Predictions for neutrinoless double-beta decay in the 3+1 sterile neutrino scenario

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Abstract. In this proceeding we present predictions of the effective Majorana mass $|m_{\beta\beta}|$ in neutrinoless double-$\beta$ decay for the standard $3\nu$ mixing case and for the $3+1$ neutrino mixing case indicated by the short-baseline anomalies. We have taken into account the uncertainties of the neutrino mixing parameters determined by oscillation experiments. We obtained that the predictions for $|m_{\beta\beta}|$ in the cases of $3\nu$ and $3+1$ mixing are quite different, in agreement with previous discussions in the literature, and that future measurements of neutrinoless double-$\beta$ decay and the total mass of the three lightest neutrinos in cosmological experiments may distinguish the $3\nu$ and $3+1$ cases if the mass ordering is determined by oscillation experiments.

1. Introduction

Are neutrinos Dirac or Majorana particles? This question cannot be probed in neutrino oscillation experiments. Since the lepton number is conserved in oscillation experiments there is no difference between a Dirac and a Majorana neutrino. However, the Majorana nature of neutrinos can be investigated in experiments in which there is violation of the lepton number. The most promising type of experiment in which it can be probed are the neutrinoless double-beta decay experiments, in which the total lepton number is violated by two units (see the recent review in Ref. [1]).

Using the current measurements of neutrino squared-mass differences and mixing angles it is possible to predict the possible range of values for the effective Majorana mass $|m_{\beta\beta}|$ in neutrinoless double-beta decay for different orderings of the neutrinos masses.

The plan of this proceeding is to present some of the results obtained in Ref. [2]. We will discuss in Section 2 the predictions for $|m_{\beta\beta}|$ in the standard $3\nu$ framework, taking into account the two possible normal and inverted mass orderings and in Section 3 we will comment how these predictions are modified in the $3+1$ mixing framework. We will also discuss the possibility of distinguishing the $3\nu$ and the $3+1$ mixing framework analyzing the predictions of the effective Majorana mass in comparison with the sum of the three lightest neutrino masses, that can be obtained by cosmological measurements.
the two complex phases \( \beta \) of the neutrino with the three massive neutrinos, can have unknown complex phases, which generate the elements \( U_{\nu e} \) with the first mass eigenstate (mass orderings for the neutrinos masses). The case in which the lightest neutrino mass is associated with the third mass eigenstate \( m_3 \) is called Inverted Ordering (IO).

In this work we use the results of the global fit of solar, atmospheric and long-baseline reactor and accelerator neutrino oscillation data presented in Ref. [3] considering the two possible orderings for the neutrinos masses. The case in which CP is conserved \( (\alpha_2 = 0, \alpha_3 = 0) \) is called Normal Ordering (NO) and the case in which CP is violated \( (\alpha_2, \alpha_3 \neq 0) \) is called Inverted Ordering (IO).

The left part of Figure 1 shows the best-fit values and the 1σ, 2σ and 3σ allowed intervals of the effective Majorana mass \( |m_{\beta\beta}| \) in Eq. (1) as functions of the lightest mass \( m_1 \) for the NO scheme, where we have plotted separately the allowed bands for the four possible cases in which CP is conserved \( (\alpha_2, \alpha_3 = 0,\pi) \). The areas between the CP-conserving curves correspond to values of \( |m_{\beta\beta}| \) which are allowed only in the case of CP violation [4–9].

2. Three-Neutrino Mixing

In the standard three-neutrino \( (3\nu) \) mixing framework, the effective Majorana mass in neutrinoless double-beta decay is defined as

\[
|m_{\beta\beta}| = |m_1|U_{e1}|^2 + m_2|U_{e2}|^2e^{i\alpha_2} + m_3|U_{e3}|^2e^{i\alpha_3}.
\]  

The elements \( U_{ek} \) \((k = 1, 2, 3)\) of the mixing matrix, which quantify the mixing of the electron neutrino with the three massive neutrinos, can have unknown complex phases, which generate the two complex phases \( \alpha_2 \) and \( \alpha_3 \) in Eq. (1).

