THE MEMBRANE AT THE END OF THE (DE SITTER) UNIVERSE\textsuperscript{1}.

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\textbf{ABSTRACT}

The original \textit{membrane at the end of the universe} corresponds to a probe $M_2$-brane of signature $(2,1)$ occupying the $S^2 \times S^1$ boundary of the $(10,1)$ spacetime $AdS_4 \times S^7$, and is described by an $OSp(4/8)$ SCFT. However, it was subsequently generalized to other worldvolume signatures $(s,t)$ and other spacetime signatures $(S,T)$. An interesting special case is provided by the $(3,0)$ brane at the end of the de Sitter universe $dS_4$ which has recently featured in the $dS/CFT$ correspondence. The resulting CFT contains the one recently proposed as the holographic dual of a four-dimensional de Sitter cosmology.

Supersymmetry restricts $S, T, s, t$ by requiring that the corresponding bosonic symmetry $O(s + 1, t + 1) \times O(S - s, T - t)$ be a subgroup of a superconformal group. The case of $dS_4 \times AdS_7$ is ‘doubly holographic’ and may be regarded as the near horizon geometry of $N_2$ $M_2$-branes or equivalently, under interchange of conformal and R symmetry, of $N_5$ $M_5$-branes, provided $N_2 = 2N_5^2$. The same correspondence holds in the pp-wave limit of conventional $M$-theory.

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1 Introduction

The original membrane at the end of the universe [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] corresponds to a probe $M2$-brane of signature $(2, 1)$ occupying the $S^2 \times S^1$ boundary of the $(10, 1)$ spacetime $AdS_4 \times S^7$ and is described by an $OSp(4/8)$ superconformal field theory. However, it was subsequently generalized [6, 9] to brane worldvolumes with $s$ space and $t$ time dimensions moving in a spacetime with $S \geq s$ space and $T \geq t$ time dimensions. The brane occupies the boundary of a universe of constant curvature so that the bosonic symmetry is $O(s + 1, t + 1) \times O(S - s, T - t)$. Supersymmetry restricts the values of $s, t, S, T$ to those for which this bosonic symmetry is a subgroup of a superconformal group, and the resulting superconformal theories have $(s + t) \leq 6$. For example, as discussed in [6], the possible signatures of M-theory are $(10, 1), (9, 2), (6, 5), (5, 6), (2, 9)$ and $(1, 10)$ and the possible $M2$-branes have worldvolume signatures $(3, 0), (2, 1), (1, 2)$ and $(0, 3)$. The corresponding superconformal groups are given in Table 1.

In view of the connections [11] between the original membrane at the end of the universe and the $AdS/CFT$ correspondence [12, 13, 14], and in view of the recent interest in a possible $dS/CFT$ correspondence [15, 16, 17], it is natural to re-examine the special case of a $(3, 0)$ brane at the end of the de Sitter universe\(^5\) $dS_4$. We show that the resulting CFT contains the one recently proposed as the holographic dual of a four-dimensional de Sitter cosmology [21, 22].

In the usual signature all extended objects appear to suffer from worldvolume ghosts because the kinetic term for the $X^0$ coordinate enters with the wrong sign. These are easily removed, however (at least at the classical level) by the presence of diffeomorphisms on the worldvolume which allow us to fix a gauge where only positive-norm states propagate e.g. the light cone gauge for strings and its membrane analogues. Alternatively we may identify the $d$ worldvolume coordinates $\xi^i$ with $d$ of the $D$ space-time coordinates $X^i$ ($i = 1, 2, \cdots d$) leaving $(D - d)$ coordinates $X^I$ ($I = 1..D - d$) with the right sign for their kinetic energy [8]. Of course, this only works if we have one worldvolume time coordinate $\tau$ that allows us to choose a light-cone gauge or else set $\tau = t$.

In the same spirit, we could now require absence of ghosts (or rather absence of classical instabilities since we are still at the classical level) for arbitrary signature by requiring that the “transverse” group $SO(S - s, T - t)$ which governs physical propagation after gauge-

\(^5\)In fact the epithet membrane at the end of the universe is even more appropriate in the de Sitter case since the restaurant in Douglas Adams’ book “Restaurant at the end of the universe” [18] is located at temporal infinity.
Table 1: $M_2$-branes with world-volume signature $(s,t)$ in spacetime signature $(S,T)$. 

| $(S,T)$ | $(s,t)$ | Bosonic Symmetry | Supergroup |
|--------|--------|------------------|------------|
| $(10,1)$ | $(2,1)$ | $O(3,2) \times O(8)$ | $OSp(4/8)$ |
| $(9,2)$ | $(3,0)$ | $O(4,1) \times O(6,2)$ | $OSp^*(4/8)$ |
| $(9,2)$ | $(1,2)$ | $O(2,3) \times O(8)$ | $OSp(4/8)$ |
| $(6,5)$ | $(2,1)$ | $O(3,2) \times O(4,4)$ | $OSp(4/4,4)$ |
| $(6,5)$ | $(0,3)$ | $O(1,4) \times O(6,2)$ | $OSp^*(4/8)$ |
| $(5,6)$ | $(3,0)$ | $O(4,1) \times O(2,6)$ | $OSp^*(4/8)$ |
| $(5,6)$ | $(1,2)$ | $O(2,3) \times O(4,4)$ | $OSp(4/4,4)$ |
| $(2,9)$ | $(2,1)$ | $O(3,2) \times O(8)$ | $OSp(4/8)$ |
| $(2,9)$ | $(0,3)$ | $O(1,4) \times O(2,6)$ | $OSp^*(4/8)$ |
| $(1,10)$ | $(1,2)$ | $O(2,3) \times O(8)$ | $OSp(4/8)$ |

fixing, be compact. This requires $T = t$. It may be argued, of course, that in a world with more than one time dimension, ghosts are the least of your problems. Moreover, in contrast to strings, unitarity on the worldvolume does not necessarily imply unitarity in spacetime. This is because the transverse group no longer coincides with the little group. (For example, the $(2,1)$ object in $(10,1)$ spacetime and the $(1,2)$ object in $(9,2)$ spacetime both have transverse group $SO(8)$, but the former has little group $SO(9)$ and the latter $SO(8,1)$.) Reference [6] remained agnostic about the physical significance of non-compact transverse groups, but simply noted that in the compact case the list of superconformal groups matches those in Nahm’s classification [23].

However, an attempt to render these theories respectable has been made in a series of interesting papers by Hull and collaborators [24, 15, 25, 26, 27, 28, 29, 30], in which it is shown that these theories with unconventional signatures are related to the usual signature by means of a timelike T-duality. Hull denotes the $(10,1)$, $(9,2)$ and $(6,5)$ signatures as $M$-theory, $M^*$-theory and $M'$-theory, respectively. Unlike $M$-theory, $M^*$ and $M'$ theories can admit de Sitter vacua invariant under a de Sitter supergroup which does not have unitary highest weight representations. This lack of unitarity representations is reflected in the fact that some of the fields have kinetic terms with the wrong sign. However, since these theories are related to the conventional ones by a timelike T-duality they are no worse than conventional theories compactified in the time direction and so perhaps might make sense after all. Thus Hull’s suggestion is both radical and conservative at the same time. It is radical in invoking spacetimes with unusual signature but conservative in saying that the
only such theories we need worry about are those dual to the conventional theories.

