Higher order bulk characteristic parameters of asymmetric nuclear matter

Lie-Wen Chen¹,²

¹Department of Physics, Shanghai Jiao Tong University, Shanghai 200240, China
²Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, China

(Dated: March 25, 2011)

The bulk parameters characterizing the energy of symmetric nuclear matter and the symmetry energy defined at normal nuclear density \(\rho_0\) provide important information on the equation of state (EOS) of isospin asymmetric nuclear matter. While significant progress has been made in determining some lower order bulk characteristic parameters, such as the energy \(E_0(\rho_0)\) and incompressibility \(K_0\) of symmetric nuclear matter as well as the symmetry energy \(E_{\text{sym}}(\rho_0)\) and its slope parameter \(L\), yet the higher order bulk characteristic parameters are still poorly known. Here, we analyze the correlations between the lower and higher order bulk characteristic parameters within the framework of Skyrme Hartree-Fock energy density functional and then estimate the values of some higher order bulk characteristic parameters. In particular, we obtain \(J_0 = -355 \pm 95\) MeV and \(I_0 = 1473 \pm 680\) MeV for the third-order and fourth-order derivative parameters of symmetric nuclear matter at \(\rho_0\) and \(K_{\text{sym}} = -100 \pm 165\) MeV, \(J_{\text{sym}} = 224 \pm 385\) MeV, \(I_{\text{sym}} = -1309 \pm 2025\) MeV for the curvature parameter, third-order and fourth-order derivative parameters of the symmetry energy at \(\rho_0\), using the empirical constraints on \(E_0(\rho_0)\), \(K_0\), \(E_{\text{sym}}(\rho_0)\), \(L\), and the isoscalar and isovector nucleon effective masses. Furthermore, our results indicate that the three parameters \(E_0(\rho_0)\), \(K_0\), and \(J_0\) can reasonably characterize the EOS of symmetric nuclear matter up to \(2\rho_0\) while the symmetry energy up to \(2\rho_0\) can be well described by \(E_{\text{sym}}(\rho_0)\), \(L\), and \(K_{\text{sym}}\).

PACS numbers: 21.65.Mn, 21.30.Fe, 21.65.Ef, 21.60.Jz, 21.65.Cd

I. INTRODUCTION

The equation of state (EOS) of isospin asymmetric nuclear matter, especially its isospin dependent part which is essentially characterized by the nuclear symmetry energy, is important for understanding not only the structure of radioactive nuclei, the reaction dynamics induced by rare isotopes, and the liquid-gas phase transition in asymmetric nuclear matter, but also many critical issues in astrophysics [1–7]. The nuclear matter EOS is conventionally defined as the binding energy per nucleon as a function of the density and a number of bulk parameters defined at normal nuclear density \(\rho_0\) are usually introduced to characterize the energy of symmetric nuclear matter and the nuclear symmetry energy. For example, the energy \(E_0(\rho_0)\) and incompressibility \(K_0\) of symmetric nuclear matter are the two lowest order bulk parameters for the EOS of symmetric nuclear matter while the symmetry energy \(E_{\text{sym}}(\rho_0)\) and its slope parameter \(L\) are the two lowest order bulk parameters of the nuclear symmetry energy. The bulk parameters defined at \(\rho_0\) provide important information on sub- and supra-saturation density behaviors of the EOS of isospin asymmetric nuclear matter.

While significant progress has been made in determining some lower order bulk characteristic parameters of asymmetric nuclear matter, such as \(E_0(\rho_0)\), \(K_0\), \(E_{\text{sym}}(\rho_0)\) and \(L\) [2][11], yet the higher order bulk characteristic parameters are still poorly known. Actually, there is so far even not any direct experimental information on the third-order derivative parameter \(J_0\) of symmetric nuclear matter at \(\rho_0\) and the symmetry energy curvature parameter \(K_{\text{sym}}\). However, the higher order bulk characteristic parameters have been shown to be closely related to some important issues in nuclear physics and astrophysics, such as the determination of the isobaric incompressibility of asymmetric nuclear matter [11][12] and the core-crust transition density and pressure in neutron stars [13][15].

