Measuring growth index in a universe with sterile neutrinos

Jing-Fei Zhang,¹ Yun-He Li,¹ and Xin Zhang*¹,2,†

¹Department of Physics, College of Sciences, Northeastern University, Shenyang 110004, China
²Center for High Energy Physics, Peking University, Beijing 100080, China

Consistency tests for the general relativity (GR) can be performed by constraining the growth index γ using the measurements of redshift-space distortions (RSD) in conjunction with other observations. In previous studies, deviations from the GR expected value of γ ≈ 0.55 at the 2–3σ level were found. In this work, we reconsider the measurement of γ in a universe with sterile neutrinos. We constrain the sterile neutrino cosmological model using the RSD measurements combined with the cosmic microwave background data (Planck temperature data plus WMAP 9-yr polarization data), the baryon acoustic oscillation data, the Hubble constant direct measurement, the Planck Sunyaev-Zeldovich cluster counts data, and the galaxy shear data. We obtain the constraint result of the growth index, γ = 0.584±0.047, well consistent with the GR expected value (the consistency is at the 0.6σ level). For the parameters of sterile neutrino, we obtain $N_{\text{eff}} = 3.62^{+0.26}_{-0.25}$ and $m_{\nu_{\text{sterile}}} = 0.48^{+0.14}_{-0.11}$ eV. We also consider the BICEP2 data and perform an analysis on the model with tensor modes. Similar fit results are obtained, showing that once light sterile neutrino is considered in the universe, GR will become well consistent with the current observations.

Since the discovery of the acceleration of the current universe’s expansion, dark energy has been viewed as the mainstream for explaining the cause of this phenomenon [1, 2]. In particular, the cosmological constant (Λ) plus cold dark matter (CDM) model, called the ΛCDM model, has been achieving successes in fitting various observational data. Notwithstanding, other possibilities explaining the cosmic acceleration still exist, among which the most popular alternative is the modification to general relativity (GR). Various modified gravity (MG) models in the large-scale/weak-field limit have been proposed; for recent reviews, see, e.g., Refs. [3, 4]. Since dark energy and MG theories can in principle predict identical expansion histories, a potential way of distinguishing between them is to probe and compare the different structure growth histories of them.

In linear perturbation theory, it is possible to describe the cosmic structure’s growth history through a second-order differential equation, which depends on both the Hubble expansion rate $H(z)$ and the specific theory of gravity. Solving this differential equation, one can derive the dimensionless linear growth rate, $f(a) = d\ln D(a)/d\ln a$, describing how rapidly structure grows as a function of cosmic scale factor $a(t)$, where $D(a)$ is the growth factor depicting the growth of the matter perturbations at late times. A fitting formula of $f(a) = \Omega_m(a)^{\gamma}$ proposed by Wang and Steinhardt [5] has been proven to be an accurate description for a wide range of models [6], in which both the growth index $\gamma$ and the fractional matter density $\Omega_m(a) = \Omega_m H_0^2 H(a)^{-2} a^{-3}$ depend on the specific model. For dark energy models with slowly varying equation of state (within the framework of GR), $\gamma$ was analytically given, showing that $\gamma = 6/11 \approx 0.545$ for the ΛCDM model and actually $\gamma \approx 0.55$ corresponds to a wide range of dark energy models in GR [5, 6]. For the MG models, different values of $\gamma$ can be derived; for example, $\gamma \approx 0.68$ is obtained theoretically for the Dvali-Gabadadze-Porrati (DGP) braneworld model [7].

Redshift-space distortions (RSD) provide an important way of measuring $f(z)$ at different redshifts [8, 9]. RSD arise from peculiar velocities of galaxies on observed galaxy map. Since the coherent motions of galaxies are actually a direct consequence of the growth of structure, the measurement of anisotropy they induce in the redshift space provides information about the formation of large-scale structure. In practice, RSD measure the combination of $f(z)$ and $\sigma_8(z)$, i.e., $f(a)\sigma_8(a) = a\sigma_8(a)/d\ln a$, where $\sigma_8(z)$ is the root-mean-square mass fluctuation in spheres with radius $8h^{-1}$ Mpc at redshift $z$ [10].

