Radiating power tracking based on fractional order sliding mode controller

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Abstract
The radiating power with sine waveform is required for modulated infrared thermography nondestructive testing. Sine voltage was applied by former authors for deriving sine power waveform. The theory mode of infrared lamp is proposed. According to numerical solution and FFT analysis, high frequency components are included in the radiating power waveform, when sine voltage is set to drive the lamp. A light intensity sensor was utilized for monitoring the real-time radiating power, and a close-loop power tracking system was constructed. Fractional order sliding mode controller was selected in the tracking system, for its robustness and flexibility property. The reachability of the controller is proved and the calculation formulas of reach time are given. In experiment, the tracking results with both PID and sliding mode controller were derived. Lower tracking overshoot and better robustness was derived through the sliding mode controller, compared with the PID counterpart.

Keywords
Radiating power, fractional order, sliding mode controller, power tracking, reach time

Introduction
Infrared thermography nondestructive testing (NDT) has been rapidly developing in recent years. Its research and application are of great significance to improve the safety and reliability of aerospace vehicles, military equipment, and civilian industrial equipment. Infrared thermography NDT is an active approach, where an external stimulus is applied to the specimen in order to induce relevant thermal contrast between regions of interest. The exciting source could be divided into two types: optic and non-optic. The method discussed in this article could be applied to most optic exciting sources.

For specimen with low thermal conductivity, such as carbon fiber reinforced plastic (CFRP), the test piece needs to be fully heated for defect detection. The stimulation energy required is high and the heating time takes minutes. Modulated infrared thermography (MT) which is also referred as lock-in thermography is appropriate method. In MT, the heating power is modulated to a sine waveform. Then sine component is included in surface temperature information which is recorded by thermal camera. For defect and non-defect area, the transmission and reflection time of thermal wave is different. According to the phase difference of surface temperature between defect and non-defect area, sub-surface defect of specimen is detected.

The combination of function generator and power amplifier is generally utilized for driving the heating lamp. This is based on the assumption that a sine heating power waveform is generated, when the lamp is driven by a sine voltage. It is true when cycle of driving voltage is seconds or less. Because of thermal inertia, the temperature change of filament couldn’t follow the transient power change in one cycle. The resistance of filament keeps relatively stable within one voltage cycle. When the voltage period increases to a few minutes, the filament temperature changes dynamically over a voltage cycle. The assumption is not applicable any more. High frequency components are included in the radiating power waveform. For MT, the detection
sensitivity depends on the modulation frequency, even blind frequency was shown for defect at a special depth and thickness. The detection effectiveness is weakened when the phase information of different frequency points is mixed together. To solve this problem, an infrared light sensor was used to monitor the heating power and close-loop power tracking system was constructed. In order to deal with disturbance of environmental light change, slide mode controller (SMC) was applied for the real-time power tracking.

In recent years, SMC is widely researched and applied in science and engineering fields because of its strong robustness. The radiating power tracking system is prone to be disturbed by lighting system and changes in external ambient light. The strong anti-interference characteristic of SMC is preferred for the tracking system under discussion. Fractional order (FO) SMC has been applied in various engineering systems such as electromechanical transducers, antilock braking system, permanent synchronous motor, and wind turbines. Chakrabarty and Bandyopadhyay investigated the generalized switching law for discrete time SMC. In our previous work, fractional order sign function was introduced in switching law of SMC. Better performance was disclosed comparing with integer order counterpart. The SMC with fractional order sign function is also utilized in this radiating power tracking system.

Theory mode of the controlled object – heating lamp is proposed. Relation among driving voltage, filament temperature of heating lamp and radiating power is given by differential equation. Numerical solution was applied for evaluation of the radiating power waveform when the lamp was driven by sine voltage. Further FFT analysis disclosed harmonic distortion of the radiating power waveform, especially in the case of dual frequencies sine voltage. The analysis declares why close-loop tracking system is necessary for getting sine radiating power waveform. The sliding surface and equivalent control law is derived according to the theory mode. Filament temperature is chosen as state variable. The tracking performance is challenging because the state variable is defined in fourth-order state equation. The reachability of controller is proved by Lyapunov method and the calculation formulas of reaching time are given. An experiment platform was constructed for close-loop radiating power tracking. Some tricks of the platform such as sampling data preprocessing and communication between custom software and Simulink are introduced. PID controller is usually the first choice in engineering application because of its simplicity and mature theoretical basis. The platform was tested with both SMC involved and PID controller. Lower tracking undershoot at initial phase and better robustness for environmental light change was derived with SMC, while better tracking accuracy was with PID controller.