Theoretically, if the form of an energy density functional and its parameters are given, then the EOS of isospin asymmetric nuclear matter can be calculated and thus the bulk characteristic parameters at any orders can be easily obtained. Usually, the empirical values of some lower order bulk characteristic parameters, such as \(E_0(\rho_0)\), \(K_0\), \(E_{\text{sym}}(\rho_0)\) and \(L\), are used to constrain the parameters of an energy density functional. In such a way, while different energy density functionals usually predict similar results for the lower order bulk characteristic parameters, they could give very different predictions for the higher order bulk characteristic parameters. Therefore, the higher order bulk characteristic parameters can be sensitive to the energy density functional form and useful for constraining the energy density functional and its parameters.

In the present work, we analyze the correlations between the lower and higher order bulk characteristic parameters of asymmetric nuclear matter within the framework of Skyrme Hartree-Fock energy density functional. Using the empirical constraints on the lower order bulk characteristic parameters and other macroscopic properties of asymmetric nuclear matter, we then estimate the values of some higher order bulk characteristic parameters.

The paper is organized as follows. We discuss the general properties of asymmetric nuclear matter in Section...
and then introduce the Skyrme Hartree-Fock energy density functional in Section [III]. The results and discussions are presented in Section [IV]. A summary is then given in Section [V].

II. EQUATION OF STATE OF ASYMMETRIC NUCLEAR MATTER

The EOS of isospin asymmetric nuclear matter, given by its binding energy per nucleon, can be expanded to 2nd-order in isospin asymmetry \( \delta \) as

\[
E(\rho, \delta) = E_0(\rho) + E_{sym}(\rho)\delta^2 + O(\delta^4),
\]

where \( \rho = \rho_n + \rho_p \) is the baryon density with \( \rho_n \) and \( \rho_p \) denoting the neutron and proton densities, respectively; \( \delta = (\rho_n - \rho_p)/(\rho_p + \rho_n) \) is the isospin asymmetry; \( E_0(\rho) = E(\rho, \delta = 0) \) is the binding energy per nucleon in symmetric nuclear matter, and the nuclear symmetry energy is expressed as

\[
E_{sym}(\rho) = \frac{1}{2!} \frac{\partial^2 E(\rho, \delta)}{\partial \delta^2} |_{\delta=0}.
\]

The absence of odd-order terms in \( \delta \) in Eq. (1) is due to the exchange symmetry between protons and neutrons in nuclear matter when one neglects the Coulomb interaction and assumes the charge symmetry of nuclear forces. Neglecting the contribution from higher-order terms in Eq. (1) leads to the well-known empirical parabolic law for the EOS of asymmetric nuclear matter, which has been verified by all many-body theories to date, at least for densities up to moderate values [3]. As a good approximation, the density-dependent symmetry energy \( E_{sym}(\rho) \) can thus be extracted from the parabolic approximation of \( E_{sym}(\rho) \approx E(\rho, \delta = 1) - E(\rho, \delta = 0) \).

Around the nuclear matter saturation density \( \rho_0 \), the binding energy per nucleon in symmetric nuclear matter \( E_0(\rho) \) can be expanded, e.g., up to 4th-order in density, as

\[
E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2!} \chi^2 + \frac{J_0}{3!} \chi^3 + \frac{I_0}{4!} \chi^4 + O(\chi^5),
\]

where \( \chi \) is a dimensionless variable characterizing the deviations of the density from the saturation density \( \rho_0 \) of symmetric nuclear matter and it is conventionally defined as

\[
\chi = \frac{\rho - \rho_0}{3\rho_0}.
\]

The first term \( E_0(\rho_0) \) on the right-hand-side (r.h.s) of Eq. (3) is the binding energy per nucleon in symmetric nuclear matter at the saturation density \( \rho_0 \) and the coefficients of other terms are

\[
K_0 = 9\rho_0^2 \frac{d^2 E_0(\rho)}{d\rho^2}|_{\rho=\rho_0},
\]

\[
J_0 = 27\rho_0^3 \frac{d^3 E_0(\rho)}{d\rho^3}|_{\rho=\rho_0},
\]

\[
I_0 = 81\rho_0^4 \frac{d^4 E_0(\rho)}{d\rho^4}|_{\rho=\rho_0}.
\]

The coefficient \( K_0 \) is the incompressibility coefficient of symmetric nuclear matter and it characterizes the curvature of \( E_0(\rho) \) at \( \rho_0 \). The coefficients \( J_0 \) and \( I_0 \) correspond to the 3rd-order and 4th-order derivative parameters of symmetric nuclear matter, respectively.

