a “resolved-lepton” contribution. The distribution that was used in Ref. [1] is

\[
f_{q/l}(\xi, \mu^2) = e_q^2 \left( \frac{\alpha}{2\pi} \right)^2 \left\{ \left[ (1 - \xi)(4 + 7\xi + 4\xi^2 - 8\xi^3) \log \frac{\mu^2}{m_l^2} \right] \log \xi - \frac{3}{2}(1 + \xi) \log^2 \xi \right\},
\]

where \( m_l \) is the lepton mass, \( e_q \) is the electric charge of the quark, and \( \alpha \) is the quantum-electrodynamics (QED) coupling constant. The single and double logarithms of \( \mu \) cancel the \( \mu \)-dependence of other factors in the cross section at order \( \alpha^2 \). In Ref. [1], \( f_{q/l}(\xi, \mu^2) \) is derived by making use of the Dokshitzer, Gribov, Lipatov, Altarelli, Parisi (DGLAP) evolution equation \( \mu^2 \frac{\partial}{\partial \mu^2} f_{q/l} = P_{q\gamma} \otimes f_{\gamma/l} + P_{ql} \otimes f_{l/l}, \) in the form

\[
\mu^2 \frac{\partial}{\partial \mu^2} f_{q/l} = P_{q\gamma} \otimes f_{\gamma/l} + P_{ql} \otimes f_{l/l}, \tag{2}
\]

Here, \( f_{\gamma/l}(\xi, \mu^2) \) is the distribution of a photon in a lepton, \( f_{l/l}(\xi, \mu^2) \) is the distribution of a lepton in a lepton, \( P_{q\gamma}(z) \) and \( P_{ql}(z) \) are the DGLAP splitting functions, and \( \otimes \) denotes the convolution

\[
[P \otimes f](\xi) = \int_\xi^1 \frac{dz}{z} P(\xi) f(\xi/z). \tag{3}
\]

(In Eq. (2), we have absorbed factors of \( \alpha \) into the definitions of the splitting functions.) In Ref. [1], the splitting functions are evaluated to order \( \alpha \) and order \( \alpha^2 \), respectively, and the QED distributions on the right side of Eq. (2) are evaluated at leading order in \( \alpha \): \( f_{\gamma/l}(\xi, \mu^2) \) is the Weizsäcker-Williams distribution, and \( f_{l/l}(\xi) = \delta(1 - \xi) \). The distribution in Eq. (1) is obtained by integrating Eq. (2) with the boundary condition \( f_{q/l}(\xi, m_l^2) = 0 \).

In this comment, we point out that \( f_{q/l}(\xi, \mu^2) \) receives nonperturbative QCD contributions that invalidate the expression for the distribution of a quark in an electron defined by Eq. (1). If the lepton has a sufficiently large mass, as is the case for the \( \tau \) lepton, then \( f_{q/l}(\xi, m_l^2) \) can be computed in QCD perturbation theory, and it can be evolved perturbatively from
the scale $m_l^2$ to the scale $\mu^2$ in order to absorb logarithms of $\mu^2/m_l^2$ into $f_{q/l}(\xi, \mu^2)$. In this case, the expression in Eq. (1) is a valid approximation for $f_{q/l}(\xi, \mu^2)$ in that it captures the logarithmic contributions at leading-order in $\alpha$. However, when the lepton is an electron or a muon, $f_{q/l}(\xi, \mu^2)$ cannot be computed in QCD perturbation theory.

The nonperturbative nature of $f_{q/l}(\xi, \mu^2)$ can be seen by considering its DGLAP evolution. When one considers QCD corrections, the evolution equation for $f_{q/l}(\xi, \mu^2)$ contains additional contributions that arise from the emission of real and virtual gluons by the quark:

$$
\mu^2 \frac{\partial}{\partial \mu^2} \left( \frac{f_{q/l}}{f_g} \right) = \left( P_{q\gamma} \otimes f_{\gamma/l} \right) + \left( P_{q\ell} \otimes f_{\ell/l} \right) + \sum_{q_j} \left( P_{q_j\gamma} 2 P_{q_jg} \right) \otimes \left( f_{q_j/l} \right),
$$

(4)

where the sum over $q_j$ includes both quarks and antiquarks. Suppose that one were to follow the procedure in Ref. [1], evolving $f_{q/l}$ from the scale $m_l$ to a hard-scattering scale. The splitting functions in Eq. (4) depend on $\alpha_s$ at scales $\mu$ that range from $m_l$ to the hard-scattering scale. If $\mu$ is sufficiently large, then the splitting functions can be computed in perturbation theory. However, if $\mu$ is less than a scale of order $\Lambda_{\text{QCD}}$, then the perturbation expansion for the splitting functions fails, and the evolution of $f_{q/l}$ receives nonperturbative contributions. In the case of the electron or the muon, the range of $\mu$ includes a region in which perturbative QCD fails and nonperturbative effects dominate.

Although the computation of the short-distance part of the cross section through the order of interest in Ref. [1] requires only that collinear poles through order $\alpha^2$ be absorbed into $f_{q/l}(\xi, \mu^2)$, a reliable calculation of $f_{q/l}(\xi, \mu^2)$ requires that QCD corrections be taken into account. The concept that the short-distance part of the cross section can be computed at a fixed order in $\alpha_s$, while the parton distributions, when they are nonperturbative, cannot is, of course, familiar from other hard-scattering processes, such as deep-inelastic scattering.