Problem statement
Sine voltage \( V \) in (1) was applied for driving the heating lamp. It was inferred that the radiating power with sine waveform was derived, which is necessary for modulated infrared thermography NDT.

\[
V = E \sin \omega t
\]  

where \( E \) is the amplitude and \( \omega \) is angular frequency. If the heating lamp is considered as a constant resistance \( R \), the power consumption \( P \) is:

\[
P = \frac{V^2}{R} = \frac{E^2}{R} - \frac{1 - \cos 2\omega t}{2}
\]

The electric power \( P \) is sine wave at twice frequency of the driving voltage, in combination with a DC component. It was thought to be equal to the power radiated to specimen.

Actually mass of heating lamp filament is small and its heat capacity is pretty low. When the period of sine wave is minutes, the temperature of filament would be changing dramatically along with different transient voltage of the sine wave cycle. Equation (2) is no longer applicable.

The commonly used heating lamps are kinds of incandescent lamp. The radiating power \( P_E \) of filament is given by the Stefan-Boltzmann law:

\[
P_E = A \sigma T^4
\]

where \( A \) is the radiating surface area. \( A = 1.33 \times 10^{-4} m^2 \) is derived by measurement of the a broken lamp in experiment. \( \varepsilon \) is the emissivity. \( \varepsilon = 0.32 \) is calculated according to previous study, while the temperature is set to 2973 K. \( \sigma = 5.67 \times 10^{-8} \text{Wm}^{-2}\text{K}^{-4} \) is the Stefan-Boltzmann constant and \( T \) is the temperature. The resistance of filament depends on its temperature:

\[
R(T) = R_0 \left( \frac{T}{T_0} \right)^k
\]

where \( R_0 \) and \( T_0 \) is resistance and temperature of filament in steady state. For the lamp in experiment (Philips, model R115), the rated power is 245 W at rated voltage 220 V. \( R_0 \) could be calculated as 197.6 \( \Omega \). The cold resistance could be measured by multimeter. It is about 19 \( \Omega \) in room temperature of 20°C. Ordinary tungsten lamps typically operate around 2700°C. Exponent value \( k \) is a function of temperature. To simplify the model, a fixed value 1.01 is used in this paper. It is calculated by (4) based on the cold and rated resistance of filament. The temperature of filament in relation to time is defined by differential equation:
\[
G \frac{dT}{dt} = -Ae(\frac{T^4}{C_0} - T_R^4) + R(T) \frac{V^2}{[R(T) + r_0]^2}
\]  

(5)

where \(G\) is heat capacity of filament. \(G\) is calculated to be \(G = 4.4 \times 10^{-3} \text{JK}^{-1}\). \(T_R\) is background temperature. It is set to be 293 K. \(V\) is applied voltage and \(r_0\) is the lead resistance of lamp. The lead resistance is expected to be only tens of milliohms. Because \(r_0\) is far less than \(R_T\), equation (5) could be simplified to:

\[
G \frac{dT}{dt} = -Ae(\frac{T^4}{C_0} - T_R^4) + \frac{V^2}{R(T)}
\]  

(6)

The differential equation (6) could be numerically solved by MATLAB ode45 function. Sine voltage in (1) with amplitude \(E = 220\text{V}\) and angular frequency \(\omega = 0.04\pi\) was applied for driving the lamp. The radiating power is shown in Figure 1(a).

FFT analysis was applied for the radiating power and the spectral distribution is shown in Figure 1(b). In addition to DC and fundamental component 0.04 Hz, high frequency harmonics is shown at 0.08 Hz and above. The total harmonic distortion (THD) is 7.7%.