It seems that one is forced to this interpretation if one wants to combine dS/CFT space with supersymmetry [15]. Alternatively, one can be content to look at non-supersymmetric vacua [16, 17]. In section 3, we shall explore both possibilities, having first reviewed the AdS case in section 2.

Finally in section 4 we note that the case of $dS_4 \times AdS_7$ is ‘doubly holographic’ and may be regarded as the near horizon geometry of $N_2$ M2-branes or equivalently, under interchange of conformal and R symmetry, of $N_5$ M5-branes, provided $N_2 = 2N_5^2$. The same correspondence holds in the pp-wave limit of conventional $M$-theory.

## 2 A probe (2,1)-brane on the boundary of $AdS_4 \times S^7$

### 2.1 The AdS background

We begin by reviewing the original membrane at the end of the universe for which the spacetime is $AdS_4 \times S^7$ and for which the supermembrane occupies the $S^1 \times S^2$ boundary of the $AdS_4$. It will be useful to regard the geometry as the near-horizon limit of a stack of M2 branes.

For M-theory in the usual $(10,1)$ signature, the low-energy effective field theory is $D = 11$ supergravity with bosonic action

$$S_M = \int d^{11}x \sqrt{-g} \left( R - \frac{1}{48} F^2_4 \right) - \frac{1}{12} \int A_3 \wedge F_4 \wedge F_4, \quad (2.1)$$

where $F_4 = dA_3$. The M2-brane solution is given by [31]

$$ds^2 = H^{-2/3}(-dt^2 + dx_1^2 + dx_2^2) + H^{1/3}(dy_3^2 + \ldots + dy_9^2 + dy_{10}^2), \quad A_{012} = H^{-1}, \quad (2.2)$$

where $H(y_3, \ldots, y_{10})$ is a harmonic function in the transverse space. For a stack of $N_2$ M2 branes at $y = 0$, we take

$$H = 1 + \left( \frac{L_2}{y} \right)^6, \quad (2.3)$$

where $y^2 = \delta_{mn}y^m y^n$ with $m$ running over the spatial indices in the transverse space, and $L_2 = (2^5\pi^2N_2)^{1/6}l_p$ with $l_p$ being the eleven-dimensional Planck length. The world-volume has signature $(2,1)$ and the solution has bosonic symmetry $ISO(2,1) \times SO(8)$. It is nonsingular at $y = 0$ with near-horizon geometry $AdS_4 \times S^7$. 

3
2.2 Unitary but non-supersymmetric

In anticipation of finding in the next section a unitary but non-supersymmetric \((3,0)\) brane on the boundary of \(dS_4\), we begin by forgetting the \(D = 11\) supergravity origins of the \((2,1)\) brane and simply look at its bosonic sector in a background of \(AdS_4\) with a 4-form that follows from the \(A_{012}\) given above. We write the \(AdS_4\) metric as

\[
ds_{AdS_4}^2 = R_{AdS_4}^2 \left[ d\rho^2 + \sinh^2 \rho \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) - \cosh^2 \rho \, dt^2 \right] \tag{2.4}
\]

and the 4-form as

\[
F_4 = 3 R_{AdS_4}^3 \cosh \rho \sinh^2 \rho \, dt \wedge d\rho \wedge \epsilon_2, \tag{2.5}
\]

where \(\epsilon_n\) is the volume form of a unit \(n\)-sphere and the \(AdS_4\) radius is \(R_{AdS_4} = L_2/2\).

Now consider a probe \((2,1)\)-brane with a worldvolume action given by the bosonic sector of the \(M_2\)-brane:

\[
S_2 = T_2 \int d^3 \xi \left[ -\frac{1}{2} \sqrt{-\gamma} \gamma^{ij} \partial_i x^M \partial_j x^N g_{MN}(x) + \frac{1}{2} \sqrt{-\gamma} + q \frac{1}{3!} \epsilon^{ijk} \partial_i x^M \partial_j x^N \partial_k x^P A_{MNP}(x) \right], \tag{2.6}
\]

where \(T_2\) is the membrane tension, \(\xi^i (i = 0, 1, 2)\) are the worldvolume coordinates, \(\gamma_{ij}\) is the worldvolume metric and \(x^M(\xi)\) are the spacetime coordinates \((M = 0, 1, \ldots, 10)\). Following [35] we allow for a non-extremality parameter \(q\) where the BPS condition corresponds to \(q = 1\).

The embedding of the membrane in \(AdS_4\) is given by

\[
t = \xi^0, \quad \theta = \xi^1, \quad \phi = \xi^2, \tag{2.7}
\]

so that its worldvolume occupies the \(S^1 \times S^2\) section. Since we are temporarily ignoring the \(S^7\), we focus just on the bosonic radial mode \(\rho(\xi)\). We substitute (2.4) and (2.5) into (2.6) to find

\[
S_2 = T_2 R_{AdS_4}^3 \int_{S^1 \times S^2} d^3 \xi \left[ -\sqrt{-\det(\tilde{h}_{ij} + \partial_i \rho \partial_j \rho)} + q \sinh^3 \rho \right] \tag{2.8}
\]

where from (2.7) and (2.4)

\[
\tilde{h}_{ij} = \begin{pmatrix}
-\cosh^2 \rho & 0 & 0 \\
0 & \sinh^2 \rho & 0 \\
0 & 0 & \sinh^2 \rho \sin^2 \theta
\end{pmatrix}. \tag{2.9}
\]

We are interested in the \(\rho \to \infty\) limit, where the brane approaches the boundary of \(AdS_4\). Then the action (2.8) becomes

\[
S_2 = T_2 R_{AdS_4}^3 \int_{S^1 \times S^2} d^3 \xi \sqrt{-\tilde{h}} \left[ -\frac{1}{4} e^\rho \, \tilde{h}^{ij} \partial_i \rho \partial_j \rho + \frac{1}{4} \left( 1 - 3q \right) e^\rho - \frac{1}{8} e^{3\rho}(1-q) + \mathcal{O}(e^{-\rho}) \right] \tag{2.10}
\]
where $h_{ij}$ is the metric on $S^1 \times S^2$, $ds^2 = -dt^2 + d\theta^2 + \sin^2 \theta \, d\phi^2$. Making the change of variable

$$
\rho = \ln \frac{\varphi^2}{2T_2 R^6_{AdS_4}} + 4T_2^2 R^6_{AdS_4} \varphi^{-4}
$$

(2.11)

where $\varphi$ is the 3-dimensional scalar field with canonical dimension $1/2$, and ignoring $O(\varphi^{-2})$ terms, we have

$$
S_2 = -\int_{S^1 \times S^2} d^3 \xi \sqrt{|h|} \left[ \frac{1}{2} h^{ij} \partial_i \varphi \partial_j \varphi + \frac{R}{16} \varphi^2 + \frac{(1-q)}{64T_2^2 R^6_{AdS_4}} \varphi^6 \right],
$$

(2.12)

This is just the bosonic singleton action [3] with its scalar mass terms and $g\varphi^6/6!$ coupling. Since the scalar curvature of $S^1 \times S^2$ equals 2, we recognize the correct $R \varphi^2$ coefficient for Weyl invariance [3]. The coupling constant is given by $g = 45(1-q)/4T_2^2 R^6_{AdS_4}$. Bearing in mind that $T_2 = 1/(2\pi)^2 l_p^3$ and $T_2 R^3_{AdS_4} = (2N_2)^{1/2}/8\pi$, we have $g = 360\pi^2(1-q)/N_2$.

