How much cosmological information can be measured?

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It has become common to call this the “era of precision cosmology,” and hence one rarely hears about the finiteness of the amount of information that is available for constraining cosmological parameters. Under the assumption that the perturbations are purely Gaussian, the amount of extractable information (in terms of total signal-to-noise ratio for power spectrum measurements) is the same (up to a small numerical factor) as an accounting of the number of observable modes. For studies of the microwave sky, we are probably within a factor of a few of the amount of accessible information. To dramatically reduce the uncertainties on parameters will require three-dimensional probes, such as ambitious future redshifted 21-cm surveys. However, even there the available information is still finite, with the total effective signal-to-noise ratio on parameters probably not exceeding $10^7$. The amount of observable information will increase with time (but very slowly) into the extremely distant future.

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I. INTRODUCTION

The standard cosmological model has been confirmed with ever-increasing precision using cosmic microwave background (CMB) data, such as from the Wilkinson Microwave Anisotropy Probe [1] and now particularly from the Planck satellite [2, 3]. This model (hereafter referred to as ΛCDM), containing a cosmological constant ($\Lambda$) and cold dark matter, is built on a framework of simplifying assumptions within which only a relatively small number of free parameters are required. One can consider that the information contained in the data is compressed down to the combined constraints on these cosmological parameters. To the extent that the observable Universe comes from a Gaussian random process, all of the information is contained in the two-point functions (which are limited by cosmic variance), and not in the particular realization of the CMB sky (which can in principle be determined to arbitrary precision).

As the constraining power of the data improves, the parameters are determined more precisely, and hence for the parameter set some overall “signal-to-noise” increases with time. Currently we have tight constraints on the six-parameter ΛCDM model, with the parameters conventionally chosen to be the baryon density, $\Omega_b h^2$, with $h$ being the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$; the cold dark matter density, $\Omega_c h^2$; the angular size of the sound horizon, $\theta_s$, which is a function of the $\Omega$s; the amplitude of the power spectrum of initial density perturbations, $A_s$; the slope of this power spectrum, $n_s$; and the optical depth due to reionization, $\tau_{\text{ion}}$. Improvements in data quality are leading to tighter and tighter constraints on these parameters, and hence one is led to consider whether there is some ultimate precision that can be obtained, or alternatively one can ask how much cosmological information can be measured.

In fact, Planck has already mapped a large fraction of the primordial anisotropies in temperature [4] and hence we are starting to reach the point where we are squeezing the CMB sky for the remains of the information (although this is not yet true for polarization, see Ref. [5]). To obtain a dramatic increase in cosmological information therefore requires us to consider the observable modes of the three-dimensional power spectrum, $P_m(k)$, as measured using galaxy clustering, cosmic shear, or 21-cm correlations, for example.

II. SIZE OF THE OBSERVABLE UNIVERSE AND THE FAR FUTURE

We first investigate the maximal space that can be observed and the smallest linear-scale structure one can probe at different epochs. Then we calculate the total number of (linear) modes as a function of cosmic time, and the ultimate parameter precision one can achieve.

Figure 1 shows a spacetime diagram from $t = 0$ (neglecting an inflationary phase) to the infinite future. Due to the acceleration caused by dark energy, our comoving Hubble length increases to its maximum at $z \simeq 10$, but afterwards decreases as the dark energy starts to dominate. However, the maximal volume of observable space is not limited to the Hubble patch, since we are observing along the past light cone, and so we will have access to larger and larger regions as time passes. It was suggested in Ref. [7] that we live at a time near the maximum in the available number of modes, but in fact our observable patch has not decreased from $z \simeq 10$ until now, and will not diminish further in the future. The comoving

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size of the observable Universe at any time is an integral from the last-scattering surface \( \tau_s \) to the observed time \( \tau_{\text{obs}} \):

\[
L(\tau_{\text{obs}}) = \int_{\tau_s}^{\tau_{\text{obs}}} c \, d\tau,
\]

which monotonically increases with time [shown as the red line in Fig. 2(a)]. So even though the Hubble scale shrinks because of the accelerating expansion, in the comoving frame we will be able to see more and more volume; however, the volume increases only slowly in the far future, corresponding to a flattening of the red line in Fig. 2(a) for \( \log(a) > \text{few} \).