In recent years, a new detection method - amplitude modulated thermography (AMT) was proposed.\(^9\) In AMT, voltage including multi frequency components was applied for driving the lamp. It was expected to get radiating power components at double of these frequencies. For example, the voltage includes two frequency components 0.01 and 0.02 Hz, as shown in (7).

\[
V = 110 \sin 0.02\pi t + 110 \sin 0.04\pi t
\]  

(7)

The corresponding radiating power was also evaluated by numerical solution of (3)–(6), as shown in Figure 2(a). The spectral distribution was calculated by FFT, as shown in Figure 2(b).

With the synthetic voltage input, the radiating power at expected double frequency 0.02 and 0.04 Hz is even lower than the intermediate frequency 0.01 and 0.03 Hz. The result is far from expectation.

According to theory mode described in (3)–(5), the radiating power \(P_E\) is not directly decided by driving voltage \(V\). The temperature of filament \(T\) is an important intermediate variable and it is influenced by many other parameters such as ambient temperature \(T_R\), radiating surface \(A\) and emissivity \(\varepsilon\). The expected radiating power waveform can’t be derived by open-loop adjustment of driving voltage. The real time detection of radiating power and close-loop tracking is necessary for getting the desired waveform.

### Sliding mode controller design

The tracking system of radiating power is prone to be disturbed by environmental light change. Robustness is required for the tracking system. The SMC with fractional order switching function is constructed for radiating power tracking. The temperature of filament \(T\) is selected as state variable. In the experiment, the feedback captured by sensor Si1133-AA00 is light intensity \(I_E\). Please be mentioned that the sensitivity of the sensor is not focused on a single wavelength, but a range of wavelength, from visible to near infrared. The peak value of sensitivity is in the range of 770–850 nm. The feedback is response of the sensor to whole sensitive wavelength range of heating power. Assuming that the heating power is evenly distributed on the surface of the specimen, the light intensity \(I_E\) is defined by

\[
I_E = \frac{P_E}{S}
\]  

(8)

where \(S\) is the area of the irradiated region. In the real case, the radiating power is generally not evenly distributed. However, if the relative assembly location of the lamp and sensor is changeless, the light intensity
captured by sensor is proportional to the total radiating power. There is a fixed relation between the state variable $T$ and the feedback variable $I_e$ by (3) and (8). The feedback is transferred to state variable and then sent to corresponding SMC. The analysis of controller is based on the deduction of fractional order sign function in our previous work. It is demonstrated in lemma 3.1.

**Lemma 3.1.** For $D^\beta_t \sigma(t) = \frac{1}{\Gamma(1-\beta)} \frac{d}{dt} \int_0^t (t-r)^{\beta-1} \sigma(r) \, dr$ with $\sigma(t) \in R, 0 \leq \beta < 1$, one gets

$$D^\beta_t \text{sgn}(\sigma(t)) = \begin{cases} > 0, & \text{if } \sigma(t) > 0, t > 0, \\ < 0, & \text{if } \sigma(t) < 0, t < 0, \\ \end{cases}$$

(9)

**Remark 3.1.** Similar to the sign function, $D^\beta_t \text{sgn}(\sigma)$ is shown to be able to extract the sign of $\sigma$. The definition of $D^\beta_t \text{sgn}(\sigma)$, $0 \leq \beta < 1$ is the fractional order derivative of the sign function. The derivative represents how a function changes when its input changes. The sign of the derivative may not be the same as the sign of the function itself. Even so, the sign of the fractional order sign function $D^\beta_t \text{sgn}(\sigma)$ is proven to be the same as the sign function $\text{sgn}(\sigma)$. This is a basic property of the FO sign function and would be applied in the manuscript.

**Remark 3.2.** The fractional order sign function will be utilized to build a fractional order exponential rate switching function. According to Lemma 3.1, the fractional order switching function can guarantee the system working on the sliding surface.