As discussed by Seiberg and Witten, for $q > 1$, the system is unstable against emission of branes, which is perfectly possible in a non-supersymmetric theory.

### 2.3 Unitary and supersymmetric

When $q = 1$, the fermionic completion of the worldvolume action (2.6) is kappa symmetric. This demands that the background metric $g_{MN}$ and background 3-form potential $A_{MNP}$ obey the classical field equations of $D = 11$ supergravity. We shall now consider this case with the full is $AdS_4 \times S^7$ background:

$$
ds_{11}^2 = R^2_{AdS_4} \left[ d\rho^2 + \sinh^2 \rho \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right) - \cosh^2 \rho \, dt^2 \right] + 4R^2_{AdS_4} d\Omega_7^2.
$$

(2.13)

We denote the seven angles parameterizing the $S^7$ as $\theta^m$ with $m = 1, 2, \ldots, 7$ and $d\Omega_7^2 = d\theta^m d\theta^n g_{mn}$. We can no longer limit ourselves to a single radial mode, but the generalization to include the $S^7$ modes is straightforward. To keep things simple, we shall continue to ignore the fermions, however. Note that the Wess-Zumino, or volume term remains unchanged. This implies that the relation between $\rho$ and $\varphi$ is the same as before. Note also that the radius for the $S^7$ in the full metric is twice that of $AdS_4$. With all the above, we have, for $\varphi \to \infty$,

$$
S_2 = -\int_{S^1 \times S^2} d^3 \xi \sqrt{|h|} \left[ \frac{1}{2} h^{ij} \left( \partial_i \varphi \partial_j \varphi + \varphi^2 \partial_i \theta^m \partial_j \theta^n g_{mn} \right) + \frac{1}{8} \varphi^2 \right],
$$

(2.14)

If we define

$$
\varphi^a = \varphi^{a \alpha}, \quad (a = 1, 2, \cdots 8),
$$

(2.15)
where \( \omega^a \) are the 1-forms parameterizing the unit \( S^7 \) and \( \varphi^a \varphi^a = \varphi^2 \), then we have

\[
S_2 = - \int_{S^1 \times S^2} d^3 \xi \sqrt{|h|} \left[ \frac{1}{2} h^{ij} \partial_i \varphi^\mu \partial_j \varphi^\nu \eta_{\mu\nu} + \frac{R}{16} \varphi^a \varphi^b \delta_{ab} \right],
\]

(2.16)

where \( \eta_{\mu\nu} = \delta_{\mu\nu} = (+, +, \cdots, +) \) and which has manifest \( SO(3, 2) \) conformal symmetry and \( SO(8) \) \( R \)-symmetry. It is just the bosonic sector of the \( OSp(4/8) \) SCFT [3]. (Incidentally, this rectifies a previous inability to derive the correct bosonic mass terms for the membrane at the end of the universe, necessary to identify it with the \( OSp(4/8) \) supersingleton action [2, 8] on \( S^1 \times S^2 \). The singleton actions in [36, 37] were defined on a Minkowski background and so required no mass terms.)

3 A probe \((3, 0)\)-brane on the boundary of \( dS_4 \times AdS_7 \)

3.1 The dS background

We will now seek the analogue of the \( M2 \)-brane that occurs in the \((9, 2)\) \( M^* \) theory, whose field theory limit is a supergravity theory with bosonic action [15]

\[
S_{M^*} = \int d^{11} x \sqrt{-g} \left( R + \frac{1}{48} F_4^2 \right) - \frac{1}{12} \int A_3 \wedge F_4 \wedge F_4.
\]

(3.1)

Note that the sign of the kinetic term of \( F_4 \) is opposite\(^6\) to that of the action (2.1). As discussed in [26], the sign of the kinetic term is intimately related with the world-volume signatures that can occur. For example, if the sign of the kinetic term of \( F_4 \) were reversed in (3.1) to give a Lagrangian \( R - F_4^2 / 48 + \ldots \) in \( 9 + 2 \) dimensions, there would be a membrane solution with 2+1 dimensional world-volume, while the action (3.1) with the opposite sign for the \( F_4 \) kinetic term has brane solutions with world-volume signatures \((3, 0)\) and \((1, 2)\), as we shall see below. The sign of the \( F_4 \) kinetic term in actions (2.1), (3.1) is determined by supersymmetry.

This theory has a number of Freund-Rubin-type solutions involving the de-Sitter-type spaces, including \((d+1)\)-dimensional de Sitter space \( dS_{d+1} \), \((d+1)\)-dimensional anti-de Sitter space \( AdS_{d+1} \), the \((d + 1)\)-dimensional hyperbolic space \( H_{d+1} \), and the two-time de Sitter space \( AAdS_{d+1} \) which is a generalized de Sitter space given by (a connected component of) the coset \( SO(d, 2) / SO(d - 1, 2) \), with signature \((d - 1, 2)\) and isometry \( SO(d, 2) \). The solutions are \( dS_4 \times AdS_7 \), \( AAdS_4 \times S^7 \), \( AdS_7 \times dS_4 \) and \( AAdS_7 \times H^4 \).

\(^6\)Interestingly enough, this means that \( M^* \) theory avoids the objections that are present in \( M \)-theory to implementing Hawking’s resolution [19] of the cosmological constant problem [20].
There are two solutions of $M^*$-theory analogous to the $M2$-brane: a $(3,0)$-brane and a $(1,2)$-brane. In this paper we consider the $(3,0)$-brane with Euclidean worldvolume. Its solution is given by

\[ ds^2 = H^{-2/3}(dx_1^2 + dx_2^2 + dx_3^2) + H^{1/3}(-dt^2 - dt'{}^2 + dy_4^2 + \ldots + dy_9^2), \]

\[ A_{123} = H^{-1}, \tag{3.2} \]

where $H$ is a harmonic function on the transverse space.

For the $(3,0)$-brane, the null-cone $y^2 = t^2 + t'^2$ divides the transverse space into two regions, and there are two distinct brane solutions, in which the transverse coordinate space is restricted to the region inside or outside the null cone. In the region $y^2 > t^2 + t'^2$, a natural choice for the time-dependent harmonic function is

\[ H = 1 + \frac{L_2}{(y^2 - t^2 - t'^2)^3}, \tag{3.3} \]

which gives a real solution (3.2) for $y^2 > t^2 + t'^2$. For $y^2 < t^2 + t'^2$, we take instead

\[ H = 1 + \frac{L_2}{(t'^2 - t'^2 - y^2)^3}. \tag{3.4} \]

In either case, the solution has bosonic symmetry $ISO(3) \times SO(6,2)$. The geometry of (3.2) near $y^2 = t^2 + t'^2$ differs in the two cases.