The CMB and other signals will certainly get harder to measure as they redshift away into the Λ-dominated (de Sitter) phase, but it is important to realize that they will not disappear entirely. Nevertheless, it is worth considering what happens in the far future, since eventually a fundamental limit exists below which the CMB temperature cannot be sensibly defined. In an empty de Sitter universe one would see a thermal bath with temperature (in natural units) \( T_{\text{DS}} = H/2\pi = \sqrt{\lambda_\text{p}^3/2\pi} \) [8], and thus after the CMB redshifts to \( T_{\text{DS}} \), it becomes completely lost in the thermal noise of the de Sitter background. This is known as the “Gibbons-Hawking bound” (GH) bound [9] [10] and gives a limit in the very far future of a purely Λ model; it is equivalent to considering the time when the typical wavelength of a CMB photon becomes equal to the Hubble scale, so that one cannot sensibly define a temperature afterwards.

However, even well before this epoch, the CMB spectral peak will redshift below the plasma frequency of the interstellar medium, and thus would be screened from any observer within our Galaxy. The plasma frequency in our Galaxy is around 1 kHz, which corresponds to a wavelength of \( \lambda_p \approx 3 \times 10^5 \text{ m} \) [11].

These screening effects can limit the effectively observable size of the Universe. As an example, light with an emitted wavelength \( \lambda_e \) originating from point C on the CMB LSS in Fig. 1 can be observed at any time until the light’s wavelength is stretched to the size of the Hubble length (point B), and then it becomes invisible after \( \tau_{\text{obs}} \). If light is emitted at point \( \Lambda \), with wavelength comparable to \( cH^{-1}(a_*) \), then it is screened by the GH effect soon after, and can never be detected. Therefore, as the Universe expands, the observer can see a larger and larger volume of space along the past light cone, until they reach the turning point where \( c/H = \lambda_e(a_{\text{turn}}/a_*) \). For the plasma bound, this simply means replacing the Hubble length \( c/H \) with the plasma wavelength \( \lambda_p \). After
FIG. 2: (a) Observable length scale (comoving frame) as a function of logarithmic scale factor. (b) Effect of different cutoffs in $k$ at early times due to various possibilities for unknown small-scale physics, picking three example values.

FIG. 3: (a) Number of modes confined on the past light cone that can be measured (here taking the early cutoff scale to be $k_{\text{max}} = 7.1 \, h \, \text{Mpc}^{-1}$). (b) Sensitivity of $N_{\text{modes}}$ as a function of different cutoff scales.

this time $\tau_{\text{turn}}$, the observable comoving length evolves as

$$L(\tau_{\text{obs}}) = \int_{\tau_{e}}^{\tau_{\text{obs}}} c d\tau,$$

where $\tau_e$ is the emission time corresponding to scale factor $a_e = (\lambda_e a_{\text{obs}})/(c/H)$. In Fig. 2(a), we show the observable length scale as a function of $\log(a)$, and we see that the observable length increases until it is saturated by the effects of vacuum energy. In the far future, the screening effect from the interstellar plasma or the GH bound can decrease the observable size. For example, light with peak CMB wavelength at the LSS epoch corresponding to $\lambda \simeq 1 \, \mu$m will have a turning point at $a \simeq 10^{67}$, i.e., 1100 Gyr [blue line in Fig. 2(a)], while for a shorter wavelength (e.g., the brown dashed line), the screening effect will happen later.

Of course, real observers billions of years into the future may need to consider other events that are important for the evolution of their particular location, such as the red giant phase of the Sun, or the merging of the Milky Way and Andromeda galaxies. This latter interaction would significantly affect the ISM plasma and hence the observability of the CMB. However, we neglect all such observer-specific effects here, and simply consider the case in which the local conditions for the observer are continuously stable over the full cosmic evolution, in the same spirit as previous discussions of the future of cosmology (e.g., Ref. [7]). In the same vein, we are only considering the standard $\Lambda$CDM picture, with the dark energy being exactly a cosmological constant, and with no additional unconventional physics, such as future phase transitions.