Table 1. Ranges of \( m_1 \) for which there can be a complete cancellation of the three partial mass contributions to \( |m_{\beta\beta}| \) for the best-fit values (b.f.) of the oscillation parameters and at 1σ, 2σ and 3σ in the case of 3\( \nu \) mixing with Normal Ordering.

| \( m_1 \) [10\(^{-3}\) eV] | b.f. | 1σ | 2σ | 3σ |
|-----------------|-----|----|----|----|
| 2.3 – 6.6       | 1.9 – 7.2 | 1.6 – 8.0 | 1.3 – 9.0 |

In the case of CP violation, the 90% upper limit is explained in the main text.

Figure 1. Left part: Value of the effective Majorana mass \( |m_{\beta\beta}| \) as a function of the lightest neutrino mass \( m_1 \) in the case of 3\( \nu \) mixing with Normal Ordering. Right part: Value of the effective Majorana mass \( |m_{\beta\beta}| \) as a function of the lightest neutrino mass \( m_3 \) in the case of 3\( \nu \) mixing with Inverted Ordering. The signs in the legend indicate the signs of \( e^{i\alpha_2}, e^{i\alpha_3} = \pm 1 \) for the four possible cases in which CP is conserved. The intermediate yellow region is allowed only in the case of CP violation. The 90% upper limit is explained in the main text.
confidence intervals using the $\chi^2$ function

$$\chi^2_{3\nu} = \chi^2(\Delta m^2_{\text{SOL}}) + \chi^2(\Delta m^2_{\text{ATM}}) + \chi^2(\sin^2 \vartheta_{12}) + \chi^2(\sin^2 \vartheta_{13}),$$  \hspace{1cm} (2)

with the partial $\chi^2$'s extracted from Figure 3 of Ref. [3]. For each value of $m_1$ we calculated the confidence intervals for one degree of freedom.

Figure 1 shows also the 90\% C.L. upper limit band for $|m_{\beta\beta}|$ estimated in Ref. [1] from the results of the KamLAND-Zen experiment [10], taking into account the uncertainties of the nuclear matrix element calculations.

In the left part of Figure 1 one can see that there can be a complete cancellation of $|m_{\beta\beta}|$ for $m_1$ in the intervals that are given in Tab. 1 at different confidence levels. Furthermore, $|m_{\beta\beta}|$ is larger than about 0.01 eV, which is a value that may be explored experimentally in the near future, for values of $m_1 \gtrsim 0.008$ eV which corresponds to almost degenerate $m_1$ and $m_2$ (see Ref. [2] for more details), hence, it will be very difficult to measure $|m_{\beta\beta}|$ if there is a normal hierarchy of neutrino masses ($m_1 < m_2 < m_3$) for any value of the unknown phases $\alpha_2$ and $\alpha_3$ in Eq. (1). We also note that $|m_{\beta\beta}| \gtrsim 0.01$ eV is realized independently of the values of the unknown phases $\alpha_2$ and $\alpha_3$ for $m_1 \gtrsim 0.04$ eV, which is close to the region $m_1 \gtrsim 0.05$ eV in which all the three neutrino masses are quasi degenerate (see Ref. [2] for further details).

In the case of Inverted Ordering ($m_3 \ll m_1 < m_2$) there isn’t any cancellation region of $|m_{\beta\beta}|$ and we obtain from the right part of Figure 1 the lower bounds

$$|m_{\beta\beta}| > 1.6 (1\sigma), 1.5 (2\sigma), 1.3 (3\sigma) \times 10^{-2} \text{ eV}.$$  \hspace{1cm} (3)

In the case of an Inverted Hierarchy ($m_3 \ll m_1 < m_2$) we also have the upper bounds

$$|m_{\beta\beta}| < 4.8 (1\sigma), 4.9 (2\sigma), 4.9 (3\sigma) \times 10^{-2} \text{ eV}.$$  \hspace{1cm} (4)

The next generations of neutrinoless double-beta decay experiments aim to explore the range of $|m_{\beta\beta}|$ between the limits found in Eqs. (3) and (4) (see Refs. [11–16]), and, in case the Inverted Hierarchy being the true one, in the next few years we will have a definitive answer of the Majorana nature of neutrinos.

3. 3+1 Mixing

The 3+1 mixing is motivated by the explanation of the short-baseline anomalies [17, 18], which requires the existence of a new squared-mass difference $\Delta m^2_{\text{SBL}} \sim 1 \text{ eV}^2$.