**Sliding surface and control scheme design**

The sliding surface is selected as

$$s(t) = T/C_0 T_{r}$$

where $e = T - T_r$, $T_r$ is the reference signal, $\lambda$ is a positive gain. Take derivative of sliding surface and set $\delta = 0$. One gets

$$\lambda \left( \frac{dT}{dt} - \frac{dT_r}{dt} \right) = 0$$

(11)

Base on the mode of state variable in (6), one can get

$$\lambda \left[ \frac{1}{G} A \sigma(T^4 - T^4_r) + \frac{V^2}{R(T)} - \frac{dT_r}{dt} \right] = 0$$

(12)

Let the control law $c_a = V^2$, then one has

$$c_a = R(T) \left[ A \sigma(T^4 - T^4_r) + \frac{G}{\lambda} \frac{dT_r}{dt} \right]$$

(13)

where the equivalent control law $c_{EQ} = A e(a(T^4 - T^4_r) + G \frac{dT_r}{dt})$. Let $\eta = R(T)$, thus $c_a = \eta c_{EQ}$.

Let the exponential rate switching law be

$$c_{SW} = -a D^\beta_t \text{sgn}(\delta(t)) - b \delta(t)$$

(14)

where $a > 0$ and $b > 0$. Set the controller of system to be

$$V^2 = \eta (c_{EQ} + c_{SW})$$

$$= R(T) \left[ A \sigma(T^4 - T^4_r) + \frac{G}{\lambda} \frac{dT_r}{dt} - a D^\beta_t \text{sgn}(\delta(t)) - b \delta(t) \right]$$

(15)

**Remark 3.3.** $D^\beta_t \text{sgn}(\delta)$, $n \in [0, 1]$ is the FO sign function, while $\text{sgn}(\delta)$ is a special case of $D^\beta_t \text{sgn}(\delta)$. The classical IO exponential rate switching law is equivalent to the above FO exponential rate switching law when
\[ n = 0. \text{ Better flexibility is expected in adjusting FO, comparing with IO exponential rate switching law.} \]

**Remark 3.4.** Comparing with the standard IO switching law \( c_{\text{IO-SW}} = -a \text{sgn}(\delta(t)), a > 0 \), a proportional rate term \(-bb\delta(t)\) is included in \( c_{\text{SW}}\). This term could speed up the state to converge to the switching manifold when \( \delta(t) \) is large.

**Stability and reachability analysis**

**Theorem 3.1.** For the system defined in (6), the state trajectories under the controller (15) with the FO switching law (14) could reach in finite time.

**Proof.** Construct the Lyapunov function \( U(t) = \frac{1}{2} \delta^2(t) \), the derivative of \( U(t) \) could be derived as

\[
U(t) = \delta(t) \dot{\delta}(t)
\]

Based on (10) and (12), one gets

\[
U(t) = \delta(t) \dot{\delta}(t) = \lambda \delta(t) \left[ \dot{\delta}(t) - \frac{\dot{C}}{e^{\frac{t}{\lambda}}} - \frac{\dot{D}}{e^{\frac{t}{\lambda}}} \right]
\]

Bring the controller \( t^2 = \eta(c_{\text{EQ}} + c_{\text{SW}}) \) into it, one gets

\[
U(t) = \frac{1}{2} \delta(t) \left[ -a D_2 \text{sgn} (\delta(t)) - b \delta(t) \right] = -\frac{a}{2} \delta(t) \dot{D}_2 \text{sgn} (\delta(t)) = \frac{a}{2} \delta^2(t) \leq 0
\]

Therefore, the controlled system could reach the switching surface in finite time. The proof is completed.

Next, the computation formulas of reaching time will be discussed.

Before discussing the reaching time \( t_R \) with FO switching law, \( t_R \) with the IO exponential rate switching law \( c_{\text{SW}} = -a \text{sgn}(\delta) - bb \) will be calculated. For this switching law, one has \( \delta \dot{\delta} = -a|\delta| - bb \leq 0 \). There are two cases:

1) When the initial condition \( \delta(0) > 0 \), one has

\[
\dot{\delta}(t) = -b \delta - a = \Rightarrow \dot{\delta}(t) + b \delta = -a.
\]

Because of \( (\delta(t)e^{bt})' = e^{bt}(\dot{\delta}(t) + b \delta(t)) = -ae^{bt} \), one gets \( \delta(t)e^{bt} = \delta(0) = \frac{a}{b}(1 - e^{-bt}) \). Thus, one can conclude \( \delta(t) = (\delta(0) + \frac{a}{b}) e^{-bt} - \frac{a}{b}. \) While \( \delta(t_R) = 0 \), one gets

\[
t_R = \frac{1}{b} \ln \left( \frac{\delta(0)}{a/b} + 1 \right) = \frac{1}{b} \ln \left( \frac{b \delta(0)}{a} + 1 \right) > 0
\]