Consistency tests of GR using the RSD measurements have been performed in the literature. In Ref. [11], Samushia et al. used the BOSS CMASS DR9 measurement of growth rate in combination with cosmic microwave background (CMB) and type Ia supernova (SN) data to constrain the growth index of the ΛCDM model and obtained $\gamma = 0.75 \pm 0.09$; when other $f\sigma_8$ measurements are added (totally 9 data), the result is improved to $\gamma = 0.64 \pm 0.05$. In both cases, the constraint results are in tension with the GR expected value of $\gamma = 0.55$ at about the 2σ level. In Ref. [12], Beutler et al. used the BOSS CMASS DR11 data (including $D_V/r_s, F_{AP}$, and $f\sigma_8$ at the effective redshift $z = 0.57$) combined with the Planck data to place constraint on the growth index and obtained the result $\gamma = 0.772^{+0.124}_{-0.097}$ in tension with the GR expected result at the 2.3σ level. Similar situation can also be found in other studies, e.g., Ref. [13], in which the constraint result of $\gamma$ from the RSD measurements (10 $f\sigma_8$ data in total) combined with the SN, baryon acoustic oscillation (BAO), and Planck data is also discrepant from the GR expected value at about the 2–3σ level.

In this paper, we consider the growth rate of structure in a universe with sterile neutrinos. Considering
the parametrization $f(z) = \Omega_m(z)^{\gamma}$, we make a consistency test for GR in this case. In our previous work [14], we have shown that involving a light (eV or sub-eV mass scale) sterile neutrino species in the ΛCDM model can help reconcile the tensions between Planck and other astrophysical observations (such as the direct measurement of $H_0$ of HST observation, the Planck Sunyaev-Zeldovich cluster counts, the cosmic shear measurement of CFHTLenS survey, and the CMB polarization measurements of BICEP2). It was shown that, under the CMB+BAO constraint, the inconsistencies with Planck are improved from 2.4σ to 1.0σ for $H_0$ observation, from 4.3σ to 2.0σ for SZ cluster counts observation, and from 2.3σ to 1.7σ for cosmic shear observation; at the same time, the 95% upper limit for the tensor-to-scalar ratio $r_{0.002}$ is also enhanced to 0.20, relieving the tension between Planck and BICEP2 [14]. Furthermore, combining CMB data with growth of structure measurements could provide independent evidence for the existence of light sterile neutrinos, i.e., the current cosmological data prefer $\Delta N_{\rm eff} > 0$ at the 2.7σ level and a nonzero mass of sterile neutrino at the 3.9σ level [14]. For other relevant studies, see, e.g., Refs. [15–23]. Here we caution the reader that the systematic errors in the astrophysical observations, such as cluster counts, weak lensing, and Hubble constant measurements, could weaken these conclusions to some extent. In particular, it was shown in Ref. [24] that the standard ΛCDM model is favored over the model with sterile neutrino if the Bayesian evidence is used as a criterion for comparing models.

Since the growth of structure could be suppressed by light sterile neutrinos in the ΛCDM cosmology, perhaps the growth index constrained by the RSD measurements would become consistent with the GR expected value of $\gamma \approx 0.55$. This motivates the present work. In the following, we test GR using the RSD measurements (in conjunction with other observations) in the universe with sterile neutrinos.

We use the recent RSD measurements to constrain the growth index $\gamma$. The $f(z)\sigma_8(z)$ data we consider in this work include the measurements from 6dFGS ($z = 0.067$) [25], 2dFGRS ($z = 0.17$) [26], WiggleZ ($z = 0.22$, 0.41, 0.60, and 0.78) [27], SDSS LRG DR7 ($z = 0.25$ and 0.37) [28], BOSS CMASS DR11 ($z = 0.57$) [12], and VIPERS ($z = 0.80$) [29]. In Ref. [30], through a test of the reliability of low redshift growth of structure measurements (including RSD), it was shown that the cosmological parameter constraints are robust against changes in the power spectrum template related to neutrino mass. This justifies the use of the RSD data as well as other growth of structure data in this work.