In this section we will consider the case of 3+1 mixing with a new massive neutrino $\nu_4$ at the eV scale, mainly sterile. In this case, the effective Majorana mass in neutrinoless double-beta decay is given by

$$|m_{\beta\beta}| = |m_1 |U_{e1}| + m_2 |U_{e2}|e^{i\alpha_2} + m_3 |U_{e3}|e^{i\alpha_3} + m_4 |U_{e4}|e^{i\alpha_4}|,$$  \hspace{1cm} (5)

and the contribution of $\nu_4$ enters with an unknown new phase $\alpha_4$.

As in any extension of the standard $3\nu$ mixing, the ordering of the three lightest neutrinos are not known and then we will consider separately the two cases of Normal and Inverted Ordering of $\nu_1, \nu_2, \nu_3$.

We calculated the confidence intervals using the $\chi^2$ function

$$\chi^2_{3+1} = \chi^2_{3\nu} + \chi^2(\Delta m^2_{\text{SBL}}, \sin^2 \vartheta_{14}),$$  \hspace{1cm} (6)

with $\chi^2_{3\nu}$ defined in Eq. (2) and $\chi^2(\Delta m^2_{\text{SBL}}, \sin^2 \vartheta_{14})$ obtained from an update [19, 20] of the global fit of short-baseline neutrino oscillation data presented in Ref. [18].
Figure 2. Left part: Value of the effective Majorana mass $|m_{\beta\beta}|$ as a function of the lightest neutrino mass $m_1$ in the case of 3+1 mixing with Normal Ordering of the three lightest neutrinos. Right part: Value of the effective Majorana mass $|m_{\beta\beta}|$ as a function of the lightest neutrino mass $m_3$ in the case of 3+1 mixing with Inverted Ordering of the three lightest neutrinos. The signs in the legend indicate the signs of $e^{i\alpha_2}, e^{i\alpha_3}, e^{i\alpha_4} = \pm 1$ for the four possible cases in which CP is conserved. The intermediate yellow region is allowed only in the case of CP violation.

Figure 3. Left part: Comparison of the 3\(\sigma\) allowed regions in the $\Sigma-|m_{\beta\beta}|$ plane in the cases of 3\(\nu\) and 3+1 mixing with Normal Ordering of the three lightest neutrinos. Right part: Comparison of the 3\(\sigma\) allowed regions in the $\Sigma-|m_{\beta\beta}|$ plane in the cases of 3\(\nu\) and 3+1 mixing with Inverted Ordering of the three lightest neutrinos.

The left part of Figure 2 shows the allowed intervals for the best-fit value and the 1\(\sigma\), 2\(\sigma\) and 3\(\sigma\) of $|m_{\beta\beta}|$ for the 3+1 mixing with NO as a function of $m_1$. The corresponding intervals of $m_1$ for which there can be a total cancellation of $|m_{\beta\beta}|$ are given in Tab. 2.

In Figure 2 we have plotted separately the allowed bands for the eight possible cases in which CP is conserved ($\alpha_2, \alpha_3, \alpha_4 = 0, \pi$). The areas between the CP-conserving allowed bands correspond to values of $|m_{\beta\beta}|$ which are allowed only in the case of CP violation.

The right part of Figure 2 presents the allowed regions in the case for the 3+1 neutrino mixing with IO as a function of $m_3$. Comparing Figure 2 with Figure 1 one can see that the predictions for $|m_{\beta\beta}|$ are dramatically changed from the 3\(\nu\) scheme to the 3+1 scheme if there is an Inverted...
Ordering of the three lightest neutrinos, in agreement with the discussions in Refs. [9, 21–29]. The ranges of values of \( m_3 \) for which there can be a complete cancellation of \( |m_{\beta\beta}| \) are given in Tab. 2.

Figure 1 gives a clear view of the possible values of \( |m_{\beta\beta}| \) depending on the scale of the lightest mass \( m_{\text{min}} \) but, in practice, the investigation of the absolute values of neutrino masses is performed through the measurements of the effective electron neutrino mass in \( \beta \)-decay experiments [30, 31] and through the measurement of the sum of the neutrino masses \( \Sigma = m_1 + m_2 + m_3 \) in cosmological experiments. Hence it is interesting to compare the 3σ allowed regions of \( |m_{\beta\beta}| \) as a function of \( \Sigma \) for the 3ν and 3+1 neutrino mixing schemes, as done in Figure 3.