For $t^2 + t'^2 > y^2$, let $\tau^2 = t^2 + t'^2 - y^2$. Then near $\tau = 0$, $H^{1/3} \to L_2^{1/3}/\tau^2$. Setting $y = \tau \sinh \alpha$, $t = \tau \cosh \alpha \cos \theta$, and $t' = \tau \cosh \alpha \sin \theta$, the metric near $\tau = 0$ takes the form

\[ ds^2 = \frac{W^2}{R_{dS_4}^2}(dx_1^2 + dx_2^2 + dx_3^2) - \frac{R_{dS_4}^2 dW^2}{W^2} + 4R_{dS_4}^2 \left( d\alpha^2 - \cosh^2 \alpha d\theta^2 + \sinh^2 \alpha d\Omega_3^2 \right), \tag{3.5} \]

where $W = L_2^{-1/6}\tau^2/2$ and again $R_{dS_4} = L_2^{1/6}/2 = R_{AdS_7}/2$. This is the metric of $dS_4 \times AdS_7$. The region $t^2 + t'^2 > y^2$ of the solution (3.2) then interpolates between the flat space $\mathbb{R}^{9,2}$ and $dS_4 \times AdS_7$. Similar analysis shows that the region $y^2 > t^2 + t'^2$ of the solution (3.2) interpolates between the flat space $\mathbb{R}^{9,2}$ and $H^4 \times AAdS_7$.

### 3.2 Unitary but non-supersymmetric

We now wish to repeat the calculation given in the section (2.2) but with a $(3,0)$-brane occupying the $S^3$ boundary of $dS_4$ and with a 4-form that follows from $A_{123}$ above. In terms of global coordinates, we write the $dS_4$ metric and 4-form as

\[ ds^2_{dS_4} = R_{dS_4}^2 (-d\alpha^2 + \cosh^2 \alpha d\Omega_3^2), \]

\[ F_4 = 3R_{dS_4}^3 \cosh^3 \alpha d\alpha \wedge \epsilon_3 \tag{3.6} \]
where \( R_{dS_4} = L_2/2 \).

Now consider a probe \((3,0)\) brane whose action given by

\[
S_2 = T_2 \int d^3 \xi \left[ -\frac{1}{2} \sqrt{\gamma} \gamma^{ij} \partial_i x^M \partial_j x^N g_{MN}(x) + \frac{1}{2} \sqrt{\gamma} + \frac{q}{3!} \epsilon^{ijk} \partial_i x^M \partial_j x^N \partial_k x^P A_{MNP}(x) \right],
\]

(3.7)

where \( i \) now runs over 1, 2, 3 and we have once again allowed the presence of a non-extremality parameter \( q \). The overall sign is chosen so that the spatial derivatives are the same for \((3,0)\) as for \((2,1)\). The embedding of the membrane in \( dS_4 \) is given by

\[
\xi^1 = \theta^1, \quad \xi^2 = \theta^2, \quad \xi^3 = \theta^3.
\]

(3.8)

so that its worldvolume occupies the \( S^3 \) section of \( dS_4 \) which is located at \( \alpha(\xi) \). We denote the three angles parameterizing the unit 3-sphere of the \( dS_4 \) background as \( \theta^1, \theta^2, \) and denote its metric by \( h_{ij} \).

On the \( S^3 \) boundary of \( dS_4 \) where \( \alpha \) is large the area term is

\[
T_2 A = \frac{T_2 R^3_{dS_4}}{2^3} \int_{S^3} d^3 \xi \sqrt{h} \left[ e^{3\alpha} + 3e^{\alpha} - 2e^{\alpha} h^{ij} \partial_i \alpha \partial_j \alpha + O(e^{-\alpha}) \right]
\]

(3.9)

Further the \( A_3 \) can be solved from \( F_4 \) given above as

\[
A_3 = R^3_{dS_4} \left( 3 \sinh \alpha + \sinh^3 \alpha \right) \epsilon_3.
\]

(3.10)

so the Wess-Zumino term is

\[
q T_2 \int_{S^3} A_3 = q T_2 R^3_{dS_4} \int_{S^3} d^3 \xi \sqrt{h} (3 \sinh \alpha + \sinh^3 \alpha)
\]

\[
= \frac{q T_2 R^3_{dS_4}}{2^3} \int_{S^3} d^3 \xi \sqrt{h} \left[ e^{3\alpha} + 9e^{\alpha} + O(e^{-\alpha}) \right].
\]

(3.11)

Note that since \( \alpha \) is a time coordinate, its kinetic term enters with the opposite sign to that of the spatial coordinate \( \rho \) in section (2.2). Let us re-express \( \alpha \) in terms of a scalar field \( \varphi \) to eliminate the \( e^\alpha \) in the above Wess-Zumino term. This can be achieved via

\[
\alpha = \ln \frac{\varphi^2}{2T_2 R^3_{dS_4}} - 12T_2^2 R^6_{dS_4} \varphi^{-4}.
\]

(3.12)

Note also the different coefficient in the \( \varphi^{-4} \) term relative to that in (2.11). This reflects the fact that \( S^3 \) has a scalar curvature different from \( S^1 \times S^2 \). Ignoring \( O(\varphi^{-2}) \) terms, the brane action is now

\[
S_2 = \int d^3 \xi \sqrt{h} \left[ \frac{1}{2} (\partial \varphi)^2 + \frac{R}{16} \varphi^2 - (1 - q) \frac{\varphi^6}{64T_2^2 R^6_{dS_4}} \right],
\]

(3.13)

Interestingly enough, the above action is also the one (generalized to a curved boundary) proposed in [21, 22] for the holographic dual of a de Sitter cosmology in the context of a \( dS/CFT \) correspondence [15, 16, 17]. However, the relative sign of the coupling is opposite to that in the \( AdS/CFT \) case of section (2.2) so that the signal for instability is now \( q < 1 \).
3.3 Non-unitary and supersymmetric

In closing this section, we consider a probe super $(3,0)$-brane with $q = 1$ on the boundary of $dS_4$ but now in the full $dS_4 \times AdS_7$ background. So we consider now other modes of the brane in addition to the radial one. If we write the metric for $AdS_7$ with unit radius as

$$ds^2_{AdS_7} = d\beta^2 - \cosh^2 \beta d\phi^2 + \sinh^2 \beta d\Omega^2_5 = g_{ij}d\beta^i d\beta^j$$ (3.14)

then

$$ds^2_{11} = R_{dS_4}^2 [(-d\alpha^2 + \cosh^2 \alpha d\Omega^2_3) + 4ds^2_{AdS_7}]$$ (3.15)

Then we have

$$S_2 = \int_{S^3} d^3 \xi \sqrt{h} \left[ \frac{1}{2} (\partial \varphi)^2 - \frac{1}{2} g_{ij} \varphi^2 \partial \beta^i \partial \beta^j + \frac{3}{8} \varphi^2 + O(\varphi^{-2}) \right],$$ (3.16)

where we have used the relation between $\alpha$ and $\varphi$ given earlier.