Around the normal nuclear density \( \rho_0 \), the nuclear symmetry energy \( E_{sym}(\rho) \) can be similarly expanded, e.g., up to 4th-order in \( \chi \), as

\[
E_{sym}(\rho) = E_{sym}(\rho_0) + L\chi + \frac{K_{sym}}{2!} \chi^2
\]

\[
+ \frac{J_{sym}}{3!} \chi^3 + \frac{I_{sym}}{4!} \chi^4 + O(\chi^5),
\]

where \( L, K_{sym}, J_{sym}, I_{sym} \) are the slope parameter, curvature parameter, curvature parameter, 3rd-order and 4th-order derivative parameters of the nuclear symmetry energy at \( \rho_0 \), i.e.,

\[
L = 3\rho_0 \frac{d E_{sym}(\rho)}{d\rho}|_{\rho=\rho_0},
\]

\[
K_{sym} = 9\rho_0^2 \frac{d^2 E_{sym}(\rho)}{d\rho^2}|_{\rho=\rho_0},
\]

\[
J_{sym} = 27\rho_0^3 \frac{d^3 E_{sym}(\rho)}{d\rho^3}|_{\rho=\rho_0},
\]

\[
I_{sym} = 81\rho_0^4 \frac{d^4 E_{sym}(\rho)}{d\rho^4}|_{\rho=\rho_0}.
\]

The coefficients \( L, K_{sym}, J_{sym} \) and \( I_{sym} \) characterize the density dependence of the nuclear symmetry energy around the normal nuclear density \( \rho_0 \), and thus carry important information on the properties of nuclear symmetry energy at both high and low densities.

In the above Taylor expansions, we have kept all terms up to 4th-order in \( \chi \). The 9 coefficients, namely, \( E_0(\rho_0), K_0, J_0, I_0, E_{sym}(\rho_0), L, K_{sym}, J_{sym}, I_{sym} \), are theoretically well-defined, and they characterize the EOS of an asymmetric nuclear matter and its density dependence at the normal nuclear density \( \rho_0 \). Among these parameters, the lower order bulk characteristic parameters \( E_0(\rho_0), K_0, E_{sym}(\rho_0) \), and \( L \) have been extensively studied in the literature and significant progress has been made over past few decades. Based on the empirical constraints on the lower order bulk characteristic parameters and other macroscopic properties of asymmetric nuclear matter, we investigate in the following to what extend the higher order parameters \( J_0, I_0, K_{sym}, J_{sym}, I_{sym} \) can be constrained within the framework of Skyrme Hartree-Fock energy density functional.
III. SKYRME-HARTREE-FOCK APPROACH AND MACROSCOPIC PROPERTIES OF ASYMMETRIC NUCLEAR MATTER

In the standard Skyrme Hartree-Fock model, the nuclear effective interaction is taken to have a zero-range, density- and momentum-dependent form \[ \mathbf{10} \], i.e.,

\[
V_{12}(\mathbf{r}, \mathbf{r}) = t_0(1 + x_0 P_\sigma) \delta(\mathbf{r}) + \frac{1}{6} t_3(1 + x_3 P_\sigma) \rho^2(\mathbf{r}) \delta(\mathbf{r}) + \frac{1}{2} t_1(1 + x_1 P_\sigma)(K^2 \delta(\mathbf{r}) + \delta(\mathbf{r}) K^2) + t_2(1 + x_2 P_\sigma) K' \cdot \delta(\mathbf{r}) K + \delta W_0(\sigma_1 + \sigma_2) \cdot [K' \times \delta(\mathbf{r}) K],
\]
with \( \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2 \) and \( \mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2 \). In the above, the relative momentum operators \( K = (\nabla - \nabla)/2i \) and \( K' = -(\nabla - \nabla)/2i \) act on the wave function on the right and left, respectively. The quantities \( P_\sigma \) and \( \sigma_i \) denote, respectively, the spin exchange operator and Pauli spin matrices. The \( \sigma, t_0 - t_3, x_0 - x_3 \) are the 9 Skyrme interaction parameters and \( W_0 \) is the spin-orbit coupling constant. Within the standard form, the EOS of symmetric nuclear matter can be written as