2) When \( \delta(0) < 0 \), one has \( \dot{\delta}(t) = -b \delta + a = \Rightarrow \dot{\delta}(t) + b \delta = a. \) Similarly, one has \( \dot{\delta}(t)e^{bt} = -\delta(0) = \frac{a}{b}(e^{bt} - 1). \) Then one can get \( \delta(t) = (\delta(0) - \frac{a}{b}) e^{-bt} + \frac{a}{b}. \) While \( t = t_R \), \( \delta(t_R) = 0. \) Thus,

\[
t_R = \frac{1}{b} \ln \left( 1 - \frac{\delta(0)}{a/b} \right) = \frac{1}{b} \ln \left( 1 - \frac{b \delta(0)}{a} \right) > 0
\]

According to the analysis of cases 1) and 2), it could be concluded that \( t_R \) with IO exponential rate switching law as

\[
t_R = \frac{1}{b} \ln \left( 1 + \frac{b \delta(0)}{a} \right) = 0
\]

Similarly, \( t_R \) with FO exponential rate switching law could be derived as follows:

3) When \( \delta(0) > 0 \), one gets \( \dot{\delta}(t) = -b \delta - a D_2 \text{sgn} (\delta). \)

The FO sign function is

\[
D_2 \text{sgn} (\delta) = \frac{1}{\Gamma(1 - n)} \int_0^t \text{sgn} (\delta(\theta)) d\theta
\]

Then, one gets

\[
\dot{\delta}(t) = -b \delta(t) - \frac{a}{\Gamma(1 - n)} \int_0^t \delta(\theta) d\theta
\]

Furthermore, one gets

\[
(\delta(t)e^{bt})' = e^{bt}(\dot{\delta}(t) + b \delta(t)) = -ae^{bt} \frac{1}{\Gamma(1 - n)} \int_0^t \delta(\theta) d\theta
\]

Therefore, one has

\[
\delta(t)e^{bt} = \delta(0) - \frac{a}{\Gamma(1 - n)} \int_0^t \theta^{-n} e^{\theta} d\theta
\]

While \( \delta(t_R) = 0 \), one gets

\[
\delta(t_R)e^{bt_R} = \delta(0) - \frac{a}{\Gamma(1 - n)} \int_0^{t_R} \theta^{-n} e^{\theta} d\theta = 0
\]

Thus, the solution of (27) is \( t_R \). Let \( \zeta(t) = \int_0^t \theta^{-n} e^{\theta} d\theta = \frac{\Gamma(1 - n)}{\theta^{1 - n}} > 0 \). It is deducible that \( \zeta(t), t \in [0, \infty) \) is a monotonically increasing function, \( \zeta(0) = 0 \) and \( \zeta(\infty) = \infty \). According to the intermediate value theorem, there exists \( t' > 0 \), such that
mediate value theorem again, there exists
while \( z(t) = 0 \) and
numerical approximation.

So, the arriving time

**Figure 3.** Close loop system for radiating power tracking.

\[
\xi(t^*) = \int_0^{t^*} \theta^{-n}e^{\theta}d\theta = \frac{\delta(0)\Gamma(1-n)}{a} \quad (28)
\]

Therefore, numerical approximation could be used to obtain \( t_R = t^* \).