The above is not yet the wanted form. For this, we need to define

$$t = \varphi \cosh \beta \sin \phi, t' = \varphi \cosh \beta \cos \phi,$$

$$y^i = \varphi \sinh \beta \omega^i, (i = 1, 2, \cdots 6),$$ (3.17)

such that

$$\varphi^2 = t^2 + t'^2 - y^2,$$ (3.18)

where $y^2 = y^i y^i$. In the above, $\omega^i$ are the one-forms parameterizing the unit 5-sphere satisfying $\omega^i \omega^i = 1$.

One can check explicitly that

$$dt^2 + dt'^2 - dy^i dy^i = d\varphi^2 - \varphi^2 (d\beta^2 - \cosh^2 \beta d\phi^2 + \sinh^2 \beta d\Omega^2_5).$$ (3.19)

If we now denote $\varphi^\mu = (t, t', y^i)$, then, ignoring $O(\varphi^{-2})$ terms, we have the action

$$S = - \int_{S^3} d^3 \xi \sqrt{h} \left[ \frac{1}{2} h^{ab} \partial_a \varphi^\mu \partial_b \varphi^\nu \eta_{\mu\nu} + \frac{R}{16} \varphi^\mu \varphi^\nu \eta_{\mu\nu} \right],$$ (3.20)

where we have also set $\varphi^2 \rightarrow -\varphi^2$ and therefore the signature is now $\eta_{\mu\nu} = (-, -, +, +, +, +)$. Once again, the above action has the same form as that given in (2.16). It has manifest $SO(4,1)$ conformal symmetry and $SO(6,2)$ $R$-symmetry.
4 Dualities between $M_2$ and $M_5$-branes in $M$, $M^*$ and $M'$ theories

4.1 Double holography in $M^*$ and $M'$-theory

An interesting feature of the various $M_2$ and $M_5$ branes appearing in $M^*$ and $M'$ theories is that their near horizon geometries frequently coincide [28]. In the $(9, 2)$ spacetime of section (3.1), for example, both the $(3, 0)$ $M_2$-brane and the $(5, 1)$ $M_5$ brane tend to $dS_4 \times AdS^7$ and share the same superconformal group $OSp^*(4/8)$ whose bosonic symmetry is $O(4, 1) \times O(6, 2)$. Note that, in contrast to the usual $(10, 1)$ signature, both factors are conformal groups, and so the system is “doubly holographic” [28]. Further if we accept the duality between the bulk M-theory and the boundary conformal theory, the direct consequence of this is that the boundary 3-dimensional conformal theory describes the same physics as the boundary 6-dimensional conformal theory. The spacetime symmetry and $R$-symmetry are exchanged in the two pictures. As we shall now demonstrate, however, the requirement that the spacetime may be regarded as the near horizon geometry of $N_2$ $M_2$-branes or equivalently, under interchange of conformal and $R$ symmetry, of $N_5$ $M_5$-branes, imposes the constraint

$$N_2 = 2N_5^2 \quad (4.1)$$

(Although we focus on the concrete example of $(3, 0, -)$-branes and $(5, 1, -)$-branes in $M^*$ theory, the conclusion $N_2 = 2N_5^2$ is true also for the other examples of double holography.)

The near-horizon geometry for a stack of $N_2$ $(3, 0, -)$-branes is given by (3.6), whereas the near-horizon geometry for a stack of $N_5$ $(5, 1, -)$-branes is given by

$$ds_5^2 = R_{AdS_7}^2 (d\beta^2 - \cosh^2 \beta \, d\phi^2 + \sinh^2 \beta \, d\Omega_5^2) + \frac{R_{AdS_7}^2}{4} (-d\alpha^2 + \cosh \alpha^2 \, d\Omega_3^2),$$

$$F_7 = 6R_{AdS_7}^6 \cosh \beta \, \sinh^5 \beta \, d\beta \wedge d\phi \wedge \epsilon_5,$$

$$R_{AdS_7} = 2L_5,$$ \hspace{1cm} (4.2)

where $L_5 = (\pi N_5)^{1/3} l_p$.

To make these two geometries identical, we need to identify the two metrics and relate the 4-form $F_4$ and 7-form $F_7$ via $F_7 = *F_4$ with the * denoting the Hodge dual. This can be achieved provided

$$R_{dS_4} = R_{AdS_7}/2,$$ \hspace{1cm} (4.3)

which implies

$$L_2 = 2L_5,$$ \hspace{1cm} (4.4)
Since $L_2 = (2^5 \pi^2 N_2)^{1/6} l_p$, this gives the relation $N_2 = 2N_5^2$, as promised.

The above conclusion can be understood as the consequence of the relation between the two radii, one (denoted as $R_4$) is associated with 4-dimensional manifold and the other (denoted as $R_7$) with the 7-dimensional one. If the 11-dimensional metric $ds^2 = ds_4^2 + ds_7^2$ describes the near-horizon geometry of either M2 or M5 branes, we always have $R_4 = R_7/2$. If in a given theory, the M2 branes and the M5 branes have the same near-horizon bulk geometry, it must require $N_2 = 2N_5^2$ given the relation between $R_4$ and $N_2$ and that between $R_7$ and $N_5$.

At present, due to our lack of understanding of the theories with more than one time, it is hard for us to make further checks on this conjectured $CFT_3/CFT_6$ correspondence. So the question arises whether there exists a possibility of such a correspondence, in certain limit, from the conventional one-time $M$-theory. The answer is actually positive thanks to the recent interest in PP-wave backgrounds.

### 4.2 PP waves in M-theory

Let us recall how the pp-wave limit arises in the conventional $M$-theory. The near-horizon geometries for $M2$ and $M5$ branes are $AdS_4 \times S^7$ and $AdS_7 \times S^4$, respectively. $AdS/CFT$ correspondence tells us that $M$-theory on the $AdS_4 \times S^7$ background is dual to a $(2,1)$-dimensional conformal theory while $M$-theory on $AdS_7 \times S^4$ is dual to $(5,1)$-dimensional conformal theory. At first sight, these look very different. In global coordinates, the near-horizon geometry for $M2$ branes is

$$
    ds_2^2 = R_{AdS_4}^2 \left( d\rho^2 + \sinh^2 \rho d\Omega_2^2 - \cosh^2 \rho d\tau^2 \right) + 4R_{AdS_4}^2 d\Omega_7^2,
$$

$$
    F_4 = -3R_{AdS_4}^3 \cosh \rho \sinh^2 \rho d\rho \wedge d\tau \wedge \epsilon_2,
$$

$$
    R_{AdS_4} = L_2/2,
$$

(4.5)

while the near-horizon geometry for $M5$ branes is

$$
    ds_5^2 = R_{AdS_7}^2 \left( d\sigma^2 + \sinh^2 \sigma d\Omega_5^2 - \cosh^2 \sigma d\tau^2 \right) + \frac{R_{AdS_7}^2}{4} d\Omega_4^2,
$$

$$
    F_7 = 6R_{AdS_7}^3 \cosh \sigma \sinh^5 \sigma d\sigma \wedge d\tau \wedge \epsilon_5,
$$

$$
    R_{AdS_7} = 2L_5.
$$

(4.6)

In the above $L_2$ and $L_5$ are the radii of $S^7$ and $S^4$ and are related to the numbers of $M2$ and $M5$ branes, respectively, as before.