\[
E_0(\rho) = \frac{3\hbar^2}{10m} \left( \frac{3\pi^2}{2} \right)^{2/3} \rho^{2/3} + \frac{3}{8} t_0 \rho + \frac{3}{80} \Theta_s \left( \frac{3\pi^2}{2} \right)^{2/3} \rho^{2/3} + \frac{1}{16} t_3 \rho^{3/2} + 1, \quad (14)
\]
with \( \Theta_s = 3t_1 + (5 + 4x_2)t_2 \). Furthermore, the symmetry energy can be obtained as

\[
E_{\text{sym}}(\rho) = \frac{1}{2} \left( \frac{\partial^2 E_0}{\partial \rho^2} \right)_{\rho=0} = \frac{\hbar^2}{6m} \left( \frac{3\pi^2}{2} \right)^{2/3} \rho^{2/3} - \frac{1}{8} t_0 (2x_0 + 1) \rho - \frac{1}{24} \Theta_{\text{sym}} \rho^{2/3} - \frac{1}{48} t_3 (2x_3 + 1) \rho^{3/2} + 1, \quad (15)
\]
with \( \Theta_{\text{sym}} = 3t_1 x_1 - t_2 (4 + 5x_2) \).

As shown in Ref. \[ \mathbf{3} \] the 9 Skyrme interaction parameters, i.e., \( \sigma, t_0 - t_3, x_0 - x_3 \) can be expressed analytically in terms of 9 macroscopic quantities \( \rho_0, E_0(\rho_0) \), the incompressibility \( K_0 \), the isoscalar effective mass \( m_{*0} \), the isovector effective mass \( m_{\nu0} \), \( E_{\text{sym}}(\rho_0) \), \( L \), gradient coefficient \( G_S \), and symmetry-gradient coefficient \( G_V \), i.e.,

\[
t_0 = 4\alpha/(3\rho_0), \\
x_0 = (y - 1)E_{\text{sym}}^{\text{loc}}(\rho_0)/\alpha - 1/2, \\
t_3 = 16\beta/[(\rho_0^2(\gamma + 1)], \\
x_3 = -3g(\gamma + 1)E_{\text{sym}}^{\text{loc}}(\rho_0)/(2\beta) - 1/2, \\
t_1 = 20C/[9\rho_0(\alpha_0^2)]^2 + 8G_S/3, \\
t_2 = 4(25C - 18D)/(9\rho_0(\alpha_0^2))^2 - 8(G_S + 2G_V)/3, \\
x_1 = \left[ 12G_V + 4G_S - 6D/\rho_0(\alpha_0^2) \right]/(3t_1), \\
x_2 = \left[ 20G_V + 4G_S - 5(16C - 18D)/3\rho_0(\alpha_0^2) \right]/(3t_2), \\
\gamma = \sigma - 1 - 1, \\
\]
where the parameters \( C, D, \alpha, \beta, \gamma, E_{\text{sym}}^{\text{loc}}(\rho_0), \) and \( y \) are defined as

\[
C = \frac{m - m_{*0}^2}{m_{*0}^2} E_0^{\text{kin}}, \\
D = \frac{5}{9} E_0^{\text{kin}} \left( \frac{4m}{m_{*0}} - 3 \right), \\
\alpha = -\frac{4}{3} E_0^{\text{kin}} - \frac{10}{3} C - \frac{2}{3} E_0^{\text{kin}} + 3E_0(\rho_0) - 2C, \\
\beta = \frac{E_0^{\text{kin}}}{C} - E_0(\rho_0) - \frac{2}{3} C, \\
\gamma = \frac{K_0 + 2E_0^{\text{kin}} - 10C}{2E_0^{\text{kin}} - 9E_0(\rho_0) - E_0^{\text{kin}} - 4C}, \\
E_{\text{sym}}^{\text{loc}}(\rho_0) = E_{\text{sym}}(\rho_0) - E_{\text{sym}}^{\text{kin}}(\rho_0) - D, \\
y = \frac{L - 3E_{\text{sym}}(\rho_0) + E_{\text{sym}}^{\text{kin}}(\rho_0) - 2D}{3(\gamma - 1)E_{\text{sym}}^{\text{loc}}(\rho_0)}.
\]