To constrain other parameters and break degeneracies between parameters, we should also employ other observational data. We consider the CMB, BAO, $H_0$, cluster counts, and cosmic shear data. For the CMB data, we use the Planck temperature power spectrum data [31] in combination with the WMAP 9-yr polarization (TE and EE) power spectrum data [32]. For the BAO data, we use the measurements from 6dFGS ($z = 0.1$) [33], SDSS DR7 ($z = 0.35$) [34], WiggleZ ($z = 0.44$, 0.60, and 0.73) [35], and BOSS DR11 ($z = 0.32$ and 0.57) [36]; note that the three data from WiggleZ survey are correlated, with the inverse covariance matrix given in Ref. [35]. For the $H_0$ measurement, we use the HST result, $H_0 = (73.8 \pm 2.4)$ km s$^{-1}$ Mpc$^{-1}$ [37]. For the Planck SZ (PlaSZ) measurement, we use the result of $\sigma_8(\Omega_m/0.27)^{0.3} = 0.764 \pm 0.025$ (which is derived by allowing the bias $(1-b)$ to vary in the range of $[0, 7, 1]$) [38]. For the shear measurement, we use the CFHTLenS result, $\sigma_8(\Omega_m/0.27)^{0.6} = 0.79 \pm 0.03$ [39]. The consistency of these data sets has been tested for the universe with sterile neutrinos, and so the combination of these data is appropriate [14, 18].

Following our previous work [14], the model with sterile neutrinos considered in the paper is called the ΛCDM+$\nu_s$ model, in which the active neutrino mass is kept at 0.06 eV (minimal-mass normal hierarchy is assumed) and two additional parameters related to sterile neutrino, $N_{\text{eff}}$ and $m_{\nu_s}^{\text{eff}}$, are involved. In addition, the growth index $\gamma$ is introduced through the parametrization of the growth rate $f(z) = \Omega_m(z)^{\gamma}$. To place constraints on $\gamma$, we follow the recipe of Ref. [12] to calculate the values of $f(z)\sigma_8(z)$ in the numerical code (we modify the CosmoMC code [40]). In the following, we report the results of the parameter estimation.

We use the CMB+BAO+$H_0$+PlaSZ+Shear+RSD data combination to constrain the ΛCDM+$\nu_s$ model. The main constraint results are shown in Table I as well as Figs. 1 and 2. We obtain $\gamma = 0.584^{+0.047}_{-0.048}$, well consistent with the GR expected value of $\gamma = 0.55$. Figure 1 shows the one-dimensional posterior distribution for $\gamma$ (upper panel) and two-dimensional posterior contours for $\gamma$ and $\Omega_m$ (lower panel). One can clearly see that GR lies well inside the 1σ range; the consistency is at the 0.6σ level. In addition, the parameters of sterile neutrino can also be precisely determined in this case. The constraint result for sterile neutrino in the $m_{\nu_s}^{\text{eff}}$ plane is shown in Fig. 2. We obtain $N_{\text{eff}} = 3.62^{+0.26}_{-0.42}$ and $m_{\nu_s}^{\text{eff}} = 0.49^{+0.11}_{-0.14}$ eV, indicating the preference for $\Delta N_{\text{eff}} \equiv N_{\text{eff}} - 3.046 > 0$ at the 1.4σ level and for nonzero mass of sterile neutrino at the 3.4σ level.

| Parameter       | ΛCDM | ΛCDM+$\nu_s$ | ΛCDM+$r+\nu_s$ |
|-----------------|------|--------------|-----------------|
| $\gamma$        | 0.667^{+0.049}_{-0.053} | 0.584^{+0.044}_{-0.048} | 0.600^{+0.042}_{-0.049} |
| $N_{\text{eff}}$| ...  | 3.62^{+0.26}_{-0.42}  | 4.01^{+0.30}_{-0.33}  |
| $m_{\nu_s}^{\text{eff}}$ | ...  | 0.48^{+0.11}_{-0.14}  | 0.54^{+0.11}_{-0.14}  |
| $r_{0.002}$     | ...  | 0.221^{+0.042}_{-0.052} | 0.209^{+0.073}_{-0.062} |
| $\Omega_m$      | 0.3051^{+0.0072}_{-0.0075} | 0.3015^{+0.0076}_{-0.0077} | 0.3009^{+0.0073}_{-0.0074} |
| $\sigma_8$      | 0.822^{+0.011}_{-0.013}  | 0.746^{+0.013}_{-0.012}  | 0.745^{+0.013}_{-0.012}  |
| $H_0$           | 68.0^{+0.6}_{-0.7}  | 70.5^{+1.3}_{-1.7}  | 72.0^{+1.3}_{-1.3}  |
| $-\ln L_{\text{max}}$ | 4908.591 | 4912.801 | 4932.818 |