We are now seeking a limit such that the above two geometries have the same isometries and the unit 2-sphere in $AdS_4$ of $AdS_4 \times S^7$ can be identified with a unit 2-sphere in the
of $AdS_7 \times S^4$ and the unit 5-sphere in the $AdS_7$ with a unit 5-sphere in the $S^7$. This is nothing but the pp-wave limit. Following [42, 43], the limit for $AdS_4 \times S^7$ is
\[
\bar{s} = L_2^2 \left[ 2dx^+dx^- - \frac{1}{2} (\rho^2 + \sigma^2) (dx^+)^2 + \frac{1}{4} \left( d\rho^2 + \rho^2 d\Omega_2^2 \right) + d\sigma^2 + \sigma^2 d\Omega_5^2 \right],
\]
\[
\bar{\Phi}_4 = \frac{3\sqrt{2}}{2^3} L_2^3 \rho^2 dx^+ \wedge d\rho \wedge \epsilon_2 ,
\]
and the limit for $AdS_7 \times S^4$ is
\[
\bar{s}^5 = 4L_5^3 \left[ 2dx^+dx^- - \frac{1}{2} (\rho^2 + \sigma^2) (dx^+)^2 + \frac{1}{4} \left( d\rho^2 + \rho^2 d\Omega_2^2 \right) + d\sigma^2 + \sigma^2 d\Omega_5^2 \right],
\]
\[
\bar{\Phi}_7 = -32^6 \sqrt{L_5^6} \sigma^5 dx^+ \wedge d\sigma \wedge \epsilon_5 .
\]

In the above, the $d\Omega_2^2$ is the original one in $AdS_4$ while the $d\Omega_5^2$ is the one in $AdS_7$. With this in mind, the above two metrics can be identified provided $L_2^2 = 2L_5^2$. From $\bar{\Phi}_4 = \star \bar{\Phi}_7$ we get the same relation $L_2^2 = 2L_5^2$, which implies $N_2 = 2N_5^2$. Therefore the conclusion we draw here is that there is a correspondence between the six-dimensional $(N_+, N_-) = (2, 0)$ superconformal theory and the three-dimensional $N = 8$ superconformal theory, in the respective pp-wave limits of the conventional one-time M-theory. The advantage here is that we can make further checks beyond what we can do in the $M^*$ case. By the respective $AdS/CFT$ correspondence, this must imply that the entropy calculated from the respective bulk theory must also agree with each other in the pp-wave limit. For this purpose, we consider both the near-extremal configuration for black $M_2$ branes and that for black $M_5$ branes, in terms of their respective Poincaré coordinates [44]. For $M_2$-branes,
\[
ds_2^2 = \frac{4\rho^2}{L_2^2} \left[ -e^{2f} dt^2 + dx^i dx^i \right] + \frac{L_2^3}{4} e^{-2f} \frac{dr^2}{r^2} + L_2^2 d\Omega_7^2 ,
\]
\[e^{2f} = 1 - \frac{r_0^6}{(2L_2 r)^3} , \tag{4.9}
\]
with $i = 1, 2$, and for $M_5$-branes,
\[
ds_5^2 = \frac{r^2}{4L_5^2} \left[ -e^{2f} dt^2 + dx^i dx^i \right] + 4L_5^2 e^{-2f} \frac{dr^2}{r^2} + L_5^2 d\Omega_4^2 ,
\]
\[e^{2f} = 1 - \frac{(4L_5^2 r_0)^3}{r_0} , \tag{4.10}
\]
with $i = 1, 2, \ldots 5$.

We now calculate the black hole entropy for the above configurations, respectively, using the Bekenstein-Hawking entropy formula $S = A/4G_{11}$ with $A$ the event-horizon area and $G_{11}$ the eleven-dimensional Newton’s constant\(^7\). For $M_2$ branes, it is not difficult to find

\(^7\)The entropy per unit brane volume for both of the two cases was calculated in [45].
that the event horizon is located at \( r_+ = r_0^2/(2L_2) \) and therefore

\[
A = \int \frac{4r_+^2}{L_2^3} L_2^3 dx^1 \wedge dx^2 \wedge \epsilon_7,
\]

\[
= r_0^4 L_2^3 \int dx^1 \wedge dx^2 \wedge \epsilon_7. \tag{4.11}
\]

So we have the entropy

\[
S_2 = \frac{r_0^4 L_2^3}{4G_{11}} \int dx^1 \wedge dx^2 \wedge \epsilon_7. \tag{4.12}
\]

For M5 branes, we have from the above \( r_+ = 2\sqrt{L_5 r_0} \). By the same token, we have the entropy

\[
S_5 = \sqrt{\frac{r_0^8 L_5^3}{4G_{11}}} \int dx^1 \wedge ... \wedge dx^5 \wedge \epsilon_4. \tag{4.13}
\]

In general, it is easy to see that the entropy \( S_2 \) is quite different from the entropy \( S_5 \).

\( AdS_{p+2} \) can usually be realized as a hyperboloid in \( \mathbb{R}^{2,(p+1)} \). The metric in terms of the global coordinates covers the entire hyperboloid while the one in terms of Poincaré coordinates covers only one half of the hyperboloid. On the one hand, the metric in terms of the Poincaré coordinates appears naturally as the near-horizon limit of certain brane configuration and the corresponding near-extremal thermodynamical entropy can be calculated while the one in terms of the global coordinates does not have these properties. On the other hand, the pp-wave limit is more naturally taken in the \( AdS \) metric in terms of the global coordinates. In order to examine whether there is a possibility that the two entropies can be set to equal, we need to relate these two-coordinate descriptions for a given \( AdS \)-space in the region for which both descriptions are valid. For a given \( AdS_{p+2} \), we have

\[
r = R_{AdS} (\cosh \alpha \sin \tau + \omega_{p+1} \sinh \alpha),
\]

\[
r x^0 = R_{AdS}^2 \cosh \alpha \cos \tau,
\]

\[
r x^1 = R_{AdS}^2 \omega_1 \sinh \alpha,
\]

\[
:\quad:\quad:\quad:\quad:
\]

\[
r x^p = R_{AdS}^2 \omega_p \sinh \alpha, \tag{4.14}
\]

where \((x^\mu, r)\) with \(\mu = 0, 1, ..., p\) are the Poincaré coordinates with \((\tau, \alpha, \omega_i)\) are the global coordinates and \(\sum_i \omega_i^2 = 1\).