III. TOTAL NUMBER OF MODES

In the standard cosmological picture, nonlinear structure develops first on small spatial scales (large $k$ values), where it erases any memory of the initial conditions [12]. One is free to pick any reasonable definition for the scale of this nonlinearity; e.g., we can use $k_{\text{max}}$ corresponding to the radius $R$ in a spherical top-hat window for which the root mean square of the filtered density field is unity; i.e., $\sigma^2(R) = (1/2\pi^2) \int P_m(k) (3j_1(kR)/kR)^2 \, dk = 1$, where $P_m(k)$ is the matter power spectrum and $j_1(kR)$ is the first-order Bessel function of the first kind. The total number of modes can be calculated as $N_{\text{modes}} = (2\pi)^3 n_k k^3 P_m(k) \, dk$, where $n_k$ is the density of modes. In Fig. 3(b), we show the sensitivity of $N_{\text{modes}}$ as a function of different cutoff scales, with $k_{\text{cut}}$ and $z_{\text{cut}}$ indicating the cutoff scale and redshift, respectively. The solid line represents the past light cone, while the dashed line represents the inside the past light cone.
where $P_m(k)$ is matter power spectrum and $j_1$ is the first-order spherical Bessel function. In Fig. 2(b), we plot $k_{\text{max}} \equiv \frac{2\pi}{R}$ as a function of log$(a)$. The universe becomes more clustered as it evolves, so that the amplitude of $P_m(k)$ grows and the size of the region $R$ with rms unity fluctuations becomes bigger, until the growth of structure is frozen out by the vacuum energy. So $k_{\text{max}}$ is a decreasing function of log$(a)$.

At early times $k_{\text{max}}$ was relatively large, but complicated baryonic physics (and potentially other exotic effects) at early times on small scales can also limit our ability to probe perturbation up to very high-$k$ modes, giving an effective cutoff in $k$ at higher redshifts. These effects include the baryon gas pressure [1], the relative velocity between baryons and dark matter [13], the x-ray and photo-ionization heating of intergalactic baryons from first-light objects [14], any warm dark matter thermal velocity [15], and the potential existence of primordial black holes [16]. To investigate the effect of this uncertainty, we take three typical values of $k_{\text{cut}}$ at corresponding redshifts, as shown in Fig. 2(b), these being related to the specific nature of the dark matter and baryonic feedback processes.

We can now calculate the total number of linear modes as

$$N_{\text{modes}} = \frac{1}{(2\pi)^3} \int_0^\infty \frac{V(\tau_{\text{obs}})}{\tau_{\text{min}}} \frac{dV}{d^3k} \int_{k_{\text{min}}}^{k_{\text{max}}} \frac{d^3k}{\tau(\tau_{\text{obs}})} dV.$$  

Here, since $k_{\text{min}}L_{\text{obs}} \sim 1$, we replace the lower limit of the $k$ integral with zero, and write $k_{\text{max}}$ as a function of the light emission time. We can consider two distinct cases for observations carried out to recover the power spectrum of fluctuations.

First, we can assume that observations are confined to the surface of our past light cone. This would be valid for traditional galaxy surveys [17], gravitational lensing, and 21-cm surveys [18], for example. Thus for each observational time $\tau_{\text{obs}}$ when we integrate out the maximum observable length scale [Eqs. 1 and 2], we are integrating over all emission times out to the LSS ($\tau_e$). Figure 3(a) shows the evolution of the observable number of modes for such observations, explicitly assuming a cutoff scale $k_{\text{cut}} \simeq 7.1 h$Mpc$^{-1}$ at $z \simeq 20$. We find that, although it will certainly become harder to extract cosmological information in the future (because of the accelerated expansion), the number of measurable modes still increases monotonically with time. This will be the case until screening effects (either ISM plasma or GH) eventually prohibit observations in the very distant future.

As a second case, we can also consider observations that probe the inside of our past light cone. Potential observations of this kind include the kinetic Sunyaev-Zeldovich effect [19], spectral distortions of the CMB due to scattered light [20], or anisotropies in a massive neutrino background [21]. In practice little additional information is probably accessible [22], but in principle, all of the interior of our last-scattering surface could be observed in this way. This represents the fundamental limit of our ability to observe fluctuations due to causality and the opacity prior to last scattering. Since such observations always measure the structure back to the LSS, we can substitute $\tau_e = \tau_s$ into Eq. 3 and take $k_{\text{max}} = k_{\text{cut}}$ out of the integral, thus obtaining $N_{\text{modes}} = k_{\text{cut}}^3 P_m(\tau_{\text{obs}})/6\pi^2/2$.

Figure 3(b) shows the number of modes for the two cases (light cone surface and inside the light cone), while varying the value of $k_{\text{cut}}$. One can see that the number of modes is sensitive to the cutoff $k$ scale, and varies between about $10^{13}$ and $10^{15}$. Since (up to a small numerical factor) the number of bits of information is essentially the same as the number of accessible modes, the total amount of information within the past light cone is somewhat bigger than the information confined on the past light cone, but the two have a similar order of magnitude.