The reason of this choice is that \( \Sigma \) is a measurable quantity also in the 3+1 scheme, in cosmology, the effects of the larger mass \( m_4 \) can be disentangled from those of the smaller masses, because \( \nu_4 \) becomes non-relativistic shortly after matter-radiation equality, much earlier than \( \nu_1, \nu_2, \nu_3 \). In order to compute the confidence intervals we used the \( \chi^2 \) functions in Eq. (2) and Eq. (6) with two degrees of freedom.

In the left part of Figure 3 we show the comparison of the 3σ allowed region in the \( \Sigma-|m_{\beta\beta}| \) for the cases of 3ν and 3+1 mixing with Normal Ordering. If the Normal Ordering will be established by oscillation experiments (see Refs. [32, 33]), with measurements of \( \Sigma \) and \( |m_{\beta\beta}| \) it may be possible to distinguish 3ν mixing and 3+1 mixing if the measured values select a region which is allowed only in one of the two cases. It is interesting that there are two regions allowed only in the 3+1 mixing scheme: one with \( |m_{\beta\beta}| \) smaller than that in the case of 3ν mixing and one with \( |m_{\beta\beta}| \) larger than that in the case of 3ν mixing. Part of the second region can be probed in the next generation of neutrinoless double-beta decay experiments and it is a very promising feature for distinguishing the 3+1 mixing and the 3ν mixing schemes once the cosmological observations aim to measure the sum of the three light neutrino masses down to the lower limit of about \( 5.6 \times 10^{-2} \) eV [34].

The right part of Figure 3 shows the comparison of the 3σ allowed regions in the \( \Sigma-|m_{\beta\beta}| \) plane in the cases of 3ν and 3+1 mixing with Inverted Ordering of the three lightest neutrinos. If the Inverted Ordering will be established by oscillation experiments (see Refs. [32, 33]), it will be possible to exclude 3ν mixing in favor of 3+1 by restricting \( \Sigma \) and \( |m_{\beta\beta}| \) in the corresponding large region at small \( |m_{\beta\beta}| \) allowed only in the 3+1 case.

4. Conclusions

We have presented some of the results obtained in Ref. [2] in which we calculated the effective Majorana mass \( |m_{\beta\beta}| \) in neutrinoless double-\( \beta \) decay in the standard case of 3ν mixing and in the case of 3+1 neutrino mixing indicated by the short-baseline anomalies. We took into account the uncertainties of the standard 3ν mixing parameters obtained in the global fit of solar, atmospheric and long-baseline reactor and accelerator neutrino oscillation data presented

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**Table 2.** Ranges of \( m_1 \) \{\( m_3 \)\} for which there can be a complete cancellation of the four partial mass contributions to \( |m_{\beta\beta}| \) for the best-fit values (b.f.) of the oscillation parameters and at 1σ, 2σ and 3σ in the case of 3+1 mixing with Normal Ordering {Inverted Ordering} of the three lightest neutrinos.

|        | b.f.  | 1σ     | 2σ     | 3σ     |
|--------|-------|--------|--------|--------|
| \( m_1 [10^{-2} \text{eV}] \) | 3.5 - 10.5 | 2.8 - 12.5 | 2.0 - 16.3 | 1.3 - 20.0 |
| \( m_3 [10^{-2} \text{eV}] \) | < 9.1 | < 11.4 | < 15.5 | < 19.3 |
in Ref. [3] and the uncertainties of the additional mixing parameters in the 3+1 case obtained from an update [19, 20] of the global fit of short-baseline neutrino oscillation data presented in Ref. [18].

We conclude that the predictions for $|m_{33}|$ in the case of 3ν is dramatically changed with the addition of one sterile neutrino, in agreement with the previous discussions in Refs. [9, 21–29].

We also compared the allowed regions in the plane $\Sigma – |m_{33}|$, taking into account the two possibilities of Normal and Inverted Ordering of the three lightest neutrinos. We have shown that future measurements of these quantities may distinguish the 3ν and 3+1 cases if the mass ordering is determined by oscillation experiments (see Refs. [32, 33]).

5. Acknowledgments

E. Z. thanks the support of funding grants 2013/02518-7 and 2014/23980-3, São Paulo Research Foundation (FAPESP). The work of C. Giunti is partially supported by the research grant Theoretical Astroparticle Physics number 2012CPPYP7 under the program PRIN 2012 funded by the Ministero dell’Istruzione, Università e della Ricerca (MIUR).

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