For the near-extremal M2 branes, we write \(d\Omega_5^2 = d\sigma^2 + \cos^2 \sigma d\psi^2 + \sin^2 \sigma d\Omega_5^2\) and take \(\alpha = \rho\) in the above formulas (4.14). Taking the pp-wave limit \((\Omega \to 0), \rho \to \Omega \rho, \sigma \to \Omega \sigma\) and \(x^-(\equiv (\psi - \tau/2)/\sqrt{2}) \to \Omega^2 x^-\) at \(r = r_+\), we then have

\[
dx^1 \wedge dx^2 \wedge \epsilon_7 = \frac{\Omega^9 L_2^8}{2^3 r_0^4} \rho^2 \sigma^5 d\rho \wedge d\sigma \wedge \epsilon_2 \wedge \epsilon_5, \tag{4.15}
\]
Then the entropy $S_2$ given above becomes

$$S_2 = \Omega^9 \frac{L^3 V_9}{2^{34} G_{11}} \quad (4.16)$$

which is now independent of the extremal parameter $r_0$ and where

$$V_9 = \int \rho^2 \sigma^5 d\rho \wedge d\sigma \wedge \epsilon_2 \wedge \epsilon_5. \quad (4.17)$$

If we define the entropy in the pp-wave limit as $\bar{S}_2 = \lim_{\Omega \to 0} S_2/\Omega^9$, then we have

$$\bar{S}_2 = \frac{L^9 V_9}{2^{34} G_{11}}. \quad (4.18)$$

For the near-extremal M5 branes, we write $d\Omega^2 = d\rho^2 + \cos^2 \rho d\psi^2 + \sin^2 \rho d\Omega_5^2$ and take now $\alpha = \sigma$ in (4.14). Taking the pp-wave limit, $\sigma \to \Omega \sigma$, $\rho \to \Omega \rho$ and $x^-(\equiv (\psi/2 - \tau)/\sqrt{2}) \to \Omega^2 x^-$ at $r = r_+$, we then have

$$dx^1 \wedge dx^2 \ldots \wedge dx^5 \wedge \epsilon_4 = \Omega^9 \frac{2^6 L^{15/2}}{3^{3/2} r_0^2} \rho^2 \sigma^5 d\rho \wedge d\sigma \wedge \epsilon_2 \wedge \epsilon_5, \quad (4.19)$$

Then the entropy $S_5$ becomes

$$S_5 = \Omega^9 \frac{2^6 L^9 V_9}{4 G_{11}} \quad (4.20)$$

which is now also independent of the extremal parameter $r_0$ and where $V_9$ is also given by (4.17). By the same token, we have

$$\bar{S}_5 = \frac{2^6 L^9 V_9}{4 G_{11}}. \quad (4.21)$$

Given our previous discussion about the identification of the two metrics under the respective pp-wave limit, we can identify the two volume $V_9$ in $\bar{S}_2$ and $\bar{S}_5$. So the two entropies can be identified provided $L_2^9/2^3 = 2^6 L_5^9$ which again gives $N_2 = 2N_5^2$.

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**5 Appendix**

**5.1 Large Brane for arbitrary $d$: AdS case**

It is instructive to rewrite the action (2.6) as [1]

$$S_2 = -T_2 \int_{\partial W} d^3 \xi \sqrt{|g|} + q T_2 \int_{W} F_4, \quad (5.1)$$
where we use the same symbols \( g \) and \( F_4 \) for the metric and 4-form pullbacks onto the brane. \( W \) is a 4-manifold whose boundary is the brane worldvolume \( \partial W \). The first term in the action is actually proportional to the area \( A \) of the worldvolume \( \partial M \) of the brane. Also with \( F_4 \) given by the volume form on \( AdS_4 \) \([32, 33, 34]\) in (2.5), the second term can be recognized as proportional to the volume \( V \) of \( M \) \([1, 35]\), so that
\[
S_2 = -T_2(A - \frac{3q}{R_{ds_4}}V). \tag{5.2}
\]
Following \([35]\), the results of section (2.2) may now be generalized to an arbitrary \((d-1)\)-brane occupying the conformal boundary \( M_d \) of an arbitrary Einstein \((d+1)\)-dimensional manifold \( W_{d+1} \) of negative curvature.

We shall adapt the notation and the signature used in \([35]\) for our convenience. The boundary \( M_d \) has a natural conformal structure but not a natural metric. Let \( h_{ij} \) be an arbitrary metric on the boundary in its conformal class. Here the \( \xi^i, i = 0, \ldots, d-1 \) are an arbitrary set of local coordinates on the boundary. There is then a unique way \([38]\) to extend the \( \xi^i \) to coordinates on \( W_{d+1} \) near the boundary, adding an additional coordinate \( \rho \) that tends to infinity on the boundary, such that the metric in a neighborhood of the boundary is
\[
ds^2 = r_0^2 \left( d\rho^2 + \frac{1}{4} e^{2\rho} h_{ij} d\xi^i d\xi^j - P_{ij} d\xi^i d\xi^j + O(e^{-2\rho}) \right) \tag{5.3}
\]
where
\[
P_{ij} = \frac{2(d-1)R_{ij} - h_{ij} R}{2(d-1)(d-2)}, \tag{5.4}
\]
and \( R_{ij} \) is the Ricci tensor of \( M_d \), which implies
\[
h^{ij} P_{ij} = \frac{R}{2(d-1)}, \tag{5.5}
\]
Generalizing (5.1), a probe \((d - 1)\)-brane occupying the conformal boundary \( M(\rho) \) of the \( W_{d+1} \) with the above metric at large \( \rho \), the action for a radial mode can be written as
\[
S = -T_{d-1}(A_d - qdV_{d+1}/r_0) \tag{5.6}
\]
where \( T_{d-1} \) is the brane tension, \( A_d \) is the area of the conformal boundary \( M(\rho) \) with respect to the brane induced metric and \( V_{d+1} \) is the volume of \( W_{d+1} \) enclosed by the conformal boundary. The area \( A_d \) and the volume \( V_{d+1} \) are all calculated explicitly in \([35]\). The action for a radial mode is also given there:
\[
S = -T_{d-1}(A_d - \frac{qd}{r_0}V_{d+1}),
\]
\[
T_{d-1} \frac{r_0^2}{q^2} \int d^d \xi \sqrt{|h|} \left( (1 - q) \varphi - 2 \left( \frac{d}{d-2} \right) \varphi + \varphi R \right) - 2 \left( \frac{d}{d-2} \right) \varphi^2 + \frac{(d-2)}{(d-1)} R \varphi^2 \right) + O\left( \varphi^2 \right) \quad \text{for } d > 2, \\
- T_{d-1} \frac{r_0^2}{q^2} \int d^d \xi \sqrt{|h|} \left( (1 - q) \varphi - 2 \left( \frac{d}{d-2} \right) \varphi + \varphi R \right) - 2 \left( \frac{d}{d-2} \right) \varphi^2 + \frac{(d-2)}{(d-1)} R \varphi^2 \right) + O\left( \varphi^2 \right) \quad \text{for } d = 2.
\]

In the above, we have made the change of variable

\[
\rho = \begin{cases} 
\frac{2}{d-2} \ln \varphi + \frac{1}{(d-1)(d-2)} \varphi \varphi^\frac{d}{d-2} R & \text{for } d > 2, \\
\varphi + e^{-2\varphi} \varphi R & \text{for } d = 2,
\end{cases}
\]

where \( \varphi \) is the d-dimensional scalar field with canonical dimension \((d-2)/2\).