We use the CMB+BAO+$H_0$+PlaSZ+Shear+RSD data combination to constrain the ΛCDM+$\nu_s$ model. The main constraint results are shown in Table I as well as Figs. 1 and 2. We obtain $\gamma = 0.584^{+0.047}_{-0.048}$, well consistent with the GR expected value of $\gamma = 0.55$. Figure 1 shows the one-dimensional posterior distribution for $\gamma$ (upper panel) and two-dimensional posterior contours for $\gamma$ and $\Omega_m$ (lower panel). One can clearly see that GR lies well inside the 1σ range; the consistency is at the 0.6σ level. In addition, the parameters of sterile neutrino can also be precisely determined in this case. The constraint result for sterile neutrino in the $m_{\nu_s}^{\text{eff}}$ plane is shown in Fig. 2. We obtain $N_{\text{eff}} = 3.62^{+0.26}_{-0.42}$ and $m_{\nu_s}^{\text{eff}} = 0.49^{+0.11}_{-0.14}$ eV, indicating the preference for $\Delta N_{\text{eff}} \equiv N_{\text{eff}} - 3.046 > 0$ at the 1.4σ level and for nonzero mass of sterile neutrino at the 3.4σ level.
the growth of structure could be suppressed below its free streaming length, allowing \( \sigma_8 \) to be substantially lower. This explains why the tension with GR is reduced (i.e., the RSD constraint on the growth index \( \gamma \) becomes in good agreement with the GR prediction, as shown in the above analysis), once a light sterile neutrino is introduced into the model. Moreover, the same reason leads to the reduction of tensions between Planck and other large-scale structure observations, as indicated in Refs. [14–23]. Since the cluster counts and weak lensing data are important for determining the properties of sterile neutrino, we make a further analysis. In previous studies, e.g., [14, 16–19, 21, 23], somewhat different PlaSZ and Shear data were used, i.e., \( \sigma_8(\Omega_m/0.27)^{0.3} = 0.782 \pm 0.010 \) (derived by fixing \( 1 - b = 0.8 \)) [38] and \( \sigma_8(\Omega_m/0.27)^{0.46} = 0.774 \pm 0.040 \) [41]. Replacing with these two measurement results in our analysis, we obtain the constraint results: \( \gamma = 0.598 \pm 0.046 , \ N_{\text{eff}} = 3.65_{-0.37}^{+0.28} , \) and \( m_{\nu, \text{sterile}} = 0.42_{-0.12}^{+0.10} \) eV. In addition, due to the existence of some unknown systematic uncertainties in the PlaSZ data, we also perform an analysis without PlaSZ. We follow Ref. [30] to replace PlaSZ with the galaxy-galaxy lensing result, \( \sigma_8(\Omega_m/0.25)^{0.57} = 0.80 \pm 0.05 \) [42], and redo the analysis. In this case, we obtain the fit results: \( \gamma = 0.582_{-0.049}^{+0.049} , \ N_{\text{eff}} = 3.66_{-0.46}^{+0.24} , \) and \( m_{\nu, \text{sterile}} = 0.49_{-0.19}^{+0.12} \) eV. Comparing the three cases, we find that their results are in good agreement with each other.

Recently, the detection of the B-mode polarization of CMB was reported by the BICEP2 Collaboration [43]. Provided that the foreground model was treated correctly, the BICEP2’s result would indicate the discovery of the primordial gravitational waves (PGWs). If confirmed by upcoming experiments, the frontiers of fundamental physics will be pushed forward in an unprecedented way. Fitting to the BICEP2 data gives an un-
In the present work, we also perform an analysis for the ΛCDM+$r+\nu_s$ model. In this case, we also use the BICEP2 data [43] to carry out the joint constraints. The main constraint results are shown in Table I and Fig. 3. The fit result for the growth index is $\gamma = 0.600^{+0.044}_{-0.049}$, also consistent with the GR prediction of $\gamma = 0.55$ at the 1σ level (see the upper panel of Fig. 3). For the parameters of sterile neutrino, we obtain $N_{\text{eff}} = 4.01^{+0.40}_{-0.33}$ and $m_{\nu_s,\text{sterile}} = 0.53^{+0.14}_{-0.11}$ eV (see the lower panel of Fig. 3), indicating the preference for $\Delta N_{\text{eff}} > 0$ at the 2.9σ level and for nonzero mass of sterile neutrino at the 3.8σ level.