4) When \( \delta(t) < 0 \), one gets \( \dot{\delta}(t) = -b\delta - aD_i^e \text{sgn}(\delta) \).
Then one gets

\[
\dot{\delta}(t) = -b\delta(t) - \frac{a}{\Gamma(1-n)} \int_0^t \frac{-d\theta}{(t - \theta)^{n}} \quad (29)
\]

Similar to 3), one can get

\[
(\delta(t)e^{\theta t})' = e^{\theta t}(\dot{\delta}(t) + b\delta(t)) = \frac{ad^i}{\Gamma(1-n)} \int_0^t \frac{d\theta}{(t - \theta)^{n}} \quad (30)
\]

Furthermore, one has

\[
\delta(t_R)e^{\theta t} = \delta(0) + \frac{a}{\Gamma(1-n)} \int_0^{t_R} \theta^{-n}e^{\theta}d\theta = 0 \quad (31)
\]

Let \( \xi(t) = \int_0^{t} \theta^{-n}e^{\theta}d\theta \), \( a \Gamma(1-n) > 0 \). Since \( \xi(t) \), \( t \in [0, \infty) \) is a monotonically increasing function, meanwhile \( \xi(0) = 0 \) and \( \xi(\infty) = \infty \). According to the intermediate value theorem again, there exists \( t^* > 0 \), such that

\[
\xi(t^*) = \int_0^{t^*} \theta^{-n}e^{\theta}d\theta = -\frac{\delta(0)\Gamma(1-n)}{a} \quad (32)
\]

So, the arriving time \( t_R = t^* \) can be also obtained by numerical approximation.

From the analysis in cases 3) and 4), it could be concluded that \( t_R \) with the FO exponential rate switching law can be got through

\[
\xi(t) = \int_0^{t} \theta^{-n}e^{\theta}d\theta = \frac{\frac{\delta(0)\Gamma(1-n)}{a}}{a} \quad (33)
\]

The stability and reachability analysis ensures the convergence of the radiating power tracking system and provides theory basis for parameter adjustment in SMC.

**Experiment verification**

A close-loop system was constructed for radiating power tracking, as shown in Figure 3.

The SMC is executed on PC and the controlled object is the infrared lamp. The lamp is powered by a programmable AC power supply. The output of power supply is amplitude modulated voltage waveform. The amplitude (0–300 V) and frequency (45–500 Hz) of carrier signal is programmable. In the experiment, the frequency was fixed at 100 Hz and the amplitude depended on the modulated signal which was decided by controller output. The light intensity of the lamp is monitored by a light sensor SoC (System on Chip) Si1133-AA00. Both visible and infrared photodiodes are integrated in the sensor. The output of infrared photodiode is selected as control feedback. In addition, the photo current to voltage conversion circuit, analog-digital converter, signal processor and a micro controller are also integrated in the single 2×2 mm sensor chip. The output of sensor chip is digital sampling value. It is configured to be unsigned 16 bit integer value. The full range output is 65,535.

The sampling rate of light sensor is configurable. It was set to be 1 kS/s in the experiment. The sensor output reflected not only the modulated signal, but also the fluctuation caused by 50 Hz lighting system and 100 Hz carrier signal. A moving average filter in duration of 20 ms inside chip was configured for eliminating these fluctuations. The captured light intensity information is sent to the switch board through Inter-Integrated Circuit (I2C) interface. When the switch board received the light intensity sampling value, it sent the value to PC through cluster communication (COM) port immediately. The experiment setup is shown in Figure 4.

The radiating power of a single infrared lamp is generally not quick enough for heating the CFRP specimen. A 5×5 lamp matrix was constructed. Each lamp could be switched on or off independently by the switch board. All the switched on lamps were driven together by the power supply output voltage. The size of sensor board is 17×17 mm. It was assembled on a piece of cardboard by two screws.

A software “Thermcon” operated on PC was designed for communication with switch board and power supply. The output of SMC is a series of unsigned 16 bit integer value. 65,535 represents the 300 V maximum output voltage of power supply. It is converted to amplitude value of modulation signal by
Thermcon and sent to power supply through standard commands for programmable instruments (SCPI). The switching command of lamp matrix and sampling feedback is transmitted and received through COM1 port, which is possessed by Thermcon. A graphic interface was designed for lamp matrix switching operation. As a demonstration in Figure 5, the five lamps at position of white dots are switched on, while others are switched off. The detail of control signal loop is shown in Figure 6.

The switching command of lamp matrix needs to be specified only once after power on. It is not part of the control loop, so it is not shown in Figure 6. The light intensity feedback and power supply amplitude control information is interchanged between Thermcon and Simulink through a pair of virtual COM ports. There is no easy way for real-time data exchange between a self-designed software and Simulink. A third-party software virtual serial port driver (VSPD) is utilized for creating two interchangeable virtual COM ports: COM2 and COM3. They are possessed by Thermcon and Simulink respectively.