The nonperturbative distribution for a quark in an electron $f_{q/e}(\xi, \mu^2)$ at a scale $\mu^2$ that is in the perturbative regime of QCD could, in principle, be determined phenomenologically by fitting cross-section predictions to data. A process that is particularly sensitive to $f_{q/e}(\xi, \mu^2)$ is single-inclusive jet production in electron-electron scattering. Alternatively, with some sacrifice of sensitivity, one could make use of cross sections for single-jet inclusive production in electron-nucleon collisions. Lattice calculations might also provide informa-

---

1 We note that the expression in Eq. (1) omits constant terms that arise in standard renormalization schemes, such as modified minimal subtraction.
tion on $f_{q/e}(\xi, \mu^2)$. Once the nonperturbative distribution for a quark in an electron has been determined, it could be used to make reliable predictions for the resolved-electron contributions to hard-scattering processes.

Because of the sensitivity of $f_{q/e}(\xi, \mu^2)$ to nonperturbative QCD effects, the expression in Eq. (1) can at best be regarded as a model for the distribution. One unphysical aspect of this model is its double-logarithmic dependence on the electron mass. There is a logarithm of $m_e^2$ in the Weizsäcker-Williams distribution $f_{\gamma/e}(\xi, \mu^2)$. A second logarithm arises when one integrates Eq. (2) from $m_e^2$ to $\mu^2$ using the perturbative expressions for the splitting functions. This procedure implies that quarks in the electron are generated by perturbative evolution all the way down to virtualities of order $m_e^2$. One would not expect a probe with a virtuality that is much less than a typical hadronic scale to be able to resolve the hadronic structure of the electron. For the range of $\mu$ that is considered in Ref. [1], much of the large coefficient $\log^2(\mu^2/m_e^2)$ in Eq. (1) comes from integration over virtualities that are smaller than a typical hadronic scale of, say, 700 MeV. This feature of the model in Eq. (1) would tend to produce a significant overestimate of the contribution from quarks in the electron to the cross section for single-jet inclusive production in electron-nucleon collisions. Other nonperturbative effects that are not accounted for in the model could be substantial, as well.

We note that a sensitivity to nonperturbative QCD effects arises in the same way in the case of the distribution of a quark in a real photon $f_{q/\gamma}$. In this case, the leading-order QED expression for the logarithmic contribution to the distribution that is analogous to Eq. (1) is

$$f_{q/\gamma}(\xi, \mu^2) = e^2 q \frac{\alpha}{2\pi} \left[ \xi^2 + (1 - \xi)^2 \right] \log \frac{\mu^2}{m_\gamma^2}.$$  (5)

The inadequacy of this leading-order logarithmic approximation is manifest in the logarithm of the photon mass $m_\gamma$. Of course, it is well established that the distribution of a quark in a real photon involves contributions that cannot be calculated in perturbation theory, but must, instead, be obtained from fits to experimental data. (See, for example, Refs. [6–8].)

Acknowledgments

We thank Frank Petriello and Yuri Kovchegov for helpful discussions. The work of E.B. was supported in part by the Department of Energy under grant de-sc0011726. The work of G.T.B. is supported by the U.S. Department of Energy, Division of High Energy Physics,
under Contract No. DE-AC02-06CH11357. The submitted manuscript has been created in part by UChicago Argonne, LLC, Operator of Argonne National Laboratory. Argonne, a U.S. Department of Energy Office of Science laboratory, is operated under Contract No. DE-AC02-06CH11357. The U.S. Government retains for itself, and others acting on its behalf, a paid-up nonexclusive, irrevocable worldwide license in said article to reproduce, prepare derivative works, distribute copies to the public, and perform publicly and display publicly, by or on behalf of the Government.

[1] G. Abelof, R. Boughezal, X. Liu and F. Petriello, Phys. Lett. B 763, 52 (2016)
  \(\text{[http://dx.doi.org/10.1016/j.physletb.2016.10.022]}\) [arXiv:1607.04921].

[2] V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys. 15, 438 (1972) [Yad. Fiz. 15, 781 (1972)].

[3] L.N. Lipatov, Sov. J. Nucl. Phys. 20, 94 (1975) [Yad. Fiz. 20, 181 (1974)].

[4] Y.L. Dokshitzer, Sov. Phys. JETP 46, 641 (1977) [Zh. Eksp. Teor. Fiz. 73, 1216 (1977)].

[5] G. Altarelli and G. Parisi, Nucl. Phys. B 126, 298 (1977).

[6] F. Cornet, P. Jankowski and M. Krawczyk, Phys. Rev. D 70, 093004 (2004)
  \(\text{[http://dx.doi.org/10.1103/PhysRevD.70.093004]}\) [hep-ph/0404063].

[7] P. Aurenche, M. Fontannaz and J. P. Guillet, Eur. Phys. J. C 44, 395 (2005)
  \(\text{[http://dx.doi.org/10.1140/epjc/s2005-02355-1]}\) [hep-ph/0503259].

[8] C. Berger, J. Mod. Phys. 6 1023 [http://dx.doi.org/10.4236/jmp.2015.68107] [arXiv:1404.3551 [hep-ph]].