In summary, we have performed a consistency test for GR in a universe with sterile neutrinos through constraining the growth index $\gamma$ using the RSD data in conjunction with other observations. The observational data we used in this work include the CMB, BAO, H0, PlaSZ, Shear, and RSD data. For the ΛCDM+$\nu_s$ model, we obtained the result of $\gamma = 0.584^{+0.047}_{-0.048}$, well consistent with the GR expected value of $\gamma = 0.55$; the consistency is at the 0.6σ level. We obtained the parameters of sterile neutrino, $N_{\text{eff}} = 3.62^{+0.26}_{-0.42}$ and $m_{\nu_s,\text{sterile}} = 0.48^{+0.33}_{-0.11}$ eV, indicating the preference for $\Delta N_{\text{eff}} > 0$ at the 1.4σ level and for nonzero mass of sterile neutrino at the 3.4σ level. We also tested the ΛCDM+$r+\nu_s$ model, and the BICEP2 data were also included in the analysis for this case. The constraint results are $\gamma = 0.600^{+0.044}_{-0.049}$, $N_{\text{eff}} = 4.01^{+0.30}_{-0.49}$, and $m_{\nu_s,\text{sterile}} = 0.53^{+0.11}_{-0.14}$ eV, indicating the consistency with GR at the 1σ level as well as the preference for $\Delta N_{\text{eff}} > 0$ at the 2.9σ level and for nonzero mass of sterile neutrino at the 3.8σ level. The previous consistency tests for GR in the literature show some tension at about the 2–3σ level. In this work, we show that when light sterile neutrinos are considered in the universe, GR will become well consistent with the current observations.

Acknowledgments

We acknowledge the use of CosmoMC. JFZ is supported by the Provincial Department of Education of Liaoning under Grant No. L2012087. XZ is supported by the National Natural Science Foundation of China under Grant No. 11175042 and the Fundamental Funds for the Central Universities under Grant No. N120505003.

expectedly large tensor-to-scalar ratio, $r = 0.20^{+0.07}_{-0.05}$, in tension with the 95% upper limit, $r < 0.11$, given by Planck. In our previous work [14], we have shown that involving sterile neutrino in the model (i.e., considering the ΛCDM+$r+\nu_s$ model) could well relieve the tension between Planck and BICEP2, and meanwhile, the other tensions of Planck with other astrophysical observations can all be significantly reduced (see also Ref. [15]).

FIG. 3: Joint constraint results (68.3% and 95.4% CL) for the ΛCDM+$r+\nu_s$ model from the CMB+BAO+$H_0+$PlaSZ+Shear+RSD+BICEP2 data combination in the $\Omega_m$–$\gamma$ plane (upper) and in the $m_{\nu_s,\text{sterile}}$–$N_{\text{eff}}$ plane (lower). The horizontal red dashed line in the upper panel indicates the GR expected value of $\gamma = 0.55$.

[1] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. 75, 559 (2003) [astro-ph/0207347].
[2] J. Frieman, M. Turner and D. Huterer, Ann. Rev. Astron. Astrophys. 46, 385 (2008) [arXiv:0803.0982 [astro-ph]].
[3] T. Clifton, P. G. Ferreira, A. Padilla and C. Skordis, Phys. Rept. 513, 1 (2012) [arXiv:1106.2476 [astro-ph.CO]].
[4] A. Joyce, B. Jain, J. Khoury and M. Trodden, arXiv:1407.0059 [astro-ph.CO].
[5] L. M. Wang and P. J. Steinhardt, Astrophys. J. 508, 483 (1998) [astro-ph/9804015].
[6] E. V. Linder, Phys. Rev. D 72, 043529 (2005) [astro-ph/0507263].
[7] A. Lue, R. Scoccimarro and G. D. Starkman, Phys. Rev. D 69, 124015 (2004) [astro-ph/0401515].
[8] J. A. Peacock, S. Cole, P. Norberg, C. M. Baugh, J. Bland-Hawthorn, T. Bridges, R. D. Cannon and M. Colless et al., Nature 410, 169 (2001) [astro-