Both PID controller and SMC were used for radiating power tracking. For speeding up the defect detection of CFRP, higher effective power is preferred. The reference signal was set to be a sine waveform with high DC offset, as shown in (34).

\[ I_E = 10000 \sin (0.01 \pi t) + 35000 \quad (34) \]

Furthermore, the reference signal was set to be dual sine components with DC offset as shown in (35).

\[ I_E = 5000 \sin (0.01 \pi t) + 5000 \sin (0.02 \pi t) + 35000 \quad (35) \]

The test results of PID controller with sine and dual sine reference signal are shown in Figure 7(a) and (b) respectively. The test results of SMC are in Figure 8(a) and (b). For both waveform of SMC, the parameters in (15) were set to be: \( a = 0.1, b = 0.36 \) and \( n = 0.3 \). Other parameters were same as numerical solution in problem statement section.

The primary drawback of the PID test result is overshoot at the initial phase. The equivalent control law of SMC involves theory mode of lamp. The initial error between reference signal and feedback could be compensated. Tracking lag is shown in both test results of PID controller and SMC. The tracking ripple of SMC is comparable to PID counterpart, but the tracking accuracy is worse. Reference signals in (34) with same offset but various amplitude were tracked by SMC. The result is shown in Figure 9.

The reference signals with different amplitude could be tracked, but all are with tracking errors. The higher tracking error is derived with higher amplitude. The error is caused by defective control law in (13), which is based on theory mode defined in (3)–(6). The emissivity \( \varepsilon \) and exponent value \( k \) is variable along with...
filament temperature $T$ according to previous studies, but fixed values were applied in this article. Further research is needed to improve the theory mode and controller.

To verify robustness of the controller, a flash light was used for simulating the transient ambient light change. The flash light was used to illuminate the light sensor in tracking status for about 1 s and removed immediately. Both the illumination time and space between sensor and flash light were controlled by hand, so the repeatability of overshoot is not so good. The experiment was done for both PID controller and SMC. The test result of PID controller is shown in Figure 10(a), while SMC is in Figure 10(b).

The two overshoots at time points of 27 and 73 seconds are caused by illumination of flash light. The sensor feedback value is sum of infrared lamp and flash light illumination at these moments. The tracking waveform of SMC returns to original trajectory in 2 s
following the overshoot, while PID controller returns in 7 s. The undershoot amplitude of SMC is much smaller than PID counterpart. Advantages of SMC in robustness is shown by this experiment. The radiating power tracking system with SMC is more prone to be immune from ambient light interference.

Conclusion

Numerical solution and FFT analysis of the IR lamp theory mode disclosed why sine radiating power could not be derived with sine driving voltage. An experimental platform was constructed for radiating power tracking of IR lamp. Design details of the platform were proposed. Fractional order SMC was constructed in Simulink for close-loop tracking. The filament temperature $T$ defined in fourth-order state equation was selected as state variable. The Lyapunov method was used for proving the reachability of the system. The analytic expressions of reaching time $t_R$ were given. The SMC showed better dynamic performance, comparing with PID controller, but tracking accuracy was worse. Improvement of the theory mode is necessary in the further study. The minimum programing interval of the current AC power supply is 1 s. It decides the control interval of whole system. Power supply with lower programing interval ($< 50$ ms) would be utilized in the future. The performance of controller is expected to be further improved. Defect detection effectiveness with the tracked radiating power will be evaluated.