IV. RESULTS AND DISCUSSIONS

Based on the formulism in previous sections, one can now estimate the values of higher order bulk characteristic parameters, such as \( J_0 \) and \( K_{\text{sym}} \) by analyzing their
correlations with $\rho_0$, $E_0(\rho_0)$, $K_0$, $m_{v,0}$, $m_{s,0}$, $E_{\text{sym}}(\rho_0)$, $L$, $G_S$, and $G_V$ within the framework of Skyrme Hartree-Fock energy density functional. As a reference for the correlation analyses below, we use the MSL0 parameter set [8], which is obtained by using the following empirical values for the macroscopic quantities: $\rho_0 = 0.16$ fm$^{-3}$, $E_0(\rho_0) = -16$ MeV, $K_0 = 230$ MeV, $m_{s,0} = 0.8m$, $m_{v,0} = 0.7m$, $E_{\text{sym}}(\rho_0) = 30$ MeV, and $L = 60$ MeV, $G_V = 5$ MeVfm$^5$, and $G_S = 132$ MeVfm$^5$. And $W_0 = 133.3$ MeV-fm$^5$ is used to fit the neutron $p_{1/2}−p_{3/2}$ splitting in $^{16}$O. It has been shown [8] that the MSL0 interaction can describe reasonably (the relative deviation from the data is less than 2%) the binding energies and charge rms radii for a number of closed-shell or semi-closed-shell nuclei. It should be pointed out that the MSL0 is only used here as a reference for the correlation analyses below. Using other Skyrme interactions obtained from fitting measured binding energies and charge rms radii of finite nuclei as in usual Skyrme parametrization will not change our conclusion.

To reveal clearly the correlation of higher order bulk characteristic parameters with each macroscopic quantity, we vary one macroscopic quantity at a time while keeping all others at their default values in MSL0. Shown in Fig. 1 is the value of the higher order bulk characteristic parameter $J_0$. Within the uncertain ranges considered here, the parameter $J_0$ exhibits a very strong correlation with $K_0$ and $E_0(\rho_0)$. However, it depends only moderately on $m_{s,0}$ and very weakly on $\rho_0$ while it displays no dependence on the other parameters $E_{\text{sym}}(\rho_0)$, $L$, $G_S$, $G_V$, $m_{v,0}$, and $W_0$. From the empirical values of $E_0 = -16 \pm 1$ MeV, $K_0 = 240 \pm 20$ MeV, and $m_{s,0} = (0.8 \pm 0.1)m$, we can obtain an estimate of $J_0 = -355 \pm 95$ MeV. The results of a similar correlation analysis on the parameter $I_0$ are shown in Fig. 2 and it is seen that the parameter $I_0$ also exhibits a very strong correlation with $K_0$ and $E_0(\rho_0)$ and the empirical values of $E_0 = -16 \pm 1$ MeV and $K_0 = 240 \pm 20$ MeV lead to an extraction of $I_0 = 1473 \pm 680$ MeV. Therefore, the higher order bulk characteristic parameters $J_0$ and $I_0$ can be determined within relative uncertainties of about 27% and 46%, respectively, in the present analysis.

In Fig. 3 we make a similar correlation analysis as in Fig. 1 for the symmetry energy curvature parameter $K_{\text{sym}}$. It is clearly seen that the parameter $K_{\text{sym}}$ exhibits a very strong correlation with $L$ and $E_{\text{sym}}(\rho_0)$.
characteristic parameters, one can see how they influence the EOS of symmetric nuclear matter. Shown in Fig. 4 is the case for symmetric nuclear matter. Fig. 5 displays how the uncertainties of $\chi$, $\chi^3$, and $\chi^4$, respectively. One can see that Eq. (3) with terms up to $\chi^2$ [i.e., the parabolic approximation] already gives a convergent prediction for the EOS of symmetric nuclear matter from about $0.5\rho_0$ to $1.5\rho_0$. The higher order terms of $\chi^3$ and $\chi^4$ with the characteristic parameters $J_0$ and $I_0$ in Eq. (3) significantly improves the approximation to the EOS at low densities and that up to about $2\rho_0$. To describe reasonably the EOS of symmetric nuclear matter above $2\rho_0$, one needs to include higher order terms in $\chi$. These results are consistent with those obtained in Ref. [12] where the MDI interaction has been used. Figs. 6(b) and (c) display how the uncertainties of $J_0$ and $I_0$ affect the EOS of symmetric nuclear matter. It is seen that their uncertainties have only minor influence on the sub-saturation density behaviors but significantly affect the EOS at higher densities (above about $2\rho_0$). These results imply that the EOS of symmetric nuclear matter in densities up to about $2\rho_0$ can already be well described by the three bulk characteristic parameters $E_0(\rho_0)$, $K_0$, and $J_0$.

