Determining the quantum numbers of excited heavy mesons

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Abstract

We discuss the decays $X^* \rightarrow Xe^+e^-$ (“Dalitz decays”) of excited heavy mesons into their ground states and an electron–positron pair. We argue that the measurement of the invariant mass spectrum of the lepton pair gives clear indication on the quantum numbers of the excited meson and thus provides an experimental test of the quark model predictions.

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We investigate radiative decays of excited heavy mesons with charm and beauty, i.e. $D^{*0}$, $D_s^{*+}$, $B^{*0}$ and $B_s^{*0}$, into their ground state plus an electron–positron pair. There are several reasons that led us to investigate these decays:

1. The quantum numbers $J$ and $P$ of the excited mesons given in the Review of Particle Physics [1] are either merely predictions of the quark model, which gives for the particles discussed here $J^P = 1^-$, or at best the quantum numbers need confirmation [1].

2. The quantum numbers of some even higher heavy meson states, depend on the correct assignments for the $X^*$’s which are the focus of the present work.

3. The central values of the branching ratios for $D^{*0} \rightarrow D^0\gamma$ and $D^{*0} \rightarrow D^0\pi$ sum up to exactly 100 % [1]. Only closer inspection of both relevant experimental papers [2] reveals that this was one of the assumptions underlying the analyses. A similar assumption was made for $D_s^{*\pm}$, where again it was assumed that branching ratios for $D_s^{*+} \rightarrow D_s^+\gamma$ and $D_s^{*+} \rightarrow D_s^+\pi^0$ sum to 100 % [3]. Although the current errors on the individual branching ratios are clearly higher than the branching ratios for the decay into a Dalitz pair which, as we will see below, are of the

\[1\text{We would like to mention here, that for the } B-\text{meson even the quantum numbers of the ground state are not known experimentally; nevertheless we assume } J^P(B) = 0^-.\]
order of 0.5% of their real photon emission counterparts, in the long
term this channel cannot be neglected. Especially in light of future pre-
cision measurements in the $B$–sector, a precise tracking of all outgoing
$D$’s is required. In other words, it is important to know which decay
of $X^*$ has the closest branching ratio to the decays that have been al-
ready measured. Let us remark, that for the $B^*$ decay, for which the
dominant mode is $B^* \to B\gamma$, its decay into $Be^+e^-$ presented here has
a much larger rate (about 0.5%, see below) than the decay $B^* \to B\gamma\gamma$,
recently discussed in [4].

4. The only recent theoretical analysis of an $X^* \to Xe^+e^-$ decay was
performed for $D^* \to De^+e^-$ [5] and yielded $R \approx 0.001$, where the ratio
$R$ is defined as

$$R = \frac{\Gamma(X^* \to Xe^+e^-)}{\Gamma(X^* \to X\gamma)} = \frac{\Gamma_{ee}}{\Gamma_{\gamma}}$$

(1)

with $X = D$. As found below, we predict $R$ to be about 5 times
larger than the result of [5]. Let us note here that the suggestion to
use the ratio between the Dalitz decay and the real photon emission–
and in particular its $q^2 \equiv m_{12}^2$ dependence–to determine the quantum
numbers of the decaying particle, was made many years ago [6, 7]. Our
results are consistent with theirs.

Since we will be mainly concerned with ratios $R$, where $X = D, B$,
it is sufficient to parametrize the $X^* \to X\gamma$ transition with some effective
coupling constant $g_{X^*X\gamma}$ and a suitable form factor $\mathcal{F}$. We replace the various
form factors possible for off–shell photons by a single one, and furthermore
assume that it is independent of $q^2$. This is justified by the observation that
for all the $X^*$–$X$ combinations we consider, the mass difference $\Delta_{X^*X} \equiv m_{X^*} - m_X$ is of the order of up to 150 MeV and thus much smaller than
the $\rho$–mass which is the most relevant one in the vector dominance model
for the form factors. Therefore the influence of the form factors on the
results is indeed very small. We have confirmed that our numerical results
are practically unaffected by the assumption $\mathcal{F}(q^2) \approx \mathcal{F}(0)$.

The matrix elements we consider read [8]

$$\mathcal{M}_{1^-0^-} = g_{X^*X\gamma} \cdot \mathcal{F}(q^2) \cdot \epsilon^{a\beta\mu\nu} \epsilon^*_\alpha(\gamma) \epsilon^*_\beta(X^*) P_\mu(X^*) q_\nu(\gamma)$$

$$\mathcal{M}_{2^+0^-} = g_{X^*X\gamma} \cdot \mathcal{F}(q^2) \cdot \epsilon^{a\beta\mu\nu} \epsilon^*_\alpha(\gamma) P_\beta(X^*) \epsilon^*_\mu(\gamma) q_\nu(\gamma) q^\rho(\gamma).$$

(2)

We have concentrated here on the $1^-$ and $2^+$ quantum numbers for the excited
and $0^-$ for the ground state, since in the $D$–system the quantum numbers
of the ground states are known to be $0^-$ and because the $D^*$ decays into a $D$ and both a pion or a photon. Therefore, the $1^-\text{ and } 2^+$ are the lowest lying quantum numbers allowed for the $D^*$. In fact, for the $B-$system the situation is slightly different. Here the small mass difference of the $B^*$ and the $B$ of roughly 45 MeV does not provide enough phase space for the $B^*$ to decay into a $B$ and a $\pi$, hence the $B^*$ could in principle be a $1^+$. This would cause the relevant strong coupling constant $g_{B^*B\pi}$ to vanish. However, we consider this idea to be too far–out and will not discuss it here.