IV. INFORMATION ON COSMOLOGICAL PARAMETERS

We can also cast the information question in terms of parameter uncertainties, i.e., we can ask how the available information maps into constraints in determining the cosmological parameters. To obtain an estimate of the total signal-to-noise ratio (SNR), we can simplify to a situation where all parameters are determined except one, which we take to be the overall amplitude of primordial fluctuations, $A_s$. One can then calculate the Fisher matrix (see e.g., Ref. [23]) of the $A_s$ parameter at any observational time. Since $\Delta A_s = F_A^{-1/2}$, the total information in $A_s$ becomes [24, 25]

$$I_{A_s} = \frac{1}{4\pi^2} \int_0^{\tau_{\text{obs}}} \frac{V(\tau_{\text{obs}})}{\tau(\tau_{\text{obs}})} \frac{dV}{d^3k} \int_{k_{\text{min}}}^{k_{\text{max}}} \frac{d^3k}{\tau(\tau_{\text{obs}})} \left( \frac{\partial \ln P_m(k)}{\partial \ln A_s} \right)^2 dV.$$  

Because $\partial \ln P_m(k)/\partial \ln A_s = 1$, the above equation immediately reduces to $I = N_{\text{modes}}/2$; i.e., the amount of information (in terms of SNR$^2$) equals half the number of Gaussian modes. Therefore, the SNR$^2$ of $A_s$ measured at different cosmic epochs has a similar shape to the evolution shown in Fig. 3(a). The maximal SNR will depend on the specific value of the high-redshift cutoff and details of the observational methods. The precision in $A_s$ as a function of $k_{\text{cut}}$ gives $\delta A_s/A_s \simeq 2.5 \times 10^{-7} k_{\text{cut}}^{-1}$, with $k_{\text{cut}}$ in units of $h$Mpc$^{-1}$. The further one can probe into the small-scale regime at higher redshifts, the tighter the parameter constraints that one can achieve.

For multiple parameters one can consider that the SNR is effectively divided among them (with some being better constrained than others of course). It is possible that for a particular power spectrum measurement, two parameters might be degenerate with each other; in this case probing more fluctuation modes will improve a combined parameter constraint, rather than the individual ones.
Note that the above calculation represents the ultimate precision for parameter uncertainties, neglecting all of the practical limitations of experimental noise and foreground contamination. For example, real 21-cm measurements will suffer seriously from foreground emission (both Galactic and terrestrial), which is generally 10^5 (or more) times higher than the underlying signal [26]. Nevertheless, we can approach the ultimate SNR limit by being increasingly innovative in future experimental designs, as well as by probing further into the nonlinear regime.

V. OTHER KINDS OF INFORMATION

Astute readers may at this point be wondering why we have assumed that the only source of cosmological information is in the measurement of the amplitude of modes in the fluctuating density fields. Why are traditional observations of background cosmological quantities (like H_0, luminosity distance as a function of redshift, helium abundance, etc.) not included in the discussion? The answer is that although such observations are extremely useful cosmological probes at the present day, as the data improve dramatically, they will either provide relatively modest increments to the amount of information, or will become equivalent to counting modes, like the power spectrum measurements we have focused on. Let us illustrate this by considering a few specific examples.

A. The Hubble constant and expansion rate

The observation of type-Ia supernovae is a classical example of another kind of cosmological data [27]. However, in terms of the ideal amounts of information we have been discussing, these kinds of observation offer only minimal additional constraints, as can be seen directly from the conformal diagram in Fig. 1. At τ_{obs}, the supernova observations only provide information on H(z) along a relatively short radius (since it is hard to observe very high-z supernovae). Additionally, the cosmological information is limited by dispersion in the properties of standard candles, and so in practice measurements of H(z) will not decrease like \sqrt{N} as we add supernovae.

But more importantly, as the data become of high enough signal-to-noise ratio, one will have to appreciate that the local value of H_0 can be slightly different from the global one, because of density variations. Right now we have estimates of H_0 at the SNR \geq 50 level, and it is well known that “cosmic variance” on this number is a non-negligible fraction of the current uncertainty [28]. Hence, even an infinite number of local distance estimates will give a result similar to only measuring \sim 10^4 modes, in terms of constraints on cosmological parameters.

