Helicity sources in a rotating convection

A Teimurazov, A Sukhanovskii, A Evgrafova and R Stepanov
Institute of Continuous Media Mechanics, Academ. Korolyov, 1, Perm, 614013, Russia
E-mail: san@icmm.ru

Abstract. Convective flows from localized heater in a rotating layer were studied numerically in a three-dimensional non-stationary formulation. Distribution of helicity, its mean and fluctuating contributions were simulated for two specific regimes. In the first one the stable cyclonic vortex and intensive convective jet produce substantial amount of helicity near the axis of rotation. In the second one convective flow is more chaotic, cyclonic vortex appears at some distance from the center and as a result helicity is more dispersed in the lower layer of the fluid. Helicity in a described system is characterized by high level of pulsations. Spatial and temporal variations of helicity sources were analysed using equation for helicity balance. Time variations of viscous term and buoyancy term in helicity equation strongly exceed variations of other terms and the helicity time derivative. In a described system the buoyancy term is a source of helicity and viscous term is a sink. It was found that they are in antiphase and compensate each other.

1. Introduction
Nonlinear dynamics of fluids can lead to significant effects manifested in a particular flow structure. Explanation of these effects needs detailed consideration of physical mechanisms. Such analysis is getting especially complex when flow consists of structures of different scales. Their interactions can be local or nonlocal in the Fourier space and provide an effective transport of inviscid invariants up or down scale. Energy cascades (direct or inverse) in turbulent flows are well known examples. The theory of turbulence suggests a phenomenology which helps to consider a turbulent flow in the frame of simplified model, introducing eddy viscosity and simulate a mean flow. However a concept of homogeneous and isotropic turbulence is a very idealized view and should be used carefully because real turbulent flows are usually non-homogeneous and non-isotropic. Existence of boundaries leads to the different approaches for description of the bulk and boundary layer turbulence. In addition there are external volume forces like a buoyancy in the case of convection. This force may inject in flow not only kinetic energy but also helicity which is the second inviscid invariant. Helicity \( h = u \cdot \omega \), characterizes velocity-vorticity correlation is observed in supercell of numerically simulated convective flow under rotation [1]. Also helicity can lead to a reduction of turbulent viscosity and energy accumulation in large scales [2, 3, 4] or play a significant role in the generation of large-scale helical vortex structures in the atmospheres of planets [5, 6, 7].

In the present paper we focused on nonlinear dynamics of the convective flow from localized heating source in the rotating layer. The general structure of the flow, its dependence on governing parameters were studied experimentally and numerically [8, 9, 10]. It contains global cyclone and anti-cyclone rotation and convective circulation. It was shown [11, 12] that described flow is characterized by strongly non-homogeneous distribution of helicity. Formation
2. Formulation of the problem and the mathematical model

The parameters of the numerical model were deliberately chosen similar to the experimental ones. The computational domain is a rotating cylinder with the diameter \( D = 300 \text{ mm} \) and height \( H = 30 \text{ mm} \) (figure 1a). There is a heater with the diameter \( d = 100 \text{ mm} \) in the central part of the bottom.

The problem is considered in a three-dimensional nonstationary formulation. The numerical code solves the Oberbeck-Boussinesq equations of thermal convection. All the simulations were run using the free and open source finite volume code OpenFOAM 4.1. The mesh has a regular block structure with a total of 3.5 million nodes with additional mesh refinement near the boundaries (mesh grid step is 0.25 mm near the boundaries). As a part of verification procedure numerical results were compared with experimental data [10] and numerical calculations [12]. Velocity profiles in figure 2 show good quantitative agreement between them. The PISO (Pressure Implicit with Splitting of Operators) algorithm based on the pressure correction procedure is used to solve the system of equations. The discretization of terms with time derivatives are done using an implicit Euler scheme. The convective terms are calculated by the “limitedlinear” TVD (Total Variation Diminishing) scheme [14]. The Courant number in our computations does not exceed 0.5. The equation for helicity evolution was implemented inside the solver in order to use the same numerical procedure for calculating its terms as for governing equations of thermal convection.

Silicone oil was used as a working fluid. Its physical properties at temperature 28°C, are: Prandtl number \( Pr = 60 \), kinematic viscosity \( \nu = 5.35 \cdot 10^{-6} \text{ m}^2/\text{s} \), density \( \rho = 904 \text{ kg/m}^3 \), and thermal expansion coefficient \( \beta = 9 \cdot 10^{-4} \text{ K}^{-1} \). Rotating cylinder is considered thus the “rotating wall” boundary condition with angular velocity \( \Omega = 0.0816 \text{ 1/s} \) and \( 0.170 \text{ 1/s} \) (which correspond to period of revolution 77 s and 37 s) is specified to bottom and sidewalls of the vessel. The free-slip velocity boundary condition is applied at the top surface. Fixed heat flux \( Q = 17 \text{ W} \) is applied on the heater surface. Therefore, the heat flux density is \( q_h = Q/S_h \). The equal amount of heat is removed from the top surface of the domain that is \( q_t = -Q/S_t \). Here \( S_h \) and \( S_t \) are area of the heater and top surface of the model respectively. The sidewalls and the bottom except the area of the heater are heat insulated.
3. General structure of the flow

Detailed description of the flow formation can be found in [10] and here we present only brief description of the general structure of the large-scale flow. The heat flux in the central part of the bottom is a source of the intensive upward motion above the heater. Warm fluid cools at the free surface and moves toward the periphery where the cooled fluid moves downward along the side wall. Finally the large-scale advective flow occupies the whole vessel (figure 1b, figure 3a).