We are interested in \( \rho \to \infty \), i.e., \( \varphi \to \infty \). From the above, we find

\[
S = -T_{d-1} (A_d - \frac{qd}{r_0} V_{d+1}) = \\
= \begin{cases} 
\frac{T_{d-1} r_0^2}{q^2} \int d^d \xi \sqrt{|h|} \left( (1 - q) \varphi - 2 \left( \frac{d}{d-2} \right) \varphi + \varphi R \right) - 2 \left( \frac{d}{d-2} \right) \varphi^2 + \frac{(d-2)}{(d-1)} R \varphi^2 \right) + O\left( \varphi^2 \right) \quad \text{for } d > 2, \\
- \frac{T_{d-1} r_0^2}{4} \int d^d \xi \sqrt{|h|} \left( (1 - q) \varphi - 2 \left( \frac{d}{d-2} \right) \varphi + \varphi R \right) - 2 \left( \frac{d}{d-2} \right) \varphi^2 + \frac{(d-2)}{(d-1)} R \varphi^2 \right) + O\left( \varphi^2 \right) \quad \text{for } d = 2,
\end{cases}
\]

where we recognize the curvature term as that required for Weyl invariance of the action [3]. It is also easy to check that for \( d = 3 \), we reproduce the singleton action (2.12) derived earlier noting that \( r_0 = R_{AdS_4} \).

### 5.2 Large Brane for arbitrary \( d \): dS case

Once again we can write the action (3.7) as

\[
S_2 = -T_2 \int d^3 \xi \sqrt{g} + q T_2 \int F_4,
\]

so that once again it takes the form of “area minus volume”

\[
S = -T_2 (A - \frac{3q}{R_{dS_4}} V).
\]

Our probe \((3,0)\)-brane is actually wrapped on the 3-sphere at a given \( \alpha(\Omega) \). In the action (5.11) the kinetic term can be essentially viewed as an area integration with the induced metric while the Wess-Zumino term is proportional to the volume enclosed by the brane. Here we check this explicitly and derive the effective action for the large brane for arbitrary dimension of the de Sitter factor of the spacetime. In general, for \( dS_{d+1} \) with metric

\[
ds^2 = r_0^2 \left[ -d\alpha^2 + \cosh^2 \alpha \, d\Omega_d^2 \right],
\]

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where we have the area
\[
A_d = r_0^d \int d^d \Omega \frac{1}{\sqrt{\det h_{ab}}} \sqrt{\det(h_{ab} \cosh^2 \alpha - \partial_a \alpha \partial_b \alpha)},
\]
\[
= \frac{r_0^d}{2^d} \int d^d \Omega \left[ e^{d \alpha} + d e^{(d-2) \alpha} - 2 e^{(d-2) \alpha} h_{ab} \partial_a \alpha \partial_b \alpha + \mathcal{O}(e^{(d-4) \alpha}) \right], \quad (5.13)
\]
and the volume
\[
V_{d+1} = r_0^{d+1} \int d^d \Omega \int_0^\alpha d \alpha' \cosh^d \alpha',
\]
\[
= \frac{r_0^{d+1}}{2^d} \int d^d \Omega \int_0^\alpha d \alpha' \left[ e^{d \alpha'} + d e^{(d-2) \alpha'} + \mathcal{O}(e^{(d-4) \alpha'}) \right],
\]
\[
= \begin{cases} 
\frac{r_0^{d+1}}{2^d} \int d^d \Omega \left( \frac{1}{d} e^{d \alpha} + \frac{d}{d-2} e^{(d-2) \alpha} + \mathcal{O}(e^{(d-4) \alpha}) \right) & \text{for } d > 2, \\
\frac{r_0^d}{4} \int d^2 \Omega \left( \frac{1}{2} e^{2 \alpha} + 2 \alpha + \mathcal{O}(e^{-2 \alpha}) \right) & \text{for } d = 2.
\end{cases}
\quad (5.14)
\]
In the above the \(r_0\) is the radius of \(dS_d\) and \(h_{ab}\) is the metric of the unit \(d\)-sphere.

If we express the radial mode \(\alpha\) in terms of \(d\)-dimensional scalar field \(\varphi\) with canonical dimension \((d-2)/2\) via
\[
\alpha = \begin{cases} 
\frac{2}{d-2} \ln \varphi - \frac{1}{(d-1)(d-2)} \varphi^{-\frac{4}{d-2}} R & \text{for } d > 2, \\
\varphi - e^{-2 \varphi} \varphi R & \text{for } d = 2,
\end{cases}
\quad (5.15)
\]
where \(R\) is the curvature of unit \(d\)-sphere and \(R = d(d-1)\), we then have
\[
A_d = \begin{cases} 
\frac{r_0^d}{2^d} \int d^d x \sqrt{h} \left( \varphi^{\frac{2d}{d-2}} - \frac{8}{(d-2)^2} [ (\partial \varphi)^2 + \frac{d-2}{4(d-1)} R \varphi^2] + \mathcal{O}(\varphi^{\frac{2(d-4)}{d-2}}) \right) & \text{for } d > 2, \\
\frac{r_0^d}{4} \int d^2 x \sqrt{h} (e^{2 \varphi} - 2 [ (\partial \varphi)^2 + R \varphi] + R + \mathcal{O}(e^{-2 \varphi})) & \text{for } d = 2,
\end{cases}
\quad (5.16)
\]
and
\[
V_{d+1} = \begin{cases} 
\frac{r_0^{d+1}}{2^d} \int d^d x \sqrt{h} \left( \frac{1}{d} \varphi^{\frac{2d}{d-2}} + \mathcal{O}(\varphi^{\frac{2(d-4)}{d-2}}) \right) & \text{for } d > 2, \\
\frac{r_0^d}{4} \int d^2 x \sqrt{h} \left( \frac{1}{2} e^{2 \varphi} + \mathcal{O}(e^{-2 \varphi}) \right) & \text{for } d = 2.
\end{cases}
\quad (5.17)
\]

Finally, the action for the large radial mode \(\varphi\) of a probe \((d,0)\)-brane on the conformal boundary \(S^d\) of \(dS_{d+1}\) dimensional manifold is given as (ignoring the \(\mathcal{O}(\varphi^{\frac{2(d-4)}{d-2}})\))
\[
S = -T_{d-1}(A_d - \frac{qd}{r_0} V_{d+1}),
\]
\[
= \begin{cases} 
\frac{T_{d-1} r_0^{d}}{2^d} \int d^d x \sqrt{h} \left( -(1 - q) \varphi^{\frac{2d}{d-2}} + \frac{8}{(d-2)^2} [ (\partial \varphi)^2 + \frac{(d-2)}{4(d-1)} R \varphi^2] \right) & \text{for } d > 2, \\
\frac{T_{d-1} r_0^d}{4} \int d^2 x \sqrt{h} \left( -(1 - q) e^{2 \varphi} + 2 [ (\partial \varphi)^2 + \varphi R - R] \right) & \text{for } d = 2.
\end{cases}
\quad (5.18)
\]

In analogy with (5.9), we expect this formula to be valid for arbitrary manifold \(W_d\), not just \(dS\), but note the change of sign of the kinetic and mass terms relative to the \(AdS\) case. When \(d = 3\), we recover (3.13) as expected.
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