Figure 7 displays how the higher order bulk characteristic parameters $K_{\text{sym}}$, $J_{\text{sym}}$, and $I_{\text{sym}}$ influence the density dependence of symmetry energy. Figure 7(a) shows the results obtained by using Eq. (3) including terms up to $\chi$, $\chi^2$, $\chi^3$, and $\chi^4$, respectively. One can see that Eq. (3) with terms up to $\chi^2$ [i.e., the parabolic approximation] already gives a convergent prediction for the EOS of symmetric nuclear matter from about $0.5\rho_0$ to $1.5\rho_0$. The higher order terms of $\chi^3$ and $\chi^4$ with the characteristic parameters $J_0$ and $I_0$ in Eq. (3) significantly improves the approximation to the EOS at low densities and that up to about $2\rho_0$. To describe reasonably the EOS of symmetric nuclear matter above $2\rho_0$, one needs to include higher order terms in $\chi$. These results are consistent with those obtained in Ref. [12] where the MDI interaction has been used. Figs. 6(b) and (c) display how the uncertainties of $J_0$ and $I_0$ affect the EOS of symmetric nuclear matter. It is seen that their uncertainties have only minor influence on the sub-saturation density behaviors but significantly affect the EOS at higher densities (above about $2\rho_0$). These results imply that the EOS of symmetric nuclear matter in densities up to about $2\rho_0$ can already be well described by the three bulk characteristic parameters $E_0(\rho_0)$, $K_0$, and $J_0$. However, it depends only moderately on $m^{*}_{s,0}$ and $m^{*}_{o,0}$ and very weakly on $E_{\text{sym}}(\rho_0)$, $K_0$, and $\rho_0$ while it is independent of other parameters $G_S$, $G_V$, and $W_0$. From the empirical values of $L = 60 \pm 30$ MeV and $E_{\text{sym}}(\rho_0) = 30 \pm 5$ MeV, $m^{*}_{s,0} = (0.8 \pm 0.1)m$, and $m^{*}_{s,0} - m^{*}_{o,0} = (0.126 \pm 0.051)m$ [14], we can obtain $K_{\text{sym}} = -100 \pm 165$ MeV. Furthermore, Figs. 4 and 5 display the results from similar analysis as in Fig. 4 for the higher order bulk characteristic parameters $J_{\text{sym}}$ and $I_{\text{sym}}$, respectively. Similarly as in the case of $K_{\text{sym}}$, they exhibit very strong correlation with $L$ and $E_{\text{sym}}(\rho_0)$. The empirical values of $L = 60 \pm 30$ MeV, $E_{\text{sym}}(\rho_0) = 30 \pm 5$ MeV, $m^{*}_{s,0} = (0.8 \pm 0.1)m$, and $m^{*}_{s,0} - m^{*}_{o,0} = (0.126 \pm 0.051)m$ lead to the estimates of $J_{\text{sym}} = 224 \pm 385$ MeV and $I_{\text{sym}} = -1309 \pm 2025$ MeV. These results indicate that the higher order bulk characteristic parameters of the symmetry energy, i.e., $K_{\text{sym}}$, $J_{\text{sym}}$, and $I_{\text{sym}}$, are largely uncertain based on our present knowledge in the standard SHF energy density functional.

Using the estimated values of the higher order bulk characteristic parameters, one can see how they influence the EOS of asymmetric nuclear matter. Shown in Fig. 4 is the case for symmetric nuclear matter. Fig.
vergent result for the density dependence of symmetry energy up to about $2\rho_0$. The higher order terms of $\chi^3$ and $\chi^4$ with the characteristic parameters $J_{\text{sym}}$ and $I_{\text{sym}}$ in Eq. (8) significantly improve the approximation to the symmetry energy up to about $3\rho_0$. Figures 7(b), 7(c) and 7(d) display how the uncertainties of $K_{\text{sym}}, J_{\text{sym}}$ and $I_{\text{sym}}$ affect the density dependence of symmetry energy. One can see that the uncertainty of the parameter $K_{\text{sym}}$ significantly affects both the sub- and supra-saturation density behaviors of the symmetry energy while the uncertainties of $J_{\text{sym}}$ and $I_{\text{sym}}$ only have significant influence on the symmetry energy at higher densities (above about $2\rho_0$). These features indicate that the three bulk characteristic parameters $E_{\text{sym}}(\rho_0), L$, and $K_{\text{sym}}$ essentially determine the symmetry energy with the density up to about $2\rho_0$.