The cyclonic vortex formation in the laboratory system can be described by following scenario. Large-scale radial circulation leads to the angular momentum transport and the angular momentum exchange on the solid boundaries. Convergent flow in the lower layer brings the fluid parcels with large values of angular momentum from the periphery to the center and produces cyclonic motion (figure 1b, lower horizontal cross-section). In the upper layer situation is opposite – divergent flow takes the fluid with low values of angular momentum to the periphery resulting in anticyclonic motion (figure 1b, upper horizontal cross-section). Mean distribution of azimuthal velocity in a vertical cross-section is shown in figure 3b. The friction in the viscous boundary layers leads to the sink of angular momentum in the part of the bottom occupied by cyclonic flow and produces source of angular momentum on the sidewalls when anticyclonic flow comes to the periphery. Zero net angular momentum flux on the solid boundaries is the necessary condition for the steady-state regime [9].
The large-scale advective flow in the lower part of the layer leads to the formation of boundary layer with unstable temperature stratification above the heater and generation of the horizontal rolls and thermal plumes (figure 4a). Horizontal rolls have spiral shape because of intensive cyclonic motion. One pair of rolls brings relatively cold fluid down and another pair takes warm fluid up. Periodically overheated parcels of fluid continue to float and produce thermal plumes moving to the center with the main flow.

Specific feature of the vortex formation in a rotating layer with localized heating is its strong dependence on the dimensionless rotation rate $Re = \Omega h^2/\nu$. Up to some values of $Re (\approx 23)$ intensive cyclonic vortex is located in the center (figure 4b) and is quite robust (stable regime). At $Re > 23$ the vortex appears at larger radii and different azimuthal locations (unstable regime).

The main focus of the presented paper is a helicity generation in the described system. Instantaneous and mean helicity fields for the stable and unstable regimes are shown in figure 5. As we expected the change of the flow structure leads to the remarkable change of helicity distribution. Instantaneous helicity fields are strongly non-homogeneous as a result of small-scale structures dynamics (figure 5a,b). In the stable regime (I) mean helicity is concentrated in the central part where upgoing jet and intensive cyclonic vortex are located (figure 5c). In the unstable regime (II) mean helicity is spread in the lower part of the layer (figure 5d). The intensity of helicity pulsations can be characterised by the standard deviation (SD). Figure 5e,f shows the helicity SD fields. Helicity in a described system is characterized by high level of pulsations. For the regime I there are two distinct areas where peaks of pulsations are localized. Near the periphery of the heater where thermals are formed and near the center due to intensive upgoing jet fluctuations. The flow in regime II is less organized and more chaotic so pulsations are distributed more uniformly except periphery of the heater where secondary flows are absent. The level of pulsations is about half of the mean helicity values for the regime I and it is comparable with value of mean helicity in the regime II. It is useful to consider different contributions of helicity (parts of scalar product of velocity and vorticity vectors):

$$h = h_r + h_\phi + h_z = u_r \omega_r + u_\phi \omega_\phi + u_z \omega_z.$$  \hspace{1cm} (1)

Mean fields of helicity contributions are shown in figure 6. For the both regimes $h_r$ is essentially weaker than $h_z$ and $h_\phi$.

In this section we focused on the qualitative description of helicity fields but understanding of helicity distribution and dynamics requires analysis of helicity sources which are considered in Section 4.
Figure 5. Distribution of helicity in the $yOz$ plane (only central area is shown) at $x = 0$: (a), (b) – instant helicity field; (c), (d) – time-averaged helicity field; (e), (f) – standard deviation of the helicity field. Left column – $\Omega = 0.0816$ 1/s; right column – $\Omega = 0.170$ 1/s.

Figure 6. Distributions of different contributions of helicity in the $yOz$ plane (only central area is shown) at $x = 0$: (a), (b) – time-averaged radial contribution of the helicity field $\langle h_r \rangle$; (c), (d) – time-averaged azimuthal contribution of the helicity field $\langle h_\phi \rangle$; (e), (f) – time-averaged vertical contribution of the helicity field $\langle h_z \rangle$. Left column – $\Omega = 0.0816$ 1/s; right column – $\Omega = 0.170$ 1/s.
4. Helicity balance

The governing equation for helicity evolution has the form [15]:

$$\frac{\partial h}{\partial t} = u \cdot \nabla \times (u \times \omega) - \omega \cdot (u \cdot \nabla u) - \omega \cdot \nabla P/\rho + \nu (\omega \cdot \nabla^2 u + u \cdot \nabla \omega) + \beta g (T \cdot e_z + u \cdot \nabla \times (Te_z)),$$  \(2\)

where a gravity acceleration \(g = -ge_z\). There are several terms which are responsible for helicity balance. First two terms at the right side of Eq. (2) correspond to the advection of helicity by the flow. The third term is related to the pressure gradient. The fourth term is responsible for viscous dissipation and the last one determines the buoyancy effects.