We use the completeness relations (for the $2^+$ see e.g: [9])

\[
\sum \epsilon^\mu(p)\epsilon^\nu(p) = -\left( g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \]

\[
\sum \epsilon^{\mu\nu}(p)\epsilon^{\rho\sigma}(p) = \frac{1}{2} \left[ \left( g^{\mu\rho} - \frac{p^\mu p^\rho}{p^2} \right) \left( g^{\sigma\nu} - \frac{p^\sigma p^\nu}{p^2} \right) + \left( g^{\sigma\mu} - \frac{p^\sigma p^\mu}{p^2} \right) \left( g^{\nu\rho} - \frac{p^\nu p^\rho}{p^2} \right) \right] - \frac{2}{3} \left( g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \left( g^{\rho\sigma} - \frac{p^\rho p^\sigma}{p^2} \right). \tag{3}
\]

After squaring, summing and averaging we obtain for the the decay into a real photon

\[
3 \sum |M_{1-\rightarrow0^-\gamma}|^2 = 2g^2_{X^*X\gamma} \cdot F^2(0) \cdot (Pq)^2 \tag{4}
\]

\[
5 \sum |M_{2+\rightarrow0^-\gamma}|^2 = g^2_{X^*X\gamma} \cdot F^2(0) \cdot \frac{(Pq)^4}{P^2},
\]

where $P$ and $q$ are the momenta of the excited meson and the photon, respectively. The resulting branching ratios are given by

\[
\Gamma_{1^-\rightarrow0^-\gamma} = g^2_{X^*X\gamma} \cdot F^2(0) \cdot \frac{(m^2_{X^*} - m^2_X)^3}{96\pi m^3_{X^*}} \tag{5}
\]

\[
\Gamma_{2^+\rightarrow0^-\gamma} = g^2_{X^*X\gamma} \cdot F^2(0) \cdot \frac{(m^2_{X^*} - m^2_X)^5}{1280\pi m^3_{X^*}}.
\]

With suitable replacements we recover the known result for the width of the decay $a_2 \rightarrow \pi\gamma$ [10].

For the decay into $e^+e^-$ the polarization vector $\epsilon_\mu$ of the photon has to be replaced by the lepton–current $e\bar{u}(e^-)\gamma_\mu u(e^+)$. Squaring the matrix elements and summing and averaging over polarizations yields

\[
3 \sum |M_{1^-\rightarrow0^-e\bar{e}}|^2 = 2g^2_{X^*X\gamma} \cdot \frac{F^2(q^2)}{q^4} \cdot \left[ 4m^2_e ((Pq)^2 - P^2 q^2) + q^2 \left( 2(P\bar{e})^2 + 2(Pe\bar{e})^2 - P^2 q^2 \right) \right] \tag{6}
\]

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\left[ 4m^2_e ((Pq)^2 - P^2 q^2) + q^2 \left( 2(P\bar{e})^2 + 2(Pe\bar{e})^2 - P^2 q^2 \right) \right] \tag{6}
\]
The resulting widths $\Gamma(X^* \rightarrow X e^+ e^-)$ and their respective ratios to the real photon widths agree with the results given in [7]. Our results for the ratio $R$, defined in Eq. (1), in some meson systems are displayed in Table 1. All of them are of the order of $5 \cdot 10^{-3}$. As can be observed from the table, the ratio

$$5 \sum |\mathcal{M}_{2+0-ee}|^2 = g_{X^*X\gamma}^2 \frac{F^2(q^2)}{q^4} \frac{(Pq)^2 - P^2 q^2}{P^2} \cdot \left[ 4m_e^2 \left( (Pq)^2 - P^2 q^2 \right) + q^2 \left( 2(P^2 p_e)^2 + 2(Pp_e)^2 - P^2 q^2 \right) \right]. \quad (7)$$

$R$ may serve as an indicator for the $J^P$ quantum numbers of $X^*$ mesons, which are believed to have $J^P = 1^-$. At present, the only ratio for which an experimental number exists is $R = (4.7 \pm 1.1 \pm 0.9) \times 10^{-3}$, for $B^* \rightarrow 35$ MeV. Although the central value agrees well with the quark model quantum numbers $J^P(B^*) = 1^-$, it is premature to claim, in view of the large error, a clear-cut rejection of the $2^+$ possibility.