In order to expand the amount of information, one has to go to higher redshifts. But then there are additional issues to consider, such as the cosmic peculiar velocity field [29] and the effects of gravitational lensing [30]. Precision measurements of luminosity distance will thus determine different expansion histories in different directions. This will degrade our ability to improve the SNR of such measurements, although with sufficiently ambitious surveys the correlated fluctuations can be considered as a cosmological signal [31]. Hence endeavors to constrain the cosmological background parameters then become effectively measurements of cosmological power spectra, where the information is determined by mode counting.

B. Other direct constraints on background cosmology

Another example of an astrophysical measurement of a “background” parameter is the determination of helium or deuterium abundance at z = 0. One might naively imagine that it can be measured with infinite accuracy, and hence provide a high-quality constraint on the baryon-to-photon ratio \eta_b, or cosmic baryon density \Omega_b h^2, independent of any modes. However, once one recalls that the Universe at the time of big bang nucleosynthesis (BBN) contains perturbations in density (and hence \Omega_b h^2, for example) and that one measures the abundance in different regions of today’s Universe (actually along the light cone) where conditions will vary from place to place (e.g., Ref. [35]), then it becomes clear that the precision of a single number is limited, and to make further progress we are back to considering modes. In addition, astration in astrophysical systems (i.e., destruction in stars of light elements such as deuterium), and experimental systematics may limit the precision of determining primordial abundance [35, 36].

In fact the same general argument applies to all such measurements, with the exception of a direct determination of the cosmological constant, \Lambda. If we could devise a laboratory experiment to determine the value of \Lambda, then we could in principle make a measurement of this cosmological parameter to arbitrary precision. The reason such a measurement would be different is, of course, because the cosmological constant is a pure number, which does not permit perturbations (and in that sense is more like a physical constant, like the fine structure constant \alpha, than a cosmological parameter). However, we know of no way to measure \Lambda without making measurements over cosmological scales.

C. CMB distortions

In the early Universe, energy stored in small-scale density perturbations is dissipated through Silk damping, producing \mu- and y-type distortions of the CMB spectrum [37, 38]. Observations with future experiments, such as PIXIE [39], will enable us to measure such distor-
tions and thereby place constraints on the amplitude and shape of the primordial power spectrum at wave numbers \( k \lesssim 10^4 \text{ Mpc}^{-1} \). The two types of distortion produced from the small-scale power spectrum of primordial curvature perturbation \( [P_\zeta(k) \propto P(k)/k] \) are \(^{38}\)

\[
\begin{align*}
\mu &\simeq 2.2 \int_{k_{\text{min}}}^{\infty} P_\zeta(k) \left[ e^{-k/5400} - e^{-(k/31.6)^2} \right]^2 d \ln k; \\
y &\simeq 0.4 \int_{k_{\text{min}}}^{\infty} P_\zeta(k) e^{-(k/31.6)^2} d \ln k. \tag{5}
\end{align*}
\]

One can see that measurements of the distortions provide integral constraints on the primordial power spectrum in the regime \( 1 \lesssim k \lesssim 10^4 \text{ Mpc}^{-1} \), and hence a limit on cosmological parameters that affect this power. Although constraints on nonstandard models are interesting, it is much harder to detect a signal from the standard cosmology, where \( \mu \) and \( y \) distortions are expected to be only a few \( \times 10^{-10} \). Moreover, the integral constraint on small-scale power seems unlikely to be competitive with direct measurements of power spectra, and additionally there are other sources of spectral distortion that will have to be dealt with first. Nevertheless, this is a promising way of probing a range of scales that is inaccessible to current CMB anisotropy and large-scale structure observations. Similar “integral” constraints on the small-scale power spectrum may also be available from BBN \(^{40}\).

D. Neutrinos and gravitational waves

Despite the fact that electromagnetic experiments overwhelmingly dominate present-day astronomy, one may also consider information coming from other messengers, such as gravitational waves or neutrinos. However, this does not change our conclusions fundamentally. For neutrinos, unless they are genuinely massless and travel at the speed of light, this allows us to see some extra volume of space (see Fig.\(^{1}\)). The neutrino LSS is actually closer to us than the CMB LSS (again see Fig.\(^{1}\)\(^{41}\)) and therefore we do not see more space using massive neutrinos, but just fill in some of the observable volume. Realistically speaking of course, we have no expectation that measuring neutrino anisotropies will ever be easy.