Due to complexity of helicity equation we decided to analyse helicity production at two specific points: at \(z = 0.003\) m which is located in the boundary layer and in the middle of the layer at \(z = 0.015\) m. Time series for variations of the helicity time derivative and all the terms in the right side of the helicity equation (buoyancy term, viscous term, advection term, pressure term) at \(\Omega = 0.0816\) 1/s at different points on the central axis of the cavity are shown in figure 7. Time
variations of viscous term and buoyancy term (two last terms in equation 2) strongly exceed variations of other terms and the helicity time derivative. The most interesting part is that they are in antiphase and compensate each other. In a described system the buoyancy term is a source of helicity and viscous term is a sink. The time evolution of helicity sources and sinks is very complex and its analyses requires long-time calculations. The spatial distributions of the buoyancy term for two regimes are shown in figure 8. In a stable regime the helicity generation by buoyancy term is concentrated in the center and in the unstable regime with more chaotic convective flows it is spread over larger area.

5. Conclusion
Convective flows from localized heater in a rotating layer were studied numerically using open source finite volume code OpenFOAM 4.1. Distribution of helicity, its contributions and pulsations were reconstructed for two specific regimes. In the first one the stable cyclonic vortex and intensive convective jet produce substantial amount of helicity near the axis of rotation. In the second one convective flow is more chaotic, cyclonic vortex appears at some distance from the center and as a result helicity is more dispersed in the lower layer of the fluid. Helicity in a described system is characterized by high level of pulsations. The level of pulsations is about half of the mean helicity values for the regime I and it is comparable with value of mean helicity in the regime II. Analysis of helicity sources using equation for helicity balance was carried out for two locations, in the boundary layer and in the middle of the layer. Time variations of viscous term and buoyancy term in helicity equation strongly exceed variations of other terms and the helicity time derivative. The amplitude of viscous and buoyancy terms are higher in the boundary layer but resulting helicity time derivative is larger in the middle of the layer. The buoyancy term produces helicity and viscous term efficiently devours most of it. This result is very interesting and deserves special attention. It is unclear how variation of dimensional parameters such as kinematic viscosity or coefficient of thermal expansion would change the ratio of buoyancy and viscous terms.

Acknowledgments
This work was supported by the Russian Science Foundation (grant RSF-16-41-02012). Computing resources of the supercomputer “Triton” were provided by Center of Shared Facilities (Institute of Continuous Media Mechanics UrB RAS).

References
[1] Marino R, Mininni P D, Rosenberg D and Pouquet A 2013 Phys. Rev. E 87 033016 (Preprint 1211.3159)
[2] Yokoi N and Yoshizawa A 1993 Phys. Fluids 5 464–477
[3] Stepanov R, Golbraikh E, Frick P and Shestakov A 2015 Phys. Rev. Lett. 115 234501 (Preprint 1508.07236)
[4] Kessar M, Plunian F, Stepanov R and Balarac G 2015 Phys. Rev. E 92(3) 031004
[5] Moiseev S S, Sagdeev R Z, Tur A V, Khomenko G A and Shukurov A M 1983 Soviet Physics Doklady 28 926
[6] Ivanov M F, Gal’Burt V A and Fortov V E 1996 JETP Lett. 63 813–817
[7] Lilly D K 1986 J. Atmos. Sci. 43 126–140
[8] Bogatyrev G P 1990 Soviet Journal of Experimental and Theoretical Physics Letters 51 630
[9] Batalov V, Sukhanovsky A and Frick P 2010 Geophysical and Astrophysical Fluid Dynamics 104 349–368
[10] Sukhanovskii A, Evgrafova A and Popova E 2016 Quarterly Journal of the Royal Meteorological Society 142 2214–2223
[11] Sukhanovskii A, Evgrafova A and Popova E 2016 Journal of Physics: Conference Series vol 754 (IOP Publishing) p 072005
[12] Sukhanovskii A, Evgrafova A and Popova E 2017 arXiv:1705.00892 [physics.flu-dyn]
[13] Sukhanovskii A, Evgrafova A and Popova E 2016 Phys. D 316 23–33
[14] Versteeg H K and Malalasekera W 2007 An Introduction to Computational Fluid Dynamics: The Finite Volume Method (Pearson Education, Harlow)
[15] Kurgansky M V 2006 Meteorologische Zeitschrift 15 409–416