A better indicator for the quantum numbers of $X^*$ is the distribution of the invariant mass squared ($q^2 = m_{ee}^2$) of the $e^+ e^-$ pair. To illustrate the effect of different quantum numbers on $d\Gamma(X^{*0} \rightarrow X^0 e^+ e^-)/dq^2$, scaled by

| $X$ | $R (1^-)$ | $R (2^+)$ | $\mathcal{B}(X^* \rightarrow X\gamma)$ | $m_{X^*} - m_X$ |
|----|-----------|-----------|--------------------------------|----------------|
| $B_s^0$ | $4.65 \times 10^{-3}$ | $4.34 \times 10^{-3}$ | dominant | $45.78 \pm 0.35$ MeV |
| $B^0$ | $4.69 \times 10^{-3}$ | $4.38 \times 10^{-3}$ | dominant | $45.78 \pm 0.35$ MeV |
| $D_s^0$ | $6.45 \times 10^{-3}$ | $6.14 \times 10^{-3}$ | $0.942 \pm 0.025$ | $143.8 \pm 0.4$ MeV |
| $D^0$ | $6.44 \times 10^{-3}$ | $6.13 \times 10^{-3}$ | $0.381 \pm 0.029$ | $142.12 \pm 0.07$ MeV |
| $D^+$ | $6.42 \times 10^{-3}$ | $6.11 \times 10^{-3}$ | $0.011^{+0.021}_{-0.007}$ | $140.64 \pm 0.10$ MeV |
| $K^0$ | $7.99 \times 10^{-3}$ | $(7.68 \times 10^{-3})$ | $0.0023 \pm 0.0002$ | $398.42 \pm 0.28$ MeV |

Table 1: Predicted ratios $R = \Gamma(X^* \rightarrow X e^+ e^-)/\Gamma(X^* \rightarrow X\gamma)$, for some mesonic systems $X$. The branching ratios $\mathcal{B}(X^* \rightarrow X\gamma)$ and mass differences $m_{X^*} - m_X$ are from [11]. We have assumed $m_{B_s} - m_{B_s} = m_{B_d} - m_{B_d}$. The number for the $K^*(892)$ decay is included for completeness only, since here form factors might become important, and of course, the $K^*(892)$ is a $1^-$ particle. Note however that $K^*(892) \rightarrow Ke^+ e^-$ has not been observed. The only ratio for a heavy system $X$ observed so far is $B^* \rightarrow 11$, where $R = (4.7 \pm 1.1 \pm 0.9) \times 10^{-3}$. 


$m_{12}^2$–distribution of the electron–positron pairs in the Dalitz decay $B^* \rightarrow B e^- e^+$ with $q$ the invariant mass of the pairs. Clearly, the quantum numbers affect the tail of the distribution.

Figure 1: $m_{12}^2$–distribution of the electron–positron pairs in the Dalitz decay $B^* \rightarrow B e^- e^+$ with $q$ the invariant mass of the pairs. Clearly, the quantum numbers affect the tail of the distribution.

To summarize, we have presented results for the decay widths of $D^* \rightarrow De^+ e^-$, $D_s^{\pm} \rightarrow D_s^{\pm} e^+ e^-$ and $B^{*0} \rightarrow B^0 e^+ e^-$. All of them are of the order of roughly 0.5% of the corresponding decays into real photons, as expected from a QED–like calculation. The difference between the two $J^P$ assignments $1^-$ and $2^+$ for the the decaying particle, is about 5%. A significant difference arises for the $q^2$–distribution, especially at high $q^2$. In the light of the recent experimental results by [11] on the invariant mass spectrum in the Dalitz decay of $B^*$’s we are very optimistic that the ambitious measurements we advertise are feasible.

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References

[1] C. Caso et al.; Europ. Phys. J. C3 (1998) 1.

[2] H. Albrecht et al., ARGUS collab., Z. Phys. C66 (1995) 63; J. Adler et al., MARK III collab., Phys. Lett. B208 (1988) 152.

[3] J. Grondberg et al.; CLEO collab., Phys. Rev. Lett. 75 (1995) 3232.

[4] D. Guetta, P. Singer; Phys. Rev. D61 (2000) 054014.

[5] T. M. Aliev, E. Iltan, N. K. Pak, M. P. Rekalo; Zeit. f. Phys. C64 (1994) 683.

[6] G. Feinberg; Phys. Rev. 109 (1958) 1019.

[7] For reviews on the decays $X^* \rightarrow X\pi^\pm\pi^\mp$ for light mesons see e.g: L. G. Landsberg; Phys. Rep. 128 (1985) 301, and A. Faessler, C. Fuchs and M.I. Krivoruchenko; Phys. Rev. C61 (2000) 035206 and references therein.

[8] See e.g: H.M. Pilkuhn; *Relativistic Particle Physics*, Springer, 1979.

[9] T. Han, J.D. Lykken and R-J. Zhang; Phys.Rev. D59 (1999) 105006.

[10] H. Högaasen, J. Högaasen, R. Keyser, B. E. Y. Svenson; Nuovo Cim. 42A (1966) 323.

[11] G. Eigen; hep-ex/9901007 and references therein.