Gravitational wave detectors will have the potential to measure absolutely calibrated distances to individual black hole binary sources, which again is mainly measuring the expansion of the Universe \(^{41}\). Perturbations in the primordial gravity waves, potentially detectable through CMB observations of the so-called \( B \)-modes (see, e.g., Ref.\(^{42}\)), simply add a small fraction of the amount of information coming from the other CMB power spectra \(^{3}\).

VI. DISCUSSION

Within the simplest cosmological picture the information available to constrain the cosmology is dominated by the count of modes in power spectra. Approximately a \( 10^9 \) CMB modes have already been measured (see Ref.\(^{3}\)) and the total number accessible is only a factor of a few times larger than this. To provide tighter constraints on cosmological models one therefore has to go to three dimensions, where it should be possible to measure more than \( 10^{12} \) modes.

Our current view of the large-scale Universe is that it is described by just six parameters—but of course no one expects that things will ultimately be that simple. Certainly we expect neutrino density to be a seventh parameter, and departures from the standard cosmology will also presumably be found eventually (perhaps curvature, tensors, dark energy equation of state, isocurvature perturbations, running spectral index, etc.), including extensions that we have not yet imagined. Adding additional parameters to the background cosmology would not change what we have described, but just add details to the story. It could also be that the primordial perturbations break the Gaussian assumption that we have made—i.e., that we will eventually measure \( f_{\text{NL}} \)—but if the non-Gaussianity is weak, then it also does not alter the basic idea that mode counting dominates our ability to define the cosmology that we live in.

However, there is a whole other side of cosmological constraints that we have been neglecting, which concerns itself with structure formation; i.e., how baryons populate dark matter halos and turn into galaxies. When one uses large three-dimensional surveys, one has to consider phenomenological descriptors of nonlinear structure (bias parameters, halo model functions, etc.) as well as the background parameters. But in practice, this distinction may be rather fuzzy. Already, one of the standard cosmological parameters, namely the reionization optical depth \( \tau_{\text{ion}} \), is unrelated to fundamental processes, and in principle can be predicted from known physics. The split is also complicated in practice because of the way that the parameters are related to specific observables. We expect this distinction to become less clear in the future as more parameters are required to describe the structure formation part of the cosmological picture.

These kinds of complications are highlighted in an interesting idea proposed in Ref.\(^{43}\), using two distinct tracers of the same modes in order to make apparently cosmic-variance-free determinations of cosmological parameters. This is certainly a promising technique that will be exploited to obtain more information from large-scale surveys; however, it is ultimately limited by the applicability of the purely linear bias approximation. In practice, it should enable us to (i) use redshift-space distortions to estimate the velocity divergence power spectrum to its cosmic variance limit, rather than having an error dominated by the uncertainty on the parameter \( \beta \) (a combination of the growth rate and the bias of the
tracers (the information in the picture); and (ii) simultaneously give a model-by-mode estimate of the ratio of the biases of the two tracers and the parameter $\beta$, which are free of cosmic variance. The measurement of a velocity power spectrum is a pure background cosmology determination, whose SNR comes from mode counting, and could one day be very large, so that finding ways to reach the cosmic variance limit will improve our ability to probe more cosmological modes. On the other hand, it seems doubtful that the SNR in the measurement of $\beta$ will ultimately be competitive, and moreover requires a full understanding of the bias of galaxies, including an assumption that there is no stochastic element.

Despite these reservations, we are sure that further methods will be developed to use multiple tracers to directly probe more fluctuation modes, while at the same time generating more complex models for structure formation within the background and linear perturbations picture. This is the only way that we will fully exploit the information in the $>10^{12}$ modes that should be accessible.

VII. CONCLUSIONS

We have presented here a calculation of the total number of linear modes that can be measured at different epochs of cosmic evolution. We separately consider $N$ from the light confined on the past light cone and the light scattering within past light cone. Our results show that the total number of modes (which is equal to twice the amount of information on parameter precision) increases monotonically; however, it will reach a saturation point in the distant future due to $\Lambda$ domination, and in the very far future will drop due to plasma or Gibbons-Hawking screening effects. The detailed precision obtainable on cosmological parameters depends on the smallest scales that can be probed in the early Universe, with the ultimate value for SNR being around $10^7$. Cosmologists today appear to live at the epoch where dark energy starts to dominate, so that measuring cosmological information from structures might be easiest, and will become harder as we move billions of years into the future. Nevertheless, future cosmologists will always be able to do better if they are inventive enough.

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