V. SUMMARY

We have analyzed the correlations between the lower and higher order bulk characteristic parameters of asymmetric nuclear matter within the framework of Skyrme Hartree-Fock energy density functional. Based on these correlations, we have estimated the values of some higher order bulk characteristic parameters. In particular, we have obtained $J_0 = -355 \pm 95$ MeV, $I_0 = 1473 \pm 680$ MeV, $K_{\text{sym}} = -100 \pm 165$ MeV, $J_{\text{sym}} = 224 \pm 385$ MeV, and $I_{\text{sym}} = -1309 \pm 2025$ MeV using the empirical constraints on $E_0(\rho_0), K_0, E_{\text{sym}}(\rho_0), L$, and the isoscalar and isovector nucleon effective masses. Our results indicate that the three bulk characteristic parameters $E_0(\rho_0), K_0,$ and $J_0$ essentially determine the EOS of symmetric nuclear matter in densities up to about $2\rho_0$ while the three bulk characteristic parameters $E_{\text{sym}}(\rho_0), L,$ and $K_{\text{sym}}$ well characterize the symmetry energy in densities up to about $2\rho_0$. For higher density (above $2\rho_0$) behaviors of the EOS of asymmetric nuclear matter, the higher order bulk characteristic parameters become important.

In the present work we have estimated the higher order bulk characteristic parameters only based on the standard SHF energy density functional. It will be interesting to see how our results change if different energy-density functionals are used and these studies are in progress. The experimental information or model independent prediction on higher order bulk characteristic parameters of asymmetric nuclear matter is expected to put important constraints on the nuclear energy density functional form and its parameters.

Acknowledgments

The author thanks Che Ming Ko, Bao-An Li, Chang Xu, and Jun Xu for helpful discussions. This work was supported in part by the NNSF of China under Grant No. 10975007, Shanghai Rising-Star Program under Grant No. 06QA14024, the National Basic Research Program of China (973 Program) under Contract No. 2007CB815004.

[1] B.A. Li, C.M. Ko, and W. Bauer, Int. Jour. Mod. Phys. E 7, 147 (1998).
[2] P. Danielewicz, R. Lacey, and W.G. Lynch, Science 298, 1592 (2002).
[3] J.M. Lattimer and M. Prakash, Science 304, 536 (2004); Phys. Rep. 442, 109 (2007).
[4] W. Steiner, M. Prakash, J.M. Lattimer, and P.J. Ellis, Phys. Rep. 411, 325 (2005).
[5] V. Baran, M.Colonna, V. Greco, and M. Di Toro, Phys. Rep. 410, 335 (2005).
[6] L.W. Chen, C.M. Ko, B.A. Li, and G.C. Yong, Front. Phys. China 2, 327 (2007) [arXiv:0704.2340].
[7] B.A. Li, L.W. Chen, and C.M. Ko, Phys. Rep. 464, 113 (2008).
[8] D.H. Youngblood, H.L. Clark, and Y.-W. Lui, Phys. Rev. Lett. 82, 691 (1999).
[9] L.W. Chen, C.M. Ko, B.A. Li, and J. Xu, Phys. Rev. C 82, 024321 (2010).
[10] C. Xu, B.A. Li, and L.W. Chen, Phys. Rev. C 82, 054607 (2010).
[11] L.W. Chen, Sci. China Ser. G 52, 1494 (2009) [arXiv:0911.1092].
[12] L.W. Chen, B.J. Cai, C.M. Ko, B.A. Li, C. Shen, and J. Xu, Phys. Rev. C 80, 014322 (2009).
[13] J. Xu, L.W. Chen, B.A. Li, and H.R. Ma, Phys. Rev. C 79, 035802 (2009).
[14] J. Xu, L.W. Chen, B.A. Li, and H.R. Ma, Astrophys. J. 697, 1549 (2009).
[15] C. Ducoin, J. Margueron, and C. Providência, Europhys. Lett. 91, 32001 (2010).
[16] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, Nucl. Phys. A627, 710 (1997).