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![Figure 10. Robustness test result of: (a) PID controller and (b) SMC.](image-url)
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**References**

1. Qu Z, Jiang P and Zhang W. Development and application of infrared thermography non-destructive testing techniques. *Sensors* 2020; 20(14): 3851.
2. Ibarra-Castanedo C, Galmiche F, Darabi A, et al. Thermographic nondestructive evaluation: overview of recent progress. *Proc SPIE* 2003; 5073: 450-459.
3. Yang R and He Y. Optically and non-optically excited thermography for composites: a review. *Infrared Phys Technol* 2016; 75: 26–50.
4. Maldague X and Marinetti S. Pulse phase infrared thermography. *J Appl Phys* 1996; 79(5): 2694–2698.
5. Chatterjee K, Tuli S, Pickering SG, et al. A comparison of the pulsed, lock-in and frequency modulated thermography nondestructive evaluation techniques. *NDT E Int* 2011; 44(7): 655–667.
6. Busse G, Wu D and Karpen W. Thermal wave imaging with phase sensitive modulated thermography. *J Appl Phys* 1992; 71(8): 3962–3965.
7. Rajic N and Antolis C. An investigation of noise performance in optical lock-in thermography. *Infrared Phys Technol* 2017; 87: 1–10.
8. Song H, Lim HJ, Lee S, et al. Automated detection and quantification of hidden voids in triplex bonding layers using active lock-in thermography. *NDT E Int* 2015; 74: 94–105.
9. Wu X and Peters K. Non-destructive inspection of adhesively bonded joints using amplitude modulated thermography. *Exp Mech* 2015; 55(8): 1485–1501.
10. Liu JY, Tang QJ and Wang Y. Research on the quantitative analysis of subsurface defects for nondestructive testing by ultrasound lock-in thermography. *Adv Mater Res* 2011; 301–303: 635–640.
11. Liu J, Yang W and Dai J. Research on thermal wave processing of lock-in thermography based on analyzing image sequences for ndt. *Infrared Phys Technol* 2010; 53(5): 348–357.
12. Chatterjee K and Tuli S. Prediction of blind frequency in lock-in thermography using electro-thermal model based numerical simulation. *J Appl Phys* 2013; 114(17): 174905.
13. Utkin VI. *Sliding modes in control and optimization*. Berlin, Heidelberg: Springer, 1992.
14. Hua J, An LX and Li YM. Bionic fuzzy sliding mode control and robustness analysis. *Appl Math Model* 2015; 39(15): 4482–4493.
15. Juang JG and Yu ST. Disturbance encountered landing system design based on sliding mode control with evolutionary computation and cerebellar model articulation controller. *Appl Math Model* 2015; 39(19): 5862–5881.
16. Aghababa MP. A fractional sliding mode for finite-time control scheme with application to stabilization of electrostatic and electromechanical transducers. *Appl Math Model* 2015; 39(20): 6103–6113.
17. Tang Y, Zhang X, Zhang D, et al. Fractional order sliding mode controller design for antilock braking systems. *Neurocomputing* 2016; 111: 122–130.
18. Zhang B, Pi Y and Luo Y. Fractional order sliding-mode control based on parameters auto-tuning for velocity control of permanent magnet synchronous motor. *ISA Trans* 2012; 51(5): 649–656.
19. Ebrahimkhani S. Robust fractional order sliding mode control of doubly-fed induction generator (DFIG)-based wind turbines. *ISA Trans* 2016; 63: 343–354.
20. Chakraborty S and Bandyopadhyay B. A generalized reaching law for discrete time sliding mode control. *Automatica* 2015; 52: 83–86.
21. Yin C, Huang X, Chen Y, et al. Fractional-order exponential switching technique to enhance sliding mode control. *Appl Math Model* 2017; 44: 705–726.
22. Gershenson M. Use of quartz halogen lamp in transient thermography imaging. *Proc SPIE* 2018; 10661: 106610O.
23. Lassner E and Schubert WD. *Tungsten: properties, chemistry, technology of the element, alloys, and chemical compounds*. PhD Thesis, Springer, Berlin, 1999.
24. Ben-Yaakov S, Peretz MM and Hesterman B. A spice compatible behavioral electrical model of a heated tungsten filament. In: *Twentieth annual IEEE applied power electronics conference and exposition, 2005. APEC 2005*, Austin, TX, 6–10 March 2005. New York, NY: IEEE.
25. Furfari FA. A different kind of chemistry: a history of tungsten halogen lamps. *IEEE Ind Appl Mag* 2001; 7(6): 10–17.
26. Yin C, Chen Y and Zhong SM. Fractional-order sliding mode based extremum seeking control of a class of nonlinear systems. *Automatica* 2014; 50(12): 3173–3181.